Stationary state volume fluctuations in a granular medium

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(Dated: March 23, 2022)

A statistical description of static granular material requires ergodic sampling of the phase space spanned by the different configurations of the particles. We periodically fluidize a column of glass beads and find that the sequence of volume fractions $\phi$ of post-fluidized states is history independent and Gaussian distributed about a stationary state. The standard deviation of $\phi$ exhibits, as a function of $\phi$, a minimum corresponding to a maximum in the number of statistically independent regions. Measurements of the fluctuations enable us to determine the compactivity $X$, a temperature-like state variable introduced in the statistical theory of Edwards and Oakeshott [Physica A 157, 1080 (1989)].

PACS numbers: 45.70.-n, 05.40.-a, 64.30.+t, 47.55.Kf

Granular materials consist of a large number $N$ (typically more than $10^6$) dissipative particles that are massive enough so that their potential energy is orders of magnitude larger than their thermal energy. The large number suggests that a statistical description might be feasible. Edwards and coworkers [1] developed such a description with the volume $V$ of the system, rather than the energy, as the key extensive quantity in a static granular system. The corresponding configuration space contains all possible mechanically stable arrangements of grains.

Brownian motion is insufficient for a granular system to explore its configuration space, so energy must be supplied by external forcing such as tapping [2], shearing [3], or both [4]. The theory of Edwards requires that the forcing assures ergodicity; all mechanically stable configurations must be equally probable and accessible. A necessary condition for ergodicity is history independence: the physical properties of the system must not depend on the way a specific state was reached. History independence has previously been demonstrated only by Nowak et al. [2] for tapped glass beads at volume fractions $\phi > 0.625$.

In this paper we explore the configuration space using a periodic train of flow pulses in a fluidized bed. A stationary column of glass beads in water is expanded by an upward stream of water until it reaches a homogeneously fluidized state [5], and then the flow is switched off. The fluidized bed collapses [6] and forms a sediment of volume fraction $\phi$, which we find depends in a reproducible way on the flow rate $Q$ of the flow pulse. This forcing results in a history independent steady state where the volume exhibits Gaussian fluctuations around its average value.

A central postulate of the Edwards theory is the existence of a temperature-like state variable called compactivity $X = \partial V / \partial S$. The entropy $S$ is defined in analogy to classical statistical mechanics as $S(V,N) = \lambda \ln \Omega$, where $\Omega$ is the number of mechanically stable configurations of $N$ particles in $V$, and $\lambda$ is an unknown analog to the Boltzmann constant. The assumption that $X$ is a relevant control parameter in granular systems has found support in simulations of segregation in binary mixtures [7], compaction under vertical tapping [8], and shearing [9]. However, no measurements of $X$ have been reported. In this paper we determine $X$ from the measured volume fluctuations using a method suggested in [2].

Experiment. The apparatus is shown in Fig. 1(a). A square bore glass tube (24.1 mm $\times$ 24.1 mm) contains about $3.6 \times 10^6$ beads (soda-lime glass, $d = 250 \pm 50 \mu$m) dissipative particles that are massive enough so that their potential energy is orders of magnitude larger than their thermal energy. The large number suggests that a statistical description might be feasible. Edwards and coworkers [1] developed such a description with the volume $V$ of the system, rather than the energy, as the key extensive quantity in a static granular system. The corresponding configuration space contains all possible mechanically stable arrangements of grains.

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The beads were fluidized with pulses of temperature-controlled (23.0 ± 0.1 °C) de-ionized water.

Flow pulses were generated by a computer-controlled syringe pump. During a flow pulse of length $\tau_{\text{pulse}}$ (Fig. 1(b)), the bed expanded until its height reached a stable value where the time-averaged viscous drag force on each particle corresponded to its weight; $\tau_{\text{pulse}}$ was chosen so $Q\tau_{\text{pulse}} = 30\text{ mL}$. After each flow pulse, the bed was allowed to settle into a mechanically stable configuration, which was determined by measuring the time-resolved correlation of laser speckle patterns. A waiting time $\tau_{\text{wait}}$ of 30 s was found to be ample for achieving a jammed state. After $\tau_{\text{wait}}$ the volume fraction $\phi$ was determined by measuring the bed height $h$ with two CCD cameras at a 90° angle. Averaging over the width of the bed yielded height values with a standard deviation of only 0.008 d for a bed at rest (cf. inset of Fig. 1(a)).

**History independent steady state.** Starting with an initial $\phi = 0.581$ prepared with a single flow pulse of $Q = 60\text{ mL/min}$, the volume fraction quickly approaches a $Q$-dependent constant value $\phi_{\text{avg}}$, as shown in Fig. 2(a). Then with successive flow pulses, $\phi$ fluctuates about $\phi_{\text{avg}}$. The history independence is demonstrated by ramping up and down in flow rate (Fig. 2(b)); $\phi$ depends only on $Q$ of the last flow pulse, not the earlier history of the bed. The remaining differences in $\phi_{\text{avg}}$ for increasing and decreasing $Q$ are on average only $1.6 \times 10^{-4}$. These small variations are correlated with the viscosity changes due to the temperature drift of ±0.1 °C during the course of the experiment (35 h).

