The DIS cross-sections ratio \( R = \sigma_L/\sigma_T \) at small \( x \) from HERA data.

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Abstract

We extract the deep inelastic scattering cross-sections ratio \( R = \sigma_L/\sigma_T \) in the range \( 10^{-4} \leq x \leq 10^{-1} \) from \( F_2 \) HERA data using very simple relations based on perturbative QCD. The result depends on only one parameter \( \delta \), being \( x^{-\delta} \) the behavior of the parton densities at low \( x \), which has been determined recently with a good accuracy by the H1 group.
In recent years the behaviour of deep inelastic lepton-hadron scattering (DIS) at the small values of Bjorken variable \(x\), has been intensively studied. The present letter is devoted to the behaviour of the ratio of cross-sections of the absorption of a longitudinal- and transverse-polarized photon by hadron: \(R = \sigma_L/\sigma_T\), at small values of \(x\). The ratio \(R\), which may be represented as the combination of the longitudinal \(F_L(x, Q^2)\) and transverse \(F_T(x, Q^2)\) DIS structure functions (SF):

\[
R(x, Q^2) = \frac{F_L(x, Q^2)}{F_T(x, Q^2) - F_L(x, Q^2)}
\]

is a very sensitive QCD characteristic because it is equal to zero in the parton model with spin\(-1/2\) partons (it is very large with spin\(-0\) partons). However, modern DIS experimental data (see the review in [1]) are not accurate enough to determine \(R(x, Q^2)\). In addition, at small values of \(x\), \(R\) data are not yet available, as they require a rather cumbersome procedure (see [2], for example) for the extraction from the experiment.

We study the behaviour of \(R(x, Q^2)\) at small values of \(x\), using the H1 data [3, 4] and the method [5] of replacement of the Mellin convolution by ordinary products. By analogy with the case of the gluon distribution function (see [1, 3, 4, 5, 6]), it is possible to obtain the relation between \(F_L(x, Q^2)\), \(F_T(x, Q^2)\) and \(dF_T(x, Q^2)/d\ln Q^2\) at small \(x\). Thus, the small \(x\) behaviour of the ratio \(R(x, Q^2)\) can be extracted directly from the measured values of \(F_T(x, Q^2)\) and its derivative. These extracted values of \(R\) may be well considered as new small \(x\) “experimental data” [6]. Moreover, when experimental data for \(R\) at small \(x\) become available with a good accuracy, a violation of this exactly perturbative relation will be an indication of the importance of other effects as higher twist contribution and/or of non-perturbative QCD dynamics at small \(x\).

We follow the notation of our previous work in refs. [8, 9]. The singlet quark \(s(x, Q_0^2)\) and gluon \(g(x, Q_0^2)\) parton distribution functions (PDF) [7] at some \(Q_0^2\) are parameterized by (see, for example, [10]):

\[
p(x, Q_0^2) = A_p x^{-\delta}(1 - x)^{\nu_p}(1 + \epsilon_p \sqrt{x} + \gamma_p x) \quad \text{(hereafter } p = s, g).\]

The value of \(\delta\) is a matter of controversy. The “conventional” choice is \(\delta = 0\), which leads to a non-singular behaviour of the PDF when \(x \to 0\). Another value, \(\delta \sim \frac{1}{2}\), was obtained in the studies performed in ref. [11] as the sum of the leading powers of \(\ln(1/x)\) in all orders of perturbation theory. Experimentally, recent NMC data [12] favor small values of \(\delta\). This result is also in agreement with present data for \(pp\) and \(\bar{p}p\) total cross-sections (see [13]) and corresponds to the model of Landshoff and Nachtmann pomeron [14] with the exchange of a pair of non-perturbative gluons, yielding \(\delta = 0.086\). However, the new HERA data [9, 15] prefer \(\delta \geq 0.2\). For example, the \(\delta\) value obtained recently by H1 group [3] seems to depend slowly on \(Q^2\) values. Its average value increases from \(\delta = 0.228\) at \(Q^2 = 8.5\) GeV\(^2\) to \(\delta = 0.503\) at \(Q^2 = 800\) GeV\(^2\).

