Heavy flavours production in deeply inelastic scattering and gluon density in the proton

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Abstract

Heavy flavours production in e-p DIS is studied at intermediate values of the transferred four-momentum square, under the assumption of boson-gluon-fusion mechanism dominance (no intrinsic heavy flavours contributions).

In this framework different expressions for the splitting functions in the gluon density evolution equation, with respect to the standard (DGLAP) ones, are explicitly derived.
Lepton-proton deeply inelastic scattering can be studied perturbatively in the single-photon-exchange approximation neglecting electroweak corrections ($Z_0$ exchange) when $\Lambda_{QCD}^2 \ll Q^2 \ll M_Z^2$ ($q^2 = -Q^2$ is the square of the transferred four momentum).

The following expression for the differential cross section, with respect to $Q^2$ and the Bjorken scaling variable $x$, can be obtained by Lorentz-invariance and current conservation arguments:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ 2xy^2 F_1(x, Q^2) + 2(1-y) F_2(x, Q^2) \right]$$

(1)

$\alpha$ is the e.m fine structure constant ($\alpha \simeq \frac{1}{137}$) and $y$ is the fraction of energy lost by the lepton, during the collision, in the proton rest frame.

The proton electromagnetic structure functions $F_{1,2}(x, Q^2)$ satisfy the following factorization formulas [1] [2] [3]

$$F_1(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi} C_1^{(i)} \left( \frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \rho_i(\xi, \epsilon, \mu^2)$$

(2)

$$F_2(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi} C_2^{(i)} \left( \frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \rho_i(\xi, \epsilon, \mu^2)$$

(3)

where $\mu$ is the renormalization scale and $\rho_i$ is the density of the partonic species $i$ in the proton with respect to the fraction of longitudinal momentum carried by the parton itself ($\xi$), while $\epsilon$ stands for the infrared cutoff used to regularize collinear divergences.

The coefficients $C_{1,2}^{(i)} \left( \frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right)$ take into account the short-distance effects, they are infrared safe because of a Bloch-Nordsieck type compensation of soft divergences [4] [4] and can be computed in perturbative QCD.

The long-distance dynamics is entirely factorized in the parton densities $\rho_i(\xi, \epsilon, \mu^2)$.

The scale independence condition for the structure function $F_2$

$$\frac{\partial F_2(x, Q^2)}{\partial \log \mu^2} = 0$$

leads to the renormalization group equations for the quarks densities, id est

$$\frac{\partial \rho_f(x, \epsilon, \mu^2)}{\partial \log \mu^2} = \alpha_s(\mu^2) \int_1^x \frac{d\xi}{\xi} \left[ P_{ff} \left( \frac{x}{\xi} \right) \rho_f \left( \xi, \epsilon, \mu^2 \right) + \right.$$

$$\left. \right.$$
\[ P_{fg} \left( \frac{x}{\xi} \right) \rho_g \left( \xi, \epsilon, \mu^2 \right) \]  

(4)

where \( \alpha_s (\mu^2) \) is the strong running coupling constant.

The explicit expressions of the splitting functions \( P_{ff} (z) \) and \( P_{fg} (z) \) are found to be [5] [6]

\[ P_{ff} (z) = C_2 (F) \left( \frac{1 + z^2}{1 - z} \right) , \quad P_{fg} (z) = T (F) \left[ z^2 + (1 - z)^2 \right] \]  

(5)

where in general \([F (z)]_+\) is a distribution defined as

\[ \int_x^1 dz [F (z)]_+ \phi (z) = \int_x^1 dz F (z) [\phi (z) - \phi (1)] \]

and

\[ C_2 (F) = \frac{N^2 - 1}{2N} = \frac{4}{3} , \quad T (F) = \frac{1}{2} \]

Charm production in \( e^+p \) DIS has been extensively studied by ZEUS and H1 collaborations at HERA electron-proton collider [7] [8] [9] [10].

