Fitting dwarf galaxy rotation curves with conformal gravity

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ABSTRACT

We continue our study of the application of the conformal gravity theory to galactic rotation curves. Previously we had studied a varied 111 spiral galaxy sample consisting of high surface brightness galaxies, low surface brightness galaxies and dwarf galaxies. With no free parameters other than galactic mass-to-light ratios, we had found that the theory is able to account for the systematics that are observed in the entire set of galactic rotation curves without the need for any dark matter whatsoever. In this paper, we extend our study to an additional set of 27 galaxies of which 25 are dwarf galaxies, and provide updated studies of three additional galaxies that had been in the original sample, and again without dark matter find fully acceptable fits, save only for just a few galaxies that we find to be somewhat troublesome. Our current study brings to 138 the number of rotation curves of galaxies that have been accounted for by the conformal gravity theory. Since one of the primary ingredients in the theory is a universal contribution to galactic motions coming from matter exterior to the galaxies, and thus independent of them, our study reinforces one of the central concepts of the conformal gravity studies, namely that invoking dark matter should be viewed as being nothing more than an attempt to describe global physics contributions in purely local galactic terms.

Key words: galaxies: dwarf – galaxies: fundamental parameters – galaxies: general – galaxies: kinematics and dynamics.

1 INTRODUCTION

As a possible alternative to standard Einstein gravity, Weyl introduced conformal gravity in the very early days of general relativity. It is an attractive theory in that it is a pure metric theory of gravity that possesses all of the general coordinate invariance and equivalence principle structure of standard gravity while augmenting it with an additional symmetry, local conformal invariance, in which the action is left invariant under local conformal transformations on the metric of the form $g_{\mu\nu}(x) \to e^{2\alpha(x)}g_{\mu\nu}(x)$ with arbitrary local phase $\alpha(x)$. Under such a symmetry, the gravitational action is uniquely prescribed to be of the form (see e.g. Mannheim 2006)

$$I_W = -\alpha_g \int d^4x \left(-g\right)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$$

$$\equiv -2\alpha_g \int d^4x \left(-g\right)^{1/2} \left[ R_{\mu\nu} R^{\mu\nu} - (1/3) (R^{\gamma\delta})^2 \right] ,$$

(1)

where

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2} \left( g_{\lambda\kappa} R_{\mu\nu} - g_{\kappa\nu} R_{\mu\lambda} - g_{\mu\lambda} R_{\nu\kappa} + g_{\nu\kappa} R_{\lambda\mu} \right)$$

$$+ \frac{1}{6} R^\rho_{\nu\kappa} \left( g_{\lambda\mu} g_{\rho\nu} - g_{\lambda\nu} g_{\rho\mu} \right)$$

(2)

is the conformal Weyl tensor and the gravitational coupling constant $\alpha_g$ is dimensionless. With the conformal symmetry forbidding the presence of any $\int d^4x \left(-g\right)^{1/2} \Lambda$ term in the action, the conformal theory has a control over the cosmological constant that the standard Einstein theory does not, and through this control one is able to both address and resolve the cosmological constant problem (Mannheim 2011a). Similarly, with the coupling constant $\alpha_g$ being dimensionless, unlike standard gravity conformal gravity is renormalizable, and with it having been shown (Bender & Mannheim 2008a,b; Mannheim 2011a) to be unitary at the quantum level, the theory is offered (Mannheim 2011b) as a consistent theory of quantum gravity in four space–time dimensions.

With the conformal theory being a consistent, renormalizable quantum theory at the microscopic level, then just as with electrodynamics, one is assured that its macroscopic classical predictions are reliable and will not be ruined by quantum corrections. Consequently, application of the theory to astrophysical phenomena allows one to test the theory. Early work in this direction was provided in Mannheim (1997) where the theory was used to fit the rotation curves of a set of 11 spiral galaxies, with the mass-to-light ratio ($M/L$) of the luminous optical disc of each galaxy being the only free parameters, and with no dark matter being required. More recently, Mannheim & O’Brien (2010, 2011) conducted a systematic, broad-based study of the rotation curves of a varied set of 111 galaxies (consisting of high surface brightness galaxies, low
surface brightness (LSB) galaxies and dwarf galaxies; the set included the original 11 galaxies and updates where available, and again acceptable fitting was obtained with the optical disc $M/L$ of each galaxy being the only free parameter, and again with no dark matter being required. In this paper, we extend the study of Mannheim & O’Brien (2010, 2011) to an additional set of 30 galaxies that contains 20 of the dwarf galaxies described in Swaters et al. (2009), two LSB galaxies, five of the dwarf galaxies described in Oh et al. (2011), and updates of three galaxies (two dwarf galaxies and an LSB galaxy) that were in the 11-galaxy sample, and report findings of the same quality as before. With this latest study, the sample of galactic rotation curves that can be accounted for by the conformal theory now runs to 138.

