Flavored Non-Minimal Left-Right Symmetric Model
Fermion Masses and Mixings

E. A. Garcés,1 Juan Carlos Gómez-Izquierdo,2,1 and F. Gonzalez-Canales3,4

1 Instituto de Física, Universidad Nacional Autónoma de México,
Apdo. Postal 20-364, CDMX 01000, México.
2 Departamento de Física, Centro de Investigación y de Estudios Avanzados del I. P. N.,
Apdo. Post. 14-740, 07000, Ciudad de México, México.
3 Fac. de Cs. de la Electrónica, Benemérita Universidad Autónoma de Puebla, Apdo. Postal 542,
Puebla, Pue. 72000, México.
4 Centro Internacional de Física Fundamental, Benemérita Universidad Autónoma de Puebla.

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A complete study on the fermion masses and flavor mixing is presented in a non-minimal left-right symmetric model (NMLRMS) where the $S_3 \otimes Z_2 \otimes Z_2^e$ flavor symmetry drives the Yukawa couplings. In the quark sector, the mass matrices possess a kind of the generalized Fritzsch textures that allow us to fit the CKM mixing matrix in good agreement to the last experimental data. In the lepton sector, on the other hand, a soft breaking of the $\mu \leftrightarrow \tau$ symmetry provides a non zero and non maximal reactor and atmospheric angles, respectively. The inverted and degenerate hierarchy are favored in the model where a set of free parameters is found to be consistent with the current neutrino data.

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I. INTRODUCTION

In particle physics, flavor symmetries [1–4] have played an important role in the understanding of the quark and lepton flavor mixings through the CKM [5, 6] and PMNS [7, 8] mixing matrices, respectively. According to the experimental data, the values for the magnitudes of all CKM entries obtained from a global fit are [9]:

$$V_{\text{CKM}} = \begin{pmatrix}
0.97434^{\pm 0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\
0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\
0.00875^{\pm 0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005
\end{pmatrix}. \tag{1}
$$
The Jarlskog invariant is $J = (3.04_{-0.20}^{+0.21}) \times 10^{-5}$. In the lepton sector, on the other hand, we know that active neutrinos have a small, but not negligible mass which can be understood by type I see-saw mechanism \cite{10}. The mixings turn out to be non-trivial, so in the theoretical framework of three active neutrinos, the numerical values for the squared neutrino masses and flavor mixing angles obtained from a global fit to the current experimental data on neutrino oscillations \cite{13} are, at Best Fit Point (BFP) ±1σ and 3σ ranges, are \cite{16,19}

$$\Delta m^2_{21} \ (10^{-5} \text{eV}^2) = 7.60^{+0.19}_{-0.18}, \ 7.11 - 8.18; \ \quad |\Delta m^2_{31}| \ (10^{-3} \text{eV}^2) = \begin{cases} \ 2.48^{+0.05}_{-0.07}, \ 2.30 - 2.65 \\ \ 2.38^{+0.05}_{-0.06}, \ 2.20 - 2.54 \end{cases},$$

$$\sin^2 \theta_{12}/10^{-1} = 3.23 \pm 0.16, \ 2.78 - 3.75, \quad \sin^2 \theta_{23}/10^{-1} = \begin{cases} \ 5.67^{+0.32}_{-1.24}, \ 3.93 - 6.43 \\ \ 5.73^{+0.25}_{-0.39}, \ 4.03 - 6.40 \end{cases}, \quad (2)$$

$$\sin^2 \theta_{13}/10^{-2} = \begin{cases} \ 2.26 \pm 0.12, \ 1.90 - 2.62 \\ \ 2.29 \pm 0.12, \ 1.93 - 2.65 \end{cases}.$$ 

The upper and lower rows are for a normal and inverted hierarchy of the neutrino mass spectrum, respectively. At the same time, there is not yet solid evidence on the Dirac CP-violating phase. So, from these data it is obtained (for inverted ordering) that the magnitude of the leptonic mixing matrix elements have the following values at 3σ \cite{17}

$$\begin{pmatrix} 0.799 - 0.844 & 0.516 - 0.582 & 0.141 - 0.156 \\ 0.242 - 0.494 & 0.467 - 0.678 & 0.639 - 0.774 \\ 0.284 - 0.521 & 0.490 - 0.695 & 0.615 - 0.754 \end{pmatrix}. \quad (3)$$

Understanding the contrasted values between the CKM and PMNS mixing matrices is still a challenge in particle physics. In this line of thought, many flavor models such as $S_3$ \cite{20,62}, $A_4$ \cite{63,91}, $S_4$ \cite{92,103}, $D_4$ \cite{104,111}, $Q_6$ \cite{112,122}, $T_7$ \cite{123,131}, $T_{13}$ \cite{132,135}, $T'$ \cite{136,141}, $\Delta(27)$ \cite{142,159}, and $A_5$ \cite{160,170} have been proposed to face this open question.

