We study the relaxation of ultra-cold bosons in one dimension. In a setting similar to the Newton’s cradle experiment by Kinoshita et al. [Nature (London) 440, 900 (2006)], we excite the atoms to oscillate and collide in a 2D optical lattice of 1D tubes and observe the oscillations for up to 4.8 seconds (400 oscillations) with very little heating and loss. Tuning the initial distribution, we can probe the onset of relaxation in the cross over between 1D and 3D. Investigating the population of the transverse excited state by band mapping, we show that even for samples where the energy put into the system is initially not sufficient to excite transverse state, relaxation starts through a minimal (1%) seed of atoms in transverse excited states.

The problem of thermalization and equilibration in an isolated many-body quantum system [1-3] has a long history beginning with the pioneering work by von Neumann [4]. An integrable system will not fully relax [5, 6], but dephase towards a Generalised Gibbs Ensemble (GGE) [7, 8], reflecting its many conserved quantities. If the system is not exactly integrable, the dephased state is pre-thermal [9, 10], and will slowly relax.

Bosons in one dimension (1D) [11, 12] are a model system to study these fundamental questions at the interface between microscopic quantum evolution and of statistical physics. Hard-core bosons in 1D are described by the integrable Lieb-Liniger model [13, 14]. In any realistic experimental setting this integrability will be broken at some level. Key candidates are the motion in the longitudinal confinement [15], virtual 3-body collisions [16-19] which ‘feel’ the transverse confinement even when the energy in the atom-atom collisions does not allow to excite transverse states, or long range interactions [20].

An interesting experimental setting that was first employed by Kinoshita et al. [5] resembles Newton’s cradle: A large opposite longitudinal momenta are imprinted on the atomic ensemble, which then oscillate in the trap and collide. If the momentum is much smaller then required to excite transverse excitations, the oscillations persist for hundreds of atomic collisions. In this letter we revisit this 1d atomic Newton’s cradle and study in detail the mechanisms that lead to dephasing and finally relaxation.

We start our experiment with a pure BEC with an atom number fixed between of $10^4 - 10^5$ Rubidium-87 atoms in an all-optical trap. We then adiabatically load the BEC into the transverse ground state of an optical lattice of 1D tubes, which are formed by retro-reflecting two perpendicular laser beams of $\lambda = 1064$ nm in the horizontal plane (Fig. 1). A vertical magnetic field gradient is used to balance gravity. After loading the atoms are distributed in 330 to 1000 tubes with an average number of atoms per tube from 30 to 100, determined by the atom number of the initial BEC.

Atoms are strongly confined in the transverse directions (x and y) by the optical lattice, with a transverse trap frequency of $\omega_{\perp}/2\pi = 31$ kHz, and weakly confined in the vertical direction (z), with a longitudinal trap frequency of $\omega_{z}/2\pi = 83.3$ Hz. In the optical lattice,
the tunnelling between different lattice sites can be ignored, and the atoms in each tube can be regarded as an independent 1D Bose gas. For the BEC in the lattice the measured increase in longitudinal kinetic energy is \( \sim 0.016\hbar\omega_{\perp}/s \) for transverse excitation \( \sim 0.007\hbar\omega_{\perp}/s \). The atom loss is between 4\%/s and 7\%/s depending on the atomic density.

To start the motion of the Bose gases in the 1D tubes, a sequence of standing-wave light pulses is applied along the longitudinal direction of the 1D gases. The atoms initially at rest on \( |\psi_0\rangle \) are transferred to states with \( e^{2\pi i n k z}|\psi_0\rangle \), where \( n \) is an integer, \( k = 2\pi/\lambda_p \) is the wavevector of the standing wave, \( \lambda_p = 852 \) nm. The fraction of atoms in each of the momentum components is controlled by choosing a special pulse sequence [21] [22]. In a situation with all atoms in \( e^{\pm 2\pi i k z}|\psi_0\rangle \), the total collision energy between a pair of atoms is \( 2 \times (2\hbar k)^2/2m \approx 0.8\hbar\omega_{\perp} \), which is 40\% of the minimum energy needed to create a transverse excitation.

After the initial state is prepared, we keep the lattice on for a holding time \( t \), during which the atoms oscillate in the tubes and collide. The atoms are then released from the optical lattice and imaged after time-of-flight either with transverse imaging, which measures their momenta along tubes, or with vertical imaging which allows us to study the population in transverse excited states by band mapping [23] (Fig. 1). For the former one, the lattice is ramped down in 500\,\mu s, which is fast enough to keep the longitudinal momenta unchanged, and slow enough to prevent the atomic cloud from spreading too much in transverse directions. For the vertical imaging, the lattice is ramped down in 2000\,\mu s.

