Coherent destruction of tunneling in a lattice array with controllable boundary

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We have investigated how the dynamics of a quantum particle initially localized in the left boundary site under periodic driving can be manipulated via control of the right boundary site of a lattice array. Because of the adjustable coupling between the right boundary site and its nearest-neighbor, we can realize either coherent destruction of tunneling to coherent tunneling (CDT-CT) transition or coherent tunneling to coherent destruction of tunneling (CT-CDT) transition, by driving or moving the right boundary site while keeping the left boundary site driven by a periodically oscillating field with a fixed driving parameter. In particular, the transition direction shows odd-even sensitivity to the number of lattice sites. We have also revealed that our proposed CDT-CT transition is robust against the second order coupling (SOC) between next-nearest-neighbor sites in odd-$N$-site systems, whereas localization can be significantly enhanced by SOC in even-$N$-site systems. More interestingly, it is found destruction and revival of CDT observable in non-high-frequency regimes. Our results can be readily verified within the capacity of current experiments.

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I. INTRODUCTION

Coherent control of quantum dynamics via a periodically oscillating external field has been one of the subjects of long-lasting interest in diverse branches of physics and chemistry\textsuperscript{1,2}. One seminal result of the control is coherent destruction of tunneling (CDT), a phenomenon originally discovered by Grossmann \textit{et al.} in 1991 for a periodically driven double-well system\textsuperscript{3}, upon the occurrence of which the tunneling can be brought to a standstill provided that the system parameters are carefully chosen. Due to its importance for understanding many fundamental time-dependent processes and its potential application in quantum motor\textsuperscript{4,5} and quantum-information processing\textsuperscript{6}, CDT has received growing attention from both theoretical and experimental studies. Theoretically, it has been extended in various forms such as non-degenerate CDT\textsuperscript{7}, selective CDT\textsuperscript{8}, nonlinear CDT\textsuperscript{9}, many-body CDT\textsuperscript{10,11}, instantaneous CDT\textsuperscript{12} and multiphoton CDT\textsuperscript{13}. Experimentally, it has been observed in many different physical systems like modulated optical coupler\textsuperscript{14}, driven double-well potentials for single-particle tunneling\textsuperscript{15}, three-dimensional photonic lattices\textsuperscript{16}, a single electron spin in diamond\textsuperscript{17}, Bose-Einstein condensates in shaken optical lattices\textsuperscript{18,19}, and chaotic microcavity\textsuperscript{20}.

In addition to conventional approach of modulating a lattice in a uniform fashion, modulating some certain lattice sites selectively also provides an attractive alternative for generation of CDT, in which the rescaled tunneling amplitudes for different sites can be shut off effectively, thereby some intriguing quantum manipulations are realizable such as dissipationless directed transport\textsuperscript{21} and beam splitter\textsuperscript{22}. CDT was traditionally thought to occur only at the isolated degeneracy point of the quasi-energies and is related to dynamical localization\textsuperscript{23}. However, recently a novel quantum phenomenon called dark CDT occurring over a wide range of system parameters has been introduced in odd-$N$-state systems\textsuperscript{24} which is demonstrated to be caused by localized dark Floquet state that has zero quasi-energy and negligible population at the intermediate state, rather than the superposition of degenerate Floquet states. Those advances on CDT studies offer benefits for coherent control of quantum dynamics.

