Luminous Efficiency Estimates of Meteors. II. Application to Canadian Automated Meteor Observatory Meteor Events

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Abstract

Luminous efficiency is a necessary parameter for determining meteoroid mass from optical emission. Despite this importance, it is very poorly known, with previous results varying by up to two orders of magnitude for a given speed. We present the most recent study of luminous efficiency values determined with modern high-resolution instruments, by directly comparing dynamic and photometric meteoroid masses. Fifteen non-fragmenting meteoroids were used, with a further five clearly fragmenting events for comparison. Twelve of the fifteen non-fragmenting meteoroids had luminous efficiencies less than 1%, while the fragmenting meteoroids had upper limits of a few tens of per cent. No clear trend with speed was seen, but there was a weak negative trend of luminous efficiency on meteoroid mass, implying that smaller meteoroids radiate more efficiently.

Key words: meteorites, meteors, meteoroids – methods: observational

1. Introduction

Meteoroid masses are poorly constrained. Various studies have used different experimental and observational techniques in the past to determine meteoroid masses from the light they emit, but results vary by up to two orders of magnitude for a given meteoroid speed. This has consequences for meteoroid flux estimates, which can affect satellites and spacecraft in orbit around Earth (National Research Council 2011). Because all small meteoroids ablate completely in the atmosphere, mass estimates need to be made based on observations that are typically less than one second long. There are many unknown parameters that affect calculations of the mass of a meteoroid, such as its shape and density. Assuming the meteoroid is a solid, non-fragmenting object, its mass can be determined with either of a pair of coupled differential equations that describe the mass loss and deceleration of the meteoroid. The luminous intensity equation (shown in Equation (1)) allows one to determine the mass of a meteoroid \( m \) from the change in its kinetic energy \( E_k \), given the speed \( v \) and brightness \( I \).

\[
I = -\tau \frac{dE_k}{dt} = -\tau \left( \frac{v^2}{2} \frac{dm}{dt} + m \frac{dv}{dt} \right).
\]  

(1)

A value must be chosen for the proportionality constant \( \tau \), the luminous efficiency—a measure of how much of the kinetic energy lost by the meteoroid is used for visible light production. The mass given in Equation (1) is the photometric mass and includes the mass of all fragments if the meteoroid has broken up. There is a large uncertainty associated with the photometric mass due to the uncertainty in the luminous efficiency.

The deceleration of the meteor is described by Equation (2), the drag equation, derived from conservation of momentum.

\[
\frac{dv}{dt} = -\frac{\Gamma \rho_{\text{atm}} v^2 A}{m^2 \rho_m^2}.
\]  

(2)

It provides a second way to determine the meteoroid mass. In this equation, the mass \( m \) is the dynamic mass and describes the leading (usually largest and brightest) fragment; \( \Gamma \) is the drag coefficient and describes the efficiency of momentum transfer between the atmosphere and meteoroid; \( \rho_{\text{atm}} \) is the atmospheric density; \( \rho_m \) is the meteoroid density; and \( A \) is the shape factor, defined as the cross sectional area\(^1\) divided by the object volume to the exponent 2/3. Often, the drag coefficient, the meteoroid density, and the shape factor are assumed to be constant.

Because most meteoroids fragment (Subasinghe et al. 2016), it is more practical to determine the meteoroid mass through the luminous intensity equation, assuming a suitable value can be found for the luminous efficiency.

The uncertainty in the luminous efficiency is large as it may depend on many factors: meteoroid speed and height; meteoroid and atmospheric composition; the spectral response of the detector; and possibly mass (Ceplecha et al. 1998). Whether each factor has an effect, and its effect on the luminous efficiency, is unknown.

The luminous efficiency of meteors has been investigated with numerous methods in the past. A theoretical approach was first taken by Öpik (1933) but has been disregarded by many researchers due to theoretical considerations (such as not knowing how quantum states are populated) (Thomas & Whipple 1951; Verniani 1965). A modern theoretical approach was taken by Jones & Halliday (2001), using excitation cross-sections to predict the light produced (however, they assumed that ionized atoms are not available for excitation). Verniani (1965) determined the luminous efficiency by equating the dynamic and photometric masses of Super-Schmidt meteors; this study is still the source of luminous efficiencies commonly in use (e.g., Ceplecha & McCrosky 1976). Verniani (1965) assumed that luminous efficiency is proportional to meteoroid speed to some constant exponent \( (\tau \propto v^n) \), which his data found to be \( n = 1 \). Verniani (1965) made an effort to correct his results for fragmentation as it was well known that many of the Super-Schmidt meteors crumbled during ablation, but his calibration for \( \tau \) is based on a single non-fragmenting asteroidal meteor at low altitude. Lab experiments entail charging and accelerating tiny metal particles in a Van deGraaf generator and

