Embedding DBI inflation in scalar-tensor theory

Carsten van de Bruck
School of Mathematics and Statistics, University of Sheffield, Hounsfield Road, Sheffield S3 7RH, United Kingdom

David F. Mota
Institute of Theoretical Astrophysics, University of Oslo, 0315 Oslo, Norway

Joel M. Weller
School of Mathematics and Statistics, University of Sheffield, Hounsfield Road, Sheffield S3 7RH, United Kingdom

Abstract: The Dirac-Born-Infeld (DBI) action has been widely studied as an interesting example of a model of k-inflation in which the sound speed of the cosmological perturbations differs from unity. In this article we consider a scalar-tensor theory in which the matter component is a field with a DBI action. Transforming to the Einstein frame, we explore the effect of the resulting coupling on the background dynamics of the fields and the first-order perturbations. We find that the coupling forces the scalar field into the minimum of its effective potential. While the additional scalar field contributes significantly to the energy density during inflation, the dynamics are determined by the DBI field, which has the interesting effect of increasing the number of e-folds of inflation and decreasing the boost factor of the DBI field. Focusing on this case, we show, with the benefit of numerical examples, that the power spectrum of the primordial perturbations is determined by the behaviour of the perturbations of the modified DBI field.

Keywords: Cosmology
1. Introduction

The popularity of string/M theory and its interpretation as a truly fundamental model of physical phenomena has motivated the search for a mechanism to explain the process of inflation. In canonical models of inflation, a slowly rolling scalar field, with its energy density dominated by its potential, is introduced as a phenomenological tool to account for the dramatic expansion of the universe to solve the flatness and horizon problems of the Big Bang theory and also as a source of the primordial fluctuations that give rise to structure formation and the observed anisotropies in the CMB radiation. While there is no shortage of scalar fields in stringy models, problems arise when one looks for a suitable candidate to play the role of inflaton in the canonical models [1, 2]. The abundance of light scalars (moduli) not only complicates the dynamics of inflation, but also means that models must be tuned to prevent unwanted light moduli affecting the post-inflationary universe. As well as this, although heavy fields are effectively frozen during inflation they may contribute to the potential that determines the evolution of the dynamical fields, further complicating the ‘eta problem’ in inflationary model building, in which one encounters large corrections to the flat potential required in slow-roll inflation [3, 4, 5, 6, 7].

An intriguing scenario is that of Dirac-Born-Infeld (DBI) inflation [8, 9] in which the role of inflaton is played by the radial coordinate of a D3 brane moving in a warped region
(throat) of a compactified space. Crucially, a speed limit is imposed upon the motion of the brane (which appears as a point-like object in the compact dimensions), affected by both its speed and the warp factor of the throat. The speed limit introduces a parameter $\gamma$, which plays an analogous role to the Lorentz factor in special relativity, that grows without bound as the speed limit is approached. This has the effect that in the ‘relativistic’ regime (as we shall call it), the importance of the friction and potential terms in the scalar field equation of motion is lessened, so that a DBI field can drive inflation with a relatively steep potential.

From a cosmological point of view, DBI inflation is important as it is a concrete example of k-inflation i.e. inflation driven by a scalar field with a non-canonical kinetic term $[10, 11, 12, 13, 14, 15, 16, 17, 18]$. A distinguishing feature of these models is that the field perturbations travel at a speed less than that of light, with the result that the Fourier modes freeze in at a sound horizon that is different from the curvature radius and which changes dynamically with the background evolution $[11]$. This is an exciting possibility, as the amplitude of the primordial perturbations would not be (almost) independent of wavenumber as in the standard inflationary models and the effects of the evolution of the universe during the inflationary regime could be encoded in mode correlations. In DBI inflation, the sound speed is given by $\gamma^{-1}$, and decreases to small values as the speed limit is saturated. Observational limits on the level of non-Gaussianities in the CMB radiation can thus be combined with traditional constraints arising from the power spectrum of primordial perturbations to distinguish between DBI and canonical models of inflation (see $[19, 20]$ for recent reviews).

It is to be expected that the effective action governing the inflationary phenomenon will contain other features characteristic of fundamental theories such as non-minimal coupling and multiple fields. In particular, the effect of the presence of many scalar fields on the inflationary dynamics has been the subject of intense study over the past decade, with many works focusing on the behaviour of entropic or isocurvature perturbations in these models. These can source the adiabatic fluctuations in the system so that the curvature perturbation, which in single field models is conserved outside the horizon, acquires a dependence on the inflationary dynamics. Interestingly, the presence of a coupling of the type $A(\phi) = \exp(\beta \phi)$ between two scalar fields (arising naturally in scalar-tensor theories considered in the Einstein frame) has been found to further influence the transfer between entropic modes and adiabatic modes $[21, 22]$; however, the analogous case for multifield k-inflation models has been neglected in the literature. Multiple fields arise naturally in DBI scenarios, where the extra degrees of freedom can be identified with the non-radial coordinates of the D3 brane in the compactified space, and several studies (both analytical and numerical) have been devoted to understanding the evolution of adiabatic and isocurvature perturbations and the predicted level of non-Gaussianity in these models $[23, 24, 25, 26, 27, 28, 29, 30]$. In contrast, the question of the behaviour of the perturbations in models involving many fields, each with a different intrinsic sound speed, has received relatively little attention. One such model was investigated in $[31]$, in which inflation is driven by two standard DBI fields, each corresponding to a brane with its own sound speed dependent on its evolution. The presence of two sound speeds means that the
perturbations freeze in at the sound horizon corresponding to the larger value of $c_s$ while the non-Gaussianities are at a level corresponding to the square of the larger value of $\gamma$. In order to address these issues, we propose a model in which a scalar field with a DBI action evolves in the presence of a canonical scalar field. We embed the DBI inflationary scenario into a scalar-tensor theory, in which the additional field could describe the degree of freedom associated with additional moduli fields in the higher-dimensional theory. We will not construct such a model from first principles, but rather treat it as a phenomenological model. Therefore, we consider the generalised case in which the fields are coupled, motivated by the possibility of the additional scalar field having a non-minimal coupling to gravity in the effective action. Notwithstanding the importance of non-minimal coupling from a fundamental theory standpoint, couplings of this type have often been neglected in the DBI inflation literature, partly due to the success of the standard DBI scenario as an consistent phenomenological model in their absence. However, we should point out that there have been some interesting studies such as [3], where non-minimally coupled DBI models were considered in order to circumvent the eta problem in inflationary model building. Some issues pertaining to the realisation of non-minimal coupling in DBI models of inflation from the string theory perspective were raised in [8] and also [22]. The authors of [33] performed a detailed analysis of a single field DBI scenario with non-minimal coupling of the Brans-Dicke type. Their analytical work shows that both in the small $\gamma$ and large $\gamma$ limits the effect of the coupling is negligible but in the intermediate regime, the degeneracy of observables can be broken: for a given tensor-to-scalar ratio and medium $f_{NL}$, non-minimally coupled DBI models have a redder scalar spectral index. The effect of coupling in multifield DBI models was also considered in [34, 35], focusing on the effect of particle production due to the interaction between the ‘inflaton’ brane and trapped branes in the warped throat. Although, like our model, this scenario involves a DBI action non-minimally coupled to a second field, the motivation and dynamics are very different: for example, in our model the scalar field as well as the DBI field can contribute significantly to the energy density and the background dynamics depend strongly on the evolution of its vacuum expectation value (VEV). Recently, coupled DBI inflation has also been considered in the context of a warm inflationary model [36]. This paper is organised as follows. In Sec. 2 we introduce the model in the Jordan and Einstein frames, and derive the equations of motion for the inflationary Universe. Sec. 3 is devoted to an analysis of the background dynamics, both in the non-minimally coupled and coupled cases and in Sec. 4 the equations determining the evolution of the first-order perturbations are derived and solved numerically. We conclude in Sec. 5 with a short discussion.

2. The Model

Non-minimally coupled scalar fields arise in a number of scenarios in high-energy physics, such as low-energy effective actions from higher dimensional theories, quantum field theory in curved space and $f(R)$ models of gravity. We consider a scalar-tensor theory with canonical field $\varphi$ coupled to the Ricci scalar combined with a second field $\chi$ with a DBI
action. In the Jordan frame, this is described by the action

\[ S = \int d^4x \sqrt{-g} \left[ F(\varphi) \frac{R}{2} - \frac{1}{2}(\nabla \varphi)^2 - U(\varphi) + f^{-1}(\chi) \left[ 1 - \gamma^{-1} \right] - V(\chi) \right], \tag{2.1} \]

where the function \( F(\varphi) \) determines the coupling of the canonical scalar field to gravity, \( f(\chi) \) is the warp factor of the DBI field, \( U(\varphi) \) and \( V(\chi) \) are potentials for the canonical and DBI fields respectively and \( \gamma \) is the boost factor of the DBI field, given by

\[ \gamma = \frac{1}{\sqrt{1 + f(\chi)(\nabla \chi)^2}}. \tag{2.2} \]

Varying (2.1) with respect to the metric gives the modified Einstein equations,

\[ FG_{\mu\nu} = T_{\mu\nu}^{\text{DBI}} + \left[ \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu\nu} (\nabla \varphi)^2 \right] - g_{\mu\nu} \Delta \varphi - \nabla_\mu \nabla_\nu F - g_{\mu\nu} \Box F, \tag{2.3} \]

where \( \Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \) is the d’Alembertian operator and \( T_{\mu\nu}^{\text{DBI}} \) the energy momentum tensor of the DBI field. The equations of motion for the fields are

\[ 2\varpi \Box \varphi = F' T - \varpi' (\nabla \varphi)^2 - 4UF' + 2U' F, \tag{2.4} \]

\[ \nabla_\mu \left[ \gamma g^{\mu\nu} \nabla_\nu \chi \right] = \frac{f'}{F^2} (1 - \frac{1}{2} \gamma - \frac{1}{2} \gamma^{-1}) + V', \tag{2.5} \]

where \( \varpi \equiv F + \frac{3}{2} F' \). In the Jordan Frame, the equations governing the dynamics are evidently quite complicated. In order to simplify the system and gain a different perspective we can perform a conformal transformation to the Einstein frame in which the Hilbert term in the action takes the standard form. We define

\[ \tilde{g}_{\mu\nu} = F(\varphi) g_{\mu\nu}, \tag{2.6} \]

where the tilde denotes quantities evaluated in the new frame in which the \( \varphi \) field is minimally coupled to gravity. Using the definitions,

\[ \frac{d \tilde{\varphi}}{d \varphi} = \sqrt{\frac{3}{2} \left( \frac{F'}{F} \right)^2 + 1}, \tag{2.7} \]

\[ \tilde{U}(\tilde{\varphi}) = U(\varphi) F^{-2}, \tag{2.8} \]

\[ A(\tilde{\varphi}) = F^{-\frac{1}{2}}, \tag{2.9} \]

\[ \tilde{\gamma} = \frac{1}{\sqrt{1 + A^{-2} f(\chi) g^{\mu\nu} \chi_{,\mu} \chi_{,\nu}}}, \tag{2.10} \]

the action in the Einstein frame can be written as

\[ S = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{\tilde{R}}{2} - \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\varphi}_{,\mu} \tilde{\varphi}_{,\nu} - \tilde{U}(\tilde{\varphi}) \right] + S_{\text{DBI}} \left[ A^2 \tilde{g}_{\mu\nu} \right] \tag{2.11} \]

where,

\[ S_{\text{DBI}} \left[ A^2 \tilde{g}_{\mu\nu} \right] = \int d^4x \sqrt{-\tilde{g}} A^4 \left\{ f^{-1}(\chi) \left( 1 - \tilde{\gamma}^{-1} \right) - V(\chi) \right\}. \tag{2.12} \]
The scalar field can thus be redefined to retain its canonical kinetic term in this frame; the DBI field is not redefined but its behaviour will be affected by the coupling terms \(A(\tilde{\varphi})\) present both as an overall multiplier in the action but also under the square root in the definition of the boost factor \(\tilde{\gamma}\). Throughout the rest of this paper we shall work in the Einstein frame and so will neglect the tildes for convenience of notation.

