Particle Production and Positive Energy Theorems for Charged Black Holes in DeSitter

David Kastor and Jennie Traschen

Department of Physics and Astronomy
University of Massachusetts
Amherst, MA 01003-4525

Abstract

We study quantum mechanical and classical stability properties of Reissner-Nordstrom deSitter (RNdS) spacetimes, which describe black holes with mass $M$ and charge $Q$ in a background with cosmological constant $\Lambda \geq 0$. There are two sources of particle production in these spacetimes; the black hole horizon and the cosmological horizon. A scattering calculation is done to compute the Hawking radiation in these spacetimes. We find that the flux from the black hole horizon equals the flux from the cosmological horizon, if and only if $|Q| = M$, indicating that this is a state of thermodynamic equilibrium. The spectrum, however, is not thermal. We also show that spacetimes containing a number of charge equal to mass black holes with $\Lambda \geq 0$, have supercovariantly constant spinors, suggesting that they may be minimum energy states in a positive energy construction. As a first step in this direction, we present a positive energy construction for asymptotically deSitter spacetimes with vanishing charge. Because the construction depends only on a spatial slice, our result also holds for spacetimes which are asymptotically Robertson-Walker.

1 Internet: kastor@phast.umass.edu
2 Internet: lboo@phast.umass.edu
1. Introduction

In this paper we will discuss quantum mechanical and classical aspects of the stability of Reisner-Nordstrom deSitter (RNdS) spacetimes, in which the magnitude of the electric charge $Q$ is equal to the mass $M$. In the remainder of the introduction, we give the argument, based on euclidean quantum field theory, that $|Q| = M$ RNdS black holes are stable end points of the process of Hawking evaporation. In section 2, we present a lorentzian scattering calculation of particle production in RNdS spacetimes, which shows that the flux of particles across the cosmological horizon is equal to the flux across the black hole horizon, if and only if $Q = M$. Such a calculation is useful for a number of reasons. As well as confirming the Euclidean argument, in this case, it provides information on the spectrum of particles produced. Moreover, generically a spacetime will not have a real euclidean section, and we will be limited to lorentzian techniques. The multi-black hole generalizations of the $|Q| = M$ RNdS spacetimes are examples of nonstatic spacetimes which should have interesting thermodynamic properties. In section 3, we show that $|Q| = M$ RNdS spacetimes admit spinor fields which are constant with respect to a certain super-covariant derivative operator. The choice of derivative operator is motivated by supergravity and should lead to a version of the positive energy theorem relevant to RNdS black holes. Here, we present a more limited, preliminary result relevant to positivity of a suitably defined energy in asymptotically Robertson-Walker spacetimes without charge.

RNdS spacetimes have metric and gauge field given in static coordinates by

$$\begin{align*}
ds^2 &= -f(R)dt^2 + f(R)^{-1}dr^2 + r^2 d\Omega^2, \\
A &= -\frac{Q}{R} dt,
\end{align*}$$

where the cosmological constant $\Lambda$ is assumed to be positive. For a range of values of $Q$ and $M$, the spacetime has three Killing horizons: inner and outer black hole horizons and a cosmological horizon. The extremal limit, in which the inner and outer black hole horizons become coincident, occurs for $M \leq |Q|$, with equality in the case $\Lambda = 0$.

There are two sources of particle production in a RNdS spacetime: the black hole horizon and the deSitter horizon. It is interesting to ask whether these two sources can ever be in a state of thermal equilibrium. The deSitter horizon would then be like the wall of a box containing a black hole in thermal equilibrium with a bath of radiation. Several facts suggest that such an equilibrium exists when the magnitude of the charge $|Q|$ and mass $M$ of the black hole are equal and furthermore that the equilibrium is stable, as we now review.

One can assign temperatures $T_{bh}$ and $T_{ds}$ to the black hole and deSitter horizons based on the periodicity that must be imposed on the imaginary time coordinate in order

---

3 The euclidean picture of black hole thermodynamics can also be misleading, as in the case of charged dilaton black holes.

