Chapter 4
Survey on Control Schemes for Automated Driving on Highways

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4.1 Introduction

Over the past two decades, autonomous driving has attracted the attention of several research groups. The main goal of the control design for automated or autonomous vehicles is an increased safety by means of reduced accidents. Moreover, traffic utilization and energy efficiency are considered as additional control objectives. An early publication on control issues on automated highway systems has been published in [34]. Much research has been conducted and many vehicles have been equipped with automated driver assistance functions since that time. Adaptive cruise control (ACC), which controls the longitudinal vehicle speed with respect to a target vehicle or a desired velocity, is widely used in common cars. Lane keeping assist (LKA) and lane departure warning are available lateral assistance functions. The assistance is achieved by active steering and braking. In contrast, the warning functions only alert the driver by giving haptic or acoustic signals. A major topic of recent years is a lane change assist (LCA) function. This function must compute and track a safe and comfortable trajectory for the lane change maneuver. The combination of the mentioned functions yields advanced assistance functions for highway driving, e.g., the combination of ACC and LKA allows the vehicle to drive highly automated in its own lane. Adding a function capable of lane changes results in the so-called Motorway Chauffeur or Highway Chauffeur. Another highly automated assistance function responsible for efficient and safe driving is platooning. A platoon consists of several vehicles driving with a closer
headway for traffic congestion relief. The first vehicle is the leader and may be controlled by a human driver, the following vehicles track this leader autonomously.

The control task of a highly automated vehicle is to drive safely and efficiently from one point to another while avoiding obstacles and infeasible maneuvers.

This task is usually divided into three sublevels [91]: first, the mission planning level computes the shortest or best path from point A to point B. The second level constitutes the behavioral planning, which is the decision unit of the vehicle. This level is responsible for processing environment data and computing the possible lanes and goal points for the local motion planner. Motion planning is the third and lowest level. It computes the best trajectory based on the information from the behavioral planner. The tracking controller takes care of tracking this trajectory and is the focus of this work.

This survey is organized as follows: the remainder of Sect. 4.1 states the mathematical problem of tracking a trajectory on a highway with a highly automated vehicle. Sections 4.2–4.6 treat the controller design using different control techniques, namely fuzzy control Sect. 4.2, linear state feedback control Sect. 4.3, sliding mode control Sect. 4.4, model predictive control Sect. 4.5, and other concepts Sect. 4.6. In Sect. 4.7, tracking controllers implemented in autonomous vehicle prototypes are briefly discussed. A comparison is performed in Sect. 4.8, analyzing the performance of some of the presented controllers and finally, an outlook is given.

### 4.1.1 Problem Statement

To reduce complexity of the control tasks during highway driving, the dynamics of a highly automated vehicle is usually divided into longitudinal and lateral motion. The longitudinal behavior is often modeled by a simple first order system, while the lateral behavior is of a more complex nature. There are two common models for the lateral dynamics: the kinematic bicycle model and the dynamic bicycle model. Both are summarized briefly in this section, for details, refer to [74] or [83].

The simple kinematic bicycle model assumes left and right wheels being collapsed into single wheels at the center of the front and rear axle. The wheels are assumed to have no lateral slip and only the front wheel is steerable. These assumptions are fulfilled for low driving speeds and the model can be used in case of zero velocity. Figure 4.1 shows the kinematic bicycle model.

This single track model can be written as

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) \\
\sin(\theta) \\
L^{-1}\tan(\delta)
\end{bmatrix} v ,
\]  

(4.1)
Fig. 4.1 Kinematic bicycle model with the heading $\theta$, the steering angle $\delta$, and the global coordinates of the rear wheels $x, y$. The wheels are connected by a shaft of length $L$. The velocity vector at the center of the rear wheels is $v$.

Fig. 4.2 Dynamic bicycle model, which considers forces acting at the wheels: $F_{x,f}$ and $F_{y,f}$ are the longitudinal and lateral forces of the front wheel, $F_{x,r}$ and $F_{y,r}$ describe the forces acting on the rear wheel. The slip angles of the center of gravity (CG) and the wheels are denoted by $\beta$, $\alpha_r$, and $\alpha_f$.

where the input variables $v$ and $\delta$ are the longitudinal velocity of the rear wheel and the steering angle. Symbols $x, y$ are the global coordinates and $\theta$ is the heading of the car with respect to the global x axis.

The dynamic bicycle model includes wheel slipping effects due to lateral forces on the vehicle (Fig. 4.2).
The nonlinear dynamic bicycle model is given by the relations

\[ \begin{align*}
\dot{v}_y &= \frac{-c_f \alpha_f \cos(\delta) - c_r \alpha_r}{m} - v_x r, \\
\dot{r} &= \frac{-l_f c_f \alpha_f \cos(\delta) + l_r c_r \alpha_r}{I_z}
\end{align*} \tag{4.2} \]

at the center of gravity, where \( v_x \) and \( v_y \) are the velocities in a vehicle local coordinate system in longitudinal and lateral direction. Symbol \( r \) denotes the yaw rate. The coefficients \( c_i \) correspond to the cornering stiffness and \( \alpha_i \) to the slip angle of the wheels, \( i \in \{f, r\} \). \( m \) denotes the vehicle mass and \( I_z \) its yaw inertia. \( l_f \) and \( l_r \) are the distances from the wheels to the center of gravity. Applying small angle assumptions, a linearized state-space description can be formulated as

\[ \begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2 \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & \frac{c_f + c_r}{mv_x} & 0 & -v_x \\
0 & 0 & \frac{1}{mv_x} & -\frac{c_f l_f - c_r l_r}{mv_x}
\end{bmatrix} \begin{bmatrix}
y_1 \\
y_2 \\
\theta
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{c_f}{m} \\
0 \\
\frac{c_f l_f}{I_z}
\end{bmatrix} \delta, \tag{4.3}
\]

which is of the linear form \( \dot{z} = Az + bu \) for constant velocity \( v_x \). For many tracking tasks, a reformulation in path and/or error coordinates is advantageous. For more detail, see \[74, 83\].

