SMALL SATELLITE OPTICAL COMMUNICATION NETWORKS: ANALYTICAL MODELS

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ABSTRACT. Small satellites, especially picosatellites, appear poised to play an important role in the future of space systems. Due to their size, however, integrating them with high-throughput laser-based communication systems remains a challenge. In this paper, we develop several analytical models that quantify how optical communication networks can be implemented with picosatellites. We do so with the goal of identifying design challenges that need to be addressed if picosatellites and optical communication systems are indeed to be a standard part of the future space technological landscape.

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1. INTRODUCTION

It appears that small satellite technology, and picosatellites in particular, will be a significant part of space system architectures in coming years. Architectures reliant upon so-called monolithic (or, more descriptively, aggregated) spacecraft in which a broad range of highly-performing subsystems are brought together into a complex, consolidated system are expensive (on the order of hundreds of millions to billions of dollars) and require significant development times. Small satellites, on the other hand, with dimensions on the orders of tens of centimeters typically have power requirements far lower than aggregated satellites. Small satellites that rely on off-the-shelf components or have well-defined form factors that make them amenable to modularity offer significant financial savings, although at the cost of technical capability. For instance, a remote sensing system mounted on a CubeSat is expected to have significantly lower performance (measured in terms of, say,
image stability, detector sensitivity, or on-board processing capability) than its counterpart on a Flagship-class or even a Discovery-class mission.

The tradeoff between the two types of spacecraft in terms of technical capability can in part be mitigated by the idea of “power in numbers”, in which a larger number of small satellites can be deployed and therefore make up for some of the lost signal. For instance, if the desired performance in terms of signal to noise ratio \( \text{SNR} \) is \( \text{SNR}^* \), but the actual performance is \( \text{SNR}_0 \), then the number \( N \) of deployed small satellites would be \( N \sim (\text{SNR}^*/\text{SNR}_0)^2 \). This strategy would of course only be practical if, for instance, the cost of a single small satellite was at most \( 1/N \) the cost of its aggregated counterpart.

The strategy of regaining technical performance in numbers has an important corollary: disaggregated systems enable a system to be more resilient.

Here, “resilient” is taken to mean that the marginal utility lost in a system due to a particular element being compromised in some way is such that the system is still able to retain its functionality up to a specified level. This is a natural feature of architectures in which capabilities are parallelized and disaggregated. These characteristics are obviously not desirable in every space system, but they lend themselves to missions where high throughput and persistent coverage is necessary (such as surveillance applications) and in which there are a large number of potential targets (e.g. asteroid characterization).

While small satellites (and especially picosatellites) offer many potential benefits, some of the factors that make them promising for wide-scale application in the future also make them challenging to implement. The greatest driver is their small size. If for simplicity we assume small satellites and their monolithic counterparts have a similar volumetric density, then the moment of inertia of the spacecraft scales as \( d^5 \), where \( d \) is a typical length dimension of the spacecraft. A CubeSat may have a length scale of order ten times smaller than a monolithic spacecraft, requiring the control torque to be controlled to a precision \( 10^{-5} \) it would be on a larger spacecraft. This difficulty has led to the development of multi-stage attitude control systems for CubeSats developed for precise photometric applications.

Another issue with widespread small satellite application is communication. Traditional satellite communication architectures rely on wavelengths of the same order of a small satellite’s size, \( \sim 1 – 10 \text{ cm} \). Communicating with small satellites using traditional communications systems is therefore a challenge, as the size of transmitting and receiving antennae must be of the same order of size as the signal wavelength and hence of similar size to the spacecraft itself.

While strategies to mitigate this issue have been proposed [Babuscia et al., 2013], this latter challenge has also led to proposals to develop optical-communication based small satellite networks. In particular, recent work has focused on developing architectures and carrying out testbed demonstrations of CubeSat optical communication networks [Morgan, 2017]. The purpose of this paper is to develop a simple analytical model for the mutual acquisition of satellites communicating by optical beacons and to propose a framework for the design of optical beacon networks comprising many elements and perhaps spanning interplanetary distances.

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\[\text{SNR} \sim (\text{SNR}^*/\text{SNR}_0)^2\]  

1This assumes integration times for all satellites are the same, and that the signal received has Poisson-type noise characteristics.
2. Characterizing Optical Links

Figure 1. Overview of lasercom geometry.

