Emergence of the periodic oscillatory modulation in time-like nucleon form factors

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We have studied the oscillatory behavior exhibited in the timelike electromagnetic form factors of nucleons by considering the final-state interaction effect. This mechanism introduces the Jost function of $NN$ pair into the time-like form factors with the help of the distorted-wave Born approximation. Using a simple square-well potential, the contribution from the final-state interaction in our approach is naturally damped oscillatory, which can explain the experimental data very nicely. This scenario seems to be universal considering that it also reproduces well the threshold enhancements on the cross sections for $e^+e^- \rightarrow n\bar{n}$, $\Lambda\bar{\Lambda}$, and $\Lambda_c\bar{\Lambda}$. In Ref. [25] that the qualitative feature of the oscillation can be produced with a complicated optical potential of $NN$ FSI. The role of soft rescattering on time-like proton form factors was discussed based on its physical interpretation similar to the charge distribution of space-like ones [27, 28]. Other considerations of FSI can explain the near-threshold enhancement [26, 29], which is the reflection of the first maximum of the oscillation behavior.

So far a complete and plausible understanding of the emergence of oscillation behavior is still missing. A more natural explanation is needed for the observed good periodicity.

In the present work, we focus on how FSI leads to the oscillatory behavior with proper separation between the formation and FSI processes. The FSI effect is closely related to the zero-point wave function of $NN$ due to the small interaction range for formation compared to that for rescattering. The oscillation-like behavior arises naturally from the Jost function of $NN$ caused by FSI, and the “period” of the oscillation is approximately determined by the Yukawa interaction range $1/m_\pi$. As a by-product, our approach can also interpret the threshold enhancement of time-like form factors.

FSI effect with distorted-wave Born approximation.— The separation of the short-range formation and the long-range FSI allows us to study the FSI modification to the production process $e^+e^- \rightarrow NN$. It is more intuitive to understand this effect for the time reversal process $NN \rightarrow e^+e^-$ since the attractive interaction between $NN$ will increase the probability that they meet each other and hence enhance the cross section while the repulsive interaction will suppress it.

The FSI effect can be studied with the distorted-wave Born approximation, and the cross section $\sigma$ can be expressed with an enhancement or suppression factor $|1/J|^2$ multiplied by the cross section $\sigma_0$ without...
FSI \([30]\): \[
\sigma = \frac{1}{|\mathcal{J}(p)|^2} \sigma_0. \tag{1}
\]

\(\mathcal{J}(p)\) is the Jost function of final states \(N\bar{N}\) with the pure FSI \(V\), and it is closely related to the zero-point wave function.

The radial wave function \(\psi_{\ell,p}(r)\) of \(N\bar{N}\) can be given by the radial Schrödinger equation for each \(\ell\) partial wave

\[
\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - 2\mu V + p^2\right) \psi_{\ell,p}(r) = 0, \tag{2}
\]

where \(\mu\) is the reduced mass and \(p\) is the center-of-mass momentum of \(N\bar{N}\) system. In S-wave dominant processes,

\[
\mathcal{J}(p) \approx \mathcal{J}_{\ell=0}(p) = \lim_{r \to 0} \frac{j_0(pr)}{\psi_{0,p}(r)}. \tag{3}
\]

\(j_0(pr) = \sin(pr)\) is the regular spherical Bessel function. From Eq. (3), this enhancement/suppression factor \(1/|\mathcal{J}|^2\) has a well probability interpretation, that is, the ratio of the probability density for finding the scattering state \(N\bar{N}\) near the origin with FSI to that without FSI.

The formation of the \(N\bar{N}\) pair via photon is of short-range nature, and a naive estimation is \(r_0 \sim 1/(2m_N)\). While the interaction range related to \(N\bar{N}\) rescattering is approximately given by \(a \sim 1/m_\pi\). An estimation \(a/r_0 \approx 14\) indicates an explicit separation of formation and rescattering process. We can thus very confidently use the the factorized expression in Eq. (1) to study the \(e^\alpha e^{-r} \to N\bar{N}\) reaction.

The interaction range of the annihilation and recreation processes of \(N\bar{N}\) is close to \(r_0\), and they should strongly tangle with the initial formation of the \(N\bar{N}\) pair. Thus we put their contribution also into \(\sigma_0\), that is, the FSI \(V\) does not contain the annihilation potential and therefore we neglect its imaginary part in this work.

