Theory of the ac spin-valve effect

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The spin-valve complex magnetoimpedance of symmetric ferromagnet/normal metal/ferromagnet junctions is investigated within the drift-diffusion (standard) model of spin injection. The ac magnetoresistance—the real part difference of the impedances of the parallel and antiparallel magnetization configurations—exhibits an overall damped oscillatory behavior, as an interplay of the diffusion and spin relaxation times. In wide junctions the ac magnetoresistance oscillates between positive and negative values, reflecting resonant amplification and depletion of the spin accumulation, while the line shape for thin tunnel junctions is predicted to be purely Lorentzian. The ac spin-valve effect could be a technique to extract spin transport and spin relaxation parameters in the absence of a magnetic field and for a fixed sample size.

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Recently, Rashba has generalized the spin-polarized drift-diffusion theory to the alternating current (ac) regime \([8,9]\). We apply this theory and investigate the complex impedance \(Z(\omega)\) of symmetric FNF junctions. We show that the real part of the spin-valve magnetoimpedance (we call it here ac magnetoresistance) \(\Delta Z = Z_{AP} - Z_P\) of the junctions exhibits damped oscillations as a function of frequency. The oscillation period is given by the diffusion time through the normal layer. In mesoscopic junctions (of sizes up to the spin relaxation length \(L_s\)), the ac magnetoresistance can be negative at experimentally accessible frequencies, meaning that the antiparallel configuration has a lower ac resistance than the parallel one. The negative ac magnetoresistance is a consequence of a resonant spin accumulation effect, namely a resonant spin amplification in the P configuration and a resonant spin depletion in the AP one. In nanoscale junctions (with sizes much less than \(L_s\)), with tunnel contacts, the oscillation period is large, leaving a nice Lorentzian profile with the width of the spin relaxation rate. A one-parameter fit to the line shape (either damped oscillator or Lorentzian) determines the spin relaxation time \(\tau_s\).

We present the ac spin-valve effect as an alternative to other methods that measure \(\tau_s\) of nonmagnetic conductors, such as the conduction electron spin resonance, spin pumping, or the Hanle effect, which require magnetic fields, or to the dc spin injection method (in vertical or lateral geometries), which requires studying various sample sizes (distances to electrodes) to extract the spin diffusion length \(L_s\). In a sense the ac spin-valve effect is similar to the Hanle effect, which is widely used to find spin relaxation times in metals and semiconductors \([4,18]\), but the role of the magnetic field is taken by the frequency: in the Hanle effect too the signal in general oscillates as a function of magnetic field, with a modified Lorentzian shape in the diffusive regime \([2]\).

![FIG. 1: Scheme of an FNF spin valve. The spacer N region has width \(d\) and the sizes of the ferromagnetic electrodes are assumed greater than the corresponding spin diffusion length \(L_s\). In the dc regime the parallel configurations results in smaller spin accumulation (dashed line) than in the antiparallel one, demonstrated by the positive dc spin-valve magnetoresistance. In the ac regime, this can be reversed (solid): at certain frequency ranges there can be a resonant spin amplification in the parallel and spin depletion in the antiparallel configuration, resulting in a negative ac magnetoresistance.](image)
striking case is aluminum in which \( \tau_s \) decreases by an order of magnitude as the magnetic field increases from 0.05 to 1.3 T [19]. Still, \( \tau_s \) obtained by spin resonance tend to be, for a given temperature, much greater than that obtained from transport techniques, as catalogued for Al and Cu in Ref. [20]. The case of Au is even more striking, as spin resonance shows that at low temperatures the ratio of \( \tau_s \) to the momentum relaxation time is about one, while transport techniques predict the ratio to be about 100 [21, 22]; at room temperature, at which phonons are relevant, the ratio is about 10, as measured by spin pumping [23] which requires both magnetic field and nanoscale transparent junctions. For extracting bulk spin relaxation times it may be preferable to work with tunnel contacts and mesoscopic conductance, so that spin relaxation is not strongly influenced by the interface and surface effects. (Various techniques for measuring \( \tau_s \) as well as useful data are given in the review Ref. [13].) The ac spin-valve method could potentially explore nano and mesoscopic spin valves, in both vertical and lateral geometries, at no magnetic field applied to the normal conductor, and provide the spin relaxation times at a fixed sample size [27].

