CPT Violation in String-Modified Quantum Mechanics and the Neutral Kaon System

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Abstract

We show that CPT is in general violated in a non-quantum-mechanical way in the effective low-energy theory derived from string theory, as a result of apparent world-sheet charge non-conservation induced by stringy monopoles corresponding to target-space black hole configurations. This modification of quantum mechanics does not violate energy conservation. The magnitude of this effective spontaneous violation of CPT may not be be far from the present experimental sensitivity in the neutral kaon system. We demonstrate that our previously proposed stringy modifications to the quantum-mechanical description of the neutral kaon system violate CPT, although in a different way from that assumed in phenomenological analyses within conventional quantum mechanics. We constrain the novel CPT-violating parameters using available data on $K_L \to 2\pi$, $K_S \to 3\pi^0$ and semileptonic $K_{L,S}$ decay asymmetries. We demonstrate that these data and an approximate treatment of interference effects in $K \to 2\pi$ decays are consistent with a non-vanishing amount of CPT violation at a level accessible to a new round of experiments, and further data and/or analysis are required to exclude the extreme possibility that they dominate over CP violation. Could non-quantum-field theoretical and non-quantum-mechanical CPT violation usher in the long-awaited era of string phenomenology?

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1 Introduction

The $CPT$ theorem is one of the deepest results of Quantum Field Theory \[1\]. It is a consequence of Lorentz invariance and locality, as well as quantum mechanics. The best experimental test of $CPT$ invariance so far has been in the neutral kaon system, where the equality of particle and antiparticle masses has been confirmed to better than one part in $10^{17}$ \[1\]. On the other hand, $CP$ invariance, which is by no mean sacred in Quantum Field Theory, appears to be violated in the neutral kaon system \[2\] at a level higher than any possible violation of $CPT$ invariance \[3\].

The possibility of a violation of $CPT$ invariance has been raised in various theoretical contexts that go beyond conventional local Quantum Field Theory. One important example is quantum gravity: it has been argued that conventional quantum mechanics and the axioms of Quantum Field Theory cannot be maintained once one considers topologically non-trivial quantum fluctuations in the space-time background \[4\]. It has been argued that a mixed-state description must be used, because of the inevitable loss of information across event horizons, even at the microscopic level. As a corollary of this observation, it has been argued \[5\] that $CPT$ invariance must be abandoned in quantum gravity, or at least re-expressed in a weakened form \[6\].

String theory is the only candidate we have for a consistent quantum theory of gravity, and serves as a unique laboratory for studying and quantifying these suggestions. We have argued in a recent series of papers that whilst the usual axioms of Quantum Field Theory apply on the string world-sheet, and quantum coherence is maintained by the complete spectrum of massive string states \[7\], the effective truncated theory of light particles observed in laboratory experiments must obey a modified form of quantum mechanics which allows for the evolution of pure states into mixed states \[9\]. With this new string motivation, we have also revived \[8\] a non-quantum-mechanical density matrix description of the neutral kaon system that two of us (J.E. and D.V.N.) proposed several years ago \[10\] together with J.S. Hagelin and M. Srednicki in the general context of quantum gravity, and confronted it with some of the available experimental data on the neutral kaon system \[3\].

We pointed out in this recent paper some possible tests of this non-quantum-mechanical density matrix formalism in the neutral kaon system, including for example comparisons between the experimental rate for $K_L \to 2\pi$ decay and semileptonic decay asymmetries for $K_L$ and $K_S$. As we also pointed out, similar comparisons had been discussed previously as probes of a possible violation of $CPT$ \[11\]. However, those previous proposals were in the context of a conventional quantum-mechanical pure state vector description of the neutral kaon system, and the correspondence

\[1\] For comparison, the equality of proton and antiproton masses has been checked to one part in $10^{10}$. 
with the parameters of our modified mixed-state density matrix description was neither direct nor obvious.

The possibility that $CPT$ might be violated in string theory has been discussed by several authors. It has been pointed out that $CPT$ invariance is linked to the conservation of charges on the string world-sheet, and it has been shown that these charges are indeed conserved in a flat space-time background. It has been conjectured that $CPT$ might be spontaneously violated in string field theory, but charge conservation on the world-sheet and $CPT$ invariance in space-time have not been studied in the type of non-trivial space-time background which, according to the quantum gravity literature, might lead to $CPT$ violation.

We have developed in previous papers a theoretical framework suitable for addressing the problem of $CPT$ violation in topologically non-trivial space-time backgrounds in string theory. It has been shown that physics in the neighbourhood of a spherically-symmetric stringy black hole singularity can be represented by an Abelian $U(1)$ Chern-Simons gauge theory with a monopole configuration on the world-sheet. The time evolution of a black hole is described by a homotopic extension of the $1+1$-dimensional gauge theory to $2+1$ dimensions. The quantum foam of microscopic topologically non-trivial space-time fluctuations consists of a gas of monopole-antimonopole pairs. Furthermore, the infinite symmetries coupling observable light particles to unobserved massive solitonic string states associated with these world-sheet monopoles implied that the observable particles should be described as an open quantum system described by the modified quantum-mechanical formalism mentioned above.