An important prerequisite for these tracking tasks is a trajectory generation. The following subsection tackles one way to generate such reference trajectories.

\subsection*{4.1.2 Trajectory Generation}

For automated driving, trajectory generation is essential for maneuvering safely and efficiently on a highway or in urban scenarios. In contrast to the basic assistance functions such as ACC and LKA, which can be executed without reference trajectory (they have other references, i.e., target vehicles or target lanes), LCA requires a smooth path in order to guarantee a safe lane change.

A survey on trajectory generation can be found in \[28\]. One can distinguish between two main areas where trajectory generation arises:

1. Robotics: in the navigation task of robots in known or unknown environments, grids are usually used to decompose the sensor data with information of obstacle-free and obstacle-occupied space into cells. Neighboring cells are connected by weighted edges and a graph is constructed with the initial condition as starting node. A graph-based algorithm is then applied to find the path with the lowest cost from the starting point to an end point. This path consists of several segments that are tracked one after another, which yields non-smooth behavior. A smoothing algorithm thus needs to be applied for the use in autonomous driving, in order to avoid discontinuities in trajectories.
The involved kinematics and dynamics of the system are ignored by most algorithms, which may result in infeasible solutions for motorway driving. Another drawback of these approaches is the memory storage required for building the graph and the necessity of post-smoothing. Moreover, the lanes of the highway are typically available in the form of “drive corridors” from sensor fusion, which makes the search for a possible path unnecessary.

2. Control theory: optimal theory is used to find trajectories of a dynamical system that minimize a given cost function given an initial condition and an end condition. The dynamics of the system is taken into account explicitly, the generated solutions are feasible with respect to the model. Obstacles can be considered by formulating constraints or by rejecting trajectories that result in collisions.

The control theory domain offers several possibilities to deal with the trajectory generation task. Polynomial approaches or model predictive control (MPC) are employed in many publications, e.g., [46, 96] and [18]. Since the polynomial approach is simple and very popular, its main points are described in brief below.

4.1.2.1 Polynomial Approach

One common straightforward approach for lane changing trajectories is the calculation of fifth order polynomials, since they can establish smooth transitions between the vehicle’s actual and desired final positions. In most cases, the lateral behavior is defined by the polynomial and the longitudinal trajectories are chosen based on simpler methods.

Assuming constant longitudinal velocity for simplicity, the lateral trajectory generation process consists of three main steps:

1. End point definition:
   In order to avoid obstacles and to allow efficient driving, a set of end positions on the road is defined as goals of the local trajectories, as described, for instance, in [91]. The lane centers of the possible lanes are typically included. Additionally, other end points may be considered for safety purposes, e.g., if a truck overlaps the lane markings slightly, the vehicle shall keep a safe distance and prefer driving more on the far side of the lane (but still in the lane). This non-centered driving when adjacent to large traffic participants also resembles human driving strategy.

2. Generation of trajectories:
   Given initial and end conditions for position, velocity, and acceleration, one can choose a fifth order polynomial in order to minimize a cost function consisting of the lateral jerk, which is the third time derivative of the position. The initial conditions are provided by sensor data or observers, while the end conditions are chosen as desired. As described, e.g., in [41], the coefficients of the polynomials can be computed based on this problem formulation and the trajectories from the starting point to each end point are generated. Changing the cost function
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results in a different computation of the coefficients of the polynomial. Since each generated trajectory is evaluated by another cost function, several aspects such as safety and efficiency can be incorporated in the last step.

3. Evaluation of trajectories:

The best choice in respect to a specific cost function is selected from the set of generated trajectories. The cost function typically includes obstacle avoidance, choice of lane, and lateral position in the desired lane, among others. Obstacle avoidance is certainly the crucial task in this step. If a trajectory causes a collision, it is rejected by setting the cost to infinity. A prediction of the ego-vehicle and all obstacles is thus needed. If no collision is detected, the cost can be computed based on the distance, given, e.g., by an ellipsoid around the obstacle as in [41]. For the other cost components, the trajectory is evaluated at certain time stamps with respect to certain aspects, e.g.,

- desired lane (defined by the driver, default is the rightmost lane in Europe)
- lateral position in current lane
- jerk and/or acceleration
- desired longitudinal velocity
- stationarity

Finally, all cost components are summed up and the trajectory with the least cost is chosen. However, this may result in high-frequency switching if the computation is executed at every time step. A filter can be added in order to ensure smooth and efficient behavior.

In many trajectory generation approaches, the longitudinal behavior is not constant but defined by the so-called velocity profile. This profile allows combinations of constant velocities, acceleration or deceleration, or other fixed functions of time to be taken into consideration. The evaluation process is then executed based on this velocity, see, e.g., [56]. Other approaches consider the generation of longitudinal and lateral behavior simultaneously, e.g., [100].

