SOME CHARACTERIZATIONS OF HOM-LEIBNIZ ALGEBRAS

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ABSTRACT. Some basic properties of Hom-Leibniz algebras are found. These properties are the Hom-analogue of corresponding well-known properties of Leibniz algebras. Considering the Hom-Akivis algebra associated to a given Hom-Leibniz algebra, it is observed that the Hom-Akivis identity leads to an additional property of Hom-Leibniz algebras, which in turn gives a necessary and sufficient condition for Hom-Lie admissibility of Hom-Leibniz algebras. A necessary and sufficient condition for Hom-power associativity of Hom-Leibniz algebras is also found.

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1. Introduction

The theory of Hom-algebras originated from the introduction of the notion of a Hom-Lie algebra by J.T. Hartwig, D. Larsson and S.D. Silvestrov [6] in the study of algebraic structures describing some q-deformations of the Witt and the Virasoro algebras. A Hom-Lie algebra is characterized by a Jacobi-like identity (called the Hom-Jacobi identity) which is seen as the Jacobi identity twisted by an endomorphism of a given algebra. Thus, the class of Hom-Lie algebras contains the one of Lie algebras.

Generalizing the well-known construction of Lie algebras from associative algebras, the notion of a Hom-associative algebra is introduced by A. Makhlouf and S.D. Silvestrov [13] (in fact the commutator algebra of a Hom-associative algebra is a Hom-Lie algebra). The other class of Hom-algebras closely related to Hom-Lie algebras is the one of Hom-Leibniz algebras [13] (see also [9]) which are the Hom-analogue of Leibniz algebras [10]. Extending the Loday’s construction ([10]) of Leibniz algebras from dialgebras, D. Yau [14] introduced Hom-dialgebras and proved that every Hom-dialgebra gives rise to a Hom-Leibniz algebra. Roughly, a Hom-type generalization of a given type of algebras is defined by a twisting of the defining identities with a linear self-map of the given algebra. For various Hom-type algebras one may refer, e.g., to [3,8,11,12,16,17]. In [15] D. Yau showed a way of
constructing Hom-type algebras starting from their corresponding untwisted algebras and a self-map.

In [10] (see also [4,5]) the basic properties of Leibniz algebras are given. The main purpose of this note is to point out that the Hom-analogue of some of these properties holds in Hom-Leibniz algebras (section 3). Considering the Hom-Akivis algebra associated to a given Hom-Leibniz algebra, we observe that the property in Proposition 3.3 is the expression of the Hom-Akivis identity. As a consequence we found a necessary and sufficient condition for the Hom-Lie admissibility of Hom-Leibniz algebras (Corollary 3.5). Generalizing power-associativity of rings and algebras [2], the notion of the (right) n-th Hom-power of an element x in a Hom-algebra is introduced by D. Yau [18], as well as Hom-power associativity of Hom-algebras. We found that \( x^n = 0 \), \( n \geq 3 \), for any \( x \in \mathcal{L} \) in a left Hom-Leibniz algebra \((\mathcal{L}, \cdot, \alpha)\) and that \((\mathcal{L}, \cdot, \alpha)\) is Hom-power associative if and only if \( \alpha(x)x^2 = 0 \), for all \( x \in \mathcal{L} \) (Theorem 3.8). Then we deduce, as a particular case, corresponding characterizations of left Leibniz algebras (Corollary 3.9). Apart of the (right) n-th Hom-power of an element of a Hom-algebra [18], we consider in this note the left n-th Hom-power of the given element. This allows to prove the Hom-analogue (see Theorem 3.11) of a result of D.W. Barnes ([5], Theorem 1.2 and Corollary 1.3) characterizing left Leibniz algebras. In section 2 we recall some basic notions on Hom-algebras. Modules, algebras, and linearity are meant over a ground field \( \mathbb{K} \) of characteristic 0.

2. Preliminaries

In this section we recall some basic notions related to Hom-algebras. These notions are introduced in [6,8,11,13,15].

