Gauged $U(1)_R$ supergravity on orbifold 1

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Abstract. We discuss a gauged $U(1)_R$ supergravity on five-dimensional orbifold ($S^1/Z_2$) in which a $Z_2$-even $U(1)$ gauge field takes part in the $U(1)_R$ gauging, and show the structure of Fayet-Iliopoulos (FI) terms allowed in such model. Some physical consequences of the FI terms are examined.

Keywords: Supergravity, Supersymmetric models

PACS: 04.65.+e, 12.60.Jv

INTRODUCTION

Recently five-dimensional (5D) supergravity (SUGRA) on the orbifold $S^1/Z_2$ has been studied as an interesting theoretical framework for physics beyond the SM. It has been noted that 5D orbifold SUGRA with a $U(1)_R$ symmetry gauged by the $Z_2$-odd graviphoton can provide the supersymmetric Randall-Sundrum (RS) model [1] in which the weak to Planck scale hierarchy can arise naturally from the geometric localization of 4D graviton [2], and/or Yukawa hierarchy can be generated by the quasi-localization of the matter zero modes in extra dimension where we generically have an interesting correlation between the flavor structure in the spectra and the hierarchical Yukawa couplings [3]. In the former case, the bulk cosmological constant and brane tensions which are required to generate the necessary $AdS_5$ geometry appear in the Lagrangian as a consequence of the $U(1)_R$ FI term with $Z_2$-odd coefficient.

In this talk we consider a more generic orbifold SUGRA which contains a $Z_2$-even 5D gauge field $A_{1R}^X$ participating in the $U(1)_R$ gauging [4]. If 4D $N=1$ SUSY is preserved by the compactification, the 4D effective theory of such model will contain a gauged $U(1)_R$ symmetry associated with the zero mode of $A_{1R}^X$, which is not the case when the 5D $U(1)_R$ is gauged only through the $Z_2$-odd graviphoton. Based on the known off-shell formulation [5], we formulate a gauged $U(1)_R$ SUGRA on $S^1/Z_2$ in which both $A_{1R}^X$ and the graviphoton take part in the $U(1)_R$ gauging and then analyze the structure of FI terms allowed in such model. As expected, introducing a $Z_2$-even $U(1)_R$ gauge field accompanies new bulk and boundary FI terms in addition to the known integrable boundary FI term which could be present in the absence of any gauged $U(1)_R$ symmetry [6]. As we will see, those new FI terms can have interesting implications to the quasi-localization of the matter zero modes in extra dimension and the SUSY breaking [3] and also to the radion stabilization.

FORMULATION

For a minimal setup, we introduce two vector multiplets and two hypermultiplets in the off-shell formulation of 5D (conformal) SUGRA [5]:

$\mathbf{Y}_X = (M^X = \alpha, A_{1R}^X, \Omega^{Zi}, Y_{Zij}),$

$\mathbf{Y}_c = (M^2 = \beta, A_{1R}^c, \Omega^{Xi}, Y_{Xij}),$

and

$\mathcal{H}_c = (\sigma_i^c, \eta^c, \Xi_i^c),$

$\mathcal{H}_p = (\Phi^c, \xi^c, F^c),$

with the norm function

$N = \alpha^3 - \frac{1}{2} \alpha \beta^2,$

and the hypermultiplet gauging

$\left( t_z, t_x \right) \Phi = \left( e \varepsilon(y), q \right) i \sigma_3 \Phi,$

$\left( t_z, t_x \right) \mathcal{A} = \left( -\frac{3}{2} e \varepsilon(y), -r \right) i \sigma_3 \mathcal{A},$

where we adopt the $2 \times 2$ matrix notations omitting $x = 1, 2$ index and $SU(2)$ indices $i, j = 1, 2$, and the hyper-scalars satisfy the reality condition $\mathcal{A} = i \sigma_2 \Phi_i \sigma_2^T$. The $Z_2$-even bosonic (non-axial) components are $\alpha, A_{1R}^X, A_{1R}^c, \alpha^{X}_{i=1,2}$ and $\Phi^{Z}_{i=1,2}$, and $Y_1, \mathcal{H}_c$ are the graviphoton vector multiplet and the compensator hypermultiplet respectively. The $Z_2$-odd coefficient $\varepsilon(y)$ in the hypermultiplet gauging is consistently introduced by the mechanism proposed in [7]. The nonzero value of

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1 Talk presented at the 11th International Symposium on Particles, Strings and Cosmology, May 30 - June 4, 2005, Gyeongju, Korea
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the charge $r$ corresponds to the $U(1)_R$ symmetry gauged by $Z_2$-even vector field $A^Y_2$.

