Shrinkage Deformations of Composite Slabs with Open Trapezoidal Sheeting

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Abstract

Composite steel-concrete slabs that have integral steel shuttering at the soffit of a reinforced concrete slab are commonplace in building infrastructure. Inevitable shrinkage of the concrete represents an indirect (or non-mechanical) strain that results in deformations of the composite slab, resulting in long-term effects in which creep of the concrete also plays a role. Because the impervious steel sheeting prevents moisture egress at the slab soffit whereas at the top of the slab such moisture egress can occur, there is a variation of the shrinkage strain through the depth of the slab, resulting in warping-type deformations. In addition, the composite action between the steel and concrete results in partial shear interaction between these elements, and this interacts with the shrinkage response. Surprisingly little appears in the published literature on rational modelling of the behaviour of composite slabs incorporating partial interaction that are subjected to shrinkage straining, despite its practical importance and significance. Based on fundamental principles of mechanics, a theoretical model of this shrinkage behaviour that includes partial interaction is developed, and prescriptive equations are derived. These may be used to assess the deflections and stresses in the concrete due to restrained shrinkage which may lead to cracking of the slab. It is also shown that the two effects of partial interaction and shrinkage straining counteract each other when slab deflections are considered.

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Keywords: Composite slabs; Partial interaction; Shrinkage straining; Warping; Time-effects

1. Introduction

Composite slabs having a thin-walled profiled steel deck at the soffit of a reinforced concrete slab are used widely in steel and composite framed buildings, acting as one-way slabs of the type shown in Figure 1, or as two-way slabs in which they are composite with steel beams with headed stud shear connectors as shown in Figure 2. The focus of this paper is on one-way slabs (Figure 1). The shear connection in this type of slab is known as rib-shear connection (Oehlers and Bradford 1995), with the embossments in the sheeting, chemical bonding and aggregate interlock contributing to provide longitudinal shear resistance at the...
steel/concrete interface that enhances the strength and stiffness of the composite slab above that of the reinforced concrete component alone. Re-entrant profiles (Figure 1) have found traditional use for the steel decking, but deep trapezoidal profiles are now being utilised because of their ability to span large distances, providing an economic design solution.

Figure 1: One-way composite profiled deck.  

Although design codes provide guidance for the strength design of composite slabs (Bode and Sauerborn 1993, Johnson 2004), the important issue of service deformations caused by shrinkage straining has not been addressed to the extent to which designers can make useful quantitative assessments. Indeed, neither the Australian composite structures code AS2327 (SA 2003) nor the Australian concrete structures code AS3600 (SA 2009) consider the issue, and many designers have incorrectly applied the deflection rules in AS3600 for shrinkage in composite slabs.

After casting the concrete on the steel deck, the development of drying shrinkage is not uniform through the depth of the slab because moisture egress at the soffit through the impervious steel layer is retarded relative to the top of the slab, resulting in a gradient of the shrinkage strain $\varepsilon_{sh}$ through its depth. This phenomenon can lead to “shrinkage warping” which exists in reinforced concrete members because of the eccentric placement of the reinforcement relative to the centroid of the cross-section. Composite slabs also exhibit partial interaction (Oehlers and Bradford 1995) and so the stiffness of the shear connection between the concrete and steel decking needs to be considered at service load levels, since it is well-known that restraint in a reinforced concrete slab has considerable influence on its time-dependent behaviour. Surprisingly little research has addressed these two issues in composite slabs, apart from a generic non-mechanical straining model proposed recently by the author (Bradford 2010). Because of this, the current paper proposes a technique of analysis for composite slabs that leads to a description of the service-load behaviour in closed form. The equations developed can be used for design purposes, providing insight into this important grey area of structural mechanics.

2. Material properties

The concrete is treated as being uncracked throughout, with its time-varying response being developed by drying shrinkage and creep. Because the decking provides an impervious surface and the top of the concrete is exposed to the ambient environment, the shrinkage strain $\varepsilon_{sh}$ necessarily varies through the depth of the slab. Concrete creep, on the other hand, is taken without loss of generality to be thickness-independent. Herein, the total strain in the concrete $\varepsilon_c$ is taken as the sum of its instantaneous value $\varepsilon_i$ and the time-varying creep ($\varepsilon_{cr}$) and shrinkage ($\varepsilon_{sh}$) strains, so that
\[ \varepsilon_c(t,t_0) = \varepsilon_i + \varepsilon_{cr}(t,t_0) + \varepsilon_{sh}(t), \]

