CP-violation phases and Majorana neutrino magnetic moments in left-right models

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Abstract. An implication of nonzero neutrino masses is the existence of neutrino magnetic moments, which arise in extensions of the Standard Model. Among the whole set of electromagnetic properties, these physical quantities have received much attention, both theoretically and experimentally. In the present paper we review the contributions to neutrino magnetic moments from new physics described by a left-right model, with Majorana neutrinos, which might be as large as $10^{-11} \mu_B$. These electromagnetic moments depend on Majorana phases. It turns out that, in presence of CP violation, specific sets of values of these phases can cancel up to two magnetic moments, while the remaining one must necessarily be nonzero and large.

1. Brief chronicle of neutrino mass

Originally proposed by W. Pauli in 1930, and called neutrons at the time, neutrinos were assumed to be electrically neutral fermions with a tiny mass. It was at the famous experiments in the Savannah River Plant, leaded by Clyde Cowan and Frederick Reines, where the existence of these elusive particles was finally confirmed by the observation of antineutrinos through the process $\bar{\nu} + p \rightarrow e^+ + n$ [1], in 1956. A year later, Bruno Pontecorvo proposed, for the first time, the phenomenon of neutrino oscillations [2], according to which a neutrino produced by a source with a definite flavor has a nonzero probability of being measured, after traveling some distance, with a different flavor. For instance, in the two-family formalism oscillations of ultrarelativistic neutrinos in vacuum are described by the transition probability [3, 4]

$$P(\nu_e \rightarrow \nu_\mu) = \sin 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right),$$

where $E$ is the energy of a massless neutrino, $L$ is the distance traveled by it, $\theta$ is a mixing angle and $\Delta m^2$ is a difference of squared neutrino masses. The presence of the mixing angle $\theta$ in Eq. (1) illustrates that neutrino mixing is a necessary ingredient for neutrino oscillations to

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1 Even so, note that neutrinos are assumed to be massless in the Standard Model.
occur. In the case of three families such mixing is described by the unitary transformation [3, 5]

\[ |\nu_\alpha\rangle = \sum_{j=1}^{3} U_{\alpha j}^* |\nu_j\rangle, \quad (\alpha = e, \mu, \tau) \] (2)

that relates neutrino mass eigenstates, \( |\nu_j\rangle \), to flavor states, \( |\nu_\alpha\rangle \), which participate in charged currents involving charged leptons of definite flavors. Another significant element of Eq. (1) is the squared-mass difference \( \Delta m^2 \), because it sets the requirement that neutrino masses must be different of each other, and particularly nonzero, for neutrino oscillations to exist.

From around 1967 and for about three decades, Raymond Davis Jr. headed an experiment in the Homestake mine at Lead, South Dakota, with the purpose of measuring neutrinos produced by nuclear fusion in the Sun [6, 7]. Following the solar model, John Bahcall performed a theoretical calculation of the expected rate of solar electron-neutrinos that should be detected\(^2\). The result was that the experiment of Davis and collaborators only measured 1/3 of the solar neutrinos predicted by Bahcall. Such disagreement, known as the solar neutrino problem, was solved by the confirmation that neutrinos oscillate. The first experimental evidence of neutrino oscillations was observed in 1998 by the Super-Kamiokande Collaboration, which aimed at atmospheric neutrinos [9]. Under direction of Takaaki Kajita, this collaboration was able to establish the bounds \( \sin^2 2\theta_{23} > 0.82 \) and \( 4 \times 10^{-4} \, \text{eV} < \Delta m^2 < 6 \times 10^{-3} \, \text{eV} \) on the atmospheric mixing angle \( \theta_{23} \) and the squared-mass difference \( \Delta m^2 \). Shortly after, Arthur B. McDonald leaded measurements, at the Sudbury Neutrino Observatory, of fluxes of neutrinos coming from the Sun [10]. It was found that the sum of all the flux components, corresponding to the different neutrino flavors, was consistent with the solar model, used in Bahcall’s calculations, thus finally settling the discrepancy. In 2012 the Daya Bay and RENO Collaborations measured the mixing angle \( \theta_{13} \) [11, 12], which is the last piece of the confirmation that neutrinos oscillate and thus that they mix and they are massive.