From a phenomenological point of view, the CKM mixing matrix may be accommodated very well by the Fritzsch \cite{171,173} and the Nearest Neighbour Interaction textures (NNI) \cite{174,177}, however, only the latter can fit with good accuracy the CKM matrix. On the other hand, as can be seen from the PMNS values, the lepton sector seems to obey approximately the $\mu \leftrightarrow \tau$ symmetry \cite{103,178,181} since that $|V_{\mu i}| \approx |V_{\tau i}| \ (i = 1, 2, 3)$. At present, the Long-baseline energy experiment NOvA has disfavored the exact $\mu \leftrightarrow \tau$ symmetry, in this line of though some works have explored the breaking and other ideas on this appealing symmetry \cite{120,182,201}.

Along with this, $\mu \leftrightarrow \tau$ reflection symmetry has gained relevance since it predicts the CP violating Dirac phase ($\delta_{CP} = -90^\circ$), the atmospheric and the reactor angles are 45° and non-zero respectively \cite{202,209}.

Even though the quark and lepton sectors seem to obey different physics, we proposed a framework \cite{56} to simultaneously accommodate both sectors under the $S_3 \otimes Z_2 \otimes Z_2^5$ discrete symmetry within the left-right theory. So that, we will recover the fermion mass matrices, that were obtained previously \cite{56}, to make a complete study on fermion masses and mixings. In the present work, the quark sector will be studied in detail since this was only mentioned in \cite{54}. As we will see, the up and down mass matrices possess the generalized Fritzsch textures \cite{210} (which are not hierarchical \cite{211}), so that the CKM mixing matrix is parametrized by the quark masses and some free parameters that will be tuned by a $\chi^2$ analysis in order to fit the mixings. In the lepton sector, on the other hand, the mixing angles can be understood by a soft breaking of the $\mu \leftrightarrow \tau$ symmetry in the effective neutrino mass matrix that comes
from type I see-saw mechanism. In the current analysis, we found a set of the free parameters that fit the PMNS mixing matrix for the inverted and degenerate hierarchy.

The paper is organized as follows: the fermion mass matrices will be introduced in Sec. II. The CKM and PMNS mixing matrices will be obtained in Sec. III and IV, respectively, besides of a $\chi^2$ analysis is presented to fit the free parameters in the relevant mixing matrices for the quark and lepton sectors separately. Finally, in Sec. V, we present our conclusions.

II. FERMION MASSES

The following mass matrices were obtained in a particular model [56] where left-right theory [12, 212–215] and a $S_3 \otimes Z_2 \otimes Z_2$ symmetry are the main ingredients.

- Pseudomanifest left-right theory (PLRT).

\[ M_q = \begin{pmatrix} a_q + b_q & b_q & c_q \\ b_q & a_q - b_q & c_q \\ c_q & c_q & g_q \end{pmatrix}, \quad M_\ell = \begin{pmatrix} a_\ell & 0 & 0 \\ 0 & b_\ell + c_\ell & 0 \\ 0 & 0 & b_\ell - c_\ell \end{pmatrix}, \quad M_{(L,R)} = \begin{pmatrix} a_{(L,R)} & b_{(L,R)} & b_{(L,R)} \\ b_{(L,R)} & c_{(L,R)} & 0 \\ b_{(L,R)} & 0 & c_{(L,R)} \end{pmatrix}. \] (4)

- Manifest left-right theory (MLRT).

\[ M_q = \begin{pmatrix} a_q + b_q & b_q & c_q \\ b_q & a_q - b_q & c_q \\ c_q^* & c_q^* & g_q \end{pmatrix}, \quad M_\ell = \begin{pmatrix} a_\ell & 0 & 0 \\ 0 & b_\ell + c_\ell & 0 \\ 0 & 0 & b_\ell - c_\ell \end{pmatrix}, \quad M_{(L,R)} = \begin{pmatrix} a_{(L,R)} & b_{(L,R)} & b_{(L,R)} \\ b_{(L,R)} & c_{(L,R)} & 0 \\ b_{(L,R)} & 0 & c_{(L,R)} \end{pmatrix}. \] (5)

where $q = u, d$ stands for the label of up and down quark sector, and $\ell = e, D$ for the charged leptons and Dirac neutrinos. On the other hand, as was stated in [56], the fermion mass matrices are complex in the PLRT. In MLRT, the charged lepton and the Dirac neutrino mass matrices are reals and the Majorana neutrino is complex.

Let us point out that an analytical study on the lepton mixing, in the PLRT, was already made in detail in the particular case where the Majorana phases are CP parities, this means, these can be 0 or $\pi$ [56]. In what follows, the theoretical PMNS mixing mass matrix is recovered but the Majorana phases can take any values, in general. At the same time, for the MLRT the neutrino mass matrix is easily included in the above framework as we will see below.

