Q-ball dark matter and baryogenesis in high-scale inflation

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We investigate the scenario that one flat direction creates baryon asymmetry of the universe, while Q balls from another direction can be the dark matter in the gauge-mediated supersymmetry breaking for high-scale inflation. Isocurvature fluctuations are suppressed by the fact that the Affleck-Dine field stays at around the Planck scale during inflation. We find that the dark matter Q balls can be detected in IceCube-like experiments in the future.

I. INTRODUCTION

The Affleck-Dine baryogenesis \cite{AD} is one of the promising mechanisms for creating the baryon asymmetry of the universe. It utilizes the scalar field which carries the baryon number. In the minimal supersymmetric standard model (MSSM), there are a lot of such scalar fields called flat directions which consist of squarks (and sleptons). The scalar potential of the flat directions vanishes in the supersymmetry (SUSY) limit, and is lifted by SUSY breaking effects.

It is well known that the Q balls form during the course of the Affleck-Dine baryogenesis \cite{AD1,AD2,AD3}. In particular, Q balls with large enough charge Q are stable against the decay into baryons (nucleons) in the gauge-mediated SUSY breaking. In this case, Q balls would be dark matter of the universe, and could be detectable. The baryon asymmetry of the universe is explained by the baryon numbers that are not remained inside the Q balls. It has been considered that the single flat direction transforms into dark matter Q balls, and the baryon numbers are emitted through the Q-ball surface in the thermal bath of the universe \cite{AD1,AD2,AD3}. Unfortunately, this simple scenario is observationally excluded \cite{11}.

However it is natural to consider one direction which take part in the Affleck-Dine mechanism. There are at most 29 complex degrees of freedom for D-flat and renormalizable F-flat directions which do not involve Higgs field \cite{12}. Most of the directions are lifted by \( n = 4 \) nonrenormalizable superpotential of the form \( W = \phi^n \), which applies to the even \( n \) case, where \( \phi \) is the field of a flat direction. For the odd \( n \) case, the potential is lifted by \( W = H_i \phi^{n-1} \) superpotential, where \( H_i \) is the Higgs field \( H_u \) or \( H_d \). In Ref. \cite{11}, we considered \( n = 5 \) and \( n = 6 \) directions, where the former direction has responsible for the baryon asymmetry of the universe while the latter forms dark matter Q balls.

Recently, the BICEP2 team reported the detection of the primordial B-mode polarization of the cosmic microwave background \cite{13}. It could be originated from gravitational waves produced during inflation, which leads to the large tensor-to-scalar ratio \( r \approx 0.2 \) and the large Hubble parameter during inflation \( H_{\text{inf}} \approx 10^{14} \) GeV. In such high-scale inflation, in general, large baryonic isocurvature fluctuations will be induced in the Affleck-Dine mechanism \cite{14,15}. In order to suppress isocurvature fluctuations sufficiently, the amplitude of the flat direction must be as large as the Planck scale, \( M_P = 2.4 \times 10^{18} \) GeV, for \( H_{\text{inf}} \approx 10^{14} \) GeV, which can be achieved if the coupling in the nonrenormalizable superpotential is small enough \cite{16}.

In the present Letter, we investigate the possibility that one flat direction creates baryon asymmetry of the universe, while Q balls from another direction can be the dark matter in the gauge-mediated SUSY breaking for high-scale inflation. It is realized for two \( n = 4 \) directions with different sizes of the couplings. The direction with larger coupling disintegrates into the gauge-mediation type Q balls, which decay into baryons before the big bang nucleosynthesis (BBN). The direction with smaller coupling transforms into the new type Q balls, which are stable against the decay into baryons and become the dark matter of the universe. These dark matter Q balls could be detected by the IceCube-like experiment in the future.

