Pentaquark $\Theta^+$, constituent quark structures, and prediction of charmed $\Theta_c^0$ and bottomed $\Theta_b^+$.

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The newly observed $\Theta^+$ resonance is believed to be a pentaquark with the constituent quarks $uudd\bar{s}$. There are a few options for the constituent quark structure. Some advocate diquark-diquark-antiquark $(ud)-(ud)-\bar{s}$ while others favor diquark-triquark $(ud)-(ud\bar{s})$. We use the color-spin hyperfine interaction to examine the energy levels of these structures, and we find that the diquark-diquark-antiquark structure is slightly favored. We proceed to write down the flavor triplet and antiseptet of the charmed or bottomed exotic baryons with internal $qqqQ$ quarks. We also estimate the mass of $\Theta_c^0$ and $\Theta_b^+$. 

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I. INTRODUCTION

The recent discovery of the $\Theta^+(1540)$ resonance $\bar{\Theta}^{3}\Theta^{0}\Theta$ has revived an old interest in bound states with more than 3 constituent quarks (see e.g. Refs. [8,9,10].) The resonance has been observed in the reaction $\gamma^{12}C \rightarrow K^-\Theta^+ \rightarrow K^-K^+n$ by LEPS [1], in $K^+Xe \rightarrow Xe'\Theta^+ \rightarrow Xe'K^0p$ by DIANA [2], in $\gamma d \rightarrow K^-p\Theta^+ \rightarrow K^-pK^+n$ by CLAS [3], and in $\gamma p \rightarrow K_s\Theta^+ \rightarrow K_sK^+n$ by SAPHIR [4]. The mass of the resonance is at around 1540 MeV with a width of order 20 MeV and an isospin $I = 0$. The spin-parity is $\frac{1}{2}^+$. Such a narrow width can be explained by an isospin-violating decay.

The interpretation of the $\Theta^+$ has been made in the constituent quark model [8,9,10] and in the Skyrmion or chiral soliton models [11]. There are also other studies [13] related to the newly discovered $\Theta^+$. In this work, we concentrate on the quark constituent model. Since $\Theta^+$ has the internal quarks $uudd\bar{s}$, there are various possible configurations for this complicated system. The naive $K-N$ molecular interpretation involves only a weak van-der-Waals-like force between the $K$ and $N$. In general, the color triplet, sextet, and octet interactions are much more attractive than the color singlet bond. In view of this, Jaffe and Wilczek (JW) [8] interpreted the bound state as a diquark-diquark-antiquark. Each diquark pair is in the $3_f$ representation of SU(3), and therefore the system is like $3_f \times 3_f \times 3_f$, similar to a normal antibaryon. Of course, the spin of each diquark pair is different from a normal quark. Its spin $S = 0$. Thus, the two diquark pairs combine in a $P$-wave orbital angular momentum to form a state with $3_f$, in color, spin $S = 0$, and $6_f$ in flavor. Then, combining with the antiquark to form a flavor antidecuplet and octet, with spin $S = 1/2$. The $\Theta^+$ is at the top of the antidecuplet and has an isospin $I = 0$.

On the other hand, Karliner and Lipkin (KL) [8,11] interpreted the bound state as a diquark-triquark $(ud)-(ud\bar{s})$. The first stand-alone $(ud)$ diquark pair is in a state of spin $S = 0$, color $3_v$ and flavor $3_f$ while the second $(ud)$ diquark pair inside the cluster $(ud\bar{s})$ is in a state of spin $S = 1$, color $6_v$ and flavor $3_f$. The triquark cluster is then in a state of spin $S = 1/2$, color $3_v$ and flavor $6_f$. So the overall configuration will give a color singlet, spin $S = 1/2$, and a flavor octet and antidecuplet. The $\Theta^+$ is at the top of the antidecuplet and thus has $I = 0$. This internal configuration of KL is different somewhat from that of JW. The differences are (i) both the diquark pairs have the same configuration in JW while in KL they are asymmetric, (ii) the order of combining: in JW the diquark pairs are first combined to form the diquark-diquark subsystem before combining with the antiquark while in KL the second diquark pair first combines with the antiquark to form the triquark cluster, then combine with the diquark cluster, and (iii) the color-spin hyperfine interaction would be different (we shall explain next.)