III. QUARK SECTOR

In this model, the quark mass matrices can be rotated to a basis in which these mass matrices acquire a form with some texture zeros. Also, in the PLRT and MLRT framework the quark mass matrices can be expressed in the following polar form

\[ M_{qj} = U_{\pi/4}^T Q_{qj} \left( \mu_{qj} I_{3 \times 3} + M_{qj} \right) P_{qj} U_{\pi/4}, \] (6)

where

\[ U_{\pi/4} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \quad \text{and} \quad M_{qj} = \begin{pmatrix} D_{qj} & B_{qj} & 0 \\ B_{qj} & A_{qj} & C_{qj} \\ 0 & C_{qj} & 0 \end{pmatrix}. \] (7)
The $P_{qj}$ and $Q_{qj}$ are diagonal matrices, whose explicit form depends on the theoretical framework in which we are working. In the above expressions, the $j$ subscript denote to the PLRT and MLRT frameworks. Concretely, $j = 1$ refers to the PLRT framework, where we have that $\mu_{q1} = |g_q|$, 

$$
Q_{q1} = P_{q1}^\dagger, \quad \text{and} \quad P_{q1} = \text{diag} \left( e^{i\alpha_{q1}}, e^{i\beta_{q1}}, e^{i\gamma_{q1}} \right).
$$

The phase factors in the $P_{q1}$ matrix must satisfy the relations

$$
2\alpha_{q1} = \arg (a_q + b_q), \quad 2\beta_{q1} = \arg (a_q - b_q), \quad 2\gamma_{q1} = \arg (g_q),
$$

$$
\alpha_{q1} + \beta_{q1} = \arg (b_q), \quad \beta_{q1} + \gamma_{q1} = \arg (c_q).
$$

The entries of the $M_{q1}$ matrix have the form $A_{q1} = |a_q + b_q| - |g_q|, B_{q1} = |b_q|, C_{1q} = \sqrt{2}|c_q|$, and $D_{q1} = |a_q - b_q| - |g_q|$. On the other hand, $j = 2$ refers to the MLRT framework in which $\mu_{q2} = |g_q|$, 

$$
Q_{q2} = P_{q2}^\dagger, \quad \text{and} \quad P_{q2} = \text{diag} \left( 1, 1, e^{i\gamma_{q2}} \right),
$$

where $\gamma_{q2} = \arg (c_q)$. The entries of the $M_{q2}$ matrix have the form $A_{q2} = a_q + b_q - g_q, B_{q2} = b_q, C_{q2} = \sqrt{2}|c_q|, D_{q2} = a_q - b_q - g_q$.

The real symmetric matrix $M_{q3}$ in eq. [4], with $j = 1, 2$, can be brought to its diagonal shape by means of the following orthogonal transformation

$$
M_{q1} = \mathbf{O}_{q1} \Delta_{q1} \mathbf{O}_{q1}^T,
$$

where $\mathbf{O}_{q1}$ is a real orthogonal matrix, while

$$
\Delta_{q1} = \text{diag} \left( \sigma_{q1}^{(1)}, \sigma_{q2}^{(1)}, \sigma_{q3}^{(1)} \right).
$$

In the last matrix the $\sigma_{q1}^{(i)}$, with $i = 1, 2, 3$, are the shifted quark masses [11]. Now, it is easy conclude that quark mass matrices in both frameworks can be brought to its diagonal shape by means of the following transformations

$$
U_{q1} M_{q1} U_{q1}^T = \text{diag} \left( m_{q1}, m_{q2}, m_{q3} \right), \quad \text{for PLRT},
$$

$$
U_{q2} M_{q2} U_{q2}^T = \text{diag} \left( m_{q1}, m_{q2}, m_{q3} \right), \quad \text{for MLRT}.
$$

In the above expressions the $m_{qi}$ are the physical quark masses, while

$$
U_{q1} \equiv \mathbf{O}_q^T P_{q1}^* U_{\pi/4} \quad \text{and} \quad U_{q2} \equiv \mathbf{O}_q^T P_{q2} U_{\pi/4}.
$$

The relation between the physical quark masses and the shifted masses is [38] [11]:

$$
\sigma_{q2}^{(i)} = m_{q1} - \mu_{qj}.
$$

From the invariants of the real symmetric matrix $M_{q3}$, tr $\{M_{q3}\}$, tr $\{M_{q3}^2\}$ and det $\{M_{q3}\}$, the parameters $A_{q1}$, $B_{q1}$, $C_{q1}$ and $D_{q1}$ can be written in terms of the quark masses and two parameters. In this way, we get that the entries of the $M_{q1}$ matrix take the form

$$
\bar{A}_{q1} = \frac{A_{q1}}{\sigma_{q3}^{(j)}}, \quad \bar{B}_{q1} = \frac{B_{q1}}{\sigma_{q3}^{(j)}}, \quad \bar{C}_{q1} = \frac{C_{q1}}{\sigma_{q3}^{(j)}}, \quad \bar{D}_{q1} = \frac{D_{q1}}{\sigma_{q3}^{(j)}} = 1 - \delta_q,
$$

$$
\bar{A}_{q2} = \frac{A_{q2}}{\sigma_{q3}^{(j)}}, \quad \bar{B}_{q2} = \frac{B_{q2}}{\sigma_{q3}^{(j)}}, \quad \bar{C}_{q2} = \frac{C_{q2}}{\sigma_{q3}^{(j)}}, \quad \bar{D}_{q2} = \frac{D_{q2}}{\sigma_{q3}^{(j)}} = 1 - \delta_q.
$$

