Research Article
Input-to-State Stabilization of a Class of Uncertain Nonlinear Systems via Observer-Based Event-Triggered Impulsive Control

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This article concerns the problem of input-to-state stabilization for a group of uncertain nonlinear systems equipped with nonabsolutely available states and exogenous disturbances. To appropriately cope with these partially measurable state variables as well as dramatically minimize controller updating burden and communication costs, an event-triggered mechanism is skillfully devised and an observer-based impulsive controller with the combination of sample control is correspondingly presented. By resorting to the iterative method and Lyapunov technology, some sufficient criteria are established to guarantee the input-to-state stability of the newly uncertain controlled system under the employed controller, in which an innovative approximation condition as to the uncertain term is proposed and the linear matrix inequality technique is utilized for restraining sophisticated parameter uncertainties. Furthermore, the Zeno behavior in the proposed event-triggered strategy is excluded. The control gains and event-triggered mechanism parameters are conjointly designed by resolving some inequalities of linear matrix. Eventually, the availability and feasibility of the achieved theoretical works are elucidated by two simulation examples.

1. Introduction

Since it is originally put forward by [1, 2], input-to-state stabilization has caught widespread attention [3–5], attributing to its performance in characterizing dynamical systems reaction to exogenous disturbances with bounded magnitude. The property of input-to-state stabilization, crudely speaking, symbolizes that the system state will ultimately approach the origin neighborhood whose dimension is in direct proportion to the size of the system input regardless of the magnitude of the initial state. With this characteristic, a system is asymptotically stabilizable under disturbance-free condition and has the evolution of bounded state in the bounded perturbation circumstance. Indeed, input-to-state stability behavior can characterize robustness and stability on dynamic systems possessed disturbances, in which the corresponding stabilization problem has a greatsignality for the control issue of [5–7]. Input-to-state stability is incipiently introduced for continuous systems to evaluate dynamical behaviors, which is especially a fundamental conception for investigating robust dynamics on nonlinear systems influenced by noise, inputs, or interferences [8]. Afterwards, it is diffusely capitalized for stabilizing controller synthesis and stabilization analysis of diverse discontinuous systems, to name a few, switched systems [9, 10], stochastic systems [5, 11], and fuzzy systems [12, 13].

Accompanied by the prompt development of some technologies such as digital control for resource-limited models and sensors incorporated embedded microprocessors, event-triggered impulsive control strategy, more recently, has been highly valued. On the one hand, the impulsive system, composed of discrete dynamics and continuous dynamics, is an important hybrid system in which the uncontinuous behavior is a momentary state jump occurring at given moments, while the consecutive behavior is usually expressed as differential equation. Correspondingly, impulsive control is a control approach that the control signals are transmitted to a system only at certain moments. In comparison with continuous control [14, 15], it
has the advantages of only discrete control which is required for deriving the desired performance, discontinuity, and stronger robustness. Consequently, the control approach is extensively applied in practice, such as ecosystem management [16, 17], satellite orbit transfer [18], secure communication [19], pharmacokinetics [20], and complex switched network [21]. Furthermore, controlling the operation of systems all the while is unnecessary or even impossible in practice. In population model [16], for example, it is merited to release predators at appropriate discrete circumstances, rather than the continuous instances for controlling the amount of a category insect. Moreover, as [22] amply demonstrated, impulsive control allows utilizing small control impulses as much as possible to stabilize a type of chaotic system. Not merely does it reduce redundant information transmission, but it increases the robustness of disturbances rejection. On the other hand, event-based control, as the name implies, is the strategy that event is triggered by some elaborate state-based or output-based event conditions to update the control input, which compared to the conventional time-triggered control is capable of avoiding unnecessary communication since a system adjusts the sampling rate adaptively according to the current situation [23, 24]. Specifically, the issue of self-learning optimal supervision on discrete systems via event-driven formulation is investigated in [24]. And the critics learning standard is improved for the design of nonlinear $H_{∞}$ state-feedback control based on events [25]. Distinct from the extant achievements involving sectionally continuous or consecutive control inputs, event-triggered impulsive control is able to dramatically minimize communication load and communication cost as well as enhance robustness, which, for these reasons, is deserving increasing attention. Simultaneously, the integration of two control strategies also creates tremendous challenges in designing appropriate controller.

As yet, some (but few) significant accomplishments about the event-triggered impulsive control such as [26–29] have been reported. Taking [26] as an instance, the synchronization issue on multiple neural networks with disconnected switching topology and delay under this control strategy is studied. Nevertheless, systems are generally affected by some uncertain factors such as human error, random disturbance, information loss, inherent deviation, or environmental noise. The uncertainty caused by these factors is referred to as the parametric uncertainty that is perhaps foremost provenance of model uncertainty [30].

Without taking model uncertainty into account, it seems to be far-fetched and preposterous in reality for analyzing performances of various systems like estimating the property indexes on steady state. In this condition, none of the before-mentioned results are valid. Besides, in the control engineering application, when it comes to the fact that the system states may not be fully available because of implementation costs or physical restrictions, it becomes crucial and inevitable to formulate the event-triggered impulsive control strategy according to practical observer measurements. At this juncture, once the incomplete testability of states and the uncertainty of parameters are incorporated into the characterization of nonlinear systems, then these uncertainties may give rise to a totally new rule with more uncertain antecedents and results. What is exhilarating is that there is no work on the observer-based event-triggered impulsive control strategy to achieve the input-to-state property of uncertain nonlinear systems. After all, it is of more difficulty to find a feasible analytical framework compared with the nominal nonlinear systems. Moreover, in comparison with the previous methods, the robust handling for uncertain parameters during the course of system performance implementation becomes increasingly tricky as the number of uncertain parameters surges. Therefore, the theoretical challenges and technical deficiencies urge us to explore the actual performance evaluation for nonlinear systems with parametric uncertainties under observer-based control.

The abovementioned analysis motivates us to focus on issues of both input-to-state stability and event-triggered impulsive control scheme design on a type of uncertain nonlinear systems with incomplete measurable state variables and exogenous disturbances in this paper. Firstly, we establish a category of newly uncertain nonlinear systems, where the uncertainty terms are legitimately estimated by capitalizing on a creationary approximation condition of uncertainties, matrix synthesis method, and some inequalities of linear matrix. Secondly, a novel observer is constructed on the uncertain nonlinear system, in which the information between plant and observer is transmitted as impulses. In particular, the impulsive controllers are dependent upon the partial measurement output of observer and plant, which can eliminate the adverse effects of output data loss attributed to the external environment. Thirdly, an applicable observer-based event-triggered mechanism is designed and an event-triggered impulsive control strategy is correspondingly constructed, which could lessen burden of sampling and information transmission. At last, several sufficient criteria on excluding the Zeno behavior and analyzing the input-to-state stability property are developed, meanwhile, which suggest that a more extensible framework in complex dynamics can be explored through taking full advantage of a range of the employed ideas and methods.

The content of the remaining sections is summarized as follows. Section 2 puts forward the model and preparatory works for a kind of uncertain nonlinear systems. Section 3 furnishes primary research results. In addition, Section 4 corroborates the validity of the derived results by two numerical simulations. Finally, conclusion is exhibited in Section 5.

2. Preliminaries and Model Description

2.1. Notations. Throughout this article, $\mathbb{R}^{p \times p}$, $\mathbb{R}^{p}$, and $\mathbb{N}^{+}$ are separately the set of all $q \times p$ real matrices and $q$-dimensional Euclidean space and the set of positive integers. $I$ stands for an identity matrix with matched dimensionality in matrices or matrix inequalities. $I$ stands for an identity matrix with matched dimensionality in matrices or matrix inequalities. $0$ in matrices is a zero matrix of appropriate dimensions. Let $\|\mathcal{D}\|$ and $\|\mathcal{D}\|_{p}$ denote the 2-norm of matrix $\mathcal{D}$ and the supremum of $\|\mathcal{D}\|$ on the interval $\delta$, respectively. For a matrix $\mathcal{D}$, $\mathcal{D}^{-1}$,
\( \mathcal{D}^T \), \( \lambda_{\text{max}}(\mathcal{D}) \), and \( \lambda_{\text{min}}(\mathcal{D}) \) represent several its inverse, transposition, maximum eigenvalue, and minimum eigenvalue. The symbol \( * \) is defined as the symmetric term in a matrix. \( s/vd \) and \( s/vd \) represent the maximum and minimum of \( s \) and \( d \), respectively. \( \mathcal{D} > 0 \) and \( \mathcal{D} < 0 \) mean that \( \mathcal{D} \) are symmetric positive definite and symmetric negative definite separately. Let \( He(\mathcal{D}) = \mathcal{D} + \mathcal{D}^T \). \( \mathcal{X} = \{ \phi \in \mathcal{C}(\mathbb{R}^+, \mathbb{R}^n) \mid \phi(0) = 0, \lim_{t \to \infty} \phi(s) = \infty \) and \( \phi(s) \) is strictly increasing in \( s \), and \( \mathcal{Y}^T = \{ \psi \in \mathcal{C}(\mathbb{R}^n \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \) \} \psi(s, j), for each fixed \( j \geq 0 \), belongs to the function of class \( \mathcal{X} \) as regards \( s \), but \( \psi(s, j) \), for each fixed \( s \geq 0 \), is strictly decreasing to \( 0 \) as \( j \to \infty \).

