Two-dimensional full-wave simulation of propagation and absorption of a microwave beam in magnetized plasma

A S Sakharov

Prokhorov General Physics Institute of the Russian Academy of Sciences, Moscow, 119991 Russia

E-mail: sakharov_as@mail.ru

Abstract. Results from solving a full-wave two-dimensional model problem on the propagation and absorption of a microwave beam in plasma for the magnetic configuration of the L-2M stellarator are presented. The coefficients of transmission and reflection are calculated. The distributions of the microwave power absorbed in plasma are obtained. It is found that, under conditions typical of L2-M experiments on ECR plasma heating at the second harmonic of the electron gyrofrequency, an appreciable fraction (about 10%) of the incident microwave power can be deflected downward from the plasma axis, not reaching the absorption region. The fraction of the downward-deflected microwave power is shown to increase considerably at central plasma densities close to the cutoff density \( n_e \approx (0.8-0.9) n_{cut} \).

1. Introduction

One of the main methods of plasma heating in toroidal magnetic confinement systems is electron cyclotron resonance (ECR) heating at the second harmonic of the electron gyrofrequency (\( \omega_0 = 2 \omega_{ce} \)) [1, 2]. As a rule, an extraordinarily (X) polarized microwave beam is launched from the low-field side of the torus, propagates across the magnetic field, and is then absorbed in a narrow resonance region in the center of the plasma column. Since the characteristic scale lengths of the magnetic field and plasma density are usually much larger than the heating radiation wavelength (\( L_B > L_n >> \lambda_0 \)), various versions of the ray-tracing (eikonal) method are traditionally used to calculate the propagation and absorption of microwave beams in toroidal fusion plasmas (see, e.g., [3–6]). As the beam propagates from the plasma boundary to the resonance region, thermal effects in weakly relativistic plasma usually play a minor role and can thus be ignored, i.e., the ray trajectories can be calculated in the cold plasma approximation, while the thermal effects are taken into account only in the coefficient of microwave absorption in the ECR region. As a whole, the ray-tracing method adequately describes the spatial distribution of the microwave power deposited in plasma; nevertheless, some fine effects, such as microwave reflection from the ECR region [7], cannot be described in terms of this method. A small (but detectable [8, 9]) fraction of microwave radiation can be reflected from the ECR region, because the inhomogeneity scale lengths of the plasma permittivity and electric field amplitude in the narrow region (\( \Delta x \sim L_B T_e/m_e c^2 \) [4]) near the electron cyclotron resonance can be comparable with the radiation wavelength, i.e., the applicability conditions of the ray-tracing method are violated there. The ray-tracing method can also be poorly applicable in systems with a complicated structure of the magnetic field (such as stellarators), where the injected microwave beam can propagate toward the...
plasma axis nearly parallel to the ECR surface, so that the peripheral rays graze along this surface (see, e.g., [10]). To correctly describe these effects, it is necessary to solve a full-wave problem with allowance for the nonlocal (operator) thermal correction to the cold plasma permittivity tensor.

2. Nonlocal plasma permittivity tensor near ECR

Earlier, a one-dimensional differential expression for the thermal correction to the plasma permittivity tensor near the ECR at the second harmonic of $\omega_{ce}$ was derived in [11, 12]. Such an operator expression can be used to calculate absorption and reflection of an X-polarized microwave beam propagating nearly along the normal to the plane resonance surface—a situation close to the actual conditions of ECR plasma heating in conventional (unrippled) tokamaks [2, 4]. However, in a more complicated stellarator field, the one-dimensional approximation is poorly applicable and at least two-dimensional (2D) model is required to describe the wave effects of microwave beam–plasma interaction in the ECR region (a model adopting results of the one-dimensional theory to full-wave simulations of ECR plasma heating in 3D magnetic configurations was earlier proposed in [13]).

