SPIN DENSITY MATRIX ELEMENTS IN VECTOR MESON LEPTOPRODUCTION *

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It is reported on an analysis of vector meson leptoproduction at small Bjorken-$x$ and large photon virtuality within an approach that bases on generalized parton distributions (GPDs). This approach leads to results for cross sections and spin density matrix elements for $\rho$ and $\phi$ leptoproduction in fair agreement with experiment.

Vector meson leptoproduction at small Bjorken-$x$ and large photon virtuality $Q^2$ factorizes into a hard partonic subprocess, vector meson leptoproduction off gluons, and GPDs describing the collinear emission and reabsorption of gluons from the proton. The process of interest is dominated by transitions from longitudinally polarized photons to longitudinally polarized vector mesons, $\gamma^*_L \rightarrow V_L$, other transitions are suppressed by inverse powers of $Q^2$. For experimentally accessible values of $Q^2$, however, the $L \rightarrow L$ transition amplitude, calculated to leading-twist accuracy, leads to a cross section that exceeds experiment typically by order of magnitude. Moreover, for $Q^2$ of the order of, say, $10 \text{ GeV}^2$ other transitions can not be ignored as data on spin density matrix elements reveal. A calculation of the transition amplitudes involving transversely polarized photons and/or vector mesons ($T$) in collinear factorization is problematic since infrared

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singularities occur indicating a breakdown of factorization.

We are reporting here on work in progress in which the GPD approach is modified by employing the modified perturbative approach in the calculation of the hard subprocess: the transverse degrees of freedom of the quarks building up the vector meson are retained and Sudakov suppressions are taken into account. This method provides a regularization scheme for the infrared singularities mentioned above. In passing we note that our approach bears resemblance to models proposed in where, up to occasional corrections, the GPDs are replaced by the usual gluon distributions.

The gluon contribution to the lepton production amplitude, calculated at the momentum transfer \( t \simeq t_{\text{min}} \simeq 0 \) for a proton with positive helicity, reads

\[
M_{\mu',\mu+} = \frac{e}{2} C_V \int_0^1 \frac{d\xi}{(\xi_0 + \xi)(\xi_0 - \xi + i\varepsilon)} \times [A_{\mu',\mu+}^V + A_{\mu',-\mu}^V] H^g(\xi, t) \simeq 0,
\]

where \( \mu (\mu') \) denotes the helicity of the photon (meson). Proton helicity flip and contributions from the GPD \( \tilde{H}^q \) which is much smaller than the GPD \( H^g \) at small \( x \), are neglected in (1). \( \bar{x} \) is the average momentum fraction of the gluons and the skewness \( \xi \) is related to Bjorken-\( x \) by \( \xi \simeq x_{BJ}/2 \). The flavor factors are \( C_\rho = 1/\sqrt{2} \) and \( C_\phi = -1/3 \).

The hard scattering amplitudes for the various helicity configurations are (the signs now label gluon helicities)

\[
A_{\mu',\mu+}^V = \frac{2\pi \alpha_s(\mu_R)}{N_c} \int d\tau \int \frac{dk_{\perp}^2}{16\pi^2} \Psi_V(\tau, k_{\perp}^2) K_{\mu',\mu}^V(\tau, \bar{x}, \xi, k_{\perp}^2, Q^2).
\]

The hard scattering kernels \( K \) are calculated from the relevant Feynman graphs at \( t \simeq 0 \) (see for instance), only the factors of \( \sqrt{-t} \) required by angular momentum conservation, are kept. The amplitude (2) is Fourier transformed to the impact parameter space where the variable \( b \) is canonically conjugated to \( k_{\perp} \). Multiplication with the \( b \)-space version of the Sudakov factor completes the specification of the subprocess amplitude.

In the calculation of the subprocess a spin wave function for the vector meson is used which takes into account the quark transverse momentum, \( k_{\perp} \), linearly, and, hence, one unit of orbital angular momentum in a covariant manner. For the momentum-space meson wavefunction \( \Psi_V \) a simple Gaussian form is adopted

\[
\Psi_V(\mathbf{k}_{\perp}, \tau) = 8\pi^2 \sqrt{2N_c} f_V a_V^2 \exp \left[ -a_V^2 \frac{k_{\perp}^2}{\tau \bar{\tau}} \right],
\]
where \( \tau \) is the fraction of the meson momentum the quark carries \( (\bar{\tau} = 1 - \tau) \). In the numerical evaluation of meson leptoproduction the following values for the decay constants and the transverse size parameters are used: \( f_\rho = 0.216 \text{ GeV}, \ f_\phi = 0.237 \text{ GeV}, \ a_\rho = 0.52 \text{ GeV}^{-1} \) and \( a_\phi = 0.45 \text{ GeV}^{-1} \).

The GPD is modelled from a double distribution \(^9\) for which the following ansatz is exploited:

\[
 f^g(\beta, \alpha, t \simeq 0) \sim \frac{(1 - |\beta|)^2 - \alpha^2}{}^{n(1 - |\beta|)^2 + 1} g(\beta). \tag{4}
\]

An appropriate integral over \( f^g \) leads to the GPD \(^9\). For the gluon distribution \( g(\beta) \) we take the CTEQ5M parameterization \(^\text{10}\). The cases \( n = 1 \) and \( 2 \) lead to nearly the same results.

We have evaluated the amplitudes for \( L \to L, T \to L \) and \( T \to T \) transitions at small \( t \) and neglected the \( L \to T \) and \( T \to -T \) ones since they are strongly suppressed in our approach. The three amplitudes are multiplied by exponentials \( \exp[-B_i t/2] \) in order to make contact with experiment (or to extrapolate to \( t = 0 \)). The integrated cross sections for \( \gamma^* p \to V p \) are shown in Fig. 1. Good agreement with experiment is to be observed for \( \rho \) and \( \phi \) mesons.

In summary, the GPD approach combined with the modified perturbative approach applied to the partonic subprocess allows for a calculation of various cross section and spin density matrix elements at large \( Q^2 \) and small Bjorken-\( x \). Fair agreement between theory and experiment for light vector

![Figure 1](image_url)

Figure 1. The integrated cross section for \( \gamma^* p \to \rho p \) (left) and \( \gamma^* p \to \phi p \) (right) at \( W \simeq 75 \text{ GeV} \). Data taken from \(^\text{11}\) (filled squares) and \(^\text{12}\) (open squares). The solid lines represent our results.
meson leptoproduction is found. It is important to extend our results to lower energies but larger Bjorken-$x$. This is the HERMES and COMPASS region where the quark GPDs have be taken into account. Preliminary data for the spin density matrix elements have been presented by COMPASS at this conference.  

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