Visual reasoning and the perception of forms

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Abstract. Whenever a new device enables our senses to access an uncharted sensible world, our experience needs to be widened to be able to embrace it. Many animals don’t recognize themselves in mirrors. Thus, mirrors found in ancient Egyptian tombs bear witness to a device that involved a widening of human experience. While by then reflectors were commonly believed to hold the spirit of their beholder, they also provided the motivating force for use of geometry as a logical framework, rather than the form of the outer world. Euclid of Alexandria conceived of geometric constructions and their rules as the connecting link between visual world and hypothetical-deductive reasoning. Yet, he didn’t have a clue about receivers. In our view the problem of linking received information in an image format to a mathematical space cannot be solved once and for all, but rather needs to be posed and understood afresh once in a while. All the more so in an information and telecommunication era, when the techniques of acquisition and rendering of visual information have been extended well beyond the domain of optical instruments, and the language of mathematics has advanced to a different level of proficiency.

1. The camera and the eye
The analogy between the physical functioning of the eye and that of a *camera obscura* is very old [1]. However, while modern cameras successfully avail of diffractive optics designs to capture permanent images on a flat physical medium, progress in the clarification of the overall processes involved in vision lags behind. The inquiry on the mechanisms governing eye coordination and those regulating the microsaccades occurring during fixation is still very young, and the path to understand conversion of binocular images into a perception of the shape of our world is scantily delineated. As regards space, even the most advanced branch of artificial intelligence dealing with the human visual system, computer vision, refers back to geometry when accounting for it. Surely, *Euclid’s Elements* – briefly, geometry – have regard to *visual reasoning*. In the Alexandrian epoch the interest in sight had already strongly emerged, indeed [2]. Furthermore, Euclid’s so called minor works testify to his deeper engagement in the logical description of visual perception, presumably owing to the concomitant anatomo-physiological studies of Herophilus [3]. In the oldest surviving formal study in optics he defines the eye gaze direction by resorting to the straight line as an element, and envisages the scene on which an eye casts a glance as limited by the base of a cone of lines whose vertex is at the eye. Avowedly mammalian eyes, like mirrors, reconstruct a whole three dimensional scene *at a glance*. The visible world in front of each eye is firstly imagined internally, then that fleeting real image is seized by both retinas. Directly visualized things and their images are sensed by the animals with foveal vision by means of tiny movements of the eyes.
2. The six postulates of Catoptrics and the laws of reflection

Euclid chooses to deal with the property of things of being visible while remaining outside the psycho-physiological doctrines of visual perception debated by then. Possibly sharing Aristotle’s belief that, under solar lighting, colors and contrast are directly sensed by the eyes, while perception of forms and their discernment (appereception) require further processing, he disregards colors altogether and focuses on the interpretive levels of sight. While in the Elements geometric constructions support logical deductions, reciprocally, in his Optics and Catoptrics reasoning is the key to interpret the viewed things as extended in space. The interpretation of the world we see and the development of geometric reasoning go hand in hand. In reviewing this ancient landmark, we highlight some kind of mix-up involving the location of reflected images [4], believing that the more deeply a stereotype is rooted, the more difficult it is to become aware of it.

Figure 1: Euclid’s cathetus rule & reflection law. As the eye V sees itself in B, so does the Ankh O reflect itself in A (perpendicular ray)
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\frac{BC}{BC} = \frac{VB}{OA}, \quad VCB \text{ and } OCA \text{ are similar triangles, thus the angles in } C \text{ are equal.}
\]

Figure 2: Bronze mirror N. 2164. Musée du Louvre.

Euclid’s Catoptrics begins by stating six observational postulates that have been discussed in full in the literature [3]. Possibly, so far back in time there was still no sharp distinction between reflection and refraction [5]. Whereas Euclid’s inquiry is aimed at locating images in diffuse lighting, his wording of the postulates refers to a material thing being gazed through the mirror (postulate 2), and carefully avoids the notion of image (εἰδωλον, eidolon). Figure 1 shows the symmetrical layout he conceived to describe reflection. His approach is as follows:

(i) Mindfulness that the visual experience is fundamental, but not exhaustive: the relationship between a thing and what is seen in the mirror is nonreciprocal. An image can be concealed from view while the thing is still there (postulate 1), not the other way around.

(ii) Belief that, looking at a mirror with a single eye, just the thing itself is seen. The explanation correlated to that belief assumes sight lines to be projected from the tip of the visual cone at the eye to the surface of the mirror. Binocular vision would require dealing with a more general ruled surface.

(iii) Wish to describe the seen thing like a trompe l’œil perspective clinging to the mirror or transparent material surface (postulate 3). Over the following centuries, the restatement of the ensuing reflection law as a rebounding of light off the mirror would wipe out his attempt to flatten the image on the surface of the mirror (postulates 4, 5,) by comparison with the surfacing of the image of a coin in a bowl when water is poured in (postulate 6) [6].

