Light Front Theory Of Nuclear Matter

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A relativistic light front formulation of nuclear dynamics is applied to infinite nuclear matter. A hadronic meson-baryon Lagrangian, consistent with chiral symmetry, leads to a nuclear eigenvalue problem which is solved, including nucleon-nucleon NN correlations, in the one-boson-exchange approximation for the NN potential. The nuclear matter saturation properties are reasonably well reproduced, with a compression modulus of 180 MeV. We find that there are $\approx 0.05$ excess pions per nucleon.
An important class of high-energy experiments involving nuclear targets is most easily understood using light-front variables. Consider the lepton-nucleus deep inelastic scattering experiments \[ \text{Refs. [5,6].} \] which found a significant difference between the parton distributions of free nucleons and nucleons in a nucleus. This is interpreted as a $\sim 10\%$ shift in the momentum distribution of valence quarks towards smaller values of the Bjorken variable $x_{Bj}$. The Bjorken variable is a ratio of the plus-momentum $p^+ = p^0 + p^3$ of a quark to that of the target. Treating the nucleus as a collection of nucleons gives $x_{Bj} = p^+/k^+$, where $k^+$ is the plus momentum of a bound nucleon. The spatial variable canonical to $k^+$ is $x^- = x^0 - x^3$, and the time variable is $x^0 + x^3 = x^+$. To use these variables is to use the light front variables of Dirac. See the recent reviews [3].

Deep inelastic scattering depends on the probability $f(k^+)$ that a bound nucleon has a momentum $k^+$. Other nuclear reactions, such as $(e,e')$ and $(p,2p)$ depend also on this very same probability $f$. The quantity $f(k^+)$ is simply related to the square of the ground state wave function, provided light front dynamics are used. In the equal-time formulation $f(k^+)$ is obtained as a response function that depends on matrix elements between the ground and all excited states.

The use of light front variables is convenient for interpreting experiments, but one is faced with computing the nucleon field equations using light front variables. We introduce here a theory of infinite nuclear matter, using light front dynamics, in which the effects of correlations are taken into account. This is a light front Brueckner theory of nuclear matter.

Light front dynamics is a three-dimensional Hamiltonian formalism. A Lagrangian is chosen and one derives field equations which allow the elimination of dependent degrees of freedom. The light front Hamiltonian $P^-$ is obtained from $T^{+-} (T^{\mu\nu}$ is the energy momentum tensor), and $P^+$, the plus-momentum operator is obtained from $T^{++}$. We use a chiral model, detailed in Refs. [3,4], in which the nucleon constituents are nucleons $\psi$, pions, scalar mesons, vector mesons, and $\eta$ mesons.

We start by briefly reviewing some of the necessary formal details presented in Ref. [5] (which presented a mean field calculation of the properties of nuclear matter). Our specific equations employ the rest frame of nuclear matter. The derivation of $T^{+-}$ is accomplished by solving for the dependent nucleon fields in terms of the independent ones. The simple completion of this task requires setting the $+\mu$ components of the vector fields $V^\mu$ to zero. This is accomplished using a transformation \[ \text{on the nucleon fields, in which one replaces } V^\mu \text{ with other fields } V^\mu (V^+ = 0) \text{ which enter in the nucleon field equations. We carry this feature through in our calculations (including the related mean fields), but drop the bar to simplify the notation. More detailed explanations are given in Refs. [3] and [4].} \]

We now confront the eigenvalue problem for the ground state of infinite nuclear matter:

$$P^- |\Psi\rangle = M_A |\Psi\rangle. \quad (1)$$

For a nuclear system at rest we must also satisfy $P^+ |\Psi\rangle = M_A |\Psi\rangle$. The operator $P^-$ can be divided into kinematic $P_0^-(N)$ and interaction $J$ terms:

$$P^- = P_0^-(N) + J \quad (2)$$

in which $P_0^-(N)$ is the kinetic contribution to the $P^-$ operator, giving $\frac{p^2 - m^2}{p^0}$ for the minus-momentum of free nucleons. The operator $J$ is the sum of three terms $J \equiv v_1 + v_2 + v_3 : v_1$ gives the single meson-nucleon vertex functions; $v_2$ accounts for meson emission followed by instantaneous nucleon propagation followed by another meson emission; and, $v_3$ accounts for the instantaneous propagation of vector mesons.

