**Δ contribution in \( e^+ + e^- \rightarrow p + \bar{p} \) at small \( s \)**

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**Abstract**

Two-photon annihilate contributions in the process \( e^+ + e^- \rightarrow p + \bar{p} \) including \( N \) and \( \Delta \) intermediate are discussed in a simple hadronic model. The corrections to the unpolarized cross section and polarized observables \( P_x, P_z \) are presented. The results show the two-photon annihilate correction to unpolarized cross section is small and its angle dependence becomes weak at small \( s \) after considering the \( N \) and \( \Delta(1232) \) contributions simultaneously, while the correction to \( P_z \) is enhanced.

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**1 Introduction**

The Two-Photon-Exchange (TPE) effect has attracted many interests after its success in explaining the un-consistent measurements of \( R = \frac{\mu_p G_E/G_M}{G_E/G_M} \) from \( ep \rightarrow ep \) by Rosenbluth technique and polarized methods [1–4]. It is found that the TPE corrections play an important role in extracting the proton’s form factors due to its explicit angle dependence. Later some other processes [5–7] are suggested to measure the TPE like effects. The \( e^+ + e^- \rightarrow p + \bar{p} \) is one of such processes and the two-photon annihilate corrections in this process have been discussed by [8] where only the \( N \) intermediate was included. The estimate by [8] showed the two-photon annihilate corrections are about a few percent in the magnitude but strongly depend on the hadron production angle. On another hand, the calculation in [9] showed the \( \Delta(1232) \) intermediates also unneglectable in the TPE corrections in the simple hadronic model [2,4,9]. These researches prompt us to extent the estimate of the two-photon annihilate corrections in [8] to include \( \Delta \) intermediate state. In this work, we present such results.

**2 Two-Photon Annihilate Corrections including \( N \) and \( \Delta(1232) \) as Intermediate State**

Considering the process \( e^+(k_2) + e^-(k_1) \rightarrow p(p_2) + \bar{p}(p_1) \), the Born diagram is showed as Fig.1. The differential cross section for this process at the tree level can be written as [10]

\[
\frac{d\sigma}{d\Omega_{CM}} = \frac{\alpha^2 \sqrt{1 - 4M_N^2/q^2} |G_M|^2 (1 + \cos^2 \theta) + \frac{1}{\tau} |G_E|^2 \sin^2 \theta}{4q^2}.
\]  

\[ (1) \]

\[ \]
\[ p_1 \rightarrow p + \bar{p} \] 

Figure 1: One photon annihilating diagram for \( e^+ + e^- \rightarrow p + \bar{p} \).

\[ N(k) \rightarrow p_1 + p_2 \] 

(a) \hspace{2cm} \[ \Delta(k) \rightarrow p_1 + p_2 \] 

(b) 

Figure 2: Two-photon annihilating diagrams (a) with \( N \) as intermediate state, (b) with \( \Delta(1232) \) as intermediate state. Corresponding cross-box diagrams are implied.

where \( q = k_1 + k_2, \tau = q^2/4M^2 > 1 \) and \( \theta \) is the angle between the momentum of final antiproton and initial electron in the center of mass frame. The Sachs form factors have been used as

\[
G_M(q^2) = F_1(q^2) + F_2(q^2), \quad G_E(q^2) = F_1(q^2) + \tau F_2(q^2).
\]

(2)

In principle, the form factors at certain \( s = q^2 \) can be extracted from the measurement of the unpolarized differential cross section at different angle. To extract the form factors more precisely, the radiative corrections should be considered. Among the one loop radiative corrections, the box and crossed box diagrams play special role due to their strong angle dependence. This leads us restrict our discussions on the two-photon annihilate correction firstly.

Using the simple hadronic model developed in [2, 4, 9] and including \( N \) and \( \Delta \) as the intermediate state like Fig.2, the unpolarized cross section can be written as

\[
d\sigma = d\sigma_0(1 + \delta^{2\gamma}_N + \delta^{2\gamma}_\Delta) \propto \sum_{\text{helicity}} |M_0 + M^{2\gamma}_N + M^{2\gamma}_\Delta|^2,
\]

(3)

where \( M_0 \) is the contribution of one-photon annihilate diagram and \( M^{2\gamma}_{N,\Delta} \) denote the contribution from two-photon annihilate diagrams with \( N \) and \( \Delta \) as intermediate state. The corrections to the unpolarized cross section can be defined as

\[
\delta^{2\gamma}_{N,\Delta} = \frac{\sum_{\text{helicity}} 2Re\{M^{2\gamma}_{N,\Delta}M_0^\dagger\}}{\sum_{\text{helicity}} |M_0|^2}.
\]

(4)