2.2. Some Preliminaries and Problem Formulation. A class of uncertain nonlinear systems incorporating exogenous disturbances is of the following form:

\[
\begin{align*}
\dot{x}(t) &= (B + \Delta B)x(t) + (B_d + \Delta B_d)f(x(t)) + (A + \Delta A)u(t) + (C + \Delta C)v(t), \\
y(t) &= (D + \Delta D)x(t),
\end{align*}
\]

in which \( t \geq t_0 \), \( x(t) \in \mathbb{R}^n \), \( y(t) \in \mathbb{R}^p \), and \( v(t) \in \mathbb{R}^n \) are the system state, the measurement output, and measurable locally bounded exogenous disturbances, respectively; \( u(t) = u_t(t) + u_s(t) \) means the control input in which \( u_t(t) \) is the sample control input and \( u_s(t) \) is the Dirac delta control input; a nonlinear vector-valued function \( f: \mathbb{R}^n \to \mathbb{R}^n \) satisfies some conditions that will be provided in the sequel, and \( \dot{x}(t) \) represents the right-hand derivative of \( x(t) \). \( B, B_d, A, C, \) and \( D \) are constant matrices, and \( \Delta B, \Delta B_d, \Delta A, \Delta C, \) and \( \Delta D \) are the norm-bounded uncertain parameters.

Given that the incompletely procurable system states can generate the ineffectiveness of state-feedback controllers, an observer-based controller is considered in this paper, and the state observer for uncertain system (1) is constructed by

\[
\begin{align*}
\dot{x}(t) &= Bx(t) + B_d f(x(t)) + Au(t) + Cv(t), \\
y(t) &= Dx(t),
\end{align*}
\]

where \( t \geq t_0 \); \( x(t) \in \mathbb{R}^n \); and \( y(t) \in \mathbb{R}^p \) are the estimated state and the estimated output. The control input \( u_2(t) \) of observer is described as

\[
u_2(t) = \sum_{a=1}^{\infty} \lambda(t) + K\gamma(t) - \mathcal{K}\gamma(t))\delta(t - t_a),
\]

where \( t \in [t_a, t_{a+1}) \); \( K \) and \( \mathcal{K} \) are control gains; \( \delta \) is the Dirac delta function, which is also called the impulsive control function. And the impulsive time sequence \( \{t_a\}_{a=1}^{\infty} \) satisfies \( t_1 < t_2 < \cdots < t_a < \cdots \) and \( \lim_{a \to \infty} t_a = +\infty \). It is well-known that the Dirac delta function has two properties: for any constants \( c \) and \( \Delta > 0 \) and function \( g(t) \), \( (1) \delta(t - c) = 0 \) only when \( t \neq c \); \( (2) \int_{c-\Delta}^{c+\Delta} g(t)\delta(t - c) dt = g(c) \). Then, by virtue of (2) and (3) and the properties of function \( \delta \), what we can see is that \( u_2(t) = 0 \) at \( t \neq t_a \) and an \( a \in \mathbb{N}^+ \), and for any constant \( \Delta(> 0) \) that is small enough,

\[
\mathcal{X}(t_a + \Delta) - \mathcal{X}(t_a - \Delta) = \int_{t_a}^{t_a+\Delta} [Bx(t) + B_d f(x(t)) + Cv(t)] dt
\]

\[
+ \int_{t_a}^{t_a+\Delta} \sum_{a=1}^{\infty} \lambda(t) + K\gamma(t) - \mathcal{K}\gamma(t)\delta(s - t_a) ds
\]

\[
+ A(K\gamma(t_a) + K\gamma(t_a) - \mathcal{K}\gamma(t_a)),
\]

where \( \int_{t_a}^{t_a+\Delta} \sum_{a=1}^{\infty} A(K\gamma(t) + K\gamma(t) - \mathcal{K}\gamma(t)) \delta(s - t_a) ds \), which can be regarded as the convolution in the interval \([t_a - \Delta, t_a + \Delta]\) based on the properties of function \( \delta \), represents the sum of the effects of all unit impulses on the observer state over \([t_a - \Delta, t_a + \Delta]\).

Let \( \Delta \to 0 \) and \( \Delta x(t) = x(t) - \mathcal{X}(t_a) \); then we can infer that

\[
\Delta x(t) = A(K\gamma(t_a) + K\gamma(t_a) - \mathcal{K}\gamma(t_a)),
\]

where \( \Delta x(t) \to \mathcal{X}(t_a) \); and all the impulsive effects can be regarded as the convolution in the interval \([t_a - \Delta, t_a + \Delta]\). It is well-known that the convolution in the interval \([t_a - \Delta, t_a + \Delta]\) can be considered in the sum of the effects of all unit impulses on the observer state over \([t_a - \Delta, t_a + \Delta]\).

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\begin{align}
\dot{e}(t) &= B(e(t) - f(x(t))) + (A + \Delta A)\mathbf{x}(t) \\
&\quad + (C + \Delta C)v(t) + \Delta Bx(t) + \Delta B_d f(x(t)), \quad t \neq t_a, \\
\Delta e(t) &= (\Delta AKD - \Delta K\Delta D)e(t^-) \\
&\quad + (\Delta AKD - \Delta K\Delta D)x(t^-), \quad t = t_a. \\
\end{align}

(9)

**Remark 1.** Since uncertain system (1) is a category of impulsive systems, coupled with the incomplete measurability of the system state, we need to construct an appropriate observer and subsequently establish an applicable error system related to plant and observer. Based on the prerequisite of ensuring real-time monitoring, fault-tolerant control, easy realization, and so on, as a result, it is necessary and natural to construct observer (2) which is only influenced by impulsive. One more point needs noting that the controllers \(u_2(t)\) and \(u_3(t)\) designed by us can exert positive effects on the unstable systems and meanwhile control them only at the impulsive instant. In this way, \(u_2(t)\) and \(u_3(t)\) can stabilize systems (1) and (2), respectively, for ages, while reducing unnecessary computing costs.

Let

\begin{align}
\varphi(t) &= (x^T(t), e^T(t))^T, \\
\zeta(t) &= (v^T(t), v^T(t))^T, \\
\mathcal{F}(t) &= (f^T(x(t)), f^T(x(t)) - f^T(\mathbf{x}(t)))^T,
\end{align}

(10)

and then the argument system can be deduced as

\begin{align}
\dot{\varphi}(t) &= \bar{B}\varphi(t) + \bar{B}_d\mathcal{F}(t) + \bar{A}\bar{L}\varphi(t_a) + \bar{C}\zeta(t), \quad t \neq t_a, \\
\varphi(t) &= \bar{K}\varphi(t^-), \quad t = t_a,
\end{align}

(11)

where

\begin{align}
B &= \begin{bmatrix} B + \Delta B & 0 \\ \Delta B & B \end{bmatrix}, \\
\bar{B}_d &= \begin{bmatrix} B_d + \Delta B_d & 0 \\ \Delta B_d & B_d \end{bmatrix}, \\
\bar{A} &= \begin{bmatrix} A + \Delta A & 0 \\ 0 & A + \Delta A \end{bmatrix}, \\
\bar{L} &= \begin{bmatrix} L & -L \\ L & -L \end{bmatrix}, \\
\bar{C} &= \begin{bmatrix} C + \Delta C & 0 \\ 0 & C + \Delta C \end{bmatrix}, \\
\bar{K} &= \begin{bmatrix} I + (A + \Delta A)KD & -(A + \Delta A)KD \\ \Delta AKD - \Delta K\Delta D & I - \Delta AKD - \Delta K\Delta D \end{bmatrix}.
\end{align}

(12)

Furthermore, \(\forall t \in [t \!, t_{a+1}], a \in \mathbb{N}^+\), argument system (11) is rewritten as

\begin{align}
\dot{\varphi}(t) &= A_1\varphi(t) + A_2\mathcal{F}(t) + A_3\varphi(t) + A_4\zeta(t), \quad t \neq t_a, \\
\varphi(t) &= \bar{K}\varphi(t^-), \quad t = t_a,
\end{align}

(13)

where \(\varphi(t) = \varphi(t_a) - \varphi(t)\);

\begin{align}
A_1 &= \begin{bmatrix} B + \Delta B + (A + \Delta A)L & -(A + \Delta A)L \\ \Delta B + (A + \Delta A)L & B - (A + \Delta A)L \end{bmatrix}, \\
A_2 &= \bar{B}_d = \begin{bmatrix} \bar{B}_d & 0 \\ \Delta B_d & \bar{B}_d \end{bmatrix}, \\
A_3 &= \bar{A}\bar{L} = \begin{bmatrix} (A + \Delta A)L & -(A + \Delta A)L \\ (A + \Delta A)L & -(A + \Delta A)L \end{bmatrix}, \\
A_4 &= \bar{C} = \begin{bmatrix} C + \Delta C & 0 \\ 0 & C + \Delta C \end{bmatrix}.
\end{align}

(14)

An adaptive event-triggered mechanism, determining the continuously updated controller works at the instants \(\{t_a | a \in \mathbb{N}^+\}\) known as the triggered time sequence, is introduced to decrease the burden of updating and communication in control. It is notable that the system states are imperfectly accessible, so the event-triggered mechanism included exogenous disturbances as well as the system and observer output is designed. By defining

\begin{align}
\mathfrak{S}(t) &= (y^T(t), (y(t) - \bar{y}(t))^T)^T, \\
\bar{D} &= \begin{bmatrix} D + \Delta D & 0 \\ \Delta D & D \end{bmatrix}, \\
\bar{g}(t) &= \bar{D}\varphi(t),
\end{align}

(15)

the event-triggered mechanism is formulated as

\begin{align}
t_{a+1} = t_a + \ell, \\
t_{a+1} = \inf \{t \geq t_a | \mathfrak{S}(t) \geq 0\},
\end{align}

(16)

where the event generator function \(\mathcal{H}(t) = \|\bar{\mathfrak{S}}(t)\|^2 - \eta\mathfrak{S}(t_a)\|^2 - \rho\|\zeta(t)\|^2\), \(t \in [t_a, t_{a+1}]\); parameters \(\eta > 0\), \(\rho > 0\), and \(\ell \in (0, \infty)\) is a forced triggered constant. Denote by \(\{t_{a+1}^1 | a \in \mathbb{N}^+\}\) the event-triggered time sequence that it is determined by function \(\mathcal{H}(t)\). For \(t \geq t_{a+1}(a \in \mathbb{N}^+)\), the next event \(t_{a+1}^1\) will be triggered only when the cumulative measurement reaches or surpasses the stated threshold, and then, the next triggered instant (impulsive instant) will be generated by comparing the obtained event-triggered time with the forced triggered time. In addition, it is worth mentioning that the aforesaid two sequences may differ depending on the selected parameters \(\ell\), \(\eta\), and \(\rho\).

