Influence of Boundary Conditions on the Stress-Strain State of a Corrugated Sheet Under Its Weight

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Abstract. In this paper, the calculation of the corrugated sheet is reduced to the calculation of the strength and stiffness of the corrugated plate under its weight. The plate is made structurally orthotropic with different cylindrical stiffnesses in two mutually orthogonal directions. The curved middle surface is described by a fourth-order differential equation and is solved by the Bubnov-Galerkin method. As approximating functions in the expansion of the deflection, beam functions are also used to compare the Legendre function. It is shown that the beam functions give much worse convergence than the Legendre functions. The influence of the location of the rigid and hinged support, located in the direction of the guided line of the wave and perpendicular to it, was investigated. Rigid attachment along the side coinciding with the guiding wave reduces the deflections and the bending moments. The stress-strain state of a wavy and flat plate is compared with the same mechanical and geometric characteristics. The influence of the amplitude of the wave amplitude on the stress-strain state of a wavy plate is analyzed. It is shown that an increase in the wave amplitude leads to an increase in the strength and stiffness of a wavy plate.

1. Introduction

Corrugated plates are widely used in various designs of mechanical engineering and construction. In construction, corrugated plates are used as profiled sheets. Decking is a versatile building material that finds extensive and diverse use in construction. Profiled sheeting are corrugated sheet profiles with corrugated sheets of various shapes repeating across the width of the sheet: corrugated sheets, corrugated sheets with trapezoid corrugations, cassette profiled sheets. Wavy corrugated sheets, rolled from sheet steel, belong to the earliest group of this type of metal products. They appeared at the end of the XIX century. The cross-sectional shape of these profiles has the form of a sinusoid or mating circles. One of the critical properties of corrugated flooring is high mechanical strength (flexural rigidity) in the direction of sheet corrugations with a certain lightness (weight of 1 m² sheet is 5-16 kg), which ensures the safety of operation and durability of building structures made using profiled sheeting.

The complexity of the corrugated plates calculation using exact methods of flat and three-dimensional theories of elasticity leads to the search for methods that simplify the study of the stress-strain state under various parameters of the load, the geometry of the plate, as well as boundary conditions. The method of converting the calculation of corrugated and ribbed plates to the calculation of constructive orthotropic plates is given in S.G. Lehnitsky [1]. There is also a need to develop refined
methods for the theory of thin structural orthotropic plates and shells. There are many methods, without claiming to be a complete list, we note only a few studies in this area [2–7].

2. Materials and methods

In work, the wavy professional flooring in the form of a thin steel wavy plate under the influence of body weight is considered. (figure 1)

![Figure 1. The scheme of the wavy plate](image)

The plate waveform is 

\[ z = f \sin \frac{\pi x}{l} \]

The calculation of corrugated board is reduced to the calculation of a thin flat constructive orthotropic plate with different cylindrical stiffnesses along with the wave and the direction perpendicular to it by the Bubnov-Galerkin method [8–10]. For a thin structurally orthotropic flat plate, the differential equation of a curved middle surface is:

\[
D_1 \frac{\partial^4 w(\xi,\eta)}{\partial \xi^4} + 2D_2 \lambda^2 \frac{\partial^4 w(\xi,\eta)}{\partial \xi^2 \partial \eta^2} + D_3 \lambda^4 \frac{\partial^4 w(\xi,\eta)}{\partial \eta^4} = \left( \frac{a}{2} \right)^4 q(\xi,\eta)
\]

where

\[
D_1 = \frac{l}{s} \frac{Eh^3}{12(1-\nu^2)}, \quad D_2 = 0.5 \frac{Ehf^2}{12.5 \left( \frac{f^2}{2l} \right)^2}, \quad D_3 = 2D_h = \frac{s}{l} \frac{Eh^3}{12(1+\nu)},
\]

\[ q(\xi,\eta) = \gamma h, \quad \gamma \text{ is the density of the material, } E \text{ and } \nu \text{ - elastic constants of the plate material, } h \text{ – plate thickness, } s = l \left( 1 + \frac{\pi^2 f^2}{4l^2} \right) \text{ - arc length, } f \text{ - wave amplitude, } \lambda \text{ – elastic foundation modulus.}
\]