We study the flat FRW metric in this frame, describing an expanding universe,

\[
\begin{align*}
ds^2 &= -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \\
\end{align*}
\]

(2.13)

with scale factor \(a(t)\). The background equation of motion for the homogeneous field \(\chi\) is,

\[
\ddot{\chi} + 3H\gamma^{-2} \dot{\chi} + \frac{1}{2} A^2 \frac{f}{f^2}(1 - 3\gamma^{-2} + 2\gamma^{-3}) + A^2 \gamma^{-3} V\chi = -\beta \dot{\chi} \dot{\varphi} (3\gamma^{-2} - 1),
\]

(2.14)

where dots indicate derivatives with respect to the time coordinate in the Einstein frame, \(f_\chi \equiv df/d\chi\), \(H = \dot{a}/a\) and \(\beta = d\ln A/d\varphi\). Throughout this paper we will assume that \(\beta\) is constant, so the coupling \(A(\varphi)\) is given by

\[
A(\varphi) = \exp(\beta \varphi). 
\]

(2.15)

Eqn. (2.14) reduces to the standard DBI equation of motion when \(\beta = 0\), although it is important to note that in the coupled case, the boost factor \(\gamma\) acquires a factor of \(A\) under the square root that affects the dynamics of the field,

\[
\gamma = \frac{1}{\sqrt{1 - A^{-2} f_\chi^2}}.
\]

(2.16)

The \(\varphi\) equation takes the simple form,

\[
\ddot{\varphi} + 3H \dot{\varphi} + U_\varphi = \beta T_{DBI},
\]

(2.17)

in terms of the trace of the stress-energy tensor of the DBI field \(T_{DBI} = -\rho_{DBI} + 3p_{DBI}\). The energy density and pressure for \(\chi\) are

\[
\rho_{DBI} = A^4 \left[ f^{-1}(\gamma - 1) + V \right] \\
p_{DBI} = A^4 \left[ f^{-1}(1 - \gamma^{-1}) - V \right].
\]

(2.18)

so \(T_{DBI}\) can be written in terms of \(\gamma\), \(A\) and \(V\) as

\[
T_{DBI} = A^4 \left[ f^{-1}(4 - 3\gamma^{-1} - \gamma) - 4V \right].
\]

(2.19)

The Friedmann equations are

\[
3H^2 = \frac{1}{2} \dot{\varphi}^2 + U + \rho_{DBI},
\]

\[
-2\dot{H} = \dot{\varphi}^2 + \gamma A^2 \dot{\chi}^2.
\]

(2.20)

(2.21)

From these we can define the slow-roll parameter, \(\epsilon\),

\[
\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2} \frac{\dot{\varphi}^2 + \gamma A^2 \dot{\chi}^2}{\rho_{DBI}} + \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} U,'
\]

(2.22)

in the standard way, so that \(\ddot{a}/a = H^2(1 - \epsilon)\).
Figure 1: DBI inflation, plotted against $N = \ln a$. The upper left panel shows the log of the field value of $\chi$; the upper right panel shows the increase in the boost factor $\gamma$. The lower left panel shows the logarithm of the slow-roll parameter $\epsilon$, and the bottom right shows the equation of state parameter $w_{\text{DBI}} = p/\rho$, which remains close to $-1$ until $\chi$ starts to oscillate about its minimum. A reasonable number of efolds of inflation are generated in this case, but the boost factor gets very large. Parameter Values: $\lambda = 10^{12}$, $m = 5 \times 10^{-5}$, $\mu = 0.1$.

3. Background Dynamics

3.1 DBI inflation

Before discussing the two-field system, it will be useful to remind the reader of some of the features of single-field DBI inflation [8]. Setting $\beta = 0$ in (2.14) the equation of motion for the minimally coupled DBI field can be written as

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{2} f' \gamma^{-2} \dot{\chi} + \frac{1}{2} f'' (1 - 3\gamma^{-2} + 2\gamma^{-3}) + \gamma^{-3} V'_\chi = 0. \quad (3.1)$$

As the $\chi$ field rolls down its potential, $\gamma$ gradually increases from 1, driven by the potential. As $|\dot{\chi}|$ increases, so does the boost factor, with the result that terms in $1/\gamma$ are suppressed. The potential and friction terms then drop out and the equation reduces to,

$$\ddot{\chi} + \frac{1}{2} f' \dot{\chi} \approx 0 \quad \Rightarrow \quad \ddot{\chi} - \frac{2}{\lambda} (\chi^3 + \mu^2 \chi) \approx 0, \quad (3.2)$$

where in the second equation we have used the standard 'mass gap' form for the warp factor, given by

$$f(\chi) = \frac{\lambda}{(\chi^2 + \mu^2)^2}. \quad (3.3)$$

This is an approximation to the Klebanov-Strassler throat (see [37] and references therein) used to describe the warped conifold in which the DBI field moves that reproduces phenomenologically the form of $f(\chi)$ in both the IR and UV limits. In the limit of an AdS
throat ($\chi^2 \gg \mu^2$) the solution is $\chi \sim \sqrt{\lambda}/t$; towards the tip of the throat where $\chi^2 \ll \mu^2$, $\chi$ decreases faster as $\chi \sim \exp(-\mu \sqrt{2/\lambda t})$.

The slow-roll parameter $\epsilon$ is given by,

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2} \frac{f^{-1}(\gamma - \gamma^{-1})}{f^{-1}(\gamma - 1) + V} \approx \frac{3}{2} \frac{\gamma}{\gamma + fV}.$$  \hfill (3.4)

Note that although $\gamma$ gets large, the large value of $f$ required to satisfy observational constraints means that the potential term is the dominant contribution to the energy density and the pressure, implying that $\epsilon$ is less than 1 when $\chi$ is small and the equation of state $w_{\text{DBI}} = p_{\text{DBI}}/\rho_{\text{DBI}}$

$$w_{\text{DBI}} = \frac{f^{-1}(1 - \gamma^{-1}) - V}{f^{-1}(\gamma - 1) + V} \approx -\frac{fV}{\gamma + fV}$$ \hfill (3.5)

comes very close to $-1$. This is plotted in Fig. 1.

Single field DBI inflation is normally treated analytically using the Hamilton-Jacobi formalism, in which the field value $\chi$ assumes the role of the time-coordinate. Differentiating $H = H(\chi)$ with respect to cosmic time and using the second Friedmann equation, one finds,

$$\dot{\chi} = -2 \frac{H'(\chi)}{\gamma(\chi)}.$$ \hfill (3.6)

Therefore, the definition of $\gamma$ can be rewritten as,

$$\gamma(\chi) = \sqrt{1 + 4f(\chi)H'(\chi)^2},$$ \hfill (3.7)

and the potential,

$$V(\chi) = 3H(\chi)^2 - f(\chi)^{-1}(\gamma(\chi) - 1).$$ \hfill (3.8)

The slow-roll parameter \footnote{One can generally construct a 'tower' of flow (slow-roll) parameters (the first two being $\epsilon$ and $\eta$) defined in terms of derivatives of $H(\chi)$ with respect to $\chi$ \cite{38}. In addition, it has been shown in \cite{39} that one can additionally construct a second 'tower' in terms of derivatives of $\gamma(\chi)$ with respect to $\chi$. The first flow parameter in this 'tower', $s$, enters the expression for the spectral index of curvature perturbations as a measure of the variation of the speed of sound i.e. $n_s - 1 = -4\epsilon + 2\eta - 2s$.} can then be written

$$\epsilon = \frac{2}{\gamma} \left(\frac{H'(\chi)}{H}\right)^2 \approx \sqrt{\frac{3}{\lambda m}},$$ \hfill (3.9)

where the second equality holds for the AdS throat with potential $m^2\chi^2$ when $\epsilon \ll 1$. An unusual feature of DBI inflation is that an increased mass term decreases $\epsilon$, unlike in the standard slow-roll case. The advantage of the Hamilton-Jacobi approach is that, given a potential $V(\chi)$, eqns. \cite{38} (using the definition of $\gamma(\chi)$ in \cite{37}) and \cite{36} can be solved sequentially. One can also choose an anzatz such as $H(\chi) \propto \chi$ in order to calculate $V$.

In much of the DBI literature $m^2\chi^2$ potentials similar to those in large field inflation are used, following the analysis in \cite{38}. This showed not only could a mass term be generated...
in the DBI scenario without the setup being ruined by back-reaction of the probe brane, but it could drive the power-law inflation discussed above. Although it has since been found [40, 41, 42] that the class of potentials that lead to successful DBI inflation is much wider, in this work we focus on modifications to the standard DBI scenario and will restrict ourselves to the potential

\[ V = \frac{1}{2} m^2 \chi^2. \]  

(3.10)

In the cutoff throat, the \( \gamma \) increases to a maximum then decreases slightly as the friction term starts to become more important. Generally, \( \gamma \) remains large even after \( \chi \) has passed through the origin, whereupon the other terms in the equation start to combine to stop the field, giving \( \gamma = 1 \). After this point, the field exhibits oscillations about its minimum. The presence of one or more D3 branes in the tip region of many throat constructions, means that DBI inflation is often taken to end with the probe brane annihilating at the point \( \chi_{\text{end}} \), rather than with a violation of the condition for acceleration \( \epsilon < 1 \). The mechanism by which the energy released in this scenario is transferred to the standard model brane is discussed in [43]. If \( \chi \) can reach the bottom of its potential, it can oscillate, which allows for the possibility of energy transfer in a period of (p)reheating [44, 45], however, this is beyond the scope of the present paper.

As discussed in the introduction, DBI inflation is a particular case of k-inflation in which the squared sound speed of scalar perturbations is equal to \( c_s^2 = \gamma^{-2} \). The non-Gaussianity of the primordial perturbations is determined by the bispectrum (the three-point correlation function of the scalar curvature perturbation \( \zeta \)), which in general is a non-trivial function of the momentum configuration. The bispectrum can be decomposed into several parts (shapes), each with a magnitude parameterised by an estimator (or non-linearity parameter) \( f_{NL} \) defined by an ansatz of the form [46]

\[ \zeta = \zeta_L - \frac{3}{5} f_{NL} \zeta_L^2, \]  

(3.11)

where \( \zeta_L \) is the linear part of the scalar curvature perturbation. In DBI inflation, the leading contribution is estimated by\(^2\)

\[ f_{NL} = \frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right). \]  

(3.12)

Therefore,

\[ f_{NL} \approx 0.32 \gamma^2. \]  

(3.13)

Conservative estimates of the observational value\(^3\) of the non-linearity parameter have \( |f_{NL}| \lesssim O(100) \), leading to an upper bound on the value of the boost factor \( \gamma \lesssim 20 \).