The output of the trajectory planning algorithm is the references $x(t), y(t), \theta(t), \kappa(t)$, where $t \in [t_0, t_f]$ and $\kappa$ denotes the path’s curvature. Tracking of these reference trajectories is a control objective and is described in the next section.

4.1.3 Control Concepts

Controlling a vehicle’s motion is a crucial task for advanced driving assistance functions. In the context of a highly automated vehicle many control tasks need to be considered: e.g., lateral stability control and driving at the limits for collision avoidance. The focus in this survey, however, lies on comfortable driving on highways only. In order to concentrate on control approaches it is additionally assumed that a smooth and feasible reference trajectory is available and that the
vehicle is able to track this in respect to dynamic constraints such as maximum steering angle and maximum steering rate.

In the subsequent comparison of the different approaches that will be summarized below, important requirements from the implementation perspective are introduced. We pay special attention in the discussion that follows to the following controller properties:

- real-time capability: the control law must be executed on an embedded control unit within a defined and guaranteed calculation time.
- parametrization: tuning of parameters should be straightforward.
- structure (specific vs. general): the controller should work on different vehicles.
- robustness: since parameter uncertainties exist such as unknown load or road surfaces, and external disturbances such as side wind force and inclination of road, robust performance must be ensured.
- nonlinearities/dependence on vehicle speed: the controller must work from 0 (in the event of traffic jams) to at least 130 km/h.

These requirements typically result in a two degree of freedom controller consisting of a feed-forward term based on the reference trajectory, and a feedback controller responsible for disturbance rejection. Not all approaches will satisfy the requirements stated above. Nevertheless, some approaches can be used for simulation purposes only.

In some control concepts, the so-called look-ahead distance (LAD) is introduced in order to improve controller performance. Not the actual position with respect to the reference, but the position a certain distance ahead is considered for the control task. The choice of this distance is not a trivial matter: by choosing a greater value, the vehicle tends to cut corners, which is not desirable. However, for greater values of LAD robustness and smoother behavior can be achieved.

There are many different control schemes that are applied to the tracking task. In this survey, we want to highlight the benefits and drawback of these schemes. Before listing previous work, a short introduction to the different controllers is given.

### 4.1.3.1 PID Control

The PID controller is a simple control law which takes into account the error variable (P as “proportional”), the integral (I as “integral”), and the derivative of the error variable (D as “derivative”) see, e.g., [2]. Various adjustment rules exist for the parameters of such controllers, e.g., [69]. The control law in time-domain is given by

\[
    u(t) = k_p e(t) + k_i \int_{t_0}^{t} e(\tau) d\tau + k_d \frac{d}{dt} e(t) .
\]

The parameters \( k_p, k_i, k_d \) can either be fixed or computed by a scheme called “gain scheduling,” which updates the parameters based on the velocity or other
scheduling variables. An ideal PID controller cannot be implemented in reality, therefore the derivative part is replaced by a DT1-transfer function in practice. The main advantage of this controller is its generic applicability, i.e., it is not necessary to know the mathematical model of the plant. However, the integral part may be troublesome, and the D-part of the controller may be sensitive to noise in measurement. In many cases, this simple controller can be outperformed by other control approaches.

### 4.1.3.2 Fuzzy Control

This heuristic scheme has been introduced in [35] and is similar to a PID controller since it employs the error, its integral, and derivative. Fuzzy control is usually applied in systems where no mathematical model is known or where models are difficult to obtain. It is thus possible to use the controller for nonlinear dynamics and multiple-input multiple-output systems. The input variables are transformed into linguistic variables by using membership functions. The output of the controller is chosen based on fuzzy rules which take an “if-then”-like form. Such a rule can be designed as

\[
\text{if "lateral position error" is left then "steering" is right.}
\]

In this case, “lateral position error” is the linguistic input variable and “steering” the output variable. Tuning of the membership functions for these variables is done manually during driving tests. The controller acts in a manner similar to human behavior, because of the human-like rules. However, the tuning is not straightforward, and stability analysis is hardly possible without mathematical models. Moreover, depending on the number of variables, the rules can become unmanageable.

### 4.1.3.3 Neural Networks

Artificial neural networks have first been investigated for vehicle dynamic control in [73]. They are typically represented by a system of interconnected neurons, where each connection is assigned a certain weight that is tuned based on training data or on-line. This procedure results in an adaptive net that is capable of learning. The network can be taught to imitate driver reaction if the driver makes an allowance for a specific training phase. A controller is designed based on the model to emerge from this. The main drawbacks of this approach are the need for training data and the fact that no explanation for failure can be given.
4.1.3.4 Linear Quadratic Regulator

The linear quadratic regulator (LQR) uses a linear plant model and optimal control theory to obtain an optimal state feedback controller as described, for instance, in [50]. The control input $u$ is computed by the relation $u(t) = -k^T x(t)$, where $x \in \mathbb{R}^n$ is the states of the system and $k \in \mathbb{R}^n$ is computed in such a manner that the cost function

$$J[u] = \int_0^\infty (x^T Q x + u^T R u) dt$$  \hspace{1cm} (4.5)

is minimized with respect to an infinite time horizon. In contrast to the already mentioned control structures, this approach needs the information of a plant model in advance and actual signals of all states during operation. Since this is not given by default, a state observer needs to be implemented.

4.1.3.5 Feedback Linearization

Feedback linearization is a common technique rendering the closed loop system linear with the help of nonlinear compensation, see [33, 49]. Flatness-based approaches are closely related to this linearization, for further details, refer to [53].

