The planar perfect lens: physical requirements and possible alternative realizations

Stanislav Maslovski, Sergei Tretyakov
Radio Laboratory, SMARAD, Helsinki University of Technology
P.O. Box 3000, FIN-02015 HUT, Finland
E-mails: stanislav.maslovski@hut.fi, sergei.tretyakov@hut.fi

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Abstract

Several alternative possibilities of how to create an electromagnetic device being able to reconstruct near-field distribution of a source with sub-wavelength resolution (so-called perfect lens) are considered. It is shown that there is a variety of such means not involving double-negative (left-handed, or Veselago) materials or periodical backward-wave structures. It is demonstrated that devices working in a similar manner can be constructed using planar grids or material sheets imposing necessary boundary conditions at two parallel planes in air.

1 Introduction

It is known that using materials with simultaneously negative permittivity and permeability (at a given frequency) focusing of divergent homocentric electromagnetic beams by “planar lenses” becomes possible because of the negative refraction [1]. As it was found in [2], such lenses are also able to “amplify” evanescent fields carrying information about sub-wavelength details of a source near field. There were also a few experimental papers [3, 4] demonstrating negative refraction effects in microwave composite materials. These facts received a lot of attention (and criticism) in the recent literature (see, e.g., [5, 6]). The “amplification” of evanescent waves in a Veselago’s slab lens is a phenomenon that easily contradicts with intuition and common sense, especially for those who are used to associate the word “amplification” with an active device that amplifies the signal power. As it was recently shown in [7, 8], this “amplification” is simply a resonant excitation of waveguide modes of a slab waveguide filled by the Veselago medium. Misunderstanding of the nature of this phenomenon comes, in our opinion, from a not quite appropriate and clear terminology used in the first papers on this subject. The terms “resonance” or “resonant growth” are probably more appropriate for this phenomenon.

In this paper, we will not discuss problems and difficulties in theoretical interpretation and practical realization of backward-wave materials and the perfect lens based on
these media. Only briefly, we note that from our point of view, the physical existence of the negative refraction phenomenon as well as the evanescent field “amplification” phenomenon is beyond any theoretical doubt today. Still, there maybe some questions regarding the accuracy of the experimental approach used in [4] and the degree of the performance deterioration because of inevitable losses and the finite size of the lens, but at least theoretically the problem may be considered now as well-understood. Indeed, the causality question mentioned in [5] has been solved in [9]. The negative influence of the medium dispersion and losses discussed in [6] can, in principle, be overcome by using metamaterials involving active devices [10].

However, these difficulties call for a study of alternative realizations of a device that can restore near fields. Thus, despite the fact that there is now a direct and in principle well-understood way to realizing new lenses by means of backward-wave materials, we will make a step aside in this paper. Starting from the analysis of an ideal Veselago slab lens, we will formulate a couple of equivalent problems and show that the special electromagnetic property of the material filling, namely, existence of backward waves, is not, in fact, crucial for a planar device operating as a lens or, moreover, as a perfect lens. What makes a Veselago lens to behave as a perfect lens, are the properties of the two slab interfaces. We will show that devices working in a similar manner can be constructed using planar grids or material sheets imposing necessary boundary conditions at two parallel planes in air.

2 The ideal Veselago lens and an equivalent problem

Let us start from considering an ideal Veselago slab lens operation. Its well-known structure is depicted in Figure 1. We work in the frequency domain, and the time dependence is of the form $e^{+j\omega t}$. We suppose that the lens is positioned in space with the relative permittivity and permeability equal to 1. The corresponding relative parameters of the slab material both equal to $-1$ at the working frequency. The boundary conditions at the lens interfaces are the usual Maxwellian boundary conditions (the tangential components of the fields are continuous across the interfaces). The corresponding field equations are also shown in Figure 1 for all three regions.

$$
\begin{align*}
\nabla \times E_1 &= -j\omega \mu_0 H_1 \\
\nabla \times H_1 &= j\omega \epsilon_0 E_1 \\
air &
\end{align*}
\begin{align*}
\nabla \times E_2 &= j\omega \mu_0 H_2 \\
\nabla \times H_2 &= -j\omega \epsilon_0 E_2 \\
\text{Veselago medium} &
\end{align*}
\begin{align*}
\nabla \times E_3 &= -j\omega \mu_0 H_3 \\
\nabla \times H_3 &= j\omega \epsilon_0 E_3 \\
air &
\end{align*}

Figure 1: Veselago slab lens: a planar slab of a backward-wave material with the medium parameters $\epsilon = -\epsilon_0$ and $\mu = -\mu_0$ in free space.