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1 Mistakenly given as the surface area in Subasinghe et al. (2017).
Table 1

| Initial Speed (km s⁻¹) | Initial Mass (kg) | Shape Factor | Drag Coefficient | Meteoroid Density (kg m⁻³) | Zenith Angle (degrees) |
|------------------------|------------------|--------------|------------------|-----------------------------|------------------------|
| 30                     | 10⁻⁵             | 1.21         | 1                | 2000                        | 30                     |

Note. These are the parameters used in the Campbell-Brown & Koschny (2004) ablation model to simulate our standard event for testing purposes.

observing as they ablate in a low-pressure air chamber. A common difficulty in completing these lab studies is accurately recreating atmospheric compositions and conditions. A number of studies were carried out in the late sixties and early seventies (see Friichtenicht et al. 1968; Becker & Friichtenicht 1971; Becker & Slattery 1973), and work in this area has recently been revived, though no results for luminous efficiency have been published yet (Thomas et al. 2016). Another method of determining luminous efficiency is to use artificial meteoroids, in which objects of known mass, composition, and density are launched into the atmosphere and observed as they ablate. This was done by Ayers et al. (1970) for iron and nickel projectiles, at relatively low speeds compared to meteors. Simultaneous radar and optical studies, which use the ionization efficiency to determine the luminous efficiency, were carried out by Saidov & Simek (1989, who assumed the optical data was more correct) and Weryk & Brown (2013, who assumed the radar data was more correct), but the results of the two studies do not agree. A combination of theoretical and lab work was analyzed by Hill et al. (2005), and the result (corrected for bandpass and composition) is presented in Weryk & Brown (2013).

Many of these studies are summarized in Subasinghe et al. (2017), who investigated the precision of using the dynamic mass to determine the photometric mass using meteor data simulated with the ablation model of Campbell-Brown & Koschny (2004). Data with sub-meter scale resolution from the Canadian Automated Meteor Observatory (CAMO) has the potential to redo the work of Verniani much more accurately. In testing the method, we found that uncertainties in the meteoroid density (which cannot be uniquely determined from observations) and atmospheric density account for a factor of two uncertainty, each, in the calculated luminous efficiency. A similar factor of two uncertainty was found when investigating simulated meteoroids of different masses and speeds. Good agreement was found between the luminous efficiency value used in the simulations and the derived values for all of the mass-speed groups except for the lowest speed group for each of the three masses, which had slightly poorer agreement. Subasinghe et al. (2017) used simulated data free from measurement noise, so the effects of observed noise were not considered, or the possibility that parameters in Equation (2) may not be constant over time.

In this work, we continue that investigation by applying a modified method to real meteor events recorded with the high-precision Canadian Automated Meteor Observatory.

2. Method Refinement

The simulated data used in Subasinghe et al. (2017) was free of noise, which is unrealistic, but this made it easy to evaluate various functional fits to the meteor lag, or distance by which the meteor lags behind an object travelling at constant speed. That study found that a two-term exponential fit to the lag matched the luminous efficiency most closely. The real data used in this study have residuals in position of about 2 m: we added scatter of this order to the positions generated by the ablation model and again tried to recover the luminous efficiency used in the model. A simplification from the two-term exponential \( \text{lag} = a \exp(bx + c \exp(dx)) \) to a single-term exponential \( \text{lag} = a \exp(bx) \) was necessary, because when noise was added to the model the two-term exponential produced unphysical values of the speed and deceleration. A single-term exponential is a relatively poor fit for the full lag curve but does well when fitting only the last half of the curve, which is where the deceleration is high enough to calculate a meaningful luminous efficiency. The final method used in this work fits a single-term exponential to the second half of the meteor lag data and uses the fit parameters to determine the speed and deceleration profiles used for the luminous efficiency determination.

To illustrate the process, we present a standard, non-fragmenting simulated event (with the parameters given in Table 1) run through our method and the resulting luminous efficiency profile. The event was simulated using the meteoroid ablation model of Campbell-Brown & Koschny (2004) with an assumed luminous efficiency of 0.7%, constant over the entire ablation time. The derived luminous efficiency was calculated using Equations (1) and (2), with values for the drag coefficient \( \Gamma \), meteoroid density \( \rho_m \), shape factor \( A \), and atmospheric density \( \rho_{\text{atm}} \) coming from the simulation (for real events, these will be estimates except for the atmospheric density, which will come from a model). A single-term exponential (given in Equation (3)) was fit to the second half of the lag data of the standard event, and the fitted parameters were \( a = 1.112 \) and \( b = 6.84 \).

\[
\text{lag} = a \exp(bx).
\] (3)

The speed and deceleration values are based on these fitted parameters. The fit to the lag is shown in Figure 1, and the corresponding speed and deceleration plots are shown in Figure 2, with the simulated values plotted against the curves based on the fit parameters from the lag.

