Optimization of control parameters of a boost converter for energy harvesting

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Abstract. In order to maximize the energy efficiency for energy harvesting, a simplified circuit model using a boost converter and a capacitor is proposed. The circuit model allows the analysis of the whole system's theoretical energy efficiency and determination of the optimum control parameters. The optimum duty ratio is determined analytically and the optimum switching period numerically. This is important information for designing a switching controller that achieves both low power consumption and accurate maximum power point tracking. Moreover, the proposed theoretical energy efficiency is helpful for component selection when designing boost converters.

1. Introduction
Energy harvesting with small solar cells or thermoelectric generators requires an efficient storage method. Compared to large-scale generation, power consumption in an energy harvester is a fatal energy loss for achieving high efficiency. The main source of power consumption is the switching controller of the DC-DC converter that operates the generating device at its maximum power point (MPP). Two methods of maximum power point tracking (MPPT) for energy harvesting have typically been used: fractional open-circuit voltage (FOCV) [1]-[3] and perturb and observe (P&O)[4]. The FOCV method achieves low power consumption for only one comparator and a few discrete components but has low tracking accuracy. On the other hand, the P&O method exploits very accurate MPP calculations but requires a high power consumption for complex circuits such as microcontrollers or digital signal processors. Therefore, an ideal switching controller can improve energy efficiency by achieving both low power consumption and accurate MPPT. The optimum control parameters, which are output by the ideal switching controller, should be clarified theoretically before the design stage. In the present study, in order to achieve high efficiency, a capacitor is used as a storage device with a low equivalent series resistance (ESR). Additionally, a boost converter is used for MPPT because of its excellent transmission efficiency for electric power [3]. The objective is to introduce an equivalent energy harvester model and obtain the optimum control parameters for maximizing the energy efficiency.

2. Modeling an energy harvester
In order to analyze the electric behavior of an energy harvester for different switching periods, the equivalent circuit model shown in Figure 1 is used. The power source should be operated
around its MPP. Therefore, the i-v characteristics can be assumed to be a tangential line at the
MPP. The i-v characteristics can be expressed as

\[ v = 2V_{MP} - R_{MP}i, \]  

where \( V_{MP} \) and \( I_{MP} \) are the power source output voltage and current at the MPP, and \( R_{MP} \) is
defined as \( \frac{V_{MP}}{I_{MP}} \).

For the same reason, the diode forward voltage \( v_f \) for a current \( i \) is assumed to be

\[ v_f = V_D + R_Di. \]

Additionally, the storage capacitor has a large capacitance so that its voltage can be assumed
to be a constant \( V_C \) during one switching period.

Moreover, the components for the boost converter and capacitor should have low resistance
compared to \( R_{MP} \) in order to improve energy efficiency. Thus, the total resistance of the current
path in the on-state can be assumed to be nearly equal to that in the off-state. This can be
expressed as

\[ R = R_{MP} + R_L + R_{DS} \approx R_{MP} + R_L + R_D + R_C. \]

Kirchhoff’s voltage law for the on- and off-states can be written as

\[ L \frac{di_{on}}{dt} + Ri_{on} = 2V_{MP}, \]

when the switch is on and

\[ L \frac{di_{off}}{dt} + Ri_{off} = 2V_{MP} - V_C - V_D, \]

when the switch is off.

In order to analyze the current flow, a constant switch-on time of \( T_{on} \) and a switch-off time
of \( T_{off} \) are considered. If the number of switching operations is defined as \( n \), the current in the
on-state \( i_{on(n-1)} \) is expressed as

\[ i_{on(n-1)} = (I_{2n-2} - \frac{2V_{MP}}{R})e^{-\frac{R}{L}} + \frac{2V_{MP}}{R}, \]

where \( I_{2n-2} \) is a boundary condition when switching from off to on. Likewise, the current in the
off-state \( i_{off(n)} \) is expressed as

\[ i_{off(n)} = (I_{2n-1} - \frac{2V_{MP} - V_{C,D}}{R})e^{-\frac{R}{L}} + \frac{2V_{MP} - V_{C,D}}{R}, \]

where \( I_{2n-1} \) is a boundary condition when switching from on to off and \( V_{C,D} = V_C + V_D \). The boundary conditions \( I_{2n-2}, I_{2n-1} \) and \( I_{2n} \) are related by the recurrence equations

\[ I_{2n-1} = I_{2n-2}e^{-\frac{RT_{on}}{L}} + \frac{2V_{MP}}{R}(1 - e^{-\frac{RT_{on}}{L}}), \]

and

\[ I_{2n} = I_{2n-1}e^{-\frac{RT_{off}}{L}} + \frac{2V_{MP} - V_{C,D}}{R}(1 - e^{-\frac{RT_{off}}{L}}). \]

When the initial condition \( I_0 \) is 0, the recurrence equations can be expressed as
Figure 1. Equivalent circuit model of an energy harvester.

