Galaxy filaments as pearl necklaces

E. Tempel\textsuperscript{1,2}, R. Kipper\textsuperscript{1,3}, E. Saar\textsuperscript{1,4}, M. Bussov\textsuperscript{5}, and J. Pelt\textsuperscript{1}

\textsuperscript{1} Tartu Observatory, Observatooriumi 1, 61602 Tõravere, Estonia
e-mail: elmo.tempel@to.ee
\textsuperscript{2} National Institute of Chemical Physics and Biophysics, Rävala pst 10, 10143 Tallinn, Estonia
\textsuperscript{3} Institute of Physics, University of Tartu, 51010 Tartu, Estonia
\textsuperscript{4} Estonian Academy of Sciences, Kohtu 6, 10130 Tallinn, Estonia
\textsuperscript{5} Institute of Mathematical Statistics, University of Tartu, 50409 Tartu, Estonia

ABSTRACT

Context. Galaxies in the Universe form chains (filaments) that connect groups and clusters of galaxies. The filamentary network includes nearly half of the galaxies and is visually the most striking feature in cosmological maps.

Aims. We study the distribution of galaxies along such a filamentary network, trying to find specific patterns.

Methods. Our galaxy filaments are defined using the Bisous process. We use the two-point correlation function and the Rayleigh $Z$-squared statistic to study how the galaxies are distributed along the filaments.

Results. We show that galaxies and galaxy groups are not uniformly distributed along filaments, but tend to form a regular pattern. The characteristic length of the pattern is 7 \(h^{-1}\) Mpc. A slightly smaller characteristic length 4 \(h^{-1}\) Mpc can also be found, using the $Z$-squared statistic.

Conclusions. One can say that galaxy filaments in the Universe are like pearl necklaces, where the pearls are galaxy groups distributed more or less regularly along the filaments. We propose that this well defined characteristic scale could be used as a cosmological test.

Key words. Methods: numerical – methods: observational – large-scale structure of Universe.

1. Introduction

Many cosmological probes have been developed and used to derive the values of the parameters of the cosmological models describing our Universe. Most of them rely on some aspects of the large-scale structure. One of the most common probes to quantify galaxy clustering is the two-point correlation function which has been used already decades ago (Davis & Geller 1976; Groth & Peebles 1977; Davis et al. 1988; Hamilton 1988). Some recent examples include Connolly et al. (2002), Zehavi et al. (2005), Contreras et al. (2013), de Simoni et al. (2013), Wang et al. (2013), and references in these papers. Other examples of cosmological probes are three-point correlation functions (e.g. McBride et al. 2011; Marin et al. 2013; Guo et al. 2014) and the power spectrum analysis (Hütsi 2006; 2010; Blake et al. 2010; Balaguera-Antolínez et al. 2011).

It is well known that galaxy filaments are visually the most dominant structures in the galaxy distribution. Presumably, nearly half of the galaxies are located in filaments (e.g. Jusche et al. 2010; Cautun et al. 2014). The properties of the three-point correlation function also indicate that galaxies tend to populate filamentary structures (Guo et al. 2014). Tempel et al. (2014a) shows that filaments extracted from the spatial distribution of galaxies/haloes are also dynamical structures that are well connected with the underlying velocity field. Galaxy filaments that connect groups and clusters of galaxies are also affecting the evolution of galaxies (e.g. Tempel & Libeskind 2013; Tempel et al. 2013). So, galaxy filaments are structures that are determined by the underlying cosmology and galaxy filaments also influence the formation and evolution of galaxies in them.

In this paper we study the distribution of galaxies along galaxy filaments to search for regularities in galaxy and group distributions. Such a regularity exists at least in some filaments as shown decades ago (e.g. Joeveer et al. 1978; Einasto et al. 1980). In the current study, we use the two-point correlation function and the Rayleigh $Z$-squared statistics to check for the regularity in the galaxy distribution. We suggest that the measured regularities in the galaxy distribution could be used as cosmological probes for dark energy and dark matter, the mysterious components in the dark energy dominated cold dark matter ($\Lambda$CDM) cosmological models.

Throughout this paper we assume the Wilkinson Microwave Anisotropy Probe (WMAP) cosmology: the Hubble constant \(H_0 = 100h\ \text{km}\ \text{s}^{-1}\text{Mpc}^{-1}\), with \(h = 0.697\), the matter density \(\Omega_m = 0.27\) and the dark energy density \(\Omega_\Lambda = 0.73\) (Komatsu et al. 2011).

