Evaluation of Network Security Service Provider Using 2-Tuple Linguistic Complex \(q\)-Rung Orthopair Fuzzy COPRAS Method

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In recent years, network security has become a major concern. Using the Internet to store and analyze data has become an integral aspect of the production and operation of many new and traditional enterprises. However, many enterprises lack the necessary resources to secure information security, and selecting the best network security service provider has become a real issue for many enterprises. This research introduces a novel decision-making method utilizing the 2-tuple linguistic complex \(q\)-rung orthopair fuzzy numbers (2TLC\(q\)-ROFNs) to tackle this issue. We propose the 2TLC\(q\)-ROF concept by combining the complex \(q\)-rung orthopair fuzzy set with 2-tuple linguistic terms, including the fundamental definition, operational rules, scoring, and accuracy functions. Aggregation operators are the fundamental mathematical approach used to combine various inputs into a single output. Taking into account the interaction between the attributes, we develop the 2TLC\(q\)-ROFH operators by using the innovative operational rules. These operators include the 2TLC\(q\)-ROFH weighted average (2TLC\(q\)-ROFHW), 2TLC\(q\)-ROFH ordered weighted average (2TLC\(q\)-ROFHOW), 2TLC\(q\)-ROFH hybrid average (2TLC\(q\)-ROFHH), 2TLC\(q\)-ROFH weighted geometric (2TLC\(q\)-ROFHWG), 2TLC\(q\)-ROFH ordered weighted geometric (2TLC\(q\)-ROFHOWG), and 2TLC\(q\)-ROFH hybrid geometric (2TLC\(q\)-ROFHHG) operators. In addition, we talk about the properties of 2TLC\(q\)-ROFH operators such as idempotency, commutativity, monotonicity, and boundedness and also examine their spatial cases. To tackle the problems of the 2TLC\(q\)-ROF multiattribute group decision-making (MAGDM) environment, we develop a novel approach according to the COPRAS (complex proportional assessment) model. Finally, to validate the feasibility of the given strategy, we employ a quantitative example related to select the best network security service provider. In comparison with existing approaches, the developed decision-making algorithm is most extensively used and reduces the loss of information.

1. Introduction

The use of the computer network has spread to every industry as a result of its popularization and advancement. People in the information society are becoming increasingly reliant on the network, and as the network has grown in size and complexity, network security has become a major concern. Although the Internet and other information technologies empower businesses, financial institutions, and even government agencies with the ease of data storing and processing to efficiently serve society, network security risks also present a threat to enterprises and the entire society. In the information era, data losses are certainly fatal. Many noncyber security organizations find it impractical to maintain a long-term professional network security technology staff due to the extremely professional aspects of network security technology. As a result, many enterprises want to entrust network security defense to professional network security service providers for technical help. This means that choosing the proper network security service
providers has much impact on ordinary enterprises. As a result, selecting the best network security company is a MADM issue. MAGDM is an integrative research field that combines MADM with the group of decision-makers, particularly analyzing different alternatives through different decision-making (DM) approaches. MAGDM usually provides structures to fuse individual preference information into group preference information. Due to the increasing complexity in economics and management, it is almost impossible for decision-makers (DMs) to collect all information about optimal alternatives associated with MAGDM problems. Hence, uncertainty and fuzziness occur in real-life issues, and how to effectively deal with such kind of fuzziness is crucial to select the best alternative. Many scholars and researchers have worked hard to develop different methods to represent fuzzy DM information in the MAGDM process. Recently, for expressing vagueness and uncertainty, various tools have been developed. For some MAGDM problems, DMs experience problems in describing attribute values of alternatives by using crisp numbers. To describe the uncertainties, Zadeh [1] introduced the fuzzy set (FS) as a generalization of the crisp set, and the value of FS lies between [0, 1]. However, FS has only a membership degree (MD) and ignores the nonmembership degree (NMD) in DM problems. Furthermore, intuitionistic FS (IFS) [2], Pythagorean FS (PF S) [3], and Fermatean FS (FFS) [4], whose elements are pairs of fuzzy numbers, have been introduced. All of the above described FSs demonstrate the MD and NMD. The limitation of MD and NMD is that the sum, square sum, and cube sum of both would belong to [0, 1]. Yager [5] realized that the current IFS, PFS, and FFS frameworks are unable to represent human opinion more realistically and developed the q-rung orthopair FS (q-ROFS), which effectively enhances the scope of information by establishing novel subjective constraints where the qth sum of MD and NMD lies between [0, 1]. If \( q = 1 \), \( q = 2 \), and \( q = 3 \), and then, the q-ROFS is reduced into the IFS, PFS, and FFS, respectively.

The q-ROFS theory deals only with one dimension at a time, which sometimes destroys information. However, in real life, we encounter complex natural phenomena in which it becomes significant to integrate the second dimension for the representation of MD and NMD. The development of the second dimension allows complete information to be projected into a set, avoiding any information loss. With the unit disc, Ramot et al. [6] extended the MD range from real number to complex number and proposed the concept of a complex FS (CFS). Furthermore, representing the complex-valued NMD, Alkouri and Salleh [7, 8] extended an idea of CFS to complex IFS (CIFS) and also put forward the concept of CIF relations and a distance measure in CIF circumstances. Ullah et al. [9] developed various distance measures of the complex PFS (CPFS) and an algorithm for addressing pattern recognition problems. Liu et al. [10] put forward an innovative, effective, and powerful tool to describe uncertain phenomena named q-ROFSSs and introduced the Cq-ROF weighted average operator and Cq-ROF weighted geometric operator. To aggregate complex q-rung orthopair fuzzy numbers, Liu et al. [11] extended the Einstein operations to Cq-ROFSs and proposed a family of Cq-ROF Einstein averaging operators, such as the Cq-ROF Einstein weighted averaging, the Cq-ROF Einstein ordered weighted averaging, the generalized Cq-ROF Einstein weighted averaging, and the generalized Cq-ROF Einstein ordered weighted averaging operators. The newly proposed Cq-ROFSs are incredibly flexible and efficient, as opposed to many existing FS theories, which can clearly describe the DM perspectives of experts in a complex environment. The amplitude term implies the extent to which an object belongs in a Cq-ROFS, while the phase terms are frequently associated with periodicity. The Cq-ROFS differs from typical q-ROFS theories because of these phase terms. Akram et al. developed novel decision-making methods based on complex Pythagorean fuzzy [12] and complex Fermatean fuzzy N-soft circumstances [13].

The above FSs can only represent information from a quantitative perspective. And it is difficult for DMs to provide precise numerical values to describe their point of view. As a result, Zadeh [14] developed the linguistic variable (LV) as a tool to express qualitative information in DM problems. Following that, various innovative concepts based on the LV and FS were proposed, including intuitionistic linguistic numbers [15], single-valued neutrosophic linguistic set [16], and linguistic q-ROF numbers [17]. Furthermore, Herrera and Martnez [18] introduced the concept of a 2-tuple linguistic FS (2TLFS) established by LV and numerical value to reduce information loss in the DM procedure. Zhao et al. [19] presented an advanced TODIM strategy based on 2-tuple linguistic neutrosophic sets and cumulative prospect theory as a novel approach to MAGDM problems. Based on previous research, Zhang et al. [20] improved dramatically the TODIM technique as well as the cumulative prospect theory under the 2TL Pythagorean fuzzy sets. Naz and Akram [21, 22] developed a new DM approach to deal with the MADM problems based on the graph theory. Recently, many research studies [23–27] have developed several DM methods under generalized fuzzy scenario.

Later, many researchers integrated the 2TL model with several FSs and proposed 2TLIFS [28], 2TLIFS [29], and so forth. These extensions can effectively describe uncertain fuzzy information in addressing DM problems. The Cq-ROFS and the 2TL terms, as previously noted, are two strategies for describing the quantitative and qualitative assessment information. Motivated by the concept of a 2TLIFS, Rong et al. [30] introduced the novel concept of 2TLCq-ROFS. The 2TLCq-ROFS is the more universal than existing FSs because we can obtain multiple specific examples by considering some particular circumstances. In the context of 2TLCq-ROFS, the parameters \( q = 1 \) and \( q = 2 \) degenerate into the 2TLCIFS and the 2TLCIFS, respectively. Furthermore, if the imaginary part of 2TLCq-ROFS is set to zero, it is reduced to a 2TLq-ROFS. From the previous linguistic set research, the 2TLCq-ROFS is set to zero, it is reduced to a 2Tlq-ROFS. From the previous linguistic set research, the 2TLCq-ROFS is stronger because: (1) it can prevent information distortion throughout the linguistic information procedure; (2) it can avoid information loss by expressing assessment information through complex-valued MD and complex-valued NMD;
and (3) in real-life applications, it can tackle problems with two dimensions of information.

An aggregation operator (AO) is a well-known approach in the field of information fusion, and it has provided lots of new research results on a variety of topics. To design the MAGDM method, Liu and Wang [31] developed a weighted average and geometric operator for \( q \)-ROFS. However, in DM problems, these operators fail to evaluate the interrelationship of attributes. Hamacher product and Hamacher sum were first presented by Hamacher [32] as part of the Hamacher operations. Furthermore, as a generalization of the algebraic and Einstein t-norm and t-conorm, the Hamacher t-norm and t-conorm are more general and flexible. According to a review of the 2TLC\( q \)-ROF-AOs, there is limited research by using Hamacher operations to propose new operators. Therefore, it is necessary to research AOs utilizing Hamacher operations with 2TLC\( q \)-ROF information. Moreover, in decision analysis, selecting the appropriate alternative(s) is critical. As a result, it is crucial to use Hamacher operations to develop 2TLC\( q \)-ROF-AOs for solving MAGDM problems. Akram et al. [33] introduced the complex intuitionistic fuzzy Hamacher-weighted averaging operator, complex intuitionistic fuzzy Hamacher ordered weighted averaging operator, complex intuitionistic fuzzy Hamacher weighted geometric operator, and complex intuitionistic fuzzy Hamacher ordered weighted geometric operator. With the use of Hamacher operations and 12TL elements, Faizi et al. [34] developed the intuitionistic 2-tuple linguistic Hamacher weighted average (12TLMWA) and intuitionistic 2-tuple linguistic Hamacher weighted geometric (12TLHGW) operators. Rawat [35] introduced \( q \)-rung orthopair fuzzy Hamacher Muirhead mean aggregation operators and developed a decision-making approach utilizing proposed operators. Pamucar et al. [36] introduced a novel weighted aggregated sum product assessment approach for advantage prioritization of the electric ferry's sustainable supply chain based on the fuzzy Hamacher weighted averaging function and weighted geometric averaging function.

In recent years, a wide range of methods such as AHP, VIKOR, TOPSIS, and COPRAS that can effectively deal with the ranking procedure has been introduced. The basic purpose of these methods is to select the best alternative by aggregating the information and ranking the objectives according to their significance. Zavadskas et al. [37] proposed the COPRAS method, which compares each alternative and computes their priorities based on attribute weights. COPRAS method is one of the most appropriate methods for ranking the alternatives among all of these methods, and it is widely used for both quantitative and qualitative analyses. The COPRAS method considers direct and proportional reliance of the weights and the utility degree of examined adaptations on a framework of the attributes. To explain logistic regression, boosted regression trees, and random forest, Arabamiri et al. [38] built three new ensemble models and assessed them using the COPRAS method. A comparative analysis of COPRAS and the other existing methods such as AHP, TOPSIS, and VIKOR is conducted by Chatterjee et al. [39] and concluded that the COPRAS method indicates good transparency, less calculation time, and a high possibility of graphical understanding of their counterpart strategies. Alipour et al. [40] provided an integrated approach for fuel cell combined with hydrogen supplier selection based on entropy, step-wise weight assessment ratio analysis, and COPRAS methods in a Pythagorean fuzzy environment. Balali et al. [41] utilized the COPRAS approach for risk assessment and the analytic network process technique for determining the weights of each risk assessment criteria. Narang et al. [42] introduced a new hybrid multicriteria decision-making method comprised of group fuzzy COPRAS and fuzzy BCM, followed by a strategy based on the combination of the fuzzy set theory and the COPRAS to rank alternatives in uncertain and ambiguous contexts. This paper extends the COPRAS method to the 2TLC\( q \)-ROF environment, considering the flexibility of 2TLC\( q \)-ROFS and the quality of the COPRAS method. The crucial properties of the COPRAS method are (1) during the execution of the process, it evaluates the proportions of the ideal and worst solutions at the same time; (2) this method evaluates the direct and relative dependencies of the significance and the utility degree of the alternatives under the contrary attribute values; and (3) this method is designed to obtain the decision much more effective and sensible. Thus, considering the advantages of the AOs and the COPRAS method, this article intends to establish an innovative MAGDM approach for managing the information associated with the 2TLC\( q \)-ROFS and some new information measures.

