Jörg Eschmeier’s Mathematical Work

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Abstract
An outline of Jörg Eschmeier’s main mathematical contributions is organized both on a historical perspective, as well as on a few distinct topics. The reader can grasp from our essay the dynamics of spectral theory of commutative tuples of linear operators during the last half century. Some clear directions of future research are also underlined.

Introduction
Jörg Eschmeier grew up mathematically in the vibrant atmosphere of late XX-th Century German Modern Analysis. His doctoral advisor, Heinz Günther Tillmann was a scientific grandson of Otto Toeplitz. He instilled with high competence in Jörg a life-long fascination with distribution theory and general duality theory in locally convex spaces. This happened around the late 1970-ies at the University of Münster, where also George Maltese offered Jörg the challenge of an active research group. At the same time and in the same place, function theory of several complex variables was blooming, turning Münster into one of the leading world centers on the subject. Jörg was exposed early on during his studies to analytic techniques, homological algebra methods and geometric interpretations of the intricate nature of holomorphic functions of several complex variables. Throughout his brilliant career, he masterfully combined these two main streams of his student years. We collect below a few pointers to Jörg
Eschmeier’s highly original discoveries. Our text merely touches upon his deep impact on contemporary operator theory, conveying a fraction of his superb scientific orientation and elegant style of pursuing research.

1 Duality and Spectral Decompositions

The reader of this essay has unquestionably been fascinated by some sort of spectral decompositions. It is said that John von Neumann was asked by reporters on his death bed what the highest scientific discovery of his outstanding career was. To the stupefaction of all he mentioned the spectral theorem for unbounded self-adjoint operators, an esoteric concept for the layman, leaving aside his contributions to mathematical economics, game theory, nuclear weapons, electronic computers, fluid mechanics and much more.

The ubiquitous eigenvector of a finite, self-adjoint matrix has to be replaced when dealing with infinite self-adjoint matrices carrying a continuous spectrum by a subspace of vectors representing a localized window in the spectrum. This necessary step in mathematical spectral analysis stirred countless discussions among quantum physicists. Generalized eigenfunctions were proposed by them as a natural substitute, obtained however with the price of stepping outside the original Hilbert space. Dirac’s generalized function $\delta$ stands aside in this respect.

On a totally perpendicular direction and in full resonance with the Bourbaki tendencies of the day, groups of mathematicians explored in the second part of 20th Century several comprehensive axiomatic approaches to spectral decomposition. The third volume of Dunford and Schwartz monumental monograph [112] contains ample references on this forgotten chapter of operator theory. We owe to the genius of Erret
Bishop [99] the leap forward and to the unmatched insight of Foiaş [119] the foundational concept of decomposable operator.

Let $X$ be a Banach space over the complex numbers and let $T \in L(X)$ be a linear bounded operator acting on $X$. The operator $T$ is called decomposable if, for every finite open cover of its spectrum,

$$\sigma(T) = \sigma(T, X) \subset U_1 \cup U_2 \cup \ldots \cup U_n$$

there are $T$-invariant subspaces $X_1, X_2, \ldots, X_n$ with localized spectra of the respective restrictions of $T$:

$$\sigma(T, X_j) \subset U_j, \quad 1 \leq j \leq n,$$

and spanning the whole space: $X = X_1 + X_2 + \cdots + X_n$. Normal operators on Hilbert space, compact operators, and representations of functions algebras carrying a partition of unity are all decomposable.

1988 Timisoara Operator Theory Conference. From left to right: Sasha Helemskii, Mihai Putinar, Jörg Eschmeier, Henry Helson

At the beginning of a theory, variants of the definitions and permanence problems usually have to be clarified. In the original definition of a decomposable operator Foiaş in [119] required the subspaces $X_j$ to be spectral maximal and in the last chapter of their monograph [104] Colojoară and Foiaş ask if the restrictions of decomposable operators are again decomposable. As Jörg told one of us (E.A.), he first came in touch with the theory of decomposable operators in a student seminar of Tillman in 1977/78, which was announced by his assistant Erich Marschall as follows: “Here is a Lecture Notes volume [114] with a positive answer to that question and a manuscript [90] with a negative answer. Let us find out which one is correct.” In [73] Jörg published a further class of examples which are much more natural and less technical than the example in [90]. After this seminar he wrote his (unpublished) Diploma Thesis [60] on local decomposability (in the sense of Vasilescu [133, 134]) and functional calculi for closed linear operators on Banach spaces. In particular (using methods from [91]) he
already obtained most of the characterization of such operators described in Theorem IV.4.26 of [135].