4.1.3.6 Sliding Mode Control

The sliding mode control (SMC) approach relies on a variable structure controller and is robust with respect to a specific class of modeling uncertainties and external disturbances [20]. It consists of two main design parts: in a first step, a desired dynamics is defined by the so-called sliding variable, e.g., $s = \dot{e} + \lambda e$, which ensures for $s = 0$ that the error $e$ converges to 0 in finite time, with the weighting factor $\lambda$. In the second step, a controller is defined so that this desired dynamics is obtained, for example, $\dot{s} = -k \text{sign}(s)$ with the control parameter $k$. This switching law may generate chattering in the control input, which can be prevented by using higher-order SMC. A well-known second-order sliding mode controller is the “super-twisting algorithm,” [52]. A downside of this approach is that it is derived in continuous-time, but its discrete-time behavior strongly depends on the sampling frequency [62].

4.1.3.7 Model Predictive Control

At each time step an internal model is used to predict the system behavior over a predefined horizon, where an optimal control input sequence is generated that minimizes a certain cost function. This approach allows the consideration of
different types of constraints for states and inputs, which is a major advantage of MPC. The main disadvantage is the high computational complexity, which is the reason for its scarce utilization in real-time applications. Additionally, there is need for an explicit model for prediction, and external inputs and disturbances need to be known fairly accurately in advance for the entire prediction horizon [6]. A simplified MPC formulation similar to the one derived in [14] can be represented by

\[
\begin{align*}
\min_{u_0, \ldots, u_{N_C-1} | t} & \sum_{k=0}^{N_p-1} x_k^T | t Q x_k | t + \sum_{k=0}^{N_C-1} u_k^T | t R u_k | t \\
\text{subject to} & \quad x_{k+1} | t = A x_k | t + B u_k | t, \quad k = 0, \ldots, N_p - 1, \\
& \quad x_{k+1}^\min | t \leq x_{k+1} | t \leq x_{k+1}^\max | t, \quad k = 0, \ldots, N_p - 1, \\
& \quad u^\min \leq u_k | t \leq u^\max, \quad k = 0, 1, \ldots, N_C - 1, \\
& \quad u_k = 0, \quad N_C \leq k \leq N_p.
\end{align*}
\] (4.6)

where \( N_C \) is the control horizon, \( N_p \) the prediction horizon, \( N_C \leq N_p \), and \( x_{k+1}^\min | t \), \( x_{k+1}^\max | t \), \( u^\min \), \( u^\max \) the constraints for states and input, respectively. The subscript \( k + 1 | t \) denotes the value of the variable \( k + 1 \) steps ahead of the current time \( t \).

### 4.1.3.8 \( H_\infty \) Control

This robust approach attempts to control a plant affected by modeling uncertainties and parameter variations [82, 102]. An optimization problem needs to be solved once, minimizing the so-called \( H_\infty \)-norm of a particular transfer function \( T \) of the control system. The transfer function \( T \) is defined by the control objective (noise rejection, tracking, etc.), the plant, additional uncertainty models, weighting transfer functions, and the feedback control matrix, which is the optimization parameter. For a stable single-input-single-output system, the \( H_\infty \)-norm is the largest value of the frequency response magnitude. For a stable multi-input-multi-output system it is the largest singular value \( \tilde{\sigma} \) across all frequencies \( \omega \)

\[
\|T\|_\infty := \sup_{\omega \in \mathbb{R}} \tilde{\sigma}[T(j\omega)].
\] (4.7)

The advantages are inherent robust stability and robust performance. Depending on the approach (choice of weighting/shaping transfer functions), this design may result in dynamic controllers of high order, which are complex to realize in practice.
4.1.4 Partitioning the Problem

The control design of automated vehicles can be partitioned into the following tasks:

(a) Longitudinal control:
   A common approach to control the longitudinal behavior is to divide the controller level into an inner loop for throttle and brake control, and an outer loop for velocity or acceleration tracking. This approach is exploited in many of the listed publications, for an introduction, see [74].

   There are typically two fields of research: the first one tackles the ACC functionality, where a desired velocity is to be maintained or a target is to be followed at a safe distance. In [43], safety issues in real world contexts and vehicles equipped with ACC in 2003 are discussed. Since the control task of the ACC is more or less completed, we refer to a review on earlier works on ACC in [8].

   The second field deals with platooning, where a leader vehicle shall be followed by the other vehicles. A survey on platooning can be found in [6]. With the assumption of communication between the vehicles in the platoon, the so-called cooperative ACC (CACC) arises, as treated in [81].

(b) Lateral control:
   For comfortable automated driving, the central lateral control tasks are lane keeping and lane changing. From a control point of view, the main difference of these two functions is that the LKA tries to keep the vehicle in the middle of the lane, i.e., a stabilization task, while LCA tries to follow a reference trajectory computed by another entity (trajectory planning), which corresponds to a tracking task. In publications from the late 1990s, e.g., [38, 68], magnetic strips are considered to allow localization of the vehicle. In the California PATH program, the lateral sensing system consisted of magnetic markers embedded along the road center line and two sets of magnetometers installed under the front and rear bumpers of the vehicle, see, for example, [40]. At the present time the algorithms are either vision-based [87] or GPS-based [45, 64]. However, the control task for lateral control stays the same: minimize the lateral displacement and the angular error with respect to the reference trajectory (either lane center or polynomial).

