The curvaton scenario in Brane cosmology: model parameters and their constraints

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Abstract

We have studied the curvaton scenario in brane world cosmology in an intermediate inflationary scenario. This approach has allowed us to find some constraints on different parameters that appear in the model.

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1. INTRODUCTION

Inflation as the most promising framework for understanding the physics of the very early universe, ties the evolution of the universe to the properties of one or more scalar inflaton fields, responsible for creating an accelerating expanding universe. This then creates a flat and homogeneous universe which later evolves into the present universe. In general, the identity of the scalar inflaton fields are currently unknown and lies outside the standard model for particle physics [1]. It is well known that inflation is also the most requiring solution to many other deep-rooted problems of the cosmological models such as horizon and flatness problems. Another success of the inflationary universe model is that it provides a causal interpretation of the origin of the observed anisotropy of the cosmic microwave background radiation (CMBR), and also the distribution of large scale structures [2]–[6].

Intermediate inflation is a class of cosmological models where the scale factor of the universe behaves in a certain extend intermediate between a power law and exponential expansion [7] where the expansion rate is faster than power law and slower than exponential ones [8]–[10]. In this class, the slow-roll approximation conditions are well satisfied with time and therefore similar to power-law inflation, there is no natural end to inflation within the model [11].

The problem of inflation in the Randall-Sundrum model of a single brane in an AdS bulk successfully incorporates the idea that our universe lies in a three-dimensional brane within a higher-dimensional bulk spacetime [12]. All the brane-world inflationary models in the high energy limit possess correction terms in their Friedmann equations. Although these terms have important consequences in the inflationary dynamics, as the energy density decreases, these corrections become unimportant, and the inflaton field enters a kinetic energy dominated regime and brings inflation to an end. An alternative reheating mechanism might be required since the inflaton may survive this process without decay [13]. While during the inflationary period the universe becomes dominated by the inflation scalar potential, at the end of inflation the universe represents a combination of kinetic and potential energies related to the scalar field, which is assumed to take place at very low temperature [14].

In the standard reheating mechanism while the temperature grows in many orders of magnitude, most of the matter and radiation of the universe was created, via the decay of the inflaton field and the Big-Bang universe is recovered. This reheating temperature is
of particular interest. In this era the radiation domination, in which there exist a number 
of particles of different kinds, begins. In the standard mechanism of reheating, the stage 
of oscillation of the scalar field is very fundamental in which a minimum in the inflaton 
potential is something crucial for the reheating mechanism. However, since the scalar field 
potential in these models do not present a minimum, the usual mechanism introduced to 
bring inflation to an end becomes ineffective \[15\]. These models are known in the literature as 
non-oscillating or simply NO Models \[16\]. To overcome this problem, one of the mechanism 
of reheating in these kind of models is the introduction of the curvaton field \[17\]–\[19\].

The curvaton scenario was in the first place suggested as an alternative mechanism to 
generate the primordial scalar perturbation responsible for the structure formation \[13\]. Its 
decay into conventional matter offers an efficient mechanism of reheating, and its field has 
the property whose energy density is not diluted during inflation therefore the curvaton 
may be responsible for some or all the matter content of the universe at the present time. 
Alternatively, the large scale structure of the universe may also be explained by the curvaton 
field. However, in here, in the context of brane intermediate inflation, we would like to 
present the curvaton field in a mechanism to bring inflation to an end and explain the 
reheating mechanism, similar to the work adopted by the authors in \[17\] and \[20\]. In 
\[17\], the authors have used the curvaton mechanism for intermediate inflation in standard 
cosmology and in \[20\] the authors have studied steep inflation and not intermediate inflation 
model in brane cosmology.