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1 If the charge of the Q ball is small enough, the lightest SUSY particles, such as the gravitino \cite{5,6} or the axino\cite{8}, decayed from the Q ball, may be the dark matter, while the baryons are produced by the Q-ball decay as well.
2 The Affleck-Dine baryogenesis and the lightest SUSY particle (LSP) dark matter production after high-scale inflation was considered in Ref. \cite{18}, where all the Q balls, if it form, decay into baryons and LSPs in both the gravity and gauge mediation.
3 The dark matter new type Q ball has a good feature that avoids astrophysical constraints by the neutron star destruction \cite{11}.
The structure of the Letter is as follows. In the next section, we review how the Affleck-Dine mechanism works in the SUSY setup. Isocurvature fluctuations are considered in Sec.III. In Sec.IV, we show the properties of the $Q$ balls in the gauge mediation. We estimate the baryon number and the dark matter of the universe in Sec.V, and seek for successful scenarios to explain both the baryon asymmetry and the dark matter $Q$ balls in Sec.VI. Sec.VII is devoted to our conclusions.

II. AFFLECK-DINE MECHANISM

The flat direction is a scalar field $\Phi$ which consists of squarks (and sleptons) in MSSM. The potential of the flat directions is flat in SUSY limit, and lifted by SUSY breaking effects and nonrenormalizable superpotential of the form

$$W_{NR} = \frac{\lambda \Phi^n}{n M_P^{n-3}},$$

where $\lambda$ is a coupling constant which will be determined later. In the gauge-mediated SUSY breaking, the potential is lifted as

$$V = V_{SUSY} + V_{NR} = V_{gauge} + V_{grav} + V_{NR},$$

where

$$V_{gauge} = \begin{cases} \frac{1}{2} m^2 \phi^2 & (\phi \ll M_m), \\ M_P^4 \left( \log \frac{\phi^2}{2 M_m^2} \right)^2 & (\phi \gg M_m), \end{cases}$$

$$V_{grav} = \frac{1}{2} m_{3/2}^2 \phi^2 \left( 1 + K \log \frac{\phi^2}{2 M_*^2} \right),$$

$$V_{NR} = \frac{\lambda^2 \phi^{2(n-1)}}{2^{n-1} M_P^{2(n-3)}}.$$

Here, $\Phi = \frac{1}{\sqrt{2}} \phi e^{i \theta}$, $m_\phi \sim O(\text{TeV})$ is a soft breaking mass, $M_m$ is the messenger scale, $m_{3/2}$ is the gravitino mass, $K(<0)$ is a coefficient of one-loop corrections, and $M_*$ is the renormalization scale. $M_F$ ranges as

$$10^3 \text{GeV} \lesssim M_F \lesssim \frac{g^{1/2}}{4 \pi} \sqrt{m_{3/2} M_F}. \quad (6)$$

Since the gravitino mass is much smaller than TeV scale, the potential is dominated by $V_{gauge}$ for $\phi < \phi_{eq}$, while $V_{grav}$ overcomes $V_{gauge}$ for $\phi < \phi_{eq}$, where

$$\phi_{eq} = \sqrt{2} \frac{M_F^2}{m_{3/2}},$$

derived from $V_{gauge}(\phi_{eq}) = V_{grav}(\phi_{eq})$.

In the Affleck-Dine mechanism, the flat direction has large VEV during inflation. This is usually realized by the so-called negative Hubble-induced mass term [1]

$$V_H = -c H^2 |\Phi|^2 = -\frac{1}{2} c H^2 \phi^2,$$

which stems from the SUSY breaking effect by the finite inflaton energy density. Here $H$ is the Hubble parameter, and $c$ is a constant of $O(1)$. During inflation the flat direction stays at the minimum determined by the balance of $V_H$ and $V_{NR}$:

$$\phi_{\text{min}} = \sqrt{2} \left( \frac{1/2 H_{\text{inf}} M_P^{n-3}}{\lambda} \right)^{1/(n-2)}.$$

Well after inflation when the Hubble parameter decreases as large as $m_{\phi, \text{eff}} \simeq \sqrt{V''(\phi_{\text{osc}})}$, the field begins oscillations. The field actually rotates due to the usual soft $A$ terms, and the baryon number of the universe is created. If the field
starts to oscillate at the amplitude smaller than $\phi_{\text{eq}}$, the potential is dominated by $V_{\text{gauge}}$ so that the gauge-mediation type $Q$ balls eventually form. On the other hand, the new-type $Q$ balls are created for $\phi_{\text{osc}} > \phi_{\text{eq}}$ where $V_{\text{grav}}$ is larger than $V_{\text{gauge}}$. It is determined by the size of $\lambda$ whether the field starts its oscillation smaller or larger than $\phi_{\text{eq}}$. The gauge-mediation type $Q$ balls form for the flat direction with larger $\lambda$, while the new-type $Q$ balls are created for that with smaller $\lambda$. See Fig.1. We will consider the situation that the gauge-mediation type $Q$ balls decay to produce the baryon asymmetry of the universe, while the new type $Q$ balls are stable and become the dark matter of the universe.