For slow sedimentation, corresponding to $Q \to \infty$, $\phi$ should converge to its random loose packing value, $\phi_{\text{RLP}}$. In the absence of a theory, we fit the data to a phenomenological equation,

$$\phi(Q) = \phi_{\text{RLP}} + \frac{a}{Q - b}$$

(Fig. 2(b)). This yields $\phi_{\text{RLP}} = 0.573 \pm 0.001$ ($a = 0.365 \text{ mL/min}$, $b = 12.9 \text{ mL/min}$), where the uncertainty arises not from the very small uncertainty in bed height measurement (corresponding to 0.0003 in $\phi$) but from variations in the cross section of the square tube.

Volume fluctuations. Gaussian fluctuations in $\phi$ are observed for the whole range of $\phi$ studied; examples are shown in Fig. 3. The standard deviation of these fluctuations $\sigma_\phi$, like $\phi_{\text{avg}}$, was found to be history independent. The variation of $\sigma_\phi$ with $\phi$ fits a parabola with
The medium, the number of contacts per grain increases, and the probability that the displacement of one grain will mechanically destabilize other grains decreases. Consequently \( N_{ppr} \) decreases. But with increasing \( \phi \), the free volume per particle decreases. The latter effect is dominant for \( \phi > \phi_m \), where geometrical constraints allow movement only as collective process of an increasing number of beads. Hence \( N_{ppr} \) and \( \sigma_\phi \) increase. At the maximum random jammed packing, \( \phi_{MRJ} = 0.64 \), any motion would require a rearrangement of the whole system.

Knowing the dependency of \( \sigma_\phi \) on \( \phi \) enables us to determine Edward’s compactivity \( X \) using a granular version of the fluctuation dissipation theorem derived by Nowak et al. \(^2\). We obtain

\[
\frac{\sigma_\phi}{\phi_{avg}} = \frac{k}{\sqrt{N_1}}
\]

(2)

where \( k \approx 0.2 \), estimated by Nowak et al. \(^2\) from the maximal volume fraction fluctuations a single spatial region could undergo. Using \(^2\) and the parabolic fit in Fig. 4(a), we estimate the average number of particles per statistically independent region, \( N_{ppr} = N/N_1 \) (where \( N \) is the number of particles). Figure 4(b) shows \( N_{ppr} \) to be 40 for random loose packing, it reaches a minimum of about 1.8 at \( \phi_m \) and then increases again. This contrasts with the roughly constant value of \( N_{ppr} \) found in previous experiments \(^2\) for \( \phi \) in the range 0.625-0.634.

We suggest that the minimum of \( N_{ppr} \) at \( \phi_m \) corresponds to a crossover of competing mechanisms. With increasing \( \phi \) above \( \phi_{RLP} \), where force chains stabilize the surface asperities, as illustrated in Fig. 5. The data we have presented show no aging during the 12,000 pulses in the experiment; the beads had been previously used.
for more than 45,000 flow pulses. Just as for aged beads, the volume fraction standard deviation for new beads fits a parabola, as Fig. 5 illustrates; the minimum is at \( \phi_m = 0.587 \). The shift of the minimum to a value lower than lower \( \phi_m = 0.596 \) for aged beads is consistent with our argument about statistically independent regions: a higher values of \( \mu \) makes dilute states less fragile and lowers the onset of the necessity of collective motion. For new beads a fit to (1) results in \( \phi_{RLP} = 0.566 \pm 0.004 \) interpolated from the measurements (presumably with new beads) in [12]. This shows that \( \phi_{RLP} \) depends on the frictional properties of the beads, as predicted in [13].

Conclusions. We have shown that the configuration space of a granular medium can, by using flow pulses, be explored in a history independent way, which is essential for the statistical theory [1] to be applicable. Fluctuations of the volume fraction are Gaussian with a parabolic minimum, which corresponds to a maximum in the number of statistically independent regions. The minimum arises as a consequence of competing mechanisms; the location of the minimum depends on the frictional properties of the beads. The compactivity can be determined from the volume fraction fluctuations; hence compactivity is a well-defined parameter representative of the material. This opens the door for new experiments that would, for example, investigate the use of compactivity as a control parameter for segregation in binary mixtures [7, 15].

We thank Antonio Coniglio for rousing our interest in this problem; Massimo Pica Ciamarra, Karen Daniels, Sungilwan Jung, Salvatore Torquato, and Thomas Truskett for helpful discussions; and Michael Schmerling for help with the SEM. This work was supported by the Engineering Research Program of the Office of Basic Energy Sciences of the U.S. Department of Energy (Grant No. DE-FG03-93ER14312), by the Robert A Welch Foundation, and by the Office of Naval Research Quantum Optics Initiative (Grant No. N00014-03-1-0639).

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