A Unique Method to Determine SNe Initial Explosion Energy

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ABSTRACT

There are several different methods to determine the individual supernovae (SNe) initial explosion energy, here we derive the average or typical explosion energy of shell-type supernova remnants (SNRs) in a particular way. By solving a group of equations pertaining to shell-type SNRs at the same stage we obtained some physical parameters, e.g., the distance \((d)\), evolved age \((t)\), etc.. Assuming series of different SN initial explosion energies ranging from \(10^{48}\) ergs to \(10^{53}\) ergs, we derived series of distance and age parameters with which compared already known ones. Thus the most likely value of the SNe initial explosion energy is obtained when the deviation is least, which equals to about \(10^{51}\) ergs, in good agreement with the undertook value.

Subject headings: supernovae: general — energy — value

1. Introduction

We have already known the initial kinetic energy \((E_0)\) of lots of Galactic SNRs through various estimate methods (table I). These SNRs embody shell-type and composite-type ones. Fig. II denotes the number distribution of 44 such remnants. It seems that they concentrate to about \(10^{50} \sim 10^{51}\) ergs. Therefore we can significantly take the average initial energy \((E_0)\) as a typical SNR physical parameter which is also able to be determined by our unique means.

Our mathematical method to derive SNe initial energy is rather different from others. One first method of others, for example, by \(E = \frac{1}{2} M_{su} v^2\), \(M_{su}\) is the swept-up mass of remnant shell expanding into interstellar media (ISM), \(v\) is the velocity of shock wave of remnant, the initial explosion energy of SNR G180.0−1.7 is thus obtained (Braun et al. 1989). Sun et al. (1999) calculating their detected ASCA data plus ROSAT data has derived the initial energy of G327.1−1.1 as the fitting result. After knowing the SNR G299.2−2.9 radius \((R)\) value, the particle density \((n_0)\) and the age \((t)\), Slane et al. (1996) get \(E_0\) value by \(E_0 \approx 340 R^2 n_0 t^{-2} \times 10^{51}\) ergs. Bamba et al. (2001) have got the \(E_0\) value by assuming a thin thermal NEI plasma model plus standard Sedov model. And so on. All of them obtain the initial energy of a individual SNR through physical means. Here in the paper we get SNe average initial explosion energy by statistical method.

Galactic supernova remnants are classified into three types: shell-type, Plerion-type and composite-type. Our work here merely include the shell-type remnants. Moreover, shell type SNRs usually have four evolution stages: the free expansion phase, the Sedov or adiabatic phase, the radiative or snowplough phase and the dissipation phase. Nearly all the observed SNRs are in the adiabatic phase, or in the 3rd. And almost none is detected in the 1st and 4th phases.

In the paper, numerical analysis for the most likely value of the SN initial energy is described in section 2 of which they are made separately at adiabatic-phase and radiative-phase for shell-type SNRs, and both made comparison by distances and ages. In the last section we discuss and
Table 1

List of the initial explosion energy $E_0$ of 44 Galactic shell-type or composite-type SNRs of which their value was somewhat well determined by various methods.

