Generalised Penrose Limits and PP-Waves

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ABSTRACT: In this paper we construct generalised Penrose limits for the solutions of massive type IIA supergravity. We consider the Freund-Rubin type solution and apply these massive Penrose limits and obtain supersymmetric pp-wave which is standard type IIA background. The results in this paper can be easily generalised for the cases of gauged supergravities.

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1. Introduction

The maximally supersymmetric plane-fronted parallel (pp) wave configurations in string theory can generically be obtained by applying the Penrose limits \cite{1, 2, 3, 4} on the solutions of the type $AdS_p \times S^q$. Although, unlike the flat spacetime these supersymmetric pp-waves are (asymptotically) non-flat geometries, nevertheless string theory in these backgrounds becomes exactly solvable theory in lightcone gauge\cite{5, 6}. Also the pp-wave spacetime in bulk has useful consequences for dual conformal field theories on boundary \cite{7}. Under BMN correspondence \cite{8} the operators in the CFT with large $U(1)$ R-charge, $J$, are dual to type IIB closed string excitations in a pp-wave background spacetime.\footnote{More precisely the Penrose limits of Blau et. al. \cite{3, 4} where we zoom in onto the null geodesics involving a direction along the sphere. If the null geodesic is such that it does not involve the sphere then we obtain plane-waves after taking the Penrose limit.} The supersymmetric pp-wave vacua have been obtained in lower dimensional supergravities as well \cite{9, 10}. In general, there is an enhancement of the supersymmetries after taking Penrose limits on the anti-de Sitter vacua.

Unlike standard supergravities, the massive or gauged supergravities cannot admit pp-wave vacua due to the presence of cosmological constant (or a potential). Several massive and gauged supergravities admit $AdS \times S$ type solutions, yet the

\begin{itemize}
\item\footnote{Several useful works have already been carried out in the references \cite{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36}.}
\end{itemize}
Penrose limits for them are unexplored. The Penrose limits \[2, 4\] map anti-de Sitter solutions of a supergravity theory to the pp-wave solutions of the same supergravity, generically accompanied with the enhancement of supersymmetry. The problem with massive/gauged supergravities is that they admit \(AdS \times S\) vacua but cannot have pp-wave solutions and hence there cannot be pp-wave limits which interpolate between them. Our aim in this work is to understand the map between anti-de Sitter solutions of massive/gauged supergravities and the pp-waves under some \textit{generalised} Penrose limits.

We choose an explicit example of massive type IIA supergravity \[37\] in ten dimensions which fits well to exploit our proposal which is the following. First we make two observations about this theory;

i) \textit{The theory admits \(AdS \times S\) solutions, although they are not supersymmetric.}

ii) \textit{The action of this supergravity theory has a scaling symmetry under which mass parameter \(m\) also gets scaled.}

We exploit this scaling to generalise the Penrose limits \[4, 1, 2\] such that when limits are applied on the \(AdS \times S\) solutions of a gauged supergravity we get pp-wave solutions of the standard (ungauged) supergravity.

The paper is organised as follows. We discuss the Penrose limits for massive type IIA supergravity in section-2. We consider non-supersymmetric anti-de Sitter solution of massive type IIA supergravity and apply \textit{massive} Penrose limits to obtain pp-wave background. In this way we obtain 1/2 supersymmetric pp-wave of type IIA supergravity. We also remark on the corresponding Matrix model. In section-3 we discuss some cases of gauged supergravities where such an exercise could be carried out. The last section section-4 is for the conclusions.

