A new formulation of the EPR argument is presented, one which uses John Bell’s mathematically precise local causality condition in place of the looser locality assumption which was used in the original EPR paper and on which Niels Bohr seems to have based his objection to the EPR argument. The new formulation of EPR bears a striking resemblance to Bell’s derivation of his famous inequalities. The relation between these two arguments – in particular, the role of EPR as part one of Bell’s two-part argument for nonlocality – is also discussed in detail.

I. INTRODUCTION

Eugene P. Wigner summed up a widely held view of the implications of Bell’s Theorem[1] when he stated: “In my opinion, the most convincing argument against the theory of hidden variables was presented by J. S. Bell. ....The...argument shows that any theory of hidden variables conforming to the postulate of locality is in conflict with quantum mechanics.”[2]

N. David Mermin echoed this view in his review article on “Hidden Variables and the Two Theorems of John Bell”:

“Bell’s theorem establishes that the value assigned to an observable must depend on the complete experimental arrangement under which it is measured, even when two arrangements differ only far from the region in which the value is ascertained – a fact that Bohm theory exemplifies, and that is now understood to be an unavoidable feature of any hidden-variables theory.

To those for whom nonlocality is anathema, Bell’s Theorem finally spells the death of the hidden-variables program.”[3]

In a nutshell, this common viewpoint seems to be based on the following apparently straightforward sort of reasoning: Bell proved that hidden-variables theories have to be nonlocal (in order to agree with the empirically correct predictions of quantum mechanics); nonlocality conflicts with relativity’s prohibition on super-luminal causation; relativity is true; so hidden-variables theories must be false.

Yet somehow this argument failed to compell the discoverer of the theorem in question. As Mermin concedes, Bell himself “did not believe that either of his no-hidden-variables theorems excluded the possibility of a deeper level of description than quantum mechanics.” How strange! Bell himself did not believe that what Mermin refers to as Bell’s two “no-hidden-variables theorems,” actually exclude hidden-variables! Why didn’t Bell accede to the interpretation of his own theorems offered by Wigner, Mermin, and so many other learned commentators on the foundations of quantum physics? Was this universally-recognized genius really so obtuse?

Mermin provides a clue in the continuation of the above block-quote:

“But not for Bell. None of the no-hidden-variables theorems persuaded him that hidden-variables were impossible. What Bell’s Theorem did suggest to Bell was the need to reexamine our understanding of Lorentz invariance...”

Thus Bell believed that his theorems brought out a conflict not merely between relativity and hidden-variables theories, but, rather, between relativity and the predictions of quantum theory as such, in any interpretation. Mermin briefly mentions this possible view in a footnote: “Many people contend that Bell’s Theorem demonstrates nonlocality independent of a hidden-variables program, but there is not general agreement about this.”

Evidently the “many people” referred to here by Mermin include in their ranks Bell himself.

Given the unique clarity and forthrightness of Bell’s writings, it is not surprising that we needn’t undertake extensive detective work to infer Bell’s views. He tells us quite explicitly both that and why he believes his theorems call into question our understanding of fundamental space-time structure, and not merely the attempt to supplement quantum mechanics with additional variables.

Here is the that: “...the nonlocality of quantum mechanics cannot be attributed to incompleteness, but is somehow irreducible.”[1, pg 244] Also: “The obvious definition of ‘local causality’ does not work in quantum mechanics, and this cannot be attributed to the ‘incompleteness’ of that theory.”[1, pg 256] And: “For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory...”[1, pg 172]

And here is the why: “That ordinary quantum mechanics is not locally causal was pointed out by Einstein, Podolsky, and Rosen, in 1935.”[1, pg 24] That is, according to Bell, the reason Bell’s Theorem spells trouble
locality implies
in the form:
tum theory referred to in (A), the EPR argument takes
imentally well-confirmed) empirical predictions of quan-
cannot be correct. Or, taking as indubitable the (exper-
then the claim that
– locality – that Bell claims EPR disproved: according
to EPR, if

(A) the quantum-mechanical predictions for
certain correlations are correct

and if

(B) quantum theory is required to respect the
principle of locality

then the claim that

(C) the theory is complete

cannot be correct. Or, taking as indubitable the (exp-
imentally well-confirmed) empirical predictions of quan-
tum theory referred to in (A), the EPR argument takes
the form:

(B) \rightarrow \neg(C). \quad (1)

That is, locality implies incompleteness. (We use the
symbol \neg to denote negation: e.g., \neg(X) should be read
“it is not the case that (X)”.)

