Automatic Tuner for PID Controllers with Elements of Artificial Intelligence

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Abstract. The vast majority of automatic control systems (ACS) currently in operation use the PID algorithm. However, there are still many problems in the adjustment of PID regulators parameters that provide the required quality of control. In this connection, it is proposed to use for this task an algorithm with two reference models for self-tuning of automatic control systems, which use continuous processes that accumulate information about the dynamics of the controlled object. It is used for determining optimal values of tunable parameters, assessing the performance of ACS in the course of modeling and automatic change of the ACS adjustment factors, in the case of approaching the performance of ACS to the limits of admissible values of the quality criteria, defined in the specifications.

1. Introduction

The overwhelming majority of automatic control systems (ACS) currently in operation use the PID algorithm. The reason for such high popularity is the ease of implementation and industrial use, clarity of functioning, suitability for solving most practical problems and low cost. Nevertheless, there are still many problems with the adjustment of the PID regulators parameters, which provide the required quality of control.

In this connection, it is proposed to use for this task a self-tuning algorithm for automatic control systems with two reference models [1], which uses continuous processes of accumulating information on the dynamics of the controlled object, its use for determining optimal values of tuned parameters, mathematical modeling and automatic change of the ACS adjustment factors, in the case of approaching the performance of ACS to the limits of admissible values of quality criteria defined in the specifications.

In the proposed algorithm for tuning the PID controller in addition to the reference model, a second reference model, called a tunable model (TM), is introduced. Using the TM, a parametric identification of the ACS is carried out, in which the TM is adjusted to a configurable ACS. After that, the TM is tuned to the specified performance requirements for ACS using a reference model (RM) of special type (parametric RM set).

2. Mathematical description

The considered ACS tuning scheme, along with the use of algebraic methods of ACS analysis, makes it possible to solve the identification problem using the self-adjustment algorithm. Methods of non-linear programming are used to find the area of acceptable settings. Moreover, as the sought-for values of the coefficients of the given zone, the point at which the domain of permissible settings degenerates with a proportional increase in the requirements for transient processes (for example, reduction of the...
maximum permissible overshoot, reduction of the time of entry into the tube of accuracy, etc.) is chosen as the required values of the coefficients of this zone. Mathematically, the description of the TM is formed as a set of fractional-rational (polynomial) expressions from the adjustable parameters that determine the response of the ACS to the corresponding test signal.

The operation diagram shown in Fig. 1 explains the interaction of the parts of the program that implements the algorithm of the tuning automaton. The order of the algorithm is as follows.

![TM Functional diagram](image)

**Figure 1. TM Functional diagram**

The test signal is fed to the input of the system being tuned and the tunable model. After passing the test signal, a divergence error occurs in the identification unit. In accordance with the identification algorithm in the PM, the values of the parameters $p_i$ are calculated, which are the corresponding estimates of these parameters in the real system. Estimates of the parameters $p_1,...,p_z$ are calculated by minimizing the expressions:

$$
\min_{\mathcal{W}_1,...,\mathcal{W}_n} \|y_{\mathcal{I}}(t) - y_{\mathcal{IM}}(t)\|, \quad i = 1, 2, ..., n,
$$

where $n$ - the number of test signals, $y_{\mathcal{I}}(t)$ - response of tuned ACS to the test signal $g_{\mathcal{I}}(t)$, and $y_{\mathcal{IM}}(t)$ - TM response to the test signal. In case the configurable ACS is linear, it is sufficient to consider only one test signal at the identification stage, for example, stepwise, as some broadband, a compelling system to show its dynamic qualities. Thus, the TM is adjusted to the real system, and later the tunable model is tuned, and the real system is disconnected from the tuning process.

