States of Strongly Interacting Matter

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Abstract:
I discuss the phase structure of strongly interacting matter at high temperatures and densities, as predicted by statistical QCD, and consider in particular the nature of the transition of hot hadronic matter to a plasma of deconfined quarks and gluons.

1. Prelude
To speak about quark matter in a meeting dedicated to Heisenberg is somewhat problematic. In one of his last talks, Heisenberg noted: “There exists the conjecture that the observable hadrons consist of non-observable quarks. But the word ‘consist’ makes sense only if it is possible to decompose a hadron into these quarks with an energy expenditure much less than the rest mass of a quark” \[1\]. Therefore I asked myself what arguments might have convinced Heisenberg to revise his opinion. On philosophical level, which after all played a significant role in Heisenberg’s argumentation, one might remember what Lucretius pointed out more than two thousand years earlier: “So there must be an ultimate limit to bodies, beyond perception by our senses. This limit is without parts, is the smallest possible thing. It can never exist by itself, but only as primordial part of a larger body, from which no force can tear it loose” \[2\]. This must be one of the earliest formulations of confinement; curiously enough, it was generally ignored by all ‘atomists’ before the advent of QCD. Lucretius argues that the ultimate building blocks of matter cannot have an independent existence, since otherwise one could ask what they are made of.

On more physical grounds, we note that the energy density of an ideal electromagnetic plasma, consisting of electrons, positrons and photons, is given by the Stefan-Boltzmann law
\[
\epsilon_{\text{QED}} = \frac{\pi^2}{30} \left[ 2 + \frac{7}{8} \times 2 \right] T^4,
\]
which counts the number of constituent species and their degrees of freedom (two spin orientations each for electrons, positrons and photons). For a hot and hence asymptotically free quark-gluon plasma, the corresponding form is
\[
\epsilon_{\text{QCD}} = \frac{\pi^2}{30} \left[ 2 \times 8 + \frac{7}{8} \times 2 \times 3 \times 3 \right] T^4,
\]
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which again counts the number of otherwise confined constituents and their degrees of freedom (eight colors of gluons, three colors and three flavors for quarks and antiquarks, and two spin orientations for quarks and gluons). The energy density of the hot QGP thus provides direct information on what it is made of.

A similar argument was in fact used by Zel’dovich even before the advent of the quark model [3]. He notes that if dense stellar or pre-stellar media should not obey the equation of state of neutron matter, this might be an indication for other types of elementary particles: “It will be necessary to consider as many Fermi distributions as there are elementary particles. The problem of the number of elementary particles may be approached in this way, since if some particle is in reality not elementary, it would not give rise to a separate Fermi distribution”. So the behavior of matter in the limit of high constituent density seems to be a good way to address the question of its ultimate building blocks.

2. Hadronic Matter and Beyond

Hadrons have an intrinsic size, with a radius of about 1 fm. Hence a hadron needs a volume \( V_h = \frac{4\pi}{3}r_h^3 \approx 4 \text{ fm}^3 \) to exist. This implies an upper limit \( n_c \) to the density of hadronic matter, \( n_h < n_c \), with \( n_c = \frac{V_h}{r_h^3} \approx 0.25 \text{ fm}^{-3} \approx 1.5 n_0 \), where \( n_0 \approx 0.17 \text{ fm}^{-3} \) denotes standard nuclear density. Fifty years ago, Pomeranchuk pointed out that this also leads to an upper limit for the temperature of hadronic matter [4]. An overall volume \( V = NV_h \) causes the grand canonical partition function to diverge when \( T \geq T_c \approx \frac{1}{r_h} \approx 0.2 \text{ GeV} \).

This conclusion was subsequently confirmed by more detailed dynamical accounts of hadron dynamics. Hagedorn proposed a self-similar composition pattern for hadronic resonances, the statistical bootstrap model, in which the degeneracy of a given resonant state is determined by the number of ways of partitioning it into more elementary constituents [5]. The solution of this classical partitioning problem is a level density increasing exponentially with mass, \( \rho(m) \sim \exp\{am\} \), which leads to a diverging partition function for an ideal resonance gas once its temperature exceeds the value \( T_H = 1/a \), which turns out to be close to the pion mass. A yet more complete and detailed description of hadron dynamics, the dual resonance model, confirmed this exponential increase of the resonance level density [6, 7]. While Hagedorn had speculated that \( T_H \) might be an upper limit of the temperature of all matter, Cabbibo and Parisi pointed out that \( T_H \) could be a critical temperature signalling the onset of a new quark phase of strongly interacting matter [8]. In any case, it seems clear today that hadron thermodynamics, based on what we know about hadron dynamics, contains its own intrinsic limit [9].

