Photon-assisted electron-hole shot noise in multi-terminal conductors

Valentin S. Rychkov,* Mikhail L. Polianski, and Markus Büttiker
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland
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Motivated by a recent experiment by L.-H. Reydellet et al., Phys. Rev. Lett. 90, 176803 (2003), we discuss an interpretation of photon-assisted shot noise in mesoscopic multiprobe conductors in terms of electron-hole pair excitations. AC-voltages are applied to the contacts of the sample. Of interest are correlations resulting from the fact that electrons and holes are generated in pairs. We show that with two out-of-phase ac-potentials of equal magnitude and frequency, applied to different contacts, it is possible to trace out the Hanbury Brown Twiss exchange interference correlations in a four probe conductor. We calculate the distribution of Hanbury Brown Twiss phases for a four-probe single channel chaotic dot.

I. INTRODUCTION

Theoretical and experimental investigations of the current and charge noise properties of small conductors are an important frontier of mesoscopic physics. The aim is to analyze noise as an additional source of information on the quantum statistical properties of small conductors. Interest in quantum communication and quantum computation have further drawn attention to this subject. We refer the interested reader to reviews, a conference book with extended articles on the subject, a special issue of a journal, and to original work on shot noise in mesoscopic conductors. Investigations have predominantly considered samples subject to stationary applied voltages which drive a dc-current through the sample. The work reported here is motivated by a recent experiment by Reydellet et al. [13] which applies only an ac-voltage to a contact of a two-terminal sample. There is no dc-current linear in voltage. However, the ac-voltage leads to the generation of electron-hole pairs and the members of this pair are subsequently transmitted through the sample or reflected back into the excited contact.

For several reasons, it is of interest to analyze the noise in terms of electron-hole excitations. First, if an ac-voltage is applied to a conductor that is otherwise at equilibrium, electron-hole pairs are in fact the natural elementary excitations of the system. This is in contrast with most of the literature on shot noise which even in the presence of ac-excitations uses an electron picture only. The understanding and interpretation of the results can differ dramatically depending on whether one relies on an all electron picture or on an electron-hole picture. A second reason to investigate electron-hole pair generation is that recently it was realized that such pairs are a source of entanglement, similar to the optical process in which a pair of photons with entangled polarization state is generated through down conversion. Indeed an electron-hole pair creation event leads to an electron and hole who’s spins are entangled. However, it is not only the spin degree which can be useful, but electron-hole sources can be used to generate orbital quasi-particle entanglement. This leads to simple and controllable geometries which permit the investigation of entanglement in electrical conductors without the need of superconducting/semiconductor/ferromagnetic hybrid structures. Indeed the dynamic generation of electron-hole pairs through the periodic modulation of potentials in the interior of conductors or through the application of pulses has recently been discussed.

Fig. 1 shows the type of structures which we analyze in this work. Electron-hole pairs are excited in contact 1. The rate with which carriers are excited is much smaller than the inverse time taken by an electron or hole to be transmitted through the conductor. Fig. 1 depicts a particular scattering event in which the pair is split and the electron leaves the conductor through contact 2 and the hole leaves the conductor through contact 3. An essential point of our work is the fact the correlation of currents between contacts 2 and 3 is entirely determined by scattering events in which an electron-hole pair is split. Other scattering processes, for instance one in which only one
carrier of the pair leaves the conductor through contact 2 and the other is scattered back into contact 1, obviously give no contribution to the current correlation. Thus in the limit of rare excitations of electron-hole pairs, the current-correlation between contact 2 and 3 is a direct measurement of the coincident creation of an electron and hole.

The term 'photon-assisted transport' is applied here in the following sense: For weak coupling of the photon density with the electrons in the lead, the photon field appears in the many-body electron Hamiltonian as a weak periodic potential $V(t)$. We can neglect the feedback of the weak currents on the photon source. Thus in the following, we consider all photon sources as weak ac-bias potentials applied to the leads of the conductor.\(^{21}\)

Photon-assisted current in quantum dots have been experimentally investigated in Ref. [22]. But only a few experiments have thus far investigated the noise properties of photon-assisted transport. Following the theoretical work of Lesovik and Levitov,\(^{23}\) Schoelkopf \textit{et al.}\(^{24}\) illuminated a normal diffusive conductor and found features in the zero-frequency shot noise at the photon energies $V = nh\nu/e$. Theoretical work on hybrid superconducting system by Lesovik \textit{et al.}\(^{25}\) lead similarly to the experimental observation of features at energies $V = nh\nu/2e$ by Kozhevnikov \textit{et al.}\(^{26}\) In both of these experiments a dc-voltage was applied in addition to the radiation. A similar regime for the systems in the fractional quantum Hall state is discussed in Ref. [27]. The theoretical works\(^{23,25}\) consider photon-assisted transport purely as a particle transport problem. However, despite the fact that one measures a dc-current or a zero-frequency noise spectrum, the radiation actually also generates a charge and current response at the excitation frequency. The important role of displacement currents and their rectification effects have been emphasized in the work of Pedersen and Buttiker.\(^{28}\) In the work presented here we will also only consider non-interacting particles and derive expressions for electron and hole currents and their correlations. The role of Coulomb interactions, inelastic scattering and dephasing on the photon assisted noise of electron-hole pairs is treated in the Ref. [29].

In contrast, to the above mentioned experiments, the recent shot noise measurements by Reydellet \textit{et al.} [13] report results for the case that only an ac-voltage is applied to the leads. Reydellet \textit{et al.} [13] also discuss their results in terms of electron-hole excitations assuming that the electrons and holes generate a partition noise independently from each other and add incoherently. As indicated above our interest is in the \textit{correlated} nature of the electron-hole processes and the manifestation of this correlation in shot-noise spectra. We first derive general expressions for electron and hole currents and subsequently apply them to two, three and four terminal conductors. Of particular interest is an arrangement which uses two out-of-phase ac-voltages of the same magnitude and frequency. The phase difference between the applied voltages provides an additional degree of control. It turns out that the exchange contribution to the current correlations\(^{7,8,17,30}\) (the Hanbury Brown Twiss (HBT) correlations\(^{31}\)) are sensitive to the phase difference between the applied voltages. The HBT-correlations reflect indistinguishable two-particle processes in the shot noise.\(^{17}\) Sweeping the phase shift allows to maximize (minimize) shot-noise correlations. The positions of these extrema yield information about combinations of phases of the scattering matrix, which we call the HBT-phase. If the phase-difference of the ac-potentials is set to the HBT-phase, the correlation is maximal. This permits to explore the statistical properties of the HBT-phase. We analyze the distribution of the HBT-phase for a chaotic dot coupled to single channel leads.

