A Phenomenological Interpretation of Atmospheric
and Solar Neutrino Oscillations

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Abstract

We give a phenomenological interpretation of atmospheric and solar neutrino oscillations by considering two scenarios for the neutrino mass spectrum: (a) $m_1 \ll m_2 \ll m_3$ and (b) $m_1 \approx m_2 \approx m_3$. A new parametrization of the flavor mixing matrix, which can naturally reflect the hierarchy of lepton masses in scenario (a) and the approximate decoupling of solar and atmospheric neutrino oscillations, is highlighted. For scenario (b) an ansatz starting with flavor democracy for charged leptons and mass degeneracy for neutrinos is proposed, and two different symmetry breaking possibilities with “maximal calculability” are discussed. Finally we point out that possible $(\sin^2 2\theta, \Delta m^2)$ parameter-space correlation should seriously be taken into account in future analyses of neutrino oscillations.
1 Introduction

The recent preliminary results of the Superkamiokande experiment \cite{1} have provided a more credible hint that the atmospheric neutrino anomaly should be attributed to the oscillation of massive neutrinos. The possible oscillation scenario could \textit{a priori} be either $\nu_\mu \leftrightarrow \nu_e$ or $\nu_\mu \leftrightarrow \nu_\tau$, but the former appears disfavored by the result of the CHOOZ experiment \cite{2}. It turns out that the most likely solution to the atmospheric neutrino problem comes from the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation with the following ranges of two oscillation parameters \cite{1,3}:

$$\Delta m^2_{\text{atm}} \approx (0.3 - 8) \times 10^{-3} \text{ eV}^2,$$

$$\sin^2 2\theta_{\text{atm}} \approx 0.7 - 1.$$ (1.1)

The large mixing angle here implies that the physics responsible for neutrino masses and lepton flavor mixings may be essentially different from that for the quark sector.

The long-standing problem of the solar neutrino deficit has been evidenced again by the preliminary data from Superkamiokande \cite{1}. In the assumption of the resonant Mikheyev-Smirnov-Wolfenstein (MSW) effect \cite{4}, the recent analysis of all available solar neutrino data gives the ranges of two oscillation parameters as follows \cite{4}:

$$\Delta m^2_{\text{sun}} \approx (0.3 - 1.2) \times 10^{-5} \text{ eV}^2,$$

$$\sin^2 2\theta_{\text{sun}} \approx (0.3 - 1.2) \times 10^{-2}.$$ (1.2)

It is also possible to interpret the solar neutrino deficit by invoking the pure vacuum oscillation mechanism (the “Just-so” solution). In this case the oscillation parameters read \cite{4}:

$$\Delta m^2_{\text{sun}} \approx (0.6 - 1.1) \times 10^{-10} \text{ eV}^2,$$

$$\sin^2 2\theta_{\text{sun}} \approx 0.7 - 1.$$ (1.3)

for the survival probability of electron neutrinos.

Note that in current analyses $\Delta m^2$ and $\sin^2 2\theta$ are treated as two independent parameters. This might be misleading in some models of lepton mass generation, where the mixing angle $\theta$ depends sensitively on the neutrino mass ratios – the oscillation parameters $\sin^2 2\theta$ and $\Delta m^2$ are correlated with each other.

A deeper insight into the yet unknown dynamics of the lepton mass generation requires nontrivial steps beyond the standard model. Phenomenologically the proper approach might

\footnote{The large-angle MSW solution \cite{5}, which allows $\Delta m^2_{\text{sun}} \approx (0.8 - 3.0) \times 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_{\text{sun}} \approx 0.42 - 0.74$, will not be considered in this work. Also we shall not consider the result from the LSND experiment \cite{6}, since a complete interpretation of all existing neutrino oscillation data requires the introduction of a sterile neutrino.}
be first to identify the patterns of lepton mass matrices from some kinds of symmetries, and then to interpret the neutrino oscillation data. For this purpose, two distinct possibilities of the neutrino mass spectrum are of particular interest and have attracted some attention [7]:

(a) Three neutrino masses perform a clear hierarchy: \( m_1 \ll m_2 \ll m_3 \);

(b) Three neutrino masses are almost degenerate: \( m_1 \approx m_2 \approx m_3 \).

