Numerical investigation of heat transfer on the building insulation materials with successive layers of polystyrene and various inert gases

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Abstract
Savings from heating and cooling energy use while not compromising indoor comfort condition is essential to reduce the energy costs on buildings. In order to accomplish this goal, an alternative is to search for a new insulation material that is an effective insulator and inexpensive at the same time. In the present work, various insulation materials created with successive layers of polystyrene and either of the following inert gases: carbon dioxide, argon, nitrogen, or air were investigated. Radiation effect on the polystyrene surfaces was included. Based on 1.0 cm of gas layer size, $R_{value}$ (thermal resistance) of carbon dioxide exceeded $R_{value}$ for argon, nitrogen, or air. This result identified the carbon dioxide as a potential medium to be used inside insulation materials. Insulation effectiveness of carbon dioxide relative to single polystyrene without any radiation effect was calculated for gas layer sizes of 1.0 cm, 0.86 cm, and 0.6 cm on 3 cm-thick insulation material, and it was, respectively, 31.8%, 28.6%, and 22.0%. Radiation effect on the polystyrene surfaces needs to be kept at minimum for a better insulation. An insulation material with carbon dioxide layered structure is expected to pay off its implementation cost in regard to energy savings in a reasonable amount of time (2.39 yr). Argon was the next alternative to carbon dioxide gas.

Key words : Insulation effectiveness, Insulation material, Thermal resistance, Energy savings

1. Introduction

Insulation materials are often implemented in order to prevent the heat exchange between the inside and outside environments. The purpose is to reduce the heating and cooling costs. Polymer-based porous materials are usually preferred. Extruded polystyrene foam (XPS), for example, is such an insulation material. Buildings are extensively fitted with XPS type insulation materials.

In the literature related to the insulation materials, thermal design optimization (Del Coz Díaz et al., 2009) and thermal analysis (Del Coz Díaz et al., 2011) of concrete blocks were investigated with FEA. Overall thermal conductivity is an important property indicating an effective or ineffective insulation. Effective thermal conductivity was given for a medium which incorporated dispersed spherical and cylindrical pores and overlapping solid spheres (Helte, 1993). Effective thermal conductivity of a composite material was determined to be dependent on the volume fraction as well as distribution of the fillers (Zhou et al., 2007). Various techniques were implemented to ensure effective insulation: In addition to the development of low-thermal-conductivity polymer foams, light metal foams, and various foam manufacturing techniques (Banhart, 2001), low-thermal-conductivity bricks (Topçu and Işıkdağ, 2007) were studied to reduce the energy usage. In the latter by Topçu and Işıkdağ (2007), bricks incorporating perlite were tested for their compressive strength ($MPa$) and thermal conductivity [$kcal \cdot m^{-1} \cdot h^{-1} \cdot °C^{-1}$] values. It was shown that for an increase of the perlite ratio, the compressive strength and thermal conductivity of the bricks decreased. Insulation capability and manufacturing techniques of aerogels were given (Baetens et al., 2011). Vacuum insulation
panels (VIPs) were evaluated in relation to their performance, mechanical durability, and life-time (Baetens et al., 2010). In their work, thermal performances that are 3 to 6 times better than still air have been demonstrated. The authors stated that a vacuum space performed better compared to a stagnant air space. Heat transfer was modeled on insulation materials (Dukhan et al., 2005): Effect of conjugated convection and conduction was studied on the open-cell metal foams. Heat transfer was measured on the rectangular metal foams with constant heat flux presence at one side (Dukhan et al., 2007). In their work, temperature was given for different porosities and pore densities of the foam. Energy savings of $65 – 77\%$ were realized by using fiberglass-urethane, roof-deck urethane, extruded polystyrene, perlite, and rigid fiberglass insulation materials where air gap sizes of $2\text{cm}$, $4\text{cm}$, and $6\text{cm}$ were realized (Mahlia and Iqbal, 2010). Optimum insulation material thickness was studied (Ucar, 2010). In the work of Ucar (2010), application of optimum insulation thickness resulted in an energy saving of $80.6\%$ in the province of Erzurum in Turkey for a building which maintained an indoor temperature of $22\degree\text{C}$. Several criteria in selection of the proper insulation material were cited (Al-Homoud, 2005). It was mentioned that type of the building, outdoor climate, and type of the insulation material affected the amount of energy savings in the buildings. Convection-conduction and convection-conduction-radiation conjugate effects were investigated by a number of researchers (Turkoglu and Yuçel, 1996), (Kangni et al., 1991), (Baig and Antar, 2008), (Antar and Baig, 2009), (Antar, 2010). A method was described on the flat surfaces, ducts, and pipes for the optimum insulation material thickness which was based on the specific amount of heat flow or the surface temperature (Bahadori and Vuthaluru, 2010a). An easy-to-use correlation was included in its relation to the economic insulation thickness of process equipment and piping (Bahadori and Vuthaluru, 2010b).

