Dancing with Venus in the shadow of the Earth: a pair of genetically related near-Earth asteroids trapped in a mean-motion resonance

C. de la Fuente Marcos$^1$ and R. de la Fuente Marcos$^2$

$^1$Universidad Complutense de Madrid, Ciudad Universitaria, E-28040 Madrid, Spain
$^2$AEGORA Research Group, Facultad de Ciencias Matemáticas, Universidad Complutense de Madrid, Ciudad Universitaria, E-28040 Madrid, Spain

Abstract

YORP-induced fission events may form dynamically coherent pairs or even families of asteroids. The outcome of this process is well documented among members of the main asteroid belt, but not in the case of the near-Earth asteroid (NEA) population because their paths randomize very efficiently in a short time-scale. Mean-motion resonances (MMRs) may stabilize the orbits of small bodies by making them avoid close encounters with planets. In theory, YORP-induced fission of asteroids trapped in MMRs can preserve evidence of this process even in near-Earth space. Here, we show that two NEAs, 2017 SN$_{16}$ and 2018 RY$_7$, are currently following an orbital evolution in which their relative mean longitude does not exhibit any secular increase due to the stabilizing action of the 3:5 MMR with Venus. The mechanism that makes this configuration possible may be at work both in the Solar system and elsewhere. Our analysis suggests that the pair 2017 SN$_{16}$–2018 RY$_7$ may have had its origin in one out of two mechanisms: YORP-induced splitting or binary dissociation.

Key words: celestial mechanics – minor planets, asteroids: general – minor planets, asteroids: individual: 2017 SN$_{16}$ – minor planets, asteroids: individual: 2018 RY$_7$ – planets and satellites: individual: Venus – planets and satellites: individual: Earth.

1 INTRODUCTION

The Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect (see e.g. Bottke et al. 2006) can induce spin-up of asteroids and mass shedding. Dynamically coherent pairs or groups of asteroids probably produced by this process have been found among members of the main asteroid belt (see e.g. Vokrouhlický & Nesvorný 2008; Pravec et al. 2018). Near-Earth asteroids (NEAs) should also fission via the YORP mechanism (see e.g. Jacobson & Scheeres 2011), but any dynamically coherent pairs resulting from this process are difficult to identify because the orbits randomize very quickly in near-Earth space (see e.g. Schunová et al. 2012, 2014).

Mean-motion resonances (MMRs; see e.g. Gallardo 2006, 2019) can make orbits long-term stable by protecting small bodies against close encounters with planets, in our case the Earth–Moon system (see e.g. Milani et al. 1989). In theory, YORP-induced fission of asteroids trapped in MMRs may preserve the evidence of this process even in near-Earth space. Resonant confinement has been previously discussed within the context of cometary dust dynamics (see e.g. Asher, Bailey & Emel’Yanenko 1999) and ring dynamics (see e.g. Namouni & Porco 2002). Here, we show that two NEAs, 2017 SN$_{16}$ and 2018 RY$_7$, are currently trapped in the 3:5 MMR with Venus and following an unusual mutual orbital evolution, which may be consistent with an origin in a YORP-induced fission event. This Letter is organized as follows. Section 2 presents the available data on this pair of NEAs. Their orbital evolution is studied in Section 3 and their origin discussed in Section 4. In Section 5, we compare with predictions from a new four-dimensional orbit model of the NEA population. Our results are discussed in Section 6. Section 7 summarizes our conclusions.

2 THE NEA PAIR 2017 SN$_{16}$–2018 RY$_7$: DATA

The orbit determinations used in this work have been obtained from Jet Propulsion Laboratory’s (JPL) Small-Body Database (SBDB). Minor body 2017 SN$_{16}$ was discovered by A. R. Gibbs working for the Mount Lemmon Survey in Arizona (1.5-m reflector + 10K CCD) on 2017 September 24 (Schwartz et al. 2017). Additional data have been obtained during the last favourable observation window (Gilmore et al. 2018). It is a relatively small object with an absolute magnitude, $H = 23.3$ mag (assumed $G = 0.15$), which suggests a diameter close to 80 m, but with a possible range of values of 38–170 m for an assumed albedo in the range 0.60–0.03. This Apollo asteroid has a semimajor axis $a = 1.0161$ au, and moves in a low-eccentricity, $e = 0.1455$, and moderate-inclination, $i = 13.38$, orbit that keeps it confined to the neighbourhood of the Earth–Moon system (see Table 1); its Minimum Orbit Intersection

