A Fundamental Solution to the CCC equations
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Abstract
We display a simple solution to the Penrose CCC scenario. For this solution we chose for the late stages of the previous aeon a FRW, $k=0$, universe with a both a cosmological constant and radiation (no mass) while for the early stages of the 'present' aeon we have again a FRW universe, $k=0$, with the same cosmological constant and again with radiation but with mass not yet present. The Penrose conditions force the parameters describing the radiation of the former and present aeons to be equal and the transition metric in the overlap region turns out to be flat.
We further study how different rest-mass zero fields transition between the different conformally related regions. These (test) fields appears to easily allow perturbations of the geometry within the CCC scenario.

1 Introduction
Several years ago Penrose proposed a detailed, rather radical - and certainly contentious - idea into cosmology. It has been referred to as Cycle Conformal Cosmology, (CCC)[1,2]. The idea is that there exists a non-ending sequence of aeons, each beginning with a Big-Bang and ending (after a very long time) with an exponentially expanding universe. The space-time metric, $\tilde{g}_{ab}$, of one aeon, (referred to as the previous aeon), is conformally connected to the metric, $\hat{g}_{ab}$, of the next aeon (referred to as the present aeon) by the conformal factor $\Omega$. The relationship between them is given in the transition or overlap region via the transition metric $g_{ab}$, by

$$\tilde{g}_{ab} = \Omega^2 g_{ab}$$
$$\hat{g}_{ab} = \omega^2 g_{ab}$$
$$\omega = -\Omega^{-1}.$$ 

As the transition surface (or Big Bang), denoted by $\chi$, is approached from the previous aeon, both $\tilde{g}_{ab}$ and $\Omega$ tend to infinity so that $\hat{g}_{ab}$ is regular while $\omega$ and $\tilde{g}_{ab}$ vanish at the start, $\chi$, the Big Bang of the present aeon.
The metrics of the two aeons satisfy the Einstein equations with positive cosmological constant scaled to be

$$\Lambda = 3.$$  \hspace{1cm} (4)

The matter content of the late stages of the previous aeon and the very early stages of the new aeon are taken to be pure radiation. (It is assumed - a potential weak point of the theory - that all rest mass disappears in the late stages of every aeon and returns shortly after the new Big Bang.)

In order to make this scenario into a predictive physical theory, Penrose chooses a dynamical equation, with initial conditions, for the evolution of the conformal factor $\Omega$. The equation chosen, the Yamabe equation,

$$\Box \Omega + 2\varepsilon \Omega = 2\Omega^3$$ \hspace{1cm} (5)

is the special case of the transformation of the Ricci scalar under a conformal transformation when both Ricci scalars are constant. $\Box$ is the wave operator with the transition metric, $g_{ab}$. Though $\varepsilon$ is usually chosen to be one, we will allow it to be either one or zero.

Penrose chooses the initial conditions for $\Omega$ so that near $\chi$, the norm of $\nabla_a \omega$ (using the transition metric $g_{ab}$ for the norm) is given by

$$\nabla_a \omega \nabla^a \omega = 1 + (Q - 2)\omega^2 + 0(\omega^3)$$ \hspace{1cm} (6)

$Q$ is to be a given universal positive constant.

Though there are a variety of foundational theoretical questions and fascinating real predictions and observational issues associated with this scheme of Penrose, since it is not the thrust of this note, for completeness and continuity of ideas we only briefly mention three of the most important:

1. The CCC scheme seems to be the first reasonable means of discussing the so-call Big Bang of standard cosmology without invoking religion. A well-known Astrophysicist once called me on the phone to ask "if I did not think that the Big Bang was the proof of the existence of God.” He even wrote a book on the subject. In addition, the default standard cosmological model with the inflationary scenario simply avoids the difficulties of the Big Bang by forgetting that the difficulties exist or that it even is an issue.

2. The CCC allows for an explanation of the origin of the 2nd law of thermodynamics.$^1$

3. A generic CCC scenario predicts that in the CMB sky of the present aeon there will be families of concentric circles with lower and higher than background temperatures. Though it is not yet generally agreed on, there appears to be good evidence that such families of circles do exist.$^2$ Our present simple model does not allow for such concentric circles.

