Statefinder Parameters for Tachyon Dark Energy Model

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In this paper we study the statefinder parameters for the tachyon dark energy model. There are two kinds of stable attractor solutions in this model. The statefinder diagrams characterize the properties of the tachyon dark energy model. Our results show that the evolving trajectories of the attractor solutions lie in the total region and pass through the LCDM fixed point, which is different from other dark energy model.

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I. INTRODUCTION

Current astrophysical observations indicate that our universe has entered a phase of accelerated expansion in the recent past.1,2 The accelerated expansion has been attributed to the existence of mysterious dark energy with negative pressure. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment tells us that the usual baryonic matter, dark matter and dark energy occupy about 4%, 23% and 73% of the total energy of the universe, respectively. At present, candidates for dark energy have been widely studied. With the exclusion of the cosmological constant Λ, most dark energy modelling using scalar field has followed the quintessence3,4 paradigm of a slowly rolling canonical scalar field. However, there has been increasing interest in loosening the assumption of a canonical kinetic term. In its most general form, this idea is known as k-essence.5 A more specific choice is the tachyon,6 which can be viewed as a special case of k-essence models with Dirac-Born-Infeld (DBI) action.7 This kind of scalar field is motivated by string theory as the negative-mass mode of the open string perturbative spectrum. Sen6 showed that the decay of D-branes produces a pressureless gas with finite energy density that resembles classical dust. A rolling tachyon has an interesting equation of state whose parameter smoothly interpolates between −1 and 0. This has led to flurry of attempts being made to construct viable candidate for the inflaton at high energy. Meanwhile the tachyon can also act as a source of dark energy depending on the form of tachyon potential. Tachyon dark energy has been explored by many authors, for example Refs [8,9,10]. One of these studies is to investigate whether there are any distinctive signatures of non-canonical actions available to be probed by observations.

Since there are more and more models proposed to explain the cosmic acceleration, it is very desirable to find a way to discriminate between the various contenders in a model independent manner. In Ref [11], Sahni et al proposed a cosmological diagnostic pair \{r, s\} called statefinder, which is defined as

\[ r \equiv \frac{\ddot{a}}{aH^3}, \quad s \equiv \frac{r - 1}{3(q - 1/2)}. \] (1)

to differentiate among different forms of dark energy. Here \( q \) is the deceleration parameter. The statefinder is a geometrical diagnostic that it depends on the scalar factor \( a \). Since different cosmological models involving dark energy exhibit qualitatively different evolution trajectories in the \( s-r \) plane, this statefinder diagnostic can differentiate these dark energy models.

In this paper, we will study the statefinder parameters for the tachyon field dark energy model. It is found that the evolving trajectories of this model are different from other dark energy models. The paper is organized as follows: In Sec. 2, we specify the form of tachyonic potential and obtain the autonomous equations of tachyon dark energy model. In Sec. 3, it is analyzed to the evolving trajectories of this model in the statefinder parameter plane. Section 4 is our conclusions.

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II. THE TACHYON DARK ENERGY MODEL

The evolution equations for a flat FRW cosmological model filled with a tachyon field $T$ and a barotropic perfect fluid with equation of state $p_\gamma = (\gamma - 1)\rho_\gamma$ are

$$\begin{align*}
\frac{\ddot{T}}{1 - T^2} + 3H\dot{T} + \frac{1}{V(T)} \frac{dV(T)}{dT} &= 0, \\
\dot{\rho}_\gamma + 3\gamma H \rho_\gamma &= 0,
\end{align*}$$

which are subject to the Friedmann equation

$$3H^2 = \frac{V(T)}{\sqrt{1 - T^2}} + \rho_\gamma,$$

where dots denote differentiation with respect to cosmic time $t$ and $H$ is the Hubble parameter.

