Spontaneous CP Violation in Models with Anomalous U(1)

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Abstract

We examine a class of Froggatt-Nielsen models with an anomalous $U(1)$ as the flavor dependent symmetry. Anomaly cancellation and unbroken supersymmetry impose constraints on the $U(1)_X$ charges of the fermions and the vacuum expectation values of the symmetry-breaking scalars. We show by example that it is possible to find models that reproduce the observed masses and mixings of the standard model fermions, and exhibit a realistic amount of CP violation.
1 Introduction

Froggatt-Nielsen models [1] offer an elegant explanation for the observed hierarchy in fermion masses and mixings. They have recently attracted much interest in the context of string theory. One hopes that a more fundamental theory can provide more details on how the rather general Froggatt-Nielsen mechanism is implemented, and give some predictions. One such attempt is to assume that the requisite broken symmetry is the anomalous $U(1)$ of a compactified string theory [2]. The anomalies are cancelled by the Green-Schwarz mechanism [3], which imposes constraints on the $U(1)_X$ charges of the standard model particles.

In an earlier paper [4], we examined such a model with the additional assumption that supersymmetry must not be broken at the string scale. In the simplest case where the $U(1)_X$ symmetry is broken by the vacuum expectation value (VEV) of only one field, this determines the symmetry breaking scale and hence the hierarchy parameter $\lambda$. We found it was possible to find models ($U(1)_X$ charge assignments for the standard model particles) that would produce phenomenologically viable masses and mixings, while at the same time satisfying all the anomaly and supersymmetry constraints.

Although the results were encouraging, our simplest model was not rich enough to include CP violation. This could be achieved by allowing the order one coefficients in the Yukawa mass matrices to be complex; in this paper, we examine the case of spontaneous CP violation — we assume that the coefficients are real, but there are two scalar fields breaking the the $U(1)_X$ symmetry. We find, without any fine tuning or additional assumptions, a number of models that satisfy all anomaly and supersymmetry constraints, and include acceptable masses, mixings, and the right amount of CP violation. The result does not strongly depend on the particular choice of the VEVs of the symmetry-breaking fields or the coefficients in the mass matrices. We conclude that the existence of such models is a generic phenomenon, rather than an exception. This allows more room for string model building, since the underlying fundamental string theory may not produce complex couplings.

Froggatt-Nielsen models are built on the assumption that a flavor-dependent $U(1)$ gauge symmetry is broken by the VEV of a field that is a singlet under the standard model gauge interactions. Charge conservation disallows direct Yukawa couplings of quarks and leptons; the interactions required by
the standard model between the up and down type quarks and leptons and
the Higgs boson proceed through higher-order tree diagrams such as the one
shown in Fig. 1. With all the three-point couplings of the same order of
magnitude, one can obtain the hierarchy of the standard model couplings re-
quired by experiment if the VEV of the symmetry-breaking field \( \theta \) is slightly
below the mass of the intermediate fermions—so that \( \lambda = f_\theta \langle \theta \rangle / M \simeq 0.2 \).
We assume that the Yukawa coupling \( f_\theta \) of the \( \theta \) field is of order one; we will
set it to one for simplicity. If \( q_\theta = -1 \), the resulting effective Yukawa mass
matrix is

\[
Y_u = f_u \lambda^{q_{H1}} \begin{pmatrix}
\lambda^{q_{Q1}+q_{u1}} & \lambda^{q_{Q1}+q_{u2}} & \lambda^{q_{Q1}+q_{u3}} \\
\lambda^{q_{Q2}+q_{u1}} & \lambda^{q_{Q2}+q_{u2}} & \lambda^{q_{Q2}+q_{u3}} \\
\lambda^{q_{Q3}+q_{u1}} & \lambda^{q_{Q3}+q_{u2}} & \lambda^{q_{Q3}+q_{u3}}
\end{pmatrix}.
\]

(1)

Throughout this paper, \( q_{Qi}, q_{ui}, q_{di}, q_{Li}, \) and \( q_{ei} \) will denote the \( X \) charges
of the left-handed fields: quark doublets, up-type antiquarks, down-type
antiquarks, lepton doublets and positrons (\( i = 1, 2, 3 \) is the family index).
We use \( q_Q = \sum_{i=1}^{3} q_{Q_i} \), etc. as an abbreviation for the sum of charges over
families, but \( q_H = q_{H1} + q_{H2} \) is the sum of the \( X \) charges of the two Higgs
doublets. With the freedom to choose the flavor symmetry charges for all the
standard model fields, we can approximate the experimental results. Thus,
very small ratios of masses and mixings are explained as powers of a not-so-
small number \( \lambda \simeq 0.2 \).

