Data-driven stochastic optimization for power grids scheduling under high wind penetration

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Abstract
To address the environmental concern and improve the economic efficiency, the wind power is rapidly integrated into smart grids. However, the inherent uncertainty of wind energy raises operational challenges. To ensure the cost-efficient, reliable and robust operation, it is critically important to find the optimal decision that can correctly and rigorously hedge against all sources of uncertainty. In this paper, we propose data-driven stochastic unit commitment (SUC) to guide the power grids scheduling. Specifically, given the finite historical data, the posterior predictive distribution is developed to quantify the wind power prediction uncertainty accounting for both inherent stochastic uncertainty of wind power generation and input model estimation error. For complex power grid systems, a finite number of scenarios is used to estimate the expected cost in the planning horizon. To further control the impact of finite sampling error induced by using the sample average approximation (SAA), we propose a parallel computing based optimization solution methodology, which can quickly find the reliable optimal unit commitment decision hedging against various sources of uncertainty. The empirical study over six-bus and 118-bus systems demonstrates that our approach can provide more cost-efficient and robust performance than the existing deterministic and stochastic unit commitment approaches.

Keywords Stochastic programming · Unit commitment · Parallel computing · Wind power · Power grids scheduling · Renewable energy

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1 Introduction

Wind power is rapidly incorporated into power grids in an effort to combat the climate change and improve power system resilience [1–3]. In the past few years, the wind energy capacity expanded explosively [1, 3, 4]. It is projected that the wind power penetration will continue to grow in the near future [2]. However, the inherent volatility of wind energy has a significant impact on the system operation [1, 4, 5]. To ensure a cost-efficient and reliable power grid scheduling, the stochastic unit commitment (SUC) model is widely used in the literature, especially under the situations with high wind penetration [6–9]. Decision makers seek the unit commitment (UC) decision minimizing the total expected cost of the power production to meet the demand, which explicitly accounts for the inherent stochastic uncertainty of wind power generation [2, 6–8].

However, the existing SUC approaches, tend to ignore two sources of uncertainty, which can lead to inferior and unreliable unit commitment decisions. First, the underlying true statistical input model characterizing the wind power generation uncertainty is unknown and estimated by finite historical data, which can induce the input model estimation uncertainty, called model risk. The existing SUC approaches, called the empirical approach, tend to take the estimated statistical model as the true one [7–9] and ignore the input model estimation error [10]. Second, to solve SUC, given a first-stage unit commitment decision, we often use the sample average approximation (SAA) or finite number of scenarios to estimate the expected cost in the planning horizon, which can introduce the finite sampling error. This source of uncertainty is also typically ignored in the existing SUC approaches [7–9].

In this paper, we introduce a data-driven SUC model and further develop a parallel computing based optimization solution methodology. Our study can lead to the optimal unit commitment decision which can appropriately hedge against all sources of uncertainty. Basically, both parametric and nonparametric statistical models can be used to characterize the inherent stochastic uncertainty of wind power generation. Since the underlying input model is unknown and estimated by finite historical data, this induces the model estimation uncertainty and we quantify it with the posterior distribution. Then, we utilize the posterior predictive distribution to quantify the wind power generation prediction uncertainty, which accounts for both stochastic uncertainty and model estimation error. Thus, driven by the scenarios generated by the posterior predictive distribution, we propose the data-driven SUC, which leads to the optimal decision simultaneously hedging against both wind power generation stochastic uncertainty and input model estimation error.

Then, we introduce a parallel computing based optimization approach, called the optimization and selection (OPSEL). This approach is built based on ranking-and-selection techniques or statistical comparison approaches introduced in the simulation community [11–14]. In the proposed OPSEL procedure, we consider Optimal Computing Budget Allocation (OCBA) [11, 15–17], because it has several advantages, including good convergence rate and finite sample performance.
[18–20]. Specifically, to solve SUC problems, we observe that the computational effort is heavily invested in searching for the optimal unit commitment decision. Compared with the optimization search, given a candidate unit commitment decision, it takes much less time to assess its performance. Thus, the OPSEL approach includes: using the parallel optimization search to obtain multiple candidates and then efficiently selecting the best candidate decision. In the optimization step, we utilize the parallel computing to simultaneously solve a sequence of finite sample approximated data-drive SUC problems and obtain candidate solutions. Then, in the selection step, we use the OCBA-based rank and selection approach to efficiently evaluate these candidate decisions and select the best decision. The proposed OPSEL approach can be used by general SUC to control the impact of finite sampling error and quickly find the optimal unit commitment decision, which is critically important for guiding the dynamic scheduling decision for complex power grids with high wind power penetration.

The main contributions of this paper are listed as follows.

1. To the authors’ best knowledge, there is no existing approach explicitly accounting for all three sources of uncertainty: (1) inherent stochastic uncertainty of wind power generation, (2) SUC input model estimation uncertainty, and (3) finite sampling error induced by using the sample average approximation to estimate the expected cost occurring in the planning horizon. We propose a data-driven stochastic optimization framework that can appropriately hedge against all sources of uncertainties and quickly deliver a reliable cost-efficient optimal dynamic unit commitment decision.

2. The proposed data-driven SUC leads to the optimal unit commitment decision simultaneously hedging against both the inherent stochastic uncertainty of wind power generation and the SUC input model estimation uncertainty. It can be applied to cases with either parametric or nonparametric wind power forecast models.

3. The proposed OPSEL approach can utilize the parallel computing and quickly solve for the optimal unit commitment decisions hedging against the impact of finite sampling error induced by SAA, which could be large especially for complex power grids with high wind power penetration. The OPSEL can be used for general SUC approaches to reduce the impact of finite sample approximation error. We provide the comprehensive empirical study to show that the proposed framework can provide better and more robust unit commitment decisions for power grid systems with: (1) different scales (i.e., 6-bus and 118-bus power grids); (2) different levels of wind power penetration; (3) different levels of wind power generation variability or volatility; (4) different SUC decision planning horizons (i.e., day-ahead and intra-day energy market); and (5) parametric and nonparametric input models characterizing the wind power generation inherent stochastic uncertainty. The empirical study demonstrates that the proposed data-driven SUC demonstrates better performance, especially for the situations with high wind power penetration and complex power grids.
The organization of this paper is as follows. We review the related literature in the next section. We formally state our problem and introduce the proposed data-driven SUC modeling in Sect. 3. In Sect. 4, we introduce the OPSEL approach that can quickly solve SUC and control the impact of SAA finite sampling error. Both six-bus and 118-bus test cases are used to study the performance of our approach in Sect. 5 and the results demonstrate that the proposed data-driven SUC framework has clear advantages. We conclude this paper in Sect. 6. The code of the proposed data-driven SUC framework is available on GitHub at https://github.com/kw48792/data-driven-suc.

2 Literature review

In this section, we review various optimization approaches developed for the SUC problem. The first one and the most commonly used one is called the empirical SUC. Given the historical data, it first estimates the underlying statistical input model characterizing the wind power generation variation, and then takes the estimate as the true one. Stochastic programming was first introduced to solve the unit commitment problem with uncertainty from load [21]. This approach has frequently been applied in recent research, as renewables are integrated into the power system on a large scale; see the examples in [8, 9, 22, 23]. While the empirical SUC accounts for the inherent stochastic uncertainty of renewable energies, it fails to account for statistical input model estimation uncertainty and finite sampling error induced by using SAA to estimate the expected cost in the planning horizon, which could lead to inferior decisions.

The second stream is the robust optimization (RO). Without assuming the distribution modeling wind power generation variability, this approach focuses on the worst-case scenario, with the objective of minimizing the worst-case cost. The studies in [4, 24] employed RO to smart grids with high wind power penetration. [25] further extended RO to multi-stage cases, and [26] included the transmission line constraints in RO. However, RO is too conservative; see [27]. While some efforts have been made to reduce the conservativeness [4, 28–30], since RO only considers the worst-case without taking into consideration of the likelihood of all scenarios, the conservativeness issue persists.