In particular the charm contribution to the electromagnetic structure functions of the proton is estimated from the measurements of the cross sections for inclusive \( D^{*+} \) production, given the value for the hadronization fraction of a quark charm in \( D^{*+} \) by OPAL Collab. \((e^+ e^- \) annihilation [11]):

\[ f \left( c \rightarrow D^{*+} \right) = 0.222 \pm 0.014 \pm 0.014 \]

Experimental data can be explained in perturbative QCD by assuming that the proton wave function does not possess any heavy flavour content; under this hypothesis heavy quarks production in e-p DIS is thought to be governed by boson-gluon-fusion (BGF) pair production according to the partonic subprocess (figure 1)

\[ \gamma^* g \rightarrow q_h \bar{q}_h X \]

That kind of experiments provides important informations on the gluonic structure of the proton; actually the most precise measurements of the gluon density in the proton available nowadays are coming from HERA experiments [10] [12].
The differential cross section for inclusive production of a pair of heavy quarks in virtual photon-proton scattering
\[ \gamma^* (q) \ p (P) \rightarrow q_h \bar{q}_h \left( M^2 \right) \ X \]
in pair invariant mass kinematics, enjoys the following factorization theorem
\[ \frac{d\sigma^{(hh)}}{dM^2} (x, M^2, Q^2, m_h^2) = \frac{x^2}{Q^4} \sum_i \int_x^1 \frac{d\xi}{\xi} \rho_i (\xi, \epsilon, \mu^2) \]
where \( m_h \) represents the heavy flavour mass, \( M \) is the invariant mass of the pair while the sum is understood to be over all massless partonic species (gluon and light (anti)quarks).
The coefficient functions \( \hat{\omega}_i \left( \frac{x}{\xi}, \frac{Q^2}{\mu^2}, \frac{M^2}{\mu^2}, \frac{m_h^2}{\mu^2}, \alpha_s (\mu^2) \right) \) include, as usual, the short-distance dynamics; they turn out to be infrared safe (compensation of soft divergences) and can be computed in perturbative QCD.
The collinear divergences are regularized and factorized in the parton densities \( \rho_i (\xi, \epsilon, \mu^2) \).

\[ \text{Figure 1: BGF mechanism} \]

All physical quantities turn out to be scale independent, then from the condition
\[ \frac{\partial}{\partial \log \mu^2} \left( \frac{d\sigma^{(hh)}}{dM^2} (x, M^2, Q^2, m_h^2) \right) = 0 \]
one may obtain the renormalization group equation for the density of gluons; at leading order it gives

\[
\frac{\partial \rho_g(x, \epsilon, \mu^2)}{\partial \log \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left[ \sum_f P_{gf} \left( \frac{x}{\xi} \right) \rho_f \left( \xi, \epsilon, \mu^2 \right) + P_{gg} \left( \frac{x}{\xi} \right) \rho_g \left( \xi, \epsilon, \mu^2 \right) \right]
\]

(7)

The Born term of the lepton-proton cross sections for heavy flavours production in deeply inelastic scattering coincides with the lowest order contribution of the transition

\[ e \ g \rightarrow e \ q_h \ \bar{q}_h \]

It comes from the partonic diagrams in figure 2:

![Figure 2: Born terms for e g \rightarrow e q_h \ \bar{q}_h](image)

The calculations are straightforward and have already been performed in references [13] [14].

This contribution to the double-differential cross section \( \frac{d^2\sigma}{dxdQ^2} \) can be written in the same way as in eq.(1):

\[
\frac{d^2\sigma_B^{(hh)}}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ 2xyF_1^{hh}(x, Q^2, m_h) + 2(1-y)F_2^{hh}(x, Q^2, m_h) \right]
\]

(8)
where $F_{1,2}^{hh}(x, Q^2, m_h)$ are the lowest order contributions of the heavy flavour $h$ to the e.m structure functions of the proton.