In this paper we use this framework to show that $CPT$ violation is generic in the effective low-energy theory derived from string theory, once topologically non-trivial space-time fluctuations are taken into account. We show that the above-mentioned Chern-Simons monopoles violate charge conservation on the world-sheet, leading to non-quantum-field-theoretical effects in the truncated low-energy space-time theory that violate $CPT$ but conserve energy. We attempt to quantify the possible magnitude of such effects, and confirm our previous estimate that they could be suppressed by just one power of the light particle mass scale divided by the Planck mass, i.e., possibly of order $10^{-19}$ in the dynamics of strongly-interacting particles. Since this estimate is very close to the present experimental upper limits on $CPT$ violation in the neutral kaon system, we re-examine these previous analyses in the context of our density matrix formalism for effectively-open quantum systems. We demonstrate the existence of three extra $CPT$-violating parameters distinct from particle-antiparticle mass and lifetime differences that appear in the conventional state vector description of the neutral kaon system. We then make a preliminary analysis of various aspects of the available data within this modified quantum framework, including measurements of the $K_L \to 2\pi$ decay rate, published
(preliminary [18]) data on $K_L (K_S)$ semileptonic decay asymmetries, the experimental upper bound on $K_S \rightarrow 3\pi^0$ decays, and an estimate of the likely sensitivity of intermediate-time interference measurements [19] of the phases of $K \rightarrow 2\pi$ amplitudes. A full analysis of the latter constraint would require a global fit to all the available experimental data within the density matrix formalism, which would take us beyond the scope of this paper.

Our analysis does not exclude the possibility that the apparent $CP$ violation observed in $K_L \rightarrow 2\pi$ decays and semileptonic decay asymmetries could be accompanied (or even replaced) by non-quantum-mechanical $CPT$-violating effects. This radical possibility could be constrained by a complete fit to the intermediate-time data, or by improvements in the present measurements of the $K_S$ semileptonic decay asymmetry, or in the present bounds on the $K_S \rightarrow 3\pi^0$ decay rate. The latter could be achieved by the CPLEAR experiment at CERN or the forthcoming DAΦNE project [20] at Frascati.

2 Spontaneous $CPT$ Violation and Modifications of Quantum Mechanics in String Theory

We are concerned in this paper with the truncated effective theory of the light degrees of freedom in a generic string theory, and in particular with their quantum time evolution. These light string degrees of freedom are linked to the massive string modes by an infinite set of gauge symmetries that mix mass-levels [21]. Among these, we have identified [7] a $W_\infty$ symmetry that is responsible for the maintenance of quantum coherence in the presence of stringy black holes [15], precisely as a result of this coupling of different mass-levels. Since the effective low-energy theory is a truncation of the full string theory, it is in general subject to non-trivial renormalization effects. For example, shifts in the “tachyon”, i.e. light particle, background are not exactly marginal deformations of the conformal Wess-Zumino coset theory that describes a spherically-symmetric black hole in space-time [22]. This means that the couplings $g^i$ of the tachyons have non-trivial renormalization group coefficients $\beta^i$. We have proposed that target time $t$ be identified with the corresponding world-sheet renormalization scale.

A conventional laboratory experiment, measuring for example neutral kaon decays, does not detect massive string states, although these could in principle be observed by an infinite set of generalized Aharonov-Bohm phase measurements [23]. In the absence of such a measurement, any observation in the effective low-energy theory will appear as a non-conformal deformation, which will lead to a renormalization group flow of the truncated light-mode system. Unitarity of the effective
light-particle theory implies that this flow is irreversible, and the identification of the flow variable with $t$ provides an arrow of time, i.e. spontaneous T-violation, in the string universe [17]. Time is a statistical parameter that measures the interaction (gravitational friction) of the light particles with the massive string modes in the presence of singular space-time backgrounds (foam).

The associated quantum time evolution of the light-mode density matrix is given by a modified Liouville equation [10, 8]:

$$\dot{\rho} = i[\rho, H] + iG_{ij}\beta^j$$

where $H$ is the light-mode Hamiltonian and $G_{ij}$ is the Zamolodchikov metric in the coupling space $\{g^i\}$. Associated with the evolution (1) is a monotonic increase in entropy

$$\dot{S} = Tr\beta^i G_{ij}\beta^j \frac{\partial \rho}{\partial H} \ln \rho$$

It has been observed [10, 24] that such a modification (1) of the Liouville equation will in general lead to the non-conservation of quantities associated with symmetries, and this has been held against [25] such a modification of conventional quantum mechanics. We take this opportunity to point out that energy is indeed conserved by the modification (1), for specifically stringy reasons. It is easy to derive from (1) the following expression for the time-variation of the expectation value of the light-system Hamiltonian $H$:

$$\partial_t <\langle H \rangle >= Tr(\dot{\rho} H) + Tr(\partial_t H \rho) = Tr(i[g^i, H]\beta^j G_{ij} \rho) + \dot{H} \rho)$$

where $<\langle O \rangle >= Tr(O \rho)$ denotes the average value of the observable $O$ in this non-quantum mechanical framework. Identifying $-i[g^i, H]$ with the time derivative $\beta^i$ of the coupling $g^i$, equation (3) becomes

$$\partial_t <\langle H \rangle >= Tr(\dot{H} \rho - \beta^i G_{ij} \beta^j \rho) = <\langle \partial_t (H + C) \rangle$$

where $C$ is the Zamolodchikov C-function [26]. Since this can be identified with the string effective action [27], $H + C$ must be a constant, since $\partial_t (H + C) \propto \partial_t (p_i \beta^i) = 0$, as a result of the fact that neither $p_i$ nor $\beta^i$ has any explicit cutoff dependence, because the world-sheet theory is finite.