It is easy to notice that the equations in region 2 differ from that in regions 1 and 3 only by complex conjugation. Substitution

$E_{(\text{old})}, H_{(\text{old})} \Rightarrow E_{(\text{new})}^*, H_{(\text{new})}^*$ (1)
(here and thereafter * denotes the complex conjugation operation) into the field equations in region 2 results in a new but equivalent problem written for new field vectors, as it is shown in Figure 2.

\[
\begin{array}{ccc}
\text{region 1} & | & \text{region 2} & | & \text{region 3} \\
E_{t_1} & | & E_{t_2}^* & | & E_{t_3}^* \\
\epsilon_{r_1} = 1 & | & \epsilon_{r_2} = 1 & | & \epsilon_{r_3} = 1 \\
\mu_{r_1} = 1 & | & \mu_{r_2} = 1 & | & \mu_{r_3} = 1 \\
H_{t_1} & | & H_{t_2}^* & | & H_{t_3}^* \\
\text{air} & | & \text{air} & | & \text{air} \\
\end{array}
\]

Figure 2: Two conjugating planes in free space. This system is equivalent to that shown in Figure 1.

In the new formulation the field equations are the same in all three regions, and they are simply the Maxwell equations in free space:

\[
\nabla \times \mathbf{E} = -j\omega \mu_0 \mathbf{H}, \quad \nabla \times \mathbf{H} = j\omega \epsilon_0 \mathbf{E} \tag{2}
\]

The boundary conditions on the two interfaces, however, are no more the standard continuity conditions, but they involve complex conjugation:

\[
E_{t_{(1,3)}} = E_{t_{(2)}}, \quad H_{t_{(1,3)}} = H_{t_{(2)}}^* \tag{3}
\]

Let us discuss the physics of these boundary conditions involving complex conjugation a bit later. At this stage we see that an ideal Veselago slab refraction problem is mathematically equivalent to the refraction at two conjugating planes in free space. Hence, for the system of two such planes in free space the field solutions are the same as for a Veselago material slab: propagating plane waves are refracted negatively, and the evanescent modes are “amplified”, which are, obviously, the conditions for a perfect lens.

To understand how these conjugating planes operate consider a plane wave incidence problem for a single conjugating plane. The problem geometry for the TM incidence is shown in Figure 3. The incident wave comes from the left. The magnetic field is orthogonal to the picture plane and is not shown. Because of the specific nature of the boundary conditions (3), the solution for the transmitted wave must depend on the tangential coordinate \(x\) as \(e^{+j k_t x}\) (\(k_t\) denotes the tangential component of the wave vector), if the incident wave phasor is \(e^{-j k_t x}\), otherwise one cannot satisfy the boundary conditions. We can say that \(k_t\) changes sign when a plane wave passes across the interface. In this process, the transmitted field energy in the second region must propagate to the right. This determines the direction of the Poynting vector normal component \(S_n\). The second region is now an air region that means the normal component of the wave vector in the second region is along the same direction as \(S_n\). A similar relation holds for \(k_t, S_t\) pair (see the picture). Clearly, negative refraction takes place.

It is possible to write simple equations for the complex field amplitudes of the reflected and transmitted fields. If \(A, B, \) and \(C\) denote the amplitudes of the incident, transmitted, and reflected wave electric field tangential components, respectively, then for the
considered TM case one can write:

\[
A + C = B^* \quad \frac{(A - C)}{\eta} = B^*/\eta^* 
\]

(4)

Here \( \eta = \eta(k_t) \) is the wave impedance connecting tangential components of electric and magnetic fields of a wave. It is obviously a function of \( k_t \). For the propagating modes \( \eta \) is purely real, for the evanescent ones it is purely imaginary. The solution of (4) is

\[
C = \frac{1 - \eta/\eta^*}{1 + \eta/\eta^*} A, \quad B = \frac{2A^*}{1 + \eta^*/\eta} 
\]

(5)