But this is logically equivalent to the claim that

(C) \rightarrow \neg(B) \quad (2)

(i.e., completeness implies nonlocality) and also to the
claim

\neg(B) \text{ or } \neg(C). \quad (3)

(i.e., either locality or completeness must fail). Einstein
himself stated the conclusion of the EPR argument in
this last form:

“By this way of looking at the matter, it be-
comes evident that the paradox [EPR] forces
us to relinquish one of the following two as-
sertions:
(1) the description by means of the \psi-function
is complete.
(2) the real states of spatially separated ob-
jects are independent of each other.”\[4\]

Evidently, then, this is the basis for Bell’s assertion
that EPR showed that “ordinary quantum mechanics is
not locally causal.” For if we grant the premise (and
it is one that has been insisted on by advocates of the
Copenhagen approach ever since the 1930’s) that QM is
complete, it follows from the EPR argument that ordi-
ary QM itself is nonlocal. So if the EPR argument is
sound – if it is correct that quantum mechanics, if com-
plete, is nonlocal – then Bell’s own interpretation of the
significance of his theorems (and not the more widely-
held interpretation put forward by Wigner and Mermin)
would be validated. For this would show that, whether or
not one wishes to supplement QM with hidden variables,
one must advocate a nonlocal theory in order to account
for the empirical data.

But is the EPR argument sound? The standard view in
the physics community has been that Niels Bohr refuted
the EPR argument in 1935 by pointing out an “essen-
tial ambiguity” in the famous EPR criterion of reality:
“If, without in any way disturbing a system, we can pre-
dict with certainty (i.e., with probability equal to unity)
the value of a physical quantity, then there exists an el-
ement of physical reality corresponding to this physical
quantity.”\[6\]

Bohr claimed that “the wording of the above men-
tioned criterion...contains an ambiguity as regards the
meaning of the expression ‘without in any way distur-
bng a system’.” That is, Bohr seems to have objected to
the formulation of locality which entered into the EPR
reality criterion. In particular, he argued that there was
a type of non-mechanical disturbance which EPR had ne-
eglected and that the criterion was therefore inapplicable
to the very example on which they base their argument:
“Of course there is in a case like that just considered
no question of a mechanical disturbance of the system
under investigation during the last critical stage of the
measuring procedure. But even at this stage there is es-
entially the question of an influence on the very condi-
tions which define the possible types of predictions regard-
ing the future behavior of the system. Since these condi-
tions constitute an inherent element of the description
of any phenomenon to which the term ‘physical reality’
can be properly attached, we see that the argumenta-
tion of [EPR] does not justify their conclusion that [, if
local, the] quantum-mechanical description is essentially
incomplete.”\[8\]

Many commentators (including, not surprisingly, Bell)
have questioned the validity, clarity, and relevance of
Bohr’s reply. (See in particular pages 155-6 of Ref. \[1\]
for Bell’s lucid analysis of Bohr’s reply to EPR.) Nev-
etheless, it is true that the exact definition of locality
used as a crucial premise in the EPR argument – and
also the exact role of that premise in the argument – are
less than crystal clear. It would be desirable, therefore,
if the condition of local causality could be clarified, and
the EPR argument reformulated in terms of this clearer
concept. This is the goal of the present paper.

Happily, there is almost no work to do to achieve
this goal – for in the course of establishing his so-called
“no-hidden-variables” theorems, John Bell introduced an
intuitive and mathematically precise definition of local
causality. So the goal at hand can be achieved simply
by replacing EPR’s somewhat vague language about not disturbing a distant system with the quantitative requirement of Bell Locality. We will perform this replacement in Section III after first, in Section II, briefly reviewing the original EPR argument. Finally, in Section IV we discuss the relation of the re-formulated EPR argument to Bell’s Theorem – in particular, the role of the EPR argument in Bell’s two-part argument for nonlocality.