We now first look at the dephasing process (Fig. 3). To probe the influence of interactions we conduct our experiments with different average densities (30 to 100 atoms per tube). We characterised the weighted interaction strength by \( \gamma = 2a_{1D}/a_{1D} \) [12] [23] where \( n_{1D} \) is the 1D atomic density, \( a_{1D} = a_s/[l^2_\perp (1 - C_{\perp}/l^2_\perp)] \) is the 1D scattering amplitude with \( l_\perp = \sqrt{\hbar/ma_{\perp}} \). \( a_s = 5.3 \) nm is the 3D s-wave scattering length for \( ^{87}\)Rb, \( C \approx 1.06 \ldots \) and \( m \) is the atomic mass of \( ^{87}\)Rb. At the beginning of the oscillation, \( \gamma_0 = 0.9 \) for \( N_{tot} = 1 \times 10^5 \), and \( \gamma_0 = 1.9 \) for \( N_{tot} = 1 \times 10^4 \) initial atom number in the BEC. As the dephasing process proceeds, the atoms spread along the tube and the mean 1D density \( n_{1D} \) is reduced, increasing \( \gamma \). When the system is fully dephased, \( \gamma = 3.3 \) for \( N_{tot} = 1 \times 10^5 \), and \( \gamma = 7.0 \) for \( N_{tot} = 1 \times 10^4 \).

FIG. 2. Basic observations: (a) Evolution of the longitudinal momentum distribution of the atomic Newton’s cradle in the first 30 periods (360ms) for \( N_{tot} = 1 \times 10^5 \). (b) Long time evolution of the period average momentum distribution for \( N_{tot} = 1 \times 10^4 \). For long times the momentum distribution relaxes towards a Gaussian shape.

FIG. 3. Dephasing of the atomic Newton’s cradle for three different total atom numbers. The dephasing rate is almost independent of the atomic density. The insets show the period average momentum distribution at the indicated time point, the border colors indicate the corresponding curve. At about \( t=0.8 \) s (green dashed line) the dephasing is basically complete.
We quantify the dephasing process by characterising the variance of the momentum distribution within a given period as given by:

$$\mathcal{D}(t) = \frac{1}{N_{\text{pix}}} \frac{\Delta t}{\tau} \sum_{i=1}^{N_{\text{pix}}} \sum_{j=0}^{\Delta t-1} [f_{t+j}(z_i) - F_i(z_i)]^2,$$

(1)

where $f_t(z_i)$ is the momentum distribution at pixel $z_i$ with holding time $t$, $F_i(z_i)$ is the average value over the whole period around $t$, $\Delta t$ is the measurement step, $\tau$ is the period of the oscillation, and $N_{\text{pix}}$ is the number of pixels. For more details about evaluating dephasing, see Supplemental Material [25]. The dephasing is plotted in Fig. 3 for different total atom number $N_{\text{tot}}$. From the figure, we find the dephasing rate to be almost independent of the initial atomic density, even though the dephased distributions are significantly different. The momentum distributions are fully dephased at around 0.8 s. The final value of $\mathcal{D}$ is given by the imaging noise (photon and atom shot noise).

The second process is relaxation towards a Gaussian (thermalized) distribution of longitudinal momenta, as shown in Fig. 2. To quantify how different the measured (period average) momentum distributions are from a thermal equilibrium we evaluate the non-Gaussianity of the momentum distribution. Calculating the mean squared distance from the closest Gaussian curve

$$\mathcal{R}(t) = \frac{1}{N_{\text{pix}}} \sum_{i=1}^{N_{\text{pix}}} [F_i(z_i) - G_i(z_i)]^2,$$

(2)

emerged as the most robust way to quantify the distance from a relaxed thermal equilibrium state (see also [20]). Thereby $F_i(z)$ is the period average momentum distribution at time $t$, and $G_i(z)$ is the corresponding best Gaussian approximation (fit).

The time evolution of the relaxation processes for different atom numbers are shown in Fig. 4. The blue dotted curves in both branches show the situations of a Newton’s Cradle with $+2\pi k i|\psi_0\rangle$ excitation. Even though initially no atoms have energies above the threshold to create transverse excitations the momentum distributions approach a Gaussian. The relaxation is faster for stronger interactions (more atoms per tube). As seen from the inserts in Fig. 3 this relaxation towards a Gaussian distributions already proceeds in parallel with the dephasing, and continues after the system is fully dephased.

To illuminate the physics behind the relaxation, we study the evolution of the transverse excited state by band mapping [24]. Fig. 5a shows time of flight pictures for the band mapping experiments with a pure BEC in the lattice of 1D tubes right after loading ($t = 0$) and the end ($t = 4.8$ s), Fig. 5b the same for the Newton’s cradle experiment. From these pictures we extract the population in the first excited state by summing over the regions of the second Brillouin zone as indicated by the 4 rectangles. Fig. 5(c) then shows, evolution of the relative population in the first excited state $\eta_1$ over the full time of the experiment.

![Image](image_url)

FIG. 4. Relaxation process for different initial conditions and different atom number: (a) $N = 1 \times 10^4$ (30 atoms per tube), (b) $N = 1 \times 10^5$ (100 atoms per tube). The dots are calculated from period average experimental measurements, and the solid curves are from theoretical simulations. The green dashed line indicates the dephasing time from Fig. 3.
distribution of the Newton’s Cradle which is consistent with the expected $\eta_1 \sim 8\%$ calculated for a fully thermalized Bose-gas in the elongated trap with the same total energy as introduced by the initial momentum pulses.