(c) Combined control:
   Assistance functions for both longitudinal and lateral motion are in general more complex than the control strategies discussed so far. The separate design of the steering angle happens mostly under the assumption that the velocity is constant. This of course is not always the case and may lead to problems. For example, at higher speeds, a smaller steering angle is needed for the same lateral displacement at a certain LAD. However, many publications handle the steering and the velocity inputs separately, using the current velocity of the vehicle in the lateral control design.
The control techniques described in the next sections are partitioned according to problems (a)–(c).

### 4.2 Fuzzy Control

(a) In [63], a fuzzy controller for throttle and brake control is proposed for a combination of ACC and Stop&Go. Car-following behavior based on fuzzy control is also discussed in [101] and in [90], where the latter proved stability based on the so-called linear matrix inequalities (LMI) conditions.

The longitudinal controller is implemented by a neuro-fuzzy controller in [13]. The output layer is updated on-line via a gradient algorithm. According to the authors, an attractive feature of this method is that it does not require training data or the vehicle longitudinal dynamic model. Simulations show that in comparison to a PID controller, the neuro-fuzzy controller exhibits a smoother performance and hence provides a more comfortable feeling for passengers onboard. In thesis [11], more details on this approach can be found.

The authors of [72] introduced a fuzzy control strategy for CACC in order to achieve human-like driving behavior in a cooperative environment and produce satisfactory results.

(b) In [12], lateral control is conducted via fuzzy PD control. A generic algorithm method is used to optimize the parameters of the fuzzy controller.

A comparison of a fuzzy controller and a control law derived by using the nonlinear kinematics model with a Lyapunov function has been published in [58]. The fuzzy controller demonstrates effective performance.

A simple lane change fuzzy controller with four rules is presented in [64], where two different control strategies are designed: one for straight road driving, the other one for lane change maneuvers. Simulations indicate that the switching between the controllers is smooth and the overtaking maneuver can be executed similar to human performance.

In [71], a cascade architecture for lateral control has been presented. The low-level is governed by a discrete PID controller. The high-level is governed through fuzzy logic, which computes desired steering and angular speed for the PID controller. This control architecture gives good results for different vehicle speeds and curves. The high-level output signals are smoothed out by the lower level PID controller, which avoids undesirable oscillations of the steering wheel that could be particularly dangerous at higher speeds.

A genetic algorithm has been applied to adjust a fuzzy controller automatically in [70], which overcomes the tuning problem of the fuzzy control scheme.

In [93], two controllers based on fuzzy logic are designed for the lateral offset and angle error, respectively. The speed of the vehicle is taken into account and the two outputs are finally weighted and summed up to obtain the steering wheel angle for the actuator.
4.3 Linear State Feedback Control

(b) The lateral dynamics is controlled by using LQR design for the state feedback controller in [74]. A feed-forward term is computed considering the road curvature. A similar LQR approach incorporating constraints on the steering angle has been discussed in [48].

In [40], an output-feedback controller has been derived by the LQR method when only position feedback is available, preserving certain robustness properties such as phase margin and gain margin.

In [87], lane keeping performance on a curved road is improved by adding the integral of the lateral offset error to the state feedback controller. The authors implement a multi-rate Kalman Filter for state estimation in order to allow the LQR controller to operate at the fast update rate of the microprocessor. This multi-rate control scheme can reduce the inter-sample ripples in the yaw rate.

Bilinear matrix inequality (BMI) optimization is presented in [7] to compute a state feedback controller, which is capable of lane keeping as well as obstacle avoidance assistance.

4.4 Sliding Mode Control

(a) In [75], a sliding mode-based longitudinal control design for platoons is presented using a time-variant LAD. Similarly, a sliding mode controller for platooning with wireless communication is demonstrated in [74]. The experiments indicate that the spacing performance and ride quality are superior to human driving skills.

Cruise and longitudinal tracking control of vehicles are considered in [24] and [25]. The first publication exploited suitably designed observers for SMC. The second presented an additional collision avoidance functionality.

SMC for hybrid electric vehicles has been applied for cruise control in [27]. The authors claim that this approach yields good performance, especially when compared to a PID controller.

The authors of [98] focus on a sliding mode controller for platooning due to its robustness to parameter uncertainties and external disturbances.

(b) In [1], the first sliding mode controller for steering a city bus has been published. The author introduced the so-called Ackermann-model for vehicle steering, which is used quite extensively. The sliding mode approach is extended in [38] to a chatter-free performance for automated highway systems. In thesis [39], loop shaping, $H_{\infty}$ control, and SMC for lateral motion on a tractor semi-trailer are compared. It is shown that the performance of the controllers strongly depends on the choice of the LAD. The robustness of a sliding mode controller has also been mentioned by Hatipoglu et al. [31], where lane changing and lane keeping maneuvers are implemented as different modes. Sliding mode
controllers for trajectory tracking or path tracking can also be found in [84–86]. In these papers, the lateral displacement and the heading error of the vehicle are combined in the sliding manifold for the steering input. This combination demands that both variables converge and have been implemented using the kinematic bicycle model as well as four-wheel steering vehicles. In [32], active front steering and dynamic stability control are coordinated. For the steering functionality, a sliding surface for yaw rate tracking is employed. The dynamic stability control is developed independently for emergency maneuvers. A rule-based integration scheme using a fuzzy membership function chooses between the two subsystems. The authors state that this integrated approach leads to improved vehicle stability.

In [88], lateral dynamics are controlled by a super-twisting controller, which is robust to time-varying speed, curvature, and parameter uncertainties, while chattering is avoided. A terminal sliding mode method based on a look-ahead scheme has been presented in [99]. An adaptive algorithm is used to tune the controller parameters.