In [14], the following 2D operator expression for the thermal correction to the plasma permittivity tensor near the $\omega_0 = 2\omega_{ce}$ resonance for an X-polarized wave propagating perpendicular to the magnetic field was obtained:

$$\delta\hat{\varepsilon}_\perp = \begin{bmatrix} \delta\varepsilon_{\perp} & -i\hat{\varepsilon}_{\perp} \\ i\hat{\varepsilon}_{\perp} & \delta\varepsilon_{\perp} \end{bmatrix},$$

(1)

where

$$\delta\varepsilon = k_0^{-2} \nabla^2 (F^{X^2}(x, y)) \nabla^{(-)},$$

(2)

$$F^{X^2} = \left( \frac{\omega_0^2}{2\omega_{ce}^2} \right) F_{\gamma/2} \left( \mu_e (\omega - 2\omega_{ce0}(x, y))/\omega \right),$$

with $k_0 = \omega/c = 2\pi/\lambda_0$, $\nabla^{(-)} = \partial/\partial x \pm i \partial/\partial y$, $F_{\gamma/2}(\xi)$ is the Dnestrovskii function [15], $\mu_e = m_ec^2/T_e \gg 1$ and $\omega_{ce0}$ is the nonrelativistic electron gyrofrequency. For $\partial/\partial y = 0$, expressions (1) and (2) are reduced to the one-dimensional case considered in [11, 12].

3. Wave equation

The 2D wave equation for the electric field $E = \{E_x, E_y, 0\}$ of an X-polarized wave propagating perpendicular to the magnetic field has the form

$$\nabla \times (\nabla \times \hat{\varepsilon}_0 \hat{\varepsilon}_0 \varepsilon) = 0,$$

(3)

where $\hat{\varepsilon}_0$ is the permittivity tensor of cold magnetized plasma [15] and $\delta\hat{\varepsilon}_\perp$ is thermal correction (1). Equation (3) has the integral [14]

$$\nabla \cdot P + Q = 0,$$

which has the sense of the energy conservation law. Here,

$$Q = -\frac{c}{8\pi k_0} \left| \nabla^{(-)} \varepsilon^{(-)} \right| \d{F^{X^2}}$$

is the specific power absorbed by plasma and $P = S + G$ is the power flux density, which is the sum of the Poynting vector $S$,

$$S = \frac{c}{8\pi k_0} \d{ \left[ E^* \times (\nabla \times E) \right]}.$$

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and the thermal correction $\mathbf{G}$ to the power flux density [11],

$$
\mathbf{G} = \frac{c}{8\pi k_0} \text{Im} \left[ F^{\times 2} E^{(-)} \left( \nabla^{\times} E^{(-)} \right) (\mathbf{e}_x + i\mathbf{e}_y) \right],
$$

where $E^{(-)} = E_x - iE_y$.

4. Formulation of the numerical problem

Equation (3) was solved numerically for a 2D magnetic configuration imitating the actual magnetic configuration of the L2-M stellarator in its standard cross section used for ECR plasma heating [8–10]. Although our 2D model is rather simplified and disregards the radial and poloidal components of the magnetic field (the magnetic field is assumed to be directed along the $z$ axis, i.e., $\mathbf{B}_0 = \{0,0,B_0(x,y)\}$), it allows one to examine specific features of microwave beam–plasma interaction under conditions of ECR plasma heating at the L2-M stellarator.

For our simulations, we used the radial profiles of the plasma density and electron temperature typical of L2-M experiments [10] (see Figure 1). Figure 2 shows the corresponding structure of the magnetic surfaces (with allowance for their outward shift in hot plasma [16]) and magnetic field $\mathbf{B}_0$ in the standard cross section of L2-M [10, 17].

![Figure 1](image1.png)  
**Figure 1.** Radial profiles (in flux variables) of the electron density and temperature used in numerical simulations ($n_e(0) = 1.75 \times 10^{13} \text{ cm}^{-3}$, $T_e(0) = 1 \text{ keV}$).

![Figure 2](image2.png)  
**Figure 2.** Magnetic surfaces and contour lines of $B_0(x, y)$ ($\Delta B_0 = 0.04B_{\text{res}}$) in the standard cross section of L2-M. The heavy red and blue lines show the positions of the plasma boundary and resonance surface $\omega = 2\omega_c(x,y)$, respectively. The major axis of the torus is on the left. The magnetic field on the plasma axis ($x = y = 0$) increases to the left upward.

