Cavitation induced by shock wave focusing in eye-like experimental configurations

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Abstract: During laser-induced, breakdown-based medical procedures in human eyes such as posterior capsulotomy and vitreolysis, shock waves are emitted from the location of the plasma. A part of these spherically expanding transients is reflected from the concave surface of the corneal epithelium and refocused within the eye. Using a simplified experimental model of the eye, the dominant secondary cavitation clusters were detected by high-speed camera shadowgraphy in the refocusing volume, dislocated from the breakdown position and described by an abridged ray theory. Individual microbubbles were detected in the preheated cone of the incoming laser pulse and radially extending cavitation filaments were generated around the location of the breakdown soon after collapse of the initial bubble. The generation of the secondary cavitation structures due to shock wave focusing can be considered an adverse effect, important in ophthalmology.

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1. Introduction

Many laser medical procedures in human eyes are based on tightly focusing laser light within transparent ocular tissues to achieve optical breakdown followed by a rapid absorption of light [1]. The therapeutic effect of such ionization is formation of plasma in a localized focal volume that causes disintegration of tissue within this volume [2]. However, this highly perturbed state of matter is relaxed by emitting a shock wave and by a generation of a cavitation bubble [3]. Both phenomena may contribute to the therapy, but are often rather unwanted and may lead to possible complications [1,4–9]. This paper deals with the propagation and refocusing of the breakdown-induced shock wave due to its reflection from the corneal epithelium. Experiments supported by a simple ray theory show that the dominant secondary cavitation cloud occurs if the shock wave emitted during the first collapse of the main bubble refocuses in the anterior laser cone. Additional cavitation structures were discovered due to the interplay between the refocusing shock wave and the oscillating cavitation bubble.

Ophthalmic surgery based on laser-induced breakdown is predominantly used in iridotomy [10], posterior capsulotomy [11] and vitreolysis [12]. While iridotomy is performed away from the optical axis of the eye, in posterior capsulotomy and vitreolysis, laser-induced breakdown takes place on or near this axis. In posterior capsulotomy, laser-induced breakdown is used to perforate the central part of the posterior lens capsule to treat posterior capsule opacification. Deeper within the eye, in the vitreous humor, vitreolysis is performed to disintegrate vitreous strands. Both, in posterior capsulotomy and vitreolysis some already reported complications can be explained by the shock wave refocusing effect explained in this paper. For example, several papers report on the acute cataract formation or posterior lens capsule injury following laser vitreolysis ([6] and references therein).

Similar laser systems are used for posterior capsulotomy and vitreolysis. A typical laser is a Q-switched Nd:YAG emitting laser pulses with a wavelength of 1064 nm and time duration of a few nanoseconds with clinical energies of a few millijoules [1,13]. When, for example, a 10-mJ laser pulse is focused within the vitreous of the eye with a convergence angle of 22° in water, it takes about 170 µs for the initial cavitation bubble to reach its maximum diameter of 3.6 mm (15% of the diameter of a human eye!) [4]. Employing such pulses, 25% of the laser
pulse energy is acquired by the bubble and nearly twice this amount by the emitted shock wave [4]. Around 5% of the energy is transmitted, less than 1% reflected and around 0.5% scattered [14]. After reaching its maximum size, the bubble collapses, emits another shock wave, similar to the initial one, and the bubble grows again, this time to a smaller diameter [3]. A few bubble oscillations can be achieved, but the first two are always detectable [15].

When posterior capsulotomy and vitreolysis are performed, the ophthalmologist may or may not use an ophthalmic lens which is brought in contact with the patient’s cornea [13]. Such lens fits the outer curvature of the cornea, which has a shape of a spherical cap with a radius of approximately 7.8 mm in an adult human.

The most important aspects of the shock wave refocusing mechanism in a human eye are sketched in Fig. 1. In Fig. 1(a), a simplified side view of the eye depicts the optional ophthalmic lens placed on the cornea. A laser pulse is focused within the eye either directly or through the transparent ophthalmic lens. Its path forms two cones, one in front of the focus (anterior laser cone) and the other one behind it (posterior laser cone). Breakdown occurs in the optical focus. Laser light is transmitted past the focus only until dense plasma is formed in the very beginning of the pulse, therefore more laser energy is absorbed in the anterior cone (darker magenta) compared to the posterior cone (paler magenta) [14].

![Fig. 1. Illustration of the shock wave refocusing mechanism at four time instants. (a) Laser-induced breakdown takes place at the focus. (b) After the breakdown, a cavitation bubble starts to grow and a compressional shock wave (solid blue circle) is launched spherically into the surrounding tissue. (c) The shock wave travelling posteriorly continues its propagation into soft tissues, while a portion of its anterior wavefront is reflected from the corneal epithelium as a tensile wave (dashed blue line). (d) The reflected shock wave is refocused reaching negative pressure amplitudes that exceed the threshold for secondary acoustic (inertial) cavitation. After the first collapse of the bubble, steps (b)–(d) are repeated with the following differences: smaller isolated cavitation bubbles induced predominantly in the anterior laser cone are now accompanied by larger secondary cavitation clouds near the acoustic focus and radial cavitation filaments are formed in the vicinity of the collapse.](image-url)
Figure 1(b) shows the emitted spherical shock wave and the expanding cavitation bubble, generated in the optical focus. The temporal shape of the shock wave is composed of a steep rise in pressure with a gentler fall. To obtain a general sense of the magnitude of the shock wave parameters used in the present work, one can compare with the results presented in [4]. Take for example a 10-mJ laser pulse with a convergence angle of 22° to achieve the breakdown in water. Measured 10 mm from the breakdown, the maximum pressure of the shock wave is about 2.6 MPa and has a FWHM (full width at half maximum) of about 150 ns [4].

When the shock wave reaches the cornea-air or cornea-lens interface, the compressional wave reflects either as a rarefaction and changes its phase (example: cornea-air) or it remains a compression (example: cornea-lens) depending on the acoustic impedances of both media [16]. Figure 1(c) shows the reflection of the shock wave from the cornea-air interface, where the positive pressure is marked by a solid line and negative pressure by a dashed line. Only a smaller solid angle of the shock wave is reflected from the cornea. The rest of the shock wave continues to propagate into soft tissues surrounding the eye. Due to comparable acoustic impedances of eye tissues, negligible reflections are expected at tissue boundaries [16]. Isolated bubbles are formed inside the frontal part of the anterior laser cone already overpassed by the reflected shock wave.