The traditional path of using a two-nucleon potential and temporarily eliminating the meson degrees of freedom by adding and subtracting a two-nucleon potential to the Hamiltonian is followed. The terms involving the difference between $J$ and the two-nucleon potential are handled later using perturbation theory. Then the effective Lagrangian is $L_K \equiv \bar{\psi}(i\gamma \cdot \partial - M)\psi - \frac{K}{2}$ with the two-nucleon interaction density $K/2$. Using $L_K$, gives the corresponding $P^\pm$ operators, with $P_K^- = P_0^-(N) + K$. $K$ is the volume integral of $K$, and

$$P^- = P_K^- + J - K + P_0^-(m), \quad (3)$$

where $P_0^-(m)$ accounts for the non-interacting mesonic contribution to $P^-$. The purely nucleonic part of the full wave function is defined as $|\Phi\rangle$, and is the solution of the light front Schrödinger equation

$$P_K^- |\Phi\rangle = (P_0^-(N) + K) |\Phi\rangle = M_0 |\Phi\rangle. \quad (4)$$

The interaction $K$ is obtained using second-order perturbation theory. Consider the scattering process $1+2 \rightarrow 3+4$. We find

$$\langle 3,4 |K| 1,2 \rangle = \langle 3,4 |v_1 g(P_{ij}^-) v_1 + v_3 | 1,2 \rangle, \quad (5)$$

with $g_0(P_{ij}^-) \equiv \frac{1}{P_{ij}^- - P_0^-}$, where $P_{ij}^-$ is the negative component of the total initial (12) or final (34) momentum which here are the same.

We discuss the energy denominator $(P_{ij}^- - P_0^-) k^+$. Suppose that $k_{ij}^+ > k_3^+$. Then the emitted meson of mass $\mu$ and momentum $k$, with $k^+ = k_{ij}^- - k_3^+$, $k_\perp = k_{ij\perp} - k_3\perp$, $k^- = k_3^+ + \mu^2$, and $k^+(P_{ij}^- - P_0^-) = k^+(k_{ij}^+ - k_3^+) - k_\perp^2 + \mu^2 = q^2 - \mu^2$, where $q \equiv k_1 - k_3$. That this form is the Lorentz-invariant inverse of the Klein-Gordon propagator indicates that $K$ accounts for the nucleon-nucleon...
one-boson exchange potential. Note that \( q^2 = q_0^2 - q^2 \), with \( q_0^2 \) accounting for retardation effects.

The interaction \( K \) is strong, so that we sum the unitary set of iterations to obtain the two-nucleon integral equation for the transition matrix \( T \):

\[
\langle 3, 4|T|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \int \sum_{\lambda_5, \lambda_6} \langle 3, 4|V|5, 6 \rangle \times \frac{2M^2}{\alpha(1 - \alpha)} \frac{d^3p_{1\perp} \, da}{p_{1\perp}^2 - p^2 + M^2 + i\epsilon} \langle 5, 6|T|1, 2 \rangle. \tag{6}
\]

The plus-momentum variable is expressed in terms of a momentum fraction \( \alpha \) such that \( p_{5\perp}^\alpha = \alpha p_{1\perp}^\alpha \), and \( p_{\perp} = (1 - \alpha)p_{5\perp} - \alpha p_{6\perp} \). The potential \( V \) is a kinematic factor times \( K \).

We mention again that \( V \) is derived using the one boson exchange approximation, so that effects of multimesons in flight are not included in Eq. (6). The effects of the stretched box diagram has been studied in the light front calculions of Ref. [3], and can be important in maintaining the full Lorentz invariance of the scattering amplitude. The present results (6) for the on-shell \( T \)-matrix are invariant under boosts in the \( z \)-direction.

