Proper use of technical risk assessment tools as an element of the railway safety culture

O I Verevkina¹ and I A Veprinyak²

¹ Rostov State Transport University, 2, Lenina Str., Rostov-on-Don, 344038, Russia
² Military Academy of logistics named after General of the army A.V. Khrulev, 1, Suvorovskaya str., Peterhof, St. Petersburg, 198504, Russia

E-mail: ov18111966@mail.ru

Abstract. The work is aimed at developing ideas and improving the technical means of risk assessment – the risk matrix. A typical risk matrix defined by regulatory sources is considered. An original approach to evaluating the effectiveness of the risk matrix with associated scales is proposed. The definition of the risk matrix with minimum scale spacing is introduced and the existence of this matrix is proved. It is shown by example that a matrix with minimum scale spacing is a more accurate tool for determining the risk category compared to matrices with wider scale spacing. The problem of finding the parameters of such a matrix from the initial data in a mathematical statement is formulated. One of the branches of the developed simplified algorithm for constructing such a matrix is given.

1. Introduction

The current time in railway transport is a time of increasing the culture of traffic safety management. One of the important elements of traffic safety management is risk assessment, and the technical tool used for risk assessment is the risk matrix. The use of this tool for interval risk assessment is widespread in railway transport [1-5]. However, the issue of correctly setting the problem of determining the parameters of a typical risk matrix, as well as correctly calculating these parameters, has not been raised before. Below, the problem is formulated in a strict mathematical statement and compared with the solution in the normative source.

2. Preliminary statement of the problem

The problem statement is given for a typical risk matrix with the size of 6×4.

2.1 Definition 1

The risk plane (Fig. 2) is the I-th quadrant of the plane in which the axis scale is set in an exponential manner:

\[ f = f_n \cdot K^x \]  \hspace{1cm} (1)

\[ c = c_n \cdot K^y \]  \hspace{1cm} (2)

where \( f_n, c_n, K \) – positive constants,
\( x, y \) – non-negative coordinates (x – abscissa, y – ordinate).
2.2 Definition 2
A risk matrix is a rectangular region in the plane of risk, with the coordinates of the vertices: \((c_{\text{min}}, f_{\text{min}}), (c_{\text{max}}, f_{\text{min}}), (c_{\text{min}}, f_{\text{max}}), (c_{\text{max}}, f_{\text{max}})\) satisfying the conditions (3-4):

\[
\left( \frac{f_{\text{max}}}{f_{\text{min}}} \right)^{1/2} = \left( \frac{c_{\text{max}}}{c_{\text{min}}} \right)^{3/8} \tag{3}
\]

\[
f_{\text{max}} \cdot (c_{\text{min}} \cdot c_{\text{max}})^{1/2} = R \tag{4}
\]

where \(R\) – positive number assigned by the level of "permissible risk", the left and right part of the ratio (3) is called the scale spacing \(K\) of the risk plane:

\[
K = \left( \frac{f_{\text{max}}}{f_{\text{min}}} \right)^{1/2} = \left( \frac{c_{\text{max}}}{c_{\text{min}}} \right)^{3/8}. \tag{5}
\]

The problem in this statement is formulated in [3, 4]

3. The problem of determining the parameters of the risk matrix
Let us set the frequency limits \((a_{\text{min}}, a_{\text{max}})\) of adverse events, and the limits of consequences \((b_{\text{min}}, b_{\text{max}})\), as well as the level of permissible risk: \(R_{\text{perm}}\).

Based on the data presented above, it is necessary to find such a risk matrix that in addition to condition (3), the following conditions are met:

\[
f_{\text{max}} \cdot (c_{\text{min}} \cdot c_{\text{max}})^{1/2} = R_{\text{perm}} \tag{6}
\]

\[
f_{\text{min}} \leq a_{\text{min}} \tag{7}
\]

\[
f_{\text{max}} \geq a_{\text{max}} \tag{8}
\]

\[
c_{\text{min}} \leq b_{\text{min}} \tag{9}
\]

\[
c_{\text{max}} \geq b_{\text{max}}. \tag{10}
\]

