Fractal evaluation aspects in characterizing the roughness of a driving wheel from a locomotive

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Abstract. For comprehending tribological phenomena such as wear, friction, contact deformation and tightness of contact joints is the essential characterisation of the multiple scale topography of roughness surfaces. Wheel roughness obtained through direct measurement and then a description of wheel roughness has been obtained using the fractal function. The statistical parameters and fractal sizes for drive wheels which ran 2000 km and another ones new wheels have been analysed. By registering the results and programmable automaton of the roughness, the principal statistical characteristics were determined to be stands out the Abbott-Firestone curve and respectively, the fractal character.

1. Introduction
Nowadays, one of the essential sources of vibration in urban areas is the railway transports. It has been observed that aspects regarding the vibrations, can be harmful to the human body, in particular in sectors like rail and construction [3].
In the railway field, vibrations caused by the passing of trains arise as one of the primary sources of potential disturbance for the environment near railway lines. Roughness plays a crucial role in the generation of vibration. The vibrations levels of surroundings and its nearby buildings induced by running trains considered in relation with speed [4] rail roughness [5, 6] and general of vibratory behaviour of the superstructure [7]. Studies have been made concerning the similarity of the state of the surface, in particular, geometry roughness, comparing of multiple scale length in fractal terms. A discrete structure-function gives a suitable description. Majority of surfaces used this structure functions, and it was characterised with the r.m.s.-value of the roughness, transition length between fractal behaviour at high wavenumbers and the fractal dimension in the fractal region, respectively [8].

The first works in which the asperities were treated with the help of the Gaussian distribution were those of Greenwood and Williamson [9] followed by those of Nayak [10], Bush et al. [11], Greenwood [12] and McCool [13]. The roughness determined by the fractals taking into account an elastic deformation was presented by Ciavarella et al. [14], Persson [15] and Archard [16].
On the other hand, Ciavarella and Demelio [17, 18] and subsequently Gao and Bower [19] extended their calculations taking into account a plastic deformation and analysed the contact between a rigid surface and an elastic surface, both surfaces with roughness with fractal character. In Romania, D. Pavelescu, together with A. Tudor [20] dealt with the micro-metric roughness of the engineering surfaces.
of some machine elements. The proposed paper describes, for the first time, the roughness of a driving wheel with the help of fractal theory.

2. Roughness parameters of wheel
Surface roughness is formed by fluctuations of short wavelength in the surface, characterised by hills or valleys of varying amplitude and spacing. The irregularities of rail (roughness and corrugation) and wheel (roughness and plane defect) induce the vibrations, mainly, as an effect of contact load. The corrugation could have values typically 50 µm and the wavelength of 50 mm. The amplitude of corrugation can be higher than wavelengths when these are superior to 50 mm [21]. To describe the roughness of a surface, in general, it is the possibility to choose the amplitude parameters, spacing parameters and hybrid parameters [22]. To characterise surface topography, to measure the vertical characteristics there are used the amplitude parameters. For horizontal aspects, there are spacing parameters and the combination of the first two, are hybrid parameters.

3. Fractal geometry of wheel roughness
To characterise the multiscale self-affine topography for a railway wheel, the fractal geometry was used, using the fractal parameters that are independent of the scale. Also, the fractal character of the roughness of the wheel surfaces was investigated by analysing the structure-function using the fractal parameters DwandGw[1, 2, 19, 20]. The fractal approach to other approaches such as the power of spectral density has the potential to predict the behaviour of a surface phenomenon at any length scale. This paper aims to analyse the roughness of the wheel surface through different methods: fractal parameters, bearing area curve and amplitude parameters.

Analysing the surface of the wheel, the method algorithm was similar to the one used in our paper [1] where has been developed for the rail. Based on measurements made with m | wheel device which offered the dates of amplitude parameters, it has been developed the fractal character derived from surface topography. According to the vertical characteristics of the surface of the system wheel-rail, the vibrations occur in vertical coordinated in movement.

4. Roughness measurement and results
Roughness on the wheel surface performed with m | wheel device (Figure 1). For the roughness of the wheel, the bogie of the LE-MA locomotive from the Softronic Craiova was used. (figure 2).