2. A massive Penrose limit

The Penrose limits \[4, 1, 2\] of anti-de Sitter spacetimes \(AdS_p \times S^q\) along any null geodesic with a generic orbit (i.e. with a non-zero component along sphere) leads to a pp-wave geometry. The procedure is well described in \[4\] where maximally supersymmetric pp-wave solutions are obtained. Our interest here is to study Penrose limits for the solutions of massive type IIA supergravity for which there exist anti-de Sitter solutions but has no supersymmetry \[37\]. This theory has an explicit cosmological constant which is proportional to the square of the mass parameter \(m\). In this supergravity we could define a scaling limit on the supergravity fields as,

\[
g_{\mu\nu} \rightarrow \xi^{-2} g_{\mu\nu} \\
\phi \rightarrow \phi \\
A_{(p)} \rightarrow \xi^{-p} A_{(p)}
\]

accompanied with the scaling

\[
m \rightarrow \xi m
\]
under which the action scales homogeneously, see Appendix. The scale parameter $\xi$ has to be strictly positive. This scaling symmetry could be used to tune the mass parameter $m$ in the theory. The generalised or massive Penrose limits could then be described as

\begin{align*}
\bar{g}_{\mu\nu} &= \lim_{\Omega \to 0} \Omega^{-2} g_{\mu\nu}(\Omega) \\
\bar{\phi} &= \lim_{\Omega \to 0} \phi(\Omega) \\
\bar{A}_{(p)} &= \lim_{\Omega \to 0} \Omega^{-p} A_{(p)}(\Omega) \\
\bar{m} &= \lim_{\Omega \to 0} \Omega m
\end{align*}

where parameter $\Omega$ is strictly positive. It can be easily noted that such a pp-wave limit takes mass parameter to zero value and hence will give us pp-wave solutions of ordinary type IIA string theory starting from $AdS \times S$ solutions of massive type IIA supergravity. Note that the limits (2.3) are well defined when the massive IIA supergravity action is written in such a form that massless limit could be taken. Thus if there is a solution with the local data $(AdS, g, \phi, A_p, m)$ by implementing the limits (2.3) we get to new data $(W, \bar{g}, \bar{\phi}, \bar{A}_p; \bar{m} = 0)$ which is a pp-wave solution of the ordinary type IIA string theory. In the next section we will explicitly cover this aspect by taking an example of massive type IIA supergravity solution. These massive Penrose limits can be generalised in a straightforward manner for the case of gauged supergravities as well. It should however be noted that the limits (2.3) are the same as in the case of Blau et al [3, 4] except in the last equation which scales the mass parameter of the massive or gauged supergravity.

### 2.1 The Freund-Rubin solution

The massive type IIA supergravity, action (A.1), has no maximally supersymmetric flat vacua. Due to the cosmological constant this theory cannot have pp-wave solutions either. However, it admits half-supersymmetric D8-brane solutions [38] and D8 bound states with $B$-field [39, 40]. The field equations do also have non-supersymmetric Freund-Rubin type solution $AdS_4 \times S^6$ [37] describing the compactification to four dimensions. This solution is given by

\begin{align*}
g := l_1^2[- dt^2 + sin^2 t \left( \frac{dr^2}{1 + r^2} + r^2 d\Omega_2^2 \right)] + l_2^2 [d\psi^2 + sin^2 \psi d\Omega_5^2] \\
F_{(4)} = \sqrt{5} m \text{Vol}(AdS_4(l_1)), \quad \phi = 0
\end{align*}

where $m$ is the mass parameter,

\begin{align*}
l_1 = \sqrt{2}/m, \quad l_2 = \sqrt{5}/m
\end{align*}

are the radii of the $AdS_4$ and $S^6$ respectively, and $d\Omega_n^2$ is the unit $n$-sphere metric. The volume

\begin{align*}
\text{Vol}(AdS_4(l_1)) = l_1^3 sin^3 t \frac{r^2}{\sqrt{1 + r^2}} dt dr d\omega_2
\end{align*}
with $\omega_2$ being the volume element of a unit 2-sphere. The rest of the background fields are vanishing in the above. Now we would like to change coordinates in the $(\psi, t)$ plane to

