The work of Judita Cofman on didactics of mathematics

Abstract
Judita Cofman was the first generation student of mathematics and physics at Faculty of Philosophy in Novi Sad, Serbia, and the first holder of doctoral degree in mathematics sciences at University of Novi Sad. Her PhD thesis as well as her scientific works till the end of 70's belongs to the field of finite projective and affine planes and the papers within this topic were published in prestigious international mathematical journals. The aim of this paper is to draw attention to Cofman’s contribution in didactics and teaching of mathematics through the activities with young mathematicians, to whom she devoted the second part of her life and scientific work. Her reflections on importance of geometry based on her experiences with high school students are specially pointed out.

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Key words and phrases: Judita Cofman, teaching of mathematics, didactics of mathematics.
1 Introduction

Mathematician Judita Cofman\(^2\) was born in Vršac on 4th June 1936. She came from a well-known and formerly wealthy family of Zoffmanns whose arrival in Vršac is put at the time of the reign of Maria Theresa of Austria (1717-1780) where they came from a German region with a strong beer brewing tradition (Kuručev, D., 2007; 157-163). Although the Zoffmanns were originally German, they gradually adopted Hungarian identity, so Judita declared herself as a Hungarian from Vojvodina. An environment of material and cultural wealth marked the life of Judita's father Ákos Zoffmann (1910-1974). Having received wide education in Germany, he became a great expert in beer brewing, and wine growing and storing industries. Judita's mother Lujza (1910-2000), born Kozics, comes from a Hungarian family of lawyers from her father's side, while her mother came from Vršac. Lujza's grandfather was a mathematics teacher at the Vršac Grammar School, and her uncle was the mayor of the town of Vršac. Despite the incertitude and horrors of World War II, Judita enjoyed a happy childhood at her family home. She went to primary school in Hungarian and later to Serbian Grammar School in her hometown. The family home, full of love and harmony, installed in her a great feeling that work, study, reading, as well as the knowledge of foreign languages are necessary preconditions for success in life. Besides being gifted for mathematics, Judita had a talent for languages, so, besides her mother tongue of Hungarian and the official Serbian language, as a child she learned German, Russian and, which was rare at that time, English. She later learned French and Italian.

Judita Cofman's PhD thesis as well as her scientific work till the end of 70's belong to the theory of finite projective planes, Möbius planes and Sperner's spaces a very up-to-date and lively mathematical field, closely related to algebra and group theory. Her results within this topics were published in prestigious international mathematics journals (Mathematische Zeitschrift, Archives Mathematica, Canadian Journal of Mathematics, for example) and were presented at high ranking conferences devoted to these field of projective geometry. Her results complemented the results of many great geometricians of the early 20th century on the one hand, and on the other, the active follow-up and advancement of certain subfields of projective geometry rest upon her results (Nikolić, A., 2012). In the second period of 20 years, from 1980

\(^2\) Her name was entered into the Official Register as Judit Zoffmann, but in the documents of Yugoslavia of that time her name was written in its Serbian rendition as Judita Cofman, and that was the name she used for the rest of her life.
till 2001, Judita was completely devoted to the mathematical education and didactics of mathematics and improvements in teaching process, especially working with gifted teenage mathematicians.