In comparison with the well-known mass spectrum of the charged leptons \( m_e \ll m_\mu \ll m_\tau \), we expect that the charged lepton and neutrino mass matrices \( M_l \) and \( M_\nu \) in scenario (a) may both take the hierarchical form, similar to the quark mass matrices. The neutrino mass matrix in scenario (b), however, must take a form different from the charged lepton mass matrix.

Phenomenologically a popular approach to describing hierarchical neutrino masses might be to start from the Fritzsch-like ansätze [8, 9] with or without the see-saw mechanism [10]. In the assumption of the charged lepton flavor democracy and the neutrino mass degeneracy, a model has also been proposed to accommodate the almost degenerate neutrinos [11, 12]. In this talk we shall have a further look at these two neutrino mass scenarios and to confront them with the updated results of neutrino oscillations. We highlight a new parametrization of the \( 3 \times 3 \) flavor mixing matrix for scenario (a), and find that its three mixing angles can get apparent (physical) meanings from the Fritzsch-like ansätze as well as from the approximately decoupled atmospheric and solar neutrino oscillations. For scenario (b) we first point out a few more possibilities to break the mass degeneracy of neutrinos which respect the “maximal calculability” requirement but were not considered in Refs. [11, 12], and then discuss their consequences on neutrino oscillations. Finally we comment briefly on the possible problem of \((\sin^2 2\theta, \Delta m^2)\) parameter-space correlation in current analyses of neutrino oscillations.

2 Hierarchical neutrino masses

An analogy between the lepton mass hierarchy and the quark mass hierarchy is attractive in phenomenology, and this could naturally be obtained from a grand unified theory responsible for the generation of both quarks and leptons. It has been noticed, among a variety of parametrizations of the quark flavor mixing matrix, that there is a particular parametrization which can naturally reflect the hierarchy of quark masses and is favored by the \( B \)-meson physics [13]:

\[
V = \begin{pmatrix}
c_u & s_u & 0 \\
-s_u & c_u & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
e^{-i\phi} & 0 & 0 \\
0 & c & s \\
0 & -s & c
\end{pmatrix} \begin{pmatrix}
c_d & -s_d & 0 \\
s_d & c_d & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
where \( s_u \equiv \sin \theta_u, \ s_d \equiv \sin \theta_d, \ c \equiv \cos \theta \), etc. One can see

\[
\tan \theta_u = \frac{|V_{ub}|}{|V_{cb}|},
\]

\[
\tan \theta_d = \frac{|V_{td}|}{|V_{ts}|},
\] (2.2)

and

\[
\sin \theta = \sqrt{|V_{ub}|^2 + |V_{cb}|^2};
\] (2.3)

i.e., all three mixing angles can be directly measured from \( B \) decay or \( B^0-\bar{B}^0 \) mixing experiments. An analysis of current data on flavor mixing and \( CP \) violation gives \( \theta = 2.30^\circ \pm 0.09^\circ, \ \theta_u = 4.87^\circ \pm 0.86^\circ, \ \theta_d = 11.71^\circ \pm 1.09^\circ, \) and \( \varphi = 91.1^\circ \pm 11.8^\circ \) \[^{[14]}\]. For a variety of quark mass matrices, each of the four parameters gets a definite (physical) meaning \[^{[13]} \), \[^{[13]} \]: \( \varphi \) amounts approximately to the phase difference between the up and down mass matrices, \( \theta \) essentially describes the heavy quark flavor mixing, and \( \theta_u \) and \( \theta_d \) are related to the light quark mass ratios to a good degree of accuracy \[^{[14]} \):

\[
\tan \theta_u = \sqrt{\frac{m_u}{m_c}},
\]

\[
\tan \theta_d = \sqrt{\frac{m_d}{m_s}}.
\] (2.4)

These simple results make the parametrization (2.1) uniquely useful for the study of \( B \)-meson physics and quark mass matrices. In contrast, we argue that the parametrization advocated by the Particle Data Group \[^{[16]}\] indeed has little merit to be “standard”.

