Quantum discreteness and fundamental 1/f noise in tunnel junctions, nano-composites and other many-electron systems

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Abstract

It is shown, with citing tunnel junction as an example, that mutual interplay of the electron quantum transfers in a conducting system can be the fast mechanism for generation fundamental low-frequency flicker conductance fluctuations (1/f noise) without composing Lorentzians. This effect is lost in a theory which neglects the actual discreteness of electron energy levels. The analytical estimates of fluctuations of tunnel conductance are obtained, and the strong 1/f-noise sensibility to the discreteness as observed in nano-composites is explained both qualitatively and quantitatively.

I. INTRODUCTION

1. The low-frequency flicker noise (1/f-noise) known in a large variety of systems is actual problem of applied and theoretical physics [1-5]. This noise manifests itself as fluctuations in rates of relaxation and transport processes. In electronics that is conductance fluctuations in practice often representing most bad type of noise. As a rule, these fluctuations are much higher sensitive to a structure of materials and external influences than conductivity itself and hence may bring a delicate information about mechanisms of charge transport. But the problem is that there is no adequate comprehension of the problem.

For many years theory tries to reduce flicker noise to large temporal scales (the concepts of hierachykal kinetics [5] and self-organized criticality [21] involve also large spatial scales). Usually, flicker noise is thought be result of slow thermally activated fluctuations in structural disorder or in occupancy of electron traps, etc. [1-3,5-15,29,37]. Spectrum 1/f is composed by Lorentzians corresponding to "fluctuators" with different activation energies, under assumption that sufficiently broad and uniform distribution of the energies takes place [1,5-15,37]. But in fact either qualitative origin of fluctuators remains unclear or quantitative treatment of experiments remains made at a stretch. Many important facts in no way could be kept within this theory, for instance, 1/f mobility fluctuations of carriers in clear intrinsic semiconductors [2,10], or 1/f-noise in liquid metals [4].

At present, the noise in systems with bad (narrow-band, hopping, tunnel, percolative) conductivity is under common attention, namely, in strongly doped or defected or amorphous semiconductors [5,7,11-14] and metals [8,42], in oxides [6,9], in manganites with
colossal magneto-resistance [16-20], etc. Since in such systems long-range Coulombian forces can be significant, the charge redistributions were suggested to the role of slow fluctuators [5,11-15]. But again it is hard to explain the absence of saturation of real 1/f spectra at low frequencies [7,15]. Perhaps, it was the true prophecy [22] that the game "1/f from Lorentzians" may turn to a disaster for 1/f-noise theory.

2. With no doubts of the importance of thermally activated "slow" fluctuations, it would be reasonable to separate them from the fundamental 1/f-noise whose direct source is nothing but the same "fast" kinetic events (interactions and collisions of particles and quanta) which produce the electric resistivity itself [4,23-27].

Let us recall that generally the phenomenon of relaxation and irreversibility (in particular resistivity) in one or another dynamical system is equivalent to its ability to constantly forget its own history. Most easy the oblivion spreads to the number of kinetic events which took place in the past (for instance, more than a few inelastic free-path times ago). Hence, the system loses both stimulus and means for keeping some definite "number (or probability) of events per unit time" (more accurately, only the ratio of numbers of time-reversed events is under thermodynamical control, but not their sum or difference) [23,26,27]. Therefore, the fluctuations in "thickness" (the term from [28]) of kinetic events and correspondingly in the rates of relaxation and transport do not cause some back compensating reaction, hence, they themselves do not relax and have no upper time scale.

Such statistical behaviour results just in scale-invariant non-saturating spectrum 1/f (for details see [4,27]). Thus, the extremes converge: long-living correlations associated with 1/f spectrum reflect not a long memory but, in opposite, indifference with respect to the past (it is useful, as recommended by Krylov [8], to get rid of the prejudice that any statistical correlation testifies some actual causal correlation).

The kinetic theory throws out this child at the same time as turbid water if postulating quite certain "probabilities per unit time" (i.e. collision integrals etc.). But if the path from statistical mechanics of a gas to gas kinetics is overcome without such an ansatz [24,27] then it leads to 1/f fluctuations of diffusivities and mobilities of molecules. The origin of these fluctuations is not long relaxation times (surely absent here) but merely indifference of the system with respect to impact parameters of previous collisions (and thus to their effective cross-section). The flicker fluctuations in dissipation and in light scattering in quartz also can be explained as statistical property of phonon kinetics [25]. The latter does not reduce to a certain marginal collision integrals, because phononic kinetic events are strongly entangled in space and time and parametrically interplay one with another.