Figure 2. Current oscillation obtained from equations (6) and (7) using boundary conditions set by equations (10) and (11) with available specifications.

\[ I_{2n-2} = \left( \frac{2V_{MP}}{R} - \frac{V_{C,D}(1-e^{-\frac{RT_{off}}{L}})}{R(1-e^{-\frac{RT_{off}}{L}})} \right) \left( 1 - e^{-\frac{(n-1)RT}{L}} \right) \]

and

\[ I_{2n-1} = \left( \frac{2V_{MP}}{R} - \frac{V_{C,D}e^{-\frac{RT_{on}}{L}}(1-e^{-\frac{RT_{off}}{L}})}{R(1-e^{-\frac{RT_{off}}{L}})} \right) \left( 1 - e^{-\frac{(n-1)RT}{L}} \right) + \frac{2V_{MP}}{R} \left( 1 - e^{-\frac{RT_{on}}{L}} \right) e^{-\frac{(n-1)RT}{L}}, \]

where \( T = T_{on} + T_{off} \).

The resulting current oscillation is shown in Figure 2 based on equations (6),(7),(10) and (11). Figure 2 is based on the experimental specifications shown in Table 1, with \( T_{on} = 8.09 \) [\( \mu \text{s} \)], \( T_{off} = 1.91 \) [\( \mu \text{s} \)] and \( V_{C} = 7.5 \) [V]. Figure 2 indicates that the top and bottom boundary conditions converge immediately from the initial conditions. Thus, the steady state current can be expressed using the boundary conditions of equations (10) and (11) at \( n \to \infty \) as

\[ i_{on} = -\frac{V_{C,D}(1-e^{-\frac{RT_{off}}{L}})}{R(1-e^{-\frac{RT_{off}}{L}})} e^{-\frac{R_{L}}{L}} + \frac{2V_{MP}}{R}, \]

and

\[ i_{off} = \frac{V_{C,D}(1-e^{-\frac{RT_{on}}{L}})}{R(1-e^{-\frac{RT_{on}}{L}})} e^{-\frac{R_{L}}{L}} + \frac{2V_{MP} - V_{C,D}}{R}. \]
The energy stored in the capacitor during one switching period can be calculated using equations (12) and (13). The energy $E_{ct}$ is expressed as

$$E_{ct} = V_C \int_0^{T_{off}} i_{off} dt.$$  \hspace{1cm} (14)

However, the field effect transistor (FET) operates as an ideal switch without $R_{ds}$ in the model. The switching loss $E_{sw}$ and gate drive loss $E_{gd}$ at the FET should be considered for energy harvesting. The switching loss $E_{sw}$ is approximated as

$$E_{sw} = \frac{I_{on} V_{off} T_{r,f}}{6},$$  \hspace{1cm} (15)

where $I_{on}$ is the current when the switch is changing from on to off, $V_{off}$ is the voltage across the diode and the capacitor, and $T_{r,f}$ is the total current rise or fall time required for switching.

Additionally, the gate drive loss $E_{gd}$ can be expressed as

$$E_{gd} = Q_g V_{gs},$$  \hspace{1cm} (16)

where $Q_g$ is the gate charge and $V_{gs}$ is the gate-source voltage.

On the other hand, an ideal boost converter for energy harvesting must operate the power source at its MPP and store the energy in the capacitor without loss. Thus, the energy efficiency of the energy harvester model during one switching period can be defined as

$$\eta = \frac{E_{ct} - E_{sw} - E_{gd}}{E_{MP}},$$  \hspace{1cm} (17)

where $E_{MP} = I_{MP} V_{MP} T$.

Using equations (12),(13),(14),(15) and (16), equation (17) can be expressed as

$$\eta = \frac{R_{MP} V_C}{RV_{C,D}} \left( \frac{\tilde{g}_{C,D}^2 (1 - e^{-(1-d)\tilde{T}})(1 - e^{-d\tilde{T}})}{\tilde{T} (1 - e^{-\tilde{T}})} - \tilde{g}_{C,D} (1 - d)(\tilde{g}_{C,D} - 2) \right) - \frac{I_{on} V_{off} T_{r,f} + 6Q_g V_{gs}}{6I_{MP} V_{MP} T},$$  \hspace{1cm} (18)