The third stream, called the distributionally robust optimization (DRO), is proposed to overcome the limitation of RO; see for example [27, 31, 32]. The distributional robust unit commitment model minimizes the worst-case expected cost over a set of probability distributions, called an ambiguity set, accounting for the input model estimation uncertainty. DRO is a special case of the composite measure approach where separate measures are used to quantify the input model estimation uncertainty and stochastic uncertainty, and then the composite of these measures is used in the objective [27]. Even although this approach could produce less conservative decisions than RO, it fails to take into account the possibility of distribution candidates being the true one. Hence, the resulting scheduling decision is still too conservative and costly.
The fourth stream is called the minimax regret optimization. The regret is defined by the objective difference between the current solution without knowing the uncertain parameters and the perfect-information solution. Jiang et al. [33] introduced an innovative minimax regret unit commitment model aiming to minimize the maximum regret of the day-ahead decision over all possible realizations of the uncertain wind power generation. While the minimax regret optimization can deliver less conservative results than RO, like DRO, it also fails to take into account the possibility of distribution candidates being the true one. Hence, it suffers the similar drawback as DRO.

Additionally, there is another stream called the chance-constrained programming. It was first introduced to model the UC problem with random wind power generation in [34, 35]. The objective is to satisfy the net load (load minus wind) with a specified high probability level over the entire time horizon while minimizing the operating cost. The original SUC problem is decomposed to a sequence of deterministic versions of the UC problem that converge to the solution of the chance-constrained program. In [36], the SUC problem with uncertain wind power generation is formulated as a chance-constrained two-stage stochastic program. Similar to the empirical SUC, this method relies on the assumption that the underlying input model of uncertain variables can be accurately estimated with historical data.

The two-stage SUC problem is the stochastic mixed integer program with discrete unit commitment decision variables. To efficiently solve it, the authors in [37, 38] provide a great comprehensive review on the decomposition optimization solution algorithms. The design of these methods depends on where the uncertainty appears and where the continuous and discrete decision variables are. The authors provide computational evidence to show that the decomposition methods have desirable theoretical properties and computational performance. In addition, [39, 40] propose stochastic dual dynamic integer programming and a new type of decomposition algorithm for multistage SUC problem. Since it can handle uncertainty dynamically in the sense that the UC decision is a function of the realization of renewable energy generation, this approach can increase system flexibility and reduce operating costs compared to the two-stage approaches. The proposed methodology in [39, 40] can efficiently solve large-scale multi-stage SUC problems. However, these methodologies fail to consider the impact of input model estimation uncertainty and finite sampling error.

### 3 Problem statement and data-driven SUC

In this section, we first describe the two-stage SUC problem in Sect. 3.1. Since the underlying input model, characterizing the inherent stochastic uncertainty of wind power generation, is unknown, it is estimated by using finite historical data. This introduces the input model estimation uncertainty. In Sect. 3.2, we review the existing empirical SUC approach which takes the estimated model as the true one and ignores the input model estimation uncertainty. We also discuss the existing deterministic unit commitment (UC) model, which ignores the prediction risk induced from both wind power inherent stochastic uncertainty and model risk. Then, in
Sect. 3.3, we propose the data-driven SUC accounting for both sources of uncertainties. Our proposed stochastic unit commitment approach can be applied to various cases with either parametric or nonparametric forecast model for wind power generation.

### 3.1 Stochastic unit commitment model

Let $\xi$ denote the random wind power generation, and let $F^c$ represent the underlying “correct” statistical input model for SUC with $\xi \sim F^c$. Here, we consider a general formulation of two-stage stochastic unit commitment problem [41, 42]

$$\min_u G(u) \equiv C^{su} u + E_{\xi \sim F^c} \left[ \min_y C^{fuel} y(u, \xi) \right]$$

subject to

$$A u \leq B$$

$$Hu + Q y(u, \xi) \leq M(\xi)$$

where $C^{su}$ is the first-stage cost coefficient, consisting of various startup and shutdown costs, and the coefficient $C^{fuel}$ represents the fuel cost; see more information in [41, 43, 44]. The first-stage unit commitment decision for thermal generators, denoted by $u$, is made prior to the realization of renewable energy generation $\xi$. The second-stage economic dispatch decision, denoted by $y(u, \xi)$, is made after the unveiling of $\xi$ and it depends on $u$ as well.

Objective (1) includes the cost incurred in the first stage and the expected dispatch cost incurred in the planning horizon. The general constraints for the first- and second-stage decisions can be expressed in (2) and (3). Denote the optimal unit commitment decision by $u^*$ and the optimal objective by $G(u^*) \equiv C^{su} u^* + E_{\xi \sim F^c} [\min_y C^{fuel} y(u^*, \xi)]$.

### 3.2 Review of existing empirical stochastic unit commitment and deterministic unit commitment approaches

In this section, we briefly summarize the existing deterministic UC and SUC modeling approaches and then discuss their limitations. Traditionally, scheduling and dispatch in power system operations have been done by using deterministic methods, and this is still the industry practice in most regions [45]. Some recent studies also use the deterministic UC to analyze the impact of renewable resources on power system operations [46, 47].

Basically, given the historical data, various methods can be used for wind power forecasting, such as nonparametric persistence-based forecasting method [48–50] and parametric Autoregressive Moving Average (ARMA) approach [46]. The deterministic UC makes the optimal decision, denoted by $u^{*d}$, based on the point predictor of future wind power generation in the planning horizon, denoted by $\hat{\xi}$, by solving the deterministic optimization,
Thus, deterministic UC does not consider the prediction risk induced by both wind power generation inherent stochastic uncertainty and input model estimation uncertainty. It could deliver an inferior and unreliable decision, especially as the penetration of renewable energy increases.

Differing with the deterministic UC, the empirical SUC considers the impacts from stochastic uncertainty of wind power generation on electric power systems [8, 9, 23]. Basically, given the historical data $D$, to find the optimal decision, the empirical SUC takes the input model estimate, denoted by $F_e$, as the true one, and then solves the stochastic optimization,

$$
\min_{\mathbf{u}} G^p(\mathbf{u}) \equiv C^{su} \mathbf{u} + \min_{y} C^{fuel} y(\mathbf{u}, \xi)
\text{s.t. } A\mathbf{u} \leq B
H\mathbf{u} + Q y(\mathbf{u}, \xi) \leq M(\xi).
$$

Then, a Monte Carlo approach can be used to generate a finite number of scenarios from $F_e$, use the sample mean to approximate the expected future cost in the planning horizon, and obtain the optimal decision, denoted by $u^*$. Thus, the empirical UC ignores the input model estimation uncertainty and finite sampling error from SAA. It could lead to an inferior and unreliable unit commitment decision, especially under the situations when the wind power penetration is high and the amount of representative real-world wind power historical data is limited; see as shown in the case studies in Sect. 5.

### 3.3 Data-driven stochastic unit commitment model

In this paper, we propose a data-driven SUC accounting for both stochastic uncertainty of wind power generation and input model estimation uncertainty. Given the historical data $D$, the posterior distribution of underlying input model $F^e$ is used to quantify the model estimation uncertainty. Then, the posterior predictive distribution, denoted by $F^p$, can quantify the prediction risk induced from both sources of uncertainties. Thus, the proposed data-driven SUC model becomes,

$$
\min_{\mathbf{u}} G^p(\mathbf{u}) \equiv C^{su} \mathbf{u} + E_{\xi \sim F^p} \left[ \min_{y} C^{fuel} y(\mathbf{u}, \xi) \right]
\text{s.t. } A\mathbf{u} \leq B
H\mathbf{u} + Q y(\mathbf{u}, \xi) \leq M(\xi).
$$

Our empirical study shows that the proposed data-driven SUC approach can lead to cost-efficient, reliable and robust optimal unit commitment decision, denoted by
\(\mathbf{u}^{*p}\), which can hedge against the prediction risk induced by stochastic uncertainty of wind power generation and input model estimation uncertainty.

Given the historical data \(\mathcal{D}\), the posterior distribution characterizing the input model estimation uncertainty can be obtained by the Bayes’ rule, 
\[ p(F|\mathcal{D}) \propto p(F)p(\mathcal{D}|F), \]
where \(p(F)\) denotes the prior and \(p(\mathcal{D}|F)\) denotes the likelihood of historical data. Then, the density of posterior predictive distribution \(F^p\),
\[ f^p(\xi) \equiv \int p(\xi|F)p(F|\mathcal{D})dF, \tag{7} \]
can quantify the overall prediction uncertainty of wind power generation with \(p(F|\mathcal{D})\) characterizing the input model estimation uncertainty and \(p(\xi|F)\) characterizing the prediction uncertainty induced by wind power generation inherent volatility or stochastic uncertainty.