2. Dirac and Majorana neutrinos
The confirmation that neutrinos oscillate, which requires the existence of neutrino mixing and neutrino mass, is a quite relevant conclusion, since it incarnates experimental evidence of new physics, beyond the reach of the Standard Model. Moreover, it opens the question of whether neutrinos are Dirac or Majorana fermions. The Dirac equation can be split into two coupled equations for chiral spinor fields, \( \psi_L \) and \( \psi_R \):

\[ i\gamma^\mu \partial_\mu \psi_L = m \psi_R, \quad i\gamma^\mu \partial_\mu \psi_R = m \psi_L, \] (3)

that are decoupled if \( m = 0 \), in which case the corresponding fermion field is described by a two-component spinor. It was Ettore Majorana who realized that it is also possible to describe a massive fermion by using a two-component spinor if the chiral fermions \( \psi_L \) and \( \psi_R \) are not independent, but they fulfill the condition \( \psi_R = C \psi_L^T \) [3, 13], where \( C \) is the charge-conjugation matrix. This yields the equations

\[ i\gamma^\mu \partial_\mu \psi_L = m C \psi_L^T, \quad \text{(Majorana equation)} \] (4)

\[ \psi = C \psi^T, \quad \text{(Majorana constraint)} \] (5)

with Eq. (5) indicating that Majorana fermions \( \psi \) coincide with their antiparticles [13]. For the last statement to be true, note that Majorana fermions must be electrically neutral particles,

\(^2\) A discussion can be found in Ref. [8].
because otherwise electric charge would set a clear difference among particles and antiparticles. Since neutrinos are massive and neutral particles, they might be either Dirac or Majorana fermions. While this issue is nowadays an open question and theoretical and experimental investigations on the matter are being carried out, simplicity in the sense of the number of degrees of freedom has been used as an argument favoring the scenario of Majorana neutrinos [14].

In general, a main difference between Dirac and Majorana fermions is the number of degrees of freedom characterizing them. If $CPT$ conservation is assumed, the former have 4 degrees of freedom (2 helicities, particle-antiparticle), whereas Majorana fermions have only 2 (2 helicities) [3]. Lepton number violation is currently being used to establish the nature of neutrinos. Experiments that are nowadays looking for the elusive neutrinoless double beta decay are well motivated, since the observation of this rare process, forbidden in the Standard Model, would be evidence pointing towards a world in which neutrinos are Majorana particles. A claim of a measurement of this physical process was made some years ago [15], but up to now this discussion has not arrived at a conclusion [16]. Dirac neutrinos are also different from Majorana ones in the parametrization of the matrix describing neutrino mixing, also known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. The parametrization of the PMNS mixing matrix for Dirac neutrinos looks the same as that used for the Cabibbo-Kobayashi-Maskawa (CKM) matrix [17], which mixes quarks in the Standard Model [3]. The PMNS matrix is conveniently written in terms of three mixing angles, $\theta_{12}, \theta_{23}, \theta_{13}$, and one complex $CP$-violating phase, $\delta$, which is known as the $Dirac\ phase$ [3, 5]. In the case of Majorana neutrinos, the mixing matrix, $U$, can be written as $[3, 5] \ U = V \ P$, where $V$ is a CKM-like matrix, with the aforementioned parametrization, and $P$ is a diagonal matrix that is given by $P = diag(1, e^{i\varphi_2}, e^{i\varphi_3})$, where $\varphi_2$ and $\varphi_3$ are the so-called Majorana phases, which are sources of nonconservation of $CP$ symmetry.