In this paper, we propose two schemes for the control of quantum tunneling in a periodically driven lattice array with controllable boundary through combination of characteristics of both normal CDT and dark CDT. In the first scheme, we consider the coherent motion of a quantum particle in a lattice array, in which the harmonic oscillating external fields act on only the two boundary sites of chain. We find an interesting result that a single particle initially occupying the left boundary site experiences coherent destruction of tunneling to coherent tunneling (CDT-CT) transition or coherent tunneling to coherent destruction of tunneling (CT-CDT) transition, when the driving amplitude of the external periodic field applied to the right boundary site is increased from zero. In particular, the transition direction shows odd-even sensitivity to the number of lattice sites. We also reveal that the CDT-CT transition in odd-$N$-site lattices is robust against the second order coupling (SOC) between next-nearest-neighbor sites, whereas the CT-CDT transition in even-$N$-site systems is significantly affected by SOC. In the second scheme, we consider quantum control in a similar lattice array by moving the right boundary site while keeping the left boundary site driven with fixed driving parameters as the first scheme. In such a scheme, we reach some parameter regimes not accessible in the first scheme, and discover that the system exhibits a similar CDT-CT or CT-CDT transition. Interestingly, destruction and revival of CDT is observable in non-high-frequency regimes. Moreover, we present a good understanding of the results with help of the

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II. CDT CONTROL BY DRIVING THE RIGHT BOUNDARY SITE

A. Model system for the first scheme

We begin with the first scheme by considering a single particle in an array of lattice sites with only two boundary sites driven by external periodic field, as shown in Fig. (1). In this case, the left boundary site is driven with fixed driving amplitude and driving frequency, while the right boundary site is driven with varied driving amplitude and fixed driving frequency. In the tight-binding approximation and assuming a coherent dynamics, the single-particle motion can be generally described by the tight-binding Hamiltonian

\[ H_I = E_1 |1\rangle \langle 1| + E_N |N\rangle \langle N| + \sum_{j=2}^{N-1} \Omega_0 (|j-1\rangle \langle j| + H.c.) + \sum_{j=2}^{N-1} \nu_0 (|j-1\rangle \langle j+1| + H.c.), \]

where \(|j\rangle\) represents the Wannier state localized in the \(j\)th site, \(\Omega_0\) is the coupling strength connecting nearest-neighboring sites, and \(\nu_0\) is the second-order coupling (SOC) strength between next-nearest-neighboring sites. Instead of modulating a lattice in a uniform fashion, we modulate the on-site energies selectively. In this scheme, we assume a harmonic oscillating field applied in form of \(E_1(t) = A_1 \cos(\omega t)\) for site 1, and \(E_N(t) = A_2 \cos(\omega t)\) for site \(N\). Here \(A_1\) and \(A_2\) are the driving amplitudes and \(\omega\) is the driving frequency, respectively.

We expand the quantum state of system (1) as \(|\psi(t)\rangle = \sum_j a_j(t) |j\rangle\), where \(a_j(t)\) represents occupation probability amplitudes at the \(j\)th site, with the normalization condition \(\sum_j |a_j(t)|^2 = 1\). From the Schrödinger equation \(i\hbar \partial_t |\psi(t)\rangle = H |\psi(t)\rangle\), the evolution equation for the probability amplitudes \(a_j(t)\) reads

\[ i \frac{da_1}{dt} = E_1(t) a_1 + \Omega_0 a_2 + \nu_0 a_3, \]
\[ i \frac{da_j}{dt} = \Omega_0 (a_{j-1} + a_{j+1}) + \nu_0 (a_{j-2} + a_{j+2}) \quad (j = 2, 3, ..., N-1), \]
\[ i \frac{da_N}{dt} = E_N(t) a_N + \nu_0 a_{N-2} + \Omega_0 a_{N-1}. \]

Since Hamiltonian (1) is periodic in time, \(H_I(t + T) = H_I(t)\), where \(T = 2\pi/\omega\) is the period of the driving, the Floquet theorem allows us to write solutions of the Schrödinger equation in the form \(a_j(t) = \tilde{a}_j(t) \exp(-i\epsilon t)\). Here \(\epsilon\) is the quasi-energy, and \(\tilde{a}_j(t)\) are periodic with modulation period \(T\).

In this section, we mainly illustrate how the quantum dynamics of a single particle initially localized at the left boundary site can be controlled by driving the other boundary site, for both zero and non-zero SOC cases.