4 It has come to our attention that work on a positive energy construction in asymptotically deSitter spacetimes has recently been done by T. Shiromizu.
to obtain a regular euclidean section of the metric (1) at the given horizon [4]. The
temperatures defined in this way are given by

\[ 2\pi k T_{bh} = \kappa_{bh}, \quad 2\pi k T_{dS} = \kappa_{dS}, \]

(2)

where \( k \) is Boltzmann’s constant and \( \kappa_{bh} \) and \( \kappa_{dS} \) are the magnitudes of the surface gravities
at the black hole and deSitter horizons respectively. For \( |Q| = M \), one finds \( \kappa_{bh} = \kappa_{dS} \)
and the two temperatures are equal. Further, if \( M > |Q| \) then one finds that \( \kappa_{bh} > \kappa_{dS} \)
(and vice versa), suggesting that the black hole radiates (or absorbs) energy to reach
equilibrium at \( |Q| = M \). This is also the configuration of maximal gravitational entropy.
The gravitational entropy is given in terms of the areas \( A_{bh} \) and \( A_{dS} \) of the black hole and
deSitter horizons by

\[ S = \frac{1}{4} (A_{bh} + A_{dS}). \]

For an infinitesimal perturbation between RNdS solutions with fixed charge the first law of thermodynamics
[4] states that \( -\kappa_{dS} \delta A_{dS} = \kappa_{bh} \delta A_{bh} \).

These arguments based on euclidean quantum field theory suggest that if quantum
mechanical processes are taken into account, so that for example the area of the black hole
horizon can decrease, then a charged black hole with \( |Q| \neq M \) will evolve to a state with
\( |Q| = M \). In this paper, we give the results of a Lorentzian scattering calculation of the
particle production in Hawking radiation. We compare the flux of particles crossing the
cosmological horizon at late times (i.e., particles “coming from the black hole”) to the flux
which crosses the black hole horizon (i.e., particles “coming from the deSitter horizon”).
We find that the fluxes are equal if and only if \( |Q| = M \). In this sense, a \( |Q| = M \) black hole
is in equilibrium with the deSitter background, in agreement with the Euclidean picture.

2. Particle Production

We compute the rate of particle production in the RNdS spacetimes (1), for a massless
scalar field \( \phi \), by finding the mode mixing coefficients \( \beta_{\omega, \omega'}^{bh} \) and \( \beta_{\omega, \omega'}^{dS} \). The conformal
diagram for the relevant portion of RNdS is shown in fig. [4]. The region is bounded by the
white hole, black hole, past and future deSitter horizons, which have replaced past and
future null infinity in the case with zero cosmological constant. The first step is
to define what is meant by particles, i.e., we must choose a time coordinate near each
boundary, which defines the positive frequency modes there. Define the following Kruskal
type coordinates which are well behaved near the appropriate horizons

\[ U_{bh} = -\frac{1}{\kappa_{bh}} e^{-\kappa_{bh} u}, \quad V_{bh} = \frac{1}{\kappa_{bh}} e^{\kappa_{bh} v}, \]

\[ U_{dS} = \frac{1}{\kappa_{dS}} e^{\kappa_{dS} u}, \quad V_{dS} = -\frac{1}{\kappa_{dS}} e^{-\kappa_{dS} v} \]

(3)

where, in standard notation \( u \) and \( v \) are defined in terms of the tortoise coordinate \( r^* \)
according to

\[ u = t - r^*, \quad v = t + r^*, \quad dr^* = \frac{dr}{f^2}. \]

5 This can be generalized to include arbitrary, not necessarily stationary, stress energy perturbations, by the methods of [5]. Then there appears an additional volume integral of the perturbed energy density on the right hand side.
Near the black hole horizon, the metric then has the limiting form
\[ ds^2 \approx \kappa_{bh} dU_{bh} dV_{bh}, \]  
showing that these are good coordinates near the black hole horizon. One can similarly show that the coordinates are also good near the deSitter horizon.

For \( \Lambda = 0 \), one defines positive frequency near \( I^- \) by \( \phi \sim e^{-i\omega v} \), and positive frequency on the white hole horizon by \( \phi \sim e^{-i\omega U_{bh}} \). This choice of positive frequency on the white hole horizon correctly gives the outgoing thermal flux at \( I^+ \), allowing one to replace the collapsing body in Hawking’s original calculation with this boundary condition on the field (see e.g. [8],[9]). This suggests that for RNdS spacetimes, positive frequency at each horizon should be defined with respect to the appropriate Kruskal coordinate there. One can check that this is equivalent to choosing the affine parameter on e.g., the white hole horizon to define positive frequency there.

