The isoscalar transition strengths of the cluster states of $^{12}$C

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Abstract. An elaborate folding model + coupled channel analysis of the inelastic $\alpha+^{12}$C scattering has been done using the inelastic form factors based on the nuclear transition densities predicted by the antisymmetrized molecular dynamics. The form factors for the observed isoscalar states of $^{12}$C were fine-tuned against the measured ($\alpha$, $\alpha'$) cross sections to estimate the corresponding $E\lambda$ transition strengths. A significant $E2$ transition strength has been allocated for the second $2^+$ state of $^{12}$C from our consistent coupled channel analysis of the ($\alpha$, $\alpha'$) data measured at $E_\alpha = 240$ and 386 MeV. The $E\lambda$ transition strengths of the $0^+_2$, $3^-_1$, $3^+_3$, and $1^-_1$ states were also carefully deduced from the same folding model + coupled channel analysis.

The excited states of $^{12}$C at energies near the $\alpha$-decay threshold are extensively studied recently [1, 2] because of the interesting $\alpha$-cluster structure established in several cases, like the isoscalar $0^+_2$ state at 7.65 MeV in $^{12}$C, known as the Hoyle state that is vital for the carbon synthesis in cosmos. Along with a three $\alpha$-cluster structure of the Hoyle state shown about three decades ago by the Resonating Group Method (RGM) calculations [3, 4, 5], an interesting $\alpha$-condensate scenario [2] for this state has been suggested [6, 7], where three $\alpha$ clusters are shown to condense into the lowest $S$ state of their potential.

Given a nonspherical shape of the Hoyle state, an excited rotational band with the angular momentum $J^\pi = 2^+, 4^+, ...$ built upon this state was suggested long ago by Morinaga [8]. In the $\alpha$-condensate scenario, it is also natural that the next level in the potential containing three $\alpha$-particles should be a $2^+$ state formed by promoting an $\alpha$-particle from the $S$ to $D$ level. The second $2^+$ state of $^{12}$C has been predicted by the antisymmetrized molecular dynamics (AMD) [9, 10] and some other structure studies at the excitation energy around 10 MeV, i.e., about 2 MeV above the $\alpha$-decay threshold, with a pronounced $^8$Be+$\alpha$ structure [1]. Although some evidence for a broad $2^+$ resonance was found in several experiments that might be assigned to the $2^+_2$ state of $^{12}$C, a clear identification of this state could be made just recently in the high-precision ($\alpha$, $\alpha'$) experiment at $E_\alpha = 386$ MeV [11] and the photodissociation of carbon [12, 13].

To validate conclusion made in the structure calculation, the wave functions must be carefully tested in the study of nuclear reactions. Since the spin- and isospin zero $\alpha$-particle is a very good projectile to excite the isoscalar (IS) states, the nuclear transition densities predicted by AMD [9] for the known IS excitations of $^{12}$C have been used in the present work for a detailed folding model + coupled channel analysis of the ($\alpha$, $\alpha'$) data taken at $E_\alpha = 240$ [14] and 386 MeV [11]. The generalized folding model of Ref. [15] was used to evaluate the complex optical potential (OP) and inelastic scattering form factor (FF) from the (complex) effective nucleon-nucleon
(NN) interaction between the projectile nucleon $i$ and target nucleon $j$ as

$$U_{A \rightarrow A^*} = \sum_{i \in \alpha, j \in \alpha'} [(ij'|v_D|ij) + (ij'|v_{EX}|ji)],$$

(1)

where $A$ and $A^*$ denote the target in the entrance- and exit channel of the ($\alpha, \alpha'$) scattering, respectively. The direct ($v_D$) and exchange ($v_{EX}$) parts of the central NN interaction are those of the density dependent CDM3Y6 interaction [16], supplied by an imaginary part [17] determined from the JLM results for the complex nucleon OP in the nuclear matter limit [18]. Eq. (1) gives the OP if $A^* = A$ and inelastic scattering FF if otherwise. The AMD nuclear transition densities enter the folding calculation (1) such that the IS transition strength for a $2^+$-pole nuclear transition $|J_i \rightarrow J_f|$ is given by the reduced nuclear transition rate as $B(IS\lambda; J_i \rightarrow J_f) = |M(IS\lambda; J_i \rightarrow J_f)|^2$, with the $2^+$-pole transition moment determined from the corresponding nuclear transition density (see explicit expressions in Ref. [19]). Because the exit channel contains $^{12}\text{C}^*$ being in an excited state that is generally more dilute, with the predicted $\langle r^2 \rangle^{1/2}$ radius larger than that of the ground state (see Table 1), the OP of each exit channel ($U_{A^* \rightarrow A^*}$) has been computed separately using the diagonal ($\lambda = 0$) density of $^{12}\text{C}^*$ given by the AMD. Such a treatment of the exit OP lead to a better agreement of the calculated

![Figure 1](image-url)
cross sections with the \((\alpha, \alpha')\) data, which helped to deduce accurately the \(B(E\lambda)\) values for the excited states under study [19].

