Towards Parameterized Regular Type Inference Using Set Constraints

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Abstract. We propose a method for inferring parameterized regular types for logic programs as solutions for systems of constraints over sets of finite ground Herbrand terms (set constraint systems). Such parameterized regular types generalize parametric regular types by extending the scope of the parameters in the type definitions so that such parameters can relate the types of different predicates. We propose a number of enhancements to the procedure for solving the constraint systems that improve the precision of the type descriptions inferred. The resulting algorithm, together with a procedure to establish a set constraint system from a logic program, yields a program analysis that infers tighter safe approximations of the success types of the program than previous comparable work, offering a new and useful efficiency vs. precision trade-off. This is supported by experimental results, which show the feasibility of our analysis.

1 Introduction

Type inference of logic programs is the problem of computing, at compile time, a representation of the terms that the predicate arguments will be bound to during execution of the program. This kind of type inference involves not only assigning types to procedure arguments out of a predefined set of type definitions, as in traditional type inference, but also the more complex problem of inferring the type definitions themselves, similarly to what is done in shape analysis. Although most logic programming languages are either untyped or allow mixing typed and untyped code, inferring type information for the entire program is important since it allows the compiler to generate more efficient code and it has well-known advantages in detecting programming errors early. For instance, simple uses of such information include better indexing, specialized unification, and more efficient code generation. More advanced uses include compile-time garbage collection and non-termination detection. There are also other areas in which type information can be useful. For example, during verification and debugging it can provide information to the programmer that is not straightforward to obtain by manual inspection of the program.

In this paper we use the set constraint-based approach [14,15]. We propose an algorithm for solving a set constraint system that relates the set of possible values of the program variables, by transforming it into a system whose solutions provide the type definitions for the variables involved in parameterized regular form. We focus on types
which are conservative approximations of the meaning of predicates, and hence, over-approximations of the success set (in contrast to the approach of inferring well-typings, as in, e.g., [3,21], which may differ from the actual success set of the program). Type inference via set constraint solving was already proposed in [12,15]. However, most existing algorithms [13,14,9] are either too complicated or lacking in precision for some classes of programs. We try to alleviate these problems by, first, generating simple equations; second, using a comparatively straightforward procedure for solving them; and, third, using a non-standard operation during solving that improves precision by “guessing” values. At the same time, we attack a more ambitious objective since our resulting types, that we call parameterized, are more expressive than in previous proposals.

Consider for instance the append/3 program in Fig. 1a, and assume the type descriptors A1, A2, and A3 for each predicate argument. Most state-of-the-art analyses will simply infer that the first argument (type A1) is a list, leaving open the types of the other two arguments. In Fig. 1b we show classical parametric types for append/3. We are unaware of an existing proposal that is able to infer them as an approximation to the success set of the predicate (see Sec. 5). Even if we had an analysis that inferred them, they are still less expressive (or need more elaboration for the same expressiveness) than our proposal, as we will discuss.

The parameteric types in Fig. 1b denote the expected list type for A1 which is parametric on some X. Note that A2 is unbound since it may be instantiated to any term. On the other hand, A3 is an open-ended list of elements of some type Y whose tail is of some type Z. A key observation is that while there is a clear relation between the type of the elements of A1 and A3, and between the type of the tails in A2 and A3, these relations are not captured. In Fig. 1c, we show a desirable, more accurate type for append/3. It denotes that the type of A3 is that of open-ended lists of elements of the same type as A1 with tail of the same type as A2.

The relations between the types of arguments can be captured with parametric types only if type parameters are instantiated. For example, the first typing in Fig. 1d captures the desired relations for append/3. Instead, by using global type parameters, as we propose, the parametric type definitions may exhibit the required exact relations right from the inference of the type definitions alone. Although the absence of typings to express the same relations is a small advantage, a bigger one might be expected with regard to the analysis. By using parametric types, an analysis should not only infer type

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**Fig. 1.** Parametric vs parameterized regular types

| (a) Program: | (b) Parametric regular types: |
|---|---|
| `:- typing append(A1, A2, A3).` | `:- type A1(X) -> [] | [X | A1(X)].` |
| `append([|], L, L).` | `:- type A2(W) -> W.` |
| `append([X|Xs], Ys, [X|Zs]):-` | `:- type A3(Y, Z) -> Z | [Y | A3(Y, Z)].` |

| (c) Parameterized regular types: |
| `:- type A1(X) -> [X | A1].` |
| `:- type A2 -> .` |
| `:- type A3 -> A2 | [X | A3].` |

| (d) Local parameters: |
| `:- typing append(A1(E), A2(T), A3(E, T)).` |
| `:- typing nrev(N1(E), N2(E)),` |
| `append(A1(E), A2([|], A3(E, [|])).` |

| (e) Global parameters: |
| `:- typing nrev(R1, R2).` |
| `:- type R1 -> [] | [X | R1].` |
| `:- type R2 -> [] | [X | R2].` |
definitions but also typings showing the exact values for the type parameters. It is not clear at all how this might be done (note that typings are not the types of calls). Our proposal infers type definitions alone, as usual, and yet are at least as expressive as parametric types with typings.

Consider a program construct where two different predicates share a variable, so that the corresponding arguments have the same type. This happens, for example, with the arguments of `nrev/2` and `append/3` in the classical naive reverse program (see Ex. [1]). This property cannot be captured with (standard) typings for the predicates if parametric type definitions are used. One would need something like the second typing in Fig. [4d]. However, this is not usually a valid typing (it types several predicate atoms at the same time), and it is also not intuitive how it could be inferred. In contrast, the parameterized type definition of Fig. [4e], together with those of (c), easily capture the property, by sharing the type variable global parameter \( x \). Therefore, more precise types (called parameterized regular types [19]) can be produced if the scope of each type variable in a type definition is broader than the definition itself, so that the types of different arguments can be related.