In view of our wealthy knowledge on quark mass matrices and quark flavor mixings, we believe that the most appropriate description of the \( 3 \times 3 \) lepton flavor mixing matrix, in terms of three Euler angles (\( \theta_l, \ \theta_\nu, \ \theta \)) and one \( CP \)-violating phase \( \phi \), should read as follows \[^{[17]}\):

\[
V = \left( \begin{array}{ccc}
c_l & s_l & 0 \\
-s_l & c_l & 0 \\
0 & 0 & 1
\end{array} \right) \left( \begin{array}{ccc}
e^{-i\phi} & 0 & 0 \\
0 & c & s \\
0 & -s & c
\end{array} \right) \left( \begin{array}{ccc}
c_\nu & -s_\nu & 0 \\
s_\nu & c_\nu & 0 \\
0 & 0 & 1
\end{array} \right),
\] (2.5)

where \( s_l \equiv \sin \theta_l, \ s_\nu \equiv \sin \theta_\nu, \ c \equiv \cos \theta \), etc. This parametrization has naturally reflected the hierarchical properties of lepton masses, and its mixing angles may also have very instructive
(physical) meanings. To see this point more clearly, we consider two reasonable limits for the sake of the assumed neutrino mass hierarchy.

(1) The “light lepton” limit: \( m_e/m_\mu \to 0 \) and \( m_1/m_2 \to 0 \). In this case both the charged lepton mass matrix \( M_l \) and the neutrino mass matrix \( M_\nu \) have texture zeros in the \((1, i)\) and \((i, 1)\) positions for \( i = 1, 2 \) and 3. Therefore the mixing angles \( \theta_l \) and \( \theta_\nu \) vanish, and \( V \) exclusively describes the mixing between the 2nd and 3rd lepton families. The resultant probability for \( \nu_\mu \to \nu_\tau \) transition reads

\[
P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \left( 1.27 \frac{\Delta m^2_{23} L}{|P|} \right),
\]

where \( \Delta m^2_{23} \equiv m^2_3 - m^2_2 \) (in unit eV^2), \( L \) is the distance from the production point of the muon neutrino to its interaction point (in unit km), and \( P \) is the momentum of the neutrino beam (in unit GeV). The transition probabilities for \( \nu_\tau \to \nu_\mu \) and \( \nu_\mu \to \nu_\tau \) both vanish in this “light lepton” limit. One can see that \( \theta = \theta_{\text{atm}} \) and \( \Delta m^2_{23} = \Delta m^2_{\text{atm}} \), if the atmospheric neutrino anomaly is ascribed to \( \nu_\mu \to \nu_\tau \).

(2) The “heavy lepton” limit: \( m_\mu/m_\tau \to 0 \) and \( m_2/m_3 \to 0 \). In this case the \((\tau, \nu_\tau)\) system is strongly decoupled from the light \((\mu, \nu_\mu)\) and \((e, \nu_e)\) systems due to the dominance of the 3rd family in the lepton mass spectrum. The transition probabilities for \( \nu_\mu \to \nu_\tau \) and \( \nu_\tau \to \nu_\mu \) vanish, while the survival probability \( P(\nu_e \to \nu_e) \) is given as

\[
P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_C \sin^2 \left( 1.27 \frac{\Delta m^2_{12} L}{|P|} \right),
\]

where \( \Delta m^2_{12} \equiv m^2_2 - m^2_1 \), and the “leptonic” Cabibbo angle \( \theta_C \) reads

\[
\sin \theta_C = \sqrt{s^2 l c^2 c_\nu s^2 \nu - 2s_l s_c c_\nu c_\nu \cos \phi}.
\]

Considering the mass hierarchy \( m_e/m_\mu \ll 1 \) and \( m_1/m_2 \ll 1 \), we expect \( \theta_C \) to be small enough to match the small-angle MSW solution to the solar neutrino deficit. Then \( \theta_C = \theta_{\text{sun}} \) and \( \Delta m^2_{12} = \Delta m^2_{\text{sun}} \) hold.

