Tailoring quantum gases by Floquet engineering

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Floquet engineering is the concept of tailoring a system by a periodic drive, and it is increasingly employed in many areas of physics. Ultracold atoms in optical lattices offer a particularly large toolbox to design a variety of driving schemes. A strong motivation for developing these methods is the prospect to study the interplay between topology and interactions in a system where both ingredients are fully tunable. We review the recent successes of Floquet engineering in realizing new classes of Hamiltonians in quantum gases, such as Hamiltonians including artificial gauge fields, topological band structures and density-dependent tunnelling. The creation of periodically driven systems also gives rise to phenomena without static counterparts such as anomalous Floquet topological insulators. We discuss the challenges facing the field, particularly the control of heating mechanisms, which currently limit the preparation of many-body phases, as well as the potential future developments as these obstacles are overcome.

T he philosophy of quantum simulation with ultracold atoms is to engineer the Hamiltonian of interest by adding the relevant terms step by step, for example, tailored potentials, additional internal states or controlled interactions. A new dimension is opened by time-dependent control of the system, which allows adding terms such as artificial gauge fields or density-dependent tunnelling. Modifying a system by periodic driving is called Floquet engineering and has been proven to be a very powerful tool\textsuperscript{1,2,3}. Periodic driving has been employed in quantum gas experiments since the early days as a tool to probe the excitation spectrum. Floquet engineering radically changes the perspective and uses the same technique to modify the system. When periodically shaking an optical lattice, instead of performing spectroscopy of the band structure, one can hybridize the bands to produce bands with new physical properties. Periodic driving thus allows the engineering of new, effective static Hamiltonians with properties that are otherwise not accessible in the specific system or fundamentally impossible in any static system. In classical physics, Kapitza's pendulum is one such example, where the upright state emerges as a new, stable equilibrium position due to the fast drive of the pivot point\textsuperscript{4}. Periodic driving is also used in ion traps, where an oscillating saddle-point potential provides the stable trapping of particles\textsuperscript{5}.

Floquet engineering as a general concept is used in many areas including solid-state physics\textsuperscript{6} and synthetic systems such as photonic waveguides\textsuperscript{7}, and it stimulates intense exchange among these areas. In cold-atom research, this approach is particularly fruitful due to the high control over these systems as well as easily accessible timescales. Driving a solid-state crystal at the relevant scales requires an oscillating force from the electric field of terahertz radiation with extremely high intensity, which can only be provided by pulsed lasers and requires suitably fast measurement schemes\textsuperscript{8}. In cold atoms, the driving frequency is typically in the kilohertz regime and the driving is realized, for example, by simple phase modulation of the laser beams forming the optical lattice, which allows large displacement amplitudes of many lattice constants as well as very versatile driving schemes. Therefore, in cold-atoms systems, the limitation is not the accessibility of driving regimes, but the heating inherently associated with Floquet systems.

In this Review, we want to give an overview of the techniques known as Floquet engineering and discuss the new physics that has become accessible to ultracold atoms with a focus on optical lattices. We take an experimental perspective and introduce a few basic concepts in sufficient detail for researchers entering the field to understand the central ideas, referring the reader to other reviews for a more comprehensive treatment of theoretical ideas\textsuperscript{9–12}. Naturally, this short review cannot cover all phenomena related to periodic driving such as quantum turbulence\textsuperscript{13}, Bose fireworks\textsuperscript{14} or pattern formation\textsuperscript{15}, and we restrict the discussion to systems that are explicitly described within a Floquet picture.

In designing Floquet protocols, cold-atom experiments can choose from a large toolbox of experimental techniques, including lattice shaking, amplitude modulation, additional Raman beams, modulation of interaction strength, modulation of external field gradients or a combination of these methods. These modulation schemes can be employed to renormalize tunnel elements, hybridize Bloch bands or induce laser-assisted tunnelling in tilted lattices. These different methods will be discussed together with the scientific goals that they can address.

