The pygmy dipole strength, the neutron radius of $^{208}\text{Pb}$ and the symmetry energy

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Abstract. The accurate characterization of the nuclear symmetry energy and its density dependence is one of the outstanding open problems in nuclear physics. A promising nuclear observable in order to constrain the density dependence of the symmetry energy at saturation is the neutron skin thickness of medium and heavy nuclei. Recently, a low-energy peak in the isovector dipole response of neutron-rich nuclei has been discovered that may be correlated with the neutron skin thickness. The existence of this correlation is currently under debate due to our limited experimental knowledge on the microscopic structure of such a peak. We present a detailed analysis of Skyrme Hartree-Fock (HF) plus random phase approximation (RPA) predictions for the dipole response in several neutron-rich nuclei and try to elucidate whether models of common use in nuclear physics confirm or dismiss its possible connection with the neutron skin thickness. Finally, we briefly present theoretical results for parity violating electron scattering on $^{208}\text{Pb}$ at the conditions of the PREx experiment and discuss the implications for the neutron skin thickness of $^{208}\text{Pb}$ and the slope of the symmetry energy.

1. Introduction

Experimental studies [1, 2, 3] on the low-energy isovector dipole response or Pygmy Dipole Resonance (PDR) in neutron rich nuclei are of crucial relevance because they determine reaction rates in the $r$-process [4] and, in addition, the PDR has been related to the neutron skin thickness of the studied nuclei in Refs. [5, 6, 7]. However, the existence of this correlation is currently under debate [8] due to our limited experimental knowledge of the microscopic structure of such a peak. If the underlying dynamics giving rise to the PDR is, eventually, confirmed to be strongly correlated to the formation of a neutron skin in neutron-rich nuclei, the density dependence of the nuclear symmetry energy at saturation will be immediately constrained [9]. This is of broad interest since the symmetry energy and its density dependence impact on a variety of physical systems, such as the composition and structure of the crust in a neutron star [10, 11], the neutron skin thickness of a heavy nucleus [9, 12, 13], atomic-parity violation [14] and heavy ion collisions [15].

For all these reasons, we have performed in Ref. [16] a detailed analysis of Skyrme Hartree-Fock (HF) plus random phase approximation (RPA) predictions for the isovector and isoscalar...
dipole response in $^{68}\text{Ni}$, $^{132}\text{Sn}$ and $^{208}\text{Pb}$ nuclei — representative of different mass regions — in order to elucidate the nature and possible connection of the PDR with the slope of the symmetry energy. The strategy adopted to understand if such a connection may exist, is very simple. Giant resonances are collective excitations of atomic nuclei that have allowed, in the past, to determine some nuclear saturation properties such as the nuclear incompressibility (strongly related to the Giant Monopole Resonance [17]), the effective nucleon mass (which affects the Giant Quadrupole Resonance [18]), or the nuclear symmetry energy at sub-saturation density (which act as a restoring force in the Giant Dipole Resonance [19]). Hence, collective phenomena inform us about general properties of the nuclear effective interaction and all realistic models should predict such a collectivity even though some differences on details may appear. The additional fact that the energy weighted sum rule for the PDR has been correlated with the $L$ parameter [6], defined as $L = 3\rho_0 \frac{\partial \text{sym}(\rho)}{\partial \rho_0}$ where $\text{sym}(\rho)$ is the symmetry energy, $\rho$ the baryon density and $\rho_0$ the nuclear saturation density, has motivated our selection of studied interactions. Specifically, we use three Skyrme interactions widely used for nuclear structure calculations and that differ in their predictions of the $L$ parameter to study the collectivity displayed by the RPA states giving rise to the pygmy dipole strength, or RPA-pygmy state, as well as its possible relation with the neutron skin thickness. Based on our experience, we present here the common features found in different mass regions by using $^{208}\text{Pb}$ as a representative example.

Currently, the PREx collaboration [20] aims to determine the neutron radius of $^{208}\text{Pb}$ within a 1% error by parity violating electron scattering (PVES) [21]. Such a measurement is very important for three basic reasons. First, it measures the neutron distribution in a heavy nucleus free from most of the strong interaction uncertainties. Second, it paves the way for further measurements of neutron densities by PVES [23, 24]. Third, it may allow one to derive a significant constraint on the $L$ parameter [22, 25] and, therefore, it may help in constraining the isovector channel of the nuclear effective interaction. Here, we will shortly discuss mean-field model predictions for PVES at the kinematics of PREx [22].

The work is organized as follows. In Sec. 2 we briefly present the basic formalism employed in our analysis. For further information we address the reader to Refs. [21, 26]. In Sec. 3, we show the main results of our works [16, 22]. Finally, our conclusions are laid in Sec. 4.