II. THE EPR ARGUMENT

Before presenting the updated version, let us briefly recap the original EPR argument. We will use the scenario introduced by Bohm in which different spin components of two spin-1/2 particles take the place of the position and momentum variables used in the original EPR paper. The two versions, however, are identical in terms of logical structure, so we will refer freely to the original EPR paper as if they had based the argument on Bohm’s example. Our goal in this section is simply to lay out the logical structure of the EPR argument, so that we can recognize a rigorously similar structure in the next section (but with Bell Locality in place of EPR’s looser locality assumption).

Consider two spin-1/2 particles which are spatially separated and in the spin singlet state:

$$\psi_0 = \frac{1}{\sqrt{2}} (|-z>_{1} - |+z>_{2} - |-z>_{1} + |+z>_{2})$$

where $|-z>_{1}$ is the state in which particle 1 has spin + along the z-axis, etc. Equation 4 attributes no definite spin values to either of the two particles separately, but it does imply a definite relation between the spins: whatever the spin of particle 1 is (measured to be) along the z-axis, the spin of particle 2 along the z-axis will be (measured to be) opposite.

The same singlet state can also be written in other bases, e.g., the basis of eigenstates for spin along the x-axis:

$$\psi_0 = \frac{1}{\sqrt{2}} (|+x>_{1} - |x>_{2} - |-x>_{1} + |+x>_{2}).$$

This rewriting makes manifest the fact that, in addition to the perfect (anti-) correlation of (measured) spin values along the z-axis mentioned above, there is also a perfect (anti-) correlation of (measured) spin values along the x-axis: if we measure the spin of particle 1 along the x-axis and find the result +, we can be certain that the spin of particle 2 along the x-axis (if/when measured) will be −, and vice versa.

This perfect anti-correlation of outcomes when the spins are measured along parallel axes is the first assumption of the EPR argument. We will henceforth refer to it as “EPR(A)”.

The EPR argument then proceeds as follows.

According to the EPR criterion of reality, “If, without in any way disturbing a system, we can predict with certainty ... the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.” Let us simply assume that, since particles 1 and 2 are spatially separated, the act of measuring the spin of particle 1 along the z-axis doesn’t disturb particle 2 in any way, and likewise for a measurement of the spin of particle 1 along the x-axis. (This assumption is obviously motivated by relativity’s prohibition on causal relations between space-like separated events – a relationship that the two measurement events in question here can simply be stipulated to have.) Let us call this locality assumption “EPR(B)”.

EPR now argue: by measuring the z-axis spin of particle 1, we can infer the z-axis spin of particle 2 by using EPR(A) – without, evidently, in any way disturbing particle 2. There exists, therefore, an element of reality corresponding to the z-axis spin of particle 2. Why? Because after the measurement on particle 1, particle 2 is known to be in a state with a definite value of spin along the z-axis. This follows from a trivial application of EPR(A). But, by EPR(B), the measurement on particle 1 could not have caused particle 2 to acquire this property, for particle 2 was not disturbed in any way by the measurement on 1. Thus if particle 2 is known to have this property after the measurement, it must evidently have possessed it all along, independent of the measurement made on particle 1. The measurement on 1, if/when performed, permits us to learn something about the z-axis spin of particle 2. But the fact we learn about exists (i.e., is an element of reality) independent of that measurement.

The same argument obviously goes through for the x-axis spin as well: by measuring the x-axis spin of particle 1, we can determine without in any way disturbing particle 2, the x-axis spin of particle 2. There exists, therefore, an element of reality corresponding to this property of particle 2 as well.

The reader might perhaps worry that either one or the other of these inferences can be validly made in a given experimental situation, but both cannot be, since we can measure at most one of the two relevant properties of particle 1 (and hence infer using EPR(A) only one of the two relevant properties of particle 2). EPR answer this possible worry in their paper:

“One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously measured or predicted. On this point of view, since either one or the other, but not both simultaneously, of the quantities ... can be predicted, they are not simultaneously real. This makes the reality of [the two properties of particle 2] depend upon the process of measurement carried out on [particle 1], which does not disturb the sec-
ond [particle] in any way. No reasonable [i.e., local] definition of reality could be expected to permit this."