On one hand, the quark infrastructure of hadrons provides a natural explanation of such a limit; on the other hand, it does so in a new way, different from all previous reductionist approaches: quarks do not have an independent existence, and so reductionism is at the end of the line, in just the way proposed by Lucretius.

The limit of hadron thermodynamics can be approached in two ways. One is by compressing cold nuclear matter, thus increasing the baryon density beyond values of one baryon per baryon volume. The other is by heating a meson gas to temperatures at which collisions produce further hadrons and thus increase the hadron density beyond values allowing each hadron its own volume. In either case, the medium will undergo a transition from a state in which its constituents were colorless, i.e., color-singlet bound
states of colored quarks and gluons, to a state in which the constituents are colored. This end of hadronic matter is generally referred to as deconfinement.

The colored constituents of deconfined matter
• could be massive constituent quarks, obtained if the liberated quarks dress themselves with gluon clouds;
• or the liberated quarks could couple pairwise to form bosonic colored diquarks;
• or the system could consist of unbound quarks and gluons, the quark-gluon plasma (QGP).

One of the tasks of statistical QCD is to determine if and when these different possible states can exist.

In an idealized world, the potential binding a heavy quark-antiquark pair into a color-neutral hadron has the form of a string,

\[ V(r) \sim \sigma r, \]  

where \( \sigma \) specifies the string tension. For \( r \to \infty \), \( V(r) \) also diverges, indicating that a hadron cannot be dissociated into its quark constituents: quarks are confined. In a hot medium, however, thermal effects are expected to soften and eventually melt the string at some deconfinement temperature \( T_c \). This would provide the string tension with the temperature behavior

\[ \sigma(T) = \begin{cases} \sigma(0) \left[ T_c - T \right]^a & T < T_c, \\ 0 & T > T_c, \end{cases} \]  

with \( a \) as critical exponent for the order parameter \( \sigma(T) \). For \( T < T_c \), we then have a medium consisting of color-neutral hadrons, for \( T > T_c \) a plasma of colored quarks and gluons. The confinement/deconfinement transition is thus the QCD version of the insulator/conductor transition in atomic matter.

In the real world, the string breaks when \( V(r) \) becomes larger than the energy of two separate color singlet bound states, i.e., when the ‘stretched’ hadron becomes energetically more expensive than two hadrons of normal size. It is thus possible to study the behavior of eq. (4) only in quenched QCD, without dynamical quarks and hence without the possibility of creating new \( q \bar{q} \) pairs. The result [11] is shown in Fig. 1, indicating that \( a \simeq 0.5 \). We shall return to the case of full QCD and string breaking in section 4.

The insulator-conductor transition in atomic matter is accompanied by a shift in the effective constituent mass: collective effects due to lattice oscillations, mean electron fields etc. give the conduction electron a mass different from the electron mass in vacuum. In QCD, a similar phenomenon is expected. At \( T = 0 \), the bare quarks which make up the hadrons ‘dress’ themselves with gluons to form constituent quarks of mass \( M_q \simeq 300 - 350 \) MeV. The mass of a nucleon then is basically that of three constituent quarks, that of the \( \rho \) meson twice \( M_q \). With increasing temperature, as the medium gets hotter, the quarks tend to shed their dressing. In the idealized case of massless bare quarks, the QCD Lagrangian \( \mathcal{L}_{\text{QCD}} \) possesses chiral symmetry: four-spinors effectively reduce to two independent two-spinors. The dynamically created constituent quark mass at low \( T \) thus corresponds to a spontaneous breaking of this chiral symmetry, and if at some high \( T = T_\chi \) the dressing and hence the constituent quark mass disappears, the chiral symmetry of \( \mathcal{L}_{\text{QCD}} \) is restored. Similar to the string tension behavior of Eq. (4) we thus expect

\[ M_q(T) = \begin{cases} M_q(0) \left[ T_\chi - T \right]^b & T < T_\chi, \\ 0 & T > T_\chi. \end{cases} \]  

3
for the constituent quark mass: $T_\chi$ separates the low temperature phase of broken chiral symmetry and the high temperature phase in which this is restored, with $b$ as the critical exponent for the chiral order parameter $M_q(T)$.

