Formatting physical fields and pseudometric manifolds. The dark matter

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Abstract. It is shown that from the equations of conservation laws for energy, linear momentum, angular momentum, and mass, which are conservation laws for material media (such as thermodynamic, gas dynamical, cosmic systems, systems of charged particles, systems of elementary particles and others), it follows the evolutionary relation. This relation, which appears to be nonidentical due to the noncommutativity of conservation laws, describes the evolutionary processes proceeded in material media and accompanied by the origination of physical structures. The physical fields and relevant manifolds are formed by such physical structures. Since, due to the noncommutativity of the conservation laws, the nonpotential inconsistent external actions upon material medium do not converge into the quantities of the material medium itself, this leads to appearance of immeasurable quantities in the material medium. As it is shown, under realization of any degrees of freedom of material medium (to what the degenerate transformation corresponds) the immeasurable quantity locally partly converts into observable and measurable formations and physical structures forming physical fields and corresponding manifolds. However, since this occurs only locally, only a part of measurable quantity converts into physical structures. This means that a certain immeasurable quantity remains in material medium. The dark energy and dark matter are such immeasurable and nonobservable quantities (essence) that emerges due to various nonpotential actions and, due to the noncommutativity of conservation laws, cannot directly convert into own quantities of material medium.

1. Introduction

As it is known, the Einstein equation connects the space-time with the matter and its motion. From the Einstein equation one can find the metric tensor of the pseudo-Riemann manifold corresponding to the energy-momentum $T_{\mu\nu}$ of the matter.

It turns out the from the equations of mathematical physics, which describe material media, it follows the evolutionary relation that not only describes the pseudomanifolds corresponding to the matter, but also discloses a mechanism of formatting physical fields and relevant pseudometric manifolds.

The present investigation bases on the properties of conservation laws. It turns out that there exists two types of conservation laws, namely, the conservation laws for material media and the conservation laws for physical fields.

The conservation laws for material media are conservation laws for energy, linear momentum, angular momentum, and mass. Such conservation laws are described by differential equations. These equations establish a balance between physical quantities of material media and external actions. (Such conservation laws can be named the balance or differential conservation laws.)
The conservation laws for physical fields are conservation laws that state the presence of conserving quantities or objects (structures). Such conservation laws, which can be named exact ones, are described by closed exterior skew-symmetric forms (the Noether theorem is an example).

One can see that the conservation laws for material media and the conservation laws for physical fields are different conservation laws. However, as it follows from the mathematical physics equations composed of conservation law equations for material media, there exists a connection between the conservation laws for material media and those for physical fields. A peculiarity consists in the fact that this connection is realized discretely in the evolutionary process proceeded in material media and accompanied by discrete transitions and the emergence of physical structures that made up physical fields. And this is due to a non-commutativity of conservation laws for material media.

The results of present paper were obtained with the help of skew-symmetric differential forms that correspond to the properties of conservation laws. In this case, to the conservation laws for physical fields there correspond closed exterior forms. And to the conservation laws for material media there correspond skew-symmetric differential forms obtained from the conservation law equations and possess the evolutionary properties. Such skew-symmetric form, whose basis are deforming non-integrable manifolds, possess an unique property, namely, they can generate closed exterior forms. That is, from evolutionary skew-symmetric forms, which correspond to conservation laws for material media, there are obtained closed exterior forms that correspond to conservation laws for physical fields. Such properties of skew-symmetric forms describe the processes of generation of physical fields by material media and the processes of forming pseudometric manifolds.

One can read about the properties of skew-symmetric forms in [1,2].

2. Non-commutativity of conservation laws for material media
The material media composed of moving material particles (elements) having internal structure are subject to various external (with respect to elements, local domains of material medium) actions (energetic, forced and so on). These actions, which are non-potential ones, cannot convert directly into quantities of material medium itself (such as a pressure, density, energy) and acts as internal force that is a moving force of evolutionary processes. And this is due to a non-commutativity of conservation laws.

