The Stability Analysis of the Exoplanetary Systems

Jianghui Ji$^{1,2}$, Lin Lin$^3$, H. Kinoshita$^4$, Guangyu Li$^{1,2}$, H. Nakai$^4$

1. Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing, 210008
2. National Astronomical Observatory, Chinese Academy of Sciences, Beijing 100012, China
3. Department of Astronomy, Nanjing University, Nanjing, 210093
4. National Astronomical Observatory, Mitaka, Tokyo 181-8588, Japan

Abstract. To date, more than 100 giant Jupiter-like planets have been discovered in Doppler surveys of solar-type stars. In this paper, we perform simulations to investigate three systems: GJ 876, HD 82943 and 55 Cnc. The former two systems both have a pair of planets in the 2:1 Mean Motion Resonance (MMR), while the inner two companions of the later is close to 3:1 MMR. By integrating hundreds of the planetary orbits of three systems for million years, we find that for GJ 876 and HD 82943, the critical argument $\lambda_1 - 2\lambda_2 + \bar{\omega}_1$ and $\lambda_1 - 2\lambda_2 + \bar{\omega}_2$ librate about $0^\circ$ or $180^\circ$, indicating 2:1 MMR can play an important role in stabilizing the motion of the planets so that they are protected from frequent close encounters. As for 55 Cnc, we further show the three resonant arguments for 3:1 MMR execute librations for millions of years respectively, which reveals the evidence of the resonance for this system. Additionally, we should emphasize another vital mechanism is the apsidal phase-locking between a couple of planets for a certain system. For GJ 876 and HD 82943, we discover the relative apsidal longitudes $\bar{\omega}_1 - \bar{\omega}_2$ move about $0^\circ$ or $180^\circ$, respectively; but for 55 Cnc, we find that there exists an asymmetric apsidal libration between two inner planets. Finally, we made a brief discussion about the Habitable Zones in the exoplanetary systems.

1. Introduction

The discovery of the extrasolar planets is opening a new world beyond our solar system. Ever since 1995, Mayor & Queloz (1995) detected the first extrasolar giant Jupiter–51 Peg and as of August 5, 2003, there are 111 exoplanets discovered (Butler et al. 2003 and references therein; also see http://exoplanets.org) using the radial velocity technique in the surveys of the solar-type stars. Nowadays, a dozen of multiple planet systems – HD 82943, GJ 876, HD 168443, HD 74156, 47 Uma, HD 37124, HD 38529, HD 12661, 55 Cnc, Ups And (see Table 8 in Fischer et al. 2003), HD 169830 (Mayor et al. 2003), and HD 160691 (Jones et al. 2003; Gozdziewski, Konacki & Maciejewski 2003; Kiseleva-Eggleton et al. 2002) were detected in recent years. And it is necessary to classify the dis-
covered multiple planetary systems according to their statistical characteristics (such as planet masses, semi-major axes, eccentricities and metallicity) (Marcy et al. 2003; Santos et al. 2003; also see Figure 1a-b) and to study the correlation between mass ratio and period ratio (Mazeh & Zucker 2003). And the other important point is to investigate the possible stable configurations for the multiple systems, especially for the couples in the mean motion resonance.

Table 1. The astrocentric orbital parameters for three systems in MMR

| Planet   | Mass (Mjup) | Period (day) | a (AU) | ecc. | MMR  |
|----------|-------------|--------------|--------|------|------|
| GJ 876 b | 1.06        | 29.995       | 0.1294 | 0.314| 2:1  |
| GJ 876 c | 3.39        | 62.092       | 0.2108 | 0.051| 2:1  |
| HD82943 b| 1.63        | 444.6        | 1.16   | 0.41 | 2:1  |
| HD82943 c| 0.88        | 221.6        | 0.73   | 0.54 | 2:1  |
| 55 Cnc b | 0.83        | 14.65        | 0.115  | 0.03 | 3:1  |
| 55 Cnc c | 0.20        | 44.27        | 0.241  | 0.41 | 3:1  |
| 55 Cnc d | 3.69        | 4780.0       | 5.461  | 0.28 |      |

2. Multiple Planetary Systems in Mean Motion Resonance

In the multiple planetary systems are increasingly found the existence of the MMR, such as HD 82943 (Gozdziewski & Maciejewski 2001), GJ 876 (Lee & Peale 2002), and possibly HD 160691 (Gozdziewski et al. 2003) in a 2:1 MMR, 55 Cnc in a 3:1 MMR (Marcy et al. 2002; Ji et al. 2003a). Fischer D.A. (private communication) pointed out that more than half of the solar-like stars with
one detected planet show velocity trends indicating the presence of a second or multiple planets. Thus, we turn to the interesting topic of studying the stable geometry for two planets locking into a resonance. And our investigations are helpful in an understanding of the dynamics and possible mechanisms of sustaining the stability of such systems. In Table 1 are listed the orbital parameters of HD 82943, GJ 876 and 55 Cnc. Another important feature for the resonant systems is the apsidal lock between the orbiting companions, indicating that the relative apsidal longitudes of two orbits librate about a constant, such that two planets have common time-averaged rate of apsidal precession (Kinoshita & Nakai 2000; Chiang & Murray 2002; Malhotra 2002).

