Neutron/proton ratio of nucleon emissions as a probe of neutron skin

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Abstract

The dependence between neutron-to-proton yield ratio ($R_{np}$) and neutron skin thickness ($\delta_{np}$) in neutron-rich projectile induced reactions is investigated within the framework of the Isospin-Dependent Quantum Molecular Dynamics (IQMD) model. The density distribution of the Droplet model is embedded in the initialization of the neutron and proton densities in the present IQMD model. By adjusting the diffuseness parameter of neutron density in the Droplet model for the projectile, the relationship between the neutron skin thickness and the corresponding $R_{np}$ in the collisions is obtained. The results show strong linear correlation between $R_{np}$ and $\delta_{np}$ for neutron-rich Ca and Ni isotopes. It is suggested that $R_{np}$ may be used as an experimental observable to extract $\delta_{np}$ for neutron-rich nuclei, which is very significant to the study of the nuclear structure of exotic nuclei and the equation of state (EOS) of asymmetric nuclear matter.

Key words: isospin, neutron-proton ratio, neutron skin

Nuclear radius is one of the basic quantities of a nucleus. The proton root-mean-square (RMS) radius can be determined to very high accuracy via the charge radius measured by electromagnetic interactions, typically with an error of 0.02 fm or better for many nuclei \cite{1}. In contrast, it is much more difficult to accurately determine the neutron density distribution of a nucleus experimentally \cite{2}. Thus, the accuracy of experimental neutron radius is much lower than that of the proton radius. However, the information of neutron density is very important to the study of nuclear structure for neutron-rich nuclei, atomic parity non-conservation, iso-vector interactions, and neutron-rich matter in astrophysics etc. It is remarkable that a single measurement has so many applications in the research fields of atomic, nuclear and astrophysics \cite{3,4}.

A nucleus with neutron number ($N$) being larger than proton number ($Z$) are expected to have a neutron skin (defined as the difference between the neutron and proton RMS radii: $\delta_{np} \equiv (\langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2})$). The neutron skin thickness $\delta_{np}$ depends on the balance between various aspects of the nuclear force. The actual proton and neutron density distributions are determined by the balance between the isospin asymmetry and Coulomb force. $\delta_{np}$ is found to be related with a constraint on the equation of state (EOS) of asymmetric nuclear matter. Strong linear correlation between $\delta_{np}$ and $L$ (the slope of symmetry energy coefficient $C_{sym}$), the ratio $L/J$ ($J$ is the symmetry energy coefficient at the saturation density $\rho_0$), $J - a_{sym}$ ($a_{sym}$ is the symmetry energy coefficient of finite nuclei) are demonstrated \cite{3}. This constraint is important for extrapolation of the EOS to high density and hence useful for studying properties of neutron star \cite{5,6,7,8,9,10,11,12}. Neutron skin thickness can yield a lot of information on the derivation of volume and surface symmetry energy, as well
as nuclear incompressibility with respective to density. \( \delta_{np} \) is significant to the study of the EOS in different theoretical models such as Skyrme Hartree-Fock (SHF) [8, 11, 12], relativistic mean-field (RMF) [9, 10, 13], BUU model [14, 15, 16] and Droplet model [17]. Furthermore, neutron skin thickness helps to identify a nucleus with exotic structure. Thus the precise determination of \( \delta_{np} \) for a nucleus becomes an important research subject in nuclear physics.

Several attempts have been made or suggested to determine the neutron density distribution such as using proton scattering, interaction cross sections in heavy ion collisions at relativistic energies [18], parity violating measurements [3, 4, 2, 8, 9], neutron abrasion cross sections in heavy ion collisions [19]. On the other hand, the neutron and proton transverse emission and double neutron-proton ratios have been studied as a sensitive observable of the asymmetry term of the nuclear EOS in the experiment [20] and different kinds of simulations [21, 22, 23, 24, 25]. In this Letter, the relationship between \( \delta_{np} \) and the ratio of the emitted neutron and proton yields \( R_{np} = Y_n/Y_p \) in neutron-rich projectile induced reactions is, for the first time, presented within the framework of Isospin-Dependent Quantum Molecular Dynamics (IQMD) model. The possibility of extracting \( \delta_{np} \) from \( R_{np} \) is investigated.

The QMD approach is a many-body theory that describes heavy-ion reactions from intermediate to relativistic energies [26]. It includes several important factors: initialization of the projectile and target, nucleon propagation in the effective potential, nucleon-nucleon (NN) collisions in a nuclear medium and the Pauli blocking effect. A general review of the QMD model can be found in [27]. The IQMD model is based on the QMD model with consideration of the isospin effect.

