Quantum Information and Elementary Particles

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Abstract

Motivated by string theory and standard model physics, we discuss the possibility of other particles-based quantum information. A special attention is put on the consideration of the graviton in light of the gravitational wave detection. This may offer a new take in approaching quantum information using messenger particles. The construction is readily extended to higher dimensional qubits where we speculate on possible connections with open and closed string sectors in terms of quiver and graph theories, respectively. In particular, we reveal that the vectorial qubits could be associated with skeleton diagrams considered as extended quivers.

Key words: Quantum Information; String Theory; Graphs; Gauge bosons; Graviton.
1 Introduction

Quantum Information Theory (QIT) has attracted recently much attention mainly in connection with many subjects including condensed matter, particle physics, string theory, graph theory, black holes and communication tools [1, 2, 3, 4, 5]. This theory is considered as a bridge between computer science and quantum mechanics. In particular, it is based on a fundamental component known by qubit. This piece has been investigated using certain mathematical operations corresponding to tensor-product of Hilbert vector spaces. Precisely, qubits have been extensively dealt with by applying various methods including type II superstrings, D-branes and graphic representations. Concretely, a nice link between the stringy black holes and qubit systems have been studied by exploiting the compactification scenario. In these activities, it has been shown that the supersymmetric STU black hole obtained from the type II superstrings has been related to 3-qubits using the hyperdeterminant concept [1]. This correspondence has been enriched by many generalizations associated with toric and superqubit calculations. All these works have lead to a classification of qubit systems in terms of black objects in type II superstrings using D-brane physics.

Recently, a strong interest has been devoted to apply graphs in different aspects of QIT. In particular, models have been developed from Adinkras explored in the study of the supersymmetric representation theory. These graphs have been used to classify a class of qubits in terms of extremal black branes resulted from abelian gauge theories involving photons which are a particular case of the fundamental messengers, belonging to the electromagnetic interaction. This comes up with the idea that other interaction messengers could be considered in quantum information processing.

The main objective of this paper is to discuss the possibility of other messenger particles-based QI by combining symmetry and geometry in the context of string theory and standard model of particle physics (SM). Motivated by the recent detection of gravitational waves (GWs), a particular attention is devoted to the consideration of the graviton in QI. This may offer a new take on studying QI using the known elementary particles. The construction is readily extended to higher dimensional qubits where we speculate on possible connections with open and closed string sectors in terms of quiver and graph theories, respectively. Concretely, we show that the vectorial qubits could be associated with skeleton diagrams considered as extended quivers in string theory and related topics.

The present work is organized as follows: In section 2, we outline how the bosons, according to SM, can be implemented in QI. Section 3 concerns the spin two particle providing gravitational qubits supported by the recent detection of GWs. In section 4, multiple qubits are speculated in terms of extended quivers and graphs. The last section contains some concluding remarks and perspectives.
2 Vectorial quantum information

In this section, we bridge a relation between QIT and bosonic fields appearing either in SM or in string theory. To do so, let us recall that the qubit is a two-state system [6]. A superposition of a single qubit is generally given by the following Dirac notation

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$ (1)

where $a_i$ are scalars belonging to a field $F$ satisfying the normalization condition

$$|a_0|^2 + |a_1|^2 = 1.$$ (2)

The analysis can be extended to more than one-qubit, given in (1), which have been used to discuss entangled states. For $n$-qubits, the general state has the following form

$$|\psi\rangle = \Sigma_{i_1...i_n=0,1} a_{i_1...i_n} |i_1...i_n\rangle$$ (3)

where $a_{i_1...i_n}$ satisfy the normalization condition

$$\Sigma_{i_1...i_n=0,1} |a_{i_1...i_n}|^2 = 1.$$ (4)

It has been remarked that, in almost all quantum activities, the single qubit has been associated with the polarization of the photon producing photonic qubits. The later has been considered as the channel for sending QI in four dimensional universe. A close inspection, however, in particle physics and string theory, shows that the photon, $\gamma$, is not the only particle having two-polarization states. In particular, there are other bosons which could play a similar role as photons. Indeed, the QI could be transferred using other bosonic particles living in four dimensions. These particles belong to three types.

