Triquark-Diquark Models of a $\Theta(1540)$?

Jozef Dudek
Theory Division, Jefferson Lab, 12000 Jefferson Ave., Newport News, VA 23606, USA
E-mail: dudek@jlab.org

Abstract. The enhancement in the flavour exotic $NK$ channel seen by a number of experiments has been interpreted as a resonant pentaquark state, the $\Theta^+(1540)$. Theoretical efforts to understand its structure, with the aim of explaining both the low mass and small decay width, have thrown up the possibility of a triquark-diquark $P$-wave correlation. We will discuss the constituent quark model study of Karliner & Lipkin and suggest that their $P$-wave excitation energy is an underestimate, a more plausible value raising the $\Theta$ mass in this model to more than 1700 MeV. We also propose on rather general grounds that these correlated quark models give rise to not only exotic baryon states, but also exotic meson states, in particular the strangeness +2, isoscalar $\vartheta^+$ which will decay to $K^+K^0$.

1. Karliner & Lipkin (KL)
One of the first models to attempt to describe the $\Theta^+$ is due to Karliner and Lipkin[1]. They applied the colour-magnetic interaction model, which has some success describing the ground state mesons and baryons, to the $q^4\bar{q}$ system. They proposed that the structure which maximises the colour-magnetic binding energy is one consisting of a well separated diquark, triquark pair. The diquark has flavour $ud$, is a 3 of colour and has spin-0, while the triquark has flavour $uds(\text{in a flavour 6})$, colour 3 and spin-1/2 with its $ud$ component in a colour 6, spin-1 state. A $P$-wave is included between the diquark and triquark which prevents colour-magnetic interactions between quarks in different clusters, the effect being to keep apart the identical pairs $uu, dd$ which have a repulsive interaction. This $P$-wave also has the effect of rendering the $\Theta$ parity positive, in line with the predictions of chiral soliton models.

Subsequent investigation in [2] found that in fact a lower energy correlation comes from a mixture of this configuration and one with colour-spin structure akin to the Jaffe-Wilczek[3] correlation with a colour $\bar{3}$, spin-0 quark pair inside the triquark.

KL evaluate the colour-magnetic interaction energy (in the $SU(3)_F$ limit) and express the diquark-triquark mass as

$$m_N + m_K - m_\Delta - m_N$$

where the $u\bar{s}$ interaction energy has been eliminated by using the mass of the kaon. As we now show this introduces a large error, as this model as presented is not able to simultaneously fit the meson and baryon spectrum. With colour-magnetic interaction Hamiltonian

$$\delta H = -\sum_{i>j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j \hat{S}_i \cdot \hat{S}_j \frac{v_{ij}}{m_i m_j},$$

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the $\Sigma^*, \Sigma$ mass difference is $\frac{m_u - m_s}{m_u m_s}$ and the $K^*, K$ mass difference is $\frac{4}{3} \frac{m_u - m_s}{m_u m_s}$. If we set $\frac{m_u - m_s}{m_u m_s}$ using the experimental $\Sigma$ masses and use the experimental $K^*$ mass as input, this model predicts a kaon mass of $\sim 620$ MeV, over 100 MeV greater than the experimental value. Thus by replacing $m_u + m_s - \frac{v_{us}}{m_u m_s}$ with $m_K$ in their equations, KL have lowered the pentaquark mass by over 100 MeV relative to the value it would have been if we set $\frac{v_{us}}{m_u m_s}$ using baryon masses.

The $P$-wave excitation energy for the $\Theta$ is determined by KL by noting that the diquark-triquark reduced mass is rather similar to that in the $D_s$ meson. They use experimental masses in the $D_s$ sector to estimate $\omega_P = 207$ MeV. This “small” value is critical for them to obtain a low mass $\Theta$. They outline a few mild caveats in their paper, but we have some more serious objections to this estimate, which we now detail.