The above explanation is attractive but by itself not satisfactory: no the-
oretical principle determines \( \langle \theta \rangle \) or \( M \), we only know that for the mechanism
to work, their ratio must be about 0.2. We have to assume the existence
of some intermediate heavy fermions that will carry color and hypercharge.
Unless $M$ is above the unification scale, those fermions will interfere with gauge coupling unification.

The gauge groups of compactified string models often include an anomalous $U(1)$ factor. Taking such a model as our underlying theory brings a number of benefits. The energy scale of the problem is around the string scale $M_s$, and the symmetry-breaking scale should be somewhat lower, so having $M = M_s$ and $\langle \theta \rangle = 0.2M_s$ is quite acceptable. This avoids the problem with gauge coupling unification. Cancelling the anomalies via the Green-Schwarz mechanism imposes constraints on the otherwise unrestricted Froggatt-Nielsen model. Requiring that supersymmetry remain unbroken at the high scale gives another constraint, which may be used to predict the value of the ratio $\langle \theta \rangle/M$. Finally, it gives the Froggatt-Nielsen construction a fundamental background by setting it in the context of string theory.

2 Supersymmetry and anomaly constraints

In theories with anomalous $U(1)$, the $D$ term corresponding to $U(1)_X$ is modified by a Fayet-Iliopoulos term [4, 5, 7]:

$$D = \frac{g_s M_s^2}{192\pi^2} \text{tr} \ Q + \sum_i q_i |\phi_i|^2,$$

(2)

where $M_s$ is the string scale, $g_s$ is the string coupling constant, and $\phi_i$ are all the scalars of the theory. In order for supersymmetry to be left unbroken at high energies, we must have $\langle D \rangle = 0$. Assuming that $\theta$ is the only field that develops an expectation value, this determines the VEV of $\theta$, and the hierarchy parameter $\lambda$, in terms of the $X$ charges of all the fermions:

$$\lambda = \frac{\langle \theta \rangle}{M_s} = \sqrt{\frac{-g_s}{192\pi^2} \frac{\text{tr} \ Q}{q_\theta}}.$$  

(3)

We are looking for models with $X$ charge assignments such as to give $\lambda \simeq 0.2$.

We need the Green-Schwarz mechanism to cancel mixed anomalies of $U(1)_X$ with the standard model gauge groups. The anomaly coefficients are

$$C_1 = \frac{1}{6} (q_Q + 8q_u + 2q_d + 3q_L + 6q_e + 3q_H).$$

3
\[
C_2 = \frac{1}{2}(3q_Q + q_L + q_H)
\]
\[
C_3 = \frac{1}{2}(2q_Q + q_u + q_d)
\]
\[
C_{\text{grav}} = \frac{1}{24} \sum_{\text{all fields}} q_i = \frac{1}{24}(6q_Q + 3q_u + 3q_d + 2q_L + q_e + 2q_H + q_\theta + q_X)
\]

where \(C_1 = \text{tr} [Q(Y/2)^2]\) is the coefficient of the \(U(1)_X [U(1)_Y]^2\) anomaly, and \(C_{2,3} = \frac{1}{2}\text{tr}_{2,3} Q\) are the \(U(1)_X [SU(2)_L]^2\) and \(U(1)_X [SU(3)_c]^2\) anomalies. (The trace is over fermions with \(SU(2)_L\) and \(SU(3)_c\) charges, respectively.) \(C_{\text{grav}}\) is the mixed gravitational anomaly, and \(C_X = \text{tr} Q^3\) is the cubic anomaly \([U(1)_X]^3\). In order for anomalies to be cancelled, we must have \(C_1 : C_2 : C_3 : C_X : C_{\text{grav}} = 5 : 3 : 1 : 1 : 1\). (5)