We consider a symmetric \( FNF \) junction as comprising two \( FN \) junctions in series, see Fig. 1. Each \( FN \) junction has a contact (c) region with a spin-dependent conductance; otherwise spin is assumed to be preserved at the contact. The spin valve dc magnetoresistance \( \Delta R = \Delta Z(\omega = 0) \) of a symmetric \( FNF \) junction, whose \( N \) region has width \( d \) and the \( F \) regions have widths much greater than the spin diffusion lengths, can be expressed analytically within the drift–diffusive regime [18, 19, 20]. This dc formula has a straightforward extension to the harmonic ac regime, and we write the complex magnetoresistance as

\[
\Delta Z(\omega, d) = \frac{8 r_N(\omega)[r_F(\omega) P_{sF} + r_c P_{\Sigma_c}]^2 e^{d/L_N(\omega)}}{[r_F(\omega) + r_c + r_N(\omega)]^2} e^{2d/L_N(\omega)} - \left[ r_F(\omega) + r_c - r_N(\omega) \right]^2,
\]

by indicating the complex frequency-dependent quantities (labeled by the region \( N \) and \( F \)),

\[
\tau_s(\omega) = \tau_s(1 - i\omega \tau_s),
\]

\[
L_s(\omega) = L_s/\sqrt{1 - i\omega \tau_s},
\]

\[
r(\omega) = r/\sqrt{1 - i\omega \tau_s}.
\]

Here \( \tau_s \) is the spin relaxation time, \( L_s = \sqrt{D \tau_s} \) is the spin diffusion length, \( D \) is the diffusivity, and \( r = L_s/\sigma \) is the effective resistance, with \( \sigma \) denoting the conductivity. The effective contact resistance is \( r_c = (\Sigma_t + \Sigma_i)/4\Sigma_t \Sigma_i \), with \( \Sigma_t \) the contact conductance of spin \( \lambda \). Finally, \( P_{sF} \) and \( P_{\Sigma_c} \) denote the spin polarization of the conductivity and conductance of the \( F \) and contact region, respectively. Driving \( ac \) is assumed to be harmonic with the angular frequency \( \omega = 2\pi f \), i.e. \( j(t) \propto e^{-i\omega t} \).

We analyze the spin-valve impedance, based on Eq. (1), for a realistic model Py/Cu/Py junction, with the following experimentally obtained data [17, 20] at the temperature \( T = 4.2 \) K: \( L_{sN} = 1 \) \( \mu \)m, \( \tau_{sN} = 42 \) ps, \( D_N = 238 \) cm\(^2\) s\(^{-1} \), \( r_N = 14 \) \( \Omega \) m\(^2 \), \( L_{sF} = 5.5 \) nm, \( \tau_{sF} = 0.6 \) ps, \( D_F = 0.5 \) cm\(^2\) s\(^{-1} \), \( r_F = 0.42 \) \( \Omega \) m\(^2 \), \( P_{F N} = 0.22 \). For the contact characteristics we employ [23]: \( r_c(\approx r_F) = 0.5 \) \( \Omega \) m\(^2 \) and \( P_{\Sigma_c} = 0.4 \), so the contact interface is generic, neither tunnel nor transparent. The specific spin resistivities \( r_F, r_N \) and \( r_c \) and hence the spin valve \( \Delta Z(\omega, d) \) are evaluated for a unit cross section. In the experiment one divides these resistivities by the actual conductor cross sections, which could be \( 10^{-3} - 1 \) \( \mu \)m\(^2 \).

Figure 2 presents the calculated magnetoresistance. In Fig. 2(b) we show the dc magnetoresistance as a function of the \( d \). With increasing \( d \) the magnetoresistance exponentially decreases, as the injected spin accumulation is damped. The plot in Fig. 2(c) shows the ratio of the ac to the dc magnetoresistance, \( \text{Re}[\Delta Z(f, d)]/\Delta R(d) \), as a function of \( d \) and frequency \( f = \omega/2\pi \). For a given \( d \), the ac magnetoresistance oscillates as a function of \( f \), between positive and negative values. In Fig. 2(a) the oscillations are shown for \( d = 4 \) \( \mu \)m. The negative peaks are considerable fractions (tens of percents) of the dc values. On the \( d-f \)-plot the oscillations show hyperbolic stripes. For thin samples, the dependence on \( f \) is rather weak for this generic junction. We will see below that for tunnel junctions the dependence becomes Lorentzian.

![Figure 2](image)

**FIG. 2:** (color online) ac spin-valve effect in a model Py/Cu/Py junction. (a) Calculated ac/dc ratio of the spin-valve magnetoresistance as a function of \( f = \omega/2\pi \) for \( d = 4 \) \( \mu \)m. (b) Calculated dc spin-valve magnetoresistance as a function of \( d \). (c) Calculated ac/dc ratio of the spin-valve magnetoresistance as a function of \( d \) and driving frequency \( f \). The visible light and dark bands of equal signs are separated by the node lines, \( \text{Re}[\Delta Z(f, d)] = 0 \).