The AMD has been proven to give a realistic description of the shell-model like \(0^+_1\), \(2^+_1\) and \(3^-_1\) states as well as the cluster \(0^+_2\), \(0^+_3\) and \(2^+_2\) states. Given very strong \(E\lambda\) strengths predicted for IS transitions between the cluster states, we have calculated all the corresponding FF’s for the coupled channel (CC) calculation. In total, 39 diagonal OP and inelastic FF’s were obtained in the folding calculation using the AMD transition densities.

### Table 1. Excitation energies and \(E\lambda\) transition strengths of the IS states of \(^{12}\text{C}\) under study.

| \(J^\pi\) | \(\langle r^2/2 \rangle_{\text{calc}}\) (fm) | \(E_{\text{calc}}\) (MeV) | \(E_{\text{exp}}\) (MeV) | Transition | calc. \(B(E\lambda)\) (\(e^2\text{fm}^2\)) | best-fit \(B(E\lambda)\) (\(e^2\text{fm}^2\)) | exp. \(B(E\lambda)\) (\(e^2\text{fm}^2\)) | Ref. |
|---|---|---|---|---|---|---|---|---|
| \(2^+_1\) | 2.668 | 4.5 | 4.44 | \(B(E2; 2^+_1 \rightarrow 0^+_1)\) | 8.4 | 8.4 ± 1.5 | 7.4 ± 0.2 | [11] |
| \(0^+_2\) | 3.277 | 8.1 | 7.65 | \(M(0_2^+ \rightarrow 0^+_1)\) | 6.6 | 4.5 ± 0.5 | 3.7 ± 0.2 | [14] |
| \(3^-_1\) | 3.139 | 10.8 | 9.64 | \(B(E2; 3^-_1 \rightarrow 0^+_1)\) | 74.4 | 59.5 ± 3.2 | 35.9 ± 1.4 | [11] |
| \(0^+_3\) | 3.985 | 10.7 | 10.3 | \(M(0^-_3 \rightarrow 0^+_1)\) | 2.3 | 2.9 ± 0.3 | 3.0 ± 0.2 | [14] |
| \(2^+_2\) | 3.993 | 10.6 | 9.84 | \(B(E2; 2^+_2 \rightarrow 0^+_1)\) | 0.4 | 0.6 ± 0.1 | 0.37 ± 0.02 | [11] |
| \(1^-_1\) | 3.424 | 12.6 | 10.84 | \(M(1^-_1 \rightarrow 0^+_1)\) | 1.58 | 0.34 ± 0.04 | 0.31 ± 0.04 | [14] |

The present CC analysis (see the coupling scheme shown in Fig. 1) has included also the \(E2\) reorientation of the \(2^+, 3^-\) and \(1^-\) states, with the reorientation FF’s \(F_{A^* \rightarrow A^-}\) given by the diagonal transition \((\lambda = 2)\) densities of \(^{12}\text{C}^*\) (see the corresponding \(B(E2)\) values in Table 1). Thus, the present CC scheme contains three more couplings compared to that of our previous CC
the best-fit $E\lambda$ transition strengths presented in Table 1.

![Figure 2. DWBA and CC descriptions of the $(\alpha, \alpha')$ data for the Hoyle state, measured at $E_\alpha = 240$ MeV [14] and 386 MeV [11]. The DWBA1 results were obtained using the same OP for both the entrance and exit channels, and the DWBA2 and CC results were obtained with the OP of the exit channel computed separately at the energy $E_\alpha - Q$, using the diagonal $(\lambda = 0)$ density of $^{12}$C*.](image)

