Detection of initial strain of main ribs of concrete beam (or slab) by bending moment increment method

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Abstract. In the inspection and identification of engineering structures, it is often necessary to understand the force state of the main ribs of the components. Based on the structural performance test, the relationship between the bending moment of the concrete beam (or slab) in the elastic phase and the plastic phase and the strain of the main rib is analyzed. The initial moment of the main beam of the concrete beam (or slab) is proposed by the bending moment increment method. By adding a small amount of bending moment and measuring the strain increment of the main rib, the initial strain of the main rib of the flexural member is estimated to better understand the structural performance of the member. During the bending moment increment detection process, the increase in bending moment is small and does not adversely affect the concrete structure.

1. General Introduction

For engineering projects, structural performance tests are often done to examine bearing capability of flexural components. In this paper, however, for components in use, just a little load will be imposed and the structural performance will be examined basing on deflection change. Flexural components adopted for engineering projects generally refer to reinforced concrete beam (or slab). In this paper, the elastic or plastic state of the reinforced beam (or slab) is verified by the test. The strain and load of the steel bar are approximately linear at a certain stage, so that the initial rib initial deformation of the flexural member can be estimated by measuring the change of the main rib strain under a small load. So that to gain a deeper understanding of the structural properties of the component.

2. Tests

2.1 Preparation of Test Specimen

Table 1: Basic parameters of components

|          | strength grade of concrete | scantlings | main reinforcement |
|----------|---------------------------|------------|---------------------|
| specimen 1# | C25                       | 2400×250×150 | 2 12                |
| specimen 2# | C25                       | 2400×250×150 | 3 12                |
2.2 Tests Preparation

Before the test, seven strain gauges were placed evenly in vertical direction on the section in the position of mid-span of the beam (as shown P1 to P7 in Figure 1). The gauges placed were used for measuring position of neutral axis during the process the beam was bent. P8 in Figure 1 shows the position of the reinforcement strain gauges. To eliminate negative impacts raised due to bending of reinforcement, two strain gauges were glued in this position at each end of the section's diameter.

2.3 Test Methods

Reinforcement concrete components were tested according to Standard for Test Method of Concrete Structures (GB/T 50152-2012). Reaction frame, distributive girder and jack were adopted to carry out stage loading. It was hold for 30 minutes under effect of standard load and for 10 minutes under load of other levels. After holding test components for specified time under corresponding loads, read the dial indicator for inflection of the components. At the same time, static strain mater was adopted for an automatic recording of strain of concrete and reinforcement under specific load.

![Diagram](image)

**Figure 1.** Schematic for the test and distribution of meters and strain gauges

2.4 Test Result

It can be seen from the following two figures that state of mid-span of the beam complied with the plane-section assumption. Position of neutral axis under each load could be determined basing on relationship interpolation of strain value of each strain gage and their positions.

![Diagram](image)

**Figure 2.** Diagram of concrete strain at each point each point under each level of load for test component #1

![Diagram](image)

**Figure 3.** Diagram of concrete strain at each point each point under each level of load for test component #2
Table 2. Calculation results (distance from the neutral axis to the beam top surface was a (mm)).

|            | specimen 1# | specimen 2# |
|------------|-------------|-------------|
| Load (kN)  | value of “a”| strain of main reinforcement (με) | Load (kN)  | value of “a”| strain of main reinforcement (με) |
| 0          | 130         | 0.55        | 0          | 133         | 0.20        |
| 1.34       | 130         | 13.4        | 3.91       | 133         | 24          |
| 7.18       | 110         | 73.23       | 12.31      | 129         | 121.88      |
| 13.02      | 47          | 278.03      | 20.72      | 99          | 390.25      |
| 18.86      | 48          | 554.23      | 29.12      | 72          | 716.75      |
| 24.70      | 47          | 834.23      | 37.53      | 66          | 997.75      |
| 27.62      | 47          | 946.73      | 41.73      | 59          | 1112.88     |
| 30.54      | 47          | 1081.00     | 45.94      | 56          | 1239.75     |
| 33.46      | 47          | 1211.08     | 50.139     | 55          | 1362.50     |
| 36.38      | 47          | 1347.05     | 54.34      | 54          | 1519.00     |
| 38.16      | 47          | 1428.28     | 56.89      | 53          | 1606.62     |
| 40.02      | 48          | 1501.90     | 59.56      | 53          | 1689.12     |
| 41.88      | 47          | 1597.80     | 62.22      | 53          | 1787.00     |
| 43.73      | 43          | 1687.33     | 66.72      | 53          | ---         |
| 45.58      | 39          | 1779.38     | 80.07      | 52          | ---         |

