Analysis of Self-similar Nature Vehicle Arrival Data Pattern on Arterial Roads

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Abstract: Traffic congestion is a main criterion of increasing the disturbance of the movements of traffic on roads. In various metropolitan areas managing the traffic congestion is a difficult task, it was investigated that busy traffic reveals self-similarity, influenced by this aspect, real-time road traffic arrival data pattern proven as self-similar paradigm by means of disparate Hurst index methods. The current article furnishes various proficiency used to measure Hurst index which is popular demarcate of self-similar behavior traffic. In this paper, real time data is used to investigate the pattern of vehicular arrival data traffic flow at Mancherial Chowrasta intersection of Karimnagar City. Traffic arrival data patterns are recorded from location is processed and analyzed using Hurst index method named as correlogram. Quantitative results finally reveal that analysis shown in this study is supportive to manage the traffic signals at that junction, and to discover methods to avoid congestion traffic streams.

1. Introduction

In the transport system congestion road traffic is an occurrence of increased disturbance of the movement of traffic in urbanized areas. It is also supposed that the inadequate infrastructure, huge vehicles on road and the irrational distribution of the development are the foremost reasons of increasing traffic. For instance, traffic flow at unsignalized intersections [1], modelling the urban traffic streams [2]. Traffic congestion leads to drivers become more irritated and engaging in road rage. There are number of situations which worsen congestion traffic explore still can’t completely predict, under which condition traffic jam may suddenly occur.[3] they are investigated the quantitative individuality of vehicle influx pattern on highways. This study shows to discover if the arrival form of vehicles on highway road shows the self-similar also the ensuing time development distribution is carried out. The study has been done to model the road traffic using Poisson process, regardless of intensity of the traffic. This is used to test in the case of Ethernet traffic, LAN, WAN,
Local area networks. But [4] IP packet traffic in the LAN, WAN, Local area networks inclines to be bursty in time scale, this bursty traffic can be measured by Long-Range Dependence or Self-similarity. [5] investigated that poisson process can’t challenging the network traffic in self-similar. [6] Markovian arrival process discloses that by associating the second order statistics self-similar network traffic is suitable over required time scales. [7] studies on the single vehicle passing through the sequence of traffic lights exhibits self-similar characteristics. [8] found that network data packets are in self-similar this shows that supposition of poisson process is inappropriate subsequently it fails to model the arrival data pattern precisely. [9] Vehicle arrival pattern at toll plaza shown as self-similar characteristics.

Fortified by these novel discoveries, we study whether the arterial road vehicle arrival pattern at Mancherialchowrasta intersection of Karimnagar city can be characterized as a self-similar process too. The main criteria of this paper lie in the scheming of the Hurst Index which express the measurement of self-similarity and employed various models which are exactly reflecting the self-similar traffic.

The base of this paper is, we observed the nature of real time arterial road traffic data is self-similar. this investigation examines the continued survival of self-similar characteristics lying about these arrival data of vehicles. It is ended by calculating the Hurst index, which is the measurement of self-similarity testing. Hypothesis examination from Hurst index evaluation displays that the arrival pattern of vehicles on the highway via restrained to congested traffic conditions shows the self-similarity properties.

2. Experimental Program

2.1. Data Collection and Area of Study

Ordinary In this paper we present the real time arterial road traffic data for week days by taking the videos from intersection Area. Then the data is processed and analyzed to find the vehicular arrival volume of the particular interval of times. The data of road traffic between intersection of Choppadandi and Hospital Street, and intersection between Peddapalli Highway and Jagityal Road. Both are located in Karimnagar city, Telangana, India. There are 8 Lanes and the traffic data collected from all these lanes of different approaches. The traffic arrival data collection carried out from 15 July 2019 to 21 July 2019 we set up the video cameras at 4 intersection regions, due to the blurry recordings we choose the recorded traffic data for 8am to 8pm per a week period. Vehicular traffic flow for the intersections R1 and R2 and there are 4 approaches named as A1, A2, A3 and A4. The vehicle flow rate for each intersection approximately 280 vehicles per hour.

