NLO perturbative QCD predictions for $\gamma\gamma \rightarrow M^+M^-$ ($M = \pi, K$)

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We report the first complete leading-twist next-to-leading-order perturbative QCD predictions for
the two-photon exclusive channels $\gamma\gamma \rightarrow M^+M^-$ ($M = \pi, K$) at large momentum transfer. The
asymptotic distribution amplitude is utilized as a candidate form for the nonperturbative dynamical
input. Comparison of the obtained results with the existing experimental data does not provide
sufficiently clear evidence to support the applicability of the hard-scattering approach at currently
accessible energies.

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Exclusive processes at large momentum transfer provide a particularly suitable testing ground for pertur-

bative QCD (PQCD). In the hard-scattering approach (HSA) the amplitudes of these processes are given
in terms of a convolution of a hard-scattering amplitude of collinear hadron constituents, and distribution am-

plitudes summarizing soft physics. Although the HSA is widely used, its applicability at currently accessible
energies is still being questioned.

The leading-order (LO) PQCD predictions have been obtained for many exclusive processes, but owing to their
sensitivity to the choice of the renormalization scale and scheme, they do not have much predictive power. To
stabilize the LO results, and to achieve a complete confrontation between theoretical predictions and experi-
mental data, it is crucial to obtain higher-order contributions. However, owing to great technical difficulties
involved, the complete next-to-leading-order (NLO) predictions were obtained only for the pion electromagnetic
form factor and the pion transition form factor.

The two-photon annihilation into flavor-nonsinglet helicity-zero mesons, $\gamma\gamma \rightarrow M^+M^-$ ($M = \pi, K$), is one of
the simplest hadronic processes. Owing to the point-like structure of the photon, the initial state is simple,
with strong interactions present only in the final state. The energy behavior, angular behavior, and normaliza-
tion of the cross section of this process are nontrivial predictions of PQCD. This process has been the subject
of a number of experimental investigations. The most recent data (DELPHI, ALEPH, and Belle) are obtained
for the center-of-mass energies $W < 6$ GeV and the scattering angles in the range $|\cos\theta| < 0.6$.

In the HSA, at large momentum transfer (high $W$ and large $\theta$), the process is described by the helicity
amplitude

\[ M(\lambda; W, \theta) = \int_0^1 dx \int_0^1 dy \, \Phi^*_M(x, \mu^2_F) \Phi_M(y, \mu^2_F) \times T_H(\lambda'; \lambda; x, y; W, \theta; \mu^2_F, \mu^2_R) \]

(1)

where $\lambda$ and $\lambda'$ are photon helicities, $\mu^2_F$ is the factorization and $\mu^2_R$ the renormalization scale.

The function $\Phi_M(x, \mu^2_F)$ is the meson distribution amplitude (DA), representing the amplitude for the meson
consisting of a $q\bar{q}$ pair, with the quark and antiquark collinear and on shell, and sharing fractions $x$ and $1-x$
of the meson’s total momentum. There is a long on-
going debate in the literature regarding the correct
form of the DA at currently accessible energies. For
the purpose of this work we approximate the DA by
its asymptotic form and choose $\Phi_M(x, \mu^2_F) = \Phi_M^{as}(x) = f_M \sqrt{3/2} x(1-x)$, where $f_M$ is the meson decay constant ($f_\pi = 0.131 (0.160)$ GeV).

The function $T_H(\lambda'; \lambda; x, y; W, \theta; \mu^2_F, \mu^2_R)$ in Eq. (1) is the hard-scattering amplitude for producing collinear me-

son constituents from the initial photon pair, is free of collinear singularities, and has a well-defined perturba-
tive expansion.

At the NLO the helicity amplitude in Eq. (1) is given by the NLO result for $T_H$, obtainable from the LO and
the NLO contributions to the $\gamma\gamma \rightarrow (q\bar{q})+(q\bar{q})$ amplitude, with massless quarks collinear with outgoing mesons.
These contributions arise from 20 tree and 422 one-loop Feynman diagrams, respectively. Distinct diagrams are shown in Fig. (1). The other diagrams are obtained from these using various symmetries.