We note that there are an infinite number of risk matrices that solve this problem, which differ in the magnitude of the scale spacing, as well as the coordinates that define the rectangle of the risk matrix, for the same level of permissible risk \(R_{\text{perm}}\). This is due to the fact that 2 (two) equations and 4
We will prove the following statement: there is the smallest of all \( K \) belonging to the set \( \{ K \} \) of all risk matrices defined by the data: \( a_{\text{min}}, a_{\text{max}}, b_{\text{min}}, b_{\text{max}}, R_{\text{perm}} \).

\[ (11) \]

4. Evidence

Since, according to (5) \( K = \left( \frac{c_{\text{max}}}{c_{\text{min}}} \right)^{3/8} \), and according to the requirements for the risk matrix (9)

\[ c_{\text{max}} \geq b_{\text{max}}, \text{ then:} \]

\[ \left( \frac{c_{\text{max}}}{c_{\text{min}}} \right)^{3/8} \geq \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)^{3/8}. \]

\[ (12) \]

At the same time, according to (9) \( c_{\text{min}} \leq b_{\text{min}} \):

\[ \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)^{3/8} \geq \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)^{3/8}. \]

\[ (13) \]

Combining (11) and (12) we get: \( K \geq \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)^{3/8} \).

\[ (14) \]

Similarly, using (7)-(8), we obtain: \( K \geq \left( \frac{a_{\text{max}}}{a_{\text{min}}} \right)^{1/2} \).

\[ (15) \]

Choosing the minimum of the two values on the right side (13) and (14):

\[ K_{\text{min}} = \max \left\{ \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)^{3/8}, \left( \frac{a_{\text{max}}}{a_{\text{min}}} \right)^{1/2} \right\} > 0, \]

we get that for any \( K \):

\[ K \geq K_{\text{min}}. \]

Since the set of all \( K \) is a limit from below, according to the theorem [6, 7] on the lower bound of the set, the set \( \{ K \} \) has a lower limit that we will consider the smallest value, or the smallest spacing of the risk matrix defined by fixed initial data, and thus the statement is proved.

Since it is known [4] that the accuracy of estimation using the risk matrix decreases with increasing \( K \), it is natural to consider the risk matrices with the smallest scale spacing as the best one (we can hypothesize that these risk matrices are real).

Here is an example of the fact that when manipulating an arbitrary extension of the initial data (that is, essentially increasing \( K \)), errors may occur in determining the risk category: undesirable, permissible, not taken into account. As an example, let us take the data from [4]:

\[ a_{\text{min}} = 293 \text{ 1/year}; \ a_{\text{max}} = 1031 \text{ 1/year}; \]

\[ b_{\text{min}} = 0.081 \text{ train-hour}; \ b_{\text{max}} = 0.727 \text{ train-hour}. \]

\[ R_{\text{perm}} = 150 \text{ train-hour/year}. \]

Figures 2 and 3 below show 2 risk matrices, the parameters of the first one are calculated based on the minimum scale spacing requirement (without expanding the initial data), and the second one is
calculated taking into account the initial scale expansion, which is a part of the calculation algorithm presented in [4]. The sloping lines are lines of the equal level of risk $R_{\text{perm}}$, $R_{\text{perm}}/K$, $R_{\text{perm}}/K^2$ respectively.

The risk category is determined not by belonging to a rectangle of a particular color, but by its location relative to these lines, according to table 1. Such matrices are called refined according to [4].

Risk matrix for loss of train hours because of technical equipment failures
(2013, communications industry)

| Event frequency | Specific damage - loss of train hours per 1 failure |
|-----------------|-----------------------------------------------------|
| frequent        | minor                                                |
| likely          | insignificant                                         |
| random          | significant                                           |
| rare            | critical                                              |
| extremely rare  |                                                     |
| unlikely        |                                                     |

Figure 2. The risk matrix with the minimum scale spacing

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| rare            | critical                                              |
| extremely rare  |                                                     |
| unlikely        |                                                     |

Figure 3. Risk matrix with an expanded scale spacing
Let us consider the differences in the possibilities of the criterion risk assessment, taking into account that the risk category is assigned in accordance with Table 1 [3.4].

We will estimate the category of hypothetical risk consisting in the fact that there are 210 failures with the damage of 0.163 train-hours per year. To do this we shall note the coordinates of the point on both risk matrices. Obviously, whilst assessing risk with a minimum scale spacing matrix (Figure 2), the risk category is permissible. When assessing risk with an expanded scale spacing matrix, the same risk falls into the category undesirable. Calculated category boundaries for both risk matrices are presented in Table 1.