The constituent quark model has been successful in describing the meson and baryon spectra, with the mesons in the flavor singlet and octet, and the baryons in the flavor singlet, octet, and decuplet. The chromo-magnetic (color-spin) hyperfine interaction was shown to be the dominant mechanism in the determination of the mass splittings in the $S$-wave $qq$ mesonic and $qqq$ baryonic systems [14]. It was widely believed that the same is true in 4-quark and 5-quark systems [8,9].

In this note, we shall employ the color-spin hyperfine interaction to investigate the hyperfine energy levels of various quark configurations.\footnote{There is another approach using flavor-spin hyperfine interaction [15] to study the stability of various configurations.} We found that the picture of diquark-diquark-antiquark of JW [8] will give the most favorable
hyperfine interaction while the picture of KL [9, 10] has a slightly higher hyperfine interaction, but it does not mean that it is unstable. The difference in hyperfine interaction is less than 100 MeV, which is in the same order as the uncertainty in the estimation. We shall also point out that the naive treatment of KL that there is no color-spin hyperfine interaction between the diquark and triquark clusters is not adequate. If we took their assumption, we found that the difference in hyperfine interaction between the configurations of diquark-diquark-antiquark and diquark-triquark would be of order 200 MeV, which is too large compared with the uncertainty. We shall also extend to the charmed and bottomed baryons with the replacement of $\bar{s} \rightarrow \bar{c}, \bar{b}$. We give the estimate for the mass of $\Theta^+_c$ and $\Theta^+_b$.

II. COLOR-SPIN HYPERFINE INTERACTION

The color-spin hyperfine interaction [14] was shown to be dominant in the determination of the mass splittings in the $S$-wave mesons and baryons. Extensions to more complicated quark systems were also performed (see e.g. [3, 6]). The Hamiltonian describing the color-spin hyperfine splitting of a multi-quark system is given by [5]

$$ H_{hf} = -V \sum_{i>j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) , $$

where $\vec{\lambda}$ and $\vec{\sigma}$ denote, respectively, the matrices for the color SU(3)$_c$ and the spin SU(2), and $i, j$ are the quark labels. The color SU(3)$_c$ and the spin SU(2) of the quarks can be combined in a SU(6) color-spin symmetry. For example, the fundamental representation for a quark in SU(6) is $6 = (3, 1/2)$, where the first label inside the parenthesis is the representation for the SU(3)$_c$ and the second label is the spin. We use the following notation to denote a particular quark configuration

$$ |D_6, D_{3c}, S, N, D_{3f} \rangle , $$

where $D_6, D_{3c}, D_{3f}$ are representations in SU(6) of color-spin, in SU(3)$_c$, and in SU(3)$_f$, respectively, $S$ is the spin of the system, and $N$ is the total number of quarks or antiquarks in the configuration. A quark will be denoted by $|6, 3, 1/2, 1, 3_f \rangle$.

The general expression for the color-spin hyperfine splitting in systems with quarks and antiquarks is given by [5]

$$ V_{hf} = \frac{v}{2} \left[ \tilde{C}(\text{total}) - 2\tilde{C}(Q) - 2\tilde{C}(\bar{Q}) + 16N \right] , \quad (1) $$

where $\tilde{C}(\text{total})$ refers to the whole system, $\tilde{C}(Q)$ refers to the subsystem of quarks only while $\tilde{C}(\bar{Q})$ refers to the subsystem of antiquarks only. Here

$$ \tilde{C} = C_6 - C_3 - \frac{8}{3}S(S+1) $$

(2)

where $C_6(D_6)$ and $C_3(D_3)$ denote the Casimir of the representation $D_6$ in SU(6) and of $D_3$ in SU(3)$_c$, respectively. The constant $v$ can be determined using the hyperfine splitting between $N$ and $\Delta$:

$$ V_{hf}(\Delta) - V_{hf}(N) = M_\Delta - M_N = 16v = 293.08 \text{ MeV} , \quad (3) $$

where we have used the average mass of $p$ and $n$ for $M_N$ [15].

A. Diquark-diquark-antiquark

First, we look at the possible structures of a diquark with their hyperfine splittings

$$ |21, 3, 0, 2, 3_f \rangle : -8v $$

$$ |21, 6, 1, 2, 3_f \rangle : -\frac{4}{3}v $$

$$ |15, 3, 1, 2, 6_f \rangle : \frac{8}{3}v $$

$$ |15, 6, 0, 2, 6_f \rangle : 4v , $$
where we have required the combined wavefunction to be antisymmetric under the interchange of the two quarks. It is obvious that the first diquark is the most stable, followed by the second one. We shall keep these two structures in the following discussion.