By all accounts, Kepler put optics on a firmer ground than Euclid by stating that a real image is generated by bringing light to focus point by point on a screen placed where it forms, no matter where the observer is [7]. In our opinion, except for the different choice of the screen, being it the

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1 Those preliminary statements are listed here, for example: Euclid, Catoptrics, Definitions

2 Alexandrian philosophers were fascinated by problems like how to show an image while concealing the thing, or else how to make one’s image change in a specified manner, e.g. multiplying one’s nose, or eyes.
mirror itself for Euclid, the functioning of a reflector receiver is misjudged: it doesn’t perform like the projector screen facing an image projector. In hindsight, perhaps Euclid’s unwillingness to try his hand at a piece of information received from the mirror has a shallow explanation: manufacturing accurate optical mirrors was still a long way in the future. As shown in figure 2, Egyptian mirrors of polished bronze, or silver, simply didn’t evoke the idea of one being able to climb through the mirror into the space beyond it. Although the ultimate seat of perceptions is – and was known to be – the encephalon, the relationship between a directly seen thing and its observed mirror-image can be represented as a geometric transformation of space in itself. Except that the latter notion was only formalized in mathematics starting from the 17th century, and then as a new, non-Greek conception of geometry [8].

3. The geometric properties of the space of everyday life and the eidolon

After Euclid (about 300 B.C.), the Platonic tradition gradually prevails, that identifies Euclidean geometry with the idea of the space we live in. Despite meanwhile the path of visual ray has been justified by extremal principles, Ptolemy (about 100-170 A.D.) sticks to Euclid’s rules and undertakes a painstaking quantitative analysis of location, size, shape, motion and rest of the eidola with respect to the mirror and the beholder [9]. To perform the experiment depicted in figure 3 he takes a peg whose color can be spotted by watching through a slit. In the scheme of his experimental layout the outer circle has center $A$ and radius $AD$. Let’s label a flat, a convex, and a concave mirror of the same radius respectively $GAE$, $TAK$, and $ZAH$ (graphed by a solid line). For all mirrors, if the eye aiming at $A$ through the slit spots a color, this is aligned with the eye and with $A$. Let the eye be above the center-point of the mirror, then an unusual place, figure 4 shows how to locate the geometric locus of the eidolon at the intercept of the visual ray and the perpendicular ray from the object to the mirror through the center of curvature, possibly produced. Since, by experiment, the thing is arranged according to the reflection law, any intangible thing seen through the slit is only a sham. Knowledge of that law allows correcting for visual distortions.

Figure 3: Ptolemy’s experimental layout for a concave mirror $ZAH$. The eye is in the location where it should spot the eidolon because of the broken visual ray.

Figure 4: Extension to a concave mirror of Euclid’s cathetus rule. It is $i = r$, and the eidolon is on the intercept of the visual with the perpendicular ray.

Ptolemy’s achievements were canonized in the perspectivist tradition of optics. Nevertheless, whether the eidolon was observed with one or both eyes, the information it provides became immaterial [10]. After the mid 1600s ray optics led to a new concept of image: the real picture of a thing is projected point by point at focus on a material screen, which definitely belongs to our world. The light-ray concept, too, can be reified by resorting to a directional source, such as a laser-pointer. But, when much is said and done on structured light, there is no reason why the forms observed by ancient Greeks should not re-enjoy objective acknowledgment. In figure 5 a picture of Pinocchio meeting its eidolon in the light of a torch was taken from a distance, staring into the mirror. If the concave mirror is well made, there is no way to visually distinguish between the two Pinocchios, although when the observer moves each image is displaced (and resized) according to its own rules. When the image clears away from its source marionette, or
the observer approaches the mirror, he needs to keep accommodating on the shifting image on its own to form a sharp picture of it, and may see the mirror surface twice, once for each eye. This fact, besides vouching for the strength of the visual sensation originated by the received signal, also gives evidence that it is perceived in front of the mirror. Last but not least, Pinocchio’s surroundings, too, are involved in mirror reception, and need to be taken into account.

Although the transfer characteristics that qualify mirrors for spatial rendition of received signals don’t guarantee that they function likewise for bee-observers, say, the very fact that our scientific endeavor largely relies on sight ought to motivate a reassessing of received images as such. An inquiry may also make sense on how to form images in other frequency ranges and with different pass bands. Provided that received image signals can be dealt with uniformly, in a communication and information age one can also wonder whether one and the same conception of geometry can be extended to cover, and render, the information on our world that can be gathered all along the electromagnetic spectrum. This move would unify the approach intended to spatially interpret what can be seen, keeping in mind that the consequences of the assumed postulates should be tested, because explanations proceed linearly while phenomena involving radiating properties of things seldom are either self explanatory or linear. To give an example, the aim at spatially interpreting signals requires to weigh up carefully whether to keep alive the currently drawn distinction between real and virtual optical images, that involves embedding an inaccessible region in space, or to drop the notion of distance altogether, and rejuvenate projective geometry as the most general geometry [11]. In order to use computer for visualization and interpretation we need an analytic correlate to the geometric notions. Still, algorithms should not replace constructions, if we learn from Euclid.

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