Definition 2.1. A Hom-algebra is a triple \((A, \cdot, \alpha)\) in which \( A \) is a \( \mathbb{K} \)-vector space, “\( \cdot \)” a binary operation on \( A \) and \( \alpha : A \to A \) is a linear map (the twisting map) such that \( \alpha(x \cdot y) = \alpha(x) \cdot \alpha(y) \) (multiplicativity), for all \( x, y \in A \).

Remark 2.2. A more general notion of a Hom-algebra is given (see, e.g., [11], [13]) without the assumption of multiplicativity and \( A \) is considered just as a \( \mathbb{K} \)-module. For convenience, here we assume that a Hom-algebra \((A, \cdot, \alpha)\) is always multiplicative and that \( A \) is a \( \mathbb{K} \)-vector space.

Definition 2.3. Let \((A, \cdot, \alpha)\) be a Hom-algebra.

(i) The Hom-associator of \((A, \cdot, \alpha)\) is the trilinear map \( as : A \times A \times A \to A \) defined by \( as(x, y, z) = (x \cdot y) \cdot \alpha(z) - \alpha(x) \cdot (y \cdot z) \), for all \( x, y, z \in A \).

(ii) \((A, \cdot, \alpha)\) is said to be Hom-associative if \( as(x, y, z) = 0 \) (Hom-associativity), for all \( x, y, z \in A \).
Remark 2.4. If $\alpha = \text{Id}$ (the identity map) in $(A,\cdot,\alpha)$, then its Hom-associator is just the usual associator of the algebra $(A,\cdot)$. In Definition 2.1, the Hom-associativity is not assumed, i.e. $as(x,y,z) \neq 0$ in general. In this case $(A,\cdot,\alpha)$ is said non-Hom-associative [8] (or Hom-nonassociative [15]; in [12], $(A,\cdot,\alpha)$ is also called a nonassociative Hom-algebra). This matches the generalization of associative algebras by the nonassociative ones.

Definition 2.5. (i) A (left) Hom-Leibniz algebra is a Hom-algebra $(A,\cdot,\alpha)$ such that the identity
\begin{equation}
\alpha(x) \cdot (y \cdot z) = (x \cdot y) \cdot \alpha(z) + \alpha(y) \cdot (x \cdot z)
\end{equation}
holds for all $x,y,z$ in $A$.

(ii) A Hom-Lie algebra is a Hom-algebra $(A,[-,-],\alpha)$ such that the binary operation $"[-,-]"$ is skew-symmetric and the Hom-Jacobi identity
\begin{equation}
J_\alpha(x,y,z) = 0
\end{equation}
holds for all $x,y,z$ in $A$, and $J_\alpha(x,y,z) := [[[x,y],\alpha(z)] + [[y,z],\alpha(x)] + [[z,x],\alpha(y)]$ is called the Hom-Jacobian.

Remark 2.6. The original definition of a Hom-Leibniz algebra [13] is related to the identity
\begin{equation}
(x \cdot y) \cdot \alpha(z) = (x \cdot z) \cdot \alpha(y) + \alpha(x) \cdot (y \cdot z)
\end{equation}
which is expressed in terms of (right) adjoint homomorphisms $Ad_y x := x \cdot y$ of $(A,\cdot,\alpha)$. This justifies the term of “(right) Hom-Leibniz algebra” that could be used for the Hom-Leibniz algebra defined in [13]. The dual of (2.3) is (2.1) and in this note we consider only left Hom-Leibniz algebras. For $\alpha = \text{Id}$ in $(A,\cdot,\alpha)$ (resp. $(A,[-,-],\alpha)$), any Hom-Leibniz algebra (resp. Hom-Lie algebra) is a Leibniz algebra $[4], [10]$ (resp. a Lie algebra $(A,[-,-])$). As for Leibniz algebras, if the operation $"-"$ of a given Hom-Leibniz algebra $(A,\cdot,\alpha)$ is skew-symmetric, then $(A,\cdot,\alpha)$ is a Hom-Lie algebra (see [13]).