2. THE MODEL

We consider a five-dimensional brane cosmology in which the modified Friedmann equations 
are given by \[21, 22\],

\[
H^2 = \kappa \rho_\phi [1 + \frac{\rho_\phi}{2\lambda}] + \frac{\Lambda_4}{3} + \frac{\xi}{a^4},
\]

\[
2\dot{H} + 3H^2 = -3\kappa (\rho_\phi + p_\phi)[1 + \frac{\rho_\phi}{\lambda}] + 3\kappa \rho_\phi[1 + \frac{\rho_\phi}{2\lambda}] + \Lambda_4 - \frac{\xi}{a^4}
\]

where \(H = \dot{a}/a\), \(\rho_\phi\), \(p_\phi\) and \(\Lambda_4\) represent respectively the Hubble parameter, the energy and 
pressure of the matter field confined to the brane and the four-dimensional cosmological 
constant. In here we assume \(\kappa = 8\pi G/3 = 8\pi/(3m_p^2)\). The last term in equation \[1\] or \[2\]
is called the radiation term and represents the influence of the bulk gravitons on the brane, with $\xi$ as an integration constant. It has been noted in [23] that while during inflation this term can be neglected, but it might play an important role at the beginning of inflation. The brane tension $\lambda$ which relates the four and five-dimensional Planck masses via the expression $m_p = \sqrt{3M_5 \beta / (4\pi \lambda)}$, is constrained by nucleosynthesis to satisfies the inequality $\lambda > (1\,MeV)^4$. In the following, for the inflation epoch, we assume that the universe is in the high energy regime, i.e. $\rho_\phi \gg \lambda$. We also assume that the four-dimensional cosmological constant is vanished and just when the inflation begins, the last term in (1) and (2) will rapidly become unimportant which, in turn, leaves us with the following effective equations

$$H^2 \simeq \beta \rho_\phi^2,$$  

$$2\dot{H} + 3H^2 = -3\beta (\rho_\phi + 2p_\phi)\rho_\phi,$$  

where $\beta = \kappa / (2\lambda)$, with dimension of $m_p^{-6}$.

We also assume that the inflaton field is confined to the brane, thus its field equation is in the standard form:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0,$$  

where $V(\phi)$ is the effective scalar potential. The dot means derivative with respect to the cosmological time and prime means derivative with respect to scalar field $\phi$. In addition, the conservation equation is,

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0,$$  

where $\rho_\phi = (\dot{\phi}^2/2) + V(\phi)$, and $p_\phi = (\dot{\phi}^2/2) - V(\phi)$. In here, for convenience we also take units in which $c = \hbar = 1$.

For intermediate inflationary universe models, the exact solution can be found by assuming that the scale factor $a(t)$ expands as,

$$a(t) = \exp(t^f),$$  

where $f$ is a constant parameter with range $0 < f < 1$.

From equations (3), (5) and (7) the expressions for the scalar potential $V(\phi)$ and the scalar field $\phi(t)$ are respectively

$$V(\phi) = \frac{2(1-f)}{9\beta} \left[ 6f \left( \frac{3\beta^{1/2}}{4(1-f)} \right) ^f \phi^{2(f-1)} - \frac{(1-f)}{\phi^2} \right].$$
\[ \phi(t) = \left[ \frac{4(1 - f)}{3\beta^{1/2}} \right]^{1/2}. \tag{9} \]

Then, the Hubble parameter as a function of the inflaton field becomes

\[ H(\phi) = f \left( \frac{3\beta^{1/2}}{4(1 - f)} \right)^{f-1} \phi^{-2(1-f)}. \tag{10} \]

The form of the scale factor expressed in equation (7) also arises when we solve the field equations in the slow roll approximation, where a simple power law scalar potential is considered as

\[ V(\phi) = f \beta^{(f-2)/2} \left[ \frac{3}{4(1 - f)} \right]^{f-1} \phi^{-2(1-f)}. \tag{11} \]

The solutions for \( \phi(t) \) and \( H(\phi) \) obtained with this potential in the slow roll approximation are similar to those obtained in the exact solution, expressed by (9) and (10). Note also that for this kind of potential a minimum does not exist.

The dimensionless slow roll parameters \( \varepsilon \) and \( \eta \) which are defined by \( \varepsilon \simeq V''/(3\beta V^3) \) and \( \eta \simeq V''/(3\beta V^2) \), respectively, in our case reduce to

\[ \varepsilon \simeq \frac{4(1 - f)^2}{3f\beta^{1/2}\left[ \frac{3}{4(1 - f)} \right]^{f-1}} \phi^{-2f}, \tag{12} \]

and

\[ \eta \simeq \frac{2(1 - f)(3 - 2f)}{3f\beta^{1/2}\left[ \frac{3}{4(1 - f)} \right]^{f-1}} \phi^{-2f}. \tag{13} \]

