Horizons of Strong Field Physics

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Abstract. Discussing the limitations on the validity of classical electrodynamics, we show that present day laser pulse technology applied to head-on-collisions with relativistic electrons generates fields strong enough to permit experimentation at the limits of validity of the Lorentz force, and the development of experimental tests of Mach’s principle. We also discuss more distant opportunities for exploring the nature of laws of physics and the vacuum structure. We then conclude that the predictions of quantum electrodynamics in the presence of critical fields are not completely satisfactory and argue that the study of Laser materialization into particle pairs opens a new domain of quantum electrodynamics.

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CRITICAL ACCELERATION AND MACH’S PRINCIPLE

Strong fields produce strong acceleration, and thus are a probe of inertia. It is widely agreed that acceleration requires as reference a frame against which inertia is measured, and this is true as much in a Lorentz covariant theory as in nonrelativistic dynamics. More than 100 years ago Ernst Mach pointed out the need to quantify the inertial force with reference to what we call ‘an equivalence class comprising all inertial frames of reference’. Mach chose the background of fixed stars, i.e. cosmologically, the Universe at large[1]. Given that Mach connected inertia to stars afar, there is no lack of misunderstandings about the meaning of Mach’s principle, and thus inertial force, in context of Einstein’s or Newton’s gravity. These have been recently explained and the meaning of ‘Mach’s Principle’ categorized [1].

In our context we see two concepts contained in Mach’s statement that play a role.

a) The measured value of inertial mass depends on all mass in the Universe (Mach1).

Could inertial forces thus depend on the contact of a body with the Universe at large and thus be subject to control? This is a question which fascinates public at large. The reader will find googling ‘Mach’s principle’ many millions of hits. Similarly, there are many web notes, and bona-fide research papers with ‘Mach’s principle’ in the title.
b) Acceleration is measured with reference to a select universal (fixed star) reference frame (Mach3).
Here the numbering in parentheses follows Bondi and Samuel [1] who further present nine other positions one can take regarding Mach’s principle. A review of the subject goes beyond scope of this report and our discussion remains focused on the two items: case a) is addressed by modern quantum field theory, with the Higgs field filling the Universe and providing the scale of masses, probably the scale of all luminiferous matter, perhaps the scale of gravity, in a yet-to-be-understood way.

The Brans-Dicke [2] extension to Einstein’s Gravity (GR) was formulated with this goal and in that sense it includes Mach in the theory of Gravity even at the Newtonian level by creating a framework for computing the gravitational constant $G$ as the amplitude of a scalar field having many properties similar to the Higgs field but employing totally different scales. Note that as long as $G$ remains fixed and is not a dynamical field, Newtonian gravity and any theory which reduces to Newtonian gravity include some non-Machian contents. Hence the leading Newtonian component in Einstein’s geometric theory of gravity (GR) must also be non-Machian. However, if we agree that Einstein’s theory is an effective theory in the sense we describe below, with the dimensioned matter-gravity coupling constant $G$ to be computed from a more foundational approach, then we can have Mach’s principle fully implemented. This is not the objective of this work, but a point of view which we keep in memory as we address the shortcomings of the current understanding.

Direct contact to light pulse high acceleration physics is made when considering the second conceptual statement b) contained in Mach’s principle. Today, we can study laser-electron interactions involving ultra high accelerations achieving in special situations values which rival those expected at the event horizon of black holes. Einstein’s equivalence principle requires physics at high acceleration to be identical with physics in a strong gravitational field. Thus in ultra high acceleration experiments we are addressing the understanding of gravity, and more importantly, of inertia.

An acceleration of unit strength measured in natural units is achieved when the particle attains energy equivalent to its mass over the distance of its Compton wavelength. For comparison, imagine that we accelerate an electron initially at rest over a distance of one meter to an energy of 1 MeV. In natural units, and in human experience units, this amounts to an acceleration

$$\dot{v} = 7.54 \times 10^{-13} m_e = 1.79 \times 10^{16} \text{g}$$

We see that in natural units accelerators do not accelerate much, yet expressed in terms of Earth’s surface gravity, the scale is larger than one can imagine.

Can one ever reach in laboratory ‘critical’ acceleration limit, $\dot{v} \to 1m_e$? The answer, amazingly, is yes, and it could be the next big experimental project. Using an intense laser pulse for which the normalized vector potential is $a_0 = eA_0/m_e$ we can generate a field to accelerate an electron of 100 MeV/micron by acting within the space of a quarter wavelength (0.25µ) given $a_0 = 50$. This corresponds to a gain in field strength of 8 orders of magnitude over the conventional accelerator case we considered above. Now relativity comes to help: if we look at the laser pulse from the frame of reference of a moving electron we gain a factor $\gamma(1 + v^2) \simeq 2\gamma$. For exactly $\gamma = 7000$, that is
an electron of 3.5 GeV, an observer riding on the electron experiences a unit value of acceleration.

