Abstract—A transparent digital twin (DT) is designed for output control using the belief rule base (BRB), namely, DT-BRB. The goal of the transparent DT-BRB is not only to model the complex relationships between the system inputs and output but also to conduct output control by identifying and optimizing the key parameters in the model inputs. The proposed DT-BRB approach is composed of three major steps. First, BRB is adopted to model the relationships between the inputs and output of the physical system. Second, an analytical procedure is proposed to identify only the key parameters in the system inputs with the highest contribution to the output. Being consistent with the inferencing, integration, and unification procedures of BRB, there are also three parts in the contribution calculation in this step. Finally, the data-driven optimization is performed to control the system output. A practical case study on the Wuhan Metro System is conducted for reducing the building tilt rate (BTR) in tunnel construction. By comparing the results following different standards, the 80% contribution standard is proved to have the highest marginal contribution that identifies only 43.5% parameters as the key parameters but can reduce the BTR by 73.73%. Moreover, it is also observed that the proposed DT-BRB approach is so effective that iterative optimizations are not necessarily needed.

Index Terms—Belief rule base (BRB), building tilt rate (BTR), output control, transparent digital twin (DT).

I. INTRODUCTION

The digital twin (DT) has been a powerful tool in modeling the complex and hidden relationships between system inputs and the output [1], [2]. Ever since it was proposed, DT has been successfully applied in solving many practical problems [3]. DT provides an interface between the practical conditions, which are mostly represented by data, and the theoretical model that does not physically exist [5]. Among many advantages, DT has shown superior expandability in varied conditions with different requirements because DT does not designate the specific model which can be either an existing model [5], [6] or a customized one [7] as long as the requirements of the practical problems are met. Either way, DT would inherit the characteristics from the model it adopts or customizes.

Besides modeling the hidden relationships between the system inputs and output, the more important goal of this study is to control the system output, which means to automatically maintain, adjust, or optimize the system output through a deep understanding of the nonsmooth nonlinearity of complex systems [8], [9]. Comparatively, modeling the hidden relationships for complex systems is a goal on a relatively lower and fundamental level while output control is a more important goal on a higher level. For example, the building tilt rate (BTR) in tunnel-induced damages is a typical hazard that can cause large economic losses or even safety concerns [10]. Naturally, it is important to monitor and predict the BTR, which, however, is only a lower level goal. Comparatively, it is more important to reduce the BTR as much as possible, which is the goal on a higher level.

To achieve the goal of output control, it requires optimizing the key parameters with maximum contributions to the system output. In engineering practices, field engineers normally first tune an operational parameter, then the construction would go on for a while, and finally, the BTRs are measured again to check if it initially tuned correctly. If not, then the field engineer may further tune the same parameter or another parameter. This process repeats until the BTRs stay within the safety limits [11], [12]. The above example indicates that output control is achieved through optimizing key parameters. From a theoretical perspective, it is also noticeable that all parameters cannot be optimized simultaneously for two reasons: 1) optimization resources are limited and 2) it would not be consistent with practical conditions. Therefore, the problem becomes: how to identify key parameters.

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Leilei Chang is with the School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China, and also with the School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639798 (e-mail: leileichang@hotmail.com).

Limao Zhang is with the School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639798 (e-mail: limao.zhang@ntu.edu.sg).

Chao Fu is with the School of Management, Hefei University of Technology, Hefei 230009, China, and also with the Ministry of Education Engineering Research Center for Intelligent Decision-Making & Information System Technologies, Hefei University of Technology, Hefei 230009, China (e-mail: wls_fuchao@163.com).

Yu-Wang Chen is with the Manchester Business School, The University of Manchester, Manchester M15 6PB, U.K. (e-mail: yu-wang.chen@manchester.ac.uk).

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Normally, there are two solutions for identifying the key parameters.

1) One is to use simulation-based sensitivity analysis [13], [14]. Simulation-based sensitivity analysis does not have specific requirements on the adopted model as long as it fits the problem characteristics and produces accurate results. However, it could invite unnecessary uncertainty from two sources: a) the simulation model and b) parameter settings for sensitivity analysis, for example, scales, intervals, etc. As there is unavoidable randomness, the results are not deterministic. As many simulation approaches are black-box approaches, it is inaccessible which could greatly jeopardize the interpretability of the produced results.

2) The other is to use analytical deductions to identify the key parameters for further optimization. The advantage is that a deterministic solution could be produced without inviting unnecessary uncertainty. Consequently, the burden of the adopted model is high because only a model with analytical and deductible procedures can meet this requirement, that is, a transparent white-box model.

To satisfy this requirement, the belief rule base (BRB) is adopted to construct the DT for two reasons: 1) BRB has a superior nonlinear modeling ability and 2) more important, BRB is a white-box model with analytical inferencing, integration, and unification procedures [15]–[17]. Due to this feature, the contribution made by key parameters in the system inputs to the output can be calculated in an analytical, deducible, and deterministic fashion. Finally, only the identified key parameters are optimized for output control via a data-driven optimization process.

The remainder of this article is organized as follows. Section II introduces the background of DT and BRB. The DT-BRB approach is proposed in Section III. Section IV studies a practical case on reducing the BTR in tunnel construction. The model effectiveness is further discussed in Section V. This article is concluded in Section VI.

II. BACKGROUND

A. Digital Twin

DT has been a hot topic since it was proposed [1], [2]. Initially, it was motivated by problems in the industry and engineering fields [3]. Later, DT has demonstrated great advantages in 1) constructing a virtual operational interface to connect the theoretical model that does not exist and the practical system that can provide a large quantity of data, as well as in 2) providing accurate prediction and more balanced solutions for decision makers [3], [5]. In this process, the theoretical foundations have also been extended to multiple disciplines, for example, information sciences, engineering, data, computer sciences, etc. [6], [7]. Although DT does not have a widely accepted definition yet, it has been successfully applied in solving a wide range of problems, for example, product design, production, prognostic and health management (PHM), structural health monitoring (SHM), etc. [1]–[3].

Many studies from different fields have concluded that DTs consists of elements in either three dimensions (i.e., physical product, virtual product, and their connections) or five dimensions (i.e., the physical part, virtual part, connection, data, and service) [3]. In general, this is consistent with the commonly accepted understanding that DT connects the physical aspects with the theoretical aspects using data and models in which the exact makeup of DT may be different according to different problem backgrounds. This character grants DT with extreme feasibility and expandability to a wider range of problems with varied requirements. In other words, DT does not explicitly designate a specific method or model. In practice and also recent studies, different methods have been used to meet the special requirements of different problems, for example, neural network (NN) [6], etc. In other practices, a new model can also be customized to satisfy the special requirements of certain problems [7]. The goal of either adopting an existing model or designing a new one is to satisfy the requirements of a specific problem. Both are feasible as long as the problem requirements are met.

