The Tangential Velocity Estimation Algorithm for Space-borne SAR

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Abstract. This article presents a discussion of one of the important areas of application of synthetic aperture radars (SAR) – the moving target indication (MTI) mode. The principle of trajectory signal processing in MTI systems is based on the estimation of the Doppler frequency or phase shift of the echo signals due to the movement of objects. The paper describes an algorithm for estimating the tangential velocity of moving targets for the MTI system based on the use two apertures displaced along trajectory of SAR spacecraft. The structure of the MTI system based on the analysis of phase and amplitude radar images is considered. Further, the amplitude and phase characteristics of the moving target are obtained. The block diagram of the tangential velocity estimation algorithm is given. Simulation has been performed and the features of the operation of this algorithm are presented. The errors in estimating of the target tangential velocity are determined.

1. Introduction
In recent decades, Earth remote sensing systems have been intensively developing, in particular, synthetic aperture radar systems, which are characterized by high resolution and have wide capabilities [1-3]. The main areas of their application are monitoring of natural resources and emergency situations, environmental monitoring, mapping, etc.

An important feature of synthetic aperture radars (SAR) is the ability to operate in the moving target indication (MTI) mode. The main tasks of this mode are detecting moving targets, measuring their coordinates and motion parameters [3-11]. The principle of processing trajectory signals in MTI systems is based on the estimation of the additional Doppler frequency (phase) shift of the echo signals obtained due to the movement of objects [3-4, 11].

The paper proposes and investigates the tangential velocity estimating algorithm of a moving object for a moving target indication system based on the use of two apertures displaced along trajectory of the spacecraft. The algorithm is simulated for a point moving target and obtained characteristics are analyzed.

2. The tangential velocity MTI algorithm
Let us consider a tangential velocity moving targets indication algorithm based on the use of two apertures displaced by a distance \( d \) along trajectory of the synthetic aperture radar (SAR) platform. The displacement value \( d \) determines the characteristics and capabilities of the MTI system. We will consider the case of side-looking mode when the viewing angle \( \theta_v = 90^\circ \) respect to the SAR platform velocity vector \( V_0 \).
Let a point target \( O \) move with a constant tangential velocity \( V_t \) (figure 1). The expression for the trajectory signal reflected from the target in one element of the range resolution for each aperture will be written in the form [1-3, 10]

\[
s_j(t) = A_{0j} \exp \left[ i \frac{4\pi}{\lambda} \left( \frac{V_0^2 + 2V_0V_t + V_t^2}{2R_0} t^2 \right) + i \varphi_{0j} \right],
\]

where \( A_{0j} \) and \( \varphi_{0j} \) are amplitude and random phase shift of the echo signal of the \( j \)-th aperture \( (j = 1, 2) \), \( V_0 \) is orbital velocity of the SAR platform, \( R_0 \) is a slant range at \( t = 0 \), \( \lambda \) is wavelength of radiated signal.

In the system for processing the trajectory signal, the quadratic phase shift caused by the movement of the carrier is compensated. This is done by multiplying the signal by the reference function [2, 3].

The complex amplitude \( \tilde{\rho}_j \) at the output of the processing system matched with the signal of a stationary object in the synthesis interval \( [-T_s/2; T_s/2] \) is written in the form

\[
\tilde{\rho}_j = A_{0j} \exp(i\varphi_{0j}) \int_{-T_s/2}^{T_s/2} \exp \left[ i \frac{4\pi}{\lambda} \frac{V_0V_t}{R_0} t^2 \right] dt. \tag{1}
\]

Expression (1) takes into account the smallness of the term \( V_t^2 t^2 / (2R_0) \).

As follows from expression (1), information on the tangential velocity of the target can be obtained by analyzing the phase characteristic of the differential complex signal. The complex amplitude \( \rho_j \) in each channel can be represented exponentially

\[
\rho_j = |\tilde{\rho}_j| \exp(i\varphi_j),
\]

where \( |\tilde{\rho}_j| \) is amplitude of the radar image, \( \varphi_j \) is signal phase.