An obvious basic problem for statistical QCD is thus the clarification of the relation between $T_c$ and $T_\chi$. In atomic physics the electron mass shift occurs at the insulator-conductor transition; is that also the case in QCD?

The deconfined QGP is a color conductor; what about a color superconductor? In QED, collective effects of the medium bind electrons into Cooper pairs, overcoming the repulsive Coulomb force between like charges. These Cooper pairs, as bosons, condense at low temperatures and form a superconductor. In contrast to the collective binding effective in QED, in QCD there is already a microscopic $qq$-binding, coupling two color triplet quarks to an antitriplet diquark. A nucleon can thus be considered as a bound state of this diquark with the remaining third quark,

$$[3 \oplus 3 \oplus 3]_1 \sim [(3 \oplus 3 \rightarrow \overline{3}) \oplus 3]_1,$$

leading to a color singlet state. Hence QCD provides a specific dynamical mechanism for the formation of colored diquark bosons and thus for color superconductivity. This possibility [12] has created much interest and activity over the past few years [13].

We thus have color deconfinement, chiral symmetry restoration and diquark condensation as possible transitions of strongly interacting matter for increasing temperature and/or density. This could suggest a phase diagram of the form shown on the left in Fig. 2, with four different phases. The results of finite temperature lattice QCD show that at least at vanishing baryochemical potential ($\mu = 0$) this is wrong, since there deconfinement and chiral symmetry restoration coincide, $T_c = T_\chi$, as the corresponding transitions in atomic physics do. In section 4 we shall elucidate the underlying reason for this.

A second guess could thus be a three-phase diagram as shown on the right in Fig. 2, and this is in fact not in contradiction to anything so far. In passing, we should note, however, that what we have here called the diquark state is most likely more complex and may well consist of more than one phase [13].
After this conceptual introduction to the states of strongly interacting matter, we now turn to the quantitative study of QCD at finite temperature and vanishing baryochemical potential. In this case, along the $\mu = 0$ axis of the phase diagram, the computer simulation of lattice QCD has provided a solid quantitative basis.

3. Statistical QCD

The fundamental dynamics of strong interactions is defined by the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} \left( \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} - g f_{bc}^{a} A_{\mu}^{b} A_{\nu}^{c} \right)^{2} - \sum_{f} \bar{\psi}^{f}_{\alpha} \left( i \gamma_{\mu} \partial^{\mu} + m_{f} - g \gamma_{\mu} A^{\mu} \right) \psi^{f}_{\beta},$$

in terms of the gluon vector fields $A$ and the quark spinors $\psi$. The corresponding thermodynamics is obtained from the partition function

$$\mathcal{Z}(T,V) = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\{-S(A,\psi,\bar{\psi};T,V)\},$$

here defined as functional field integral, in which

$$S(A,\psi,\bar{\psi};T,V) = \int_{0}^{1/T} d\tau \int_{V} d^{3}x \mathcal{L}_{\text{QCD}}(\tau = ix_{0}, x)$$

specifies the QCD action. As usual, derivatives of $\log \mathcal{Z}$ lead to thermodynamic observables; e.g., the temperature derivative provides the energy density, the volume derivative the pressure of the thermal system.

Since this system consists of interacting relativistic quantum fields, the evaluation of the resulting expressions is highly non-trivial. Strong interactions (no small coupling constant) and criticality (correlations of all length scales) rule out a perturbative treatment in the transition regions, which are of course of particular interest. So far, the only ab initio results are obtained through the lattice formulation of the theory, which leads to something like a generalized spin problem and hence can be evaluated by computer simulation. A discussion of this approach is beyond the scope of this survey; for an overview, see e.g. [14]. We shall here just summarize the main results; it is to be noted that for computational reasons, the lattice approach is so far viable only for vanishing baryochemical potential, so that all results given in this section are valid only for $\mu = 0$. 

Figure 2: Four-phase and three-phase structure for strongly interacting matter
As reference, it is useful to recall the energy density of an ideal gas of massless pions of three charge states,

\[ \epsilon_\pi(T) = \frac{\pi^2}{30} 3 T^4 \simeq T^4, \tag{10} \]
to be compared to that of an ideal QGP (see Eq. (2)), which for three massless quark flavors becomes

\[ \epsilon_{\text{QCD}}(T) \simeq 16 T^4. \tag{11} \]

The corresponding pressures are obtained through the ideal gas form \( 3P(T) = \epsilon(T) \). The main point to note is that the much larger number of degrees of freedom of the QGP as compared to a pion gas leads at fixed temperatures to much higher energy densities and pressures.

