Classical resolution of black hole singularities via wormholes

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Introduction. The blow up of curvature scalars or of components of the Riemann tensor is typically used in the literature to put forward the existence of space-time singularities. Indeed, it is common to use those divergences to argue that when curvature scalars get close or surpass Planckian scales it is time to replace Einstein’s theory by an improved description, opening a door to quantum theories of gravity. This intuitive view has even shaped numerous approaches in the search of nonsingular space-times by trying to build theories with bounded curvature scalars [1–6]. However, from a formal perspective, the most widely accepted criterion to characterize a singular space-time is through the existence of incomplete paths (inextendible time-like or null geodesics) [7–9] (see also [10] and references therein).

Given that geodesics are classical geometrical entities associated with idealized (structureless) physical observers, it is unclear what a quantum theory of gravity should say about them. The focus is thus typically turned back to curvature divergences as one of the disturbing elements an improved theory of gravity should get rid of. A combined approach to better understand the correlation observed between the incompleteness of geodesics and the appearance of curvature divergences consists on exploring the impact of curvature divergences on physical observers represented by geodesic congruences. This has led to a classification of the strength of curvature divergences [11–15] according to whether an object going through them is crushed to zero volume or ripped apart by infinite tidal forces (strong case), or if it is not severely affected (weak case). In the end, however, one must accept that, to the best of our knowledge, matter and energy have quantum properties and a fundamental wave-like behavior, which demands for a characterization of curvature divergences and space-time singularities by means of quantum scattering experiments [10].

In this work we provide an example of black hole space-time in which a wormhole gets rid of the central singularity without necessarily avoiding curvature divergences. This geometry is an exact electrovacuum solution of certain high-energy extensions of general relativity (GR) formulated in metric-affine spaces (see [17, 19] for details). The requirement of a metric-affine geometry is essential to avoid ghosts and higher-order derivative equations. The resulting space-time roughly consists on two copies of the Reissner-Nordström or Schwarzschild black hole solution, connected by a wormhole in a small region near the center. This wormhole is supported by an electric field and, as such, satisfies all the classical energy conditions.

Incomplete geodesics that in GR end (or start) at the central singularity can now go through the wormhole, thus avoiding incompleteness [20]. The wormhole throat, however, generically exhibits divergent curvature scalars. Nonetheless, we find that physical observers (described as time-like congruences) do not experience any pathological behavior upon crossing the wormhole [21]. We also find that a scalar field propagating in this background is everywhere well-behaved, and compute the transmission coefficients and cross section of scalar waves as they interact with the wormhole [22]. The resolution of black hole singularities in this model provides new insights on how this disturbing aspect of classical gravitation may be cured at high energies.

Background geometry and geodesics. Our geometry is described by the line element (see [17, 19] for details)

\[
\begin{align*}
    ds^2 &= -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2(x)dx^2 ,
\end{align*}
\]

(1)

Depending on particular parameters two copies of Minkowski are also possible.
with \( B(r) = A(r)(1 + r_4^4/r^4)^2, \) \( d\Omega^2 = d\theta^2 + \sin \theta^2 d\varphi^2, \)

\[
r^2(x) = \frac{x^2 + \sqrt{x^4 + 4r_4^2}}{2}
\]

\[
A(r) \approx \begin{cases} \frac{1 - \frac{x}{r_c} + \frac{x^2}{r_c^2}}{N_c (\delta_1 - \delta_2)} \sqrt{\frac{1}{r - r_c}} + \frac{N_r - N_c}{2N_c} & \text{if } |x| \gg r_c \\ \frac{N_r - N_c}{2N_c} & \text{if } x \to 0 \end{cases}
\]