Using the speed and deceleration profiles based on the lag fit, and all other values taken from the simulation, we find the luminous efficiency profile shown in Figure 3. Recall that in the simulation, a constant luminous efficiency of 0.7% was used. Figure 3 shows two profiles: the blue asterisks show the luminous efficiency determined using all the simulation parameters, including the simulated deceleration and simulated speed. The red line shows the luminous efficiency found using the speed and deceleration profiles based on the single-term exponential fit to the lag. The resulting profile from the simulated data is not able to perfectly reproduce the constant 0.7% luminous efficiency used in the simulation due to the simplicity of this method compared to the physics in the ablation model that was used. The average luminous efficiency of the profile based on the fitted lag is 0.598%, and 0.699% for
the profile resulting from using all the simulated data. Subasinghe et al. (2017) explored the deviation from an assumed luminous efficiency due to different functional fits and each parameter in the drag and luminous intensity equations.

3. Equipment and Data Reduction

3.1. Equipment

The meteor events in this study were recorded by the Canadian Automated Meteor Observatory (CAMO), which is located in Ontario, Canada. CAMO is a two-station image intensified optical system which records meteors nightly, under appropriate weather conditions. One station is located in Tavistock, Ontario, Canada (43°265 N, 80°772 W), and the other is about 45 km away in Elginfield, Ontario, Canada (43°193 N, 81°316 W). For our work, we are using the guided system, which is a two-part system, consisting of a wide-field camera (with a field of view of 28°) and a narrow-field camera (1.5° FOV). The wide-field camera detects meteors in real time with the All Sky and Guided Automatic Realtime Detection (ASGARD) software (Weryk et al. 2008), which guides a pair of mirrors to direct the meteor light into the narrow-field camera. The cameras run at different frame rates (80 and 110 fps for the wide-field and narrow-field, respectively), and each camera is used for different scientific goals. The wide-field camera typically captures the full meteoroid ablation profile, which allows for light curve calculation, as well as orbit determination. The narrow-field camera does not capture the entire meteoroid ablation profile as collection only begins after the meteor has been detected in the wide-field camera over 4 to 7 frames, and it typically takes a few frames of observation in the narrow-field camera before the meteor is tracked smoothly by the mirrors. However, the narrow-field camera is a high-resolution camera able to resolve up to 3 m per pixel at 100 km range, which allows the fragmentation behavior of the object to be observed. These observations allow us to find objects that show single-body ablation, which is required when using the classical ablation equations, and to measure the deceleration very precisely. More details about the cameras can be found in Weryk et al. (2013).

3.2. Data Reduction

CAMO records meteors each night if certain sky conditions are met and has been in operation since 2007 (Weryk et al. 2013). This large database of meteors was searched for meteor events recorded at both stations in the narrow-field cameras that showed either single-body ablation, or a leading fragment with no wake. The classical meteoroid ablation equations apply to objects that are solid, single bodies that do not fragment. Any object that does fragment will have a smaller dynamic mass (this is the mass of the largest and brightest fragment) than the photometric mass (this is the mass of the entire meteoroid), and therefore an artificially large luminous efficiency. Despite CAMO having recorded thousands of meteor events, finding meteors that showed next to no fragmentation proved to be a difficult task, as more than 90% of CAMO meteors show some form of fragmentation in the form of wake (Subasinghe et al. 2016). A meteor with a leading fragment is a special case where a fragment of the main body decelerates less than the others, and this piece shows little to no fragmentation. Examples of these non-fragmenting morphologies are shown in Figure 4. Meteors that met the requirements of showing single-body ablation (either the entire body, or a leading fragment) were then analyzed with two software packages: METAL (Weryk & Brown 2012) and mirfit.

METAL allows for orbit determination and light curve analysis of a meteor in the wide-field camera. Astrometric and photometric plates are computed using a minimum of 10 stars from each station: each stellar pixel centroid and brightness are calibrated against those from the SKY2000v4 catalog. Once this is complete, the head of the meteor is picked in each frame, for both stations, for which the entire meteor is visible, and a trajectory solution is determined using an implementation of the least squares method (Borovicka 1990) called MILIG. By masking out pixels containing light from the meteor, the meteor apparent magnitude can be determined and converted to an absolute magnitude using the photometric calibration plate and meteor range.