The energy density and pressure have been studied in detail in finite temperature lattice QCD with two and three light dynamical quark species, as well as for the more realistic case of two light and one heavier species. The results are shown in Fig. 3, where it is seen that in all cases there is a sudden increase from a state of low to one of high values, as expected at the confinement-deconfinement transition. To confirm the connection between the transition and the increase of energy density or pressure, we make use of the order parameters for deconfinement and chiral symmetry restoration; these first have to be specified somewhat more precisely than was done in the more conceptual discussion of section 2.

![Figure 3: Energy density and pressure in full QCD with light dynamical quarks](image)

In the absence of light dynamical quarks, for \( m_q \to \infty \), QCD reduces to pure SU(3) gauge theory; the potential between two static test quarks then has the form shown in Eq. (3) when \( T < T_c \) and vanishes for \( T \geq T_c \). The Polyakov loop expectation value defined by

\[ \langle |L(T)| \rangle \equiv \lim_{r \to \infty} \exp\{-V(r,T)/T\} = \begin{cases} 0, & \text{confinement} \\ L(T) > 0, & \text{deconfinement} \end{cases} \tag{12} \]

thus also constitutes an order parameter for the confinement state of the medium, and it is easier to determine than the string tension \( \sigma(T) \). In lattice QCD, \( L(T) \) becomes very similar to the magnetization in spin systems; it essentially determines whether a global
$Z_3 \in SU(3)$ symmetry of the Lagrangian is present or is spontaneously broken for a given state of the medium.

In the other extreme, for $m_q \to 0$, $\mathcal{L}_{\text{QCD}}$ has intrinsic chiral symmetry, and the chiral condensate $\langle \bar{\psi}\psi \rangle$ provides a measure of the effective mass term in $\mathcal{L}_{\text{QCD}}$. Through

$$\langle \bar{\psi}\psi \rangle = \begin{cases} K(T) > 0, & \text{broken chiral symmetry,} \\ 0, & \text{restored chiral symmetry.} \end{cases}$$

we can determine the temperature range in which the state of the medium shares and in which it spontaneously breaks the chiral symmetry of the Lagrangian with $m_q = 0$.

There are thus two bona fide phase transitions in finite temperature QCD at vanishing baryochemical potential.

For $m_q = \infty$, $L(T)$ provides a true order parameter which specifies the temperature range $0 \leq T \leq T_c$ in which the $Z_3$ symmetry of the Lagrangian is present, implying confinement, and the range $T > T_c$, with spontaneously broken $Z_3$ symmetry and hence deconfinement.

For $m_q = 0$, the chiral condensate defines a range $0 \leq T \leq T_\chi$ in which the chiral symmetry of the Lagrangian is spontaneously broken (quarks acquire an effective dynamical mass), and one for $T > T_\chi$ in which $\langle \bar{\psi}\psi \rangle(T) = 0$, so that the chiral symmetry is restored. Hence here $\langle \bar{\psi}\psi \rangle(T)$ is a true order parameter.

In the real world, the (light) quark mass is small but finite: $0 < m_q < \infty$. This means that the string breaks for all temperatures, even for $T = 0$, so that $L(T)$ never vanishes. On the other hand, with $m_q \neq 0$, the chiral symmetry of $\mathcal{L}_{\text{QCD}}$ is explicitly broken, so that $\langle \bar{\psi}\psi \rangle$ never vanishes. It is thus not clear if some form of critical behavior remains, and we are therefore confronted by two basic questions:

- how do $L(T)$ and $\langle \bar{\psi}\psi \rangle(T)$ behave for small but finite $m_q$? Is it still possible to identify transition points, and if so,
- what if any relation exists between $T_c$ and $T_\chi$?

In Fig. 4, we show the lattice results for two light quark species; it is seen that $L(T)$ as well as $\langle \bar{\psi}\psi \rangle(T)$ still experience very strong variations, so that clear transition temperatures can be identified through the peaks in the corresponding susceptibilities, also shown in the figure. Moreover, the two peaks occur at the same temperature; one thus finds here (and in fact for all small values of $m_q$) that $T_c = T_\chi$, so that the two ‘quasi-critical’ transitions of deconfinement and chiral symmetry restoration coincide.