2 GENERAL FORMALISM

The general formalism for applying the conformal theory to galactic rotation curves has been described in detail in Mannheim & O’Brien (2010, 2011), and we recall the main results. For the Weyl action $I_W$ given in equation (1), functional variation with respect to the metric $g_{\mu\nu}(x)$ generates a gravitational equation of motion of the form (Mannheim 2006)

$$4\alpha g^{\mu\nu} W_{\mu\nu} = 4\alpha g^{\mu\nu} \left[ 2C^{\mu\nu\lambda\kappa} g_{\lambda\kappa} - C^{\mu\nu\lambda\kappa} R_{\lambda\kappa} \right] - \frac{1}{3} W_{\mu\nu} = T_{\mu\nu},$$

(3)

where

$$W_{\mu\nu} = 2g^{\mu\nu}(\partial_{\alpha} R)_{\beta} - 2(\partial_{\alpha} g)_{\beta\mu
u\lambda} - 2R_{\alpha}^{\mu
u} + \frac{1}{2} g^{\mu
u}(R_{\alpha}^{\beta\lambda})^2,$$

$$W_{\mu\nu} = \frac{1}{2} g^{\mu\nu}(R_{\alpha}^{\beta\lambda})^{\beta} + (\partial_{\beta} g)_{\mu\nu\lambda} - (\partial_{\beta} R_{\mu\nu})_{\lambda} - R_{\beta
\mu
\nu\lambda} + \frac{1}{2} g^{\mu\nu} R_{\lambda\beta\alpha} R^{\lambda\beta\alpha} - 2R_{\beta\alpha
\mu\nu}^{\alpha
\lambda} + \frac{1}{2} g^{\mu\nu} R_{\alpha\beta\lambda\kappa} R^{\alpha\beta\lambda\kappa}.$$

(4)

For the case of a static, spherically symmetric geometry, it was shown in Mannheim & Kazanas (1989, 1994) that without loss of generality, the exact, all-order classical line element could be brought to the form

$$ds^2 = -B(r)c^2 dr^2 + \frac{dr^2}{B(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

(5)

and that in terms of this line element the exact fourth-order equation of motion equation (3) could be reduced to the remarkably simple fourth-order Poisson equation form

$$\frac{3}{B(r)} (W_0^0 - W_r^r) = \nabla^2 B = \frac{d^4 B}{dr^4} + \frac{4}{r} \frac{d^3 B}{dr^3} = \frac{3}{4\alpha g_{\lambda\kappa} B(r)} (T_0^0 - T_r^r) = f(r)$$

(6)

without any approximation whatsoever.

Solutions to equation (6) are of the form

$$B(r) = \frac{c^2}{2} \int_0^r dr' r'^2 f(r') - \frac{c^2}{6r} \int_0^r dr' r'^4 f(r'),$$

$$\frac{1}{2} \int_0^r dr' r'^3 f(r') - \frac{r^2}{6} \int_0^r dr' r' f(r') + \hat{B}(r),$$

(7)

where $\hat{B}(r)$ is the general solution to $\nabla^2 \hat{B}(r) = 0$. With $B(r)$ evaluating to

$$B(r) = \frac{1}{2} \int_0^r dr' r'^2 f(r') + \frac{1}{6r} \int_0^r dr' r'^4 f(r') - \frac{r}{3} \int_0^r dr' r' f(r') + \hat{B}(r),$$

(8)

and with the non-relativistic gravitational force being given by $\nabla(r)c^2/2$, we recognize two classes of contribution to the force, a local contribution due to the first two terms in equation (8) and a global one due to the last two terms.

For a localized system such as a star whose source function $f(r)$ is restricted to its interior $0 \leq r \leq r_0$ region, only the first two integrals in equation (7) contribute to $B(r > r_0)$, and with $B(r) = 1$ yield

$$B(r > r_0) = 1 - \frac{2\beta}{r} + \gamma r,$$

(9)

where

$$2\beta = \frac{1}{6} \int_0^r dr' r'^4 f(r'), \gamma = \frac{1}{2} \int_0^r dr' r'^2 f(r').$$

(10)

With the luminous material in a spiral galaxy typically being distributed with a surface brightness of the form $\Sigma(R) = e^{-R/R_0}/L/2\pi R_0^2$, where $L$ is the total luminosity, and with the potential produced by a single star being of the form $V^r(r) = -\beta c^2/r + \gamma c^2 r/2$ as per equation (9), the local galactic potential $V_{LOC}(r)$ is readily computed (Mannheim 2006), with the contribution of the material in a galaxy to rotation velocities then being given by the very compact formula

$$v^2_{LOC}(R) = \frac{N_0^* \gamma^* c^2 R^2}{2 R_0^4} \left[ I_0 \left( \frac{R}{2R_0} \right) K_1 \left( \frac{R}{2R_0} \right) - I_1 \left( \frac{R}{2R_0} \right) K_0 \left( \frac{R}{2R_0} \right) \right] + \frac{N_0^* \gamma^* c^2 R^2}{2 R_0^4} I_1 \left( \frac{R}{2R_0} \right) K_1 \left( \frac{R}{2R_0} \right),$$

(11)

where $N_0^* M_0^* = M = (M/M)c/2 \pi R_0^2$ is the total mass of the galaxy.