The ratio between \( \varepsilon \) and \( \eta \) is \( \varepsilon/\eta = 2(1 - f)/(3 - 2f) \) and thus for \( 0 < f < 1 \), \( \eta \) is always larger than \( \varepsilon \). Note also that \( \eta \) reaches unity earlier than \( \varepsilon \) does. Therefore, one can represent the end of inflation is governed by the condition \( \eta = 1 \) in place of \( \varepsilon = 1 \). From this condition, for the inflaton field at the end of inflation we obtain

\[ \phi_e = \left[ \frac{2(1 - f)(3 - 2f)}{3f\beta^{1/2}\left[ \frac{3}{4(1 - f)} \right]^{f-1}} \right]^{1/2f}, \tag{14} \]

where, the subscript ”\( e \)” is used to denote the end of the inflationary period. Also, the number of the e-folds corresponding to the cosmological scales, i.e. the number of remaining inflationary e-folds at the time when the cosmological scale exits the horizon defines as

\[ N_\ast = \int_{t_e}^{t_\ast} H(t')dt' = A \left[ \frac{3\beta^{1/2}}{4(1 - f)} \right]^{f/2} (\phi_e^2 - \phi_*^2). \tag{15} \]
3. THE CURVATON FIELD DURING THE KINETIC EPOCH

By neglecting the term $V'$ in the field equation (5) in comparison to the friction term $3H\dot{\phi}$, the model begins a new period which is called the kinetic epoch or kination. Hereafter, we will use the subscript (or superscript) "$k$", to label different quantities at the beginning of this era. During the kination era we have $\ddot{\phi}^2/2 > V(\phi)$ which could be seen as a stiff fluid since $p_\phi = \rho_\phi$. Also, at the beginning of the kination we assume the low energy limit, i.e. $\rho_\phi \ll \lambda$. In this regime two Friedmann equations become:

$$H^2 = \kappa \rho_\phi,$$
(16)

$$2\dot{H} + 3H^2 = -3\kappa p_\phi.$$  
(17)

In the kinetic epoch, the field equations (1) and (5) become, $H^2 = \kappa \dot{\phi}^2/2$ and $\ddot{\phi} + 3H \dot{\phi} = 0$, where the second equation gives,

$$\dot{\phi} = \dot{\phi}_k \left(\frac{a_k}{a}\right)^3.$$  
(18)

Then, the energy density and Hubble parameter respectively become

$$\rho_\phi(a) = \rho_\phi^k \left(\frac{a_k}{a}\right)^6,$$
(19)

and

$$H(a) = H_k \left(\frac{a_k}{a}\right)^3,$$
(20)

where $H_k$ and $\rho_\phi^k$ are the values of the Hubble parameter and energy density associated to the inflaton field at the beginning of the kinetic epoch.

The curvaton field obeys the Klein-Gordon equation and in here we assume that the scalar potential associated to this field is given by $U(\sigma) = m^2 \sigma^2/2$, with $m$ to be the curvaton mass.

We now assume that $\rho_\phi$ to be the dominant component when compared to the curvaton energy density, $\rho_\sigma$. In addition, the curvaton field oscillates around the minimum of its effective potential $U(\sigma)$. During the kination, the inflaton remains dominated and the curvaton density evolves as a non-relativistic matter, i.e. $\rho_\sigma \propto a^{-3}$. Then, the curvaton field decay into radiation and the standard Big-Bang cosmology is recovered.

In the inflation era, it is assumed that the curvaton field is effectively massless. In the same period the curvaton rolls down its potential until its kinetic energy is weakened by the exponential expansion that its kinetic energy has almost vanished, and so it becomes frozen.
The curvaton field then assumes roughly to be a constant, $\sigma_* \approx \sigma_e$, where, the subscript "*" refers to the era when the cosmological scale exits the horizon.

In here, we also assume that during the kination the Hubble parameter decreases so that its value is equivalent with the curvaton mass, i.e. $m \approx H$, where at this stage, the curvaton field becomes effectively massive. Then, from equation (20), we obtain

$$\frac{m}{H_k} = \left( \frac{a_k}{a_m} \right)^3,$$

where the subscript "m" represents quantities at the time when the curvaton mass $m$ during kination is of the order of $H$.