The motivation and objectives of this study are to find the best network security service provider. After conducting several experiments, the MAGDM method is applied to make the final decision. A significant component of MAGDM is the selection of attributes. Attributes are divided into two types: benefit attribute and cost attribute, to select the best alternative in the application based on whether they are beneficial or not. Existing CFS theories fail to depict uncertain information through the 2TL representation model, which has a higher capability to express linguistic information and can avoid information distortion loss while dealing with linguistic decision problems. The 2TLC\( q \)-ROFS and related fundamental theories are developed to enhance CFS theories and provide a reliable tool for experts to express assessment information. Using the 2TLC\( q \)-ROFS in this type of MAGDM method gives rise to the clear thinking of DMs who assigns value to complex membership and complex nonmembership functions. Information fusion is essential for aggregating the opinions of DMs. In addition, in a range of practical problems, the correlation of selected attributes is essentially addressed. Several 2TLC\( q \)-ROFH operators are presented to address two-dimensional fuzzy information in the light of the excellent superiority of the Hamacher operator. The COPRAS method establishes to rank the given 2TLC\( q \)-ROFNs, to develop two algorithms based on COPRAS and AOs to understand DM problems. The approach is described with a numerical illustration to examine the research study.

The main contributions of this research work are as follows:
(i) We introduce the 2TL terms into the complex q-rung orthopair fuzzy environment and propose the construction process of 2TLq-ROFNs.

(ii) The 2TLq-ROFHWG and 2TLq-ROFHWA operators are proposed combining 2TL terms with complex q-rung orthopair fuzzy set, Hamacher weighted average, and Hamacher weighted geometric operators.

(iii) We propose some operational properties and special cases of 2TLq-ROF Hamacher AOs.

(iv) Based on 2TLq-ROFNs, we improve the COPRAS method and develop a 2TLq-ROF-COPRAS method to solve the MAGDM problem for ranking of alternatives.

(v) We apply the 2TLq-ROF-COPRAS method to the assessment of the network security service provider. This method is verified to provide a new idea for the assessment of the network security service provider.

To achieve the cognitive approach, the overall framework of this article is as follows: In Section 2, we give several fundamental concepts and definitions including 2TL term, Cq-ROFS, and Hamacher operator. Section 4 presents some new 2TLq-ROF aggregation operators, that is, 2TLq-ROFHWA operator, 2TLq-ROFHHOWA operator, 2TLq-ROFHWG operator, and 2TLq-ROFHGW operator, and also discussed some desirable properties and particular cases of them. In Section 5, we design an extended COPRAS method for the MAGDM problem based on the 2TLq-ROFHWA and 2TLq-ROFHGW operators. Section 6 employs an example of the best network security service provider to show the application of the proposed method. Some sensitive and comparative analysis is also provided. Finally, Section 7 presents the conclusions, remarks, and also future directions.

2. Preliminaries

In this section, some correlative basic concepts of LTS, 2TL, and Cq-ROFS, are recapitulated to facilitate the next sections.

2.1. 2TL Representation Model and Cq-ROFS

Definition 1 (see [43]). Let there exist a linguistic term set (LTS) \( S = \{s_\epsilon | \epsilon = 0, 1, \ldots, \tau \} \) with odd cardinality, where \( s_\epsilon \) indicates a possible linguistic term for a linguistic variable. For instance, an LTS \( S \) having seven terms can be described as follows:

\[ S = \{ s_0 \text{ no influence}, s_1 \text{ very low influence}, s_2 \text{ low influence}, s_3 \text{ same influence}, s_4 \text{ high influence}, s_5 \text{ very high influence} \} \]

If \( s_\epsilon, s_\kappa \in S \), then the LTS meets the following characteristics:

(i) The set is ordered: \( s_\epsilon > s_\kappa \), iff \( \epsilon > \kappa \)

(ii) Max operator: \( \max(s_\epsilon, s_\kappa) = s_\kappa \), iff \( \epsilon \geq \kappa \)

(iii) Min operator: \( \min(s_\epsilon, s_\kappa) = s_\kappa \), iff \( \epsilon \leq \kappa \)

(iv) Negative operator: \( \text{Neg}(s_\kappa) = s_\epsilon \) such that \( \kappa = \tau - \epsilon \)

The 2TL representation model based on the idea of symbolic translation, introduced by Herrera and Martínez [18, 44], is useful for representing the linguistic assessment information by means of a 2-tuple of labels, \( (s_\epsilon, v_\epsilon) \), where \( s_\epsilon \) is a linguistic label from predefined LTS \( S \) and \( v_\epsilon \) is the value of symbolic translation, and \( v_\epsilon \in [-0.5, 0.5] \)

Definition 2 (see [18, 44]). Let \( \vartheta \) be the result of an aggregation of the indices of a set of labels assessed in an LTS \( S \), that is, the result of a symbolic aggregation operation, \( \vartheta \in [1, \tau] \), where \( \tau \) is the cardinality of \( S \). Let \( \epsilon = \text{round}(\vartheta) \) and \( v = \vartheta - \epsilon \) be two values, such that \( \epsilon \in [1, \tau] \) and \( v \in [-0.5, 0.5] \), then \( v \) is called a symbolic translation.

Definition 3 (see [18, 44]). Let \( S = \{ s_\epsilon | \epsilon = 1, \ldots, \tau \} \) be an LTS and \( \vartheta \in [1, \tau] \) is a number value representing the aggregation result of linguistic symbolic. Then, the function \( \Delta \) used to obtain the 2TL information equivalent to \( \vartheta \) is defined as

\[ \Delta: [1, \tau] \rightarrow S \times [-0.5, 0.5], \]

\[ \Delta(\vartheta) = \begin{cases} s_\epsilon, & \epsilon = \text{round}(\vartheta) \\ v = \vartheta - \epsilon, & v \in [-0.5, 0.5] \end{cases} \]  

(1)

Definition 4 (see [18, 44]). Let \( S = \{ s_\epsilon | \epsilon = 1, \ldots, \tau \} \) be an LTS and \( (s_\epsilon, v_\epsilon) \) be a 2-tuple; there exists a function \( \Delta^{-1} \) that restore the 2-tuple to its equivalent numerical value \( \vartheta \in [1, \tau] \subset R \), where

\[ \Delta^{-1}: S \times [-0.5, 0.5] \rightarrow [1, \tau], \]

\[ \Delta^{-1}(s_\epsilon, v) = \epsilon + v = \vartheta. \]  

(2)

Definition 5 (see [10]). A Cq-ROFS \( \mathcal{F} \) is defined as

\[ \mathcal{F} = \{ (l, \mu_{\mathcal{F}}(l), \nu_{\mathcal{F}}(l)) | l \in L \}, \]

where \( \mu_{\mathcal{F}}, \nu_{\mathcal{F}} \) are the complex-valued membership and nonmembership functions, respectively, and are defined as

\[ \mu_{\mathcal{F}}(l) = \eta_{\mathcal{F}}(l)e^{2\pi i \omega_{\eta}(l)}, \nu_{\mathcal{F}}(l) = \gamma_{\mathcal{F}}(l)e^{2\pi i \omega_{\gamma}(l)} \]  

(4)

where \( 0 \leq \eta_{\mathcal{F}}(l), \gamma_{\mathcal{F}}(l), \eta_{\mathcal{F}}(l) + \gamma_{\mathcal{F}}(l) \leq 1 \) and \( 0 \leq \omega_{\eta}(l), \omega_{\gamma}(l) \). \( \omega_{\eta}(l) + \omega_{\gamma}(l) \leq 1 \). Furthermore, \( \pi_{\mathcal{F}}(l) = (1 - (\eta_{\mathcal{F}}(l) + \gamma_{\mathcal{F}}(l)))^{1/q} \) and \( \omega_{\eta}(l) = (1 - (\omega_{\eta}(l) + \omega_{\gamma}(l)))^{1/q} \) are complex hesitancy degree of \( l \). For simplicity, the pair \( \tau = ((\eta, \omega_{\eta}), (\gamma, \omega_{\gamma})) \) is called the Cq-ROF number (Cq-ROFN), where \( 0 \leq \eta, \gamma, \eta + \gamma \leq 1 \), and \( 0 \leq \omega_{\eta}, \omega_{\gamma}, \omega_{\eta} + \omega_{\gamma} \leq 1 \).

2.2. Hamacher t-Norm and Hamacher t-Conorm. To extend the existing operations of t-norm and t-conorm, Hamacher [32] introduced the Hamacher product t-norm and Hamacher sum t-conorm as generalizations of t-norms and t-conorms, respectively, as follows:
Complexity

\[ T^H_\varrho (r, s) = \begin{cases} \frac{rs}{\rho + (1 - \rho)(r + s - rs)} & \text{if } \varrho > 0, \\ \frac{rs}{r + s - rs} & \text{if } \varrho = 0. \end{cases} \]

\[ (T^*)^H_\varrho (r, s) = \begin{cases} \frac{r + s - rs - (1 - \rho)rs}{1 - (1 - \rho)rs} & \text{if } \varrho > 0, \\ \frac{r + s - 2rs}{1 - rs} & \text{if } \varrho = 0. \end{cases} \]

Clearly, when \( \varrho = 1 \), the Hamacher t-norm and t-conorm change into the algebraic t-norm and t-conorm as follows:

\[ P(r, s) = rs, \]
\[ P^*(r, s) = r + s - rs. \]

Again, when \( \varrho = 2 \), the Hamacher t-norm and t-conorm reduce to the Einstein t-norm and t-conorm [45] as follows:

\[ I(r, s) = \frac{rs}{1 + (1 - r)(1 - s)}, \]
\[ I^*(r, s) = \frac{r + s}{1 + rs}. \]

3. 2-Tuple Linguistic Complex \( q \)-Rung Orthopair Fuzzy Set

**Definition 6.** Let \( S = \{s_1, s_2, \ldots, s_r\} \) be a LTS with odd cardinality \( r + 1 \). The 2TLCq-ROFS is defined as

\[ T = \{ (r, v_T(r), s_T(r)) : r \in R \}, \]

where \( v_T, s_T \) are termed as 2TL complex-valued membership and nonmembership functions, respectively, and are defined as

\[ v_T(r) = \left( s_{p_r}, q \right) e^{i2\pi q (s_{p_r}, r)}, \]
\[ s_T(r) = \left( s_{r_r}, q \right) e^{i2\pi q (s_{r_r}, r)}, \]

where

\[ 0 \leq (\Delta^{-1}(s_{p_r}, q))_T(r) \leq r, \]
\[ 0 \leq (\Delta^{-1}(s_{r_r}, q))_T(r) \leq r, \]
\[ 0 \leq (\Delta^{-1}(s_{p_r}, q))_T^q(r) + (\Delta^{-1}(s_{r_r}, q))_T^q(r) \leq r^q, \]
\[ 0 \leq (\Delta^{-1}(s_{p_r}, q))_T(r) + (\Delta^{-1}(s_{r_r}, q))_T(r) \leq r^q. \]

For simplicity, the pair \( ((s_{p_r}, q), (s_{r_r}, q)), ((s_{p_r}, q), (s_{r_r}, q)) \) is called the 2TLCq-ROF number and is defined for \( s_{p_r}, s_{r_r} \in S \) and \( p, q \in [-0.5, 0.5] \), where

\[ 0 \leq (\Delta^{-1}(s_{p_r}, q))_T \leq r, \]
\[ 0 \leq (\Delta^{-1}(s_{r_r}, q))_T \leq r, \]
\[ 0 \leq (\Delta^{-1}(s_{p_r}, q))_T^q + (\Delta^{-1}(s_{r_r}, q))_T^q \leq r^q, \]
\[ 0 \leq (\Delta^{-1}(s_{p_r}, q))_T(r) + (\Delta^{-1}(s_{r_r}, q))_T(r) \leq r^q. \]