The algorithm for indication of targets, moving with tangential velocities, when using apertures displaced along the trajectory can be represented as block diagram (figure 2). In the figure 2 \( \Delta \varphi \) is the phase difference of the target signals, \( |\bullet| \) and \( \text{Arg}(\bullet) \) are blocks for calculating the modulus and phase of the signals, respectively [2]; \( m \) and \( n \) are sampling time; \( h_x(m,n) \) and \( h_y(m) \) are the reference functions.
When simulating the resulting MTI system [10] for S-band SAR at $V_0 = 7.61$ km/s, $\gamma = 40.2^\circ$ and spatial resolution of 2.7 m, the following target responses (figure 3) and the phase difference of the MTI system (figure 4) were obtained. The markers on these figures correspond to the initial azimuth position of the target.

**Figure 2.** Block diagram of the tangential velocity MTI system.

**Figure 3.** Amplitude responses from the simulated point target with $V_t = 20$ m/s at the output of the processing units (1) and at the output of the MTI system (2) at $d = 450$ m.

**Figure 4.** Phase difference $\Delta \phi$ in the range channel with the detected point target with $V_t = 20$ m/s at $d = 450$ m.

As shown by the modeling of the considered MTI algorithm with the tangential motion of a point target, its response on the radar image expands in azimuth and shifts by the value of the $\Delta x$ towards
the target movement. These effects are more intense with an increase in $V_t$ and SAR resolution [10].

In this case, the center of the response does not coincide with the azimuthal position of the stationary object (marked in figures 3 and 4).

The MTI system response has a deep falling in the center indicating the coincidence (in phase) of the complex signals $\rho_1$ and $\rho_2$ in this sample. The phase difference in the current range channel is linear (figure 4). In this case, the phase of the azimuthal coordinate corresponding to the initial position of the target lags behind or ahead (depending on the direction of the target movement) the phase of the response center.

The amplitude and phase characteristics of the MTI system are shown in figures 5-6.

**Figure 5.** Amplitude of the responses on the radar image (dashed line) and amplitude of the responses of the MTI system (solid line) at $d=180$ m (1), $d=450$ m (2) and $d=900$ m (3).

**Figure 6.** Phase difference at $d=180$ m (1), $d=450$ m (2) and $d=900$ m (3).

From the analysis of figure 5 it follows that with an increase in $V_t$, the amplitude of the response of the MTI system increases. Moreover, with increasing displacement $d$, the response amplitude increases with greater steepness, however, the range of an unambiguous estimate of the target tangential velocity decreases. The phase difference is proportional to both velocity $V_t$ and
displacement $d$ (figure 6). The shift $\Delta x$ in the position of the response of the moving target relative to the true target position depends only on the tangential velocity $V_t$ and does not depend on $d$ and is determined by

$$
\Delta x = \text{int}\left[ \frac{V_t T_s}{\delta_x} \right],
$$

where $\delta_x$ is SAR azimuthal resolution.

3. The tangential velocity estimation algorithm

An analysis of the above amplitude and phase characteristics of the MTI system responses and their behavior when $V_t$ changes indicate that the target tangential velocity can be estimated by analyzing the phase characteristic of the differential complex signal. However, for such a definition to be unambiguous, information about the target response width and its azimuth position is required. This information can be obtained from the radar images of the MTI system. Thus, the following algorithm for estimating the tangential velocity of a moving object can be proposed (figure 7).

![Block diagram of the tangential velocity estimation algorithm](image)

**Figure 7.** Block diagram of the tangential velocity estimation algorithm.

The block diagram contains $k$ channels of signal weight processing each of which is configured to detect the responses of moving targets with a certain tangential velocity. Suppose, if the channel is configured to detect the MTI system response $\rho_0$ created by an target that moves with tangential velocity $V_t = 10$ m/s (let’s call such channel K.10) then the amplitude $A$ at its output will be maximum in the presence of response $\rho_0$. The Max block implements the algorithm for finding the maximum value.

Weighting is performed using window functions $a_k W_k(n)$, where $a_k$ is a scaling factor. The shape and duration of the window functions is determined by the type of the MTI system response at a particular tangential velocity value. In the simplest case, the function $W_k(n)$ can consist of two rectangular pulses with the duration of $t_w = (N_w - n_w)/2$ samples (figure 8).

