Double $J/\psi$ production as a test of parton correlations in double parton scattering

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Abstract

Using the GS09 model we predict the possible impact of the parton correlation on the $J/\psi$-pair production at the Spin Physics Detector at the Nuclotron-based Ion Collider Facility. The double $J/\psi$ production and the effective cross sections are calculated.

1 Motivation

With the advent of high luminosity accelerators, multi parton scattering (MPS) physics at high-energy hadron colliders like the Tevatron at Fermilab and the Large Hadron Collider (LHC) at CERN, in particular those observing double parton scattering (DPS), has become one of the hottest topics of the modern experimental particle physics [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21]. DPS results are usually interpreted in terms of the so-called “pocket formula” [22]

$$\sigma_{DPS}^{AB} = \frac{m \sigma_{SPS}^{A} \sigma_{SPS}^{B}}{\sigma_{eff}},$$

where the combinatorial factor is $m = 1$ for identical final states $A = B$ and $m = 2$ for $A \neq B$. This formula is derived under the assumption of independent parton scatterings, with

$$\sigma_{eff} = \left[ \int d^2 b \, T^2(b) \right]^{-1},$$

where $T(b)$ is the overlap function that characterizes the transverse area occupied by the interacting partons in the impact parameter space $b$. Such a definition leads to the
assumption that the value of $\sigma_{\text{eff}}$ should be universal, independent of the final states or the phase space. On the other hand, the DØ collaboration provided two different measurement of DPS for $\gamma + 3$ jet events, namely $\langle \sigma_{\text{eff}} \rangle = 16 \pm 0.3 \pm 2.3$ in 2010 [5] and $\sigma_{\text{eff}}^{\text{incl}} = 12.7 \pm 0.2 \pm 1.3$ in 2014 [9]. It is easy to see that not only the central values are different, but the errors bars do not overlap neither. In contrast to many other results, the only difference between these two measurements is the kinematic region (cf. Tab. 1). This definitely contradicts the “universality” assumption for $\sigma_{\text{eff}}$.

In Refs. [23, 24, 25] the importance of the effect of the evolution of the double parton distribution function (dPDF) is emphasized. Most of the phenomenological analysis of a possible impact of the dPDF effect are focused on the LHC energies (cf. Refs. [26, 27] and references therein). Indeed, standard wisdom tells us that DPS should be more preferable at high energies. On the other hand, possible dPDF effect is expected at higher values of the Bjorken-$x$ or with a significant gap between the $x$ values. However, even for double W-boson production the typical value for the Bjorken-$x$ is $\langle x \rangle \sim 0.01$. In contrast to that, preferable conditions can be easily achieved at lower energy experiments, where the DPS contribution could be relatively small but still expected to be far from zero [36, 37].

The Spin Physics Detector at the Nuclotron-based Ion Collider fAcility (NICA) collider (JINR, Dubna) is a universal facility to investigate the spin structure of the proton and deuteron and the other spin-related phenomena with polarized proton and deuteron beams at a collision energy up to 27 GeV and a luminosity up to $10^{32}$ cm$^{-2}$ s$^{-1}$ [29]. It is easy to see that the production threshold of the pair of $J/\psi$ is already more than 20% of the NICA energies, leading to the typical value for the Bjorken-$x$ of $\langle x \rangle > 0.1$. The relatively small cross section can be compensated by the high luminosity.

In this paper we phenomenologically investigate the possible impact of the evolution of the dPDF on the production cross section of double $J/\psi$ events and the measurements of $\sigma_{\text{eff}}$ at NICA energies.
Table 1: The effective cross section measured in $\gamma + 3$ jets process in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV measured by the DØ experiment at the Tevatron.

| Year | Final State          | $p_T^{\text{min}}$ (GeV/$c$) | $\eta(y)$ range               | $\sigma_{\text{eff}}$ (mb) |
|------|----------------------|------------------------------|-----------------------------|-----------------------------|
| 2010 | $\gamma + 3$ jets    | $60 < p_T^\gamma < 80$       | $|y^\gamma| < 1.0$          | $\langle \sigma_{\text{eff}} \rangle = 16 \pm 0.3 \pm 2.3$ |
|      |                      | $p_T^{\text{jet}1} > 25$     | $1.5 < |y^\gamma| < 2.5$    | $\sigma_{\text{eff}}^{\text{incl}} = 12.7 \pm 0.2 \pm 1.3$ |
|      |                      | $p_T^{\text{jet}2,3} > 15$   | $|y^\text{jet}| < 3.0$      | $\sigma_{\text{eff}}^{\text{HF}} = 14.6 \pm 0.6 \pm 3.2$ |
| 2014 | $\gamma + 3$ jets    | $p_T^\gamma > 26$            | $|y^\gamma| < 1.0$          | $\sigma_{\text{eff}}^{\text{incl}} = 12.7 \pm 0.2 \pm 1.3$ |
|      | $\gamma + c/b + 2$ jets | $p_T^{\text{jet}1} > 15$ | $1.5 < |y^\gamma| < 2.5$ | $\sigma_{\text{eff}}^{\text{HF}} = 14.6 \pm 0.6 \pm 3.2$ |
|      |                      | $15 < p_T^{\text{jet}2,3} < 35$ | $|y^\text{jet}| < 2.5$     |                                           |

