Effect of $K^0 - \bar{K}^0$ Mixing on $CP$ Asymmetries in Weak Decays of $D$ and $B$ Mesons

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Abstract

Within the standard electroweak model we carry out an instructive analysis of the effect of $K^0 - \bar{K}^0$ mixing on $CP$ asymmetries in some weak decay modes of $D^\pm$ and $B_d^0$ mesons. We point out that a clean signal of $CP$ violation with magnitude of $2\text{Re}(\epsilon) \approx 3.3 \times 10^{-3}$ can manifest in the semileptonic decays $D^+ \to l^+ \nu_l K_S (K_L)$ vs $D^- \to l^- \bar{\nu}_l K_S (K_L)$. The $CP$ asymmetries are also dominated by $2\text{Re}(\epsilon)$ in the two-body nonleptonic decays $D^\pm \to (\pi^\pm, \rho^\pm, a_1^\pm) + (K_S, K_L)$ and in the multi-body processes $D^\pm \to \pi^\pm + n_0 \pi^0 + n_\pm (\pi^\pm \pi^\mp) + (K_S, K_L)$, where $n_0$ and $n_\pm$ are integers. It is possible to observe such $CP$-violating signals with about $10^7 D^\pm$ events at $\tau$-charm factories or hadron machines. Finally we show that the $CP$ asymmetry induced by $\text{Re}(\epsilon)$ may compete with those from $B_d^0 - \bar{B}_d^0$ mixing and final-state interactions in the semi-inclusive and exclusive decays $B_d \to X(c\bar{c}) + (K_S, K_L)$ on the $\Upsilon(4S)$ resonance.

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It has been known for 30 years that there exists \( CP \) asymmetry in the \( K^0 \Leftrightarrow \bar{K}^0 \) transitions \([1]\). This effect, conveniently parametrized by \( \epsilon \), is only of order \( 10^{-3} \) \([2]\). So far, no other evidence for \( CP \) violation has been unambiguously established. Many sophisticated experimental efforts, such as the programs of \( B \) factories \([3]\), \( \tau \)-charm factories \([4]\) and higher-luminosity hadron machines \([5]\), are being made to discover new signals of \( CP \) asymmetries beyond the neutral kaon system.

\( CP \) violation in the \( Q = 2/3 \) quark sector, particularly in the \( D \)-meson system, is complimentary to that in the \( K \) - and \( B \)-meson systems. The standard electroweak model predicts rather small \( D^0 - \bar{D}^0 \) mixing \( (\Delta m_D/\Gamma_D \leq 10^{-4}) \) and \( CP \)-violating effects in weak decays of \( D \) mesons (at the level of \( 10^{-3} \) or smaller) \([4, 6-8]\). Since many exclusive \( D \) decay modes are Cabibbo-favored, there are still possibilities to observe small \( CP \) asymmetries with about \( 10^7 \) events, e.g., at the forthcoming \( \tau \)-charm factories. If new physics were to enhance \( D^0 - \bar{D}^0 \) mixing or penguin-mediated processes, then large \( CP \) violation could also manifest in the \( D \)-meson system.

In this note we shall examine the effect of \( K^0 - \bar{K}^0 \) mixing on \( CP \) asymmetries in some semileptonic and nonleptonic \( D^\pm \) decays with \( K_S \) or \( K_L \) in the final states. Although this effect was noticed in a few previous works (see, e.g., refs. \([4,7,8]\)), an instructive study of it has been lacking. We point out that \( K^0 - \bar{K}^0 \) mixing can give rise to a clean signal of \( CP \) violation with magnitude of \( 2 \text{Re}(\epsilon) \) in the semileptonic transitions \( D^+ \rightarrow l^+ \nu_l K_S \) (\( K_L \)) vs \( D^- \rightarrow l^- \bar{\nu}_l K_S \) (\( K_L \)). The \( CP \) asymmetries are also dominated by \( 2 \text{Re}(\epsilon) \) in the two-body nonleptonic decays \( D^\pm \rightarrow (\pi^\pm, \rho^\pm, a_1^\pm) + (K_S, K_L) \) and in the multi-body processes \( D^\pm \rightarrow \pi^\pm + n_0 \pi^0 + n_\pm (\pi^\pm \pi^\mp) + (K_S, K_L) \), where \( n_0 \) and \( n_\pm \) are integers. Measurements of such \( CP \)-violating signals can be carried out with about \( 10^7 \) \( D^\pm \) events at either \( \tau \)-charm factories or hadron machines. Finally we carry out an analysis of \( CP \) violation in the semi-inclusive and exclusive decays \( B_d \rightarrow X(c\bar{c}) + (K_S, K_L) \) on the \( \Upsilon(4S) \) resonance. It is shown that the \( \text{Re}(\epsilon) \)-induced \( CP \) asymmetry may compete with those from \( B_0^d - \bar{B}_0^d \) mixing and final-state interactions, and thus cannot be neglected \( a \) priori.