The Weinberg-type [4] equation [4], can be re-expressed using the variable transformation [4]: \( \alpha = \frac{E(p) + p^2}{2E(p)} \), with \( E(p) \equiv \sqrt{p \cdot p + M^2} \). The result is:

\[
\langle 3, 4|T|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \int \sum_{\lambda_5, \lambda_6} \langle 3, 4|V|5, 6 \rangle \times \frac{M^2}{E(p) p_{1\perp}^2 - p^2 + i\epsilon} \langle 5, 6|T|1, 2 \rangle, \tag{7}
\]

which shows a manifest rotational invariance.

The one-boson exchange potential \( V \) obtained with our formalism is constructed elsewhere [4]. We choose the meson-nucleon coupling constants and include the effects of form factors to reproduce the NN data. This light front work yields potentials and transition matrix elements essentially identical to the Bonn OBEP potentials [12] because the differences arising from including retardation effects, and using Eq. (6) are very small. Thus we find a good description of the deuteron and the two-nucleon scattering data below the inelastic threshold [6].

Now that \( V \) is determined, we turn to solving Eq. (6). It is worthwhile to begin by explaining how our light front quantized calculation of the energy and density of nuclear matter includes the effects of two-nucleon NN correlations. We employ a light front version of the standard technique of obtaining the G-matrix (see Eqs. (11) and (12) below) which is a nuclear matter generalization of the T-matrix of Eq. (6). We then determine the nuclear density by finding the Slater determinant \( |\phi > \) which minimizes the expectation value of \( P^- \) subject to the constraint that the expectation values of \( P^+ \) and \( P^- \) are equal. The single particle wave functions that make up the determinant \( |\phi > \) are to be given in Eq. (9), which is not the Dirac equation. The self-consistent fields \( U \) which enter are to be obtained from the expectation value of the G-matrix in \( |\phi \rangle \). The use of symmetries [13] causes the only non-vanishing mean-fields to be scalar \( U_S \) and vector fields \( U^\mu_V \). Neglect of the very small spatial component of \( U^\perp_V \) leaves us with \( U^\perp_V \).

We now obtain the Slater determinant \( |\phi \rangle \) and the related scalar \( U_S \) and vector \( U^\perp_V \) potentials. The eigenvalues and eigenvectors of the resulting light-front mean field Hamiltonian \( P^-_{MF} \) are easy to obtain, and can be chosen to best approximate the effects of the two-nucleon interaction. The mean field light front Hamiltonian density \( T^+_{MF} \) is obtained and its volume integral is a single-nucleon operator \( P^-_{MF} \). If one expands \( \phi \) in terms of the single particle wave equation including \( U \) as the only interaction, one finds that \( P^-_{MF} = P^-_0 \). The ground state eigenvector of this operator is a Slater determinant denoted as \( |\phi \rangle \): \( P^-_0(N)|\phi \rangle = m_0|\phi \rangle \), and

\[
|\Phi \rangle = |\phi \rangle + \frac{1}{M_0 - P^-_0(N) - K} \Lambda \Phi |\phi \rangle, \tag{8}
\]

where \( \Lambda = 1 - |\phi \rangle \langle \phi | \). The single nucleon orbitals \( |i \rangle \) are obtained from [6]:

\[
(P^-_{MF} - U^-_V)|i \rangle^+ = (\alpha_{1\perp} \cdot \hat{k}_{1\perp} + \beta(M + U_S))|i \rangle^+, \quad k^+ |i \rangle^- = (\alpha_{1\perp} \cdot \hat{k}_{1\perp} + \beta(M + U_S))|i \rangle^-, \quad (P^-_{MF} - U^-_V)|i \rangle^+ = \frac{k_{1\perp}^2 + (M + U_S)^2}{k_i^2} |i \rangle^+. \tag{9}
\]

The subscripts \( \pm \) refer to the independent (+) and dependent (−) projections of the nucleon fields.

