Electroweak dipole moment form factors of the top quark in supersymmetry

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We present a complete analysis of the electric and weak dipole moment form factors of the top quark within the Minimal Supersymmetric Standard Model with complex parameters. We include gluino, chargino, and neutralino exchange in the loops of the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices.

1. Introduction

The large mass of the top quark allows one to probe physics at a high energy scale, where new physics might show up. In the last years a number of papers \cite{1,2,3} considered CP violating observables in top quark production as tests for new physics. In $e^+e^-$ annihilation these effects are due to the weak and electric dipole moment form factors $d^Z(s)$ and $d^γ(s)$ of the top quark. In general, the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices including the CP violating form factors are

\begin{align}
e(\nu^\gamma_t)_\mu &= e \left( \frac{2}{3} \gamma_\mu - \frac{d^γ(s)}{m_t} \gamma_5 \right), \\
gZ(\nu^Z_t)_\mu &= gZ \left( \gamma_\mu (g_V + g_A \gamma_5) - \frac{d^Z(s)}{m_t} \gamma_5 \right),
\end{align}

where $d^Z(s)$ and $d^γ(s)$ are the weak and electric dipole moment form factors of the top quark, and $g_V = (1/2) - (4/3) \sin^2 \Theta_W$, $g_A = -(1/2)$, $\gamma_\mu = p_t^\mu - p_{\bar{t}}^\mu$, $gZ = g/(2 \cos \Theta_W)$, and $g = e/\sin \Theta_W$ with $e$ the electro–magnetic coupling constant and $\Theta_W$ the Weinberg angle.

In the Standard Model (SM) CP violation can appear only through the phase in the CKM–matrix. The dipole moment form factors $d^γ(s)$ and $d^Z(s)$ for the quarks are at least two–loop order effects and hence very small. In the Minimal Supersymmetric Standard Model (MSSM)\textsuperscript{4} additional complex couplings can be introduced that lead to CP violation within one generation only \textsuperscript{5}, and which occur at one–loop level. If the masses of the SUSY particles are not very much higher than the mass of the top, one expects SUSY radiative corrections to induce larger values for $d^γ(s)$ and $d^Z(s)$. According to the particle content in the loop (see Fig. 1a, b) we distinguish the following three contributions:

1. gluino contribution $d^γ-Z$ with $(\tilde{t}\tilde{g})$ in the loop,
2. chargino contribution $d^γ-Z$ with $(\tilde{\chi}^+\tilde{\chi}^-\tilde{b})$ and $(\tilde{t}\tilde{b}\tilde{\chi}^+)$ in the loop,
3. neutralino contribution $d^{0-Z}$ with $(\tilde{\chi}^0\tilde{\chi}^0\tilde{t})$ and $(\tilde{t}\tilde{t}\tilde{\chi}^0)$ in the loop.

The gluino contribution $d^γ-Z$ was considered in \textsuperscript{3}. The chargino and neutralino contributions were calculated in \textsuperscript{3}. Although the gluino contribution $d^γ-Z$ is proportional to $\alpha_s$, it turns out...
that the chargino contribution \( d_{\gamma,Z}^\chi \), which is proportional to \( \alpha_w \) (\( \alpha_w = g^2/(4\pi) \)) can be equally important (see also [3]). This is due to threshold enhancements which can occur in the contributions of the diagrams in Fig. 1a, and the large Yukawa couplings: \( Y_t = m_t/(\sqrt{2}m_W \sin \beta) \) and \( Y_b = m_b/(\sqrt{2}m_W \cos \beta) \). In general the neutralino contribution turns out to be smaller. However, there are cases where it is important.

2. Complex couplings in the MSSM

In the MSSM the Higgs–higgsino mass parameter \( \mu \) and the trilinear scalar coupling parameters \( A_t \) and \( A_b \) can be complex, and provide the CP violating phases. The calculation requires the diagonalization of the squark, chargino, and neutralino mass matrices. We use the singular value decomposition [8] to diagonalize the complex neutralino and chargino mass matrices.

The size of the dipole moment form factors \( d_{\gamma(s)} \) and \( d_{Z(s)} \) depends strongly on the phases of the SUSY parameters. There are constraints [3,4] on some phases from the measurement of the electric dipole moment (EDM) of the neutron. Usually, one concludes [3,4] that either the phases involved in the EDM of the neutron are very small or the masses of the first generation of squarks are in the TeV range. By using supergravity (SUGRA) with grand unification (GUT) there are attempts to constrain also the phases entering the dipole moments of the top. In our analysis we want to be more general and we do not make any additional assumptions about GUT except for the unification of the gauge couplings and gaugino masses. In particular, we do not assume unification of the scalar mass parameters and of the trilinear scalar coupling parameters \( A_q \) of the different generations.