The real symmetric matrix $M_{q3}$ in eq. [4], with $j = 1, 2$, can be brought to its diagonal shape by means of the following orthogonal transformation
where
\[
\xi^{(i)}_{q_1} = 1 - \sigma^{(i)}_{q_1} - \delta_q, \quad \xi^{(i)}_{q_2} = 1 + \sigma^{(i)}_{q_2} - \delta_q,
\]
\[
\bar{\sigma}^{(i)}_{q_1} = \sigma^{(i)}_{q_1} \frac{m_{q_1} - m_{q_2}}{1 - \mu_{q_1}}, \quad \bar{\sigma}^{(i)}_{q_2} = \sigma^{(i)}_{q_2} \frac{|m_{q_2} - m_{q_1}|}{1 - \mu_{q_1}}.
\]

In order to obtain the above parametrization we considered \(\sigma^{(i)}_{q_2} = -|\sigma^{(i)}_{q_2}|\). With the aid of the expressions in eqs. (16) and (17), we obtain that the parameters \(\delta_q\) and \(\bar{\mu}_{q_3}\) must satisfy the following relations
\[
\bar{m}_{q_1} > \bar{\mu}_{q_3} \geq 0 \quad \text{and} \quad 1 - \frac{\bar{m}_{q_1}}{1 - \bar{\mu}_{q_3}} > \delta_q > 0.
\]

From the conditions above, we conclude that parameter \(\bar{\mu}_{q_3}\) must be positive and smaller than one. As \(m_{q_3} > 0\) and \(\bar{\mu}_{q_3} \geq 0\) we have \(|g_q| = g_q\) which implies that \(\bar{\mu}_{q_1} = \bar{\mu}_{q_2}\).

Therefore, in this parameterization the difference between the quark flavor mixing matrix obtained in the PLRT framework and that obtained in the MLRT framework lies in the \(P_{q_3}\) matrix, which is a diagonal matrix of phase factors. From here we will suppress the \(j\) index in the expressions of eqs. (16) and (17), whereby \(\sigma^{(i)}_{q_1} = \sigma_{q_1}, \bar{\sigma}^{(i)}_{q_1,2} = \bar{\sigma}_{q_1,2},\) and \(\bar{\mu}_{q_1} = \bar{\mu}_{q_2} = \bar{\mu}_q\), thus \(\xi^{(i)}_{q_1,2} = \xi_{q_1,2}\). The real orthogonal matrix \(O_{q_3} \equiv O_q\) in terms of the physical quark mass ratios has the form:

\[
O_{q_3} = \begin{pmatrix}
\sqrt{\sigma_{q_1} \delta_{q_2} \xi_{q_1}} & -\sqrt{\sigma_{q_2} \delta_{q_1} \xi_{q_1}} & \sqrt{\xi_{q_1} \xi_{q_2}} \\
\sqrt{\sigma_{q_1} (1-\delta_{q_2}) \xi_{q_1}} & \sqrt{\sigma_{q_2} (1-\delta_{q_1}) \xi_{q_2}} & \sqrt{\sigma_{q_1} \sigma_{q_2} \delta_{q_3}} \\
-\sqrt{\sigma_{q_1} \xi_{q_2}} & -\sqrt{\sigma_{q_2} \xi_{q_1}} & \sqrt{\sigma_{q_1} \sigma_{q_2} \delta_{q_3}}
\end{pmatrix},
\]

where
\[
D_{q_1} = (1 - \bar{\sigma}_{q_1}) (\bar{\sigma}_{q_1} + \bar{\sigma}_{q_2}) (1 - \delta_q),
D_{q_2} = (1 + \bar{\sigma}_{q_2}) (\bar{\sigma}_{q_1} + \bar{\sigma}_{q_2}) (1 - \delta_q),
D_{q_3} = (1 - \bar{\sigma}_{q_1}) (1 + \bar{\sigma}_{q_2}) (1 - \delta_q).
\]

**Quark Flavor Mixing Matrix**

The quark flavor mixing matrix CKM emerges from the mismatch between the diagonalization of \(u\) - and \(d\)-type quark mass matrices. So, this mixing matrix is defined as \(V_{CKM} = U_u U_d\), where \(U_u\) and \(U_d\) are the unitary matrices that diagonalize to the \(u\) - and \(d\)-type quark mass matrices, respectively.

From eqs. (14) we obtain
\[
V_{CKM} = O_{u_1} P_{s_{u_1}} U_{\pi/4} (O_{d_{q_1}}^T P_{s_{d_{q_1}}} U_{\pi/4})^T = e^{i\zeta_1} O_{u_1} P_{s_{u_1}} U_{\pi/4}^T = e^{i\zeta_1} O_{u_1} P_{s_{u_1}}^u U_{\pi/4}^T = O_{u_1} P_{s_{u_1,2}}^u e^{i\zeta_1} O_{u_2} P_{s_{u_2}}^d U_{\pi/4}^T = O_{u_1} P_{s_{u_1,2}}^u O_{d_1},
\]

