From Traces To Proofs: Proving Concurrent Programs Safe

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Abstract—Nondeterminism in scheduling is the cardinal reason for difficulty in proving correctness of concurrent programs. A powerful proof strategy was recently proposed\textsuperscript{5} to show the correctness of such programs. The approach captured data-flow dependencies among the instructions of an interleaved and error-free execution of threads. These data-flow dependencies were represented by an inductive data-flow graph (iDFG), which, in a nutshell, denotes a set of executions of the concurrent program that gave rise to the discovered data-flow dependencies. The iDFGs were further transformed in to alternative finite automata (AFAs) in order to utilize efficient automata-theoretic tools to solve the problem. In this paper, we give a novel and efficient algorithm to directly construct AFAs that capture the data-flow dependencies in a concurrent program execution. We implemented the algorithm in a tool called ProofTranPar to prove the correctness of finite state cyclic programs under the sequentially consistent memory model. Our results are encouraging and compare favorably to existing state-of-the-art tools.

I. INTRODUCTION

The problem of checking whether or not a correctness property (specification) is violated by the program (implementation) is already known to be challenging in a sequential set-up, let alone when programs are implemented exploiting concurrency. The central reason for greater complexity in verification of concurrent implementations is due to the exponential increase in the number of executions. A concurrent program with $n$ threads and $k$ instructions per thread can have $\frac{(nk)!}{(k!)^n}$ executions under a sequentially consistent (SC)\textsuperscript{12} memory model. A common approach to address the complexity due to the exponential number of executions is trace partitioning.

In\textsuperscript{6}, a powerful proof strategy was presented which utilized the notion of trace partitioning. Let us take Peterson’s algorithm in Figure\textsuperscript{1} to convey the central idea behind the trace partitioning approach. In this algorithm, two processes, \(P_i\) and \(P_j\), coordinate to achieve an exclusive access to a critical section (CS) using shared variables. A process \(P_i\) will busy-wait if \(P_j\) has expressed interest to enter its CS and \(t\) is \(j\).

In order to prove the mutual exclusion (ME) property of Peterson’s algorithm, we must consider the boolean conditions of the while loops at control locations 3 and 8. The ME property is established only when at most one of these conditions is false under every execution of the program, \textit{i.e.}, ME must be shown to hold true on unbounded number of traces (trace is “a sequence of events corresponding to an interleaved execution of processes in the program”\textsuperscript{9}) generated due to unbounded number of unfoldings of the loops. Notice that events at control locations 3 and 8 are data-dependent on events from control locations 2, 6, 7 and 1, 2, 7, respectively. In any finite prefix of a trace of \(P_i\parallel P_j\) (interleaved execution of \(P_i\) and \(P_j\)) up to the events corresponding to control location 3 or 8, the last instance of event at control location 2, 1, 2, 2, and the last instance of event at control location 7, 1, 7, can be ordered in only one of the following two ways: either 1, 2 appears before 1, 7 or 1, 2 appears after 1, 7. This has resulted in partitioning of an unbounded set of traces to a set with mere two traces.

When 1, 2 appears before 1, 7, then the final value of the variable \(t\) is \(i\), thus making the condition at control location 8 to be true. In the other case, when 1, 2 appears after 1, 7, the final value of the variable \(t\) is \(j\), thereby making the condition at control location 3 evaluate to true. Hence, in no trace both the conditions are false simultaneously. This informal reasoning indicates that both processes can never simultaneously enter in their critical sections. Thus, proof of correctness for Peterson’s algorithm can be demonstrated by picking two traces, as mentioned above, from the set of infinite traces and proving them correct. In general, the intuition is that a proof for a single trace of a program can result in pruning of a large set of traces from consideration. To convert this intuition to a feasible verification method, there is a need to construct a formal structure from a proof of a trace \(\sigma\) such that the semantics of this structure includes a set of all those traces that have proof arguments equivalent to proof of \(\sigma\). Inductive Data Flow Graphs (iDFG) was proposed in\textsuperscript{6} to capture data-dependencies among the events of a trace and to perform trace partitioning. All traces that have the same iDFG

\begin{verbatim}
flagi = false, flagj = false, t = 0;
\end{verbatim}

\begin{verbatim}
P1
While(true){
  1. flagi=true;
  2. t=j;
  3. while(flagj = true & t = j);
  4. //Critical Section
  5. flagj=false;
}
\end{verbatim}

\begin{verbatim}
P2
While(true){
  6. flagj=true;
  7. t=i;
  8. while(flagi = true & t = i);
  9. //Critical Section
  10. flagi=false;
}
\end{verbatim}

Fig. 1: Peterson’s algorithm for two processes \(P_i\) and \(P_j\)
must have the same proof of correctness. In every iteration of their approach, a trace is picked from the set of all traces that is yet to be covered by the iDFG. An iDFG is constructed from its proof. The process is repeated until all the traces are either covered in the iDFG or a counter-example is found. An intervening step is involved where the iDFG is converted to an alternating finite automaton (AFA). While we explain AFA in later sections, it suffices to understand at this stage that the language accepted by this AFA and the set of traces captured by the corresponding iDFG is the same. Their reason for this conversion is to leverage the use of automata-theoretic operations such as subtraction, complement etc., on the set of traces.

Though the goal of paper [6] is verification of concurrent programs which is the same as in this work, our work has some crucial differences: (i) An AFA is constructed directly from the proof of a trace without requiring the iDFG construction, (ii) the verification procedure built on directly constructed AFA is more efficient (by a factor of magnitude better than THREADER and 3 times better than Lazy-CSeq), (iii) the AFA is constructed from its proof. The process is repeated until all the traces are covered, (iv) the verification method does not comment on the performance and the feasibility of their approach due to the lack of an implementation.

The second contribution of this paper is an implementation in the form of a tool, ProofTraPar. We compare our implementation against other state-of-the-art tools in this domain, such as THREADER [10] and Lazy-CSeq [11] (winners in the concurrency category of the software verification competitions held in 2013, 2014, and 2015). ProofTraPar, on average, performed an order of magnitude better than THREADER and 3 times better than Lazy-CSeq.

The paper is organized as follows: Section [1] covers the notations, definitions and programming model used in this paper; Section [II] presents our approach with the help of an example to convey the overall idea and describes in detail the algorithms for constructing the proposed alternating finite automaton along with their correctness proofs. This section ends with the overall verification algorithm with the proof of its soundness and completeness for finite state concurrent programs. Section [IV] presents the experimental results and comparison with existing tools namely THREADER [10] and Lazy-CSeq [11]. Section [V] presents the related work and Section [VI] concludes with possible future directions.