The dynamics of heavy ion collision at intermediate energies is governed mainly by three components: the mean field, two-body collisions, and Pauli blocking. Therefore, for an isospin-dependent reaction dynamics model, it is important to include isospin degrees of freedom with the above three components. In addition, the sampling of phase space of neutrons and protons in the initialization should be treated separately because of the large difference between neutron and proton density distributions for nuclei that far from the \( \beta \)-stability line.

In the IQMD model, each nucleon \( i \) is represented by a Gaussian wave packet with definite width \( (L = 2.16 \text{ fm}^2) \) centered around the mean position and the mean momentum:

\[
\Psi_i(\mathbf{r}, t) = \frac{1}{(2\pi L)^{3/4}} \exp\left[-\frac{(\mathbf{r} - \mathbf{r}_i(t))^2}{4L^2}\right] \times \exp\left[-\frac{1}{n}(\mathbf{r} - \mathbf{p}_i(t))^2\right]
\]

The nuclear mean field in the IQMD model is determined as following

\[
U(\rho, \tau_z) = \alpha(\frac{\rho_n}{\rho_0}) + \beta(\frac{\rho_n}{\rho_0})^\gamma + \frac{1}{2}(1 - \tau_z)V_c + C_{sym}\frac{\rho_n - \rho_p}{\rho_0}\tau_z \ + U_{Yuk}
\]

with the normal nuclear matter density \( \rho_0 = 0.16 \text{ fm}^{-3} \). \( \rho, \rho_n \) and \( \rho_p \) are the total, neutron and proton densities, respectively. \( \tau_z \) is \( z \)-th component of the isospin degree of freedom, which equals 1 or \(-1 \) for neutron or proton, respectively. The coefficients \( \alpha, \beta \) and \( \gamma \) are parameters for the nuclear EOS. \( C_{sym} \) is the symmetry energy strength due to the difference between neutron and proton. In the present work, we take \( \alpha = -356 \text{ MeV}, \beta = 303 \text{ MeV} \) and \( \gamma = 1.17 \) which correspond to the so-called soft EOS with \( C_{sym} = 32 \text{ MeV} \) and incompressibility of \( K = 200 \text{ MeV} \). \( V_c \) is the Coulomb potential and \( U_{Yuk} \) is Yukawa (surface) potential. In the phase space initialization of the projectile and target, the density distributions of proton and neutron are distinguished from each other. The neutron and proton density distributions for the initial projectile and target nuclei in the present IQMD model are taken from the Droplet Model [28].

In the Droplet model, we can change the diffuseness parameter to get different density,

\[
\rho_i(r) = \frac{\rho_0^i}{1 + \exp\left(\frac{r - r_i}{t_i/4}\right)}, i = n, p
\]

where \( \rho_0^i \) is the normalization constant which ensures that the integration of the density distribution equals to the number of neutrons \((i=n)\) or protons \((i=p)\); \( t_i \)
is the diffuseness parameter; $C_i$ is the half density radius of neutron or proton determined by the Droplet model \[28\]

$$C_i = R_i[1 - (b_i/R_i)^2], \ i = n, p \quad (4)$$

here $b_i = 0.413f_i t_i$, $R_i$ is the equivalent sharp surface radius of neutron and proton. $R_i$ and $t_i$ are given by the Droplet model. A factor $f_i$ is introduced by us to adjust the diffuseness parameter. In [29], Trzeińska et al. found that the half density radii for neutrons and protons in heavy nuclei are almost the same, but the diffuseness parameter for neutron is larger than that for the proton. So we introduce the factor $f_i$ to adjust the diffuseness parameter for neutron. In the calculation for neutron-rich nucleus, $f_p = 1.0$ is used in Eq.(3) for the proton density distribution as in the Droplet model, while $f_n$ in Eq.(3) is changed from 1.0 to 1.6. Different values of $\delta_{np}$ will be deduced from Eq.(3). Using the density distributions of the Droplet model, we can get the initial coordinate of nucleons in nuclei in terms of the Monte Carlo sampling method. The momentum distribution of nucleons is generated by means of the local Fermi gas approximation:

$$P_{\text{F}}(\mathbf{r}) = \hbar[3\pi^3 \rho_i(\mathbf{r})]^{1/3}, \ (i = n, p) \quad (5)$$

In the IQMD model, the nucleon’s radial density can be written as

$$\rho(r) = \sum \frac{1}{(2\pi \hbar)^3/2} \exp\left(-\frac{r^2 + r_i^2}{2\hbar}\right) \frac{L_i}{2\pi \hbar} \left[\exp\left(\frac{r_i}{L_i}\right) - \exp\left(-\frac{r_i}{L_i}\right)\right] \quad (6)$$

Figure 1: (Color online.) The evolution time dependence of $R_{np}$ for $^{50}\text{Ca}^{12}\text{C}$ at 50A MeV.

