Asymptotic Capacity of Massive MIMO With 1-Bit ADCs and 1-Bit DACs at the Receiver and at the Transmitter

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ABSTRACT In this paper, we investigate the capacity of massive multiple-input multiple-output (MIMO) systems corrupted by complex-valued additive white Gaussian noise (AWGN) when both the transmitter and the receiver employ 1-bit digital-to-analog converters (DACs) and 1-bit analog-to-digital-converters (ADCs). As a result of 1-bit DACs and ADCs, the transmitted and received symbols, as well as the transmit- and receive-side noisy channel state information (CSI) are assumed to be quantized to 1-bit of information. The derived results, applicable to both single-user and multi-user MIMO, show that the capacity of the considered massive MIMO system is $2^N$ and $2^M$ bits per channel use when $N$ is fixed and $M$ goes to infinity and when $M$ is fixed and $N$ goes to infinity, respectively, where $M$ and $N$ denote the number of transmit and receive antennas, respectively. These coincide with the respective capacities with full and noise-free CSI at both the transmitter and the receiver. In both cases, we showed that the derived capacities can be achieved with noisy 1-bit CSI at the massive-side and without any CSI at the other end. Moreover, we showed that the capacity can be achieved in one channel use without employing channel coding, which results in a latency of one channel use.

INDEX TERMS Massive MIMO, capacity, 1-bit CSI, 1-bit ADC, achievability scheme.

I. INTRODUCTION
Massive antenna arrays at the transmitter and at the receiver have shown to be extremely useful for improving the capacity of next generation wireless networks. Among the main design problems for the massive multiple-input multiple-output (MIMO) technology is the power consumption of analog-to-digital converters (ADCs) and digital-to-analog converters (DACs) associated with each radio frequency (RF) chain. This problem is especially significant in massive MIMO systems employing large bandwidths since these systems require ADCs and DACs with large sampling rates, which results in high-resolution, high-speed ADCs and DACs that consume large amounts of energy and are costly [1]–[3]. Since in standard deployments an RF chain is connected to each antenna, this issue limits the number of antennas at the transmitter and at the receiver.

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the transmitter/receiver [8]. Therefore, it is important to investigate how 1-bit ADCs/DACs influence the capacity of massive MIMO systems. Motivated by this, in this paper, we investigate the capacity of the additive white Gaussian noise (AWGN) massive MIMO systems with 1-bit ADCs and DACs at the receiver and at the transmitter.

B. RELATED WORKS

One of the earliest works on the problem of finding the capacity of MIMO systems, where low resolution ADCs are employed is [7]. In [7], the capacity of a MIMO channel with 1-bit ADCs at the receiver is studied at low signal-to-noise ratios (SNRs) under the assumption of perfect channel state information (CSI) at the receiver (CSIR). This problem shows that, at low SNRs, the capacity is smaller by a factor of 2/\pi, which corresponds to a gap of -1.96 dB in $E_b/N_0$, as compared to a system with infinite resolution. In [9], the results in [7] have been extended to the case where the additive Gaussian noise is mutually correlated across the receive antennas. Bussgang decomposition has been used in [9] to model the corresponding MIMO channel and it has been demonstrated that certain conditions on the channel and the noise covariance matrices result in lower performance loss compared to the case of uncorrelated noise. In [10] and [11], achievable rates have been presented for MIMO systems with 1-bit ADCs at the transmitter and full-precision DACs at the transmitter, where the antenna outputs are processed by analog combiners, and full CSI is available both at the transmitter and the receiver.

In [12], it has been shown that at high SNRs, under the assumption of perfect CSIT and CSIR, the capacity of MIMO systems with high-resolution inputs and 1-bit quantized outputs is lower bounded by the rank of the channel matrix. On the other hand, in [1], when the channel matrix has full-row rank, a tight upper bound on the capacity has been derived for the MIMO systems at finite SNRs. In [1], the results of [12] have been extended, where under the assumption of full CSIT and CSIR, the capacity of MIMO systems with 1-bit quantization at the receiver only has been derived for infinite SNR. In [13], an achievable rate has been derived for a massive MIMO system with high resolution DACs at the transmitter and 1-bit ADCs at the receiver under the assumption of imperfect CSIT, where the wideband frequency-selective channel is being estimated by a linear low-complexity algorithm. The authors in [14] investigated the achievable rate of a massive MIMO system when low-resolution ADCs are used at the receiver only and perfect CSIT is available, where it has been shown that the performance loss caused by using low resolution ADCs can be compensated by increasing the number of antennas at the receiver.

In [15]–[18], under the assumption of full CSI, the effects of non-linearities of employing 1-bit DACs at the transmitter or 1-bit ADCs at the receiver have been investigated in massive MIMO systems. The main finding of these works is that by increasing the number of antennas on the side with low-resolution DACs/ADCs, the negative impacts of non-linear distortions can be decreased. In [19], these results have been extended to the case where the CSI is imperfect. It is shown that in a MIMO system with 1-bit DACs at the transmitter and full precision ADCs at the receiver, the performance penalty associated to the nonlinear distortion can be tolerable if the number of transmit antennas is greater than the number of independent data streams.

In [20], the authors analyze the mutual information of a MIMO system with high resolution DACs at the transmitter and 1-bit ADCs at the receiver, where CSI is not available at the transmitter (CSIT) nor at the receiver. Achievable rates in [20] are provided only for the quantized single-input single-output (SISO) channels, where on-off QPSK signaling is shown to be capacity achieving. In [21], the mutual information of a MIMO system with high-resolution DACs at the transmitter and 1-bit ADCs at the receiver is derived in the low SNR regime under the assumption of no CSIT/CSIR. Achievable rates in [21] are provided only for the SISO channel and only in the asymptotic case of low SNRs. Finally, in [22], the capacity of a massive multiple-input single-output (MISO) channel with 1-bit transceivers is investigated, where perfect CSI is available at both the transmitter and the receiver. The main finding in [22] is that capacity achieving strategy is to uniformly use multi-dimensional constellation points.

C. MAIN CONTRIBUTIONS

In prior works, the assumption of perfect CSI has been leveraged for deriving the achievable rates and/or capacity results for MIMO systems. Although different algorithms have been proposed for estimating the channel while 1-bit ADCs are used at the receiver, they are associated with large estimation errors, high computational complexity, and require extremely long training sequences [23]–[30]. Therefore, the assumption of having full CSI when 1-bit ADCs/DACs are present, is unrealistic. To the best of our knowledge, this is the first work on MIMO systems with 1-bit DACs/ADCs at the transmitter and at the receiver, that adopts the practical assumption of noisy 1-bit CSI at the transmitter and/or at the receiver.

In this paper, we investigate two system models of massive MIMO, one when the number of transmit antennas, denoted by $M$, is very large and goes to infinity, and the second when the number of receive antennas, denoted by $N$, is very large and goes to infinity. Next, the results are derived for complex-valued AWGN MIMO channels. The derived results, applicable to both single-user and multi-user MIMO, show that the capacity of the considered massive MIMO system is $2N$ and $2M$ bits per channel use when $N$ is fixed and $M \to \infty$ and when $M$ is fixed and $N \to \infty$ hold, respectively. These coincide with the respective capacities with full and noise-free CSI at both the transmitter and the receiver. In both cases, we showed that the derived capacities

1The achievable rate in [20] is for SISO and in [21] for SIMO, but in the asymptotic low-SNR regime only.
can be achieved with noisy 1-bit CSI at the massive-end, and without any CSI at the other end. Moreover, we showed that the capacity can be achieved in one channel use without employing channel coding, which results in a latency of one channel use. Therefore, massive MIMO systems with 1-bit DACs/ADCs maybe a practical approach for achieving ultra reliable low latency communication (URLLC).

This paper is organized as follows. In Section II, the system model of a MIMO system with 1-bit quantization at both the transmitter and the receiver is described. In Section III, the asymptotic capacities and respective achievability schemes for the considered massive MIMO systems with massive antenna array at the transmitter side and the receiver side are presented. Next, the results found in Section III are simulated and numerically evaluated in Section IV. Finally, conclusion is brought in Section V.