2. Data and methods

2.1. Galaxy and filament samples for the SDSS data

The present work is based on the Sloan Digital Sky Survey (SDSS, York et al. 2000) data release 10 (DR10; Ahn et al. 2014). We use the galaxy and group samples as compiled in Tempel et al. (2014c) that cover the main contiguous area of the survey (the Legacy Survey, approximately 17.5% from the full sky). The flux-limited catalogue extends to 574 \(h^{-1}\) Mpc and includes 588193 galaxies and 82458 groups with two or more members. In Tempel et al. (2014c) the finger-of-god (FoG) effect is suppressed using the detected galaxy groups.

The catalogue of filaments is built by applying an object point process with interactions (the Bisous process; Stoica et al. 2014).
to the distribution of galaxies. The method and parameters are exactly the same as in Tempel et al. (2014b), where the Bisous model was applied to the SDSS DR8 (Aihara et al. 2011) data. Since the datasets in the SDSS DR8 and DR10 are basically the same, the extracted filaments in DR10 are statistically the same as presented in Tempel et al. (2014b). The assumed scale (radius) for the extracted filaments is roughly $0.5 \, h^{-1}\text{Mpc}$. Because of the flux-limited survey, the sample is very diluted farther away. Hence, we are only able to detect filaments in this scale up to $450 \, h^{-1}\text{Mpc}$.

A detailed description of the Bisous model is given in Stoica et al. (2007), 2010 and Tempel et al. (2014b). In the Bisous algorithm, random segments (thin cylinders) based on the positions of galaxies are used to form the filamentary network according to the connection and alignment rules between these segments. The morphological and quantitative characteristics of these complex geometrical objects can be obtained by following a straightforward procedure: constructing a model, sampling the probability density describing the model, and, finally, applying the forward procedure: constructing a model, sampling the probability and filament orientation fields. The filament probability field is detected using a Markov-Chain Monte-Carlo scheme that effectively samples a large parameter space. A deterministic filamentary pattern spine can be extracted based on the detection probability and filament orientation fields (see Tempel et al. 2014b). Using this method, we extract single filaments in the survey. The longest filaments in our sample are up to $70 \, h^{-1}\text{Mpc}$ long.

The filaments are extracted using a flux-limited galaxy sample, hence, the completeness of extracted filaments decreases with distance. In Tempel et al. (2014b) we showed that the volume filling fraction of filaments is roughly uniform if filaments longer than $15 \, h^{-1}\text{Mpc}$ are considered. Therefore, in the current study we are using only filaments longer than this limit. In addition, this choice is justified since longer filaments allow us to study the distribution of galaxies along the filaments, which is the purpose of this paper. The remaining incompleteness of filaments in our sample is not a problem, because we are analysing single filaments. The filaments we use are the strongest filaments (or segments of filaments) in the sample.

To suppress the flux-limited sample effect, we volume-limit the galaxy content for single filaments. For that, we find for every filament the maximum distance (from observer) of its galaxies and the corresponding magnitude limit and use only galaxies brighter than that. Since the majority of filaments extend over a relatively narrow distance interval (always narrower than the length of the filament), the number of excluded galaxies in every filament is small.

To study the galaxy spacing along the filaments, we project every galaxy to the filament spine. Hence, the distance along the filament is measured along the filament spine. Figure 1 illustrates our galaxy filaments and the filament spines in the field of galaxies.

For our analysis, we use only filaments that include at least 10 galaxies. When studying galaxy groups, we use only filaments where the number of groups per filament is at least 5. We use the groups given in Tempel et al. (2014b). The upper panel in Fig. 2 shows the cumulative filament length and the cumulative number of galaxies in filaments as a function of filament length.

The lower panel in Fig. 2 shows the filament length distribution. In this study, a galaxy is considered to belong to a filament if it is closer than $0.5 \, h^{-1}\text{Mpc}$ to the filament spine. This is also the scale of detected filaments in our Bisous model. In addition, we study the distribution of galaxies that are closer than $0.25 \, h^{-1}\text{Mpc}$ to the filament spine. From Fig. 2 we see that the total filament length in our study is $50000 \, h^{-1}\text{Mpc}$ and half of it comes from filaments shorter than $23 \, h^{-1}\text{Mpc}$. The number of filaments and galaxies in filaments for studied subsamples are given in Table 1.