As discussed in detail in Sec. [5] we believe that no previous proposal exists for inferring types for logic programs with the expressive power of parameterized regular types. In addition, our proposal fits naturally in the set constraint-based approach. In this context we also define a number of enhancements to the solving procedure. The result is a simple but powerful type inference analysis. Our preliminary experiments also show that our analysis runs in a reasonable amount of time.

2 Preliminaries

Let \( V \) be a non-terminal which ranges over (set) variables \( \nu' \), and let \( f \) range over functions (constructors) \( \nu' \) of given arity \( n \geq 0 \). Set expressions, set expressions in regular form, and in parameterized form are given by \( E \) in the following grammars:

Expressions: \[ E ::= \emptyset \mid V \mid f(E_1, \ldots, E_n) \mid E_1 \cup E_2 \mid E_1 \cap E_2 \]

Regular form: \[ E ::= \emptyset \mid N \mid N ::= V \mid f(V_1, \ldots, V_n) \mid N_1 \cup N_2 \]

Parameterized form: \[ E ::= \emptyset \mid N \mid N ::= N_1 \cup N_2 \mid R \mid K ::= V \mid f(V_1, \ldots, V_n) \mid V \cap R \]

The meaning of a set expression is a set of (ground, finite) terms, and is given by the following semantic function \( \mu \) under an assignment \( \sigma \) from variables to sets of terms:

\[
\begin{align*}
\mu(\emptyset, \sigma) &= \emptyset \\
\mu(V, \sigma) &= \sigma(V) \\
\mu(E_1 \cup E_2, \sigma) &= \mu(E_1, \sigma) \cup \mu(E_2, \sigma) \\
\mu(E_1 \cap E_2, \sigma) &= \mu(E_1, \sigma) \cap \mu(E_2, \sigma) \\
\mu(f(E_1, \ldots, E_n), \sigma) &= \{ f(t_1, \ldots, t_n) \mid t_i \in \mu(E_i, \sigma) \}
\end{align*}
\]

Let \( E_1 \) and \( E_2 \) be two set expressions, then a set equation (or equation, for short) is of the form \( E_1 = E_2 \). A set equation system is a set of set equations. A solution \( \sigma \) of a system of equations \( S \) is an assignment that maps variables to sets of terms which satisfies: \( \mu(e_1, \sigma) = \mu(e_2, \sigma) \) for all \( (e_1 = e_2) \in S \). We will write \( S \models e_1 = e_2 \) iff every solution of \( S \) is a solution of \( e_1 = e_2 \). A set equation system is in top-level form if in all expressions of the form \( f(x_1, \ldots, x_n) \), all the \( x_i \) are variables. The top-level variables of a set expression are the variables which occur outside the scope of a constructor.
A standard set equation system is one in which all equations are of the form \( V = E \) (i.e., lhs are variables and rhs set expressions) and there are no two equations with the same lhs. Equations where the rhs is also a variable will be called aliases. In a standard set equation system variables which are not in the lhs of any equation are called free variables (since they are not constrained to any particular value). Variables which do appear in lhs are called non-free.

A regular set equation system is one which is standard, all rhs are in regular form, has no top-level variables except for aliases, and also no free variables. A regular set equation system is in direct syntactic correspondence with a set of regular type definitions. Regular types are equivalent to regular term grammars where the type definitions are the grammar rules. In a regular set equation system the set variables act as the type symbols, and each equation of the form \( x = e_1 \cup \ldots \cup e_n \) acts as \( n \) grammar rules of the form \( x ::= e_j, 1 \leq j \leq n \). By generalizing regular set equation systems to allow free variables what we obtain is the possibility of having parameters within the regular type definitions. However, when free variables are allowed intersection has to be allowed too: given that free variables are not constrained to any particular value, intersections cannot be “computed out.”

A leaf-linear set equation system [19] is one which is standard, all rhs are in parameterized form, and all top-level variables are free. Note that leaf-linear set equation systems are the minimal extension of regular equation systems in the above mentioned direction, in the sense that intersections are reduced to a minimal expression: several free variables and only one (if any) constructor expression. More importantly, a leaf-linear set equation system is more expressive than parametric regular types, since parameters have the scope of the whole system, instead of the particular type definition in which they occur, as in parametric type definitions.

3 Type inference

In this section, we present the different components of our analysis method for inferring parameterized regular types. First, a set equation system is derived from the syntax of the program, then, the system is solved, and finally it is projected onto the program variables providing the type definitions for such variables. The resulting equation system is in solved form, i.e., it is leaf-linear. Such a system will be considered a fair representation of the solution to the original set equation system, since it denotes a set of parameterized regular types. When the system is reduced to solved form, the parameters of such a solution are the free variables.

