The Resonant Effect of an Annihilation Channel in the Interaction of the Ultrarelativistic Electron and Positron in the Field of an X-ray Pulsar

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Abstract: We investigated the effects that occur during the circulation of ultrarelativistic electrons and positrons in the field of an X-ray pulsar. A resonant process in annihilation and the subsequent production of the electron–positron pairs were studied theoretically. Under the resonance, the second-order process in an original fine-structure constant process effectively decays to two first order processes of the fine-structure constant: single-photon annihilation of the electron–positron pair stimulated by the external field, and the Breit–Wheeler process (single-photon birth of the electron–positron pair) stimulated by the external field. We show that resonance has a threshold energy for a certain combinational energy of the initial electron and positron. Furthermore, there is a definite small angle between initial ultrarelativistic particles’ momenta, in which resonance takes place. Initial and final electron–positron pairs fly in a narrow cone. We noticed that electron (positron) emission angle defines the energy of the final pair. We show that the resonant cross-section in the field of the X-ray pulsar may significantly exceed the corresponding cross-section without the field (Bhabha cross-section).

Keywords: Oleinik resonances; X-ray field; electron; positron; Bhabha scattering

1. Introduction

Astroparticle physics lies between particle physics and cosmology and is rapidly evolving. In addition, for the development of an astrophysical object (for example, neutron stars such as pulsars, X-ray pulsars and magnetars), quantum processes play the key roles [1]. There is a strong electromagnetic field near a pulsar [2] and it can have a great impact on such processes. The process focused on in this study—the resonant effect of an annihilation channel interacting with the ultrarelativistic electron and positron in the field of the X-ray pulsar—can also contribute to the electron–positron fluxes near the pulsar. Initial electron–positron pairs appear due to the interactions of photons with the magnetic field [3,4] and also due to the resonant photoproduction of high energy electron–positron pairs in the field of a nucleus and a weak electromagnetic wave [5] near the polar cap of the pulsar. A final electron–positron pair makes a contribution to the pulsar wind, which gives luminosity to the pulsar wind nebulae.

In the present communication we study the second-order process in the fine-structure constant. One of the features of the process is that in an external field it can take place in two modes: nonresonant and resonant. Resonance (or Oleinik resonances [6,7]) means that the intermediate virtual particle falls within the mass shell and becomes real. A second-order process in the fine-structure constant decays into two first-order processes. It can happen because lower order processes are allowed in the external field [8]. In the case of the present study, these are a single-photon annihilation of the electron–positron
pair stimulated by the field of the X-ray pulsar and the Breit–Wheeler process (single-photon birth of the electron–positron pair) stimulated by the field of the X-ray pulsar (see Figure 1). Another important characteristics of Oleinik resonances is that the probability of the resonant processes in the external field can greatly surpass the probability of the process without an external field. The study of these processes is among the highest priorities and the most intensively developing directions [5–23] due to the development of high-power laser radiation sources [24–26].

![Feynman diagrams](image)

**Figure 1.** Feynman diagrams of the electron–positron scattering (annihilation channel) in the field of the X-ray pulsar: (a) nonresonant and (b) resonant cases. Dual lines correspond to the Volkov functions of an electron and a positron (initial and final states). A wavy line represents an intermediate virtual photon.

Previous works [12–14,17,19–22] included a study of only the scattering channel of the Oleinik resonance for the interactions of an electron with another electron and the electron with a positron in the field of the electromagnetic wave. There were no articles dedicated to the annihilation channel and resonant case of the electron–positron interaction in the corresponding wave [21]. It was shown [20,21] that in a weak electromagnetic field, interference in the two resonant channels can be ignored due to the fact that they lie in different kinamitcs regions.

In this communication the resonant theory of the annihilation channel in the interaction between the ultrarelativistic electron and positron in the field of an X-ray pulsar is developed. There is a characteristic parameter in such processes in the external field—the classical relativistic-invariant parameter [8,18–21]:

$$\eta = \frac{eF\lambda}{mc^2}$$  \hspace{1cm} (1)

which numerically equals the ratio of the work done by the field over an electron on the wavelength to the electron rest energy (\(e\) and \(m\) are the electron’s charge and mass; \(F\) and \(\lambda = c/\omega\) are the field strength and wavelength; \(\omega\) is the wave frequency). The problem was examined in the weak fields, when one-photon emission and absorption processes are most probable. Weak field means that

$$\eta << 1.$$  \hspace{1cm} (2)

From the works [27–29] it can be concluded that the classical parameter and \(\eta << 1\) near the magnetic poles of X-ray pulsar and \(F \sim 10^{13} \text{ V/cm}\). Hereafter, we will use the natural units \(\hbar = c = 1\).
2. Resonance of the Annihilation Channel in the Field of the X-ray Pulsar