The problem was solved in a $24.96 \times 24$-cm simulation box divided into $616 \times 600$ cells with dimensions $\Delta x = \Delta y = 0.04$ cm. A Gaussian microwave beam with a vacuum wavelength of 0.4 cm ($f_0 = 75 \text{ GHz}$) and half-width of 2 cm at a level of $e^{-1}$ in amplitude was launched through the right boundary (i.e., from the outer side of the torus) along the $x$ axis. Outgoing conditions were imposed on the transmitted and reflected waves at the left and right boundaries, respectively, and smooth absorbing layers were introduced near the lower and upper boundaries to suppress reflection from them. Equation (3) with these boundary conditions was solved by the matrix sweep method [18].
For comparison, the propagation and absorption of a similar microwave beam in the same magnetic configuration was also calculated by the simplest version of the ray-tracing method, in which the ray trajectories are found in the cold plasma approximation, while the thermal effects are taken into account via the absorption coefficient [3].

5. Simulation results

Figures 3a and 3b show the distributions of the microwave field amplitude squared, \(|E^2| = |E_x^2| + |E_y^2|\), in the \((x, y)\) plane, calculated by the ray-tracing method and by solving the full-wave equation, respectively, for a microwave beam propagating in cold plasma (without thermal correction to the permittivity tensor). It is seen that the results obtained by both methods almost coincide, except for a slight diffraction divergence of the full-wave beam on the rear side of the plasma column. The maxima in the field intensity correspond to the humps of the plasma density profile (see Figure 1).

![Figure 3](image)

**Figure 3.** Distributions of the microwave field amplitude squared \(|E^2|\) in the \((x, y)\) plane for a microwave beam propagating in cold plasma, calculated (a) by the ray-tracing method and (b) by solving the full-wave equation. Here and in the subsequent figures, the red and yellow lines show the positions of the plasma boundary and resonance surface, respectively. The envelope of the incident microwave beam power is shown on the right of each panel.

Figure 4 compares the distributions of \(|E^2|\) calculated by both methods for hot plasma with the density and temperature profiles shown in Figure 1. It is seen that the distributions differ substantially near the resonance surface. The beam calculated by the ray-tracing method (Figure 4a) simply “disappears” after crossing the resonance surface, the fraction of the transmitted microwave power being about 1%. In contrast, the full-wave beam (Figure 4b) undergoes refraction and self-interference as it approaches the plasma axis. About 0.5% of the incident microwave power is transmitted through the resonance region. The fraction of radiation that leaves the simulation region through the right boundary (i.e., is reflected) is as low as \(\approx 2 \times 10^{-4}\) (in this case, only \(\approx 2 \times 10^{-5}\) of the incident microwave power is reflected back into the aperture of the injected microwave beam).

An appreciable fraction of the microwave power (\(\approx 13\%\)) is deflected downward. This effect is caused by the specific shape of the ECR surface in the injection plane. Before reaching the absorption region, the microwave beam propagates nearly parallel to the resonance surface. Analysis of thermal correction (1) to the cold permittivity tensor shows that the effective (“local”) refractive index of plasma in the upper part of the beam is negative and increases in magnitude toward the ECR surface until it reaches a local minimum close to this surface. In other words, there is a negative vertical gradient of the effective refractive index, due to which the upper part of the beam is deflected
downward and begins to interfere with its lower part. As the ECR surface gradually turns down, a fraction of radiation is “guided” along this surface and finally escapes downward.

Figure 4. Distributions of $|E|^2$ in the $(x, y)$ plane, calculated (a) by the ray-tracing method and (b) by solving the full-wave equation for a microwave beam propagating in hot plasma with the density and temperature profiles shown in Figure 1.

Figures 5a and 5b show the distributions of the absorbed microwave power density $Q$ calculated by the ray-tracing method and by solving the full-wave equation, respectively, for the same plasma parameters as in Figure 4. It is seen that, due to the downward deflection of the upper part of the beam, the absorption region of the full-wave beam is somewhat more compact than that calculated by the ray-tracing method.

Figure 5. Distributions of the absorbed microwave power density $Q$ in the $(x, y)$ plane, calculated (a) by the ray-tracing method and (b) by solving the full-wave equation for the same plasma parameters as in Figure 4.

