DO SUPERMASSIVE BLACK HOLES EXIST AT THE CENTER OF GALAXIES?

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Abstract

Models of superdense star clusters at the center of galaxies are investigated to see whether such objects can be stable and long-lived based on evaporation and collision time-scales and stability criteria. We find that physically reasonable models of massive clusters of stellar remnants can exist with masses $\geq 10^6 M_\odot$, which could simulate black holes at the center of galaxies with large $M/L$ ratios and gas motions of order $\geq 10^3\text{ km s}^{-1}$. It follows that the evidence is not conclusive for massive dark objects at the center of galaxies being black holes.

Subject headings: black hole physics - galaxies: individual (NGC 4258) - galaxies: nuclei - masers
I. INTRODUCTION

There has been a recent show of exuberance about the supporting evidence for the existence of $\sim 10^6 - 10^{9.5} M_\odot$ supermassive black holes at the center of galaxies (Kormendy 1993; Kormendy & Richstone 1995 and references therein, Kormendy et al., 1997). However, as emphasized by Kormendy and Richstone, large $M/L$ ratios and gas motions of order $\geq 10^3$ km s$^{-1}$ do not provide a unique signature for supermassive black holes. Could these central dark objects in galaxies be massive clusters of stellar remnants, brown dwarfs, or halo dark matter? The arguments used to support the evidence that the massive dark objects are supermassive black holes are of an indirect astrophysical nature. The event horizons of the black holes would reside $\sim 10^4 - 10^5$ Schwarzschild radii below the spatial resolution of the Hubble space telescope.

Maoz (1995) has used the measurement of the rotation curve of maser emission sources at the center of NGC 4258 (Miyoshi et al., 1995) to support the conclusion that this dynamical system cannot be a central cluster, unless the cluster consists of extremely dense objects with mass $\leq 0.03 M_\odot$, e.g., low-mass black holes or elementary particles.

We shall show that it is possible to construct models of clusters with central masses in excess of $10^6 M_\odot$, which have reasonably long evaporation and collision time-scales for such galaxies and can be expected to be stable objects. Miyoshi et al., obtained data using the Very Long Baseline Array, confirming that for NGC 4258 ($M \approx 4 \times 10^7 M_\odot$) the rotation curve is Keplerian to a high precision. If the rotation is circular, then the mass interior to $0''.005 = 0.18$ pc is $M \sim 4 \times 10^7 M_\odot$. This gets closer than is normally possible to a potential black hole at the center.

Supermassive stars with arbitrarily large redshifts were first studied by Zel’dovich and Poduretz (1965) and subsequently by Ipser & Thorne (1968), Bisnovatyi-Kogan & Zel’dovich (1969), Ipser (1969, 1970), Fackerell (1970), Bisnovatyi-Kogan & Thorne (1970) and by Weinberg (1972). These studies were motivated by the suggestion of Hoyle and Fowler (1967) and Zapolsky (1968) that the emission lines of QSOs might come from the centers
of supermassive star clusters, whose gravitational fields would produce all or most of the observed redshift. Since then evidence strongly supports that QSOs are at cosmological distances and that the redshifts are due to the expansion of the universe. However, there is now a renewed interest to study dark massive objects at the center of galaxies, because of the possibility that they could be either black holes or supermassive clusters.

II. EQUILIBRIUM CONFIGURATIONS OF SUPERMASSIVE CLUSTERS

Let us give an overview of the supermassive cluster equilibrium problem. We shall assume that the core cluster is mainly supported by the pressure of radiation rather than of matter and that it is in convective equilibrium and has a uniform chemical composition. For radiation the energy density is \( \epsilon \sim 3p \) and the cluster can be described by a Newtonian polytrope with

\[
\epsilon = \rho - mn = (\gamma - 1)^{-1}p,
\]

where \( m \) and \( n \) denote the mass of a star in the cluster and the number density of stars, respectively, \( \rho \) and \( p \) denote the mass density and pressure, respectively, and \( 1/(\gamma - 1) \) is a constant of proportionality. The condition of uniform entropy per star then yields the polytrope equation of state

\[
p = K\rho^\gamma,
\]

where \( K \) is a constant of proportionality. The standard polytrope equilibrium equation is (Chandrasekhar 1939):