Further, we restrict the analysis to the case of large \(\delta\) values (i.e. \(x^{-\delta} \gg 1\)) following recent H1 data [3]. The more complete analysis concerning to the extraction of the longitudinal SF \(F_L(x, Q^2)\), may be found in [4], where we took into account also the case \(\delta \sim 0\) corresponding to the standard pomeron.

Assuming the Regge-like behaviour for the gluon distribution and \(F_2(x, Q^2)\) at \(x^{-\delta} \gg 1\):

\[
g(x, Q^2) = x^{-\delta} \tilde{g}(x, Q^2), \quad F_2(x, Q^2) = x^{-\delta} \tilde{s}(x, Q^2),
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Assuming the Regge-like behaviour for the gluon distribution and \(F_2(x, Q^2)\) at \(x^{-\delta} \gg 1\):

\[g(x, Q^2) = x^{-\delta} \tilde{g}(x, Q^2), \quad F_2(x, Q^2) = x^{-\delta} \tilde{s}(x, Q^2), \]
we obtain the following equation for the $Q^2$ derivative of the SF $F_2$:

$$
\frac{dF_2(x, Q^2)}{dlnQ^2} = -\frac{1}{2} x^{-\delta} \sum_{p=s,g} \left( r_{sp}^{1+\delta}(\alpha) \, \tilde{p}(0, Q^2) + r_{sp}^{\delta}(\alpha) \, x \tilde{p}'(0, Q^2) + O(x^2) \right),
$$

$$
F_L(x, Q^2) = x^{-\delta} \sum_{p=s,g} \left( r_{Lp}^{1+\delta}(\alpha) \, \tilde{p}(0, Q^2) + r_{Lp}^{\delta}(\alpha) \, x \tilde{p}'(0, Q^2) + O(x^2) \right),
$$

where $r_{sp}^{\eta}(\alpha)$ and $r_{Lp}^{\eta}(\alpha)$ are the combinations of the anomalous dimensions of Wilson operators $\gamma_{sp}^{\eta} = \alpha \gamma_{sp}^{(0),\eta} + \alpha^2 \gamma_{sp}^{(1),\eta} + O(\alpha^3)$ and Wilson coefficients $f \alpha B_{Lp}^{\eta,\xi} \left( 1 + \alpha R_{Lp}^{\eta,\xi} \right) + O(\alpha^3)$ and $\alpha B_{Lp}^{\eta,\xi} + O(\alpha^2)$ of the $\eta$ "moment" (i.e., the corresponding variables extended from integer values of argument to non-integer ones):

$$
r_{Ls}^{\eta}(\alpha) = \alpha B_{Ls}^{\eta,\xi} \left[ 1 + \alpha \left( R_{Ls}^{\eta,\xi} - B_{Ls}^{\eta,\xi} \right) \right] + O(\alpha^3),
$$

$$
r_{Lg}^{\eta}(\alpha) = \frac{e}{f} \alpha B_{Lg}^{\eta,\xi} \left[ 1 + \alpha \left( R_{Lg}^{\eta,\xi} - B_{Lg}^{\eta,\xi} / B_{Lg}^{0,\eta} \right) \right] + O(\alpha^3),
$$

$$
r_{ss}^{\eta}(\alpha) = \alpha \gamma_{ss}^{(0),\eta} + \alpha^2 \left( \gamma_{ss}^{(1),\eta} + B_2^{\eta,\xi} \gamma_{gs}^{(0),\eta} + 2 \beta_0 B_2^{\eta,\xi} \right) + O(\alpha^3),
$$

$$
r_{sg}^{\eta}(\alpha) = \frac{e}{f} \left[ \alpha \gamma_{sg}^{(0),\eta} + \alpha^2 \left( \gamma_{sg}^{(1),\eta} + B_2^{\eta,\xi} \gamma_{gs}^{(0),\eta} + B_2^{\eta,\xi} (2 \beta_0 + \gamma_{gg}^{(0),\eta} - \gamma_{ss}^{(0),\eta}) \right) \right] + O(\alpha^3),
$$

and

$$
\tilde{p}'(0, Q^2) \equiv \frac{d}{dx} \tilde{p}(x, Q^2) \text{ at } x = 0,
$$

where $e = \sum_f e_i^2$ is the sum of squares of quark charges.