II. PRELIMINARIES

A. Program Model

We consider shared-memory concurrent programs composed of a fixed number of deterministic sequential processes and a finite set of shared variables SV. A concurrent program is a quadruple $P = (P, A, I, D)$ where $P$ is a finite set of processes, $A = \{A_p \mid p \in P\}$ is a set of automata, one for each process specifying their behaviour, $I$ is a set of constants appearing in the syntax of processes and $D$ is a function from variables to their initial values. Each process $p \in P$ has a disjoint set of local variables $LV_p$. Let $Exp_p$ ($BExp_p$) denote the set of expressions (boolean expressions), ranged over by $\phi$, and constructed using shared variables, local variables, $D$, and standard mathematical operators. Each specification automaton $A_p$ is a quadruple $(\Sigma_p, q^{init}_p, \delta_p, Assn_p)$, where $\Sigma_p$ is a finite set of control states, $q^{init}_p$ is the initial state, and $Assn_p \subseteq \Sigma_p \times BExp_p$ is a relation specifying the assertions that must hold at some control state. Each transition in $\delta_p$ is of the form $(q, op_p, q')$ where $op_p \in \{x := \text{exp}, \text{assume}(\phi), \text{lock}(x)\}$. Here $x := \text{exp}$ evaluates $\exp$ in the current state and assigns the value to $x$ where $x \in SV \cup LV_p$, $\text{assume}(\phi)$ is a blocking operation that suspends the execution if the boolean expression $\phi$ evaluates to false otherwise it acts as nop. This instruction is used to encode control path conditions of a program, $\text{lock}(x)$, where $x \in SV$, is a blocking operation that suspends the execution if the value of $x$ is not equal to 0 otherwise it assigns 1 to $x$. Operation unlock is achieved by assigning 0 to this shared variable. Each of these operations are deterministic in nature,
i.e. execution of any two same operations from the same states always give the same behaviour. In all examples of this paper, we use symbolic labels to succinctly represent program operations. For example, in Peterson’s algorithm, Labels \{a, b, p, q\} denote operations in the program. Variable res is introduced to specify the mutual exclusion property as a safety property. A process \(P_i\) sets this variable to 1 inside its critical section. Assertions assert(res = 1) is checked in \(P_i\) before leaving its critical section. If these assertions hold in every execution of these two processes then the mutual exclusion property holds.

Fig. 3: Specification of Peterson’s algorithm

of the operation \(op\) terminates and the resulting program state \(s’\) satisfies \(\phi\).

Given a formula \(\phi\), variable \(X\) and expression \(e\), let \(\phi[X/e]\) denote the formula obtained after substituting all free occurrences of \(X\) by \(e\) in \(\phi\). We assume an equality operator over formulas that represents syntactic equality. Every formula is assumed to be normalized in a conjunctive normal form (CNF). We use true (false) to syntactically represent a logically valid (unsatisfiable) formula. Weakest precondition axioms for different program statements are shown in Figure 6. Here empty sequence of statements is denoted by \(\emptyset\). We have the following properties about weakest preconditions.

Property 1: If \(wp(op, \phi_1) = \psi_1\) and \(wp(op, \phi_2) = \psi_2\) then,
- \(wp(op, \phi_1 \land \phi_2) = \psi_1 \land \psi_2\)
- \(wp(op, \phi_1 \lor \phi_2) = \psi_1 \lor \psi_2\).

We say that a formula \(\phi\) is stable with respect to a statement \(S\) if \(wp(S, \phi)\) is logically equivalent to \(\phi\). In this paper, we use weakest preconditions to check the correctness of a trace with respect to some safety assertion. A trace \(\sigma\) reaching up to a safety assertion \(\phi\) is safe if the execution of \(\sigma\) starting from the initial state \(I\) either 1) blocks (does not terminate) because of not satisfying some path conditions, or 2) terminates and the resulting state satisfies \(\phi\). The following lemmas clearly define the conditions, using weakest precondition axioms, for declaring a trace \(\sigma\) either safe or unsafe. Detailed proofs of these are given in Appendix A and B. Here \(\sigma[\text{assert/assume}]\) denote the trace obtained by replacing every instruction of the form \(\text{assert}(\phi)\) by \(\text{assume}(\phi)\) in \(\sigma\).

Lemma 1: For a trace \(\sigma\), an initial program state \(I\) and a safety property \(\phi\), if \(wp(\sigma[\text{assert/assume}], -\phi) \land I\) is unsatisfiable then the execution of \(\sigma\), starting from \(I\), either does not terminate or terminates in a state satisfying \(\phi\).

Lemma 2: For a trace \(\sigma\), an initial program state \(I\) and a safety property \(\phi\), if \(wp(\sigma[\text{assume/assume}], -\phi) \land I\) is satisfiable then the execution of \(\sigma\), starting from \(I\), terminates in a state not satisfying \(\phi\).

C. Alternating Finite Automata (AFA)

Alternating finite automata [11, 12] are a generalization of nondeterministic finite automata (NFA). An NFA is a five tuple \((S, \Sigma, \delta, s_0, F)\) with a set of states \(S\), ranged over by \(s\), an initial state \(s_0\), a set of accepting states \(F\) and a transition function \(\delta : S \times \Sigma \to P(S)\). For any state \(s\) of this NFA, the
set of words accepted by \( s \) is inductively defined as \( \text{acc}(s) = \{ a.\sigma \mid a \in \Sigma \cup \{ \epsilon \}, \sigma \in \Sigma^*, \exists s'. s' \in \delta(s,a), \sigma \in \text{acc}(s') \} \) where \( \epsilon \in \text{acc}(s) \) for all \( s \in S_F \). Here, the existential quantifier represents the fact that there should exist at least one outgoing transition from \( s \) along which \( a.\sigma \) gets accepted. An AFA is a six tuple \((S_F, S_\Sigma, \Sigma \cup \{ \epsilon \}, \delta, s_0, S_F)\) with \( \Sigma, s_0 \) and \( S_F \subseteq S \) denoting the alphabet, initial state and the set of accepting states respectively.

The set of words accepted by a state of an AFA depends on whether that state is an existential state (from the set \( S_3 \)) or a universal state (from the set \( S_4 \)). For an existential state \( s \in S_3 \), the set of accepted words is inductively defined in the same way as in NFA. For a universal state \( s \in S_4 \), the set of accepted words are \( \text{acc}(s) = \{ a.\sigma \mid a \in \Sigma \cup \{ \epsilon \}, \forall s' \in \delta(s,a), \sigma \in \text{acc}(s') \} \) where \( \epsilon \in \text{acc}(s) \) for all \( s \in S_F \). Notice the change in the quantifier from \( \exists \) to \( \forall \). In the diagrams of AFA used in this paper, we annotate universal states with \( \forall \) symbol and existential states with \( \exists \) symbol. For a state \( s \), let \( \text{succ}(s,a) = \{ s' \mid (s,a,S) \in \delta \} \) be the set of \( a \)-successors of \( s \). For an automaton \( A \), let \( \mathcal{L}(A) \) be the language accepted by the initial automaton. For any \( \sigma \in \Sigma^* \) \(|\sigma|\) denote the length of \( \sigma \) and \( \text{rev}(\sigma) \) denote the reverse of \( \sigma \).