To achieve the goal of output control, DT must not only provide accurate modeling between the system inputs and output but also be able to support analytical analysis, so that the key parameters with the largest contributions to the output can be identified. Considering that DT only inherits the characteristics of the adopted model, DT must adopt an appropriate model so that the goal of output control can be achieved.

In previous studies, the simulation-based sensitivity analysis has been used to identify the key parameters [13]. The results of simulations in different runs are different, that is, uncertain, which are used to provide a holistic view of the problem for decision makers. This characteristic of simulations cannot guarantee a deterministic solution as there would be unavoidable uncertainty [18]. Comparatively, a deterministic result obtained through analytical deduction is the optimal solution. To do so, a white box model should be adopted in DT for output control.

B. Belief Rule Base

In this study, BRB is used as the DT because it is essentially a white box as well as its superior nonlinear modeling ability [15]. BRB is an expert system that can handle different types of information under uncertainty. BRB consists of multiple belief rules in the same belief structure [15]. The kth rule in a BRB system is expressed as

\[
R_k : \text{if } (x_1 \text{ is } A^k_1) \land (x_2 \text{ is } A^k_2) \land \cdots \land (x_M \text{ is } A^k_M) \tag{1}
\]

then \(\{D_1, \beta_{1,k}, \ldots, D_N, \beta_{N,k}\}\) with rule weight \(\theta_k\)

where \(x_m(m = 1, \ldots, M)\) denotes the nth parameter, \(A^k_m(m = 1, \ldots, M; k = 1, \ldots, K)\) denotes the reference values of the nth parameter, \(M\) denotes the number of parameters, \(\beta_{n,k}(n = 1, \ldots, N)\) denotes the belief at the nth degree, \(D_n\), and \(N\) denotes the number of degrees. The nth degree, \(D_n\), has
the utility of $U(D_n)$ if it needs to be transformed into a continuous value. “$\land$” indicates that a conjunctive assumption is adopted. Otherwise, it is presented as “$\lor$” if a disjunctive assumption is used [19].

The superior nonlinearity modeling ability of BRB is built upon its inferencing, integration, and unification procedures [16], [17], as illustrated in Fig. 1. By using belief rules, different types of information under uncertainty are transformed into the same belief structure. For an input with $M$ parameters, $I^b = (I_1, I_2, \ldots, I_M)$, multiple correlated belief rules are activated $R_k$ (see Part 1 in Fig. 1). Then, the activated rules are integrated using the evidential reasoning algorithm [17]. After integration, the result is in a belief distribution, $(D_1, \beta_1), \ldots, (D_N, \beta_N)$ (see Part 2 in Fig. 1). The result in a belief distribution can also be unified into a continuous result $y = \sum_{n=1}^{N} \beta_n U(D_n)$ by considering the utility of each scale (see Part 3 in Fig. 1). Moreover, all of the inferencing, integration, and the unification procedures are analytical and deductible so that the produced results are deterministic. The detailed procedures can be found in [16] and [17] and also Appendix A in the supplementary material.

Moreover, BRB can provide good interpretability because it is essentially a white box. In the belief rules, the complicated nonlinearity that is often hidden in many complex systems can be well represented and easily understood, which normally cannot be modeled using traditional analytical methods, for example, physical models. Thus, human knowledge can also be used as input. As introduced above on the BRB inferencing, integration, and unification procedures, they are also transparent and accessible. Moreover, this characteristic provides an opportunity for further exploration of the inner mechanism of BRB, so that how the input is transformed into the output can be analytically investigated. Due to the above two characteristics, BRB has been successfully applied in multiple theoretical and practical problems, for example, classification [20], medical diagnosis [21], facility evaluation [22], pipeline leakage detection [23], etc.

Due to the superior nonlinearity modeling ability, BRB can support constructing the DT with high accuracy. Due to the analytical procedures of BRB inferencing, integration, and unification procedures, it also fully supports identifying key parameters by calculating the contribution made by each parameter to the system output. Therefore, BRB is adopted in this study as the model engine of DT for output control.

### III. METHODS

#### A. Framework

To achieve the goal of output control via optimizing the key parameters, the challenges are: 1) it is unknown about how the parameters as the system inputs are correlated with the output and 2) optimization resources are limited to support simultaneously optimizing all parameters in the system inputs. Therefore, the following three-step framework is proposed. To conquer the challenge 1), a DT-BRB is constructed in step 1. To conquer the challenge 2), step 2 identifies the key parameters that are then optimized in step 3. The DT-BRB approach with three major steps is given in Fig. 2.

**Step 1 (Transparent Digital Twin Construction):** After dividing the complete dataset into the training and testing datasets, the DT is initialized by employing the BRB which is called DT-BRB. Then, an optimization model of the initial DT-BRB is constructed. Specifically, the optimization objective is to minimize the error between the actual system output and the estimated output of DT-BRB. Then, the restraints of the decision variables are defined accordingly. Finally, an optimization algorithm is designed to solve the optimization model. The optimization efficiency of the constructed DT-BRB is validated using the testing dataset.

The input of step 1 is the historically collected data that characterize the physical system while the output of step 1 is the constructed DT-BRB as the theoretical model. More details are presented in Section III-B.

**Step 2 (Key Parameter Identification):** The inferencing, integration, and unification procedures of DT-BRB are dissected to trace back to its source. Specifically, the contribution of a parameter in the system input to the output can be dissected into three sequential parts, namely: 1) the contribution of a parameter to the belief rule; 2) the contribution of a belief rule to the integrated output in the belief distribution; and 3) the contribution of the integrated output in the belief distribution to the unified output. The three parts are consistent with the BRB inferencing, integration, and unification procedures (see Appendix A in the supplementary material). Finally, the contribution of a parameter can be calculated by combining the results of the three parts.

The inputs of step 2 are the newly collected data and the constructed DT-BRB in step 1 while the output of step 2 is the key parameters. More details can be found in Section III-C.

**Step 3 (Data-Driven Optimization):** After the key parameters are identified, they are optimized. The optimization objective is to control the output, that is, to maximize or minimize the system output. This process is limited to the identified key parameters in step 2 and, therefore, they are the only decision variables. Then, the restraints of the decision variables are defined. Finally, an optimization algorithm is also designed to solve the optimization model.

The inputs of step 3 are the newly collected data, the constructed DT-BRB in step 1, and key parameters identified in

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**Fig. 1.** Inferencing, integration, and unification procedures of BRB.

![Diagram](image-url)
Fig. 2. Framework of the transparent DT-BRB.

Step 2 while the outputs of step 3 are the optimized key parameters and corresponding optimized system outputs. More details can be found in Section III-D.