The dimensionless variables \( \xi, \eta \) are related to \( x, y \) by the following substitution

\[
x = \frac{a}{2}(\xi + 1); \quad y = \frac{b}{2}(\eta + 1); -1 \leq \xi \leq 1; -1 \leq \eta \leq 1; \quad \lambda = \frac{a}{b}.
\]

The formulas determine the bending moments arising in the plate

\[
M_x(\xi,\eta) = -4D_1 a^2 \left( \frac{\partial^2 w(\xi,\eta)}{\partial \xi^2} + \nu \lambda^2 \frac{\partial^2 w(\xi,\eta)}{\partial \eta^2} \right); \quad \text{при } \xi = \pm 1, -1 \leq \eta \leq 1
\]
The stress-strain state of the corrugated plate was investigated and compared with two kinds of fixing.

[Figure 2. The scheme of the wavy plate boundary conditions]

The first case: the plate is rigidly constrained along two opposite sides with $x = 0, x = a$ the other two sides with $y = 0, y = h$ are simply supported.

In this case, the boundary conditions are

$$w(-1, \eta) = \frac{\partial w(-1, \eta)}{\partial \xi} = 0; w(\xi, 1) = \frac{\partial w(\xi, 1)}{\partial \eta} = 0;$$

$$w(\xi, -1) = M_y (\xi, -1) = 0; w(\xi, 1) = M_y (\xi, 1) = 0;$$

The second case: at $x = 0, x = a$, the sides are pivotally supported, the other two sides, at $y = 0, y = b$, are rigidly clamped.

For the second case, the boundary conditions take the form:

$$w(\xi, -1) = \frac{\partial w(\xi, -1)}{\partial \eta} = 0; \quad w(\xi, 1) = \frac{\partial w(\xi, 1)}{\partial \eta} = 0;$$

$$w(-1, \eta) = M_x (-1, \eta) = 0; \quad w(\xi, 1) = M_x (1, \eta) = 0;$$

The functional equation of the Bubnov-Galerkin method of the corrugated plate under the action of the distributed load $q(\xi, \eta)$, into account taking Eq. 1 will take the form of

$$\iint_A \left\{ D_1 \frac{\partial^4 w(\xi, \eta)}{\partial \xi^4} + 2D_2 \lambda^2 \frac{\partial^4 w(\xi, \eta)}{\partial \xi^2 \partial \eta^2} + D_2 \lambda^4 \frac{\partial^4 w(\xi, \eta)}{\partial \eta^4} - a^4 q(\xi, \eta) \right\} W_{\xi \eta} (\xi, \eta) dA = 0. \quad (9)$$

The solution is made in the form of a double row

$$w(\xi, \eta) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} W_{mn}(\xi, \eta),$$

The calculation of the wavy plate was carried out for two different types of decomposition function $W_{mn}(\xi, \eta)$

Case 1. Special orthogonal polynomials of degree $i$

$$W_{mn}(\xi, \eta) = S_{m+3} (\xi) S_{n+3} (\eta) \quad (11)$$
where $A_{mn}$ - unknown coefficients, $S_i(\beta)$ - special orthogonal polynomials of degree $i$, which can be represented as a combination of three classical Legendre polynomials $P_n(\beta)$, $P_n(\beta) = \frac{1}{2^n n!} \frac{d^n}{d\beta^n}(\beta^2 - 1)^n \int_{-1}^{1} P_n(\beta) P_m(\beta) d\beta = 0$ at $m \neq n$ - classical Legendre orthogonal polynomial.