Unlike the standard second-order gravitational Poisson equation where only local material within a galaxy contributes to the gravitational force, in the fourth-order conformal case, there are contributions from material outside the galaxy as well. These global contributions are due to the homogeneous cosmological background and the inhomogeneities in it. With the $\beta^*$-dependent term in equation (11) being the standard galactic luminous Newtonian term, it is these global contributions together with the $\gamma^*$-dependent term in equation (11) that are to replace dark matter, with dark matter potentially being nothing more than an attempt to describe global physics contributions in purely local galactic terms.

Since $W_{\mu\nu}$ given in equation (3) is zero in solutions that obey $\nabla^2 B = 0$, and since $W_{\mu\nu}$ vanishes in the conformal to flat Robertson–Walker (RW) geometry, the $B(r)$ contribution is due to the background cosmology itself. For the specific form of the contribution, we recall (Mannheim & Kazanas 1989; Mannheim 1997) that a coordinate transformation of the form

$$\rho = \frac{4r}{2(1 + \gamma_0 r)^{1/2} + 2 + \gamma_0 r}, \quad r = \frac{\rho}{(1 - \gamma_0 \rho/4)^2},$$

$$\tau = \int dt a(t)$$

(12)

effects the metric transformation

$$- \frac{1 + \gamma_0 r}{c^2} d\tau^2 + \frac{dr^2}{(1 + \gamma_0 r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 = \frac{1}{\sigma^2} \left( \frac{1 + \gamma_0 \rho/4}{1 - \gamma_0 \rho/4} \right)^2 \left[ c^2 d\tau^2 + \frac{\alpha^2(t)}{(1 - \gamma_0 \rho^2/16)^2} \right.$$

$$\times \left( d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2 \theta d\phi^2 \right).$$

(13)

With the bracketed term on the right-hand side of equation (13) being recognized as a RW geometry with an expressly negative
definite $K = -\gamma_0^2/4$, we see that in the rest frame of a galaxy, a $K < 0$ cosmological background acts as an universal linear potential $\gamma_0$ whose strength is independent of the galaxy of interest. Since as noted in Mannheim (2006) conformal gravity precisely produces such a $K < 0$ RW background, its effect on galactic rotation curves is to produce an effective universal linear potential.

As regards the contribution due to inhomogeneities, it was noted in Mannheim & O'Brien (2010, 2011) that if the exterior $f(r)$ distribution starts at some typical cluster radius $r_{clus}$ then in the $r_0 \leq r \leq r_{clus}$ region the contribution of the fourth integral in equation (7) acts like a quadratic de Sitter like potential with a strength $\kappa = 1/6 \int_{r_{clus}}^\infty dr' r' f(r')$. (14)

Since the integral in equation (14) is independent of the galaxy of interest, its contribution is universal for all galaxies, and since it is due to all the inhomogeneous material in the Universe, it would equally act on galaxies that are within clusters as well.

Finally, for weak gravity, and on scales $r < r_{clus}$, we can augment equation (11) with the linear and quadratic potential terms, and obtain a total contribution $v_{TOT}^2(R)$ to the rotational velocity of the form

$$v_{TOT}^2(R) = v_{LOC}^2(R) + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2,$$  \hspace{1cm} (15)

with an associated asymptotic limit

$$v_{TOT}^2(R) \rightarrow \frac{N^* \beta^* c^2}{R} + \frac{N^* \gamma^* c^2 R}{2} + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2.$$  \hspace{1cm} (16)

Equation (15) is our key result, with the $M/L$ of the luminous disc in $v_{LOC}^2(R)$ being the only free parameter in any given galaxy, and with everything else being universal.

In Mannheim (1997) and Mannheim & O'Brien (2010, 2011), equation (15) was used to fit the galactic rotation curve data of a sample of 111 galaxies, and good fits were found, with the three universal parameters being given by

$$\gamma^* = 5.42 \times 10^{-41} \text{cm}^{-1}, \quad \gamma_0 = 3.06 \times 10^{-30} \text{cm}^{-1},$$
$$\kappa = 9.54 \times 10^{-54} \text{cm}^{-2}.$$  \hspace{1cm} (17)

The value obtained for $\gamma^*$ entails that the linear potential of the Sun is so small that there are no modifications to standard Solar system phenomenology, with the values obtained for $N^* \gamma^*$, $\gamma_0$ and $\kappa$ being such that one has to go all the way to galactic scales before their effects can become as big as the Newtonian contribution. The value obtained for $\gamma_0$ shows it to indeed be of cosmological magnitude, with the value of $\kappa$ being a typical 100-Mpc inhomogeneity scale, just as desired. Armed with the above analysis and the values of the three universal parameters as given in equation (17), we now proceed to apply equation (15) to the 30-galaxy sample.