2 Phenomenological basement

If the final states $A$ and $B$ are produced in a DPS process independently, Eq. (1) can be phenomenologically cast into the form

$$\sigma_{\text{DPS}}^{AB} = \frac{m}{2} \frac{1}{\sigma_{\text{eff}}} \int dx_1 \cdots dx_4 \ f(x_1, Q_A) f(x_2, Q_A) \hat{\sigma}_A(x_1, x_2) \times$$

$$f(x_3, Q_B) f(x_4, Q_B) \hat{\sigma}_B(x_3, x_4) \theta(1 - x_1 - x_3) \theta(1 - x_2 - x_4),$$

where $f(x, Q)$ denotes the parton distribution function, $\hat{\sigma}$ is the cross section at parton level, and $\theta$ is the Heaviside step function.

As we already mentioned, some models predict the correlation between partons of a same hadron [23 [25]. In this case, Eq. (1) should be written in the form

$$\sigma_{\text{DPS}}^{AB} = \frac{m}{2} \frac{1}{\sigma_{\text{eff}}} \int dx_1 \cdots dx_4 \ D(x_1, x_3, Q_A, Q_B) D(x_2, x_4, Q_A, Q_B) \hat{\sigma}_A(x_1, x_2) \hat{\sigma}_B(x_3, x_4),$$

where e.g. the dPDF $D(x_1, x_3, Q_A, Q_B) \neq f(x_1, Q_A) f(x_3, Q_B) \theta(1 - x_1 - x_3)$ is not the product of the two sPDFs. For the possible impact of the dPDFs we can write the ratio

$$R_\Delta(x_1, x_2, x_3, x_4, Q_A, Q_B) = \frac{D(x_1, x_3, Q_A, Q_B) D(x_2, x_4, Q_A, Q_B)}{f(x_1, Q_A) f(x_2, Q_A) f(x_3, Q_B) f(x_4, Q_B)}.$$  

We use odd indexes for the “first” proton and even indexes for the “second” proton.
In single parton scattering, the evolution of the sPDF is described by the DGLAP QCD evolution equations, formerly known as the Altarelli–Parisi equations. As shown in Ref. [28], these renormalisation group equations can be solved by imposing two sum rules, namely the momentum and the number sum rule. While for single parton scattering the sums over momenta fractions and the number of valence partons stay constant, the sum rules for double parton scattering are more involved and, as it turns out, more restrictive.

Corresponding to the (s)DGLAP for single parton scattering, Gaunt and Stirling postulate a dDGLAP which is based on a couple of principles explained in detail in Ref. [23]. As for the sDGLAP, an increasing scale \( t = \ln(Q^2) \) allows for the splitting of partons (by, e.g., the emission of a gluon), described by the splitting functions \( P_{i \rightarrow j}(x) \) which denote a parton \( i \) splitting to \( j \) with a momentum fraction \( x \). For double parton scattering, the splitting functions \( P_{i \rightarrow jk}(x) \) can be understood as the parton \( i \) splitting to a parton \( j \) with momentum fraction \( x \), and a parton \( k \) with momentum fraction \( 1 - x \). This latter is certainly true in leading order (LO) scattering but has to be modified in next-to-leading order (NLO). For this reason, in Ref. [23] Gaunt and Stirling deal with the LO scattering only, postponing the NLO analysis to a future publication. In LO, the sum rules to be postulated have to take into account two very basic correlations, namely

1. having found a quark with a given flavour, the probability is smaller to find a second quark with the same flavour, and

2. having found a parton with momentum fraction \( x \), the probability is smaller to find a second parton with momentum faction \( 1 - x \).

Based on three splitting diagrams for increasing and two diagrams for decreasing population of partons, the dDGLAP equation is justified in detail (cf. Eq. (2.1) and Fig. 3 in Ref. [23]). The new sum rules are postulated, and a procedure is developed to solve the dDGLAP on a grid of momentum fractions and scale, known as the implementation of the GS09 model.
3 \( J/\psi \)-pair production from DPS with the “pocket formula”

As a first step, it is interesting to estimate the DPS effect with the “pocket formula” approximation [1]. In order to provide such an estimate, we are going to use experimentally measured \( J/\psi \) production cross sections at similar energies. Using the CERN proton beam at 400 GeV/c to produce charm particles with incident on different nuclear targets, the NA3 experiment provided data on the production of \( J/\psi \) pairs on a platinum target with the production cross sections of \( 27 \pm 10 \text{ pb per nucleon} \) [30], and the NA50 experiment measured single \( J/\psi \) production \( \text{Br}(J/\psi \rightarrow \mu^+\mu^-) \times \sigma(J/\psi) \) on Be, Al, Cu, Ag, W, and Pb targets in the range between \( 4.717 \pm 0.026 \text{ nb} \) and \( 3.715 \pm 0.016 \text{ nb per nucleon} \) [31].