Thus – although the operators for these two observables don’t commute and therefore cannot according to quantum mechanics possess simultaneous definite values – the x-axis and z-axis spins of particle 2 do possess simultaneous definite values. There are elements of reality corresponding to both quantities. And that means the descriptive limitations imposed by QM (expressed most pointedly by the Heisenberg Uncertainty Principle) can be beaten: there are more facts out there in the world than can be squeezed into the quantum mechanical description. We thus arrive at the negation of “EPR(C)” – the claim that the quantum mechanical description of reality can be considered complete.

The EPR argument thus takes the symbolic form:

\[ \text{EPR(A) and EPR(B)} \rightarrow \neg \text{EPR(C)} \]  

which is logically equivalent to

\[ \text{EPR(A) and EPR(C)} \rightarrow \neg \text{EPR(B)} \]  

and also to

\[ \neg \text{EPR(A)} \text{ or } \neg \text{EPR(B)} \text{ or } \neg \text{EPR(C)} \]  

which is Einstein’s formulation quoted above: (given the empirically well-verified quantum mechanical expressions for certain correlations) the advocate of orthodox quantum theory is “forced to relinquish” either the locality claim [EPR(B)] or the completeness claim [EPR(C)].

\section*{III. THE EPR-BELL ARGUMENT}

Like the EPR argument just considered, the new version of EPR (let us call it the EPR-Bell argument, since it is modeled after the reasoning of Bell) begins with Bohm’s example of a system consisting of two spatially separated spin-1/2 particles in the spin-singlet state, Equation 4.

Let us first introduce the analog of EPR(A) for the EPR-Bell argument. We will use here a slightly-expanded set of empirical predictions compared to the simple perfect anti-correlation used in the original EPR argument. But, like the assumption EPR(A) above, these predictions will all be straightforward, uncontroversial predictions of QM that are well-confirmed by experiment.

First, we introduce the probability for joint outcomes \( A \) and \( B \) for spin measurements along arbitrary directions \( a \) and \( b \) on the two particles (respectively) in the singlet state \( \psi_0 \) of Equation 4:

\[
P(A=+, B=+ | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \sin^2(\theta/2) \\
P(A=+, B=- | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \cos^2(\theta/2) \\
P(A=-, B=+ | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \cos^2(\theta/2) \\
P(A=-, B=- | \hat{a}, \hat{b}, \psi_0) = \frac{1}{2} \sin^2(\theta/2)
\]

where \( \theta \) is the angle between \( \hat{a} \) and \( \hat{b} \). Call this set of expressions “EPR-Bell(A1)”, and also to

\[
P(A=+, B=+ | \hat{a}, \psi_0) = 1/2 \\
P(A=-, B=+ | \hat{a}, \psi_0) = 1/2 \\
P(B=+, \hat{b}, \psi_0) = 1/2 \\
P(B=+, \hat{b}, \psi_0) = 1/2.
\]

This set of expressions – which we shall refer to as “EPR-Bell(A2)” – simply states that with the particles in the state \( \psi_0 \), we are equally likely to get a + or – outcome for any single measurement on a single particle, independent of the angles \( \hat{a} \) and \( \hat{b} \).

With that set of assumptions on the table, let us proceed with the argument.

Consider now a general expression for the joint probability for the two outcomes \( A \) and \( B \), when the spin values along directions \( \hat{a} \) and \( \hat{b} \), respectively, are measured:

\[
P(A, B | \hat{a}, \hat{b}, \lambda).
\]

Here \( \lambda \) is a complete specification of the physical state of the particle pair prior to measurement.