A sliding mode controller is compared in [16] to a driver-model-based controller, which consists of a PI controller with LAD. Two sliding surfaces are designed: one for the lateral displacement and one for the heading error. Since only one control input is available, these two sliding surfaces are then combined to one sliding variable. The SMC achieves higher accuracy in path tracking, but needs a larger steering input. With a saturation on the steering angle, both controllers yield similar performance. It is also demonstrated that a SMC using preview control does not outperform the conventional SMC.

(c) In [30], a sliding mode controller is implemented by the super-twisting algorithm for the tracking problem of a car-like system called “Robucar.” The derived control laws are of discontinuous type and may lead to discontinuous velocities in practice. This difficulty is overcome by taking into account the actuator dynamics. The approach works well for the robot, but needs to be adapted for autonomous vehicles.

In thesis [21], an integrated high-level control for longitudinal and lateral dynamics has been introduced. A higher-dimensional sliding surface is chosen to compute the reference for the low-level controller. In the higher level, the dynamics of the longitudinal and lateral control are coupled, whereas the low-level control treats the dynamics separately. A simple PI controller is used in the feedback loop, a feed-forward block is in charge of compensating for the estimated disturbances.

Discrete-time and continuous-time sliding mode controllers have been investigated in [19] on a four-wheel-robot. The work points out that the discrete-time controller has a lower longitudinal error, while the continuous-time controller has a lower lateral error.

The authors of [97] tested a sliding mode based control on the autonomous vehicle “Kuafu-II,” where the driving control system integrates both the longitudinal and lateral controllers. The longitudinal controller is designed as a main speed controller and a space controller for obstacle avoidance. The
parameters of the speed controller are computed by an adaptive law, while the space controller is implemented as PI controller with gain scheduling. For low velocities, the lateral controller will use Stanley’s law that is discussed in Sect. 4.7 and published in [89]. In higher speed ranges, the controller switches to a sliding mode controller in order to improve tracking accuracy.

### 4.5 Model Predictive Control

The main advantage of the MPC approach is the possibility for explicitly handling constraints. Moreover, the control design for nonlinear models or combined dynamics is easier than with multiple-input multiple-output control schemes. Since MPC is able to consider highly dynamical systems, many lateral controllers are also combined with longitudinal control. The controllers from the lateral control (b) are thus moved to the combined problem (c). The main drawback has long been the computational complexity, which is why a computation time is stated in many publications. Efficient solvers exist, however, that make use of MPC in real-time applications possible. A little more computation time is not critical, especially in comfortable scenario. Due to the constraint handling, many publications on MPC also treat collision avoidance maneuvers. In these maneuvers, driving at the limits occurs and constraints become active, which induces longer computation times. However, the collision avoidance maneuvers are practically “built-in,” which is a very nice property of this control scheme.

(a) Early works using MPC for ACC control tasks have been presented in [5] and [17]. In [55], an MPC strategy has been used for ACC. A low-level controller compensates for vehicle dynamics and tracks the desired trajectory. The high-level MPC minimizes a quadratic cost function that consists of minimal tracking error, low fuel consumption, and accordance with driver dynamic car-following characteristics. Driver longitudinal ride comfort, driver permissible tracking range, and rear-end safety are formulated as linear constraints which are softened to avoid computational infeasibility.

In [79], a nonlinear MPC approach is investigated using only one control loop (instead of an inner and outer control loop). The overall model is augmented in order to combine distance and speed tracking control.

(c) In [9], a nonlinear model predictive approach (NLMPC) is described, where its performance is checked for a double-lane change maneuver on snow. The performance depends on the horizons of the control scheme and the vehicle speed. The computation time has been investigated using the NPSOL software package and is given for inactive constraints by 0.1 to 0.3 s, for active constraints by 0.1 to 1.6 s. An extension of this work that examines side wind rejection has been published in [47], with an average NPSOL computational time 0.13 s, and in the worst case 0.38 s.
In [23], another extension has been proposed for combined braking and steering, where a linear time-varying (LTV) MPC is recast into a quadratic program. The performance is enhanced when compared to steering only.

NLMPC has also been applied in [3]. Additionally, a cruise speed-profile generator is responsible for adapting the cruise speed at lateral maneuvers. The proposed control law allows the consideration of variable longitudinal speed and sliding during lateral maneuvers. The same author presented a nonlinear longitudinal control strategy considering powertrain dynamics in [4]. A control architecture is introduced which combines the steering and the longitudinal controllers so as to ensure the simultaneous control of longitudinal and lateral motions.

In [51], the authors aim to reduce the computational complexity of the MPC approach. An approximate explicit MPC scheme is demonstrated, where suboptimal controls are used with its computation time being drastically reduced and its accuracy being significantly improved. The authors generate grid points to build nodal state parameter vectors in the state space, for which optimal solutions are to be computed off-line. The nodal state parameter vectors construct polytopes, for which equivalent suboptimal feedback control gains are computed off-line.

Automated lane change maneuvers have also been treated in [67], where the longitudinal and the lateral control is handled by two loosely coupled low complexity quadratic programs.

In [57], an LTV-MPC using sparse clothoid-based path description is implemented. Only a few waypoints are computed to represent the road, which makes the cost function minimization more efficient by allowing larger prediction distances.