| Intersection | Investigation period | Vehicles per hr/lane | No.of Vehicles per Investigation Period |
|--------------|----------------------|----------------------|----------------------------------------|
| R1A1         | 8AM- 8PM             | 36 - 245             | 2448                                   |
| R1A2         | 8AM- 8PM             | 67 - 313             | 3578                                   |
| R1A3         | 8AM- 8PM             | 78 - 350             | 3683                                   |
| R1A4         | 8AM- 8PM             | 43 - 278             | 2611                                   |
| R2A1         | 8AM- 8PM             | 88 - 342             | 3675                                   |
| R2A2         | 8AM- 8PM             | 72 - 237             | 2486                                   |
| R2A3         | 8AM-8PM              | 54 - 270             | 2747                                   |

Table 1. Statistics for the road traffic flow
Table 1. shows the statistics for the traffic data of intersection regions R1 and R2 with the four individual approaches A1, A2, A3 and A4 and their traffic flow of minimum number to maximum number vehicles per hour from morning 08:00:00AM to evening 08:00:00PM, also showed the total number of vehicles per investigation period.

Table 2. Representation of Vehicular Traffic data at Different times

| Time   | No.of Vehicles | Time   | No.of Vehicles | Time | No.of Vehicles | MAE  | Chemical compositions |
|--------|----------------|--------|----------------|------|----------------|------|-----------------------|
| 08:02:21 | 10             | 08:04:34 | 36             | 08:02:21 | 184           | 64112.5062 | SiO2                   |
| 08:05:44 | 08             | 08:11:28 | 49             | 09:02:34 | 302           | 65991.8400 | Al2O3                  |
| ....... | .......         | ....... | .......        | ....... | .......        | 63805.4018 | Fe2O3                  |
| 20:57:05 | 16             | 20:40:56 | 99             | 19:01:17 | 245           | CaO            |
| 20:58:55 | 16             | 20:54:14 | 102            | 20:01:28 | 299           | MgO            |

Table 2. Shows the primary data concerning number of vehicle arrivals on arterial road at the intersection areas on different time scales of traffic data arrivals during one-minute interval of time, 10 minutes and 1-hour interval of time are displayed individually for both the intersections.

2.1.1. **self-similarity property R1 intersection lane:**
The vehicle arrival pattern for conventional traffic flow versus one-minute, 10 minute, 1-hour interval of time represented graphically in fig. 1. It was observed that there was a congestion traffic flow and it’s clearly resembled the definition of self-similarity characteristics this observation was helpful for further investigation of testing the intensity of self-similarity nature of vehicular traffic flow mathematically using distribution method like correlogram method.
2.1.2. **Self-similarity property R2 intersection lane**

Vehicular traffic volume pattern versus time interval for one minute, 10 minutes, 1-hour has demonstrated in the fig. 2 also reflects the self-similar characteristics. This justifies the self-similarity property and it permits the necessity of further investigation of the existence self-similar nature vehicular traffic flow on arterial roads.

![Graphical Representation of vehicular traffic vs. Time.](image)

**Figure 1.** Graphical Representation of vehicular traffic vs. Time.

2.2. **Self-similar process**

**Definition (1):** A stochastic process \( \{ x(t), t \geq 0 \} \) is known as self-similar if whichever \( a > 0 \), there subsists \( b > 0 \) such that

\[
X(at) = d(bX(t)) \tag{1}
\]

Let \( d \) denotes the parity of finite dimensional distributions.

