Alternative integrated criteria of quality by optimization of processes in control systems

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Abstract. Several groups of nonconventional integrated indicators quality and their use for dynamic tuning systems automatic control are considered. Application search algorithms allowed to receive values of the tuning the regulator providing both the minimum value integrated indicators quality, and a sufficient stock stability for single-loop system. Results a research allow to recommend concrete alternative integrated criteria quality and a technique of their application for in single-loop control systems.

1. Introduction
Automatic control of continuous technological processes in power and other industries is carried out on the basis of the modern microprocessor controllers allowing to realize rather difficult laws of control, one of which is PID the law of regulation [1].

During the developing and adjustment of systems automatic control an actual problem parametrical synthesis is. In case regulation of technological processes is result of the solution this problem optimum, in a sense, values the tuning of the used regulator. The standard technique of tuning of industrial regulators provides the solution of two tasks:

- definition in space parameters the regulator border of the set stock of stability ACS;
- search on this border a point of the minimum chosen integrated criterion of quality step responses of system.

The step responses is understood as the reaction of the customized system referred to the size of step entrance influence.

2. Assessment of control quality. For the generalized assessment of quality step responses of system the greatest application was received by linear and square integrated criteria in which as subintegral functions either the error of regulation, or its square are used.

\[ J_L = \int_{t_p}^{t_g} \varepsilon(t) \, dt \quad \text{and} \quad J_{Kv} = \int_0^{t_g} [\varepsilon(t)]^2 \, dt \] (1)
These criteria give only an indirect idea of quality step responses therefore after minimization the chosen criterion the developer reproduces the step responses of system and analyzes, what direct indicators of quality have step responses in optimum adjusted ACS. Direct indicators of quality it is considered to be \([2]\) maximum dynamic deviation of an error of regulation \(\varepsilon_{\text{max}}\) and duration of step responses \(T_{\text{pr}}\).

Wide use of the criteria given above for optimization values of tuning regulators, is explained by a possibility of analytical definition their values and evidence of physical sense. However, performance of analytical calculations parameters of the regulator, especially PID of the law regulation, is accompanied by considerable difficulties \([4]\). Therefore optimum values of tuning regulators most often find method of numerical modeling on the computer.

Use of the computer allows to perform optimization of tuning regulators not only on standard, but also by other integrated criteria significantly differing by the form subintegral functions. Let's call these criteria of quality alternative.

The purpose of the report is search of alternative integrated criteria, which minimization in space of parameters the regulator at the same time provides optimum, by the accepted criterion, the step responses of system and a sufficient stock of stability of the tuning system of regulation.

3. Control system model
We will carry out search of alternative integrated criteria by numerical modeling of the single-loop control systems consisting of an object, the regulator and blocks of formation integrated criteria. The block diagram of model for the studied single-loop control systems considered in work is provided on figure 1.

![Block Diagram](image)

**Figure 1.** The settlement block diagram at a control system research
Here: 1 - object of control; 2 - regulator; 3,4 - blocks of formation of step signal; 5 - block of calculation of criterion of an optimality; 6 - the block of calculation of the area under a curve of transition process.

The procedure of search is performed by minimization of value the chosen criterion of an optimality in space of tuning parameters of the regulator. As an indicator for comparison of the considered options of integrated criteria we will use the area of the received optimum step responses of an error regulation. Assessment of the step responses of system regulation received as a result of optimization, we will make, using the following indicators:
- maximum dynamic deviation of an error regulation, \(\varepsilon_{\text{max}}\);
- duration of the step responses, \(T_{\text{pr}}\);
- the module of the amplitude-frequency characteristic of the closed system on the channel of a set, \(A_{\text{res}}\);
- value of resonant frequency of system, \(\omega_{\text{res}}\);
- the coefficient, characterizing extent of input a derivative in the law of regulation, \(\alpha\);
Object of control. As object of control the boiler superheater of TGMP-204 of the power unit 800 Mw on the channel a water consumption on injection - temperature of superheated steam is chosen.

Transfer function an object of control was received by approximation of the experimental step responses of object at loading of 100% by expression:

\[ W_{ob}(s) = \frac{0.9}{(10 \cdot s + 1)^5}, \frac{\circ C}{T/h} \]

Object represents five consistently connected A-links, there is no delay. It should be noted that at the choice of structure and values coefficients of model an object the system approach considering the purpose of creation of model [3] has to be observed.

Regulator. As the control device we use ideal PID, the law of control which transfer function consists of three components: proportional, integrated and differential.

\[ W_r(s) = K_r \left(1 + \frac{1}{T_i s} + T_d s\right), \]

where \( K_r, T_i, T_d \) - parameters of dynamic tuning of the control device; \( s \) - argument of transformation of Laplace.