Below we consider the conservation law equations for energy and momentum. They describe the state of a local domain of material medium, namely, an element (particle) and its vicinity. The conservation law for energy describes an element motion along trajectory. If to use the functionals specifying the material system state [3], then, in the accompanying frame of reference connected with the manifold made up by the trajectories of material medium elements, the conservation law equation for energy can be written in the form [4]:

$$\frac{\partial \psi}{\partial \xi^1} = A_1$$ \hspace{1cm} (1)

here $\psi$ is the state functional, $A_1$ is a quantity that depends on specific features of material medium and, correspondingly, external energy actions onto material medium, $\xi^1$ is coordinate along the trajectory. The state functional is a characteristics of material medium that unites the quantities of material medium in a single whole. [The wave function, entropy, the action functional, the Pointing’s vector, the Einstein tensor and so on are such functionals. In the case of gas dynamic medium such a functional is entropy. The energy equation for ideal gas can be presented in the form: $\partial s/\partial \xi^1 = A_1$, where $s$ is entropy. The energy equation expressed in terms of the action functional $S$ has the following form: $\partial S/\partial \xi^1 = L$, where $L$ is the Lagrange function.]
In the accompanying frame of reference the equation of conservation law for linear momentum can be written in the form \[4\]:

\[
\frac{\partial \psi}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, \ldots
\] (2)

Here \( A_\nu \) are the quantities that depend on specific features of material medium and on force actions onto local domain of material medium, \( \xi^\nu \) are the coordinates in the direction normal to the trajectory.

It is evident that under the energetic and force actions energy and momentum of local domain of material medium will change. But changed quantities of energy and momentum cannot become quantities of a material medium itself since they are of different nature and cannot redistributed in such a way to become consistent (potential) quantities of a material medium itself.

And this is governed by conservation laws for energy and momentum, namely, by their non-commutativity. The non-commutativity of the conservation laws does not allow a direct transition of the external actions into the physical quantities of the material medium. As a result itarises an immeasurable quantity that acts as an internal force and gave rise of a local domain of material medium to non-equilibrium state.

This is described by evolutionary relation obtained from the analysis of the conservation law equations for energy and momentum.

### 2.1. Evolutionary relation

Equations (1) and (2) can be convoluted into the relation

\[
d\psi = A_\mu d\xi^\mu, \quad \mu = 1, \nu
\] (3)

that can be written in the form

\[
d\psi = \omega
\] (4)

Here \( \omega = A_\mu d\xi^\mu \) is the skew-symmetric differential form of the first degree.

The relation (4), which is evolutionary one (since it was obtained from evolutionary equations) appears to be non-identical and to be selfvariable.

A relation can be identical one if it is composed of differentials, i.e., closed exterior forms. Relation (4) is not an identical since it contains the skew-symmetric differential form \( \omega = A_\mu d\xi^\mu \), which isn’t closed. The differential of this form is nonzero. This differential can be written as \( d\omega = K_{\alpha\beta} d\xi^\alpha d\xi^\beta \), where \( K_{\alpha\beta} \) is a commutator of skew-symmetric form (see \[2\]). Here the peculiarity consists in the fact that the skew-symmetric form \( \omega \) is defined on accompanying deforming manifold. For this reason, the commutator of such a form will contain the commutator of manifold metric form that is nonzero (this relates to manifold deformation). As it has been show in a series of author’s papers \[1,2\], if it is possible to define connectednesses, the commutator of skew-symmetric form \( \omega \) can be written as

\[
K_{\alpha\beta} = \left( \frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta}) A_\sigma
\] (5)

The first term in the commutator is nonzero since the coefficients \( A_mu \) describe actions of different nature, and the second term is nonzero since it is a commutator of a metric form of a manifold that is deforming one (connectednesses of deforming non-integrable manifold are not symmetric ones \[5\]).

