I. INTRODUCTION

In the last two decades, there has been dramatic theoretical progress in our understanding of RVB ideas and spin liquids. We now know that there are many different kinds of spin liquids. Gapped topological spin liquids are best understood and have been shown to exist in model systems. Gapless spin liquids are also possible and have been shown to exist in model systems. Since then, there have been many studies of Kitaev-type spin-1/2 model on the honeycomb lattice with a spin liquid phase paved an exciting road for the study of spin liquids. Gapped topological spin liquids with are also possible. and recently realized in experiments, but are understood to a lesser degree, particularly when both the emergent parton and gauge field excitations are gapless.

From the early days, slave particle approaches have played an important role in studying such phases. The discovery by Kitaev of an exactly solvable, interacting two-dimensional spin-1/2 model on the honeycomb lattice with a spin liquid phase paved an exciting road for the study of spin liquids. Since then, there have been many studies of Kitaev-type models. In the last year, several of them realized gapless spin liquid phases with parton Fermi surface, (and gapped gauge fields). In this work, we want to directly detect the presence of a surface of low-energy excitations. Note that the parton Fermi surface itself is gauge-dependent and is not accessible via local observables. However, there is a geometric surface information that is physical and can be detected using gauge-invariant local energy observables. We take up a very recent model by Chua et al. to illustrate this point.

Chua et al. proposed an exactly solvable spin-3/2 model on the kagome lattice and found a regime with a gapless spin liquid with a stable Fermi surface. Motivated by such a spin liquid phase and known techniques to characterize situations with gapless partons, we propose to study gauge-invariant operators such as bond energy operators. Our main results show that, unlike any spin correlations which are ultra-short ranged, the bond energy correlations have power-law behavior with a $1/|r|^3$ envelope and oscillations at incommensurate wavevectors. We determine the corresponding singular surfaces in momentum space, which provide a gauge-invariant characterization of this gapless spin liquid.

II. CHUA-YAO-FIETE KITAEV-TYPE HAMILTONIAN

We begin by formulating the Hamiltonian in the parton language. The model is defined on the kagome lattice, see Fig. 1. On each site $i$ of the kagome lattice, there is a phys-
tical four-dimensional Hilbert space realized using six Majorana fermions $\xi^1, \xi^2, \xi^3, \xi^4, c_i,$ and $d_i,$ with the constraint $D_1 \equiv -i\xi^1 \xi^2 \xi^3 \xi^4 c_i d_i = 1$ (namely, for any physical state $|\Phi\rangle$, we require $D_1 |\Phi\rangle_{\text{phys}} = |\Phi\rangle_{\text{phys}}$). The Chua-Yao-Fiete Kitaev-type Hamiltonian is

$$\mathcal{H} = i \sum_{(ij)} u_{ij} [J_{ij} c_i c_j + J'_{ij} d_i d_j] + i J_5 \sum_i c_i d_i$$

where $\langle ij \rangle$ represents nearest neighbor links and $u_{ij} = -u_{ji},$ with $u_{ij} = -i\xi^1 \xi^2$ if $\langle ij \rangle \in \triangle$ and $u_{ij} = -i\xi^3 \xi^4$ if $\langle ij \rangle \in \nabla$ for bond directions chosen to go counter-clockwise around the triangles. Plackett operators $W_p = \prod_{\langle ij \rangle \in p} u_{ij},$ with $p = \triangle, \nabla, \circ,$ are gauge-invariant (i.e., act in the physical Hilbert space) and are conserved by the Hamiltonian. The terms in Eq. (2) with $\alpha > 0$ and $\beta > 0$ are added to stabilize particular ground states with $W_\triangle = 1, W_\nabla = W_\circ = \pm 1.$ Since in the Kitaev-type model, $[u_{ij}, \mathcal{H}] = [u_{ij}, u_{ij}'] = 0,$ we can treat the $\mathbb{Z}_2$ gauge fields $u_{ij}$ as static background and replace by their eigenvalues $\pm 1.$ We then have free Majorana fermions $c_i$ and $d$ hopping on the lattice in the presence of “fluxes” $\phi_p$ defined via $e^{-i\phi_p} \equiv \prod_{\langle ij \rangle \in p} e^{i u_{ij}}.$