3. Hence, the general principle of "1/f-noise from lossing memory" realizes by different concrete ways. In this work (representing development of the preliminary study [46]) we argue that in many-electron systems it may realize like in phononic systems by means of mutual entangling kinetic events, i.e. electron transfers and jumps. We will see that flicker conductance fluctuations naturally come to light if a theory takes into account factual finite duration of kinetic events and besides real discreteness of electron energy states.

Because of its finite duration any one-electron transfer becomes the part of a complex many-particle process. If cutting off this part across bosonic lines one gets that quan-
tum amplitude of given transfer evolves under influence by effective time-varying fields representing other components of the process. For instance, currently realizing electron jumps through a tunnel junction, as well as thermal charge motion in its sides, induce fast voltage fluctuations at the junction. In its turn, this voltage noise causes random phase changes in the increments of quantum amplitude of the maturing transfer. Such a picture with mutual influence of one-electron events mediated by many soft photons was considered in the theory of Coulomb blockade and low-temperature anomalies of current-voltage characteristics (CVC) of small tunnel junctions [31-33]. The mathematically close interpretation of many-phonon processes was used in the theory of mobility of strongly coupled polarons [34].

Obviously, this picture must foresee not only renormalization of transport characteristics but in addition their specific fluctuations. If quantum amplitude of any electron transfer becomes a functional of environment noise (electric, elastic, exchange fields, etc.) then corresponding quantum probabilities become random values, and eventually the conductance as well. As far as we know, previously this phenomenon was not under consideration. It must be well expressed if correlation time of the noise is small as compared with typical duration (expectation time) of electron jumps. We will demonstrate that such a situation is easily realizable, and the result of fast environmental noise, regardless of its concrete mechanisms and statistics, is just flicker scaleless fluctuations of transfer rates (probabilities) and conductance (therefore, flicker noise what accompanies hopping conductivity does not need in slowness of charge redistributions, instead such thasts are sufficient which are as fast as diffusion of carriers).

As the experiments prompt, in relative units conductance fluctuations are inversely proportional to a number of movable carriers in a noisy region, that is to its volume. From here the principal statement does follow: flicker noise can not be cathed by a theory which underestimates role of the discreteness and too hurries to turn sums over energy states into integrals.

4. For visuality, we will concentrate at "ideal" tunnel junction. Previously various aspects of many-particle processes and the discreteness in tunnel junctions were touched (see e.g. [35,36] and refs. therein), but with no respect to the effect of our interest. This effect, i.e. conductance fluctuations caused by fast thermal noise, principally differs of the low-temperature conductance fluctuations in tunnel micro-junctions due to quasi-elastic electron scattering by static structural disorder. [44,45]. The 1/f-noise observed in "material" junctions [37,41] usually is attributed to structural fluctuators, so called two-level systems. The possibilities of this theory (developing general approach [1]) were analysed in [30]. This theory surely explains Lorentzian contributions to the whole noise but avoids an interpretation of the "residual" 1/f component observed in [37]. Unlikely one might categorically say that this component (and then almost all high-temperature noise possessing smooth 1/f spectrum) can not have another origin. Such a suspicion is supported by the experiment [41] as discussed below.

II. TIME SCALES OF TUNNELLING

1. If a voltage $U < T/e$ ($T$ is temperature) is applied to a tunnel junction then the mean charge transported during time $\Delta t$ and corresponding conductance can be expressed
as
\[ \Delta Q = e \cdot \frac{U e}{\delta E} \cdot \frac{\Delta t}{\tau_t}, \quad G = \frac{\Delta Q}{U \Delta t} = e^2 \nu \gamma, \quad \nu = \frac{1}{\delta E}, \quad \gamma = \frac{1}{\tau_t} \]  \hfill (1)

Here \( \delta E \) is mean spacing of electron energy levels, thus \( \nu \) is density of states; \( U e/\delta E \) is the number of "active" levels effectively contributing to electron transport between outward leads; \( \tau_t \) is mean transmission time required for one electron jump from a given level at one side to wherever at opposite side, in other words, this is the time of accumulation of quantum jump probability to a value \( \sim 1 \) (though quantum jumps are momentary, the moment is unpredictable and may be expected up to \( \tau_t \) or longer); \( \gamma \) plays the role of mean jump probability per unit time (thus \( e/\tau_t \) is the mean current per active level).

Any real junction has a finite capacity \( C \) and thus characteristic time \( \tau_c \approx RC \equiv C/G \). Its physical sense is the relaxation time of charge captured by \( C \) and the correlation time of thermal voltage fluctuations at the junction. Compare the defined time scales and let be convinced of that tunneling is very long event, that is
\[ \frac{\tau_t}{\tau_c} = \frac{e^2 \nu}{C} = E_c = \frac{\delta E}{\delta E} >> 1 \]  \hfill (2)

even if Coulombian effects are weak in the trivial sense \( E_c << T \). For certainty, consider flat metallic electrodes with thickness \( w \), correspondingly flat barrier with thickness \( d \) and typical dielectricity \( \epsilon \sim 20 \). Then taking from textbooks standard estimates for capacity and Fermi energy and velocity one obtains
\[ \frac{e^2 \nu}{C} \approx \frac{e^2}{\hbar v_F} \cdot \frac{4 \pi d w}{\epsilon a^2} \approx \frac{d w}{a^2} , \]

where \( a \) is atomic size, \( a \sim 3 \cdot 10^{-8} cm \).