A four-potential of the external field was chosen in the form of an external circular polarized light wave propagating along the z axis:

$$A(\phi) = (F/\omega) \cdot (e_x \cos \phi + \delta \cdot e_y \sin \phi), \quad \phi = kx = \omega (t - z),$$  \hspace{1cm} (3)

where $\delta$ is the wave ellipticity parameter; $\delta = \pm 1$; $(e_x, e_y)$ are the polarization four-vectors; $k = \omega n = \omega (1, n)$ is the wave four-vector; and $k^2 = 0, e_x^2 = -1, e_x e_y k = 0$. The field near X-ray pulsar is not a plain monochromatic wave, but it was chosen to obtain a qualitative evaluation of the electron and positron interaction probability. Considering those, we obtained the expression for an amplitude of the process of the annihilation channel of the electron–positron scattering in the field of the X-ray pulsar.

$$S = \sum_{l=-\infty}^{+\infty} S_{(l)}. \hspace{1cm} (4)$$

Here $S_{(l)}$ is a partial amplitude of the process with the radiation (absorption) of $l$ photons of the external X-ray field.

$$S_{(l)} = \frac{i e^2 (2\pi)^5}{4\sqrt{E_- E_+ E'_- E'_+}} \frac{(\bar{\Sigma}_2 H_+ \Sigma_2)(\bar{\Sigma}_1 H_+ u_1)}{q^2} \delta^{(4)}(p'_- + p'_- - p_- - p_+ + lk) \hspace{1cm} (5)$$

$$H_1 = \gamma_1 e^{-i\chi_1} - \frac{\eta m}{4} \left(\hat{\gamma}_1 \hat{k} \hat{e}_- \left(\frac{\hat{k} \hat{e}_-}{kp_-}\right) - \hat{e}_- \hat{k} \hat{\gamma}_1 \left(\frac{\hat{k} \hat{e}_+}{kp_+}\right)\right) \hspace{1cm} (6)$$

Cap means $\hat{a} = a^\mu \gamma^\mu$. The parameters $\gamma_1$ and $\chi_1$ are defined in the following way:

$$\gamma_1 = \eta m \sqrt{-Q^2_1}, \quad Q_1 = \frac{p_-}{(kp_-)} - \frac{p_+}{(kp_+)} \hspace{1cm} (7)$$

$$\tan \chi_1 = \frac{Q \epsilon_y}{(Q, \epsilon_x)} = \epsilon_\pm = \epsilon_x \pm i \delta \epsilon_y \hspace{1cm} (8)$$

As stated above, the intermediate photon with the four-momentum $q$ comes to the mass shell and becomes real. Additionally, amplitude tends to infinity when

$$q^2 = 0. \hspace{1cm} (9)$$

The most probable process is the process with the emission and absorption of one photon under Equation (2) and resonant conditions Equations (11)–(16). As a result, at the first vertex there is the process of the one-photon annihilation of the electron–positron pair, stimulated by the external field of the X-ray pulsar, and at the second vertex there is the Breit–Wheeler process (single-photon birth of the electron–positron pair) stimulated by the external field of the X-ray pulsar. Additionally, the four-momentum conservation laws in all vertices can be written as:

$$p_+ + p_- = q + k, \quad q + k = p'_+ + p'_-, \hspace{1cm} (10)$$

where $p_\pm = (E_\pm, p_\pm)$ is the four-momentum of the initial electron and positron and $p'_\pm = (E'_\pm, p'_\pm)$ is the four-momentum of the final electron and positron.

An analysis of the conservation laws Equation (10) using Equation (9), demonstrates that in the weak fields Equation (2) resonance happens when initial electron and positron have ultrarelativistic energies and a small angle between their momenta. The final electron–positron pair also should be ultrarelativistic, have small angle between their momenta and be emitted in a narrow cone along the direction of momentum of the initial electron–positron pair:
\[ E_\pm >> m, \quad E'_\pm >> m, \] (11)

\[ \theta_i = \angle \left( \mathbf{p}_+, \mathbf{p}_- \right) << 1, \quad \theta_f = \angle \left( \mathbf{p}'_+, \mathbf{p}'_- \right) << 1, \quad \theta_\pm = \angle (\mathbf{k}, \mathbf{p}_\pm) \sim 1. \] (12)

By applying equalities Equations (9)–(12), one can obtain the apex angle of the initial electron–positron pair in resonance

\[ \delta_i^2 = \delta^2_{\text{res}}, \quad \delta^2_{\text{res}} = \left( \frac{\epsilon_i}{\epsilon_c} \right)^2 (\epsilon_c - 1) \geq 0. \] (13)