Combining the mean values of the single \( J/\psi \) cross section for proton-W and proton-Pb collisions, \( \sigma(J/\psi) \approx 12.5 \text{ \( \mu \)b} \), and the effective cross section \( \langle \sigma_{\text{eff}} \rangle = 4.6 \text{ mb} \) for double \( J/\psi \) production (cf. Tab. 2), we can calculate the contribution of DPS. However, \( \sigma(J/\psi) \) as measured by the NA50 collaboration includes \( J/\psi \) from the feed-down effect. To consider this effect, we cast formula (1) into the form

\[
\sigma_{\text{DPS}}(J/\psi J/\psi) = \sigma(J/\psi)^2 \frac{\left(r_{J/\psi}^2 + r_{\psi'}^2 + r_{\chi_c}^2\right) + 2 \cdot (r_{J/\psi}r_{\psi'} + r_{J/\psi}r_{\chi_c} + r_{\psi'}r_{\chi_c})}{2}
\]

(cf. also the discussion in Ref. [32]), where \( r_{J/\psi} = 0.62 \pm 0.04 \), \( r_{\psi'} = 0.08 \pm 0.02 \), and \( r_{\chi_c} = 0.30 \pm 0.08 \) are feed-down fractions [33]. Performing this calculation, we obtain \( \sigma_{\text{DPS}}(J/\psi J/\psi) = 2.6 \text{ pb} \). Utilizing the double \( J/\psi \) production cross section measured by the NA3 experiment, the fraction of DPS events can be estimate to be \( f_{DP} \approx 9.6\% \).
Table 2: The effective cross section values measured in double and triple $J/\psi$ production at DØ, ATLAS, and CMS experiments.

| Experiment | $\sqrt{s}$, TeV | Colliding Mode | $\sigma_{\text{eff}}$, mb |
|------------|-----------------|----------------|--------------------------|
| DØ [10]   | 1.96            | $p\bar{p}$     | $4.8 \pm 0.5\text{(stat)} \pm 2.5\text{(syst)}$ |
| ATLAS [17] | 8               | $pp$           | $6.3 \pm 1.6\text{(stat)} \pm 1.0\text{(syst)}$ |
| CMS [21]  | 13              | $pp$           | $2.7^{+1.4}_{-1.0}\text{(exp)}^{+1.5}_{-1.0}\text{(th)}$ |

4 Possible effects of double PDF within the GS09 model

In order to distinguish between single and double PDF predictions, we use a Pythia 8 Monte-Carlo simulation [34], where the sPDFs are calculated with MSTW2008LO [35] and the dPDFs are calculated in the GS09 model employing the ratio (2), where $R_{\Delta}$ was calculated for every single event.

The effect of double PDF for double $J/\psi$ production cross section at NICA, differential in the normalized angular difference for the polar angle, $\Delta \theta_{\pi} = (\theta(J/\psi_1) - \theta(J/\psi_2))/\pi$, is presented in Fig. [1]. In case of the single PDF prediction, the first bin $\Delta \theta_{\pi} < 0.25$ contains more than 40% of the statistics. In contrast to that, double PDF predicts almost equal statistics for all bins, except for the bin $\Delta \theta_{\pi} > 0.75$, where almost no events are found.

It is also interesting to calculate $\sigma_{\text{eff}}$ in the style of a “pocket formula” analysis. Our calculation gives $\langle R_{\Delta} \rangle \approx 0.37$ that leads us to the effect of the dPDFs, namely the amplification of $\sigma_{\text{eff}} = \langle \sigma_{\text{eff}} \rangle / \langle R_{\Delta} \rangle \approx 12.4\text{ mb}$. This value is much higher than the value previously measured by DØ, ATLAS and CMS at low Bjorken-$x$. Obviously, our result also reduces the DPS fraction to $f_{DP} \approx 3.6\%$. 
Figure 1: Differential cross section distributions for double $J/\psi$ production, as predicted by single PDF (squared points) and double PDF (circular points).

5 Discussion and Summary

Current theoretical and experimental knowledge about parton distribution functions do not provide clear evidence for the parton correlation. To resolve this issue, we propose to measure the double $J/\psi$ production cross section at NICA energies, differential in the normalized angular difference for the polar angle, $d\sigma_{DPS}(J/\psi J/\psi)/d\Delta\theta_\pi$. As it turns out, the significant difference in distributions predicted in sPDF and dPDF does not require a precision fit of data. Therefore, in the analysis we can rely on counting of events in different $\Delta\theta_\pi$ bins only.

In order to distinguish DPS from SPS, we proposed to use the (normalized) angular difference for the azimuthal angle of radiation of the $J/\psi$ pair, given by $\Delta\phi_\pi = (\phi(J/\psi_1) - \phi(J/\psi_2))/\pi$. Having taken into account the fact that $\Delta\phi_\pi$ has a peak near 1 for SPS but a flat shape for DPS, we were able to exclude the region $\Delta\phi_\pi \sim 1$ in order to maximize the DPS/SPS ratio.
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