A Review on Fractal Analysis and its Applications in Structural Engineering

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Abstract. Fractals refer to rough or fragmented geometric shape in which each sub-section is a reduced-size copy of the whole. Its potential to describe the complex and irregular phenomena leads to wider applicability, and its enormous unexplored potential in the field of structural engineering is to be attended by the research community. This paper first presents a brief theoretical background of the most popular algorithms, such as the box-counting method, Multifractal Detrended Fluctuation Analysis (MFDFA), and Multifractal Detrended Cross-Correlation Analysis (MFDCCA). This paper further presents a comprehensive review of the potential of fractals and multifractals in four specific domains, such as crack identification, pore structure analysis and chloride resistance, and structural damage detection. The review of literature further reveals that 2 dimensional and multi-dimensional extension of fractal theory can be effectively coupled with image analysis for micro-level examination of pore structures, which opens the scope for abundant applications in research in the field of structural materials. The applications of MFDFA and cross-correlation analysis structural health monitoring are very recent research contributions in the field, and they are in the infant stage of applications. The review of research works performed in this study shows that the complete potential of fractals in the structural engineering field is not yet explored by the researchers.

1. Introduction

A fractal is a rough or fragmented geometric shape that can be split into parts, each of which is a reduced-size copy of the whole [1]. Fractals are distinguished over traditional Euclidean geometry as they define irregular and complex phenomena and paved a new research path on fractal applications [2]. Most of the complex time series exhibits fractal behaviour including medical and physiological time series [3], geophysics [2], financial time series [4], and many more. If the scaling behaviour is described by a single exponent, it is referred to as mono-fractals, and if scaling behaviour is described by multiple scaling exponents in different parts of the series, it is referred to as multi-fractals. Fractal objects are commonly characterised by fractal dimension and express the degree of complexity involved. The various approaches for computing fractal dimensions include the box-counting method, power spectrum method, and fracture island method [3]. Among the various methods for estimation of fractal dimension estimation, the box-counting method is the most popular one [5]. This method outperforms the other methods due to its simplicity in the calculation. The scaling behaviour of many time series records are complicated and do not exhibit monofractal behaviour. Thus, different scale exponents are required to explain different parts of the series. Hence, the multifractal analysis should be opted in such situations. Multifractal detrended fluctuation analysis (MFDFA) proposed by Kantelhardt et al. [6] is the most popular algorithm to describe it. Even though fractality and multifractality were analysed successfully...
in many scientific disciplines, the applications of the fractal theory are not much explored in the field of structural engineering except few studies in the domains of pore structure analysis, crack identification and structural health monitoring.

The main objective of this paper is to give an insight into the present researches and applications of fractal and multifractal analysis in structural engineering, thereby, provide assistance to future researches in this domain. First, the algorithms of most commonly used methods of fractal/multifractal analysis are presented briefly, then the detailed review on applications of fractal/multifractal analysis in structural engineering is presented by categorising into four parts. The conclusions drawn from the review are presented in the final section.

2. Algorithms of popular methods for fractal and multifractal analysis

2.1. Box-counting method

In this method, a grid of squared boxes is superimposed on the object of interest, and the number of boxes that cover the object is counted. The procedure is repeated for different grid sizes. The fractal dimension is obtained from the following relation:

\[ N(r) \propto \left( \frac{1}{r} \right)^D \]  

(1)

where \( D \) represents the fractal dimension. The slope of the log-log plot \( \left( \frac{1}{r} \right) \) against log \( N(r) \) gives the fractal dimension \( D \). The concept of the box-counting process for the determination of crack pattern is illustrated in figure 1.

![Figure 1. Box counting method applied for the analysis of crack pattern [20]](image)

2.2. Multifractal detrended fluctuation analysis

The multifractal detrended fluctuation analysis (MF-DFA) proposed by Kantelhardt et al. [6] is one of the most popular tools for the detection of the scaling behaviours and multifractal properties of non-linear and nonstationary time series. The procedure for MF-DFA is described below:

1. Consider a time series \( X \), having length \( N \). The time series is converted into a random walk by subtracting the mean value of the time series.

\[ Y(i) = \sum_{k=1}^{i} [x_k - \bar{x}] \]  

(2)

where \( i = 1, 2, ..., N \)

2. The profile \( Y(i) \) is divided into \( N_s \) non-overlapping segments, where \( N_s = \text{int}(N/s) \) and \( s \) stands for scale. \( N \) may not always be a multiple of \( s \), so the procedure is done for a total of \( 2N_s \) segments.

3. In order to find the local trend in each segment, least-square fitting is done for each segment for variance
\[ \nu = 1, 2, \ldots, N, \]
\[ F^2(s, \nu) = \frac{1}{s} \sum_{i=1}^{s} \left\{ y[N-(\nu-N_s)s+i]-y_{s}(i) \right\}^2 \]

for \( \nu = N_s + 1, \ldots, 2N_s \). Here \( y_{s}(i) \) is the fitting polynomial in segment \( \nu \).