To be specific, consider \( d = L_{sN} = 1 \) \( \mu \)m. The ac
magnetoresistance remains positive for $f < 34.8$ GHz. Further increase in the driving frequency leads to a negative ac spin-valve magnetoresistance: $\text{Re}(\Delta Z) < 0$. For $d = 3L_{\text{ss}}N = 3 \mu m$ the spin-valve magnetoresistance remains positive up to the frequency $f \approx 6$ GHz, then it becomes negative for $6$ GHz $\lesssim f \lesssim 26.1$ GHz. There should be more oscillations observable at larger values of $d$, but at the cost of exponentially reducing the magnitude, see Fig. [2a]. We will see below that the relevant time scale parameter for the oscillations is the diffusion time through the spacer layer. For our model junction, a reasonable parameter range for measuring the ac oscillations would be the sample sizes $L_{\text{ss}}N \lesssim d \lesssim 4L_{\text{ss}}N$. The involved frequency, $f = \omega/2\pi$, ranges are $1$ GHz$-50$ GHz, experimentally well accessible.

Mathematically, the spin valve oscillations appear naturally. The real dc transport parameters become in the ac case complex, see Eqs. (2) - (11). The imaginary part of $d/L_{nnN}(\omega)$ gives rise to the complex exponential $e^{d/L_{nnN}(\omega)}$ in Eq. (11) with the trigonometric character and hence a certain oscillatory behavior of the complex spin valve impedance $\Delta Z(\omega)$. For the frequencies $\omega \ll \tau_s^{-1}(\ll \tau_F^{-1})$ the imaginary part of $d/L_{\text{ss}}N(\omega)$ plays no role, see Eq. (5). The ac magnetoresistance exhibits changes on the scales of the relaxation rate $1/\tau_s$ or the diffusion rate through the spacer. These provide the practical limit for the use of microwaves in the experiment.

We now give a qualitative picture of the predicted oscillatory behavior, including the negative ac spin-valve magnetoresistance. First, we show that the spin valve impedance $\Delta Z(\omega)$ is related to the contact values of the spin accumulations in $N$, for P and AP configurations. From the standard spin injection model for a symmetric $FNF$ junction we derive the following formula

$$\mu_{\text{ss}}^{P/\text{AP}}(c, t) - \mu_{\text{ss}}^{\text{AP}}(c, t) = \frac{r_F(\omega) + r_c}{r_F(\omega)P_{\sigma_F} + r_cP_{\Sigma_c}} \Delta Z(\omega).$$

(5)

Here $\mu_{\text{ss}}^{P/\text{AP}}(c, t)$ represent the actual nonequilibrium spin accumulation in the $N$ spacer for P and AP configurations respectively, at the left $FN$ contact interface $c$ (see Fig. [1]) and $j(t)$ is the driving harmonic ac. To understand the ac magnetoresistance oscillations, one needs to look at the contact spin accumulation only.

The qualitative picture is in Fig. [2] which shows P and AP configurations at three times $t = 0$, $t = T_N/4$, and $t = T_N/2$, where $T_N = \tau_s N d^2/L_{\text{ss}}N^2 = d^2/D_N$ is the diffusion time through $N$. The resonant spin amplification and depletion effect happens if the driving current $j(t)$ is close to the $N$ spacer diffusion time $T_N$, this case is shown in Fig. [3].

At time $t = 0$ the current $j$ is negative and electrons are injected from the left and extracted to the right electrodes, leaving behind positive and negative spin accumulations, indicated in Fig. [3] by diffusive packets (the sample is locally charge neutral, only spin is redistributed nonuniformly). The dynamics of these spin packets is governed by diffusion and relaxation, but by not bias voltage. This is because in the $N$ spacer, there is no spin-charge coupling and spin and charge transports are decoupled, see [2]. At time $t = T_N/4$ the current vanishes, $j = 0$, as well as the spin injection and extraction. In the meantime the spin packets diffusively spread and reach the center of the $N$ spacer. At $t = T_N/2$ the spin packets reach the other contacts. Now the current is fully reversed: in the P configuration the new spin packet is injected at the right electrode, amplifying the initial injected spin packet that has traveled from the left. In the AP configuration, the new spin packet of the opposite sign is injected at the right, depleting the initial injected spin. Similarly at the left electrode. The left contact difference $\mu_{\text{ss}}^{\text{P}}(c) - \mu_{\text{ss}}^{\text{AP}}(c)$ at $t = T_N/2$ becomes negative, the actual current $j > 0$ and, according to Eq. (6), we get negative ac magnetoresistance, $\text{Re}(\Delta Z) < 0$.

