Magnetic field dependence of superfluid density in cuprate superconductors

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Abstract. Within the kinetic energy driven superconducting mechanism, the doping and magnetic field dependence of the superfluid density in cuprate superconductors is studied. The electromagnetic response kernel is evaluated by considering both couplings of the electron charge and electron magnetic momentum with an external magnetic field and employed to calculate the superfluid density based on the specular reflection model for a purely transverse vector potential, then the main features of the doping and magnetic field dependence of the superfluid density in cuprate superconductors are well reproduced.

The superfluid density $\rho_s$ is proportional to the squared amplitude of the macroscopic wave function, and therefore describes the superconducting (SC) charge carriers. It is thus a fundamental parameter whose variation as a function of doping and magnetic field provides important information about the SC state [1]. Experimentally, by virtue of systematic studies using the muon-spin-rotation measurement technique, some essential features of the evolution of the superfluid density in cuprate superconductors with doping and magnetic field have been established now for all the temperature $T \leq T_c$ throughout the SC dome: (1) the superfluid density is a linear temperature dependence at higher temperature except for the low temperatures where a strong deviation from the linear characteristics (a nonlinear effect) appears [2]; (2) the superfluid density exhibits a peak around the critical doping $\delta \approx 0.19$, and then decreases at both lower doped and higher doped regimes [3, 2]. This in turn gives rise to the domelike shape of the doping dependence of the SC transition temperature; (3) the magnetic field dependence of the superfluid density is observed [4], where a weak magnetic field can induce a reduction of the low-temperature superfluid density.

Theoretically, it has been shown [5] that the simple d-wave pairing state gives the linear temperature dependence of the superfluid density at higher temperature in the local limit, while the nonlinear effect in the low temperatures is closely related to the divergence of the coherence length at the d-wave gap nodes on the Fermi surface. Furthermore, it has been argued that the weak magnetic field induced reduction of the low-temperature superfluid density also arises from this nonlinear response [6, 7]. In this paper based on the kinetic energy driven SC mechanism [8], we [9, 10] study the doping and magnetic field dependence of the superfluid density in cuprate superconductors for all the temperature $T \leq T_c$ throughout the SC dome by considering both couplings of the electron charge and electron magnetic momentum with a weak magnetic field, where one of our main results is the striking behavior of the weak magnetic field induced reduction of the low-temperature superfluid density is intriguingly related to both depairing due to the Pauli spin polarization and nonlocal response in the vicinity of the d-wave gap nodes on...
the Fermi surface to a weak magnetic field.

It has been argued that the essential physics of cuprate superconductors is properly accounted by the $t$-$J$ model on a square lattice [11]. However, for discussions of the doping and magnetic field dependence of the superfluid density, the $t$-$J$ model can be extended by including the exponential Peierls factor and Zeeman term as,

$$H = -t \sum_{l \tilde{\eta}} e^{-i \Phi A(l) \tilde{\eta}} C_{l \sigma}^\dagger C_{l+\tilde{\eta} \sigma} + t' \sum_{l i' \sigma} e^{-i \Phi A(l) \tilde{\eta}'} C_{l \sigma}^\dagger C_{l+\tilde{\eta}' \sigma} + \mu \sum_{l \sigma} C_{l \sigma}^\dagger C_{l \sigma},$$