Table 1. The boundaries of the criterion values

| Criterion | $R \leq R_{perm}K^2$ | $R_{perm}/K^2 < R \leq R_{perm}/K$ | $R_{perm}/K < R \leq R_{perm}$ | $R > R_{perm}$ |
|-----------|----------------------|---------------------------------|---------------------------------|----------------|
| matrix with a scale spacing of 3.34 | $R \leq 13.4$ | $13.4 < R \leq 44.9$ | $44.9 < R \leq 150$ | $R > 150$ |
| matrix with a scale spacing of 8.68 | $R \leq 2.0$ | $2.0 < R \leq 17.3$ | $17.3 < R \leq 150$ | $R > 150$ |

Let us consider the differences in the possibilities of the criterion risk assessment, taking into account that the risk category is assigned in accordance with Table 1 [3].

It is obvious that the second matrix misjudges the "permissible and undesirable risk" categories for all risks at the boundary, (17.3; 44.9). The 34.6 risk we are considering falls within this interval. The length of the interval is 27.6, which is about 25% of the length of the interval in the "undesirable risk" category.

The analysis clearly and numerically shows the significance of the fact that risk matrix should be derived from initial data without any preliminary arbitrary scale changes. After all, classifying a risk as "undesirable" means the need for increased attention to risk and the development of measures to reduce the risk to the "permissible category in many cases.

All of the above forces us to formulate the problem of finding the parameters of the risk matrix in the following statement, which provides the minimum scale spacing for the initial data:

We find such a risk matrix that converting a rectangle with borders $(b_{min}, a_{max})$, $(b_{max}, a_{min})$, $(b_{min}, a_{max})$, $(b_{max}, c_{max})$ to a rectangle of the risk matrix provides minimal changes in coordinates.

Mathematical record of the last condition:

$$\min \left( \begin{array}{c} \frac{a_{min}}{f_{max}} + \frac{1 - c_{min}}{b_{min}} + \frac{1 - f_{max}}{a_{max}} + \frac{1 - c_{max}}{b_{max}} \end{array} \right) \to \min. \quad (16)$$

In addition to condition (3), the above conditions (6)-(10) must be met. Thus, we come to the problem of minimizing a certain function, which is reduced to the problem of linear programming [8-11] when a number of restrictions are met.

The problem stated above can also be solved using a constructive algorithm. Next we will give one of its branches that works under the condition:

$$R_{perm} > f_{max} \cdot (c_{min} \cdot c_{max})^{1/2}. \quad (17)$$

Table 2 shows the actions necessary to be performed depending on the ratio of parameters. Condition (16) is fulfilled in this case due to minimal changes in the parameters of the initial data rectangle, when it is reduced to the risk matrix.

Symbols used in calculating the parameters of the risk matrix are:

$S_a$ – scale spacing of the initial data by frequency, calculated by the formula:
\[ S_a = \left( \frac{a_{\text{max}}}{a_{\text{min}}} \right)^{1/2}; \]  

(18)

\[ S_b \text{ – scale spacing of initial data on damages, calculated by the formula:} \]

\[ S_b = \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)^{3/8}; \]  

(19)

\[ S_b^u \text{ – new spacing in the damage scale;} \]

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\[ b_0 \text{ – center of the initial damage data scale: } b_0 = (b_{\text{min}} \cdot b_{\text{max}})^{1/2}; \]

\[ b_{\text{max}}^u \text{ – new right limit of the damage scale;} \]

\[ b_0^u \text{ – new center of the damage scale;} \]

\[ \alpha_1 \text{ – coefficient of increase of the right limit of the damage scale;} \]

\[ \alpha_2 \text{ – coefficient of increase in the damage scale in both directions;} \]

\[ \beta_1 \text{ – coefficient of reduction of the lower limit of the frequency scale;} \]

\[ \beta_2 \text{ – coefficient of increasing the upper limit of the frequency scale.} \]

