Transient heat transfer characterization of impinging hot / cold jets by analytical IHCP

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Abstract: Unknown transient surface temperature and heat flux distribution at the impingement side (front side) is estimated from known temperature distribution at non-impingement side (back side) using analytical solution to three dimensional inverse heat conduction problem (IHCP). Back side input temperature data are obtained by running forward code in Fluent. Hot as well as cold impinging jets are characterized with the help of this solution. Laminar and turbulent jets at nozzle to plate spacing (Z/d) equal to 4 are considered. Hot gas at temperature 500 K and 3000 K is impinged on a 1 mm flat plate which considered initially at temperature 300 K in case of heating application. Whereas in cooling application, flat plate is initially assumed at temperature 673 K and isothermal air jet at 293 K is impinged on it. The temperature and heat flux estimation data by present technique is in very good agreement with the simulation data.

1. Introduction
Impinging jets have been preferred since decades in heating, cooling and drying applications due to their high rate of heat transfer ability. In case of impinging jets, rate of heat transfer depends on many parameters like Reynolds number, nozzle to plate spacing, inclination of target plate, nozzle geometry and turbulence intensity at the exit of nozzle. Effect of these parameters on rate of heat transfer are reported in numerous studies in the past. Review [1] is on experimental work carried out in case of single circular turbulent jet impingement. Whereas another review [2] is on experimental studies of single and multiple turbulent isothermal and impinging flame jets. Exclusive work in jet impingement heat transfer in gas turbine systems is reviewed in [3]. Most of the reported heat transfer distribution data to and from impinging jets is at steady state. However, there are very limited studies on transient-state condition of jet impingement heat transfer. The knowledge of transient heat transfer is of great significance to study the material behavior and to control the thermal stresses developed in the material which may cause thermal fatigue [4, 5 & 6]. Estimation of unknown parameters like temperature, heat flux or material properties from known parameters like temperature is an Inverse problem. The solution to these inverse heat conduction problem can help in estimation of transient heat transfer to or from impinging jets [4, 5, 6 & 7]. Method in [4], makes use of analytical solution to Fourier equation for one dimensional problem with time and temperature independent material properties. This approach requires the transient temperature history at impingement side which is sometimes very difficult to get it without disturbing the flow process. In studies [5, 6 & 7], numerical techniques have been used to estimate transient heat transfer from impinging jets.

In this paper, analytical solution presented in [8] to three dimensional inverse conduction problem is used to estimate unknown transient temperature and improved further with the help of Duhamel theorem to get heat flux distribution at the impinging side (front side) from known temperature distribution at non-impingement side (back side). Back side input temperature data are obtained by running forward
code in Fluent. Hot as well as cold impinging jets are characterized with the help of this solution. Laminar and turbulent jets with varying nozzle to plate spacing (Z/d) and Reynolds numbers are considered.

2. Multidimensional Inverse Heat conduction problem (IHCP) and its solution

![Figure 1. Schematic diagram of the three dimensional plate impinged by a jet.](image)

The front surface temperature and flux can be determined inversely by solving an inverse heat conduction problem (IHCP) based on the transient temperature measured on the back surface. The generalized multi-dimensional heat conduction equation for jet impinging on a flat plate (figure 1) can be mathematically expressed as given in equation (1).

$$\rho C \frac{\partial T}{\partial \tau} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

(1)

At the sections, planes normal to x and y axes passing through the impingement point, the symmetry condition is applied. The magnitude of convective heat transfer coefficient at the non-impinging side due to natural convection is very less than the impinging side, hence neglected. The radiation loss at the impinging as well as non-impinging side is considerable only at higher surface temperature. The side surfaces of the plates can be considered as insulated on account of their thinness. The transient inverse heat conduction problem can be formulated by assumption of time dependent flux boundary condition at the impinging side. The solution to this transient inverse heat conduction problem is discussed in subsequent section.

2.1 Mathematical model of Feng et al. [8]

The method of Feng et al. assumes temperature independent material properties (k, ρ & C) to solve transient three dimensional heat conduction equation. The boundary conditions assumed are as mentioned in Table (1).