Weight processing is performed for each range line of the MTI system output $|\hat{\rho}(n)|$. The need to input signal normalization is caused by the dependence of the MTI system response amplitude on the target tangential velocity (figure 5). Thus, the described operation algorithm can be expressed by

$$
\max(A_k(n)) = \max \left\{ \sum_{n=1}^{N_w} |\hat{\rho}(n)| \cdot a_k W_k(n) \right\}.
$$
To select the $n_w$ and $N_w$ parameters of the window function, compare it with the response of the MTI system (figure 9). Obviously, the windowing pulses must match the maxima of the MTI system response. Three variants of window functions were investigated in the paper: function number 1 is function designated as $W_{0.5}$ in which the parameters $n_w$ and $N_w$ correspond to half the response magnitude; function number 2 is function $W_{0.7}$ in which these parameters correspond to the magnitude values of the response at a level of 0.7 from its maximum; function number 3 is function $W_n$, the parameter $N_w$ of which corresponds to the width of the response at a level of 0.7 from the maximum and the parameter $n_w$ is determined by the azimuthal position of the response of the moving target (in figure 9, the position is indicated by a marker).

From the figure 9 it follows that in the SAR implementation, the specific values of $n_w$ and $N_w$ of the window function will be determined by the parameters of the survey. As studies have shown, the $N_w(V_t)$ and $t_w(V_t)$ are almost linear functions of the tangential velocity and can be calculated with some error from two values which were obtained, for example, at the stage of testing and tuning the SAR system.

**Figure 8.** Window function $W_n(n)$ consisting of two rectangular pulses.

**Figure 9.** Explanation of selecting parameters of the window function.
The results of simulation of the weight processing unit operation for two window functions \((W_{0.5}(n)\) and \(W_{о}(n)\)) are shown in figure 10. This figure shows the responses at the output of the K.10 channel in the presence of a point moving target with \(V_t = 10\) m/s. It follows from the figure 10 that the response formed in the channel is symmetric and contains three humps: the central one with the maximum magnitude and two lateral ones, the values of which are almost two times less than the central one. This type of response is due to the shape of the windowing and MTI system response pulses. It can also be noted that an increase in the duration \(t_w\) of the impulses of window functions leads to a proportional increase in the magnitude (energy) of the response at the output of the weight processing channel.

Thus, the channel of weight processing K.\(k\) at the output of which the response has the greatest value will indicate the magnitude of the target tangential velocity \(V_k\). The maximum absolute estimation error \(\hat{V}_K\) equal to half the tuning step of adjacent channels \(\Delta\hat{V}_K\) will occur if the true tangential velocity of the target has the value 

\[V_t = \Delta\hat{V}_K/2 = (\hat{V}_{K,k} + \hat{V}_{K,(k+1)})/2.\]

Figure 10. Responses at the output of a signal weight processing unit for window functions \(W_{0.5}(n)\) (1) and \(W_{о}(n)\) (2).

In the Velocity estimation block (figure 7), the tangential velocity of the target is calculated based on the phase characteristics of the MTI system response using the expression:

\[
\hat{V}_t(n_0) = \frac{\Delta\varphi(n_0 + n_w/2) - \Delta\varphi(n_0 - n_w/2)}{2 \cdot b} = \\
= \frac{1}{2 \cdot b} \left[ \arctg \left[ \frac{\text{Im}\{\hat{\rho}_2(n_0 + n_w/2)\}}{\text{Re}\{\hat{\rho}_2(n_0 + n_w/2)\}} \right] - \arctg \left[ \frac{\text{Im}\{\hat{\rho}_1(n_0 + n_w/2)\}}{\text{Re}\{\hat{\rho}_1(n_0 + n_w/2)\}} \right] \right] - \\
- \arctg \left[ \frac{\text{Im}\{\hat{\rho}_2(n_0 - n_w/2)\}}{\text{Re}\{\hat{\rho}_2(n_0 - n_w/2)\}} \right] + \arctg \left[ \frac{\text{Im}\{\hat{\rho}_1(n_0 - n_w/2)\}}{\text{Re}\{\hat{\rho}_1(n_0 - n_w/2)\}} \right],
\]

where \(\Delta\varphi\) is the phase difference between the images \(\hat{\rho}_1\) and \(\hat{\rho}_2\) at points \((n_0 + n_w/2)\) and \((n_0 - n_w/2)\), \(n_0\) is azimuth coordinate corresponding to the center of the response, \(b\) is coefficient of proportionality depending on the value of the apertures displacement (base) \(d\) (see figure 1).