| Source | $E_0 \times 10^{51}$ ergs | Ref. | Source | $E_0 \times 10^{51}$ ergs | Ref. |
|--------|----------------------|------|--------|----------------------|------|
| G0.0+0.0 | $\geq 40^a$ | KF96 | G180.0−1.7 | 0.24 | BGL89 |
| G4.5+6.8 | 0.4/0.5$^b$ | BKV05 | G184.6−5.8 | 0.015 | GAT04a |
| G18.8+0.3 | 0.01 | D99 | G261.9+5.5 | 0.29 | CD80 |
| G18.9−1.1 | 0.08−0.18 | H04 | G263.9−3.3 | 1−2 | GAT04a |
| G24.7+0.6 | 0.066 | L89 | G290.1−0.8 | 0.8 | S02 |
| G28.6−0.1 | 0.9 | BUK01 | G291.0−0.1 | 0.25? | HHS98 |
| G29.7−0.3 | 2 | HCG03 | G292.0+1.8 | 0.18 | GAT04b |
| G31.9+0.0 | 0.3−1.4$^c$ | C05 | G292.2−0.5 | 1.0 | GAT04b |
| G39.7−2.0 | 1 | MS96 | G296.1−0.5 | 0.23 | HM94 |
| G41.1−0.3 | 1.2 | SDP05 | G296.5+10.0 | 0.2−0.6 | GAT04b |
| G74.0−8.5 | 0.24 | BGL89 | G299.2−2.9 | 0.12 | SVH96 |
| G82.2+5.3 | 0.17 | M04 | G312.4−0.4 | 0.6 | CB99 |
| G93.3+6.9 | 0.39 | LRR99 | G315.4−2.3 | 0.66 | GAT04b |
| G94.0+1.0 | $\geq 0.27$ | F05 | G320.4−1.2 | 1−2 | GAT04b |
| G106.3+2.7 | $>0.07$ | KUP01 | G326.3−1.8 | 1.0 | GAT04b |
| G109.1−1.0 | 1−10 | GAT04a | G327.1−1.1 | 0.23 | SWC99 |
| G111.7−2.1 | 2−3 | V06 | G327.6+14.6 | 1.0 | GAT04b |
| G116.9+0.2 | 0.1 | CHP97 | G347.3−0.5 | 1.0 | MTT05 |
| G119.5+10.2 | 0.03 | GAT04a | G349.7+0.2 | 0.5 | L05 |
| G120.1+1.4 | 1.16 | WHB05 | G352.7−0.1 | 0.2 | K98a |
| G126.2+1.6 | $>7$ | B05 | G357.7+0.3 | 0.11 | L89 |
| G132.7+1.3 | 0.31 | GAT04a | G359.0−0.9 | 0.21 | L89 |

$^a$Notes: In this case we just make use of the minimum value for our purpose.

$^b$Notes: In the case we take the average value.

$^c$Notes: The same as above.
summarize our results.

2. Numerical Analysis

2.1. At adiabatic phase

Let us list the group of equations for shell-type supernova remnants at the second stage as follow (Wang & Seward 1984, Koyama & Meguro-Ku 1987, Bignami & Caraveo 1988, Xu et al. 2005),

\[ D_{pc} = 4.3 \times 10^{-11} \left( \frac{E_0}{n_{cm-3}} \right)^{1/5} t_{yr}^{2/5} \]  \hspace{1cm} (1)

\[ \Sigma(D) = 1.505 \times 10^{-19} \frac{S_{1GHz}}{\theta_{arcmin}^2} \]
\[ = 2.88 \times 10^{-14} D_{pc}^{-3.8} n_{cm-3}^2 \]  \hspace{1cm} (2)

\[ \left( \frac{E_0}{10^{48} \text{ erg}} \right) = 5.3 \times 10^{-7} n_{cm-3}^{1.12} v_{km s^{-1}}^{1.4} \]
\[ \times \left( \frac{D_{pc}}{2} \right)^{3.12} \]  \hspace{1cm} (3)

Here, \( D_{pc} \) is the SNR diameter in unit of pc, \( t_{yr} \) is the remnant age in year, \( n \) is the ISM electron density in \( cm^{-3} \), \( S_{1GHz} \) is the detected fluxes of an SNR in Jy at 1 GHz, \( \theta_{arcmin} \) is the observational angle in arcmin, \( v \) is the velocity of shock waves in \( Km s^{-1} \). And we know \( \tan \left( \frac{\theta_{arcmin}}{2} \right) = \frac{D_{pc}}{2n} \), where, \( D_{pc} \) is the distance to a remnant in pc.

From these parameters above, the fluxes \( S_{1GHz} \) and observational angle \( \theta_{arcmin} \) is the detected value for each SNR (table 2) of which we regard these remnants evolving at the Sedov-phase since their diameter less than 36 pc. And the SNRs explosion energy \( E_0 = \xi \times E_{48} \) (\( \xi = 1, 2, 3, ..., 10^5 \)) is an assumed value. But the diameter \( D_{pc} \) (and distance \( d_{pc} \)), age \( t_{yr} \), velocity \( v_{kms^{-1}} \) and electron density \( n_{cm-3} \) is unknown and to be derived. Parameters of the distance \( d_{pc} = d_{our}(\xi, i) \), and the age \( t_{yr} = t_{our}(\xi, i) \) (\( \xi = 1, 2, 3, ..., 10^5 \), \( i = 1, 2, 3, ..., 37 \)) will be adopted in our paper. But we do nothing with \( v \) and \( n \).