However, space-time foam does lead to the apparent violation of certain global symmetries on the world-sheet, which leads in turn to CPT violation in the effective low-energy theory. This can be seen using the Hall fluid picture of space-time foam discussed in ref. [17]. According to this picture, space-time foam can be represented as a statistical population of topological defects (spikes) on the world-sheet, which correspond to monopoles of a 2 + 1-dimensional Abelian $U(1)$ Chern-Simons gauge theory introduced as a homotopic extension of the underlying 1 + 1-dimensional world-sheet theory. The effective 2 + 1-dimensional action is

$$\int_{\Sigma \otimes S^1} \left[ |D(a)\phi|^2 + \frac{k}{4\pi} \varepsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho + V(\phi) \right]$$
where \( a_\mu \) is the Chern-Simons gauge field coupled to a complex scalar field \( \phi \) that represents deviations from the singularity, and hence the generation of space-time through symmetry breaking provided by a suitable form of the effective potential \( V(\phi) \). The parameter \( k \) is the Wess-Zumino level parameter, and the string black hole corresponds to the adiabatic limit in which the pseudo-“temperature” \( \tau = \beta^{-1} \), where \( \beta \) is the radius of the compactified \( S^1 \) in (5), goes to zero. Non-critical deformations correspond to deviations of \( k \) from its critical value \( 9/4 \). The black hole is a non-topological soliton in this picture, which can be regarded as a monopole-instanton in the \( 2 + 1 \)-dimensional effective theory (5). It is known [28] that the charge of the monopole is not in general conserved in \( 2+1 \)-dimensional Chern-Simons theory, in the sense that there are tunnelling processes between states with different monopole charges. Rather like the (more?) familiar instantons in 4-dimensional gauge theory [29], the \( a_\mu \) monopole-instantons lead to charge-violating effective interactions that look like

\[
<f|e^{-HT}|i> \propto \int e^{-S_{\text{eff}}}; \quad S_{\text{eff}} = \int Ke^{-B_i+\alpha \Phi_{em}} + h.c
\]

in the low-energy limit. Here \( \Phi_{em} \) is a gauge-invariant charge- and magnetic-flux-changing operator in the Chern-Simons theory, \( q \) denotes the monopole charge, \( B \) is the classical instanton action suppression term, \( K \) is the one-loop quantum correction, and \( \alpha \) is an arbitrary phase.

The charge violation due to this type of tunnelling process is similar to that expected from wormholes [30] in 4-dimensional Euclidean gravity. In our case, a tunnelling event between states with monopoles of different masses, which are the same as the monopole charges \( q \) since the monopoles are always extremal, leads locally to apparent charge violation by the space-time foam. In the wormhole case, it is possible to transfer charge through the wormhole from one region of space-time to another, also with a local, but not global, violation of charge. In our case, the charge is transferred to to the black hole, i.e. to massive string modes according to the selection rules discussed in ref. [23].

The apparent charge violation found above manifests itself as a violation of \( CPT \) in the effective low-energy theory, as follows from the general discussion in ref. [12]. From the point of view of the world-sheet, this is spontaneous \( CPT \) violation. It appears only associated with the non-quantum-mechanical open-system term in the quantum evolution equation (1), and not within the context of conventional quantum mechanics and Quantum Field Theory, as assumed in previous phenomenological analyses [13]. It is allowed because the conditions used to prove the \( CPT \) theorem in Quantum Field Theory are not all met in string theory: specifically, string theory is non-local. This deviation from locality is sufficiently weak to preserve \( CPT \) in flat target space-times [12, 14]. However, the non-locality rises up to violate \( CPT \) in a black hole background, and hence once space-time foam is taken into account, as has been argued on general grounds in the context of quantum gravity [1].
The next problem is to estimate the possible order of magnitude of CPT-violating effects. The non-quantum-mechanical open-system term in equation (1) contains an explicit coordinate factor $q^i$, which one would naively expect to be of order $m_P^{-1}$, corresponding to the scale of fluctuations in the space-time foam. Are there any other Planckian suppression factors? This inverse linear dependence on $m_P$ mirrors the inverse linear dependence on the scales of the “environmental” oscillators in the Feynman-Vernon [31, 32] formulation of open systems, to which our formalism is very similar [8]. In our case, the key issue is the scale size of the microscopic black hole fluctuations in the space-time foam, which is in turn related to the dominant range of values of the pseudo-“temperature” $\beta^{-1}$. We recall that the world-sheet has both spikes and dual vortices, which are viewed [33] as statistical excitations of the system characterized by an extra scale, the pseudo-“temperature” $\beta^{-1}$, which does not have a literal thermal interpretation, but parametrizes the phase diagram for the two-dimensional gas of topological defects. Minkowski black holes correspond to pseudo-“temperatures” below the Berezinskii-Kosterlitz-Thouless (BKT) temperature above which the monopoles become free. The conformal dimensions of the operators that describe deformations due to monopoles are

$$4\Delta_m = \frac{e^2}{2\pi\beta}$$

where the critical temperature corresponding to the BKT transition is that associated with $\Delta_m = 1$. For $e = 1$, which is energetically preferred this implies a critical temperature $T_c = 8\pi M_P$. The charge quantization condition for a monopole with charge $q_m$

$$2q_m\pi\beta = e = 1$$

then allows for black hole configurations with masses up to $\simeq 8\sqrt{2}M_P$ in the space-time foam, since $M_{bh} \propto 2\sqrt{2}q_m$. Tunneling effects then restrict the physics of space-time foam to the region of black-hole masses $1 \leq M_{bh}/M_P \leq 8\sqrt{2}$. The correlation functions that appear implicitly in (1) via the Zamolodchikov metric $G_{ij}$ and the renormalization group coefficients contain logarithmic dependences on the black hole mass that could vanish for some particular $M_{BH} = O(M_P)$, but will be $O(1)$ in general. We have not yet identified any exponential or power Planckian suppression beyond the single factor of $M_P^{-1}$ already mentioned. This means that CPT violation is generically maximal: non-quantum mechanical effects should be able to violate CPT as much as the weak interactions violate C and P.

