Deterministic and stochastic influences on Japan and US stock and foreign exchange markets. A Fokker-Planck approach

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Summary. The evolution of the probability distributions of Japan and US major market indices, NIKKEI 225 and NASDAQ composite index, and JPY/DEM and DEM/USD currency exchange rates is described by means of the Fokker-Planck equation (FPE). In order to distinguish and quantify the deterministic and random influences on these financial time series we perform a statistical analysis of their increments $\Delta x(\Delta(t))$ distribution functions for different time lags $\Delta(t)$. From the probability distribution functions at various $\Delta(t)$, the Fokker-Planck equation for $p(\Delta x(t), \Delta(t))$ is explicitly derived. It is written in terms of a drift and a diffusion coefficient. The Kramers-Moyal coefficients, are estimated and found to have a simple analytical form, thus leading to a simple physical interpretation for both drift $D^{(1)}$ and diffusion $D^{(2)}$ coefficients. The Markov nature of the indices and exchange rates is shown and an apparent difference in the NASDAQ $D^{(2)}$ is pointed out.

Key words. Econophysics; Probability distribution functions; Fokker-Planck equation; Stock market indices; Currency exchange rates

1 Introduction

Recent studies have shown that the power spectrum of the stock market fluctuations is inversely proportional to the frequency on some power, which points to self-similarity in time for processes underlying the market [1, 2]. Our knowledge of the random and/or deterministic character of those processes is however limited. One rigorous way to sort out the noise from the deterministic components is to examine in details correlations at different scales through the so called master equation, i.e. the Fokker-Planck equation (and the subsequent Langevin equation) for the probability distribution function (pdf) of signal increments [3]. This theoretical approach, so called solving the inverse problem, based on rigorous statistical principles [4, 5], is often the first step in sorting out the best model(s). In this paper we derive FPE,
directly from the experimental data of two financial indices and two exchange rates series, in terms of a drift $D^{(1)}$ and a diffusion $D^{(2)}$ coefficient. We would like to emphasize that the method is model independent. The technique allows examination of long and short time scales on the same footing. The so found analytical form of both drift $D^{(1)}$ and diffusion $D^{(2)}$ coefficients has a simple physical interpretation, reflecting the influence of the deterministic and random forces on the examined market dynamics processes. Placed into a Langevin equation, they could allow for some first step forecasting.

2 Data

We consider the daily closing price $x(t)$ of two major financial indices, NIKKEI 225 for Japan and NASDAQ composite for US, and daily exchange rates involving currencies of Japan, US and Europe, JPY/DEM and DEM/USD from January 1, 1985 to May 31, 2002. Data series of NIKKEI 225 (4282 data points) and NASDAQ composite (4395 data points) and are downloaded from the Yahoo web site (http://finance.yahoo.com/). The exchange rates of JPY/DEM and DEM/USD are downloaded from http://pacific.commerce.ubc.ca/xr/ and both consists of 4401 data points each. Data are plotted in Fig. 1(a-d). The DEM/USD case was studied in [3] for the 1992-1993 years. See also [6], [8-10] and [11] for some related work and results on such time series signals, some on high frequency data, and for different time spans.

3 Results and discussion

To examine the fluctuations of the time series at different time delays (or time lags) $\Delta t$ we study the distribution of the increments $\Delta x = x(t + \Delta t) - x(t)$. Therefore, we can analyze the fluctuations at long and short time scales on the same footing. Results for the probability distribution functions (pdf) $p(\Delta x, \Delta t)$ are plotted in Fig. 2(a-d). Note that while the pdf of one day time delays (circles) for all time series studied have similar shapes, the pdf for longer time delays shows fat tails as in [2] of the same type for NIKKEI 225, JPY/DEM and DEM/USD, but is different from the pdf for NASDAQ.

More information about the correlations present in the time series is given by joint pdf’s, that depend on $N$ variables, i.e. $p^N(\Delta x_1, \Delta t_1; \ldots; \Delta x_N, \Delta t_N)$. We started to address this issue by determining the properties of the joint pdf for $N = 2$, i.e. $p(\Delta x_2, \Delta t_2; \Delta x_1, \Delta t_1)$. The symmetrically tilted character of the joint pdf contour levels (Fig. 3(a-c)) around an inertia axis with slope $1/2$ points out to the statistical dependence, i.e. a correlation, between the increments in all examined time series.

The conditional probability function is
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Fig. 1. Daily closing price of (a) NIKKEI 225, (b) NASDAQ, (c) JPY/DEM and (d) DEM/USD exchange rates for the period from Jan. 01, 1985 till May 31, 2002

\[
p(\Delta x_{i+1}, \Delta t_{i+1}|\Delta x_i, \Delta t_i) = \frac{p(\Delta x_{i+1}, \Delta t_{i+1}; \Delta x_i, \Delta t_i)}{p(\Delta x_i, \Delta t_i)}
\]

(1)

for \( i = 1, ..., N - 1 \). For any \( \Delta t_2 < \Delta t_i < \Delta t_1 \), the Chapman-Kolmogorov equation is a necessary condition of a Markov process, one without memory but governed by probabilistic conditions

\[
p(\Delta x_2, \Delta t_2|\Delta x_1, \Delta t_1) = \int d(\Delta x_1)p(\Delta x_2, \Delta t_2|\Delta x_1, \Delta t_i)p(\Delta x_1, \Delta t_i|\Delta x_1, \Delta t_1).
\]

(2)

