Neutron elastic scattering on calcium isotopes from chiral nuclear optical potentials

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We formulate microscopic neutron-nucleus optical potentials from many-body perturbation theory based on chiral two- and three-body forces. The neutron self energy is first calculated in homogeneous matter to second order in perturbation theory, which gives the central real and imaginary terms of the optical potential. The real spin-orbit term is calculated separately from the density matrix expansion using the same chiral interaction as in the self energy. Finally, the full neutron-nucleus optical potential is derived within the improved local density approximation utilizing mean field models consistent with the chiral nuclear force employed. We compare the results of the microscopic calculations to phenomenological models and experimental data up to projectile energies of $E = 200$ MeV. Experimental elastic differential scattering cross sections and vector analyzing powers are generally well reproduced by the chiral optical potential, but we find that total cross sections are overestimated at high energies.

I. INTRODUCTION

Nucleon-nucleus optical potentials describe the interaction of a projectile nucleon with a target nucleus by reducing the complicated many-body interactions to an average single particle potential that is complex and energy-dependent. Global phenomenological optical potentials [1,2] are able to describe scattering processes for a large range of nuclei and projectile energies. These potentials are developed by optimizing the model parameters to best reproduce experimental data. Phenomenological potentials yield remarkably good results when interpolating within these ranges, but may not reliably extrapolate to regions where there are no experimental data. Since microscopic optical potentials are built up from fundamental nuclear interactions without tuning to data, they may have greater predictive power in regions of the nuclear chart that are unexplored experimentally.

There has been much interest recently in the development of microscopic optical potentials [3,11] based on chiral effective field theory (EFT) [12,13], which implements realistic microphysics including multi-pion exchange processes and three-body interactions all within a framework that allows for the assessment of theoretical uncertainties. Chiral optical potentials are well suited to describe low-energy scattering processes but are expected to break down for energies approaching the breakdown scale of the theory. In practice, the presence of the cutoff constrains nucleon projectile energies to $E \lesssim 200$ MeV.

In the present study, we compute neutron-nucleus optical potentials along the lines of our previous work in [11] that focused exclusively on the description of proton elastic and total reaction cross sections. Since proton elastic scattering at forward angles approaches the well known Rutherford cross section, the microscopic description of neutron scattering presents a novel challenge that has not yet been addressed in our work. Ultimately our goal is to develop a new microscopic global optical potential for nucleon-nucleus scattering across a large range of isotopes including unstable ones, up to projectile energies of 200 MeV in support of current and future experiments at radioactive ion beam facilities. Presently we consider differential elastic and total cross sections for $^{40,48}$Ca scattering at energies ranging from 3-200 MeV. Additionally, in the first direct test of our microscopic spin-orbit term, the vector analyzing power is calculated at selected energies for $^{40}$Ca scattering. The choice of isotopes and energies is limited by the availability of experimental data for comparison. We also compare the microscopically calculated scattering observables to the results of the global phenomenological optical potential of Koning and Delaroche [2]. Scattering observables are calculated using the TALYS [15] reaction code. While the vector analyzing power by is not output directly by TALYS, it can be extracted from the output files of ECIS-06, a program that runs in the background of TALYS.

We take as the foundation of our calculations a particular high-precision 2N + 3N chiral nuclear potential with momentum-space cutoff $\Lambda = 450$ MeV. The low-energy constants of the potential are fitted to nucleon-nucleon (NN) scattering phase shifts, deuteron properties, and in the case of three-body contact terms, the triton binding energy and lifetime [16]. The nucleon-nucleon interaction is calculated to next-to-next-to-leading order (N3LO), while the three-nucleon force is only calculated at N2LO. Work towards the inclusion of three-nucleon N3LO interactions is in progress [17,22] and we plan to implement them in future works. The chiral nuclear potential employed in the present work reproduces known values for nuclear matter properties, such as saturation energy and density [16], thermodynamics [23,24], and Fermi liquid parameters [25] when calculated to at least second order in many-body perturbation theory. In future works we also plan to calculate the nucleon-nucleus optical potential with a selection of high-precision chiral nuclear forces [26,27] to better assess theoretical uncer-
nuclear matter approach well suited to constructing a nucleon-nucleus optical potential, making the calculated, only the nuclear density distribution is needed to derive optical potentials is its adaptability to many nuclei. Once the nuclear matter optical potential is computed \[30\] by folding the nucleus optical potential is computed \[30\] by folding the nuclear matter optical potential with a nuclear density distribution. The LDA is known to underestimate the surface diffuseness of the optical potential in finite nuclei and requires a modification known as the improved local density approximation (ILDA) \[30\] \[31\] that accounts for the nonzero range of the nuclear interaction.

