Baryons in the Field Correlator Method

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Abstract
The ground and \(P\)-wave excited states of \(nnn\), \(nns\) and \(ssn\) baryons are studied in the framework of the field correlator method using the running strong coupling constant in the Coulomb–like part of the three–quark potential. The string correction for the confinement potential of the orbitally excited baryons, which is the leading contribution of the proper inertia of the rotating strings, is estimated.

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1 INTRODUCTION

Quantum chromodynamics (QCD) has been established as the theory describing the strong interaction but its application to low–energy hadron phenomenology is still far from a routine deduction. Various approximations, whose connection to the underlying theory remains sometimes obscure, are presently used to describe baryon spectroscopy. It has become an attractive program to develop model independent methods which are firmly based in fundamental theory.

The field correlator method (FCM) in QCD [1] provides promising perspective. The FCM is a formulation of the nonperturbative QCD that gives additional support to the quark model assumptions. Progress was recently made [2, 3] towards placing the computation of baryon masses within the FCM on the same level as that of mesons. Nevertheless, this work can be refined. In Refs. [2] a freezing value of the strong coupling constant in the
perturbative Coulomb–like potential has been employed. This choice appears to be a reasonable approximation and gives rise to a good description of heavy quarkonia and heavy–light mesons. Note that for light baryons the Coulomb–like force does not play a crucial role and produces only a marginal (∼10%) correction. Nevertheless, it is important to include into the FCM approach the modern knowledge about the one–gluon exchange forces that still represent a fundamental concept which might give us a deeper understanding of baryon spectroscopy.

In this talk, we explore the FCM for baryons in a more regular way by considering the effects of the running strong coupling constant in the Coulomb–like part of the three–quark potential. We use the background perturbation theory (BPTh) [4] for the coupling constant to avoid the infrared singularities. Below are considered the baryons composed of valence light quarks, the up, down, and strange flavors. We present the new results for the masses of the ground states and P–wave excited states of nnn, nns and ssn baryons. We also estimate the string correction to the baryonic orbital excitations.

2 EFFECTIVE HAMILTONIAN in FCM

In the FCM three–quark dynamics in a baryon is encoded in gluonic field correlators which are responsible for quark confinement. Starting from the Feynman–Schwinger representation for the quark and gluon propagators in the external field, one can extract hadronic Green’s functions and calculate the baryon spectra. The key ingredient of the FCM is the use of the auxiliary fields (AF) initially introduced in order to get rid of the square roots appearing in the relativistic Hamiltonian [3]. Using the AF formalism allows one to derive a simple local form of the effective Hamiltonian (EH) for the three-quark system, which comprises both confinement and relativistic effects, and contains only universal parameters: the string tension \( \sigma \), the strong coupling constant \( \alpha_s \), and the bare (current) quark masses \( m_i \).

The baryon mass is given by

\[
M_B = M_0 + \Delta M_{\text{string}} + C, \quad M_0 = \sum_{i=1}^{3} \left( \frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + E_0(\mu_i)
\]

where \( E_0(\mu_i) \) is an eigenvalue of the Schrödinger operator \( H_0 + V \), \( V \) is the sum of the string potential \( V_Y(r_1, r_2, r_3) = \sigma r_{\text{min}}, \) \( r_{\text{min}} \) being the minimal string length corresponding to the Y–shaped configuration\(^1\) and a Coulomb interaction term \( V_C(r_1, r_2, r_3) = \sum_{i<j} V_C(r_{ij}) \), arising from the one-gluon exchange. The constant auxiliary fields \( \mu_i \) finally acquire the meaning of the dynamical quark masses. They are defined by minimization condition. The quark self–energy correction \( C \) that is created by the color magnetic moment of a quark propagating through the vacuum background field. This correction, which can be calculated perturbatively, adds an overall negative constant to the hadron masses [6].

Note that the string potential \( V_Y(r_1, r_2, r_3) \) represents only the leading term in the expansion of the QCD string Hamiltonian in terms of angular velocities [5]. The leading correction in this expansion is known as a string correction. This is the correction totally missing in relativistic equations with local potentials. Its sign is negative, so the contribution

\(^1\)This potential naturally arises from the consideration of the Wilson loop depicted in Fig. 1a.
of the string correction lowers the energy of the system, thus giving a negative contribution to the masses of orbitally excited states, leaving the S–wave states intact.

The calculation of the string correction is greatly simplified, if the string junction point is chosen to coincide with the center–of–mass coordinate \( R_{cm} \). In this case, the complicated string junction potential is approximated by a sum of the one–body confining potentials. The accuracy of this approximation for the \( P \)–wave baryon states is better than 1% [3]. As the result one obtains

\[
\Delta M_{\text{string}} = -\frac{\sigma}{6} <\Psi| \sum_i \frac{(r_i \times p_i)^2}{\mu_i^2 r_i} |\Psi> ,
\]

where \( \Psi \) is the eigenfunction of the unperturbed EH.