In order to achieve our goal of probing critical acceleration in a laboratory experiment other combinations of $a_0$ and $\gamma$ are possible. All it takes is placing an intense laser near an accelerator or using a second intense laser pulse to form a relativistic electron beam. In fact, there has been an experiment aiming to study strong field effects organized just in this way. The SLAC 46.6 GeV ($\gamma = 10^5$) electron beam was collided with most intense lasers available 13 years ago. Light pulses were still much less intense, offering $a_0 \simeq 0.5$ and as result the exploration of strong fields occurred at well below critical acceleration limit [3].

Today we can reach the strong acceleration limit in the laboratory and explore experimentally the limits of our understanding of inertia and Maxwell-Lorentz electromagnetism. The study of ultra high acceleration outside of the realm of GR opens a new physics frontier in that we deal with ‘real’ acceleration and inertial resistance to it. Acceleration is absent in the geometric general theory of relativity: though we observe motion of a satellite as if there were a force, there is none; a satellite is free-falling. Einstein employed the equivalence principle to eliminate acceleration from his theory – Newton’s force arises from geodesic motion.

Since Mach3 is a statement about acceleration, Mach3 becomes irrelevant in the ‘classical’ Newtonian limit of GR. A solution of the geodesic equation of motion for a probe particle of negligible mass in any gravitational field of ‘external’ character (a field unperturbed by the probe particle) is by definition acceleration-free dynamics. That is why a ‘good’ theory such as Einstein’s gravity, which is Machian as much as it can be, reduces easily to non-Machian Newtonian gravity (non-Machian as long as $G$ is non-dynamical and not rooted in properties of space-time).

Only when we probe GR beyond the Newtonian limit can the question be posed: is Machian physics involved in GR? It helps to remember that a free fall is interrupted by the presence of matter. A Machian effect arises already when we have dynamics of many bodies or one extended material body. The best studied example is the rotation of the Earth dragging the nearby space-time manifold, leading to the Lense-Thirring effect [4]. The measurement of frame dragging amounts, paraphrasing here Francis Everitt of Gravity Probe B [5], to the measurement of a missing inch in the orbit of a satellite. The Gravity Probe B project refers the orientation of a satellite to a fixed star in order to measure that extra inch, thus directly implementing Mach’s suggestion to employ the fixed stars as the frame of reference.

It is probably true that any effect in GR beyond the Newtonian theory is Machian, requiring for its evaluation reference to the space-time manifold on which matter exists. Einstein pointed this out to Mach before completing his theory [6]. Mach’s Principle would seem to be addressed at this point. The issue remains that to the best of our understanding, forces acting between, and on material particles, were not formulated in a form respecting a relation to Mach3. Most vexing in our context is the fact that Maxwell-Lorentz electromagnetism is non-Machian3. Paraphrasing Mach and Newton, how can we be sure that it is not the Universe that accelerates when we presume to measure acceleration? The only reason we can do physics is that in natural units the (electromagnetic) accelerations we encounter in normal life are negligible.

On this basis, one could argue that Mach3 is not satisfied by the fundamental forces we
study in the microscopic world. Moreover, we know that these forces operate in the realm of quantum physics which is inconsistent beyond the leading classical limit with the one exceptional force (Gravity) that does seem to be consistent with Mach3. However, the situation is much more complex. In modern thinking, fundamental interactions are ‘effective’, arising from the behavior and properties of the quantum vacuum state, the modern aether. In some way we have not yet grasped, this means that just like with Brans-Dicke or Higgs approach, these interactions are Machian, being a part of the modern aether theory.

AETHER, VACUUM, LAWS OF PHYSICS, AND MATTER

Aether, the carrier of light waves, fell into disrepute 100 years ago due both to the absence of the effect of an aether drag in the Michelson-Morley experiment, and to Einstein taking the position in his 1905 papers that aether is unobservable. However, as is often the case, scientific positions evolve, and 15 years later Einstein wrote:

According to the general theory of relativity, space without aether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this aether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it. [Concluding paragraph of: Aether und die Relativitaetstheorie (Berlin, 1920)].

The last phrase is of particular relevance here and we will soon return to the question in what sense aether is a ponderable medium, with parts which can be tracked in time.

Einstein in effect postulated in 1920 the existence of a relativistically invariant aether. In this way he could connect matter present at any place in the Universe with a common inertial frame of reference against which acceleration is measured. This creates a foundation for the implementation of Mach3. Moreover, the aether is the carrier of physical qualities and thus Einstein directly implemented Mach1.

