In the past few years, both experimental \[\text{[1,2]}\] and theoretical \[\text{[3,4]}\] work has emphasized the importance of pinning in type II superconductors. Although much of this work has focused on random defects, e.g. twin boundaries, the layered structure of the copper–oxide materials itself provides a non–random source of pinning \[\text{[5]}\]. At low temperatures, since the c-axis coherence length \(\xi_0 \approx 4\AA \lesssim s \approx 12\AA\), the lattice constant in this direction, vortex lines oriented in the ab plane are attracted to the regions of low electron density between the \(\text{Cu}_2\) layers. In this paper, we discuss the phase diagram of intrinsically pinned vortices in near perfect alignment with the layers. In contrast to previous work, we focus on the \(\text{Cu}_2\) layers. In this paper, we discuss the phase diagram of intrinsically pinned vortices in near perfect alignment with the layers. In contrast to previous work, we focus on the

To understand thermal fluctuations, we employ the boson mapping \[\text{[6]}\]. Consider an isolated flux line oriented along the \(a\)–\(b\) plane. For \(T \gtrsim 80K\), \(\xi_c \approx \xi_0(1 - T/T_c)^{-1/2} \gtrsim s\), and the intrinsic pinning barrier energy per unit length is \[\text{[6]}\]

\[U_p \approx 5 \times 10^2 \frac{\xi_0}{\gamma} \left(\frac{\xi_c}{s}\right)^{5/2} e^{-15.8\xi_c/s},\]

where the energy scale \(\xi_0 = (\phi_0/4\pi\lambda_{ab})^2\), with \(\phi_0\) the flux quantum and \(\lambda_{ab}\) the \(a\)–\(b\) plane penetration depth, and where \(\gamma = \sqrt{m_c/m_{ab}}\) is the anisotropy. With coordinates \(\hat{y} \parallel B \perp \hat{c}\), \(\hat{z} \parallel \hat{c}\), the free energy of the vortex (described by \(x(y)\) and \(z(y)\)) is \[\text{[3]}\]

\[F_v = \int_0^L dy \left\{ \frac{\xi_0}{2} \left| \frac{dx}{dy} \right|^2 + \frac{\xi_c}{2} \left| \frac{dz}{dy} \right|^2 - U_p \cos 2\pi z/s \right\},\]

where the stiffness constants obtained from anisotropic Ginzburg–Landau (GL) theory are \(\tilde{\xi}_0 = \xi_0/\gamma\) and \(\tilde{\xi}_c = \xi_0\gamma\) (see, e.g. Ref. \[\text{[3]}\]). Upon integrating out the \(x\)–displacement, the remainder of Eq. \[\text{[3]}\] maps to the Euclidean action of a quantum particle in a one–dimensional periodic potential. In the quantum–mechanical analogy, the particle tunnels between adjacent minima of the pinning potential, leading to the well–known completely delocalized Bloch wavefunctions even for arbitrarily strong pinning. The “time” required for this tunneling maps to the distance, \(L_{\text{kink}}\), in the \(y\)–direction between kinks in which the vortex jumps across one \(\text{Cu}_2\) layer. The WKB approximation gives

\[L_{\text{kink}} \sim s \sqrt{\frac{\xi_c}{U_p}} \sqrt{\frac{\xi_c U_p s}{k_B T}},\]

where \(k_B T\) plays the role of \(\hbar\). When the sample is larger than \(L_{\text{kink}}\) along the field axis, the flux line will wander as a function of \(y\), with

\[(|z(y) - z(0)|)^2 \sim D_y y,\]

where the “diffusion constant” \(D \approx s^2/L_{\text{kink}}\).

For \(\sqrt{\xi_c U_p s} \lesssim k_B T\), the pinning is extremely weak, and the WKB approximation is no longer valid. Instead, the diffusion constant \(D \approx k_B T/\tilde{\xi}_c\), as obtained from Eq. \[\text{[3]}\] with \(U_p = 0\). At much lower temperatures, when \(\xi_c \ll s\), the energy in Eq. \[\text{[3]}\] must be replaced by the cost of creating a “pancake” vortex \[\text{[3]}\] between the \(\text{Cu}_2\) planes. In this regime, \(L_{\text{kink}} \sim \xi_c / \xi_c \sim s/\xi_c k_B T\).