Generating a set equation system for a program. Let \( P \) be a program, \( \Pi_P \) the set of predicate symbols in \( P \), ranked by their arity, and \( P \models_P \) the set of rules defining predicate \( p \) in program \( P \). Our analysis assumes that all rules in \( P \) have been renamed apart so that they do not have variables in common. For each \( p \in \Pi_P \) of arity \( n \) we associate a signature of \( p \) defined as \( \Sigma(p) = p(x_1, \ldots, x_n) \) where \( \{x_1, \ldots, x_n\} \) is an ordered set of \( n \) new variables, one for each argument of \( p \). For atom \( A \), let \( [A]_j \) denote its \( j \)-th argument. For a predicate \( p \in \Pi_P \) with arity \( n \) we define \( C_p \) and the initial set equation system, \( E \), for \( P \) as follows. In order to avoid overloading symbol \( \cup \), to clarify the presentation, we
will use ∪ for the usual set union, while ∪ will stand for the symbol occurring in set expressions.

\[ E = \{ C_p \mid p \in \Pi_P \} \quad C_p = C_{\text{Head}} \cup C_{\text{Body}} \quad (1) \]

\[ C_{\text{Head}} = \{ x_j = \bigcup \{ [H]_j \mid (H):-B \in P[p] \} \mid x_j \in \text{vars}(\Sigma(p)) \} \]

\[ C_{\text{Body}} = \{ y = \bigcap \{ [\Sigma(A)]_i \mid [A]_i = y, A \in B \} \mid (H):-B \in P[p], y \in \text{vars}(B) \} \]

Example 1. Take signatures `append(A1,A2,A3)` and `nrev(N1,N2)` in the following program for naive reverse. The equation system for `nrev/2` is \[ C_{nrev/2} = C_H \cup C_B. \]

\[ \text{nrev}([],[]). \]
\[ \text{nrev}([X|Xs],Ys) :- \text{nrev}(Xs,Zs), \text{append}(Zs,[X],Ys). \]

\[ C_H = \{ N1 = [], Xs = Xs, N2 = [], Ys = Ys \} \]
\[ C_B = \{ Xs = N1, Ys = A3, Zs = N2 \cap A1, W = [X] \cap A2 \} \]

Note that a system \( E \) which results from Eq. 1 is in standard form. Moreover, to put it also in top-level form we only need to repeatedly rewrite every subexpression of every equation of \( E \) of the form \( f(e_1, \ldots, e_j, \ldots, e_n) \) into \( f(e_1, \ldots, y_j, \ldots, e_n) \), whenever \( e_j \) is not a variable, adding to \( E \) equation \( y_j = e_j \), with \( y_j \) a new fresh variable, until no further rewriting is possible. The new equations added to \( E \) are in turn also rewritten in the same process. We call the resulting system \( Eq(P) \). Obviously, \( Eq(P) \) is equivalent to \( E \) in Eq. 1.

**Analysis of the program.** In order to analyze a program and infer its types, we follow the call graph of the program bottom-up, as explained in the following. First, the call graph of the program is built and its strongly connected components analyzed. Nodes in the same component are replaced by a single node, which corresponds to the set of predicates in the original nodes. The (incoming or outgoing) edges in the original nodes are now edges of the new node. The new graph is partitioned into levels. The first level consists of the nodes which do not have outgoing edges. Each successive level consists of nodes which have outgoing edges only to nodes of lower levels. The analysis procedure processes each level in turn, starting from the first level. Predicates in the level being processed can be analyzed one at a time or all at once.

Each graph level is a subprogram of the original program. To analyze a level, equations are set up for its predicates as by Eq. 1 and copies of the solutions already obtained for the predicates in lower levels are added. To do this, the signatures and solutions of the predicates in lower levels are renamed apart. The new signatures replace the old ones when used in building the equations for the subprogram. The new copies of the solutions are added to the set equation system. For a given predicate, there is a different copy for each atom of that predicate which occurs in the subprogram being analyzed.

Example 2. Consider the following (over-complicated) contrived predicate to declare two lists identical, which calls predicate `nrev/2` of Ex. 1 twice.

\[ \text{same}(L1,L2) :- \text{nrev}(L1,L), \text{nrev}(L,L2). \]
Predicate \textit{nrev/2}, which is in a lower scc, would have been analyzed first. Taking signatures \textit{same}(S1, S2) and \textit{nrev}(N1, N2), the solution for \textit{nrev/2} would be:

\[
\{ N1 = [ ] \cup [X|N1], \, N2 = [ ] \cup [X|N2] \}
\]

Since there are two calls to this predicate in the above definition for \textit{same/2}, two copies of the above equations (renaming the above equations for \langle N1, N2 \rangle into \langle N11, N12 \rangle and \langle N21, N22 \rangle) would be added. The initial equation system for \textit{same/2} is thus:

\[
\{ S1 = L1, \, S2 = L2, \, L1 = N11, \, L2 = N22, \, L = N12 \cap N21, \\
N11 = [ ] \cup [X1|N11], \, N21 = [ ] \cup [X2|N21], \\
N12 = [ ] \cup [X1|N12], \, N22 = [ ] \cup [X2|N22] \}
\]

Note that the two arguments of \textit{same/2} have, in principle, list elements of different type (\textit{X1} and \textit{X2}). That they are in fact of the same type will be recovered during the solving of the equations, in particular, the equation for \textit{L}, the rhs of which is an intersection (unification). This will be done by an operation we propose (BIND) which will make an appropriate “guess”, as explained later.

Note that the copies are required because the type variables involved might get new constraints during analysis of the subprogram. Such constraints are valid only for the particular atom related to the given copy. Also, as the example above shows, using a single copy would impose constraints, because of the sharing of the same parameter between equations referring to the same and single copy (as it would have been the case with \textit{X} in the example, had we not copied the solution for \textit{nrev/2}). Such constraints might be, in principle, not true (though in the example it is finally true that both lists have elements of the same type). Thus, copies represent the types of the program atoms occurring at the different program points (i.e., different call patterns).

Note also that equations with intersection in the expression in the rhs will be used to capture unification, as it is the case also in the example. This occurs sometimes by simplification of intersection (procedure SIMP below), but most of the times from operation BIND, that mimics unification. Such equations are particularly useful. This is also true even if the equation is such that its lhs is a variable which does not occur anywhere else in the program (as it is the case of \textit{W} in Ex. 1). Additionally, in these cases, if the rhs finally becomes the empty set, denoting a failure, this needs to be propagated explicitly, since the lhs is a variable not related to the rest of the equations.