The lower limit of the amplitude of the flat direction, which is responsible for the dark matter $Q$ balls, is highly model-dependent. It might be at most $\Omega_{\text{CDM}}/\Omega_b$ times larger than Eq. (12) for $\varepsilon = 1$.  

\[ V(\phi) \]

\[ M^4_{\phi} \]

\[ \lambda_G \]

\[ \lambda_N \]

\[ V_{\text{gauge}} \]

\[ V_{\text{grav}} \]

\[ M_\text{m} \]

\[ \phi_G \]

\[ \phi_{\text{eq}} \]

\[ \phi_N \]

\[ \phi \]

FIG. 1: Potential of the flat direction. One direction is lifted by the nonrenormalizable term $V_{\text{NR}}$ with $\lambda_G$ (orange), while another direction is lifted by that with $\lambda_N$ (brown).

### III. AVOIDING TOO LARGE ISOCURVATURE FLUCTUATIONS

During high-scale inflation the flat direction acquires quantum fluctuations along the phase direction if there is no sizable $A$ terms, and it may lead to too large baryonic and CDM isocurvature fluctuations in this case. Actually, the Hubble-induced $A$ terms do not appear for most inflation models in supergravity [17]. The amplitude of the baryonic isocurvature fluctuations can be estimated as

\[ S_b = \frac{\delta \rho_b}{\rho_b} \simeq \frac{\delta n_b}{n_b} \simeq \frac{\delta n_\phi}{n_\phi} \simeq \delta \theta \simeq \frac{H_{\text{inf}}}{2\pi \phi_{\text{inf}}} \times \frac{H_{\text{inf}}}{2\pi \phi_{\text{inf}}} = 5.0 \times 10^{-5}, \]

(10)

On the other hand, observational upper bound is calculated as

\[ S_b^{\text{(obs)}} \simeq \frac{\Omega_{\text{CDM}}}{\Omega_b} (\beta_{\text{iso}} A_s)^{1/2} = 5.0 \times 10^{-5}, \]

(11)

where $\beta_{\text{iso}} = 0.039$ and $\ln(10^{10} A_s) = 3.098$ are respectively the primordial isocurvature fraction and the amplitude of the curvature power spectrum obtained in the Planck results [20]. It should be $S_b < S_b^{\text{(obs)}}$, which results in

\[ \phi_{\text{inf}} > \frac{H_{\text{inf}}}{2\pi S_b^{\text{(obs)}}} = 3.5 \times 10^{17} \text{GeV} \left( \frac{H_{\text{inf}}}{1.1 \times 10^{14} \text{GeV}} \right) \left( \frac{S_b^{\text{(obs)}}}{5.0 \times 10^{-5}} \right)^{-1}. \]

(12)

Thus the field should stay at around $M_P$ during inflation in order to have small enough isocurvature fluctuations. This can be achieved for small enough $\lambda$:

\[ \lambda \lesssim 2^{n/2-1} c^{1/2} \frac{H_{\text{inf}}}{M_P} \left( c = 9.3 \times 10^{-5} \right) \left( \frac{H_{\text{inf}}}{1.1 \times 10^{14} \text{GeV}} \right). \]

(13)
Here and hereafter, \(_{(n=4)} = \) means that we evaluate values for \(n = 4\). Moreover, if the amplitude of the flat direction is as large as \(M_P\) during inflation, the terms such as
\[
V_{HA} = a_H \frac{H^2 \Phi^m}{M^4} + \text{h.c.,}
\]
would come from a nonminimal Kähler potential of the form \(K = I^I \Phi^m / M^m\) (the \(F\) term of the \(I\) field is responsible for inflation) and results in the mass scale of \(O(H)\) for the phase direction [18], which further suppresses the isocurvature fluctuations. Therefore, we adopt \(\lambda < \sim 10^{-4}\), and investigate the scenario that the gauge-mediation type \(Q\) balls from one flat direction decay into baryons, while another direction forms new-type \(Q\) balls to be dark matter of the universe in this situation.