II. SYSTEM MODEL

We consider a MIMO system comprised of $M$ transmit and $N$ receive antennas. Each antenna element is equipped with two\(^2\) 1-bit quantizers, 1-bit ADC and 1-bit DAC, that quantize the received and transmitted signals, respectively, as shown in Fig. 1. Let $x \in \mathcal{X}^M$ denote the complex-valued $M \times 1$ transmit vector after the 1-bit quantization at the transmitter. We assume that each element of $x$ has unit energy. Let $P$ be the total transmit power and let $y$ denote the $N \times 1$ received vector before the 1-bit quantization at the receiver. Then, $y$ is given by

$$y = \sqrt{\frac{P}{M}} Hx + w,$$

where $H$ denotes the $N \times M$ MIMO complex-valued channel matrix and $w$ is the $N \times 1$ complex-valued Gaussian noise vector with independent and identically distributed (i.i.d.) entries having zero mean and unit variance. Following standard Rayleigh fading, the channel matrix $H$ is also assumed to have complex-valued Gaussian i.i.d. entries with zero mean and unit variance. Vector $y$ undergoes 1-bit quantization at the receiver, yielding the $N \times 1$ quantized received vector $z \in \{1 + j, 1 - j, -1 + j, -1 - j\}^N$, given by

$$z = \text{sign}(y) = \text{sign}\left(\frac{P}{M} Hx + w\right),$$

where

$$\text{sign}(a + jb) = \begin{cases} 1 + j, & \text{if } a \geq 0, b \geq 0 \\ -1 + j, & \text{if } a < 0, b \geq 0 \\ 1 - j, & \text{if } a \geq 0, b < 0 \\ -1 - j, & \text{if } a < 0, b < 0 \end{cases}.$$  \hspace{1cm} (3)\)

Due to the 1-bit ADCs/DACs at both the receiver and the transmitter, the transmitter and the receiver can only have access to 1-bit CSI corrupted by noise. More precisely, let $G$ denote the noisy 1-bit estimate of the channel matrix $H$, which is obtained by $K$ pilot transmissions per antenna, given by

$$G = \text{sign}\left(\sum_{k=1}^{K} \text{sign}\left(\sqrt{P_p} H + W_k\right)\right),$$

where $W_k$ is an $N \times M$ noise matrix with i.i.d. zero-mean unit variance complex-valued Gaussian elements, and $P_p$ is the pilot power. In practice, assuming channel reciprocity, the noisy 1-bit CSI channel matrix $G$ can be obtained at the receiver (transmitter) by sending $K$ orthogonal training symbols per antenna from the receiver (transmitter) to the transmitter (receiver) in a TDD fashion. The transmitter (receiver) collects the $K$ 1-bit quantized pilot signals received on each of its antennas, then makes a 1-bit estimate on the channel based on majority rule and thereby obtains $G$ in (4).
The capacity of the considered MIMO system with 1-bit quantized inputs and outputs, and noisy 1-bit CSI is given as

$$C = \max_{p(x|G)} \ I(z; x|G),$$

(5)

where $p(x|G)$ is the probability mass function (PMF) of the transmit signal $x \in \mathcal{X}^M$ given the available 1-bit noisy CSI matrix $G$. In this paper, we derive the capacity in (5) by focusing on the massive MIMO regime, where the number of either transmit or receive antennas goes to infinity, i.e., either $M \to \infty$ and $N < \infty$ is fixed or $N \to \infty$ and $M < \infty$ is fixed. To this end, we assume that $P > 0$ and $P_n > 0$ hold. Moreover, we assume that the size of the transmitter/receiver grows as the number of antennas grows. As a result, we can assume that the signals on the different antenna elements are uncorrelated.

### III. ASYMPTOTIC CAPACITIES

In the following, we present the asymptotic capacities and their respective achievability schemes for two cases of massive antenna arrays at the transmitter side and the receiver side.

#### A. ASYMPTOTIC CAPACITY OF THE MIMO SYSTEM WITH MASSIVE ANTENNA ARRAY AT THE TRANSmitter-SIDE

In this subsection, the capacity of a massive MIMO systems with 1-bit quantized inputs and outputs, and noisy 1-bit CSI is presented for the case when the massive antenna array is at the transmit-side. The following theorem establishes the capacity when $N$ is fixed and $M \to \infty$ holds.

**Theorem 1:** The capacity of the massive MIMO system with 1-bit quantized inputs and outputs, and noisy 1-bit CSI satisfies the limit

$$\lim_{M \to \infty} C = 2N,$$

(6)

where $N < \infty$ is fixed. The capacity can be achieved in one channel use by a scheme that uses the noisy 1-bit CSI available at the transmitter only, neglecting any CSI at the receiver.

We now present a scheme that is asymptotically able to communicate at this rate with vanishing probability of error by using the noisy 1-bit CSI only at the transmitter.

**Achievability Scheme 1:** In order to communicate $2N$ bits per channel use, the transmitter constructs a codebook comprised of $2^{2N}$ codewords $s_i \in \mathcal{S}^N$, for $i = 1, \ldots, 2^{2N}$, where $\mathcal{S} = \{-1-j, -1+j, 1-j, 1+j\}$. Without loss of generality, assume that codeword $s$ is selected to be transmitted in the considered channel use. Let $s_m^R$ and $s_m^I$ denote the real-valued and imaginary-valued parts of the $n$-th complex-valued element of $s$. Assume that $s_m^R$ and $s_m^I$ belong to the $n$-th group of transmit antennas, for $n = 1, 2, \ldots, N$. Then, $s_m^R$ and $s_m^I$ are constructed as

$$\left[ s_m^R \ s_m^I \right] = \frac{1}{2} \begin{pmatrix} z_{nm} \ g_{nm}^R \ g_{nm}^I \ g_{nm}^R \ g_{nm}^I \end{pmatrix}.$$

(7)

Note that $s_m^R$ and $s_m^I$, constructed using (7), are such that either $s_m^R$ or $x_m^I$ is always zero while the other element is always 1 or -1. Thereby, in a given channel use, the $m$-th transmit antenna is always silent on either the real-valued or the imaginary-valued channel/carrier. Next, $s_m^R$ and $s_m^I$ are amplified by $\sqrt{P/2}$, and then transmitted in one channel use from the $m$-th transmit antenna for $m = 1, \ldots, M$. The receiver receives $z$ and it will then decide that $z$ has been the transmitted codeword in the given channel use. As a result, an error happens at the receiver if $z \neq s$ occurs. In Appendix B, we prove that the error rate goes to zero when $N$ is fixed and $M \to \infty$ holds.

Achievability Scheme 1 works by splitting the $M \to \infty$ antennas at the transmitter into $N$ groups of antennas, where each group is comprised of $M/N \to \infty$ antennas. Then, the $n$-th group of antennas, for $n = 1, 2, \ldots, N$, uses the 1-bit CSI vector, obtained from the pilots sent by the $n$-th receive antenna, to beamform towards the direction of the $n$-th receive antenna. Thereby, for $M/N \to \infty$, the beamformed signal is amplified when received at the $n$-th receive antenna and completely attenuated at any other receive antenna. Hence, Achievability Scheme 1 splits the considered MIMO system into $N$ parallel complex-valued Gaussian channels, where each parallel channel has an SNR that satisfies $SNR \to \infty$ as $M/N \to \infty$. As a result, instantaneous error-free decoding is possible.