Remark 1: There are two roles on two levels of DT. The first role is to connect the physical system to a theoretical model. The second role is to achieve output control based on the theoretical model. The first role is on a fundamental level as it provides the basis for the second role that is on a higher level. The second role provides characteristics that must be taken into consideration in achieving the first role.

In this study, the first role of DT is implemented by constructing a DT-BRB model (see step 1) for best modeling practical conditions of complex systems. The second role is achieved by identifying (see step 2) and optimizing (see step 3) key parameters for output control.

From a generic perspective, “the two roles on two levels of DT” also stand. It is true that DT is an effective tool for numerically modeling or simulation, but there is always a higher level goal of DT, either to mitigate unpredictable system behavior [2], to ensure industrial engineering safety [3], or to improve manufacturing effectiveness [4].

Remark 2: The ability of the proposed DT-BRB approach to handle the large-scale data that needs to be analyzed. Concerning the modeling construction stage that uses the historic data, that is, step 1 of DT-BRB, it has the ability of handling data on the scale of $10^3$ to $10^4$ [16], [19], [20]. A more practical means is to construct a representative training dataset at a fixed size using effective sampling techniques.

Concerning the output control stage that uses the newly collected data, it mainly involves steps 2 and 3 of DT-BRB. For step 2, it is an analytical process. Therefore, its computational time can be omitted. For step 3, it is very efficient, for example, the computational time is in the scale of seconds, because it only requires optimizing the identified key parameters in step 2.

It is also worthy of pointing out that practical conditions often cannot produce such “large-scale data.” For example, in the tunnel construction practices, only a dozen sets of data can be collected per day. For another pipeline detection case, a total of 2007 sets of data are collected for over 3 h [23]. In medical diagnosis, only a dozen or even less medical data can be collected per day [21], [22]. For such practical cases, the proposed DT-BRB approach can almost produce results instantly.

Remark 3: Building on Remark 2, the ability of DT-BRB being implemented in real time or online needs to be discussed. For the modeling construction stage with historical data, step 1 of DT-BRB is not in real time because it is built upon historical data during a long period. Therefore, the modeling construction stage normally does not require being implemented in real time. For the output control stage with newly collected data, step 2 of DT-BRB is analytical, therefore it can be implemented in real time. For step 3, theoretically, it cannot be implemented in real time since EAs are used as the optimization engine. However, considering that there is only a limited number of identified key parameters the optimization speed is highly efficient. In this sense, output control in step 3 can be recognized as being implemented in a real-time fashion, especially in practical conditions.

Moreover, the discussion can also be extended to the interaction with the physical system. In the modeling construction stage, that is, step 1 of DT-BRB, there is normally no need for interaction with the physical system unless the initial model needs updating. In the output control stage, that is, steps 2 and 3 of DT-BRB, it is interacting with the physical system as each set of newly collected data is handled separately.

B. Transparent Digital Twin Construction

Step 1.1: Divide the initial dataset into training and testing datasets.

Step 1.2: Construct an initial DT-BRB using both the professional knowledge of the experts and their understanding of the training data.
Step 1.3: Construct an optimization model for the initialized DT-BRB. The optimization objective is to minimize the error between the actual and estimated output by DT-BRB, for example, the root mean-square error (RMSE), as in

$$\text{RMSE} = \sqrt{\frac{1}{P} \sum_{p=1}^{P} (y_p^a - y_p^e)^2}$$

(2)

where $y_p^a$ and $y_p^e$ denote the actual and estimated output by the DT-BRB for the $p$th set of data, respectively. $y_p^e$ is obtained from the fitness calculation process in step 1.4.

The optimization model is expressed as follows:

$$\min \text{RMSE}(A^k_m, \theta_k, \beta_{n,k})$$

s.t.

$$lb_m \leq A^k_m \leq ub_m$$

$$0 \leq \theta_k \leq 1$$

$$0 \leq \beta_{n,k} \leq 1$$

$$\sum_{n=1}^{N} \beta_{n,k} = 1$$

(3)

(4)

(5)

(6)

(7)

where (4) denotes that the reference value of the $m$th parameter in the $k$th rule should be within its lower and upper bounds, that is, $lb_m$ and $ub_m$, respectively. Equations (5) and (6) denote that the range of the initial weight of the $k$th rule and beliefs of the $n$th scale in the $k$th rule, that is, $\theta_k$ and $\beta_{n,k}$, respectively. Equation (7) denotes that the sum of the beliefs in the $k$th rule should be “1” if there is no incomplete information or this expression should hold: $\sum_{n=1}^{N} \beta_{n,k} < 1$.

Step 1.4: Design the optimization algorithm. Multiple algorithms can be used to solve the optimization model, including the deterministic approaches, for example, the Newton approach, or the evolutionary algorithms (EAs), for example, the genetic algorithm [24], the particle swarm optimization [25], or the differential evolutionary algorithm [26], etc. Fig. 2 shows the procedure using an EA as the optimization engine, including the initialization, operation, fitness calculation, selection, and stop-criterion check. Detailed procedures using EAs can be found in Appendix B in the supplementary material.

Step 1.5 (Validation): The optimized DT-BRB is validated using the testing dataset.

Remark 4: Data plays an important role in the proposed DT-BRB approach aside from experts’ knowledge. In step 1, experts’ knowledge is used in determining the system inputs and output as well as their approximate (not 100% accurate) ranges. Comparatively, almost all of the parameters of DT-BRB are optimized using historical data [see (3)–(7)]. Therefore, the initial parameters do not need to be 100% accurate since they can be optimized using the collected data. In further steps, data plays a more important role as experts’ involvement is not needed. Step 2 is an analytical process that is automatically implemented with newly collected data while step 3 also relies on the newly collected data because DT-BRB has already been constructed in step 1. Furthermore, objectivity is preserved by widely using data to represent the physical system.

Remark 5: Although faults are not explicitly considered in model construction in step 1, the ability to improve the modeling accuracy [i.e., reducing the RMSE in (3)] is used as a generic ability to make up for the ability to tolerate faults. If the constructed DT-BRB has the ability to achieve higher accuracy, it means that the system output can be accurately predicted, regardless of whether there are faults in the inputs or the system outputs. Multiple past studies [27]–[29] have specifically discussed this topic and demonstrated that BRB (or its theoretical basis, the D-S evidence theory) has the ability to dealing with faults.

Moreover, we should pay more attention to the conditions and number of faulty parameters in the system inputs, for example, the total number of parameters versus the number of faulty parameters. Furthermore, as the contribution of parameters is a major topic of this study, whether faults occur in a parameter with a higher level or lower level contribution should also be considered. Remark 8 gives more in the contexts of the case study.