This expression clearly shows that inequality (2) is always satisfied. Consequently, while tunneling an electron has time to virtually feel many variations of the inter-side voltage noise, \( u(t) \), produced by many other jumps in both directions. At one-electron language, this means that quantum-mechanical transfer probabilities behaves as random processes. From the point of view of rigorous field theory of many particles, the description of corresponding excess fluctuations of transport current would need in at least four-particle Green functions \[4\]. Since the necessary formal technique is still absent, we will try to understand the essence of the matter by means of archaic tunnel Hamiltonian method and quantum-mechanical non-stationary perturbation theory. We confine ourselves by the simplest variant: the sides are identical, all tunnel matrix elements \( g_{kq} \approx g \) are approximately equal and transparency is small.

2. Let us recall the underlying reasons of standard tunnel Hamiltonian scheme (for a time forgetting voltage noise). Let \( p_{kq}(\Delta t, U) \) be the probability of electron transfer from a state "k" at left side to a state "q" at right side during an observation time \( \Delta t \) (or probability of the time-reversed event). The probability of electron jump from left level "k" to wherever at right-hand side is given by sum of these elementary probabilities, and jump duration is the inverse jump probability per unit time:
\[ \gamma = \tau_t^{-1} = p_k(\Delta t, U)/\Delta t, \quad p_k(\Delta t, U) \equiv \sum_q p_{kq}(\Delta t, U) \]
Further, the standard scheme recommends (as usually when constructing kinetics) to resort to the Fermi "golden rule",

\[ p_{kq}(\Delta t, U) \rightarrow 2\pi g^2 \delta(E_{kq})\Delta t/\hbar, \quad E_{kq} \equiv E_q^+ - E_k + eU \]

(plus relates to right-hand side). This recipe ensures the time-linear increase of the jump probability and thus the existence of certain rate of jumps,

\[ \gamma = \tau_t^{-1} \approx 2\pi g^2 \nu/\hbar \]  \hspace{1cm} (3)

The using golden rule (in other words, continuous spectrum approximation) implies that the duration of watching on the evolution of quantum amplitudes, \( \Delta t \), what is sufficient for adequate evaluation of jump probabilities is in the frames \( \hbar/T \ll \Delta t \ll \tau_g \). Here the characteristic time is introduced, \( \tau_g = 2\pi/\delta E \), involved by discreteness of energy spectrum.

Clearly, if one wants to account for the influence of voltage noise \( u(t) \) then the adequate watching time should be much greater than its correlation time, and in view of (2) it is desirable to continue the watching time up to a value of order of factual duration of jumps. Hence, the standard scheme needs in the condition \( \tau_g > \tau_t \). But the Eqs.1 and 3 lead to the relation

\[ \frac{\tau_t}{\tau_g} = \frac{R}{R_0} \approx \left( \frac{\delta E}{2\pi g} \right)^2, \quad R_0 \equiv \frac{2\pi \hbar}{e^2}, \]  \hspace{1cm} (4)

which shows that in a bad transparent junction just the opposite situation takes place,

\[ \tau_t/\tau_g >> 1 \]  \hspace{1cm} (5)

Thus it comes out that the golden rule is inapplicable, and one runs into the problem: "probabilities per unit time" remain uncertain.

We may take in mind the case of well expressed discreteness (5), since this is principally most interesting case and besides at \( R < R_0 \) compatibility of the tunnel Hamiltonian method and perturbation theory could be under question [38] (nevertheless, junctions with \( R < R_0 \) also will be described if thats can be represented as parallel sum of autonomous high-resistance junctions). Under condition (5), the perturbation theory is undoubtly applicable even in presence of voltage noise.

III. FLUCTUATIONS OF PROBABILITIES

1. Consider quantum transfers influenced by time-varying potential difference, \( u(t) \), between initial and final states. According to modern theory of quantum chaos, stochastic behaviour is typical for quantum systems regardless of energy discreteness [39,40]. Therefore, we will treat \( u(t) \) like classical random process (though remembering that in rigorous theory \( u(t) \) is an operator entangled with particle operators). At \( \Delta t \sim \tau_t \), the perturbation theory is sufficient to approximately solve the Schroedinger equation for wave function of tunneling particle. The result reads as
\[ p_{kq} \approx |A_{kq}|^2, \quad A_{kq} = \frac{g_{kq}}{\hbar} \int_0^{\Delta t} \exp(iE_{kq}t/\hbar) Z(t) dt, \quad Z(t) = \exp[i\varphi(t)], \quad (6) \]

where the diffusively accumulating random phase is introduced,

\[ \varphi(t) = \frac{e}{\hbar} \int_0^t u(t') dt' \]

At \( u(t) = 0 \) these formulas reduce to usual ones, otherwise they describe chaotic parametrical excitation or damping of probabilities by the random phase.

