Research on the Hand-eye calibration Method Based on Monocular Robot

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Abstract-The measurement system is composed of a single camera and a manipulator, and the conversion relationship between the manipulator tool coordinate system and the camera coordinate system is one of the key technologies of the system. This paper proposes a unit octet hand-eye calibration algorithm, which has better accuracy and anti-noise ability. This algorithm combines the characteristics of data on SO(4) on the basis of traditional dual quaternion, and establishes the conversion equation of AX=XB in combination with robot kinematics, and realizes the eye calibration of the manipulator. Compared with the traditional dual quaternion algorithm, the speed is faster, the robustness is good, and it has better stability and practicability.

1. Introduction

The basic task of manipulator hand eye calibration is to realize the mapping relationship between the vision sensor and the robot coordinate system. In this article, the vision sensor is combined with the manipulator to estimate the conversion matrix from the end tool coordinate system of the manipulator to the coordinate system of the vision sensor. The vision sensor is mainly composed of a camera. The camera determines the two-dimensional coordinates of the object to be measured and realizes the measurement of the object's coordinates. The commonly used model AX=XB for robot hand-eye calibration was first proposed by Shiu et al. in 1989[1], where X is the conversion matrix from the end coordinate system to the visual sensor, and A is the end coordinate system of the robot. Relative transformation matrix, B is the transformation matrix corresponding to the coordinate system of the vision sensor.

In the past 30 years, a large number of methods for solving the hand-eye calibration equation AX=XB have appeared. Generally speaking, the solution method can be divided into two categories: one is the stepwise method, which first solves the rotation matrix, and then solves the translation matrix. For example, quaternion based method for solving hand-eye calibration conversion matrix proposed by Lu and Chou in 2002[2]; the dual quaternion method proposed by Daniilidis in 1998[3]; Andreff et al proposed a solution method based on the Silvester equation in 2001[4]. The other is synchronization. Simultaneously solving the rotation and translation matrix. For example, the gradient/Newton method proposed by Gwak et al in 2003[5]. The linear matrix inequality method proposed by Heller et al in 2014[6]. Different mathematical methods are used to solve the hand-eye calibration equation. Due to the introduction of errors in robot kinematics modeling and linear structured light vision sensor calibration, it is more complicated to solve the conversion matrix from...
the robot end coordinate system to the vision sensor coordinate system. Based on the quaternion method, this paper proposes a unit octonion method to estimate the conversion matrix from vision sensor to manipulator.

2. Solution of hand-eye calibration matrix equation

The hand-eye calibration equation $AX=XB$ is transformed into the homogeneous matrix[7], as shown in Formula (1)

$$\begin{bmatrix} R_a & T_a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_x & T_x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_b & T_b \\ 0 & 1 \end{bmatrix}$$

(1)

After decoupling, the rotation formula and translation formula are respectively (2) and (3).

$$R_a \ast R_a = R_x \ast R_b$$

(2)

$$R_a \ast T_a + T_a = R_b \ast T_b + T_x$$

(3)

In the formula, $R_a$ is the 3*1 translation matrix corresponding to the manipulator end 3*3 rotation matrix $T_a$, $R_b$ is the 3*1 translation matrix corresponding to the camera coordinate system 3*3 rotation matrix $T_b$, $R_x$ is the rotation matrix and translation matrix of the manipulator end solved by substitution, $T_x$ is the rotation matrix and translation matrix of the camera coordinate system solved by substitution.

2.1. Optimal estimation based on 3-D rotation

Use $a_i$ and $b_i$ to represent the rotation of the manipulator and the camera in only when a group of transformation matrix, by rotating formula (2), available:

$$\arg \min_{R_x \in SO(3)} \sum_{i=1}^{N} \left( || R_x a_i - b_i ||^2 \right)$$

(4)

Among them

$$a_i = \left[ \log(R_a) \right]^i$$

(5)

$$b_i = \left[ \log(R_b) \right]^i$$

(6)

By using SVD to solve formula (4), the rotation matrix can be obtained $R_x$. And then put $R_x$ back into equation (3), you get flat matrix $T_x$. However, by using the step-by-step solution method, the error of the rotation matrix will be substituted into the translation matrix [8]. Therefore, an iterative approach can also be used to get both $T_x$ and $R_x$.

$$\arg \min_{R_x \in SO(3)} \sum_{i=1}^{N} \left( || R_x a_i - b_i ||^2 + || R_x t_a + t_x - R_x t_b - t_a || \right)$$

(7)

However, equation (6) cannot reach the local optimal solution [9]. We used a new perspective to perform 4-D analysis on the hand eye calibration equation $AX=XB$.

