We analyze the consistency of electroweak breaking within the simplest “dark matter completions” of the high-scale type-I seesaw mechanism. We derive the full two-loop RGEs of the relevant parameters, including the quartic Higgs self-coupling $\lambda$ of the Standard Model. For the simplest type-I seesaw with bare “right-handed” neutrino mass terms, we find that with sizeable Yukawa couplings, the Higgs quartic self-coupling $\lambda$ becomes negative much before reaching the seesaw scale. For “large” Yukawa couplings the type-I seesaw may be inconsistent even as an effective theory. We further show that simple extensions of the canonical type-I seesaw involving a viable dark matter candidate can indeed fix this problem rendering the Higgs vacuum stable up to Planck scale. We examine two such extensions, the type-I seesaw with spontaneous lepton number violation and the recently proposed scoto-seesaw mechanism. Both have better stability properties due to the new scalars required.

1. INTRODUCTION

The discovery of a scalar particle with 125 GeV mass plays a central role within particle physics [1, 2]. In particular, the precise Higgs boson mass measurement determines the value of the quartic coupling in the scalar potential at the electroweak scale and allows one to study its behavior all the way up to high energies. Given today’s measured values of Standard Model parameters such as the top quark and Higgs boson masses, we know that the Higgs quartic coupling remains perturbative after renomalization group equations
(RGEs) are used to evolve it to high energies. However, the stability of the fundamental vacuum is likely to fail at mass scales below the fundamental Planck scale [3].

Another most important milestone in the landscape of particle physics has been the discovery of neutrino oscillations [4, 5]. This implies the existence of new physics [6] and forces us to amend the Standard Model by adding new particles in order to provide nonzero masses to neutrinos [7].

Electroweak symmetry breaking acquires a new perspective in dynamical theories of neutrino mass generation. In fact, these extensions can affect the profile of the Higgs boson in an important way. Indeed, it has been argued that the presence of new couplings associated to neutrino mass generation can have an important impact upon the vacuum stability issue [8]. Models with low-scale violation of lepton number [11] deserve special interest in this context, because of their potential in changing the structure of the vacuum as well as their phenomenological impact upon Higgs boson decays, specially the presence of invisible Higgs decays [11]. An analysis of the resulting sensitivities of Higgs Boson searches at the ATLAS and CMS experiments at the LHC was also performed in the presence of the Higgs boson triplet associated to the type-II seesaw mechanism [12, 13].

Here we examine more closely the issue of the consistency of the Higgs vacuum within the more conventional high-scale type-I seesaw extensions of the Standard Model and their dark matter completions. We focus specifically on SU(3)⊗SU(2)⊗U(1) type-I seesaw extensions both with explicit [14] as well as spontaneous violation of lepton number [15, 16]. The latter contains a physical Nambu-Goldstone boson, dubbed Majoron. We show how the associated scalar couplings can easily restore stability of the electroweak symmetry breaking even if the lepton number violation scale is high.

We stress that SU(3)⊗SU(2)⊗U(1) seesaw extensions are the most general ones, as they can be formulated with any number of “right-handed” neutrinos, as noted in Ref. [14], who proposed seesaw models with an arbitrary number of “right-handed” neutrinos, either more or less than the number of left neutrinos. Here for illustration we first consider the minimalistic (3,1) model containing only one right-handed neutrino, in addition to the 3 known left-handed neutrinos. It has better stability properties than the fully sequential (3,3) seesaw mechanism which is then considered to further highlight the vacuum instability problem in pure type-I seesaw. We then show that vacuum stability can be improved by adding extra scalars as required, for example, in order to implement spontaneous violation of lepton number or to implement the scotoseesaw mechanism [17].

We analyse the scale at which instability sets in as a function of the magnitude of the Yukawa coupling relevant for generating neutrino mass in (3,1) as well as the conventional (3,3) seesaw schemes. We also study the role of extra scalars required in Majoron and scotogenic completions [17] and study its improved stability properties.

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1 Recent stability analyses in the context of the seesaw mechanism are given in Ref. [9, 10].
This work is organized as follows: In Section 2 we revisit the vacuum stability problem in the Standard Model showing that the Higgs quartic coupling becomes negative when RG-evolved to high scales. We then show in Section 3 that the vacuum stability problem becomes much worse in type-I seesaw extensions of the Standard Model. In Section 4 we look at the Majoron extension of the canonical type-I seesaw. In addition to providing a viable dark matter candidate, we show how it helps stabilize the Higgs vacuum, which can be made stable all the way up to Planck scale. In Section 5 we look at another simple extension of type-I seesaw, namely the recently proposed scoto-seesaw. We show that, apart from providing a WIMP dark matter candidate, this simple extension can also improve vacuum stability up to Planck scale. Finally, we conclude and summarize our main results in Section 6.

2. HIGGS VACUUM IN THE STANDARD MODEL

Let’s start with briefly revisiting the status of the electroweak vacuum within the Standard Model. For a long time the Higgs boson was the “last” missing piece of the theory. The discovery of a scalar particle with mass $m_H \approx 125$ GeV at the Large Hadron Collider (LHC) is very suggestive that it could be the long-awaited Standard Model Higgs boson. While further work is still needed to unambiguously establish this, current data so far indicates that its couplings and decay properties are close to the Standard Model Higgs expectations. If, indeed, this is the case, the next obvious question is, given that so far we have not seen any evidence for new particles at the LHC, whether the Standard Model can be the final theory all the way up to Planck scale. The answer is obviously no, since the Standard Model predicts neutrinos to be massless and there is no viable Standard Model candidate for dark matter.

For the moment we put these two issues aside and ask ourselves whether there are other compelling reasons to argue that the Standard Model cannot be the final theory up to Planck scale. Indeed, there are several other theoretical and aesthetical arguments against it being the final theory such as the unification of forces and the hierarchy/fine-tuning/naturalness problem. However, the “Higgs discovery” has facilitated us to study the high energy behavior of the Standard Model up to the Planck scale. In particular the study of electroweak phase transition, possibly linked to baryogenesis, and the stability of Higgs vacuum at energies far above the electroweak scale. Its the latter question that we address in this work.

The detailed analysis of Higgs vacuum within the Standard Model has of course been carried out before us [18, 19]. For completeness here we revisit this analysis. This serves us to calibrate our Renormalization Group (RG) analysis against known results. Although for the Standard Model case there are some partial 3-loop results in dedicated studies[18]...
here we perform the RG analysis at the two-loop level. We perform our analysis in the $\overline{\text{MS}}$ scheme, taking the parameter values at low scale as the input values \cite{3}. In particular, the Higgs pole mass is taken as the current best fit value of $m_H = 125.18 \pm 0.16$ GeV, the pole mass of top quark is taken as $m_t = 173 \pm 0.4$ GeV and the strong coupling constant $\alpha_s(M_Z) = 0.1184 \pm 0.0007$. Using these experimental values, we adopt the “On-Shell” renormalization scheme (OS) to directly express the renormalized parameters in terms of the physical observables and then relate the OS parameters to the $\overline{\text{MS}}$ parameters in a way similar to \cite{18}. In Table. I we list the $\overline{\text{MS}}$ input values of the relevant parameters at the top mass $m_t$ scale.

| Parameter | $g_1$ | $g_2$ | $g_3$ | $y_t$ | $\lambda$ |
|-----------|-------|-------|-------|-------|----------|
| $\mu(m_t)$ | 0.462607 | 0.647737 | 1.16541 | 0.93519 | 0.126115 |

TABLE I: $\overline{\text{MS}}$ values of the main input parameters at the top quark mass scale, $m_t = 173 \pm 0.4$ GeV.

Taking the initial $\overline{\text{MS}}$ values of Table. I as input values, we then RG-evolve the Standard Model parameters to higher scales as shown in Fig. 1.

![Fig. 1: The RG evolution of the Standard Model gauge couplings $g_1$, $g_2$, $g_3$, top Yukawa coupling $y_t$ and Higgs self-quartic coupling $\lambda$.](image)

Our two-loop results are in good agreement with earlier results up to very small differences due to higher loops and updated initial values compared to the earlier papers. The latter are mainly due to the increased precision of the experimental numbers which has now become available. We stress that an in-depth reanalysis of the Standard Model Higgs is not the
main goal of our paper, hence we will not carry out a sensitivity analysis of Higgs vacuum stability and its dependence on the input parameter errors. In fact, in the seesaw scenarios of interest to us, such tiny effects are negligible when compared to the effects of the new Yukawa couplings.