They can be expressed as follows

$$
F_1^{hh}(x, Q^2, m_h) = \frac{\alpha_s Q_h^2}{4\pi} \int_{z_{\text{min}}}^{z_{\text{max}}} \frac{dz}{z} \rho_g \left( \frac{x}{z}, \mu^2 \right) \left[ G_1(z, \frac{m_h^2}{Q^2}) - G_2(z, \frac{m_h^2}{Q^2}) \right]
$$

$$
F_2^{hh}(x, Q^2, m_h) = \frac{\alpha_s Q_h^2 x}{2\pi} \int_{z_{\text{min}}}^{z_{\text{max}}} \frac{dz}{z} \rho_g \left( \frac{x}{z}, \mu^2 \right) \left[ G_1(z, \frac{m_h^2}{Q^2}) - 3G_2(z, \frac{m_h^2}{Q^2}) \right]
$$

with $z_{\text{min}} = x$, $z_{\text{max}} = \frac{Q^2}{Q^2 + 4m_h^2}$ and

$$
G_1(z, \frac{m_h^2}{Q^2}) = -\left[ z^2 + (1 - z) \left( 1 - z + \frac{4zm_h^2}{Q^2} \right) \right] \sqrt{1 + \frac{4z - m_h^2}{z - 1} \frac{Q^2}{Q^2}}
$$

$$
+ \left[ z^2 + (1 - z)^2 + \frac{4zm_h^2}{Q^2} \left( 1 - \frac{2zm_h^2}{Q^2} \right) \right] \ln \left( \frac{1 + \sqrt{1 + \frac{4z - m_h^2}{z - 1} \frac{Q^2}{Q^2}}}{1 - \sqrt{1 + \frac{4z - m_h^2}{z - 1} \frac{Q^2}{Q^2}}} \right)
$$

$$
G_2(z, \frac{m_h^2}{Q^2}) = 2z^2 \left\{ \frac{2m_h^2}{Q^2} \ln \left( \frac{1 + \sqrt{1 + \frac{4z - m_h^2}{z - 1} \frac{Q^2}{Q^2}}}{1 - \sqrt{1 + \frac{4z - m_h^2}{z - 1} \frac{Q^2}{Q^2}}} \right) + \frac{z - 1}{z} \left[ 1 + \frac{4z - m_h^2}{z - 1} \frac{Q^2}{Q^2} \right] \right\}
$$

while $Q_h$ is the heavy quark electric charge (in e units).

The aim of this paper is to derive the explicit form of the splitting functions $P_{gf}(z)$ and $P_{gg}(z)$ which appear in the evolution equation for the gluon density, directly from the study of heavy flavours production in e-p DIS.

Then these results are compared with the corresponding quantities obtained by Altarelli and Parisi in a different framework [5] [6].

For this purpose we start by considering the next-to-leading order (NLO), quark initiated corrections to the Born term, namely the lowest order contributions to

$$
e q_f \to e q_f q_h \bar{q}_h$$
where $q_f$ is here any light quark. They are given by the diagrams shown in figures 3 and 4 (all the calculations in the present paper are made in Feynman Gauge).

The corresponding contribution to the differential electron-proton cross section is expressed, in the framework of free parton model, as a convolution over the free (light)quark density

$$\frac{d^2\sigma_f^{(hh)}}{dx dQ^2} = \int_x^1 \frac{d\xi}{\xi} \rho_f(\xi) \frac{d^2\hat{\sigma}_f^{(hh)}}{dz dQ^2}$$

where $z = \frac{x}{\xi}$ represents the “partonic” Bjorken variable and $\frac{d^2\hat{\sigma}_f^{(hh)}}{dz dQ^2}$ is the corresponding lepton-parton cross section.