2.2. Optimization estimation based on 4-D rotation

The rotation matrix can be represented by the unitary quaternions $q_L=(a,b,c,d)^T$ and $q_R=(p,q,r,s)^T$ [10]. The relationship between $R$, $q_L$ and $q_R$ can be expressed as:

$$R = R_L(q_L) \ast R_R(q_R)$$

(8)

In the above formula, $R_L$ and $R_R$ are represented by the product of quaternions

$$R_L(q_L) = \begin{pmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{pmatrix}$$

(9)
The rotation matrix $R$ is parameterized by the formula (8)

$$\sigma = \frac{1}{\sqrt{2}} (q^T, q^T) \in R^8$$

(11)

The rotation matrix $R$ of equation (11) can be expressed as

$$R = (c_1, c_2, c_3, c_4)$$

(12)

If $c_1, c_2, c_3$ and $c_4$ similar factorization are established for the columns $c_1, c_2, c_3$ and $c_4$ of the matrix $R$, then

$$c_1 = \begin{pmatrix} ap & -bq & -cr & -ds \\ aq & bp & cs & -dr \\ ar & cp & -bs & dq \\ as & br & -cq & dp \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} p & -q & -r & -s \\ q & p & s & -r \\ r & -s & p & -b \\ s & r & q & p \end{pmatrix} \frac{1}{\sqrt{2}} (q^T, q^T) = P_1(\sigma)^* \sigma$$

$$c_2 = \begin{pmatrix} -qa & -bq & cs & -dr \\ -bp & ap & ds & cr \\ -cp & dp & -as & -br \\ -dp & -cp & -bs & ar \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -q & -p & s & -r \\ q & -q & r & s \\ -s & -r & -q & p \\ s & r & q & -p \end{pmatrix} \frac{1}{\sqrt{2}} (q^T, q^T) = P_2(\sigma)^* \sigma$$

$$c_3 = \begin{pmatrix} -ar & -bs & cp & -bq \\ -br & as & dp & cq \\ -cr & ds & ap & -bq \\ -dr & -cs & bp & aq \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -r & -s & -p & q \\ s & -r & -q & p \\ p & q & -r & s \\ -s & r & -q & -p \end{pmatrix} \frac{1}{\sqrt{2}} (q^T, q^T) = P_3(\sigma)^* \sigma$$

$$c_4 = \begin{pmatrix} -bs & ar & -cq & -dq \\ -cs & dr & aq & -bp \\ -ds & cr & bq & ap \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -s & r & -q & -p \\ -r & s & -p & q \\ q & -p & -s & r \\ p & q & r & s \end{pmatrix} \frac{1}{\sqrt{2}} (q^T, q^T) = P_4(\sigma)^* \sigma$$

(13)

According to expressions (11) and (13), $P_1(\sigma), P_2(\sigma), P_3(\sigma)$ and $P_4(\sigma)$ are matrices[11] of $4*8$, which satisfy

$$P_1(\sigma)^T P_1(\sigma)^T = P_2(\sigma)^T P_2(\sigma)^T = P_3(\sigma)^T P_3(\sigma)^T = P_4(\sigma)^T P_4(\sigma)^T$$

$$= \frac{1}{2} (a^2 + b^2 + c^2 + d^2 + p^2 + q^2 + r^2 + s^2)^* E = E$$

(14)

In the same way $R$ can be factored out into rows and factored out, with the results shown below $R = (\sigma^T Q_1(\sigma), \sigma^T Q_2(\sigma), \sigma^T Q_3(\sigma), \sigma^T Q_4(\sigma))^T$

$$Q_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} p & -q & -r & -s & a & -b & -c & -d \\ -q & -p & s & -r & -b & a & -d & c \\ -r & s & -p & q & -c & d & -a & -b \\ -s & r & -q & -p & -d & -c & b & a \end{pmatrix}$$

(13)
In order to solve the problem that equation (7) cannot reach the local optimal solution, the hand-eye calibration model $AX=XB$ is transferred from SO(3) to SO(4) to solve the hand-eye calibration equation. Since the solution of $AX=XB$ in SO(4) is very similar to the QDR that was used to solve the rotation quaternion decomposition, therefore, the rotation octonions can be decomposed on the basis of QDR.