The capacity result in Theorem 1 is asymptotic and holds when $N$ is fixed and $M \to \infty$. However, Achievability Scheme 1 is general and can be applied to the cases where $M$ and $N$ are both finite. The performance of Achievability Scheme 1 in terms of BER when $M$ and $N$ are both finite is found in (42) and is given by

$$BER = \sum_{k=0}^{L} \Pr(K_k^R = k) \cdot \int_{\hat{h}_k^R} \cdots \int_{\hat{h}_L^R} Q \left( \sqrt{\frac{P}{2N}} \sum_{m=k+1}^{L} \left| \hat{h}_m^R \right| - \sqrt{\frac{P}{2N}} \sum_{m=1}^{k} \left| \hat{h}_m^R \right| \right) \cdot \prod_{m=1}^{L} f(\hat{h}_m^R) d\hat{h}_m^R,$$

(8)

where $k$ is the number of incorrect 1-bit CSIs and $\Pr(K_k^R = k)$, given in (44), is the probability of receiving $k$ incorrect 1-bit CSIs. On the other hand, when $M \to \infty$, the BER of the considered MIMO system was found in (46).
as

$$\text{BER} = \sum_{k=0}^{L} \Pr \left( K_1^R = k \right) \times Q \left( \frac{\sqrt{P_M} (L - 2k) E \{ |\hat{h}_m^R| \}}{\sqrt{P(N-1)+N} \ 2N} \right),$$ (9)

where \( L = \frac{M}{N} \). The BER in (9) can be written equivalently as

$$\text{BER} = E_k \left\{ Q \left( \frac{\sqrt{P_M} (L - 2k) E \{ |\hat{h}_m^R| \}}{\sqrt{P(N-1)+N} \ 2N} \right) \right\} = E_k \left\{ Q \left( \frac{\sqrt{P_M} L (1 - 2p_e)}{\sqrt{P(N-1)+N} \ 2N} \right) \right\}.$$

Now, by taking the expectation inside the \( Q(\cdot) \) function, we obtain an approximation of the BER as

$$\text{BER} \approx Q \left( \frac{\sqrt{P_M} L (1 - 2E(k))}{\sqrt{P(N-1)+N} \ 2N} \right) = Q \left( \frac{\sqrt{P_M} L (1 - 2p_e)}{\sqrt{P(N-1)+N} \ 2N} \right) = Q \left( \frac{\sqrt{PM}}{\sqrt{N}} (1 - 2p_e) \right),$$ (11)

where \( p_e \) is the probability of receiving an incorrect 1-bit CSI and is given in (19).

**B. ASYMPTOTIC CAPACITY OF THE MIMO SYSTEM WITH MASSIVE ANTENNA ARRAY AT THE RECEIVER-SIDE**

We now turn to the opposite case when the massive antenna array is at the receiver’s side. The following theorem provides the capacity of this massive MIMO system for the asymptotic case when \( N \to \infty \) holds and \( M \) is fixed.

**Theorem 2:** The capacity of a massive MIMO system with 1-bit quantized inputs and outputs, and noisy 1-bit CSI satisfies the limit

$$\lim_{N \to \infty} C = 2M,$$ (12)

where \( M < \infty \) is fixed. The capacity can be achieved in one channel use by a scheme that uses the noisy 1-bit CSI available at the receiver only, neglecting any CSI at the transmitter.

**Achievability Scheme 2:** In order to communicate \( 2M \) bits per channel use, the transmitter constructs a codebook comprised of \( 2^{2M} \) codewords \( s_i \in S^M \), for \( i = 1, \ldots, 2^{2M} \), where \( S = \{-1-j, -1+j, 1-j, 1+j\} \). Without loss of generality, assume that codeword \( s \) is selected to be transmitted in the considered channel use. Let \( s_m^R \) and \( s_m^I \) denote the real-valued and imaginary-valued parts of the \( m \)-th complex-valued element of \( s \), respectively, for \( m = 1, 2, \ldots, M \). Then, the transmit vector \( x \in \mathcal{X}^M \), where \( \mathcal{X} = \frac{1}{\sqrt{2}} S \), is constructed as

$$x = \frac{1}{\sqrt{2}} s.$$ (13)

Next, the elements of \( x \) are amplified by \( \sqrt{P/M} \) and then are transmitted in one channel use from the \( M \) transmit antennas. The receiver receives \( z \). Assume that \( G \) given by (4) is known at the receiver. Then the receiver decides that

$$\hat{s} = G z$$ (14)

has been the transmitted codeword. As a result, an error happens at the receiver if \( \hat{s} \neq s \). In Appendix C, we prove that the error rate goes to zero as \( N \to \infty \) when \( M < \infty \) is fixed.

Achievability Scheme 2 works since the receiver uses its \( N \to \infty \) antennas to direct its reception such that it receives only from the \( M \) directions characterized by the \( M \) column vectors of the 1-bit CSI matrix \( G \). On a given direction, the receiver receives a complex-valued symbol on each of its \( N \) receive antennas, which can be either equal or not equal to the actual transmitted symbol. For \( N/M \to \infty \), the number of received symbols which are equal to the transmitted symbol on a given direction is always larger than the number of received symbols which are not equal to the transmitted symbol, leading to an error-free transmission.

The derived capacity result in Theorem 2 is asymptotic and holds when \( M \) is fixed and \( N \to \infty \). However, Achievability Scheme 2 is general and can be applied to the cases when \( M \) and \( N \) are both finite. The performance of Achievability Scheme 2 in terms of BER when \( M \) and \( N \) are both finite is found in (73) and is given by

$$\text{BER} = \sum_{b=0}^{Np_e} \sum_{a=\text{max} \left\{ \frac{a}{2}, 0 \right\}}^{N-j} \binom{N}{j} P_c \left( 1 - P_c \right)^{N-j} \times \binom{N-j}{a} \binom{N-j-a+b}{b} \times \left( \frac{Np_e}{b} \right)^{Np_e-b+a} \left( P_c \right)^{N(1-p_e) - a + b},$$ (15)

where \( P_c \) and \( P_w \) are given in (71) and (72), respectively, and \( p_e \) is given in (19). According to the law of large numbers, when \( N \to \infty \), the number of incorrect 1-bit CSIs will be close to \( Np_e \). Hence, (15) can be approximated as

$$\text{BER} \approx \sum_{b=0}^{Np_e} \sum_{a=\text{max} \left\{ \frac{a}{2}, 0 \right\}}^{N(1-p_e)} \binom{N(1-p_e)}{a} \times \left( \frac{Np_e}{b} \right)^{Np_e-b+a} \left( P_c \right)^{N(1-p_e) - a + b}.$$ (16)

**C. OBSERVATIONS**

Given the 1-bit quantizer at the receiver (transmitter) side, it is immediate to conclude that the capacity in (6)/(12) is
upper bounded by $2N (2M)$ bits/channel use independent of the level of quantization at the other end and of the CSI knowledge. In other words, this upper bound holds also when the full-precision symbols are transmitted (received), as well as when full-precision CSI is present at both the transmitter and the receiver.

Since each receive/transmit antenna receives/transmits its information without requiring coordination with the other receive/transmit antennas, Achievability Scheme 1/2 can also be applied to multi-user massive MIMO, i.e., to a MIMO system where the receive/transmit antennas are implemented on non-cooperating distinct devices.

Opposite to capacity achievability schemes for the SISO channel where the capacity is achieved with one transmit antenna but the number of channel uses goes to infinity, here, in the considered MIMO system model, the capacity is achieved with one channel use but the number of transmit or receive antennas goes to infinity. More specifically, for the considered MIMO systems, instead of protecting the transmit information in time by encoding it into a codeword of length $n \to \infty$, one can protect it by sending (receiving) it from $M \to \infty$ (at) antennas.

Achievability Schemes 1 and 2 do not require channel coding at the transmitter. As a result, the latency that Achievability Schemes 1 and 2 achieve is one channel use. Hence, massive MIMO systems with 1-bit ADCs and 1-bit DACs may be a practical approach for achieving URLLC.

**IV. NUMERICAL ANALYSIS**

In this section, we numerically evaluate the derived BERs in Section III and compare them with the simulation results. Moreover, we show that both simulated and derived results are aligned with the results of Theorems 1 and 2. For the case of massive antenna arrays at the transmitter side, since the BER found in (8) contains $M$ integrals, which imposes a high level of computational complexity, the theoretical BERs used in the following figures have been obtained from (11).