KL used the mass of the recently observed $D_s(2317)$ meson - proposing that it is a $P$-wave $c\bar{s}$ excitation (presumably the $0^+$). The structure of this state is the subject of some discussion, with many authors (including Lipkin himself[4]) suggesting that it is a four-quark state, an idea related to the fact that it is very close to the $D_K$ kinematic threshold and much lighter than the anticipated mass of a $c\bar{s}$ 0$^+$ state.

Furthermore, even if the $D_s(2317)$ is the $^3P_0$ $c\bar{s}$ state, the mass difference KL use is not the $P$-wave excitation energy in any sensible quark model. Figure 1 shows the $D_s$ experimental spectrum where the gap used by KL is indicated. With colour-magnetic interactions (exactly what KL are exploiting in their paper), the $0^+, 1^-$ system is split from its zeroth-order value which we can find from the spin-averaged mass, $\tilde{m} = \frac{3}{4}m(1^-) + \frac{1}{4}m(0^-) \sim 2080$ MeV and which is shown in Figure 1 by the lower dashed line. This mass should have been the lower value in the KL estimate, but the difference here is relatively small. A much larger difference appears in the upper value. The experimental states are clearly split by a large spin-orbit force (if we accept the $^3P_0$ proposal for the $D_s(2317)$) and we should use the pre-splitting (“zeroth-order”) mass to compute the $P$-wave excitation energy if we wish to be consistent. In this case KL’s estimate can only be correct if the $0^+$ state has not been moved by the spin-orbit splitting, while the $2^+, 1^+$ states have been pushed up. We can examine this possibility:

Eichten and Quigg[5] parameterise within a quark-potential model, the spin-orbit splittings
of $P$-wave meson states in their equations(2.16, 2.20). We display below the $2^+$ and $0^+$ shifts,

\[
\delta m(2^+) = \frac{1}{2} \left( \frac{1}{m_u^2} + \frac{1}{m_c^2} \right) \tilde{T}_1 + \frac{1}{m_s m_c} \tilde{T}_2 - \frac{2}{5} \frac{1}{m_s m_c} \tilde{T}_4
\]

\[
\delta m(0^+) = -\frac{1}{2} \left( \frac{1}{m_u^2} + \frac{1}{m_c^2} \right) \tilde{T}_1 - \frac{2}{m_s m_c} \tilde{T}_2 - 4 \frac{1}{m_s m_c} \tilde{T}_4. \tag{3}
\]

Setting $\delta m(0^+) = 0$ (as KL require) and using the resulting equality to eliminate the first two terms from $\delta m(2^+)$ we obtain $\delta m(2^+) = -\frac{12}{5} \frac{1}{m_s m_c} \tilde{T}_4$, so that to agree with experiment we require $\tilde{T}_4 < 0$. In terms of one-gluon-exchange, Eichten and Quigg have $\tilde{T}_4 = \frac{8s}{3} \langle r^{-3} \rangle$ which is clearly positive. If $\tilde{T}_4$ generalises to $\sim \langle \frac{1}{r^3} \rangle$ as expected from the Breit-Fermi Hamiltonian, then $\tilde{T}_4$ negative would require the potential to have negative slope over the range of $r$ where the wavefunction is peaked - this is highly unlikely for an attractive, confining potential.

We conclude that the use by KL of the $0^+$ state mass as the upper value is badly flawed. We can limit the splitting using equations(3). We use only the experimental $0^+, 2^+$ masses (assuming that the $0^+$ is simply the $3P_0$ state) to avoid the complication of the $1^+$ state mixing. Assuming that $\tilde{T}_4$ is positive then puts a lower limit of 2480 MeV on the zeroth order $P$-wave state mass and thus $\omega_p \geq 400$ MeV.

We are forced to conclude that the KL estimate for the mass of the $\Theta$ in their model is a considerable under-estimate and that their prediction should have been at least 200 MeV larger. This undermines the original aim of trying to explain the lightness of a positive parity $\Theta$.