Notice from Fig. 1 that the Standard Model Higgs quartic coupling $\lambda$ becomes negative at $\mu \simeq 10^{10}$ GeV. This would imply that the Higgs potential is unbounded from below and the Higgs vacuum is unstable. A dedicated analysis shows that Standard Model Higgs vacuum is not unstable, but rather metastable$^2$ with very long lifetime [18, 19].

However, as already mentioned, the Standard Model cannot be the final theory up to the Planck scale as it has massless neutrinos and no viable candidate for dark matter. Hence the vacuum stability analysis must be carried out within Standard Model extensions which can address these issues. We now move on to consider simple seesaw extensions of Standard Model accounting for neutrino masses (and/or dark matter) and show that, the Higgs vacuum stability in these simple and realistic scenarios can be completely dominated by new couplings. Thus for our purposes it suffices to discuss electroweak vacuum stability at two loops, without going into fine details about input parameters$^3$.

3. HIGH-SCALE TYPE-I SEESAW MECHANISM

We begin with Standard Model extensions accounting for naturally small neutrino masses, and in latter sections we consider schemes which also include viable dark matter candidates. It is well known that, in the Standard Model neutrinos are massless, while the observation of neutrino oscillations has conclusively proven that at least two of the three “active” neutrinos are massive [4, 5].

In the most general “type-I seesaw” mechanism, proposed in [14], neutrino masses arise from the exchange of an arbitrary number of singlet “right-handed neutrinos”, $\nu_{R_i}$, $i = 1, 2, \cdots n$, added to the Standard Model for this purpose. Since these right-handed neutrinos are gauge singlets from the point of view of the Standard Model gauge symmetries, they need not be added “sequentially”, so as to match in number the left-handed ones. For the general case, the relevant part of the Lagrangian is written as,

$$-\mathcal{L} = \sum_{a,i} Y_{\nu}^{ai} \bar{\ell}_L^i \tilde{H} \nu_{R_i} + \frac{1}{2} \sum_{i,j} M_{ij}^{R} \nu_{R_i}^c \nu_{R_j} + \text{H.c.}$$

where $\ell_L^a = (\nu_L^a, l_L^a)^T$ with $a = 1, 2, 3$ denotes the three families of left lepton doublets, while $i, j = 1, 2, \cdots n$ labels the right-handed singlet neutrinos, and $H$ is the Standard Model Higgs

$^2$ Standard Model vacuum stability is sensitive to input parameter values, in particular the top-quark mass.

$^3$ It is straightforward to refine the seesaw analysis along the lines presented in [18] for the Standard Model.
doublet. After the electroweak symmetry breaking, the Higgs gets vacuum expectation value (vev) \( \langle H \rangle = \frac{v}{\sqrt{2}} \) with \( v = 246 \text{ GeV} \). Then the full neutrino mass matrix is expressed as,

\[
\mathcal{M}_\nu = \begin{pmatrix}
0 & m_D \\
M_D^T & M_R
\end{pmatrix}
\]  

(2)

where \( m_D = \frac{Y_\nu}{\sqrt{2}} v \) is the “Dirac mass matrix” generated after spontaneous electroweak symmetry breaking.

Being invariant under the SU(3)_c \otimes SU(2)_L \otimes U(1)_Y gauge symmetry, the \( n \times n \) right-handed neutrinos “Majorana mass matrix” \( M_R^{ij} \) is independent of the electroweak scale and can be large, \( |M_R^{ij}| >> v \) implying \( \left| \frac{m_R^{ij}}{M_R^{ij}} \right| << 1 \). Hence the mass matrix in Eq.(2) can be block-diagonalized perturbatively [16]. To leading order the mass matrix elements for light neutrinos \( m_{ab}^{\nu} \) are given as [16]

\[
m_{ab}^{\nu} \simeq -m_D^{ai}(M_R^{-1})^{ij}(M_D^{T})^{jb} + \text{higher order terms}
\]  

(3)

The expression for the full diagonalizing matrix was first given in [16]. The light neutrino mass matrix in (3) is further diagonalized by a unitary matrix \( U_\nu \) in the light neutrino sector \( \nu^a; \ a = 1, 2, 3 \). This is the famous type-I seesaw formula linking the smallness of the light neutrinos to the heaviness of the right-handed neutrinos \( \nu_R \). Depending on the value of “n” many possibilities can be envisaged. For example, first consider \( n \leq 3 \) of “high-scale” constructions. Current experimental and cosmological observations indicate that the neutrino masses are \( \nu_m^a \leq \mathcal{O}(0.1) \text{ eV} \) [20, 21] with recent Planck data indicating that sum of neutrino masses \( \sum_a \nu_m^a = 0.23 \text{ eV} \) [22]. Hence, for “sizeable” Yukawa coupling, \( Y_\nu \sim \mathcal{O}(1) \), in order to satisfy the neutrino mass bound, heavy neutrino mass \( M_N \) will be order of \( \mathcal{O}(10^{14} \text{ GeV}) \). This characterizes the case of genuine “high-scale” seesaw constructions. For \( n=2 \) one has the minimal realistic scenario in which two small neutrino masses arise naturally at the tree-level [14]. These will be identified with the “atmospheric scale” and the “solar” mass scales. The case \( n=1 \) leads the minimal prescription that generates only one small neutrino mass at the tree level [14] that can be identified with the “atmospheric scale”. Realistic values for the “solar” mass scale could arise, for example, from adding a scotogenic “dark” sector [17].

Turning to the case \( n \geq 3 \) we have interesting schemes for \( n=3 \), that corresponds to the canonical type-I seesaw mechanism [7]. The cases \( n=6, 4 \) and 2 also provide the setting for “low-scale” seesaw mechanisms in which “quasi”-Dirac heavy neutrinos act as neutrino mass generation messengers. Their masses can be much lower than in the “high-scale” seesaw, possibly at the TeV scale, as neutrino masses are “protected” in ’t-Hooft’s sense. For \( n=6 \) we have the template for the sequential “low-scale” seesaw mechanisms, both of the inverse [23, 24] and linear seesaw type [25–27]. The cases \( n=4 \) and \( n=2 \) correspond to more economical constructions analogous to \( n=2,1 \) of the high-scale seesaw.
In this paper we will be mostly concerned with the effects of sizable Yukawa couplings in the context of high-scale seesaw constructions and their significant impact on the stability of the Higgs vacuum. Here we have studied the effect of such large Yukawa couplings $Y_\nu$ on the stability of the electroweak vacuum.

### 3.1. Higgs Vacuum Stability in High-scale Type-I Seesaw

For definiteness and simplicity here we focus on the simplest type-I seesaw mechanism in which only one right-handed neutrino, $n = 1$, is added to the Standard Model particle content, the scenario called (3,1) in [14]. Since at least two light neutrino masses are required to explain the current neutrino oscillation data, the minimal (3,1) seesaw scheme is not phenomenologically viable by itself. However, it gives us a good starting point since it provides a clear picture of the impact of seesaw extensions on the Higgs vacuum stability in the simplest possible setting. Moreover, it provides an adequate template for the formulation of a fully realistic scotogenic dark matter completion [17], to be discussed in latter sections.

Let us start by looking at the impact of the right-handed neutrinos on the stability of the Higgs vacuum. As we discussed at length in Section 2, the Standard Model RG running of the Higgs quartic scalar coupling $\lambda$ is dominated by the top Yukawa, which is the largest coupling present in the theory. As we saw, in this case the Standard Model $\lambda$ coupling becomes negative around the scale $\mu \sim 10^{10}$ GeV. However, within the seesaw completion, the neutrino Yukawa couplings $Y_\nu$ of (1) can completely dominate the RG behavior of $\lambda$ as shown in Fig. 2.

![Fig. 2](image-url)

**FIG. 2**: Evolution of the Higgs quartic self-coupling within the minimal (3,1) Type I seesaw scheme (continuous (red) curve). The gauge and Yukawa couplings $g_1$, $g_2$, $g_3$, $y_t$, and $Y_\nu$ are also indicated by the dashed lines. The light neutrino mass is fixed in both panels at $m_\nu = 0.1$ eV, corresponding to a heavy neutrino mass $M_R$ of $2.7 \times 10^{13}$ GeV and $7.5 \times 10^{13}$ GeV, respectively.

Fig. 2 illustrates the effect of the new neutrino Yukawa coupling $Y_\nu$ on various other
couplings. One sees that the RG running of $\lambda$ can be completely dominated by the Yukawa coupling $Y_\nu$, particularly for $Y_\nu \approx \mathcal{O}(1)$. For illustration we have taken two representative values of Yukawa couplings $Y_\nu = 0.3, 0.5$. One sees from Fig. 2 that for $Y_\nu = 0.3, 0.5$, the Higgs quartic coupling $\lambda$ becomes negative at $\mu = 10^9, 10^8$ GeV respectively i.e. faster than the Standard Model case, Fig. 1.