For optical terminals to communicate with one another, they must first acquire one another and then maintain mutual acquisition in order to transfer data. Given picosat inertial characteristics, acquisition over large distances is challenging. In this section, we calculate the probability of acquisition based on pointing error.

The geometry of the problem is summarized in Fig. 1 wherein we have two spacecraft (optical terminals) identified by indices $i$ and $j$ and separated by a distance $d_{ij}$. Assume each spacecraft’s optical emitter is a Gaussian beam. Then the power $P_{0,i}$ emitted by the $i$th spacecraft is attenuated as a function of the mutual distance $d_{ij}$ between the two spacecraft due to diffraction. Explicitly, the flux density profile of a monochromatic Gaussian beam of wavelength $\lambda_i$ is given by

$$F_i(d_{ij}) = P_{0,i} \frac{\pi w_0^2}{d_{ij}^2 \lambda_i^2} \exp \left[ -\frac{(\Delta \vartheta_i)^2}{2\sigma_{\vartheta_i}^2} \right],$$

where $\Delta \vartheta_i$ is the angular displacement from the beam’s center line, $\sigma_{\vartheta_i}$ is the spreading factor due to diffraction, and $w_{0,i}$ is the beam diameter at the emission point. The diffraction...
spreading factor is on the order of $\sim \lambda_i/w_{0,i}$; for a Gaussian beam it is

$$\sigma_{\vartheta_i} = \frac{\lambda_i}{2\pi w_{0,i}}.$$  

(2)

The beam flux can be rewritten in a more illuminating form:

$$F_i(d_{ij}) = \frac{P_{0,i}}{4\pi w_{0,i}^2} \left[ \frac{w_{0,i}}{w(d_{ij})} \right]^2 \exp \left[ -\frac{(\Delta \vartheta_i)^2}{2\sigma_{\vartheta_i}^2} \right],$$

(3)

where $w$ is the beam width as a function of distance $d_{ij}$ from the emitter. At distances more than $\sim w_{0,i}$ from the emitter, it is given by $w \approx \sigma_{\vartheta_i} d_{ij} = \lambda_i d_{ij}/(2\pi w_{0,i})$. The equation above reflects the conservation of energy via the standard $\sim 1/d_{ij}^2$ rule: the prefactor $4/(\pi w_{0,i}^2)$ is the beam area at emission, and the flux is attenuated by a factor $(w_0/w)^2$ over a distance $d_{ij}$. To recover the power received, we multiply by the detector area $A_j$ at the receiver: $F_i(d_{ij}) A_j$.

2.1. The Optical Beacon Link Budget. The total power received can be written in the following form (see also Marshall and Burk [1986]):

$$P_{Rx} = P_0 \left( \frac{\lambda^2}{4\pi d_{ij}^2} \right) \left( \frac{\pi^2 w_{0,i}^2}{\lambda^2} \right) \left( \frac{4A_{det}}{\lambda^2} \right) \exp \left[ -\frac{(\Delta \vartheta_i)^2}{2\sigma_{\vartheta_i}^2} \right],$$

(4)

where

$$\frac{\lambda^2}{4\pi d_{ij}^2} \equiv L_s \text{ (space loss)},$$

(5)

$$\frac{\pi^2 w_{0,i}^2}{\lambda^2} \equiv G_{Tx} \text{ (Tx gain)},$$

(6)

$$\frac{4A_{det}}{\lambda^2} \equiv G_{Rx} \text{ (Rx gain)},$$

(7)

$$\exp \left[ -\frac{(\Delta \vartheta_i)^2}{2\sigma_{\vartheta_i}^2} \right] \equiv L_p \text{ (pointing loss)}.$$
then the signal to noise ratio SNR is

$$
\text{SNR} = \frac{P_{Rx} R_{PD}}{2qBF_{EN}} = \frac{P_{0} L_{s} G_{Tx} G_{Rx} L_{p} R_{PD}}{2qBF_{EN}},
$$

with the value in decibels, SNR$_{\text{dB}}$, given by $10 \log_{10} \text{SNR}$. Typically, a floor is set for detection at a certain SNR$_{\text{dB}}$, say SNR$_{\text{dB}} \geq 3$ dB.