After the reflected shock wave passes the acoustic focus, secondary cavitation takes place in its vicinity (Fig. 1(d)). To discriminate the initial laser-induced cavitation bubble (primary cavitation) from the acoustically generated bubbles that are generated later, the term secondary cavitation refers to the gaseous structures that are formed by acoustic nucleation at locations where pressure amplitude of the shock wave surpasses the pressure-threshold for the incipience of acoustic cavitation or to the expansion of the already present nuclei that are expanded to become visible after they had been overpassed by the shock wave. The secondary cavitation is primarily confined to the anterior laser cone, because the threshold for the incipience of acoustic cavitation is lower in this volume due to prior absorption of laser light on the impurities [3]. The threshold for secondary cavitation depends largely on the peak negative pressure [17]. Close inspection of the shock wave after it has already passed the acoustic focus reveals a bipolar shape. This is a consequence of a Gouy phase shift the wave experiences after passing the focus [16].

After the first collapse of the main bubble, the dynamics is repeated (Figs. 1(b)–1(d)). After the passage of the reflected shock wave over the anterior laser cone, smaller bubbles reappear at the same locations as initially, this time larger in size. Additional radially extending cavitation filaments are formed in the vicinity of the collapse (Fig. 1(d)).

Indications that the refocused shock waves may play an important role in laser-induced, breakdown-based medical procedures in human eyes are scattered over the literature. It is well known that when laser breakdown is generated in water close to its free surface, smaller secondary bubbles appear within the laser path after the shock wave reflected from the free surface overpasses the volume irradiated by the laser pulse [18,19]. Additional focusing of shock waves from concave surfaces, such as from the internal walls of round flasks [20], rising free water surfaces [21], free surfaces of liquid jets [22] and water drops [23] make the secondary cavitation even more pronounced. In cylindrical water jets and in spherical water drops, a prominent secondary cavitation occurs even far from the laser breakdown, at the location of the acoustic focus of the concave surface [22,23]. Due to comparable geometry, dislocated secondary cavitation in water drops is hinting towards similar effects in human eyes. However, this effect has so far been neglected in the extensive studies of photodisruption of ocular tissues [24], because the experimental pulse entering surface was either flat or convex and thus did not reproduce the effects of the actual concave geometry when pulses are delivered into the eye.

The experimental configuration will be presented first. It is followed by a ray theory that predicts the location on the axis where secondary cavitation is expected to occur. Then, we present the experimental results accompanied by a discussion of the importance of secondary cavitation in ophthalmology.
2. Experimental setup

The clinical conditions of laser-induced, breakdown-based laser procedures in human eyes were experimentally modelled by a simplified setup sketched in Fig. 2.

![Fig. 2. Schematics of the experimental setup in two perspectives (side and top view). The dimensions of the water tank and ophthalmic lens are in right proportions.](image)

The laser used was a Q-switched Nd:YAG laser intended for capsulotomy and iridotomy. It emits 4 ns FWHM pulses at the wavelength of 1064 nm with a maximum energy of 15 mJ per pulse. Pulses are air-delivered by a cone angle of 16° into a water tank through a focusing poly(methyl methacrylate) (PMMA) ophthalmic lens (Capsulotomy lens K30-1120, Katena Products, USA) [25] which increases the focal cone angle in water to 29°. This lens has an antireflection (AR) coating on the entering surface and no coatings on the contacting surface with a 7.8 mm concave curvature. The black plastic button holding the lens is removed so that only the molded PMMA lens is inserted into the water tank wall through a matching hole. To prevent leakage, the lens is wrapped by a PTFE (polytetrafluoroethylene) Teflon tape around its lateral surface before it is inserted into the hole.

The water container is made of 5-mm-thick PMMA plates to form a five-faceted cube with an inner edge of 100 mm. The upright sides are glued on the bottom plate by an adhesive based on alpha-cyanoacrylate ester (Pasco Fix, Pasco, Germany). The water tank is filled by a 0.9 dm³ of a non-degassed distilled water. The temperature of water was 20 °C. The ophthalmic lens is centered on the front facet with its center 45 mm below the water surface and 45 mm above the bottom plate.

The laser-induced breakdown is located within the water chamber on the axis of the lens, so that the geometry locally retains cylindrical symmetry. The location of the optical focus/breakdown along the axis can be varied by a triaxial translation stage by moving the water tank closer or farther away from the laser.

Horizontally and perpendicularly to the laser beam, a high-speed camera shadowgraphy captures the dynamics near the optical breakdown [27]. The system consists of a continuous, collimated halogen cold light source (KL 2500 LCD, Olympus, Japan) and a high-speed camera (FASTCAM SA-Z 2100K-M-64GB, Photron, Japan) with a 105-mm objective (AF-S VR Micro-Nikkor 105mm f/2.8G IF-ED, Nikor, Japan) that images the plane with the breakdown onto the camera’s detector. A glass ruler scale is used for dimensional calibration of the acquired images.
The light emitted during the breakdown is captured by a biased Si detector with a 1-ns rise time (DET10A/M, Thorlabs, USA) and used as a trigger to start the acquisition of the high-speed camera. The energy of the excitation laser pulse is measured by a power/energy meter (Vega 7Z01560, Ophir, Israel) and pyroelectric sensor (PE25BF-C 7Z02935, Ophir, Israel) in air just before the light enters the ophthalmic lens.

3. Theory

We have developed a ray model of shock wave refocusing which predicts the location on the axis where the secondary cavitation is expected to occur. The geometry of the problem is reduced and simplified as shown in Fig. 3. The ophthalmic lens (index 2) made of PMMA is modelled as a homogeneous and isotropic elastic solid, while water (index 1) is modelled as a liquid medium. Axial symmetry is assumed with the axial coordinate \( x \), extending from the apex of the posterior concave surface of the lens towards the water, and the radial coordinate \( y \). The surface of the lens in contact with water acts as a concave, spherical acoustic mirror—a spherical cap with the radius \( R \), the base of the cap \( h \) and its height \( g \).

At 20 °C, water has a mass density of \( \rho_1 = 998.2 \text{ kg/m}^3 \) [28] and acoustic velocity of \( \alpha_1 = 1482 \text{ m/s} \) [29]. At the same temperature, PMMA has a mass density of \( \rho_2 = 1183 \text{ kg/m}^3 \), P-wave velocity of \( \alpha_2 = 2746 \text{ m/s} \) and S-wave velocity of \( \beta_2 = 1392 \text{ m/s} \) [30].