With the proper definition of joint spectrum \( \sigma(\tau, X) \) of a tuple of commuting linear bounded operator \( \tau \) proposed by Taylor [131], the notion of decomposability carries over, but the challenging questions and complications multiply.

Jörg Eschmeier’s dissertation of 1981 [61], summarized in his *Inventiones* article [63], is devoted to the study of spectral localization within the novel, at that time, Taylor analytic functional calculus. His lucid insight builds on solid ground well cultivated in Germany at that time, notably an earlier contribution of one of us (E.A.) [89], who is incidentally a mathematical nephew of Jörg via the filiation Tillmann-Gramsch. Jörg navigates there with ease and efficiency through the formidable Cauchy-Weil integral representation formulas invoked by Taylor. He would then continue for two good decades to explore the subject.

But progress was not possible without Bishop’s ideas. In a nutshell, Bishop claimed that it is not generalized eigenvectors à la Dirac’s distribution that illuminates general spectral decomposition behavior, but analytic functionals. Fortunately, the necessary passage from Schwartz distributions to analytic functionals or hyperfunctions was much explored by mathematical analysts of the 1980-s decade. To be more specific, in order to better understand the spectral characteristics of the linear bounded operator \( T \in L(X) \), Bishop points to the space of \( X \)-valued analytic functions defined on an open set \( U, \mathcal{O}(U)^X \), and the linear pencil:

\[
zI - T : \mathcal{O}(U)^X \longrightarrow \mathcal{O}(U)^X.
\]

The resolvent of \( T \), restricted to values of \( z \) outside the spectrum, is lurking around. The operator \( T \) is said to have the single valued extension property if the map \( zI - T \) above is injective for every open set \( U \). In that case a localized spectrum of \( T \) with respect to a vector makes sense. The operator \( T \) satisfies Bishop’s condition \((\beta)\) if the map \( zI - T \) is injective with closed range for every open set \( U \). A timely 1983 observation of one of us (M.P.) [125] interprets these notions in terms of sheaf theory:

\[
\mathcal{F}_T(U) = \text{coker}(zI - T : \mathcal{O}(U)^X \longrightarrow \mathcal{O}(U)^X) = \mathcal{O}(U)^X / (zI - T) \mathcal{O}(U)^X,
\]

is an analytic sheaf if \( T \) has the single valued extension property and it is an analytic sheaf of Fréchet spaces if \( T \) satisfies property \((\beta)\). Note that the canonical identification

\[
\mathcal{F}_T(\mathbb{C}) = X, \quad T = M_z,
\]

takes place, with the operator \( T \) represented by multiplication by the complex variable. Moreover, the spectrum of \( T \) in this case equal to the support of the sheaf model \( \mathcal{F}_T \), the support of a section (a.k.a. vector) \( x \in X \) is the local spectrum, and so on. The crucial insight came from the proof that \( T \) is decomposable if and only if the canonical sheaf model (and as a matter of fact any sheaf model) is soft [125]. Now, analytic sheaves were at home both at Münster and Bucharest. A fruitful collaboration between Jörg and Mihai was built on this rich ground, leading to a dozen joint publications.
The rest is technique, sheaf cohomology back and forth, in the treacherous topological homological setting.