Next, we combine the diquark-diquark. Based on the fact that the diquark-diquark must be in $3_c$ in order to give a color singlet with the antiquark, there are two possibilities to combine the diquark-diquark:

$$|21, \bar{3_c}, 0, 2, 3_f\rangle \otimes |21, \bar{3_c}, 0, 2, 3_f\rangle \supset |210, 3_c, 0, 4, 3_f + \bar{6}_f\rangle \quad (4)$$

$$|21, \bar{3_c}, 0, 2, 3_f\rangle \otimes |21, \bar{6}_c, 1, 2, 3_f\rangle \supset |210, 3_c, 1, 4, 3_f + \bar{6}_f\rangle . \quad (5)$$

Note that in the second combination it is also possible to have the $105$ that contains $\binom{3_c}{1}$, but however with a smaller Casimir. Also note that the flavor in the diquark-diquark configuration can either be a $3_f$ or a $\bar{6}_f$, which is antisymmetric and symmetric, respectively, under the interchange of the diquark pair. Since the diquark-diquark system is a totally symmetric state, the $\bar{6}_f$ must be combined with a spatially antisymmetric state (i.e. a $P$-wave), while the $3_f$ has to combine with a spatially symmetric state (i.e. a $S$-wave). Jaffe and Wilczek \cite{26} argue that the blocking repulsion may raise the energy of the spatially symmetric states, and so the $P$-wave state is preferred. We use Eq. (1) to evaluate the color-spin hyperfine splitting between these two diquark-diquark states:

$$V(|210, 3_c, 0, 4, 3_f + \bar{6}_f\rangle) = -16v, \quad V(|210, 3_c, 1, 4, 3_f + \bar{6}_f\rangle) = -\frac{40}{3}v. \quad (6)$$

The first diquark-diquark configuration is relatively more stable, but however the second configuration is only slightly higher in hyperfine level. The next step is to combine the diquark-diquark with the antiquark, using the diquark-diquark state in Eq. (4), we have

$$|210, 3_c, 0, 4, 3_f\rangle \otimes |6, \bar{3_c}, 1/2, 1, 3_f\rangle \supset |70, 1_c, 1/2, 5, 8_f + \bar{10}_f\rangle , \quad (7)$$

which has a hyperfine splitting of

$$V_{hf}(|70, 1_c, 1/2, 5, 8_f + \bar{10}_f\rangle) = -40v . \quad (8)$$

On the other hand, combining the diquark-diquark state in Eq. (5) with the antiquark we have

$$|210, 3_c, 1, 4, 3_f\rangle \otimes |6, \bar{3_c}, 1/2, 1, 3_f\rangle \supset |70, 1_c, 1/2, 5, 8_f + \bar{10}_f\rangle , \quad (9)$$

which has a hyperfine splitting of

$$V_{hf}(|70, 1_c, 1/2, 5, 8_f + \bar{10}_f\rangle) = -\frac{104}{3}v . \quad (10)$$

Note that the spin $S = 3/2$ state has to go with the $1134$ of the SU(6), which would give a much less favorable configuration. Although the final configurations in Eqs. (7) and (9) are the same, one of the diquark pairs in Eq. (4) is different in the spin, which induces the difference in the final hyperfine energy levels. Thus, in the picture of Jaffe and Wilczek the most favorable configuration is (i) both the diquark pairs are in $|21, \bar{3_c}, 0, 2, 3_f\rangle$, (ii) the diquark pairs are in a $P$-wave state, and (iii) combining with the antiquark to produce $|70, 1_c, 1/2, 5, 8_f + \bar{10}_f\rangle$, which has a positive parity.

**B. Diquark-triquark**

Karliner and Lipkin \cite{27, 28} suggested that the $\Theta^+$ has an internal $(ud)$-$(uds)$ quark structure, in which the $(ud)$ diquark is an $I = 0$ color antitriplet and the $(uds)$ triquark is an $I = 0$ color triplet, with a $P$-wave orbital angular momentum between the two clusters. They also assumed that the color-spin hyperfine interaction only operates within each cluster, but is negligible between the two clusters. However, we will show below that the hyperfine interaction energy will be further minimized if we consider the hyperfine interaction between the clusters.