supplemented by an important on-site local constraint $\sum_\sigma C_{l \sigma}^\dagger C_{l \sigma} \leq 1$ to remove the double occupancy, where $\tilde{\eta} = \pm \tilde{x}, \pm \tilde{y}$, $\tilde{\eta}' = \pm \tilde{x} \pm \tilde{y}$, $C_{l \sigma}$ ($C_{l \sigma}^\dagger$) is the electron creation (annihilation) operator, $S_l = (S_l^x, S_l^y, S_l^z)$ are spin operators, and $\mu$ is the chemical potential. The exponential Peierls factors account for the coupling of the electron charge to an external magnetic field in terms of the vector potential $A(l)$, while the Zeeman magnetic energy $\varepsilon_B = g \mu_B B$ accounts for the coupling of the electron magnetic momentum $g \mu_B$ with the magnetic field $B = \text{rot} A$, with the Lande factor $g$ and Bohr magneton $\mu_B$. To incorporate the electron single occupancy local constraint, the charge-spin separation (CSS) fermion-spin theory [12] has been proposed, where the physics of no double occupancy is taken into account by representing the electron as a composite object created by $C_{l \sigma}^\dagger = h_{l+\tilde{\eta}}^\dagger S_{l \sigma}^-$ and $C_{l \sigma} = h_{l+\tilde{\eta}} S_{l \sigma}^+$, with the spinful fermion operator $h_{l \sigma} = e^{-i \Phi A} h_l$ that describes the charge degree of freedom of the electron together with some effects of spin configuration rearrangements due to the presence of the doped hole itself (charge carrier), while the spin operator $S_l$ represents the spin degree of freedom of the electron, then the electron single occupancy local constraint is satisfied in analytical calculations. In this CSS fermion-spin representation, the $t$-$J$ model (1) can be expressed as,

$$H = t \sum_{l \tilde{\eta}} e^{-i \Phi A(l) \tilde{\eta}} (h_{l+\tilde{\eta}}^\dagger h_{l \tilde{\eta}} S_l^+ S_{l+\tilde{\eta}}^- + h_{l+\tilde{\eta}}^\dagger h_{l+\tilde{\eta}'} S_l^+ S_{l+\tilde{\eta}'}^- - \mu \sum_{l \sigma} h_{l \sigma} h_{l \sigma}^\dagger + J_{\text{eff}} \sum_{l \tilde{\eta}} S_l \cdot S_{l+\tilde{\eta}} - 2 \varepsilon_B \sum_l S_l^z,$$

where $J_{\text{eff}} = (1 - \delta)^2 J$, and $\delta = \langle h_{l \sigma}^\dagger h_{l \sigma} \rangle = \langle h_l^\dagger h_l \rangle$ is the doping concentration.

For discussions of the SC state properties of cuprate superconductors, the kinetic energy driven SC mechanism [8] has been developed based on the CSS fermion-spin theory, where the charge carrier-spin interaction from the kinetic energy term in the $t$-$J$ model (2) induces a charge carrier pairing state with the d-wave symmetry by exchanging spin excitations, then the SC transition temperature is identical to the charge carrier pair transition temperature. In particular, this kinetic energy driven SC state is the conventional BCS-like with the d-wave symmetry, so that main low energy features of the SC coherence of the quasiparticle peaks have been quantitatively reproduced [13, 14]. Following these previous discussions [8, 13, 14], the full charge carrier Green’s function in the presence of both couplings of the electron charge and electron magnetic momentum with a weak magnetic field can be obtained explicitly in the Nambu representation as [9, 10],

$$g(k, i \omega_n, B) = Z_h^{(B)} \frac{i \omega_n \tau_0 + \xi_\mathbf{k} \tau_3 - A_{h,k}^{(B)}(k) \tau_1}{(i \omega_n)^2 - E_{h,k}^{(B)}},$$

where $\tau_0$ is the unit matrix, $\tau_1$ and $\tau_3$ are Pauli matrices, while the renormalized charge carrier excitation spectrum $\xi_\mathbf{k}$, the magnetic field dependence of the renormalized charge carrier d-wave
pair gap $\Delta^{(B)}_{\uparrow\downarrow}(k)$, the magnetic field dependence of the quasiparticle coherent weight $Z_{hF}^{(B)}$, and the charge carrier quasiparticle spectrum $E_{hk}^{(B)}$ have been given in Ref. [10].

