Weibel instability in relativistic asymmetric electron positron plasma

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Abstract

We consider a situation in when the interaction of relativistically intense EM waves with an isotropic electron positron plasma takes place, i.e., we consider short pulse lasers with intensity up to $10^{21}$ W/cm$^2$, in which the photon density is of the order of $10^{30}$ cm$^3$ and the strength of electric field $E = 10^9$ statvolt/cm. Such a situation is possible in astrophysical and laboratory plasmas which are subject to intense laser radiation, thus leading to nonthermal equilibrium fields. Such interaction of the superstrong laser radiation with an isotropic pair plasma leads to the generation of low frequency electromagnetic EM waves and in particular a quasistationary magnetic field. When the relativistic circularly polarized transverse EM wave propagates along z-axis, it creates a ponderomotive force, which affects the motion of particles along the direction of its propagation. On the other hand, motion of the particles across the direction of propagation is defined by the ponderomotive potential. Moreover, dispersion relation for the transverse EM wave using a special distribution function, which has an anisotropic form, is derived and is subsequently investigated for a number of special cases. In general, it is shown that the growth rate of the EM wave strongly depends upon its intensity.

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I. INTRODUCTION

As proposed by Stephan Hawking in his path breaking paper [1], that the objects like black hole would emit radiations, now famously known as Hawking radiations. These radiations are very high energetic photons, which can be converted into electron positron pair for a short time though via the well-known pair production. In which gamma rays photons are converted into the electron and it’s anti particle positron via Einsteins mass energy equation \( E = Mc^2 \). However it is still unclear how the pair is formed in astrophysical environment since there are no atomic nuclei as is believed to be the necessary for the recoil in the pair production. At adequate temperature near \( 10^9 \) K, electron and positrons appear spontaneously i.e.,

\[
\gamma + \gamma \rightarrow e^+ + e^-
\]

A pulsar is an object that emits extremely regular pulses of radio waves. It is generally believed that a pulsar is a rapidly rotating Neutron Star which is extremely hot along with intense magnetic field. Because of the pulsar co-rotation with its magnetic field an electric field is generated, which has non-zero component along magnetic field. The electric field ejects particles from pulsar surface and accelerates them up to relativistic velocities. The particle moving along curved magnetic field lines radiates \( \gamma \) quanta and when its kinetic energy, \( \varepsilon_\gamma \), exceeds twice the electron rest energy \( 2m_0c^2 (\varepsilon_\gamma > 2m_0c^2) \), \( \gamma \) quanta decays into an electron -positron pair. This pair is also accelerated in the electric field and \( \gamma \) quanta appears again, which decays in to an electron-positron pair. This fills the magnetosphere of the pulsar with the relativistic electron-positron plasma, which in turn screens out the electric field generated by the pulsar rotation.

Electron-positron plasmas are believed to be an important ingredient of the early universe and in astrophysical objects such as pulsars, supernova remnants, active galactic nuclei and in gamma-ray bursts. In such extreme environments, the electron-positron pairs may be created by collisions between particles that are accelerated by electromagnetic and electrostatic waves and/or by gravitational forces. In pulsar environments, there is also possibility of pair creations via high energy curvature radiation photons that are triggered by charged particles streaming along the curved magnetic field, with a resulting collection of positrons at the polar cups of the pulsar. High energy laser plasma interactions and fusion devices on Earth also constitute a source of electron-positron plasmas. Experiments with petawatt lasers (with intensities exceeding \( 10^{20} \) \( W/cm^2 \)) have demonstrated the production of MeV
electrons and evidence of positron production via electron collisions. Positrons are also believed to be created in post-disruption plasma in large tokamaks through collisions between MeV electrons and thermal particles.

Recently, there has been a great deal of interest in the study of electron-positron plasma. In fact, in the entire astrophysics the matter is usually at a very high temperature. When the plasma temperature becomes of the order of (or larger than) the rest energy of electrons $m_0c^2$, it becomes relativistic. In this relativistic regime the processes of creation and annihilation of electron-positron pairs become important. The equilibrium is established on account of the physical mechanism quite similar to that of the Rayleigh Jeans law black-body radiation. Furthermore, at thermodynamical equilibrium the chemical potential of and electron-positron gas is identically zero. As we have already mentioned, at sufficiently high temperatures, say from $10^9$ K, the electrons and positrons appear spontaneously in any region of the space. They are created by the temperature radiation itself, i.e.