To prevent a period of curvaton-driven inflation the universe must still be dominated by the inflaton field, i.e. $\rho_\phi^m \gg \rho_\sigma (\sim U(\sigma_e) \simeq U(\sigma_*))$. This inequality permits us to find a constraint on the values of the curvaton field $\sigma_*$. Now when $H \approx m$, we get $\frac{m^2 \sigma^2_*}{2 \rho_\phi} \ll 1$, which means that the curvaton field $\sigma_*$ satisfies the constraint,

$$\sigma^2_* \ll \frac{2}{\kappa}.$$  \hspace{1cm} (22)

At the end of inflation, the ratio of the potential energies becomes,

$$\frac{U_e}{V_e} = \frac{m^2 \sigma^2_* \beta^{1/2}}{2 H_e} < 1,$$

and, thus, the curvaton energy becomes subdominant at the end of inflation. The curvaton mass then should comply the constraint,

$$m^2 < H^2_e = f^2/f \left( \frac{3 - 2f}{2} \right)^{2(f-1)/f}.$$ \hspace{1cm} (24)

The constraint (24) imposed by the fact that the curvaton field must be effectively massless during the inflationary era and thus $m < H_e$. When the mass of the curvaton field becomes important, i.e. $m \approx H$, its energy density decays like a non-relativistic matter in the form of $\rho_\sigma = m^2 \sigma^2_* a^3_m/(2a^3)$. This is because, as the curvaton undergoes quasi-harmonic oscillations, the potential and kinetic energy densities become comparable.

The decay of the curvaton field may happen in two different situations. One, when the curvaton field first dominates the cosmic expansion and then decays. Two, when the decay of the curvaton field occurs before it dominates the cosmological expansion. In the following section we will investigate these situations in more details.
4. CONSTRAINTS ON MODEL PARAMETERS

Case 1: curvaton decay after domination

If the curvaton field dominates the cosmic expansion (i.e. \( \rho_\sigma > \rho_\phi \)), then, at a distance, (say \( a = a_{eq} \)), the inflaton and curvaton energy densities must become equal. Therefore, from (19) and (20) and using \( \rho_\sigma \propto a^{-3} \), we obtain,

\[
\left( \frac{\rho_\sigma}{\rho_\phi} \right)_{a=a_{eq}} = \frac{\kappa m^2 \sigma^*_2}{2} \frac{a_m^3 a_{eq}^3}{a_k^6 H_k^2} = 1. \tag{25}
\]

in addition, from equations (20), (21) and (25), we find a relation for the Hubble parameter, \( H_{eq} \), in terms of the curvaton parameters,

\[
H_{eq} = H_k \left( \frac{a_k}{a_{eq}} \right)^3 = \frac{\kappa m \sigma^2_*}{2}. \tag{26}
\]

Since the decay parameter \( \Gamma_\sigma \) is constrained by nucleosynthesis, it is required that the curvaton field decays before nucleosynthesis, i.e., \( H_{nucl} \sim 10^{-40} < \Gamma_\sigma \) (in units of Planck mass \( m_p \)). By the requirement that \( \Gamma_\sigma < H_{eq} \), we acquire a constraint on the decay parameter \( \Gamma_\sigma \), as

\[
10^{-40} < \Gamma_\sigma < \frac{\kappa m \sigma^2_*}{2}. \tag{27}
\]

Following the argument given by the authors in [17], we constrain the parameters appearing in our model by revising the scalar perturbations related to the curvaton field \( \sigma \). At the time when the decay of the curvaton field occurs, the Bardeen parameter, \( P_\zeta \), whose observed value is about \( 2.4 \times 10^{-9} \) [24], becomes [19],

\[
P_\zeta \simeq \frac{1}{9\pi^2} \frac{H_*^2}{\sigma^2_*}. \tag{28}
\]

Since the spectrum of fluctuations is automatically gaussian for \( \sigma^2_* \gg H_*^2/(4\pi^2) \), and also is independent of \( \Gamma_\sigma \), the analysis of space parameter in our model is simplified. With the help of equations (24) and (27) we obtain

\[
\Gamma_\sigma < \frac{\kappa \sigma^2_*}{2} f^{1/f} \left( \frac{3 - 2f}{2} \right)^{(f-1)/f}, \tag{29}
\]

that imposes an upper limit on \( \Gamma_\sigma \).