Let us introduce phase correlation function, corresponding coherence time of quantum amplitudes and "coherence region" by formulas

\[ K(t_1 - t_2) = \langle Z(t_1)Z^*(t_2) \rangle, \quad \tau_{coh} = \int_0^\infty |K(\tau)| d\tau, \quad \Delta E = 2\pi\hbar/\tau_{coh}, \quad (7) \]

where angle brackets denote the averaging with respect to \( u(t) \). Generally, because of multiplicative structure of \( Z(t) \) calculations of even simplest statistical characteristics of the solution require a vast statistical information about \( u(t) \). But if the coherence time is much smaller than observation time then factor \( Z(t) \) under integral in (6) acts as complex fast ("white") noise. Consequently, the transfer amplitudes \( A_{kq} \) behave as (complex) Brownian walks, and the minimal information presented by characteristics (7) is already sufficient.

2. Discuss briefly the coherence time, assuming for simplicity that shunting influence by external circuit on \( \tau_c \) is negligible. Note that \( K(t) \) represents characteristic function of the phase, in the sense of probability theory. This function can be easy written at \( E_c << T \) if one treats voltage noise as Gaussian random process. Corresponding calculations give

\[ \tau_{coh} \sim (\hbar/e)(C/T)^{1/2} << \tau_g \]

(perhaps, that is minimally possible value of \( \tau_{coh} \)). At \( E_c \sim T \), charge quantization is essential and it is better to treat \( u(t) \) like triple-valued random process, \( u(t) = 0, \pm e/C \). Corresponding analysis finishes with simple estimate

\[ \tau_{coh} \sim \tau_c \]

In this case coherence time may be comparable with \( \tau_g \).

Hence, there are all grounds to consider \( Z(t) \) as white noise and consequently the amplitudes as Brownian walks. This allows to write

\[ \langle p_{kq}^2 \rangle = \langle |A_{kq}|^4 \rangle \approx 2 \langle |A_{kq}|^2 \rangle^2 = 2 \langle p_{kq} \rangle^2, \quad \langle p_{kq}, p_{kq} \rangle \approx \langle p_{kq} \rangle^2 \quad (8) \]

We use Malakhov cumulant brackets,

\[ \langle x, y \rangle \equiv \langle xy \rangle - \langle x \rangle \langle y \rangle \]

At first, we see that after a time longer than coherence time the quantum amplitudes and probabilities acquire 100-percent uncertainty. Secondly, the mean probabilities grow linearly with time:
\[ \langle p_{kq} \rangle \approx \Delta t (g_{kq}/\hbar)^2 \int K(\tau) \exp(iE_{kq}\tau/\hbar) d\tau \propto \Delta t, \quad (9) \]

Now, the fixed scale \( \Delta E \) instead of \( 2\pi \hbar/\Delta t \) determines energy region available for transfers from a given level.

3. We are most interested in the summary jump probability. It can be represented as

\[ p_k \equiv \sum_q p_{kq} = \int \int_0^{\Delta t} \Gamma_k(t_1 - t_2)Z(t_1)Z^*(t_2)dt_1dt_2, \quad (10) \]

with the integral kernel

\[ \Gamma_k(\tau) = \sum_q \left( \frac{g_{kq}}{\hbar} \right)^2 \exp(i\tau E_{kq}/\hbar) \]

Analytical properties of such kernels dictated by quantum discreteness play a key role in the theory. In the continuous spectrum approximation this kernel turns into a function which quickly (integrably) and irreversibly decays to zero, for instance, delta-function. But in reality this kernel is quite non-local and never decays (instead, sometimes it returns to a value of order of its value at \( \tau = 0 \)). In principle, this property is nothing but reflection of the unitarity of quantum dynamics. If take, for visuality, the equidistant spectrum at side accepting jumps, \( E_{kq} = n\delta E + \varepsilon_k \), with integer \( n \), then

\[ \Gamma_k(\tau) = \frac{1}{\tau_1} \exp(i\varepsilon_k\tau/\hbar) \sum_n \delta(\tau - n\tau_g) \quad (11) \]

Now the third point is obvious: if \( \Delta E > \delta E \) then the mean jump probability \( \langle p_k \rangle \approx \Delta t/\tau_1 \) practically coincides with what is used in kinetics. Hence, due to the noise transfers into discrete spectrum realize as successfully as to continuous spectrum.