III. OUR APPROACH

The overall approach of this paper can be described in the following steps: (i) Given a concurrent program \( P \), construct all its interleaved traces represented by automaton \( A(P) \), as defined in Subsection IIA; (ii) Pick a trace \( \sigma \) and a safety property, say \( \phi \), to prove for this trace; (iii) Prove \( \sigma \) correct with respect to \( \phi \) using Lemma 1 and Lemma 2 and generate a set of traces which are also provably correct. Let us call this set \( T_{\sigma'} \); (iv) Remove set \( T_{\sigma'} \) from the set of traces represented by \( A(P) \) and repeat from Step (ii) until either all the traces in \( P \) are proved correct or an erroneous trace is found.

Step (iii) of this procedure, correctness of \( \sigma \), can be achieved by checking the unsatisfiability of \( \text{wp}(\sigma[\text{assume}][\text{assert}], \neg \phi) \land \mathcal{I} \). However, we are not only interested in checking the correctness of \( \sigma \) but also in constructing a set of traces which have a similar reasoning as of \( \sigma \). Therefore, instead of computing \( \text{wp}(\sigma[\text{assume}][\text{assert}], \neg \phi) \) directly from the weakest precondition axioms of Figure 4 we construct an AFA from \( \sigma \) and \( \neg \phi \). Step (iv) is then achieved by applying automata-theoretic operations such as complementation and subtraction on this AFA. Notion of universal and existential states of AFA helps us in finding a set of sufficient dependencies used in the weakest precondition computation so that any other trace satisfying those dependencies gets captured by AFA. Subsequent subsections covers the construction, properties and use of this AFA in detail.

A. Constructing the AFA from a Trace and a Formula

Definition 1: An AFA constructed from a trace \( \sigma \) of a Program \( P \) and a formula \( \phi \) is \( A_{\sigma,\phi} = (S_F, S_3, OP, s_0, S_F, \delta, A_{\text{Map}}, R_{\text{Map}}) \), where,

\[
\delta(s, op) =
\begin{cases}
1. & A_{\text{Map}}(s') = \text{wp}(op[assume][assert], A_{\text{Map}}(s)), \\
2. & s \in \text{an existential state}, \text{and} \\
3. & R_{\text{Map}}(s') = R_{\text{Map}}(s') \cup op'. \\
\text{where } op' \text{ is the longest sequence s.t.} \\
\text{wp}(\sigma'[assume][assert], A_{\text{Map}}(s')) = A_{\text{Map}}(s) & (\text{LITERAL-ASSN}) \\
\text{wp}(\sigma'[assume][assert], A_{\text{Map}}(s')) = A_{\text{Map}}(s) & (\text{LITERAL-SELF-ASSN}) \\
\text{1. } & A_{\text{Map}}(s) = \bigvee \phi_k, \text{or } A_{\text{Map}}(s) = \bigvee \phi_k, \\
\text{2. } & R_{\text{Map}}(s_0) = \phi_k, \\
\text{3. } & \exists k, R_{\text{Map}}(s) = R_{\text{Map}}(s_k), \\
\text{4. } & \text{op} = \epsilon & (\text{COMPOUND-ASSN}) \\
\text{otherwise}
\end{cases}
\]

Fig. 5: Transition function used in the Definition 1

1) \((OP_\epsilon = OP \cup \{\epsilon\})\) is the alphabet ranged over by \( op \). Here \( OP \) is the set of instructions used in program \( P \). Symbol \( \epsilon \) acts as an identity element of concatenation and \( \text{wp}(\epsilon, \phi) = \phi \).
2) \( S = S_4 \cup S_3 \) is the largest set of states, ranged over by \( s \).

a) Every state is annotated with a formula and a prefix of \( \sigma \) denoted by \( A_{\text{Map}}(s) \) and \( R_{\text{Map}}(s) \) respectively. State \( s_0 \) is the initial state such that \( A_{\text{Map}}(s_0) = \phi, R_{\text{Map}}(s_0) = \sigma \).

b) \( s' \in S \) iff either of the following two conditions hold,

- \( \exists s \in S \) such that \( A_{\text{Map}}(s') \) is \( \text{wp}(op[assume][assert], A_{\text{Map}}(s)) \). \( R_{\text{Map}}(s) = R_{\text{Map}}(s') \cup op' \) and \( \sigma' \) is the largest suffix of \( R_{\text{Map}}(s) \) such that formula \( A_{\text{Map}}(s) \) is stable with respect to \( \sigma'[assume][assert] \).

- \( \exists s \in S \) such that \( A_{\text{Map}}(s) = \bigwedge \{\phi_1, ..., \phi_k\} \) or \( A_{\text{Map}}(s) = \bigvee \{\phi_1, ..., \phi_k\} \), \( R_{\text{Map}}(s) = R_{\text{Map}}(s') \), \( A_{\text{Map}}(s') = \phi' \) and \( \phi' \in \{\phi_1, ..., \phi_k\} \).

a) A state \( s \in S \) is an existential state (universal state) iff \( A_{\text{Map}}(s) \) is a literal (compound formula).

3) \( S_F \subseteq S \) is a set of accepting states such that \( s \in S_F \) iff \( R_{\text{Map}}(s)[\text{assume}[assert], A_{\text{Map}}(s)] \) is same as \( A_{\text{Map}}(s) \), i.e. \( A_{\text{Map}}(s) \) is stable with respect to \( R_{\text{Map}}(s)[\text{assume}[assert], A_{\text{Map}}(s)] \), and

4) Function \( \delta : S \times OP_\epsilon \rightarrow \mathcal{P}(S) \) is defined in Figure 5

Following Point 2a any state added to \( S \) is either annotated with a smaller \( R_{\text{Map}} \) or a smaller formula compared to the states already present in \( S \). Further, every formula and trace \( \sigma \) is of finite length. Hence the set of states \( S \) is finite. By Point 2a of this construction, a state \( s \) where \( A_{\text{Map}}(s) \) is a compound formula, is always a universal state irrespective of whether \( A_{\text{Map}}(s) \) is a conjunction or a disjunction of clauses. The reason behind this decision will be clear shortly when we will use this AFA to inductively construct the weakest precondition \( \text{wp}(\sigma[\text{assume}][\text{assert}], \phi) \). Note that we assume every formula is normalized in CNF.

