Generation of four-photon polarization entangled state based on EPR entanglers

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We show how to prepare four-photon polarization entangled states based on some Einstein-Podolsky-Rosen (EPR) entanglers. An EPR entangler consists of two single photons, linear optics elements, quantum non-demolition measurement using a weak cross-Kerr nonlinearity, and classical feed forward. This entangler which acts as the most primary part in the construction of our scheme allows us to make two separable polarization qubits entangled near deterministically. Therefore, the efficiency of the present device completely depends on that of EPR entanglers, and it has a high success probability.

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I. INTRODUCTION

Entanglement plays an important role in quantum information processing (QIP). Although most of the research in QIP are concerned with bipartite systems, multipartite entanglement has also attracted increasing interest since it is more stable and has a longer decoherence time [1–15]. In Ref. [1], Briegel et al. introduced a special type of multipartite entangled states, i.e., the so-called cluster states, which have some special features. Cluster states share the properties of both Greenberger-Horne-Zeilinger (GHZ) and W class entangled states. While they have a high persistence of entanglement and can be regarded as an entanglement source for the GHZ state [1], but are more immune to decoherence than GHZ states [2]. It has been shown that a new inequality is maximally violated by the four-particle cluster states but not by the four-particle GHZ states, and the cluster states can also be used to test nonlocality without inequalities [3]. In addition, the cluster states have been recognized as the basic building blocks for one-way quantum computation proceeding only by local measurements and feed forward of their outcomes [4]. Since its particular properties, the cluster states have attracted much more attention in theoretical research and practical applications [5–11].

Recently, experiments with entangled photons open a broad field of research. Several schemes have been proposed for generation of polarization entangled cluster states. In particular, Walther et al. [16] experimentally generated four-photon cluster states and demonstrated the feasibility of the one-way quantum computation. Zou and Mathis [17] proposed an experimentally feasible scheme for preparing a four-photon polarization entangled cluster state of the form

\[
|\text{cluster}\rangle = \frac{1}{2}( |H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4 + |H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4 + |V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4 - |V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4 ),
\]

where \( H \) and \( V \) denote horizontal and vertical linear polarizations respectively, and subscripts \( i \) (\( i = 1, 2, 3, 4 \)) denote the spatial modes of photons.

Another example of multipartite entangled states is a four-qubit entangled state of the polarization form

\[
|\chi\rangle = \frac{1}{2}([|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2]|H\rangle_3|H\rangle_4 + (|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2)|V\rangle_3|V\rangle_4],
\]

whose importance has been pointed out by Gottesman and Chuang [18] that it can be used as a resource for teleportation of two qubits. The state \( |\chi\rangle \) is also important in the sense that it is equivalent to the cluster state described by Eq.(1) under a local unitary transformation. Several schemes [19–23] can be utilized to prepare \( |\chi\rangle \).

In this paper, we propose a simple experimental scheme for the preparation of the four-photon polarization entangled states \( |\text{cluster}\rangle \) and \( |\chi\rangle \) based on EPR entanglers described in Ref. [24], which involves some basic elements: the polarizing beam splitters (PBSs), the cross-Kerr nonlinearity and quantum non-demolition detection (QND) with feed forward. In Sec. II, we first review the cross-Kerr nonlinear interaction, then describe how the EPR entangler works. In Sec. III, we show an experimental scheme for generation of four-photon polarization entangled states. Finally, we describe our conclusions in Sec. IV.

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II. CROSS-KERR NONLINEARITY AND NEAR DETERMINISTIC EPR ENTANGLER

In the process of preparing the entangled states in EPR entanglers, one of the basic elements is cross-Kerr nonlinearity, which was first used by Chuang and Yamamoto to realize the simple optical quantum computation [25]. Let us briefly review the cross-Kerr nonlinearity interaction between a signal mode and a probe mode. The interaction Hamiltonian has the form $\hat{H}_k = -\hbar \kappa \hat{a}_s \hat{a}_p$, where the signal (probe) mode has the creation and annihilation operators given by $\hat{a}_s, \hat{a}_p$ respectively, and $\kappa$ is the strength of the nonlinearity. If we consider the signal state to have the form $|\psi\rangle = c_0 |0\rangle_s + c_1 |1\rangle_s$ with the probe beam initially in a coherent state $|\alpha\rangle_p$, the cross-Kerr nonlinearity interaction causes the combined signal-probe system to evolve as

$$e^{-i\hat{H}_k t/\hbar} |\psi\rangle_s |\alpha\rangle_p = c_0 |0\rangle_s |\alpha\rangle_p + c_1 |1\rangle_s |\alpha e^{i\theta}\rangle_p,$$

