Sub-gap conductance in ferromagnetic-superconducting mesoscopic structures

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We study the sub-gap conductance of a ferromagnetic mesoscopic region attached to a ferromagnetic and a superconducting electrode by means of tunnel junctions. In the absence of the exchange field, the ratio \( r = \gamma/\epsilon_T \) of the two tunnel junction resistances determines the behaviour of the sub-gap conductance which possesses a zero-bias peak for \( r \gg 1 \) and for \( r \ll 1 \) a peak at finite voltage. We show that the inclusion of the exchange field leads to a peak splitting for \( r \ll 1 \), while it shifts the zero-bias anomaly to finite voltages for \( r \gg 1 \).

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In the last few years there has been a great deal of activity in the study of transport through hybrid superconducting mesoscopic structures (S/N) \(^1\). Amongst the many interesting findings it is worth mentioning the observation of a sub-gap conductance \(^3\) in S/N structures, long-range \(^4\) and anomalous proximity effects \(^\|\) in N-S systems and giant oscillations in the electrical conductance \(^6\) and thermopower \(^7\) in S/N structures containing superconducting and/or normal loops.

In parallel with these discoveries, ferromagnetic-normal hybrid systems (F/N) have stimulated the curiosity of the scientific community \(^8\) and more recently combined structures involving both ferromagnetic and superconducting materials like F/S or F/N/S have been studied both theoretically \(^9\) and experimentally \(^10\). We note here that, in general, the introduction of the exchange field leads to a decrease of the Andreev scattering due to the change in the ratio of the two spin populations. For our case, where the exchange energy is of order the superconducting energy gap, the density of states is almost constant and this reduction effect will be small.

In this paper we study how the presence of a finite exchange field affects well-known coherent phenomena in disordered N(F)/S systems. We shall focus attention upon the F/S structure shown in Fig.1, calculating its electrical conductance in the diffusive regime by means of both analytical and numerical techniques. The former is based on the use of the quasi-classical Green’s function approach, whereas the latter solves the Bogoliubov de Gennes equations on a tight-binding lattice. Both of these methods have been used extensively in the study of N/S systems \(^6\). For our structure, we recall that in the ballistic limit for a simple NIS system the conductance is suppressed at voltages below the superconducting energy gap and the conductance-voltage curve is featureless. A sub-gap anomaly in the conductance appears only in the presence of two barriers. In the diffusive regime the sub-gap conductance exhibits a peak below the superconducting energy gap even in the absence of a second barrier, because the disorder itself provides the mechanism for multiple scattering. The position of this peak depends on the particular geometry and relevant parameters \(^2\). In the absence of the exchange field, the ratio \( r = \gamma/\epsilon_T \) of the two tunnel junction resistances controls different regimes for the sub-gap conductance: a zero-bias peak is present for \( r \gg 1 \) and a finite bias peak for \( r \ll 1 \). Here \( \gamma \) and \( \epsilon_T \) are proportional to the conductances of the N’/N and N/S interfaces respectively. In what follows we show that the inclusion of the exchange field (we denote the exchange energy by \( h \)) leads to a peak splitting for \( r \ll 1 \) for small \( h \) (weak ferromagnetism) while the zero-bias anomaly is shifted to a finite voltage \( eV = h \) for \( r \gg 1 \).

To numerically model the structure of Fig.2 we consider a dirty normal or ferromagnetic region (denoted N or F) connected to external reservoirs by a clean semi-infinite normal or ferromagnetic lead (denoted N’ or F’) and a clean superconductor (denoted by S). For our numerical calculation we use a tight-binding version of the Bogoliubov-de Gennes Hamiltonian:

\[
\begin{pmatrix}
    H_0 - \sigma h & \sigma \Delta \\
    \sigma \Delta^* & -H_0 - \sigma h
\end{pmatrix}
\begin{pmatrix}
    u_{\sigma} \\
    v_{\sigma}
\end{pmatrix} = \epsilon
\begin{pmatrix}
    u_{\sigma} \\
    v_{\sigma}
\end{pmatrix}
\]