Figure 4 shows an example trace \( \sigma = abAbpPrcc \) of Peterson’s algorithm. This trace is picked from the Peterson’s
Fig. 6: AFA of trace given in Figure 5(b) and \( \phi = \neg(\ell_2 = 2) \)

specification in Figure 5. To prove \( \sigma \) correct with respect to the safety formula \( \phi \stackrel{\text{def}}{=} (\ell_2 = 2) \) we first construct \( \hat{A}_{\sigma, \neg \phi} \) which will later help us to derive \( \wp(\sigma[\text{assume/assert}], \neg \phi) \). This AFA is shown in Figure 5. For a state \( s \), \( \hat{M}_{\text{map}}(s) \) is written inside the rectangle representing that state and \( \hat{R}_{\text{map}}(s) \) is written inside an ellipse next to that state. We show here some of the steps illustrating this construction.

1. By Definition 1 we have \( \hat{M}_{\text{map}}(s_0) = \neg(\ell_2 = 2) \) and \( \hat{R}_{\text{map}}(s_0) = s = ab\text{app}P_{\text{rc}} \) for initial state \( s_0 \).
2. In a transition \( \delta(s, op) = \{s'\} \) created by Rule LITERAL-ASSN the state \( s' \) is annotated with the weakest precondition of an operation \( op \), taken from \( \hat{R}_{\text{map}}(s) \), with respect to \( \hat{M}_{\text{map}}(s) \). Operation \( op \) is picked in such a way that \( \hat{M}_{\text{map}}(s') \) is stable with respect to every other operation present after \( op \) in \( \hat{R}_{\text{map}}(s) \). Such transitions capture the inductive construction of the weakest precondition for a given \( \phi \) and trace \( \sigma \). Transition \( \delta(s_0, a) = \{s_1\} \) in Figure 6 is created by this rule as \( \wp(a[\text{assume/assert}], \hat{M}_{\text{map}}(s_0)) = \hat{M}_{\text{map}}(s_1) \), and \( \hat{R}_{\text{map}}(s_0) = \hat{R}_{\text{map}}(s_1) \).
3. In any transition created by Rule COMPOUND-ASSN say from \( s \) to \( s_1, \ldots, s_k \), the states \( s_1, \ldots, s_k \) are annotated with the subformulae of \( \hat{M}_{\text{map}}(s) \). For example, transitions \( \delta(s_1, e) = \{s_4, s_5\} \) and \( \delta(s_7, e) = \{s_8, s_9\} \).
4. Transition \( \delta(s_8, a) = \{s_{12}\} \) follows from the rule LITERAL-ASSN. Note that \( \hat{R}_{\text{map}}(s_{12}) \) is empty and hence by Point 4 of Definition 1 \( s_{12} \) is an accepting state. Following the same reasoning, states \( s_{6}, s_{10} \) and \( s_{13} \) are also set as accepting states.
5. Rule LITERAL-SELF-ASSN adds a self transition at a state \( s \) on a symbol \( op \in \hat{O}_{\text{PC}} \), such that \( \hat{M}_{\text{map}}(s) \) is stable with respect to \( op[\text{assume/assert}] \). For example, transitions \( \delta(s_9, op) = \{s_9\} \) where \( op \in \hat{O}_{\text{PC}} \setminus \{s, A, P\} \).

The following lemma relates \( \hat{R}_{\text{map}}(s) \) at any state to the set of words accepted by \( s \) in this AFA.

**Lemma 3:** Given a \( \sigma \in L(\hat{A}(P)) \) and \( \phi \), let \( \hat{A}_{\sigma, \phi} \) be the AFA satisfying Definition 1. For every state \( s \) of this AFA, the condition \( \text{rev}(\hat{R}_{\text{map}}(s)) \cap \text{acc}(s) \) holds.

A detailed proof of this lemma is given in Appendix C. This lemma uses the reverse of \( \hat{R}_{\text{map}}(s) \) in its statement because the weakest precondition of a sequence is constructed by scanning it from the end. This can be seen in the transition rule LITERAL-ASSN. As a corollary, \( \text{rev}(\sigma) \) is also accepted by this AFA because by Definition 1 \( \hat{R}_{\text{map}}(s_0) \) is \( \sigma \).

**B. Constructing the weakest precondition from \( \hat{A}_{\sigma, \phi} \)**

After constructing \( \hat{A}_{\sigma, \phi} \) the rules given in Figure 8 are used to inductively construct and assign a formula, \( \hat{M}_{\text{map}}(s) \), to every state \( s \) of \( \hat{A}_{\sigma, \phi} \). Figure 7 shows the AFA of Figure 6 where states are annotated with formula \( \hat{M}_{\text{map}}(s) \). This formula is shown in the ellipse beside every state. For better readability we do not show \( \hat{R}_{\text{map}}(s) \) in this figure.

Following Rule BASE-CASE \( \hat{M}_{\text{map}} \) of \( s_6, s_{12} \) and \( s_{13} \) are set to false whereas \( \hat{M}_{\text{map}}(s_{10}) \) is set to \( \text{flag}_2 = \text{false} \). By Rule LITERAL-ASSN \( \hat{M}_{\text{map}} \) of \( s_5, s_8 \) and \( s_{11} \) are also set to false. After applying Rule DISJ-CASE for transition \( \delta(s_9, e) = \{s_{10}, s_{11}\} \), \( \hat{M}_{\text{map}}(s_9) \) is set to \( \text{flag}_2 = \text{false} \). Similarly, using Rule CONJ-CASE we get \( \hat{M}_{\text{map}}(s_7) \) as false. Finally, \( \hat{M}_{\text{map}}(s_{10}) \) is also set to false. \( \hat{M}_{\text{map}} \) constructed inductively in this manner satisfies the following property:

**Lemma 4:** Let \( \hat{A} \) be an AFA constructed from a trace and a post condition as in Definition 1 then for every state \( s \) of this AFA and for every word \( \sigma \) accepted by state \( s \), \( \hat{M}_{\text{map}}(s) \) is logically equivalent to \( \wp(\text{rev}(\sigma)[\text{assume/assert}], \hat{M}_{\text{map}}(s)) \).

Here we present the proof outline. Detailed proof is given in Appendix C. First consider the accepting states of \( \hat{A} \). For example, states \( s_6, s_{10}, s_{12} \) and \( s_{13} \) of Figure 7. Following the definition of an accepting state and by the self-loop adding transition rule LITERAL-SELF-ASSN ev-
∀ \sigma \in \sigma_1, \ldots, \sigma_k \end{align*}

Every word \sigma accepted by such an accepting state \sigma must satisfy \wp(\sigma[\text{assume/\text{assert}}], \text{AMap}(\sigma)) = \text{AMap}(\sigma). Therefore, setting \text{HMap}(\sigma) as \text{AMap}(\sigma) for these accepting states, as done in Rule \text{BASE-CASE} completes the proof for accepting states.