2 Studies and career

Judita Cofman started mathematical studies in 1954, graduating with the highest grades in 1958. She was among 66 students enrolled as the first generation of students of mathematics at Faculty of Philosophy in Novi Sad. They were all studying to be teachers of mathematics. At that time the majority of classes were given by professors from the University of Belgrade, academicians Miloš Radojičić, Anton Bilimović, Radivoje Kašanin and Jovan Karamata (professor in Geneva at the time). Among the mathematicians from Novi Sad, there were Mirko Stojaković and Bogoljub Stanković, while the first assistants were Vojislav Marić and Mileva Prvanović. They were all to become eminent Serbian scientists and members of Serbian Academy of Sciences and Arts. The first elected Assistant Professor at the Mathematics Department was Mileva Prvanović (1956), and it was for the field of geometry. Judita Cofman was appointed as her Assistant in 1960. Judita had been the best student of mathematics for generations. Her younger colleagues, later professors at the University of Novi Sad, Irena Čomić and Danica Nikolić Despotović, remember that students had great respect for professors but also some kind of fear for them. Despite all efforts of professors to travel from Belgrade, they were not always accessible to their students. The professional literature in Serbian language was still insufficient at that time, and students could not use foreign titles because their knowledge of English, French, German or Russian was modest. The only person who was able to answer at any moment a variety of questions by curious students was Judita Cofman. As her knowledge of foreign languages was high, she was almost the only one among the students who could use German, English and Russian textbooks and widen her knowledge of mathematics, which she used to unselfishly share with her colleagues. They felt that she knew all there was to know about mathematics! As soon as she was made Assistant, in collaboration with students she published the lecture notes Ruler-and-compass Constructions. This was the first publication in the field of mathematics issued at the Faculty of Philosophy in Novi Sad and it heralded what was to become an abundant publishing activity at the University of Novi Sad.
The following year, in 1961, she left for postgraduate studies in Roma. There she studied with well-known Italian mathematician Professor Lucio Lombardo-Radice (1916-1982). As Lombardo-Radice contributed to finite geometry and geometric combinatorics together with Guido Zappa (1915) and Beniamino Segre (1903-1977), and wrote and published important papers concerning the Non-Desargues Plane, Judita Cofman chose the same field of mathematics for her scientific work. In 1963 she returned to Novi Sad and defending her PhD thesis under the title *Finite Non-Desargues Projective Planes Generated by Quadrangle*, she took the first doctoral degree in mathematical sciences from the University of Novi Sad. The committee for her thesis defense consisted of Lombardo-Radice and professors Mirko Stojaković (mentor) and Mileva Prvanović.

As the holder of the Alexander von Humboldt scholarship she spent 1964/65 school year at the University of Frankfurt/Main. The following six years she spent as a lecturer professor at Imperial College in London (University of London), where she was also engaged in her research work. In 1970 she was a visiting professor at the University of Perugia (Italy). From 1971 to 1978 she taught mathematics at the universities in Tübingen and Mainz. During this period she took part in three major conferences - International Colloquio on Combinatorial Theory held in Rome, Combinatorial Geometry and Applications held at the University of Perugia and the International Conference on Projective Planes held at the Washington State, which hosted all the important mathematicians of that time whose field of work involved Projective Planes.

From 1978 till 1993 she worked at the state Putney High School in London and began to interest in teaching and methodology of mathematics. The academic year 1985-86 she stayed at St. Hilda's College, Oxford, where she enjoyed a teacher fellowship in Trinity term. She spent these six years professionally absolutely dedicated to the pedagogical-methodological work with teachers of mathematics and talented students. She also taught on several advanced master classes held at the City of London School, together with teachers Terry Heard and Martin Perkins. In 1987 she was the member of training staff of British Olympic team at 28th International Mathematical Olympiad held on Cuba. In April 1993 she participated at Second

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Report by Mr. Robert Lyness, leader of the British team at 28th International Mathematical Olympiad, Cuba 1987: Our team was selected by means of the National Mathematics Contest and the British Mathematical Olympiad, followed by some postal tuition and a residential selection/training session which included a further test. This session was held at the Ship Hotel, Reading from Friday 8th May to Sunday 10th May 1987. It was staffed by Judita Cofman, David Cundy, Terry Heard, John Hersee, Paul Woodruff, and myself. The training programme consisted of short lectures and tutorial periods during which the participants had opportunities to expound their own solutions to
German meeting of the European Woman in Mathematics in Tübingen with a talk *On the role of problem solving in math classes*. She collaborated with several foundations and associations (Sir John Cass Foundation, Advanced Royal Institution Mathematical Classes, Association of Gifted Children in Great Britain). She was organiser, and active participant and lecturer of several inspirational International summer camps for young mathematicians. The number of people that attended the maths camps was also coached by Judita in preparation for the Maths Olympiads (Jeroen Nijhof, Anders Bjorn, Alex Selby for example).

She was the editor of journal Hypotenuse whose contents was in relation with her seminars and these camps. She also gave regular seminars in Germany for teachers of mathematics and students preparing to become teachers, as well as lectures within didactics seminars (Didaktik der Mathematik - Seminar der Universität Freiburg). From 1993 till her retirement in August 2001 she worked as the professor of didactics of mathematics at University of Erlangen, Nürnberg and became the head of the Mathematics Teaching Methods Department (Prvanović, M., 2002; 57). In her spare time she conducts Maths-Workshops for 13-19 years old youngsters. About her experience gained during these Workshops it could be heard and seen from the video being recorded at the FAU College Alexandrinum (Collegium Alexandrinum) as a part of Projekt Uni-TV. Judita Cofman gave the talk *Mathematik macht Spass! - Über Workshops für Gymnasial Schülerinnen und Schüler am Mathematischen Institut on 24th of June 1999.* In September 2001 she was invited to participate in the work of the Postgraduate Studies Department for Mathematics Teaching Methods and appointed a professor at the Faculty of Natural Sciences at University Kossuth Lajos in Debrecen, Hungary, where she passed away on 19th December of the same year.