In the presence of the coupling of the electron magnetic momentum with a weak magnetic field, the magnetic field dependence of the response current density $J_\mu$ and the vector potential $A_\nu$ are related by a nonlocal kernel of the response function $K_{\mu\nu}$ as [9],

$$J_\mu(q, \omega, B) = -\sum_{\nu=1}^3 K_{\mu\nu}(q, \omega, B) A_\nu(q, \omega),$$  

with the Greek indices label the axes of the Cartesian coordinate system. This response kernel (4) can be separated into two parts as $K_{\mu\nu}(q, \omega, B) = K_{\mu\nu}^{(d)}(q, \omega, B) + K_{\mu\nu}^{(p)}(q, \omega, B)$, where $K_{\mu\nu}^{(d)}(q, \omega, B)$ and $K_{\mu\nu}^{(p)}(q, \omega, B)$ are the corresponding diamagnetic and paramagnetic parts, respectively. In the CSS fermion-spin representation [12], the vector potential $A$ has been coupled to the electron charge, which are now represented by $C_{\uparrow\sigma} = h_{\uparrow\sigma}^{\dagger} S_{\sigma}^+ I_\uparrow$ and $C_{\downarrow\sigma} = h_{\downarrow\sigma}^{\dagger} S_{\sigma} I_\downarrow$. In this case, the electron polarization operator is expressed as $\mathbf{P} = -e \sum_i \mathbf{R}_i C_{\uparrow\sigma}^\dagger C_{\downarrow\sigma} = e \sum_i h_{\uparrow\sigma}^{\dagger} h_i$, then the corresponding electron current operator is obtained by evaluating the time-derivative of this polarization operator as $\mathbf{j} = j^{(d)} + j^{(p)}$, with $j^{(d)}$ and $j^{(p)}$ are the corresponding diamagnetic and paramagnetic components of the electron current operator. Following our previous discussions [9], these diamagnetic and paramagnetic parts of the magnetic field dependence of the response kernel $K_{\mu\nu}^{(d)}(q, \omega, B)$ and $K_{\mu\nu}^{(p)}(q, \omega, B)$ can be obtained in the the static limit as,

$$K_{\mu\nu}^{(d)}(q, 0, B) = -\frac{4e^2}{\hbar^2} (\chi_1 \phi_1 t - 2\chi_2 \phi_2 t') \delta_{\mu\nu},$$  

$$K_{\mu\nu}^{(p)}(q, 0, B) = \frac{1}{N} \sum_k \gamma_\mu(k + q, k) \gamma_\nu^*(k + q, k)[L_1^{(B)}(k, q) + L_2^{(B)}(k, q)] = K_{\mu\nu}^{(p)}(q, 0, B) \delta_{\mu\nu},$$

where the charge carrier particle-hole parameters $\phi_1 = \langle h_{\uparrow\sigma}^{\dagger} h_{\downarrow\sigma} \rangle$ and $\phi_2 = \langle h_{\downarrow\sigma}^{\dagger} h_{\uparrow\sigma} \rangle$, the spin correlation functions $\chi_1 = \langle S_{\sigma}^+ S_{\sigma}^{+\uparrow} \rangle$ and $\chi_2 = \langle S_{\sigma}^+ S_{\sigma}^{+\downarrow} \rangle$, $\lambda_L^2 = -4e^2 (\chi_1 \phi_1 t - 2\chi_2 \phi_2 t')/\hbar^2$ is the London penetration depth, and now is doping, temperature, and magnetic field dependent, while the bare current vertex $\gamma_\mu(k + q, k)$, $L_1^{(B)}(k, q)$ and $L_2^{(B)}(k, q)$ have been given in Ref. [10]. In this case, we have shown [9] that in the long wavelength limit, i.e., $|q| \to 0$, $K_{yy}^{(p)}(q \to 0, 0, B) = 0$ at temperature $T = 0$, then the long wavelength electromagnetic response is determined by the diamagnetic part of the system only. On the other hand, at the SC transition temperature $T_c$, $K_{yy}^{(p)}(q \to 0, 0, B) = -(1/\lambda_L^2)$, which exactly cancels the diamagnetic part of the response kernel (5), and then the Meissner effect in the presence of the coupling of the electron magnetic momentum with a weak magnetic field is obtained for all $T \leq T_c$ throughout the SC dome.