The wheel roughness measuring device m|wheel used to perform the tests belongs to the laboratories Romanian Railway Notified Body (RRNB). This equipment has been used to record the acoustically relevant roughness for the investigation of the polygonization (out-of-roundness), wheel flat and the exact wheel diameter to examine the wear behaviour. To determine the roughness of the wheel, scanning in running direction of the wheel of 1 mm was carried out (determination of the diameter
with an accuracy of ± 0.15 mm. For the roughness of the wheel, the bogie of the LE-MA locomotive from the Softronic Craiova used. Measurements for half of the bogie mounted with roughness wheels and another half of the bogie attached with new wheels were carried out. The obtained results were processed with one soft developed in Mathcad program.

4.1. Hybrid parameter- Bearing Area Curve (BAC)

The roughness parameters of the wheel allow describing the topography of the surface. The analyzes based on the BAC for tribological characterization of the surface topography revealed information regarding the wheel's rolling capabilities. BAC shows us the dependence of the height of the roughness of the real contact area [1]. The BAC curves are shown in Figure 3 and 4 for the roughness of a new wheel, respectively, for used wheel.

![Figure 3. BAC (za1) for roughness of used wheel](image)

![Figure 4. BAC (za1) for roughness of new wheel](image)

4.2. Amplitude parameters

The amplitude parameters are useful for the qualitative and quantitative assessment of roughness wheel. These parameters have been analysed (ISO 4287-1:1984 Surface roughness) are listed in table 1. The results are obtained for three new wheels and three used wheels, respectively.

| Parameter/Wheel type          | \(^{1}\text{nw}_1\) | \(\text{nw}_2\) | \(\text{nw}_3\) | \(^{2}\text{uw}_1\) | \(\text{uw}_2\) | \(\text{uw}_3\) |
|------------------------------|---------------------|----------------|----------------|---------------------|----------------|----------------|
| Ra [μm]                      | 0.697               | 11.769         | 14.055         | -24.181             | -46.624        | -127.400       |
| Rq [μm]                      | 45.470              | 19.055         | 20.879         | 124.529             | 133.051        | 178.165        |
| Ten points ISO [μm]          | 1.000               | 0.100          | 4.080          | 54.260              | 27.360         | -24.140        |
| Ten points DIN [μm]          | 0.620               | 0.170          | 2.120          | 27.130              | 27.140         | -12.070        |
| Skewness (Rsk) -             | 0.175               | 2.500          | 2.168          | -1.261              | -1.499         | -1.370         |
| Kurtosis (Rku) -             | 2.114               | 4.177          | 8.390          | 3.738               | 4.177          | 2.116          |

\(^{1}\text{new wheel (nw)}\)

\(^{2}\text{used wheel (uw)}\)

5. Characterisation geometry of surface topography by fractal

The characterisation of surface topography with fractal geometry it is an option for many researchers in the study the contact surfaces more detailed[1, 12-15]. The fractal theory has been applied to characterise the surface of the wheel, and also in the algorithm from [1] where has been developed for the rail. The fractal analysis methods can replace the disadvantages of the surface analysis methods by the conventional method. Lately, various methods have been developed to characterise the size of a fractal set, such as the size of the box, the size of the compass etc. [1, 11, 12]. To calculate the size of the fractal parameters for the roughness of a wheel (\(D_w\))
and the topothesy length \( L_{tw} \), we used the structure function method. Theroughness wheel signal \( y_w(x_i) \) is investigated as a function to delay \( t \).

The structure-function for the wheel roughness is presented in equation 1 and it is in continuum form:

\[
S_w(t) = E[(y_w(x) - (y_w(x + t))^2] = c_w(|t|^{4-2D_w})
\]

(1)

It is known that the topothesy length of roughness wheel \( L_{tw} \) is defined that displacement which r.m.s. slope has a unity statistically:

\[
S_w(L_{tw}) = L_{tw}^2.
\]

(2)

The structure-function for wheel fractal roughness is presented in equation 3:

\[
S_w(t) = L_{tw}^{2D_w-1}(|t|)^{4-2D_w}
\]

(3)

where \( D_w \) and \( L_{tw} \) are fractal parameters of wheel roughness. These parameters can be determined by graphical form (log-log) of the structure-function, similarly to roughness of rail [1].