The CC analysis of the $(\alpha, \alpha')$ data for the Hoyle state (see Fig. 2) has revealed important coupling effects that are best seen in the results obtained at the $\alpha$ energy of 240 MeV. As discussed in Ref. [25], the original multipole decomposition analyses (MDA) of the $(\alpha, \alpha')$ data at different energies consistently gave a much weaker $E0$ transition strength of the Hoyle state, with $M(E0; 0^+_2 \rightarrow 0^+_1) \approx 3.6 \sim 3.8$ e fm$^2$ [14] that is about 30% weaker than the experimental value $M(E0)_{\text{exp}} \approx 5.4$ e fm$^2$ deduced from the $(e, e')$ data [22]. The DWBA1 calculation using the (rescaled) AMD transition density would give the best-fit $M(E0) \approx 3.65$ e fm$^2$, about the same as that given by the RGM transition density rescaled to fit the $(\alpha, \alpha')$ data in the DWBA [25]. The present CC calculation included all strong $E\lambda$ transitions from the Hoyle state to the neighboring cluster states (see Fig. 1), and the best-fit $E0$ strength given by the folding model + CC analysis of the 240 MeV data is $M(E0) \approx 4.5$ e fm$^2$ which is about 20% stronger than
that given by the standard DWBA analysis. We expect that a full coupled reaction channel analysis of the \((\alpha, \alpha')\) data including different breakup channels would yield the \(M(E0)\) value closer to the \((e, e')\) data, and thus explain the missing monopole strength of the Hoyle state in \((\alpha, \alpha')\) scattering that can only be accounted for in the DWBA by an enhanced absorption in the exit channel [25, 26].

![Figure 3](image_url)

**Figure 3.** DWBA (a) and CC (b) descriptions of the \((\alpha, \alpha')\) data measured at \(E_\text{in} = 386\) MeV for the \(0^+_3\) and \(2^+_2\) states [11]. The DWBA2 and CC results were obtained in the same way as described in the caption of Fig. 2. The FF for the \(2^+_2\) state has been scaled to the best CC fit to the data, giving \(B(E2 \downarrow) \approx 0.6\, \text{e}^2\text{fm}^4\).

The MDA of the \((\alpha, \alpha')\) data measured at \(E_\text{in} = 386\) MeV has shown a broad \(0^+_3\) resonance and a narrower \(2^+_2\) state centered at the excitation energies \(E_x \approx 9.93\) and \(9.84\) MeV, respectively. The \((\alpha, \alpha')\) angular distribution deduced for the wide bump centered at \(E_x \approx 10\) MeV [11] contains mainly the contributions from the \(2^+_2\) and \(0^+_3\) states (see Fig. 3). Given the \(E0\) strength of the \(0^+_3\) state accurately determined in the CC analysis [19] of the 240 MeV data, the \(E2\) strength of the \(2^+_2\) state remains the only parameter in the CC analysis of the 386 MeV \((\alpha, \alpha')\) data. After the FF for the \(2^+_2\) state is scaled to the best CC fit to the \((\alpha, \alpha')\) data, we obtained \(B(E2; 2^+_2 \rightarrow 0^+_1) \approx 0.6\, \text{e}^2\text{fm}^4\), in a good agreement with \(B(E2 \downarrow)_{\text{exp}} \approx 0.73\, \text{e}^2\text{fm}^4\) deduced from the photodissociation data [12]. Note that this value is lower than \(B(E2 \downarrow) \approx 1.57\, \text{e}^2\text{fm}^4\), deduced from the revised analysis of the \(^{12}\text{C}(\gamma, \alpha)\text{^9Be}\) data [13].

A question now is why the \(2^+_2\) state has not been found at \(E_x \approx 10\) MeV by the MDA of the \((\alpha, \alpha')\) data taken at \(E_\text{in} = 240\) MeV [14]. Given the realistic \(E\lambda\) strengths of the IS states found in our folding model + CC analysis of both data sets, we have analysed again the 240 MeV \((\alpha, \alpha')\) data measured for several energy bins around 10 MeV. The best-fit \(E\lambda\) transition strengths of the \(2^+_2\), \(0^+_3\), and \(1^-_1\) states (Table 1) were distributed over the considered energy bins by an averaging procedure [19] to estimate the corresponding inelastic FF. From the CC results obtained with and without the contribution from the \(2^+_2\) state one can see in Fig. 4 that this state is indeed present in the energy bins around \(E_x \approx 10\) MeV. Such a subtle effect could not be resolved by the original MDA of the 240 MeV \((\alpha, \alpha')\) data.

We are indebted to M. Itoh, B. John, T. Kawabata, and W.R. Zimmerman for their helpful communications. This work has been supported, in part, by VINATOM and the LIA project of


Figure 4. Differential ($\alpha, \alpha'$) cross sections measured at $E_\alpha = 240$ MeV [14] for the 475 keV-wide energy bins centered at $E_x = 9.69$ MeV (a), 10.17 MeV (b), and 10.65 MeV (c), in comparison with the CC results given by different $2^+$-pole transition strengths. The total cross sections obtained with and without the contribution from the $2^+_2$ state are shown as the thick (blue) and thin (red) solid lines, respectively.

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the Ministry of Science and Technology of Vietnam.

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