Our view of physics laws has evolved vastly since 1920 and the realization of Einstein’s Machian objectives may be achievable today, partially because of the development of new experimental tools discussed in this note. The main new insight since Einstein’s times is the recognition that with quantum mechanics and quantum field theory we acquire a structured vacuum state which has measurable physical properties. This vacuum structure arises from quantum fluctuations in the vacuum permitted by the uncertainty relation.

To make these vacuum fluctuations concrete in the early stages of development of quantum field theory, one imagined a network of points connected by ideal springs, and these could undergo oscillations which were the vacuum fluctuation modes. In the continuum limit of infinitely dense points, the transition from the quantum oscillator picture to a non-interacting quantum theory with particles occurs, and the zero-point oscillations of the oscillators become the (divergent) vacuum energy. In the numerical effort to solve quantum field theory on the lattice, we in fact return to this quantum lattice
of a three-dimensional chain of harmonic oscillators, but introduce a gauge invariant action.

The energy of the quantum ground state diverges in two ways: since it is extensive, any finite energy density diverges with the volume size. Moreover, when we allow the distance of lattice points to shrink, we allow fluctuations of arbitrarily large momentum and thus in the continuum limit we arrive at:

$$E/V = \frac{\pm g^2}{2} \int \frac{d^3 p}{(2\pi)^3} \sqrt{m^2 + p^2}$$

where $g$ is the degeneracy, due to two spin states of fluctuations at same momentum, and also due to fluctuations of both particles and antiparticles – thus for electrons, positrons $g_e = 4$. The factor $1/2$ originates in the zero point energy of each single harmonic oscillator, $E_0 = \omega/2$.

The overall sign changes between Fermions (-), and Bosons (+). It should be remembered that for photons we have $g_\gamma = 2$ since there are only two (transverse) degrees of freedom of free photons, and no antiparticles. Thus, there is a partial cancellation of leading infinity from photons with that from electrons contributing with opposite sign, and the dominant vacuum energy remains divergent in quantum electrodynamics (QED). In general, to cancel the zero point energy by symmetry, the additional ‘super’-symmetry is required, which must be badly broken in nature. Still, this symmetry gives birth to new particles, including candidates for dark matter.

We can measure the energy in the vacuum against this infinite value. The best known example is the Casimir energy which arises between two conductive plates. Since photon fluctuations have to end on the plates, fewer quantum fluctuations can exist between the plates, and the greater fluctuations outside press the plates together. Measurement of the Casimir effect is routinely possible today. Because the effect requires presence of matter, some argue that the Casimir effect does not require a change in vacuum structure of the fluctuations. While we do not share this opinion, we note that a much simpler effect exists which directly relates to vacuum fluctuations, namely the vacuum polarization.

When we apply an external electromagnetic field to the vacuum, the fluctuations of electrons and positrons separate and we find that the vacuum has a dielectric polarization property. In order to arrive at a finite observable value we need to redefine electron charge (charge renormalization). This means that the measured electrical charge, $e = \sqrt{hc/137}$ arises from a bare charge $e_0 \simeq \sqrt{hc/40}$ which we would observe were we able to probe at Planck length scale $L_P$, the shortest physical length in our Universe, $L_P \equiv \sqrt{\hbar G/c^3} = 1.61 \times 10^{-35}$ m. Because the leading effect of the polarization is to alter the charge, in QED we observe polarization effects that strengthen the magnitude of the applied field. For example, the Coulomb potential near to the atomic nucleus is stronger by a few parts in a thousand compared to our expectations.

While vacuum structure effects in QED are relatively subtle in normal laboratory environment, the situation is drastically different for quantum chromodynamics (QCD), the theory of strong interactions between nucleons. The much larger color charge of gluons, the ‘photons’ of QCD, alters the nature of the vacuum state completely, in a way that makes it impenetrable to the motion of color charges of quarks and gluons and so generating quark confinement. This also implies that the vacuum must have a physical
property related to this radical change in its structure. Indeed, the vacuum expectation value of the square of the gluon field fluctuations is non-zero, and we give this effect the name ‘gluon condensate’, a dimensioned quantity associated with every point in our Universe. We see that by the way of the quantum vacuum, Einstein’s relativistically invariant aether is back.

Long ago, at the beginning of time, when the Universe was much hotter, at a temperature that exceeded 30 000 times that in the core of the sun, quarks and gluons could roam free. Since the quark-gluon interaction charge, the color, is ‘ionized’, the new state of matter is referred to as the quark-gluon plasma or for short, quark matter. The expansion of the quark-gluon Universe cooled the plasma until the free color charges were frozen into the hadrons we find in our Universe today. However, in each hadron containing quarks a ‘piece’ of the non-confining ‘vacuum’ is captured, allowing quarks to exist there. In colloquial language this is the quark-bag. In this way each proton, or neutron, is a carrier of a piece of the ‘wrong’ aether from the early Universe.