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \xi^2 \frac{d\theta}{d\xi} + \theta^{1/(\gamma - 1)} = 0,
\]

where the radial variable \( r \) is related to \( \xi \) by

\[
r = \left( \frac{K\gamma}{4\pi G(\gamma - 1)} \right)^{1/2} \rho_0^{(\gamma - 2)/2} \xi,
\]

and
\[ \rho = \rho_0 \theta^{1/(\gamma - 1)}, \quad (5a) \]
\[ p = K \rho_0^\gamma \theta^{\gamma/(\gamma - 1)}. \quad (5b) \]

Boundary conditions on the Lane-Emden function \( \theta \) are given by \( \theta_0 = 1 \) and \( \theta'_0 = 0 \).

The total energy for the core Newtonian cluster is \( E = T + V \), where \( T \) and \( V \) denote the thermal energy and gravitational potential energy, respectively, and for Newtonian polytropes the total energy is \( E = -(3\gamma - 4)GM^2/(5\gamma - 6)R \).

Since we have assumed that the supermassive cluster is mainly supported by thermal radiation, we can choose \( \gamma \sim 4/3 \) and its mass is given by (Chandrasekhar 1939):
\[ M = 25.3620 \left( \frac{K}{\pi G} \right)^{3/2} \quad (6) \]
and its radius by
\[ R = 6.89685 \left( \frac{K}{\pi G} \right)^{1/2} \rho_0^{-1/3}, \quad (7) \]
where \( \rho_0 \) is the central density.

The structure of the supermassive cluster can be described by a Newtonian polytrope with \( \gamma \sim 4/3 \) (for NGC4258 \( GM/R \sim 1.6 \times 10^{-5} \) which is locally Newtonian), but to settle the question of stability we need General Relativity (GR), for a cluster with \( \gamma \sim 4/3 \) is sensitively balanced between stability and instability, so small effects due to GR and matter pressure must be accounted for, although they play little role in the structure calculations (Chandrasekhar 1964).

We characterize the spherically symmetric cluster by a perfect fluid with the metric (Weinberg 1972):
\[ ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (8) \]
where
\[ A(r) = \left[ 1 - \frac{2GM(r)}{r} \right]^{-1}, \quad (9a) \]
\[ B(r) = \exp \left\{ -2G \int_r^\infty \frac{dr'}{r'} [M(r') + 4\pi r'^3 p(r')] \left[ 1 - \frac{2GM(r')}{r'} \right]^{-1} \right\}. \quad (9b) \]
Moreover, we have

\[ M(r) = 4\pi \int_0^r dr' r'^2 \rho(r'). \tag{10} \]

Outside the cluster \( p(r) \) and \( \rho(r) \) vanish and the metric is described the Schwarzschild solution

\[ B(r) = A^{-1}(r) = 1 - \frac{2GM(R)}{r} \quad \text{for } r \geq R. \tag{11} \]

The general relativistic equation is given by the Oppenheimer-Volkoff equation (Oppenheimer & Volkoff 1939):

\[ p'(r) = -\frac{GM(r)\rho(r)}{r^2} \left[ 1 + \frac{p(r)}{\rho(r)} \right] \left[ 1 + \frac{4\pi r^3 p(r)}{M(r)} \right] \left[ 1 - \frac{2GM(r)}{r} \right]^{-1}. \tag{12} \]

The thermal and gravitational energies are given by

\[ T = 4\pi \int_0^R dr r^2 \left[ 1 - \frac{2GM(r)}{r} \right]^{-1/2} \epsilon(r), \tag{13a} \]

\[ V = 4\pi \int_0^R dr r^2 \left\{ 1 - \left[ 1 - \frac{2GM(r)}{r} \right]^{-1/2} \right\} \rho(r), \tag{13b} \]

where \( \epsilon(r) \) denotes the internal energy density.