This suggests that the main reason for the observed relaxation lies in the two-body collisions with the transversely excited atoms, which allow to redistribute the longitudinal momenta. Although all atoms are initially below the threshold for transverse excitations, the relaxation is triggered by the very small initial population in the first excited state produced most probably by the loading or by imperfect excitation pulses. Energy is also transferred from the longitudinal to the transverse direction according to the total energy in the system. In this way, the Bose gas achieves thermal equilibrium in the elongated trap.

To further evaluate the relaxation in the 1D-3D cross over, we study different initial conditions which include also higher-momentum ($e^{\pm i4kz}|\psi_0\rangle$ and $e^{\pm i6kz}|\psi_0\rangle$) atoms, prepared by choosing the specific designed pulse sequences [23]. The collision energy between atoms with momentum larger than or equal to $4\hbar k$ is large enough to excite the atoms in the transverse directions, and as shown in Fig. 4 the relaxation proceeds much faster even for a very small proportion ($\sim 10\%$) of $e^{\pm i4kz}|\psi_0\rangle$ atoms.

To test the above interpretation we developed a semi-classical model for a Monte Carlo (MC) simulation of the experiments. The atoms move in the longitudinal direction as classical point-like objects. Transverse excitations are treated as discrete quantum levels. Scattering and transitions between these transverse states are simulated within the MC model. The transition matrix elements determining the transition probabilities are obtained from quantum-mechanical calculations similar to those of Ref. [24]. In calculations the initial longitudinal momenta of the atoms are sampled according to the probability distribution derived from the experimentally measured initial momentum distribution. The change of the radial state of an atom due to the heating is also considered [20]. The results of the MC simulations are in qualitative agreement with the experimental data as shown in Fig. 4. The only free parameter of our theory is the 1D scattering length. Its value corresponding to the best fit of experimental data in the whole range of parameters exceeds by 26% the value predicted by [24]. This discrepancy may stem from two factors: (i) the otherwise quite involved scheme of possible transitions for atoms colliding in their transversely excited states is simplified for the sake of reasonable computational time. For more details see Supplemental Material [25]. (ii) our semiclassical model of the longitudinal motion of atoms neglects their correlations arising in the limit $\gamma \gtrsim 1$ [27].

In conclusion we have investigated the relaxation of 1D bosons in the cross over regime from 1D to 3D. If the initial energy in atom-atom collisions is not sufficient to excite transverse states, a very small initial population of transverse excited states is sufficient to start thermalization much faster than other integrability breaking mechanisms like the longitudinal trapping confinement [15] or virtual 3-body collisions [16][19]. Never the less the observed relaxation times for close to ideal starting conditions are still very long, and there is a sizeable window where 1D integrable physics can be observed. To investigate the real integrability breaking terms for short range interacting Bosons new experimental techniques need to be developed that completely eliminate all transverse excitations during the whole experimental time.

We thank J.S. Caux, B. Lev, and M. Rigol for enlightening discussions. X.C. acknowledges support by the National Key Research and Development Program of China (Grant No. 2016YFA0301501), and the National Natural Science Foundation of China (Grants Nos. 91736208, 61475007, 11334001). I.M., H.P.S. and J.S. acknowledge support by the European Research Council, ERC-AdG, QuantumRelax and the WWTF project MA16-066 SEQUEX.

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[1] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalat-
tore, Rev. Mod. Phys. 83, 863 (2011).
[2] C. Gogolin and J. Eisert, Reports on Progress in Physics 79, 056001 (2016).
[3] L. D’Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Advances in Physics 65, 239 (2016).