In this paper, we are only interested in \(X\) charges of the fields that either are part of the standard model or break the \(U(1)_X\) symmetry. We will refer to such charge assignments as “models” or “examples”. Our framework allows any number of fields that are singlets under the standard model gauge groups and do not break \(U(1)_X\). Except for (5), we make no assumptions about their \(X\) charges. Although we cannot evaluate \(C_X\) and \(C_{\text{grav}}\) independently of Eq. (5), we can now write \(\lambda\) as a function of \(C_3\), that is, in terms of the \(X\) charges of the quark fields:

\[
\lambda = \sqrt{-\frac{g_s}{8\pi^2} \frac{C_3}{q_\theta}}.
\]

The order of magnitude of \(C_3/q_\theta\) can be estimated from the determinants of quark mass matrices [9]:

\[
\prod_{\text{all quarks}} m_q = |\det Y_u| |\det Y_d| \sim f_u f_d^3 \lambda ^{-2q_Q + q_u + q_d + 3q_H}/q_\theta.
\]

If we take the mass ratios at the unification scale to be [10]

\[
\frac{m_u}{m_t} = \mathcal{O}(\lambda^8) ; \quad \frac{m_d}{m_b} = \mathcal{O}(\lambda^4) ; \quad \frac{m_e}{m_\tau} = \mathcal{O}(\lambda^4)
\]
\begin{align*}
\frac{m_c}{m_t} &= \mathcal{O}(\lambda^4) ; \\
\frac{m_s}{m_b} &= \mathcal{O}(\lambda^2) ; \\
\frac{m_\mu}{m_\tau} &= \mathcal{O}(\lambda^2) ,
\end{align*}
(8)
the product of the quark masses becomes
\begin{align*}
\prod_{\text{all quarks}} m_q &\sim f_u^3 f_d^3 \lambda^{18}.
\end{align*}
(9)
For \( q_H = 0 \), Eqs. (7) and (9) give \( C_3 \simeq 9, \quad \lambda \simeq 0.28 \).
(10)
Another constraint we need to take into account comes from \( C_{YXX} \), the \([U(1)_X]^2 U(1)_Y \) mixed anomaly. \( C_{YXX} \) depends only on the charges of the standard model fields, and it cannot be cancelled by the Green-Schwarz mechanism, so for every example we have to make sure that it vanishes:
\begin{align*}
C_{YXX} &= \sum_{\text{all fields}} Y_i q_i^2 = \sum_{i=1}^3 \left( q_{Qi}^2 - 2 q_{ui}^2 + q_{di}^2 - q_{Li}^2 + q_{ei}^2 \right) \\
&\quad - q_{H1}^2 + q_{H2}^2 = 0.
\end{align*}
(11)

3 Two symmetry-breaking fields

In the case of one symmetry-breaking field, the VEV could always be rotated by a gauge transformation so that it would be real. This is no longer true for two symmetry-breaking fields with different \( X \) charges. (Two fields with the same \( X \) charges and all other couplings will always appear together in the formulae and can be treated as a single field \( \theta = \theta_1 + \theta_2 \).) We can gauge away the imaginary part of one field, \( \theta_1 \), and are left with three parameters: \( |\langle \theta_1 \rangle|, \quad |\langle \theta_2 \rangle|, \quad \) and the angle between them, \( \alpha \). The powers of the complex VEV will make the Yukawa matrices and the CKM matrix complex, and consequently may lead to CP violation.

The anomaly conditions stay the same as in the model with one \( \theta \), but the supersymmetry condition becomes
\begin{align*}
q_{\theta_1} |\langle \theta_1 \rangle|^2 M_s^2 + q_{\theta_2} |\langle \theta_2 \rangle|^2 M_s^2 + \frac{g_s}{8\pi^2} C_3 = 0
\end{align*}
(12)
and the element of the Yukawa mass matrix, for the excess charge \( x = q_Q + q_u \) is

\[
(Y_u)_{ij} = \sum_{n_1 q_{Q1} + n_2 q_{Q2} = -x} C(n_1, n_2) \frac{\langle \theta_1 \rangle^{n_1} \langle \theta_2 \rangle^{n_2}}{M_\psi^{n_1 + n_2}}
\]

(13)

where \( C(n_1, n_2) \) is the combinatorial factor that in field theory sums all the diagrams with \( n_1 \) insertions of \( \theta_1 \) and \( n_2 \) insertions of \( \theta_2 \). In the context of a specific string model, it will be determined by a single string tree diagram, but here we use the field-theory value

\[
C(n_1, n_2) = \frac{(n_1 + n_2)!}{n_1!n_2!} f^{n_1+n_2} \theta.
\]

(14)