In what follows, two assumptions are proposed around the uncertain terms and the nonlinear function.

**Assumption 1**

\begin{align}
[\Delta B \Delta B_d \Delta A \Delta C \Delta D] &= ME(t)[F_1 F_2 F_3 F_4 F_5],
\end{align}

(17)
in which $E(t)$ is the unknown time-varying matrix with $a^1 \leq E^T(t)E(t) \leq I$, the adjustment coefficient of the uncertain term $\alpha \in (0,1]$, and $F_1, F_2, F_3, F_4$, and $F_5$ are constant matrices with compatible dimensionality.

Remark 2. Different from existing achievements, such as [21, 30, 31], this paper has more uncertain parameters. Note that, in the practical application, each program will inevitably be subjected to the actual limitation of imprecise modeling for controlled plant and affected by external factors like environmental noise. Therefore, it is favorable and urgent for increasing the number of uncertainties to describe a larger range nonlinear system.

Remark 3. Only the norm-bounded uncertainties are taken into account in this article to efficaciously avoid needlessly intricate notations and restrain parameter uncertainty. In accordance with Assumption 1 and several linear matrix inequalities, the uncertainties $\Delta B, \Delta A, \Delta C,$ and $\Delta D$ can be reasonably eliminated. Moreover, compared with the conventional constraint conditions of uncertain terms, the adjustment coefficient $\alpha$ is added in this paper, which not merely does not change the norm value range of the uncertain terms but also can ingeniously resolve the input-to-state stability problem of fairly sophisticated system. Even though the uncertainty parameters are also present at other singular structures, the subsequent results could be popularized to this circumstance in parallel.

Assumption 2. Suppose that there exists a scalar $\beta > 0$ such that the nonlinearity $f$ satisfies $|f(y_1) - f(y_2)| \leq \beta |y_1 - y_2|$, $\forall y_1, y_2 \in \mathbb{R}^n$. Particularly, $f(0) = 0$

Hereafter, a definition and several lemmas are introduced for latter use.

Definition 1. For every initial condition $(t_0, \varphi_0)$ and each measurable locally bounded exogenous disturbance $\zeta(t)$ (see [1]), system (13) is said to be input-to-state stabilizable under the given event-triggered mechanism (16) if there exist functions $\Psi \in \mathcal{K}$ and $\mathcal{I} \in \mathcal{K}_{\infty}$ such that the solution $\varphi(t)$ satisfies

$$\|\varphi(t)\| \leq \Psi(\|\varphi_0\|, t - t_0) + \mathcal{I}(\|\zeta(t)\|_{[t_0, t]}), \quad t \geq t_0. \quad (18)$$

Lemma 1. Given constant matrices $\mathcal{U}, \mathcal{P}$, and $\mathcal{V}$ with suitable dimensionality and a matrix function $\mathcal{M}(t)$ (see [23, 32]),

(1) $\forall \varepsilon_1 > 0$ and $\mathcal{M}^T(t) \mathcal{M}(t) \leq I$, then

$$\mathcal{M}(t)\mathcal{V} + \mathcal{V}^T \mathcal{M}^T(t) \mathcal{P} \leq \frac{1}{\varepsilon_1} \mathcal{P} + \varepsilon_1 \mathcal{V}^T \mathcal{V}. \quad (19)$$

(2) $\forall \varepsilon_2 > 0$ such that $\varepsilon_2 \mathcal{V}^T \mathcal{V} < I$ and $\mathcal{M}(t) \mathcal{M}(t) \leq I$,

$$\mathcal{U} + \mathcal{M}(t) \mathcal{V} \left(\mathcal{U} + \mathcal{M}(t) \mathcal{V}\right)^T \leq \mathcal{U} \left(1 - \varepsilon_2 \mathcal{V}^T \mathcal{V}\right)^{-1} \mathcal{U} + \frac{1}{\varepsilon_2} \mathcal{P} \mathcal{P}^T. \quad (20)$$

Particularly, when $\mathcal{U} \equiv 0$, we obtain

$$\mathcal{M}(t) \mathcal{V} \left(\mathcal{M}(t) \mathcal{V}\right)^T \leq \frac{1}{\varepsilon_2} \mathcal{P} \mathcal{P}^T. \quad (21)$$

Lemma 2. $\forall s_1, s_2 \in \mathbb{R}^d$ (see [33]), the inequality

$$s_1^T s_2 + s_2^T s_1 \leq s_1^T R s_1 + s_2^T R^{-1} s_2, \quad (22)$$

holds, where $R \in \mathbb{R}^{d \times d}$ is a positive definite matrix.

Lemma 3. Given constant matrices $\mathcal{B}_1, \mathcal{B}_2$, and $\mathcal{B}_3$ (see [34]), where $\mathcal{B}_1 = \mathcal{B}_1^T$ and $\mathcal{B}_2 > 0$, then

$$\mathcal{B}_1 + \mathcal{B}_2 \mathcal{B}_2^{-1} \mathcal{B}_3 < 0, \quad (23)$$

if and only if

$$\begin{bmatrix} \mathcal{B}_1 & \mathcal{B}_3^T \\ \mathcal{B}_3 & -\mathcal{B}_2 \end{bmatrix} < 0, \quad (24)$$

or

$$\begin{bmatrix} -\mathcal{B}_2 & \mathcal{B}_3 \\ \mathcal{B}_3^T & \mathcal{B}_1 \end{bmatrix} < 0.$$

3. Main Results

This section is devoted to the following tripartite through theoretical analysis and demonstrates the following:

(T1) The presence of the lower bound of adjacent impulse instants is testified, whereafter, the Zeno behavior can be excluded.

(T2) The resultant augmented system which is equipped with parameter uncertainties and exogenous disturbances is input-to-state stabilizable where the uncertainties are tactfully subdued.

(T3) The control gains and event-triggered scheme parameters are devised without strong constrained condition under system (13) corresponding stability.

Before verifying the above statements, it is necessary to introduce some symbols:
According to Assumption 1, we have
\[
\begin{bmatrix}
\Delta \mathcal{B} & \Delta \mathcal{B}_d & \Delta \mathcal{A} & \Delta \mathcal{C} & \Delta \mathcal{D}
\end{bmatrix}
= M(t) \begin{bmatrix}
F_1 & F_2 & F_3 & F_4 & F_5
\end{bmatrix},
\]
(26)
where
\[
F_1 = \begin{bmatrix}
F_1 & 0 \\
F_2 & 0 \\
F_3 & 0 \\
F_4 & 0 \\
F_5 & 0
\end{bmatrix},
\]
(27)

**Theorem 1.** Under event-triggered scheme (16), then (T1) holds, where the positive lower bound \( J \) of adjoining impulsive moments conforms to
\[
J = \left( \ln \left( \frac{\varphi_1 (\eta \wedge \rho) \vee \varphi_2 + 1}{\varphi_1} \right) + 1 \right) / \varphi_1 \wedge \ell, \text{ constants } \eta, \rho, \text{ and } \ell \text{ are specified in (16), and}
\]
\[
\varphi_1 = 2 \| \mathcal{B} + \mathcal{A} L \| + 2 \| M \| \cdot \| F_1 \| + 2 \| M \| \cdot \| F_3 L \| + 2 \beta \| \mathcal{B}_d \| + 2 \beta \| M \| \cdot \| F_2 \| + 2 \| \mathcal{A} L \|
\]
\[
+ 2 \| M \| \cdot \| F_3 L \| \varphi_3,
\]
\[
\varphi_2 = \| \mathcal{B} + \mathcal{A} L \| + \| M \| \cdot \| F_1 \| + \| M \| \cdot \| F_3 L \| + \beta \| \mathcal{B}_d \|
\]
\[
+ \beta \| M \| \cdot \| F_2 \|,
\]
\[
\varphi_3 = \| C \| \cdot \| D \| + \| C \| \cdot \| M \| \cdot \| F_5 \| + \| D \| \cdot \| M \| \cdot \| F_4 \|
\]
\[
+ \| M \|^2 \cdot \| F_4 \| \cdot \| F_5 \|.
\]
(28)

