Exploring the order parameter symmetry of $p$-wave Fermi condensates

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We discuss the time-of-flight expansion of dilute $p$-wave Fermi condensates on the BEC side of Feshbach resonances, as a way to extract information about the order parameter symmetry for superfluidity. We show that the cloud profile is in general sensitive to the interaction strength between fermions, the magnitude and direction of external magnetic fields, and to the angular momentum projection of the order parameter. In particular, due to the anisotropic nature of $p$-wave interactions we show that the time-of-flight expansion of a $p$-wave superfluid is anisotropic even if the superfluid is confined to a completely isotropic trap, unlike the case of Bose or $s$-wave Fermi condensates, which under the same circumstances expand isotropically. Furthermore, we demonstrate that expanding $p$-wave superfluids released from axially symmetric traps experience anisotropy inversions, where the aspect ratio between the axial and radial directions change during expansion, while the axial symmetry can also be lost reflecting the spatial anisotropy of the underlying interaction.

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One of the next frontiers of research in ultra-cold atoms is the search for superfluidity in higher angular momentum channels, which are quite rare in nature. With the exception of liquid $^3$He, there are no known neutral superfluids with angular momentum in the $p$-wave channel or higher. In addition, there are very few known examples of $p$-wave and $d$-wave superconductors (charged superfluids) in standard condensed matter physics, where the degree of control of the interactions is not existent. In contrast, for ultra-cold atoms, interactions in higher angular momentum channels can be tuned via the appropriate choice of Feshbach resonances.

In general, a key point for the understanding of superfluidity is the determination of the order parameter symmetry, and some ingenuity is usually necessary to extract this information experimentally in standard condensed matter systems. However, for ultra-cold atoms, time-of-flight measurements can provide valuable information about their superfluidity and the corresponding order parameter symmetry, as discussed below.

Time-of-flight experiments are powerful methods to study ultra-cold atoms, and have been applied to confirm the existence of Bose-Einstein condensation (BEC) and to explore superfluid dynamics of Bose systems [1] [2] [3] [4] [5] [6]. Furthermore, this type of measurements were used to probe signatures for superfluidity of fermions with $s$-wave interactions [3] [4] [5].

Time-of-flight techniques were also used to detect $p$-wave Feshbach resonances of ultra-cold fermions in harmonic traps [2] [8] [9] [10] and in optical lattices [11], and paved the way for the exploration of $p$-wave Fermi superfluidity, where fermions are paired in the angular momentum channel $\ell = 1$. Using these observed resonances, $p$-wave molecules have been produced in the laboratory [12] [13] [14] for fermion isotopes $^{40}$K and $^6$Li, bringing the experimental community a step closer to realizing $p$-wave Fermi superfluidity of ultra-cold atoms. In addition, other Fermi atoms like $^{173}$Yb have also been cooled to quantum degeneracy [15], and are likely to possess $p$-wave Feshbach resonances, thus permitting the study of $p$-wave molecules or superfluids in non-alkali fermions.

In contrast to their $s$-wave counterparts, $p$-wave Feshbach resonances have characteristic features including splitting of peaks depending on hyperfine (pseudospin) states as in $^6$Li and $^{40}$K (i.e., $|11\rangle$, $|12\rangle + |21\rangle$, and $|22\rangle$) [2] [8] [9] [10] [11], and splitting of peaks depending on angular momentum projections as in $^{40}$K (i.e., the magnetic quantum number $m_\ell = 0$ or $\pm 1$) [8] [11]. Hence, these splitting may allow the separate tuning of $p$-wave scattering parameters in different pseudospin and/or $m_\ell$ channels, such that $p$-wave interactions may be anisotropic in both pseudospin and angular momentum states.