where \( \sigma = 1(-1) \) for spin-\( \uparrow(\downarrow) \). In this equation \( H_0 = \sum_i |i\rangle \epsilon_i \langle i| - t \sum_{\langle ij \rangle} |i\rangle \langle j| \) is the standard single-particle Anderson model with \( \langle ij \rangle \) denoting pairs of nearest neighbour sites, \( h = \sum_i |i\rangle \lambda_i \langle i| \) models the exchange energy and \( \Delta = \sum_i |i\rangle \Delta_i \langle i| \) is the superconducting order parameter. The width of the whole structure is \( W \) and the length of the disordered ferromagnet is \( d \) (in units of the lattice constant \( a \)). The off-diagonal matrix elements \( t \), which determine the width of the energy band, are equal to 1 throughout the system except at the F/S and F’/F interfaces where the value of \( t = t_{F\text{ barrier}} \) is chosen to model a barrier. Such barriers may be due to either the presence of an insulating layer or to a mismatch of the electronic parameters of the adjacent materials. Within the ferromagnetic lead and the superconducting region,
the diagonal matrix elements are \( \epsilon_i = \epsilon_0 \) (with \( \epsilon_0 = 1 \), which keeps the Fermi level away from the van Hove singularity in the band centre). For simulating bulk disorder in the ferromagnet, the \( \epsilon_i \) in the scattering region are taken uniformly from the interval \(-U/2 < \epsilon_i - \epsilon_0 < U/2\) where \( U \) is the disorder strength. A numerical decimation technique \([13]\) is used to compute the Greens function for each realization of disorder. From this one obtains the Andreev reflection coefficients \( R_{\sigma \sigma'}^{\uparrow \downarrow} \) and the conductance \( G = R_{\uparrow \downarrow}^{\uparrow \downarrow} + R_{\downarrow \uparrow}^{\downarrow \uparrow} \), in units of \( \frac{e^2}{h} \).

Our analytical treatment is based on the transport theory described in \([13]\). There equations for the quasi-classical Green’s functions were employed in the diffusive regime with the corresponding boundary conditions at the interfaces \([13]\). We begin by recalling the main results for non ferromagnetic structures. The normalised conductance \( S(V) = (dI/dV)/G_n \) (where \( G_n \) is the conductance of the isolated normal wire) of the system \( N'/N/S \) (exchange energy \( h \) is zero) is equal to (see \([12]\))

\[
S(V) = \int_0^\infty d\epsilon F^\nu_\epsilon \left[ m^{-1}(\epsilon) + \frac{1}{\nu_d(\epsilon)} \frac{\epsilon}{\gamma} \frac{\epsilon_d}{M(\epsilon)\epsilon_T} \right]^{-1} \text{e}^{-\frac{\epsilon}{\epsilon_T}} \text{d}\epsilon
\]

Here \( F^\nu_\epsilon = (\beta/2)[\cosh^{-2}(\epsilon + \epsilon V)\beta + \cosh^{-2}(\epsilon - \epsilon V)\beta] \) is the derivative of the distribution function, \( V \) is the voltage difference between reservoirs, \( \beta = 1/(2T) \). \( \nu_d(\epsilon) = Re((\epsilon + i\gamma)/(\sqrt{(\epsilon + i\gamma)^2 - \epsilon^2}^+) \) is the density-of-states in the central \( N \) region, \( \gamma = \rho D/2R_{N\Sigma d}, \epsilon_T = \rho D/2R_{S\Sigma d}, \) \( \rho \) and \( D \) are the specific resistivity and diffusion constant in the middle region, \( d \) is the length of the middle region, \( R_{N,S\Sigma} \) are the \( N'/N \) or \( N/S \) resistances per unit area in the normal state and \( \epsilon_d = D/d^2 \) is the Thouless energy. We assume here that the length \( d \) is small enough and the condition \( \epsilon_d > T \) is fulfilled (short contact). The coefficient \( m(\epsilon) \) determines the conductance of the normal wire and is given by

\[
m^{-1}(\epsilon) = \frac{1}{2} \left[ 1 + \frac{[\epsilon + i\gamma]^2 + \epsilon_T^2}{[\epsilon + i\gamma]^2 - \epsilon_T^2} \right].
\]

The coefficient \( M(\epsilon) \) in eq.(2) determines the energy dependence of the \( N/S \) interface conductance. For example, from eq.(107) of Ref. \([13]\), we have

\[
M(\epsilon) = \text{Re} \frac{\epsilon + i\Gamma}{\sqrt{(\epsilon + i\Gamma)^2 - \Delta^2}} \text{Re} \frac{\epsilon + i\gamma}{\sqrt{(\epsilon + i\gamma)^2 - \epsilon_T^2}} + \text{Re} \frac{\epsilon_T}{\sqrt{\Delta^2 - (\epsilon + dT)^2}} \text{Re} \frac{\epsilon}{\sqrt{(\epsilon + i\gamma)^2 - \epsilon_T^2}}
\]

Here \( \Delta \) and \( \Gamma \) are the gap and the damping rate in the superconductor, respectively. The first term in eq.(1) is due to the quasi-particle contribution (at zero temperature it is not zero only if \( eV > \Delta \)). The second term in \( M(\epsilon) \) gives the so-called interference current in SIS Josephson junctions and leads to a sub-gap conductance in SIN contacts.