$$U = \psi + \rho t, \quad V = \psi - \rho t,$$

(2.6)
in terms of which the background becomes,

$$l_2^{-2}g := dUdV + \rho^2 \sin^2 \left( \frac{U - V}{2\rho} \right) \left( \frac{dr^2}{1 + r^2} + r^2 d\Omega^2_2 \right) + \sin^2 \left( \frac{U + V}{2} \right) d\Omega^2_5$$

$$l_2^{-3}F_{(4)} = \frac{2}{\rho l_1} Vol(AdS_4(U, V)), \quad \phi = 0$$

(2.7)

where $\rho = l_1/l_2 = \sqrt{2}/5$ is the ratio of the radii of curvatures. We now rescale the coordinates as

$$U = u, \quad V = \frac{v}{(l_2)^2}, \quad Y^a = \frac{y^a}{l_2}$$

(2.8)

and take the Penrose limits (2.3), $l_2 \to \infty$ i.e. large radius limit, along the null geodesic parametrised by $U$.\(^3\) This consists in dropping the dependence on the coordinates other than $u$. In this way we get the pp-wave solution written in Rosen coordinates and depending only on $u$,

$$\bar{g} := dudv + \rho \sin^2 \left( \frac{u}{2\rho} \right) \sum_a (dy^a)^2 + \sin^2 \left( \frac{u}{2} \right) \sum_{a=4}^8 (dy^a)^2$$

$$\bar{F}_{(4)} = \rho \sin^3 \left( \frac{u}{2\rho} \right) dudy^1dy^2dy^3, \quad \bar{\phi} = 0$$

(2.9)

with $\bar{m} = 0$ which implies that it is a solution of type IIA instead of the massive theory. We can switch to the new set of (Brinkman) coordinates $(dx^+, dx^-, x^a)$ as in [3]

$$x^- = u/2, \quad x^+ = v - \frac{1}{4} \sum_a \sin(2\lambda_a u)y^a y^a, \quad x^a = y^a \sin(\lambda_a u)$$

(2.10)

we get to the familiar form of the pp-wave metric (a Cahen-Wallach spacetime)

$$g := 2dx^+dx^- + W(x)(dx^-)^2 + \sum_{a=1}^8 (dx^a)^2$$

$$F_{(4)} = 5dx^-dx^1dx^2dx^3$$

(2.11)

\(^3\)Since the curvature of spacetime is coupled to the mass parameter $m$, see (2.5), taking large radius limit involves simultaneously taking $m \to 0$ limit. Note that $m$ is the mass parameter in the action. This is the essence of the massive Penrose limits defined in (2.3). Since taking these limits involves zooming in onto a null geodesic with component along sphere, the generalised limits at the basic level are the Penrose limits in [3, 4].
with $W(x) = -\frac{5}{2}[(x^1)^2 + \cdots + (x^3)^2] - [(x^4)^2 + \cdots + (x^8)^2]$. It can be calculated that the only nonvanishing component of the Ricci tensor for the pp-wave metric (2.11) is

$$R_{--} = \frac{25}{2}.$$ 

As it is now obvious that the background (2.11) satisfies the field equations of ordinary type IIA supergravity and not of the Romans’ theory. This is because under the Penrose limits (2.3), which effectively takes $m \to 0$, the massive modes have decoupled from the theory.

### 2.2 $16 + 0$ supersymmetries

We now aim to find the amount of supersymmetries preserved by the pp-wave solution in (2.11). Let us write down the tangent space metric as

$$ds^2 = 2e^+e^- + e^ae^a + e^\alpha e^\alpha,$$

where tangent space indices are taken same as the space-time indices. The basis elements are given by

$$e^+ = dx^+ + (W/2)dx^-, \quad e^- = dx^-, \quad e^a = dx^a. \quad (2.12)$$

Only non-vanishing spin connections are

$$\omega^{+a} = \frac{1}{2} \partial_a W dx^-.$$ 

Then the Killing spinors are obtained by solving the vanishing type IIA supersymmetry variations [41, 37]