Remark 6: The rules in the DT-BRB can be generated automatically. There are two procedures of rules generation, that is: 1) rule initialization and 2) further optimization. For rule initialization, it can be implemented either by experts or randomly, preferably evenly distributed in the solution space, which is by definition an automatic process. For the rule optimization, it is implemented by learning from the training dataset, which is by definition also an automatic process. Moreover, by employing EAs as the optimization engine, randomly initialized rules are also helpful in exploring the solution space, avoiding local optimality, and saving optimization resources [30], [31].

C. Key Parameter Identification

Definition 1: One or multiple key parameter(s) in the $p$th input $\mathbf{I}_m^p$ denotes the one(s) with the biggest contribution $\text{ctr}(\mathbf{I}_m^p, y_p)$ to the unified $p$th output $y_p$.

By exploring the BRB inferencing, integration, and unification procedures (see Section II-B and Appendix A in the supplementary), the contribution of the $m$th parameter in the $p$th input $\mathbf{I}_m^p$ to the $p$th output $y_p$, $\text{ctr}(\mathbf{I}_m^p, y_p)$, can be divided into three parts: 1) the contribution of the $m$th parameter to an activated rule $\text{ctr}(\mathbf{I}_m^p, R_k)$; 2) the contribution of the activated rule to the $n$th scale in the output in the belief distribution $\text{ctr}(R_k, \beta_n)$; and 3) the contribution of the output in the belief distribution to the unified output $\text{ctr}(\beta_n, y_p)$. According to the above analysis, and to be consistent with the three parts in Fig. 3, Fig. 4 shows the three parts of the key parameter identification procedure.

Definition 2: The contribution of the $m$th parameter in the $p$th input $\mathbf{I}_m^p$ to the $k$th belief rule is calculated based on the matching degree concerning the $m$th attribute between the input and $k$th belief rule, $a_{mk}^p$, in comparison with the integrated matching degree of $M$ parameters, $a_k$. 
Step 2.1: The matching degree of the $m$th parameter $\alpha^k_m$ is calculated by comparing the $m$th parameter in the input and the $k$th rule. By repeating $M$ times for $M$ parameters, the integrated matching degree $ak$ is then derived. More details can be found in Appendix A in the supplementary material.

Step 2.2: According to Definition 2, the contribution of the $m$th parameter $Ip^m_k$ in the $m$th input to the $k$th rule $R_k$ can be calculated by

\[
ctr(Ip^m_k, R_k) = \frac{\alpha^k_m}{ak}\]

where $\alpha^k_m$ denotes the matching degree of the $m$th parameter in the input $Ip^m_k$ and the $k$th rule, and $ak$ denotes the integrated matching degree of $M$ parameters.

Definition 3: The contribution made by the $k$th rule to the belief of the $n$th scale in the integrated result, namely, $\beta_n$, is calculated by the weight of the $k$th rule $wk$ and the belief of the $n$th scale in the $k$th rule $\beta_{n,k}$.

According to Definition 3 and the integration procedures using the ER algorithm (see Appendix A in the supplementary material), the contributions made by $wk$ and $\beta_{n,k}$ need to be calculated, specifically by calculating their respective partial derivatives in step 2.5. Then, based on the addition law of probability [32], their contributions are combined as the contribution made by the $k$th rule $R_k$ to the $n$th integrated belief $\beta_n$ in step 2.6.

Step 2.3: The weight of the $k$th rule $wk$ is calculated by multiplying the initial rule weight $\theta_k$ and the activated rule weight $wk_{\text{activated}}$. And the activated rule weight for the $k$th rule $wk_{\text{activated}}$ is calculated by normalizing the integrated matching degree $ak$. More details can be found in Appendix A in the supplementary material.

Step 2.4: By considering the weight of $K$ activated rules and their belief distributions, the integrated result in the same belief distribution can be calculated using the evidential reasoning algorithm [17] in

\[
\beta_n = \frac{\sum_{k=1}^{K} \prod_{n=1}^{N} \left( wk \beta_{n,k} + 1 - wk \right) - \prod_{k=1}^{K} \left( 1 - wk \beta_{n,k} \right)}{\sum_{k=1}^{K} \prod_{n=1}^{N} \left( 1 - wk \sum_{n=1}^{N} \beta_{n,k} \right) - \prod_{k=1}^{K} \left( 1 - wk \right)}
\]

(9)

where $wk$ denotes the weight of the $k$th rule and $\beta_{n,k}$ denotes the belief of the $n$th scale in the $k$th rule.

By assuming that there is no incomplete information, that is, $\sum_{n=1}^{N} \beta_{n,k}=1$, (9) is transformed into

\[
\beta_n = \frac{f(w_k, \beta_{n,k})}{g(w_k, \beta_{n,k})} = \frac{\prod_{k=1}^{K} (wk \beta_{n,k} + 1 - wk) - \prod_{k=1}^{K} (1 - wk)}{\sum_{n=1}^{N} \prod_{k=1}^{K} (wk \beta_{n,k} + 1 - wk) - N \prod_{k=1}^{K} (1 - wk)}
\]

(10)

where $f(k, \beta_{n,k})$ and $g(w_k, \beta_{n,k})$ are used as $f$ and $g$ in the following for convenience.

Step 2.5: Calculate the partial derivatives of the rule weights $wk$ and beliefs in the belief distribution $\beta_{n,k}$ with respect to $\beta_n$.

Based on (10), the partial derivatives of the rule weights and beliefs, $[(\partial \beta_n)/(\partial wk)]$ and $[(\partial \beta_n)/(\partial \beta_{n,k})]$, are calculated in (11) as follows:

\[
\frac{\partial \beta_n}{\partial wk} = \frac{\delta f}{\delta wk} \quad \frac{\partial \beta_n}{\partial \beta_{n,k}} = \frac{\delta g}{\delta \beta_{n,k}}
\]

(11)

where

\[
\frac{\delta f}{\delta wk} = (\beta_{k,n} - 1) \prod_{k'=1}^{K} (1 - \beta_{k',n}) (wk \beta_{n,k'} + 1 - wk') + \prod_{k'=1}^{K} \beta_{k',n} (1 - wk')
\]

\[
\frac{\delta g}{\delta wk} = wk \sum_{n=n', n'=1}^{N} \left( \beta_{k,n'} - 1 \right) \prod_{k'=1}^{K} (1 - \beta_{k',n'}) (wk \beta_{n',k'} + 1 - wk')
\]

\[
\frac{\delta g}{\delta \beta_{n,k}} = \frac{1}{\prod_{k'=1}^{K} (1 - \beta_{k',n,k'})}
\]

(12)

Step 2.6: According to Definition 3 and also the addition law of probability [32], the contribution made by the $k$th rule $R_k$ to the $n$th integrated belief $\beta_n$ is calculated by

\[
\text{ctr}(R_k, \beta_n) = \frac{\partial \beta_n}{\partial wk} + \frac{\partial \beta_n}{\partial \beta_{n,k}} \cdot \frac{\partial \beta_n}{\partial wk} \cdot \frac{\partial \beta_n}{\partial \beta_{n,k}}
\]

(13)

where $[(\partial \beta_n)/(\partial wk)]$ and $[(\partial \beta_n)/(\partial \beta_{n,k})]$ are calculated in (11).