In terms of Hom-associators, the identity (2.1) is written as
\begin{equation}
as(x,y,z) = -\alpha(y) \cdot (x \cdot z)
\end{equation}
Therefore, from Definition 2.3 and Remark 2.4, we see that Hom-Leibniz algebras are examples of non-Hom-associative algebras.

Definition 2.7. [8] A Hom-Akivis algebra is a quadruple $(A,[-,-],[-,-,-],\alpha)$ in which $A$ is a vector space, $"[-,-]"$ a skew-symmetric binary operation on $A$, $"[-,-,-]"$ a ternary operation on $A$ and $\alpha : A \rightarrow A$ a linear map such that the Hom-Akivis identity
\begin{equation}
J_\alpha(x,y,z) = \bigcirc_{(x,y,z)}[x,y,z] - \bigcirc_{(x,y,z)}[y,x,z]
\end{equation}
holds for all $x,y,z$ in $A$, where $\bigcirc_{(x,y,z)}$ denotes the sum over cyclic permutation of $x,y,z$. 

Note that when $\alpha = \text{Id}$ in a Hom-Akivis algebra $(A, [-, -], [-, -], \alpha)$, then one gets an Akivis algebra $(A, [-, -], [-, -], \alpha)$. Akivis algebras were introduced in [1] (see also references therein), where they were called $W$-algebras. The term “Akivis algebra” for these objects is introduced in [7].

In [8], it is observed that to each non-Hom-associative algebra is associated a Hom-Akivis algebra (this is the Hom-analogue of a similar relationship between nonassociative algebras and Akivis algebras [1]). In this note, we use the specific properties of the Hom-Akivis algebra associated to a given Hom-Leibniz algebra to derive a property characterizing Hom-Leibniz algebras.

3. Characterizations

In this section, Hom-versions of some well-known properties of left Leibniz algebras are displayed. Considering the specific properties of the binary and ternary operations of the Hom-Akivis algebra associated to a given Hom-Leibniz algebra, we infer a characteristic property of Hom-Leibniz algebras (Proposition 3.3). This property in turn allows to give a necessary and sufficient condition for the Hom-Lie admissibility of these Hom-algebras (Corollary 3.5). The Hom-power associativity of Hom-Leibniz algebras is considered.

Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra and consider on $(A, \cdot, \alpha)$ the operations
\begin{align*}
[x, y] &:= x \cdot y - y \cdot x \\
[x, y, z] &:= as(x, y, z)
\end{align*}
(3.1)
(3.2)

Then the operations (3.1) and (3.2) define on $A$ a Hom-Akivis structure ([8]). We have the following proposition:

Proposition 3.1. Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra. Then

(i) $(x \cdot y + y \cdot x) \cdot \alpha(z) = 0$,
(ii) $\alpha(x) \cdot [y, z] = [x \cdot y, \alpha(z)] + [\alpha(y), x \cdot z]$, for all $x, y, z$ in $A$.

Proof. The identity (2.1) implies that $(x \cdot y) \cdot \alpha(z) = \alpha(x) \cdot (y \cdot z) - \alpha(y) \cdot (x \cdot z)$. Likewise, interchanging $x$ and $y$, we have $(y \cdot x) \cdot \alpha(z) = \alpha(y) \cdot (x \cdot z) - \alpha(x) \cdot (y \cdot z)$.

Then, adding memberwise these equalities above, we come to the property (i). Next we have
\begin{align*}
[x \cdot y, \alpha(z)] + [\alpha(y), x \cdot z] & = (x \cdot y) \cdot \alpha(z) - \alpha(z) \cdot (x \cdot y) \\
& + \alpha(y) \cdot (x \cdot z) - (x \cdot z) \cdot \alpha(y) \\
& = \alpha(x) \cdot (y \cdot z) - \alpha(z) \cdot (x \cdot y) - (x \cdot z) \cdot \alpha(y) \quad \text{(by (2.1))} \\
& = \alpha(x) \cdot (y \cdot z) - (z \cdot x) \cdot \alpha(y) - \alpha(x) \cdot (z \cdot y) \\
& - (x \cdot z) \cdot \alpha(y) \quad \text{(by (2.1))} \\
& = \alpha(x) \cdot (y \cdot z) - \alpha(x) \cdot (z \cdot y) \quad \text{(by (i))} \\
& = \alpha(x) \cdot [y, z]
\end{align*}
and so we get (ii).

