The Effect of Cancellation in Neutrinoless Double Beta Decay

Silvia Pascoli\(^a\), Manimala Mitra\(^a\), Steven Wong \(^b\)

\(^a\) Institute for Particle Physics Phenomenology, Department of Physics, Durham University, Durham DH1 3LE, United Kingdom
\(^b\) Physics Department, Chinese University of Hong Kong, Shatin, N.T., Hong Kong

Abstract

In light of recent experimental results, we carefully analyze the effects of interference in neutrinoless double beta decay, when more than one mechanism is operative. If a complete cancellation is at work, the half-life of the corresponding isotope is infinite and any constraint on it will automatically be satisfied. We analyze this possibility in detail assuming a cancellation in \(^{136}\)Xe, and find its implications on the half-life of other isotopes, such as \(^{76}\)Ge. For definiteness, we consider the role of light and heavy sterile neutrinos. In this case, the effective Majorana mass parameter can be redefined to take into account all contributions and its value gets suppressed. Hence, larger values of neutrino masses are required for the same half-life. The canonical light neutrino contribution can not saturate the present limits of half-lives or the positive claim of observation of neutrinoless double beta decay, once the stringent bounds from cosmology are taken into account. For the case of cancellation, where all the sterile neutrinos are heavy, the tension between the results from neutrinoless double beta decay and cosmology becomes more severe. We show that the inclusion of light sterile neutrinos in this set up can resolve this issue. Using the recent results from GERDA, we derive upper limits on the active-sterile mixing angles and compare it with the case of no cancellation. The required values of the mixing angles become larger, if a cancellation is at work. A direct test of destructive interference in \(^{136}\)Xe is provided by the observation of this process in other isotopes and we study in detail the correlation between their half-lives. Finally, we discuss the model realizations which can accommodate light and heavy sterile neutrinos and the cancellation. We show that sterile neutrinos of few hundred MeV or GeV mass range, coming from an Extended seesaw framework or a further extension, can satisfy the required cancellation.
I. INTRODUCTION

In the past fifteen years, the experimental evidence of neutrino masses and mixing has opened up a new window on the physics beyond the standard model. The solar, atmospheric and reactor neutrino oscillation (see [1–6] for recent reviews) experiments [7–13] of the past decades confirmed that the standard neutrinos have very small masses in the eV range. Neutrino mixing data [14–17] is well described by the unitary PMNS matrix $U$, parameterized by three real mixing angles, one CP violating Dirac phase and two Majorana phases. So far, the oscillation parameters, namely the solar, atmospheric mass square differences $\Delta m^2_{12}$ and $\Delta m^2_{13}$ and the three oscillation angles $\theta_{12}, \theta_{13}$ and $\theta_{23}$, have been measured [7–20] up to a good accuracy. The current 3$\sigma$ allowed ranges of the oscillation parameters are [19–17]

$$6.99 \times 10^{-5} \text{eV}^2 \leq \Delta m^2_{21} \leq 8.18 \times 10^{-5} \text{eV}^2,$$

$$2.17 \times 10^{-3} \text{eV}^2 \leq \Delta m^2_{31} \leq 2.62 \times 10^{-3} \text{eV}^2,$$

$$0.259 \leq \sin^2 \theta_{12} \leq 0.359,$$

$$0.331 \leq \sin^2 \theta_{23} \leq 0.663,$$

$$0.016 \leq \sin^2 \theta_{13} \leq 0.031 .$$

Although a lot of information on neutrino masses and mixing have been unveiled in the past decade, yet many neutrino properties remain to be determined. We still do not know the neutrino mass hierarchy, if the CP symmetry is violated in the lepton sector, and most importantly, the nature of neutrinos - whether neutrinos are Dirac or Majorana particles. The neutrino nature is strictly related to the violation of global leptonic number and, hence, experiments in which lepton number violation can manifest itself could unveil the Majorana nature of neutrinos.

Among the different lepton number violating experiments, neutrinoless double beta decay, searching for $(A, Z) \rightarrow (A, Z + 2) + 2e^-$, [21–33] is the most sensitive one. In the minimal extension of the Standard Model, augmented by massive neutrinos, this process is mediated by light neutrino exchange [35]. In this case the observation of $(\beta\beta)_{0\nu}$-decay can shed some light on i) the mass hierarchy, the neutrino mass scale and, possibly, on one of the Majorana CP-violating phases, although this will be very challenging [36, 37]. However, in general other mechanisms could play a role in neutrinoless double beta decay. In fact, Majorana neutrino masses require further extensions of the standard model, with a new physics scale, new particles and a source of lepton number violation. The simplest realization comes from the dimension 5 operator $L \cdot H L \cdot H_{\Lambda}$ [38], which can arise as the low energy effective term from a higher energy theory with lepton number violation. The latter will typically also induce neutrinoless double beta decay directly. In most cases, such contributions are suppressed due to the heavy scale of the new mediators, but many exceptions exist [39]. Several detailed studies have been carried out [40–44] regarding Type-I [44, 50, Extended [51, 52] and Inverse seesaw [53–57], Left-Right symmetric [58–65], R-parity violating supersymmetric models [66–70]. It is found that in the Type-I and Extended seesaw scenario sterile neutrinos with few GeV masses can give a contribution comparable to the light neutrino ones or even dominant [12, 41]. For Left-Right symmetric models, the right handed current contribution can be significantly large, if the new gauge boson and right handed neutrino masses are in the TeV scale [59, 65]. In the case of R-parity violating supersymmetry, different lepton number violating states e.g. neutralino, squark and gluino can mediate this process, and their contributions have been analyzed in detail [66–70]. The different lepton number violating states can also originate from an extra dimensional framework [71] or other possible new physics scenario [72–74].

Several experiments on neutrinoless double beta decay [24–34] have been carried out using different type of nuclei, e.g. $^{76}$Ge, $^{136}$Xe, $^{100}$Mo, $^{130}$Te. The bounds coming from Heidelberg-Moscow [29] and IGEX [30] experiments apply to the $^{76}$Ge isotope and are given by $T^{0\nu}_{1/2} > (1.9, 1.55) \times 10^{25}$yrs at 90% C.L., respectively, but the most stringent bound has been recently reported by the GERDA collaboration: $T^{0\nu}_{1/2} > 2.1 \times 10^{25}$yrs at 90% C.L. [28]. Combining the latter with the Heidelberg-Moscow and IGEX experiments, the limit improves to $T^{0\nu}_{1/2} > 3.0 \times 10^{25}$yrs at 90% C.L. [28]. It should be pointed out that a part of the Heidelberg-Moscow collaboration, led by Klapdor-Kleingrothaus and collaborators, reported evidence of the observation of this process corresponding to the half-life $T^{0\nu}_{1/2}(^{76}\text{Ge}) = 1.19^{+0.37}_{-0.23} \times 10^{25}$yrs [23], which was updated later to $T^{0\nu}_{1/2}(^{76}\text{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25}$yrs [26]. This claim has been constrained significantly by the recent results from GERDA [28] but at present neither the individual nor the combined limit from GERDA [24] can conclusively rule out the updated claim [20]. Using the $^{136}$Xe isotope, the bounds on half-life from EXO-200 and KamLAND-Zen experiments are $T^{0\nu}_{1/2} > 1.6 \times 10^{25}$yrs [32] and $T^{0\nu}_{1/2} > 1.9 \times 10^{25}$yrs [31] at 90% C.L., respectively. The KamLAND-Zen collaboration has combined the two limits obtaining $T^{0\nu}_{1/2} > 3.4 \times 10^{25}$yrs at 90% C.L. [31]. According to the KamLAND-Zen collaboration this combined
bound rules out the claim in [26] at 99.7% C.L. but, as pointed out in [63], this conclusion depends on the nuclear matrix elements (NME) used. Future experiments will conclusively confirm or disprove the positive claim and can improve the sensitivity to the half-life by more than an order of magnitude [27, 31, 75, 81].

The light neutrinos, if Majorana particle, will mediate the neutrinoless double beta decay. Their contribution can saturate the present limits of half-lives only in the quasi-degenerate limit. As pointed out in Ref. [13, 63, 82], the bounds from cosmology put stringent constraint on neutrino masses and consequently on the interpretation of neutrinoless double beta decay mediated by light neutrino masses to satisfy the claim in [26], or to saturate the experimental limits from Heidelberg-Moscow, GERDA, EXO-200 and KamLAND-Zen [28, 29, 31, 32]. The conclusion remains the same, after including the stringent cosmological bound on the sum of light neutrino masses from Planck [33], as it has been explicitly shown in [63].

In the light of the recent experimental results, in this work we carefully analyze lepton number violation in neutrinoless double beta decay for the cases in which more than one mechanism is operative [44]. In presence of several left-current processes, if their contributions are comparable, they can sum up constructively in neutrinoless double beta decay or even partially or completely cancel out, making the half-life much longer than naively expected. Establishing if cancellations are at play could be of importance to conclusively determine the nature of neutrinos.

In fact, if future experiments do not find neutrinoless double beta decay in contradiction with the theoretical prediction, the conclusion that neutrinos are Dirac particles is valid only if the possibility of cancellations between different mechanisms is excluded. For instance, this would be the case if no positive evidence is found down to an effective Majorana mass parameter of 10 meV and an inverted hierarchy is established in reactor, atmospheric and/or long baseline neutrino oscillation experiments. Here, we show that if both light and heavy neutrinos, compared to the momentum exchange of the process, are at work, it might be possible to test the presence of such a cancellation.

While individual contribution from different underlying mechanisms: e.g. the most popular light neutrinos, sterile neutrinos in Type-I, Extended seesaw and Inverse seesaw, gluino and squark exchange for R-parity violating supersymmetry, have been carefully analyzed in the literature, the interference effects have been neglected to a large extent (see [84–87] for the few discussions on the interference). In this work, we discuss the effect of interference in detail and present simple model realizations in which such cancellations can emerge. Although our analysis is general, one immediate application would be to solve the mutual inconsistency between the positive claim in [26], for $^{76}\text{Ge}$ and the bounds from [31, 32] in $^{136}\text{Xe}$. If the found evidence [26] is finally refuted by future experiments, the possibility of cancellations remains open and should be tested by using different nuclei.

The paper is organized as follows. In Section II we review the different bounds on neutrinoless double beta decay; we discuss the contribution from light neutrino exchange, the stringent bounds on neutrino masses from cosmology as well as the future bound from KATRIN [88]. Following that, we discuss the contribution from sterile neutrinos in Section III. We discuss the cancellations in Section IV where we carefully consider the interference between two dominant mechanisms in neutrinoless double beta decay, e.g. light neutrino-heavy sterile neutrino exchange or light neutrino-gluino/squark exchange. We show how this possibility is further constrained from beta decay as well as cosmology. Next, we consider the case in which both light and heavy sterile neutrinos are operative in neutrinoless double beta decay. This possibility allows to overcome the constraints from cosmology. We discuss the correlation of half-lives between two different isotopes in Section V. In Section VI we discuss simple model realizations which can accommodate sterile neutrinos. Finally, in Section VII we draw our conclusions.