\(^2\)Eqn. (3.12) is calculated on an equilateral triangle using momenta of equal amplitude. This is relevant because the scalar perturbations are frozen on large scales and oscillate (averaging out) on small scales. Thus, the dominant contribution comes from modes with similar wavelengths that exit the (sound) horizon at the same time.

\(^3\)We note in passing that the measured value of \( f_{NL} \) can from receive relevant corrections from the post-inflationary evolution of the perturbations and differ from the expressions given above. [19]. The curvature perturbation, however, is conserved on large scales so one can connect the observable signal to the primordial perturbations.
Minimal coupling with a quadratic potential $U(\varphi) = U_0 \varphi^2$ with $U_0 = 10^{-14}$ (black, dotted), $10^{-12}$ (blue, dot-dashed), $2 \times 10^{-12}$ (red, dashed) and $5 \times 10^{-10}$ (cyan, solid). The upper panel shows the boost factor $\gamma$, the middle panel show the DBI density parameter $\Omega_{\text{DBI}} = \rho_{\text{DBI}} / 3H^2$ and the lower panel the slow-roll parameter $\epsilon$. Plots are against efold number $N = \ln(\max(a)/a)$ so $N = 0$ corresponds to the end of inflation. Other parameter values: $\lambda = 2 \times 10^{12}$, $m = 5 \times 10^{-5}$ and $\mu = 0.2$.

assumed to apply during the first 10 of the last 60 efolds of inflation which directly affect the CMB radiation. Since the boost factor increases with the number of efolds of inflation $N_{\text{max}}$, many DBI models with $N_{\text{max}}$ sufficiently large predict a high level of non-Gaussianity exceeding current observational bounds (cf. [47, 48] and also [9, 49, 37]).

3.2 Minimal Coupling $\beta = 0$

Let us now include the $\varphi$ field. A priori, there appear to be five possible ways in which the system can behave, viz. $\chi$ dominated inflation, $\varphi$ dominated inflation, a period of double inflation with a changeover from $\chi$ to $\varphi$ (or vice-versa) or a setup in which $\chi$ and $\varphi$ contribute roughly equally to the energy density.

In the minimally coupled case, the fields $\chi$ and $\varphi$ only interact gravitationally and since they evolve on different time scales in this case, it is difficult to get a situation in which both contribute equally to the energy density for a considerable length of time. Even if one fine-tunes the initial conditions so the fields have similar energy densities, there are problems. $\chi$ becomes small quickly unless its potential is large, but if we require $\Omega_\chi \approx \Omega_\varphi$, (where $\Omega_i = \rho_i / 3H^2$ is the density parameter for the $i^{\text{th}}$ field) the potential for $\varphi$ is steep and the field quickly rolls to its minimum. The opposite is true if $V \approx U$ is smaller: since the DBI contribution to the slow-roll parameter depends explicitly on the DBI field mass, it decays quickly, leaving $\varphi$ to slowly roll to its minimum.

If the total energy density is dominated by the DBI potential, the evolution of $\chi$ proceeds in the usual fashion: $\gamma$ increases steadily to large values, until $\chi \ll \mu$, whereupon
the field exhibits oscillations. If the difference in energy densities between the two fields is not great, the \( \phi \) field rolls in the background. If, however, \( 3H^2 \approx V(\chi) \), Hubble damping of the \( \phi \) field is disproportionally large, so the field evolves slowly until the DBI field decays. When this occurs, the \( \phi \) field rolls to its minimum as in canonical slow-roll inflation. An example of this behaviour is the black (dotted) curve in figs. 2 and 3.

The opposite occurs when the \( \phi \) field is initially dominant (i.e., \( 3H^2 \approx U(\phi) \)): one finds a period of standard slow-roll expansion followed by DBI inflation (cf. solid cyan curve in figs. 2 and 3). At first \( \Omega_\phi > \Omega_\chi \) and \( \gamma \sim 1 \) so the DBI field is dynamically unimportant, simply frozen or evolving slowly to smaller values. The timescale for \( \phi \) inflation is rapid compared to that of DBI inflation so \( \gamma \) does not increase much during this stage if \( \chi \) is rolling. When the \( \phi \) field decays, the slow-roll parameter grows, only to decrease again as the boost factor increases. In both cases, if one field is dominant, there are two distinct periods of inflation.

Perturbations in two-field inflationary models can often be decomposed into modes tangential (adiabatic) and orthogonal (entropic) to the background trajectory in field space.

**Figure 3:** The evolution of the canonical and DBI fields in the \((\phi, \chi)\) plane for the plots shown in fig. 3.
The trajectories for the two aforementioned examples exhibit a sharp turn in field space (fig. 3) as the dominant field reaches its minimum. In canonical scalar field models, behaviour of this type is of particular interest as it can lead to a transfer between entropic (isocurvature) and adiabatic modes, so that the curvature perturbation is not constant on super-Hubble scales. If one fine-tunes the mass scales so that both fields contribute roughly equally to the total energy density for a considerable number of e-folds, the dynamics of the DBI field can be significantly affected by the additional contribution to the Hubble damping, leading to smaller values of $\gamma$ (relative to the DBI dominant case). As both fields are evolving, the interplay between the field perturbations suggests that isocurvature fluctuations could be important. However, minimally coupled case is extremely sensitive to small changes in the field masses (as can be seen in fig. 3) and the initial conditions of both fields so will be analysed in more detail in future work.

In the rest of this paper we will focus on the non-minimally coupled case, in which the $\varphi$ field moves in an effective potential dependent on the DBI field.

### 3.3 Non-minimal Coupling

If we consider the non-minimally coupled model interactions between the fields can become more complex as the extra mass term $-\beta T_{DBI}$ in (2.17) starts to play an important role. The combined effect of this and the bare potential means that the effective potential $U_{\text{eff}} = U - \frac{1}{4} T_{DBI}$ can have a minimum value at which the field value $\varphi = \varphi_{\text{min}}$ satisfies,

$$
\left. \frac{dU}{d\varphi} \right|_{\varphi=\varphi_{\text{min}}} - \beta e^{4\beta \varphi_{\text{min}}} T_{DBI}^b = 0,
$$

where $T_{DBI}^b = f^{-1}(4 - \gamma - 3 \gamma^{-1}) - 4V \approx -4V$ is the trace of the ‘bare’ DBI stress-energy tensor and the $\varphi$ dependence of $A(\varphi)$ has been written explicitly using (2.15).

The existence of a minimum is dependent on the form of the bare potential: the exponential form of the coupling means that it is necessary to have a potential that slopes to the right$^4$ i.e $dU/d\varphi < 0$.

#### 3.3.1 Offset quadratic potential

In this paper, we will consider two choices of potential, dependent on two free parameters $U_0$ and $\eta$. First, the offset quadratic potential

$$
U(\varphi) = U_0(\varphi - \eta)^2,
$$

for which the true minimum is $\varphi = \eta$. (The advantages of the offset potential over the standard quadratic $\eta = 0$ case will become apparent.) In this case $dU/d\varphi = 2U_0(\varphi - \eta)$; writing $v = -4\beta \varphi_{\text{min}}$, the condition for the minimum is

$$
ve^v = -\frac{2\beta^2 T_{DBI}^b e^{4\beta \eta}}{U_0} \approx \frac{8\beta^2 V(\chi) e^{4\beta \eta}}{U_0},
$$

$^4$Alternatively, if $\beta$ were defined to be negative, for a minimum to exist one would need a potential with positive slope $dU/d\varphi < 0$. In this case, one would need negative values of $\varphi$ to ensure that $A > 1$, but the phenomenology of the model would be unchanged.
Figure 4: Non-minimal coupling with $A \lesssim 1$ for the quadratic potential. Black dot-dash curves have $\beta = 0$, blue dashed curves have $\beta = 0.25$ and solid red curves have $\beta = 0.5$. Note that in the latter case the effective potential is steep and there is a brief kination stage during which there is no inflation ($\epsilon > 1$) as $\dot{\varphi}$ is large. Parameter values: $\lambda = 2 \times 10^{12}$, $m = 5 \times 10^{-5}$, $U_0 = 2 \times 10^{-11}$, $\mu = 0.2$, $\eta = 0$.

where the approximation is valid as, generally, the pressure and energy density of the DBI field are dominated by its potential. $\varphi_{\text{min}}$ can then be expressed in terms of Lambert’s $W$ function $W(x)$ (satisfying $W(x)e^{W(x)} = x$) as

$$\varphi_{\text{min}} \approx \eta - \frac{1}{4\beta} W \left( \frac{8\beta^2 V e^{4\beta \eta}}{U_0} \right).$$

(3.16)

The effect of the coupling on the dynamics of the system depends strongly on the sign of $\varphi$, as this affects whether the coupling $A = \exp(\beta \varphi)$ is greater or less than one. Even if $\varphi$ is initially positive, the field will be attracted to its minimum, which can take negative values if $\eta$ is small. As can be seen in fig. 4, while $\varphi$ is being dragged to smaller values the extra friction term on the RHS of (2.14), which is approximately $\beta \dot{\varphi} \dot{\chi}$, acts to further slow the rate of decrease of $\chi$. When $\varphi$ reaches its minimum, depending on the steepness of the effective potential it will either overshoot (which in extreme cases can lead to values of $A$ substantially less than one) or track the minimum value, which evolves slowly toward $\eta$ at a rate determined by the decrease in $V(\chi)$.

In the non-minimally coupled case, we find numerically that for a wide range of parameters $\varphi$ quickly settles in the minimum of its effective potential, even when substantially displaced (cf. fig. 4). This considerably simplifies the analysis as one can make the assumption $\varphi = \varphi_{\text{min}}$ for most of the time during inflation, so the background dynamics are largely independent of the initial conditions for the scalar field $\varphi$. To better understand the dynamics in this case, consider the condition for the minimum (3.14) for this potential,

$$2U_0(\varphi - \eta) \simeq -4\beta A^4 V.$$

(3.17)
Using this, we can rewrite the potentials in terms of \( W \equiv W(x) \) with \( x \approx 8\beta^2 V e^{4\beta n}/U_0 \) as
\[
A^4V \simeq \frac{U_0 W}{8\beta^2}, \quad U = \frac{U_0 W^2}{16\beta^2},
\]
so the Hubble parameter becomes
\[
3H^2 \simeq (1 + \frac{1}{2}W)A^4V.
\]
(3.18)