Note that can define the SNR (and the associated threshold) in other ways. For instance, if can define a wavelength-dependent quantum efficiency of the detector QE$_{\lambda}$ such that the number of photoelectrons generated by the detector per unit time is $P_{Rx} \times$ QE$_{\lambda}$. If the number of electrons generated by thermal and other sources of noise per integration period is $\dot{n}_{e}$, then we can define an SNR

$$
\text{SNR} = \frac{P_{Rx} \times \text{QE}_{\lambda}}{\dot{n}_{e}}.
$$

We also define for future use, a signal to noise ratio value that represents the SNR in the case for which there are no pointing losses, i.e. $L_{p} = 1$. We denote this SNR with a tilde, $\tilde{\text{SNR}} = \text{SNR}/L_{p}$, and the corresponding value in decibels as $\tilde{\text{SNR}}_{\text{dB}}$.

2.2. Acquisition Probability. Consider the case where the signal is not calculated in decibels. Terminal $i$ is seeking to acquire terminal $j$; they are at a distance $d_{ij}$ from one another. The maximum off-pointing angle $\Delta \vartheta$ at which $j$ can still detect $i$ occurs when the pointing loss is such that SNR = SNR$^{*}$. Solving for $\Delta \vartheta$ in this case gives a maximum allowable excursion $\Delta \vartheta_{\text{max}}$ of

$$
(\Delta \vartheta_{\text{max}})^2 = 2\sigma_{\vartheta_{i}}^2 \log \left( \frac{P_{0} L_{s} G_{Tx} G_{Rx} \times \text{QE}_{\lambda}}{\dot{n}_{e} - \text{SNR}^{*}} \right)
$$

Define a quantity $\tilde{\text{SNR}} = \text{SNR}/L_{p} = P_{0} L_{s} G_{Tx} G_{Rx} \times \text{QE}_{\lambda}/\dot{n}_{e}$. Then we can define the ratio of $\Sigma = \tilde{\text{SNR}}/\text{SNR}^{*}$, representing the ratio of the signal to noise in the on-pointing case (with no pointing loss) to that of the detection threshold. Then we have

$$
\Delta \vartheta_{\text{max}} = \pm \sqrt{2\sigma_{\vartheta_{i}}^2 \log \Sigma},
$$

where $\sigma_{\vartheta_{i}}$ parametrizes the beam width of terminal $i$.

Now suppose the angle $\Delta \vartheta$ is distributed according to a probability distribution function $p_{i}(\Delta \vartheta_{i})$, which is accordingly normalized. Then the total probability of a single spacecraft being acquired is

$$
P_{i} = \int_{-\Delta \vartheta_{\text{max}}}^{\Delta \vartheta_{\text{max}}} p_{i}(\Delta \vartheta_{i})d(\Delta \vartheta)
$$

and if the misalignment of both spacecraft is independent of one another then

$$
P_{ij} = \int_{-\Delta \vartheta_{\text{max}}}^{\Delta \vartheta_{\text{max}}} p_{i}(\Delta \vartheta)d(\Delta \vartheta_{i}) \times \int_{-\Delta \vartheta_{\text{max}}}^{\Delta \vartheta_{\text{max}}} p_{j}(\Delta \vartheta)d(\Delta \vartheta_{i})
$$

Note that this is a conservative estimate. In practice, one terminal should base its acquisition strategy on...
Suppose \( p_i(\Delta \vartheta) \) is Gaussian with variance \( \zeta_i^2 \); \( \zeta_i^2 \) comprises the errors in the system due to both jitter and attitude knowledge error, both control and estimation errors. These two effects would not necessarily be orthogonal to one another, but for simplicity we may add them in quadrature so that \( \zeta_i^2 = \zeta_{i,\text{con}}^2 + \zeta_{i,\text{kno}}^2 \), where the individual terms represents control and knowledge errors. Then

\[
p_i(\Delta \vartheta_i) = \frac{1}{\sqrt{2\pi} \zeta_i} \exp \left[ -\frac{(\Delta \vartheta_i)^2}{2\zeta_i^2} \right]
\]

and

\[
P_i = 2 \int_0^{\Delta \vartheta_{\text{max},i}/\sqrt{2} \zeta_i} \frac{1}{\sqrt{2\pi} \zeta_i} \exp \left[ -\frac{(\Delta \vartheta_i)^2}{2\zeta_i^2} \right] d(\Delta \vartheta_i)
\]

Define the error function \( \text{erf}(x) \) using the standard form:

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp \left( -t^2 \right) dt.
\]