It is worth emphasizing that neutrino oscillations are not sensitive to Majorana phases [3]. On the other hand, measurements of oscillations of solar, atmospheric and reactor neutrinos have been translated into estimations of the values of all other parameters of neutrino mixing [17, 18]. The current best values of so-called $atmospheric\ angle$ $\theta_{23}$, which is the largest mixing angle, are so far compatible with $\pi/4$. In 2015 the T2K Collaboration reported that normal hierarchy of neutrino mass is weakly favored over inverted hierarchy [19]. Then this Collaboration provided the values $\sin^2\theta_{23} = 0.528^{+0.055}_{-0.038}$ and $|\Delta m^2_{32}| = (2.51 \pm 0.11) \times 10^{-3} \ eV^2$. One of the objectives of current and future experimental investigations at experimental facilities such as T2K and NOvA is the determination of the deviation, with sign, of $\theta_{23}$ from $\pi/4$ [18]. In Ref. [20] recent results on solar neutrinos from the Super-Kamiokande experiment are presented, featuring the value $\Delta m^2_{21} = (7.49^{+0.19}_{-0.17}) \times 10^{-5} \ eV^2$ and an estimation of the $solar\ angle$ $\theta_{12}$ given by $\sin^2\theta_{12} = 0.305 \pm 0.013$. Future improvements on the precision of these values are planned for experiments like JUNO, to start in 2020 [21]. It is considered that 2012 was quite important for neutrino physics because in this year the last mixing angle, $\theta_{13}$, was measured [11, 12] at a relatively large value. Furthermore, the determination of a nonzero value for this angle implied that $CP$ violation may be measured in the leptonic sector. Recently, the Daya Bay Collaboration updated its value of $\theta_{13}$, improving it to $\sin^22\theta_{13} = 0.084 \pm 0.005$ [22]. In contrast with the Majorana phases, the Dirac phase does introduce effects in neutrino oscillations, which are characterized by the $CP$ asymmetry $A_{CP}^{(\nu)} = P(\nu_l \rightarrow \nu_r) - P(\nu_l \rightarrow \nu_r)$ [3]. The T2K Collaboration has excluded values of $\delta$ within the ranges $(0.19\pi, 0.80\pi)$, for normal hierarchy, and $(-\pi, -0.97\pi)$ and $(-0.04\pi, \pi)$, for inverted hierarchy, at $90\%$ C.L. This collaboration pointed out that the value $\delta = -\pi/2$ is favored by data [23].

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3 Of course, the values of the parameters of neutrino mixing are quite different from those of the CKM matrix [17].
3. Electromagnetic properties of neutrinos

Even though neutrinos are at present known to be neutral particles, they may possess electromagnetic properties that are generated by quantum loops’ effects. The electromagnetic neutrino vertex $\nu_j \gamma_k \gamma_\mu$ is parametrized by [24, 25]

$$\Lambda^{jk}_\mu(q) = \left( \gamma_\mu - \frac{q_\mu q}{q^2} \right) \left[ f^{jk}_Q(q^2) + f^{jk}_A(q^2) q^2 \gamma_5 \right] - i\sigma_{\mu\nu} q^\nu \left[ f^{jk}_M(q^2) + i f^{jk}_E(q^2) \gamma_5 \right],$$  \hspace{1cm} (6)

where $q$ is the outgoing momentum of the external photon. The electromagnetic form factors $f^k_Q$, $f^k_A$, $f^k_M$ and $f^k_E$ are respectively named charge form factor, anapole form factor, magnetic form factor, and electric form factor. Those form factors with $j \neq k$ are called transition form factors, whereas those in which $j = k$ are known as diagonal form factors. Upon taking the photon on shell, which comes along with the condition $q^2 = 0$, such form factors define the electromagnetic moments. On general grounds [14, 24] and at the phenomenological level [26] it has been shown that the only nonzero electromagnetic form factor of a Majorana neutrino is the anapole form factor. This differs from the case of Dirac neutrinos, whose diagonal electromagnetic form factors can all be nonzero. In both cases, all transition form factors are allowed to exist [24].

