Parity Violating Photoproduction of $\pi^\pm$ on the $\Delta$ Resonance

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We analyze the real photon asymmetry $A_{\gamma N}^{\pm}$ for the parity violating (PV) $\pi^\pm$ production on the $\Delta$ resonance via the reactions $\gamma + p \rightarrow \Delta^+ \rightarrow \pi^+ + n$ and $\gamma + d \rightarrow \Delta^0 + p \rightarrow \pi^+ + p + p$. This asymmetry is nonvanishing due to a PV $\gamma N \Delta$ coupling constant, $d_{\Delta}^\pm$. We argue that an experimental determination of this coupling would be of interest for hadron dynamics, possibly shedding light on the S-wave/P-wave puzzle in the hyperon nonleptonic decays and the violation of Hara’s theorem in weak radiative hyperon decays.

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Despite years of scrutiny, the strangeness-changing ($\Delta S = 1$) hadronic weak interaction (HWI) continues to confront hadron structure physicists with a number of largely unsolved puzzles: e.g., the origin of the $\Delta I = 1/2$ rule, the S-wave/P-wave problem in hyperon nonleptonic weak decays, and the surprisingly large violation of SU(3) symmetry in hyperon weak radiative decays. These puzzles involve the breakdown of symmetries associated with light quark and gluon strong interactions when applied to hadronic weak observables. Indeed, hadronic weak matrix elements involve a complex interplay of strong and weak interactions, and the degree to which QCD-based symmetries are relevant in this context remains an open question for the field.

Our understanding of the strangeness-conserving ($\Delta S = 0$) HWI remains no less opaque, and similar questions about the applicability of strong interaction symmetries arise in this case. Recently, there has been resurgence of interest in studying the $\Delta S = 0$ HWI via measurements of parity-violating (PV), strangeness-conserving observables \cite{1}. The use of parity violation allows one to filter out the $\Delta S = 0$ HWI from much larger, strangeness-conserving strong and electromagnetic effects. While these PV experiments focus on nucleonic systems, there have yet to be any studies of strangeness-conserving weak transitions between the nucleon ($N$) and other $qqq$ states. In this note, we propose such a probe of the $\Delta S = 0$ HWI involving the nucleon and its lightest spin-$\frac{1}{2}$ partner, the $\Delta(1230)$ resonance: $A_{\gamma N}^{\pm}$, the PV $\pi^\pm$ photoproduction asymmetry at the $\Delta$ resonance using polarized photons.

In the limit where the number of quark and gluon colors ($N_c$) becomes large, the $N$ and $\Delta$ form a degenerate multiplet under spin-flavor SU(4) symmetry, making this system an interesting window on the hadronic dynamics of light $qqq$ hadrons. Measurements of the parity-conserving (PC) electromagnetic (EM) $N \rightarrow \Delta$ transition amplitudes (Fig. 1a) have challenged hadron structure theorists, as the experimental results differ substantially from both lattice QCD and QCD-inspired model predictions \cite{2}. A determination of the axial vector neutral current $N \rightarrow \Delta$ amplitude via PV pion electroproduction planned at Jefferson Laboratory \cite{3} could, in principle, shed additional light on the present shortcomings of hadron structure theory in this context. In a separate communication, we show that a parallel measurement of $A_{\gamma N}^{\pm}$ could sharpen the theoretical interpretation of the PV electroproduction asymmetry, $A_{\gamma N}^{\pm}$ \cite{4}. Here, we concentrate on the hadron structure implications of $A_{\gamma N}^{\pm}$ – whose measurement also appears to be feasible at Jefferson Lab \cite{5} – and show how it could provide new insight into the physics of the poorly understood HWI in both the $\Delta S = 0$ and $\Delta S = 1$ sectors.