In order to compare any two 2TLCq-ROFNs, their score value and accuracy value are defined as follows:

**Definition 7.** Let \( \bar{\varphi} = ((s_{p_r}, q), (s_{r_r}, q)), ((s_{p_r}, q), (s_{r_r}, q)) \) be a 2TLCq-ROFN. Then, the score value \( \bar{\varphi} \) of a 2TLCq-ROFN \( \bar{\varphi} \), can be represented as

\[ \bar{\varphi} = \Delta \left( \frac{r}{2} \left( 1 + \left( \Delta^{-1}(s_{p_r}, q) \right)^{\frac{q}{r}} - \left( \Delta^{-1}(s_{r_r}, q) \right)^{\frac{q}{r}} \right) + \frac{1}{\omega \left( \Delta^{-1}(s_{p_r}, q) \right)^{\frac{q}{r}} - \omega \left( \Delta^{-1}(s_{r_r}, q) \right)^{\frac{q}{r}}} \right). \]

\[ \bar{\varphi} \in [0, r], \text{ and its accuracy function } \bar{\varphi} \text{ is defined as } \]

\[ \bar{\varphi} = \Delta \left( \left( \frac{\Delta^{-1}(s_{p_r}, q)}{r} \right)^{\frac{q}{r}} + \left( \frac{\Delta^{-1}(s_{r_r}, q)}{r} \right)^{\frac{q}{r}} + \omega \left( \Delta^{-1}(s_{p_r}, q)^{\frac{q}{r}} \right) + \omega \left( \Delta^{-1}(s_{r_r}, q)^{\frac{q}{r}} \right) \right). \]
Definition 8. Let \( \tau_1 = (((s_{p_1}, \varphi_1), \omega_{(s_{p_1}, \varphi_1)}), ((s_{r_1}, R_1), \omega_{(s_{r_1}, R_1)})) \) and \( \tau_2 = (((s_{p_2}, \varphi_2), \omega_{(s_{p_2}, \varphi_2)}), ((s_{r_2}, R_2), \omega_{(s_{r_2}, R_2)})) \) be two 2TLCq-ROFNs; then, these two 2TLCq-ROFNs can be compared according to the following rules:

1. If \( f(\tau_1) > f(\tau_2) \), then \( \tau_1 \succ \tau_2 \)
2. If \( f(\tau_1) = f(\tau_2) \), then
   1. If \( \exists \tau_1 \succ \tau_2 \), then \( \tau_1 \succ \tau_2 \)
   2. If \( \exists \tau_1 \prec \tau_2 \), then \( \tau_1 \prec \tau_2 \)

3.1. Operational Laws for 2TLCq-ROFNs. Some operational laws are put forward to compute the 2TLCq-ROFNs like complex numbers:

Definition 9. Let \( \tau = (((s_{p}, \varphi), \omega_{(s_{p}, \varphi)}), ((s_{r}, R), \omega_{(s_{r}, R)})) \), \( \tau_1 = (((s_{p_1}, \varphi_1), \omega_{(s_{p_1}, \varphi_1)}), ((s_{r_1}, R_1), \omega_{(s_{r_1}, R_1)})) \), and \( \tau_2 = (((s_{p_2}, \varphi_2), \omega_{(s_{p_2}, \varphi_2)}), ((s_{r_2}, R_2), \omega_{(s_{r_2}, R_2)})) \) be three 2TLCq-ROFNs, then

(1)

\[
\tau \oplus \tau_1 = \left( \Delta \left( \tau \sqrt{q} \right), \Delta \left( \tau \sqrt{q} \right) \right)
\]

(2)

\[
\tau \oplus \tau_2 = \left( \Delta \left( \tau \sqrt{q} \right) \left( \Delta \left( \tau \sqrt{q} \right) \right), \Delta \left( \tau \sqrt{q} \right) \left( \Delta \left( \tau \sqrt{q} \right) \right) \right)
\]

(3)

\[
\lambda \tau = \left( \Delta \left( \tau \sqrt{q} \right) \left( \Delta \left( \tau \sqrt{q} \right) \right), \Delta \left( \tau \sqrt{q} \right) \left( \Delta \left( \tau \sqrt{q} \right) \right) \right)
\]

(4)

4. Some 2TLCq-ROFH Aggregation Operators

In this section, we present Hamacher operational laws of 2TLCq-ROFS, and based on these Hamacher operational laws, we propose some 2TLCq-ROFH AOs by using weighted average and weighted geometric operators.

Definition 10. Let \( \tau_1 = (((s_{p_1}, \varphi_1), \omega_{(s_{p_1}, \varphi_1)}), ((s_{r_1}, R_1), \omega_{(s_{r_1}, R_1)})) \), \( \tau_2 = (((s_{p_2}, \varphi_2), \omega_{(s_{p_2}, \varphi_2)}), ((s_{r_2}, R_2), \omega_{(s_{r_2}, R_2)})) \), and \( \tau_3 = (((s_{p_3}, \varphi_3), \omega_{(s_{p_3}, \varphi_3)}), ((s_{r_3}, R_3), \omega_{(s_{r_3}, R_3)})) \) be two 2TLCq-ROFNs, with \( \varphi > 0 \); then, the basic Hamacher operations between \( \tau_1 \) and \( \tau_2 \) are given as follows:
Definition 11. Let $\zeta_e = ((s_{p,e}, \varphi_e), (\omega_{(s_{p,e}, \varphi_e)})), ((s_{r}, \mathcal{R}_e), (\omega_{(s_{r}, \mathcal{R}_e)})) (e = 1, 2, \ldots, n)$ be a collection of 2TLCq-ROFNs; then, the 2TLCq-ROFHWA operator is defined as

$$2\text{TLC}_q - \text{ROFHWA}_p(\zeta_1, \zeta_2, \ldots, \zeta_n) = \Phi_{\omega_{\zeta}}(\varphi_{\zeta})$$

where $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^T$ be the weight vector of $\zeta_e (e = 1, 2, \ldots, n)$, and $\varphi_{\zeta} > 0$, $\sum_{e=1}^{n} \varphi_{\zeta} = 1$.

We derive the following theorem from Def. 11 using the 2TLCq-ROFHWA operations.

**Theorem 1.** Let $\zeta_e = ((s_{p,e}, \varphi_e), (\omega_{(s_{p,e}, \varphi_e)})), ((s_{r}, \mathcal{R}_e), (\omega_{(s_{r}, \mathcal{R}_e)})) (e = 1, 2, \ldots, n)$ be a collection of 2TLCq-ROFNs, where $q > 0$. Then, for any $q > 0$, the aggregated value by utilizing 2TLCq-ROFHWA operator is also a 2TLCq-ROFN, and
where $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^T$ be the weight vector of $\zeta_\epsilon (\epsilon = 1, 2, \ldots, n)$, and $\varphi_\epsilon > 0$, $\sum_{\epsilon=1}^{n} \varphi_\epsilon = 1, \varphi > 0$.

Proof. We use the mathematical induction principle, to prove (20). For $n = 2$, utilizing the operational laws (1) and (3) of Def. 10, we obtain the following result:

(21)
The result in (20) holds true for \( n = 2 \). Suppose the result is true for \( n = k \), and we can get the result:

\[
\Phi_{e=1}^k (\varphi_{\bar{e}e}) = \left( \Phi_{e=1}^k (\varphi_{\bar{e}e}) \right) \Phi (q_{k+1}, \bar{e}_{k+1})
\]

When \( n = k + 1 \), by using the operational laws of Def. 10, we have

\[
\Phi_{e=1}^{k+1} (\varphi_{\bar{e}e}) = \left( \Phi_{e=1}^k (\varphi_{\bar{e}e}) \right) \Phi (q_{k+1}, \bar{e}_{k+1})
\]
Hence, (20) is also true for \( n = k + 1 \), and therefore, (20) holds for all \( n \).

Based on the parameter \( q \), we can derive the following special cases of Theorem 1.

**Remark 1.** When \( q = 1 \), the 2TLCq-ROFHW operator transforms to the 2TLCq-ROF (2TLCq-ROFWA) operator as follows:
Theorem 2. Let $\bar{\zeta}_c = (((s_{p_c}, \varphi_c), (s_{\varphi_c}), (\omega_{(s_{\varphi_c})})), (s_{\varphi_c}), (\omega_{(s_{\varphi_c})}))$ and $\bar{\zeta}'_c = (((s_{p_c}, \varphi_c), (s_{\varphi_c}), (\omega_{(s_{\varphi_c})})), (s_{\varphi_c}), (\omega_{(s_{\varphi_c})}))$ (e = 1, 2, . . . , n) be two sets of 2TLCq-ROFNs; then, the 2TLCq-ROFWA operator has the following properties:

1. Idempotency. If all 2TLCq-ROFNs $\bar{\zeta}_c = (((s_{p_c}, \varphi_c), (s_{\varphi_c}), (\omega_{(s_{\varphi_c})})), (s_{\varphi_c}), (\omega_{(s_{\varphi_c})}))$ (e = 1, 2, . . . , n) are equal, that is, $\bar{\zeta}_e = \bar{\zeta} = (((s_{p_e}, \varphi_e), (s_{\varphi_e}), (\omega_{(s_{\varphi_e})})), (s_{\varphi_e}), (\omega_{(s_{\varphi_e})}))$, then

\[
2TLCq - ROFWA_p(\bar{\zeta}_1, \bar{\zeta}_2, \ldots, \bar{\zeta}_n) = \bar{\zeta}.
\]

2. Boundedness. Let $\bar{\zeta}_e (e = 1, 2, \ldots, n)$ be a collection of 2TLCq-ROFNs, and let $\bar{\zeta} = \min \bar{\zeta}_e$ and $\bar{\zeta}' = \max \bar{\zeta}_e$; then,

\[
\bar{\zeta} \leq 2TLCq - ROFWA_p(\bar{\zeta}_1, \bar{\zeta}_2, \ldots, \bar{\zeta}_n) \leq \bar{\zeta}'.
\]

3. Monotonicity. Let $\bar{\zeta}_e$ and $\bar{\zeta}'_e (e = 1, 2, \ldots, n)$ be two collections of 2TLCq-ROFNs, if $\bar{\zeta}_e \leq \bar{\zeta}'_e$, for all e; then,

\[
2TLCq - ROFWA_p(\bar{\zeta}_1, \bar{\zeta}_2, \ldots, \bar{\zeta}_n) \leq 2TLCq - ROFWA_p(\bar{\zeta}'_1, \bar{\zeta}'_2, \ldots, \bar{\zeta}'_n).
\]
Definition 12. Let \( \bar{\zeta} = (((s_{P_{c}}, \varphi_{c}), (\omega_{(s_{i_{e}}, \varphi_{i_{e}})}), (s_{e}, \mathcal{R}_{e}), (\omega_{(s_{e}, \mathcal{R}_{e})}))(e = 1, 2, \ldots, n) \) be a collection of 2TLC\( q\)-ROFHs, then the 2TLC\( q\)-ROFHOWA operator is defined as

\[
2\text{TLC}_{q} - \text{ROFHOWA} A_{p}(\bar{\zeta}_{1}, \bar{\zeta}_{2}, \ldots, \bar{\zeta}_{n}) = \Phi_{e = 1}^{n}(\psi e\bar{\zeta}(e)),
\]

where \((b(1), b(2), \ldots, b(n))\) is a permutation of \((1, 2, \ldots, n)\), such that \( \bar{\zeta}_{e}(e-1) \geq \bar{\zeta}_{e}(e) \) for all \( e = 2, \ldots, n \), and \( \psi = (\psi_{1}, \psi_{2}, \ldots, \psi_{n})^{T} \) is the aggregation-associated weight vector such that \( \psi_{e} \in [0, 1] \) and \( \sum_{e=1}^{n} \psi_{e} = 1, q > 0 \).

The 2TLC\( q\)-ROFHOWA operator has the same properties as Theorem 2.