The additional degree of control generated by two potentials which are out of phase has its analog in quantum pumping where a scatterer is modulated with two out-of-phase parameters.\(^{32–36}\) Adiabatic quantum pumping uses in an essential way this additional degree of control. The difference is that here we deal with potentials applied to contacts and we analyze a situation in which the frequency to voltage ratio is large.

The paper is organized as follows: In Sec. II we define the geometry of the mesoscopic conductors, present the model assumptions and the theoretical approach used. Sec. III describes the electron-hole picture for photon-excited multiprobe conductors and presents sample-specific results for the shot-noise spectra. Sec. IV discusses an arrangement to measure the Hanbury Brown Twiss exchange interference correlations. We conclude in Sec. V. Appendix A gives the quantum state which leads to the HBT-exchange interference correlations, and Appendix B presents a probability theory argument to show the existence of electron-hole correlations.

\section{II. SCATTERING APPROACH}

We consider a multi-terminal mesoscopic conductor connected to metallic contacts. The leads are subject to time-dependent voltages. We briefly recall the main aspects of the scattering approach and introduce the basic notations. We follow Refs.\(^{28,37}\) who distinguish external potentials (applied to leads) from internal potentials (generated in the interior of the conductor). This distinction is particularly useful to a theory formulated in terms of the scattering matrix: external potentials leave the scattering matrix invariant whereas internal potentials lead to emission and absorption of energy inside the mesoscopic conductor. A realistic description of time-dependent transport involves both external and internal oscillating potentials. Here we focus on the effect of external potentials, keeping the internal potential fixed.

Metallic reservoirs are considered as emitters (absorbers) of charge carriers incident on (exiting from) the mesoscopic conductor. Motion of electrons in the leads can be described by their energy, transverse mode number and direction of momenta. Thus we introduce oper-
ator of incoming and outgoing states: $a_{\lambda}(\varepsilon)$ and $b_{\lambda}(\varepsilon)$ are vectors of operators $a_{\lambda n}(\varepsilon)$ and $b_{\lambda n}(\varepsilon)$. The operator $a_{\lambda n}(\varepsilon)$ annihilates a carrier incident in the $\lambda$-th lead with energy $\varepsilon$ in the $\lambda$-th transverse channel. The operator $b_{\lambda n}(\varepsilon)$ annihilates an outgoing carrier in the $\lambda$-th lead with energy $\varepsilon$ in the $\lambda$-th transverse channel. Here and in the following greek indices run over all contacts of the conductor. For a stationary scatterer the connection between incoming and outgoing carriers is governed by scattering matrices which depend on one energy only. We consider the case of spinless electrons. The scattering matrix transforms the operators $a_{\lambda}(\varepsilon)$ into $b_{\lambda}(\varepsilon)$ according to $b_{\lambda}(\varepsilon) = \sum_\beta S_{\alpha\beta}(\varepsilon)a_\beta(\varepsilon)$. In a multichannel conductor, the matrix $S_{\alpha\beta}(\varepsilon)$ is a sub-block of the scattering matrix $\mathcal{S}$ for scattering from lead $\beta$ ($N_\beta$ channels) to lead $\alpha$ ($N_\alpha$ channels) with energy $\varepsilon$. It has dimensions $N_\alpha \times N_\beta$.

The current operator in contact $\lambda$ is

$$I_\lambda(t) = \frac{e}{2\pi\hbar} \int d\varepsilon_1 d\varepsilon_2 e^{i(\varepsilon_1 - \varepsilon_2)t/\hbar} \sum_{\alpha\beta} a_{\lambda}^\dagger(\varepsilon_1) A_{\alpha\beta}(\lambda, \varepsilon_1, \varepsilon_2) a_{\beta}(\varepsilon_2).$$

Using the projector matrix $\mathbb{1}_\lambda$ which is a unit matrix of size $N_\lambda \times N_\lambda$ in the $\lambda$-th lead, we introduce the current matrix

$$A(\lambda, \varepsilon_1, \varepsilon_2) = \mathbb{1}_\lambda - \mathcal{S}^\dagger(\varepsilon_1) \mathcal{S}(\varepsilon_2).$$

In the following we use the abbreviated notation $A(\lambda, \varepsilon)$ if the energies coincide and $A(\lambda)$ if the energy dependence is unimportant. Electrons obey Fermi statistics. In order to transfer charge across the mesoscopic conductor the incoming charge state has to be filled and at the same time the outgoing state has to be open. A dc-voltage applied across the sample opens an energy window where both conditions are fulfilled and transport is possible.

Consider now a time-dependent potential $\epsilon V_{\alpha}(t) = \epsilon V_{\alpha} \cos(\omega t + \phi_{\alpha})$ applied to the $\alpha$-th lead. This potential can be absorbed in the phase of the wave function. The single particle wave function in the presence of the perturbation is: $\psi_{\alpha\varepsilon}(\varepsilon, t) = \phi_{\alpha\varepsilon}(\varepsilon, t) \exp\{-i \epsilon V_{\alpha} \sin(\omega t + \phi_{\alpha})/\hbar \omega\}$. Here $\phi_{\alpha\varepsilon}(\varepsilon, t)$ is the stationary wave function describing incoming (outgoing) carriers in the $n$-th transverse channel with energy $\varepsilon$. This wave function can be expressed as a series in Bessel functions:

$$\psi_{\alpha\varepsilon}(\varepsilon, t) = \phi_{\alpha\varepsilon}(\varepsilon, t) \sum_l J_l \left( \frac{\epsilon V_{\alpha}}{\hbar \omega} \right) e^{-il(\omega t + \phi_{\alpha})}.$$ 

We see, that wave function $\psi_{\alpha\varepsilon}(\varepsilon, t)$ in Eq. (3) has the same coordinate dependence as $\phi_{\alpha\varepsilon}(\varepsilon, t)$ but in energy space it is a superposition of sideband states with amplitudes $J_l(\epsilon V_{\alpha}/\hbar \omega) \exp\{-il\phi_{\alpha}\}$ and energies $\varepsilon - l\hbar \omega$.