We now have two distinct cases: one when the signs of \( q_{\theta 1} \) and \( q_{\theta 2} \) are the same (negative by our convention) and the other when they are different. When the signs are the same, we can roughly estimate \( |\langle \theta_1 \rangle| \) and \( |\langle \theta_2 \rangle| \). There are two limit cases: when the VEV of one field dominates \( (|\langle \theta_1 \rangle| \gg |\langle \theta_2 \rangle| \) and \( |\langle \theta_1 \rangle|^{1/|q_{\theta 1}|} \gg |\langle \theta_2 \rangle|^{1/|q_{\theta 2}|} \), the dominant VEV will be about the same as in a single-\( \theta \) model with \( \langle \theta \rangle = |\langle \theta_1 \rangle| \). The other case is when \( |\langle \theta_1 \rangle| = |\langle \theta_2 \rangle| \).

Then,

\[
\frac{|\langle \theta_1 \rangle|}{M_s} = \frac{-g_s}{8\pi^2} \frac{C_3}{q_{\theta 1} + q_{\theta 2}}.
\]

(15)

In this case, \( C_3 \) can be estimated by following the same logic as in the single-\( \theta \) case (but with a greater margin of error). If \( |q_{\theta 1}| < |q_{\theta 2}| \), then the Yukawa matrix element will be of order \( \lambda^{n_1+n_2} \), where \( n_1 + n_2 \) will be no smaller than \( (q_{Qi} + q_{uj})/|q_{\theta 2}| \). The product of the determinants of mass matrices becomes

\[
\prod_{\text{all quarks}} m_q \sim f_u f_d \lambda^{-2C_3/|q_{\theta 2}|}.
\]

(16)

and the resulting hierarchy parameter is

\[
\lambda = \lambda_{\text{single-\( \theta \)}} \sqrt{q_{\theta 2}} / (q_{\theta 1} + q_{\theta 2}).
\]

(17)

For \( \lambda_{\text{single-\( \theta \)}} = 0.28 \), \( q_{\theta 1} = -1 \) and \( q_{\theta 2} = -2 \), we get \( \lambda = 0.23 \).

Since the superpotential must be holomorphic in \( \theta_1 \) and \( \theta_2 \), we must have \( n_1, n_2 \geq 0 \). This will give rise to texture zeroes when the equation \( n_1 q_{\theta 1} + n_2 q_{\theta 2} = -x \) has no solution such that \( n_1, n_2 \geq 0 \).
The case where \( q_{\theta_1} \) and \( q_{\theta_2} \) have opposite signs is different in many aspects. First, we have no easy way to estimate the magnitude of the VEVs. If \(|q_{\theta_1}|\) and \(|q_{\theta_2}|\) are relatively prime, the equation \( n_1 q_{\theta_1} + n_2 q_{\theta_2} = -x \) always has infinitely many solutions, so there are no texture zeroes. The sum (13) becomes an infinite series which may or may not converge. In the following section, we generate some numerical examples for the same-sign case.

4 Numerical results

We now examine the two-theta model numerically through an exhaustive search of all \( X \) charge assignments for the standard model fields, where the charges are in the range from \(-10\) to \(10\). (We adopt a normalization such that the charges will be integers.) For the purpose of the search, we take the VEVs of the \( \theta \) fields to be equal (\(|\langle \theta_1 \rangle| = |\langle \theta_2 \rangle|\)), the angle between them \( \alpha = \pi/2 \), and the charges \( q_{\theta_1} = -1 \), \( q_{\theta_2} = -2 \). We use the tree-level value of the string coupling \([11, 12]\)

\[ g_s^2 = \frac{g_{GUT}^2}{k_{GUT}}, \]

with the Kac-Moody level \( k_{GUT} = 1 \). The unified gauge coupling constant \( \alpha_{GUT} = \frac{g_{GUT}^2}{4\pi} \simeq 1/25 \) gives \( g_s \simeq 0.7 \). The Yukawa coupling \( f_{\theta} = 1 \) as before.

