Students’ proportional reasoning in solving non-routine problems based on mathematical disposition

M Izzatin¹,²*, SB Waluya¹, Rochmad¹, Kartono¹, N Dwidayati¹, NR Dewi¹

¹ Doctoral Program of Mathematics Education, Universitas Negeri Semarang, Indonesia
² Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Borneo Tarakan, Indonesia

*Corresponding author: maharanizzatin@borneo.ac.id

Abstract. Proportional reasoning is the important process in solving social arithmetic problems. A students’ successes in solving mathematical problem is also determined by affective abilities, one of which is a mathematical disposition. This study aims to describe the students’ proportional reasoning in solving non-routine problems based on mathematical disposition. This research is a qualitative study by conducting an in-depth analysis of the proportional reasoning ability of students in students with high, medium, and low mathematical dispositions. The research instrument used was a problem solving ability test, a mathematical disposition scale, and an interview guide. The result shows that in solving non-routine problems, students with high mathematical disposition have mathematical reasoning abilities at level five (Functional and Scalar Relationships), students with moderate mathematical disposition at level four (Accomodating Covariance and Invariance), and students with low mathematical disposition at level five one (Qualitative). Thus it can be concluded that the higher the mathematical disposition of students, the proportional reasoning in solving non-routine problems will be better.

1. Introduction

The purpose of learning mathematics according to the National Council of Teachers of Mathematics, there are five basic mathematical abilities that are standard namely problem solving, reasoning and proof, communication (communication), connection (connection), and representation [1]. This shows that problem solving is one of the important things in learning mathematics. Gagne defined problem solving as the process of synthesizing various concepts, rules or formulas to solve problems [2]. Therefore, the problem solving process is closely related to students’ way of thinking to apply previously owned knowledge to the new situation at hand. In solving a mathematical problem, it takes a strategy or systematic steps so that problems can be solved appropriately.

Reasoning is inseparable from the problem solving process. Permatasari, et al [3] states that reasoning is an activity of thinking that organizes knowledge to draw conclusions. Proportional reasoning is one reason that is important in learning mathematics and also in everyday life. Silvestre [4] defines proportional reasoning as the basis for solving many routine problems in the real world, and is a prerequisite of many advanced mathematics subjects. However, students’ abilities in proportional reasoning are very limited. In his study, Silvestre reviewed the difficulty of sixth graders in solving proportional problems before formal teaching. They have reported misdiagnosis of the multiplicative nature of proportional reasoning as a major problem for students.
Apart from the cognitive aspects, students' success in solving mathematical problems is also determined by affective abilities. One of them is a mathematical disposition. According to NCTM, mathematical disposition is a link with mathematics learning as a goal to think and act positively [1]. The mathematical disposition of students is manifested through students' attitudes and behavior in choosing strategies to solve problems.

In junior high schools, students' problem solving abilities are still relatively low. When faced with problem solving problems, students still find it difficult to find solutions. Students have difficulty using the concepts they have learned to further use them to solve problems. The results of observations of learning mathematics so far, students' difficulties in solving problems are less of a concern. Learning still does not support students to train and improve their abilities in problem solving.

Erinosho and Ogunkola suggest that problem solving techniques are an excellent way to stimulate intellectual curiosity, which will ultimately lead to the acquisition of new knowledge [5]. This technique involves identifying and selecting mathematical problems in the experience of students, placing these problems in front of students and guiding them towards their solutions. According to Özcan, problems are divided into two types namely routine and non-routine problems. Routine problems can also be called exercise. Exercises are questions that test students' knowledge of what has recently been ‘discussed’. The problem may be difficult, but it never confuses, and is similar to what students have learned. Conversely, non-routine problems cannot be answered immediately, and unlike problems they have solved before [6].

According to Berk, proportional reasoning is found in materials such as congruence, opportunities or social arithmetic [7]. Behr and Lesh argue that proportional reasoning is related to sensitivity to covariation, multiple comparisons, and the ability to remember and process several parts of information [8]. Whereas disposition is defined as a tendency to view mathematics as something that can be understood, something useful in understanding mathematics, believe that diligent and tenacious efforts in learning mathematics will produce results, and act as effective students [9].

In mathematics learning, it was found that many students had difficulty solving problems related to ratios and comparisons. Sometimes students are still confused about when to use addition or multiplication to solve it. Students are still lacking in proportional reasoning and tend to memorize the appearance of the steps to get the solution of a problem. Based on this problem, it is necessary to conduct research to examine more deeply the students' proportional reasoning process in terms of mathematical disposition.