We follow Refs. [28,37] and assume that the oscillating potential exists in a region of the lead between the reservoir and the conductor. The potential vanishes as we approach the conductor. Matching wave functions in the regions with and without oscillating potential leads to

$$a_{\alpha n}(\varepsilon) = \sum_l a'_{\alpha n}(\epsilon - l\hbar \omega) J_l \left( \frac{\epsilon V_{\alpha}}{\hbar \omega} \right) e^{-il\phi_{\alpha}}.$$ 

Here and in the following $a_{\alpha n}(\varepsilon)$ are operators corresponding to annihilation of carrier incident on the conductor, $a'_{\alpha n}(\varepsilon - l\hbar \omega)$ are operators corresponding to annihilation of carriers in the reservoir. This expression has an accuracy of order $\hbar \omega/\epsilon F$. We neglect corrections which arise from the difference of momenta of the particles with different sideband energies $\varepsilon$ and $\varepsilon - l\hbar \omega$.

The reservoir is at thermal equilibrium and the statistical average of the operators $(a'_{\alpha})^\dagger a'$ corresponds to the Fermi distribution of electrons in the absence of the time-dependent voltage: $(a'_{\alpha})^\dagger a' = \frac{\epsilon}{\epsilon - \epsilon_{k,\alpha}}$, where $\epsilon_{k,\alpha}$ are the Fermi energies.

Let us next discuss the noise. At zero frequency, the noise spectral density is defined as follows:

$$S_{\lambda\mu} = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \int_{-\infty}^{\infty} d\tau \langle \Delta I_{\lambda}(t + \tau) \Delta I_{\mu}(t) \rangle.$$ 

In general, in the case of a time-dependent perturbation, the current correlator depends only on the time difference, hence the integral over time $t$ in Eq. (5) is trivial and leads to the stationary state expression for shot noise.\(^1\) In general, in the case of a time-dependent perturbation, the current correlator depends on both times $t$ and $\tau$. One can think about $\tau$ as the time difference and $t$ as the moment at which the measurement starts. In the steady state situation, a shift of the time $t$ by an integer number of periods changes nothing. This is why the averaging over time $t$ can be performed only over one period of the perturbation $T_0 = 2\pi/\omega$.

$$S_{\lambda\mu} = \frac{1}{T_0} \int_0^{T_0} dt \int_{-\infty}^{\infty} d\tau \langle \Delta I_{\lambda}(t + \tau) \Delta I_{\mu}(t) \rangle.$$ 

Below, we will write this current noise in terms of pure electron and pure hole noise contributions and their correlations.

### III. ELECTRON-HOLE PICTURE OF NOISE

In the presence of ac-potentials it is appropriate to consider excitations away from the global equilibrium state and these excitations are electron-hole pairs. Our goal, motivated by the recent experiment of Reydellet et al. [13], is to develop an electron-hole description of photon-assisted shot noise. The creation of an electron-hole pair is a correlated process. These correlations have been used
in proposals for the generation and detection of orbital quasi-particle entanglement.\textsuperscript{15–17} Therefore the manifestations of electron-hole correlations in the noise properties are of particular interest.

To distinguish between different contributions to the shot noise we express the incident states in terms of electron and hole operators,

\[
e_\alpha(e) = a_\alpha(e)\theta_\alpha(e), \quad h_\alpha(e) = a^\dagger_\alpha(e)\theta_\beta(e),
\]

where \(\theta_\alpha(e) = \theta(e)\) and \(\theta_\beta(e) = \theta(\varepsilon)\) are step functions for electrons and holes. Note that \(h_\alpha(e)\) is a transposed vector (row). Equation (6) for the shot noise can now be converted to energy space. Using the Fourier transform of the current operator \(I_\lambda(t) = \int d\varepsilon d\Omega \exp(i\Omega t) I_\lambda(\varepsilon + \Omega, \varepsilon)\) and recalling the fact that the condition (4) implies \(\varepsilon_1 - \varepsilon_2 = \hbar\omega\) for the current operators we obtain the following result for the zero-frequency shot noise:

\[
S_{\lambda\mu} = 2\pi\hbar \int \int d\varepsilon_1 d\varepsilon_2 \langle \Delta I_\lambda(\varepsilon_1)\Delta I_\mu(\varepsilon_2) \rangle.
\] (8)

The current operator \(I_\lambda(\varepsilon) \equiv I_\lambda(\varepsilon, \varepsilon)\) at equal energies can be represented as a sum of electron and hole currents \(I_\lambda(\varepsilon) = I^e_\lambda(\varepsilon) + I^h_\lambda(\varepsilon)\). Using Eq. (7) and Fourier transform of (1) we obtain:

\[
I^e_\lambda(\varepsilon) = -\frac{e}{2\pi\hbar} \sum_{\alpha\beta} e^\dagger_\alpha(\varepsilon) A_{\alpha\beta}(\lambda, \varepsilon) e_\beta(\varepsilon),
\] (9)

\[
I^h_\lambda(\varepsilon) = -\frac{e}{2\pi\hbar} \sum_{\alpha\beta} h^\dagger_\alpha(\varepsilon) A^T_{\alpha\beta}(\lambda, \varepsilon) h_\beta(\varepsilon),
\] (10)

where the electron charge is \(e = -|e|\). It is now interesting to calculate explicitly the correlations between the same or different types of particles:

\[
S_{\lambda\mu} = \sum_{ij=e,h} S_{ij}^{\lambda\mu} = 2\pi\hbar \sum_{ij=e,h} \int \int d\varepsilon_1 d\varepsilon_2 \langle \Delta I^i_\lambda(\varepsilon_1)\Delta I^j_\mu(\varepsilon_2) \rangle.
\]

