Stationary Entanglement and Quantum State Transfer in Opto-magnomechanics

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We show how to prepare a steady-state entangled state between magnons and optical photons in an opto-magnomechanical configuration, where a mechanical vibration mode couples to a magnon mode in a ferrimagnet by the dispersive magnetostrictive interaction, and to an optical cavity by the radiation pressure. We find that, by appropriately driving the magnon mode and the cavity to simultaneously activate the magnomechanical Stokes and the optomechanical anti-Stokes scattering, a stationary optomagnonic entangled state can be created. We further show that, by activating the magnomechanical state-swap interaction and by subsequently sending a weak red-detuned optical pulse to drive the cavity, the magnonic state can be read out in the cavity output field of the pulse via the mechanical transduction. The demonstrated entanglement and state readout protocols in such a novel opto-magnomechanical configuration allow us to optically control, prepare, and read out quantum states of collective spin excitations in solids, and provide promising opportunities for the study of quantum magnonics, macroscopic quantum states, and magnonic quantum information processing.

I. INTRODUCTION

The past decade has witnessed a significant development and the formation of cavity magnonics [1,5]. One of the pronounced advantages of the system is that the collective magnetic excitations (magnons) exhibit an excellent ability to interact coherently with a variety of distinct systems, including microwave photons [6-11], optical photons [12-18], vibration phonons [19-22], and superconducting qubits [23-25], etc. The composite architecture based on magnons promises broad application prospects in quantum information processing, quantum sensing, and quantum networks [26-28]. In particular, the coupling between magnons and microwave or optical photons offers the possibility to optically control, engineer, detect, and transmit magnonic states [6-18,27-39]. This coupling is an indispensable component to build a magnonic quantum network with remote quantum nodes connected by light [26]. The mature and handy optical means also allows us to prepare non-classical magnonic states [27-29,33-36].

Among a variety of non-classical states, entangled states find a particularly wide range of applications, e.g., in quantum information processing [40-41], quantum teleportation [42], quantum metrology [43], and fundamental tests of quantum mechanics [44,45]. Magnons and microwave cavity photons can get entangled by exploiting the nonlinear magnetostrictive coupling to vibration phonons [20,46]. Alternatively, they can also be entangled by utilizing the dissipative coupling between magnons and photons [47]. In the optical frequency, magnons and optical photons can be entangled by using optical pulses to activate the optomagnonic Stokes scattering [26,28,34-36]. Typical optomagnonic systems, either a whispering-gallery-mode [12,14], or a waveguide [17], or a microcavity [18] configuration, suffer a significantly large cavity decay rate $\kappa_\text{c}$ (typically in GHz) and a much smaller effective optomagnonic coupling strength $G_{\text{om}} \ll \kappa_\text{c}$, thus putting the system well within the weak coupling regime. Therefore, in all of the aforementioned protocols [26,28,34-36], fast optical pulses are adopted to generate a transient optomagnonic entangled state. This evades the stringent requirement on the optomagnonic cooperativity $C_{\text{om}} = \frac{G_{\text{om}}^2}{\kappa_\text{c}} > 1$ ($\kappa_\text{c}$ is the magnon decay rate) that is required for preparing a stationary entangled state.

Here we provide a novel approach to prepare a stationary entangled state between magnons and optical photons by using a mechanical vibration mode being an intermediary, which couples to the magnons and the photons by the nonlinear magnetostriction and radiation-pressure interaction, respectively. This indirect-coupling configuration [49] takes advantage of the low damping rate $\gamma_\text{b}$ of the mechanical mode, and can easily achieve both the magnomechanical and optomechanical cooperativities $C_{\text{om}} = \frac{G_{\text{om}}^2}{\kappa_\text{c}} > 1$, and $C_{\text{om}} = \frac{G_{\text{om}}^2}{\gamma_\text{b}} > 1$ under current technology [19,22,48]. This enables us to generate a stationary optomagnonic entangled state without requiring the stringent condition $C_{\text{om}} > 1$ in current directly-coupled optomagnonic systems. Specifically, the magnons and optical photons are prepared in a steady entangled state by simultaneously activating the magnomechanical Stokes and the optomechanical anti-Stokes scattering, realized by properly driving the magnon and the optical cavity. The steady-state optomagnonic entanglement is more useful as it can provide a stable entanglement source to be applied to the quantum information science.

We further show that, by activating the magnomechanical state-swap operation realized by driving the magnon with a red-detuned microwave field, the magnonic state can be transferred to the mechanical mode. We then switch off the microwave drive and, after a short period, send a weak red-detuned optical pulse to the cavity to activate the optomechanical beam-splitter interaction. The magnonic state is then read out in the output field of the pulse via the mechanical transduction. Taking squeezed states as an example, we show that the magnon squeezing can be transferred to the optical pulse via the mechanical coupling, reflected in their squeezed Wigner...
distributions. This offers an optical means to read out localized magnonic states in solids.

The paper is organized as follows. In Sec. II, we introduce the system of opto-magnomechanics, which is used for our entanglement and state readout protocols. We derive in detail the Hamiltonian of the tripartite system and, in particular, the magnomechanical dispersive interaction Hamiltonian, which provides key nonlinearity for producing the entanglement. We then show how to use this hybrid system to obtain a stationary optomagnonic entanglement in Sec. III, and how to use optical pulses to read out the magnonic states in Sec. IV. Finally, we summarize the paper in Sec. V.

II. OPTO-MAGNOMECHANICAL SYSTEM

The magnon-phonon dispersive coupling is a vital component of the entanglement protocol. It provides essential nonlinearity for creating the magnomechanical entanglement, which gives rise to the optomagnonic entanglement if the state-swap operation between the phonons and the photons is activated. In Sec. II A we show how to quantize the magnetization, the strain displacement, and the magnetoelastic energy. This allows us to derive the Hamiltonian of the magnomechanical dispersive interaction, and specify under what conditions such a dispersive coupling is dominant in the magnetoelastic coupling. We then introduce, in Sec. II B, the complete model, namely the opto-magnomechanical system, that is adopted in the entanglement and state readout protocols.