2. Formalism

2.1. Random Phase Approximation

The RPA method is well-known from textbooks [26]. In short, once the HF equations are solved self-consistently for the given Hamiltonian, we build accordingly the residual interaction — considered to be a small perturbation of the HF mean-field potential — and, then, we solve the RPA coupled equations by means of the matrix formulation. The important quantities to define for our study are the following. The main one is the reduced transition strength or probability,

$$B(EJ, \tilde{0} \rightarrow \nu) \equiv \left| \sum_{ph} A_{ph}(EJ, \tilde{0} \rightarrow \nu) \right|^2 = \left| \sum_{ph} (X^{(\nu)}_{ph} + Y^{(\nu)}_{ph}) \langle p|\hat{F}_{JM}|h \rangle \right|^2$$

(1)

where $A_{ph}(EJ, \tilde{0} \rightarrow \nu)$ is the reduced amplitude, $|\tilde{0}\rangle$ is the RPA ground state, $|\nu\rangle$ is a generic RPA excited state, $J$ is the angular momentum carried by the operator, $\hat{F}_{JM}$, meant to modelize the experimental probe and $\langle p|\hat{F}_{JM}|h \rangle$ is the reduced matrix element of such an operator between a hole ($h$) state (occupied state) and a particle ($p$) state (unoccupied state). The sum of all $ph$ states that contribute to an RPA transition is weighted by the $X^{\nu}$ and $Y^{\nu}$ amplitudes, eigenvectors of the RPA secular matrix [26]. For further details we also refer to [27].
2.2. Parity Violating Elastic Electron Scattering
Parity violating electron-nucleus scattering (PVES) probes neutrons in a nucleus via the electroweak interaction \[28, 29\]. Electrons interact with the protons and neutrons of the nucleus by exchanging a photon or a \(Z^0\) boson. The former mainly couples to protons while the latter basically couples to neutrons. In the experiment, one measures the parity-violating asymmetry, \[ A_{pv} \equiv \frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega} \frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega} \] (2)
where \(d\sigma_\pm/d\Omega\) is the elastic electron-nucleus differential cross section for incident electrons with positive and negative helicity states. For a realistic calculation of the parity violating asymmetry, we solve the Dirac equation via the distorted wave Born approximation (DWBA) \[30\] where the main input are the electric and weak charge distributions of the studied target \[22, 25\].

3. Results
3.1. Low-energy dipole response of \(^{208}\)Pb
In our recent work \[16\] we have studied in detail the dipole response of \(^{68}\)Ni, \(^{132}\)Sn and \(^{208}\)Pb by means of the formalism explained in Section 2.1 and by using different Skyrme interactions. Here we will present the case of \(^{208}\)Pb as a representative example of the common trends found in different mass regions.

![Figure 1](image_url)

**Figure 1.** Strength function calculated by convoluting the corresponding reduced transition probability [Eq. (1)] with a Lorenzian of 1 MeV width for the isovector (left) and isoscalar (right) dipole response of \(^{208}\)Pb as a function of the excitation energy. In both figures the predictions of SGII, SkI3 and SLy5 are depicted. Black arrows indicate the experimental centroid energies for the PDR \((E = 7.37\ \text{MeV})\) \[1\] and for the IVGDR \((E = 13.43\ \text{MeV})\) \[34\].

In Fig. 1 we show the isovector (IV; left panel) and isoscalar (IS; right panel) averaged strength functions for the dipole in \(^{208}\)Pb predicted by the Skyrme models SGII \[31\] with an \(L = 37.6\ \text{MeV}\), SLy5 \[32\] with an \(L = 48.3\ \text{MeV}\) and SkI3 \[33\] with an \(L = 100.5\ \text{MeV}\) as a function of the excitation energy. Experimental data for the centroid energies of the PDR \((E = 7.37\ \text{MeV})\) \[1\] and the IV Giant Dipole Resonance (GDR) \[34\] are also depicted (black arrows). The theoretical predictions lie within the experimental error and, therefore, the Skyrme-HF plus RPA approach may constitute a good starting point for a detailed analysis of the microscopic structure of the PDR in those nuclei. An interesting feature that can be also observed in different mass regions \[16\] is the ratio between the low- and high-energy strength exhausted by the IS peaks: of the same order; and
by the IV peaks: one order of magnitude. This already indicates the importance of the probe used to excite a certain RPA state and reveals also its nature. In other words, the IS dipole operator excite more efficiently the RPA-pygmy state than the IV dipole operator. This means that a perfect probe for exciting such a state would be mostly isoscalar. Moreover, we see from both figures that as the predicted value for the $L$ parameter increases —in going from SGII to SLy5 and, finally, to SkI3, the low-energy peak is shifted to larger excitations energies and to larger strengths, in perfect agreement with Ref. [6].

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Neutron (black) and proton (red) $ph$ contributions to the isovector (left) and isoscalar (right) reduced amplitude [Eq. (1)] corresponding to the RPA-pygmy state of $^{208}$Pb (RPA excitation energy is indicated) predicted by the different models as a function of the $ph$ excitation energy. The single particle levels involved in the most important $ph$ transitions are indicated.