This theory describes the propagation and focusing of a single shock wave, either the one emitted during the laser-induced breakdown or one of those released during the consecutive cavitation bubble collapses. A spherical shock wave is emitted at a distance \( a \), defined as the distance from the apex of the concave mirror to the location of shock wave emission. Due to the attraction of the bubble by the nearby solid boundary, the distance \( a \) is in general larger for the initial shock wave, emitted during laser-induced breakdown, compared to the shock wave released during one of the subsequent collapses. An arbitrary chosen ray, an idealized line that...
is perpendicular to the initial spherical shock wavefront, directed towards the spherical cap by an angle \( \phi \) if the ray is reflected by the acoustic mirror and crosses the axis at a distance \( b \). This distance depends on the angle \( \phi \). In optics, this effect is known as spherical aberration. The length of the ray from the origin to the axis is \( s = l + t \). The angle \( \theta \) denotes the angle of incidence of the ray on the acoustic mirror and the angle \( \phi_{\text{max}} \) is the largest angle of the ray entering the spherical aperture.

To further simplify mathematical expressions, all geometrical variables are normalized to the radius of the concave spherical mirror \( R \). The normalized variables are denoted by a superscript tilde (~). The intersection of an arbitrary ray with a length \( \tilde{s}(\tilde{a}, \phi) = \tilde{l} + \tilde{t} \) with the axis takes place at

\[
\tilde{b}(\tilde{a}, \phi) = \frac{\tilde{a}(1 - \tilde{a}) + \tilde{l}^2}{\tilde{l}^2 - (\tilde{a} - 1)^2},
\]

where

\[
\tilde{l}(\tilde{a}, \phi) = (\tilde{a} - 1) \cos \phi + \sqrt{(\tilde{a} - 1)^2 \cos^2 \phi - \tilde{a}(\tilde{a} - 2)} \quad \text{and} \quad \tilde{t}(\tilde{a}, \phi) = \frac{\tilde{l}^2}{\tilde{l}^2 - (\tilde{a} - 1)^2}.\]

The maximum angle is

\[
\phi_{\text{max}} = \arctan \left( \frac{\tilde{h}}{\tilde{a} - \tilde{g}} \right) \quad \text{if} \quad \tilde{g} \leq \tilde{a} \quad \text{or} \quad \phi_{\text{max}} = \arctan \left( \frac{\tilde{h}}{\tilde{a} - \tilde{g}} \right) + \pi \quad \text{if} \quad \tilde{a} < \tilde{g}.\]

In the paraxial approximation (\( \phi \ll 1 \)), the above equations reduce to the results of the Gaussian mirror equation \( 1/f = 1/a + 1/b \) with the focal length \( f = R/2 \).

The experimental geometrical parameters of the acoustic mirror are: \( R = 7.8 \) mm, \( h = 6.9 \) mm (\( \tilde{h} = 0.885 \)), \( g = 4.16 \) mm (\( \tilde{g} = 0.533 \)). The length of the eye from the corneal epithelium to the retina is about 23 mm which sets the upper limit of the interval of interest to 3. Since there is no visual access to values \( \tilde{x} < \tilde{g} \approx 0.5 \), the results presented will be limited to \( 0.5 < \{\tilde{a}, \tilde{b}\} < 3 \).

Figure 4(a) shows all possible values of \( \tilde{b}_{\text{min}} \leq \tilde{b} \leq \tilde{b}_{\text{max}} \) where the rays intersect the axis after the reflection from the acoustic mirror for the experimental size of the cup defined by \( \tilde{g} = 0.533 \). The upper boundary (dashed black line) is given by the paraxial approximation \( \tilde{b}_{\text{max}}(\tilde{a}) = \tilde{b}(\tilde{a}, \phi = 0) \) and the lower (solid black line) by \( \tilde{b}_{\text{min}}(\tilde{a}) = \tilde{b}(\tilde{a}, \phi = \phi_{\text{max}}) \). If the shock wave is emitted farther from the center of curvature (\( \tilde{a} < 1 \)), the refocusing occurs closer to the acoustic mirror (\( \tilde{b} < 1 \)). On the contrary, if the shock wave is emitted closer than the center of curvature (\( \tilde{a} < 1 \)), the refocused image occurs farther from the acoustic mirror (\( 1 < \tilde{b} \)). As expected, if the source of the spherical shock wave matches the center of the curvature of the spherical cap (\( \tilde{a} = 1 \)), after reflection, all rays meet again at the same location (\( \tilde{l} = 1 \)). Only at this location, there is no smearing due to spherical aberration and the expected pressure is the highest.

The time of flights (ToFs) of rays emitted at any angle \( 0 \leq \phi \leq \phi_{\text{max}} \) entering the cup and for all locations \( \tilde{a} \) of practical interest are located within the dark gray shaded area in Fig. 4(b) bounded by the ToF curve of the paraxial rays (dashed black line) and the ToF curve of the rays reflected from the edge of the cup (solid black line). The nondimensional ToF is equal to the nondimensional path length \( \tilde{s}(\tilde{a}, \phi) \). Even though in the initial 100 ns shock waves propagate faster than the acoustic velocity [4], they spend the majority of their propagation from the emission spot back to the axis travelling at constant asymptotic velocity \( a_1 \). In this analysis, the relative error made by neglecting their rapid start is less than 1%, since they always travel a path longer than 2\( R \).

Assuming that the largest the magnitude of the local pressure in the interval \( [\tilde{b}_{\text{min}}, \tilde{b}_{\text{max}}] \) the more likely the secondary cavitation occurs, one should seek the pressure distribution \( p(\tilde{b}) \). This can be achieved by assigning to each ray an appropriate weight \( w \) that depends on the following factors: (1) As the angle \( \phi \) increases, the density of rays goes as \( w_\phi = \sin \phi \). (2) Since initially, the shock wave
has a spherical shape, the pressure falls as the inverse of the distance travelled, hence \( w_s = 1/\tilde{s} \). (3)

The reflection coefficient for pressure \( w_{R_p} = R_p(\theta, \rho_1, \alpha_1, \rho_2, \alpha_2, \beta_2) \) from the liquid (water)-solid (PMMA) boundary depends on the angle of incidence \( \theta(\tilde{a}, \phi) = \arccos((l^2 - \tilde{a}(\tilde{a} - 2))/2l) \) and on the acoustic parameters of each medium. It is calculated using the Zoeppritz equations for reflection/refraction at a liquid/solid interface [31]. (4) As the shock wave propagates in water, it is exponentially attenuated. We used the attenuation coefficient of \( \mu = 0.025 \text{ cm}^{-1} \) \( (\tilde{\mu} = \mu R = 0.0196) \) for the deionized water at 10 MHz and 20 °C [32]. This frequency roughly corresponds to the inverse of the shock wave duration. The corresponding weight is \( w_\mu = e^{-\tilde{\mu}\tilde{s}} \).

