Heterotic String Theory on non–Kähler Manifolds
with $H$–Flux and Gaugino Condensate

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Abstract: We discuss compactifications of heterotic string theory to four dimensions
in the presence of $H$–fluxes, which deform the geometry of the internal manifold, and a
gaugino condensate which breaks supersymmetry. We focus on the compensation of the
two effects in order to obtain vacua with zero cosmological constant and we comment on
the effective superpotential describing these vacua.

1 Introduction

String compactification in the presence of fluxes has been revived recently as an appeal-
ning way to address the moduli problem. Turning on fluxes in the ten–dimensional string
theories produces, at the level of the effective four–dimensional action, a potential. Upon
minimization of this potential one finds new vacua with (generically) less moduli. More-
over, one expects to find Minkowski vacua only when the deformation of the internal
manifold balances the presence of the fluxes.

Supersymmetric compactifications of the heterotic string in the presence of three–form
fluxes need non–Kähler six–manifolds described in [1, 2, 3, 4]. More precisely one finds
that a non–vanishing flux is associated to a non–zero exterior derivative of the complex
structure $J$ [5]

$$H = -\frac{1}{2}e^{-8\phi} \ast d(e^{8\phi}J).$$

This condition, which follows from the analysis of the supersymmetry rules, can also
be understood from the perspective of rewriting the ten–dimensional effective action of
heterotic string theory as a sum of squares [6]. The relevant term for such a purpose is
given by

$$S = \frac{1}{2} \int e^{8\phi} \left[ H + \frac{1}{2}e^{-8\phi} \ast d(e^{8\phi}J) \right]^2.$$

In this way it is clear that in order to obtain a vanishing four–dimensional effective poten-
tial $V = 0$, the condition (1) has to be imposed. Therefore, one obtains supersymmetric
Minkowski vacua by fixing the $(2,1) + (1,2)$ Hodge components of the three–form flux, $H^{(2,1)}$ and $H^{(1,2)}$, in terms of the internal geometry. The other Hodge components, i.e. $H^{(3,0)}$ and $H^{(0,3)}$, must vanish in order to have unbroken supersymmetry.

There is another but different physical effect in heterotic string theory which is naturally connected with the appearance of an $H$–flux, namely gaugino condensation. It is indeed known that an interesting mechanism leading to supersymmetry breaking while preserving a zero cosmological constant at the tree–level uses both a gaugino condensate and a non–vanishing flux. More precisely, one finds the following terms in the action,

$$S = \frac{1}{2} \int e^{8\phi} \left[ H - \alpha' (\bar{\chi}^A \Gamma_{(3)} \chi^A) \right]^2,$$

(3)

(where $\Gamma_{(3)} = 1/3! e^a e^b e^c \Gamma_{abc}$) and assigning an appropriate expectation value to $\text{Tr} \bar{\chi} \Gamma_{mnp} \chi$ yields

$$\Sigma_{mnp} \equiv < \text{Tr} \bar{\chi} \Gamma_{mnp} \chi > \rightarrow \Lambda^3 \Omega_{mnp} + \text{c.c.}, \quad \Lambda^3 = \frac{1}{4} < \bar{\lambda} (1 + \gamma_5) \lambda >,$$

(4)

where $\Omega$ is a $(3,0)$–form on the internal manifold and $\Lambda^3$ denotes the expectation value of the gaugino condensate in the four–dimensional space–time. Minkowski vacua follow then for Calabi–Yau compactifications if

$$H = \alpha' \left( \Lambda^3 \Omega + \bar{\Lambda}^3 \bar{\Omega} \right).$$

(5)

Therefore, one obtains flat vacua breaking supersymmetry by fixing the $(3,0) + (0,3)$ components of the three–form flux.

