Attributes of Gravitational Lensing Parallax

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1. Introduction

The cosmological density of compact objects in the range $10^{-15}$ - $10^{10}$ is currently uncertain. Potentially important components to the density of the universe include stars, MAssive Compact Halo Objects (MACHOs), and massive black holes. Here one hypothetical path to estimating the space density of these objects is proposed: the simultaneous measurement by separated observers of compact sources at cosmological distances. A population of compact objects of $\Omega_L$ would cause a slight gravitational lens induced brightness difference that would be recoverable by sufficiently sensitive photometry. As this method hinges on well calibrated observations by separated observers, it will be referred to as “gravitational lensing parallax” or just “lensing parallax.”

Lensing parallax observations of cosmological sources were first suggested by Refsdal (1966), who discussed what generic information about lensing events could be obtained by two platforms separated in the solar system. That lensing parallax observations of QSOs could determine the transverse velocity of galaxies was pointed out by Grieger, Kayser & Refsdal (1986). Gould (1992, 1994, 1995) suggested the method could better constrain the transverse velocities of lenses in MACHO searches.

GRBs were discovered in the late 1960s (Klebesadel, Strong, & Olson, 1973). In 1997, GRBs were found to have counterparts in wavelength bands from the radio to the X-ray (Costa et al., 1997; van Paradijs et al., 1997). For simplicity, these event will all be referred to as GRBs, however. In general, GRB emission lasts longer and peaks later at longer wavelengths. At the time of this writing, one GRB was found to have an absorption line placing the event at cosmological distances (Metzger et al. 1997).

Microlensing of GRBs had been proposed by Paczynski (1986). A search for gravitational lensing effects in GRB data has been ongoing since 1993 (Nemiroff et al., 1993; Marani, 1997; Marani et al., 1998). Nemiroff & Gould (1995) first suggested lensing parallax observations of gamma-ray bursts (GRBs), and showed how limits could be placed on a cosmological abundance of lens masses from $10^{-15} M_\odot$ to $10^{-7} M_\odot$. Loeb & Perna (1998) recently discussed how lensing parallax observations of GRB afterglows could confirm a microlensing amplification bump or a lens-induced polarization signature.
In this paper the initial lensing parallax analysis of Refsdal (1966) is expanded to focus on its ability to detect a potential cosmological abundance of compact dark matter candidates. In particular, the paper explores whether lensing parallax might lead to a useful probe of a newly discovered type of sources: GRB afterglows, by a newly reported type of lenses: MACHOs (Alcock et al., 1996).

2. The Theory and Practice of Gravitational Lensing Parallax

The amplification of a point source by a compact lens is

\[ A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}. \tag{1} \]

where \( u \) is the angular separation between the lens and source on the sky of the observer, in units of \( \theta_E \), the angular Einstein ring size of the lens. It is also well known that

\[ \theta_E = \sqrt{\frac{2R_S D_{LS}}{D_{OL} D_{OS}}}, \tag{2} \]

where \( R_S \) is the Schwarzschild radius of the lens, \( D \) is angular diameter distance, and \( O, L, \) and \( S \) subscripts designate observer, lens, and source, respectively. However, equation 1 also holds when \( u \) is taken to be projected distance in the observer’s plane, measured in terms of \( E_O \) the Einstein ring projected into the observer’s plane (Nemiroff & Gould, 1995), where

\[ E_O = \sqrt{\frac{2R_S D_{OL} D_{OS}}{D_{LS}}}. \tag{3} \]

Following Refsdal (1966), differential amplification is investigated. From equation 1 it is straightforward to show that

\[ \left| \frac{dA}{du} \right| = \frac{8}{u^2(u^2 + 4)^{3/2}}. \tag{4} \]

Suppose two observers are separated in the solar system by an amount \( \Delta u \). They will detect a lensing parallax event when the difference in the apparent luminosity each measures is greater than \( \Delta A \). Here two detection scenarios will be considered: small \( u \) events, and large \( u \) events.

Small \( u \) events corresponds to high \( A \). Equation 4 can then be approximated as

\[ \Delta A \sim \frac{dA}{du} \Delta u \sim \frac{\Delta u}{u^2}. \tag{5} \]

Therefore

\[ u^2 \sim \frac{\Delta u}{\Delta A}. \tag{6} \]
when \( u < 1 \).