The main purpose of this note is to present a simple case of a CCC scenario - probably the most basic or fundamental one that captures most of the principle ideas of a CCC scenario. It also allows for the development of a perturbation procedure for the development of more general CCC scenarios.
Rather than following the Penrose scheme of beginning with the metric of the previous aeon, then solving the evolution Yamabe equation with specific initial conditions to find the $\Omega$ and thus obtaining the present aeon metric, we instead, chose a different procedure. We chose a previous aeon model universe with metric and then guess what the associated present universe and metric should be. This allows us to construct a conformal factor $\Omega$ from the two metrics and check if the Yamabe equation, (with initial conditions), is satisfied or if the parameters of the present metric (or parameters in the $\Omega$) must be adjusted.

More specifically we chose for both the previous and the present aeons FRW universes with $k = 0$ and cosmological constant, $\Lambda = 3$. (We could have chosen without much effort, $k=1,-1$.) The matter in both cases are pure radiation since it is assumed in the late stages of the previous aeon (the period of our interest) all the mass has disappeared, while for the present aeon, in the early stages, (again the time of our interest) mass has not yet made its appearance. There are only two adjustable parameters in this scenario, the radiation parameters for each of the aeons. It turns out that the Yamabe equation, with the initial conditions, are satisfied when the two parameters are set equal. The parameter $Q$ for the initial conditions turns out to be two.

## 2 Construction of the metrics

We begin with the conformally flat form of the $k = 0$, FRW metric

$$ds^2 = R^2(\tau)[d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2)] \quad (7)$$

where $\tau$ is the conformal time (which runs from $\tau = 0$ to $\tau = \tau_\infty = \text{finite}$) and $R(\tau)$ is the scale factor.

[Though we do not need it, cosmic physical time is found via $dt = R(\tau)d\tau$.]

The reduced form of the FRW differential equation, $[5]$, for $\Lambda = 3$, pure dust and radiation, with

$$S = R/R_0, \quad (8)$$

($R_0$ being the ‘present’ value (arbitrary) of the scale factor) becomes

$$\frac{dS}{d\tau} = R_0(b + aS + S^4)^\frac{1}{2}. \quad (9)$$

The parameters $a$ and $b$, (which describe the matter and radiation sources), are given by the ratio of the density parameters at the ‘present’ time,

$$a = \frac{\Omega_M}{\Omega_\Lambda}, \quad b = \frac{\Omega_\gamma}{\Omega_\Lambda}. \quad (10)$$

Taking $a$, the mass parameter to vanish, by assumption, in the two regions of interest, Eq.(10) can be formally solved by

$$\int_{S_0}^{S} \frac{dS}{(b + S^4)^\frac{1}{2}} = R_0(\tau - \tau_0). \quad (11)$$
Though the integral can be evaluated in terms of elliptic functions, it is far easier and more useful to evaluate it by approximations in the two different domains - the late previous aeon where $S$ tends to infinity and the early stages of the present domain where $S$ begins with zero.

To keep the notation for the two aeons clear and separate the variables of the previous aeon will be written with a hat, e.g., $\hat{S}, \hat{b}$, etc., while for the present the same variables will have an inverted hat, (hachek) e.g., $\check{S}, \check{b}$.

### 2.1 Previous Aeon

By rewriting Eq. (11) as

$$\int_{\hat{S}_0}^{\hat{S}} \frac{d\hat{S}}{\hat{S}^2(1 + \hat{b}\hat{S}^{-4})^{1/2}} = \hat{R}_0(\hat{\tau} - \hat{\tau}_0) \quad (12)$$

and expanding the denominator for large $S$, we have

$$\int_{S_0}^{S} d\tilde{S}(\tilde{S}^{-2} - \frac{1}{2}b\tilde{S}^{-6}) = \tilde{R}_0(\tilde{\tau} - \tilde{\tau}_0). \quad (13)$$

This is easily integrated as

$$-\tilde{S}^{-1} + \frac{1}{10}\tilde{b}\tilde{S}^{-5} + \tilde{S}_0^{-1} - \frac{1}{10}b\tilde{S}_0^{-5} = \tilde{R}_0(\tilde{\tau} - \tilde{\tau}_0). \quad (14)$$

Choosing $\tilde{S}_0$ as infinity and $\tilde{\tau}_0 = \tilde{\tau}_\infty$, we have

$$\tilde{S}^{-1} - \frac{1}{10}\tilde{b}\tilde{S}^{-5} = -\tilde{R}_0(\tilde{\tau} - \tilde{\tau}_\infty). \quad (15)$$