* E-mail: nbplanet@ucm.es

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1 https://ssd.jpl.nasa.gov/sbdb.cgi
Distance (MOID) with our planet is 0.093 au. These orbital properties make it relatively easy to access from the Earth and it is part of the Near-Earth Object Human Space Flight Accessible Targets Study (NHATS)\(^2\) list (Abell et al. 2012). Asteroid 2018 RY\(_7\) was first observed on 2018 September 14 by B. M. Africano also working for the Mount Lemmon Survey (Ries et al. 2018). Its orbit determination still requires improvement, but the values of its orbital elements are markedly similar to those of 2017 SN\(_{16}\) (see Table 1); with \(H = 24.4\) mag it could be \(\sim 45\) m wide, its MOID with the Earth is 0.094 au, and it is also listed by NHATS.

Neglecting binaries and higher multiplicity systems, the pair 2017 SN\(_{16}\)--2018 RY\(_7\) shows the highest degree of orbital coherence ever observed among NEAs. Although they are not binary companions, the asteroids happen to be rather close to each other, far closer than might be attributed to chance. Such an arrangement has never before been observed among low-mass minor bodies in near-Earth space. Being small NEAs, they may be pieces of larger asteroids and there are a number of processes that can make this possible. In addition to the YORP mechanism pointed out above, subcatastrophic collisions in which a small body hits a larger object can produce fragments (see e.g. Durda et al. 2007), but they can also be released as a result of tidal disruption events during very close encounters with planets (see e.g. Schunov et al. 2014). On the other hand, the present-day values of their semimajor axes are close to 1.0168037 au, the location of the 3:5 MMR with Venus, and they are both strong candidates to being locked in this planetary resonance. It is however possible that being locked in resonance with Venus plays a major role in preserving their high degree of orbital coherence. In order to validate these theoretical expectations, a representative set of control orbits must be integrated forward and backwards in time to confirm that the dynamical evolution of this pair of NEAs over a reasonable amount of time is consistent with not being close by chance and that the MMR with Venus is actually responsible for what is being observed. The critical angles relevant to such a numerical exploration are the relative mean longitude, \(\lambda_t\), or difference between the mean longitudes of the NEAs (to study the mutual evolution of the pair), and the resonant angle between one NEA and Venus, \(\sigma_V = 5\lambda - 3\lambda_V - 2(\Omega + \omega)\) — to study the 3:5 MMR with Venus. In celestial mechanics, the mean longitude of an object — planet or minor body — is given by \(\lambda = M + \Omega + \omega\), where \(M\) is the mean anomaly, \(\Omega\) is the longitude of the ascending node, and \(\omega\) is the argument of perihelion (see e.g. Murray & Dermott 1999). Resonance happens when the value of \(\sigma_V\) oscillates or librates over time. The 3:5 MMR with Venus is not the strongest or traditionally most populated (Gallardo 2006).

### 3 THE NEA PAIR 2017 SN\(_{16}\)--2018 RY\(_7\): EVOLUTION

In order to explore the details of the orbital evolution of the pair 2017 SN\(_{16}\)--2018 RY\(_7\), we use a direct \(N\)-body code that implements a fourth-order version of the Hermite integration scheme (Makino 1991; Aarseth 2003). The standard version of this software is publicly available from the IoA web site.\(^3\) Our calculations use the latest orbit determinations and include perturbations by the eight major planets, the Moon, the barycentre of the Pluto–Charon system, and the three largest asteroids. Further details of the code and of our overall approach and physical model can be found in de la Fuente Marcos & de la Fuente Marcos (2012a). Data to generate initial conditions as well as other input data have been obtained from JPL’s SBDB. Figure 1 shows the short-term orbital evolution of the pair (nominal orbits in Table 1) and confirms that 2017 SN\(_{16}\) and 2018 RY\(_7\) are engaged in an unusual dance that keeps them not far from each other for an extended period of time. As suspected, the 3:5 MMR of the pair with Venus keeps them together (see Fig. 2). When the pair leaves the planetary resonance their dancing engagement ends abruptly. It can however be argued that the orbit of 2018 RY\(_7\) is too uncertain to confirm this analysis.