For each set of charges that satisfies the anomaly constraints, we obtain the VEVs from the supersymmetry condition, then compute the Yukawa mass matrices using Eq. (13), and diagonalize them by singular value decomposition to obtain the masses and the CKM matrix. We reject the examples where (a) any of the masses is zero, or (b) two masses within the same sector are equal, e.g. \( m_u = m_c \), or (c) the mass ratios are too far away from the experimental values \([8]\), or (d) the CKM matrix is too different from the measured mixing matrix. To implement condition (c), we introduce a “badness” score, for which a difference by a factor of 0.22 from a mass ratio in \([8]\) is worth one point; we sum those points for all the ratios \([8]\). Any example with badness greater than three, or more than two badness points in any one sector, is rejected; our best examples have badness around one. It should be noted that our method is not dependent on the particular choice of the mass ratios. If at some point a slightly different choice turns out to be better (e.g. because it solves another problem \([9]\)), it may change which examples will be
picked, but it will not affect our conclusions. (We don’t even need to write (8) in terms of powers of $\lambda$: any set of mass ratios will do.)

For condition (d) we reject all examples where $|V_{12}|$ or $|V_{21}|$ is not in the range 0.17–0.25. The Cabibbo angle is the most precisely measured element of the CKM matrix and it is also almost invariant when renormalized to the unification scale [13]. We will see in the examples below that the remaining CKM matrix elements are within an order of magnitude of the experimental values. The 90% confidence experimental limits on the magnitude of the CKM matrix elements [14], renormalized to the GUT scale [13], are

$$
\begin{pmatrix}
0.9745 \text{ to } 0.9757 & 0.219 \text{ to } 0.224 & 0.001 \text{ to } 0.003 \\
0.218 \text{ to } 0.224 & 0.9736 \text{ to } 0.9750 & 0.023 \text{ to } 0.030 \\
0.002 \text{ to } 0.009 & 0.022 \text{ to } 0.032 & 0.9995 \text{ to } 0.9997
\end{pmatrix}.
$$

For each example we calculate $J^{CP}$, the invariant measure of CP violation [15]. $J^{CP}$ is defined as

$$
J^{CP} = \left| \text{Im} \left( V_{i\alpha} V_{j\beta} V_{i\beta}^* V_{j\alpha}^* \right) \right| \tag{18}
$$

(no summation, $i \neq j$, $\alpha \neq \beta$), which in the Kobayashi-Maskawa parametrization of the CKM matrix [16] becomes

$$
J^{CP} = c_1 c_2 c_3 s_1^2 s_2 s_3 \sin \delta, \tag{19}
$$

and in the Chau-Keung parametrization [17] used by the Review of Particle Properties [14] it is

$$
J^{CP} = c_{12} c_{13} c_{23} s_{12} s_{13} s_{23} \sin \delta_{13}. \tag{20}
$$

An important property of $J^{CP}$ is that it can be written in terms of the absolute values of the CKM matrix elements:

$$
(J^{CP})^2 = |V_{ub}|^2 |V_{cb}|^2 |V_{ud}|^2 |V_{cd}|^2 - \frac{1}{2} \left( 1 - |V_{ud}|^2 - |V_{cd}|^2 - |V_{ub}|^2 \right)
+ |V_{ud}|^2 |V_{cb}|^2 + |V_{ub}|^2 |V_{cd}|^2 \right)^2. \tag{21}
$$

That means that if we can generate the correct magnitudes of the CKM matrix elements, we will automatically generate the correct amount of CP
violation. Conversely, in a single-theta model without CP violation \[4\], we
cannot correctly generate the small elements of the CKM matrix. To be
consistent with recent measurements, $J^{CP}$ should be about $10^{-5}$ or less.

In order to generate the examples, we had to introduce a parameter,
the “texture factor” (TF). We have assumed that all the coefficients in the
Yukawa mass matrices are of order one; however, setting them all to one does
not produce acceptable examples. Since there is no reason to believe that
they are all equal to one, we arbitrarily decided to multiply the (2,3), (3,2)
and (3,3) entries of $Y_u$, and to divide the same entries of $Y_d$, by TF. (We
chose to do it this way to avoid introducing many additional parameters.)