A non-zero answer for \(S_{ij}^{\lambda\mu}\) will show us the presence of intrinsic correlations between electrons and holes. Using the expressions for the currents (9), (10) and Eqs. (4) and (7) we find the correlations of the electron and the hole currents:

\[
S_{ij}^{\lambda\mu} = \frac{e^2}{2\pi\hbar} \sum_{k\lambda,\mu,\beta} \int \int d\varepsilon_1 d\varepsilon_2 \delta(\varepsilon_1 - \varepsilon_2 + m\hbar\omega)
\]

\[
\times J_k \left( \frac{eV_\alpha}{\hbar\omega} \right) J_{k+m} \left( \frac{eV_\beta}{\hbar\omega} \right) J_1 \left( \frac{eV_\gamma}{\hbar\omega} \right) J_1 \left( \frac{eV_\delta}{\hbar\omega} \right)
\]

\[
\times e^{im(\phi - \phi_0)} f_\alpha(\varepsilon_2 - k\hbar\omega)(1 - f_\beta(\varepsilon_2 - \hbar\omega))
\]

\[
\times \text{tr} (A_{\alpha\beta}(\lambda, \varepsilon_1) A_{\beta\alpha}(\mu, \varepsilon_2)) \theta_\alpha(\varepsilon_1) \theta_\beta(\varepsilon_2).
\] (11)

Here \(i = e, h\) stands for electron or hole. For example, in the limit of in-phase applied voltages the sum of Eq. (11) over electron and hole indices gives the result which for a two-probe sample with energy-independent scattering matrix can be found in Ref. [23] and for a multi-probe conductor is given in Ref. [28].

To probe correlations of electron-hole pairs we impose two conditions. First, the ac-potentials are taken to be weak enough in order to exclude processes of multiple absorption or emission of photons which implies \(eV_\alpha \ll \hbar\omega\). As a consequence, electron-hole pairs are generated one by one. In this case one can prove that the many body wave function of the system will correspond to incoming single electron-hole pairs (see [16,18] and Appendix A). Second, electrons and holes have different energies (the typical energy scale of an electron-hole pair is \(\hbar\omega\)). We neglect the energy dependence of scattering matrices on the scale of \(\hbar\omega\). The energy dependence of the scattering matrix starts to play a role when the frequency becomes comparable to the inverse of the dwell time \(\tau_d\) such that \(\omega\tau_d \sim 1\). When screening is taken into account the frequency must even be comparable to the inverse charge relaxation time.\textsuperscript{29} As a consequence of these conditions, the correlations of \(ee\) and \(hh\), as well as \(eh\) and \(he\) correlations, are identical. Neglecting corrections of the order of \((eV/\hbar\omega)^4\), we find the correlations between electron and hole currents:

\[
S_{\lambda\mu}^{ee} = -\frac{e^2\omega}{2\pi} \text{tr} A(\mu) P A(\lambda),
\] (12)

\[
S_{\lambda\mu}^{hh} = -\frac{e^2\omega}{2\pi} \text{tr} A(\mu) \sqrt{P} e^{i\phi} A(\lambda) \sqrt{P} e^{-i\phi}.
\] (13)

Similarly to the experimental work\textsuperscript{13} we introduce the probability to create an electron-hole pair in the \(\alpha\)-th channel, \(P_\alpha = (eV_\alpha)^2/(2\hbar\omega)^2\) and diagonal matrices \(P = \text{diag}(P_1,...,P_N)\) and \(\phi = \text{diag} (\phi_1,...,\phi_N)\) are defined by the amplitudes and phases of applied voltages. We next consider a number of different set-ups.

Consider a multi-lead conductor and consider an alternating voltage applied to only one lead, say lead 1. The probability to create an electron-hole pair is \(P_1 = (eV_1/2\hbar\omega)^2\). All other leads are grounded. The auto-correlations and cross-correlations are

\[
S_{\lambda\mu}^{ee} = -\frac{e^2\omega}{2\pi} P_1 \text{tr} A(\mu) I_1 A(\lambda),
\] (14)

\[
S_{\lambda\mu}^{hh} = -\frac{e^2\omega}{2\pi} P_1 \text{tr} A(\mu) I_1 A(\lambda) I_1.
\] (15)

Next consider the 2-terminal case. Charge current conservation and particle (electron and hole) current conservation imply that the auto-correlations and the cross-correlations are equal in magnitude and differ only by a sign. Thus it is sufficient to give the auto-correlations,

\[
S_{11}^{ee} = -\frac{e^2\omega}{2\pi} P_1 \sum_n T_n, \quad S_{11}^{hh} = -\frac{e^2\omega}{2\pi} P_1 \sum_n T_n^2.
\] (16)

Here \(T_n\) is transmission probability of the \(n - \text{th}\) eigenchannel, \(i.e.\) an eigenvalue of \(S^1 I_2 S^1\). Summing up all
four terms gives a shot noise of the total measured charge current proportional to \( \sum_n T_n(1 - T_n) \). However, unlike the case of a dc-biased conductor, where the \( T_n(1 - T_n) \) is proportional to the quantum partition noise of a *fully filled* incident channel, here \( T_n(1 - T_n) \) has a completely different origin. The auto-correlations of the electron \( S^{ee}_{11} \) and hole noise \( S^{eh}_{11} \) are simply proportional to \( T_n \), reflecting Poissonian shot noise of a nearly *empty channel* of incident particles. Electron and hole particle currents are correlated and it is this two particle correlation which is proportional to \( T_n^2 \). The existence of electron-hole correlations can also be proved using a simple probability theory argument, which is demonstrated in Appendix B.

