Compact Supersymmetry

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Introduction. — Supersymmetry has widely been regarded as the prime candidate for physics beyond the standard model [1]. It can explain the dynamical origin of electroweak symmetry breaking through renormalization group effects and provide a natural candidate for the cosmological dark matter in its simplest incarnations. In particular, it stabilizes the large hierarchy between the cosmological dark matter in its simplest incarnations. In

Supersymmetry broken geometrically in extra dimensions naturally leads to a nearly degenerate spectrum for superparticles, ameliorating the bounds from the current searches at the LHC. We present a minimal such model with a single extra dimension, and show that it leads to viable phenomenology despite the fact that it essentially has two less free parameters than the conventional CMSSM. The theory does not suffer from the supersymmetric flavor or CP problem because of universality of geometric breaking, and automatically yields near-maximal mixing in the scalar top sector with \(|A_t| \approx 2m_t| \) to boost the Higgs boson mass. Despite the rather constrained structure, the theory is less fine-tuned than many supersymmetric models.

values in visible and missing energies. The last option, however, has been discussed only phenomenologically [6], lacking theoretical justifications based on simple and explicit models of supersymmetry breaking.

In this Letter, we point out that the third possibility of a nearly degenerate superparticle spectrum is quite automatic when supersymmetry is broken by boundary conditions in compact extra dimensions, the so-called Scherk–Schwarz mechanism [7, 8]. With the simplest extra dimension—the S²/Z₂ orbifold—the mechanism has a rather simple structure [9]. In particular, locating matter and Higgs fields in the bulk and on a brane, respectively, and forbidding local-parity violating bulk mass parameters for the matter fields, the theory has only four parameters relevant for the spectrum of superparticles: the compactification scale \(1/R\), the 5D cutoff scale \(\Lambda\) (\(> 1/R\)), the supersymmetry-breaking twist parameter \(\alpha (\in [0, 1/2))\), and the supersymmetric Higgs mass \(\mu\).

Using the common notation in the MSSM, the spectrum of superparticles is given at the compactification scale \(\approx 1/R\) as

\[
M_{1/2} = \frac{\alpha}{R}, \quad m_{\tilde{Q}, \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}} = (\frac{\alpha}{R})^2, \quad m_{\tilde{H}_u, \tilde{H}_d} = 0, \quad A_0 = -\frac{2\alpha}{R}, \quad \mu \neq 0, \quad B = 0, \quad (1)
\]

at tree level. While these masses receive radiative corrections from physics at and above \(1/R\), they are under control because of the symmetries in higher-dimensional spacetime, and thus can naturally be small. Therefore, in this limit, the theory essentially has only three free parameters:

\[
\frac{1}{R}, \quad \frac{\alpha}{R}, \quad \mu. \quad (2)
\]

This rather compact set of parameters gives all the superparticle as well as the Higgs boson masses.

Even though Eq. (2) gives two less parameters than in the traditional CMSSM framework, we show that it still leads to viable phenomenology. In addition, it solves
the flavor problem that often plagues models of supersymmetry breaking, because the geometry is universal to all scalar particles and hence respect a large flavor symmetry. The problem of accommodating a large enough Higgs boson mass is ameliorated by the near-degeneracy between $t_L$ and $t_R$, and $|A_t| \approx 2m_t$. And the degenerate spectrum at tree level automatically achieves a compact spectrum that allows superpartners to be hidden from the current searches even when they are below TeV.

This Letter is organized as follows. We first review the basics of supersymmetry breaking by boundary conditions in the $S^1/\mathbb{Z}_2$ orbifold, and present the simplest model we study. We then present the low-energy spectrum of superparticles and discuss its phenomenology. We also provide benchmark points useful for further phenomenological studies of the model.