$$\delta \lambda = 0 = -\frac{1}{2\sqrt{2}} \partial_\mu \phi \Gamma^\mu \epsilon - \frac{1}{2(96)\sqrt{2}} e^{\frac{1}{4}\phi} F_{\mu\nu\lambda \sigma} \Gamma^{\mu\nu\lambda \sigma} \epsilon, \quad (2.13)$$

$$\delta \Psi_\mu = 0 = \nabla_\mu \epsilon + \frac{1}{256} e^{\frac{1}{4}\phi} F_{\mu\nu\lambda \sigma} (\Gamma^{\mu\nu\lambda \sigma}_{\mu} - \frac{20}{3} \delta^\rho_{\mu} \Gamma^{\nu\lambda \sigma}) \epsilon. \quad (2.14)$$

For the pp-wave background (2.11) the Killing equataion (2.13) implies

$$\gamma_+ \epsilon = 0 \quad (2.15)$$

while the Killing equations (2.14) reduce to the following set of equations

$$\partial_+ \epsilon = 0,$$

$$[\partial_+ + \frac{1}{4} \partial_a W \gamma_+ \gamma^a + \frac{15}{32} \Theta(\gamma_+ \gamma_- - \frac{8}{3})] \epsilon = 0$$

$$[\partial_a + \frac{25}{32} \gamma_\alpha \Theta \gamma_+] \epsilon = 0, \quad \text{for} \ a = 1, 2, 3$$

$$[\partial_a - \frac{15}{32} \gamma_\alpha \Theta \gamma_+] \epsilon = 0, \quad \text{for} \ a = 4, \ldots, 8 \quad (2.16)$$
where $\Theta = \gamma_1 \gamma_2 \gamma_3$ and all small $\gamma$ matrices are undressed. Now there is only one type of solutions of the above equations. These correspond to taking $\gamma_+ \psi = 0$ and are called ‘standard’ Killing spinors. This condition can be satisfied only when 16 spinors out of the set of total 32 are vanishing. For these spinors except for $\partial_- \epsilon + \cdots = 0$ all other equation are trivially satisfied when we take $\epsilon$ to be independent of $x^+, x^a$. These sixteen standard killing spinors are

$$\epsilon = e^{\overline{2} x^+ \psi}, \quad \gamma_+ \psi = 0,$$

(2.17)

which depend only on $x^-$. Other set of Killing spinors in the pp-waves are usually those for which $\gamma_+ \psi \neq 0$ and are known as ‘super-numerary’ Killing spinors. But these spinors do not exist for this pp-wave solution of type IIA.

### 2.3 M-theory pp-waves; Matrix model

The half-supersymmetric type IIA pp-wave in (2.11) can be easily lifted to 11-dimensional pp-wave solution on a circle which is

$$g := 2 dx^+ dx^- + W(x)(dx^-)^2 + \sum_{a=1}^{9} (dx^a)^2$$

$$G_{(4)} = 5 dx^- dx^1 dx^2 dx^3,$$

(2.18)

with $W(x) = -\frac{5}{2} [(x^1)^2 + \cdots + (x^3)^2] - [(x^4)^2 + \cdots + (x^8)^2]$. This pp-wave in (2.18) has an overall translational isometry direction $x^9$ along with $SO(3) \times SO(5)$ rotational isometries. The number of Killing spinors remains sixteen. Compare this with maximally supersymmetric 11-dimensional pp-wave solution [4]

$$g := 2 dx^+ dx^- + H(x)(dx^-)^2 + \sum_{a=1}^{9} (dx^a)^2$$

$$G_{(4)} = 5 dx^- dx^1 dx^2 dx^3,$$

(2.19)

where the function $H = -\frac{25}{9} [(x^1)^2 + \cdots + (x^3)^2] - \frac{25}{36} [(x^4)^2 + \cdots + (x^9)^2]$. This pp-wave has larger rotational isometry group $SO(3) \times SO(6)$. The sixteen Killing spinors depend upon all the transverse coordinates. While the Killing spinors in 1/2-supersymmetric pp-wave in (2.18) do not depend on overall isometry direction $x^9$. Thus it appears that getting rid of an overall coordinate, $x^9$, say, in the pp-wave (2.19) kills half of the supersymmetries. M-theory pp-wave solutions have been previously studied also [22, 19, 26].