In Fig. 2, the derived and simulated BER of the considered MIMO system with massive antenna arrays at the transmit-side have been illustrated for fixed SNR = 20 dB and $N = [2, 3, 4]$ as a function of $M$. Similarly, in Fig. 3, the derived and simulated BER of the considered MIMO system with massive antenna arrays at the receiver-side have been illustrated for fixed SNR = 20 dB and $M = [2, 3, 4]$ as a function of $N$. In both Figs. 2 and 3, it has been assumed that only noisy 1-bit CSI is available at the massive side and no CSI is present at the other side. Furthermore, it has been assumed that only one pilot has been used for 1-bit CSI acquisition, i.e., $K = 1$ in (4). As it can be seen from Fig. 2, when $M$ increases, the BER tends to zero which complies with the result in Theorem 1. On the other hand, it can be seen that when $N$ increases, the BER also increases for a fixed $M$ due to higher inter-user interference. Fig. 3 shows similar results, with the following two differences. First, when the massive antenna array is on the receiver side, the BER decays more linearly compared to the case when the massive array is at the transmitter side. This is due to the fact that the signals combine after going through the 1-bit quantization, which adds a huge quantization noise, compared to the case when the massive antenna array is on the transmitter side and signals combine before going through the 1-bit ADCs. The second difference is that in the case of massive antenna array at the transmitter side, when $M$ increases, the performance degradation is much higher compared to the case of massive antenna array at the receiver side and $M$ increasing. This is due to the fact that the ratio of the signal strength to noise plus interference is much lower in the case of massive antenna array at the receiver side, and thereby adding additional interference will not have a huge effect on the performance.

In Figs. 4 and 5, assuming $K = 1$ in (4), we evaluate the effect of increasing SNR on the BER of the considered MIMO system as the number of antennas grows on the massive side. As it can be seen in Figs. 4 and 5, by increasing the SNR, the BER decreases. Moreover, similar to the first two figures, due to the effect of quantization noise, differences in regards to the slope of the BER curves and performance degradation as the SNR increases can be seen between the two scenarios of massive antenna array at the transmitter and receiver-side. Furthermore, one interesting observation is that the effect of increasing the number of antennas at the massive-side is the same as increasing the SNR which is inline with the achievability schemes for Theorems 1 and 2.
where we explain that increasing the number of antennas on the massive-side equals increasing the SNR.

Lastly, in Fig. 6, we evaluate the effect of increasing the number of pilots on the BER in the 1-bit CSI acquisition process. In particular, the simulated and theoretical BER of the considered MIMO system with massive antenna arrays at the transmitter side has been depicted for $M = 100$, $N = 2$, and $K = \{1, 3, 5, 7\}$ in (4) versus SNR. As it can be seen, as the number of transmitted pilots, i.e., $K$, increases, the BER decreases. This is due to the fact that for higher $K$, the probability of obtaining an incorrect 1-bit CSI decreases, which results in a lower BER. On the other hand, it can be seen that as $K$ increases, the gain obtained from sending higher number of pilots diminishes. This is because $P_e$ in (19) does not decrease linearly with $K$. Hence, for large $K$, the probability of obtaining an incorrect 1-bit CSI is already low and will not change significantly by increasing $K$. Ultimately, as $K \rightarrow \infty$, the performance of the considered MIMO system converges to the case with noise-free 1-bit CSI.

V. CONCLUSION

In this paper, we presented the asymptotic capacity of the complex-valued AWGN massive MIMO system when the inputs, outputs, and noisy CSI are quantized to one bit of information, for the cases when $M \rightarrow \infty$ and $N$ is fixed, and $N \rightarrow \infty$ and $M$ is fixed. We showed that the capacities of the considered single-user and multi-user MIMO systems are $2N$ and $2M$ (bits per channel use) when $M \rightarrow \infty$ and $N \rightarrow \infty$ hold, respectively. In both cases, we showed that the capacity can be achieved in one channel use without using channel coding, which results in a latency of one channel use. Moreover, the derived capacities can be achieved with noisy 1-bit CSI at the massive-end, and without any CSI at the other end.

APPENDIX A

PROBABILITY OF RECEIVING AN INCORRECT 1-BIT CSI

Without loss of generality, we find the probability of receiving an incorrect pilot, when the symbol $x = \sqrt{P_p}$ is transmitted from one of the transmit/receive antennas.

The probability of receiving $z_p \neq \text{sign}(h^\alpha)$ after one pilot transmission, where $\alpha \in \{R, I\}$, is given by

$$\Pr (z_p \neq \text{sign}(h^\alpha)) = \Pr (\text{sign}(h^\alpha) \neq \text{sign}(\sqrt{P_p}h^\alpha + w))$$

$$= \int_{-\infty}^{+\infty} \Pr (\text{sign}(h^\alpha) \neq \text{sign}(\sqrt{P_p}h^\alpha + w) \mid h^\alpha) f(h^\alpha) dh^\alpha$$

$$= \int_{-\infty}^{+\infty} Q \left( \sqrt{2P_p|h^\alpha|} \right) f(h^\alpha) dh^\alpha,$$

(17)

where $h^\alpha$ and $w$ are Gaussian RVs with mean zero and variance $\frac{1}{2}$. Now, by expanding the $Q(\cdot)$ function, (17) can be written as

$$\Pr (z_p \neq \text{sign}(h^\alpha))$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}(u^2 + v^2)} dudh^\alpha$$

$$= \frac{1}{\pi} \int_{0}^{+\infty} \int_{0}^{+\infty} e^{-\frac{1}{2}(u^2 + v^2)} dudh^\alpha$$

$$= \frac{1}{\pi} \int_{0}^{+\infty} \frac{1}{\tan^{-1}(\sqrt{P_p})} e^{-\frac{1}{2}r^2} r dr d\theta$$

$$= \frac{1}{2} - \frac{\tan^{-1}(\sqrt{P_p})}{\pi},$$

(18)
where $h^{mt} = \sqrt{2}n^{mt}$. On the other hand, when a receive (transmit) antenna sends a pilot to the transmit (receive) antennas $K$ times, each transmit antenna decodes the received symbols according to the majority rule. Hence, after $K$ pilot transmissions, an error happens when half or more than half of the received symbols are incorrect. On the other hand, the probability of receiving an incorrect symbol after one transmission was found in (18). Therefore, after $K$ pilot transmissions, the probability of obtaining an incorrect 1-bit CSI at the a given transmit (receive) antenna is given by

$$P_e = \sum_{j=K}^{K} \left( \frac{1}{2} - \frac{\text{arctan}(\sqrt{P_f})}{\pi} \right)^j \times \left( 1 + \frac{\text{arctan}(\sqrt{P_f})}{\pi} \right)^K - j. \quad (19)$$

**APPENDIX B**

**PROOF OF THEOREM 1**

*Proof:* (Converse) For the considered MIMO system, we have

$$I(x; z|G) = H(z|G) - H(z|x, G) \leq H(z|G) \leq 2N, \quad (20)$$

where the last inequality follows from the 1-bit quantized outputs. Hence, the capacity of this system cannot be larger than $2N$. \(\square\)

*Proof:* (Achievability) Using Achievability Scheme 1, the transmitter sends $2N$ bits of information at each channel use. Hence, the rate of Achievability Scheme 1 is $2N$ bits per channel use. Now, we are only left to prove that the symbols received at the receiver can be decoded with probability of error that goes to zero when $N$ is fixed and $M \to \infty$ holds.