In order to obtain a better thermal barrier, a variety of approaches, for example, substituting some environmentally harmful and explosive agents were tried (Brodt, 1995); an experimental sorption technique was included which focused on the cellular solubility, diffusion, and permeability of the gaseous agents inside polyurethane foams. A cellular polyurethane foam structure was experimentally tested (Brodt, 1995). In the literature, heat transfer models after filling, between XPS layers, various types of inert gas layers have not been studied. A performance study of various inert gas usage between polystyrene layers was absent. In fact, this kind of layered insulation material provides better insulation than what is currently available. Hence, the aim of the present heat transfer study is to better the thermal insulation capability of XPS type insulation materials in an effective, cheap, durable, and hazard-free way. An adaptive insulation material (with a low $R_{\text{value}}$ and a high $R_{\text{value}}$) was proposed (Kimber et al., 2014). In this paper, we numerically analyze the insulation effectiveness of various insulation materials with successive layers of XPS and either of the inert gases; carbon dioxide, argon, nitrogen, or air. Based on the three gas layer sizes, selected due to the conduction-to-convection transitioning on the gas layers, numerical heat transfer analysis results were given. Thermal resistance ($R_{\text{value}}$) of each insulation material was calculated from the developed theoretical model. Subsequently, effective thermal conductivity ($k_{\text{eff}}$) and insulation effectiveness ($\epsilon$) were given on the insulation materials with types of inert gases used.

**Nomenclature**

**Letter**

| Symbol | Description | Unit |
|--------|-------------|------|
| $A$    | area        | m$^2$ |
| $dx$   | layer size  | m    |
| $g$    | gravitational acceleration | m·s$^{-2}$ |
| $i$    | rate of return | % |
| $k$    | thermal conductivity | W·m$^{-1}$·K$^{-1}$ |
| $L$    | total insulation material thickness | m |
| $N$    | number of periods, dimensionless | |
| $NPV$  | net present value | $\$ |
| $q$    | power | W |
| $R$    | cash flow in minus cash flow out | $\$ |
| $R$    | thermal resistance | m$^2$·K·W$^{-1}$ |
| $Ra$   | Rayleigh number | $\frac{g\beta\Delta T}{\nu a}(\Delta x)^3$, dimensionless |
| $T$    | temperature | K |
| $t$    | period | mo, yr |

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Greek symbol
\( \alpha \) thermal diffusivity, m\(^2\) \cdot s\(^{-1}\)
\( \beta \) thermal expansion coefficient, K\(^{-1}\)
\( \Delta \) finite difference, dimensionless
\( \varepsilon \) emissivity, dimensionless
\( \varepsilon \) insulation effectiveness, dimensionless
\( \nu \) kinematic viscosity, m\(^2\) \cdot s\(^{-1}\)
\( \sigma \) Stefan-Boltzmann constant (= 5.67 \cdot 10\(^{-8}\)), W \cdot m\(^{-2}\) \cdot K\(^{-4}\)

Subscript
\( ef \) effective
\( g \) gas
\( gf \) glass-fiber
\( m \) mean, average
\( poly \) polystyrene
\( s \) surficial
\( sp \) single polystyrene
\( value \) value

Superscript
\( " \) flux or per unit area

2. Modeling

Rayleigh number \((Ra)\) on enclosures was given by the following (Bergman et al., 2011)

\[
Ra = \frac{g \beta \Delta T}{\nu \alpha} (\Delta x)^3
\]

where \( g \) is the gravitational acceleration \([m \cdot s^{-2}]\), \( \beta \) is the thermal expansion coefficient \([K^{-1}]\), \( \Delta T \) is the temperature difference across the enclosure \([K]\), \( \Delta x \) is the characteristic length in transverse direction (the gas layer size) \([m]\), \( \nu \) is the kinematic viscosity \([m^2 \cdot s^{-1}]\), and \( \alpha \) is the thermal diffusivity \([m^2 \cdot s^{-1}]\). \( Ra \) as prescribed in Eq. (1) relates to the degree of natural convection in the heat transfer process. Eq. (1) can also be used to assess the extent of heat conduction in the heat transfer process. In other words, lower the value of \( Ra \), lower the degree of advection currents in a heat transfer process that sets the onset of convection. A low value for \( Ra \) renders the heat transfer process pure conduction by suppressing the extent of heat transfer being due to convection.