The main advantage of the nuclear matter approach to deriving optical potentials is its adaptability to many nuclei. Once the nuclear matter optical potential is calculated, only the nuclear density distribution is needed to produce a nucleon-nucleus optical potential, making the nuclear matter approach well suited to constructing a microscopic global optical potential. However, the drawback is that some physical processes present in scattering with finite nuclei are not captured by nuclear matter calculations. Among these are collective surface modes, shell structure effects, and the fact that the spin-orbit term is not present in homogeneous nuclear matter. We therefore include a microscopic spin-orbit term from the improved density matrix expansion \[32\] \[34\] based on chiral interactions that provides a better description of the spin-dependent part of the energy density functional compared to the standard density matrix expansion of Negele and Vautherin \[35\].

The paper is organized as follows. In Section II we compute neutron-nucleus elastic differential scattering cross sections up to a projectile energy \(E = 185\) MeV and total cross sections up to \(E = 200\) MeV. We also calculate the vector analyzing power for elastic \(^{40}\)Ca scattering as a test of our spin-orbit term. These results are compared to empirical data and predictions from the KD phenomenological optical potential. We end with a summary and conclusions.

II. OPTICAL POTENTIAL FROM CHIRAL EFFECTIVE FIELD THEORY

A. Real and imaginary central terms

The nucleon self-energy is calculated as a function of density and momentum in homogeneous nuclear matter of arbitrary isospin asymmetry using a nuclear potential derived from chiral EFT. The expressions for the first- and second-order perturbative contributions to the nucleon self-energy are given by

\[ \Sigma_{2N}^{(1)}(q; k_f) = \sum_1 \langle \vec{q} | \bar{h}_1 s_{s_1} t_{t_1} | V_{2N}^{\text{med}} | \vec{q} | h_1 s_{s_1} t_{t_1} \rangle n_1, \] (1)

\[ \Sigma_{2N}^{(2a)}(q, \omega; k_f) = \sum_{123} \frac{\langle \vec{p}_1 \vec{p}_2 s_{s_1} s_{s_2} t_{t_1} t_{t_2} | V_{2N}^{\text{med}} | \vec{q} \vec{q}_2 s_{s_2} t_{t_2} \rangle^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 + i\eta} n_1 n_2 \bar{n}_3, \] (2)

\[ \Sigma_{2N}^{(2b)}(q, \omega; k_f) = \sum_{123} \frac{\langle \vec{h}_1 \vec{h}_2 s_{s_1} s_{s_2} t_{t_1} t_{t_2} | V_{2N}^{\text{med}} | \vec{q} \vec{q}_2 s_{s_2} t_{t_2} \rangle^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 - i\eta} n_1 n_2 \bar{n}_3, \] (3)

and shown diagrammatically in Fig. 1. In the above expressions \(n_i = \theta(k_i - k_f)\) is the occupation probability for a filled state with momentum \(k_i < k_f\) below the Fermi momentum, the occupation probability for particle states is \(n_i = \theta(k_i - k_f)\), the summation is over intermediate-state momenta for particles \(p_i\) and holes \(\bar{h}_i\), their spins \(s_i\), and isospins \(t_i\). The overbar on the potential indicates that it is properly antisymmetrized. The in-medium effective nuclear potential \(V_{2N}^{\text{med}}\) represents the two-body interaction consisting of the bare nucleon-nucleon (NN) potential \(V_{NN}\) together with an effective, density-dependent (and isospin-asymmetry-dependent) NN interaction \(V_{NN}^{\text{med}}\) derived from the N2LO chiral three-nucleon force by averaging one particle over the filled Fermi sea of noninteracting nucleons \[22\] \[36\] \[38\]. In the first-order Hartree-Fock contribution, Eq. (1), the effective interaction is given by \(V_{2N}^{\text{med}} = V_{NN} + V_{NN}^{\text{med}}\), while for the higher-order contributions, Eqs. (2) and (3), the effective interaction is given by \(V_{2N}^{\text{med}} = V_{NN} + V_{NN}^{\text{med}}\). The Hartree-Fock contribution is nonlocal, energy independent, and purely real, while the second-order contributions are in general nonlocal, energy dependent, and...
where Ψ
subscript occupied orbitals of the non-relativistic many-body system, Negele-Vautherin [35].