The configuration of the stand-alone diquark is $|21, 3_c, 0, 2, 3_f\rangle$, while the diquark inside the triquark system has a configuration $|21, \bar{6}_c, 1, 2, 3_f\rangle$. Combining with the antiquark $|6, 3_c, 1/2, 1, 3_f\rangle$, the triquark $(uds)$ has a configuration $|6, 3_c, 1/2, 3, \bar{6}_f\rangle$. Then they evaluate the hyperfine splitting as

$$V_{hf} = V_{hf}({\text{diquark}}) + V_{hf}({\text{triquark}}) = -8v - \frac{56}{3}v = -\frac{80}{3}v . \quad (11)$$
With this value of $V_{hf}$ the diquark-triquark configuration is not as stable as the diquark-diquark-antiquark configuration of Eq. (7).

However, we can evaluate more carefully the hyperfine energy of the diquark-triquark system, including the interaction between the clusters, with Eq. (11):

$$V_{hf} = \frac{V}{2} \left[ \tilde{C}(ud-ud\bar{s}) - 2\tilde{C}(ud-ud) - 2\tilde{C}(\bar{s}) + 80 \right],$$

where

$$
\begin{align*}
\tilde{C}(ud-ud\bar{s}) &= \tilde{C}(|70, 1_c, 1/2, 5, 8 + 10f_f\rangle) = 64 \\
\tilde{C}(ud-ud) &= \tilde{C}(|105 + 210, 3_c, 1, 4, 6_f\rangle) = \frac{224}{3} \text{ or } \frac{272}{3} \\
\tilde{C}(\bar{s}) &= \tilde{C}(|6, 3_c, 1/2, 1, 3_f\rangle) = 16.
\end{align*}
$$

Then

$$V_{hf} = -\frac{56}{3}v \text{ or } -\frac{104}{3}v,$$

which depends on whether one takes 105 or 210, respectively, for the $ud-ud$ system. It happens that the naive assumption that there is no hyperfine interaction between the two clusters [13] gives the average of the two hyperfine splittings obtained in Eq. (13). At this point, we can compare the hyperfine levels of the configurations of JW and LK. According to Eq. (8) and the more negative one in Eq. (13), the diquark-diquark-antiquark configuration is slightly more favorable than the diquark-triquark configuration.

C. Mass of $\Theta^+$

Overall, the most favorable configuration is the diquark-diquark-antiquark picture, in which both diquark pairs are in color and flavor antitriplet, spin $S = 0$, in a $P$-wave orbital angular momentum. The diquark-diquark then combine with the antiquark to form a spin $S = 1/2$, color singlet, and flavor octet or antidecuplet. However, the quark configuration that the diquark-diquark is in a spin $S = 1$ state of Eq. (9) is just slightly higher in energy level.

We should keep this state as well in the following discussion. In fact, this configuration has the same hyperfine energy as the diquark-triquark picture, the hyperfine energy of which is given by the smaller value in Eq. (13).

We shall next estimate the mass of $\Theta^+$, using these two favorable configurations. We use the approach of Karliner and Lipkin [8, 10]. We separate the total hyperfine interaction into two portions: one from the $(ud)$-$(ud)$ and another one from the interaction with the antiquark. Thus, from Eq. (8) and Eq. (13),

$$V_{hf} = -16v - 24 \left( \frac{m_u}{m_Q} \right) v$$

where $m_Q$ is the mass of the antiquark inside the $\Theta^+$. Now we can evaluate the hyperfine interactions of a nucleon $N$ and a meson $(q\bar{Q})$:

$$\begin{align*}
V_{hf}(N) &= -8v \\
V_{hf}(q\bar{Q}) &= -16 \left( \frac{m_u}{m_Q} \right) v.
\end{align*}$$

We then take the difference in the hyperfine splitting as the mass difference:

$$V_{hf}(ud-ud\bar{Q}) - V_{hf}(N) - V_{hf}(q\bar{Q}) = M_{ud-ud\bar{Q}} - M_N - M_{q\bar{Q}} = -\frac{1}{2} \left( 1 + \frac{m_u}{m_Q} \right) (M_\Delta - M_N).$$

On the other hand, if we use the quark configuration that the diquark-diquark is in a spin $S = 1$ state, the above equation becomes

$$V_{hf}(ud-ud\bar{Q}) - V_{hf}(N) - V_{hf}(q\bar{Q}) = M_{ud-ud\bar{Q}} - M_N - M_{q\bar{Q}} = -\frac{1}{3} \left( 1 + \frac{m_u}{m_Q} \right) (M_\Delta - M_N).$$
We also have to estimate the $P$-wave excitation energy of $\Theta^+$. Instead of using the $D_s$ system, here we employ the mass difference between $\Lambda(\frac{3}{2}^+)$ and $\Lambda(\frac{1}{2}^-)$:

$$\delta P_s = M_{\Lambda(\frac{1}{2}^-)} - M_{\Lambda(\frac{3}{2}^+)} = 290.3 \text{ MeV}.$$  \hspace{1cm} (19)