It is seen that for propagating modes (real wave impedance) the interface is perfectly matched: reflected field amplitude \( C = 0 \), and the transmitted field amplitude \( B = A^* \). For evanescent modes (imaginary wave impedance) the denominator of (5) is zero, and a surface wave (surface polariton) resonance occurs: \( B, C \to \infty \). As is known [7, 10], an interface between free space and a backward-wave material with \( \epsilon = -\epsilon_0 \) and \( \mu = -\mu_0 \) has similar properties. Another interesting feature of the obtained result is that it is impossible to introduce the usual transmission coefficient because the transmitted field is proportional to \( A^* \), but not directly to \( A \). This is because the boundary conditions (3) are no more linear in the sense of multiplication by a complex number.

An important conclusion from this analysis is that all the phenomena necessary for perfect reconstruction of the entire wave spectrum take place at the two interfaces of the Veselago slab. If one can realize a sheet such that traveling waves refract negatively when crossing this sheet, a system of two such sheets in free space will focus propagating modes of a source just like a Veselago slab. If this sheet also supports surface waves for any \( k_t > k_0 = \omega/\sqrt{\epsilon_0\mu_0} \), then two such sheets in free space will reconstruct the entire evanescent spectrum as well.
3 Physical restrictions

Let us now discuss the physical meaning of the boundary conditions (3) and the restrictions on their physical realization. Complex conjugation in the frequency domain corresponds to time reversal in the time domain. If the boundary conditions (3) were true for every spectral frequency component in the range \((-\infty, +\infty)\), then this condition would obviously violate the causality principle. Speaking in simple words, one could in this case say that transformation (1) forces the time to go in the opposite directions inside sub-regions of a single physical system. But if we are interested only in steady-state operations at a given frequency, then the complex conjugation becomes possible, at least using nonlinear or active devices, e.g., mixers. Conceptually, if a thin sheet of a nonlinear material is illuminated by a signal plane wave with the harmonic time dependence \(\cos(\omega_0 t + \phi)\) and a reference plane wave \(\cos(2\omega_0 t)\), among the output harmonics there is a plane wave \(\cos(\omega_0 t - \phi)\), which corresponds to the complex conjugation of the original field.

Another important point is the following. Applying transformation (1) and concluding that the middle region has become a true free-space region we have silently assumed that all electromagnetic quantities and relations have been kept in their original free-space form after such a transformation. Let us consider an illustrative example. A simpler idea of how to transform the field equations of an ideal Veselago slab to free-space equations is changing the sign of the electric (or magnetic) field in the Maxwell equations. The (sourceless) field equations take the necessary form after such transformation, but the problem is that the Poynting vector \(S = \text{Re}(E \times H^\ast)\) in the old variables becomes \(S = -\text{Re}(E \times H^\ast)\) in the new ones. From the other hand, it is easy to see that substitution (1) preserves the Poynting vector expression in the original form. The physical reason of this is, of course, the time reversal invariance of a reciprocal electromagnetic system.

4 Possible realizations not involving complex conjugation of the fields

In the previous section we have considered a potential device that is able to ideally imitate the operation of a Veselago slab lens. Now we will concentrate on other possibilities providing additional freedom in realization of sub-wavelength resolution lenses without involving the complex conjugation operator. In this section we will make use of a powerful synthesis method based on so-called transmission matrices, known in the microwave circuit theory. These matrices connect the complex amplitudes of waves traveling in the opposite directions and measured at two reference planes:

\[
\begin{pmatrix}
E_2^- \\
E_2^+
\end{pmatrix}
= 
\begin{pmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{pmatrix}
\cdot 
\begin{pmatrix}
E_1^- \\
E_1^+
\end{pmatrix}
\tag{6}
\]

Here, \(E_1^\pm\) and \(E_2^\pm\) denote the tangential components of the electric field complex amplitudes of waves at the first (input) and the second (output) interfaces of a device, respectively (we restrict ourselves by plane structures and plane waves). The signs \(\pm\) correspond to the signs in the propagator exponents \(e^{\pm jk_0 z}\) of these waves, and \(z\) is the
axis orthogonal to the interfaces. It is known that the T-matrix of a serial connection of several devices described by their T-matrices is simply a multiplication of the matrices in the order determined by the connection.