Throughout, we work in the ground state with $W_\triangle = 1, W_\nabla = W_\circ = 1$ which breaks time reversal symmetry; this translates to fluxes $\langle \phi_\triangle, \phi_\nabla, \phi_\circ \rangle = \{\pi, \pi/2, \pi/2\}$ as shown in Fig. 1. We fix the gauge by taking $u_{ij} = 1$ with bonds $i \rightarrow j$ directed counter-clockwise around the triangles. There are three physical sites per unit cell and six remaining Majoranas per unit cell. We replace the labeling $\{c_i, d_i\}$ with $\Psi^M_r = (r, a),$ where $r$ runs over the Bravais lattice of unit cells of the kagome network and $a$ runs over the six Majoranas in each unit cell (three $c$ Majoranas and three $d$ Majoranas). The Hamiltonian can be written in a concise form,

$$\mathcal{H} = \sum_{\langle (r, a), (r', a') \rangle} \Psi^M_{r, a} A_{r, a; r', a'} \Psi^M_{r', a'}$$

$$(3) \quad \mathcal{H} = \sum_{\langle IJ \rangle} \Psi^M_I A_{IJ} \Psi^M_J.$$  

The Majorana field satisfies the usual anticommutation relation, $\{\Psi^M_{r, a}, \Psi^M_{r', a'}\} = 2\delta_{rr'} \delta_{a a'} = 2 \delta_{IJ}.$ In the chosen gauge, there is translational symmetry between different unit cells; hence, $A_{r, a; r', a'} = A_{a a'}(r - r').$

In order to give a concise long wavelength description, it will be convenient to use familiar complex fermion fields. To this end, we can proceed as follows. For a general Majorana problem specified by matrix $A_{IJ}$, we diagonalize $A_{IJ}$ for spectra, but only half of the bands are needed while the rest of the bands can be obtained by a specific relation and are redundant. Explicitly, for a system with $2m$ bands, we can divide them into two groups. The first group contains bands from $1$ to $m$ with eigenvector-eigenenergy pairs $\{v_{b, k}, \epsilon_{b, k}\},$ where $b = 1, 2, \ldots, m$ are band indices, and the second group contains bands from $m + 1$ to $2m$ related to the first group, $\{v_{b', m+b, k}, v_{b'根本不\'} = m+b, k\} = \{v_{b, k}, -\epsilon_{b, k}\}.$

In the present case, $2m = 6$ and therefore three bands are sufficient to give us a full solution of the Majorana problem.

![FIG. 1. Kagome lattice with three sites (labelled as 1, 2, and 3) per unit cell. We consider Chua et al. model with the ground state flux configuration as shown. $\{\phi_\triangle, \phi_\nabla, \phi_\circ\} = \{\pi, \pi/2, \pi/2\}.$ We fix the gauge by taking $u_{ij} = 1$ with bonds directed counter-clockwise around the triangles. We also show the bond energy operator $B_{12}$ whose correlations are presented in this paper, while other bond energy operators have qualitatively similar correlations.](image)
working gauge. We will study bond energy correlator defined

\[ G_B(r) = \langle B_{12c}(0) B_{12c}(r) \rangle - \langle B_{12c}(0) \rangle \langle B_{12c}(r) \rangle. \]  

Power-law correlations in real space correspond to singularities in momentum space, which we can study by considering the structure factor

\[ S_B(q) = \sum_r G_B(r) e^{-i q \cdot r}. \]

We will present exact numerical calculation of the bond energy correlations in Sec. III B using the definitions in Eqs. (7)-(9). Before showing the numerical data, we present a long wavelength analysis of such correlations due to the gapless Fermi sea of partons.

### A. Long wavelength analysis

Focusing on the long distance behavior and therefore retaining only the contribution from band-3, the bond operator, Eq. (7), can be written approximately as

\[ B_{12c}(r) \simeq \sum_{k, k'} M_{kk'} f_3(k) f_3(k') e^{ij(k+k') \cdot r} + \text{H.c.} \]

\[ + \left[ N_{kk'} f_3^i(k) f_3(k') e^{-ij(k-k') \cdot r} + \text{H.c.} \right]. \]  

where \( M_{kk'} = 2iJ_\Delta v_3(k) v_3(k')/N_{uc} \), \( N_{kk'} = 2iJ_\Delta v_3^2(k) v_3(k')/N_{uc} \).