To derive optical potentials of neutron- or proton-rich nuclei, it is necessary to calculate the self-energy for arbitrary isospin-asymmetry, δ
rho = (ρ_n - ρ_p)/(ρ_n + ρ_p). The resulting optical potentials for nucleons propagating in homogenous matter with proton and neutron Fermi momenta k_f^n and k_f^p are given by

\[ U_p(E; k_f^n, k_f^p) = V_p(E; k_f^n, k_f^p) + iW_p(E; k_f^n, k_f^p), \]
\[ U_n(E; k_f^n, k_f^p) = V_n(E; k_f^n, k_f^p) + iW_n(E; k_f^n, k_f^p), \]

with

\[ V_i(E; k_f^n, k_f^p) = \Re \Sigma_i(q, E(q); k_f^n, k_f^p), \]
\[ W_i(E; k_f^n, k_f^p) = \frac{M_{i^{Ks}}}{M} \Im \Sigma_i(q, E(q); k_f^n, k_f^p), \]

where the subscript \( i \) denotes a propagating proton or neutron. To relate the microscopically derived imaginary part of the nucleon self-energy to the imaginary term of the optical potential used in phenomenology, the non-locality must be accounted for [39, 40]. This is achieved by multiplying the imaginary term of the effective k-mass \( M_{i^{Ks}} \) defined by

\[ \frac{M_{i^{Ks}}}{M} = \left( 1 + \frac{M}{k} \frac{\partial}{\partial k} V_i(k, E(k)) \right)^{-1}. \]

B. Spin-orbit optical potential

The effective one-body spin-orbit interaction vanishes in homogenous nuclear matter due to translational invariance and thus cannot be computed within the framework described above. To account for the spin-orbit interaction, we employ an improved density matrix expansion [33, 34, 41] to construct the one-body spin-orbit interaction from chiral two- and three-body forces. By utilizing the improved density matrix expansion that takes advantage of phase space averaging, a more accurate spin-dependent energy density functional can be derived compared to the standard density matrix expansion of Negele-Vautherin [35].

From the definition of the density matrix

\[ \rho(\vec{r}_1\sigma_1; \vec{r}_2\sigma_2) = \sum_\alpha \Psi_\alpha^*(\vec{r}_2\sigma_2)\Psi_\alpha(\vec{r}_1\sigma_1), \]

where \( \Psi_\alpha \) are energy eigenfunctions associated with occupied orbitals of the non-relativistic many-body system, the energy density functional for \( N = Z \) even-even nuclei in the Hartree-Fock approximation expanded up to second order in spatial gradients is given by

\[ \mathcal{E}[\rho, \tau, \vec{J}] = \rho \mathcal{E}(\rho) + \left[ \tau - \frac{3}{5} \rho \kappa_f^2 \right] \left( \frac{1}{2M_N} + F_\tau(\rho) \right) + (\vec{\nabla}\rho)^2 F_\tau(\rho) + \vec{\nabla} \rho \cdot \vec{J} F_{SO}(\rho) + \vec{J}^2 F_J(\rho), \]

where \( \rho(\vec{r}) = 2k_f^3(\vec{r})/3\pi^2 = \sum_\alpha \Psi_\alpha^*(\vec{r})\Psi_\alpha(\vec{r}) \) defines the local density with \( k_f(\vec{r}) \) the local Fermi momentum, \( \tau(\vec{r}) = \sum_\alpha \vec{\nabla}\Psi_\alpha^*(\vec{r}) \cdot \vec{\nabla}\Psi_\alpha(\vec{r}) \) is the kinetic energy density, and \( \vec{J}(\vec{r}) = i \sum_\alpha \Psi_\alpha^*(\vec{r})\sigma \times \vec{\nabla}\Psi_\alpha(\vec{r}) \) is the spin-orbit density. These terms are multiplied by the strength functions \( \mathcal{E}(\rho), F_\tau(\rho), F_\tau(\rho), F_{SO}(\rho), F_J(\rho) \). This calculation yields the spin-orbit term \( F_{SO}(\rho) \) of the optical potential for \( N = Z \) nuclei to first order in many-body perturbation theory. Higher-order perturbative contributions [42] to the microscopic nuclear energy density functional will be investigated in future works. We do not include the isovector part [33] of the spin-orbit interaction for \(^{48}\text{Ca}\) in this study since it is known to be small compared to the isoscalar part [34]. In the context of nucleon-nucleus scattering, the spin-orbit term of the optical potential determines the polarization of scattered nucleons. One such polarization observable is the vector analyzing power, which we will calculate microscopically and compare to experimental data and phenomenological results.