There are two types of relativistic regimes in plasma. When a plasma is subjected to a strong electromagnetic field, the plasma particles may acquire relativistic velocities or the thermal energy of the plasma particles become of the order of or larger than the rest mass energy. Objects like pulsars, quasars, active galactic nuclei and black holes are characterized by such relativistic effects. In laboratories, such plasmas may be created either by heating a gas to very high temperatures or by the impact of a high-energy particle beam or by high-intensity ultrashort laser pulses. Plasmas hot enough for electrons to be relativistic exist in different astrophysical environments but for particles heavier than electrons such environments are rare since more energy is required to accelerate them to a significant fraction of the speed of light. For relativistic plasmas the thermal energy of particles is greater than the rest mass energy i.e., $3/2KBT > m_0c^2$. So a relativistic plasma with thermal distribution has temperature greater than 511 keV.

Weibel instabilities in electron-ion plasmas have been widely studied for both unmagnetized and magnetized plasmas. Such instabilities are usually referred to as whistler instabilities in magnetized electron-ion plasmas, when the wave vectors are parallel to the background magnetic field $B_0$, and the temperature perpendicular to $B_0$ is higher than that parallel to $B_0$. The Weibel instabilities in relativistic electron-ion plasmas have also been investigated by many authors. While the analyses for weakly relativistic and unmagnetized plasmas are not relevant for our purposes, detailed properties of Weibel- like instabilities in
fully relativistic magnetized plasmas have been investigated, to our knowledge, only for a particular choice of distribution function, which allows no spread in perpendicular momentum. Furthermore, due to mass symmetry, the stability properties for the electron-positron plasmas can be significantly different from those for the electron-ion plasmas. For example, when the electrons and positrons have identical distribution function, the dispersion relation is the same for the right-hand and left-hand circularly polarized modes; therefore, the waves and instabilities can have arbitrary polarizations. In this paper, we study the linear stability properties of the Weibel instability in relativistic magnetized electron-positron-pair plasmas, with wave vector parallel to $B_0$. The instability in the ultrarelativistic regime, with the typical Lorentz factor

The interaction of a plasma with short high intensity laser pulses can lead to the magnetic part of the Lorentz force on the electrons to become as important as the electric part leading in turn to a self-generated magnetic field. Subsequently, it is expected that this self-generated magnetic field will significantly change the pattern of the nonlinear laser pulse interaction with the plasma. The relativistic non-linearities introduced by the magnetic field interaction are of general interest in relation to the field of an ultrastrong EM wave propagating in plasmas and in pulsar like media.

Recent developments in astronomical and astrophysical observations have revealed that our universe is full of enigmatic phenomena, such as jets, bursts, flares, etc. It is possible now to study and simulate extremely complex astrophysical phenomena supernova explosion, etc. in laboratories using intense and ultraintense lasers. Intense lasers have been used to investigate hydrodynamics radiation flows, opacities, etc., related to supernova explosions, giant planets, and other astrophysical systems. Thus the study of the properties of such radiation strong laser pulse, nonthermal equilibrium, cosmic field radiation, etc. in plasmas is of vital importance. In the field of superstrong femtosecond pulses having power of $10^{23} - 10^{24}$ $W/cm^2$, it is expected that the character of the nonlinear response of the medium will radically change due to relativistic effects.

Recently the properties of a relativistically intense EM wave in electron-positron plasmas were investigated. Interest in such investigations arises because after the Big Bang, matter constituted of electrons, positrons, and photons almost in thermal equilibrium with one another at a temperature much higher than $mc^2$. On the other hand, we now know that electron-positron plasmas constitute pulsar atmospheres, accretion disks, active galactic
nuclei, and black holes.