In order to determine long-distance behavior at separation \( r \), we focus on patches near the Fermi surface of band 3 where the group velocity is parallel or antiparallel to the observation direction \( \hat{n} = r/|r| \), because at long distance \( |r| \gg k_F^{-1} \), the main contributions to the bond energy correlations come precisely from such patches. Specifically, we introduce Right(R) and Left(L) Fermi patch fields and the corresponding energies

\[ f_P^{(n)}(\delta k) = f_3(k_P^{(\hat{n})} + \delta k), \]

\[ \epsilon_P^{(n)}(\delta k) = |\psi_P^{(\hat{n})}|^2 \left( P \delta k || + \frac{\alpha_P^{(\hat{n})}}{2} \delta k_\perp \right). \]

where the superscript \((\hat{n})\) refers to the observation direction and \( P = R/L = +/−; \psi_P^{(\hat{n})}\) is the corresponding group velocity (parallel to \( \hat{n} \) for the Right patch and anti-parallel for the Left patch); \( \alpha_P^{(\hat{n})} \) is the curvature of the Fermi surface at the Right/Left patch; \( \delta k_\parallel \) and \( \delta k_\perp \) are respectively components of \( \delta k \) parallel and perpendicular to \( \hat{n} \). It is convenient to define slowly varying fields in real space

\[ f_P^{(\hat{n})}(r) \sim \sum_{\delta k \in \text{Fermi Patch}} f_P^{(\hat{n})}(\delta k) e^{i \delta k \cdot r}, \]

which vary slowly on the scale of the lattice spacing [and from now on we will drop the superscript \((\hat{n})\)]. Therefore, in this
long wavelength terms of the relevant bond terms in the bond operator are
\begin{align}
B_{12c}(r) & \sim \left[ (N_{RR} + N_{RR}^*) f_R^*(r) f_R(r) + (R \rightarrow L) \right] \quad (14) \\
& + \left[ (N_{LR} + N_{LR}^*) f_L^*(r) f_L(r) e^{i(k_{FR} - k_{FL}) \cdot r} + \text{H.c.} \right] \quad (15) \\
& + \left[ (M_{RL} - M_{LR}) f_R(r) f_L(r) e^{i(k_{FR} + k_{FL}) \cdot r} + \text{H.c.} \right], \quad (16)
\end{align}

where we dropped terms such as $f_R(r) f_R(r)$ due to Pauli exclusion principle. The above long wavelength expression for the bond energy operator implies that the corresponding correlation function defined in Eq. (8) contains contributions with $q = 0, \pm(k_{FR} - k_{FL})$, and $\pm(k_{FR} + k_{FL})$.

More explicitly, for a patch specified by $\epsilon_P(\delta k)$ above, Eqs. (11)-(12), we can derive the Green’s function for the continuum complex fermion fields as
\[
\langle f_R^\dagger f_L(0) f_R f_L(r) \rangle = \frac{\exp[i\frac{2\pi}{3} r \cdot \epsilon_P]}{2^{3/2} \pi^{3/2} \alpha_{LR}^{1/2} |r|^{3/2}}. \quad (17)
\]

Using this and the long-wavelength expression for bond energy operators, we can obtain the bond energy correlation, Eq. (8).

\[
G_B(r) \sim -\frac{(N_{RR} + N_{RR}^*)^2}{\alpha_R^2 |r|^3} - \frac{(N_{LL} + N_{LL}^*)^2}{\alpha_L^2 |r|^3} + \frac{2|N_{RL} + N_{LR}^*|^2 \sin[(k_{FR} - k_{FL}) \cdot r]}{\alpha_R^{1/2} \alpha_L^{1/2} |r|^3} \quad (18)
\]

\[
+ \frac{2|M_{RL} - M_{LR}|^2 \cos[(k_{FR} + k_{FL}) \cdot r]}{\alpha_R^{1/2} \alpha_L^{1/2} |r|^3}. \quad (19)
\]

\[
+ \frac{2|M_{RL} - M_{LR}|^2 \cos[(k_{FR} + k_{FL}) \cdot r]}{\alpha_R^{1/2} \alpha_L^{1/2} |r|^3}. \quad (20)
\]

Therefore, the above low energy description can be used to analyze the numerical data we obtain by exact calculations. Here we also note that the model does not have inversion symmetry (and the time reversal is broken in the ground state), so the location of the corresponding R-L patches which are parallel or antiparallel to the observation direction can not be determined easily and need to be found numerically.