https://doi.org/10.1080/00018732.2016.1198134
[4] J. v. Neumann, Zeitschrift für Physik 57, 30 (1929).
[5] T. Kinoshita, T. Wenger, and D. S. Weiss, Nature (Lon-
don) 440, 900 (2006).
[6] F. H. L. Essler and M. Fagotti, Journal of Statistical Me-
chanics: Theory and Experiment 2016, 064002 (2016).
[7] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, Phys.
Rev. Lett. 98, 050405 (2007).
[8] T. Langen, S. Erne, R. Geiger, B. Rauer, T. Schweigler,
M. Kuhnert, W. Rohringer, I. E. Mazets, T. Gasenzer,
and J. Schmiedmayer, Science 348, 207 (2015).
[9] J. Berges, S. Borsányi, and C. Wetterich, Phys. Rev. Lett.
93, 142002 (2004).
[10] M. Gring, M. Kuhnert, T. Langen, T. Kitagawa,
B. Rauer, M. Schreitl, I. E. Mazets, D. A. Smith, E. Dem-
ler, and J. Schmiedmayer, Science 337, 1318 (2012).
[11] T. Giamarchi, Quantum physics in one dimension
(Clarendon Press, Oxford, 2004).
[12] M. A. Cazalilla, R. Citro, T. Giamarchi, E. Orignac, and
M. Rigol, Rev. Mod. Phys. 83, 1405 (2011).
[13] E. H. Lieb and W. Liniger, Phys. Rev. 130, 1605 (1963).
[14] E. H. Lieb, Phys. Rev. 130, 1616 (1963).
[15] I. E. Mazets, The European Physical Journal D 65, 43
(2011).
[16] I. E. Mazets, T. Schumm, and J. Schmiedmayer, Phys.
Rev. Lett. 100, 210403 (2008).
[17] I. E. Mazets and J. Schmiedmayer, Phys. Rev. A 79,
061603 (2009).
[18] I. E. Mazets and J. Schmiedmayer, New Journal of
Physics 12, 055023 (2010).
[19] S. Tan, M. Pustilnik, and L. I. Glazman, Phys. Rev.
Lett. 105, 090404 (2010).
[20] Y. Tang, W. Kao, K.-Y. Li, S. Seo, K. Mallayya,
M. Rigol, S. Gopalakrishnan, and B. L. Lev, ArXiv e-
prints (2017), arXiv:1707.07031 [cond-mat.quant-gas].
[21] S. Wu, Y.-J. Wang, Q. Diot, and M. Prentiss, Phys. Rev.
A 71, 043602 (2005).
[22] C. Li, T. Zhou, Y. Zhai, X. Yue, J. Xiang, S. Yang,
W. Xiong, and X. Chen, Physical Review A 95, 033821
(2017).
[23] M. Greiner, I. Bloch, O. Mandel, T. W. Hänsch,
and T. Esslinger, Phys. Rev. Lett. 87, 160405 (2001).
[24] M. Olshanii, Phys. Rev. Lett. 81, 938 (1998).
[25] See Supplemental Material for details of the experimental
setup, data analysis, and theoretical calculations.
[26] J.-F. Riou, L. A. Zundel, A. Reinhard, and D. S. Weiss,
Phys. Rev. A 90, 033401 (2014).
[27] I. Brouzos and P. Schmelcher, Phys. Rev. Lett. 108,
045301 (2012).
Supplementary Materials

EXPERIMENTAL PROCEDURE

Loading into optical lattice

A $^{87}$Rb BEC is produced on the Zeeman sublevel $F = 1$, $m_F = -1$ by evaporative cooling in a crossed optical dipole trap. On the final stage of the evaporative cooling, the atom cloud is levitated by switching on a magnetic field gradient in the vertical direction ($z$), and decompressed by reducing the trap laser power. By holding the BEC in the dipole trap with trap depth of $\sim 100$ nK for different time, the total atom number $N_{tot}$ can be varied between $1 \times 10^5$ and $1 \times 10^6$. As soon as the final BEC is prepared, it is adiabatically transferred from the optical dipole trap to a 2D square optical lattice, which lies in the horizontal plane. The lattice lasers are derived from a 1064nm fiber laser, and are detuned 220 MHz from each other to avoid interference. The polarization of two beams is perpendicular to each other. The beam waist of the lattice laser is 145 $\mu$m (see Fig. 1 in the main text).

During the loading procedure, the lattice depth $V_0$ is exponentially ramped to the maximum value $70 E_r$ in 250 ms, with a time constant of 62.5 ms, where $E_r = (\hbar k_l)^2 / 2m$, $k_l = 2\pi / \lambda_l$, and $\lambda_l$ is the wavelength of the lattice laser. The optical dipole trap is ramped down at the same time. The atoms are confined by the red-detuned lattice laser both in the vertical (longitudinal) direction, with a trap frequency of 83.3 Hz, and in the horizontal (transverse) direction, with a trap frequency of 31 kHz. The atoms in each tube can be regarded as independent 1D gases.

![FIG. S1. The experimental sequence. Green: optical lattice; blue: standing-wave light pulses; yellow: imaging pulse.](image)

For the transverse imaging, the atomic gases are detected by a standard absorption imaging along the angular bisector of $x$ and $y$. The lattice is exponentially ramped down in 0.5 ms, with a time constant of 0.125 ms. By integrating the image along the transverse direction in the region of interest (ROI), the longitudinal momentum distribution $f(z)$ is observed. The longitudinal range of the ROI is proportional to the TOF chosen for different cases. To analyze the impact of the imaging noise, which mainly comes from the photon and atom shot noise on the CCD camera, two background regions with same size besides the ROI are chosen to be used in data analysing.

![FIG. S2. An example of the transverse imaging. The atoms are all contained in the ROI, and the background regions are chosen on both sides of ROI with same size. The pixel size is 6.45 $\mu$m $\times$ 6.45 $\mu$m. To get the longitudinal momentum direction and fluctuation distributions, images in different regions are integrated in the transverse direction separately.](image)

Excitation of longitudinal motion

To start the oscillation, the 1D gases are exposed to a sequence of standing wave light pulses ($\lambda_p = 852$ nm) in the longitudinal direction. In this way, atoms are transferred to states with $e^{2\pi i k z} |\psi_0\rangle$, where $k = 2\pi / \lambda_p$ is the wave-vector of the standing wave. The design of the pulse sequence follows the method described in [1]. The sequences used in our experiments are listed in Table S1, and the corresponding momentum distributions are shown in Fig. S3. The momenta $\pm 2\hbar k$ correspond to

Imaging the atoms

The atomic cloud is imaged either in transverse direction or in vertical direction after time-of-flight (TOF). To keep the signal-to-noise ratio (SNR) of images in the same order of magnitude, the expansion time for $N_{tot} = 1 \times 10^4$, $1 \times 10^5$ are 10 ms and 30 ms, respectively.