Some important conclusions can be drawn from this simple $(3,1)$ seesaw scenario.

- The problem of Higgs vacuum instability becomes more acute in a type-I seesaw completion of the Standard Model. As seen from the example of the minimal $(3,1)$ seesaw mechanism illustrated in Fig. 2, the quartic coupling $\lambda$ runs faster to negative values. This was expected, since the addition of new fermions tends to destabilize the Higgs vacuum.

- More importantly, the quartic coupling becomes negative much before the mass scale characterizing the right-handed neutrinos, making the theory inconsistent. Indeed, for $Y_\nu \gtrsim \mathcal{O}(10^{-2})$ and $m_\nu \sim 0.1$ eV, one has $M_R \gtrsim 10^{12}$ GeV, so that the simplest $(3,1)$ seesaw cannot even be considered a consistent effective theory. In contrast, in Standard Model the quartic coupling $\lambda$ becomes negative at $\mathcal{O}(10^{10})$ GeV, so the instability problem can be avoided by embedding the theory in a larger one at or below this scale. Thus, the Standard Model still provides a consistent effective description of physics at the electroweak scale while the $(3,1)$ seesaw with $M_R \gtrsim 10^{10}$ GeV cannot be considered even as a consistent effective theory.

- As we will further elaborate, the instability problem only gets worse within higher $(3,n)$ seesaw schemes, with $n \geq 2$, as the extra right-handed neutrinos contribute to the running and tend to further destabilize $\lambda$, as we discuss in the next Section, 3.3.2.

Of course as the value of the Yukawa coupling $Y_\nu$ decreases, so does its impact on the RG running of $\lambda$. Also, for a given fixed value of the light neutrino mass $m_\nu$, say 0.1 eV, the required mass of right-handed neutrinos $M_R$ will also decrease as the Yukawa coupling decreases. Combining these two effects we find that, for low enough $Y_\nu$, the $(3,1)$ type-I would still be consistent as an effective theory, see Fig. 3.

### 3.2. Stability in higher $(3,n)$ type-I seesaw mechanism, $n \geq 2$

Since right-handed neutrinos appear as $\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes U(1)_Y$ singlets, there are no theory constraints on their number, so the $(3,n)$ type-I seesaw mechanism, with $n \geq 2$ is perfectly viable. Let us now briefly discuss this case. The $(3,2)$ case leads to two light neutrinos getting small masses [14], while in the “sequential” $(3,3)$ case, all three light neutrinos get small masses. Hence these schemes are phenomenologically viable even in
FIG. 3: Zoomed view of the evolution of the Yukawa coupling $Y_\nu$ and the quartic Higgs self-coupling $\lambda$ within the minimal $(3,1)$ seesaw. The Yukawa $Y_\nu$ is chosen small enough so that the seesaw provides at least a self-consistent effective theory.

the absence of radiative corrections present, for example, in the scotogenic dark matter completion proposed in [17].

As already mentioned, the problem of Higgs vacuum stability in type-I seesaw extensions only gets worse with the addition of extra right-handed neutrinos. This fact is clearly illustrated in Fig. 4, where we compare the Higgs quartic self coupling $\lambda$ evolution within the Standard Model with $(3,n)$ seesaw completions, with $n = 1$ and $n = 3$. For simplicity, in Fig. 4, we have fixed the benchmark value of $Y_{\nu}^{aj} = 0.5$; $a = j = 1, 2, 3$ and taken the off-diagonal terms to be zero for the $(3,3)$ case.

As can be seen from Fig. 4, keeping all other parameters fixed, for a given value of $Y_\nu$, the problem of vacuum stability becomes more acute with increasing number of right-handed neutrino species. This is expected since fermions contribute negatively to the evolution (see RG equations in Appendix B), so extra right-handed neutrinos makes the vacuum more unstable. Fig. 4 also shows that a type-I seesaw extension will, at best, reach the Standard Model curve (continuous line). Thus any $(3,n)$ type-I seesaw extension of the Standard Model with $n \geq 1$ will:

I. Necessarily be an inconsistent model for any $M_R^i \geq 10^{10}$ GeV; $i = 1, \cdots , n$

II. For lower $M_R^k$ values the theory will still be inconsistent if the Higgs quartic self-
coupling $\lambda$ goes negative before the largest mass scale $M^4_i$ in the theory $^4$.

III. In the best case scenario, i.e. for small enough Yukawas $Y^{\nu}_{\alpha j}$ and masses $M^{ij}_R$, the type-I seesaw extensions can be self-consistent effective theories, but only in the regime where $\lambda$ is still positive. They should be embedded in a bigger theory.

Thus, in seesaw scenarios the stability properties of the electroweak vacuum will at best be those of the Standard Model Higgs vacuum. In order to enhance Higgs vacuum stability it is quite desirable to further extend or embed the type-I seesaw. A natural way to do this is to include a candidate for dark matter intimately connected with the neutrino mass generation mechanism. In the next two sections, we consider two such scenarios and analyze the vacuum stability properties for both cases.

4. THE MAJORON COMPLETION OF THE TYPE-I SEESAW

We now consider the type-I seesaw extensions of the Standard Model, in which lepton number is promoted to a spontaneously broken symmetry within the $\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$ gauge framework $^{15, 16}$. In addition to the right-handed neutrinos $\nu_R$ we add a complex singlet $\sigma$ carrying two units of lepton number.

$^4$ Though potentially viable, a long-lived vacuum would open up the allowed parameters but only slightly.
The relevant Lagrangian is given by
\[
\mathcal{L} = - \sum_{a,i} Y_{\nu}^{ai} \bar{\nu}_L^a \tilde{H} \nu_R^i - \frac{1}{2} \sum_{i,j} Y_{R}^{ij} \sigma \nu_{R}^i \nu_{R}^j + \text{H.c.}
\] (4)

The resulting neutrino mass matrices in $\nu_L$ and $\nu_R$ basis is given by
\[
\mathcal{M}_\nu = \begin{pmatrix} 0 & \frac{Y_{\nu} \nu v}{\sqrt{2}} \\ \frac{Y_{R}^{\nu} v}{\sqrt{2}} & \frac{Y_{R}^{\nu} \nu \sigma}{\sqrt{2}} \end{pmatrix}
\] (5)

The effective light neutrino mass obtained by perturbative diagonalization of the above mass matrix is of the form
\[
m_\nu \simeq Y_\nu Y_R^{-1} Y_\nu^T \frac{v_H^2}{\sqrt{2} v_\sigma}
\] (6)

In the presence of the complex scalar singlet $\sigma$, the most general Higgs potential which is capable of driving electroweak and lepton number symmetry breaking is given by [11]
\[
V(\sigma, H) = -\mu_H^2 H^\dagger H - \mu_\sigma^2 \sigma^\dagger \sigma + \lambda (H^\dagger H)^2 + \lambda_\sigma (\sigma^\dagger \sigma)^2 + \lambda_{H\sigma} (H^\dagger H)(\sigma^\dagger \sigma).
\] (7)

This potential is bounded from below if $\lambda_\sigma$, $\lambda_H$ and $\lambda_{H\sigma} + 2\sqrt{\lambda_\sigma \lambda_H}$ are all positive. In addition to the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge invariance, the theory is also invariant under lepton number. The above potential can develop a minimum for non-zero vacuum expectation values of both $H$ and $\sigma$ if $\lambda$, $\lambda_\sigma$ and $4\lambda_\sigma - \lambda_{H\sigma}^2$ are all positive. The vevs break both the electroweak and lepton number symmetries, three of the degrees of freedom in $H$ being eaten by the massive gauge bosons, while the imaginary part of the $\sigma$ corresponds to the Majoron $J$. The real parts of $H$ and $\sigma$ will mix with each other to give two CP-even mass eigenstates $h_1$ and $h_2$. The lighter of these is identified with Standard Model Higgs boson.

**Vacuum stability in type-I seesaw with Majoron**

For simplicity we take again the simplest dark extension of the type-I seesaw mechanism based on the Majoron extension of the (3,1) scheme considered above. From Eq. 6 one finds that
\[
Y_R \approx \frac{10^{14} \text{GeV}}{v_\sigma}.
\]

for a Yukawa coupling $Y_\nu$ of $O(1)$ and a neutrino mass parameter of order $10^{-1}$ eV i.e. of the order of the atmospheric scale [6]. Hence, with $v_\sigma \approx 10^{14}$ GeV, $Y_R$ will be of $O(1)$. But for large $Y_R$, $\lambda_\sigma$ would become negative very quickly when RG-evolved, so the scalar potential will not be bounded, as $\beta_\lambda \simeq -Y_R^4$. Hence, we have chosen $v_\sigma \approx 10^{16}$ GeV i.e. at the Grand
FIG. 5: Evolution of Yukawa coupling $y_t$ and $Y_\nu$, Higgs self-quartic coupling $\lambda$ and $\lambda_\sigma$, $\lambda_{H\sigma}$ in type-I seesaw with Majoron. See text for details.