The relationship between $A_{\gamma N}^{\pm}$ and its $\Delta S = 1$ partners follows from the QCD-based symmetry properties – chiral, flavor SU(N), large $N_c$ – of the relevant transition matrix elements. Chiral perturbation theory ($\chi$PT) provides a natural and systematic framework for applying these symmetries to hadronic observables. At leading order in the chiral expansion, $A_{\gamma N}^{\pm}$ is dominated by the PV $\Delta \rightarrow N \gamma$ electric dipole (E1) amplitude, which is the $\Delta S = 0$, SU(4) analog of the SU(3)-forbidden E1 amplitude responsible for PV $\Delta S = 1$ hyperon weak radiative decays. In the limit of exact SU(3) symmetry, the asymmetry parameters $\alpha^{BB'}$ associated with the latter must vanish, a result known as Hara’s theorem \cite{6}. The non-zero splitting between the strange and light quark masses breaks the SU(3) symmetry, leading one to expect $\alpha^{BB'} \sim m_s/m_N \sim 0.15$, where $m_s$ is the strange quark mass. Experimentally, however, one finds $\alpha^{\Xi^+} = -0.76 \pm 0.08$, $\alpha^{\Sigma^0} = -0.63 \pm 0.09$, roughly four to five times the naive expectation.

In an analogous manner, the PV $\Delta \rightarrow N \gamma$ E1 transition – and thus the resonant asymmetry $A_{\gamma N}^{\pm}$ – must vanish in the limit of exact SU(4) symmetry. It is natural to ask, then, whether the dynamics underlying the surprisingly large SU(3)-violation in the $\Delta S = 1$ sector also produce enhanced SU(4)-breaking in the $\Delta S = 0$ weak radiative transition. Using a specific mechanism for both the $\Delta S = 1$ and $\Delta S = 0$ transitions, we show that one might reasonably expect enhancements to be generated in both sectors by similar dynamics. More generally, a measurement of $A_{\gamma N}^{\pm}$ could help determine...
whether anomalously large symmetry breaking is a general feature of low-energy hadronic weak interactions or simply a peculiarity of $\Delta S = 1$ hyperon decays.

Our result for the $A_{\gamma}^\pm$ leading to these conclusions has the simple form:

$$A_{\gamma}^\pm \approx - \frac{2d_5^\Delta}{C_3^\Delta} \frac{M_N}{\Lambda_X} + \cdots$$

where $A_{\gamma}^\pm$ is the PV asymmetry on the $\Delta$ resonance, $\Lambda_X = 4\pi F_\omega \sim 1$ GeV is the scale of chiral symmetry breaking, $C_3^\Delta \sim 2$ is the dominant $N \to \Delta$ vector transition form factor (Fig. 1a) as defined in Ref. \ref{4}, $d_5^\Delta$ is a low-energy constant (LEC) characterizing the PV $\gamma N \Delta$ coupling in Fig. 1b, and the ellipses denote non-resonant, higher order chiral, and $1/M_N$ corrections. Several features of $A_{\gamma}^\pm$ merit comment:

(i) The photoproduction asymmetry coincides with the $q^2 \to 0$ limit of $A_{\gamma}$. The nonvanishing $A_{\gamma}(q^2 = 0)$ results from $\gamma$-exchange between the electron and hadronic target, with the $\gamma$-hadron coupling given in Fig. (1b) corresponding to matrix elements of the transverse electric multipole operator $T_{F=1\lambda}$. The constraints of current conservation, as expressed in Siegert’s theorem \ref{14,15}, allow one to rewrite matrix elements of this operator in terms of the corresponding charge density matrix elements:

$$\langle f|T_{F=1\lambda}^\pm|i\rangle = -\frac{\sqrt{2}}{3}\omega(f)\int d^3x xY_{1\lambda}(\Omega)\rho(x)|i\rangle + O(q^2),$$

where the $\omega$ is the energy difference between the initial and final states and vanishes in the $N_c \to \infty$ limit wherein the $N$ and $\Lambda$ become degenerate \ref{3}. The leading term is $q^2$-independent and proportional to $\omega$ times the electric dipole matrix element. Up to numerical factors, this matrix element is simply $d_5^\Delta/\Lambda_X$. The remaining terms of $O(q^2)$ and higher contain matrix elements of the anapole operator \ref{14,15}, which do not contribute to $A_{\gamma}^\pm$ but generate contributions to $A_{\gamma}$ that vanish at the photon point.

(ii) In the context of $\chi$PT, one expects the “natural” scale for SU(4) breaking effects associated with $d_5^\Delta$ to be $\sim few \times 10^{-8}$ (see below). However, the magnitude of observed $\Delta S = 1$ PV asymmetries suggest that $d_5^\Delta$ could be significantly enhanced over this scale, yielding a potentially relatively large photon asymmetry $A_{\gamma}^\pm \sim few \times 10^{-6}$, which would be accessible using polarized photons at Jefferson Lab.

