DETECTING THE ABERRATION OF THE COSMIC MICROWAVE BACKGROUND

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1. INTRODUCTION

The motion of the solar system barycenter with respect to the cosmic microwave background (CMB) induces a very large apparent dipole component into the CMB brightness map at the 3 mK level. In this Letter, we discuss another kinematic effect of our motion through the CMB: the small shift in apparent angular positions due to the aberration of light. The aberration angles are only of order $\beta \approx 0.001$, but this leads to a potentially measurable compression (expansion) of the spatial scale in the hemisphere toward (away from) our motion through the CMB. In turn, this will shift the peaks in the acoustic power spectrum of the CMB by a factor of order $1 \pm \beta$. For current CMB missions, and even those in the foreseeable future, this effect is small but should be taken into account. In principle, if the acoustic peak locations were not limited by sampling noise (i.e., the cosmic variance), this effect could be used to determine the cosmic contribution to the dipole term.

Subject heading: cosmic microwave background

2. THE ABERRATION EFFECT ON THE CMB

The geometric properties of celestial aberration were extensively studied by Calvão et al. 2005. Here we briefly review the geometry relevant for our purposes. If we use spherical coordinates and define $\theta$ as the polar angle with respect to the direction of motion, and $\phi$ as the azimuthal angle, then the transformation of angles from the CMB to the barycenter frame is

$$\sin \theta = \frac{\sin \theta'}{\gamma(1 - \beta \cos \theta')}, \quad \phi = \phi',$$  \hspace{1cm} (1)

where the unprimed and primed frames are the CMB and barycenter frames, respectively. $\beta$ is the velocity of the barycenter with respect to the CMB ($\approx 0.001$), $\gamma$ is the usual Lorentz factor, and $\theta = 0$ corresponds to the “forward” direction. For small $\beta$, the $\theta$ transformations can be expanded in a Taylor series to lowest order in $\beta$ as

$$\sin \theta = \sin \theta'(1 + \beta \cos \theta').$$  \hspace{1cm} (3)
or compression of the scales in the forward and backward-looking regions where the angular scale stretching factors are approximately constant, e.g., for $\theta' < 45^\circ$ and $\theta' > 135^\circ$, then the $l$-values of the spherical harmonics at the acoustic peaks should shift (in opposite directions) for the segments of the celestial sphere located in the forward and backward directions. The shift in $l$-values is expected to be $\delta l = \pm \beta l$, where $\beta$ represents the average value of $\beta \left| \cos \theta' \right|$ over the regions of interest ($\beta \approx 0.001$). Thus, detecting this effect requires an accuracy in determining the centroids of the peaks in the CMB harmonic structure to about one part in 1000.

The uncertainty in measuring the amplitude $C_l$ due to the finite number of modes on the celestial sphere is given by

$$\frac{\delta C_l}{C_l} = \sqrt{\frac{2}{(2l + 1)f_{\text{sky}}}} \approx \frac{1}{\sqrt{n_{\text{sky}}}}. \quad (7)$$

(Scott et al. 1994), where $f_{\text{sky}}$ is the fraction of the celestial sphere included in the analysis and where the experimental noise contribution is assumed to be negligible compared to the cosmic variance. In the $n$th harmonic peak, centered at $l = l_n$, there are $\sim l_n/n$ values of $C_l$, all of which can be determined with roughly comparable fractional accuracy—given by equation (7). This implies that the peak’s centroid can be determined in an ideal measurement with an uncertainty of approximately

$$\delta l \approx \frac{1}{n \sqrt{n_{\text{sky}}/l_n}} \approx \frac{1}{\sqrt{n_{\text{sky}}}}. \quad (8)$$

For observational hemispherical caps in the forward and backward directions of $\theta' \sim 45^\circ$, the uncertainty in $l_n$ is about $(1/5f_{\text{sky}})^{1/2} \approx 1$ for the fifth harmonic. And, since the expected shift in $l$ due to the aberration effect is of order $\delta l = \beta l$, this implies that by $l \sim 1000$ we may expect to detect a significant shift in $l_n$ between the forward and backward hemispheres. If we utilize all the peaks up to the fifth harmonic, then we gain a small additional factor in signal-to-noise ratio. A more formal error estimate is discussed below.

Figure 1 illustrates how the aberration effect would quantitatively affect the CMB power spectrum peaks in the forward versus the backward hemispheres. What is shown is a plot of the quantity

$$x_{\text{aberr}} = \ln \left[ \frac{C_l/l(l + 1)}{C_l/l(l + 1)} \right] \quad (9)$$

which is simply a convenient way of visualizing the effects of the aberration shifts. A $\Lambda$CDM model was assumed in Figure 1, and was obtained through the package of CAMBFAST (Doran 2005), but the generic nature of the derivative of the angular power spectrum in the presence of adiabatic fluctuations would be very similar for other models. We have carried out a formal statistical analysis of the uncertainties in determining $\beta$ using the function given by equation (9) (see Fig. 1) and the assumed uncertainties in the individual values of $C_l$, given by equation (7). We assumed that $C_l^f$ and $C_l^b$ were both determined over $45^\circ$ cones on the celestial sphere in the forward and backward directions, respectively. The result is that the formal $1 \sigma$
uncertainty in $\tilde{\beta}$ is $3 \times 10^{-4}$, and thus the aberration effect should be detectable at a confidence level above 99.9%. A direct estimation of the significance of the ratio in Figure 1 gives a 2 $\sigma$ detection out to $l = 1000$ and a 3.3 $\sigma$ detection including all multipoles out to $l = 1500$. This is a slight overestimate as we have only included noise due to cosmic variance, but an analysis over two full opposing hemispheres would provide a measurement at slightly higher significance.