C. Improved local density approximation

The improved local density approximation (ILDA) is used to construct the nucleon-nucleus optical potential from the nucleon self energy in nuclear matter. The nucleon-nucleus optical potential is derived by folding the density-dependent self energy with the radial density distribution of a target nucleus and then smeared by integrating over the radial dimension with a Gaussian factor to account for the nonzero range of the nuclear force. The nuclear density distributions are calculated within mean field theory from the Skyrme interaction [29]. The Skyrme interaction is fit to both properties of finite nuclei as well as theoretical calculations of the asymmetric nuclear matter equation of state from the N3LO chiral potential with cutoff scale \( \Lambda = 450 \text{ MeV} \) used to calculate the self-energy. In Fig. 2 we show the resulting nucleon density distributions for \(^{40,48}\text{Ca}\). In order to benchmark these density distributions with experiment we show the charge density distribution for \(^{48}\text{Ca}\) calculated from mean field theory compared to an empirical charge density [43] obtained from electron scattering data. The theoretical charge density for \(^{48}\text{Ca}\) slightly underestimates experimental results from \( 1 \text{ fm} < r < 3 \text{ fm} \) and slightly overestimates in the surface region. We have verified as well that the charge density of \(^{40}\text{Ca}\) from mean field theory has a qualitatively similar comparison to experiment.
In the local density approximation, the nucleon-nucleus optical potential at a given radial distance $r$ is evaluated as

$$V(E; r) + iW(E; r) = V(E; k^p_f(r), k^n_f(r)) + iW(E; k^p_f(r), k^n_f(r)), \quad (11)$$

where $k^p_f(r)$ and $k^n_f(r)$ are the local proton and neutron Fermi momenta. This approximation does not account for the nonzero range of nuclear forces, and when applied to nucleon-nucleus optical potentials it is known to underestimate the surface diffuseness. For this reason, the standard LDA provides an inadequate description of nuclear scattering processes. To account for the range of the nuclear force and obtain a more realistic nuclear optical potential, the improved local density approximation is employed. The ILDA applies a Gaussian smearing

$$V(E; r)_{ILD} = \frac{1}{(t/\sqrt{\pi})^2} \int V(E; r') e^{-r^2/(t/r')^2} \, dt, \quad (12)$$

characterized by an adjustable length scale $t$ associated with the nonzero range of the nuclear force. In Ref. [31] it is found that for the central part of the optical potential $t_C = 1.3$ fm gives the best fit to experimental neutron total cross sections for 10 MeV $< E < 200$ MeV and targets ranging from $^{40}$Ca to $^{208}$Pb. In the present work we vary the range parameter over $1.25 \, fm < t_C < 1.35 \, fm$ to estimate the theoretical uncertainty associated with the choice of length scale $t_C$. As in [31], we find the spin-orbit range parameter to be $t_{SO} = 1.07 \, fm$ and vary it across the range $1.0 \, fm < t_{SO} < 1.1 \, fm$ to estimate the uncertainty.

In Fig. 2 we show the real central, imaginary central, and real spin-orbit terms of the ILDA chiral optical potential compared to the analogous terms of the KD phenomenological optical potential for $n$-$^{40}$Ca at projectile energies $E = 3.2, 30, 85$ MeV. The width of the blue band representing the chiral terms shows the relatively small effect of varying the distance parameter in the ILDA. In the left column of plots, the optical potential terms are shown at $E = 3.2$ MeV. The microscopic real volume term has a very similar depth and a slightly larger diffuseness compared to the KD term. At this low energy, the microscopic imaginary term has a surface peak and a nonzero central depth, whereas the KD imaginary term has virtually no central depth and a relatively large surface peak. The microscopic spin-orbit term has a very similar radial profile compared to KD, but with a larger depth across all energies. The density matrix expansion calculated at the Hartree-Fock level is known [34] to produce a stronger spin-orbit interaction than is required from traditional mean field theory studies of finite nuclei by about 20-50%. The inclusion of multi-pion-exchange processes has been shown [37] to reduce the strength of the one-body spin-orbit interaction in finite nuclei. In future works we intend to account for these processes by including $G$-matrix correlations in the density matrix expansion as outlined in Ref. [12].