However, we cannot rule out the possibility that there might be extra suppressions, at least for monopole matrix elements (6) between specific final states, in the same way as 4-dimensional Euclidean instanton effects are suppressed by extra factors of light fermion masses $m_f$ if the external states do not correspond precisely to zero modes of the instanton [29]. Specifically, there could be some selection rule, which we cannot yet identify, that suppresses CPT-violating effects in neutral kaons. However, we find it sufficiently interesting that we have identified a possible process
in string theory whereby CPT and T could be violated spontaneously at similar rates, and possibly within reach of present and future kaon experiments, that we now go on to explore further the phenomenology \cite{3} of our non-quantum-mechanical mechanism for CPT-violation.

3 CPT Violation in the Quantum-Mechanical Density Matrix Formalism for Neutral Kaons

Fortified by the above argument that CPT should indeed be violated in the effective low-energy theory derived from string, and the possibility that CPT violation parameters might not be much smaller than the present experimental limits from the neutral kaon system, we now review the density matrix formalism \cite{10} for neutral kaons, and analyze the possibility of CPT violation, initially within the conventional quantum-mechanical framework. The time evolution of a generic density matrix is determined by the equation

$$\partial_t \rho = -i(H\rho - \rho H^\dagger) + \delta H \rho$$

(9)

where the open-system $\delta H$ term is absent in conventional quantum mechanics. The conventional phenomenological Hamiltonian for the neutral kaon system contains hermitian (mass) and antihermitian (decay) components:

$$H = \begin{pmatrix} (M + \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma + \frac{1}{2}\delta \Gamma) & M_{12}^* - \frac{1}{2}i\Gamma_{12} \\ M_{12} - \frac{1}{2}i\Gamma_{12} & (M - \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma - \frac{1}{2}\delta \Gamma) \end{pmatrix}$$

(10)

in the ($K^0, \bar{K}^0$) basis.

The $\delta M$ and $\delta \Gamma$ terms violate CPT \cite{11}. As in ref. \cite{10}, we define components of $\rho$ and $H$ by

$$\rho \equiv \frac{1}{2}\rho_\alpha \sigma_\alpha \quad ; \quad H \equiv \frac{1}{2}h_\alpha \sigma_\alpha$$

(11)

in a Pauli $\sigma$-matrix representation: the $\rho_\alpha$ are real, but the $h_\beta$ are complex. The CPT transformation is represented by

$$\text{CPT}|K^0> = e^{i\phi}|K^0> \quad , \quad \text{CPT}|\bar{K}^0> = e^{-i\phi}|K^0>$$

(12)

for some phase $\phi$, which is represented in our matrix formalism by

$$\text{CPT} \equiv \begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix}$$

(13)

Since this matrix is a linear combination of $\sigma_{1,2}$, CPT invariance of the phenomenological Hamiltonian, $H = (\text{CPT})^{-1}HC\text{CPT}$, clearly requires that $H$ contain no term proportional to $\sigma_3$, i.e., $h_3 = 0$ so that $\delta M = \delta \Gamma = 0$. 
Conventional quantum-mechanical evolution is represented by
\[ \partial_t \rho = h_{\alpha\beta} \rho_{\beta} \], where, in the \((K^0, \bar{K}^0)\) basis and allowing for the possibility of CPT violation,

\[
h_{\alpha\beta} \equiv \begin{pmatrix} Imh_0 & Imh_1 & Imh_2 & Imh_3 \\ Imh_1 & Imh_0 & -Reh_3 & Reh_2 \\ Imh_2 & Reh_3 & Imh_0 & -Reh_1 \\ Imh_3 & -Reh_2 & Reh_1 & Imh_0 \end{pmatrix} \] (14)

Now is an appropriate time to transform to the \(K_1, K_2 = \frac{1}{\sqrt{2}}(K^0 \mp \bar{K}^0)\) basis, corresponding to \(\sigma_1 \leftrightarrow \sigma_3, \sigma_2 \leftrightarrow -\sigma_2\), in which \(h_{\alpha\beta}\) becomes

\[
h_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2} \delta \Gamma & -Im\Gamma_{12} & -Re\Gamma_{12} \\ -\frac{1}{2} \delta \Gamma & -\Gamma & -2ReM_{12} & -2ImM_{12} \\ -Im\Gamma_{12} & 2ReM_{12} & -\Gamma & -\delta M \\ -Re\Gamma_{12} & -2ImM_{12} & \delta M & -\Gamma \end{pmatrix} \] (15)

The corresponding equations of motion for the components of \(\rho\) in the \(K_{1,2}\) basis are

\[
\partial_t \rho_{11} = -(\Gamma + Re\Gamma_{12})\rho_{11} - (2ImM_{12} + \frac{1}{2} \delta \Gamma)Re\rho_{12} - (Im\Gamma_{12} - \delta M)Im\rho_{12}
\]

\[
\partial_t \rho_{12} = -(\Gamma - 2iReM_{12})\rho_{12} + (ImM_{12} - \frac{1}{2} iIm\Gamma_{12} - \frac{1}{2} \delta \Gamma - i\delta M)\rho_{11}
\]

\[
\partial_t \rho_{22} = -(\Gamma - Re\Gamma_{12})\rho_{22} + (2ImM_{12} - \frac{1}{2} \delta \Gamma)Re\rho_{12}
\]

\[
\partial_t \rho_{22} = -(\Gamma + Re\Gamma_{12})\rho_{22} - (2ImM_{12} + \frac{1}{2} \delta \Gamma)Im\rho_{12}
\]