II. LIGHT NEUTRONI EXCHONGE IN $(\beta\beta)_{0\nu}$-DECAY AND ITS CONNECTION TO BETA DECAY AND COSMOLOGY

Below we review the most stringent constraints on $T_{1/2}^{0\nu}$ for the isotopes of interest $^{76}\text{Ge}$, $^{136}\text{Xe}$, $^{130}\text{Te}$, $^{100}\text{Mo}$ and $^{82}\text{Se}$. All bounds are reported at 90% C.L. unless otherwise specified.

1. The claim of observation of $(\beta\beta)_{0\nu}$-decay by H. V. Klapdor-Kleingrothaus and collaborators for the $^{76}\text{Ge}$ isotope corresponds to the half-life: $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yrs (the range correspond to 68% C.L.) [26]. This has been challenged by the previous results from Heidelberg-Moscow [29] and by the recent result from GERDA [28]. The lower limit of half-life of $^{76}\text{Ge}$ that comes from GERDA [28] is $T_{1/2}^{0\nu}(^{76}\text{Ge}) > 2.1 \times 10^{25}$ yrs.

When combined with the limits from Heidelberg-Moscow (HDM) [29] and IGEX [30] experiments, the limit is
\[ T_{1/2}^{\beta\beta}(76\text{ Ge}) > 3.0 \times 10^{25} \text{ yrs} \]. Note that, as pointed out in Ref. [63, 89], the individual as well as the combined limit from GERDA does not conclusively rule out the positive claim [26].

2. The bounds from EXO-200 [32] and KamLAND-Zen [31] experiments for \(^{136}\text{Xe}\) are \( T_{1/2}^{\beta\beta}(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yrs} \) and \( T_{1/2}^{\beta\beta}(^{136}\text{Xe}) > 1.9 \times 10^{25} \text{ yrs} \), respectively. Combining the two, the lower limit becomes \( T_{1/2}^{\beta\beta}(^{136}\text{Xe}) > 3.4 \times 10^{25} \text{ yrs} \) [31].

3. The bound on the half-life of \(^{130}\text{Te}\) coming from CUORICINO is \( T_{1/2}^{\beta\beta}(^{130}\text{Te}) > 2.8 \times 10^{24} \text{ yrs} \) [33].

4. The lower limit on half-life of \(^{100}\text{Mo}\) from NEMO 3 is \( T_{1/2}^{\beta\beta}(^{100}\text{Mo}) > 1.1 \times 10^{24} \text{ yrs} \) [34].

5. The half-life of \(^{82}\text{Se}\) is bounded from below as \( T_{1/2}^{\beta\beta}(^{82}\text{Se}) > 3.6 \times 10^{23} \text{ yrs} \) [34].

Among these different bounds, those on the half-life for \(^{76}\text{Ge}\) and \(^{136}\text{Xe}\) are in particular quite stringent. As pointed out in Ref. [63], the claim of observation of \((\beta\beta)_{0\nu}\)-decay in \(^{76}\text{Ge}\) is compatible with the individual limits from KamLAND-Zen and EXO-200 for few NME calculations, and it is in contradiction with the combined bound for most of the NME calculations, except of the calculation corresponding to Ref. [90]. For the discussion on the mutual compatibility between the positive claim [26] and the bounds on the half-lives, see also Ref. [91]. It should be noted that, for a given value of \( m_{\nu e}^\nu \), the predicted value of the half-life \( T_{1/2}^{\beta\beta} \) depends strongly on the NME uncertainty. Taking this variation into account, the correlation between half-lives for two different isotopes can be used to test the positive claim [26], as it has been done in Refs. [31, 63].

If light neutrinos are Majorana particles [42], they will mediate neutrinoless double beta decay [35]. The observable in \((\beta\beta)_{0\nu}\)-decay is the ee element of the mass matrix \( m_{\nu e}^\nu \), known as the effective Majorana mass parameter of neutrinoless double beta decay, see e.g. Ref. [6, 36, 93, 95]. Explicitly written in terms of the elements of the PMNS mixing matrix, this reads

\[ m_{\nu e}^\nu = m_1t_{12}^2t_{13}^2 + m_2s_1^2s_{13}^2e^{2\imath\alpha_2} + m_3s_{13}^2e^{2\imath(\alpha_3+\delta)}, \] (3)

where \( \alpha_{2,3} \) are the Majorana phases and \( \delta \) is the Dirac phase. The half-life \( T_{1/2}^{\beta\beta} \) of \((\beta\beta)_{0\nu}\)-decay and the effective mass \( m_{\nu e}^\nu \) are related through the nuclear matrix element \( M_{\nu\nu} \), the phase-space factor \( G_{0\nu} \) and electron mass \( m_e \) as

\[ \frac{1}{T_{1/2}^{\beta\beta}} = G_{0\nu}|M_{\nu\nu}|^2 \left| \frac{m_{\nu e}^\nu}{m_e} \right|^2. \] (4)

In Fig. 1 we show the variation of \( |m_{\nu e}^\nu| \) with the lightest neutrino mass \( m_{\nu\text{lightest}} \), which we have used the 3\sigma range of oscillation parameters from [17]. The blue and green areas correspond to \( \alpha_{2,3} \) taking CP conserving values, while the red regions correspond to the violation of the CP symmetry. The dashed and dotted horizontal purple lines represent the required effective mass that will saturate the GERDA and GERDA+HDM+IGEX limits, respectively [28]. The orange lines correspond to the positive claim (90\% C.L.) [26]. The bands represent the NME uncertainty, taken from the compilation in Ref. [63]. As the plot suggests, a measurement of \( |m_{\nu e}^\nu| \) will give information on masses correlated with the CP violating phases, under the assumption that light neutrino exchange is the only underlying mechanism in \((\beta\beta)_{0\nu}\)-decay.

In addition, the light neutrino mass is also bounded from beta decay studies as well as from cosmology. The mass probed in beta-decay is \( m_\beta = \sqrt{\Sigma_{i}|U_{ei}|^2 m_i^2} \) [94] and the present 95\% C.L. limit on this observable is \( m_\beta < 2.3 \text{ eV} \) from MAINZ [87] and \( m_\beta < 2.1 \text{ eV} \) from Troitsk [95] collaborations, respectively. This bound can be improved by one order of magnitude down to \( m_\beta < 0.2 \text{ eV} \) from the beta decay experiment KATRIN [88], which is currently under commissioning. The sum of light neutrino masses \( m_\Sigma = \Sigma m_i \) is constrained from cosmology. In the quasi-degenerate regime \( m_{\nu\text{lightest}} > \sqrt{\Delta m_{\text{atm}}^2} \), that is of particular interest for \((\beta\beta)_{0\nu}\)-decay, beta decay as well cosmological searches, we have \( \Sigma m_i/3 \sim m_{\nu\text{lightest}} \sim m_\beta \geq m_{\nu e}^\nu \). The recent upper bounds on the sum of light neutrino masses coming from Planck [83], which we consider in our studies, are the following: i) \( m_\Sigma < 0.23 \text{ eV} \), derived from the Planck+WP+highL+BAO data (Planck1) at 95\% C.L. and ii) \( m_\Sigma < 1.08 \text{ eV} \) from Planck+WP+highL (AL) (Planck2) at 95\% C.L. [83]. As pointed out in Refs. [42, 83, 93, 82] and is evident from Fig. 1, after imposing the bounds from cosmology (assuming standard cosmology), the light neutrino contribution itself can not satisfy the claim in [26] or saturate the current bounds [28, 29, 31].
III. STERILE NEUTRINO EXCHANGE IN $\langle \beta \beta \rangle_{0\nu}$-DECAY

Sterile neutrinos can also give large contributions to neutrinoless double beta decay as analyzed in detail in Refs. [40–46]. We assume here sterile neutrinos with a mass $M_i$ and which mix with $\nu_e$. The half-life $T^{0\nu}_{1/2}$ is

$$\frac{1}{T^{0\nu}_{1/2}} = K_{0\nu} \left| \frac{\Theta_{ei}^2}{p^2 - M_i^2} M_i \right|^2,$$

(5)

where $K_{0\nu} = G_{0\nu}(m_p M_N)^2$ and $p^2 = -m_e m_p M_N M_{\nu_e}$. Here $M_\nu$ is the NME for the light neutrino exchange and $M_N$ is for the heavy neutrino exchange, $|p| \sim 100$ MeV is the exchanged momentum scale in $(\beta\beta)_{0\nu}$-decay, $\Theta_{ei}$ is the active-sterile neutrino mixing and $m_p$ is the mass of the proton. In the subsequent discussions, we denote $\Theta_{ei}$ by $U_{ei}$ and $M_i$ by $m_i$ for light sterile neutrinos, i.e. when $M_i^2 << |p|^2$. For the heavy sterile case $M_i^2 >> |p|^2$, and we denote them by $V_{eN_i}$ and $M_{N_i}$, respectively. For light sterile neutrinos the above equation simplifies to

$$\frac{1}{T^{0\nu}_{1/2}} \simeq G_{0\nu} M_\nu^2 \left| \frac{U_{ei}^2 m_i}{m_e} \right|^2,$$

(6)

while for the heavy sterile one we have

$$\frac{1}{T^{0\nu}_{1/2}} \simeq G_{0\nu} M_N^2 \left| \frac{V_{ei}^2 m_p}{M_{N_i}} \right|^2.$$

(7)

For simplicity, we call the massive states mainly in the sterile neutrino direction simply "sterile neutrinos" as commonly done in the literature.
Using the above equations and the recent result from GERDA [28], we derive the bound on the active-sterile mixing angle, assuming only one light or heavy sterile neutrino participates in \((\beta\beta)_{0\nu}\)-decay. In all our subsequent analysis, we use the values of NMEs \(M_1\) and \(M_N\) from Ref. [31], corresponding to the axial vector cut-off \(g_A = 1.25\). We use the phase-space for \(^{76}\text{Ge}: G_{0\nu}^{\text{Ge}} = 5.77 \times 10^{-15}\) yrs [100]. In Fig. 2 we show the upper bound on the active-light sterile neutrino mixing angle \(|U_{e4}|^2\) from \((\beta\beta)_{0\nu}\)-decay, that saturates the individual limit \(T_{1/2}^{\text{Ge}} = 2.1 \times 10^{25}\) yrs from GERDA [28]. The gray region is due to the uncertainty introduced by the NME \(M_0\) corresponding to the light neutrino exchange. For comparison, we also show the other different bounds, first compiled in Ref. [101].