Now consider a state \sigma with transition \delta(\sigma, \epsilon) = \{s_1, \ldots, s_k\}, created using Rule \text{COMPOUND-ASSN} and let \sigma be a word accepted by \sigma. By construction, \sigma must be a universal state and hence \sigma must be accepted by each of \sigma_1, \ldots, \sigma_k as well. Using this lemma inductively on successor states \sigma_1, \ldots, \sigma_k (induction on the formula size) we get \wp(\sigma[\text{assume/\text{assert}}], \text{AMap}(\sigma_i)) = \text{HMap}(\sigma_i) for all i \in \{1, \ldots, k\}. Now we can apply Property \text{1} depending on whether \text{AMap}(\sigma) is a conjunction or a disjunction of \text{AMap}(\sigma_i). By replacing \text{AMap}(\sigma) with \bigvee_k \text{AMap}(\sigma_i)(\land_k \text{AMap}(\sigma_i)) and \text{HMap}(\sigma) with \bigvee_k \text{HMap}(\sigma_i)(\land_k \text{HMap}(\sigma_i)) completes the proof. Note that, making \sigma as a universal state when \text{AMap}(\sigma) is either a conjunction or a disjunction allowed us to use Property \text{1} in this proof. Otherwise, if we make \sigma an existential state when \text{AMap}(\sigma) is a disjunction of formulae then we can not prove this lemma for states where \text{HMap}(\sigma) is constructed using Rule \text{DISJ-CASE}.

This lemma serves two purposes. First, it checks the correctness of a trace \sigma w.r.t. a safety property for which this AFA was constructed. If \text{HMap}(s_0) \land I is unsatisfiable, as in our Peterson’s example trace, then \sigma is declared as correct. Second, it guarantees that every trace accepted by this AFA, that is present in the set of all traces of \mathcal{P}, is also safe and hence we can skip proving their correctness altogether. Removing such traces is equivalent to subtracting the language of this AFA from the language representing the set of all traces. Then a natural question to ask is if we can increase the set of accepted words of this AFA while preserving Lemma 4.

C. Enlarging the set of words accepted by \text{AMap} of Figure 10 shows an example trace \sigma = abcd\epsilon obtained from the parallel composition of some program P. Figure 11 shows the AFA constructed for \sigma and \phi as \mathcal{S} < t \land z < x. From Lemma 4 we get \wp(\sigma, \phi) as false. Note that the \wp(\sigma, \epsilon < t) and \wp(\sigma, z < x) are unsatisfiable, i.e. we have two ways to derive the unsatisfiability of \wp(\sigma, \phi); one is due to the operation d, and the other is due to the operation a followed by operation e. In this example, any word that enforces either of these two ways will derive false as the weakest precondition. For example, the sequence \sigma’ = a \text{dec} be is not accepted by the AFA of Figure 11 but the condition \wp(\delta(\sigma’), \neg\phi) = false follows from \wp(\delta(d, \neg\phi), e) = false which is already captured in the AFA of Figure 11. Note that states s_1 and s_2 in Figure 11 are annotated with unsatisfiable \text{HMap} assertion. It seems sufficient to take any one of these branches to argue the unsatisfiability of \text{HMap}(s_0) because \text{HMap}(s_0), by definition, is a conjunction of \text{HMap}(s_1) and \text{HMap}(s_2). Therefore, if we convert s_0, a universal state, to an existential state then the modified AFA will accept a\text{dec}be. Let us look at Algorithm 1 to see the steps involved in this transformation. This algorithm picks a universal state \sigma such that \text{AMap}(\sigma) is a conjunction of clauses and only a subset of its successors are sufficient to make \text{HMap}(\sigma) unsatisfiable. State \sigma_0 of Figure 11 is one such state. For each such minimal subsets of its successors, this algorithm creates a universal state, as shown in Line 5 of this algorithm. It is easy to see that \text{HMap}(s_0) is also unsatisfiable. Before adding \delta(s_0, \epsilon) = \{s_1', \ldots, s_n\} transition in AFA this algorithm sets \text{AMap}(s_0) as \land_k \text{AMap}(s_i). By construction, every word accepted by \sigma must be accepted by \sigma_1', \ldots, s_n'. Each of these states s_1', \ldots, s_n' satisfy Lemma 4. Hence Lemma 4 continues to hold for these newly created universal states as well. Now consider a newly created transition (s, \epsilon, U) in Line 12 for any state \sigma’ \in U, \text{AMap}(\sigma’) logically implies \text{AMap}(\sigma’’) because \sigma’’ represents a subset of the original successors.
of $s$, viz. $s_1, \ldots, s_8$. As $s$ is now an existential state, any word accepted by $s$, say $\sigma'$, is accepted by at least one state in $U$, say $s'$. Using Lemma 4 on $s'$, $\text{HM}(s')$ is logically equivalent to $\wp(\text{rev}(\sigma')[\text{assume/assert}], \text{HM}(s'))$. Using unsatisfiability of $\text{HM}(s)$ and $\text{HM}(s')$ and the monotonicity property of the weakest precondition, Property 4 we get that $\text{HM}(s)$ is logically equivalent to $\wp(\text{rev}(\sigma')[\text{assume/assert}], \text{HM}(s))$. This transformation is formally proved correct in Appendix E.

Adding More transitions to $A_{\sigma, \phi}$ using the Monotonicity Property of the Weakest Precondition

We further modify $A_{\sigma, \phi}$ by adding more transitions. For any two states $s$ and $s'$ such that $\text{HM}(s)$ and $\text{HM}(s')$ are literals, both $\text{HM}(s)$ and $\text{HM}(s')$ are unsatisfiable, and there exists a symbol $a$ (can be $\epsilon$ as well) such that $\wp(a[\text{assume/assert}], \text{HM}(s))$ logically implies $\text{HM}(s')$, an edge labeled $a$ is added from $s$ to $s'$. This transformation also preserves Lemma 4 following the same monotonicity property. Property 2 used in the previous transformation. Similar argument holds when $\text{HM}(s)$ and $\text{HM}(s')$ are valid and $\text{HM}(s') \Rightarrow \wp(a, \text{HM}(s))$ holds. The rules of adding edges are shown in Figure 13.

Figure 10 shows the AFA of Figure 9 modified by above transformations. Rule [RULE-UNSAT] adds an edge from $s_4$ to $s_8$ on symbol $\epsilon$ because $\text{HM}(s_4)$ and $\text{HM}(s_8)$ are unsatisfiable and $\wp(\epsilon, \text{HM}(s_4))$ logically implies $\text{HM}(s_8)$. Same rule also adds a self loop at $s_8$ on operation $A$ and a self loop at $s_2$ on operation $P$. Transformation by Algorithm 1 removes the transition from $s_7$ to $s_9$ and all other states reachable from $s_9$. Now consider a trace $\text{rev}(abpq\text{Arcs})$ that is accepted by this modified AFA in Figure 12 but was not accepted by the original AFA of Figure 9. Note that $\wp(abpq\text{Arcs}, \neg(\ell_2 = 2))$ is unsatisfiable and this is a direct consequence of Lemma 4. Because of the transformations presented in this sub-section we do not need to reason about this trace separately. This transformation is formally proved correct in Appendix E.