Effective Hamiltonian and renormalized tunnelling

Floquet systems are periodically driven systems described by a Hamiltonian $H(t + T) = H(t)$ with time $t$, driving period $T = 2\pi / \Omega$ and driving frequency $\Omega$. The primary interest in Floquet systems is the fact that despite being time dependent, they can be described by a time-independent effective Hamiltonian $H_{\text{eff}}$, if one probes at multiples of the driving period, that is, the time-evolution operator is given by $U(t_0 + T, t_0) = e^{-iH_{\text{eff}} T}$ (refs. 1,2), where $i$ is the imaginary unit and $h$ is Planck’s constant divided by $2\pi$. The dynamics within one driving period—the so-called micromotion—can often be separated from long-time dynamics. For the calculation of the effective Hamiltonian, one typically resorts to a high-frequency expansion in powers of $1/\Omega$ (refs. 1,2). When the driving frequency is larger than the relevant energy scales, for example, the bandwidth in the case of an optical lattice, a high-frequency approximation containing only the leading terms of the expansion often yields a suitable description of the relevant physics. The lowest order is simply given by the average over one driving period $H_{\text{eff}} = \langle H(t) \rangle_T$.

As a simple but insightful example, let us consider ultracold atoms in a driven one-dimensional optical lattice, that is, a periodic potential formed by the interference of two laser beams. In the tight-binding description, the dispersion relation is given by $E(k) = -2 \cos(kd)$, with tunnelling element $J$ determining the bandwidth $4J$, quasi-momentum $k$ and lattice constant $d$. Lattice shaking can be easily realized by periodically changing the frequency of one laser beam, resulting in an oscillating inertial force of the form $F \cos(\Omega t)$, that is, with zero average force. For an
atom prepared in a certain quasi-momentum state, lattice shaking leads to a periodic oscillation in quasi-momentum space. In a high-frequency approximation, the effective energy is given by the average over the varying energy explored during one oscillation (Fig. 1a). The effective Hamiltonian is now described by the tight-binding dispersion $E_{\text{eff}}(k)$ with a renormalized tunnelling element $\tilde{J}_{\text{eff}}(k)$, where $J_0$ is the Bessel function of the first kind of order zero and $K_0$ is the driving strength. This renormalization of $J$ with the Bessel function was experimentally mapped out by studying the rate of expansion of an atomic cloud, which is directly related to $J_{\text{eff}}$ (Fig. 1a). The phenomenon of complete suppression of tunnelling at the zero crossing of the Bessel function is known as dynamical localization. The same effect as lattice shaking can also be produced in a static lattice by applying an oscillating external force, for example, from an oscillating magnetic field gradient.

For sufficiently large driving amplitudes where $J_0(K_0) < 0$, a negative tunnelling element is obtained corresponding to an inverted band structure with new minima at the edges of the Brillouin zone. A Bose–Einstein condensate (BEC) in a shaken lattice will recondense at the new minima, which can be directly observed in the quasi-momentum distribution obtained by a time-of-flight expansion (Fig. 1a, bottom, inset). This sign inversion of the tunnelling elements is particularly interesting in a triangular lattice, where it can lead to frustration and intriguing magnetic phases of the classical XY model.

The scheme discussed so far considers the renormalization of the lowest Bloch band via periodic driving. Such a single-band picture is justified when the shaking frequency is off-resonant from the higher bands, that is, it lies well below the bandgap. A complementary, powerful approach is to utilize the higher bands by resonantly coupling to them and consequently hybridizing the lowest band with a chosen higher band. A resonant coupling between the bands can be realized either by lattice shaking or by amplitude modulation. In this scheme, an intuitive picture is the dressing of bands, that is, mixing of the Bloch states of the bands for each quasi-momentum, analogous to the dressing of internal atomic states by light fields. In contrast to tunnel renormalization, this scheme allows to drastically modify the dispersion of the lowest band beyond the cosine shape of an s-band, for example, realizing a double-well dispersion in a one-dimensional lattice by dressing the s-band with the p-band. The tunability from single-well to double-well dispersion can be mapped to a ferromagnetic spin model, allowing the observation of many phenomena such as quantum critical scaling. Resonant shaking is also relevant between two separated s-bands in a non-Bravais lattice with a sublattice offset, linking to the creation of artificial gauge fields (as discussed below).
Artificial gauge fields and topological band structures