Unified Theory (GUT) scale, so that $Y_R \approx 10^{-2}$ and its effects will be small in the running of $\lambda_\sigma$. For such very large $v_\sigma \approx 10^{16}$ GeV the light and heavy Higgs sectors will be almost decoupled, though we can still allow appreciable $\lambda_{H\sigma}$ with very small mixing angle $\alpha$, see Appendix A.

Our results for the (3,1) type-I seesaw mechanism with Majoron are shown in Fig. 5, where we have taken $\lambda_\sigma = 0.1$ at the initial mass scale $m_t$. The RG evolution in Fig. 5 is shown for four Yukawa coupling values $Y_\nu = 0.1, 0.3, 0.4$ and 0.5. It shows that, indeed, the stability properties can substantially improve due to the presence of the new scalar. In fact, for appreciable Yukawa couplings, one can have positive $\lambda$ all the way up to Planck scale. For the Yukawa couplings $Y_\nu = 0.1, 0.3, 0.4$ and 0.5, the required values of minimum $\lambda_{H\sigma}$ are 0.2, 0.22, 0.30 and 0.34 respectively. Note, however, that a large Yukawa coupling $Y_\nu \geq 0.6$ is not allowed. The reason is two-fold: $\lambda$ becomes negative for $\lambda_{H\sigma} \leq 0.34$, while non-perturbative effects arise for $\lambda_{H\sigma} \geq 0.34$.

To sum up, these results show how, in contrast to the type-I seesaw with explicit breaking of lepton number, this Majoron version can have stable electroweak vacuum all the way upto
Planck scale for adequate Yukawa coupling choices. Thus, the Majoron completion of the (3,1) type-I seesaw can be considered as a full theory, unlike the simplest version which at best can be considered as an effective theory.

Going to the (3,3) Majoron type-I seesaw with sequential right-handed neutrino assignment, we find that the Higgs vacuum can be still kept stable up to Planck scale for appreciable Yukawa couplings. Of course, the presence of two more new fermions means that the maximum values of $Y^a_i$, for which Higgs vacuum stability can be achieved up to Planck scale, is somewhat reduced. In Fig. 6 we compare the Higgs vacuum stability of the (3,3) Majoron seesaw case with the (3,1) analogue as well as with Standard Model.

![FIG. 6: Zoomed view of the evolution of the quartic Higgs self-coupling $\lambda$ in the Standard Model (solid) and (3,1) and (3,3) Majoron seesaw (blue dot-dashed and green dotted, respectively).](image)

In plotting Fig. 6 we have taken $Y_\nu = 0.3$ for (3,1) case and for (3,3) case we have taken $Y^{ai}_\nu = 0.3; a = i = 1, 2, 3$ while all the off-diagonal entries are taken to be zero. The rest of the parameters are kept same as described previously for the (3,1) Majoron seesaw case. Thus, Majoron models phenomenologically consistent with the neutrino oscillation data, can also have a completely stable vacuum all the way up to Planck scale.

Before concluding we should note the cosmological advantages of the Majoron completion. The first is that it can also provide a dark matter candidate, namely the Majoron [28], providing an alternative to the ΛCDM paradigm. The Majoron is assumed to get mass from gravitational effects that explicitly violate the global lepton number [29]. Assuming that its mass lies in the keV range one can show that it can be a viable warm dark matter candidate, decaying to neutrinos, with a strength proportional to their tiny mass [16]. Hence, it is naturally long-lived on a cosmological scale, as required, with lifetime $\tau_J$ larger than the age of the Universe $t_0 = 13.8$ Gyr $\simeq 4 \times 10^{17}$ s. Indeed, the massive Majoron dark
matter scenario has been shown to be consistent with cosmic microwave background data for adequate choices of the relevant parameters [30–32], the Majoron decay lifetime constraints ranging from $\tau_J > 50 – 160$ Gyr. Using N-body simulations one can show that Majoron as a dark matter leads to a viable alternative to the $\Lambda$CDM scenario, with predictions that can differ substantially on small scales [33].

Finally we mention that, in addition to dark matter, the Majoron picture may also provide a solution to other cosmological drawbacks of the Standard Model, such as inflation. The role of the Majoron has also been discussed in the context of leptogenesis [34]. Moreover, in Ref. [35], it was shown that in seesaw Majoron models, inflation and dark matter can have a common origin, connected to the neutrino mass generation scheme.

5. THE MINIMAL SCOTOGENIC SEESAW

The minimal scotogenic seesaw model proposed in Ref. [17] is based on the combination of the simplest (3,1) version of the seesaw mechanism [14] with the minimal scotogenic model [36]. The scotogenic seesaw model has a viable WIMP dark matter candidate and can also account for the observed neutrino masses, both features being closely related. For example, the “atmospheric” mass scale arises at the tree level from the (3,1) seesaw mechanism, while the “solar” oscillation scale emerges radiatively, through a loop involving the “dark sector” exchange. We now briefly describe the minimal scotogenic seesaw scenario [17]. The new particles and their charges are given in Table II.

|       | $\nu_R$ | $\eta$ | $f$ |
|-------|---------|--------|-----|
| $SU(2)_L$ | 1       | 2      | 1   |
| $U(1)_Y$   | 0       | $\frac{1}{2}$ | 0   |
| $Z_2$     | +       | -      | -   |
| Multiplicity | 1       | 1      | 1   |

TABLE II: New particles in the minimal scotogenic seesaw mechanism.

In Table II the additional $Z_2$ symmetry is the “dark parity” which is responsible for the stability of the dark matter candidate. The full Yukawa sector can be split as

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{ATM} + \mathcal{L}_{DM,SOL}$$

(8)

where $\mathcal{L}_{SM}$ is the Standard Model Lagrangian and

$$\mathcal{L}_{ATM} = - \sum_a Y^a_\nu L^a_i \sigma_2 H^* \nu_R + \frac{1}{2} M_{R\nu_R} \nu_R + h.c,$$

(9)
which induces the tree level neutrino mass (atmospheric neutrino mass scale) after the electroweak symmetry breaking. The Lagrangian responsible for the solar and dark sector is given by

$$\mathcal{L}_{\text{DM,SOL}} = Y_f \bar{L} \sigma^2 \eta^* f + \frac{1}{2} M_f \bar{f} f + \text{h.c.} \quad (10)$$

The dark sector consists of one fermion $f$ and one scalar $\eta$, both of which are odd under the dark $\mathbb{Z}_2$ parity. All the Standard Model particles and $\nu_R$ are even under this dark $\mathbb{Z}_2$ parity.

The scalar sector is given by

$$V = -\mu_H^2 H^\dagger H + m_\eta^2 \eta^\dagger \eta + \lambda (H^\dagger H)^2 + \lambda_3 (H^\dagger \eta)(\eta^\dagger H) + \lambda_4 (H^\dagger \eta)(\eta^\dagger \eta) + \lambda_5 ((H^\dagger \eta)^2 + \text{h.c.}) \quad (11)$$

As the $\mathbb{Z}_2$ symmetry is exactly conserved, the mixing between the Higgs doublet and $\eta$ are forbidden. The lightest field in the dark sector can play the role of WIMP dark matter candidate, that can be detected by nuclear recoil experiments such as XENON-1T [37]. The mass spectrum for the components of the $\eta$ doublet are given by

$$m_{\eta_R}^2 = m_\eta^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + 2\lambda_5) \quad (12)$$

$$m_{\eta_I}^2 = m_\eta^2 + \frac{1}{2} (\lambda_3 + \lambda_4 - 2\lambda_5) \quad (13)$$

$$m_{\eta^+}^2 = m_\eta^2 + \frac{1}{2} \lambda_3 v^2. \quad (14)$$

The total neutrino mass has the structure

$$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$$

$\eta^R$ $\eta^I$ $\eta^R$ $\eta^I$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\eta^R$ $\eta^I$ $\eta^R$ $\eta^I$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

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$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$