Assume that $s$ is transmitted and $z$ received. Then, the probability of error can be bounded as

$$P_e = \text{Pr}(z \neq s) \leq \sum_{n=1}^{N} \left( \text{Pr}(z^R_n \neq s^R_n) + \text{Pr}(z^I_n \neq s^I_n) \right) \quad (21)$$

$$= 2N \text{Pr}(z^R_1 \neq s^R_1) \quad (22)$$

where (21) follows from the union bound and (22) follows from symmetry, i.e., since $Pr(z^R_n \neq s^R_n) = Pr(z^I_n \neq s^I_n) = Pr(z^I_n \neq s^R_n) = Pr(z^R_n \neq s^I_n)$. In the following, we derive a simplified expression for $z^R_1$. To this end, we assume that the 1-bit CSI is noiseless, i.e., that $g^R_{m} = \text{sign}(h^R_{m})$ and $g^I_{m} = \text{sign}(h^I_{m})$ hold, $\forall n, m$. Later on we extend the results to noisy 1-bit CSI. From (2), we can obtain the real-valued and imaginary-valued parts of the quantized received symbol at the first receive antenna, $z^R_1$ and $z^I_1$, as

$$z^R_1 = \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=1}^{M/N} (h^R_{1m}^{R} - h^I_{1m}^{I}) + w^R_1 \right) \quad (23)$$

$$z^I_1 = \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=1}^{M/N} (h^R_{1m}^{I} + h^I_{1m}^{R}) + w^I_1 \right). \quad (24)$$

Only transmit symbols from transmit antennas $m = 1, 2, \ldots, M/N$ are intended for the first receive antenna, and the symbols coming from all other antennas act as interference. Having this in mind, (23) and (24) can be written as

$$z^R_1 = \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=1}^{M/N} (h^R_{1m}^{R} - h^I_{1m}^{I}) + w^R_1 \right) \quad (25)$$

$$z^I_1 = \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=1}^{M/N} (h^R_{1m}^{I} + h^I_{1m}^{R}) + w^I_1 \right). \quad (26)$$

where

$$v^R_1 = \sqrt{\frac{P}{M}} \sum_{m=M/N+1}^{M} (h^R_{1m}^{R} - h^I_{1m}^{I}) \quad (27)$$

$$v^I_1 = \sqrt{\frac{P}{M}} \sum_{m=M/N+1}^{M} (h^R_{1m}^{I} + h^I_{1m}^{R}) \quad (28)$$

are the interference at the first receive antenna. Since $v^R_1$ and $v^I_1$ are zero-mean Gaussian distributed with variance $\frac{P}{M} \frac{N}{2}$, we can write (25) and (26) equivalently as

$$z^R_1 = \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=1}^{M/N} (h^R_{1m}^{R} - h^I_{1m}^{I}) + \hat{w}_1^R \right) \quad (29)$$

$$z^I_1 = \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=1}^{M/N} (h^R_{1m}^{I} + h^I_{1m}^{R}) + \hat{w}_1^I \right) \quad (30)$$

where $\hat{w}_1^R$ and $\hat{w}_1^I$ are independent zero-mean additive white Gaussian noises with variance

$$\sigma_w^2 = \frac{P}{N} \frac{N-1}{2} \frac{1}{N} + \frac{P}{N} = \frac{P(N-1) + N}{2N}. \quad (31)$$

Now, for clarity of presentation, for a given $m$, we represent $h^R_{1m}^{R} - h^I_{1m}^{I}$ and $h^R_{1m}^{I} + h^I_{1m}^{R}$ in (25) and (26), respectively, in a matrix form as

$$\begin{bmatrix} h^R_{1m}^{R} - h^I_{1m}^{I} \\ h^R_{1m}^{I} + h^I_{1m}^{R} \end{bmatrix} = \begin{bmatrix} h^R_{1m} \\ h^I_{1m} \end{bmatrix} \begin{bmatrix} h^R_{1m} \\ h^I_{1m} \end{bmatrix}^{T} \end{bmatrix} \begin{bmatrix} x^R_1 \\ x^I_1 \end{bmatrix}. \quad (32)$$

By inserting $x^R_1$ and $x^I_1$ from (7) into (32), we obtain

$$\begin{bmatrix} h^R_{1m}^{R} - h^I_{1m}^{I} \\ h^R_{1m}^{I} + h^I_{1m}^{R} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} h^R_{1m} \\ h^I_{1m} \end{bmatrix} \begin{bmatrix} g^R_{1m} \\ g^I_{1m} \end{bmatrix} = \frac{1}{2} A \begin{bmatrix} x^R_1 \\ x^I_1 \end{bmatrix}. \quad (33)$$

where $A$ is given by

$$A = \begin{bmatrix} g^R_{1m}h^R_{1m} + g^I_{1m}h^I_{1m} \\ -g^I_{1m}h^R_{1m} + g^R_{1m}h^I_{1m} \\ g^R_{1m}h^R_{1m} - g^I_{1m}h^I_{1m} \\ -g^I_{1m}h^R_{1m} + g^R_{1m}h^I_{1m} \end{bmatrix}. \quad (34)$$

Now, there are four cases depending on whether $g^R_{1m} = g^I_{1m}$ or $g^R_{1m} = -g^I_{1m}$ holds, and depending on whether $s^R_1 = s^I_1$ or $s^R_1 = -s^I_1$ holds. If $g^R_{1m} = g^I_{1m}$ and $s^R_1 = s^I_1$ hold, or if $g^R_{1m} = -g^I_{1m}$ and $s^R_1 = -s^I_1$, (33) simplifies to

$$\begin{bmatrix} h^R_{1m}^{R} - h^I_{1m}^{I} \\ h^R_{1m}^{I} + h^I_{1m}^{R} \end{bmatrix} = \begin{bmatrix} x^R_1 s^R_{1m} \\ x^I_1 s^I_{1m} \end{bmatrix}. \quad (35)$$
If \( g'^{R}_{1m} = g'^{I}_{1m} \) and \( s'^{R}_{1} = -s'^{I}_{1} \) hold, or if \( g'^{R}_{1m} = -g'^{I}_{1m} \) and \( s'^{R}_{1} = s'^{I}_{1} \) hold, (33) simplifies to
\[
\begin{bmatrix}
H'^{R}_{1m} - h'^{I}_{1m} x'^{R}_{m} \\
H'^{I}_{1m} + h'^{I}_{1m} x'^{R}_{m}
\end{bmatrix}
= \begin{bmatrix}
\gamma^{R}_{1m} \gamma^{I}_{1m} \\
\gamma^{I}_{1m} \gamma^{R}_{1m}
\end{bmatrix}
\]
(36)

Now, we have the following depending on whether the estimation is correct or not
\[
g'^{R}_{1m} h'^{I}_{1m} = \begin{cases}
|\hat{h}'^{R}_{1m}|, & \text{if } C_1 \text{ holds} \\
-|\hat{h}'^{R}_{1m}|, & \text{if } C_2 \text{ holds} \\
-|\hat{h}'^{I}_{1m}|, & \text{if } C_3 \text{ holds},
\end{cases}
\]
(37)

where \( \alpha \in \{R, I\} \). Using this, we can write (33) equivalently as
\[
\frac{1}{2} A \begin{bmatrix}
|\hat{h}'^{R}_{1m}| s'^{R}_{1} \\
|\hat{h}'^{I}_{1m}| s'^{I}_{1}
\end{bmatrix} = \begin{cases}
|\hat{h}'^{R}_{1m}| s'^{R}_{1} & \text{if } C_1 \text{ holds} \\
-|\hat{h}'^{I}_{1m}| s'^{I}_{1} & \text{if } C_2 \text{ holds} \\
-|\hat{h}'^{I}_{1m}| s'^{I}_{1} & \text{if } C_3 \text{ holds},
\end{cases}
\]
(38)

where \( \hat{h}'^{R}_{1m} \) and \( \hat{h}'^{I}_{1m} \) are zero-mean real-valued Gaussian RVs with variance 1/2. \( C_1 \) : when \( g'^{R}_{1m} \) and \( g'^{I}_{1m} \) are both correctly estimated, \( C_2 \) : when either \( g'^{R}_{1m} \) is correctly estimated and \( g'^{I}_{1m} \) is incorrect, or \( g'^{I}_{1m} \) is incorrectly estimated and \( g'^{R}_{1m} \) is correct, and \( C_3 \) : when \( g'^{R}_{1m} \) and \( g'^{I}_{1m} \) are both incorrectly estimated.