When the driving scheme breaks the time-reversal symmetry (TRS), it can give rise to new effects such as artificial gauge fields. The tunnel renormalization for off-resonant driving can then generate a complex tunnelling element $J_{\text{eff}} \propto e^{i\theta}$ with Peierls phase $\theta$ (refs. 31–32; Fig. 1b). TRS can be broken, for example, by the multistep scheme shown in Fig. 1b or by the circular shaking of a two-dimensional lattice $^{26,20,21}$. In a one-dimensional lattice, the Peierls phase leads to an effective band structure with the minimum shifted to finite quasi-momentum $\theta/2d$, which can again be directly revealed via the condensation of bosons at this minimum (Fig. 1b). This shift in band structure, however, disappears for a description in a moving reference frame and is therefore not a gauge-invariant effect. Gauge-invariant effects appear either by implementing time-dependent Peierls phases, which correspond to an artificial electric field $^{14}$, or by going to higher dimensions where spatially dependent Peierls phases give rise to artificial magnetic fluxes $^{4,15,25–27}$ as explained in the following. In higher dimensions, the TRS breaking of the driving scheme indeed induces a TRS breaking of the effective Floquet Hamiltonian.

The engineering of artificial gauge fields can be understood as follows: in quantum mechanics, a particle with charge $e$ acquires an Aharonov–Bohm phase $\Phi = e/\hbar \phi_B$ when encircling a magnetic flux $\phi_B$. In the Peierls substitution on a lattice, this phase is attached to the tunnelling elements and the sum of the Peierls phases around a plaquette yields a phase $\Phi = \sum_{\text{plaquettes}} \theta_i$, which corresponds to a net magnetic flux of $\Phi_B = \frac{\Phi}{2\pi}$ through the plaquette. Using Floquet engineering, these Peierls phases are directly implemented independently of a real magnetic field. This means that the effects of a magnetic field on a charged particle become accessible for neutral particles such as cold atoms. Furthermore, one can easily imprint any value of the Peierls phases and thereby reach $\Phi = \pi$ corresponding to a magnetic flux quantum $\Phi_B = \Phi_{2\pi} = h/2e$ per plaquette, which would require a magnetic field of thousands of teslas in a solid-state crystal. Often, $\Phi$ itself is called the magnetic flux, which corresponds to setting $\hbar = e = 1$, and we adopt this convention in the following discussion. For real magnetic fields, a central concept is the gauge freedom, which means that different vector potentials describe the same magnetic field. In Floquet engineering, however, one experimentally implements a specific gauge by directly imprinting specific Peierls phases on certain bonds around the plaquette. For this reason, one usually speaks of artificial gauge fields as opposed to artificial magnetic fields. This gauge choice of the Floquet protocol can, in fact, make a difference, for example, in the adiabaticity timescale for ramping up a magnetic flux $^{20}$. The same considerations also hold true for artificial gauge fields in bulk systems $^{25}$.