Theoretical studies of $p$-wave systems focusing on the evolution from Bardeen-Cooper-Schrieffer (BCS) to BEC superfluidity have emphasized the existence of quantum phase transitions [16] [17]. However, these works dealt only with thermodynamic properties, while here we study the time-of-flight expansion of ultra-cold fermions with $p$-wave interactions and focus on the BEC side of Feshbach resonances, where the critical temperatures are higher. Our main conclusions are as follows. We show that the cloud profile reveals the anisotropic nature of the $p$-wave interaction and provide valuable information about the order parameter for $p$-wave superfluidity. In particular, we find that a completely isotropic cloud experiences anisotropic expansion driven by $p$-wave interactions, in contrast to the isotropic expansion of Bose and $s$-wave Fermi systems under the same circumstances. For a cigar-shaped cloud, we find that the aspect ratio...
between the axial and radial directions of a p-wave Fermi condensate undergoes an anisotropy inversion driven by the anisotropy of the harmonic potential, like in Bose and s-wave Fermi systems. However, the axial symmetry of p-wave Fermi condensates may be broken during the expansion reflecting the spatial anisotropy of the underlying interaction, in sharp contrast to Bose and s-wave Fermi systems, where the axial symmetry is always preserved.

To describe the time-of-flight expansion of p-wave Fermi condensates, we consider dilute fermions with mass \( m \) in a single hyperfine state (pseudospin). The time-dependent Hamiltonian is (with \( \hbar = k_B = 1 \))

\[
\mathcal{H}(t) = \int d\mathbf{r} \left\{ \psi_\dagger(t, \mathbf{r}) \left[ -\frac{\nabla_\mathbf{r}^2}{2m} + U_{\text{ext}}(\mathbf{r}, t) \right] \psi(t, \mathbf{r}) - \frac{1}{2} \int d\mathbf{r}' \psi_\dagger(t, \mathbf{r}') \psi(t, \mathbf{r}') V(\mathbf{r} - \mathbf{r}') \psi(t, \mathbf{r}')(t, \psi(t, \mathbf{r}')(t, \psi(t, \mathbf{r}'),(1))
\]

where \( \psi_\dagger (\psi) \) are creation (annihilation) operators, \( U_{\text{ext}}(\mathbf{r}, t) \) is the time dependent trapping potential, and \( V(\mathbf{r} - \mathbf{r}') \) denotes the fermion-fermion interaction. The superfluid state in a harmonic trap is characterized by two length scales: the coherence length \( \xi \) and the length scale \( R_U \) over which \( U_{\text{ext}} \) varies. In the limiting case where the trapping potential varies slowly in comparison to the coherence length (\( R_U \gg \xi \)), a semiclassical approximation is applicable such that one can first consider a dilute Fermi gas in free space and add the harmonic potential later. With this approximation, the free space Hamiltonian takes the form

\[
\mathcal{H} = \sum_{\mathbf{k}} \xi(\mathbf{k}) \psi_\dagger(\mathbf{k}) \psi(\mathbf{k}) + \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V(\mathbf{k}, \mathbf{k}') B_\mathbf{k,q}^\dagger B_{\mathbf{k}, \mathbf{q}},
\]

where \( \xi(\mathbf{k}) = k^2/(2m) - \mu \) is the fermion dispersion shifted by the chemical potential \( \mu \), and \( B_{\mathbf{k}, \mathbf{q}} = \psi_{\mathbf{k}+\mathbf{q}/2}^\dagger(\mathbf{k}) \psi_{\mathbf{k}+\mathbf{q}/2} \) is the creation operator of a pair with center of mass momentum \( \mathbf{q} \) and relative momentum \( 2\mathbf{k} \). Here, the momentum space interaction \( V(\mathbf{k}, \mathbf{k}') \) is the Fourier transform of \( V(\mathbf{r} - \mathbf{r}') \), and can be approximated by a separable form \( V(\mathbf{k}, \mathbf{k}') = -4\pi \sum_{\mathbf{p}} g_m \Gamma(k) \Gamma(k') Y_{1,m_1}^*(\mathbf{k}) Y_{1,m_2}^*(\mathbf{k}') \), where \( \Gamma(k) = k k_0 / (k_0^2 + k^2) \) is the symmetry function and \( Y_{1,m_1}, Y_{1,m_2} \) are spherical harmonics. The momentum scale \( k_0 \) is determined by the effective range \( R_0 \approx 1/k_0 \) of the interaction and the diluteness condition \( (nR_0 e^3) \ll 1 \) requires \( (k_0/k_F)^3 \gg 1 \), where \( n \) is the particle density and \( k_F \) is the Fermi momentum. The interaction can be related to the scattering volume \( a_p \) through the T-matrix \( \frac{1}{i}. \)