The density $\rho_T$ and pressure $p_T$ of tachyon field are given by, then

$$\begin{align*}
\rho_T &= \frac{V(T)}{\sqrt{1 - T^2}}, \\
p_T &= -V(T)\sqrt{1 - T^2}.
\end{align*}$$

We introduce the following dimensionless quantities:

$$x \equiv \dot{T}, \quad y \equiv \frac{V(T)}{3H^2}, \quad z \equiv \frac{\rho_\gamma}{3H^2}.$$ 

Thus the fractional densities of the two fluids are defined as, respectively

$$\begin{align*}
\Omega_T &= \frac{\rho_T}{3H^2} = \frac{y}{\sqrt{1 - x^2}}, \\
\Omega_\gamma &= \frac{\rho_\gamma}{3H^2} = z.
\end{align*}$$

Consider the inverse square potential $V(T) = \beta T^{-2}$ now. We can cast the evolution equations in the following autonomous form:

$$\begin{align*}
\frac{dx}{dN} &= 3(x^2 - 1)(x - \sqrt{\alpha y}), \\
\frac{dy}{dN} &= 3y[x(x - \sqrt{\alpha y}) + z(\gamma - x^2)], \\
\frac{dz}{dN} &= 3z(z - 1)(\gamma - x^2),
\end{align*}$$

where the number of e-folds $N \equiv \ln a$ and the constant $\alpha \equiv 4/3\beta > 0$. From the definitions of these new variables, we obtain the equation $\omega_T$ of state of tachyon field

$$\omega_T = \frac{p_T}{\rho_T} = x^2 - 1.$$ 

III. THE STATEFINDER PARAMETERS FOR TACHYON DARK ENERGY MODEL

In order to discriminate between different forms of dark energy, Sahni et al$^{11}$ proposed a geometrical diagnostic method—statefinder papameter. Since different cosmological models involving dark energy exhibit qualitatively different evolution trajectories in the $s - r$ plane, this statefinder diagnostic can differentiate various kinds of dark energy models. By far some models, including the cosmological constant, quintessence, phantom, quintom, the Chaplygin gas, holographic models, interacting and coupling dark energy models$^{11,14-20}$ have been successfully differentiated. For example, the statefinder parameters correspond to a fixed point $\{ r = 1, s = 0 \}$ for the spatially flat LCDM cosmological model which is a good fit for the observation$^{21,22}$, the quintessence model with inverse power law potential, the phantom model with power law potential and the chaplygin gas model all tend to approach the LCDM fixed
In this paper, we study the statefinder parameters of the tachyon dark energy model. We analyze two cases of the attractor solutions: tachyon dominated solution and tracking solution. The statefinder diagrams characterize the difference from the other dark energy models. In Fig. 1 the statefinder parameter pairs \(\{r, s\}\) and \(\{r, q\}\) for different values of \(\gamma\) are shown. The equations of state for the tachyon field model. To begin with, statefinder parameters (1) can be rewritten as

\[
\begin{align*}
  r &= 1 + \frac{9}{2} \gamma^2 - \frac{9}{2} \gamma + \frac{1}{2} (9 \gamma - 9 \gamma^2) \Omega_T + \frac{9}{2} \omega_T (1 + \omega_T) \Omega_T - \frac{3}{2} \omega'_T \Omega_T, \\
  s &= \frac{(3 \gamma^2 - 3 \gamma)(1 - \Omega_T) + 3 \Omega_T \omega_T (1 + \omega_T) - \Omega_T \omega'_T}{3(\gamma - 1)(1 - \Omega_T) + 3 \Omega_T \omega_T},
\end{align*}
\]

which \(\omega'_T = d\omega_T/dN\), and the deceleration parameter is also given

\[
q = \frac{3}{2} \gamma^2 - 1 - \frac{3}{2} (\gamma - 1) \Omega_T + \frac{3}{2} \omega_T \Omega_T.
\]