A. Magnomechanical coupling

The magnetoelastic coupling describes the interaction between the magnetization and the elastic strain of the magnetic material. There are three kinds of interactions, depending on the distance between magnetic atoms (or ions): The spin-orbital interaction, the exchange interaction, and the magnetic dipole-dipole interaction. For a cubic crystal, the magnetoelastic energy density is given by

\[ f_{\text{me}} = \frac{B_1}{M_s^2} \left( M_x^2 \epsilon_{xx} + M_y^2 \epsilon_{yy} + M_z^2 \epsilon_{zz} \right) \]

where \( B_1 \) and \( B_2 \) are the magnetoelastic coupling coefficients, \( M_s \) is the saturation magnetization, and \( M_x, M_y, M_z \) are the magnetization components. \( \epsilon_{ij} = \frac{1}{V} \left( \partial u_i / \partial x_j + \partial u_j / \partial x_i \right) \) denotes the strain tensor of the magnetic crystal, with coordinate indices \( i, j \in \{ x, y, z \} \), and \( u_{x,y,z} \) are the components of the displacement vector \( \vec{u} \).

Using the Holstein-Primakoff transformation, the magnetization can be quantized as

\[ m = \sqrt{\frac{V}{2 \hbar y M_s}} \left( M_x - i M_y \right), \]

where \( m \) denotes the magnon mode operator, \( V \) is the volume of the crystal, and \( y \) is the gyromagnetic ratio. Thus, we obtain

\[ M_x = \sqrt{\frac{\hbar y M_s}{2V}} (m + m^\dagger), \]

\[ M_y = i \sqrt{\frac{\hbar y M_s}{2V}} (m - m^\dagger), \]

and

\[ M_z = \left( M_x^2 - M_y^2 + M_z^2 \right)^{1/2} = M_s - \frac{\hbar y}{V} m^\dagger m. \]

Substituting the above expressions into the magnetoelastic energy density \( f_{\text{me}} \) and integrating over the whole volume of the crystal, we obtain the semiclassical magnetoelastic Hamiltonian. The first line in Eq. (1) gives rise to

\[ H_1 \approx \frac{B_1}{M_s^2} \frac{\hbar y}{V} m^\dagger m \int d^3l \left( \epsilon_{xx} + \epsilon_{yy} - 2 \epsilon_{zz} \right) \]

\[ + \frac{B_1}{M_s^2} \frac{\hbar y}{2V} \left( m^2 + m^\dagger m^\dagger \right) \int d^3l \left( \epsilon_{xx} - \epsilon_{yy} \right) \]

\[ + \frac{B_1}{M_s^2} \frac{\hbar y}{V} m^\dagger m^\dagger m \int d^3l \epsilon_{zz}, \]

and the second line yields the Hamiltonian

\[ H_2 \approx \frac{B_2}{M_s^2} \frac{\hbar y}{V} \left( m^2 - m^\dagger m^\dagger \right) \int d^3l \epsilon_{yy} + \frac{2B_2}{M_s^2} \frac{\hbar y M_s}{2V} \int d^3l \left( m^\dagger m \right) \int d^3l \left( \epsilon_{zz} + i \epsilon_{yz} \right) + \text{H.c.}. \]

The magnetoelastic Hamiltonian, \( H_{\text{me}} = H_1 + H_2 \), implies diverse magnon-phonon interactions, depending on their frequency relation. For given magnon and phonon frequencies, a certain type of coupling can be dominant in the magnetoelastic coupling, while other coupling mechanisms play only a negligible role. This can be seen more clearly in the fully quantized interaction Hamiltonian by further quantizing the strain displacement.

The magnetoelastic displacement can be decomposed and expressed as the superposition form

\[ \vec{u}(x, y, z) = \sum_{n,m,k} d^{(n,m,k)}(n,m,k)(x, y, z), \]

where \( \chi^{(n,m,k)}(x, y, z) \) represents the dimensionless displacement eigenmode, and \( d^{(n,m,k)} \) is the corresponding amplitude, with the mode indices \( n, m, k \). The displacement amplitude \( d^{(n,m,k)} \) can be quantized as

\[ d^{(n,m,k)} = d^{(n,m,k)}_{\text{ph}} (b_{n,m,k} + b_{n,m,k}^\dagger), \]

where \( d^{(n,m,k)}_{\text{ph}} \) denotes the amplitude of the zero-point motion, and \( b_{n,m,k} \) (\( b_{n,m,k}^\dagger \)) is the annihilation (creation) operator of the corresponding phonon mode.

By quantizing the strain displacement in the semiclassical Hamiltonians of Eqs. (2) and (3), the fully quantized magnetoelastic Hamiltonian can be derived, which accounts for
dissimilar magnon-phonon interactions when their frequencies vary. To be specific, when the phonon frequencies are much lower than the magnon frequency, \( \omega_{(n,m)} \ll \omega_m \) (typically the case for a large-size crystal \([19, 22]\)), by neglecting the fast-oscillating terms, we obtain the dominant dispersive-type interaction

\[
H_{\text{me}} \approx \sum_{n,m,k} \hbar g_{\text{disp}}(n,m,k) m^1 m (b_{n,m,k} + b_{n,m,k}^\dagger),
\]

where the dispersive coupling strength

\[
g_{\text{disp}}(n,m,k) = \frac{B_1 \gamma}{M_S V} \int d^3 r \, d^3 q \, \gamma m^1 m \left( \frac{\partial^2 \chi_x(n,m,k)}{\partial x^2} + \frac{\partial^2 \chi_y(n,m,k)}{\partial y^2} - 2 \frac{\partial \chi_z(n,m,k)}{\partial z} \right).
\]

Note that the above Hamiltonian is derived under the assumption that the magnon-phonon linear coupling of kHz or lower is much smaller than the magnon frequency, \( \omega \), implying that the system can be approximated as a coupled cavity. Moreover, in analogy to the generation of entangled pairs of magnons, in analogy to the generation of entangled pairs of magnons, such a magnon PA coupling can be used to generate entangled pairs of magnons, or in analogy to the generation of entangled photon pairs by optical parametric down-conversion.