Definition 13. Let \( \bar{\zeta} = (((s_{P_{c}}, \varphi_{c}), (\omega_{(s_{i_{e}}, \varphi_{i_{e}})}), (s_{e}, \mathcal{R}_{e}), (\omega_{(s_{e}, \mathcal{R}_{e})}))(e = 1, 2, \ldots, n) \) be a collection of 2TLC\( q\)-ROFHs, then the 2TLC\( q\)-ROFHHHA operator is defined as

\[
2\text{TLC}_{q} - \text{ROFHHHA} A_{p}(\bar{\zeta}_{1}, \bar{\zeta}_{2}, \ldots, \bar{\zeta}_{n}) = \Phi_{e = 1}^{n}(\psi e\bar{\zeta}(e)),
\]

where \( \psi = (\psi_{1}, \psi_{2}, \ldots, \psi_{n}) \) is the associated weighting vector such that \( \psi_{e} \in [0, 1] \) and \( \sum_{e=1}^{n} \psi_{e} = 1, \) and \( \bar{\zeta}_{e}(e) \) is the \( e \)-th largest element of 2TLC\( q\)-ROF arguments \( \bar{\zeta}_{e}(e) = (n_{p_{e}}, \bar{\zeta}_{e}, e = 1, 2, \ldots, n) \).

Theorem 3. Let \( \bar{\zeta} = (((s_{P_{c}}, \varphi_{c}), (\omega_{(s_{i_{e}}, \varphi_{i_{e}})}), (s_{e}, \mathcal{R}_{e}), (\omega_{(s_{e}, \mathcal{R}_{e})}))(e = 1, 2, \ldots, n) \) be a collection of 2TLC\( q\)-ROFHs, then for any \( \varphi > 0 \), the aggregated value by utilizing 2TLC\( q\)-ROFHOWA operator is also a 2TLC\( q\)-ROF, and
where \( \psi = (\psi_1, \psi_2, \ldots, \psi_n) \) is the aggregation-associated weighting vector such that \( \psi_e \in [0, 1] \) and \( \sum_{e=1}^{n} \psi_e = 1 \), and \( \tau_{(e)} \) is the e-th largest element of 2TLCq-ROF arguments \( \tau_{(e)}(\xi_e) = (n\varphi_e \xi_e, \epsilon = 1, 2, \ldots, n) \), \( \varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n) \) is the weighting vector of 2TLCq-ROF arguments \( \xi_e \), with \( \varphi_e \in [0, 1] \) and \( \sum_{e=1}^{n} \varphi_e = 1 \), and \( n \) is the balancing coefficient.

The 2TLCq-ROFHHHA operator has the same properties as Theorem 2.

**Definition 14.** Let \( \xi = ((s_{p_e}, \varphi_e), (\omega_{(s_{p_e}, \varphi_e)}), ((s_{r_e}, \varpsilon_e), (\omega_{(s_{r_e}, \varpsilon_e)}))) (e = 1, 2, \ldots, n) \) be a collection of 2TLCq-ROFNs; then, the 2TLCq-ROFHWG operator is defined as

\[
2TLCq - ROFHWG_{\varphi_e}(\xi_1, \xi_2, \ldots, \xi_n) = \omega_{(\xi_1)}^{\varphi_e},
\]

where \( \varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n) \) be the weight vector of \( \xi_e (e = 1, 2, \ldots, n) \), and \( \varphi_e > 0 \), \( \sum_{e=1}^{n} \varphi_e = 1, \varphi > 0 \).

Based on the parameter \( \varphi \), we can derive the following special cases of Theorem 5.
Remark 3. When \( q = 1 \), the \( 2TLC_q \)-ROFHWG operator transforms to the \( 2TLC_q \)-ROF weighted geometric (\( 2TLC_q \)-ROFWEWG) operator as follows:

\[
2TLC_q - ROFWEWG_q(\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n) = \left( \Delta \left( \tau \left( \sum_{i=1}^{n} \left( \Delta^{-1}(s_{i}, \varphi_q) / \tau \right)^{\psi_q} \right) \right) \right)
\]

Remark 4. When \( q = 2 \), the \( 2TLC_q \)-ROFHWG operator reduces to the \( 2TLC_q \)-ROF Einstein weighted geometric (\( 2TLC_q \)-ROFEWG) operator as follows:

\[
2TLC_q - ROFEWG_q(\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n) = \left( \Delta \left( \tau \left( \sum_{i=1}^{n} \left( \Delta^{-1}(s_{i}, \varphi_q) / \tau \right)^{\psi_q} \right) \right) \right)
\]

Theorem 6. Let \( \tilde{c}_e = ((s_{p_{e}}, \varphi_e), (\omega_{s_{p_{e}}, \varphi_{e}})), (s_{r_{e}}, \varphi_{e}), (\omega_{s_{r_{e}}, \varphi_{e}})) \) and \( \tilde{c}_e' = ((s_{p_{e}'}, \varphi_{e}'), (\omega_{s_{p_{e}'}, \varphi_{e}'})), (s_{r_{e}'}, \varphi_{e}'), (\omega_{s_{r_{e}'}, \varphi_{e}'})) \) \( (e = 1, 2, \ldots, n) \) be two sets of \( 2TLC_q \)-ROFNS; then, the \( 2TLC_q \)-ROFHGW operator has the following properties:

1. Idempotency. If all \( 2TLC_q \)-ROFNS \( \tilde{c}_e = ((s_{p_{e}}, \varphi_{e}), (\omega_{s_{p_{e}}, \varphi_{e}})), (s_{r_{e}}, \varphi_{e}), (\omega_{s_{r_{e}}, \varphi_{e}})) \) \( (e = 1, 2, \ldots, n) \) are equal, then \( \tilde{c}_e \) \( \tilde{c}_e'. \) Then,

\[
2TLC_q - ROFHGW_q(\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n) = \tilde{c}_e. \quad (37)
\]

2. Boundedness. Let \( \tilde{c}_e (e = 1, 2, \ldots, n) \) be a collection of \( 2TLC_q \)-ROFNS, and let \( \tilde{c}_e' = \min \tilde{c}_e (e) \) and \( \tilde{c}_e'' = \max \tilde{c}_e (e) \); then,

\[
\tilde{c}_e' \leq 2TLC_q - ROFHGW_q(\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n) \leq \tilde{c}_e''. \quad (38)
\]

3. Monotonicity. Let \( \tilde{c}_e \) \( (e = 1, 2, \ldots, n) \) be the two collections of \( 2TLC_q \)-ROFNS, if \( \tilde{c}_e \leq \tilde{c}_e' \) for all \( e \); then,

\[
2TLC_q - ROFHGW_q(\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n) \leq 2TLC_q - ROFHGW_q(\tilde{c}_1', \tilde{c}_2', \ldots, \tilde{c}_n'). \quad (39)
\]
Definition 15. Let \( \vec{c} = (((s_{\phi_1}, \varphi_1)), ((s_{\phi_2}, \varphi_2)), ((s_{\phi_n}, \varphi_n)), (\omega_{(s_{\phi_n}, \varphi_n)}))(e = 1, 2, \ldots, n) \) be a collection of 2 TLCq-ROFNs, then the 2 TLCq-ROFHOG operator is defined as

\[
2\text{TLC}_q - \text{ROFHOG}_\psi(\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n) = \phi_{e=1}^n(\vec{c}_{(e)})^{\psi_e},
\]

where \((b(1), b(2), \ldots, b(n))\) is a permutation of \((1, 2, \ldots, n)\), such that \(\vec{c}_{(e-1)} \geq \vec{c}_{(e)}\) for all \(e = 2, \ldots, n\), and \(\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T\) is the aggregation-associated weighted vector such that \(\psi_e \in [0, 1]\) and \(\sum_{e=1}^n \psi_e = 1\).

\[
\Psi(\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n) = \phi_{e=1}^n(\vec{c}_{(e)})^{\psi_e},
\]

where \((b(1), b(2), \ldots, b(n))\) is a permutation of \((1, 2, \ldots, n)\), such that \(\vec{c}_{(e-1)} \geq \vec{c}_{(e)}\) for all \(e = 2, \ldots, n\), and \(\Psi = (\psi_1, \psi_2, \ldots, \psi_n)^T\) is the aggregation-associated weighted vector such that \(\psi_e \in [0, 1]\) and \(\sum_{e=1}^n \psi_e = 1\).

The 2 TLCq-ROFHOG operator has the same properties as Theorem 2.

Definition 16. Let \( \vec{c} = (((s_{\phi_1}, \varphi_1)), ((s_{\phi_2}, \varphi_2)), ((s_{\phi_n}, \varphi_n)), (\omega_{(s_{\phi_n}, \varphi_n)}))(e = 1, 2, \ldots, n) \) be a collection of 2 TLCq-ROFNs; then, the 2 TLCq-ROFHOG operator is defined as

\[
2\text{TLC}_q - \text{ROFHOG}_\psi(\vec{c}_1, \vec{c}_2, \ldots, \vec{c}_n) = \phi_{e=1}^n(\vec{c}_{(e)})^{\psi_e},
\]

where \(\psi = (\psi_1, \psi_2, \ldots, \psi_n)\) be the associated weighting vector with \(\psi_e \in [0, 1]\), \(\sum_{e=1}^n \psi_e = 1\), and \(\vec{c}_{(e)}\) is the \(e\)-th largest element of 2 TLCq-ROF arguments \(\vec{c}_e(\vec{c}_e = (n\varphi), \vec{c}_1, \psi_1, \psi_2, \ldots, \psi_n)^T\) is the aggregation-associated weighted vector such that \(\psi_e \in [0, 1]\) and \(\sum_{e=1}^n \psi_e = 1\).

Theorem 7. Let \( \vec{c} = (((s_{\phi_1}, \varphi_1)), ((s_{\phi_2}, \varphi_2)), ((s_{\phi_n}, \varphi_n)), (\omega_{(s_{\phi_n}, \varphi_n)}))(e = 1, 2, \ldots, n)\) be a collection of 2 TLCq-ROFNs, where \(\varphi > 0\). Then, its aggregated value by utilizing the 2 TLCq-ROFHOG operator is also a 2 TLCq-ROFN, and

\[
\phi_{e=1}^n(\vec{c}_{(e)})^{\psi_e},
\]

where \((b(1), b(2), \ldots, b(n))\) is a permutation of \((1, 2, \ldots, n)\), such that \(\vec{c}_{(e-1)} \geq \vec{c}_{(e)}\) for all \(e = 2, \ldots, n\), and \(\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T\) is the aggregation-associated weighted vector such that \(\psi_e \in [0, 1]\) and \(\sum_{e=1}^n \psi_e = 1\).
where \( \psi = (\psi_1, \psi_2, \ldots, \psi_n) \) be the associated weighting vector with \( \psi_e \in [0, 1] \), \( \sum_{e=1}^{n} \psi_{e} = 1 \), and \( \hat{\xi}_{(c)} \) is the \( e \)-th largest element of 2TLCq-ROF arguments \( \xi (\xi = (\xi)_{n}) \) \( (e = 1, 2, \ldots, n) \), \( \varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n) \) is the weighting vector of 2TLCq-ROF arguments \( \xi (\xi = (\xi)_{n}) \) \( (e = 0, 1 \) and \( \sum_{e=1}^{n} \psi_{e} = 1 \), and \( n \) is the balancing coefficient, \( \varphi > 0 \).

The 2TLCq-ROFHHG operator has the same properties as Theorem 2.

5. An Extended COPRAS Method for MAGDM Approach

In this section, the 2TLCq-ROFHW and 2TLCq-ROFHWG operators are used to integrate the evaluation values of the 2TLCq-ROF-MAGDM problem and develop a ranking procedure based on the COPRAS method for the 2TLCq-ROF-MAGDM problem. Firstly, we demonstrate the MAGDM problem with the 2TLCq-ROF. For the 2TLCq-ROF-MAGDM problem, we build an extended COPRAS method, and then, with the assistance of the 2TLCq-ROFHW and 2TLCq-ROFHWG operators, we fuse the individual input arguments into a combined viewpoint and also describe the DM algorithm.

5.1. An Extended COPRAS Method with 2TLCq-ROF Aggregation Operators. In this section, we will extend COPRAS method to solve a MAGDM problem within the 2TLCq-ROF environment. According to the work of Zhang [46], group decision-making problems can be solved from two angles: (1) aggregation stage and (2) exploitation stage.