Returning to Bishop [99], his clear message was to consider in parallel the non-pointwise behavior of the linear pencils $z I - T$ and $z I - T'$, where $T'$ is the topological dual of $T$. This is for the simple reason that elements of the dual of $\text{coker}(z I - T)$ on $\mathcal{O}(U)^X$ are analytic functionals $\phi \in \mathcal{O}(U)^{X'}$ which fulfill the generalized eigenvector equation $(z I - T')\phi = 0$. Playing this distribution theory game at the abstract multidimensional level bares fruit:

1. A commutative tuple $\tau$ of linear bounded operators satisfies Bishop’s property $(\beta)$ if and only if admits a quasi-coherent analytic, Fréchet sheaf model;
2. A commutative tuple $\tau$ of linear bounded operators is decomposable if and only if both $\tau$ and $\tau'$ satisfy property $(\beta)$;
3. A commutative tuple $\tau$ of linear bounded operators is the restriction of a decomposable tuple to a joint invariant subspace if and only if $\tau$ satisfies property $(\beta)$;
4. Two quasi-similar tuples of operators subject to property $(\beta)$ have equal joint spectra and equal essential joint spectra;
5. Division of distributions by complex analytic functions follows from a Bishop’s type property with respect to smooth functions;
6. The functoriality of the sheaf model of a commutative tuple of linear bounded operators implies Riemann-Roch Theorem on singular analytic spaces.

Some of the definitive results enumerated above were obtained in several increments, each a source of joy and bewilderment [2, 30, 32, 79, 127]. The monograph [37] collects such advances obtained until early 1990-ies and unified by the concept of analytic sheaf model. A decade later, the monograph by Laursen and Neumann [123] offered a complementary, cohomology free account of local spectral theory (in the case of single operators).
2 Invariant Subspaces

The famous results and in particular the methods of proof in Scott Brown’s work [100, 101] on invariant subspaces for subnormal operators and hyponormal operators with thick spectrum on Hilbert spaces had a great impact for the further development in the research on the invariant subspace problem. In [101], Scott Brown used the fact (due to [126]) that a hyponormal operator $T$ is subscalar (and hence subdecomposable) to obtain nontrivial invariant subspaces for $T$ when the spectrum $\sigma(T)$ is thick in the sense that for some non empty set $G \subset \mathbb{C}$ the set $\sigma(T) \cap G$ is dominating in $G$. He also noticed that subdecomposability is sufficient for his result. In [75] Jörg obtained a first result for general Banach spaces: If $T \in L(X)$ is subscalar with $\text{int}(\sigma(T)) \neq \emptyset$, then $T$ has nontrivial invariant subspaces.

By [2], subdecomposability is equivalent to Bishop’s property ($\beta$), which is a local property and has a nice duality theory. Using these results Jörg removed restrictions on the operators or on the underlying Banach spaces to obtain some first localized invariant subspace results. Finally, in joint work with Bebe Prunaru [34], he proved that every continuous linear operator $T$ on an arbitrary Banach space $X \neq \{0\}$ which, for some compact set $S \subset \mathbb{C}$, satisfies ($\beta$) or the dual property ($\delta$) on $\mathbb{C}\setminus S$ and for which there is a bounded open set $V \subset \mathbb{C}$ with $S \cap V = \emptyset$ or $S \subset V$, then the following holds:

1. If $\sigma(T)$ is dominating in $V$ than $T$ has a nontrivial invariant subspace.
2. If the essential spectrum of $T$ is dominating in $V$ then the lattice $\text{Lat}(T)$ of invariant subspaces is rich.

Even more generally, the authors showed in [27] a corresponding result for operators $T$ for which the localizable spectrum of $T$ or of $T'$ is thick (in the above mentioned sense), has a non-trivial invariant subspace. Notice, that some thickness condition on the spectrum is necessary, as there exists an example due to Charles Read [128] of a quasinilpotent and hence even decomposable operator on some Banach space without any non-trivial invariant subspaces.