After the identification phase, the test signal is fed to the input of the tuned model and the reference model. The divergence error between the output signals (TM) and RM goes to the tuner, in which the expression is minimized:

$$
\min_{k_1, k_2, k_3} \sum_{i=1}^{n} |y_{\mathcal{PM}} (k_1, k_2, k_3, t) - y_{\mathcal{EM}} (p_1, p_2, ..., p_r)|,
$$

where $k_1, k_2, k_3$ are the adjustable parameters of the TM, and $p_1, p_2, ..., p_r$ are the coefficients of the ACS.
where $k_1, k_2, k_3$ are tunable parameters; $y_{pm}(k_1, k_2, k_3, t)$ are output signals of the tunable model for test signal $g(t)$: $y_{rm}(p_1, p_2, ..., p_r)$ are output signals of the reference model with set parameters values $p_i$: $K_0$ - area of permissible changes of tunable parameters $k_i$ (for instance, the range of such $k_i$ parameters leading to stability of the tuned system); $P_{pl}$ - area of permissible changes of parameters $p_i$, i.e. such $p_i$ parameters, where the reference model’s response satisfies set requirements. Thus, not only one fixed RM, but the whole set is determined by specifying the range of changes in the parameters of the RM transfer function.

Let us suppose that for the considered TM and the input abrupt test signal, an output signal is obtained in the form of the expression:

$$y_{pm}(t) = \sum_{j=0}^{N} p_j(k_1, k_2, k_3)T_1(t),$$

where according to the algorithm specified in [2], $p_1(k_1, k_2, k_3)$ polynomials relating to $k_1, k_2, k_3$ parameters and $\{T_1(t)\}_0^N$ is the system of Chebyshev 1 type orthogonal polynomials determined at segment $[0, T]$, where the transition process is being analyzed. $T$ values should be selected as equal to $3\tau_{tp}$, where $\tau_{tp}$ is transition process time for the considered ACS. If in case of initial values of adjusted $k_1, k_2, k_3$ parameters (we denote them further by the vector $\vec{K}$) primary quality indicators are not satisfied, that is, the overshoot $\frac{\sigma}{\sigma_0}$, $T > T_{TDD}$ PID controller should be tuned.

We will review the tuning algorithm based on the example of ACS with such quality indicators as $\sigma > \sigma_0$, $T > T_{TDD}$. As the transfer function of RM, we consider the oscillatory link. By the way, despite the variety and complexity of real control components, two structures of mathematical models are generally used in PID controllers tuning: a first-order model with a delay and a second-order model.

Let the time constant $T$ and the damping coefficient $\xi$ be the transfer function of the RM such that $y_{et}(t)$ satisfies the imposed quality control requirements. Further, we represent $y_{et}(t)$ in the form of an expansion in a series of Chebyshev polynomials:

$$y_{et}(t) = \sum_{j=0}^{N} \mu_j T_j(t) + \xi_k(t).$$

$N$ value is chosen such that $|k(t)| \leq \frac{\pi}{10}$. Then:

$$y_{pm}(t) - y_{et}(t) = \sum_{j=0}^{N} (p_j(\vec{K}) - \mu_j)T_j(t),$$

and the solution of the non-linear algebraic equations system being represented as:

$$\begin{align*}
p_0(\vec{K}) - \mu_0 &= 0 \\
p_1(\vec{K}) - \mu_1 &= 0 \\
p_N(\vec{K}) - \mu_N &= 0
\end{align*}$$

relating to $\vec{K}$ is the desired TM settings. However, taking into account the problems of proving the existence and searching for a solution of non-linear systems of equations, we weaken the condition and look for a vector $\vec{K}$ that minimizes the expression:
\[
\min_{\mathcal{K}} F_N(\mathcal{K}) = \sum_{j=0}^{N} (\rho_j(\mathcal{K}) - \mu_j)^2.
\]

If as a result of minimizing the functional \( F_N(\mathcal{K}) \) we obtain \( \mathcal{K} \) such that:

\[
y_{pm}(t) = \sum_{j=0}^{N} \rho_j(\mathcal{K}) T_j(t)
\]

satisfies the requirements, then the problem is considered as solved.