![Fig. 1 Direction of heat flow and volumetric gaps for inert gas fill shown on the insulation material.](image-url)
In the model, heat conduction was modeled as a 1D process across the insulation material, or the successive layers of the polystyrene and inert gases. Figure 1 shows the structure of the insulation material with alternating polymer and gas layers. Uniform surface temperatures were taken on both sides of the insulation material. In fact, any non-uniformities of the outdoor or indoor air temperature would cause temperature non-uniformities on the side walls presenting local patches with different temperatures.

XPS material has a surface emissivity to emit certain thermal radiation. Radiation from the XPS surfaces is an important factor accompanying the conductive heat transfer through these layers. The effect of thermal radiation across each gas layer was incorporated. The developed model was a coupled 1D radiation-conduction model.

2.1 Multiple gas layers

The insulation material was constructed with successive layers of the XPS material and either of the inert gases; carbon dioxide, argon, nitrogen, or air (See Figure 2(a)). The XPS material serves as the matrix of the insulation materials. Energy conservation equation was written on the control surfaces shown in Figure 2(b) without energy generation and energy storage.

The heat transfer equations were acquired on the control surfaces:

First layer (polystyrene):

\[ k_{\text{poly}} A \left( \frac{T_1 - T_2}{dx_{\text{poly},1}} \right) + k_g A \left( \frac{T_2 - T_3}{dx_{g,1}} \right) + \varepsilon_s \sigma A (T_2^4 - T_1^4) = 0 \] (2)

Second layer (inert gas):

\[ k_g A \left( \frac{T_2 - T_3}{dx_{g,1}} \right) + k_{\text{poly}} A \left( \frac{T_3 - T_4}{dx_{\text{poly},2}} \right) + \varepsilon_s \sigma A (T_3^4 - T_2^4) = 0 \] (3)

Third layer (polystyrene):

\[ k_{\text{poly}} A \left( \frac{T_3 - T_4}{dx_{\text{poly},2}} \right) + k_g A \left( \frac{T_4 - T_5}{dx_{g,2}} \right) + \varepsilon_s \sigma A (T_4^4 - T_3^4) = 0 \] (4)

Fourth layer (inert gas):

\[ k_g A \left( \frac{T_4 - T_5}{dx_{g,2}} \right) + k_{\text{poly}} A \left( \frac{T_5 - T_6}{dx_{\text{poly},3}} \right) + \varepsilon_s \sigma A (T_5^4 - T_4^4) = 0 \] (5)

The results were primarily given for boundary surfaces kept at uniform surface temperatures of \( T_{s,1} = 20^\circ C \) and \( T_{s,2} = 0^\circ C \). Later, \( T_{s,1} = 20^\circ C \) and \( T_{s,2} = -10^\circ C \) and then \( T_{s,1} = 25^\circ C \) and \( T_{s,2} = 0^\circ C \) were taken reflecting...
changes in the indoor and outdoor environment conditions (temperatures). Inert gas layer sizes of 1.0 cm \( \left( dx_{\text{poly}} = \frac{dx_g}{3} \right) \), 0.86 cm \( \left( dx_{\text{poly}} = \frac{dx_g}{2} \right) \), 0.6 cm \( \left( dx_{\text{poly}} = dx_g \right) \) were used throughout the study. \( L \) (total insulation material thickness) was 3 cm.

The heat transfer equations, Eqs. (2-5), were set up after considering the effective modes of heat transfer from neighboring surface nodes toward the surface node under consideration. The surface node 1 \( (T_1) \), for instance, included the conductive heat transfer from the boundary surface \( s,1 \) \( (T_s) \) and the combined conductive and radiative heat transfer from the neighboring surface node 2 \( (T_2) \). The same held for the second surface node 2 \( (T_2) \) with combined conductive and radiative heat transfer from the surface node 1 \( (T_1) \) and conductive heat transfer from the neighboring surface node 3 \( (T_3) \).