B. CDT control in odd-\(N\)-site system

We start our considerations from the three-site lattice array, the minimal one for odd-\(N\)-site system. According to Eq. (2), the evolution equations for the probability amplitudes reads,

\[ i \frac{da_1}{dt} = A_1 \cos(\omega t) a_1 + \Omega_0 a_2 + \nu_0 a_3, \]
\[ i \frac{da_2}{dt} = \Omega_0 a_1 + \nu_0 a_3 + \Omega_0 a_3, \]
\[ i \frac{da_3}{dt} = A_2 \cos(\omega t) a_3 + \nu_0 a_1 + \Omega_0 a_2. \]

We numerically solve the time-dependent Schrödinger equation (3) with \(\nu_0 = 0\), i.e., neglecting next-nearest-neighbor tunneling in the chain. The initial state is \((1, 0, 0)^T\), and the driving parameters of site 1 fixed as \(A_1 = 22, \omega = 10\). The evolution of the probability distribution \(P_1 = |a_1|^2\) is presented in figure (2) for three typical driving conditions of site \(N\). For \(A_2/\omega = 0\), \(P_1\) remains near unity, signaling suppression of tunneling. This is the quantum phenomenon well known as CDT. For \(A_2/\omega = 2.0\), \(P_1\) oscillates between 1 and \(\sim 0.4\), showing partial suppression of tunneling. At \(A_2/\omega = 2.4\), \(P_1\) oscillates between zero and one, demonstrating no suppression of tunneling. The numerical results clearly indicate that the system undergoes a CDT-CT transition when \(A_2/\omega\) is increased from zero. Such a CDT-CT transition is more clearly demonstrated in Fig. (2)(b), where the minimum value of \(P_1\) is used to measure the suppression of tunneling. When there is large suppression of tunneling, \(\text{Min}(P_1)\) is close to 1; when there is no suppression, \(\text{Min}(P_1)\) is zero. As clearly shown in Fig. (2)(b), \(\text{Min}(P_1)\) slowly falls from its initial value to zero with \(A_2/\omega\) increasing from zero to 2.4. It is observed that the value of \(\text{Min}(P_1)\) drops to zero in narrow intervals around the zeros of \(J_0(A_2/\omega)\), indicating that the particle is able to tunnel freely from site to site in this regime.

We explain the above numerical results through some analytic deduction based on high-frequency approximation. In the high-frequency limit, we introduce the transformation \(b_1 = \exp(-i \int A_1 \cos(\omega t) dt) a_1, \quad b_2 = a_2,\)
The minimum value of $P_1$ as a function of $A_2/\omega$. The initial condition is $a_1 = 1, a_2 = 0, A_2 = 0$. The other parameters are chosen as $A_1 = 22, \omega = 10, \Omega_0 = 1, \nu_0 = 0$.

$$b_3 = \exp[-i \int A_2 \cos(\omega t) dt] a_3,$$

where $b_j(t) (j = 1, 2, 3)$ are slowly varying functions. Using the expansion $\exp[\pm i A \sin(\omega t) / \omega] = \sum_k J_k(A/\omega) \exp[\pm i k \omega t]$ in terms of Bessel functions and neglecting all orders except $k = 0$ in the high frequency region, we arrive at the effective equation of motion

$$\frac{d b_1}{dt} = \Omega_0 J_0(A_1/\omega) b_2,$$

$$\frac{d b_2}{dt} = \Omega_0 J_0(A_1/\omega) b_1 + \Omega_0 J_0(A_2/\omega) b_3,$$

$$\frac{d b_3}{dt} = \Omega_0 J_0(A_2/\omega) b_2.$$

(4)

Here we have dropped the SOC terms. It is easy to derive the analytical solutions of Eq. (4).

$$b_1 = \frac{J_{01}}{J_{02}} C_1 + i \frac{J_{01}}{\sqrt{J_{01}^2 + J_{02}^2}} [C_2 \sin(Kt) - C_3 \cos(Kt)]$$

$$b_2 = C_2 \cos(Kt) + C_3 \sin(Kt)$$

$$b_3 = C_1 + i \frac{J_{02}}{\sqrt{J_{01}^2 + J_{02}^2}} [C_2 \sin(Kt) - C_3 \cos(Kt)],$$