**Proof.** For the sake of checking on (T1), we consider the following three cases:

Case (i). The generation of the triggered time sequence \( \{ t_a \}_{a=1}^{\infty} \) depends entirely on the event-triggered time sequence \( \{ t_d \}_{a=1}^{\infty} \). Based on \( \varrho(t) = D_q(t) = \mathfrak{F}(t_a) - \mathfrak{F}(t) \), in this case, the upper right Dini derivative of \( \varrho(t) \) in the interval \( [t_a, t_{a+1}) \) is calculated as
Using (16), we have
\[
\eta \| \mathbf{F} (t_a) \|^2 + \rho \| \xi (t) \|^2 \leq \| \mathbf{F} (t_{a+1}) \|^2.
\]
Together with \( \varphi_a = \varphi_2 \| \mathbf{F} (t_a) \|^2 + \varphi_3 \| \xi (t) \|^2 \), it follows that
\[
\beta \eta \rho \| \mathbf{F} (t_a) \|^2 + \rho \| \xi (t) \|^2 \leq \frac{\beta \eta \rho}{\varphi_a} (t_{a+1} - t_a) \leq \frac{\beta \eta \rho}{\varphi_a} (t_{a+1} - t_a),
\]
which indicates that
\[
t_{a+1} - t_a \geq \frac{\ln ((\eta \rho/\varphi_a) + 1)}{\varphi_a} > 0, \ a \in \mathbb{N}^+.
\]

Case (ii). Only the forced triggered time sequence \( t_a + n \epsilon \mathbb{N}^+ \) exists in the sequence \( t_a \mathbb{N}^+ \). Obviously, in this case, \( t_{a+1} - t_a = \epsilon > 0, \ a \in \mathbb{N}^+ \).

Case (iii). The sequence \( t_a \mathbb{N}^+ \) is composed of the event-triggered instants \( t_a^* \) (s \( \in \mathbb{N}^+ \)) and the forced triggered instants \( t_j \) (j \( \in \mathbb{N}^+ \)). If the Zeno behavior lives in argument system (13), it must be that a finite time interval owns infinite impulse jumps in this case. To exhibit this phenomenon, suppose that \( T \) presents the accumulation time (or Zeno time) on the finite time interval \([t_0, T]\). By defining \( \zeta = (T - (\epsilon / 2)) / \epsilon \), it is apparent that countless of impulse instants appears in the interval \([\zeta, T]\). Denote by \( \{t_{N_{a+1}}\} \subset [\zeta, T] \) the subsequence of \( \{t_a\} \) satisfying \( t_{N_{a+1}} \rightarrow T \) as \( s \rightarrow \infty \), where integer \( N_0 \geq 0 \). If there is no forced triggered moment in \([\zeta, T]\), similar to the discussion of Case (i), we conclude that \( t_{N_{a+1}} \rightarrow \infty \) as \( s \rightarrow \infty \), which can be deducted that the definition of the accumulation time \( T \). If there exists \( t_j \in \{t_{N_{a+1}}\} \) for some \( j \in \mathbb{Z}^+ \) over the interval \([\zeta, T]\), recalling the definition of \( \zeta \), it can be deduced that only one \( t_j \in \{t_{N_{a+1}}\} \) which implies that the triggered moments totally consist of the event-triggered moments in \( (t_j, T) \). Then it follows from Case (i) that \( t_{N_{a+1}} \rightarrow \infty \) as \( s \rightarrow \infty \). Hence, the Zeno behavior is precluded in Case (iii).
to take the supremum of $\|z(t)\|^2$ in $[t_0, t]$. In addition, the study in [29] assists us in detecting the indispensability of the forced trigger condition in (16).

**Remark 5.** As described earlier, mechanism (16) is presented to select the optimal triggered moment according to the steady state of resulting system (13). Based on the results of Theorem 1, it can be proved that the designed event-triggered mechanism is effective. By comparison with the existing results of the uncertain models, such as [21, 35, 36], although they reduce the transmission of information, impulsive controller with fixed impulsive moments, in design, is still conservative. Now, in this paper, the controller based on event-triggered mechanism only is updated at the triggered moment. By this means, the burden of controller update can be decreased without affecting accurate control. Moreover, the sampling control adopted in this paper is only dependent on the state information of observer at the triggered moment, which can reduce the communication between the equipment under test and the observer.

**Assumption 3.** $\bar{K}$ is a nonsingular matrix, and there exists a constant $\gamma > 0$ such that $\lambda_{\text{max}}(\bar{K}) \leq e^{-(1/2)\gamma}$.

**Theorem 2.** Uncertain system (13) is input-to-state stable via the event-triggered scheme (16); suppose that for given parameters $\eta \in (0, (1/2)), \rho > 0$, and $\ell > 0$ and the control gains $K, \bar{K} \in \mathbb{R}^{n \times p}$, and $L \in \mathbb{R}^{n \times m}$, there exist matrices $P > 0$ and $H_i > 0 (i = 1, 2, 3)$ and constants $\bar{\theta} > 0$, $\vartheta > 0$, $\gamma > 0$, and $\varepsilon_k > 0 (b = 1, 2, 3, \ldots, 58)$, such that

\[
\begin{bmatrix}
\Omega_1 & \mathcal{D}^T \mathcal{A} \mathcal{D}^T \mathcal{A} \mathcal{D} \mathcal{D}^T \mathcal{A} \mathcal{D} \mathcal{D}^T \mathcal{A} \\
\ast & -H_1 & 0 \\
\ast & \ast & -H_2 \\
\ast & \ast & \ast & -H_3
\end{bmatrix}
> 0, \quad (35)
\]

\[
\begin{bmatrix}
\Omega_2 & \mathcal{M} \mathcal{M}^T \mathcal{F} \mathcal{F}^T \mathcal{F} \\
\ast & -\varepsilon_{57} I & 0 & 0 \\
\ast & \ast & \frac{1}{\varepsilon_{57}} I & 0 & 0 \\
\ast & \ast & \ast & \ast & -\varepsilon_{58} I \\
\ast & \ast & \ast & \ast & \ast & \frac{1}{\varepsilon_{58}} I
\end{bmatrix}
\leq 0, \quad (36)
\]

\[
\begin{bmatrix}
\Psi_1 & 0 & 0 \\
\ast & \Psi_2 & 0 \\
\ast & \ast & \Psi_3
\end{bmatrix}
> 0,
\]

\[
\gamma - \bar{\theta} \ell > 0, \quad (37)
\]

where

\[
\Psi_1 = \begin{bmatrix}
1 & \mathcal{F}_\mathcal{A}^T \\
\ast & \frac{1}{\varepsilon_{57}} I
\end{bmatrix}
\]

\[
\Psi_2 = \begin{bmatrix}
1 & \Lambda \mathcal{F}_\mathcal{A} \\
\ast & \frac{1}{\varepsilon_{57}} I
\end{bmatrix}
\]

\[
\Psi_3 = \begin{bmatrix}
1 & \mathcal{M} \mathcal{F}_\mathcal{A} \\
\ast & \frac{1}{\varepsilon_{57}} I
\end{bmatrix}
\]

\[
\begin{align*}
\Omega_1 &= \mathcal{D}^T \mathcal{P} \mathcal{D}_\mathcal{A} \mathcal{D}^T \mathcal{P} \mathcal{D}_\mathcal{A} \\
\Omega_2 &= \mathcal{M} \mathcal{M}^T \mathcal{F} \mathcal{F}^T \mathcal{F} \\
\Psi_1 &= \mathcal{F}_\mathcal{A}^T \mathcal{P} \mathcal{P} \mathcal{D}_\mathcal{A} \\
\Psi_2 &= \mathcal{M} \mathcal{P} \mathcal{P} \mathcal{D}_\mathcal{A} \\
\Psi_3 &= \mathcal{F}_\mathcal{A} \mathcal{P} \mathcal{P} \mathcal{D}_\mathcal{A} \\
\end{align*}
\]
Proof. Suppose that $\varphi(t) = \varphi(t, t_0, \varphi(0))$ is the solution of system (13) with the initial value $(t_0, \varphi(0))$. Due to the imperfect measurability of the system states, the Lyapunov functional related to the relevant outcomes is considered as $V(t) = \mathfrak{F}^T(t)P\mathfrak{F}(t). \forall t \in [t_\alpha, t_{\alpha+1}), \alpha \in \mathbb{Z}^+$, the derivative of $V(t)$ can be calculated that

$$
\dot{V} = 2\mathfrak{F}^T(t)P\mathfrak{F}(t)
= 2\varphi^T(t)D^TP\mathfrak{F}(t)
\leq 2\varphi^T(t)D^TP\Lambda_1\varphi(t) + \tilde{\varphi}^T(t)D^TP\mathfrak{F}(t)
+ 2\varphi^T(t)D^TP\Lambda_2\varphi(t)
+ 2\varphi^T(t)D^TP\Lambda_4\tilde{\varphi}(t)
\leq 2\varphi^T(t)D^TP\Lambda_1\varphi(t) + \tilde{\varphi}^T(t)D^TP\mathfrak{F}(t)
+ 2\varphi^T(t)D^TP\Lambda_2\varphi(t)
+ 2\varphi^T(t)D^TP\Lambda_4\tilde{\varphi}(t).
$$