It is now natural to ask whether these two balancing mechanisms between the flux and the geometry on one side and the fermion condensate on the other can be combined. Since the two effects talk to different Hodge sectors of the $H$–flux, it is natural to expect that they can be combined in the action in a unique square,

$$S = \frac{1}{2} \int e^{8\phi} \left[ H - \alpha' \Sigma + \frac{1}{2} \ast e^{-8\phi} d(e^{8\phi} J) \right]^2.$$

(6)

In the following we will first discuss to what extent (6) is valid and then analyze what is the modification of the superpotential needed in order to describe these vacua.

### 2 Gaugino condensate, fluxes and torsion

The bosonic part of the Lagrangean up to second order in $\alpha'$, including the gaugino condensate $\Sigma$, is given by

$$S = \int d^{10}x \sqrt{g} e^{8\phi} \left[ \frac{1}{4} R - \frac{1}{12} (H_{MNP} - \alpha' \Sigma_{MNP})^2 + 16 (\partial M \phi)^2 \right.\left. - \frac{1}{4} \alpha' \left( F^I_{MN} F^{I, MN} - R^+_{MNPQ} R^+_{MNPQ} \right) \right].$$

(7)

This action is written in the string frame and its fermionic completion makes it supersymmetric using the three–form Bianchi identity given by

$$dH = \alpha' \left( \text{tr} R^+ \wedge R^+ - \text{tr} F \wedge F \right),$$

(8)
where the curvature $R^+$ is the generalized Riemann curvature built from the generalized connection $\nabla^+$ (i.e. from $\omega^\pm = \omega \mp H$). Note that since we work at first order in $\alpha'$, corrections to $\nabla^+$ or $\nabla^-$ by the gaugino condensate $\Sigma$ can be neglected. Also note that it is the combination $H - \alpha'\Sigma$ that enters in the kinetic term for $H$, whereas it is only $H$ that enters in the lhs of the Bianchi identity. This asymmetry will result in the presence of an additional term $d\Sigma \wedge (e^{8\phi} J)$ in the BPS rewriting of action $[7]$.

In the search for a BPS rewriting of $[7]$, and in the same spirit as in $[8]$, we will assume that the space is given by the warped product of four–dimensional Minkowski spacetime with an internal space admitting an $SU(3)$ structure. In order to consistently obtain that setting to zero the BPS–like squares implies a solution to the equations of motion, we also impose that the only degrees of freedom for the various fields are given by expectation values on the internal space and are functions only of the internal coordinates.

To simplify the discussion we limit ourselves to the case with dilaton and warp factor identified, i.e. $\phi = \Delta$, but the generalization of the following results is straightforward. After various manipulations, the action $[7]$ can be written as

$$ S = \int d^6x \sqrt{g_4} \left\{ -\frac{1}{2} \int_{M_6} e^{8\phi} (8d\phi + \Theta) \wedge (8d\phi + \Theta) + \frac{1}{8} \int_{M_6} e^{8\phi} J \wedge J \wedge \hat{R}_{ab} J_a J_b - \frac{1}{4} \int d^6y \sqrt{g_6} e^{8\phi} N_{mn}^p \gamma^m \gamma^n \gamma^p \gamma^q - \frac{\alpha'}{2} \int_{M_6} d\Sigma \wedge (e^{8\phi} J) + \frac{1}{2} \int_{M_6} e^{8\phi} \left( H - \alpha'\Sigma + \frac{1}{2} e^{-8\phi} d(e^{8\phi} J) \right) \wedge \left( H - \alpha'\Sigma + \frac{1}{2} e^{-8\phi} d(e^{8\phi} J) \right) - \frac{\alpha'}{2} \int d^6y \sqrt{g_6} e^{8\phi} \left[ \text{tr}(F^{(2,0)})^2 + \text{tr}(F^{(0,2)})^2 + \frac{1}{4} \text{tr}(J_{mn} F_{mn})^2 \right] + \frac{\alpha'}{2} \int d^6y \sqrt{g_6} e^{8\phi} \left[ \text{tr}(R^{+(2,0)})^2 + \text{tr}(R^{+(0,2)})^2 + \frac{1}{4} \text{tr}(J_{mn} R_{mn}^{+})^2 \right] \right\}. \tag{9} $$