(5)

where $C_j (j = 1, 2, 3)$ are constants to be determined by the initial states and normalization condition. $J_{01} = J_0(A_1/\omega)$, $J_{02} = J_0(A_2/\omega)$ and $K = \Omega_0 \sqrt{J_{01}^2 + J_{02}^2}$. Applying the initial conditions $b_1(0) = 1, b_2(0) = 0, b_3(0) = 0$ to Eq. (5) yields the undetermined constants $C_j (j = 1, 2, 3)$ in the forms

$$C_1 = -\frac{J_{02} J_{00}}{J_{01}^2 + J_{02}^2}, C_2 = 0, C_3 = i \frac{J_{02}}{\sqrt{J_{01}^2 + J_{02}^2}}$$

and the occupation probability at site 1 as

$$|b_1|^2 = \frac{J_{02}^4}{J_{01}^2 + J_{02}^2} + \frac{J_{01}^4}{J_{01}^2 + J_{02}^2} \cos(Kt)^2.$$

(6)

From expression (6), we immediately have two observations: (i) when $A_2/\omega \approx 2.4, 5.02$, i.e., the zeros of $J_{02}$, the minimal value of $|b_1|^2$ is zero; (ii) when $A_1/\omega$ is tuned near to the zeros of $J_{01}$ and $A_2/\omega$ tuned far away from those values, $|b_1|^2$ remains near unity. The analytical results of $P_1$ based on formula (6) are plotted in Fig. 2(d), which agrees well with the theoretical results obtained from the original model (3) as shown in Fig. 2(c).

For a deep insight into the tunneling dynamics obtained in Fig. 2, we numerically compute the quasi-energies and Floquet states of this system as shown in Fig. 3. There are three Floquet states with quasi-energies $\tilde{\epsilon}_1, \tilde{\epsilon}_2$ and $\tilde{\epsilon}_3$. We immediately notice from Fig. 3(a) that there exists a dark Floquet state with zero quasi-energy for all of the values of $A_2/\omega$. For this three-site system, there is no degeneracy in the quasi-energy levels. Therefore, the occurrence of suppression of tunneling should be further explored through probe into the Floquet states. We display the time-averaged population probability $\langle P_j \rangle = \langle \int_0^T dt |a_j|^2 / T \rangle$ for a given Floquet state $(a_1, a_2, a_3)^T$ in Figs. 3(b)-(d). The Floquet state with $\langle P_j \rangle > 0.5$ is generally regarded as a state localized at the $j$th site. As seen in Figs. 3(c), the dark Floquet state has negligible population at site 2 while the population at site 1 is near unity. These numerical results obtained from the original model (3) as shown in Fig. 2(c).

Computer simulations of the system (1) with more than three sites verify the generality of the above obtained physical
results in all odd-$N$-site systems. Dynamics for five sites is depicted in Fig. (4), which shows a similar CDT-CT transition as that of three-site system. Like the case of three-site system, this five-site system possesses a dark Floquet state with zero quasi-energy and negligible population at all of the even sites, as illustrated in Figs. (4) (c)-(d). The reason for CDT-CT transition in the five-site system lies in the fact that population distribution $\langle P_{j} \rangle$ for the dark Floquet state also experiences a localization-delocalization transition (see Fig. (4)(d)). It can be seen that all properties for five sites closely resemble those of the three-site system.

![Figure 4](image_url)

**FIG. 4:** (Color online) Five-site model. (a) Time evolution of the probability at site 1, $P_1 = |a_1|^2$, with different values of $A_2$; (b) The minimum value of $P_1$ as a function of $A_2/\omega$; (c) Quasi-energies versus $A_2/\omega$; (d) The time-averaged populations $\langle P_{j} \rangle$ of the dark Floquet state corresponding to $\epsilon_1$. The other parameters are $A_1 = 22$, $\omega = 10$, $\Omega_0 = 1$, $v_0 = 0$.