Supersymmetry Breaking by Boundary Conditions. — We consider a single compact extra dimension with the coordinate $y$ identified under $T : y \rightarrow y + 2\pi R$ and $P : y \rightarrow -y$. These two operations satisfy the algebra $PTP = T^{-1}$ and $P^2 = 1$, and the resulting extra dimension is an interval $y \in [0, \pi R]$; the $S^1/\mathbb{Z}_2$ orbifold.

We consider a supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory in this spacetime, with the gauge and matter supermultiplets yielding three generations of quarks and leptons as the zero modes, while their superpartners obtain the common soft mass of their zero modes leads to nontrivial cancellations of the corrections to Eq. (1) from physics above 1/R. (The same can also be seen in the 4D picture. In this picture, $N = 2$ supersymmetry existing for the $n > 0$ modes leads to nontrivial cancellations of the corrections to $M_{1/2}$, $m_f^2$ and $A_0$ from these modes. In order to see the cancellations for the gauge multiplets, the effect of anomalies must be taken into account correctly. The explicit demonstration of these nontrivial cancellations will be given elsewhere.)

The corrections from physics at 1/R arise from nonlocal operators in 5D. They affect all the supersymmetry-breaking masses, and are of order 1/16$\pi^2$. Here we calculate only the contributions to the Higgs mass parameters, which could potentially affect the analysis of electroweak symmetry breaking. By choosing the renormalization scale to be 1/(2$\pi R$), we find these corrections are

\begin{align}
\delta m_H^2 &= \frac{-3\alpha y_1^2 + 9(g_3^2 + g_4^2/5)}{16\pi^2} \left( \frac{\alpha}{\Lambda R} \right)^2, \\
\delta m_H^2 &= \frac{9(g_3^2 + g_4^2/5)}{16\pi^2} \left( \frac{\alpha}{\Lambda R} \right)^2, \\
\delta B &= \left( \frac{9g_1^2}{8\pi^2} - \frac{3(g_3^2 + g_4^2/5)}{8\pi^2} \right) \frac{\alpha}{\Lambda R},
\end{align}

where we have included only the contributions from the top-Yukawa coupling, $y_t$, and $SU(2)_L$ and $U(1)_Y$ gauge couplings, $g_2$ and $g_1$ (in the $SU(5)$ normalization).

1. The twist parameter $\alpha$ in the boundary conditions is equivalent to an $F$-term vacuum expectation value of the radion superfield [11], which can be generated dynamically through a radion stabilization mechanics and hence can be naturally small.

2. This assumption can be justified by a local parity in the bulk; see, e.g., [11].
In summary, the low-energy superparticle masses are obtained by evolving down Eqs. (1, 6) defined at the renormalization scale

\[ \mu_{\text{RG}} = \frac{1}{2\pi R}, \]

using the MSSM renormalization group equations. Incidentally, the gravitino mass is \( m_{3/2} = \alpha/R \), generated by the supersymmetry-breaking twist in the fifth dimension.

**Superparticle Spectrum.** — Following the procedure described above, we calculate the MSSM mass spectrum using SOFTSUSY 3.3.1 [12] and the lightest Higgs boson mass using FeynHiggs 2.8.6 [13]. In Fig. 1 we plot the contours of the mass of the lightest Higgs boson, \( M_H \), and the fine-tune parameter, defined by

\[ \Delta^{-1} = \min_x | \partial \ln m_\chi^2 / \partial \ln x |^{-1} \]

with \( x = \alpha, \mu, 1/R, y_1, y_2, \ldots, \) in the \( 1/R-\alpha/R \) plane. (The fine-tuning parameter is determined mostly by \( x = \mu \).) In the calculation, we have used the top-quark mass of \( m_t = 173.2 \text{ GeV} \) [14]. Varying it by \( 1\% \), \( \Delta m_\alpha = \pm 0.9 \text{ GeV} \), affects the Higgs boson mass by \( \Delta M_H \approx \pm 1 \text{ GeV} \). Also, theoretical errors in \( M_H \) are expected to be about \( \Delta M_H \approx 2 - 3 \text{ GeV} \) [15], so that the regions with \( M_H \gtrsim 121 - 123 \text{ GeV} \) in the plot are not necessarily incompatible with the 125 GeV Higgs boson hinted at the LHC [3]. Indeed, using the recently-released program HSIM [16], which includes a partial three-loop effect, we find that the corrections to \( M_H \) from higher order effects are positive and of order a few GeV in most of the parameter region in the plot.

