OBSERVATION OF PLANAR THREE-JET EVENTS IN $e^+e^-$ ANNIHILATION AND EVIDENCE FOR GLUON BREMSSTRAHLUNG

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Topological distributions of charged and neutral hadrons from the reaction $e^+e^- \rightarrow$ multihadrons are studied at $\sqrt{s}$ of about 30 GeV. An excess of planar events is observed at a rate which cannot be explained by statistical fluctuations in the standard two-jet process. The planar events, mostly consisting of a slim jet on one side and a broader jet on the other, are shown actually to possess three-jet structure by demonstrating that the broader jet itself consists of two collinear jets in its own rest system. Detailed agreement between data and predictions is obtained if the process $e^+e^- \rightarrow q\bar{q}g$ is taken into account. This strongly suggests gluon bremsstrahlung as the origin of the planar three-jet events. By comparison of the data with the $q\bar{q}g$-model we obtain a value for the strong coupling constant of $\alpha_s(q^2) = 0.17 \pm 0.04$.

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In a previous paper [1], we studied the topological distribution of hadrons produced in the reaction $e^+e^- \rightarrow \text{multihadrons}$. The main features of the data suggested a dominant two-jet structure, which would be expected from the process $e^+e^- \rightarrow q\bar{q}$, with subsequent fragmentation of the quarks into hadrons. A search for spherical events produced no evidence for the production of massive particles.

The perturbative QCD theory of strong interactions predicts [2] a specific type of deviation from two-jet structure due to the gluon bremsstrahlung process $e^+e^- \rightarrow q\bar{q}g$. Soft bremsstrahlung of gluons at small angles to the quark direction will cause a broadening of one of the two jets, whereas hard bremsstrahlung at large angles will produce a well-defined event plane containing the quark, antiquark and gluon, which then fragment into three distinct jets. Therefore, in addition to the predominant two-jet topology, the data should contain a subset of events with a planar configuration [3,4], of which a fraction should exhibit a distinct three-jet like structure. This letter describes the search for such effects and discusses the evidence for the planar and three-jet-like events.

The experiment was carried out at the $e^+e^-$ storage ring PETRA using the JADE detector. The apparatus, triggers and selection of multihadron events have been described elsewhere [5]. The results described here are based on the combined sample of 89 hadronic events obtained at $\sqrt{s} = 27.7$ GeV and 198 events obtained at $\sqrt{s} = 30$ GeV. The data were analysed as follows. Firstly, a comparison was made between the data and a two-jet model [6] which incorporated qq production with subsequent fragmentation. Secondly, a model-independent analysis was carried out to look for three-jet topology by analysing the broader of the two primary jets in its own rest frame to see whether it actually consisted of two jets. Finally, a comparison was made with a jet model [7] which included gluon bremsstrahlung $q\bar{q}g$ in addition to $q\bar{q}$.

The $q\bar{q}g$ model [6,8] included production of $u$, $d$, $s$ and $c$ quarks which were assumed to decay with fragmentation function [6] $f(z) \sim 1 - a + 3a(1 - z^2)$, where $a = 0.77$, and production of $c$ and $b$ quarks with fragmentation function $f(z) = \text{const}$. The transverse momentum ($q_\perp$) distribution of secondary quarks in the hadron-jet cascades was described [6] by $dN/dq_\perp \sim \exp(-q_\perp^2/2\sigma_q^2)$ where two values of $\sigma_q$ were used, a standard value of 250 MeV/c, with which the $q\bar{q}$ model adequately describes low energy data [9], and an arbitrarily large value of 350 MeV/c, which was used for comparison purposes.

In the $q\bar{q}g$ model [7] the $u$, $d$, $s$ and $b$ quarks were produced in the same way, but the $u$, $d$, $s$ and $c$ quarks were then allowed to radiate vector gluons with a probability given by QCD [10]. A value $\alpha_s(q^2)$ = 0.17 was used for the $q\bar{q}$ coupling constant. The quark fragmentation functions were as defined above and the gluon fragmentation function was taken to be $f(z) \sim (1 - z)^2$. The events were generated according to these models by a Monte Carlo program in which hard photon radiation by the initial $e^+$ was taken into account [5]. The final state particles were then followed through the detector and analysed in exactly the same way as the real data.