### Table 1. The numbers of filaments ($N_{fil}$) and galaxies in filaments ($N_{gal}$) in various subsamples. Only filaments longer than $15 \, h^{-1}\text{Mpc}$ and containing at least 10 galaxies (or 5 groups) are considered. $N_{rich}$ denotes the group richness (number of galaxies in groups) and $d_{fil}$ marks the galaxy distance from filament spine.

| Sample | $N_{fil}$ | $N_{gal}$ |
|--------|-----------|-----------|
| All galaxies ($d_{fil} < 0.5 \, h^{-1}\text{Mpc}$) | 2150 | 41543 |
| All galaxies ($d_{fil} < 0.25 \, h^{-1}\text{Mpc}$) | 1752 | 26830 |
| Groups ($N_{rich} \geq 1; \, d_{fil} < 0.5 \, h^{-1}\text{Mpc}$) | 1943 | 15874 |
| Groups ($N_{rich} \geq 1; \, d_{fil} < 0.25 \, h^{-1}\text{Mpc}$) | 1493 | 10492 |
| Groups ($N_{rich} \geq 2; \, d_{fil} < 0.5 \, h^{-1}\text{Mpc}$) | 759 | 4667 |
| Groups ($N_{rich} \geq 2; \, d_{fil} < 0.25 \, h^{-1}\text{Mpc}$) | 401 | 2323 |
| Groups ($N_{rich} \geq 3; \, d_{fil} < 0.5 \, h^{-1}\text{Mpc}$) | 211 | 1196 |
| Groups ($N_{rich} \geq 3; \, d_{fil} < 0.25 \, h^{-1}\text{Mpc}$) | 111 | 606 |

2.2. Estimating the pair correlation function

To study the galaxy correlations in galaxy filaments, we use the two-point correlation function $\xi(r)$ that measures the excess probability of finding two points separated by a vector $r$ compared to that probability in a homogeneous Poisson sample (Peebles 1980; Martínez & Saar 2002). In statistics, this quantity is called the pair correlation function (Stoyan & Stoyan 1994). For galaxy filaments, the correlation function along the filament can be expressed simply in terms of the distances between the galaxies along the filaments. Note that this is not exactly the case when studying a sample from a galaxy redshift survey. The line-of-sight component of the position of a galaxy is derived from the observed redshift, hence, the distance along the line of sight is influenced by redshift distortions. However, since filaments are defined in comoving coordinates and we are interested on scales larger than the group/cluster scale, we can ignore this effect. The distances between galaxies along filaments in this study are measured along the filament spines that are defined in comoving coordinates. Below (see Fig. 3) we divide our filament sample into parallel to the line of sight and perpendicular to the line of sight subsamples and show that the redshift space distortions do not affect our results.

We estimate $\xi(r)$ following the Landy-Szalay border-corrected estimator (Landy & Szalay 1993). We generated a random distribution of points for each filament considered, and estimated the correlation function $\tilde{\xi}(r)$:

$$\tilde{\xi}(r) = 1 + \frac{DD(r)}{RR(r)} - 2 \frac{DR(r)}{RR(r)},$$

where $DD(r)$, $RR(r)$, and $DR(r)$ are the probability densities for the galaxy-galaxy, random-random, and galaxy-random pairs, respectively, for a pair distance $r$. In this work, we fix the size of the random point set for each filament $N_{rad} = 50N$ ($N$ is the number of galaxies in each filament). We tested that our results do not change if we increase the number of random points used to $N_{rad} = 100N$.

We estimate the probability densities by the kernel method, summing the box spline $B_1(\cdot)$ kernels (Saar et al. 2007) centred...
at each pair distance, and sampling the distributions at smaller intervals than the kernel width. The kernel width is 1.0 $h^{-1}$Mpc (for groups it is twice as large). In [Martínez et al. 2009] the same method is applied to detect baryon acoustic peaks. In general, this method does not require binning the data and with a good choice of the kernel width, can optimally recover the probability distribution. In practice, the choice of the kernel width (if it is reasonable) does not affect the results, and most importantly it does not introduce any bias in the correlation estimator. If the kernel is too wide, the signal is lost, and in the contrary, if the kernel is too narrow, the noise dominates.

The integral constraint is a bias in the estimation of the correlation function due to the use of a finite volume sample (a finite length of a filament in our case). The integral constraint does not affect the shape of the correlation function, it only changes the normalisation. Since the lengths of the filaments are comparable with the studied scale of the correlation function and the length of each filament is different (hence the bias is different), we cannot ignore the integral constraint. It is shown that the bias introduced by the integral constraint is given, at first order, by (Bernardeau et al. 2002; Labatie et al. 2012)

$$\xi(r) = \xi^{\text{true}}(r) - K,$$

where following Roche et al. (1999) the bias $K$ can be computed using the auxiliary Poisson sample.

