Form-Finding Using Nonlinear Analysis Method in Tensioned Fabric Structure in The Form of Handkerchief Surface

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Abstract. Tensioned fabric structure with different surface form could be realized. Their variations as possible choice form of minimal surface for tensioned fabric structure have been studied. The form of used in TFS is Handkerchief Surface. Handkerchief Surface used in TFS because Handkerchief Surface is the form of minimal surface and Handkerchief Surface has not been studied by other researcher. Besides, no other work on Handkerchief Surface as idea in tensioned fabric structure has been found. The aim of the study is to propose converged shape of Handkerchief TFS model with variable $u=v=0.4$ and $u=v=1.0$. The method used for Form-Finding is nonlinear analysis method. From the result, the surface of Handkerchief TFS model, $u=v=0.4$ and $u=v=1.0$ show the total warp and fill stress deviation is less than 0.01. The initial equilibrium shape of Handkerchief tensioned fabric structure model, $u=v=0.4$ and $u=v=1.0$ is corresponding to equal tension surface. Tensioned fabric structure in the form of Handkerchief Surface is a structurally viable surface form to be considered by engineer.

1. Introduction
Tensioned fabric structure is the lightweight structure was composed fabric surface as the main material. The materials used for fabric surface generally consist of a woven fabric coated with a polymeric resin. Figure 1 shows example of tensioned fabric structure. Two types of fabric surface is generally used is poly tetra fluoro ethylene (PTFE) coated plain weave glass-fibre fabrics and PVC coated plain weave nylon or polyester fabrics.
Figure 1. Tensioned Fabric Structure

Fabric is made of woven yarn. Warp and fill yarns that are twist together. Figure 2 shows Basic structure of the coated fabric. The structures contained warp and fill yarn, also contain coating. The yarns are weaved in a such way that treads are perpendicular to one another and they are alternately passing over and under each other. From the Figure 2, long straight yarns are called warp yarns and the direction parallel to the warp yarns are called warp direction.

Figure 2. Basic structure of the coated fabric

Tensioned fabric structure with different surface form could be realized. Their variations as possible choice form of minimal surface for tensioned fabric structure have been studied. The form of used in TFS is Handkerchief Surface. Handkerchief Surface used in TFS because Handkerchief Surface is the form of minimal surface and Handkerchief Surface has not been studied by other researcher. Besides, no other work on Handkerchief Surface as idea in tensioned fabric structure has been found.

[1] has revealed, shape finding procedure for lightweight tension structure. According for this study, a computational method based procedure for finding the shape of tensioned fabric structure is the method is quicker, more accurate and cheaper than that using model as specified in [1]. Models it is not easy to investigate several loading cases and is uneconomic to build new models or modify existing ones. [2] investigated the actual surface geometry is a key parameter in the initial equilibrium problem. In finite element, the surface is described through the nodal coordinate and the element. The surface geometry used to determine the stresses in the structure at various times. Curvature of surface geometry are affects the structural behavior of membrane structures.

From the previous study, [3] has discussed the architects and engineers are involved in the study of tensioned fabric structure. The architects are focus on the geometry shape and external appearance of tensioned fabric structure and engineer are focus on internal stress distribution and viability of tensioned
fabric structure. The technique used in this study for form-finding of tensioned fabric structure is nonlinear analysis method based on [4].

[4] and [5] has proposed the applicability of computational strategies, this models has been verify by form-finding in tensioned fabric structure. A convergence criteria is used to measure the accuracy of converged shape in Form-Finding. The criteria adopted for checking of convergence of form-finding is least square error (LSE) of total warp and fill stress deviation less than 0.01.

[6], [7],[8],[9],[10],[11],[12],[13],[14],[15],[16],[17] and [18] has shown previous studied to carried out form-finding using nonlinear analysis method also in Catenoid, Helicoid, Scherk, Enneper, Oval, Costa, Moebius Strip, Monkey Saddle and Chen-Gackstatter tensioned fabric structure models.

Understanding of the possible form of Handkerchief Surface, the converged shape to be obtained will provide alternative shape for designers to be considered. Form-finding using nonlinear analysis method can be used for the converged shape and load analysis for new form of Handkerchief Surface. Therefore, The aim of study is to determine converged shape of Handkerchief Surface applied in tensioned fabric structure. Tensioned fabric structure is highly suited to be used for realizing surface of new forms. However, none of the new example mentioned present any result on Handkerchief Surface as load carrying members. In this study, form-finding of Handkerchief Surface with variable $u=v=0.4$ and $u=v=1.0$ used.

2. Generation of Handkerchief Surface
Figure 3 shows the form of Handkerchief surface. The form of Handkerchief Surface can be obtained from [19] and [20]. Equation (1) shows the equation for Handkerchief Surface [21].

$$X = u, \ Y = v, \ Z = \frac{1}{3} u^3 + uv^2 + 2(u^2 - v^2)$$

Equation (1) shows the equation for Handkerchief Surface [21].