We begin with the generic decay modes \( D^\pm \rightarrow X^\pm K_S \) or \( X^\pm K_L \), where \( X \) denotes a semileptonic or nonleptonic state. To be more specific, we concentrate upon the transitions occurring only through the tree-level quark diagrams (see, e.g., fig. 1). Some such transitions have large branching ratios (\( \geq 1\% \)) and are measurable in the near future. In the presence of
CP violation $|K_S\rangle$ and $|K_L\rangle$ are given by

$$
|K_S\rangle = (1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle,
|K_L\rangle = (1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle,
$$

(1)

where the complex parameter $\epsilon$ signifies deviation of the mass eigenstates from the CP eigenstates. Due to mixing, $K_S$ in the decay $D^+ \to X^+ K_S$ may come from either $K^0$ or $\bar{K}^0$, or both of them. The transition amplitudes of this type of decays are symbolically written as

$$
A(D^+ \to X^+ K_S) = (1 + \epsilon^*) T_a e^{i\delta_a} + (1 - \epsilon^*) T_b e^{i\delta_b},
A(D^- \to X^- K_S) = (1 - \epsilon^*) T_a^* e^{i\delta_a} + (1 + \epsilon^*) T_b^* e^{i\delta_b},
$$

(2)

where $T_a$ and $T_b$ denote the hadronic amplitudes containing the weak phases, and $\delta_a$ and $\delta_b$ are the corresponding strong phases. The signal of CP violation in the decay rates is obtained, to lowest order of $\epsilon$, as

$$
A(X^\pm K_S) \equiv \frac{\Gamma(D^- \to X^- K_S) - \Gamma(D^+ \to X^+ K_S)}{\Gamma(D^- \to X^- K_S) + \Gamma(D^+ \to X^+ K_S)}
= \frac{\Delta_{K_S} - 2\text{Im}(T_a T_b^*) \cdot \sin(\delta_b - \delta_a)}{|T_a|^2 + |T_b|^2 + 2\text{Re}(T_a T_b^*) \cdot \cos(\delta_b - \delta_a)},
$$

(3a)

where

$$
\Delta_{K_S} = 2\text{Re}(\epsilon) \cdot \left(|T_b|^2 - |T_a|^2\right) + 4\text{Im}(\epsilon) \cdot \text{Re}(T_a T_b^*) \cdot \sin(\delta_b - \delta_a)
$$

(3b)

stands for the effect of $K^0 - \bar{K}^0$ mixing on the CP asymmetry $A(X^\pm K_S)$. In a similar way one can obtain the CP asymmetry in $D^\pm \to K^\pm X_L$:

$$
A(X^\pm K_L) \equiv \frac{\Gamma(D^- \to X^- K_L) - \Gamma(D^+ \to X^+ K_L)}{\Gamma(D^- \to X^- K_L) + \Gamma(D^+ \to X^+ K_L)}
= \frac{\Delta_{K_L} + 2\text{Im}(T_a T_b^*) \cdot \sin(\delta_b - \delta_a)}{|T_a|^2 + |T_b|^2 - 2\text{Re}(T_a T_b^*) \cdot \cos(\delta_b - \delta_a)},
$$

(4a)

with

$$
\Delta_{K_L} = 2\text{Re}(\epsilon) \cdot \left(|T_b|^2 - |T_a|^2\right) - 4\text{Im}(\epsilon) \cdot \text{Re}(T_a T_b^*) \cdot \sin(\delta_b - \delta_a).
$$

(4b)

Note that $\delta_b \neq \delta_a$ is a necessary condition for nonvanishing direct CP violation in the decay amplitude. In some previous studies, $\Delta_{K_S} = \Delta_{K_L} = 0$ was assumed. Subsequently we shall take a few decay modes for example to illustrate the significant role of $\text{Re}(\epsilon)$ in the CP asymmetries.

