Using plasmonic cloaking method on infinite cylindrical structures and its applications

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Abstract
In recent years, cloaking using materials with negative electric permittivity or magnetic permeability has been studied and researched. It has been demonstrated that covering an object with a cloak having an electric permittivity or magnetic permeability that is negative or less than unity can cause a reduction of the scattering cross-section (SCS) of the object. In this paper, we solve the scattering problem for an object with a single- or multilayer cylindrical cloak and thus obtain the fundamental equations necessary to design such cloaks under two conditions, viz. with and without consideration of the effects of coupling when solving the scattering problem. Using the obtained equations we demonstrate that this technique can indeed reduce the visibility of the object.

Keywords Plasmonic · Metamaterial · Plasmonic cloak · Reverse polarization

1 Introduction
Humans have long been interested in becoming invisible, as found in stories from different nations. This human ambition has not been abandoned over the centuries. Nowadays, cloaking is no longer an unreachable dream as a result of the advancement of science. Researchers have looked for an answer to the question of whether a cloaking layer can be achieved to make an object invisible, at least in a limited frequency range. In 1961, Dolin investigated anisotropic nonhomogeneous structures and showed that they enabled an electromagnetic wave to pass through an object without perturbation [1]. Other structures were introduced by Pendry in 2006 [2] and Greenleaf in 2009 [3]. Also, Kerker published a paper entitled “Cloaked Objects” in 1975, which inspired Engheta and Alú to investigate invisibility based on the scattering cancellation method [4]. The advent of metamaterials and the advancement of fabrication technology for such artificial materials with extraordinary electromagnetic properties opened a new avenue for the achievement of cloaking, and many researchers have been attracted to the use of metamaterials for this purpose.

An electromagnetic cloaking layer is a device that can make an object that is exposed to electromagnetic waves become invisible in a given frequency range. In physics, cloaking means that the cloaked object does not produce any changes in the fields around it. In other words, a cloaked object does not produce reflected waves towards the source or scattered waves in other directions. Moreover, it does not absorb waves. In electromagnetic wave scattering theory, invisibility is the same as achieving a near-zero or ideally exactly zero SCS. Since the SCS is defined as the ratio of the scattered power density from the object to the incident power density, zero SCS means invisibility.

Several methods to achieve invisibility have been introduced to date, each offering some advantages and disadvantages. Some of the most famous methods are the coordinate transformation technique [5], transmission line techniques [6, 7], parallel-plates cloaking [8], the hybrid method [9], and active schemes [10]. Since anisotropic and inhomogeneous materials are used in the majority of these methods, the realization of a cloaking layer has been impossible in practice. However, this all changed with the introduction of plasmonic cloaking and the feasibility of fabricating such
cloaking layers. In 2005, Engheta and Alú presented a new technique in their paper entitled “Achieving Transparency with Plasmonic and Metamaterial Coatings,” which is realizable in practice through the use of isotropic and homogeneous materials [11]. In this method, which is based on the scattering cancellation method and is called “plasmonic cloaking,” invisibility is obtained by using materials with an electric permittivity or magnetic permeability that is less than unity or negative. For instance, some metals near their plasma frequencies or metamaterials with negative parameters can be used [12]. It is worth noting that this idea of using layers with near-zero electric permittivity to achieve invisibility in quasistatic conditions has been introduced before for very small spheres (with a radius much smaller than the wavelength) [13]. However, in the method known as plasmonic cloaking, the scattering problem can be solved for more general cases (dynamic and nonstatic).

The main idea behind the achievement of invisibility using a plasmonic cloak is a significant reduction of the SCS of the combination of the object and cloaking layer. It is known that, the larger the size of an object, the larger its SCS. Therefore, it is expected that the SCS will increase when adding a plasmonic layer to an object. However, in the plasmonic cloaking method, the SCS is reduced by adding a properly designed layer with a negative or near-zero electric permittivity or magnetic permeability. Therefore, by optimizing the design of the cloaking layer parameters, the SCS of the combination can reach zero, resulting in invisibility.

A physical justification of the plasmonic cloaking operation can be presented as follows: According to Fig. 1, consider a spherical object with a dielectric constant of \( \varepsilon_1 > \varepsilon_0 \), covered by a layer with a dielectric constant of \( \varepsilon_2 < \varepsilon_0 \) which is illuminated by an electromagnetic wave.