The LO prediction for the process $\gamma\gamma \rightarrow M^+M^-$ ($M = \pi, K$) is due to Brodsky and Lepage (BL). Making use of the similarity of the LO expressions of the helicity amplitudes for the process in question and the timelike meson electromagnetic form factor $F_M$, they obtained the approximate relation

\[ \frac{d\sigma}{d\cos\theta}(\gamma\gamma \rightarrow M^+M^-) \approx \frac{8\pi\alpha^2}{W^2} \frac{|F_M(W^2)|^2}{\sin^4\theta} \]  

(2)

Despite the fact that the complete LO prediction is given in Ref. 10, the approximate relation of Eq. (2) is what is usually referred as the Brodsky-Lepage prediction.

The first attempt to perform the NLO analysis was done by Nizic. Lacking powerful analytical meth-
ods for calculating one-loop Feynman diagrams with five and six internal lines, the NLO prediction in Ref. 10 was obtained with the simplest equipartition meson DA $\Phi_M \propto \delta(x-1/2)$. Also, the contribution of the diagrams
of group E was omitted. Therefore, the prediction of Ref. [10] cannot be considered complete.

Here we present the results of a complete NLO analysis of the process $\gamma\gamma \to M^+M^-$ ($M = \pi, K$). Using the method of Refs. [11], we evaluated all the one-loop diagrams analytically. It should be pointed out that the computation of one-loop diagrams with five and six internal lines represents a highly demanding task. The details will be given elsewhere.

Our complete NLO PQCD result for the cross section, in the angular range $|\cos \theta| < 0.6$, assuming the asymptotic DA and using the $\overline{MS}$ renormalization scheme, is

$$
\sigma(\gamma\gamma \to M^+M^-) = f_M^4 \frac{1.035}{W^6} \alpha_s^2(\mu_R^2) 
\times \left\{ 1 + \frac{\alpha_s(\mu_R^2)}{\pi} \left[ -3.828 + \frac{\beta_0}{2} \left( 3.563 + \ln \frac{\mu_R^2}{W^2} \right) \right] \right\},
$$

where $\alpha_s(\mu_R^2) = 4\pi/(\beta_0 \ln(\mu_R^2/\Lambda^2))$ is the coupling constant renormalized at $\mu_R^2$ (function of $W^2$ only), with $\beta_0 = 11 - 2n_f/3$ ($n_f = 3$) and $\Lambda = 0.2$ GeV. As a consequence of using the asymptotic DA, the result does not depend on $\mu_R^2$. The corresponding result for the differential cross section is too lengthy to be presented here.

As it is seen from Eq. (3), the NLO corrections, at accessible energies ($W < 6$ GeV), are substantial. Thus, for $\mu_R^2 = W^2$ (pragmatical choice), the ratio of the NLO to the LO contributions is 0.9 for $W = 4$ GeV, where $\alpha_s(\mu_R^2) = 0.23$, and becomes less than 0.5 only for $W > 45$ GeV, where $\alpha_s(\mu_R^2) = 0.13$. Obviously, the problem of convergence relates to the size of the NLO coefficient, not to the coupling constant.

It is important to observe that the large contribution to the NLO coefficient in Eq. (3) comes from the $\beta_0$ term, arising from the vacuum-polarization diagrams, and it is desirable to resum them into the running coupling constant. This can be achieved using the Brodsky-Lepage-Mackenzie (BLM) scale setting procedure [12].

However, the fact that the form of the effective coupling at low energies, as well as its behavior at timelike scales are not determined, together with the fact that the process at hand contains both spacelike and timelike scales, does not allow one to perform the BLM procedure in a completely satisfactory way. For the sake of clarity, we discuss only the BLM improved LO prediction LO(BLM), which is of the broadest interest. The BLM scales were determined for each LO diagram separately, for different helicities, angles, and quark momenta. The mean-value theorem for integration was used to avoid too low scales in the integration indicated in Eq. (1). To deal with the timelike scales, the continuation $\ln(\mu^2/\Lambda^2) \rightarrow \ln(\mu^2/\Lambda^2) - i\pi$ was used in the coupling constant.