The details of the solving procedure are explained below. The procedure is based on the usual method of treating one equation at a time, the lhs of which is a variable, and replacing every (top-level) occurrence of the variable by the expression in the rhs everywhere else. Once the system is set up as explained, it is first normalized and then solved as described in the following.

\textbf{Normal form.} The normal form used is \textit{Disjunctive Normal Form (DNF)}, plus some simplifications based on equality axioms for \textit{\cap}, \textit{\cup}, and \textit{\emptyset}, which transform expressions into parameterized form. Once the system is in top-level form, two auxiliary algorithms are used to rewrite equations to achieve normal form. These algorithms are based on semantic equivalences, and therefore preserve solutions.
The first algorithm, DNF, puts set expressions in a set equation in disjunctive normal form. Note that if all expressions of an equation \( q \) are in top-level form then those of equation \( \text{DNF}(q) \) are in parameterized form, except for nested occurrences of \( \emptyset \) and possibly several occurrences of constructor expressions in conjuncts. This is taken care of in the second algorithm, SIMP.

SIMP simplifies set expressions in an equation system \( E \) by repeatedly rewriting every subexpression based on several equivalences until no further rewriting is possible, as follows:

1. \( e \cap \emptyset \sim \emptyset \) INTER. SIMPLIFICATION
2. \( e \cap e \sim e \) INTER. ABSORPTION
3. \( e \cup \emptyset \sim e \) UNION SIMPLIFICATION
4. \( e \cup e \sim e \) UNION ABSORPTION
5. \( e_1 \cup (e_1 \cap e_2) \sim e_1 \) SUBSUMPTION
6. \( f(e_1, \ldots, e_m) \cap g(d_1, \ldots, d_n) \sim \emptyset \) if \( f \neq g \) or \( n \neq m \) CLASH
7. \( f(e_1, \ldots, e_n) \cap f(d_1, \ldots, d_n) \sim f(y_1, \ldots, y_n) \) if \( \forall j. 1 \leq j \leq n, y_j = e_j \cap d_j \) INTER. DISTRIBUTION
8. \( f(y_1, \ldots, y_n) \sim \emptyset \) if \( \exists i. 1 \leq i \leq n, E \vdash y_i = \emptyset \) EMPTINESS

Note that line 8 makes use of the check \( E \vdash y_i = \emptyset \). For this test a straightforward adaptation of the type emptiness test of [7] to parameterized definitions is used. Note also that the rewriting in line 7 deserves some explanation. If the set equation system \( E \) does not contain an equation \( y_j = e_j \cap d_j \) for some \( j \) then the equation is added to \( E \), with \( y_j \) a new fresh variable. Otherwise, \( y_j \) from the existing equation is used. If equations were not added, it would prevent full normalization of the set expressions. If variables were not reused, but instead new variables added each time, it would prevent the global algorithm from terminating.

**Solving recurrence equations.** Since the set equation system is kept in standard form at all times, the number of different equations that need to be considered is very small. The procedure CASE reduces a given equation into a simpler one. It basically takes care only of recurrences. We call recurrence an equation \( x = e \) where \( x \) occurs top-level in \( e \). There are four cases of recurrences that are dealt with:

1. \( x = x \sim x = \emptyset \)
2. \( x = x \cap e \leftrightarrow x \subseteq e \sim x = \emptyset \)
3. \( x = x \cup e \leftrightarrow e \subseteq x \sim x = e \)
4. \( x = (x \cap e_1) \cup e_2 \leftrightarrow e_2 \subseteq x \subseteq e_1 \sim x = e_2 \)

Note that in all cases we have chosen for variable \( x \) the least solution of all possible ones allowed by the corresponding recurrence. We are thus taking a minimal sufficient solution, in the sense that the resulting set of terms would be the smallest possible one that still approximates the program types. This is possible because we keep equations in standard form, so that their lhs is always a variable and there is only one equation per variable. If the equation turns into a recurrence, then the program contains a recursion which does not produce solutions (e.g., an infinite failure).

**Example 3.** Consider the following program with signatures \( p(P) \), \( q(Q) \), and \( r(R) \):
Algorithm 1 SOLVE

**Input:** a set equation system $E$ in standard form.

**Output:** a set equation system $S$ in solved form.

1: initialize $S$
2: repeat
3: initialize $C$
4: repeat
5: subtract $q$ from $E$
6: $\langle q', C \rangle \leftarrow$ SIMP(DNF($q$),$C$,$S$)
7: $q' \leftarrow$ CASE($q'$,$S$)
8: let $q''$ be of the form $x = e$
9: replace every top-level occurrence of $x$ in $E$ and in $S$ by $e$
10: add $q''$ to $S$
11: until $E$ is empty
12: repeat
13: for all $q \in S$ do
14: $\langle q, C \rangle \leftarrow$ SIMP(DNF($q$),$C$,$S$)
15: if $q$ is of the form $x = e$, $e \neq \emptyset$, and $S \models x = \emptyset$ then
16: replace $q$ by $x = \emptyset$ in $S$
17: end if
18: end for
19: until no equation $q$ is replaced
20: for all $(x = \emptyset) \in S$ do
21: if $x$ occurs in $Eq(P)$ for some clause $c$ then
22: for all variable $y$ in the head of $c$ do
23: replace the equation in $S$ with lhs $y$ by $y = \emptyset$
24: end for
25: end if
26: end for
27: for all $(x = e) \in S$ do
28: replace every top-level occurrence of $x$ in $C$ by $e$
29: end for
30: add $C$ to $E$
31: if $E$ is empty then
32: $E \leftarrow$ BIND($S$)
33: end if
34: until $E$ is empty

$p(X) :- p(X)$, $q(a)$, $r(b)$.
$q(Y) :- q(Y)$, $r(Z) :- q(Z)$, $r(Z)$.