Plasmonic cloaking offers several advantages compared with previous methods, such as its nonresonant nature, wider bandwidth [14], the robustness of the layer, i.e., that a small change in the geometry or frequency will not degrade the cloaking effect [15], obtaining transparency in both the near and far field for one or various particles [16], the penetration of the field into the object, and the lack of isolation from the environment, which allows the cloaked object to be used in measurement sensors [17].

In 2013, Farhat and colleagues presented a paper describing the possibility of achieving a controllable cloaking layer using graphene. They showed that, by biasing the graphene layers using an external direct-current (DC) voltage, it is possible to obtain a tunable cloaking layer for operation at different frequencies [18]. Another advantage of using graphene in the plasmonic cloaking technique is its easy realization in practice.

So far, most research has focused on achieving invisibility for spherical objects, while less work has been done on cylindrical structures [19]. This paper tries to address the issue of invisibility for infinite cylindrical structures.

### 2 Infinite cylindrical structures with a multilayer cloak

Engheta and Alú introduced a two-layer plasmonic cloak to achieve invisibility at some different frequencies [14]. Our goal is to investigate the scattering problem of a multilayer plasmonic cloak for cylindrical structures under plane-wave excitation. We first consider the desired number of layers and obtain the scattering coefficients in the general case. It is worth noting that the use of a multilayer cloak can result in a multifrequency effect as well as reduced scattering coefficients at higher harmonics, because the addition of each layer to the structure leads to an increase in the degrees of freedom in the design procedure, thus enabling the higher harmonics to be minimized simultaneously. Figure 2 shows a cross-section of such a multilayer cloak to make a cylindrical structure invisible.

The number of cloaking layers is \( N = 1 \), and their axis is along the \( z \) direction. The radius of the considered invisible cylinder is \( a \), while its electric permittivity and magnetic permeability are \( \varepsilon \) and \( \mu \), respectively. The radius of the \( r \)th cylinder is \( a_r \), while its electric permittivity and magnetic permeability are \( \varepsilon_r \) and \( \mu_r \), respectively.

This problem can be solved for either transverse electrical (TE) or transverse magnetic (TM) polarization. We continue...
our investigation by considering scattering of TM polarization by such cylinders, as this effect is much higher than for TE polarization.

Consider a TM$_z$-polarized plane wave with time dependence of $e^{-j\omega t}$ incident on the structure shown in Fig. 2. The incident wavevector can be written in the form of Eq. (1) [20].

$$E^i = E_0 \hat{a}_z e^{-jk_0 x},$$

(1)

where $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$ is the wavenumber in air.

By transforming from Cartesian to cylindrical coordinates, the tangential incident electric field can be written in the form of Eq. (2) [20].

$$E^i_z = E_0 \sum_{-\infty}^{+\infty} j^{-n} J_n(k_0 \rho) e^{jnp},$$

(2)

The total tangential electric field in air consists of two parts, corresponding to the incident wave and the scattered wave. The electric field of the scattered wave in the $z$ direction can be written according to Eq. (3).

$$E^{\text{scat}}_z = E_0 \sum_{-\infty}^{+\infty} C^{\text{TM}}_{z,n} J_n(k_0 \rho) e^{jnp},$$

(3)

where $J_n$ is the Hankel function of the second kind. For the special case of normal incidence, the electric field in all the layers can be written as

$$E^1_z = E_0 \sum_{-\infty}^{+\infty} j^{-n} C_{1,n}^{\text{TM}} J_n(k \rho)e^{jnp},$$

$$E^2_z = E_0 \sum_{-\infty}^{+\infty} j^{-n} [C_{2,n}^{\text{TM}} J_n(k_{c2} \rho) + C_{2,n}^{\text{TM}} Y_n(k_{c2} \rho)] e^{jnp},$$

$$E^3_z = E_0 \sum_{-\infty}^{+\infty} j^{-n} [C_{3,n}^{\text{TM}} J_n(k_{c3} \rho) + C_{3,n}^{\text{TM}} Y_n(k_{c3} \rho)] e^{jnp},$$