In Eqs. (2-5), \( k_{\text{poly}} \) is the thermal conductivity of the polystyrene material \( [W \cdot m^{-1} \cdot K^{-1}] \), \( k_g \) is the thermal conductivity of the inert gas layer \( [W \cdot m^{-1} \cdot K^{-1}] \), \( dx_{\text{poly}} \) is the thickness of the polystyrene layer \( [m] \), \( dx_g \) is the thickness of the inert gas layer \( [m] \), \( \varepsilon_g \) is the emissivity of the polystyrene material (in the study, \( \varepsilon_g = 90\% \), \( \varepsilon_g = 30\% \), \( \varepsilon_g = 10\% \), \( \varepsilon_g = 0\% \)), \( \sigma \) is the Stefan-Boltzmann constant \( (\sigma = 5.67 \cdot 10^{-8} \ W \ m^{-2} \ K^{-4}) \), \( A \) is the area for the heat transfer \( [m^2] \), and \( T \) is the temperature \([K]\). In Eqs. (2-5), thermal conductivities of the gas \( (k_g) \) and polymer \( (k_{\text{poly}}) \) layers were evaluated at their mean or average surface temperatures.

The steady-state solution gives the final state temperatures. The transient-state solution was not considered since it was not aimed to present the time-varying surface temperatures on the insulation materials.

It was calculated with Eq. (1) that when \( dx_{\text{poly}} > \frac{dx_g}{3} \) for an inert gas layer, mode of the heat transfer transformed into pure conduction \( (Ra \lesssim 10^3) \) on that gas layer. For the all inert gas types and inert gas layers between the layers of the XPS, extent of advection was calculated on each gas layer and the effect was found to be more significant as moved from the first inert gas layer from the \( T_{S,1} \) side toward the second inert gas layer toward the \( T_{S,2} \) side (Table 1). The aim here is to suppress the effect of natural convection inside the gas layers and to make it pure conduction. Therefore, appropriate gas layer sizes needed to be calculated on the inert gas layers with \( Ra \) (Eq. (1)) to suppress any advection effect. The term advection means, in that sense, incorporating macroscopic bulk fluid motion inside the inert gas-filled layers.

### Table 1

| Inert filler | \( dx_g = 1.0 \text{ cm} \) | \( dx_g = 0.86 \text{ cm} \) | \( dx_g = 0.6 \text{ cm} \) |
|-------------|----------------|----------------|----------------|
|             | \( dx_{g,1} \) | \( dx_{g,2} \) | \( dx_{g,1} \) | \( dx_{g,2} \) | \( dx_{g,1} \) | \( dx_{g,2} \) |
| carbon dioxide | 766 | 941 | 430 | 524 | 113 | 136 |
| argon | 757 | 914 | 422 | 506 | 109 | 129 |
| nitrogen | 687 | 823 | 371 | 441 | 91 | 106 |
| air | 686 | 822 | 371 | 441 | 90 | 106 |

\( dx_{g,1} \) is the first gas layer from the \( T_{S,1} \) side and \( dx_{g,2} \) is the second gas layer from the \( T_{S,1} \) side.

### Table 2

| Inert filler | \( dx_g = 1.0 \text{ cm} \) | \( dx_g = 0.86 \text{ cm} \) | \( dx_g = 0.6 \text{ cm} \) |
|-------------|----------------|----------------|----------------|
|             | \( dx_{g,1} \) | \( dx_{g,2} \) | \( dx_{g,1} \) | \( dx_{g,2} \) | \( dx_{g,1} \) | \( dx_{g,2} \) |
| carbon dioxide | 1,185 | 1,623 | 667 | 904 | 176 | 234 |
| argon | 1,173 | 1,569 | 655 | 867 | 170 | 221 |
| nitrogen | 1,068 | 1,409 | 578 | 754 | 142 | 181 |
| air | 1,066 | 1,407 | 577 | 753 | 141 | 180 |

\( dx_{g,1} \) is the first gas layer from the \( T_{S,1} \) side and \( dx_{g,2} \) is the second gas layer from the \( T_{S,1} \) side.
Table 3 Ra of the insulation materials of Figure 2 with \( dx_g \) versus different inert gas. (\( \varepsilon_s = 10\% \)), (\( L = 3\text{cm} \)), (\( T_{s,1} = 25^\circ\text{C} \)), and (\( T_{s,2} = 0^\circ\text{C} \)).