One can differentiate (3.17) with respect to time to get \( \dot{\phi} \) in terms of \( \dot{\chi} \)
\[
\dot{\phi} \simeq -\frac{1}{2\beta} \left( \frac{W}{1 + W} \right) \frac{\dot{\chi}}{\chi},
\]
(3.20)
where we have used the quadratic form of the \( \chi \) potential to write \( V_\chi/V = 2\chi \). Thus, the rate of change of the scalar field \( \dot{\phi} \) has an explicit dependence on \( \dot{\chi} \). Focusing on the rate of change of the Hubble parameter, it can be seen from eqn. (2.21) that for the kinetic term of the DBI field to dominate over that of the scalar field, we need the condition \( A^2 \gamma \dot{\chi}^2 \gg \dot{\phi}^2 \) to be satisfied i.e.
\[
A^2 \gamma \dot{\chi}^2 \gg \frac{1}{4\beta^2} \left( \frac{W}{1 + W} \right)^2 \frac{\dot{\chi}^2}{\chi^2},
\]
\[
\Rightarrow \gamma \gg \frac{A^2 m^2}{8\beta^2} \left( \frac{W}{1 + W} \right)^2 \frac{1}{A^4V},
\]
\[
\Rightarrow \gamma \gg \left( \frac{(2 + W)W^2}{48\beta^2(1 + W)^2} \right) \left( \frac{A^2 m^2}{H^2} \right),
\]
(3.21)
where (3.19) was used in the third step. Notice that since \( W/4\beta \) is equal to the dislocation of \( \phi \) from its minimum, \( W/2 \) is fairly small, around \( O(1) \). Since the effective mass of the DBI field is \( m_A \) [cf. (2.14)], which should be (at least) the order of the Hubble scale, we find that the \( \phi \) field is dynamically subdominant (i.e. \( -\dot{H} = \frac{1}{2}(\dot{\phi}^2 + A^2 \gamma \dot{\chi}^2) \approx A^2 \gamma \dot{\chi}^2 \)) if \( \gamma \gg O(1) \). So, when \( \chi \) is affected by the DBI terms in the action, the slow-roll parameter becomes
\[
\epsilon \simeq \frac{3A^2 \gamma \dot{\chi}^2}{(2 + W)A^4V} \simeq \frac{1}{A^2(1 + \frac{1}{2}W)} \left( \frac{3\gamma \dot{\chi}^2}{2V} \right).
\]
(3.22)

The case in which \( \phi_{\text{min}} > 0 \) is thus of particular interest as this gives \( A \gtrsim 1 \), yielding
\[
\epsilon \simeq \left( \frac{3\gamma \dot{\chi}^2}{2V} \right) \approx \epsilon_{\text{DBI}}
\]
where \( \epsilon_{\text{DBI}} \) is the slow-roll parameter for a standard DBI field of mass \( m \). Since \( \epsilon \) is inversely related to the number of e-folds of inflation \( N_{\text{max}} = \ln(a_f/a_i) \) by
\[
N_{\text{max}} = \int_{t_i}^{t_f} Hdt = \int_{t_i}^{t_f} \frac{1}{\sqrt{-H}} \sqrt{-H}dt \simeq \int_{\chi_{\text{ini}}}^{\chi_{\text{ini}}} A \sqrt{\frac{\gamma}{2\epsilon}} d\chi,
\]
where (3.22) is used in the third step. It should be noted that the factor of \( A^{-2} \) in the definition of \( \gamma \) in the Einstein frame means that \( \gamma \) is smaller in the coupled case compared to the standard DBI scenario so even discounting the small factor \( A^{-2}(1 + \frac{1}{2}W)^{-1} \) in (3.22), \( \frac{3}{2}\gamma \dot{\chi}^2/V \) is generally less than \( \epsilon_{\text{DBI}} \).
Figure 5: The effect of increasing $\beta$ from 0.1 to 1.5 for the offset potential case. The upper-left panel shows the trajectory in field space (starting at $\chi = 1.5$) with $\varphi$ tracking the minimum, the upper-right panel shows the DBI boost factor $\gamma$, the lower-left panel shows the slow-roll parameter $\eta$ and the lower-right panel shows the density parameter of the DBI field. It can be seen that increasing the coupling increases the duration of inflation and decreases the maximum boost factor. Parameter values: $\lambda = 8 \times 10^{12}$, $m = 8 \times 10^{-6}$, $U_0 = 10^{-10}$, $\mu = 0.15$, $\eta = 3$.

this results in a increase in the number of efolds of inflation. This is evident in fig. 5, in which the the evolution of the fields is plotted for several values of $\beta$. One can see that increasing the coupling from 0.1 to 1.5 decreases the slow-roll parameter by over an order of magnitude, causing the the number of efolds of inflation to increase from $\sim 30$ to $\sim 140$. Also, the maximum value of the DBI boost factor is dramatically reduced by a factor of 10, which could lead to interesting observational deviations from the standard DBI model (discussed in the final section).

If $A \ll 1$, the effective mass of the DBI field is decreased, which has the twofold effect of increasing the slow-roll parameter and decreasing the number of efolds of inflation, as well as affecting the DBI speed limit via the factor of $A^{-2}$ under the square root in eqn. (2.16). This can be seen in the cases presented in fig. 4. As $\varphi_{\min}$ is negative, $A \lesssim 1$ and the number of efolds of inflation is decreased compared to the minimally coupled case.

One can then ask, what conditions need to be fulfilled for in order for the coupling $A$ to be greater than one? Substituting (3.16) into $\varphi_{\min} > 0$ gives the condition

$$W < 4 \beta \eta,$$

which is plotted in the left-panel of fig. 4 for several values of the ratio $V/U_0$. As one might expect, it is easier to satisfy this condition with larger values of the bare minimum $\eta$, as

\[ \text{Note that the mexican hat potential with minima at positive and negative values of } \varphi \text{ is not suitable here as, if the field starts at small values, it is dragged in the direction of the negative minimum by the term on the RHS of eqn. (2.17) giving } A < 1. \]
Figure 6: Conditions for a positive minimum and DBI dominance for the offset quadratic (left panel) and exponential (right panel) potentials. Left panel: regions above the solid lines satisfy the condition for $\varphi_{\text{min}} > 0$ (3.23) using $V/U_0 = 0.01, 0.1, 1, 10$ (starting from the bottom). Shaded regions indicate where the DBI energy is dominant (3.24) with the lightest region corresponding to $V/U_0 = 0.01$. Right panel: the equivalent conditions for the exponential potential. In this case the region satisfying the condition for DBI dominance (3.30), represented by the shaded region, is independent of the ratio $V/U_0$.

the minimum of the effective potential is shifted to larger (more positive) values. Using (3.19), the condition for DBI dominance, $\Omega_{\text{DBI}} > \Omega_\varphi$ is

$$W < 2,$$

which is also plotted in fig. 3. Naively, one might expect the situation wherein the DBI plays a dominant role to be characterised by $V/U_0 \gg 1$. However, this means that the second term in (3.16) is dominant unless the offset is large so the net effect is $\varphi_{\text{min}} < 0$, leading to smaller values of the energy density $\rho_\chi \sim A^4(\varphi)V$. One can see in fig. 3 that the region of the $\beta, \eta$ parameter space that can satisfy both (3.23) and (3.24) is fairly small meaning that for a large number of cases for which $A > 1$ the DBI energy density is subdominant. However, this does not mean that $\chi$ is dynamically unimportant: the opposite is the case. The large contribution of the energy density of the scalar field arises as $\varphi$ is trapped in a false vacuum away from its true minimum, so if $\varphi$ tracks the minimum, its evolution is determined solely by the DBI field. This can be seen in the examples shown in fig. 3. In each of the cases presented here $\Omega_{\text{DBI}} \lesssim 1$, but the field is never completely subdominant. However, despite the significant contribution of $\Omega_\varphi$ to the energy budget, the dynamics are controlled by the DBI field and $\varphi$ is trapped in the $\chi$-dependent minimum of its effective potential. (This behaviour is similar to hybrid inflation [52], see also [53] for a recent discussion of hybrid inflation in scalar-tensor theory.)
3.3.2 Exponential potential

As well as the offset quadratic potential, we also consider an exponential potential

\[ U(\varphi) = U_0 \exp(-\eta \varphi), \quad (3.25) \]

in which the parameter \( \eta \) determines the slope and, under the influence of the bare potential, \( \varphi \) moves to larger values. The condition for the minimum in this case reads

\[ U \simeq \frac{4\beta}{\eta} A^4 V, \quad (3.26) \]

so the minimum is given by the logarithmic function

\[ \varphi_{\text{min}} = \frac{1}{4\beta + \eta} \log \left( \frac{\eta U_0}{4\beta V} \right). \quad (3.27) \]

Note that in this case, as the fields are dominated by their potentials, the energy densities of the fields are proportional to each other. Thus, we have a tracker solution for the fields, in which the density parameters are constant \( \Omega_{\text{DBI}} \propto \Omega_\varphi \simeq \text{const} \) (as can be seen in fig. [7]). The condition for \( \varphi_{\text{min}} > 0 \) is therefore

\[ \eta > 4\beta(V/U_0). \quad (3.28) \]

This is plotted in the right-panel of fig. [3], and one can see that the condition is similar to that for the offset quadratic potential, where \( \eta \) was the position of the true minimum. This is simply because in both cases increasing the parameter \( \eta \) steepens the bare potential at small values of \( \varphi \).

Using (3.17), the Friedmann equation can be written,

\[ 3H^2 = (1 + 4\beta/\eta) A^4 V, \quad (3.29) \]

so the condition for DBI dominance is independent of the ratio \( V/U_0 \):

\[ \eta > 4\beta. \quad (3.30) \]

This is also shown in the right-panel of fig. [3]. Differentiating (3.26) with respect to time and repeating the steps in the previous section yields a simple expression for the rate of change of \( \varphi \) in the minimum

\[ \dot{\varphi} \simeq -\frac{1}{\eta(1 + 4\beta \eta)} \frac{\dot{\chi}}{\chi}, \quad (3.31) \]

so the condition \( \dot{\varphi}^2 \ll A^2 \gamma \dot{\chi}^2 \) implies that

\[ \gamma \gg \left[ \frac{(1 + 4\beta/\eta)}{6\eta(1 + 4\beta \eta)} \right] \left( \frac{A^2 m^2}{H^2} \right) \gtrsim \mathcal{O}(1), \quad (3.32) \]

must be satisfied, which is true when \( \chi \) is affected by the DBI terms in the action. As with the case the of the offset potential, we find that the slow-roll parameter

\[ \epsilon \simeq \frac{3A^2 \gamma \dot{\chi}^2}{2(1 + 4\beta/\eta) A^4 V} \simeq \frac{1}{A^2(1 + 4\beta/\eta)} \left( \frac{3\gamma \dot{\chi}^2}{2V} \right) < \epsilon_{\text{DBI}}, \quad (3.33) \]

is, as in the previous case, smaller than the standard DBI model when \( A > 1 \), leading to a greater amount of efolds of inflation (see fig. [3]). Also, as in the example of the offset potential, one can see in fig. [3] a decrease in the boost factor of the DBI field.
Figure 7: The effect of increasing $\beta$ from 0.1 to 1.5 for the exponential potential case. Quantities plotted are the same as fig. [3]. Parameter values: $\lambda = 2 \times 10^{13}$, $m = 10^{-7}$, $U_0 = 5 \times 5^{-10}$, $\mu = 0.32$, $\eta = 2.9$.

