Next-to-leading order QCD corrections to
hadron+jet production in \( pp \) collisions at RHIC

Daniel de Florian

Departamento de Física
Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires
Pabellón I, Ciudad Universitaria
(1428) Capital Federal
Argentina

Abstract

We compute the next-to-leading order (NLO) QCD corrections to the spin-independent and spin-dependent cross sections for the production of a single-hadron accompanied by an opposite jet in hadronic collisions. This process is being studied experimentally at RHIC, providing a new tool to unveil the polarized gluon distribution \( \Delta g \). We perform a detailed analysis of the phenomenological impact of the observable at NLO accuracy and show that the preliminary data by the STAR collaboration confirms the idea of a small gluon polarization in the \( 0.05 \lesssim x \lesssim 0.3 \) range.

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I. INTRODUCTION

In the past two decades, measurements of the spin asymmetries $A_1^N$ ($N = p, n, d$) in longitudinally polarized deep-inelastic scattering (DIS) have provided new information on the spin structure of the nucleon. One of the most surprising results is that only a small fraction of the spin of the proton can be attributed to the spin of the quarks. The main goal of the spin program, besides obtaining the partonic share to the total spin of the nucleon, is the extraction of the full set of the $x$-dependent polarized quark ($\Delta q = q^1 - q^\dagger$) and gluon ($\Delta g = g^1 - g^\dagger$) densities of the nucleon. Many phenomenological analyses demonstrate, however, that available DIS data alone is not sufficient for this purpose. This is true in particular for $\Delta g(x, Q^2)$ since it contributes to DIS in leading-order (LO) only via the $Q^2$ dependence of $g_1$ (or $A_1$) which could not yet be accurately studied experimentally. As a result of this, it turns out that the $x$ shape of $\Delta g$ seems to be hardly constrained by the DIS data.

The precise extraction of $\Delta g$ thus remains one of the most interesting challenges for spin physics experiments. The RHIC collider at BNL, running in a proton–proton mode with longitudinally polarized beams provides the ideal tool for that purpose. The observables measured so far include single-pion production at center-of-mass energy $\sqrt{s} = 62$ and 200 GeV and jet production at $\sqrt{s} = 200$ GeV. Unlike DIS, those processes have a direct gluonic contribution already at the lowest order.

A next-to-leading order (NLO) global analysis that includes all available data from inclusive and semi-inclusive polarized deep-inelastic scattering, as well as from polarized proton-proton scattering at RHIC has been recently performed. The main outcome of the analysis is the indication of a rather small gluon polarization in the nucleon over the limited region of momentum fraction $0.05 \lesssim x \lesssim 0.2$.

Recently the STAR collaboration at RHIC has presented preliminary data corresponding to the 2006 run on a less inclusive observable, involving the production of a charged hadron accompanied by a back-to-back jet. From the pure experimental point of view, counting with an opposite jet allows one to use it as a trigger for the hadron, reducing the bias in the selection. Furthermore, having a more exclusive observable, and particularly counting with the transverse momentum of both the hadron and the jet, permits one to perform a more detailed study to extract $\Delta g$.

In order to make reliable quantitative predictions for a high-energy process, it is crucial to determine the NLO QCD corrections to the Born approximation. In general, in hadronic collisions,
cross sections computed at the lowest order in perturbation theory are severally affected by the dependence on the ‘unphysical’ factorization and renormalization scales, dependence that can be partially cured only by including the NLO corrections. Furthermore, the appearance of one extra final-state parton in the NLO from the $2 \rightarrow 3$ real corrections allows one to improve the matching between the theoretical calculation and the realistic experimental conditions, particularly when jets are present.

The calculation of the NLO QCD corrections to hadron+jet production by unpolarized and polarized hadrons is the purpose of this paper. Several modern versions of the subtraction method to calculate any infrared-safe quantity in unpolarized collisions are at present available in the literature [8, 9]. The formalism of Ref. [9] has been used in Ref. [10] to construct a Monte Carlo code that can calculate any jet infrared-safe observable in hadron–hadron unpolarized collisions and generalized, in Refs. [11, 12], to the polarized case. In [13], the method was extended to the case of single-hadron inclusive observables.