In both limits taken above, \( C P \) violation vanishes. To examine how far these limits are from the reality, we have to assume the explicit pattern of \( M_l \) and \( M_\nu \), and to express the mixing angles \((\theta_l, \theta_\nu, \theta)\) in terms of the lepton mass ratios. Only if we impose the phenomenological constraints \( \Delta m^2_{23} = \Delta m^2_{\text{atm}} \) and \( \Delta m^2_{12} = \Delta m^2_{\text{sun}} \), however, we can always obtain \( m_3 \approx (1.7 - 9) \times 10^{-2} \text{ eV} \) from (1.1) and \( m_2 \approx (1.7 - 3.5) \times 10^{-3} \text{ eV} \) from (1.2) because of the assumed hierarchy \( m_1 \ll m_2 \ll m_3 \). The large gap between \( \Delta m^2_{\text{atm}} \) and \( \Delta m^2_{\text{sun}} \) generally allows the oscillations of solar and atmospheric neutrinos to decouple as a good approximation, provided the flavor mixing matrix element \( V_{e3} = s_\tau s \) is sufficiently small [18].
As we can see from some reasonable Fritzsch-like ansätze of lepton mass matrices \cite{8, 9}, the mixing angle $\theta_l$ is essentially given by \cite{17}

$$\theta_l = \arctan \left( \sqrt{\frac{m_e}{m_\mu}} \right) \approx 4^\circ.$$  \hspace{1cm} (2.9)

Therefore the decoupling condition for the solar and atmospheric neutrino oscillations can be satisfied, and the instructive formulas obtained in the above two limits should hold to a good degree of accuracy.

In this talk we shall not go into the details of the neutrino mass matrix with hierarchical mass eigenvalues (for a recent study with extensive references, see, e.g., Ref. \cite{17}).

### 3 Nearly degenerate neutrino masses

The dominance of $m_\tau$ in the hierarchical ("H") mass spectrum of charged leptons implies a plausible limit in which the mass matrix takes the form

$$M_l^H = c_l \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \hspace{1cm} (3.1)$$

with $c_l = m_\tau$. This mass matrix is equivalent to the following matrix with $S(3)_L \times S(3)_R$ symmetry or flavor democracy ("D"):

$$M_l^D = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \hspace{1cm} (3.2)$$

through the orthogonal transformation $O^\dagger M_l^H O = M_l^D$, where

$$O = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \hspace{1cm} (3.3)$$

For either $M_l^H$ or $M_l^D$, further symmetry breaking terms can be introduced to generate masses for muon and electron. Recently the possible significance of the approximate democratic mass matrices has been emphasized towards understanding fermion masses and flavor mixings \cite{19}.

The similar picture is however invalid for the neutrino sector, if three neutrino masses are almost degenerate. Provided the netrino masses persist in an exact degeneracy ("D")
symmetry, the corresponding mass matrix should take the diagonal form:

\[ M^D_\nu = c_\nu \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_2 & 0 \\ 0 & 0 & \eta_3 \end{pmatrix}, \]  

(3.4)

where \( c_\nu \equiv m_0 \) measures the mass scale of three neutrinos, and \( \eta_i (= \pm 1) \) denotes the \( CP \)-parity of the Majorana neutrino \( \nu_i \). By breaking the mass degeneracy of \( M^D_\nu \) slightly, one may get the realistic neutrino masses \( m_1, m_2 \) and \( m_3 \) (at least two of them are different from \( m_0 \) and different from each other).