For the mass of sterile neutrino \(m_4 < 1\) MeV, the kink searches in \(\beta\)-decay spectrum is a sensitive probe of sterile neutrinos. The excluded regions with contours that are labelled by \(^{187}\text{Re}, ^{3}\text{H}, ^{63}\text{Ni}, ^{35}\text{S}, ^{20}\text{F}\) and \(\text{Fermi}_2\) refer to the bounds from kink searches [102][106]. Note that, in addition, we have also included the bound coming from beta decay experiment of \(^{64}\text{Cu}\) [107], which was not reported in Ref. [101]. The reactor and solar experiments Bugey and Borexino [108][109] are sensitive in the region \(m_4 \sim \) few MeV. Exclusion contours have been drawn by looking into the decay of sterile neutrino into electron-positron pairs. On the other hand, for mass \(m_4 > \) few MeV, the sensitive probe is the peak search in \(\pi \rightarrow e\nu\) [110], where the region inside the dot-dashed black contour is excluded. As can be seen from the figure, the bound on the active-light sterile neutrino mixing coming from \((\beta\beta)_{0\nu}\)-decay is the most stringent for most of the parameter spaces in \(U_{e4} - m_4\) plane. For the mass of the light sterile neutrino \(m_4 \lesssim 10^{-4}\) GeV, the bounds from different beta decay searches are close to the ones from \((\beta\beta)_{0\nu}\)-decay and possibly can be improved by the future beta decay experiments. In the range \(10^{-4} \lesssim m_4 \lesssim 0.01\) GeV, the bound from \((\beta\beta)_{0\nu}\)-decay is the most stringent, while around \(m_4 \sim 0.1\) GeV, the bound from peak searches, \(\pi \rightarrow e\nu\) [110], can almost compete with the bound from \((\beta\beta)_{0\nu}\)-decay.

Similarly, the upper limit on the mixing angle \(|V_{eN}|^2\) is shown in Fig. 3. The gray region is due to the uncertainty in the NME \(M_N\) corresponding to the heavy neutrino exchange. In addition, we also show the other different bounds, from Ref. [101]. The regions inside the brown dot-dashed line is excluded from the beam dump experiment PS191 [111]. For mass of sterile neutrino \(M_N \sim O(100)\) MeV, the stringent bound is obtained from the electron spectrum in meson decay \(K \rightarrow e\nu\) decay [112]. For heavier masses \(M_N \sim O(\text{GeV})\), the \(Z^0\) decays into sterile neutrinos can be used to obtain exclusion contours, labelled as DELPHI and L3 [113][114]. See Ref. [101] and the references therein for the detail description of other different bounds [115][116]. Also in this case, for most of the parameter space, the \((\beta\beta)_{0\nu}\)-decay gives the most stringent limit. For the mass of the heavy sterile neutrino \(M_N \sim O(100)\) MeV, the bound from the beam dump experiment PS191 is competitive with the one from \((\beta\beta)_{0\nu}\)-decay. For the positive claim [26], the results are very similar and we do not show the corresponding region explicitly.

**IV. CANCELLATIONS AMONG DIFFERENT CONTRIBUTIONS IN \((\beta\beta)_{0\nu}\)-DECAY**

The discussion of the previous section on the effective Majorana mass relies on the assumption that either the light or heavy neutrino exchange is the only underlying mechanism in \((\beta\beta)_{0\nu}\)-decay. However, in an extension of the standard model leading to light Majorana masses, the lepton number violating mechanism responsible for it will also contribute to neutrinoless double beta decay directly and could potentially interfere with the light neutrino one. Below we consider this possibility in detail. This is of particular interest, as it can solve the mutual inconsistency between the positive claim [26] and the results from KamLAND-Zen [31].

If more than one mechanism is operative at \((\beta\beta)_{0\nu}\)-decay, the half-life \(T_{1/2}^{0\nu}\) of \((\beta\beta)_{0\nu}\)-decay for a particular isotope will receive different contributions as

\[
\frac{1}{T_{1/2}^{0\nu}} = G_{0\nu}(|\eta_1|^2|M_1|^2 + |\eta_2|^2|M_2|^2 + 2\cos \alpha |\eta_1||\eta_2||M_1|M_2),
\]

where \(G_{0\nu}\) is the phase-space factor, \(M_{1,2}\) are the NMEs for the two different exchange processes. Here, \(\eta_1\) and \(\eta_2\) are the two dimensionless quantities which contain all the information from the particle physics parameters associated with the two exchange mechanisms and \(\alpha\) is the relative phase factor between them. The different exchange mechanisms can be for e.g. light neutrino and sterile neutrino exchange, or light neutrino and squark/gluino exchange. If a complete cancellation takes place between two exchange mechanisms, then the phase \(\cos \alpha = -1\) and \(|\eta_1|M_1 = |\eta_2|M_2\). Consequently the half-life \(T_{1/2}^{0\nu}\) in Eq. 8 would be infinite, and this process in a specific
nucleus would never be observed. However, this does not need to be the case for another isotope. Between two isotopes (A, B), if this cancellation is effective for isotope A, then the half life for isotope B is

$$\frac{1}{T_{1/2}^{0\nu}(B)} = G_{0\nu}^B|\eta_1|^2((M_{1,B} - \frac{M_{1,A}}{M_{2,A}}M_{2,B})^2, \tag{9}$$

where $M_{1,A}, M_{2,A}$ are the NMEs for the two exchange processes in isotope A and $M_{1,B}, M_{2,B}$ are for isotope B. As an example we consider the case when the cancellation is effective in $^{136}\text{Xe}$. In this case, the bound on half-life $T_{1/2}^{0\nu} > 3.4 \times 10^{25}$ yrs \cite{31} is automatically satisfied, irrespective of the absolute magnitude of $|\eta_{1,2}|$. Denoting the nuclear matrix elements for $^{76}\text{Ge}$ and $^{136}\text{Xe}$ by $M_{1,\text{Ge}}, M_{2,\text{Ge}}$ and $M_{1,\text{Xe}}, M_{2,\text{Xe}}$ and the phase space of $^{76}\text{Ge}$ by $G_{0\nu}^\text{Ge}$, the half-life of $^{76}\text{Ge}$ is

$$\frac{1}{T_{1/2}^{0\nu}(^{76}\text{Ge})} = G_{0\nu}^\text{Ge}|\eta_1|^2((M_{1,\text{Ge}} - \frac{M_{1,\text{Xe}}}{M_{2,\text{Xe}}}M_{2,\text{Ge}})^2. \tag{10}$$

The value of $|\eta_1|$ that saturates the lower limit of half-life from GERDA \cite{28} and GERDA+HDM+IGEX \cite{28} are

$$|\eta_1| \leq \frac{(2.87, 2.40) \times 10^{-6}}{|(M_{1,\text{Ge}} - \frac{M_{1,\text{Xe}}}{M_{2,\text{Xe}}}M_{2,\text{Ge}})|}, \tag{11}$$

while the range of $|\eta_1|$ that satisfies the positive claim (90\% C.L.) in \cite{26} is

$$|\eta_1| = \frac{(2.42 - 3.18) \times 10^{-6}}{|(M_{1,\text{Ge}} - \frac{M_{1,\text{Xe}}}{M_{2,\text{Xe}}}M_{2,\text{Ge}})|}. \tag{12}$$
As stressed before, note that, the individual or the combined limit from GERDA [28] does not conclusively rule out the positive claim in [26]. Hence, in addition to the GERDA, GERDA+HDM+IGEX limits [28], we also carry out the discussion on the positive claim [26]. If the above mentioned cancellation is operative for $^{136}\text{Xe}$, it would be possible to automatically satisfy the bounds obtained by EXO-200, KamLAND-Zen collaboration [31, 32] for $^{136}\text{Xe}$ isotope and yet to satisfy the claim in [26], irrespective of any NME uncertainty. Hence, it is possible to reconcile any mutual conflict between the results of $^{136}\text{Xe}$ and $^{76}\text{Ge}$.

A. Light active and heavy sterile neutrinos

We first discuss the case when the two interfering mechanisms correspond to light active and heavy sterile neutrino exchange. We also include the discussion when the cancellation is operative between light neutrino exchange and squark/gluino exchange mechanisms, for e.g. in R-parity violating supersymmetry.

First, we study the case of light active neutrinos $\nu_i$ and heavy sterile $N_j$ with mass $M_{N_j}$, larger than the typical momentum exchange $|p|$ in $(\beta\beta)_{0v}$-decay: $M_{N_j}^2 \gg |p|^2 \sim (100)^2 \text{MeV}^2$. We consider maximum destructive interference between the two mechanisms, i.e. $\cos \alpha = -1$. A cancellation in isotope A will lead to the following relation,

$$|\eta_N| = |\eta_\nu| \frac{M_{\nu,A}}{M_{N,A}},$$

(13)

Here, we have replaced $\eta_{1,2}$ of the previous section by $\eta_{\nu,N}$, respectively, where $\eta_\nu$ correspond to light neutrino exchange and $\eta_N$ correspond to the heavy sterile neutrino exchange. The nuclear matrix elements $M_{1,A}$ and $M_{2,A}$ in this case correspond to light and heavy neutrino exchange and have been denoted as $M_{\nu,A}$ and $M_{N,A}$, respectively. In the above, the particle physics dimensionless parameters $\eta_\nu$ and $\eta_N$ are given by

$$\eta_\nu = \frac{m_{ee}}{m_e},$$

(14)
FIG. 4. Variation of redefined effective mass $|m_{ee}^{\text{eff}}|$ with the lightest neutrino mass $m_{\text{lightest}}$ for $^{76}\text{Ge}$. The effect of cancellation is operative between light active and heavy sterile neutrino. The horizontal purple lines represent the required $|m_{ee}^{\text{eff}}|$ that will saturate the limits of half-lives from GERDA [28]. The horizontal dashed orange lines represent the ranges of $|m_{ee}^{\text{eff}}|$ for which the half-life for $^{76}\text{Ge}$ is in agreement with the positive claim (90% C.L.) [26]. The vertical black solid line represents the future sensitivity of KATRIN [33]. The dashed and dot-dashed vertical lines represent the limits obtained from cosmology [33]. The horizontal brown and black lines show the future sensitivity of the effective mass for $^{76}\text{Ge}$, assuming a half-life $T_{1/2}^{0\nu} = 1.5 \times 10^{26}$ yr (GERDA Phase-II) [118] and $T_{1/2}^{0\nu} = 6 \times 10^{27}$ yr [24], respectively.

The half life for any other isotope B is predicted to be

$$\frac{1}{T_{1/2}^{0\nu}(B)} = G_{B0\nu} \frac{m_{ee}^{\nu}}{m_e} |M_{\nu,B}|^2 \left( 1 - \frac{M_{\nu,A}}{M_{N,A}} M_{N,B} \right)^2.$$  \hspace{2cm} (16)

It can be rewritten in terms of an effective mass, where the redefined effective mass is

$$|m_{ee}^{\text{eff}}| = |m_{\nu}^{\nu}(1 - \frac{M_{\nu,A}}{M_{N,A}} M_{N,B})|.$$  \hspace{2cm} (17)

Hence, if the light and heavy exchange contributions cancel each other for isotope A, for any other isotope B the effect would manifest itself by increasing the half-life. Below, as a relevant example, we again focus on the case in which the cancellation is present in $^{136}\text{Xe}$ and we explore its effect on the half-life of $^{76}\text{Ge}$.