For a certain supernova remnant \( (i) \) \( (i = 1, 2, 3, ..., 37) \) (table 2) and assumed series of SNe initial explosion energies \( E_0 = \xi \times E_{48}(\xi = 1, 2, 3, ..., 10^5) \) in the unit of \( 10^{48} \) ergs, we can obtain the remnant distance \( d_{our}(\xi, i) (= d_{pc}) \) and the age \( t_{our}(\xi, i)(= t_{yr}) \) values by solving the group of equations above. Then we compare them with the already known parameters \( d_{true}(i) \) and \( t_{true}(i) (i = 1, 2, 3, ..., 37) \) listed in table 2 in order to derive the most likely original energy of remnants. Thus the explosion energy was derived when the deviation in comparison is least.

The group of equations are not strictly correct as not to be figured out mathematically, but they are correct enough for us to confirm the SNe initial energy \( (E_0) \).

Figure 2 shows that when \( S_{1GHz,2}/S_{1GHz,1} = \theta_2^2/\theta_1^2 = 1 \) for both SNR1 and SNR2, here \( \theta(i) = 1, 2 \) is the visual area of the remnant, their radio surface brightness (\( \Sigma \)) can be the same to each other. Thus the remnant diameter (\( D \)) and distance (\( d \)) value will be uncertain according to equation (2). But fortunately the true reality will never take on this case because one can see \( \Sigma(D) \sim D_{pc}^{-3.8} \) from formula (2) and not \( \sim D_{pc}^{-2} \). Therefore we are able to uniquely determine the SNe initial kinetic energy \( (E_0) \).

Many of the radio SNRs have more than one published value for distance and age in table 2 and 3. For these, we either chose the most recent estimates, or the most commonly adopted value. We can compare these resolved distances \( (d_{our}(\xi, i)) \) above with the already known ones \( (d_{true}(i)) \) listed in table 2 by

\[ \Phi(\xi, d) = \sum_{i=1}^{37} (d_{our}(\xi, i) - d_{true}(i))^2 \]
\[ \div \sum_{i=1}^{n} d_{true}^2(i) \]

(\( \xi = 1, 2, 3, ..., 10^5 \)) \hspace{1cm} (4)

For example, one gets \( \Phi(10, d) = 0.126 \), when \( \xi = 10 \), and \( \Phi(100, d) = 0.184 \), when \( \xi = 100 \).

Figure 3 shows that the most likely value of supernova initial explosion energy \( (E_0) \) derived by this method for the shell-type remnants at Sedov-phase equals nearly to \( 0.23 \times 10^{50} \) ergs.

There are some different methods to derive
Table 2

List of the distance ($d$), Age ($t$) and some other physical parameters of 37 shell-type Galactic SNRs of which their diameter is less than 36 pc.