Meanwhile, from Big Bang Nucleosynthesis (BBN) temperature, \( T_{BBN} \), we constrain our model parameter \( f \). The reheating occurs before BBN, with \( T_{BBN} \sim 10^{-22} \) (in unit of
$m_p$), and thus we have $T_{reh} > T_{BBN}$. By knowing that $T_{reh} \sim \Gamma^{1/2}_{\sigma^2} > T_{BBN}$ we obtain the constraint,

$$H^2_* = f^2 \left[ \frac{3 - 2f}{2f} - N_* \right]^{2(f-1)/f} > (540\pi/8)^{2/3} (P_\zeta T_{BBN}^2)^{2/3} \sim 10^{-34}, \quad (30)$$

where, for the later, we have used the scalar spectral index $n_s$, close to one ($m \approx H_*/10$).

In FIG.1 (left panel), the number of e-folds, $N_*$, with respect to $f$ is shown by fitting the constraint (30) in its lower limit. Alternatively, when the upper limit, $H_* \leq 10^{-5}$, is used, the number of e-folds against the parameter $f$ is shown in the FIG.1 (right panel).

**Case 2: curvaton decay before domination**

We assume that the curvaton decays before its energy density becomes greater than the inflaton one. Moreover, the mass of the curvaton is comparable with the Hubble expansion rate, such that we can use $\rho_H \propto a^{-3}$. Following [17], we also assume that the curvaton field decays at a time when $\Gamma = H(a_d) = H_d$ and therefore from equation (20) we get,

$$\Gamma = H_d = H_k \left( \frac{a_k}{a_d} \right)^3, \quad (31)$$

where "d" stands for quantities at the time when the curvaton decays.

If we assume that the curvaton field decays after its mass becomes important, (so that $\Gamma < m$) and also before the curvaton field dominates the expansion of the universe (i.e., $\Gamma > H_{eq}$), we then obtain a new constraint as,

$$\frac{\kappa \sigma^2_*}{2} < \frac{\Gamma}{m} < 1. \quad (32)$$

Again, following [17], for this scenario, we find that the Bardeen parameter becomes,

$$P_\zeta \simeq \frac{r_d^2}{16\pi^2} \frac{H_*^2}{\sigma_*^2}, \quad (33)$$

where $r_d = \frac{\rho_\sigma}{\rho_m} |_{a=a_d}$. By using $\rho_\sigma = m^2 \sigma_*^2 a_m^3/(2a^3)$ and equations (19), (21) and (31) we obtain,

$$r_d = \frac{\kappa m \sigma_*^2}{2\Gamma_*}. \quad (34)$$

Now, From equations (21) and (32), we get the inequality,

$$\Gamma_{\sigma} < f^{1/f} \left( \frac{3 - 2f}{2} \right)^{(f-1)/f}, \quad (35)$$

...
which gives an upper limit on $\Gamma_\sigma$. Finally, since the reheating temperature satisfies $T_{reh} > T_{BBN}$, and also $\Gamma_\sigma > T_{BBN}^2$, we derive a new constraint as

$$H_*^2 = f^2 \left[ \frac{3-2f}{2f} - N_* \right]^{2(f-1)/f} > \frac{(120\pi)^{2/3}}{(P_\zeta T_{BBN})} \approx 10^{-34},$$

where, as before, we have used the scalar spectral index $n_s$ close to one. Note that apart from the coefficient, the expression is similar to the one obtained for the decay of curvaton field after domination, as expressed by equation (30). Therefore, the graph of the number of e-folds, $N_*$, against $f$ is similar to the one obtained in case of the curvaton decay after domination.

FIG. 1: The number of the e-folds with respect to $f$ in both curvaton decay after and before domination, fitted from the lower limit of the $T_{BBN}$ (left panel) and upper limit of the $T_{BBN}$ (right panel).

5. CONSTRAINTS ON CURVATON MASS

Similar to the case of constraint on $\Gamma_\sigma$ parameter, one may constrain on the value of the curvaton mass, using tensor perturbation methods. In these methods, the corresponding gravitational wave amplitude can be written as [22],

$$h_{GW} \simeq C_1 H_*,$$

(37)
where $C_1$ is an arbitrary constant. Therefore, one may take $H \ll 10^{-5}$, which means that inflation may occur at an energy scale smaller than the grand unification. This, in turn, is an advantage of the curvaton approach in comparison with the single inflaton field one.