B. Exact numerical calculation

We calculate the bond energy correlations, Eq. (8), for any real-space separation $r$ and confirm that they have power law envelope $1/|r|^3$. For an illustration, we show the bond energy correlations for $r$ along a specific direction, e.g. $x$-axis, calculated on a $300 \times 300$ lattice. In Fig. 4 the log-log plot of $|G_B(r)|$ along the $x$-axis clearly shows the $1/|r|^3$ envelope. In addition, the irregular behavior of the data is due to oscillating components. For certain directions, the oscillating parts are sufficiently strong that $G_B(r)$ also changes signs. The wavevectors of the real-space oscillations form some singular surfaces in the momentum space, which we will analyze next.

Shifting our focus on the structure factor $S_B(q)$ defined in Eq. (9), we calculate the bond energy correlation at each site within a $100 \times 100$ lattice and numerically take Fourier transformation. Figure 5(a) gives a three-dimensional (3D) view of the structure factor. We can clearly see cone-shaped singularity at $q = 0$, which is expected from Eq. (18).

\[
S_B(q \sim 0) \sim |q|. \quad (21)
\]

A closer look at the structure factor also reveals singular surfaces at $k_{FR} - k_{FL}$ and $k_{FR} + k_{FL}$, as expected from Eqs. (19) and (20). In order to see the location of the singular surfaces more clearly and compare it with our long wavelength analysis, we show top view of $S_B(q)$ in Fig. 5(b). We also numerically calculate $Q_{\pm} = k_{FR} \pm k_{FL}$ (by first finding corresponding Right and Left Fermi points with anti-parallel group velocities) and superpose these lines on the figure. We can see that the lines we get from the long wavelength analysis match the singular features in the exact structure factor. Note that the singularities are expected to be one-sided,

\[
S_B(Q_{\pm} + \delta q) \sim |\delta q||^3/2 \Theta(-\delta q||), \quad (22)
\]

\[
S_B(Q_{\pm} + \delta q) \sim |\delta q||^3/2 \Theta(-\delta q|| \text{sign}(\alpha_R - \alpha_L)). \quad (23)
\]

The first line is singular from the inner side of the “ring” in Fig. 5(b) and the second line from the inner side of the “triangles”.

IV. CONCLUSION

We studied bond energy correlation functions in the Chua et al.\cite{50} Kitaev-type model with a parton Fermi surface. Unlike spin correlations, we found that the local energy correlations have power-law behavior in real space with an envelope of $1/|r|^3$ and oscillations at incommensurate wavevectors that form singular surfaces in momentum space. By
We conclude by speculating about some interesting similarity with recent experiments in EtMe$_3$Sb[Pd(dmit)$_2$]$_2$. While the thermal conductivity measurements$^{22}$ are consistent with the presence of a Fermi surface of fermionic excitations down to the lowest temperatures, very recent NMR experiments$^{21}$ show a drastic reduction in spin relaxation below temperature of the order 1 K, almost as if a spin gap is opened. This reminds of the present situation where the spin operators have short-range correlations, which occurs because some of the constituent partons have a gap (here are ultralocalized), while there remain partons that are metallic and give rise to metal-like thermodynamics and manifestly gapless properties such as the discussed local energy correlations. Of course, the present model is on a different lattice and is very differently motivated. However, in a recent paper$^{22}$ working in a setting closer to the EtMe$_3$Sb[Pd(dmit)$_2$]$_2$ experiments, we discussed the following scenario in magnetic Zeeman field: Upon writing the spin operator as $S^+ = f^+_\uparrow f^-\downarrow$, we considered a state where one spinon species (say, $f^+_\uparrow$) becomes gapped due to pairing, while the other species retains the Fermi surface. In this case, $S^+$ spin correlations are short-range while the thermodynamics is metal-like. Furthermore, just as in the present paper, there are other properties that are manifestly gapless, e.g., $S^z$ spin correlations and transverse spin-2 correlations. It would be interesting to explore such scenarios in more realistic settings further.

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