All matter particles of finite size therefore break Einstein’s rule that specifically forbids tracking vacuum pieces in time. However, this feat is achieved in a Lorentz invariant way, since the matter particles are characterized by mass and helicity, (projection of spin onto axis of motion) which are the two Casimir operators of the Poincare group of all space-time symmetry (translation, rotation) transformations. There are further characteristic properties, such as charge, which imply that a group greater than the four dimensional Poincare group characterizes our space-time manifold.

We extend the concept of Einstein’s relativistic aether by adding matter and allowing the coexistence of several forms of aether captured within matter particles, which can be tracked and are ponderable. In this way the problem of finite size of massive matter particles is resolved and the magnitude of their large mass understood as being due to the zero point energy of more fundamental particles captured within. The first step of Pauli’s ‘what is matter’ problem has been resolved. The second step, the relationship of matter to fields, remains.

Much of the above insight about QCD and vacuum participation in the explanation of structure of matter is a paradigm, a way of thinking about what we see in nature, supported by elaborate interpretation of experimental data. The example of the Geocentric vs. Heliocentric systems of celestial motion shows that without more direct and drastic experimental evidence, quark confinement could, in principle, be differently interpreted another day. For this reason it is necessary to demonstrate directly the role of confinement in particle structure.

To achieve this we would like to recreate in a laboratory experiment the conditions of the early Universe, that is, the other type of aether in which quarks roam freely. The formation of quark-gluon plasma seems possible in high energy heavy ion collisions. It is believed that we can smash the boundaries between the individual hadrons and fuse into one the quark content of individual nucleons. In the reaction all particles are heated tremendously and the confining structure of the vacuum that surrounds us today is melted. An extended—albeit very small—domain of space is created for an exceedingly short time in which quarks can roam freely as they did in the early Universe.

The draw-back in this program of research is that the laboratory micro-bang is indeed very small since the energy content we can deliver with particle accelerators is below 10 erg ($10^{-6}$ J). This energy restricts the size of QGP we form to nuclear length scales.
\( R \approx 6 \text{ fm}. \) Since such a small drop of early Universe can expand unconstrained, with edges moving near velocity of light, its lifespan is bounded to \( 3R/c \approx 6 \times 10^{-23} \text{ s}. \) These relatively small values invite new efforts using light pulses, which are known to be much more energetic than particle collisions. The challenge is here to learn how to focus a good fraction of the MJ (megajoule) energy of a laser pulse into nuclear size. Aside from present efforts to focus and/or compress light wavelength, we can also explore multi-ion collisions once laser intensity is at the level to directly accelerate heavy ions to relativistic energies. Ion acceleration at this scale will be possible just above \( a_0 \approx 5,000, \) still two orders of magnitude below the field strength needed to reach critical acceleration. While this objective is as yet beyond today’s technology horizon, it constitutes a worthy challenge for the future, with considerable pay-off in terms of study of the quantum vacuum, the modern aether.

An interesting element of the discussion presented is that elementary particle properties and thus their interactions are subject to change. In that sense, they are not truly elementary. The question which comes to mind is, if elementary particles melt and change, can laws of physics melt too? Many if not all elementary interactions are effective interactions. A well-known QED example is light-light scattering, impossible in Maxwell theory but present in the quantum vacuum due to the effective action of Euler and Heisenberg, which is essentially an evaluation of Eq. (2) with vacuum fluctuation energies modified by the presence of the electromagnetic field. Further discussion of this effective action is offered below, see Eqs.(7,8).

Clearly, if properties of the quantum aether generate new interactions, we can expect that the nature of the interactions we hold to be fundamental, and more generally all laws of physics, depends on the nature, condition and type of the vacuum state. ‘Melting’ the QCD vacuum on a relatively-speaking macroscopic scale using light pulses will help us to understand these questions. In fact, there is a hierarchy of vacuum structure states, and beyond QCD we have the Higgs vacuum, which could perhaps be melted if we were able to compress a 10 kJ light pulse into the volume of an elementary particle such as a proton. According to current thinking, in this state the Higgs vacuum properties would dissolve and the masses of all particles would go to zero or have the neutrino mass scale. We are looking forward to learning more about the topic in the context of the forthcoming study of the Higgs particle at LHC.