2. Methods
This research was conducted on students of class VII SMPN 2 Undaan academic year 2019-2020 a total of 41 students. The subject selection is limited to only 3 students who come from different levels of mathematical disposition abilities, namely high, medium, and low. Of the 3 selected students, the results of the test of the ability of the problem related to student proportional reasoning on social arithmetic material were analyzed. In-depth interviews were conducted to confirm student answers and get more in-depth information about the students' reasoning process. Based on the steps in selecting the subject, the subject selection technique in this study is purposive sampling.

The instruments in this study consisted of the main instruments of the researchers themselves and supporting instruments in the form of: 1) The problem solving ability test on social arithmetic material, which was used to measure students' proportional reasoning abilities in solving non-routine problems, 2) The students' mathematical disposition scale, which used to determine the categories of mathematical dispositions students have, and 3) interview guidelines, which are used to dig deeper about students' proportional reasoning abilities in solving non-routine problems.

Mathematical disposition scale data analysis is done by calculating the score on the results of student answers, then determine the category of mathematical disposition based on the criteria that have been prepared. While the data analysis of problem solving abilities and interviews refers to the student proportional reasoning component which aims to see students' proportional reasoning. The steps include (1) data reduction, (2) presentation of data, and (3) drawing conclusions.
3. Results and Discussion

3.1. Students’ Mathematical Disposition
After obtaining student data in the form of answers to students' mathematical disposition scale, a score calculation is then performed which is categorized based on the criteria that have been prepared. The following are the results of mathematical disposition categorization of 41 students:

| Score Categories | Frequency | Percentage |
|------------------|-----------|------------|
| \( x \geq \bar{x} + SB_x \) | 15 | 36.58% |
| \( \bar{x} - SB_x \leq x < \bar{x} + SB_x \) | 4 | 9.76% |
| \( x < \bar{x} - SB_x \) | 22 | 53.66% |

Table 1. Categories of Student Mathematical Disposition

Based on the table, it can be seen that the most mathematical disposition of students is in the low category that is equal to 53.66%. The second most are in the medium category, and only a few students are in the medium category. This shows that students' mathematical disposition is still low. The results of student answers indicate that students' confidence in their abilities and beliefs about the importance of mathematics are still lacking. Then three subjects were chosen to be further investigated in Proportional Reasoning in solving non-routine problems. Subjects are students with high (S1), moderate (S2), and low (S3) mathematical dispositions.

3.2. Proportional reasoning in solving non-routine problems

Table 2. Proportional reasoning stages

| Stage | Observable Performance |
|-------|------------------------|
| 1. Qualitative | Students knowing quantity is shown by being able to answer questions about more and less (for example, which drink is sweeter?) or about fairness (for example, dividing the bread so that everyone gets a fair share). |
| 2. Early Attempts at Quantifying | Early attempts at measurement often involve constant additive differences rather than multiplicative relationships. Students still rely on calculations using addition or subtraction. |
| 3. Recognition of Multiplicative Relationship | Students have the intuition that the ratio is two numbers that change together but the change may be caused by addition or multiplication. Students more often use the addition strategy when multiplicative reasoning is expected to be used. Situations that involve absolute change cannot always be distinguished from situations that involve change in relative terms. |
| 4. Accomodating Covariance and Invariance | Students begin to develop a multiplicative change model. Students realize that when some quantities may change, relationships between these quantities remains invariant. Students view ratios as possible units of units. Students can distinguish between situations that involve absolute change and situations that involve relative change. The multiplication strategy is used in certain contexts or problems, but if students are faced with a difficult context the students will return to using addition reasoning. |
| 5. Functional and Scalar Relationships | Students know the nature of the invariant relationship between the changing quantities. Students have a general model for solving problems and choose efficient strategies to use. Students understand well about the concept of covariance and invariance. |

Assessment of students' proportional reasoning ability in solving non-routine problems refers to five stages based on the theory of building proportional reasoning by Bexter and Junker, namely (1) qualitative, (2) early attempts at quantifying, (3) recognition of multiplicative relationships, (4)
accommodating covariance and invariance, and (5) functional and scalar relationships. The following is an explanation related to the five stages in building proportional reasoning [11].

Based on the results of students’ answers on the test of problem solving skills, the results of the analysis of the feasibility of the proportional reasoning stage in solving no-routine problems can be seen in Table 3.