Using Eq. 16 the different values of redefined effective mass $|m_{ee}^{\text{eff}}|$ that is required to saturate the individual and combined limits of half-life from GERDA [28] and to satisfy the positive claim (90% C.L.) [26] are given in Table I. The redefined effective mass $|m_{ee}^{\text{eff}}|$ is smaller than the true effective mass $|m_{ee}^{\nu}|$, as expected. We show the variation of the effective mass $|m_{ee}^{\text{eff}}|$ with the lightest neutrino mass scale $m_{\text{lightest}}$ in Fig. 4. The horizontal purple bands show the effect of NME uncertainties and correspond to the two different ranges of required effective masses $|m_{ee}^{\text{eff}}| = (0.25 - 0.31)$ eV (dashed purple band) and $|m_{ee}^{\text{eff}}| = (0.21 - 0.26)$ eV (dotted purple band) to saturate the GERDA and GERDA+HDM+IGEX [28] limits, respectively. The horizontal dashed orange lines represent the minimum and maximum of the required ranges of effective mass $|m_{ee}^{\text{eff}}| = (0.21 - 0.34)$ eV that satisfies the positive claim [26]. In both of the figures, the vertical black solid line represents the future sensitivity...
TABLE I. The upper limits of the redefined effective neutrino mass $|m_{ee}^{\text{eff}}|$ that saturate the lower limits of half-life of 76Ge from GERDA [28] and GERDA+HDM+IGEX [28]. The NMEs have been taken from [84]. Also shown are its required ranges corresponding to the positive claim (90% C.L.) [26].

| NME                  | $|m_{ee}^{\text{eff}}|$ (eV) |
|----------------------|-----------------------------|
| SRQRPA               | GERDA                       |
| Argonne intm         | 0.31                        | 0.26-0.34 | 0.26-0.34 |
| Argonne large        | 0.27                        | 0.23-0.30 | 0.23-0.30 |
| CD-Bonn              | 0.29                        | 0.24-0.32 | 0.24-0.32 |
| CD-Bonn              | 0.25                        | 0.21-0.28 | 0.21-0.28 |

FIG. 5. Variation of the redefined effective mass $|m_{ee}^{\text{eff}}|$ with the lightest neutrino mass $m_{\text{lightest}}$ for 76Ge. Left Panel: the cancellation is effective between light neutrino and gluino exchange. The horizontal purple lines represent the required $|m_{ee}^{\text{eff}}|$ that will saturate the limits from GERDA [28]. The horizontal dashed orange lines represent the ranges of $|m_{ee}^{\text{eff}}|$ for which the half-life for 76Ge is in agreement positive claim (90% C.L.) [26]. See text for details. Right panel: for the case when the cancellation is effective between light neutrino and squark exchange. All other descriptions remain same as in Fig. 4.

of KATRIN $m_{\text{lightest}} < 0.2$ eV [88] and the other two vertical lines represent the bound $m_{\text{lightest}} < 0.077$ eV and $m_{\text{lightest}} < 0.36$ eV, following the two extreme bounds from Planck data set $m_{\Sigma} < 0.23$ eV (Planck1) and $m_{\Sigma} < 1.08$ eV (Planck2) [89], respectively.

As can be seen from the figure, the effective mass $|m_{ee}^{\text{eff}}|$ can saturate the required values only in the quasi-degenerate regime. However, this possibility can be severely constrained by the future sensitivity from KATRIN [88], which does not depend on any particular physics model. In particular, for the bound $m_{\beta} < 0.2$ eV from KATRIN [88], the effective mass can not reach the required value of $|m_{ee}^{\text{eff}}|$. The bound from cosmology is even more stringent compared to the case when the light neutrinos are the only mediators and therefore the tension between cosmology and the possible claim in neutrinoless double beta decay is more severe. We also show the future sensitivity for 76Ge by the horizontal brown and black lines that correspond to half-lives $T_{1/2}^{\nu} = 1.5 \times 10^{26}$ yrs for GERDA Phase-II [118] and $T_{1/2}^{\bar{\nu}} = 6 \times 10^{27}$ yrs [24], respectively. It is evident from Fig. 4 that the effective mass can saturate the future limit from GERDA Phase-II around $m_{\text{lightest}} \sim 0.25$ eV. This possibility is unconstrained from the most stringent limit from Planck and marginally constrained by the future sensitivity of KATRIN. For the half-life $T_{1/2}^{\nu} = 6 \times 10^{27}$ yrs, the effective mass can saturate the limit even for $m_{\text{lightest}}$ as low as $10^{-5}$ eV. This possibility is not at reach for future cosmological observations and beta decay experiments.

The cancellation between light contribution and heavy contribution can also be realized in other new physics scenarios, for e.g. R-parity violating supersymmetry. In this framework, the gluino and squarks can give large contribution in $(\beta\beta)_{0\nu}$-decay. Below, we discuss the case when the cancellation is effective between light neutrino...
exchange and gluino/squark exchange. We denote the NMEs corresponding to the gluino exchange by $\mathcal{M}_{\lambda'}$ and the squark exchange by $\mathcal{M}_q$ and parameterize their contributions by $\eta_{\lambda'}$ and $\eta_q$, respectively. The detail description of $\eta_{\lambda'}$ and $\eta_q$ on the fundamental parameters of the theory has been described in detail in Ref. [20, 87], and we do not repeat them here. Like the previous case, the cancellation between light neutrino exchange and squark/gluino exchange in isotope A will result in a reduction of effective mass for any other isotope. The left and right panels of Fig. 5 corresponds to the two different cases, when the cancellation is effective between light neutrino-gluino and heavy masses, respectively. The NMEs have been used from Ref. [84]. The horizontal dashed and dotted purple lines represent the required effective mass that will saturate GERDA and GERDA+HDM+IGEX [28] limits. They have been derived using Eq. 16 and includes the effect of cancellation in $^{136}\text{Xe}$. The horizontal orange lines correspond to the required ranges of effective mass $|m_{\text{ee}}^{\text{eff}}|$, that will satisfy the positive claim [26].

**B. Light and heavy sterile neutrinos**

The tension discussed above between cosmology and neutrinoless double beta decay can be avoided if, in addition to the heavy sterile neutrinos, we also have light sterile neutrinos. The latter, depending on their mass and mixing, can give a large contribution even compared to the light active ones and can in fact saturate the required value of $|m_{\text{ee}}^{\text{eff}}|$. On the other hand, the bounds from cosmology is only relevant if the masses of the sterile neutrinos are very small $m_4 \sim \text{eV}$ and they were copiously produced in the Early Universe contributing to hot dark matter. For heavier masses, $m_4 > \text{keV}$, the mixing angles of interest are very large and would lead to fast decays of sterile neutrinos and consequently to no bounds from cosmology. Hence, adding light sterile neutrinos in addition to heavy sterile ones can solve the mutual inconsistency between the positive claim in [26] and KamLAND-Zen [31], can saturate the upper limits of effective masses for GERDA and GERDA+HDM+IGEX [28] and can be in accordance with the bounds coming from cosmology. Here, we study in detail this case.

We assume both Majorana light sterile neutrinos $\nu_{4k}$ with mass $m_4^2 << |p^2| \sim (100)^2 \text{MeV}^2$ and heavy sterile neutrinos $N_j$ with mass $M_{N_j}^2 >> |p^2| \sim (100)^2 \text{MeV}^2$. In this case, the half-life of any isotope is

$$\frac{1}{T_{1/2}^{\nu}} = G_0(|\eta_l|^2\mathcal{M}_l^2 + |\eta_N|^2\mathcal{M}_N^2 + 2 \cos \alpha |\eta_l||\eta_N|\mathcal{M}_l\mathcal{M}_N),$$

where the parameters $\eta_l$ and $\eta_N$ correspond to the contributions from light and heavy neutrinos as

$$\eta_l = \left(\Sigma_i m_i U_{ei}^2 + \Sigma_k m_{4k} U_{e4k}^2\right)/m_e, \quad \eta_N = \sum_j \frac{m_p V_{eNj}^2}{M_{Nj}}.$$  \hspace{1cm} (19)

For simplicity we consider the case in which only one light sterile and one heavy sterile neutrinos are present. If the cancellation between light and heavy neutrino contribution is effective for isotope A, then following the discussions of previous sections, $\eta_l$ and $\eta_N$ are related as $|\eta_N| = |\eta_l|\mathcal{M}_N/\mathcal{M}_A$. For any other isotope B, the redefined effective mass $m_{\text{ee}}^{\text{eff}}$ is

$$m_{\text{ee}}^{\text{eff}} = (m_{\text{ee}}^\nu + m_4 U_{e4}^2) \times \left(1 - \frac{\mathcal{M}_{\nu,A} \mathcal{M}_{N,B}}{\mathcal{M}_{N,A} \mathcal{M}_{\nu,B}}\right).$$

In the above we have dropped the generation index and $m_4$ denotes the mass of the light sterile neutrino, while $U_{e4}$ is the active-light sterile mixing. We again assume a cancellation for $^{136}\text{Xe}$ and we examine its implications on $^{76}\text{Ge}$. From Table I, it is evident that to satisfy/saturate either the positive claim [20] or the limits from GERDA [28], a large effective mass $|m_{\text{ee}}^{\text{eff}}| \sim \mathcal{O}(0.1) \text{eV}$ is required. We denote the limiting values of effective masses $|m_{\text{ee}}^{\text{eff}}|$ of Table I by $\kappa$ for GERDA, GERDA+HDM+IGEX [28] and the minimum and maximum values of the required $|m_{\text{ee}}^{\text{eff}}|$ by $\kappa_1$ and $\kappa_2$ for the positive claim [20]. Following the stringent constraint from cosmology, the effective neutrino mass $|m_{\text{ee}}^{\text{ee}}|$ corresponding to the light neutrino exchange is extremely small $|m_{\text{ee}}^{\text{ee}}| < 0.09 \text{eV}$ (see Fig. 1) and we will neglect it in the following. Hence, if the total contribution saturates the limits from GERDA and GERDA+HDM+IGEX [28], the active-light sterile neutrino mixing $|U_{e4}^2|$ is bounded as follows

$$|U_{e4}^2| \lesssim \kappa_4 m_4 \left(1 - \frac{\mathcal{M}_{\nu,\text{Ge}} \mathcal{M}_{N,\text{Ge}}}{\mathcal{M}_{N,\text{Xe}} \mathcal{M}_{\nu,\text{Xe}}}\right).$$  \hspace{1cm} (21)
FIG. 6. Upper bounds of $|U_{\alpha e}|^2$ that saturate the limits from GERDA [28]. The different color coding corresponds to the NME uncertainty. See text for details. For comparison, we also show the different bounds from beta decay, solar and reactor experiments, peak search and beam dump experiment, first compiled in Ref. [101].