**IV. \(Q\) Balls in the Gauge Mediation**

The \(Q\)-ball solution exists in those potential that is flatter than \(\phi^2\), which is the case for SUSY. In the gauge-mediated SUSY breaking there exist two types of the \(Q\) ball. One is the gauge-mediation type [2, 9], and the other is the new type [25].

The former forms when the potential is dominated by \(V_{\text{gauge}}\) when the field starts oscillations and fragments into lumps. The charge of the formed \(Q\) ball is estimated as [9]
\[
Q_G = \beta_G \left( \frac{\phi_{\text{osc}}}{M_F} \right)^4,
\]
where \(\phi_{\text{osc}}\) is the amplitude of the field at the onset of the oscillation (rotation). \(\beta_G \simeq 6 \times 10^{-4}\) for a circular orbit \((\varepsilon = 1)\), while \(\beta_G \simeq 6 \times 10^{-5}\) for an oblate case \((\varepsilon \ll 0.1)\). Here \(\varepsilon\) is the ellipticity of the field orbit. The features of this type are represented as
\[
M_Q \simeq \frac{4\sqrt{2}\pi}{3} \zeta M_F Q_G^{3/4},
\]
\[
R_Q \simeq \frac{1}{\sqrt{2}} \zeta^{-1} M_F^{-1} Q_G^{1/4},
\]
\[
\omega_Q \simeq \sqrt{2}\pi \zeta M_F Q_G^{-1/4},
\]
\[
\phi_Q \simeq \frac{1}{\sqrt{2}} \zeta M_F Q_G^{1/4},
\]
where \(M_Q\) and \(R_Q\) are the mass and the size of the \(Q\) ball, respectively, and \(\omega_Q\) and \(\phi_Q\) are respectively the rotation speed and the amplitude of the field inside the \(Q\) ball, and \(\zeta\) is the \(O(1)\) parameter [21, 22].

On the other hand, the new type of the \(Q\) ball forms when the potential is dominated by \(V_{\text{grav}}\) at the onset of the oscillation. The charge of the new-type \(Q\) ball is given by [25, 26]
\[
Q_N = \beta_N \left( \frac{\phi_{\text{rot}}}{m_{3/2}} \right)^2,
\]
where \(\beta_N \simeq 0.02\) [23]. The properties of the new type \(Q\) ball are as follows:
\[
M_Q \simeq m_{3/2} Q_N,
\]
\[
R_Q \simeq |K|^{-1/2} m_{3/2}^{-1},
\]
\[
\omega_Q \simeq m_{3/2},
\]
\[
\phi_Q \simeq m_{3/2} Q_N^{1/2}.
\]

The charge \(Q\) is in fact the \(\Phi\)-number, and relates to the baryon number of the \(Q\) ball as
\[
B = bQ,
\]
where \(b\) is the baryon number of \(\Phi\)-particle. For example, \(b = 1/3\) for the \(u dd\) direction.
The Q ball is stable against the decay into nucleons for large field amplitude when the charge is very large. Since \( \omega Q \) corresponds to the effective mass of the field inside the Q ball, the stability condition is given by \( \omega Q < m_D \), where \( m_D \) is the mass of the decay particles. The new type Q ball is generally stable against the decay into nucleons, except for the gravitino mass larger than that of nucleons. On the other hand, the gauge-mediation type Q ball is stable for \( Q_G > Q_D \), where

\[
Q_D \equiv 4\pi^4 \zeta^4 \left( \frac{M_F}{bm_N} \right)^4 \simeq 1.2 \times 10^{30} \left( \frac{\zeta}{2.5} \right)^4 \left( \frac{b}{1/3} \right)^{-4} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^4,
\]

with \( m_N \) being the nucleon mass. Since we need unstable Q ball to produce the baryon number of the universe, we consider the gauge-mediation type Q ball with the charge smaller than \( Q_D \).