where \( x \in -\infty, +\infty \), but \( r(x) \geq r_c > 0 \). The wormhole throat is at \( r = r_c = \sqrt{r_4 q} \), \( l_4 \) is a length scale characterizing the departures from GR [17, 18], \( r_4^2 \equiv 2Gq^2 \) is the charge radius, \( N_q = |q/e| \) the number of charges, \( N_c \approx 16.55, r_S \) the Schwarzschild radius, \( \delta_1 \equiv (2r_S)^{-1} \sqrt{r_4^2/L_c} \) the charge-to-mass ratio, and \( \delta_c \approx 0.572 \). The GR geometry is quickly recovered as soon as \( |x| \) is slightly greater than \( r_c \) (recovering \( B(r) \approx A(r) \)) and also in the limit \( l_4 \to 0 \). As \( x \to 0 \), however, the area function \( r^2(x) \approx r_c^2 + x^2/2 \) reaches a minimum, and \( A(r) \) dramatically departs from GR, turning the Kretschmann scalar from \( R_{\alpha\beta\gamma\delta}^2 \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \approx 14r_4^4/r^6 \) in the Reissner-Nordström solution into \( \sim (\delta_1 - \delta_2)^2 K_2/(r - r_c)^3 + (\delta_1 - \delta_c) K_1/(r - r_c)^2 + K_0 \) in our case, with the \( K_1 \) constants.

If \( \delta_1 = \delta_c \), the metric and all curvature scalars are regular everywhere and, naturally, all time-like and null geodesics smoothly extend across the wormhole. In this case, if \( N_q > N_c \), the wormhole is hidden in between two event horizons symmetrically located around \( x = 0 \) \((r = r_c)\), connecting in this way two Schwarzschild-like geometries. If \( N_q < N_c \), there are no event horizons, and the wormhole connects two Minkowski-like spaces.

In terms of an affine parameter \( \lambda \), the geodesics satisfy \[1] (\( \kappa = 0, 1 \) for null and time-like, respectively)

\[
\frac{1}{(1 + r_4^2/x^2)} \left( \frac{dx}{d\lambda} \right)^2 = E^2 - A(r) \left( \kappa + \frac{L^2}{r^2(x)} \right),
\]

where \( E \) and \( L \) are constants which, in the time-like case, represent the total energy and angular momentum per unit mass. For null geodesics with \( L = 0 \), we get

\[
\frac{1}{(1 + r_4^2/x^2)} \left( \frac{dx}{d\lambda} \right)^2 = E^2,
\]

whose solution is plotted and compared with the GR prediction in Fig.\[1\]. Given that \[5\] is independent of \( \delta_1 \), all configurations (with or without curvature divergences) have an identical behavior, which confirms the extendibility of null geodesics across \( x = 0 \) in these space-times.

For null and time-like geodesics with \( L \neq 0 \), near the wormhole the effective potential

\[
V(x) = A \left( \kappa + \frac{L^2}{r^2(x)} \right)
\]

can be approximated as \( V(x) \approx -a/|x| \), with \( a = \left( \kappa + \frac{L^2}{r_c^2} \right) N\delta_1 - \delta_1 r_c \). One readily sees that if \( \delta_1 > \delta_c \)

\[
\lambda = x + 2x_0
\]

\[
\lambda = x
\]

\[
\lambda = x + 2x_0
\]

\[
\lambda = x
\]

**Figure 1.** Affine parameter \( \lambda(x) \) as a function of the radial coordinate \( x \) for radial null geodesics (outgoing in \( x > 0 \)). In the GR case (green dashed curve in the upper right quadrant), \( \lambda = x \) is only defined for \( x \geq 0 \). In this plot \( E = 1 \) and the horizontal axis is measured in units of \( r_c \).

(Reissner-Nordström-like case), an infinite potential barrier arises which prevents any geodesic from reaching \( x = 0 \). This behavior is also observed in GR, where such geodesics can never reach the singularity. In the Schwarzschild-like case, \( \delta_1 < \delta_c \), the potential becomes infinitely attractive as \( x \to 0 \) and turns \[4\] into

\[
\frac{d\lambda}{dx} \approx \frac{1}{2} \left( \frac{x}{a} \right)^{1/2} \to \lambda(x) \approx \frac{x}{3} \left( \frac{x}{a} \right)^{1/2},
\]

where \( \pm \) is for outgoing/ingoing geodesics. In the case of GR, time-like radial geodesics near the singularity behave as \( \lambda(r) \approx \pm \frac{x}{3} \left( \frac{x}{a} \right)^{1/2} \). Given that \( r > 0 \), ingoing/ outgoing geodesics in GR end/start at \( \lambda(0) = 0 \) and, therefore, are incomplete. In our case, on the contrary, the fact that \( x \in -\infty, +\infty \) allows to smoothly extend the affine parameter \( \lambda \) across \( x = 0 \) to the whole real axis. It is also worth noting that geodesics with \( L \neq 0 \) in GR have an ill defined angular velocity \( \frac{dx}{d\lambda} = L/r^2 \) as \( r \to 0 \), which would prevent their extension once \( r = 0 \) has been reached. In the wormhole case, given that \( r \to r_c \) at the throat, \( d\varphi/d\lambda \) there is finite, thus allowing a smooth extension across it.