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Inert filler} & dx_{g,1} = 1.0\text{cm} & dx_{g,2} & dx_{g,1} & dx_{g,2} \\
\hline
\text{carbon dioxide} & 895 & 1,152 & 502 & 641 & 132 & 166 \\
\text{argon} & 887 & 1,120 & 494 & 619 & 128 & 158 \\
\text{nitrogen} & 806 & 1,007 & 436 & 539 & 106 & 130 \\
\text{air} & 804 & 1,005 & 435 & 538 & 106 & 129 \\
\hline
\end{array}
\]

\((1) dx_{g,1}\) is the first gas layer from the \( T_{s,1} \) side and \( dx_{g,2} \) is the second gas layer from the \( T_{s,1} \) side.

Effect of changing boundary surface temperatures \( T_{s,1} \) and \( T_{s,2} \) on \( Ra \) is seen in Tables (2-3). Decreasing \( \varepsilon_s \) (\( \varepsilon_s = 0\% \)) increased the values of \( Ra \) listed in these tables and increasing \( \varepsilon_s \) (\( \varepsilon_s = 90\% \)) decreased the values of \( Ra \). For a better \( Ra \) value, \( \varepsilon_s \) needs to be set to \( \varepsilon_s = 0\% \). Tables (2-3) pointed out that the gas layer sizes needed to be small (\( dx_g = 0.6\text{cm} \)) for conduction to entirely dominate through the gas layers; \( dx_g = 0.86\text{cm} \) was moderately suitable while \( dx_g = 1.0\text{cm} \) was not viable.

2.2 Single gas layer

Eqs. (2-5) were modified for an insulation material having one single inert gas layer. It was found that \( Ra \) changed with the size of the gas layer. Table 4 tabulates the results.

The single gas layer insulation material can be modeled with pure conduction and the advection effects can be suppressed for \( dx_g \leq 1.0\text{cm} \). Increasing \( dx_g \) (\( dx_g > 1.0\text{cm} \)) initiates the onset of advection inside the inert gas layer. Decreasing \( \varepsilon_s \) (\( \varepsilon_s = 0\% \)), caused an increase on the \( Ra \) values in Table 4 and increasing \( \varepsilon_s \) (\( \varepsilon_s = 90\% \)), caused a decrease on the \( Ra \) values in Table 4.

Table 4 Ra of the insulation materials with one single gas layer with \( dx_g \) versus different inert gas (\( \varepsilon_s = 10\% \)), (\( L = 3\text{cm} \)), (\( T_{s,1} = 20^\circ\text{C} \)), and (\( T_{s,2} = 0^\circ\text{C} \)).

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Inert filler} & dx_g = 1.0\text{cm} & dx_g = 1.5\text{cm} & dx_g = 1.8\text{cm} \\
\hline
\text{carbon dioxide} & 918 & 4,231 & 8,464 \\
\text{argon} & 877 & 4,101 & 8,262 \\
\text{nitrogen} & 716 & 3,548 & 7,370 \\
\text{air} & 714 & 3,541 & 7,357 \\
\hline
\end{array}
\]

Although \( Ra \) of carbon dioxide and argon were larger than that of nitrogen and air in the tables, it was the small values of \( k_g \) for the gases that dominated the results here. Thus, \( Ra \) was set up only to differentiate the mode of heat transfer either as a pure conduction or a natural convection one.

In the environments that undergo random and rapid change, these insulation materials may have onset of convection on the gas layers which adversely decreases \( Ra \) value of the insulation materials. Therefore, the proposed insulation materials are to be used in environments not prone to rapid and random (extreme) temperature variations.

3. Results

In order to assess the insulation effectiveness of the insulation materials, \( Ra \) value (Bergman et al., 2011) were calculated

\[
Ra = \frac{T_{s,1} - T_{s,2}}{q''}
\]

where \( Ra \) is the thermal resistance of an insulation material [\( m^2 \cdot K \cdot W^{-1} \)] and \( q'' \) is the heat flux across the insulation material [\( W \cdot m^{-2} \)]. \( k_{poly} \) and \( k_g \) in Eqs. (2-5) were taken from (Bergman et al., 2011) and (Touloukian et al., 1970), respectively. \( k_{poly} \) and \( k_g \) in Eqs. (2-5) were temperature dependent thermal conductivity values.
evaluated at average surface temperatures. Figure 3 compares $k_{eff}$ on the insulation materials.

![Graph comparing $k_{eff}$ values for different materials](image)

Fig. 3 $k_{eff}$ ($\varepsilon = 10\%$) of the insulation materials, ($T_{s,1} = 20^\circ C$) and ($T_{s,3} = 0^\circ C$).

