Scheduling quality of precise form sets which consist of tasks of circular type in GRID systems

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Abstract. Users’ demand in computer power and rise of technology favour the arrival of Grid systems. The quality of Grid systems’ performance depends on computer and time resources scheduling. Grid systems with a centralized structure of the scheduling system and user’s task are modeled by resource quadrant and re-source rectangle accordingly. A Non-Euclidean heuristic measure, which takes into consideration both the area and the form of an occupied resource region, is used to estimate scheduling quality of heuristic algorithms. The authors use sets, which are induced by the elements of square squaring, as an example of studying the adaptability of a level polynomial algorithm with an excess and the one with minimal deviation.

1. Introduction
Users’ demand in computer power and rise of technology favour the arrival of Grid systems [1, 2]. The quality of Grid systems’ performance depends on computer and time resources scheduling. The implementation of optimal resource scheduling has no practical value because of its exponential completeness. An environment of resource rectangles is developed in [3-7], as a polynomial completeness scheduling theory tool, for the purpose of computer and time resources distribution. In the resource rectangles environment, the operations on resource rectangles were introduced and the polynomial heuristic algorithms of resource distribution based on the presented operations were suggested. The quadratic type of user’s task was introduced in [8].

2. Materials and methods
Grid systems with a centralized structure of the scheduling system and resource co-allocation are modeled by resource quadrant [3, 9]. User’s task, which is to be served by Grid system’s scheduler, is presented as a resource rectangle with its horizontal and vertical dimensions respectively equal to the number of time resource units and processors required to process the task [10]. Scheduling quality of the heuristic algorithms is estimated by the Non-Euclidean heuristic measure which takes into consideration both the area and the form of an occupied resource region:

\[ \frac{1}{2} \left( \frac{LH+(L-H)^2}{2} \right) + \sum_{j=0}^{a} a(j)b(j), \]

where L denotes length, H - vertical level of the resource enclosure, a(j) stands for time units, b(j) - processor units which task j needs. The heuristic measure reaches its minimum of \( \frac{1}{2} \) in square packing with no empty space.
In [11] a resource rectangle set was defined as the set of precise form, which has its square resource enclosure with no any empty spaces. Scheduling quality of a level polynomial algorithm with level unattainability for a set of precise form, which consists of the resource squares, was the point of study in [11]. The level polynomial algorithm with the exceeding of the level and the level algorithm with minimal deviation were introduced in [12]. Scheduling quality of level polynomial algorithms for a set of precise form, which consists of the tasks of circular and hyperbolic type, was the point of study in [13].

In this paper, the question that is posed concerns the adaptability of a level polynomial algorithm with an excess and the one with minimal deviation for sets of precise form, which consist of circular type tasks and hyperbolic type tasks.

3. Scheduling of a set of precise form with the tasks of circular type by level algorithms

In accordance with the definitions from [8], a square relates to the circular type.

Let us denote the set of the resource squares ordered by a decrease of their heights, which is induced by perfect simple squaring of a square of the 21-st order [14], as set I.

Let us denote the sets of the resource squares ordered by a decrease of their heights, which are induced by perfect simple squaring of a square of the 22-nd order [15], as sets II and III.

Let us denote the set of the resource squares ordered by a decrease of their heights, which is induced by perfect simple squaring of a square of the 23-rd order [15], as set IV.

Let us denote the set of the resource squares ordered by a decrease of their heights, which is induced by perfect simple squaring of a square of the 24-th order [16], as set V.

The results of packing of sets I-V by the level algorithm with exceeding of the level are presented in Figures 1-5. Square’s side value is indicated in the center of a square.
The heuristic measure values of the resource enclosures of the level algorithm with the exceeding of the level for the sets which consist of the tasks of the circular quadratic type are presented in Table 1.

| Set's number | Heuristic measure value |
|--------------|-------------------------|
| I            | 0.64                    |
| II           | 0.65                    |
| III          | 0.66                    |
| IV           | 0.65                    |
| V            | 0.64                    |

It is possible to see that resource enclosures’ heuristic measure values of the level algorithm with the exceeding of the level do not exceed the value of $\frac{1}{2} + 0.16$. The results of packing of sets I-V by the level algorithm with minimal deviation are presented in Figures 6-10.
The heuristic measure values of the resource enclosures of the level algorithm with minimal deviation for the sets which consist of the tasks of the circular quadratic type are presented in Table 2.

| Set’s number | Heuristic measure value |
|--------------|-------------------------|
| I            | 0.64                    |
| II           | 0.61                    |
| III          | 0.60                    |
| IV           | 0.64                    |
| V            | 0.67                    |

It is possible to see that resource enclosures’ heuristic measure values of the level algorithm with minimal deviation do not exceed the value of \( \frac{1}{2} + 0.17 \). The graphs of the resource enclosures’ heuristic measure values, which were obtained with the use of the level algorithm with the exceeding of the level and the one with minimal deviation when scheduling sets I to V, are presented in Figure 11.
It is evident that the level algorithm with the exceeding of the level has the smallest maximum value of the heuristic measure equaled to $\frac{1}{2} + 0.16$ when considering tested sets of resource squares. The research allows recommending the polynomial algorithms, which were considered here, for implementation in Grid systems with the centralized structure and resource co-allocation for serving sets which consist of tasks of circular quadratic type.

4. Conclusion
Having some sets of precise form consisted of tasks of the circular quadratic type as an example, the resource enclosures’ heuristic measure values were calculated for the level algorithms presented in the paper. It was shown that the developed polynomial algorithms were suitable for mentioned class of sets of user’s tasks in Grid systems. Therefore, the authors can recommend these algorithms for implementation for scheduling in Grid systems.

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