Case 2: Beam expansion

For a plate with rigid sealing along the edges at $x = 0$, $x = a$ and pivotally supported at the edges at $y = 0$, $y = b$, the expansion in beam functions has the form

$$W_{mn}(\xi,\eta) = \left(1 - \cos(\pi j (\xi + 1))\right) \sin\frac{\pi k (\eta + 1)}{2}$$

For a plate with rigid sealing along the edges at $y = 0$, $y = b$ and simply supported at the edges at $x = 0$, $x = a$, the expansion in beam functions has the form

$$W_{mn}(\xi,\eta) = \left(1 - \cos(\pi j (\eta + 1))\right) \sin\frac{\pi k (\xi + 1)}{2}$$

3. Results and discussion

3.1. First case. Calculation of a wavy plate rigidly embedded at $x = 0$, $x = a$ and simply supported at $y = 0$, $y = b$

Plate characteristics $a=1.1$ m, $b= 0.5$ m, $q= 4.6 \times 10^{-2}$ kN/m, $E=2 \times 10^5$ MPa, $\nu =0.3$, $h= 0.6$ mm, $f=9.0$ mm, $l=45.84$ mm

A qualitative picture of the distribution of the deflections $w(x,y)$ and bending moments $M_x(x,y), M_y(x,y)$ over the area of the plate is presented in figure 3, figure 4, figure 5. The bending moments reach the maximum value in pinching, and the deflections reach the maximum value on the axis of symmetry parallel to the x axis.

![Figure 3. Dependence of deflections $w(x,y)$ of a wavy plate](image)
Determining the characteristics of the stress-strain state of a wavy plate when \( w(x, y) \) is expanded by beam functions, and Legendre polynomials showed that beam functions give worse convergence (Table 1, Table 2). Therefore, all other calculations were carried out using Legendre functions.

| Number of row members | Beam functions | Special polynomials |
|-----------------------|----------------|---------------------|
| \( N \) | \( M \) | \( w_{\text{max}} \) cm | \( M_{\text{max}} 10^4 \) kNm | \( M_{\text{max}} 10^4 \) kNm | \( w_{\text{max}} \) cm | \( M_{\text{max}} 10^4 \) kNm | \( M_{\text{max}} 10^4 \) kNm |
| 4 | 4 | 0.003797 | 2.35 | 2.57 | 0.00357 | 8.11 | 6.84 |
| 8 | 8 | 0.00358 | 4.66 | 3.94 | 0.00358 | 8.37 | 7.06 |
| 12 | 12 | 0.00357 | 5.77 | 4.87 | 0.00358 | 8.37 | 7.06 |
| 16 | 16 | 0.00358 | 6.38 | 5.39 |
| 20 | 20 | 0.00358 | 6.76 | 5.71 |
| 26 | 26 | 0.00358 | 7.129 | 6.01 |
| 32 | 32 | 0.00358 | 7.35 | 6.20 |
Table 2. Characteristics of the stress-strain state and the study of convergence in the expansion of \( w(\xi, \eta) \) in the Legendre polynomials and beam functions for a flat plate

| Number of row members | Beam functions | Special polynomials |
|-----------------------|----------------|---------------------|
| \( N \) | \( M \) | \( w_{\text{max}}, \text{cm} \) | \( M_{\text{max}} 10^3 \text{kNm} \) | \( M_{\text{ymax}} 10^3 \text{kNm} \) | \( w_{\text{max}}, \text{cm} \) | \( M_{\text{max}} 10^5 \text{kNm} \) | \( M_{\text{ymax}} 10^5 \text{kNm} \) |
| 6 | 6 | 0.63524 | 1.086 | 1.061 | 0.6356 | 1.365 | 1.061 |
| 8 | 8 | 0.6355 | 1.150 | 1.065 | 0.6356 | 1.365 | 1.061 |
| 12 | 12 | 0.6356 | 1.218 | 1.063 |
| 20 | 20 | 0.6356 | 1.275 | 1.060 |
| 26 | 26 | 0.6356 | 1.295 | 1.061 |
| 32 | 32 | 0.6356 | 1.308 | 1.061 |

3.2. Second case. Calculation of a wavy plate rigidly embedded at \( y = 0, y = b \) and hingedly supported at \( x = 0, x = a \).

