Fractional Charge Experiments: What quantity is measured in the quantum Hall effect or calculated by Laughlin?

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We have examined the experiments performed by Goldman and Su, de-Picciotto et al, Saminadayar et al and Comforti et al, in which it is claimed that a fractional charge of $e/3$ is found. In all of the measurements, the quantity measured is the product of the charge and the magnetic field but not the charge. It is possible to interpret that charge per unit area has been measured where area is the square of the magnetic length. This type of correction to Laughlin’s result does not affect the exactness of the calculation. Anderson has suggested the extension of Laughlin’s state to particles of charge $2/m$ or $3/m$ with $m=$ odd integer. We find that the quasiparticle charge is given by angular momenta,

$$e_{\text{eff}}/e = \frac{l + (1/2)\pm s}{2l + 1}$$

which agrees with the data. Therefore, Laughlin’s $1/\text{odd}$ becomes an angular momentum so that the charge depends on spin, $s$, and zero charge has spin $1/2$.

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1. Introduction

Several authors have claimed that they measure a fractional charge of $e/3$. The question here is, that whether such a charge occurs in vacuum in absence of a magnetic field? If not, then what is the quantity which is measured and yet, it is thought to be charge. If the electrons by themselves, due to Coulomb repulsion, produce a fractional charge then why in the first instance, it comes out to be $e/3$ and not $e/2$ or $e/10$ and how the charge is conserved. Then what determines that it should split into three equal parts and what happened to the mass? Is it possible to have three parts with one of mass $m$ and the other two of mass zero each or all parts have mass $m/3$ each. Is it possible to split the charge but not the mass? All such questions can best be answered by reexamination of the experimental data. We have, therefore, reexamined all of the data and find that in all of the experiments, the quantity measured is not the charge but the product of the charge and the magnetic field. This necessitates, rechecking of the Laughlin’s theory of fractional charge. In this case also, it is found that charge per unit area is the quantity where the area is $\phi_o/B$. Hence, the experimentalists, have not measured the pure charge.

In this paper, we explain all of the experimental data in relation to the quantity measured. It is found that the product of the charge and the magnetic field has been
measured.

2. Experiments

(a): Goldman and Su[1-3] claim to measure the fractional charge. We have no objection to the measurement of fractional charge but it will be nice to know the quantity which is being measured. The resonant tunneling (RT) conductance peak is measured as a function of (a) magnetic field and (b) the back gate voltage and it is claimed that “a combination of these two measurements for a given ν in the quantum Hall effect constitutes a direct measurement of the charge of the tunneling quasiparticles”. At some condition between $B^m$ and $V^m_{BG}$, the lowest unoccupied state is aligned with magnetic moment $\mu$. At these values of field and voltage, the resonant tunneling is possible with a peak in the tunneling conductance, $G_{tun}$. At a fixed voltage $V_{BG}$, when $B$ is increased, the area $S_m$ decreases because the product $B.S_m$ is a constant due to quantization condition,

$$B^m.S_m = m\phi_o.$$  \hspace{1cm} (1)

When $m$ changes to $m+1$, the field should also change,

$$B^{m+1}.S_{m+1} = (m + 1)\phi_o.$$  \hspace{1cm} (2)

The difference between the two fields should be quantized,

$$B^{m+1} - B^m = \Delta B.$$  \hspace{1cm} (3)

Apparently, there is no objection to the quantization of the difference but the difference can not have the same area as $S_m$. In any case, if we substract the above two equations, we find,

$$B^{m+1}S_{m+1} - B^mS_m = \phi_o.$$  \hspace{1cm} (4)

Goldman and Su write this equation as

$$\Delta B S_m = \phi_o.$$  \hspace{1cm} (5)

This is possible if,

$$S_{m+1} \approx S_m$$  \hspace{1cm} (6)

so that

$$(B^{m+1} - B^m)S_m = \phi_o$$  \hspace{1cm} (7)

but it is dangerous to take such an approximation. Let us proceed with what we have. The voltage is written in terms of charge and capacitance as,

$$\Delta V_{BG} = \frac{q}{C S_m}.$$  \hspace{1cm} (8)

where the capacitance is,

$$C = \frac{\epsilon \epsilon_o}{d_{BG}}.$$

\hspace{1cm} (9)
Here $d_{BG}$ is the distance of the back gate from where the two-dimensional electron gas, (2 DEG), is. In GaAs, $\epsilon = 13.1$ and $d_{BG} = 428 \pm 5 \mu m$. From (8), we find the charge and substitute the value of $C$ from (9) to obtain,

$$
q = \Delta V_{BG} C S_m = \Delta V_{BG} S_m \frac{\epsilon \epsilon_o}{d_{BG}}.
$$

(10)