This weight has negligible effect in water at the dominant frequency of 10 MHz and distances of a few cm, but has a larger effect at higher frequencies as the attenuation coefficient in water increases as a square of the frequency. In ocular tissues, the attenuation coefficient depends on the tissue [33]. The cumulative weight is

\[
  w = \prod_i w_i = R_p \frac{\sin \phi}{\tilde{s}} e^{-\tilde{\mu}\tilde{s}}. \tag{4}
\]

The pressure distribution is calculated as follows. For a chosen \( \tilde{a} \in [0.5, 3] \), the intersection of the ray with the axis \( \tilde{b} \) is calculated for discrete values of \( \phi \in [0, \phi_{\text{max}}] \) using a preset constant angle increment \( \Delta \phi \ll \phi_{\text{max}} \). The corresponding weights are also calculated. Since the density of rays is not constant in the interval \([\tilde{b}_{\text{min}}, \tilde{b}_{\text{max}}]\), this interval is divided into subintervals of equal widths \( \Delta \tilde{b} \ll (\tilde{b}_{\text{max}} - \tilde{b}_{\text{min}}) \) and all weights that fall into each subinterval are accumulated. This procedure is repeated for each \( \tilde{a} \in [0.5, 3] \) keeping the values of \( \Delta \phi \) and \( \Delta \tilde{b} \) unchanged.
Figure 5(a) shows relative pressure profiles in the refocusing interval $[\tilde{b}_{\text{min}}, \tilde{b}_{\text{max}}]$ for various locations of shock wave origin $\tilde{a}$. To display all profiles of interest on a single graph, the relative pressure $p$ is presented on a logarithmic scale and the values of $\tilde{b}$ are shifted by the paraxial value $\tilde{b}_{\text{max}}$. For a chosen $\tilde{a}$, the pressure in the refocusing interval is relatively constant. For $\tilde{a} < 1$, the maximum pressure $p_{\text{max}}$, shown in Fig. 5(b), is expected at $\tilde{b}_{\text{min}}$ and for $1 < \tilde{a}$ at $\tilde{b}_{\text{max}}$, except for larger values of $\tilde{a}$ where the effect of the angular dependence of the reflection coefficient plays an important role (see for example the log$p$ curve for $\tilde{a} = 2.5$ and $\tilde{a} = 3$). This effect is seen as an abrupt change in pressure as the rays reach the angle of total reflection. The position of the expected maximum pressure within the focusing interval is marked by the red line in Fig. 4(a) while its value is presented in Fig. 5(b).

Fig. 5. (a) Relative pressure profiles $\log[p(\tilde{b} - \tilde{b}_{\text{max}})]$ in the refocusing interval for various values of the shock wave origin $\tilde{a}$. The chosen values of $\tilde{a}$ are labeled in red for $\tilde{a} < 1$ in blue for $\tilde{a} = 1$ and in black for $1 < \tilde{a}$. (b) Maximum pressure $\log[p_{\text{max}}(\tilde{a})]$ as a function of shock wave origin.

Based on the presented theoretic results, it is expected that the secondary cavitation occurs in the interval $[\tilde{b}_{\text{min}}, \tilde{b}_{\text{max}}]$, and to be more pronounced where the pressure is larger. The refocusing pressure is the largest when the shock wave is emitted at the center of curvature of the acoustic mirror ($\tilde{a} = 1$).

When applying this simple ray theory to the experimental results, one has to keep in mind that a cavitation bubble is present at the shock wave origin. It can enhance the secondary cavitation in front of the bubble [34,35] and decrease the cavitation potential behind the bubble by shielding the refocusing acoustic wavefront [36]. Moreover, laser energy is primarily deposited in the water cone between the acoustic mirror and the laser breakdown site, because once plasma in the focal volume becomes opaque, laser light is no longer absorbed in the laser cone behind the focal volume. After their first reflection, some rays may cross the axis again, but are not considered in the presented theoretic description.

4. Results and discussion

4.1. Temporal aspects of shock wave refocusing

Figures 6(a)–6(k) (left column) show 11 selected single-frame shadowgraphs (out of 100) acquired by a high-speed camera with a framerate of 100 kfps (kilo frames per second), shutter speed of 250 ns, resolution of 640×280 pixels, pixel size 20 $\mu$m×20 $\mu$m and total acquisition time of 1 ms. The exposure of the first frame starts 1.50 $\mu$s after the breakdown.

Each frame presents a cross-section along the optical axis. Features closer or farther away from this plane appear out-of-focus. Only cavitation structures are seen on the frames. Shock waves cannot be visualized with this type of illumination, since they travel a distance of approximately 0.4 mm during the exposure of a frame and are thus blurred over the image. Between consecutive frames, shock waves make a distance of approximately 15 mm. It takes about 60 $\mu$s for the shock wave to reflect from the free water surface and the bottom wall of the water tank and return back
Fig. 6. (a)–(k) Dynamics of the cavitation structures following the breakdown at a distance $a = 9.53 \text{ mm}$ ($\tilde{a} = 1.222$) from the apex of the lens (left column). (l)–(v) Secondary cavitation structures acquired soon after the first collapse of the initial bubble generated at various locations along the optical axis (right column). Laser pulse energy was 15 mJ. The scale (1 mm) is given by the white bar. The meaning of the overlaying lines and labels is described in the main text.
to the initial breakdown site, 67 µs from the lateral walls, and 120 µs for the share of the shock wave initially travelling towards to back wall.