Now the probability of lensing above any amplitude is given by Nemiroff (1989) for a smooth \( \Omega = 1 \) universe as

\[
P = 6 \Omega_L \Phi \Psi
\]

(7)

where \( \Phi = \sqrt{A^2/(A^2 - 1)} - 1 \), and

\[
\Psi = \left( \frac{4 + S^{1/2} \ln S - 4S^{1/2} + \ln S}{S^{1/2} - 1} \right),
\]

(8)

where \( S = (1 + z_S) \). Nemiroff (1989) also showed that \( u^2 = 2\Phi \), so that

\[
P = 3u^2 \Omega_L \Psi.
\]

Using equation 6 it is found that

\[
P \sim 3 \Omega_L \Psi \left( \frac{\Delta u}{\Delta A} \right),
\]

(9)

To translate this relation to a ratio of more intuitive values, it will be assumed that the source is at redshift \( z_S = 1 \), so that \( \Psi \sim 0.04 \) and, taking \( H_0 = 65 \text{ km sec}^{-1} \text{ Mpc}^{-1}, D_{OS} = (2c/H_0)(S^{-1} - S^{-3/2}) = 1.35 \text{ Gpc} \). It will also be assumed here that the lens is placed at \( D_{OL}/D_{LS} = 1 \), near its most probable position, and that \( \Delta u = D_{sep}/E_O \). From \( N \sim 1/P \), it is found that

\[
N \sim 3 \times 10^3 \Omega_L^{-1} \left( \frac{1 \text{ AU}}{D_{sep}} \right) \left( \frac{M_L}{M_\odot} \right)^{1/2} \left( \frac{\Delta A}{0.1} \right),
\]

(10)

where \( D_{sep} \) is the separation of satellites, and \( M_L \) is the mass of the lenses, and \( N \) is the number of objects that must be observed before one object would be expected to show a parallax lensing effect above \( \Delta A \).

Optimistically, there are scenarios involving high densities, low lens masses, and high precision measurements where detection is virtually assured. Also, universes involving high \( \Lambda \) may yield small \( N \). Although these universes might not be preferred, their existence can be tested with this method. Pessimistically, millions of simultaneous measurements of cosmological objects to moderate precision must be made with expensive spacecraft to detect a single parallax lensing effect created by the known star field, and/or a comparable density of MACHOs.

Let’s reverse the above question and ask how accurate does relative simultaneous photometry have to be to see lensing parallax with a single observation? Here it is assumed that \( u \) is large so that amplification is low, very near unity. At very large \( u \), it is clear from equation 4 that, in magnitude,

\[
\left| \frac{dA}{du} \right| \sim \frac{8}{u^5}.
\]

(11)
To find a probable measurement, set $P \sim 1$ in equation 7, substitute $\Phi = u^2/2$, solve for $u$, and substitute the result in the above equation, so that

$$\Delta A \sim 8 \frac{3^{5/2}}{\Omega_L^{5/2}} \left( \frac{D_{\text{sep}}}{E_O} \right),$$

which becomes, when $z_S = 1$ and stated in more intuitive relative quantities,

$$\Delta A \sim 10^{-5} \frac{5^{5/2}}{\Omega_L^{5/2}} \left( \frac{D_{\text{sep}}}{1 \text{ AU}} \right) \left( \frac{M_o}{M_L} \right)^{1/2}.$$  \hspace{1cm} (13)

$\Delta A$ would be about a factor of 10 more likely were the source at $z_S = 2$, and an additional factor of 3 more likely were the source at $z_S = 3$. Still, the above equation confirms that lensing parallax is practically undetectable, for present technology, for paired observations of a single source. Therefore, this method hinges on many sources being observed.

How could two separated observers best measure a small differential magnitude of a variable source? One possible route is for each observer to simultaneously monitor the same field, which includes a common set of comparison objects. Simultaneous observations should minimize the effect of target intrinsic variability. Comparison observations should minimize systematic errors of independent magnitude estimation. The precise effectiveness of this minimization will determine the usefulness of this method.

Which sources are best candidates for lensing parallax? To assess this, the angular size of a source will be estimated from its minimum variability time scale, and by assumption of a canonical distance. When the source is angularly smaller than the size of the Einstein ring of the lens, large parallax amplifications are possible. The relationship between source variability and the Schwarzschild radius of the lens is therefore given by

$$R_S \geq \frac{(c\Delta t_{\text{var}})^2 D_{OL}}{2 D_{LS} D_{OS}}.$$  \hspace{1cm} (14)

Again applying our canonical values, a more intuitive equation is

$$M_L \geq \left( \frac{\Delta t_{\text{var}}}{1 \text{ sec}} \right)^2 10^{-12.5} M_\odot.$$  \hspace{1cm} (15)

