Some methods to evaluate complicated Feynman integrals

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I discuss a progress in calculations of Feynman integrals based on the Gegenbauer Polynomial Technique and the Differential Equation Method.

1. Introduction

Last years there was an essential progress in calculations of Feynman integrals. It seems that most important results have been obtained for two-loop four-point massless Feynman diagrams: in on-shell case (see [1,2]) and for a class of off-shell legs (see [3]). A review of the results can be found in [4]. Moreover, very recently results for a class of these diagrams have been obtained [5] in the case when some propagators have a nonzero mass.

In the letter, I review very shortly results obtained with help of two methods for calculations of Feynman diagrams (for details, see [6]): the Gegenbauer Polynomial Technique [7] (see also [8,9]) and the Differential Equation Method (DEM) [10]. The additional information about a modern progress in calculations of Feynman integrals can be found, for example, also in recent articles [11].

2. Applications of the Gegenbauer Polynomial Technique

The Gegenbauer Polynomial Technique has been used for evaluation of very complicated Feynman diagrams (see also [9]) which contribute mostly in calculations based on various type of $1/N$ expansions:

- In the calculation (in [12]) of the next-to-leading (NLO) corrections to the value of dynamical mass generation (see [13]) in the framework of three-dimensional Quantum Electrodynamics.
- In the evaluation (in [14]) of the correct value of the $\beta$-function of the $\theta$-term in Chern-Simons theory. The $\beta$-function is zero in the framework of usual perturbation theory but it takes nonzero values in $1/N$ expansion (see [15]).
- In the evaluation (in [16]) of NLO corrections to the value of gluon Regge trajectory (see discussions in [16] and references therein).
- In the calculation (in [17]) of the next-to-leading corrections to the BFKL intercept of spin-dependent part of high-energy asymptotics of hadron-hadron cross-sections.
- In the calculation (in [17,18]) of the next-to-leading corrections to the BFKL equation at arbitrary conformal spin.
- In the evaluation (in [19]) of the most complicated parts of $O(1/N^3)$ contributions to critical exponents of $\phi^4$-theory, for any spacetime dimensionality $D$.

3. The recent progress in calculation of Feynman integrals by the DEM.

- The articles [20] and [21]:
  a) The set of two-point two-loop Feynman diagrams with one- and two-mass thresholds has been evaluated by DEM (see Fig.1). The results are presented in Ref [20,21] and in the review [6]. Some of them have been known before (see [21]). The check of the results has been done by Veretin programs (see discussions in [20] and references therein).
Figure 1. Two-loop self-energy diagrams. Solid lines denote propagators with the mass $m$; dashed lines denote massless propagators.

b) The set of three-point two-loop Feynman integrals with one- and two-mass thresholds has been evaluated (the results of some of them has been known before (see [20])) by a combination of DEM and Veretin programs for calculation of first terms in small-moment expansion of Feynman diagrams (see discussions in [20] and references therein).

- The article [22]: The full set of two-point two-loop on-shell master diagrams has been evaluated by DEM. The check of the results has been done by Kalmykov programs (see discussions in [22] and references therein).
- The article [2]: The set of three-point and four-point two-loop massless Feynman diagrams has been evaluated.

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REFERENCES

1. V.A. Smirnov, Phys.Lett. B460 (1999) 397; V.A. Smirnov and O.L. Veretin, Nucl.Phys. B566 (2000) 469; J.B. Tausk, Phys.Lett. B469 (1999) 225; C. Anastasiou, et al., Nucl.Phys. B (Proc.Suppl.) 89 (2000) 262.
2. T. Gehrmann and E. Remiddi, Nucl.Phys. B580 (2000) 485; hep-ph/0101147
3. V.A. Smirnov, Phys.Lett. B491 (2000) 130; Phys.Lett. B500 (2001) 3;
4. V.A. Smirnov, hep-ph/0209177
5. V.A. Smirnov, Phys.Lett. B524 (2002) 129.
6. A.V. Kotikov, hep-ph/0112347
7. K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, Nucl.Phys. B174 (1980) 345.
8. D.I. Kazakov and A.V. Kotikov, Theor. Math.Phys. 73 (1987) 1264. Nucl.Phys. B307 (1988) 721; B345 (1990) 299(E); Phys. Lett. B291 (1992) 171. A.V.Kotikov, Theor.Math.Phys. 78 (1989) 134.
9. A.V. Kotikov, Phys.Lett. B375 (1996) 240; hep-ph/0102177
10. A.V. Kotikov, Phys.Lett. B254 (1991) 158; Phys.Lett. B259 (1991) 314; Phys.Lett. B267 (1991) 123; hep-ph/0102178.
11. G. Passarino, hep-ph/0101299; G. Passarino and S. Uccirati, hep-ph/0112004.
12. A.V. Kotikov, JETP Lett. 58 (1993) 731.
13. T. Appelquist et al., Phys.Rev. D33 (1986) 3774; Phys.Rev.Lett. 60 (1988) 2575; D. Nash, Phys.Rev. Lett. 62 (1989) 3024.
14. I.N. Kondrashuk and A.V. Kotikov, Phys. Rev. D53 (1996) 2260.
15. S.H. Park, Phys.Rev. D45 (1992) 3332.
16. V.S. Fadin, R. Fiore and M.I. Kotsky, Phys. Lett. B387 (1996) 593.
17. A.V. Kotikov and L.N. Lipatov, Nucl.Phys. B582 (2000) 19.
18. A.V. Kotikov and L.N. Lipatov, hep-ph/0112346; hep-ph/0208220.
19. D.J. Broadhurst and A.V. Kotikov, Phys. Lett. B441 (1998) 345.
20. J. Fleischer, A.V. Kotikov, and O.L. Veretin, Nucl.Phys. B547 (1999) 343.
21. J. Fleischer, A.V. Kotikov, and O.L. Veretin, Phys.Lett. B417 (1998) 163; Acta Phys.Polon. B29 (1998) 2611.
22. J. Fleischer, M.Yu. Kalmykov, and A.V. Kotikov, Phys.Lett. B462 (1999) 169; B467 (1999) 310(E).
23. J. Fleischer, M.Yu. Kalmykov, and A.V. Kotikov, hep-ph/9905379; J. Fleischer and M.Yu. Kalmykov, Comput.Phys.Commun. 128 (2000) 531; Phys.Lett. B470 (1999) 168.