### C. CDT control in even-$N$-site system

We are now in the position to investigate the quantum control of the even-$N$-site systems. In our discussion, we focus on the dynamics for four sites. However, similar behaviors will appear in other even-$N$-site lattice systems as well. For a four-site system, the coupled equation (2) reads

$$
\begin{align*}
    i \frac{da_1}{dt} &= A_1 \cos(\omega t)a_1 + \Omega_0 a_2 + v_0 a_3 \\
    i \frac{da_2}{dt} &= \Omega_0 a_1 + \Omega_0 a_3 + v_0 a_4 \\
    i \frac{da_3}{dt} &= v_0 a_1 + \Omega_0 a_2 + \Omega_0 a_4 \\
    i \frac{da_4}{dt} &= A_2 \cos(\omega t)a_4 + v_0 a_2 + \Omega_0 a_3.
\end{align*}
$$

We plot in Fig. (5)(a) the minimum value of $P_1$ versus $A_2$ by direct numerical simulations of the Schrödinger equation (7) with $v_0 = 0$. We start a particle at site 1 and fix the driving parameters of site 1 as $A_1 = 22$, $\omega = 10$. As shown in Fig. (5)(a), Min($P_1$) takes extremely low values about zero except at a series of very sharp peaks. The quasi-energies of this system are shown in Fig. (5)(b), where we find that a pair of quasi-energies cross at the zeros of $J_0(A_2/\omega)$. This is the origin of the extremely sharp peaks in localization seen in Fig. (5)(a). Figs. (5)(c)-(f) show that there is no localized Floquet state. These numerical results demonstrate the possibility of inducing CT-CDT transition in even-$N$-site systems (e.g., four-site system) through tuning the rescaled driving amplitude $A_2/\omega$ from zero to the points of quasi-energy crossing.

![Figure 5](image_url)

**FIG. 5:** (Color online) Four-site model described by Eq. (7). (a) The minimum value of $P_1$ as a function of $A_2/\omega$. The extremely narrow peaks appear at the zeros of $J_0(A_2/\omega)$. (b) Quasi-energies versus $A_2/\omega$. At the zeros of $J_0(A_2/\omega)$, a pair of quasi-energies degenerate. (c)-(f) The time-averaged populations $\langle P_{j} \rangle$ of the Floquet states corresponding to $\epsilon_1-\epsilon_4$. The other parameters are $A_1 = 22$, $\omega = 10$, $\Omega_0 = 1$, $v_0 = 0$.

### D. Effects of second-order coupling on CDT control

In the above discussion, the influence of second order coupling (SOC), generally thought to be detrimental to CDT, is neglected. In this subsection, We have checked the robustness of our proposed scheme by direct numerical simulations of the Schrödinger equation (2) in the presence of SOC effects. As before, the left boundary site is initially occupied and its driving parameters is fixed as $A_1 = 22$, $\omega = 10$.

The influence of second order coupling on three-site system is plotted in Fig. (6). When SOC effects are taken into account, the numerical results in Fig. (6)(a)-(b) show that the three-site system displays similar dynamical behaviors (CDT-CT transitions) as the case without considering SOC. The numerically computed quasi-energies and Floquet states are depicted in Fig. (6)(c)-(f). Correspondingly, a localization-delocalization transition can be seen for the population distribution $\langle P_1 \rangle$ of the Floquet state corresponding to a quasi-
energy $e_3$ close to zero, when $A_2$ is increased from zero. The numerical results with more than three sites (not shown here) confirm that the CDT-CT transition persists in all odd-$N$-site systems even if the SOC effects are considered.