The software package mirfit allows video observations taken with the high-resolution narrow-field cameras to be analyzed. Because the field of view is so small (1.5°) compared to video taken with the wide-field cameras (28°), stellar astrometric plates cannot be used due to the lack of visible stars; in addition, the field of view is moving during the observations. To determine meteor positions in the narrow-field cameras, the mirror positions need to be taken into account. These are recorded every 0.5 ms, so the position of the center of the field at the beginning of the exposure can be interpolated. Then, the distance (or offset) between the position of the meteor on the narrow-field image and the image center is determined and converted into an offset in mirror encoder coordinates. This offset is then added to the mirror position at that time and mapped onto the celestial sphere. This is done using two plates: (1) a scale plate that determines the offset and converts pixel position to mirror encoder coordinates, and (2) an exact plate that maps the mirror encoder coordinates into celestial coordinates. A calibration for the exact plate is done at the beginning of any night’s observations and every two hours throughout the night.
If the program that creates the exact plate calibration makes an error, for example, by attempting to calibrate a star in the field of view with a different star in the catalog, errors with the plates can occur, so plates are verified prior to meteor analysis, as bad calibration data cannot be replaced. Stars visible in the field of view will be trailed across each frame, with the predicted location of the initial position indicated by the software. If the plates are functioning correctly, the predicted star positions will not drift across the star trails, but will have the same position relative to the star streak in each frame. The method we are using to determine meteor luminous efficiency is very sensitive to the position measurements (as deceleration values are needed), which means for the most reliable results, we should only use meteor events with accurate plates. The number of useful meteors is then restricted not only to the few meteors that show no fragmentation, but to those non-fragmenting meteors that also have accurate plates. Meteor events that pass both of these requirements are then analyzed in mirfit. The meteor astrometry is done through centroiding for each frame in both stations; centroiding works particularly well on meteors with no visible wake. Photometry was also done for each meteor. mirfit is not able to calibrate the meteor brightness because of the lack of calibration stars in the field, but it can calculate the log of the sum of the brightness of the meteor pixels, which is proportional to the meteor magnitude. This relative light curve is then calibrated using the photometry for the meteor in the wide-field camera.

Data reduction was completed for 13 meteors showing a leading fragment and 2 meteors showing single-body ablation. Each image is from an individual frame, with the meteor cropped, rotated, and stacked such that time increases downwards. The image on the left has the leading fragment centered (clearly showing the deceleration of the rest of the body behind it), while the meteor on the right has the entire object centered. The 100 m scale bar for the leading fragment example (on the left) is for a height of 80 km, and for the single-body example on the right, is for a height of 85 km.
efficiency values. The mirfit analysis provides high-precision meteor positions, with an average random position uncertainty of 1.6 m for the 15 non-fragmenting events (with a maximum uncertainty of 2.5 m, and a minimum of 0.9 m). The position data was turned into lag values (i.e., the distance the meteoroid would fall behind an identical object moving with a constant speed). The speed used was the initial meteoroid speed, found by fitting the first half of the distance-time data, though the exact value is not important, as only derivatives of the lag are used. The second half of this lag was fit by a single-term exponential function, and the derivatives were used for the speed and deceleration profiles. Typical values were used for the drag coefficient ($\Gamma = 1$), shape factor ($A = 1.21$), meteoroid density (values taken from either Kikwaya et al. 2011 or Kikwaya Eluo 2011), and the atmospheric density profile was taken for the event date from the NRLMSISE-00 Atmosphere model (Picone et al. 2002). An analysis of how sensitive this method is to each of the parameters can be found in Subasinghe et al. (2017).

### Table 2

| Mass (kg) | Speed (km s$^{-1}$) | Meteoroid density (kg m$^{-3}$) | Zenith Angle (degrees) | Shape factor |
|----------|----------------------|-------------------------------|-----------------------|-------------|
| 10$^{-5}$ | 11                   | 1000                          | 30                    | 0.5         |
| 10$^{-6}$ | 20                   | 2000                          | 60                    | 0.8         |
| 10$^{-6}$ | 30                   | 3000                          | ...                   | 1.21        |
| ...      | 40                   | 5000                          | ...                   | 2           |
| ...      | 50                   | 8000                          | ...                   | 4           |
| ...      | 60                   | ...                           | ...                   | ...         |
| ...      | 70                   | ...                           | ...                   | ...         |

4. Results

4.1. Noise Analysis

Prior to evaluating the luminous efficiency for each of our 15 events, we investigated the effect that noise has on our method. In Subasinghe et al. (2017), we investigated a set of simulated meteor events that covered the entire physical phase space of mass, speed, meteoroid density, zenith angle, and shape factor. There were 21 mass–speed groups (three different masses and seven different meteor speeds), each of which had 50 possible meteors (all combinations of five possible meteoroid densities, two possible zenith angles, and five possible shape factor values; the possible values are given in Table 2); however, not all meteors produced enough light that the CAMO optical system would detect it. This left 18 mass–speed groups, with up to 50 meteors, to study. Each meteor was simulated with a luminous efficiency of 0.7%, constant over time. Five uncertainty levels (standard deviations of 0.1, 0.5, 1, 2, 5 m) were selected to showcase the effect that position precision would have on the results, with the 2 m uncertainty being closest to our measured uncertainties. Noise was randomly added 500 times at each uncertainty level to each meteor in the 18 mass–speed groups, and the luminous efficiency was calculated. The mean value of each profile was found, and the 500 mean luminous efficiency values for each event were averaged. These results are presented in Figure 6, separated by mass.