For \(T \approx 90K\), as in the experiments of Kwok et. \[\text{[6]}\], \(\xi_c / s \approx 2.3\), and Eq. \[\text{[3]}\] gives \(\sqrt{\xi_c U_p s / k_B T} \ll 1\), indicative of weak pinning and highly entangled vortices in the liquid state. The transverse wandering in this anisotropic liquid is described by a boson “wavefunction” with support over an elliptical region of area \(k_B T L_y / \sqrt{\xi_c} \perp \Delta x / \Delta z = \gamma \approx 5\) for \(\text{YBa}_2\text{Cu}_3\text{O}_7\) \((L_y \approx 1\text{mm})\) the sample dimension along \(\hat{y}\) \[\text{[3]}\].

To explain the observed transition, the interplay between inter–vortex interactions \[\text{[3]}\] and thermal fluctuations must be taken into account in an essential way. The experiments of Ref. \[\text{[3]}\] rule out conventional freezing, which is first order in all known three–dimensional cases. A vortex/Bose glass (VG/BG) transition also appears untenable, due to the purity of the sample (only six

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twin planes are present, and first order melting is seen for \( \theta > 1^\circ \). The dynamical scaling exponents are also inconsistent with VG/BG values. Instead, we postulate freezing into an intermediate “smectic” phase between the flux liquid (high temperature) and crystal/glass (low temperature). Such smectic freezing, as first discussed by de Gennes for the nematic–smectic A transition \([2]\), can occur via a continuous transition in three dimensions. Changes in the applied magnetic field tune the commensurability of the vortex system with the underlying periodic structure \([3]\).

Our analysis leads to the phase diagram shown in Fig.1. Upon lowering the temperature for \( H_c = 0 \) and a commensurate value of \( H_b \), the vortex liquid (L) freezes first at \( T_s \) into the pinned smectic (S) state, followed by a second freezing transition at lower temperatures into the true vortex crystal (X). When \( H_c \neq 0 \), tilted smectic (TS) and crystal (TX) phases also exist. The TS–L and TX–TL transitions are XY-like, while the TS–S and TX–X phase boundaries are commensurate–incommensurate transitions (CITs) \([4]\). At larger tilts, the TX–TS and TX–L lines merge into a single first order melting line. As \( H_b \) is changed, incommensurate smectic (IS) and crystal (IX) phases appear, again separated by CITs from the pinned phases, and an XY transition between the IS and L states. Note that the commensurate smectic order along the \( c \) axis is stable to point disorder because phonon excitations are massive (see below). This stability should increase the range of smectic behavior relative to the (unstable) crystalline phases when strong point disorder is present.

![FIG. 1. Phase diagram for the pure vortex system as a function of temperature, \( T \), and \( c \)-axis field, \( H_c \). The fuzzy lines indicate first order transitions. A similar phase diagram holds in the \( \delta H_b–T \) plane with the TX and TS phases replaced by IX and IS states, respectively. The merging of the IX–IS and IS–L lines need not occur in this case.](image)

We now proceed with the derivation of these results; further details will be given in Ref. \([1]\). The \( a,b \), and \( c \), crystallographic axes are parameterized by the coordinates \( x,y \), and \( z \), respectively. The CuO\(_2\) layers are thus perpendicular to the \( z = c \) axis. The magnetic field is primarily along the \( y = b \) direction. When \( H_c = 0 \) and \( H_b \) is tuned to commensuration, de Gennes’ freezing theory applies near \( T_s \). Smectic order appears as a density wave,

\[
n(r) \approx n_0 \Re \{ 1 + \Phi(r)e^{iqz} \},
\]

where \( n_0 \) is the background density, and \( q = 2\pi/a \) is the wavevector of the smectic layering (with wavelength \( a \)). The complex translational order parameter \( \Phi(r) \) is assumed to vary slowly in space. The superconductor is invariant under translations and inversions in \( x \) and \( y \), and has a discrete translational symmetry under \( z \rightarrow z + s \), where \( s \) is the CuO\(_2\) double–layer spacing. From Eq.5, these periodic translations correspond to the phase shifts \( \Phi \rightarrow \Phi e^{iqs} \). In the commensurate limit, \( m \) an arbitrary integer. The most general free energy consistent with these symmetries is

\[
F = \int d^3r \left\{ \frac{K}{2} |(\nabla - iA)r|^2 + \frac{r}{2} |\Phi|^2 + \frac{v}{4} |\Phi|^4 - \frac{g}{2} (\Phi^m + \Phi^{-m} + \cdots) \right\},
\]

where the coordinates have been rescaled to obtain an isotropic gradient term. The “vector potential” \( A \) represents changes in the applied field \( \delta H = \delta H_c \hat{y} + H_b \hat{z} \), with \( A_x = 0 \), \( A_y = qH_c/H_b \), and \( A_z = q\delta H_b/H_b \). The form of this coupling follows from the transformation properties of \( \Phi \). Additional interactions with long wavelength fluctuations in the density and tangent fields \([4]\) are irrelevant to the critical behavior \([1]\).