**Table 2. Branch of the algorithm for calculating the parameters of the risk matrix**

| Ratio of scale spacings | Action (step 1) | Checking the ratio | Action (step 2): Equalizing of the spacing by scales | The result – the parameters of the risk matrix: |
|-------------------------|----------------|-------------------|-------------------------------------------------|-----------------------------------------|
| 1. \( S_a \geq S_b \)  | \( \alpha_1 = \left( \frac{R}{b_0 a_{\text{max}}} \right)^2 \) | \( 1.1 S_a > S_b^u \) \( \alpha_2 = \left( \frac{S_a}{S_b^u} \right)^{4/3} \) | Scale spacing: \( K = S_a = S_b^{u1} \); the borders: \( a_{\text{min}}, a_{\text{max}} \); \( b_{\text{max}}^u, b_{\text{min}}^u \); |
|                         | \( b_{\text{max}}^u = \alpha_1 b_{\text{max}} \) |                              |                                                 |                                         |
|                         | \( b_0^u = \sqrt{\alpha_1 b_0} \)                              | \( b_{\text{max}}^{u1} = \alpha_2 b_{\text{max}} \) |                                                 |                                         |
|                         | \( S_b^u = \alpha_1^{3/8} S_b^u \)                              | \( b_{\text{min}}^u = \frac{b_{\text{min}}}{\alpha_2} \); \( S_b^u = \alpha_2^{3/4} S_b^u \) |                                                 |                                         |
|                         | explanation: by expanding the right border of the damage range, we achieve equality: \( R b_0^u = a_{\text{max}} \) |                              |                                                 |                                         |
| 1.2 \( S_a < S_b^u \)   | the lower border of the frequency range decreases: \( \beta_1 = \left( \frac{S_b^u}{S_a} \right)^2 \) | \( \alpha_1 = \frac{S_b^u}{S_a^u} \) \( \alpha_2 = \frac{S_b^u}{S_a^u} \) | scale spacing: \( K = S_b^u \); the borders: \( a_{\text{min}}, a_{\text{max}} \); \( b_{\text{max}}^u, b_{\text{min}}^u \); |
|                         | \( a_{\text{min}}^u = a_{\text{min}} / \beta_1 \); \( b_{\text{max}}^u, b_{\text{min}}^u \); |                              |                                                 |                                         |
| 1.3 \( S_a = S_b^u \)   | no action;                                                 |                                                 | scale spacing: \( K = S_b^u \); the borders: \( a_{\text{min}}, a_{\text{max}} \); \( b_{\text{max}}^u, b_{\text{min}}^u \); |
In conclusion it should be noted that rejection of the above-mentioned features of the problem statement, careless development of algorithms can result in errors not only in the assessment of significant risks, but also in the regulatory literature. For example, despite the significance, usefulness and achievements of the document [4], the calculation of the risk matrix parameters was done with mistakes: the ratio (6) was not fulfilled (error – 250%), as a result, the risk related to the "undesirable" category was evaluated as "permissible" by this matrix. Below there is a matrix based on the minimum spacing rule with same initial data that shows that the risk is in the "undesirable" category. By the way, the spacing for the matrix is calculated correctly, if the risk category was evaluated according to Table 1, the result would be correct, hence the risk of 67.1 train hours / year belongs, according to table 1, to the "undesirable" category.

The conducted research allows us to draw the following conclusions.

The definition of a risk matrix with minimum scale spacing is introduced and the example shows that the matrix with minimum scale spacing is a more accurate tool for determining the risk category.

It is shown by examples and numerically that the matrix with the smallest spacing gives better results in comparison with matrices constructed using an arbitrary scale extension.

The problem of finding the parameters of such a matrix from the initial data in a mathematical statement is formulated.

One of the branches of the developed simplified algorithm for constructing such a matrix is given.

**Risk matrix for loss of train hours due to technical equipment failures**

(2013, communications industry)
5. Conclusion

This article reflects the development of ideas about the risk matrix as a tool for interval risk assessment for assigning it to a particular category. The possibility of using the risk matrix is related to how effectively this tool will work. Therefore, the given method of matrix constructing that most accurately determines the risk category; the constructed algorithm for calculating parameters is relevant. Another aspect of the relevance of the work is the upcoming digitalization of risk assessments in railway transport with the resulting need to have effective algorithms for calculating the parameters of optimal risk matrices.

Reference

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