Then we get

\[
P_i = \frac{2}{\sqrt{\pi}} \int_0^{\Delta \vartheta_{\text{max},i}/\sqrt{2} \zeta_i} \exp \left( -t^2 \right) dt
\]

\[
= \text{erf} \left( \frac{\Delta \vartheta_{\text{max},i}}{\sqrt{2} \zeta_i} \right)
\]

\[
= \text{erf} \left[ \frac{\sigma_{\vartheta_i}}{\zeta_i} (\log \Sigma_i)^{1/2} \right].
\]

The total probability of acquisition is

\[
P_{ij} = \text{erf} \left( \frac{\sigma_{\vartheta_i}}{\zeta_i} \sqrt{\log \Sigma_i} \right) \times \text{erf} \left( \frac{\sigma_{\vartheta_j}}{\zeta_j} \sqrt{\log \Sigma_j} \right)
\]

and for identical terminals

\[
P_{ij} = \text{erf} \left( \frac{\sigma_{\vartheta}}{\zeta} \sqrt{\log \Sigma} \right)^2.
\]

If we choose instead to define the signal to noise ratio metric using decibels, we rewrite \( \Sigma \) as

\[
\Sigma = \frac{\text{SNR}}{\text{SNR}^*} = 10^{(\text{SNR}_{\text{dB}} - \text{SNR}^*_{\text{dB}})/10}.
\]

3. Optimizing the Beam Width for Acquisition

3.1. The Concept of Operations. Given an expression for \( P_{ij} \), we can seek its stationary values in order to determine conditions that enable us to maximize the acquisition probability. A realistic acquisition scheme might be the following:
(1) Knowing the expected position of the receiving satellite from its orbit, the transmitter aligns itself towards the same orientation based on star-tracker data.

(2) The transmitter then performs a “search” for the receiver using one or both of the following methods:
   (a) The transmitter and receiver both perform scans of the sky centered about their expected positions.
   (b) The beacons each defocus their beams to increase the beams’ solid angle “footprint” and hence the likelihood of being detected by the other spacecraft.

(3) Once initial, mutual acquisition is achieved, the terminals progressively fine-tune their pointing (at the same time reducing beam width for increased transmission gain) and commence data transfer.

Once the initial acquisition has been made, regardless of whether a scanning procedure is implemented or a defocusing procedure is implemented, fine tuning in mutual pointing can be achieved through a combination of expected position, star tracker data, and detection of beam power received (which indicates how far off the transmitter is off-pointing).

3.2. Beam Width Modulation for Initial Acquisition. Here, we explore the option of modulating beam width for initial acquisition based on fixed design parameters and \( \sigma \). Beam width could be modulated randomly, or in some fixed pattern, but as we see below, there is an optimum value that depends on the detection threshold and pointing error. We note that it is not possible to defocus the beam to arbitrarily high values, since then the transmitted flux would fall below the detectability threshold. In principle, then, focusing would be such that acquisition is marginally achieved, from which then the beam would be progressively focused to increase gain and data rates.

To determine the optimum \( \sigma_\theta \) we calculate

\[ \frac{\partial P_{ij}}{\partial \sigma_\theta} = 0. \]  

(21)

For simplicity, we assume both terminals are identical, so that we seek \( \sigma_\theta \) such that

\[ \frac{\partial}{\partial \sigma_\theta} \text{erf} \left( \frac{\sigma_\theta}{\zeta} \sqrt{\log \Sigma} \right)^2 = 0. \]  

(22)

Keeping in mind that \( \Sigma \) is a function of \( \sigma_\theta \) itself (through the \( G_{Tx} \) term), we find that the optimum beam width for detection is such that

\[ \log \Sigma = 1 \text{ or } \Sigma = \tilde{\text{SNR}}/\text{SNR}^* = e, \]  

(23)

where \( e \) is the base of the natural logarithm. Since

\[ \tilde{\text{SNR}} = \frac{P_0 L_s G_{Tx} G_{Rx} R_{PD}}{2\eta BF_{EN}}, \]  

(24)

and \( G_{Tx} = \pi w_\theta^2/\lambda^2 = 1/(4\sigma_\theta^2) \), we solve for the optimum \( \sigma_\theta^* \):

\[ \sigma_\theta^* = \sqrt{\frac{P_0 L_s G_{Rx} R_{PD}}{4e\text{SNR}^* BF_{EN}}}. \]  

(25)

If our threshold acquisition SNR is expressed in decibels, replace \( \text{SNR}^* \) with \( 10^{\text{SNR}^*_\text{dB}/10} \).
3.3. **Active Beam Modulation.** The previous calculation for optimum beam width demodulation calculated $\sigma_\theta$ by maximizing a probability distribution function integrated over all off-pointing angles $\Delta \theta$. Suppose now we wish to actively change the beamwidth $\sigma_\theta$. While technically more expensive, it would enable greater flexibility and have particular application to beam focusing after initial acquisition.