Table 3. The implementation of Student Proportional Reasoning stages

| No. | Stage                                      | S1 | S2 | S3 |
|-----|--------------------------------------------|----|----|----|
| 1   | Qualitative                                | √  | √  | √  |
| 2   | Early Attempts at Quantifying              | √  | √  | √  |
| 3   | Recognition of Multiplicative Relationship | √  | √  | -  |
| 4   | Accommodating Covariance and Invariance    | √  | √  | -  |
| 5   | Functional and Scalar Relationships        | √  | √  | -  |

Based on the table, S1 is a student with a high disposition has good proportional reasoning with proportional reasoning stages have reached level 5. While subjects S2 to level 4, but in question number 2 is still at level 2. Subject S3 shows the ability of reasoning that is not good, which in the results of the third answer the questions are still at level 1.

Next will be discussed in more depth proportional reasoning in solving no-routine problems based on the results of test answers and deepened with the results of interviews with students.

3.2.1. S1 subject. S1 subjects are students with high mathematical disposition. Examples of answers S1 in problem number 1 can be seen in figure 1.

![Figure 1. Results of S1 students' answer](image)

Based on the answer of S1 subjects, students have reached the level of Functional and Scalar Relationships. Based on Figure 1, it appears that students write down the quantities that are known and asked about the problem. Covariance and invarian conceptions develop well, where students know well the information provided on the problem and choose efficient strategies to solve it. The results of the interviews showed that students were able to arrange known quantities by registering information in the questions. While in determining the strategy, students use the multiplicative concept of multiplication of fractions and integers to determine the selling price of two types of oranges, and use the concept of additives to determine the remaining oranges, the overall selling price, and the amount of profit.

3.2.2. S2 subject. S2 subjects are students with moderate mathematical disposition. The results of student answers to question number 1 can be seen in Figure 2.
Based on the answers of S2 subjects, students reach level four namely Accommodation Covariance and Invariance. Based on Figure 1, it appears that students write down the quantities that are known and asked about the problem. Students know well the information provided on the problem. Students also choose efficient strategies to solve them. An error occurred in the concept of covariance, which is when determining the number of remaining oranges (broken oranges). Students write the remaining orange for ¾ which should be 1/4 which is the result of subtraction from 1- ¾. This error causes an error in the results obtained in the next step. At the time of the interview, S2 subjects said that the subjects were not careful when determining the amount of orange remaining. When working on students do not think that the number of broken oranges is the difference between many oranges as a whole and oranges that are still good. Students are only focused on prices on two different types of oranges.

3.2.3. S3 subject. Students with low mathematical disposition categories are represented by S3 subjects. The results of the answers to S3 subjects are in Figure 3.

Based on Figure 1, it appears that students do not write down quantities that are known and asked about the problem. Students are also not right in determining strategies to solve them. After the interview, students are able to mention information or quantities that are known but have difficulty in developing their problem solving strategies. Students are still able to compare the selling prices of oranges that are still good and those that have been damaged. This shows that the reasoning of S3 subjects is at the Qualitative level.

The results of the qualitative analysis showed that subjects with high mathematical disposition had better proportional reasoning abilities than students with moderate mathematical dispositions. The
results of this study concluded that the higher the mathematical disposition of students, the proportional reasoning of students in solving non-routine problems would be better. These results agree with Rahayu and Kartono (2014), if students have a positive disposition then the ability to solve mathematical problems will increase [12]. Mathematical disposition must be increased because it is the most important factor in determining student success [9]. Kusmaryono, et al (2019) revealed that there is an influence of mathematical disposition on the power of mathematics. If the disposition of mathematics increases, the power of mathematics will increase [13]. With confidence in mathematics and also one's own abilities, it will encourage positive behavior to better understand mathematics. As stated by Mahmudi (2016), when students solve problems, students must be confident in using mathematics, flexible in investigating ideas. Students must also have perseverance, high interest, curiosity, meeting ability, tend to monitor and reflect on themselves, and like to assess and appreciate the role of mathematics [14].

4. Conclusion

Based on the results of the study, it can be concluded that in solving non-routine problems, students with high mathematical disposition have mathematical reasoning abilities at level five (Functional and Scalar Relationships), students with moderate mathematical disposition at level four (Accommodating Covariance and Invariance), and students with a low mathematical disposition at level one (Qualitative). Thus it can be concluded that the higher the mathematical disposition of students, the proportional reasoning in solving non-routine problems will be better.

Based on these results, it is recommended to teachers to improve students' mathematical disposition so that the proportional reasoning ability becomes better. In learning, teachers should practice a lot of students' proportional reasoning abilities, especially in mathematics related to proportions. Further research is expected to be able to examine learning strategies that can improve mathematical disposition and students' reasoning abilities.

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