**WHAT IS WRONG WITH ELECTROMAGNETISM**

After this grand tour of modern particle and field concepts we are ready to reconsider the theory of electromagnetic interactions and to describe its shortcomings. The covariant form of the Lorentz force is

\[
m\dot{u}^\mu = qF^{\mu\alpha}u_\alpha g_{\alpha\beta}, \quad u^\mu = (1, \gamma, v), \quad g_{\alpha\beta} = \text{diag}(1, -1, -1, -1).
\]

(3)

The extension of the Lorentz force in the presence of strong acceleration has been a topic of intense research effort for the past 100 years, beginning as soon as the form of the force was written. The deficiency is easily seen: the Lorentz force does not ‘know’ that the accelerated charged particle radiates.
We all know of synchrotron radiation and compute it as students and set it as an exercise when we teach. But few of us reached the last section of David Jackson’s third edition of Classical Electrodynamics which reveals that the radiation emitted alters in principle the dynamics of the charged particle source. A search in literature produces several distinct methods of accounting for this effect (see table 1). Each of these new force equations produces a different outcome and we recognize that all are not more than a patch which is only meaningful when the acceleration is tiny. For a recent comprehensive and rigorous first order study of the effect, see [7].

| TABLE 1. Models of radiation-reaction extensions of the Lorentz force |
|---------------------------------------------------------------|
| **LAD [8]** | $\mathbf{m}\ddot{\mathbf{u}} = qF^\alpha\beta\mathbf{u}_\beta + m\tau_0\left[\dot{\mathbf{u}}^\alpha - u^\beta\dot{u}_\beta u^\alpha\right]$ |
| Landau-Lifshitz [9] | $\mathbf{m}\ddot{\mathbf{u}} = qF^\alpha\beta\mathbf{u}_\beta + q\tau_0\left\{F^\alpha\beta u^\gamma\dot{u}_\gamma + \frac{m}{c^2} \left[F^\alpha\beta F^0_\gamma\dot{u}^\gamma - (u_jF^\gamma\beta)(F^\beta_0\dot{u}^\gamma)\right]u^\alpha\right\}$ |
| Caldirola [10] | $0 = qF^\alpha\beta(\tau)\mathbf{u}_\beta(\tau) + \frac{m}{c^2} \left[u^\alpha(\tau - \tau_0) - u^\alpha(\tau)\right]u^\beta(\tau - \tau_0)$ |
| Mo-Papas [11] | $\mathbf{m}\ddot{\mathbf{u}} = qF^\alpha\beta\mathbf{u}_\beta + q\tau_0\left[F^\alpha\beta\dot{u}_\beta + F^\beta\gamma\dot{u}_\gamma u^\alpha\right]$ |
| Eliezer [12] | $\mathbf{m}\ddot{\mathbf{u}} = qF^\alpha\beta\mathbf{u}_\beta + q\tau_0\left[F^\alpha\beta\dot{u}_\beta + F^\beta\gamma\dot{u}_\gamma u^\alpha\right]$ |
| Caldirola-Yaghjian [13] | $\mathbf{m}\ddot{\mathbf{u}} = qF^\alpha\beta(\tau)\mathbf{u}_\beta(\tau) + \frac{m}{c^2} \left[u^\alpha(\tau - \tau_0) - u^\alpha(\tau)\right]u^\beta(\tau - \tau_0)$ |

Equations of motion are usually obtained by means of an action principle. Thus it is important to note that the action principle of electromagnetism does not implement at all the ability of accelerated charged particles to radiate. The action comprises three elements: Maxwell field action, Matter-Field interaction (gauge invariant), and charged matter dynamics. These are written in the covariant form

$$\mathcal{S} = -\frac{1}{4} \int d^4x F^{\alpha\beta} F_{\alpha\beta} + q \int_{\text{path}} dx \cdot A + \frac{mc}{2} \int_{\text{path}} d\tau (u^2 - 1). \quad (4)$$

Two natural constants are introduced: ‘$q$’ describing the relation of matter to field and ‘$m$’ describing the inertia of matter. Many books write $mc \int d\tau$ for the last term and struggle with the constraint $u^2 = 1$, and some books write the middle term in the form $\epsilon \int_{\text{path}} d\tau u \cdot A$ introducing explicitly the 4-velocity $u^\mu \equiv dx^\mu / d\tau$.

The two first terms in Eq. (4) assure that, upon variation with respect to the field, accelerated charges radiate according to the Maxwell equations with accelerated sources. The second and third term, when varied with respect to the form of the material particle world line, produce the Lorentz force Eq. (3) (the first two terms in the table 1). One of probably several reasons the standard action fails is thus that we add matter and specifically inertia in an ad hoc fashion to the action. Without doubt, inertia represented by the $m$-term, is from the theoretical point of view, the least satisfactory of the three terms in Eq. (4). It is constrained merely by the nonrelativistic limit. The other two action terms are constrained also by gauge invariance. If, for example, mass were made out of field energy, this would introduce a relation of fields and velocities, helping to create radiation reaction terms in matter dynamics. In fact all studies of radiation reaction must address the renormalization challenges originating in the electromagnetic energy component in the material mass.