In order to make a direct measurement of electron-hole correlations we now analyze cross-correlations in three terminal mesoscopic conductors (see Fig. 1). For a weak ac-perturbation electron-hole pairs are rarely injected into the mesoscopic conductor. If we measure the current-correlation at the grounded leads, then if electron and hole exit through the same lead or if one of the particles of the pair is reflected back then their contribution to the cross-correlation is zero. Thus current cross-correlations are determined only by electron-hole pairs for which one of the particles exits through contact 2 and the other through contact 3 (see Fig. 1). In fact a cross-correlation measurement is a coincidence measurement run over long times.\(^{16}\)

For a three terminal conductor we find for the particle cross-correlations,

\[
S^{eh}_{23} = -\frac{e^2\omega}{2\pi} P_1 \sum_n \frac{T_{21n}T_{31n}}{2}, \quad S^{ee}_{23} = 0. \tag{17}
\]

Here \( T_{21n} \) and \( T_{31n} \) are transmission eigenvalues of \( S^\dagger \mathbb{1}_2 S \) and \( S^\dagger \mathbb{1}_3 S \). This demonstrates that the current-cross-correlations are a direct measure of electron-hole correlations.

The considerations made above are valid at zero-temperature. It is thus important to consider the effect of thermal noise. After all, thermal noise can be viewed as another mechanism which generates electron-hole pairs. It should not matter how exactly electron-hole pairs are created. From the above consideration we see that electron-hole correlations are of second order in the transmission probability both for auto-correlations and for cross-correlations. We know from FDT that equilibrium noise is proportional to conductance and hence it is proportional to transmission probability. Consequently we do not expect any electron-hole correlations in thermal equilibrium.

Equilibrium noise is given by Eq. (11); only the terms \( k = l = m = 0 \) contribute at \( V = 0 \). In Eq. (11) the expression \( \theta_\epsilon(\epsilon_1)\theta_\epsilon(\epsilon_2)\delta(\epsilon_1 - \epsilon_2) \equiv 0 \), so the only contribution to the equilibrium noise comes from electron-electron and hole-hole correlations, both are proportional to the dc-conductance matrix \( G_{\lambda \mu} = (e^2/\hbar)(N_\lambda\delta_{\lambda \mu} - \epsilon_\lambda\delta_{\lambda \mu}) \). The existence of these terms is the quantum mechanical indistinguishability of particles. A particle from source contact 1 can be transmitted to either 3 or 4 and is indistinguishable form a particle injected through contact 2 (see Fig 2). The quantum state is given explicitly in Appendix A.

This exchange interference effect reflects the fact that two particles can be simultaneously in the conductor. It has long been proposed that this can be used to generate a two-particle Aharonov-Bohm effect which exists only in the current correlations while under the same conditions conductance exhibits no Aharonov-Bohm effect at all.\(^{8}\) Recently a geometry which demonstrates this explicitly has been proposed and analyzed.\(^{17}\) The magnitude of the current cross-correlations is shown to depend on the phase

\[
\chi = \arg \left( S^\dagger_{13} S^\dagger_{32} S^\dagger_{24} S_{41} \right), \tag{19}
\]

![FIG. 2: Electron-hole pairs originating from the lead 1 or 2 contribute to cross-correlations measured in 3 and 4. Indistinguishability of the sources leads to the dependence of the shot noise \( S_{44} \) on the relative phase \( \Delta \phi \) of the voltage sources](image-url)
which we call Hanbury Brown Twiss (HBT) phase.

For simplicity, let us consider a four terminal mesoscopic conductor with one mode contacts. AC-voltages of equal magnitude are applied to leads 1 and 2 and generate electron-hole pairs with probability $P = (eV/2\omega)^2$. The phase difference between the oscillating potentials $V_1(t)$ and $V_2(t)$ is $\Delta \phi$. Using (12) we calculate the charge current correlation at contact 3 and 4:

$$S(\Delta \phi) = -\frac{e^2\omega}{2\pi} P \left| S^{\dagger}_{13} S_{41} e^{-i\Delta \phi} + S^{\dagger}_{23} S_{42} \right|^2. \quad (20)$$

Once created in the lead 1 or 2 an electron-hole pair contributes to the shot noise only if it is split to the leads 3 and 4. The relative phase of pairs injected from 1 and 2 is $\Delta \phi$. Indistinguishability of particles emitted from the sources 1 and 2 leads to the phase dependence of the shot noise. From Eq. (20) we calculate the phase shift $\Delta \phi$ which corresponds to the maximum or minimum in the shot noise, $\phi_\pm$, and the extremal values of the cross-correlations $S^\pm$:

$$\Delta \phi_+ = \text{arg} \left( \langle S^{\dagger}_{13} S_{32} S^{\dagger}_{24} S_{41} \rangle \right), \quad \Delta \phi_- = \Delta \phi_+ + \pi. \quad (21)$$

$$S^\pm = -\frac{e^2\omega}{2\pi} P \left| \langle S^{\dagger}_{13} S_{41} \rangle \pm \langle S^{\dagger}_{23} S_{42} \rangle \right|^2. \quad (22)$$

Notice, that when the phase shift becomes equal to the HBT-phase of Eq. (19), $\Delta \phi = \chi$, the shot noise reaches its maximum value $S^+$. For a dc-biased 4 terminal conductor one can extract information about $\Re \langle S^{\dagger}_{13} S_{32} S^{\dagger}_{24} S_{41} \rangle$ using results of three different measurements.\cite{9,30} In contrast, the ac-perturbation gives an additional degree of freedom to vary the phase shift between the voltages. Periodic dependence of the noise on the phase shift allows us to reproduce the whole function $S(\Delta \phi)$ using measurements at three different values of the phase shift $\Delta \phi$. Maximal $S^+$ and minimal $S^-$ values of the shot noise, as well as the corresponding phase shifts $\Delta \phi_{1,2}$, yield not only the real part of $\langle S^{\dagger}_{13} S_{32} S^{\dagger}_{24} S_{41} \rangle$ but also the full exchange interference correlation, i.e. its imaginary part.

We next investigate the statistical properties of the HBT-phase of four-terminal conductors. First, it is useful to mention that especially simple structures like a Mach-Zehnder (MZ) interferometer which was recently realized in a 2DEG\cite{40} exhibit only a trivial HBT-phase. A Mach-Zehnder interferometer has the property that there exists only forward scattering, i.e. every incoming particle is transmitted in one of two output arms. As a consequence the transmission sub-matrix is a unitary matrix itself. It follows that $S_{41} S^{\dagger}_{13} + S_{42} S^{\dagger}_{23} = 0$. Here 1 and 2 are contacts under ac-excitation and 3 and 4 are measurement contacts. As a consequence the HBT-phase is given by $\chi = \pi$. In particular there are no correlations if two in-phase voltages are applied to two input contacts of a Mach-Zehnder interferometer.