in which \( t = t_0 \) is the time at first loading. The mechanical or stress-producing concrete strain is \( \varepsilon_{cm}(t,t_0) = \varepsilon_c(t,t_0) - \varepsilon_{cr}(t,t_0) - \varepsilon_{sh}(t) \), resulting in

\[ \varepsilon_{cm}(t,t_0) = \frac{\varepsilon_c(t,t_0) - \varepsilon_{sh}(t)}{1 + \phi(t,t_0)}, \]

in which \( \phi(t,t_0) = \varepsilon_c(t,t_0)/\varepsilon_i \) is the creep coefficient. The time-dependent concrete stress \( \sigma_c \) is then

\[ \sigma_c(t,t_0) = E_c(t_0)\varepsilon_{cm}(t,t_0) = E_c(t_0)\left[\varepsilon_c(t,t_0) - \varepsilon_{sh}(t)\right] \]

in which \( E_c(t,t_0) = E_c(t_0)/(1 + \phi(t,t_0)) \) is the effective modulus and \( E_c(t_0) \) the short-term elastic modulus of the concrete. Empirical models for the shrinkage strain and creep coefficient are reported widely in design codes and elsewhere, but useful data for both the magnitude and depth-wise distribution of \( \varepsilon_{sh}(z) \) are hitherto unavailable in the open literature, with current experimental research at The University of New South Wales focusing on enhancing this knowledge. Typical shrinkage and creep data obtained from the ACI Standard (1982) representations are

\[ \phi(t,t_0) = \left[\frac{(t-t_0)^\nu}{d + (t-t_0)^\nu}\right]\phi_a(t_0) \quad \text{and} \quad \varepsilon_{sh}(t) = \varepsilon_{sh}(t) = \frac{t^\beta}{f + t^\beta}\varepsilon_{sh}(\infty), \]
in which \( \phi(t_0) = 1.25 \times t_0^{0.118} \phi(\infty, 7) \) is the empirical ultimate creep strain at \( t \to \infty \), and the data \( d = 10 \) days, \( \phi(\infty, 7) = 2.35 \), \( \psi = 0.6 \), \( f = 35 \) days, \( \beta = 1 \) and \( \varepsilon_{3h}(\infty) = 500 \times 10^{-6} \) are taken from tests reported by Wang et al. (2005).

Chemical bonding, mechanical friction and embossments in the decking are often assumed to fully restrict slip at the decking-concrete interface, but in reality a condition of partial interaction is realised. This can be quantified by a relative slip deformation \( s \), which is related to the shear flow force at the interface \( F_i \) by

\[
F_i = ks \, ,
\]  

where \( k \) is the empirically-derived shear connection stiffness with units of \((\text{force})/(\text{length})^2\), taken as being constant in this paper. The steel decking and conventional reinforcement are assumed to be in the elastic range of structural response, both with an elastic modulus of \( E_s \).

3. Strains and stress

The composite slab in one-way bending is assumed to be simply supported of length \( L \) and subjected to a sustained uniformly distributed load of intensity \( q \), as well as being propped during construction. It is further assumed that the member is symmetric with respect to the \( z \) axis, that the deformations are sufficiently small so that the curvature is \( \kappa = \frac{d^2 \psi}{dx^2} \), that the strain-displacement relationship is linear and that the curvatures in the concrete, reinforcement and steel decking are the same. The depths of the geometric centroids of the concrete, reinforcement and decking are \( d_c, d_r \) and \( d_s \) respectively as shown in Figure 3, and the profile of the interface between the concrete and decking is defined by the function \( h_i(z) \) (this is taken as piecewise linear for profiled sheeting).

With respect to the elastic centroids of the concrete, reinforcement and sheeting, the strains are

\[
\varepsilon_c = u'_c - y_c v'', \quad \varepsilon_r = u'_r - y_r v'' \quad \text{and} \quad \varepsilon_s = u'_s - y_s v''
\]  

respectively, where \( y_r = d_c - d_r \) is the depth of the reinforcement below the centroid of the concrete, \( u_c \) and \( u_r \) are the axial deformations at the centroids of the concrete and steel respectively, the variables \( t \) and \( t_0 \) have been dropped for convenience and primes denote differentiation with respect to \( x \). For kinematic compatibility at the interface location in Figure 3,

\[
s = \left\{ u_r + \left[ d_s - h_i(z) \right] v' \right\} - \left\{ u_r - \left[ h_i(z) - d_c \right] v' \right\} = u_r - u_c + hv'
\]  

