An iterative FEM reconstructor for the mammography geometry

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Abstract. Electrical Impedance Tomography applied in the mammography geometry has shown promise in the early detection of breast cancer. However, improvements are continually sought to improve the accuracy of static reconstructions. For this reason we present an initial framework outlining the implementation of a 3D FEM reconstructor for the mammography geometry using the complete electrode model.

1. Introduction

Electrical Impedance Tomography (EIT) is a method which allows the admittivity distribution on the interior of a body to be calculated based solely on voltage and current measurements on the surface of the body. Due to Jossinet’s findings that cancerous breast tissue displays significant admittivity differences from non-cancerous breast tissue [1], EIT shows promise as a modality for the early detection of breast cancer. At Rensselaer, we have focused on the implementation of the mammography geometry, which allows for the simultaneous application of EIT and X-Ray mammography. There have been a number of algorithms created for this geometry ([2] [3] [4]) and initial patient testing has shown some promise in correctly classifying cancerous tissue [5].

One of the difficulties in applying EIT to breast cancer detection is that difference images are not available and we must strive for accurate static reconstructions. For this reason we are continuously attempting to improve the accuracy of our algorithms. This paper represents the first attempt at implementing a three-dimensional finite element model (FEM) reconstruction algorithm, which we can apply iteratively to improve the accuracy of our reconstructions.

2. Forward model

In X-ray mammography, the breast is compressed between two plates and an x-ray image of the breast is taken. We attach radiolucent arrays of electrodes [6] to both the top and bottom plates. Thus, the breast can be modeled as a rectangular parallelepiped with zero current density on all the boundaries except for electrodes located on the top and bottom faces, on which a desired current is applied. Please see Figure 1.

We model the electrodes using the complete electrode model (CEM). The CEM takes into account both the shunting effect of the electrodes and the contact impedances between the electrodes and tissue. Our goal is to compute the potential distribution ($u$) for a known...
admittivity distribution \( (\gamma) \) and applied current density. The problem is represented by the following set of equations:

\[
\begin{align*}
\nabla \cdot (\gamma \nabla u) &= 0, \quad x \in \Omega, \quad (a) \\
u + z_l \gamma \frac{\partial u}{\partial \nu} &= U_l, \quad x \in e_l, \quad l = 1, 2, \ldots, L, \quad (b) \\
\int_{e_l} \gamma \frac{\partial u}{\partial \nu} dS &= T_l, \quad x \in e_l, \quad l = 1, 2, \ldots, L, \quad (c) \\
\gamma \frac{\partial u}{\partial \nu} &= 0, \quad x \notin \bigcup_{l=1}^L e_l, \quad (d)
\end{align*}
\]

where \( z_l \) is effective contact impedance between the \( l^{th} \) electrode and tissue, \( T_l \) and \( U_l \) are, respectively, the applied current and measured voltage for the \( l^{th} \) electrode. In addition, the following two conditions for the injected current and measured voltages are needed to ensure existence and uniqueness of the result:

\[
\begin{align*}
\sum_{l=1}^{L} T_l^n &= 0, \\
\sum_{l=1}^{L} U_l^n &= 0. \quad (2)
\end{align*}
\]

We follow the implementation of a 3D FEM for the conductivity equation with CEM given in [7]. It states that for any \((v, V)\), \( v \in H^1(\Omega) \), \( V \in \mathbb{R}^L \),

\[
B_s((u, U), (v, V)) = \sum_{l=1}^{L} T_l V_i, \quad (3)
\]

where \( B_s \) is the bilinear form associated with the complete model, given by

\[
B_s((u, U), (v, V)) = \int_{\Omega} \gamma \nabla u \cdot \nabla v dx + \sum_{l=1}^{L} \frac{1}{z_l} \int_{e_l} (u - U_l)(v - V_l) dS. \quad (4)
\]

We wish to compute a solution to the FEM problem, \((u^h, U^h)\), which approximates the true potential distributions, \((u, U)\). The approximate solution is defined as:

\[
u^h = \sum_{i=1}^{N_n} \alpha_i \varphi_i \quad (5)
\]
and
\[ U^h = \sum_{j=1}^{L-1} \beta_j n_j, \]  
where \( \varphi_i \) is assumed to be a linear basis element and defined
\[ \varphi_j(N_i) = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases} \]
\( N_i \) is assumed to be the \( i \)th node of the FEM mesh, and \( n_1 = [1, -1, 0, \ldots, 0]^T, \) \( n_2 = [1, 0, -1, 0, \ldots, 0]^T \in \mathbb{R}^L \), etc. This guarantees that (2) is not violated.

After a few steps one arrives at a linear system given by
\[ Ab = \left( \begin{array}{cc} B & C \\ C^T & D \end{array} \right) b = \left( \sum_{l=1}^{L} T_l(n_j) \right) = f, \]
where \( b = (\alpha, \beta)^T \in \mathbb{R}^{N_n+L-1} \) and \( A \in \mathbb{R}^{(N_n+L-1) \times (N_n+L-1)} \) where \( 0 = (0, 0, \ldots, 0)^T \in \mathbb{R}^{N_n}. \)
For details on how the contents of \( A \) are calculated please see [7].