In the present paper, we apply the method of Refs. [9, 10, 13] to the case of hadron+jet observables. As a result, we will present a customized code, with which it will be possible to calculate any infrared-safe quantity corresponding to one-hadron+jet production to NLO accuracy, for both polarized and unpolarized collisions. With the technique introduced in [6] the code is suited to allow the inclusion of the observable in a global fit analysis.

With the code at hand, we analyze in detail the phenomenological implications of the observable. The key point here is that, being a more exclusive measurement, it is possible to analyze the data in terms of a new set of variables and, also, to impose experimental cuts that can enhance the contribution of some partonic subprocesses over others. That is fundamental in order to increase the sensitivity on the spin-dependent gluon distribution in polarized collisions.

This paper is organized as follows: in section 2 we describe the main ingredients of the calculation and study the perturbative stability of the NLO results, by looking at the scale dependence and ‘$K$-factors’. In section 3 we discuss some phenomenological aspects of hadron+jet production in hadronic collisions and in section 4 we present the results for the asymmetries at RHIC. Finally, section 5 contains the conclusions.
II. NLO CORRECTIONS AND PERTURBATIVE STABILITY

The factorization theorem [14] allows one to write the cross section for one-hadron production in hadronic collisions as

$$d\sigma_{pp \rightarrow h + jetX} = \sum_{f_1, f_2, f} \int dx_1 dx_2 dz \ f_1^{H_1}(x_1, \mu_{FI}) \ f_2^{H_2}(x_2, \mu_{FI}) \times d\hat{\sigma}_{f_1 f_2 \rightarrow f X'}(x_1 p_1, x_2 p_2, p_h/z, \mu_{FI}, \mu_{FF}, \mu_R) \ D_f^h(z, \mu_{FF}^2) \times S(p_i)$$

(1)

where, $H_1$ and $H_2$ are the colliding particles with momentum $p_1$ and $p_2$, respectively, $h$ is the outgoing hadron with momentum $p_h$ and the sum in Eq.(1) runs over all possible initial and final partonic states. The parton distributions $f_{H_i}^H$ are evaluated at the factorization scale $\mu_{FI}$, the fragmentation functions at the scale $\mu_{FF}$ [23] and the coupling constant, appearing in the perturbative expansion of the partonic cross section, at the renormalization scale $\mu_R$. The measurement function $S(p_i)$ accounts for possible experimental cuts applied to the cross section, and in this particular case, for the definition of the jet in terms of the kinematics of the final-state partons. The analogous of Eq.(1) for polarized cross section is obtained by replacing the parton distributions and the partonic cross section by its polarized expressions, $\Delta f_i^{H_i}$ and $d\Delta\hat{\sigma}_{f_1 f_2 \rightarrow f X'}$, respectively.

As usual, the (longitudinally polarized) asymmetry is defined by the ratio between the polarized and unpolarized cross sections

$$A_{LL}^h = \frac{d\Delta\sigma}{d\sigma}.$$

(2)

In order to evaluate the QCD corrections to the process we rely on the version of the subtraction method introduced and extensively discussed in Refs. [9, 10], and in Ref. [11]. We refer the reader to those references for the details. The implementation is performed in a MonteCarlo like code, profiting from the one available for the computation of single-hadron production in Ref. [13]. Since the calculation in [13] provides the full kinematics for the final-state partons it is possible, after some modifications, to build the jet kinematics in terms of them and obtain the required cross section. It is worth noticing that the same code computes both unpolarized and polarized cross sections, since the formal structure of the corrections is exactly the same.

In this work, we concentrate on the phenomenology of pion production accompanied by a back-to-back jet for the kinematics of the STAR experiment at RHIC with a center-of-mass energy of $\sqrt{S} = 200$ GeV. Unless otherwise stated, we require the pion transverse momenta to be larger than 2 GeV and the one for the jet to $10 \text{GeV} < p_T^{jet} < 25$ GeV. The rapidities of the pion and the jets are limited to the range $|\eta| < 1$. They have to be separated in the azimuthal angle.
by $\Delta \phi \equiv |\phi^\pi - \phi^{jet}| > 2$ to ensure the pion and jets are produced from ‘opposite-side’ partons. Finally, the jets are defined according to the cone algorithm with $R=0.7$.