(4)

$$\vdots$$

$$E^r_z = E_0 \sum_{-\infty}^{+\infty} j^{-n} [C_{r,n}^{\text{TM}} J_n(k_{cr} \rho) + C_{r,n}^{\text{TM}} Y_n(k_{cr} \rho)] e^{jnp},$$

$$\vdots$$

$$E^N_z = E_0 \sum_{-\infty}^{+\infty} j^{-n} [C_{N,n}^{\text{TM}} J_n(k_{cN} \rho) + C_{N,n}^{\text{TM}} Y_n(k_{cN} \rho)] e^{jnp},$$

where $J_n$ and $Y_n$ are Bessel functions of the first and second kind, respectively, $C_{z,n}^{\text{TM}}$ are scattering coefficients, $k = \omega \sqrt{\varepsilon \mu}$ and $k_{cr} = \omega \sqrt{\varepsilon_c \mu_c}$ are the wavenumbers in the cylindrical layers.

It is worth noting that, because we neglect the mutual coupling, the magnetic field of the scattered wave in the $z$ direction is zero: $H^{\text{scat}}_z = 0$.

Fig. 2 A cross-section of a multilayer cloak for an infinite cylinder.
To obtain the unknown coefficients in Eq. (4), continuity of the tangential electric and magnetic fields should be applied at all the intersections. The tangential magnetic fields are calculated from Maxwell’s equations as follows [20]:

\[
H_\varphi = -\frac{1}{j\omega \mu} \left[ \frac{\partial}{\partial z} \left( -\frac{1}{j\omega \varepsilon} \frac{\partial H_\varphi}{\partial z} \right) - \frac{\partial E_z}{\partial \varphi} \right].
\]  

(5)

By applying the boundary conditions at the intersections \(\rho = a, a_2, a_3, \ldots, a_N\), the following 2N equations are obtained:

\[
\begin{align*}
C_{TM}^n J_n^1(k_o a) + C_{TM}^n Y_n^1(k_o a) &= C_{TM}^n J_n^1(k_o a_2) + C_{TM}^n Y_n^1(k_o a_2)
\end{align*}
\]

(6)

\[
\begin{align*}
C_{TM}^{i+1, n} J_n^1(k_o a_3) + C_{TM}^{i+1, n} Y_n^1(k_o a_3) &= C_{TM}^{i+1, n} J_n^1(k_o a_2) + C_{TM}^{i+1, n} Y_n^1(k_o a_2)
\end{align*}
\]

\[
\begin{align*}
&\vdots
\end{align*}
\]

\[
\begin{align*}
C_{TM}^{N, n} J_n^1(k_o a_N) + C_{TM}^{N, n} Y_n^1(k_o a_N) &= J_n^1(k_o a_N) + C_{TM}^n H_n^2(k_o a_N)
\end{align*}
\]

(7)

where \(J_n^1, Y_n^1\) are the derivative of Bessel functions of the first and second kind, respectively, \(H_n^{(2)}\) is the derivative of the Hankel function of the second kind with respect to the argument, and \(\eta = \sqrt{\frac{\mu}{\varepsilon}}\) and \(\eta_{c i} = \sqrt{\frac{\mu_{c i}}{\varepsilon_{c i}}}\) are the intrinsic impedances in the layers. Equations (6) form a system of linear and inhomogeneous equations with 2N unknown coefficients. By solving these equations, the scattering coefficients are obtained as

\[
\Delta' = \frac{[\Delta]}{[\Delta']}
\]

(8)

\[
\Delta = \begin{bmatrix}
A_1 \\
A_2 \\
A_3 \\
\vdots \\
A_r \\
A_{N-1} \\
A_N
\end{bmatrix}
\]

(9)

where the coefficients \(A_1, A_2, \ldots, A_r, \ldots, A_N\) and \(B_N\) are 2 × 2N matrices that are evaluated as follows:
\[ A_1 = \begin{bmatrix} -J_n(ka) & J_n(k_c a_c) & Y_n(k_c a_c) & 0 & 0 & \ldots & 0 \\ -\frac{1}{\eta_2} J_n'(ka) & \frac{1}{\eta_2} J_n'(k_c a_c) & -J_n(k_c a_c) & Y_n(k_c a_c) & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 1 \\ \end{bmatrix} \]

\[ A_2 = \begin{bmatrix} 0 & J_n(k_c a_c) & Y_n(k_c a_c) & -J_n(k_c a_c) & Y_n(k_c a_c) & 0 & \ldots & 0 \\ 0 & \frac{1}{\eta_2} J_n'(k_c a_c) & \frac{1}{\eta_2} Y_n'(k_c a_c) & -\frac{1}{\eta_2} J_n'(k_c a_c) & \frac{1}{\eta_2} Y_n'(k_c a_c) & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 1 \\ \end{bmatrix} \]