On the other hand, in order to explain the positive claim in Ref. [26] we need,

$$\kappa_1 \frac{1}{m_4} \left| \frac{1}{M_{136Xe}} - \frac{1}{M_{76Ge}} \right| \lesssim \frac{\kappa_2}{m_4} \left| \frac{1}{M_{136Xe}} - \frac{1}{M_{76Ge}} \right|,$$

(22)

In Fig. 6 we show the upper bound on the active-light sterile neutrino mixing angle $|U_{\alpha e}|^2$ corresponding to the individual (solid lines) and combined (dashed lines) limits of half-life for $^{76}$Ge from GERDA [28]. The area in the $|U_{\alpha e}| - m_4$ plane, that is above this line is excluded. The red, blue lines have been derived using the NMEs corresponding to the Argonne potential (intermediate and large size single-particle spaces, respectively) between two different nucleons and the purple and orange lines are using the NMEs corresponding to the CD-Bonn potential (intm and large, respectively). For the positive claim [26], the variation of the active sterile mixing with mass of the sterile neutrino is quite similar and hence we do not show it separately. In this case, the cancellation for $^{136}$Xe is operative mostly between the light sterile and heavy sterile neutrino contributions. For comparison we also show the other different bounds, first compiled in Ref. [101]. By comparing Fig. 6 with Fig. 2 it is evident that in the presence of cancellation, a larger mixing $U_{\alpha e}$ is required to give the same value of the half-life. Also, as compared to Fig. 2 in this case the bound on active-sterile mixing angle from $\pi \rightarrow e\nu$ can compete with the bound from $(\beta\beta)_{0\nu}$-decay.

As we are assuming a cancellation in $^{136}$Xe, the heavy sterile neutrino contribution is also constrained and a bound in the mass-mixing plane can be obtained. Using the cancellation relation $|\eta_1 M_{136Xe} = |\eta_N| M_{N,Xe}$ and the values of $\kappa, \kappa_{1,2}$ as given in Table I, the active-heavy sterile neutrino mixing angle $V_{eN}$ corresponding to the GERDA and GERDA+HDM+IGEX limits [28] is bounded as

$$|V_{eN}^2| \lesssim \kappa \frac{M_N}{m_e m_p} \frac{M_{136Xe}}{M_{N,Xe}} \left| \frac{1}{M_{N,Xe}} - \frac{1}{M_{76Ge}} \right|,$$

(23)
while for the positive claim [29], it is
\[
\kappa_1 \frac{M_N}{m_e m_p} \frac{M_{\nu,Xe}}{M_{N,Xe}} \left(1 - \frac{1}{M_{N,Xe} M_{N,Ge}}\right) \lesssim |V_{eN}| \lesssim \kappa_2 \frac{M_N}{m_e m_p} \frac{M_{\nu,Xe}}{M_{N,Xe}} \left(1 - \frac{1}{M_{N,Xe} M_{N,Ge}}\right).
\]

Note that the equations Eqs. 21, 22, 23 and 24 are only valid for the light and heavy sterile masses smaller and larger than the exchange momentum scale, |p| ≈ 100 MeV, respectively. We show the generic equation that is valid for all mass scales in the Appendix. Following Eq. 23 and the formalism given in Appendix, we show in Fig. 7 the upper bound on the active-heavy sterile mixing |V_{eN}| corresponding to the individual and combined limits of half-life from GERDA [28]. The description of the different color coding is the same as in Fig. 4. The region above the different contours is excluded by (ββ)_{0ν}-decay. In this figure, for comparison we also show the bounds coming from other experiments [101]. Again comparing Fig. 7 with Fig. 3 one can see a larger mixing angle V_{eN} required to saturate the limits on the half-life from (ββ)_{0ν}-decay for the case of cancellation. For the mass of the heavy sterile neutrino M_N ≈ O(100) MeV, the bound from the beam dump experiment PS191 [111] is even stronger than the (ββ)_{0ν}-decay one. In the range M_N ≈ (1−2) GeV, the bound from CHARM [116] can compete with the bound from (ββ)_{0ν}-decay. For the positive claim [29], the result is similar and we do not show the corresponding region explicitly.

V. CORRELATION BETWEEN HALF-LIVES

In this section, we extend our discussion of the effects of cancellations to other isotopes. To this aim, for definiteness, we investigate how the cancellation between active and sterile neutrinos in $^{136}$Xe would influence the half-life of other isotopes, such as $^{100}$Mo, $^{130}$Te and $^{82}$Se as well as $^{76}$Ge. The ratio of half-lives in two isotopes,
as discussed above, the cancellation between light and heavy contributions to neutrinoless double beta decay in one isotope requires very specific values of neutrino masses and mixing angles. In this section we discuss how such values can emerge from theoretical models. The most natural framework embedding sterile neutrinos is the Type-I seesaw mechanism. Typically, heavy sterile neutrinos are introduced at or just below the GUT scale leading to light mass values can emerge from theoretical models. The most natural framework embedding sterile neutrinos is the Type-I one isotope requires very specific values of neutrino masses and mixing angles. In this section we discuss how such

\[
\nonumber
\frac{T_{1/2}^{\nu\nu}(A)}{T_{1/2}^{\nu\nu}(B)} = \frac{G_{0e}^{B}(\mathcal{M}_{\nu,B} - \mathcal{M}_{\nu,Xe} \mathcal{M}_{N,Xe})^2}{G_{0e}^{A}(\mathcal{M}_{\nu,A} - \mathcal{M}_{\nu,Xe} \mathcal{M}_{N,Xe})^2}.
\]

Using Eq. 25, we show the correlations between half-lives of $^{76}\text{Ge}$-130$\text{Te}$, 82$\text{Se}$-130$\text{Te}$, 76$\text{Ge}$-100$\text{Mo}$ and 76$\text{Ge}$-82$\text{Se}$ in Figs. 8 and 9, respectively. We use different values of the NMEs which correspond to the various lines in the figures, as specified in the captions. The region within the two horizontal black dashed lines correspond to positive claim (90% C.L.) [26]. The horizontal black solid line corresponds to the individual limit from GERDA [28], where the region below this line is excluded. We also show the combined GERDA+HDM+IGEX limit [28] by the green horizontal line. The gray shaded region is disallowed by the results from GERDA [28] and CUORICINO experiments [33]. The red and blue lines correspond to the SRQRPA [84] and IBM-2 [117] NME calculations, while the different numerical values represent the effective mass of light neutrino exchange in eV. The red dot-dashed, dashed, solid and dotted lines correspond to the NMEs that have been derived using Argonne and CD-Bonn potential, respectively. Right Panel: Variation of half-life of $^{82}\text{Se}$ with the half-life of $^{130}\text{Te}$. The color coding is the same as for the left panel.

isotope A and isotope B, is

\[
\nonumber
|m_{ee}| = \frac{m_{ee}}{\sqrt{G_{0e}^{A}T_{1/2}^{\nu\nu}(A)\mathcal{M}_{\nu,A}^2}} = m_{ee}'(1 - \frac{\mathcal{M}_{\nu,Xe} \mathcal{M}_{N,A}}{\mathcal{M}_{N,Xe} \mathcal{M}_{\nu,A}}).
\]

and similarly for the isotope B. The different numerical values shown in the figures represent the required effective mass $|m_{ee}|$ in eV for a particular set of half-lives ($T_{1/2}^{\nu\nu}(A)$, $T_{1/2}^{\nu\nu}(B)$) of the two isotopes. Finally, we conclude this section by showing the individual prediction of half-lives of $^{130}\text{Te}$ and $^{100}\text{Mo}$ in Table II and Table III, respectively.

VI. MODEL-SEESAW REALIZATIONS

As discussed above, the cancellation between light and heavy contributions to neutrinoless double beta decay in one isotope requires very specific values of neutrino masses and mixing angles. In this section we discuss how such values can emerge from theoretical models. The most natural framework embedding sterile neutrinos is the Type-I seesaw mechanism. Typically, heavy sterile neutrinos are introduced at or just below the GUT scale leading to light
neutrino masses. If their mass is larger than few tens of TeV, the contribution in $(\beta\beta)_{\nu}$-decay would be negligibly small [41, 42, 44]. However, sterile neutrino can have much smaller masses, even well below the electroweak scale,

### FIG. 9

Left Panel: Variation of the half-life of $^{76}$Ge with the one of $^{100}$Mo, assuming a cancellation between light and heavy neutrino contributions in $^{136}$Xe. The region in between the horizontal dashed black lines corresponds to the positive claim (90% C.L.) [26]. The black solid line correspond to the lower limit of half-life of $^{76}$Ge from GERDA [28] and the region below this line is excluded. The combined bound GERDA+HDM+IGEX [28] is shown by the green horizontal line. The red and blue lines correspond to the SRQRPA [84] and IBM-2 [117] NME calculations, while the different numerical values represent the effective mass $m_{\nu}^e$ in eV. The red dot-dashed, dashed, solid and dotted lines correspond to the NMEs that have been derived using Argonne and CD-Bonn potential, respectively. The vertical gray region is the excluded region from NEMO 3 [32] experiment. Right Panel: Variation of half-life of $^{76}$Ge with the one of $^{82}$Se. The color coding is the same as for the left panel.

### TABLE II

Predictions of the half-life $T_{1/2}^{\nu}(130\mathrm{Te})$ that corresponds to the i) positive claim in $^{76}\mathrm{Ge}$: $T_{1/2}^{\nu} = 2.23^{+0.73}_{-0.53} \times 10^{25}$ (90% C.L.) [26]; ii) saturates the GERDA and iii) the GERDA+HDM+IGEX (Combined) limits $T_{1/2}^{\nu} > (2.1, 3.0) \times 10^{25}$yrs of half-life [28], while satisfying the limit of half-life from EXO-200, KamLAND-Zen [31, 32] as an artifact of cancellation between light and heavy states. We have used the NMEs from [84]. Following [100], the phase space factors that have been used are $G_{\nu}(^{76}\mathrm{Ge}) = 5.77 \times 10^{-12}$yrs, $G_{\nu}(^{136}\mathrm{Xe}) = 3.56 \times 10^{-14}$yrs and $G_{\nu}(^{130}\mathrm{Te}) = 3.47 \times 10^{-14}$yrs.

| NME | $T_{1/2}^{\nu}(130\mathrm{Te}) (10^{25} \mathrm{yr})$ |
|-----|---------------------------------|
| | Positive claim | GERDA | Combined |
| $M_{\nu}(^{76}\mathrm{Ge})$ | $M_{\nu}(^{136}\mathrm{Xe})$ | $M_{\nu}(^{130}\mathrm{Te})$ | $M_{\nu}(^{100}\mathrm{Mo})$ | $M_{\nu}(^{136}\mathrm{Xe})$ | $M_{\nu}(^{130}\mathrm{Te})$ | $M_{\nu}(^{100}\mathrm{Mo})$ |
| 4.75 | 232.8 | 163.5 | 234.1 | 0.82-1.40 | 0.997 | 1.42 |
| 5.44 | 264.9 | 159.7 | 239.7 | 80.2-137.97 | 98.0 | 140.0 |
| 5.11 | 351.1 | 166.7 | 364.3 | 0.10-0.18 | 0.13 | 0.18 |
| 5.82 | 411.5 | 172.1 | 384.5 | 0.18-0.31 | 0.22 | 0.31 |

### TABLE III

The same as Table II but for $^{100}$Mo. Following [100], the phase space factors that we have used are $G_{\nu}(^{76}\mathrm{Ge}) = 5.77 \times 10^{-15}$yrs, $G_{\nu}(^{136}\mathrm{Xe}) = 3.56 \times 10^{-14}$yrs and $G_{\nu}(^{100}\mathrm{Mo}) = 3.89 \times 10^{-14}$yrs.

| NME | $T_{1/2}^{\nu}(100\mathrm{Mo}) (10^{25} \mathrm{yr})$ |
|-----|---------------------------------|
| | Positive claim | GERDA | Combined |
| $M_{\nu}(^{76}\mathrm{Ge})$ | $M_{\nu}(^{136}\mathrm{Xe})$ | $M_{\nu}(^{130}\mathrm{Te})$ | $M_{\nu}(^{100}\mathrm{Mo})$ | $M_{\nu}(^{136}\mathrm{Xe})$ | $M_{\nu}(^{130}\mathrm{Te})$ | $M_{\nu}(^{100}\mathrm{Mo})$ |
| 4.75 | 232.8 | 163.5 | 249.8 | 0.71-1.23 | 0.87 | 1.24 |
| 5.44 | 264.9 | 159.7 | 259.7 | 1.95-3.35 | 2.38 | 3.40 |
| 5.11 | 351.1 | 166.7 | 388.4 | 0.07-0.13 | 0.09 | 0.13 |
| 5.82 | 411.5 | 172.1 | 404.3 | 0.17-0.29 | 0.20 | 0.29 |
e.g. in low energy see-saw models \cite{119,120}. A lot of attention has been recently devoted to sterile neutrino states with masses lighter than TeV scale in $(\beta\beta)_{0\nu}$-decay in Refs. \cite{111,122,144,146}. Below we discuss specific models which can accommodate light as well as heavy sterile neutrinos and lead to the cancellations we are interested in.