The BMN matrix model [8] on a fully supersymmetric pp-wave background admits fuzzy sphere and 5-branes as supersymmetric solutions. Having obtained the 1/2 supersymmetric 11-dimensional pp-wave background it would be instructive to

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4We are in the frame where $(\gamma_+)^2 = (\gamma_-)^2 = 0$ and $[\gamma_+, \gamma_-]_+ = 2$ and the projector is $\gamma_+ \gamma_-$. 

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discuss the BMN model in this background. For that let us assume 
\( x^+ \sim x^+ + 2\pi R \), being periodic over a circle of radius and \( R \equiv R_{BFSS} \). Then the dynamics of the theory in the momentum sector \( p^- = p_+ = N/R \) is given by \( U(N) \) matrix model action \( S = S_{BFSS} + S' \),

\[
S_{BFSS} \sim \int dt \, \text{Tr} \left[ \frac{1}{2} (D_t x^i)^2 + \frac{1}{4} [x^i, x^j]^2 + \text{fermionic terms} \right]
\]

\[
S' \sim -\int dt \, \frac{1}{2} \text{Tr} \left[ \frac{5}{2} (x_1^2 + \cdots + x_3^2) + (x_4^2 + \cdots + x_8^2) \right] + \frac{i}{3} \epsilon_{rst} \text{Tr} (x^r x^l x^s)
\]

\[\text{fermionic terms}\]

(2.20)

where \( i, j = 1, 2, \cdots, 9; \, r, l = 1, 2, 3 \) and we have set \( R = 1, \, l_p = 1 \) and time \( t \) is identified with \( x^- \). Comparing it to the BMN matrix model, the quadratic mass term for the bosonic field \( x^9 \) is absent here. The isometry group has also been reduced from \( SO(6) \times SO(3) \) to \( SO(5) \times SO(3) \). We conclude that a fuzzy 2-sphere solution still exists but with 1/2 supersymmetry. Its relationship with "giant gravitons" will presumably be interpreted as delocalised M2-branes wrapping the \( S^2 \). The matrix models of the type (2.20) with nonconstant fluxes have been considered in [42].

3. Penrose limits in gauged supergravities

We learned in the previous section that pp-wave backgrounds can be obtained by applying the generalised plane-wave limits on the \( AdS \) vacua of massive IIA supergravity. However, that particular anti-de Sitter solution happened to be non-supersymmetric one and the pp-wave had 16 supersymmetry. In this sense there has been an enhancement of supersymmetries, from zero for anti-de Sitter space to sixteen for the pp-wave, under massive Penrose limits. We mention here the well known fact that pp-waves always have 16 or more supersymmetries. The fact that some kind of plane-wave limits exists for massive type-IIA vacua gives us motivation that these limits should also be explored for the gauged supergravities (supergravities with cosmological constant). Several of the gauged supergravity theories admit Freund-Rubin type \( \text{electro-vac} \) and \( \text{magneto-vac} \) solutions. Again note that the gauged supergravities do have cosmological constant and therefore cannot have pp-wave solutions in them.\(^5\) The idea is the same as before, the application of generalised Penrose limits on the gauged supergravity \( AdS \)-solutions should give us pp-waves of their ungauged counter parts in an easy and simple manner. Some of these pp-wave vacua could turn out to be the new solutions.