1. The first type contains the vector particles appearing in the three fundamental interactions: electromagnetic, weak and strong interactions. These particles are noted by $\gamma$, $W^{\pm,-0}$, and $g^a$ ($a = 1, \ldots, 8$) respectively. It is recalled that $g^a$ are the eight gluons associated with the Cartan decomposition of $su(3)$ Lie algebra [7]. This class of particles can be supported from the fact that in four dimensions the massless vector bosons have two-polarization states. It is recalled that on-shell massless vectors in $d$ dimensions have $d - 2$ degrees of freedom. In fact, it has been shown that one degree of freedom is removed by the field equation and another one by the gauge condition.

2. The second type is the Higgs boson $H$ involving two degree of freedom [8]. It is recalled that this particle is the only known discovered scalar associated with many sectors in
3. The third type is the massless graviton $g$ which in four dimensions involves also two-polarization states. This can be motivated from the fact that in such a dimension a GW has two-polarization states. This can be seen also from the fact that a massless graviton in $d$ dimensions have $\frac{(d-2)(d-1)}{2} - 1$ degrees of freedom. It is mentioned also that this particle appears naturally in the quantization of the closed string theory. Recent detection of GWs might be exploited to support the idea that graviton could play a relevant rôle in communication and quantum information systems.

In this vision, it seems important to support this statement inspired by particle physics and string theory [9]. Investigations show that we can provide some arguments by combining symmetry, geometry, and physics. At first sight, it has been not clear how to approach such a problem. However, it has been remarked that symmetry in physics can be exploited to discuss the present proposition. Focusing on the state algebraic equation and keeping the two dimensional vector space analysis associated with two-state polarization in particle physics, one has only a freedom to inspect the field $F$ on which the vectors are built. An inspection shows that this filed involves certain symmetries similar to the ones appearing in particle physics and related topics including SM and string theory.

In what follows, we will show that these symmetries are hidden in the division structure of the field $F$ [10]. In this way, the state $|\psi\rangle$ physics should be invariant by the Lie symmetry corresponding to $F$. In photonic qubit, the state is invariant by the phase transformation $e^{i\alpha}|\psi\rangle$ where $\{e^{i\alpha}\}$ form the U(1) symmetry. The latter is the electromagnetic interaction symmetry corresponding to the two-polarization state photons in QIT. This phase transformation, which produces projective spaces required by the probability condition, is associated with the Bloch sphere in photonic qubits. According to fundamental interactions, one may propose the similar vectors of the weak interaction corresponding to SU(2) transformation $X|\psi\rangle$, where $X$ is a SU(2) element. It has been inspected that the field $F$ should be associated with the quaternionic projective geometries required by the probability condition. The involved bosons are massive leading to more than two-polarization states, which could be evinced from the present discussion. However, the gluons $g^a$ associated with the su(3) Lie symmetry can be implemented in QI. It is observed the octonionic nature of SU(3) color. Indeed, it is recalled that $G_2$ is the automorphism group of the octonions [11]. The latter acts transitively in the $S^6$ sphere of unit imaginary octonions implying that the $S^6$ acquires a quasi-complex structure assured by the sequence of inclusion $SU(3) \rightarrow G_2 \rightarrow S^6$. This can be considered as an octonionic representation of SU(3) symmetry. The appearance of it in QI is the more remarkable one. We remember the original group SU(3) flavour of Gell-Mann, mixing the
first three quarks flavors (up \( u \), down \( d \) and strange \( s \)). It is recalled that Gell-Mann and Fritzsch introduced also SU(3) as the gauge (color) group of the strong interactions, giving rise to QCD[12]. However, the connection with QI needs more reflexing ideas based on QCD activities.

3 Gravitational qubits

Having discussed the vector bosons in QI, one may consider other particles supported by models going beyond SM. It is worth noting that the bosonic vectorial spectrum can be derived from open string sector and related topics. While, the graviton comes from the closed string sector and should considered differently. As mentioned in the previous section, one may consider the graviton in light of the confirmed GWs observation [13]. In general relativity (GR)\(^1\), the metric \( g_{\mu\nu} \) describing the geometry of spacetime relates the spacetime coordinate \( dx^\mu \) to the spacetime interval \( d\ell^2 \) through

\[
d\ell^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{5}
\]