The proposed data-driven SUC can be applied to situations where the parametric family of underlying input model \(F^c\) is known, e.g., [9, 51, 52]. In Sects. 5.1.1 and 5.2.1, when we study the six- and 118-bus power grid systems, we use the normal distribution assumption for illustration. The proposed data-driven SUC in (6) can also apply to the situations where there is no strong prior information on the distribution family for \(F^c\). In Sects. 5.1.2 and 5.2.2, we suppose that there is no strong parametric assumption on the underlying input model \(F^c\) and use the Bayesian nonparametric probabilistic forecast introduced in our previous study [10] as an illustration.

# 4 Optimization and selection for SUC

When we solve the SUC, the sample average approximation (SAA) is typically used to estimate the expected cost in the planning horizon. It induces the finite sampling error, which can be large especially for complex power grids with high wind power penetration. In Sect. 4.1, we provide the description on why this source of error exists for general SUC problem. Then, to reduce the impact of finite sampling approximation error, we develop an optimization and selection (OPSEL) approach in Sect. 4.2, which can utilize the parallel computing to quickly solve the SUC problem and find the optimal reliable UC decision.

## 4.1 Finite sampling approximation error induced by SAA

When we solve any general SUC, there often exists the finite sample approximation error. Specifically, we solve the SUC as follows,
\[ \min_{\mathbf{u}} G(\mathbf{u}) \equiv C^{\text{str}}\mathbf{u} + E_{\xi \sim F^e} \left[ \min_{\mathbf{y}} C^{\text{fuel}}\mathbf{y}(\mathbf{u}, \xi) \right], \tag{8} \]
where \(F^e = F^c, F^e\) or \(F^p\) can represent the underlying true input model, the estimated empirical input distribution, or the posterior predictive distribution.
accounting for both input model estimation uncertainty and wind power stochastic uncertainty. Since we do not have the closed analytical form for the expected cost $E_{\xi \sim F_{\Phi}} \left[ \min_y C_{\text{fuel}}^y(u, \xi) \right]$, the sample average approximation (SAA) is typically utilized during the stochastic programming optimization solution methodologies; see the introduction of SAA in [53].

That means, given any feasible unit commitment decision $u$, we estimate the expected economic dispatch cost $E_{\xi \sim F_{\Phi}} \left[ \min_y C_{\text{fuel}}^y(u, \xi) \right]$ by using the sample average approximation. We generate $S$ scenarios, $\xi_s \sim F_{\Phi}$ for $s = 1, 2, \ldots, S$, and use the sample average of second-stage optimization outputs to estimate the expected cost in the planning horizon. Then, the SAA approximated SUC problem in (8) becomes

$$\min_u \hat{G}^p(u) = C^u u + \frac{1}{S} \sum_{s=1}^{S} \left[ \min_y C_{\text{fuel}}^y(u, \xi_s) \right].$$

Since $\frac{1}{S} \sum_{s=1}^{S} \left[ \min_y C_{\text{fuel}}^y(u, \xi_s) \right] \neq E_{\xi \sim F_{\Phi}} \left[ \min_y C_{\text{fuel}}^y(u, \xi) \right]$, the finite sampling approximation error always exists and it could be large when the power grid is complex and wind power penetration increases. Even though a large number of scenarios can reduce this source of error, it becomes computationally infeasible to solve the SUC in a tight decision time. Thus, we develop the OPSEL approach in Sect. 4.2 to reduce the impact from finite sampling approximation error from SAA.

### 4.2 Optimization and selection (OPSEL) approach

Based on our observation that the time used to assess the performance of any given unit commitment decision is much less than the optimization search, the proposed OPSEL includes two parts: parallel optimization search and best candidate decision selection. For the optimization search part, we simultaneously solve $L$ independent SAA approximated SUC problems through parallel computing. It returns a set of optimal candidate solutions quantifying the impact of finite sampling error induced by SAA. For the best candidate decision selection part, we apply the OCBA-based ranking-and-selection approach to efficiently allocate more computational budget to the most promising candidate decisions, improve the assessment of their performance, and quickly select the best decision. Thus, the combination of data-driven SUC and OPSEL can lead to the optimal and reliable unit commitment decision, which can hedge against: (1) inherent stochastic uncertainty of wind power generation, (2) input model estimation uncertainty, and (3) finite sampling error induced by SAA.

To efficiently employ the computational resource and quickly deliver the optimal reliable UC decision, we exploit the parallel computing with $L$ available CPUs. With each $\ell$-th CPU, we first generate an independent set consisting of $S$ scenarios, $\xi^1, \xi^2, \ldots, \xi^S$, drawn from $F_{\Phi}$ (i.e., $F^c, F^e$ or $F^p$), and then we solve the corresponding SAA approximated SUC problems in (9). In the case study, we use the L-Shaped algorithm for optimization [41, 43]. Thus, based on the parallel optimization search, we obtain the optimal unit commitment candidate decisions, denoted by $\hat{u}^\ell_s$ with $\ell = 1, 2, \ldots, L$, quantifying the impact of finite sampling error.
To reduce the impact of finite sampling error quantified by candidates \( \hat{u}_1^*, \hat{u}_2^*, \ldots, \hat{u}_L^* \), we efficiently utilize the computational resource to assess the performance of candidate decisions, \( G^p(\hat{u}_1^*), G^p(\hat{u}_2^*), \ldots, G^p(\hat{u}_L^*) \), and select the best one,

\[
u_{b^{op}}^* \equiv \arg\min_{\hat{u}^* \in \{\hat{u}_1^*, \ldots, \hat{u}_L^*\}} G^p(\hat{u}_\ell^*),
\]

where the subscript \( b^{op} \) denotes the index of the best unit commitment decision. The number of CPUs, \( L \), could be large. For each candidate, the performance \( G^p(\hat{u}_\ell^*) \) with \( \ell = 1, 2, \ldots, L \) can be assessed by using Monte Carlo approach. We sequentially allocate the computational budget to the most promising candidates \( \hat{u}_\ell^* \) so that we can provide more accurate estimation of their expected cost \( G^p(\hat{u}_\ell^*) \) and efficiently select out the best solution.

Basically, built on the results from parallel optimal search, we further use the OCBA-based ranking and selection [11] to guide the sequential allocation of computational resource to most promising candidates. Here, the each unit of computational budget is measured by the cost of solving one second-stage economic dispatch problem for each scenario. Let \( N_{k,\ell} \) be the accumulated number of scenarios assigned to the candidate solution \( \hat{u}_\ell^* \) for \( \ell = 1, 2, \ldots, L \) until the \( k \)-th iteration of sequential candidate selection, and the objective estimate is

\[
\bar{G}_k^p(\hat{u}_\ell^*) = C^{\text{fuel}} \hat{u}_\ell^* + \frac{1}{N_{k,\ell}} \sum_{s=1}^{N_{k,\ell}} \min_{y} C^{\text{fuel}}(y(\hat{u}^*_\ell, \xi^s)).
\]  

(10)

With more scenarios, we can estimate the performance \( G^p(\hat{u}_\ell^*) \) more accurate. The number of initial scenarios is \( N_{0,\ell} = S \) since the data-driven SUC approximated with \( S \) samples is solved by the \( \ell \)-th CPU to obtain \( \hat{u}_\ell^* \); see Eq. (9). At the \( k \)-th iteration, we allocate \( \Delta M \) additional scenarios to the candidates \( \hat{u}_\ell^* \) with \( \ell = 1, 2, \ldots, L \). Then, we solve the corresponding economic dispatch problems for the new generated scenarios and update the objective estimate for \( G^p(\hat{u}_\ell^*) \) by using Eq. (10).