But, of course, the noise can not make the "probabilities per unit time" be better certain. The most important point is that due to the discreteness the summary jump probabilities also have non-zero fluctuations. With accounting for the white-noise character of \( Z(t) \), variances of jump probabilities can be expressed as

\[ \langle p_k, p_k \rangle \approx \int \ldots \int_0^{\Delta t} \Gamma_k(t_1 - t_2)\Gamma_k(t_3 - t_4)K(t_1 - t_4)K(t_3 - t_2)dt_1...dt_4 \quad (12) \]

Here under condition \( \Delta E > \delta E \) only regions \( t_1 \approx t_4, t_3 \approx t_2 \) are significant but many delta functions from (11) contribute. The estimate of this integral yields

\[ \langle p_k, p_k \rangle \approx \frac{\Delta t^2}{\tau_1^2\tau_g} \int |K(\tau)|^2 d\tau \approx \frac{\tau_{coh}}{\tau_g} \langle p_k \rangle^2 \approx \frac{\delta E}{\Delta E} \langle p_k \rangle^2 \quad (13) \]

(we took into account that "width" of delta functions determined by inverse energy band is wittingly smaller than \( \tau_{coh} \)). Notice that contributions to \( p_k(\Delta t, U) \) from different parts of the observation time are completely statistically correlated although produced by independent pieces of realization \( u(t) \). Such unusual transport statistics were considered in [4,23,24,27]. Here, it is due to that transport is governed by subsequent time summation of the amplitudes, not probabilities.
Below mutual correlation between fluctuations of probabilities of jumps from different levels will play an important role. It reads as
\[
\langle p_{k_1}, p_{k_2} \rangle \approx \langle p_{k_1} \rangle \langle p_{k_2} \rangle \frac{\delta E}{\Delta E} S(E_{k_1} - E_{k_2}),
\]
where function
\[
S(E) = \int \exp(iE\tau/\hbar) |K(\tau)|^2 d\tau \left[ \int |K(\tau)|^2 d\tau \right]^{-1}
\]
describes its dependence on energy distance between two levels, \( E \). According to these formulas, the "uncertainty principle" takes place: with increasing \( \Delta E \) all fluctuations of jump probabilities decrease but instead their correlations spread to more and more wide energy distances. The rates of charge injection from levels belonging to the same "coherence region" \( \Delta E \) are closely correlated.

At \( \Delta E < \delta E \) fluctuations of jump probabilities may increase up to 100-percent value or even higher. In this extremal case quantitative estimates are sensible to details (commensurability etc.) of energy spectra, therefore one needs in some statistics of energy levels. Besides, in this case the noise induced renormalization of mean jump probabilities and consequently average transport current and CVC becomes significant. All the above results can be extended to when inequality (5) is not satisfied.

IV. CONDUCTANCE FLUCTUATIONS

1. Consider fluctuations of the charge transported through the junction between outward leads. We will confine ourselves by the case of not too low temperature, \( T >> \delta E \), and not too high external voltage, \( U < T/e \).

Below let \( \Delta Q \) denote the random value. It consists of two parts,
\[
\Delta Q = \Delta Q_{th} + \Delta Q_{ex}
\]
Here first term is contribution of fast thermal (shot) current noise caused by uncertainty of jump instants. This part contributes to variance of \( \Delta Q \) even in equilibrium (at \( U = 0 \)) and can be estimated with the help of fluctuation-dissipation theorem (Nyquist formula),
\[
\langle \Delta Q^2_{th} \rangle \approx 2TG\Delta t
\]
We are more interested in the second term which besides mean value of transport current includes also excess transport fluctuations caused by the above discussed uncertainty of jump probabilities.

Within statistical language, \( \Delta Q_{ex} \) may be defined as conditional average value of \( \Delta Q \) under fixed \( p_{k_q} \). It is clear, from simple thermodynamical reasonings, that this conditional average has the same sign as \( U \) and disappears at \( U = 0 \). Therefore, it can be represented as the sum of definitely directed jumps (for instance, from left to right at \( U > 0 \)). This circumstance allows us to concentrate at the excess contribution without special analysing correlations between oppositely directed electron transfers. These correlations create no obstacles to arbitrary low-frequency transport fluctuations (distributed at times longer
Thus we can assume that all the correlations are already included into statistics of voltage noise $u(t)$. All the more this is reasonable in view of the appointed fact that details of the noise statistics are unimportant.