4.2. Atmospheric Density Variations

As an extension to the work presented in Subasinghe et al. (2017), we investigated the influence of atmospheric density changes on derived luminous efficiency values. It was found that changes in the model atmospheric density profiles over the course of one year could affect derived luminous efficiency values by at most a factor of two (with the model atmospheric density profiles varying at different heights by at most a factor of two). Figure 7 (top panel) shows how seasonal variations over the course of 2016 would affect the derived luminous efficiency profile of our standard event. Everything was identical in each run except for the atmospheric profile used; in the standard profile for comparison, the atmospheric density profile used in the simulation was used to find luminous efficiency. All other values (meteoroid density and so on) matched those used in the simulation. The middle and bottom panels instead show how the solar cycle affects the luminous efficiency for two dates over the course of 2006–2012. At the time of writing, the NRLMSISE-00 model provides data up to 2017 April 17. However, some of our meteor events were recorded in 2017 July and August. The results of Figure 7 indicate that atmospheric density data from the same day but different years will result in very similar derived luminous...
efficiency results: the May 17 derived mean luminous efficiency results vary at most by a factor of 1.4, and the November 17 derived mean luminous efficiency values vary at most by a factor of 1.3. Therefore, for our meteor events recorded in 2017 July and August, atmospheric density profiles from the previous year were used.

4.3. Photometry Calibration

When determining the luminous efficiency with the drag and luminous intensity equations, the meteoroid brightness is needed, as seen in Equation (1). This information can be obtained from both the narrow-field and wide-field cameras; however, if the fragment of interest is a leading fragment, the wide-field photometry will include the brightness of all fragments (the resolution is not high enough to separate the fragment brightness from the rest of the object). Thus, the narrow-field photometry is necessary for determining the luminous efficiency of the relevant fragment only. The method of obtaining the meteor photometry from both the wide-field and narrow-field cameras is described below.

The calibration of meteor photometry for meteors observed with the CAMO guided wide-field system was discussed in Weryk et al. (2013). As we are using positions derived from the narrow-field analysis, we investigated the possibility of calibrating the narrow-field meteor photometry with stars observed in the narrow-field camera, to eliminate the need for the wide-field cameras in this work. In METAL, pixels are masked out in each frame to select the light from the meteor (giving the instrumental apparent magnitude) and converted to the absolute magnitude using both the previously determined photometric plate and range to the meteor in each frame. The uncertainty in METAL photometry is close to 0.2 mag (Subasinghe et al. 2016). Similarly, in mirfit, pixels can be masked out in each frame to give the instrumental apparent magnitude from the log of the sum of the pixel values (lsp); however, there is no photometric plate, due to the small number of visible stars in the small field of view.

To calibrate the narrow-field instrumental apparent magnitudes, we compared those log-sum-pixel values to the absolute magnitudes determined in the wide-field observations and used them as a calibration to determine the absolute magnitude of the meteor in the narrow-field camera. To verify these narrow-field absolute magnitudes, we investigated the method of calibrating against stars visible in the narrow-field camera, in spite of their small number.

Photometry calibrations were investigated for seven of the meteor events in our data set (four events had no visible stars, and nine events were added after this investigation was

![Figure 6](image_url)
completed). Each investigated event had at least one visible star, but no more than two—more than two visible stars occurred multiple times, but stars were eliminated from the study if they were only visible for a few frames or if they were binary stars. The average instrumental apparent magnitude of each star was determined, and the offset from the SKY2000 catalog R magnitude was found. This offset was applied to the meteor log-sum pixel values to correct them to an apparent magnitude. The average difference between the mir and the meteor log-sum pixel values to correct them to an apparent magnitude. If a station had two visible stars, the average offset was applied to the meteor log-sum pixel values to correct them to an apparent magnitude. The average difference between the mir calibrated photometry and the METAL calibrated photometry was −0.3098 and −0.2666 mag, for Tavistock and Elginfield, respectively. For our luminous efficiency analysis, we used the METAL calibrations simply because there are many more stars to calibrate against, but our result here indicates that there is only a minor difference between the METAL and mir calibrated photometry, which are typically based on two stars.

Equation (1) uses the meteor intensity, rather than the meteor magnitude. To convert between magnitude and intensity, we use the results of Weryk & Brown (2013) who determined that for the Gen III image intensified video cameras we are using, a zero magnitude meteor emits 820 W.