Using standard manipulations on Eq. (8), and keeping correlations between pairs of nucleons only leads to:

\[
M_0 - m_0 \approx \langle \phi \rangle \frac{1}{2} \sum_{i,j} \Gamma_{i,j}(P^-_{ij})|\phi \rangle, \tag{10}
\]

where \( \Gamma_{i,j} \) is a two-nucleon operator:

\[
\Gamma_{i,j}(P^\perp_{ij}) = K_{ij} + K_{ij} \frac{1}{P_{ij} - \Lambda P^-_0(N) \Lambda} \Gamma_{i,j}(P^\perp_{ij}). \tag{11}
\]

The notation \( i, j \) refers to the nucleon orbitals of Eq. (6). The approximation that \( U \) is independent of orbital \( i \) causes the vector potential to cancel out in the denominators e.g. \( P_{ij} - (p_{5\perp} + p_{6\perp}) \). Starting with Eq. (11), we obtain an equivalent three-dimensional, manifestly rotationally invariant integral equation by using the medium-modified version of the variable transformation [4]: \( \alpha = \frac{E^\perp + p^\perp}{2E_p} \), with \( E_p^\perp \equiv \sqrt{p \cdot p + M^2} \). The result is:

\[
\langle 3, 4|G|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \int \sum_{\lambda_5, \lambda_6} \langle 3, 4|V|5, 6 \rangle \times \frac{M^2}{E_p^\perp p^\perp} \frac{d^3p \, Q}{p^2} \langle 5, 6|G|1, 2 \rangle, \tag{12}
\]
where \( G \) is related to \( \Gamma \) by a kinematic factor. The operator \( Q \) is the two-body version of \( A \) and projects the momenta \( p_5 \) and \( p_6 \) above the Fermi sea. The evaluation of \( V \) using the basis of Eq. 4 is the essential difference between relativistic and non-relativistic Brueckner theory.

We now specify the Slater determinant \( | \phi \rangle \). The occupied states are to fill up a Fermi sea, defined in terms of a Fermi momentum, \( k_F \), that is the magnitude of a three-vector. This three vector can be defined \( k^3 \) as: \( k^+ = E^+(k) + k^3 \), which specifies \( k^3 \). Computing the energy and plus momentum then proceeds by taking the expectation values of \( T^{+-} \) and \( T^{++} \). Note that the resulting formulae can be cast in a more familiar form by using a discrete representation of the single nucleon states, a set of spinors: \( | \alpha \rangle \), with \( \alpha \equiv k, \lambda \) such that \( \langle \alpha | \alpha \rangle = 1 \). These light front spinors of Eqs. (9) can be related to equal time spinors. Multiply the first of (9) by \( \gamma^+ \) and the second by \( \gamma^- \). Then add the two equations to obtain a new equation. Its solution turns out to be a spinor \( | x | \alpha \rangle \) which differs from the equal time (ET) spinor by only a phase factor, \( \exp(-i\frac{U_\alpha x^-}{2}) \), which cancels out of matrix elements under our approximation that \( U_\alpha \) is a constant. Then the Brueckner light front Hartree-Fock (BHF) approximation is to the mean-field is given by

\[
U(\alpha) = \sum_{\beta < F} Re\langle \alpha \beta | G | \alpha \beta \rangle_\alpha . \tag{13}
\]

The nuclear matter \( G \)-matrix is the solution of the integral equation, \( \langle x | \Phi \rangle \) in a plane wave basis. We use the implicit definition of \( k^3 \) to obtain the total \( P \) and relative three-momenta of two nucleons interacting in nuclear matter. An angle averaging procedure \( \langle x | \Phi \rangle \), is used for \( P \) and to obtain the Pauli operator \( Q \). The solution of the nuclear matter \( G \) matrix involves a self-consistency, since the solution of Eq. (12) for \( G \) requires knowledge of \( M^* \) which, in turn, is determined from \( G \) via Eq. (13). Solutions are obtained using an iterative procedure. We obtain the nuclear mass from the average of \( P^2 \): \( M_0 = \frac{1}{2} \sum_{\alpha < F} \epsilon_\alpha + \frac{1}{2} \sum_{\alpha, \beta < F} \langle \alpha \beta | G | \alpha \beta \rangle_\alpha \).