4. Cosmological perturbations

4.1 First order perturbation equations

We now discuss the linear perturbations in our model. The model under consideration is part of a much larger class of multifield models with a Lagrangian of the form $P = P(X^{IJ}, \phi^K)$, where $X^{IJ} \equiv -\frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J$ is the kinetic term involving derivatives of the scalar fields $\phi^I$ and $\phi^J$. The equations of motion for the linear perturbations in the general case are calculated in [26], but in order to render transparent the phenomenology that results from the presence of the coupling and to compare to the standard case, we shall perturb the field equations resulting from the action (2.11) in the standard way. We decompose the fields $\varphi$ and $\chi$ into a homogeneous and perturbed part

$$\varphi(t, \mathbf{x}) = \varphi(t) + \delta \varphi(t, \mathbf{x}), \quad \chi(t, \mathbf{x}) = \chi(t) + \delta \chi(t, \mathbf{x}).$$

We shall work with the Fourier components $\delta \varphi_k(t)$ and $\delta \chi_k(t)$ so that spatial derivatives can be written $\nabla^2 \delta \varphi = -k^2 \delta \varphi$. Here, as below, we omit the subscript $k$ to simplify the notation. In the longitudinal gauge and in the absence of anisotropic stress, the scalar perturbations of the FRW metric can be expressed as

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Psi)\delta_{ij}dx^i dx^j. \quad (4.1)$$
The perturbed Einstein equations give

\[
3H(\dot{\Psi} + H\Psi) + \frac{k^2}{a^2}\Psi = -\frac{1}{2M_{Pl}^2}\delta\rho, \quad \text{(4.2)}
\]

\[
\dot{\Psi} + H\Psi = -\frac{1}{2M_{Pl}^2}\delta q, \quad \text{(4.3)}
\]

\[
\dot{\Psi} + 4H\dot{\Psi} + (3H^2 + 2\dot{H})\Psi = \frac{1}{2M_{Pl}^2}\delta p, \quad \text{(4.4)}
\]

where \(\delta\rho, \delta q\) and \(\delta p\) are perturbations of the total energy density, momentum potential and pressure respectively, given by

\[
\delta q = -\left[\dot{\varphi}(\delta\varphi) + \gamma A^2\dot{\chi}(\delta\chi)\right], \quad \text{(4.5)}
\]

\[
\delta\rho = \left[\dot{\varphi}(\delta\varphi) - \varphi^2\Psi + A^2\gamma^3\dot{\chi}(\delta\chi) - \dot{\chi}^2\Psi + \left[U_\varphi + 4\beta \rho_{DBI} - \beta A^2\gamma^3\chi^2\right](\delta\varphi) +
\right.
\]

\[
\left.\quad + \left[A^4V_\chi + \frac{1}{2}A^4\frac{f_\chi}{f^2}(2 - 3\gamma + 3\gamma^3)\right](\delta\chi), \quad \text{(4.6)}\right.
\]

\[
\delta p = \left[\dot{\varphi}(\delta\varphi) - \varphi^2\Psi + 2A^2\dot{\chi}(\delta\chi) - \dot{\chi}^2\Psi + \left[-U_\varphi + 4\beta p_{DBI} - \beta A^2\gamma^2\chi^2\right](\delta\varphi) -
\right.
\]

\[
\left.\quad - \left[\right. A^4V_\chi + \frac{1}{2}A^4\frac{f_\chi}{f^2}(2 - \gamma - 1 - \gamma)\left.\right](\delta\chi). \quad \text{(4.7)}\right.
\]

The perturbed field equations are

\[
(\delta\varphi)'' + 3H(\delta\varphi)' + \left[\frac{k^2}{a^2} + U_\varphi - 4\beta^2 T_{DBI} + \beta^2 A^2(3c_s^{-1} - c_s^{-3})\dot{\chi}^2\right](\delta\varphi) -
\]

\[
- \beta A^2(3c_s^{-1} - c_s^{-3})\dot{\chi}(\delta\chi)' + \beta A^4\left[\frac{f_\chi}{f^2}(4 - \frac{3}{2}c_s - 3c_s^{-1} + \frac{1}{2}c_s^{-3}) + 4V_\chi\right](\delta\chi) +
\]

\[
+ \left[2U_\varphi - 2\beta T_{DBI} + \beta A^2(3c_s^{-1} - c_s^{-3})\dot{\chi}^2\right]\Psi - 4\dot{\varphi}\dot{\Psi} = 0 \quad \text{(4.8)}
\]

and

\[
(\delta\chi)'' + \left[3H + 2\beta\dot{\varphi} - 3\frac{\dot{c_s}}{c_s}\right](\delta\chi)' + \left[c_s^2\frac{k^2}{a^2} + A^2c_s^3V_\chi - \frac{f_\chi}{f}c_s\dot{\chi} - \frac{1}{2}\dot{\chi}^2c_s f_\chi V_\chi +
\right.
\]

\[
\left.\quad + \frac{1}{2}A^2(1 - c_s)^2\left(c_s\left(\frac{f_\chi}{f^2}\right) + (1 + c_s)f^{-1}\left(\frac{f_\chi}{f}\right)\right)\right](\delta\chi)' +
\]

\[
+ \beta(3c_s^2 - 1)\dot{\chi}(\delta\varphi)' + \beta\left[A^2V_\chi c_s(1 + c_s^2) + A^2f_\chi c_s(1 - c_s^2) + 2\frac{\dot{c_s}}{c_s}\dot{\chi}\right](\delta\varphi) -
\]

\[
- (3c_s^2 + 1)\dot{\chi}\Psi + \left[A^2V_\chi c_s(1 + c_s^2) + A^2f_\chi c_s(1 - c_s)^2 + 2\frac{\dot{c_s}}{c_s}\dot{\chi}\right]\Psi
\]

\[
= 0 \quad \text{(4.9)}
\]

where we have rewritten factors of \(\gamma\) using the expression for the sound speed for the DBI field \(c_s^2 = \gamma^{-2}\). In terms of the gauge invariant variables,

\[
Q_\varphi \equiv (\delta\varphi) + \frac{\varphi}{H}\Psi, \quad \text{(4.10)}
\]

\[
Q_\chi \equiv (\delta\chi) + \frac{\chi}{H}\Psi, \quad \text{(4.11)}
\]
the equations form a closed system,
\begin{align}
\ddot{\varphi} + 3H \dot{\varphi} + B_\varphi \dot{\varphi} + \left( \frac{k^2}{a^2} + C_{\varphi \varphi} \right) Q_\varphi + C_{\varphi \chi} Q_\chi &= 0, \\
\ddot{\chi} + \left[ 3H + 2\beta \dot{\varphi} - \frac{3\dot{c}_s}{c_s} \right] \dot{\chi} + B_\chi \dot{\varphi} + \left( \frac{k^2}{a^2 c_s^2} + C_{\chi \chi} \right) Q_\chi + C_{\chi \varphi} Q_\varphi &= 0
\end{align}
where we have defined the following coefficients
\begin{align}
B_\chi &= \left[ \beta (3c_s^2 - 1) - \frac{\varphi}{2H} (1 - c_s^2) \right] \chi, \\
B_\varphi &= -A^2 c_s^{-3} B_\varphi, \\
C_{\varphi \varphi} &= \beta \left( \frac{\varphi}{H} \right) A^4 f^{-1} (3c_s + 1)(1 - c_s^{-1})^3 - \beta^2 A^4 f^{-1} (16 - 8c_s^{-1} - 9c_s + c_s^{-3}) + \\
&\quad + 3\varphi^2 - c_s^{-3} (1 + c_s^2) A^2 \varphi^2 \chi^2 - \frac{\varphi^4}{4H^2} + \frac{2\varphi V_T,\varphi}{H} + V_T,\varphi \varphi; \\
C_{\varphi \chi} &= \frac{A^4 \chi}{4H} f_x f_x c_s (1 - c_s^{-1})^2 (c_s^2 + 2c_s - 1) + 3c_s^{-1} A^2 \chi \dot{\varphi} - c_s^{-4} (1 + c_s^2) A^4 \frac{\varphi^3 \chi}{4H^2} - \\
&\quad - \frac{1}{2} \beta A^4 f^{-1} (3c_s + 1)(1 - c_s^{-1})^3 \left[ \frac{f_x}{f} - \frac{A^2 \chi}{H c_s} \right] - c_s^{-1} A^2 \frac{\varphi^3 \chi}{2H^2} + \frac{V_T,\chi \varphi}{H} + \\
&\quad + \frac{c_s^{-1} A^2 \chi}{H} V_T,\varphi + V_T,\chi \varphi; \\
C_{\chi \chi} &= \frac{A^4 \chi}{H} f_x f_x (1 - c_s^2) - \left[ \frac{f_x}{f} - \frac{A^2 \chi}{H c_s} \right] \dot{c}_s \chi - \frac{1}{2} c_s f_x A^4 \chi^2 V_T,\chi + \\
&\quad + \frac{1}{2} A^2 (1 - c_s^2) \left[ c_s \left( \frac{f_x}{f^2} \right)_{,\chi} + (1 + c_s) f^{-1} \left( \frac{f_x}{f} \right)_{,\chi} \right] + \frac{3}{2} A^2 \chi^2 c_s^{-1} (1 + c_s^2) - \\
&\quad - A^4 c_s^{-2} \chi^2 \frac{1}{2H^2} - A^4 c_s^{-1} (1 + c_s^2) \chi^2 \frac{1}{4H^2} + \frac{\chi V_T,\chi}{H} (1 + c_s^2) + c_s^3 A^{-2} V_T,\chi \chi; \\
C_{\chi \varphi} &= \left( 2\beta + \frac{\varphi}{H} \right) \left[ \frac{1}{2} A^4 f_x f_x c_s (1 - c_s^2) + \frac{\dot{c}_s}{c_s} \right] - \frac{2\beta}{H} A^4 \frac{\varphi^3 \chi}{H} f^{-1} (1 - c_s^2) - c_s^{-1} A^2 \varphi^3 \chi^3 + \\
&\quad + \frac{1}{2} (1 + c_s^2) \left[ 3\varphi \chi - \frac{\varphi^3 \chi}{2H^2} + 2\beta c_s A^{-2} V_T,\chi + c_s \varphi A^2 \frac{V_T,\varphi}{H} + \frac{V_T,\varphi}{H} \right].
\end{align}
In these expressions, $V_T(\varphi, \chi) = U(\varphi) + A^4(\varphi) V(\chi)$ is the total potential. Comparing the form of these equations to [24], where the authors treated the case of two coupled canonical fields, there are marked similarities. The principle differences are due to factors of $c_s$ that arise due to the non-canonical kinetic term of the DBI field. Let us define the auxiliary fields $\nu_\varphi$ and $\nu_\chi$, by
\begin{align}
\nu_\varphi &= r_\varphi Q_\varphi, \\
\nu_\chi &= r_\chi Q_\chi
\end{align}
with $r_\chi = a A c_s^{-3/2}$ and $r_\varphi = a$, which correspond to the perturbation variables used in the analysis of the perturbations in single-field canonical and DBI inflation respectively. The
Figure 8: Perturbation quantities for the exponential case with \( \gamma \gg 1 \) shown in fig. 7. The upper panels show the evolution of the amplitude of the variables \( Q_\varphi \) (solid) and \( Q_\chi \) (dashed) for the first run (top-left) with \( Q^{\text{ini}}_\varphi = 0 \) and the second run (top-right) with \( Q^{\text{ini}}_\varphi = 0 \) for the pivot scale with \( \beta = 1 \). (Perturbations are normalised so that \( |Q^{\text{ini}}_\varphi| = 0.1 \) for the first run.) The centre plots show the evolution of the comoving curvature perturbation \( R \) (centre-left) and the non-adiabatic contribution to its time derivative \( H/(2H)\delta p_{\text{nad}} \) for the first (blue, dashed) and second (red, solid) runs in this case. Vertical dotted lines indicate the sound and curvature horizons for this mode. The lower panels show the variation in the resulting value of the spectral index \( n_s \) (bottom-left) and power spectrum amplitude \( P_{\text{amp}} \) (bottom-right) with the coupling \( \beta \). The shaded regions shows observational values of these quantities (WMAP+ BAO+H0) at 68% c.l [54].


\begin{align}
\nu''_\varphi - B \nu'_\varphi + \left[ k^2 + a^2 C_{\varphi\varphi} - \frac{r''_\varphi}{r'_\varphi} \right] \nu_\varphi + \left[ \left( \frac{r'_\varphi}{r_\chi} \right) a^2 C_{\varphi\chi} + B \frac{r'_\varphi}{r'_\psi} \right] \nu_\psi &= 0, \quad (4.21) \\
\nu''_\chi + B \nu'_\varphi + \left[ k^2 c_s^2 + a^2 C_{\chi\chi} - \frac{r''_\chi}{r'_\chi} \right] \nu_\chi + \left[ \left( \frac{r'_\chi}{r'_\psi} \right) a^2 C_{\chi\varphi} - B \frac{r'_\varphi}{r'_\psi} \right] \nu_\psi &= 0, \quad (4.22)
\end{align}

with \( B = r_\chi B_\chi \). In this form the symmetry between the equations is more clear: each field perturbation exchanges energy with the other via the friction term \( B \) as well as the mass terms. The obvious complexity of these equations makes an analytical treatment difficult, so in the following section we shall integrate the perturbation equations numerically.