Before, we assumed that the probability distribution function of acquisition was a Gaussian, with the probability of pointing off-axis dependent upon the pointing error $\zeta$. Now, we take a different approach. We assume that $\Delta \theta$ varies as a function of time, and that the probability of acquisition at any given time depends monotonically on the received SNR. Thus we need to find the $\sigma_\theta = \sigma_{\theta,\text{def}}$ such that

$$\frac{\partial}{d\sigma_\theta} \left[ \frac{1}{\sigma_\theta^2} \exp \left( \frac{(\Delta \theta)^2}{2\sigma_\theta^2} \right) \right] = 0,$$

which follows directly from Eq. (1). From this, we find the optimum defocusing factor $\sigma_{\theta,\text{def}}$ to be

$$\sigma_{\theta,\text{def}} = \frac{\Delta \theta}{\sqrt{2}}.$$  \hspace{1cm} (27)

These results are summarized in Fig. 2 where we have normalized both $\Delta \theta$ and $\sigma_\theta$ to an arbitrary reference value. In Fig. 3 we show the results of a number of Monte Carlo simulations of mutual acquisition for identical optical terminals. In these simulations, the beam width was modulated sinusoidally, and the markers indicate the defocusing factors and off-pointing at which mutual acquisition occurred. The locus indicated by Eq. (27) sets the practical bound at which mutual acquisition occurred in these simulations.

4. **Developing Optical Satellite Constellations**

To demonstrate the potential practical application of the calculations above, we consider a simple constellation of optical terminals extending out from (say) 1 AU to interplanetary distances on the order of $\sim 1$ AU. The constellation comprises the following:

- A set of $n_k$ satellites on a circular heliocentric orbit at semimajor axis $a_k$ with corresponding Keplerian mean motion $\Omega_k = \sqrt{GM_\odot/a_k^3}$, where $G$ is Newton’s constant and $M_\odot$ is the mass of the Sun.
- The satellites are equally-spaced on each ring, with an angle between each terminal of $\Delta \theta = 2\pi/n_k$ subtended from the Sun.
- The spacing between each ring is $d_{ij}$, and the distance between closest pairs of terminals on each ring is $d_{ij}$, such that $\Delta \theta = \pi/\arcsin[d_{ij}/(2a_k)] \approx 2\pi a_k/d_{ij}$.

In this constellation, we have the benefit that all satellites on the same ring are within $d_{ij}$ of one another. The next ring of terminals at $a_{k+1} = a_k + \Delta a = a_k + d_{ij}$ has $n_{k+1}$ terminals equally spaced. The total number of terminals is then given by the sum of the $n_k$ over all rings. If we take the (good) approximation that $n_k \approx 2\pi a_k/d_{ij}$, we can calculate the total number of terminals $N(a_i, a_f)$ in a constellation that spans from semimajor axes $a_i$ to $a_f$. 

Figure 2. Optimum normalized defocusing factor based on given off-center pointing $\Delta \vartheta$ (normalized to initial beam width $\sigma_{\vartheta,0}$). The solid curves indicate the functional form $\exp\left[-(\Delta \vartheta)^2/(2\sigma^2_{\vartheta})\right]/\sigma^2_{\vartheta}$, which follows from the expression for a Gaussian beam for various combinations of normalized $\Delta \vartheta$ and over a range of $\sigma_{\vartheta}$. The dotted line indicates the locus of optima found from Eq. (27).

On the $k$th ring, there the $n_k$ satellites are indexed by $i = 1, 2, ..., n_k$. If the terminals in the $k$th ring have a phase offset of $\theta_{0,k}$, then the angle subtended by the $i$th satellite is

$$
\theta^{(i)}_k(t) = \theta_{0,k} + \frac{2\pi}{n_k} i + \Omega_k t.
$$
FIGURE 3. Optimum normalized defocusing factor based on given off-center pointing $\Delta \vartheta$ (normalized to initial beam width $\sigma_{\vartheta,0}$). Mutual acquisitions are shown, with each individual acquisition indicated by color; markers differentiate spacecraft 1 and spacecraft 2. The defocusing at acquisition is bounded the relationship given by Eq. (27). 3,000 simulations were run here.