We have also investigated the impact of SOC on the dynamics of even-$N$-site systems (e.g., four-site system), as shown in Fig. (7). Fig. (7)(a) shows the minimum value of $P_1$ as a function of the driving amplitude $A_2$. As $A_2$ is increased from zero, $\text{Min}(P_1)$ steadily decreases from a value of ~0.5 to zero before it peaks at $A_2/\omega \approx 2.4$. Contrary to our expectation, SOC facilitates rather than hindering the localization. This result is somewhat counter-intuitive. From Fig. (7)(b) it can be seen that a pair of the quasi-energies make a series of close approaches to each other as $A_2$ increases. Detailed examination of the close approaches reveals that they are in fact avoided crossings. At the points of close approach, the tunneling is suppressed and the localization peaks. Compared with the zero SOC case shown in Fig. (5), the population distributions of Floquet states change dramatically for four-site system in the presence of SOC effects; see Fig. (7)(c)-(f). The modification of the even-$N$-system's dynamics due to SOC, such as enhancement of localization, is the consequence of the Floquet states localized at site 1 instead of the quasi-energy degeneracy.

III. CDT CONTROL BY MOVING THE RIGHT BOUNDARY SITE

Instead of driving the right boundary site, another scheme, as shown in Fig. (8), is to move the position of the right boundary site towards its neighbor along the array. In this scheme, the system can be modeled well by a single-band Hamiltonian

$$H_{II} = A_1 \cos(\omega t) |1\rangle \langle 1| + \sum_{j=2}^{N-1} \Omega_0 \langle j-1 \rangle \langle j \rangle + H.c.$$

$$+ \sum_{j=2}^{N-2} v_0 \langle j-1 \rangle \langle j+1 \rangle + H.c.$$  

$$+ \Omega \langle |N-1\rangle \langle N \rangle + H.c. \rangle + \nu \langle |N-2\rangle \langle N \rangle + H.c. \rangle, \quad (8)$$

where $\Omega$ is the coupling strength between site $N$ and site $N-1$, $\nu$ is the coupling strength between site $N$ and site $N-2$, and the other parameters have the same physical meaning as the ones in Hamiltonian (1). The values of $\Omega$ and $\nu$ are adjustable when the spacing between the right boundary site and its neighbor is tuned. Note that here $\Omega > \Omega_0$ can be achieved, on contrary to its inaccessibility in the first scheme.

In this section, we address how the quantum dynamics of a single particle initially localized at the left boundary site can be manipulated by moving the right boundary site, in both high-frequency and non-high-frequency regimes.

A. Odd-$N$-site system

We first focus our attention on the dynamics for odd-$N$-site systems. As the simplest example, we consider the case of
three site where the dynamical equations are
\[ i\frac{da_1}{dt} = A_1 \cos(\omega t) a_1 + \Omega_0 a_2 + \nu a_3 \]
\[ i\frac{da_2}{dt} = \Omega_0 a_1 + \Omega a_3 \]
\[ i\frac{da_3}{dt} = \nu a_1 + \Omega a_2. \]  
(9)

Due to the fact that SOC has little influence on the dynamics of odd-\(N\)-site systems, we focus on the case of \(\nu = 0\) for simplicity and present some typical numerical results based on Eq. (9).

Fig. (2)(a)-(b) shows the tunneling dynamics for different values of \(\Omega\). In our numerical simulations, we have fixed \(\Omega_0 = 1\), \(A_1 = 22\) and \(\omega = 10\). When \(\Omega = 0\), the occupation of the left boundary site oscillates from 1 to zero, indicating no suppression of tunneling. In this case, the model is in fact a two-level system and the driving parameters are chosen to be slightly off the degenerate point of quasi-energy. When \(\Omega = 0.4\), the occupation of the left boundary site oscillates between 1 and 0.7, showing partial suppression of tunneling. When \(\Omega = 2\), the occupation of the left boundary site remains near unity, signaling coherent destruction of tunneling. Such a coherent tunneling (CT)-coherent destruction of tunneling (CDT) transition is more clearly demonstrated in Fig. (2)(b).