The resonance condition, $\omega T_N \approx \pi$, is equivalent to $L_{\text{ss}}N/d \approx \sqrt{\omega \tau_s N/\pi}$. In practice, to see the negative ac magnetoresistance one prefers $d \approx L_{\text{ss}}N$, so that $\omega \tau_s N \approx \pi$, which is the microwave regime. If $d > 2L_{\text{ss}}N$, as in our model shown in Fig. [2] then the oscillations can be observed at lower frequencies, but at the cost of decreasing the magnitude of the ac magnetoresistance due to spin relaxation. This need not be an issue with tunnel contacts, as the precision of measuring higher resistances is higher. On the other hand, no oscillations (within the GHz regime) should be seen for nanoscale junctions, for $d \ll L_{\text{ss}}N$. We will show below that this important regime
Since $\omega / \tau_s N$ is small compared to 1, the spin relaxation time according to (6) becomes a Lorentzian:

$$\Delta Z \approx \frac{4 r_N^2 P_{\Sigma}^2}{\sqrt{1 - i \omega \tau_s N} \sinh \left[ \frac{d}{L_{SN}} \sqrt{1 - i \omega \tau_s N} \right]},$$  

(6)

where $L_{SN} = \sqrt{D_N \tau_s N}$. A single parameter ($\tau_s N$, knowing $d$ and $D_N$) fit of a measurement of the $\omega$ dependence of the tunnel spin valve impedance (relative to the dc value) to Eq. (6) can determine the spin relaxation time of the normal region. The shape is illustrated in Fig. 4. Since $\tau_{sF}$ is typically an order or two magnitudes smaller than $\tau_s N$, the ac effects do not play significant in the $F$ electrodes.

For $d \gtrsim |L_{SN}(\omega)|$ we can approximate $\Delta Z$ as follows:

$$\Delta Z(\omega, d, \tau_s N) \approx 8 r_N P_{\Sigma}^2 \sqrt{1 - i \omega \tau_s N} e^{-d \sqrt{1 - i \omega \tau_s N} / D_N}. $$  

(7)

Suppose we know the experimental value of the frequency $\omega_0$ at which $\Re[\Delta Z(\omega_0, d)]$ vanishes. As an alternative to the fitting, the spin relaxation time can be given by the equation:

$$\frac{1 + \sqrt{1 + \omega_0^2 \tau_s^2}}{\omega_0 \tau_s N} = \tan \left[ \frac{\tau_s N}{\sqrt{2 D_N}} \sqrt{\frac{1 + \sqrt{1 + \omega_0^2 \tau_s^2}}{1 + 1 + \omega_0^2 \tau_s^2}} \right],$$  

(8)

which can be solved for $\tau_s N$ with simple numerics.

In the opposite important case of $d \ll L_{SN}$, Eq. (6) becomes a Lorentzian:

$$\Delta Z(\omega, d, \tau_s N) \approx 4 r_N P_{\Sigma}^2 \sqrt{d / \sqrt{D_N \tau_s N}} \frac{1 + i \omega \tau_s N}{1 + \omega_0^2 \tau_s^2}.$$  

(9)

The half-width frequency $\omega_{1/2}$ at which $\Re[\Delta Z(\omega_{1/2}, d)] = \frac{1}{2} \Delta R(d)$ determines the spin relaxation time according to $\tau_s N = 1 / \omega_{1/2}$. This Lorentzian shape is rather robust for tunnel junctions, illustrated in Fig. 4 which also shows an intermediate case of $d \approx L_{SN}$.

For transparent contacts ($r_c \ll r_F, r_N$) and nanoscale junctions, $d \ll L_{SN}$, the magnetooimpedance is $\Delta Z(\omega, d) \approx 2r_F(\omega)P_{\Sigma}^2$, the square root of the Lorentzian, but with the width of $1 / \tau_F$. The shape is therefore featureless in the microwave regime, unless the ferromagnetic contacts have a relatively large spin relaxation time, in which case $\tau_F$ could be determined from the measured shape. In mesoscopic junctions oscillations should be visible.

In summary, we have presented a simple but robust theory of the ac spin-valve effect in symmetric $FNF$ junctions and predict negative ac magnetoresistance due to resonant amplification and depletion of the spin accumulation in the normal metal region. The oscillating line shape allows a single-parameter (spin relaxation time) fitting for mesoscopic and nanoscale spin valves; for the latter a Lorentzian shape should be seen with tunnel contacts.

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[27] Spurious effects are eliminated since one takes the resistance difference for the parallel and antiparallel magnetizations of the ferromagnetic layers. The skin effect at the relevant frequencies will not be an issue, if the junction sizes are micrometers or less, even for perfect conductors such as Cu. As there is no external magnetic field, transmission spin resonance/spin pumping effects will also not be present.