It should however be made sure that there exist scaling symmetries of the kind we considered in equation (2.1). Practically almost all gauged supergravities are found

\(^5\)However, there might exist gauged supergravities with potentials (superpotentials) which admit flat vacua in which case they will also have pp-wave solutions in them.
to have an appropriate scaling symmetry under which mass parameter (cosmological constant) gets scaled. To emphasize let us note that the gauged supergravities, this including the gauged supergravities obtained via flux-compactifications, have special couplings of the gauge fields introduced through covariant derivative \(D\phi \sim \partial\phi + gA\) and/or field strength \(F = dA + g[A,A]\). Such couplings have well defined scaling \(A \rightarrow \xi^{-1}A\), \(g \rightarrow \xi g\) under which \(D\phi \rightarrow D\phi\) and \(F \rightarrow \xi^{-1}F\) similar to the scalings in (2.1). For these gauged theories generalised Penrose limits like in eq. (2.3) could be defined.  

The following examples are worth noting in this direction.

- The 6D \(F(4)\) gauged supergravity \([44]\) which admits \(AdS_2 \times S^4\), \(AdS_2 \times S^2 \times S^2\) and \(AdS_3 \times S^3\) will have massive Penrose limits defined. There exists scaling under which gauge coupling \(g\) as well as mass parameter will scale to zero value. These vacua under the generalised Penrose limits will produce pp-wave solutions.

- Similarly, the 5D \(N = 4\) \(SU(2) \times U(1)\) gauged supergravity \([43]\) admits \(AdS_3 \times S^2\) and \(AdS_2 \times S^3\) type electrovac or magnetovac vacua. Some of the magnetovacs are supersymmetric with \(N \geq 1\). There exists scaling of the action under which gauge coupling \(g\) as well as mass parameter scales to zero value. The massive Penrose limits of these vacua would give pp-waves of the ungauged 5D supergravity with varying supersymmetries with \(N \geq 2\). These pp-waves then can be oxidised to ten dimensions.

- Other interesting cases would include the case of 4D \(N = 4\) gauged \(SU(2)_A \times SU(2)_B\) supergravity \([45]\) which admits non-supersymmetric \(AdS_2 \times S^2\) ‘Freund-Rubin’ electrovac/magnetovac solutions. One can write down Penrose limit which will take the gauge couplings \(g_A\), \(g_B\) to zero. These Penrose limits (accompanied with the scalings of the gauge couplings) will give us the pp-wave vacua of the ungauged \(SU(2) \times SU(2)\) theory.  

The above examples are only some randomly chosen cases. The gauged supergravities in \(D = 7\), 8 and 9 dimensions also consist interesting examles of anti-de Sitter vacua which will lead to pp-waves.

\(^6\)If in a gauged supergravity the gauge parameter, \(g\), could not be scaled, then it would not be possible to apply the limits we have discussed in this paper. The problem is that even after taking the limits, the cosmological constant term in the action (or equations of motion) will survive, and there exist no pp-wave solution for the equations of motion in presence of cosmological constant. Though, at the moment, I do not know of an example of such a gauged supergravity to offer.

\(^7\)We note that Freedman-Schwarz model \([45]\) can be embedded into type-I string model compactified on \(S^3 \times S^3\) \([46]\). Under the pp-wave limit this internal manifold should become a flat compact six-manifold.

\(^8\)In this connection let us note that the Penrose limits of the Chamseddine-Volkov monopole background \([46]\) have been studied in \([47]\) where the authors discuss the need of scaling of the cosmological constant. This is quite in line with the proposal in the present paper.
4. Conclusions

In summary, we started with a completely non-supersymmetric $AdS_4 \times S^6$ solution of massive type-IIA supergravity. After taking appropriate massive Penrose limits we have got half-supersymmetric pp-wave solution of the ordinary type IIA supergravity theory.