In the aggregation stage, collective evaluation values are obtained from the individual evaluation values of the alternatives. Therefore, we employ the 2TLCq-ROFHW and 2TLCq-ROFHWG operators to combine the individual decision matrices into a group decision matrix. In the exploitation stage, the best alternative(s) is selected according to the priorities of the cumulative evaluation values. We will develop an extended COPRAS method based on the 2TLCq-ROF aggregation operator, named the 2TLCq-ROF-COPRAS method, to tackle the information in the group decision matrix.

By utilizing the 2TLCq-ROFHW operator, the overall value of alternative \( \xi_{e} \) based on attributes \( h_{e} \) is calculated, the result is computed as follows.

Phase 1. Establish the attributes as well as the alternatives.

The goal of the MADM process is to choose the best alternative from a set of \( m \) alternatives \( \xi \) \( (\xi = (\xi)_{m}) \) under the attributes set \( h = \{h_1, h_2, \ldots, h_n\} \). Assume a group of DESs appointed to serve on a panel \( E = \{e_1, e_2, \ldots, e_e\} \), which was formulated in order to find the optimal alternative(s). Let \( \xi = (\xi)_{e} \), \( e = 1(1)m \) be the linguistic decision matrix provided by the DESs, where \( \xi_{e} \) shows the assessed values of an alternative \( \xi_{e} \) over attributes \( h_{e} \) in the form of linguistic values for \( \lambda^{th} \) the expert.
Phase 2. Construct the aggregated 2TLCq-ROF decision matrix.

We stimulate the 2TLCq-ROFHWA and 2TLCq-ROFHWG operators to obtain the decision matrix subsequently and achieve \( R = (\eta_{x_{\kappa}})_{\kappa \in \Lambda} \) from (20) and (34).

Phase 3. Calculate the assessment values of the favorable-type and nonfavorable-type attributes.

Each alternative is defined throughout the designed model in terms of its total of maxima \( \bar{\beta}_x \) (favorable-type) and minima \( \tilde{\beta}_x \) (nonfavorable-type); that is, maxima and minima, respectively, produce the optimal outcomes. In such circumstances, \( \bar{\beta}_x \) and \( \tilde{\beta}_x \) can be obtained as described below.

Let \( \Delta = \{1, 2, \ldots, \ell\} \) be a favorable-type attribute. Afterward, for every alternative, we assess the index value in contexts of 2TLCq-ROFNs, as follows:

\[
\tilde{\alpha}_x = \Phi_{\kappa = 1}^{\ell} \Phi_{\kappa \in \Delta} \eta_{x_{\kappa}}, \quad \kappa = 1 (1) \ell. \tag{44}
\]

Let \( V = \{1 + 1, 1 + 2, \ldots, n\} \) be a nonfavorable-type attribute. Afterward, for every alternative, we assess the index value in contexts of 2TLCq-ROFNs as follows:

\[
\tilde{\beta}_x = \Phi_{\kappa = 1}^{n} \Phi_{\kappa \in V} \eta_{x_{\kappa}}, \quad \kappa = 1 (1) n, \tag{45}
\]

where \( I \) represents favorable types and \( n \) represents the attributes.

Phase 4. Furthermore, we calculate the relative degree \( \Gamma_x \) of each alternative \( Z_x, (\kappa = 1 (1) m) \). Obviously, the bigger the value of \( \Gamma_x \), the higher the importance of the alternative. \( \Gamma_x \) can be obtained as follows:

\[
\Gamma_x = \frac{\min_{\kappa} \tilde{\beta}_x \cdot \sum_{\kappa = 1}^{\ell} F(\tilde{\beta}_x)}{F(\tilde{\beta}_x) \cdot \sum_{\kappa = 1}^{\ell} \min_{\kappa} F(\tilde{\beta}_x)} = \frac{\tilde{\beta}_x}{F(\tilde{\beta}_x)}, \quad \kappa = 1 (1) \ell, \tag{46}
\]

where \( F(\tilde{\beta}_x) \) is the score value of \( \tilde{\alpha}_x \) and \( F(\tilde{\beta}_x) \) is the score value of \( \tilde{\beta}_x \).

Equation (46) can be simplified as

\[
\Gamma_x = \frac{\sum_{\kappa = 1}^{\ell} F(\tilde{\beta}_x)}{F(\tilde{\beta}_x) \cdot \sum_{\kappa = 1}^{\ell} 1/F(\tilde{\beta}_x)}, \quad \kappa = 1 (1) \ell. \tag{47}
\]

\( \Gamma_x \) from (47) reflects the satisfaction measure of each alternative. Based on the \( \Gamma_x \), maximal value \( F \) can be determined.

Phase 5. Calculate the summary of priority.

\[
F = \max_{x} \Gamma_x, \kappa = 1 (1) \ell. \tag{48}
\]

Thus, the alternative(s) with the associated maximal relative degree is selected among the possible alternatives. Moreover, we can ascertain the utility degree \( \mathcal{U}_x \) of each alternative with the aid of the \( \Gamma_x \). \( \mathcal{U}_x \) can be determined by using the following formula:

\[
\mathcal{U}_x = \left( \frac{\Gamma_x}{\Gamma_{\text{max}}} \right) \times 100\%, \kappa = 1 (1) \ell. \tag{49}
\]

Hence, the bigger the value \( \mathcal{U}_x \), the higher the rank of the alternative \( Z_x \).

6. Numerical Example

In this part, we present a numerical example to evaluate how well our strategy works. With the increased reliance on technology, it is becoming increasingly important to protect all aspects of Internet information and data. Data integrity has become one of the most critical issues for enterprises to address, as the Internet and computer networks increase over time. No matter how little or large your enterprises is, network security is one of the most crucial factors to consider while working over the Internet, LAN, or other technology. While no network is immune to cyber threats, a solid and effective network security solution is critical for securing client data. An effective network security solution minimizes the danger of data theft and tampering in the workplace. Workstations will be protected from malicious software, thanks to network security service provider. It also assures the safety of shared information. In this section, we illustrate the application of 2TLCq-ROF COPRAS method on the choice of network security service provider.

By the above analysis, let \( L = \{1, 2, 3, 4, 5, 6, 7\} \) be a set of seven network security service providers (see Table 1) and let \( A = \{h_1, h_2, h_3, h_4\} \) be a set of four attributes with weighting vector \( \xi = (0.17, 0.31, 0.27, 0.25)^T \). Suppose, seven network security service providers are evaluated by three experts \( E = \{e_1, e_2, e_3\} \), with weighting vector \( \varphi = (0.2, 0.5, 0.3)^T \) for choosing the best network security service provider. To quantify each LTS, three experts provide their opinions. Based on their experience, each decision expert has an opinion for the selection of the best network security service provider according to four attributes, including

(1) \( h_1 \): web security
(2) \( h_2 \): data loss prevention
(3) \( h_3 \): antivirus and anti-malware software
(4) \( h_4 \): mobile device security

Experts should evaluate the effectiveness of network security service provider concerning all attributes in

| Network security service providers (alternatives) | \( \mathcal{U}_x \) |
|-----------------------------------------------|--------|
| Delta tech                                    |         |
| Catalytic security                            |         |
| PakCERT                                       |         |
| Tranchulas                                    |         |
| Tier 3 cyber security services                |         |
| Cyber security consultancy company            |         |
| Institute of cyber security                   |         |
accordance with their interaction and identify the most suitable one. Each decision expert uses the 2TLCq-ROFNs to assess each network security service provider's ability to control each attribute.

6.1. The Outcomes of a Case Study

6.1.1. Decision-Making Procedure Based on the 2TLCq-ROFHW A Operator. On the basis of 2TLCq-ROFNs matrix
(see Tables 2–4) and by utilizing (34), the collective 2TLC-ROF-COPRAS method.

Table 6: Assessing values of favorable- and nonfavorable-type attributes by the 2TLCq-ROFHWA operator.

| Alternatives | Assessed values of favorable attributes | Assessed values of nonfavorable attributes |
|--------------|----------------------------------------|-----------------------------------------|
| 21           | ((s1, −0.0223), (s2, 0.0357), (s3, 0.4604), (s4, −0.4488)) | ((s1, −0.4918), (s2, 0.1659), (s3, −0.4243), (s4, −0.1948)) |
| 22           | ((s1, −0.1058), (s2, −0.1778), (s3, 0.4078), (s4, −0.0561)) | ((s1, 0.3583), (s2, 0.0065), (s3, 0.3236), (s4, −0.1917)) |
| 23           | ((s1, −0.2054), (s2, 0.1860), (s3, −0.3941), (s4, 0.0175)) | ((s1, −0.4656), (s2, −0.1525), (s3, 0.2148), (s4, −0.0104)) |
| 24           | ((s1, −0.1086), (s2, −0.4202), (s3, 0.3485), (s4, −0.2301)) | ((s1, −0.3919), (s2, 0.1491), (s3, −0.3007), (s4, 0.2438)) |
| 25           | ((s1, −0.1620), (s2, 0.3348), (s3, 0.0219), (s4, −0.4349)) | ((s1, −0.3433), (s2, 0.2963), (s3, 0.4429), (s4, −0.3404)) |
| 26           | ((s1, −0.3514), (s2, 0.3253), (s3, 0.1586), (s4, 0.2394)) | ((s1, −0.3057), (s2, −0.0024), (s3, 0.2002), (s4, 0.2980)) |
| 27           | ((s1, −0.3787), (s2, 0.2501), (s3, 0.2304), (s4, −0.2224)) | ((s1, 0.1682), (s2, 0.0718), (s3, 0.2776), (s4, −0.0865)) |

Table 7: Assessing outcomes of alternatives and ranking order by utilizing the 2TLCq-ROF-COPRAS.

| Alternatives | Benefit attributes scores \( F(\alpha_s) \) | Cost attributes scores \( F(\beta_s) \) | Relative degree \( \Gamma_e \) | Utility degree \( \mathcal{U}_e \) | Ranking |
|--------------|--------------------------------|-----------------|-----------------|-----------------|--------|
| 21           | 0.4083                          | 0.3871          | 0.8749          | 1.0000          | 1      |
| 22           | 0.3291                          | 0.4362          | 0.7432          | 0.8494          | 4      |
| 23           | 0.3737                          | 0.4236          | 0.8002          | 0.9146          | 3      |
| 24           | 0.3894                          | 0.3938          | 0.8481          | 0.9693          | 7      |
| 25           | 0.3727                          | 0.4656          | 0.7607          | 0.8695          | 5      |
| 26           | 0.3380                          | 0.4343          | 0.7540          | 0.8617          | 6      |
| 27           | 0.3521                          | 0.4400          | 0.7626          | 0.8716          | 2      |

Table 8: Collective 2TLCq-ROF assessing matrix by the 2TLCq-ROFHWG operator.

| Alternatives | Collective assessment matrix |
|--------------|-----------------------------|
| 21           | ((s1, −0.1510), (s2, −0.2137)) | ((s1, 0.2476), (s2, 0.3791)) |
| 22           | ((s1, −0.2581), (s2, 0.3599), (s3, 0.1984), (s4, −0.2088)) | ((s1, −0.1147), (s2, −0.2137), (s3, −0.2766), (s4, 0.1377)) |
| 23           | ((s1, 0.3691), (s2, −0), (s3, 0.3791), (s4, −0.0669)) | ((s1, 0.2596), (s2, −0.4478), (s3, 0.2507), (s4, 0.2600)) |
| 24           | ((s1, 0.3533), (s2, 0.2588), (s3, 0.3572), (s4, 0.3951)) | ((s1, −0.0989), (s2, 0.4961), (s3, −0.0669), (s4, −0.2770)) |
| 25           | ((s1, 0.1438), (s2, 0.0278), (s3, 0.2084), (s4, 0.0368)) | ((s1, 0.0278), (s2, 0.1438), (s3, 0.0368), (s4, 0.2084)) |
| 26           | ((s1, −0.4227), (s2, 0.1313), (s3, −0.4991), (s4, 0.3488)) | ((s1, 0.1313), (s2, −0.4227), (s3, 0.3488), (s4, −0.4991)) |
| 27           | ((s1, −0.3423), (s2, −0.2137), (s3, −0.2419), (s4, −0.1528)) | ((s1, −0.4655), (s2, −0.3273), (s3, −0.3231), (s4, 0.1377)) |

Construct the assessing matrix (see Table 6) of favorable- and nonfavorable-type attributes by utilizing the (15) and (44), and (45).