In all cases studied below, we checked that the integral constraint correction does not change the features of our measured correlation function, it mostly affects only the general scaling of the correlation function. Figure 2 shows the Landy-Szalay correlation function estimator without (red) and with (black) integral constraint correction. The differences are irrelevant, hence, our results are not sensitive to the details of the estimation of $K$.

Additionally, we checked how the estimated correlation function depends on used estimator. In Fig. 3 we show the correlation function estimated using the Double Poisson sample (Davis & Peebles 1983) and Hamilton (1993) estimators. The differences between various estimators are very small, and they do not affect the features in correlation function that we are studying.

Using Eq. (1), we estimate the correlation function for each filament separately. The final pair correlation function, averaged over all filaments, is estimated as a weighted sum

$$\xi^\text{fil}(r) = \frac{\sum_{i=1}^{N_{\text{fil}}} \mathbb{I}\{L_{\text{fil}} > r\}L_{\text{fil}} \xi_i(r)}{\sum_{i=1}^{N_{\text{fil}}} \mathbb{I}\{L_{\text{fil}} > r\}L_{\text{fil}}},$$

where summation is over all filaments, $\xi_i$ is the correlation function for a single filament, $L_{\text{fil}}$ is the length of $i$th filament, and $\mathbb{I}\{\cdot\}$ is the indicator function that selects only filaments longer than $r$.

We estimate the statistical error on our $\xi^\text{fil}(r)$ measurements with the standard jackknife method (see e.g. Norberg et al. 2009), where we omit one filament under consideration at a time. In our figures we show the 95% confidence intervals.

Fig. 1. Examples of filaments and their spines. Red points show galaxies in filaments (closer than 0.5 $h^{-1}$Mpc to the filament axis) that are located in groups with 5 or more members. The group richness is taken from Tempel et al. (2014c). Blue points show other galaxies in filaments. Grey points are background galaxies that are not located in these filaments. Note that some of the background galaxies project to the filament spines, but are actually located farther than 0.5 $h^{-1}$Mpc from the spine. The thick green line shows the spine of a filament. To show the scale of structures, cartesian coordinates are shown around each filament.

Fig. 2. Upper panel: the cumulative filament length and cumulative number of galaxies in filaments as a function of filament length: the lines show the sum of filament length (red line) and number of galaxies in filaments (blue line) summed over filaments shorter than indicated in abscissa. On average there is slightly less than one galaxy per $h^{-1}$Mpc. Lower panel: filament length distribution in our sample.
2.3. Rayleigh (Z-squared) statistic

To test whether the galaxy distribution might have some regularity along the filaments we use the Rayleigh (or Z-squared) statistics (see e.g. Buccheri et al. 1983 [11], Muno et al. 2003). It is an excellent method when the event rate (in our case, the number of galaxies per filament) are low. The method has been used to detect periodicity in time series for the data in the form of discrete events (photon arrival times). However, it can be applied to detect periodicity in the galaxy distribution along filaments, where the galaxy positions can be considered as events.

The algorithm works as following. For each filament, we produce a periodogram using the Z^2_1 (Rayleigh statistics),

\[
Z^2_1 = \frac{2}{N} \left[ \left( \sum_{j=1}^{N} \cos \phi_j \right)^2 + \left( \sum_{j=1}^{N} \sin \phi_j \right)^2 \right]
\]

where N is the number of galaxies in a filament and \( \phi_j = 2\pi l_j/d \) is the phase value for a galaxy \( j \) for a fixed period \( d \); \( l_j \) is a galaxy \( j \) distance along the filament spine from the beginning of the filament.

To measure the Z-squared statistics for a period \( d \), we are using only filaments longer than 2d. This assures that there are at least two periods for each filament. For a signal resulting purely from a Poisson noise, \( Z^2_1 \) has a \( \chi^2 \) distribution with two degrees of freedom. However, this is only in the case if the number of events is high enough. If the number of events is lower than 100 (as usual in our case), \( Z^2_1 \) does not have a \( \chi^2 \) probability distribution. So, we are deriving a null-hypothesis probability function using Monte Carlo simulations with \( N \) (the number of galaxies in a filament) data points. This allows us also to estimate the confidence intervals for our measured signal.