Although all lattice calculations are performed for non-vanishing bare quark mass in the Lagrangian, results obtained with different $m_q$ values can be extrapolated to the chiral limit $m_q = 0$. The resulting transition temperatures are found to be $T_c(N_f = 2) \approx 175$ MeV and $T_c(N_f = 3) \approx 155$ MeV for two and three light quark flavors, respectively. The order of the transition is still not fully determined. For $N_f = 3$ light quark species, one obtains a first order transition. For two light flavors, a second order transition is predicted [15], but not yet unambiguously established.

4. The Nature of Deconfinement

In this last section I want to consider in some more detail two basic aspects which came up in the previous discussion of deconfinement:

- Why do deconfinement and chiral symmetry restoration coincide for all (small) values of the input quark mass?
Is there still some form of critical behavior when \( m_q \neq 0 \)?

Both features have recently been addressed, leading to some first and still somewhat speculative conclusions which could, however, be more firmly established by further lattice studies.

In the confined phase of pure gauge theory, we have \( L(T) = 0 \), the Polyakov loop as generalized spin is disordered, so that the state of the system shares the \( Z_3 \) symmetry of the Lagrangian. Deconfinement then corresponds to ordering through spontaneous breaking of this \( Z_3 \) symmetry, making \( L \neq 0 \). In going to full QCD, the introduction of dynamical quarks effectively brings in an external field \( H(m_q) \), which in principle could order \( L \) in a temperature range where it was previously disordered.

Since \( H \to 0 \) for \( m_q \to \infty \), \( H \) must for large quark masses be inversely proportional to \( m_q \). On the other hand, since \( L(T) \) shows a rapid variation signalling an onset of deconfinement even in the chiral limit, the relation between \( H \) and \( m_q \) must be different for \( m_q \to 0 \). We therefore conjecture \[16, 17] that \( H \) is determined by the effective constituent quark mass \( M_q \), setting

\[
H \sim \frac{1}{m_q + c \langle \bar{\psi} \psi \rangle},
\]

since the value of \( M_q \) is determined by the amount of chiral symmetry breaking and hence by the chiral condensate. From Eq. (14) we obtain

- for \( m_q \to \infty \), \( H \to 0 \), so that we recover the pure gauge theory limit;
- for \( m_q \to 0 \), we have

\[
\langle \bar{\psi} \psi \rangle = \begin{cases} 
\text{large, } H \text{ small, } L \text{ disordered}, & \text{for } T \leq T_\chi; \\
\text{small, } H \text{ large, } L \text{ ordered}, & \text{for } T > T_\chi.
\end{cases}
\]

In full QCD, it is thus the onset of chiral symmetry restoration that drives the onset of deconfinement, by ordering the Polyakov loop at a temperature value below the point of
spontaneous symmetry breaking \[17\]. In Fig. 5 we compare the behavior of \(L(T)\) in pure gauge theory to that in the chiral limit of QCD. In both cases, we have a rapid variation at some temperature \(T_c\). This variation is for \(m_Q \to \infty\) due to the spontaneous breaking of the \(Z_3\) symmetry of the Lagrangian at \(T = T_c^\infty\); for \(m_q \to 0\), the Lagrangian retains at low temperatures an approximate \(Z_3\) symmetry which is explicitly broken at \(T_\chi\) by an external field which becomes strong when the chiral condensate vanishes. For this reason, the peaks in the Polyakov loop and the chiral susceptibility coincide and we have \(T_\chi = T_c < T_c^\infty\).

![Figure 5: Temperature dependence of the Polyakov loop in the chiral and the pure gauge theory limits](image)

A quantitative test of this picture can be obtained from finite temperature lattice QCD. It is clear that in the chiral limit \(m_q \to 0\), the chiral susceptibilities (derivatives of the chiral condensate \(\langle \psi \bar{\psi} \rangle\)) will diverge at \(T = T_\chi\). If deconfinement is indeed driven by chiral symmetry restoration, i.e., if \(L(T, m_q) = L(H(T), m_q)\) with \(H(T) = H(\langle \psi \bar{\psi} \rangle(T))\) as given in Eq. (14), than also the Polyakov loop susceptibilities (derivatives of \(L\)) must diverge in the chiral limit. Moreover, these divergences must be governed by the critical exponents of the chiral transition.