![Graph](image)

**FIG. 1:** Unpolarized (solid) and polarized (dashes) NLO K factors. The choice of the factorization and renormalization scales corresponds to $\mu_F = \mu_R = (p_T^\pi + p_T^{jet})/2$.

The size of radiative QCD corrections to a given hadronic process is often displayed in terms of a $K$-factor which represents the ratio of the NLO over LO results. In the calculation of the numerator of $K$, one obviously has to use NLO-evolved parton densities. As far as the denominator is concerned, a natural definition requires the use of LO-evolved parton densities. In the polarized case, a problem arises for such a definition: since the polarized pdfs are not as well constrained as the unpolarized ones, it might happen that quite different results for the $\Delta f$'s can emerge when the fit is performed at LO or at NLO. This is particularly enhanced by the fact that the polarized pdfs have nodes. Therefore, the $K$-factor for a given process, defined using LO parton densities in the denominator could be largely affected by the fact that some polarized densities are at present not well constrained. In order to avoid this problem, we define the $K$-factor as the ratio between the NLO and the ‘Born’ cross section, where the latest corresponds to the use of NLO-evolved parton densities (and two-loop expression for $\alpha_s$) when evaluating the lowest-order partonic cross sections in the denominator. Nevertheless, it is important to remember that the $K$-factor is not a physical quantity and just provides a number to ‘quantify’ the effect of the higher-order corrections.

In the unpolarized case, we use the MRST2002 parton distributions [15]. Differences of the
order of percent in the cross section are observed when more recent distributions are considered. Nevertheless, since the DSSV [6] distribution we use in the polarized case set is obtained from a global analysis that relies on the unpolarized MRST2002 set as a reference, we will restrict the analysis to the MRST2002 densities. For the fragmentation functions we rely on the DSS [16] set that provides full flavor and charge separation at NLO. Similar results for pions are obtained with the latest AKK set [17]. Considering that the process depends on two different hard scales, the transverse momentum of the hadron and the one of the jet, we define the ‘default’ scale to be the average of both of them, and, unless otherwise stated we set all the factorization and renormalization scales to $\mu_F = \mu_R = (p_T^\pi + p_T^{jet})/2$.

Fig. 1 shows the result for the unpolarized (solid) and polarized (dashes) ‘$K$—factors’ computed at the default scales in terms of $p_T^\pi/p_T^{jet}$, the ratio between the transverse momentum of the pion and the jet [7]. The relevance of this dimensionless ratio will become clear in the next section. As can be observed, the NLO corrections are sizable in the unpolarized case, ranging from 50% to more than 100% of the Born result. The corrections are smaller in the polarized case, as usual, resulting in an important decrease in the corresponding spin asymmetry. In that sense the situation is quite similar to the one found for single-hadron production [13, 18]. In some extreme kinematical regimes, the QCD corrections tend to be dominated by the contributions arising from soft-gluon emission that need to be resummed to all orders in the coupling constant $\alpha_s$ to allow for a quantitative study. It has been shown [19] that for inclusive single-hadron
production the resummation of the dominant terms is required in the case of fixed-target energy experiments, where the ‘K-factors’ largely exceed those found here, but not for a collider running at \( \sqrt{s} = 200 \) GeV. Particularly, the effect of the resummation over the corresponding asymmetries is rather small \(^{20}\). Considering that the dominant soft contributions for the hadron+jet observable originate from the same Sudakov form factors as in the inclusive case, we believe those effects can also be neglected in a first approach here.