By the fact of the extreme weakness of the GWs at the earth, the metric can be approached as that of the Minkowski flat metric \( \eta_{\mu\nu} \) by ignoring the background curvature. In the case of small perturbations, the equation of GR can be linearized. Thus, these perturbations can be approximated as the sum of the flat metric and a small perturbation induced by the GWs as

\[
d\ell^2 = \eta_{\mu\nu} + h_{\mu\nu} \tag{6}
\]

where \( |h_{\mu\nu}| \ll 1 \) is the GWs induced small perturbation. The small changes in the spacetime interval \( \delta\ell^2 \) can be expressed by the small perturbations and the spacetime coordinate like

\[
\delta\ell^2 = h_{\mu\nu} dx^\mu dx^\nu \tag{7}
\]

where \( \pm \delta\ell^2 \) represent the small contractions and dilations changes in the length \( \ell \) of the spacetime structure. In the most useful choice, the solution of the GR equation becomes a system of linear equations, in particular a system of wave equations corresponding to a three dimensional wave equation traveling at the speed of light \( c = 1 \) [14]. With the symmetry of \( h_{\mu\nu} \) and in the sinusoidal case, the physical part of these waves can be written here in the \( z \)-direction by the wave equation

\[^1\text{For simplicity, natural units with } \hbar = c = 1 \text{ are considered.}\]
\[ h_{\mu\nu} = \delta \ell \varepsilon_{\mu\nu} \cos (\omega t - kz) \] (8)

where \( \omega \) and \( k \) being the angular frequency and the wave vector respectively. The elements \( \varepsilon_{\mu\nu} \) are the so-called unit polarization tensors \( \varepsilon^+_{\mu\nu}, \varepsilon^\times_{\mu\nu} \) with the signs \(+,\times\) saying that there exist just two possible independent polarization states such as \( \delta \ell \varepsilon_{\mu\nu} = \delta \ell^{+,:\times} \) which represent the strain amplitudes of each polarization. Naively, the absence of continuous gauge symmetry associated with the gravity and completeness of division algebra push us to think about other symmetries associated with the metric calculations. Concretely, the corresponding qubit should be associated with a real geometry involving discrete gauge symmetries. It is noted that such symmetries have been investigated in the context of gravitational theories supported by black hole activities. A straightforward way to look for such a symmetry is to handle the algebraic qubit equation. This suggests that the continuous symmetries of SM can be replaced by the \( Z_2 \) gauge symmetry in the gravitational qubits. Going through this argument, it is possible to convince ourself to propose such a such symmetry for developing real Bloch spheres as geometric representations of gravitational qubits. This goes beyond the projective spaces appearing in photonic qubits. In this way, two-polarization states can be associated with a real vector space derived from the real condition \( a_0^2 + a_1^2 = 1 \), providing the circle equation. This equation is invariant under the \( Z_2 \) symmetry acting as \( a_i \to -a_i \).

The reader might be puzzled with this argument. To make it more transparent, it would be important to provide a contact with concrete observations. In this way, it is interesting to note that GWs have now been confirmed by the recent successful detection in the LIGO experiment and hence the detection of the graviton becomes now more likely and thus graviton-based QI seems possible.

4 Higher dimensional qubits

Before ending this statement, let us discuss higher dimensional qubits in the present language. It has been remarked that this is not an easy task. Borrowing ideas from the relation between D-branes in type II superstrings and gauge theories, one may approach higher dimensional vectorial qubits using the quiver method associated with open string sector [15]. In string/M-theory, this method has been used to study four dimensional D-brane gauge theories derived from the compactification either on singular Calabi-Yau manifolds or on \( G_2 \) manifolds. The matter content of the resulting models can be obtained from the geometric deformations and the topological invariant data of the internal manifolds. These models are usually refereed to quiver gauge theories. In this way, the physical content of a model with several continuous gauge group factors can be encoded in a quiver. As in graph theory, the
quiver is formed by nodes and edges. For each node, one associates a gauge factor $G_{\ell}$, where $\ell$ indicates the node on which the physical information is put. However, edges between two nodes are associated with the charged matter transforming either in bi-fundamental or fundamental representations of the gauge group $G_{\text{quiver}} = \prod_{\text{nodes}} G_{\ell}$. In such theories, various models of such quivers have been elaborated and built in connection with toric graphs and Dynkin diagrams. Roughly speaking, the $n$-qubits, discussed in the present work, may be associated with a quiver theory with $G_{\text{quiver}} = G^n$ where $G$ is the continuous symmetry corresponding to polarization states of the vector particles obtained from the open string sector. In this way, the multiple photonic qubits corresponds to a quiver diagram of a particular symmetry defined by $n$ pieces of the U(1) gauge group associated with the electromagnetic interaction. In string theory, the quantum states are $2^n$ ways of $n$ D-branes that wrap $n$ distinguishable cycles in the internal Calabi-Yau space. In this graph language, the nodes correspond to qubits and edges exhibit the presence of entanglement between a pair of qubits $\{i_1, i_2\}$. Precisely, the nodes which are not connected by edges correspond to no entangled two qubits. It is recalled that the bipartite entanglement of $i_1$ and $i_2$ qubits is given in terms of the 2-tangle $\tau_{i_1i_2}$

$$\tau_{i_1i_2} = 4|\det a_{i_1i_2}|^2.$$  

(9)