\section{Model A - Light Active and Heavy Sterile Neutrinos}

We consider first the case in which all sterile neutrinos are heavy, having masses larger than the momentum exchange scale $|p| \sim 100$ MeV, see Section \ref{4.1}. We consider $n$ generations of sterile neutrinos $(\hat{N}_i, \hat{N}'_i)$ denoted in the flavor basis. In the $(\nu, \hat{N}, \hat{N}')$ basis, the mass matrix of active+sterile neutrinos has the following form

$$
M_n = \begin{pmatrix}
0 & \alpha_T & m_D^T \\
\alpha & \mu & m_S^T \\
m_D & m_S & m_R
\end{pmatrix},
$$

(27)

where $\mu$ and $m_R$ are two lepton number violating parameters \footnote{Depending on the choice of the lepton number assignment for the $\hat{N}, \hat{N}'$ fields, different parameters in the mass matrix will be lepton number violating. Here, we adopt a common choice in which $m_D, m_S$ and $m_R$ are large masses and $\mu$ is very small.}. Particularly interesting phenomenology arises for the hierarchy $m_R > m_S > m_D \gg \mu, \alpha$ and $m_S^2 / m_R \gg \mu, \alpha$ which will lead to the Extended seesaw scenario \cite{51,52}. We denote the mass basis as $(\nu_m, N, N')$. The mass of the sterile neutrinos $N, N'$ are obtained by diagonalizing Eq. (27) and are given by

$$
m_N \simeq -m_S^T m_R^{-1} m_S,
$$

(28)

$$
m_{N'} \simeq m_R.
$$

(29)

Let us note that for simplicity we call sterile neutrinos both the flavor states and the massive states which are mainly in the sterile neutrino direction. From the inequality $m_R > m_S$ it follows that $m_{N'} > m_N$. In the following discussion we consider the simplest case in which $\alpha$ is negligibly small. The mass matrix of the active neutrino depends on the small lepton number violating parameter $\mu$ and is

$$
m_{\nu} \simeq m_D^T (m_S^T)^{-1} \mu (m_S)^{-1} m_D.
$$

(30)

Depending on the the values of $m_S / m_R$ and $\mu$, light neutrino of eV mass can be obtained. The mixings of sterile neutrinos $N$ and $N'$ with active neutrinos are

$$
U_{eN} \simeq (m_D^T m_S^T m_S^{-1})_{eN},
$$

(31)

$$
U_{eN'} \simeq (m_D^T m_R^{-1})_{eN'}.
$$

(32)

Note that, while the light neutrino mass depends on the lepton number violating parameter $\mu$, the active-sterile neutrino mixing is independent of this parameter to leading order. Hence, in this case, one can have large active-sterile neutrino mixing while neutrino masses are kept small thanks to the $\mu$ parameter.

This seesaw scenario has been explored previously in \cite{51,52}. In this work we are interested to study if the heavy neutrinos in this model can satisfy the cancellation conditions and give large contributions in $(\beta\beta)_{0\nu}$-decay, thanks to not-too large masses and large mixings \cite{122}. For simplicity, we drop the active and sterile neutrino indexes and focus on the order of magnitude of the parameters of the mass matrix. Interesting flavor effects could be present but they are beyond the scope of the present analysis. For heavy sterile neutrinos of masses $m_N, m_{N'} \gg 100$ MeV, the amplitudes corresponding to the $N$ and $N'$ contributions to $(\beta\beta)_{0\nu}$-decay are \cite{122,144}

$$
|A_N| = \frac{|U_{eN}^2|}{m_N} \sim \frac{m_D^2}{m_S m_N},
$$

(33)

$$
|A_{N'}| = \frac{|U_{eN'}^2|}{m_{N'}} \sim \frac{m_D^2}{m_R m_{N'}}.
$$

(34)
FIG. 10. Variation of \( m_S \) and the mass \( m_N \) of the sterile neutrino \( N \) vs \( \mu \) for \( m_R = 3 \times 10^6 \text{ GeV} \) and \( m_D = 0.1 \text{ GeV} \). The gray shaded region is disallowed from GERDA experiment. The brown line corresponds to the cancellation condition between the light active and heavy sterile neutrinos. The red and blue lines correspond to the half-life of GERDA and GERDA+HDM+IGEX limits [28].

As \( m_R \gg m_S \), the role of \( N' \) is suppressed both by the large mass and small mixing and we neglect it with respect to \( N \) in the following discussion.

Including the contributions from light neutrinos and the sterile state \( N \), the half-life of neutrinoless double beta decay for a particular isotope \( A \) is given by

\[
\frac{1}{T_{1/2}^{\text{nu}}(A)} \approx G_{\nu \nu}^A \left| \frac{\mu_{\nu}}{m_\nu} \right|^2 \left( \frac{m_{\nu_e}^2}{m_\nu^2} M_{\nu,A} + \frac{m_p^2}{m_S^2 m_N} \right) \left( M_{N,A}^2 + 2 \cos \alpha \left| \frac{m_\nu}{m_\nu} \right| \frac{m_p^2 m_p}{m_S^2 m_N} M_{\nu,A} M_{N,A} \right) .
\]

Again, for definiteness, we consider the case in which a cancellation is operative for \(^{136}\text{Xe} \) isotope and find its implication for \(^{76}\text{Ge} \). Using the cancellation condition \( |\eta_\nu| M_{\nu,A} = |\eta_{N,e}| M_{N,Xe} \), we get the following relation between different parameters

\[
\frac{\mu}{m_\nu} \left( \frac{m_D}{m_S} \right)^2 M_{\nu,Xe} = \left( \frac{m_D}{m_S} \right)^2 \frac{m_p m_R}{m_S^2} M_{N,Xe} ,
\]

which simplifies to

\[
\mu = \left( \frac{m_R}{m_S} \right) \left( \frac{M_{N,Xe}}{M_{\nu,Xe}} \right) m_{\nu} m_p .
\]

Using the definition \( p_{Xe}^2 \equiv -m_e m_p \left( \frac{M_{N,Xe}}{M_{\nu,Xe}} \right) \), we get \( \mu = \left( \frac{m_R}{m_S} \right) |p_{Xe}| \approx |p_{Xe}|/m_N \). We recall that \( m_N \gg \sqrt{|p_{Xe}|} \), implying that \( \mu \ll m_N \) in agreement with the original assumption of the hierarchy of the neutrino mass parameters. Taking the typical range for \( m_N \) given by \( 100 \text{ MeV} \text{–} 10^6 \text{ GeV} \), we find that the \( \mu \) parameter will be typically small, \( \mu \sim 0.1 \text{–} 10^{-8} \text{ GeV} \), as originally assumed.

Using the above [35] and Eqs. [37], we can express the lepton number violating parameter \( \mu \) as a function of the half-life time of \(^{76}\text{Ge} \)

\[
\mu = \frac{m_S^2}{m_D^2} \sqrt{G_{\nu \nu}^A T_{1/2}^{\text{nu}}(^{76}\text{Ge})} \left( \frac{1}{M_{\nu,Ge} - \frac{M_{N,Xe}}{M_{N,Xe}} M_{N,Ge}} \right) .
\]

Below we discuss the different constraints on the parameters of the model, that satisfy the cancellation in \(^{136}\text{Xe} \) and the lower limit of half-life from GERDA [28]. In the left panel of Fig. 10 the blue and red lines represent the variation of \( m_S \) with the lepton number violating parameter \( \mu \) for representative values of the parameters,
Corresponding variation of the physical mass has been computed in Ref. [44]. For the positive claim [26], the variation of \( m_\nu \) is similar and we do not show it explicitly. In the right panel of Fig. 10, we show the corresponding variation of the physical mass \( m_N \) of the sterile neutrino \( \nu \). The intersection of the blue, red line and brown line represents the point in the parameter space where the active neutrino, together with the heavy sterile neutrinos, can simultaneously saturate the bounds from GERDA and GERDA+HDM+IGEX [28], as well as the bound from EXO-200, KamLAND-Zen and the combined one [31, 32]. The light neutrino mass in the intersection region is \( m_\nu \sim 0.66 \text{ eV} \).

In principle one can also consider the limit \( \mu \to 0 \). Although light neutrinos would be massless at tree level, a nonzero mass is generated at loop level. The finite one loop correction to the light neutrino mass for this model has been computed in Ref. [44].

\[
\delta m_\nu = \frac{1}{2(4\pi)^2 \alpha} \left\{ \left( \frac{3m_N \ln \left( m_N^2/M_Z^2 \right)}{m_N^2/M_Z^2 - 1} + \frac{m_N \ln \left( m_N^2/M_H^2 \right)}{m_N^2/M_H^2 - 1} \right) \cos^2 \theta \right. \\
+ \left. \left( \frac{3m_{N'} \ln \left( m_{N'}^2/M_Z^2 \right)}{m_{N'}^2/M_Z^2 - 1} + \frac{m_{N'} \ln \left( m_{N'}^2/M_H^2 \right)}{m_{N'}^2/M_H^2 - 1} \right) \sin^2 \theta \right\}, \tag{39}
\]

where \( \theta \) is the mixing angle between the sterile states \( N \) and \( N' \).

\[
\tan \theta = \frac{m_R - \mu + \sqrt{4m_S^2 + (m_R - \mu)^2}}{2m_S}. \tag{40}
\]

For \( \mu = 0 \), the cancellation between light active and heavy sterile state leads to the following relation

\[
\delta m_\nu = \frac{m_R^2 m_p}{m_S m_N} M_{\nu,Xe}. \tag{41}
\]

Using Eq. [16] and the NMEs given above, \( \delta m_\nu \sim 0.66 \text{ eV} \) is required to satisfy the individual limit from GERDA [28]. As for the previous case, it is possible to identify the range of parameters that satisfy the cancellation condition and the constraints from GERDA [28]. Here, we present a simple numerical example: \( m_R = 10^8 \text{ GeV} \), while \( m_D = 0.75 \text{ GeV}, m_S = 6.73 \times 10^{13} \text{ GeV} \). In this case, the mass of the two sterile neutrinos are \( m_N \sim 0.45 \text{ GeV} \) and \( m_{N'} \sim 10^8 \text{ GeV} \). For this choice of parameters, both the cancellation condition Eq. [41] and the required value of light neutrino mass \( \delta m_\nu = 0.66 \text{ eV} \) to saturate the limit from GERDA [28] can be achieved. Similar discussion holds for the positive claim [26].