When \( \delta H = A = 0 \), Eq.5 is the free energy of an XY model with an \( m \)-fold symmetry breaking term. A second order freezing transition occurs within Landau theory when \( v > 0 \) and \( r \propto T - T_s \) changes sign from positive (in the liquid) to negative (in the smectic). The renormalization group (RG) scaling dimension, \( \lambda_m \), of the symmetry breaking term is known experimentally in three dimensions as \( \lambda_m \approx 3 - 0.515m - 0.152m(m-1) \) \([3]\). For \( m > m_c \approx 3.41 \), the field \( g \) is irrelevant (\( \lambda_m < 0 \)), and the transition is in the XY universality class. The magnetic fields used by Kwok et al. \([5]\) correspond to \( m = 9 \rightarrow 11 \) \([6]\), well into this regime. The static critical behavior is characterized by the correlation length exponent \( \nu \approx 0.671 \pm 0.005 \) and anomalous dimension \( \eta \approx 0.040 \pm 0.003 \) \([7]\).

Deep in the ordered phase \((r < 0)\), amplitude fluctuations of \( \Phi \) are frozen out. Writing \( \Phi = \sqrt{|r|/ve^{2\pi lu/a}} \), Eq.5 becomes, up to an additive constant,

\[
F_{\text{smectic}} = \int d^3r \left\{ \frac{\kappa}{2} (\nabla u - A)^2 - \tilde{g} \cos 2\pi u/s \right\},
\]

where \( \kappa = 4\pi^2|v|K/a^2v \), \( \tilde{g} = g(|r|/v)^{m/2} \), and the reduced vector potential is \( A = A/g \). The displacement field \( u \) describes the deviations of the smectic layers from their uniform state. The sine–Gordon term is an effective periodic potential acting on these layers. As is well known from the study of the roughening transition \([8]\), such a perturbation is always relevant in three dimensions. The smectic state is thus pinned at long distances.
(i.e. the displacements $u$ are localized in a single minima of the cosine) \cite{19}.

At lower temperatures, provided point disorder is negligible, the vortices will order along the $x$ axis as well \cite{20}. This is once again a freezing transition at a single wavevector, and is described by a Landau theory like Eq.\ref{eq:Landau} (with $g = 0$ since the modulating effect of the underlying crystal lattice is much weaker in the $x$ direction), with XY critical behavior.

Next, we consider the effects of $\delta H \neq 0$. When the fields are small, $A$ is an irrelevant operator in the smectic phase (as can be easily seen by replacing the periodic potential by a “mass” term $\propto u^2$). This implies that the smectic layers do not tilt under weak applied fields, i.e. $\partial (\partial y u) / \partial H_c |_{H_c=0} = 0$. The full susceptibility $c_{44} \equiv \partial (B_c) / \partial H_c |_{H_c=0}$ is obtained from the expression

$$B_c = \frac{K q}{H_b} \ln (\Phi^* \partial y \Phi) + \left( c_{44,0} - r'' |\Phi|^2 \right) H_c,$$

where $c_{44,0}$ is the tilt modulus obtained from anisotropic GL theory (without accounting for the discreteness of the layers) and $r'' \equiv 2 \partial^2 r / \partial H^2 |_{H_c=0}$. Eq.\ref{eq:Landau} has a simple physical interpretation. The first term is the contribution to $B_c$ from tilting of the layers (described by a phase rotation of $\Phi$). Transverse field penetration at fixed layer orientation contributes via the second term. Such motion arises microscopically from a non–zero equilibrium concentration of vortices with large kinks extending between neighboring smectic layers. Eq.\ref{eq:Landau} predicts a non–divergent singularity $c_{44}(T) - c_{44}(T_c) \sim |T - T_c|^{-\alpha}$ at the critical point, where $\alpha$ is the specific heat exponent. At low temperatures in the smectic phase this crosses over to the much larger value $c_{44} \approx \sqrt{\epsilon_{\perp} / U_p k_b T} \exp (E_R / k_b T)$, where the energy of a large kink is $E_R \approx \sqrt{\epsilon_{\perp} U_p m s}$.