\[\text{For simplicity, we neglect the overall normalization factor } 1/\sqrt{2(1 + |\epsilon|^2)} \text{ for } |K_S\rangle \text{ and } |K_L\rangle.\]
**Example 1.** We first consider the semileptonic decays $D^+ \to l^+ \nu_l K_S (K_L)$ vs $D^- \to l^- \bar{\nu}_l K_S (K_L)$. In this case, $T_a = 0$ and the $CP$ asymmetries turn out to be

$$ A(l^\pm K_S) = A(l^\pm K_L) = 2\text{Re}(\epsilon) = 2|\epsilon| \cos \phi_\epsilon. \quad (5) $$

Clearly these asymmetries are equal in magnitude to that in $K_L \to l^+ \nu_l \pi^- \text{ vs } K_L \to l^- \bar{\nu}_l \pi^+$. The current experimental data [2] give $|\epsilon| \approx 2.27 \times 10^{-3}$ and $\phi_\epsilon \approx 43.6^0$, unambiguously leading to $A(l^\pm K_S) = A(l^\pm K_L) \approx 3.3 \times 10^{-3}$.

From the averaged branching ratios of $D^+ \to l^+ \nu_l \bar{K}^0$ ($l = e$ or $\mu$) [2], one estimates $\text{Br}(D^+ \to l^+ \nu_l K_S) \approx \text{Br}(D^+ \to l^+ \nu_l K_L) \approx 3.4\%$. Thus observation of $A(l^\pm K_S)$ to 3 standard deviations needs about $2.4 \times 10^7 D^\pm$ events, if we assume perfect detectors or 100% tagging efficiencies. In practice, there are two possibilities to reduce half of the total $D^\pm$ events needed for measuring the $CP$ asymmetry. One way is to sum over the final states $e^+ \nu_e K_S$ and $\mu^+ \nu_\mu K_S$ as well as their charge-conjugated counterparts. The other is to sum over the final states $l^+ \nu_l K_S$ and $l^+ \nu_l K_L$ as well as their charge-conjugated counterparts. The result in eq. (5) implies that both ways should not induce dilution of the $CP$-violating signal. A combination of these two ways is in principle possible too, then only about $6 \times 10^6$ events of $D^\pm$ mesons are required to observe $A(l^\pm K_S)$.

**Example 2.** Now we consider $CP$ violation in the two-body nonleptonic decays $D^\pm \to (\pi^\pm, \rho^\pm, a_1^\pm) + (K_S, K_L)$. These transitions can occur through four tree-level quark diagrams, as shown in fig. 1. One expects that the annihilation-type diagram fig. 1(d) is formfactor-suppressed and thus negligible. Regardless of final-state interactions, the relative size of $T_a$ and $T_b$ in each decay can be roughly estimated by using the effective weak Hamiltonian and factorization approximation [9]. Respectively for $X = \pi, \rho$ and $a_1$, we obtain

$$ \frac{T_b}{T_a} = \frac{V_{cs} V_{us}^*}{V_{cd} V_{us}^*} \times \begin{cases} 1 + \frac{a_1}{a_2} \cdot \frac{f_\pi}{f_K} \cdot \frac{F_{BK}^{\pi}(m_{2\pi}^2)}{F_{BK}^{\pi}(m_{2}^2)} \cdot \frac{m_{D}^2 - m_{K}^2}{m_{D}^2 - m_{\pi}^2}, \\ 1 + \frac{a_1}{a_2} \cdot \frac{g_\rho}{f_K} \cdot \frac{F_{BK}^{\rho}(m_{2\rho}^2)}{A_{0}^{BK}(m_{2}^2)} \cdot \left(\varepsilon^*_\rho \cdot p_{D}\right) \left(\varepsilon_{\rho}^* \cdot p_{K}\right), \\ 1 + \frac{a_1}{a_2} \cdot \frac{g_{a_1}}{f_K} \cdot \frac{F_{BK}^{a_1}(m_{2a_1}^2)}{A_{0}^{BK}(m_{2}^2)} \cdot \left(\varepsilon^*_{a_1} \cdot p_{D}\right) \left(\varepsilon_{a_1}^* \cdot p_{K}\right), \end{cases} \quad (6) $$