Here we denote:

\[ \delta_i = \frac{E_i \theta_i}{m}, \quad \epsilon_i = \frac{E_i}{E_{\text{thr}}}, \quad \epsilon_c = \frac{E_c}{E_{\text{thr}}}, \] (14)

where \( E_{\text{thr}} \) is the threshold energy of both vertices, \( E_i \) is the initial energy of the electron–positron pair and \( E_c \) is the combination energy of the electron–positron pair:

\[ E_{\text{thr}} = \frac{m^2}{4 \omega \sin^2 (\theta_/2)}, \quad E_i = E_+ + E_-, \quad E_c = \frac{E_+ E_-}{E_i}. \] (15)

From the equality equation (Equation (13)) it can be seen that the parameter \( \epsilon_c \geq 1 \). This means that the resonance of the process in the field of the X-ray pulsar has a reaction threshold for combination energy of the electron–positron pair \( E_c \geq E_{\text{thr}} \). Equation (15). This threshold energy \( E_{\text{thr}} \), Equation (15), is defined by the electron rest energy, the wave frequency and also the angle between the momentum of the initial positron (electron) and the external wave. \( E_{\text{thr}} \sim 10^2 \text{ MeV} \) for the X-ray range. The parameter \( \epsilon_i \) shows the surplus of the primary energy of the electron-positron pair over the threshold energy. Parameters \( \epsilon_i \) and \( \epsilon_c \) define the primary pair apex angle in the resonance equation (Equation (13)) and the energies of the primary electron and positron:

\[ E_+ = \frac{1}{2} \epsilon_i E_{\text{thr}} \left[ 1 \pm \sqrt{1 - \frac{4 \epsilon_c}{\epsilon_i}} \right], \quad E_- = \frac{1}{2} \epsilon_i E_{\text{thr}} \left[ 1 \mp \sqrt{1 - \frac{4 \epsilon_c}{\epsilon_i}} \right]. \] (16)

For example, when \( \epsilon_i = 200 \) and \( \epsilon_c = 50 \cdot E_+ = E_- = 10 \text{ GeV} \). From the expressions Equation (16) it can be seen that there is an inequality for \( \epsilon_i \) and \( \epsilon_c \)

\[ \epsilon_i \geq 4 \epsilon_c \geq 4. \] (17)

From here and Equation (13) we have \( \epsilon_i \geq 4 \). Let us introduce an angle between the final positron momentum and a total momentum \( \theta_+ = \angle (\mathbf{p}'_+, \mathbf{p}_i) \ll 1 \). An analysis of the relation equation (Equation (9)) (resonant condition), taking into consideration Equations (10)–(12), demonstrates that there are two possible values of the positron (electron) energy for each value of \( \delta^2_+ \):

\[ x'_+ = \frac{1}{2(\epsilon_i + \delta^2_+)} \left[ \epsilon_i \pm \sqrt{\epsilon_i (\epsilon_i - 4) - 4 \delta^2_+} \right], \quad x'_- = 1 - x'_+. \] (18)

Here indicated:

\[ \delta'_+ = \frac{E_i \theta'_+}{m}, \quad x'_+ = \frac{E'_+}{E_i}. \] (19)

Figure 2 represents this dependency for \( \epsilon_i = 6 \) and \( \epsilon_i = 8 \). This means that if \( E_{\text{thr}} = 10^2 \text{ MeV} \) for \( \epsilon_i = 6 \) we have \( E_i = 6 \cdot 10^2 \text{ MeV} \) and for \( \epsilon_i = 8 \) we have \( E_i = 8 \cdot 10^2 \text{ MeV} \). Solutions to Equation (18) describe a physical picture only on the gap

\[ 0 \leq \delta^2_+ \leq \frac{\epsilon_i}{4} (\epsilon_i - 4). \] (20)
3. Resonant Differential Cross-Section in the Field of the X-ray Pulsar

The resonant differential cross-section of the annihilation channel in interaction of the ultrarelativistic electron and positron in the field of the X-ray pulsar under the conditions Equations (9), (11), (12) and (20) was derived in a standard manner. After straightforward calculations, the resonant differential cross-section was derived in the following form:

$$d\sigma_{\text{res}} = r_2^2 \frac{m^2}{E_i^4} \eta^4 \left( \frac{\epsilon_i}{\epsilon_c} \right)^{\frac{1}{2}} \left( \frac{x'_+}{1 - x'_+} \right) W_1 \left( \frac{G}{(\delta^2 - \delta^2_{\text{res}})^2 + \Gamma^2} \right) W'_1 d\delta'_{\text{res}} d\varphi_+.$$  \hspace{1cm} (21)

Here $\varphi_+$ is a polar outgoing angle of the positron, $W_1$ is the function which defines the probability of the single-photon electron–positron annihilation in the X-ray field and $W'_1$ defines the probability of the X-ray field-stimulated Breit–Wheeler process (single-photon birth of the electron–positron pair):

$$W_1 = 2u - 1 + 2 \frac{u}{u_1} (1 - \frac{u}{u_1}).$$ \hspace{1cm} (22)