With accuracy of $O(x^{2-\delta})$, we have for Eq.(3)

$$
\frac{dF_2(x, Q^2)}{dlnQ^2} = -\frac{1}{2} \left[ r_{sg}^{1+\delta}(\xi_{sg})^{-\delta} g(x/\xi_{sg}, Q^2) + r_{ss}^{1+\delta} F_2(x, Q^2) + (r_{ss}^{\delta} - r_{ss}^{1+\delta}) x^{1-\delta} \tilde{\xi}(x, Q^2) \right] + O(x^{2-\delta}),
$$

$$
F_L(x, Q^2) = r_{Lg}^{1+\delta}(\xi_{Lg})^{-\delta} g(x/\xi_{Lg}, Q^2) + r_{Ls}^{1+\delta} F_2(x, Q^2) + (r_{Ls}^{\delta} - r_{Ls}^{1+\delta}) x^{1-\delta} \tilde{\xi}(x, Q^2) + O(x^{2-\delta}),
$$

with $\xi_{sg} = r_{sg}^{1+\delta}/r_{sg}^{\delta}$ and $\xi_{Lg} = r_{Lg}^{1+\delta}/r_{Lg}^{\delta}$.

From Eq.(3) and (4) one can obtain $F_L$ as a function of $F_2$ and the derivative

$$
F_L(x, Q^2) = -\xi^{\delta} \left[ 2 \frac{r_{Lg}^{1+\delta} dF_2(x, Q^2)}{dlnQ^2} + \left( r_{Ls}^{1+\delta} - r_{Ls}^{\delta} \right) F_2(x, Q^2) \right] + O(x^{2-\delta}, \alpha x^{1-\delta}),
$$

where the result is restricted to $O(x^{2-\delta}, \alpha x^{1-\delta})$. To arrive to the above equation we have performed the substitution

$$
\xi_{sg} / \xi_{Lg} \rightarrow \xi = \gamma_{sg}^{(0),1+\delta} B_{L}^{\eta,\xi} / \gamma_{sg}^{(0),\xi} B_{L}^{\eta,1+\delta}
$$
and neglected the term \( \sim s'(x_{sg}, Q^2) \).

This replacement is very useful. The NLO anomalous dimensions \( \gamma_{sp}^{(1),n} \) are singular in both points, \( n = 1 \) and \( n = 0 \), and their presence into the arguments of \( \tilde{p}(x, Q^2) \) makes the numerical agreement between this approximate formula and the exact calculation worse (we have checked this point using some MRS sets \( \text{MS} \) of parton distributions).

Using NLO approximation of \( r_{sp}^{1+\delta} \) and \( r_{Lp}^{1+\delta} \) for concrete values of \( \delta = 0.5 \) and \( \delta = 0.3 \) we obtain (for \( f=4 \) and \( \text{MS} \) scheme):

\[
\text{if } \delta = 0.5 \\
F_L(x, Q^2) = \frac{0.87}{1 + 22.9\alpha} \left[ \frac{dF_2(0.70x, Q^2)}{d\ln Q^2} + 4.17\alpha F_2(0.70x, Q^2) \right] + O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}), \ (8)
\]

\[
\text{if } \delta = 0.3 \\
F_L(x, Q^2) = \frac{0.84}{1 + 59.3\alpha} \left[ \frac{dF_2(0.48x, Q^2)}{d\ln Q^2} + 3.59\alpha F_2(0.48x, Q^2) \right] + O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}). \ (9)
\]

With the help of Eqs. (1) and (8)-(9) we have extracted the ratio \( R(x, Q^2) \) from H1 1994 data \( \text{[1]} \), determining the slopes \( dF_2/d\ln Q^2 \) from straight line fits as in ref. \( \text{[4, 6]} \). In the present calculation only statistical errors have been taken into account, and we have used \( \Lambda^{(4)}_{\text{MS}} = 225 \text{MeV} \) in the calculation of the running coupling constant \( \alpha_s(Q^2) \) at two loops.