The Klein-Gordon equation for \( \phi \) near any of the horizons reduces to the free wave equation. As in the case with no cosmological constant, the potential term due to the background gravitational field decays exponentially in \( r^* \) near a horizon. Consider then a pure positive frequency outgoing wave near the deSitter horizon at late conformal time, \( \phi_\omega \sim e^{-i\omega U_{dS}} \) (see fig. [1]). In the geometrics optics limit (as with \( \Lambda = 0 \)), finding the form of this wave propagated back to the white hole horizon reduces to finding the dependence of the coordinate \( U_{dS} \) on the coordinate \( U_{bh} \). Using (3), it then follows that on the white hole horizon
\[ G_\omega (U_{bh}) \equiv \phi_\omega (U_{dS}(U_{bh})) \sim \begin{cases} e^{-i\omega \xi^2 \left( \frac{1}{\kappa_{bh}} \right) \eta}, & U_{bh} < 0 \\ 0, & U_{bh} > 0 \end{cases} \]
where \( \eta \equiv \kappa_{dS}/\kappa_{bh} \) and \( \xi^2 \equiv \frac{1}{\kappa_{dS}} \left( \frac{1}{\kappa_{bh}} \right) \eta \). The particle production follows from determining the negative frequency portion of this wave on the white hole horizon,
\[ \beta_{\omega,\omega'}^{bh} = \left| \frac{\omega'}{\omega} \right|^{\frac{1}{2}} \int dU_{bh} e^{-i\omega' U_{bh}} G_\omega (U_{bh}) \]  

Similarly, there is emission “from” the deSitter horizon as seen by an observer outside the black hole horizon at late times. Consider a positive frequency wave which is entering the black hole horizon (see fig. [1]), \( \phi_\omega \sim e^{-i\omega V_{bh}} \). Then in the geometrics optics approximation, on the past deSitter horizon, the wave is given by
\[ F_\omega (V_{dS}) \equiv \phi_\omega (V_{bh}(V_{dS})) \sim \begin{cases} e^{-i\mu^2 \left( \frac{1}{\kappa_{dS}} \right) \eta^2}, & V_{dS} < 0 \\ 0, & V_{dS} > 0 \end{cases} \]
where \( \mu^2 = 1/\kappa_{bh} \left( 1/\kappa_{dS} \right)^{\frac{1}{2}} \). Similarly to (3), \( \beta_{\omega,\omega'}^{dS} \) is given in terms of the fourier transform of (8). For general values of \( Q \) and \( M \), the functions \( F_\omega \) and \( G_\omega \) appearing in (3) and (8) are related according to
\[ G_\omega (x) = F_\omega (\frac{x^{1/2}}{\eta^2}). \]
We see that the two functions are equal for \( \eta = 1 \), which occurs when \(|Q| = M\). We then have the main result of this section

\[ \beta_{\omega, \omega'}^{bh} = \beta_{\omega, \omega'}^{dS} \quad \text{if and only if} \quad |Q| = M. \tag{10} \]

This implied that for each horizon, the flux of particles absorbed is equal to the flux of particles absorbed.

The spectrum of emitted particles is given by \( N_\omega = \int d\omega' |\beta_{\omega, \omega'}|^2 \). We can estimate the above integrals using the stationary phase approximation. It is simpler to work with the coefficients \( \alpha_{\omega, \omega'} = -\beta_{\omega, -\omega'} \). The stationary phase approximation can be used to evaluate this integral, and \( \beta_{\omega, \omega'} \) is then gotten by analytically continuation, after noticing that (6) implies that \( \alpha_{\omega, \omega'} \) is analytic in the lower half \( \omega' \) plane.