In quiver theory, the edges however are associated with $Q_{i_1i_2}$ bi-fundamental matter charged under the $G \times G$ gauge symmetry. It seems that the connection between $\tau_{i_1i_2}$ and $Q_{i_1i_2}$ is possible using some form applications. The link pushes us to discussion higher dimensional qubit systems. The curiosity shows that these systems should be dealt with differently since they will be associated with matter having more than two charges going beyond the bi-fundamental fields. To incorporate such a matter, the quivers should be replaced by skeleton diagrams, as done in [16]. In these diagrams, the gauge factors are represented by edges and matter by nodes. This can be done by thinking on a dual operation which exchanges the rôle of the node and the edges in the quiver method. It has been observed that the skeleton diagrams involve polyvalent vertices being connected to more than two other ones. In string theory, the associated graphs have been used to represent indefinite Lie algebras generalizing the finite and affine symmetries explored in the geometric engineering method of four dimensional gauge theories obtained from type II superstrings compactified on Calabi-Yau manifolds[17]. It is remarked that the tri-vertex appears in the finite so(8) Lie algebra while the tetra-vertex arises naturally in the affine so(8) Dynkin diagram. Roughly, the poly-vertex of order $n$ ($n$-vertex), in skeleton activities, should correspond to a field $Q_{i_1...i_n}$ transforming in the poly-fundamental representation of $G \times G \times \cdots \times G$ going beyond bi-fundamental matters associated with 2-qubits. Based on these observations, one might attempt to link
$n$-tangle $\tau_{i_1...i_n}$ in $n$-qubit systems with the poly-vertex of order $n$ in skeleton diagrams

$$\tau_{i_1...i_n} \leftrightarrow |\det Q_{i_1...i_n}|^2.$$  \(10\) It is believed that the link is at least not a direct one, and several aspects of it need more reflexing ideas.

Developed investigations in generalized quivers could provide lights on building interesting qubits by considering different symmetries placed at edges. This involves the symmetries appearing in particle physics including the one of SM involving photons and other similar particles. We refer to these systems as mixed qubits which would be dealt with using quiver and skeleton methods.

Investigations show that multiple gravitational qubits could be associated with four dimensional multi graviton theories being non-interacting spin two massless fields described by a sum of Pauli-Fierz actions. The latter is a linearized form of a sum of Einstein-Hilbert actions which is the free action for GR. This model has been approached using graph theory [18]. The latter can be exploited to study QI in which the gravitational $n$-qubits are associated with graph containing $n$ nodes and a sent of edges connecting such nodes. The graph provides a mathematical framework for approaching the quantum states and related concepts including the entanglement between gravitational qubits. Using similarities between multi-gluons and multi-gravitons scattering, we expect that graph theory can be used to study multi gravitational qubits in terms of the physics of spin two massless fields in four dimensions.

\section{Conclusion and perspectives}

In this work, we have investigated the possibility of implementing messenger particles in QIT by combining symmetry and geometry in the context of open and closed string sectors. In the open string sector associated with vector particles, we have approached qubits in terms of the corresponding division algebra. In particular, we have proposed how one can go beyond photonic qubits by exploring the role of fundamental interaction of particle physics in communication systems. The closed string sector has been introduced to discuss the gravitational qubits supported by the recent observation of GWs. Multi-qubit systems have also speculated using quiver and graph methods in terms of D-brane and multi-graviton theories in four dimensions. In particular, we have shown that the vectorial qubits could be linked with skeleton diagrams explored in the study of four dimensional gauge theories.

This work comes up with many open questions. The natural question is to think about communications in terms of GWs. In this way, the geometric modifications of the physical space-time could be relevant. It is worth noting that quantum geometry is the appropriate
modification of ordinary classical geometry to make it suitable for describing the physics of quantum gravity associated with string theory. It would therefore be of interest to try to use quantum information geometry in terms of size and shape modifications. We believe that this approach deserves to be studied further. Moreover, the direct link between vectorial qubits and poly-matter in skeleton diagrams would be investigated elsewhere in future.

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