**Definition (2):** Arrival patterns are modeled as point process. Assume that \( X = \{ X_t / t = 1, 2 \ldots \} \) are the arrivals in \( t^\text{th} \) interval, segregate the time axis into disjoint intervals of unit length. Let \( X \) be a second order stationary process with autocorrelation function \( \gamma(k); k \geq 0 \) and variance \( \sigma^2 \) is given by
\[
\gamma(k) = \frac{\text{Cov}(X_t, X_{t+k})}{\text{Var}(X_t)}
\]  

(2)

The process \(X\) is known to be exactly second order self-similar with the Hurst index \(H\) and variance \(\sigma^2\) if

\[
\gamma(k) = \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}], \forall k \geq 1
\]  

(3)

Definition (3): For each \(m = 1, 2, 3…\) assume a new time series as \(X^{(m)} = \{X_t^{(m)}\}, t = 1, 2, \ldots\) is determined averaging the unique time series process \(X\) over non-overlapping blocks of size \(m\)

\[
X_t^{(m)} = \frac{1}{m} \sum_{l=1}^{m} X_{(t-l)m+1}, t = 1, 2, \ldots
\]  

(4)

For each \(m\), this new series \(X_t^{(m)}\) is also a second order stationary process with Auto Correlation Function of \(\gamma^{(m)}(k)\). Hurst index \(H\) and variance if the process \(X\) is said to be asymptotically second order self-similar with in terms of variance of the averaged process, we describe similar process as

\[
\gamma^{(m)}(k) = \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (K-1)^{2H}], \forall k \geq 1
\]  

(5)

Definition (4): The process \(X\) is known to be exactly second order self-similar with Hurst index \(H = 1 - \frac{\beta}{2}\) and variance \(\sigma^2\) if Long-Range Dependence

\[
\text{Var}(X^{(m)}) = \sigma^2 m^{-\beta}, \forall m \geq 1
\]  

(6)

The differentiation between Long Range Dependence and Short-Range Dependence processes. For \(H \neq 0.5\), from the Equation (3), we get

\[
\gamma(k) = H(2H-1)k^{2H-2}\text{as } k\to \infty
\]  

and we have

\[
\sum_k \gamma(k) \sim c \sum_k k^{-\beta}, c = H(2H - 1)
\]  

(7)

The series \(c\sum_k k^{-\beta}\) is divergent if \(0 < \beta < 1\) (or) \(0.5 < H < 1\) or else they are convergent being a positive series. Consequently, the L.H.S \(\sum_k \gamma(k)\) is divergent if \(0 < \beta < 1\) (or) \(0.5 < H < 1\) or else they are convergent. The Auto Correlation Function decays gradually, that is hyperbolically for \(0.5 < H < 1\). In such case \(X\) is known as Long-Range Dependence, otherwise it is Short-Range Dependence for \(0 < H < 0.5\), in this case the Auto Correlation Function is summable.

2.3. Self-similarity Testing

Measuring the self-similarity mathematically is not so easy to depict and interpret due to its extreme level of complexity. The Hurst [10] parameter is enchanting since it narrates too many areas of
mathematics example: auto correlation, fractals, wavelets, etc. Hurst index proposes an extensive measure whether a time series has Long-Range Dependence or not. This has been beneficial in survey on network traffic analysis and modeling. The range of Hurst exponent is $0.5 < H < 1$. Measuring of exponent $H$ is complicated task. We have many techniques for measuring Hurst exponent, the methods illustrate considerably different results some of the methods are rescaled adjusted range Statistics, periodogram method, correlogram method, residuals of regression method, absolute moment’s method, Higuchi’s method. Regardless of identical speculative fundamentals, the practical application of the Correlogram method is used to compute Hurst Index.

2.3.1. Hurst Index
Hurst exponent (H) is the classical parameter of measuring the intensity of self-similarity. The development of Hurst is traced before in 1951, the hydrologist H.E Hurst [11] with his team investigate the optimum dam sizing of water storage and also to determine the drought conditions of the Nile River. Hurst parameter is used in Financial market to make decisions about trading securities. It can also be applied in ecology to increase and decrease populations. The parameter $H$ has range $0.5 < H < 1$ is a measure of self-similarity. There are Several methods for Hurst index evaluation in a time-series Roughness [12].