As is well known from the study of the sine–Gordon model \cite{13}, a larger incommensurability can be compensated for by energetically favorable “solitons”, or walls across which $u \to u + s$. Solitons begin to proliferate when their field energy per unit area $\sigma_{\text{field}} \sim -\kappa A s$ exceeds their cost at zero field, $\sigma_0 \sim \sqrt{\kappa g q s}$ (estimated from Eq.\ref{eq:Landau}).

Physically, these solitons correspond to extra/missing flux line layers and walls of aligned “jogs” for $\delta H$ along the $b$ and $c$ axes, respectively. In the former case, this leads to an incommensurate smectic (IS) phase, whose periodicity is no longer a simple multiple of $s$. For $\delta H \parallel \hat{z}$, the solitons induce an additional periodicity along the $y$ axis. This tilted smectic (TS) phase has long range translational order in two directions \cite{21}. The analogous tilted crystal (TX) phase is qualitatively similar, but has long range order in 3 directions.

Unlike the corresponding CIT in two–dimensional adsorbed monolayers \cite{13}, we find that energetic interactions between widely separated solitons dominate over entropic contributions \cite{14}. The free energy density in the incommensurate phases is thus

$$f_{\text{soliton}} \sim -\frac{\sigma}{l} + \frac{\Delta}{l} e^{-l/w},$$

where $\sigma \equiv \sigma_{\text{field}} + \sigma_0 < 0$ is the total areal free energy of the soliton; $\Delta$ and $w$ set the energy and length scales of the soliton interactions. Minimizing Eq.\ref{eq:Landau} gives a soliton separation $l \sim w \ln (|\Delta|/|\sigma|)$ near the CIT.

As the temperature is increased within the IS or TS phases, the system melts into the liquid. The IS–L and TS–L transitions are described by Eq.\ref{eq:Landau} with $g = 0$ (as can be seen by reversing the dilation discussed earlier and discarding oscillatory contributions to $F$ proportional to $g$), and are thus XY–like.

The shape of the CIT phase boundary is of particular experimental interest. In the mean field regime, this is obtained from the condition $\sigma = 0$ as $\delta H \sim |r|^T$, with $T_{\text{MF}} = (m^2 - 2)/4$. By the usual Ginzburg criterion, mean field theory breaks down for $|r| \lesssim (k_b T v / K^{3/2})^2$. To determine the shape of the phase boundary in this critical regime, we follow the RG flows until $|r|$ is order one. Then $\delta H_R \sim \xi^{\lambda_g} \Delta H$ and $g_{\nu} \sim \xi^{\lambda_r} g_r$, with $\xi \sim |r|^{-\nu}$. Rotational invariance at the rescaled fixed point ($g = 0$) implies that the field exponent is exactly $\lambda_g = 1$ \cite{11}. Using these renormalized quantities, we find $T_{\text{crit}} = (|\lambda_{\nu} n| + 2) \nu/2 \approx 4.9 - 7.2$ for the fields used in Ref.\cite{14}. The IS–L and TX–L phase boundaries are non–singular and are determined locally by the smooth $\delta H$ dependence of $r$.

We next discuss transport measurements using a coarse–grained approach similar to flux line hydrodynamics \cite{14}. The time evolution of the fluxon density $n(r, t)$ is determined by the continuity equation

$$\partial_{\nu} n + \nabla \cdot j_n = 0,$$

where $\nabla \cdot \nabla = (\partial_x, \partial_y)$, and $j_n$ is the vortex current, determined by the constitutive equation \cite{14}

$$\Gamma j_n = -n \nabla \cdot \frac{\delta F}{\delta n} + n \partial_y \frac{\delta F}{\delta \tau} - \tau \partial_x \frac{\delta F}{\delta \tau} + n \mathbf{f},$$

where $\tau$ is the coarse–grained tangent density, $\Gamma$ is related to the Bardeen–Stephen friction coefficient $\gamma_{\text{BS}} = n_0 \Gamma$, and the driving force is