We introduce the bosonic field operator \( b_{\mathbf{p} m}^\dagger = \sum_{\mathbf{k}} \Gamma(k) Y_{1,m}^*(\mathbf{k}) \psi_{\mathbf{k}+\mathbf{q}/2}^\dagger(\mathbf{k}) \psi_{\mathbf{k}+\mathbf{q}/2} \) for its corresponding auxiliary field \( \Delta_{m} \) and integrate out the fermions to derive the Ginzburg-Landau (GL) effective action

\[
S_{\text{GL}} = S_0 + \sum_{\mathbf{q}, \nu, m, m'} L^{-1}_{m, m'}(\mathbf{q}, \nu) \Delta_{m}^*(\mathbf{q}, \nu) \Delta_{m'}(\mathbf{q}, \nu) + S_4,
\]

around the trivial saddle point \( \Delta_{m} = 0 \) near the transition temperature \( T_c \), where \( S_4 \) denotes the quartic terms. The long wavelength behavior of the system can be described by the expansion of the static part of \( L^{-1} \) in powers of \( \mathbf{q} \), leading to

\[
L_{m, m'}^{-1}(\mathbf{q}, \nu) = a_{m, m'} + \sum_{i,j} c_{m, m'}^{i,j} \frac{q_i q_j}{2m} + \ldots \quad (3)
\]

Similarly, the low frequency time evolution of the system can be described by the frequency dependent part of \( L^{-1} \) in powers of \( \omega \) after analytic continuation \( i\nu \to \omega + i\delta^+ \). For simplicity, we choose to investigate the cases where the order parameter \( \Delta \) is nonzero only for the channel \( \nu = 0 \) for simplicity, and is not discussed here. In the BEC limit, the relation \( c_{m, m'}^{i,j} / d_{m, m'} = 1/2 \), such that Eq. (3) reduces the conventional Gross-Pitaevskii (GP) form for a dilute gas of paired fermions with mass \( 2m \). For simplicity and definiteness, we consider the case of a static harmonic trap \( U_{\text{ext}}(\mathbf{r}, t) = \sum_{\nu=1}^3 m \omega_{\nu}^2 \mathbf{q}_\nu^2 / 2 \) for \( t < 0 \) which is turned off at \( t = 0 \). For \( t < 0 \), the system is described by

\[
\frac{\partial}{\partial t} \Delta_m(\mathbf{r}, t) = - \sum_{i} c_{m, m'}^{i,i} \frac{\nabla_i^2}{2m} + \frac{a_{m, m'}}{d_{m, m'}} + 2U_{\text{ext}}(\mathbf{r}, t) + \frac{b_{m, m'}}{d_{m, m'}} |\Delta_m(\mathbf{r}, t)|^2 \Delta_m(\mathbf{r}, t), \quad (4)
\]

where the coefficients \( a_{m, m'}, b_{m, m'}, c_{m, m'} \), and \( d_{m, m'} \) can be obtained accordingly. The trivial expansion of the normal component is included in the evolution of \( S_0 \), and is not discussed here. In the BEC limit, the ratio \( c_{m, m'}^{i,i} / d_{m, m'} = 1/2 \), such that Eq. (3) reduces the conventional Gross-Pitaevskii (GP) form for a dilute gas of paired fermions with mass \( 2m \). For simplicity and definiteness, we consider the case of a static harmonic trap \( U_{\text{ext}}(\mathbf{r}, t) = \sum_{\nu=1}^3 m \omega_{\nu}^2 \mathbf{q}_\nu^2 / 2 \) for \( t < 0 \) which is turned off at \( t = 0 \). For \( t < 0 \), the system is described by

\[
\Delta_m(\mathbf{r}) = \sum_{i} c_{m, m'}^{i,i} \frac{\nabla_i^2}{2m} + \frac{a_{m, m'}}{d_{m, m'}} + 2U_{\text{ext}}(\mathbf{r}) + \frac{b_{m, m'}}{d_{m, m'}} |\Delta_m(\mathbf{r})|^2 \Delta_m(\mathbf{r}), \quad (5)
\]