We now investigate the statistical properties of current cross-correlations and HBT-phase in the chaotic quantum dots, connected to four single channel leads. Chaotic scattering inside the dot leads to substantial back-scattering and we expect therefore nontrivial HBT-phase behavior. Scattering properties of an open quantum dot are very sensitive to external conditions such as shape (which can be changed by gate voltages), impurity distribution or applied magnetic fields. This allows one to explore the total ensemble of quantum dots of a proper symmetry (presence or absence of time-reversal symmetry, TRS). Current cross-correlations and HBT-phase are complicated functions of transmission amplitudes and phases of the scattering matrix elements, so the distributions can be obtained only by numerical integration.

Using convenient ‘polar decomposition’ of the matrix $S$:

$$S = \begin{pmatrix} u' & 0 \\ 0 & v' \end{pmatrix} \begin{pmatrix} \sqrt{1-T} & i\sqrt{T} \\ i\sqrt{T} & \sqrt{1-T} \end{pmatrix} \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix}, \quad (23)$$

where $u, v, u', v'$ are unitary $2 \times 2$ block matrices ($u' = u^T, v' = v^T$ in time-reversal symmetric case) and $T = \text{diag}(T_1, T_2)$ with $T_1 \in [0, 1]$ distributed according to the relevant symmetry class $\beta = 1(2)$ with (without of) the TRS,\cite{38} we express the noise correlation $S$ of Eq. (20) in the form,

$$S(\Delta \phi) = -\frac{e^2\omega}{2\pi} P (v \sqrt{T} u \exp(i\sigma_{\Delta \phi} u^\dagger \sqrt{T} u)) 12| 12. \quad (24)$$

Numerical integration over random matrix ensembles gives the mesoscopic distribution $P_\beta(\chi)$, $\beta = 1, 2$ of the HBT-phase Fig. 3 presents the results for these distributions $P_\beta(\chi)$ in the range $0 \leq \chi \leq \pi$, since they are symmetric with respect to $\chi \rightarrow -\chi$. The distribution $P_1(\chi)$ has peaks at $\chi = 0, \pi$, and the distribution $P_2(\chi)$ is monotonic for $0 \leq \chi \leq \pi$. Below we present a qualitative explanation of these differences.

The matrices $u, v$ are randomly distributed over the unitary group, so the distributions $P_\beta(\chi)$ differ only because of different joint distributions of the transmission eigenvalues, $P_\beta(T_1, T_2)$. If one analyzes the distribution $P_\beta(\chi)$, two simple limits are readily calculated. First, take the limit $T_1 = T_2$ (unreachable for chaotic quantum dots, since the joint distribution is $P_\beta(T_1, T_2) \propto |T_1 - T_2|^\beta (T_1 T_2)^{-1+\beta/2}$, but still instructive). It is easy to see that in this case $\chi = \pi$. One would expect that for sufficiently close $T_1, T_2$ the phase $\chi$ does not differ much from $\chi = \pi$ (for the Mach-Zender interferometer $T_1 = T_2 = 1$ and the HBT-phase $\chi$ is locked at $\pi$). The relative statistical weight of this limit in the mesoscopic distribution $P_\beta(\chi)$ might be characterized by the average of $\sqrt{T_1 T_2}/|T_1 - T_2|$, this average is $3/4$ for $\beta = 1$ and $4/5$ at $\beta = 2$. This could explain why $P_1(\pi)$ is slightly smaller then $P_2(\pi)$.

Another example which provides some understanding is the case of $T_1 = 0$ and arbitrary $T_2$. At $\Delta \phi = 0$ the noise $S$ reaches its maximal value $|S^+| = e^2\omega P/\pi$, so that $\chi = 0$. The statistical weight of such cases could
be characterized by the average of \( \sqrt{T_1/T_2} \) which is divergent for \( \beta = 1 \) and finite for \( \beta = 2 \). Thus for \( \beta = 1 \) a peak at \( \chi = 0 \) could be expected. The monotonic almost uniform \( P_2(\chi) \) is hard to explain. We expect that in the multi-channel limit \( N \to \infty \), when the transmission value distribution reaches its symmetry-insensitive shape \( P(T) \propto 1/\sqrt{T(1-T)} \), the differences between \( \beta = 1, 2 \) are washed out and the distributions \( P_1(\chi) \) become uniform. Possibly, the distribution \( P_2(\chi) \) is much more uniform then \( P_1(\chi) \) in the four-mode quantum dot because the \( S \) matrix is characterized by a much larger number of independent variables (16 for \( \beta = 2 \) vs. 10 for \( \beta = 1 \)). The complicated behavior of the distribution of the HBT-phase \( P_3(\chi) \) can be contrasted with that of conductance or concurrence,\(^{39}\) which depend only on transmission eigenvalues of channels.

We next consider the mesoscopic distribution of the maximal (minimal) values of the noise correlations \( S \), which could be reached in an experiment by tuning the phase shift \( \Delta \phi \) between applied voltages. Using numerical integration, we find a distribution of \( S^+ \) (main figure in Fig. 4) and \( S^- \) (shown in the inset), normalized by a factor \( e^2 \omega P/2\pi \). From the numerical integration we conclude that the distribution of the maximal values \( S^- \) diverges at small arguments as \( P(S^-) \propto (S^-)^{-1/2} \) for both \( \beta = 1, 2 \). The monotonic distributions of \( S^- \) displayed in the inset quickly decay, and the averaged values of \( S^-_\beta \) are \( S^-_1 = 0.022, S^-_2 = 0.027 \). The distributions of \( S^+ \) are broad, as expected from mesoscopic distributions in few-channel systems, and the averaged values are also comparable, \( S^+_1 = 0.121, S^+_2 = 0.173 \).