The results for the binding energy per nucleon \( E/A \) in nuclear matter as a function of density, are shown in Fig. 1 as the solid line. The curve has a minimum at \( E/A = -14.71 \text{ MeV} \) and \( k_F = 1.37 \text{ fm}^{-1} \), and the incompressibility (compression modulus) is \( K = 180 \text{ MeV} \) at the minimum. These values agree well with the empirical values \( E/A = -16 \pm 1 \text{ MeV} \), \( k_F = 1.35 \pm 0.05 \text{ fm}^{-1} \), and \( K = 210 \pm 30 \text{ MeV} \). Note also that the effect of using non-relativistic Brueckner theory is that the saturation density is predicted too high, but the incompressibility is about the same as ours \( \approx 0.7 \). Comparison with the results of an earlier relativistic calculation shows that the only significant difference occurs for the incompressibility which is reduced from the earlier value of 250 MeV. This is due to using medium-modified single-particle energies in the evaluation of the retardation term \( \bar{M} \), and also to using new values of the mean fields: \( M^*(k_F) = M_N + U_S = 718 \text{ MeV} \) and \( U_V = 164.4 \text{ MeV} \). These potentials have magnitudes that are considerably smaller than those obtained using mean field theory \( \bar{M} \).

![FIG. 1. Binding Energy per nucleon in nuclear matter, E/A as a function of Fermi momentum kF. The solid line is our result using light-front Brueckner theory. The dotted curve is obtained when the medium effect on meson retardation is omitted. The dashed line is the result from nonrelativistic Brueckner theory. The box represents the range of values allowed by extrapolation from data.](image-url)
ground state of nuclear matter. Using $M^*(k_F) = 744$ MeV [20] and neglecting the influence of two-particle-two-hole states to approximate $f(k^+)$ [3] shows that nucleons carry 81% (as opposed to the 65% of mean field theory [2]) of the nuclear plus momentum. This represents a vast improvement in the description of nuclear deep inelastic scattering. The minimum value of the ratio $F_{2A}/F_{2N}$, obtained from the convolution formula

$$F_{2A}(x) = \frac{x}{A} \int_0^A dy f(y) F_{2N}(x/y), \quad (16)$$

where $y = A k^+/(M_N + E/A)$, and $x = x_{Bj} M_N/M_A$, is increased by a factor of twenty towards the data as extrapolated in Ref. [23]. But this calculation provides only a lower limit of the nucleon contribution because of the neglect of effects of the two-particle-two-hole states [22].

Turn now to the experimental information about the nuclear pionic content. The Drell-Yan experiment on nuclear targets [24] showed no enhancement of nuclear pions within an error of about 5%-10% for their heaviest target. No substantial pionic enhancement is found in $(p,n)$ reactions [25]. Understanding this result is an important challenge to the understanding of nuclear dynamics [26]. Here we have a good description of nuclear dynamics, and our 5% enhancement is consistent [27], within errors, with the Drell-Yan data.

This paper contains a new relativistic light-front theory of nuclear matter which leads to a good description of the binding energy, density, and incompressibility of nuclear matter. The use of a meson-nucleon Lagrangian enables us to also compute the mesonic content of the wave function using a consistent approach represented by Eqs. (1), (3), (4) and (14). The results are not in conflict with extrapolations of deep inelastic scattering and Drell-Yan data to nuclear matter.

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[20] $U$ depends on momentum, so to use a constant value is to take an average. Eqs. (12) and (13) are non-linear, and the solutions have a spread of values. Indeed, $M^*(k_F) = 744$ also leads to good saturation properties.
[21] The relatively small value of $N_p$ indicates that the effects of $J - K$ in Eq. (3) are small, and that $M_0$ is a good approximation to $M_A$.
[22] We estimate that nucleons with momentum greater than $k_F$ would substantially increase the computed ratio $F_{2A}/F_{2N}$ because $F_{2N}(x)$ decreases very rapidly with increasing values of $x$ and because $M^*$ would increase at high momenta.
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