where \( \mu_0 \) is the effective chemical potential. For dominant effective boson interactions the Thomas-Fermi approximation leads to \( |\Delta_m(\mathbf{r}, 0)| = \left| d_{m, m'}(\mu_0 - 2U_{\text{ext}}(\mathbf{r}))/b_{m, m'}^{1/2} \right| \) for \( \mu_0 \geq 2U_{\text{ext}}(\mathbf{r}) \), and \( |\Delta_m(\mathbf{r}, 0)| = 0 \) otherwise. When this approximation fails the initial condition for the time-of-flight expansion can be obtained by solving the static equation (5) numerically. Furthermore, since the trapping potential is separable in the spatial variables \( r_j \), a scale transformation \( R_j \to r_j \sqrt{d_{m, m'}}/c_{m, m'} \) and \( \Omega_j \to \omega_j c_{m, m'}/d_{m, m'} \) can be introduced to remove the anisotropy in the gradient term of Eq. (5), leading to an equation with the conventional GP form.
For $t > 0$, we introduce the transformation $R_j(t) = b_j(t)R_j(0)$, where the scaling factors $b_j(t)$ satisfy $d^2b_j(t)/dt^2 = \Omega^2_j/[A(t)b_j(t)]$, with $A(t) = b_x(t)b_y(t)b_z(t)$ and initial conditions $b_j(0) = 1$. Under the approximation of collisionless hydrodynamics, the time evolution of the condensate wavefunction

$$\Delta_{m\ell}(\mathbf{R}, t) \approx \frac{\exp[S(\mathbf{R}, t)\mathbf{R}]}{\sqrt{b_x(t)b_y(t)b_z(t)}} \Delta_{m\ell}(\mathbf{R}, 0)$$

is contained in the scaling factors $b_j(t)$, scaled coordinates $\mathbf{R} = \mathbf{R}k/b_k(t)$ and phase factor $S(\mathbf{R}, t) = S_0(t) + 2m \sum_x [R_k^2(t)/b_k(t)]b_\perp(t)/dt$. Now, we apply the time evolution results to an axially symmetric cigar shaped trap with $\omega_x = \omega_y \equiv \omega_\perp \gg \omega_z$, and consider first the strongly interacting BEC limit. In the limit where $\varepsilon \equiv \omega_z/\omega_\perp \ll 1$, the scaling factor $b_j(t)$ can be obtained in powers of $\varepsilon$, leading to $b_z(t) = 1 + \varepsilon^2(\tau \tan^{-1} \tau - (1/2) \ln(1 + \tau^2)) + O(\varepsilon^4)$ and $b_\perp(t) = \sqrt{1 + \tau^2} + O(\varepsilon^2)$, where $\tau = \omega_z t$ is the dimensionless time. Therefore, according to Eq. (6) the aspect ratio of the cloud between the radial ($L_x$, $L_y$) and axial ($L_z$) widths is given by

$$L_x(t) = b_\perp(t)\omega_x = \frac{\varepsilon b_z(t)}{b_\perp(t)}.$$  

In Fig. 1 the aspect ratio $r_{xz} \equiv L_x/L_z$ is shown for $\varepsilon = 0.1$ and $\varepsilon = 0.3$, indicating that the anisotropy of the cloud is inverted during expansion and that the aspect ratio reaches an asymptotic value significantly larger than one, similar to the expansion of Bose [19, 20] and s-wave Fermi [21] condensates.

Notice that in the strong attraction (BEC) limit, fermions form tightly bound molecules and the internal degrees of freedom of fermion pairs do not play an important role. Thus, it is not surprising that the anisotropic nature of the $p$-wave interaction between fermions does not manifest itself in the behavior of the condensate. However, as one moves away from the BEC limit towards unitarity, the average fermion pair size increases, and the internal structure of pairs can dramatically change the condensate properties as the pair size becomes comparable to the inter-molecular spacing. In order to study this regime, we consider a similar case where a $p$-wave Fermi condensate is initially trapped in an axially symmetric cigar-shaped potential with $\omega_x = \omega_y \gg \omega_z$. Furthermore, we impose a magnetic field applied along the $x$-direction (chosen as the quantization axis) to tune through the Feshbach resonances. Since resonances in $^{40}$K are split [11] for different $m_\ell$ states, it may be possible to adjust the magnetic field such that fermions are paired in the $m_\ell = 0$ (p$_z$) state only. In this case, the $p$-wave interaction leads to the formation of p$_x$ symmetry pairs, which are more strongly correlated along the $x$-direction. As a consequence, the coefficients $c_{m_\ell=0}$ in Eq. (1) satisfy $c_{m_\ell=0} > c_{m_\ell=0} = c_{m_\ell=0}$, hence breaking the axial symmetry in the radial ($x$-$y$) plane. Thus, upon free expansion the cloud looses its axial symmetry which is initially imposed by the harmonic potential.