Figure 1a shows the extracted ratio \( R \) at \( Q^2 = 20 \text{ GeV}^2 \) for two different values of the parameter \( \delta \). It also shows BCDMS \( \text{[15]} \) and preliminary CCFR (see \( \text{[16]} \)) data points where the errors are very much larger. For comparison we have also plotted various predictions for \( R \) using QCD formulas at \( O(\alpha_s^2) \) \( \text{[17]} \) and parton densities extracted from fits to HERA data. The large difference between the result from MRS(G) and the latest set MRS(R1) \( \text{[18]} \) shows, as it is expected, the large effect on \( R \) of the unknown of the gluon distribution at small \( x \).

In figure 1a one can also see that the result from MRS(R1) fits very well the points obtained with \( \delta = 0.5 \) for the lowest \( x \) data, although it fails to account for the highest \( x \) bins. The calculation with MRS(D-) is also statistically compatible with our data.

By other part recent theoretical predictions on \( R \) based on conventional NLO DGLAP evolution analysis of HERA data (LBY) \( \text{[19]} \) and on the dipole picture of BFKL dynamics (NPRW) \( \text{[20]} \), both finding values \( \delta = 0.3 \), lie closer to the data points obtained with \( \delta = 0.3 \) Eq. (9).

Finally Fig. 1b shows \( R \) for \( \delta = 0.3 \) (the value favoured by H1 data \( \text{[3]} \)) and at three different \( Q^2 \) values in comparison with the SLAC R(1990) parametrization \( \text{[21]} \). One can see the very good agreement at \( x \leq 10^{-2} \) even if only the statistical errors are taken into account.

Notice that the points at the same \( x \) and different \( Q^2 \) are correlated by the form in which the derivative term \( dF_2/d\ln Q^2 \) is determined.

In summary, we have presented Eqs. (1) and (7)-(9) for the extraction of the ratio \( R = \sigma_L/\sigma_T \) at small \( x \) from the SF \( F_2 \) and its \( Q^2 \) derivative. These equations provide the possibility of the non-direct determination of \( R \). This is important since the direct
extraction of $R$ from experimental data is a cumbersome procedure (see [2]). Moreover, the fulfillment of Eqs. (1), (7)-(10) by DIS experimental data is a cross-check of perturbative QCD at small values of $x$. Our formulas can also be used as a parametrization of $R$ as a function of the most widely used phenomenological $F_2$.

We have found that the results depend on the concrete value of the slope $\delta$. In the case $\delta = 0.3$, which is very close to the values obtained by H1 group [3] at the considered $Q^2$ interval, we found very good agreement with the SLAC parametrization [21] and also a relatively good agreement with the studies based on NLO DGLAP and BFKL dynamics (see [19] and [20], respectively). However the calculation performed with the latest sets of HERA parton densities using perturbative QCD at second order (see MRS(R1) curve in Fig. 1a) predicts an slightly higher value for $R$.

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c) Although with the theoretical prejudice contained in the starting QCD relation.
d) We use PDF multiplied by $x$ and neglect the nonsinglet quark distribution at small $x$.
e) Hereafter we use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.
f) Because we consider here $F_2(x, Q^2)$ but not the singlet quark distribution.

