Regret Minimization with Noisy Observations

MOHAMMAD MAHDIAN, Google Research, USA
JIEMING MAO, Google Research, USA
KANGNING WANG, Stanford University, USA

In a typical optimization problem, the task is to pick one of a number of options with the lowest cost or the highest value. In practice, these cost/value quantities are often not accurately known: they can come from physical measurements with imperfect tools, estimations of machine learning algorithms, or observations from differentially private mechanisms. In many of these situations, the cost/value quantities are noisy, but with quantifiable noise distributions. To take these noise distributions into account, one approach is to assume a prior distribution for the values, use it to build a posterior, and then apply standard stochastic optimization to pick a solution. However, in many practical applications, such prior distributions may not be available. In this paper, we study such scenarios using a regret minimization model.

In our model, the task is to pick the highest one out of \( n \) values. The values are unknown and chosen by an adversary, but can be observed through noisy channels, where additive noises are stochastically drawn from known distributions. The goal is to minimize the regret of our selection, defined as the expected difference between the highest and the selected value on the worst-case choices of values.

The most natural algorithm is perhaps to pick the highest observed value minus its expected noise. We show that this naïve algorithm can have regret arbitrarily worse than the optimum, even when \( n = 2 \). Additionally, any near-optimal algorithm must be randomized and any near-optimal deterministic algorithm cannot satisfy a natural monotonicity property. These negative results demonstrate the difficulty of this optimization problem and ask for a simple approximation algorithm.

Fortunately, on the positive side, we propose an algorithm that gives a constant-approximation to the optimal regret for any \( n \). This algorithm outputs the one with the highest observed value minus a quantity computed using its associated noise distribution. Our algorithm is conceptually simple, computationally efficient, and requires only minimal knowledge of the noise distributions.

Our proofs apply the idea of convolution in a novel way. Using it, we show that two different input instances are partially indistinguishable to any algorithm, hence giving lower bounds for the regret of the optimal algorithm that match the upper bounds for our simple algorithm up to a constant.

An immediate future direction is to generalize our framework to more complex settings than selecting one from \( n \) items. For example, it is an interesting question to find an approximately shortest path when the edge lengths can only be observed through noisy channels. This is motivated, for example, by applications in map navigation in presence of (stochastic) traffic.

A full version of this paper can be found at https://arxiv.org/abs/2207.09435.

Additional Key Words and Phrases: regret minimization, approximation algorithms, stochastic optimization

ACM Reference Format:
Mohammad Mahdian, Jieming Mao, and Kangning Wang. 2023. Regret Minimization with Noisy Observations. In Proceedings of the 24th ACM Conference on Economics and Computation (EC ’23), July 9–12, 2023, London, United Kingdom. ACM, New York, NY, USA, 1 page. https://doi.org/10.1145/3580507.3597789