Let us introduce now “EPR-Bell(B)” – the requirement of Bell Locality – according to which the joint probability \( P(A, B | \hat{a}, \hat{b}, \lambda) \) should factor into a product of individual probabilities for the two spatially separated systems, with each factor containing conditionalization only on local variables. (See also Bell’s lengthier and clearer discussion of this result in the essay “La Nouvelle Cuisine” in Ref. 1.)

\[
P(A, B | \hat{a}, \hat{b}, \lambda) = P(A | B, \hat{a}, \hat{b}, \lambda) \times P(B | \hat{a}, \hat{b}, \lambda) = P(A | \hat{a}, \hat{b}, \lambda) \times P(B | \hat{a}, \hat{b}, \lambda).
\]

The equality in the first line is standard conditional probability, and should be completely uncontroversial. The move from here to the second line involves an application of what Abner Shimony has dubbed “Parameter Independence” (PI). This principle asserts that the probabilities associated with particular outcomes for each particle should be independent of which property is measured on the distant particle – e.g., \( P(B | \hat{a}, \hat{b}, \lambda) = P(B | \hat{b}, \lambda) \). The physical intuition motivating PI is that any such stochastic dependence could only be accounted for by a “spooky nonlocal action at a distance” by which the setting of the distant instrument somehow causally influenced the (probability distribution of) results of the nearby experiment.
Finally, the move to the third line utilizes what Shimony calls “Outcome Independence” (OI), according to which the probabilities associated with particular outcomes for each particle should be independent of the outcome (+ or -) of the distant experiment. Again, this seems to be an aspect of the more general (locality) requirement that what happens here should be independent of what happens over there – or, more precisely, that the (probability distribution for) outcomes of a given experiment can be completely accounted for by facts in the past light-cone of the detection event – here, by the pre-measurement, complete, joint state of the particles: \( \lambda \).

Bell summarizes the motivation for this mathematical statement of locality as follows:

“A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region... are unaltered by specification of values of local beables in a space-like separated region..., when what happens in the backward light cone of [the first region] is already sufficiently specified, for example by a full specification of local beables [in that backward light cone].” [1, pg 240]

Bell specifically argues for the validity of the factorization of probabilities (our equation 15 above) as follows: “Invoking local causality, and the assumed completeness of... \( \lambda \)... we declare redundant certain of the conditional variables in the last expression [our first line above] because they are at space-like separation from the result in question.” That is, for example, \( P(B|\hat{a}, \hat{b}, \psi_0) = P(B|\hat{b}, \lambda), \) and so forth.

(It should also be noted that Jon Jarrett [10] was the first to point out that Bell Locality was entailed by the conjunction of the two principles PI and OI, though Jarrett referred to these principles by different names. Tim Maudlin, however, has quite reasonably criticized Jarrett’s analysis of Bell’s factorization principle. [11] As Maudlin points out, Jarrett’s parsing is not unique, so the relevance of the distinction between PI and OI is called into question. Moreover, Jarrett originally argued that a failure of PI would mean a violation of relativity, while a failure of OI would not. But, as Maudlin makes clear, this makes no sense: if relativity is taken to prohibit causal dependency between space-like separated events, violations of PI and OI are equally at odds with it. The reader is urged to consult Maudlin’s text for a much more extensive and highly enlightening discussion.)

What concerns us here, however, is not primarily the validity of Bell’s mathematical formulation of local causality, i.e., whether the sort of dependence prohibited by the factorization of the joint probability is equivalent to the sort of dependence that is supposed to be prohibited by relativity. Rather, our goal is merely to show that this condition (widely known to be required in Bell’s demonstration that hidden variable theories respecting the condition cannot agree with experiment) can also be used to reformulate the EPR demonstration that the orthodox version of QM (in which the wave function alone is considered a complete description of reality) must disagree with experiment if it respects the locality condition.

Let us then simply name the Bell Locality condition, Equation 18 as one of the premises – “EPR-Bell(B)” – and proceed with that argument.

The third principle relevant to our derivation is the completeness assumption. We have previously introduced the symbol \( \lambda \) to refer to, in Bell’s words, “a full specification of local beables” in the backwards light cones of the two detection events. In typical derivations of Bell’s theorem, this symbol refers to the wave function and/or whatever hidden-variables are needed to complete the (in that context, assumed incomplete) description provided by the wave function alone. But we are not here reproducing Bell’s theorem. We are instead aiming to reproduce the EPR argument, so we will assume with Bohr that quantum mechanics itself is already complete, without the addition of any supplementary “hidden” variables. We will then demonstrate that there is a contradiction implied by the four assembled principles, and hence arrive at the EPR conclusion that either locality or completeness – or, less plausibly, one of EPR-Bell(A1) or EPR-Bell(A2) – must fail.