It was seen from Figure 3 that even if the gas layer size changed, $k_{eff}$ change was almost flat for carbon dioxide. The onset of advection effect, however, which manifests itself by an increase of $Ra$ takes a negative turn on $R_{value}$ by replacing the pure conduction by convection. Beyond $Ra > 10^3$, the heat transfer inside the inert gas layers can not be modeled with pure conduction. For this figure, calculated $R_{value}$ with carbon dioxide was 1.363 $m^2K/W$ ($dx_{poly} = \frac{dx_g}{3}, dx_g = 1.0cm$), 1.365 $m^2K/W$ ($dx_{poly} = \frac{dx_g}{2}, dx_g = 0.86cm$), 1.347 $m^2K/W$ ($dx_{poly} = dx_g = 0.6cm$). It was seen from Figure 3 that argon worked better than air yet no better than carbon dioxide. Effective thermal conductivity ($k_{eff}$) were calculated

$$k_{eff} = \frac{\dot{q}''}{T_{s,2} - T_{s,1}}$$  \hspace{1cm} (7)

where $k_{eff}$ values calculated were found to decrease in the order of air, nitrogen, argon, and carbon dioxide for a specific gas layer size. $R_{value}$ of the single polystyrene (without any gas layer) was calculated as $1.154 \frac{m^2K}{W}$ with $k_{sp} = 0.026 \frac{W}{mK}$ evaluated at the mean of the two surface temperatures, $T_m = \frac{T_{s,1} + T_{s,2}}{2} = \frac{293.15K + 273.15K}{2} = 283.15K$. Insulation effectiveness ($\varepsilon$) of the insulation materials were calculated

$$\varepsilon = \left(\frac{k_{gf} - k_{eff}}{k_{gf}}\right)100(\%)$$  \hspace{1cm} (8)

where $k_{gf}$ is the thermal conductivity of the glass-fiber insulation material, $k_{gf} = 0.033 \frac{W}{mK}$ (Bergman et al., 2011) evaluated at $T_m = \frac{T_{s,1} + T_{s,2}}{2} = \frac{293.15K + 273.15K}{2} = 283.15K$. It is reported (Bergman et al., 2011) that $k_{gf}$ increases with $T$.

![Graphs showing insulation effectiveness for different materials](image)

Fig. 4 $\varepsilon$ of the insulation materials relative to the single glass-fiber insulation material ($k_{gf} = 0.033 \frac{W}{mK}$), ($T_{s,1} = 20^\circ C$), and ($T_{s,2} = 0^\circ C$). a) ($\varepsilon_s = 0\%$). b) ($\varepsilon_s = 30\%$).
Figure 4 shows the result of $\epsilon$ calculations. As can be inferred from this figure, independent of the layer sizes studied, carbon dioxide provided the most favorable insulation capability. Inclusion of severe radiation ($\epsilon_s = 90\%$) on the polystyrene surfaces resulted in negative $\epsilon$ values indicative of no improvement. Therefore, radiation on the polystyrene surfaces needed to be kept at a minimum.

Effect of changing boundary surface temperatures $T_{s,1}$ ($T_{s,1} = 20^\circ C$) and $T_{s,2}$ ($T_{s,2} = -10^\circ C$) on $\epsilon$ of the insulation materials can be seen in Figs. (5) and (6) relative to the single glass-fiber insulation material.

The other $\epsilon$ was calculated between the insulation materials and the single XPS material

$$\epsilon = \left( \frac{k_{sp} - k_{eff}}{k_{sp}} \right) 100\% \quad (9)$$

where $k_{sp}$ is the thermal conductivity of the single polystyrene, $k_{sp} = 0.026 \text{ W m}^{-1} \text{K}^{-1}$. Figure 7 shows the results at different $\epsilon_s$ values.
Fig. 7 $\epsilon$ of the insulation materials relative to the single polystyrene insulation material ($k_{sp} = 0.026 \text{ W/mK}$), ($T_{s,1} = 20^\circ\text{C}$), and ($T_{s,2} = 0^\circ\text{C}$). a) ($\epsilon_s = 0\%$). b) ($\epsilon_s = 30\%$). Omitted gases showed no improvement ($\epsilon < 0$). ($T_{s,1} = 20^\circ\text{C}$), and ($T_{s,2} = -10^\circ\text{C}$). c) ($\epsilon_s = 0\%$). d) ($\epsilon_s = 30\%$). Omitted gases showed no improvement ($\epsilon < 0$).

Although it may be obvious that a lower Rayleigh number, lower thermal conductivity of the inert gas, and lower emissivity is better for thermal insulation, transition into natural convection decides the final outcome. Thus, the calculations need repetition on case basis.