It turns out that the commutator of skew-symmetric form \( \omega \), and hence its differential, are nonzero. This means that the skew-symmetric form \( \omega \) isn’t closed, i.e. this form isn’t a differential. Thus, the evolutionary relation, which contains unclosed skew-symmetric form, cannot be identical.
The non-identity of evolutionary relation point to the fact that the conservation law equations for energy and momentum, from which this relation was obtained, appear to be inconsistent. And this means that the conservation laws for energy and momentum turn out to be non-commutative.

As one can see from evolutionary relation, to the non-commutativity of conservation laws leads a quantity that is described by the commutator of evolutionary form $\omega$ (if the commutator be equal to zero, the conservation law equations would be consistent, and this would point to a commutativity of conservation laws).

2.2. Non-equilibrium state of material media
The quantity described by the evolutionary form commutator cannot be transformed into quantities of material medium itself. For this reason it cannot be measurable observable quantity. Such a quantity acts in local domain of material medium as internal force that leads local domain of material medium to non-equilibrium state. This follows from evolutionary relation.

The evolutionary relation contains a differential of functional $d\psi$ that specifies a state of material medium. But since evolutionary relation turns out to be non-identical, from that it is impossible to obtain the differential $d\psi$ that would point to equilibrium state of material medium. This means that a local domain of material medium is in non-equilibrium (under the action of internal forces) state. From the evolutionary relation, which appears to be selfvariable, it follows that the state of a local domain of material medium changes but remains to be non-equilibrium one since the evolutionary relation remains to be non-identical.

2.3. Transition of material medium to locally-equilibrium state. Emergence of observable formations
From evolutionary relation it follows that under degenerate transformation a closed inexact exterior form is obtained. This form is defined only on a certain structure $\pi$ (more precisely on the pseudostructure), described by the condition of degenerate transformation that is due to degrees of freedom of material medium and can be realized under a change of non-equilibrium state of material medium. In this case, from non-identical evolutionary relation an identical relation on pseudostructure may be obtained. (The degenerate transformation corresponds to a transition from the frame of reference connected with the accompanying manifold to the inertial frame of reference on pseudostructure.) Since from identical relation one can obtain the differential $d_\pi \psi$, this points out to a transition of local domain of material medium into equilibrium (but only on pseudostructure) state. In this case, immeasurable quantity, which is described by the evolutionary form commutator and act as internal force, converts, but only partially, into a measurable quantity of material medium. In material medium this reveals in emergence of some observable formations (waves, vortices, fluctuations, turbulent pulsations are such observable formations) [6].

Due to the fact that immeasurable quantity converts into a measurable quantity of material medium only partially (since this proceeds only locally under realization of additional condition, i.e. on pseudostructure), part of an immeasurable quantity remains in material medium. Such a remained in material medium immeasurable non-observable quantity (which emerges due to energy or force actions to material medium and cannot convert into a quantity of material medium itself) is a so called dark energy.

Below the non-commutativity of conservation laws for energy and momentum was studied and a relevant evolutionary relation (4) has been obtained.

In general case under studying a consistence of equations for energy, linear momentum, angular momentum and mass the evolutionary relation will be written as:

$$d\psi = \omega^p$$  \hspace{1cm} (6)
Here $\omega^p$ is the form degree $p$, where $p$ takes the values $p = 0, 1, 2, 3$. (The evolutionary relation for $p = 0$ was obtained from the equations for energy and time.) [A concrete form of relation (6) is presented in papers [1,2]. In the paper [2] relation (6) for $p = 2$ was considered for electromagnetic field. In this case the functional $\psi$ is the Pointing' vector. The relation for the Einstein tensor is obtained when integrating the evolutionary relation for $p = 3$.]

Evolutionary relation (6), as well as relation (4), is a non-identical selfvariable relation. From non-identity of relation (6) it follows that all conservation laws of material media are non-commutative ones. And this leads to that in local domains of material media due to non-consistence and non-potentiality of external (with respect to local domains) actions it is emerged an immeasurable non-observable quantity that cannot convert into quantities of material medium itself and acts as internal force. [It should be noted that for the evolutionary relations of various degree $p$ the local domains will be different, namely, the local domain for the form of $p \text{-} 1$ degree will be an element of local domain for the evolutionary relation with the form of $p$ degree.]