In Fig. 2 the masses of selected superparticles (the lightest neutralino \( \chi^0_1 \), the lighter top squark \( \tilde{t}_1 \), and the gluino \( \tilde{g} \)) are shown. The masses of the first and second generation squarks are almost the same as the gluino mass. The masses of the electroweak superparticles are close to \( \alpha/R \), except for the lightest two neutralinos \( \chi^0_{1,2} \) and the lighter chargino \( \chi^+_1 \), which are Higgsino-like (and thus close in mass) in most of the parameter space. We find that the masses of the superparticles are degenerate at a 10% level, except possibly for the Higgsinos which can be significantly lighter (up to a factor of \( \approx 2 \)).

**Experimental Limits.** — As we have seen, the model naturally predicts a degenerate mass spectrum for superparticles. This has strong implications on supersymmetry searches at the LHC. Because of the mass degeneracy, production of high \( p_T \) jets and large missing energy is suppressed. Therefore, typical searches, based on high \( p_T \) jets and large missing energy, are less effective for the present model.

To estimate the number of supersymmetric events, we have used ISAJET 7.72 [17] for the decay table of superparticles, Herwig 6.520 [18] for the generation of supersymmetric events, AcerDET 1.0 [19] for the detector simulation, and NLL-fast [20] for estimation of the production cross section including next-to-leading order QCD corrections and the resummation at next-to-leading-logarithmic accuracy. To constrain the parameter space, we compare the obtained event numbers with the results of ATLAS searches for multi-jets plus large missing energy with and without a lepton at \( L = 4.7 \text{ fb}^{-1} \) at \( \sqrt{s} = 7 \text{ TeV} \) [21, 22]. In Fig. 3 we show the resulting LHC constraint on the model. Other searches such as those for \( b \)-jets and/or multi-leptons are less effective. We find that for \( 1/R \gtrsim 10^5 \text{ GeV} \), the case that \( m_\tilde{g} \approx m_\tilde{q} \lesssim 1 \text{ TeV} \) is still allowed. This constraint is significantly weaker than that on the CMSSM, which excludes \( m_\tilde{g} \lesssim 1.4 \text{ TeV} \) for \( m_\tilde{q} \approx m_\tilde{q} \) [21]. (We have checked that our naive method of estimating the LHC constraints adopted here reproduces this bound for the CMSSM spectra.)

We note that since \( B \) is not a free parameter in the present model, \( \tan \beta \) is determined by the electroweak symmetry breaking condition. We typically find \( \tan \beta \sim 4 - 10 \). This allows for the model to avoid the constraint from \( b \to s\gamma \), despite the large \( A \) terms.

The contribution of the Kaluza-Klein states to the electroweak precision parameters bounds \( 1/R \gtrsim \text{few TeV} \) [23]. Since we consider the region \( 1/R \gtrsim 2 \text{ TeV} \),
dance is assumed. It must be produced nonthermally to component of dark matter if only the thermal relic abundance, \( \Omega_{\chi} \), is much smaller than the observed dark matter density \( \Omega_{\chi} \). In Fig. 4, we show the thermal relic abundance, \( \Omega_{\chi} \), for \( 1/R = 10^4 \) GeV (lower, red shaded) and \( 10^5 \) GeV (upper, blue shaded). In each region, the upper and lower boundaries correspond to \( f_s = 0.26 \) and 0.02, respectively, and the dots represent the corresponding values of \( \alpha/R \). (The very light shaded regions are those in which the thermal abundance exceeds \( \Omega_{\chi} \).) The solid (black) line shows the current upper bound from XENON100.