The three terms in square brackets in eq.(2) represent the normalized resistances: the first term is the resistance of the central \( N \) region, the second is the resistance of the \( N'/N \) interface and the third that of the \( N/S \) interface. The quantity \( \epsilon_T \) is a pseudo-gap induced in the middle region by the proximity effect. One can see from the expression for \( \nu_d \) that in the case of a small \( \gamma \) (high resistance of the \( N'/N \) interface) \( \nu_d \) has a form typical for a superconductor with the energy gap \( \epsilon_T \). If \( \gamma \) is large compared with \( \epsilon_T \), the singularity in the density of the states in the \( N \) film disappears due to the strong damping caused by the \( N' \) electrode. In Fig.2 we plot the dependence of the normalized conductance \( S(V) \) on the applied voltage \( V \) at zero temperature assuming that the resistance of the \( N \) region is small (\( \epsilon_d/\epsilon_T = 10 \)). One can see that in the case of large \( \gamma \), the main contribution to \( S(V) \) is due to the last term in brackets (\( N/S \) interface resistance) and the sub-gap conductance has a zero-bias peak. In this case there is a gapless state in the central \( N \) region. When \( \gamma \) is decreased the main contribution to \( S(V) \) is given by the second term in brackets; the sub-gap appears in the central \( N \) region and the peak is shifted to \( \epsilon_T \): the conductance is just that expected for tunneling into a superconductor with a energy gap \( \epsilon_T \). Similar behaviour of the sub-gap conductance was discussed in Ref. \([16]\).

The numerical results in the absence of an exchange field are shown in Fig.3. To produce this curve we chose \( W = d = 20 \) sites. The barrier at the \( N/S \) interface was chosen by setting \( t_{NS} = 0.4 \). The different curves correspond to \( t_{N'/N} = 0.0, 0.1, \ldots, 1.0 \). The result was obtained by averaging over 500 realisations of the random potential with \( U = 2 \) (elastic mean free path is 8.5). One can clearly identify the transition from finite to zero-bias peak. One should note that for these parameters we cannot reach the regime of \( \gamma > \epsilon_T \) even in the absence of the barrier at the \( N'/N \) interface and so we do not obtain the Lorentzian line shape expected in that limit. This can easily be obtained, however, by using a smaller value for \( t_{NS} \).

Consider now the case when the central region is a ferromagnet. We describe the ferromagnet by adding to the Hamiltonian an exchange term \( H_{ex} = \hbar \sum_k (c^\dagger_{k,\uparrow} c_{k,\uparrow} - c^\dagger_{k,\downarrow} c_{k,\downarrow}) \). One can write the quasi-classical equations for the Green’s functions taking into account this term. One can easily show \([17]\) that the energy should be replaced by \( \epsilon - h \) in all terms corresponding to the ferromagnetic regions. Therefore eq.(3) remains unchanged if we replace \( \epsilon \) by \( \epsilon - h \) in all such terms (the terms corresponding to the condensate functions in the superconductor are unchanged as we do not consider an exchange field in these regions). Alternatively one can obtain the same expressions for the \( F'/F \) and \( F/S \) interface resistances as in eq.(2) using the standard tunnel Hamiltonian \( H_T = U_T \sum_{k,q,\alpha} (c^\dagger_{k,\alpha} a_{q,\alpha} + a^\dagger_{q,\alpha} c_{k,\alpha}) \), where \( a_{q,\alpha} \) are the
destruction operators of the reservoirs. We find the condensate functions $F^{R(A)}_F$ induced in the ferromagnet due to the proximity effect

$$F^{R(A)}_F = \frac{\epsilon_T}{\sqrt{(\epsilon - h) \pm i \gamma}^2 - \epsilon_T^2}$$  \hspace{1cm} (5)