To sum up, in this section we provide a strict derivation of the dominant effective magnomechanical Hamiltonian for different situations of the magnon and phonon frequencies: Specifically, i) a dispersive coupling for lower-frequency phonons, \( \omega_{(n,m)} \ll \omega_m \), ii) a linear coupling for nearly resonant magnons and phonons, \( \omega_{(n,m)} \approx \omega_m \); and iii) a magnon PA coupling for the phonon frequency being twice of the magnon frequency, \( \omega_{(n,m)} \approx 2\omega_m \). The effective Hamiltonian is obtained by neglecting inappreciable additional fast-oscillating terms in the derivation, which is a good approximation when the magnon and (or) phonon frequency are (is) much larger than their coupling strength and dissipation rates.

B. Hamiltonian and Langevin equations

The derivation of the magnomechanical Hamiltonian indicates that the mechanical frequency should be much lower than the magnon frequency in order to have a dispersive interaction. The magnomechanical system can be a yttrium-iron-garnet (YIG) three-dimensional magnon nanoresonator \([54]\), in which long-lived spin wave excitations are coupled to mechanical vibrations. In Ref. \([54]\), a micron-sized YIG bridge supports a magnon mode with the frequency in GHz and a mechanical vibration mode with the frequency ranging from tens to hundreds of MHz. Therefore, such a system possesses a dominant magnon-phonon dispersive coupling. Another advantage of the system is that it lies in the narrow linewidth (several MHz) of the magnetic excitations. By attaching a small high-reflectivity mirror pad to the surface of the YIG bridge, as depicted in Fig. 1, the magnetostriction induced mechanical displacement can be coupled to an optical field via radiation pressure \([49, 55, 56]\), forming an optomechanical cavity with another fixed mirror. Such a tripartite opto-magnomechanical configuration was adopted to optically measure the magnon...
population \cite{49}. We note that the attached mirror pad should be fabricated sufficiently small and light, such that it will not appreciably affect the mechanical properties (i.e., the mechanical displacement) of the micro bridge.

The Hamiltonian of the hybrid opto-magnomechanical system reads

\[
H/\hbar = \sum_{j=m,a} \omega_j j^2 + \frac{\omega_b}{2}(q^2 + p^2) + g_m m a^\dagger a + \hbar \Omega \left( a^\dagger e^{-i\omega_b t} - a e^{i\omega_b t} \right),
\]

where \(a\) and \(a^\dagger\) (\(m\) and \(m^\dagger\)) are the annihilation and creation operators of the cavity (magnon) mode, satisfying the canonical commutation relation \([j, j^\dagger] = 1\) \((j = m, a)\), and \(q\) and \(p\) denote the dimensionless mechanical position and momentum, \([q, p] = i\), and \(\omega_m = \omega_m (k = a, m, b)\) are the resonance frequencies of the cavity, magnon, and mechanical modes, respectively. The magnon mode is activated by placing the YIG micro bridge in a uniform bias magnetic field \((H_0)\) and applying a microwave field with its magnetic component \((H_\Omega)\) perpendicular to the bias field, see Fig. 1. The magnon frequency is tunable by varying the bias magnetic field. \(g_m\) \((g_a)\) is the bare magnomechanical (optomechanical) coupling rate, which can be improved by driving the magnon (cavity) mode with a microwave (laser) field. The Rabi frequency associated with the microwave drive is \(\Omega = \frac{\sqrt{\gamma}}{\sqrt{\hbar}} H_\Omega\) \((\gamma, \text{the gyromagnetic ratio}; N, \text{the total number of spins})\), and the cavity-laser coupling strength is \(E = \sqrt{\kappa_L P_L/\hbar\omega_L}\), with \(P_L\) \((\omega_L)\) being the power (frequency) of the laser, and \(\kappa_a\) the cavity decay rate.

Including the dissipation and input noise of each mode and working in the interaction picture with respect to \(\hbar \omega_m a^\dagger a + \hbar \omega_m m^\dagger m\), we obtain the following quantum Langevin equations (QLEs):

\[
\dot{q} = \omega_a p, \quad \dot{p} = -\omega_0 q - \gamma_p p - g_m^2 m + g_0 a^\dagger a + \xi_m,
\]

\[
\dot{a} = -i\Delta_0 a - \frac{\kappa_m}{2} a + ig_a a q + E + \sqrt{\kappa_a} a_m,
\]

\[
\dot{m} = -i\Delta_m m - \frac{\kappa_m}{2} m - ig_m m q + \Omega + \sqrt{\kappa_m} m_m,
\]

where \(\Delta_0 = \omega_a - \omega_L\), \(\Delta_m = \omega_m - \omega_0\), and \(\gamma_p, \kappa_m, \text{and } \kappa_m\) are the dissipation rates of the mechanical, cavity, and magnon modes, respectively. The corresponding zero-mean input noise operators \(\xi_m, a_m, \text{and } m_m\) obey the following correlation functions: \(\langle \xi(t) \xi(t') + \xi(t') \xi(t) \rangle/2 = \gamma_p /\{2N_0(\omega_m) + 1\}\).\(\delta(t - t')\) under the Markovian approximation, which is valid for a large mechanical quality factor \(Q = \omega_m /\gamma_p \gg 1\) \cite{57}.

\[
\langle a_m(t) a_m^\dagger(t') \rangle = \delta(t - t'),\langle m_m(t) m_m^\dagger(t') \rangle = [N_0(\omega_m) + 1]\delta(t - t'), \text{and } [m_m^\dagger(t) m_m(t)] = N_0(\omega_m)\delta(t - t'),\]

where the mean thermal excitation number \(N_0(\omega_m) = [\exp(\hbar \omega_m/k_B T) - 1]^{-1}\) \((j = b, m)\) at an environmental temperature \(T\).