Calculate the scoring outcomes of alternatives for favorable \( (F(\bar{\alpha}_s)) \)- and nonfavorable \( (F(\bar{\beta}_s)) \)-type attributes by utilizing (12), and establish the ranking order by using the 2TLCq-ROF-COPRAS method. The evaluation outcomes are listed in Table 7.

6.1.2. Decision-Making Procedure Based on the 2TLCq-ROFHWG Operator. On the basis of 2TLCq-ROFN matrix (see Tables 2–4) and by utilizing (34), the collective 2TLCq-ROF assessing matrix is computed. The aggregated outcomes are listed in Table 8.

Construct the assessing matrix (see Table 9) of favorable- and nonfavorable-type attributes by utilizing the (34) and (44), and (45).

Calculate the scoring outcomes of alternatives for favorable \( (F(\bar{\alpha}_s)) \)- and nonfavorable \( (F(\bar{\beta}_s)) \)-type attributes by utilizing (12) and establish the ranking order by using the 2TLCq-ROF-COPRAS method. The evaluation outcomes are listed in Table 10.

6.2. Parameter Analysis. Surely, the parameter \( q \) has a great influence on the ranking results. The influence of parameters on score values and ranking results based on 2TLCq-
Table 9: Assessing values of favorable- and nonfavorable-type attributes by the 2TLC$q$-ROFHWG operator.

| Alternatives | Assessed values of favorable attributes | Assessed values of nonfavorable attributes |
|--------------|----------------------------------------|-------------------------------------------|
| $\gamma_1$   | $\langle(c_s, 0.2971), (s_s, -0.0599)\rangle$ | $\langle(s_s, 0.3090), (s_s, -0.0370)\rangle$ |
| $\gamma_2$   | $\langle(c_s, 0.1244), (s_s, -0.2031)\rangle$ | $\langle(s_s, 0.3839), (s_s, -0.0397)\rangle$ |
| $\gamma_3$   | $\langle(c_s, 0.1372), (s_s, -0.4356)\rangle$ | $\langle(s_s, -0.4634), (s_s, -0.1616)\rangle$ |
| $\gamma_4$   | $\langle(c_s, 0.0442), (s_s, -0.2335)\rangle$ | $\langle(s_s, 0.4866), (s_s, -0.3619)\rangle$ |
| $\gamma_5$   | $\langle(c_s, -0.0879), (s_s, -0.0028)\rangle$ | $\langle(s_s, -0.4945), (s_s, 0.0610)\rangle$ |
| $\gamma_6$   | $\langle(c_s, -0.3032), (s_s, -0.4044)\rangle$ | $\langle(s_s, -0.3116), (s_s, -0.3189)\rangle$ |
| $\gamma_7$   | $\langle(c_s, 0.3865), (s_s, 0.2536)\rangle$ | $\langle(s_s, -0.2756), (s_s, -0.4764)\rangle$ |

Table 10: Assessing outcomes of alternatives and ranking order by utilizing the 2TLC$q$-ROF-COPRAS.

| Alternatives | Benefit attributes scores | Cost attributes scores | Relative degree $\Gamma_k$ | Utility degree $\Upsilon_k$ | Ranking |
|--------------|---------------------------|------------------------|---------------------------|---------------------------|---------|
| $\gamma_1$   | 0.6620                    | 0.5227                 | 1.2354                    | 1.0000                    | 1       |
| $\gamma_2$   | 0.5917                    | 0.5699                 | 1.1176                    | 0.9047                    | 3       |
| $\gamma_3$   | 0.6344                    | 0.5344                 | 1.1953                    | 0.9675                    | 4       |
| $\gamma_4$   | 0.6447                    | 0.5467                 | 1.1930                    | 0.9656                    | 6       |
| $\gamma_5$   | 0.5988                    | 0.5530                 | 1.1407                    | 0.9234                    | 7       |
| $\gamma_6$   | 0.6047                    | 0.5336                 | 1.1665                    | 0.9441                    | 5       |
| $\gamma_7$   | 0.6349                    | 0.5739                 | 1.1571                    | 0.9366                    | 2       |

Table 11: Score values and ranking outcomes according to the parameter $q$ by the 2TLC$q$-ROFHWHA operator.

| Parameters | $g(\beta_1)$ | $g(\beta_2)$ | $g(\beta_3)$ | $g(\beta_4)$ | $g(\beta_5)$ | $g(\beta_6)$ | $g(\beta_7)$ | Ranking |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|---------|
| $q = 2$    | 1.0000       | 0.7877       | 0.8865       | 0.9286       | 0.8096       | 0.8246       | 0.8228       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 3$    | 1.0000       | 0.8177       | 0.8975       | 0.9513       | 0.8371       | 0.8394       | 0.8457       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 5$    | 1.0000       | 0.8783       | 0.9323       | 0.9819       | 0.8096       | 0.8855       | 0.8962       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 7$    | 1.0000       | 0.9236       | 0.9615       | 0.9950       | 0.9445       | 0.9278       | 0.9359       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 11$   | 0.9999       | 0.9712       | 0.9901       | 1.0000       | 0.9845       | 0.8246       | 0.9776       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 13$   | 1.0000       | 0.9821       | 0.9953       | 0.9999       | 0.9916       | 0.9866       | 0.9865       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 17$   | 1.0000       | 0.9924       | 0.9990       | 0.9993       | 0.9970       | 0.9955       | 0.9945       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 19$   | 1.0000       | 0.9948       | 0.9996       | 0.9993       | 0.9981       | 0.9973       | 0.9962       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 23$   | 1.0000       | 0.9973       | 0.9999       | 0.9994       | 0.9991       | 0.9988       | 0.9981       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |
| $q = 29$   | 1.0000       | 0.7877       | 0.8865       | 0.9286       | 0.8096       | 0.8246       | 0.8228       | $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4 > \gamma_5 > \gamma_6 > \gamma_7$ |

ROFHWA and 2TLC$q$-ROFHWGA operators are evaluated in this subsection. We fix the several values of $q$ and evaluate the scores of the overall aggregation. Furthermore, scores are used to rank the alternatives. In Tables 11 and 12 score values are evaluated by varying $q$ and $\varphi$, respectively, based on the 2TLC$q$-ROFHWGA operator. Then, these scores are used to rank the alternatives. Ranking results are used to select the best alternative, and $\gamma_1$ is the best alternative based on the 2TLC$q$-ROFHWGA operator. As the values of $q$ and $\varphi$ vary, the scores of the seven alternatives change as well, resulting in an irregular change accordingly, based on the 2TLC$q$-ROFHWGA operator. The change in values of $q$ and $\varphi$ have a significant influence on the results of the alternative ranking. Tables 11 and 12 demonstrate that when $q$ and $\varphi$ are changed, the ranking results are relatively stable, and the best alternative remained unchanged. The decision preference can be represented in the actual DM process by varying the values of $q$ and $\varphi$ to obtain the best decision result.

We fix the several values of $q$ and $\varphi$ and evaluate the scores of the overall aggregation. Furthermore, scores are used to rank the alternatives. In Tables 13 and 14 score values are evaluated by varying $q$ and $\varphi$, respectively, based on the 2TLC$q$-ROFHWGA operator. Then, these scores are used to rank the alternatives. The ranking results are used to select the best alternative, and $\gamma_1$ is the best alternative based on the 2TLC$q$-ROFHWGA operator. As the values of $q$ and $\varphi$ vary, the scores of the seven alternatives change as well, resulting in an irregular change accordingly, based on the 2TLC$q$-ROFHWGA operator. The change in values of $q$ and $\varphi$ has a significant influence on the results of the alternative
6.3. Comparative Analysis. In this subsection, we compare our proposed work with existing work to demonstrate its reliability and efficiency. Basically, we fused the concept of 2TLq-ROFS with the concept of the complex set to present HWA, HOWA, HHA, HWG, HOWG, and HHG in the context of 2TLq-ROFS. In real-world problems, the 2TLq-ROFS effectively deals with uncertain and linguistic information. Determining the weight of the criteria is seen as an important aspect of dealing with the MAGDM challenges. Different criteria may have different weights, and different weights of criteria may provide different results. It is challenging for experts to obtain accurate and objective weight values from real-world data. Because the method of obtaining weight values is complicated, the experts’ knowledge and biases may impact their decisions. As a result, as an objective method, the COPRAS method is an excellent solution for addressing these problems. The COPRAS method has been investigated extensively in different MAGDM issues. We use various MAGDM strategies to address the network security service provider selection problem to verify the feasibility and superiority of our proposed method. To synthesize the individual assessments of the DMs, we use the 2TLq-ROFHWA and 2TLq-
ROFHWG operators. Comparative analysis with different FSs (2TLCIFH, 2TLCPFH, 2TLCFFH), aggregation operators (2TLC$q$-ROFWA, 2TLC$q$-ROFEWA, 2TLC$q$-ROFWG, 2TLC$q$-ROFEWG), and methods (CODAS, EDAS, TOPSIS) is illustrated as follows.

6.3.1. Comparative Analysis with Different Aggregation Operators. Our proposed operator is more realistic and superior because it can evaluate the interrelationship of fused arguments and scientifically consider the complexities of humans in practical MAGDM problems, whereas the 2TLC$q$-ROFWA (2TLC$q$-ROFWG) and 2TLC$q$-ROFEWA (2TLC$q$-ROFEWG) operators are unable to evaluate the interrelationships between fused arguments and cannot consider the complexity of DMs. As a result, proposed operators are more general in expressing fuzzy information.

### Table 15: Evaluation outcomes by utilizing different aggregation operators based on the 2TLC$q$-ROFHWA operator.

| Alternatives | 2TLC$q$-ROFH | Ranking | 2TLC$q$-ROFWA | Ranking | 2TLC$q$-ROFEWA | Ranking |
|--------------|--------------|---------|----------------|---------|----------------|---------|
| ℷ₁            | 1.0000       | I       | 1.0000         | I       | 1.0000         | I       |
| ℷ₂            | 0.8494       | IV      | 0.8771         | IV      | 0.8607         | IV      |
| ℷ₃            | 0.9146       | III     | 0.9413         | III     | 0.9254         | III     |
| ℷ₄            | 0.9693       | VII     | 0.9724         | V       | 0.9709         | V       |
| ℷ₅            | 0.8695       | V       | 0.9006         | VII     | 0.8822         | VII     |
| ℷ₆            | 0.8617       | VI      | 0.8925         | VI      | 0.8739         | VI      |
| ℷ₇            | 0.8716       | II      | 0.8970         | II      | 0.8820         | II      |

### Table 16: Evaluation outcomes by utilizing different aggregation operators based on the 2TLC$q$-ROFHWG operator.

| Alternatives | 2TLC$q$-ROFH | Ranking | 2TLC$q$-ROFWG | Ranking | 2TLC$q$-ROFEWG | Ranking |
|--------------|--------------|---------|----------------|---------|----------------|---------|
| ℷ₁            | 1.0000       | I       | 1.0000         | I       | 1.0000         | I       |
| ℷ₂            | 0.9047       | III     | 0.9138         | IV      | 0.9075         | IV      |
| ℷ₃            | 0.9675       | IV      | 0.9741         | III     | 0.9693         | III     |
| ℷ₄            | 0.9656       | IV      | 0.9784         | VI      | 0.9698         | VI      |
| ℷ₅            | 0.9234       | VII     | 0.9321         | VII     | 0.9260         | VII     |
| ℷ₆            | 0.9441       | V       | 0.9486         | V       | 0.9451         | V       |
| ℷ₇            | 0.9366       | II      | 0.9447         | II      | 0.9392         | II      |

### Table 17: Evaluation outcomes by utilizing different FSs based on the 2TLC$q$-ROFHWA operator.

| Alternatives | 2TLC$q$-ROFH | Ranking | 2TLCFFH | Ranking | 2TLCPFH | Ranking | 2TLCIFH | Ranking |
|--------------|--------------|---------|---------|---------|---------|---------|---------|---------|
| ℷ₁            | 1.0000       | I       | 1.0000  | I       | 1.0000  | I       | 1.0000  | I       |
| ℷ₂            | 0.8494       | IV      | 0.8177  | IV      | 0.7877  | IV      | 0.7833  | IV      |
| ℷ₃            | 0.9146       | III     | 0.8975  | III     | 0.8685  | III     | 0.8990  | III     |
| ℷ₄            | 0.9693       | VII     | 0.9513  | VII     | 0.9286  | VI      | 0.9121  | VI      |
| ℷ₅            | 0.8695       | V       | 0.8371  | VI      | 0.8096  | VII     | 0.8134  | VII     |
| ℷ₆            | 0.8617       | VI      | 0.8394  | V       | 0.8246  | V       | 0.8387  | V       |
| ℷ₇            | 0.8716       | II      | 0.8457  | II      | 0.8228  | II      | 0.8252  | II      |