To simplify further, the non-dimensional parameters are defined as equation (2).

$$X = x / l_z, \ Y = y / l_z, \ Z = z / l_z, \ \text{and} \ \tau = a t / l_z^2$$

Where \(l_z\) is plate thickness

(2)

and the heat flux and temperature are assumed with dimensions of temperature as given in equations (3 and 4)

$$q(x, y, t) = \frac{k}{l_z} f(X, Y, \tau)$$

(3)

$$\Theta(X, Y, Z, \tau) = T(X, Y, Z, \tau) - T_\infty$$

(4)

Thus, the three dimensional heat conduction equation (1) becomes equation (5)
\[ \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \]  \hfill (5)

**Table 1.** Boundary conditions for IHCP solution of Feng et al.

| Location | Boundary condition | Description |
|----------|--------------------|-------------|
| At \( t = 0 \) | \( T(x, y, z, t) = T_0 \) | Initial |
| At \( x = 0, y = 0, z \) | \( \frac{\partial T}{\partial n} = 0 \) | Symmetry |
| At \( x = l_x, y = l_y, z \) | \( \frac{\partial T}{\partial n} = 0 \) | Insulated |
| At \( x, y, z = 0 \) | \(-k \frac{\partial T}{\partial z} = q(x, y, t) \) | Heat flux |
| At \( x, y, z = l_z \) | \( \frac{\partial T}{\partial z} = 0 \) | Insulated |

\(* n \) is the direction along x and y axes

This three dimensional and transient heat conduction problem is further reduced to one dimensional heat conduction problem via modal representation given in equations (6-9).

\[ \theta(X, Y, Z, \tau) = \sum_{m,n=0,1,...} \theta_{mn}(\tau, Z) \cos \frac{m\pi X}{L_x} \cos \frac{n\pi Y}{L_y} \]  \hfill (6)

\[ f(X, Y, Z, \tau) = \sum_{m,n=0,1,...} f_{mn}(\tau) \cos \frac{m\pi X}{L_x} \cos \frac{n\pi Y}{L_y} \]  \hfill (7)

\[ \frac{\partial^2 \theta_{mn}}{\partial Z^2} - C^2_{mn} \theta_{mn} - \frac{\partial \theta_{mn}}{\partial \tau} = 0 \text{ for } m,n = 0,1,2,... \]  \hfill (8)

Where \( C^2_{mn} = \left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2 \)  \hfill (9)

Heat flux boundary condition at \( Z = 0 \) for this one dimensional problem is given in equation (10).

\[ -\frac{\partial \theta_{mn}}{\partial Z} = f_{mn}(\tau) \]  \hfill (10)

and the adiabatic boundary condition on the back surface \( Z = 1 \) is given in equation (11).

\[ \frac{\partial \theta_{mn}}{\partial Z} = 0 \]  \hfill (11)

The function \( \theta_{mn}(\tau, Z) \) represents modal temperature. Through thermal quadrupole, the relationship between temperature and heat flux on the front and back surface is established in Laplace domain. These related functions are further obtained in time domain through inverse Laplace transform of these simple polynomials. To calculate modal flux and temperature on the front side from the back side temperature an iterative procedure is employed. The successive iterations (K) is carried out as given in equations (12 and 13).

\[ \theta^{(K+1)}_{mn}(\tau, 0) = \left[ \theta^{(K)}_{mn}(\tau, 0) + \frac{1}{\left[ (2K+1)\pi / 2 \right]^2 + C^2_{mn}} \frac{d\theta^{(K)}_{mn}(\tau, 0)}{d\tau} \right] \]  \hfill (12)

\[ f^{(K+1)}_{mn}(\tau) = f^{(K)}_{mn}(\tau) + \frac{1}{C^2_{mn} + (K\pi)^2} \frac{df^{(K)}_{mn}(\tau)}{d\tau} \]  \hfill (13)

To use this iterative scheme, the modal temperature \( \theta_{mn}(\tau, Z) \) on the back surface is required.

The backside modal temperature can be obtained as given in Equation (14).
θ_{mn}(x, t) = \frac{2^{2-(α_{mn}+δ_{mn})}}{L_x L_y} \sum_{i} \sum_{j} \theta(X_i, Y_j, 1, \tau) \cos \left( \frac{mnX_i}{L_x} \right) \cos \left( \frac{mnY_j}{L_y} \right) \delta X_r \delta Y_j \quad (14)

Where \((X_i, Y_j)\) is the center of the \((i, j)\)th grid and \(δX_r\) and \(δY_j\) are the sides of the grid. \(δ_{mn}\) and \(δ_{bn}\) are Kronecker delta functions, \(m=0, 1, 2, \ldots M\) and \(n=0, 1, 2, \ldots N\).