Based on [11], the optimal budget allocation \( N_{k,\ell} \) is obtained by solving

\[
\frac{N_{k,\ell}}{N_{k,\ell'}} = \left( \frac{\delta_{k-1,\ell'}}{\delta_{k-1,\ell}} \right)^2 \text{ for } \ell, \ell' \neq b^{op}
\]

\[
N_{k,b^{op}} = \hat{\sigma}_{k-1,b^{op}} \sqrt{\sum_{\ell=1,\ell \neq b^{op}}^{L} \frac{N_{k,\ell}^2}{\hat{\sigma}_{k-1,\ell}^2}}
\]

\[
\sum_{\ell=1}^{L} N_{k,\ell} = L \times S + k \times \Delta M
\]

(11)

where \( N_{k,b^{op}} \) denotes the number of scenarios assigned to the current best candidate selected from the \((k - 1)\)-th iteration

\[
\hat{u}_{k-1,b^{op}}^* \equiv \arg\min_{\hat{u}^* \in \{\hat{u}_1^*, \ldots, \hat{u}_L^*\}} G^p_{k-1}(\hat{u}_\ell^*).
\]  

(12)
The estimate of variance, \( \sigma^2_k = \text{Var} [\min_y \mathbf{C}^{\text{fuel}} \mathbf{y}(\mathbf{u}, \xi)] \), is the sample variance obtained from the \((k - 1)\)-th iteration,

\[
\hat{\sigma}^2_{k-1,\ell} = \frac{1}{N_{k-1,\ell} - 1} \sum_{s=1}^{N_{k-1,\ell}} \left[ \mathbf{C}^{\text{op}} \hat{\mathbf{u}}^*_\ell + \min_y \mathbf{C}^{\text{fuel}} \mathbf{y}(\hat{\mathbf{u}}^*_\ell, \xi^s) - \bar{G}^p_{k-1}(\hat{\mathbf{u}}^*_\ell) \right]^2
\]

and \( \delta_{k-1,\ell} \) denotes the standardized distance between \( \hat{\mathbf{u}}^*_\ell \) with the current estimated best candidate \( \hat{\mathbf{u}}^*_{k-1,\ell} \),

\[
\delta_{k-1,\ell} = \frac{\bar{G}^p_{k-1}(\hat{\mathbf{u}}^*_\ell) - \bar{G}^p_{k-1}(\hat{\mathbf{u}}^*_{k-1,\ell})}{\hat{\sigma}_{k-1,\ell}}.
\]  

By applying Eqs. (11) and (13), the budget allocation to any candidate \( \hat{\mathbf{u}}^*_\ell \) is directly related to its standardized distance with the current best estimate, which can allocate more computational resource to the promising candidates and efficiently select out the best decision. Thus, in the \( k \)-th iteration, the number of new scenarios allocated to the candidate \( \hat{\mathbf{u}}^*_\ell \) for \( \ell = 1, 2, \ldots, L \) is,

\[
\Delta N_{k,\ell} = \max(0, N_{k,\ell} - N_{k-1,\ell}).
\]  

Then, we solve the additional \( \Delta N_{k,\ell} \) second-stage economic dispatch problems and update the objective estimate \( \bar{G}^p_k(\hat{\mathbf{u}}^*_\ell) \).

The OPSEL procedure is provided in Algorithm 1. We denote the overall computational budget in terms of number of scenarios allocated for the candidate selection by \( M \). In Step (1), we utilize \( L \) number of CPUs to simultaneously solve the sample average approximated SUC problems with form (9) and obtain the optimal candidate decisions \( \hat{\mathbf{u}}^*_\ell \) with \( \ell = 1, 2, \ldots, L \). Then, in Steps (2) and (3), we sequentially allocate the computational resource to \( \hat{\mathbf{u}}^*_1, \hat{\mathbf{u}}^*_2, \ldots, \hat{\mathbf{u}}^*_L \) and select the best candidate decision hedging against the finite sampling error.

**ALGORITHM 1: The Optimization and Selection Procedure**

Step (0) Specify \( S \) and the total budget \( M \), the total number of scenarios used for the candidate solution selection.

Step (1) Utilize \( L \) CPUs to simultaneously solve the SUC problems (9) approximated by \( S \) scenarios, and obtain the optimal candidate decisions \( \hat{\mathbf{u}}^*_1, \hat{\mathbf{u}}^*_2, \ldots, \hat{\mathbf{u}}^*_L \).

Step (2) At the \( k \)-th iteration of selection, allocate \( \Delta M \) new scenarios to \( \hat{\mathbf{u}}^*_1, \hat{\mathbf{u}}^*_2, \ldots, \hat{\mathbf{u}}^*_L \) by using (14) and solve the additional second-stage dispatch problems. Update \( \bar{G}^p_k(\hat{\mathbf{u}}^*_\ell) \) for \( \ell = 1, 2, \ldots, L \) and \( \hat{\mathbf{u}}^*_{k,\ell} \) by applying (10) and (12).

Step (3) Repeat Step (2) until reaching to the budget \( M \). Return \( \hat{\mathbf{u}}^*_{k,\ell} = \arg\min_{\mathbf{u}_\ell \in \{\hat{\mathbf{u}}^*_1, \hat{\mathbf{u}}^*_2, \ldots, \hat{\mathbf{u}}^*_L\}} \bar{G}^p_k(\hat{\mathbf{u}}^*_\ell) \).
5 Empirical studies

We use the six-bus system from [54] in Sect. 5.1 and the derivative 118-bus system from [55] in Sect. 5.2 to compare the performance of proposed data-driven SUC framework with that of the deterministic unit commitment (UC) and the empirical SUC under the situations when the parametric input model for wind power generation is known or not. In the empirical studies, we consider the two-stage SUC problem,

\[
\min \ G(\mathbf{u}) = \sum_{t=1}^{T} \sum_{i=1}^{I} \left[ C^i F_{mi}(P_{i,min}) u_{i,t} + SU_{i,t} + SD_{i,t} \right] \\
+ E \left[ \min_{P_{i,t}, P_{w,t}, P_{ens}} \sum_{t=1}^{T} \sum_{i=1}^{I} C^i F_{ai}(P_{i,t}) + \sum_{t=1}^{T} \sum_{b=1}^{B} C^{ens}_{b,t} + \sum_{t=1}^{T} \sum_{w=1}^{W} C^{wc}_{w,t} \right] \\
\text{s.t.} \quad u_{i,t} - u_{i,t-1} \leq u_{i,k} \quad \forall k = t, \ldots, \min(T, t + T_{i}^{on} - 1) \\
\quad u_{i,k} \leq 1 + u_{i,t} - u_{i,t-1} \quad \forall k = t, \ldots, \min(T, t + T_{i}^{off} - 1) \\
\quad \sum_{i=1}^{I} P_{i,t} + \sum_{w=1}^{W} (P_{w,t}^{c} - P_{w,t}^{wc}) = \sum_{b=1}^{B} (P_{b,t}^{D} - P_{b,t}^{ens}) \\
\quad -PL_{\epsilon,max} \leq \sum_{b=1}^{B} SF_{\epsilon-b} P_{b,t}^{D} + \sum_{i=1}^{I} SF_{\epsilon-i} P_{i,t} + \sum_{w=1}^{W} SF_{\epsilon-w} (P_{w,t}^{c} - P_{w,t}^{wc}) \leq PL_{\epsilon,max} \\
\quad u_{i,t} P_{i,min} \leq u_{i,t} P_{i,t} \leq u_{i,t} P_{i,max} \quad \forall i, \forall t \\
\quad u_{i,t} \quad \text{binary.}
\]