At boundary surface temperatures ($T_{s,1}$ and $T_{s,2}$) different from the studied, the inert gas layers may start to behave more like a fluid rather than a solid medium after onset of natural convection on the gas layers. This transitioning reduces both $R_{value}$ and $\epsilon$. With the current parameters $L$, $T_{s,1}$, $T_{s,2}$, $dx_{poly}$, and $dx_g$, the mode of heat transfer remains conduction through the inert gas layers, however, with a change on one of the parameters, convection can be adversely onset resulting in a less desirable insulation material. Thermal properties of the inert gases will change somewhat with temperature and $L$, $T_{s,1}$, $T_{s,2}$, $dx_{poly}$, and $dx_g$ will determine the mode of the heat transfer through the inert gas layers and the ultimate outcomes of $R_{value}$ and $\epsilon$.

Effect of changing boundary surface temperatures $T_{s,1}$ ($T_{s,1} = 25^\circ\text{C}$) and $T_{s,2}$ ($T_{s,2} = 0^\circ\text{C}$) on $\epsilon$ of the insulation materials can be seen in Figure (8) relative to the single polystyrene insulation material. In Figures 7 and 8, omitted negative $\epsilon$ values meant no improvement over the use of a single polystyrene insulation material. Not included in Fig. 7, $\epsilon_s = 50\%$, $\epsilon_s = 70\%$, and $\epsilon_s = 90\%$ resulted in negative $\epsilon$ for the all gases.

Not included in Fig. 8, $\epsilon_s = 50\%$, $\epsilon_s = 70\%$, and $\epsilon_s = 90\%$ resulted in negative $\epsilon$ for the all gases.

In Figs. (4-8), when $\epsilon_s = 0\%$, $\epsilon$ was linearly correlated to the gas layer size. Inclusion of the radiation effect in the results of Figs. (4-8) made the $\epsilon$ values uncorrelated with the gas layer size.
Increasing $\varepsilon_s$ enhances the conductive heat transfer on the gas layers although it has a negative effect on the calculated $\varepsilon$ values. Thus, $\varepsilon_s$ should be considered with $\varepsilon$ and $Ra$ on the gas layers. The foregoing analysis can be repeated with different $L$, $T_{x,1}$, $T_{x,2}$, $dx_{poly}$, and $dx_g$ but Eqs. (2-9) will require a recalculation under the light of Eq. (1). Also, different inert gases like neon or helium may be substituted.

### 4. Economics

The insulation materials are anticipated to be economically competitive to be purchased. Price of $1.00$ on (1mx1m) $L = 3cm$ piece of glass-fiber insulation material ($k_{gf} = 0.033 \frac{W}{mK}$) was obtained from the local retailers. (1mx1m) $L = 3cm$ piece of single polystyrene material, on the other hand, costs around $3.12$. Insulation sheets made with (1mx1m) $L = 3cm$ sizes can be expected to incur an overall manufacturing cost of around $5.00$ excluding the purchasing costs of the inert gases. Each gas may incur a separate cost, thus, may give a different overall manufacturing cost for an insulation material. Under the cost estimate, a payback analysis relative to the single polystyrene use and based on 1yr of energy replacement, a manufacturing cost of $250$ ($5/m^2$), $A = 50m^2$, ($dx_{poly} = dx_g/3, dx_g = 1.0cm$), $T_{x,1} = 20^\circC$, $T_{x,2} = 0^\circC$, $\varepsilon_s = 10\%$, and electricity cost of $0.089723$/kWh indicated that after nearly 40.5mo ($3.8\text{yr}$), all insulation materials will pay off. Table 5 lists the payback times versus inert gas types of the insulation materials. A manufacturing cost of $150$ ($3$/m$^2$) and $A = 50m^2$ were used for the single glass-fiber insulation material. The energy saving capability of an insulation material with a specific inert gas was reflected in the payback analysis. An insulation material that used carbon dioxide was able to compensate for its implementation cost in a short term (0.52yr or 6.3mo). That is, carbon dioxide provided a better insulation compared to the other inert gas types relative to the single glass-fiber usage. Argon was the next preferable. It was: Better the insulation capability of an insulation material, shorter the payback time. In the payback analysis, one single manufacturing cost was associated with each insulation material. These payback times may extend further as inert gases will incur different purchasing and filling prices.