The corrections from \( N \) have been discussed in [8]. To discuss the correction from \( \Delta \), we take the following
matrix elements as \[9, 11\]

\[
\langle N(p_2)|J_{\mu}^m|\Delta(k)\rangle = -\frac{F_\Delta(q^2)}{M_\Delta^2} \pi(p_2)|g_1(q_1^\alpha \not{k_1} - k_\mu \gamma^\alpha \not{q_1} - \gamma_\mu \gamma^\alpha k \cdot q_1 + \gamma_\mu q_1^2)|
\]

\[
+ g_2(k_\mu q_1^\alpha - k \cdot q_1 g_1^\alpha) + \frac{g_3}{M_N} (q_1^\alpha (k_\mu \gamma^\alpha - g_1^\alpha \not{k}) + q_1 \mu (q_1^\alpha \not{k} - \gamma_\mu k \cdot q_1))|\gamma_5 T_3 u_\alpha^\Delta(k),
\]

\[
\langle \Delta(k)|\overline{N}(p_1)|J_{\mu}^m\rangle|0\rangle = -\frac{F_\Delta(q^2)}{M_\Delta^2} \pi(\not{k} + M_\Delta) T_\gamma^+ \gamma_5 [g_1(q_3^\alpha \not{q_2} - k_\nu \gamma^\alpha - \gamma_\nu \gamma^\alpha k \cdot q_2 + \gamma_\nu q_2^\alpha) + g_2(k_\nu q_1^\alpha - k \cdot q_1 g_1^\alpha) - \frac{g_3}{M_N} (q_1^\alpha (k_\nu \gamma^\alpha - g_1^\alpha \not{k}) + q_2 \nu (q_1^\alpha \not{k} - \gamma_\nu k \cdot q_2))]|v(p_1),
\]

where \(q_1 = p_2 - k, \quad q_2 = k + p_1\) and \(T_3\) is the third component of the \(N \rightarrow \Delta\) isospin transition operator and is \(-\sqrt{2/3}\) here. The effective vertexes of \(\gamma N\Delta\) are defined as \(\pi(p_2)\Gamma^\alpha_{\mu}(\gamma \rightarrow N)u_\alpha^\Delta(k) = -i e\langle N(p_2)|J_{\mu}^m|\Delta(k)\rangle, \quad \pi(k)\Gamma^\beta_\gamma(\gamma \rightarrow \overline{N}\Delta)v(p_1) = -i e\langle \Delta(k)|\overline{N}(p_1)|J_{\mu}^m\rangle|0\rangle\). Both the two vertexes satisfy the conditions \(q_{1,2}^\mu \Gamma^\mu = 0\) and \(k_\alpha \Gamma^\alpha = 0\), the first condition ensure the gauge invariance of the result and the second condition ensure to select only the physical spin3/2 component \[9\].

For the propagator of \(\Delta\), the same form is employed as \[9\]

\[
S_{\alpha\beta}^\Delta(k) = \frac{-i(k + M_\Delta)}{k^2 - M_\Delta^2 + i\varepsilon} P^{3/2}_{\alpha\beta}(k),
\]

\[
P^{3/2}_{\alpha\beta}(k) = g_{\alpha\beta} - \gamma_\alpha \gamma_\beta/3 - (k_\beta \gamma_\alpha k_\alpha + k_\alpha \gamma_\beta k_\beta)/3k^2.
\]

Such propagator is different with the usual R.S one which read as

\[
S_{\alpha\beta}^{R.S}(k) = \frac{k + M_\Delta}{k^2 - M_\Delta^2 + i\varepsilon} \left[-g_{\alpha\beta} + \frac{1}{3} \gamma_\alpha \gamma_\beta + \frac{1}{3m} (\gamma_\alpha k_\beta - \gamma_\beta k_\alpha) + \frac{2}{3m^2} k_\alpha k_\beta \right].
\]

After using the properties of the vertexes, these two forms result in the same amplitude.

By this effective interaction, the amplitude of box diagram Fig.2(b) can be written as

\[
M^{(2b)} = -i \int \frac{d^4k}{(2\pi)^4} \pi(k_2)(-i e\gamma_\mu) \frac{i(p_1 + k - k_2 + m_e)}{(p_1 + k - k_2)^2 - m_e^2 + i\varepsilon} (-i e\gamma_\mu)v(k_1) \frac{-i}{(p_1 + k)^2 + i\varepsilon} \frac{-i}{(p_2 - k)^2 + i\varepsilon}
\]

\[
\pi(p_2)\Gamma^{\alpha\mu}_{\gamma\Delta \rightarrow N} \frac{-i(k + M_\Delta)}{k^2 - M_\Delta^2 + i\varepsilon} P^{3/2}_{\alpha\beta}(k)\Gamma^{\beta\nu}_{\gamma \rightarrow \overline{N}\Delta}v(p_1),
\]

where Feynman gauge invariance has been used. Similarly one can get the amplitude of crossed box diagram with \(\Delta\) intermediate state.