### 4.2 The evolution of the perturbations

In order to integrate the perturbation equations, it is important to specify the initial conditions. In the case of single-field k-inflation, the approach introduced by Garriga and Mukhanov [11] is to construct the auxiliary variable \( \nu = z R \), with \( z = a (\rho + p)^{1/2}/(c_s H) \). One then quantises \( \nu \), substituting \( \nu \propto \nu_k e^{i k \cdot x} \) to obtain,

\[
\frac{d^2 v_k}{d\tau^2} + \left( c_s^2 k^2 - \frac{1}{z} \frac{d^2 z}{d\tau^2} \right) v_k = 0. \quad (4.23)
\]
Assuming the quantum fluctuations are in the trivial vacuum state, the solutions for $k^2c_s^2 \gg a^2H^2$ are oscillatory as in the standard case, but with a frequency reduced by a factor of $c_s$ that also affects the normalisation:

$$\nu_k = \frac{1}{\sqrt{2kc_s}}e^{-ikc_s\tau}. \quad (4.24)$$

The above procedure can be applied in the single field case, but when there are two fields present, one has to be to careful when choosing initial conditions for the numerical run. As in the standard two-field case we can neglect correlations between the perturbations for modes well within the horizon, so the short wavelength solutions for the auxiliary variables $\nu_\phi$ and $\nu_\chi$ are

$$\nu_\phi = \frac{1}{\sqrt{2k}}e^{-ik\tau}, \quad \nu_\chi = \frac{1}{\sqrt{2kc_s}}e^{-ikc_s\tau}, \quad (4.25)$$

so the relative normalisation of the two perturbations is dependent on the sound speed of the DBI field. However, in order to avoid introducing correlations between the fields at early times when integrating numerically (in which case the Bunch-Davies vacua would be inappropriate) the perturbations must be integrated twice, and the results combined to form the curvature power spectrum (see Appendix for a more in-depth description of the numerical method). In the first run, shown in the upper-left panel of fig. 8, the scalar field perturbations are set in the Bunch-Davies vacuum with $Q^{\text{ini}}_\chi = 0$ and in the second run (upper-right panel of fig. 8) $Q^{\text{ini}}_\phi = 0$ and the perturbations of the DBI field are set in the trivial vacuum.

The comoving curvature perturbation is given by,

$$\mathcal{R} = \Psi - \frac{H}{\rho+p} \delta q \quad (4.26)$$

where $\delta q$ is the momentum potential given in eq. (4.5) and $p$ and $\rho$ are the total background pressure and energy density respectively. In terms of the gauge invariant variables $Q_\phi$ and $Q_\chi$ this yields the simple expression,

$$\mathcal{R} = \frac{H}{(-2H)} \left[ \dot{\phi}Q_\phi + A^2\gamma\chi Q_\chi \right]. \quad (4.27)$$

where $(-2H) = \dot{\varphi}^2 + A^2\gamma\chi^2$.

When the fields are minimally coupled and both fields are evolving, as in the standard DBI case, the sound speed $c_s$ affects the evolution of the perturbations so that $Q_\chi$ freezes in at the sound horizon ($kc_s = aH$) while $Q_\phi$ freezes in at the curvature horizon $k=aH$. However, in the non-minimally coupled case, discussed in Sec. 3.3, the homogeneous field $\varphi$ is in the minimum of its effective potential and its perturbations are suppressed, so the primary contribution to the comoving curvature perturbation $\mathcal{R}$ (middle-left in fig. 8) is the second run, in which the $Q_\chi$ is dominant. Thus in the coupled case, the curvature perturbation freezes in at the sound horizon of the DBI field, as in a single-field DBI system. The lower panels in fig. 8 show the variation in the resulting value of the spectral index.
Figure 9: Background dynamics with small \( \gamma \) (exponential potential). In this case, as \( \beta \) increases from 0.5 to 4, the large coupling decreases the warp factor so \( \chi \) slow-rolls before reaching smaller values so \( \gamma \approx 1 \) for a considerable period. Quantities plotted are the same as fig. 7. Parameter values: \( \lambda = 7.8 \times 10^{11}, m = 2.8 \times 10^{-6}, U_0 = 7 \times 10^{-9}, \mu = 0.07, \eta = 2.8 \).

\( n_s \) (bottom-left) and power spectrum amplitude \( P_{\text{amp}} \) (bottom-right) with the coupling \( \beta \). In single field DBI, the power spectrum amplitude increases with the number of e-folds of inflation. Here, the increase in \( \beta \) is accompanied by an increase in \( N_{\text{max}} \), leading to a monotonic increase of \( P_{\text{amp}} \) with \( \beta \). The spectral index is much affected by the small sound speed of the DBI field in this example, which counteracts the red-tilt due to the deviation from a pure de Sitter stage, so that \( n_s \) exhibits a slightly blue tilt.

If the coupling is large, or the mass of the DBI field small, the value of \( \gamma \) can be close to 1 when the modes that affect the CMB freeze in. An example of this is shown in fig. 9, where increasing the value of \( \beta \) not only decreases \( \gamma \) and increases \( N_{\text{max}} \) but gives rise to a period of slow-roll at the start of the run. This can be seen by comparing the behaviour of the slow-roll parameter \( \epsilon \) and the boost factor \( \gamma \) at the start and end of the run. At first \( \gamma \sim 1 \); afterwards increasing to values \( \lesssim 20 \) when the effective DBI warp factor \( A^2 f \) becomes significant. In this case, the sound and curvature horizons are almost identical and the field perturbations behave as in the case of canonical fields, freezing in at \( k = aH \) (see fig. 10). As in the previous case, the perturbations of the \( \varphi \) field are subdominant, and only give small corrections to \( \mathcal{R} \). It can be seen in fig. 10, that, due to this effect, the spectral index displays a red-tilt putting it within current observational limits.

The rate of change of the curvature perturbation can be written,

\[
\dot{\mathcal{R}} = \frac{H}{-2H} \left[ (\delta p_{\text{nad}}) + \frac{\dot{\rho}}{\rho} (\delta \rho_m) \right]
\]  

(4.28)
Figure 10: The evolution of the perturbation quantities for the small $\gamma$ case plotted in fig. 9 for the pivot scale with $\beta = 3.5$. Quantities plotted are the same as in fig. 8. Note that, as $\gamma \approx 1$, the sound and curvature horizons (dotted, vertical lines) are indistinguishable and modes freeze-on as in slow-roll inflation, giving a red-tilted spectrum.

where non-adiabatic pressure is defined by,

$$ (\delta p_{\text{nad}}) \equiv (\delta p) - \frac{\dot{\rho}}{\rho} (\delta \rho). $$

The quantity $\delta \rho_m \equiv \delta \rho - 3H\delta q$ is the gauge invariant comoving density perturbation. This appears on the RHS of the Poisson equation,

$$ \frac{k^2}{a^2} \Psi = -\frac{1}{2} \delta \rho_m, $$

and thus decays in the long-wavelength/late-time limit. It is given explicitly by,

$$ \delta \rho_m = \left[ \dot{\phi} \dot{Q}_\phi + A^2 \gamma^3 \dot{Q}_X \right] + \frac{2\dot{H}}{H} \left[ \frac{1}{2H} (\dot{\phi}^2 + A^2 \gamma^3 \dot{\chi}^2) - 3H \right] \mathcal{R} $$

$$ + \left[ V_{T,\phi} - \beta A^4 f^{-1}(4 - 5\gamma + \gamma^3) \right] Q_\phi + \left[ V_{T,\chi} + \frac{1}{2} \beta A^4 f \chi^2 (2 - 3\gamma + \gamma^3) \right] Q_X. $$

In the standard two-field case, the non-adiabatic pressure perturbation consists of a contribution proportional to $\delta \rho_m$ and another proportional to the entropy field perturbation $\delta s$. The former is negligible at late times, but the latter can act as a source for the curvature perturbation even after the fluctuations grow larger than the curvature horizon. In order to obtain an explicit form for $\delta p_{\text{nad}}$, we can rewrite (4.31) as

$$ \delta \rho_m = -\frac{2\dot{H}}{H} \dot{\mathcal{R}} + A^2 (\gamma^3 - \gamma) \dot{\chi} \dot{Q}_X + \left[ A^2 \chi^2 \frac{\dot{H}}{H} (\gamma^3 - \gamma) - \frac{2\dot{H}}{H} \right] \mathcal{R} $$

$$ + \left[ \beta A^2 f^{-1}(4\gamma - 3\gamma^{-1} - \gamma^3 - 2\dot{\phi}) \right] Q_\phi + \left[ \frac{1}{2} f \gamma^2 \chi^4 - 2(A^2 \gamma \dot{\chi}) \right] Q_X. $$
and combine with (4.28) to get,

$$
\delta p_{\text{nad}} = \left[1 - \frac{\ddot{\rho}}{\rho}\right] \delta \rho_m + \frac{2}{\sqrt{-2H}} \left[\sqrt{-2H} P_s + \beta A^3 \gamma^{3/2} \dot{\chi}^3\right] (\delta s) + \left[\frac{\dot{\chi}}{\sqrt{-2H}} Q_s - Q_\gamma\right] A^2 \dot{\chi}^2.
$$

(4.33)

Here, we have introduced adiabatic and entropic fields, defined as

$$
Q_\sigma = \frac{1}{\sqrt{-2H}} [\dot{\phi} Q_\phi + A^2 \gamma \dot{\chi} Q_\chi]
$$

(4.34)

$$
\delta s = A^{\gamma^{1/2}} \frac{\dot{\chi}}{\sqrt{-2H}} [\dot{\phi} Q_\phi - \dot{\chi} Q_\phi]
$$

(4.35)

in a similar manner to the coupled two-field system [21], so that the adiabatic field is related to the comoving curvature perturbation by

$$
\mathcal{R} = \frac{H}{\sqrt{-2H}} Q_\sigma,
$$

(4.36)

and the partial derivative of the total pressure with respect to the entropy field is,

$$
P_s = \frac{1}{\sqrt{-2H}} \left[\frac{\dot{\phi}}{A^{\gamma^{1/2}}} \frac{\partial p}{\partial \chi} - A^{\gamma^{1/2}} \dot{\chi} \frac{\partial p}{\partial \phi}\right] = \frac{1}{\sqrt{-2H}} \left[V_{T,\phi} A^{\gamma^{1/2}} + \frac{\dot{\phi}}{A^{\gamma^{1/2}}} V_{T,\chi} + \frac{1}{2} A^3 \frac{f}{f^2} \gamma^{1/2} (1 - \gamma^{-1})^2 \dot{\phi}
\right.
\left. - \beta A^3 f^{-1} \gamma^{1/2} (4 - 3 \gamma^{-1} - \gamma) \dot{\chi}\right].
$$