Figure 4. Load—Reinforcement strain curve

Figure 5. Relationship between strain and position of concrete and steel

It can be seen from the test mentioned above that the neutral axis was relatively stable during the elastic stage before concrete cracks; after concrete cracks at the tensile region, the neutral axis moved fast upward; during the plastic stage, the neutral axis still moved upward but movement was not obvious. In this paper, with less increment of load (bending moment), upward movement of the neutral axis was neglected and action spots of resultant force of stresses in the pressure zone of concrete was assumed as unchanged.

3. Calculation and Derivation of Height of the Neutral Axis in Elastic Working State

In the test, the neutral axis was relatively stable in the plastic stage and these were mostly reflected through test data. However, less data was obtained during the elastic stage. Following derivation is given for proving:

Section of flexural component was in height h as shown in the figure. Due to that no cracks appeared in the component, concrete within the tensile region was under action as well (Figure 5).

Basing on deformation compatibility conditions:
Tension and pressure was in balance in horizontal direction:
\[
\frac{\varepsilon_y}{h-a} = \varepsilon_y
\]

Substituting equation (1) and equation (2) into equation (3), we can get:
\[
\frac{1}{2} E_s \varepsilon_s ab = \frac{1}{2} E_s \varepsilon_s (h-a) b + \varepsilon_y E_y A
\]

In equations (3) and (4), “b” refers to width of the component, “A” refers to cross-sectional area of reinforced bars.

By equation (4), it could be obtained:
\[
E_s a^2 b = E_s (h-a)^2 b + 2(h - a_s - a) E_y A
\]

Solve equations and “a” could be calculated as:
\[
a = \frac{1}{2} E_s b h^2 + (h - a_s) E_y A
\]

The parameters in (5) are all fixed values, so for any bent member, the neutral axis position is fixed before cracking.

4. Calculation of Existing Strain of Flexural Components

Based on all of the above, it can be considered that the neutral axis of the bending component is in a relatively fixed position in the elastic phase and the plastic phase in the case where the load increase is not large. (Position of neutral axis changed greatly when got into plastic stage from elastic stage, it was not discussed in this paper).

4.1 Elastic Stage

Assuming that initial bending moment of flexural component is “\(M_x\)” and the initial strain is “\(\varepsilon_s\)”. Before the structural performance test, the steel strain gauge is attached, and the strain increment “\(\Delta \varepsilon\)” of the steel bar is measured. In the initial state, the moment of the tension material on the neutral axis should be equal to the bending moment of the section:
\[
E_s \varepsilon_s A(h-a-a_s) + \frac{2(h-a)^2 \varepsilon_s E_s}{3(h-a-a_s)} = M_x
\]

After adding certain load, assume the increment of reinforcement as “\(\Delta \varepsilon\)”, and increment of bending moment as “\(\Delta M\)”. Then,
\[
(\varepsilon_s + \Delta \varepsilon) E_s A(h-a-a_s) + \frac{2(h-a)^2 (\varepsilon_s + \Delta \varepsilon) E_s}{3(h-a-a_s)} = M_x + \Delta M
\]

\[
\Delta \varepsilon E_s A(h-a-a_s) + \frac{2(h-a)^2 \Delta \varepsilon E_s}{3(h-a-a_s)} = \Delta M
\]

\[
\frac{\Delta M}{\varepsilon_s} = C
\]

In equation 8, each term including “a” is of fixed but not explicit value. Value of \(\Delta \varepsilon/\Delta M\) was calculated through the test. Therefore, \(\varepsilon_s=M_x C\). Initial strain of reinforcement could be known basing on strain increment of reinforcement and bending moment increment of components and initial strain of reinforcement could be calculated then.
4.2 Plastic Stage

Taking “b” as the value of distance from action point of resultant force on concrete in pressure zone to peak point of the pressure zone, then:

\[ M_x = \varepsilon_x E_x A (h - a_x - b) \]  \hspace{1cm} (10)

After adding a certain load, assume the increment of reinforcement as “\( \Delta \varepsilon \)”, and increment of bending moment as “\( \Delta M \)”. At this moment, tensile stress of concrete material would not be taken into consideration. Therefore,

\[ M_x + \Delta M = (\varepsilon_x + \Delta \varepsilon) E_x A (h - a_x - b) \]  \hspace{1cm} (11)