### Table 18: Evaluation outcomes by utilizing different FSs based on the 2TLC$q$-ROFHWG operator.

| Alternatives | 2TLC$q$-ROFH | Ranking | 2TLCFFH | Ranking | 2TLCPFH | Ranking | 2TLCIFH | Ranking |
|--------------|--------------|---------|---------|---------|---------|---------|---------|---------|
| ℷ₁            | 1.0000       | I       | 1.0000  | I       | 1.0000  | I       | 1.0000  | I       |
| ℷ₂            | 0.9047       | III     | 0.8984  | IV      | 0.9017  | IV      | 0.9171  | IV      |
| ℷ₃            | 0.9675       | IV      | 0.9632  | III     | 0.9616  | III     | 0.9639  | III     |
| ℷ₄            | 0.9656       | VI      | 0.9635  | VI      | 0.9661  | VI      | 0.9745  | VI      |
| ℷ₅            | 0.9234       | VII     | 0.9198  | VII     | 0.9231  | VII     | 0.9330  | VII     |
| ℷ₆            | 0.9441       | V       | 0.9410  | V       | 0.9413  | V       | 0.9464  | V       |
| ℷ₇            | 0.9366       | II      | 0.9274  | II      | 0.9254  | II      | 0.9341  | II      |
2TLC\(_q\)-ROFHWG and 2TLC\(_q\)-ROFHWA operators are highly suitable for dealing with MAGDM in a 2TLC\(_q\)-ROF environment.

### 6.3.2. Comparative Analysis with Different Fuzzy Sets.
We compared our suggested operators to the 2TLCIFH (for \( q = 1 \)), 2TLCPFH (for \( q = 2 \)), and 2TLCFFH (for \( q = 3 \)) operators in this subsection. When we set the parameter \( q = 1, 2, 3 \), we can see that the 2TLCIFH, 2TLCPFH, and 2TLCFFH operators are all special cases of our approach. Clearly, our technique can represent more fuzzy information and is applicable in a wide range of real-world MAGDM situations. Furthermore, in a complicated DM environment, the DM’s risk attitude is an important factor to consider; our method can accomplish this goal by changing the parameter \( q \), whereas 2TLCIFH, 2TLCPFH, and 2TLCFFH operators cannot dynamically adjust the parameter based on the DM’s risk attitude.

The characteristic of the proposed set is that the sum of \( q \) th power of MD and NMD is constrained to unit disc instead of real numbers in the range \([0, 1]\). As a result, our proposed work is more effective in solving MAGDM problems. The comparison of the proposed work with existing work is shown in Tables 17 and 18. Utilizing 2TLC\(_q\)-ROFH, 2TLCFFH, 2TLCPFH, and 2TLCIFH operators the optimal alternative is \( L_1 \) based on the 2TLC\(_q\)-ROFHW (2TLC\(_q\)-ROFHWA) operator.

### 6.3.3. Comparative Analysis with Different MAGDM Methods.
This subsection compares the proposed method to some existing methodologies in order to highlight the superiority of our method. To begin, our developed method is compared to CODAS, EDAS, and TOPSIS methods. We carefully calculate the results of these methods and compare them with our proposed method. For the CODAS, EDAS, and TOPSIS methods, the ranking order is \( L_6 > L_7 > L_2 > L_1 > L_3 > L_4 > L_5 \), and \( L_6 > L_7 > L_2 > L_1 > L_3 > L_4 > L_5 \), and \( L_6 > L_7 > L_2 > L_1 > L_3 > L_4 > L_5 \), respectively, based on the 2TLC\(_q\)-ROFHW operator in Table 19. Therefore, the best alternatives are \( L_6, L_7, \) and \( L_2 \) according to CODAS, EDAS, and TOPSIS methods, respectively. However, in our purposed method (COPRAS), the ranking order is \( L_1 > L_2 > L_3 > L_4 > L_5 > L_6 > L_7 \), and the best (worst) alternative is \( L_1 (L_2) \), based on the 2TLC\(_q\)-ROFHWA operator as shown in Table 19.

In Table 20 the ranking order is \( L_6 > L_7 > L_2 > L_1 > L_3 > L_4 > L_5 \), \( L_6 > L_7 > L_2 > L_1 > L_3 > L_4 > L_5 \), and \( L_6 > L_7 > L_2 > L_1 > L_3 > L_4 > L_5 \), respectively, based on the 2TLC\(_q\)-ROFHWG operator. Therefore, the best alternatives are \( L_6, L_7, \) and \( L_2 \) according to CODAS, EDAS, and TOPSIS methods, respectively. In our purposed method (COPRAS), the ranking order is \( L_1 > L_2 > L_3 > L_4 > L_5 > L_6 > L_7 \), based on the 2TLC\(_q\)-ROFHWG operator as shown in Table 20. However, the best and the worst alternative remains the same that are \( L_1 \) and \( L_2 \) that show the reliability and effectiveness of our purposed method.

### 6.4. Advantages of the Proposed Work.
Different aggregation operators perform different functions, and the decision expert can select appropriate aggregation operators based on the real-world DM situation. In this subsection, we try to express how the presented approach is superior. The merits of our proposed method are summarized as follows:

(i) The presented method is superior to other existing methods because they effectively handle the interdependence of the multi-input arguments. Moreover, they have monotonicity for the parameter \( q \) and can impact the risk perspective of the DMs. Thus, we can conclude that the proposed methods

### Table 19: Evaluation outcomes by utilizing different methodologies based on the 2TLC\(_q\)-ROFHWA operator.

| Alternatives | COPRAS Ranking | CODAS Ranking | EDAS Ranking | TOPSIS Ranking |
|--------------|----------------|---------------|--------------|----------------|
| \( L_1 \)    | 1.0000 I        | -0.0439 IV    | 0.5277 VI    | -1.3229 V      |
| \( L_2 \)    | 0.8494 IV       | 0.0339 VII    | 0.5331 VII   | -1.9971 III    |
| \( L_3 \)    | 0.9146 III      | -0.2050 II    | 0.2846 V     | -1.1370 IV     |
| \( L_4 \)    | 0.9693 VII      | -0.5126 I     | 0.0000 II    | -1.1445 I      |
| \( L_5 \)    | 0.8695 V        | -3.3930 III   | 0.5648 I     | 0.0000 II      |
| \( L_6 \)    | 0.8617 VI       | 2.7821 IV     | 0.7304 III   | -2.7610 VII    |
| \( L_7 \)    | 0.8716 II       | 1.3185 V      | 0.5820 IV    | -2.2180 VI     |

### Table 20: Evaluation outcomes by utilizing different methodologies based on the 2TLC\(_q\)-ROFHWG operator.

| Alternatives | COPRAS Ranking | CODAS Ranking | EDAS Ranking | TOPSIS Ranking |
|--------------|----------------|---------------|--------------|----------------|
| \( L_1 \)    | 1.0000 I        | -1.4587 VI    | 0.2452 VII   | -0.2422 VII    |
| \( L_2 \)    | 0.9047 III      | 1.4557 II     | 0.4219 V     | -0.5476 IV     |
| \( L_3 \)    | 0.9675 IV       | 0.8752 III    | 0.2846 IV    | -0.6408 I      |
| \( L_4 \)    | 0.9656 VI       | -1.6952 VII   | 0.4511 II    | -0.1472 II     |
| \( L_5 \)    | 0.9234 VII      | 0.4201 V      | 0.4725 III   | -0.6044 VI     |
| \( L_6 \)    | 0.9441 V        | 1.6987 I      | 0.0913 I     | -0.5873 V      |
| \( L_7 \)    | 0.9366 II       | -1.2959 IV    | 0.9024 VI    | -0.0653 III    |
are significantly preferable and have more comprehensive applications. The extended 2TLCq-ROFH-COPRAS method utilizes 2TLCq-ROFS as the information representation, and 2TLCq-ROF can provide more comprehensive assessment details as it combines the excellent aspects of the Cq-ROFS and 2TL terms. Furthermore, 2TLCq-ROFS can tackle realistic problems both quantitatively and qualitatively. Therefore, the proposed approach is clear and has less loss of data.

(ii) The 2TLCq-ROF-H-COPRAS method can provide more versatile and robust information fusion and make it more feasible to tackle risk MAGDM problems. It is based on the Hamacher operator so that the attribute’s interrelationship can be interpreted and it can be widely applied in various cases by assigning preference values with different parameter \( t \). Thus, the 2TLCq-ROFH-COPRAS method has an excellent ability to describe the interaction among multi-input parameters and is efficiently applied in the information fusion process.

7. Conclusions

In this work, we contributed to the development of MAGDM by analyzing difficulties in a 2TLCq-ROF context. The theoretical basis of aggregation operators must be carefully addressed in preparation for their use in decision-making. The inadequacies of existing methods, together with the advantageous properties of Hamacher aggregation operators, led us to evaluate their capacity to generate optimal combinations of 2TLCq-ROFNs. The following conclusions can be drawn in summary:

1. The 2TLCq-ROFS idea has been used to describe uncertain data. The Cq-ROFS is an extended version of the CIFS and CPyFS. The Cq-ROFS is defined by two functions that express the degree of complex-valued membership and nonmembership. A flexible parameter \( q \) in \( C_q \)-ROFS will influence its values to reflect information in a broader domain.

2. The 2TL terms can better reflect the human perception and \( C_q \)-ROFSs are more reliable due to the \( q \)-th power of MD and NMD. So, we developed a new concept of the 2TLCq-ROFS by incorporating the \( C_q \)-ROFS with 2TL terms.

3. We expanded the arithmetic mean, geometric mean, and hybrid operators into the 2TLCq-ROF environment and utilized Hamacher operational rules to propose six novel aggregation operators, the 2TLCq-ROFHWA, 2TLCq-ROFHOWA, 2TLCq-ROFHHA, 2TLCq-ROFHGW, 2TLCq-ROFHOWG, and 2TLCq-ROFHGG operators. Several novel characteristics of these proposed operators are being considered.

4. Additionally, we developed a novel DM technique entitled the 2TLCq-ROF-COPRAS approach for solving 2TLCq-ROF-MAGDM problems.

5. Finally, we solved a problem for selecting the best network security service provider by using our newly developed MAGDM approach.

In a nutshell, the fundamental contribution of this research is that it consolidates both the role of Hamacher aggregation operators and the favorable properties of 2TLCq-ROFNs. This model of uncertain knowledge demonstrates its versatility in presenting vague and imprecise data in complex situations. So, our proposed approach is more generic and versatile than previous approaches.

Moreover, there are some limitations to this approach that should be taken into account in future studies. Firstly, this research focuses entirely on the aggregation of the 2TLCq-ROFNs by utilizing Hamacher AOs. Future research will include a variety of assessment knowledge, such as triangular fuzzy numbers, interval-valued IF number, trapezoidal fuzzy number, and interval-valued picture fuzzy number with the integration of Hamacher operators. Therefore, this research will continue to expand. Secondly, the one parameter in our developed approach may present DMs with a quandary regarding how to choose the proper values of parameter in parameter analysis. A methodological combination with alternative techniques may therefore be justified for determining the quantities of parameters objectively. The DM approach based on the 2TLCq-ROF-COPRAS method, which have complicated assessments due to the involvement of two-step aggregation: (1) for benefit attributes and (2) for cost attributes. Therefore, the DM approach can be expanded further with other MAGDM methods, namely, MABAC, EDAS, CODAS, and TOPSIS. In addition, no details were given of the parameter estimation of the attribute weighting values. In the future, more attention may receive on the development of innovative types of AOs, DM strategies [47–51], and distance measures for interval-valued Cq-ROFSs. Furthermore, Hamacher weighted average and geometric operators will be extended to (1) 2TL complex interval-valued \( q \)-ROFSs; (2) 2TL complex simplified interval-valued \( q \)-ROFSs; and (3) 2TL complex hesitant \( q \)-ROFSs.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] K. T. Atanassov, “Intuitionistic fuzzy sets,” *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[3] R. R. Yager, “Pythagorean membership grades in multicriteria decision making,” *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 958–965, 2013.