Since the magnon and cavity modes are strongly driven, resulting in large steady-state amplitudes \([m], [a] \gg 1\), the nonlinear opto- and magnomechanical dynamics can be linearized around stationary classical averages. This is realized by writing each mode operator as the sum of its classical average and a quantum fluctuation operator, \(k = (k) + \delta k (k = a, m, q, p)\), and neglecting small second-order fluctuation terms. Consequently, the QLEs \cite{16} are separated into two sets of equations for classical averages and quantum fluctuations, respectively. By solving the former set of equations for the classical averages in the steady state, we obtain the following solutions:

\[
\langle q \rangle = \left( g_a \langle |a|^2 \rangle - g_m \langle |m|^2 \rangle \right)/\Omega, \quad \langle p \rangle = 0,
\]

\[
\langle a \rangle = \frac{E}{i\Delta_m + \frac{\kappa_m}{2}}, \quad \langle m \rangle = \frac{\Omega}{i\Delta_m + \frac{\kappa_m}{2}},
\]

where \(\Delta_m = \Delta_a + g_a \langle q \rangle\), \(\Delta_m = \Delta_a + g_m \langle q \rangle\) is the effective cavity (magnon)-drive detuning by including the frequency shift jointly caused by the opto- and magnomechanical interactions.

The linearized QLEs for the quantum fluctuations are given, in the quadrature form, by

\[
\delta \ddot{q} = \omega_m \delta p, \quad \delta \ddot{p} = -\omega_0 \delta q - \gamma_p \delta p + \text{Im} G_a \delta X_a - \text{Re} G_a \delta Y_a
\]

\[
+ \text{Im} G_m \delta X_m - \text{Re} G_m \delta Y_a + \frac{\sqrt{\kappa_m}}{\sqrt{\hbar}} X_{a,m},
\]

\[
\delta \ddot{X}_a = -\frac{\kappa_m}{2} \delta X_a + \Delta_a \delta Y_a + \text{Re} G_a \delta q + \sqrt{\kappa_m} X_{a,m},
\]

\[
\delta \ddot{Y}_a = -\Lambda_a \delta X_a - \frac{\kappa_m}{2} \delta Y_a + \text{Im} G_a \delta q + \sqrt{\kappa_m} Y_{a,m},
\]

\[
\delta \ddot{X}_m = -\frac{\kappa_m}{2} \delta X_m + \Delta_m \delta Y_m + \text{Re} G_m \delta q + \sqrt{\kappa_m} X_{m,m},
\]

\[
\delta \ddot{Y}_m = -\Lambda_m \delta X_m - \frac{\kappa_m}{2} \delta Y_m + \text{Im} G_m \delta q + \sqrt{\kappa_m} Y_{m,m},
\]

where the quadrature fluctuations \(\delta X_k = (\delta k + \delta k')/\sqrt{2}, \delta Y_k = i(\delta k - \delta k')/\sqrt{2}\), and the quadratures of the input noises \(X_{a,m} = (k_m + k_m')/\sqrt{2}\), and \(Y_{a,m} = i(k_m' - k_m)/\sqrt{2}\) \((k = a, m)\). The effective opto- and magnomechanical coupling strengths are \(G_a = i\sqrt{2} g_a \langle a \rangle\), and \(G_m = -i\sqrt{2} g_m \langle m \rangle\), respectively.

The QLEs \cite{18} can be rewritten in a compact matrix form of

\[
u(t) = Au(t) + n(t),
\]

where \(u(t)\) is the vector of the fluctuation quadratures, \(u(t) = [\delta q(t), \delta p(t), \delta X_a(t), \delta Y_a(t), \delta X_m(t), \delta Y_m(t)]^T\); \(n(t)\) is the vector of the input noises, \(n(t) = [0, \xi(t), \sqrt{\kappa_m} \delta X_{a,m}(t), \sqrt{\kappa_m} \delta Y_{a,m}(t), \sqrt{\kappa_m} \delta X_{m,m}(t), \sqrt{\kappa_m} \delta Y_{m,m}(t)]^T\), and the drift matrix \(A\) is given by

\[
A = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_b & -\gamma_b & 0 & 0 & -G_a & 0 \\
G_a & 0 & -\frac{\kappa_m}{2} & \Delta_a & 0 & 0 \\
0 & 0 & -\Delta_a & -\frac{\kappa_m}{2} & 0 & 0 \\
G_m & 0 & 0 & 0 & -\frac{\kappa_m}{2} & \Delta_m \\
0 & 0 & 0 & 0 & -\Delta_m & -\frac{\kappa_m}{2}
\end{pmatrix}.
\]

Note that the above drift matrix is given under the optimal condition for the entanglement, \(\Delta_m = \Delta_a + g_m \langle q \rangle\), \(\kappa_m \gg \kappa_a, \kappa_m \) \cite{20}. This gives rise to approximately pure imaginary steady-state amplitudes \(a\) and \(m\), and thus real opto- and magnomechanical coupling strengths \(G_a\) and \(G_m\).
Under the condition $\omega_h \gg G, \kappa$, which validates the rotating wave approximation, the dispersive opto- and magnomechanical couplings enable the realization of distinct opto- and magnomechanical operations by properly driving the system, i.e., the parametric down-conversion for a blue-detuned drive, $\Delta \approx -\omega_h$, and the beam-splitter (state-swap) operation for a red-detuned drive, $\Delta \approx \omega_h$. These operations are the building blocks for realizing the two protocols to be discussed in the following sections.