for PLRT,
\[
V_{CKM} = O_{u_2} P_{s_{u_2}} U_{\pi/4} (O_{d_{q_2}}^T P_{s_{d_{q_2}}} U_{\pi/4})^T = O_{u_2} P_{s_{u_2}} U_{\pi/4}^T = O_{u_2} P_{s_{u_2}}^u O_{d_2},
\]

for MLRT,

where
\[
P_{s_{j}}^{u-d} = \text{diag}(1, e^{i\Theta_j}, e^{i\Gamma_j}), \quad j = 1, 2,
\]

with
\[
\Theta_1 = -(\beta_{u_1} - \beta_{d_1} + \alpha_{d_1} - \alpha_{u_1}), \quad \Gamma_1 = -(\gamma_{u_1} - \gamma_{d_1} + \alpha_{d_1} - \alpha_{u_1})
\]
\[
\Theta_2 = 0, \quad \Gamma_2 = \gamma_{u_2} - \gamma_{d_2}, \quad \zeta_1 = -(\alpha_{u_1} - \alpha_{d_1}).
\]
For performing the likelihood test we define the \( \chi \) as a particular case of the matrix obtained in\( \sqrt{ } \) of phase factors which each one contains. From the model-independent point of view, the mixing matrix obtained in \( \sqrt{ } \) where \( \varepsilon \), \( \varepsilon \), \( \varepsilon \) and \( \varepsilon \) are values for the masses and flavor mixing in the quark sector are correctly reproduced by the model. To carry out the above, we perform a likelihood test \( \chi^2 \), in which we consider the values of the quark masses reported in Ref \( \sqrt{ } \) and using the \( \text{RunDec} \) program \( \sqrt{ } \), we obtain the following values for the quark mass ratios at the top quark mass scale:

\[
\begin{align*}
\tilde{m}_u &= (1.33 \pm 0.73) \times 10^{-5}, \\
\tilde{m}_c &= (3.91 \pm 0.42) \times 10^{-3}, \\
\tilde{m}_d &= (1.49 \pm 0.39) \times 10^{-3}, \\
\tilde{m}_s &= (2.19 \pm 0.53) \times 10^{-2}.
\end{align*}
\]

For performing the likelihood test we define the \( \chi^2 \) function as:

\[
\chi^2 = \sum_{i=d,s,b} \frac{\left( |V_{ui}^h| - |V_{ui}^{eR}| \right)^2}{\sigma_{V_{ui}}^2} + \frac{\left( |V_{cb}^h| - |V_{cb}^{eR}| \right)^2}{\sigma_{V_{cb}}^2}.
\]
The Jarlskog invariant is

\[ J_{CP} = Im (V_{ud}V_{cs}V_{us}^*V_{cd}^*) = (2.92^{+0.38}_{-0.29}) \times 10^{-5}. \]
FIG. 1: Allowed regions in the parameter space at 70% CL (blue line) and 95% CL (red dashed line). Here, the black asterisk correspond to the BFP, while the Θ, $\bar{\mu}_u$, and $\bar{\mu}_d$ parameters are fixed to the values given in the first row of the table.

All these values are in good agreement with experimental data. Also, the results of the above likelihood test can be considered as predictions of the PLRT and MLRT theoretical frameworks. Because when $\Theta_1 = \Theta_2 = 0$ both schemes are equivalent.

IV. LEPTON SECTOR

As it can verified straightforward, the $M_e$ charged lepton mass matrix is diagonalized by $U_{eL} = S_{23}P_e$ and $U_{eR} = S_{23}P_e^\dagger$ in the case of PLRT and $U_{eL} = S_{23}$ and $U_{eR} = S_{23}$ in the MLRT

$$S_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad P_e = \text{diag}\left(e^{i\eta_e/2}, e^{i\eta_\mu/2}, e^{i\eta_\tau/2}\right)$$

with $|m_e| = |a_e|$, $|m_\mu| = |b_e - c_e|$ and $|m_\tau| = |b_e + c_e|$ for the former framework and $m_e = a_e$, $m_\mu = b_e - c_e$ and $m_\tau = b_e + c_e$ in the second one.

The $M_\nu$ neutrino mass matrix, that comes from the type I see-saw mechanism, is parametrized as

$$M_\nu \approx \begin{pmatrix} A_\nu & -B_\nu(1 - \epsilon) & -B_\nu(1 + \epsilon) \\ -B_\nu(1 - \epsilon) & C_\nu(1 - 2\epsilon) & D_\nu \\ -B_\nu(1 + \epsilon) & D_\nu & C_\nu(1 + 2\epsilon) \end{pmatrix}$$