| Source           | $t_{true}(i)$ | $d_{true}(i)$ | Dia.$^a$ | $S_{1GHz}$ | Ref. |
|------------------|---------------|---------------|----------|-----------|------|
|                  | yr            | pc            | pc       | arcmin    | Jy   |
| G4.5+6.8         | 380           | 2900          | 3        | 3         | 19   |
| G7.7−3.7         | −             | 4500          | 29       | 22        | 11   |
| G27.4+0.0        | 2700          | 6800          | 8        | 4         | 6    |
| G31.9+0.0        | 4500          | 7200          | 13       | 7x5       | 24   |
| G32.8−0.1        | −             | 7100          | 35       | 17        | 11   |
| G33.6+0.1        | 9000          | 7800          | 23       | 10        | 22   |
| G39.2−0.3        | 1000          | 11000         | 22       | 8x6       | 18   |
| G41.1−0.3        | 1400          | 8000          | 8        | 4.5x2.5   | 22   |
| G43.3−0.2        | 3000          | 10000         | 10       | 4x3       | 38   |
| G53.6−2.2        | 15000         | 2800          | 24       | 33x28     | 8    |
| G73.9+0.9        | 10000         | 1300          | 8        | 22?       | 9    |
| G74.0−8.5        | 14000         | 400           | 23       | 230x160   | 210  |
| G78.2+2.1        | 50000         | 1500          | 26       | 60        | 340  |
| G84.2−0.8        | 11000         | 4500          | 23       | 20x16     | 11   |
| G89.0+4.7        | 19000         | 800           | 24       | 120x90    | 220  |
| G93.3+6.9        | 5000          | 2200          | 15       | 27x20     | 9    |
| G93.7−0.2        | −             | 1500          | 35       | 80        | 65   |
| G109.1−1.0       | 17000         | 3000          | 24       | 28        | 20   |
| G111.7−2.1       | 320           | 3400          | 5        | 5         | 2720 |
| G114.3+0.3       | 41000         | 700           | 15       | 90x55     | 6    |
| G116.5+1.1       | 280000        | 1600          | 32       | 80x60     | 11   |
| G116.9+0.2       | 44000         | 1600          | 16       | 34        | 9    |
| G120.1+1.4       | 410           | 2300          | 5        | 8         | 56   |
| G260.4−3.4       | 3400          | 2200          | 35       | 60x50     | 130  |
| G272.2−3.2       | 6000          | 1800          | 8        | 15?       | 0.4  |
| G284.3−1.8       | 10000         | 2900          | 20       | 24?       | 11   |
| G299.2−2.9       | 5000          | 500           | 2        | 18x11     | 0.5  |
| G309.2−0.6       | 2500          | 4000          | 16       | 15x12     | 7    |
| G315.4−2.3       | 2000          | 2300          | 28       | 42        | 49   |
| G327.4+0.4       | −             | 4800          | 29       | 21        | 30   |
| G327.6+14.6      | 980           | 2200          | 19       | 30        | 19   |
| G332.4−0.4       | 2000          | 3100          | 9        | 10        | 28   |
| G337.2−0.7       | 3250          | 15000         | 26       | 6         | 2    |
| G337.8−0.1       | −             | 12300         | 27       | 9x6       | 18   |
| G346.6−0.2       | −             | 8200          | 19       | 8         | 8    |
| G349.7+0.2       | 14000         | 14800         | 9        | 2.5x2     | 20   |
| G352.7−0.1       | 2200          | 8500          | 17       | 8x6       | 4    |

$^a$Notes: Diameters were calculated by using the distances together with the angular sizes in Green (2006) catalogue.
Fig. 1.— the number distribution of Galactic supernova remnants corresponding to their progenitor initial kinetic energy ($E_0$).

Fig. 2.— The plot shows for both SNRs their radio surface brightness ($\Sigma$) can be equal to each other when $S_{1\,GHz,2}/S_{1\,GHz,1} = \theta_2^2/\theta_1^2 = 1$. Here $\theta_i^2 (i = 1, 2)$ is the visual area of the remnant. Thus we are not able to uniquely derive the SNR diameter ($D$) and distance ($d$) value. But fortunately this case will never occur since $\Sigma(D) \sim D_{pc}^{-3.8}$ and not $D_{pc}^{-2}$. Therefore we can obtain the just remnant distance ($d$) by solving the group of equations offered in text.

2.1.2. Comparison by ages

Similarly we can compare these resolved ages ($t_{our}(\xi, i)$) above with the already known ones ($t_{true}(i)$) in table 2 by

$$\Phi(\xi, t) = \sum_{i=1}^{37} (t_{our}(\xi, i) - t_{true}(i))^2 \div \sum_{i=1}^{n} t_{true}^2(i)$$

$$\left(\xi = 1, 2, 3, ..., 10^5\right) \quad (5)$$

For example, one gets $\Phi(10, t) = 0.532$, when $\xi = 10$, and $\Phi(100, t) = 0.507$, when $\xi = 100$.

