Studying the Higgs Potential at the $e^+e^-$ Linear Collider

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The determination of the shape of the Higgs potential is needed to complete the investigation of the Higgs profile and to obtain a direct experimental proof of the mechanism of electro-weak symmetry breaking. This can be achieved, at a linear collider, by determining the Higgs triple self-coupling $g_{HHH}$ in the processes $e^+e^- \rightarrow H^0H^0Z^0$ and $H^0H^0
\nu\bar{\nu}$, and, possibly, the quartic coupling. This paper summarises the results of a study of the expected accuracies on the determination of $g_{HHH}$ at a TeV-class LC and at a multi-TeV LC. The statistical dilution arising from contributions not sensitive to the triple Higgs vertex, can be reduced by means of variables sensitive to the kinematics and the spin properties of the reactions.

I. INTRODUCTION

The detailed study of the Higgs potential represents a conclusive test of the mechanism of symmetry breaking and mass generation. After the discovery of an elementary Higgs boson and the test of its couplings to quarks, leptons and gauge bosons, a further proof of the Higgs mechanism will be the experimental evidence that the Higgs field potential has the properties required for breaking the electro-weak symmetry. In the Standard Model (SM), the Higgs potential can be written as $V(\Phi^\dagger\Phi) = \lambda(\Phi^\dagger\Phi - \frac{1}{2}v^2)^2$. A probe of the shape of this potential comes from the determination of the triple and quartic Higgs self-couplings [1, 2, 3, 4]. The triple coupling can be expressed, in the SM, as $g_{HHH} = \frac{3M_H^2}{2}$ where $v=246$ GeV and $M_H$ can be determined to $O(100$ MeV) accuracy. An accurate test of this relation may reveal the extended nature of the Higgs sector. This can be achieved by observing the deviations, arising in a generic 2HDM or in a SUSY scenario, from the SM prediction above. Accurate data can be analysed in terms of an effective Lagrangian to establish the relationships between the Higgs self-couplings and the size of anomalous terms of other nature.

This paper summarises the findings of a comparative study of the potential of a TeV-class $e^+e^-$ linear collider (LC) and of a second-generation multi-TeV LC in the study of the Higgs potential through the analysis of the $e^+e^- \rightarrow H^0H^0Z^0$ and $H^0H^0\nu\bar{\nu}$ processes in SM. The opportunity, offered by a LC, to study the Higgs potential may be unique, as no evidence that the SM triple Higgs vertex is experimentally accessible at hadron colliders has been obtained so far [5].

At the linear collider the triple Higgs coupling $g_{HHH}$ can be accessed by studying multiple Higgs production in the processes $e^+e^- \rightarrow H^0H^0Z^0$ and $H^0H^0\nu\bar{\nu}$ that are sensitive to the triple Higgs vertex. The first process is more important at lower values of the centre-of-mass energy, $\sqrt{s}$, and for Higgs boson masses in the range 115 GeV < $M_H$ < 130 GeV, with a cross section of the order of 0.15 fb for $M_H = 120$ GeV. The second process becomes sizeable at collision energies above 1 TeV and ensures sensitivity to the triple Higgs vertex for heavier Higgs masses. It also provides a cross section larger by a factor $\approx 7$. The quartic Higgs coupling remains elusive.

The cross sections for the $HHHZ$ and $HH\nu\bar{\nu}$ processes are reduced by three order of magnitude compared to those for the double Higgs production. In the most favourable configuration, a 10 TeV $e^+e^-$ collider operating with a luminosity of $10^{35}$ cm$^{-2}$s$^{-1}$ would be able to produce only about five such events in one year ($=10^7$ s) of operation for $M_H = 120$ GeV (see Table I).

A $\gamma\gamma$ collider has also access to the triple Higgs couplings through the processes $\gamma\gamma \rightarrow HH$ and $\gamma\gamma \rightarrow HHW^+W^-$. However the cross section of the first process is suppressed by the effective $H\gamma\gamma$ coupling, compared to $e^+e^-$ collisions, and that of the second is only 0.35 fb at $\sqrt{s_{ee}} = 1$ TeV for $M_H = 120$ GeV. A muon collider
does not offer significant advantages, compared to a high energy \(e^+e^-\) LC. The most favourable production cross sections are comparable: the \(\mu^+\mu^-\rightarrow HH\nu\bar{\nu}_\mu\) process has \(\sigma=0.8 \text{ fb}^{-1} 2.7 \text{ fb}\) for \(\sqrt{s} = 3 \text{ TeV}-7 \text{ TeV}\). On the contrary, the \(\mu^+\mu^-\rightarrow HH\) reaction, which would be unique to a muon collider, is strongly suppressed.