where $A_\nu$, $B_\nu$, $C_\nu$ and $D_\nu$ are complex parameters; $\epsilon$ is a complex and real free parameter in the LRSM and MLRT frameworks, respectively. Along with this, the $\epsilon$ parameter was considered as a perturbation to the effective mass matrix such that $|\epsilon| \leq 0.3$ in order to break softly the $\mu \leftrightarrow \tau$ symmetry. So that, the $|\epsilon|^2$ quadratic terms were neglected in the above matrix. Let us remark that the above neutrino mass matrix has been already rotated by the $S_{23}$ orthogonal matrix. As it was shown in [56], the $M_\nu$ effective neutrino mass matrix is diagonalized by $U_\nu \approx S_{23}\mathcal{U}_\nu$ such that $\hat{M}_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \approx U_\nu^\dagger M_\nu U_\nu^* = U_\nu^0 \mathcal{M}_\nu \mathcal{U}_\nu^*$ where $U_\nu \approx U_\nu^0$ and $\mathcal{U}_\nu$ diagonalizes the $M_\nu^0$ neutrino mass matrix with exact $\mu \leftrightarrow \tau$ symmetry ($|\epsilon| = 0$) this means $U_\nu^{0\dagger} M_\nu^0 U_\nu^0 = \hat{M}_\nu^0 = \text{diag}(m_{\nu_1}^0, m_{\nu_2}^0, m_{\nu_3}^0)$. 
Along with this, the $\epsilon$ parameter breaks the $\mu \leftrightarrow \tau$ symmetry so that its contribution to the mixing matrix is contained in $U'_\nu$.

$$U'_\nu = \begin{pmatrix}
\cos \theta_\nu \ e^{i(\nu + \pi)} & \sin \theta_\nu \ e^{i(\nu + \pi)} & 0 \\
-\sin \theta_\nu \ e^{i(\nu + \pi)} & \cos \theta_\nu & -\frac{1}{\sqrt{2}} \\
-\sin \theta_\nu & \cos \theta_\nu & \frac{1}{\sqrt{2}}
\end{pmatrix}, \quad U'_\nu \approx \begin{pmatrix}
N_1 & 0 & -N_3 \sin \theta_1 \epsilon \\
0 & N_2 & N_3 \cos \theta_\nu \epsilon \\
N_1 \sin \theta_\nu \epsilon & -N_2 \cos \theta_\nu \epsilon & N_3
\end{pmatrix}$$ (34)

where $r_{(1,2)} \equiv (m_{\nu_3}^0 + m_{\nu_{(1,2)}}^0)/(m_{\nu_3}^0 - m_{\nu_{(1,2)}}^0)$ and the $N_i$ the normalization factors are given as

$$N_1 = (1 + \sin^2 \theta_\nu |r_{1}\epsilon|^2)^{-1/2}, \quad N_2 = (1 + \cos^2 \theta_\nu |r_{2}\epsilon|^2)^{-1/2}, \quad N_3 = (1 + \sin^2 \theta_\nu |r_{3}\epsilon|^2 + \cos^2 \theta_\nu |r_{2}\epsilon|^2)^{-1/2}.$$ (35)

Let us emphasize that two relative Majorana phases will be considered along this work in which the $m_{\nu_3}^0$ neutrino mass is kept positive. Explicitly, we have $M'_\nu = \text{diag}(m_{\nu_{v1}}^0, m_{\nu_{v2}}^0, m_{\nu_{v3}}^0) = \text{diag}(|m_{\nu_{v1}}^0 e^{i\alpha}|, |m_{\nu_{v2}}^0 e^{i\beta}|, |m_{\nu_{v3}}^0|)$ where the associate Majorana phase of $m_{\nu_{v3}}^0$ has been absorbed in the neutrino field.

**Lepton Flavor Mixing Matrix**

In the PLRT (MLRT) case, we found that $V_{PMNS} \approx P^\dagger \nu U'_\nu \nu \approx U'^\dagger \nu U'^\dagger \nu _\nu$. Explicitly,

$$V_{PMNS} \approx P^\dagger \nu \begin{pmatrix}
\cos \theta_\nu N_1 & \sin \theta_\nu N_2 & \sin 2\theta_\nu N_3 (r_{2} - r_{1}) \epsilon \\
-\sin \theta_\nu N_1 (1 - r_{1} \epsilon) & \cos \theta_\nu N_2 (1 + r_{2} \epsilon) & -N_3 \epsilon (1 - r_{3}) \\
-\sin \theta_\nu N_1 (1 + r_{1} \epsilon) & \cos \theta_\nu N_2 (1 - r_{2} \epsilon) & N_3 \epsilon (1 + r_{3})
\end{pmatrix}$$ (36)

with $r_3 \equiv r_2 \cos^2 \theta_\nu + r_1 \sin^2 \theta_\nu$ and $P^\dagger \nu = \text{diag}(e^{i(\nu_\beta/2 - \nu_\alpha - \pi)}, e^{i\nu_\alpha/2}, e^{i\nu_\beta/2})$. On the other hand, comparing the magnitude of entries $V_{PMNS}$ with the mixing matrix in the standard parametrization of the PMNS, we obtain the following expressions for the lepton mixing angles

$$\sin^2 \theta_{13} = |V_{13}|^2 = \frac{\sin^2 2\theta_\nu N_3^2 |\epsilon|^2}{4} |r_2 - r_1|^2,$$

$$\sin^2 \theta_{23} = |V_{23}|^2 = \frac{N_3^2}{1 - |V_{13}|^2} \frac{|1 - r_3|^2}{2} \frac{1 - \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}},$$

$$\sin^2 \theta_{12} = |V_{12}|^2 = \frac{N_3^2 \sin^2 \theta_\nu}{1 - \sin^2 \theta_{13}}.$$ (37)