Without loss of generality, assume that there are \( K'^{R} \) incorrect estimates that influence \( h'^{R}_{1m} x'^{R}_{m} \) and \( K'^{I} \) incorrect estimates that influence \( h'^{I}_{1m} x'^{I}_{m} \). Then, we can write (25) and (26) equivalently as
\[
\begin{align*}
\hat{s}'^{R}_{1} &= \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=K'^{R}+1}^{M/N} |\hat{h}'^{R}_{m}| - \sqrt{\frac{P}{M}} s'^{R}_{1} + \hat{w}'^{R}_{n} \right), \\
\hat{s}'^{I}_{1} &= \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=K'^{I}+1}^{M/N} |\hat{h}'^{I}_{m}| - \sqrt{\frac{P}{M}} s'^{I}_{1} + \hat{w}'^{I}_{n} \right).
\end{align*}
\]
(39)

Since the received real-valued symbol at the first antennas, \( s'^{R}_{1} \), is given by (39), \( \Pr\{s'^{R}_{1} \neq s'^{I}_{1}\} \) is given by
\[
\Pr\{s'^{R}_{1} \neq s'^{I}_{1}\} = \Pr \left( \text{sign} \left( \sqrt{\frac{P}{M}} \sum_{m=K'^{R}+1}^{M/N} |\hat{h}'^{R}_{m}| - \sqrt{\frac{P}{M}} s'^{R}_{1} + \hat{w}'^{R}_{n} \right) \neq s'^{I}_{1} \right),
\]
(40)

where \( \hat{w}'^{R}_{n} \) is a zero-mean Gaussian RV with variance given by (31). Setting \( L = M/N \), (41) can be written as
\[
\Pr\{s'^{R}_{1} \neq s'^{I}_{1}\} = \sum_{k=0}^{L} \Pr \left( K'^{R} = k \right)
\]
\[
\times \int_{\hat{h}'^{R}_{m}} \cdots \int_{\hat{h}'^{R}_{L}} \text{Pr} \left( \text{sign} \left( \sqrt{\frac{P}{M}} s'^{R}_{1} + \sum_{m=K'^{R}+1}^{L} |\hat{h}'^{R}_{m}| - \sqrt{\frac{P}{M}} s'^{R}_{1} + \hat{w}'^{R}_{m} \right) \neq s'^{I}_{1} \right) \\
\times \prod_{m=1}^{L} f(\hat{h}'^{R}_{m}) d\hat{h}'^{R}_{m}.
\]
(42)

Since \( \hat{w}'^{R}_{n} \) is a Gaussian RV, (42) simplifies to
\[
\Pr\{s'^{R}_{1} \neq s'^{I}_{1}\} = \sum_{k=0}^{L} \Pr \left( K'^{R} = k \right)
\]
\[
\times \int_{\hat{h}'^{R}_{1}} \cdots \int_{\hat{h}'^{R}_{L}} \text{Pr} \left( \text{sign} \left( \sqrt{\frac{P}{M}} s'^{R}_{1} + \sum_{m=K'^{R}+1}^{L} |\hat{h}'^{R}_{m}| - \sqrt{\frac{P}{M}} s'^{R}_{1} + \hat{w}'^{R}_{m} \right) \neq s'^{I}_{1} \right) \\
\times \prod_{m=1}^{L} f(\hat{h}'^{R}_{m}) d\hat{h}'^{R}_{m}.
\]
(43)

In (42) and (43), \( \Pr \left( K'^{R} = k \right) \) is the probability of receiving \( k \) incorrect 1-bit CSIs that affects \( z'^{R}_{1} \), and can be found as
\[
\Pr \left( K'^{R} = k \right) = \binom{L}{k} p_{\epsilon}^{k} (1 - p_{\epsilon})^{L-k},
\]
(44)

where \( p_{\epsilon} \) is given in (19) and has been derived in Appendix A. Due to the law of large numbers, as \( L \rightarrow \infty \), we have the following asymptotic equality
\[
\frac{1}{L} \left( \sum_{m=K'^{R}+1}^{M/N} |\hat{h}'^{R}_{m}| - \sum_{m=1}^{K'^{R}} |\hat{h}'^{R}_{m}| \right) \\
\rightarrow \frac{1}{L} (L-k) E(|\hat{h}'^{R}_{m}|) - \frac{1}{L} k E(|\hat{h}'^{R}_{m}|) \\
= \frac{L-2k}{L} E(|\hat{h}'^{R}_{m}|), \text{ as } L \rightarrow \infty.
\]
(45)

As a result of (45), (42) transforms to
\[
\Pr\{s'^{R}_{1} \neq s'^{I}_{1}\} \\
\rightarrow \sum_{k=0}^{L} \Pr \left( K'^{R} = k \right) \int_{\hat{h}'^{R}_{1}} \cdots \int_{\hat{h}'^{R}_{L}} Q \left( \sqrt{\frac{P}{M}} (L-2k) E(|\hat{h}'^{R}_{m}|) \right) \\
\times \prod_{m=1}^{L} f(\hat{h}'^{R}_{m}) d\hat{h}'^{R}_{m} = \sum_{k=0}^{L} \Pr \left( K'^{R} = k \right) \\
\times Q \left( \sqrt{\frac{P}{M}} (L-2k) E(|\hat{h}'^{R}_{m}|) \right).
\]
(46)
Next, \( \Pr[z^R_1 \neq x^R_1] \) in (46) can be expressed as
\[
\Pr[z^R_1 \neq x^R_1] \to \sum_{k=0}^{L/2} \Pr(K^R_1 = k) \\
\times Q \left( \sqrt{\frac{P}{M} (L - 2k)E[|\hat{r}^R_m|]} \right) \\
+ \sum_{k=L/2+1}^{L} \Pr(K^R_1 = k) \\
\times Q \left( \sqrt{\frac{P}{M} (L - 2k)E[|\hat{r}^R_m|]} \right) (47)
\]
Now, since \( Q(x) \leq 1, \forall x \), (47) is upper bounded by
\[
\Pr[z^R_1 \neq x^R_1] \leq \sum_{k=0}^{L/2} \Pr(K^R_1 = k) \\
\times Q \left( \sqrt{\frac{P}{M} (L - 2k)E[|\hat{r}^R_m|]} \right) \\
+ \sum_{k=L/2+1}^{L} \Pr(K^R_1 = k) \\
\leq \sum_{k=0}^{L/2} \Pr(K^R_1 = k) e^{-\beta^2(L-2k)^2/2} \\
+ \sum_{k=L/2+1}^{L} \Pr(K^R_1 = k), (48)
\]
where (b) follows since for \( x \geq 0 \)
\[
Q(x) \leq \frac{e^{-x^2}}{2} (49)
\]
holds, and \( \beta \) is given by
\[
\beta = \sqrt{\frac{P}{M} E[|\hat{r}^R_m|]} \sqrt{\frac{P(N-1)+N}{2N}} (50)
\]
Now, by substituting (44) into (48), we obtain for \( L \to \infty \)
\[
\Pr \left\{ z^R_1 \neq x^R_1 \right\} \leq \sum_{k=0}^{L/2} \left( \frac{L}{k} \right) p^k_\epsilon (1-p_\epsilon)^{L-k} e^{-\beta^2(L-2k)^2/2} \\
+ \sum_{k=L/2+1}^{L} \left( \frac{L}{k} \right) p^k_\epsilon (1-p_\epsilon)^{L-k} (51)
\]
where \( p_\epsilon \) is given in (19). Next, since \( (\beta^2) \leq 2L \) holds, (51) is bounded by
\[
\Pr \left\{ z^R_1 \neq x^R_1 \right\} \leq \sum_{k=0}^{L/2} \Pr(K^R = k) e^{-\beta^2(L-2k)^2/2} \\
\leq \sum_{k=0}^{L/2} 2^k p^k_\epsilon (1-p_\epsilon)^{L-k} e^{-\beta^2(L-2k)^2/2} (52)
\]
We can upper bound \( O_2 \) as
\[
O_2 = \sum_{k=L/2+1}^{L} 2^k p^k_\epsilon (1-p_\epsilon)^{L-k} \\
< 2^L \frac{L}{2} \max \left\{ p^L_\epsilon (1-p_\epsilon)^{L-k} \right\} \\
< 2^L \frac{L}{2} p^L_\epsilon (1-p_\epsilon)^{L/2} \\
= \frac{L}{2} \left( 2^2 p_\epsilon (1-p_\epsilon) \right)^{L/2} \to 0, (53)
\]
since \( 4 p_\epsilon (1-p_\epsilon) < 1 \) for \( p_\epsilon < 1/2 \). To upper bound \( O_1 \), we consider two cases. Let \( E_1 \) and \( E_2 \) be defined as
\[
E_1 = \left\{ k : 0 \leq k \leq L/2; \ k/L \to 1/2 \text{ as } L \to \infty \right\}. (54)
\]
\[
E_2 = \left\{ k : 0 \leq k \leq L/2; \ k/L \to 1/2 \text{ as } L \to \infty \right\}. (55)
\]
Then, using (44), \( O_1 \) is upper bounded as
\[
O_1 \leq \sum_{k \in E_1} \Pr(K^R = k) e^{-\beta^2(L-2k)^2/2} \\
+ \sum_{k \in E_2} 2^k p^k_\epsilon (1-p_\epsilon)^{L-k} e^{-\beta^2(L-2k)^2/2}. (56)
\]
Now, the first sum in (56) is upper bounded as
\[
\sum_{k \in E_1} \Pr(K^R = k) e^{-\beta^2(L-2k)^2/2} \\
\leq \sum_{k \in E_1} e^{-\beta^2 L^2 (1-\frac{L}{2})^2} \to 0 \text{ as } L \to \infty. (57)
\]
On the other hand, the second sum in (56) is upper bounded as
\[
\sum_{k \in E_2} 2^k p^k_\epsilon (1-p_\epsilon)^{L-k} e^{-\beta^2(L-2k)^2/2} \\
= \sum_{k \in E_2} 2^k L^2 p^L_\epsilon (1-p_\epsilon)^{L(1-k)/L} \\
\times e^{-\beta^2 L^2 (1-2k)^2/2} \\
= \sum_{k \in E_2} 2^k L^2 p^L_\epsilon (1-p_\epsilon)^{L(1-1/2)} e^{-\beta^2 L^2 (1-1/2)^2/2} \\
= |E_2| (2^2 p_\epsilon (1-p_\epsilon))^{L/2} \\
\leq \frac{L}{2} (2^2 p_\epsilon (1-p_\epsilon))^{L/2} \to 0 (58)
\]
since \( 4 p_\epsilon (1-p_\epsilon) < 1 \) for \( p_\epsilon < 1/2 \). Combining (52), (53), (56), (57), and (58), we obtain \( \Pr \left\{ z^R_1 \neq x^R_1 \right\} \leq 0 \) as \( L \to \infty \).
APPENDIX C
PROOF OF THEOREM 2