Figure 2: (Color online.) The dependence of $R_{np}$ on neutron skin thickness of the projectile for $^{50}\text{Ca}^{12}\text{C}$ (a) and $^{68}\text{Ni}^{12}\text{C}$ (b) at 50A MeV under the condition of $Y > 0$. Different symbols are used for different range of the reduced impact parameter as shown in the legend. The dotted lines just guide the eye.

To avoid taking an unstable initialization of projectile and target in the IQMD calculation, we only select the initialization samples of those nuclei that meet the required stability conditions. The binding energies and root mean square (rms) radii of the initial configuration for the colliding nuclei are required to be stable. Using the selected initialization phase space of nuclei in IQMD to simulate the collisions, the nuclear fragments are constructed by a modified isospin-dependent coalescence model, in which nucleons with relative momentum smaller than $P_0 = 300 \text{ MeV}/c$ and relative distance smaller than $R_0 = 3.5 \text{ fm}$ will be combined into a cluster.

The collision processes of some Ca and Ni isotopes with $^{12}\text{C}$ target at 50A MeV are simulated using the IQMD model. The fragments including neutrons and
Figure 3: (Color online.) The dependence of $R_{np}$ from 0.8 to 1.0 for $^{50}\text{Ca}+^{12}\text{C}$ (a) and $^{68}\text{Ni}+^{12}\text{C}$ (b) at 50A MeV. The results without any gate on the rapidity ($Y$) are shown as solid squares. The results with $Y > 0$ are plotted as solid circles. The dotted lines just guide the eye.

protons that formed during the evolution of the collision are constructed by the coalescence method. The yield ratio $R_{np}$ of the emitted neutrons and protons can be calculated from the yields of the produced neutrons and protons. By changing the factor $f_n$ in the neutron density distribution of the Droplet model for the projectile, different values of $\delta_{np}$ and the corresponding $R_{np}$ are obtained. Thus we can obtain the correlation between $R_{np}$ and $\delta_{np}$. In the calculation, the time evolution of the dynamical process was simulated until $t = 200$ fm/c. The calculated $R_{np}$ is stable after 150 fm/c as shown in Fig. 3 so we accumulate the emitted neutrons and protons between 150 fm/c and 200 fm/c in order to improve statistics. The $R_{np}$ from different range of the reduced impact parameter for $^{50}\text{Ca}+^{12}\text{C}$ and $^{68}\text{Ni}+^{12}\text{C}$ are plotted in Fig. 3. The reduced impact parameter is defined as $b/b_{\text{max}}$ with $b_{\text{max}}$ being the maximum impact parameter. From Fig. 2 we can see that $R_{np}$ rises as $\delta_{np}$ increases. With $\delta_{np}$ being fixed, $R_{np}$ also becomes larger with the increasing of the reduced impact parameter. It means that $R_{np}$ from peripheral collisions is usually larger than that from central collisions. The main purpose of the present study is to investigate the relationship between $R_{np}$ and the neutron skin thickness of the projectile. In order to minimize the target effect on $R_{np}$, we use rapidity ($Y$) cut to select neutrons and protons from the projectile. The rapidity of the fragment which is normalized to the incident projectile rapidity is defined as:

$$Y = \frac{1}{2}\log\left(\frac{E + p_z}{E - p_z}\right)/Y_{\text{proj}},$$

where $E$ is the energy of the fragment, $p_z$ is the momentum of $z$ direction, $Y_{\text{proj}}$ is the rapidity of the projectile. We choose $Y > 0$ to strip away fragments coming from the target. From the results shown in Fig. 3 we can see that $R_{np}$ with $Y > 0$ are larger than $R_{np}$ without $Y$ cut. Since the projectile is neutron-rich nucleus, the correlation between $R_{np}$ and $\delta_{np}$ is stronger for nucleons coming from the projectile than those from both the projectile and target.