We now proceed to the examples. With the $X$ charges

| $i$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|-----|------|------|------|------|------|
| 1   | 8    | 9    | 0    | 3    | 10   |
| 2   | 5    | 2    | −1   | −3   | 5    |
| 3   | 1    | −1   | −1   | −6   | 3    |

we have $C_3 = 18$ and $\lambda_1 = \lambda_2 = 0.23$. With the texture factor TF = 2, we
find the fermion mass ratios (corresponding to badness 0.98)

\[
\frac{m_u}{m_t} = 7.0 \times 10^{-6}, \quad \frac{m_d}{m_b} = 2.2 \times 10^{-3}, \quad \frac{m_e}{m_\tau} = 1.9 \times 10^{-3}, \\
\frac{m_c}{m_t} = 2.1 \times 10^{-3}, \quad \frac{m_s}{m_b} = 2.6 \times 10^{-2}, \quad \frac{m_\mu}{m_\tau} = 6.1 \times 10^{-2}, \\
\frac{m_t}{f_u} = 1.5, \quad \frac{m_b}{f_d} = 0.97, \quad \frac{m_\tau}{f_d} = 1.0,
\]

the CKM matrix (we show the absolute values of the elements)

\[
V = \begin{pmatrix}
0.97 & 0.24 & 3.6 \times 10^{-3} \\
0.24 & 0.97 & 2.2 \times 10^{-2} \\
8.9 \times 10^{-3} & 2.1 \times 10^{-2} & 1.0
\end{pmatrix},
\] (22)

and the CP violation invariant

\[
J^{CP} = 1.2 \times 10^{-6}.
\] (23)

While not all values of the texture factor work equally well (TF = 2 is a
good choice, and TF = 1 is a poor one), we were able to find examples for
a range of TF between 1 and 3. This shows that TF = 2 is not a necessary condition for the existence of good examples. Here is one for TF = 1.8:

\[
\begin{array}{c|cccccc}
 i & q_i & q_{u_i} & q_{d_i} & q_{L_i} & q_{e_i} \\
 1 & 9 & 10 & 0 & 0 & 8 \\
 2 & 5 & 3 & 0 & -1 & 7 \\
 3 & 0 & 0 & -2 & -2 & 0 \\
\end{array}
\]

gives \( C_3 = 19.5 \) and \( \lambda_1 = \lambda_2 = 0.24 \), the fermion mass ratios (badness 0.85) are

\[
\begin{align*}
\frac{m_u}{m_t} &= 6.0 \times 10^{-6}, \quad \frac{m_d}{m_b} = 1.6 \times 10^{-3}, \quad \frac{m_e}{m_{\tau}} = 3.5 \times 10^{-3}, \\
\frac{m_c}{m_t} &= 2.3 \times 10^{-3}, \quad \frac{m_s}{m_b} = 5.9 \times 10^{-2}, \quad \frac{m_\mu}{m_{\tau}} = 5.5 \times 10^{-2}, \\
\frac{m_t}{f_u} &= 1.8, \quad \frac{m_b}{f_d} = 1.1, \quad \frac{m_{\tau}}{f_d} = 1.0,
\end{align*}
\]

the CKM matrix (absolute values) is

\[
V = \begin{pmatrix} 0.98 & 0.22 & 2.9 \times 10^{-3} \\ 0.22 & 0.98 & 8.4 \times 10^{-3} \\ 1.2 \times 10^{-3} & 8.8 \times 10^{-3} & 1.0 \end{pmatrix}.
\]

and the CP violation invariant

\[ J^{CP} = 1.5 \times 10^{-6}. \]

To see that the results do not depend on the way the texture factors were introduced, we “randomly” picked by hand the coefficients of the up and down mass matrices. For the following choice of coefficients

\[
Y_u \sim \begin{pmatrix} 1.2 & 1.1 & -0.7 \\ 0.95 & 1.5 & -2.0 \\ 1.6 & 0.5 & 1.2 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} 0.8 & -0.9 & -1.3 \\ 0.9 & 1.4 & 2.0 \\ 2.0 & 0.7 & 0.9 \end{pmatrix}
\]

we get

\[
\begin{array}{c|cccccc}
 i & q_i & q_{u_i} & q_{d_i} & q_{L_i} & q_{e_i} \\
 1 & 7 & 5 & 1 & 10 & 10 \\
 2 & 4 & 4 & 0 & -3 & 8 \\
 3 & 0 & 0 & 0 & -8 & -5 \\
\end{array}
\]
The fermion mass ratios (badness 2.33) are
\[ \begin{align*}
\frac{m_u}{m_t} &= 5.6 \times 10^{-6}, \quad \frac{m_d}{m_b} = 5.0 \times 10^{-3}, \quad \frac{m_e}{m_\tau} = 2.5 \times 10^{-3}, \\
\frac{m_c}{m_t} &= 1.0 \times 10^{-2}, \quad \frac{m_s}{m_b} = 1.1 \times 10^{-1}, \quad \frac{m_\mu}{m_\tau} = 3.1 \times 10^{-2}, \\
\frac{m_t}{f_u} &= 1.2, \quad \frac{m_b}{f_d} = 1.2, \quad \frac{m_\tau}{f_d} = 1.0,
\end{align*} \]