(4.37)

An interesting feature of eqn. (4.33) is that, unlike the canonical two-field case and the standard coupled case, this expression cannot be decomposed into purely entropic and adiabatic perturbation terms. The extra terms are expressed in the gauge-invariant combination $Q_\gamma$,

$$
Q_\gamma = \delta \gamma + \frac{\dot{\gamma}}{H} \Psi
= \gamma^3 A^{-2} f \dot{\chi} Q_\chi + (\gamma^3 - \gamma) \left(\frac{1}{2} \frac{f}{f} Q_\chi - \beta Q_\phi - \frac{\sqrt{-2H}}{2H} Q_\gamma\right).
$$

(4.38)

The fact that this term appears like this suggests that the field decomposition is not appropriate in this case. We can also see this if we consider the form of the equation of motion for $Q_\sigma$:

$$
\ddot{Q}_\sigma = \frac{1}{\sqrt{-2H}} \left[\dot{\phi} Q_\phi + A^2 \gamma \dot{\chi} Q_\chi\right] + ...
= - \frac{k^2}{a^2} \frac{1}{\sqrt{-2H}} \left[\dot{\phi} Q_\phi + A^2 \gamma^{-1} \dot{\chi} Q_\chi\right] + ...
$$

(4.39)

where the ... represent terms in $\dot{Q}_\phi$ and $\dot{Q}_\chi$ as well as terms proportional to $Q_\phi$ and $Q_\chi$ not proportional to $k^2/a^2$. From this, we can see that the $\ddot{Q}_\sigma$ equation cannot be recast in
the form, \( \ddot{x} + c_{\text{ad}}\frac{k^2}{a^2}x + \ldots = 0 \), as a scalar perturbation can. The same is true of the entropy field. Even if the definition of the adiabatic field were changed to include a non-canonical kinetic term itself, i.e.

\[
\dot{\sigma} \sqrt{P_Z} Q_{\sigma} = \dot{\phi} \sqrt{P_X} Q_{\phi} + \dot{\chi} \sqrt{P_Y} Q_{\chi},
\]

(4.40)

where \( P_Z \dot{\sigma}^2 \equiv -2\dot{H} \), no extra terms proportion to \( k^2/a^2 \) would arise, so the original conclusion would stand. The description of non-adiabatic pressure perturbations in such a system as this could perhaps be better treated by the formalism introduced in [55], considering separate potential and kinetic fluids. However, as in this paper we focus on the coupled case in which the perturbations of \( \phi \) are suppressed, non-adiabatic perturbations are negligible. We will thus discuss this issue in more detail in future work focusing on the non-minimally coupled case discussed briefly in Sec. 3.3.

In the non-minimally coupled case under consideration, we have found that the coupled DBI model is effectively a single-field system, and so the contribution of non-adiabatic fields is negligible. This can be seen in the middle-right panels of figs. 8 and 10, in which the non-adiabatic pressure contribution to \( \dot{R} - \frac{H^2}{2\dot{H}} (\delta p_{\text{had}}) \), is shown to decrease exponentially as inflation proceeds. Thus, the curvature perturbation is constant to a high degree of accuracy on large scales.

5. Discussion

In this paper we have studied the consequences of coupling the DBI action to a canonical scalar field in a scalar-tensor theory. The motivation for this work is purely phenomenological: the DBI inflationary scenario has its motivation in string theory, but it is an interesting example of k-inflation in its own right. The additional scalar field might have its origin as a moduli field in a higher-dimensional setup, but we leave the explicit construction for future work (although, see [34] on how DBI inflation could be related to scalar tensor theories).

We have investigated the case in which the canonical scalar field is heavy during inflation. We found this to be the case even for couplings of order a tenth of the strength of gravity (\( \beta \approx 0.1 \)). Thus, we believe this to be a realistic scenario. With couplings larger than this, the additional scalar field is forced into the minimum of its effective potential. Thus, the parameters of the DBI field, which are dependent on the additional scalar field, vary during inflation. As a consequence, we find that the number of e-folds is extended and the boost factor is decreased (when compared to standard DBI inflation with the same bare parameters). The energy density during inflation comes mostly from the additional scalar field that sits in the minimum of the effective potential, which varies slowly as the DBI field evolves. But because the canonical field is heavy during inflation, its perturbations are negligible and only the DBI perturbations contribute to the curvature perturbation. Since the boost factor is smaller than in the standard case, the level of non-Gaussianity (which is proportional to \( \gamma^2 \)) is smaller than the standard DBI case. In fact, we found that the presence of the canonical scalar field alleviates the problems of the DBI inflationary scenario.

We have considered two potentials for the canonical scalar field: a quadratic potential (with a minimum \( \varphi_{\text{min}} > 0 \)) and an exponential potential. The difference between the
phenomenology of these two models is slight, mainly due to the presence of a minimum in the bare potential in the offset quadratic case, which affects the background dynamics towards the end of inflation, when the DBI contribution to the effective potential is less important. In the case of the exponential potential, the energy densities of the scalar fields (dominated by their potential energies) are proportional to each other (as in a tracker solution). In both cases we find that the behaviour of the perturbations is extremely similar, affected only indirectly by presence of the $\varphi$ field in its minimum and the details of the DBI dynamics; thus, we venture to suggest that our conclusions may hold for any choice of potential steep enough to allow $\varphi$ to closely track the minimum of its effective potential.

Finally, we have investigated two possible couplings. In the minimally coupled case we find find that if one fine-tunes the mass scales so that both fields contribute roughly equally to the total energy density for a considerable number of e-foils, the dynamics of the DBI field can be significantly affected by the additional contribution to the Hubble damping, leading to smaller values of $\gamma$ (relative to the DBI dominant case). As both fields are evolving, the interplay between the field perturbations suggests that isocurvature fluctuations could be important. However, the minimally coupled case is extremely sensitive to small changes in the field masses and the initial conditions of both fields. In the non-minimally coupled case, we find that the coupled DBI model is effectively a single-field system, and so the contribution of non-adiabatic fields is negligible. Moreover, the curvature perturbation is constant to a high degree of accuracy on large scales.

We have not discussed in detail the range of allowed parameter for the model but will study this question in future work. A range of parameters affect the prediction for the spectral index (for both the exponential and the quadratic potential) and we are currently scanning the parameter space numerically. Much has to be learned in this model, such as the decoupling limit $\beta \to 0$. In this case, the canonical scalar field will become light and isocurvature modes become important. We expect that the signatures of non-gaussianity will be very different.

Acknowledgments

We are grateful to Philippe Brax for useful discussions and comments on an earlier draft of this paper. CvdB is partly supported by STFC. DFM thanks the Research Council of Norway for the FRINAT grant 197251/V30. DFM is also partially supported by project CERN/FP/109381/2009 and PTDC/FIS/102742/2008. JW is supported by EPSRC and a Yggdrasil Grant from the research council of Norway, and would like to thank the members of the Institute of Theoretical Astrophysics at the University of Norway for their kind hospitality while parts of this work were completed.

A. Numerical method

In this short appendix, we devote a few words to an explanation of the general numerical method used to integrate the perturbations. The pivot scale for the CMB ($k_{\text{pivot}} = \ldots$
0.002Mpc\(^{-1}\) at which the power spectrum amplitude is calculated is identified with the mode that crosses the curvature horizon \((k = aH)\) 55 e-folds before the end of inflation (cf. [56]), when the slow-roll parameter \(\epsilon\) ceases to be smaller than 1. The spectral index is calculated by integrating the perturbation equations for \(k\)-values in the range \(k \in [k_{\text{pivot}} \times 10^{-2}, k_{\text{pivot}} \times 10^2]\), starting when \(kc_s = 50aH\) (so the wavelength of the perturbation is much smaller than the sound horizon of the DBI field) and ending 45 e-folds from the end of inflation, at which time the modes are beyond the horizon and have frozen in. We do not integrate the equations (4.12) and (4.13) with respect to conformal time or \(N = \log a\), as is commonly done, but instead use logarithmic time \(x = \log t\), which is efficient in dealing with both the background and perturbation variables. In this case the oscillatory initial conditions (4.25) for \(Q_\phi\) and \(Q_\chi\) are

\[
Q_\phi = \frac{1}{L^{3/2}} \frac{1}{aV} \frac{1}{\sqrt{2k}} e^{-\frac{ik}{2a}t}, \quad Q_\chi = \frac{1}{L^{3/2}} \frac{c_s}{aA} \frac{1}{\sqrt{2k}} e^{-\frac{ik}{2a}c_st},
\]

where \(L\) is a normalisation factor chosen so that \(Q_\phi^{\text{ini}} = 0\). Representing \(d/dx\) by \(\dot{}\) and writing \(\mathcal{H} = a'/a\) the derivatives are

\[
Q_\phi' = -\left(\mathcal{H} + \frac{k}{a} \right) Q_\phi, \quad Q_\chi' = -\left(\mathcal{H} + \beta \varphi' - \frac{c_s'}{c_s} + \frac{k}{a} \right) Q_\chi,
\]

where as in the previous expressions, background quantities are evaluated at \(t_{\text{ini}} = \exp(x_{\text{ini}})\).

As in the standard two-field case (cf. [57]) in order to make sure that the modes are uncorrelated deep within the horizon, we perform two numerical runs. In the first run \(Q_\phi\) is set in the Bunch-Davies vacuum (A.1) with \(Q_\chi^{\text{ini}} = 0\) and in the second run \(Q_\phi^{\text{ini}} = 0\) and the perturbations of the DBI field are set in the (quasi) Bunch-Davies vacuum. Evaluating the curvature perturbation (4.27) for each run yields the solutions \(\mathcal{R}_1\) and \(\mathcal{R}_2\), which can be combined to form the curvature power spectrum

\[
\mathcal{P}_\mathcal{R} = \frac{k^3}{2\pi^2} \left(|\mathcal{R}_1|^2 + |\mathcal{R}_2|^2\right).
\]

The power spectrum amplitude (for the pivot mode \(k_{\text{pivot}}\)) and the spectral index can then be calculated at the end of the numerical run.

References

[1] L. McAllister and E. Silverstein, *String Cosmology: A Review*, *Gen. Rel. Grav.* 40 (2008) 565–605, [arXiv:0710.2951](https://arxiv.org/abs/0710.2951).

[2] D. Baumann and L. McAllister, *Advances in Inflation in String Theory*, *Ann. Rev. Nucl. Part. Sci.* 59 (2009) 67–94, [arXiv:0901.0265](https://arxiv.org/abs/0901.0265).

[3] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, *False vacuum inflation with Einstein gravity*, *Phys.Rev.* D49 (1994) 6410–6433, [astro-ph/9401011](https://arxiv.org/abs/astro-ph/9401011).

[4] E. D. Stewart, *Inflation, supergravity and superstrings*, *Phys.Rev.* D51 (1995) 6847–6853, [hep-ph/9405385](https://arxiv.org/abs/hep-ph/9405385).
[5] J. McDonald, *F-term hybrid inflation, the eta-problem and extra dimensions*, JHEP **12** (2002) 029, [hep-ph/0201016](http://arxiv.org/abs/hep-ph/0201016).

[6] D. A. Easson and R. Gregory, *Circumventing the eta problem in building an inflationary model in string theory*, Phys. Rev. **D80** (2009) 083518, [arXiv:0902.1798](http://arxiv.org/abs/0902.1798).