It was shown that in the case of relativistically intense circularly polarized EM waves propagating into a plasma, the perpendicular momentum \( p = e \alpha A / c A \) is the perpendicular component of the vector potential of the pump EM waves relative to the propagation direction; \( \alpha \) stands for the particle species can be much larger than perpendicular component of the thermal momentum of particles. However the momentum of particles along the direction of propagation of EM waves remains thermal. This effect modifies the distribution function which must be relativistic, and should take into account the fact that in the perpendicular direction the particle momentum due to the electromagnetic field be dominant over the thermal momentum in this direction. Such a distribution is valid for arbitrary parallel temperature and takes into account the anisotropy in the parallel and perpendicular direction.

The manuscript is organized as follows: basic equations and the dispersion relations for the weible instability in electron-positron ions are presented in section II. quantitative analysis and conclusions are given in Sec. III.

II. DISPERSION RELATION AND DISTRIBUTION FUNCTION

We present a new physical concept for the case when the interaction of relativistically intense EM waves with an isotropic plasma take place, i.e., we consider short pulse lasers with intensity up to \( 10^{21} \) \( W/cm^2 \), in which the photon density is of the order of \( 10^{30} \) \( cm^{-3} \) and the strength of electric field \( E \) \( 10^9 \) statvolt/cm. Such a situation is possible in astrophysical plasmas and in laboratory plasmas which are subject to intense laser radiation, thus leading to nonthermal equilibrium field radiations. Here we consider the generation of low frequencies of the transverse EM wave by taking stationary ions \( (m_i \to \infty) \) and take only a single active component of relativistic electrons. However, the relativistic EM wave interaction with the plasma leads to the plasma becoming anisotropic in the manner described in the preceding. The linear dispersion relation for circularly polarized electromagnetic wave in an unmagnetized relativistic plasma propagating along the z-axis is given by

\[
0 = 1 - \frac{\epsilon^2 k_z^2}{\omega^2} - \sum_j \frac{\omega_{p_j}^2}{\omega^2 \gamma_j} \int \frac{d^3p}{\gamma_j} \left( \frac{p_{\perp(j)}}{2} \right) \times \left[ \frac{\partial}{\partial p_{\perp(j)}} + \frac{k_z p_{\perp(j)}}{m} \frac{1}{\gamma \omega - \frac{k_z p_{\perp(j)}}{m}} \frac{\partial}{\partial p_{||}} \right] F \left( p_{\perp(j)}^2, p_{||}^{(j)} \right)
\]

(1)
where \( j \) represents electron and positron species. Nonrelativistic plasma frequency is defined as \( \omega_{pj} = \left( \frac{4 \pi ne_j^2}{m_j} \right)^{1/2} \), and relativistic mass factor is given as \( \gamma_j = 1 + \left( p_{\perp(j)}/m_j^2 \right) + \left( p_{\parallel(j)}/m_j^2 \right) \)^{1/2} \). In (1) \( F(p_{\perp(j)}, p_{\parallel(j)}) \) is the normalized distribution function of the relativistic \( j \) species.

Here we consider propagation of a superstrong circularly polarized EM wave along z-axis and \( \vec{E}(E_x, E_y, 0) \). In such scanrio, the lighter charged particles electrons and positrons obtain an additional perpendicular (relative to the propagation of waves) momentum which is defined by the amplitude of EM waves and is given as \( \vec{p}_\perp(j) = (e_j/c) \vec{A}_\perp \), where \( \vec{A}_\perp \) is the vector potential. The total perpendicular momentum can be expressed as

\[
\vec{p}_\perp(j) = \vec{p}_{\perp(j)}^{th} + (e/c) \vec{A}_\perp \tag{2}
\]

here \( \vec{p}_\perp(j) \) and \( \vec{p}_{\perp(j)}^{th} \) denote the field and perpendicular thermal momentum of the \( j \) particles, respectively, which for nonrelativistic temperature are given as

\[
\left\langle \left( \vec{p}_\perp(j) \right)^2 \right\rangle \sim m_j^2 U_{pond} \tag{3}
\]

\[
\left\langle \left( \vec{p}_{\perp(j)}^{th} \right)^2 \right\rangle \sim 2m_j T_{\perp(j)} \tag{4}
\]