It should be noted that the success of the solution of the problem depends to a large extent on the successful choice of the curve \( y_{et}(t) \), that is, the parameters of the reference oscillatory link. Therefore, it makes sense from the given requirements for the quality of regulation, to determine the region \( G_{et} \) such that the transition function of the oscillatory link with the values of \( \xi \) and \( T \) from this region satisfies the required quality. Let there be a decomposition of the transition function of the oscillatory link in the form of an explicit dependence of the coefficients of the expansion on \( \xi \) and \( T \), that is:

\[
y_{et}(t) = \sum_{j=0}^{N} \mu_j(\xi, T) T_j(t).
\]

Then function \( F_N(\mathcal{K}) \) will also depend on \( \xi, T (F_N(\mathcal{K}, \xi, T)) \) variables and its minimal values will be determined considering such variables:

\[
\min_{\mathcal{K}, \xi, T \in G_{et}} F_N(\mathcal{K}, \xi, T).
\]

Thus, from among the whole set of admissible reactions one chooses the one that most corresponds to the dynamics of the system under consideration. To solve the problem, it is necessary to select \( N \) and define the range of admissible values for the given quality control requirements. Let us consider the technique for determining \( G_{et} \) for the given quality control requirements, that is, \( \mathcal{G}_{et} \) for the oscillatory link, whose transition function has the form:

\[
y_{et}(t) = \left[ 1 - e^{-\beta t} (\cos \omega_1 t + \frac{\beta}{\omega_1} \sin \omega_1 t) \right] l(t),
\]

where \( \beta = \frac{\xi}{\sigma} = \omega_0 \xi \) is damping coefficient, \( \omega_1 = \omega_0 \sqrt{1 - \xi^2} = \frac{\sqrt{1 - \xi^2}}{T} \) is own frequency of the oscillatory link. We will calculate \( \mu_j(\xi, T) \), using the method of trigonometric interpolation. We redefine the function \( y_{et}(t) \) for segment \([-1,1]\), by replacement of variables \( t = \frac{1}{2} (t' + 1) \), and obtain:

\[
y_{et} \left( \frac{1}{2} (t' + 1) \right) = 1 - e^{-\frac{\beta (t' + 1)}{2}} \left( \cos \left( \frac{\omega_1 (t' + 1)}{2} \right) + \frac{\beta}{\omega_1} \sin \left( \frac{\omega_1 (t' + 1)}{2} \right) \right),
\]

or via corresponding transformations:

\[
\tilde{y}_{et}(t') = y_{et} \left( \frac{1}{2} (t' + 1) \right) = 1 - e^{-\frac{\beta t'}{2}} \left( \cos \left( \frac{\omega_1 (t' + 1)}{2} \right) + \frac{\beta}{\omega_1} \sin \left( \frac{\omega_1 (t' + 1)}{2} \right) \right).
\]

In accordance with the triangle interpolation, the coefficients of expansion are determined based on the formula:

\[
\mu_0(\beta, \omega_1) = \rho^2 \frac{1}{\pi} \sum_{k=1}^{N} \tilde{y}_{et}(\lambda_k),
\]

\[
\mu_j(\beta, \omega_1) = \rho^2 \frac{2}{\pi} \sum_{k=1}^{N} \tilde{y}_{et}(\lambda_k) \cos \left( j \frac{2k - 1}{2N} \pi \right),
\]

where

\[
\rho^2 = \left\{ \left( \frac{1}{2} + \sum_{k=1}^{N} \cos(k \frac{\pi}{2N})^2 \right)^{-1} \right\}.
\]
For a selected segment and function \( \tilde{y}_{\text{et}}(t) \) the coefficient \( \rho^2 = 0.4 \) for \( N = 5 \), and
\( \lambda_k = \cos \pi \frac{2k-1}{2N} \), \( k = 1, 2, 3, 4, 5 \). Then we obtain the following expressions for \( \mu_j(\beta, \omega_1) \):