Following the success of the Scott Brown technique for single operators, Jörg set out to extend these ideas to tuples of commuting operators. The natural goal is to prove that any contractive tuple of commuting operators with sufficiently rich spectrum has non-trivial invariant subspaces. In the multivariate setting, several difficulties arise. As mentioned before, spectral theory becomes substantially more difficult. Moreover, just like there is more than one generalization of the unit disc to higher variables, there are several reasonable notions of contractive tuples of operators. Finally, the Sz.-Nagy–Foiaş $H^\infty$-functional calculus, which is a crucial ingredient in the classical Scott Brown technique, is generally not available in the multivariable setting. In [82], Jörg proved that every commuting row contraction that possesses a spherical dilation and whose Harte spectrum is dominating in the ball admits non-trivial invariant subspaces. Along the way, he established an $H^\infty(\mathbb{D})$-functional calculus, which itself inspired further research approximately 20 years later [98, 103]. He also established a version of this theorem on the polydisc [44].

In the one variable setting, the Scott Brown technique was refined by Olin and Thomson [124] to show that a subnormal operator $T$ not only admits nontrivial invari-
ant subspace, but is even reflexive. This means that the weak operator topology closed algebra generated by $T$ can be completely recovered from its invariant subspace lattice. Jörg successfully established variants of this result in the multivariable setting \[9, 43, 46, 48, 84\]. The difficulty of this subject can perhaps be appreciated from the fact that the question of whether every subnormal operator tuple is reflexive remains open to this day.

Jörg in 2009, at the occasion of EA’s 65th birthday

3 Multivariable Operator Theory

The interplay between functional analysis and function theory of several complex variables entered around 1970-ies into a new era thanks to Joseph Taylor’s innovative insights, in particular his novel joint spectrum and the topological-homological approach to functional calculi \[131, 132\]. Jörg was a central figure and inspired contributor to this new chapter of modern analysis. His many notable contributions would fill an entire volume, way beyond the size of the present biographical note. We outline a couple of snapshots.

In duality with the fibre product of complex spaces, the tensor product of analytic modules is inviting at analyzing the blend of spectral behaviors of its factors. Two traditional constructions of operator theory stand aside in this respect. More specifically, consider a pair of Banach spaces $X, Y$ and commutative tuples of linear bounded operators acting on them $S \in L(X)^m$ and $T \in L(Y)^n$. With a fixed cross-norm, the completed tensor product $X \otimes Y$ carries the commutative $(n + m)$-tuple $\tau = (S \otimes I, I \otimes T)$. One of Jörg’s early works \[74\] (see also the third chapter in his Habilitation Thesis \[72\]) offers a complete evaluation of the joint spectrum of $\tau$, its joint essential spectrum, and the values of the Fredholm index. A second natural path of melting tuples of linear transforms into new ones has to do with the so-called elementary operators. Namely, let $J$ be a bilateral ideal in the space of linear bounded operators $L(K, H)$ acting between two Hilbert spaces $K, H$. Let $S \in L(H)^n$ and
$T \in L(K)^n$ be tuples of commuting elements. The operator

$$R : J \to J, \quad R(A) = \sum_{j=1}^n S_j A T_j,$$

is an elementary operator. A complete spectral picture of $R$ has quite surprising implications, for instance to solving non-commutative operator equations relevant to control and stability theory of systems. The spectrum, essential spectrum, and Fredholm index of $R$ were first computed by one of us (R.E.C.) together with Lawrence A. Fialkow [105, 106], and further refined under the assumption of an intricate geometric condition, known as the “finite fibre property” [118]. The note [36] shows, employing cohomology and analytic localization techniques, that the finite fibre property is always satisfied. The investigations of tensor products of analytic modules and separately of elementary operators continue to flourish.

