Bounding the $Z'\mu\tau$ coupling from lepton flavor violating transitions

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Abstract. We present the calculation for the strength of the lepton flavor violating $Z'\mu\tau$ coupling, in which the experimental result of the muon magnetic dipole moment was used to bound this vertex, by means of a model-independent approach. The $Z'\mu\tau$ vertex is represented by the $\Omega_{\mu\tau}$ parameter, which is of the order of $10^{-2}$ for a $Z'$ boson mass of 1 TeV.

1. Introduction

The presence of a new neutral massive gauge boson ($Z'$) is predicted in the context of numerous extensions of the Standard Model (SM). The $SU_C(3) \times SU_L(2) \times U_Y(1) \times U'(1)$ extended electroweak gauge group is the simplest model capable to predict the existence of the $Z'$ boson [1].

Although absent in the SM, the phenomena of lepton flavor violation can arise in many of its well-motivated extensions. One interesting feature of most models beyond the SM is the presence of generalized Yukawa sectors, which favor nondiagonal transitions mediated by neutral massive spin-1 particles [1, 2].

Currently, the experimental data have proven the existence of neutrino oscillations, which in simple words, tell us that lepton flavor conservation is violated in nature. However, the only signal of lepton flavor violation comes from transitions between neutral leptons. Lepton flavor violating transitions between charged leptons constitute an interesting object of study, since if they occur in nature are further evidence of lepton flavor violation.

We are interested in study the impact of lepton flavor violation on the $\tau \to \mu\gamma$ decay mediated by a $Z'$ gauge boson. To this end, we estimate the strength of the $Z'\mu\tau$ coupling in the context of the simplest extension of the SM.

2. The extended model

We consider the neutral current lagrangian contained in the simplest model based on the $SU_C(3) \times SU_L(2) \times U_Y(1) \times U'(1)$ electroweak gauge group [2, 3]

$$L_{NC} = -eJ_{EM}^a A_\alpha - g_1 J_1^a Z_{\alpha,1} - g_2 J_2^a Z_{\alpha,2},$$

(1)
where $e$ is the electromagnetic coupling, $J_α^{EM}$ is the electromagnetic neutral current, $g_1$ is the gauge coupling of the SM, $J_α^Z$ is the weak neutral current of the SM, $g_2$ is the gauge coupling of the $U′(1)$ group and $J_α^Z$ is the new weak neutral current given as

$$J_2^Z = \sum_{i,j} \overline{\psi}_i^j \gamma^\alpha (\epsilon_{Lij}^\nu P_L + \epsilon_{Rij}^\nu P_R) \psi_j^\alpha,$$

where $\epsilon_{L,Rij}$ are the chiral couplings of $Z_2$ with $i,j$ running over all quarks and leptons, $\psi_i^j$ represents a fermion in the gauge interaction basis and $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$ are the chiral projectors. The interaction between $Z_1$ and $Z_2$ is so weak that it can be neglected. Thus, the mass eigenstates are $Z^0$ and $Z'$, respectively [1].

The $\epsilon_{L,Rij}$ matrix elements for the charged leptons are represented in the gauge eigenstates basis. This matrix is transformed into the mass eigenstates one by diagonalizing the mass matrix in the Yukawa sector [2]. Therefore, the couplings in the mass eigenstates basis ($\Omega_L, \Omega_R$) can be written as [4]

$$\Omega_{Lij} = g_2 (V_L \epsilon_{ij}^L V_L^\dagger)_{ij},$$

$$\Omega_{Rij} = g_2 (V_R \epsilon_{ij}^R V_R^\dagger)_{ij},$$

where $V_{L,R}$ are the unitary matrices that diagonalize the mass matrix in the Yukawa sector of the SM. The lepton current is transformed into the mass eigenstates basis as

$$J^\alpha = \sum_{i,j} \overline{\ell}_i \gamma^\alpha (\Omega_{Lij} P_L + \Omega_{Rij} P_R) \ell_j,$$

where $\ell_i = e, \mu, \tau$.