## 3 COULOMB-LIKE INTERACTION in the BPTh

Details of our treatment of the string junction confinement potential can be found in Ref. [2]. We now concentrate on the Coulomb–like part of interaction, \( V_C(r) \). It is convenient to write the Coulomb–like potential in QCD in momentum space as

\[
V_C(q^2) = -C_F \frac{\alpha_V(q^2)}{q^2} ,
\]

where \( C_F \) is the color factor. A running constant \( \alpha_V(q^2) \) controls the behavior of standard perturbation theory (SPTh) at low momentum scales. The formal expression for \( V_C(r) \) in position space is written as

\[
V_C(r) = -C_F \frac{\alpha_s(r)}{r} , \quad \alpha_s(r) = 2 \pi \int_0^\infty dq \frac{\sin qr}{q} \alpha_V(q^2) .
\]

Up to two loops

\[
\alpha_V(q^2) = \frac{4\pi}{\beta_0 t} \left( 1 - \frac{\beta_1}{\beta_0^2} \ln t \right) , \quad t = \frac{q^2}{\Lambda_V^2} ,
\]

where \( \beta_i \) are the coefficients of the QCD \( \beta \)-function.

The conventional coupling (5) is analytically singular at a scale \( q^2 = \Lambda_V^2 \), so the SPTh itself is not well defined in the infrared domain where the coupling become large. This problem can be traced back to the fact that the integral over the running coupling which appears in Eq. (4) is ill defined. As a result, \( \alpha_s(r) \) is known only in the perturbative region, \( r \lesssim 0.1 \) fm. Estimates of the average interquark distances in light baryons are in the vicinity of 0.7 fm which is certainly outside of the perturbative region.

There also exist the possibilities of defining a running coupling which stays finite in the infrared. For our purposes, we find it convenient, as a useful approximation, to define the strong coupling constant \( \alpha_B(r) \) in the BPTh [4]. The logic behind this approach is that the perturbative gluon propagator is modified strongly at \( q \lesssim m_B \) by the physics of large distances. In momentum space, \( \alpha_B(q^2) = \alpha_s(q^2 + m_B^2) \), where \( m_B \sim 1 \) Gev has the
meaning of the lowest hybrid excitation; from comparison with the lattice static potential \( m_B \sim 1 \text{ GeV} \). The result can be conventionally viewed as arising from the interaction of a gluon with background vacuum fields.

We define \( \alpha_B(r) \), as well as the Coulomb–like potential in the configuration space via expressions similar to \( \Sigma + 2 \Sigma \) with the substitution \( \alpha_s(q^2) \rightarrow \alpha_B(q^2) \) and \( \alpha_s(r) \rightarrow \alpha_B(r) \). For \( \alpha_B(q^2) \), we use the standard two–loop result with the substitution \( t \rightarrow t_B = \ln \frac{q^2 + m_B^2}{\Lambda_V^2} \). The resulting coupling \( \alpha_B(q^2) \) is finite in the infrared, \( q^2 \rightarrow 0 \). In the ultraviolet region, \( q^2 \gg m_B^2 \) one recovers the SPTh result. In configuration space the background coupling constant \( \alpha_B(r) \) exists for all distances and saturates at some critical, or freezing, value for \( r \gg 1/m_B \).

4 RESULTS and DISCUSSION

The EH contains five parameters: the current quark masses \( m_n \) and \( m_s \), the string tension \( \sigma \), and two parameters \( \Lambda_V \) and \( m_B \) defining \( \alpha_B(r) \). Let us underline that they are not the fitting parameters. In our calculations we use \( \sigma = 0.15 \text{ GeV}^2 \) found in the SU(3) QCD lattice simulations. We employ the current light quark masses \( m_u = m_d = 7 \text{ MeV} \) and the bare strange quark mass \( m_s = 175 \text{ MeV} \) found previously from the fit to \( D \) spectra. This value of the strange quark mass is consistent with the QCD sum rules estimation \( m_s(2 \text{ GeV}) = (125 \pm 40) \text{ MeV} \), much lower scale. For the remaining two parameters \( \Lambda_V \) and \( m_B \) we employ the values \( \Lambda_V = (0.36 \pm 0.02) \text{ GeV} \), \( m_B = (1 \pm 0.05) \text{ GeV} \), determined previously in Ref. [7]. The result is consistent with the freezing of \( \alpha_B(r) \) at \( r \gg m_B^{-1} \) with a magnitude \( \sim 0.5 - 0.6 \). The behavior of \( \alpha_B(r) \) for \( \Lambda_V = 0.36 \text{ GeV} \) is shown in Fig. 1b for three different values of \( m_B \). Note that \( \alpha_B(r) \) increases with \( \Lambda_V \) and, for fixed \( \Lambda_V \), decreases with \( m_B \).