However, the result we have obtained the response kernel in Eqs. (5) and (6) can not be used for a direct comparison with the corresponding experimental data of cuprate superconductors because the response kernel derived within the linear response theory describes the response of an infinite system, whereas in the problem of the penetration of the field and the system has a surface, i.e., it occupies a half-space $x > 0$. In such problems, it is necessary to impose boundary conditions for charge carriers. This can be done within the simplest specular reflection model [15] with a two-dimensional (2D) geometry of the SC plane. Taking into account the 2D geometry of cuprate superconductors within the specular reflection model [15], we [9] can obtain the magnetic field penetration depth as,

$$\lambda(T, B) = \frac{1}{B} \int_0^\infty h_z(x, B) \, dx = \frac{2}{\pi} \int_0^\infty \frac{dq_z}{\mu_0 K_{yy}(q_x, 0, 0, B) + q_z^2}.$$
Now the superfluid density $\rho_s(T, B)$ can be obtained in terms of this magnetic field penetration depth $\lambda(T, B)$ as $\rho_s(T) = 1/\lambda^2(T)$.

In cuprate superconductors, although the values of $J$, $t$, and $t'$ are believed to vary somewhat from compound to compound, however, as a qualitative discussion, the commonly used parameters in this paper are chosen as $t/J = 2.5$, $t'/t = 0.3$, and $J = 1000K$. Furthermore, a characteristic length scale $a_0 = \sqrt{\hbar^2a/\mu_0e^2J}$ is introduced. Using the lattice parameter $a \approx 0.383$nm for YBa$_2$Cu$_3$O$_{7-\delta}$, this characteristic length is obtain as $a_0 \approx 97.8$nm. In this case, we have performed a calculation for the doping dependence of the zero-field superfluid density $\rho_s(0, 0)$ at temperature $T = 0$ for all levels of doping, and the result is plotted in Fig. 1 in comparison with the corresponding experimental data [3] for $Y_{0.8}Ca_{0.2}Ba_2(Cu_{1-\varepsilon}Zn_\varepsilon)_3O_{7-\delta}$ and $Tl_{1-\varepsilon}Pb_{\varepsilon}Sr_2Ca_{1-x}Y_xCu_2O_7$ (inset). It is shown clearly that $\rho_s(0, 0)$ increases with increasing doping in the lower doped regime, and reaches a maximum (a peak) around the critical doping $\delta \approx 0.195$, then decreases in the higher doped regime, in good agreement with the experimental results of cuprate superconductors [3]. Since $\rho_s(T, B)$ is related to the current-current correlation function, the charge carrier pair gap parameter is relevant, i.e., the variation of $\rho_s(T, B)$ with doping and magnetic field is coupled to the doping and magnetic field dependence of the charge carrier pair gap parameter $\Delta_{h\sigma}$. In this case, our present domelike shape of the doping dependent $\rho_s(0, 0)$ also is a natural consequence of the domelike shape of the doping dependent $T_c$ in the kinetic energy driven SC mechanism [8], where the the maximal $T_c$ occurs in the optimal doping, and then decreases in both underdoped and overdoped regimes. However, the calculated $T_c$ exhibits the maximal value at the optimal doping $\delta_{\text{optimal}} \approx 0.15$ [13], therefore there is a difference between the optimal doping $\delta_{\text{optimal}} \approx 0.15$ (the maximal $\Delta_{h\sigma}(0, 0)$ value) and the critical doping $\delta_{\text{critical}} \approx 0.195$ [the highest $\rho_s(0, 0)$ value]. This difference can be understood in terms of the competition for different order parameter correlation functions. In the domelike shape of the doping dependent gap parameter $\Delta_{h\sigma}(0, 0)$, the gap parameter $\Delta_{h\sigma}(0, 0)$ reaches its maximal value at the optimal doping $\delta_{\text{optimal}} \approx 0.15$, where the doping-derivative of $\Delta_{h\sigma}(0, 0)$ can be obtained as $(d\Delta_{h\sigma}(0, 0)/d\delta)|_{\delta=\text{optimal}} = 0$. On the other hand, at the critical doping $\delta_{\text{critical}} \approx 0.195$, the peak of $\rho_s(0, 0)$ appears, where the doping-derivative of $\rho_s(0, 0)$ can be obtained as $(d\rho_s(0, 0)/d\delta)|_{\delta=\text{critical}} = 0$. According to the definition of $\rho_s(T, B)$, $(d\rho_s(0, 0)/d\delta)|_{\delta=\text{critical}} = 0$ is equivalent to $(d\lambda(0, 0)/d\delta)|_{\delta=\text{critical}} = 0$. In this case, $(d\lambda(0, 0)/d\delta)|_{\delta=\text{critical}} = 0$ can be expressed from Eq. (7) as,