In absence of a theoretical framework two different approaches have been pursued: modifications of inertia or modification of field dynamics.
I: We can modify the Lorentz Force Eq. (3) as is shown in table 1 exploiting the known expression for the power radiated ($P \propto \int a^2 dt$). Considering further that the power radiated depends at least in principle on the form of the generalized Lorentz Force used to obtain the world lines of particles, there has been considerable freedom in introducing different modifications which only agree with each other at first order in $\tau_0$.

   a) Given $P$ for radiation emitted, the unique result is the Lorentz-Abraham-Dirac (LAD) equation \cite{8}. This equation has not been widely accepted, since among its solutions are unphysical components which need to be eliminated using the knowledge of the dynamics at an infinite future time.

   b) The Landau-Lifshitz (LL) equation \cite{9} is of particular interest since this generalization of the Lorentz Force is using velocities and fields, and for this reason does not introduce the LAD problems. It has gained the status of a valid theory. However, absence of an action principle from which this, or any other similar form, can be derived renders these claims \cite{14,15} wanting, showing that these are also ad-hoc modifications.

II: Since strong acceleration is related to gravity there were two efforts to arrive at unification of electromagnetic and gravitational field phenomena, and a third effort simply introduces a limit to acceleration by limiting the physical field strength:

   a) Weyl’s electromagnetism derives from the requirement that the metric vanishes under double covariant differentiation (gravitationally covariant and gauge covariant in EM theory). For a better insight into reasons this theory fallen out of favor we recommend the excellent review of O’Raifeartaigh and Straumann \cite{16}. Weyl adapted his framework to quantum theory and did not claim its relation to gravity later in his life. We use his gauge invariant derivative daily and yet few remember that Weyl introduced the concept of gauge invariance and covariant derivative to field theory.

   b) The Kaluza-Klein Theory remains a candidate for a unification of Gravity and Electromagnetism. This theory has never been abandoned, for a recent review see \cite{17,18}. However, only once this field theory can be consistently complemented with matter will it be suitable for our purpose. For this reason and because it is the entry point to string theory, theorists marched into greater KK string dimensionality seeking the solution of what Pauli termed ‘the matter problem.’ The Kaluza-Klein theory is still a theory without matter, and in a theory lacking entirely the material electron, we cannot explore its response to acceleration.

   c) The Born-Infeld theory of electromagnetism \cite{19} included a manifest limit to acceleration modeled after the limit to velocity seen in relativity: the Lorentz-force was bounded from above by an explicit limit to the field strength. This approach also allows to interpret the mass of an electron as entirely electromagnetic, however this produces radiation effects that can be very large. A more serious practical problem with this approach is that one can see deviations from linear electromagnetism in precision study of high $Z$ atomic spectra, and the resulting high limiting limiting value found for the Born-Infeld field in order that this effect is invisible is at least 50 times greater than the critical acceleration field \cite{20,21}.

To close this general discussion we once more note that there is no action available for any of the radiation reaction forces we show in table 1. In general all patches of force equations not originating from an action violate energy conservation. Therefore if energy conservation is implemented this becomes a further patch of a patch, and we are losing further control of the missing physics. The long development of the
best known patches to Lorentz force, the LAD effective radiation reaction force and of the related Landau-Lifshitz radiation reaction force are testimonials to the fact that combination of Maxwell equations with Lorentz force acting between charged particles, even if solved self-consistently, does not provide a full theoretical framework describing strongly radiating charged particles.

This quite stunning insight is not new—one can see a trail of research reaching back to beginning of Lorentz and Maxwell theory. It is also clear that perhaps as long as 50 years ago, as soon as lasers were invented, someone somewhere noticed that the best opportunity to experimentally test radiation reaction theory is to combine intense lasers with moving electrons. 50 years of laser technology development finally allows the planning of experiments in this domain.

The trail of publications is thick, and thus we will present only the head of the trail, the latest reports at this meeting [22, 23]. This said, the new element which we address in this paper is that exploration of radiation reaction is a research program on inertia, Mach’s principle and strong gravity, the development of new fundamental action principle, and not merely a measurement of an obscure and not uniquely defined radiation effect.

We stand before an enormous scientific opportunity.

**EXPERIMENTS ON RADIATION REACTION**

In order to gain quantitative insight about the experimental conditions needed for the study of inertia, we ask when the radiation reaction becomes important in the description of dynamics of charged massive particles. For an electron traveling against an electromagnetic laser field, the radiation-reaction effects dominate the dynamics of the electron when

\[
(\omega \tau_0) \gamma_0 a_0^2 \sim 1
\]

where \(\omega\) is the frequency of the laser wave, \(\gamma_0\) is the initial \(\gamma\)-factor of the electron and

\[
\tau_0 = \frac{2e^2}{3mc^3}
\]

is a constant with dimensions of time, whose numerical value for the electron is \(\tau_0 = 6.24 \times 10^{-24}\) s. The origin of the radiation-reaction criterion Eq. (5) is recognized by inspecting the relative scale of Lorentz and Landau-Lifshitz radiation correction force in table 1.