The structure-function of roughness wheel, in discrete form, \( S_w(N,k) \), of a signal \( y_w(x_i) \) is:

\[
S_w(N,k) = \frac{1}{N-k} \sum_{i=1}^{N-k} (y_w(i+k) - y_w(i))^2.
\]

(4)

For example, in Figures 5 a and b show the structure function of the roughness profile corrected for the used and new wheel and figure 6 highlighting the comparing structure-function for both wheel.

**Figure 5a.** Structure function for used wheel  
**Figure 5b.** Structure function for new wheel  
**Figure 6.** Comparing structure-function for new and used
In Table 2 are presented the results for fractal parameters. It can be seen that the slope of the curve for the two wheels is different up to around 1.5, both increasing exponentially and then overlapping them to the value of 3.5. It is furthermore noticeable for the new wheel a sharp decrease for structure-function in discrete form and an increase for the used wheel. In terms of the topological length $L_{tw}$ and the fractal parameter, $D_w$ has a higher value for the used wheel compared to the new wheel. It is observed that structure function in log-log coordinate \((\log(S(N,k)) - \log(x(k)))\) has a linear function, having slope $\beta$. In this case, can be nominate that roughness wheel has a fractal character [1, 9-16].

**Table 2. Fractal parameters**

| Parameter | wheel type | $^1$nw1 | nw2 | nw3 | $^2$uw1 | uw2 | uw3 |
|-----------|------------|---------|------|------|---------|------|------|
| $B_w$     |            | 0.816   | 0.872| 0.737| 0.369   | 0.43 | 0.559|
| $D_w$     |            | 1.592   | 1.564| 1.632| 1.815   | 1.785| 1.720|

$^1$New wheel (nw)

$^2$Used wheel (uw)

For the fractal character roughness (parameters $D_w$, $L_{tw}$, $G_w$ (function to $L_{tw}$)), it was associated with a continuous and undifferentiated Weierstrass-Mandelbrot (W-M) function $y_w(x)$ [1,9, 14, 16, 18]:

\[
y_w(x, D_w, G_w, \gamma_w) = G_w D_w^{-1} \sum_{n=1}^{\infty} \frac{\cos(2\pi \gamma_w^n x)}{y_w^{(2-D_w)n}}, \quad 1 < D_w < 2 \text{ and } \gamma_w > 1,
\]

with frequency modes $\gamma_w^n$. This frequency is a function to wavelength ($\lambda_{nw}$) of wheel roughness and be calculated similarly to our paper [1].

Figures 7 a and b exemplify the fractal evaluation of the wheel roughness by the Weierstrass-Mandelbrot (W-M) function for small part of wheel.

![Figure 7 a. W-M ($y(x, D_w, G_w, \gamma)$) for roughness of used wheel](image1)

![Figure 7 b. W-M ($y(x, D_w, G_w, \gamma)$) for roughness of new wheel](image2)

**6. Conclusions**

The roughness, corrugations and circularity of features wheel are the fundamental cause of rolling noise and vibrations. The amplitude parameters, through its six two-dimensional statistical parameters, provided us with information regarding the average statistical properties, the shape of the height distribution histogram, and about the extreme properties (peak and valley). BAC shows how the profile bearing length ratio varies with a level in both situations for a new and used wheel and also could be observed that the area. Structure-function comes to rest on towards the end of increment size for each scan length. The WM function presents for both types of wheels, new and respectively used, profiles using three values of the gamma scaling parameter Mathcad's computational program has been allowed statistical analysis, correction of aberrant spikes and valley, deduction of the BAC, deduction of power spectral density, and fractal parameters of the microgeometry of new and used wheels.
Acknowledgements
The authors feel thankful for the support from Softronic Craiova, Romania and Romanian Railway Authority.

7. References

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