\[
\mu_0(\beta, \omega_1) = 1 - \frac{1}{5} \sum_{k=1}^{5} e^{-\alpha_k \beta} \left( \cos(\alpha_k \omega_1) + \frac{\beta}{\omega_1} \sin(\alpha_k \omega_1) \right),
\]

where
\[
\alpha_k = \frac{1}{2} \left( \cos \frac{2k-1}{10} \pi + 1 \right), \, k = 1, 2, 3, 4, 5;
\]
\( \alpha_1 = 0.975529 \), \( \alpha_2 = 0.793893 \), \( \alpha_3 = 0.5 \), \( \alpha_4 = 0.206108 \), \( \alpha_5 = 0.024472 \);
\[
\mu_j(\beta, \omega_1) = \sum_{k=1}^{5} \gamma_j^k + \sum_{k=1}^{5} \gamma_j^k e^{-\beta \alpha_k} \left( \cos(\alpha_k \omega_1) + \frac{\beta}{\omega_1} \sin(\alpha_k \omega_1) \right),
\]

Further, we obtain restrictions on \( \beta, \omega_1 \) under the given restrictions on the transient process. Let the maximum permissible time of the transient process \( T_{\text{ppd}} \) and the maximum allowable overshoot \( \sigma_\text{d} \) be set. Let us find the expression for the extremal values \( y_{\text{et}}(t) \):

\[
\frac{dy_{\text{et}}(t)}{dt} = (\beta^2 + \omega_1) e^{-\beta \pi} \sin \omega_1 t = 0,
\]

wherefrom \( \omega_1 t = \pi n \),

thus, the extremal value is as follows:

\[
\max_{t} y_{\text{et}}(t) = y_{\text{et}}\left(\frac{\pi}{\omega_1}\right) = 1 + e^{-\frac{\beta \pi}{\omega_1}}.
\]

i.e. \( \sigma = e^{-\frac{\beta \pi}{\omega_1}} \) and in order that \( \sigma \leq \sigma_\text{d} \) the coefficients \( \beta \) and \( \omega_1 \) must satisfy the following inequality:

\[
\frac{1}{\pi} \ln \frac{1}{\sigma_\text{d}} + \beta \leq 0.
\]

Now we find the conditions under which the time of the transient process, that is, the time of entry into the tube of the accuracy given by the static error \( \varepsilon_{\text{st}} \), is less than maximum permissible value \( T_{\text{ppd}} \). Since the values of \( y_{\text{et}}(t) \) are bounded above and below, namely:

\[
1 - e^{-\beta t} \leq y_{\text{et}}(t) \leq 1 + e^{-\beta t},
\]

therefore, if \( e^{-\beta t} \leq \varepsilon_{\text{st}} \), then the transition process is inside the precision tube:

\[
T_{\text{ppd}} \leq \frac{1}{\beta} \ln \frac{1}{\varepsilon_{\text{st}}}.
\]

As a result we obtain restrictions on \( \beta \) and \( \omega_1 \), under which the transient process \( y_{\text{et}}(t) \) satisfies the requirements for the quality of control from the point of view of the given quality indicators, therefore \( G_{\text{em}} \) is determined for the considered example.

3. Conclusion
In conclusion, it should be noted that the construction of an explicit approximate dependence of the transient process on a given test signal on the parameters being tuned (in the form of a polynomial expression with respect to these parameters) allows us to obtain explicit dependences of the quality gradients on the parameters being tuned, which provides additional possibilities for solving the problem. The presence of modern packages, such as LabVIEW PID Control Toolset by National Instrument, MATLAB, implementing algebraic transformations in symbolic form, greatly simplifies the development of software for the proposed algorithm for setting PID regulators. In addition, the proposed algorithm for adapting ACS accumulates information about the dynamics of a managed object and provides a forecast of the state of the system in the space of custom parameters. Thus, in the ACS contour, in fact, an action acceptor is predicted that forecasts in real time the best option for tuning ACS within the specified quality criteria.

References
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