Proof: (Converse) For the considered 1-bit quantized MIMO system, we have
\[ I(x; z(G)) = H(x|G) - H(x|z, G) \leq H(x|G) = 2M, \] (59)
where the last inequality follows from the 1-bit quantized inputs. Hence, the capacity of this system cannot be larger than \( 2M \). In the following, we prove that (59) is asymptotically achievable. \( \square \)

Proof: (Achievability) Using Achievability Scheme 2, the transmitter sends \( 2M \) bits of information at each channel use. Hence, the rate of Achievability Scheme 2 is \( 2M \) bits per channel use. Now, we are only left to prove that the symbols received at the receiver can be decoded with probability of error vanishing when \( N \to \infty \) and \( M < \infty \) is fixed.

Assume that \( s \) is transmitted and \( z \) received. From \( z \), we find \( \hat{s} \) using (14). Then, the probability of error can be bounded as
\[ P_e = \Pr[\hat{s} \neq s] \leq \sum_{m=1}^{M} \Pr[\hat{s}_m \neq s_m] = 2M \Pr[\hat{s}_1 \neq s_1], \] (60)
where (60) follows from the union bound and (61) follows from symmetry. Now, \( \Pr[\hat{s}_1 \neq s_1] \) is given by
\[ \Pr[\hat{s}_1 \neq s_1] = \Pr\left( \sum_{n=1}^{N} g_{1n}^R s_n \neq s_1 \right) = \Pr\left( \left| \sum_{n=1}^{N} g_{1n}^R s_n - h_{n1}^R s_1 \right| + \left| w_n^R \right| \neq s_1 \right), \] (61)
where \( h_{n1}^R \) is correct with probability \( 1 - p_e \), in which case \( g_{n1}^R = \text{sign}(h_{n1}^R) \) holds, and is incorrect with probability \( p_e \), in which case \( g_{n1}^R = -\text{sign}(h_{n1}^R) \) holds. Without loss of generality, assume that \( h_{n1}^R \) for \( n = 1, 2, \ldots, N-j \) is correctly estimated and \( h_{n1}^R \) for \( n = N-j+1, \ldots, N \) is incorrectly estimated, where \( j \) is an RV that takes values from zero to \( N \). Then, (62) can be written as
\[ \Pr[\hat{s}_1 \neq s_1] = \sum_{j=0}^{N} \binom{N}{j} p_e^j (1-p_e)^{N-j} \times \Pr\left\{ \text{sign}\left( \sum_{n=1}^{N-j} \text{sign}(h_{n1}^R) s_n \right) + \sqrt{\frac{P}{2M}} \left( h_{n1}^R s_1 + \sum_{m=2}^{M} h_{nm}^R s_n + w_n^R \right) + \sum_{m=2}^{M} h_{nm}^R s_n \right\} \neq s_1^{R}, \] (63)
where \( p_e \), derived in Appendix A and given in (19), is the probability of estimating \( h_{n1}^R \) incorrectly. Since \( \text{sign}(a)\text{sign}(b) = \text{sign}(ab) \) holds, (63) can be written as
\[ \Pr[\hat{s}_1^{R} \neq s_1^{R}] = \sum_{j=0}^{N} \binom{N-j}{j} p_e^j (1-p_e)^{N-j} \times \Pr\left\{ \text{sign}\left( \sum_{n=1}^{N-j} \text{sign}(h_{n1}^R) s_n \right)+ \sqrt{\frac{P}{2M}} \left( h_{n1}^R s_1 + \sum_{m=2}^{M} h_{nm}^R s_n + w_n^R \right) \right\} \neq s_1^{R}, \] (64)
where
\[ \hat{w}_n^R = \text{sign}(h_{n1}^R) \left[ \sqrt{\frac{P}{2M}} \sum_{m=2}^{M} h_{nm}^R s_n + w_n^R \right] \] (65)
is a zero-mean Gaussian RV with variance
\[ \sigma_w^2 = \frac{P}{2M} \frac{2M - 1}{2} + \frac{1}{2}. \] (66)
On the other hand, (64) can be expressed as
\[ \Pr[\hat{s}_1^{R} \neq s_1^{R}] = \sum_{j=0}^{N} \binom{N-j}{j} p_e^j (1-p_e) \times \Pr\left\{ \text{sign}\left( \sum_{n=1}^{N-j} \text{sign}(h_{n1}^R) s_n \right)+ \sqrt{\frac{P}{2M}} \left( h_{n1}^R s_1 + \hat{w}_n^R \right) \right\} \neq s_1^{R} \]
\[ = \Pr\left( C (N^-_1) + C (N^+_2) \right) \geq C (N^+_1) + C (N^-_2) \] (67)
where $C(\cdot)$ denotes cardinality, $\mathcal{N}^+_1$ and $\mathcal{N}^+_2$ contain the elements in $1 \leq n \leq N-j$ and $N-j+1 \leq n \leq N$, respectively, for which

$$\text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right] = s_1^R.$$  

(68)

Moreover, $\mathcal{N}^-_1$ and $\mathcal{N}^-_2$ contain the elements in $1 \leq n \leq N-j$ and $N-j+1 \leq n \leq N$, respectively, for which

$$\text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right] \neq s_1^R.$$  

(69)

Now, the probability of the event in (68) is given by

$$P_c = \Pr\left\{\text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right] = s_1^R\right\}$$

$$= \frac{1}{\sqrt{\pi}} \int_{h_{n1}^R} Q\left(\frac{-\sqrt{\frac{P}{2M}}|h_{n1}^R|}{\sqrt{\frac{P}{2M}(M - \frac{1}{2}) + 1}}\right) \times e^{-(h_{n1}^R)^2} dh_{n1}$$

$$= \frac{1}{\sqrt{\pi}} \int_{h_{n1}^R} Q\left(\frac{\sqrt{\frac{P}{2M}}|h_{n1}^R|}{\sqrt{\frac{P}{2M}(2M - 1) + 1}}\right) \times e^{-(h_{n1}^R)^2} dh_{n1}$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{+\infty} \int_{-\tan^{-1}(\sqrt{\frac{P}{2M}})}^{+\infty} e^{-\frac{(h_{n1}^R)^2}{2} + \frac{h_{n1}^R}{2}r} \text{d}r \text{d}h_{n1}^R$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\tan^{-1}(\sqrt{\frac{P}{2M}})}^{+\infty} \int_0^{+\infty} e^{-\frac{(h_{n1}^R)^2}{2} + \frac{h_{n1}^R}{2}r} \text{d}r \text{d}h_{n1}^R$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{\pi} \tan^{-1}\left(\frac{1}{\sqrt{2M-1+\frac{4P}{2M}}}\right).$$  