The reason we used this mass difference as the $P$-wave excitation energy is this is the closest known system to the $\Theta^+$ that both systems contain exactly one strange antiquark and the rest being light $u, d$ quarks. The hyperfine splitting between $\Lambda(\frac{1}{2}^-)$ and $\Lambda(\frac{3}{2}^+)$ is zero. Thus, the mass of $\Theta^+$ is estimated to be, with $m_u/m_s \approx 2/3$, \begin{equation}
M_{\Theta^+} = \begin{cases} 
1481 \text{ MeV & if using Eq. (17)} \\
1562 \text{ MeV & if using Eq. (18)}.
\end{cases} \hspace{1cm} (20)
\end{equation}

The observed mass of $\Theta^+$ is closer to the second value. Some comments are in order.

(i) The $P$-wave excitation energy estimated here is almost 100 MeV larger than that in Ref. [10]. It means that there is an intrinsic uncertainty of order 100 MeV in the estimation.

(ii) Although the observed mass of $\Theta^+$ is closer to the second favorable configuration, it does not mean that the most favorable configuration is wrong. There are perhaps some unknown nonperturbative effects involved in the five-quark bound states that may affect the most favorable configuration and the second most favorable configuration. Also, the $P$-wave excitation energy may be different in these two configurations, perhaps due to some orbital-spin interactions, which we naively ignore.

(iii) Nevertheless, we believe the assumption taken by Karliner and Lipkin [11] that there is no hyperfine interaction between clusters is somewhat inadequate. We found that if we took their naive assumption the difference in hyperfine interaction between the most favorable configuration and their configuration is $v(8 + 16/(m_u/m_s)) \approx 200 \text{ MeV}$, which is too large compared with the uncertainty.

(iv) Straightly speaking, the hyperfine interaction formula in Eq. (11) is only applicable to $S$-wave hadronic systems. Here we have taken the assumption that the color-spin hyperfine interaction in $P$-wave hadronic systems is the same as in the $S$-wave systems. When we compared the hyperfine energy levels of the JW and KL configurations, they are in the same orbital angular momentum.

(v) There is also a possible mixing between these two configurations. Let us denote the diquark-diquark-antiquark configuration by $|a\rangle$ and diquark-triquark by $|b\rangle$. Allowing a mixing between these two states we write the hyperfine interactions as

\begin{equation}
\langle a| \delta h \delta h |b\rangle, \hspace{1cm}
\end{equation}

where $H_a, H_b$ denote the hyperfine interaction of the state $|a\rangle$ and $|b\rangle$, respectively, and $\delta h$ denotes the mixing. We can diagonalize the states through a mixing angle $\theta_{\text{mix}}$

\begin{align*}
|a\rangle &= \cos \theta_{\text{mix}} |1\rangle + \sin \theta_{\text{mix}} |2\rangle \\
|b\rangle &= -\sin \theta_{\text{mix}} |1\rangle + \cos \theta_{\text{mix}} |2\rangle
\end{align*}

\begin{equation}
\tan 2\theta_{\text{mix}} = \frac{2\delta h}{H_b - H_a},
\end{equation}

Assuming $\delta h \ll H_b - H_a$ the mixing angle $\theta_{\text{mix}} \approx \delta h/(H_b - H_a)$, and we obtain the eigen-masses

\begin{align*}
m_1 &= H_a - \frac{(\delta h)^2}{H_b - H_a} \\
m_2 &= H_b + \frac{(\delta h)^2}{H_b - H_a}
\end{align*}

Therefore, the mass splitting between $m_1$ and $m_2$ is

\begin{equation}
m_2 - m_1 = H_b - H_a + \frac{2(\delta h)^2}{H_b - H_a}, \hspace{1cm} (21)
\end{equation}

which implies that the splitting between the two configurations is increased by a factor, which depends on the parameter $\delta h$. Based on this mixing argument the mass estimates in Eq. (20) will be modified such that the smaller one is lowered while the larger one is raised by an amount $(\delta h)^2/(81 \text{ MeV})$. 