where \( h = d_c - d_s \) is the distance between the centroids of the steel and concrete. Using elementary elasticity theory, the stresses in the concrete, reinforcement and steel decking are

\[
\sigma_c = E_c \left( u'_c - y_c v'' - \varepsilon_{sh} \right), \quad \sigma_r = E_r \left( u'_r - y_r v'' \right) \quad \text{and} \quad \sigma_s = E_s \left( u'_s - y_s v'' \right)
\]  

respectively, and these produce axial forces in the concrete, reinforcement and decking given by

\[
N_c = \int_{A_c} \sigma_c \, dA_c = E_c A_c \left( u'_c - \Lambda_c \right), \quad N_r = A_r \sigma_r = E_r A_r \left( u'_r - y_r v'' \right) \quad \text{and} \quad N_s = \int_{A_s} \sigma_s \, dA_s = E_s A_s u'_s
\]  

in which
\[ \Lambda_{sh} = \frac{1}{A_c} \int \varepsilon_{sh}(y) dA_c \] (10)

is an area-shrinkage property of the concrete, \( A_c, A_r \) and \( A_s \) are the areas of the concrete, reinforcement and decking respectively, and which makes use of the centroidal properties of the \( y_c \) and \( y_s \) axes. The counterpart bending moments produced by the stresses in Equations (8) with respect to their centroids in the concrete, axial force and steel decking are

\[ M_c = \int_{A_c} y_c \sigma_c dA_c = -E_c I_c (\varepsilon'' + \Psi_{sh}), \quad M_r = 0 \quad \text{and} \quad M_s = \int_{A_s} y_s \sigma_s dA_s = -E_s I_s \varepsilon'' \] (11)

respectively, in which

\[ \Psi_{sh} = \frac{1}{I_c} \int_{A_c} y_c \varepsilon_{sh}(y) dA_c \] (12)

is a first moment of area shrinkage property of the concrete, and \( I_c \) and \( I_s \) are the second moments of area of the concrete and steel decking respectively. The transverse shear forces in the concrete, reinforcement and steel decking are

\[ V_c = -d M_c / dx = E_c I_c \varepsilon''', \quad V_r = 0 \quad \text{and} \quad V_s = -d M_s / dx = E_s I_s \varepsilon''' \] (13)

respectively.

4. Virtual work formulation

The principle of virtual work can be invoked to provide a convenient analysis of the shear connection problem with shrinkage, which for member equilibrium requires the virtual work functional

\[ \delta W = \delta W_c + \delta W_r + \delta W_s + \delta W_q \] (14)

to vanish for all perturbations of the deformations \( \delta u_c, \delta u_r \) and \( \delta v \), in which

\[ \delta W_c = \int_{-L/2}^{L/2} \int_{A_c} \sigma_c \cdot \delta \varepsilon_c dA_c dx \] (15)

is the virtual work in the concrete related to a virtual perturbation \( \delta \varepsilon_c \) of the strain in the concrete,

\[ \delta W_r = A_r \int_{-L/2}^{L/2} \sigma_r \cdot \delta \varepsilon_r dx \] (16)

is the virtual work in the reinforcement related to a virtual perturbation \( \delta \varepsilon_r \) of the strain in the reinforcement,

\[ \delta W_s = \int_{-L/2}^{L/2} \int_{A_s} \sigma_s \cdot \delta \varepsilon_s dA_s dx \] (17)
is the virtual work in the steel deck related to a virtual perturbation $\delta \varepsilon_s$ of the strain in the deck,

$$\delta W_i = \int_{-L/2}^{L/2} F_i \cdot \delta s \, dx$$

(18)

is the virtual work at the interface related to a virtual perturbation $\delta s$ of the interface slip and

$$\delta W_q = \int_{-L/2}^{L/2} q \delta v \, dx$$

(19)

is the virtual work associated by a virtual perturbation $\delta v$ applied to the external loading $q$, in which

$$\delta \varepsilon_c = \delta u'_c - y_c \delta v'', \quad \delta \varepsilon_r = \delta u'_r - y_r \delta v'' \quad \text{and} \quad \delta \varepsilon_s = \delta u'_s - y_s \delta v''$$

(20)

are the virtual perturbations of the strains in the concrete, reinforcement and steel deck respectively. Defining the stress resultants

$$N = N_c + N_r + N_s, \quad V = V_c + V_s \quad \text{and} \quad M = M_c + M_s,$$