In performing a consistent derivation of the photoproduction asymmetry, one must consider all contributions to the PV amplitudes through a given chiral order. However, while one may readily identify the formal chiral order of various contributions to $A_{\gamma}^\pm$, the physical significance of chiral counting is complicated by the dominance of the $\Delta$ intermediate state at resonant kinematics. In particular, we do not include various non-resonant background contributions, even though some may be formally of lower chiral order than those involving the $\Delta$ intermediate state (see, e.g. the studies of PV threshold $\pi$ production in Refs. \ref{12,13}). The reason is that for resonant kinematics, the contribution of the $\Delta$ is enhanced relative to the non-resonant (NR) background contributions by $\sigma^\Delta/\sigma^{NR} \sim (2M_\Delta/\Gamma_\Delta)^4 \sim 2 \times 10^4$. This enhancement factor more than compensates for the relative chiral orders of the $\Delta$ and NR contributions. Indeed, from a blind application of chiral power counting to $A_{\gamma}^\pm$, one might erroneously truncate the chiral expansion at $O(p)$, retaining only the NR background contributions to the resonant asymmetry. Hence, we use chiral power counting as a means of organizing various resonant contributions instead of using it to delineate the relative importance of resonant and NR amplitudes.

To that end, we employ heavy baryon chiral perturbation theory (HB$\chi$PT) \ref{19,20} and adopt the $p$-counting scheme, where $p$ denotes a small external momentum or mass or the photon field. The leading PV $\Delta \to N\gamma$ transition operator is then $O(p^2)$ \ref{21,24}:

$$\mathcal{L}_{PV}^\Delta N\gamma = i\epsilon_{\lambda\mu\nu}\Delta_{\mu\nu}^\pm \gamma_\lambda p + d_5^\Delta \Delta_{\mu\nu}^\pm \gamma_\lambda n F_{\mu\nu} + h.c.$$  \tag{2}

and we truncate our expansions of $d_{5,\Delta}^\pm$ at this order. The $O(p^4)$ corrections— including loop effects—are generally small and can be found in \ref{15,24}.

Separate determinations of $d_5^\Delta$ and $d_5^\Delta$ could be achieved using proton and deuterium targets, respectively. In the latter case, the resonant $\pi^-$ production process $\gamma + d \to \pi^- + p + p$ is dominated by the subprocess $\gamma + n \to \Delta^0 \to \pi^- + p$ since two body meson exchange currents are always higher order due to the presence of an additional loop \ref{24}. The asymmetry derivation is the same for $\pi^+$ so we take $\pi^\pm$ resonant production as a specific example. Defining the kinematic variables as

$$\gamma(q) + p(p) \to \Delta^\pm(p_\Delta) \to n(p') + \pi^\pm(k),$$  \tag{3}

we have in the laboratory frame $s = (k + p)^2, q = p_\Delta - p, p_\Delta = p' + p_w$ where $p = 0$ and $p_w^2 = m_\Delta^2$. The PV asymmetry is defined as $A_{\gamma} = (N_+ - N_-)/(N_+ + N_-)$, where $N_+$ ($N_-$) is the number of detected $\pi^+$ ($\pi^-$) produced via the reaction \ref{3} for a beam of left (right) handed circularly polarized photons.

To compute $A_{\gamma}^\pm$, we require the PC and PV response functions, generated by photon helicity ($h$) amplitudes $M_{PC}^h$ (Fig. 1a) and $M_{PV}^h$ (Fig. 1b), respectively: $W_{PC} = |M_{PC}^h|^2 + |M_{PC}^{-h}|^2$ and $W_{PV} = 2Re(M_{PC}^h M_{PV}^{-h} - M_{PV}^{-h} M_{PC}^h)$. A straightforward calculation leads to