In these mixing angles there are four free parameters namely, the absolute neutrino masses, two relative Majorana phase, the $\epsilon$ parameter and the $\theta_\nu$ angle. Some parameters could be reduced under certain considerations as follows: the $\theta_\nu$ parameter, in good approximation, coincides with the solar angle $\theta_{12}$ since we are in the limit of a soft breaking $\mu \leftrightarrow \tau$ symmetry so the normalization factors, $N_i$, are expected to be of the order 1, then $\theta_{12} = \theta_\nu$. Along with this, the mixing angles may be written in terms of one relative Majorana phase to do so we just have to observe that the reactor angle is non negligible when $|r_2 - r_1|^2$ is large.

$$|r_2 - r_1|^2 = \frac{4 |m_{\nu_3}^0|^2 |m_{\nu_2}^0 - m_{\nu_1}^0|^2}{|m_{\nu_3}^0 - m_{\nu_1}^0|^2 |m_{\nu_3}^0 - m_{\nu_2}^0|^2}.$$ (38)

This happens if $\beta - \alpha = \pi$, then we have

$$|m_{\nu_2}^0 - m_{\nu_1}^0|^2 = |m_{\nu_2}^0| + |m_{\nu_1}^0|^2,$$

$$|m_{\nu_3}^0 - m_{\nu_1}^0|^2 = |m_{\nu_3}^0|^2 + |m_{\nu_1}^0|^2 - 2 |m_{\nu_1}^0| |m_{\nu_3}^0| \cos \alpha,$$

$$|m_{\nu_3}^0 - m_{\nu_2}^0|^2 = |m_{\nu_3}^0|^2 + |m_{\nu_2}^0|^2 + 2 |m_{\nu_2}^0| |m_{\nu_3}^0| \cos \alpha.$$ (39)
where the last two factors enhance the former one in order to get allowed values for the reactor angle. In addition, the factors $r_2$ and $r_1$ can be written in terms of the only relative Majorana phase, $\alpha$. Then,

$$r_1 = \frac{|m_{\nu_1}^0| + |m_{\nu_3}^0| e^{i\alpha}}{|m_{\nu_1}^0| - |m_{\nu_3}^0| e^{i\alpha}}, \quad r_2 = \frac{|m_{\nu_1}^0| - |m_{\nu_2}^0| e^{i\alpha}}{|m_{\nu_1}^0| + |m_{\nu_2}^0| e^{i\alpha}}. \tag{40}$$

In this way, the Majorana phases are related by the expression already mentioned, $\beta - \alpha = \pi$. This analysis is valid for PLRT and MLRT, however, in the latter framework the $\epsilon$ parameter is real.

**Likelihood Test $\chi^2$**

Once we fixed the $\theta_2$ parameter to the solar neutrino mixing angle $\theta_{12}$, the $\chi^2$ analysis is carried out to find allowed values of the three remaining free parameters $\epsilon$, the Majorana phase $\alpha$ and the mass of the lightest (common) neutrino $m_3(m_0)$. Two of the absolute neutrino masses can be written as a function of the the lightest mass and $\Delta m_{ij}^2$ as follows

$$|m_{\nu_2}^0| = \sqrt{\Delta m_{13}^2 + \Delta m_{21}^2 + |m_{\nu_3}^0|^2}, \quad |m_{\nu_2}^0| = \sqrt{\Delta m_{13}^2 + |m_{\nu_3}^0|^2}, \quad \text{Inverted Hierarchy}$$

$$|m_{\nu_3}^0| = \sqrt{\Delta m_{21}^2 + m_0^2}, \quad |m_{\nu_2}^0| = \sqrt{\Delta m_{21}^2 + m_0^2}, \quad \text{Degenerate Hierarchy} \tag{41}$$

where $|m_{\nu_3}^0|$ and $m_0 (> 0.1 \text{ eV})$ are the lightest and common neutrino masses for the inverted and degenerate ordering, respectively.

In this analysis, the normal hierarchy will be left out since this was discarded, for the NMLRMS model, in the previous analytical study [56]. The inverted and the degenerate hierarchies will be discussed next.

The $\chi^2$ function is built as

$$\chi^2(\epsilon, \alpha, m_{0,3}) = \frac{\left(\sin^2 \theta_{13}^{\text{th}} - \sin^2 \theta_{13}^{\text{ex}}\right)^2}{\sigma_{13}^2} + \frac{\left(\sin^2 \theta_{23}^{\text{th}} - \sin^2 \theta_{23}^{\text{ex}}\right)^2}{\sigma_{23}^2}, \tag{42}$$

where the experimental data and theoretical expressions for the mixing angles are given in Eq. [2] and Eq. [37], respectively. We use the absolute neutrino masses in Eq. [41] as a function of $m_{0,3}$, fixing $\Delta m_{ij}^2$ to the central values of the global fit [16] and letting $m_{0,3}$ as a free parameter. For $\sigma_{13}$ and $\sigma_{23}$ we take the one sigma upper and lower uncertainties using summation in quadrature.