Jet characteristics were studied by determining the quantity thrust [11] $T = \max_{\alpha}(\sum_i p_i^\alpha/\sum_i |p_i^\alpha|)$ for each event. The axis along which the value of $T$ is maximized provides a good determination of the jet axis, and the magnitude of $T$ indicates how jet-like the event is. The limiting values of $T$ are $T_{\text{min}} = 0.5$ (spherical) and $T_{\text{max}} = 1.0$ (extreme jet). Fig. 1 shows our measured thrust distribution compared with the predictions of the simple $q\bar{q}$ model. The data show an excess of low thrust events compared with the $q\bar{q}$ model expectation. The production of a new flavour (e.g. top) has already been excluded as a possible origin of this broadening on the basis of the total cross section measurement [5] and the absence of spherical events [1].

A more thorough analysis can be made by studying the detailed event shapes. For this purpose the normalized sphericity tensor [12] was constructed and diagonalized for each event: $T_{\alpha\beta} = \sum_i p_{i\alpha} p_{i\beta} / \sum_i p_i^2$, where $\alpha = 0.77$, and production of $c$ and $b$ quarks with fragmentation function $f(z) = \text{const}$. The transverse momentum ($q_\perp$) distribution of secondary quarks in the hadron-jet cascades was described [6] by $dN/dq_\perp \sim \exp(-q_\perp^2/2\sigma_q^2)$ where two values of $\sigma_q$ were used, a standard value of 250 MeV/c, with which the $q\bar{q}$ model adequately describes low energy data [9], and an arbitrarily large value of 350 MeV/c, which was used for comparison purposes.

Early indications for, and preliminary results on jet broadening and planar events were presented at the Geneva and FNAL conferences [3]. While writing this paper we learned that papers by the TASSO, MARK J and PLUTO Collaborations are being published on the same subject [4].

\footnote{We did not consider $q_\perp$ distributions with arbitrary non-gaussian tails. The physical interpretation of such modifications, which might possibly simulate some of the effects of gluon bremsstrahlung, is not clear to us.}
where \( P_{\alpha i} \) is the \( \alpha \)-component (\( \alpha = x, y, z \)) of the momentum of the \( i \)th particle. The sum runs over all charged and neutral particles. The resulting eigenvalues \( Q_1, Q_2, Q_3 \) (\( Q_1 < Q_2 < Q_3 \)) correspond, respectively, to the lengths of the orthogonal axes \((n_1, n_2, n_3)\) of the momentum ellipsoid and they satisfy the constraint \( Q_1 + Q_2 + Q_3 = 1 \). Each event is then represented by a point in a triangle plot as shown in fig. 2a. The two independent variables can be chosen either as \( Q_1 \) and \((Q_3 - Q_2)/\sqrt{3}\) or sphericity = \( \frac{3}{2}(Q_1 + Q_2) \) and planarity = \( Q_2 - Q_1 \). Fig. 2a shows the data and can be compared with the \( q\bar{q} \) model prediction shown in fig. 2b. A substantial deviation from the two-jet model is clearly apparent: the data contain an excess of planar events with small \((Q_3 - Q_2)/\sqrt{3} < 0.35, Q_1 < 0.07\) compared to a prediction of 6 from the \( q\bar{q} \) model (see table 1).

Fig. 3 shows the projection of the \( Q \) plot onto the \( Q_2 - Q_1 \) (planarity) axis. The data show a considerable excess of events with high values of \( Q_2 - Q_1 \) (planar events) compared with the prediction of the \( q\bar{q} \) model. For the region \( Q_2 - Q_1 > 0.07 \), we observe 78 events, whereas 24 events would be expected on the basis of the simple \( q\bar{q} \) model with \( \sigma_q = 250 \text{ MeV/c} \). It can be seen from the curves in fig. 3 that an arbitrary increase in \( \sigma_q \) from 250 to 350 MeV/c still does not account for the excess of events (see table 2).

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**Fig. 1.** The observed thrust distribution compared with \( q\bar{q} \) (\( \sigma_q = 250 \text{ MeV/c} \)) and \( q\bar{q}g \) model predictions.

**Fig. 2.** "Event shape" \( Q \)-plot as defined in the text. (a) High energy data, (b) \( q\bar{q} \) model prediction with \( \sigma_q = 250 \text{ MeV/c} \), (c) \( q\bar{q}g \) model prediction. The planarity \((Q_2 - Q_1)\) axis is orthogonal to the sphericity axis. The dotted line indicates planarity = 0.07.