As defined, Taylor’s joint spectrum of a commutative tuple of linear operators is quite elusive, requiring a grasp of higher torsion spaces of a pair of analytic modules. Cohomologically trivial cases are in this sense a treat, as much as, keeping the proportion, pseudoconvex domains are among all domains in $\mathbb{C}^d$. For this reason, a collection of simple examples is both precious and inspiring. Take for instance a bounded pseudoconvex domain $\Omega$ in $\mathbb{C}^d$ and a finite system of bounded analytic functions defined on $\Omega$: $f = (f_1, f_2, \ldots, f_n)$. The note [35] contains a description of Taylor’s joint spectrum of $f$, as acting on Bergman space $L^2_a(\Omega)$:

$$\sigma(f, L^2_a(\Omega)) = \overline{f(\Omega)}.$$

In the same setting, assuming the boundary of $\Omega$ is smooth, strictly pseudoconvex, the joint essential spectrum is

$$\sigma_e(f, L^2_a(\Omega)) = \bigcap_U \overline{f(U \cap \Omega)},$$

where $U$ runs over all open neighborhoods of $\partial \Omega$. These results can be interpreted as solutions to division problem with $L^2$-bounds, in the spirit of the celebrated Corona Problem.

Bounded analytic interpolation in several complex variables is significantly more challenging than the classical Nevanlinna-Pick or Carathéodory-Fejér 1-dimensional problems. A major breakthrough was recorded in the 1990-ies with the isolation of the Schur-Agler class of analytic multipliers on a Hilbert space of analytic functions with a reproducing kernel. It was Jim Agler who recognized at that time that this new algebra provides the correct analog for stating and proving multivariate analogs of the quasi-totality of known bounded analytic interpolation results in one dimension [88]. Jörg entered into the first line of avant-garde researchers charting this new territory, as for instance, the articles [28, 29, 39] amply illustrate.

A commuting tuple $T = (T_1, \ldots, T_n)$ of operators is said to be Fredholm if the cohomology groups $H^p(T)$ of the Koszul complex are all finite dimensional. The dimensions of $H^p(T)$ and related objects naturally carry operator theoretic meaning. It turns out that they can also be related to certain analytical quantities. In his paper
[49], Jörg showed that the limits \( \lim_{k \to \infty} \dim H^p(T^k)/k^n \) exist and in fact agree with the so-called Samuel multiplicities of stalks of cohomology sheaves; moreover, they agree with the generic dimension of \( H^p(z - T) \) near \( z = 0 \). This result employs a beautiful blend of operator theory, commutative algebra and analytic geometry. Jörg continued his investigations in [47, 51].

The model theory of Sz.-Nagy and Foiaş yields a complete unitary invariant for (completely non-unitary) contractions, namely the characteristic function. This is closely related to the theory of the Hardy space on the disc and classical inner functions. Jörg, together with several coauthors, made advances into a corresponding multivariable theory [6] and into setting of more general reproducing kernel Hilbert spaces [4, 58, 85].

Another topic in which Jörg was active is the characterization of the essential commutant of the analytic Toeplitz operators. A theorem of Davidson [107] shows that, on the Hardy space, an operator \( T \) commutes modulo the compacts with every analytic Toeplitz operator if and only if \( T \) is a compact perturbation of a Toeplitz operator with symbol in \( H^\infty + C \). Very general extensions to the multivariable setting were given by Eschmeier and coauthors in [16, 17, 20].

Jörg in 2019

4 Arveson-Douglas Conjecture

Classically, there is an intimate relationship between the study of contraction operators on Hilbert space and the Hardy space \( H^2 \) on the unit disc. In the theory of tuples of commuting operators on a Hilbert space, the appropriate generalization of the Hardy space is the Drury–Arveson space \( H^2_d \), see [93]. This is the reproducing kernel Hilbert space of analytic functions on the Euclidean unit ball \( \mathbb{B}_d \) in \( \mathbb{C}^d \) with reproducing kernel

\[
K(z, w) = \frac{1}{1 - \langle z, w \rangle}.
\]
Given Jörg’s experience in multivariable operator theory and in several complex variables, it is no surprise that he made important contributions to this subject early on [28].

Since the coordinate functions $z_1, \ldots, z_d$ are multipliers of $H^2_d$, the space $H^2_d$ becomes a module over the polynomial ring. To each (closed) submodule $M \subset H^2_d$, one associates the operators

$$S_j : M^\perp \to M^\perp, \quad f \mapsto P_{M^\perp}(z_j f),$$

which are commuting linear operators on $M^\perp$. A very influential conjecture, made by Arveson [92] and refined by Douglas [110], asserts that if $M$ is the closure of a homogeneous ideal $I$ of polynomials, then the cross commutators $[S_j, S^*_k]$ belong to the Schatten class $S^p$ for all $p > \dim Z(I)$, where $Z(I)$ denotes the zero set of $I$. This conjecture has attracted a large amount of attention.