(16)

It is easy to check that \(\rho\) decays at large \(t\) to

\[
\rho \simeq e^{-\Gamma_L t} \begin{pmatrix} 1 & \epsilon^* + \delta^* \\ \epsilon + \delta & |\epsilon + \delta|^2 \end{pmatrix}
\] (17)

corresponding to a pure long-lived mass eigenstate \(K_L\), with the CP-violating parameter \(\epsilon\) given by [20]

\[
\epsilon = \frac{\frac{1}{2} iIm\Gamma_{12} - ImM_{12}}{\frac{1}{2} \Delta \Gamma - i\Delta M}
\] (18)

where \(\Delta M = M_L - M_S\) is positive and \(\Delta \Gamma = \Gamma_L - \Gamma_S\) is negative, and the CPT-violating parameter \(\delta\) by

\[
\delta \simeq -\frac{1}{2} \frac{\delta \Gamma - i\delta M}{\Delta \Gamma - i\Delta M}
\] (19)

Conversely, in the short-\(t\) limit a \(K_S\) state is represented by

\[
\rho \simeq e^{-\Gamma_S t} \begin{pmatrix} |\epsilon - \delta|^2 & \epsilon^* - \delta^* \\ \epsilon - \delta & 1 \end{pmatrix}
\] (20)
where we see that the relative signs of the $\delta$ terms have reversed: this is the signature of $CPT$ violation in the conventional quantum-mechanical formalism. Note that the density matrices (17, 20) correspond to the state vectors

$$|K_{L(S)} > \propto (1 + \epsilon \mp \delta)|K^0 > \mp (1 - \epsilon \pm \delta)|\overline{K}^0 >$$

and are both pure, as should be expected in conventional quantum mechanics, even if $CPT$ is violated.

\section{4 $CPT$ Violation in the String Modification of the Density Matrix Formalism}

We now extend the above formalism to include the non-quantum-mechanical term $\delta H$ in equation (1). This can be parametrized by a $4 \times 4$ matrix $\eta_{\alpha\beta}$ analogous to the matrix $h_{\alpha\beta}$ discussed above. We work in the $K_{1,2}$ basis. As discussed in ref. [10], we assume that the dominant violations of quantum mechanics conserve strangeness, so that $\eta_{1\beta} = 0$, and hence that $\eta_{0\beta} = 0$ so as to conserve probability. Since $\eta_{\alpha\beta}$ is a symmetric matrix, it follows that also $\eta_{\alpha 0} = \eta_{0\alpha} = 0$. Moreover, $\eta_{\alpha\beta}$ must be a negative matrix, so we arrive at the general parametrization

$$\eta_{\alpha\beta} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -2\alpha & -2\beta \\
0 & 0 & -2\beta & -2\gamma
\end{pmatrix}$$

(22)

where $\alpha, \gamma > 0$, $\alpha \gamma > \beta^2$ [10].

We recall that the $CPT$ transformation, which is [see (13)] a linear combination of $\sigma_{1,2}$ in the $(K^0, \overline{K}^0)$ basis, becomes in the $K_{1,2}$ basis a linear combination of $\sigma_{3,2}$. It is apparent that none of the non-zero terms $\propto \alpha, \beta, \gamma$ in $\eta_{\alpha\beta}$ commutes with the $CPT$ transformation. In other words, each of the three parameters $\alpha$, $\beta$, $\gamma$ violates $CPT$, leading to a much richer phenomenology than in conventional quantum mechanics. This is because the symmetric $\eta$ matrix has three parameters in its bottom right-hand $2 \times 2$ submatrix, whereas the antisymmetric $h$ matrix has only one complex $CPT$-violating parameter. This means that the experimental constraints [3] on $CPT$ violation have to be rethought, as we discuss in section 5.

The equations of motion for the components of $\rho$ in the $K_{1,2}$ basis are [10, 9]

$$\partial_t \rho_{11} = -(\Gamma + Re \Gamma_{12})\rho_{11} - \gamma(\rho_{11} - \rho_{22}) - 2Im M_{12} Re \rho_{12} - (Im \Gamma_{12} + 2\beta) Im \rho_{12}$$
\[
\begin{align*}
\partial_t \rho_{12} &= -(\Gamma - 2i\text{Re}M_{12})\rho_{12} - 2i\alpha \text{Im}\rho_{12} + (\text{Im}M_{12} - \frac{1}{2}i\text{Im}\Gamma_{12} - i\beta)\rho_{11} \\
&\quad - (\text{Im}M_{12} + \frac{1}{2}i\text{Im}\Gamma_{12} - i\beta)\rho_{22} \\
\partial_t \rho_{22} &= -(\Gamma - \text{Re}\Gamma_{12})\rho_{22} + \gamma(\rho_{11} - \rho_{22}) + 2\text{Im}M_{12}\text{Re}\rho_{12} \\
&\quad - (\text{Im}\Gamma_{12} - 2\beta)\text{Im}\rho_{12}
\end{align*}
\]

which are to be compared with the corresponding quantum-mechanical equations (16). We see that the parameters \(\delta M\) and \(\beta\) play similar roles, although they appear with different relative signs in some places, because of the symmetry of \(\hat{h}\) as opposed to the antisymmetry of \(\hat{h}\).