Definition 4: The contribution of the $n$th scale in the output in the belief distribution $\beta_n$ to the unified output $yp$ is calculated by the belief $\beta_n$ and utilities $U(D_n)$ of $n$th scale.

Step 2.7: According to Definition 4, the contribution of $\beta_n$ to the unified output $yp$ can be calculated using

\[
\text{ctr}(\beta_n, yp) = \frac{U(D_n) \beta_n}{yp} = \frac{U(D_n) \beta_n}{\sum_{n=1}^{N} U(D_n) \beta_n}
\]

(14)

where $U(D_n)$ denotes the utility of the $n$th scale and $\beta_n$ denotes the belief of the $n$th scale in the unified belief distribution obtained by (9). More details can be found in Appendix A in the supplementary material.
Step 2.8: According to Definition 1, the contribution of the $m$th parameter in the $p$th input $I_{mp}$ to the unified output $y_p$ can be calculated by

$$\text{ctr}(I_{mp}, y_p) = \text{ctr}(I_{mp}, R_k)\text{ctr}(R_k, \beta_n)\text{ctr}(\beta_n, y_p)$$

(15)

where $\text{ctr}(I_{mp}, R_k)$, $\text{ctr}(R_k, \beta_n)$, and $\text{ctr}(\beta_n, y_p)$ are derived from steps 2.3, 2.7, and 2.8, respectively.

Step 2.9: Create a list of parameters with respective contributions in the descending order, that is, $\text{List} = [I_{mp, \max(\text{ctr})}, \ldots, I_{mp, \min(\text{ctr})}]$. By following a standard that sets a threshold of the overall contribution to the system output, the parameter(s) $I_{mp}$ can be identified as key parameters if their overall contribution is over the threshold set in the standard.

D. Data-Driven Optimization

The identified key parameter(s) is/are optimized in the following two steps.

Step 3.1 (Construct the Key Parameter Optimization Model): The optimization objective is to control the output, for example, minimize the output, and the decision variables are only limited to the identified key parameters. As a result, the optimization model is expressed as follows:

$$\min y(I_{mp})$$

s.t.

$$lb_{mp} \leq I_{mp} \leq ub_{mp}$$

(16)

(17)

where (17) shows that the identified key $m$’th parameter should be within its lower and upper bounds.

Step 3.2 (Design the Key Parameter Optimization Algorithm): With the EAs as the optimization engine, the optimization algorithm still follows the major steps but with new revisions. In the initialization step, it is only limited to the identified key parameters. In the fitness-function calculation step, the optimized parameters are combined with the fixed parameters as the input in the fitness calculation. The rest steps are the same. More can be found in Appendix B in the supplementary material.

The theoretical contribution and features of the proposed DT-BRB approach are that:

1) objectivity is preserved by widely using data to represent the physical system. Data that represents the physical system is widely used in optimizing DT-BRB, identifying key parameters, conducting output control, etc.;
2) the key parameter identification process is completely analytical and deducible, which guarantees that the produced results are completely quantitative. Due to the analytical and deducible feature, each procedure produces a deterministic result, and therefore, the final output control result is also deterministic rather than randomized;
3) output control is highly efficient. Different from mainly depending on experts who iteratively tune the parameters to control the output, the output control of this study is achieved via the constructed DT-BRB by optimizing only the key parameters. Comparatively, DT-BRB is more efficient. With the above features, the proposed DT-BRB approach is transparent. This is especially important for complex systems with high economic or strategic values that have a high requirement of transparency.

Remark 7: Concerning the optimization models, they are different. For the optimization model in step 1.3, its goal is to construct a DT that can best model the actual conditions. For the optimization model in Section III-A, its goal is output control. Subsequently, the decision variables and the constraints in Sections I-C and III-A are also different. To solve the optimization models in steps 1.3 and 3.1, the EAs are adopted as the optimization engine. Therefore, the algorithms in steps 1.4 and 3.2 share the same framework, that is, initialization, crossover/mutation, fitness calculation, selection, etc. The major difference lies in individual makeup in the initialization step and parameter settings.

IV. CASE STUDY: BUILDING TILT RATE REDUCTION IN TUNNELING CONSTRUCTION

A. Background

Tunnels are important components of many infrastructural constructions, for example, metro lines and bridges. However, tunnel construction in cities may cause safety concerns in nearby buildings. The direct influence of tunnel construction through multiple parameters is called tunnel–soil–building interaction [10]. Specifically, the operating process of heavy machinery may cause severe damages to buildings that are relatively close with relatively shallow foundation [11], [12]. Therefore, it is very important to closely monitor the tunnel construction as well as its impact on the nearby buildings [34], [36]. Many countries have issued strict regulations on limiting the BTRs within a very small range, for example, China has issued two national standards in GB 50007-2011 and GB 50715-2011 that require the BTRs to be within 5‰ (“‰” is implied and omitted in the remainder of this article) during the tunnel construction [13].

BTR is determined according to four types of parameters, that is: 1) tunnel design; 2) geological; 3) operational; and 4) building parameters [10], [12]. The tunnel-design parameters represent the tunnel conditions, for example, the tunnel cover depth. The geological parameters represent the practical geological conditions where the tunnel is constructed, for example, the soil cohesion condition. The operational parameters denote the configurations of certain heavy machinery, for example, cutter torque, which are determined by engineers and technicians. The building parameters are obtained using special devices and sensors on the target buildings adjacent to the tunnels, for example, the relative horizontal, vertical, and longitudinal distances between the building and tunnel. Each type of parameter can be further disintegrated into more parameters, as in Fig. 5.

Based on the understanding of the parameters and also the engineering practices, three types of parameters are normally...
first measured, the tunnel design, geological, and building parameters. Then, engineers would determine the operational parameters after careful deliberation based on their experiences and understanding of the gathered data of the three types of parameters [35]. To conclude, the tunnel design, geological, and building parameters are fixed while only the operational parameters are adjustable. In other words, only the operational parameters can be adjusted.

The Wuhan Metro System (WMS) in the city of Wuhan, Hubei, China, is studied. WMS was initially under engineering construction since 2004, and a Web-based system early warning has been developed in 2008 to collect and monitor the conditions of WMS to ensure the construction safety, especially the tunneling construction safety [34]. A total of 500 sets of data have been collected using the WMS. Each set of data had a total of 16 parameters of four types, namely: 1) tunnel design; 2) geological; 3) operational; and 4) building parameters, and the BTR as the output.