The initial motivation for the Arveson–Douglas conjecture came from Arveson’s work on the curvature invariant of certain operator tuples [92], but the conjecture turned out to be interesting for other reasons as well. For instance, if $I$ is a homogeneous ideal of infinite co-dimension so that the Arveson–Douglas conjecture holds for $M = \overline{I}$, then the quotient of the Toeplitz $C^*$-algebra associated with $M$ modulo the compacts is commutative. In fact, the quotient is isomorphic to $C(Z(I) \cap \partial B_d)$. As Douglas observed, this gives rise to a $K$-homology element of the space $Z(I) \cap \partial B_d$ [110].

After the Arveson–Douglas conjecture was formulated, it was verified in a number of special cases. Jörg reduced the conjecture to a certain operator inequality [53]. This led to a unified proof of all cases in which the conjecture was known to hold at the time. A few years later, Jörg Eschmeier and Miroslav Engliš achieved a spectacular breakthrough by showing that the conjecture holds if the homogeneous ideal $I$ is the vanishing ideal of a homogeneous variety that is smooth away from the origin [19]. A similar, related result was independently obtained by Douglas, Tang and Yu around the same time [113]. These works have inspired a lot of research, and the area remains very active to this day.

### 5 Teaching and Mentoring

Jörg Eschmeier was a very highly regarded teacher and mentor. His lectures were widely known for their clarity and precision. They were frequently attended by a large number of students, and even his more advanced courses in functional analysis and complex analysis drew in students from outside of mathematics. At Saarland University, he won the award for the best lecture in mathematics five times, more often than any of his colleagues in the department.

In addition, Jörg was an extremely dedicated mentor for bachelor’s, master’s and doctoral theses. He was very generous with his time and invested considerable effort into advising students.

A particular gem among Jörg’s lectures were those about several complex variables. His lecture notes formed the basis for his book [57]. The book stands out in that it provides a self-contained treatment of important theorems in several complex vari-
ables, assuming only basic undergraduate analysis as a prerequisite. The book starts with basic properties of holomorphic functions in several variables, treats analytic sets and domains of holomorphy, proves Oka’s theorems and explains the solution of the Levi problem. The last chapter contains beautiful applications to functional analysis, namely the Arens-Calderón functional calculus, Shilov’s idempotent theorem and the Arens-Royden theorem. Through his lectures and his book, Jörg made this fascinating subject accessible to many students.

Another very nice example of informative and well structured writing are his lectures on invariant subspaces contained in Part III of [7].

Already during his period as an Alexander von Humboldt Fellow, Jörg was involved in the mentoring of Roland Wolff, a doctoral student of George Maltese, whom he introduced to the field of Bergman and Hardy spaces in several variables. Roland Wolff finished his thesis Spectral theory on Hardy spaces in several complex variables [137] in Saarbrücken, where he became the first assistant of Jörg. Part of this theses is also published in [138].

The following is a list of all completed doctoral theses written under the mentorship of Jörg at Saarbrücken:

(1) Michael Didas, On the structure of von Neumann n-tuples over strictly pseudoconvex sets [108]. Michael Didas considers operator tuples $T = (T_1, \ldots, T_n)$ on a Hilbert space $H$ admitting a contractive functional calculus $\Phi_T : A(D) \to L(H)$, where $D$ is a strictly pseudoconvex subset of a Stein submanifold of $\mathbb{C}^n$ which admit a $\partial D$-unitary dilation. Using dual algebra methods he obtains reflexivity results and invariant subspace results for large classes commuting operator tuples $T = (T_1, \ldots, T_n)$ on a Hilbert space with dominating Harte- or Taylor spectra in $D$. The main parts of this thesis have been published in [109]. Though Michael Didas left the university he always stayed in contact with Jörg and they published seven joint articles.