In this expression the traces are taken with respect to the fiber indices $a, b, \ldots$, whereas the Hodge type refers to the base indices $m, n, \ldots$ of the curvatures. The other geometrical objects appearing in the above expression are the Lee–form

$$ \theta \equiv J_{\ldots} dJ = \frac{3}{2} J_{mn} \partial_{[m} J_{np]} dx^p, \tag{10} $$

the Nijenhuis tensor

$$ N_{mn}^p = J_m \partial_{[q} J_n] - J_n \partial_{[q} J_m] + J_{[q} \partial_{m]} J_n] - J_{[q} \partial_{n]} J_m] \tag{11} $$

and the generalized curvature $\hat{R}$, which is constructed using the Bismut connection built from the standard Levi–Civita connection and a totally antisymmetric torsion $T^B$ proportional to the complex structure,

$$ T^B_{mnp} = \frac{3}{2} J_m \partial_{[q} J_n] J_{r]} \partial_{[r} J_{ps]} = - \frac{3}{2} J_{[m} \nabla_{[q]} J_{np]} \tag{12} $$

The action $[7]$ will now be used to find the conditions determining the background geometry and the form of the condensate $\Sigma$ by demanding the vanishing of $[3]$. Setting the squares to zero yields

- the vanishing of the Nijenhuis tensor

$$ N_{mn}^p = 0, $$
• the vanishing of some components of the generalized Riemann curvature constructed from the $\nabla^+$ connection,

$$R^{+}(2,0) = R^{+}(0,2) = J^{mn} R_{mn}^+ = 0,$$

• the vanishing of

$$d\phi + \frac{1}{8} \theta = 0 ,$$

(13)

• the vanishing of

$$H - \alpha' \Sigma + \frac{1}{2} \ast e^{-8\phi} d(e^{8\phi} J) = 0 ,$$

(14)

• the vanishing of

$$F^{(2,0)} = F^{(0,2)} = J^{mn} F_{mn} = 0.$$

The vanishing of the Nijenhuis tensor states that the internal manifold is complex. The conditions on the $R^+$ curvature can be translated into the requirement of $SU(3)$ holonomy for the $\nabla^-$ connection. The proof requires the identity

$$R^+_{abcd} = R^-_{cdab} - (dH)_{abcd} ,$$

(15)

which relates the $R^+$ and $R^-$ curvatures with the base and fiber indices swapped (again, terms proportional to $\Sigma$ can be neglected because they are of higher order in $\alpha'$). Using this identity and the fact that $dH$ gives higher order terms in $\alpha'$ the conditions on the base indices of $R^+$ become conditions on the $R^-$ fiber indices, to lowest order in $\alpha'$,

$$R^{-}(2,0) = R^{-}(0,2) = J^{ab} R_{ab}^- = 0 .$$

(16)

These conditions precisely state that the generalized curvature $R^-$ is in the adjoint representation of $SU(3) \subset SO(6)$ and therefore its holonomy group is contained in $SU(3)$.

The conditions in the gauge sector are that the gauge field strength is of type $(1,1)$ and $J$ traceless.

On a complex manifold, the condition (14) yields

$$H^{(2,1)+(1,2)} = -\frac{1}{2} \ast e^{-8\phi} d(e^{8\phi} J) = \frac{1}{2} i(\partial - \bar{\partial})J ,$$

$$H^{(3,0)+(0,3)} = \alpha' \Sigma ,$$

(17)

where we also used (13).