In Figs. 2 and 3 we compare our predictions with
FIG. 3: Angular dependence $\sigma^{-1} \sigma d|/\cos \theta|$ for the $\gamma\gamma \rightarrow M^+ M^-$ processes. The experimental points are from Ref. 6.

the recent experimental data and the BL estimate in Eq. (2). To assess the theoretical uncertainty related to the choice of the scale $\mu_R^2$ in Eq. (3), in Fig. 2 we show the range of the NLO predictions (shaded region) obtained by varying $\mu_R^2$ from $\mu_R^2 = W^2$ down to the scale where the NLO prediction reaches maximum, $\mu_R^2 \approx W^2/15$ (the scale determined by the fastest-apparent-convergence principle [13]). The NLO prediction (not shown in Fig. 2) for $\mu_R^2 \approx W^2/20$, the scale set by the principle of minimal sensitivity [14], is located slightly below the upper limit. The solid line in Fig. 2 is our LO(BLM) prediction, while the dotted line is the BL estimate based on Eq. (2) with the form factors approximated by $F_{\gamma\gamma}(W^2) \approx 0.4 \text{GeV}^2/W^2$ and $F_{\pi K}(W^2) \approx f_{\pi}^2/2 F_{\pi}(W^2) = 0.6 \text{GeV}^2/W^2$, as in the original paper [9]. However, the recent measurements [17], suggesting that $F_{\pi}(K)(W^2) \approx 1.01(0.85) \text{GeV}^2/W^2$ (around $W^2 = 13.48 \text{GeV}^2$) put the success of the widely cited BL prediction (dotted line) to question. Figure 2 shows that our results for the cross section are still significantly, if less than an order of magnitude, below the data.

Next consider the $W$ dependence of the cross section. Using the parametrization $\sigma(\gamma\gamma \rightarrow M^+ M^-) \propto W^{-n_K}$ the results obtained by Belle 7 give $n_K = 7.9 \pm 0.4 \pm 1.5$ and $n_K = 7.3 \pm 0.3 \pm 1.5$, for $3.0 \text{ GeV} < W < 4.1 \text{ GeV}$. On the other hand, we find from Eq. (4) that the power $n_{\pi} (= n_K)$ takes the values $6.9(7.4)$ for $\mu_R^2 = W^2(W^2/15)$. The LO(BLM) and BL predictions are 6.7 and 6, respectively. As for the ratio of kaon to pion cross sections, our prediction equals $f_K/f_\pi = 2.23$, coinciding with the BL prediction, while the Belle results 7 for $3.0 \text{ GeV} < W < 4.1 \text{ GeV}$ give $0.89 \pm 0.04 \pm 0.15$.

In Fig. 3 we show the angular dependence of the ratio $\sigma^{-1} \sigma d|/\cos \theta|$. The range of the NLO and LO(BLM) predictions is obtained by varying $W$ from 2.4 GeV to 4.1 GeV (range covered by the Belle experiment 7). The BL curve, based on Eq. (2), corresponds to $1.227/\sin^4 \theta$. To keep figure synoptic, experimental points are given only for $W = 3.2 - 3.3 \text{ GeV}$ (middle of the Belle range 7).

As it is seen from Figs. 2 and 3 the energy and the angular dependence of our NLO and LO(BLM) predictions for the cross section are in very good agreement with the data. As for the absolute normalization, our results are still significantly below the data. Obviously, the normalization represents a problem.

This problem has been the subject of a long debate, and critics use it as an argument to discard, at experimentally available energies, either the HSA [16] or the asymptotic DA as an appropriate distribution amplitude [17]. The strength of criticism depends on the size of higher-order corrections. Here we have shown that the NLO corrections can be substantial, so a part of the normalization problem can be attributed to the size of uncalled higher-order corrections. However, assuming reasonable scale variation of the NLO prediction, or controlled perturbative series (LO > NLO term > NNLO term) it is not to be expected that inclusion of higher-order corrections would lead to a solution of the normalization problem. On the other hand, good agreement between theory and experiment for angular dependence (Fig. 3), and the disagreement for the ratio of the kaon to pion cross sections suggest that the specific form of the DA certainly plays a relevant role. An analysis of various choices of the DA, however, is outside the scope of this letter.