![FIG. 3: Mesoscopic distribution \( P_3(\chi) \) of HBT-phase \( \chi \) that maximizes the value of cross-correlations \( S_{34} \) in quantum dots. Angle \( \chi \) is normalized by \( \pi \) and \( P_3(-\chi) = P_3(\chi) \). The difference between the presence of TRS (solid) and absence of TRS (dashed) is due to different distributions \( P_3(T) \) in Eq. (23).](image1)

![FIG. 4: Distribution functions of \( S^+ \) and \( S^- \) (inset) for quantum dots with (solid) and without (dashed) time-reversal symmetry. Mesoscopic averages are similar in both cases, but the fluctuations are large. At small arguments \( S^- \), the distributions on the inset are divergent as \( P(S^-) \propto 1/\sqrt{S^-} \).](image2)

V. CONCLUSIONS

We constructed the scattering theory for electron-hole transport in mesoscopic systems photon-excited at contacts. In the limit of a weak ac-excitation we investigate the correlation between electrons and holes. In different geometries signatures of such correlations appear in a different way. In a two terminal mesoscopic conductor subjected to a weak ac-voltage at one of the contacts, the electron-hole correlation effect in the shot noise co-exists with electron-electron correlations. In a three terminal geometry we investigate the correlations between currents at different terminals and find that they are pure electron-hole correlations.

In contrast to dc-biased systems, in the ac-biased systems investigated here we can vary not only the magnitude of the applied voltages but also the phases of the applied voltages. This provides additional controllable parameters. Shot noise measurements in a four terminal mesoscopic conductor provide information about two particle exchange interference in the sample. The shot noise depends on the relative phase between applied voltages, because the particles from different sources are indistinguishable.

We illustrate our theory on the example of a four probe chaotic dot coupled to four single channel leads. At two leads the conductor is subject to ac-voltages. When the phase shift \( \Delta \phi \) of the applied voltages coincides with the HBT-phase \( \chi \) of the sample, the correlations reach their maximal value \( S^+ \). One might expect the phase to be uniformly distributed. However the quantum dot coupled to single channel leads exhibits a strongly non-uniform mesoscopic distributions \( P(\chi) \) of the HBT-phase and of \( P(S^\pm) \) of the extremal values \( S^\pm \). Recent advances in high-frequency measurement techniques\(^{33,23,26}\)
will make it possible to measure these distributions. The close link between the two-particle HBT-effect$^{17}$ and quasi-particle entanglement$^{15,17}$ and recent proposals for dynamic generation of quasi-particle entanglement$^{18-20}$ make such experiments highly desirable.

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APPENDIX A: QUANTUM STATE AND EXCHANGE INTERFERENCE

In this appendix we construct the quantum state which leads to the Hanbury Brown Twiss exchange interference effect discussed in Sec. IV. We consider the conductor shown in Fig. 2 with four one channel leads. In particular we want to show that only electron hole pairs which are created in either lead 1 or 2 and are subsequently split into lead 3 or 4 generate the HBT-exchange interference term.

Each state incident from a reservoir subject to an oscillating voltage can gain or lose modulation energy quanta. The many particle state incident from the $\alpha$-th reservoir, $\prod_{\epsilon<0} a_\alpha^\dagger(\epsilon)|\rangle$, can be transformed using Eq. (4) into the state $|\text{in}⟩$ incident on the mesoscopic conductor. $|\rangle$ is the true vacuum. Taking only the first sidebands into account, we find

$$|\text{in}⟩ = \prod_{E<0} \left( a_\alpha^\dagger(\epsilon) + e^{i\phi}\frac{a_\alpha^\dagger(\epsilon^+)}{\hbar^2} + e^{-i\phi}\frac{a_\alpha^\dagger(\epsilon^-)}{\hbar^2} \right) |\rangle.$$  \hspace{1cm} (A1)

Here $V_\alpha^{\pm} = eV_\alpha e^{\pm i\phi_\alpha}/2\hbar\omega$ is the probability amplitude for the creation of an electron-hole pair in the $\alpha$-th lead and $e^{\pm} = \epsilon \pm \hbar\omega$. Using the commutation rules of free fermion operators we find the state incident on the four-terminal mesoscopic conductor in the presence of oscillating potentials at contacts 1 and 2:

$$|\text{in}⟩ = |0⟩ + \int_0^{\hbar\omega} d\epsilon a_\alpha^\dagger(\epsilon)\tilde{V}a(\epsilon^-)|0⟩.$$  \hspace{1cm} (A2)

Here the vacuum state $|0⟩ = \prod_{\epsilon<0,\alpha=-13} a_\alpha^\dagger(\epsilon)|\rangle$ is the filled Fermi sea at equilibrium in all four leads. The second term on the r.h.s. of Eq. (A2) represents the superposition of the electron-hole pairs coming from leads 1 and 2. The matrix $\tilde{V}$ is diagonal $\tilde{V} = \text{diag}(eV_1 e^{i\Delta\phi}/2\hbar\omega, eV_2/2\hbar\omega, 0, 0)$. Using the relation between incoming and outgoing states $a_\alpha = \sum_\beta S_\alpha^\dagger S_\beta b_\beta$ we find for the outgoing state:

$$|\text{out}⟩ = |\bar{0}⟩ + \int_0^{\hbar\omega} d\epsilon b_\alpha^\dagger(\epsilon)\tilde{S}\tilde{V}\tilde{S}^\dagger b(\epsilon^-)|\bar{0}⟩.$$  \hspace{1cm} (A3)

Here the new vacuum $|\bar{0}⟩ = \prod_{\epsilon<0,\alpha=-13} \text{det}(\tilde{S})b_\alpha^\dagger(\epsilon)|\rangle$ describes the equilibrium state in the basis of out-going states.