where $\tilde{g}_{C,D} = \frac{V_{C,D}}{V_{MP}}$, $\tilde{T} = \frac{RT}{L}$ and $d$ is the duty ratio expressed as $\frac{T_{on}}{T}$. The duty ratio can be optimized analytically by differentiating equation (18) with respect to $d$. The optimized duty ratio $d_{op}$ is then written as

$$d_{op} = 1 + \frac{1}{\tilde{T}} \log \left( \frac{(\tilde{g}_{C,D} - 2)(1 - e^{-\tilde{T}})}{2\tilde{g}_{C,D}} + \sqrt{\frac{(\tilde{g}_{C,D} - 2)^2 (1 - e^{-\tilde{T}})^2}{4\tilde{g}_{C,D}^2} + e^{-\tilde{T}}} \right).$$  \hspace{1cm} (19)

Using equation (18) with $d$ set to $d_{op}$, as given by equation (19), the relationship between the switching period and energy efficiency is plotted in Figure 3 for the experimental specifications shown in Table 1 and $V_C=7.5 \text{ [V]}$. The calculation for Figure 3 used a power source MPP of 150 [mW], which is similar to the output often used in solar harvesting. The optimum switching period for achieving both a higher power source output and a lower FET energy loss is 9.29 [µs] from Figure 3, which gives an energy efficiency of 92.7%.
3. Experimental results
To validate the theoretical energy efficiency, the experimental energy efficiency is calculated by measuring the time $\Delta t$ for charging the capacitor from $V_L = 7.45 \text{ [V]}$ to $V_H = 7.55 \text{ [V]}$. Figure 4 shows the experimental circuit for measuring the capacitor voltage as a function of time, which is designed using the components shown in Table 1.

Table 1. Specifications for the experiment.

| Components | Manufacturer | Model number | Symbols | Values |
|------------|--------------|--------------|---------|--------|
| Power source | MATSUSADA KOA | PL-36-2.2 MOS2CT52A15R0F | $V_{MP}$, $R_{MP}$ | 1.5 [V], 15 [Ω] |
| Inductor | Panasonic | ELC18B471L | $L$, $R_L$ | 470 [$\mu$H], 210 [mΩ] |
| FET | TOSHIBA | TPC6006-H | $R_{DS}$, $T_{ref}$, $Q_g$ | 78 [mΩ], 7.0 [ns], 2.4 [nC] |
| Diode | TOSHIBA | CMS16 | $V_D$, $R_D$ | 0.32 [V], 200 [mΩ] |
| Capacitor | BHC | ALS3-224NP040 | $C$, $R_C$ | 225 [mF], 6.0 [mΩ] |

The stored energy $E_{ce}$ in the capacitor during one switching period is experimentally obtained to be

$$E_{ce} = \frac{1}{2} C (V_H^2 - V_L^2) \frac{T}{\Delta t}. \quad (20)$$

Since $E_{ce}$ is influenced by the FET switching loss, equation (17) can be equivalently expressed as

$$\eta = \frac{E_{ce} - E_{gd}}{E_{MP}}, \quad (21)$$

where $E_{gd}$ is calculated using equation (16), with $Q_g = 2.4$ [nC] and $V_{gs} = 5.0$ [V]. Table 2 shows the values of $d_{op}$ calculated using equation (19) for different switching periods.

Table 2. Calculated values of $d_{op}$ using equation (19) for different $T$ in the experiment.

| $T$ [μs] | 0.316 | 1.00 | 3.16 | 10.0 | 31.6 | 100 | 316 |
|----------|-------|------|------|------|------|-----|-----|
| Frequency [MHz] | 3.16 | 1.00 | 316 | 100 | 31.6 | 10.0 | 3.16 |
| $d_{op}$ | 0.808 | 0.808 | 0.808 | 0.809 | 0.816 | 0.869 | 0.953 |

The experimental results are plotted in Figure 3. Approximation errors for the FET switching loss occur for low switching periods, but this is not important for determining the optimum
Switching period: $T$ [s]
Energy efficiency: $\eta$

| Theoretical energy efficiency | Experimental result |
|-------------------------------|---------------------|
| $E_{ci}$ | Achieving 92.7% at 9.29[$\mu$s] |
| $E_{sw} + E_{gd}$ | |

Figure 3. Relationship between switching period and energy efficiency.

switching period. The theoretical energy harvesting model and energy efficiency are validated experimentally in the high efficiency region.

4. Conclusion
This paper focuses on the optimum control parameters for a boost converter for energy harvesting. The optimum duty ratio is determined analytically and the optimum switching period numerically. The results provide important information for the design of an ideal switching controller for energy harvesting. Additionally, the proposed energy harvesting model and theoretical energy efficiency are helpful for component selection when designing boost converters.

References
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