D. Putting All Things Together For Safety Verification

In Algorithm 2 all the above steps are combined to check if all the SC executions of a concurrent program $P$ satisfy the safety properties specified as assertions. Proof of the following theorem is given in Appendix G.

**Theorem 1:** Let $P = (p_1, \ldots, p_n)$ be a finite state program (with or without loops) with associated assertion maps $\text{Assr}_{p_i}$. All assertions of this program hold iff Algorithm 2 returns yes. If the algorithm returns a word $\sigma$ then at least one assertion fails in the execution of $\sigma$. (Note that $\text{Assr}_{p_i}$ is a set of formulas corresponding to the assertion maps of $p_i$)
IV. EXPERIMENTAL EVALUATION

We implemented our approach in a prototype tool, ProofTraPar. This tool reads the input program written in a custom format. In future, we plan to use off-the-shelf parsers such as CIL or LLVM to remove this dependency. Individual processes are represented using finite state automata. We use an automata library, libFAUDES [6] to carry out operations on automata. As this library does not provide operations on AFA, mainly complementation and intersection, we implemented them in our tool. After constructing the AFA from a trace we first remove ε transitions from this AFA. This is followed by adding additional edges in AFA using proposed transformations. Instead of reversing this AFA (as in Line 11 of Algorithm 2) we subtract it with an NFA that represents the reversed language of the set of all traces. This in turn allows us to subtract it from a trace but unlike CTP it only captures those different interleavings which guarantee the same proof outline. Recently in [9], a formalism called HB-formula has been proposed to capture the set of happens-before relations in a set of executions. This relation is then used for multiple tasks such as synchronization synthesis[2], bug summarization and predicate refinement. Since the AFA constructed by our algorithm can also be represented as a boolean formula (universal states correspond to conjunction and existential states correspond to disjunction) that encodes the ordering relations among the participating events, it will be interesting to explore other usages of this AFA along the lines of [9].

VI. CONCLUSION AND FUTURE WORK

We presented a trace partitioning based approach for verifying safety properties of a concurrent program. To this end, we introduced a novel construction of an alternating finite automaton to capture the proof of correctness of a trace in a program. We also presented an implementation of our algorithm which compared competitively with existing state-of-the-art tools. We plan to extend this approach for parameterized programs and programs under relaxed memory models. We also plan to investigate the use of interpolants with weakest precondition axioms to incorporate abstraction for handling infinite state programs.

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Appendix

A. Proof of Lemma

We prove it by induction on \( n \).

1) Base case \( |\sigma| = 0 \): If \( |\sigma| = 0 \) then \( \wp(\sigma[\text{assume/assert}], \neg\phi) \neq \neg\phi \). If \( \neg\phi \land \mathcal{I} \) is unsatisfiable then \( \mathcal{I} \) satisfies \( \phi \). Hence proved.

2) Induction step, \( |\sigma| = n + 1 \): Let \( \sigma = \sigma'.a \). If \( \wp(\sigma',a[\text{assume/assert}], \neg\phi) \land \mathcal{I} \) is unsatisfiable then following cases can happen based on \( a \).

- \( a : x \leftarrow E \): If \( \wp(\sigma',a[\text{assume/assert}], \neg\phi) \land \mathcal{I} \) is unsatisfiable then \( \wp(\sigma'[\text{assume/assert}], \wp(a, \neg\phi)) \land \mathcal{I} \) is also unsatisfiable. By substituting \( \wp(a, \neg\phi) \) with \( \neg\phi[E/x] \) we get that \( \wp(\sigma'[\text{assume/assert}], \neg\phi[E/x]) \land \mathcal{I} \) is unsatisfiable. Using IH on \( \sigma' \) it implies that after executing \( \sigma' \) from \( \mathcal{I} \) the resultant state either does not terminate or terminates in a state satisfying \( \phi[E/x] \). If \( \sigma' \) does not terminate then so does the execution of \( \sigma \) starting from \( \mathcal{I} \). If \( \sigma' \) terminates in a state satisfying \( \phi[E/x] \) then by the definition of the weakest precondition, execution of \( a \) from this state will satisfy \( \phi \). Hence proved.

- \( a : \text{assume}(\phi') \): If \( \wp(\sigma',a[\text{assume/assert}], \neg\phi) \land \mathcal{I} \) is unsatisfiable then \( \wp(\sigma'[\text{assume/assert}], \wp(a, \neg\phi)) \land \mathcal{I} \) is also unsatisfiable. By substituting \( \wp(a, \neg\phi) \) with \( \phi' \land \neg\phi \) we get that \( \wp(\sigma'[\text{assume/assert}], \phi' \land \neg\phi) \land \mathcal{I} \) is unsatisfiable. Using IH on \( \sigma' \) it implies that after executing \( \sigma' \) from \( \mathcal{I} \) the resultant state either does not terminate or terminates in a state satisfying \( \neg\phi \lor \phi' \). If \( \sigma' \) does not terminate then the execution of \( \sigma \) from \( \mathcal{I} \) does not terminate as well. If \( \sigma' \) terminates in a state satisfying \( \neg\phi \) then the execution of \( a \) blocks and hence the execution of \( \sigma \) does not terminate. If \( \sigma' \) terminates in a state satisfying \( \phi' \) but \( \neg\phi \) does not hold then \( \phi \land \phi' \) must hold. Execution of \( \text{assume}(\phi') \) acts as nop instruction and the resultant state satisfies \( \phi \). Hence proved.

- \( a : \text{lock}(x) \): As weakest precondition of \( \text{lock}(x) \) is obtained from the weakest precondition of assignment and assume instruction hence the similar reasoning works for this case.

B. Proof of Lemma

Proof: Let us prove it by induction on the length of \( \sigma \).

1) Base case, \( |\sigma| = 0 \): When the length of \( \sigma \) is 0 and \( \mathcal{I} \land \neg\phi \) is satisfiable then \( \mathcal{I} \) does not satisfy \( \phi \). Hence proved.

2) Induction Step, \( |\sigma| = n + 1 \): Let \( \sigma = \sigma'.a \). Following case can happen based on the type of \( a \).

- \( a : x \leftarrow E \): If \( \wp(\sigma[\text{assume/assert}], \neg\phi) \land \mathcal{I} \) is satisfiable then \( \wp(\sigma'[\text{assume/assert}], \wp(a, \neg\phi)) \land \mathcal{I} \) is also satisfiable. By substituting \( \wp(a, \neg\phi) = \neg\phi[E/x] \) we get that...
C. Proof of Lemma 3

Proof: We use induction for this proof. Let us use the following ordering on the states of $A_{φ}$. For any two states $s$ and $s'$, $s < s'$ if $|RMap(s)| < |RMap(s')|$ or if lengths are same then $AMap(s)$ is a sub formula of $AMap(s')$. Any two states which are not related by this order, put them in any order to make $<$ as a total order. It is clear that the smallest state in this total order must be one of the accepting state. Now we are ready to proceed by induction using this total order.