Thus, let us now formally make the completeness assumption – i.e., replace \( \lambda \) with the appropriate quantum mechanical wave function, \( \psi_0 \). Call this replacement “EPR-Bell(C)”.

We may now combine EPR-Bell(B) with EPR-Bell(C). The result is:

\[
P(A, B | \hat{a}, \hat{b}, \psi_0) = P(A | \hat{a}, \psi_0) \times P(B | \hat{b}, \psi_0).
\]

(19)

This leaves us in a position to utilize the expressions in EPR-Bell(A2). Plugging in yields:

\[
P(A = +, B = + | \hat{a}, \hat{b}, \psi_0) = 1/4
\]

(20)

and

\[
P(A = +, B = - | \hat{a}, \hat{b}, \psi_0) = 1/4
\]

(21)

and

\[
P(A = -, B = + | \hat{a}, \hat{b}, \psi_0) = 1/4
\]

(22)

and

\[
P(A = -, B = - | \hat{a}, \hat{b}, \psi_0) = 1/4.
\]

(23)

These expressions have been deduced by straightforwardly combining EPR-Bell(A2), (B), and (C) – i.e., the quantum expressions for marginal probabilities, Bell Locality, and the assumption that the quantum mechanical description of physical reality is complete. And, as should be obvious, these predictions conflict with the expressions in EPR-Bell(A1). We have thus proved that all four of these principles cannot be simultaneously correct. At least one member of the set must be false.
And since EPR-Bell(A1) and EPR-Bell(A2) are both directly supported by experiment, we must evidently reject either EPR-Bell(B) – (Bell) Locality – or EPR-Bell(C) – completeness – on pain of contradiction. This of course matches the conclusion reached by EPR.

IV. DISCUSSION

We have shown that it is possible to reformulate the EPR argument by using Bell’s mathematically precise local causality requirement, and that doing so permits the EPR argument to go through as intended by its authors. That is, we have shown that orthodox QM (to the extent that it is consistent with experiment and regarded as providing a complete description of physical reality) violates Bell Locality. In some sense, this is very old news. It has been known for some time that orthodox QM violates the condition of Outcome Independence (OI), and that Bell Locality is equivalent to the conjunction of OI and PI. Yet the full implications of this do not seem to have been widely appreciated.

Opponents of the hidden-variables program tend to side with Bohr in dismissing the EPR argument (considered as an argument against the completeness doctrine), and simultaneously to regard Bell’s Theorem as a valid proof of the non-viability of hidden-variables theories. The real point of the present paper is to demonstrate the inconsistency of these two views, by highlighting the similarity between the EPR argument (as reformulated here) and Bell’s Theorem. There seems to be no way of rejecting the argument in Section III that does not simultaneously commit one to rejecting the applicability of Bell’s Theorem. For example, one might object that the factorizability condition (Equation 18) fails to capture relativity’s prohibition on super-luminal causation. (See, for example, [12].) If granted, this objection would indeed preclude the need to regard orthodox quantum theory as in conflict with relativity; but it would also remove the ground from those who use Bell’s Theorem to argue that hidden variable theories must conflict with relativity (and are thus not viable).

Simply put, the EPR argument (as recast in Section III and the well-known arguments leading to Bell’s Theorem are are completely parallel: each shows that a certain type of theory, if required to respect the Bell Locality condition, fails to reproduce certain empirical facts – or, equivalently, each shows that the only way the theory in question can maintain consistency with experiment is to violate Bell Locality.