$\langle H \rangle \quad \langle H \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle \quad \langle \eta^R \rangle \quad \langle \eta^I \rangle$
for inducing the solar mass scale. The loop function $\mathcal{F}$ is expressed as the difference of two $B_0$-Veltman functions, namely,

$$
\mathcal{F}(m_{\eta R}, m_{\eta I}, M_f) = \frac{1}{32\pi^2} \left( m_{\eta R}^2 \log \left( \frac{M_f^2/m_{\eta R}^2}{M_f^2 - m_{\eta R}^2} \right) - m_{\eta I}^2 \log \left( \frac{M_f^2/m_{\eta I}^2}{M_f^2 - m_{\eta I}^2} \right) \right)
$$

(16)

Both terms in Eq. 15 have a projective nature. Hence one out of the three neutrinos remains massless. From the eigenvalues of the neutrino mass matrix $M_{\nu}^{ab}$ one can estimate the solar and the atmospheric square mass differences as,

$$
\Delta m_{\text{ATM}}^2 \sim \left( \frac{v^2 M_N^2}{\lambda_5^2 v^2} \right)^2, \quad \Delta m_{\text{SOL}}^2 \sim \left( \frac{2\lambda_5 v^2}{M_f^2 - m_{\eta R}^2} M_f \frac{\lambda_5^2}{2} \right)^2.
$$

(17)

where we take $M_N^2$, $m_{\eta R}^2$, $M_f^2 - m_{\eta R}^2 \gg \lambda_5 v^2$ and $\lambda_5^2 = (Y_\ell^e)^2 + (Y_\ell^\mu)^2 + (Y_\ell^\tau)^2$ for $\ell = \nu, f$.

From Eq. 17, it is clear that one can fit the observed atmospheric and solar mass square differences in many ways.

Here our aim is to study whether the vacuum can be stable for large Yukawa coupling or not. Hence we consider only large Yukawa coupling. For example, with Yukawa coupling $Y_\nu = Y_f = \mathcal{O}(1)$, taking large value for $M_N$, $M_f$, $m_{\eta R}$ we can easily fit the solar and atmospheric scale as long as one takes an adequately small value for $\lambda_5$. Even with this small value of $\lambda_5$, in the minimal scotogenic seesaw model, one can have stable vacuum for large Yukawa coupling due to the presence of other quartic couplings, as shown in Fig. 8.

![FIG. 8: The RG-evolution of $\lambda$ and the Yukawas $Y_\nu$ and $Y_f$. The evolution of other quartic couplings $\lambda_\eta$, $\lambda_3$, $\lambda_4$ and $\lambda_4$ is not shown to avoid cluttering the plot. The values given in the boxes are the initial values at the top mass scale. During the RG evolution, couplings remain well within the perturbative range and all quartic couplings remain positive up to Planck Scale. See text for more details.](image)

To plot Fig. 8, we computed the two-loop RG-equations for all the quartic scalar couplings as well as the Yukawa couplings which are given in Appendix (D). To avoid overcrowding
the plot, only the RG evolution of the quartic scalar coupling $\lambda$ is shown in Fig. 8. The results are for two set of Yukawa couplings $Y_{\nu} = Y_f = 0.3, \ Y_{\nu} = Y_f = 0.5$ and they show that the quartic coupling $\lambda$ can remain positive all the way up to Planck scale for the initial benchmark values of $\lambda_3$ and $\lambda_4$ at $m_t$ taken to be $\lambda_3 = \lambda_4 = 0.14$ in the left panel and $\lambda_3 = \lambda_4 = 0.18$ in the right panel. In both panels $\lambda_\eta = 0.1$ is taken as the initial value at the $m_t$ mass scale. Note that, although the RG evolution of the other quartic couplings is not shown in Fig. 8, we checked that none of these couplings becomes negative or non-perturbative all the way up to the Planck scale.

One sees that the minimal scotogenic seesaw model can explain both solar and atmospheric neutrino mass scales as well as dark matter, upgrading the (3, 1) type-I seesaw, which can only generate atmospheric neutrino mass scale. In addition, the minimal scotogenic seesaw model leads to stable electroweak vacuum all the way up to the Planck scale, hence can be considered as a full consistent theory for neutrino masses and dark matter.

6. SUMMARY AND DISCUSSION

We have examined the consistency of electroweak symmetry breaking within the context of the simplest “dark matter completions” of the high-scale type-I seesaw mechanism. We have derived the full two-loop RGEs for the relevant parameters, such as the quartic Higgs self-coupling $\lambda$ of the Standard Model within the schemes of interest. These are compared, for calibration, with the Standard Model results. We find that adding a fermionic field like “right-handed” neutrino, can have a destabilizing effect on the Higgs boson vacuum. For the simplest type-I seesaw with bare mass term for the right-handed neutrinos, one finds that for sizeable Yukawa couplings the Higgs quartic self-coupling $\lambda$ becomes negative much before reaching the seesaw scale. For “large” Yukawas the type-I seesaw may be inconsistent even as an effective theory. We have taken as our simplest benchmark the “incomplete” (3,1) seesaw scheme with a single right-handed neutrino, as it has the “best” stability properties within the class of high-scale type-I seesaw schemes. We compared this case, in which only the atmospheric scale is generated at tree level, with the “higher” (3,2) type-I seesaw, in which the solar mass scale also arises at the tree level leaving one massless neutrino. We did the same for the canonical sequential (3,3) type-I seesaw, in which all three neutrinos get tree-level mass. We also showed how the stability properties improve in the case of spontaneous lepton number violating due to the presence of a Nambu-Goldstone boson, the Majoron. We referred to the relevant literature that shows how this can provide viable warm dark matter.

As an alternative benchmark we have also examined the recently proposed scoto-seesaw mechanism [17]. This is a very simple yet fully realistic scotogenic completion of the type-I seesaw. Its stability properties are improved with respect to those of the (3,1) scenario due
to the new scalars needed to produce the solar mass scale radiatively, even in the absence of spontaneous lepton number violating and a Majoron. Both scenarios can have viable dark matter candidates, either the warm dark matter Majoron or conventional WIMP dark matter. These simple extensions of the Standard Model not only yield adequate neutrino masses and a viable dark matter candidate, but can also render the Higgs vacuum stable all the way up to Planck scale.

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Appendix A: Higgs Sector in Majoron Model

The scalar potential for the Majoron type-I seesaw is given by,

\[
V = -\mu_H^2 H^\dagger H - \mu_\sigma^2 \sigma^\dagger \sigma + \lambda (H^\dagger H)^2 + \lambda_\sigma (\sigma^\dagger \sigma)^2 + \lambda_H \sigma (H^\dagger H)(\sigma^\dagger \sigma).
\]  
(A1)

The Standard Model gauge singlet scalar \(\sigma\) carries two units of lepton number and its vev \(\langle \sigma \rangle = \frac{v_\sigma}{\sqrt{2}}\) breaks the lepton number symmetry \(U(1)_L\) to a \(\mathbb{Z}_2\) subgroup. After symmetry breaking one has, in the unitary gauge

\[
H \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H + h' \end{pmatrix}, \quad \sigma \rightarrow v_\sigma + \sigma'.
\]  
(A2)

The \(h'\) and \(\sigma'\) will mix with each other and the mass eigenvalues are given by,

\[
m_{h_1}^2 = \lambda v_H^2 + \lambda_\sigma v_\sigma^2 - \sqrt{(\lambda v_H^2 - \lambda_\sigma v_\sigma^2)^2 + (\lambda_H \sigma v_H v_\sigma)^2},
\]  
(A3)

\[
m_{h_2}^2 = \lambda v_H^2 + \lambda_\sigma v_\sigma^2 + \sqrt{(\lambda v_H^2 - \lambda_\sigma v_\sigma^2)^2 + (\lambda_H \sigma v_H v_\sigma)^2}.
\]  
(A4)

The mass eigenstates \(h_1, h_2\) are related to the fields \(h', \sigma'\) by the mixing matrix parameterized by the angle \(\alpha\) and is given by

\[
\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h' \\ \sigma' \end{pmatrix},
\]  
(A5)

where the mixing angle \(\alpha\) is given by,

\[
\sin 2\alpha = \frac{\lambda_H \sigma v_H v_\sigma}{\sqrt{(\lambda v_H^2 - \lambda_\sigma v_\sigma^2)^2 + (\lambda_H \sigma v_H v_\sigma)^2}},
\]  
\[
\cos 2\alpha = \frac{\lambda v_H^2 - \lambda_\sigma v_\sigma^2}{\sqrt{(\lambda v_H^2 - \lambda_\sigma v_\sigma^2)^2 + (\lambda_H \sigma v_H v_\sigma)^2}}.
\]  
(A6)
One can see from (A6) that in the limit $v_{\sigma} \gg v_H$ the mixing angle $\alpha \to 0$, irrespective of the value of the quartic couplings.