\[ \square \]

**Remark 3.2.** If set \( \alpha = \text{Id} \) in Proposition 3.1, then one recovers the well-known properties of Leibniz algebras: \((x \cdot y + y \cdot x) \cdot z = 0\) and \(x \cdot [y, z] = [x \cdot y, z] + [y, x \cdot z]\) (see [4], [10]).

**Proposition 3.3.** Let \((A, \cdot, \alpha)\) be a Hom-Leibniz algebra. Then
\[
J_\alpha(x, y, z) = \mathcal{C}_{(x,y,z)}(x \cdot y) \cdot \alpha(z),
\]
(3.3)
for all \(x, y, z\) in \(A\).

**Proof.** Considering (2.5) and then applying (3.2) and (2.4), we get
\[
J_\alpha(x, y, z) = \mathcal{C}_{(x,y,z)}(-\alpha(y) \cdot (x \cdot z)) = \mathcal{C}_{(x,y,z)}[\alpha(x) \cdot (y \cdot z) - \alpha(y) \cdot (x \cdot z)] = \mathcal{C}_{(x,y,z)}(x \cdot y) \cdot \alpha(z) \quad \text{(by (2.1))}.
\]

One observes that (3.3) is the specific form of the Hom-Akivis identity (2.5) in case of Hom-Leibniz algebras.

**Definition 3.4.** [13] A Hom-algebra \((A, \cdot, \alpha)\) is said to be Hom-Lie admissible if \((A, [-, -], \alpha)\) is a Hom-Lie algebra, where \([x, y] := x \cdot y - y \cdot x\) for all \(x, y\) in \(A\).

The skew-symmetry of the operation “\(-\)” of a Hom-Leibniz algebra \((A, \cdot, \alpha)\) is a condition for \((A, \cdot, \alpha)\) to be a Hom-Lie algebra [13]. From Proposition 3.3 one gets the following necessary and sufficient condition for the Hom-Lie admissibility [13] of a given Hom-Leibniz algebra.

**Corollary 3.5.** A Hom-Leibniz algebra \((A, \cdot, \alpha)\) is Hom-Lie admissible if and only if \(\mathcal{C}_{(x,y,z)}(x \cdot y) \cdot \alpha(z) = 0\), for all \(x, y, z\) in \(A\).

In [18] D. Yau introduced Hom-power associative algebras which are seen as a generalization of power-associative algebras. It is shown that some important properties of power-associative algebras are reported to Hom-power associative algebras.

Let \(A\) be a Hom-Leibniz algebra with a twisting linear self-map \(\alpha\) and the binary operation on \(A\) denoted by juxtaposition. We recall the following definition.

**Definition 3.6.** [18] Let \(x \in A\) and denote by \(\alpha^m\) the \(m\)-fold composition of \(m\) copies of \(\alpha\) with \(\alpha^0 := \text{Id}\).

(1) The \(n\)th Hom-power \(x^n \in A\) of \(x\) is inductively defined by
\[
x^1 = x, \quad x^n = x^{n-1}\alpha^{n-2}(x) \quad \text{(3.4)}
\]
for \(n \geq 2\).

(2) The Hom-algebra \(A\) is \(n\)th Hom-power associative if
\[
x^n = \alpha^{n-i-1}(x^i)\alpha^{i-1}(x^{n-i}) \quad \text{(3.5)}
\]
for all \(x \in A\) and \(i \in \{1, \ldots, n - 1\}\).

(3) The Hom-algebra \(A\) is up to \(n\)th Hom-power associative if \(A\) is \(k\)th Hom-power associative for all \(k \in \{2, \ldots, n\}\).
The Hom-algebra $A$ is $n$th Hom-power associative if $A$ is $n$th Hom-power associative for all $n \geq 2$.