At $E = 30$ MeV the middle column of plots in Fig. 3 shows a microscopic real volume term that has a slightly larger central depth and similar diffuseness compared to phenomenology. The microscopic imaginary term has a large central depth with almost no surface peak, while its phenomenological counterpart has a small central depth and moderate surface peak. This feature has been observed in other microscopic optical potentials calculated from nuclear matter [48-51]. To mitigate this discrepancy, some semi-microscopic optical potentials apply an energy-dependent scaling factor to the imaginary term [31, 52], but in the present work we employ no such factors. As the energy increases to $E = 85$ MeV, the real volume term becomes more shallow for both the microscopic and phenomenological potentials while qualitatively remaining the same relative to each other. At such high energy, the imaginary surface peak is no longer present in either the microscopic or phenomenological potentials. However, at this energy the central depth of the microscopic imaginary term is very large compared to phenomenology. This results in a chiral optical potential that is overly absorptive at high energy.

D. Parameterization of the chiral optical potential

In order to facilitate the implementation of our microscopic optical potential into nuclear reaction codes, we fit our optical potential to the phenomenological form of Koning and Delaroche. Our aim is to eventually construct a global microscopic optical potential and make it available in a convenient form for the nuclear reaction community. The Koning-Delaroche phenomenolog-
FIG. 3. The real, imaginary, and spin-orbit terms of the n-^40^Ca optical potential at projectile energies \( E = 3.20, 30, 85 \text{ MeV} \). The blue bands represent the microscopic chiral optical potential after applying the improved local density approximation with a varied length scale. The green dashed lines represent the analogous terms of the Koning-Delaroche global optical potential.

The phenomenological imaginary spin-orbit term is not considered in the current work since it has a negligible effect on elastic scattering cross sections at relatively low energies due to its very small magnitude and cannot be extracted within the present microscopic approach. The energy and radial dependence of the terms in the phenomenological optical potential are assumed to factorize according to

\[
U(r, E) = V_V(r, E) + iW_V(r, E) + iW_D(r, E) + V_{SO}(r, E)\mathbf{\ell} \cdot \mathbf{s} + iW_{SO}(r, E)\mathbf{\ell} \cdot \mathbf{s},
\]

consisting of a real volume term, imaginary volume and surface terms, and real and imaginary spin-orbit terms. The phenomenological imaginary spin-orbit term is not considered in the current work since it has a negligible effect on elastic scattering cross sections at relatively low energies due to its very small magnitude and cannot be extracted within the present microscopic approach. The energy and radial dependence of the terms in the phenomenological optical potential are assumed to factorize according to

\[
V_V(r, E) = V_V(E)f(r; r_v, a_V),
\]

\[
W_V(r, E) = W_V(E)f(r; r_w, a_W),
\]

\[
W_D(r, E) = -4a_DW_D(E)\frac{d}{dr}f(r; r_D, a_D),
\]

\[
V_{SO}(r, E) = V_{SO}(E)\frac{1}{m_s^2} \frac{1}{r} \frac{d}{dr}f(r; r_{SO}, a_{SO}),
\]

where

\[
f(r; r_i, a_i) = \frac{1}{1 + e^{(r - A^{1/3}r_i)/a_i}}
\]

is the Woods-Saxon shape factor with \( A \) the mass number and \( r_i, a_i \) the energy-independent geometry parameters that represent the size and diffuseness of a given target nucleus respectively. In phenomenological and microscopic optical potentials, these shape parameters vary weakly with the target nucleus. The chiral optical potential is fit to the KD form at a given energy thus there is no explicit parameterization of the energy dependence. We note that the microscopic real spin-orbit optical potential is calculated from the density matrix expansion at the Fermi energy \( E_F \) and has no energy dependence. We therefore incorporate a phenomenological energy dependence that is small and constant across all nuclei into our parameterization of the spin-orbit optical potential.