Table 5 Payback times of the insulation materials with different inert gases.

| Inert filler   | Payback time¹, mo (yr) | Payback time², mo (yr) |
|---------------|------------------------|------------------------|
| carbon dioxide| 28.7 (2.39)            | 6.3 (0.52)             |
| argon         | 40.5 (3.38)            | 7.0 (0.58)             |
| nitrogen      | -47.4 (-3.95)          | 15.0 (1.25)            |
| air           | -46.0 (-3.83)          | 15.2 (1.27)            |

¹Relative to the single polystyrene insulation material. The negative values are the indicative of the fact that these insulation materials are not able to provide better insulation than the single polystyrene insulation material, or $\varepsilon < 0$. ²Values are relative to the single glass-fiber insulation material.
Using the net present value (NPV), 
\[ \text{NPV} = \sum_{t=1}^{N} R_t / (1 + i)^t \]  
where \( R_t \) is the cash flow in minus cash flow out, \( i \) is the rate of return (%), \( t \) is the time period (mo), \( N \) is the number of periods (–), the payback times were calculated for the insulation materials. The results were included in Table 6. The expected financing scenario requirement here is to have \( \text{NPV}(i) > 0 \). Present \( \text{NPV} \) economic analysis indicated with \( i = 10\% \), a manufacturing cost of 250$ for the XPS panels, and a manufacturing cost of 150$ for the glass-fiber panels that carbon dioxide and argon were favorable with 11.0\( \times \)mo (0.92\ yr) and 13.0\( \times \)mo (1.08\ yr) of payback times, respectively. Calculation with the \( \text{NPV} \) method taking into account the single polystyrene insulation material use did not produce \( \text{NPV}(i) > 0 \). Even after 72\( \times \)mo (6\ yr), \( \text{NPV}(i) < 0 \). It was seen that there was not considerable savings (or cash flow in \( \equiv \) cost of electricity saved at 0.089723$/kWh) relative to the single glass-fiber insulation material and carbon dioxide, argon, nitrogen, and air payback times relative to the single polystyrene insulation material use were too long to make these gases applicable between the polystyrene layers.

Table 6 Payback times of the insulation materials using the \( \text{NPV} \) method.

| Inert filler | \( \text{NPV value} \) | \( \text{Payback time} \), mo (yr) | \( \text{NPV value} \) | \( \text{Payback time} \), mo (yr) |
|--------------|----------------|----------------|----------------|----------------|
| carbon dioxide | \( \text{NPV} < 0 \) | – | 3.722 | 11.0 (0.92) |
| argon | \( \text{NPV} < 0 \) | – | 0.283 | 13.0 (1.08) |
| nitrogen | \( \text{NPV} < 0 \) | – | \( \text{NPV} < 0 \) | – |
| air | \( \text{NPV} < 0 \) | – | \( \text{NPV} < 0 \) | – |

1 Relative to the single polystyrene insulation material. 2 Relative to the single glass-fiber insulation material. Whenever \( \text{NPV} < 0 \), insulation materials with those gases were not able to provide a payback time using the \( \text{NPV} \) method. The method is useful when the cash flow in (saving) sums big numbers.

5. Conclusions

Several insulation materials constructed with successive layers of the polystyrene material and either of the inert gases carbon dioxide, argon, nitrogen, or air were modeled and solved and evaluated. Incorporation of the successive carbon dioxide and argon layers between the polystyrene layers was promising. The filler inert gases are expected to be well sealed-off to prevent their leakage.

Any advection inside the inert gas layers needs to be suppressed for better insulation capability, which limits the size of the inert gas layers on the insulation materials.

Temperature gradient acted favorable on the insulation material effectiveness, in general. High emissivity at the polystyrene surfaces is unwanted rendering the insulation materials less favorable compared to the condition of absence of any surface emissivity. Carbon dioxide provided the most favorable insulation capability followed by argon, nitrogen, and air in respective order. Carbon dioxide, if it is acquired cheaply, can also be economically viable. A large gas gap size is favorable, which is, however, restricted by the corresponding high value of \( Ra \).

The insulation materials were found to pay their implementation costs relative the single glass-fiber insulation material usage in a reasonably short period of time. Carbon dioxide and argon were the most favorable among the other inert gases. There seemed to have been a limited improvement over the use of the single polystyrene material as this polystyrene material had a relatively low thermal conductivity. Nevertheless, even in the case of XPS material, carbon dioxide and argon use showed promise. Neon gas may be equally considered as a choice for the filler inert gas but with argon still performing better than neon. Helium is not seen a viable choice as the filler gas.

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