As a quantity which is expected to be largely insensitive to both the form of the effective coupling as well as to the choice of the DA, and as such representing a nontrivial PQCD prediction, we next consider the ratio of $\sigma(\gamma\gamma \rightarrow M^+ M^-)$ and $|F_M(W^2)|^2$ at the NLO.

The result of our NLO calculation of the timelike meson form factor is of the form

$$|F_M(W^2)|^2 = f_M^4 \frac{64 \pi^2}{W^4} \alpha_s^2(\mu_R^2) \times \left\{ 1 + \frac{\alpha_s(\mu_R^2)}{\pi} \left[ -7.833 + \frac{\beta_0}{3} \left( \frac{14}{3} + \ln \frac{\mu_R^2}{W^2} \right) \right] \right\}, \tag{4}$$

and is consistent with the NLO result for the spacelike form factor 2. On the basis of Eqs. 3 and 1, and using the same $\mu_R^2$, we find

$$W^2 \sigma(\gamma\gamma \rightarrow M^+ M^-) \frac{|F_M(W^2)|^2}{W^2} = 638 \left[ 1 - \alpha_s(\mu_R^2) 0.306 \right] \text{nb GeV}^2. \tag{5}$$

A comparison of this result with the only two reliable experimental points, based on the data from 5 and 15, is given in Fig. 4. The solid line corresponds to Eq. 3, with $\mu_R^2 = W^2$, the dotted line is the BL estimate following from Eq. 2, the dot-dashed line is the complete LO prediction (equal to 638 nb GeV$^2$, according to Eq. 4), while the dashed line is the ratio of the LO(BLM) results with the BL procedure performed separately for the cross section and the form factor. As it is seen, our
results for the ratio are above the data by approximately a factor of 2 to 3 which is in much better agreement with the data than the prediction for the cross sections based on Eq. 3. As Fig. 1 shows, the usage of the BLM scales leads to much better agreement with the data. This, together with the fact that the LO(BLM) prediction lies inside the range of the NLO prediction (shaded area in Fig. 2), supports the idea behind the BLM procedure according to which the BLM determined scales represent the relevant physical scales (putting aside understatements regarding the effective coupling).

In conclusion, the NLO results obtained for the process \( \gamma \gamma \to M^+ M^- \) \((M = \pi, K)\) are not conclusive enough to allow a definite statement regarding the applicability of the HSA at the currently accessible energies above 3 GeV. However, improvements are possible in two directions: the first, the more appropriate choice of the DA (possibly in the direction suggested in Ref. [17]), and the second, a better understanding of the resummation for the processes containing both spacelike and timelike scales (possibly in the directions suggested in Refs. [18] [19]).

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[1] G. P. Lepage and S. J. Brodsky, Phys. Lett. B 87, 359 (1979); Phys. Rev. Lett. 43, 545 (1979) [Erratum-ibid. 43, 1625 (1979)]; Phys. Rev. D 22, 2157 (1980); A. V. Efremov and A. V. Radyushkin, Theor. Math. Phys. 42, 97 (1980) [Teor. Mat. Fiz. 42, 147 (1980)]; Phys. Lett. B 94, 245 (1980); A. Duncan and A. H. Mueller, Phys. Lett. B 90, 159 (1980); Phys. Rev. D 21, 1636 (1980).

[2] R. D. Field, R. Gupta, S. Otto and L. Chang, Nucl. Phys. B 186, 429 (1981); F. M. Dittes and A. V. Radyushkin, Sov. J. Nucl. Phys. 34, 293 (1981) [Yad. Fiz. 34, 529 (1981)]; M. H. Sarmadi, Phys. Lett. B 143, 471 (1984); R. S. Kahlmuradov and A. V. Radyushkin, Sov. J. Nucl. Phys. 42, 289 (1985) [Yad. Fiz. 42, 458 (1985)]; E. Braaten and S. M. Tse, Phys. Rev. D 35, 2255 (1987); B. Melic, B. Nizic and K. Passek, Phys. Rev. D 60, 074004 (1999); FizikaB 8, 327 (1999).