Challinor & van Leeuwen (2002) extensively analyzed the effects of aberration on individual harmonics in the CMB power spectrum. However, they considered only the net distortion of the CMB power spectrum averaged over the entire celestial sphere. They showed that aberration results in a narrow effective blurring kernel (in $l$-space) and has little effect on the overall measured CMB power spectrum or cosmological inferences drawn from all-sky maps. In contrast, we have examined how the aberration can be measured via differential measurements of forward and backward hemispheres.

There are two competing effects in trying to choose an optimum fractional hemispherical cap size in the forward and backward directions in order to search for the kinematic aberration effect. In the first, the larger the solid angle of the cap size (as characterized by $\theta_{\text{max}}$) the greater is the available backward directions in order to search for the kinematic aberration. For instance, the greater is the available backward directions in order to search for the kinematic aberration, the larger is the available backward directions in order to search for the kinematic aberration. For instance, the greater is the available backward directions in order to search for the kinematic aberration.

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4. SUMMARY AND CONCLUSIONS

We have shown that the aberration of the detected CMB radiation, due to the motion of the barycenter relative to the CMB, can shift the centroids of the acoustic peaks in the forward versus the backward directions by a measurable amount. This effect may already be marginally detectable with the WMAP data and should be detectable with the data to be acquired with the Planck mission (cf. Challinor & van Leeuwen 2002). To put the magnitude of this aberration effect into perspective, it is anticipated that the Planck CMB maps will be constructed with ~1° angular bins (Sandri et al. 2004). Locations in CMB features will be shifted by 2′, 3′, 4′, and 6′ at $\theta = 30°$, 45°, and 90°, respectively (see also Bottani et al. 1992; Challinor & van Leeuwen 2002; Calvão et al. 2005). In addition, other statistical measurements of the CMB could be applied differentially to forward and backward hemispheres to measure the anisotropy induced by the aberration. For instance, the topological measures of the CMB (e.g., Colley & Gott 2003) could be applied to count 1 $\sigma$ hot and cold spots in the forward and backward hemispheres.

If there is, in fact, a significant measurable difference between the locations of the peaks in the CMB power spectrum for the forward and backward directions, then this will serve to demonstrate the expected kinematic effect due to motion of the barycenter. At present, it does not appear possible to measure this shift with better than about one part in 4000 accuracy due to the cosmic variance. It may be worth noting, nonetheless, that if it were somehow possible to achieve a factor of ~100 better in accuracy, this would be sufficient to allow the kinematic contribution to the dipole term to be determined independently with ~1% accuracy. This, in turn, would enable a determination of the cosmic contribution to the dipole term. A complete treatment designed to measure the cosmic dipole would also need to account for contributions from gravitational lensing (e.g., Lewis & Challinor) as well as proper modeling of the aberration of CMB foregrounds. In the case of the foregrounds, the extragalactic populations (outside the local supercluster) will add to the aberration signal of the CMB, while the remaining local foregrounds will contribute differential aberrations that arise from a different net peculiar velocity. In future work, one could perform realistic simulations of the sky, as it will be observed by Planck, with known populations of foregrounds and distortions from weak lensing to test the differential aberration measurement directly.

Also, we would like to emphasize that a detection and measurement of the aberration of the CMB is complementary to other effects that arise due to the motion of the solar system. The intensity quadrupole, which has a distinct frequency dependence (Kamionkowski & Knox 2003; Bottani et al. 1992), represents an independent observable to constrain the contribution of the kinematic temperature dipole. Non-CMB measurements of the motion of the solar system barycenter, including counts of gamma-ray bursts (Maoz 1994), radio galaxies (Ellis & Baldwin 1984), and galaxy clusters (Chluba et al. 2005), also hold the potential to independently constrain the contribution of the kinematic dipole in the CMB temperature power spectrum.

Finally, we remark that in the same spirit of searching for direction-dependent aberration effects, it might be generally worthwhile to measure the CMB power spectrum over a number (~10) of independent regions of the sky and intercompare the results. The isotropy of the temperature power spectrum has been investigated (Eriksen et al. 2004; Hansen et al. 2004), and regions of excess asymmetry have been reported. The next generation of isotropic tests should account for the aberration of the CMB to avoid systematic errors over the sky.

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