In Fig. (2)(c)-(d), we show the quasi-energies and Floquet states corresponding to \(\epsilon_2\) for \(\Omega = 0.4\) and \(\Omega = 2\), respectively. It is clearly seen that: (i) for the three-site system with no identical coupling constants between the neighboring sites, there still exist a dark Floquet state with zero quasi-energy and negligible population at the central site; (ii) the time-averaged population distribution \(\langle P_j \rangle\) of dark Floquet state exhibits two peaks when \(A_1\) is varied from zero to 60, and the local minimum between two peaks becomes more shallow with increasing \(\Omega\). It means that increase of \(\Omega\) enhances degree of localization.

Under the high-frequency approximation, the system described by Eq. (9) behaves like the undriven one with coupling strength rescaled by a factor of \(J_0(A_1/\omega)\). We have presented the analytical results of \(\text{Min}(P_1)\) on the basis of the high-frequency approximation analysis in Fig. (2)(b), which is in good agreement with the numerical results obtained from Eq. (9).

Finally, we turn to the case of the four-site system. Here the dynamical equations are
\[ i\frac{da_1}{dt} = A_1 \cos(\omega t) a_1 + \Omega_0 a_2 + \nu_0 a_3 \]
\[ i\frac{da_2}{dt} = \Omega_0 a_1 + \Omega_0 a_3 + \nu a_4 \]
\[ i\frac{da_3}{dt} = \nu_0 a_1 + \Omega a_2 + \Omega a_4 \]
\[ i\frac{da_4}{dt} = \nu a_2 + \Omega a_3. \]  
(10)

On basis of a numerical analysis of equation (10) with the parameters \(\nu_0 = 0, \nu = 0, A_1 = 22, \omega = 10\) and the initial condition \(\{a_1(0) = 1, a_2(0) = 0, a_3(0) = 0, a_4(0) = 0\}\), we show in Fig. (10) (a) the time evolution of \(P_1\) with three typical values of \(\Omega\), and the behavior of \(\text{Min}(P_1)\) versus \(\Omega\), respectively. As we can see, with the increase of the coupling strength between the right boundary and its nearest neighbor, the value of \(\text{Min}(P_1)\) will rapidly drop from one to zero, indicating occurrence of a CDT-CT transition. To better understand the CDT-CT transition, we have examined the corresponding quasi-energies and Floquet states. In Fig. (10) (c), we observe that there exist two zero quasi-energy levels: one

![FIG. 8: (Color online) Schematic of model (8) with adjustment of the spacing between the right boundary site and its neighbor. The dynamical properties of a single particle in the left boundary site driven with fixed amplitude \(A_1\) and frequency \(\omega\), can be controlled by moving the right boundary site with varied \(\Omega\) and \(\nu\).](image1)

![FIG. 9: (Color online) Three-site system described by Eq. (9). (a) The time evolution of \(P_1\) with different coupling strength \(\Omega\). \(A_1 = 22, \omega = 10\). (b) The minimum value of \(P_1\) as a function of the coupling strength \(\Omega\). \(A_1 = 22, \omega = 10\). (c) Quasi-energies versus \(A_1/\omega\) at \(\Omega = 0.4\). (d) The time-averaged populations \(\langle P_j \rangle\) of the dark Floquet state corresponding to quasi-energy \(\epsilon_j = 0\) in (c). (e)-(f) Quasi-energies and the corresponding localized dark Floquet state at \(\Omega = 2\). The other parameters are chosen as \(\Omega_0 = 1, \nu = 0, \omega = 10\).](image2)
is virtual because our considered system is in fact a three-state system on condition of $\Omega = 0$, the other is the dark Floquet state which is localized at site 1; see Fig. 11 (d). The behavior of the quasi-energies and Floquet states changes dramatically if $\Omega \neq 0$. The two examples at $\Omega = 0.4$ and $\Omega = 2$ are illustrated in Figs. 11 (e)-(f) and their respective neighbor column, in which we witness that two of the four quasi-energy levels become degenerate at isolated points of parameters and no localized dark Floquet state exist at all in this case. The modification of structure seen in the quasi-energies and Floquet states indeed corresponds to a great change of the system’s dynamics.