Figure 3: NLO quark-initiated corrections; the e.m interaction involves the light quark

Figure 4: NLO quark-initiated corrections with the virtual photon probing a heavy flavour
One can easily realize that the collinear divergent part of \( \frac{d^2\sigma^{(hh)}}{dx dQ^2} \) comes from the last two diagrams in figure 4; it can be written in the same form as the Born contribution

\[
\frac{d^2\sigma_B^{(hh)}}{dx dQ^2} = \int_0^1 \frac{d\xi}{\xi} \rho_g (\xi) \frac{d^2\hat{\sigma}_B^{(hh)}}{dz dQ^2}
\]

(9)
simply by substituting the “free” gluon density \( \rho_g (\xi) \) with its next-to-leading order correction \([15]\)

\[
\delta \rho_g^{(1)} (\xi, \frac{Q^2}{Q_0^2}) = \frac{\alpha_s}{2\pi} \log \left( \frac{Q^2}{Q_0^2} \right) \int_\xi^1 \frac{d\lambda}{\lambda} P_{gf} \left( \frac{\xi}{\lambda} \right) \rho_f (\lambda)
\]

where \( Q_0 \) is the infrared cutoff used to regularize the light quark-gluon collinear singularities (we choose \( Q \) as factorization/renormalization scale).

Therefore we can regularize these NLO collinear divergences and reabsorb them in the Born term through the following redefinition of the gluon density in eq.(9)

\[
\rho_g (\xi) \Rightarrow \rho_g (\xi) + \delta \rho_g^{(1)} (\xi, \frac{Q^2}{Q_0^2})
\]

(10)

The splitting function \( P_{gf} (z) \) is given by

\[
P_{gf} (z) = C_2 (F) z = \frac{4}{3} z
\]

(11)
It should be noticed that this expression vanishes linearly in the limit \( z \to 0 \) and that it is different from the following formula obtained in references \([3]\) \([3]\)

\[
P_{gf} (z) = C_2 (F) \frac{1 + (1 - z)^2}{z}
\]

To compute now the explicit expression for the gluon-gluon splitting function \( P_{gg} (z) \), the next-to-leading order, gluon initiated corrections, due to real gluon emission, have been derived in the collinear limit.

Essentially the following partonic process has to be studied (at lowest order):

\[
e g \to e g q_h \bar{q}_h
\]

There are in total 8 diagrams to consider, they are shown in figures 5 and 6:
Figure 5: NLO gluon-initiated corrections involving only quark-gluon vertices
The contribution of those diagrams to the electron-proton \( \frac{d^2 \sigma}{dx dQ^2} \) cross section can be written, in the free parton model, as a convolution over the free gluon density:

\[
\frac{d^2 \sigma_g^{(hh)}}{dx dQ^2} = \int_x^1 \frac{d\xi}{\xi} \rho_g(\xi) \frac{d^2 \hat{\sigma}_g^{(hh)}}{dz dQ^2}
\]

where \( \frac{d^2 \hat{\sigma}_g^{(hh)}}{dz dQ^2} \) is the corresponding lepton-gluon cross section.

The leading term in the gluon-gluon collinear limit comes from the last 2 diagrams in Figure 6; furthermore it can be written in the same form as the Born contribution if one replaces the free gluon density \( \rho_g(\xi) \) in eq.(9) with the next-to-leading order correction

\[
\delta \rho_g^{(2)}(\xi, \frac{Q^2}{Q_0^2}) = \frac{\alpha_s}{2\pi} \log \left( \frac{Q^2}{Q_0^2} \right) \int_\xi^1 \frac{d\lambda}{\lambda} P_{gg} \left( \frac{\xi}{\lambda} \right) \rho_g(\lambda)
\]

where the explicit expression for the splitting function \( P_{gg}(z) \), valid for \( z < 1 \), is given by \[15\]

\[
P_{gg}(z) = 2C_2(A) \left( \frac{z}{1-z} + z(1-z) \right), \quad C_2(A) = N = 3
\]

