Non-Parallel Electric and Magnetic Fields in a Gravitational Background, Stationary Gravitational Waves and Gravitons

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Abstract

The existence of an electromagnetic field with parallel electric and magnetic field components in the presence of a gravitational field is considered. A non-parallel solution is shown to exist. Next, we analyse the possibility of finding stationary gravitational waves in nature. Finally, we construct a $D = 4$ effective quantum gravity model. Tree-level unitarity is verified.

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1 Electric and Magnetic Field in a Gravitational Background

Based on a series of papers by Brownstein [1] and Salingaros [2], we consider here the possibility of the existence of an electromagnetic field whose electric and magnetic field components are parallel in the presence of a gravitational field. The coupling between the electromagnetic sector and the gravitational backgrounds is accomplished by means of the action

\[ S = \int \sqrt{-\tilde{g}} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) d^4x , \quad (1.1) \]

where

\[ \tilde{g} = \text{det} (g_{\mu\nu}) , \]

and

\[ F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} . \]

From the above action, the following field-equations follow:

\[ \mathcal{D}_\mu F^{\mu\nu} = \partial_\mu F^{\mu\nu} + \Gamma^\beta_{\beta\lambda} F^{\lambda\nu} + \Gamma^\nu_{\mu\lambda} F^{\mu\lambda} = J^{\nu} , \quad (1.2) \]

\[ \mathcal{D}_\mu F_{\nu\beta} + \mathcal{D}_\nu F_{\beta\mu} + \mathcal{D}_\beta F_{\mu\nu} = 0 . \quad (1.3) \]

Choosing the background to be described by the F.R.W metric,

\[ dS^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - Ar^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) , \quad (1.4) \]

the Maxwell equations in the absence of electromagnetic sources are

\[ \tilde{\nabla} \cdot \tilde{E} = g \tilde{\nabla} f \cdot \tilde{E} , \quad (1.5) \]

\[ \tilde{\nabla} \cdot \tilde{B} = 0 , \]

\[ \tilde{\nabla} \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t} , \]

\[ \tilde{\nabla} \times \tilde{B} = \frac{\partial \tilde{E}}{\partial t} - g \frac{\partial f}{\partial t} \tilde{E} + g \tilde{\nabla} f \times \tilde{B} - \Gamma^\mu_{\mu\beta} F^{\mu\beta} , \]

where

\[ g = \frac{1 - Ar^2}{a^3 r^2 \sin \theta} , \quad f = \frac{a^3 r^2 \sin \theta}{\sqrt{1 - Ar^2}} , \quad (1.6) \]
and $A = +1, 0, -1$.

The wave-equations for $\vec{E}$ and $\vec{B}$ are found to be:

$$\nabla^2 \vec{E} - \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{\nabla} (g \vec{\nabla} f \cdot \vec{E}) - \frac{\partial g}{\partial t} \frac{\partial f}{\partial t} \vec{E} - g \frac{\partial^2 f}{\partial t^2} \vec{E} - g \frac{\partial f}{\partial t} \frac{\partial \vec{E}}{\partial t} + \left(1.7\right)$$

$$\frac{\partial g}{\partial t} \vec{\nabla} f \times \vec{B} + g \frac{\partial}{\partial t} \vec{\nabla} f \times \vec{B} + g \vec{\nabla} f \times \frac{\partial \vec{B}}{\partial t} - \frac{\partial}{\partial t} \left(\Gamma^i_{\mu \beta} F^{\mu \beta}\right),$$

$$\nabla^2 \vec{B} - \frac{\partial^2 \vec{B}}{\partial t^2} = \vec{\nabla} \times \left(g \frac{\partial f}{\partial t} \vec{E} - g \vec{\nabla} f \times \vec{B} + \Gamma^i_{\mu \beta} F^{\mu \beta}\right). \quad \left(1.8\right)$$

Now, due to the presence of the gravitational background, we have explicitly built up a solution for $\vec{E}$ and $\vec{B}$ that is different from the one obtained by Brownstein [1] and Salingaros [2]. These authors state that it is always possible to find solutions for parallel $\vec{E}$ and $\vec{B}$ in plasma physics or in an astrophysics plasma. However, contrary to their result, we have found non-parallel solutions due to the non-flat background of gravity:

$$\vec{E} = i \left(\sin \theta G(r, t, \theta) - \cos \theta F(r, t)\right) ka \cos(kz) \cos(\omega t) +$$

$$+ j \left(\cos \theta F(r, t) - \sin \theta G(r, t, \theta) ka \sin(kz) \cos(\omega z)\right) +$$

$$+ k \left(\sin \theta \cos \varphi F(r, t) + \cos \theta \cos \varphi G(r, t, \theta) ka \cos(kz) \cos(\omega t) +$$

$$- \left(\sin \theta \sin \varphi F(r, t) + \cos \theta \sin \varphi G(r, t, \theta) ka \sin(kz) \cos(\omega t)\right)\right)$$

and

$$\vec{B} = ka \left[i \sin(kz) + j \cos(kz)\right] \cos(\omega t) \quad \left(1.10\right)$$

where the functions $G(r, t, \theta) = \frac{a \cot g \theta}{3a r}$ and

$$F(r, t) = \frac{2a}{3a r} + \frac{Aa r}{3a(1 - Ar^2)} \quad \left(1.11\right)$$

are the metric contribution.
2 Stationary Gravitational Waves and Gravitons

Now, we analyse the possibility of finding stationary gravitational waves. From a phenomenological viewpoint, a distribution of black holes could play the role of knots for the non-propagating gravitational waves. We postulate the equation that may lead to this sort of waves to be of the form

\[ R_{\mu\nu} = \kappa \Lambda h_{\mu\nu} , \]  