Expanding in powers of \( GM(r)/r \) we get

\[ T = 4\pi \int_0^R dr r^2 \left[ 1 + \frac{GM(r)}{r} + \ldots \right] \epsilon(r), \tag{14a} \]

\[ V = -4\pi \int_0^R dr r^2 \left[ \frac{GM(r)}{r} + \frac{3G^2M^2(r)}{2r^2} + \ldots \right] \rho(r). \tag{14b} \]

A cluster described by a perfect fluid may pass from stability to instability with a radial normal mode at a value of the central density \( \rho_0 \) for which the energy \( E \) and the number of stars \( N \) is stationary (Chandrasekhar 1964):

\[ \frac{\partial E}{\partial \rho_0} = 0, \tag{15a} \]

\[ \frac{\partial N}{\partial \rho_0} = 0. \tag{15b} \]

The energy \( E \) has the approximate form

\[
E \simeq 4\pi \int_0^R dr r^2 \epsilon(r) + 4\pi G \int_0^R dr r M(r) \epsilon(r) - 4\pi G \int_0^R dr r M(r) \rho(r)
- 6\pi G^2 \int_0^R dr M^2(r) \rho(r). \tag{16}
\]
The total pressure is $p = p_r + p_m$ where $p_r$ and $p_m$ denote the radiation and matter pressure, respectively, and the internal energy density is

$$\epsilon = 3p_r \left[ 1 + \frac{\beta}{3(\Gamma - 1)} \right], \quad (17)$$

where $\Gamma$ is the specific heat ratio of the matter and $\beta = p_m/p_r$. The total pressure is $p = p_r(1 + \beta)$, so that to first order in small $\beta$, the ratio of energy density to pressure gives

$$\epsilon \simeq 3p \left[ 1 - \frac{(3\Gamma - 4)}{3(\Gamma - 1)} \beta + O(\beta^2) \right]. \quad (18)$$

We now integrate by parts and to first order in $\beta$ and employing the approximate Newtonian equation

$$p'(r) \simeq -\frac{GM(r)\rho(r)}{r^2} \quad (19)$$

to evaluate $\rho, p$ and $M(r)$, we get (Weinberg 1972):

$$E \simeq -\frac{(3\Gamma - 4)}{2(\Gamma - 1)} \beta \frac{GM^2}{R} + 5.1 \frac{G^2M^3}{R^2}. \quad (20)$$

The stability criterion

$$\frac{\partial E}{\partial r} = \frac{\partial E}{\partial \rho_0} \frac{\partial \rho_0}{\partial r} = 0 \quad (21)$$

yields the minimum radius for stability

$$R_{\text{min}} = \frac{20.4(\Gamma - 1) GM}{(3\Gamma - 4) \beta}. \quad (22)$$

For a core cluster with $M = 3.6 \times 10^7 M_\odot, \beta \sim 0.1$ and $\Gamma \sim 5/3$ we get the minimum radius

$$R_{\text{min}} = 2.3 \times 10^{-4} \text{pc}, \quad (23)$$

which is deep inside a typical supermassive core residing in a galaxy (NGC4258). The ratio of $R_{\text{min}}$ to the Schwarzschild radius $R_S = 2GM$ is $R_{\text{min}}/R_S = 6.8/\beta$.

Although we have used a perfect fluid to investigate the stability properties of a supermassive cluster, we consider that it is a good approximation and that it is safe to conclude that a kinetic particle description of the supermassive cluster will yield similar results.
III. TIMESCALES OF EVAPORATION AND COLLISIONS

The time-scale of evaporation of a bound system of objects with a single mass can be determined to be $t_{\text{evap}} \approx 300 t_{\text{relax}}$ (Spitzer & Thuan 1972; Spitzer & Hart 1971; Binney & Tremaine 1987) with

$$t_{\text{relax}} = \left[ \frac{0.14N}{\ln (0.4N)} \right] \left( \frac{R_{1/2}^3}{GM} \right)^{1/2},$$

(24)

where $N$ is the number of objects in the cluster, $M$ is the cluster mass, and $R_{1/2}$ is the cluster radius within which lies half of the cluster’s mass.