III. RESULTS

In a continuation of Ref. [11], we calculate cross sections and the vector analyzing power of neutrons scattering on calcium isotopes from a microscopic optical potential based on chiral forces and compare to experiment and phenomenology. Both the differential elastic scattering
cross sections and total cross sections are calculated for n-\(^{40}\)Ca at energies where experimental data are available. Specifically, we compute differential elastic scattering cross sections for n-\(^{40}\)Ca at projectile energies \(E = 3.2, 5.3, 6.52, 11.9, 16.9, 21.7, 25.5, 30, 40, 65, 85, 107.5, 155, 185\) MeV. In order to test the spin-orbit term, vector analyzing powers are also calculated at \(E = 11.9, 16.9\) MeV. Differential elastic scattering cross sections are calculated for n-\(^{48}\)Ca at \(E = 7.97, 11.9, 16.9\) MeV. The total cross sections for n-\(^{40,48}\)Ca scattering are also calculated. Energies exceeding 200 MeV are not considered since the chiral expansion is expected to break down near that energy scale [22]. Experimental data are taken from Refs. [55, 63]. The TALYS reaction code is used to calculate all scattering observables. In all cases we employ the microscopic optical potential calculated using the ILDA and parameterized to the Koning-Delaroche phenomenological form at a specific energy. Presently the only theoretical uncertainties considered are those for the ILDA length scales \(l_C\) and \(l_{SO}\). In future works we will consider a wider class of chiral nuclear potentials in order to more accurately assess the complete theoretical uncertainty. We also include results from the KD global phenomenological optical potential [2].

A. Microscopic optical potential at low energy

Low-energy nuclear reactions are important for a wide range of astrophysical applications. These reactions play an important role in cold \(r\)-process environments [66, 67] such as neutron star mergers where freeze-out is achieved rapidly and neutron capture plays an enhanced role. Neutron capture rates on exotic, neutron-rich isotopes have large theoretical uncertainties [69]. These neutron-capture rates are included as inputs for most modern \(r\)-process reaction network codes. The neutron-nucleus optical potential, and especially the imaginary part of the optical potential at low energies [52], is a key ingredient in calculating neutron capture rates. One of the primary motivations for the construction of a new global microscopic optical potential is to better understand (and potentially reduce) these theoretical uncertainties. In the future, we will directly implement the developed microscopic optical potentials to applications including neutron-capture cross sections. In the present work, we benchmark to microscopic optical potentials at low energies.

In Fig. 4 we show microscopic and phenomenological elastic scattering cross sections for neutron projectiles on a \(^{40}\)Ca target at energies of \(E = 3.2, 5.3, 6.52\) MeV as well as \(^{48}\)Ca at \(E = 7.97\) MeV and compare to experimental data [55, 60, 64]. Interestingly, there is very little difference between the chiral optical potential predictions and those of phenomenology. We find that the Koning-Delaroche global optical potential is in very good agreement with experimental data in this energy regime when the direct and compound contributions to the elastic scattering cross section are accounted for (cf. Ref. [6]). The compound contribution to the elastic scattering cross section is experimentally indistinguishable to the shape elastic contribution and must be included when comparing to experimental data. In the top plot of Fig. 4, we provide a comparison to the results found in Ref. [51] for elastic n-\(^{40}\)Ca scattering at \(E = 3.2\) MeV. The results by Idini et al. are obtained through an ab initio calculation of the optical potential using a self consistent Green function approach. We see that the nuclear matter approach in the improved local density approximation gives better agreement with data than the fully ab initio approach of...
B. Microscopic optical potential at medium-low energy