A reliable error estimate on our NLO results requires some knowledge on the size of the uncalculated higher-order terms. The best we can do, before higher-order terms are computed, is to study the dependence of the full NLO results on the renormalization and factorization scales. Although physical observables are obviously independent of the scales, theoretical predictions do have such a dependence, arising from the truncation of the perturbative expansion at a fixed order in the coupling constant \( \alpha_s \). A large dependence on the scales, therefore, implies a large theoretical uncertainty. In order to show how the scale dependence is substantially reduced once the next-to-leading order corrections are included we will compare to the Born result. For the sake of presentation we set all the scales to be equal, and vary them by a factor of 2 up and down with respect to the default choice, i.e., \( \mu_F = \mu_R = \mu \left( p_T^\pi + p_T^{jet} \right) / 2 \) with \( \mu = 1/2, 1, 2 \). Fig. 2 plots the corresponding scale dependence of the unpolarized (left) and polarized (right) cross sections, were we observe a considerable reduction when the NLO corrections are included. Nevertheless, it is worth noticing that the scale dependence is still rather large at NLO for the unpolarized cross section, of the order of ±20% or more (compared to about ±80% at the Born level). The scale dependence is much smaller in the polarized case, even reaching the stage in which at some kinematics the NLO cross section evaluated at \( \mu = 2 \) is larger than the one at \( \mu = 1/2 \), opposite to the LO expectation. Since the uncertainty in the unpolarized cross section directly contributes to the one for the asymmetry, one might consider the convenience of using directly \( \Delta\sigma \), instead of the asymmetry, to extract the polarized parton distributions with a considerably better theoretical accuracy.

III. PHENOMENOLOGY

As discussed in the Introduction, counting with the jet kinematics allows one to impose cuts that enhance the relevance of some kinematical region in the momentum fractions \( x_{1,2} \) and \( z \). That can be observed in Fig. 3 where we plot the average value of the momentum \( \langle x \rangle \) \(^{24}\) and hadronization \( \langle z \rangle \) fractions for unpolarized single-\( \pi^+ \) production (solid) and the corresponding
one for $\pi^+ + \text{jet}$ (dashes), in both cases in terms of the transverse momentum of the pion. While the averages are relatively similar for $x$, the change is quite clear for the fragmentation fraction $z$. Whereas for single-hadron production $\langle z \rangle \sim 0.6 - 0.7$ remains almost constant over the full kinematical range, it shows a large variation, from 0.2 to 0.8, when the opposite jet is required. The rise of $\langle z \rangle$ with the transverse momentum of the pion can be easily understood from simple physical considerations. At the Born level, only two final-state partons are produced, with opposite transverse momentum. For that kinematics, the pion is produced by the fragmentation of one of the partons, while the jet is just formed by the other one. Therefore, the ratio between the transverse momentum of the pion and the one of the jet is exactly the hadronization fraction $z$. Once a jet cut is applied, selecting the transverse momentum of the pion is equivalent to selecting the fraction of momentum that is transferred from the parton in the hadronization process. It is worth noticing that at the Born level, counting with the jet and hadron kinematics allows one to fully reconstruct all the momentum fractions as

$$z \equiv \frac{p_T^h}{p_T^{\text{jet}}},$$

$$x_1 \equiv \frac{(p_T^{\text{jet}} \exp(\eta_{\text{jet}}) + p_T^{\text{jet}} \exp(\eta_h))}{\sqrt{s}}$$

$$x_2 \equiv \frac{(p_T^{\text{jet}} \exp(-\eta_{\text{jet}}) + p_T^{\text{jet}} \exp(-\eta_h))}{\sqrt{s}}.\quad(3)$$

While those relations are not valid at NLO, since one more parton can be radiated, still one can observe that there is a strong correlation between the ‘real’ momentum fractions (the arguments of the parton distributions and fragmentation functions in Eq.(1)) and those obtained from the

FIG. 3: Average of the partonic momentum fractions $x$ and $z$ for single pion production (solid) and pion accompanied by a jet with $p_T^{\text{jet}} > 10$ GeV (dashes).
measured observables in Eq. (3). The correlations found for $\pi^+$ production in unpolarized collisions are plotted in Fig. 4. Considering a bin of size 0.05 (0.1) for $x$ ($z$), we find that about 90% (60%) of the generated (weighted) events in the MonteCarlo implementation of the NLO corrections give the same value for the ‘real’ and ‘measured’ momentum fractions, at least in the kinematical range where their contribution to the cross section is dominant.

The situation is also visible when the cross section is plotted in terms of the same variables, as shown in Fig. 5. In the dominant range of $0.05 \lesssim x \lesssim 0.3$, the agreement between the ‘measured’ cross section and the one obtained in terms of the ‘real’ momentum fraction is at the percent level. The use of the variables in Eq. (3) can therefore allow for an accurate reconstruction of the initial
state momentum fractions $x_{1,2}$. In the case of the $z$ distributions, the differences between the ‘real’

\[ |\eta| < 1 \]

and the ‘measured’ quantities can reach up to $10\% - 15\%$. However, it still becomes quite useful

![Graph](image)

**FIG. 6:** Contribution to the cross section from the $gg$ (solid), $qg$ (dashes) and $qq$ (dots) channels in terms of the transverse momentum of the pion (left) and the variable $z = \frac{p_T^\pi}{p_T^{jet}}$ (right).