Let us assume that all the mass in the dense cluster is confined within a spherically symmetric core, mantle and halo. The core is described by (Bisnovatyi-Kogan & Thorne 1970):

$$\rho_c(r) = \rho_0 \left[ 1 - \frac{2\pi G \rho_0 r^2}{3\gamma} \right],$$

(25)

where $\rho_0$ is the central density. This core is joined smoothly to a mantle with the density profile

$$\rho_m(r) = \left( \frac{\gamma}{1 + 6\gamma + \gamma^2} \right) \left( \frac{1}{2\pi G r^2} \right),$$

(26)

with the join point in a region just outside the core radius,

$$r_c = \left( \frac{\gamma}{2\pi \rho_0 G} \right)^{1/2}.$$

(27)

Outside the mantle a Newtonian envelope is constructed which is assumed to be convectively stable and has a finite radius $r_e$ at which $\rho$ goes to zero in a polytropic fashion,

$$\rho_e(r) \propto (r_e - r)^N, \quad N > 0.$$

(28)

We now assume that almost all the mass is confined within the core and the mantle. Then, we have $\rho \approx \rho_c + \rho_m$ and the mass is

$$M \approx 4\pi \int_0^{r_c} dr r^2 \rho_c(r) + \int_{r_c}^{r_m} dr r^2 \rho_m(r) \approx 4\pi r_c^3 \rho_0 + \frac{8\pi^2 G \rho_0^3}{15\gamma} r_c^5 + \frac{2\gamma}{G} (r_m - r_c).$$

(29)
From (27) and with $\gamma < 1$ and $r_m >> r_c$, it follows that we can ignore the first two terms on the right-hand side of (29). The evaporation time for $R_{1/2} \sim 0.2$ pc, $M \sim 3.6 \times 10^7 M_\odot$ and a dense cluster consisting of neutron stars with $m \sim 1.4 M_\odot$ is

$$t_{\text{evap}} \sim 10 \text{ Gyr},$$

which is an adequate lifetime for the superdense cluster.

Maoz (1995) has imposed constraints on the mass distribution of a dense stellar cluster by using the observational findings for NGC 4258. The high-velocity maser emission data obtained for NGC 4258 describe a nearly planar structure, and the velocity decreases from $v_{\text{in}} = 1080 \pm 2$ km s$^{-1}$ at a distance $r_{\text{in}} = 0.13$ pc to $v_{\text{out}} = 770 \pm 2$ km s$^{-1}$ at a distance $r_{\text{out}} = 0.25$ pc (Miyoshi et al., 1995). Assuming circular motion and a perfectly planar disk, it was found that it can be fitted very well by a Keplerian relation. The systematic deviation of the velocity profile from a Keplerian relation is $\Delta v \leq 3$ km s$^{-1}$ (Maoz 1995) or a fractional deviation of $\Delta v/\bar{v} \leq 4 \times 10^{-3}$, where $\bar{v}$ is the average rotational velocity. Maoz assumed that the entire mass is within a radius $r_{\text{in}}$ with a mass density profile described by a Plummer model (Binney & Tremaine 1987):

$$\rho(r) = \rho_0 \left[1 + \frac{r^2}{r_c^2}\right]^{-5/2}, \quad M(< r) = \frac{4\pi\rho_0}{3} r^3 \left(1 + \frac{r^2}{r_c^2}\right)^{-3/2}.$$  \hspace{1cm} (31)

The ratio of the cluster mass enclosed between the spheres of radii $r_{\text{in}}$ and $r_{\text{out}}$ to its mass within $r_{\text{in}}$ is given by

$$\delta_{\text{Kep}} = [M(< r_{\text{out}}) - M(< r_{\text{in}})]/M(< r_{\text{in}}).$$ \hspace{1cm} (32)

Then, for $\delta_{\text{Kep}} \sim 0.01$ and solving for $r_c$ using (31) and (32), it follows that $r_c \leq 0.012$ pc and with $R_{1/2} = 1.3 r_c$ and $r_{\text{in}} \gg r_c$, we get for a hypothetical cluster of neutron stars ($m \sim 1.4 M_\odot$) at the center of NGC 4258 the evaporation time $t_{\text{evap}} \sim 10^8$ yr. Since this is a period of time much shorter than the age of the galaxy, Maoz ruled out the possibility that NGC 4258 is a cluster of stars with mass $\approx 1.4 M_\odot$. A cluster of stars with mass $\sim 0.03 M_\odot$ would yield $t_{\text{evap}} \sim 6$ Gyr, which would not be ruled out, but would be difficult to reconcile.
with collision timescales, unless the objects are extremely dense, e.g., light black holes or
elementary particles, which are difficult to reconcile with any known theory of structure
formation or stellar evolution.