In Fig. 11(b), we compare the numerical values of $\text{Min}(P_1)$ obtained from the original model (10) and the analytical results obtained from the effective model in which the coupling constants connecting the left boundary site and other sites are renormalized by a factor of the Bessel function $J_0$ through application of the high-frequency approximation. A good agreement between them is found. Moreover, we have numerically simulated the system (3) with other numbers of sites. The numerical results, which are not displayed here, suggest that the CT-CDT transition occurs in odd-$N$-site systems and the CDT-CT transition occurs in even-$N$-site systems, when the coupling strength $\Omega$ is increased from zero.

C. CDT control in non-high-frequency regimes

Generally, when driving frequency $\omega$ is roughly equal to or smaller than all the coupling constants between different sites, the high-frequency approximation may not be regarded as safe. In the large $\Omega$ regime, the high-frequency approximation becomes invalid and SOC effects may be included. In what follows, we will numerically elaborate the dynamical properties of system (8) for $\Omega$ up to 25, as shown in Fig. 11. It is seen from Fig. 11 that: (i) a destruction of CDT appears in the three-site system when the coupling strength $\Omega$ matches integer multiples of the driving frequency; (ii) in the large $\Omega$ limit, a perfect coherent tunneling is observable in the four-site system even in the presence of SOC. As expected, the impact of SOC on the three-site system is negligible while localization is strongly enhanced by SOC coefficient in the four-site system. These remarkable features may offer an additional possibility to coherent control of quantum tunneling dynamics.

IV. CONCLUSION AND DISCUSSION

In conclusion, we have studied how the dynamics of a single quantum particle initially localized in the left boundary site under periodic driving can be controlled by only driving or moving the right boundary site of a lattice array. Our main results are summarized as follows. First, in the odd-$N$-site system, there exists a dark Floquet state which has a zero quasi-energy. By raising the driving amplitude of the external periodic field applied to the right boundary site from zero, we can induce a CDT-CT transition, caused by a localization-delocalization transition of the dark Floquet state; In the even-$N$-site system, there is no localized Floquet state. In the same way, we can realize a CT-CDT transition, thanks to the fact that a pair of quasi-energies become degenerate at isolate points of parameters. The transition direction shows odd-even sensitivity to the number of lattice sites. Second, in the odd-$N$-site system, the CDT-CT transition persists when the SOC effects are considered. Long-range interactions in the lattice are important and non-negligible in some systems like biomolecules [27], polymer chains [28], coupled waveguides [24], and charge transport in a quantum dot array [29, 30]. Therefore, our proposed scheme provides a new route to possible application of CDT in such systems. However, in the even-$N$-site system, CT-CDT transition is considerably affected by the SOC effects. We find a counter-intuitive phenomenon that SOC can enhance rather than hinder local-
FIG. 11: (Color online) The minimum value of $P_1$ as a function of $\Omega$ with different SOC coefficients. (a) three-site system; (b) four-site system. The other parameters are $\Omega_0 = 1, A_1 = 22, \omega = 10, \nu_0 = 0$.

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[35] T. Yamamoto, M. Watanabe, J. Q. You, Yu. A. Pashkin, O. Astafiev, Y. Nakamura, F. Nori, and J. S. Tsai, Phys. Rev. B 77, 064505 (2008).
[36] I. L. Garanovich, S. Longhi, A. A. Sukhorukov, and Y. S. Kivshar, Phys. Rep. 518, 1 (2012).
[37] S. Longhi, Laser and Photon. Rev. 3, 243 (2009).

[38] S. Longhi, F. Dreisow, M. Heinrich, T. Pertsch, A. Tünnermann, S. Nolte and A. Szameit, Phys. Rev. A 82, 053813 (2010).
[39] A. Szameit, I. L. Garanovich, M. Heinrich, A. A. Sukhorukov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann, S. Longhi and Y. S. Kivshar, Phys. Rev. Lett. 104, 223903 (2010).