$$
\frac{d\lambda(0, 0)}{d\delta} \bigg|_{\delta=\text{critical}} = -\frac{2\mu_0}{\pi} \int_0^\infty d\kappa_x \frac{1}{[\mu_0K_{yy}(\kappa_x, 0, 0, 0) + \kappa_x^2/\delta]} = 0,
$$

then with the help of Eqs. (5) and (6), it is straightforward to find that when $(d\rho_s(0, 0)/d\delta)|_{\delta=\text{critical}} = 0$, $(d\Delta_{h\sigma}(0, 0)/d\delta)|_{\delta=\text{critical}} \neq 0$, since the spin correlation functions $\chi_1 = \langle S^+_iS^-_{i+\delta} \rangle$ and $\chi_2 = \langle S^+_iS^-_{i+q_\parallel} \rangle$, which describe the short-range antiferromagnetic correlation, and the charge particle carrier-hole parameters $\phi_1 = \langle h^+_{i\sigma}h_{i+\delta\sigma} \rangle$ and $\phi_2 = \langle h^+_{i\sigma}h_{i+q_\parallel\sigma} \rangle$ in the kernel $K_{yy}(q_x, 0, 0, B)$ are also doping dependent. In this case, there are competitions between different order parameter correlation functions, which leads to the difference between the optimal doping $\delta_{\text{optimal}} \approx 0.15$ for the zero-field gap parameter (then $T_c$) and the critical doping $\delta_{\text{critical}} \approx 0.195$ for the zero-field $\rho_s(0, 0)$. In particular, it is found that $(d\Delta_{h\sigma}(0, 0)/d\delta)|_{\delta=\text{critical}} < 0$, indicating that the critical doping locates at the slightly overdoped regime.

Now we turn to discuss the weak magnetic field induced reduction of the low-temperature superfluid density $\rho_s(T, B)$. In this case, we have made a series of calculations for $\rho_s(T, B)$ at differently weak magnetic fields, and the result of $\rho_s(T, B)$ as a function of temperature at $\delta = 0.09$ for the magnetic field $B = 0$ T (solid line), $B = 0.5$ T (dashed line), and $B = 1.0$ T (dash-dotted line) is plotted in Fig. 2 in comparison with the corresponding
Figure 1. The doping dependence of the zero-field superfluid density at \( T = 0 \) with \( t/J = 2.5, t'/t = 0.3, \) and \( J = 1000K \). Inset: the corresponding experimental result for \( Y_{0.8}C_{0.2}Ba_2(Cu_{1-\delta}Zn_{\delta})_3O_7-\delta \) (open circles) and \( Tl_{1-y}Pb_ySr_2Ca_{1-x}Y_xCu_2O_7 \) (solid triangles) taken from Ref. [3].

Figure 2. The temperature dependence of the superfluid density at \( \delta = 0.09 \) for \( B = 0 \) T (solid line), \( B = 0.5 \) T (dashed line), and \( B = 1.0 \) T (dash-dotted line) with \( t/J = 2.5, t'/t = 0.3, \) and \( J = 1000K \). Insets: the corresponding experimental results for \( YBa_2Cu_3O_{6.95} \) taken from Ref. [4].