In the case of the (reformulated) EPR argument, the relevant theory is the orthodox interpretation of quantum mechanics, according to which the wave function alone is regarded as providing a complete description of physical reality. We may thus state the upshot of the argument as follows: if you maintain that QM is complete (and that its empirical predictions are correct) you are forced to concede that the theory violates Bell Locality. Thus, the completeness assumption entails the failure of Bell Locality:

\[ \text{EPR: Completeness } \rightarrow \neg \text{Bell Locality.} \]  \hspace{1cm} (24)

Bell’s Theorem, on the other hand, tells us that a certain type of local hidden variable theory cannot agree with experiment – or, equivalently, the only way a hidden variable theory (i.e., a theory in which the wave function alone is regarded as an incomplete description of physical reality) can be made to agree with experiment is to violate the Bell Locality condition:

\[ \text{Bell: Incompleteness } \rightarrow \neg \text{Bell Locality.} \]  \hspace{1cm} (25)

Combining these two arguments forces us to conclude (without qualification, for surely QM either is or is not complete\[15\] ) that Bell Locality fails:

\[ \text{EPR + Bell : } \neg \text{Bell Locality.} \]  \hspace{1cm} (26)

We can now see the extent to which the widely held view described in Section II is confused and misleading. Mermin is, strictly speaking, correct when he says: “to those for whom nonlocality is anathema, Bell’s Theorem finally spells the death of the hidden-variables program.” But he seems to have forgotten that, to those same people (for whom nonlocality is anathema), the EPR argument spells the death of the non-hidden-variables program – i.e., the orthodox interpretation of QM which upholds the completeness doctrine. For orthodox QM itself violates Bell Locality, the same locality condition that empirically-viable hidden-variable theories must, according to Bell’s Theorem, violate.

The choice between orthodox QM and hidden variables theories is thus not (as is so often suggested\[14\] ) a choice between a local theory and a nonlocal theory; it is a choice between two non-local theories, two theories that violate Bell Locality. What Bell’s Theorem (combined with the reformulated EPR argument) spells the death of is thus the principle of Bell Locality – nothing more and nothing less. People “for whom [such] nonlocality is anathema” are therefore simply out of luck.

This should clarify exactly why Bell understood his theorem not as ruling out the hidden-variables program, but rather as evidencing a deep conflict between the predictions of quantum theory as such, in any interpretation, and the locality principle suggested by relativity. This seems to have been misunderstood largely because it has not been grasped that Bell’s Theorem is the second part of a two-part argument for the conclusion.\[20\] The necessary first part of that argument is nothing but EPR, which generations of physicists have claimed was refuted by Bohr. But, simply put, it wasn’t – as the new formulation presented in Section III should help make clear.

In order to stress the parallel structure of the EPR argument and Bell’s Theorem, we have up to this point...
framed them as both arguments showing the inevitability of nonlocality for a certain type of (empirically viable) theory: EPR shows that the price of regarding QM as complete is rendering the theory manifestly nonlocal; Bell shows that if we regard standard QM as incomplete, the hidden variable theory which replaces it will have to be nonlocal. It is possible, however, to frame the same argument in a slightly different logical form, as follows: according to EPR, if we want to insist that quantum theory respect the Bell Locality principle (and agree with experiment) we must conclude that it is incomplete (i.e., that there exist variables which supplement the wave function description and determine experimental outcomes in a local manner); but then, Bell’s Theorem shows that this project cannot succeed – a theory which uses such variables to explain the EPR correlations simply cannot yield the correct predictions for more general correlations. This means that the goal of interpreting quantum mechanics in a way that respects the locality principle, cannot be reached. No local theory – orthodox QM most certainly included – can be consistent with experiment.

This is the form in which Bell himself elaborated his complete two-part argument for nonlocality:

“Let me summarize once again the logic that leads to the impasse. The EPRB [i.e., EPR-Bell] – the EPR argument using Bohm’s example] correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting. [That is the EPR argument – part 1 of Bell’s 2-part argument.] But this has implications for non-parallel settings which conflict with those of quantum mechanics. [That is Bell’s Theorem – part 2.] So we cannot dismiss intervention on one side as a causal influence on the other.”[1, pg 149]

It is hoped that the current paper will begin to overturn a truly unfortunate historical injustice – namely, the idea (implied by the widely-held misinterpretation of the meaning of Bell’s Theorem described in Section I) that John Bell failed to understand his own most important insight.