3 Work in the Field of Mathematical Education

The second period of Judita Cofmans life and scientific work, from 1980 till 2001, was marked by theory and practice in the field of pedagogy and didactics of mathematics dedicated to students and their teachers. In this scientific engagement there were no momentary ascents of problems. It proved extremely helpful. All these activities are the responsibility of the Mathematical Associations "National Committee for Mathematical Contests".

4 "Mathe mal anders", Freizeitaktivitäten für Schüler und Studenten, lecture held on the 5th of December 1995.
5 See the Web page http://www.university-tv.de/ca.html or directly the video of her talk http://giga.rrze.uni-erlangen.de/movies/collegium_alexandrinum/ss99/19990624.mpg
mind and creation of new systems, no new theories nor proving theorems or conjectures; that what mattered was an overall understanding of teaching mathematics, approach to students and different teaching methods through a well-balanced proportion between theory and problems designed to motivate students to think and work independently on solving them. Judita Cofman possessed all the preconditions to be successful in the methods of teaching mathematics she was a mature personality, proven mathematician, and most importantly, she cherished deep and true love for children.

Judita Cofman collaborated with a number of universities around the world, which prepare future mathematical educators, and she also had an intensive cooperation with institutes for mathematics and associations of mathematicians. She constantly emphasized the importance of the quality of teaching of mathematics, from the lower grades of elementary school to university level. She was known for being an exceptionally good teacher, who had a very responsible attitude towards this profession, which was the result of her great respect for her audiences and science, and prepared thoroughly for her lectures. In 1984 Judita started an intensive collaboration with associations and methodological centers dedicated to teaching in the Hungarian language in Vojvodina. Her work is known in Hungary as well. She maintained a close contact with the universities in Budapest, Szeged and Debrecen in Hungary, and took part in the work of camps for talents and future teachers of mathematics.

The central problem of her engagement in didactics of mathematics was how to motivate pupils to think and work independently on their solutions even at an early age. Judita Cofman thought that one way of introducing youngsters to independent study was to get them involved in work on projects. One such example was her well known International Camps for young mathematicians held in England and Germany (See Cofman, J., 1990; preface and Cofman, J., 1986. Also see Gardiner, T., Jones, L., 1985; 35-37). The contents and organization of her well-known book *What to solve? - Problems and suggestions for young mathematicians*, as a compilation of problems and solutions discussed during seminars and sessions on problem solving in these camps. The organization of the text, selecting and grouping of questions, comments, references to related mathematical topics and instructions on teaching represent the core of Judita's ideas of teaching mathematics. In the aim of problem solving the campers-pupils (and readers of her book) were led gradually step by step through encouraging independent

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6 Cofman, J., 2000; 84.
investigation in finding an answer to a question and demonstrating different approaches to problem solving.

The atmosphere at the camp held in the summer of 1984, near Chelmsford in Essex is described well by Heather Cordell, at the time a fifteen-year-old girl from London.\(^7\) Here it is her account:

"Like its predecessors, the camp in 1984 proved to be a resounding success. This was in no small way helped by the fact that its participants came from all over Europe, from nine different countries altogether, each bringing his/her own way of looking at mathematics and tackling mathematical problems. An average working day at the camp would start at 9:00 a.m. with a demonstration of problem-solving techniques. These problems varied from day to day in both standard and topic, so that a wide variety of interest and ability could be catered for. Examples included a proof of the existence of an infinite number of primes, ways of solving problems by the "pigeon hole principle", and many others. Next came a session of project work. The projects were stimulated by particular problems (from a list provided) or were chosen by the students themselves. A practical interest in bell-ringing inspired one participant to investigate the various permutations possible on a certain set of bells. Many of the projects were a result of combined efforts; a more advanced student could often use his/her knowledge to work with a less advanced but equally dedicated student. In the second week, the students themselves led discussions about their results and difficulties with their projects. The first afternoon session lasted from 3:00 p.m. to 4:00 p.m., and consisted of a lecture a guest speaker or one of the tutors. The topics again varied, but usually involved some less traditional subjects such as codes and ciphers, non-conventional geometries, topology and ways to win "Nim". The lecture contents would be expanded in the second afternoon session, from 5:00 p.m. to 6:00 p.m., to give more insight for the advanced participants. The fourth session was not compulsory - but many younger students did attend and enjoy what they could understand. Of course, there were many other opportunities for both relaxation and study. These included a visit to London (alternatively a country walk for those who preferred), an invitation watch the local bell-ringing and day spent..."

