Hidden but real: novel relativistic “paradox” exposing the ubiquity of hidden momentum

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The tight connection between mass and energy unveiled by Special Relativity, summarized by the iconic formula $E = mc^2$, has revolutionized our understanding of nature and even shaped our political world over the past century through its military application. It is certainly one of the most exhaustively-tested and well-known equations of modern science. Although we have become used to its most obvious implication — mass-energy equivalence —, it is surprising that one of its subtle — yet, inevitable — consequences is still a matter of confusion: the so-called hidden momentum. Often considered as a peculiar feature of specific systems or as an artifact to avoid paradoxal situations, here we present a novel relativistic “paradox” which exposes the true nature and ubiquity of hidden momentum.

I. INTRODUCTION

Einstein’s iconic mass-energy relation, $E = mc^2$, is arguably the most famous formula in modern science. It expresses the equivalence between total mass $m$ and total energy $E$ of a system ($c$ being the speed of light in vacuum), with wide-ranging consequences: from the unattainability of the speed of light for massive objects, to particle production in high-energy accelerators; from the origin of the energy of stars — less than 0.1% of the star’s mass, converted into radiation over its entire existence —, to violent bursts of gravitational waves from merging black holes — some of them sourced by several solar masses converted into energy in a fraction of a second. Given the importance and generality of mass-energy equivalence, it may strike as a surprise that one of its subtle — but inevitable — consequences is still a matter of confusion: the concept of hidden momentum [1] — here generalized as the (purely relativistic) part of total momentum which is not encoded in the motion of the center of mass-energy (CME) of the system.

In Newtonian mechanics, the total momentum $P$ of a closed mechanical system — one which does not exchange matter with “the rest of the universe” — is always given by $P = MV_{cm}$, where $M$ is the total mass of the system and $V_{cm}$ is the velocity of its center of mass. This result, known as the center-of-mass theorem, holds true regardless whether the system is subject to external forces or not. In contrast to that, a variety of (relativistic) systems possessing nonzero total momentum in the rest frame of their CME has been identified over the past decades (see, e.g., Refs. [1][3]). Such rest-frame momentum has been termed hidden momentum (HM) [1], which now seems to be somewhat unfortunate because apparently this has misled many to interpret its nature as somehow distinct from “regular” momentum — as we argue here, from the relativity-theory perspective, it is not. Adding confusion to the story, all systems in which HM had been identified, until now, involved interaction with electromagnetic fields and/or moving inner parts subject to some external force field. This masked the generic nature of HM as if it were a feature — undesired by some — of peculiar interaction laws or specific systems.

Here, we present a novel relativistic “paradox” which shows that this view is limited and that HM is ubiquitous in a relativistic world. Moreover, the general definition of HM we propose leads to a new formula to compute its value. The present analysis is intended to put an end to decades of confusion about the nature and reality of the so-called HM.

The paper is organized as follows. In Sec. II we present the novel relativistic “paradox” involving a heat-conducting bar analyzed from different inertial-frame perspectives. In order to make the presentation clearer, textbook-level relativistic calculations which support statements made in this Section are described in Appendix A. In Sec. III we put the heat-conducting-bar (HCB) “paradox” in context with other known pseudo-paradoxes, stressing their origin in our intuition based on space and time as separate entities rather than in inconsistencies with known theories. In Sec. IV we argue that HM is just another inevitable consequence of Einstein’s mass-energy relation $E = mc^2$, while in Sec. V we discuss the resolution of the HCB “paradox” using our formula for HM obtained in Appendix B.