Now write (5) as $S_m \Delta B = \phi_o / p_\nu$ with an extra number $p_\nu$ and substitute this value of $S_m$ in (10) to obtain,

$$
q = \Delta V_{BG} S_m \frac{\epsilon \epsilon_o}{d_{BG}} = \Delta V_{BG} \frac{\epsilon \epsilon_o}{d_{BG}} \frac{\phi_o}{p_\nu \Delta B}
$$

(11)

so that the charge becomes,

$$
q = \frac{\epsilon \epsilon_o \phi_o \Delta V_{BG}}{p_\nu d_{BG} \Delta B}.
$$

(12)

When $p_\nu$ was introduced, it must be the inverse of an integer so that as long as $p_\nu = 1$, there is no trouble at all but $p_\nu$ can not be 2 because 2 is not the inverse of a number but 1/2 can be a component in $(n+1/2)=1/p_\nu$ so that when $n=0$, $p_\nu=2$ is not much objectionable. So there is no objection to $p_\nu=1$ or 2. However, it is clear that eq.(12) measures the product $q \Delta B$ and not the charge, $q$, and this is a serious matter.

We conclude that the experiment of Goldman and Su does not measure the charge of a quasi-particle but it measures the product of charge and a magnetic field. Let us write the field as the inverse area. Then the experiment measures the charge per unit area and not the charge,

$$
\frac{e^*}{area} \frac{area}{e} = 0.331.
$$

(13)

This is the quantity measured and it is claimed that,

$$
\frac{e^*}{e} = 0.331.
$$

(14)

Therefore Goldman and Su’s experiment does not measure the fractional charge. The area in the above is $\phi_o / B$. Hence the quantity measured is the product of charge and the magnetic field, $B$.

(b): de-Picciotto et al [4] report that a two-dimensional electron gas, subjected to a strong perpendicular magnetic field, $B$, consists of highly degenerate Landau levels with a degeneracy per unit area,

$$
p = B / \phi_o
$$

(15)

with

$$
\phi_o = h / e
$$

(16)

the flux quantum with $h$ as the Planck’s constant. Actually in both these expressions, the dimensions are incorrect. The correct formula for $p$ is,

$$
p = B.A / \phi_o
$$

(17)
and the correct value of $\phi_o$ is,

$$\phi_o = \frac{hc}{e}.$$  \hfill (18)

This is not a serious error because it can be corrected when necessary. The area $A$ is $p\phi_o/B$ which defines the magnetic length, $\sqrt{A} = (p\phi_o/B)^{1/2}$. The areal density is $n_s$, which is the number of electrons per unit area,

$$n_s = n/A.$$ \hfill (19)

The integer number $\nu$ is the filling factor,

$$\nu = n_s/p.$$ \hfill (20)

This is the number per unit area, so it can not be correct. The correct value is obtained only when $p$ is divided by the area $A$ so that,

$$\nu = \frac{n_s.A}{p}.$$ \hfill (21)

is dimensionless. The Hall resistivity is defined as,

$$\rho_{xy} = \frac{h}{\nu e^2}$$ \hfill (22)

and hence the conductivity,

$$\sigma_{xy} = \frac{\nu e^2}{h}$$ \hfill (23)

is correct with $\nu$ as a number. It has been argued that the resistivity observed in the fractional quantum Hall effect, can be explained in terms of quasiparticles of a fractional charge,

$$Q = \frac{e}{q} \quad (q = \text{number}).$$ \hfill (24)

We have no objection to this charge equation where $q$ is a number but it will be interesting to see that there are other factors in the problem so that even if the fractional charge is observed, it does not necessarily mean that there is a fractional charge. Such a fraction can arise from the angular momentum so that the effective charge appears to be fractional. This means that the observed charge is fractional but the fraction can arise without fractionalization of charge. De Picciotto et al suggest that “the FQH effect cannot be explained and is believed to result from interactions between the electrons brought about by the strong magnetic field”. In this case the “electrons brought about by the strong magnetic field” may be correct but this is not born out from Laughlin’s paper. De Picciotto et al have argued against the paper of Goldman and Su. We have seen above that their experiment measures the product of the magnetic field and the charge but not charge alone. De Picciotto et al report that “Quantum shot noise, probes the temporal behaviour of the current and thus offers a direct way to measure the charge”. We wish to examine whether it is correct to say that “quantum shot noise” measures the charge.
Actually, the noise measures the resistivity not the charge. The fractional filling factor is the inverse of the fraction which measures the charge as in $Q$ (eq.11), so that,

$$\nu = \frac{1}{q}. \quad (25)$$

The thermal noise is,

$$S_i = 4k_B T G \quad (26)$$

where $G$ is the conductance, so that a plot of $S_i$ versus $G$ determines the temperature. The quantum shot noise, $S_i$, generated by the backscattering of the current, $I_B$, is proportional to the charge of the quasiparticle so that,

$$S_i = 2Q I_B \quad (27)$$

The magnetic field is swept from 0 to 14 Tesla, which means that product of field and charge is measured but not the charge. The transmission, $t$, is given by the ratio of conductance $G$ and the quantum conductance $g_o = e^2/h(area')$ as,

$$t = (\frac{G}{area})(\frac{area'}{g_o}). \quad (28)$$

The charge $Q = e/3$, when measured from the transmission, $t$, without regard to areas involved, will not be the true charge of the quasiparticles. Besides, there is nonlinearity in the $I-V$ characteristics but only a small voltage range is shown.