10 TeV in this paper, however, the model is not constrained by the electroweak precision data.

Dark Matter. — In the present model, the dark matter candidate is the lightest neutralino \( \tilde{\chi}_1^0 \), whose dominant component is the Higgsino. In Fig. 4, we show the thermal relic abundance, \( \Omega_{\chi} h^2 \), and the spin-independent cross section with a nucleon, \( \sigma_{Nucleon} \), of \( \tilde{\chi}_1^0 \). The solid (black) lines are the contours of \( \Omega_{\chi} h^2 \), while the dotted (red) lines are those of \( \sigma_{Nucleon} \).

For further phenomenological studies, we presented the simplest such model in the \( S^1/Z_2 \) orbifold, and showed that it leads to viable phenomenology despite the fact that it essentially has two less free parameters than the CMSSM: \( 1/R, \alpha/R \), and \( \mu \). In Table I we give two representative points in the parameter space, which can serve benchmark points together with superpotential interactions on the boundary.

Final Remarks. — In this Letter, we pointed out that supersymmetry broken by boundary conditions in extra dimensions leads naturally to a nearly degenerate superparticle spectrum, ameliorating the limits from experimental searches. We presented the simplest such model in the \( S^1/Z_2 \) orbifold, and showed that it leads to viable phenomenology despite the fact that it essentially has two less free parameters than the CMSSM: \( 1/R, \alpha/R \), and \( \mu \). In Table I we give two representative points in the parameter space, which can serve benchmark points for further phenomenological studies.

The theory presented here can be extended in several different ways. An interesting one is to introduce a singlet field \( S \) together with superpotential interactions on the

\[ \sigma_{Nucleon}^\text{eff} \equiv \sigma_{Nucleon} \frac{\min\{\Omega_{\chi}, \Omega_{\chi}^{\text{DM}}\}}{\Omega_{\chi}^{\text{DM}}} \quad \text{(8)} \]

which is the quantity to be compared with the dark matter-nucleon cross section in the usual direct-detection exclusion plots (which assume \( \Omega_{\chi} = \Omega_{\chi}^{\text{DM}} \)). In Fig. 5 we plot \( \sigma_{Nucleon}^\text{eff} \) as a function of \( m_{\chi_1^0} \) for \( 1/R = 10^4 \) GeV and \( 10^5 \) GeV. To represent the uncertainty from the nucleon matrix element, we show both the \( f_s = 0.02 \) and 0.26 cases. We also present the current upper bound on \( \sigma_{Nucleon}^\text{eff} \) from XENON100 [26]. We find that improving the bound by one or two orders of magnitude will cover a significant portion of the parameter space of the model.
TABLE I. Phenomenologically viable mass spectrum of the benchmark points (in GeV). Point1: $1/R = 10^5$ GeV, $\alpha/R = 1400$ GeV and Point2: $1/R = 10^8$ GeV, $\alpha/R = 800$ GeV.

| Particle | Point1 | Point2 |
|----------|--------|--------|
| $\tilde{g}$ | 1494 | 949 |
| $\tilde{t}_L$ | 1467 | 939 |
| $\tilde{d}_L$ | 1469 | 942 |
| $\tilde{b}_2$ | 1460 | 924 |
| $\tilde{\chi}^0_1$ | 767 | 630 |
| $\tilde{\chi}^0_2$ | 1384 | 755 |
| $\tilde{\chi}^0_3$ | 771 | 642 |
| $\tilde{h}^0$ | 125 | 120 |
| $A^0$ | 819 | 717 |

$y = 0$ brane: $\lambda S H_d + f(S)$, where $f(S)$ is a polynomial of $S$ with the simplest possibility being $f(S) = -\kappa S^3/3$.

This allows for an extra contribution to the Higgs boson mass from $\lambda$, and can make the lightest neutralino (which would now contain a singlino component as well) saturate the observed dark matter abundance without resorting to nonthermal production. Detailed studies of this possibility will be presented elsewhere.

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