The Coulomb interaction between the \(N\bar{N}\) pair is weaker and has a much longer interaction range compared to the strong interaction for \(N\bar{N}\) rescattering, and this leads to additional well-known Sommerfeld factor \(C = |1/S|^2\) with [31]

\[
\mathcal{S} = \left(\frac{y}{1-e^{-y}}\right)^{-1/2}, \quad y = \frac{\pi \alpha \sqrt{1-\beta^2}}{\beta}. \tag{4}
\]

Here, \(\alpha\) is the electromagnetic fine-structure constant, \(\beta = \sqrt{1-4m_N^2/s}\) is the center-of-mass velocity of nucleon, and \(s\) is the Mandelstam variable. Actually, we can repeat this Coulomb enhancement factor very easily in our approach. Substituting the Coulomb potential in Eq. (2) for \(V\), Eq. (3) is reduced to \(\mathcal{S}\) by using the non-relativistic approximation \(\sqrt{1-\beta^2} \approx 1\) in the near-threshold region.

| \(N\bar{N}\) | \(a_e\) (fm) | \(V_r\) (MeV) | \(a\) (fm) | \(V_a\) (MeV) |
|---|---|---|---|---|
| \(p\bar{p}\) | 0.5 | 50 | 1.6 | 90 |
| \(n\bar{n}\) | 0.5 | 400 | 1.4 | 650 |

FSI factor with damped oscillation.— As an illustration of the overall FSI effect, firstly we consider the simple case of a rectangular potential well:

\[
V(r) = \begin{cases} -V_a & \text{for } 0 \leq r < a, \\ 0 & \text{for } r \geq a. \end{cases} \tag{5}
\]

We can easily adjust the interaction range \(a\) and the potential depth \(V_a\) to glimpse some general features of a somehow complicated potential. The Schrödinger equation with this potential have analytical solution. The general solution for \(\ell = 0\) is

\[
\psi_{0,p}(r) = \begin{cases} e^{i\delta_0} \sin(p_inr) & \text{for } 0 \leq r < a, \\ \sqrt{\sin^2(p_inr) + \frac{p^2}{p_{in}^2}} \cos(p_inr) & \text{for } r \geq a. \end{cases}
\]

We have the enhancement factor

\[
|\mathcal{J}(p)| = \sqrt{\frac{p^2}{p_{in}^2}} \sin^2(p_ina) + \cos^2(p_ina). \tag{6}
\]

The attractive potential gives an enhancement factor as one can easily check that \(|1/|\mathcal{J}|^2 > 1\) in this model.

Surprisingly this enhancement factor \(|1/|\mathcal{J}|^2\) is of “oscillation” nature: it reaches its local maximum \(1/|\mathcal{J}|^2_{\text{max}} = 1 + 2\mu V_a/p^2\) when \(p_ina = n\pi + \pi/2\) and local minimum \(1/|\mathcal{J}|^2_{\text{min}} = 1\) when \(p_ina = n\pi\). The energy gaps among the 1st, the 2nd, the 3rd and the 4th minimums are:

\[
\frac{3\pi^2}{2\mu a^2}, \quad \frac{5\pi^2}{2\mu a^2}, \quad \frac{7\pi^2}{2\mu a^2}. \tag{7}
\]

The energy gaps, which show a manifestation of the observed oscillation period, are approximately determined by the interaction range \(a\). Concerning the \(N\bar{N}\) scattering, we have \(\mu = m_N/2\) and \(a \approx 1/m_\pi\), and this gives the 1st gap \(\Delta E_1 \approx 0.6\text{ GeV}\). It is approximately close to the observed value in experiment.

In addition, the peaks of \(|1/|\mathcal{J}|^2\) decrease with the increasing energies. This feature describes the damping behavior of the oscillation observed in experiment.

Description for the effective form factor \(G_{\text{eff}}\).— It is commonly to express the cross section data in terms of the
effective form factor which can be obtained as

$$|G_{\text{eff}}(s)| = \sqrt{\frac{3s}{4\pi\alpha^2\beta C(1 + 2m_N^2/s)\sigma_{e^+e^-\to N\bar{N}}}}.$$  \hfill (10)

It can be factorized with the $NN$ rescattering correction $1/|J|$ and the form factor $G_0$ without FSI:

$$|G_{\text{eff}}(s)| = \frac{1}{|J|} G_0(s),$$  \hfill (11)

where $G_0$ describes the short range production and determines the main feature of $G_{\text{eff}}$ in time-like region. Following Ref. [5], it can be parameterized as

$$G_0(s) = \frac{\mathcal{A}}{(1 + s/m_N^2)(1 - s/(0.71 \text{ GeV}^2))^2},$$  \hfill (12)

where $m_N^2 = 7.72$ GeV$^2$ and $\mathcal{A}$ is a constant. With fitted $\mathcal{A}_p = 9.37$ and $\mathcal{A}_n = 5.8$, $G_0$ can give excellent overall descriptions of effective form factors for nucleons over a wide range. Subtracting the smooth continuum $G_0$ from $|G_{\text{eff}}|$ we can get the oscillation behavior

$$G^{osc}(s) = |G_{\text{eff}}| - G_0 = \left(\frac{1}{|J|} - 1\right) G_0(s).$$  \hfill (13)