These differences are important for the asymptotic limits of the density matrix, and its impurity. It is easy to check that, for large \(t\), \(\rho\) decays exponentially to

\[
\rho_L \propto \left( \frac{1}{\frac{1}{2}i(\text{Im}\Gamma_{12} + 2\beta) - \text{Im}M_{12}} \right) \\
\frac{-\frac{1}{2}i(\text{Im}\Gamma_{12} + 2\beta) - \text{Im}M_{12}}{\frac{1}{2}\Delta \Gamma + i\Delta M} \\
\frac{|\epsilon|^2 + \frac{\gamma}{\Delta \Gamma}}{-\frac{4\beta \text{Im}M_{12}(\Delta M/\Delta \Gamma) + \beta^2}{\frac{1}{2}\Delta \Gamma^2 + \Delta M^2}} \left( \frac{\epsilon^*}{\epsilon^* + \frac{\beta}{\frac{1}{2}\Delta \Gamma + i\Delta M}} \right) \\
1
\]

where the \(CP\) impurity parameter \(\epsilon\) is given by equation (18) as usual. The density matrix (24) describes a mixed state corresponding to a mixture of conventional \(K_L\) and \(K_S\) states, and not a pure state as in equation (17). Conversely, if we look in the short-time limit for a solution of the equations (23) with \(\rho_{11} \ll \rho_{12} \ll \rho_{22}\), we again find a mixed state:

\[
\rho_S \propto \left( \frac{|\epsilon|^2 + \frac{\gamma}{\Delta \Gamma}}{-\frac{4\beta \text{Im}M_{12}(\Delta M/\Delta \Gamma) + \beta^2}{\frac{1}{2}\Delta \Gamma^2 + \Delta M^2}} \right) \\
\left( \frac{\epsilon}{\epsilon^* + \frac{\beta}{\frac{1}{2}\Delta \Gamma + i\Delta M}} \right) \\
1
\]

to be contrasted with the conventional pure \(K_S\) state (20).

5 Phenomenological Analysis of Possible \(CPT\) and Apparent \(CP\) Violation in the Neutral Kaon System

The framework for treating experimental observables in our density matrix formalism for neutral \(K\) decays was introduced in ref. [10] and reviewed in ref. [9]. The experimental value of any observable \(O\) is given by the expectation value

\[<< O >> = \text{Tr}\rho O \]
where $O$ is represented by a suitable hermitian $2 \times 2$ matrix. We express the matrices $O_{ij}$ in the $K_{1,2}$ basis introduced earlier. The most commonly measured observables are the $K \rightarrow 2\pi$ decay rate $O_{2\pi}$:

$$O_{2\pi} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (27)$$

and the semileptonic $K$ decay rates:

$$O_{\pi^-l+\nu} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad (28)$$

and

$$O_{\pi^+l-\nu} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad (29)$$

out of which the semileptonic decay asymmetry observable $\delta$ can be constructed

$$\delta \equiv \frac{\Gamma(\pi^-l+\nu) - \Gamma(\pi^+l-\nu)}{\Gamma(\pi^-l+\nu) + \Gamma(\pi^+l-\nu)} \quad (30)$$

Another variable which we discuss here for the first time in this framework is the $K \rightarrow 3\pi^0$ decay rate $O_{3\pi}$:

$$O_{3\pi} = (0.22) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (31)$$

where the prefactor is determined by the measured $3\pi^0$ branching ratio for $K_L \rightarrow 3\pi^0$. (Strictly speaking, there should be a corresponding prefactor of 0.998 in the formula (27) for the $O_{2\pi}$ observable.)

It is a simple matter to combine the above formulae for $K$ decay observables with the asymptotic solutions (24, 25) to the non-quantum-mechanical equations of motion for the density matrix to obtain parametrizations of the values of the observables $R_{2\pi}^L \equiv << O_{2\pi} >>_L$, $\delta_{L,S}$, and $R_{3\pi}^S \equiv << O_{3\pi} >>_S /0.22$:

$$R_{2\pi}^L = |\epsilon|^2 + \frac{\gamma}{\Delta \Gamma} + \frac{4\beta}{|\Delta \Gamma|^2} |\epsilon| \sin \phi_\epsilon - \frac{4\beta^2}{|\Delta \Gamma|^2} \cos^2 \phi_\epsilon$$

$$\delta_L = 2 \text{Re}[\epsilon(1 - \frac{i\beta}{Im M_{12}})]$$

$$\delta_S = 2 \text{Re}[\epsilon(1 + \frac{i\beta}{Im M_{12}})]$$

$$R_{3\pi}^S = |\epsilon|^2 + \frac{\gamma}{\Delta \Gamma} - \frac{4\beta}{|\Delta \Gamma|^2} |\epsilon| \sin \phi_\epsilon - \frac{4\beta^2}{|\Delta \Gamma|^2} \cos^2 \phi_\epsilon \quad (32)$$

that can be compared with experiment.
We are now in a position to confront the above formalism with salient aspects of the available data [3]. These include the well-measured rate for $K_L \to 2\pi$:

$$\sqrt{R_{2\pi}^L} \simeq (2.265 \pm 0.023) \times 10^{-3}$$

(33)

the observed $K_L$ semileptonic decay asymmetry:

$$\delta_L = (3.27 \pm 0.12) \times 10^{-3}$$

(34)

and an upper bound on the rate for $K_S \to 3\pi^0$:

$$\sqrt{R_{3\pi}^S} < 1.3 \times 10^{-2}$$

(35)