Another way to induce Peierls phases is laser-assisted (or Raman-assisted) tunnelling $^{26,29}$. In this scheme, tunnelling is suppressed by applying a tilt with an energy shift of $\triangle$ per lattice site and then resonantly restored by a Raman transition of two laser beams with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and frequencies $\omega_1$ and $\omega_2$, where the resonance is ensured via the two-photon detuning $\delta \equiv \omega_1 - \omega_2$, for $h\delta = \Delta$ (Fig. 1c). The effect of the Raman laser beams can also be viewed as a secondary moving lattice, which induces time-periodic amplitude modulation at lattice site $l$ at position $\mathbf{r}_l$ with locally varying phase $\phi_l = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}_l$. The effective tunnelling is described by a Bessel function of the first kind of order one $J_1(\mathbf{K})$ and a Peierls phase on a bond between the sites $l$ and $l'$, which stems from the local phases of...
the two laser beams, that is, the local phase of amplitude modulation at the respective lattice sites as \( \theta_{ij} = (\phi_i + \phi_j)/2 \). For a small driving amplitude \( K \), where the Bessel function can be linearized, the tunneling element is, therefore, given by \( Ke^{i\theta_{ij}} \) (ref. 7).

In a two-dimensional square lattice, one can arrange the laser-assisted tunneling such that the Peierls phases yield a net magnetic flux \( \Phi \), the value of which depends on the angle between the laser beams (Fig. 1c). This model is known as the Hofstadter model, which has topologically non-trivial bands with non-zero Chern numbers, leading to the quantum Hall effect (Fig. 2a). Experiments realizing this model have revealed the extended periodicity of the magnetic unit cell (Fig. 2b) and the Chern number of the lowest band was measured via the transverse Hall drift in an accelerated lattice (Fig. 2c). Interestingly, this quantized response originally predicted for fermionic band insulators also appears for a homogeneous filling of the lowest band with thermal bosons. Finite Hofstadter ribbons were also realized with one artificial dimension formed by internal spin states of the atoms. To realize spatially dependent Peierls phases, these experiments use Raman transitions that couple the internal atomic states within the same lattice site. The artificial dimension has sharp edges, which allows for the observation of skipping orbits at the edges.

Another important model is the Haldane model on the honeycomb lattice, which also has Peierls phases, but no net magnetic flux (Fig. 2d). It can be realized by elliptical lattice shaking or similarly by illuminating graphene sheets with circularly polarized terahertz radiation, as well as in helically propagating photonic waveguides. Experiments have mapped out the phase diagram via closing of the bandgap as well as the Chern number via the transverse Hall drift in an accelerated lattice (Fig. 2c). Experiments have also demonstrated the expected renormalization of superexchange coupling \( J_s \approx \ell^2 / U \) due to the renormalization of \( J \). For a driving in near resonance with \( U \), the behaviour is radically different: red-detuned driving \( (\hbar \Omega < U, \text{ red diamonds}) \) enhances the superexchange coupling \( J_s \), increasing driving amplitude \( K \), while blue-detuned driving \( (\hbar \Omega > U, \text{ blue and white diamonds}) \) turns \( J_s \) negative for larger driving amplitudes. The errors denote the standard deviation.

Topological systems are an active area of current research and many models and phenomena are explored with cold atoms using Floquet engineering, such as non-Abelian gauge fields and one-dimensional and two-dimensional synthetic spin–orbit coupling both in theory and experiment. Cold-atom research has developed various new detection techniques complementary to those of solid-state physics, which allow directly revealing fundamental topological concepts such as Berry phases, Berry curvature, skipping orbits, and quantized Thouless pumping, as well as completely new concepts such as linking numbers or Hopf invariants, which appear in quench dynamics. Current efforts aim at understanding the interplay between topology and interactions and challenges in the context of Floquet realizations of the latter.