The unstable gauge-mediation type Q ball decays into baryons through its surface. The decay rate is reestimated in Refs. [6, 21] as

\[
\Gamma_Q \simeq N_q \frac{\omega_Q^3}{Q_G 12\pi^2 4\pi R_Q^2},
\]

where \( N_q \) is the number of the decay quarks. Then the temperature at the decay is obtained as

\[
T_D = \left( \frac{90}{4\pi^2 N_D} \right)^{1/4} \sqrt{\frac{\Gamma_Q}{\rho M_F}} \simeq 16 \text{ MeV} \left( \frac{\zeta}{2.5} \right)^{1/2} \left( \frac{N_q}{18} \right)^{1/2} \left( \frac{N_D}{10^{17}} \right)^{-1/4} \left( \frac{M_F}{10^6 \text{ GeV}} \right)^{1/2} \left( \frac{Q_G}{10^{24}} \right)^{-5/8},
\]

where \( N_D \) is the relativistic degrees of freedom at the decay time. Since the decay must take place before the BBN, the charge of the gauge-mediation type Q ball has the upper limit.

V. BARYON ASYMMETRY AND THE Q BALL DARK MATTER

We consider two \( n = 4 \) flat directions with different values of \( \lambda \). The direction with smaller \( \lambda \) forms the new type Q balls to be the dark matter of the universe, while that with larger \( \lambda \) becomes the unstable gauge-type Q balls to produce baryon asymmetry in the universe. The fields start their oscillations when the Hubble-induced mass term \( V_H \) becomes smaller than \( V_B^{SLAVER} \). The amplitudes at the onset of oscillations are thus calculated as

\[
\phi_G,\text{osc} = \sqrt{2} \lambda_G^{-1/2} \left( M_F^2 M_F^{n-3} \right)^{1/2} \frac{1}{(n=4)} \sqrt{2} \lambda_G^{-1/2} \left( M_F^2 M_F \right)^{1/2},
\]

\[
\phi_N,\text{osc} = \sqrt{2} \lambda_N^{-1/2} \left( m_{3/2} M_{3/2}^{n-3} \right)^{1/2} \frac{1}{(n=4)} \sqrt{2} \lambda_N^{-1/2} \left( m_{3/2} M_{3/2} \right)^{1/2},
\]

for the gauge-mediation type and the new type, respectively. Therefore, the charges of both types of the Q balls are obtained as

\[
Q_G = \beta_G \left( \frac{\phi_G,\text{osc}}{M_F} \right)^4 = \frac{4\beta_G}{\lambda_G^{4/3}} \left( \frac{M_F}{M_F} \right)^{4(n-3)/n} = \frac{4\beta_G}{\lambda_G^{4/3}} \left( \frac{M_F}{M_F} \right)^{4} \quad (\text{Gauge-mediation type}),
\]

\[
Q_N = \beta_N \left( \frac{\phi_N,\text{osc}}{m_{3/2}} \right)^2 = \frac{2\beta_N}{\lambda_N^{2/3}} \left( \frac{M_F}{m_{3/2}} \right)^{2(n-3)/n} = \frac{2\beta_N}{\lambda_N^{2/3}} \left( \frac{M_F}{m_{3/2}} \right) \quad (\text{New type}).
\]

Since all the charges (the baryon numbers) are released by the decay of the gauge-mediation type Q balls, the baryon number is given by

\[
Y_b = \frac{9}{8\sqrt{2}} b \frac{T_{RH} \phi_G,\text{osc}}{M_F^2 M_F} = \frac{9}{4} b \lambda_G^{-3/7} T_{RH} M_F^{2(n-4)/n} M_F^{n-7} M_F^{-1} (n=4) = \frac{9}{4} b \lambda_G^{-1} T_{RH} M_F^{-1},
\]

\[^5\] The abundance of next-to-LSPs produced by the Q-ball decay is many orders of magnitude smaller than the BBN bound in our successful scenario.
where $T_{RH}$ is the reheating temperature after inflation. In this $n = 4$ case, $T_{RH}$ is independent of $M_F$ and solely determined by $\lambda_G$ as

$$T_{RH} = \frac{4}{9} b^{-1} \lambda_G Y_b M_P = 3.2 \times 10^4 \text{ GeV} \left( \frac{b}{1/3} \right)^{-1} \left( \frac{\lambda_G}{10^{-4}} \right) \left( \frac{Y_b}{10^{-10}} \right). \quad (34)$$