From this study, the application of [22] has been used for the purpose of model generation. Aspect of modeling of surface of Handkerchief surface and form as well pre-stress pattern of the resulting tensioned fabric structure through form-finding using nonlinear analysis method proposed by [4] is studied.
3. Computational Method using Nonlinear Analysis Method

The principle of nonlinear analysis method is based on [4]. The large displacement finite element formulation used for analysis of structural behaviour under external loads. Since the method can be used for both the converged shape problem and load analysis, the approach using nonlinear analysis is quite common.

\[
(0^L_K + 0^G_K)u = t^P F - 0^f
\]  

Equation (2) used for formulation of nonlinear analysis method. Where \(0^L_K\) is linear strain incremental stiffness matrix, \(0^G_K\) is nonlinear strain incremental stiffness matrix, \(0^f\) is vector internal forces, \(t^P F\) is load vector and \(u\) is vector of increment in displacement.

A nonlinear analysis method by [4] for the analysis of tensioned fabric structures has been used in this study. The procedure adopted is based on the work as specified in [4]. 3-node plane stress element has been used as element to model the surface of tensioned fabric structure. All x, y and z translation of nodes lying along the boundary edge of the Handkerchief surface have been restrained. The member pretension in warp and fill direction, is 2000N/m, respectively. The shear stress is zero or 0N/m.

Two stages of analysis were involved in the procedures of form-finding in one cycle proposed by [4]. First stage (denoted as SF1) is analysis which starts with an initial guess shape in order to obtain an updated shape for converged shape. The initial guess shape can be obtained from any pre-processing software and reference [4] is chosen for this study. This is then followed by the second stage of analysis (SS1) aiming at checking the convergence of updated shape obtained at the end of stage (SF1). During stage (SF1), artificial tensioned fabric properties, E with very small values are used. Both warp and fill tensioned fabric stresses are kept constant. In the second stage of (SS1), the actual values of tensioned fabric properties are used. Resulting warp and fill tensioned fabric stresses are checked at the end of the analysis against prescribed tensioned fabric stresses. Then, iterative calculation has to be carried out in order to achieve convergence where the criteria adopted is that the average of warp and fill stress deviation should be < 0.01. (The criteria adopted for checking of convergence of form-finding is least square error (LSE) of total warp and fill stress deviation with tolerance of 0.01.) The resultant shape at the end of iterative step n (SSn) is considered to be in the state of converged shape under the prescribed warp and fill stresses and boundary condition if difference between the obtained and the prescribed membrane stresses relative to the prescribed stress is negligibly small. Such checking of difference in the obtained and prescribed stresses has been presented in the form of total stress deviation in warp and fill direction versus analysis step. As a first shape for the start of form-finding procedure adopted in this study, initial guess shape is needed. For the generation of such initial guess shape, knowledge of the requirement of anti-clastic nature of tensioned fabric structure is used. The incorporation of anti-clastic feature into the model will help to produce a better initial guess shape.

4. Computational Result

Form-Finding on tensioned fabric structure models in the form of Handkerchief Surface have been carried out. Two variable of Handkerchief Surface with variable \(u=v=0.4\) and \(u=v=1.0\). In computational result, converged shape is determined. The numbers of nodes and triangular element for Handkerchief Surface is 225 and 392, respectively.

4.1. Handkerchief Surface, \(u=v=0.4\)

The initial guess shape of Handkerchief Surface, \(u=v=0.4\) as shown in Figure 4. Figure 5 shows the different view converged shape of Handkerchief Surface, \(u=v=0.4\). Figure 6 shows the total warp and fill stress deviation of Handkerchief Surface, \(u=v=0.4\) is less than 0.01.
Figure 4. Initial Guess Shape of Handkerchief Surface, $u=v=0.4$

Figure 5. Converged Shape of Handkerchief Surface, $u=v=0.4$
Figure 6.  Convergent curve of Handkerchief Surface, $u=v=0.4$

4.2. Handkerchief Surface, $u=v=1.0$

Figure 7 shows initial guess shape for Handkerchief Surface, $u=v=1.0$. Figure 8 shows the different view of converged shape for Handkerchief Surface, $u=v=1.0$. Figure 9 shows LSE of total warp and fill stress deviation for Handkerchief Surface, $u=v=1.0$ has been found to satisfy the tolerance of LSE <0.01.

The computational results show that error in term of LSE is less than 0.01 for Handkerchief Surface, $u=v=0.4$ and 1.0 where convergence has been achieved. The resultant shape at the end of iterative step has been considered to be in the state of initial equilibrium under the prescribed warp and fill stresses and boundary condition. From the results, the converged shape of Handkerchief Surface, $u=v=0.4$ and $u=v=1.0$ is corresponding to equal tension surface.
Figure 7. Initial Guess Shape of Handkerchief Surface, $u = v = 1.0$

Figure 8. Converged Shape of Handkerchief Surface, $u = v = 1.0$
5. Conclusion

For structural analysis, the equilibrium configuration of a structure is generally known in advance. Due to the low flexural stiffness of the fabric these structures have to be constructed so that they will experience significant prestress all times. Form-finding is the problem of finding a configuration that satisfies the laws of equilibrium. Thus the new form of form-finding minimal surface in the form of Handkerchief Surface with variable $u=v=0.4$ and $u=v=1.0$ from this study can be utilized as a useful reference for structural engineers in the design of an efficient tensioned fabric structure. It is expected to reduce the construction cost, shorten the construction time and improve the construction quality. Therefore, this result would enhance the understanding on the suitable choice of minimal surface for tensioned fabric structure among practicing engineers.

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