2.3.2 Correlogram Method
To estimate the Hurst index H using autocorrelation function in time series analysis is known as correlogram method, where the expected correlation can be specified in terms of auto-covariance function $\gamma(k)$. In this analysis, Hurst parameter H is related to autocorrelation function by means of slope, it is the coefficient of the estimate of log autocorrelation function versus the log of frequencies. To apply this method for the given data series, one should find out the auto correlation function of a series until the ACF is -ve, use all the positive values of a data series and run a regression line on log of ACF values versus natural log of the lags of the ACF values. It is used to find the Hurst exponent $H$, which is the slope of the coefficient by:

$$H=1+\alpha/2 \quad (9)$$

Where, $\alpha$ is the slope of the regression. Some pitfalls of sample correlation can be found in Mandelbrot [13] and Beran [14].

3. Results and Discussion

3.1. Intensity of self-similarity
Correlogram method were chosen to find the intensity of self-similar traffic flow for real time data has been taken from the intersection lanes of arterial roads in the Karimnagar city. Using SPSS tool, we put the effort on results of the self-similarity existence traffic data on urban roads. For the convenience we take log log correlogram plot has been drawn by taking log lag k (k=1,2, 3…) along x-axis and log log Auto correlation function on y-axis. The real time data series is exhibited a significant degree of trend line and the curve depicted in the fig. 3, and the predicted regression line $Y=-0.2315X+0.0376$. The $R^2$ value specifies the total variation in the explanatory variable (traffic data) able to describe by the predictor variable and the calculated Hurst value is 0.8845 for the first intersection. While the estimated Hurst index for the second intersection lane is 0.9135. It is clearly showing that the estimated degree of self-similarity vehicular traffic of the first intersection lane is less compared to the second intersection lane traffic flow. From the results Hurst value is greater than 0.5 as mentioned by the definition of Hurst index and it is worthwhile. As well the
testing of hypothesis is significant statistically at p=0.05. Therefore, the vehicle arrivals on arterial road does not follow Poisson distribution.

Figure 3: Autocorrelation Plot

Table 3 shows the calculated Hurst exponent values using correlogram analysis for the two intersections R1 and R2 with corresponding lanes A1, A2, A3 and A4. It was observed that the average Hurst value measured by correlogram method satisfies the (0.5<H<1) condition of self-similarity and is more than 0.5 for the eight lanes of intersections. It is clear that the degree of self-similarity for R1 intersection is more than the intersection R2.

Table 3. Estimated Hurst Index for the different lanes.

| Intersection | Lane 1     | Lane 2     | Lane 3     | Lane 4…….. | MAE   |
|--------------|------------|------------|------------|--------------|-------|
| R1A1         | 0.8845     | 0.8678     | 0.8655     | 0.9032       | 64112.5062 |
| R1A2         | 0.8312     | 0.9012     | ……         | ……           | 63991.8400 |
| R1A3         | 0.8765     | 0.8789     | ……         | ……           | 63805.4018 |
| R1A4         | 0.7996     | 0.8989     | ……         | ……           | ……     |
| R2A1         | 0.9135     | 0.9545     | 0.9615     | 0.9587       | ……     |
| R2A2         | 0.9317     | 0.9637     | ……         | ……           | ……     |
| R2A3         | 0.8974     | 0.9216     | ……         | ……           | ……     |
| R2A4         | 0.9292     | 0.9434     | ……         | ……           | ……     |

4. Conclusion
In this study, real time arterial road traffic data on a busy national highway proved to be self-similar, and to test the self-similarity correlogram method is used. The obtained values of Hurst parameter H show the measurement of self-similarity. The present study will help the design of highways in terms of cost and time etc., and also useful for forthcoming “smart city” Karimnagar which is a dream project of the people Karimnagar. Though this investigation is helpful to revise the webster method at the congestion traffic stream on urban streets.