![FIG. S1. The experimental sequence. Green: optical lattice; blue: standing-wave light pulses; yellow: imaging pulse.](image)

For the vertical imaging, the lattice is exponentially ramped down in 2 ms, with a time constant of 0.5 ms, after the same loading procedure, excitation pulses, oscillation time and TOF. The crystal momentum is mapped to the free particle momentum and the Brillouin zones are imaged, from which we can get the information of the energy-band population (see Fig. 5 in the main text).
an energy which is $\sim 40\%$ of the excitation energy, and $\pm 4\hbar k$ are above the excitation threshold.

FIG. S3. Initial momentum states with different longitudinal momentum excitation. The integral of the distributions is normalized to 1. The indices correspond to the different pulse sequences shown in Table. [S1] Red lines indicate the threshold of transverse excitation.

DATA ANALYSIS

Quantification of dephasing

As discussed in the main text, the dephasing mainly comes from the anharmonicity within each tube and the inhomogeneity among tubes. Both of them are caused by the Gaussian beam profile of the lattice laser. The anharmonicity within the tube is caused by a Gaussian shape of the longitudinal confinement. The further away from the center the atoms go, the longer the oscillation period. The oscillation periods for $\pm 2\hbar k$, $\pm 4\hbar k$, $\pm 6\hbar k$ is 12 ms, 13.2 ms, 15.2 ms, respectively (see Fig. [S4]). The inhomogeneity of the traps is caused by the different local laser power on each tube. The longitudinal trap frequency of the central tube is $\sim 2\%$ larger than the tubes close to the edge of the atom cloud.

To evaluate the dephasing process, we study the longitudinal momentum distributions with a time step $\Delta t = 1$ ms, that is 12 pictures in a period of 12 ms, and estimate the remaining oscillations within a period by calculating the mean square distance of the different longitudinal profiles from the period averaged longitudinal profile. This can be quantified by evaluating:

$$\mathcal{D}(t) = \frac{1}{N_{\text{pix}}} \frac{\Delta t}{\tau} \sum_{i=1}^{N_{\text{pix}}} \sum_{j=0}^{\tau/\Delta t-1} [f_i(t_i) - F_i(z_i)]^2 \ , \ (S1)$$

Here, $f_i(z)$ is the longitudinal momentum distribution at time $t$, $F_i(z)$ is the period average longitudinal momentum distribution: $F_i(z) = \sum_{i=0}^{\tau/\Delta t} f_i(z_i)$. $\Delta t$ is the measurement step, $\tau$ is the period of the oscillation, and $N_{\text{pix}}$ is the number of pixels.

We further illustrate the dephasing in Fig. [S5] for three different times $t_0 = 0$, 0.4 and 0.8 s as an example. We first calculate average momentum distribution $F_{t=t_0}$ (upper graphs in Fig. [S5]) and then the variance of the lon-

![Table S1. Pulse sequences for generating initial momentum state. ($E_r = (\hbar k)^2/2m$, $k = 2\pi/\lambda_p$)](data:image/png;base64,iVBORw0KGgoAAAANSUhEUgAAAgAAAAAbCAYAAAB5MqXWAAAAGXRFWHRTb2Z0d2FyZQBBZG9iZSBJbWFnZVJlYWR5ccllPAAAAE3RSTlMAQObgAAATpJREFUeNrs2bQsT/3AEGwD+y3579sJkZBjIbM3Ri+7h6/tKv5P3TJ-motion.png)
We found that calculating the mean square distance
\[ \mathcal{R}(t) = \frac{1}{N_{\text{pix}}} \sum_{i=1}^{N_{\text{pix}}} [F_i(z_i) - G_t(z_i)]^2, \]
\[ \text{(S2)} \]
of a period average momentum distribution from a thermalized Gaussian distribution is a much more robust estimator in the experimental situation of background noise and significant tails.

To calculate \( \mathcal{R}(t) \), the period average momentum distribution \( F_t(z) \) is fitted to a Gaussian \( G_t(z) \). The differential

\[ F_t(z) = G_t(z). \]

Quantification of relaxation

As the system relaxes, the longitudinal momentum distribution approaches a Gaussian. To quantify the relaxation we evaluate the Gaussianity of the longitudinal momentum distribution at different evolution times. However, the standard methods of Gaussian test, like the Kolmogorov-Smirnov test, Lilliefors test, and Kurtosis, are sensitive to tails and noise in both ends of the distribution, which come from the imaging in the experiment.