2.3. The hand-eye calibration equation is solved

Let $E_i$ and $Z_i$ be the transformation matrix of the coordinate system of the manipulator and the linear structured light Angle sensor under 4-d, from Equation (2), can be obtained

$$\arg \min_{R_{i} \in SO(4)} \sum_{i=1}^{N} \| E_i R_{i} - R_{i} Z_{i} \|^2$$

In the top formula, $E_i \in SO(4)$ and $Z_i \in SO(4)$, Its exponent maps to a 6*6 skew symmetric matrix. Let's define $E_i$ and $Z_i$ as follows

$$E_i = (e_{i,1}, e_{i,2}, e_{i,3}, e_{i,4})$$

$$Z_i = (z_{i,1}, z_{i,2}, z_{i,3}, z_{i,4})$$

(17)

Bring equation (17) to $E_i R_{i} - R_{i} Z_{i}$, according to the properties of the matrix, formula (18) can be obtained as follows

$$E_i R_{i} - R_{i} Z_{i} = \begin{pmatrix} e_{i,1} \\ e_{i,2} \\ e_{i,3} \\ e_{i,4} \end{pmatrix} \begin{pmatrix} P_1(\sigma) \sigma, P_2(\sigma) \sigma, P_3(\sigma) \sigma, P_4(\sigma) \sigma \end{pmatrix} \begin{pmatrix} \sigma^T Q_1(\sigma) \\ \sigma^T Q_2(\sigma) \\ \sigma^T Q_3(\sigma) \\ \sigma^T Q_4(\sigma) \end{pmatrix} \begin{pmatrix} z_{i,1}, z_{i,2}, z_{i,3}, z_{i,4} \end{pmatrix}$$

(18)

$E_i R_{i} - R_{i} Z_{i} = (M_{i,1} \sigma, M_{i,2} \sigma, M_{i,3} \sigma, M_{i,4} \sigma)$ can be obtained from formula (18), then
Each $G_{j,k,i}$, $(j,k=1,2,3,4)$ in the equation is shown in the following equation:

$$
M_{i,j} = \begin{pmatrix}
\sigma^T G_{11,i} \\
\sigma^T G_{12,i} \\
\sigma^T G_{13,i} \\
\sigma^T G_{14,i}
\end{pmatrix},
M_{i,2} = \begin{pmatrix}
\sigma^T G_{21,i} \\
\sigma^T G_{22,i} \\
\sigma^T G_{23,i} \\
\sigma^T G_{24,i}
\end{pmatrix},
M_{i,3} = \begin{pmatrix}
\sigma^T G_{31,i} \\
\sigma^T G_{32,i} \\
\sigma^T G_{33,i} \\
\sigma^T G_{34,i}
\end{pmatrix},
M_{i,4} = \begin{pmatrix}
\sigma^T G_{41,i} \\
\sigma^T G_{42,i} \\
\sigma^T G_{43,i} \\
\sigma^T G_{44,i}
\end{pmatrix}
$$

(19)

The parameter matrix $J_{j,k,i} \in \mathbb{R}^{4*8}$. By formula (17), (18), (19) and (20), the optimal unit octonions can be solved.

$$
G_{j,k,i} = \begin{pmatrix}
0 & J_{j,k,i} \\
J_{j,k,i}^T & 0
\end{pmatrix}
$$

(20)

The parameter matrix $J_{j,k,i}$ is obtained through measurement, and the hand-eye calibration model is constructed, the feature vectors $q_L$ and $q_R$ obtained from formula (22) are substituted into formula (8) to determine the transformation matrix $R$ between the end of the manipulator and the coordinate system of the linear structured light sensor.

2.4. The hand-eye calibration equation $SO(4)$ was parameterized

The solution method of the hand-eye calibration model $AX=XB$ under $SO(4)$ is determined. Next, consider how the conversion matrix of the obtained coordinate system can be solved from $SO(3)$ to $SO(4)$. Thomas [12] proposed the conversion relationship of the matrix from $SO(3)$ to $SO(4)$ as follows

$$
T_{SO(3)} = \begin{pmatrix}
R & t \\
0 & 1
\end{pmatrix} \leftrightarrow R_{SO(4)} = \begin{pmatrix}
R & \varepsilon t \\
\varepsilon t^T & 1
\end{pmatrix}
$$

(27)

In the above formula, $\varepsilon$ satisfies $\varepsilon^2 = 0$. In order to satisfy the equation, we actually set $\varepsilon = 1/d$.

When $d >> 1$, it satisfies the equation. After actual experiments, we set $d = 1*10^7[13]$. 

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3. Experiments and Results

3.1. Hand-eye calibration method

According to the above algorithm to start the experiment, the hand-eye calibration experiment is carried out through the robotic hand shown in Figure 1. The experimental steps are:

1. Obtain the coordinates of the end of the N groups of manipulators and the coordinate system of the vision sensor by moving the manipulator, in order to ensure stability and experiment accuracy should be N>3.