III. STATIONARY OPTOMAGNONIC ENTANGLEMENT

In this section, we show how to achieve the stationary entanglement between magnons and optical photons via the mediation of phonons. Specifically, we drive the magnon (cavity) mode with a blue-detuned microwave field (a red-detuned laser field), and the detuning is equal to the mechanical frequency, i.e., $-\Delta_m = \Delta_a = \omega_b$, such that the magnon (cavity) mode is resonant with the magnomechanical (optomechanical) Stokes (anti-Stokes) sideband, see Fig. 2. We consider the case of resolved sidebands ($\omega_h > \kappa_m, \kappa_a$), which can be achieved for a micron-sized YIG bridge [54] and a typical optomechanical cavity [48]. The simultaneous activation of the magnomechanical parametric down-conversion (leading to magnon-phonon entanglement) and the optomechanical beam-splitter interaction (yielding photon-phonon state swapping) results in magnon-phonon entanglement. The optomagnonic entanglement can be prepared in the steady state of the system quantum fluctuations is a three-mode Gaussian state, which can be fully characterized by the covariance matrix (CM) $V$, with the entries defined as $V_{ij} = \langle u_i(t)u_j(t') + u_j(t')u_i(t) \rangle / 2$ ($i, j = 1, 2, ..., 6$). The steady-state CM of the system can be obtained by directly solving the Lyapunov equation [53]

$$AV + VA^T = -D,$$

where $D = \text{diag}[0, 2\gamma_b(N_b + \frac{1}{2}), \gamma_\text{opt}, \gamma_\text{opt}, \kappa_m(N_m + \frac{1}{2}), \kappa_m(N_m + \frac{1}{2})]$ is the diffusion matrix, which is defined by $D_{ij} = \langle n_i(t)n_j(t') + n_j(t')n_i(t) \rangle / 2$. The Gaussian bipartite entanglement is quantified by the logarithmic negativity [59], defined as

$$E_N = \max [0, -\ln(2\eta^-)].$$

where $\eta^- \equiv 2^{-1/2}[\Sigma - (\Sigma^2 - 4\det V_4)^{1/2}]^{1/2}$, and $V_4 = [V_a, V_{am}, V_{am}^T, V_m]$ is the $4 \times 4$ CM associated with the optomagnonic subsystem, with $V_a, V_{am}, V_{am}^T, V_m$ being the Lyapunov equation [58] for the optimal detunings $\Delta_m = \Delta_a = \omega_b$. When $\eta^- > 0$, we confirm our preceding analyses on the mechanism of entanglement generation, as depicted in Fig. 3. The stationary entanglement is governed by the negative eigenvalues (real parts) of the drift matrix $A$. We use experimentally feasible parameters [48][54]: $\omega_m/2\pi = 5 \text{ GHz}$, $\omega_b/2\pi = 40 \text{ MHz}$, an optical wavelength $\lambda = 1064 \text{ nm}$, $\kappa_m/2\pi = 3 \text{ MHz}$, $\kappa_a/2\pi = 2 \text{ MHz}$, $\gamma_b/2\pi = 10 \text{ Hz}$, $g_{am}/2\pi = 10 \text{ Hz}$, $g_{am}$, $2\pi = 1 \text{ kHz}$, $T = 10 \text{ mK}$, and the effective coupling strengths $G_a/2\pi = 4 \text{ MHz}$ and $G_m/2\pi = 1 \text{ MHz}$, corresponding to the laser power $P_L = 7.51 \text{ mW}$ and the microwave drive power $P_m = 9.24 \text{ mW}$ for a $5.0 \times 0.6 \times 0.1 \mu\text{m}$ YIG bridge [60]. Note that a stronger optomechanical coupling strength $G_a > G_m$ is used to keep the system stable, where the optomechanical anti-Stokes process by absorbing phonons outperforms the magnomechanical Stokes process by emitting phonons.
After a short period, the environment temperature, as shown in Fig. 3(b) that the output field of the pulse.

phonons, then from phonons to cavity photons, and eventually to the microwave field [61, 62], see Fig. 4. When the magnomechanical state is weak (m ≪ g_m), the magnonic state is essentially decoupled from the magnon mode is prepared in a squeezed vacuum state, and see how the squeezing can be transferred to the mechanics by applying a red-detuned drive field. The magnonic squeezed states can be realized by activating the magnomechanical state-swap interaction realized by driving the magnon mode with a red-detuned microwave field [61] [62], see Fig. 4. When the magnomechanical system reaches a steady state, we then switch off the drive. After a short period, the magnon excitations completely dissipate, while the mechanical state remains almost unchanged due to a much longer mechanical lifetime, we then send a weak red-detuned optical pulse to the cavity to activate the optomechanical state-swap interaction. The two successive state-swap operations (magnon-to-phonon and phonon-to-photon) transfer the magnonic state to the mechanical mode.

IV. OPTICAL READOUT OF MAGNONIC STATES

Optical readout and transmission of localized magnonic states in solids play a crucial role in building a future magnonic quantum network [26], and implementing remote quantum operations. Although it was proved that an arbitrary magnonic (either classical or quantum) state can be read out by using fast optical pulses [26] in cavity optomagnonic systems [12–14], the protocol suffers a low optomagnonic state-conversion efficiency because of the currently weak optomagnonic coupling and the much larger cavity decay rate.

Here we provide a new approach using the indirectly coupled opto-magnomechanical system, which can evade the low conversion efficiency problem existing in the directly coupled optomagnonic systems. The protocol consists of two steps. We first transfer the magnonic state to the mechanical mode by activating the magnomechanical state-swap interaction realized by driving the magnon mode with a red-detuned microwave field [61] [62], see Fig. 4. When the magnomechanical system reaches a steady state, we then switch off the drive. After a short period, the magnon excitations completely dissipate, while the mechanical state remains almost unchanged due to a much longer mechanical lifetime, we then send a weak red-detuned optical pulse to the cavity to activate the optomechanical state-swap interaction. The two successive state-swap operations (magnon-to-phonon and phonon-to-photon) transfer the magnonic state to the cavity output field of the pulse, which can be readily measured by optical means, e.g., homodyne detection or state tomography. Note that in each step, we consider a two-mode model, in view of the fact that the magnomechanical coupling is weak (g_m ≪ g_e), the magnon drive is switched off and the magnons die out. This allows us to assume, in the second step, that the magnon mode is essentially decoupled from the optomechanical system when the pulse is applied. The Hamiltonian of the magnomechanical subsystem is given by

$$H_m = \sum_{j=n,b} \omega_j \hat{b}_j^\dagger \hat{b}_j + \frac{g_m}{\sqrt{2}} m^\dagger \hat{m} + i \Omega \left( m^\dagger e^{-i\omega_{cb} t} - H.c. \right),$$