The bosonic part of the Lagrangian is given by

$$\mathcal{L}_{\text{bosonic}} = \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\partial \epsilon} + \mathcal{L}_{N=1},$$

$$e^{-1}\mathcal{L}_{\text{bulk}} = -\frac{1}{4}R - \frac{1}{4}a_{ij} F^i_{\mu} F^{\mu \nu \rho} + \frac{1}{4}a_{ij} \nabla M^i \nabla M^j + \frac{1}{8}e^{-1}C_{ijkl} e^{\phi \mu \nu \rho \sigma} A^k_{\lambda} F^i_{\mu \nu \rho} F^j_{\sigma} + \text{tr} \left[ \nabla M^i - \nabla M^j \right]^2 - |M|^2 + \text{tr} \left[ \nabla^2 M^i - \nabla^2 M^j \right]$$

$$- M^i M^j \left( \Phi^i \Theta^i M^j - \Phi^j \Theta^j M^i \right)$$

$$\times - 2 \left[ \nabla M^i Y^i Y^j - 4 Y^i \left( \Phi^i \Theta^j - \Phi^j \Theta^i \right) \right]$$

$$e^{-1} \mathcal{L}_{\partial \epsilon} = -2e^{-1} \epsilon_{(4)} \partial_j \beta \left( \lambda_0 \delta(y) + \lambda_2 \delta(y - \pi R) \right)$$

$$+ \left( \lambda_0 \delta(y) + \lambda_2 \delta(y - \pi R) \right),$$

$$e^{-1} \mathcal{L}_{N=1} = M^2 \left[ - 2 \epsilon(2 Y^i Y^j - e^{-1} \epsilon_{(4)} \partial_i \beta) - \frac{1}{2} R^{(4)} \right]$$

$$\times \left( \lambda_0 \delta(y) + \lambda_2 \delta(y - \pi R) \right),$$

where the matrix notation is employed again, $I,J = (Z,X)$, $a_{ij} = \left\langle X \big| A_M \big| X \right\rangle$, $M^2 = (1 + \text{tr} \left[ \Phi^i \Theta^j \right])^{2/3}$, and $V_M = \frac{1}{2} \left( \Phi^i (\nabla M^i) - (\nabla M^i) \Phi^i \right)$.

For $k = 0$ that results in $K(y) \simeq 0$, the vacuum values of the scalar fields are given by

$$\phi = 0, \quad v = v_0 \equiv \pm \sqrt{-\frac{1}{4r^2}}.$$

and $\phi$ is the physical gauge scalar field parameterizing the (very special) manifold of vector multiplet determined by $\mathcal{N} = \alpha^3(\phi) - \alpha(\phi)\beta^2(\phi)/2 = 1$ with the metric $g_{\phi \phi} = a_{ij} M^i_0 M^j_0$. We choose $\alpha(\phi) = \cosh^{2/3}(\phi)$ and $\beta(\phi) = \sqrt{2} \cosh^{2/3}(\phi) \tan(\phi)$ in the following. The real and diagonal component of the quaternionic hypermultiplet field $\Phi$ is represented by $v$ in the Killing parameters, and zero vacuum values are assumed for the other components for simplicity. In terms of these Killing parameters, the 4D energy density is found to be

$$E = \int dy \, e^{4k} \left[ \frac{1}{2} g_{\phi \phi} D^2 + \frac{2}{1 + v^2} |F|^2 - 6 |\kappa|^2 \right],$$

and it is obvious that the Killing condition $\kappa = D = F = 0$ determines a stationary point of the 4D scalar potential if the solution exists.