**Congruences.** We have just seen that there are no restrictions for null, time-like or space-like geodesics to be extended across the wormhole (when they can reach it). Nonetheless, given that physical observers can be more accurately described as geodesic congruences, one should consider the impact of curvature divergences in the evolution of nearby geodesics. For this purpose, we consider the line element \[1\] written in coordinates adapted to an observer in free fall and study if causal contact between nearby time-like observers is lost at any time of the transit across the hole. In freely falling coordinates, \[4\] turns into

\[
ds^2 = -d\lambda^2 + (u^y)^2 d\xi^2 + r^2(\lambda, \xi) d\Omega^2,
\]
where $\lambda$ is the proper time (affine parameter), $\xi$ measures the radial separation between geodesics, and $u^\nu \equiv dy/d\lambda$, with $dy = dx/(1 + r^2/r^2(x))$. Focusing on the Schwarzschild-like case, which is the only configuration in which time-like geodesics can go through the wormhole, $(u^\nu)^2$ can be approximated as $(u^\nu)^2 \approx a/|x| \approx (\frac{3}{a}|\lambda - E\xi|)^{-\frac{2}{3}}$, which turns (8) into

$$ds^2 \approx -d\lambda^2 + \left(\frac{3}{a}|\lambda - E\xi|\right)^{-2/3} d\xi^2.$$  \hfill (9)

One should note that as the wormhole throat is approached, $\lambda - E\xi \rightarrow 0$, the physical spatial distance between any two infinitesimally nearby geodesics diverges: $dl_{phys} = \left(\frac{3}{a}|\lambda - E\xi|\right)^{-1/3} d\xi$ (see [21] for a more elaborate discussion of this point using Jacobi fields). However, for any finite comoving separation $\Delta \xi \equiv \xi_1 - \xi_0$, the physical spatial distance $l_{phys} \equiv \int |u^\nu| d\xi$ is always finite:

$$l_{phys} \approx \left(\frac{a}{3}\right)^{1/3} \frac{1}{E} \left|\lambda - E\xi_0\right|^{2/3} - \left|\lambda - E\xi_1\right|^{2/3}.$$  \hfill (10)

It is thus necessary to clarify if the constituents making up an object that reaches the wormhole lose causal contact because of the divergent spatial stretching that affects its infinitesimal elements (infinite tidal forces). In such a case, the interactions that keep the object cohesioned would no longer be effective, resulting in disruption or disintegration of the body. However, as shown in Fig[2] a fiducial observer at $\xi = 0$ never loses causal contact with its neighbors. Indeed, in [21] we show that the proper time a light ray takes in a round trip from $\xi = 0$ to any nearby geodesic is always finite as the wormhole is crossed. We can thus conclude that extended objects going through the wormhole, where curvature scalars generically diverge, do not undergo destructive deformations and, therefore, can effectively cross this apparently troublesome region.

**Scattering experiments.** As a third test to verify that curvature divergences do not affect the well-posedness of physical laws in our space-time (1), we consider the propagation of scalar waves in this background. According to the cosmic censorship conjecture, naked singularities should not exist in nature. This idea is traditionally invoked in the literature to interpret naked singularities as unphysical artifacts of Einstein’s theory. From this physical perspective, therefore, one can say that naked divergences are the worst-case scenario. From a technical viewpoint, this turns out to be the simplest situation because there exists a time-like Killing vector over the whole space, which facilitates the choice of coordinates and the separation of variables. Our approach is also valid for the region inside the inner horizon, where the wormhole is a time-like hypersurface. We will thus focus on configurations with $\delta_1 > \delta_c$ (Reissner-Nordström-like). Taking the scalar field equation $(\Box - m^2)\phi = 0$, we decompose it in modes of the form $\phi_{\omega,lm} = e^{-i\omega t}Y_{\omega,l}(\theta, \phi)f_{\omega,l}(x)/|r(x)|$, where $Y_{\omega,l}(\theta, \phi)$ represents spherical harmonics. Using the radial coordinate $y = \int dx/A(1 + r^2/r^4)$, the $f_{\omega,l}(x)$ are governed by a Schrödinger-like equation of the form

$$-f_{yy} + V_{eff}f = \omega^2 f,$$

where

$$V_{eff} = \frac{r_{yy}}{r} + A(r) \left( m^2 + \frac{l(l+1)}{r^2} \right).$$  \hfill (11)