\( U_{pond} = (e_j A_\perp/m_j c^2)^2 \) is the ponderomotive potential. Under the condition \( U_{pond} \gg (2T_{\perp(j)}/m_j c^2) \), the perpendicular thermal momentum of electrons and positrons can be ignored. So thermal distribution function of the \( j \) species is given by \( p_{\parallel(j)} \) only and we have \( \left( \vec{p}_\perp(j) \right) = (e_j/c) \vec{A}_\perp \). For such a case the full equilibrium distribution function can be given as

\[
F\left( p_{\perp(j)}, p_{\parallel(j)} \right) = \frac{1}{2m_j c K_1(\beta_j)} \delta \left( \vec{p}_\perp(j) - \frac{e_j}{c} \vec{A}_\perp \right) \exp \left( \frac{-c \left( m_j^2 c^2 + p_{\perp(j)}^2 + p_{\parallel(j)}^2 \right)^{1/2}}{T_{\perp(j)}} \right) \tag{5}
\]

where \( K_1 \) is the MacDonald function and \( \beta_j = m_j c^2 / T_{\perp(j)} \). Substitution of Eq. (5) into (1), and integration over \( p_\perp \) gives us

\[
0 = 1 - \frac{c^2 k^2}{\omega^2} - \sum_j K \frac{\omega_p}{\omega^2} \int_{-\infty}^{+\infty} d^3 \vec{p}_\perp(j) \frac{dp_{\parallel(j)}}{\gamma_{0(j)}} \times \left[ 1 - \frac{p_{\perp 0(j)}^2 \left( \omega^2 - c^2 k^2 \right)}{2m_j^2 c^2 \left( \gamma_{0(j)} \omega - \frac{k_x p_{\parallel(j)}}{m_j} \right)^2} \right], \tag{6}
\]

where \( p_{\perp 0(j)} = \frac{e_j}{c} \vec{A}_\perp \) and \( \gamma_{j0} = \left[ 1 + \left( p_{\perp 0(j)}/m_j^2 \right) + \left( p_{\parallel(j)}/m_j^2 \right) \right]^{1/2} \). In Eq.(6) we have used
\[
\frac{\partial \gamma_j}{\partial p_{\parallel(j)}} = p_{\parallel(j)}/\gamma_j m_j^2 c^2. \]
The parallel distribution function \( F(p_{\parallel}) \) is given as
\[
F(p_{\parallel}) = \frac{1}{2m_j c K_1(\beta_j)} \exp \left( \frac{-c \left( m_j^2 c^2 + p_{\perp 0(j)}^2 + p_{\perp 1(j)}^2 \right)^{1/2}}{T_{\parallel(j)}} \right) \quad (7)
\]
Let us define the following identity
\[
\hat{\gamma}_{\perp 0(j)} = \left[ 1 + \frac{p_{\perp 0(j)}^2}{m_j^2 c^2} \right]^{1/2} \quad (8)
\]
and so
\[
\gamma_{\perp 0(j)} = \hat{\gamma}_{\perp 0(j)} \left[ 1 + \frac{p_{\parallel(j)}^2}{(m_j^2 c^2) \hat{\gamma}_{\perp 0}^2} \right]^{1/2} \quad (9)
\]
introducing the new variables such as
\[
p_{\parallel(j)} = (\hat{\gamma}_{\perp 0(j)} m_j c) \sinh \theta \quad (10)
\]
Eqs. (8) - (9) give us
\[
\gamma_{0(j)} = (\hat{\gamma}_{\perp 0(j)}) \cosh \theta; \quad \frac{dp_{\parallel(j)}}{\gamma_{0(j)}} = (m_j c) d\theta \quad (11)
\]
and by using the identity
\[
\int_0^\infty \tau d\tau \exp \left[ i \left( \gamma_{0(j)} \omega - \frac{k_z p_{\perp 0(j)}}{m_j} \right) \tau \right] = -\frac{1}{\left( \gamma_{0(j)} \omega - \frac{k_z p_{\perp 0(j)}}{m_j} \right)^2} \quad (12)
\]
substitution of Eqs. (7) and (10) - (12) into (6) and integrating over \( \theta \) gives us the dispersion relation
\[
0 = 1 - \frac{c^2 k_z^2}{\omega^2} - \sum_j \frac{\omega_{p_j}^2}{\omega^2 \hat{\gamma}_{\perp j}} \left[ \frac{K_0 \left( \frac{m_j c^2}{T_{\parallel(j)}} \hat{\gamma}_{\perp} \right)}{K_1 \left( \frac{m_j c^2}{T_{\parallel(j)}} \right)} + \frac{p_{\perp 0(j)}^2}{K_1 \left( \frac{m_j c^2}{T_{\parallel(j)}} \right)} \left( \omega^2 - c^2 k_z^2 \right) \right] \int_0^\infty K_0(\eta) \tau d\tau \quad (13)
\]
where argument \( \eta = \left[ \left( \frac{m_j c^2}{T_{\parallel(j)}} \hat{\gamma}_{\perp 0(j)} - i \omega \tau \right)^2 + (c k_z \tau)^2 \right]^{1/2} \). For deriving (13), we have used the identity
\[
\frac{1}{\pi} K_0 \left( a^2 + b^2 \right)^{1/2} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\theta \exp(-ib \sinh \theta - a \cosh \theta) \quad (14)
\]
condition, allows us to Taylor expansion of \( K_0 \)
\[
\int_0^\infty \tau d\tau K_0(\eta) = \int_0^\infty \tau d\tau K_0(\beta_{\perp(j)}^2 + \sigma^2 \tau^2)^{1/2} \quad (15)
\]
where we have denoted \( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} = \beta_{\perp(j)} \); \((c^2 k_z^2 - \omega^2)^{1/2} = \sigma \) and have used the identity \( dK_0/dx = -K_1(x) \). Substituting (15) into (13) and performing the integrations gives us the following general dispersion relation for the propagation of electromagnetic waves in electron positron ion plasma where electrons and positrons are active whereas ions form a neutralizing background.