It follows from Lemma 1 and conditions (26) and (35) that

$$
Hd\left(\mathfrak{F}^T P\mathfrak{D}[\Delta B \Delta \mathfrak{aL}]\right) \\
\leq \left[\frac{1}{\epsilon_1}\mathfrak{F}_1^T \mathfrak{F}_1 + \frac{1}{\epsilon_2}L^T \mathfrak{F}_3^T \mathfrak{F}_3 \right] + \Lambda^T_3 \Lambda_3 \left[ \epsilon_1 I \ \epsilon_2 I \right],
$$

$$
Hd\left(\mathfrak{F}^T P\mathfrak{D}[\Lambda_1 \Delta B \Delta \mathfrak{aL}]\right) \\
\leq \left[\frac{1}{\epsilon_3}\Lambda_3^T \mathfrak{F}_3^T \mathfrak{F}_3 \mathfrak{F}_5 \mathfrak{F}_5 \Lambda_1 \frac{1}{\epsilon_6}L^T \mathfrak{F}_3^T \mathfrak{F}_3 \right]
+ \Lambda^T_3 \Lambda_3 \left[ \epsilon_3 I \ \epsilon_4 I \ \epsilon_6 I \right],
$$

$$
Hd(\Delta \mathfrak{a}^T P\mathfrak{D}\Lambda_1) \leq \epsilon_6 \mathfrak{F}_5^T \mathfrak{F}_5 + \frac{1}{\epsilon_8}\Lambda^T_3 \Lambda_3 \Lambda_1 \Lambda_1,
$$

$$
Hd(\Delta \mathfrak{a}^T P\mathfrak{D}[\Delta B \Delta \mathfrak{aL}]\right) \leq \left[\frac{1}{\epsilon_9}\epsilon_{10} \mathfrak{F}_1^T \mathfrak{F}_1 \left[ \frac{1}{\epsilon_1}L^T \mathfrak{F}_3^T \mathfrak{F}_3 \right] \right]
+ \mathfrak{F}_5^T \mathfrak{F}_5 \left[ \epsilon_9 I \ \epsilon_{11} I \right],
$$

$$
Hd(\Delta \mathfrak{a}^T P\mathfrak{D}[\Lambda_1 \Delta B \Delta \mathfrak{aL}]\right) \leq \left[\frac{1}{\epsilon_{12}}\Lambda^T_3 \mathfrak{F}_5^T \mathfrak{F}_5 \mathfrak{F}_5 \mathfrak{F}_5 \Lambda_1 \frac{1}{\epsilon_{13}}L^T \mathfrak{F}_3^T \mathfrak{F}_3 \right]
+ \mathfrak{F}_5^T \mathfrak{F}_5 \left[ \epsilon_{13} I \ \epsilon_{14} I \ \epsilon_{16} I \right].
$$

$$
2\varphi^T(t)D^TP\mathfrak{D}\Delta B_1 \mathfrak{F}(t)
\leq 4\mathfrak{F}^T(t)\mathfrak{D}_2^T \mathfrak{F}_2 \mathfrak{F}(t) + \epsilon_{21}\mathfrak{F}^T(t)\Lambda^T_3 \Lambda_3 \mathfrak{F}(t),
$$

$$
2\varphi^T(t)D^TP\mathfrak{D}[\mathfrak{D}_2 \mathfrak{F}(t) \Delta B_1 \mathfrak{F}(t)]
\leq \left[\frac{1}{\epsilon_{22}}\mathfrak{F}^T(t)\mathfrak{D}_2^T \mathfrak{F}_3 \mathfrak{F}_5 \mathfrak{D}_2 \mathfrak{F}(t) \frac{1}{\epsilon_{23}}\mathfrak{F}^T(t)\mathfrak{F}_3 \mathfrak{F}_2 \mathfrak{F}(t) \mathfrak{F}(t) \mathfrak{F}_2 \right] + \mathfrak{F}^T(t)\Lambda^T_3 \Lambda_3 \mathfrak{F}(t) \left[ \epsilon_{22} I \ \epsilon_{23} I \right],
$$

$$
2\varphi^T(t)\Delta \mathfrak{a}^T P\mathfrak{D}[\mathfrak{D}_2 \mathfrak{F}(t) \Delta B_1 \mathfrak{F}(t)]
\leq \left[\frac{1}{\epsilon_{25}}\mathfrak{F}^T(t)\mathfrak{D}_2^T \Lambda^T_2 \Lambda_2 \mathfrak{F}(t) \frac{1}{\epsilon_{26}}\mathfrak{F}^T(t)\mathfrak{F}_2 \mathfrak{F}(t) \mathfrak{F}(t) \mathfrak{F}_2 \right] + \mathfrak{F}^T(t)\Lambda^T_3 \Lambda_3 \mathfrak{F}(t) \left[ \epsilon_{25} I \ \epsilon_{26} I \right].
$$
\[\begin{align*}
&+ \rho^T(t)F^T_s \mathcal{F}_s \mathcal{P}(t)[\epsilon_{35}I \ \epsilon_{26}I], \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{A} \left[ \mathcal{B}_d \mathcal{F}(t) \ \Delta \mathcal{B}_d \mathcal{F}(t) \right] \\
&\leq \left[ \frac{1}{\epsilon_{38} \epsilon_{39}} \mathcal{F}^T(t) \mathcal{B}_d \mathcal{F}_s^T \mathcal{F}_s \mathcal{B}_d \mathcal{F}(t) \right] \mathcal{F}^T(t) \mathcal{F}_s^T \mathcal{F}_s \mathcal{F}(t) \\
&\quad + \rho^T(t)\mathcal{F}^T_s \mathcal{P}(t)[\epsilon_{38}I \ \epsilon_{39}I], \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{D} \left[ \mathcal{A}_s \mathcal{L}_q(t) \right] \\
&\leq \frac{1}{\epsilon_{33}} \mathcal{D}^T(t) \mathcal{L}_q^T \mathcal{L}_q(t) + \epsilon_{33} \rho^T(t) \mathcal{A}_s \mathcal{L}_q(t), \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{D} \left[ \mathcal{A}_s \mathcal{L}_q(t) \ \Delta \mathcal{L}_q(t) \right] \\
&\leq \left[ \frac{1}{\epsilon_{34}} \mathcal{D}^T(t) \mathcal{L}_q^T \mathcal{L}_q(t) \right] \mathcal{D}^T(t) \mathcal{L}_q(t) \\
&\quad + \rho^T(t)\mathcal{A}_s \mathcal{L}_q(t)[\epsilon_{34}I \ \epsilon_{35}I], \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{D} \left[ \mathcal{A}_s \mathcal{L}_q(t) \ \Delta \mathcal{L}_q(t) \right] \\
&\leq \left[ \frac{1}{\epsilon_{37}} \mathcal{D}^T(t) \mathcal{L}_q^T \mathcal{L}_q(t) \right] \mathcal{D}^T(t) \mathcal{L}_q(t) \\
&\quad + \rho^T(t)\mathcal{D}_s \mathcal{P}(t)[\epsilon_{37}I \ \epsilon_{38}I], \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{D} \left[ \mathcal{A}_s \mathcal{L}_q(t) \ \Delta \mathcal{L}_q(t) \right] \\
&\leq \frac{1}{\epsilon_{40} \epsilon_{41}} \mathcal{D}^T(t) \mathcal{L}_q^T \mathcal{L}_q(t) \\
&\quad + \rho^T(t)\mathcal{D}_s \mathcal{P}(t)[\epsilon_{40}I \ \epsilon_{41}I], \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{D} \left[ \mathcal{A}_s \mathcal{L}_q(t) \ \Delta \mathcal{L}_q(t) \right] \\
&\leq \frac{1}{\epsilon_{44} \epsilon_{45}} \mathcal{D}^T(t) \mathcal{L}_q^T \mathcal{L}_q(t) \\
&\quad + \rho^T(t)\mathcal{D}_s \mathcal{P}(t)[\epsilon_{44}I \ \epsilon_{45}I], \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{D} \left[ \mathcal{A}_s \mathcal{L}_q(t) \ \Delta \mathcal{L}_q(t) \right] \\
&\leq \frac{1}{\epsilon_{48} \epsilon_{49}} \mathcal{D}^T(t) \mathcal{L}_q^T \mathcal{L}_q(t) \\
&\quad + \rho^T(t)\mathcal{D}_s \mathcal{P}(t)[\epsilon_{48}I \ \epsilon_{49}I], \\
&2\rho^T(t)\Delta \mathcal{T}^T \mathcal{P} \Delta \mathcal{D} \left[ \mathcal{A}_s \mathcal{L}_q(t) \ \Delta \mathcal{L}_q(t) \right] \\
&\leq \frac{1}{\epsilon_{52} \epsilon_{53}} \mathcal{D}^T(t) \mathcal{L}_q^T \mathcal{L}_q(t) \\
&\quad + \rho^T(t)\mathcal{D}_s \mathcal{P}(t)[\epsilon_{52}I \ \epsilon_{53}I].
\end{align*}\]
and from Assumption 2 and Lemma 2, we derive
\[
\mathcal{F}(t) \leq \beta \phi(t)
\]
\[
2 \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{A}_\Delta \phi(t) \leq \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{A}_\Delta \mathcal{R}_4 \mathcal{D}^T P \mathcal{D} \phi(t) + \beta \phi^T(t)\mathcal{H}_4 \beta \phi(t),
\]
\[
2 \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{A}_\Delta \phi(t) \leq \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{H}_4^{-1} \mathcal{D}^T P \mathcal{D} \phi(t) + \chi^T(t)H_4 \chi(t).
\]

