Quantum Discrete Symmetry and the Strong CP Problem

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ABSTRACT

We study a $2N$-flavor effective theory of $N$-flavor QCD. With the axial anomaly accounted for in the effective theory by a ’t Hooft interaction, only QCD conserved currents survive. However, there is a residual discrete symmetry with interesting properties. With non-vanishing quark masses, this $S_2$ symmetry is broken unless $\bar{\theta}=0$ or $\pi$. We further show that there is a sense in which hadrons in the effective theory fall into $S_2$ multiplets. Surprisingly, predictions of this multiplet structure in the four-flavor effective theory are in good agreement with experiment and have been found previously by imposing Regge asymptotic constraints on pion-hadron scattering amplitudes.

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1. Introduction

In this letter we consider the possibility that there exists an effective theory with $2N$ flavors that describes the same low-energy physics as QCD with $N$ flavors \[1\]. The $2N$-flavor effective theory can be sensible only if it has the same continuous global symmetries as QCD since more or fewer conserved currents would almost certainly do violence to the particle data group. We show that when the axial anomaly is accounted for in the effective theory by a ’t Hooft interaction, only QCD conserved currents survive. However, there remains a discrete symmetry which interchanges quark multiplets of opposite chirality. This $S_2$ symmetry is the focus of this letter. We demonstrate that if $S_2$ is unbroken, P and CP are also unbroken. Therefore, the $2N$-flavor effective theory does not have a strong CP problem in the usual sense. This immediately raises the question of whether there are other testable consequences of $S_2$. We show that $S_2$ makes several familiar and successful predictions which were obtained long ago by assuming soft asymptotic behavior in forward pion-hadron scattering.

Our letter is organized as follows. In section 2, as a warm up, we review the strong CP problem in $N$-flavor QCD with a ’t Hooft interaction. In section 3 we construct the $2N$-flavor effective theory and show that the effective theta angle, $\tilde{\theta}$, is a natural parameter in the sense that when it vanishes there is an enhanced $S_2$ symmetry. In section 4 we show that $S_2$ has algebraic consequences that have been found previously by imposing Regge asymptotic constraints on pion-hadron scattering amplitudes. The predictions of $S_2$ are in good agreement with experiment. Finally, in section 5 we summarize and comment on the significance of our results.

2. The Strong CP Problem in $N$-flavor QCD

In order to provide contrast for our results, we first give a simple review of the strong CP problem in $N$-flavor QCD. We do not include explicit gauge fields and color indices are suppressed. The matter content of QCD consists of $N$ Dirac fermions assembled into the vector, $q$, in the fundamental representation of $SU(N)$, which transforms with respect to $SU(N)_L \times SU(N)_R$ as

\[
(N, 1) : \quad q_L \rightarrow Lq_L \\
(1, N) : \quad q_R \rightarrow Rq_R.
\]

The most general $SU(N)_L \times SU(N)_R$ invariant free lagrangian one can build with $q$ is

\[
\mathcal{L}_0 = \bar{q}_L i \not\!{\partial} q_L + \bar{q}_R i \not\!{\partial} q_R = \bar{q} i \not\!{\partial} q.
\]
This free lagrangian also admits $U(1)_B$ and $U(1)_A$ transformations. We therefore add a $U(1)_A$ violating quark interaction to take into account the effect of the axial anomaly. Consider the $U(1)_A$ violating, $SU(N)_L \times SU(N)_R \times U(1)_B$ preserving ’t Hooft interaction

$$\mathcal{L}''(\bar{\theta}) = -\bar{\kappa}\{e^{i\bar{\theta}} \det \bar{q}(1-\gamma_5)q + e^{-i\bar{\theta}} \det \bar{q}(1+\gamma_5)q\}, \quad (3)$$

where the determinant acts on $SU(N)$ matrices and $\bar{\kappa}$ is a parameter of mass dimension $4-3N$. We have included a P and CP violating phase, $\bar{\theta}$, which includes pure QCD effects as well as quark mass matrix effects; i.e. $\bar{\theta}=\theta_{QCD}+\theta_{EW}$.