The objective in (15) is to minimize the total expected cost, including the start-up cost \(SU_{i,t}\), the turn-off cost \(SD_{i,t}\) and minimal thermal operation cost \(C^i F_{mi}(P_{i,min}) u_{i,t}\) incurring in the first stage, where \(C^i\) is the fuel price and \(F_{mi}(P_{i,min})\) is the amount of fuel consumption for the minimal power production \(P_{i,min}\) at generator \(i\). The start-up cost \(SU_{i,t}\), incurring when the generator \(i\) is turned on at time \(t\), is the product of the start-up fuel consumption \(Su_{i,t}\) and the fuel price \(C^i\) (i.e., \(SU_{i,t} = Su_{i,t} \cdot C^i\)). Similarly, the shut down cost \(SD_{i,t}\) incurring when generator \(i\) is shut down at time \(t\), is the product of \(C^i\) and the shut down fuel consumption \(Sd_{i,t}\) (i.e., \(SD_{i,t} = Sd_{i,t} \cdot C^i\)). In addition, once the thermal generator \(i\) is committed (e.g., \(u_{i,t} = 1\)), it has to produce above the minimal production level, which incurs the minimal thermal operation cost \(C^i F_{mi}(P_{i,min}) u_{i,t}\). To produce the power \(P_{i,t}\), the actual amount of fuel consumption follows \(F_{i} = a_i + b_i \cdot P_{i,t} + c_i \cdot P_{i,t}^2\). Thus, the minimal production fuel consumption is \(F_{mi}(P_{i,min}) = a_i + b_i \cdot P_{i,min} + c_i \cdot P_{i,min}^2\).
There are three additional costs incurring at the second-stage. Since the thermal generator consumes the extra fuel to produce actual power \( P_{i,t} \) at time \( t \), the additional production cost in the second-stage is \( C_{i}^{F}(P_{i,t}) \), where \( F_{ai}(P_{i,t}) = a_i + b_i \cdot P_{i} + c_i \cdot P_{i}^2 - F_{mi}(P_{i,\text{min}}) \). Linearization techniques are used to transfer \( F_{ai} \) into a piecewise linear function [56]. If the smart grid could not produce enough energy to satisfy the load demand, a shortage penalty incurs. The penalty cost of non-satisfied demand for bus \( b \) at time \( t \) is \( C_{\text{ens}}^{P_{\text{ens}}} b,t \), where \( C_{\text{ens}} \) is the unit load shedding price and \( P_{\text{ens}} b,t \) is the amount of unmet load at bus \( b \) in time \( t \). Lastly, for the wind power, we may not use up all its capacity \( P_{c,w,t} \) and there exists a wind farm curtailment \( P_{wc,w,t} \). The wind curtailment cost at the \( t \)-th hour for wind farm \( w \) is \( C_{wc}^{P_{wc}} w,t \), where \( C_{wc} \) is the per unit monetary reward for the wind production and \( P_{wc,w,t} \) is the amount of wind curtailment.

Constraints (16)–(17) formulate the minimum up and down time requirements. Constraint (18) is for the nodal power balance. Constraint (19) is the DC power flow constraint on the transmission lines and \( SF \) is the shift factor matrix which can be obtained by using reactance; see [57]. Constraint (20) enforces the minimum and maximum generator output limits.

### 5.1 Empirical study with six-bus system

For the six-bus system, it consists of three thermal units, a wind farm and seven transmission lines as depicted in Fig. 1. The three thermal units are located in No.1, No.2 and No.6 buses, while No.4 Bus hosts a wind farm; see Table 1 for the description of the six-bus system. Table 2 describes the characteristics of the thermal generators. The Min Off (h) and Min On (h), denoted by \( T_{\text{off}} \) and \( T_{\text{on}} \), represent the minimum off and on time requirements; see Constrains (16) and (17).

Table 3 describes the cost of three thermal generators. The characteristics of the transmission lines are provided in Table 4.

Since the uncertainty in wind supply typically dominates, in this study, we assume deterministic loads and stochastic wind supply [2]. Following [54], we use the 2006 data of the U.S. Illinois power system for the load \( P_{d,t} \) and the wind supply \( P_{w,t} \). Then, the wind penetration level measured by the ratio of wind power generation to actual demand, \( R = \sum_{w=1}^{W} \sum_{t=1}^{T} P_{w,t} / \sum_{b=1}^{B} \sum_{t=1}^{T} P_{d,t} \), is 37.8\%. Additionally, the wind curtailment price \( C_{wc} \) is fixed at 50$/$MWh and the load shedding price \( C_{\text{ens}} \) is set at 3500$/$MWh [54].

### 5.1.1 Case study with parametric forecast model

In this section, suppose that the input model of wind power generation \( F^c \) follows the normal distribution [6, 9, 52] with unknown parameters estimated by valid historical data. Here, we consider the day-ahead unit commitment with the planning horizon equal to 24 h. In each day \( d \), suppose that the wind power generation at the \( t \)-th hour follows a normal distribution, \( \xi_{d,t} \sim N(\mu_{d,t} ^{\ddagger}, \phi_{d,t} ^{\ddagger}) \), where \( \mu_{d,t} ^{\ddagger} \) and \( \phi_{d,t} ^{\ddagger} \) are mean and variance. Thus, the underlying true input model \( F^c \) for \( \xi_{d,t} \) in
SUC (1) is \(N(\mu_{d,t}^c, \phi_{d,t}^2)\). To evaluate the performance, we pretend that the true parameter \(\mu_{d,t}^c\) is unknown. Suppose that wind power at the \(t\)-th hour in the past \(m\) days follows the same distribution. To predict \(\xi_{d,t}\), we use the valid historical observation \(D_{d,t} = \{\xi_{(d-m),t}, \ldots, \xi_{(d-1),t}\}\) with \(t = 1, 2, \ldots, 24\) h, where \(\xi_{(d-i),t}\) denotes the real wind power observation at the \(t\)-th hour in day \((d-i)\) with \(i = 1, 2, \ldots, m\). Here, we set \(\mu_{d,t}^c\) equal to the 2006 wind power generation.

---

**Fig. 1** Six-bus system

![Six-bus system](image)

**Table 1** Bus data

| Bus ID | Type   | Thermal unit | Wind farm | Load share (%) |
|--------|--------|--------------|-----------|----------------|
| NO 1   | Thermal| G1           |           |                |
| NO 2   | Thermal| G2           |           |                |
| NO 3   |        |              |           | 20             |
| NO 4   | Wind   | W1           | 40        |                |
| NO 5   |        |              |           | 40             |
| NO 6   | Thermal| G3           |           |                |

**Table 2** Thermal generator data

| Unit | \(P_{\text{max}}\) (MW) | \(P_{\text{min}}\) (MW) | Min Off \(T_{\text{on}}\) (h) | Min On \(T_{\text{off}}\) (h) |
|------|--------------------------|--------------------------|-------------------------------|-------------------------------|
| G1   | 220                      | 90                       | 4                             | 4                             |
| G2   | 100                      | 20                       | 3                             | 2                             |
| G3   | 30                       | 10                       | 1                             | 1                             |

**Table 3** Thermal generator data

| Unit | Fuel consumption function | Start up fuel | Shut down fuel | Fuel price \(C\) ($) |
|------|----------------------------|---------------|----------------|----------------------|
|      | \(a\) (MBtu) | \(b\) (MBtu/MWh) | \(c\) (MBtu/MW2h) | \(Sd'\) (MBtu) | \(Sd''\) (MBtu) |
| G1   | 176.9          | 13.5          | 0.0004          | 180            | 50               | 1.2469          |
| G2   | 129.9          | 32.6          | 0.001           | 360            | 40               | 1.2461          |
| G3   | 137.4          | 17.6          | 0.005           | 60             | 0                | 1.2462          |
The empirical SUC takes the estimated input model coefficient, sample mean, as the true one. Thus, the input model $F_{e,d}^c$, for SUC is $N(\bar{\gamma}_{d,t}^r, \phi_{d,t}^2)$, where the sample mean of the historical data $\bar{\gamma}_{d,t}^r = \frac{1}{m} \sum_{i=1}^{m} \gamma_{(d-i),t}^r$ is the plug-in estimate of unknown parameter $\mu_{d,t}^c$. For the proposed data-driven SUC, the model estimation uncertainty is characterized by the posterior distribution. Without strong information about the mean $\mu_{d,t}^c$, we use the non-informative prior, a normal distributed with mean zero and infinite variance [58]. The posterior distribution is $p(\mu_{d,t}^c | \mathcal{D}_{d,t}) = N(\bar{\gamma}_{d,t}^r, \phi_{d,t}^2/m)$. Then, the resulting posterior predictive distribution $F_{d,t}^p$ is $N(\bar{\gamma}_{d,t}^r, (1 + \frac{1}{m})\phi_{d,t}^2)$, which quantifies the prediction risk accounting for both input model estimation uncertainty and wind power generation inherent stochastic uncertainty.