\[
0 = 1 - \frac{c^2 k_z^2}{\omega^2} - \sum_j \frac{\omega_{pj}^2}{\omega_{pj}^2} \left[ \frac{K_0(\beta_{\perp(j)})}{K_1(\beta_{\perp(j)})} - \frac{\sigma^2 \tau^2}{2m_j T_{z(j)} \gamma_{\perp(j)} K_1(\beta_{\perp(j)})} \right]
\]

(16)

To obtain above relation, we have used the following MacDonald function integrals [ ]

\[
\int_0^\infty \tau d\tau K_0(\beta_{\perp(j)}^2 + \sigma^2 \tau^2)^{1/2} = \frac{\beta_{\perp(j)}^2}{(c^2 k_z^2 - \omega^2)} K_1(\beta_{\perp(j)})
\]

(17)

and

\[
\int_0^\infty \tau d\tau \frac{i \beta_{\perp(j)}^2 \tau}{(\beta_{\perp(j)}^2 + \sigma^2 \tau^2)^{1/2}} K_1(\beta_{\perp(j)}^2 + \sigma^2 \tau^2)^{1/2}
\]

\[
= \left[ \frac{i \omega(\beta_{\perp(j)})^{1/2}(\frac{\sigma^2}{2})^{1/2}}{(c^2 k_z^2 - \omega^2)(c^2 k_z^2 - \omega^2)^{1/2}} \right] K_1^{1/2}(\beta_{\perp(j)})
\]

(18)

### III. ANALYSIS OF ELECTROMAGNETIC INSTABILITY

The dispersion relation (16) is complex or purely imaginary, there is a possibility of obtaining unstable situation. In the subsequent section we investigate various situations for the interaction of relativistic EM pump wave with electron and positron particles. ELECTRON POSITRON

#### a. Case 1: EMW wave with large relativistic transverse energy

We first investigate a situation in which the relativistic transverse energy of the electromagnetic pump wave is larger than its thermal energy \( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \gg 1 \) or \((\beta_{\perp(j)} \gg 1)\). This allows us to expand the MacDonald function \( K_0, K_1, \) and \( K_{1/2} \) in the following manner.