1.1. Kochelev, Lee & Vento (KLV)

These authors have built a model which uses in addition to one-gluon-exchange, instanton motivated interactions[6]. They find that the lightest state comes (as in [2]) from a mixture of two correlations of the triquark-diquark type. Again the clusters are separated by a $T$-wave, but their estimate of the excitation energy comes from the hidden strangeness meson sector and is a more reasonable 400 MeV. They suggest a $\Theta$ near 1600 MeV.

2. Exotic pentaquarks imply exotic mesons

The models of Karliner & Lipkin and Kochelev, Lee & Vento propose a triquark-diquark structure for the $\Theta^+$ whose flavour clustering is $(ud \bar{s}) - (ud)$. The diquark in these models is in a $3$ of colour and of flavour and as such resembles an antiquark apart from its spin. Let us consider replacing the diquark with an $\bar{s}$; to the extent that the $P$-wave between triquark and diquark suppresses short-range spin-spin interactions, and noting that in the model of KLV $m(di) \approx m(s)$, we would conclude that this state also has mass $\sim 1600$ MeV. This flavour exotic meson state, which we can label $\vartheta^+$, has strangeness +2 and is isoscalar, living at the tip of a meson $10_F$.

In actual fact since both the triquark and the antiquark are spin-1/2 and with a $P$-wave in the system, we expect a supermultiplet of spin states $(0,1,1,2)^-$, degenerate before triquark-antiquark spin-dependent interactions. The vector states are expected to decay to $K^+K^0$, while for the $0^-, 2^-$ this channel is not available with the largest phase space decays being $KK\pi$ or $K^*K$.

KLV propose that the $\Theta$ has a narrow width into $KN$ owing to the assumed inability of a triquark with mass less than $m(K) + m(u)$, where $m(u)$ is the mass of a constituent light quark, to decay into $K + u$. The $P$-wave is believed to suppress rearrangement between the triquark and diquark. This same mechanism can be proposed to apply in the $\vartheta^+$ and will have the effect of suppressing the $K^+K^0$ decay. The increased phase space for this decay will likely raise the width from $\mathcal{O}(1)$ MeV for the $\Theta$ to $\mathcal{O}(10)$ MeV; with other possible decay modes included we might expect $\vartheta^+$ states with widths around 100 MeV. This is contrary to our usual expectation.
of “fall-apart” decays of multiquark states and is only possible because of the connection to the surprisingly narrow Θ.

Thus we expect that if the Θ has a triquark-diquark structure there should be at least one relatively narrow strangeness +2 meson with a comparable mass. One might expect that it would be possibly to discount such a state on the basis of existing kaon beam experimental data, however it appears that there is very little information available. See for example [7] to observe the lack of data in the $K^+K^0$ channel.

The possibility of a phenomenology of triquark-antiquark mesons consistent with experiment is discussed in some detail in [8], where it is found that the triquark-antiquark structure favours only the vector meson possibility with the $(0, 2)^-$ states having higher mass and much broader widths; this seems to circumvent another common problem of multiquark models - the large multiplicity of states, many of which can be exotic in either flavour or $J^{PC}$ and which usually oversaturate the number of observed exotic candidates.

3. Conclusions

Diquark-triquark models have been proposed to explain the Θ state. The model of Karliner & Lipkin makes an estimate of the mass which is rather close to the experimental candidate mass, however this has been shown to be fortuitous, mostly due to an underestimate of the $P$-wave excitation energy between the triquark and diquark.

Replacing a diquark by an antiquark in these models yields exotic meson states, notably the strangeness +2, isoscalar $\vartheta^+$ which will decay into $K^+K^0$ with a canonical width. Such states seem to be unavoidable in these correlated models and we would welcome a dedicated search especially considering there is virtually no previous data on the exotic channel $K^+K^0$.

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