It is easy to see that its initial set equation system will progress towards the following form: \{ $P = P$, $Q = a \cup Q$, $R = b \cup (Q \cap R)$ \}.

**The global algorithm.** The global algorithm for solving a set equation system is given as Algorithm SOLVE. In order to facilitate the presentation, the set equation system is partitioned into three: $E$, $C$, and $S$. $E$ is the initial equation system (generated from the program as in Eq. 1), $S$ is the solved system (i.e., the output of the analysis), and $C$ is
an auxiliary system with the same form as $E$. This forces the use of SIMP in lines 6 and 14 with two input equation systems instead of one. SIMP may add new equations, and these are added to $C$. Also, in SIMP the test in line 8 ($E \vdash y_j = \emptyset$) is carried along in $S$ (i.e., it should be read as $S \vdash y_j = \emptyset$ when called from SOLVE).

Once more, we use the test $S \vdash x = \emptyset$ (line 15 of Algorithm SOLVE). This is related to the other use of this test in SIMP. Note that SIMP is invoked again at line 14 of Algorithm SOLVE. Together, the loop in lines 12-19 of Algorithm SOLVE and line 8 of SIMP perform a complete type emptiness propagation. Because of this, the effect of the rest of cases of SIMP, which propagate symbol $\emptyset$ in subexpressions, is to achieve an implicit check for non-emptiness of intersections. The rationale behind the loop in lines 20-26 is different. It solves a lack of propagation of failure for some programs, due to the form of the initial equation system.

Example 4. In the program below, failure of $p/1$ would not be detected unless step 23 is performed, even if failure of $q/2$ is detected because of type emptiness.

```
p(X) :- q(b,X).
q(a,a).
```

Finally, the BIND procedure invoked at line 32 of Algorithm SOLVE is not needed for obtaining the solved equation system, since $S$ is already in solved form when BIND is called, but it may improve the precision of the analysis.

Example 5 (SOLVE with binding of free variables). We show the analysis of a contrived example which exposes the strength of our approach in propagating types through the type variables that act as parameters. We consider signature $\text{append}(A_1, A_2, A_3)$ already solved with solution: $A_1 = [\ ] \cup [X|A_1]$, $A_3 = A_2 \cup [X|A_3]$; and concentrate on the analysis of:

```
\text{appself}(A,B) :- \text{append}(A,\ [],B).
```

We show the contents of $E$ at the beginning of Algorithm SOLVE and of $S$ at line 32 ($C$ and $E$ are empty). We only show the equations for the program variables of $\text{appself}/2$. The equations missing are those for the solution of $\text{append}/3$—note that $A_2$ is free—and aliases of variables $A$ and $B$ for the arguments of the signature of $\text{appself}/2$:

$$E = \{ A = A_1, B = A_3, W = A_2 \cap [\ ] \}$$

$$S = \{ A = [\ ] \cup [X|A_1], B = A_2 \cup [X|A_3], W = A_2 \cap [\ ] \}$$

Now, BIND can give a more precise value to the only free variable that appears in an intersection, $A_2$ ($A_2 = [\ ]$). We now include the equation for $A_3$, which is relevant:

$$E = \{ A_2 = [\ ] \}$$

$$S = \{ A = [\ ] \cup [X|A_1], B = A_2 \cup [X|A_3], W = A_2 \cap [\ ], A_3 = A_2 \cup [X|A_3] \}$$

After a second iteration of the main loop (lines 2-34) we have $E$ empty again, and:

$$S = \{ A = [\ ] \cup [X|A_1], B = [\ ] \cup [X|A_3], W = [\ ], A_3 = [\ ] \cup [X|A_3] \}$$

Finally, after projection on the variables of interest (which are in fact those of the signature of $\text{appself}/2$, but we use its program variables $A, B$ for clarity instead):

$$S = \{ A = [\ ] \cup [X|A], B = [\ ] \cup [X|B] \}$$
**Binding free variables to values.** Since the equation system is already solved, i.e., it is in solved form, when BIND is applied, free variables can at this moment take any value. BIND will then mimic unification by binding free variables to the minimal values required so that the expressions involving them have a solution. In order to do this, all subexpressions with the form of an intersection (i.e., unification) with a free variable are considered. These are called *conjuncts*.

Since the set expressions are in parameterized form, conjuncts are all of the form $e_1 \cap e_2$, where $e_1$ is a, possibly singleton, conjunction of free variables, and $e_2$ a, possibly non-existing, constructor expression. Let $e_1$ be of the form $x_1 \cap \ldots \cap x_n$, $n \geq 1$, and $e$ be $e_2$ if it exists or a new variable, otherwise. A set of candidates is proposed of the form \[ \{ x_i = e \mid 1 \leq i \leq n \} \] . For an equation $q$, let $\text{Candidates}(q)$ denote the set of sets of candidates for each conjunct in $q$. For defining procedure BIND we need to consider the following relation between equations. Let $q \rightarrow q'$ when equation $q$ is the equation taken by SIMP at a given time, and $q'$ the new equation added at step 7 of Algorithm SIMP from $q$. Let $\rightarrow^*$ denote the transitive closure of $\rightarrow$. BIND constructs the formula below, and synthesizes from it a suitable set of set equations.

\[
\bigwedge_{q \in \text{Eq}(P)} \bigvee q \rightarrow^* q' \bigvee \bigwedge_{c \in \text{Candidates}(q')} e \quad (2)
\]