The Chapman-Kolmogorov equation when formulated in differential form yields a master equation, which can take the form of a Fokker-Planck equation [4]. For \( \tau = \log_2(32/\Delta t) \),

\[
\frac{d}{d\tau}p(\Delta x, \tau) = \left[ -\frac{\partial}{\partial \Delta x} D^{(1)}(\Delta x, \tau) + \frac{1}{2} \frac{\partial^2}{\partial (\Delta x)^2} D^{(2)}(\Delta x, \tau) \right] p(\Delta x, \tau)
\]

(3)
in terms of a drift \( D^{(1)}(\Delta x, \tau) \) and a diffusion coefficient \( D^{(2)}(\Delta x, \tau) \) (thus values of \( \tau \) represent \( \Delta t_i, i = 1, ... \)).
The coefficient functional dependence can be estimated directly from the moments $M^{(k)}$ (known as Kramers-Moyal coefficients) of the conditional probability distributions:

$$M^{(k)} = \frac{1}{\Delta \tau} \int d\Delta x' (\Delta x' - \Delta x)^k p(\Delta x', \tau + \Delta \tau|\Delta x, \tau)$$  \hspace{1cm} (4)$$

$$D^{(k)}(\Delta x, \tau) = \frac{1}{k!} \lim_{\Delta \tau \to 0} M^{(k)}$$  \hspace{1cm} (5)$$

for $\Delta \tau \to 0$. The functional dependence of the drift and diffusion coefficients $D^{(1)}$ and $D^{(2)}$ for the normalized increments $\Delta x$ is well represented by a line and a parabola, respectively. The values of the polynomial coefficients are summarized in Table 1 and Fig. 4.

The leading coefficient ($a_1$) of the linear $D^{(1)}$ dependence has approximately the same values for all studied signals, thus the same deterministic noise (drift coefficient). Note that the leading term ($b_2$) of the functional dependence of diffusion coefficient of the NASDAQ closing price signal is about
Fig. 3. Typical contour plots of the joint probability density function \( p(\Delta x_2, \Delta t_2; \Delta x_1, \Delta t_1) \) of (a) NIKKEI 225, (b) NASDAQ closing price signal and (c) JPY/DEM and (d) DEM/USD exchange rates for \( \Delta t_2 = 1 \) day and \( \Delta t_1 = 3 \) days. Contour levels correspond to \( \log_{10} p = -1.5, -2.0, -2.5, -3.0, -3.5 \) from center to border.

Table 1. Values of the polynomial coefficients defining the linear and quadratic dependence of the drift and diffusion coefficients

|            | \( a_1 \) | \( a_0 \) | \( b_2 \) | \( b_1 \) | \( b_0 \) | \( \sigma \) |
|------------|---------|---------|---------|---------|---------|---------|
| NIKKEI 225 | -0.54   | 0.002   | 0.18    | -0.004  | 0.003   | 1557.0  |
| NASDAQ     | -0.49   | -0.0004 | 0.30    | -0.010  | 0.001   | 198.11  |
| JPY/DEM    | -0.55   | 0.002   | 0.17    | -0.002  | 0.004   | 2.9111  |
| DEM/USD    | -0.49   | -0.001  | 0.16    | -0.004  | 0.004   | 0.0808  |

twice the leading, i.e., second order coefficient, of the other three series of interest. This can be interpreted as if the stochastic component (diffusion coefficient) of the dynamics of NASDAQ is twice larger than the stochastic components of NIKKEI 225, JPY/DEM and DEM/USD. A possible reason for such a behavior may be related to the transaction procedure on the
Fig. 4. Functional dependence of the drift and diffusion coefficients $D^{(1)}$ and $D^{(2)}$ for the pdf evolution equation (3); $\Delta x$ is normalized with respect to the value of the standard deviation $\sigma$ of the pdf increments at delay time 32 days: (a,b) NIKKEI 225 and (c,d) NASDAQ closing price signal, (e,f) JPY/DEM and (g,h) DEM/USD exchange rates.
Fig. 5. Equal probability contour plots of the conditional pdf $p(\Delta x_2, \Delta t_2 | \Delta x_1, \Delta t_1)$ for two values of $\Delta t$, $\Delta t_3 = 8$ days, $\Delta t_2 = 1$ day for NASDAQ. Contour levels correspond to $\log_{10} p = -0.5, -1.0, -1.5, -2.0, -2.5$ from center to border; data (solid line) and solution of the Chapman Kolmogorov equation integration (dotted line); (b) and (c) data (circles) and solution of the Chapman-Kolmogorov equation integration (plusses) for the corresponding pdf at $\Delta x_2 = -50$ and $+50$ NASDAQ. Our numerical result agrees with that of ref. [3] if a factor of ten is corrected in the latter ref. for $b_2$.

The validity of the Chapman-Kolmogorov equation has also been verified. A comparison of the directly evaluated conditional pdf with the numerical integration result (2) indicates that both pdf’s are statistically identical. The more pronounced peak for the NASDAQ is recovered (see Fig. 5). An analytical form for the pdf’s has been obtained by other authors [6, 10] but with models different from more classical ones [8].

4 Conclusion

The present study of the evolution of Japan and US stock as well as foreign currency exchange markets has allowed us to point out the existence of deterministic and stochastic influences. Our results confirm those for high frequency
The Markovian nature of the process governing the pdf evolution is confirmed for such long range data as in [3, 7, 8] for high frequency data. We found that the stochastic component (expressed through the diffusion coefficient) for NASDAQ is substantially larger (twice) than for NIKKEI 225, JPY/DEM and DEM/USD. This could be attributed to the electronic nature of executing transactions on NASDAQ, therefore to different stochastic forces for the market dynamics.

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