For the case \(|Q| = M\), when the surface gravities are equal, one finds in the stationary phase approximation

\[ |\beta_{\omega, \omega'}|^2 = \frac{\pi}{2\kappa |\omega\omega'|^{1/2}} e^{-\frac{2}{\kappa} |\omega\omega'|^{1/2}} \tag{11} \]

This approximation can be checked, because in the \(|Q| = M\) case the integral can be done exactly. One can show that \( \alpha_{\omega, \omega'} \) is only a function of the combination \( x \equiv \frac{1}{\kappa} |\omega\omega'|^{1/2} \). The function \( \alpha(x) \) can then be shown to satisfy the differential equation

\[ \alpha''(x) + \frac{1}{x} \alpha' + \left(4 - \frac{1}{x^2}\right) \alpha = 0. \tag{12} \]

Hence \( \alpha \) is proportional to the Bessel function \( J_1(x) \). Again analytically continuing, we then have \( \beta_{\omega, \omega'} \propto K_1(\frac{2}{\kappa} |\omega\omega'|^{1/2}) \). Using the asymptotic behavior of the modified Bessel function \( K_1(x) \), this agrees with our previous expression for large values of the argument \( x \), and fixes the constant of proportionality to be \( \sqrt{2}/\kappa \). For small values of the argument, one then finds \(|\beta_{\omega, \omega'}|^2 \approx \frac{1}{2\kappa x} \). However, the geometrical optics approximation which was used in doing the calculation means that the results are only valid at high frequencies. We would expect that the wavelength of the propagating wave must be small compared to the length scales \( r_{bh} \) and \( r_{dS} \). Using a lower limit of \( 2\pi/r_{bh} \) in the integration over frequencies, one finds for the spectrum

\[ N_\omega \approx \frac{\pi}{4\omega} e^{-\frac{4\sqrt{2\pi} x}{\kappa \sqrt{\omega}}} \tag{13} \]

This is not a thermal spectrum, so the situation is probably best described as a thermodinamic equilibrium. If the lower cutoff were taken to be proportional to \( \omega \), rather than \( 1/r_{bh} \), the spectrum would be thermal.

There are several limits one can take in order to check this result. Letting \( \kappa \to 0 \) above, corresponds to keeping \(|Q| = M \) and letting the cosmological constant \( \Lambda \) approach zero, so that the spacetime approaches extremal Reissner-Nordstrom. In this limit \( N_\omega \) goes to zero, as it should. Secondly, one can consider setting \( Q = 0 \), and then letting \( \Lambda \to 0 \), so that the metric approaches Schwarzschild. This is equivalent to letting \( r_{dS} \) approach zero, while keeping \( r_{bh} \) finite. The particle production (7) from the black hole can again be evaluated in the stationary phase approximation. One finds that the coefficients \( \alpha_{\omega, \omega'} \) approach those for a Schwarzschild black hole.
3. Supercovariantly Constant Spinors for \( \Lambda > 0 \)

We now turn to issues relevant to the classical stability of \( |Q| = M \) RN\( dS \) spacetimes. First, recall that for \( \Lambda = 0 \) a generalization of the positive energy theorem \([10],[11]\) implies that the ADM mass \( M \) of an asymptotically flat spacetime satisfies the inequality

\[
M \geq |Q|,
\]

where \( Q \) is the total electric charge of the spacetime. This inequality is saturated for spacetimes which admit a spinor field \( \epsilon \) satisfying \( \hat{\nabla}_\mu \epsilon = 0 \), where \( \hat{\nabla}_\mu \) is a certain supercovariant derivative operator arising naturally in ungauged \( N = 2 \) supergravity. In the context of supergravity \( \hat{\nabla}_\mu \epsilon \) gives the variation of the gravitino field under an infinitesimal local supersymmetry transformation. Spacetimes having supercovariantly constant spinors have unbroken supersymmetries. Amongst asymptotically flat electrovac solutions, the bound (14) is saturated by the Majumdar-Papapetrou multi-black hole solutions \([12]\), which represent collections of extremal (\( |Q| = M \)) charged black holes. These spacetimes each admit two supercovariantly constant spinors. Calculations of the effective potential in supergravity have shown that these spacetimes remain ground states at the quantum level as well \([13]\).