Strong linear correlation between $R_{np}$ and $\delta_{np}$ of the projectile, especially for $0.8 < b/b_{\text{max}} < 1.0$, is exhibited in Fig. 2. It indicates that $R_{np}$ is very sensitive to $\delta_{np}$ of the projectile, especially in peripheral collisions. For systematic study, reactions of other Ca and Ni isotopes such as $^{52,54,56}\text{Ca}$ and $^{60,62,64,66,70}\text{Ni}$ are also calculated. The results with the reduced impact parameter from 0.8 to 1.0 and the rapidity being positive are shown in Fig. 3. Strong linear relationship between $R_{np}$ and $\delta_{np}$ are also observed for all projectiles. From Fig. 3 a linear function ($R_{np} = a + b \cdot \delta_{np}$) can describe the correlation between $R_{np}$ and $\delta_{np}$ well for both Ca and Ni isotopes. The dependence between $R_{np}$ and $\delta_{np}$ is fitted using this linear function. The mean slopes for Ca and Ni isotopes are 1.06 and 0.83, respectively.

From the above discussions, $R_{np}$ could be viewed as a sensitive observable of $\delta_{np}$ for the projectile. If the produced neutrons and protons are measured by experiment, it is possible to extract $\delta_{np}$ from the
neutron-to-proton yield ratio $R_{np}$. From the fitted slope values, the estimated error of $\delta_{np}$ may be less than 0.08 fm if the uncertainty of $R_{np}$ is less than 5%. It should be pointed out that this uncertainty does not take into account the error arising from the determination of impact parameter, which is always an important issue in studies of nuclear reactions in the energy range of our calculation. We estimate the importance of the error arising from the determination of the centrality of the reaction is required for this method.

In our calculation, different $\delta_{np}$ is obtained by changing the factor $f_n$ in Eq.(3). We can also study the mass dependence of $R_{np}$ with the same $f_n$ as shown in Fig. 5. For $f_n = 1.0$, it refers to the neutron skin thickness of the Droplet model’s prediction. In this figure, the linear dependence of $R_{np}$ with the mass number $A$ is also seen at different value of $f_n$.

Due to the acceptance and efficiency in measurement of the emitted neutron and proton, the uncertainty of $R_{np}$ may not be easy to be estimated which makes it difficult to obtain the error of $\delta_{np}$. There is one method to minimize these effects: we can measure $R_{np}$ of one isotope chain with different mass number. In this case, the acceptance and efficiency of neutron and proton will not change the mass dependence of $R_{np}$. If $\delta_{np}$ of some nuclei is larger than the normal value of the Droplet model, it will deviate from the line of $f_n = 1.0$. By studying the mass dependence of $R_{np}$, it is possible to extract $\delta_{np}$ and identify nucleus with abnormal neutron skin thickness.

The neutron excess ($I = (N-Z)/A$) dependence of...
neutron skin thickness for four parameter sets (NL3, NL-SH, SkM*, SIII) in the Droplet model and self-consistent extended Thomas-Fermi (ETF) calculations have been investigated by Warda et al. [30]. Differences of the neutron skin thickness among different predictions increase with the increase of $I$. To distinguish which potential parameter set or method is more close to the experimental data, higher resolution measurement on neutron skin thickness is required for stable nuclei having small $I$ in comparison with nuclei far from the stability line having large $I$ value. From their results, the neutron skin thickness by different calculation varies from 0.01 fm to 0.11 fm at $I = 0.11$. In this sense, it will be capable of distinguishing different theoretical predictions when $I > 0.11$ by extracting $\delta_{np}$ from $R_{np}$ measurement which the present paper proposes. In other words, the present method will be more feasible for heavy nuclei or intermediate mass nuclei far from the $\beta$-stability line.

In summary, we have made calculation on the relationship between $R_{np}$ and the neutron skin thickness for the first time. The simulated data for Ca and Ni isotopes induced reactions show a good linear correlation between $R_{np}$ and $\delta_{np}$ for neutron-rich projectile. It is suggested that $R_{np}$ could be used as an experimental observable to extract the neutron skin thickness for neutron-rich nucleus. When the neutron skin thickness of one isotope is obtained, we can get some information on the EOS in different theoretical models such as Skyrme Hartree-Fock, relativistic mean-field and BUU model. From the isotope dependence of neutron skin thickness, a lot of information on the derivation of volume and surface symmetry energy, as well as nuclear incompressibility with respective to density can be extracted. This is very important to the study of the EOS of asymmetric nuclear matter.

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