### 3 Conformal Gravity Fitting to the 30-Galaxy Sample

The sample of galaxies we study is composed of a 24 spiral galaxy sample and a six-galaxy sample. The 24-galaxy sample consists of 23 of the 27 galaxies that were studied in Swaters, Sanders & McGaugh (2010) as augmented by UGC 12732, with six dwarf galaxies being taken from the THINGS collaboration as reported in Oh et al. (2011). Of the 24 galaxies in the Swaters et al. (2010) sample, 21 galaxies (UGC 731, UGC 3371, UGC 4173, UGC 4325, UGC 4499, UGC 5414, UGC 5721, UGC 7232, UGC 7323, UGC 7399, UGC 7524, UGC 7559, UGC 7577, UGC 7603, UGC 8490, UGC 9211, UGC 11707, UGC 11861, UGC 12060, UGC 12632 and UGC 12732) belong to the late-type dwarf galaxy sample described in Swaters et al. (2009), two are LSB galaxies to which we had not previously applied conformal gravity fitting (F568-V1, F574-1), with the remaining galaxy being the LSB galaxy UGC 5750. Even though UGC 4325 and UGC 5750 had both been contained in our previously studied 111-galaxy sample, the data we use for them now are sufficiently different to warrant their inclusion here. [The remaining four galaxies studied in Swaters et al. (2010) (F583-1, F583-4, UGC 5005, UGC 6446) were all contained in our 111-galaxy sample and are not included here as there is no significant change in either the data or our fits to them.] For the 21 dwarf galaxies, we use the rotation curve data reported in Swaters (1999) and Swaters et al. (2009). For F568-V1 and F574-1, we use the rotation curve data reported in Swaters et al. (2003).

For UGC 5750, we use the rotation curve data reported in de Blok, McGaugh & Rubin (2001), data that extend to much larger distances of significance to conformal gravity than had been considered in Mannheim & O'Brien (2010). For the fitting, we take $B$-band absolute luminosities and $R$-band optical disc scalelengths from Swaters & Balcells (2002) and Swaters et al. (2002, 2010), and references therein [using $B - R = 0.8$ to extract the $B$-band luminosity of UGC 11861 from the $R$-band luminosity value listed in Swaters et al. (2002)], except that for UGC 5750 we take the $B$-band luminosity from de Blok & McGaugh (1997). For the 24 galaxies, we take HI gas masses from Swaters et al. (2002, 2010).

The six-galaxy sample presented in Oh et al. (2011) consists of UGC 3851 (NGC 2366, DDO 42), UGC 4305 (Holmberg II, DDO 50), UGC 4459 (DDO 53), UGC 5139 (Holmberg I, DDO 63), UGC 5423 (M81dwB) and UGC 5666 (IC2754, DDO 81). The galaxy UGC 5666 had been included in our earlier 111-galaxy study, but the data have changed sufficiently to warrant its being included here. [The remaining galaxy studied in Oh et al. (2011) (DDO 154) was also contained in our 111-galaxy sample and is not included here as there has been no significant change in the data since then.]

For the six dwarf galaxies, we use the rotation curve data reported in Oh et al. (2011), and take $B$-band absolute luminosities and HI gas masses from Walter et al. (2008). For UGC 3851, UGC 4305 and UGC 5139, we take $R$-band disc scalelengths from Swaters et al. (2002), for UGC 4459 and UGC 5423, we follow Oh et al. (2011) and take disc scalelengths to be five times the $z_0$ scaleheights they report, and for UGC 5666, we take the disc scalelength from Pasquali et al. (2008).

In our theory, the place where there is the most sensitivity to parameters is in the adopted distances to the individual galaxies, since the parameters $\gamma^*$, $\gamma_0$ and $\kappa$ that appear in equation (15) are given as absolute quantities. Moreover, for dwarf galaxies the parameter $N^*$ is usually so small that the $N^* \gamma^*\kappa^2$-dependent term in equations (15) and (16) cannot compete with the $\gamma_0^2$ term, while the $N^* \beta^*\kappa^2$-dependent terms in equations (15) and (16) are falling off at largest distances in galaxies where the mass discrepancy problem is the most severe, so they do not compete with the $\gamma_0$ term either. In addition, except for only a few cases, the dwarf galaxy data do not go out far enough from galactic centres for the $\kappa$-term contribution to be that significant. Hence, the $\gamma_0^2 c^2 R^2$ term in equation (15) is the most relevant for our study, and it requires a good determination of the adopted distance to each galaxy.