(42)

And then we infer that
\[
2 \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{A}_\Delta \phi(t) + 2 \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{A}_\Delta \mathcal{F}(t)
\]
\[
= 2 \phi^T(t)(\mathcal{D} + \Delta \mathcal{D})^T P(\mathcal{D} + \Delta \mathcal{D})(\mathcal{A}_1 + \Delta \mathcal{A} + \Delta \mathcal{A} \mathcal{L}) \phi(t) + 2 \phi^T(t)(\mathcal{D} + \Delta \mathcal{D})^T P(\mathcal{D} + \Delta \mathcal{D})(\mathcal{A}_4 + \Delta \mathcal{A}_4) \mathcal{F}(t)
\]
\[
\leq \phi^T(t)\mathcal{R}_4 \phi(t) + \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{H}_4^{-1} \mathcal{D}^T P \mathcal{D} \phi(t),
\]

(43)

\[
2 \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{A}_\Delta \zeta(t)
\]
\[
= 2 \phi^T(t)(\mathcal{D} + \Delta \mathcal{D})^T P(\mathcal{D} + \Delta \mathcal{D})(\mathcal{C} + \Delta \mathcal{C}) \zeta(t)
\]
\[
\leq \zeta^T(t)\mathcal{R}_3 \zeta(t) + \phi^T(t)\mathcal{D}^T P \mathcal{D} \mathcal{H}_4^{-1} \mathcal{D}^T P \mathcal{D} \phi(t),
\]

(44)

where
\[
\mathcal{R}_3 = \left( \frac{1}{\varepsilon_{45}} + \frac{1}{\varepsilon_{47} \varepsilon_{48}} + \frac{1}{\varepsilon_{50} \varepsilon_{51}} + \frac{1}{\varepsilon_{54} \varepsilon_{55} \varepsilon_{56}} \right) \mathcal{F}_4^T \mathcal{F}_4 + \left( \frac{1}{\varepsilon_{46}} + \frac{1}{\varepsilon_{52} \varepsilon_{53}} \right) \mathcal{F}_5^T \mathcal{F}_5 \mathcal{C} + \frac{1}{\varepsilon_{49}} \mathcal{D} \mathcal{A}_2 \mathcal{A}_2 \mathcal{C} + H_3.
\]

(46)

Consequently, under Lemma 3 and (35), substituting (43)–(45) into (40) gives that, \( \forall t \in [t_0, t_{a+1}], a \in \mathbb{N}^+ \),
\[
\dot{V} \leq \delta \phi^T(t) \phi(t) + \beta \phi^T(t)\mathcal{D}^T P \mathcal{D} \phi(t)
\]
\[
+ \phi^T(t)\mathcal{R}_2 \phi(t) + \chi^T(t) \mathcal{R}_3 \chi(t)
\]
\[
\leq \delta \mathcal{V}(t) + \| \phi(t) \|^2 + \lambda_{\max}(\mathcal{R}_2) \| \phi(t) \|^2 + \lambda_{\max}(\mathcal{R}_3) \| \chi(t) \|^2.
\]

(47)

\( \forall t \in [t_0, t_{a+1}] \), it holds from Assumption 1, (26), and the fact \( \| \phi(t) \|^2 \leq \eta \| \mathfrak{F}(t) \|^2 + \rho \| \chi(t) \|^2 \), that
\[
\| \mathfrak{F}(t) \|^2 \leq \frac{2\eta}{1 - 2\eta} \left( \| \mathcal{D} \| + \alpha \| \mathcal{M} \| \right)^2 \mathcal{I}(t) \| \mathcal{I}(t) \|^2
\]
\[
+ \frac{\rho}{(1 - 2\eta) \left( \| \mathcal{D} \| + \alpha \| \mathcal{M} \| \right)^2} \| \mathcal{I}(t) \|^2 \| \mathcal{I}(t) \|_{[t_0, t_1]}^2.
\]

(48)

which supplied into (47) yields that
\[
\dot{V}(t) \leq \delta \mathcal{V}(t) + \psi \left( \| \mathfrak{F}(t) \|_{[t_0, t_1]} \right), \quad \forall t \in [t_0, t_{a+1}],
\]

(49)

where
\[
\psi(c) = \left( \lambda_{\max}(\mathcal{R}_3) + \frac{\rho \lambda_{\max}(\mathcal{R}_2)}{(1 - 2\eta) \left( \| \mathcal{D} \| + \alpha \| \mathcal{M} \| \right)^2} \right)^2.
\]

(50)

By virtue of condition (38), a positive constant \( \bar{\delta} > \delta \) could be spotted with ease such that \( \gamma - \bar{\delta} > 0 \), and it could be derived that
\[
\dot{V}(t) \leq \bar{\delta} \mathcal{V}(t),
\]

(51)

whenever \( \mathcal{V}(t) \geq \bar{\delta} \mathcal{V}(t) \), \( \sigma = 0 \). When \( t = t_{a+1} \), it is deduced from (13) that
\[
\mathcal{V}(t) = \mathcal{H}^T(t) \mathcal{P} \mathcal{S}(t) = \phi^T(t)\mathcal{D}^T P \mathcal{D} \phi(t)
\]
\[
= \phi^T(t)\mathcal{K} \mathcal{D}^T P \mathcal{D} \phi(t) = \phi^T(t)\mathcal{K} \phi(t).
\]

(52)

By Assumption 3, there exists an invertible matrix \( \mathcal{R} \) such that
\[
\mathcal{R}^{-1} \mathcal{K} \mathcal{R} \leq e^{-\gamma (1/2)} I.
\]

(53)

In accordance with Lemma 1, we discover that
\[
\mathcal{H}(\mathcal{A} \mathcal{D} \mathcal{H}_2) \leq \frac{1}{\varepsilon_{57}} \mathcal{M} \mathcal{M}^T + \epsilon_{57} \mathcal{F}_3 \mathcal{F}_3 \mathcal{K}_2 \mathcal{F}_2 \mathcal{K}_2 \mathcal{F}_2
\]
\[
\mathcal{H}(\mathcal{H}_4 \mathcal{A} \mathcal{D}) \leq \frac{1}{\varepsilon_{58}} \mathcal{H}_4 \mathcal{M} \mathcal{M}^T \mathcal{H}_4 \mathcal{F}_3 \mathcal{F}_3 \mathcal{H}_4 \mathcal{F}_3 \mathcal{H}_4.
\]

(54)

Afterwards, combined with Lemma 3, (37), (52), and (53), we obtain
\[
\mathcal{V}(t) \leq e^{-\gamma(t - t_0)} \mathcal{V}(t_0).
\]

(55)

Pay attention that \( \mathcal{V}(t_0) \geq \sigma \mathcal{V}(\| \mathfrak{F}(t_0) \|) \). Accordingly, we can define \( \tilde{t}_1 = \inf \{ t \geq t_0 | \mathcal{V}(t) \leq \sigma \mathcal{V}(\| \mathfrak{F}(t) \|) \} \), which symbolizes that \( \mathcal{V}(t) \geq \sigma \mathcal{V}(\| \mathfrak{F}(t) \|) \), \( \forall t \in [t_0, \tilde{t}_1) \). Consider this case that \( \tilde{t}_1 < \infty \) initially. If there is no impulse over \([t_0, \tilde{t}_1)\), it gives from (51) that
\[
\mathcal{V}(t) \leq \delta \mathcal{V}(t - t_0) \mathcal{V}(t_0), \quad \forall t \in [t_0, \tilde{t}_1).
\]

(56)

If there exist a few impulsive moments \( t_{11} < t_{12} < \cdots < t_{1m} < \cdots \) on \([t_0, \tilde{t}_1)\), we can deduce that
\[
\mathcal{V}(t) \leq \delta \mathcal{V}(t - t_0) \mathcal{V}(t_0), \quad t \in [t_0, \tilde{t}_1).
\]

(57)

Based on (55), we get
which illustrates that
\[ V(t) \leq e^{-\gamma (t-t_1)} V(t_1) \leq e^{-\gamma \tau (t-t_s)} V(t_s), \quad t \in [t_1, t_2). \]  
(59)

Equally, the inequality \( V(t_1) \leq e^{-2\gamma (t-t_1)} V(t_0) \) holds. Paralleling to the above deduction, we could get an inference that, \( \forall t \in [t_0, t_1) \),
\[ V(t) \leq e^{-\gamma \tau(t-t_s)} V(t_0) \leq e^{-\gamma \tau(t-t_s)} V(t_0), \quad t \in [t_1, t_2). \]  
(60)

where \( n (t, t_0) \) denote the number of impulsive moments in the corresponding interval and \( \bar{\gamma} = \gamma - \overline{\delta} > 0 \).