B. Transparent DT-BRB Construction

Based on Section IV-A, there are 16 parameters in the DT-BRB model. For the input part, upon analyzing the collected 500 sets of data, the lower and upper bounds, \( lb \) and \( ub \), of the 16 parameters are listed in Table I.

For the output part, the lower and upper bounds of the BTRs are 0.74 and 2.75, respectively. To cover a wider range, five scales are assumed in the output

\[
\{ \text{Low} (L), \text{LowMedium} (LM), \text{Medium} (M), \text{MediumHigh} (MH), \text{High} (H) \}
\]

and the utilities are 0.4, 1, 1.6, 2.2, and 2.8, respectively.

Among the 500 sets of data, Data No. 1-300 are used as the training dataset, Data No. 301-400, and Data No. 401-500 are used as the testing dataset. In this sense, the training and validation datasets are used as historical data while the testing dataset is the newly collected data. By using the optimization approach in Section III-B, the number of rules is set as five, the number of individuals is set as 20, and the generation is set as 1000. The RMSE is used as the optimization objective. A total of 30 runs are conducted. After optimization, the minimum, average, and variance of the RMSEs for the testing dataset are 0.1245, 0.1748, and 0.0003, respectively, indicating that the proposed approach is highly stable. Fig. 6 presents validation results on the testing dataset using the optimized DT-BRB. Table II lists the optimized DT-BRB with the RMSE of 0.1245.

C. Key Operational Parameter Identification

After deriving the optimal DT-BRB, its key parameters need to be identified following the key parameter identification procedure of Definition 1 in Section III-C. As discussed in Section IV-A, the key parameter identification process is limited only to the operational parameters because only they can be adjusted. Nonetheless, the contribution calculation is conducted for all of the 16 parameters.

Next, the first set of data in the testing dataset is used as an example to illustrate this process. The operational
parameters are \((x_6, 37.7875), (x_7, 2800.5), (x_8, 5520.1), (x_9, 3.2484), (x_{10}, 2.7099),\) and \((x_{11}, 4.1271)\). With the constructed DT-BRB in Table II, the estimated BTR in belief distribution is \((L, 0.0958), (LM, 0.0081), (M, 0.0403), (MH, 0.2140), (H, 0.6418)\). Recall the utilities in (18), the estimated BTR is 2.3787\((=0.4*0.0958 + 1*0.0081+1.6*0.0403 + 2.2*0.2140 + 2.8*0.6418)\). Comparatively, the actual BTR is 2.4394. Therefore, the absolute error for the first set of data is 0.0607. The standard of identifying the key parameters is that their combined contribution should be over 80% to the BTR based on the advice of the field engineers.

Following Definition 2 and steps 2.1 and 2.2, the matching degrees between the six operational parameters and five rules are listed in Table III. By considering the integrated matching degrees, Table IV lists the respective contribution of each operational parameter to each rule.

Remember that Tables III and IV only present the results of the operational parameters, that is, \(x_6\)–\(x_{11}\), but the calculation is conducted among all of the 16 parameters. Consequently, it leads to highly varied contributions in Table IV. For example, there is a very small matching degree between \(x_{11}\) and Rule 5, being 0.1231. However, since Rule 5 is activated only by one parameter that is \(x_{11}\), the contribution made by \(x_{11}\) to Rule 5 is 100%. In contrast, although the matching degree between \(x_{11}\) and Rule 3 is very high, being 0.8769, the contribution made by \(x_{11}\) to Rule 3 is relatively small, being 0.1599, because the fixed parameters, that is, \(x_1\)–\(x_5\), \(x_{12}\)–\(x_{16}\), also make contributions to Rule 3.

For a brief conclusion, the matching degree of one parameter to a rule cannot be directly recognized as its contribution to the rule. The integrated matching degree of \(M\) parameters has to be taken into consideration because the fixed parameters also make contributions although they cannot be identified as key parameters.

Following Definition 3 and steps 2.3–2.6, the partial derivatives of the weights and beliefs of Rules 1–5 to the BTR in belief distribution are listed in Tables V and VI, respectively. Based on step 2.6, the differentials of the weights and beliefs are combined into the contributions made by the rules to each scale of the BTRs in belief distribution as in Table VII.

Following Definition 4 and steps 2.7 and 2.8, the contribution made by each scale of the BTR in belief distribution to the unified BTR is listed in Table VIII.

According to Definition 1 and (15), the contributions made by the parameters to the unified BTR are listed in Table IX.
using the results in Tables IV, VII, and VIII. According to Table IX, \(x_9, x_{11}, \text{and } x_8\) should be selected as key parameters because they have contributed over 80% to the unified BTR.

Note that the summed combination of identified key parameters is expected to be higher than the preset standard, for example, the summed contribution of key parameters \(x_9, x_{11},\) and \(x_8\) for the first set of testing data is 85.87% that is higher than 80%.

Remark 8: According to the contributions made by different parameters to the BTR in Table IX and also Remark 5, some parameters with a lower level of contribution are less relevant to the BTR, for example, \(x_6, x_7,\) and \(x_{10}:\) their contribution levels are all within 6%. For such parameters, it is safe to infer that their impacts on the BTR must be limited even if faults occur to them. Comparatively, \(x_9\) is with a contribution level of 47.17%, indicating that it is much more relevant to the BTR. It can be inferred that the impact on the BTR should be substantial if faults occur to \(x_9.\) Furthermore, the number of faulty parameters is also worthy of notice. Besides \(x_9\) with a significantly bigger contribution, there should not be a heavy negative impact on the BTR even if faults occur to any other of the remaining five parameters. However, if all of the remaining five parameters are faulty, considering that their combined contribution is as high as 52.63%, the impact would be substantial. Therefore, it is important to consider the conditions of faulty parameters in the system input when discussing the ability to tolerate faults.

D. Data-Driven Optimization for Reducing the BTR

The first testing data are also used as an example to illustrate the key parameter optimization process. Following step 3.1, the optimization model is given as follows:

\[
\begin{align*}
\min & \quad \text{BTR}(I_9, I_{11}, I_8) \\
\text{s.t.} & \quad lb_9 \leq I_9 \leq ub_9 \\
& \quad lb_{11} \leq I_{11} \leq ub_{11} \\
& \quad lb_8 \leq I_8 \leq ub_8
\end{align*}
\]  

where (19) denotes that the optimization objective is to minimize the BTR through the constructed DT-BRB in step 1. Equations (20)–(22) denote the reference values of the identified key operational parameters, that is, \(x_9, x_{11},\) and \(x_8\), should be within their lower and upper bounds, respectively.