A purely phenomenological assumption, that a more fundamental theory of lepton interactions might simultaneously accommodate the charged lepton mass matrix \( M^D_l \) and the neutrino mass matrix \( M^D_\nu \) in the symmetry limit, has been made first in Ref. [11] and then in Ref. [12] to discuss the lepton flavor mixing and to interpret the neutrino oscillation data. In this case one can find that the constant matrix \( O \) may play an important role in the flavor mixing matrix, obtained from the diagonalization of \( M^D_l \) and \( M^D_\nu \). The realistic flavor mixing matrix \( V \) depends on the explicit pattern of the perturbative corrections to \( M^D_l \) and \( M^D_\nu \). For simplicity we shall neglect the small correction from nonvanishing \( m_e/m_\mu \ll 1 \) and \( m_\mu/m_\tau \ll 1 \), and concentrate on the degeneracy-breaking pattern of \( M_\nu \).

The forms of the perturbative correction to \( M^D_\nu \), denoted as \( \Delta M_\nu \), can be classified into two categories.

(i) \( \Delta M_\nu \) is a completely diagonal matrix with two or three parameters. In this case \( M^D_\nu + \Delta M_\nu \) remains diagonal, then the flavor mixing matrix \( V \) turns out to be \( O \). As shown in Ref. [11], this ansatz results in the survival probability of the electron neutrino as

\[ P(\nu_e \to \nu_e) \approx 1 - \sin^2 \left( 1.27 \frac{\Delta m^2_{12} L}{|P|} \right); \]  

(3.5)

i.e., the mixing factor \( \sin^2 2\theta_{\text{sun}} \) is almost maximal. Obvioulsy this result is favored by the vacuum oscillation solution to the solar neutrino deficit, which requires the mass-squared difference \( |\Delta m^2_{12}| = \Delta m^2_{\text{sun}} \sim 10^{-10} \text{ eV}^2 \) (see (1.3) for the value). The transition probability of \( \nu_\mu \to \nu_\tau \), on the other hand, reads as follows:

\[ P(\nu_\mu \to \nu_\tau) \approx \frac{8}{9} \sin^2 \left( 1.27 \frac{\Delta m^2_{23} L}{|P|} \right); \]  

(3.6)

where the oscillating term of \( \Delta m^2_{12} \) has been ignored due to its tiny effect, and \( \Delta m^2_{23} \approx \Delta m^2_{13} \) is a good approximation because of \( \Delta m^2_{\text{atm}} \gg |\Delta m^2_{12}| \). We conclude that both the atmospheric and solar neutrino oscillations can be interpreted with this lepton mass ansatz, which allows a remarkable mass degeneracy between \( \nu_1 \) and \( \nu_2 \) states as well as a relatively weaker mass degeneracy between \( \nu_2 \) and \( \nu_3 \) states.
For Majorana neutrino masses, it is necessary to fulfill the bound \( \langle m \rangle \leq 0.7 \text{ eV} \) for neutrinoless \( \beta\beta \)-decay \[20\], where \( \langle m \rangle \) is an effective mass factor. It is known that the \( (\beta\beta)_{0\nu} \)-decay amplitude depends on the masses of Majorana neutrinos \( m_i \) and on the elements of the lepton mixing matrix \( V_{ei} \):

\[
\langle m \rangle \sim \sum_{i=1}^{3} \left( V_{ei}^2 m_i \eta_i \right),
\]

where \( \eta_i \) is the \( CP \)-parity of the Majorana field for \( \nu_i \). If \( \eta_1 = +1 \) and \( \eta_2 = -1 \) (or vice versa), one finds that \( \langle m \rangle \sim (m_1 - m_2)/2 \), which is considerably suppressed due to the near degeneracy of \( m_1 \) and \( m_2 \). This implies that there may exist two Majorana neutrinos with opposite \( CP \) eigenvalues, and their relative \( CP \) parities are in principle observable in \( (\beta\beta)_{0\nu} \)-decay. Within our approach this possibility exists. Thus a degenerate Majorana mass of about \( m_0 \sim 2.5 \text{ eV} \) for all three neutrinos, favored by the hot dark matter, need not be in conflict with the data on neutrinoless \( \beta\beta \)-decay.