FIG. S6. Quantification of relaxation (Non-Gaussianity test). Examples are for \( N_{\text{tot}} = 1 \times 10^4 \) at (a) \( t = 0.8 \text{ s} \), (b) \( t = 4.8 \text{ s} \). The average momentum distributions \( F(z) \) (solid black curves) are fitted to the closest Gaussian curves \( G(z) \) (dashed red curves). The fitting residues \( F(z) - G(z) \) that are shown below in blue are used to quantify the relaxation. The fluctuations in the background regions around ROI are shown as an estimator of the contribution of the image noise.

FIG. S5. Quantification of dephasing. Examples are for \( N_{\text{tot}} = 1 \times 10^4 \) at (a) \( t = 0 \text{ s} \), (b) \( t = 0.4 \text{ s} \), (c) \( t = 0.8 \text{ s} \). The solid black curves in the upper branches show the period average distributions \( F(z) \) and the dashed red curves show \( F(z) \pm \sqrt{\text{Var}(z)} \) as an illustration of the change of the longitudinal momentum distributions during one period. The variances of the distributions \( \text{Var}(z) \) are shown as blue curves below. The green and purple curves correspond to the background regions which around the the region of interest.
ference between $F_l(z)$ and $G_l(z)$ (fitting residue) gives the information how far $F_l(z)$ is away from the Gaussian distribution. We give two examples in Fig. S6. The average momentum distribution just after dephasing at $t = 0.8$ s is still far away from a thermal Gaussian distribution. The fitting residue, shown in blue in the lower part of the figure, are significantly larger than the background noise. The relaxation as quantified by Eq. S2 is the mean sum of the square of the fitting residue. In contrast, at $t = 4.8$ s, the distribution is almost Gaussian, and the fitting residue is approaching the same level with the background noise.

To estimate the noise floor, we choose the same background areas as shown in Fig. S2. In Fig. 4 of the main text, the background noise is subtracted from the result obtained in ROI.

**HEATING AND ATOM LOSS**

The heating process in our experiment is quantified in two methods. Firstly, we observe the evolution of the longitudinal momentum distribution of a BEC in the tubes without excitation pulses. The distributions for $N_{tot} = 1 \times 10^4$ and $N_{tot} = 1 \times 10^5$ are shown both in Fig. S7. From that, we estimate the average longitudinal kinetic energy of the atoms at different evolution times and are calculate the rate of increase of the longitudinal kinetic energy as $\sim 0.010 \hbar \omega_{z}/s$ for $N_{tot} = 1 \times 10^4$ and $\sim 0.016 \hbar \omega_{z}/s$ for $N_{tot} = 1 \times 10^5$, respectively.

Secondly, we can also estimate the heating rate by evaluating the excitation rate to the first transverse excited state for the 1D gases without excitation pulses. We see up to $0.007 \hbar \omega_{z}/s$ excitation for $N_{tot} = 1 \times 10^5$ (see Fig. S7(b) in the main text).

The heating mainly results from spontaneous emission and trap fluctuations. In this way the atoms are excited to transverse states, and the energy may be restored into the longitudinal kinetic energy through collisions between them. In general, the heating is larger for a system with higher density (larger atom number). This suggests that the heating rate for a BEC in 2D optical lattice should give an upper limit to the Newtons cradle system with excitation pulses, where the atoms are spread out more and the density is lower.

The atom loss rate is $\sim 4\%$/s (for $N_{tot} = 1 \times 10^4$) to $\sim 7\%$ (for $N_{tot} = 1 \times 10^5$), which also increases the $\gamma$ during the evolution. The loss rate is nearly uniform during the evolution.

**THEORETICAL MODEL**

A numerical model, to be practicable and efficient in terms of computational resources and time, must contain certain simplifications and also disregard less relevant properties of a system in question. Therefore we summarize first, what is not included into our model.

The relaxation mechanisms due to three-body elastic collisions mediated by virtual excitations of the radial degrees of freedom [2, 3] and the interplay between the longitudinal motion of atoms in a harmonic potential and the Wigner delay time associated with a collision [4] are neglected since they are extremely inefficient at the low 1D densities typical for the experiment.

Our model is semiclassical. The atoms move in the longitudinal direction as classical point-like objects. A harmonic longitudinal potential is assumed in order to make calculations simple and fast. We tested also an anharmonic potential $U_0 \tanh^2(z/\Delta z)$ that admits analytic integration of the equations of motion. The parameter $U_0$ was taken equal to the lattice depth and the typical length scale $\Delta z$ was chosen to provide the harmonic potential $\frac{1}{2}m \omega^2_0 z^2$ for $|z| \ll \Delta z$, where $\omega_0$ is the frequency of the longitudinal oscillations from the ex-
periment. With these parameters the effect of the anharmonicity of the potential was found to be small and therefore we restricted ourselves to the harmonic model.

The radial state of an atom is treated in a quantum way and specified by a number \( n \) of radial excitation quanta, \( n = 0 \) corresponding to the ground state of the radial motion. We do not resolve the degenerate sublevels, but invoke the statistical weight (i.e., degeneracy)

\[
w_n = n + 1
\]

of the corresponding state of an isotropic 2D harmonic oscillator.