**Example 6.** Let the following contrived program for alternate/2, which resembles the way the classical program for the towers of Hanoi problem alternates the arguments representing the pegs that hold the disks across recursions:

\begin{verbatim}
alternate(A1,B1).
alternate(A2,B2):- alternate(B2,A2).
\end{verbatim}

It is easy to see that, for a signature alternate($P_1,P_2$), the solution will be the system of the following two equations:

\[ P_1 = A_1 \cup B_1, \quad P_2 = A_1 \cup B_1 \]

The analysis of a program containing an atom of the form alternate($a,b$) in a clause body, will be faced with the equations:

\[ W_1 = (A_1 \cup B_1) \cap a, \quad W_2 = (A_1 \cup B_1) \cap b, \]

which at line 32 of Algorithm SOLVE would be solved as:

\[ W_1 = (A_1 \cap a) \cup (B_1 \cap a), \quad W_2 = (A_1 \cap b) \cup (B_1 \cap b), \]

so that BIND will find two original equations (in $\text{Eq}(P)$; not related by $\rightarrow^*$) with two candidates each ($\{A_1=a\},\{B_1=a\}$ for the equation for $W_1$ and $\{A_1=b\},\{B_1=b\}$) for the equation for $W_2$). Thus, BIND obtains the formula:

\[ (A_1 = a \lor B_1 = a) \land (A_1 = b \lor B_1 = b), \]

which is equivalent to:

\[ (A_1 = a \land B_1 = b) \lor (A_1 = b \land B_1 = a), \]

and results in the following two set equations, which “cover” all solutions of the above formula:

\[ A_1 = a \cup b, \quad B_1 = a \cup b, \]
and (imply expressions for $P_1$ and $P_2$ which) correctly approximate the success types of alternate/2 when called as in alternate(a,b).

In the synthesis of set equations from the formula in Eq. 2 $\lor$ turns into $\cup$ and $\land$ turns into $\cap$. The details of how to do this in a complete synthesis procedure are the subject of current work. Our implementation currently deals with the simpler cases, including those similar to the example above and those in which all candidates involve only one free variable (as in Ex. 5).

| Benchmark | # Desc. | NFTA | PTA |
|-----------|---------|------|-----|
|           | Prec. | Time (s) | Error | Prec. | Time (s) | Error | Bad call type |
| append    | 3     | 1     | 0.000 | n     | 3     | 0.000 | y     | append(A,a,A) |
| blanchet  | 1     | 1     | 0.052 | y     | 1     | 0.032 | y     | attacker(s) |
| dnf       | 2     | 2     | 1.156 | n     | 2     | 1.892 | n     | dnf(X,a(z1, o(z2,z3))) |
| fib       | 2     | 0     | 0.000 | n     | 2     | 0.000 | y     | fib(a,X) |
| grammar   | 1     | 0     | 0.016 | y     | 1     | 0.124 | y     | parse([boxes,fly],S) |
| hanoi     | 5     | 1     | 0.020 | n     | 5     | 0.028 | n     | hanoi(5,a,b,c,[mv(e,f)]) |
| mmatrix   | 3     | 3     | 0.008 | y     | 3     | 0.008 | y     | mmult([[1,2],[1,2],[3,4]],X) |
| mv        | 3     | 1     | 0.020 | n     | 3     | 0.036 | y     | mv([[1,3,1],[b,c,a]],X) |
| pvgabriel | 2     | 0     | 0.184 | n     | 2     | 0.124 | n     | pvgabriel([[1,2],X]) |
| pvqueen   | 2     | 1     | 0.020 | n     | 2     | 0.028 | y     | queens(4,[a,b,c,d]) |
| revapp    | 2     | 2     | 0.004 | y     | 2     | 0.004 | y     | rev([[1,2],[a,b]]) |
| serialize | 2     | 2     | 0.044 | n     | 2     | 0.164 | y     | serialize(’’hello’’,[a,b,c]) |
| zebra     | 7     | 1     | 0.880 | n     | 7     | 0.372 | y     | zebra(E,S,J,U,second,2,W) |
| Total     | 35    | 13    | 1.604 | 2     | 35    | 2.812 | 10    |

Table 1. Experimental results for NFTA and parameterized type analysis (PTA)

4 Preliminary experimental evaluation

In order to study the practicality of our method we have implemented a prototype analyzer in Ciao (http://www.ciaohome.org [4]) and processed a representative set of benchmarks taken mostly from the PLAI (the Ciao program analyzer) and GAIA [22] sets. We chose the Non-deterministic Finite Tree Automaton (NFTA)-based analysis [9] for comparison. We believe that this is a fair comparison since it also over-approximates the success set of a program in a bottom-up fashion (inferring regular types). Its implementation is publicly available (http://saft.ruc.dk/Tattoo/index.php) and it is also written in Ciao. We decided not to compare herein with top-down, widening-based type analyses (e.g. [16][22][23]), since it is not clear how they relate to our method. We leave this comparison as interesting future work.

The results are shown in Table 1. The fourth and seventh columns show analysis times in seconds for both analyses on an Intel Core Duo 1.33GHz CPU, 1GB RAM, and Ubuntu 8.10 Linux OS. The time for reading the program and generating the constraints is omitted because it is always negligible compared to the analysis time. Column # Desc. shows the number of type descriptors (i.e., predicate argument positions)
which will be considered for the accuracy test (all type descriptors are considered for the timing results). For simplicity, we report only on the argument positions belonging to the main predicate. Note that execution of these predicates often reaches all the predicates in the program and thus the accuracy of the types for those positions is often a good summary of precision for the whole program. Columns labeled Prec. show the number of type descriptions inferred by NFTA and our approach that are different from type any. To test precision further, we have added to each benchmark a query that fails, which is shown in the last column (Bad call type). Columns labeled Error show whether this is captured by the analysis or not.