We compare the performance of unit commitment decisions obtained from the data-driven SUC with the empirical SUC under various settings with standard deviation $\phi_{d,t} = 5%\mu_{d,t}^c, 10%\mu_{d,t}^c, 20%\mu_{d,t}^c$. Let $n_d$ denote the total number of days used for the evaluation. Let $\tilde{\mathbf{u}}_{d}^{*p}$ and $\tilde{\mathbf{u}}_{d}^{*e}$ denote the 24-h optimal unit commitment decisions obtained by data-driven and empirical SUCs with $d = 1, \ldots, n_d$. Then, the total expected costs obtained by these approaches are

$$\sum_{d=1}^{n_d} G^p = \sum_{d=1}^{n_d} G(\tilde{\mathbf{u}}_{d}^{*p})$$

and

$$\sum_{d=1}^{n_d} G^e = \sum_{d=1}^{n_d} G(\tilde{\mathbf{u}}_{d}^{*e}). \tag{22}$$

Since there is no closed form, the sample average approximations, $\tilde{G}(\tilde{\mathbf{u}}_{d}^{*p})$ and $\tilde{G}(\tilde{\mathbf{u}}_{d}^{*e})$, are used to estimate the true objectives. To determine a proper scenario size $S_e$ so that we can estimate $G(\tilde{\mathbf{u}}_{d}^{*p})$ and $G(\tilde{\mathbf{u}}_{d}^{*e})$ accurately, we conduct a side experiment and consider the high uncertainty case with $\phi_{d,t} = 20%\mu_{d,t}^c$. In addition, since the empirical approach ignores the input model parameter estimation uncertainty, the unit commitment decisions highly fluctuate with the random wind power observations and its solution quality is more volatile. Thus, to decide the proper sample size that can ensure the accurate estimation of the expected total cost, we consider the empirical approach. Specifically, we first apply the L-shaped algorithm to solve the empirical SUC with the expected cost approximated by SAA having $S = 50$ and obtain a unit commitment decision $\tilde{\mathbf{u}}_{d}^{*e}$.

### Table 4  Transmission line data

| Line no. | From bus | To bus | Reactance X (p.u) | Flow limit $P_{Lmax}$ (MW) |
|----------|----------|--------|-------------------|--------------------------|
| 1        | 1        | 2      | 0.17              | 200                      |
| 2        | 1        | 4      | 0.258             | 100                      |
| 3        | 2        | 4      | 0.197             | 100                      |
| 4        | 5        | 6      | 0.14              | 100                      |
| 5        | 2        | 3      | 0.037             | 200                      |
| 6        | 4        | 5      | 0.037             | 200                      |
| 7        | 3        | 6      | 0.018             | 200                      |

The empirical SUC takes the estimated input model coefficient, sample mean, as the true one. Thus, the input model $F_{e,d}^c$ for SUC is $N(\bar{\gamma}_{d,t}^r, \phi_{d,t}^2)$, where the sample mean of the historical data $\bar{\gamma}_{d,t}^r = \frac{1}{m} \sum_{i=1}^{m} \gamma_{(d-i),t}^r$ is the plug-in estimate of unknown parameter $\mu_{d,t}^c$. For the proposed data-driven SUC, the model estimation uncertainty is characterized by the posterior distribution. Without strong information about the mean $\mu_{d,t}^c$, we use the non-informative prior, a normal distributed with mean zero and infinite variance [58]. The posterior distribution is $p(\mu_{d,t}^c | \mathcal{D}_{d,t}) = N(\bar{\gamma}_{d,t}^r, \phi_{d,t}^2/m)$. Then, the resulting posterior predictive distribution $F_{d,t}^p$ is $N(\bar{\gamma}_{d,t}^r, (1 + \frac{1}{m})\phi_{d,t}^2)$, which quantifies the prediction risk accounting for both input model estimation uncertainty and wind power generation inherent stochastic uncertainty.

We compare the performance of unit commitment decisions obtained from the data-driven SUC with the empirical SUC under various settings with standard deviation $\phi_{d,t} = 5%\mu_{d,t}^c, 10%\mu_{d,t}^c, 20%\mu_{d,t}^c$. Let $n_d$ denote the total number of days used for the evaluation. Let $\tilde{\mathbf{u}}_{d}^{*p}$ and $\tilde{\mathbf{u}}_{d}^{*e}$ denote the 24-h optimal unit commitment decisions obtained by data-driven and empirical SUCs with $d = 1, \ldots, n_d$. Then, the total expected costs obtained by these approaches are

$$\sum_{d=1}^{n_d} G^p = \sum_{d=1}^{n_d} G(\tilde{\mathbf{u}}_{d}^{*p})$$

and

$$\sum_{d=1}^{n_d} G^e = \sum_{d=1}^{n_d} G(\tilde{\mathbf{u}}_{d}^{*e}). \tag{22}$$

Since there is no closed form, the sample average approximations, $\tilde{G}(\tilde{\mathbf{u}}_{d}^{*p})$ and $\tilde{G}(\tilde{\mathbf{u}}_{d}^{*e})$, are used to estimate the true objectives. To determine a proper scenario size $S_e$ so that we can estimate $G(\tilde{\mathbf{u}}_{d}^{*p})$ and $G(\tilde{\mathbf{u}}_{d}^{*e})$ accurately, we conduct a side experiment and consider the high uncertainty case with $\phi_{d,t} = 20%\mu_{d,t}^c$. In addition, since the empirical approach ignores the input model parameter estimation uncertainty, the unit commitment decisions highly fluctuate with the random wind power observations and its solution quality is more volatile. Thus, to decide the proper sample size that can ensure the accurate estimation of the expected total cost, we consider the empirical approach. Specifically, we first apply the L-shaped algorithm to solve the empirical SUC with the expected cost approximated by SAA having $S = 50$ and obtain a unit commitment decision $\tilde{\mathbf{u}}_{d}^{*e}$. Then, we estimate the
expected cost by using $S_e$ scenarios to obtain $\bar{G}(\hat{\mathbf{u}}_d^{*,e})$ and calculate the relative error,
\[
\text{relativeError} = \left| \frac{\bar{G}(\hat{\mathbf{u}}_d^{*,e}) - G(\hat{\mathbf{u}}_d^{*,e})}{\bar{G}(\hat{\mathbf{u}}_d^{*,e})} \right|, \quad \text{where } G(\hat{\mathbf{u}}_d^{*,e}) \text{ denotes the objective value obtained by using } 10^5 \text{ scenarios.}
\]
Suppose that $10^5$ is large enough so that the estimation error of $G(\hat{\mathbf{u}}_d^{*,e})$ is negligible. The maximum relative error obtained from 10 day-period is recorded in Table 5. We observe that $S_e = 1000$ achieves accuracy with the maximum relative error not exceeding 1.0%. Balancing the computational cost and the accuracy, we use $S_e = 1000$ to evaluate the true expected cost.

The wind power data in 2006 October are used for evaluation. Let $m = 1$. In the study, we set the scenario size $S = 50$ and get the optimal decision estimates $\hat{\mathbf{u}}_d^{*,e}$ and $\hat{\mathbf{u}}_d^{*,p}$ by solving the sample average approximated empirical and data-driven SUCs. For cases with $\phi_{d,t} = 5\% \mu_{d,t}^{e}$, $10\% \mu_{d,t}^{e}$, $20\% \mu_{d,t}^{e}$, the results of $\sum G^p$ and $\sum G^e$ in (22) for the one-month period are recorded in Table 6. We also record the relative expected saving obtained by our method, denoted by $r\Delta G$,
\[
r\Delta G = \frac{\sum G^e - \sum G^p}{\sum G^p}. \tag{23}
\]

The results in Table 6 demonstrate that the proposed data-driven SUC significantly outperforms the empirical SUC. When $\phi_{d,t} = 5\% \mu_{d,t}^{e}$, the total expected cost-saving by our approach is 208, 730, which represents a 8.8% lower cost than the empirical SUC. As $\phi_{d,t}$ increases, the advantages of data-driven SUC tend to be larger. When $\phi_{d,t} = 20\% \mu_{d,t}^{e}$, our approach outperforms the empirical approach by 15.4% savings.

5.1.2 Case study with nonparametric forecast model

In the real application, we often do not have any strong prior knowledge about the underlying input model $F^e$ and its distribution family is typically unknown. Thus, we consider a unit commitment problem with nonparametric forecast models. Here, we compare the performance of various approaches, including (1) the deterministic unit commitment (UC); (2) the empirical SUC accounting for wind power stochastic uncertainty; and (3) the data-driven SUC accounting for both wind power stochastic uncertainty and input model estimation uncertainty. Specifically, for the data-driven SUC, we use the Bayesian nonparametric wind power forecast model.
proposed in our previous study [10]. For the empirical SUC and deterministic UC, we use probabilistic and deterministic persistence models [48–50]. At the time $h_i$, the probabilistic persistence model is based on the empirical predictive distribution for $\xi_{h_i+i}$ specified by
\[
\{\xi_{h_i}^r - \xi_{h_i-j}^r + \xi_{h_i-j-i}^r : j = 0, \ldots, h_i - i - 1\}
\]
where $\xi_{h_i}^r$, $\xi_{h_i-j}^r$ and $\xi_{h_i-j-i}^r$ are wind power observations at time $h_i$, $h_i-j$ and $h_i-j-i$, respectively. The deterministic persistence model simply takes the previous historical observation as the point estimator, i.e., $\xi_{h_i+i} = \xi_{h_i}^r$.