This quantitative criterion is verified in figure 1 where we show the relative deviation in the energy predicted by an equation of motion including radiation reaction (the Landau-Lifshitz equation) compared to the Lorentz force.

The quantum field theory of charged particles, quantum electrodynamics, displays a similar pathology near to where acceleration turns to unity in presence of strong uniform fields: this quantum critical field renders the vacuum state unstable to conversion into a gas of electron-positron pairs. We will review this matter in more detail in next sections. This is only an apparent limit to acceleration strength since we can explore behavior of charged particles in collisions with non-uniform fields as discussed above.
FIGURE 1. Demonstration of the criterion in Eq. (5) which determines in what conditions are the radiation-reaction effects important. The density presents the relative deviation in energy of a particle obeying the Landau-Lifshitz equation compared to a particle subject to only the Lorentz force. One sees that above the critical line the radiation-reaction effects completely dominate the Lorentz dynamics of the particle. For an electron at rest this requires fields at the Schwinger limit ($E \rightarrow 1m^2/e$ corresponding to $a_0 \approx 500000$), but the required field ($a_0$) drops rapidly as we exploit the large relativistic $\gamma$ factor.

The question rings loud at this point of our discussion: is there a radiation-reaction effect when we scatter strongly interacting particles from each other? Strong interactions can impart large acceleration on electromagnetically charged particles and thus one would expect that there is a serious violation of conventional theoretical expectations for production of radiation. In fact, in $pp$ and $\pi p$ collisions a very serious photon excess was identified in painstaking analysis by Martha Spyropoulou-Stassinaki of Athens University [25]. It has remained without explanation.

In heavy ion collisions excess of both photons and lepton (electron, muon) pairs have been seen and remained largely unexplained – these radiation reaction effects have obscured the usefulness of photons and leptons as signatures of quark-gluon plasma. However, because collisions involving many nucleons (of type $A$) in nuclei are much more difficult to interpret, the situation is not as experimentally clean and clear as it is in the more elementary $pp$ and $\pi p$ collisions. Moreover, these results are often scaled in cascade programs to extract the expected heavy ion backgrounds, and thus any measured $pp$ photon and lepton pair enhancement becomes part of the $AA$ background.
The story does not end here; the theory of interacting quarks and gluons, quantum chromodynamics is patterned after quantum electrodynamics and thus any shortcoming that one finds in QED will be present in QCD, especially at high energies where the classical limit prevails. Moreover, since the quark-gluon coupling is as much as 50 times stronger, in suitable circumstances the strong acceleration and radiation reaction effects could appear much more easily in QCD. A very pertinent effect is the observation of the strong stopping power of quarks and gluons in quark-gluon plasma, the effect of ‘jet quenching’ [26]. Theories can be developed to explain this within the conventional theoretical framework, yet the fact remains that one must stretch all parameters and reaction mechanisms in order to describe these effects. Thus it is safe to say that there would be no contradiction with strong interaction physics if a theoretical framework were to appear in which radiation reaction of gluons and photons (both couple to quarks) acts much stronger than inferred within small-acceleration Lorentz-type theory.

This short section and foregoing discussion show that there is some experimental and theoretical progress and an inkling presence of radiation-reaction effects. However, clearly a dedicated effort must be made to understand critical acceleration or simply, inertia. As we have argued, within the near future light pulse collisions with relativistic electrons will provide the experimental opportunity for exploring this most challenging question from the experimental perspective.

**CRITICAL ACCELERATION, QED AND TEMPERATURE**

The classic prediction of the Euler-Heisenberg-Schwinger analysis of QED in strong constant external fields (for a recent review see [27]) is that at the unit field strength, \( E_s \equiv m_e^2/e = 1 \) in natural units, the electrical field becomes massively unstable, collapsing via materialization into electron-positron pairs on a microscopic time scale [28]. This is a fascinating result and the measurement of production of matter from fields in vacuum by ultra intense laser fields is the signature experiment in strong field physics [29, 30, 31].

Though pair production goes under the premise of a test of QED in strong fields, nobody ever questions the theoretical framework when the fields become strong. The general assumption is that QED is correct, and thus exploration of the pair production mechanism is the mainstream of the current study. The problem is that we already know that the classical theory of electromagnetism is not complete, as has been remarked in preceding sections. Can the quantum field theory built upon it be complete? This is the question that the fields-to-matter experiment can answer.

In our opinion, QED at the critical field strength cannot be complete, and it is very likely that before laser fields strong enough to break the vacuum are created, theoretical clarity will be reached on this issue. In order to see what is the problem we inspect the effective action of Euler and Heisenberg, which has the privilege of being one of a few exact results in this area of physics. Indeed, one does not need to look far to see that something is missing in this expression.