(ii) \( \Delta M_{\nu} \) is not a completely diagonal matrix but consists of at least two parameters. In this case a variety of patterns for \( \Delta M_{\nu} \) is allowed. To ensure the “maximal calculability” for the neutrino mass matrix, we require a special form of \( \Delta M_{\nu} \) which has two unknown parameters (to break the mass degeneracy of \( M_{\nu}^D \)) and can be diagonalized by a \textit{constant} orthogonal transformation (independent of the neutrino masses). Then we find that only three patterns of \( \Delta M_{\nu} \), as listed in Table 1, satisfy these strong requirements. They can be diagonalized by three Euler rotation matrices \( R_{ij} \) with the rotation angles \( \theta_{ij} = 45^\circ \) in the (1,2), (2,3) and (3,1) planes respectively (see also Table 1). In Ref. \[12\] only pattern (I) was proposed and discussed. For each pattern the resultant flavor mixing matrix reads as \( V \approx OR_{ij} \), which remains a constant matrix before the introduction of small corrections from \( m_e/m_\mu \) and \( m_\mu/m_\tau \). We calculate the transition probability for \( \nu_\mu \rightarrow \nu_\tau \) and find the mixing factor to be \( 8/9 \), \( 2/9 \) or \( 2/9 \), respectively, for pattern (I), (II) or (III). Therefore only pattern (I) can survive when confronting the atmospheric neutrino data. The survival probability \( P(\nu_e \rightarrow \nu_e) \) amounts to unity in the limit \( m_e = m_\mu = 0 \) for pattern (I). It can interpret the small-angle MSW solution to the solar neutrino deficit, however, after a proper perturbative term \( \Delta M_l \) is introduced to \( M_l^D \). For example, \( \sin^2 2\theta_{\text{sun}} \sim m_e/m_\mu \) may be obtained if \( \Delta M_l \) is taken to be of the diagonal form \[11\], \[12\]. Also this ansatz has no conflict with requirements of neutrinoless \( \beta\beta \)-decay and hot dark matter on neutrino masses.

Note that the situation will change if one assume a theory of lepton interactions to accommodate the charged lepton mass \( M_l^H \) (instead of \( M_l^D \)) and the neutrino mass matrix \( M_{\nu}^D \) simultaneously in the symmetry limit. In this case the large mixing angle(s) of \( V \) can only come from diagonalizing \( M_{\nu}^D + \Delta M_{\nu} \), where \( \Delta M_{\nu} \) may take the forms listed in Table 1 to guarantee the “maximal calculability”. It is obvious that \( V \approx R_{ij} \) holds. We find that only
Table 1: Three patterns of $\Delta M_\nu$ and their consequences on $V$ and $P(\nu_\mu \rightarrow \nu_\tau)$

| Pattern | $\Delta M_\nu$ | $R_{ij}$ | $V$ | $P(\nu_\mu \rightarrow \nu_\tau)$ |
|---------|----------------|----------|-----|----------------------------------|
| (I)     | $\begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 0 & 0 \\ 0 & 0 & \delta \end{pmatrix}$ | $\begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}$ | $8/9 \sin^2 \left(1.27 \frac{\Delta m_{23}^2 L}{|P|}\right)$ |
| (II)    | $\begin{pmatrix} \delta & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ | $\begin{pmatrix} 1/\sqrt{2} & -1/2 & -1/2 \\ 1/\sqrt{6} & 3/2\sqrt{3} & -1/2\sqrt{3} \\ 1/\sqrt{3} & 0 & 2/\sqrt{6} \end{pmatrix}$ | $2/9 \sin^2 \left(1.27 \frac{\Delta m_{23}^2 L}{|P|}\right)$ |
| (III)   | $\begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 0 & 0 \\ 0 & \delta & 0 \end{pmatrix}$ | $\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$ | $\begin{pmatrix} 1/2 & -1/\sqrt{2} & -1/2 \\ -1/2\sqrt{3} & 1/\sqrt{6} & -3/2\sqrt{3} \\ 2/\sqrt{6} & 1/\sqrt{3} & 0 \end{pmatrix}$ | $2/9 \sin^2 \left(1.27 \frac{\Delta m_{23}^2 L}{|P|}\right)$ |

