Static \(\bar{Q}-Q\) Potential from \(N_f=2\) Dynamical Domain-Wall QCD

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We calculate the static quark and anti-quark potential both in quenched and two-flavor dynamical quark lattice QCD using DBW2 gauge and domain-wall quark actions. Lattice spacings from Sommer scale are determined. We find (i) mixing of excited states is different in between quenched and dynamical, (ii) lattice spacing \(a_{r_0} \sim a_{m_\rho}\) in dynamical and (iii) coefficient of Coulomb term being \(\alpha_{N_f=0} < \alpha_{N_f=2}\) at \(a^{-1} \sim 2\) GeV.

1. Introduction

This study has the following purposes: (i) determination of lattice spacing \(a\) from Sommer scale \(r_0 = R_0a \approx 0.5\) fm \textsuperscript{[1]}, (ii) observation of dynamical quark effects in the static potential, such as larger coefficient of Coulomb term and string breaking. All quoted errors are statistic only, and systematical errors will be reported elsewhere \textsuperscript{[2]}. 

2. Calculation

The static potential \(V(\vec{R})\) between infinitely heavy quark and anti-quark separated by \(\vec{R}\) in spatial direction is obtained from the Wilson loop \(\langle W(\vec{R}, T)\rangle\):

\[\langle W(\vec{R}, T)\rangle = C(\vec{R})e^{-V(\vec{R})T} + \langle \text{"excited states"}\rangle, \quad (1)\]

where \(C(\vec{R})\) is the overlap with the "ground state" which is normalized with \(C(\vec{R} = 0) = 1\).

We calculate \(\langle W(\vec{R}, T)\rangle\) both in quenched and dynamical QCD. We employ quenched DBW2 gauge action \textsuperscript{[3]} with \(\beta=0.87, 1.04\) on \(16^3 \times 32\) and \(\beta=1.22\) on \(24^3 \times 48\) lattices (100, 405, 106 configurations respectively), corresponding to \(a_{m_\rho}^{-1} \sim 1.3, 2\) and 3 GeV \textsuperscript{[1,5]}, and DBW2 and \(N_f = 2\) domain-wall fermion \textsuperscript{[6]} dynamical action with \(\beta = 0.80, L_\pi=12, M_\pi=1.8, m_{\text{dyn}}a=0.02, 0.03, 0.04\) (941, 559, 473 configurations respectively), corresponding to \(m_\pi/m_\rho=0.53(1), 0.60(1), 0.65(1)\) on \(16^3 \times 32\) lattices. Inverse of the lattice spacing from rho meson mass in the chiral limit is \(a_{m_\rho}^{-1} \sim 1.7\) GeV in dynamical \textsuperscript{[7]}.

We implement Bresenham algorithm \textsuperscript{[8]}, which allows us to obtain the geodesic path connecting the ends of \(\vec{R} = (N_1, N_2, N_3)\) on lattice, where \(N_i\) \((i = 1, 2, 3)\) are any three integers, and APE smearing \textsuperscript{[9]} for spatial links. The smearing coefficient and iteration time are tuned: \((c, a) = (0.50, 20 \sim 25)\) for both dynamical and quenched to maximize \(C(\vec{R})\).

The static potential \(V(\vec{R})\) and the overlap with the ground state \(C(\vec{R})\) are obtained from \textsuperscript{[10]}:

\[V(\vec{R}, T) = \ln \left[\langle W(\vec{R}, T)\rangle/\langle W(\vec{R}, T + 1)\rangle\right], \quad (2)\]

\[C(\vec{R}, T) = \langle W(\vec{R}, T)\rangle^{T+1}/\langle W(\vec{R}, T + 1)\rangle^{T}, \quad (3)\]

by neglecting the excited states. \(T\) is selected as the smallest time on which the excited states contribution becomes negligible in \(V(\vec{R})\).

The physical parameters are obtained from a fitting function:

\[V(\vec{R}) = V_0 - \frac{\alpha}{R} + \sigma R, \quad R = |\vec{R}|, \quad (4)\]

\[R_0 = \sqrt{\frac{1.65 - \sigma}{\sigma}}, \quad (5)\]
3. Results

Figure 1 shows the overlap with the ground state $C(\mathbf{r})$, where $\mathbf{r}=\mathbf{R}a$, in dynamical and quenched at $T=5$. In quenched $r$ dependence is small at $r \geq r_0$. On the other hand $C(\mathbf{r})$ decreases as $r$ increases in dynamical. This suggests the mixing of excited states is different for dynamical and quenched. Further investigation about the excited states in dynamical case is needed.

Note that $C(\mathbf{r}) > 1$ may mean the existence of excited states with negative norm which is possible because improved gauge actions break reflection positivity [10].

Figure 2 shows the static potential for quenched (upper) and dynamical (lower), normalized with Sommer scale $r_0$.

$K^{-1}=1.69(2)$ GeV with Sommer scale $r_0=0.5$ fm.

We compare lattice spacing from Sommer scale and that from rho meson mass (Table 1). $a_{r_0}^{-1}$ is a few percent larger than $a_{m_\rho}^{-1}$ for quenched. On the other hand $a_{r_0}^{-1}$ is consistent with $a_{m_\rho}^{-1}$ within the error in dynamical. The lattice spacings from $r_0$ and $m_\rho$ become much closer by dynamical quark effect.

In Figure 4 we compare the static potential in between quenched and dynamical at short range, normalized with $r_0$. When the scale is fixed by $r = r_0$, we see a deeper Coulomb potential for dynamical than that of quenched. Also the coefficient of Coulomb term $\alpha$ increases, i.e. $\alpha_{N_f=0} < \alpha_{N_f=2}$. This relation is consistent with the per-
3. $r_0/a$ vs. $(m_{\text{dyn}} + m_{\text{res}})a$ with linearly extrapolate the chiral limit.

$$\begin{array}{|c|c|c|}
\hline
\text{dyn. in the chiral limit} & a_{r_0}^{-1} [\text{GeV}] & a_{m_{\rho}}^{-1} [\text{GeV}] \\
\hline
\text{quenched } \beta = 0.87 & 1.69(2) & 1.69(5) \\
\hline
\text{quenched } \beta = 1.04 & 2.15(1) & 1.98(3) \\
\hline
\text{quenched } \beta = 1.22 & 3.09(2) & 2.91(5) \\
\hline
\end{array}$$

Table 1: $a_{r_0}^{-1}$ vs. $a_{m_{\rho}}^{-1}$. Quenched $\beta = 1.22$ is preliminary. $\beta = 0.87$ and 1.04 are reanalyzed in this work after [4].

4. Conclusion

We calculated the static potential both in quenched and two-flavor dynamical domain-wall QCD and obtained lattice spacings estimates (Table 1). We saw the following dynamical quark effects: (i) $a_{r_0}^{-1} > a_{m_{\rho}}^{-1}$ for quenched and $a_{r_0}^{-1} \sim a_{m_{\rho}}^{-1}$ for dynamical in the chiral limit, (ii) deeper Coulomb potential in dynamical than in quenched due to screening effect: $\alpha_{N_f=0} < \alpha_{N_f=2}$.

String breaking was not observed. However we observed the different behaviors of $C(\vec{R})$ between quenched and dynamical. This suggests the existence of the heavy-light excited states.

To study string breaking, we may have to treat the excited states more carefully. Multi-state calculation [8,11] is one of the possibilities.

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