To establish a common baseline for determining the needed adopted distances, for all the galaxies in our sample we follow our earlier studies (Mannheim & O'Brien 2010, 2011) and use the distances listed in the NASA/IPAC Extragalactic Database (NED). In this data base, distances are obtained either via direct visual
measurements (typically Cepheids or the Tully–Fisher relation) or indirectly via redshift measurements. For the directly determined distances, a world average mean value and its 1 standard deviation (1σ) uncertainty are listed. The redshift-based determinations depend on how one models both the peculiar velocity with respect to the Hubble flow of the Milky Way galaxy and the peculiar velocity of the galaxy of interest. For definitiveness, for redshift-based distance determinations (as needed for F568-V1, F574-1 and UGC 5750) we have opted to use the mean values associated with galactocentric distances.

In Tables 1 and 2, we list the mean NED values for adopted distances to the 30 galaxies in our sample, and assemble all the optical and H I gas input data as determined at the NED mean distances. In Tables 1 and 2, we also list the values of the last data points $R_{\text{last}}$ in each of the 30 galaxies as determined at the NED mean distances. In the fitting, we multiply the H I gas masses given in Tables 1 and 2 by 1.4 to account for primordial helium.

A second place in our theory where there is sensitivity to parameters is in the values of galactic inclination angles that are extracted from the rotation curve data, since changes in inclination angle entail changes in the extracted values of rotational velocities and their errors, with the $\gamma R^2/R^2$ term needing to account for whatever the extracted values are. Specifically, with the inclination angle being the angle between the normal to the plane of the galactic disc and our line of sight, a measured Doppler shift $v(\text{meas})$ is actually a measurement of the projection $v(\text{meas}) \sin i$ at inclination angle $i$ along our line of sight [so that for edge-on galaxies $v(90^\circ) = v(\text{meas})$. At two different inclinations, the associated velocities would thus be related as $v(\text{meas}) = v(i_1) \sin i_1 = v(i_2) \sin i_2$, with a decrease in inclination leading to an increase in inferred rotation velocity. While we will consider varying inclination angles for a few of the galaxies in our sample in the discussion below, the rotation curve data that are displayed in Figs 1 and 2 were obtained using the original inclinations given in Swaters et al. (2010) and Oh et al. (2011), as listed in Tables 1 and 2.

If we were to use the actually observed line-of-sight surface density $\sigma(\text{obs})$ for the gas profile, the associated contribution of the H I gas to rotation velocities would then be inclination-dependent as well. Specifically, on assuming optical thinness, one would infer a surface density $\sigma(i) = \sigma(\text{obs}) \cos i$ that would have been obtained had the galaxy been face-on, with the contribution of the gas to the square of the rotation velocity then behaving as $v^2(i) \sim \cos i$ (see e.g. Swaters et al. 2010). However, since in our study here we define the...
Figure 1. Fitting to the rotational velocities (in $\text{km s}^{-1}$) of the 24-galaxy sample with their quoted errors plotted as a function of radial distance (in kpc). For each galaxy, we have exhibited the contribution due to the luminous Newtonian term alone (dashed curve), the contribution from the two linear terms alone (dot–dashed curve), the contribution from the two linear terms and the quadratic term combined (dotted curve), with the full curve showing the total contribution. No dark matter is assumed.

gas profile to be given by a model exponential, that profile already is the effective face-on value. Thus, when we consider varying some inclinations below, we will not make any inclination-dependent correction to the gas contribution to rotation velocities. [As a practical matter, while the $v(\text{meas}) = v(i_1)\sin i_1 = v(i_2)\sin i_2$ modification can have a quite significant effect on the velocity values that are extracted from data, since dwarf galaxies generally exhibit large mass discrepancies, the $\text{H}_1$ gas contribution to rotation velocities is not
significant, with any overall $v^2(i_2)/v^2(i_1) = \cos i_2/\cos i_1$ modification to it having little effect on the rotation velocity fits (something we actually checked in those cases where we allowed the inclination angle to vary).