A nonzero cosmological constant \( \Lambda = -3g^2 \) arises in gauged \( N = 2 \) supergravity, where \( g \) is the coupling constant of the gravitino with the \( U(1) \) gauge field. If the coupling \( g \) is real, as one would require in a quantum theory, then the cosmological constant is negative. Romans \([14]\) has written down the supercovariant derivative operator \( \hat{\nabla}_\mu \), which arises in gauged \( N = 2 \) supergravity, with real coupling \( g \) and showed that \( |Q| = M \) RN\( dS \) spacetimes each admit two supercovariantly constant spinors. At the classical level one can also consider imaginary values of \( g \) and hence positive values of the cosmological constant. With the substitution \( g = iH \), the supercovariant derivative operator is given by

\[
\hat{\nabla}_\mu \epsilon = \left( 4\nabla_\mu + HA_\mu + \frac{i}{2} H\gamma_\mu + \frac{i}{4} F_{\rho\sigma}\gamma^\rho\gamma^\sigma\gamma_\mu \right) \epsilon
\]

We are then interested whether or not \( |Q| = M \) RN\( dS \) black holes admit spinor fields satisfying \( \hat{\nabla}_\mu \epsilon = 0 \). If this is the case, then it seems promising that a version of the positive energy theorem can be proved, showing the classical stability of these spacetimes.

In fact we can find supercovariantly constant spinors for a more general class of spacetimes, which includes the \( |Q| = M \) RN\( dS \) spacetimes. These are the deSitter analogues of the MP multi-black hole solutions, which were found in \([1]\). In spatially flat Robertson-Walker type coordinates, these spacetimes are given by

\[
\Omega = 1 + \sum_i \frac{M_i}{a|\vec{x} - \vec{x}_i|}, \quad a(t) = \exp(Ht).
\]

These spacetimes have naked singularities, because for \( \Lambda < 0 \) the extremal limit occurs with \( |Q| < M \).
We find that these spacetimes each have two supercovariantly constant spinors, which are given by
\[ \epsilon = \frac{1}{\sqrt{a\Omega}} \bar{\epsilon} = e^{-\frac{Ht}{2}} \sqrt{\Omega} \bar{\epsilon} \] (17)
where \( \bar{\epsilon} \) is a constant spinor satisfying the projection7. \( (1 - i\gamma^5)\bar{\epsilon} = 0 \)

4. Positive Energy Constructions

As stated above, the supercovariant derivative operator (15) should be the starting point for a positive energy construction, showing that a suitably defined mass for asymptotically deSitter spacetimes is minimized for fixed charge by the spacetimes (16). The relevant mass in the asymptotically deSitter case should turn out to be that given by Abbott and Deser in [15]. In the present note, we will address only the case \( Q = 0 \). Such a construction for \( \Lambda < 0 \) is given in [11]. We will return to the \( Q \neq 0 \) case in a future publication [3]. The result we present here, however, will be valid for spacetimes which are asymptotic to spatially flat Robertson-Walker, not necessarily asymptotically deSitter. This is because the construction takes place only on a spatial slice, and a spatial slice of Robertson-Walker can be embedded in a deSitter spacetime. In this case the asymptotic mass will be related to the boundary term given in [6], in the context of perturbations of Robertson-Walker spacetimes, and discussed further in [16].

Let the metric \( g_{\mu\nu} \) of a spacetime asymptotically approach the metric for a Robertson-Walker spacetime \( g_{\mu\nu}^{RW} \) on each \( t = constant \) spatial slice, where
\[ ds^2_{RW} = g_{\mu\nu}^{RW} dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j. \] (18)
With vanishing gauge field the supercovariant derivative operator (15) becomes
\[ \hat{\nabla}_\mu \epsilon = \nabla_\mu \epsilon + i H(t)\gamma_6 \epsilon, \] (19)
where \( H(t) \equiv \dot{a}(t)/a(t) \) may depend on time.