(2) Eric Réolon, Zur Spektraltheorie vertauschender Operatortupel: Fredholmtheorie und subnormale Operatortupel [129]. In his thesis Eric Réolon obtains a Banach space variant of a Fredholm index formula for essentially normal tuples given in [37], Theorem 10.3.15. He also shows that an operator tuple on a Hilbert space is essentially subnormal if and only if it has an essentially normal extension and if and only if it has an extension to a compact perturbation of a normal tuple. For single operators this had been shown by N.S. Feldman in [117].

(3) Christoph Barbian, Beurling-Type Representation of Invariant Subspaces in Reproducing Kernel Hilbert Spaces [94]. Christoph Barbian studies invariant subspaces of reproducing kernel Hilbert spaces, including the difficult case of the Bergman space on the unit disc. He introduces the notion of Beurling decomposability, and obtains criteria for this property to hold. Among other things, this has implications for multivariable spectra of multiplication tuples. This thesis as well as the following ones is available for download. For further developments see also [95–97].

(4) Dominik Faas, Zur Darstellungs- und Spektraltheorie für nichtvertauschende Operatortupel [116]. This thesis is related to Jörg’s work on Samuel multiplica-
ties. A local closed range theorem for semi-Fredholm valued functions was later improved to a global version by Dominik Faas and Jörg in [21].

(5) Kevin Everard, *A Toeplitz projection for multivariable isometries* [115]. For compact sets $K \subset \mathbb{C}^n$ and closed subalgebras $A$ of $C(K)$ Jörg introduced in [46] the notion of an $A$-isometry. This class of commuting $n$-tuples of operators includes spherical isometries and $n$-tuples of commuting isometries. This allowed Jörg together with Michael Didas and Kevin Everard to introduce associated analytic Toeplitz operators [16]. The thesis of Kevin Everard completes that approach (see also [20]).

(6) Michael Wernet, *On semi-Fredholm theory and essential normality* [136]. Michael Wernet contributes to four areas of Jörg’s interests: He extends Jörg’s results of [50], he shows that a number of positive results on the Arveson-Douglas conjecture can be extended to arbitrary graded Hilbert modules and that the validity of the conjecture is equivalent for a large class of analytic functional Hilbert spaces, he generalizes an essential von Neumann inequality of Matthew Kennedy and Orr Shalit ([121], Theorem 6.1), and using a result of [21] answers a question by Ronald Douglas ([111], Question 1) and (extending some results of Jörg and Johannes Schmitt [40]) gives a partial answer to another question of Ronald Douglas in [111], Question 3.

(7) Dominik Schillo, *K-contractions, and perturbations of Toeplitz operators* [130]. For many analytic Hilbert function spaces with reproducing kernels $K$ Dominik Schillo obtains model theorems for $K$-contractions. In a second part he studies Toeplitz operators associated with regular $A$-isometries and uses methods from [16] to characterize finite rank and Schatten-p-class perturbations of analytic Toeplitz operators. Part of these results have also been published in [17].

(8) Sebastian Langendörfer, *On unitarily invariant spaces and Cowen-Douglas theory* [122]. A rather general version of a characterization of Toeplitz operators with pluriharmonic symbols on unitarily invariant with appropriate reproducing kernel and an extension of results by Chang, Chen and Fang [102] to the several variable case are given. See also the joint works [22–24] of Sebastian Langendörfer with Jörg.

(9) Daniel Kraemer, *Toeplitz operators on Hardy spaces* [120]. A several variable Toeplitz operator theory is developed on Hardy type $H^p(G)$ spaces which is applicable for bounded symmetric domains and bounded strictly pseudoconvex domains. In particular, in the situation of strictly pseudoconvex domains he obtains a generalization of Jörg’s spectral mapping theorem from [55].

The high quality of these theses reflects the outstanding quality of Jörg Eschmeier as an academic teacher.

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**Declarations**

**Conflict of interest** The authors declare no competing interests.
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