$$\mathbf{f} = \frac{\phi_0}{c} \mathbf{J} \wedge \mathbf{y} + \mathbf{\eta}.$$

Here $\mathbf{J}$ is the applied transport current density and $\mathbf{\eta}(r)$ is a random thermal noise. Upon projecting the critical smectic density modes near $\pm q \hat{z}$ out of Eq.\ref{eq:Landau}, we find

$$\gamma_{\text{BS}} \partial_t \Phi = -4 q^2 F_{\text{crit}} \frac{\delta F_{\text{crit}}}{\delta \Phi^*} - i \mu J_y \Phi - \eta,$$

where $\gamma_{\text{BS}}$ is the tilt modulus obtained from anisotropic GL theory (without accounting for the discreteness of the layers). The analogous tilted crystal (TX) phase is qualitatively similar, but has long range order in 3 directions.
where $\mu = \varphi_0 n_0 / e$ and $\tilde{\eta}(k) = ini_0 q n_2 (q^2 + k^2)$.

Remarkably, Eq.13 has the same form as the model E dynamics [23] for the complex “superfluid” order parameter $\Phi$, where now $J_x$ plays the role of the “electric field” in the Josephson coupling. The actual electric field is $E_x = j_x \varphi_0 / \mu$, leading via Eq.14 to

$$E_x \approx \frac{n_0 \varphi_0}{2 e c} \text{Im} \left[ \Phi^* \partial_x \Phi \right]$$

$$+(1 - |\Phi|^2 / 2) \left( \frac{B}{\hbar c} \right) \rho_{xx,n} J_x,$$

where $\rho_{xx,n}$ is the normal state resistivity in the $x$ direction, whose appearance in the last term follows from the relation $(n_0 \varphi_0 / c)^2 / \gamma_{BS} \approx (B / H_2) \rho_{xx,n}$. Eq.14 is interpreted in close analogy with Eq.8. The first term is the contribution to vortex flow from translation of the layers, while motion of equilibrium vortex kinks yields the second term. Such flow at “constant structure” is analogous to the permeation mode in smectic liquid crystals [23].

The presence of this defective motion implies a small but non-zero resistivity at the L–S transition. Near $T_s$, Eq.14 predicts a singular decrease of the form $\rho_{xx}(T) - \rho_{xx}(T_s) \sim (|T_s - T|)^{1 - \alpha}$. At lower temperatures (but still within the $S$ phase) transport occurs via two channels. The permeation mode gives an exponentially small linear resistivity $\rho_{xx} \sim \exp(-E_{Bk}/k_B T)$ (above $T_s$, single layer kinks give $\rho_{xx} \sim \exp(-E_{Bk}/k_B T)$, with $E_{Bk} \approx E_{Bk}/m$). Non-linear transport occurs via thermally activated liberation of vortex droplets, inside which $u$ (or $u_z$ in the crystal phase) is shifted by $s$. Balancing the soliton energy on the boundary with the Lorentz energy in the interior gives $E_{nl} \approx e^{-(J_s / B)}$, where $J_s \sim (c / B)^{5/4} (s / k_B T)^{1/2}$. A more detailed discussion of the full scaling form of $E(J)$ will be given in Ref. [11].

In the TS phase, net vortex motion along the $c$ axis occurs by sliding soliton walls along the $b$ direction. The resulting electric field is proportional to $J$ and the soliton density, leading to an additional contribution to the resistivity which vanishes at the CIT like $\rho_{xx}^{\text{soliton}} \sim \rho_{xx}^{\text{soliton}} / \ln(\Delta / \sigma_0)$. The situation in the IS phase is more complex, due to the possibility of a roughening transition for a single soliton wall, and will be deferred to Ref. [11].

Lastly, we consider the effects of weak point disorder, which enters the free energy as a random field $F_2 = \int d^3 \varphi_0 V_d(\varphi) \text{Re} \left\{ \Phi(\varphi) e^{i \varphi z} \right\}$, where $V_d(\varphi)$ is a quenched random white noise potential. Such a perturbation alters the universality classes of the phase transitions in Fig.1, and renders all but the L and S phases glassy [11]. The stability of the S phase to randomness (which takes the form $F_2 = \int d^3 \varphi_0 V_d(\varphi) / \sqrt{V_d(\varphi)} \cos 2 \pi (u + z) / a$ in this regime) is a consequence of the phonon “mass” $g / s^2$. The nature of the glassy phases and altered critical behavior will be discussed in Ref. [11].

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