\*Another possible resonant subleading correction comes from PV $\pi N \Delta$ D-Wave interaction. However, a careful analysis shows it does not contribute to the total real photon asymmetry \ref{3}.
\[ W_{PC} = \frac{32\pi\alpha}{9} S_\Delta \left( \frac{g_{\pi N A} C_\pi V}{F_\pi} \right)^2 |q|^2 |k|^2 \left( \frac{5}{3} - \cos^2 \theta \right) \]

\[ W_{PV} = -\frac{64\pi\alpha}{9} m_N S_\Delta \left( \frac{g_{\pi N A}}{F_\pi} \right)^2 \frac{d^+}{A_\chi} C_\pi V \times |q|^2 |k|^2 \left( \frac{5}{3} - \cos^2 \theta \right) \]

where \( S_\Delta = \left( s - m_\Delta^2 + \Gamma_\Delta^2 / 4 \right)^{-1} \) and \( g_{\pi N A} \) is the \( \pi N \Delta \) strong coupling constant. The asymmetry is given by the ratio \( W_{PV}/W_{PC} \), yielding the result in Eq. (4). It is interesting to note that the leading asymmetry in HBCPT is isotropic.

In order to assess the potential size of this contribution to the asymmetry, we need to estimate the size of \( d_\Delta^+ \). Because LEC's such as \( d_\Delta^+ \) are governed in part by short-distance \( (r > 1/\Lambda_\chi) \) strong interactions, they are difficult to compute from first principles in QCD. A standard alternative is to employ “naive dimensional analysis” (NDA) \[23\], according to which effective weak interaction operators should scale as \[17\]

\[ \left( \frac{\psi \bar{\psi}}{\Lambda_\chi F_\pi^2} \right)^k \left( \frac{f_\pi}{F_\pi} \right)^\ell \left( \frac{D}{\Lambda_\chi} \right)^m \left( \Lambda_\chi F_\pi^2 \right)^2 \times g_\pi \]  

where \( g_\pi \sim G_F F_\pi^2 / 2 \sqrt{2} \sim 5 \times 10^{-8} \) is the scale of weak charged current hadron processes. The SU(4)-violating E1 operator of Eq. (4) corresponds to \( k = 1, \ell = 0 \) and \( m = 2 \) and should scale as \( g_\pi / \Lambda_\chi \), so that \( d_\Delta^+ \sim g_\pi \). A more detailed consideration of hadron dynamics, however, suggests that \( d_\Delta^+ \) may be considerably larger.

In the purely mesonic sector of \( \chi PT \), one knows that low energy constants are well reproduced by assuming resonance saturation for the short distance physics. In the baryon sector, a particularly intriguing application of resonance saturation involves the electric dipole transitions responsible for the anomalously large asymmetries \( \alpha^{BB'} \) discussed above. The theoretical challenge has been to account for these enhanced values of \( \alpha^{BB'} \) in a manner consistent with the corresponding nonleptonic pion decay rates.

While a number of approaches have been attempted, the inclusion of \( \frac{1}{2}^- \) resonances as in Fig. 2a appears to go further in enhancing the theoretical predictions for the asymmetries while simultaneously resolving the S-wave/P-wave problem in the pion decay channel \[24\]. Here, the pseudoscalar, nonleptonic weak interaction \( \mathcal{H}_{\pi V} \) mixes states of the same spin and opposite parity into the initial and final baryon states, and if \( \frac{1}{2}^- \) resonance saturation is indeed the correct explanation for the enhanced \( \Delta S = 1 \) PV radiative asymmetries, then one might naturally expect a similar mechanism to play an important role in the \( \Delta S = 0 \) PV electric dipole transition examined in this paper. The relevant diagrams appear in Figs. 2b and 2c, where the intermediate states have \( J^P = \frac{1}{2}^- \) and \( \frac{3}{2}^- \), respectively, and where the \( \gamma \) vertex brings about the change in spin. In using this picture, we note that:

(i) At present, one has detailed information on the \( \frac{1}{2}^- \leftrightarrow \frac{3}{2}^+ \Delta S = 1 \) amplitudes from fits to the S-wave \( B \pi \) decay channel. In contrast, no experimental information exists on the \( \Delta S = 0 \Delta S = \frac{1}{2}^- \leftrightarrow \frac{3}{2}^- \) or \( \frac{1}{2}^- \leftrightarrow \frac{1}{2}^- \Delta S = 1 \) amplitudes. Since we seek only to provide an estimate for \( d_\Delta^+ \) and not to perform a detailed treatment of the underlying quark dynamics, we use the results of Ref. \[25\] for the \( \Delta S = 1 \Delta S = \frac{1}{2}^- \leftrightarrow \frac{3}{2}^- \) amplitudes for guidance in setting the scale of the \( \Delta S = 0 \) weak matrix elements, but recognize that there can be considerable uncertainty in these numbers.