This current allow us to analyze the $Z'\mu\tau$ coupling, which is given by the strength of the $\Omega_{L,R\mu\tau}$ matrix elements. Therefore, the lagrangian containing the relevant information is

$$\mathcal{L}_{NC}^{Z'\ell_i\ell_j} = - \left[ \overline{\ell}_i \gamma^\alpha (\Omega_{Lij} P_L + \Omega_{Rij} P_R) \ell_j \right] Z^\alpha,$$

where $\ell_i = e, \mu, \tau$. After some algebra, the last equation becomes

$$\mathcal{L}_{NC}^{Z'\ell_i\ell_j} = - \left[ \overline{\ell}_i \gamma^\alpha (Re(\Omega_{Lij}) + i Im(\Omega_{Lij}) \gamma_5) \ell_j \right] + \left[ \overline{\ell}_j \gamma^\alpha (Re(\Omega_{Lij}) - i Im(\Omega_{Lij}) \gamma_5) \ell_i \right] Z^\alpha.$$

The $Z'\tau\mu$ vertex that emerges from the $\mathcal{L}_{NC}$ induces the lepton flavor violating $Z' \rightarrow \mu\tau$ decay at the one-loop level. Thus, the estimation of the strength of the $\Omega_{\mu\tau}$ parameter is a crucial information to compute the branching ratio for the $Z' \rightarrow \mu\tau$ process. We present two different methods in order to obtain the $\Omega_{\mu\tau}$ strength. The first one consists in bounding from the experimental result for the muon anomalous magnetic dipole moment in a model-independent way. The second one relies on bounding from the experimental constraint for the $\tau \rightarrow \mu\gamma$ decay, by using several extended models.
3. Calculation of the $\Omega_{\mu\tau}$ parameter

The contribution of the $Z'\mu\tau$ vertex to the anomalous magnetic dipole moment of the muon is calculated using the Feynman diagram shown in Figure 1(a), which corresponds to the $\mu\mu\gamma$ electromagnetic vertex at the one-loop level. To define the magnetic dipole moment contribution $i(a_\mu/2m_\mu)\sigma^{\alpha\beta}q_\beta$, we must calculate the one-loop amplitude for the on-shell $\mu\mu\gamma$ vertex. The contribution to the magnetic dipole moment is free of ultraviolet divergences. We find that contribution to the $a_\mu$ form factor can be written as

$$a_\mu = \frac{\sqrt{2}x_\mu x_\tau}{\pi^{3/2}} \text{Re}(\Omega^2_{\mu\tau}) g(x_\mu, x_\tau),$$

(5)

where $\alpha$ is the fine structure constant and

$$g(x_\mu, x_\tau) = \int_0^1 dx \int_0^{1-x} dy \frac{1 - 2x - 2y}{(x + y)(1 - x^2 - x^2_\mu (x + y - 1))}.$$ 

(6)

Here, we introduced the dimensionless variables $x_i = m_i/m_{Z'}$, $i = \mu, \tau$.

![Feynman diagrams](image)

**Figure 1.** Feynman diagrams contributing to the muon anomalous magnetic dipole moment (a) and lepton flavor violating $\tau \to \mu\gamma$ decay (b).

Nowadays, the anomalous magnetic dipole moment of the muon is one of the physical observables best measured. Therefore, we employ the experimental uncertainty $|\Delta a_{\text{Exp}}|$ [5] to get a bound on the $\text{Re}(\Omega^2_{\mu\tau})$ parameter, where

$$|\Delta a_{\mu}^{\text{Exp}}| < 6 \times 10^{-10}.$$ 

(7)

In Figure 2 is shown with a solid line, the behavior of the maximum of $\text{Re}(\Omega^2_{\mu\tau})$ as a function of the $Z'$ boson mass. It is important to notice that the growth of this parameter starts at $10^{-3}$ and reaches values of $10^{-2}$ for a $Z'$ mass interval of $500 \text{ GeV} < m_{Z'} < 1.5 \text{ TeV}$.