Magnetic moments are the most widely studied static electromagnetic properties ($q^2 = 0$) of neutrinos. They have been calculated in the minimally extended Standard Model [27], in which the Standard Model is endowed with sterile right-handed neutrinos, yielding nonzero Dirac neutrino masses [3]. In this context, magnetic moments are of order $10^{-19} \mu_B$ [24], where $\mu_B$ is the Bohr magneton. Among a variety of searches for neutrino magnetic moments, the GEMMA experiment, which uses a Germanium detector that is placed near a reactor in the Kalinin Nuclear Power Plant, has established the best bound so far [28]: $\mu_\nu < 2.9 \times 10^{-11} \mu_B$, at 90\% C.L. The huge difference among the experimental sensitivity and the theoretical prediction of the minimal Standard Model extension, not likely to improve greatly in the short term [25], has been a motivation to explore different extensions of the Standard Model that generate large neutrino magnetic moments, with hope in measuring a large neutrino magnetic moment and thus evidence of new physics and its proper description. For instance, calculations of neutrino magnetic moments have been performed in the 331 model [29, 30, 31], Supersymmetry [32, 33], and in left-right extensions [34, 35, 36, 37, 38, 39].

4. Majorana neutrino magnetic moments in left-right

The defining characteristic of left-right models is the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [34, 35, 36], which is an extension of the electroweak group of the Standard Model. Originally meant to describe parity violation as a remnant of a spontaneously broken high-energy symmetry, a variety of versions of left-right models have been realized and explored. In particular, models with nonmanifest left-right symmetry, in which left and right coupling constants are assumed to be different of each other ($g_L \neq g_R$), are interesting because they keep the lower bound on the mass of heavy charged gauge bosons around the relatively light value 0.3 TeV [40], in contrast with minimal versions, in which lower bounds on such masses range within the TeV scale [41, 42]. A feature that distinguishes different left-right formulations is the scalar sector [37]. While scalar sectors made of Higgs doublets yield Dirac neutrinos, much attention has been paid to scalar sectors that involve scalar triplets, since they lead to Majorana neutrinos. Typically, in these models there are two scalar triplets, $\Delta_L$ and $\Delta_R$, and one scalar bidoublet, $\Phi$. In a first stage of symmetry breaking, the right triplet acquires a vacuum expectation value $\langle \Delta_R \rangle$ which is characterized by a high-energy scale $\nu_R$. This breaks the left-right symmetry group into the Standard Model electroweak symmetry, which is then spontaneously broken by the vacuum expectation
values $\langle \Delta L \rangle$ and $\langle \Phi \rangle$, of the remaining scalar structures, into the electromagnetic group. Then, a seesaw mechanism \[36\] drives the definition of light and heavy Majorana neutrino masses.

The enlargement of the Standard Model electroweak symmetry group, by the addition of an SU(2), introduces new gauge degrees of freedom. Among the new fields, an SU(2)$_R$ right charged boson $W_R$ is defined, which mixes, by an angle $\zeta$, with the left boson $W_L$, corresponding to the left group SU(2)$_L$ to define two charged bosons, $W_1$ (light) and $W_2$ (heavy), with definite masses:

$$W_{L}^{\pm\mu} = \cos \zeta W_1^{\pm\mu} - \sin \zeta W_2^{\pm\mu},$$

$$W_{R}^{\pm\mu} = e^{i\omega} \left( \sin \zeta W_1^{\pm\mu} + \cos \zeta W_2^{\pm\mu} \right),$$

where $e^{i\omega}$ is a complex phase. Such $W$ bosons couple to neutrinos and charged leptons through charged currents \[39\]

$$\sum_{a=1,2} \sum_{j=1,2,3} \sum_{a=e,\mu,\tau} \left[ W_{a\mu}^+ \gamma_\mu (v_{a\alpha} - a_{a,ja} \gamma_5) l_\alpha + W_{a\mu}^- \gamma_\mu (v_{a\alpha}^* - a_{a,ja}^* \gamma_5) l_\alpha \right],$$