II. HEAT-CONDUCTING-BAR PARADOX

Consider the system depicted in Fig. 1(a), composed by a free bar connecting two thermal reservoirs at temperatures $T_1$ and $T_2$ — with, say, $T_1 \geq T_2$ —, at rest in an inertial frame. In order to avoid unnecessary subtleties, we consider that (i) the stationary heat-flow regime has been established, (ii) thermal contact between the bar and each reservoir is symmetric (for instance, through...
the lateral surface of the bar), and (iii) the bar is coated with a thermal insulator all over the parts which are not in contact with the reservoirs. Conditions (i) and (ii) ensure that the CME of the bar stays at rest in the inertial rest frame of the reservoirs without the need for any mechanical constraint; the heat-conducting bar in the stationary regime is in static mechanical equilibrium. (Condition (iii) only serves to keep the system simple.) From the rest-frame perspective, the effect of the reservoirs on the bar is merely exchange of heat, with no net forces or torques being applied. Let \( W > 0 \) represent the (constant) rate at which heat is exchanged between the bar and the reservoirs — flowing into (respectively, out of) the bar from (resp., to) the reservoir at temperature \( T_1 \) (resp., \( T_2 \)). (Side note: for any given heat-exchange rate \( W \), we can consider the temperature difference \( T_2 - T_1 \) to be arbitrarily small by choosing bars with arbitrarily large thermal conductivities. Therefore, although unnecessary, one can simplify further the setup considering the mass-energy and temperature distributions along the bar to be arbitrarily close to homogeneous.)

Now, let us analyze the same setup from the perspective of another inertial frame, with respect to (w.r.t.) which the bar (and the whole system) moves with velocity \( \mathbf{V} \) perpendicular to itself. Although it may sound odd at first, it follows directly from Einstein’s Special Relativity that, in this new frame, the reservoirs apply opposite net forces \( \pm \mathbf{F} = \pm W \mathbf{V}/c^2 \) at the moving-bar’s ends [see Fig. 1(b)]. The proof of this fact is actually quite simple (a textbook-level exercise) and is explained in detail in Appendix A.

Once the reader is convinced of the existence of such forces, he/she promptly realizes that they lead to a torque on the heat-conducting moving bar, which (neglecting the spatial extension of the thermal contacts) is given by

\[
\mathbf{T} = W(\mathbf{V} \times \mathbf{D})/c^2,
\]

where \( \mathbf{D} \) is the separation vector between the thermal contacts (see Fig. 1): although the opposite forces have no net effect on the total momentum of the bar as time passes, they do change the bar’s angular momentum, apparently trying to rotate it. But this is obviously in conflict with the fact that in the reservoirs’ rest frame the bar is in static mechanical equilibrium: there is absolutely no reason for rotation. We have stumbled on a novel relativistic “paradox.”

### III. NOT REAL PARADOXES

Relativistic “paradoxes” — more precisely, situations whose descriptions from different inertial perspectives seem paradoxical when compared to each other — are numerous and even serve as teaching tools in relativity. Rather than pointing to inconsistencies in fundamental theories, they reveal how our perception of space and time as separate entities, instead of interwoven in an absolute four-dimensional spacetime, can be deceiving. Their nature can be loosely classified as kinematical — those which involve only time-interval and spatial-distance measurements — and dynamical — those which involve forces. The twins’, the barn-pole, and the Bell’s spaceship “paradoxes” are well-known textbook samples of the kinematical type — see, e.g., Ref. [14] —, whereas the Trouton-Noble [15], the right-angle-levier [16], and the submarine [17–20] “paradoxes” are representative of the dynamical type. The heat-conducting-bar (HCB) “paradox” presented above clearly fits into this latter class. Contrary to kinematical “paradoxes,” the dynamical ones are rarely addressed in relativity textbooks and introductory courses. This may explain why many of them are unknown to nonrelativists or, when known, concepts involved in their resolution are seen with suspicion.

