The Nucleon and Roper Resonance in a Chiral Quark Diquark Model

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Abstract. A description of the nucleon and Roper resonance in a quark-diquark approach is presented. We show that two states with the quantum number of the nucleon can appear in the ground state of the spatial configuration, when there are two types of diquarks: scalar-isoscalar and axial-vector-isovector diquarks. The mass difference between the two states is generated by the mass difference between the two diquarks, which is due to the spin-spin interaction between the two quarks in the diquarks. The two states are then identified with the nucleon and Roper resonance.

INTRODUCTION

Chiral symmetry with its spontaneous breaking is a powerful tool to investigate the low energy dynamics of QCD. The internal structure of hadrons is also an important ingredient, as we investigate phenomena at finite momentum transfer. The Nambu-Jona-Lasinio (NJL) model is one of the effective theories inspired by chiral symmetry[1], and has been used to study the dynamics of mesons from the vacuum to finite density/temperature system. A chiral quark-diquark model is an effective approach, which is an extension of the NJL model for the study of baryons by the inclusion of the diquark degrees of freedom. This model incorporates not only the mesons but also baryons as composite particles with respecting chiral symmetry, where mesons and baryons are quark-antiquark and quark-diquark bound states, respectively. It was used to study not only one particle properties of the nucleon[2, 3, 4], but also meson-baryon[2, 3] and baryon-baryon interactions[5]. The importance of the diquark correlation has been also suggested from the recent study of exotics[6, 7]. Current authors suggested the possibility that the Roper resonance is described together with the nucleon by the mixing of two types of quark-diquark channels[8], where the correlation between quarks plays an important role.

It was shown that two local operators can be independently used for the study of the nucleon[9]. Inspired by this, here we prepare a model that can generate two types of nucleon states from two independent diquarks: the scalar and axial-vector diquarks. In naive quark models(NQM) with uncorrelated quarks in the \((0s)^3\) configuration, one of the two states is forbidden due to spin-flavor SU(6) symmetry. Therefore, in NQMs the second state with the quantum number of the nucleon \(I(J)^P = 1/2(1/2)^+\), which is known as the Roper resonance, must be a radially excited state with \(N = 2\) and the excitation energy \(2\hbar\omega\). If, however, the diquark correlation becomes significant, two states appear as active degrees of freedom, generating the two nucleon states approximately as bound states of a quark and a diquark. This is the case we consider in the present paper, where we identify the higher state with the Roper resonance. In this case the mass
difference of the nucleon and Roper is approximately dictated by the mass difference of the two diquarks, the origin of which is the spin-spin interaction between quarks, and is about the same order as the mass splitting of the nucleon and delta.

**FRAMEWORK**

We employ the chiral quark-diquark model[3, 8], which is given by

\[
\hat{\mathcal{L}} = \bar{\chi}_c (i \partial - m_q) \chi_c + D_c^\dagger (\partial^2 + M_S^2) D_c + \bar{D}_c^{\dagger \mu} \left( (\partial^2 + M_A^2) g_{\mu\nu} - \partial_{\mu} \partial_{\nu} \right) \bar{D}_c^\nu + L_{\text{int}},
\]

where \( \chi_c, D_c \) and \( \bar{D}_{\mu c} \) are the constituent quark, scalar diquark and axial-vector diquark fields, and \( m_q, M_S \) and \( M_A \) are the masses of them. The indices \( c \) represent the color. The term \( L_{\text{int}} \) is the quark-diquark interaction, which is written as

\[
L_{\text{int}} = G_S \bar{\chi}_c D_c^\dagger D_{c'} \chi_{c'} + v (\bar{\chi}_c D_c^\dagger \gamma^\mu \bar{\tau} \cdot \bar{D}_{\mu c} \chi_{c'} + \bar{\chi}_c \gamma^\mu \bar{\tau} \cdot \bar{D}_{\mu c} D_{c'} \chi_{c'}) \tag{2}
\]

\[
+ G_A \bar{\chi}_c \gamma^\mu \bar{\tau} \cdot \bar{D}_{\mu c} \gamma^\nu \bar{\tau} \cdot \bar{D}_{\nu c'} \chi_{c'}, \tag{3}
\]

where \( G_S \) and \( G_A \) are the coupling constants for the quark and scalar diquark, and for the quark and axial-vector diquark. The coupling constant \( v \) causes the mixing between the scalar and axial-vector channels. It is important that there are two quark-diquark channels and they are independent. Hence the nucleon and Roper states appear as physical states. An effective Lagrangian for them is then derived by the path-integral hadronization method [3].

**MASSES OF TWO STATES**

In this work we concentrate on the masses of the two nucleon states, which are obtained from the quark-diquark self-energies shown in figure 1. After the calculation of the self-energies and diagonalization of the \( 2 \times 2 \) mass matrix for the scalar and axial-vector channels, the masses of physical states are obtained as follows[8],

\[
M_{1,2} = \frac{1}{2} \left[ a_S + a_A \pm \sqrt{(a_S - a_A)^2 + 4Z_SZ_A \left( \frac{v}{|\hat{G}|} \right)^2} \right],
\]

\[
\tag{4}
\]

\[
\tag{5}
\]

where \( a_S \) and \( Z_S \) are the mass and wave-function renormalization constant of the self-energy for the scalar channel, \( a_A \) and \( Z_A \) are those for the axial-vector channel, obtained by

\[
\Sigma_S(p_0) - \frac{1}{|\hat{G}|} G_A = Z_S^{-1} (p_0 \gamma^0 - a_S), \tag{6}
\]

\[
\Sigma_A(p_0) - \frac{1}{|\hat{G}|} G_S = Z_A^{-1} (p_0 \gamma^0 - a_A), \tag{7}
\]
FIGURE 1. A diagrammatic representation of the quark-diquark self-energy for the scalar diquark (left), axial-vector diquark (middle) and mixing between them (right). The single, double and triple lines represent the quark, diquark and nucleon respectively. The blobs represent the three point quark-diquark-baryon interactions.

FIGURE 2. \( M_1 \) and \( M_2 \) as functions of \( v \) for fixed \( G_1 \) and \( G_2 \) (left) and for fixed \( a_S \) and \( a_A \) (right).

and \( |\hat{G}| \) is defined by \( |\hat{G}| = G_S G_A - v^2 \). The masses of \( M_1 \) and \( M_2 \) as functions of the mixing strength \( v \) are shown for two cases in Fig.2: for fixed \( G_1 \) and \( G_2 \) (left) and for fixed \( a_S \) and \( a_A \) (right).

For both cases the mass difference is about several hundreds MeV, which is significantly smaller than the values of NQMs. The reason is that it is generated by the mass difference of the scalar and axial-vector diquark and their mixing. The order of the amount is expected to be that of the spin-spin interaction between the quarks. Hence the mass difference is almost the same order as that of the nucleon and delta.

For further extension of the present study, it is important to investigate various transition amplitudes such as the decay rate and helicity amplitude of the Roper resonance. These works are in progress.

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