Any two terminals indexed by $i$ and $j$ on rings $k$ and $k+1$ are separated by an angle

$$\Delta_{k,k+1} + 2\pi \left( \frac{i}{n_k} + \frac{j}{n_{k+1}} \right) + (\Omega_k - \Omega_{k+1}) t.$$  

Solving for $t$, we get

$$t = \frac{1}{\Omega_k - \Omega_{k+1}} \left[ \Delta_{k,k+1} + 2\pi \left( \frac{i}{n_k} + \frac{j}{n_{k+1}} \right) \right],$$

and this quantity can be minimized by choosing $i$ and $j$ such that the term in the brackets is minimized. Creating links between adjacent rings would then mean transmitting data along one ring ($k$, say), bridging to ring $k+1$, and then moving back across to the desired terminal.
The production cost is approximated by the following expression [Larson and Wertz, 1992]:

$$\text{Cost} = \text{TFU} \times N^B,$$

where TFU is the cost of the theoretical first unit and $B = 1 - \log[(100\% / S) - 2]$ is a factor associated with the learning curve of manufacturing. $B$ is in general a smaller number if more units are manufactured. Larson and Wertz [1992] recommend setting $S = 85\%$ if the number of units manufactured is greater than 50. Since an interplanetary constellation would require far more terminals, we set $S = 80\%$. The cost determined in this way as a function of inter-ring spacing and constellation extent is shown in Fig. 5.

To determine the number of terminals in the first ring, we use the following geometric criterion. Defining an angle $\Delta \phi$ such that $n_1 \Delta \phi = 2\pi$, we should have $2a_1 \sin(\Delta \phi/2) = d_{ij}$, so that $n_1 = \lceil \pi / \arcsin(d_{ij}/(2a_1)) \rceil$.

5. **Feasibility and Technology Frontiers**

For a TFU $\sim 10^5 - 10^6$, if an available budget is on the order of $10^9 - 10^{10}$, a value for Total cost/TFU $\sim 10^3 - 10^5$ suggests that with this constellation design, a network
to Mars might be feasible with pointing capabilities on the order of $0.1 - 1$ arcsec. How realistic is this? Currently, the CubeSat ASTERIA (formerly ExoplanetSat) can achieve pointing on the order of $\sim 1$ arcsec [Smith et al., 2010], which it does so using a multistage attitude control system comprising a set of reaction wheels for “coarse attitude control”, and a piezoelectrically-controlled focal plane that maintains fine control of the image. As mentioned above, the parameter $\zeta$ reflects error in both attitude knowledge and control.

As it stands, based on the results shown in the previous section (Fig. 5), it seems very likely that the manufacturing and cost benefits of nano- and picosatellites will lead to their forming the backbone of the network. The question we wish to ask is whether current or future technologies will enable the construction of interplanetary optical terminals. In order to do this, we first choose some baseline parameters for an optical communication terminal. The values shown in Table I are taken from Kingsbury [2014] and Morgan [2017]. In Fig. 8, we show mutual acquisition probabilities based on these hardware parameters, with varying interterminal distances $d_{ij}$ and attitude errors $\zeta$. We have assumed that the threshold for acquisition is $\text{SNR}^*_\text{dB} = 3$.

The results in Fig. 8 have been calculated without optimizing the beamwidth for acquisition. In Fig. 7, on the other hand, we show the effect of optimizing the beamwidth for acquisition.
acquisition. We show the $\zeta$ dependence of $P_{ij}$ for a range of interterminal distances between $2 \times \text{GEO}$, and $200 \times \text{GEO}$. We see that in the “close” case, optimizing the beamwidth can increase acquisition probability by factors of up to 8 or 9. For the “far” cases, we see that acquisition would not be possible with the baseline system variables—the lines corresponding to the non-optimized beamwidths are at 0. On the other hand, optimizing beamwidths leads to reasonable acquisition probabilities for attitude errors $\sim 0.01 - 1 \text{ arcsec}$. The ranges for which this becomes possible are to be compared with those in Fig. 5.