In 2012, M. Mansuripur [5] analyzed in detail an ingenious dynamical “paradox” — previously discussed in Ref. [21] — which, in a simplified but equivalent version, can be realized by a neutral magnet at rest in an inertial frame, where there exists a uniform (external) electric field \( \mathbf{E} \) perpendicular to the magnet’s magnetic dipole moment \( \mathbf{m}_0 \) (see Fig. 2). In the magnet’s rest frame [Fig. 2(a)], the magnet “seems” oblivious to the presence of the electric field — apart from induced polarization, which can be made negligible. However, looking at the same system from another inertial frame, w.r.t. which the magnet moves with velocity \( \mathbf{V} \) along the electric-field’s direction [Fig. 2(b)], the magnet now also bears an electric dipole moment \( \mathbf{d} = \mathbf{V} \times \mathbf{m}_0/c^2 \) — since \( \mathbf{m}_0 \) is ultimately due to electric currents, not pairs of magnetic monopoles [22]. Thus, according to this inertial frame, there must exist a torque \( \mathbf{T} = \mathbf{d} \times \mathbf{E} = (\mathbf{V} \cdot \mathbf{E}) \mathbf{m}_0/c^2 \) acting on the magnet, which would supposedly make it spin — in gross contradiction with the fact that in its
inertial rest frame the magnet stands still. Mansuripur concludes that this contradiction is an “incontrovertible theoretical evidence of the incompatibility of the Lorentz law [of force] with the fundamental tenets of special relativity” \[5\].

It is evident that the HCB “paradox” presented earlier is a close thermal analogue of Mansuripur’s, with the thermal reservoirs playing the role of the external electric field, the heat-conducting bar substituting the magnet, and the heat exchange rate \(W\) playing the same role as the energy exchange rate, per volume, \(j \cdot E\) between the current density \(j\) in the magnet and the external electric field. However, in the thermal analogue, there is no specific “law of force” to blame for the apparent contradiction between different inertial-frame descriptions; the torque on the bar seen from the “moving-frame” perspective is a direct consequence of the “tenets of special relativity,” in particular of \(E = mc^2\) (plus locality and causality). Certainly, no one would hold that Einstein’s mass-energy relation is “incompatible with the fundamental tenets of special relativity.” Therefore, there is no logical reason for taking this stand regarding the Lorentz force.

**IV. REAL HIDDEN MOMENTUM**

Relativistic thermodynamics has its own history of subtleties and controversies. The most emblematic of them is probably the question of how temperature transforms from one inertial frame to another. It took about 90 years for this to be recognized as an ill-posed question — hence, the conflicting answers given during this period (see Refs. \[23\] and references therein). Fortunately, none of these subtleties — not even temperature transformation — concerns us; the purpose of thermal reservoirs in the setup of Fig. 1 is only to guarantee an eventual stationary situation in the rest frame of the system.

The resolution of the HCB “paradox” — as well as Mansuripur’s — consists in taking mass-energy equivalence to its ultimate consequences. As heat (i.e., energy) flows through the bar, it contributes to momentum in very much the same way as would a flow of matter. In fact, distinguishing contributions to the total momentum coming from “different forms” of energy flows is quite contrary to the spirit of relativity theory. Therefore, the total momentum of the bar in its rest frame [Fig. 1(a)] does not vanish — a purely relativistic effect. For the same reason, according to the inertial frame w.r.t. which the bar moves with velocity \(V\) perpendicular to itself [Fig. 1(b)], there is a momentum contribution along the bar. Consequently, the bar’s total momentum \(P\) and the CME velocity \(V\) are misaligned; and dragging momentum \(P\) along a spatial direction which is not aligned to it inevitably leads to a time-varying angular momentum \(L\) (with \(dL/dt = V \times P\)) and, therefore, demands a torque — which, as we shall see in the next section, is precisely the one supplied by the thermal reservoirs in the moving frame.

As mentioned earlier, several systems with nonvanishing total momentum in their rest frames have been found and discussed in the literature (see, e.g., Refs. \[11\] — including Mansuripur’s setup \[6\]). All such systems involved interaction with electromagnetic fields and/or moving inner parts subject to some external force field, which led many to view it as a feature of peculiar interaction laws or systems. Mansuripur, for instance, considered HM to be an \textit{ad hoc} addition to materials interacting with electromagnetic fields, with no justification other than artificially avoiding paradoxical situations \[11\] —\[13\]. A better law of electromagnetic force, he reasoned, should be one which leads to no torque on the moving magnet in an electric field — hence, doing away with HM. In this sense, the HCB “paradox” we discuss in this work is unique, for it does not depend on the inner details of the system (the bar and the heat/energy flow) and of the interaction with “the rest of the world” (the thermal/energy reservoirs).