Finally, defining

$$\Gamma = \hat{\tau} - \hat{\tau}_\infty, \quad (16)$$

(which is negative since $\hat{\tau} < \hat{\tau}_\infty$) and iterating for small $\Gamma$, (starting from $\hat{S}_{start}^{-1} = -\hat{R}_0\Gamma$) we have our solution near $\Gamma = 0$:

$$\hat{S}^{-1} = -\hat{R}_0\Gamma + \frac{1}{10}\tilde{b}\hat{S}^{-5} \quad (17)$$

$$\hat{S}_{start}^{-1} = -\hat{R}_0\Gamma \quad (18)$$

$$\hat{S}^{-1} = -\hat{R}_0\Gamma (1 + \frac{1}{10}\hat{b}\hat{R}_0^4\Gamma^4). \quad (19)$$

Using $\hat{R} \equiv \hat{S}\hat{R}_0$ and inverting, we finally have our scale function for the late stages of the previous aeon.

$$\hat{R} \equiv -\frac{\Gamma^{-1}}{(1 + \frac{1}{10}\hat{b}\hat{R}_0^4\Gamma^4)}, \quad (20)$$

$$= -\Gamma^{-1}(1 - \frac{1}{10}\hat{b}\hat{R}_0^4\Gamma^4).$$
2.2 Present Aeon

Returning to the integral, Eq. (11), and applying it to the very early stages of the present aeon, i.e., for small $\dot{S}$, the integral is approximated by

\[
\int_{S_0}^{\dot{S}} \frac{d\dot{S}}{\sqrt{b(1 + S^4/6)}} = R_0(\tau - \tau_0), \tag{21}
\]

\[
\int_{S_0}^{\dot{S}} d\dot{S}(1 - \frac{S^4}{2}) = \sqrt{b}R_0(\tau - \tau_0). \tag{22}
\]

Performing the integration and using $\dot{S}_0 = 0$ at $\tau_0 = 0$, followed by the iteration, leads to

\[
\dot{S} = \sqrt{b}R_0\tau + \frac{1}{10}b^{3/2}\hat{R}_0^{5}\tau^5. \tag{23}
\]

Using $\dot{R} = \dot{S}\hat{R}_0$, with the definition, $\Gamma = \tau$, gives us the scale factor for the present aeon,

\[
\dot{R} = \sqrt{b}\hat{R}_0^2\Gamma + \frac{1}{10}b^{3/2}\hat{R}_0^5\Gamma^5. \tag{24}
\]

The conformal time parameter $\Gamma$, which is positive for the present aeon is simply the continuation of the negative valued $\Gamma$ of the previous aeon. $\Gamma = 0$ is the time of the transition or Big Bang.

2.3 The Conformal Factor and the Metrics

Introducing the parameters $\hat{J}$ and $J$ by

\[
\hat{J} = \sqrt{b}\hat{R}_0^2 \quad \text{and} \quad J = \sqrt{b}\hat{R}_0^2 \tag{25}
\]

the two scale factors of the past and present aeon are

\[
\hat{R} = -\Gamma^{-1}(1 - \frac{1}{10}J^2\Gamma^4), \tag{26}
\]

\[
\hat{R} = J\Gamma(1 + \frac{1}{10}J^2\Gamma^4).
\]

The associated metrics are thus

\[
\hat{g}_{ab} = \hat{R}^2\eta_{ab}, \tag{27}
\]

\[
\hat{g}_{ab} = \hat{R}^2\eta_{ab}.
\]

By eliminating $g_{ab}$ from the two relationships, Eqs. (1) and (2), with (3), we obtain

\[
\hat{g}_{ab} = \Omega^4\hat{g}_{ab}, \tag{28}
\]

so that, from (27),

\[
\Omega^4 = \frac{\hat{R}^2}{\hat{R}^2}.
\]
Our conjecture is that this \( \Omega \), perhaps with the adjustment of the parameters, \((\tilde{J}, \hat{J})\), satisfies the Penrose conditions for the transition from the previous aeon to the present one.