![Graph](image)

**FIG. 7:** Contribution to the cross section from the $gg$ (solid), $qg$ (dashes) and $qq$ (dots) channels in terms of $x$

to plot the cross section in terms of it. This is mainly because, by selecting a range in $z$, one can enhance or decrease the contribution from some partonic channel due to the particular behavior of the fragmentation functions. This feature can be observed in Fig. 6 where we show the fractional contribution to the NLO unpolarized cross section from the $gg$, $qg$ and $qq$ initial state partonic
If the cross section is analyzed in terms of the transverse momentum of the hadron, as it happens for the single-inclusive case, the pure gluonic channel contribution $gg$ becomes only sizable at small values of $p_T^2$ and then decreases rapidly, making the cross section less sensitive on the gluonic content of the proton. The situation changes when the same results are studied in terms of the variable $z$. Here the $gg$ channel shows a larger fractional contribution over the entire kinematical range at expenses of a suppression of the pure quark channels, that at most account for only 20% of the cross section, providing an ideal scenario to extract $\Delta g$ in polarized collisions. Something similar occurs for the same observable plotted in terms of the variable $x$, as shown in Fig. 7. The analyses confirm that hadron+jet production in hadronic collisions, in terms of both dimensionless variables $x$ and $z$, provides a clear source of information on the gluon distribution. In the next section we will look directly at the correspondent sensitivity on $\Delta g$ in polarized $pp$ collisions.

IV. ASYMMETRIES AT RHIC AND SENSITIVITY ON $\Delta g$

In order to analyze the sensitivity of the process on the polarized gluon distribution, we will compute the NLO asymmetries with three different sets of spin-dependent densities: DSSV [6], GRSV (standard) [21], and GS-C [22]. The corresponding NLO distributions at $Q^2 = 50$ GeV$^2$, a typical scale for this process, are shown in the left-hand side of Fig. 8. As can be observed, the expectations from the three sets are quite different. While the DSSV distribution corresponds to

![Figure 8](image_url)

**FIG. 8:** Polarized gluon density at $Q^2 = 50$ GeV$^2$ from different sets of polarized pdfs (left) and their ratios to the unpolarized distribution (right).
the best fit from the latest global analysis of all polarized data\cite{6}, the GRSV set can be considered as an ‘upper bound’ for the allowed range of gluon densities. The GS-C set provides a distribution compatible with the requirement of a small gluon polarization in the range $0.05 \lesssim x \lesssim 0.3$ but with a node in that region and a very different behavior at smaller $x$ compared to the DSSV set. The right-hand side in Fig. 8 shows the ratio between the corresponding polarized distribution and the unpolarized MRST2002 set. In the quark sector, the dominant distributions are very similar among the different sets. Therefore, since the variations in the quark sector are much smaller than the ones for gluons, we can expect that any differences between predictions for the asymmetries that are found when using different polarized parton density sets are to be attributed to the sensitivity of the observable to $\Delta g$.

![Graph](image)

**FIG. 9:** Expected asymmetries for $\pi^-$ (left) and $\pi^+$ (right) production at RHIC from different sets of polarized pdfs in terms of $z$.