In order to investigate if the uncertainties have an impact on our results, we have applied the covariance matrix methodology described in de la Fuente Marcos & de la Fuente Marcos (2015); the covariance matrices necessary to generate initial conditions have been obtained from JPL’s SBDB. The results of the evolution of 500 control orbits generated using this approach are presented in Fig. 3; in general, the dispersions (in pink and red) are too small to play any role. Our analysis also suggests that the dynamical age of this pair is younger than about 60,000 yr (see Fig. 2), although the most likely value is around 14,600 yr (not shown in the figures). If we focus on the critical angles, Fig. 4 shows the results of 1000 control orbits and the dispersions are consistently small. Therefore, we can confirm that our numerical results are robust and the orbital evolution of this pair is well characterized within the time window explored here.
The NEA pair 2017 SN$_{16}$–2018 RY$_7$: ORIGIN

Regarding the origin of the pair 2017 SN$_{16}$–2018 RY$_7$, four scenarios may be considered: accidental proximity induced by differential precession in $\Omega$ and $\omega$, tidal disruption after a close flyby with our planet, binary dissociation, and YORP-induced rotational disruption. Although the first mechanism appears to be behind the orbital evolution of 15810 Arawn (1994 JR$_1$) —see de la Fuente Marcos & de la Fuente Marcos (2012b, 2016b) and Porter et al. (2016)— it can be easily discarded in the present case because Fig. 2 shows that the orbital engagement between 2017 SN$_{16}$ and 2018 RY$_7$ is not of a recurrent nature, but a configuration that remains relatively stable for an extended period of time (see the value of the relevant critical angle, in purple); Arawn could be a quasi-satellite of Pluto, but this is not a plausible dynamical status for the pair of NEAs under analysis here (more on this in Section 6). Tidal disruption after a planetary close encounter must be discarded as well because flybys at rather short planetary distances, in the range 2–5 planetary radii (see e.g. Sridhar & Tremaine 1992), are required and this is not observed during the stable phase of the simulations. As for the third scenario, binary dissociation requires the presence of a pre-existing binary system, but 2017 SN$_{16}$ has $H = 23.2$ mag and one may wonder if asteroids that small may host long-term binary companions. At the time of this writing, the faintest known asteroid with a companion is the NEA 2015 TD$_{144}$. 

Figure 1. The top panel shows the evolution of the values of the critical angles over the time span (−300, 300) yr according to the nominal orbit determinations in Table 1—in purple, the relative mean longitude of the pair 2017 SN$_{16}$–2018 RY$_7$, in orange, the resonant angle associated with the 3:5 MMR of 2017 SN$_{16}$ with Venus, and in teal, the one of 2018 RY$_7$. The mutual distance is displayed in the middle panel. The bottom panel shows the orbital arrangement projected on to the ecliptic plane in a frame of reference centred at the Sun that rotates with 2017 SN$_{16}$—2017 SN$_{16}$ in blue, 2018 RY$_7$ in red. The orbit of 2017 SN$_{16}$ is indicated in black. All the control orbits investigated in this work evolve in a similar fashion within this time interval. The zero-point in time corresponds to epoch JD 2458600.5 TDB, 27-April-2019.

Figure 2. Same as Fig. 1, top panel, but showing the evolution of the critical angles over a longer period of time.

Figure 3. Time evolution of the dispersions of the values of the orbital elements of 2017 SN$_{16}$ (grey/pink) and 2018 RY$_7$ (black/red): semimajor axis (top panel), eccentricity (second to top panel), inclination (middle panel), longitude of the ascending node (second to bottom panel), and argument of perihelion (bottom panel). Average values are displayed as thick grey/black curves and their ranges (1σ uncertainties) as thin pink/red curves.