Each numerical realization corresponds to a single tube. The number of atoms per tube \( N_{th} \) is an input parameter. The momenta of the atoms are sampled according to the probability distribution derived from the experimentally measured initial momentum distribution (with peaks at 0, \( \pm 2\hbar k_L \), and \( \pm 4\hbar k_L \)). The co-ordinate distribution is Gaussian, its width is chosen such that the equidistribution of the energy between its potential and kinetic parts takes place if the momentum distribution contains only a single central peak.

Now we describe how atomic collisions are modeled. Since the system is 1D and atoms are indistinguishable, we always can consider an ordered array of atoms, \( z_1 < z_2 < \ldots < z_{N_{th}-1} < z_{N_{th}} \). In what follows, it is convenient to introduce the scaled co-ordinates \( \bar{z}_j = z_j/l \) and momenta \( \bar{k}_j = p_j/l/\hbar \), where \( l = \sqrt{\hbar/(m\omega_\perp)} \).

For a given configuration of \( N_{th} \) atoms in the phase space we calculate the time of the first collision, i.e. the first (smallest) time when the co-ordinate of any two neighboring atoms atoms coincide. The oscillatory motion of the \( j \)th atom is described by

\[
\begin{align*}
\bar{x}_j(t + \tau) &= \bar{x}_j(t) \cos \omega_\parallel \tau + \bar{k}_j(t) \sin \omega_\parallel \tau, \\
\bar{k}_j(t + \tau) &= -\bar{x}_j(t) \sin \omega_\parallel \tau + \bar{k}_j(t) \cos \omega_\parallel \tau.
\end{align*}
\]

Then we calculate the collision time \( \tau_j \) for the \( j \)th and \( (j + 1) \)th atoms:

\[
\tan \tau_j = -\omega_\parallel^{-1} \bar{x}_j(t) - \bar{x}_{j+1}(t) / \bar{k}_j(t) - \bar{k}_{j+1}(t), \quad \tau_j > 0,
\]

and find the smallest one,

\[
\tau = \min_{1 \leq j \leq N_{th}-1} \tau_j.
\]

We propagate the atoms until the time \( t + \tau \) according to Eq. (S3) and then decide, according to the probabilities (see below) and using a pseudorandom number generator, what happens to the radial states of the involved atoms. The probabilities of the change of the radial state are based on the standard quantum mechanical expressions, which can be easily derived for a pair of colliding atoms with the initial state of their relative motion in the \((x, y)\)-plane is the radial ground state \([5]\). However, for the sake of simplicity, we neglect any dependence of the radial transition probabilities on the radial quantum states of colliding atoms. Also the scheme of radial transitions is simplified.

To be definite, consider a collision of the atoms 1 and 2. Their co-ordinates at the collision time are \( z_1 = z_2 \) and the respective momenta are \( \hbar k_1 \) and \( \hbar k_2 \). The momentum of the relative motion is canonically conjugate to the interatomic distance \( 2z_2 - z_1 \) and defined as

\[
\hbar k = \frac{1}{2} h(k_2 - k_1).
\]

The total momentum of the pair is denoted by \( hK = h(k_1 + k_2) \).

As concerns the radial quantum numbers, we begin with the option \( n_1 = n_2 \).

Because of the parity conservation, the radial energy of a pair of atoms in the course of collision can change by a multiple of \( 2\hbar \omega_\perp \). If the kinetic energy of the relative motion is less than \( 2\hbar \omega_\perp \), then the increase of the radial energy is impossible. In the opposite case,

\[
\frac{\hbar^2 q^2}{m} = \frac{\hbar^2 k^2}{m} - 2\hbar \omega_\perp > 0,
\]

the increase of the radial energy by \( 2\hbar \omega_\perp \) is possible. The probability of such an event is

\[
P_\tau = \frac{2k q a_{1D}^2}{k^2 q^2 + a_{1D}^2 (k + q)^2},
\]

where

\[
a_{1D} = \frac{a_s}{l_\perp}
\]

is the 1D scattering amplitude, \( a_s \) is the 3D s-wave scattering length, and \( l_\perp = \sqrt{\hbar/(m\omega_\perp)} \). A more exact formula [24] taking into account the effects of the strong radial confinement on the scattering amplitude yields

\[
a_{1D} = \frac{a_s}{l_\perp (1 - C \frac{a_s}{l_\perp})},
\]

where \( C \approx 1.06 \ldots \). A pseudorandom number \( \zeta \) uniformly distributed between 0 and 1 is generated. If \( \zeta < P_\tau \) then we raise the radial excitation energy by 2 quanta. To preserve the ordering of atoms in the course of the subsequent evolution, we assign the new (primed) momenta to them as follows:

\[
\hbar k_1' = h\left(\frac{1}{2} K - q\right), \quad \hbar k_2' = h\left(\frac{1}{2} K + q\right).
\]
With the help of a new pseudorandom number we assign the new radial quantum numbers with the following probabilities:

\[ n'_1 = n_1, \quad n'_2 = n_1 + 2 \quad (25\%), \]
\[ n'_1 = n_1 + 2, \quad n'_2 = n_1 \quad (25\%), \]
\[ n'_1 = n_1 + 1, \quad n'_2 = n_1 + 1 \quad (50\%). \]