**PHYSICAL CONSEQUENCES**

Now we examine some physical consequences of the 5D gauged $U(1)_R$ supergravity on $S^1/Z_2$ which can have the bulk and the boundary FI term, for the supersymmetric vacuum configurations, $\kappa = D = F = 0$.

First we consider the case that we have a charged hypermultiplet $\Phi$ with the charge satisfying $q/r < -1$.

We are interested in the 4D Poincaré invariant background geometry,

$$ds^2 = e^{2K(y)} \eta_{ij} dx^i dx^j - dy^2,$$

and the gravitino-, hyperino- and gaugino-Killing parameters on this background are given respectively by

$$\kappa = \partial_i K - \mathcal{P}/3,$$

$$F = \partial_\epsilon K - (q \Phi + c \epsilon(y) \alpha - \mathcal{P}/2) v,$$

$$D = \partial_\epsilon \phi + g_{\phi \phi} \mathcal{P}\phi - 2rM^2 \epsilon_{(4)} \partial_\epsilon \beta \left( \lambda_0 \delta(y) - \lambda_2 \delta(y - \pi R) \right),$$

where

$$\mathcal{P} = -2 \left[ \frac{3}{4} e(y) \alpha + r \beta \right] + \left\{ \left( \frac{3}{4} k + e \right) e(y) \alpha + (r + q) \beta \right\} v^2.$$
We have studied a 5D gauged $U(1)_R$ supergravity on $S^1/Z_2$ in which both a $Z_2$-even $U(1)$ gauge field and the $Z_2$-odd graviphoton take part in the $U(1)_R$ gauging. Based on the off-shell 5D supergravity of Ref. [5], we examined the structure of Fayet-Iliopoulos (FI) terms allowed by such theory. As expected, introducing a $Z_2$-even $U(1)_R$ gauging accompanies new bulk and boundary FI terms in addition to the known integrable boundary FI term which could be present in the absence of any gauged $U(1)_R$ symmetry. The new (non-integrable) boundary FI terms originate from the $N=1$ boundary supergravity, and thus are free from the bulk supergravity structure in contrast to the integrable boundary FI term which is determined by the bulk supergravity [6].

We have examined some physical consequences of the $Z_2$-even $U(1)_R$ gauging in several simple cases. It is noted that the FI terms of gauged $Z_2$-even $U(1)_R$ can lead to an interesting deformation of vacuum structure which can affect the quasi-localization of the matter zero modes in extra dimension and also the SUSY breaking and radion stabilization. Thus the 5D gauged $U(1)_R$ supergravity on orbifold has a rich theoretical structure which may be useful for understanding some problems in particle physics such as the Yukawa hierarchy and/or the supersymmetry breaking [3]. For such phenomenological study and for the analysis of the radion stabilization, the $N=1$ superfield description [8] will be useful. When one tries to construct a realistic particle physics model within gauged $U(1)_R$ supergravity, one of the most severe constraint will come from the anomaly cancellation condition. In some cases the Green-Schwarz mechanism might be necessary to cancel the anomaly, which may introduce another type of FI term into the theory [9]. These issues will be studied in future works.

ACKNOWLEDGMENTS

The author would like to thank Kiwoon Choi for the collaboration [4] which forms the basis of this talk. This work was supported by KRF PBRG 2002-070-C00022.

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FIGURE 1. The profiles of $\phi$ and the matter zero mode $\Phi(0)$ for some cases with $k = 0$ and $\lambda_0 = 0$. Here we choose $\lambda_{\pi} = (r - 1)/2$. For the matter zero mode profile, the solid-, dotted- and dashed-curves represent the case with $(q_c) = (0.5, 0), (0, 0.5)$ and $(0.5, 0.5)$, respectively. All the curves are shown within $|y| \leq \pi R$.

FIGURE 2. The profiles of $\phi$ and $\Phi(0)$ for $r, k \neq 0, \lambda_0 = 0$ and $\lambda_{\pi} = (r - 1)/2$. Again the solid-, dotted- and dashed-curves represent the case $(q_c) = (0.5, 0), (0, 0.5)$ and $(0.5, 0.5)$, respectively. Note that $K \simeq -ky$ in this supersymmetric solution.