Since the Bayesian nonparametric forecast proposed in [10] and persistent models focus on the short term prediction, we consider the unit commitment problem for the intra-day market; see [59]. The 1-h ahead intra-day unit commitment problem has the planning horizon with $n_h = 4$ h and we make the unit commitment decision of the $t$-th hour at hour $h_t$, where $h_t \equiv \left(\left\lceil \frac{t}{n_h}\right\rceil - 1\right) \cdot n_h$. It means that we make the 1-h ahead intra-day unit commitment decisions every 4 h. For example, we make $t = 1, \ldots, 4$ h intra-day unit commitment decisions at $h_t = 0$. Suppose that all three generators in the six-bus system are fast start generators that can be committed/decommitted at the intra-day market.

At any time $h_t + 1$, we can use $m$ latest historical data for the wind power prediction, $D = \{\xi_{h_t-m+1}^r, \ldots, \xi_{h_t}^r\}$, where $\xi_{h_t-i}^r$ is the hourly wind power observation happened $i$ hours prior to $h_t$. By following [10], we set $m = 100$. For the Bayesian nonparametric forecast approach, we apply the sampling procedure developed in [10]. For the probabilistic persistence model, we apply the sampling procedure developed in [49].

Denote the optimal unit commitment decisions between hours $h_t + 1$ and $h_t + n_h$ on day $d$ obtained from data-driven SUC, empirical SUC and deterministic UC by $\hat{\mathbf{u}}_{d,h_t}^{*,\rho}$, $\hat{\mathbf{u}}_{d,h_t}^*$ and $\hat{\mathbf{u}}_{d,h_t}^{*,d}$ for $d = 1, \ldots, n_d$ and $h_t = 0, 4, \ldots, 20$. Let the set $\mathcal{H} \equiv \{0, 4, \ldots, 20\}$. Since $F^\rho$ is unknown, we evaluate the performance of these solutions by the actual incurred cost; see the details in [2]. Denote $\xi_{d,h_t}^r \equiv (\xi_{d,(h_t+1)}^r, \ldots, \xi_{d,(h_t+n_h)}^r)$ as the real wind power realizations between hours $h_t + 1$ and $h_t + n_h$ on day $d$. Then, the real cost $G_{d,h_t}^r$ including both commitment and economic dispatch costs is,
\[
G_{d,h_t}^r(\mathbf{u}_{d,h_t}) \equiv C^{\text{mu}}_{d,h_t} + \min_y C^{\text{fuel}}_{d,h_t}(y(\mathbf{u}_{d,h_t}, \xi_{d,h_t}^r)).
\]

Thus, the total cost occurring in the $n_d$ days is
\[
\sum_{d=1}^{n_d} \sum_{h_t \in \mathcal{H}} G_{d,h_t}^r(\hat{\mathbf{u}}_{d,h_t}^{*,\gamma})
\]
(24)
where $\gamma = \rho$ is for the data-driven SUC with the Bayesian nonparametric forecast model [10], $\gamma = e$ is for the empirical SUC with the probabilistic persistent model, and $\gamma = d$ is for the deterministic UC with the persistent model.
The wind power data of 2006 October are used to evaluate the performance of these approaches. The results are recorded in Table 7. We also report the relative savings with respect to deterministic UC,

\[
\Delta G = \frac{\sum G_r^d - \sum G_r^p}{\sum G_r^d} \quad \text{and} \quad \Delta G = \frac{\sum G_r^d - \sum G_r^e}{\sum G_r^d},
\]

obtained by the data-driven and empirical SUC approaches. The data-driven SUC leads to the total aggregated cost 2,822,705, while the empirical SUC has a total incurred cost 2,969,178. Thus, our proposed method has a total saving 146,473, which represents a 5% savings. Compared with the deterministic UC, the advantage of our data-driven SUC is even more substantial and it leads to a total saving 638,756. Since the proposed data-driven SUC rigorously considers the wind power generation inherent stochastic uncertainty and input model estimation uncertainty, the results in Table 7 indicate that our approach outperforms the empirical SUC and deterministic UC.

5.1.3 Performance of OPSEL approach

In this section, we use the intra-day unit commitment problem described in Sect. 5.1.2 to study the performance of the OPSEL approach developed to hedge against the finite sampling uncertainty induced by using SAA. We consider the data-driven SUC with the nonparametric Bayesian forecast model [10]. To illustrate the impact of finite sampling uncertainty, we use a representative day, October 2nd, for demonstration. Figure 2 plots the incurred cost \(\sum_{h_t \in H} G_r^{d,h_t,\ell}(\tilde{u}_r^{p,\ell})\) for \(\ell = 1, 2, \ldots, L\) obtained by utilizing \(L = 20\) CPUs to independently solve the SAA approximated data-driven SUC optimization problem in (9), where \(\tilde{u}_{d,h_r,\ell}\) is obtained from the \(\ell\)-th CPU. Each decision is obtained by using SAA with the number of scenarios, \(S = 50\). In the plot, each dot represents one optimal decision, the horizontal axis provides the CPU index \(\ell\), and the vertical axis provides the incurred cost. We can observe that the cost is widespread, ranging from about 55,000 to as much as around 120,000. Thus, Fig. 2 demonstrates the obvious impact of finite sampling error induced by SAA.

Then, we study the performance of OPSEL approach introduced in Sect. 4.2. Suppose there are \(L\) CPUs available for parallel computing. Specifically, for each intra-day unit commitment problem at the \(h_t\)-th hour on day \(d\), we consider the sample average approximated data-driven SUC in (9) with \(S = 50\). By solving \(L\) intra-day unit commitment problems in parallel, we obtain the optimal unit commitment

| Table 7 | Total costs obtained by data-driven SUC, empirical SUC, and deterministic UC approaches when there is no strong prior knowledge on wind power generation input model |
|---------|-----------------|--------|
|         | Total cost      | \(r\Delta G\) |
| Data-driven SUC: \(\sum G_r^p\) | 2,822,705 | 18.4% |
| Empirical SUC: \(\sum G_r^e\) | 2,969,178 | 14.2% |
| Deterministic UC: \(\sum G_r^d\) | 3,461,462 | 0 |
decisions for the next \( n_h \) hours, denoted by \( \hat{u}_{d,h_i,L}^{*} \). After that, the OPSEL is used to estimate the objective values \( G^p(\hat{u}_{d,h_i,L}^{*}) \) and efficiently select the best unit commitment decision, denoted by \( \hat{u}_{d,h_i,bop}^{*} \).

We compare the proposed OPSEL with the sample average approximated data-driven SUC approach that ignores the finite sampling error by using the wind power data occurring in October 1st–31st. We evaluate these two approaches with the actual incurred cost. If we ignore the finite sampling error, the total cost is

\[
\sum G^p_r = \sum_{d=1}^{n_d} \sum_{h_t \in H} G^p_{d,h_t}(\hat{u}_{d,h_t}^{*}).
\]

For the OPSEL approach, the total incurred cost is

\[
\sum G^p_{r, bop} = \sum_{d=1}^{n_d} \sum_{h_t \in H} G^p_{d,h_t}(\hat{u}_{d,h_t,bop}^{*}),
\]

where \( G^p_{d,h_t}(u_{d,h_t,bop}) \equiv C^w u \hat{u}_{d,h_t,bop}^{*} + \min_y C^{fuel} y(\hat{u}_{d,h_t,bop}^{*}, \xi_{d,h_t}) \) and it is the cost occurring between hours \( h_t + 1 \) and \( h_t + n_h \) on day \( d \).