4.4. Meteoroid Density

One parameter we have control over in our analysis is the initial meteoroid density. In our study, we assume for simplicity that this density is constant over time. The density for each meteoroid was determined using the results from Kikwaya et al. (2011) and Kikwaya Eluo (2011), in which meteoroid densities were found after searching the entire parameter space in an attempt to match each meteor deceleration and light curve shape using the ablation model of Campbell-Brown & Koschny (2004). Kikwaya et al. (2011) used the classification of Borovička et al. (2005), which considers meteoroid physical composition, in their meteoroid density analysis, and uses not only the meteoroid Tisserand parameter, but individual orbital elements to help classify objects. The Tisserand parameter with respect to Jupiter is based on an object’s orbital elements, and is given by

$$T_J = \frac{a_J}{a} + 2 \sqrt{\frac{a}{a_J}(1 - e^2)} \cos(i).$$

(4)

Where $a$, $e$, and $i$ are the semimajor axis, eccentricity, and inclination of the meteoroid, respectively, and a subscript $J$ indicates an orbital element belonging to Jupiter. A Tisserand parameter greater than three suggests that an object has an asteroidal orbit; between two and three is associated with Jupiter-family comets; and $T_J$ less than two describes a Halley-type orbit.

The meteoroid orbit classification described by Borovička et al. (2005) and the associated meteoroid densities from Kikwaya et al. (2011) used in this work are given in Table 3 (note: $q$ is the perihelion distance and $Q$ is the aphelion distance).

In our set of non-fragmenting meteor events, we have eight meteors with Halley-type orbits, five with Jupiter-family orbits,
and two with asteroidal-chondritic orbits. Of our five fragmenting meteor events, two are Halley-type, one has a Jupiter-family orbit, and two have asteroidal-chondritic orbits. Kikwaya et al. (2011) give a mean density for objects with Jupiter-family orbits and objects with asteroidal-chondritic orbits. They give a minimum and maximum density for objects on Halley-type orbits: the mean density for Halley-type objects comes from the raw data from Kikwaya Eluo (2011).

### 4.5. Error Analysis

In Table 4, we present the luminous efficiency values determined for each of the 20 events. Each luminous efficiency value is presented with an associated uncertainty that takes into account the uncertainty from assuming the drag coefficient, meteoroid density, shape factor, and random errors in the position. A random error of up to halff pixel was added to each analyzed position pick, 100 times. Half a pixel corresponds to approximately 2 m at a range of 110 km, which was found to be close to the average error in our measured positions. The entire parameter space of drag coefficient, meteoroid density, and shape factor was then tested with each of the 100 variations, and the luminous efficiency was found. Table 4 presents the mean luminous efficiency and corresponding standard deviation. The range of drag coefficient values tested was from 0.8 to 1.2 (with 1.0 being an inelastic collision); shape factor varied from 0.71 to 1.71 (with 1.21 being a sphere); and meteoroid density taken from Kikwaya et al. (2011) with an uncertainty of ±500 kg m⁻³. The distribution of luminous efficiency values spanning the entire parameter space was more skewed than normal; however, Table 4 presents the mean, median, and standard deviation values.

### 4.6. Meteor Event 20161009_064237

In Figure 8, we present the analysis of one event, from fitting the lag, to the final determined luminous efficiency profile. To reduce the noise that comes from finite differencing the values, we smoothed the speed and deceleration values by finite differencing over larger separations. Despite this, there is still considerable scatter in the speed and deceleration points, emphasising the importance of both precise position measurements for this work and using a suitable fit for the meteor lag.

The lag fit parameters for this event are $a = 0.8908$ and $b = 23.82$, and the following parameters were assumed: meteoroid density of 3190 kg m⁻³, drag coefficient of 1, and a shape factor of 1.21. The fitted parameters are based on the original data analysis: they do not take into account searching the parameter space or the uncertainty of half a pixel in position. The average luminous efficiency over the second half of this meteoroid’s ablation was $0.101 \pm 0.111\%$.

### 4.7. Final Meteor Results

The calculated luminous efficiency values for each meteor event analyzed, as a function of initial speed, are presented in Figure 9, plotted over a few past studies for comparison. The error bars are based on searching the entire parameter space of appropriate meteoroid density, drag coefficient, and shape factor, and the downward arrows indicate that the determined luminous efficiency values are upper limits (due to fragmentation), or that the lower bound of the luminous efficiency is less than zero. Some meteor events moved out of the narrow-field camera’s field of view at one station: these events are plotted as either blue squares or red diamonds in Figure 9. The single-term exponential was fit to single station data rather than two-station data in those
cases; however, data from both stations was used to calculate the meteoroid trajectory.