In Fig. 5 we plot microscopic and phenomenological differential elastic scattering cross sections for neutrons on \(^{40,48}\text{Ca}\) targets at \(E = 11.9, 16.9\) MeV and compare to experimental data \([50, 62 - 63]\). At the neutron projectile energy of 11.9 MeV, we find a significant discrepancy between the microscopic results and experimental data at certain scattering angles. In particular, for \(E = 11.9\) MeV the \(^{40,48}\text{Ca}\) cross sections from the chiral optical potential have a sharp dip around \(\theta = 45^\circ\) which is not present in the experimental data. For larger scattering angles, the chiral optical potential results have better agreement with experiment than the KD potential, whose predictions are uncharacteristically departed from experimental data. At \(E = 16.9\) MeV the phenomenological and microscopic optical potentials both predict a dip just below \(\theta = 40^\circ\) that is partly confirmed by experiment. At larger scattering angles, results from the chiral optical potential tend to overestimate the elastic scattering cross sections, while phenomenological optical potentials moderately underestimate them. The large disagreement between microscopic calculations and experimental results in this narrow energy range may be due to resonances and surface effects that are not accounted for in the nuclear matter approach. One such resonance present in the relevant energy range is the giant dipole resonance (GDR). The cross section for \(^{40}\text{Ca}(n,\gamma)^{41}\text{Ca}\) is shown in Ref. \([65]\) to be enhanced around \(E = 12 - 20\) MeV due to the GDR. This resonance could be in part responsible for the large discrepancies between experimental data and our micro-

Idini et al., which might be due to different theoretical nuclear density distributions or the density of states in the two approaches.
scopic nuclear matter calculations.

We also plot the vector analyzing power for $^{40}$Ca at $E = 11.9, 16.9$ MeV in Fig. [6]. The vector analyzing power is a spin observable defined by

$$ A_y(\theta) = \frac{1}{p_y} \frac{\sigma(\theta) - \sigma_0(\theta)}{\sigma_0(\theta)}, $$

where $\sigma$ and $\sigma_0$ correspond to the scattering cross sections for a polarized and unpolarized beam respectively and $p_y$ is the beam polarization in the direction normal to the scattering plane. This quantity is largely determined by the spin-orbit term of the optical potential. In this first direct test of our chiral spin-orbit potential we find that overall it reproduces experimental data well. In particular, the angles at which the polarized cross section $\sigma$ is equal to the unpolarized cross section $\sigma_0$ are reproduced very well.

C. Microscopic optical potential at medium-high energy

In Fig. [7] we plot microscopic and phenomenological differential elastic scattering cross sections for neutrons on $^{40}$Ca targets at $E = 21.7, 25.5, 30, 40$ MeV and compare to experimental data [58, 59]. For relatively low scattering angles in the range of $0^\circ < \theta < 80^\circ$, the microscopic optical potential produces cross sections that are consistent with experiment and the phenomenological KD optical potential. However, at larger scattering angles the microscopic calculations of the cross sections exhibit a weaker interference pattern, that persists as the energy increases. Overall, the microscopic elastic scattering cross sections are larger than experiment at high scattering angles.

From Fig. [3] we expect that the underlying cause of these discrepancies is due to the imaginary part of the microscopic optical potential. At these intermediate projectile energies, the imaginary volume integral is close to phenomenology. However, the microscopic surface imaginary peak is too small, as can be seen in Fig. [3] leading to larger elastic scattering cross sections. In contrast the imaginary volume part is much larger than phenomenology at higher projectile energies. We have verified that replacing only the microscopic imaginary part with the Koning-Delaroche phenomenological imaginary part leads to significantly improved angular distributions for $\theta > 80^\circ$.

D. Microscopic optical potential at high energy

In Figs. [6], we plot microscopic and phenomenological differential elastic scattering cross sections for neutrons on $^{40}$Ca targets at $E = 65, 85, 107.5, 155, 185$ MeV and compare to experimental data [67, 61]. In Fig. [6], we see that the cross sections from chiral effective field theory exhibit the same angular dependence as the experimental data, but microscopic many-body theory systematically underestimates the cross section across all scattering angles. In contrast, the KD phenomenological optical potential reproduces the experimental cross section up to $\theta = 25^\circ$ well. For larger scattering angles, however, the phenomenological cross sections are smaller than experiment but very similar to those from chiral effective field theory.

In Fig. [5] we compare experimental, phenomenological, and microscopic differential elastic scattering cross sections for n-$^{40}$Ca at $85 \text{ MeV} < E < 185 \text{ MeV}$. For these projectile energies, the experimental data span only a small set of scattering angles $\theta \leq 25^\circ$ with associated large uncertainties up to a factor of $2 - 5$ in the cross section. The results from chiral effective field theory are
consistent with data up to experimental error bars in most cases, but the tendency is again for the microscopic optical potential to underestimate the cross sections. In all cases the KD results are within or very close to experimental data.