[1] John S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, 2nd ed., Cambridge University Press, 2004.
[2] Eugene P. Wigner, “Interpretation of Quantum Mechanics”, 1976, reprinted in “Quantum Theory and Measurement”, John A. Wheeler and Wojciech H. Zurek, editors, Princeton University Press, 1983.
[3] N. David Mermin, “Hidden Variables and the Two Theorems of John Bell”, Rev. Mod. Phys., 65, No. 3, July 1993, pp. 803-815.
[4] Albert Einstein, “Reply to Criticisms” in “Albert Einstein: Philosopher Scientist”, P.A. Schilpp, ed., Harper and Row, 1959, pg 681.
[5] Albert Einstein, Boris Podolsky, and Nathan Rosen, “Can Quantum-Mechanical Description of Physical Reality be Considered Complete?”, Phys. Rev. 47, pp. 777-80 (1935). Also reprinted in Wheeler and Zurek, op cit.
[6] Niels Bohr, “Can Quantum-Mechanical Description of Physical Reality be Considered Complete?”, Phys. Rev. 48, pp. 696-702; reprinted in Wheeler and Zurek, op cit. (Emphasis in original)
[7] David Bohm, “Quantum Theory”, Prentice-Hall, Inc., 1951, pages 611-23.
[8] Travis Norsen, “Einstein’s Boxes”, Am. J. Phys. 73 (2), February 2005, pp. 164-176.
[9] Abner Shimony, “Our Worldview and Microphysics” in “Philosophical Consequences of Quantum Theory”, James T. Cushing and Ernan McMullin, editors, University of Notre Dame Press, 1989.
[10] John Jarrett, “On the Physical Significance of the Locality Conditions in the Bell Arguments”, Nous, 18, pp. 569-89 (1984)
[11] Tim Maudlin, *Quantum Non-Locality and Relativity*, 2nd ed., Blackwell Publishing, 2002, Chapter 4.
[12] Arthur Fine, “Do correlations need to be explained?”, in *Philosophical Consequences of Quantum Theory*, edited by J. Cushing and E. McMullin (University of Notre Dame Press, Notre Dame, 1989), pp. 175-194; see also Arthur Fine, “The Shaky Game”, University of Chicago Press, 1996, pp. 59-63.
[13] Goldstein’s article on Bohmian Mechanics at the Stanford Internet Encyclopedia at http://plato.stanford.edu/entries/qm-bohm
[14] D. Durr, S. Goldstein, and N. Zanghi, “Quantum Equilibrium and the Role of Operators as Observables in Quantum Theory”, Journal of Statistical Physics, 116, pp. 959-1055 (2004), quant-ph/0308038
[15] F. Laudisa, “The EPR Argument in a Relational Interpretation of Quantum Mechanics”, Foundations of Physics Letters, 14 (2), pp. 119-132 (2001).
[16] Although the conclusion of incompleteness in the EPR paper was framed around the notion of beating the uncertainty principle, this seems not to have been the argument that Einstein himself favored. See Ref. 3.
[17] Perhaps it is worth noting that one currently fashionable interpretation of QM — the many-worlds interpretation (MWI) — regards QM as both complete and local, eluding EPR’s Locality-Completeness dilemma by denying EPR-Bell(A1) and EPR-Bell(A2). In particular, according to MWI these probabilities are all zero since they are probabilities for the experiments to have definite, specific outcomes — something which, according to MWI, never occurs in these situations.
[18] Note that what we are here calling “Incompleteness” is, specifically, the assumption that there exist local random variables which, along with the wave function and the relevant facts about the detection apparatus, determine the probability distributions for the possible outcomes of each separate experiment. “Completeness” is simply the denial of this – i.e., the claim that such variables do not exist. So it is fully warranted to assert their disjunction. Thanks to Arthur Fine (private communication) for suggesting this clarification.

[19] It boggles the mind that this can continue to be suggested, even though it is abundantly evident that QM itself violates OI and hence Bell Locality.

[20] The idea and phraseology of a “two-part argument” is due to Sheldon Goldstein. I was motivated to write the current essay after struggling to come to grips with the arguments outlined in Section II of Ref. [13]. My goal with the current paper is both to flesh out the validity of part one of Bell’s two-part argument for nonlocality – that is, the validity of EPR – and also to shed some new light on the similarities between the two parts, which I think the reformulation of EPR in Section III does. For additional perspectives on Bell’s two-part argument, see also [14] and [15].