As a result, when \( \tau_1 < \infty \), whether impulse exists in \([t_0, \tau_1)\) or not, the following inequality is always attainable:
\[ V(t) \leq e^{-\gamma \tau(t-t_s)} V(t_0), \quad \forall t \in [t_0, \tau_1). \]  
(61)

When \( \tau_1 = \infty \), executing the similar argument as the case of \( \tau_1 < \infty \), we could reason out
\[ V(t) \leq e^{-\gamma \tau(t-t_s)} V(t_0), \quad \forall t \in [t_0, \infty). \]  
(62)

Associating with the situation of \( \tau_1 < \infty \), let \( \tau_1 = \inf\{t \geq \tau_1 \mid V(t) \geq -T \| \zeta(t) \|_{\|t\|_{1,t}} \} \); the priority concern is the case of \( \tau_1 < \infty \). Evidently, whether impulse appears on \([\tau_1, \tau_2)\) or not, it can be invariably obtained that
\[ V(t) \leq -\gamma \tau(t-t_s) V(t_0), \quad \forall t \in [t_1, \tau_1). \]  
(63)

Similarly, when \( \tau_1 = \infty \); \( V(t) \leq -\gamma \tau(t-t_s) V(t_0), \quad \forall t \in [t_1, \infty). \)

Combined with the circumstance that \( \tau_1 < \infty \), it follows that
\[ V(t) \leq e^{-\gamma \tau(t-t_s)} V(t_0) + \gamma \tau(t-t_s) V(t_0), \quad \forall t \in [t_1, \tau_1). \]  
(64)

and then let \( \tau_2 = \inf\{t \geq \tau_1 \mid V(t) \geq -T \| \zeta(t) \|_{\|t\|_{1,t}} \} \); when \( \tau_2 < \infty \), we give priority to the case of no impulse over \([\tau_1, \tau_2)\); we could derive from \( V(t) = -\gamma \tau(t-t_s) V(t_0) \) that
\[ V(t) \leq -\gamma \tau(t-t_s) V(t_1) \leq e^{-\gamma \tau(t-t_s)} V(t_1), \quad t \in [t_1, \tau_2). \]  
(65)

If there exist impulsive moments \( t_{21} < t_{22} < \cdots < t_{2m} < \cdots \), it holds that
\[ V(t) \leq e^{-\gamma \tau(t-t_s)} V(t_{21}) \leq e^{-\gamma \tau(t-t_s)} V(t_{21}), \quad \forall t \in [t_1, t_{21}). \]  
(66)

According to (55), we have
\[ V(t_{21}) \leq e^{-\gamma \tau(t_{21})} V(t_{21}) \leq e^{-\gamma \tau(t_{21})} \sigma \| \zeta(t) \|_{\|t\|_{1,t}} \).  
(67)

which suggests that
\[ V(t) \leq e^{-\gamma \tau(t-t_s)} \sigma \| \zeta(t) \|_{\|t\|_{1,t}} \), \quad \forall t \in [t_{21}, t_{22}). \]  
(68)

On account of \( \gamma - \overline{\delta} > 0 \), performing semblable operation, we get
\[ V(t) \leq e^{-\overline{\delta} \tau(t-t_s)} V(t_0), \quad \forall t \in [t_1, t_2). \]  
(69)

When \( \tau_2 = \infty \), it can be inferred in repeating similar iterations that
\[ V(t) \leq e^{-\overline{\delta} \tau(t-t_s)} V(t_0) + e^{-\overline{\delta} \tau(t-t_s)} V(t_0), \quad \forall t \in [t_0, \tau_2). \]  
(70)

Together with \( \tau_2 < \infty \), we derive
\[ V(t) \leq e^{-\overline{\delta} \tau(t-t_s)} V(t_0) + e^{-\overline{\delta} \tau(t-t_s)} V(t_0), \quad \forall t \in [t_0, \tau_2). \]  
(71)

Arguing in the identical manner, we conclude that
\[ V(t) \leq e^{-\overline{\delta} \tau(t-t_s)} V(t_0) + e^{-\overline{\delta} \tau(t-t_s)} V(t_0), \quad \forall t \in [t_0, \tau_2). \]  
(72)

which fulfills the proof of (T2) based on Definition 1.

Remark 6. Obviously, Assumption 3 and condition (37) play a complementary role in the proof, which simultaneously explains the rationality and significance of Assumption 3.

Next, for the purpose of achieving (T3), the following hypothesis needs to be proposed.

Assumption 4. Matrices \( D \) and \( A \) with appropriate dimensionality are of full-row rank and full-column rank separately.

Theorem 3. Assume that, for given constants \( \varepsilon_t (t = 1, 2, 3) \), there exist matrices \( P > 0 \), \( S > 0 \), \( \Sigma \), and \( \Theta, t = 1, 2 \), and constants \( \delta > 0 \), \( \delta > 0 \), \( \gamma > 0 \), and \( \varepsilon_t > 0 \) \( (t = 1, 2, 3, \ldots, 58) \) such that (35) and

\[
\begin{bmatrix}
H(t, t) & \mathcal{M} & 
\Sigma_1^T \mathcal{S}_1^T \\
\mathcal{M}^T & \mathcal{S}_1 \mathcal{S}_1 & 
\mathcal{M}^T \\
\mathcal{M}^T & \mathcal{S}_1 \mathcal{S}_1 & 
\mathcal{M}^T
\end{bmatrix}
\begin{bmatrix}
* & -\varepsilon_{57} I & 0 & 0 & 0 \\
* & * & -\varepsilon_{57} I & 0 & 0 \\
* & * & * & -\varepsilon_{58} I & 0 \\
* & * & * & * & -\varepsilon_{58} I \\
\end{bmatrix}
< 0,
\]
(73)

\[
\begin{bmatrix}
\Omega & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{S} & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{A} \mathcal{S} & 
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{O} \mathcal{S} & 
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{F} \mathcal{S} \\
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{S} & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{A} \mathcal{S} & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{O} \mathcal{S} & 
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{F} \mathcal{S} \\
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{O} \mathcal{S} & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{O} \mathcal{S} & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{O} \mathcal{S} & 
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{F} \mathcal{S} \\
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{F} \mathcal{S} & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{F} \mathcal{S} & \mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{F} \mathcal{S} & 
\mathcal{S}^T \mathcal{P} \mathcal{D} \mathcal{F} \mathcal{S}
\end{bmatrix}
< 0,
\]
(74)
and then system (13) is input-to-state stabilizable under event-triggered strategy (16). In addition, the control gains $K$, $\overline{K}$, and $L$ and parameters $\eta$, $\rho$, and $\ell$ are jointly devised by

$$K = \Xi_1 D^T (DD^T)^{-1},$$
$$\overline{K} = (A^T A)^{-1} A^T \Xi_2,$$
$$L = \Theta S_1$$

(76)

$$0 < \eta \leq \frac{\kappa}{2 + 2\kappa},$$

$$0 < \ell < \frac{\lambda_{\text{min}}(P)\gamma}{\lambda_{\text{min}}(P)\bar{\delta} + \bar{\delta}},$$

$$\rho > 0,$$

where

$$\kappa = \frac{\lambda_{\text{min}}(P)\lambda_{\text{min}}(S)((\gamma/\kappa) - (8/\lambda_{\text{min}}(P))(\|\overline{\mathcal{M}}\| + a\|\mathcal{M}\| - \|\mathcal{F}\|)^2}{\lambda_{\text{max}}(\mathcal{H}_2)}$$

(77)

$$\mathcal{H}_2 = \mu_6 \Theta^T P \mathcal{M}^T \mathcal{M} \mathcal{P} \mathcal{D} \overline{\mathcal{P}} \mathcal{S} + \mu_7 \Theta^T \mathcal{M} \mathcal{D} \mathcal{P} \mathcal{D} \overline{\mathcal{P}} \mathcal{S}$$

$$+ \mu_8 \Theta^T \mathcal{P} \mathcal{D} \mathcal{P} \mathcal{D} \overline{\mathcal{P}} \mathcal{S} + \mu_9 \Theta^T \mathcal{D} \mathcal{P} \mathcal{D} \overline{\mathcal{P}} \mathcal{S}$$

$$+ \mu_10 \Theta^T \mathcal{P} \mathcal{D} \mathcal{P} \mathcal{D} \overline{\mathcal{P}} \mathcal{S} + \mu_11 \Theta^T \mathcal{P} \mathcal{D} \mathcal{P} \mathcal{D} \overline{\mathcal{P}} \mathcal{S}$$

Proof. The controller gains $K$ and $\overline{K}$, under Theorem 2, could be determined by (73), which leads to (37). Meanwhile, let $\mathcal{H}_1 = \mathcal{E} S^T (i = 1, 2, 3)$; then we use $\text{diag}[S^{-1}, S^{-1}, S^{-1}, \mathcal{S}^{-1}]$ to postmultiplication and premultiplication in inequality (74); condition (36) can be derived. Furthermore, condition (38) is linearized to (76). This accomplishes the proof.

Remark 7. More recently, the design problems of adaptive controller are studied for uncertain nonlinear systems with diverse structures, such as lower triangular structure [37], nonlower-triangular structure [38, 39], and nonstrict-feedback structure [40, 41]. Inspired by these interesting and pioneering works, the event-triggered impulsive control scheme designed and the approach adopted in this paper may be extended in these areas. In addition, through the enlightenment of some excellent works like [42–44], whether our model can be extended to various memristive neural network models would be a probable and rewarding research topic.
4. Illustrative Examples

For the purpose of corroborating the merit and effectiveness of the developed results, two numerical examples are considered in this section.