where $V_{ij}$ ($i = u, c, t; j = d, s, b$) are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements; $\varepsilon$ denotes the polarization vector of $1^\pm$ mesons, $p$ stands for the 4-momentum of a particle,
and \(a_{1,2}\) are the effective Wilson coefficients. The decay constants and formfactors in eq. (6) are self-explanatory and their values can be found from refs. [2,9]. From the above results we observe \(|T_b/T_a| \sim 1/\lambda^2 \approx 20\) and \(\text{Im}(T_b/T_a) \sim A^2\lambda^2\eta < 1/20\), where \(\lambda, A\) and \(\eta\) are the Wolfenstein parameters of the CKM matrix [10]. This implies that the \(\text{Re}(\epsilon)\) term of \(\Delta_{K_S}\) plays the dominant role in \(\mathcal{A}(X^+K_S)\), even though the size of \(\sin(\delta_b - \delta_a)\) might be maximal \((\pm 1)\). To a good degree of accuracy, one finds

\[
\mathcal{A}(\pi^+K_S) \approx \mathcal{A}(\rho^+K_S) \approx \mathcal{A}(a^+_1K_S) \approx 2|\epsilon| \cos \phi_\epsilon . \tag{7}
\]

The same result is obtainable for the \(CP\) asymmetries in \(D^\pm \to (\pi^\pm, \rho^\pm, a^\pm_1) + K_L\).

With the help of experimental data [2] on \(D^+ \to (\pi^+, \rho^+, a^+_1) + \bar{K}^0\), the branching ratios of \(D^\pm \to (\pi^\pm, \rho^\pm, a^\pm_1) + K_S (K_L)\) are estimated to be 1.4\%, 3.3\% and 4.0\%, respectively. As discussed for the case of semileptonic \(D^\pm\) decays, here one can also consider the possibility to sum over the available final states in order to obtain a statistically significant signal of \(CP\) violation. Optimistically speaking, only about \(10^{6-7}\) \(D^\pm\) events are needed to establish \(\mathcal{A}(\pi^+K_S)\) under a perfect experimental environment.

**Example 3.** Let us briefly discuss the effect of \(K^0 - \bar{K}^0\) mixing on \(CP\) asymmetries in multi-body nonleptonic decays of the type \(D^\pm \to \pi^\pm + n_0\pi^0 + n_\pm(\pi^\mp \pi^\pm) + (K_S, K_L)\), where the integers \(n_0\) and \(n_\pm\) are not simultaneously vanishing. Such decay modes can occur through the same quark diagrams as \(D^\pm \to \pi^\pm + (K_S, K_L)\) do, but \((u\bar{u})\) or \((d\bar{d})\) pair(s) need be created from the vacuum to form the additional \(\pi^0\) or \((\pi^+\pi^-)\) meson(s) in the final state. It is easy to show that the \(CP\) asymmetries in this type of decays are given by \(\mathcal{A}(\pi^\pm K_S)\) or \(\mathcal{A}(\pi^\pm K_L)\), i.e., \(2\text{Re}(\epsilon)\). Current experimental data have reconstructed the modes \(D^+ \to \bar{K}^0\pi^+\pi^0\), \(K^0\pi^+\pi^+\pi^-\) and \(K^0\pi^+\pi^-\pi^-\pi^0\), whose branching ratios are 9.7\%, 7.0\% and 5.4\% respectively [2]. Thus there exists large potential to isolate a signal of \(CP\) violation in these multi-body processes. For this purpose, the required number of \(D^\pm\) events might be of order \(10^{6-7}\).

The above results show that the effect of \(K^0 - \bar{K}^0\) mixing on \(CP\) asymmetries in weak \(D^\pm\) decays is not negligible \textit{a priori}. In a similar way one may discuss \(\text{Re}(\epsilon)\)-induced \(CP\) violation in the decay modes of \(D^+_s\) or \(D^0\) mesons. A \(\tau\)-charm factory running at the \(\psi^{''}(3770)\) resonance with an integrated luminosity \(L = 10^{33}\) cm\(^{-2}\)s\(^{-1}\) could produce about \(4 \times 10^7\) \(D^\pm\) pairs [11], just the number of events needed for probing \(CP\) violation at the level of \(10^{-3}\). This amount of \(D^\pm\) events could also be accumulated at the forthcoming \(B\) factories, \(Z\) factory, or hadron
machines. Thus it is very promising to observe Re(\(\epsilon\))-induced CP violation in exclusive decays of \(D^\pm\) mesons in the near future. To finally explore CP violation in the \(c\)-quark sector or to pin down new physics at the level of \(10^{-3}\) in the \(D\)-meson system, one has to study those weak \(D\) transitions where \(K^0 - \bar{K}^0\) mixing is absent or its effect can be safely neglected.