A major problem arising in the extraction of \(g_{HHH}\) from the measurement of the double Higgs production cross section comes from the existence of diagrams that lead to the same final states but do not include a triple Higgs vertex. The resulting dilution \(\frac{g_{HHH}}{g_{SM}}\), is significant for all the processes considered. It is therefore interesting to explore means to enhance the experimental sensitivity to the signal diagrams with additional variables. In this study the \(H\) decay angle in the \(HH\) rest frame and the \(HH\) invariant mass have been considered.

The cross section computation and the event generation at parton level have been performed using the CompHEP program where the \(g_{HHH}\) coupling and the \(M_H\) mass have been varied. Partons have been subsequently fragmented according to the parton shower model using PYTHIA 6. The resulting final state particles, for the \(HH\nu\bar{\nu}\) channel, have been processed through the Simdet parametrised detector simulation and the reconstruction analysis performed. Background processes for the \(HH\nu\bar{\nu}\) channel have also been simulated and accounted in the analysis. The centre-of-mass energies of 0.5 \(\text{ TeV} / 0.8 \text{ TeV}\) and 3 \(\text{ TeV}\), representative of the anticipated parameters for a \(\text{TeV}\)-class LC, such as \(\text{TESLA}\) or a \(X\)-band collider, and a multi-TeV collider, such as \(\text{Clic}\), have been chosen with an integrated luminosity of 1 \(\text{ ab}^{-1}\) and 5 \(\text{ ab}^{-1}\) respectively. In this study, unpolarised beams are assumed. Polarizing the beams brings a gain of a factor of two for the \(HHZ\) cross section and of four for \(HH\nu\bar{\nu}\).

II. THE HHZ PROCESS

The \(g_{HHH}\) determination from the measurement of the \(e^+e^-\rightarrow HHZ\) cross-section has been already extensively discussed in its theoretical and experimental issues. A \(e^+e^-\) collider operating at \(\sqrt{s} = 500 \text{ GeV}\) can measure the \(HHZ\) production cross section to about 15% accuracy if the Higgs boson mass is 120 \(\text{ GeV}\), corresponding to a fractional accuracy of 23% on \(g_{HHH}\). At higher \(\sqrt{s}\) energies the dilution increases and it becomes interesting to combine a discriminating variable. The invariant mass of the \(HH\) system provides with a significant discrimination of the \(H\rightarrow HH\) process from other sources of \(HHZ\) production not involving the triple Higgs vertex. In fact, in the first case the distribution is peaked just above the \(2M_H\) kinematic threshold and dumped by the virtuality of the \(H^*\) boson while the background processes exhibit a flatter behaviour extending up to the upper kinematic limit \(\sqrt{s} - M_Z\) (see Figure 3). These characteristics have been tested to discriminate between the signal triple Higgs vertex contribution and the background processes in genuine \(HHZ\) final states. As the phase space increases with increasing \(\sqrt{s}\), for a fixed \(M_H\) value, this variable is most effective at larger \(\sqrt{s}\) values, i.e. as the dilution effect increases. A likelihood fit to the total number of events and to the normalised binned distribution of the \(M_{HH}\) distribution has been performed on \(HHZ\) events at generator level for \(M_H = 120 \text{ GeV}\) and \(\sqrt{s} = 500 \text{ GeV}\) and \(800 \text{ GeV}\) (see Figure 3). Results are given in Table 4. Work is in progress for improving the measurement further by including an additional \(e^+e^-\rightarrow HHZ\), where \(Z\rightarrow \nu\bar{\nu}\) in the final states.

III. THE HH\nu\bar{\nu} PROCESS

The \(e^+e^-\rightarrow HH\nu\bar{\nu}\) process exhibits a significant larger cross-section, compared to \(HHZ\), if the collision energy exceeds 1 \(\text{ TeV}\). In addition, being the interference of the signal triple Higgs vertex and the other diagrams, leading to \(HH\nu\bar{\nu}\), destructive, the total cross section decreases with increasing \(g_{HHH}\), contrary to the case of the \(HHZ\) channel. This may represent an important cross-check of the Higgs self coupling contribution, in the case any deviations would be observed in the \(HHZ\) cross-section. Since the \(HH\nu\bar{\nu}\) production is peaked in the forward region, it is important to ensure that an efficient tagging of the \(HH\rightarrow bb\bar{b}\), \(WW^*W^-W^*W^-\) decay can be achieved. Therefore a full reconstruction of \(HH\nu\bar{\nu}\) and \(ZZ\nu\bar{\nu}\) events has been performed. The \(4b+E_{miss}\) and \(4W+E_{miss}\) final states have been considered for the Higgs boson masses of 120 \(\text{ GeV}\) and 180 \(\text{ GeV}\). The sensitivity to the triple Higgs vertex has been enhanced by studying the angle \(\theta^*\) made by the \(H\) boson, boosted back in the \(HH\) rest frame, with the \(HH\) direction. Due to the Higgs boson scalar nature, the \(cos\theta^*\) distribution is flat for the signal \(H^*\rightarrow HH\) process, while it has been found to be forward and backward peaked for the \(HH\nu\bar{\nu}\) contributions from other diagrams. This characteristics is preserved after event reconstruction (see Figure 6) and a fit including the measured cross section and the normalised shape of the \(|cos\theta^*|\) distribution has been performed, similarly to the case of the \(HHZ\) channel (see Figure 4). Results are given in Table 11. Beam polarisation has not been taken into account. This may provide the ultimate means for pushing the accuracy on \(g_{HHH}\) below 5%.
Table I: Production cross sections for $e^+e^- \rightarrow HHH\bar{\nu}\bar{\nu}$