The frames are equipped by dashed lines which help the reader to visualize experimentally important features. The horizontal white line delineates the axis of symmetry. The vertical lines mark: the edge of the ophthalmic lens (white line), the apex of the concave surface of the lens (yellow line), the center of curvature of this lens (black line), the position of the laser breakdown \( a \) (blue line), the interval of the theoretical refocusing interval \( [b_{\min}, b_{\max}] \) for shock waves emitted during the breakdown (two red lines), during the first collapse of the main bubble (two magenta lines) and during the second collapse (two brown lines), and the anterior and posterior positions of the ocular lens’ surface if the same experiment had been performed in a human eye (two green lines). The solid yellow curve depicts the hidden concave surface of the lens that serves as a concave acoustic mirror. The solid magenta lines designate the anterior laser cone. The white asterisk marks the position of the first collapse of the main bubble and the open black circles indicated the dislocated position of the dominant secondary cavitation generated soon after the first collapse of the initial bubble.

The breakdown using 15-mJ laser pulse was generated at a distance \( a = 9.53 \text{ mm (} \tilde{a} = 1.222) \) from the apex of the lens or 5.37 mm if measured from the edge of the lens. The shock wave launched at this distance at time zero reaches the acoustic focus in the interval between 11.2 µs and 11.3 µs after the breakdown (see Fig. 4(b)). Frame 1 (Fig. 6(a)) presents the shadow of the initial cavitation bubble which is not symmetric due to optical aberrations influencing the focusing of the laser pulse [37]. This is a common feature of ophthalmic laser systems. The extrusion on the left of the main bubble originates from occasional multiple breakdown along the axis near the optical focus. Since the image is captured from 1.50 µs to 1.75 µs after the breakdown, plasma radiation is not seen in the bubble’s center. The position of the shock wave (not seen in the image) is blurred in the annulus from 2.2 mm to 2.6 mm radially away from the breakdown.

Frame 2 (not shown) looks almost the same as frame 3 (Fig. 6(b)), expect for a smaller initial cavitation bubble. In frame 3, one can see isolated secondary bubbles being formed in the anterior laser cone and none in the posterior cone. The white dot within the large bubble is the image of the light source used for illumination.

The smaller oscillating bubbles in the anterior cone disappear in frame 5 (Fig. 6(c)) and reappear again at the same locations in frame 6 (Fig. 6(d)). The shock waves reflected from the water surface or side walls arrive back to the volume of interest around 60 µs after the breakdown and are thus returning too late to cause this reappearance. On the other hand, the shock wave entering the lens and reflected from its anterior surface arrives too soon. Even the reflection of the returning shock wave from the main bubble’s surface happens too soon. No obvious explanation for this observation is currently at hand.

The largest bubble reaches its maximum diameter of 3.79 mm in frame 18 (Fig. 6(e)) and then starts to shrink. It collapses between frame 35 and frame 36 (Fig. 6(f)) and emits another spherical shock wave although closer to the lens, because the bubble is attracted by the solid boundary. The theoretical refocusing interval of this shock wave moves away from the apex (dashed magenta lines). In frame 37 (Fig. 6(g)), the new shock wave had already been reflected from the acoustic mirror and has overpassed the anterior laser cone. Small cavitation bubbles of 100-µm size appear again in the anterior cone, but this time they are accompanied with a large cavitation structure in front of the large bubble.

In frame 43 (Fig. 6(h)), the largest bubble reaches its maximum size and collapses in frame 48 (Fig. 6(i)). Soon after, in frame 49 (Fig. 6(j)), tiny bubbles are seen again in the anterior cone close to the largest bubble that is experiencing its third oscillation. The last frame, frame 100 (Fig. 6(k)) shows the remnant gas pockets that can be seen by a physician soon after the laser breakdown event.
A short recapitulation of the presented movie clearly shows additional cavitation structures such as isolated bubbles in the anterior laser cone. These bubbles appear whenever a strong shock wave is emitted either during the breakdown or later, during the subsequent collapses of the largest bubbles. As opposed to the shock wave emitted during the optical breakdown which does not produce visible congregations of cavitation clouds near the acoustic focus, the shock wave released during the first collapse of the main bubble yields clearly distinguishable high-density cavitation clouds in the vicinity of the theoretically predicted acoustic focus (Eq. (1)). This observation will be revisited in subsection 4.3.

### 4.2. Effects of breakdown distance

The intricate effect of shock wave refocusing was additionally researched by changing the location of the breakdown along the optical axis. Figures 6(l)–6(v) (right column) show 11 selected shadowgraphs, each taken at a different distance, from $a = 3.88$ mm ($\tilde{a} = 0.497$) to $a = 14.09$ mm ($\tilde{a} = 1.806$), and each taken soon after the first collapse of the initial cavitation bubble (360 $\mu$s - 400 $\mu$s after the breakdown) when the secondary cavitation in best seen in the frame. From top frame to bottom frame, the distance from the apex of the posterior lens surface to the breakdown increases and the predicted location of the dominant secondary cavitation moves closer to the lens.

If the breakdown occurs farther away from the center of curvature of the concave mirror (Figs. 6(p)–6(v), $1 < \tilde{a}$), the secondary acoustic cavitation of largest density is generated in the anterior laser cone, close to the focal interval predicted by the simply ray theory which assumes shock wave reflection from a spherical surface. The theoretical refocusing interval bound by the dashed magenta lines in Fig. 6 is slightly closer to the apex of the lens compared to the experimental location of the dominant secondary cloud (white asterisk). This is likely due to the effectively larger actual curvature of the lens than the value of 7.8 mm reported by the manufacturer of the lens and used to predict the theoretical refocusing interval. Moreover, due to increasing light intensity closer to the optical focus, we assume that larger and denser gaseous seeds are formed by evaporation at the impurities. This also supports the experimentally observed shift of the dominant secondary cavitation cluster from the predicted refocusing interval towards the optical focus.

Producing the breakdown in the vicinity of the center of curvature of the acoustic mirror (Figs. 6(n)–6(p)), complex cavitation structures are formed at the breakdown location. For example, at $\tilde{a} = 1.051$, the second oscillation of the largest bubble is displaced towards the lens, as expected for a collapse near a rigid boundary. Additionally, the refocused shock wave produces cavitation filaments extending radially out from the breakdown site, rather than from the center of the main bubble in the beginning of its second oscillation. These cavitation filaments (streamers) match the locations of satellite microbubbles that extend normally out from the surface of the largest bubble in its final stage before the collapse. Similar phenomenon was detected by Supponen et al. [19], due to the interplay of the shock wave with the fast flowing water, revealing the streamlines of the flow [38]. An already known phenomenon, dubbed counter jet, is clearly visible in Fig. 6(n). This is another secondary cavitation entity (a gaseous phase, not a liquid jet) generated by the shock wave emitted at the bubble collapse [38]. It extends along the axis of symmetry from the point of collapse in the direction away from the solid boundary. It light of the previous discussion, it can be thought as the dominant streamer, marking the flow with the largest velocity. Again, smaller acoustically driven bubbles are also present in the anterior laser cone.