FIG. 1: The flavor triplet and antisextet of the charmed baryons with a charm antiquark and 4 light quarks \((u, d, s)\). The triplet consists of the three antisymmetric pairs \([ud][ds]_-, [ud][us]_-, \text{and} [ds][us]_-\), while those with \([ud][ds]_+, [ud][us]_+, \text{and} [ds][us]_+\) belong to the symmetric antisextet.

III. CHARMED PENTAQUARK \(\Theta^0_c\)

The diquark-diquark-antiquark picture of Jaffe and Wilczek [8] can be easily extended to charmed pentaquark, with the replacement \(\bar{s} \rightarrow \bar{c}\). Since the charm quark does not belong to the \(SU(3)_f\) of \((u, d, s)\), the internal quark configuration of \(\Theta^0_c\) will follow the configuration of the diquark-diquark subsystem, which is \(\{210, 3c_\pi, 0, 4, 3f + \bar{6}_f\}\).

For the \(3f\) the diquark-diquark will be in a \(S\)-wave state while for \(\bar{6}_f\) it will be in a \(P\)-wave state. The flavor triplet and antisextet are shown in Fig. 1. Pentaquark baryons in the framework of Skyrme model were considered before [16]. There also exist upper limits on the production of the isodoublet \(|\bar{c}suud\rangle\) and \(|\bar{c}sdud\rangle\) reported by the E791 collaboration [17].

We can also estimate the mass of \(\Theta^0_c \equiv (ud)-(ud)-\bar{c}\). The formula is similar to the one for \(\Theta^+\) (analogous to Eq. (18))

\[
M_{\Theta^0_c} = M_N + M_D - \left(\frac{1}{2} \frac{M_N + M_D}{M_N - M_D}\right) + \delta P_c ,
\]

where the \(P\)-wave excitation energy is estimated by the mass difference between \(\Lambda_c(\frac{1}{2}^+)\) and \(\Lambda_c(\frac{1}{2}^-)\). The mass of \(\Theta^0_c\) is then given by, with \(m_u/m_c \approx 0.21\),

\[
M_{\Theta^0_c} = \begin{cases} 2938 \text{ MeV} \\ 2997 \text{ MeV} \end{cases} .
\]

Again, the major uncertainty comes from the estimation of the \(P\)-wave excitation energy, which is of order of 100 MeV. Similarly, we can estimate the bottomed baryon \(\Theta^+_b\) using

\[
M_{\Theta^+_b} = M_N + M_B - \left(\frac{1}{2} \frac{M_N + M_B}{M_N - M_B}\right) + \delta P_b \quad \text{for} \quad \Lambda_b(\frac{1}{2}^-) \text{ not found experimentally} .
\]

Since the \(\Lambda_b(\frac{1}{2}^-)\) has not been found experimentally, we use \(\delta P_b \approx \delta P_c\), which is reasonable because \(\delta P_c \approx \delta P_s\). Therefore, we obtain with \(m_u/m_b \approx 0.071\)

\[
M_{\Theta^+_b} = \begin{cases} 6370 \text{ MeV} \\ 6422 \text{ MeV} \end{cases} .
\]

2 Jaffe and Wilczek [8] also estimated the mass of \(\Theta^0_c\) and \(\Theta^+_b\). They used \(M_{\Theta^0_c} - M_{\Theta^+_b} = M_{\Lambda_c} - M_{\Lambda} = 1170 \text{ MeV}\), and similarly for \(\Theta^+_b\). They obtained substantially lower masses than our estimates. Our estimates are close to those by Karliner and Lipkin [10].

3 This is analogous to what we used in Eq. (19), the \(\Lambda_c(\frac{1}{2}^+)\) and \(\Lambda_c(\frac{1}{2}^-)\) system contains exactly one charm antiquark and the rest being \(u, d\) light quarks.
IV. CONCLUSIONS

We have used the color-spin hyperfine interaction to examine the hyperfine energy levels of various quark configurations. We found that the picture of diquark-diquark-antiquark of Jaffe and Wilczek [8] gives the most favorable hyperfine interaction while the picture of Karliner and Lipkin [9, 10] has a slightly higher hyperfine interaction, but it does not mean that it is unstable. The observed mass of Θ⁺ is in between the two hyperfine levels of our estimation, but the uncertainty in the estimation is of order of 100 MeV. We have also predicted a flavor triplet and antisextet for the charmed and bottomed pentaquark baryons. The mass of the Θ+c is between 2938 and 2997 MeV whereas the mass of Θ+b is between 6370 and 6422 MeV.

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