In the absence of explicit chiral symmetry breaking effects the field redefinition, $q \rightarrow e^{i\bar{\theta}\gamma_5/2N}q$, removes $\bar{\theta}$ from the lagrangian, and then P and CP are manifest discrete symmetries of the theory.

Assuming $N$ degenerate flavors, we can include an explicit chiral symmetry breaking mass term, $-m_q\bar{q}q$, and again perform a field redefinition which removes $\bar{\theta}$ from the ’t Hooft interaction. However, now $\bar{\theta}$ cannot be removed from the lagrangian. Generally, redefining a single quark flavor is sufficient to remove $\bar{\theta}$ from the ’t Hooft interaction. It is therefore sufficient that one quark flavor be massless in order to render $\bar{\theta}$ unphysical. For small $\bar{\theta}$, $\bar{q}q \rightarrow \bar{q}q + i(\bar{\theta}/N)\bar{q}\gamma_5q$, which induces the P and CP violating operator,

$$-i\frac{m_q}{N} \bar{\theta} \bar{q}\gamma_5q. \quad (4)$$

If nonvanishing, this operator will contribute to low-energy physics, e.g. the neutron electric dipole moment. Experimentally one finds $|\bar{\theta}| \leq 10^{-9}$ [2]. Small parameters are considered natural if their vanishing implies enhanced symmetry [3]. The fact that $\bar{\theta}$ is so small and yet the symmetry of the standard model is not increased by taking $\bar{\theta}=0$ is known as the strong CP problem. It is true that CP becomes a good symmetry of the strong interactions when $\bar{\theta}=0$. However, CP is violated by the weak interactions, as observed in kaon decays, and is therefore not a symmetry of the standard model.

3. Absence of a Strong CP Problem in a 2N-flavor Effective Theory of QCD

We now repeat the analysis in a 2N-flavor low-energy effective theory of N-flavor QCD. The matter content of the effective theory consists of Dirac fermions, $q$ and $p$, each in the fundamental representation of $SU(N)$, which transform with respect to $SU(N)_L \times SU(N)_R$ as

$$(N, 1) : \quad q_L \rightarrow Lq_L \quad p_R \rightarrow Lp_R$$

$$(1, N) : \quad p_L \rightarrow Rp_L \quad q_R \rightarrow Rp_R.$$ (5)

These quarks are also assumed to carry a color charge which will be suppressed. The most general $SU(N)_L \times SU(N)_R$ invariant free lagrangian one can build with $q$ and $p$ is
\[ \mathcal{L}_0 = \bar{q}i\not{\partial}q + \bar{p}i\not{\partial}p - M_0\bar{q}_Lp_R - M_0'\bar{q}_Rp_L + \text{h.c.} \]  

(6)

Parity conservation implies \( M_0 = M_0' \) which gives

\[ \mathcal{L}_0 = \bar{q}i\not{\partial}q + \bar{p}i\not{\partial}p - M_0 (\bar{q}p + \bar{p}q). \]  

(7)

This lagrangian clearly has symmetries beyond those assumed. In order to see the full symmetry structure it is convenient to define a new field,

\[ \Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} q + p \\ \gamma_5(q - p) \end{pmatrix}. \]  

(8)

In terms of this field the lagrangian, Eq. (7), becomes

\[ \mathcal{L}_0 = \bar{\psi}i\not{\partial}\psi - M_0\bar{\psi}\psi. \]  

(9)

Since \( \Psi \) is a \( 2N \)-component vector, the full symmetry of the lagrangian is \( U(2N) \). We choose to work in the original basis because it allows a convenient identification of \( U(2N) \) subgroups with QCD symmetries [1]. So we assemble \( q \) and \( p \) into the \( 2N \)-component vector, \( \psi = (q \ p)^T \). The lagrangian, Eq. (7), then takes the form

\[ \mathcal{L}_0 = \bar{\psi}i\not{\partial}\psi - M_0\bar{\psi}\sigma_1\psi \]  