The following result provides a characterization of third Hom-power associativity of Hom-Leibniz algebras.

Lemma 3.7. Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra. Then

(i) $x^3 = 0$, for all $x \in A$;
(ii) $(A, \cdot, \alpha)$ is third Hom-power associative if and only if $\alpha(x)x^2 = 0$, for all $x \in A$.

Proof. From (3.4) we have $x^3 := x^2\alpha(x)$. Therefore, the assertion (i) follows from Proposition 3.1(i) if set $y = x = z$. Next, from (3.5) we note that the $i = 2$ case of $n$th Hom-power associativity is automatically satisfied since this case is $x^3 = \alpha^0(x^2)\alpha^1(x^1) = x^2\alpha(x)$, which holds by definition. The $i = 2$ case says that $x^3 = \alpha^1(x)\alpha^0(x^2) = \alpha(x)x^2$. Therefore, since $x^2\alpha(x) = 0$ naturally holds by Proposition 3.1 (i), we conclude that the third Hom-power associativity of $(A, \cdot, \alpha)$ holds if and only if $\alpha(x)x^2 = 0$ for all $x \in A$, which proves the assertion (ii). □

The following result shows that the condition in Lemma 3.7 is also necessary and sufficient for the Hom-power associativity of $(A, \cdot, \alpha)$. To prove this, we rely on the main result of [18] (see Corollary 5.2).

Theorem 3.8. Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra. Then

(i) $x^n = 0$, $n \geq 3$, for all $x \in A$;
(ii) $(A, \cdot, \alpha)$ is Hom-power associative if and only if $\alpha(x)x^2 = 0$, for all $x \in A$.

Proof. The proof of (i) is by induction on $n$: the first step $n = 3$ holds by Lemma 3.7(i); now if suppose that $x^n = 0$, then $x^{n+1} := x^{n+1} \alpha^{(n+1)-2}(x) = x^n\alpha^{n-1}(x) = 0$ so we get (i).

Corollary 5.2 of [18] says that, for a multiplicative Hom-algebra, the Hom-power associativity is equivalent to both of the conditions

$$x^2\alpha(x) = \alpha(x)x^2 \quad \text{and} \quad x^4 = \alpha(x^2)\alpha(x^2). \quad (3.6)$$

In the situation of multiplicative left Hom-Leibniz algebras, the first equality of (3.6) is satisfied by Lemma 3.7(i) and the hypothesis $\alpha(x)x^2 = 0$. We have the following from (3.5):

Case $i = 1$: $x^4 := \alpha^{4-2}(x)\alpha^0(x^3) = \alpha^2(x)x^3$,
Case $i = 2$: $x^4 := \alpha(x^2)\alpha(x^2)$,
Case $i = 3$: $x^4 := \alpha^0(x^3)\alpha^2(x) = x^3\alpha^2(x)$.

Because of the assertion (i) above, only the case $i = 2$ is of interest here. From one side we have $x^4 = 0$ (by (i)) and, from the other side we have $\alpha(x^2)\alpha(x^2) = \alpha(x\alpha(x))$. Therefore, we have $\alpha(x\alpha(x)) = 0$ which proves the assertion (ii). □
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\[ \alpha(x)^2 \alpha(x^2) = 0 \] (by multiplicativity and Proposition 3.1(i)). Therefore, Corollary 5.2 of [18] now applies and we conclude that (3.6) holds (i.e. \((A, \cdot, \alpha)\) is Hom-power associative) if and only if \(\alpha(x)x^2 = 0\), which proves (ii).

Let \(A\) be an algebra (over a field of characteristic 0). For an element \(x \in A\), the \textbf{right powers} are defined by

\[ x^1 = x, \quad x^{n+1} = x^n x \]  
(3.7)

for \(n \geq 1\). Then \(A\) is power-associative if and only if

\[ x^n = x^{n-i} x^i \]  
(3.8)

for all \(x \in A, n \geq 2, \) and \(i \in \{1, \ldots, n-1\}\). By a theorem of Albert [2], \(A\) is power-associative if only if it is third and fourth power-associative, which in turn is equivalent to

\[ x^2 x = x x^2 \quad \text{and} \quad x^4 = x^2 x^2. \]  
(3.9)

for all \(x \in A\).