References

[1] R. G. Roberts and M. R. Whalley, J. of Phys. G17, D1 (1991).
[2] A. M. Cooper-Sarkar et al., Z. Phys. C39, 281 (1988); M. W. Krasny et al., Z. Phys. C53, 687 (1992); L. Favart et al., Report No. LAL 96-32, IIHE 96-01 (hep-ph/9606465) (unpublished).
[3] T. Ahmed et al., H1 Collab., Nucl. Phys. B470, 3 (1996).
[4] T. Aid et al., H1 Collab., Phys. Lett. B354, 494 (1995).
[5] A. V. Kotikov, Yad. Fiz. 57, N1, 142 (1994); Phys. Rev. D49, 5746 (1994).
[6] M. Derrick et al., ZEUS Collab., Phys. Lett. B345, 576 (1995); Z. Phys. C65, 379 (1995).
[7] K. Prytz, Phys. Lett. B311, 286 (1993); B332, 393 (1994) 393; R. K. Ellis, Z. Kunszt and E. M. Levin, Nucl. Phys. B420, 517 (1994).
[8] A.V. Kotikov and G. Parente, Phys. Lett. B379, 195 (1996).
[9] A. V. Kotikov and G. Parente, Report No. US-FT-19-96 (hep-ph/9605207) (unpublished).

[10] A. D. Martin, W. S. Stirling, and R. G. Roberts, Phys. Lett. B306, 145 (1993); Phys. Lett. B354, 155 (1995).

[11] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, ZHETF 53, 2018 (1976), 54, 128 (1977); Ya. Ya. Balitzki and L. N. Lipatov, Yad. Fiz. 28, 822 (1978); L. N. Lipatov, ZHETF 63, 904 (1986).

[12] P. Amaudruz et al, NMC Collab., Phys. Lett. B295B, 159 (1992); M. Arneodo et al., NMC Collab., Report No. CERN-PPE/95-138.

[13] A. Donnachie and P. V. Landshoff, Nucl. Phys. 244B, 322 (1984); 267B, 690 (1986).

[14] P. V. Landshoff and O. Nachmann, Z. Phys. C35, 405 (1987).

[15] A. C. Benvenuti et al., BCDMS Collab., Phys. Lett. B237, 592 (1990).

[16] H. Abramowicz, “Test of QCD at low x”, talk given at the EPS Conference, Warsaw 1996.

[17] D.I. Kazakov, A.V. Kotikov, G. Parente, O.A. Sampayo and J. Sánchez Guillén, Phys. Rev. Lett. 65 (1990) 1535, 2921 (E); J. Sánchez Guillén, J.L. Miramontes, M. Miramontes, G. Parente and O.A. Sampayo, Nucl. Phys. B353 (1991) 337; D.I. Kazakov and A.V. Kotikov, Phys. Lett. B291 (1992) 171; G. Parente and J. Sánchez Guillén in Proc. of the XXII Int. Symposium on Multiparticle Dynamics, Santiago de Compostela, July 1992; E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. B383 (1992) 525.

[18] A. D. Martin, R. G. Roberts and W. S. Stirling, Report No. RAL-TR-96-037, DTP/96/44.

[19] C. Lopez, F. Barreiro and F. J. Yndurain, Report No. hep-ph/9605395 (unpublished).

[20] H. Navelet et al., Report No. SPhT T96/043, DAPNIA/SPP 96-08 (hep-ph/9605389) (unpublished).

[21] L. W. Whitlow et al., SLAC Collab., Phys. Lett. B250, 193 (1990).

Figure captions

Figure 1: The ratio $R = \sigma_L/\sigma_T$ at small $x$. The points were extracted from Eqs. (1), (8) and (9) using H1 [3, 4] data. The dashed-dotted line (NPRW) is the prediction of Saclay group [20] based on the dipole picture of BFKL dynamics. The band represent the uncertainty from the DGLAP analysis of HERA data by [19]. It is also shown BCDMS data [15] points at high $x$ and the preliminary CCFR data point from [16]. The solid lines in Fig. 1b are the SLAC R1990 parametrization [21] at $Q^2 = 8.5, 20$ and 35 GeV$^2$ (lower curve corresponds to lower $Q^2$ value).