**V. DISCUSSION**

Although Mansuripur’s speculation on alternative laws of electromagnetic force is a valid inquiry — which can only be definitely settled by experiments —, the generic nature of the HCB “paradox” — with electromagnetism and moving inner parts playing no explicit essential role — shows that the existence of torques acting on spinless, uniformly-moving objects is a ubiquitous feature of relativistic dynamics. As stated earlier, this torque \((T = V_{\text{cme}} \times P)\) is responsible for translating the CME of the system (with velocity \(V_{\text{cme}}\)) along a direction which is not aligned to its total momentum \((P)\) — which exposes the existence of HM. In the Appendix \[B\] we propose and show that a generalized definition of HM as

\[P_h := P - MV_{\text{cme}}\]  \hspace{1cm} (2)

— which makes sense not only in the rest frame of the system (where \(V_{\text{cme}} = 0\)) —, leads to a formula relating HM...
with asymmetric (w.r.t. the system’s CME) exchange of energy with “the rest of the universe”:

\[ \mathbf{P}_h = -\frac{1}{c^2} \sum_j (x_j - \mathbf{X}_{\text{cme}}) \mathbf{W}_j, \tag{3} \]

where \( \mathbf{X}_{\text{cme}} \) is the CME position and \( x_j \) is the position where energy exchange occurs at a rate \( \mathbf{W}_j \) (\( \mathbf{W}_j > 0 \), if energy enters the system; \( \mathbf{W}_j < 0 \), if energy leaves the system). The interpretation is simple: this asymmetry leads to energy flows in the system which, regardless of their nature, contribute to momentum in very much the same way as do matter flows — thanks to mass-energy equivalence. As stressed earlier, distinguishing contributions to the total momentum coming from “different forms” of energy flows is quite contrary to the spirit of relativity theory — reason why a covariant, observer-independent definition of HM does not (and cannot) exist. Applying Eq. (3) to the bar in Fig. 2(a) leads to \( \mathbf{P}_h = W\mathbf{D}/c^2 \), which, as stated earlier, precisely accounts for the torque \( \mathbf{T} = \mathbf{V} \times \mathbf{P} = \mathbf{V} \times \mathbf{P}_h = W(\mathbf{V} \times \mathbf{D})/c^2 \) applied by the reservoirs on the moving bar [see Eq. (1)].

Obviously, only experiments can decide on the correctness of candidate laws of Nature. However, aiming at substituting a law of force solely on the basis that it leads to HM is a misguided effort. As made explicit by the HCB “paradox” and Eq. (3), HM is simply an inevitable consequence of \( E = mc^2 \) when seen from an arbitrary inertial frame.

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**Appendix A: Energy-momentum transfer in inelastic collisions and force exchange with thermal reservoirs**

Consider the symmetric process depicted in Fig. 3(a), where two identical particles with opposite momenta \( (\mathbf{p}_1' = -\mathbf{p}_1) \) and vanishing total angular momentum collide with a surface at rest. We shall allow the collisions to be inelastic, each delivering an energy \( \Delta E/2 \) into the surface. The symmetry of the setup makes it clear that, in this frame, the net momentum transferred to the surface is zero \( (\Delta \mathbf{P}' = -(\Delta \mathbf{p}_1' + \Delta \mathbf{p}_2') = 0) \). In Fig. 3(b), the same process is depicted as seen from an inertial frame w.r.t. which the surface moves with velocity \( \mathbf{V} \). Obviously, the whole process is determined from its description above; all one has to do is to Lorentz transform the primed quantities to this new frame. By doing so — which is a textbook exercise —, one realizes that the momentum exchanges between the particles and the surface are no longer symmetric \( (\Delta \mathbf{p}_2 \neq -\Delta \mathbf{p}_1) \) and that a net momentum \( \Delta \mathbf{P} = \Delta EV/c^2 \) is transferred to the surface, where \( \Delta E = \gamma \Delta E' \) is the net energy delivered into the surface in this frame \( (\gamma \) is the Lorentz factor). The implication is clear: inelastic collisions which are symmetric in the rest frame of the surface exert a net force on the surface when analyzed from inertial frames w.r.t. which the surface is moving.