Some consequences of the results above are the following simple characterizations of (left) Leibniz algebras.

**Corollary 3.9.** Let \((A, \cdot)\) be a left Leibniz algebra. Then

(i) \(x^n = 0, \ n \geq 3, \) for all \(x \in A;\)

(ii) \((A, \cdot)\) is power-associative if and only if \(xx^2 = 0, \) for all \(x \in A.\)

**Proof.** The part (i) of this corollary follows from (3.7) and Theorem 3.8(i) when \(\alpha = \text{Id}\) (we used here the well-known property \((xy + yx)z = 0\) of left Leibniz algebras). The assertion (ii) is a special case of Theorem 3.8(ii) (when \(\alpha = \text{Id}\)), if keep in mind the assertion (i), (3.8), and (3.9). \(\square\)

**Remark 3.10.** Although the condition \(xx^2 = 0\) does not always hold in a left Leibniz algebra \((A, \cdot)\), we do have \(xx^2 \cdot z = 0\) for all \(x, z \in A\) (again, this follows from the property \((xy + yx)z = 0\)). In fact, \(b \cdot z = 0, \ z \in A, \) where \(b \neq 0\) is a left \(m\)th power of \(x (m \geq 2), \) i.e. \(b = x(x(...(x...))))) \) ([5], Theorem 1.2 and Corollary 1.3).

Let us call the \(n\)th right Hom-power of \(x \in A\) the power defined by (3.4), where \(A\) is a Hom-algebra. Then one may consider the \(n\)th left Hom-power of \(a \in A\) defined by

\[ a^1 = a, \quad a^n = \alpha^{n-2}(a)a^{n-1} \]  
(3.10)

for \(n \geq 2.\) In this setting of left Hom-powers, we have the following theorem.

**Theorem 3.11.** Let \((A, \cdot, \alpha)\) be a Hom-Leibniz algebra and let \(a \in A.\) Then \(L_a^n \circ \alpha = 0, \ n \geq 2,\) where \(L_z\) denotes the left multiplication by \(z\) in \((A, \cdot, \alpha),\) i.e. \(L_z x = z \cdot x, \ x \in L.\)

**Proof.** We proceed by induction on \(n\) and the repeated use of Proposition 3.1(i). From Proposition 3.1(i), we get \(a^2 \alpha(z) = 0, \ \forall a, z \in A\) and thus the first step \(n = 2\)
is verified. Now assume that, up to the degree \( n \), we have \( a^n \alpha(z) = 0, \forall a, z \in A \). Then Proposition 3.1(i) implies that \( (a^n \alpha(a^{n-1}(a) + a^{n-1}(a)a^n)\alpha(z) = 0, \) i.e. \( (a^n \alpha(a^{n-2}(a)) + a^{n-1}(a)a^n)\alpha(z) = 0. \) The application of the induction hypothesis to \( a^n \alpha(a^{n-2}(a)) \) leads to \( (a^{n-1}(a)a^n)\alpha(z) = 0, \) i.e. \( (a^{n-1}(a)a^{n+1} - 1)\alpha(z) = 0 \) which means (by (3.10)) that \( a^{n+1} \alpha(z) = 0. \) Therefore, we conclude that \( a^n \alpha(z) = 0, \forall n \geq 2, \) i.e. \( L_{a^n} \circ \alpha = 0, \) \( n \geq 2. \)

\[ \square \]

**Remark 3.12.** We observe that Theorem 3.11 above is an \( \alpha \)-twisted version of a result of D.W. Barnes ([5], Theorem 1.2 and Corollary 1.3), related to left Leibniz algebras. Indeed, setting \( \alpha = \text{Id} \) in Theorem 3.11 we get the result of Barnes.

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