In Fig. 2, we plot the cloud aspect ratio in the radial plane $r_{xy} = L_x/L_y$ for the $m_\ell = 0$ state as a function of the $p$-wave scattering parameter $1/(k_F^3a_p)$ on the BEC side of the Feshbach resonance. In this plot, we choose the parameter $k_0/k_F = 5$, which is compatible with a dilute gas of $^{40}$K with density $n = 10^{14}$cm$^{-3}$. Notice that the anisotropy disappears in the BEC limit as it must, but becomes more evident with decreasing interaction strength, leading to a 5% anisotropy near unitarity for sufficiently long times. Investigations on the BCS side [$1/(k_F^3a_p) < 0$] and at unitarity require the inclusion of Landau damping which leads to the decay of Cooper pairs, as well as collisional dynamics of the normal flow [22, 23], which are beyond the scope of the present theory. In addition to the anisotropic expansion in the radial plane which is characteristic of the $p$-wave case, we also find an anisotropy inversion of the aspect ratio between the axial and radial directions, which is similar to the cases of Bose and s-wave Fermi condensates.
induced by the harmonic potential and the anisotropic expansion induced by the $p$-wave interactions is found considering the time-of-flight expansion for a completely isotropic trap. In this case, for a $p$-wave Fermi condensate with either $m_\ell = 0$ or $m_\ell = \pm 1$ pairing, the cloud becomes axially symmetric during expansion and takes a cigar shape (for $m_\ell = 0$) or pancake shape (for $m_\ell = \pm 1$). This is in sharp contrast to the case of Bose and $s$-wave Fermi condensates, where an initially isotropic cloud remains always isotropic throughout the expansion.

Before concluding, we would like to stress that although Eq. (1) is obtained within the Ginzburg-Landau scheme for temperatures not too far below $T_c$, our qualitative results remain valid down to lower temperatures. In particular, the anisotropy inversion between axial and radial directions is just a direct consequence of superfluid hydrodynamics, while the anisotropic expansion in the radial plane is due to anisotropic $p$-wave interactions. These two qualitative features are characteristic of $p$-wave Fermi condensates, and are not sensitive to temperature. Therefore, the qualitative aspects of our results become more evident at lower temperatures, where a more sizable condensate fraction emerges (larger bimodal distribution).

In summary, we considered a single component $p$-wave Fermi condensate and studied the time-of-flight expansion of a cloud initially trapped in a harmonic potential. We obtained the time evolution of the condensate on the BEC side of the Feshbach resonances for temperatures close to the critical temperature. Starting from a cigar-shaped cloud with axial symmetry, we showed that the aspect ratio between axial and radial directions presents an anisotropy inversion during expansion, indicating a characteristic feature of condensate physics. Furthermore, we found an anisotropic expansion in the radial plane depending selectively on the magnetic quantum numbers $m_\ell = 0$ and $m_\ell = \pm 1$ of the $p$-wave interaction. We also emphasized that this anisotropic expansion is a direct consequence of anisotropic $p$-wave interactions and occurs most clearly when the harmonic trap is completely isotropic. In addition, we would like to stress that this anisotropic expansion should occur not only for $p$-wave, but also for any higher angular momenta ($d$-wave, $f$-wave, etc...) as Feshbach resonances are approached from the BEC side. Finally, we proposed that the orbital symmetry of the order parameter for $p$-wave condensates can be directly probed through time-of-flight expansions of harmonically trapped clouds. Potential candidates for such experiments are $^{60}$K, $^6$Li, and $^{171}$Yb.

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FIG. 2: Aspect ratio in the radial plane $r_{xy} = L_x/L_y$ as a function of scattering parameter $1/(k_F^2a_p)$ at various time $\tau$ (solid lines) for a $p$-wave Fermi condensate with (a) $m_\ell = 0$ and (b) $m_\ell = \pm 1$ orbital symmetry. Notice that $r_{xy}$ saturates to an asymptotic value (dashed lines) for $\tau \to \infty$. Insets show the corresponding pair wave functions.

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