The results of the minimization of the $\chi^2$ function are shown in Figures [2], [3] and [4], we show the allowed regions at 90% and 95% CL in the plane of pairs of the three parameters marginalizing the $\chi^2$ function for the parameter not shown. In the left (right) panel is shown the case of degenerate (inverted) hierarchy for each figure. We can notice that the $\alpha$ parameter is more constrained in the case of inverted hierarchy than in the degenerate hierarchy case, and that the fit prefers smaller values of the $\epsilon$ parameter in the case of inverted hierarchy. For illustration purposes only we show the BFP in each case as a black dot.

From comparison of our $\chi^2$ analysis with the qualitative analysis in [56] we find that a wide region of the parameter space is still statistically compatible with experimental data.

**Prediction on the Effective Majorana Mass of the Electron Neutrino**

From the neutrino oscillation experiments, we get information on the mass squared differences, but these experiments cannot say anything about the absolute neutrino mass scale. However, there are three processes that can address
FIG. 2: Allowed regions in the sin(\(\alpha\))-\(\epsilon\) plane, at 90\% CL (blue) and 95\% CL (red) for degenerate (left) and inverted (right) hierarchy. In this case the \(\theta_\nu\) parameter is fixed to the solar angle, and \(m_{0,3}\) is marginalized.

FIG. 3: Allowed regions in the sin(\(\alpha\))-\(\epsilon\) plane, at 90\% CL (blue) and 95\% CL (red) for degenerate (left) and inverted (right) hierarchy. The \(\theta_\nu\) parameter is fixed to the solar angle and \(\epsilon\) is marginalized.

directly the determination of this important parameter: 

i) analysis of CMB temperature fluctuations \[217\], 
ii) the single \(\beta\) decay \[218\] and 
iii) neutrinoless double beta decay \(0\nu\beta\beta\) \[219\].

In here, we only focus on the last process which occurs if neutrinos are Majorana particles. With this decay process we can probe the absolute neutrino mass scale by measuring of the effective Majorana mass of the electron neutrino, which is defined as:

\[
|m_{ee}| = \left| \sum_{i=1}^{3} m_{\nu_i} V_{ei}^2 \right|.
\] (43)

The lowest upper bound on \(|m_{ee}| < 0.22\) eV was provided by GERDA phase-I data \[222\]. That value has been
FIG. 4: Allowed regions in the $m_0$-$\epsilon$ plane, at 90% CL (blue) and 95% CL (red) for degenerate (left) and inverted (right) hierarchy. Again, the $\theta_\nu$ parameter is fixed to the solar angle and the $\alpha$ Majorana phase is marginalized.

FIG. 5: Effective mass $|m_{ee}|$ as a function of the common mass $m_0$ in the case of Degenerate Hierarchy or of the lightest neutrino mass $|m_{\nu_3}|$ for Inverted Hierarchy. The horizontal regions defined by the blue dotted and purple dashed lines correspond to the limits by GERDA phase II [220] and KamLAND-Zen [221] respectively.

Significantly reduced by GERDA phase-II data [223], see Fig. (5). According to our model, the above quantity can be performed directly using the fitted free parameters. Therefore, the plot in Fig. (5) shows the predicted regions for the effective Majorana mass of the electron neutrino.
V. CONCLUSIONS

We performed a complete study on the fermion masses and flavor mixing in the non-minimal left-right symmetric model where the scalar sector was extended by three Higgs bidoublets, three right-handed (left-handed) triplets. We do this analysis for the first time in the quark sector where the quark mass matrices comes out being symmetric and hermitian in the PLRT and MLRT framework, respectively. In the hadronic sector of PLRT (MLRT) framework, we write the quarks flavor mixing matrix, CKM, in terms of quark mass ratios, two shifted mass parameters $\tilde{\mu}_d$ and $\tilde{\mu}_u$, two parameters $\delta_d$ and $\delta_u$, two (one) phase factors. So, the difference between the CKM matrices obtained in the PLRT and MLRT framework lies in the number of phase factors, namely in PLRT we have two phase factors, $\Gamma_1$ and $\Theta_1$, while in MLRT only one, $\Theta_2$. Whereby the quarks flavor mixing matrix in MRLT is a particular case of the CKM matrix obtained in PRLT, since we only need take $\Theta_2 = 0$. We performed a likelihood test $\chi^2$, in which the $\Theta_2$, $\tilde{\mu}_u$, and $\tilde{\mu}_d$ parameters are fixed to the values given in the first row of the table 1, thus the $\chi^2$ function has one degree of freedom. All values obtained in this $\chi^2$ analysis are in good agreement with experimental data. Also, these values can be considered as predictions of the PLRT and MLRT theoretical frameworks, because when $\Theta_1 = \Theta_2 = 0$ both schemes are equivalent.

In the lepton sector our results are in good agreement with the study previously reported in [56], with slight differences attributed to the new neutrino oscillation data used for the likelihood test in the present work. The rich phenomenology of the NMLRSM provides a region of the parameter space that is statistically compatible with experimental data.

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