E. Total cross section

The total cross section is written as the sum of the elastic scattering and reaction cross section:

$$\sigma_T = \sigma_{el} + \sigma_{re}. \quad (20)$$

The reaction cross section in particular is expected to be very sensitive to the strength of the imaginary part of the optical potential. Consequently, we expect chiral optical potentials, with their large imaginary volume parts, to produce a large reaction cross section and hence a large total cross section at high energies. At low and moderate energies, the picture is more complicated as demonstrated in Ref. [11]. At low energies the microscopic surface imaginary part is small and the volume imaginary part is large compared to phenomenological optical potentials. Depending on the energy, the volume integral of the microscopic imaginary part is therefore either larger or smaller than phenomenology and the reaction cross section behaves analogously.

In Fig. 10 we show the total cross sections for neutron scattering on $^{40,48}$Ca from the chiral optical potential and the KD phenomenological optical potential. The chiral results are shown as the blue band, while the KD results are shown as dashed green lines. Experimental data [65] are shown with red circles. In both plots of Fig. 10 the microscopic optical potential overestimates the total cross section for low energy then underestimates the cross section for medium energy. Past $E = 100 \text{ MeV}$ the total cross section from chiral nuclear optical potentials is systematically too large. As mentioned above, this can be traced to the overly absorptive imaginary term. Overall, the phenomenological optical potential of Koning and Delaroche gives a good description for both isotopes at most energies. The only exception is the $n^{40,48}$Ca total cross section for projectile energies in the range $10 \text{ MeV} < E < 50 \text{ MeV}$, where the KD total cross sections are small compared to experiment. The experimental data in Ref. [65] were not included in the KD potential since the experiment was performed more recently. Additionally, for the previously mentioned energy range, these experimental data are in slight disagreement with previous experimental results [69] that the KD potential is fit to. We choose to plot only the more recent results are shown as dashed green lines. Experimental data in Ref. [65] were not included in the KD global phenomenological optical potential. The cross section calculated from the Koning Delaroche phenomenological optical potential is given by the blue band. The cross section calculated from the chiral optical potential is given by the green dashed curve, and experimental data are represented by red circles with error bars.
part into the microscopic optical potential. We see that indeed there is a significant improvement in the description of the total cross section, which motivates the need to improve the imaginary part of the microscopic optical potential.

IV. CONCLUSIONS

This work represents a continuation of an effort to construct a microscopic global optical potential based on nucleon interactions from chiral effective field theory. By calculating the nucleon optical potential in nuclear matter for arbitrary density and isospin-asymmetry, one can derive an optical potential for many isotopes across the nuclear chart by utilizing the improved local density approximation. In previous works the optical potential was calculated in nuclear matter \cite{3,4} and more recently proton optical potentials were calculated for a chain of calcium isotopes \cite{11}. New to this work are calculations of the neutron optical potential for $^{40,48}$Ca and a direct test of the microscopic spin-orbit term by calculating spin observables.

Overall, we find good agreement with experimental differential elastic scattering data, except in energy regions where unresolved resonances are expected to be important. At the highest energies ($E \approx 80 - 200$ MeV) we also find that the large imaginary volume contribution from microscopic optical potentials tends to suppress elastic scattering compared to experimental data. This feature is enhanced in microscopic calculations of the total cross section, which are too large at high energies due to the large reaction cross section induced by the strongly absorptive imaginary part. We have also computed for the first time in our improved local density approximation the vector analyzing power. We find that the analyzing power for n-$^{40}$Ca at medium energies is well described by our microscopic optical potentials, validating in particular its spin-orbit part.

We emphasize that no parameters in the model were tuned to experimental reaction data, and therefore the present work demonstrates the viability of using microscopic optical potentials in regions of the nuclear chart that are unexplored experimentally. In the future we plan to compute neutron-capture cross sections on exotic isotopes and more thoroughly explore theoretical uncertainties \cite{27,70} associated with the isovector part of the nuclear optical potential. We also plan to consider higher-order perturbative contributions to the self energy that may improve the description of the imaginary part of the optical potential and the overall spin-orbit strength.

Acknowledgments

We thank F. Nunes, G. Potel, and J. Rotureau for helpful discussions. Work supported by the National Science Foundation under Grant No. PHY1652199 and by the U.S. Department of Energy National Nuclear Security Administration under Grant No. [de-na0003841]. Portions of this research were conducted with the advanced computing resources provided by Texas A&M High Performance Research Computing.

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