Preliminary lattice studies support our picture \[17\]. In Fig. 6 we see that the peaks in the Polyakov loop susceptibilities as function of the effective temperature increases as \(m_q\) decreases, suggesting divergences in the chiral limit. Further lattice calculations for smaller \(m_q\) (which requires larger lattices) would certainly be helpful. The question of critical exponents remains so far completely open, even for the chiral condensate and its susceptibilities.

Next we want to consider the nature of the transition for \(0 < m_q < \infty\). For finite quark mass neither the Polyakov loop nor the chiral condensate constitute genuine order parameters, since both are non-zero at all finite temperatures. Is there then any critical behavior? For pure \(SU(3)\) gauge theory, the deconfinement transition is of first order, and the associated discontinuity in \(L(T)\) at \(T_c\) cannot disappear immediately for \(m_q < \infty\). Hence in a certain mass range \(m_q^0 < m_q \leq \infty\), a discontinuity in \(L(T)\) remains; it vanishes for \(m_q^0\) at the endpoint \(T_c(m_q^0)\) in the \(T - m_q\) plane; see Fig. 6. For \(m_q = 0\), we have the genuine chiral transition (perhaps of second order \[15\]) at \(T_\chi\), which, as we just saw, leads to critical behavior also for the Polyakov loop, so that here \(T_c(0) = T_\chi\) is a true critical temperature. What happens between \(T_c(m_q^0)\) and \(T_c(0) = T_\chi\)? The dashed line in Fig. 7 separating the hadronic phase from the quark-gluon plasma is
not easy to define unambiguously: it could be obtained from the peak position of chiral and/or Polyakov loop susceptibilities [18], or from maximizing the correlation length in the medium [19]. In any case, it does not appear to be related to thermal critical behavior in a strict mathematical sense.

An interesting new approach to the behavior along this line could be provided by cluster percolation [20]. For spin systems without external field, the thermal magnetization transition can be equivalently described as a percolation transition of suitably defined clusters [21, 22]. We recall that a system is said to percolate once the size of clusters reach the size of the system (in the infinite volume limit). One can thus characterize the Curie point of a spin system either as the point where with decreasing temperature spontaneous symmetry breaking sets in, or as the point where the size of suitably bonded like-spin clusters diverges: the critical indices of the percolation transition are identical to those of the magnetization transition.

For non-vanishing external field $H$, there is no more thermal critical behavior; for the 2d Ising model, as illustration, the partition function now is analytic. In a purely geometric description, however, the percolation transition persists for all $H$, but the critical indices now are those of random percolation and hence differ from the thermal (magnetization) indices. For the 3d three state Potts’ model (which also has a first order magnetization transition), the resulting phase diagram is shown on the right of Fig. 7; here the dashed line, the so-called Kertész line [23], is defined as the line of the geometric critical behavior obtained from cluster percolation. The phase on the low temperature side of the Kertész line contains percolating clusters, the high temperature phase does not [24]. Comparing this result to the $T - m_q$ diagram of QCD, one is tempted to speculate that deconfinement for $0 < m_q < \infty$ corresponds to the Kertész line of QCD [24]. First studies have shown that in pure gauge theory, one can in fact describe deconfinement through Polyakov loop percolation [26, 27]. It will indeed be interesting to see if this can be extended to full QCD.
5. Summary

We have seen that at high temperatures and vanishing baryon density, hadronic matter becomes a plasma of deconfined colored quarks and gluons. In contrast, at high baryon densities and low temperatures, one expects a condensate of colored diquarks. The quark-gluon plasma constitutes the conducting, the diquark condensate the superconducting phase of QCD.

For vanishing baryon density, the deconfinement transition has been studied extensively in finite temperature lattice QCD. In pure $SU(N)$ gauge theory (QCD for $m_q \to \infty$), deconfinement is due to the spontaneous breaking of a global $Z_N$ symmetry of the Lagrangian and structurally of the same nature as the magnetization transition in $Z_N$ spin systems. In full QCD, deconfinement is triggered by a strong explicit breaking of the $Z_N$ symmetry through an external field induced by the chiral condensate $\langle \bar{\psi} \psi \rangle$. Hence for $m_q = 0$ deconfinement coincides with chiral symmetry restoration.

For finite quark mass, $0 < m_q < \infty$, it does not seem possible to define thermal critical behavior in QCD. On the other hand, spin systems under similar conditions retain geometric cluster percolation as a form of critical behavior even when there is no more thermal criticality. It is thus tempting to speculate that cluster percolation will allow a definition of color deconfinement in full QCD as genuine but geometric critical behavior.

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