![Diagram](image)

**FIG. 3:** (a) Symmetric inelastic collisions of two identical particles with a surface at rest. The symmetry of the setup leads to no net momentum transfer to the surface. (b) The same process analyzed from an inertial frame w.r.t. which the surface is moving with velocity \( \mathbf{V} \). In this new frame, the collisions are no longer symmetric and a net momentum \( \Delta \mathbf{P} = \Delta EV/c^2 \) is transferred to the surface, where \( \Delta E \) is the energy delivered into the surface during the process. Assuming processes like this occurring at a constant rate, a net force given by \( \mathbf{F} = \mathbf{WV}/c^2 \) would be exerted on the surface, where \( \mathbf{W} \) is the rate at which energy is delivered into the surface. By modeling a thermal reservoir as an isotropic bath of particles, this implies that an object at rest in a thermal reservoir is subject to a net force \( \mathbf{F} = \mathbf{WV}/c^2 \) when seen from a reference frame w.r.t. which the whole system (object and reservoir) is moving — with \( \mathbf{W} \) being the rate at which energy (heat) is absorbed by the object.

Modeling a thermal reservoir as an isotropic bath of particles, the result above inevitably leads to the conclusion that an object static (and symmetrically immersed) in a thermal reservoir is subject to a (purely relativistic) net force \( \mathbf{F} = \mathbf{WV}/c^2 \) when seen from an inertial frame w.r.t. which the whole system (object and reservoir) is moving with velocity \( \mathbf{V} \), with \( \mathbf{W} \) being the rate at which energy (heat) is absorbed by the object. Although this may sound odd at first, it becomes quite obvious when one realizes the need of an external force in order to keep the constant velocity of an object with increasing rest energy (i.e., rest mass). In fact, the existence of this relativistic force \( \mathbf{F} = \mathbf{WV}/c^2 \) can be inferred from this more general argument, independent of microscopic modeling of the reservoir (see Fig. 4). The importance of the microscopic collisional model is explicitly showing that the existence of such a force does not depend on the fate of the absorbed energy \( \Delta E \) — for instance, whether it is accumulated in the object or constantly drained to sustain a heat flow (as in Fig. 4).
FIG. 4: Force applied on a moving object by a comoving thermal reservoir. (a) An object at rest in the reservoir’s frame exchanges an amount ΔE’/c of energy in a time interval Δt’, with no momentum transfer (due to the symmetry of the reservoir in its rest frame). According to mass-energy equivalence, this corresponds to a (rest-)mass variation ΔM’ = ΔE’/c². (b) The same situation seen from another reference frame: a variation ΔM = γΔM’ in the object’s mass, at constant velocity V, corresponds to a momentum transfer ΔP = ΔMV = γΔE’V/c² from the reservoir, in a time interval Δt = γΔt’. Therefore, in this frame, the reservoir must (and does) apply a net force \( F = \Delta P/\Delta t = \Delta E’V/(c^2\Delta t') = WV/c^2 \) on the object, where \( W = \Delta E'/\Delta t' = \Delta E/\Delta t \) is the energy exchange rate. Causality/locality ensures that this final result cannot depend on whether the energy exchange \( \Delta E’ \) is accumulated in the object or if it is used to sustain a stationary heat flow, as in Fig. 1.

Appendix B: Generalized hidden momentum in terms of energy-injection-rate dipole moment

Let \( \mathcal{A} \) be a system characterized by the energy-momentum tensor whose components in inertial Cartesian coordinates \( \{t, \mathbf{x}\} \) are given by \( T^\mu_\nu_{\mathcal{A}} \). For technical simplicity, we assume that at each instant \( t \), \( T^\mu_\nu_{\mathcal{A}} = 0 \) only in a bounded spatial region (i.e., \( T^\mu_\nu_{\mathcal{A}} \) has compact spatial support). If \( \partial_\mu T^\mu_\nu_{\mathcal{A}} = 0 \) everywhere, then \( \mathcal{A} \) is said to be isolated.