• Base case: For every accepting state $s \in S_F$, by Point 3 of Definition 1 the condition $wp(op, AMap(s)) = AMap(s)$ holds for every $op \in E\overline{L}(RMap(s))$. Further, By transition rule $[\text{LITERAL-Self-Assn}]$ of this AFA, a self transition must be there for all such $op \in E\overline{L}(RMap(s))$ and hence the condition $\text{rev}(RMap(s)) \in acc(s)$ holds (because these transitions can be taken in any order to construct the required word).

• Induction step; Following possibilities exist for the state $s$.
  - $s$ is a universal state; By construction, there should be states $s_1, \ldots, s_k$ such that $(s, e, \{s_1, \ldots, s_k\})$ is a transition. By our induction ordering, $s_1, \ldots, s_k$ are smaller than $s$ and hence we apply IH on them to get that $\text{rev}(RMap(s_i)) \in acc(s_i)$ for $i \in \{1 \ldots k\}$. However, by the transition rule $[\text{Compound-Assn}]$ $\text{RMap}(s) = RMap(s_1) = \ldots = RMap(s_k)$ and hence $\text{rev}(RMap(s)) \in acc(s_i)$ for $i \in \{1 \ldots k\}$. By the definition of $acc(s)$ for a universal state, $acc(s)$ is intersection of the sets $acc(s_i)$ for $i \in \{1 \ldots k\}$ and hence we get the required result, viz. $\text{rev}(RMap(s)) \in acc(s)$.
  - $s$ is an existential state; If $s$ is an accepting state then Base case holds here. Consider the case when $s$ is not an accepting state. It should have a successor state $s'$ such that $(s, op, \{s\})$ is a transition. By transition rule $[\text{LITERAL-Assn}]$ $RMap(s) = RMap(s').op.s''$ such that $wp(s'')[\text{assume/assert}, AMap(s)] = AMap(s)$. By transition rule $[\text{LITERAL-Self-Assn}]$ $s$ will have self loop transitions on all symbols in $s'"(*)$. Applying IH on $s'$ gives that $\text{rev}(RMap(s')) \in acc(s')$(*). Because of the transition $(s, op, \{s'\})$, $op.acc(s') \subseteq acc(s)$. This along with (#) gives us $\text{rev}(RMap(s').op.s'') \in acc(s)$ or equivalently $\text{rev}(RMap(s)) \in acc(s)$. Hence proved.

D. Proof of Lemma 4

Proof: We use induction for this proof. Same as in the previous proof, let us use the following ordering on the states of $A$. For any two states $s$ and $s'$, $s < s'$ if $|RMap(s)| < |RMap(s')|$ or if lengths are same then $AMap(s)$ is a sub formula of $AMap(s')$. Any two states which are not related by this order, put them in any order to make $<$ as a total order. It is clear that the smallest state in this total order must be one of the accepting state. Now we are ready to proceed by induction using this total order.

• Base case, By definition of the accepting state in AFA construction, Point 3 of Definition 1 and the self loop transition rule, Rule $[\text{LITERAL-Self-Assn}]$ we know that for every word $s' \in acc(s)$, $wp(s')[\text{assume/assert}, AMap(s)] = AMap(s)$. Rule $[\text{Base-CASE}]$ of Figure 3 sets $HMap(s)$ same as $AMap(s)$ for such states hence the statement of this lemma follows for the accepting states.

• Induction step; We pick a state $s$ such that one of the following holds,
  1) $s$ is a universal state; By construction, there should be states $s_1, \ldots, s_k$ such that $(s, e, \{s_1, \ldots, s_k\})$ is a transition. Let $s$ be a word accepted by $s$ then by the definition of accepting set of words of a universal states, $s$ must be accepted by each of $s_1, \ldots, s_k$. By our induction ordering, $s_1, \ldots, s_k$ are smaller than $s$ and hence we apply IH on them to get that $\text{wp}(\text{rev}(σ)[\text{assume/assert}, AMap(s_i)]) = HMap(s_i)$ for $i \in \{1 \ldots k\}$. Two cases arise based on whether
    - $AMap(s_i)$ is a conjunction of $AMap(s_i)$ for $i \in \{1 \ldots k\}$; Following Rule $[\text{Conj-Case}]$ we set $HMap(s) = \bigwedge_i HMap(s_i)$ and $wp(\text{rev}(σ)[\text{assume/assert}, AMap(s)]) = HMap(s)$ then follows from the Property 1 using conjunction, of the weakest precondition.
    - $AMap(s_i)$ is a disjunction of $AMap(s_i)$ for $i \in \{1 \ldots k\}$; Following Rule $[\text{Conj-Case}]$ we set $HMap(s) = \bigwedge_i HMap(s_i)$ and $wp(\text{rev}(σ)[\text{assume/assert}, AMap(s)]) = HMap(s)$ then follows from the Property 1 using disjunction, of the weakest precondition.
2) $s$ is an existential state; If $s$ is an accepting state then the same argument as used in the Base case holds. If $s$ is not an accepting state then the only outgoing transition from $s$ is of the form $(s, op, \{s'\})$. By rule [LITERAL-ASSN(*)]. Now consider a word $\sigma \in acc(s)$. $\sigma$ must be of the form $\sigma'' . op . \sigma'$ where $wp(\sigma''[\text{assume/assert}], AMap(s)) = AMap(s)(*)$ (because of the self transitions constructed from Rule [LITERAL-SELF-ASSN] and $\sigma' \in acc(s')$). Therefore, $wp(\sigma)[\text{assume/assert}, AMap(s)) = \equiv wp(\sigma'' . op . \sigma'[\text{assume/assert}])$. $wp(\sigma'' . op . \sigma'[\text{assume/assert}]) = \equiv wp(\sigma'[\text{assume/assert}])$. $wp(\sigma'[\text{assume/assert}]) = \equiv wp(\sigma'[\text{assume/assert}])$ (using (*)). $wp(\sigma'[\text{assume/assert}]) = \equiv wp(\sigma'[\text{assume/assert}])$ (using weakest precondition definition) $wp(\sigma'[\text{assume/assert}]) = \equiv wp(\sigma'[\text{assume/assert}])$. $wp(\sigma'[\text{assume/assert}])$ (using Transition rule [LITERAL-ASSN].

As $\sigma' \in acc(s')$ this is same as $HMap(s')$ by applying IH on $s'$. As $HMap(s)$ is same as $HMap(s')$, as done in Rule [LITERAL-ASSN] we prove this case as well.

E. Proof of Correctness of Transformation-I

**Lemma 5:** Let $\tilde{A}$ be an automaton constructed from a trace and a post condition as defined in Definition 11 and further modified by Algorithm 11 then for every state $s$ of this AFA and for every word $\sigma$ accepted by state $s$, $HMap(s)$ is logically equivalent to $wp(\sigma)[\text{assume/assert}, AMap(s))$.