Figure 4. Evolution of the dispersions (in red, mean in black) of the value of the relative mean longitude (top panel) of the pair 2017 SN$_{16}$–2018 RY$_7$ and that of the resonant angle (bottom panel) of 2018 RY$_7$ with Venus.
with $H = 22.6$ mag; therefore, it is in principle possible that 2018 Ry$_7$ could be a former binary companion of 2017 SN$_{16}$ that became unbound at some point in the past as a result of an external action (e.g. small impact). The asteroidal YORP effect — which is the result of anisotropic reemission of sunlight from the surfaces of the affected minor bodies — can slowly increase their rotational speed, leading them to reach their fission limit and eventual disruption (see e.g. Walsh, Richardson & Michel 2012; Jacobson et al. 2016). This mechanism is behind the fourth scenario and it can produce binaries as well as unbound asteroid pairs. With the available data, it is virtually impossible to decide: 2018 Ry$_7$ may well be a YORPlet as described by Christou et al. (2017) that came from the putative disrupted progenitor of 2017 SN$_{16}$, but it may also be a former binary companion of 2017 SN$_{16}$ formed by YORP-induced fission, or some other mechanism, that became unbound at some point.

5 NEO ORBIT MODEL PREDICTIONS

If the NEA pair 2017 SN$_{16}$–2018 Ry$_7$ (and perhaps other NEAs in similar orbits) is the result of a recent fragmentation or binary dissociation event, such orbits must be largely absent from synthetic, debiased data from a near-Earth object (NEO) population model that includes both asteroid and comets. By comparing observational and synthetic data, we may be able to understand better the circumstances surrounding the formation of this unusual NEA pair. The NEO orbit model developed by the Near-Earth Object Population Observation Program (NEOPOP) and described by Granvik et al. (2018) is the state-of-the-art tool fit for the purpose; this new four-dimensional model provides debiased steady-state distributions of $a$, $e$, $i$, and $H$ for $H < 25$ mag. The software that implements this model is publicly available$^5$ and it has been successfully validated (Granvik et al. 2016, 2017; Granvik & Brown 2018). We have used the list of NEOs with $H < 25$ mag currently catalogued (as of 2018 November 4, 14390 objects) to estimate how likely is that the pair 2017 SN$_{16}$–2018 Ry$_7$ could be explained by the NEO orbit model. In order to do this, we have applied a randomization test (Fisher 1935). As test statistics, we use the differences between the observed number of NEOs in orbits close to those of the pair 2017 SN$_{16}$–2018 Ry$_7$; and the number predicted by the NEOPOP software. Following de la Fuente Marcos & de la Fuente Marcos (2018), we consider two $D$-criteria, $D_{LS}$ and $D_{R}$, to count two NEOs as dynamically similar: $D_{LS}$ as in equation 1 of Lindblad (1994) and the $D_{R}$ from Valsecchi, Jopek & Froeschle (1999). We find nine NEOs that follow 2017 SN$_{16}$–like orbits and eight that follow 2018 Ry$_7$–like ones; in contrast, NEOPOP predicts 1.1±0.9 and 1.3±1.1, respectively, for a synthetic sample of the same size. With these results, the respective differences (our test statistics) are 7.9±0.9 and 6.7±1.0. If we extract two random samples, we can compute the number of synthetic NEOs in orbits similar to each member of the pair for both samples and calculate the differences. In order to obtain statistically significant results, we have repeated this experiment 10000 times and our results are summarized in Fig. 5 where our test statistics are plotted as vertical black lines. The probability of obtaining a difference $> 7.0$ for 2017 SN$_{16}$-like orbits is 0.0001 and that of getting a value $> 7.9$ is 0.0001; for 2018 Ry$_7$, the probability of obtaining a value $> 5.7$ is 0.0011 and that of getting one $> 6.7$ is 0.0001. Our analysis using NEOPOP strongly suggests that this pair of NEOs may have not followed the conventional dynamical pathways that populate near-Earth orbital parameter space. The pair must have been produced within near-Earth space.