### 2.1. HD 82943 and GJ 876

The critical argument $\theta_1, \theta_2$ for the 2:1 MMR is

$$\begin{align*}
\theta_1 &= \lambda_1 - 2\lambda_2 + \varpi_1, \\
\theta_2 &= \lambda_1 - 2\lambda_2 + \varpi_2,
\end{align*}$$

where $\lambda_{1,2}$ are, respectively, the mean longitudes of the inner and outer planets, and $\varpi_{1,2}$, respectively, denote their periastron longitudes.

And the relative apsidal longitude is

$$\theta_3 = \Delta \varpi = \varpi_1 - \varpi_2.$$

There are three possible stable configurations for a system in a 2:1 MMR: (I) Only $\theta_1$ librates about 0°; (II) Case of alignment, $\theta_1 \approx \theta_2 \approx \theta_3 \approx 0^\circ$; (III) Case of antialignment, $\theta_1 \approx 180^\circ$, $\theta_2 \approx 0^\circ$, $\theta_3 \approx 180^\circ$. The simulation results show that HD 82943 could occupy the above three stable configurations, while GJ 876 does Classes I and II. Figure 2a displays an antialignment case for HD 82943. The librations of the relative apsidal longitudes and mean motion resonant variables about 0 or 180 degrees show that two important dynamical factors to characterize the multiple planetary systems.

As for HD 82943, we established a semi-analytical model on the averaged Hamiltonian (Ji et al. 2003b; also see Figure 2b) to explore the apsidal motion for this system and found that although the two planets have high eccentricities to 0.54 and 0.41, the system remain stable for 10 Myr because of both dynamical mechanisms of the 2:1 MMR and apsidal lock. This is also true for the HD 160691 system (see Bois et al. 2003, where the stable configuration is involved in a 2:1 mean motion resonance with an anti-aligned in two apsidal lines, with higher eccentricity of the outer planet).

### 2.2. 55 Cnc

The work of Ji et al. (2003a) confirmed that the inner two companions of 55 Cnc (Marcy et al. 2002) are in a 3:1 resonance and still experience the apsidal phase-locking. Two characterized arguments for resonances are:

$$\theta_1 = \lambda_1 - 3\lambda_2 + 2\varpi_2, \Delta \varpi = \varpi_1 - \varpi_2.$$
Figure 2. Left Panel (a): Antialignment case for HD 82943. The figure exhibits that $\theta_1$ (filled circle) and $\theta_2$ (empty circle) librate about $180^\circ$ and $0^\circ$, respectively, for millions years. The relative apsidal longitude of $\theta_3$ (solid line) move about $180^\circ$. Right panel (b): The eccentricity versus $\Delta\varpi$ evolution. The theoretical Hamiltonian contour (thin lines) concords with the numerical simulations (thick line).

where $\lambda_{1,2}$ are, respectively, the mean longitudes of 55 Cnc b and 55 Cnc c, and $\varpi_{1,2}$, individually, denote their apsidal longitudes. In Figure 3a, $\theta_1$ librate about $90^\circ$ for million years. The relative apsidal longitude of $\Delta\varpi$ walks about $250^\circ$, which indicates that there exists an asymmetric apsidal libration between two inner planets. Here two dynamical mechanisms are responsible for the stability of the 55 Cnc system. In addition, the numerical results further show that a wider stable region between 55 Cnc c and 55 Cnc d, implying an Earth-like planet may survive (see §3) in the triple-planet system.

3. Possible Habitable Zones (HZ) in the exoplanetary systems

Do the terrestrial planets exist in the Habitable Zones (Kasting, Whitmore, & Reynolds 1993) for the multiple systems? Is there possibly a stable liquid water environment for other civilizations? With the means of the direct integrations, we performed many tests in 55 Cnc by placing an Earth-mass planet at $\sim 1$ AU with low eccentricity and found that such orbits can survive for 10 Myr or even longer (Figure 3b) and there are wider stable zones for this system. And the stable orbits can be still reserved even when we gradually increased the mass of the assumed planet. Similar cases occur in the system of GJ 876 (Ji et al., in preparation) and other exosystems, especially for the host star that harbors two well-separated planets. Although the present ground-based observations set limitations on the discovery of the Earth-like planets, Space Interferometer Mission (SIM) may detect our Earth twins in the future task.

Due to the limited numbers of the exosystems involved in low mean motion resonance at this stage, thus it is too early to conclude that the planets tend to be captured into the resonance in the early planetary formation. However, the resonant geometry may become a suitable dynamical environment related to the stability of these systems, and future more observations on the exosystems will elucidate more clues on this.
Figure 3. Left Panel (a): Evidence for a 3:1 resonance for 55 Cnc. The argument of $\theta_1$ librates about 90° for million years. The relative apsidal longitude of $\Delta \varpi$ walks about 250°. Right panel (b): Orbital evolution of an Earth-mass planet orbiting 55 Cnc at 1 AU. Such orbits may exist and be stable in the planetary systems.

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