Consider now our model of a superdense cluster of stars with a mass profile given by
("29"). Assuming that \( r_c \ll r_{in} \sim r_m \) we get

\[
    r_{in} \leq \frac{r_{out}}{1 + \delta_{Kep}} \tag{33}
\]

and for the observational value \( \delta_{Kep} \sim 0.01 \) we obtain \( r_{in} \sim r_{out} \sim 0.2 \) pc, which
yields an evaporation time given by \( (30) \) consistent with the age of the galaxy NGC 4258.
The structure of the supermassive cluster at the centre of the galaxy consists of a massive
core and mantle with a tenuous gas envelope. We can conclude that such a compact object
should not produce a deviation of the Keplerian rotation curve which exceeds the observed
value \( \Delta v \leq 3 \text{ km s}^{-1} \).

Let us now consider the physical collision time-scale. This can be estimated from the
formula (Binney & Tremaine 1987):

\[
    t_{coll} = \left[ 16\pi^{1/2} n \sigma_{r*}^2 \left( 1 + \frac{Gm}{2\sigma^2 r_{*}} \right) \right]^{-1} \tag{34}
\]

where \( n \) is the number density of stars, \( r_{*} \) is the radius of the star, \( m \) is the mass of the star,
and \( \sigma \) is the velocity dispersion. For zero-temperature brown dwarfs and low-mass stars the
mass-radius relation can be taken to be (Zapolsky & Salpeter 1969; Stevenson 1991):

\[
    r_{*} = 2.2 \times 10^9 \left( \frac{m}{M_{\odot}} \right)^{-1/3} \left[ 1 + \left( \frac{m}{0.0032M_{\odot}} \right)^{-1/2} \right]^{-4/3} \text{ cm.} \tag{35}
\]

From \((8)\) and \((7)\), we can estimate the core density to be

\[
    \rho_0 = \frac{12.93M}{R^3}. \tag{36}
\]

For \( R \sim 0.2 \) pc, \( M \sim 3.6 \times 10^7M_{\odot}, m \sim 1.4M_{\odot}, r_{*} \sim 1.97 \times 10^8 \) cm and \( \sigma \sim 1500 \text{ km s}^{-1} \),
we get \( \rho_0 \sim 5.9 \times 10^{10} M_{\odot} \text{ pc}^{-3}, n = \rho_0/m \sim 4.2 \times 10^{10} \text{ pc}^{-3} \) and

\[
    t_{coll} \sim 1 \text{ Gyr}. \tag{37}
\]
which is long enough to offset a rapid evolution of the massive stellar cluster through coalescence of stars. Of course, as we decrease the radius $R$ of the massive core, then the time-scale of physical collisions of stars will decrease and lead to an unstable configuration.

In a recent article (Moffat 1997) the fate of a dense cluster of stars which is undergoing a final stage of gravithermal catastrophe (Lynden-Bell & Wood 1968) was analysed. Since this phase of the evolution of a superdense core of stars is far from thermodynamic equilibrium, nonlinear cooperative contributions are expected to be important in the transport equations describing the last stage of evolution. It was found that such nonlinear contributions can prevent the core redshift from increasing without limits as the core becomes increasingly dense, preventing the collapse to a black hole. In particular, the redshift can remain less than the critical value for relativistic collapse, resulting in a stable, massive dark object at the center of a galaxy with a Newtonian core, mantle and thin halo.

IV. CONCLUSIONS

We see that it is not possible to rule out the hypothesis that NGC 4258 or other supermassive galaxies are dense clusters of star-like objects with a mass $m \sim 1 M_\odot$, since the structure of the theoretical mass profile of such superdense clusters is not known with certainty. Therefore, we must conclude that there is presently no conclusive evidence for the existence of black holes at the center of galaxies such as M31, M32, M87, NGC 4594, NGC 4258 and other potential black hole candidates. However, if these dark massive objects with masses in excess of $10^6 M_\odot$ are superdense clusters of stars with relatively large central redshifts, then they would be of considerable theoretical interest to the astrophysics community.

Since it is difficult in the foreseeable future to obtain an observational spatial resolution less than $10^4$ Schwarzschild radii, it is not clear how the dark object at the center of galaxies can be proved to be a black hole. Black holes may exist at the center of galaxies but it cannot be claimed without further conclusive evidence that they have been detected
by current observational data.

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