Reconsidering maximum luminosity

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Abstract:
The suggestion that there is a maximum luminosity (maximum power) in nature has a long and somewhat convoluted history. Though this idea is commonly attributed to Freeman Dyson, he was actually much more circumspect in his views. What is certainly true is that dimensional analysis shows that the speed of light and Newton’s constant of gravitation can be combined to define a quantity $P^* = \frac{c^5}{G_N}$ with the dimensions of luminosity (equivalently, power). Then in any physical situation we must have $P_{\text{physical}} = \phi P^*$, where the quantity $\phi$ is some dimensionless function of dimensionless parameters. This has lead some authors to suggest a maximum luminosity/maximum power conjecture. Working within the framework of standard general relativity, we will re-assess this conjecture, paying particular attention to the extent to which various examples and counter-examples are physically reasonable. We focus specifically on Vaidya spacetimes, and on an evaporating version of Schwarzschild’s constant density star. For both of these spacetimes luminosity can be arbitrarily large. We argue that any luminosity bound must depend on delicate internal features of the radiating object.

Date: Tuesday 30 March 2021; \LaTeX-ed May 17, 2021

Keywords:
maximum luminosity; maximum power; Dyson luminosity; general relativity.

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Honourable mention in the 2021 Gravity Research Foundation essay contest.
1 Introduction

One starts by noting that in (3+1) dimensions the quantity

\[ P_* = \frac{c^5}{G_N} = 1 \text{ Dyson} \approx 3.6 \times 10^{52} \text{ W} \]  

(1.1)

has the dimensions of luminosity (equivalently, power). Here \( c \) is the speed of light in vacuum, and \( G_N \) is Newton’s gravitational constant. Thereby, using straightforward dimensional analysis, any physical luminosity can always be written in the form

\[ P_{\text{physical}} = \varphi P_*, \]  

(1.2)

where the quantity \( \varphi \) is some dimensionless function of dimensionless parameters.

The suggestion that \( \varphi \) is bounded, that is, \( \varphi = \mathcal{O}(1) \), is commonly misattributed to Freeman Dyson. See reference [1]. Some relevant historical background is reported in reference [2]; see particularly footnote 5 in reference [2]:

“It is not true that I proposed the formula \( c^5/G \) as a luminosity limit for anything. I make no such claim. Perhaps this notion arose from a paper that I wrote in 1962 with the title, “Gravitational Machines”, published as Chapter 12 in the book, “Interstellar Communication” edited by Alastair Cameron, [New York, Benjamin, 1963]. Equation (11) in that paper is the well-known formula \( 128V^{10}/5Gc^5 \) for the power in gravitational waves.
emitted by a binary star with two equal masses moving in a circular orbit with velocity $V$. As $V$ approaches its upper limit $c$, this gravitational power approaches the upper limit $128c^5/5G$. The remarkable thing about this upper limit is that it is independent of the masses of the stars. It may be of some relevance to the theory of gamma-ray bursts.”¹²

—Freeman Dyson

Indeed, the fact that $P_\ast = \frac{\dot{P}}{G_N}$ has units of power can be traced back to the 1880s, to the development of the classical Stoney units, which pre-date Planck units by some 20 years [3–5]. The classical Stoney units use $G_N$, $c$, and Coulomb’s constant $\frac{e^2}{4\pi\epsilon_0}$, (instead of $G_N$, $c$, and Planck’s constant $\hbar$), to set up a universal physically motivated system of units.

Some of the classical Stoney units are equal to the corresponding better-known quantum-inspired Planck units — those where the factors of Coulomb’s constant or $\hbar$ cancel. Specifically we have $P_\ast = P_{\text{Planck}} = P_{\text{Stoney}} = \frac{\dot{P}}{G_N}$. Similarly we have natural units of force $F_\ast = F_{\text{Planck}} = F_{\text{Stoney}} = \frac{c^4}{G_N}$, mass-loss-rate $(\dot{m})_\ast = (\dot{m})_{\text{Planck}} = (\dot{m})_{\text{Stoney}} = \frac{c^3}{G_N}$, and mass-per-unit-length $(m')_\ast = (m')_{\text{Planck}} = (m')_{\text{Stoney}} = \frac{c^2}{G_N}$. Based ultimately on simple dimensional analysis, any one of these natural units might be used to advocate for a maximality conjecture: maximum luminosity [1, 2, 6–9], maximum force [7, 9–14], maximum mass-loss-rate, or maximum mass-per-unit-length. We have recently argued for a certain amount of caution regarding the conjectured bound on maximum force [15], and in this essay we will now turn attention to the maximum luminosity conjecture.