On the solution, $R^+ \wedge R^+$ and $F \wedge F$ are of type $(2,2)$. Therefore, the Bianchi identity (8) implies that $(dH)^{(3,1)+(1,3)} = 0$, and hence

$$d\Sigma = 0 \longrightarrow \Sigma = \Lambda^3 \Omega + \bar{\Lambda}^3 \bar{\Omega} ,$$

(18)

where $\Omega$ is now a holomorphic $(3,0)$–form.

Finally, let us discuss the term $d\Sigma \wedge (e^{8\phi} J)$ in the action (9). This term vanishes on the solution, since $\Sigma$ is closed (18). Let us now consider its variation. We obtain

$$\delta \int_{M_6} d\Sigma \wedge (e^{8\phi} J) = \int_{M_6} d\Sigma \wedge \delta(e^{8\phi} J) + \int_{M_6} d\delta \Sigma \wedge (e^{8\phi} J) ,$$

(19)

where we assumed that $\delta$ and $d$ commute. The first term, when evaluated on the solution, vanishes due to (18). The second term, however, is more problematic, because $d\delta \Sigma$ may
contain a piece of \((2,2)\)-type for non complex variations, unless one can show that it vanishes when evaluated on the solution. Though this may seem reasonable if variations of a closed form result in closed forms, it is not clear to us that (19) vanishes on the solution. Notice that for the subclass of Calabi–Yau compactifications, the second term doesn’t contribute, since then \(dJ = 0\).

We conclude our discussion with some remarks about the possible superpotential describing such vacua in the effective theory. It has been argued that a candidate superpotential describing the \(N = 1\) vacua of the heterotic theory in the presence of fluxes is given by \[ W = \int_{\mathcal{M}_6} \mathcal{H} \wedge \Omega = \int_{\mathcal{M}_6} (H + \text{id}J) \wedge \Omega. \] (20)

In the presence of a gaugino condensate, the supergravity potential obtained from (9) contains a new contribution (6) proportional to the gaugino condensate \(\Sigma\). Therefore it is natural to expect that such a contribution will be captured by a shift in the superpotential \(W\) in the following way,

\[ W \rightarrow W + \int_{\mathcal{M}_6} \Sigma \wedge \Omega. \] (21)

On the other hand, it is known that in the four-dimensional effective \(N = 1\) theory the gaugino condensate produces a non–perturbative contribution to the effective superpotential which schematically is given by

\[ W_{\text{eff}} = c + e^{-S}, \] (22)

where \(S\) denotes the four-dimensional dilaton–axion field. On a Calabi–Yau threefold, \(c\) is related to the three–form flux \(H\), whereas \(e^{-S}\) arises from \(\int_{\mathcal{M}_6} \Sigma \wedge \Omega \sim \Lambda^3 \sim e^{-S}\).

Inspection of (21) now suggests that the microscopic description of \(W_{\text{eff}}\) should be given by

\[ W = \int_{\mathcal{M}_6} (H + \text{id}J + \Sigma) \wedge \Omega. \] (23)

A comparison of (23) with (22) then schematically yields

\[ c \sim \int_{\mathcal{M}_6} (H + \text{id}J) \wedge \Omega, \quad e^{-S} \sim \int_{\mathcal{M}_6} \Sigma \wedge \Omega. \] (24)

The potential of the \(N = 1\) effective theory is given by (neglecting D-terms)

\[ V = e^K \left( |DW|^2 - 3|W|^2 \right). \] (25)

Inspection of the potential (6) indicates that the resulting model is of the no-scale type. Using a tree-level Kähler potential \(K = -3 \log(T + \bar{T}) - \log(S + \bar{S})\) one then finds that

\[ V = \frac{1}{(S + \bar{S})(T + \bar{T})^4} - W + (S + \bar{S})W_S|_S^2, \] (26)

where \(W_S = \partial_S W\). This does not fully coincide with (6) due to the presence of \((S + \bar{S})\) in (20). This discrepancy was already noticed in the case of Calabi-Yau threefold compactifications with gaugino condensates (see the second reference in [7]).
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