From evolutionary relation (6) (as well as from relation (4)) it follows that under a realization of any degree of freedom immeasurable quantity partially converts into a measurable quantity of material medium. In this case the remained part of immeasurable quantity conserves in material medium. It is evident that immeasurable non-observable quantity which appears in material cosmic medium due to external action and cannot be converted into the quantities of cosmic medium, is so called dark matter.

2.4. Emergence of physical structures
The process of transition (partial) of immeasurable quantity to observable formation of material medium itself relates to emergence of physical structures. As it follows from evolutionary relation this it correspond a degenerate transformation under which a closed inexact on pseudostructure exterior form is obtained from evolutionary skew-symmetric form. The closed inexact exterior form (which is a conserving quantity since its differential is equal to zero) and relevant dual form (a pseudostructure which is a metric form of a manifold) made up a differential-geometrical structure that describes physical structure, namely, a pseudostructure with conserving quantity. (Massless particles, structures made up by eikonal surfaces, wave fronts, and so on are examples of physical structures.) [Physical structures and the observed formations of material medium are a manifestation of the same phenomena. The light is an example of such a duality. The light manifests itself in the form of a massless particle (photon) and as a wave. On the other hand, the observed formation and the physical structure are not identical objects. If the wave be such a formation, the element of wave front made up the physical structure while its motion.]

The structures that made up physical fields are exactly such physical structures. This follows from the properties of conservation laws. The physical structures, i.e. pseudostructures with conserving quantity, are conserving objects. This is, they subject to conservation laws for physical fields.

As one can see, from the conservation law equations for material media it is obtained evolutionary relation from which closed inexact exterior forms describing physical structures, which correspond to conservation laws for physical fields, are realized. This points to a connection between physical fields and material media. From evolutionary relation it follows that physical structures, which made up physical fields, are generated by material media.

3. Forming pseudometric manifolds
The evolutionary processes in material media, the result of which is an emergence of physical structures, discloses the processes of realization and forming pseudometric and metric manifolds.

As it was shown above, physical structures emerged in material media are a representation of differential-geometrical structure, namely, a pseudostructure, which is a geometrical object, and
a conserving quantity, which is an algebraic object. It turns out that the pseudostructure and conserving quantity exist as a whole.

A manifold also possesses such a property. Manifolds, which are geometrical objects, and quantities defined on these manifolds, which are algebraic objects, make a whole.

And such a unity is described by a unity of closed inexact exterior skew-symmetric and dual forms. In this case, a quantity defined on manifold is described by closed skew-symmetric form, whereas a dual form describes a manifold metric form.

Closed inexact exterior form and relevant closed dual form disclose a unity of pseudometric manifolds and quantities conserved on manifold and describe them as a whole. Physical fields are such a form of the matter. (It can be noted that physical structures and pseudometric manifolds with conserving quantities obey conservation laws for physical fields.)

One can see that physical structures, which made up physical fields, represent a conjugation of objects of different nature, namely, a physical (i.e. algebraic) quantity described by closed exterior form, and some spatial (or pseudospatial) structure described by relevant dual form.

The processes of emergence of physical structures and the realization of pseudometric manifolds and physical fields are processes of realization the conjugation of physical quantity and spatial structure.

And this process is described by evolutionary relation and the commutator of evolutionary skew-symmetric form incoming into evolutionary relation.

As it was shown above, the evolutionary skew-symmetric form obtained from the mathematical physics equations is defined on accompanying manifold made up of trajectories material medium elements (particles). This manifold is a deforming manifold. As it was noted, the commutator of skew-symmetric form defined on such manifold contains the commutator of manifold metric form. [Here it should be noted that the metric forms of manifold and their commutators [2,5] determine the metric and differential characteristics of the manifold. The closed metric forms define the manifold structure, whereas the metric forms commutators define such characteristics of manifold as bending, torsion, curvature that corresponds to deforming (nonintegrable) manifolds.]