The detailed balance condition should be satisfied: the number of transitions up and down per unit time must be the same on average. Therefore, if

\[ n_1 = n_2 > 0 \]

then we allow for the transition to the state characterized by

\[ n'_1 = n_1 - 1, \quad n'_2 = n_1 - 1, \]

and

\[ \hbar k'_1 = \hbar \left( \frac{1}{2} K - Q \right), \quad \hbar k'_2 = \hbar \left( \frac{1}{2} K + Q \right), \]

where

\[ Q = \sqrt{q^2 + L^2}. \]

The probability of this process is

\[ P_{n_1, n_1 \rightarrow n_1 - 1, n_1 - 1} = \frac{1}{2} \frac{w_{n_1 - 1}}{w_{n_1}^2} P_\perp, \]

where

\[ P_\perp = \frac{2kq\alpha_{1D}^2}{k^2Q^2 + \alpha_{1D}^2(k + Q)^2}. \]

The prefactor in front of \( P_\perp \) in Eq. (S6) ensures the detailed balance. The condition of the downward radial transition corresponds to the pseudorandom number \( \zeta \) falling between \( P_\parallel \) and \( P_\perp + P_{n_1, n_1 \rightarrow n_1 - 1, n_1 - 1}. \)

If, finally, \( \zeta > P_\parallel + P_{n_1, n_1 \rightarrow n_1 - 1, n_1 - 1} \) then no change of the radial states takes place. To maintain the ordering of atoms in this case, we set

\[ \hbar k'_1 = \hbar k_2, \quad \hbar k'_2 = \hbar k_1. \]

This is always the case when two atoms in the ground radial states collide with the energy insufficient for excitation by two radial quanta. Consider now another possibility

\[ n_1 \neq n_2. \]

Here an important simplification of the model comes into play. If the radial states of colliding atoms are different, we neglect, except of a special case described below, the change of the set of the radial excitation numbers, allowing for the exchange process only, when the radial excitation numbers associated with the two momenta \( \hbar k_1 \) and \( \hbar k_2 \) are interchanged:

\[ \hbar k'_1 = \hbar k_2, \quad \hbar k'_2 = \hbar k_1, \]
\[ n'_1 = n_1, \quad n'_2 = n_2. \]

The probability of the exchange process is given by

\[ P_{ex} = \frac{\alpha_{1D}^2}{k^2 + \alpha_{1D}^2}. \]

A special case is given by

\[ n_2 = n_1 + 2 \quad \text{or} \quad n_1 = n_2 + 2. \]

In this case, in addition to the exchange process, the decrease of the larger of the radial excitation numbers by 2 can happen, as it is required by the detailed balance:

\[ n'_1 = n'_2 = \min(n_1, n_2) \]

and the momenta after collision are given by Eq. (S6). The respective probability is given by

\[ P_{|n_1 - n_2|=2\rightarrow n_1=n_2} = \frac{1}{4} \frac{w_{\min(n_1, n_2)}}{w_{\max(n_1, n_2)}^2} P_\perp, \]

where \( P_\perp \) is again given by Eq. (S8).

The last ingredient of our model is the change of the radial state of an atom due to the heating in the optical lattice. Its probability per unit time per atom is denoted by \( \Gamma \). We assume \( N_{ib}\Gamma \tau \ll 1 \), where \( \tau = \pi/(\omega_{ib}N_{ib}) \) is the typical time between two subsequent atomic collisions. Each time we have a collision (i.e., when co-ordinates of two neighboring atoms coincide) we do not only check for the possibility of the change of the radial states of the colliding pair of atoms, but also check the possibility of the change of the radial state for all the atoms. We generate a pseudorandom number \( \zeta' \) uniformly distributed between 0 and 1. If \( \zeta' < \exp(-N_{ib}\Gamma \tau) \), where \( \tau \) is the time elapsed since the previous collision, then no state change occurs. Otherwise we pseudorandomly select one of the \( N_{ib} \) atoms and change its radial excitation number \( n_j \) to \( |n_j + 1| \) or \( |n_j - 1| \), each channel having the probability of 50%. Since atoms are predominantly in the ground state, the most probable process \( n_j = 0 \rightarrow n'_j = 1 \) leads to the energy supply to the system (heating).

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[1] C. Li, T. Zhou, Y. Zhai, X. Yue, J. Xiang, S. Yang, W. Xiong, and X. Chen, Physical Review A 95, 033821 (2017).

[2] J. E. Mazets, T. Schumm, and J. Schmiedmayer, Physical Review Letters 100, 210403 (2008).
[3] I. E. Mazets and J. Schmiedmayer, *New Journal of Physics* **12**, 055023 (2010).

[4] I. E. Mazets, *The European Physical Journal D* **65**, 43 (2011).

[5] M. Olshanii, *Physical Review Letters* **81**, 938 (1998).