Figure 4 shows that the most likely value of supernova initial explosion energy ($E_0$) derived by this method for the shell-type remnants at Sedov-phase equals nearly to $7.0 \times 10^{50}$ ergs.

There are many measures to obtain the age values ($t$) of remnants (Xu et al. 2005), and a majority of these ages are not made use of the $E_0$ value derived before. For example, if a remnant is associated with a pulsar, we can estimate its age by using the neutron star characteristic age obtained from the rotation period of the pulsar ($P$) and the rate of change of period ($\dot{P}$) by $t = P/2\dot{P}$ (Gotthelf et al. 2000). For SNRs with a known radius ($R$) and thermal temperature ($T$) taken from X-ray data, one can obtain the age by $t = 3.8 \times 10^2 R_{pc} (kT)^{-1/2}$ yr (Seward et al. 1995). We can also calculate the SNR age by $t \approx 40000 B^{-1.5} \nu_0^{-0.3}$ yr, when a remnant has its spectrum showing the usual break at frequency $\nu_b$ due to synchrotron losses in a magnetic field $B$ (Bock et al. 2001). Therefore through compar-
### Table 3

List of the distance \((d)\), Age \((t)\) and some other physical parameters of 20 shell-type Galactic SNRs of which their diameter is larger than 36 pc.

| Source     | \(t_{\text{true}}(i)\) | \(d_{\text{true}}(i)\) | Dia.\(^a\) | size\((\theta)\) | \(S_{\text{1GHz}}\) | Ref.          |
|------------|-------------------------|-------------------------|-----------|-----------------|----------------|--------------|
| G8.7−0.1   | 15800                   | 3900                    | 51        | 45              | 80             | G96          |
| G18.8+0.3  | 16000                   | 14000                   | 57        | 17x11           | 33             | D99,G04a     |
| G49.2−0.7  | 30000                   | 6000                    | 52        | 30              | 160            | KKS95, G04a  |
| G55.0+0.3  | 1100000                 | 14000                   | 71        | 20x15?          | 0.5            | MWT98        |
| G65.3+5.7  | 14000                   | 1000                    | 78        | 310x240         | 52             | LRH80, R81   |
| G119.5+10.2| 24500                   | 1400                    | 37        | 90?             | 36             | M00          |
| G127.1+0.5 | 85000                   | 5250                    | 69        | 45              | 13             | FRS84        |
| G132.7+1.3 | 21000                   | 2200                    | 51        | 80              | 45             | GTG80, G04a  |
| G156.2+5.7 | 26000                   | 2000                    | 64        | 110             | 5              | RFA92        |
| G160.9+2.6 | 7700                    | 1000                    | 38        | 140x120         | 110            | LA95         |
| G166.0+4.3 | 81000                   | 4500                    | 57        | 55x35           | 7              | L89, KH91, G04a |
| G166.2+2.5 | 150000                  | 8000                    | 186       | 90x70           | 11             | RLV86        |
| G182.4+4.3 | 3800                    | 3000                    | 44        | 50              | 1.2            | KFR98        |
| G205.5+0.5 | 500000                  | 1600                    | 102       | 220             | 160            | CB99         |
| G206.9+2.3 | 600000                  | 7000                    | 102       | 60x40           | 6              | L86          |
| G226.2−1.2 | 680                     | 1500                    | 52        | 120             | 50             | K02, AIS99   |
| G296.5+10.0| 20000                   | 2000                    | 44        | 90x65           | 48             | MLT88        |
| G296.8−0.3 | 1600000                 | 9600                    | 47        | 20x14           | 9              | GJ95, G04a   |
| G321.9−0.3 | 200000                  | 9000                    | 70        | 28              | 13             | SFS89, S89   |
| G330.0+15.0| −                       | 1200                    | 63        | 180?            | 350            | K96          |

\(^a\)Notes: Diameters were calculated by using the distances together with the angular sizes in Green (2006) catalogue.
Fig. 3.— The most likely value of supernova initial explosion energy ($E_0$) derived by comparison with already known distance ($d$) of the shell-type remnants at Sedov-phase equals nearly to $0.23 \times 10^{50}$ ergs.