Now it is certainly true that in very many specific situations [10–13] explicit calculations yield $\varphi \leq \frac{1}{4}$, though sometimes numbers such as $\varphi \leq \frac{1}{2}$ also arise [6]. Specifically, consider strong, medium, and weak versions of the maximum luminosity conjecture:

1. Strong form: $\varphi \leq \frac{1}{4}$.
2. Medium form: $\varphi \leq \frac{1}{2}$.
3. Weak form: $\varphi = \mathcal{O}(1)$.

The question we wish to address is whether or not these conjectured bounds are truly universal. See particularly the cautionary comments in [8].

¹Dyson’s article was, in its original essay form, also awarded 4th prize in the 1962 Gravity Research Foundation essay contest, now some 59 years ago.

²The fact that $128/5 > 25 \gg 1$ should perhaps encourage a certain amount of caution regarding any precise numerical bound being placed on the dimensionless number $\varphi$. 

2
Let us first consider Vaidya’s spacetime, which is most typically interpreted as the exterior geometry of a shining star \([16–18]\). We find it convenient to use \((t, r, \theta, \phi)\) coordinates, set \(c \to 1\), and to represent the line element in Kerr–Schild form:

\[
ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \frac{2G_N m(t-r)}{r}(dr - dt)^2. \tag{2.1}
\]

Equivalently

\[
g_{ab} = \eta_{ab} + \frac{2G_N m(t-r)}{r} \ell_a \ell_b, \tag{2.2}
\]

where the vector \(\ell_a = (-1, 1; 0,0)\) is null both with respect to the flat metric \(\eta_{ab}\) and the physical metric \(g_{ab}\). It is then a straightforward and standard calculation to check that

\[
G_{ab} = -\frac{2G_N \dot{m}(t-r)}{r^2} \ell_a \ell_b. \tag{2.3}
\]

Applying the Einstein equations, \(G_{ab} = 8\pi G_N T_{ab}\), one has

\[
T^{ab} = -\frac{\dot{m}(t-r)}{4\pi r^2} \ell^a \ell^b. \tag{2.4}
\]

Checking that

\[
R_{\hat{\theta} \hat{\phi} \hat{\theta} \hat{\phi}} = \frac{2G_N m(t-r)}{r^3}, \tag{2.5}
\]

verifies that the function \(m(t-r)\) is indeed the Misner–Sharp quasilocal mass \([19]\). Then the luminosity, as seen by a static observer at some fixed value \(r_*\) of the radial coordinate, is simply

\[
P(t; r_*) = (\text{flux}) \times (\text{area}) = -\left(\frac{\dot{m}(t-r_*)}{4\pi r_*^2}\right) \times (4\pi r_*^2) = -\dot{m}(t-r_*). \tag{2.6}
\]

(Positive luminosity corresponds to mass loss by the central object.)

Reinstating SI units

\[
P(t; r_*) = -\dot{m}(t-r_*/c) c^2. \tag{2.7}
\]

In terms of the Schwarzschild radius \(r_{\text{Schwarzschild}}(t,r_*) = 2G_N m(t-r_*/c)/c^2\) one has

\[
P(t; r_*) = -\frac{1}{2} \frac{c^5}{G_N} \frac{\dot{r}_{\text{Schwarzschild}}(t-r_*/c)}{c} = -\frac{1}{2} P_* \frac{\dot{r}_{\text{Schwarzschild}}(t-r_*/c)}{c} \tag{2.8}
\]
That is
\[ \varphi(t; r_s) = -\frac{1}{2} \frac{\dot{r}_{\text{Schwarzschild}}(t - r_s/c)}{c}. \]  

(2.9)

The point is that the mass function \( m(t - r) \), and consequently \( r_{\text{Schwarzschild}}(t - r_s/c) \), is completely arbitrary — the exterior Vaidya spacetime by itself places no constraint on the luminosity. (That is, there is no plausibly justifiable physics reason for demanding \( |\dot{r}_{\text{Schwarzschild}}(t - r_s/c)| < c \).) Consequently, the only hope one has for possibly deriving a general purpose maximum luminosity bound must depend on the interior geometry of the source, not the exterior geometry of the radiating object.