In this paragraph we proceed to compute the contribution of the lepton flavor violating $Z'\mu\tau$ vertex to the $\tau \to \mu\gamma$ decay. The $\tau \to \mu\gamma$ process corresponds to the Feynman diagrams shown in Figure 1(b). Notice that this transition is of dipolar type, finite and model dependent since the $Z'\tau\mu$ vertex is found in terms of the $g_2$ coupling constant and the $Q_R$ ($Q_L$) chiral couplings [2, 3, 6]. After having calculated the amplitude of the $\tau \to \mu\gamma$ process and then apply the Passarino-Veltman reduction scheme, we arrive to the associated branching ratio

$$\text{Br}(\tau \to \mu\gamma) = \frac{\alpha g^2 Q_{\mu\tau}^2}{2^{11/4}} \left[|Q_L + Q_R|^2 |F_V|^2 + |Q_L - Q_R|^2 |F_A|^2\right] \frac{m_\tau}{\Gamma_\tau},$$

(8)
where $\Gamma_\tau$ is the total decay width of the tau lepton and

$$F_V = 1 + 2 m_\tau^2 C_0(1) + \frac{m_{Z'}^2}{m_\tau^2} (B_0(1) - B_0(2)), \quad (9)$$

$$F_A = 1 + \frac{m_\tau^2}{m_{Z'}^2} + 2 \left( m_\tau^2 C_0(1) + \left( \frac{m_{Z'}^2}{m_\tau^2} - 3 \right) \left( B_0(1) - B_0(2) \right) \right), \quad (10)$$

where $B_0(1) \equiv B_0(0, m_\tau^2, m_{Z'}^2)$, $B_0(2) \equiv B_0(m_\tau^2, m_\tau^2, m_{Z'}^2)$ and $C_0(1) \equiv C_0(m_\tau^2, 0, m_\tau^2, m_{Z'}^2, m_{Z'}^2)$. Here, the muon mass has been neglected.

The branching ratio in Eq. (8) must be less than the corresponding experimental constraint $\text{Br}_{\text{Exp}}(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$ [5], so this restriction allows us to bound the $|\Omega_{\mu\tau}|^2$ parameter as a function of the $Z'$ boson mass. The behavior of $|\Omega_{\mu\tau}|^2$ for different models can be appreciated in Figure 2. It can be seen from this figure that the sequential model offers the most restrictive constraint and its strength varies between $10^{-1}$ to $10^{0}$ for a $Z'$ mass in the range of 500 GeV < $m_{Z'}$ < 1.5 TeV.

4. Conclusions

In this work we resort to the simplest extension of the SM to estimate the strength of the $Z'\mu\tau$ coupling, represented by the $\Omega_{\mu\tau}$ parameter. For this purpose, we used two different methods: by using the experimental result for the anomalous magnetic dipole moment of the muon and by means of the experimental constraint for the $\tau \rightarrow \mu \gamma$ transition. By using the experimental uncertainty of the muon anomalous magnetic moment, which is measured with amazing precision, we obtain the maximum allowed value for Re($\Omega_{\mu\tau}^2$) as a function of $Z'$ boson mass, which ranges from $10^{-3}$ to $10^{-2}$. The results indicate that the bound for the Re($\Omega_{\mu\tau}^2$) parameter is more restrictive than that obtained by the $\tau \rightarrow \mu \gamma$ dipolar transition calculation.

![Figure 2. Behavior of Re($\Omega_{\mu\tau}^2$), $|\Omega_{\mu\tau}|^2$ as a function of the $Z'$ boson mass. Notice that Re($\Omega_{\mu\tau}^2$) is shown with continuous line, while the dotted lines corresponds to the different models.](image-url)
Acknowledgments

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