The commutator of evolutionary skew-symmetric form defined on deforming manifold possesses topological properties which enable one to describe the process of conjugation of physical quantities described by skew-symmetric forms and spatial structures described by dual forms, as well as the process of emergence of physical structures. This relates to the fact that the commutator of evolutionary skew-symmetric form is composed of two terms. The first one of which is made up of derivatives of evolutionary form coefficients, and the second term is a commutator of manifold metric form (see the example of the commutator of the first degree form (5)). The interaction between these two terms under selfchanging non-identical evolutionary relation describes the process of physical structure emergence and the realization of pseudometric manifolds with conserving quantities that made up physical fields.

3.1. Characteristics of physical structures
Since closed inexact exterior forms corresponding to physical structures are obtained from the evolutionary relation for a material medium, it follows that physical structures are generated by material media. (This is controlled by the conservation laws.)

The physical structures are generated by numerous local domains of material medium and at numerous instants of realizing various degrees of freedom of material medium. By combining with one another they can form large-scale structures making up pseudomanifolds and physical fields. Here it should be underlined that physical fields and relevant manifolds are made up by the structures, which in turn are formed by microstructures. This points out to the fact that physical fields and relevant manifolds are formed quantum objects.
To every physical field it is assigned its own material medium. As examples of material media it may be cosmic systems, systems of elementary particles and others.

The type of physical structures and accordingly of physical fields generated by the evolutionary relation depends on the parameters $p$, $k$ and $n$, where $p$ is the degree of the evolutionary form in the evolutionary relation that is connected with a number of interacting balance conservation laws, and $k$ is the degree of a closed form generated by the evolutionary relation, and $n$ is the dimension of initial inertial space. By introducing the classification by numbers $p$, $k$, $n$ one can understand the internal connection between various physical fields. Since the physical fields are the carriers of interactions, such classification enables one to see the connection between interactions. This is reflected in the Table of correspondence to elementary particles, which is presented in papers [7].

3.2. What can be said concerning the pseudo-Riemann manifold and the Riemann space

The basic property of the Einstein equation (as well as another field theory equations) consists in the fact that it consists of covariant tensors, i.e. closed exterior forms. This means that it describe pseudometric and metric manifolds which are pseudo-Riemann or Riemann manifolds. What does follow from the evolutionary relation.

The evolutionary relation for cosmic material medium $d\psi = \omega^3$ obtained from the analysis of a consistence of conservation law equations for energy, linear momentum, angular momentum, and mass [1], possesses the properties of the Einstein equation. From the evolutionary form entered into this relation the closed exterior forms, which corresponds to covariant tensors of the Einstein equation, are obtained. But here there are differences.

As it is known, when deriving the Einstein equation [5,8] it was supposed that the following conditions to be satisfied: the Bianchi identity is fulfilled, the connectedness coefficients are symmetric ones (the connectedness coefficients are the Christoffel symbols), and there exists a transformation under which the connectedness coefficient becomes zero.

As it follows from evolutionary relation this conditions are conditions of realization of the degenerate transformations for the non-identical evolutionary relation and the transition to the identical relation, from which closed exterior forms assigned to covariant tensor of the Einstein equation are obtained. As it was note above, such conditions do not fulfill identically. They correspond to degrees of freedom of cosmic medium and can be realized only in evolutionary process.

This is, from the evolutionary relation it follows that the closed inexact exterior forms, which are covariant tensors, are obtained only under degenerate transformation (under realization of any degrees of freedom of material media). And this points to emergence of physical structure, namely, a pseudostructure (dual form) with conserving quantity (closed inexact exterior form).

In other words, the transitions from initial tensor of curvature to covariant tensors of lower order are non-identical, as it is embedded in the Einstein equation, but they are realized discretely and describe the emergence of physical structure. This is, the Einstein equation, which contains only covariant tensors, as opposed to evolutionary relation, cannot disclose a quantum character of pseudometric manifolds corresponding to physical fields.

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