(10)

where \( \sigma_1 \) is a Pauli matrix acting in the \( q-p \) space. It is easy to show that in this basis the algebra of \( U(2N) \), or \( SU(2N) \times U(1)_B \), arises from the embedding \( SU(2) \times SU(N)_V \rightarrow SU(2N) \) [1]. With \( \bar{\sigma}_i = \sigma_i/2 \), the generators in the defining representation of \( SU(2N) \) can be written as \( \{ \bar{\sigma}_1, \bar{\sigma}_2\gamma_5, \bar{\sigma}_3\gamma_5 \} \otimes 1, 1 \otimes T_a \) and \( \{ \bar{\sigma}_1, \bar{\sigma}_2\gamma_5, \bar{\sigma}_3\gamma_5 \} \otimes T_a \). In particular, \( 1 \otimes T_a, \bar{\sigma}_3\gamma_5 \otimes T_a \) and \( \bar{\sigma}_3\gamma_5 \otimes 1 \) are identified with \( SU(N)_V, SU(N)_A \) and \( U(1)_A \), respectively. This is consistent with the chiral symmetry assignments in Eq. (5).

As in the \( N \)-flavor QCD analysis, we now add a \( U(1)_A \) violating quark interaction to take into account the effect of the axial anomaly. Consider the \( U(1)_A \) violating, \( SU(N)_L \times SU(N)_R \times U(1)_B \) preserving ’t Hooft interaction

\[ \mathcal{L}''(\bar{\theta}) = -\kappa \{ e^{i\bar{\theta}} \det \bar{\psi}(1 - \sigma_3\gamma_5)\psi + e^{-i\bar{\theta}} \det \bar{\psi}(1 + \sigma_3\gamma_5)\psi \}, \]  

(11)

where the determinant acts on \( SU(N) \) matrices and \( \kappa \) is a parameter of mass dimension \( 4 - 3N \). We have included a P and CP violating phase, \( \bar{\theta} \), which is again assumed to contain both pure QCD effects and quark mass matrix effects. Note that

\[ \bar{p}_R p_L + \bar{q}_L q_R = \frac{1}{2} \bar{\psi}(1 - \sigma_3\gamma_5)\psi \text{ and } \bar{p}_L p_R + \bar{q}_R q_L = \frac{1}{2} \bar{\psi}(1 + \sigma_3\gamma_5)\psi. \]

Although the ’t Hooft interaction is constructed to taken into account the effect of the axial anomaly, one can check that all of the continuous subgroups of \( U(2N) \) that are not
identified with QCD symmetries are broken by the ’t Hooft interaction. That is, the two 
$U(1)$’s, $\bar{\sigma}_1 \otimes 1$ and $\bar{\sigma}_2 \gamma_5 \otimes 1$, and the two $SU(N)$’s, $\bar{\sigma}_1 \otimes T_a$ and $\bar{\sigma}_2 \gamma_5 \otimes T_a$ are broken by
the ’t Hooft interaction. However, consider the discrete transformation $\psi \to \pm i\sigma_1 \psi$. This
transformation generates an $S_2 (=Z_2)$ subgroup of the $U(1)$ generated by $\bar{\sigma}_1 \otimes 1$. $S_2$, the
group of permutations of two objects, has the effect of interchanging $q$ and $p$. In the ’t
Hooft interaction, $S_2$ interchanges $\bar{\psi}(1 - \sigma_3 \gamma_5)\psi$ and $\bar{\psi}(1 + \sigma_3 \gamma_5)\psi$, which is equivalent to
the transformation $L \leftrightarrow R$. That is

$$S_2 \mathcal{L}''(\bar{\theta}) S_2^{-1} = \mathcal{L}''(-\bar{\theta}).$$

In the absence of explicit chiral symmetry breaking effects we can perform the field redefinition, $\psi \to e^{i\bar{\theta}\sigma_3 \gamma_5/2N} \psi$, which removes $\bar{\theta}$ from the problem, and then P, CP and $S_2$ are manifest discrete symmetries of the theory.