In the following we discuss Re(\(\epsilon\))-induced CP violation in the semi-inclusive and exclusive decays \(B_d \rightarrow X(c\bar{c}) + (K_S, K_L)\), whose quark diagrams are shown in fig. 2. The tree-level amplitude is dominant over the loop-induced penguin (hairpin) amplitude, since the latter can only occur through three-gluon exchanges. With the help of unitarity of the CKM matrix, we obtain the ratio of transition amplitudes for \(\bar{B}_d^0 \rightarrow X(c\bar{c}) + K_S\):

\[
\xi = \frac{A(\bar{B}_d^0 \rightarrow X(c\bar{c}) + K_S)}{A(B_d^0 \rightarrow X(c\bar{c}) + K_S)}
\]

\[= \frac{1 - \epsilon^*}{1 + \epsilon^*} \cdot \frac{(V_{cb}V_{cs}^*)A_c + (V_{tb}V_{ts}^*)A_t}{(V_{cb}V_{cs}^*)A_c + (V_{tb}V_{ts}^*)A_t},\]

where \(A_{c,t}\) are the hadronic amplitudes containing the strong phases. Because \(|A_t/A_c|\) is expected to be rather smaller than unity, \(\xi\) can be further written as

\[
\xi \approx 1 - 2\epsilon^* - 2\chi
\]

with \(\chi \approx i\lambda^2\eta(A_t/A_c)\) to lowest-order approximation. The nonvanishing \(\chi\) implies direct CP violation in the decay amplitude. We shall see later on that only Re(\(\chi\)) \(\approx -\lambda^2\eta\text{Im}(A_t/A_c)\) enters the CP asymmetries of our interest. For an exclusive decay mode like \(B_d \rightarrow J/\psi K_S\), a rough estimate gives Re(\(\chi\)) \(\leq 10^{-3}\) [12]. It is likely that the semi-inclusive decay rate of \(B_d \rightarrow X(c\bar{c}) + K_S\) should hardly be affected by final-state interactions (signified by \(\chi\)), since a sum over many available \(X(c\bar{c})\) states may give rise to dilution of different \(\chi\).

Specifically we assume that the experimental scenario is at the \(\Upsilon(4S)\) resonance, the basis of the forthcoming \(B\)-meson factories. On the \(\Upsilon(4S)\) resonance, the \(B\)'s are produced in a two-body (\(B_u^+B_u^-\) or \(B_d^0\bar{B}_d^0\)) state with odd charge-conjugation parity. Since the two neutral \(B\) mesons mix coherently until one of them decays, one can use the semileptonic decay of one meson to tag the flavor of the other meson decaying to \(X(c\bar{c})K_S\) or \(X(c\bar{c})K_L\). A generic formalism for the time-dependent or time-integrated decays of any coherent \(P^0\bar{P}^0\) system \((P^0 = K^0, D^0, B_d^0\text{ or } B_s^0)\) has been given by the author in ref. [13]. For our present purpose we only consider the time-integrated \(B_d^0\bar{B}_d^0\) transitions, which can be measured at either symmetric
or asymmetric $B$ factories. The joint decay rates of $(B^0_d \bar{B}^0_d)_{\Upsilon(4S)} \to (X(c\bar{c}) + K_S)_{B_d}(l^+ + \ldots)_{B_d}$ are given as [13]

$$
\begin{align*}
R(l^+, X(c\bar{c})K_S) &\propto |p/q|^2 + |\xi|^2 - a \left(|p/q|^2 - |\xi|^2\right), \\
R(l^-, X(c\bar{c})K_S) &\propto |q/p|^2 \cdot \left[|p/q|^2 + |\xi|^2 + a \left(|p/q|^2 - |\xi|^2\right)\right],
\end{align*}
$$