5. References
[1] Lieberman E and Rath A K 2007 Traffic Flow Theory: A State-of-Art Report Available from www.tfhrc.gov/its/tft/tft.htm
[2] Sullivan D P and Troutbeck R J 1994 The use of Cowan’s M3 headway distribution for modelling urban traffic flow Traffic Engineering and Control 35 445-50
[3] Qiang Meng and Hooi Ling Khoo 2009 Self-similar characteristics of vehicle arrival pattern on Highways Journal of Transportation Engineering 135 11
[4] Leland W E, Taqqu M S, Willinger W and Wilson D V 1994 On the self-similar nature of ethernet traffic IEEE/ACM Transactions on Networking 2 1-15
[5] Paxson V and Floyd S 1995 Wide-area traffic the failure of Poisson modelling IEEE/ACM Transactions on Networking 3 226-44
[6] Andersen A T and Nielsen BF 1998 A Markovian approach for modelling packet traffic with long-range dependence IEEE Journal of Select Areas in Commun 16719-32
[7] Nagatani T 2005 Self-similar behaviour of a single vehicle through periodic traffic lights Physical A 347 673-82
[8] Crovella M and Bestavros A 1996 Self-similarity in world wide web traffic evidence and possible causes International Conference on Measurement and Modelling of Computer System (ACM SIGMETRICS) Philadelphia
[9] Perati MR, Raghavendra K, Koppula HKR, Doodipala M R and Dasari R 2012 Self-similar Behavior of Highway Road Traffic and Performance Analysis at Toll Plazas Journal of Transportation Engineering 138 1233-38
[10] Hurst H 1951 Long-Term storage of reservoirs an experimental study, Trans of the American Society of Civil Engineers, 770-99
[11] Hurst H 2005 Hurst parameter of self-similar network traffic International Conference on Computer Systems and Tech
[12] Roughness 2003 Length Method for Estimation Hurst Exponent and Fractal Dimension of Traces Help Benoit 1.3 version Software TruSoft International Inc
[13] Beran J, Taquq MS and Willinger W 1995 Long-range dependence in variable bit rate traffic IEEE Trans on Communications 43 pp 1566-79
[14] Pushpalatha Sarla and Mallikarjuna Reddy D 2017 Linear Regression Model Fitting and Implication to Self-Similar Behavior Traffic Arrival Data Pattern at Web Centers, IOSR Journal of Computer Engineering 19 01-05
[15] Sarla P, Mallikarjuna Reddy D and Manohar Dingari 2016 Self-Similarity Analysis of Web Users Arrival Pattern at Selected Web Centers American Journal of Computational Mathematics 6 17-22
[16] Jerzy Wawszczak 2005 Methods for estimating the Hurst exponent the analysis of its value for fracture surface research Materials Science-Poland 23
[17] Beran J Statistics for Long-Memory Processes (Chapman and Hall) 1994
[18] Chaudhry ML and Gupta UC 1997 Queue-Length and Waiting-Time Distributions of Discrete Time GIX/Geom/1 Queueing Systems with Early and Late Arrivals Queueing Systems 25 307-24
[19] Kendall D G 1953 Stochastic Processes Occurring in the Theory of Queues and their Analysis by the Method of the Imbedded Markov Chain the Annals of Mathematical Statistics 24 338-54
[20] Chaudhry, ML and Gupta UC 1997 Queue-Length and Waiting-Time Distributions of Discrete Time GIX/Geom/1 Queueing Systems with Early and Late Arrivals Queueing Systems 25 307-324
[21] Zhao YQ and Campbell LL 1996 Equilibrium Probability Calculations for a Discrete-Time Bulk Queue Model Queueing Systems 22 189-98
[22] Mallikarjunareddy D, Vamshi Krishna and Pushpalatha Sarla 2020 Markovian model for Internet Router Employing Partial Buffer Sharing Mechanism under Self-similar Traffic 020064, AIP conference Proceedings 2246

[23] Haight FA 1963 Mathematical Theories of Traffic Flow (New York: Academic Press)

[24] Mallikarjuna Reddy D, A.M Girija and Pushpalatha Sarla 2020 An Application of Queuing System to patient satisfaction at a selected hospital - A field Study AIP conference Proceedings 2246