\[
K_0 \left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right) \approx \left( \frac{\pi}{2 \beta_{\perp(j)}} \right)^{1/2} e^{\left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right)} \approx K_1 \left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right) \approx K_{1/2} \left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right)
\]

(19)

Neglecting the higher order terms we have

\[
\frac{K_{1/2} \left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right)}{K_1 \left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right)} \approx \frac{K_0 \left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right)}{K_1 \left( \frac{m_e c^2}{T_{z(j)}} \gamma_{\perp} \right)} \approx \frac{1}{e^{\frac{m_e c^2(1-\gamma_{\perp})}{T_{z(j)}}}}
\]

(20)
Inserting Eq. 21 in Eq. 20 we obtain

\[
0 = 1 - \frac{c^2 k_z^2}{\omega^2} - \sum_j \frac{\omega_{pj}^2}{\omega^2 \gamma_{j+1}^2} \left[ \frac{m_j c^2 (1 - \gamma_{j+1})}{T_{j+1}^2} (1 - \frac{m_j c^2 (\gamma_{j+1}^2 - 1)}{2 T_{j+1} \gamma_{j+1}}) \right]
\]

\[
- i \omega \left( \frac{m_j c^2}{T_{j+1}^2} \right)^{3/2} \left( \frac{\pi}{c \kappa_z - \omega} \right)^{1/2} \left( \frac{\gamma_{j+1}^2 - 1}{\gamma_{j+1}} \right) \frac{m_j c^2 (1 - \gamma_{j+1})}{T_{j+1}^2}
\]

here we have defined effective momentum of the pumping wave

\[
m_j c^2 (\gamma_{j+1}^2 - 1) = p_{j+1}^2
\]

Below we discuss above equation for the the low and high frequency cases, respectively.

**b. Low frequency regime** In the present section, we solve Eq. 21 for low frequency case, i.e., when \((c^2 k_z^2 - \omega^2)^{1/2} \approx c k_z\) Under these conditions, Eq. 22 can be simplified and we obtain for a purely imaginary expression given by

\[
\text{Im} \omega = -\frac{8 k_z \left( \frac{T_+}{m_+} \right)^{3/2}}{\pi^{1/2} c^2 (\gamma_+^2 - 1)} \left[ \frac{c^2 k_z^2 \gamma_+^{3/2}}{\omega_{pp}^2 e^{\gamma_+ (1 - \gamma_+)}} + \left( 1 + \left( \frac{m_+}{n_+} \right) \right) e^{(\beta_+ - \beta_-) (1 - \gamma_+)} \right]
\]

\[
\left( 1 + \left( \frac{m_+}{n_+} \right) \left( \frac{T_-}{T_+} \right)^{3/2} e^{(\beta_+ - \beta_-) (1 - \gamma_+)} \right)
\]

let \(\omega_{pp}^2 = \omega_{pp}^2 e^{\gamma_+ (1 - \gamma_+)}\)

\[
\text{Im} \omega = -\frac{k_z \left( \frac{2 T_+}{m_+} \right)^{3/2}}{\pi^{1/2} c^2 (\gamma_+^2 - 1)} \left[ \frac{c^2 k_z^2}{\omega_{pp}^2} \right] + \left( \frac{m_+}{n_+} \right) e^{(\beta_+ - \beta_-) (1 - \gamma_+)} \left( 1 - \frac{\beta_+ (\gamma_+^2 - 1)}{2 \gamma_+} \left( \frac{T_-}{T_+} \right)^{3/2} e^{(\beta_+ - \beta_-) (1 - \gamma_+)} \right)
\]

\[
\left( 1 + \left( \frac{m_+}{n_+} \right) \left( \frac{T_-}{T_+} \right)^{3/2} e^{(\beta_+ - \beta_-) (1 - \gamma_+)} \right)
\]

\[
\text{High frequency regime}
\]