(21)

integration of Equation (14) by parts results in the statement of virtual work given as

$$\left( N_c + N_r \right) \delta u'_c \bigg|_{-L/2}^{L/2} + N_s \delta u'_s \bigg|_{-L/2}^{L/2} - M \delta v' \bigg|_{-L/2}^{L/2} - V \delta v \bigg|_{-L/2}^{L/2}$$

$$- \int_{-L/2}^{L/2} \left( M'' + F'_i h + q \right) \delta v \, dx - \int_{-L/2}^{L/2} \left( N'_c + N'_r + F'_i \right) \delta u'_c \, dx - \int_{-L/2}^{L/2} \left( N'_s - F'_i \right) \delta u'_s \, dx = 0.$$  

(22)

Equation (22) is a statement of the statical boundary conditions that $(N_c + N_r)(x = \pm L/2) = 0$, $N_r(x = \pm L/2) = 0$ which supplement the kinematic boundary conditions that $\nu(x = \pm L/2) = 0$ and $u_r(x = 0) = u_r(x = 0) = 0$. Because the virtual variations $\delta v$, $\delta u_c$ and $\delta u_s$ in Equation (22) are arbitrary, the differential equations of equilibrium are

$$N'_c + N'_r + F'_i = 0, \quad N'_s - F'_i = 0 \quad \text{and} \quad M'' + hF'_i + q = 0.$$  

(23)

Manipulation of these equations (Bradford 2010) leads to the third order linear differential equation for the shear flow force at the interface given by

$$F''_i - \alpha^2 F'_i = \frac{k h q}{E I},$$

(24)

in which

$$\frac{1}{EA} = \frac{1}{E A} + \frac{1}{E_s A_s}$$

and

$$\alpha^2 = k \left( \frac{1}{EA} + \frac{h h}{EI} \right), \quad \bar{h} = h - \frac{y_s E_c A_c}{E A}.$$
and which has the solution that satisfies the boundary conditions given by

\[ F_i = k \left[ \frac{h_q}{\alpha^2EI} \left( \frac{\sinh(\alpha x)}{\alpha \cosh(\alpha L/2)} - x \right) \frac{\beta_{sh} \sinh(\alpha x)}{\alpha \cosh(\alpha L/2)} \right], \]  \hspace{1cm} (26)

in which

\[ \beta_{sh} = \frac{E_c A_n}{EA} + \left( \frac{h E_c I_c}{EI} \right) \Psi_{sh} \]  \hspace{1cm} (27)

is a dimensionless shrinkage property. From the third of Equations (23), the lengthwise variation of the bending moment is

\[ M = \frac{qL^2}{\gamma} \left[ \frac{\xi^2}{2} + \frac{1}{\Theta^2} \left( 1 - \frac{\cosh \theta}{\cosh(\Theta/2)} \right) \frac{1}{8} \right] + \frac{\beta_{sh} E_c I_c}{\gamma h} \left( \frac{\cosh \theta}{\cosh(\Theta/2)} - 1 \right) \frac{qL^2}{8} \left( 1 - 4\xi^2 \right) \]  \hspace{1cm} (28)

in which \( \gamma = 1 + \frac{h}{\bar{r}}/\bar{r}^2, \xi = x/L, \theta = \alpha x \) and \( \bar{r} = \sqrt{(EI/EA)} \), while the lengthwise variation of the deflection is

\[ v = \frac{qL^4}{\gamma EI} \left\{ \frac{1}{8\Theta^2} \left( 1 - 4\xi^2 \right) + \frac{1}{\Theta^4} \left[ \frac{\cosh \theta}{\cosh(\Theta/2)} - 1 \right] \right\} + \frac{\beta_{sh} E_c I_c L^2}{h} \left( \frac{\xi^2}{2} - \frac{1}{8} \right) + \frac{\beta_{sh} E_c I_c L^2}{h} \left( \frac{\xi^2}{2} - \frac{1}{8} \right) \left( \frac{\gamma - 1}{\gamma} \right) + \frac{\Psi_{sh} E_c I_c L^2}{EI} \left( \frac{\xi^2}{2} - \frac{1}{8} \right) \]  \hspace{1cm} (29)

in which \( \Theta = \alpha L. \) The midspan deflection \( v(0) \) for a very stiff connection \( (\alpha \to \infty) \) is

\[ v(0)_{\text{stiff}} = \lim_{\Theta \to \infty} v(0) = \frac{5}{384} \frac{qL^4}{\gamma EI} - \frac{\beta_{sh} E_c I_c L^2}{8\bar{h} \gamma} - \frac{\Psi_{sh} E_c I_c L^2}{8EI}, \]  \hspace{1cm} (30)

while that for a flexible connection \( (\alpha \to 0) \) is

\[ v(0)_{\text{flex}} = \lim_{\Theta \to 0} v(0) = \frac{5}{384} \frac{qL^4}{EI} - \frac{\Psi_{sh} E_c I_c L^2}{8EI}. \]  \hspace{1cm} (31)