[3] E. P. Kadantseva, S. V. Mikhailov and A. V. Radyushkin, Yad. Fiz. 44, 507 (1986) [Sov. J. Nucl. Phys. 44, 326 (1986)].

[4] F. del Aguila and M. K. Chase, Nucl. Phys. B 193, 517 (1981); E. Braaten, Phys. Rev. D 28, 524 (1983); B. Melic, B. Nizic and K. Passek, Phys. Rev. D 65, 053020 (2002); B. Melic, D. Muller and K. Passek-Kumericki, Phys. Rev. D 68, 014013 (2003).

[5] K. Grzela (DEPHI Collaboration), Published in *Ascona 2001, The structure and interactions of the photon* 279-282.

[6] A. Heister et al. [ALEPH Collaboration], Phys. Lett. B 569, 140 (2003).

[7] H. Nakazawa et al. [BELLE Collaboration], Phys. Lett. B 615, 39 (2005).

[8] V. L. Chernyak and A. R. Zhitnitsky, Nucl. Phys. B 201, 492 (1982) [Erratum-ibid. B 214, 547 (1983)]; Phys. Rept. 112, 173 (1984); A. V. Radyushkin, Acta Phys. Polon. B 26, 2067 (1995); S. Ono, Phys. Rev. D 52, 3111 (1995); P. Kroll and M. Raulfs, Phys. Lett. B 387, 845 (1996); V. M. Braun and I. E. Halperin, Phys. Lett. B 328, 457 (1994); R. Jakob, P. Kroll and M. Raulfs, J. Phys. G 22, 45 (1996); A. Schmeding and O. I. Yakovlev, Phys. Rev. D 62, 116002 (2000); A. P. Bakulev, S. V. Mikhailov and N. G. Stefanis, Phys. Lett. B 508, 279 (2001) [Erratum-ibid. B 590, 309 (2004)]; Phys. Rev. D 67, 074012 (2003); Phys. Rev. D 73, 056002 (2006); S. S. Agaev, Phys. Rev. D 72, 074020 (2005); P. Ball and R. Zwicky, Phys. Lett. B 633, 289 (2006); JHEP 0602, 034 (2006); A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D 70, 094002 (2004); V. M. Braun and A. Lenz, Phys. Rev. D 70, 074020 (2004); M. Gockeler et al., arXiv:hep-lat/0510089; V. M. Braun et al., arXiv:hep-lat/0606012.

[9] S. J. Brodsky and G. P. Lepage, Phys. Rev. D 24, 1808 (1981).

[10] B. Nizic, Phys. Rev. D 35, 80 (1987).

[11] G. Duplancic and B. Nizic, Eur. Phys. J. C 20, 357 (2001); Eur. Phys. J. C 24, 385 (2002); Eur. Phys. J. C 35, 105 (2004).

[12] S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D 28, 228 (1983).

[13] G. Gluung, Phys. Lett. B 95, 70 (1980) [Erratum-ibid. B 110, 501 (1982)]; Phys. Rev. D 29, 2315 (1984).

[14] P. M. Stevenson, Phys. Lett. B 100, 61 (1981); Phys. Rev. D 23, 2916 (1981); Nucl. Phys. B 231, 65 (1984); P. Stevenson, Nucl. Phys. B 203, 472 (1982).

[15] T. K. Pedlar et al. [CLEO Collaboration], Phys. Rev. Lett. 95, 261803 (2005).

[16] M. Diehl, P. Kroll and C. Vogt, Phys. Lett. B 532, 99 (2002).

[17] M. Benayoun and V. L. Chernyak, Nucl. Phys. B 329, 285 (1990); V. L. Chernyak, arXiv:hep-ph/0605072.

[18] S. J. Brodsky and H. J. Lu, Phys. Rev. D 51, 3652 (1995); S. J. Brodsky, C. R. Ji, A. Pang and D. G. Robertson, Phys. Rev. D 57, 245 (1998).

[19] D. V. Shirkov, AIP Conf. Proc. 806, 97 (2006).
arXiv:hep-ph/0510247.