When the metric is exactly Robertson-Walker, there exist four supercovariantly constant spinors \( \epsilon_o \) satisfying \( \hat{\nabla}_\mu \epsilon_o = 0 \). This can be seen as follows. In five dimensional Minkowski spacetime there are four constant spinor solutions to \( \nabla_\mu \psi = 0 \). Projecting this equation onto the four dimensional deSitter hyperboloid, one finds that there are four solutions to the equation \( \hat{\nabla}_\mu \psi \equiv (\nabla_\mu + \frac{H}{2} \gamma_6 \gamma^5)\psi = 0 \). Letting \( \epsilon_o = (1 - i\gamma^5)\psi \), we have \( \hat{\nabla}_\mu \epsilon_o = 0 \). A \( t = constant \) spatial slice in a general Robertson-Walker spacetime (18) can be embedded in the deSitter hyperboloid with the appropriate value of \( H(t) \). On a given spatial slice then, there are four solutions to \( \hat{\nabla}_j \epsilon_o = 0 \), where \( j \) is a spatial index. Explicitly, these are given by
\[ \epsilon_o = (1 - i\gamma^5)(1 - \frac{H}{2} x^i \gamma_i (\gamma^i + \gamma^5))\bar{\epsilon} \] (20)

7 In our conventions hatted indices denote frame indices and unhatted indices are coordinate indices. Hence \( \{\gamma^\mu, \gamma^\rho\} = 2\eta^{\mu\rho} \) and \( \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \).

8 Note that \( (\gamma^5)^2 = +1 \), so this is not a projection.
where $\bar{\epsilon}$ is any constant spinor.

Since the positive energy construction will take place on a spatial surface, this is actually sufficient to construct a positive energy statement, for any spatial slice in the coordinates of equation (18). However, there are solutions which satisfy the full equation $\hat{\nabla}_\mu E = 0$. These are conveniently given by $E = \exp(\pm \int H dt')\epsilon_o$, where $\gamma^i \epsilon_o = \mp i \epsilon_o$. The latter projection is equivalent to $(\gamma^i \pm \gamma^5) \bar{\epsilon} = 0$.

Let $g_{ij}$ and $K_{ij}$ be the metric and extrinsic curvature on a spatial slice, which approach the metric and extrinsic curvature of a Robertson-Walker spatial slice with Hubble constant $H$. Assume that $g_{ij}$ and $K_{ij}$ solve the Einstein constraint equations with sources $T_{\hat{t}\hat{t}}$ and $T_{\hat{t}\hat{k}}$. Let $\epsilon$ be a solution to the contracted equation

$$\gamma^j \hat{\nabla}_j \epsilon = 0$$

on this spatial slice (where the index $j$ runs over spatial values only) with the boundary condition that $\epsilon$ approaches one of the constant spinors $\epsilon_o$ at large $r$. Let $V$ be a volume in the spatial slice. Then following the analysis of [17] we find the following relation

$$\int_{\partial V} da_i \epsilon^i \hat{\nabla}_i \epsilon = \int V d^3x \sqrt{g} \left( 4\pi \epsilon^\dagger (T_{\hat{t}\hat{t}} - T_{RW}^{\hat{t}\hat{t}} + T_{k\hat{t}}^\dagger k^\dagger \gamma^i) \epsilon + (\hat{\nabla}_i \epsilon)^\dagger \hat{\nabla}_i \epsilon \right),$$

where $T_{RW}^{\hat{t}\hat{t}} = 3H(t)^2/8\pi$ is the energy density of a Robertson-Walker spatial slice. If black holes are present in $V$, then the boundary $\partial V$ has components at the black hole horizons. These boundary terms can be made to vanish by imposing a suitable projection on $\epsilon$, as shown in [11]. This leaves the outer boundary of $V$, which we will take to be spatial infinity. In the case $\Lambda = 0$, this would be related to the ADM 4-momentum [17]. The volume integral on the right hand side of (22) can, in the context of asymptotically Robertson-Walker spacetimes, have both positive and negative contributions. The second term, involving derivatives of the spinor field, is positive definite. However, the first term, which is a difference between the matter stress energy and the background Robertson-Walker energy density, can be either positive or negative, depending on whether there are mass overdensities or underdensities. If the matter stress energy satisfies the dominant energy condition, however, this term cannot be more negative than minus the energy density of the background itself. Furthermore, for asymptotically deSitter spacetimes it is natural to consider this difference to be positive.