where, to more transparently express the physics, we use the operator \(b\) to denote the mechanical mode, which is related to the displacement \(q\) via \(q = (b + b^\dagger)/\sqrt{2}\). In the frame rotating at the drive frequency \(\omega_d\), we obtain the following QLEs:

$$\dot{b} = -i\omega_b b - \frac{\gamma_b}{2} b + i \frac{g_m}{\sqrt{2}} m^\dagger + \sqrt{2} \gamma_b \hat{b}_{in},$$

$$\dot{m} = -i \Delta_m m - \frac{\kappa_m}{2} m - i \frac{g_m}{\sqrt{2}} (b^\dagger + b) + \Omega + \sqrt{2} \kappa_{m,\text{in}} m_{\text{in}}^\dagger,$$

where \(b_{in}\) denotes the mechanical input noise, which obeys the correlation functions:

$$\langle \hat{b}^\dagger(t) \hat{b}(t') \rangle = (N_b(\omega_{cb}) + 1) \delta(t - t'),$$

and \(\langle b(t) b^\dagger(t') \rangle = N_b(\omega_{cb}) \delta(t - t').$$ Using the same linearization treatment, we get the QLEs for the fluctuations

$$\dot{\delta b} = -i\omega_b \delta b - \frac{\gamma_b}{2} \delta b + \frac{G_m}{2} (\delta m^\dagger - \delta m) + \sqrt{2} \gamma_b \delta b_{in},$$

$$\dot{\delta m} = -i \Delta_m \delta m - \frac{\kappa_m}{2} \delta m + \frac{G_m}{2} (\delta b + \delta b^\dagger) + \sqrt{2} \kappa_{m,\text{in}} \delta m_{\text{in}},$$

where the definition of the effective coupling strength \(G_m\) is the same as in Eq. (18).

Moving to another interaction picture by introducing the slowly moving operators \(\delta b = \delta b e^{i\Omega t}, \delta m = \delta m e^{i\Omega t}\), and by further taking the optimal detuning \(\Delta_m = 0\) for realizing state transfer and neglecting nonresonant fast-oscillating terms (valid when \(\omega_d \gg G_m, \kappa_m, \gamma_b\)), we obtain

$$\dot{\delta b} = -\frac{\gamma_b}{2} \delta b - \frac{G_m}{2} \delta m + \sqrt{2} \gamma_b \delta b_{in},$$

$$\dot{\delta m} = -\frac{\kappa_m}{2} \delta m + \frac{G_m}{2} \delta b + \sqrt{2} \kappa_{m,\text{in}} \delta m_{\text{in}},$$

which correspond to an effective beam-splitter interaction Hamiltonian, accounting for the magnon-phonon state-swap operation.

We now consider a specific example where the magnon mode is prepared in a squeezed vacuum state, and see how the squeezing can be transferred to the mechanics by applying a red-detuned driven field. The magnonic squeezed vacuum can be characterized by the following squeezed noise correlations [61] [62]:

$$\langle \hat{m}_u(t) \hat{m}_u^\dagger(t') \rangle = (N + 1) \delta(t - t'),$$

$$\langle \hat{m}_u(t) \hat{m}_v(t') \rangle = N \delta(t - t'),$$

and \(\langle \hat{m}_u^\dagger(t) \hat{m}_v^\dagger(t') \rangle = N \delta(t - t'),\) where \(N = \sinh^2 r, M = \sinh \text{csinh} r, r\) being the squeezing parameter, characterizing the degree of the magnon squeezing.

The CM of the mechanical mode \(V_b\) can be achieved by solving the QLEs [26]. The steady-state \(V_b\) can be obtained more conveniently by solving the Lyapunov equation (see the Appendix). Given the CM \(V_b\), one can calculate the Wigner function of the mechanical mode [63]

$$W(u_b) = \frac{\exp(-u_b V_b^{-1} u_b^\dagger)}{\pi \sqrt{\det V_b}},$$

FIG. 4: Mode frequencies and linewidths used in the state readout protocol. When the magnon (cavity) mode is resonant with the anti-Stokes sideband of the microwave drive field (the optical pulse) at the frequency of \(\omega_m (\omega_c)\), the magnonic state can be transferred to phonons, then from phonons to cavity photons, and eventually to the output field of the pulse.
where \( u_0 = (\delta q, \delta p) \) denotes the phase-space variables associated with the fluctuations of the mechanical quadratures. Similarly, one can get the Wigner function \( W(\delta X_m, \delta Y_m) \) of the squeezed magnon mode at the initial time. The squeezed Wigner distribution \( W(\delta q, \delta p) \) in phase space clearly shows the squeezing is transferred from magnons to phonons by applying a red-detuned magnon drive, c.f. Figs. 5(a) and 5(b). The squeezing is reduced, to some extent, due to the presence of thermal noises and dissipations of the system. We use the same magnomechanical parameters as in Fig. 3 but take a smaller coupling rate \( G_m/2\pi = -0.1 \, \text{MHz} \), corresponding to the microwave drive power \( P_m \approx 0.09 \, \text{mW} \).

When the system reaches its steady state, we switch off the magnon drive. After a short period, \( \kappa_m^{-1} < \tau_0 \ll \gamma^{-1} \), during which the magnon excitations completely decay whereas the mechanical state remains virtually unchanged, we then send a weak red-detuned pulse to the optical cavity to activate the optomechanical beam-splitter interaction, which transfers the mechanical state to the cavity output field. The pulse duration is much shorter than the mechanical lifetime, such that the mechanical damping within the pulse duration is negligible and thus can be neglected for simplicity [26].