where $\epsilon_T = N_S |U_F|/\gamma$ and $\gamma = N_F |U_F'/F|^2$. $N_S, N_F, F'$ are the density of states in the $S$ and $F'$ reservoirs in the normal state. Using eq.(2) we come again to the expressions for the resistances of the $F/S$ and $F'/F$ interfaces (the third and second terms in eq. (2)). In Fig. 4b we present the voltage dependence of the conductance $S(V)$ for $h = 0.5\epsilon_T$ and $h = 5\epsilon_T$. In the former case for all the curves with $\gamma > 0.6\epsilon_T$, the tunnelling rate due to the ferromagnetic lead is greater than the exchange energy, and no peak splitting appears. When $\gamma$ becomes less than the exchange energy, a splitting of the peak starts to appear. In Fig. 5b, due to larger exchange energy, the zero-bias peak is shifted to finite bias $\epsilon V = h$ and for small $\gamma$ the splitting of the two peaks is now $2\epsilon_T$ reflecting the shift to finite bias of the induced gap in the density of states (the gap is now that for one spin–species only).

To compare with numerical results, we repeat the calculation used to obtain Fig. 3 but this time there is a finite exchange energy in the normal lead and central region. The results are shown in Fig. 6 where $h = 0.025$. We again see that we have good qualitative agreement with the whole curve being shifted to finite bias.

Finally we show that this agreement can be made almost exact. In Fig. 7 we show results for the case $\Delta \gg \epsilon$ with $\Gamma = 0$.

$$S(V) = \int_0^\infty d\epsilon F'_F \left[ \frac{\Delta + \epsilon_T}{\sqrt{\Delta^2 + \epsilon_T^2} (\epsilon - h)^2 + \gamma^2} \right]$$  \hspace{1cm} (6)

We see that the sub-gap conductance is just a Lorentzian: at $T = h = 0$ for $\Delta \gg \epsilon$ equation (5) is simply $S(V) = S(0) \gamma^2/((\epsilon V)^2 + \gamma^2)$. In Fig. 8 we show the sub-gap conductance obtained numerically with $t_{FF} F' = 1.0$ and $t_{FS} = 0.2$ averaged over 500 realisations. Each curve corresponds to various values of $h = 0.00, 0.01, \ldots, 0.19$. The value of the gap was $\Delta = 0.17$. Each curve has been translated vertically by an amount 0.15 for clarity. The curve for $h = 0$ can be fitted well by a Lorentzian plus a constant $C$ to take into account the fact that in the numerical result we cannot neglect completely the other terms in equation (2). From this we obtain a value of $\gamma \approx 0.015$ and $S(0) = 0.23$ and the constant was $C = 0.075$. Putting $S(0)$ and $\gamma$ into the above formula at $T = 0$ for the same values of $h$ and $\Delta$ used in the numerics gives us Fig. 8b.

In conclusion, we have studied the sub-gap conductance of a $F'/F/S$ structure for varying ratio of the interface resistances. The presence of an exchange field yields:

i) a shift to finite bias of the zero-bias peak when the $F/S$ resistance dominates; ii) a splitting of the finite-bias peak when the two resistances are comparable.

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FIG. 2. The normalized conductance $S = (dI/dV)/G_n$ as obtained from quasiclassical theory, for varying $\gamma$, for $\gamma/\epsilon_T = 0.05$ (lower most curve at large $V$), 0.25, 0.45, 0.65, 1.0 and 2.5 (the uppermost curve at large $V$). The applied voltage is in units of the $\epsilon_T$. Here the exchange energy $h = 0$.

FIG. 3. The conductance $G$ as obtained from numerical simulations, for varying $\gamma$, from $\gamma > \epsilon_T$ uppermost curve to $\gamma < \epsilon_T$ lowermost curve. Here the exchange energy $h = 0$.

FIG. 4. The same as in figure 2 for finite exchange energy, (a) $h = 0.5\epsilon_T$ and (b) $h = 5\epsilon_T$.

FIG. 5. The same as in figure 3 for finite exchange energy, $h = 0.025$. 
FIG. 6. (a) The conductance $G$ vs voltage as obtained from numerical simulations, for varying exchange energy, $h$, from $h = 0$ (lowermost curve) to $h = 0.19$ (uppermost curve). These results are in the regime $\gamma \gg \epsilon_T$. Each subsequent curve was translated vertically by an amount 0.15. A value of $\Delta = 0.17$ was used. (b) The normalised conductance $S$ vs voltage as obtained from quasiclassical theory, for varying exchange energy, $h$, from $h = 0$ (lowermost curve) to $h = 0.19$ (uppermost curve). Each curve is shifted by 0.15 vertically. Values for the zero voltage conductance and gamma were obtained by fitting to the numerical result for $h = 0$ and the same value of $\Delta$ was used.