Assuming $2N$ degenerate flavors, we can include an ($S_2$ invariant) explicit chiral symmetry breaking mass term, $-m_q \bar{\psi}\psi$, and we can again perform a field redefinition which removes $\bar{\theta}$ from the ’t Hooft interaction. For small $\bar{\theta}$, $\bar{\psi}\psi \to \bar{\psi}\psi + i(\bar{\theta}/N)\bar{\psi}\sigma_3 \gamma_5 \psi$, which induces the P, CP and $S_2$ violating operator,

$$-i \frac{m_q}{N} \bar{\theta} \bar{\psi}\sigma_3 \gamma_5 \psi. \quad (13)$$

From the point of view of the effective theory, this operator contributes to the neutron electric dipole moment. However, in general, P, CP and $S_2$ will be broken unless $\bar{\theta}=0$ or $\pi$. So in the $2N$-flavor effective theory, $\bar{\theta}$ is a natural parameter since its vanishing enlarges the symmetry of the standard model to include $S_2$. This is to be contrasted with the operator, [Eq. (4)], which one has in QCD.

A natural question to ask at this point is whether one wants the symmetry of the standard model to be enlarged to include $S_2$. In practical terms, does $S_2$ have other consequences beyond those discussed above? If $S_2$ is unbroken, one would expect that the low-energy spectrum in the $2N$-flavor effective theory in some sense reflects $S_2$. In the next section we will see that this is the case.

### 4. $S_2$ Doublets in the Hadron Spectrum

Consider $N = 2$ QCD. We have the pattern of symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_V$. In the four-flavor effective theory this will occur if the condensate $\langle \bar{\psi}\psi \rangle$ is non-vanishing. We will assume that $\langle \bar{\psi}\sigma_3 \psi \rangle$ vanishes since otherwise $S_2$ is spontaneously broken. The simplest and most convincing way of finding consequences of $S_2$ in the low-energy theory is to construct meson states directly out of four quarks [1]. A general symmetry argument is given in [Ref. 1], and an equivalent argument using superconvergent sum rules is given in [Ref. 4]. It is convenient to work in the diagonal basis:
\[ \phi_{\pm} \equiv \frac{1}{\sqrt{2}}(q \pm p). \]  

These states transform as an \( S_2 \) doublet with chiral transformation properties given by Eq. (5). Since the doublet is the only non-trivial representation of \( S_2 \), we expect that the product of two doublets gives two doublets; i.e. \( 2 \otimes 2 = 2 \oplus 2 \). However, invariance under charge conjugation unfolds one of the doublets since \( \bar{\phi}_+ \phi_- \) and \( \bar{\phi}_- \phi_+ \) are not states of definite charge conjugation sign. The meson states of definite charge conjugation sign and their associated chiral transformation properties are:

\[
|I\rangle \sim \bar{\phi}_+^1 \phi_-^1 = \frac{1}{2} (\bar{q}_1 q_2 + \bar{p}_1 p_2) - \frac{1}{2} (\bar{q}_1 p_2 + \bar{p}_1 q_2) \\
|II\rangle \sim \bar{\phi}_+^1 \phi_+^2 = \frac{1}{2} (\bar{q}_1 q_2 + \bar{p}_1 p_2) + \frac{1}{2} (\bar{q}_1 p_2 + \bar{p}_1 q_2) \\
|III\rangle \sim \bar{\phi}_+^1 \phi_-^2 + \bar{\phi}_-^1 \phi_+^2 = \bar{q}_1 q_2 - \bar{p}_1 p_2 \\
|IV\rangle \sim \bar{\phi}_+^1 \phi_-^2 - \bar{\phi}_-^1 \phi_+^2 = \bar{p}_1 q_2 - \bar{q}_1 p_2
\]