Next, following step 3.2, the settings of the optimization algorithm are designed as follows: the number of individuals is set to 20, and the generation is set to 500. This process is repeated for 30 runs. Note that only \(x_9, x_{11},\) and \(x_8\) are optimized while rest parameters stay the same in the fitness function calculation step. After optimization, \(x_9, x_{11},\) and \(x_8\) are optimized by increasing/reducing from the original 3.2484, 4.1271, and 5520.83 to 3.5000, 10.000, and 5026.01, respectively. As a result, the BTR is reduced from the original 2.4394 to 0.8087 by 1.6307 (66.85%).

E. Comprehensive Results of the Testing Dataset

By repeating this process for the rest 99 sets of data in the testing dataset, the contributions made by both the fixed and operational are calculated and listed in Table C.1 in Appendix C in the supplementary material and the contributions of only the operational parameters are in Table C.2. There are two questions to answer: 1) how many key parameters have been identified in 100 sets of testing data and 2) how BTRs have been reduced?

To answer 1), Fig. 7 compares the number of key parameters among 100 sets of testing data. It shows that 70 (28 sets of data require only one key parameter, 17 require two, and 25 require three) sets of data only need three or fewer operational parameters to meet the standard of 80% contribution. Comprehensively, the average key parameter is 2.61 (43.5%) out of six operational parameters.

Table X lists five examples with a varied number of identified key parameters. It is easy to find that, by adopting the 80% contribution standard, the summed contribution of key parameters for each set of data must be over 80%. In fact, the range of summed contributions of 100 sets of testing data is [80.09%, 95.75%] with the mean contribution of 86.00%.

To answer 2), Fig. 8(a) shows the original and reduced BTRs, and Fig. 8(b) and (c) shows the reduced BTRs in terms of value and percentage, respectively. Based on the results in Fig. 8, it can be found that the average original BTR, being 1.8275, has been reduced to 0.4802. In other words, the reduction is 1.3473 (73.73%).

By answering 1) and 2), and specifically according to Figs. 7 and 8, the BTRs can be reduced by over 70% by optimizing less than 45% of the original parameters.

F. Investigations Into Contributions of Parameters

Fig. 9 presented the average contribution made by each parameter in 100 sets of testing data, as well as the times...
Fig. 8. Results of the testing dataset. (a) Original and after-reduction BTRs. (b) Reduced BTRs. (c) Reduced BTRs in percentages.

Fig. 9. Average contributions of parameters, and times of being identified as key parameters.

Fig. 10. Contributions of parameters in four categories.

of each parameter being identified as a key parameter, which has presented an orderly consistency.

According to Fig. 9 and more detailed results in Table C.1, the sixth operational parameter (x11) is the most identified key parameter, that is, 96 times out of 100 sets of testing data, as well as with the highest average contribution being 55.41%. Following by x9(46/100, 13.40%) and x0(43/100, 11.03%), while x6(27/100, 6.78%), x7(27/100, 6.96%), and x10(22/100, 6.43%) share relatively close probability.

Next, the discussion is extended to all of the sixteen parameters in four categories. Fig. 10 presents the contributions of 100 sets of data, indicating that the tunnel design, geological, and building parameters also make contributions to the BTRs. In fact, according to Fig. 10 and more detailed results in Table C.1, the building parameters (x12–x15) actually make even higher contributions than the operational parameters (x6–x11).

Based on Figs. 9 and 10, and also Table C.1, it can be concluded that only optimizing key parameters cannot reach the goal of reducing the BTR to zero because: 1) not all of the operational parameters are identified as key parameters for all testing data (see Fig. 7) and 2) the fixed parameters, that is, the tunnel design, geological, and building parameters, also make contributions to the BTR, but they cannot be identified as key parameters, and thus cannot be optimized.

V. DISCUSSION ON MODEL EFFECTIVENESS

In Section V-A, multiple approaches are compared with DT-BRB to test the modeling ability that is formulated in step 1 in Section III-B. Note that further steps 2 and 3 cannot be held using those approaches since their procedures are not accessible. In Section V-B, more standards of identifying the key parameters are tested to identify the optimal standard.

A. Partial Comparison With Other Approaches

Seven approaches in various settings are compared, that is, the backpropagation NN (BPNN) [37]; support vector machine (SVM) [38], [39]; adaptive neuro-fuzzy inference system (ANFIS) [40]; genetic neuro-fuzzy inference system (GENFIS) [41]; radial basis function NN (RBFNN) [42]; generalized regression NN (GRNN) [43]; and Gaussian process regression (GPR) [44].

Among the seven approaches, only SVM is implemented via the libsvm [38]. The other six approaches are implemented in the respective toolbox or functions in MATLAB. Moreover, there are various parameter settings in BPNN, SVM, ANFIS, and GENFIS while RBFNN, GRNN, and GPR automatically determine their parameters. Appendix D in the supplementary material gives detailed results of the seven approaches with their respective parameter settings. Table IV gives the summarized results.

Fig. 11(a) and (b) compares the results of DT-BRB, BPNN (with the optimal results), and ANFIS (with the least optimal results). It is clear that BPNN and DT-BRB have produced rather accurate results while ANFIS is relatively inferior.

According to Table XI and Appendix D in the supplementary material, the following conclusions can be drawn.

1) BPNN and DT-BRB have produced optimal results compared with other approaches. Concerning the RMSE, only BPNN and DT-BRB have produced far smaller RMSEs. It should also be noted that BPNN is superior
to DT-BRB within a very small margin: the difference between BPNN and DT-BRB, that is, 0.0106, is far smaller smaller than the average actual BTR of the 100 sets of testing data, that is, 1.8275, indicating that they both have both have demonstrated very superior modeling ability.

2) Except for ANFIS with an RMSE of 0.2983 and GENFIS with an RMSE of 0.5098, the RMSEs of the other approaches are within [0.1761, 0.2239] that is about twice the RMSEs of BPNN and DT-BRB. On the one hand, it is admitted that their inferiority is not trivial. On the other hand, the difference is also not as high as cross-scale: all approaches have produced results on the same scale, that is, 10\(^{-1}\), even including ANFIS and GENFIS.

3) Nonetheless, it should be reminded that a model with the highest modeling accuracy or the lowest modeling error is not the only requirement of the proposed DT-BRB approach. The other requirement is that the adopted model must also be a white-box approach to support identifying key parameters in an analytical, deducible, and deterministic fashion. Comparatively, the latter is a more important requirement for DT-BRB. In this sense, BPNN although with slightly superior performance still cannot be adopted since it is not a white-box approach.