Is it possible to achieve the $\zeta \sim 0.01 - 1 \text{ arcsec}$ attitude errors that would enable interplanetary lasercom networks? Here we explore whether the contribution from attitude knowledge can be decreased to sufficiently low values. We assume the attitude knowledge comes from star trackers and gyroscopes. Using a range of star tracker and gyroscope data and using an iterative Kalman filter model [Hemmati, 2006], in Fig. 9 we plot the contribution to $\zeta$ from star tracker and gyroscope performance. In Fig. 10 we show gyroscope performance over time as a function of gyroscope type and in Fig. 11 we show how physics-limited MEMS gyroscope performance may evolve due to manufacturing capabilities. In this particular case, we show how MEMS gyroscope performance varies as a function of quality factor $Q$ (for which larger values represent smaller losses from dissipation) and system temperature (which leads to thermal noise; see Zotov et al. [2014] and Leland [2005] for further details). Individual results from the Kalman filter model over the range of star tracker and gyroscope data points that lead to Fig. 9 are shown in Fig. 12.

6. DISCUSSION AND CONCLUSIONS

6.1. Summary and Discussion. In this paper, we have considered several topics relevant to the development of interplanetary optical communication networks for small satellites. We have

- Constructed an analytical model for the mutual acquisition probability between two optical terminals,
- Calculated the optimum beam width for acquisition over all angles,
Figure 6. Mutual acquisition probabilities as a function of interterminal distance and for varying attitude errors $\zeta$. Hardware parameters are summarized in Table I and the threshold for acquisition is $\text{SNR}_{\text{dB}}^* = 3$.

- Calculated the optimum beam width for a given off-pointing $\Delta \vartheta$.
- Constructed a notional interplanetary optical communication network and calculated its cost, and
- Identified regimes of the parameter space whose future development would enable the implementation of interplanetary optical networks.

In recent years, picosats have seen increasing use for academic and commercial purposes, and their relatively low cost, set form factor, and reliance on commercial/off-the-shelf hardware makes them attractive for further development, especially where large numbers of assets are involved. At the same time, however, these same qualities make their implementation difficult in circumstances where precision is involved. The purpose of this paper has been to highlight the quantities that are important to consider if picosats are to be implemented for optical communication networks at the interplanetary scale. Given the push to characterize the bodies of interplanetary space for both resource identification and security purposes, as well as the possibility of forming human habitats on these bodies or Mars, it seems worthwhile to further consider.
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6.2. Recommendations and Conclusions. In this paper, we have constructed a series of simple analytical models to aid us in characterizing optical terminals. These models can be used to inform more sophisticated models, as well as to motivate the exploration of further technological development to enable interplanetary-scale optical networks—especially those in the challenging part of the design space occupied by the otherwise attractive picosat. Future work should explore optimizing interplanetary networks further. The geometry can be accounted for more precisely, and more economical constellations be explored. In particular, the “resiliency” and reliability of such networks from a data integrity and concept of operations point of view should be explored; one possible means is through graphical analysis. In this paper, we focused on MEMS gyroscopes as a potential enabling
FIGURE 8. Signal to noise ratios (SNR) as a function of interterminal distance and for varying attitude errors $\zeta$. Hardware parameters are those summarized in Table I and the threshold for acquisition is $\text{SNR}_{\text{dB}}^* = 3$ (dash-dotted green line).

technology, but attitude control technologies for picosats should also be further explored. Future work should also explore other hardware options, and look more in depth at the technology frontiers (and potential disruptive technologies) that would enable interplanetary picosat networks.
Figure 9. Attitude knowledge error as a function of gyroscope and star tracker performance.
Figure 10. Gyroscope performance over time as a function of gyroscope type. Typical MEMS gyroscope performance (blue line) is about an order of magnitude worse than those for widely-used fiber optic gyroscopes. On the other hand, MEMS gyros are more amendable to miniaturization and future performance capabilities may enable the requisite performance Fig. [1].
Figure 11. MEMS gyroscope performance as a function of quality factor $Q$ (a smaller value of which represents greater mechanical losses due to damping), and as a function of system temperature, reflecting thermal noise. Lines representing current capabilities (black) and future capabilities based on a literature review (blue and red lines) are shown. For reasonable system temperatures, thermal noise-limited performance may be achievable in the near future.
Figure 12. Least-squares Kalman filter models for all combinations of star tracker and gyroscope data. The minima from these iteratively-calculated curves [Hemmati, 2006] are used to generate Fig. 9.
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