where $q/p \equiv (1 - \epsilon_B)/(1 + \epsilon_B)$ is a mixing parameter of the $B^0_d - \bar{B}^0_d$ system, and $a = (1 - y_d^2)/(1 + x_d^2)$ with $x_d \equiv \Delta m_B/\Gamma_B$ and $y_d \equiv \Delta \Gamma_B/(2\Gamma_B)$. The current experimental data give $x_d \approx 0.71$ [2], while a model-independent analysis shows $y_d \lesssim 10^{-2}$ [3]. In the context of the standard model one estimates $2\text{Re}(\epsilon_B) \sim O(10^{-3})$ [6], which is of the same order as $|\epsilon|$. Thus $y_d^2$ is negligible in the decay-rate difference between $R(l^+, X(c\bar{c})K_S)$ and $R(l^-, X(c\bar{c})K_S)$. To lowest-order approximations of $\epsilon_B$, $\epsilon$ and $\chi$, we obtain the time-independent $CP$ asymmetry

$$
\mathcal{A}_{CP} = \frac{R(l^-, X(c\bar{c})K_S) - R(l^+, X(c\bar{c})K_S)}{R(l^-, X(c\bar{c})K_S) + R(l^+, X(c\bar{c})K_S)}
$$

$$
\approx 2\text{Re}(\epsilon) - 2x_d^2\text{Re}(\epsilon_B) + 2\text{Re}(\chi)
\approx \frac{2\text{Re}(\epsilon) - 2x_d^2\text{Re}(\epsilon_B) + 2\text{Re}(\chi)}{1 + x_d^2}.
$$

As pointed out before, $\text{Re}(\chi)$ might be comparable in magnitude with $\text{Re}(\epsilon)$ and $\text{Re}(\epsilon_B)$ in the exclusive decay modes such as $B_d \to J/\psi K_S$. For the semi-inclusive decay $B_d \to X(c\bar{c}) + K_S$, the contribution of $\text{Re}(\chi)$ should be less important due to possible dilution effects. If this is true, then the semi-inclusive asymmetry $\mathcal{A}_{CP}$ mainly measures $CP$ violation in the $K^0 - \bar{K}^0$ and $B^0_d - \bar{B}^0_d$ mixing matrices. Note that the same $CP$ asymmetry ($\mathcal{A}_{CP}$) is obtainable for $B_d \to X(c\bar{c}) + K_L$.

In principle $\text{Re}(\epsilon_B)$ can be measured from the charge asymmetry of semileptonic $B_d$ decays on the $\Upsilon(4S)$ resonance [6]: $\mathcal{A}_{SL} \approx 4\text{Re}(\epsilon_B)$. In practice to measure this signal to 3 standard deviations requires $10^7$ like-sign dilepton events, corresponding to about $10^{9-10}$ $B^0_d \bar{B}^0_d$ events. So many events are only reachable within the second-round experiments at a $B$ factory [3]. In comparison, to measure the $CP$ asymmetry $\mathcal{A}_{CP}$ needs a bit more $B^0_d \bar{B}^0_d$ events due to the cost for flavor tagging. It has been shown that the semi-inclusive branching ratio of $B_d \to X(c\bar{c})K_S$ is about 1% [14]. Practically one may use the semileptonic decays $B^0_d \to D^{*-}l^+\nu_l$ and $\bar{B}^0_d \to D^{*-}l^-\bar{\nu}_l$, which have branching ratios of 4.4% [2], to tag the flavor of $B_d$ mesons decaying to $X(c\bar{c}) + (K_S, K_L)$. Thus it is possible to establish $\mathcal{A}_{CP}$ to 3 standard deviations with about $10^{10-11}$ $B^0_d \bar{B}^0_d$ events on the $\Upsilon(4S)$ resonance. Here the interesting point is that smallness of $\mathcal{A}_{CP}$ implies its high sensitivity to new physics in $\Delta B = 2$ transitions [15] or from $CPT$
violation [16]. For this reason, a more detailed study of $CP$ violation in the semi-inclusive and exclusive decays $B_d \rightarrow X(c\bar{c}) + (K_S, K_L)$ is worthwhile.

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Figure 1: Quark diagrams for $D^+ \rightarrow \pi^+ K_S$ or $\pi^+ K_L$ in the standard model. Here $\pi^+$ can be replaced by $\rho^+$ or $a_1^+$. 

Figure 2: Tree-level and penguin (hairpin) diagrams for the semi-inclusive (or exclusive) decay modes $B_d^0 \rightarrow X(\bar{c}c) + (K_S, K_L)$. 