Regarding accuracy, our experiments show that parameterized types allow inferring type descriptors with significant better precision. Our approach inferred type descriptions different from any for every one of the 35 argument positions considered, while NFTA inferred only 13. Moreover, those type descriptors were accurate enough to capture type emptiness (i.e., failure) in 10 over 13 cases, whereas NFTA could only catch two errors. The cases of dnf, hanoi, and pvgabriel deserve special attention, since our method could not capture emptiness for them. For dnf the types inferred are not precise enough to capture that the second argument is not in disjunctive normal form. This is due to the lack of inter-variable dependencies in our set-based approach. However, emptiness is easily captured for other calls such as dnf(X,b). A similar reasoning holds for hanoi since the analysis infers that the types of the pegs are the union of a, b, c, e, and f. Finally, the success of pvgabriel depends on a run-time condition, and thus no static analysis can catch the possible error.

Regarding efficiency, we expected our analysis to be slower than other less expressive methods (like NFTA), since our method is more expressive than previous proposals. Table 1 shows that this is indeed the case, but the differences are reasonable for the selected set of benchmarks, specially considering the improvement in accuracy. Further research is of course needed using larger programs (see Sect. 6), but we find the results clearly encouraging.

5 Related work

Type inference, i.e., inferring type definitions from a program, has received a lot of attention in logic programming. Mishra and Reddy [18,19] propose the inference of ground regular trees that represent types, and compute an upper approximation of the success set of a program. The types inferred are monomorphic. Zobel [24] also proposes a type inference method for a program, where the type of a logic program is defined as a recursive (regular) superset of its logical consequences. However, the inference procedure does not derive truly polymorphic types: the type variables are just names for types that are defined by particular type rules.

In [17] an inference method called type reconstruction is defined, which derives types for the predicates and variables of a program. The types are polymorphic but they are fixed in advance. On the contrary, our inference method constructs type definitions during the analysis. In [8] the idea is that the least set-based model of a logic program can be seen as the exact Herbrand model of an approximate logic program. This approximate program is a regular unary program, which has a specific syntactic form which
limits its computational power. For example, type parameters cannot be expressed. The work of [11,20] also builds regular unary programs, and their type inference derives such programs. However, the types remain monomorphic.

Heintze and Jaffar [12] defined an elegant method for semantic approximation, which was the origin of set-based analysis of logic programs. The semantics computes, using set substitutions, a finite representation of a model of the program that is an approximation of its least model. Unfortunately, the algorithm of [12] was rather complicated and practical aspects were not addressed. Such practical aspects were addressed instead in subsequent work [14,15], simplifying the process. However, our equations are still simpler: for example, we do not make use of projections, i.e., expressions of the form \( f^i(x) \), where \( f \) is a constructor, the meaning of, e.g., \( y = f^1(x) \) being that \( x \supseteq f(y,\ldots) \). Also, and more importantly, these approaches do not take advantage of parameters, and the types obtained are again always monomorphic.

Type inference has also been approached using the technique of abstract interpretation. A fixpoint is computed, in most cases with the aid of a widening operation to limit the growth of the type domain. This technique is also used in [11]. However, none of the analyses proposed to date (e.g., [16,22,23]) is parametric. It is not clear how the approach based on abstract domains with widening compares to the set constraints-based approach. An interesting avenue for future work, however, is to define our analysis as a fixpoint. This is possible in general due to a result by Cousot and Cousot [6] which shows that rewriting systems like ours can be defined alternatively in terms of a fixpoint computation.

Directional types [1,5] are based on viewing a predicate as a procedure that maps a call type to a success type. This captures some dependencies between arguments, but they are restricted to monomorphic types. It would be interesting to build this idea into our approach. We believe that some imprecision found in analyzing recursive calls with input arguments could be alleviated by using equations which captured both calls and successes. The relation of these with directional types might be worth investigating.

Bruynooghe, Gallagher, Humbeeck, and Schrijvers [3,21] develop analyses for type inference which are also based on the set constraints approach. However, their analyses infer well-typings, so that the result is not an approximation of the program success set, as in our case. In consequence, their algorithm is simpler, mainly because they do not need to deal with intersection. In [21] the monomorphic analysis of [3] is extended to the polymorphic case, using types with an expressiveness comparable to our parameterized types.

The set constraints approach to type analysis is also taken in NFTA [9], which is probably the closest to ours. However, the types inferred in this analysis, which are approximations of the program success set, are not parametric and as a result the analysis is less precise than ours, as shown in our experiments. The work of [2] aims at inferring parametric types that approximate the success set. Although independently developed, it includes some of the ideas we propose. However, the authors do not use the set-based approach, and their inference algorithm is rather complex. The latter made the authors abandon that line of work and the approach was not developed, switching instead to well-typings.
6 Concluding remarks and future work

To conclude, using the set constraints approach we have proposed a simple type inference based on the rewriting of equations which, despite its simplicity, allows enhanced expressiveness. This is achieved by using type variables with a global scope as true parameters of the equations. The improved expressiveness allows for better precision, at an additional cost in efficiency, which is, nevertheless, not high.