Since the HI gas distributions of the galaxies in our sample are found to extend well beyond the associated optical disc regions (Swaters et al. 2002), just as in Mannheim & O’Brien (2010, 2011) we have approximated the gas profiles as single exponential discs with scalelengths equal to four times those of the corresponding optical discs. For our purposes here this is very convenient as it allows us to use equation (11) as is for the gas contribution to $v^2_{LOC}(R)$. Moreover, in the region in a galaxy where $R$ is much greater...
than the gas scalelength, the asymptotic (16) will then hold with $N^*$ now being understood to connote the stars and gas masses combined. Since for dwarf galaxies such a combined $N^* \gamma_0 c^2 R/2$ term does not compete in equation (16) with the $\gamma_0 c^2 R/2$ term while the $N^* \beta c^2 R/2$ term is falling off, the $\gamma_0 c^2 R/2$ term will dominate in the outer region, regardless of what specific values for the gas scalelengths we might actually use. With the use of single exponential discs for the gas distributions, we can then directly monitor the outer regions of rotation curves, the regions where the galactic mass discrepancy problem is at its most pronounced. As regards the inner rotation curve regions, even though there could be some sensitivity to the forms we might use for the gas distributions, in practice we found fits of comparable quality using a variety of smaller gas-to-optical scalelength ratios, and in this paper only report the fits we obtain in which the gas-to-optical scalelength ratios are taken to be equal to 4 in each galaxy. [To the extent that there is any sensitivity to gas scalelengths at all, we note that for an exponential disc the standard Freeman formula (the Newtonian term in equation 11) peaks at $R = 2.15R_0$ with a contribution to the square of the rotation velocity of the form $\nu^2(2.15R_0) = 0.39N/\beta R_0$. For a fixed number of sources $N$, the peak value of $\nu^2$ varies inversely with $R_0$, and even in cases where the gas profile might be better fitted with a much smaller scalelength, since the contribution of the H I gas to rotation velocities falls well below the measured rotation velocities; in practice, changing $R_0$ has little effect.]

To get an immediate sense of the expectations of the conformal theory, we fit the rotation curves of the 30 galaxies in our sample using the input data listed in Tables 1 and 2 as is, without regard to any uncertainties in adopted distances or galactic inclinations. In Figs 1 and 2, we present the resulting rotational velocities with their quoted errors (in km s$^{-1}$) for all of the 30 galaxies as plotted as functions of radial distances from galactic centres (in kpc). For each galaxy, we have exhibited the contribution due to the luminous (stars plus gas) Newtonian term alone (dashed curve), the contribution from the two linear terms alone (dot–dashed curve), and the contribution from the two linear terms and the quadratic term combined as per equation (15) (dotted curve), with the full curve showing the total contribution.

In Tables 1 and 2, we list the fitted optical disc masses and $M/L$ values that we obtained in the conformal gravity fitting. As described in Mannheim & O’Brien (2010), we constrained the fits so that the $M/L$ values would not be less than 0.2 nor larger than 10.0. The $M/L$ values that we obtain are typical of the values that one ordinarily obtains in galactic rotation curve fitting, and are reasonably close to the $M/L$ value found in the local solar neighbourhood.

As we see, use of the tightly constrained equation (15) and the tightly constrained input values given in Tables 1 and 2 captures the general trend of the data to a remarkably good degree, and does so without needing any dark matter whatsoever. The fact that our fits work so well even though the velocities are dominated by a single galaxy-independent universal $\gamma_0 c^2 R/2$ term in equation (15) is thus a quite noteworthy achievement for our theory.

In the fitting, the only real concern we have is for UGC 5721, UGC 7577 and UGC 4305, as their overall normalizations are not well accounted for. Improved fitting for these three galaxies can be obtained by using NED distances as modified by 1 standard deviation distance uncertainties, and by using different inclination angles. For UGC 5721, we found it advantageous to increase the adopted distance by 1 standard deviation and increase the inclination by $10^\circ$. In addition, for UGC 5721 we took advantage of the uncertainty in the disc scalelength reported in Spano et al. (2008), and also incorporated an optical disc thickness correction of the form $f_2 = \text{sech}^2(z_0^2/2z_0)$ with $z_0 = R_0/5$ as per the formalism given in Mannheim (2006). For UGC 7577, we found it
advantageous to decrease the adopted distance by 1 standard deviation, and on adding (in quadrature) an overall asymmetric drift error of 3 km s$^{-1}$ that was reported in Swaters et al. (2009) but not allowed for in the velocity errors shown in Fig. 1, we obtained good fitting if we reduced the inclination by 15$^\circ$. [With this asymmetric drift correction we could obtain a comparable fit (not shown) with an inclination reduction of 20$^\circ$.] For UGC 4305 we found it advantageous to decrease the adopted distance by 1 standard deviation and reduce the inclination by 5$^\circ$, to then be mid-way between the inclinations of 50$^\circ$ and 40$^\circ$, which were, respectively, reported in Oh et al. (2011) and Swaters et al. (2009).

For four other of the galaxies also we have concerns, though they are mild, and making analogous adjustments proved beneficial. For UGC 7399, we found it advantageous to increase the adopted distance by 1 standard deviation. For UGC 4173, we found it advantageous to reduce the inclination by 5$^\circ$, for UGC 8490, we increased the inclination by 10$^\circ$, and for UGC 3851, we increased the adopted distance by 1 standard deviation. With all the above changes we obtained the fits given in Fig. 3, with the associated parameters and fitted 1σ distance changes ($\Delta D$) and inclination changes ($\Delta i$) being listed in Table 3. The fitting to the galaxy UGC 5666 is of interest in that while we had found it beneficial to increase its adopted distance by 1 standard deviation in our earlier study of it (Mannheim & O’Brien 2010), with the new data we can now use the NED mean distance as is.