The strawberry-shaped feature at $\tilde{a} = 0.926$ (Fig. 6(o)) is a typical shadowgraph of a bubble penetrated by a liquid jet. In the next 60 $\mu$s, the spike elongates and this feature evolves into a shape of horse shoe nail. Jet formation near a solid boundary is known to cause damage to the surface of the solid boundary in its center, the closest the breakdown to the boundary. Not
seen in figures, it is expected that another erosive feature of the bubble dynamics, namely the cavitation torus accompanying and embracing the jet, develops close to the solid surface and causes damage at its collapse in the shape of a ring [39,40].

The collapse of the initial bubble near the edge of the lens (Figs. 6(l)–6(m)) additionally expels smaller bubbles from the cup at the lateral edge. These bubble are out of optical focus and appear blurred.

In summary, experiments presented in Figs. 6(l)–6(v) demonstrate the consequences of shock wave refocusing back towards the axis. Three important features were identified on top of what would be expected if there were no shock wave refocusing. The first is the generation of multiple, isolated smaller cavitation bubbles in the anterior laser cone and a larger bubble cloud near the theoretically predicted location of the acoustic focus. These two effects were present when the breakdown was generated further from the center of curvature (Figs. 6(p)–6(v), $1 < \tilde{a}$) and are highly reproducible. The third feature are the radial cavitation filaments, centered at the breakdown (Figs. 6(m)–6(p)). These filaments depend largely on the preexisting satellite microbubbles surrounding the large bubble and their distribution depends on the shape of the initial plasma. The filaments retain the cylindrical symmetry of the experiment, but their distribution and intensity differs from shot to shot.

It has to be noted that laser pulse energy at the breakdown is smaller than the energy measured by the energy meter in air just before the lens, because of light absorption in the lens and water. The absorption coefficient for water at 1064 nm is 0.144 cm$^{-1}$ [41]. Using the minimum and maximum values of $a$, the laser pulse energy at the breakdown is not 15 mJ, but 14.2 mJ at $a = 3.88$ mm ($\tilde{a} = 0.497$, Fig. 6(l)) and 12.2 mJ at $a = 14.09$ mm ($\tilde{a} = 1.806$, Fig. 6(v)). Smaller energy at the breakdown produces the main bubble of smaller maximum diameter and shorter duration of its first oscillation. Apart from the influence of the vicinity of the solid boundary, which prolongs the duration of the first bubble oscillation [38], this is the main reason why earlier frames in Figs. 6(l)–6(v) were chosen for presentation at larger breakdown distances.

Comparing frame 1 (Fig. 6(a)) and frame 36 (Fig. 6(f)), it is evident that the shock wave released during the breakdown and the one launched during the first collapse take place at different locations. In turn, this also affects the refocusing site (compare the interval presented by the dashed red lines and the one given by the dashed magenta lines): the more the smaller the breakdown distance $a$. This can be seen in Figs. 6(l)–6(v) as a displaced position of the first collapse compared to the location of the breakdown (vertical dashed blue lines).

To facilitate the comparison between the positions of the experimentally determined secondary cavitation clouds and the theoretically predicted positions, the locations of the densest parts of the clouds (black open circles in Figs. 6(q)–6(v)) were mapped to the theoretical results in Fig. 4(a). It can be seen that the measurements lay close to the solid red line representing the theoretical maximum pressure curve of the refocused shock waves.

### 4.3. Energy dependence

Energy dependence of the secondary cavitation structures is presented in Fig. 7. The location of the breakdown is the same as in Figs. 6(a)–6(k) ($\tilde{a} = 1.222$). The measurements were carried out at three different excitation energies: 5 mJ, 10 mJ and 15 mJ. The first row (Figs. 7(a)–7(c)) presents the formation of small gas bubbles in the anterior laser cone 10 $\mu$s after the optical breakdown (exposure from 11.50 $\mu$s to 11.75 $\mu$s after the breakdown). The second row (Figs. 7(d)–7(f)) shows the main bubble at its largest extent and the third row (Figs. 7(g)–7(i)) the frames acquired just after the first collapse, before the refocusing of the emitted shock wave. The fourth row (Figs. 7(j)–7(l)) gives those frames taken after the first collapse where the dominant secondary acoustic cavitation is best seen near the acoustic focus. The small secondary bubbles seen just after the optical breakdown were generated at all three excitation energies. According to the Rayleigh model of spherical bubble dynamics [3,42], the maximum diameter (2.55 mm at 5 mJ,
3.32 mm at 10 mJ and 3.79 mm at 15 mJ) of the bubble scales linearly with the duration of the first oscillation (240 µs at 5 mJ, 310 µs at 10 mJ and 350 µs at 15 mJ). This is exactly the behavior observed in Figs. 7(d)–7(f). The shadowgraphs Fig. 7(d) and 7(e) taken just after the first collapse indicate no visible bubbles in the anterior cone, while there is a cluster of small gas bubbles present in Fig. 7(f). This is the remnant cluster of cavities originating from occasional multiple breakdown in the optical focus (see the extrusion on the left of the main bubble in Fig. 6(a)).

The main conclusion drawn from the shadowgraphs in Figs. 7(j)–7(l) is the presence of the dominant secondary cavitation near the calculated acoustic focus at all energies tested, with an expected reduction of the size of the cavitation clusters at smaller energies. This is also true for all tested locations of the breakdown expect for the farthest two locations at the smallest energy of 5 mJ where no visible bubbles were formed in the anterior laser cone. Interestingly, the dominant secondary cavitation cloud is not generated by the shock wave emitted during the optical breakdown (Figs. 7(a)–7(c)), but by the one released during the first collapse of the main bubble (Figs. 7(j)–7(l)).

4.4. Implications in ophthalmology

The presented results have immediate implications on laser-induced, breakdown-based laser medical procedures in ophthalmology. The experiments performed at a breakdown distances $\tilde{a} = 0.926$ (Fig. 6(o)) and $\tilde{a} = 1.051$ (Fig. 6(p)) offer some insights of what is likely happening during posterior capsulotomy [11] in the first millisecond after the application of the laser pulse. Exactly how the present experiments can be mapped to the real medical procedure is a proposed topic of further research.