From the modified Friedmann equation we have $H_*^2 = \beta V_*$, and thus by using equation (36) we obtain,

$$ h_{GW}^2 \simeq C_1^2 f^2 \left[ \frac{3 - 2f}{2f} - N_* \right]^{2(1-f)/f}. \tag{38} $$

From equations (24) and (38), we yield the inequality

$$ m^2 < \frac{h_{GW}^2}{C_1^2} \left[ 1 - \frac{2fN_*}{3 - 2f} \right]^{2(1-f)/f}, \tag{39} $$

which imposes an upper limit to the curvaton mass.

From [27], the constraint on the density fraction of the gravitational wave is given by

$$ I \equiv h^2 \int_{k_{BBN}}^{k_*} \Omega_{GW}(k)dk \simeq 2h^2\epsilon\Omega_\gamma(k_0)h_{GW}^2 \left( \frac{H_*}{H} \right)^{2/3} \leq 2 \times 10^{-6}, \tag{40} $$

where $k_{BBN}$ and $\Omega_{GW}(k)$ are respectively the physical momentum corresponding to the horizon at BBN and the density fraction of the gravitational wave with physical momentum $k$. The density fraction of the radiation at present on horizon scales is $\Omega_\gamma(k_0) = 2.6 \times 10^{-5} h^{-2}$. Also, $\epsilon \sim 10^{-2}$ and $h = 0.73$ is the Hubble constant in which $H_0$ is in units of 100 km/sec/Mpc. The parameter $\tilde{H}$ is either $\tilde{H} = H_{eq}$ or $\tilde{H} = H_d$, for the curvaton decays after or before domination.

**Case 1: curvaton decay after domination**

In this case, using equations (26), (28) and (37), the constraint on the density fraction of the gravitational wave, expressed by equation (40), becomes

$$ \frac{m}{\sigma_*^2} \geq 2.5 \times 10^{-13} P_\zeta^2 \sim 10^{-30}. \tag{41} $$

A second constraint is obtained by incorporating (27) and (41):

$$ m^2 > 5.96 \times 10^{-54} P_\zeta^2 \simeq 3.43 \times 10^{-71} \tag{42} $$

If the decay rate is of gravitational strength, then, $\Gamma_\sigma \sim m^3$ [28, 31] and equation (27) imposes a third constraint as

$$ \frac{m^2}{\sigma_*^2} < \frac{4\pi}{3} \approx 4.19. \tag{43} $$
From the previous sections, we also have the constraints (22) and (27). One may also find another constraint by using equations (24) and (28) on the curvaton mass for the curvaton decay after domination as

$$\frac{m^2}{\sigma_*^2} < 9\pi^2 P_\zeta \sim 2.13 \times 10^{-7}. \tag{44}$$

Altogether, there are six constraints (22), (27), (41)–(44) on either $m$ or $\sigma_*$, for the curvaton decay after domination. In FIG. 2(left panel), the allowed region for curvaton mass against curvaton field is shown in shaded color.

**Case 2: curvaton decay before domination**

In case 2, the constraint on the density fraction of the gravitational wave, expressed by equation (40), with regards to $\Gamma_{\sigma^2}^{1/2} > T_{BBN}$, becomes,

$$m\sigma_* > 3.2 \times 10^{-4} P_\zeta^{1/2} T_{BBN}^{3/2} \sim 10^{-41}, \tag{45}$$

by using equations (31), (33) and (34). A second constraint by incorporating (32) and (45) is given by,

$$m^2 > 4.3 \times 10^{-7} P_\zeta T_{BBN}^3 \simeq 1.04 \times 10^{-81}. \tag{46}$$

By using equation (32), a pair of new constraints can be obtained as,

$$m^2 < 1, \tag{47}$$

and

$$\frac{m^2}{\sigma_*^2} > 4.19. \tag{48}$$

Furthermore, from equation (45) one can get another constraint as,

$$\frac{\sigma_*}{m^{5/4}} > 3.2 \times 10^{-4} P_\zeta^{1/2} \simeq 1.57 \times 10^{-8}. \tag{49}$$

The constraint (22) can also be imposed on the curvaton field in the case of curvaton decay before domination. In addition, one may find another constraint by using (24), (32), (33) and (34) as,

$$m^2 \sigma_*^2 < 9 P_\zeta \sim 2.16 \times 10^{-8}. \tag{50}$$

There are seven constraints (22), (45)–(50) on either $m$ or $\sigma_*$ if the curvaton decay before domination. In Fig. 2(right panel), The shaded region shows the allowed region for $m$ which is bounded by the above constraints.