It is known that taking the Penrose limits [4] in no-scale supergravities leads to the pp-wave vacua. The pp-wave solutions always have sixteen or more supersymmetries. In this work we have seen that the massive Penrose limits in massive/gauged supergravities gets coupled with taking space-time (bulk) cosmological constant to zero value. In this regard the massive Penrose limits are the generalisation of the pp-wave limits of standard (no-scale) supergravity. This could also be understood as follows. The Penrose limit of anti-de Sitter space amounts to taking large radius limit, which effectively means scaling the curvature of spacetime to zero. Note that such a limit will then simultaneously lead to the flatness of the potential (including the potential on the moduli space as well). Effectively speaking by taking generalised pp-wave limit we go from a massive case to the massless one, a gauged to ungauged one and from flux to the non-flux one. This process leads to the recovery of the certain amount (minimum sixteen) of supersymmetries into the pp-wave solutions.\(^9\)

It would be useful to extend this work for a large class of gauged supergravities at our disposal. This may provide us with some totally new pp-wave solutions unaccounted so far though a large class of plane-wave solutions have already been found. It would also be interesting to study generalised Penrose limits of gauged supergravities obtained via flux-compactifications. We intend to report more on this in subsequent investigations [49].

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A. Scaling in massive type IIA supergravity

The Romans’ supergravity theory [37] is a generalization of the type IIA supergravity to include a mass term for the NS-NS $B$-field without disturbing the supersymmetry content of the theory. The bosonic action for Romans’ theory in the string frame can be written as (after appropriate rescalings)

\[
S = \int \left[ e^{-2\phi} \left\{ R + 4d\phi \cdot d\phi - \frac{1}{2} H(3) \cdot H(3) \right\} - \frac{1}{2} F(2) \cdot F(2) - \frac{1}{2} F(4) \cdot F(4) - \frac{m^2}{2} \right] \]

\[9\]I thank Ashoke Sen for an illuminating discussion on this point.
\[ + \frac{1}{2} dC(3) dC(3) B(2) + \frac{1}{2} dC(3) dA(1) B^2(2) + \frac{1}{3!} dA(1) dA(1) B^3(2) + \frac{1}{3!} m dC(3) B^3(2) \\
+ \frac{1}{8} m dA(1) B^4(2) + \frac{1}{40} m^2 B^5(2) \right] , \]  

(A.1)

where \( m \) is the mass parameter.\(^{10}\) The field strengths in the action (A.1) are given by

\[
H(3) = dB(2), \quad F(2) = dA(1) + mB(2), \quad F(4) = dC(3) + B(2) dA(1) + \frac{m}{2} B^2(2) .
\]  

(A.2)

Note that potentials \( A \) and \( C \) appear only through their derivatives in the action (A.1) and thus obey the standard \( p \)-form gauge invariance \( A(p) \to A(p) + d\lambda(p-1) \). The two-form \( B \) on the other hand also appears without derivatives but nevertheless the ‘Stueckelberg’ gauge transformation

\[
\delta A = -m\lambda(1), \quad \delta B = d\lambda(1), \quad \delta C = -\lambda(1) dA
\]  

(A.3)

leaves the action invariant. This action is presented in a form where one can directly implement a massless limit \( m \to 0 \). The action does scale homogeneously as

\[
S \to \xi^{-8} S
\]  

(A.4)

under the scaling

\[
g_{\mu\nu} \to \xi^{-2} g_{\mu\nu}, \quad \phi \to \phi, \quad A(1) \to \xi^{-1} A(1), \quad B(2) \to \xi^{-2} B(2), \quad C(3) \to \xi^{-3} C(3)
\]  

(A.5)

accompanied with the scaling of the mass parameter

\[
m \to \xi m .
\]  

(A.6)

The scale parameter \( \xi \) has to be strictly positive. This scaling property of the action could be used to tune the mass parameter \( m \) in the theory through Penrose limits.

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\(^{10}\)Our conventions are same as in [48] where every product of forms is understood to be a wedge product. We denote a \( p \)-form with a lower index like \( (p) \) which later on is dropped.
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