In the absence of shrinkage (\( \beta_{sh} = \Psi_{sh} = 0 \)) and noting that \( \gamma EI \) is the flexural rigidity of a cross-section comprising of the concrete slab of area \( A_c \) with reinforcement of area \( A_r \) and of the decking of area \( A_s \), these results reduce to the familiar deflection of a simply supported beam under a uniformly distributed load \( q \) with flexural rigidities \( \gamma EI \) and \( EI \) respectively.
5. Numerical study

The composite slab considered has the profile and properties shown in Figure 4. Under short-term loading without shrinkage ($\varepsilon_{sh} = \phi = 0$) and with $A_r = 1000$ mm$^2$ and $y_r = 0$, the deflections of this one-way slab are presented in Figure 5 as a function of the degree of partial interaction represented in dimensionless form by the parameter $\Theta$. The deflections range from $L/920$ for flexible shear connection ($\Theta = 0.1$) to $L/6930$ for stiff shear connection ($\Theta = 100$); by comparison the deflection of the reinforced slab without the profiled decking is $L/885$.

The same slab has been analysed at a time when the shrinkage strain $\varepsilon_{sh} = -400 \times 10^{-6}$ is uniform through the depth, using $\phi = 1.5$. For this, $\Psi_{sh} = 0$ and the results are shown in Figure 6 as a function of $\Theta$. It can be seen that the deflections are significantly greater than the short-term deflections, and that the effects of shrinkage offset the reduction in deflection that results from a stiff shear connection. This occurs because the shrinkage takes place in the concrete, which is 42 mm above the centroid of the steel decking, and the shrinkage therefore creates a curvature in the member; a stiffer shear connection allows for greater transfer of actions to the steel and thus predominates the curvature caused by shrinkage straining. When the shrinkage varies linearly from $\varepsilon_{sh}$ at the top defined by $y_c = -D/2$ to $\eta \varepsilon_{sh}$ at the soffit defined by $y_c = D/2$, the parameters $\Lambda_{sh}$ and $\Psi_{sh}$ are

$$\Lambda_{sh} = \varepsilon_{sh} (\eta + 1)/2 \quad \text{and} \quad \Psi_{sh} = \varepsilon_{sh} (\eta - 1)/D.$$  \hspace{1cm} (32)

Figure 7 shows the deflection for $\eta = 0$ (zero shrinkage at the soffit), while Figure 8 plots the variation of the central deflection $v(0)$ as a function of $\eta$, using $\varepsilon_{sh} = -420 \times 10^{-6}$. By comparison with Figure 5, it can be seen that the deflections decrease with an increase of the shrinkage gradient (i.e. when $\eta$ decreases from 1 to 0) when the shear connection is stiff (high values of $\Theta$), but that the deflections increase with an increase in the shrinkage gradient when the connection is flexible (low values of $\Theta$); for the member under analysis this cross-over is around $\Theta = 4$. 

![Properties of composite slab](image-url)
Figure 5: Short-term deflections.

Figure 6: Long-term deflections (uniform shrinkage: $\eta = 1$).

Figure 7: Long-term deflections (non-uniform shrinkage: $\eta = 0$).
6. Conclusions

This paper has addressed the behaviour of composite slabs subjected to strain caused by concrete shrinkage, including the effects of rib shear partial interaction between the concrete slab and the steel decking. A technique based on the principle of virtual work was proposed which, in the uncracked range of structural response, results in solutions for the deflection and stresses in the composite slab in closed form, allowing for quantification of the influences of the shrinkage strain and partial interaction. Importantly, it allows for any distribution of the shrinkage strain through the depth of the section, as the straining would be non-uniform because of the presence of the soffit steel sheeting. The results show that the interaction between the parameters governing the problem is complex, in particular various combinations of partial interaction stiffness and the shrinkage strain profile lead to situations which may cause cracking or cause small concrete stresses. These must be addressed through the analytical modelling proposed in the paper. Despite composite slabs being widely used, little guidance is available on their service-load behaviour and design, and importantly more research is needed (particularly experimental) to shed light on this application of structural mechanics.

Figure 8: Deflection as a function of shrinkage gradient $\eta$ and partial interaction $\Theta$.

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