We start the presentation of the expected asymmetries by looking first at $\pi^-$ and $\pi^+$ production in terms of the variable $z$, as shown in Fig. 9. As expected, the asymmetries for the DSSV and GS-C distributions turn out to be small. When a set with a larger gluon distribution, like GRSV, is considered, the asymmetries increase to the 1%-2% level. The asymmetries for positive pions show a stronger sensitivity on the polarized gluon distribution at large $z$, in line with similar findings for single pion production at large transverse momentum\cite{13}. The differences and similitudes between negative and positive pion asymmetries can be easily understood: at small $z$, the ‘favored’ (like $u \rightarrow \pi^+$ and $d \rightarrow \pi^-$) and ‘unfavored’ (as $\bar{u} \rightarrow \pi^+$ and $\bar{d} \rightarrow \pi^-$) fragmentation functions are rather similar and therefore the cross section is almost charge invariant in that range. On the
contrary, at larger $z$, favored distributions overcome the unfavored ones. The asymmetries reflect the differences between the positive $\Delta u$ and negative $\Delta d$ parton distributions in the polarized proton that contribute with a different weight to the cross section. In this regime, the $\Delta u\Delta g$ channel becomes dominant for $\pi^+$ production resulting in a larger sensitivity on the polarized gluon density. A similar analysis can be performed in terms of the variable $x$, as defined in Eq. (3). Here, since $z$ is integrated out, one can expect closer results for $\pi^-$ and $\pi^+$ production. This is observed in Fig. 10, where both asymmetries reflect the shape and order of the curves for the $\Delta G/G$ ratio plotted in the right-hand side of Fig. 8. Notice that the sensitivity on the gluon distribution for $\pi^-$ production is increased when the asymmetry is analyzed in terms of the variable $x$ instead of $z$.

The STAR collaboration at RHIC has recently presented preliminary data on $\pi^+$+jet production. Even though the data has been obtained with very similar cuts to those used along this work, the experimental results can not be directly compared to the expected NLO asymmetries shown above because the data has not been corrected by the jet trigger efficiency. In order to allow for a comparison, we have recomputed the corresponding asymmetries by incorporating in the theoretical calculation the trigger efficiency parametrized as in Ref. [7] and modifying the experimental cuts accordingly. The only modification with respect to the previous cuts are those applying to the jet, which is required to have a transverse momentum in $9.5 \text{ GeV} < p_T^{\text{jet}} < 25 \text{ GeV}$ and rapidity in the range $-0.7 < \eta < 0.9$. The result is presented in Fig. 11. The main effect

![Figure 10: Expected asymmetries for $\pi^-$ (left) and $\pi^+$ (right) production at RHIC from different sets of polarized pdfs in terms of $x$.](image-url)

![Figure 11:](image-url)
FIG. 11: Asymmetries measured by STAR at RHIC for \( \pi^- \) (left) and \( \pi^+ \) (right) compared to the prediction from different sets of polarized pdfs. The theoretical predictions were corrected to account for the jet trigger efficiency.

of the trigger is to enhance the contribution from large \( p_T^{\text{jet}} \) with respect to the small \( p_T^{\text{jet}} \) events and, therefore, increase the average \( \langle x \rangle \) resulting in larger asymmetries.

With the present experimental accuracy it is not yet possible to perform a precise extraction of the polarized gluon density from this observable. Nevertheless, the data can already rule out any possible scenario with a large gluon polarization in the range \( 0.05 \lesssim x \lesssim 0.3 \). For that purpose we include in Fig. 11 the prediction from the set GRSV-max set, where the polarized gluon distribution is assumed to be equal to the unpolarized density at the very low initial scale of \( \mu^2 = 0.4 \) GeV\(^2\). That set completely overestimates the experimental data at small \( z \). Therefore, in line with other measurements performed at RHIC, the preliminary data confirms the results from the global analysis in [6] and points out to a small gluon polarization in the proton.

V. CONCLUSIONS

It is shown that the perturbative stability of the hadron+jet cross section improves considerably after including the NLO contributions. The corrections are found to be nontrivial: \( K \)-factors are larger for the unpolarized cross section than for the polarized one, resulting in a reduction of the asymmetry at NLO. The possibility of looking at charged pions accompanied by a back-to-back jet is studied phenomenologically in detail, finding that the asymmetries for \( \pi \) production, in terms
of both dimensionless variables $x$ and $z$, are sensitive to the polarized gluon density in the range $0.05 \lesssim x \lesssim 0.3$. Furthermore, we show to NLO accuracy that the available data collected by the STAR collaboration at RHIC for this observable confirms the idea of a rather small gluon polarization.

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[23] We chose equal factorization scales $\mu_{FF} \equiv \mu_{FI} \equiv \mu_F$
[24] The results are symmetric in the $x_1$ and $x_2$ distributions, we denote it generally by $x$
[25] We include all different quark-(anti)quark channels under the common label $qq$.
[26] The asymmetries are almost insensitive to the precise values of those cuts.