4. Types of integrated criteria

For the purpose identification of the possibility application in practice optimization parameters of the regulator several groups of alternative criteria were considered.

When forming subintegral functions of different types of integrated criteria the main properties of variables characterizing step responses in system were considered. Treat them:

- a regulation error (a mismatch on an entrance to the regulator), \( \varepsilon \);
- speed of change of an error of regulation \( \frac{d\varepsilon}{dt} \);
- current time of transition process, \( t \);
- change of the regulating influence, \( \mu \);
- speed of change of the regulating influence \( \frac{d\mu}{dt} \).

Thus, set of the studied integrated criteria can be presented in the following generalized view:

\[ J = \int_{0}^{\infty} \left(t, \varepsilon, \mu, \frac{d\varepsilon}{dt}, \frac{d\mu}{dt}\right) dt \]  

In an extensive set of possible modifications of a formula (2) in work several characteristic groups were allocated (see table 1).

The first group is formed by criteria which subintegral functions depend on an error of regulation \( \varepsilon \) and time \( t \).

The second group is formed by criteria, subintegral functions which depend on a square of an error regulation \( \varepsilon^2 \) and time \( t \).

The third group is formed by criteria with subintegral functions, each of which represent set of some dependences on change of the regulating influence \( \mu \), an error of regulation \( \varepsilon \) and time \( t \).

Minimum of values of integrated criteria decided on the help of a search method of a deformable polyhedron in the program Mathcad environment.

5. Results of calculations

It was specified above that in practice various integrated indicators of quality control are widely applied to the choice of values tuning parameters PID-regulator. The integrated criteria quality of control given in the table were used for direct search in a time domain an absolute minimum of indicators quality control at change of values tuning parameters of the control unit. Results calculations for each group of criteria are given in the table 1.
Table 1. The values of parameters corresponding to a minimum integrated criterion of quality control.

| Integrated criteria quality of control | Values of the tuning regulator | Maximum of step responses and AChH of a control system. |
|---------------------------------------|-------------------------------|---------------------------------------------------|
| $J = \int_0^\infty |e(t)| dt$ | 15.8 | 2.45 | 2.23 | 2.20 | 0.984 | 0.31 | 210 | 3.31 | 0.106 |
| $J = \int_0^\infty |e(t)| \cdot t dt$ | 15.9 | 2.25 | 2.78 | 1.77 | 0.635 | 0.349 | 133 | 2.18 | 0.093 |
| $J = \int_0^\infty \sqrt{\frac{d|e(t)|}{dt}} dt$ | 17.6 | 2.00 | 2.98 | 1.65 | 0.553 | 0.378 | 180 | 1.66 | 0.086 |
| $J = \int_0^\infty \sqrt{|e(t)| \cdot t} dt$ | 22.1 | 1.62 | 3.32 | 1.25 | 0.377 | 0.447 | 130 | 1.17 | 0.053 |
| $J = \int_0^\infty \sqrt{|e(t)|} dt$ | 17.4 | 2.02 | 2.96 | 1.66 | 0.561 | 0.377 | 181 | 1.68 | 0.086 |
| $J = \int_0^\infty |e(t)| \cdot t^2 dt$ | 18.7 | 1.91 | 3.17 | 1.47 | 0.464 | 0.40 | 187 | 1.51 | 0.080 |
| $J = \int_0^\infty \sqrt{|e(t)| \cdot t} dt$ | 27.0 | 1.32 | 3.36 | 1.24 | 0.369 | 0.485 | 118 | 1.043 | 0.061 |

Group 2.

| $J = \int_0^\infty |e(t)|^2 dt$ | 21.3 | 2.73 | 1.37 | 2.85 | 2.08 | 0.256 | 591 | 8.54 | 0.116 |
| $J = \int_0^\infty |e(t)|^2 \cdot t^2 dt$ | 15.2 | 2.59 | 2.25 | 1.98 | 0.88 | 0.311 | 270 | 3.24 | 0.101 |
| $J = \int_0^\infty |e(t)|^2 \cdot t^4 dt$ | 16.8 | 2.20 | 2.90 | 1.56 | 0.538 | 0.368 | 178 | 1.97 | 0.087 |
| $J = \int_0^\infty |e(t)|^2 \cdot t^5 dt$ | 18.4 | 2.00 | 3.11 | 1.43 | 0.46 | 0.395 | 186 | 1.64 | 0.08 |

Group 3.