**constraint on the reheating temperature**
FIG. 2: Allowed region of parameter space of the curvaton-brane intermediate inflation model for the case of curvaton domination after decay (left panel) and curvaton domination before decay (right panel). The allowed regions are shaded.

In a very limited case, another constraint can be imposed on the decay rate of the curvaton field as \( \Gamma_\sigma = g^2 m \),

\[
\Gamma_\sigma = g^2 m, \tag{51}
\]

where \( g \) is the coupling of the curvaton to its decay products. Then, the allowed range for the coupling constant is given by,

\[
\max \left( \frac{T_{BBN} m^{1/2}}{m}, m \right) \lesssim g \lesssim \min \left( 1, \frac{m \sigma_*^3}{T_{BBN}^2} \right), \tag{52}
\]

where the inequality \( m \lesssim g \) is due to gravitational decay. For the first case and using \( T_{reh} > T_{BBN} \) this constraint gives an upper limit given by \( g < m \sigma_*^3/T_{BBN}^2 \), and when the curvaton decays after domination, a lower limit is given by \( T_{BBN} m^{-1/2} < g \).

By using equation (52) and by considering \( H_* \simeq 10^{-17}, \sigma_* \sim 10^{-12} \) \cite{25} and \( m \sim 10^{-18} \) (from \( n_s \simeq 1 \)), we obtain that \( 10^{-13} \lesssim g \lesssim 10^{-10} \). As a result, since \( T_{reh} \sim gm^{1/2} \), the allowed range for the reheating temperature becomes \( 10^{-22} \lesssim T_{reh} \lesssim 10^{-19} \) (in units of \( m_p \)).

Alternatively, by choosing \( \sigma_* = 1 \) \cite{25}, from expression (52) the range for \( g \) becomes \( 10^{-13} \lesssim g \lesssim 1 \). Therefore, with \( T_{reh} \sim gm^{1/2} \), the allowed range for the reheating temperature becomes \( 10^{-22} \lesssim T_{reh} \lesssim 10^{-9} \) (in units of \( m_p \)). The constraints on the density fraction of gravitational waves suggest \( g \sim 1 \) \cite{25}. Thus, we obtain that the reheating temperature
becomes of the order of $T_{reh} \sim 10^{-9}$ (in units of $m_p$) which challenges gravitino constraint [34].

6. SUMMARY

We have analysed the scenario of intermediate inflation in the brane-world cosmology in the presence of curvaton field. The curvaton field, responsible for universe reheating, imposes constraints on the model parameters.

For the intermediate inflationary universe with the scale factor given by equation (7), there is one space parameter $f$. Following [17], two possible scenarios have been taken to study universe reheating via curvaton mediation, where the curvaton dominates the universe after or before it decays. As a result, we have obtained an upper limit constrain for $\Gamma_\sigma$ expressed by equation (29) in the first scenario. We have also found a lower limit constraint for the value of $\Gamma_\sigma$ which is represented by equation (35) for the second scenario. In both scenarios, we have acquired constraints for the parameters expresses by the equations (30) and (36). The plot for different values of e-folding with respect to $f$ fitted from both lower and upper limits of $T_{BBN}$ is also given in FIG. (1).

For the curvaton field and its mass we have also obtained constraints in both scenarios. In the first scenario, there are six constraints which bound variation of curvaton mass $m$ with respect to the curvaton field $\sigma_*$ to the region shown in FIG. 2(left panel). Similarly, one can find the seven constraints limited these parameters and plotted in FIG. 2(right panel). Finally, we probe the reheating scenario in our model and come across the possible constraints on the reheating temperature by talking into account the result obtained from the previous sections.

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