In the following we will discuss the statefinder for the attractor solutions in Ref [13]: tachyon dominated solution and tracking solution. Firstly, we discuss the attractor solution \(P\) at \((x, y) = (\sqrt{\alpha y_1}, y_1)\) \((y_1 = \sqrt{\frac{\alpha + 1 - \alpha}{2}})\), which is an asymptotically stable mode and corresponds to tachyon dominated solutions for \(\gamma > \gamma_1\) \((\gamma_1 = \alpha y_1)\). Nextly, there exists another attractor solution \(Q\) at \((x, y) = (\sqrt{\gamma}, \frac{1}{2})\). When \(0 \leq \gamma < \gamma_1\), \(Q\) is always asymptotically stable and corresponds to the tracking solution. In Ref [13] the constant \(\alpha = 1.5\), we can obtain \(y_1 = 0.5\) and \(\gamma_1 = 0.75\). With the condition of the above attractor solutions, we consider the constants \(\gamma = 4/3\) for the tachyon dominant solution and \(\gamma = 0.3\) for the tracking solution. In addition, the two special values \(\gamma = 0\) and \(1\) are also studied. In Fig. 1 and Fig. 2 we show the time evolution of the statefinder parameter pairs \(\{r, s\}\) and \(\{r, q\}\). The plot is for variable interval \(N \in [-5, 10]\). We see clearly that the curves pass through the LCDM fixed point \(\{0, 1\}\). And the evolving trajectories lie in the total region, which is different from other quintessence and phantom models. In Fig. 3 and Fig. 4 we plot the evolution trajectories of the statefinder parameters versus \(N\) diagram of the stable attractor solutions. It is noted that \(\gamma\) causes the deviation between statefinder parameters and LCDM scenario. In the figure of \(N - r\), the curve for \(\gamma = 0\) approaches to LCDM scenario after \(N = 0\), while for other cases the distance from the curves to LCDM scenario becomes large with the decreasing of \(\gamma\). In the figure of \(N - s\), the constant \(\gamma\) also causes the deviation between the curves and LCDM scenario. In Fig. 5 the equation of state for the tachyon field has been shown. The equations of state for \(\gamma = 0\) and \(\gamma = 0.3\) tend to \(-1\) and \(-0.7\) after oscillations, respectively, while for the other two cases \(\omega_T\) would violate the basic condition \(\omega < -1/3\), which can not lead to the acceleration. So, through the statefinder diagnostic, we not only characterize the properties of the tachyon dark energy model, but also show the difference from the other dark energy models.

IV. CONCLUSIONS

In this paper, we study the statefinder parameters of the tachyon dark energy model. We analyze two cases of the stable attractor solutions: tachyon dominated solution and tracking solution. The statefinder diagrams characterize the properties of the tachyon dark energy model. Our results show that the evolving trajectories of this model lie in the total region and pass through the LCDM fixed point. The statefinder diagnostic can differentiate the tachyon model.
FIG. 2: The figure is $q - r$ diagram of the stable attractor solutions. The curves evolve in the variable interval $N \in [-5, 10]$. Selected curves for $\alpha = 1.5$, $\gamma = 0$, $\gamma = 0.3$, $\gamma = 1$ and $\gamma = 4/3$ respectively. Dots locate the current values of the statefinder parameters.

FIG. 3: The figure is $N - r$ diagram of the stable attractor solutions in the variable interval $N \in [-5, 10]$. Selected curves for $\alpha = 1.5$, $\gamma = 0$, $\gamma = 0.3$, $\gamma = 1$ and $\gamma = 4/3$ respectively.

from other dark energy models. We hope that the future high precision observation will be capable of determining these statefinder parameters and consequently shed light on the nature of dark energy.

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FIG. 4: The figure is $N - s$ diagram of the stable attractor solutions in the variable interval $N \in [-5, 10]$. Selected curves for $\alpha = 1.5$, $\gamma = 0$, $\gamma = 0.3$, $\gamma = 1$ and $\gamma = 4/3$ respectively.
FIG. 5: The figure is the equation of state for the tachyon field $\omega_T$ versus $N$. The curves evolve in the variable $N \in [-5, 10]$. Selected curves for $\alpha = 1.5$, $\gamma = 0$, $\gamma = 0.3$, $\gamma = 1$ and $\gamma = 4/3$ respectively.

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