To find the error

\[
\Delta V = \|\hat{V}_t - V_t\|
\]
in estimating the tangential velocity $\hat{V}_t$ of the target relative to the true value $V_t$ we simulate the algorithm for various window functions $W_\alpha(n)$ under the condition $V_t = \hat{V}_{K,\alpha}$. The simulation results are shown in figure 11. It follows from the figure 11 that the minimum estimation error which for the considered survey parameters is equal $|\Delta \nu| < 5\%$ is ensured by the use of the window function $W_\alpha(n)$. The minimum value of the error is due to the discrete nature of radar images and window functions.

![Figure 11](image.png)

**Figure 11.** Errors in the target tangential velocity estimating when using various window functions.

Another task in the implementation of the algorithm for estimating the tangential velocity of the target (except for calculating the value $\hat{V}_t$) is to determine the direction of its movement. Studies have shown that for this purpose, the real $\text{Re}[\rho(m,n)]$ and imaginary $\text{Im}[\rho(m,n)]$ parts of the difference signal of the MTI system can be used (figure 12).

As can be seen from the figure 12, when the target moves along the spacecraft trajectory (figure 12a) the real and imaginary parts of the response at the points with the azimuthal coordinates of the $(n_0 + n_w/2) = 4961$ and $(n_0 - n_w/2) = 4936$ coincide in sign. If a point target moves in the direction opposite to the spacecraft movement (figure 12b) then the real and imaginary parts at the corresponding points have different signs. The markers in the figure 12 correspond to the initial azimuth position of the target.

5. **Conclusion**

The paper proposes an algorithm for estimating the tangential velocity of a moving target for the MTI system based on the use of two apertures offset displaced along the trajectory of the SAR platform. The algorithm involves weighting processing the output signal of the MTI system and the target tangential velocity calculating based on the phase characteristics. Weighting is performed using a window function which in the simplest case can consist of two rectangular pulses.

The proposed algorithm makes it possible to estimate the velocity and determine the direction of the tangential movement of the target.

The studies of the algorithm showed that the error in estimating the tangential velocity when using a rectangular window function for the considered survey parameters is less than 5%.
Figure 12. Real and imaginary parts of the response at the output of the MTI system in the presence of a point moving target with $V_t = 20$ m/s (a) and $V_t = -20$ m/s (b).

References
[1] Antonov I K, Detkov A N, Nitsak D A, Tonkikh A N and Tsvetkov O E 2021 Aerial reconnaissance. Automated decoding of radar images, ed I K Antonov (Moscow: Radiotechnika) p 296
[2] Shkolny N E et al 2008 Radar systems of aerial reconnaissance, decoding of radar images, ed N E Shkolny (Moscow: Air Force Engineering Academy named after Professor N.E. Zhukovsky) p 531
[3] Verba V S, Neronskiy L B, Osipov I G and Turuk V E 2010 Space-borne Earth Surveillance Radar Systems, ed V S Verba (Moscow: Radiotechnika) p 680
[4] Fedosov V P and Kalinovskiy P Yu 2006 Radiotechnika 2 86
[5] Xu J, Huang Z, Yan L, Zhou X, Zhang F and Long T 2016 Sensors (Basel) Oct 12; 16(10): 1676
[6] Wang X, Wang R, Li N and Zhou Ch 2016 IEEE Int Geoscience and Remote Sensing Symposium (IGARSS Beijing China 10.07 – 15.07) p 6819
[7] Kim H-S, Goodman N A, Lee C K and Yang S-I 2017 Electronics Letters 53 13 879
[8] Gao H, Li J and Zhang L 2013 Progress In Electromagnetics Research Symposium Proceedings (Taipei March 25-28) 1003
[9] Fedosov V P and Kevtun D G 2016 Izvestiya SFedU 3 (176) 43
[10] Kostrov V V, Tolstov E F and Khramov K K 2020 Modern problems of remote sensing, radar, wave propagation and diffraction (MPRSRWPD Murom Russia 23-25 June) p 1
[11] Tatarskiy B G and Yasentsev D A 2019 Radiotechnika 10(15) 74