The Hamiltonian of the optomechanical subsystem reads

\[
H_{ab}/\hbar = \sum_{j=a,b} \omega_j j - \frac{g}{\sqrt{2}} a (b + b^\dagger) + i E (a e^{-jG_
u t} - \text{H.c.}). \tag{28}
\]

Following the same approach used from Eq. (24) to Eq. (26), and neglecting the mechanical dissipation, we obtain the QLEs (the tilde signs are removed for convenience)

\[
\dot{\delta a} \approx -\frac{\kappa_a}{2} \delta a + \frac{G_a}{2} \delta b + \sqrt{\kappa_a} \delta a_{in},
\]

\[
\dot{\delta b} \approx -\frac{G_a}{2} \delta a - \frac{\kappa_a}{2} \delta b + \sqrt{\kappa_a} \delta a_{in}, \tag{29}
\]

which are derived under the conditions of the optimal detuning \( \Delta_\nu = \omega_b \), and \( \omega_b \gg G_a, \kappa_a \) for taking the rotating-wave approximation. The definition of the coupling \( G_a \) is consistent with that in Eq. (18).

To simplify the model, we consider a flattop pulse and thus a constant coupling \( G_a \) during the pulse. The pulse strength is relatively weak to have a weak coupling \( G_a \ll \kappa_a \), which allows one to adiabatically eliminate the cavity, and obtain \( \delta a \approx \sqrt{\kappa_a} \delta b + \frac{1}{2} \sqrt{\kappa_a} \delta a_{in} \). By using the cavity input-output relation, \( a_{out} = \sqrt{\kappa_a} \delta a - a_{in} \), we have

\[
a_{out} = \sqrt{2G_a} \delta b + a_{in},
\]

\[
\delta b = -G_a \delta b - \sqrt{2G_a} \delta a_{in}, \tag{30}
\]

where \( G_a = G_a^2/2\kappa_a \). Following [64], we define the normalized input and output temporal modes for the cavity driven by the pulse as

\[
A_{in}(t) = \int_0^t e^{G_a t} a_{in}(\tau) d\tau,
\]

\[
A_{out}(t) = \sqrt{\frac{2G_a}{1 - e^{-2G_a t}}} \int_0^t e^{-G_a \tau} a_{out}(\tau) d\tau. \tag{31}
\]

FIG. 5: Wigner distribution of (a) the squeezed \((r = 1)\) magnon mode before applying the microwave drive, (b) the steady-state mechanical mode under the drive, and (c) the output field of the pulse after the magnon drive is turned off. The dashed white circle denotes vacuum fluctuations, below which the corresponding quadrature is squeezed. See the text for the parameters.

By integrating Eqs. (30), we obtain the following solutions [26]

\[
A_{out}(t) = \sqrt{1 - e^{-2G_a t}} A_{in}(0) + e^{-G_a t} A_{in}(t),
\]

\[
\delta b(t) = e^{G_a t} \delta b(0) - \sqrt{1 - e^{-2G_a t}} A_{in}(t). \tag{32}
\]

From the first equation of Eq. (32), we can get the CM of the output field of the pulse, i.e.,

\[
V_{out}(t) = S V_{in}(0) + (1 - S) V_{in}(t), \tag{33}
\]

where \( S = 1 - e^{-2G_a t} (0 < S < 1) \) represents the state conversion efficiency, depending on the pulse strength and duration, \( V_{in}(t) \) is the CM of the input vacuum noise, and \( V_{in}(0) \) is approximately the steady-state CM \( V_{in} \) of the mechanical mode obtained in the first step, i.e., \( V_{in}(0) \approx V_{in} \). For \( G_a \tau \gg 1 \), \( S \rightarrow 1 \), and therefore \( V_{out}(t) \approx V_{in}(0) \), which means the mechanical state is almost perfectly transferred to the output field of the pulse.

Using a relatively weak coupling \( G_a/2\pi = 0.3 \, \text{MHz} \ll \kappa_a/2\pi = 2 \, \text{MHz} \) (corresponding to the pulse power \( P_L = 0.04 \, \text{mW} \)), and pulse duration \( t = 10 \, \mu\text{s} \), we obtain the state conversion efficiency \( S = 94 \). The rest of the parameters are the same as in Fig. 3. The Wigner function of the pulse output field is displayed in Fig. 5(c). The nonunity conversion efficiency adds additional vacuum noise into the mechanical squeezed state, thus reducing the degree of squeezing, as shown by comparing Figs. 5(b) and 5(c).
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APPENDIX

The QLEs (26) can be rewritten in quadratures and in the matrix form of

\[ \dot{u}(t) = Ru(t) + n'(t), \]

where

\[
\begin{bmatrix}
\dot{u}(t) \\
\dot{p}(t)
\end{bmatrix}
= \begin{bmatrix}
[\mathcal{H} + \gamma \mathcal{N}]
\end{bmatrix} \\
\begin{bmatrix}
[\mathcal{H} + \gamma \mathcal{N}]
\end{bmatrix}
\]

\[ n'(t) = [\mathcal{H} + \gamma \mathcal{N}]
\]

For simplicity, we remove the tilde signs on the operators. The drift matrix \( R \) is given by

\[
R = \begin{bmatrix}
-\frac{\gamma}{2} & 0 & -\frac{\gamma}{2} & 0 \\
0 & -\frac{\gamma}{2} & 0 & -\frac{\gamma}{2} \\
-\frac{\gamma}{2} & 0 & -\frac{\gamma}{2} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

The diffusion matrix \( Z \), which is defined as

\[
Z = \begin{bmatrix}
\gamma_b(N_b + + \frac{1}{2}) & 0 & 0 & 0 \\
0 & \gamma_b(N_b + \frac{1}{2}) & 0 & 0 \\
0 & 0 & Z_m^{11} & Z_m^{12} \\
0 & 0 & Z_m^{21} & Z_m^{22}
\end{bmatrix}
\]

where

\[
Z_m^{11} = \frac{\gamma_b}{2}(2N + 1 + M + M^*), \quad Z_m^{12} = \frac{\gamma_b}{2}(M + M^* - M), \quad Z_m^{21} = \frac{\gamma_b}{2}(2N + 1 - M - M^*), \quad Z_m^{22} = \frac{\gamma_b}{2}(2N + 1 - M - M^*)
\]

The steady-state \( 4 \times 4 \) CM \( V_{ab} \) of the magnomechanical system can then be achieved by solving the Lyapunov equation

\[
RV_{ab} + V_{ab}R^T = -Z
\]

Once the \( V_{ab} \) is obtained, one then gets the \( 2 \times 2 \) CM of the mechanical (magnon) mode by removing the irrelevant rows and columns in \( V_{ab} \).