The total transmission matrix from the source plane to the plane where the source field distribution is ideally reconstructed must be the identity matrix

\[ T_{\text{total}} = T_{\text{space after}} \cdot T_{\text{device}} \cdot T_{\text{space before}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \] (7)

for every spatial harmonic of the source field. Here, \( T_{\text{space before}} \) and \( T_{\text{space after}} \) represent air layers occupying the space between the source plane and the device, and the space between the device and the image plane. \( T_{\text{device}} \) is the transmission matrix of our device. From this formula it is obvious that a complete reconstruction of the field distribution in the source plane at a distant image plane must involve phase compensation for the propagating space harmonics and “amplification” for the evanescent ones. In other words, we need to synthesize a device that somehow inverts the action of a free space layer.

Condition (7) is a strict condition requiring not only the device one-way transmission to be such that it reconstructs the source field picture at the image plane, but also the matching to be ideal and the device operation to be symmetric (reversible in the optical sense). We will consider some less strict conditions a bit later. The matrices presented in (7) can be written in explicit forms. A space layer of thickness \( d/2 \) has the T-matrix

\[ T_{\text{space}} = \begin{pmatrix} e^{-jk_n d/2} & 0 \\ 0 & e^{jk_n d/2} \end{pmatrix} \] (8)

To compensate the action of two such layers before and after the device, the device T-matrix \( T_{\text{device}} \) has to be, obviously, the inverse of the transmission matrix of these space layers:

\[ T_{\text{device}} = \begin{pmatrix} e^{jk_n d} & 0 \\ 0 & e^{-jk_n d} \end{pmatrix} \] (9)

There are at least two solutions known for this idealistic case by now: an ideal Veselago slab lens and a system of two conjugating planes discussed above\(^1\). Are other solutions possible? Let us consider a device that is a combination of two “field transformers” (e.g., this sheets of certain electromagnetic properties) separated by a layer of free space. In other words, we want to study if we can replace conjugating planes by some other layers, hopefully more easily realizable. This device is modeled by the transmission matrix

\[ T_{\text{device}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e^{-jk_n d} & 0 \\ 0 & e^{jk_n d} \end{pmatrix} \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix} \] (10)

Here, the first and the last matrices with yet unknown components describe the two layers forming the device. It is easy to show that if \( a = d = 0, \ e = h = 0 \) and \( bg = cf = 1 \) then the total device T-matrix takes form (9), i.e., the necessary matrix of an ideal lens.

\(^1\)Perhaps, it is not easy to see directly why (9) holds for a pair of conjugating planes, since there is only the normal component of the wave vector in (9) which is kept untouched by (3). But considering conjugations at both planes together and taking the inner space partial wave propagators \( e^{\pm jk_n d} \) into account, equation (9) can be easily obtained.
The next question is whether there is a way to realize a layer with the transmission matrix of the form

\[ T_{\text{trans}} = \begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix} \]  

(11)

A person with experience in microwave engineering would probably say that such T-matrix can never be achieved in a physical device, because it corresponds to a scattering matrix (well-known S-matrix) having all components being infinite. However, a more careful investigation of this question will lead us to an important result presented in the next section.

5 The use of impedance grids

Here, we will show that under certain restrictions devices that “amplify” evanescent fields can be designed using only passive elements, if we do not demand that the same device reconstructs the propagating part of the spectrum. This is possible because the two main phenomena on the two interfaces in a perfect lens (negative refraction, necessary to focus propagating modes, and surface polariton resonance, necessary to reconstruct the evanescent spectrum) are fundamentally different, and the required properties of such sheets are not necessarily combined in a single design.

Let us consider a simple system: a lossless isotropic grid, e.g., a conductive wire mesh. If the grid induced current is only electric current, and there is no effective magnetic current induced in the grid (e.g., when the grid structure is completely planar), then the grid reflection coefficient \( R \) and transmission coefficient \( T \) at the grid plane are connected as

\[ T = 1 + R \]  

(12)

provided that they are defined through the electric field tangential components. The corresponding T-matrix of such a grid is

\[ T_g = \begin{pmatrix} 1 + 2R & R \\ 1 + R & 1 + R \end{pmatrix} \begin{pmatrix} 1 \\ -R \end{pmatrix} \begin{pmatrix} 1 + R \\ 1 + R \end{pmatrix} \]  

(13)