Floquet schemes in correlated systems
A completely different class of Hamiltonians can be accessed by driving correlated Hubbard systems involving an on-site interaction energy \( U \) (ref. 8). In laser-assisted tunneling, the resonance condition for restoring tunneling in a tilted lattice now becomes dependent on the occupation of the lattice sites involved, that is, dependent on whether the initial and final states involve the interaction energy \( U \). For example, starting from one atom at each site, a tunneling process in the lattice tilted by \( \Delta \) becomes resonant for a two-photon detuning of \( \hbar \delta = \Delta + U \) (Fig. 3a) (compare to the two-photon resonance in Fig. 1c). This allows restoring the tunneling processes for different occupation numbers with separate pairs of laser beams and therefore to address them separately and imprint occupation-dependent (or density-dependent) Peierls phases. Such processes have been proposed as the building block for many interesting exotic Hamiltonians. The density-dependent Peierls phases can, for example, be mapped onto a one-dimensional anyon–Hubbard model with the Peierls phase becoming the statistical exchange phase of the effective anyons (Fig. 3a). The same processes can be realized by restoring tunneling in the tilted lattice via multicolour lattice shaking.

For an appropriate choice of laser beams, laser-assisted tunneling can additionally flip the spin of the atoms. This was employed in an experimental demonstration of density-dependent tunneling in a fermionic Mott insulator. Another possibility to obtain density-dependent tunneling is to periodically modulate the interaction strength itself, taking advantage of Feshbach resonances. This strategy was employed in an experimental demonstration of correlated tunneling processes in a bosonic Mott insulator.

Density-dependent tunneling processes are essential for the implementation of dynamical gauge fields and lattice gauge theories, which include a feedback of neutral matter onto the synthetic gauge fields. Recent experiments have realized the first steps in this direction by implementing density-dependent tunneling processes in isolated double wells using sophisticated shaking schemes, for example, involving two spin states and spin-selective tilts. Density-dependent gauge fields have also been realized by combining lattice shaking and modulation of interactions. Such a combination of modulations allows engineering a broad class of unconventional Hubbard models with correlated tunneling.

Periodic driving can also be employed to modify the spin interactions in Hubbard models (Fig. 3b). Using two spin states in a regime of a Mott insulator with one atom per site, spin interactions arise as a superexchange process consisting of two virtual tunneling processes with tunneling element \( J \), where the intermediate state with both atoms at the same lattice site is detuned by \( U \). The
a mott insulator. Heating processes revealed by the formation of holes set

$$k$$

two-dimensional lattice in strip geometry as a function of quasi-momentum

$$\Omega$$

shaking frequency

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c anomalous.

Chern numbers of the bulk bands. These edge states are, therefore, called

bulk bands have a non-zero Chern number. In a Floquet system with its

edge states. In a static system (Fig. 4a), one can have two chiral edge states despite the zero

Chern numbers of the bulk bands. These edge states are, therefore, called anomalous. c Floquet thermalization in a driven interacting Bose–Hubbard system. Measurement of the density of holes as a function of driving cycles

$$N_{cy}$$

measured after adiabatically ramping the system into the atomic limit of a Mott insulator. Heating processes revealed by the formation of holes set in after $10^3$–$10^4$ cycles. Strikingly, the heating rate decreases for increasing shaking frequency $\Omega_c$, which is an indication of the exponential slowdown of the heating characteristic of Floquet prethermalization. Panel c adapted with permission from ref. 113 under a Creative Commons licence CC BY 4.0.

supercorrelation coupling is, therefore, given by $J_s \approx F/U$; renormalizing $J$ and $U$ accordingly modifies $J_s$. High-frequency lattice shaking only reduces $J$ and therefore $J_s$. However, lattice shaking with $\Omega$ in near resonance with $U$ renormalizes the interaction to $U_{\text{eff}} = U - \hbar \Omega$ and therefore reduces, enhances and even reverses the spin interactions depending on its frequency. A recent experiment has measured both the effect on the superexchange dynamics in isolated double wells (Fig. 3b) and the resulting spin correlations in a fermionic many-body system on a honeycomb lattice changing from antiferromagnetic to ferromagnetic.

Physics beyond an equilibrium description

The physics of driven systems is richer than what is captured by the effective Hamiltonian. While the slow dynamics of Floquet systems can often be mapped to static Hamiltonians, Floquet systems are inherently non-equilibrium in nature with new properties beyond static concepts and they possess new phases without a static counterpart. These situations are of particular interest in non-equilibrium quantum statistical physics and they can be assessed with cold atoms.