[7] S. Antusch, M. Bastero-Gil, K. Dutta, S. F. King, and P. M. Kostka, *Solving the η-Problem in Hybrid Inflation with Heisenberg Symmetry and Stabilized Modulus*, JCAP **0901** (2009) 040, [arXiv:0808.2423](http://arxiv.org/abs/0808.2423).

[8] E. Silverstein and D. Tong, *Scalar Speed Limits and Cosmology: Acceleration from Deceleration*, Phys. Rev. **D70** (2004) 103505, [hep-th/0310221](http://arxiv.org/abs/hep-th/0310221).

[9] M. Alishahiha, E. Silverstein, and D. Tong, *DBI in the sky*, Phys. Rev. **D70** (2004) 123505, [hep-th/0404084](http://arxiv.org/abs/hep-th/0404084).

[10] C. Armendariz-Picon, T. Damour, and V. F. Mukhanov, *k-Inflation*, Phys. Lett. **B458** (1999) 209–218, [hep-th/9904075](http://arxiv.org/abs/hep-th/9904075).

[11] J. Garriga and V. F. Mukhanov, *Perturbations in k-inflation*, Phys. Lett. **B458** (1999) 219–225, [hep-th/9904176](http://arxiv.org/abs/hep-th/9904176).

[12] F. Helmer and S. Winitzki, *Self-reproduction in k-inflation*, Phys. Rev. **D74** (2006) 063528, [gr-qc/0608019](http://arxiv.org/abs/gr-qc/0608019).

[13] G. Panotopoulos, *Detectable primordial non-gaussianities and gravitational waves in k-inflation*, Phys. Rev. **D76** (2007) 127302, [arXiv:0712.1713](http://arxiv.org/abs/0712.1713).

[14] V. Taveras and N. Yunes, *The Barbero-Immirzi Parameter as a Scalar Field: K-Inflation from Loop Quantum Gravity?*, Phys. Rev. **D78** (2008) 064070, [arXiv:0807.2652](http://arxiv.org/abs/0807.2652).

[15] N. Bose and A. Majumdar, *A k-essence Model Of Inflation, Dark Matter and Dark Energy*, Phys. Rev. **D79** (2009) 103517, [arXiv:0907.2330](http://arxiv.org/abs/0907.2330).

[16] Y.-b. Yue and B. Chen, *K-Inflation with a Dark Energy Coupling*, Phys. Rev. **D81** (2010) 023506, [arXiv:0906.0145](http://arxiv.org/abs/0906.0145).

[17] N. Bose and A. Majumdar, *Unified Model of k-Inflation, Dark Matter and Dark Energy*, Phys. Rev. **D80** (2009) 103508, [arXiv:0907.2330](http://arxiv.org/abs/0907.2330).

[18] C. Ringeval, *Dirac-Born-Infeld and k-inflation: the CMB anisotropies from string theory*, J. Phys. Conf. Ser. **203** (2010) 012056, [arXiv:0910.2167](http://arxiv.org/abs/0910.2167). * Brief entry *.

[19] N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, *Non-Gaussianity from inflation: Theory and observations*, Phys. Rept. **402** (2004) 103–266, [astro-ph/0406398](http://arxiv.org/abs/astro-ph/0406398).

[20] M. Liguori, E. Sefusatti, J. Fergusson, and E. Shellard, *Primordial non-Gaussianity and Bispectrum Measurements in the Cosmic Microwave Background and Large-Scale Structure*, Adv. Astron. **2010** (2010) 980523, [arXiv:1001.4707](http://arxiv.org/abs/1001.4707). * Temporary entry *.

[21] F. Di Marco, F. Finelli, and R. Brandenberger, *Adiabatic and Isocurvature Perturbations for Multifield Generalized Einstein Models*, Phys. Rev. **D67** (2003) 063512, [astro-ph/0211276](http://arxiv.org/abs/astro-ph/0211276).

[22] Z. Lalak, D. Langlois, S. Pokorski, and K. Turzynski, *Curvature and isocurvature perturbations in two-field inflation*, JCAP **0707** (2007) 014, [arXiv:0704.0212](http://arxiv.org/abs/0704.0212).

[23] S. Renaux-Petel, *Primordial bispectrum and trispectrum in multifield Dirac-Born-Infeld inflation*, [arXiv:1004.3693](http://arxiv.org/abs/1004.3693).
[24] S. Mizuno, F. Arroja, and K. Koyama, *On the full trispectrum in multi-field DBI inflation*, Phys. Rev. **D80** (2009) 083517, [arXiv:0907.2439](https://arxiv.org/abs/0907.2439).

[25] D. A. Easson, R. Gregory, D. F. Mota, G. Tasinato, and I. Zavala, *Spinflation*, JCAP **0802** (2008) 010, [arXiv:0709.2666](https://arxiv.org/abs/0709.2666).

[26] D. Langlois, S. Renaux-Petel, D. A. Steer, and T. Tanaka, *Primordial perturbations and non-Gaussianities in DBI and general multi-field inflation*, Phys. Rev. **D78** (2008) 063523, [arXiv:0806.0336](https://arxiv.org/abs/0806.0336).

[27] C. P. Burgess, R. Easther, A. Mazumdar, D. F. Mota, and T. Multamaki, *Multiple inflation, cosmic string networks and the string landscape*, JHEP **05** (2005) 067, [hep-th/0501128](https://arxiv.org/abs/hep-th/0501128).

[28] S. Renaux-Petel, *Combined local and equilateral non-Gaussianities from multi-field DBI inflation*, JCAP **0910** (2009) 012, [arXiv:0907.2476](https://arxiv.org/abs/0907.2476).

[29] D. Langlois, S. Renaux-Petel, D. A. Steer, and T. Tanaka, *Primordial fluctuations and non-Gaussianities in multi-field DBI inflation*, Phys. Rev. Lett. **101** (2008) 061301, [arXiv:0804.3139](https://arxiv.org/abs/0804.3139).

[30] D. Langlois, S. Renaux-Petel, and D. A. Steer, *Multi-field DBI inflation: introducing bulk forms and revisiting the gravitational wave constraints*, JCAP **0904** (2009) 021, [arXiv:0902.2941](https://arxiv.org/abs/0902.2941).

[31] Y.-F. Cai and H.-Y. Xia, *Inflation with multiple sound speeds: a model of multiple DBI type actions and non-Gaussianities*, Phys. Lett. B**677** (2009) 226–234, [arXiv:0904.0062](https://arxiv.org/abs/0904.0062).

[32] S. Kachru *et. al.*, *Towards inflation in string theory*, JCAP **0310** (2003) 013, [hep-th/0308055](https://arxiv.org/abs/hep-th/0308055).

[33] D. A. Easson, S. Mukohyama, and B. A. Powell, *Observational Signatures of Gravitational Couplings in DBI Inflation*, Phys. Rev. D**81** (2010) 023512, [arXiv:0910.1355](https://arxiv.org/abs/0910.1355).

[34] P. Brax and E. Cluzel, *Brane Bremsstrahlung in DBI Inflation*, JCAP **1003** (2010) 016, [arXiv:0912.0806](https://arxiv.org/abs/0912.0806).

[35] P. Brax and E. Cluzel, *Trapped Brane Features in DBI Inflation*, [arXiv:1010.4462](https://arxiv.org/abs/1010.4462). Temporary entry *.

[36] Y.-F. Cai, J. B. Dent, and D. A. Easson, *Warm DBI Inflation*, [arXiv:1011.4074](https://arxiv.org/abs/1011.4074). Temporary entry *.

[37] S. Kecskemeti, J. Maiden, G. Shiu, and B. Underwood, *DBI inflation in the tip region of a warped throat*, JHEP **09** (2006) 076, [hep-th/0605189](https://arxiv.org/abs/hep-th/0605189).

[38] W. H. Kinney, *Inflation: Flow, fixed points and observables to arbitrary order in slow roll*, Phys. Rev. D**66** (2002) 083508, [astro-ph/0206032](https://arxiv.org/abs/astro-ph/0206032).

[39] K. Tzirakis and W. H. Kinney, *Non-canonical generalizations of slow-roll inflation models*, JCAP **0901** (2009) 028, [arXiv:0810.0273](https://arxiv.org/abs/0810.0273).

[40] M. Spalinski, *On Power Law Inflation in DBI Models*, JCAP **0705** (2007) 017, [hep-th/0702195](https://arxiv.org/abs/hep-th/0702195).

[41] L. P. Chimento and R. Lazkoz, *Bridging geometries and potentials in DBI cosmologies*, Gen. Rel. Grav. **40** (2008) 2543–2555, [arXiv:0711.0712](https://arxiv.org/abs/0711.0712).

[42] D. Bessada, W. H. Kinney, and K. Tzirakis, *Inflationary potentials in DBI models*, JCAP **0909** (2009) 031, [arXiv:0907.1311](https://arxiv.org/abs/0907.1311).
[43] N. Barnaby, C. P. Burgess, and J. M. Cline, Warped reheating in brane-antibrane inflation, JCAP 0504 (2005) 007, [hep-th/0412040].

[44] J. Lachapelle and R. H. Brandenberger, Preheating with Non-Standard Kinetic Term, arXiv:0808.0936 (2008) [arXiv:0808.0936].

[45] N. Bouatta, A.-C. Davis, R. H. Ribeiro, and D. Seery, Preheating in Dirac-Born-Infeld inflation, JCAP 1009 (2010) 011, [1005.2425].

[46] X. Chen, M.-x. Huang, S. Kachru, and G. Shiu, Observational signatures and non-Gaussianities of general single field inflation, JCAP 0701 (2007) 002, [hep-th/0605045].

[47] R. Bean, S. E. Shandera, S. H. Henry Tye, and J. Xu, Comparing Brane Inflation to WMAP, JCAP 0705 (2007) 004, [hep-th/0702107].

[48] H. V. Peiris, D. Baumann, B. Friedman, and A. Cooray, Phenomenology of D-Brane Inflation with General Speed of Sound, Phys. Rev. D76 (2007) 103517, [arXiv:0706.1240].

[49] D. Baumann and L. McAllister, A Microscopic Limit on Gravitational Waves from D-brane Inflation, Phys. Rev. D75 (2007) 123508, [hep-th/0610285].

[50] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, Adiabatic and entropy perturbations from inflation, Phys. Rev. D63 (2000) 023506, [astro-ph/0009131].

[51] S. Groot Nibbelink and B. van Tent, Scalar perturbations during multiple field slow-roll inflation, Class.Quant.Grav. 19 (2002) 613–640, [hep-ph/0107272].

[52] A. D. Linde, Hybrid inflation, Phys. Rev. D49 (1994) 748–754, [astro-ph/9307002].

[53] S. Koh and M. Minamitsuji, Non-minimally coupled hybrid inflation, arXiv:1011.4655.

[54] E. Komatsu et. al., Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation, [arXiv:1001.4538].

[55] K. A. Malik and D. Wands, Adiabatic and entropy perturbations with interacting fluids and fields, JCAP 0502 (2005) 007, [astro-ph/0411703].

[56] A. R. Liddle and S. M. Leach, How long before the end of inflation were observable perturbations produced?, Phys.Rev. D68 (2003) 103503, [astro-ph/0305263].

[57] S. Tsujikawa, D. Parkinson, and B. A. Bassett, Correlation - consistency cartography of the double inflation landscape, Phys.Rev. D67 (2003) 083516, [astro-ph/0210322].