4.6 Other Concepts

(b) The so-called $H_2$ control design for lane keeping has been presented in [80]. LQG and $H_{\infty}$ have been investigated by Eom et al. [22], where it has been shown that $H_{\infty}$ is more robust and produces lower control input.

Low energy consumption is accomplished by $H_2$ control in [42]. Additionally, a $H_{\infty}$ controller for disturbance rejection is employed. A switching control for the two subsystems was designed for further improving system performance.

In [66], an adaptive output-feedback controller called “self-tuning regulator” is demonstrated. Adaption laws with the so-called parameter reduction are stated. All vehicle parameters are considered unknown and therefore this approach can be realized for a wide variety of vehicles. It is also robust to variations on curvature and lateral wind. The main disadvantage of this approach is the high computational complexity, as has been pointed out in [83].

The self-tuning regulator has been compared to $H_{\infty}$ control, fuzzy control, and a P controller in [15]. Three different situations are simulated in order to
investigate the robustness of the approaches: first, the road friction coefficient is varied. The P controller produces the largest error while the self-tuning regulator has the best response. Second, the longitudinal speed is changed. Similar performances as in the first case have been observed. Third, lateral wind is introduced as disturbance. The P control is less affected by the wind force than the other controllers. The authors state that the self-tuning regulator is one with the best performance, at least from a simulation point of view. However, it is also the most complicated to be implemented.

The authors of [59] propose an active front steering control based on the yaw rate tracking error. They use two PID control loops, where one loop tracks the yaw rate reference signal that is generated by the other. Seven control gains and the LAD must be chosen, where the design parameters are tuned through numerical optimization.

In [44], a steering control algorithm that does not depend on vehicle parameters has been presented. The steering angle is computed in such a manner that a desired yaw rate can be tracked and adapts the corresponding gain in order to compensate the yaw rate error. This control law can be applied for any vehicle without any information on vehicle parameters. However, real-time experiments for automated driving have not yet been tested by the authors.

(c) In one of the earlier publications on autonomous driving [68], an $H_\infty$ based lane keeping controller has been investigated that works for both curved and straight highway sections without knowledge of the radius of the road curvature. Another early approach [10] applies a gain scheduled P controller for active safety systems.

The authors of [45] propose a steering controller based on finite preview optimal control, which is basically an LQR controller plus feed-forward term. The speed controller is designed as a simple proportional control and the desired speed is set so that the lateral acceleration stays below a predefined value. This controller in combination with the steering controller significantly reduces lateral acceleration when compared to constant speed.

In [37], a longitudinal control via PID controller and a lateral control via $H_\infty$ loop shaping have been published. The authors employed a feed-forward term for curvature dependence and also drew attention to the high computational burden this involved. The same research group published an adaptive backstepping approach in [36], using the yaw rate as virtual input. The controller is robust with respect to parameter variation in the matrix $A$ of the dynamic bicycle model (4.3). However, variations in the input vector $b$ have not been investigated.

Backstepping is also mentioned in [29], where a simple trajectory planning algorithm for the lane change maneuver is presented and the tracking controller produces satisfactory performance. Coupled longitudinal and lateral backstepping control with a control robustification is discussed in [65].

Another backstepping-based approach has been studied in [95], where the authors consider forward and backward driving. A feedback law with orientation control has been investigated mainly for backward driving, since
forward driving is improved when using a feedback law without this orientation control. The same author also describes exact input-output-linearization in his thesis [94], where the nonlinear transformation yields two linear decoupled systems that are stabilized by simple state feedback controllers. The exact linearization shows good performance at higher speeds, while the backstepping-based approaches have smooth behavior also in the starting and stopping processes, since the singularities at standstill are avoided by velocity-independent feedback gains.

A modular approach has been presented in [77]. A dynamic feed-forward term is computed using a model of the system with a state feedback controller and a pre-filter. A simple P controller for lateral displacement and angular error is combined with a disturbance observer that provides steady-state accuracy. Gain scheduling for the controller parameters depending on the velocity is proposed. A lookup table is obtained using a parameter space approach by the so-called gamma and beta stability, as also described in [92].

Flatness-based control for nonlinear vehicle dynamics has been proposed in [26]. A nonlinear bicycle model is used and a flat output is derived. Given the desired trajectory of the flat output, the tracking controller computes the control input for the nonlinear state feedback control block. This block then generates the steering and velocity for the vehicle. The performance has been shown for a lane change maneuver.

In [60] and [61], lateral and longitudinal controllers are combined to perform some coupled maneuvers, such as stop-and-go control with obstacle avoidance and lane change maneuvers. A flatness-based controller computes steering and velocity for the nonlinear bicycle model. Since derivatives of reference and measured vehicle signals are needed in these computations, an algebraic nonlinear estimation is introduced for the numerical differentiation of the signals. The results indicate good performance even under sudden and sharp maneuvers.

The authors of [76] compare a simple kinematic controller consisting of a feed-forward term plus P controller with a flatness-based control approach. This approach combines a flatness-based feed-forward control with an LQR feedback control. Disturbances can be rejected in both cases, but the simple approach has higher overshoot and the flatness-based approach is faster. The main drawback of the flatness-based strategy is that it needs additional information from the vehicle, specifically the side-slip angle and the yaw rate.

4.7 Autonomous Vehicles

This section briefly lists the control structures demonstrated on autonomous vehicle prototypes. The control schemes are primarily standard approaches, since the focus has been on environment perception or trajectory planning.
The team of Stanford [89] implemented the velocity and steering controllers separately for its vehicle “Stanley” at the DARPA Grand Challenge. The velocity control is implemented as a PI controller, while the steering angle is computed by a nonlinear feedback function of the cross-track error, which is known as the “Stanley method.” The performance of this controller degrades seriously at higher speeds, as also mentioned in [83].