To get some insight into these issues, let us start with the Floquet theorem for time-periodic Hamiltonians, which states that the eigenstates are Floquet states $|\psi_n(t)\rangle$—analogous to Bloch states for spatially periodic potentials: they can be written as the product of a time-periodic wave function and a phase evolution $|\psi_n(t)\rangle = |\psi_n(0)\rangle e^{-i E_n \Omega t / \hbar}$ with Floquet modes $|\psi_n(t)\rangle = |\psi_n(t)\rangle$. The states do not have energies, but quasi-energies $\epsilon_n^\tau$, which are defined modulo $\hbar \Omega$, because the system can always exchange energy quanta of $\hbar \Omega$ with the drive: energy is not conserved in a driven system. In the extended zone scheme, this leads to many copies of the spectrum and correspondingly to new bandgaps (Fig. 4b). These new bandgaps in the quasi-energies can have important consequences such as new collision processes becoming resonant.

An illustrative example of the consequences of Floquet bandgaps is the anomalous Floquet topological insulator, where the Floquet nature of the phase gives rise to new topological properties by redefining the connection between the bulk topological index (for example, the Chern number) and the existence of chiral edge states. According to the bulk–edge correspondence of static systems, chiral edge states appear for non-trivial bulk bands. However, in the Floquet system, anomalous edge states can appear in a system with zero Chern number. The game changer comes from the additional gap between the bands around the Brillouin zone of quasi-energies (Fig. 4a,b). To characterize such gaps, new winding numbers are required, which cannot be evaluated in an effective static Floquet Hamiltonian, but only including the full time dependency of the system. These states have been realized with photonic waveguides and recently with cold atoms, which promise access to bulk and edge properties in the same system. This example shows that established principles have to be re-examined in a non-equilibrium context. In fact, a new classification of topological insulators has been introduced for Floquet systems.

A consequence of the absence of energy conservation in driven systems is that in the long-time limit, the system heats up to unconstrained temperatures. However, there is a prethermal regime at an intermediate timescale, where the system is in an equilibrium-like state and that can be employed to study the physics of the effective Floquet Hamiltonian. The timescale of this prethermal regime, which can be up to $10^4$ driving periods, critically depends on the drive frequency and interaction strength and generally increases for high-frequency driving. Therefore, experimental studies with cold atoms have investigated heating processes in various conditions and mapped out the parameter space to identify and characterize the prethermal regime, particularly for bosons in driven optical lattices where a reduction in the heating rate is exponential in the driving frequency (Fig. 4c).

Outlook

In this Review, we have summarized how the Floquet engineering of quantum gases has evolved into a very active field of research, exploring ever more complex systems. One ubiquitous challenge is heating in a driven system, which has to be controlled. Therefore, the recent systematic studies of the prethermal regime are an important benchmark. Current efforts aim at exploring ways to circumvent heating, be it by using non-ergodic systems or by employing destructive interference of excitation paths via a two-colour drive. Controlling heating would allow accessing strongly correlated phases such as fractional quantum Hall states or half-integer Mott insulators, as well as extending quantum simulation to high-energy physical concepts such as full-fledged dynamical gauge fields.
Furthermore, Floquet systems grant access to totally new concepts such as discrete time crystals, which spontaneously break the discrete time symmetry of the Floquet system to a reduced symmetry with a period that is a multiple of the driving period. Floquet techniques have been introduced to ultracold atoms as a way to tailor the systems and to add new properties such as artificial gauge fields. After more than one decade of intense research and cross-fertilization between theory and experiments, we can say that Floquet techniques do much more than that. They have opened a new avenue to exotic states of matter and to exciting non-equilibrium physics and they will continue to play an important role. 

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Competing interests

The authors declare no competing interests.

Additional information

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