Table 3. Characteristics of the stress-strain state and the study of convergence in the expansion of \( w(\xi, \eta) \) in the Legendre polynomials and beam functions for a wavy plate

| Number of row members | Beam functions | Special polynomials |
|-----------------------|----------------|---------------------|
| \( N \) | \( M \) | \( w_{\text{max}}, \text{cm} \) | \( M_{\text{max}} 10^5 \text{kNm} \) | \( M_{\text{ymax}} 10^5 \text{kNm} \) | \( w_{\text{max}}, \text{cm} \) | \( M_{\text{max}} 10^3 \text{kNm} \) | \( M_{\text{ymax}} 10^3 \text{kNm} \) |
| 4 | 4 | 0.000803 | 1.428 | 1.005 \( \times 10^{-3} \) | 0.000761 | 4.072 | 1.107 |
| 8 | 8 | 0.000725 | 3.915 | 9.765 \( \times 10^{-4} \) | 0.000708 | 1.520 | 1.020 |
| 12 | 12 | 0.000762 | 4.981 | 1.060 \( \times 10^{-3} \) | 0.000708 | 1.483 | 1.023 |
| 16 | 16 | 0.000725 | 2.633 | 1.012 \( \times 10^{-3} \) | 0.000708 | 1.484 | 1.024 |
| 20 | 20 | 0.000704 | 1.047 | 9.816 \( \times 10^{-4} \) |
| 26 | 26 | 0.000706 | 1.059 | 9.929 \( \times 10^{-4} \) |
| 32 | 32 | 0.000709 | 1.805 | 1.009 \( \times 10^{-3} \) |

The results obtained (Table 1, Table 3) show that rigid fixing of the wavy plate along the sides parallel to the plate wave reduces the deflections by 5 times, the bending moment \( M_x(x, y) \) by 18 times, the bending moment \( M_y(x, y) \) in 7 times compared with the corresponding stress-strain state values for a wavy plate rigidly fixed on the sides perpendicular to the plate wave.

If we compare the characteristics of the stress-strain state of a wavy plate at \( f = 9 \text{mm} \) and a flat plate (Table 1, Table 2, Table 4), the deflections of the flat plate are greater than the deflections of the wavy plate 897 times, the bending moment \( M_x(x, y) \) is greater than 304 times, and bending moment \( M_y(x, y) \) practically does not change.
Table 4. The influence of the magnitude of the amplitude $f$ on the deflection and bending moments

| $f$, mm | $l$, mm | $w_{max}$, cm | $M_{x_{max}}$, kNm | $M_{y_{max}}$, kNm |
|---------|---------|---------------|-------------------|-------------------|
| 0       | 0       | 0.173         | $2.955\times10^{-4}$ | $9.850\times10^{-4}$ |
| 4       | 45.84   | 0.00376       | $1.758\times10^{-5}$ | $9.975\times10^{-4}$ |
| 6       |         | 0.00168       | $6.649\times10^{-6}$ | $1.024\times10^{-3}$ |
| 8       |         | 0.000917      | $2.516\times10^{-6}$ | $1.027\times10^{-3}$ |
| 9       |         | 0.000708      | $1.484\times10^{-6}$ | $1.024\times10^{-3}$ |

Increasing the amplitude of the plate wave reduces the deflection $w(x, y)$ and the bending moment $M_y(x, y)$, and has almost no effect on the bending moment $M_x(x, y)$. So, with an increase in amplitude twice (Table 4), the deflection decreases four times, $M_y(x, y)$ decreases twelve times, $M_x(x, y)$ increases 1.03 times. The proposed models allowed the development of diagnostic and non-destructive testing methods for assessing the quality of structural elements at all stages of the product life cycle [11-14].

4. Conclusions

The method of calculating wavy corrugated sheets as a wavy plate proposed in this paper allows us to analyze the influence of the plate side supports on the characteristics of the stress-strain state of the plate under its weight. It was found that rigid fastening of the sides parallel to the guided line of the wave of the plate increases the strength by eighteen times and the rigidity of the plate five times.

The rationality of using corrugated sheets compared to flat ones with the same mechanical and geometric parameters is shown.

Analysis of the effect of amplitude on the stress-strain state of a wavy plate showed that an increase in the magnitude of the amplitude reduces the number of deflections, the bending moment in sections perpendicular to the $x$-axis, and practically does not affect the bending moments arising in sections perpendicular to the $y$-axis.

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