The many-particle state (A3) contains information about the final states in all the leads. We are specifically interested in the correlation of currents at leads 3 and 4 for a conductor subject to oscillating potentials at contacts 1 and 2. Leads 3 and 4 are grounded and at zero temperature there are no particles injected into the mesoscopic conductor through these leads. Thus the cross-correlations $S_{34}$ are determined only by that portion of the state (A3) which describes out-going particles in lead 3 and 4,

$$|\text{out}⟩_{34} = \frac{eV}{2\hbar\omega} \int_0^{\hbar\omega} d\epsilon \langle...|\bar{0}⟩,$$  \hspace{1cm} (A4)

$$[...] = \left(e^{i\Delta\phi/2}S_{31}^b S_{13}^b + S_{32} S_{23}^b \right) b_3^\dagger(\epsilon)b_4(\epsilon^-) + \left(e^{-i\Delta\phi/2}S_{31} S_{13}^b + S_{32} S_{23}^b \right) b_3(\epsilon)b_4^\dagger(\epsilon^-).$$  \hspace{1cm} (A5)

An excitation with energy above zero corresponds to an electron $b_3^\dagger(\epsilon) = e_3^\dagger(\epsilon)$ and an excitation with energy below zero to a hole $b_4(\epsilon^-) = h_4^\dagger(\epsilon^-)$. Thus the first term in Eq. (A4) represents a superposition of amplitudes for an electron-hole pair created in contact 1 or contact 2 with the electron leaving through 4 and the hole through contact 3. It is an orbitally entangled electron-hole pair state and the index of the source contact 1 and 2 is a pseudo-spin index. Similarly the second term represents orbitally entangled electron-hole pairs with the electron leaving through contact 3 and the electron through contact 4. In fact, from a formal point of view the state is identical to the one created by two oscillating potentials acting in spatially separated interior regions of a conductor investigated in Ref. [18]. One might think that the oscillating potentials in contacts 1 and 2 serve to mark particles created in these contacts, since they carry the phase factor $\exp(-i\phi_1)$ and $\exp(-i\phi_2)$ and thus to make them distinguishable. However, also for the applied oscillating voltages it is only the phase difference $\Delta\phi = \phi_2 - \phi_1$ that counts. As a consequence, the phase of the oscillating voltages only modulates the HBT-interference but does not destroy it.

We now show how the different terms in the quantum state Eq. (A4) contribute to the HBT-current-correlation between leads 3 and 4. This correlation can now be derived by using either the full state (A3) or the state (A4) which contains the out-going particles in leads 3 and 4 only. The results of these two calculations are of course identical. Using Eq. (8) we find:

$$S_{34} = 2\pi\hbar \int d\epsilon d\epsilon' \langle\text{out}|\Delta I_3(\epsilon)\Delta I_4(\epsilon')|\text{out}⟩.$$  \hspace{1cm} (A6)

Since leads 3 and 4 are grounded and there are no particles injected into the mesoscopic conductor through these leads, the current operators in 3 and 4 can be written in
terms of operators of the outgoing states only:

\[ I_\alpha(\varepsilon) = -\frac{e}{2\pi\hbar} b_\alpha^\dagger(\varepsilon) b_\alpha(\varepsilon). \]

Since \( \hbar\omega > \epsilon > 0 \), the operators \( b_\alpha^\dagger(\varepsilon - \hbar\omega) \) and \( b_\beta(\epsilon) \) acting to the right on the vacuum give zero. Applying Wick’s theorem to the quantum mechanical average in Eq. (6) and performing the integration over the energies we find:

\[ S_{34} = \frac{e^2}{2\pi} \int d\varepsilon d\varepsilon' \int_0^{\hbar\omega} d\varepsilon d\varepsilon' \sum_{\alpha,\beta,\gamma,\delta} \left[ S V^\dagger S^\dagger \right]_{\alpha\beta,\gamma\delta} \left[ S V S^\dagger \right]_{\gamma\delta} \langle \hat{0} | b_\alpha^\dagger(\varepsilon^-) b_\beta(\varepsilon) b_\delta(\varepsilon) b_\gamma^\dagger(\varepsilon') b_\delta^\dagger(\varepsilon') b_\gamma(\varepsilon') \hat{0} \rangle. \quad (A6) \]

The Poissonian distribution \( \rho_\lambda(N) = \lambda^N \exp(-\lambda)/N! \). A barrier of transparency \( T \) separates the left and right contacts of the conductor. The probability that \( m \) electrons and \( n \) holes out of \( N \) electron-hole pairs are transmitted from the left to the right contact is

\[ P(m, n|N) = \binom{m}{N} \binom{n}{N} T^{m+n}(1-T)^{2N-(m+n)}. \quad (B1) \]

The limits of summations result from the fact that to create the current \( j \) one needs at least \( j \) electrons out of \( N \geq m \) electron-hole pairs. For \( \lambda \ll 1 \) which we assume from now on, the leading contribution to Eq. (B2) is

\[ P(j) \approx \frac{(\lambda T(1-T))^j}{j!}, \quad (B3) \]

and thus \( \langle j^{2k+1} \rangle = 0 \), \( \langle j^{2k} \rangle \approx 2\lambda T(1-T) \). We conclude that the noise equals \( 2\lambda T(1-T) \). From comparison with the scattering theory, summing up the Eqs. (16), we find \( \langle j^2 \rangle = (eV/2\hbar\omega)^2 T(1-T) \), which allows us to identify \( \lambda = (eV)^2/(8\hbar\omega)^2 \ll 1 \).

If we now consider the distribution of the electron current \( j = j_e \) and hole current \( j = j_h \) separately, we find

\[ P(j) = \sum_{N=j}^{\infty} \frac{N! \rho_N}{j!(N-j)!} T^j(1-T)^{N-j} = \rho_\lambda T(j). \]

The distribution of the current \( j \) in the r.h.s. is a Poissonian distribution characterized by the parameter \( \lambda T \). However, since now the total measured current is the difference between two independent Poissonian processes, it is not a Poissonian process itself (only for two independent Gaussian processes is their difference also a Gaussian process). For \( j = j_e - j_h \) we find

\[ P(j) = I_{ji}(2\lambda T) \exp(-2\lambda T), \quad (B4) \]
with $I_{j|}(x)$ the modified Bessel function of the $j|th$ (integer) order. Thus we find $\langle j^2 \rangle = 2\lambda T + O((\lambda T)^2)$, unlike the correct result $2\lambda T (1 - T)$ found from the distribution Eq. (B3).