The numerical scripts make the permutation properties clear, and we have used the chiral transformation properties of \( q \) and \( p \) given in Eq. (5). These states have charge conjugation sign: \( \pm \epsilon \) for \( |I\rangle \), \( |II\rangle \) and \( |III\rangle \), and \( \mp \epsilon \) for \( |IV\rangle \), and are invariant (up to a phase) with respect to the \( S_2 \) transformation

\[ q_1 \leftrightarrow p_1 \quad q_2 \leftrightarrow p_2. \]

Note that the \( S_2 \) transformation

\[ q_i \leftrightarrow p_i \quad q_j, p_j \text{ fixed } i \neq j, \]

interchanges \((2, 2)\) and \((1, 1 \oplus 3) \oplus (1 \oplus 3, 1)\) representations and therefore leaves \( |I\rangle \) and \( |II\rangle \) invariant while interchanging \( |III\rangle \) and \( |IV\rangle \).

In the broken symmetry phase with \( \langle \bar{\psi}\psi \rangle \neq 0 \) it is not generally sensible to classify states by their chiral transformation properties, and so one might think that the chiral decomposition of the meson states given in Eq. (15) is useless. This is not so as there are Lorentz frames in which the condensate decouples and the full chiral algebra is useful for classification purposes [5,6]. The infinite momentum frame is one example of such a frame. In these Lorentz frames helicity is conserved and so hadrons can be classified according to the full chiral algebra for each helicity [7]. Therefore we assume that the meson states
in Eq. (15) are states of definite helicity, parity and isospin. Note that the insertion of additional gamma matrices can only change the parity of the state, or interchange the \((2,2)\) and \((1,1 \oplus 3) \oplus (1 \oplus 3,1)\) representations.

Consider the consequences of this multiplet structure for the ground state of the four-flavor effective theory. The pattern of chiral symmetry breaking determines that the lowest lying state in the spectrum is the pion and so we are interested in the chiral representation involving the pion. Since the pion is a Lorentz scalar, all states in this representation have zero-helicity. In the case of zero-helicity there is conservation of normality, \(\eta = P(-1)^J\), where \(P\) is intrinsic parity and \(J\) is spin [5]. Since \(\pi\) has \(\eta = -1\), only states of opposite normality communicate by single-pion emission and absorption. The grouping we consider here is well known [5]. The pion is joined by a scalar \(\epsilon\) (\(\eta = +1\)), and the helicity-0 components of \(\rho\) (\(\eta = +1\)) and \(a_1\) (\(\eta = -1\)). These are states with \(GP(-1)^J = +1\) where \(G\) is \(G\)-parity. From Eq. (15) we identify

\[
|\pi\rangle_a = -\cos \phi |2,2\rangle_a + \sin \phi |A\rangle_a \\
|a_1\rangle_a^{(0)} = \sin \phi |2,2\rangle_a + \cos \phi |A\rangle_a \\
|\epsilon\rangle = |2,2\rangle_4 \\
|\rho\rangle_a^{(0)} = |V\rangle_a,
\]

where \(|1,3\rangle_a - |3,1\rangle_a \equiv \sqrt{2}|V\rangle_a\) and \(|1,3\rangle_a + |3,1\rangle_a \equiv \sqrt{2}|A\rangle_a\), the superscripts denote helicity and the subscripts are isospin indices. By considering matrix elements of the vector and axialvector currents between these states one finds that

\[
f_\pi = f_\rho \sin \phi \quad \text{and} \quad f_{a_1} = f_\rho \cos \phi \quad [1].
\]

In helicity conserving frames the mass-squared matrix is a relevant quantity. All mass-squared splittings transform like \(\langle \bar{\psi}\psi \rangle\); i.e., like the fourth component of a chiral four-vector [3,4]. Setting \(M_\pi^2 = 0\) one then obtains

\[
M_\rho^2 = \cos^2 \phi M_{a_1}^2.
\]

By considering matrix elements of the pion transition operator, \(X_\lambda^a\), one also finds

\[
g_{\rho\pi\pi}^2 f_\pi^2 = M_\rho^2 \sin^2 \phi \quad [3].
\]

One can then obtain combinations of masses and decay constants that are independent of the mixing angle. In particular it is clear that

\[
f_{a_1}^2 + f_\pi^2 = f_\rho^2 \quad (19a)
\]

\[
M_\rho^2 f_\rho^2 = M_{a_1}^2 f_{a_1}^2 \quad (19b)
\]

which is precisely the content of the first and second spectral function sum rules [7], respectively, evaluated in resonance saturation approximation.