(ii) In computing the amplitudes associated with Figs. 2 b, c we require the electromagnetic (EM) \( R(\frac{1}{2}^-) \rightarrow \Delta(1232) \) and \( R(\frac{1}{2}^-) \rightarrow N(939) \) transition amplitudes. The EM decays of the \( \frac{1}{2}^- \) resonances to the \( \Delta(1232) \) have not been observed, while the partial widths for \( R(\frac{1}{2}^-) \rightarrow p_f \) have been seen at the expected rates. For purposes of estimating \( d_\Delta^+ \), we consider only the contributions from Fig. 2c involving the \( \frac{1}{2}^- \) resonances.

The lowest order weak and EM Lagrangians needed in computing the amplitudes of Fig. 2c are:

\[ \mathcal{L}^{EM}_{\pi N} = \frac{e C_\mu}{A_\chi} \bar{R}_\mu \gamma_\mu \rho F_{\mu \nu} \ + \ h.c. \]  

\[ \mathcal{L}^{PV}_{\pi N} = i W_{\pi} \bar{R}_\mu \Delta_\mu \ + \ h.c. \]  

where we have omitted labels associated with charge and isospin for simplicity and where \( R^{\mu} \) denotes the spin-3/2 field. The constants \( C_\mu \) and \( W_{\pi} \) are \textit{a priori} unknown. Using Eqs. (4, 6), we obtain

\[ d_\Delta^+ = \frac{C_\mu W_{\pi}}{M_R - M_\Delta} \]  

The experimental EM decay widths given in \[26\] imply that \( |C_{1520}| \approx 0.98 \pm 0.05, |C_{1700}| \approx 0.70 \pm 0.13 \) with the overall sign of each uncertain. For the weak amplitudes \( W_{\pi} \), we note that the analysis of Ref. \[25\] obtained \( |W_{\pi}(\Delta S = 1)| \sim 2 \times 10^{-7} \) GeV \( \approx 5 g_\pi A_\chi \). Writing our estimates for \( d_\Delta^+ \) in terms of this quantity we obtain

\[ d_\Delta^+ \sim 17 g_\pi \frac{W_{1520}}{W_R(\Delta S = 1)} + 8 g_\pi \frac{W_{1700}}{W_R(\Delta S = 1)} \]  

with an uncertainty as to the relative and overall phase. To the extent that \( |W_{\pi}(\Delta S = 0)| \sim |W_{\pi}(\Delta S = 1)| \), we would expect then \( |d_\Delta^+| \sim (10 - 25)g_\pi \). By comparison, for the \( N \rightarrow \Delta \) transition anapole moment we obtain \( g^{CT}(VMD) \sim -15g_\pi \) using the “best values” of Ref. \[27\].

Based on NDA, one might have expected \( |W_{\pi}(\Delta S = 0)| \sim A_\chi g_\pi \), and, thus, \( d_\Delta^+ \sim g_\pi \). However, the results of Ref. \[25\] give \( |W_{\pi}(\Delta S = 1)| \sim 5 g_\pi A_\chi \), while the energy denominators in Eq. (8) give additional enhancement factors of two-to-three. Since the \( \Delta S = 0 \) amplitudes are generally further enhanced by \( V_{ud}/V_{us} \) as well as neutral current contributions, our estimate of \( d_\Delta^+ \) could potentially be four to five times larger than given in Eq.
\[ W_N(\Delta S = 0) \sim W_N(\Delta S = 1) \]. Hence, we quote a “reasonable range” \[ |\Delta_A| = (1 - 100)\sigma_\gamma \] based on this possible factor of four enhancement. Our “best value” \[ A_\pm \Delta \sim 1.3 \times 10^{-6} \] as a reasonable estimate for the size of the effect.