Fig. 4.— The most likely value of supernova initial explosion energy ($E_0$) derived by comparison with already known age ($t$) of the shell-type remnants at Sedov-phase equals nearly to $7.0 \times 10^{50}$ ergs.

Fig. 5.— The most likely value of supernova initial explosion energy ($E_0$) derived by comparison with already known distance ($d$) of the shell-type remnants at Snowplow-phase equals nearly to $15.0 \times 10^{50}$ ergs.

Fig. 6.— The most likely value of supernova initial explosion energy ($E_0$) derived by comparison with already known age ($t$) of the shell-type remnants at Snowplow-phase equals nearly to $400 \times 10^{50}$ ergs.
ison by ages to determine SNe initial kinetic energy \((E_0)\) is somewhat meaningful and basically no paradox.

But this measure to determine SNe energy \(E_0\) through comparison by ages is to some extent less reliable than that through comparison by distances. Because the initial energy \(E_0\) of some individual SNRs derived from its age. One can see that the obtained remnants distance is more independent to \(E_0\) than the age be, and therefore it causes less self-contradiction in our work.

2.2. At radiative phase

Similarly we list the group of equations for shell-type remnants at the third stage (Koyama & Meguro-Ku 1987, Kitayama & Yoshida 2005, Xu et al. 2005)

\[
D_{pc} = 1.42 \left( \frac{E_0/10^{51} \text{ergs}}{n_{cm^{-3}}} \right)^{5/21} t_{yr}^{2/7} \tag{6}
\]

\[
\Sigma(D) = 1.505 \times 10^{-19} \frac{S_{1 \text{GHz}}}{\theta_{arcmin}^2} \]
\[
= 2.88 \times 10^{-14} D_{pc}^{-3.8} n_{cm^{-3}}^2 \tag{7}
\]

\[
t_{yr} = 10^5 n_{cm^{-3}}^{-3/4} \left( \frac{E_0}{10^{51} \text{ergs}} \right)^{1/8} \tag{8}
\]

Here, \(D_{pc}, t_{yr}, n_{cm^{-3}}, S_{1 \text{GHz}}\) and \(\theta_{arcmin}\) defined as in section 2.1 as well as their units, and \(\tan \left( \frac{\theta_{arcmin}}{D_{pc}} \right) = \frac{D_{pc}}{2n_{cm^{-3}}} \). The fluxes \(S_{1 \text{GHz}}\) and observational angle \(\theta_{arcmin}\) are already known to us for each remnant (table B) of which we regard these remnants evolving at the snow-plough phase since their diameter larger than 36 pc. When the initial energy \(E_0 = \xi \times E_{48}\) assumed, then the remnant diameter \(D_{pc}\) (and distance \(d_{pc}\)), age \(t_{yr}\) and electron density \(n_{cm^{-3}}\) can be obtained.

One can see the equations (6) and (8) are rather different from equations (1) and (3). But formulae (7) and (2) are completely the same.

For a certain supernova remnant \((i)\) \((i = 1, 2, 3, ..., 20)\) (table B) and assumed \(E_0 = \xi \times E_{48}\) \((\xi = 1, 2, 3, ..., 10^5)\) in units of \(10^{48}\) ergs, we can get the remnant distance \(d_{our}(\xi, i) = d_{pc}\) and the age \(t_{our}(\xi, i) = t_{yr}\) by solving the equations group above. Then we compare them with the already known parameters \(d_{true}(i)\) and \(t_{true}(i)\) \((i = 1, 2, 3, ..., 37)\) listed in table B in order to derive the most likely original energy of SNRs.

2.2.1. Comparison by distances

We can compare these resolved distances \((d_{our}(\xi, i))\) above with the already known ones \((d_{true}(i))\) listed in table B by

\[
\Phi(\xi, d) = \sum_{i=1}^{20} (d_{our}(\xi, i) - d_{true}(i))^2 \]
\[
\div \sum_{i=1}^{n} d_{true}^2(i) \tag{9}
\]

\((\xi = 1, 2, 3, ..., 10^5)\)

Figure 5 shows that the most likely value of supernova initial kinetic explosion energy \((E_0)\) derived by this method for the S-type remnants at Snowplow-phase equals nearly to \(15.0 \times 10^{50}\) ergs.