The Equation (14) is valid for all mode numbers with the exception of \((m, n) = (0, 0)\). At \((m, n) = (0, 0)\), the equation (15) which assumes constant uniform heat flux at the front surface has been used.

\[ \theta_{00}(t, Z) = \int_{0}^{t} \left[ τ + \frac{Z^2}{2} - Z + \frac{1}{3} - 2 \sum_{k=1}^{∞} \cos \left( k\pi Z \right) \frac{e^{-(iπ)^2τ}}{k^2} \right] \quad (15) \]

2.2 Duhamel Theorem

![Schematic diagram of the one-dimensional plate for the application of Duhamel theorem.](image)

**Figure 2.** Schematic diagram of the one-dimensional plate for the application of Duhamel theorem.

Duhamel theorem is used to estimate heat flux at \((m, n) = (0, 0)\) instead equation (15). The reason for its use is discussed later. Duhamel’s theorem can be applied to a problem which is completely homogeneous except for a single time-dependent forcing function appearing in a boundary condition. Hence the temperature distribution solution to the problem shown in figure (2) can be obtained by the application of Duhamel’s theorem given in equation (16).

\[ T(z, t) = T_{x_0} + \int_{0}^{t} F(t) \frac{\partial U(z, t-\tau)}{\partial t} d\tau + \sum_{n=1}^{∞} U(z, t-\tau) \times D_0 \quad (16) \]

\(U(z, t)\) is the fundamental solution as given in equation (17) and \(τ\) is dummy variable of integration.

\[ U(z, t) = 1 + \frac{4}{π} \sum_{n=1}^{∞} \frac{(-1)^n}{(2n-1)} \cos(\lambda_n z) \ e^{-\lambda_n^2 t} \quad (17) \]

\(D_0\) is the initial disturbance (jump) and \(F(t)\) is the continuous function for time dependent temperature boundary condition and \(\lambda_n\) are the infinite roots of transcendental equation \( \cos(\lambda l_z) = 0 \) and it can be obtained in general form as given in equation (18).

\[ \lambda_n = \frac{1}{2} (2n-1) \frac{π}{l_z}, \quad n=1, 2, 3, \ldots \quad (18) \]

The wall heat flux at any time \(t\) is obtained by equation (19).

\[ q(z = l_z, t) = -k \left( \frac{∂T(z, t)}{∂z} \right) \bigg|_{z = l_z} \quad (19) \]

3. Numerical Procedure

Aim of the present study is to validate and establish this transient analytical IHCP technique to characterize impinging jets. In view of that required input (back side) temperature is obtained by numerical modelling in Ansys (Fluent) CFD Software.

The present technique is applied to both laminar and turbulent jets impinging on the flat plate. Hence, numerical modelling used here for respective jets is independently validated with related published studies. For the numerical simulation of laminar impinging jets, the numerical procedure similar to [9] has been followed. While simulation technique presented in [10] is considered for turbulent impinging
jets. Later validation of numerical procedure, for all further simulations, the properties of impinging gas and plate material are assumed same as mentioned by [9]. Before going for unsteady heating/cooling of the plate, steady state solution was obtained for both the cases. Except the front surface, all other surfaces are assumed insulated. A boundary layer mesh is attached to the convection side on impingement surface for both the cases.

4. Results and discussion

At the first, the simulation procedure and then the improved analytical solution to multidimensional IHCP is validated and then its application to estimate transient heat flux for different configurations of impinging jets is discussed.

4.1 Validation of simulation procedure

The numerical procedure adopted for laminar impinging flame jets is similar as described in [9]. As mentioned in [9] impinging gas is assumed at 3000 K and initial plate temperature is considered at 300 K. The current simulation results at Re = 1333 and Z/d = 4, for transient front surface temperature and front wall heat flux are compared in figure (3a) and figure (3b) respectively with simulation results of [9] and the match is within 2%.

![Figure 3.](image)

Figure 3. Comparison of (a) Front temperature (b) wall heat flux simulation data with data [9].

Further, to validate the numerical procedure employed for turbulent impinging jets, numerical modelling mentioned in [10] is followed. Through the comparative analysis of different turbulence models, RNG $k - \varepsilon$ turbulence model was used in [10] and here same is employed, to simulate jet impingement. Instead 3-D simulation, simple 2-D axisymmetric simulation is carried out to save computational time. The comparative analysis of the current Nusselt number (Nu) distribution with simulation data published in [10] for Re = 34000 at Z/d = 4 and Z/d = 8.5 is presented in figure (4). At stagnation point, for Z/d = 4, the current simulation results predicts Nu around 3% less than the published data [10] whereas for Z/d = 8.5, it matches exactly. For both Z/d, Nu prediction is poor at larger r/d due to difference in mesh structure. The mesh used in 3-D simulation [10] is course at larger r/d whereas in current numerical modelling, fine boundary layer mesh continues till the end (r/d). It was necessary to maintain orthogonality in 2-D simulation.
4.2 Validation of analytical solution [8] and need of improvement
The three dimensional IHCP code employing the procedure described [8] has been validated with numerically simulated data using Ansys Fluent software.