2. Substitute the obtained $A_i$ and $B_i$ into formula (27), and transfer the data from SO(3) to SO(4).

3. Solve the $AX=XB$ equation by formulas (16)~(26), and transfer the calculated R to SO(3) through formula (27).

4. Put the calculated R into practical application to calculate the accuracy.

The hand-eye calibration experiment uses an 11*8 checkerboard as a fixed target for camera calibration and hand-eye calibration. During the experiment, first use checkerboard calibration on the camera to obtain the internal parameters and distortion coefficients as shown in Table 1, and then perform hand-eye calibration experiments. The calibration board is fixed, the robot moves, and record the external parameters of the camera at different positions and the base coordinates of the manipulator at different positions to the end coordinates of the end of the manipulator. In order to briefly describe the hand-eye calibration process, this paper extracts 6 robot hand poses and external camera parameters as shown in Table 2.

![FIG. 1. AX=XB hand-eye calibration experiment scene](image-url)

| Table 1  | Camera internal reference coefficient and distortion coefficient |
|----------|---------------------------------------------------------------|
| Internal parameter coefficient | Distortion coefficient |
| $F_x=5607.42,F_y=5606.08,$ | $K_1=-0.1312, K_2=-0.7283,$ |
| $C_x=1183.6259, C_y=971.474$ | $P_1=0.0025, P_2=10.7269$ |

| Table 2  | Hand-eye calibration data |
|----------|----------------------------|
| End pose of robotic arm (X,Y,Z,A,B,C) | Calibration board pose in camera (X,Y,Z,A,B,C) |
| 1 (696.769, -25.212, 195.079, -1.312, 2.452, 0.313) | (-26.2284, -13.0656, 232.1546, -2.6001, -1.6436, 0.17575) |
| 2 (684.517, -38.304, 204.9984, -1.0434, 2.5414, 0.1304) | (-21.6356, -3.3877, 238.3466, -2.6998, -1.3048, 0.1424) |
| 3 (733.242, -72.505, 222.512, 2.422, -1.692, -0.778) | (-0.4445, -30.5581, 251.2142, 1.4261, -2.6620, -0.5059) |
| 4 (786.284, 20.484, 182.298, 2.785, 18.9168, -32.1963, 240.2048, 0.7820, | |
It can be seen from Table 2 that 2 groups of any area can form a hand-eye calibration equation, and the hand-eye relationship matrix obtained by the algorithm in this paper is:

\[
\begin{bmatrix}
-0.84596038 & -0.15141467 & 0.51129700 & -396.84040433 \\
0.15293526 & -0.98745107 & -0.03938495 & 40.54927467 \\
0.51084423 & 0.04487723 & 0.85850113 & -488.50491963 \\
0.00000000 & 0.00000000 & 0.00000000 & 1.00000000
\end{bmatrix}
\]

In order to verify the reliability of the hand-eye relationship matrix, 10 sets of completely independent experiments were repeated. The coordinates of the point in the robot base coordinates were calculated through the robot end coordinates, the hand-eye conversion matrix, and the camera pose formula, and then compared with the real values. The comparison result of 3D coordinates is shown in the figure below.

FIG. 2. X coordinate comparison

FIG. 3. Y coordinate comparison
It can be observed from the figure that the coordinate error of the X and Y coordinates of the actual point and the calculated result according to the hand-eye relationship conversion matrix is within 0.5mm, which basically meets the requirements. The larger error of the Z-axis is caused by the introduction of a certain error when measuring the actual value in order to prevent the robot from colliding with the test bench.

There are two main sources of error:

First, when using a robot to measure, it cannot be 100% guaranteed to align with the measuring point, so errors will occur.

Second, the internal parameters, distortion coefficients, external parameter solution and hand-eye calibration model of camera calibration will produce errors in the calculation process, which will cause errors in the coordinates of the measuring points.

4. Conclusion
In this paper, a measurement system based on the calibration of the hand-eye relationship of a monocular robot is established. The chessboard target is used as a reference object, and the unit octonion method is used to solve the hand-eye calibration model $AX=XB$ to determine the relative posture conversion between the camera and the robot end. Finally, the accuracy is verified by measuring the checkerboard and setting the board. The experimental results show that the system has good recognition ability, high accuracy and stability, and meets actual application requirements. However, the accuracy of the single camera for detecting the three-dimensional space coordinates of the object is not high, and more sensors are needed. Related work will continue in the future.

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