With the adjustments we have made, the only galaxy for which we still have some difficulty is UGC 5721, since even though the conformal theory could readily reproduce the rotation curve in either the outer region or the inner region alone (not shown), the presented eyeball fit represents the result of trying to accommodate both regions simultaneously. However, it was noted in Swaters et al. (2009) that for this galaxy the inner rotation curve points are uncertain because of insufficient angular resolution, while in Swaters & Balcells (2002) it was noted that that there is evidence for twisting isophotes in the central region, to thus suggest additional structure in the inner region of this galaxy. We have not taken any such inner region structure into consideration in our fitting. Apart from this one inner region concern and apart from the use of a somewhat large inclination shift for UGC 7577, the conformal theory is otherwise found to account for the rotation curve data of the entire 30-galaxy sample to a quite remarkable degree.

4 GENERAL COMMENTS

The cumulative set of 138 galaxy fits presented here and in Mannheim & O’Brien (2010, 2011) is noteworthy in that the universal $\gamma_0$ and $\kappa$ terms have no dependence on individual galactic properties whatsoever and yet have to work in every single case. Our fits are also noteworthy in that we have captured the essence of the rotation curve data even though we have imposed some rather strong constraints on the input parameters. For adopted distances in most cases we have used NED mean values. We have not used actual surface brightness distributions or actual gas profiles but have treated these distributions simply as single exponentials. On the theoretical side, our fits are noteworthy in that equation (15) is not simply a phenomenological or empirical formula that was extracted solely from consideration of the systematics of galactic rotation curves, rather equation (15) was explicitly derived from first principles in a fundamental, uniquely prescribed metric-based theory of gravity, namely conformal gravity. Moreover, conformal gravity itself was not even advanced for the purposes of addressing the dark matter problem. Rather, before it was known what its static, spherically symmetric solutions might even look like, it was advanced by one of us (Mannheim 1990) simply because it had a symmetry that could control the cosmological constant. Our fitting is thus quite non-trivial.

Since the conformal gravity fits do capture the essence of the data, it is important to ask how it is that a theory with so few adjustable parameters is actually able to do so. As was already noted in Mannheim & O’Brien (2010, 2011) for the 111-galaxy sample, those data are such that the value of the quantity $(v^2/c^2 R)_{\text{last}}$ as measured at the last data point for each of those galaxies is close in value to $\gamma_0$ across the entire 111-galaxy sample. As we see from the last columns in Tables 1 and 2, this very same last data point near universality is also possessed by the additional galaxies that we study here, with all of the $(v^2/c^2 R)_{\text{last}}$ values clustering close to the numerically extracted universal value for the parameter $\gamma_0$.

With regard to the near universality of $(v^2/c^2 R)_{\text{last}}$, we should note that this is an empirical property of the raw data themselves. Moreover, while there may be some uncertainties in the adopted distances to the galaxies, such uncertainties are never more than a factor of 2 or so. With the velocities being uncertain to no more than 10–20 per cent or so, the near universality of $(v^2/c^2 R)_{\text{last}}$ is thus a genuine property of the data. It should thus be regarded as an important empirical clue for galactic dynamics.

Apart from conformal gravity, Milgrom’s MOdified Newtonian Dynamics (MOND) theory (Milgrom 1983a,b,c) and Moffat’s metric skew-tensor gravity (MSTG) theory (Moffat 2005, 2006; Brownstein & Moffat 2006) equally succeed in explaining galactic rotation curves without invoking dark matter. Each of these particular theories has an assumed universal parameter $(a_0/c^2 = 1.33 \times 10^{-20}$ cm$^{-1}$ for MOND, and $G_0 M_0/r_c^2 c^2 = 7.67 \times 10^{-29}$ cm$^{-1}$ for MSTG), and in consequence each is able to account for the data. Now these two theories and the conformal theory predict differing behaviours at larger distances, As discussed in Mannheim & O’Brien (2010), the MOND theory typically requires asymptotic flatness of rotation curves, the MSTG theory requires Kepler behaviour at large distances, while the conformal theory requires a fall in rotation velocities to zero at $R \sim \gamma_0 d$, and thus a finite size to galaxies. Given such differing behaviours, it should eventually be possible to phenomenologically distinguish between these various theories.

It is important to recognize that for conformal gravity fitting (and likewise for MOND and MSTG) the only input one needs is luminous matter distribution. Then, with only one free parameter per galaxy (viz. the galactic MIL rotation velocities are completely determined, and as Tables 1 and 2 show, by and large the MIL values are all found to be of the order of the local solar neighbourhood MIL, just as one would want. Moreover, since the MIL values are effectively determined by the inner rotation curve region data alone, the linear and quadratic terms being at their smallest there, the outer region fitting is then essentially parameter free.