4. Fluctuation function is found out by averaging over all segments
\[ F_{q}(s) = \left( \frac{1}{2N_{s}} \sum_{v=1}^{2N_{s}} \left[ F^2(s, \nu) \right]^{1/2} \right)^{1/2} \]

Fluctuation function depends on scale \( s \) and \( q \).

5. The scaling behaviour of the fluctuation functions is obtained from the slope of log-log plots of fluctuation functions and scale \( s \) and is referred to as Hurst exponent \( h(q) \).

Hurst exponent is an indication of long-range memory of time series. When \( h(q) = 0.5 \), the series is uncorrelated. \( h(q) < 0.5 \) indicates the series is negatively correlated and \( h(q) > 0.5 \), the series is positively correlated. The scale to be chosen is of critical importance while computing this method. The minimum scale should at least greater than \((k+2)\), where \( k \) is the order of the polynomial chosen, and the maximum scale \( (s_{\text{max}}) \) should be chosen such that a sufficient number of segments is produced \((s_{\text{max}} \leq L/4, \) where \( L \) is the data length). Some guidelines, along with the selection of MF DFA control parameters, are available at [7].

2.3. Multifractal detrended cross-correlation analysis (MF-DCCA)

In order to quantify the presence of correlation between different signals, various statistical measures were developed, out of which the most popular and simplest one is the Pearson correlation coefficient. For the real field data (which often possess non-linear and non-stationarity characteristics) spurious correlation may be reported as they are always characterised by trend. Because of this, an estimation measure supported by the detrending operation is advisable. Podobnik and Stanley [9] proposed the detrended cross-correlation analysis (DCCA), to investigate power-law cross-correlations between two candidate non-stationarity time series. Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) can be used to investigate the long-term cross-correlations between two nonstationary time series.

The procedure for MF-DCCA is described below:

1. For two time series \( x_i \) and \( y_i \) having equal length \( N \), The time series are converted to random walk by subtracting their respective mean values.
\[ x(k) = \sum_{i=1}^{k} [x_i - \bar{x}] \quad \text{and} \quad y(k) = \sum_{i=1}^{k} [y_i - \bar{y}] \]

where, \( k = 1, 2, 3, \ldots, N \)

2. Both the profiles are divided into \( N_s \) non-overlapping segments, where \( N_s = \text{int}(N/s) \) and \( s \) stands for scale. \( N \) may not always be a multiple of \( s \), so the procedure is done for a total of \( 2N_s \) segments.

3. In order to find the local trend for both series in each segment, least-square fitting is done for each segment and determine the covariance:
\[ f_{DCCA}^2(s, i) = \left\{ \frac{1}{s} \sum_{k=i}^{i+n} [x_k - \bar{x}] (y_k - \bar{y}) \right\} \]

Repeat the procedure for various scales \( s \) and \( q \) values.

4. Detrended covariance is obtained by summing over all overlapping all segments of length \( N \)
\[ F_{q}(s) = \left\{ \frac{1}{2N_s} \sum_{i=1}^{2N_s} f_{DCCA}^2(s, i) \right\}^{1/2} \]
The scaling behaviour of the detrended covariance is obtained from the slope of log-log plots of $F_{xy}(s)$ and scale $s$ and is referred to as cross-correlation exponent $\lambda(q)$.

3. Applications of fractal and multifractal analysis in structural engineering

In this section, the review of applications of fractal/multifractal theory in four major fields namely, pore structure analysis, crack detection, damage identification, and chloride analysis, are presented.

3.1. Pore Structure Analysis

The fractal behaviour is well exhibited by the pore structures of many natural and artificial materials. Three types of fractality are observed in pore structures, namely, pore surface fractal, solid surface fractal [10], and pore surface fractal [11]. Surface fractal characteristics were used to find out ionic diffusivity [10]. Gao et al. [12] proposed a generalised fractal approach that can be used to examine all the three types of fractality present in pore structures. Intermingled Fractal Units (IFU) model has been adopted to analyse the microstructure by many researchers as they could reproduce fractal as well as non-fractal behaviour of pore structures. The virtual representation of pore-size distribution in the IFU model increases the efficiency in predicting sorption characteristics [11].

Bednarska et al. [13] analysed the effect of freeze-thaw cycles on pore structure based on fractal dimension. The experiment was conducted on a spliced cement mortar specimen inside PVC tubes. The surface of the specimen is polished and then painted with a black stain. The image of the painted surface was analysed, and the fractal dimension was calculated by the box-counting method. As the number of freeze-thaw cycles increased, there was a rise in the fractal dimension. The permeability at each stage was executed from corresponding to the fractal dimension value. On determining the surface fractality of pore structure in high strength ground granulated blast-furnace slag (GGBS), increased fractal dimension showed an increase in pore volume [14]. Another study conducted by Yang et al. [15] on GGBS cement pastes proved that the fractal dimension had been improved on blending the cement paste with GGBS. By evaluating the fractal dimension of individual pores, the accuracy of pore structure analysis can be increased [16].