The following are a series of approximate relationships easily derived from the \( \Omega \) that are useful for checking the Penrose transition conditions.

\[
\Omega^2 = -\frac{\tilde{R}}{\hat{R}} = \tilde{J}^2 \Gamma^{-2} (1 - \frac{1}{10} (\tilde{J}^2 + \hat{J}^2) \Gamma^4), \quad (29)
\]

\[
\Omega = -\tilde{J}^{-1/2} \Gamma^{-1} (1 - \frac{1}{10} (\tilde{J}^2 + \hat{J}^2) \Gamma^4), \quad (30)
\]

\[
d\Omega = \tilde{J}^{-1/2} \{ \Gamma^{-2} + \frac{3}{20} (\tilde{J}^2 + \hat{J}^2) \Gamma^2 \} \, d\Gamma, \quad (31)
\]

\[
\omega = -\Omega^{-1} = \tilde{J}^{1/2} \Gamma (1 + \frac{1}{20} (\tilde{J}^2 + \hat{J}^2) \Gamma^4), \quad (32)
\]

\[
\Gamma = \tilde{J}^{-1/2} \omega, \quad (33)
\]

\[
g_{ab} = \Omega^{-2} \hat{g}_{ab} = -\tilde{R} \hat{R} \eta_{ab} = R^2 \eta_{ab}, \quad (34)
\]

\[
\sqrt{-g} = \sqrt{-\det g_{ab}} = R^4 \quad (35)
\]

\[
R^2 = -\tilde{R} \hat{R} = \tilde{J} (1 + \frac{1}{10} [\tilde{J}^2 - \hat{J}^2] \Gamma^4), \quad (36)
\]

\[
R = \tilde{J}^{1/2} (1 + \frac{1}{10} [\tilde{J}^2 - \hat{J}^2] \Gamma^4). \quad (37)
\]

Notice that the intermediate metric \( g_{ab} \) is flat when \( \tilde{J} = \hat{J} \). (38)

### 3 The Penrose Transition Conditions

The most important of the Penrose conditions on \( \Omega \) is the Yamabe equation

\[
\Box \Omega + 2 \varepsilon \Omega = 2 \Omega^3 \quad (40)
\]

It becomes, using the transition metric and \( \Gamma = x^0, \, \partial_{\Gamma} F = F' \),

\[
(-g)^{-1/2} \partial_a ((-g)^{1/2} g^{ab} \partial_b \Omega) + (\varepsilon 2) \Omega = 0, \quad (41)
\]

\[
\eta^{ab} R^2 \partial_a (\partial_b \Omega) + \eta^{ab} \partial_a R^2 \partial_b \Omega + R^4 (\varepsilon 2) \Omega = 2 R^4 \Omega^3, \quad (42)
\]

\[
R^2 \Omega'' + (R^2)' \Omega' + R^4 (\varepsilon 2) \Omega = 2 R^4 \Omega^3. \quad (43)
\]
Inserting Eqs. (37) and (51) into (43), leads after a lengthy calculation to

\[ [\hat{J}^2 - \tilde{J}^2] \Gamma = 0 \]

or the condition that

\[ \hat{J} = \tilde{J}. \]  

(44)

With this equality we have that the transition metric \( g_{ab} \) is flat in the neighborhood of the transition surface and the Yamabe equation is satisfied.

To examine the remaining Penrose conditions, namely the initial conditions for the Yamabe equation, we must find \( \nabla_a \omega \) and its norm, \( \nabla_a \omega \nabla^a \omega \). They are calculated with the transition metric, using the equality of the two \( J \)'s (their decorations are discarded):

\[
\nabla_a \omega = J^1 (1 + \frac{1}{2} J^2 \Gamma^4) \delta_0^a, \\
\nabla_a \omega \nabla^a \omega = 1 + J^2 \Gamma^4.
\]

(45)

Using (34), i.e., using \( \omega \) as the 'time' parameter, the norm becomes

\[ \nabla_a \omega \nabla^a \omega = 1 + \omega^4. \]  

(46)

Comparing this with the Penrose condition, Eq. (6), we see that indeed, as required, the norm at \( \Gamma = 0 \) is one and that \( Q = 2 \).

3.1 Miscellaneous

Penrose discusses several other relationships derived from the conformal factor \( \Omega \). For completeness, we briefly examine several of them, comparing the results from our model with those of Penrose [2].