We compute the \( Z^2_1 \) statistics as a function of period \( d \) for every filament and then find the average signal using the filament length \( L_{fil} \) as the weight

\[
Z^2_{1,fil}(d) = \frac{\sum_{i=1}^{N_{fil}} [L_{fil,i} > 2d] L_{fil,i} Z^2_1(j,d)}{\sum_{i=1}^{N_{fil}} [L_{fil,i} > 2d] L_{fil,i}}
\]

where \( Z^2_{1,fil} \) is the Z-squared statistic for \( i \)-th filament. The confidence limits are found using a jackknife technique similarly to correlation functions.

To illustrate how the Z-squared statistics works, we generated three datasets and calculated the Rayleigh statistics for them. The results are shown in Fig. 4. In the first case, we generated a Poisson distribution (green line). For the Poisson sample, the statistics gives an average value 2. In the second case, we added some periodicity to the Poisson sample (blue line). For each datapoint, the period was chosen from a Gaussian distribution centred at 7 h^{-1}Mpc with a standard deviation of 0.5 h^{-1}Mpc. From Fig. 4 we see that the period is well seen in the Z-squared statistics. In the third case, we generated a uniform distribution of points (red line). For that, we divided the test filament into \( N \) (the number of points) equal regions and in each region we put one point according to a uniform distribution. Doing that the points are more homogeneously distributed along the filament and the Z-squared statistics gives the value zero. We can conclude that if the value of the statistic lies above 2, there is some periodicity in the data. Contrary, if it is below 2, it refers to a more homogeneous distribution. Additionally, the peaks in the Z-squared statistic show that there is preferred periodicity in the data with the scale of the peak position.

3. Results

3.1. Correlations along galaxy filaments

Figure 5 shows the correlation function along filaments. Three specific features are seen in this correlation function: a maximum near zero pair distance, a minimum that follows it, and a bump close to 7 h^{-1}Mpc. The first maximum is caused by galaxy groups. It shows that galaxies are not distributed uniformly in the space, they form groups and clusters, as it is well known. The minimum next to the first maximum shows that groups themselves are not distributed uniformly along filaments. It shows that two groups cannot be located directly close to each other (merging groups are exceptions) and there exists some preferred minimum distance between galaxy groups. This is also expected since matter is falling into groups and there is not enough matter in a close-by neighbourhood of groups to form another group. The most interesting feature is the small maximum close to
7 $h^{-1}\text{Mpc}$. It shows that galaxies (and also groups) have tendency to be separated by this distance along the filaments. We will analyse the nature of this bump in more detail below.

Since we are using a galaxy sample where FoGs are suppressed, we checked whether this influences our results. For that we divided our filament sample into two subsamples: mostly parallel to the line-of-sight ($\cos i > 0.5$) and mostly perpendicular to the line-of-sight ($\cos i < 0.5$). Here $i$ denotes the angle between the direction of the filament and the line of sight. The correlation functions for these two subsamples are shown in the middle and lower panels in Fig. 5, respectively. We note that the bump around $7 h^{-1}\text{Mpc}$ is visible in both subsamples, hence, it is a real feature. However, the bump is slightly weaker for the filaments perpendicular to the line of sight. So, we also calculated the correlation function for filaments where $\cos i < 0.25$, and the bump remained in the same place. We conclude that the $7 h^{-1}\text{Mpc}$ maximum is not affected by the FoG effect. The most noticeable difference between the middle and lower panels in Fig. 5 is that the minimum is stronger for filaments parallel to the line-of-sight. This is probably because of our FoG suppression. When suppressing FoGs, we also suppress the line-of-sight scatter of some of the background and foreground galaxies in the group neighbourhood that creates the lack of galaxies close to the groups. To test the effect of FoG suppression, we calculated the correlation signal for galaxies closer than 0.5 and $0.25 h^{-1}\text{Mpc}$ to the filament axis (blue and red lines in Fig. 5, respectively). For the smaller radius, the effect of finger-of-god suppression should be smaller. As expected, the prominent features are visible in both subsamples.

In Fig. 5, the correlation function is shown for galaxy groups. In this figure, the smoothing scale is twice as large as for all galaxies and we analyse only filaments that contain at least five groups. The Fig. 6 shows the correlation function for groups with different minimum group richness. We see that the bump around $7 h^{-1}\text{Mpc}$ is present in every subsample, indicating that groups themselves have some preferred distance between them. Interestingly, the upper panel in Fig. 6 shows a small bump around $4 h^{-1}\text{Mpc}$ as well. This scale is also seen when using the Z-squared statistics (see Sect. 3.2).
The biggest difference when comparing the correlation function for galaxies (Fig. 5) and for groups (Fig. 6) is the fact that there is no zero-distance maximum for groups. This shows that groups themselves do not form clusters, which is expected. The minimum close to zero distance in the group correlation function shows that there is a lack of other groups around each group. The radius of the group influence extends to 4 $h^{-1}$Mpc, after that the correlation function is mostly flat.