2.2.2. Comparison by ages

Similarly we can compare these resolved ages \((t_{our}(\xi, i))\) above with the already known ones \((t_{true}(i))\) in table B by

\[
\Phi(\xi, t) = \sum_{i=1}^{20} (t_{our}(\xi, i) - t_{true}(i))^2 \]
\[
\div \sum_{i=1}^{n} t_{true}^2(i) \tag{10}
\]

\((\xi = 1, 2, 3, ..., 10^5)\)

Figure 5 shows that the most likely value of supernova initial explosion energy \((E_0)\) derived by this way for the shell-type remnants at Snowplow-phase equals nearly to \(400 \times 10^{50}\) ergs.

2.3. Final results

From Fig. 5 to Fig. 8 we get the least value of \(log_{10}E_0\) instead of \(E_0\), therefore one has

\[
log_{10}E_0 = log_{10}E_{02d} + log_{10}E_{02i}
+ log_{10}E_{03d} + log_{10}E_{03i} \tag{11}
\]

Here, \(E_{02d}, E_{02i}, E_{03d}\) and \(E_{03i}\) corresponding to the 4 \(E_0\) values in from Fig. 5 to Fig. 8. \(E_0\) is the typical explosion energy of shell type remnants.

Thus we have \(E_0 = 0.99 \times 10^{51}\) ergs.

The publicly accepted value of the SNe initial kinetic energy is \(1 \times 10^{51}\) ergs.
3. Discussion and Summary

To combine theoretical results together with observational ones, we have derived the supernova average initial explosion energy. Such value is obviously not the same as that to a individual remnant. For individual SNR the initial energy is various for different SN explosion events, but it might be near an approximation. In Fig. 1 the distribution of the initial energies of 44 SNRs range from $10^{49}$ ergs to $10^{52}$ ergs, mainly concentrate to about $10^{50} \sim 10^{51}$ ergs. Our 4 theoretical outcomes range from $0.23 \times 10^{50}$ ergs at minimum to $400 \times 10^{50}$ ergs at maximum. It seems that they are to some extent in good consistency regarding their divergence. The relative divergence of explosion energies about $10^3$ is a rather small number and acceptable.

From table 1 one can directly calculate the mean initial kinetic energy $E_0 = 1.7 \times 10^{51}$ ergs. Or $E_0 = 0.85 \times 10^{51}$ ergs after excluding SNR G0.0+0.0, namely Sgr A East which owns an extremely large initial energy value. These is also another method to obtain the initial energy by the group of equations combining the physical parameters listed in table 2 and table 3 when we regard the age and distance values as the already known ones. We gain $E_0(i) (i = 1, 2, 3, \ldots, 37)$ or $(i = 1, 2, 3, \ldots, 20)$, and the average $E_0$ is then derived. Here, in the paper our special method is different from this, and has provided one more measure to derive the SNe typical energy by taking somewhat more numbers of the SNRs with intensified credit. Furthermore, many of the supernova remnants in table 2 or table 3 are not the same as those in table 1. Namely the explosion energy $E_0$ of rather some remnants in table 2 and table 3 are unknown to us. Because for most of the individual supernova remnant in Galaxy their explosion energy is rather difficult to derive, here our statistical $E_0$ value obtained can be used to compute other physical parameters, i.e. the distance, age, ISM electron density, magnetic field, etc. of which the errors resulted in will be to some extent rather small.

Here we did not distinguish different type supernova of our adopted sample in the numerical analysis. For Type-I and Type-II SN the initial explosion energy might be not similar, but one can expect that this deviation will be somewhat small.

The average value of the most likely initial explosion energy of supernova remnants thus equals to about $0.99 \times 10^{51}$ ergs in nice consistence with the accepted value.

JWX thanks J.S. Deng and Y.Z. Ma for their assistance and help during the paper work.

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