$W'_1$ can be found from Equation (22) by substitution $u \rightarrow u'$

$$u = \frac{(kq)^2}{4(kp'_+)(kp'_-)} = \frac{\epsilon_i}{4\epsilon_c}, \quad u' = \frac{(kq)^2}{4(kp'_+')(kp'_-')} = \frac{\epsilon_i}{4\epsilon'_c}, \quad u_1 = \frac{(kq)^2}{2m^2} = \frac{\epsilon_i}{4}.$$

$$\epsilon'_c = \frac{E'_c}{E_{\text{thr}}}, \quad E'_c = \frac{E'_i E'_c}{E_i}.$$ \hspace{1cm} (23)

$\Gamma$ is a radiation width

$$\Gamma = W \frac{\epsilon_i^2}{2\epsilon'_c E'_c}.$$

Here $\epsilon_i, \epsilon'_i$ are energy of electron and positron respectively.
In the relation Equation (21), the function $G \approx 1$ and $W$ equals the total probability (per unit time) of the inverse X-ray field-stimulated Breit–Wheeler process (single-photon annihilation of the electron–positron pair) $[7,23]$.

$$W = \frac{\alpha m^2 n}{8 \pi E_i} \eta^2 \left[ \left( 2 + \frac{2}{u_1} - \frac{1}{u_1^2} \right) \text{Arth} \left( \sqrt{1 - \frac{1}{u_1}} - \left( 1 + \frac{1}{u_1} \right) \sqrt{1 - \frac{1}{u_1}} \right) \right]. \quad (26)$$

The denominator part of the relation Equation (21) takes the Breit–Wigner form. The differential Bhabha cross-section (in the absence of the external field) of the annihilation channel is

$$d\sigma_0 = r_e^2 m^2 H d\delta^2 \, d\phi_+,$$

where the function $H \sim 1$. Additionally, the differential cross-section for the scattering channel is

$$d\sigma_0 = r_e^2 L d\delta^2 \, d\phi_+,$$

where the function $L \sim 1$. When $\delta_i^2 \to \delta_{res}^2$ the resonant differential cross-section Equation (21) has an acute peak.

$$d\sigma_{res}^{\max} = r_e^2 \left( \frac{16\pi}{\alpha} \right)^2 \left( \frac{E_{thr}}{m} \right)^2 \left( \frac{\epsilon_c}{\epsilon_i} \right)^2 \frac{x'_+}{1 - x'_+} G W_i W'_i d\delta^2 \, d\phi_+.$$ \quad (29)

For example, from Equation (29) for $E_{thr} = 10^2$ MeV, $\epsilon_c = 1.5$ and $\epsilon_i = 6$ we obtain $d\sigma_{res}^{\max} \sim 10^{11} r_e^2 d\delta^2 \, d\phi_+$. For $\epsilon_c = 50$ and $\epsilon_i = 200$ we have $d\sigma_{res}^{\max} \sim 10^{11} r_e^2 d\delta^2 \, d\phi_+$. Additionally, in this case, the ratio of the resonant differential cross-section Equation (29) to the Bhabha cross-section Equation (28) takes the maximum value

$$R_{res}^{\max} = \frac{d\sigma_{res}^{\max}}{d\sigma_0} = \left( \frac{16\pi}{\alpha} \right)^2 \left( \frac{E_{thr}}{m} \right)^2 \left( \frac{\epsilon_c}{\epsilon_i} \right)^2 \frac{x'_+}{1 - x'_+} Z \sim 10^{11},$$

where the function $Z \sim 1$. That means that $R_{res}^{\max}$ is more than $10^{11}$. It is worth noting that the field near the X-ray pulsar is significantly heterogeneous in space and time. For that reason, the resonance width will be considerably greater than the radiational width. As a result, $R_{res}^{\max}$ will be several orders of magnitude smaller than $10^{11}$. The obtained estimate is the maximum possible.

4. Conclusions

The resonant effect of the annihilation channel in the interaction of the ultrarelativistic electron and positron in the field of an X-ray pulsar was researched. It is shown that the resonance for the considered X-ray intensities is possible only when the combinational energy of the primary electron and positron energies surpasses threshold energy. This combinational energy equals the product of the electron and positron energies divided by their sum. Additionally, in the resonance electron and positron fly in such a way that the angle between directions of their momentum is small and fulfills a certain resonant condition. It is defined by the electron, positron and threshold energies. The final electron–positron pair fly in the narrow cone too. The final electron (positron) energy accepts one to two values for every fixed angle between the final positron momentum and a total momentum. It can be seen from Equation (30) that the resonant differential cross-section of the annihilation channel in the interaction of the ultrarelativistic electron and positron in the field of the X-ray pulsar can surpass the cross-section without the external field (Bhabha cross-section) by more than eleven orders of magnitude.

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