4.7.1. Luminous Efficiency and Mass

It has been suggested that while the calculated meteoroid mass depends on luminous efficiency, the luminous efficiency may depend on mass according to fireball studies (Halliday et al. 1981; Ceplecha et al. 1998). To investigate this, Figure 10 illustrates the relationship between the average initial mass and luminous efficiency of each meteor. It is worth noting that these initial masses are smaller than the true initial mass of the meteoroid: this is the dynamic mass at the beginning of the
second half of the trajectory, and in some cases, the mass of the leading fragment instead of the whole meteoroid. As with the above results, these initial masses are an average determined after searching through the entire parameter space. The meteor events are colored by initial speed.

4.7.2. Fragmentation

Five meteors showing obvious fragmentation (long distinct trails) were analyzed with this method. These events served as a sanity check to see whether meteors showing fragmentation will result in unphysical luminous efficiencies, much greater than 100%. To be able to plot these results, we ignored luminous efficiency values greater than 100% when computing the average. A clear distinction between the fragmenting and non-fragmenting meteor events can be seen in Figure 9.

5. Discussion

As discussed in Subasinghe et al. (2017), our method involves fitting the second half of the lag because the luminous efficiency can only be calculated in the part of the trajectory with maximum deceleration. For ideal data, the change in functional fit from a two-term exponential to a single exponent worsened the agreement of values given in Table 1 of Subasinghe et al. (2017) with the 0.7% constant luminous efficiency used to simulate the events. In spite of this, the single-term exponential is a better choice for real data with noise.

To test the effect of the fit on the derived luminous efficiency values, and as an extension of the work done in Subasinghe et al. (2017), a simple polynomial \( \text{lag} = ax^3 + bx^2 + cx + d \), which gives a linear deceleration, was fit to meteor event 20161009_064237. The polynomial could not be of the order of two or less, as that resulted in a constant deceleration, meaning the dynamic mass was not changing. The luminous efficiency was also determined using a simple point-to-point method, in which the mean deceleration was determined for two different sections of the meteor data near the end of ablation (resulting in two deceleration values). These were used to find a dynamic mass, compared to the photometric mass lost between those points and used to calculate the luminous efficiency. These two methods found luminous efficiency values that were within a factor of three of the value determined by fitting a single-term exponential function to the second half of the meteor lag data. The exponential fit was used for this analysis as it best describes the atmospheric density that is encountered by the meteoroid.

We analyzed 15 non-fragmenting meteors, which is a very small fraction of the thousands recorded by CAMO. However, as seen in Figure 5, the images for the 13 leading fragment events show very little to no wake, suggesting fragmentation is not important and that they can be treated as single bodies validating our method and results. Two of the meteor events (20150716_052934 and 20150915_084106) analyzed show single-body ablation (but are not leading fragment events); however, they are likely undergoing fragmentation on a scale that we cannot resolve, as with the CAMO meteor in Campbell-Brown (2017), which appeared to have negligible wake but could only be modeled with significant fragmentation. Thus, those two results should be considered upper limits on the luminous efficiency for those events—the determined dynamic mass (which considers only the largest fragment) will be less than the photometric mass (which considers light production from all fragments), and for the same amount of light production, this would cause the luminous efficiency to be artificially increased.

Our simulated noise analysis shows that for low- \((10^{-6} \text{ kg})\), mid- \((10^{-5} \text{ kg})\), and high-mass meteors \((10^{-4} \text{ kg})\), low-speed meteors are most likely to produce luminous efficiency results closest to the value used in the simulation. This is likely because slower meteors decelerate more, so uncertainties affect the dynamic mass less than for fast meteors, which ablate before they significantly decelerate. For the entire range of

Figure 9. Luminous efficiency as a function of initial speed. Note that the initial meteor speed is based on the entire meteoroid ablation profile and not where the single-term exponential begins. The curves showing other studies have not been corrected to a particular bandpass. The instruments used in the Weryk & Brown (2013) study are the same ones used in this study, but their results are bolometric values.
meteor speeds, our method tends to underestimate the luminous efficiency of meteors. We used meteoroid bulk density values from Kikwaya et al. (2011) based on the Tisserand parameters of our meteor events. The results from Kikwaya et al. (2011) assumed a luminous efficiency value. To eliminate any bias this assumed luminous efficiency value may have had on our derived luminous efficiency results, we present the derived luminous efficiency results with meteoroid density 1000 kg m$^{-3}$ and 3000 kg m$^{-3}$ for each event, in Figure 11. Assuming a density of 1000 kg m$^{-3}$ has the effect of decreasing the calculated luminous efficiencies for slow meteors, while not significantly changing those of faster meteors, while assuming a density of 3000 kg m$^{-3}$ increases the calculated luminous efficiencies of fast meteors, while not significantly changing those of slower meteors. Both assumed densities show an increase in calculated luminous efficiency with speed, while the density values from Kikwaya et al. (2011) and Kikwaya Eluo (2011) show a constant relationship with speed.