\[ A_3 = \begin{bmatrix} 0 & J_n(k_c a_c) & Y_n(k_c a_c) & -J_n(k_c a_c) & Y_n(k_c a_c) & 0 & \ldots & 0 \\ 0 & \frac{1}{\eta_2} J_n'(k_c a_c) & \frac{1}{\eta_2} Y_n'(k_c a_c) & -\frac{1}{\eta_2} J_n'(k_c a_c) & \frac{1}{\eta_2} Y_n'(k_c a_c) & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 1 \\ \end{bmatrix} \]

\[ A_r = \begin{bmatrix} 0 & 0 & J_n(k_c a_c) & Y_n(k_c a_c) & -J_n(k_c a_c) & Y_n(k_c a_c) & 0 & \ldots & 0 \\ 0 & 0 & \frac{1}{\eta_2} J_n'(k_c a_c) & \frac{1}{\eta_2} Y_n'(k_c a_c) & -\frac{1}{\eta_2} J_n'(k_c a_c) & \frac{1}{\eta_2} Y_n'(k_c a_c) & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 1 \\ \end{bmatrix} \]

\[ A_{N-1} = \begin{bmatrix} 0 & 0 & J_n(k_c a_c) & Y_n(k_c a_c) & -J_n(k_c a_c) & Y_n(k_c a_c) & 0 \\ 0 & 0 & \frac{1}{\eta_{N-1}} J_n'(k_c a_c) & \frac{1}{\eta_{N-1}} Y_n'(k_c a_c) & -\frac{1}{\eta_{N-1}} J_n'(k_c a_c) & \frac{1}{\eta_{N-1}} Y_n'(k_c a_c) & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & 1 \\ \end{bmatrix} \]

\[ A_N = \begin{bmatrix} 0 & 0 & J_n(k_c a_c) & Y_n(k_c a_c) & -J_n(k_c a_c) & Y_n(k_c a_c) & 0 \\ 0 & 0 & \frac{1}{\eta_N} J_n'(k_c a_c) & \frac{1}{\eta_N} Y_n'(k_c a_c) & -\frac{1}{\eta_N} J_n'(k_c a_c) & \frac{1}{\eta_N} Y_n'(k_c a_c) & 0 \\ \end{bmatrix} \]

Fig. 3 A dielectric cylinder covered by two cloaking layers

Fig. 4 The normalized SCS of a dielectric cylinder with radius of 10 nm covered by two plasmonic layers

( Springer)
Simulation results

In this section, simulation results for a dielectric cylinder covered by two cloaking layers as shown in Fig. 3 are described.

Figures 4 and 5 show the normalized scattering cross-section (SCS) of the cloaked cylinder for two values of the radius, viz. $a = 10\,\text{nm}$ and $a = 30\,\text{nm}$, respectively, with a dielectric constant of 2. Also $a_{c2} = 1.1a$ and $a_{c3} = 1.2a$ are the radii of the first and second covering layer. The values of the electric permittivity coefficients for these layers, obtained by using the Drude model at the desired frequencies, are $\varepsilon_{c1} = 0.15\varepsilon_0$ and $\varepsilon_{c2} = 0.12\varepsilon_0$. Also the losses of the two layers lie in the acceptable range according to the Drude model.

It can be concluded that, by using two cloaking layers, invisibility is obtained at two frequencies. It is also observed that the reduction in the scattering decreases at higher frequencies as a result of the increasing $k_0a$ and because the object becomes electrically larger. Therefore, increasing the higher harmonics of the scattering coefficients leads to a reduction of the cloaking performance, which is in complete accordance with the findings in Ref. [11] for spherical cloaks.

To better understand the performance of such a plasmonic cloak, it is worth examining the behavior of the normalized SCS of the central cylinder with respect to the relative electric permittivity of region 2 in Fig. 3 by changing its electric permittivity (Fig. 6).

In this case, the radius of the central cylinder is chosen as $a = 100\,\text{nm}$, the radii of the plasmonic cloaking layers are $a_{c2} = 1.1a$ and $a_{c3} = 1.2a$, $k_0a = 0.1\,\text{rad}$, and the relative electric permittivity of the cloaked cylinder is chosen as 3, 4, 6, or 10.