Notice that eq.(12) differs from the standard expression derived in references \[3\] \[4\], id est

\[
P_{gg}(z) = 2C_2(A) \left[ \frac{z}{1-z} + z(1-z) + \frac{1-z}{z} \right]
\]
Moreover we point out that $P_{gg}(z)$ in eq.(12) vanishes linearly as $z \to 0$ and that it is not invariant under $z \leftrightarrow 1-z$ exchange contrary to the previous expression. In addition both $P_{gf}(z)$ in eq.(11) and $P_{gg}(z)$ in eq.(12) can be obtained from the corresponding Altarelli-Parisi results by neglecting all terms that do not vanish as $z \to 0$. Therefore the NLO gluon-gluon collinear divergences can be regularized and then reabsorbed in the following redefinition of the gluon density in the “Born level” term (eq.(9))

$$\rho_g(\xi) \Rightarrow \rho_g(\xi) + \delta \rho_g^{(2)} \left( \xi, \frac{Q^2}{Q_0^2} \right)$$ (14)

Finally $\delta \rho_g^{(2)} \left( \xi, \frac{Q^2}{Q_0^2} \right)$ turns out to be divergent in view of a soft singularity in the limit $\frac{\xi}{\lambda} \to 1$ (gluon emitted with arbitrarily small energy). It is a peculiar feature of gauge theories due to the fact that gauge bosons are massless. If one examines the NLO interference terms of the virtual corrections with the Born contributions for $e g \to e q_h \bar{q}_h$ transition, related to collinear divergences, soft singularities are found, which compensate the analogous divergences in the real emission contributions. This mechanism is similar to the Bloch-Nordsieck compensation well known in QED [1] [4].

The mentioned virtual corrections are given by the six diagrams in figures 7 and 8 (three more diagrams are obtained from those in figure 7 by reversing the heavy quark line). The loops in figure 7 involve only massless states; they are indeed scaleless integrals which vanish if appropriately defined in dimensional regularization [1]. Only three diagrams are left (figure 8); their interference terms with the Born contributions have been included in the calculation [15].

In conclusion the overall next-to-leading order correction to $d^2 \sigma / dx dQ^2$, for the heavy flavour $h$ production (via boson-gluon-fusion), computed in the collinear limit, can be reabsorbed in the Born contribution by means of the following redefinition of the free gluon density in eq.(9):

$$\rho_g(\xi) \Rightarrow \rho_g(\xi) + \delta \rho_g \left( \xi, \frac{Q^2}{Q_0^2} \right)$$ (15)
with
\[
\delta \rho_g \left( x, \frac{Q^2}{Q_0^2} \right) = \frac{\alpha_s}{2\pi} \log \left( \frac{Q^2}{Q_0^2} \right) \int_x^1 \frac{d\xi}{\xi} \left[ \sum_f P_{gf} \left( \frac{x}{\xi} \right) \rho_f (\xi) + P_{gg} \left( \frac{x}{\xi} \right) \rho_g (\xi) \right]
\]

The splitting functions \( P_{gf} (z) \) and \( P_{gg} (z) \) are given by
\[
P_{gf} (z) = C_2 (F) z , \quad P_{gg} (z) = 2C_2 (A) z \left[ \frac{1}{(1-z)_+} + (1-z) \right]
\]
The sum is understood to be over all light (anti)quarks.

Therefore the effective gluon density involved in heavy flavours production in e-p DIS, at intermediate values of the factorization scale, follows a different evolution equation with respect to usual DGLAP eqs.; needless to say this fact affects the (anti)quarks densities too as soon as the evolution equations are coupled.

Figure 7: Gluon self energy contributions of order \( \alpha_s \)
This feature is due to the assumption on the dominance of boson-gluon-fusion mechanism; it is indeed a consequence of the explicit flavour symmetry breaking (intrinsic heavy flavour contributions are suppressed) and it is expected to be verified at moderate values of the scale $\mu$ with respect to $m_h$.

For $\mu$ values much higher mass effects are negligible and the heavy flavour $h$ behaves as a light (massless) flavour; the intrinsic contributions to the cross sections can no longer be neglected. In that case the gluon density follows again the standard DGLAP evolution equation so as the density for $h$ quarks.

Figure 8: $e\,g \to e\,q_h\,\bar{q}_h$ virtual corrections
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