In this section we investigate the high frequency regime, when \(\omega \gg c k_z\) which implies that \((c^2 k_z^2 - \omega^2)^{1/2} = i \omega\). After simplifying Eq. 22 , we get

\[
\text{Im} \omega = \left( \frac{\omega_{pp}^2}{\gamma_+} e^{\gamma_+ (1 - \gamma_+)} \right) \left[ \left( \frac{\gamma_+ - \beta_+ (\gamma_+^2 - 1)}{2 \gamma_+^{1/2}} \right) \left( \frac{m_+}{n_+} \right) \left( \frac{2 \gamma_+ - \beta_+ (\gamma_+^2 - 1)}{2 \gamma_+ - \beta_- (\gamma_+^2 - 1)} \right) e^{(\beta_+ - \beta_-) (1 - \gamma_+)} \right]^{1/2}
\]

\[
- \left( \frac{c}{\beta_+} \right)^{3/2} \beta_+^3 (\gamma_+^2 - 1) \left[ 1 + \left( \frac{m_+}{n_+} \right) \left( \frac{m_+}{n_+} \right)^{5/2} \left( \frac{T_-}{T_+} \right)^{3/2} e^{(\beta_+ - \beta_-) (1 - \gamma_+)} \right]
\]
\[ \text{Im } \omega = \left( \frac{\omega_{pp}}{\gamma_\perp} e^{\frac{\beta_\perp(1-\delta_\perp)}{2}} \right) \left[ \frac{(2\tilde{\gamma}_\perp - \beta_\perp (\tilde{\gamma}_\perp^2 - 1))}{2\tilde{\gamma}_\perp^{1/2}} \left\{ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{m_p}{m_e} \right) \left( \frac{(2\tilde{\gamma}_\perp - \beta_\perp (\tilde{\gamma}_\perp^2 - 1))}{(2\tilde{\gamma}_\perp - \beta_\perp (\tilde{\gamma}_\perp^2 - 1))} \right) \right\}^{1/2} \right] \]

or

\[ \text{Im } \omega = \left( \frac{\omega_{pp}}{\gamma_\perp} e^{\frac{\beta_\perp(1-\delta_\perp)}{2}} \right) \left[ \frac{b}{2\tilde{\gamma}_\perp^{1/2}} \left\{ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{m_p}{m_e} \right) \right\} \right]^{1/2} \]

let

\[ \left( \frac{(2\tilde{\gamma}_\perp - \beta_\perp (\tilde{\gamma}_\perp^2 - 1))}{(2\tilde{\gamma}_\perp - \beta_\perp (\tilde{\gamma}_\perp^2 - 1))} \right) = \frac{a}{b} \]

\[ e^{(\beta_e - \beta_p)(1-\delta_\perp)} = d \]

In Fig. 2 we have considered the positive root of Eq. (25).

1. **Case: Large thermal relativistic energy**

In present section we consider the opposite limit to that investigated in Sec. III. That is the case when the thermal relativistic energy is large as compared to the transverse relativistic energy of the pump wave. Thus, here we have

\[ \beta_\perp \ll 1, \beta \ll 1 \]

and subsequently, we have the following relationships for the MacDonald functions:

\[ K_1(\beta_1) \approx \frac{1}{\beta_1}, \ K_1(\beta) \approx \frac{1}{\beta}, \ K_0(\beta_\perp) \approx -\ln \frac{\beta_\perp}{2}, \ K_1(\beta_\perp) \approx \sqrt{\frac{\pi}{2\beta_\perp}} \]  

Inserting Eq. 26 in the general dispersion relation Eq. 20, we get

\[ 0 = 1 - \frac{c^2 k_\perp^2}{\omega^2} - \frac{\omega_{pp}}{\omega^2 \gamma_\perp} \left[ -\beta_\perp \ln \left( \frac{\beta_\perp}{2} \right) - \beta_\perp \frac{(\tilde{\gamma}_\perp^2 - 1)}{2\tilde{\gamma}_\perp} - \frac{(e \omega_\perp (\tilde{\gamma}_\perp^2 - 1))}{2\tilde{\gamma}_\perp (c^2 k_\perp^2 - \omega^2)^{1/2}} \right] \]

As previously, we separately investigate the low and high frequency regimes.
2. **Low frequency regime**

Now we investigate Eq. (27) for the low frequency case which reduces to an expression for purely imaginary .