3 Schwarzschild’s constant density star: Evaporating version

As a first crude model for the interior geometry of the source, let us consider a time-dependent evaporating version of Schwarzschild’s constant density star. (For general background, see the Delgaty–Lake review [20].) In the usual Schwarzschild curvature coordinates (Hilbert–Droste coordinates) take:

\[ ds^2 = -\left(3\sqrt{1 - \frac{8\pi}{3} G_N \rho_* r_s(t)^2} - \sqrt{1 - \frac{8\pi}{3} G_N \rho_* r^2}\right)^2 \ dt^2 
+ \frac{dr^2}{1 - \frac{8\pi}{3} G_N \rho_* r^2} + r^2 (d\theta^2 + \sin^2 \theta \ d\phi^2). \]  

(3.1)

Here \( \rho_* \) will indeed prove to be the (constant) mass density of the source, while \( r_s(t) \) will prove to be the time-dependent radius of the source.

A brief calculation yields the orthonormal components of the Einstein tensor:

\[ G_{tt} = 8\pi G_N \rho_*; \]  

(3.2)

\[ G_{\theta\theta} = G_{\phi\phi}; \]  

(3.3)

\[ = 8\pi G_N \rho_* \left(3\sqrt{1 - \frac{8\pi}{3} G_N \rho_* r_s(t)^2} - \sqrt{1 - \frac{8\pi}{3} G_N \rho_* r^2}\right)^2 \left(3\sqrt{1 - \frac{8\pi}{3} G_N \rho_* r_s(t)^2} - \sqrt{1 - \frac{8\pi}{3} G_N \rho_* r^2}\right)^2 \]  

Imposing the Einstein equations, we see that \( \rho = \rho_* \) is indeed a constant as advertised.
Furthermore the internal pressure is now explicitly time-dependent

\[
p(r, t) = 4\rho_* \left[ \sqrt{1 - \frac{8\pi}{3} G N \rho_*} \frac{r^2}{\sqrt{1 - \frac{8\pi}{3} G N \rho_*} r_s(t)^2} - 1 + \frac{8\pi G N \rho_*}{3} \left( \frac{3r_s(t)^2 + r^2}{4} \right) \right].
\]

(3.4)

Note that the pressure does indeed go to zero as \( r \to r_s(t) \). Physically this geometry corresponds to a non-moving core, \( r < r_s(t) \), with the outer layers, \( r > r_s(t) \) being blown off. Granted this is a crude model for the interior structure of an evaporating star, but it is good enough to get the main issues across.

The Misner–Sharp quasi-local mass in the bulk is simply [19]

\[
m(r) = \frac{4\pi}{3} \rho_* r^3.
\]

(3.5)

while the total Misner–Sharp quasi-local mass evaluated at the surface \( r_s(t) \) is simply

\[
m(t) = \frac{4\pi}{3} \rho_* r_s(t)^3.
\]

(3.6)

(This Misner–Sharp quasi-local mass evaluated at the surface has to match the Misner–Sharp quasi-local mass for the exterior Vaidya spacetime.)

Thence the luminosity is

\[
P(t) = -\dot{m}(t) = -3m(t) \frac{\dot{r}_s(t)}{r_s(t)}.
\]

(3.7)

Reinstating SI units

\[
P(t) = -\dot{m}(t)c^2 = -3m(t)c^2 \frac{\dot{r}_s(t)}{r_s(t)} = -\frac{3}{2} \left( \frac{2G_N m(t)}{r_s(t)c^2} \right) \frac{c^5}{G_N} \left( \frac{\dot{r}_s(t)}{c} \right)
\]

\[
= \frac{3}{2} P_* \left( \frac{2G_N m(t)}{r_s(t)c^2} \right) \left( \frac{\dot{r}_s(t)}{c} \right).
\]

(3.8)

Now to prevent black hole formation we do want the compactness to be less than unity

\[
\frac{2G_N m(t)}{r_s(t)c^2} < 1.
\]

(3.9)
Thence

\[ P(t) < -\frac{3}{2} P_\ast \left( \frac{\dot{r}_s(t)}{c} \right). \]  

(3.10)

That is

\[ \varphi(t) < -\frac{3}{2} \left( \frac{\dot{r}_s(t)}{c} \right). \]  

(3.11)

But what if anything can we say about \( \dot{r}_s(t)/c \)? Naively one might wish to assert \( |\dot{r}_s(t)| < c \), but we shall soon see that such a postulated constraint falls apart upon closer inspection.

### 4 Nothing can travel faster than light?

It is a truism of special relativity that “nothing can travel faster than light”, or more precisely, as emphasized by both Taylor and Wheeler [21], and by Rothman [22], “no thing can travel faster than light”. That is, “no physical object can travel faster than light”. But does the location of the surface of an evaporating Schwarzschild constant density star (or by extension, the location of the surface of any evaporating stellar model) qualify as a “physical object”? This is a delicate question with model-dependent answers.