At the same time, the dark matter is composed by stable new-type $Q$ balls. Their abundance is estimated as

$$\frac{\rho_{Q}}{s} = \frac{9}{8} T_{RH} \left( \frac{\phi_{N, osc}}{M_P} \right)^2 = \frac{9}{4} \lambda_N^{-2} T_{RH} \left( \frac{m_{3/2}}{M_P} \right)^{\frac{2n}{2} - n} = \frac{9}{4} \lambda_N^{-1} T_{RH} \left( \frac{m_{3/2}}{M_P} \right), \quad (35)$$

which is rewritten as

$$m_{3/2} = \left( \frac{2}{3} \right)^{n-2} \lambda_N \left( \frac{\rho_{Q}}{s} \right)^{\frac{n-2}{2}} T_{RH}^{-\frac{n-2}{2}} M_P \left( \frac{2}{3} \right)^2 \lambda_N \left( \frac{\rho_{Q}}{s} \right) \frac{M_P}{T_{RH}}. \quad (36)$$

We must have

$$\frac{\rho_{Q}}{s} = 5.4 m_N Y_b = 5.4 \times 10^{-10} \text{ GeV} \left( \frac{Y_b}{10^{-10}} \right), \quad (37)$$
in order to have right amount of the dark matter. For explaining both the baryon asymmetry and the dark matter of the universe, we have

$$m_{3/2} = 1.8 \text{ MeV} \left( \frac{b}{1/3} \right) \left( \frac{\lambda_N}{10^{-7}} \right) \left( \frac{\lambda_G}{10^{-4}} \right)^{-1}, \quad (38)$$

where we insert Eqs. (34) and (37) with $n = 4$ into Eq. (36). We call this the B-DM relation in the following.

**VI. CONSTRAINTS ON MODEL PARAMETERS AND DETECTION OF DARK MATTER $Q$ BALLS**

Let us now investigate how the B-DM relation (38) is satisfied. We consider two flat directions with different values of $\lambda$. Since isocurvature fluctuations should be suppressed, it is necessary to have $\lambda_N < \lambda_G \lesssim 10^{-4}$. There are several conditions to be hold for successful scenario. (a) The flat direction with $\lambda_G$ disintegrates into the gauge-mediation type $Q$ balls, so that $\phi_{G, osc} < \phi_{eq}$, while (b) the new type $Q$ balls forms from the direction with $\lambda_N$ for $\phi_{N, osc} > \phi_{eq}$. From Eqs. (7) and (29), the condition (a) results in

$$m_{3/2} < \frac{1}{\lambda_G^{1/2}} \left( \frac{M_F^{2(n-2)}}{M_P^{n-3}} \right)^{1/2} \left( \frac{\rho_{Q}}{s} \right) \left( \frac{\lambda_N}{10^{-7}} \right) \left( \frac{\lambda_G}{10^{-4}} \right)^{1/3} \quad (n=4), \quad (39)$$

while the condition (b) leads to

$$m_{3/2} > \frac{1}{\lambda_N^{1/2}} \left( \frac{M_F^{2(n-2)}}{M_P^{n-3}} \right)^{1/2} \left( \frac{\rho_{Q}}{s} \right) \left( \frac{\lambda_N}{10^{-7}} \right) \left( \frac{\lambda_G}{10^{-4}} \right)^{1/3} \quad (n=4), \quad (40)$$

The gauge-mediation type $Q$ ball should be (c) unstable and (d) decays before the BBN. The former condition is expressed, from Eqs. (15) and (18), as

$$M_F > \left( \frac{b_{G}^{1/4}}{\pi \zeta} \right) \left( \frac{\lambda_G}{10^{-4}} \right)^{1/2} \left( \frac{m_{n-1} M_P^{n-3}}{\pi^{n-1}} \right)^{1/2} \left( \frac{\rho_{Q}}{s} \right) \left( \frac{\lambda_N}{10^{-7}} \right) \left( \frac{\lambda_G}{10^{-4}} \right)^{-1/4} \left( \frac{b}{1/3} \right)^{3/4}, \quad (41)$$

with $\beta_G = 6 \times 10^{-4}$ and $\zeta = 2.5$ being used. The latter condition comes from $T_D > 1 \text{ MeV}$, which leads to

$$M_F > 7.19 \times 10^2 \text{ GeV} \left( \frac{\lambda_G}{10^{-4}} \right)^{-5/8} \quad (42)$$
for \( n = 4 \), from Eqs.\,(28) and \,(15). Since Eqs.\,(41) and \,(42) have different dependences on \( \lambda_G \), the latter gives more stringent constraint for \( \lambda_G < 1.13 \times 10^{-7} \left( \frac{b}{1/3} \right)^{1/2} \).