Figure 4. Comparison between current simulated Nu distributions with published simulated data of [10].

Figure 5. Comparison of (a) front temperature, (b) heat flux computed by method of [8] (c) heat flux by present technique with simulation results in case of hot laminar jet impingement.
The numerical procedure adopted is same as that mentioned in [9]. Impinging gas is assumed at 3000 K. The obtained back side temperature is given as input to the analytical IHCP method and the front side wall temperature and wall heat flux is obtained. Plots of wall temperature and wall heat flux for varying r/d obtained by numerically simulated data and 3-D IHCP solution are shown in figure (5a) and figure (5b) respectively. The obtained temperature data match within 1% but the heat flux data is overestimated for larger time. Also at larger r/d, the prediction of heat flux is poor. This over estimate by the 3-D IHCP solution is due to constant heat flux assumption at (m, n) = (0, 0). Thus, 3-D IHCP solution is improved with the help of Duhamel’s theorem. The front modal heat flux at (m, n) = (0, 0) is obtained by application of Duhamel’s theorem. The time dependent continuous function F(t) at (m, n) = (0, 0) can be obtained by using appropriate fit to the transient front modal temperatures estimated by method [8] at (m, n) = (0, 0). Here, second order polynomial is used to fit the transient front modal temperatures. The heat flux then obtained is in good agreement with numerically simulated heat flux data as shown in figure (5c). Thus validates the improved solution.

4.3 Heat transfer characteristics of impinging jets

A plate of 1 mm thickness and 60 mm width having constant thermal conductivity 1.05 (W/mK), density 2250 (kg/m3) and specific heat 780 (J/kgK) is considered as target plate for both type impinging jets.

4.3.1 Laminar Jet

The laminar impinging jet at Re=1333, Z/d=4 is considered for heating as well as cooling application. Impinging gas properties are assumed same as mentioned in [9]. A case of heating application is already explained in section 4.2. For the cooling application, a plate is considered at 500 K and laminar jet at 300 K is assumed impinging on it. Figure (6) compares the results for front surface temperature and wall heat flux estimated by present technique with simulation results. Again similar agreement in the results can be observed as that in heating application.

![Figure 6. Comparison of (a) front surface temperature (b) wall heat flux computed by present technique with simulation results in case of cold laminar jet impingement. The direction of arrow indicates the increasing value of t from 0.5 to 5 sec in steps of 0.5 sec.](image)

4.3.2 Turbulent jet

The turbulent impinging jet at Re=34000, Z/d=4 is considered for heating as well as cooling application. Impinging gas properties are assumed same as mentioned in [10]. In case of heating application, the plate is considered initially at 300 K and turbulent jet at 500 K assumed impinging on it. While for the cooling application, turbulent jet at 293 K is considered impinging on a plate at 673 K. The results of heating and cooling application for front surface temperature and wall heat flux are shown in figure (7) and figure (8) respectively. The estimated data is in good match with the simulated data.
Figure 7. Comparison of (a) front surface temperature (b) wall heat flux computed by present technique with simulation results in case of heating application of a turbulent jet. The direction of arrow indicates the increasing value of t from 1 to 8 sec in steps of 1 sec.

Figure 8. Comparison of (a) front surface temperature (b) wall heat flux computed by present technique with simulation results in case of cooling application of a turbulent jet. The direction of arrow indicates the increasing value of t from 1 to 8 sec in steps of 1 sec.

5. Conclusions
The inverse solution to multidimensional heat conduction problem is successfully applied to laminar and turbulent impinging jets to obtain unknown transient surface temperature and heat flux at the impinging side from known surface temperature of non-impinging side. Estimated front surface temperatures are very accurate with simulation data. While obtained transient wall heat flux except at initial time is in very good agreement with simulated heat flux data. As suggested in [8], this method can be successfully applied to the thin slabs with presence of smaller radial temperature gradient in it and accuracy deteriorates for jets with high rate of change and high gradients.
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