As can be seen, for a cylinder with a higher electric permittivity, cloaks with a more negative electric permittivity are needed. As a result, the cloaking performance is clearly degraded. The reason for this is that, with increasing electric permittivity of the cylinder, the scattering coefficients become larger and thus invisibility becomes more difficult to achieve. Therefore, the performance of the plasmonic cloak also decreases.

At this point, it is appropriate to examine the performance of the cylindrical cloak in relation to changes in the radius of the central cylinder in Fig. 3. Figure 7 shows the values of the normalized SCS in terms of the relative electric permittivity of region 2 for radii of the central cylinder of $a = 10$, 50, 100, and 500 nm at a frequency of 100 THz.

As can be seen, the normalized SCS decreases sharply for certain values of the relative electric permittivity of the cloak, and this occurs for all radii of the central cylinder. These values of the relative electric permittivity of the cloak are all negative. It is thus possible to minimize the scattering coefficients in Eq. (6) by using plasmonic or metamaterial cloaks, thereby achieving transparency. It is also observed that the rate of drop of the normalized SCS decreases with increasing radius of the central cylinder. This phenomenon occurs due to the appearance of higher terms in the SCS relation, being consistent with the findings reported in Ref. [18] for the performance of spherical cloaks. Also, the location of the drop in the normalized SCS in Fig. 7a, b is almost the same for cylinders with small radii, but when increasing the radius of the cylinder in Fig. 7c, d, the values of $\varepsilon_c$...
of the relative electric permittivity of the cloak where the SCS drops change. The reason for this is that, for cylinders with a smaller radius, the Bessel functions in the scattering coefficient relations (Eq. 6), can be approximated by a small-argument approximation, and finally a closed form can be obtained to minimize the scattering coefficients. Since this

![Fig. 7](image_url) The normalized SCS of a dielectric central cylinder with radius of a 10 nm, b 50 nm, c 100 nm, and d 500 nm, with respect to the relative electric permittivity of the first cloak

| Ref. | Radius of the cylinder (λ) | RCS reduction (dB) | Cloaking method | Multilayer and multifrequency |
|------|---------------------------|-------------------|----------------|-------------------------------|
| [21] | 0.1                       | 10                | Plasmonic cloaking | ×                             |
| [22] | 0.13                      | 13                | Mantle cloaking   | ×                             |
| [23] | 0.13                      | 8                 | Core–shell nanoparticles | ×                             |
| [24] | 0.12                      | 12                | Graphene metasurface | ×                             |
| [25] | 0.1                       | 13.5              | Graphene monolayer | ×                             |
| [14] | 0.2                       | 10                | Plasmonic cloaking | ✓                             |
| This work | 0.17                     | 26                | Plasmonic cloaking | ✓                             |

RCS radar cross section
closed form is a function of the ratio of the radius of the cylinder to the radius of the cloak, and here this ratio is the same in all the designs in Fig. 7, it is almost the same for small cylinders, but with increasing radius of the cylinder, this approximation is not valid. The location where the SCS drops thus changes with the radius of the cylinder.

The main advantage of the cloaking structure proposed herein in comparison with other literature reports is that, in previous works, such multilayer cloaking structures were not studied analytically using precise mathematical equations, or this was done only for two-layer structures, while the current analysis of these structures is completely accurately and enjoys mathematical support. Obviously, this precise mathematical analysis could also be applied for the particular case of two-layer structures.

Table 1 compares different cloaking techniques. It shows that, considering the radius of the cylinder, the radar cross section (RCS) reduction, and whether they offer multilayer, multifrequency operation, the method presented herein offers good performance.

4 Conclusions

The use of metamaterial or plasmonic cloaks with a negative or near-zero relative electric permittivity to reduce the scattering from a structure by creating reverse polarization and thus invisibility is investigated herein.

A design procedure for the multilayer plasmonic cloak to achieve invisibility for an infinite dielectric cylinder is described. The resulting analysis is completely accurately and enjoys mathematical support. Simulation results for the SCS of cylinders cloaked using two plasmonic layers are presented, verifying the cloaking effect at two frequencies. The behavior of the normalized SCS of the central cylinder with respect to its radius and electric permittivity is also investigated.

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