(11)-(10): \[ \frac{\Delta M}{\Delta \varepsilon} = E_x A (h - a - a_x) \] \hspace{1cm} (12)

Let:

\[ \frac{\varepsilon_x}{M_x} = \frac{\Delta \varepsilon}{\Delta M} = C \] \hspace{1cm} (13)

In the same way, value of "C" could be calculated through test, namely, \( \Delta \varepsilon / \Delta M \). Then, \( \varepsilon_x = M_xC \). Initial strain of reinforcement could be known basing on strain increment of reinforcement and bending moment increment of components and initial stress of reinforcement could be calculated then.

5. Brief Summary

From the above results, it can be concluded that in the elastic and plastic phases of the concrete members, the strain and load (bending moment) of the steel bars are approximately linear in the elastic phase and the plastic phase. And in the process from the elastic phase to the plastic phase, there is no linear relationship between strain and load. The results of this test also fully illustrate this point, as shown in Figure 4 above.

From Figure 4, it could be seen that when component was in elastic stage, the fitted curve of \( \varepsilon \sim M \) (P) went through the point \((0,0)\), namely, \( \varepsilon = MC \) assumption is tenable; when component was in plastic stage, the fitted line of \( \varepsilon \sim M(P) \) passed below the point \((0,0)\), namely, \( \varepsilon = MC - a \). In this paper, “a” was valued as “0” during calculation. Therefore, the calculated value was larger due to this.

6. Application Examples

6.1 Test Study

Follows are analyses of strain of test components in a certain stage.

For the specimen 2#, it is assumed that the strain of the steel bar is unknown under the action of 56.89 kN (the bending moment is 21.33 kN-m). At this time, the strain gauge is attached and the strain gauge is zeroed. Under the action of 59.56kN (the bending moment is 22.335 kN-m), the collected strain value should be 1689.12-1606.62=82.5με, and the added bending moment of this process is 1.005kN-m.

\[ \frac{\varepsilon_x}{M_x} = \frac{\Delta \varepsilon}{\Delta M} = \frac{82.5}{1.005} = 82 \]

\[ \therefore \varepsilon_x = M_xC = 21.33 \times 82 = 1749 \mu \varepsilon \]

Comparing with the actually measured strain, \( \varepsilon_x = 1606.62 \mu \varepsilon \), relative error is +3.5%.

6.2 Field Inspection

Structural performance tests were performed on three bent members. Firstly, two steel bars are exposed, strain gauges are attached, and the strain gauges are zeroed at the beginning of the test. The strain change amount \( \Delta \varepsilon \) is measured under the first-stage load, and the steel strain is obtained by the formula (9), and then the estimated initial stress value “a” is obtained. Compare the result with the calculated initial stress value “b” obtained according to the design as shown in Table 3.
### Table 3 Engineering projects—stress comparison

| Number | 1      | 2      | 3      |
|--------|--------|--------|--------|
| Load (kN) | 1.5    | 2.0    | 2.0    |
| Increment of moment (kN•m) | 0.619  | 0.900  | 1.050  |
| Increment of strain (με) | 399    | 520    | 452    |
| value of C | 645    | 578    | 431    |
| Initial moment (kN•m) | 3.259  | 3.878  | 5.278  |
| strain (με) | 2101   | 2241   | 2275   |
| calculated initial stress a (MPa) | 399    | 426    | 432    |
| initial stress b (MPa) | 436    | 433    | 435    |

As shown in above tables, initial stress “a” and initial stress “b” obtained through calculation are in good conformance with each other. This indicates that the method adopted is practical.

### 7. Conclusions

The experimental and computational studies in this paper are based on the calculation formula of reinforced concrete beams (or slab), so the conclusions of this paper only apply to reinforced concrete beams (or slab).

In the elastic and plastic working state of the flexural member, the strain of the steel bar has a good linear relationship with the load (bending moment), which can be used to estimate the strain of the steel bar of the component under the current situation. And then the stress of the steel bar is estimated. It is provided that important data for evaluating the current structural properties of the component and further refining the design.

When the elastic working state of the member enters the plastic working state and the member approaches the limit state of the bearing capacity, the above linear relationship does not exist, so the strain of the steel bar cannot be estimated by this method at this time.

### 8. Acknowledgments

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