Suppose one is dealing with a star made of baryons, and the only energy loss is due to photons: In such a situation the 4-velocity of the surface is determined by the average 4-velocity of the baryons in the immediate vicinity of the surface. But the average of future-pointing timelike vectors is still a future-pointing timelike vector. So in this specific situation we certainly have \( |\dot{r}_s(t)| < c \). Unfortunately there are very many other physical scenarios where such a simple argument does not apply.

Suppose now that one is dealing with a star whose outermost layers are being blown off explosively. Some of the details of the explosion process are now important. The location of the surface \( r_s(t) \) is now less clearly definable as a “physical object”, it is simply a demarcation point between a more-or-less stable core and the now explosively dispersing former outer layers of the object in question. (A mathematical boundary is not necessarily a physical object.) A “detonation wavefront” has more in common with superluminal non-things, such as relativistic scissors, a relativistic searchlight sweep, or relativistic oscilloscope writing speeds [21, 22]; all of these phenomena share in common a certain delicate dependence on initial conditions. Whether or not the material in the vicinity \( r = r_s(t) \) is about to explode depends on how close the the material in the vicinity of \( r = r_s(t) \) is to some irreversible phase transition [23, 24] — there is no a priori need for a causal subluminal signal to propagate inwards to tell the star to explode.
5 Misner–Sharp quasi-local mass in general

Let us now extend the discussion beyond the highly idealized evaporating version of Schwarzschild’s constant density star. Given only spherical symmetry one can define a Misner–Sharp quasi-local mass \( m(r, t) \). Given in addition a well-defined surface \( r_s(t) \) this specializes to \( m(t) = m(r_s(t), t) \). In terms of the average density \( \bar{\rho}(t) \) one can without further loss of generality write [19]

\[
m(t) = \frac{4\pi}{3} \bar{\rho}(t) r_s(t)^3.
\]  

(5.1)

So for the luminosity we now have

\[
P(t) = -\dot{m}(t) = -m(t) \frac{\dot{\bar{\rho}}(t)}{\bar{\rho}(t)} - 3m(t) \frac{\dot{r}_s(t)}{r_s(t)}.
\]

(5.2)

Now if we assert \( \dot{\bar{\rho}}(t) \geq 0 \), then we can deduce

\[
P(t) \leq -3m(t) \frac{\dot{r}_s(t)}{r_s(t)},
\]

(5.3)

and follow through by adapting the analysis presented above for the evolving Schwarzschild constant density star:

\[
P(t) \leq -\frac{3}{2} \frac{2Gm(t)}{r_s(t)c^2} P_* \frac{\dot{r}_s(t)}{c} < -\frac{3}{2} P_* \frac{\dot{r}_s(t)}{c}.
\]

(5.4)

Ultimately

\[
\varphi(t) \leq -\frac{3}{2} \frac{\dot{r}_s(t)}{c}.
\]

(5.5)

That is, if we assume four conditions: (i) spherical symmetry, (ii) a nondecreasing average density, \( \dot{\bar{\rho}}(t) \geq 0 \), (iii) absence of horizons, \( 2Gm/r_s < 1 \), and (iv) subluminal motion of the surface, \( |\dot{r}_s| < c \), then the luminosity is indeed bounded \( \varphi < \frac{3}{2} \).

So to derive bounded luminosity requires some significant assumptions (beyond just invoking standard general relativity). The weakest of these assumptions, as argued above, is assuming \( |\dot{r}_s| < c \) — we really have no good physics reason for making this assumption. The nondecreasing condition on average density, \( \dot{\bar{\rho}}(t) \geq 0 \), is also somewhat questionable. Certainly if the stellar object ever completely disperses one should expect \( \bar{\rho}(t) \rightarrow 0 \), requiring \( \dot{\bar{\rho}}(t) < 0 \) for at least part of the object’s history.
6 Discussion and conclusions

In this essay we have reviewed and re-analyzed the maximum luminosity conjecture, with a view to clarifying just how generic such a conjecture might actually be. We have seen that within the framework of general relativity the exterior spacetime places no physical constraint on the total luminosity — the only conceivable way in which one might place a bound on the total luminosity is by investigating the interior spacetime geometry of the source; and that is a rather model-dependent project with results that seem to be less than universal.

Acknowledgments

AJ was indirectly supported by the Marsden Fund, via a grant administered by the Royal Society of New Zealand.
MV was directly supported by the Marsden Fund, via a grant administered by the Royal Society of New Zealand.

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