### B. Model B—Light and Heavy Sterile Neutrino

We discuss the simplest seesaw realization which can accommodate one light and one heavy sterile neutrinos and the cancellation in \( (\beta \beta)_{0\nu} \)-decay, corresponding to the discussion of Section IV B. This can be achieved using the mass matrix presented in Eq. [27] of the previous section with the addition of a Type-II seesaw mass term of light neutrinos. For simplicity we consider that the Majorana mass matrix of the sterile neutrinos is diagonal. We denote the sterile neutrinos as \( \tilde{N} \) and \( \tilde{N}' \) in this basis. The neutrino mass matrix in this basis is

\[
M = \begin{pmatrix}
    m_\Delta & \hat{\alpha} & \hat{m}_D \\
    \hat{\alpha} & \mu & 0 \\
    \hat{m}_D & 0 & m_R
\end{pmatrix}. \tag{42}
\]

For the sterile neutrino masses \( m_R, \mu > \hat{\alpha}, \hat{m}_D, m_\Delta \), the light neutrino mass term and its mixing with the sterile neutrino are

\[
m_\nu = m_\Delta - \frac{\hat{\alpha}^2}{\mu} - \frac{\hat{m}_D^2}{m_R}. \tag{43}
\]
and
\[ \nu \sim \nu_m + \frac{\tilde{\alpha}}{\mu} N + \frac{\tilde{m}_D}{m_R} N'. \] (44)

The other two sterile neutrino masses are \( \mu \) and \( m_{\mu} \), respectively. For \( \mu \) and \( m_R \) to be smaller and larger than the momentum exchange scale, the total contribution in \((\beta\beta)_{0\nu}\)-decay is
\[ \frac{1}{T_{1/2}^{0\nu}} = G_{0\nu} \left| (m_{\nu} + \frac{\tilde{\alpha}^2}{\mu^2} \mu) \frac{M_N}{m_e} - \frac{\tilde{m}_D^2}{m_R^2} \frac{m_{\mu}}{m_R} \right|^2. \] (45)

To discuss the simplified constraints on the parameter space, we consider the case where \( m_{\nu} \ll \frac{\tilde{\alpha}^2}{\mu} \), which requires additional fine tuning between the different terms in Eq. (43). If this is the case, then the cancellation between the light sterile and heavy sterile contribution for \(^{136}\text{Xe}\) gives the following condition
\[ \tilde{m}_D^2 = \tilde{\alpha}^2 \frac{m_{\mu}^3}{\mu} \left( \frac{M_{\nu,Xe}}{M_{N,Xe}} \right) \frac{1}{m_{\mu} m_e}. \] (46)

In addition, we consider that the light and heavy sterile contribution satisfy the bounds for \(^{76}\text{Ge}\) or the positive claim \( [26] \). Using Eq. (45), we get
\[ \tilde{\alpha}^2 = \frac{\mu m_e}{\sqrt{(M_{\nu,Ge} - \frac{M_{\mu,Xe}}{M_{N,Xe}})^2} \frac{1}{T_{1/2}^{0\nu}(76\text{Ge})G_{0\nu}^{Ge}}}. \] (47)

In this case, the bound from \(^{136}\text{Xe}\) is possible to escape. As an example we consider the sterile neutrino masses \( \mu = 0.01 \text{ GeV} \) and \( m_R = 1 \text{ GeV} \) and the different nuclear matrix elements given in the previous section. For the choice of parameters \( \tilde{\alpha} = 2.54 \times 10^{-6} \text{ GeV} \) and \( \tilde{m}_D = 1.61 \times 10^{-3} \text{ GeV} \), the positive claim \( [26] \) as well as the cancellation condition are possible to satisfied. In this case, the sterile neutrino contribution to the neutrino mass matrix would be \( 26.85 \text{ eV} \). Hence, the amount of fine tuning that is required in this case to obtain \( m_{\nu} \sim 0.1 \text{ eV} \) is of similar order, which can be achieved by adjusting \( m_\Delta \). Note that, the contribution from Higgs triplet in \((\beta\beta)_{0\nu}\)-decay depends on the Higgs triplet mass. For very large mass of Higgs triplet, their contribution will be negligibly small and can safely be avoided.

VII. CONCLUSION

In light of recent experimental results, in this work we have carefully analyzed the effect of interference in \((\beta\beta)_{0\nu}\)-decay. Most studies assume that the light Majorana neutrino exchange is the dominant mechanism mediating this process. However, any beyond-the-standard-model framework, which is required to generate light Majorana neutrino masses, will also induce neutrinoless double beta decay directly due to its lepton number violating parameters and could give a relevant contribution. If the different contributions are sizable, they can interfere either constructively or destructively. For definiteness, we consider the case of heavy sterile neutrinos with masses larger than the momentum exchange, \(|p| \sim 100 \text{ MeV} \), and light sterile neutrinos. If their masses are smaller than TeV scale and if their mixings with the electron neutrinos are sizable, they can saturate the current bounds of half-life \( [42] \).

If a complete cancellation is at work, the half-life of \(^{136}\text{Xe}\) is infinite and any constraint on it would be automatically satisfied, independently from the results for other isotopes. Due to the different nuclear matrix elements, only a partial interference will be present for other nuclei. As an example, motivated by the not-yet-completely-excluded claim of \((\beta\beta)_{0\nu}\)-decay in \(^{76}\text{Ge}\), we have studied the predictions in detail for the half-life in this isotope and the correlations with other nuclei.

A large value of the effective mass \( m_{\nu}' \sim O(0.1) \text{ eV} \) is required to satisfy the positive claim \( [26] \) or to saturate the current bounds from \((\beta\beta)_{0\nu}\)-decay experiments. For the case in which only three light active neutrinos are present, their masses are required to be in the quasidegenerate regime. However, this possibility is strongly constrained by the stringent bounds from cosmology. If the cancellation between light and heavy neutrino exchange is at work, the redefined effective Majorana mass gets suppressed. Or in other words, a larger value of the true effective mass
The redefined effective mass $(\nu_{ee})$ (0.66 eV-1.67 eV will saturate GERDA for SRQRPA calculation) is required to have the same half-life. Hence, bigger values of neutrino masses are needed. As a result, if the redefined effective mass saturates the limits from $(\beta\beta)_{0
u}$-decay, the tension with cosmological data becomes even more severe. In the next few years, quasi-degenerate values of neutrino masses will be tested by the $\beta$-decay experiment KATRIN \cite{88}, providing additional constraints on this possibility.

The tension with cosmology can be weakened if we also consider light sterile neutrinos. Depending on the mass and mixing, light sterile neutrinos can give a large contribution in $(\beta\beta)_{0
u}$-decay, and can even saturate current limits. On the other hand, the bounds from cosmology are relevant only if their masses are in the eV range for the values of mixing angles of interest. Neutrinoless double beta decay turns out to be the most sensitive probe of these sterile neutrinos. For masses in the range 10 eV-100 KeV, the bounds from beta-decay experiments are weaker than that of $(\beta\beta)_{0
u}$-decay by a factor of $U^2_{e3} \sim \mathcal{O}(10 - 100)$. For sterile neutrinos of $\mathcal{O}(100)$ MeV masses, the constraints from the peak search in $\pi \to e\nu$ and the beam dump experiment PS191 reach similar sensitivity as $(\beta\beta)_{0
u}$-decay. In the presence of cancellations, a larger value of active-sterile mixing angle is required to obtain the same value of half-life. Hence, the bounds from experiments, such as beta decays, $\pi \to e\nu$, PS191 and even CHARM become competitive with $(\beta\beta)_{0
u}$-decay, making it easier to test the parameters required for a cancellation.

A direct test of destructive interference, being at work in a certain nuclei, will be given by the measurement of the half-life in several isotopes \cite{84-87}. In the case under study, the cancellation between light active/sterile and heavy sterile neutrino exchange in $^{136}\text{Xe}$ will lead to a definite prediction of the half-lives of other isotopes. If we take $m_{\nu_ee} \sim (0.5 - 1)$ eV, depending on the choice of NME, the predicted half-life in $^{130}\text{Te}$, $^{100}\text{Mo}$, $^{82}\text{Se}$ can vary over a wide range and may be constrained by CUORICINO \cite{88} and NEMO 3 \cite{84}. However, if we consider smaller $m_{\nu_ee}$, more sensitive experiments are needed and the searches for $(\beta\beta)_{0
u}$-decay will be even more challenging than in the case of light neutrino mass only.

The existence of heavy and/or light sterile neutrinos can be easily implemented in seesaw scenarios, such as Type-I, Extended or Type-I+Type-II seesaw, in which the cancellation between light and heavy neutrino exchanges can be realized. In these models light neutrino masses can be generated either at tree or loop level. An Extended seesaw scenario allows for sterile neutrino in the 100 MeV mass range while having sufficiently large mixing angles with electron neutrinos and a cancellation between light active and heavy sterile neutrino contribution. The case in which both light and heavy sterile neutrinos are at play can be realized in a further extension of the model above in which a light neutrino mass come from a Type-II seesaw framework. In all of the cases, very precise values of masses and mixings are needed to induce a cancellation and require a high level of fine-tuning.