This potential quickly converges to the GR prediction for $r \gg r_c$ but near $r = r_c$ diverges as $V_{eff} \approx k/|y|^{1/2}$, where

$$k \equiv \sqrt{\frac{\left(\delta_1 - \delta_c\right)N_q \left(N_c[m^2r^2 + 1 + l(l+1)] - N_q\right)}{\left(\delta_1 - \delta_c\right)N_c \left(8r_c^3\right)^{1/2}}} (12)$$

Low frequency modes are almost entirely reflected by the centrifugal barrier in much the same way as it happens in GR (see [10] for details). We thus focus on high-energy modes with sufficient energy to overcome this barrier and interact with the wormhole. Considering the $m^2 = 0$ case for simplicity, it is easy to see that $V_{eff}$ may transit from an infinite well to an infinite barrier as $l$ increases if $N_q > N_c$. The divergent barrier is reminiscent of the repulsive potential seen by geodesics in this same configurations. Despite the divergence of the potential as $y \rightarrow 0$, one can verify that the modes $f_{\omega,l}(x)$ are completely regular there. Using the mode decomposition, we thus define an incoming wave packet from past null infinity (in the naked case, or from the inner horizon in the black hole case) and study its interaction with the wormhole. As noticed above, the behavior will depend dramatically on the angular momentum of the incident mode.
transmission coefficient tending to 1 as \( \infty \) (infinite well), most of the wave is transmitted, with the \( k > 0 \) (infinite barrier), a sigmoid profile arises like in \( \sigma \).

The variation of \( \alpha \omega \) with \( \omega \) is illustrated in Fig. 3, which confirms that the transmission is near 1. For bigger \( l \), the wave changes sign as \( \omega \to \infty \), approaches \( 0 \) when \( k \) changes sign as \( l \) grows, \( \alpha \to \infty \) and the transmission is near 1. For bigger \( l \)'s, \( \alpha \) begins to decrease, and around the threshold \( \alpha \sim 1.5 \), characterized by \( l = l_{\text{max}} \), the partial waves change from being almost totally transmitted to being almost totally reflected. A rough estimate of the cross section can thus be obtained by considering that the transmission is 1 for partial waves with \( l \leq l_{\text{max}} \), and 0 for \( l > l_{\text{max}} \). The transmission cross section would thus be

\[
\sigma = \frac{\pi}{\omega^2} \sum_{l=0}^{l_{\text{max}}} (2l + 1) \left(1 + l_{\text{max}}\right)^2
\]

The generic absence of space-time singularities in these theories without requiring exotic matter-energy sources puts forward the existence of an important gap in our understanding of gravitation in metric-affine geometries. Interestingly, this type of geometries seem to play an important role in the description of continuum systems with a microstructure [24], such as Bravais crystals or graphene. This provides further (empirical) motivation for their study in gravitational scenarios.

**Summary and conclusions.** Two fundamental properties typically demanded from quantum theories of gravity (often seen as one and the same) are the removal of space-time singularities and the regularization of classical curvature divergences. We have shown here that only the former is physically relevant and can be achieved in classical geometric scenarios with nontrivial topology. Though our analysis has focused on a four-dimensional setting, our results are still valid in arbitrary dimensions, where exact analytical solutions with wormhole structure, and which are geodesically complete, have been found [23].

Higher dimensional models confirm that the resolution of the black hole singularities is a highly non-perturbative phenomenon, being the GR solution an excellent approximation down to the very throat of the wormhole, where the geometry quickly changes to account for this topological structure. We also note that the theories that generate the solution [14] also replace the big bang singularity by a cosmic bounce in a very robust manner [23, 26].

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