The center of mass-energy (CME) of \( \mathcal{A} \) is given by

\[
X_{\text{cme}} := \frac{1}{M c^2} \int d^3x \ T^0_0 \mathcal{A} \ x, \quad (B1)
\]

where \( M = \int d^3x \ T^0_0/\mathcal{A} \) is the (possibly time-dependent) total mass of the system. Multiplying Eq. (B1) by \( M \) and taking the time derivative, we get:

\[
MV_{\text{cme}} = -\frac{dM}{dt} X_{\text{cme}} + \frac{1}{c} \int d^3x \partial_0 T^0_0 \mathcal{A} \ (x - X_{\text{cme}}), \quad (B2)
\]

where \( V_{\text{cme}} := dX_{\text{cme}}/dt \) is the velocity of the CME of \( \mathcal{A} \).

The fact that system \( \mathcal{A} \) may interact with another system means that \( \partial_\mu T^\mu_\nu_{\mathcal{A}} = f^\nu \), where \( f^\nu \) is the 4-force density acting on \( \mathcal{A} \) — in particular, \( f^0 \) is related to the energy exchange rate \( W \) through \( d^3x \ f^0 = dW/c \). Substituting \( \partial_0 T^0_0 = f^0 - \partial_\mu T^\mu_0 \mathcal{A} \) into Eq. (B2) leads to:

\[
MV_{\text{cme}} = -\frac{1}{c} \int d^3x \ f^0 \ (x - X_{\text{cme}}) = -\frac{1}{c} \int d^3x \partial_\nu[T^\nu_0 \mathcal{A} \ (x - X_{\text{cme}})] + \frac{1}{c} \int d^3x \ T^0_\nu_\mathcal{A} \partial_\nu x
\]

\[
= \int d^3x \ p = \mathbf{P}, \quad (B3)
\]

where \( (p)^i = T^0_i_\mathcal{A} /c \) are the components of the momentum density of the system. Note that the second term in the left-hand side of Eq. (B3) is purely relativistic, since mass conservation in Newtonian mechanics — which follows from covariance of Newton’s second law under Galilean transformations — implies \( f^0/c = 0 \) in the corresponding limit. This expresses the well-known fact that in Newtonian mechanics, the total momentum of an arbitrary system (isolated or not) is completely encoded in the motion of its center of mass and its total mass. In relativity theory, on the other hand, asymmetric (w.r.t. the CME) exchanges of energy between \( \mathcal{A} \) and “the rest of the universe” contribute to the total momentum of the system; now, \( \mathbf{P} \) cannot be assessed only by keeping track of the system’s mass-energy distribution. This motivates us to define the “hidden” part of the total momentum of the system as \( P_h := \mathbf{P} - MV_{\text{cme}} \), which can then be calculated by:

\[
P_h = -\frac{1}{c} \int d^3x \ f^0 \ (x - X_{\text{cme}}) = -\frac{1}{c^2} \int dW(x - X_{\text{cme}}), (B4)
\]

In words: the hidden momentum of a system is given by (minus \( 1/c^2 \) times) the dipole moment (w.r.t. \( X_{\text{cme}} \)) of energy injection into the system, per unit time.
In order to illustrate the connection between asymmetric energy exchange — which leads to hidden momentum — and torque in the moving frame, in Fig. 5 we represent the spacetime diagram of the scenarios depicted in Fig. 1.

FIG. 5: Spacetime depiction of the world-volume of the bar and the energy-momentum exchange (given by the 4-force densities \( \pm f^a \)) between the bar and the reservoirs. (a) From the rest-frame perspective, \( f^a \) has only component along the time direction, describing exchange of energy without net spatial forces. (b) From the moving-frame perspective, the same \( f^a \) clearly has nonvanishing spatial projection. Therefore, in this frame, there are force densities \( \pm f \) acting on the bar.

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