During our tests, we have identified potential bottlenecks in our analysis which may appear in large programs and which are worth investigating in future research. Some practical issues were already addressed by Heintze in [14], that we have not considered, concentrating instead in this paper on the precision and soundness of our approach. In particular, it is well-known that a naive DNF expansion may make the size of an expression grow exponentially. However, we use simple rules for minimizing the number of expressions (such as computing only intersections that will survive after simplification) which work quite well in practice. Even for larger programs we expect these rules to be effective, based on the experience of [14]. An efficient method for storing the new intersections generated in line 7 in SIMP is also needed for the scalability of the system. We use the same simple technique as [14]. Note that the size of the table is in the worst case $2^N$ where $N$ is the number of original variables in the program. However, the size of the table actually grows almost linearly in our experiments (we omitted these results due to space limitations). Even if the size of the table were to grow faster, we believe that we can mitigate this effect with similar techniques (e.g., use of BDDs) to [9,10], because of the high level of redundancy. We have also observed that the replacement performed in lines 9 and 28 of SOLVE may be expensive for large programs. We think that there are many opportunities for reducing this limitation (e.g., dependency directed updating [14]).

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References

1. Alexander Aiken and T. K. Lakshman. Directional type checking of logic programs. In *In Proceedings of the 1st International Static Analysis Symposium*, pages 43–60. Springer-Verlag, 1994.
2. M. Bruynooghe and J. Gallagher. Inferring Polymorphic Types from Logic Programs. In *International Symposium on Logic-based Program Synthesis and Transformation (LOPST R’04)*, Preproceedings, July 2004.
3. M. Bruynooghe, J. Gallagher, and W. Humbeeck. Inference of Well-typings for Logic Programs with Application to Termination Analysis. In *12th International Static Analysis Symposium (SAS’05)*, volume 3672 of LNCS, pages 35–51. Springer-Verlag, 2005.
4. F. Bueno, D. Cabeza, M. Carro, M. Hermenegildo, P. López-García, and G. Puebla (Eds.). The Ciao System. Ref. Manual (v1.13). Technical report, School of CS (UPM), 2006. Available at http://www.ciaohome.org.
5. W. Charatonik. Directional Type Checking for Logic Programs: Beyond Discriminative Types. In Proc. of ESOP 2000, pages 72–87. LNCS 1782, 2000.
6. P. Cousot and R. Cousot. Formal language, grammar and set-constraint-based program analysis by abstract interpretation. In Proc. ACM Conf. on Functional Programming Languages and Computer Architecture, pages 170–181. ACM Press, New York, NY, 1995.
7. P.W. Dart and J. Zobel. A Regular Type Language for Logic Programs. In Types in Logic Programming, pages 157–187. MIT Press, 1992.
8. T. Frühwirth, E. Shapiro, M.Y. Vardi, and E. Yardeni. Logic programs as types for logic programs. In Proc. LICS’91, pages 300–309, 1991.
9. J. Gallagher and G. Puebla. Abstract Interpretation over Non-Deterministic Finite Tree Automata for Set-Based Analysis of Logic Programs. In Int’l. Symp. on Practical Aspects of Decl. Languages, number 2257 in LNCS, pages 243–261. Springer-Verlag, January 2002.
10. J. P. Gallagher, K. S. Henriksen, and G. Banda. Techniques for scaling up analyses based on pre-interpretations. In ICLP, pages 280–296, 2005.
11. J.P. Gallagher and D.A. de Waal. Fast and precise regular approximations of logic programs. In Pascal Van Hentenryck, editor, Proc. of the 11th International Conference on Logic Programming, pages 599–613. MIT Press, 1994.
12. N. Heintze and J. Jaffar. A finite presentation theorem for approximating logic programs. In Proc. 17th POPL, pages 197–209, 1990.
13. N. Heintze and J. Jaffar. An engine for logic program analysis. In IEEE-LICS, pages 318–328, 1992.
14. Nevin Heintze. Practical aspects of set based analyses. In International Joint Conference and Symposium on Logic Programming, 1992.
15. Nevin Charles Heintze. Set based program analysis. PhD thesis, Pittsburgh, PA, USA, 1992.
16. G. Janssens and M. Bruynooghe. Deriving Descriptions of Possible Values of Program Variables by means of Abstract Interpretation. Journal of Logic Programming, 13(2 and 3):205–258, July 1992.
17. T.K. Lakshman and U.S. Reddy. Typed prolog: A semantic reconstruction of the Mycroft-O’Keefe type system. In International Logic Programming Symposium. MIT Press, 1991.
18. P. Mishra. Towards a theory of types in Prolog. In Int’l. Symp. on Logic Programming, pages 289–298, Silver Spring, MD, February 1984. IEEE Computer Society.
19. P. Mishra and U. Reddy. Declaration-Free Type Checking. In 12th ACM Symposium on Principles of Programming Languages, pages 7–21, January 1985.
20. H. Saglam and J. Gallagher. Approximating logic programs using types and regular descriptions. Technical Report CSTR-94-19, Department of Computer Science, University of Bristol, Bristol BS8 1TR, 1994.
21. T. Schrijvers, M. Bruynooghe, and J. Gallagher. From Monomorphic to Polymorphic Well-typings and Beyond. In LOPSTR’08. Springer-Verlag, 2008.
22. P. Van Hentenryck, A. Cortesi, and B. Le Charlier. Type analysis of prolog using type graphs. In ACM SIGPLAN’94 Conference on Programming Language Design and Implementation, pages 337–348. ACM SIGPLAN Notices Vol. 29 No. 6, 1994.
23. C. Vaucheret and F. Bueno. More Precise yet Efficient Type Inference for Logic Programs. In International Static Analysis Symposium, volume 2477 of Lecture Notes in Computer Science, pages 102–116. Springer-Verlag, September 2002.
24. J. Zobel. Derivation of Polymorphic Types for Prolog Programs. In Int’l. Conf. on Logic Programming, pages 817–838. MIT Press, May 1987.