(2.1)

\[ g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu} , \]  

(2.2)

where \( \Lambda \) is the cosmological constant. These equations yield:

\[ \partial_\beta \partial_\nu h_\beta^\mu + \partial_\beta \partial_\mu h_\beta^\nu - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h_\beta^\beta = \Lambda h_{\mu\nu} . \]  

(2.3)

Now, solutions of the form

\[ h_{\mu\nu} = C_{\mu\nu}(z)f(t) , \]  

(2.4)

\[ h_{\mu\nu} = \begin{pmatrix} A_{00} & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{12} & -A_{11} & 0 \\ 0 & 0 & 0 & A_{00} \end{pmatrix} e^{i\tilde{k}z \cos \omega t} , \]  

(2.5)

can be found, where \( A_{00}, A_{11} \) and \( A_{12} \) are free parameters, whereas \( \tilde{k} = \sqrt{\Lambda - \omega^2} \) is the wave number. Having in mind that \( \Lambda \) is a small number, the frequency \( \omega \) must be extremely small. This forces us to search for a mechanism to detect such low-frequency stationary waves.

The equations of motion are derived from the Lagrangian density

\[ \mathcal{L}_H = \frac{1}{2} H^{\mu\nu} \Box H_{\mu\nu} - \frac{1}{4} \Box H - \frac{1}{2} H^{\mu\nu} \partial_\mu \partial_\alpha H_\nu^\alpha - \frac{1}{2} H^{\mu\nu} \partial_\nu \partial_\alpha H_\mu^\alpha - \frac{1}{2} \Lambda H^{\mu\nu} H_{\mu\nu} + \frac{1}{4} \Lambda H^2 , \]  

(2.6)

where

\[ H_\nu^\alpha = h_\nu^\alpha - \frac{1}{2} \delta_\nu^\alpha h . \]
and the bilinear form operator of lagrangian (2.17) is given by
\[
\Theta_{\mu\nu,\kappa\lambda} = (\Box - \Lambda) P^{(2)} - \Lambda P^{(1)}_m + \frac{5}{2}(\Box - \Lambda) P^{(0)}_s - \frac{(\Lambda + 3\Box)}{2} P^{(0)}_w \\
+ \frac{\sqrt{3}}{2} (\Lambda - \Box) P^{(0)}_{sw} + \frac{\sqrt{3}}{2} (\Lambda - \Box) P^{(0)}_{ws},
\]
(2.7)
and \(P^{(i)}, i = 0, 1, 2\), are spin projection operators in the space of rank-2 symmetric tensors. The graviton propagator is given by:
\[
\langle T(h_{\mu\nu}(x); h_{\kappa\lambda}(y)) \rangle = i\Theta^{-1}_{\mu\nu,\kappa\lambda} \delta^4(x - y)
\]
(2.8)
where
\[
\Theta^{-1} = [XP^{(2)} + YP^{(1)}_m + ZP^{(0)}_s + WP^{(0)}_w + RP^{(0)}_{sw} + SP^{(0)}_{ws}]_{\mu\nu,\kappa\lambda}
\]
(2.9)
with
\[
X = - \frac{1}{\Lambda - \Box}, \quad Y = - \frac{1}{\Lambda}, \quad Z = - \frac{\Lambda + 3\Box}{\Lambda^2 + 8\Lambda\Box - 9\Box^2},
\]
\[
W = - \frac{5}{\Lambda - 9\Box}, \quad R = - \frac{\sqrt{3}}{\Lambda + 9\Box} \quad \text{and} \quad S = - \frac{\sqrt{3}}{\Lambda + 9\Box}.
\]
From this propagator, a current-current amplitude is obtained and the tree-level unitarity [3] is discussed. Three massive excitations are found: They are a spin-2 quantum with mass equal to \(k^2 = \Lambda\) and two massive spin-0 quanta with masses equal to \(k^2 = \Lambda\) and \(k^2 = -\frac{1}{9}\Lambda\). The spin-2 is a physical one: the imaginary part of the residue of the amplitude at the pole \(k^2 = \Lambda\) is positive, so that it does not lead to a ghost. It remains to be shown that the tachyonic pole, \(k^2 = -\frac{1}{9}\Lambda\), is non-dynamical or decouples through some constraint on the sources.

We conclude, then, that in a gravitational background it is always possible to find non-parallel electric and magnetic fields. It is the gravitational field that breaks the parallel configuration of \(\vec{E}\) and \(\vec{B}\) [1,2]. Furthermore, a stationary gravitational wave equation is postulated and a particular solution is found. We argue that such a solution is likely to be found in Black Hole distributions. Finally, we set up an effective quantum gravity model where the necessary condition for the tree-level unitarity for the spin-2 sector is respected. The model is infrared finite though non-renormalizable in the ultraviolet limit.
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