AM to PM conversion of linear filters

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Abstract—The conversion between amplitude modulation and phase modulation as a modulated signal goes through a filter is analyzed. The difference in how the modulated sideband amplitudes experience the filter, and how AM and PM has opposite signs for one of their sidebands interact. The conversion between AM and PM is modeled, providing a scatter model and evaluation of two functions based on the linear filters transfer function. The system bandwidth effects is analyzed and rule of thumb developed to ensure AM and PM isolation.

Index Terms—amplitude modulation, phase modulation, phase noise

I. INTRODUCTION

THE amplitude modulation to phase modulation interaction of system components can severely limit the achieved phase noise performance of high performance systems. This paper aims to facilitate a simplified approach to the analyzing of how such conversion can occur in filters. The concern for both AM and PM noise is illustrated in [1], which also touches on the subject of AM to PM noise conversion and therefore the need to keep AM noise within constraints. While it covers behavior of non-linear amplifier conversion and therefore the need to keep AM noise within constraints, the focus of the present article is on strictly linear behavior and a simplified approach to analyze this in order to assist on system design.

II. MODULATION BASICS

For amplitude modulation, the sidebands can be expressed using classic trigonometry [2] assuming higher order terms insignificant:

\[
X(t) = A(1 + a \cos \omega_m t) \cos \omega_c t = A_2^c \cos (\omega_c - \omega_m) t + A \cos \omega_c t
\]

(1)

Where \( A \) is the amplitude, \( a \) is the amplitude modulation index, \( \omega_c \) the angular frequency of the carrier and \( \omega_m \) the angular frequency of the modulation.

Similarly, for phase modulation we can write from Bessel function [2] assuming higher order terms insignificant:

\[
X(t) = A \cos \omega_m t + \omega_c t = A J_1(p) \cos (\omega_c - \omega_m) t + A J_0(p) \cos \omega_c t
\]

(2)

\[
+ A J_1(p) \cos (\omega_c + \omega_m) t
\]

The following analysis assumes low modulation index, such that for phase modulation effects on carrier strength can be considered insignificant. Thus, can quadratic or higher orders of the Bessel polynomials be cancelled out and the omission of second-degree or higher order sidebands also be dropped safely. The remaining approximation thus becomes:

\[
X(t) = -A_2^p \cos (\omega_c - \omega_m) t + A \cos \omega_c t + A_2^p \cos (\omega_c + \omega_m)
\]

(3)

III. COMMON AM AND PM MODEL

Giving the similarities of how AM and PM creates sidebands, only differing in the sign of the lower sideband, we can create a model for simultaneous AM and PM where by the amplitude of the lower sideband \( A_{LSB} \) and upper sideband \( A_{USB} \) can be expressed directly in the form of the respective modulation index \( a \) and \( p \) as well as the carrier amplitude \( A \).

\[
A_{LSB} = \frac{A}{2} - \frac{A_p}{2}
\]

(4)

\[
A_{USB} = \frac{A}{2} + \frac{A_p}{2}
\]

(5)

similarly can the respective modulation indexes be expressed in terms of the LSB and USB amplitudes and carrier amplitude for the same modulation frequency:

\[
a = \frac{A_{USB} + A_{LSB}}{A}
\]

(6)

\[
p = \frac{A_{USB} - A_{LSB}}{A}
\]

(7)

This thus represents an orthogonal linear transformation between either two sidebands of a carrier or the AM and PM modulations of that carrier. Depending on what we do we view it in either of these views and as long as we have both amplitudes we can transform to the other view.

IV. AM AND PM IN LINEAR FILTER

Consider that we have a linear filter \( H(s) \) and a signal that has amplitude and phase modulation, how can the modulations be considered to behave? To answer this question, one first need to convert the modulation indexes into the relative strengths of the sidebands \( A_{LSB} \) and \( A_{USB} \). With these, the filters response to the sideband frequencies produces the output strengths \( A'_{LSB} \) and \( A'_{USB} \), while the carrier produces the output strength \( A' \) which can be recalculated into the modulation indexes \( a' \) and \( p' \). This thus becomes:

\[
\omega_u = \omega_c + \omega_m
\]

(8)

\[
\omega_l = \omega_c - \omega_m
\]

(9)

\[
A_{LSB} = A \frac{a - p}{2}
\]

(10)

\[
A_{USB} = A \frac{a + p}{2}
\]

(11)

\[
A'_{LSB} = H(\omega_u) A_{LSB}
\]

(12)

\[
A'_{USB} = H(\omega_l) A_{USB}
\]

(13)

\[
a' = \frac{A'_{USB} + A'_{LSB}}{A'}
\]

(14)

\[
p' = \frac{A'_{USB} - A'_{LSB}}{A'}
\]

(15)
reducing into

\[ a' = \frac{H(j\omega_u) a + p}{H(j\omega_c)} + \frac{H(j\omega_l) a - p}{2H(j\omega_c)} \]  
\[ p' = \frac{H(j\omega_u) a + p}{H(j\omega_c)} - \frac{H(j\omega_l) a - p}{2H(j\omega_c)} \]  
\[ a' = \frac{H(j\omega_u) + H(j\omega_l) a + H(j\omega_u) - H(j\omega_l)}{2H(j\omega_c)} \]  
\[ p' = \frac{H(j\omega_u) - H(j\omega_l) a + H(j\omega_u) + H(j\omega_l)}{2H(j\omega_c)} \]

The last formulation clearly indicate that AM to AM and PM to PM conversion depends on the common sideband response where as the AM to PM and PM to AM conversion depends on the differential of the sideband response. It behaves as a scattering matrix of a linear system. Thus can further simplification be performed by defining the common and differential responses.

\[ H_c(\omega_l, \omega_u) = \frac{H(j\omega_u) + H(j\omega_l)}{2H(j\omega_c)} \]  
\[ H_d(\omega_l, \omega_u) = \frac{H(j\omega_u) - H(j\omega_l)}{2H(j\omega_c)} \]  

\[ a' = H_c(\omega_l, \omega_u) a + H_d(\omega_l, j\omega_u) p \]  
\[ p' = H_d(\omega_l, \omega_u) a + H_c(\omega_l, j\omega_u) p \]

The AM to PM and PM to AM leakage depends on the difference in response, and is equal. The AM to AM and PM to PM depends on the common response. However, both also depends on the damping and is both sidebands sufficiently damped, there will be significant damping of both. A perfectly balanced filter response will cancel cross-modulation without high reduction of modulation transfer.