\(^7\) Heather Cordell is today Professor of Statistical Genetics and a Welcome Senior Research Fellow in the Institute of Human Genetics at Newcastle University, UK. When she was once asked about who contributed to her becoming a scientist, among others, mostly professors and colleagues from University, she mentioned Judita Cofman as her high school mathematics teacher. (See her text \textit{Moving from Promise to Proficiency}, The Scientist, 17, 8, April 21 (2003), p. 56.)
in Cambridge, with two excellent lectures on "Convex sets and their applications in Economics" and "Algorithms". All in all, everyone had a most enjoyable fortnight and is looking forward to participating again next year.

In her article On the Role of Geometry in Contemporary Mathematical High School Education (1996b), Judita Cofman lists remarks concerned primarily to geometry, based on her experiences with high school students:
1. In the contemporary education, with a curriculum overburdened with details from various fields of mathematics, there is a danger that the study of mathematics can stray into memorising facts and a mechanical learning of algorithms. On the contrary to this, the efforts should be directed at pupils understanding the existing links between the phenomena they encounter in different fields of mathematical study. Geometry can play a certain role in such efforts, because the mathematical disciplines taught in high school are rich in details for which there are geometrical illustrations appropriate to the pupils’ age. The application of such illustrations, on the one hand, facilitates the process of understanding of the totality of teaching material, and on the other, presents geometry as a science of an actual importance.
2. The importance of Euclidean geometry in teaching is supported by the fact that the shapes of this geometry are encountered in our living environment. The study of space is particularly facilitated by the study of solid geometry, which is, unfortunately, often neglected in syllabus and curriculum. There is an important fact in respect to geometrical features of space which is often forgotten; the majority of children possess a lot of elementary knowledge about objects, such as the cube or the sphere from the earliest age. This elementary knowledge can be extremely useful for introducing notions such as: defined and undefined elements, axioms and theorems, necessary and sufficient conditions, etc. All these notions are important for the field of mathematics while the familiarity with space can be used to make pupils grasp the essence starting from concrete examples.
3. Mathematics is one of the earliest scientific disciplines, an important segment of human cultural heritage. This fact must be reflected on the teaching of mathematics: it is advisable to draw pupils attention, whenever an opportunity arises, to their historical background. The history of geometry is an important part of the history of mathematics, not only because geometry is one of the oldest branches of mathematics. The importance of geometry mostly lies in the fact that there were several major problems in this field, starting from the Ancient Greek age, which could
finally be solved only in the 19th century. The solutions to these problems had been sought for for ages; the attempts led to a series of new discoveries and contributed to a further development of the entire science of mathematics. One of the famous problems of geometry was the so called Delian problem of doubling the cube.

4. Teaching of geometry can also play a useful role in illustrating the achievements in the most current fields of mathematics.

5. The knowledge gained in the study of geometry can contribute to a better understanding of the phenomena from different fields of natural sciences.

6. For teaching of geometrics to be successful, teaching personnel must have a solid knowledge of this subject. However, not only at schools, but also in university courses and other pedagogical institutions for training future mathematics teachers, there is a tendency of neglecting the study of geometry. This fact can lead to a drastic deterioration in the level of geometry teaching at schools. What is needed is an effort at elevating the respectability of geometry with the students of mathematics.

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In her pedagogical work, Judita Cofman had the ability to raise simple mathematical truths onto a higher level and turn the elementary into a science. She knew historical genesis of each problem, where it originated from and how it was solved throughout history. She deemed that an important reason for teaching mathematics in schools was to promote independent pupils' thinking processes and powers of observation.\(^8\) Throughout their schooling, pupils should be made aware of the links between various phenomena and they should be given the opportunity to discover these links on their own whenever this is possible. Moreover, pupils should be motivated to search for interdependence between seemingly unrelated topics. How can this be achieved? The key answer to the above question and generally to teaching of mathematics she gave in several papers and five books dedicated to mathematics teaching methods, which represent an outstanding approach to solving non-standard mathematical problems. Her historical approach to science and mathematical problems was the focus of her books which feature problems based on famous topics from the history of mathematics and a selection of elementary problems treated by eminent twentieth-century mathematicians.

\(^8\) Cofman, J., 1998: 23.
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