In the previous section we have shown that a simple rectangular potential well already leads to an oscillation feature, and the enhancement factor is larger than 1 in that case, which means $G^{osc}$ will be positive with the attractive interaction. To obtain the observed oscillatory structure, we need the potential to be a bit more complicated by taking

$$V(r) = \begin{cases} -V_r & 0 \leq r < a_r \\ -V_a & a_r \leq r < a \\ 0 & r \geq a \end{cases},$$  \hfill (14)

where $0 < V_r < V_a$ and we take $a_r = 0.5$ fm.

We show the FSI effect with such potentials on the $p\bar{p}$ and $n\bar{n}$ effective form factors in Fig. 1. The corresponding parameters are listed in Table. I. The overall oscillatory behavior are nicely reproduced due to the FSI effect with the interaction range $a$ about $1.4 \sim 1.6$ fm.

The details of describing the effective form factors rely on the choices of the continuum part $G_0$, which we still do not understand very well because the formation process involves the complicated hadronization and other difficulties. Possibly, better descriptions can be achieved for $|G_{\text{eff}}|$ by using different $G_0$ rather than same as the experiment collaborations.

Threshold enhancement of time-like form factors— From Fig. 1, one notices that the neutron effective form factor has stronger near-threshold enhancement than that for proton. The SND measurement observed the enhancement on the neutron cross section just above threshold at $\sqrt{s} - 2m_n \approx 5$ MeV [6], which contradicts naive phase

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**FIG. 1.** The effective form factors for protons (left) and neutrons (right) in the time-like region. On the top is the effective form factors $|G_{\text{eff}}|$ while at the bottom is the oscillation part $G^{osc}$ after subtracting the smooth background part $G_0$ (gray dashed line). The threshold positions $\sqrt{s} = 2m_N$ are labeled as gray dotdashed vertical lines. In the figure, we only include BESIII [5] (green square) and BABAR [20] (blue circle) data for $p\bar{p}$ and BESIII [21] (green square) and SND [6] (blue triangle) data considering the precision and covering range.
space expectation. Our approach can easily provide such enhancement as seen in Fig. 1. The reason is that the FSI factor near the threshold \( 1/|\mathcal{J}|_{p \to 0} \to 1/\cos^2 (\sqrt{2mV_0}) \) with an attractive squared-well potential. With appropriate \( V_0 \) and \( a \), \( 1/|\mathcal{J}|_{p \to 0} \) can give rise to very large enhancement near-threshold.

In addition to \( N\bar{N} \) production, other baryon-antibaryon pair productions near thresholds have also been measured \([32-37]\). Abnormally large cross sections are observed in \( e^+e^- \to \Lambda\bar{\Lambda} \) near the threshold \( \sqrt{s} - 2m_{\Lambda} \approx 1 \) MeV \([32]\) and possibly \( e^+e^- \to \Lambda_c\bar{\Lambda}_c \) at \( \sqrt{s} - 2m_{\Lambda_c} \approx 1.58 \) MeV \([33]\). However, no such phenomenon were found in the \( \Xi \Xi \) \([34, 35]\) and \( \Sigma \Sigma \) \([36, 37]\) productions. These have been studied with various approaches \([38-46]\). Our framework can provide a naturally unified explanation for the near-threshold enhancements either large or almost none, which relies on the different FSI between these baryon-antibaryon pairs. We can fit the enhanced cross sections near thresholds for productions of \( \Lambda\bar{\Lambda} \) and \( \Lambda_c\bar{\Lambda}_c \) nicely with simple squared-well potentials as shown in Fig. 2.

**Summary.** We have introduced a simple framework to deal with the damped oscillation observed in the electromagnetic form factors of nucleons which is measured in \( e^+e^- \to N\bar{N} \). The FSI effect is important for this phenomenon and leads to a factor made of the \( N\bar{N} \) Jost function with the distorted-wave Born approximation. Using the squared-well potentials, the FSI factors are naturally damped oscillatory and the experimental data can be well explained. It can easily extend to interpret the threshold enhancements on the production cross sections for the \( \Xi\bar{\Xi}, \Lambda\bar{\Lambda}, \text{and } \Lambda_c\bar{\Lambda}_c \), and moreover, same approach can also give apparent enhancements in other channels just with a proper adjustment of FSI parameters.

The oscillation phenomenon could also exist in the productions of hyperon-antihyperon and other pairs, which is an interesting topic in both experiment and theory \([49]\). With the better understanding of the form factors in future, we hope one can furthermore disclose the related structures of nucleons and other baryons.

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