6 DISCUSSION

The study of the pair of NEAs considered here opens a window into the present-day dynamical processes that are shaping the NEO population. Even if most NEOs may have been originally delivered from the main asteroid belt, their arrival parameters do not remain frozen in time and a tangled web of mean-motion and secular resonances (see e.g. de la Fuente Marcos & de la Fuente Marcos 2016a) together with the Yarkovsky and YORP effects, and perhaps others, might continuously modify the distributions of $a$, $e$, $i$, and $H$. This is at least what can be understood from the study of the NEA pair 2017 SN$_{16}$–2018 Ry$_7$. The unusual dynamical behaviour observed may not be exclusive of this pair: in Fig. 6 we observe another episode of dynamical coherence, in this case of 2010 Af$_3$ — another NEA close to the 3:5 MMR with Venus — with respect to 2017 SN$_{16}$, in the future.

Although the value of the relative mean longitude of the pair 2017 SN$_{16}$–2018 Ry$_7$ oscillates over time around $0^\circ$, these objects are not engaged in a mutual quasi-satellite dynamical state. In the

$^4$ https://echo.jpl.nasa.gov/lance/binary.neas.html
$^5$ http://neo.ssa.esa.int/neo-population
Solar system, quasi-satellites are minor bodies that appear to travel retrograde or backwards around a host when observed in a frame of reference that rotates with the host (see e.g. Mikkola et al. 2006; de la Fuente Marcos & de la Fuente Marcos 2016b). The value of the relative mean longitude of the quasi-satellite with respect to the host librates, but the orbital evolution of the quasi-satellite is controlled by the combined action of the Sun and the host. The probable values of the masses of the NEAs studied here are too small to play any role. Because of this, the case of this pair of NEAs is very different from that of Saturn and the moons Janus and Epimetheus (see e.g. Murray & Dermott 1999). Figure 1 can be explained as the result of two asteroids being concurrently trapped in the 3:5 MMR with Venus. Here, the average angular speed of the asteroids is nearly the same and equal to that which corresponds to the resonance location. Due to this, the difference in the mean longitudes of the asteroids shown in Figs 1 (top panel), 2, and 4 does not exhibit any secular increase and thus resembles what is seen for objects trapped in the quasi-satellite state. What we have shown is that two NEAs are currently engaged in a fuzzy-binary configuration, thanks to the stabilizing action of the 3:5 MMR with Venus. The essence of this mechanism is summarized in Fig. 2, where the libration of $\lambda$ is sustained by that of $\sigma_i$. The mechanism that makes this configuration possible may be at work elsewhere as long as all the ingredients are present: small bodies trapped in a stable planetary MMR.

7 CONCLUSIONS

In this Letter, we have presented the first example of a new type of orbital configuration, a pair of asteroids kept close to each other for an extended period of time by a non-co-orbital MMR. This study has been carried out using the latest data, direct N-body calculations, a state-of-the-art NEO orbit model, and statistical analyses. Our conclusions can be summarized as follows:

(i) We have identified a pair of NEAs, 2017 SN$_{16}$–2018 RY$_7$, trapped in the 3:5 MMR with Venus that seem to orbit around a common point when viewed in a frame of reference co-rotating with the pair. Their evolution resembles that of a quasi-satellite, but they are not engaged in true quasi-satellite resonant behaviour as the values of their masses are negligible.

(ii) Extensive calculations show that the pair of NEAs 2017 SN$_{16}$–2018 RY$_7$ may have been engaged in its current orbital dance for several thousands of years and they will remain in the same dynamical state for a similar amount of time.

(iii) Mechanisms able to create such a peculiar pair include YORP-induced splitting and binary dissociation within MRMs; simple resonance trapping cannot explain the high degree of orbital coherence exhibited by the pair of NEAs 2017 SN$_{16}$–2018 RY$_7$. Given the nature of the past orbital evolution of this pair, its existence is perhaps the first piece of solid evidence in favour of YORP-induced rotational disruption (or binary dissociation) taking place in the immediate neighbourhood of our planet.

(iv) The orbital configuration studied here may also be found among other small bodies trapped in MRMs both in near-Earth space and elsewhere.

This unusual pair of NEAs will remain favourably positioned for further investigation during the next few years. Spectroscopic studies of 2017 SN$_{16}$–2018 RY$_7$ during their future approaches to our planet (2019–2022) should be able to confirm whether they have similar chemical compositions or not, and therefore shed additional light on the mechanism that led to their formation. New data (e.g. astrometry, light curves, and albedos) may also clarify the role of the Yarkovsky and YORP effects within the context of this particularly complex orbital configuration.

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