Here, we use \( L = 4 \) CPUs. The results of total operation cost in October obtained by using the data-driven SUC with and without OPSEL are recorded in Table 8. The OPSEL approach can control the impact of finite sampling error induced by SAA and lead to less total cost, \( \sum G^p_{r, bop} \leq \sum G^p_{r} \). The relative cost saving caused by using the proposed OPSEL approach, \( r\Delta G \equiv \frac{\sum G^p_r - \sum G^p_{r, bop}}{\sum G^p_r} \), is about 20%. To further check the robustness of our approach, we study the performance of OPSEL under various settings of \( \Delta M \) and \( M \). The empirical results in Table 8 show that the proposed OPSEL approach reduces the impact of finite sampling error and leads to a better and more robust unit commitment decision. In addition, we record the percentage of computational overhead induced by the OPSEL candidate selection. It is about 3.6% and negligible.
5.2 Empirical study with 118-bus system

To evaluate the scalability and robustness of our approach, in this section, we consider the derivative 118-bus system including 54 thermal units, three wind farms and 186 transmission lines; see the description in [55]. Similar to the 6-bus system case, we assume deterministic loads and stochastic wind supply [2] and use the 2006 data of the U.S. Illinois power system for the load and the wind power generation. The whole system’s wind penetration $R$ is 9.9%. In addition, we set the wind curtailment at 30$/MWh and load shedding price at 3500$/MWh. We just consider constraints (4)–(9) in [54] as in Sect. 5.1.

5.2.1 Case study with parametric forecast model

One month wind power data are used to study the performance of the proposed data-driven SUC and the empirical SUC. We use the same settings with those used in Sect. 5.1.1, and consider one day-ahead unit commitment problem here. When we solve the SUC problems for the optimal decision estimates $\hat{u}_h^{*,e}$ and $\hat{u}_h^{*,p}$, we set the scenario size to be $S = 50$. Then, we use $S_e = 1000$ to evaluate the true expected cost $\sum G^p$ and $\sum G^e$. The cases with standard deviation $\phi_{d,t} = 5\%\mu_{d,t}, 10\%\mu_{d,t}, 20\%\mu_{d,t}$ and $m = 1, 5, 10$ are analyzed and the results of the relative expected saving obtained by our method, $r\Delta G$ in (23), are recorded in Table 9. As $m$ goes big, the input model estimation uncertainty decreases. The results in Table 9 show that the advantage of proposed data-driven SUC tends to increase when: (1) the wind penetration increases, (2) the wind power generation variation becomes larger, and (3) the amount of valid historical data decreases. The expected computational times of one day-ahead decision making required proposed data-driven SUC (97.3 min) and the empirical SUC (102.2 min) are comparable.

5.2.2 Case study with nonparametric forecast model

Here we consider the case when there is no strong prior information on the underlying wind power generation distribution and the nonparametric distributions are used for wind power forecast, which is similar with that used in Sect. 5.1.2. We use the 118-bus test case to compare the performance of: (1) deterministic unit commitment, (2) empirical SUC and (3) data-driven SUC. We utilize the deterministic persistence as prediction model for the deterministic unit commitment, use the probabilistic

| Scenario | Cost | $r\Delta G$ |
|----------|------|-------------|
| Data-driven SUC without OPSEL $\sum G^e$ | 2,822,705 | 0 |
| SUC + OPSEL $\sum G^{op}_{r}$ ($M = 500$, $\Delta M = 100$) | 2,323,507 | 17.7% |
| SUC + OPSEL $\sum G^{op}_{r}$ ($M = 1000$, $\Delta M = 50$) | 2,186,869 | 22.5% |
| SUC + OPSEL $\sum G^{op}_{r}$ ($M = 1000$, $\Delta M = 100$) | 2,282,531 | 19.1% |
| SUC + OPSEL $\sum G^{op}_{r}$ ($M = 1000$, $\Delta M = 200$) | 2,134,574 | 24.4% |

Table 8 The results of total incurred costs obtained by data-driven SUC with and without the proposed OPSEL.
persistence model for the empirical SUC, and implement Bayesian nonparametric wind power forecast model [10] for data-driven SUC with prediction risk accounting for both wind generation stochastic uncertainty and model estimation error. We consider the unit commitment problem for the intra-day market with 4 h planning horizon and set the amount of historical data used for the forecast to be \( m = 100 \). The expected computational times of intra-day decision making required by the proposed data-driven SUC (5.6 min) and the empirical SUC (5.7 min) are comparable. And the time for solving the deterministic unit commitment problem is 0.51 min. One month representative wind power data are used to evaluate the performance and the results are recorded in Table 10. The empirical SUC, accounting for wind power generation stochastic uncertainty only, leads to 1.12% savings compared with the deterministic UC method. The proposed data-driven SUC, accounting for both wind power stochastic uncertainty and input model estimation uncertainty, leads to 2.58% savings.

We also record the penalty cost induced when the energy production does not meet the demand; see Eq. (15). The proportions of penalty cost obtained by the deterministic unit commitment, the empirical SUC, and the proposed data-driven SUC are shown in Table 10. The decision made by the deterministic UC method leads to the total penalty cost $ 1,156,456.31. The proportion of the penalty cost to the total cost \( \sum G^d_r \) is 2.21%. The empirical SUC method has the total penalty cost $ 537,661.31 contributing 1.04% percentage of the total cost \( \sum G^e_r \). The empirical results in Table 10 indicate that the proposed data-driven SUC reduces the demand unmet risk, and leads to more reliable, cost-efficient, and robust power grids.

### 5.2.3 Performance of OPSEL approach

Similar to Sect. 5.1.3, we investigate the performance of the proposed OPSEL approach by using the 118-bus system. Let \( L = 3, \Delta M = 50 \) and \( M = 500 \). The results of total operation cost obtained by using the data-driven SUC with and

| \( \phi_{d,t} = 5\% \mu^c_{d,t}, 10\% \mu^c_{d,t}, 20\% \mu^c_{d,t} \) and \( m = 1, 5, 10 \) | Standard deviation | \( m = 1 \%) | \( m = 5 \%) | \( m = 10 \%) |
|---|---|---|---|---|
| \( \phi_{d,t} = 5\% \mu^c_{d,t} \) | 8.22 | 1.26 | 0.11 |
| \( \phi_{d,t} = 10\% \mu^c_{d,t} \) | 13.80 | 5.51 | 0.14 |
| \( \phi_{d,t} = 20\% \mu^c_{d,t} \) | 20.59 | 12.36 | 2.60 |

| Approach | Penalty cost | Penalty ratio (%) | Total cost | \( r \Delta G \) (%) |
|---|---|---|---|---|
| Deterministic UC: \( \sum G^d \) | 1,156,456.31 | 2.21 | 52,327,089.47 | 0.00 |
| Empirical SUC: \( \sum G^e \) | 537,661.31 | 1.04 | 51,743,500.01 | 1.12 |
| Data-driven SUC: \( \sum G^p \) | 0.00 | 0.00 | 50,974,697.80 | 2.58 |
without OPSEL are recorded in Table 11. The relative cost saving by implementing our OPSEL approach is about 2.36%. The results in Table 11 show that the proposed OPSEL approach demonstrates the promising performance to hedge against the impact of finite sampling error induced by SAA on the large-scale power grids.

### Table 11 Total incurred costs of operation on 118-bus system

|                          | Cost       | $r\Delta G$ (%) |
|--------------------------|------------|-----------------|
| Data-driven SUC without OPSEL | 50,974,697.80 | 0.00            |
| SUC + OPSEL              | 49,771,552.75 | 2.36            |

#### 6 Conclusion

When we consider the energy generation for power grid systems under high wind penetration, ignoring any source of uncertainty can lead to unreliable unit commitment decisions and result in significantly higher economic cost. In this paper, we propose the data-driven SUC framework which can explicitly and appropriately account for the impacts from all sources of uncertainties, including (1) inherent stochastic uncertainty or volatility of wind power generation; (2) statistical input model estimation error; and (3) finite sample error induced by using sample average approximation (SAA) to estimate the expected system operational cost. Specifically, we first propose a data-driven stochastic optimization to guide the optimal and robust unit commitment decision, which can simultaneously hedge against the underlying stochastic uncertainty of wind power generation and input model estimation error. Then, we introduce an optimization and selection (OPSEL) approach that can efficiently utilize parallel computing to quickly find the optimal unit commitment decision, while controlling the impact of finite sampling error induced by SAA. Various case studies on the six-bus system and the 118-bus system verify the advantages of our proposed data-drive SUC with either parametric or nonparametric input models. They also demonstrate that our OPSEL procedure can further deliver the optimal unit commitment decision hedging against the impact of finite sampling approximation error.

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