1. There is the one form \( \Pi \) and its norm in the transition region given by Penrose as

\[
\Pi = \frac{d\Omega}{\Omega^2 - 1} = \frac{d\omega}{1 - \omega^2}, \\
\Pi^a \Pi_a = 1 + Q^\omega^2 + O(\omega^3),
\]

which become in our model

\[
\Pi = J^{1/2} (1 + J\Gamma^2 + \frac{3}{2} J^2 \Gamma^4) d\Gamma, \\
\Pi^a \Pi_a = 1 + 2J\Gamma^2 + 4J^2 \Gamma^4.
\]

In the norm, if we change the coordinate from \( \Gamma \) to \( \omega \), by Eq. (34), the norm becomes

\[ \Pi^a \Pi_a = 1 + 2\omega^2 + O(\omega^4), \]
yielding again that $Q = 2$.

2. Penrose has the divergence of $\Pi^a$ as

$$\nabla_a \Pi^a = 2Q\omega + O(\omega^2),$$

while we have it as

$$\nabla_a \Pi^a = 2\omega + O(\omega^3).$$

There is thus a disagreement, by the factor of 2, when taking $Q = 2$. This disagreement probably has its origin with the fact that in the Yamabe equation we take $\varepsilon$ to be zero while Penrose takes it to be one. It appears likely that if we had chosen for the FRW universes, the curvature $k = 1$ case rather than our choice $k = 0$, this disagreement would vanish.

### 3.2 Test Fields

As a prelude to studying perturbation on our simple CCC scenario, we describe the transformations of both Maxwell fields and linearized Weyl tensor fields under the conformal transformations that relate the three metrics, $\hat{g}_{ab}$, $g_{ab}$, $\check{g}_{ab}$.

Since there are a variety of different conformal transformations that we must consider, we first use generic variables, with generic decorations, $(\star, \bullet)$, and a generic conformal factor, $\tilde{\Omega}$, to describe the transformations. Afterwards they are specialized to the ones considered in the CCC discussion.

There are a large variety of conformal rescaling, including their inverses, e.g., from Minkowski space to a FRW space, or from one aeon to another with their inverses, that we must deal with. Considerable care must be used in going between these different cases.

For the metrics we use,

$$g_{ab}^{\bullet} = \tilde{\Omega}^2 g_{ab}^{\star},$$

for the Maxwell fields,

$$F_{ab}^{\bullet} = F_{ab}^{\star},$$

for the Weyl tensor components,

$$C^{\bullet}_{\ a\ b\ cd} = C^{\star}_{\ a\ b\ cd},$$

and for the NP tetrad fields,

$$l^{\bullet}_a = \tilde{\Omega}^{-2} l^{\star}_a, \quad l^\bullet_a = l^\star_a,$$

$$n^{\bullet}_a = n^{\star}_a, \quad n^\bullet_a = \tilde{\Omega}^2 n^\star_a,$$

$$m^{\bullet}_a = \tilde{\Omega}^{-1} m^{\star}_a, \quad m^\bullet_a = \tilde{\Omega} m^\star_a,$$

$$\overline{m}^{\bullet}_a = \tilde{\Omega}^{-1} \overline{m}^{\star}_a, \quad \overline{m}^\bullet_a = \tilde{\Omega} \overline{m}^\star_a.$$

Using Eq. (50) with (48) and (49), one easily calculates the spin-coefficient versions of the transformations. For the Maxwell case we have
\[
\begin{align*}
\phi_0^\bullet &= \tilde{\Omega}^{-3} \phi_0^\bullet, \\
\phi_1^\bullet &= \tilde{\Omega}^{-2} \phi_1^\bullet, \\
\phi_2^\bullet &= \tilde{\Omega}^{-1} \phi_2^\bullet,
\end{align*}
\] (51)

with
\[
\phi_0^\bullet \equiv \dot{t}^a m^b F_{ab}, \quad \phi_1^\bullet \equiv \frac{1}{2}(\dot{t}^a n^b + m^a \dot{m}^b) F_{ab}, \quad \phi_2^\bullet \equiv m^a n^b F_{ab},
\] (52)

while for the linearized Weyl tensor we have
\[
\begin{align*}
\Psi_0^\bullet &= \tilde{\Omega}^{-4} \Psi_0^\bullet, \\
\Psi_1^\bullet &= \tilde{\Omega}^{-3} \Psi_1^\bullet, \\
\Psi_2^\bullet &= \tilde{\Omega}^{-2} \Psi_2^\bullet, \\
\Psi_3^\bullet &= \tilde{\Omega}^{-1} \Psi_3^\bullet, \\
\Psi_4^\bullet &= \Psi_4^\bullet,
\end{align*}
\] (53)