**Proof:** Proof of this lemma is very similar to the proof of Lemma 4 given in Appendix . Here we only highlight the changes in the proof. Note that this transformation converts some universal states to existential states. Let $s$ be one such state that was converted from universal to existential state. Let $(s, e, \{s_1, \cdots, s_k\})$ was the original transition in the AFA which got modified to $(s, e, \{s_{u_1}, \cdots, s_{u_n}\})$ where $s_{u_i}$ are newly created universal states in Line 5 of Algorithm 11. By construction, $HMap(s_{u_i})$ is unsatisfiable for each of these $s_{u_1}, \cdots, s_{u_n}$. Let $\sigma$ be a word accepted by $s$ after converting it to existential state. By acceptance conditions, $\sigma$ must be accepted by at least one state, say $s_{u_m}$ in the set $\{s_{u_1}, \cdots, s_{u_n}\}$. By IH on $s_{u_m}$ we get $wp(\sigma[\text{assume/assert}], AMap(s_{u_m})) = HMap(s_{u_m})(**)$. Further, by construction $AMap(s)$ implies $HMap(s_{u_m})$. This fact, along with the monotonicity property of the weakest precondition, Property 2, we get that $wp(\sigma[\text{assume/assert}], AMap(s))$ is unsatisfiable and hence same as $HMap(s)$.

F. Proof of Correctness of Transformation-II

**Lemma 6:** Let $\tilde{A}$ be an automaton constructed from a trace and a post condition as defined in Definition 1 and further modified by adding edges as discussed above then for every state $s$ of this AFA and for every word $\sigma$ accepted by state $s$, $HMap(s)$ is logically equivalent to $wp(\sigma)[\text{assume/assert}, AMap(s))$.

**Proof:** As a result of adding edges in this transformation, we can not use the ordering among states as done for earlier proofs. This is because, now a transition $(s, op, S)$ does not guarantee that the states in the set $S$ are smaller then $s$ and hence it will not be possible to apply IH directly. Therefore in this proof we apply induction on the length of $\sigma'$ accepted by some state $s$.

- Induction step: Let $s \in A$ and $\sigma \in acc(s)$ such that $|\sigma| = m + 1$. Either $s \in S_3$ or $s \in S_4$. If $s \in S_3$ and $\sigma \in acc(s)$ then there exists a state $s'$ such that $(s, op, \{s'\}) \in \delta$ and $\sigma' \in acc(s')$, where $\sigma = \sigma'' . op . \sigma'$ and $wp(\sigma''[\text{assume/assert}], AMap(s)) = AMap(s')(**)$. Based on this transition $(s, op, \{s'\}) \in \delta$ we have the following sub-cases,

  - $(s, op, \{s'\})$ was added by the this transformation virtue of one of the following conditions,
    - $HMap(s)$ and $HMap(s')$ are unsatisfiable and $wp(op[\text{assume/assert}], AMap(s)) \Rightarrow AMap(s'(\text{Rule RULE-UNSAT})$. By IH on $\sigma'$ we have $wp(\sigma'[\text{assume/assert}], AMap(s'))$ is logically equivalent to $HMap(s')$. Using Property 2 (conjunction part) and the assumption $wp(op[\text{assume/assert}], AMap(s)) = AMap(s')$. $\Rightarrow AMap(s')$ we get $wp(\sigma'[\text{assume/assert}], wp(op, AMap(s)))$ is unsatisfiable and same as $HMap(s)$. Using (**), $wp(\sigma'[\text{assume/assert}], wp(op, rev(\sigma')))$. $wp(op, rev(\sigma'))(AMap(s'))$ is unsatisfiable and same as $HMap(s)$. By replacing $\sigma = \sigma'' . op . \sigma'$ we get the required proof.
    - $HMap(s)$ and $HMap(s')$ are valid and $AMap(s') \Rightarrow wp(op[\text{assume/assert}], AMap(s)) (\text{Rule RULE-VALID})$. By IH on $\sigma'$ we have $wp(\sigma'[\text{assume/assert}], AMap(s'))$ is logically equivalent to $HMap(s')$. Using property 2 (disjunction part) and the assumption $AMap(s') \Rightarrow wp(op[\text{assume/assert}], AMap(s))$ we get $wp(\sigma'[\text{assume/assert}], wp(op, AMap(s)))$ is valid and same as $HMap(s)$. Using (**), or replacing $\sigma = \sigma'' . op . \sigma'$ we get the required result and hence proved.

- If this transition was already in $\delta$; we can use the same reasoning as used in the proof of Lemma 4 to show that $wp(\sigma)[\text{assume/assert}, AMap(s))$ is logically equivalent to $HMap(s)$

- If $s \in S_4$ then similar argument goes as in the proof of Lemma 4 because no new transition gets added from these states as a result of this transformation.

G. Proof of Theorem 11

**Proof:**
Let us first prove that this algorithm terminates for finite state programs. For finite state programs the number of possible assertions used in the construction of AFA are finite and hence only a finite number of different AFA are possible. It implies the termination of this algorithm.

Following Lemma 4 and the fact that $\text{HMap}(s_0) = \neg \phi$, every word $\sigma'$ accepted by this AFA, equivalently written as $\sigma' \in \text{acc}(s_0)$, satisfies $\text{wp}(\text{rev}(\sigma')[\text{assume/assert}], \neg \phi) = \text{HMap}(s_0)$ (*). By Lemma 5 and the fact that $\text{RMap}(s_0) = \sigma$ we get $\text{rev}(\sigma) \in \text{acc}(s_0)$ (**). Combining (** and (*), we get $\text{wp}(\text{rev}(\sigma)[\text{assume/assert}], \neg \phi) = \text{HMap}(s_0)$ or equivalently $\text{wp}(\sigma[\text{assume/assert}], \neg \phi) = \text{HMap}(s_0)$.

- If $I \land \text{HMap}(s_0)$ is satisfiable (Line 6) then $I \land \text{wp}(\sigma[\text{assume/assert}], \neg \phi)$ is satisfiable as well. Following Lemma 2 we got a valid error trace which is returned in Line 8.
- If $I \land \text{HMap}(s_0)$ is unsatisfiable then by Lemma 1 this trace is provably correct. Now we apply transformations of Section III-C on the AFA to increase the set of words accepted by it. The final AFA is then reversed and subtracted from the set of executions seen so far. Lemma 4 ensures that for all such words $\sigma'$ the condition $I \not= \text{wp}(\sigma', \neg \phi)$ holds and therefore none of them violate $\phi$ starting from the initial state. Therefore in every iteration only correct set of executions are being removed from the set of all executions. Therefore when this loop terminates then all the executions have been proved as correct.