the CKM matrix
\[ V = \begin{pmatrix}
0.98 & 0.21 & 2.5 \times 10^{-3} \\
0.21 & 0.98 & 6.9 \times 10^{-2} \\
1.7 \times 10^{-2} & 6.7 \times 10^{-2} & 1.0
\end{pmatrix}. \tag{26} \]

and
\[ J^{CP} = 1.5 \times 10^{-5}. \tag{27} \]

We also looked for plausible examples in the case where the VEVs of the two \( \theta \) fields are not equal. For the ratio \( |\langle \theta_2 \rangle| / |\langle \theta_1 \rangle| = 0.1 \), the CP violation parameter was very small. For \( |\langle \theta_2 \rangle| / |\langle \theta_1 \rangle| = 0.9 \), we have

| \( i \) | \( q_0 \) | \( q_{ui} \) | \( q_{di} \) | \( q_{Li} \) | \( q_{vi} \) |
|---|---|---|---|---|---|
| 1 | 9 | 10 | -1 | 2 | 10 |
| 2 | 5 | 1 | -1 | -3 | 8 |
| 3 | 1 | -1 | -2 | -8 | 2 |

The fermion mass ratios (badness 1.12) are
\[ \begin{align*}
\frac{m_u}{m_t} &= 5.6 \times 10^{-6}, \quad \frac{m_d}{m_b} = 2.7 \times 10^{-3}, \quad \frac{m_e}{m_\tau} = 3.2 \times 10^{-3}, \\
\frac{m_c}{m_t} &= 4.9 \times 10^{-3}, \quad \frac{m_s}{m_b} = 5.6 \times 10^{-2}, \quad \frac{m_\mu}{m_\tau} = 5.9 \times 10^{-2}, \\
\frac{m_t}{f_u} &= 2.1, \quad \frac{m_b}{f_d} = 1.1, \quad \frac{m_\tau}{f_d} = 1.0,
\end{align*} \]

the CKM matrix
\[ V = \begin{pmatrix}
0.97 & 0.23 & 5.1 \times 10^{-3} \\
0.23 & 0.97 & 1.3 \times 10^{-2} \\
2.1 \times 10^{-3} & 1.4 \times 10^{-2} & 1.0
\end{pmatrix}. \tag{28} \]
and

\[ J^{CP} = 3.3 \times 10^{-6}. \]  \hspace{1cm} (29)

5 Conclusions

We have attempted to show that string-inspired Froggatt-Nielsen models can be easily extended to include CP violation. We examined compactified string models with anomalous \( U(1) \) with anomalies cancelled by the Green-Schwarz mechanism.

Two scalar fields breaking the \( U(1)_X \) symmetry are needed to spontaneously break CP. The assumption that supersymmetry remains unbroken down to low energies leads to an estimate for the VEVs of the two fields and consequently for the Froggatt-Nielsen hierarchy parameters \( \lambda_{1,2} \). The requirement of anomaly cancellation puts constraints on the \( X \) charges of the standard model fields.

We are not working within a specific string model. We focus on model-independent features with very few important assumptions: that the \( U(1) \) symmetry in the Froggatt-Nielsen mechanism is anomalous, that it comes from string theory, that it is broken by the VEVs of two scalar fields, and that the coefficients of the powers of \( \lambda \) in the Yukawa mass matrices are real and of order unity. We also assume that \( f_\theta \), the Yukawa coupling of the \( \theta \) fields, is of order unity.

In the numerical computations we have introduced many unimportant assumptions, such as the values of the texture factors, the charges of the \( \theta \) fields, the angle between their VEVs and the ratio of their magnitudes. We have also set \( f_\theta = 1 \) and used the field-theory expression \( \text{[14]} \) for \( C(n_1, n_2) \). The numbers in the examples depend on those input values, but the qualitative features such as CP violation, masses and mixings in rough agreement with experiment do not. The numerical values of the mass ratios, mixings and the CP violation parameter can be meaningfully calculated only for a specific string model, where the unimportant assumptions will become unnecessary.
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