The geometrical arrangement corresponding to vitreolysis is presented in Figs. 6(q)–6(s) for the anterior vitreolysis and in Figs. 6(t)–6(v) for the central vitreolysis. In all cases, the dominant
shock wave refocusing effects are expected within the ocular lens. As for the amplitude of pressure in the acoustic focal volume (Fig. 5), the most problematic case is anterior vitreolysis (example: Fig. 6(q)) performed where the cavitation cloud appears at the position of the posterior capsule. Several papers report on the acute cataract formation or posterior lens capsule injury following laser vitreolysis ([6] and references therein). Energies of 5 mJ or 10 mJ are often used to perform vitreolysis [7], sometimes as high as 15 mJ [8] or even 20 mJ [12]. This justifies the choice of energy range used in our studies.

Similar pulse energies are also used in the Nd:YAG-based phacoemulsification called Dodick laser phacolysis [43]. In this procedure, the laser pulse delivered inside the lens through an optical fiber illuminates a metal plate at fiber’s tip and generates plasma accompanied with a release of a shock wave. The propagation of such shock wave is well described by the presented theory. Since phacoemulsification is performed inside the ocular lens which lies between the cornea and its center of curvature ($\tilde{a} < 1$), the acoustic focus is expected deeper within the eye ($1 < \tilde{b}$). In this case, the present experiments (Figs. 6(l)–6(o)) indicate no secondary cavitation in the posterior laser cone. Emission of similar shock waves can also be expected during bubble collapses generated during Er:YAG laser phacoemulsification [44].

As seen in Fig. 6(k), the conglomerated remnant gas pockets move anteriorly and persist in water on timescale of seconds. Their motion can be observed by a naked eye and is caused by the ‘wall vortex’ induced during the asymmetric collapse [38]. In the presented experiments, they ascend to the water surface due to buoyance, but in the eye, if formed in the vitreous, they move against the gravity until they are brought to stop at the boundary of the posterior chamber. Even if the repetition rate of the excitation laser is only 1 Hz, the interaction of the shock waves generated by the subsequent laser pulses with the already existing gas bubbles located in the vicinity of (or in contact with) delicate tissues may result in cellular damage [9].

The majority of ophthalmic lenses are made either of glass or PMMA. Of all the available choices physician has at hand (glass, PMMA or no lens at all), the PMMA lens has the smallest reflection coefficient for pressure. For that reason, it is expected that glass lenses will produce more intense secondary cavitation effects. The worst case is not using any lenses. Here, no energy is lost on reflection of the shock wave and, what is more important, after reflection, the wave switches polarity, becoming a tensile wave, capable of producing even more vigorous cavitation clouds. To shield the eye from the possible destructive effects of the reflected shock wave, we propose an ophthalmic lens made of transparent, acoustic material of similar impedance as cornea [45].

5. Conclusions

The presented measurements of the secondary cavitation structures visualized by high-speed camera shadowgraphy in eye-like experimental configurations indicate that the same type of phenomena may be expected in laser-induced, breakdown-based real medical procedures in human eyes, especially because the experiments were performed with the clinical values of the laser parameters.

Three types of secondary cavitation structures were discovered. The first one are the isolated microbubbles present in the anterior laser cone that is preheated by the excitation laser pulse. The second one are the dense secondary cavitation clusters generated at the location of the acoustic focus of the concave posterior surface of the ophthalmic lens. These bubble clouds have the largest density soon after the first collapse of the main bubble. At about the same time, a third type of secondary cavitation structures is generated. These are the streamers, cavitation filaments, extending radially outwards from the location of the breakdown, indicating the direction of the fluid flow just before the first collapse of the main bubble. These secondary effects would likely not have been present or would be less pronounced if the transient shock wave had not been refocused.
The air/cornea interface is thus not only the main refractive surface for optical focusing in human eyes, but it also serves as the focusing acoustic concave mirror for the laser-generated shock waves.

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**Disclosures**

The authors T.P. and R.P. declare that they have filed a patent application PCT/SI2018/050020 for an acoustic diverter for improved safety during ophthalmic laser treatments.