| $\sqrt{s}$ (TeV) | $g_{HHH}/g_{HHH} = 0.9$ | $g_{HHH}/g_{HHH} = 1.0$ | $g_{HHH}/g_{HHH} = 1.1$ |
|------------------|-------------------|-------------------|-------------------|
| 3                | 0.400             | 0.390             | 0.383             |
| 5                | 1.385             | 1.357             | 1.321             |
| 10               | 4.999             | 4.972             | 4.970             |

Table II: Summary of relative accuracies $\delta g_{HHH}/g_{HHH}$ for $M_H = 120$ GeV and $\int L = 1$ ab$^{-1}$ with the $HHZ$ channel.

| $\sqrt{s}$ (TeV) | $\sigma_{HHZ}$ Only | $M_H$ Fit |
|------------------|---------------------|-----------|
| 0.5              | $\pm 0.23$ (stat)   | $\pm 0.20$ (stat) |
| 0.8              | $\pm 0.35$ (stat)   | $\pm 0.29$ (stat) |

IV. CONCLUSIONS

The study of the triple Higgs couplings will provide a crucial test of the Higgs mechanism of electro-weak symmetry breaking, by directly accessing the shape of the Higgs field potential. $e^+e^-$ linear colliders, where the study of the $e^+e^- \rightarrow HHZ$ and $HH\nu\bar{\nu}$ can be performed with good accuracy, represent a possibly unique opportunity for performing this study. The study of variables sensitive to the triple Higgs vertex and the availability of high luminosity will allow to test the Higgs potential structure to an accuracy of about 20% at a TeV-class collider and to 8% and better in multi-TeV $e^+e^-$ collisions. The quartic coupling appears to remain unaccessible due to its tiny cross-section and the large dilution effect from background diagrams in $HHH\nu\bar{\nu}$.

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Table III: Summary of relative accuracies $\delta g_{HHH}/g_{HHH}$ for $\int L = 5$ ab$^{-1}$ with the $HH\nu\bar{\nu}$ channel at 3 TeV.

| $M_H$ (GeV) | $\sigma_{HH\nu\bar{\nu}}$ Only | $|\cos \theta^*|$ Fit |
|------------|-------------------------------|------------------|
| 120        | $\pm 0.094$ (stat)             | $\pm 0.070$ (stat) |
| 180        | $\pm 0.140$ (stat)             | $\pm 0.080$ (stat) |
FIG. 1: Left: the dependence of the $HHZ$ cross section on the triple Higgs coupling, normalised to its SM value, for $M_H = 120$ GeV and two $\sqrt{s}$ values. Right: The distribution of the $HH$ invariant mass in $HHZ$ events originating from diagrams containing the triple Higgs vertex (light grey) and other diagrams (dark grey).

FIG. 2: The generated $HH$ invariant mass distribution for $HHZ$ events obtained for $g_{HHH}/g_{HHH}^{SM} = 1.25, 1.0, 0.75$ and 0.5 with the points with error bars showing the expectation for $1 \text{ ab}^{-1}$ of SM data at $\sqrt{s} = 0.8$ TeV.
FIG. 3: Left: the dependence of the $HH\nu\bar{\nu}$ cross section on the triple Higgs coupling, normalised to its SM value, for $\sqrt{s} = 3$ TeV and two $M_H$ values. Right: The $\cos \theta^*$ distribution in $HH\nu\bar{\nu}$ events originating from diagrams containing the triple Higgs vertex (light grey) and other diagrams (dark grey).

FIG. 4: The reconstructed $|\cos \theta^*|$ distribution for $HH\nu\bar{\nu}$ events obtained for $g_{HHH}/g_{HHH}^{SM}$ = 1.25, 1.0, 0.75 and 0.5 with the points with error bars showing the expectation for 5 ab$^{-1}$ of SM data at $\sqrt{s}=3.0$ TeV.