[4] T. Senapaty and R. R. Yager, “Fermatean fuzzy sets,” *Journal of Ambient Intelligence and Humanized Computing*, vol. 11, no. 2, pp. 663–674, 2020.

[5] R. R. Yager, “Generalized orthopair fuzzy sets,” *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1222–1230, 2016.

[6] D. Ramot, R. Milo, M. Friedman, and A. Kandel, “Complex fuzzy sets,” *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp. 171–186, 2002.

[7] A. M. D. J. S. Alkouri and A. R. Salleh, “Complex intuitionistic fuzzy sets,” in *Proceedings of the AIP Conference Proceedings*, vol. 1482, no. 1, pp. 464–470, American Institute of Physics, Antalya, Turkey, April 2012.

[8] A. U. M. Alkouri and A. R. Salleh, “Complex Atanassov’s intuitionistic fuzzy relation,” in *Abstract and Applied Analysis*, vol. 2013, Hindawi, Article ID 287382, 18 pages, Hindawi, 2013.

[9] K. Ullah, T. Mahmood, Z. Ali, and N. Jan, “On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition,” *Complex & Intelligent Systems*, vol. 6, no. 1, pp. 15–27, 2020.

[10] P. Liu, T. Mahmood, and Z. Ali, “Complex q -runq orthopair fuzzy aggregation operators and their applications in multi-attribute group decision making,” *Information*, vol. 11, no. 1, p. 5, 2020.

[11] P. Liu, Z. Ali, and T. Mahmood, “Generalized complex q-runq orthopair fuzzy Einstein averaging aggregation operators and their application in multi-attribute decision making q-rung orthopair fuzzy Einstein averaging aggregation operators and their application in multi-attribute decision making,” *Complex & Intelligent Systems*, vol. 7, no. 1, pp. 511–538, 2021.

[12] M. Akram, K. Zahid, and J. C. R. Alcantud, “A new outranking method for multicriteria decision making with complex Pythagorean fuzzy information,” *Neural Computing & Applications*, vol. 34, no. 10, pp. 8069–8102, 2022.

[13] M. Akram, U. Amjad, J. C. R. Alcantud, and G. Santos-Garcia, “Complex fermatean fuzzy N-soft sets: a new hybrid model with applications,” *Journal of Ambient Intelligence and Humanized Computing*, pp. 1–34, 2022.

[14] L. A. Zadeh, “The concept of a linguistic variable and its application to approximate reasoning-1,” *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.

[15] P. Liu, “Some generalized dependent aggregation operators with intuitionistic linguistic numbers and their application to group decision making,” *Journal of Computer and System Sciences*, vol. 79, no. 1, pp. 131–143, 2013.

[16] J.-q. Wang, Y. Yang, and L. Li, “Multi-criteria decision-making method based on single-valued neutrosophic linguistic Maclaurin symmetric mean operators,” *Neural Computing & Applications*, vol. 30, no. 5, pp. 1529–1547, 2018.

[17] P. Liu and W. Liu, “Multiple-attribute group decision-making based on power Bonferroni operators of linguistic-q-rung orthopair fuzzy numbers -runq orthopair fuzzy numbers,” *International Journal of Intelligent Systems*, vol. 34, no. 4, pp. 652–689, 2019.

[18] F. Herrera and L. Martínez, “A 2-tuple fuzzy linguistic representation model for computing with words,” *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 6, pp. 746–752, 2000.

[19] M. Zhao, G. Wei, J. Wu, Y. Guo, and C. Wei, “TODIM method for multiple attribute group decision making based on cumulative prospect theory with 2-tuple linguistic neu- trosoftic sets,” *International Journal of Intelligent Systems*, vol. 36, no. 3, pp. 1199–1222, 2021.

[20] Y. Zhang, G. Wei, Y. Guo, and C. Wei, “TODIM method based on cumulative prospect theory for multiple attribute group decision-making under 2-tuple linguistic Pythagorean fuzzy environment,” *International Journal of Intelligent Systems*, vol. 36, no. 6, pp. 2548–2571, 2021.

[21] S. Naz and M. Akram, “Novel decision-making approach based on hesitant fuzzy sets and graph theory,” *Computational and Applied Mathematics*, vol. 38, no. 1, p. 7, 2019.

[22] M. Akram, S. Naz, S. A. Edalatpanah, and M. Mehreen, “Group decision-making framework under linguistic q-rung orthopair fuzzy Einstein models q-runq orthopair fuzzy Einstein models,” *Soft Computing*, vol. 25, no. 15, Article ID 10309, 2021.

[23] P. Liu, S. Naz, M. Akram, and M. Muzammal, “Group decision-making analysis based on linguistic q-rung orthopair fuzzy generalized point weighted aggregation operators,” *International Journal of Machine Learning and Cybernetics*, vol. 13, no. 4, pp. 883–906, 2022.

[24] M. Akram, S. Naz, and F. Ziaa, “Novel decision making framework based on complex q-rung orthopair fuzzy information q -runq orthopair fuzzy information,” *Scientia Iranica*, pp. 1–34, 2021.

[25] S. Naz, M. Akram, S. Alsulami, and F. Ziaa, “Decision-making analysis under interval-valued q -runq orthopair dual hesitant fuzzy environment,” *International Journal of Computational Intelligence Systems*, vol. 14, no. 1, pp. 332–357, 2021.

[26] S. Naz, M. Akram, M. Akram, M. M. A. Al-Shamiri, M. M. Khalaf, and G. Yousaf, “A new MAGDM method with 2-tuple linguistic bipolar fuzzy Heronian mean operators,” *Mathematical Biosciences and Engineering*, vol. 19, no. 4, pp. 3843–3878, 2022.

[27] H. Garg, S. Naz, F. Ziaa, and Z. Shoukat, “A ranking method based on Muirhead mean operator for group decision making with complex interval-valued q-runq orthopair fuzzy numbers q-runq orthopair fuzzy numbers,” *Soft Computing*, vol. 25, no. 22, pp. 14001–14027, 2021.

[28] I. Beg and T. Rashid, “An intuitionistic 2-tuple linguistic information model and aggregation operators,” *International Journal of Intelligent Systems*, vol. 31, no. 6, pp. 569–592, 2016.

[29] X. Deng, G. Wei, H. Gao, and J. Wang, “Models for safety assessment of construction project with some 2-tuple linguistic Pythagorean fuzzy Bonferroni mean operators,” *IEEE Access*, vol. 6, Article ID 52105, 2018.

[30] Y. Rong, Y. Liu, and Z. Pei, “Complex q-rung orthopair fuzzy 2-tuple linguistic Maclaurin symmetric mean operators and its application to emergency program selection q-runq orthopair fuzzy 2-tuple linguistic Maclaurin symmetric mean operators and its application to emergency program selection,” *International Journal of Intelligent Systems*, vol. 35, no. 11, pp. 1749–1790, 2020.

[31] P. Liu and P. Wang, “Some q-Rung Orthopair Fuzzy Aggregation Operators and their Applications to Multiple-Attribute Decision Making q -runq orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making,” *International Journal of Intelligent Systems*, vol. 33, no. 2, pp. 259–280, 2018.
H. Hamacher, “Uber logische verknupfungen unscharfer aussagen und deren zugehrige bewertungsfunktionen,” *Progress in Cybernetics and Systems Research*, vol. 3, pp. 276–288, 1978.

M. Akram, X. Peng, and A. Sattar, “A new decision-making model using complex intuitionistic fuzzy Hamacher aggregation operators,” *Soft Computing*, vol. 25, no. 10, pp. 7059–7086, 2021.

S. Faizi, W. Salabun, S. Nawaz, A. U. Rehman, and J. Wątróbski, “Best-Worst method and Hamacher aggregation operations for intuitionistic 2-tuple linguistic sets,” *Expert Systems with Applications*, vol. 181, Article ID 115088, 2021.

S. S. Rawat and K. Komal, “Multiple attribute decision making based on q-rung orthopair fuzzy Hamacher Muirhead mean operators -rung orthopair fuzzy Hamacher Muirhead mean operators,” *Soft Computing*, vol. 26, no. 5, pp. 2465–2487, 2022.

D. Pamucar, M. Deveci, I. Gokasar, and M. Popovic, “Fuzzy Hamacher WASPAS decision-making model for advantage prioritization of sustainable supply chain of electric ferry implementation in public transportation,” *Environment, Development and Sustainability*, vol. 24, pp. 1–40, 2021.

E. K. Zavadskas, A. Kaklauskas, and V. Sarka, “The new method of multicriteria proportional assessment of projects,” *Technological and Economic Development of Economy*, vol. 1, no. 3, pp. 131–139, 1994.

A. Arabameri, M. Yamani, B. Pradhan, A. Melesse, K. Shirani, and D. Tien Bui, “Novel ensembles of COPRAS multi-criteria decision-making with logistic regression, boosted regression tree, and random forest for spatial prediction of gully erosion susceptibility,” *The Science of the Total Environment*, vol. 688, pp. 903–916, 2019.

P. Chatterjee, V. M. Athawale, and S. Chakraborty, “Materials selection using complex proportional assessment and evaluation of mixed data methods,” *Materials & Design*, vol. 32, no. 2, pp. 851–860, 2011.

M. Alipour, R. Hafezi, P. Rani, M. Hafezi, and A. Mardani, “A new Pythagorean fuzzy-based decision-making method through entropy measure for fuel cell and hydrogen components supplier selection,” *Energy*, vol. 234, Article ID 121208, 2021.

A. Balali, A. Valipour, R. Edwards, and R. Moehler, “Ranking effective risks on human resources threats in natural gas supply projects using ANP-COPRAS method: case study of Shiraz,” *Reliability Engineering & System Safety*, vol. 208, Article ID 107442, 2021.

M. Narang, M. C. Joshi, and A. K. Pal, “A hybrid fuzzy COPRAS-base-criterion method for multi-criteria decision making,” *Soft Computing*, vol. 25, no. 13, pp. 8391–8399, 2021.

F. Herrera and E. Herrera-Viedma, “Linguistic decision analysis: steps for solving decision problems under linguistic information,” *Fuzzy Sets and Systems*, vol. 115, no. 1, pp. 67–82, 2000.

F. Herrera and L. Martinez, “An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making,” *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 8, no. 5, pp. 539–562, 2000.

W. Wang and X. Liu, “Intuitionistic fuzzy geometric aggregation operators based on Einstein operations,” *International Journal of Intelligent Systems*, vol. 26, no. 11, pp. 1049–1075, 2011.

X. Zhang, “A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making,” *International Journal of Intelligent Systems*, vol. 31, no. 6, pp. 593–611, 2016.

S. Ashraf, S. Abdullah, and A. O. Almagrabi, “A new emergency response of spherical intelligent fuzzy decision process to diagnose of COVID19,” *Soft Computing*, pp. 1–17, 2020.

B. Batool, M. Ahmad, S. Abdullah, S. Ashraf, and R. Chirram, “Entropy based pythagorean probabilistic hesitant fuzzy decision making technique and its application for fog-haze factor Assessment problem,” *Entropy*, vol. 22, no. 3, p. 318, 2020.

B. Batool, S. S. Absoluliman, S. Abdullah, and S. Ashraf, “EDAS method for decision support modeling under the Pythagorean probabilistic hesitant fuzzy aggregation information,” *Journal of Ambient Intelligence and Humanized Computing*, pp. 1–14, 2021.

G. Kou, O Olgu Akdeniz, H. Dinçer, and S. Yüksel, “Fintech investments in European banks: a hybrid IT2 fuzzy multi-dimensional decision-making approach,” *Financial Innovation*, vol. 7, no. 1, pp. 39–28, 2021.

G. Kou, H. Xiao, M. Cao, and L. H. Lee, “Optimal computing budget allocation for the vector evaluated genetic algorithm in multi-objective simulation optimization,” *Automatica*, vol. 129, Article ID 109599, 2021.