The simulations have shown that the microwave power deflected downward from the plasma axis increases substantially as the central plasma density rises to $(0.8−0.9)n_{cut}$, where

$$n_{cut} = n_e (1−\omega_p/\omega) = n_e / 2 = m_e \alpha_c^2 / 8 \pi e^2$$

is the cutoff density for an X-wave propagating perpendicular to the magnetic field [15]. Figure 6 presents results of 2D full-wave simulations for the
central plasma density \( n_e(0) = 3 \times 10^{13} \text{ cm}^{-3} \approx 0.85 n_{\text{cut}} \) and central temperature \( T_e(0) = 0.5 \text{ keV} \). These values are close to those in L2-M experiments with an increased plasma density [9] (the central density in simulations was somewhat increased compared to the experiment in order to make the effect of refraction more pronounced). The corresponding profiles of \( n_e \) and \( T_e \) are shown in Figure 7.

Figure 6. Distributions of \(|E|^2\) in the \((x, y)\) plane, calculated by numerically solving full-wave equation (3) (a) without (cold plasma) and (b) with thermal correction (1) to the cold permittivity tensor for the plasma density and temperature profiles shown in Figure 7.

It is seen from Figure 6 that the microwave beam in cold plasma significantly expands and somewhat refracts upward. When thermal correction (1) to the permittivity tensor is taken into account, the beam undergoes substantial refraction. About 0.5% of the incident power is transmitted, only \(~10^{-5}~\) is reflected back, and more than 50% is deflected downward, not reaching the absorption region. The distribution of the absorbed power in this case remains generally the same as in Figure 5.

Figure 7. Radial profiles (in flux variables) of the electron density and temperature used in the simulation version with an increased plasma density \((n_e(0) = 3 \times 10^{13} \text{ cm}^{-3}, T_e(0) = 0.5 \text{ keV})\).

6. Conclusions
In this work, specific features of the propagation and absorption of a microwave beam in plasma under conditions typical of L2-M experiments on ECR plasma heating at the second harmonic of the electron gyrofrequency have been studied by numerically solving a 2D full-wave model problem with
allowance for the nonlocal (operator) thermal correction to the permittivity tensor of cold magnetized plasma. The simulations have shown that, for the given configuration of the magnetic field, an appreciable fraction of the incident microwave power can be deflected downward, not reaching the absorption region. This effect is quite pronounced and can easily be verified experimentally. It is found that the fraction of the downward-deflected microwave power increases substantially as the central plasma density approaches the cutoff density for an X-wave propagating across the magnetic field \((n_e(0) \rightarrow n_{cr}/2)\), which should be taken into account when performing ECR heating experiments with an increased plasma density at the L2-M stellarator.

The extremely low values of the reflection coefficient obtained in these simulations (much lower than those observed in L2-M experiments \([8, 9]\)) may be due to the specific features of our planar model, in which the microwave beam is assumed to remain in a given poloidal cross section as it propagates toward the plasma axis. In this case, the microwave beam is gliding along the resonance surface and no microwave power is reflected backward. In fact, the structure of the magnetic field varies along the torus in such a way that the cross section of the ECR surface rotates in the poloidal plane. The actual microwave beam (or its fraction) can deflect from the given poloidal plane and fall into the region where it propagates nearly perpendicular to the resonance surface and can thus be reflected backward. This issue requires a more thorough analysis.

Thus, it may be concluded that the proposed model allows one to more adequately describe some wave properties of the heating microwave beam in comparison with the ray-tracing method. However, to correctly describe fine effects of microwave beam–plasma interaction in complicated magnetic configurations, it is necessary to develop a more comprehensive 3D model (probably, a combined full-wave–ray-tracing one).

**Acknowledgments**

The author is grateful to S.V. Shchepetov and M.A. Tereshchenko for helpful discussions. This work was performed under State Assignment No. AAAA-A18-118013000279-8 “Fundamental Problems of Dynamics, Confinement, and Heating of Plasma in Three-Dimensional Magnetic Configurations” and the Fundamental Research Program of the Russian Academy of Sciences No. I.13 “Condensed Matter and Plasma at High Energy Densities.”

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