(c): Let us look at the measurements performed by Saminadayar et al[5]. It has been reported that 2-dimensional electrons in a high magnetic field give rise to Landau levels with one state per flux,

$$\phi_o = \frac{hc}{e}. \quad (29)$$

For integer filling factor, $\nu = n_s/n_\phi$,

$$n_\phi = \frac{eB}{hc}. \quad (30)$$

Here it is obvious that if $n_\phi$ is dimensionless an area is missing. Therefore, $n_\phi$ should be described by an area,

$$\frac{n_\phi}{A} = \frac{eB}{hc}. \quad (31)$$

This is the corrected form of the previous expression. So the flux density is $n_\phi/A$, which is the number of flux quanta per unit area. Similarly, $n_s$ is the electron density. If the distance between two electrons is equal to the magnetic length, the Coulomb energy $\Delta = e^2/\epsilon l_c$ is called a gap with,

$$l_c^2 = \frac{hc}{eB}. \quad (32)$$
According to Ohm’s law, the current \( I_B \) is related to the voltage, \( V_{ds} \) by the relation,

\[ V = I \cdot \frac{h}{\nu e^2}. \]  

(33)

If \( \nu \) is the effective charge, the cyclotron frequency is given by,

\[ \omega = \frac{\nu eB}{mc} \]  

(34)

in which we substitute \( B = (hc/eA) \) to find,

\[ \omega = \frac{\nu h}{mA}. \]  

(35)

The quantity being measured on the right hand side is \( \nu/A \) but not \( \nu \) alone. The previous formula shows that a product \( \nu eB \) occurs so that whether \( \nu \) is to multiply \( e \) or \( B \) can not be ascertained. It is stated that “we have brought evidence of \( e/3 \) Laughlin’s\[6\] quasiparticles carrying current through the 1/3 FQH state. However, a close examination \[7 \] of Laughlin’s original paper shows that the quantity that appears in the algebra is \( e/(ma_o^2) \) where \( m = 3 \) and \( a_o = 1 \) gives \( e/3 \) used by Saminadayar et al. However, \( a_o \) is not a dimensionless number and \( a_o = \hbar c/eB \) which depends on the magnetic field so that what is thought to be \( e/3 \) is actually \( eB/3 \) so that whether \( e/3 \) or \( B/3 \) has been measured, can not be determined.

**d)**: Laughlin\[6\] has found the charge density as explained by us \[7\],

\[ \rho = \frac{e}{m2\pi a_o^2}. \]  

(36)

If \( e = x \) and \( a_o = y \) then the above equation is,

\[ 2\pi m\rho y^2 = x. \]  

(37)

Laughlin substituted \( y = 1 \) and then determined the value of \( x = e/3 \). However, \( y=1 \) is not a solution and this arbitrarily fixing \( y = 1 \) called “incompressibility” is not justified. Since there is only one equation and these two variables, Laughlin could not solve this equation correctly. Obtaining the value of \( x/y^2 \) exactly is not the solution of the problem. So the exactness is of no help and hence what is an exact solution is not necessarily satisfactory.

Similarly, the experiments of Comforti et al \[8,9\] require that three quasiparticles must group so that the total charge is \( e \) and 1/3 is prepared in a field only. So the quantity measured is the field, not the charge.

Anderson\[10\] has suggested a state with extra unit of angular momentum. An effort is made to create quasiparticles of charge \( 2/m \) and \( 3/m \), etc. from a modification of Laughlin’s state. From a different approach we find that the charge of the quasiparticles is,

\[ e_{eff}/e = \frac{l + (1/2) \pm s}{(2l + 1)}. \]  

(38)
Thus the quasiparticle charge depends on spin, s, and there is an electrically neutral quasiparticle which has a spin of 1/2.

The proper theory of the quantum Hall effect is given in Ref.11 which agrees with Stormer’s data[12].

3. Conclusions.

It will be best if Laughlin’s theory is interpreted not as determining the charge but determining \( e/a_0^2 \). There is not much harm in measuring the fractional charge but the correct result helps in determining the origin of the factor of 1/3. All of the experiments prepare the 1/3 charge in the presence of a magnetic field and this field is always needed.

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