**Appendix B: RGEs: Type I seesaw**

The $\beta$ function of a given parameter $c$ is given by,

$$\frac{dc}{dt} \equiv \beta_c = \frac{1}{16\pi^2} \beta_c^{(1)} + \frac{1}{(16\pi^2)^2} \beta_c^{(2)}.$$

where $\beta_c^{(1)}$ are the one-loop RG corrections and $\beta_c^{(2)}$ are the two-loop RG corrections.

1. **Higgs quartic scalar self coupling**

For the (3,n) seesaw the one-loop and two-loop RG corrections to the Higgs quartic self-coupling are given by:

$$\beta_\lambda^{(1)} = \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 - \frac{9}{2} g_2^2 \lambda - 9 g_2^2 \lambda + 24 \lambda^2 + 12 \lambda g_1^2 + 4 \lambda \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) - 6 g t^4 - 2 \text{Tr} \left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right),$$

$$\beta_\lambda^{(2)} = -\frac{3411}{2000} g_1^6 - \frac{1677}{400} g_1^4 g_2^2 - \frac{289}{80} g_1^2 g_2^4 + \frac{305}{16} g_2^6 + \frac{1887}{200} g_1^4 \lambda + \frac{117}{20} g_1^2 g_2^2 \lambda - \frac{73}{8} g_2^4 \lambda + \frac{108}{5} g_1^2 \lambda^2 + 108 g_2^2 \lambda^2 - 312 \lambda^3 - \frac{171}{100} g_1^4 y_t^2 + \frac{63}{10} g_1^2 g_2^2 y_t^2 - \frac{9}{4} g_2^4 y_t^2 + \frac{17}{2} g_1^2 \lambda y_t^2 + \frac{45}{2} g_2^2 \lambda y_t^2 + 80 g_3^2 y_t^2 - 144 \lambda^2 y_t^2 - \frac{9}{100} g_1^4 \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) - \frac{3}{10} g_1^2 g_2^2 \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) - \frac{3}{4} g_2^4 \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) + \frac{3}{2} g_1^2 \lambda \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) + \frac{15}{2} g_2^2 \lambda \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) - 48 \lambda^2 \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) - \frac{8}{5} g_1^4 y_t^4 - 32 g_3^2 y_t^4 - 3 \lambda g_t^4 - \lambda \text{Tr} \left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right) + 30 y_t^6 + 10 \text{Tr} \left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right).$$

2. **Yukawa Couplings**

The one-loop and two-loop RG corrections to $Y_\nu$ in the (3,n) seesaw are given by

$$\beta_{y_\nu}^{(1)} = \frac{3}{2} Y_\nu Y_\nu^\dagger + \text{Tr} \left(Y_\nu Y_\nu^\dagger\right)(3 y_t^2 - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 + \text{Tr} \left(Y_\nu Y_\nu^\dagger\right)),

$$\beta_{y_\nu}^{(2)} = \frac{1}{80} \left(279 g_1^4 Y_\nu Y_\nu^\dagger Y_\nu + 675 g_1^2 g_2^2 Y_\nu Y_\nu^\dagger Y_\nu - 960 \lambda Y_\nu Y_\nu^\dagger Y_\nu + 120 Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - 540 Y_\nu Y_\nu^\dagger Y_\nu Y_t^2 - 180 Y_\nu Y_\nu^\dagger Y_\nu \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) + 2 Y_\nu \left(21 g_1^4 - 54 g_1^2 g_2^2 - 230 g_2^4 + 240 \lambda^2 + 85 g_1^2 y_t^2 + 225 g_2^2 y_t^2 + 800 g_3^2 y_t^2 + 15 g_1^2 \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) + 75 g_2^2 \text{Tr} \left(Y_\nu Y_\nu^\dagger\right) - 270 y_t^4 - 90 \text{Tr} \left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right)\right).$$
The RG corrections to top-Yukawa coupling $y_t$ are given by

$$
\beta_{y_t}^{(1)} = \frac{3}{2} y_t^3 + y_t \left( 3 g_t^2 - 8 g_q^2 - \frac{17}{8} g_1^2 - \frac{9}{4} g_2^2 + \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) \right),
$$

(B5)

$$
\beta_{y_t}^{(2)} = + \frac{1}{80} \left( 120 y_t^5 + y_t^3 \left( 1280 g_q^2 - 180 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) + 223 g_1^2 - 540 y_t^2 + 675 g_2^2 - 960 \lambda \right) \right) 
+ y_t \left( \frac{1187}{600} g_t^4 - \frac{9}{20} g_1^2 g_2^2 - \frac{23}{4} g_2^4 + \frac{19}{15} g_1^2 g_2 - 9 g_2 g_3^2 - 108 g_3^3 + 6 \lambda^2 + \frac{17}{8} g_1^2 y_t^2 + \frac{45}{8} g_2^2 y_t^2 
+ 20 g_3^2 y_t^2 + \frac{3}{8} g_t^2 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) + \frac{15}{8} g_2^2 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - \frac{27}{4} y_t^4 - \frac{9}{4} \text{Tr} \left( Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger \right) \right).
$$

(B6)

Appendix C: RGEs: Type I seesaw with Majoron

1. Quartic scalar couplings

The scalar sector of the Majoron model is given in Eq. (7). It contains three scalar quartic couplings $\lambda, \lambda_{H\sigma}, \lambda_{\sigma}$ whose one-loop and two-loop RGEs are given by

$$
\beta_{\lambda}^{(1)} = \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 + \lambda_{H\sigma}^2 - \frac{9}{5} g_1^2 \lambda - \frac{9}{2} g_2^2 \lambda + 24 \lambda^2 + 12 \lambda y_t^2 + 4 \lambda \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - 6 y_t^4 
- 2 \text{Tr} \left( Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger \right),
$$

(C1)

$$
\beta_{\lambda}^{(2)} = - \frac{3411}{2000} g_1^6 - \frac{1677}{400} g_1^4 g_2^2 - \frac{289}{80} g_1^2 g_2^4 + \frac{305}{16} g_2^6 - 4 \lambda_{H\sigma}^2 + \frac{1887}{200} g_1^4 \lambda + \frac{117}{20} g_1^2 g_2^2 \lambda - \frac{73}{8} g_2^4 \lambda 
- 10 \lambda_{H\sigma}^2 \lambda + \frac{108}{5} g_1^2 \lambda^2 + 108 g_2^3 \lambda^2 - 312 \lambda^3 - \lambda_{H\sigma}^2 \text{Tr} \left( Y_{R_{\nu}} Y_{R_{\nu}}^* \right) - \frac{171}{100} g_1^4 y_t^2 + \frac{63}{10} g_1^2 g_2^2 y_t^2 
- \frac{9}{4} g_2^4 y_t^4 + \frac{17}{2} g_1^2 \lambda y_t^2 + \frac{45}{2} g_2^2 \lambda y_t^2 + 80 g_3^2 \lambda y_t^2 - 144 \lambda^2 y_t^2 - \frac{9}{100} g_1^4 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - \frac{3}{10} g_1^2 g_2^2 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) 
- \frac{3}{4} g_2^4 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) + \frac{3}{2} g_1^2 g_2 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) + \frac{15}{2} g_2^2 \lambda \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - 48 \lambda^2 \text{Tr} \left( Y_\nu Y_\nu^\dagger \right) - 3 \lambda \text{Tr} \left( Y_{R_{\nu}} Y_{R_{\nu}}^* \right) 
- \frac{8}{5} g_1^4 y_t^4 - 32 g_2^3 y_t^4 - 3 \lambda y_t^4 - \lambda \text{Tr} \left( Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger \right) + 2 \text{Tr} \left( Y_{R_{\nu}} Y_{R_{\nu}}^* Y_\nu Y_\nu^\dagger \right) + 2 \text{Tr} \left( Y_{R_{\nu}} Y_{R_{\nu}}^* Y_{R_{\nu}} Y_{R_{\nu}}^* \right) + 30 y_t^6 + 10 \text{Tr} \left( Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger \right). 
$$