The controller of “Junior” consists of an MPC strategy with a PID block [54]. The feed-forward term has to be tuned specifically for the vehicle. Longitudinal control for “Leonie” has been presented in [78]. The work introduces the so-called Grip Value, which is an indicator for potential changes of road conditions and is applied as an additional input to the controller.

In [103], research on the autonomous vehicle “Bertha” has been published. A Luenberger observer is designed to estimate the lateral displacement and its derivative with respect to time. The lateral control consists of a feed-forward part for the desired yaw rate that is based on the road curvature at a look-ahead point, and a feedback component realized by a PI controller. Then, an inverse single-track model is used to compute the desired steering angle. An additional observer is responsible for the offset compensation of the steering angle in order to obtain steady-state accuracy.

### 4.8 Comparison

A comparison of control approaches has been conducted in [83], where different driving courses have been simulated with a maximum velocity of 20 m/s. The Stanley method [89] has been tested and the authors observed that it is not robust to a rapid lane change maneuver for collision avoidance. However, it is well suited for standard driving maneuvers. Optimal control is proposed using LQR, LQR with feed-forward term, and preview control. LQR alone fails the rapid lane change maneuver entirely at higher speed since information about the path is not included. The optimal control with feed-forward term considers curvature and speed, and therefore improves performance. Optimal preview control combines the optimal control with road information, which gives the best performance for highways at (almost) constant speed. The survey demonstrates that depending on the maneuver and the vehicle speed, the controllers show different performance.

In addition to the comparison in the literature a simple example of a highway overtaking maneuver is simulated, using different controllers for lane keeping.

#### 4.8.1 Simulation Example

The simulated overtaking maneuver is executed on a curved road (road radius $R = 1930$ m). The longitudinal velocity of the ego-vehicle is $v_x = 120$ km/h.
Fig. 4.3 Overtaking maneuver on a curved road at high speeds \( (v_x = 120 \text{ km/h}) \). The driven trajectory of the ego-vehicle is depicted by the thick black line.

while the speed of the other vehicle is 75 km/h. The maneuver is illustrated in Fig. 4.3, marking the ego-vehicle in red and its overtaking trajectory in black. For control and simulation a dynamical bicycle model has been used with the following parameters: 

\[
m = 1564 \text{ kg}, \quad c_r = c_f = 140000 \text{ Nm/rad}, \quad l_f = 1.268 \text{ m}, \quad l_r = 1.620 \text{ m}, \quad I_z = 2230 \text{ kgm}^2.
\]

The parameters of the different controllers have been chosen as follows:

- The gain of the Stanley method is \( k = 5 \), the LQR matrices are \( Q = \text{diag}(1, 0) \), \( R = 1 \). The super-twisting algorithm of the sliding mode controller has the gains \( k_1 = 0.2 \), \( k_2 = 0.0005 \). The sliding variables are defined by \( s_i = \dot{e}_i + \lambda e_i \), \( i \in \{ y, \theta \} \) with \( \lambda_y = 5 \) and \( \lambda_\theta = 1 \). The weight for the combined sliding variable \( s = s_y + \lambda s_\theta \) is \( \lambda = 0.1 \). The membership functions of the fuzzy controller are shown in Fig. 4.4. The four rules were defined according to [64]. Note that the membership functions have been found by trial and error and also that the tuning was time-consuming.

For the MPC controller, the matrix \( Q \) and input weight \( R \) are the same as in the LQR cost function. The input constraint has been set to \( \delta_{\text{max}} = 0.03 \text{ rad} \), and the control horizon to \( N_c = 30 \) with a sampling rate \( T_c = 0.01 \text{ s} \).
The lateral displacements of the different controllers are depicted in Fig. 4.5. SMC exhibits the smoothest lateral errors while tracking the dynamic parts of the trajectory, and the LQR approach has the largest error. However, the angular error of the LQR approach has no overshoot as illustrated in Fig. 4.6.

The computed steering angle is shown in Fig. 4.7. As expected, SMC applies higher input values, which has also been observed in [16]. The Stanley method exhibits the smoothest performance.

To sum up, the Stanley method is distinguished by its simplicity and smooth performance during standard driving scenarios. The LQR exhibits low computational
complexity, robustness, and simple parameter tuning. SMC is a very robust controller, but its discrete-time performance has to be checked. MPC has a high computational complexity, but good performance and has the ability to handle constraints and include collision avoidance, which make this approach very appealing. For this reason applications based on MPC-algorithms to solve automated driving assistance tasks efficiently are still subject to intensive research.

4.9 Outlook

The latest research trends on control in automated driving can be partitioned into two main research areas, namely

- MPC and efficient solvers, e.g., [14, 57], due to its capability of dynamics and constraint consideration,
- CACC and coordinated maneuvers utilizing information from other traffic partners, e.g., [81, 92], due to the ongoing research on inter-vehicle communication.

When using communication for information exchange between traffic partners, networked control strategies may be exploited to improve the overall traffic performance, see, e.g., [6]. Moreover, automated driving in urban areas poses new challenges: intersections and other traffic participants such as bicyclists and tractors need to be considered and pedestrian safety must be ensured. These topics have been investigated recently and will be of interest in the future. The authors look forward to improvements and new challenges in these fields.

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