In the MSSM with complex phases, \( d_{\gamma(s)} \) and \( d_{Z(s)} \) are generated in one–loop order, irrespective of generation mixing. We treat the chargino and neutralino contributions separately from the gluino contribution not only because different couplings are involved (electroweak and strong), but also because they are sensitive to different SUSY parameters. The contributions from the different Feynman diagrams (Fig. 1a,b) depend in a distinctive way on the SUSY parameters. The Passarino–Veltman three point functions \( C_0, C_i \), and \( C_{ii} (i = 1, 2) \) [10] appear in the loop integrations of the diagrams Fig. 1a,b.
3. Numerical results

The chargino contribution depends on the gaugino and higgsino couplings, as well as on the squark mixing angle and phase. We have included the terms proportional to the bottom Yukawa coupling $Y_b$ which are important for large $\tan\beta$. For small values of $\tan\beta$ the terms proportional to $Y_t$ dominate. As neutralinos do not couple to the photon, $d_{\chi^0}^\gamma$ receives a non-zero contribution only from the diagram with $\tilde{t}\tilde{\chi}^0$ exchanged in the loop.

In the following we give numerical results for the real and imaginary parts of $d(s)$ and $d^\gamma(s)$. Quite generally they depend on the parameters $M', M, |\mu|, \tan\beta, m_{\tilde{t}_L}, m_{\tilde{b}_L}, \cos\theta_t, \cos\theta_b$ and the phases $\varphi_{\mu}, \varphi_t, \varphi_b$, and $\varphi_\gamma$. The GUT relations

$$m_{\tilde{g}} = (\alpha_s/\alpha_2)M \approx 3M$$  \hspace{1cm} (3)

$$M' = \frac{5}{3} \tan^2 \Theta_W M$$ \hspace{1cm} (4)

imply that the gaugino mass parameters have the same phase.

We take $m_W = 80$ GeV, $m_t = 175$ GeV, $m_b = 5$ GeV, $\sqrt{s} = 500$ GeV, $\alpha_s = 0.1$, and $\alpha_{em} = \frac{1}{125}$. In order not to vary too many parameters we choose the following set of SUSY parameter values: $M = 230$ GeV, $m_{\tilde{t}_1} = 150$ GeV, $m_{\tilde{b}_1} = 270$ GeV, $|\mu| = 250$ GeV, $m_{\tilde{t}_2} = 400$ GeV, $m_{\tilde{b}_2} = 280$ GeV, $\tan\beta = 2$, $\theta_t = \frac{\pi}{6}$, $\theta_b = \frac{3\pi}{8}$, $\varphi_{\mu} = \frac{\pi}{3}$, $\varphi_t = \frac{\pi}{6}$, $\varphi_b = \frac{\pi}{3}$.

Notice that the dipole moment form factors $d(s)$ and $d^\gamma(s)$ depend on $\varphi_{\mu}$ not only through the chargino and neutralino diagonalizing matrices, but also through the chargino and neutralino mass spectra. The values of the chargino and neutralino masses can vary by about 40 percent when $\cos\varphi_{\mu}$ is varied between $-1$ and $1$.

$3m\,d(s)$ and $3m\,d^\gamma(s)$ are determined by the absorptive parts of the amplitudes. Therefore they vanish when no real production of charginos or neutralinos is possible. Local maxima occur near the thresholds of chargino or neutralino pair production, and they get bigger if the gaugino and higgsino component of the chargino are approximately equal. The neutralino contribution to $3m\,d(s)$ and $3m\,d^\gamma(s)$ is one order of magnitude smaller than the chargino contribution because the photon does not couple to the neutralino.

Figure 2. $d(s)$ and $d^\gamma(s)$ for the reference parameter set with $M = 230$ GeV: (a) $3m\,d(s)$ (full line), $3m\,d^\gamma(s)$ (dashed line) and (b) $Re\,d(s)$ (full line), $Re\,d^\gamma(s)$ (dashed line).
nos and only Fig. 1b contributes. The neutralino contribution $\Im m_{\tilde{\chi}^0}$ is smaller than the chargino contribution because the couplings are smaller. It shows the same qualitative behaviour as the chargino contributions, but it is more complicated because of the richer particle spectrum. Note that the two neutralinos in Fig. 1a have to be different.

In Fig. 2a we show $\Im m d\gamma(s)$ and $\Im m dZ(s)$ as functions of $\sqrt{s}$, where all contributions (gluino, chargino, and neutralino) are summed up. The threshold effects can be seen very clearly. There is a big enhancement in $d\gamma(s)$ and $dZ(s)$ because the threshold for $\tilde{\chi}^+ + \tilde{\chi}^- \rightarrow 1\tilde{\chi}^{\pm}$ production is reached at $\sqrt{s} = 420$ GeV for $m_{\tilde{\chi}^\pm} = 210$ GeV. At $\sqrt{s} = 590$ GeV $\tilde{\chi}^+_1 \tilde{\chi}^-_2$ production becomes possible and again there is a big contribution but with the opposite sign. The additional thresholds in $dZ(s)$ are due to the neutralino contributions. In Fig. 2b we show $\Re e d\gamma(s)$ and $\Re e dZ(s)$, which can also be understood via dispersion relations: each spike corresponds to the opening of a new production channel.

Acknowledgements

We thank Helmut Eberl for his constructive assistance in the evaluation of the loop integrals and Sabine Kraml for the discussions about the couplings in the Lagrangian. We also thank Stefano Rigolin for his helpful correspondence regarding the numerical calculations. E.C.’s work has been supported by the Bulgarian National Science Foundation, Grant Ph–510. This work was also supported by the ‘Fonds zur Förderung der wissenschaftlichen Forschung’ of Austria, project no. P10843–PHY.

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