The \(S_2\) invariance implies that the irreducible chiral representations must enter with equal weight (see Eq. (17)) and so \(\pi\) and \(a_1^{(0)}\) form an \(S_2\) doublet, \(\cos \phi = \sin \phi = 1/\sqrt{2}\), and we obtain the familiar KSRF relations [8].
\[
\frac{M_{\rho}^2}{g_{\rho\pi\pi}f_{\pi}^2} = 2 \quad (1.89 \pm 0.07) \quad \frac{f_{\rho}M_{\rho}}{g_{\rho\pi\pi}f_{\pi}^2} = 2 \quad (2.28 \pm 0.05), \quad (20)
\]

which are in very good agreement with the experimental numbers (in parentheses) extracted from the decays \(\rho^0 \to \pi^+\pi^-\) and \(\rho^0 \to e^+e^-\) \([2]\). Eq. (20) in turn implies \(f_{a_1}^2 = f_{\pi}^2\), \(2M_{\rho}^2 = M_{a_1}^2\) and \(M_{\pi}^2 = M_{\rho}^2\). We emphasize that these relations, which have been derived previously by imposing asymptotic constraints on pion-hadron scattering amplitudes \([3]\), are exact consequences of \(S_2\) in the four-flavor effective theory.

On the basis of quark model intuition one might have erroneously supposed that the four-flavor effective theory would give a pathological spectrum. The effective theory gets the ground state correctly because it respects —by assumption— the pattern of chiral symmetry breaking. From the quark point of view chiral symmetry breaking generates a mass gap which splits the four degenerate flavors so that two remain massless —giving the pion pole and a conserved axial current— and two become massive \([1]\).

5. Conclusion

We have considered an effective theory of QCD with twice the number of QCD flavors. When the axial anomaly is taken into account this effective theory has the same continuous global symmetries as QCD. However, there is a residual discrete symmetry that has several interesting consequences. In particular, if this \(S_2\) symmetry is unbroken, there is no P or CP violation in the effective theory. One might conclude that since \(\bar{\theta}\) is an unconstrained parameter in QCD, the effective theory cannot be describing the same physics as QCD. This viewpoint can be questioned because \(S_2\) has additional consequences in the effective theory. In particular, one can show that there is a sense in which hadrons in the effective theory are in \(S_2\) multiplets, leading to predictions in agreement with experiment and which have been found previously by assuming superconvergent sum rules in pion-hadron scattering \([4]\). Of course, if \(S_2\) is a global symmetry one does not expect it to be exact and so it can clearly be broken in a manner which preserves the predictions of section 4 and yet grossly violates the experimental bound on \(\bar{\theta}\).

One interesting possibility is that \(S_2\) is a discrete gauge symmetry \([9]\). If \(S_2\) is a gauge symmetry, then the \(2N\)-flavor effective theory has the same global symmetries as \(N\)-flavor QCD. In this case, \(S_2\) seems to solve the strong CP problem without conflicting with expectations that global symmetries are sacred and should therefore be shared by different descriptions of the same physics. This interpretation is consistent with the fact that \(S_2\) only seems to have consequences related to asymptotic behavior of scattering amplitudes. Although gauge symmetries are redundancies, they do have algebraic consequences when married with asymptotic constraints. In this sense gauge symmetries behave like
spontaneously broken chiral symmetries [3]. A nice example is that of the Drell-Hearn-Gerasimov sum rule which can be expressed algebraically as a statement of the (trivial) $U(1)$ algebra of electromagnetism (see the fourth entry in Ref. 5 and also Ref. 4).

We should also note that the idea of parity doublets arising non-perturbatively in the infrared as rescuers of discrete space-time symmetries has recently been considered in Ref. 10.

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