For our purposes, a more relevant quantity is the difference between $R_{2\pi}^L$ and $R_{3\pi}^S$:

$$\delta R \equiv R_{2\pi} - R_{3\pi} = \frac{8\beta}{|\Delta \Gamma|} |\epsilon| \sin \phi \epsilon \sqrt{1 + \tan^2 \phi \epsilon}$$

(36)

Now also available is a recent preliminary measurement [18] of the $K_S$ semileptonic decay asymmetry by the CPLEAR collaboration:

$$\delta_S \simeq (8.5 \pm 7.6 (\text{stat}) \pm 15.5 (\text{syst})) \times 10^{-3}$$

(37)

Instead of $\delta_S$, it is more relevant to plot the difference between $\delta_L$ and $\delta_S$:

$$\delta \epsilon \equiv \delta_L - \delta_S = -\frac{8\beta}{|\Delta \Gamma|} \sin \phi \epsilon \sqrt{1 + \tan^2 \phi \epsilon} = -\frac{8\beta}{|\Delta \Gamma|} \sin \phi \epsilon \cos \phi \epsilon$$

(38)

In addition to these pieces of information which we include exactly in our analysis, important measurements are also available [19] of the interferences between $K_S$ and $K_L$ decays into the $\pi^+ \pi^-$ and $2\pi^0$ final states. These are conventionally used to constrain the phase of the $CP$-violating mass mixing parameter $\epsilon$, which is then compared with the value predicted on the basis of the observed values of the $K_L - K_S$ mass and lifetime differences:

$$\phi \epsilon = \arctan \left( \frac{2 \Delta M}{\Delta \Gamma} \right)$$

(39)

This comparison yields

$$\delta \phi \epsilon \simeq (2.3 \pm 1.4)^0$$

(40)

which is consistent with zero and hence the absence $CPT$ violation at the 1.5$\sigma$ level. Since we do not wish to overplay any apparent discrepancy with $CPT$ invariance without presenting a global analysis of the available data [30], for the purposes of this paper we interpret the limits (41) as corresponding to $|\delta \phi \epsilon| < 4.6^0$. In our case a fit to the data is more complicated than in conventional quantum-mechanics, because we have three $CPT$-violating parameters ($\alpha, \beta, \gamma$), instead of the single quantum-mechanical parameter $\delta M$ that is usually discussed. The value of $\delta M$ is
usually obtained from a global fit to all the available data on \( K \to 2\pi \) decays, from the short-time \( K_S \) region through the intermediate-time interference region to the long-time \( K_L \) region. Such a complete analysis goes beyond the scope of this paper \[37\]. However, since these data have been used to bound the quantum-mechanical CPT-violating parameter \(|\delta M| \lesssim 2 \times 10^{-18} \text{ GeV}\), and since \( \beta \) appears (22) in similar entries in the time evolution matrix, we expect that such a fit would yield

\[
|\frac{\beta}{\Delta \Gamma}| \lesssim 10^{-4} \text{ to } 10^{-3}
\]

depending on the values of \( \alpha \) and \( \gamma \).

We will present the results of our analysis in the \((\beta, \gamma)\) plane, since the asymptotic data that we chiefly use do not depend on the third CPT-violating parameter \( \alpha \), which would need to be included in a global fit. The value of the CPT-violating parameter \( \epsilon \) is fixed at any point in the \((\beta, \gamma)\) plane by the relatively well-determined value \((33)\) of \( R_{2\pi}^L \):

\[
|\epsilon| = -2\beta \frac{\Delta \phi \epsilon}{|\Delta \Gamma|} + \frac{4\beta^2}{|\Delta \Gamma|^2} - \frac{\gamma}{|\Delta \Gamma|} + R_{2\pi}^L
\]

where we take \( \phi \epsilon \) from equation \((39)\) and we use this formula to plot contours of \( \epsilon \) in the subsequent graphs. The other constraints \((34, 35, 36)\) can be expressed in terms of \( R_{2\pi}^L, \beta \) and \( \gamma \):

\[
\frac{\gamma}{|\Delta \Gamma|} = R_{2\pi}^L - \frac{\delta L}{4\cos^2 \phi \epsilon} - 4\delta L \tan \phi \epsilon \frac{\beta}{|\Delta \Gamma|} - 16\sin^2 \phi \epsilon \frac{\beta^2}{|\Delta \Gamma|^2} + 4\frac{\beta^2}{|\Delta \Gamma|^2}
\]

\[
\frac{\gamma}{|\Delta \Gamma|} = R_{2\pi}^L - \frac{(\delta R)^2}{64\sin^2 \phi \epsilon} \frac{|\Delta \Gamma|}{\beta} - \frac{1}{2}\delta R + 4\cos^2 \phi \epsilon \frac{\beta^2}{|\Delta \Gamma|^2}
\]

where

\[
\delta R = 4\delta L \tan \phi \epsilon \frac{\beta}{|\Delta \Gamma|} + 16\sin^2 \phi \epsilon \frac{\beta^2}{|\Delta \Gamma|^2}
\]

and

\[
\delta \delta = -\frac{8\beta}{|\Delta \Gamma|} \sin \phi \epsilon \cos \phi \epsilon = (5.2 \pm 17.3) \times 10^{-3}
\]

which we plot as bands in the \((\beta, \gamma)\) plane.