### 3.2. Regularity of the galaxy distribution along filaments

Figure 7 shows the $Z^2$-squared statistic for all galaxies closer than 0.5 $h^{-1}$Mpc to the filament axis and the lower panel shows the results for galaxies closer than 0.25 $h^{-1}$Mpc. The red line shows the $Z^2$ statistics together with the jackknife 95% confidence estimate. The blue line shows the results from Monte Carlo simulations for the null hypothesis together with the 95% confidence limits.

![Figure 7. The Rayleigh ($Z^2$) statistic $Z^2$ for a given distance (period). The upper panel shows the results for galaxies closer than 0.5 $h^{-1}$Mpc to the filament axis and the lower panel shows the results for galaxies closer than 0.25 $h^{-1}$Mpc. The red line shows the $Z^2$ statistics together with the jackknife 95% confidence estimate. The blue line shows the results from Monte Carlo simulations for the null hypothesis together with the 95% confidence limits.](image)

The biggest difference when comparing the correlation function for galaxies (Fig. 5) and for groups (Fig. 6) is the fact that there is no zero-distance maximum for groups. This shows that groups themselves do not form clusters, which is expected. The minimum close to zero distance in the group correlation function shows that there is a lack of other groups around each group. The radius of the group influence extends to 4 $h^{-1}$Mpc, after that the correlation function is mostly flat.

### 3.2. Regularity of the galaxy distribution along filaments

Figure 7 shows the $Z^2$-squared statistic for all galaxies closer than 0.5 $h^{-1}$Mpc (upper panel) and 0.25 $h^{-1}$Mpc (lower panel) to the filament axis. The $Z^2$-squared statistic based on galaxies is shown with red lines, where the shaded region shows the 95% confidence limits. The blue line shows the statistic for the null hypothesis using Monte Carlo simulation for a Poisson sample. The shaded region shows the 95% confidence limits. For Monte Carlo simulation, the filaments and numbers of galaxies per filament are the same as for the real sample, but galaxies are Poisson distributed. Since we are averaging filaments with different lengths, the $Z^2$-squared statistic for the Poisson sample is not exactly around 2 as in Fig. 4. Since the deviation is small, it does not affect our conclusions.

The biggest difference when comparing the correlation function for galaxies (Fig. 5) and for groups (Fig. 6) is the fact that there is no zero-distance maximum for groups. This shows that groups themselves do not form clusters, which is expected. The minimum close to zero distance in the group correlation function shows that there is a lack of other groups around each group. The radius of the group influence extends to 4 $h^{-1}$Mpc, after that the correlation function is mostly flat.

### 4. Concluding remarks

Using the Bisous process we extracted the galaxy filaments from the SDSS spectroscopic galaxy survey. The scale (diameter) of the extracted filaments is roughly 1 $h^{-1}$Mpc. Using a marked point process we extracted the filament spines as described in Tempel et al. (2014b). Using the galaxies and groups in filaments (with a distance from the filament axis less than 0.5 $h^{-1}$Mpc) we studied how the galaxies/groups are distributed along the filament axis. The main results of our study can be summarised as following.
The galaxy and group distributions along filaments show a regular pattern with a preferred scale around $7 \ h^{-1} \text{Mpc}$. A weaker regularity is also visible at a scale of $4 \ h^{-1} \text{Mpc}$. The regularity of the distribution of galaxies along filaments is a new result that might help to understand the structure formation in the Universe.

The pair correlation functions of galaxies and groups along filaments show that around each group, there is a region where the number density of galaxies/groups is smaller than on average.

Galaxy groups in the Universe are more uniformly distributed along filaments than in the Poisson case.

The regularity of the alignment of galaxies and groups along filaments tells us that galaxy filaments are like pearl necklaces, where the pearls are galaxy groups that are distributed along the filaments in some regular pattern.

We suggest that the measured regularity of the galaxy distribution along filaments could be used as a cosmological probe to discriminate between various dark energy and dark matter cosmological models. We plan to test this in our following analysis using N-body simulations.

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