with
\[
\begin{align*}
\Psi_0^\bullet &= -\dot{t}^a m^b l^c m^d C_{abcd}, \\
\Psi_1^\bullet &= -\dot{t}^a n^b l^c m^d C_{abcd}, \\
\Psi_2^\bullet &= -\dot{t}^a m^b \dot{m}^c n^d C_{abcd}, \\
\Psi_3^\bullet &= -\dot{t}^a n^b \dot{m}^c n^d C_{abcd}, \\
\Psi_4^\bullet &= -m^a n^b l^c m^d C_{abcd}.
\end{align*}
\] (54)

Our first application of these relations is to the transition from a solution of either Maxwell’s equations or the "flat-space" Bianchi Identities for the Weyl tensor in Minkowski space, to the conformally related solutions in a FRW spacetime.

In particular, we consider transitioning the flat-space solutions to our previous aeon solutions.

The decorations then change from \(Y^\bullet\) to \(Y^{\text{flat}}\) and \(Y^\bullet\) to \(\hat{Y}\), with \(\tilde{\Omega}\) becoming \(\hat{R} = -\Gamma^{-1}(1 - \frac{1}{10}\dot{\Gamma} R^4)\).

Close to the Big Bang surface, \(\chi, \Gamma \approx 0\), so for simplicity we take \(\hat{R} = -\Gamma^{-1}\), with \(\Gamma\) negative.

A flat-space Maxwell field \((\phi_0^{\text{flat}}, \phi_1^{\text{flat}}, \phi_2^{\text{flat}})\) or linear Weyl tensor field \((\Psi_0^{\text{flat}}, \Psi_1^{\text{flat}}, \Psi_2^{\text{flat}}, \Psi_3^{\text{flat}}, \Psi_4^{\text{flat}})\) becomes from (51) and (53)
\[
\begin{align*}
\hat{\phi}_0 &= -\Gamma^3 \phi_0^{\text{flat}}, \\
\hat{\phi}_1 &= -\Gamma^2 \phi_1^{\text{flat}}, \\
\hat{\phi}_2 &= -\Gamma \phi_2^{\text{flat}},
\end{align*}
\] (55)
and the corresponding Weyl fields

\[ \hat{\Psi}_0 = \Gamma^4 \Psi_0^{\text{flat}}, \]
\[ \hat{\Psi}_1 = -\Gamma^3 \Psi_1^{\text{flat}}, \]
\[ \hat{\Psi}_2 = \Gamma^2 \Psi_2^{\text{flat}}, \]
\[ \hat{\Psi}_3 = -\Gamma \Psi_3^{\text{flat}}, \]
\[ \hat{\Psi}_4 = \Psi_4^{\text{flat}}. \]

The Maxwell fields and almost all Weyl tensor components, vanish in the previous aeon as \( \chi \) is approached.

However in the transition region where we conformally rescale via Eq. (1),
\[ g_{ab} = \Omega^{-2} \hat{g}_{ab}, \]
with (comparing with Eq. (47)), \( \hat{\Omega} = \Omega^{-1} = -J^{1/2} / (1 + \frac{1}{4} J^2 \Gamma^4) \), or approximately \( \hat{\Omega} = \Omega^{-1} \approx -J^{1/2} \Gamma \), the Maxwell and Weyl fields return to their flat-space values, modified by the numerical factor \( \sqrt{J} \). For example, from \( \phi^0 = \Omega^{-1} \phi^\flat \), we have that
\[ \hat{\phi}_2 = \hat{\Omega}^{-1} \hat{\phi}_2 = -J^{-1/2} \Gamma^{-1} \hat{\phi}_2, \]
\[ = J^{-1/2} \phi^\flat. \]

We thus have the result that in the transition region the two massless fields go thru the Big Bang smoothly.