**References**

1. M. H. Niemz, *Laser-Tissue Interactions: Fundamentals and Applications* (Springer, 2007).
2. A. Vogel, M. R. C. Capon, M. N. Asiyo-Vogel, and R. Birngruber, “Intraocular Photodisruption with Picosecond and Nanosecond Laser-Pulses - Tissue Effects in Cornea, Lens, and Retina,” Invest. Ophth. Vis. Sci. 35(7), 3032–3044 (1994).
3. W. Lauterborn and A. Vogel, “Shock wave emission by laser generated bubbles,” in Bubble Dynamics and Shock Waves C. F. Delale, ed. (Springer, 2013), pp. 67–103.
4. A. Vogel, S. Busch, and U. Parlitz, “Shock wave emission and cavitation bubble generation by picosecond and nanosecond optical breakdown in water,” J. Acoust. Soc. Am. 100(1), 148–165 (1996).
5. A. Vogel and V. Venugopalan, “Mechanisms of pulsed laser ablation of biological tissues,” Chem. Rev. 103(2), 577-644 (2003).
6. P. Hahn, E. W. Schneider, H. Tabanbeh, R. W. Wong, and G. G. Emerson, “Reported Complications Following Laser Vitreolysis,” JAMA Ophthalmol. 135(9), 973–976 (2017).
7. J. I. Lim, “YAG Laser Vitreolysis-Is It as Clear as It Seems?” JAMA Ophthalmol. 135(9), 924–925 (2017).
8. H. L. Little and R. L. Jack, “Q-Switched neodymium:YAG laser surgery of the vitreous,” Graefe’s Arch. Clin. Exp. Ophthalmol. 224(3), 240–246 (1986).
9. A. Vogel, P. Schweiger, A. Frieser, M. N. Asiyo, and R. Birngruber, “Intraocular Nd:YAG Laser-Surgery: Light-Tissue Interaction, Damage Range, and Reduction of Collateral Effects,” IEEE J. Quantum Electron. 26(12), 2240–2260 (1990).
10. P. X. Wang, V. T. C. Koh, and S. C. Loon, “Laser iridotomy and the corneal endothelium: a systemic review,” Acta Ophthalmol. 92(7), 604–616 (2014).
11. D. J. Apple, K. D. Solomon, M. R. Tetz, E. I. Assia, E. Y. Holland, U. F. C. Legler, J. C. Tsai, V. E. Castaneda, J. P. Hoggatt, and A. M. P. Kostick, “Posterior Capsule Opacification,” Surv. Ophthalmol. 37(2), 73–116 (1992).
12. F. Fankhauser and S. Kwasniewska, “Laser vitreolysis,” Ophthalmologica 216(2), 73–84 (2002).
13. F. Fankhauser, U. Durr, H. Giger, P. Rol, and S. Kwasniewska, “Lasers, optical systems and safety in ophthalmology: A review,” Graefe’s Arch. Clin. Exp. Ophthalmol. 234(8), 473–487 (1996).
14. A. Vogel, J. Noack, K. Nahen, D. Theisen, S. Busch, U. Parlitz, D. X. Hammer, G. D. Noojin, B. A. Rockwell, and R. Birngruber, “Energy balance of optical breakdown in water at nanosecond to femtosecond time scales,” Appl. Phys. B: Lasers Opt. 68(2), 271–280 (1999).
15. C. T. Wilson, T. L. Hall, E. Johnsen, L. Mancia, M. Rodriguez, J. E. Lundt, T. Colonius, D. L. Henann, C. Franck, Z. Xu, and J. R. Sukovich, “Comparative study of the dynamics of laser and acoustically generated bubbles in viscoelastic media,” Phys. Rev. E 99(4), 043103 (2019).
16. T. Požar, M. Halilović, D. Horvat, and R. Petkovšek, “Simulation of wave propagation inside a human eye: acoustic eye model (AEM),” Appl. Phys. A: Mater. Sci. Process. 124(2), 112 (2018).
17. E. A. Brujan and A. Vogel, “Stress wave emission and cavitation bubble dynamics by nanosecond optical breakdown in a tissue phantom,” J. Fluid Mech. 558, 281–308 (2006).
18. O. Supponen, “Collapse phenomena of deformed cavitation bubbles,” PhD Thesis: (Federal Institute of Technology Lausanne, 2017).
19. O. Supponen, D. Obreschkow, P. Kobel, M. Tinguely, N. Dorsaz, and M. Farhat, “Shock waves from nonspherical cavitation bubbles,” Phys. Rev. E: Stat. Phys., Plasmas, Fluids, Relat. Interdiscip. Top. 2(9), 093601 (2017).
20. W. Lauterborn, “Laser-Induced Cavitation,” Acustica 31(2), 51–78 (1974).
21. Y. Tomita, T. Kodama, and A. Shima, “Secondary Cavitation Due to Interaction of a Collapsing Bubble with a Rising Free-Surface,” Appl. Phys. Lett. 59(3), 274–276 (1991).
22. E. Robert, J. Lettry, M. Farhat, P. A. Monkewitz, and F. Avellan, “Cavitation bubble behavior inside a liquid jet,” Phys. Fluids 19(6), 067106 (2007).
23. D. Obreschkow, N. Dorsaz, P. Kobel, A. de Bosset, M. Tinguely, J. Field, and M. Farhat, “Confined shocks inside isolated liquid volumes: A new path of erosion?” Phys. Fluids 23(10), 101702 (2011).
24. A. Vogel, W. Hentschel, J. Holzfuss, and W. Lauterborn, “Cavitation Bubble Dynamics and Acoustic Transient Generation in Ocular Surgery with Pulsed Neodymium - Yag Lasers,” Ophthalmology 93(10), 1259–1269 (1986).
25. H. L. Heacock, “Molded ophthalmic lens,” patent: (US 8,303,116 B2, 2012).
26. M. P. Kummer, J. J. Abbott, S. Dinsen, and B. J. Nelson, “Artificial Vitreous Humor for In Vitro Experiments,” 29th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, 2007, pp. 6406–6409.
27. G. S. Settles, Schlieren and Shadowgraph Techniques: Visualizing Phenomena in Transparent Media (Springer, 2001).
28. F. E. Jones and G. L. Harris, “ITS-90 Density of Water Formulation for Volumetric Standards Calibration,” J. Res. Natl. Inst. Stand. Technol. 97(3), 335–340 (1992).
29. N. Bilaniuk and G. S. K. Wong, “Speed of Sound in Pure Water as a Function of Temperature,” J. Acoust. Soc. Am. 93(3), 1609–1612 (1993).
30. D. R. Christman, General motors technical center Warren MI materials and structures lab, “Dynamic Properties of Poly(methylmethacrylate) (PMMA) (Plexiglas),” (Defense Technical Information Center, 1972), pp. 1–48.
31. B. E. Treeby, E. Z. Zhang, A. S. Thomas, and B. T. Cox, “Measurement of the Ultrasound Attenuation and Dispersion in Whole Human Blood and Its Components from 0-70 MHz,” Ultrasound. Med. Biol. 37(2), 289–300 (2011).
32. T. Požar, D. Horvat, B. Starman, M. Halilovič, and R. Petkovšek, “Pressure wave propagation effects in the eye after photoablation,” J. Appl. Phys. 125(20), 204701 (2019).
33. R. Oguri and K. Ando, “Cavitation bubble nucleation induced by shock-bubble interaction in a gelatin gel,” Phys. Fluids 30(5), 051904 (2018).
34. N. Kedem, M. Vieweg, T. Gissibl, and H. Giessen, “Linear refractive index and absorption measurements of nonlinear optical liquids in the visible and near-infrared spectral region,” Opt. Mater. Express 2(11), 1588–1611 (2012).
35. L. Rayleigh, “On the pressure developed in a liquid during the collapse of a spherical cavity,” Philos. Mag. 34(200), 94–98 (1917).
36. I. H. Fine, M. Packer, and R. S. Hoffman, “New phacoemulsification technologies,” J. Cataract Refractive Surg. 28(6), 1054–1060 (2002).
37. H. Burkard Dick, M. Tehrani, and H. Höh, “Phacoemulsification and Vitrectomy with the Erbium: YAG-Laser.” Biomed. Laser: Technol. Clin. Appl. 17(4), 313–320 (2002).
38. A. Vrečko, T. Požar, R. Petkovšek, and U. Orthaber, “An acoustic diverter for improved safety during ophthalmic laser treatments,” patent application: (PCT/SG2018/050020, 2018).