One expects that the boundary term at spatial infinity gives an analogue of the ADM 4-momentum for asymptotically Robertson-Walker spacetimes. Such a quantity has been defined for asymptotically deSitter spacetimes by Abbott and Deser [15]. For each asymptotic killing vector in an asymptotically deSitter spacetime, one can construct a conserved charge given by a two dimensional boundary integral on a spatial slice. These charges all vanish for pure deSitter. Hence, as is argued in the anti-deSitter case [11], we expect that for asymptotically deSitter spacetimes, the boundary term in (22) gives the Abbott-Deser mass. In the example of Schwarzchild-deSitter, one can evaluate the boundary term explicitly, and it gives the mass parameter.

More generally, the McVittie metric [18] describes a mass embedded in a Robertson-Walker spacetime,

$$ds^2 = \left(1 - \frac{m}{2ar}\right)^2 dt^2 + a^2(t)(1 + \frac{m}{2ar}) \delta_{ij} dx^i dx^j.$$  \hspace{1cm} (23)
In this case, the exact solution to the Dirac equation (21) can be found. The boundary term can then be evaluated and shown to give the mass parameter $m$. Hence, if a metric approaches (23) at large radius up to terms of order $1/r^2$, the boundary term will continue to give the appropriate mass parameter. In principle, one can find a general expression for the boundary term in (22) by solving (21) asymptotically on a spatial surface to sufficiently high order, as in [17]. In practice, here, this is complicated by the nonzero background extrinsic curvature and we have not yet found a general expression. We expect that the final result will be related to the boundary integral which arises in the analysis of [6] on perturbations of Robertson-Walker spacetimes.

Acknowledgements

JT is supported in part by NSF grant NSF-THY-8714-684-A01.
Fig. 1. Shown is the conformal diagram for the section of RNdS relevant to the particle production calculation. The region is bounded by the white hole, black hole, past deSitter, and future deSitter horizons. Also indicated are the choices for positive frequency waves at each horizon. To compute $\beta^{bh}_{\omega,\omega'}$, one studies wave propagation along a null geodesic $U_{dS} = constant$, parallel to the blackhole horizon. The wave is taken to be positive frequency and outgoing as it crosses the future deSitter horizon. Propagating back to the white hole horizon, the wave is decomposed into positive and negative frequency parts with respect to the coordinate $U_{bh}$ on the white hole horizon. Near the white hole horizon the wave vanishes for $U_{bh} > 0$, since that is inside the black hole. Similarly, to compute $\beta^{dS}_{\omega,\omega'}$, one considers a wave propagating on a null geodesic $V_{bh} = constant$, parallel to the future deSitter horizon, for a wave which is positive frequency and ingoing at the black hole horizon.
References

[1] D. Kastor and J. Traschen, Phys. Rev. D47, 5370 (1993).
[2] C. Holzhey and F. Wilczek, Nucl. Phys. B380, 447 (1992).
[3] D. Kastor and J. Traschen, work in progress.
[4] G.W. Gibbons and S.W. Hawking, Phys. Rev. D15, 2738 (1977).
[5] F. Mellor and I. Moss, Class. Quant. Grav. 6, 1379 (1989).
[6] J. Traschen, Phys. Rev. D31, 283 (1985).
[7] L. Parker, in Asymptotic Structure of Spacetime, New York (1977).
[8] R.M. Wald, General Relativity, University of Chicago Press (1984).
[9] N.D. Birrell and P.C.W. Davies, Quantum Fields in Curved Spacetime, Cambridge University Press (1982).
[10] G.W. Gibbons and C.M. Hull, Phys. Lett. 109B, 190 (1982).
[11] G.W. Gibbons, S.W. Hawking, G. Horowitz and M. Perry, Comm. Math. Phys. 88, 295 (1983).
[12] S.D. Majumdar, Phys. Rev. 72, 930 (1947); A. Papapetrou, Proc. Roy. Irish Acad. A51, 191 (1947).
[13] R. Kallosh, Phys. Lett. 282B, 80 (1992).
[14] L. Romans, Nucl. Phys. B393, 395 (1992).
[15] L. Abbott and S. Deser, Nuclear Physics B195, 76 (1982).
[16] D. Kastor and J. Traschen, Phys. Rev. D47, 480 (1993).
[17] E. Witten, Comm. Math. Phys. 80, 381 (1981).
[18] G.C. McVittie, Mon. Not. R. Astron. Soc. 93, 325 (1933).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/gr-qc/9311025v1