We begin the discussion of our results\(^2\) by examining our predictions for the ground states of the \( nnn \), \( nns \) and \( ssn \) baryon with \( L = 0 \). First, we mention that the baryon masses decrease when \( \Lambda_V \) increases and, for fixed \( \Lambda_V \), the baryon masses decrease when \( m_B \) increases. This comes as no surprise: the effect can be easily read off from Fig. 1b. Increasing \( \Lambda_V \) for fixed \( m_B \) and decreasing \( m_B \) for fixed \( \Lambda_V \) leads to increased running constant \( \alpha_B(r) \) and respectively, to decreased baryon masses. Upon varying the parameters \( \Lambda_V \) and \( m_B \) within the error bars indicated above, we obtain the baryon masses in the interval 1161 – 1209 MeV \( (nnn) \), 1246 – 1297 MeV \( (nns) \), and 1330 – 1383 MeV \( (ssn) \). The difference in the mass values is mostly due to the difference in the running of the strong coupling in the midmomentum regime.

The baryon energies agree reasonably well with the Particle Data Group (PDG) listing particularly if we take into consideration that spin interactions are neglected. For instance, for \( L = 0 \) we get \( \frac{1}{4}(\Lambda + \Sigma + 2 \Sigma^*)_{\text{theory}} = 1276^{+23}_{-30} \text{ MeV} \), where the error bars correspond to the variation of the hyperon masses within the chosen range of \( \Lambda_V \) and \( m_B \) versus \( \frac{1}{4}(\Lambda + \Sigma + 2 \Sigma^*)_{\text{exp}} = 1267 \text{ MeV} \). For the \( \Xi \), we have \( \Xi_{\text{theory}} = 1360^{+23}_{-30} \text{ MeV} \), whereas \( \Xi_{\text{exp}} = 1315 \text{ MeV} \). However, for the nucleon we get \( \frac{1}{2}(N + \Delta)_{\text{theory}} = 1187^{+22}_{-21} \text{ MeV} \), which is about 100 MeV heavier than \( \frac{1}{2}(N + \Delta)_{\text{exp}} = 1085 \text{ MeV} \). The difference can be ascribed to the effects of

\(^2\)For more detailed presentation of the results see Ref. [8]
spin–dependent quark–quark interactions modeled after the effect of gluon exchange in QCD or arising from one–boson exchange that are completely omitted in the present approach. Another source of the discrepancy is the systematic error associated with the use of the AF formalism, which is maximal for the $S$–wave $nnn$ states [3].

Analysis of the P-wave excitations shows that string correction does not essentially depend on the baryon flavor nor on the type of excitation. As a result of the weak dependence of the string correction on a baryon flavor and the type of excitation the masses for all considered baryons become smaller by about the same value $\sim 50 - 60$ MeV.

The physical P-wave states are not pure $\rho$ or $\lambda$ excitations but linear combinations of all states with a given total momentum $J$. Most physical states are, however, close to pure $\rho$ or $\lambda$ states. For example, the masses of $N(1535)$ and $N(1520)$ resonances with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, respectively, both match with the mass of the $\lambda$–excitation for the $nnn$ baryon: $M_{\lambda}(nnn) = 1567^{+13}_{-14}$ MeV. The masses of $\Sigma(1620)$ and $\Sigma(1670)$ states with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, respectively, match very closely with the mass of the $\lambda$–excitation for the $nns$ baryon: $M_{\lambda}(nns) = 1636^{+13}_{-15}$ MeV. The masses of the $\rho$–excitations which correspond to $P$–states of the light diquarks are typically 60 - 80 MeV higher.

5 CONCLUSIONS

In this paper the study of the ground and excited states of the $qqq$, $qqs$ and $ssq$ baryons by including into the EH derived using the FCM the effects of the running strong coupling constant $\alpha_B(r)$ in the perturbative Coulomb–like part of the three–quark potential is presented. The results have refined our previous studies for the ground and excited states of the $qqq$, $qqs$ and $ssq$ baryons obtained for the freezing coupling constant. Our study shows
that a fairly good description of the $S$ and $P$–wave baryons can be obtained with spin independent energy eigenvalues corresponding to the confining along with Coulomb potentials. We emphasize that no fitting parameter were used in our calculations. This comparative study provides deeper insight into the quark model results for which the constituent masses encode the QCD dynamics.

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