with the coefficients $v_{a,ja}$ and $a_{a,ja}$ given in terms of left and right neutrino mixing matrices\(^4\), which are respectively denoted by $L_{a\alpha}$ and $R_{a\alpha}$. These factors also involve the left coupling constant, $g_L$, and the right one, $g_R$, as well, with the assumption $g_L \neq g_R$, mentioned above. Finally, the coefficients also incorporate the angles $\zeta$ and $\omega$. In the context of a left-right model with scalar triplets, the contributions of these currents to the Majorana neutrino transition magnetic moments are generated by the diagrams shown in Fig. 1\(^5\), but dominant contributions come only from the first two diagrams. The contributions to the magnetic dipole form factors can be written as \[39\]

$$f_{M}^{jk}(q^2) = \frac{me}{8\pi^2} \mu B \sum_{j=1}^{3} \sum_{a=1,2} \left[ (a_{a,ja} a_{a,ka}^* - a_{a,ka} a_{a,ja}^*) I_1 + (v_{a,ja} v_{a,ka}^* - v_{a,ka} v_{a,ja}^*) I_2 \right].$$

In this expression the factors $I_1$ and $I_2$ have been used to denote complicated parametric integrals, arisen because of the use of the technique of Feynman parameters. From Eq. (10), it can be appreciated that diagonal magnetic form factors vanish exactly, leaving only transition form factors, which is consistent with the properties of Majorana neutrinos. Note that the magnetic dipole form factors are exactly zero if the coefficients $a_{a,ja}$ and $v_{a,ja}$ are real quantities.

\(^4\) In Ref. [39], massive neutrinos were denoted by greek indices $\alpha, \beta \ldots$ and lepton flavors were labeled by lowercase indices $j, k, \ldots$. In the present paper we have swapped these labels, so $a, \beta, \ldots$ denote lepton flavors and $j, k, \ldots$ denote neutrino mass eigenstates.

\(^5\) In the case of Dirac neutrinos, only the first and the third diagrams of Fig. 1 would exist.
Such situation would correspond to charged currents that preserve \( CP \) invariance \([39]\). So, the existence of these neutrino magnetic moments requires violation of \( CP \). It must be emphasized that this result is gauge dependent, since so far the photon field has been assumed to be off shell. Indeed, a full calculation of these form factors would require scalar contributions and pseudo-Goldstone bosons contributions as well. Taking the photon on shell and using the unitary gauge reduces the number of diagrams and simplifies the expressions.

5. Magnetic moments and Majorana phases in left-right

Assuming that the left and right mixings are equal, that is \( \mathcal{L} = \mathcal{R} \), a scenario of \textit{maximal right mixing} is defined \([39]\). Considering a heavy \( W \) mass of few TeVs, the magnetic moment is expressed as \([39]\)

\[
|\mu_{jk}| \lesssim \mu_B (4 \times 10^{-11} \text{ GeV}^{-1}) |m_{1,jk} \sin \phi_{jk} + (m_{2,jk} - m_{2,kj}) \sin \delta \cos \phi_{jk} + (m_{2,jk} + m_{2,kj}) \cos \delta \sin \phi_{jk}|,
\]

which includes the Majorana phase differences \( \phi_{jk} = (\varphi_j - \varphi_k)/2 \), with \( \varphi_1 = 0 \), and the Dirac phase \( \delta \). Moreover, the factors \( m_{r,jk} \), whose definitions are given in Ref. \([39]\), involve the masses of the charged leptons and the mixing angles \( \theta_{12}, \theta_{23}, \) and \( \theta_{13} \) as well. These leading contributions are independent of small neutrino masses, which has been noticed and reported in different investigations \([37, 43]\). It is worth mentioning that any magnetic moment given by Eq. (11) depends on just one Majorana phase difference \( \phi_{jk} \). To illustrate the expression given in Eq. (11), we provide the graph in the left of Fig. 2, which shows the magnetic moment \( |\mu_{21}| \) as a function of the Majorana phase difference \( \phi_{12} \). The region colored in light green represents all the possible values of this Majorana neutrino magnetic moment, and the solid curve in the boundary of this region is the upper bound. To plot this function, we have used the value \( \delta = -\pi/2 \) for the Dirac phase (there is \( CP \) violation), as suggested by the results of the T2K Collaboration \([23]\). As it can be appreciated from this figure, the maxima of the neutrino magnetic moments lie around \( 10^{-11} \mu_B \). The horizontal red dashed line in this graph represents what the contribution would be if neutrinos were Dirac instead of Majorana. The zeros of the three transition magnetic moments correspond to the following Majorana phases \([39]\):