| $J = \int_0^\infty |u(t)| dt$ | 29.5 | 1.48 | 3.16 | 1.76 | 0.557 | 0.43 | 213 | 1.05 | 0.080 |
| $J = \int_0^\infty |u(t)|^2 dt$ | 24.6 | 1.52 | 2.24 | 3.01 | 1.35 | 0.357 | 290 | 1.72 | 0.104 |
| $J = \int_0^\infty |e(t)| + |u(t)| dt$ | 19.9 | 1.84 | 2.86 | 1.77 | 0.617 | 0.388 | 220 | 1.47 | 0.086 |
| $J = \int_0^\infty |e(t)|^2 + |u(t)|^2 dt$ | 21.0 | 1.75 | 2.03 | 2.86 | 1.41 | 0.30 | 289 | 2.08 | 0.108 |
| $J = \int_0^\infty |e(t)|^2 + |u(t)|^2 \cdot t dt$ | 21.0 | 1.68 | 3.24 | 1.44 | 0.446 | 0.426 | 192 | 1.24 | 0.074 |
| $J = \int_0^\infty |u(t)|^2 \cdot t dt$ | 24.1 | 1.56 | 3.06 | 2.02 | 0.660 | 0.404 | 224 | 1.22 | 0.089 |
The values of tuning coefficients of PID - regulator corresponding to a minimum of the specified integrated indicators of quality control, referred to an object time constant are given in the table. When using for the mathematical description an object of control transfer function as a look (4), value of tuning coefficients of PID - regulator of $K_r$, $T_r$, $T_d$ for the chosen integrated indicator of quality control, it is possible to choose from the table 1.

Follows from the results given in table 1, that the choice of values tuning parameters of the device exercising of PID control on the basis of search an absolute minimum of the considered integrated indicators allows to provide steady step responses in a control system.

Increase in degree of a mismatch of $\varepsilon$ with the first to the second $\varepsilon^2$ leads to reduction of a dynamic deviation of adjustable size, but at the same time the oscillations and duration of step responses considerably raises.

On the other hand increase in value degree of time with the first $t$ up to the second $t^2$ and even to the fourth $t^4$ leads to reduction of a oscillation and duration of transition process, but at the same time the dynamic deviation raises.

From the practical point of view from step responses the minimum dynamic deviation and duration of step responses at an admissible oscillation of adjustable size is required. As a first approximation, this compromise can be provided with estimation of the area under a curve of step responses at indigation on the channel of the regulating influence, i.e. integral from the mismatch module (a difference between set and actual values of adjustable size) on an entrance to the regulator.

6. Analysis of results

Analyzing the results given in tables it is possible to see, that in comparison with a reference indicator of quality control (a minimum of linear integrated criterion at restriction of the set stock of stability of system $M = 1.6$, $J = \int_{0}^{\infty} \varepsilon(t) dt$), the area under a curve of step responses of $S = 22.74$, the maximum dynamic deviation the $Y_{din} = 0.348$ and duration of step responses $T_{reg} = 287$ s. considerably exceed similar values for the majority of the considered criteria:

Considering the received results it is possible to note that:

1. If not oscillatory step responses is required, then preference should be given to criterion

$$ J = \int_{0}^{\infty} \sqrt{\varepsilon^2(t)} \cdot t \, dt \quad \text{or} \quad J = \int_{0}^{\infty} |\mu(t)| \cdot t \, dt $$
2. If step responses with the minimum duration of step responses is required, then preference should be given to criterion
\[ J = \int_0^\infty \sqrt{\left| \frac{d\varepsilon(t)}{dt} \right|} \cdot t \, dt. \]

3. If step responses with the minimum area under a curve of step responses or with the minimum dynamic deviation of step responses is required, then preference should be given to criterion
\[ J = \int_0^\infty [\varepsilon(t)]^2 \cdot t^2 \, dt. \]

4. If the step responses considering the majority of requirements to quality of work of a control system is required, preference should be given to criterion
\[ J = \int_0^\infty [\varepsilon(t)] \cdot t \, dt \text{ (ITAE)}. \]

7. Conclusions
1. Select values of configuration parameters of the device performing the PID control based on absolute minimum considered alternative integral criteria, allows sustained transition process in management system.

2. The research of efficiency groups alternative integrated criteria quality in comparison with linear criterion for tuning of the PID-regulator is conducted. It is shown, that use of nonlinear integrated criteria allows to tune the PID-regulator on working off of indignations with bigger efficiency, than using for tuning of linear integrated criterion.

3. The conducted research allows to recommend criterion of ITAE as the most effective for direct search of the optimum tuning of the PID-regulator providing implementation of requirements for stability and quality of processes regulation.

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