3. Reconstruction algorithm

To develop a reconstruction method we want to compute the unknown admittance distribution, \( \gamma_1 \) from the applied current (\( T \)), the measured voltage (\( U \)) and the potential distribution and voltages (\( v, V \)) that result from \( T \) being applied to known admittance distribution, \( \gamma_0 \). We note that \( \nabla \cdot (\gamma_0 \nabla v) = 0 \) and thus the following also equates to zero:
\[ \int_{\Omega} \nabla \cdot (\gamma_1 \nabla u) v - \nabla \cdot (\gamma_0 \nabla v) udP = 0, \]
\[ \int_{\Omega} \nabla \cdot (v(\gamma_1 \nabla u) - u(\gamma_0 \nabla v)) = \int_{\Omega} \nabla \gamma_1 v \cdot \nabla u - \gamma_0 \nabla u \cdot \nabla v dP, \]
\[ \int_{S} v \gamma_1 \frac{\partial u}{\partial \nu} - u \gamma_0 \frac{\partial v}{\partial \nu} = \int_{\Omega} \delta \gamma \nabla u \cdot \nabla v dP. \]

where \( \delta \gamma = \gamma_1 - \gamma_0 \). Note that \( v \) is assumed to have similar boundary conditions to that of \( u \).
Next, assume that \( \gamma_1 \) is close to \( \gamma_0 \) and that \( u(\gamma) \approx u^0(\gamma_0) + O(\delta \gamma) \). Then from (9) we get
\[ \int_{S} v \gamma_1 \frac{\partial u}{\partial \nu} - u \gamma_0 \frac{\partial v}{\partial \nu} = \int_{\Omega} \delta \gamma \nabla u \cdot \nabla v dP + O(\delta \gamma^2) \]

We ignore the term of order \( \delta \gamma^2 \) and denote our data \( D_{i,j} \) as
\[ D_{i,j} = \int_{S} v_j \gamma_1 \frac{\partial u_i}{\partial \nu} - u_i \gamma_0 \frac{\partial v_j}{\partial \nu}. \]

We can now use (1) to plug in known values as follows:
\[ D_{i,j} = \sum_{\ell=1}^{L} \int_{e_\ell} \left( V_j^\ell - z \gamma_0 \frac{\partial v_j}{\partial \nu} \right) \gamma_1 \frac{\partial u_i}{\partial \nu} - \left( U_j^\ell - z \gamma_1 \frac{\partial u_i}{\partial \nu} \right) \gamma_0 \frac{\partial v_j}{\partial \nu} dS \]

The terms containing the surface impedance, \( z \), cancel out and we are left with:
\[ D_{i,j} = \sum_{\ell=1}^{L} V_j^\ell \int_{e_\ell} \gamma_1 \frac{\partial u_i}{\partial \nu} dS - U_j^\ell \int_{e_\ell} \gamma_0 \frac{\partial v_j}{\partial \nu} dS \]
\[ = \sum_{\ell=1}^{L} V_j^\ell T_i^\ell - U_j^\ell T_i^\ell \]

...
Our Jacobian, $J_{i,j,k}$ is given by

$$\delta \gamma_k J_{i,j,k} = \delta \gamma_k \int_{\Omega_k} \delta \gamma_k \nabla u^0_i \cdot \nabla v_j dP,$$

where $B_k$ represents the integral over the $k^{th}$ element. We assume here that we are working with a FEM implementation, therefore we assume that

$$u^0_i = \sum_{r=1}^N \alpha_{i,r} \phi_r(P),$$

$$v_j = \sum_{s=1}^N \alpha_{j,s} \phi_s(P),$$

Then we can simplify $J$ to be

$$J_{i,j,k} = \int_{\Omega_k} \delta \gamma_k \nabla u^0_i \cdot \nabla v_j dP,$$

$$= \sum_{r,s=1}^N \delta \gamma_k \alpha_{i,r} \alpha_{j,s} \int_{\Omega_k} \nabla \phi_r \cdot \nabla \phi_s dP,$$

$$= \delta \gamma_k V^T F^{(r)} V,$$

where we have defined matrices $V$ of size $N \times K$, where $K$ are the number of voltage/current patterns and $F^{(k)}$ of size $N \times N$ to be the following

$$V_{i,r} = \alpha_{i,r},$$

and

$$F^{(k)}_{r,s} = \int_{\Omega_k} \nabla \phi_r \cdot \nabla \phi_s dP.$$

We can then regularize to compute $\delta \gamma$ and thus $\gamma_1$. We can then update $D$ and $J$ using $\gamma_1$ and iterate the entire process.

4. Conclusions

This paper has presented a first formulation of an iterative reconstruction algorithm for the mammography geometry using FEM and the CEM. Implementation and results are in progress and will be available by the time of the meeting.

References

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