Finally to see how these same fields behave near the Big Bang in the present aeon we use Eq. (28), i.e., \( \hat{g}_{ab} = \Omega^4 \hat{g}_{ab} \), and compare it with Eq. (47), so that \( \hat{\Omega} = \Omega^{-2} \approx J^2 \Gamma^2 \). Using this with (51) and (53), along with (55) and (56), the test fields are found in the present aeon to be

\[ \hat{\phi}_0 = J^{-6} \Gamma_0^{-3} \phi_0^{\text{flat}}, \]
\[ \hat{\phi}_1 = J^{-4} \Gamma^{-2} \phi_1^{\text{flat}}, \]
\[ \hat{\phi}_2 = J^{-2} \Gamma^{-1} \phi_2^{\text{flat}}, \]

and the corresponding Weyl fields

\[ \hat{\Psi}_0 = J^{-8} \Gamma^{-4} \Psi_0^{\text{flat}}, \]
\[ \hat{\Psi}_1 = J^{-6} \Gamma^{-3} \Psi_1^{\text{flat}}, \]
\[ \hat{\Psi}_2 = J^{-4} \Gamma^{-2} \Psi_2^{\text{flat}}, \]
\[ \hat{\Psi}_3 = J^{-2} \Gamma^{-1} \Psi_3^{\text{flat}}, \]
\[ \hat{\Psi}_4 = \Psi_4^{\text{flat}}. \]

All the fields in the present aeon are singular at \( \chi \) and diminish as \( \Gamma \) increases from its zero value.

If we consider any Maxwell or Weyl field (or any rest-mass zero field) in the conformally related flat space of the previous aeon and refer to it as \( \Phi^{\text{flat}} \), it
and all its conformally related counterparts in the different regions can be taken as functions of the flat-space null coordinate system \((u, r, \theta, \phi)\). We thus have, generically speaking,

\[
\Phi = \Phi(u, r, \theta, \phi).
\]  

(60)

Replacing \(u\), via \(u = \tau - r\) and \(\Gamma = \tau - \tau_\infty\) \([16]\) by

\[
u = \Gamma + \tau_\infty - r
\]

(61)
we have

\[
\Phi = \Phi(\Gamma + \tau_\infty - r, r, \theta, \phi).
\]  

(62)

If, as a special case, we consider \(\Phi\) to have support only on a light-cone, (e.g., a violent event at \(\tau = \tau^\#\) in the previous aeon or perhaps even a series of them following close to each other) chosen as \(u = \tau^\#\), the support of the field lies on

\[
r = \Gamma + \tau_\infty - \tau^\#.
\]  

(63)

In the limit \(\Gamma = 0\), (\(\chi\), the cross-over or Big Bang surface) the field

\[
\Phi = \Phi(\tau^\#, \tau_\infty - \tau^#, \theta, \phi)
\]  

(64)
has support only on the sphere

\[
r = r_\infty = \tau_\infty - \tau^#,
\]  

(65)
which is finite and a known function of the initial conditions for the dynamics of the previous aeon.

As an aside we remark that if this field on \(\chi\) or rather shortly afterwards (i.e., last scattering surface) were to scatter by interactions with other fields in the present aeon, the effect could potentially be seen by present observers looking back on their past-light cones. These past cones intersect the sphere \(r = \tau_\infty - \tau^#\) in a circle (or for a series of events, a concentric family of circles) thus potentially appearing to an observer as a circle (or a series of circles) on the CMB sky.

### 4 Discussion

Working essentially with the Penrose CCC scenario, we have chosen a special simple case to explore - a previous aeon late time FRW universe with radiation that transitions to a present aeon early FRW universe also with radiation. This case appeared to us as a natural example of a CCC (with perhaps some tweaking) that must exist almost a priori. There were two needed adjustments for this to work. One was that the intermediate metric had to have a vanishing cosmological constant. This was manifested in the choice, in the Yamabe equation, that \(\varepsilon = 0\) rather than Penrose’s preference, \(\varepsilon = 1\). The other adjustment
was that the two radiation parameters must coincide. A by-product of these adjustments is that the intermediate metric is flat.

We next considered the behavior of different (test) rest-mass fields (Maxwell and linear Weyl tensor fields) as they go thru the different conformally related regions.

Our intent is to take this simple CCC case and consider the test fields as sources for perturbations on the CCC background. The most obvious thing to do is to treat both Maxwell and linearized Weyl tensor fields that have their support mainly on a light-cone from some earlier origin of the previous aeon (as we have done) and follow the perturbed geometry from the past thru to the present aeon.

This hopefully will yield detailed examples of greater complexity, for analysis in the CCC scenario.

5 Acknowledgments

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