We use here the connection between Euler-Heisenberg-Schwinger (EHS) action and temperature [32], and to simplify, we consider a reference frame such that either only an electrical field is present \((B = 0)\), or only a magnetic field is present \((E = 0)\). This requires the invariant \(E \cdot B\) vanish identically, as can easily be arranged for laser-driven
experiments. However, our remarks are more generally valid for all field configurations. For these two cases, one can write [33]:

$$\mathcal{L}_{\text{eff}}(E) = \frac{m^2}{8\pi^2\beta} \int_0^\infty dv \ln \left( \frac{v^2 - m^2 + i\epsilon}{m^2} \right) \ln(1 - e^{-\beta v}),$$  \hspace{1cm} (7)

$$\mathcal{L}_{\text{eff}}(B) = \frac{m^2}{8\pi^2\beta} \int_0^\infty dv \ln \left( \frac{v^2 + m^2}{m^2} \right) \ln(1 - e^{-\beta v}),$$  \hspace{1cm} (8)

where $\beta = m\pi/eE$ or $\beta = m\pi/eB$, respectively. For comparison, the functional in statistical physics corresponding to the effective action, the free energy, is for Bosons and Fermions (upper and lower signs, respectively)

$$\mathcal{F}_{B/F} = \pm \frac{1}{\beta} \sum_k \ln(1 \mp e^{-\beta v_k}).$$  \hspace{1cm} (9)

The sum is over all modes and it is common to transit from discrete to continuous sum in which case it is understood that one normalizes dividing by the volume $V$. $\mathcal{F}$ is then the free energy density just as $\mathcal{L}$ is the (effective) action density.

We see that beginning from a microscopic theory of fermion pair vacuum fluctuations, the effective EHS action acquires a form typical for bosons in a thermal bath. Confirmation of the statistics (sign) reversal comes from inspection of the spin-0 effective action for which the same form is found but with $\ln(1 - e^{-\beta M}) \rightarrow (1/2) \ln(1 + e^{-\beta M})$, where the factor 2 accounts for reduced number of degrees of freedom.

Another difficulty is that the temperature $\beta^{-1}$ is half of the value that would correspond to the Hawking-Unruh effect:

$$T_{\text{HU}} = \frac{a}{2\pi}; \quad a = \frac{qE}{m}; \quad \beta_{\text{HU}} = \frac{1}{T_{\text{HU}}} = \frac{2\pi m}{qE} = 2\beta.$$  \hspace{1cm} (10)

There is at present no understanding of why the sign reversal occurs, nor why the temperature differs from the only comparable quantity by a factor two. These results appear also in the classical WKB limit of quantum field theory and in many related methods. Further discussion is found in Pauchy Hwang and Kim [34].

An interpretation of vacuum fluctuations in the presence of electric field as if there were a thermal bath is inviting yet not quite consistent with the interpretation of the accelerated frame observing a thermal bath as in the picture arising from the Hawking-Unruh effect. This disagreement implies that General Relativity, the Equivalence Principle, Quantum Field Theory and Quantum Electrodynamics remain startlingly inconsistent despite 50+ years of effort.

The mutually inconsistent temperature interpretations arising from both external fields and acceleration evoke another interesting and challenging problem clearly related, but somewhat to the side of our current discussion. When electrical fields decay into a multitude of particle-anti-particle pairs, a coherent, pure quantum state materializes into a multi-particle high entropy state. There is no heat bath, no coupling to an environment. Yet rapidly a lot of entropy is created as if entropy also materialized, or the system had access to a hidden entropy source.
This problem is already intensively studied considering rapid entropy production at times of formation of quark-gluon plasma but remains unresolved. One must be aware that sudden appearance of entropy in an isolated system associated with a former pure quantum state violates the principles of quantum mechanics. We note that this effect only arises in presence of critical acceleration when massive particle production ensues. Thus we learn that also quantum mechanics may need modification when critical acceleration is reached. Though our prior discussion of non-Machian physics addressed mainly forces, the entropy crisis places quantum mechanics also among theories in need of Machian extension.

Nevertheless, the difficulties of QED underline our major insight: the classical theory of electromagnetism (both classical and quantum) is incomplete and fails in the strong acceleration limit. Considering the equivalence principle and the Machian nature of Einstein’s gravity we know that an extension of the theory of electromagnetism to the domain of strong acceleration will need to be consistent with geometric gravity and Mach3. Because the problem exists in full force in deeply classical domain, it is not necessary to develop quantum gravity in order to understand electromagnetism in the strong acceleration limit. Any improvement in the action of charged matter and EM fields will trickle down both to quantum theory and quantum field theory.

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