3.2. Crack Identification

Fracture energy is a representation of crack growth. Won et al. [17] related fractal dimension with fracture energy, and they exhibited a proportional relationship. The size and location of cracks in beams were identified using fractal dimension crack detector (FDCD) and proved its potential in practical applications [18]. The beneficial performance of FDCD was extended from 1-D to 2-D structures [19]. The main attraction of the method is that it gives insight into all crack characteristics and requires only a small portion of the signal for analysis. The use of FDCD can also be extended to the crack detection of composite plates. Multifractal analysis of 2D images taken in the visible spectrum provided a quantitative measure of damage in reinforced concrete shear walls (RCSW), by retrieving surface defect patterns [20]. Flowchart showing the procedure of surface crack detection using multifractal analysis is provided in figure 2.

![Flowchart showing the procedure of surface crack detection using multifractal analysis](image)

Figure 2. The procedure of surface crack detection using multifractal analysis [20].
For this experiment, four synthetic crack patterns were considered and referred to as CASE0, CASE1, CASE2, and CASE3, as shown in figure 3. CASE0 is the reference pattern. Singularity spectrum curves were plotted for the determination of crack, as shown in figure 4. CASE1 curves almost coincided with a positive range of $q$ with CASE0 curves since the overall shape was maintained. For CASE2, the overall shape was altered by flipping the crack patterns. Hence, they show deviation from the base curve for $q<0$. A clear distinction of deviation was exhibited by CASE3 curves on both sides due to crack pattern growth.

![Figure 3. Synthetic crack patterns (a) CASE0 (b) CASE1 (c) CASE2 (d) CASE3.](image)

![Figure 4. Multifractal description of images of crack patterns (a) singularity spectrum; (b) generalised fractal dimension plot.](image)

Furthermore, the fractal and multifractal analysis were employed to detect surface cracking patterns of prestressed concrete girders [5]. Developing an automatic alarming system for crack identification makes the solution using fractal analysis more realistic.

### 3.3. Damage Detection

Fractal dimension based damage identification is effective even for strong noise-contaminated signals [21]. Moustafa and Salamone [22] proposed a Relative Fractal dimension (RFD) approach to determine the fractality of Lamb waves. They validated the same by recognising surface damage in isotropic structures. The dimension change in displacement data can be used to monitor the state of buildings [23]. Lin et al. [24] proposed a structural health monitoring system based on the multifractal analysis. The flowchart showing the procedure of Structural health monitoring using multifractal analysis is given in figure 5. A four-storey steel structure was simulated in SAP2000 software. A mass of 120 kg was assigned to each storey. The damage was simulated by the removal of bracings in respective floors. A 1MW noise signal was used to excite the four-storey steel structure. The time history response from each
storey was recorded and then, analysed for damage condition and damage location using DFA and DCCA, respectively. Compared to reference curve, figure 6(a), figure 6(b) showed the third and fourth curves have a large area in between from their previous curves. This indicates the damage in the third and fourth floors.

Fractal analysis integrated with Bayesian learning was proposed by Huang et al. [25] for damage detection. They proved the real-life potential of the proposed method by successfully detecting the damage in the Yonghe bridge. There is ample scope for utilising the potential of advanced variants of cross-correlation studies such as Multifractal Cross-Correlation analysis (MFCCA) [4] and conjunctive use of statistical evidence theory and multifractal analysis [26] for damage identification of structural systems.

As there exist enormous opportunities for multifractal analysis in developing a well defined structural monitoring, our experiments were focussed in this area. A seven-storey structure was developed in SAP2000 finite element software. The damage on each floor was simulated by the removal of bracings in each floor. The structure was excited using a 1 MW noise signal, and the corresponding displacement response of each floor was recorded. A total of 16 damage cases were considered. DFA was employed to assess the damage condition. From figure 7, it is clear that the Hurst exponent value increases as the severity of damage increases. The Hurst value due to damage in the first floor is higher compared to the damage caused by the second, third, fourth, fifth, sixth, and seventh floors. This indicates the greater potential of the first floor in contributing to global damage.
3.4. Chloride Resistance

The fractal dimension theory was utilised by Xue et al. [27] to establish the impact of fine aggregate on chloride resistance of concrete and proved its proportional relationship. The equations relating the void volume, chloride resistance, and volume fractal dimension in the case of mortar and concrete had been established by using fractal theory [28].

4. Conclusions

This study performed a review of research works on applications of fractal and multifractal analysis in the field of structural engineering. Even though the characterisation of signals from seismic ground motion and dynamical systems using fractals were explored by the researchers, the complete potential of the fractal and multifractal theory in this field is not fully explored yet. The detection of fractal properties like fractal dimension has enormous potential in micro-level examinations like crack detection, chloride resistance, and pore-structure analysis. The multifractal analysis can be effectively used for signal characterisation, and the Detrended Cross-Correlation Analysis (DCCA) and its multifractal extension enhances its applicability in structural health monitoring. The 2D variant of MFDFA, coupled with image analysis, is believed to be a potential tool in micro-level investigations in structural engineering.

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