\[
\text{Im} \omega = - \left[ \frac{e^2 k_z^2 - \omega_{pp}^2 \beta p}{\gamma_{\perp}} \beta p \ln \left( \frac{\beta p}{2} \right) \left[ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{m_p}{m_e} \right) \ln \left( \frac{\beta e}{\beta p} \right) \right] - \frac{\omega_{pp}^2 \beta e}{2 \gamma_{\perp}} \frac{\gamma_{\perp}^2 - 1}{\gamma_{\perp}^2} \left[ 1 + \left( \frac{T_p}{T_e} \right) \left( \frac{n_e}{n_p} \right)^{3/2} \left( \frac{m_p}{m_e} \right)^{5/2} \right] \right] \left[ \frac{\pi \omega_{pp}^2 \gamma_{\perp}^2 - 1}{4 \gamma_{\perp}^2} \left( 1 + \left( \frac{T_p}{T_e} \right)^2 \left( \frac{m_e}{m_p} \right)^2 \right) \right]^{1/2}
\]

\[\omega = \pm \omega_{pp} \gamma_{\perp}^{1/2} \left[ - \beta p \ln \left( \frac{\beta p}{2} \right) \left( 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right) - \frac{\omega_{pp}^2 \beta e \gamma_{\perp}^2 - 1}{2 \gamma_{\perp}^2} \left[ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right] \right]^{1/2}
\]

\[\omega_r = \text{Re}(\omega)
\]

\[\text{Re}(\omega) = \frac{\omega_{pp} \gamma_{\perp}^{1/2}}{\gamma_{\perp}^{1/2}} \left[ - \beta p \ln \left( \frac{\beta p}{2} \right) \left( 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right) - \frac{\omega_{pp}^2 \beta e \gamma_{\perp}^2 - 1}{2 \gamma_{\perp}^2} \left[ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right] \right]^{1/2}
\]

and

\[\text{Im} \omega = \frac{\omega_{pp} \gamma_{\perp}^{1/2}}{\gamma_{\perp}^{1/2}} \left[ \beta p \ln \left( \frac{\beta p}{2} \right) \left( 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right) + \frac{\beta e \gamma_{\perp}^2 - 1}{2 \gamma_{\perp}^2} \left[ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right] \right]^{1/2}
\]

3. **High frequency regime**

\[\omega = \omega_{pp} \gamma_{\perp}^{1/2} \left[ - \beta p \ln \left( \frac{\beta p}{2} \right) \left( 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right) - \frac{\omega_{pp}^2 \beta e \gamma_{\perp}^2 - 1}{2 \gamma_{\perp}^2} \left[ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right] \right]^{1/2}
\]

\[\omega_r = \text{Re}(\omega)
\]

\[\text{Re}(\omega) = \omega_{pp} \gamma_{\perp}^{1/2} \left[ - \beta p \ln \left( \frac{\beta p}{2} \right) \left( 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right) - \frac{\omega_{pp}^2 \beta e \gamma_{\perp}^2 - 1}{2 \gamma_{\perp}^2} \left[ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right] \right]^{1/2}
\]

and

\[\text{Im} \omega = \omega_{pp} \gamma_{\perp}^{1/2} \left[ \beta p \ln \left( \frac{\beta p}{2} \right) \left( 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right) + \frac{\beta e \gamma_{\perp}^2 - 1}{2 \gamma_{\perp}^2} \left[ 1 + \left( \frac{n_e}{n_p} \right) \left( \frac{T_p}{T_e} \right) \right] \right]^{1/2}
\]

IV. **CONCLUSIONS**

In this paper we have derived a general dispersion relation for electromagnetic waves in a single component relativistic plasma where the transverse energy of the wave is relativistic and also thermal energy is relativistic. The dynamics of the ions has been neglected. This general dispersion relation has been investigated in detail for three limiting cases. First when the transverse energy of the electromagnetic wave is large in comparison with the thermal energy, which in the first instance has been taken as being non-relativistic; second when the thermal energy is large and relativistic in comparison with the wave transverse energy; and third for large relativistic transverse and relativistic thermal energies. Each of these cases is investigated separately for the high and low frequency regimes. Expressions are obtained.
for the growth/damping rates of the possible instability and the dependence of the growth rates on the different defining parameters has also been investigated and plots obtained for these different cases.

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