V. 1 POLE LOWPASS FILTER

To illustrate the effect, consider a 1 pole lowpass filter

\[ H(s) = \frac{-\omega_0}{s - \omega_0} \]

To analyze is, it gets inserted into the two formulas resulting in

\[ H_c(\omega_l, \omega_u) = \frac{\omega^2_c - \omega^2_l + j2\omega_c\omega_l}{\omega^2_0 - \omega^2_c + \omega^2_l - j2\omega_c\omega_l} \]  
\[ H_d(\omega_l, \omega_u) = \frac{\omega_c\omega_m + j\omega_l\omega_m}{\omega^2_m - \omega^2_c + \omega^2_m - j2\omega_c\omega_m} \]

As these is evaluated for different relationships of carrier and frequency relationships, assuming modulation frequency is low compared to carrier frequency:

\[
\begin{array}{c|c|c|c}
\text{Condition} & |H_c| & |H_d| & |A'_{\omega}| \\
\hline
\omega_0 >> \omega_c & 1 & 0 & A \\
\omega_0 = k\omega_c & \frac{\omega_m}{\omega_0} & \frac{f_m}{k f_c} & A \\
\omega_0 = \omega_c & \frac{\omega_m}{\sqrt{2} \omega_c} & \frac{\omega_m}{\sqrt{2} f_c} & \frac{A}{\sqrt{2}} \\
\omega_0 << \omega_c & \frac{\omega_m}{\omega_c} & \frac{f_m}{f_c} & 0 \\
\end{array}
\]

The \(|H_c|\) response is essentially that of all pass for all conditions, but notice how the amplitude of carrier reduces to reflect the low-pass filter itself, so the AM to AM and PM to PM conversions both experience the same pass action that we expect. Further notice how the \(|H_d|\) reflect a high-pass filter as scaled by \(f_m/f_c\) factor.

The first condition reflect the situation where the carrier and sidebands is well within the pass frequency of the filter, and during this condition there is no cross-talk nor alteration of the carrier and thus the AM to AM and PM to PM conversion works as expected.

The third condition reflect the situation where the carrier matches that of the filter bandwidth, at which there is a 3 dB loss of amplitude, and there is a cross-talk proportional to the \(f_m/f_c\) ratio. Thus, a filter set at the carrier frequency will provide AM to PM conversion that increases proportional with the modulation frequency. Thus, far-out AM noise can convert to far-out PM noise. A 100 kHz modulation of a 10 MHz source would have a -43 dB conversion strength.

The second condition was added to reflect that increasing the system bandwidth to be some ratio \(k\) times the carrier would allow for additional isolation, which can be readily seen, such that \(k = 10\) would provide about -60 dB conversion strength of AM to PM for the same 100 kHz of 10 MHz. This allows for a simple rule of thumb approach to ensure AM to PM conversion does not impact system performance.

The fourth condition was added to reflect the extreme case where system bandwidth is far below that of the carrier. At this condition, there is AM to PM conversion, but on the other hand the damping of carrier is significant such that the gain goes towards zero and the carrier and sidebands is replaced by thermal noise.

VI. CONCLUSION

The fundamental approach to analyze AM to AM, PM to PM as well as cross-talk of AM to PM and PM to AM has been done and can be summarized by the response of the two \(H_c(\omega_c, \omega_m)\) and \(H_d(\omega_c, \omega_m)\) responses, that can analyze the effect of any linear system \(H(s)\).

For low-pass filter action, AM to PM cross-talk is found whenever the carrier frequency is near or beyond the system bandwidth, but not when the carrier frequency is much less than the system bandwidth. The cross-talk increases with modulation frequency and is proportional to the ratio \(f_m/f_c\). By letting the system bandwidth be scaled by the factor of \(k\) up from the carrier frequency, as defined by \(f_0 = k \times f_c\), the coupling factor approximate \(f_m/(k f_c)\) which can be used as a rule of thumb to ensure enough isolation between AM and PM for far-out noise.

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different signs of sideband amplitudes for AM and PM where illustrated.

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Magnus Danielson was born in Danderyd, Sweden, on April 27, 1971. Parallel to his studies he worked on professional audio systems. His EE studies at Royal Institute of Technology started in 1993, but got interrupted as he got hired by the Department of Teleinformatics in 1994 where he worked as a Research Engineer for three years. In 1997 he followed a research group to their start-up, and he has been with Net Insight since. At Net Insight he works as Senior System Architect, overseeing synchronization, time transfer, audio transports, large scale media systems, protocol design, EMC and high speed signal integrity, standardization at ETSI. He has contributed to 18 patents, mainly focusing on synchronization, time-transfer and switching mechanisms. He is a hobby researcher in the time and frequency field, including authoring Allan variance article on Wikipedia. He also works with PNT issues, GPS/GNSS and related, and has presented before PNT Advisory Board on the use and operational issues of GPS in the commercial field. In his spare time he is a ham with call sign SA0MAD, contributing to rewriting the Swedish ham educational material. He is a member of IEEE, AES, SMPTE and ION.