**TABLE XI**

| Approaches          | RMSE |
|---------------------|------|
| DT-BRB (this study) | 0.1245 |
| BPNN (trainReg/logistic) | 0.1139 |
| SVM (C=0.2)        | 0.1761 |
| ANFIS (gaussmf)    | 0.2983 |
| GENFIS (trimf)     | 0.5098 |
| RBNN               | 0.2174 |
| GRNN               | 0.2239 |
| GPR                | 0.2174 |

**TABLE XII**

| Standards | 50% | 60% | 70% | 80% | 90% | 100% |
|-----------|-----|-----|-----|-----|-----|------|
| Avg. kp   | 1.52 | 1.98 | 2.43 | 2.62 | 3.62 | 6    |
| BTR       | 0.5850 | 0.5438 | 0.5010 | 0.4802 | 0.4451 | 0.4216 |

**B. Influence of Contribution Standards**

More standards of selecting the key parameters with over 50%, 60%, 70%, 80%, 90%, and 100% contributions are compared. Naturally, different standards would identify a different number of key parameters, and then lead to different BTR reductions. Table XII presents comprehensive results.

For example, in the case study part in Section IV, the 80% contribution standard is adopted that identified an average of 2.62 key parameters for 100 sets of testing data. As a result, the average of BTR is reduced from the original 1.8275 to 0.4802. Comparatively, the 60% standard requires an average of 1.98 key parameters, and the BTR can be reduced to 0.5438. In an extreme condition that is the 100% contribution standard, all of the six operational parameters are identified as key parameters. As a result, the BTR can be reduced to 0.4216 accordingly, which is the most extreme condition that the BTR can be reduced to. Note that this result also echoes with the conclusion made by Section IV-F: the BTR cannot be reduced to zero even if all key parameters are optimized.

Fig. 12 presents the original and reduced BTRs following 60%, 80%, and 100% contribution standards. According to Table XII and Fig. 12, the 100% contribution standard has produced the optimal results that supersede the 80% contribution standard, and further the 60% contribution standard. This is a natural outcome: a more relaxed standard leads to a smaller BTR since more key parameters are identified and optimized.

To identify the most cost-effective standard, an additional criterion is calculated, namely, the marginal contribution of the \((s + 1)\)th standard, \(\Delta ctr_{s + 1}\), by

\[
\Delta ctr(s + 1) = \frac{BTR(s + 1) - BTR(s)}{kp(s + 1) - kp(s)} \tag{23}
\]

where \(BTR(s + 1)\) and \(BTR(s)\) denote the average BTRs following from the \((s + 1)\)th and sth standards, respectively, and \(kp(s + 1)\) and \(kp(s)\) denote the average number of identified key parameters following from the \((s + 1)\)th and sth standards, respectively.

Take the 80% contribution standard as an example, its marginal contribution is calculated by

\[
\Delta ctr(80\%) = \frac{BTR(80\%) - BTR(70\%)}{kp(80\%) - kp(70\%)} = 10.95\% \tag{24}
\]

where \(BTR(80\%) = 0.4802\), \(BTR(70\%) = 0.5010\), \(kp(80\%) = 2.62\), and \(kp(70\%) = 2.43\), respectively. Similarly, the marginal contributions following standards of 60%–100% can be calculated, and the results are presented in Fig. 13. According to Fig. 13, the 80% standard has produced the biggest marginal contribution, that is, 10.95%. Comparatively, none of the other standards are above 10%. It means that the 80% contribution standard is the turning point of all standards.
Before this point, not enough key parameters have been identified to sufficiently reduce the BTR. After this point, the BTRs cannot be reduced more even if more parameters are to be optimized. In other words, the 80% contribution standard is the most cost effective and should be recommended as the standard by default.

Based on the above finding, it can also be concluded that there is no need for iterative optimization (steps 2 and 3 in Section III-A). Theoretically, the proposed DT-BRB approach can be implemented iteratively if the reduction of BTR in one run does not meet the requirement. However, even by extending to the most extreme condition that is 100% contribution, there is only a small room of 0.0586 (= 0.4802 − 0.4216) further reduction of the average BTR. The cost is 3.38 (= 6 − 2.62) parameters. Then, the marginal contribution is 1.73% (= 0.0586/3.38), which is very cost effective.

VI. CONCLUSION

This study proposed a transparent DT-BRB approach for output control. The new approach takes full advantage of the BRB as a white-box model to identify the key parameters that are then optimized to control the output of the complex system. The proposed approach is composed of three major steps. First, the DT-BRB model is constructed to model the complicated relationships between the system inputs and the system output. Second, the key parameters are identified by calculating the contribution made by each parameter in the system inputs to the system output. Third, the key parameters are optimized for output control. The analytical and deducible procedures guarantee that the final results are deterministic to avoid randomness while providing full access to decision makers. Combined with the general use of data, the final results of output control are objective rather than subjective. Moreover, the output control process is more efficient and objective than iteratively tuning parameters by relying on human’ knowledge.
A practical case on reducing the BTRs in tunnel construction is studied to validate the efficiency of the proposed DT-BRB approach. The case study results show that the key operational parameters can be accurately identified using the proposed DT-BRB approach. By performing data-driven optimization, the BTRs can be greatly reduced. Case study results show that, by adopting the 80% contribution standard, it only requires optimizing an average of 2.62 parameters to produce a 73.73% reduction in the BTRs. By comparing multiple standards from 50%–100%, it is found that the 80% contribution standard has the highest cost effectiveness. Upon further investigation, it is concluded that DT-BRB is so effective that no iterative optimization is needed.

For future research, other white-box models can be tested to validate their efficiency as the DT. Moreover, other optimization engines should be tested in a separate or hybrid manner to handle problems that require the higher optimization efficiency or multiple-objective optimization.

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Leilei Chang received the B.Eng. degree from Central South University, Changsha, China, in 2008, and the M.Eng. and Ph.D. degrees from the National University of Defense Technology, Changsha, in 2010 and 2014, respectively. He is currently an Associate Professor with the School of Automation, Hangzhou Dianzi University, Hangzhou, China. He has authored three books and published over 30 articles. His research interests include BRB structure and parameter learning, digital twin construction, and applications in multiple complex system modeling problems.

Limao Zhang received the bachelor’s, master’s, and Doctoral degrees from the Huazhong University of Science and Technology, Wuhan, China, in 2009, 2012, and 2014, respectively. He is an Assistant Professor with the School of Civil and Environmental Engineering, Nanyang Technological University, Singapore. He has led research projects up to 2 million SGD, with more than 90 papers published in peer-reviewed journals. His principal research area involves artificial intelligence, deep learning, process mining, and urban resilience.

Chao Fu received the Ph.D. degree from Hefei University of Technology, Hefei, China, in 2009. He is currently a Professor with the School of Management, Hefei University of Technology. He has published more than 40 articles in international journals. His research interests include artificial intelligence, deep learning, process mining, and urban resilience.

Yu-Wang Chen received the Ph.D. degree in control theory and control engineering from Shanghai Jiao Tong University, Shanghai, China, in 2008. He is currently a Senior Lecturer of Decision Sciences with the Alliance Manchester Business School, University of Manchester, Manchester, U.K. His research interests include decision and risk analysis under uncertainties, modeling and optimization of complex systems, operational research, and data analytics.