In addition, (e) the new type \( Q \) ball must be stable to be the dark matter of the universe. From Eq.\,(23), \( \omega_Q \sim m_D = bm_N \) is nothing but the upper bound on the gravitino mass:

\[
m_{3/2} < 0.333 \text{ GeV} \left( \frac{b}{1/3} \right).
\]

There is, of course, the condition \,(6), but it is not restrictive, as can be seen below.

In Fig.\,2, we display the B-DM relation \,(38) in the cyan line with \( \lambda_G/\lambda_N = 10^3 \). Also shown are the conditions (a) in red lines, (b) in blue lines, (c) in green lines, (d) in an orange line, and (e) in a brown line. Solid (dashed) lines correspond for \( \lambda_G = 10^{-4} \left( 10^{-9} \right) \). Varying the value of \( \lambda_N \), we obtain the allowed range for B-DM relation in red (blue) hatched region for \( \lambda_G = 10^{-4} \left( 10^{-9} \right) \). Notice that the condition \,(40) depends on \( \lambda_N \) so that the allowed range of \( M_F \) for the B-DM relation \,(38) becomes narrower as \( \lambda_N \) grows larger. This is shown by black lines in the figure.

The allowed region becomes smaller as \( \lambda_G \) gets smaller, and disappears for \( \lambda_G < 1.46 \times 10^{-11} \). One can see that the allowed region appears only for \( 0.404 \text{ MeV} < m_{3/2} < 0.33 \text{ GeV} \), and typically for \( 10^4 \text{ GeV} \lesssim M_F \lesssim 10^7 \text{ GeV} \).

Now we estimate how large the new type \( Q \) ball will be for our successful scenario. The charge of the new type \( Q \) ball is calculated as

\[
Q_N \simeq 1.06 \times 10^{28} \left( \frac{b}{1/3} \right) \left( \frac{\lambda_G}{10^{-4}} \right)^{-1} \left( \frac{m_{3/2}}{0.404 \text{ MeV}} \right)^{-2},
\]

from Eqs.\,(32) and \,(38). We plot the expected region of the charge of the dark matter \( Q \) ball for \( 10^{-4} \lesssim \lambda_G \lesssim 10^{-11} \) in Fig.\,3. We display the observationally excluded region by Baksan in green lines (lower-left region is excluded) \,[25, 27]\,\cite{25, 27}. The recent IceCube experiment reported the limits on subrelativistic magnetic monopoles \,[28], which can
be applied to the $Q$-ball search. Since we do not know the detection efficiency $\eta$, we show the constraints for $\eta = 1 (0.1)$ in solid (dashed) orange lines. We can see that the observation has just stepped into the theoretically expected region, and we can imagine that the dark matter $Q$ balls could be detected directly in the future, which will be a smoking gun for the Affleck-Dine mechanism.

![Diagram](image)

**FIG. 3**: Theoretically expected region for the new type $Q$ ball as dark matter. Iso-$\lambda G$ contours are shown in red lines. Observationally excluded region by the Baksan experiment is shown in green lines, while that by IceCube is shown in orange lines.

### VII. CONCLUSION

We have investigated the variant of the Affleck-Dine mechanism in high-scale inflation. The scenario is that one flat direction is responsible for the baryon asymmetry of the universe, while another explains the dark matter in the gauge-mediated SUSY breaking in high-scale inflation. The former disintegrates into the gauge-mediation type $Q$ balls, which is unstable and decay into baryons. The latter forms the new type $Q$ balls to become dark matter of the universe.