Acknowledgements

The authors acknowledge the partial support of the ITN INVISIBLES (Marie Curie Actions, PITN-GA-2011-289442). SP thanks SISSA for hospitality, where part of this study has been conducted. MM thanks ICTP for hospitality. The authors thank S. Petcov for discussions in the initial stage of this work.
Appendix

We consider \( n_i \) generations of light sterile neutrinos of masses \( m_{4k} \) \((k = 1, 2, \ldots, n_i)\) and \( n_h \) generations of heavy sterile neutrinos of masses \( M_{N_j} \) \((j = 1, 2, \ldots, n_h)\). The active-light and active-heavy sterile mixings are \( U_{e4k} \) and \( V_{eN_j} \), respectively. The half-life of neutrinoless double beta decay is

\[
\frac{1}{T_{1/2}^{0\nu}} = G_{0\nu} \left| \mathcal{M}_\nu \eta_\nu + \mathcal{M}_N \eta_N \right|^2, \tag{48}
\]

where \( G_{0\nu} \) is the phase space factor, \( \mathcal{M}_\nu \) and \( \mathcal{M}_N \) are the nuclear matrix elements corresponding to the light and heavy neutrino exchange. In the limit when the mass of the sterile neutrinos are far from the intermediate momentum exchange, i.e. \( m_{4k}^2 << |p^2| \text{ MeV}^2 \) and \( M_{N_j}^2 >> |p^2| \text{ MeV}^2 \), the factors \( \eta_\nu \) and \( \eta_N \) are \( \eta_\nu = \eta_N = \frac{\Sigma_i m_i U_{ei}^2 + \Sigma_k m_{4k} U_{e4k}^2}{m_{4k}} \)

and \( \eta_N = \sum_j \frac{V_{eN_j}^2 m_{4j}}{M_{N_j}} \), where we include the contributions from light active, light sterile as well as heavy sterile neutrinos. An equivalent way of description is

\[
\frac{1}{T_{1/2}^{0\nu}} = K_{0\nu} \left| \frac{m_i U_{ei} + m_{4k} U_{e4k}}{p^2} \right|^2 \tag{49}
\]

where \( \theta \) is the mixing angle and \( m \) is the mass of corresponding neutrino state. Following this, we have the generic expression,

\[
\frac{1}{T_{1/2}^{0\nu}} = K_{0\nu} \left| \frac{U_{e4k}^2 m_i}{p^2} + \frac{U_{e4k}^2 m_{4k}}{p^2 - m_{4k}^2} + \frac{V_{eN_j}^2 M_{N_j}}{p^2 - M_{N_j}^2} \right|^2. \tag{50}
\]

In the limit that light and heavy sterile neutrinos have masses far from momentum exchange scale, one will obtain Eq. \((49)\) For concreteness, we consider the case of one light sterile neutrino and one heavy sterile neutrino with masses \( m_4 \) and \( M_N \), respectively. If the light and heavy neutrino contributions cancel each other in isotope A, then the half-life \( T_{1/2}^{0\nu}(A) \) is infinite and we have

\[
\frac{|V_{eN}^2 M_N|}{|p_A^2| + M_N^2} = \left| \frac{U_{e4}^2 m_i}{|p_A^2|} + \frac{U_{e4}^2 m_4}{|p_A^2 + m_4^2|} \right| \tag{52}
\]

The expression simplifies considerably, if we neglect the three light active neutrino contribution. In addition, if light sterile and heavy sterile neutrino contribution saturate the bound or claimed value of half-life \( T_{1/2}^{0\nu}(B) \) of any other isotope B, then the contour of active-light sterile neutrino mixing is

\[
|U_{e4}^2|^2 = \frac{m_i^2}{K_{0\nu}^B T_{1/2}^{0\nu}(B)} \left( \frac{1}{|p_A^2| + m_4^2} - \frac{1}{|p_A^2| + M_N^2} \right)^2. \tag{53}
\]

Using Eq. \((52)\), the contours for active-heavy sterile neutrino mixing can be obtained:

\[
|V_{eN}^2|^2 = \frac{M_N^2}{K_{0\nu}^B T_{1/2}^{0\nu}(B)} \left( \frac{1}{|p_A^2| + M_N^2} - \frac{1}{|p_A^2| + M_N^2} \right)^2. \tag{54}
\]
These generic equations can be applied for e.g. $^{136}$Xe, $^{76}$Ge, or for any other isotopes.

[1] S. M. Bilenky, S. T. Petcov, Rev. Mod. Phys. 59, 671 (1987).
[2] S. P. Mikheyev, A. Y. Smirnov, Prog. Part. Nucl. Phys. 23 (1989) 41-136.
[3] G. Gelmini, E. Roulet, Rept. Prog. Phys. 58, 1207-1266 (1995) hep-ph/9412278.
[4] M. C. Gonzalez-Garcia, Y. Nir, Rev. Mod. Phys. 75, 345-402 (2003) hep-ph/0202058.
[5] R. N. Mohapatra, A. Y. Smirnov, Ann. Rev. Nucl. Part. Sci. 56 (2006) 569-628 hep-ph/0603118.
[6] A. Strumia and F. Vissani, arXiv:hep-ph/0606054.
[7] B. T. Cleveland et al., Astrophys. J. 496, 505 (1998); J. N. Abdurashitov et al. [SAGE Collaboration], Zh. Eksp. Teor. Fiz. 122, 211 (2002) [J. Exp. Theor. Phys. 95, 181 (2002)]; W. Hampel et al. [GALLEX Collaboration], Phys. Lett. B 447, 127 (1999); S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Lett. B 539, 179 (2002); B. Aharmim et al. [SNO Collaboration], Phys. Rev. C 72, 055502 (2005); B. Collaboration, arXiv:0708.2251 [astro-ph]; B. Aharmim et al. [SNO Collaboration], Phys. Rev. C81, 055504 (2010) arXiv:0910.2984 [nucl-ex].
[8] K. Eguchi et al., [KamLAND Collaboration], Phys.Rev.Lett. 90 (2003) 021802; T. Araki et al. [KamLAND Collaboration], Phys. Rev. Lett. 94, 081801 (2005); A. Gando et al. [The KamLAND Collaboration], Phys. Rev. D 83, 035002 (2011) arXiv:1009.4771 [hep-ex].
[9] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81, 1562-1567 (1998) hep-ex/9807003; Y. Ashie et al. [Super-Kamiokande Collaboration], Phys. Rev. D 71, 112005 (2005) arXiv:hep-ex/0501064; R. Wendell et al. [Kamiokande Collaboration], Phys. Rev. D81, 092004 (2010) arXiv:1002.3471 [hep-ex].
[10] E. Aliu et al. [K2K Collaboration], Phys. Rev. Lett. 94, 081802 (2005) hep-ex/0411038; M. H. Ahn et al. [K2K Collaboration], Phys. Rev. D74, 072003 (2006) hep-ex/0606032.
[11] K. Abe et al. [T2K Collaboration], Phys. Rev. Lett. 107, 041801 (2011) arXiv:1106.2822 [hep-ex].
[12] P. Adamson et al. [The MINOS Collaboration], Phys. Rev. Lett. 106, 181801 (2011) arXiv:1103.0340 [hep-ex].
[13] M. Apollonio et al., Eur. Phys. J. C 27, 331 (2003).
[14] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 6, 122 (2004); S. Goswami, A. Bandyopadhyay, S. Choubey, Nucl. Phys. Proc. Suppl. 143, 121-128 (2005) hep-ph/0409224; A. Bandyopadhyay, S. Choubey, S. Goswami, S. T. Petcov and D. P. Roy, Phys. Lett. B 608, 115 (2005); G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A. M. Rotunno, Phys. Rev. Lett. 101, 141801 (2008) arXiv:0806.2649 [hep-ph]; M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, JHEP 1004, 056 (2010) arXiv:1001.4524 [hep-ex].
[15] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, JHEP 1212, 123 (2012) arXiv:1209.3023 [hep-ph].
[16] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, Phys. Rev. D 84, 053007 (2011) arXiv:1106.6028 [hep-ph].
[17] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, Phys. Rev. D 86, 013012 (2012) arXiv:1205.5254 [hep-ph].
[18] J. K. Ahn et al. [RENO Collaboration], Phys. Rev. Lett. 108, 191802 (2012) arXiv:1204.0626 [hep-ex].
[19] F. P. An et al. [DAYA-BAY Collaboration], Phys. Rev. Lett. 108, 171803 (2012) arXiv:1203.1699 [hep-ex]; F. P. An et al. [Daya Bay Collaboration], Chin. Phys. C 37, 011001 (2013) arXiv:1210.6327 [hep-ex].
[20] Y. Abe et al. [Double Chooz Collaboration], Phys. Lett. B 723, 66 (2013) arXiv:1301.2948 [hep-ex].
[21] O. Cremonesi, Nucl. Phys. Proc. Suppl. 118, 287-296 (2003), Int. J. Mod. Phys. A21, 1887-1900 (2006) and arXiv:1002.1437 [hep-ex].
[22] S. R. Elliott, F. Vogel, Ann. Rev. Nucl. Part. Sci. 52, 115-151 (2002) hep-ph/0202264; P. Vogel, hep-ph/0612143.
[23] S. M. Bilenky, Phys. Part. Nucl. 41, 690-715 (2010) arXiv:1001.1946 [hep-ph].
[24] W. Rodejohann, Int. J. Mod. Phys. E 20, 1833 (2011) arXiv:1106.1334 [hep-ph]; W. Rodejohann, J. Phys. G 39, 124008 (2012) arXiv:1206.2560 [hep-ex].
[25] H. V. Klapdor-Kleingrothaus, I. V. Krivosheina, A. Dietz and O. Chkvorets, Phys. Lett. B 586, 198 (2004).
[26] H. V. Klapdor-Kleingrothaus, I. V. Krivosheina, Mod. Phys. Lett. A21, 1547-1566 (2006).
[27] I. Abt, M. F. Altmann, A. Bakalyarov, I. Barabanov, C. Bauer, E. Bellotti, S. T. Belyaev, L. B. Bezrukov et al., hep-ex/0404039; J. Schonert et al. [GERDA Collaboration], Nucl. Phys. Proc. Suppl. 145, 242-245 (2005).
[28] M. Agostini et al. [GERDA Collaboration], Phys. Rev. Lett. 111, 122503 (2013) arXiv:1307.4720 [nucl-ex].
[29] H. V. Klapdor-Kleingrothaus, A. Dietz, L. Baudis, G. Heusser, I. V. Krivosheina, S. Kolb, B. Majorovits, H. Pas et al., Eur. Phys. J. A12 (2001) 147-154 hep-ph/0103036.
[30] C. E. Aalseth et al. [IGEX Collaboration], Phys. Rev. D 65, 092007 (2002) hep-ex/0202026; C. E. Aalseth, F. T. Avignone, R. L. Brodzinski, S. Cebrian, E. Garcia, D. Gonzales, W. K. Hensley and I. G. Istratorza et al., Phys. Rev. D 70, 078302 (2004) nucl-ex/0404036.
[31] A. Gando et al. [KamLAND-Zen Collaboration], Phys. Rev. Lett. 110, 062502 (2013).
[115] J. Badier et al. [NA3 Collaboration], Z. Phys. C 31, 21 (1986).
[116] F. Bergsma et al. [CHARM Collaboration], Phys. Lett. B 166, 473 (1986).
[117] J. Barea, J. Kotila and F. Iachello, Phys. Rev. Lett. 109, 042501 (2012).
[118] A. A. Smolnikov [GERDA Collaboration], arXiv:0812.4194 [nucl-ex].
[119] P. S. B. Dev and A. Pilaftsis, Phys. Rev. D 86, 113001 (2012) [arXiv:1209.4051 [hep-ph]]; P. S. B. Dev and A. Pilaftsis, Phys. Rev. D 87, 053007 (2013) [arXiv:1212.3808 [hep-ph]].
[120] X. -G. He and W. Liao, arXiv:1309.7581 [hep-ph]; H. Zhang, Phys. Lett. B 714, 262 (2012) arXiv:1110.6838 [hep-ph].