Example 1. Consider a dynamic model of a vehicle radar servo system. Following [45, 46], the nonlinear dynamic model with parametrized uncertainties can be modeled as

\[
\dot{x}_1 = x_2,
\]

\[
\dot{x}_2 = -(b + \Delta b)x_2 + (c + \Delta c)u - (d + \Delta d)(F_c + fV_f),
\]

\[
y = x_1,
\]

(78)

where \(x_1\) and \(x_2\) are the angular and the angular speed of the motor, respectively; \(b = (B_c/J + m(r^2/G^2))\), \(\Delta b = (\Delta B_c/J + m(r^2/G^2))\), \(c = (K_f/J + m(r^2/G^2))\), \(\Delta c = (\Delta K_f/J + m(r^2/G^2))\), \(d = (r/G) + m(r^2/G^2)\), and \(\Delta d = (r/G\Delta J + m(r^2/G^2))\); \(J\), \(m\), \(r\), \(G\), \(B_c\), \(K_f\), \(i_q\), \(F_c\), and \(f\) are the rotor inertia, the mass of radar antenna, the radius of the gearing wheel, the gearing ratio, the rotor shaft friction, the torque constant, the current in \(q\) axis, the dry friction force, and the wind resistance coefficient, respectively; \(V_f\) is a nonlinear function in regard to radar antenna velocity. Besides, the parameters \(\Delta B_c\), \(\Delta J\), and \(\Delta K_f\) are not precisely known, which are dependent on several factors in the nonlinear dynamics; for instance, the direction of the radar antenna changes with vehicle vibration and gusty winds. Suppose that \(F_c\) is an interfering variable that varies over time. To stabilize this system in the input-to-state sense, now, some parameters are selected as

\[
B_c = 1.7 \times 10^{-4} \text{N} \cdot \text{m} \cdot \text{s},
\]

\[
J = 48 \times 10^{-3} \text{kg} \cdot \text{m}^2,
\]

\[
m = 1000 \text{kg},
\]

\[
r = 50 \text{mm},
\]

\[
G = 1,
\]

\[
K_f = 1.2 \frac{\text{N} \cdot \text{m}}{\text{A}},
\]

(79)

\[
f = 1.4,
\]

\[
F_c = \text{tanh},
\]

\[
V_f = 0.2\text{tanh}(x_2),
\]

\[
\Delta B_c = 0.2 \times 10^{-4} \sin t \text{N} \cdot \text{m} \cdot \text{s},
\]

\[
\Delta J = 3 \times 10^{-3} \cos t \text{kg} \cdot \text{m}^2,
\]

\[
\Delta K_f = 0.2 \sin t \frac{\text{N} \cdot \text{m}}{\text{A}}.
\]

Based on Theorem 2 proposed in this paper, let \(\varepsilon_i = 20, i = 1, 2, \ldots, 29\), \(\varepsilon_f = 25, j = 30, 31, \ldots, 49\), \(\varepsilon_1 = 30\), and \(l = 50, 51, \ldots, 58\), and choose \(\beta = 0.45\), \(\delta = 3 \times 10^{-6}\), and \(\psi = 0.03\). Then, the control gains and the event-triggered mechanism parameters can be designed by utilizing MATLAB toolbox as follows:

\[
K = \begin{bmatrix} -0.2164 & -0.1922 \end{bmatrix},
\]

\[
\bar{K} = \begin{bmatrix} 0.2149 & 0.1974 \end{bmatrix},
\]

\[
L = \begin{bmatrix} -1.6450 & -1.1250 \end{bmatrix},
\]

\[
0 < \eta < 0.2253 \times 10^{-3}, \quad 0 < \ell < 0.56.
\]

In order to achieve simulation, we choose \(\eta = 0.18 \times 10^{-3}\), \(\rho = 0.1\), and \(\ell = 0.4\). When the designed
controller does not act on system (78), as shown in Figure 1, then the system is unstable. However, under control, the change trend of error between the observer state and the system state, the dynamics of the controller updates (the points on the t-axis mean that the controller does not update), and the sampling dynamics of the event triggering mechanism are illustrated in Figures 2–4, respectively. In Figure 4, the point with a value of 1 corresponds to the event trigger sampling time, and the point with a value of 0.5 corresponds to the forced trigger sampling time. Assume that the total elapsed time of system is 20 s, if the output signal is transmitted by a fixed impulsive time sequence or time-triggered scheme, and the sampling period is 0.05 s; then the number of data traffic will be 400, whereas, as shown in Table 1, the amount of data communication can be dramatically decreased by adopting the designed event-triggered scheme. In conclusion, the aforementioned simulation outcomes reflect that when the system state is not completely measurable and uncertainties and external interferences exist in the system, the vehicle radar servo system can accurately estimate the corresponding input in real time and save unnecessary communication resources.

Example 2. To ensure the participation of more uncertainties and the establishment of Assumption 4, uncertain nonlinear system (1) incorporating two subsystems is taken into consideration, whose parameters are set as follows:

\[
B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
B_d = \begin{bmatrix} -1.7 & 0 \\ 0 & -1.7 \end{bmatrix},
A = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix},
C = \begin{bmatrix} 420.3819 & 0 \\ 0 & 420.3819 \end{bmatrix},
D = \begin{bmatrix} -3526.2 & 0 \\ 0 & -3526.2 \end{bmatrix},
\]

\[
f(x) = \begin{bmatrix} 0.3 \tanh(x_1) \\ 0.2 \tanh(x_2) \end{bmatrix},
\]
\[
\nu(t) = \begin{bmatrix} \sin(t) \\ \tanh(t) \end{bmatrix},
M = \begin{bmatrix} -149.5759 & 0 \\ 0 & -149.5759 \end{bmatrix},
F_1 = \begin{bmatrix} 0.11 & 0 \\ 0 & 0.11 \end{bmatrix},
F_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
F_3 = \begin{bmatrix} -0.000545 \\ -0.000545 \end{bmatrix},
F_4 = F_5.
\]

If system (1) has no the control input, as portrayed in Figure 5, its dynamic behavior is unstable under exogenous disturbance and uncertainties. Next, without capturing the information of system states, an observer-based event-triggered impulsive controller will be designed to implement the input-to-state stabilization attribute on system (1). In accordance with Theorem 3, let \( \epsilon_i = 15, i = 1, 2, \ldots, 29; \)
Choose \( \vartheta = 0.1, \vartheta = 10^{-10}, \gamma = 0.001, \varepsilon_1 = 160, \varepsilon_2 = 36520, \varepsilon_3 = 36971, \) and \( \beta = 18.9505; \) then we devise the control gains as

\[
K = \begin{bmatrix} 2.6446 & 2.6446 \end{bmatrix}, \\
\mathbb{K} = \begin{bmatrix} -3.8464 & -3.8464 \end{bmatrix}, \\
L = \begin{bmatrix} 0.01 & 0 \end{bmatrix},
\]

and the parameter of event-triggered mechanism as \( 0 < \eta < 0.0267, \rho > 0, \) and \( 0 < \ell < 0.643, \) so as to achieve the control performance of the input-to-state stabilization. To facilitate the simulation, we select \( \eta = 0.02, \rho = 0.4, \) and \( \ell = 0.4; \) then event-triggered mechanism (16) can be given by

\[
t_{a+1} = t_{a+1}^* \land (t_a + 0.4), \\
t_{a+1}^* = \inf \left\{ t \geq t_a \mid \mathcal{H}(t) \geq 0 \right\},
\]

where

\[
\mathcal{H}(t) = \|\varpi(t)\|^2 - 0.2\|\mathfrak{S}(t_a)\|^2 - 0.4\|\zeta(t)\|^2_{[t_a,t]}.
\]

By exploiting such mechanism and controller gains, the trajectories of states and errors are separately illustrated in Figures 6 and 7, where the evolution of control inputs is exhibited in Figure 8. Moreover, viewed from the triggered dynamics of Figure 9, the event-triggered scheme lessens unnecessary sampling and avoids repeated sampling.

The advantage of the presented strategy in reducing data traffic, in the following, will be further verified. Assume that
the total elapsed time of the closed-loop system (13) is 20s, analyzing in the same way as Example 1, then we can clearly observe from Table 2 that the event-triggered scheme proposed in this paper is superior to the periodic sampling scheme put forward in [21, 35, 36]. This table also exhibits that the sampling amount varies with the parameters \( \ell \), \( \eta \), and \( \rho \). It can be seen that the greater the value of \( \ell \) or \( \rho \) is, the less the data communication would be generated and thus the more resources would be saved.

5. Concluding Remarks

This article investigates innovatively the input-to-state stabilization and controller design for a type of uncertain nonlinear systems included partially measurable states and exogenous disturbances. A suitable event-triggered mechanism is constructed which decides when the controllers are updated. With the combination of sample control, subsequently, an observer-based impulsive controller has been devised to warrant the performance of input-to-state stabilization on the uncertain controlled system. Particularly, the Zeno behavior in presented control scheme has been eliminated. With the aid of several analysis strategies and the linear matrix inequality technology, some sufficient criteria are deduced to guarantee the input-to-state stabilization. In addition, substantial uncertain parameters are reasonably estimated by exploiting some constant matrix inequalities and an innovative hypothesis. Although the reasoning process is slightly more complicated, fortunately, the result of theoretical analysis and simulations can demonstrate that the observer-based event-triggered impulsive scheme can work well. In some circumstances, the controller perhaps oversteps its physical restrictions that possibly lead to the controlled system subjected to rigorous performance degradation. Consequently, it is a probable research work for observer-based event-triggered saturated impulsive control on a category of nonlinear controlled systems in the future.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
Authors’ Contributions
This study was carried out by the Master’s student Xiangru Xing under the direction of Assistant Professor Jin-E Zhang.

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