2. Therefore, let us attribute formulas of previous Section to the probabilities of one-directed excess jumps responsible for transport. For a first, one may neglect correlations between random energy positions of ”active” levels which inject the excess jumps, and preliminarily perform the averaging with respect to the postions. Then the start expression for the excess transport looks as

$$\Delta Q_{ex} = e \sum_k \left[ f(E_k) - f(E_k + eU) \right] p_k(\Delta t, U), \quad (15)$$

with $f(E)$ being the Fermi distribution function. Below with no noticeable losses we may put on $p_k(\Delta t, U) \approx p_k(\Delta t, 0)$. If one additionally neglected the randomness of probabilities, then Eq.15 would reduce to the well known expression for mean transport current. It must be underlined that Eq.15 completely accounts for the Fermi statistics in both electrodes. The result of averaging (15) coinsides with Eq.1.

Consider fluctuations of the conductance $G = \Delta Q_{ex}/U\Delta t$ implied by Eq.15, omitting details of calculations. At small coherence time, what corresponds to ”large” junction, dense levels, and $\Delta E > \delta E$, with the help of formulas of previous section the variance of (15) can be transformed into

$$\langle \Delta Q_{ex}, \Delta Q_{ex} \rangle \approx (GU\Delta t)^2 \frac{\delta E}{\Delta E} \int W(E')W(E'')S(E' - E'')dE'dE'' \quad (16)$$

Here function

$$W(E) = [f(E) - f(E + eU)]/eU \approx -\frac{\partial f(E)}{\partial E}$$

has the sense of the ”one-particle” probability density distribution of energy of active levels. The Eq.16 yields rather universal estimate

$$\delta G^2 \equiv \frac{\langle G, G \rangle}{\langle G \rangle^2} \approx \frac{\delta E}{T}, \quad (\tau_{coh} < \tau_g) \quad (17)$$

At large coherence time, what corresponds to ”small” junction, rarefied levels, $\Delta E < \delta E$, and weakly correlated jumps, i.e. to the above mentioned extremal situation, so definite estimate is impossible, because conductance fluctuations are very sensible to statistics of energy levels, first of all to their commensurability. In this case ”everything is possible”,

$$\frac{\delta E}{T} < \delta G^2 < 1, \quad (\tau_{coh} > \tau_g), \quad (18)$$

up to relative fluctuations of order of unit.

3. As it is seen, in general the discreteness directly serves as the measure of conductance fluctuations. At the same time, factor $\Delta E$ which characterizes intensity of ”environmental noise” does not contribute to (17). This is likely if the mean number of active levels, $N \equiv eU/\delta E$, is not large and these levels make take arbitrary relative
positions, in particular, all occur in one and the same coherence region $\Delta E$. But the latter case is impossible when $N$ exceeds total number of levels in such a region, $\Delta E/\delta E$. Consequently, at $eU > \Delta E$ permissible energy distributions of active levels are more uniform, therefore jumps from them are less correlated. Under increase of applied voltage, this may result in supressing conductance fluctuations even at $eU << T$ when average conductance is still more or less constant.

This effect is not scoped by the approximation (15). Formally, as the Eq.16 shows, the matter is that it corresponds to the factored pair ("two-particle") energy distribution $W(E')W(E'')$. But it is not hard to improve our analysis. Let us enumerate active levels in the increasing energy order. The typical distance between two levels with numbers $j > i$ can not be smaller than $\approx (j - i)\delta E$. Therefore, any pair of numbers implies its own pair distribution,

$$W_{ij}(E', E'') \approx W(E')W(E'')\vartheta(|E'' - E' - |j - i|\delta E)$$

(19)

wher $\vartheta()$ is step-like function. Further, instead of (15), we should write

$$\Delta Q_{ex} = e \sum_{j=1}^{n} p_{k_j} (\Delta t, U),$$

(20)

thus taking into consideration all possible random positions of active levels at energy scale (i.e. substituting average occupancies by factual unit). Here $n$ is random total number of active levels whose mean value equals to $N$. Then calculate variance of this expression. Since at $N \sim 1$ the answer must coincide with previous result, we may put on $N >> 1$, $n = N$. Performing the averaging with respect to both random energy positions and jump probabilities yields

$$\langle \Delta Q_{ex}, \Delta Q_{ex} \rangle \approx e^2 (\gamma \Delta t)^2 \frac{\delta E}{\Delta E} \sum_{i,j=1}^{N} \int W_{ij}(E', E'')S(E' - E'')dE'dE''$$

(21)

After elementary manipulations we obtain instead of (17) the corrected estimate of conductance fluctuations,

$$\delta G^2 \approx \frac{\delta E}{T} D(eU),$$

(22)

where function

$$D(X) = \frac{1}{X} \int_{0}^{X} dE \int_{-\infty}^{\infty} S(E')dE' \cdot \left[ \int_{0}^{\infty} S(E)dE \right]^{-1}$$

(23)

describes their dependence on external voltage. For example, in the case of exponential phase correlation we have

$$K(\tau) = \exp(-|\tau|/\tau_{coh}), \quad D(X) = \frac{1}{X} \int_{0}^{X} \left(1 - \frac{2}{\pi} \arctan \frac{E}{\Delta E}\right) dE$$

The estimate (18) modifies in analogous fashion.