In Fig. 2, we show the $ph$ contributions to the IV (left) and IS (right) reduced amplitudes [see Eq. (1)] corresponding to the different RPA-pygmy states of $^{208}$Pb predicted by the different models as a function of the $ph$ excitation energy. Two results —common also to other mass regions— arise from the analysis of this quantity. First, while the isoscalar reduced amplitude ($A_{ph}(E1,IS)$) is formed by several neutron $ph$ transitions adding coherently (same sign) and only few states are contributing destructively, the isovector reduced amplitude ($A_{ph}(E1,IV)$) is formed by few neutron and proton $ph$ transitions contributing with different signs, i.e., non-coherently. Second, the most relevant $ph$ transitions in the isoscalar dipole response of $^{208}$Pb correspond to excitations of the outermost neutrons that, as a consequence, dominate the dynamics in the low-energy region (see [16]). For the case of the isovector dipole response the situation is model dependent. However, the most relevant neutron $ph$ contributions are larger in number than the proton ones and basically due to the outermost neutrons. Therefore, albeit some differences will be present, one may expect a relation between the neutron excess and the IS and IV dipole responses in neutron rich nuclei.

From Fig. 1, we have seen that the low-energy IS and IV dipole responses of $^{208}$Pb display, to different degree, a sizeable low-energy peak in the corresponding strength functions (see section III.B of Ref. [16]). This is actually one of the main characteristics one asks to a collective state. The second one is that collective phenomena (or resonances) should display coherence between the contributions of several $ph$ transitions to the reduced amplitude. Therefore, by looking at Fig. 2, we can state that while all models support a clear collective character of the low-energy peak in the IS dipole response in $^{208}$Pb (also true in other mass regions), the collectivity of its IV counterpart is model dependent.
3.2. Parity violating electron scattering on $^{208}$Pb

The Lead Radius Experiment (PREx) at the Jefferson Laboratory has recently reported first results for PVES on $^{208}$Pb [20]. In this first run statistics were not sufficient in order to achieve the desired accuracy [20]. A second run of PREx has been approved and it is intended to be performed in the future [20, 23].

As stated in the introduction, the relevance of this measurement has motivated our study of the parity violating asymmetry at the PREx kinematics [21, 22]. We have applied the formalism mentioned in Section 2.2, which is presented in more detail in Refs. [21, 22, 29, 30]. In Fig. 3 we display the linear correlation between the parity violating asymmetry and the neutron skin thickness of $^{208}$Pb ($\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$) as predicted by more than forty mean-field models of very different nature: from non-relativistic such as Skyrme or Gogny models to relativistic such as non-linear Walecka or density dependent meson-exchange and point-couling models. All of them accurately reproduce the charge radius of $^{208}$Pb without assuming a particular shape for the nucleon spatial distributions. In the right panel of Fig. 3 we show the well-known correlation between the neutron skin thickness in $^{208}$Pb and the slope of the symmetry energy predicted by the considered mean-field models. Therefore, PVES can supply new constraints on the value of the $L$ parameter.

![Figure 3](image_url)

**Figure 3.** Left panel: parity violating asymmetry (DWBA) in $^{208}$Pb for 1.06 GeV electrons at $5^\circ$ scattering angle as a function of its neutron skin thickness. MF results (black circles) — references to all these interactions can be found in [22] — and calculations form the neutron densities deduced from experiment [a] [35], [b] [36] and [c] [37] (red squares). Right panel: Neutron skin thickness of $^{208}$Pb as a function of $L$ as predicted by the same MF interactions shown in the left panel.

4. Conclusions

We have exemplified some of the common features found in the dipole response of different nuclei [16] by showing our results for the case of $^{208}$Pb. In particular, the low-energy peak in both the isoscalar and isovector dipole responses is shifted to larger excitation energies and display larger values of the strength function as the value of the $L$ parameter increases. We have also seen that the RPA-pygmy state can be more efficiently excited by an isoscalar probe than by an isovector probe. This indicates the dominant isoscalar character of such a state. We have demonstrated that the low-energy peak in the isoscalar dipole response is a collective mode — it is formed by several $ph$ contributions adding coherently and giving a contribution to the reduced transition strength comparable to that of the isoscalar giant dipole resonance. This is found to be opposite
to what happens to its isovector counterpart where its collectivity depends on the interaction. Finally, the dynamics in the low-energy region in the isoscalar dipole response of neutron-rich nuclei is clearly dominated by the outermost neutrons —those that form a neutron skin.

Our analysis of PVES applied to the conditions of the PREx experiment predicts a high-quality correlation between the parity violating asymmetry and the neutron skin thickness of $^{208}\text{Pb}$ [22]. The results suggest that one will be able to extract significant constraints on the slope of the nuclear symmetry energy at saturation if the statistics of PREx are improved.

In conclusion, the recent experimental and theoretical studies of the ground-state and excitation properties of neutron-rich nuclei aim to complement each other and are paving the way for a better knowledge of the isovector channel of the nuclear effective interaction.

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