Figures 1 and 2 show this plane on logarithmic scales for \( \beta > 0 \) and \( < 0 \) respectively. We see that consistency between \( R_{2\pi}^L \) and the relatively well-determined value of \( \delta L \) specifies a very narrow band in the \((\beta, \gamma)\) plane. The origin \( \beta = \gamma = 0 \), which corresponds to the conventional state-vector analysis with \(|\epsilon| = 2.265 \times 10^{-3}\) and without CPT violation, lies comfortably within this band. This point can be seen more clearly in figure 3, which shows smaller values of \( \beta \) on a linear scale. Also shown in figures 1, 2, 3 are the constraints \((44, 46)\), which allow relatively large values of \(|\beta|\) and are also consistent with \( \beta = \gamma = 0 \) as in conventional quantum mechanics.
The constraints (44, 46) are consistent not only with the conventional quantum-mechanical $CP$-violating and $CPT$-conserving solution $|\varepsilon| \simeq 2.265 \times 10^{-3}$, $\beta = \gamma = 0$, but also with a purely $CP$-conserving and $CPT$-violating solution $\varepsilon = 0, \beta = -0.55 \times 10^{-3}, \gamma = 0.58 \times 10^{-5}$, as seen in fig. 4. This radical solution is also compatible with the indicative version (41) of the intermediate-time constraint, though it could well be ruled out by a more detailed intermediate-time analysis [37]. As already mentioned, we do not take (40) as significant evidence of $CPT$ violation.

6 Speculations

It is with some trepidation that we develop in this section the comments made in the previous paragraph. The $CP$ violation apparently seen in the neutral kaon system all of 28 years ago [2] was at first a big surprise, and did not fit naturally within the theoretical framework then existing. However, $CPT$ violation was even more sacrosanct. Sakharov [38] pointed out that $CP$ violation could solve very neatly one of the fundamental puzzles of cosmology, and then $CP$ violation emerged naturally from the three-generation Standard Model [39]. So the conventional wisdom learned to embrace $CP$ violation, and innumerable theoretical and experimental papers have adopted it, comforted by experimental fits to the data that did not require any $CPT$ violation [3]. It would take a lot to overturn this agreeable consensus.

We are not yet equipped to do so, because we have not yet made a complete fit to all the available experimental data, including those in the intermediate-time region where (conventionally) the phase of the $CP$-violating parameter $\varepsilon$ is checked. For the time being, the measurement (40) should be treated very cautiously. We have found that all the available asymptotic (short- and long-time) data are consistent with $CP$ invariance and the generalized parametrization of $CPT$ violation that we have derived from the modification of quantum mechanics that we have proposed.

However, we think there is a chink in the armour of the conventional wisdom that needs to be explored. As already pointed out, we cannot yet even exclude the radical possibility that the $CP$-violating parameter $\varepsilon$ actually vanishes, and that the effects usually ascribed to $CP$ violation are in fact due to non-quantum-mechanical $CPT$ violation.

---

2It should be remarked that in this formalism the $T$-reversal violation that defines the arrow of target time is of the same nature in the microcosmos and the macrocosmos, and stems from the unitarity of the effective low-energy world and the associated irreversibility of the renormalization group flow at the string world-sheet level [24].

3We are fully aware of the dramatic consequences this outlandish possibility would have for searches for $CP$ violation in the $B$ system, the neutron, and atomic electric dipole moments (doomed), the axion (demotivated) and cosmological baryogenesis (unscathed).
This possibility could perhaps be excluded by refitting the available data on
$K \rightarrow 2\pi$ using our generalized parametrization, and/or reducing somewhat the
experimental errors on the $CPT$-violating quantities $\delta \delta$ and $\delta R$. One would at least
be able to refine the present constraints on the $CPT$ violating parameters $\beta$ and $\gamma$
and (using intermediate-time data) begin to constrain $\alpha$.

We are familiar with the historical fact that more discrete symmetries are violated
as one makes more precise microscopic measurements: first $C$ and $P$ violation in
the weak interactions, then $CP$ violation, and next ...? We have argued in this
paper on the basis of string theory that $CPT$ violation could show up at a level not
far below the sensitivity of present experimental limits. We cannot make precise,
quantitative estimates of the possible magnitude of $CPT$-violating effects. However,
they stand out as a possible distinctive phenomenological signature of string theory.
Could the era of string phenomenology be ushered in by such a non-perturbative,
non-quantum-field-theoretical and non-quantum-mechanical effect?

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Figure Captions

Figure 1 - The $(\beta, \gamma)$ plane on a logarithmic scale for $\beta > 0$. We plot contours of the conventional CP-violating parameter $|\epsilon|$, evaluated from the $K_L \to 2\pi$ decay rate. The dashed-double-dotted band is that allowed at the one-standard-deviation level by the comparison between measurements of the $K_L \to 2\pi$ decay rate and the $K_L$ semileptonic decay asymmetry $\delta_L$. The dashed line delineates the boundary of the region allowed by the present experimental upper limit on $K_S \to 3\pi^0$ decays $(R^L_{2\pi} - R^S_{3\pi})$ and a solid line delineates the boundary of the region allowed by a recent preliminary measurement of the $K_S$ semileptonic decay asymmetry $\delta_S$. A wavy line bounds approximately the region of $|\beta|$ which may be prohibited by intermediate-time measurements of $K \to 2\pi$ decays.

Figure 2 - As in Fig. 1, on a logarithmic scale for $\beta < 0$.

Figure 3 - As in Fig. 1, on a linear scale for the neighborhood of $\beta = 0$.

Figure 4 - As in Fig. 1, in a blown-up region around $\beta = -0.55 \times 10^{-3}$, $\gamma = 0.58 \times 10^{-5}$, corresponding to the absence of CP violation : $\epsilon = 0$. 

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