Sources with variability satisfying equation (15) make good candidates for gravitational parallax observations. The relative angular size of these objects relative to the angular Einstein-ring size of the lens makes it possible for two observers to see measurably different gravitational lens magnifications. Were the object increasing in size, differential magnification would cease when the when equations (14) and (15) were no longer valid, allowing the observers to then estimate the mass of the lens from the size and/or variability of the source.
For GRBs in the gamma-ray band, the minimum time scale of variability is unknown (Nemiroff et al., 1998), but significant fluctuation certainly exist in many GRBs above \( t_{\text{var}} \sim 0.01 \) sec. Therefore, parallax observations of GRBs could resolve, theoretically, masses above \( 10^{-16} M_\odot \) (Nemiroff & Gould, 1995). X-ray emission is also seen from GRBs over the time scale of hours, indicating that lens masses above \( 10^{-5} M_\odot \) could be resolved. Variability over the time scales of a day, a month, and a year then could theoretically resolve, through lensing parallax, lens masses greater than about \( 0.01 M_\odot \), \( 10 M_\odot \), and \( 1000 M_\odot \) respectively. Note that some QSOs and erupting supernovae are also seen to undergo variability on the time scale of a day or less, and so might make good sources for lensing parallax observations with lenses of sub-solar masses.

Could the transverse speed of the lens affect its parallax measurability? Phrased differently, the above analysis implicitly assumed that the lens, observer, and source were stationary with respect to each other, but significant relative lens motion during the observation could confound the detection of this effect. When the lens moves on order \( D_{\text{sep}} \), an amplitude change, for one observer, on order of that discussed above would occur. Projecting all observer, lens, and source motion onto transverse speed of the lens, \( v_L \), a canonical duration of this effect can be estimated as

\[
t_{\text{dur}} \sim \left( \frac{D_{\text{sep}}}{v_L} \right) \sim 3.5 \text{ days} \left( \frac{D_{\text{sep}}}{1 \text{ AU}} \right) \left( \frac{500 \text{ km/sec}}{v_L} \right),
\]

A single observer would expect lens motion to create a \( \Delta A \) on the order of that given by equation (13) over this time-scale. A measurable differential amplification between observers should persist significantly longer, however.

3. Discussion and Conclusions

Gravitational lens induced amplification patterns continually cross our solar system. For a sufficiently compact source, stellar and MACHO lenses create a literal sea of amplifications on which we are adrift. Usually the lensing waters are smooth, so that we do not perceive this sea. Even when an amplification wave comes, it is hard for us to tell it from intrinsic source variability on which it is superimposed. But two sailors adrift on this amplification sea should both measure the same intrinsic source variability, and so should be able to interpret differences in perceived brightness more clearly in terms of differential gravitational lensing.

Stated differently, this method attempts to subtract unwanted attributes from wanted attributes. If an observer determines that a source has a certain brightness, the observer does not know how much of this brightness is intrinsic, and how much is lens amplified. To subtract intrinsic brightness, separated observers are employed.
If two separated observers each determine a different brightness for the same source, the observers do not know how much of this brightness difference was caused by intrinsic variability of the source. To subtract source variability, simultaneous observations are employed.

If two simultaneous, separated observers each determine a different brightness for the same source, the observers still do not know how much of this difference was caused by systematic errors operating between the observers. To subtract systematic effects, relative photometry between the target source and nearby unlensed comparison sources is employed. It is possible that what is left is relative gravitational lens amplification.

Ideal sources for this method are bright, compact, distant sources. The more compact the source, the greater the variety of compact lenses that create a detectable effect. The brighter the source, the better the relative photometry. The more distant the source, the greater the chance a lens will fall between it and the observers.

Ideal lenses for this method have much less than a solar mass, since they create the highest amount of differential amplification across our solar system. MACHOs and/or stars from the known star field may also be detectable, depending on the accuracy of the relative photometry.

Ideal observers for this method are large, identical, highly separated observers. Large telescopes can track more sources and more accurately. The more similar the two separated observers are, the less systematic effects would likely dominate their relative photometry. The more separated the observers are, the higher the lens mass this method is sensitive to.

One possible set of satellites that might attempt measurements of this type are HST and NGST. Each would need to have identical filters to ensure that measured magnitude differences were not spectrum based. One check that measured magnitude differences are truly lens based can be done for expanding sources. When an expanding source becomes much larger than the Einstein ring of the lens, these relative magnitude differences should diminish. An independently determined expansion rate would calibrate the angular size of the Einstein ring and hence give valuable information on the mass of the lens.

4. Acknowledgements

This research was supported, in part, by grants from NASA and NSF.

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