Pattern (II) of $\Delta M_\nu$ is favored by the atmospheric neutrino data; i.e.,

$$V \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

in the neglect of the $\Delta M_l$ effect on $M^H_l$, and the corresponding probability of $\nu_\mu \rightarrow \nu_\tau$ reads

$$P(\nu_\mu \rightarrow \nu_\tau) \approx \sin^2 \left(1.27 \frac{\Delta m_{23}^2 L}{|P|}\right),$$

which implies $\sin^2 2\theta_{\text{atm}} \approx 1$. This ansatz can also fit the small-angle MSW solution to the solar neutrino deficit, if the $m_e/m_\mu$ and $m_\mu/m_\tau$ corrections to $V$ are taken into account.
4 Summary and comments

We have given a purely phenomenological interpretation of atmospheric and solar neutrino oscillations based on the scenarios of hierarchical and almost degenerate neutrino masses. It is emphasized that the hierarchy of charged lepton and neutrino masses favors a particular parametrization of the flavor mixing matrix, and the approximate decoupling of solar and atmospheric neutrino oscillations naturally appears in this scheme. If neutrinos have nearly degenerate masses, it is shown that interesting lepton mass matrices can be obtained starting from the symmetries of charged lepton flavor democracy and neutrino mass degeneracy. Two possibilities to break these symmetries, which allow the “maximal calculability” for the flavor mixing matrix, have been discussed. We find that both of them have no conflict with current requirements of hot dark matter and neutrinoless $\beta\beta$-decay. Another possible ansatz, in which $\sin^2 2\theta_{\text{atm}} \approx 1$, has also been pointed out.

Let us end this talk by giving some brief comments on the possible problem of $(\sin^2 2\theta, \Delta m^2)$ parameter-space correlation. It is known that the flavor mixing matrix elements are in general determined by mass ratios of neutrinos and charged leptons. This is true, in particular, for the case of hierarchical neutrino masses without fine tuning. Hence the oscillation parameters $\sin^2 2\theta$ and $\Delta m^2$ are both dependent on the neutrino masses, and they are expected to have correlation to some extent. This correlation will lead to a further constraint on the currently allowed ranges of $\sin^2 2\theta$ and $\Delta m^2$, which were obtained in the naive assumption of independence between these two parameters.

This possible correlation could also undermine the prospect to observe $CP$ violation in the long baseline neutrino oscillation experiments. The reason is simply that the $CP$-violating asymmetry between $P(\nu_\alpha \to \nu_\beta)$ and $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ is proportional not only to the parameter

$$J = s_l c_l s_\nu c_\nu s^2 c \sin \phi,$$

but also to a product of three oscillation terms containing $\Delta m^2_{12}$, $\Delta m^2_{23}$ and $\Delta m^2_{31}$ [17]. Certainly $J$ is in general a function of $m_i$, thus the correlation between $J$ and three oscillating terms could not allow $J \sim 1\%$ to be a reasonable estimation. At least for the case of hierarchical neutrino masses, we believe that $J$ has little chance to be at the percent level.

Of course, the above-mentioned correlation is model-dependent and cannot be solved at present. Some cautious treatment is however necessary in the future, when we have got enough knowledge about neutrino oscillations and the dynamics of neutrino mass generation.

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