According to these formulas, at $eU > \Delta E$ the fluctuations decrease approximately as $\propto U^{-1}$. One can say, that the increase of applied voltage results in increase of
effective number of statistically independent (energetical) tunneling channels, and relative conductance fluctuations decrease inversely proportionally to the number of channels. Thus the degree of discreteness, $\delta E$, as well as the degree of voltage noise, $\Delta E$, both have direct reflection in "current-noise characteristics" if not in current-voltage characteristics. The clear separation of duties of these two parameters gives hope that their significance will be maintained in a more rigorous theory.

4. What is for the transparency of junction, in relative units it slips out from the estimates. In this sense it can not be treated as "small expansion parameter", as well as in the sense of relation between excess transport noise and thermal (shot) noise. Though the shot noise contribution to variance of the transported charge is proportional to the transparency, while excess contribution is proportional to it squared, the latter grows as the observation time squared, while the former grows linearly with time. Therefore, excess transport noise inevitably dominates at large times and low frequencies. It is easy to verify that at $eU \sim T$ this takes place already after a time of order of one jump duration, $\tau_t$. In fact, in experiments and practice excess noise noticeably dominates at low frequencies at any realistic operational currents. Besides, naively formal reasonings on "powers of transparency" are wrong in view of that the random phase $\varphi(t)$ involves in some complicated non-perturbative way all orders of the transparency.

V. DISCUSSION AND COMPARISON WITH EXPERIMENTS

1. Experimental confirmation of the theory can be found in the work [41] devoted to 1/f noise in films of the cermet (nano-composite) formed by Ni nano-particles in matrix Al$_2$O$_3$. In this system the parameters of a typical elementary tunnel junction between neighbouring particles were $\delta E \approx 0.2$ meV, $d \approx 2$ nm, $C \approx 5 \cdot 10^{-6}$ cm, $E_C \sim T$ (at room temperature), $R \approx 30$ MOhm, what means $\tau_g \approx 3 \cdot 10^{-11}$ s, $\tau_c \approx 1.5 \cdot 10^{-10}$ s and $\tau_t \approx 3 \cdot 10^{-8}$ s. In this system very intensive conductance fluctuations were observed with relative spectral power $S_{\delta G}(f) \approx \alpha/N_g f$, where $\alpha \approx 6 \cdot 10^{-3}$, and $N_g$ is number of metal granules in the sample. Since $E_c \sim T$, then $N_g$ approximately coincides with the number of movable (simultaneously transported) electrons in the sample [41], therefore it was almost "standard" 1/f-noise with "classical" Hooge constant [1-4].

These data correspond to the noise $S_{\delta G}(f) \approx \alpha/f$ in a separate elementary junction. Since both the inequalities (2) and (5) are well satisfied, we may assume that this noise is due to the mechanism under consideration. For stationary 1/f noise, the above calculated conductance variance related to a definite smoothing time can be connected with 1/f spectrum as

$$\delta G^2 \approx f S_{\delta G}(f) \ln(\Delta t/\tau_c) \approx \alpha \ln(\Delta t/\tau_c)$$

At $\Delta t \sim \tau_t$ this gives the value $\sim 0.03$. Formula (17) at $\delta E = 0.2$ meV and room temperature yields $\approx 0.008$. This is good agreement, in view of that strictly speaking this case must be subjected to formula (18). Indeed, due to above estimates, here $\tau_{coh} \sim \tau_c > \tau_g$, hence $\Delta E$ is of order of $\delta E$ or even smaller what corresponds to the extremal situation.

The remarkable observation of [41] is 1/f-noise sensibility to the discreteness of electron energy spectra in metal granules. When applied voltage per elementary junction exceeds
δE/e, i.e. when more than one level in a granule is active, relative 1/f noise intensity decreases as inverse voltage, although average CVC is still Ohmic up to \( \sim T/\delta E \sim 100 \) times larger voltages. In view of \( \Delta E \sim \delta E \), it is clear that in respect to this effect our theory is in full agreement with the experiment.

2. In case of cermet the discreteness of levels is directly determined by a volume of junction sides, i.e. metal nano-particles. Obviously, in a junction with massive sides (electrodes) the quantity \( \delta E \) also must be determined by volume of the space region which is physically available for electron jumps, that is by geometry of junction and processes of electron interactions and scatterings in electrodes. If temperature is not too low then one has no many variants. The available region must be restricted by junction area, \( A \), and non-elastic free path in sides, \( \lambda \). In other words, this is the region in which level spacing has the order of broadening of levels due to non-elastic relaxation. Of course, now it would be more precise to say not about literally levels but about statistics of energy repulsion of spatially close electron states [40], which has a sense and acts also at non-zero temperatures [39]. Thus the term discreteness serves as synonym of uncertainty: nuances of electron states depend on the "environmental noise" and can not be controlled with precision better than \( \delta E \).