Large isocurvature fluctuations are usually produced in high-scale inflation. In order to suppress them, the flat directions must have amplitudes as large as $M_P$ during inflation. This is achieved by small coupling constants $\lambda$ in nonrenormalizable superpotential (1) such that $\lambda \lesssim 10^{-4}$. We have found that the scenario actually works for two $n = 4$ directions with different sizes of $\lambda$. It is necessary for successful scenario to have $10^{-11} \lesssim \lambda G \lesssim 10^{-4}$. In this case, $10^4 \text{GeV} \lesssim M_F \lesssim 10^7 \text{GeV}$, MeV $\lesssim m_{3/2} \lesssim \text{GeV}$, and $10^{23} \lesssim Q_N \lesssim 10^{31}$. Smallest $Q_N$ region may already be excluded, but the dark matter $Q$ ball with $Q \gtrsim 10^{24}$ could be detected in the future by the IceCube-like detector.

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[1] I. Affleck and M. Dine, Nucl. Phys. B 240, 361 (1985); M. Dine, L. Randall and S. D. Thomas, Nucl. Phys. B 458, 291 (1996).
[2] A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B 418, 46 (1998).
[3] K. Enqvist and J. McDonald, Phys. Lett. B 425, 309 (1998); Nucl. Phys. B 538, 321 (1999).
[4] S. Kasuya and M. Kawasaki, Phys. Rev. D 61, 041301(R) (2000).
[5] S. Kasuya and M. Kawasaki, Phys. Rev. D 84, 123528 (2011);
[6] S. Kasuya, M. Kawasaki and M. Yamada, Phys. Lett. B 726, 1 (2013).
[7] I. M. Shoemaker and A. Kusenko, Phys. Rev. D 80, 075021 (2009);
F. Doddato and J. McDonald, JCAP 1106, 008 (2011); JCAP 1206, 031 (2012); JCAP 1307, 004 (2013).
[8] S. Kasuya, E. Kawakami and M. Kawasaki, arXiv:1202.4067 [hep-ph].
[9] S. Kasuya and M. Kawasaki, Phys. Rev. D 64, 123515 (2001).
[10] M. Laine and M. E. Shaposhnikov, Nucl. Phys. B 532, 376 (1998) [hep-ph/9804237].
[11] S. Kasuya and M. Kawasaki, Phys. Rev. D 89, 103534 (2014).
[12] T. Gherghetta, C. F. Kolda and S. P. Martin, Nucl. Phys. B 468, 37 (1996).
[13] P. A. R. Ade et al. [BICEP2 Collaboration], Phys. Rev. Lett. 112, 241101 (2014).
[14] J. Yokoyama, Astropart. Phys. 2, 291 (1994).
[15] K. Enqvist and J. McDonald, Phys. Rev. Lett. 83, 2510 (1999); Phys. Rev. D 62, 043502 (2000).
[16] M. Kawasaki and F. Takahashi, Phys. Lett. B 516, 388 (2001).
[17] S. Kasuya, M. Kawasaki and F. Takahashi, JCAP 0810, 017 (2008).
[18] K. Harigaya, A. Kamada, M. Kawasaki, K. Mukaida and M. Yamada, arXiv:1404.3138 [hep-ph].
[19] A. de Gouvea, T. Moroi and H. Murayama, Phys. Rev. D 56, 1281 (1997).
[20] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]; arXiv:1303.5082 [astro-ph.CO].
[21] M. Kawasaki and M. Yamada, Phys. Rev. D 87, 023517 (2013).
[22] J. Hisano, M. M. Nojiri and N. Okada, Phys. Rev. D 64, 023511 (2001).
[23] T. Hiramatsu, M. Kawasaki and F. Takahashi, JCAP 1006, 008 (2010).
[24] A. G. Cohen, S. R. Coleman, H. Georgi and A. Manohar, Nucl. Phys. B 272, 301 (1986).
[25] S. Kasuya and M. Kawasaki, Phys. Rev. Lett. 85, 2672 (2000).
[26] S. Kasuya and M. Kawasaki, Phys. Rev. D 75, 023512 (2000).
[27] J. Arafune, T. Yoshida, S. Nakamura and K. Ogure, Phys. Rev. D 62, 105013 (2000);
A. Kusenko, V. Kuzmin, M. E. Shaposhnikov and P. G. Tinyakov, Phys. Rev. Lett. 80, 3185 (1998).
[28] M. G. Aartsen et al. [IceCube Collaboration], arXiv:1309.7007 [astro-ph.HE]; arXiv:1402.3460 [astro-ph.CO].