To estimate the statistical precision with which one might measure an asymmetry of this magnitude, we use the standard figure of merit \[ \text{[28]} \], along with experimental conditions roughly appropriate for a deuterium target and the G0 detector at Jefferson Lab \[ \text{[5]} \]: luminosity = 0.25 Mhz/nb; \[ \frac{d\sigma}{d\Omega} \pi \rho \sim 4\text{nb/(MeV/c-sr)} \]; \[ \Delta\Omega \rho \sim 0.5 \text{ (MeV/c-sr)} \]; and photon polarization \[ P_\gamma \sim 0.5 \]. With one month of running, one could achieve a 15\% (statistical) determination of \[ A_\pm \], which would be more than adequate to address the physics issues considered here.

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[1] See, e.g. E.G. Adelberger and W.C. Haxton, Ann. Rev. Nucl. Part. Sci. 35, 501 (1985); W. Haeberli and B.R. Holstein, in Symmetries and Fundamental Interactions in Nuclei, ed. W.C. Haxton and E.M. Henley, World Scientific, Singapore (1995).

[2] D. Leinweber, T. Draper and R. M. Woloshyn, Phys. Rev. D 46, 3067 (1992); T. R. Hemmert, B. R. Holstein and N. C. Mukhopadhyay, Phys. Rev. D 51, 158 (1995).

[3] G0 Collaboration, JLAB # E97-104, S.P. Wells and N. Simicevic, spokespersons.

[4] Shi-lin Zhu, C. M. Maekawa, G. Sacco, B.R. Holstein, and M.J. Ramsey-Musolf, hep-ph/0107076.

[5] K. Gustafsson, T. Ito, J. Martin and R. McKeown, private communication.

[6] Y. Hara, Phys. Rev. Lett. 12, 378 (1964).

[7] Particle Data Group, C. Caso et al., Eur. Phys. J. C 3, 55 (1998).

[8] KTeV Collaboration, A. Alvavi-Harati et al., Phys. Rev. Lett. 86, 3239 (2001).

[9] S. L. Adler, Ann. Phys. 50, 189 (1968); Phys. Rev. D 12, 2644 (1975).

[10] The possible existence of a non-vanishing, direct PV \[ \gamma \]-hadron coupling has been discussed previously in E.M. Henley, A.H. Huffman, and D.U.L. Yu, Phys. Rev. D7, 943 (1972).

[11] A.J.F. Siegert, Phys. Rev. 52, 787 (1937).

[12] J.L. Friar and S. Fallieros, Phys. Rev. C 29, 1654 (1984).

[13] R. Dashen, E. Jenkins and A. Manohar, Phys. Rev. D 49, 4713 (1994); D 51, 3697 (1995).

[14] W.C. Haxton, E.M. Henley, and M.J. Musolf, Phys. Rev. Lett. 63, 940 (1989).

[15] M. J. Musolf and B. R. Holstein, Phys. Rev. D 43, 2956 (1991).

[16] J.-W. Chen and X. Ji, Phys. Rev. Lett. 86, 4239 (2001).

[17] Shi-lin Zhu, S.J. Puglia, B.R. Holstein, and M.J. Ramsey-Musolf, Phys. Rev. C 64, 035502 (2001).

[18] J.-W. Chen and X. Ji, Phys. Lett. B 501, 209 (2001).

[19] E. Jenkins and A. V. Manohar, Phys. Lett. B 255, 558 (1991); B 259, 353 (1991).

[20] Ulf-G. Meissner, Int. J. Mod. Phys. E 1, 561 (1992).

[21] Shi-lin Zhu, S. Puglia, M. J. Ramsey-Musolf, and B. Holstein, Phys. Rev. D 62, 033008 (2000).

[22] S. R. Beane et al., nucl-th/0008064.

[23] A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984).

[24] A. Le Yaouanc et al., Nucl. Phys. B 149, 321 (1979); M. B. Gavela et al., Phys. Lett. B 101, 417 (1981).

[25] B. Borasoy and B. R. Holstein, Phys. Rev. D 59, 054019 (1999).

[26] G. B. Feldman et al., Phys. Rev. C 43, 863 (1991).

[27] B. Desplanques, J. F. Donoghue and B. R. Holstein, Ann. Phys. 124, 449 (1980).

[28] M.J. Musolf et al. Phys. Rep. 239, 1 (1994).