(C2)
$$\beta^{(2)}_{\lambda_H^2} = \frac{1671}{400} g_1^4 \lambda_H^2 + \frac{9}{8} g_1^2 g_2^2 \lambda_{H_2}^2 - \frac{145}{16} g_2^4 \lambda_{H_2}^2 + \frac{3}{5} g_1^2 \lambda_{H_2}^2 + 3 g_2^2 \lambda_{H_2}^2 - 11 \lambda_{H_2}^2 - 48 \lambda_{H_2}^2 \lambda_{\sigma} - 40 \lambda_{H_2}^2 \lambda_{\sigma}^2$$
\[\beta^{(2)}_{\lambda^2} = \frac{72}{9} g_1^2 \lambda_{H_2}^2 + 72 g_2^2 \lambda_{H_2}^2 \lambda_{\sigma} - 72 \lambda_{H_2}^2 \lambda_{\sigma}^2 - 60 \lambda_{H_2}^2 \lambda_{\sigma}^2 - 60 \lambda_{H_2}^2 \lambda_{\sigma}^2 + \frac{17}{4} g_1^2 \lambda_{H_2}^2 \lambda_{\sigma}^2 + \frac{45}{4} g_2^2 \lambda_{H_2}^2 \lambda_{\sigma}^2 - 12 \lambda_{H_2}^2 \lambda_{\sigma}^2 + \frac{3}{4} g_1^2 \lambda_{H_2}^2 \lambda_{\sigma}^2 \]}

\[\beta^{(2)}_{\lambda^2} = \beta^{(2)}_{\lambda_{H_2}^2} + \beta^{(2)}_{\lambda^2} = \frac{12}{5} g_1^2 \lambda_{H_2}^2 + 12 g_2^2 \lambda_{H_2}^2 \lambda_{\sigma} - 8 \lambda_{H_2}^2 - 20 \lambda_{H_2}^2 \lambda_{\sigma} - 240 \lambda_{H_2}^2 \lambda_{\sigma}^2 - 20 \lambda_{H_2}^2 \lambda_{\sigma}^2 - \frac{4 \lambda_{H_2}^2 \lambda_{\sigma}^2}{2} \]}

\[\beta^{(2)}_{\lambda_{H_2}^2} + \beta^{(2)}_{\lambda^2} = \frac{3}{4} \lambda_{H_2}^2 + 4 \lambda_{H_2}^2 \lambda_{\sigma} - 4 \lambda_{H_2}^2 \lambda_{\sigma} - \frac{7}{2} \lambda_{H_2}^2 \lambda_{\sigma} - 27 \lambda_{H_2}^2 \lambda_{\sigma} + 14 \lambda_{H_2}^2 \lambda_{\sigma} - \lambda_4 \lambda_{H_2}^2 + \lambda_4 \lambda_{H_2}^2 - \frac{3}{2} \lambda_{H_2}^2 \lambda_{\sigma} + 4 \lambda_{H_2}^2 \lambda_{\sigma} + \frac{13}{4} \lambda_{H_2}^2 \lambda_{\sigma} \]

\[\beta^{(2)}_{\lambda_{H_2}^2} = 20 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 - \lambda_4 \lambda_{H_2}^2 + \lambda_4 \lambda_{H_2}^2 - \frac{3}{2} \lambda_{H_2}^2 \lambda_{\sigma} + 4 \lambda_{H_2}^2 \lambda_{\sigma} + \frac{13}{4} \lambda_{H_2}^2 \lambda_{\sigma} \]

\[\beta^{(2)}_{\lambda_{H_2}^2} = 20 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 - \lambda_4 \lambda_{H_2}^2 + \lambda_4 \lambda_{H_2}^2 - \frac{3}{2} \lambda_{H_2}^2 \lambda_{\sigma} + 4 \lambda_{H_2}^2 \lambda_{\sigma} + \frac{13}{4} \lambda_{H_2}^2 \lambda_{\sigma} \]

\[\beta^{(2)}_{\lambda_{H_2}^2} = 20 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 - \lambda_4 \lambda_{H_2}^2 + \lambda_4 \lambda_{H_2}^2 - \frac{3}{2} \lambda_{H_2}^2 \lambda_{\sigma} + 4 \lambda_{H_2}^2 \lambda_{\sigma} + \frac{13}{4} \lambda_{H_2}^2 \lambda_{\sigma} \]

\[\beta^{(2)}_{\lambda_{H_2}^2} = 20 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 - \lambda_4 \lambda_{H_2}^2 + \lambda_4 \lambda_{H_2}^2 - \frac{3}{2} \lambda_{H_2}^2 \lambda_{\sigma} + 4 \lambda_{H_2}^2 \lambda_{\sigma} + \frac{13}{4} \lambda_{H_2}^2 \lambda_{\sigma} \]

\[\beta^{(2)}_{\lambda_{H_2}^2} = 20 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 + 2 \lambda_{H_2}^2 - \lambda_4 \lambda_{H_2}^2 + \lambda_4 \lambda_{H_2}^2 - \frac{3}{2} \lambda_{H_2}^2 \lambda_{\sigma} + 4 \lambda_{H_2}^2 \lambda_{\sigma} + \frac{13}{4} \lambda_{H_2}^2 \lambda_{\sigma} \]
\[ \beta^{(2)}_{\lambda} = \frac{1}{80} \left( 120g_t^5 + g_t^3 \left( 1280g_3^2 - 180Y_\nu Y_\nu^\dagger + 223g_4^2 - 540g_5^2 + 675g_2^2 - 960\lambda \right) \right) + \frac{1187}{600} g_1^4 - \frac{9}{20} g_2^2 g_2^2 - \frac{23}{4} g_3^2 + \frac{19}{15} g_1^2 g_2^2 + 9g_2^2g_3 - 108g_4^2 + \frac{1}{2} \lambda^2 H_\sigma + 6\lambda^2 + \frac{17}{8} g_1^2 y_t^2 + \frac{45}{8} g_3^2 y_t^2 + 20g_3^2 y_t^2 + \frac{3}{8} \Delta t \left( Y_\nu Y_\nu^\dagger \right) + \frac{15}{8} \Delta t \left( Y_\nu Y_\nu^\dagger \right) - \frac{3}{4} \Delta t \left( Y_R Y_\nu^\dagger Y_\nu Y_R^\dagger \right) - \frac{27}{4} \Delta t \],
\]
\[ \beta^{(1)}_{Y_R} = \frac{1}{2} Y_R \Delta t \left( Y_R Y_\nu^\dagger \right) + Y_R Y_\nu^\dagger Y_\nu + Y_R Y_R^\dagger Y_R + Y_\nu^\dagger Y_\nu Y_R, \]
\[ \beta^{(2)}_{Y_R} = \frac{1}{40} \left( -320\lambda_\sigma Y_R Y_R^\dagger Y_R + 51g_1^2 Y_\nu^\dagger Y_\nu^\dagger Y_\nu Y_\nu + 255g_2^2 Y_\nu^\dagger Y_\nu^\dagger Y_\nu Y_R - 160\lambda H_\sigma Y_\nu^\dagger Y_\nu^\dagger Y_R \right.
\]
\[ - 10Y_R Y_\nu^\dagger Y_\nu Y_\nu - 10Y_R Y_\nu^\dagger Y_\nu Y_R Y_R + 70Y_R Y_R^\dagger Y_R Y_R - 10Y_R Y_R^\dagger Y_R Y_R
\]
\[ + 160Y_\nu^\dagger Y_\nu Y_R^\dagger Y_R - 10Y_\nu^\dagger Y_\nu^\dagger Y_\nu Y_R - 100Y_R Y_\nu^\dagger Y_\nu Y_R - 180Y_\nu^\dagger Y_\nu^\dagger Y_R y_t^2
\]
\[ + Y_R Y_\nu^\dagger Y_\nu \left( -160\lambda H_\sigma - 180y_t^2 + 255g_2^2 + 51g_1^2 - 60Y_\nu^\dagger Y_\nu \right) \left( Y_R Y_\nu^\dagger \right) - 60Y_\nu^\dagger Y_\nu Y_R \left( Y_\nu Y_\nu^\dagger \right)
\]
\[ \left. + 10Y_R \left( 16\lambda_\sigma^2 - 3\Delta t \left( Y_R Y_R^\dagger Y_R \right) + 4\lambda_\sigma^3 - 6\Delta t \left( Y_R Y_\nu^\dagger Y_\nu Y_R^\dagger \right) \right) \right) \].