(70)

where $h_{n1}^R = \sqrt{2}h_{n1}$ and $\gamma = \frac{-\sqrt{\frac{P}{2M}}}{\sqrt{\frac{P}{2M}(2M - 1) + 1}}$. Since the event in (69) is the complement of the event in (68), the following holds

$$P_w = \Pr\left\{\text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right] \neq s_1^R\right\}$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{2M-1+\frac{4P}{2M}}}\right).$$  

(71)

Now, according to (67), we know that an error occurs when $C(\mathcal{N}^-_1) + C(\mathcal{N}^-_2) \geq C(\mathcal{N}^+_1) + C(\mathcal{N}^+_2)$. On the other hand, since $C(\mathcal{N}^-_1) + C(\mathcal{N}^+_1) + C(\mathcal{N}^-_2) + C(\mathcal{N}^+_2) = N$, we can conclude that an error occurs when $C(\mathcal{N}^-_1) + C(\mathcal{N}^+_1) \geq \frac{N}{2}$. Since $\mathcal{N}^-_1 + \mathcal{N}^+_1$ comprise the first $N-j$ elements of the sum in (67), and $\mathcal{N}^-_2 + \mathcal{N}^+_2$ comprise the rest of $j$ elements, we denote $C(\mathcal{N}^-_1)$ by $a$, $C(\mathcal{N}^+_1)$ by $N-j-a$, $C(\mathcal{N}^-_2)$ by $b$, and $C(\mathcal{N}^+_2)$ by $j-b$. Consequently, in order for an error to happen, $a + b > \frac{N}{2}$ needs to hold, which in turns mean that $a > \frac{N}{2} - b$. Hence, we obtain $\Pr\{s_1^R \neq s_1^R\}$ in (67) as

$$\Pr\{s_1^R \neq s_1^R\} = \sum_{j=0}^{N} \sum_{b=0}^{N} \sum_{a=\max(\frac{N}{2} - b, 0)}^{N} \left(\begin{array}{c} N \\ a \\ b \end{array}\right)$$

$$\times p_a^j(1 - p_x)^{N-j}(a b)\times (P_x)^a (P_y)^{N-j-a}(P_y)^b (P_w)^{j-b}$$

$$= \Pr\{s_1^R \neq s_1^R\}$$

$$= \sum_{j=0}^{N} \sum_{b=0}^{N} \sum_{a=\max(\frac{N}{2} - b, 0)}^{N} \left(\begin{array}{c} N \\ j \\ N-j \\ a \\ b \end{array}\right)$$

$$\times (P_x)^j (P_y)^{N-j-a}(P_y)^b (P_w)^{j-b}.$$  

(72)

where $P_c$ and $P_w$ are given in (71) and (72), respectively. We now show that when $N \to \infty$, the error probability expressed in (64) converges to zero. Since $\frac{1}{N} > 0$ and since $\text{sign}(a)\text{sign}(b) = \text{sign}(ab)$, $\Pr\{s_1^R \neq s_1^R\}$ in (64) can be written as

$$\Pr\{s_1^R \neq s_1^R\}$$

$$= \sum_{j=0}^{N} \left(\begin{array}{c} N \\ j \\ N-j \end{array}\right) p_a^j(1 - p_x)^{N-j}$$

$$\times \Pr\left\{\text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right] \neq \text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right]\right\}.$$  

(73)

Now, the sums in (74) for $N \to \infty$ can be written as

$$\frac{1}{N} \sum_{n=1}^{N} \text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right] = \frac{N-J}{N}$$

$$\times \frac{1}{N-J} \sum_{n=1}^{N} \text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right]$$

$$\rightarrow (1 - \alpha_J)E\left\{\text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right]\right\}$$

(75)

and

$$\frac{1}{N} \sum_{n=N-J+1}^{N} \text{sign}\left[\frac{P}{2M}|h_{n1}^R|s_1^R + \hat{w}_n^R\right].$$
\[
\begin{align*}
&= \frac{1}{N} \sum_{n=N-j+1}^{N} \text{sign} \left( \sqrt{\frac{P}{2M}} |h_{n1}| s_{1}^{R} + \hat{w}_{n}^{R} \right) \\
&\to \alpha_{j} E \left\{ \text{sign} \left( \sqrt{\frac{P}{2M}} |h_{n1}| s_{1}^{R} + \hat{w}_{n}^{R} \right) \right\},
\end{align*}
\]

where
\[
\alpha_{j} = \lim_{N \to \infty} \frac{j}{N}.
\]

Hence, we can write (74) for \( N \to \infty \) as
\[
\begin{align*}
&\text{Pr} \{ s_{1}^{R} \neq \hat{s}_{1}^{R} \} \\
&= \sum_{j=0}^{N} \binom{N}{j} p_{e}^{j} (1 - p_{e})^{N-j} \\
&\times \text{Pr} \left\{ \text{sign} (1 - \alpha_{j}) \right\} \\
&\times E \left\{ \text{sign} \left( \sqrt{\frac{P}{2M}} |h_{n1}| s_{1}^{R} + \hat{w}_{n}^{R} \right) \right\} \\
&- \alpha_{j} E \left\{ \text{sign} \left( \sqrt{\frac{P}{2M}} |h_{n1}| s_{1}^{R} + \hat{w}_{n}^{R} \right) \right\} \neq \hat{s}_{1}^{R} \\
&= \sum_{j=0}^{N} \binom{N}{j} p_{e}^{j} (1 - p_{e})^{N-j} \\
&\times \text{Pr} \left\{ \text{sign}(s_{1}^{R}(1 - 2\alpha_{j})) \neq \hat{s}_{1}^{R} \right\} \\
&\times p_{e}^{j} (1 - p_{e})^{N-j} \text{Pr} \{ \text{sign}(1 - 2\alpha_{j}) \neq 1 \} \\
&\overset{(a)}{=} \sum_{j=\frac{N}{2}}^{N} \binom{N}{j} p_{e}^{j} (1 - p_{e})^{N-j}, \\
\end{align*}
\]

where (a) holds since \( \text{Pr} \{ \text{sign}(1 - 2\alpha_{j}) \neq 1 \} = 0 \) for \( j < \frac{N}{2} \) and \( \text{Pr} \{ \text{sign}(1 - 2\alpha_{j}) \neq 1 \} = 1 \) for \( j = \frac{N}{2}, \ldots, N \). Now, \( \sum_{j=\frac{N}{2}}^{N} \binom{N}{j} p_{e}^{j} (1 - p_{e})^{N-j} \) in (78) can be bounded as
\[
\begin{align*}
&\sum_{j=\frac{N}{2}}^{N} \binom{N}{j} p_{e}^{j} (1 - p_{e})^{N-j} \\
&\overset{(b)}{=} \frac{N}{2} \binom{N}{j} p_{e}^{N/2} (1 - p_{e})^{N/2} \\
&\overset{(c)}{=} \frac{N}{2} 4^{N/2} p_{e}^{N/2} (1 - p_{e})^{N/2} \\
&\times \frac{N}{2} (4p_{e} (1 - p_{e}))^{N/2} \overset{(e)}{=} 0, \text{ as } N \to \infty,
\end{align*}
\]

(b) holds since \( p_{e} < \frac{1}{2} \), (c) holds since \( \left( \frac{N}{2} \right) < 2^{N} \), and (e) holds since \( 4p_{e} (1 - p_{e}) < 1 \).

\( \square \)

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