The main results of this work are presented in Figures 9 and 10. Our results are typically consistent with lower values of luminous efficiency from previous studies and seem to rule out the highest luminous efficiencies. It is difficult to directly compare our results to others because several past studies knew the composition of their meteoroids (in the case of lab meteoroids or artificial meteoroids). We attempt to account for this by exploring a large range of possible meteoroid densities.

Figure 10 shows that there is a relationship between luminous efficiency and initial mass, which is not related to meteoroid speed. Unexpectedly, it shows that meteoroids with smaller mass radiate light more efficiently than more massive meteoroids. The first plot in Figure 10 shows the linear fit (in log–log space) when including all of the analyzed meteor events, and the second figure excluded the three events that were found to be very sensitive to meteor position picks. The entire parameter space of drag coefficient, meteoroid density, shape factor, and random errors in position was searched, with
random errors of up to half a pixel being added. Three of the fifteen events were found to be very sensitive to these position pick variations, leading to wildly different luminous efficiencies for similar position picks. Even when these three events are removed, a negative linear trend is still apparent in the luminous efficiency-mass plot. The uncertainty in each point is large, however, and more meteor events may cause this to change.

It is important to keep in mind the low number of meteor events studied here. While Figure 10 shows negative linear trends, these events are not necessarily representative of the entire population of meteors observed with the CAMO system. There are no meteors in the low luminous efficiency–low mass area of the plots in Figure 10: it is unlikely that the CAMO system could observe such faint meteors. However, there are also no high luminous efficiency–high-mass meteors: the CAMO system should be able to see these bright meteors. One matter of great interest is the dependence of luminous efficiency on speed, as some previous studies predict an increase (e.g., Verniani 1965) and some a decrease with speed (e.g., Hill et al. 2005). Our results, using the densities of Kikwaya et al. (2011), show no clear trend with speed, while when using a constant density, show that there is a weak increase with speed. At low speeds, our results are even less conclusive; we have a single meteor event with an initial speed less than 20 km s⁻¹, which does not allow us to corroborate or reject the steep increase in luminous efficiency at low speeds typically found in past studies. The error in each event is typically greater than the derived luminous efficiency value, which makes it unreasonable to draw lower bounds on the luminous efficiency value with this study. This is indicated by downward pointing arrows. It is not unreasonable to assume that the trend in luminous efficiency with speed depends on the composition, with some atoms radiating more effectively at higher collision energies, and some less effectively. In this case, any trend will be masked if the meteoroids have different compositions.

An important note is that 13 of our 15 meteor events were leading (or terminal) fragments. These fragments were composed of the strongest material in the meteoroid, which is implied by the fact that they continued to ablate after most of the meteoroid had ablated. These fragments may have had a different composition (and therefore spectrum) from the rest of the meteoroid, which means they may have a different luminous efficiency than their parent meteoroid. It is therefore difficult to compare these leading fragment meteor events to other studies. For example, if the leading fragments contain little volatile sodium, they would produce less light than sodium-rich parts of the meteoroid.

**Figure 11.** Luminous efficiency as a function of initial speed, assuming each meteoroid has a bulk density of 3000 kg m⁻³ (upper plot), or 1000 kg m⁻³ (lower plot).
6. Conclusion

This paper presents the most recent study of modern high-resolution meteor observations used to determine luminous efficiency, by comparing the dynamic and photometric masses. The second half of the observed meteor lag is fit with a single-term exponential, and the resulting speed and deceleration curves are used in conjunction with best-fit values for meteoroid density, shape factor, and drag coefficient. To determine uncertainties, the entire phase space of potential values for the above parameters was searched. Fifteen meteor events were observed, thirteen of which displayed leading fragment behavior (a fragment that persisted after the majority of the meteoroid had ablated and showed essentially no fragmentation), and two that show as close to single-body ablation as we could find. While there is no obvious relationship found between luminous efficiency and initial meteor speed, this may be due to the variability of meteoroid compositions, on which we have gathered no information for this study. It is also difficult to directly compare our results to past studies (artificial meteoroids, lab studies, simultaneous radar/optical studies) as our leading fragment events are composed of the strongest material in each meteoroid, which may not necessarily be represented in past studies. There seems to be an unexpected negative linear relationship between luminous efficiency and initial meteoroid mass; however, as there are only 15 events, this may not be meaningful. In the future, we will add spectral capabilities to the CAMO system and collect more data with the necessary high-resolution calibrations. This may cause relationships between luminous efficiency and other parameters (such as mass and speed) to reveal themselves.

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Software: ASGARD (Weryk et al. 2008), METAL (Weryk & Brown 2012), mirfit.

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