**Appendix D: RGES: Minimal Scotogetic Seesaw**

1. Higgs quartic scalar self coupling

The scalar potential of scoto-seesaw model is given in Eq. (11). The model contains five quartic couplings \( \lambda, \lambda_i; i = \eta, 3, 4, 5 \). The one-loop and two-loop RG equations of the Higgs quartic self coupling \( \lambda \) are given by

\[ \beta^{(1)}_{\lambda} = \frac{27}{200} g_4^2 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_3^2 + 2\lambda^2 + 2\lambda_3 \lambda_4 + \lambda_4^2 + 4\lambda_5^2 - \frac{9}{5} g_1^2 \lambda - 9g_2^2 \lambda + 24\lambda^2
\]
\[ + 12\lambda_3 g_t^2 + 4\lambda \Delta t \left( Y_\nu Y_\nu^\dagger \right) - 6g_t^2 - 2 \Delta t \left( Y_\nu Y_\nu^\dagger \right), \]
\[ \beta^{(2)}_{\lambda} = -\frac{3537}{200} g_4^2 - \frac{1719}{400} g_1^2 g_2^2 - \frac{303}{80} g_1^2 g_4^2 + \frac{291}{16} g_2^2 + \frac{9}{10} g_4^2 \lambda + \frac{15}{2} g_2^2 \lambda + \frac{12}{5} g_1^2 \lambda + 12g_2^2 \lambda_3^2 + 12g_2^2 \lambda_4^2
\]
\[ - 8\lambda_3^3 + \frac{9}{20} g_1^2 \lambda_4 + \frac{3}{2} g_1^2 \lambda_4^2 + \frac{15}{4} g_4^2 \lambda_4 + \frac{12}{5} g_2^2 \lambda_3 \lambda_4 + 12g_2^2 \lambda_3 \lambda_4 - 12\lambda^2 \lambda_4 + 6 \lambda_3 \lambda_4^2
\]
\[ + \frac{3}{5} g_2^2 \lambda_4^2 - 16\lambda_3 \lambda_4^2 - 6\lambda_4^3 - 3\lambda_5^2 \lambda - 12\lambda_3 \lambda_5^2 - 80\lambda_3 \lambda_5^2 - 88\lambda_4 \lambda_5^2 + \frac{1953}{200} g_4^2 \lambda + \frac{117}{20} g_1^2 g_2^2 \lambda
\]
\[ - \frac{51}{8} g_4^2 \lambda - 20\lambda_3 \lambda_5^2 + 20\lambda_3 \lambda_4 \lambda - 12\lambda_4 \lambda - 56\lambda_5^2 \lambda + \frac{108}{5} g_1^2 \lambda^2 + 108g_2^2 \lambda^2 - 312\lambda^3
\]
\[ - 4\lambda_3 \Delta t \left( Y_\nu Y_\nu^\dagger \right) - 4\lambda_3 \lambda_4 \Delta t \left( Y_\nu Y_\nu^\dagger \right) - 2 \lambda_3 \Delta t \left( Y_\nu Y_\nu^\dagger \right) - 8\lambda_5 \Delta t \left( Y_\nu Y_\nu^\dagger \right) - \frac{171}{100} g_4^2 y_t^2 \]
The RG equations for other quartic couplings in the model can be written similarly but for sake of brevity we will not write them.

2. Yukawa Couplings

The one-loop and two-loop RG equations for the Yukawa couplings \( Y_f, Y_\nu \) and \( y_t \) are given by

\[
\beta_{Y_f}^{(1)} = \frac{1}{20} \left( 10 \left( 3 Y_f Y_f^\dagger + Y_\nu Y_\nu^\dagger Y_f \right) + Y_f \left( 20 \text{Tr} \left( Y_f Y_f^\dagger \right) - 9 \left( 5 g_2^2 + g_t^2 \right) \right) \right),
\]

(3.3)

\[
\beta_{Y_f}^{(2)} = \frac{1}{80} \left( 33 g_t^4 Y_\nu Y_\nu^\dagger Y_f + 165 g_2^2 Y_\nu Y_\nu^\dagger Y_f - 160 \lambda_3 Y_\nu Y_\nu^\dagger Y_f - 320 \lambda_4 Y_\nu Y_\nu^\dagger Y_f \right)

- 120 Y_f Y_f^\dagger Y_f Y_f^\dagger Y_f - 20 Y_f Y_f^\dagger Y_f Y_f^\dagger Y_f - 20 Y_f Y_f^\dagger Y_f Y_f^\dagger Y_f - 180 Y_f Y_f^\dagger Y_f Y_f^\dagger Y_f

+ 3 Y_f Y_f^\dagger Y_f \left( 225 g_2^2 - 320 \lambda_\nu - 60 \text{Tr} \left( Y_f Y_f^\dagger \right) + 93 g_t^2 \right)

+ Y_f \left( \frac{117}{200} g_1^4 - \frac{27}{20} g_t^2 g_2^2 \right)

- \frac{21}{4} g_2^4 + \lambda_3^2 + \lambda_4^2 + \lambda_5^2 + 6 \lambda^2 + 6 \lambda^2 + \frac{3}{8} \left( 5 g_2^2 + g_t^2 \right) \text{Tr} \left( Y_f Y_f^\dagger \right)

- \frac{9}{4} \text{Tr} \left( Y_f Y_f^\dagger Y_f Y_f^\dagger \right) - \frac{3}{4} \text{Tr} \left( Y_f Y_f^\dagger Y_f Y_f^\dagger \right),
\]

(3.4)

\[
\beta_{Y_\nu}^{(1)} = \frac{1}{2} \left( 3 Y_f Y_f^\dagger Y_\nu + Y_f Y_f^\dagger Y_\nu \right) + Y_\nu \left( 3 g_t^2 - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 + \text{Tr} \left( Y_f Y_f^\dagger \right) \right),
\]

(3.5)

\[
\beta_{Y_\nu}^{(2)} = \frac{1}{80} \left( 279 g_t^2 Y_\nu Y_\nu^\dagger Y_\nu + 675 g_2^2 Y_\nu Y_\nu^\dagger Y_\nu - 960 \lambda_3 Y_\nu Y_\nu^\dagger Y_\nu - 20 Y_f Y_f^\dagger Y_f Y_f^\dagger Y_\nu \right)

- 20 Y_\nu Y_\nu^\dagger Y_f Y_f^\dagger Y_\nu + 120 Y_f Y_f^\dagger Y_f Y_f^\dagger Y_\nu + Y_f Y_f^\dagger Y_f \left( - 160 \lambda_3 + 165 g_2^2 - 320 \lambda_4 \right)

+ 33 g_2^2 - 60 \text{Tr} \left( Y_f Y_f^\dagger \right) - 540 Y_\nu Y_\nu^\dagger Y_\nu g_t^2 - 180 Y_\nu Y_\nu^\dagger Y_\nu Y_\nu Y_\nu \right)

+ Y_\nu \left( \frac{117}{200} g_1^4 - \frac{27}{20} g_t^2 g_2^2 - \frac{21}{4} g_2^4 + \lambda_3^2 + \lambda_4^2 + \lambda_5^2 + 6 \lambda^2 + 6 \lambda^2 + \frac{17}{8} g_2^4 y_t^2 \right)

+ \frac{45}{8} g_2^2 y_t^2 + 20 g_3^2 y_t^2 + \frac{3}{8} g_t^2 \text{Tr} \left( Y_f Y_f^\dagger \right) + \frac{15}{8} g_2^2 \text{Tr} \left( Y_f Y_f^\dagger \right) - \frac{13}{4} \text{Tr} \left( Y_f Y_f^\dagger Y_f Y_\nu \right)

- \frac{27}{4} y_t^2 - \frac{9}{4} \text{Tr} \left( Y_f Y_f^\dagger Y_f Y_\nu \right),
\]

(3.6)
\begin{align}
\beta_{yt}^{(1)} &= \frac{3}{2}y_t^3 + y_t \left( 3y_t^2 - 8g_3^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 + \text{Tr}\left(Y\nu Y\nu^\dagger\right) \right), \\
\beta_{yt}^{(2)} &= +\frac{1}{80}\left( 120y_t^5 + y_t^3 \left( 1280g_3^2 - 180\text{Tr}\left(Y\nu Y\nu^\dagger\right) + 223g_1^2 - 540g_2^2 + 675g_2^3 - 960\lambda \right) \right) \\
&+ y_t \left( \frac{1267}{600}g_1^4 - \frac{9}{20}g_1^2g_2 + \frac{21}{4}g_2^4 + \frac{19}{15}g_1^2g_3 + 9g_2^2g_3^2 - 108g_3^4 + \lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2 + 6\lambda_5^2 \right) \\
&+ 6\lambda^2 + \frac{17}{8}g_1^2y_t^2 + \frac{45}{8}g_2^2y_t^2 + 20g_3^2y_t^2 + \frac{3}{8}g_1^2\text{Tr}\left(Y\nu Y\nu^\dagger\right) + \frac{15}{8}g_2^2\text{Tr}\left(Y\nu Y\nu^\dagger\right) \\
&- \frac{3}{4}\text{Tr}\left(Y_fY_f^\dagger Y\nu Y\nu^\dagger\right) - \frac{27}{4}y_t^4 - \frac{9}{4}\text{Tr}\left(Y\nu Y\nu Y\nu Y\nu^\dagger\right) \right) \\
\end{align}

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