It is possible to make grids supporting propagation of surface modes (also known as slow waves in radio engineering). For wire meshes, for example, this phenomenon was investigated in [11]. If the tangential component of the wave vector of an incident wave coincides with the propagation factor of a surface mode, the surface mode resonance appears. Obviously, the incident wave should be evanescent in this case to match with the propagation constant of the surface mode. At a surface mode resonance \( R \to \infty \) (for evanescent modes \( R \) is not bounded by \( |R| \leq 1 \)). Then, the grid T-matrix takes the form

\[ T_g = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \]  

(14)

It is almost of the necessary form (11). Remembering that conditions (7) and (9) are too strict in many cases, let us calculate the total device matrix using (14) directly. We
obtain

\[ T_{\text{device}} = T_g \cdot T_{\text{space}} \cdot T_g = \begin{pmatrix} 4e^{-jknd} - e^{jknd} & 2e^{-jknd} \\ -2e^{-jknd} & -e^{-jknd} \end{pmatrix} \]  

(15)

which seems to be far from the desired, but let us check what is the device S-matrix. After some algebra involving the known relations connecting the elements of T and S matrices:

\[ S = \begin{pmatrix} -t_{21}/t_{22} & 1/t_{22} \\ t_{11} - t_{12}t_{21}/t_{22} & t_{12}/t_{22} \end{pmatrix} \]  

(16)

we get

\[ S_{\text{device}} = \begin{pmatrix} -2 & -e^{+jknd} \\ -e^{-jknd} & -2 \end{pmatrix} \]  

(17)

Notice pluses in the exponents for \( s_{12}, s_{21} \). They mean that the device can “amplify” resonating evanescent modes. This is yet another indication of the fact that Pendry’s amplification [2] means the resonance growth in a surface mode resonance. The device based on simple grids considered above is not a lens, though. It cannot focus propagating modes, as equation (17) holds only at surface mode resonances \( (R = \infty) \). Another “imperfectness” of the found realization is that there is no ideal matching in this case. The incident resonant evanescent mode reflects from the device with the coefficient \(-2\).

The matrix components of (17) can lead to various possible design conditions, less strict than (7) or (9). For example, a direct analogy gives the following form of the device S-matrix

\[ S_{\text{device}} \propto \begin{pmatrix} r & e^{+jknd} \\ e^{-jknd} & r \end{pmatrix} \]  

(18)

for a symmetric reciprocal device. The corresponding T-matrix for this case reads

\[ T_{\text{device}} \propto \begin{pmatrix} e^{jknd} - r^2e^{-jknd} & r e^{-jknd} \\ -r e^{-jknd} & e^{-jknd} \end{pmatrix} \]  

(19)

There are various other possibilities involving asymmetric and nonreciprocal devices.

6 Conclusions

In this paper, general requirements and possibilities for realization of planar lenses being able to reconstruct near-field distributions of a source with sub-wavelength resolution (so-called perfect lenses) have been considered. Based on the observation that the phenomena at the two boundary surfaces of a slab lens are more critical for a lens operation than the propagation phenomena inside the lens material, we have arrived to the following main conclusions.

One does not necessarily need a composite medium possessing negative \( \varepsilon \) and \( \mu \) or another kind of backward-wave medium to produce a perfect lens. An analogous device can be constructed using various other possibilities. So far, two general possibilities have been considered. One is based on usage of two parallel artificially made surfaces or sheets imposing boundary conditions of form (3) on fields in free space. The realizability of such a device is not forbidden (at least for single-frequency, or steady-state, operations)
by physical laws and we are looking forward for using nonlinear (or active) materials to achieve this purpose. Operating as a planar lens, the device is able to focus propagating modes of a source providing in the same time an “amplification” of the evanescent modes, i.e. it is able to work as a perfect sub-wavelength resolution imaging device.

The second possibility lies in a wide class of planar structures supporting slow waves. The surface mode (polariton) resonance occurs in such structures when the incident field vector tangential component coincides with the propagation factor of a slow wave. It happens if the incident wave is an evanescent wave in free space. We have shown that using surface mode resonances in lossless grids placed in air it is possible to achieve “amplification” of evanescent modes, like in the case of Veselago’s slab lens. This phenomenon can be used not only for making optical or electromagnetic imaging devices more precise, but also for applications where the surface mode resonance allows to detect small field irregularities in a near field of an object.

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