In contrast to the lower energy (\( \sqrt{s} < 15 \text{ GeV} \)) data [9,13], which agree well with the \( q\bar{q} \) model predictions, we see in the \( \sqrt{s} \approx 30 \text{ GeV} \) region broader jets and an excess of planar events compared with the simple \( q\bar{q} \) model predictions. The existence of planar events strongly suggests a three-body primary process such

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**Table 1**

|                | \( q\bar{q} \) model | \( q\bar{q}g \) model |
|----------------|-----------------------|-----------------------|
| \( \sigma_q = 250 \text{ MeV/c} \) | 23                    | 6                     |
|                | 22                    |                        |
as $e^+e^- \to q\bar{q}g$. Since the total energy available (30 GeV) is sufficient to collimate the three resulting jets to the extent that the primary plane is still well defined, a significant fraction of the events should demonstrate a three-jet structure.

This picture was tested in the following way. A sample of planar events was chosen according to the criterion $Q_2 - Q_1 > 0.07$ which excluded the bulk of the two-jet events. Each of the remaining events was separated by a plane normal to the thrust axis into “slim” and “fat” jets by definition; $\Sigma |P_T|_{\text{slim}} < \Sigma |P_T|_{\text{fat}}$. The fat jet was then analysed to see whether it actually consisted of two jets. To do this, all momentum four vectors of the particles in the fat jet were Lorentz transformed into the rest frame of that jet. If the fat jet really consists of two sub-jets, then the effect of this transformation will be to bring the sub-jets into their c.m. frame, where they will appear collinear [12]. The direction $\hat{\beta}$ and the $\gamma$-factor of the Lorentz transformation were obtained from the invariant mass and the momenta of the fat jet system, which were calculated by using the measured momenta of charged and neutral hadrons comprising the fat jet. The hadron masses were neglected.

The thrust of the particles in the fat jet system alone, $T^*$, was then calculated in its rest system. As shown in fig. 4a the observed $T^*$ distribution peaks at

![Fig. 3. The planarity distribution compared with model predictions.](image)

![Fig. 4. The three-jet nature of the “planar” ($Q_2 - Q_1 > 0.07$) events. (a) The observed distribution of $T^*$ (the thrust of the fat jet in its rest system) for the planar events compared with the two-jet thrust distribution obtained by PLUTO at $\sqrt{s} = 9.4$ GeV (full-line histogram). The broken-line histogram shows the normalized $T^*$ distribution for all events without the planarity cut. (b) The observed invariant mass ($M^*$) distribution of the fat jet system for the planar events compared with the distributions expected from the $q\bar{q}$ model with $q_\gamma = 250$ MeV/c (shaded, broken-line histogram) and with $q_\gamma = 350$ MeV/c (dot-dashed histogram). The full-line histogram represents the $M^*$ distribution predicted by the $q\bar{q}g$ model. (c) The same observed $T^*$ distribution as shown in (a) compared with the predictions of the $q\bar{q}$ and $q\bar{q}g$ models.](image)
high $T^*$, consistent with a two-jet structure. The invariant mass of the fat jet system of hadrons, after the planarity cut, as shown in fig. 4b. The mean value of the invariant mass is about 9 GeV, which is above the energy ($\sqrt{s} = 7$ GeV) at which SPEAR and DORIS data [9] have shown that a two-jet structure, if present, becomes distinct. For comparison, the thrust distribution for $e^+e^- \rightarrow$ two jets as measured by PLUTO [13] at $\sqrt{s}$ of 9.4 GeV is also shown in fig. 4a (full line histogram). The agreement between the two distributions is good, i.e., in its own rest frame the fat jet has a two-jet structure with a thrust distribution similar to that of the two-jet events produced in $e^+e^-$ collisions at $\sqrt{s}$ equal to the average mass of the fat jet. It should be emphasized that the above conclusion that the fat jet actually consists of two jets, making three jets for the event as a whole, was reached in a completely model-independent way.

As a check, the same analysis was repeated without the planarity cut, i.e., for the dominant two-jets events. The resulting $T^*$ distribution is shown in fig. 4a as the dotted histogram normalized with a thrust distribution similar to that of the two-jet events produced in $e^+e^-$ collisions at $\sqrt{s}$ equal to the average mass of the fat jet. This demonstrates that the dominant two-jet-events do not artificially produce the high $T^*$ peak observed in the planar events. This point was further checked by selecting "planar" $(Q_2 - Q_1 > 0.07)$ events from events generated according to the $qq\bar{g}$ model with $\sigma_q = 250$ MeV/c. The resulting $T^*$ and $M^*$ distributions are shown in figs. 4b and 4c as shaded histograms normalized by the total number of events before the planarity cut. If compared with the data they show substantial disagreement both in the number of events and in the shape. If the $qq\bar{g}$ model is modified by an arbitrary increase of $\sigma_q$ to 350 MeV/c, the agreement is still poor as seen in figs. 4b and 4c.