In the practical calculation, we take the form factor \(F_\Delta\) in the monopole form as \(G_E\) in \(N\) case \[8\]

\[
F_\Delta(q^2) = G_E(q^2) = G_M/\mu_p(q^2) = \frac{-\Lambda_1^2}{q^2 - \Lambda_1^2},
\]

the coupling parameters and cut-offs are the same as \[8, 11\]

\[
g_1 = 1.91, \quad g_2 = 2.63, \quad g_3 = 1.58, \quad \Lambda_1 = 0.84 GeV.
\]

### 3 Numerical Results and Discussion

Using the above as input, the two-photon annihilate corrections can be calculated directly. We use the package FeynCalc \[12\] and LoopTools \[13\] to carry out the calculation. The IR divergence in the \(N\)
intermediate case is treated as [8] and there is no divergence in the $\Delta$(1232) case. The numerical results for $\delta_{N+\Delta}^{2\gamma}$ are showed in Fig.3. The similar calculation can be applied to the polarized quantities $P_x$ and $P_z$ as [8,14] with the definitions

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{un}}{d\Omega} [1 + P_y \xi_y + \lambda_e P_x \xi_x + \lambda_e P_z \xi_z].$$  \hspace{1cm} (11)$$

The results of the corrections to $P_x$ and $P_z$ are presented in Fig4. In our previous results [8], when discussing the TPE corrections to polarized observables, only the contributions in term $\frac{d\sigma}{d\Omega}$ are considered, while the corrections in $\frac{d\sigma}{d\Omega}$ are neglected. Here the calculations are improved to include both corrections.

As showed in Fig.3, the correction $\delta_{\Delta}^{2\gamma}$ is found to be always opposite to the corrections $\delta_{N}^{2\gamma}$ in all the angle region. This behavior is similar to the $ep$ scattering case [9]. Detailedly, at $s = 4GeV^2$ the absolute magnitude of $\delta_{\Delta}^{2\gamma}$ is so close to $\delta_{N}^{2\gamma}$ which results in the large cancelation and small total correction to unpolarized cross section. The small $\delta_{N+\Delta}^{2\gamma}$ and its weak angle dependence suggest the Rosenbluth method will work well in this region. This conclusion is some different with the $ep$ scattering case where the cancelation is much smaller and the total correction still strongly depend on the scattering angle. At $s = 5GeV^2$, the absolute magnitude of $\delta_{\Delta}^{2\gamma}$ becomes larger than $\delta_{N}^{2\gamma}$ which suggests the important roles
played by $\Delta(1232)$ intermediate state in the process of $e^+ + e^- \rightarrow p + \bar{p}$.

For the polarized observables, Fig.4 shows the correction to $P_x$ from $\Delta$ is much smaller than $N$ and the correction to $P_z$ from $\Delta$ is close to $N$. The former property suggests $\Delta(1232)$ gives no new correction than [8] while the latter property increases the two-photon annihilate corrections to $P_z$ which enhances our previous suggestion that the nonzero $P_z$ at $\theta = \pi/2$ may be a good place to measure the two-photon exchange like effects directly.

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References

[1] M.K. Jones et al., Phys. Rev. Lett. 84, (2000)1398 ; O. Gayou et al., Phys. Rev. Lett. 88,(2002) 092301.
[2] P.G. Blunden, W. Melnitchouk, and J.A. Tjon, Phys. Rev. Lett. 91,(2003) 142304.
[3] Y.C. Chen, A.V. Afanasev, S.J. Brodsky, C.E. Carlson, M. Vanderhaeghen, Phys. Rev. Lett 93,(2004) 122301.
[4] P. G. Blunden, W. Melnitchouk and J. A. Tjon, Phys. Rev. C 72, (2005)034612.
[5] M. P. Rekalo, E. Tomasi-Gustafsson and D. Prout, Phys. Rev. C 60, (1999)042202(R).
[6] G. I. Gakh, E. Tomasi-Gustafsson, Nucl. Phys. A 761,(2005) 120.
[7] E. Tomasi-Gustafsson, E. A. Kuraev, S. Bakmaev and S. Pacetti, Phys. Lett. B 659,(2008) 197.
[8] D.Y. Chen, H.Q. Zhou, Y.B. Dong, Phys. Rev. C78,(2008) 045208.
[9] S. Kondratyuk, P.G. Blunden, W. Melnitchouk and J.A. Tjon, Phys. Rev. Lett. 95, (2005) 172503.
[10] N. Cabibbo, Raoul Gatto, Phys.Rev.124,(1961)1577, A.Zichichi et al., Nuovo Cim.24 (1962) 170.
[11] Keitaro Nagata, Hai Qing Zhou, Chung Wen Kao, Shin Nan Yang, [arXiv:0811.3539]
[12] R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. 64,(1991) 345.
[13] T. Hahn, M. Perez-Victoria, Comput. Phys.Commun, 118, (1999)153.
[14] C. Adamuscin, G. I. Gakh, and E. Tomasi-Gustafsson, [arXiv:0704.3375]