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[22] C. A. Potts, E. Varga, V. Bittencourt, S. V. Kusminskiy, and J. P. Davis, Phys. Rev. X 11, 031053 (2021).
[23] Y. Tabuchi et al., Science 349, 405 (2015).
[24] D. Lachance-Quirion et al., Sci. Adv. 3, e1603150 (2017).
[25] D. Lachance-Quirion et al., Science 367, 425 (2020).
[26] J. Li, Y.-P. Wang, W.-J. Wu, S.-Y. Zhu, and J. Q. You, PRX Quantum 2, 040344 (2021).
[27] S. Sharma, Y. M. Blanter, and G. E. W. Bauer, Phys. Rev. Lett. 121, 087205 (2018).
[28] V. A. S. V. Bittencourt, V. Feulner, and G. E. W. Bauer, Phys. Rev. A 100, 013810 (2019).
[29] Y.-P. Gao, X.-F. Liu, T.-J. Wang, C. Cao, and C. Wang, Phys. Rev. A 100, 043831 (2019).
[30] E. Almpanis, G. P. Zouros, P. A. Pantazopoulos, K. L. Tsakmakidis, N. Papanikolaou, and N. Stefanou, Phys. Rev. B 101, 054412 (2020).
[31] W.-L. Xu, Y.-P. Gao, C. Cao, T.-J. Wang, and C. Wang, Phys. Rev. A 102, 043519 (2020).
[32] Z.-X. Liu and H. Xiong, Opt. Lett. 45, 5452 (2020).
[33] S. Sharma, V. A. S. V. Bittencourt, A. D. Karenowska, and S. Viola Kusminskiy, Phys. Rev. B 103, L100403 (2021).
[34] F.-X. Sun, S.-S. Zheng, Y. Xiao, Q. H. Gong, Q. Y. He, and K. Xia, Phys. Rev. Lett. 127, 087203 (2021).
[35] W.-J. Wu, Y.-P. Wang, J.-Z. Wu, J. Li, and J. Q. You, Phys. Rev. A 104, 023711 (2021).
[36] H. Xie, Z. Shi, L. He, X. Chen, C. Liao, and X. Lin, Phys. Rev. A 105, 023701 (2022).
[37] D. Mukhopadhyay, J. M. P. Nair, and G. S. Agarwal, Phys. Rev. B 105, 064405 (2022).
[38] Q. Cao, L. Tan, and W.-M. Liu, Phys. Rev. A 105, 043705 (2022).
[39] B. Wang, X. Jia, X.-H. Lu, and H. Xiong, Phys. Rev. A 105, 053705 (2022).
[40] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. 73, 565 (2001).
[41] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. 77, 513 (2005).
[42] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[43] V. Giovannetti, S. Lloyd, and L. Maccone, Nat. Photonics 5, 222 (2011).
[44] B. Hensen et al., Nature (London) 526, 682 (2015).
[45] S. Bose et al., Phys. Rev. Lett. 119, 240401 (2017); C. Marletto and V. Vedral, Phys. Rev. Lett. 119, 240402 (2017).
[46] J. Li and S.-Y. Zhu, New J. Phys. 21, 085001 (2019).
[47] H. Y. Yuan, P. Yan, S. Zheng, Q. Y. He, K. Xia, and M.-H. Yung, Phys. Rev. Lett. 124, 053602 (2020).
[48] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014).
[49] Z.-Y. Fan, R.-C. Shen, Y.-P. Wang, J. Li, and J. Q. You, Phys. Rev. A 105, 033507 (2022).
[50] A. G. Gurevich and G. A. Melkov, Magnetization Oscillations and Waves (CRC, Boca Raton, FL, 1996).
[51] C. Kittel, Rev. Mod. Phys. 21, 541 (1949).
[52] T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).
[53] F. Godejohann et al., Phys. Rev. B 102, 144438 (2020).
[54] F. Heyroth et al., Phys. Rev. Applied 12, 054031 (2019).
[55] S. Gröblacher, K. Hammerer, M. R. Vanner, and M. Aspelmeyer, Nature (London) 460, 724 (2009).
[56] T. Bagci et al., Nature (London) 507, 81 (2014).
[57] R. Benguria and M. Kac, Phys. Rev. Lett. 46, 1 (1981); V. Giovannetti and D. Vitali, Phys. Rev. A 63, 023812 (2001).
[58] D. Vitali et al., Phys. Rev. Lett. 98, 030405 (2007).
[59] J. Eisert, Ph.D. thesis, University of Potsdam, 2001; G. Vidal and R. F. Werner, Phys. Rev. A 65, 032314 (2002); G. Adesso, A. Serafini, and F. Illuminati, Phys. Rev. A 70, 022318 (2004).
[60] The time average of energy density is $\rho_E = H^2 / 2 \mu_0$ ($\mu_0$ is the vacuum magnetic permeability). Thus, the microwave drive power $P_w = \rho_E \omega$, where $\omega$ is the speed of an electromagnetic wave propagating in vacuum, and $A$ is the cross-sectional area of the YIG micro bridge ($A = L \times W$, with $L$ and $W$ being the length and width of the micro bridge). The drive magnetic field is applied perpendicular to the cross-section (Fig. 1). Therefore, we obtain the relation between the drive magnetic field $H_d$ and the power $P_w$ via $H_d = \sqrt{2 \mu_0 P_w / (LW)}$.
[61] J. Li, S. Y. Zhu, and G. S. Agarwal, Phys. Rev. A 99, 021801(R) (2019).
[62] J. Li and S. Gröblacher, Quantum Sci. Technol. 6, 024005 (2021).
[63] S. M. Barnett and P. M. Radmore, Methods in Theoretical Quantum Optics (Clarendon, Oxford, 1997).
[64] S. G. Hofer, W. Wieczorek, M. Aspelmeyer, and K. Hammerer, Phys. Rev. A 84, 052327 (2011).