More events pass through the planarity cut but the $T^*$ distribution becomes peaked at smaller values. In order to study the origin of the three-jet structure we calculated various expected distributions from the $qq\bar{g}$ model and compared them with the measured thrust distribution (fig. 1), the triangle $Q$-plot (figs. 2a, b, c and table 1) and the planarity (fig. 3 and table 2), $M^*$ and $T^*$ distributions (fig. 4). The detailed agreement observed in all of these figures and tables between data and $qq\bar{g}$ model predictions strongly suggests gluon bremsstrahlung as the origin of the planar three-jet events.

As a next step a three-jet analysis was performed in the laboratory system for the planar events in order to study the angular and energy distribution of the "gluon jet". For each event particles were divided into three jets by maximizing the triplicity [15]

$$T_3 = \left(\sum_{j} |p_j| \right)^{-1} \left| \sum_{c_1} p_j + \sum_{c_2} p_j + \sum_{c_3} p_j \right|,$$

where $c_1, c_2$ and $c_3$ are three nonintersecting sets of particles with contiguous directions. The sums were taken over charged and neutral particles. The energies and the directions of the three resulting jets were approximated by $E_i = \sum |p_j|$ and $k_i = \sum |p_j|/\sum |p_j|$, respectively. The jet with the lowest energy, $E_1$, was then chosen as the most probable candidate for the gluon jet. The angle of the "gluon jet" with respect to the parent quark was defined by $\cos \theta_1 = -\min(k_1 \cdot k_2, k_1 \cdot k_3)$.

If gluon bremsstrahlung is really the origin of the three-jet events, we expect $\theta$ and $E_1$ to peak at small values. On the other hand, if the three-jet structure is characterized by three-body phase space, for example, flatter $\cos \theta_1$ and $E_1$ distributions would be expected. The observed $\cos \theta_1$ distribution (fig. 5a) and $E_1$ distribution (fig. 5b) sharply peak toward small $\theta_1$ and $E_1$ as expected for the gluon bremsstrahlung process. The dips observed at $\theta_1 \approx 0$ and $E_1 \approx 0$ are the effect of the planarity cut. As seen in these figures, detailed agreement is again obtained between the $qq\bar{g}$ model and the data for the $\cos \theta_1$ and the $E_1$ distributions. The same analysis was repeated for the events generated by the simple $qq$ model. As seen in fig. 5b the resulting $\cos \theta_1$ distribution is flat and incompatible with data.

Bremsstrahlung spectra contain a small, very hard component. Such hard bremsstrahlung emitted at a large angle should result in a distinct three-jet structure clearly visible even in the laboratory system. A number of such events are actually found among the planar events and were published elsewhere [3].

In conclusion, topological distributions of hadrons show significant deviation from two-jet structure at high energies of $\sqrt{s} \approx 30$ GeV. The deviation appears in the form of jet broadening and an excess of planar events. A model-independent analysis shows that planar events actually possess three-jet structure. Both the qualitative nature of these planar, three-jet events and the rate at which they occur are in good agreement.
with what is expected from the process $e^+e^- \rightarrow q\bar{q}g$. This strongly suggests that gluon bremsstrahlung is the origin of the planar, three-jet events. Accepting this interpretation, the detailed quantitative agreement between the data and the $q\bar{q}g$ model prediction means that the assumed strong coupling constant $\alpha_s(q^2) = 0.17$ is approximately correct at $\sqrt{s}$ of about 30 GeV. In fact, a $\alpha_s$ value of $0.17 \pm 0.04$ is obtained by comparing the observed high planarity tail (fig. 3) with the $q\bar{q}g$ model predictions. The quoted error on $\alpha_s$ represents only the statistical error. Varying the $\alpha_q$ value in the $q\bar{q}g$ model from 250 to 350 MeV/c, the resulting $\alpha_s$ value stayed constant within 0.02. Further detailed analysis on the $\alpha_s$ determination will be published in a forthcoming paper.

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