Experimental data for \( YBa_2Cu_3O_{6.95} \) taken from Ref. [4] (inset). Obviously, the characteristic feature of the temperature dependent superfluid density is independent on a weak magnetic field, where as in the case of zero magnetic field, \( \rho_s(T, B) \) shows a linear temperature dependence at higher temperatures, and then it crosses over to a nonlinear temperature behavior at the low temperatures. However, most importantly, the magnitude of \( \rho_s(T, B) \) at the low temperatures decreases with increasing magnetic field, and then it turns to be independent on a weak magnetic field at higher temperatures, in qualitative agreement with experimental data of cuprate superconductors [4]. The present result also indicates that the nature of the quasiparticle excitations at the low temperatures is strongly influenced by a weak magnetic field. This weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors contrasts with the that observed from the conventional superconductors, where the curves of the temperature dependent superfluid density for differently weak magnetic fields were found to collapse onto a single curve since the conventional superconductors are fully gaped.

The weak magnetic field induced reduction of the low-temperature superfluid density in cuprate superconductors arises from both depairing due to the Pauli spin polarization and nonlocal response in the vicinity of the d-wave gap nodes on the Fermi surface to a weak magnetic field. In the kinetic energy driven SC mechanism, the d-wave SC state is mediated by the interaction of electrons and spin excitations [8], where the depairing can occur due to the Pauli spin polarization in the presence of an external magnetic field. This follows a fact that an applied magnetic field aligns the spins of the unpaired electrons, i.e., there is a tendency to induce the magnetic order, then the kinetic energy driven d-wave Cooper pairs can not take advantage of the lower energy offered by a spin-polarized state [8, 16]. On the other hand, the characteristic feature of the d-wave superconductors is the existence of four nodes on the Fermi surface, where the energy gap vanishes \( \Delta_h(k)|_{\text{nodes}} = \Delta_h(\cos k_x - \cos k_y)/2|_{\text{nodes}} = 0 \). In this case, even small thermal energy or externally small magnetic energy can excite excitations, then the superfluid density decreases with increasing temperature or increasing magnetic field.
Since the quasiparticles selectively populate the nodal region at the low temperatures, then the most physical properties in the SC state are controlled by the quasiparticle excitations around the nodes. In this case, the Ginzburg–Landau ratio around the nodal region is no longer large enough for the system to belong to the class of type-II superconductors, and the condition of the local limit is not fulfilled [7, 5, 9]. On contrary, the system falls into the extreme nonlocal limit, then the nonlinear characteristic in the temperature dependence of the superfluid density can be observed experimentally in cuprate superconductors at the low temperatures. However, when a weak magnetic field is applied to the system even at zero temperature, the quasiparticles around the nodal region become excited out of the condensate, and at the same time the electron attractive interaction for the Cooper pairs by exchanging spin excitations is weaken [8], both these effects lead to a decreases in the superfluid density. With increasing temperatures, the externally small magnetic energy due to the presence of a weak magnetic field is comparable with the small thermal energy at the low temperatures, therefore both small thermal energy and weak magnetic field induce an reduction of the superfluid density. However, at higher temperatures, this externally small magnetic energy is much smaller than the thermal energy, then the major contribution to a decrease of the superfluid density comes from the thermal energy. This is why a weak magnetic field only reduces an reduction of the superfluid density only at the low temperatures.

In conclusion, we have discussed the doping and magnetic field dependence of the superfluid density in cuprate superconductors based on the kinetic energy driven SC mechanism by considering both couplings of the electron charge and electron magnetic momentum with a weak magnetic field. Our results show that in analogy to the domelike shape of the doping dependent SC transition temperature, the superfluid density increases with increasing doping in the lower doped regime, and reaches a maximum around the critical doping δ ≈ 0.195, then decreases in the higher doped regime. Although the characteristic feature of the temperature dependent superfluid density is found to be independent on a weak magnetic field, this weak magnetic field induces an reduction of the low-temperature superfluid density in the Meissner state. Our results also show that the striking behavior of the weak magnetic field induced reduction of the low-temperature superfluid density can be attributed to both depairing due to the Pauli spin polarization and nonlocal response in the vicinity of the d-wave gap nodes on the Fermi surface to a weak magnetic field.

### 0.1. Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 11074023, and the funds from the Ministry of Science and Technology of China under Grant No. 2011CB921700.

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