\[
|\mu_{21}| \approx 0, \quad \varphi_2 \approx -36.86^\circ, \quad (12)
\]
\[
|\mu_{21}| \approx 0, \quad \varphi_3 \approx -31.98^\circ, \quad (13)
\]
\[
|\mu_{32}| \approx 0, \quad \phi_{32} \approx -172.87^\circ, 7.13^\circ. \quad (14)
\]

Using Eqs. (12) and (13), note that \( \phi_{32} \approx 2.44^\circ \), which does not match Eq. (14). This means that a situation in which the three magnetic moments are zero cannot occur: in presence of \( CP \) violation, there is at least one large nonzero neutrino magnetic moment.

Another scenario, named the \textit{CKM-like mixing} \([39]\), is achieved by assuming that the mixing angles are very small, so that the neutrino right mixing resembles the pattern of the CKM matrix. Again, by taking the unitary gauge and considering a mass of the heavy charged boson that is in the range of a few TeVs, the Majorana neutrino magnetic moments can be expressed as

\[
|\mu_{jk}| \lesssim \mu_B (2 \times 10^{-11} \text{ GeV}^{-1}) (|c_{r,jk}| + |c_{\mu,jk}| + |c_{\tau,jk}|), \quad (15)
\]

where the coefficients \( c_{\alpha,jk} \), explicitly given in Ref. \([39]\), depend on the charged lepton masses, on the Dirac phase and on Majorana phase differences \( \phi_{jk} = (l_j - r_k)/2 \), where \( l_j \) and \( r_k \) denote left and right Majorana phases, respectively. In contrast with the scenario of maximal mixing, each magnetic moment depends on two Majorana phase differences, and not just on one. In each case, one of such phase differences is dominant in the sense that they appear in separate
terms and each of such terms depend multiplicatively on one charged lepton mass. The size of the mass corresponding to each phase difference then determines which one has a greater impact on the numeric result. This behavior is better noticed in the case of the magnetic moment $|\mu_{32}|$, which is illustrated by the right graph of Fig. 2. Three plots of $|\mu_{32}|$, as a function of $\phi_{23}$, have been added, each one corresponding to a different phase difference $\phi_{32}$: $\phi_{32} = \pi/4$ for the solid magenta curve; $\phi_{32} = \pi/2$ for the dashed black curve, with small dashes; and $\phi_{32} = 0$ for the dashed blue curve, with large dashes. Just as in the previous scenario, maxima are located around $10^{-11}\mu_B$. For $\delta = -\pi/2$ [23], the zeros of these magnetic moments are achieved at the left and right Majorana phases that we show below:

$$|\mu_{21}| \approx 0, \quad r_2 \approx -24^\circ, \quad l_2 = 0^\circ, \quad (16)$$

$$|\mu_{31}| \approx 0, \quad r_3 \approx 28.82^\circ, \quad l_3 = 180^\circ, \quad (17)$$

$$|\mu_{32}| \approx 0, \quad \phi_{23} \approx 6.40^\circ, -173.60^\circ, \quad \phi_{32} = 0^\circ, 180^\circ. \quad (18)$$

According to Eqs. (16) and (17), we have the values $\phi_{23} = (l_2 - r_3)/2 \approx -14.41^\circ$ and $\phi_{32} = (l_3 - r_2)/2 \approx 102^\circ$, which are not in agreement with Eq. (18). Therefore, as in the other scenario that we discussed, it is concluded that violation of $CP$ enforces the presence of at least one large magnetic moment, while two of such contributions are allowed to vanish.

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