3. Consequently, in case of massive metallic electrodes one can estimate \( \delta E \sim E_F a^3/A \lambda \), with \( E_F \) being Fermi energy and \( a \) atomic size. Then, if roughly relate \( \lambda \) to conductivity in electrodes, \( \sigma \sim \lambda/a^2 R_0 \), the estimate (17) transforms into

\[
\delta G^2 \sim \frac{E_F}{T} \cdot \frac{a^2}{A} \cdot \frac{\sigma_{min}}{\sigma}
\]  

(24)

where \( \sigma_{min} \sim (aR_0)^{-1} \) is minimal metallic conductivity. In particular, let the metal be so clear that the electron-phonon mechanism of relaxation is dominating. Then, according to [43],

\[
\sigma \sim \left( \frac{\hbar e^2 n_e}{m^* T} \right) (T_D/T)^4
\]

(all notations are standard), and one may expect that at temperatures lower than Debye temperature conductance fluctuations are proportional to \( T^4 \).

If say about possible orders of value only, then at standard \( E_F \) the Eq.24 yields

\[
\delta G^2 \sim \left( \frac{a^2}{A} \right) (T/T_D)^4
\]

(25)

From the other hand, left side of (25) may be connected, like it was made above, with spectral power of 1/f-noise. Then in the case of micro-junction with area \( 10^{-9} \ cm^2 \) investigated in [37], under assumption \( T \sim T_D \) we obtain the estimate \( f_S g f (T) \sim 10^{-7} \). This value is in agreement with measurements in [37] at 260 K. Besides, the estimate (25) naturally explain sharp (approximately by two orders of magnitude) increase of 1/f-noise under increasing temperature from 100 K to 300 K as was observed in [37]. To explain this fact the authors supposed sharp step in the activation energy distribution of fluctuators (although this assumption contradicts to uniformity of the distribution wished for composing spectrum 1/f). From our look, likely in [37] two types of low-frequency noise were observed: one has structural origin and dominates below 100 K, while another is just what was considered and dominates at higher temperatures.
4. In this connection, it is useful to notice that generally summation of structural fluctuations and fundamental flicker noise under our attention may not obey trivial adding rules [4]. Clear 1/f spectrum corresponds to fast forgetting the past [4,27], while entangling between the fundamental noise and actually slow relaxation processes (for instance, heat transport) may lead to somehow deformed spectrum (for instance, $1/f^\gamma$ with $\gamma > 1$)[4]. In particular, structural defects can affect 1/f noise although not being its origin. For example, elastic electron scattering by impurities supressess mobility 1/f noise [2]. Experiments with metallic films show that 1/f noise there is strongly influenced by vacancies [42]. But this effect can not be explained in terms of thermally activated diffusion of vacancies. Indeed, unlikely separate diffusive jumps of vacancies have very large variance of activation energies. What is for the diffusion as the whole it is described by the same equations as temperature diffusion, but the latter far ago was removed as hypothetical source of 1/f spectrum [1-4]. Perhaply, vacancies promote non-elastic scattering of electrons and in this way become mediator for the ”1/f-noise from losing memory”.

VI. CONCLUSION

As it was demonstrated, the careful treatment of quantum discreteness displays that amplitudes of elementary quantum transfers being sensitive to surrounding noise of the whole system behave like random Brownian walks what leads eventually to low-frequency flicker fluctuations of transport rates. These fluctuations have no characteristic time scale and are indifferent to duration of observation and time averaging.

Formally, in our approximation restricted by time intervals $\Delta t \sim \tau_t$ we obtained fluctuations with perfectly undecaying correlations corresponding to spectrum $\propto \delta(f)$. This resembles static universal conductance fluctuations [40], although by essence thats are quite different things: one starts from inelastic relaxation and decoherence while another finishes at them. We expect that rigorous many-particle theory will extend our results (and first of all the scaleless character of conductance fluctuations) to arbitrary long time intervals. In this theory quantities kindred to above $p_k$ will be measure of number of electron jumps but will remain proportional to both time and transparency (highest orders of transparency will be absorbed by the random phase, - see notion at end of Sec.3), while spectrum $\propto \delta(f)$ must transform to a spectrum $\propto 1/f$ with the same frequency dimensionality. Formal grounds of such a confidence will be considered elsewhere. Its principal motivation is that the only source of our results is the trivial quantum mechanical rule that even at presence of noise and decoherence any kinetics is governed by summation or other play of amplitudes, not of intermediate probabilities.

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