It is important to contrast conformal gravity fitting with dark matter fitting to galactic rotation data. For dark matter fits, one first needs to know the velocities so that one can then ascertain the needed amount of dark matter, that is, in its current formulation dark matter is only a parametrization of the velocity discrepancies that are observed and is not a prediction of them. Even with the freedom to treat galactic MIL values as free parameters, dark matter theory has yet to develop to the point where it is able to determine rotation curve velocities from a knowledge of the luminous matter distribution alone. Nor is it currently able to provide an explanation for the near universality that is found for $(v^2/c^2 R)_{\text{last}}$. Thus, dark matter theories, and in particular those theories that produce dark matter haloes in the early Universe (such as the NFW theory, Navarro,
Fitting dwarf galaxy rotation curves

Figure 3. Fitting to the rotational velocities of the seven galaxies that benefited from parameter adjustments.

Table 3. Properties of the seven galaxies that benefited from parameter adjustments.

| Galaxy   | Distance (Mpc) | $\Delta D$ | $L_B$ $(10^9 L_\odot)$ | $\Delta i$ (°) | $(R_0)_{disc}$ (kpc) | $R_{last}$ (kpc) | $M_{HI}$ $(10^9 M_\odot)$ | $M_{disc}$ $(10^9 M_\odot)$ | $(M/L)_{disc}$ $(M_\odot/L_\odot)$ | $(v^2/c^2)R_{last}$ $(10^{-30} \text{ cm}^{-1})$ |
|----------|----------------|------------|-------------------------|----------------|-----------------------|------------------|---------------------------|-------------------------------|--------------------------------|----------------------------------|
| UGC 3851 | 4.85           | $+1 \sigma$| 2.33                    | -5             | 2.07                  | 11.70            | 1.32                      | 0.47                          | 0.20                           | 1.37                             |
| UGC 4173 | 16.70          |            | 0.33                    | -5             | 4.43                  | 12.14            | 2.09                      | 0.07                          | 0.20                           | 1.21                             |
| UGC 4305 | 2.34           | $-1 \sigma$| 0.41                    | -5             | 0.68                  | 4.75             | 0.28                      | 0.08                          | 0.20                           | 1.18                             |
| UGC 5721 | 7.60           | $+1 \sigma$| 0.48                    | +10            | 0.76                  | 8.41             | 0.85                      | 1.80                          | 3.75                           | 2.27                             |
| UGC 7399 | 24.66          | $+1 \sigma$| 4.61                    |                | 2.32                  | 32.30            | 6.38                      | 5.42                          | 1.18                           | 1.32                             |
| UGC 7577 | 2.13           | $-1 \sigma$| 0.05                    | -15            | 0.51                  | 1.39             | 0.03                      | 0.01                          | 0.20                           | 1.18                             |
| UGC 8490 | 5.28           |            | 0.95                    | +10            | 0.71                  | 11.51            | 0.93                      | 1.43                          | 1.51                           | 1.47                             |

Frenk & White (1996, 1997), are currently unable to make an a priori determination as to which halo is to go with which particular luminous matter distribution, and need to fine-tune halo parameters to luminous parameters galaxy by galaxy.

The lack to date of any such underlying universal structure or of any explanation for the near universality of $(v^2/c^2)R_{last}$ is more than just a challenge to dark matter theory; it runs counter to the very motivation that led to the acceptance of Newtonian gravity in the first place. Specifically, the great appeal of Newtonian gravity was not just that it explained planetary orbits via an inverse square force, but that it did so through the use of a solar potential that possessed just one universal parameter, namely the solar Schwarzschild radius $M_\odot G/c^2$, that was to control the orbits of all the planets in the Solar system. If universal Newtonian gravity is to continue to hold on galactic distance scales, then galactic orbits (and particularly those beyond the galactic optical disc region) should also be explainable in terms of just one parameter, namely the galactic Schwarzschild radius $N^* M_\odot G/c^2$. Thus, if dark matter is to be the explanation, then both the dark-to-luminous matter mass ratio and the relation between the spatial distributions of dark and luminous...
matter should be universal to all galaxies, with rotation curves then being determinable from a knowledge of the luminous mass of the galaxy and its spatial distribution alone. It is the lack of any such universality in dark matter theory that leads to the need to fine-tune its halo parameters galaxy by galaxy. In contrast, no such fine-tuning shortcomings appear in conformal gravity, and if standard gravity is to be the correct description of gravity, then a universal formula akin to the one given in equation (15) would need to be derived by dark matter theory. However, since our study establishes that global physics has an influence on local galactic motions, the invoking of dark matter in galaxies could potentially be nothing more than an attempt to describe global physics effects in purely local galactic terms.

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