where $\theta = \kappa t$ with $t$ being the interaction time. It is easy to observe that the Fock state is unaffected by the interaction but the coherent state picks up a phase shift directly proportional to the number of photons $n_s$ in the signal mode. One can exactly obtain the information of photons in the Fock state but not destroy them by detecting the probe mode. We called this an X homodyne measurement. In the following, we will illustrate the EPR entangler with the help of the cross-Kerr effect. In Ref.[24], Nemoto and Munro constructed a two qubit polarization parity QND detector, which can be configured to act as a near deterministic entangler. Therefore, here and hereafter we call it the EPR entangler for simplicity. This gate allows us to make two separable polarization qubits entangled efficiently (near deterministically). The construction of the EPR entangler has been shown in Fig.1.

Consider two polarization qubits in signal modes initially prepared in the state $|\psi\rangle_1 = |\psi\rangle_2 = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$. These qubits are split individually on PBSs into spatial modes which transmit $|H\rangle$ and reflect $|V\rangle$. Horizontal polarization of each mode then interact with the intense coherence state in the probe mode via cross-Kerr nonlinear medium. The action of the PBSs and cross-Kerr nonlinearity will evolve the combined system $|\psi\rangle_1 |\psi\rangle_2 |\alpha\rangle_p$ to

$$|\psi\rangle_T = (|HH\rangle + |VV\rangle)|\alpha\rangle_p + (|HV\rangle e^{i\beta} + |VH\rangle e^{-i\beta})|\alpha\rangle_p,$$  

where we omit the normalization coefficients. Obviously, the states $|HH\rangle$ and $|VV\rangle$ pick up no phase shift and remain coherent with respect to each other, but the states $|HV\rangle$ and $|VH\rangle$ pick up opposite phase shift $\theta$ which can be distinguished by an X homodyne measurement. More specifically, with $\alpha$ real, an X homodyne measurement conditions $|\psi\rangle_T$ to

$$|\psi_x\rangle_T = f(x, \alpha) |HH\rangle + |VV\rangle + f(x, \cos \theta) |HV\rangle e^{i\phi(x)} + |VH\rangle e^{-i\phi(x)},$$

where $f(x, \beta) = \exp[-\frac{1}{2}(x - 2\beta)^2]/(2\pi)^{1/4}$ and $\phi(x) = \alpha x \sin \theta - \alpha^2 \sin 2\theta \ (\text{Mod } 2\pi)$. $f(x, \alpha)$ and $f(x, \cos \theta)$ are two Gaussian curves with the peaks located at $2\alpha, 2\alpha \cos \theta$ respectively. The midpoint and the distance between the two

![Fig. 1. The two polarization qubit entangling gate (the EPR entangler)](image-url)
peaks are \( x_0 = \alpha(1 + \cos \theta) \) and \( x_d = 2\alpha(1 - \cos \theta) \) respectively. Hence the signal states corresponding to different measurement results \( x \) are

\[
|\psi_x\rangle_T \sim \begin{cases} |HH\rangle + |VV\rangle, & x > x_0, \\ e^{i\phi(x)}|HV\rangle + e^{-i\phi(x)}|VH\rangle, & x < x_0. \end{cases}
\]  

Here we have chosen to call the even parity state \((|HH\rangle, |VV\rangle)\) and the odd parity state \((|HV\rangle, |VH\rangle)\). We have used the approximate symbol \((\sim)\) in these equations as there is a small but finite probability that the even parity state can occur for \( x < x_0 \). For the odd parity state, a simple phase shift achieved via classical feed forward allows it to be transformed to the even parity state. Evidently, the action of this measurement method splits the even parity terms nearly deterministically from the odd parity cases. So we also call it the two-mode polarization non-demolition parity detector, which acts as a near deterministic EPR entangler. This gate is critical and forms the key element for preparing four-photon polarized-entangled states.

## III. GENERATION OF FOUR-PHOTON POLARIZED-ENTAN GLED STATE

We now move our attention to the new protocol of generating the four-photon polarized-entangled states described by Eqs.(1) and (2) based on EPR entanglers. The setup schematic for preparing \(|\chi\rangle\) is illustrated in Fig. 2.

The initial state of the system which consists of four photons in signal modes and a coherent probe beam is

\[
|\psi_{in}\rangle_{1234} = \otimes_{i=1}^{4} |+\rangle_i \otimes |\alpha\rangle, \quad |+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle).
\]

Firstly, the two photons in mode 1 and 2 pass through the EPR entangler 1, then the two photons in mode 2 and 3 come into the EPR entangler 2. The action of the two entanglers is to create the maximally entangled state \((|HHH\rangle + |VVV\rangle)\) or \((|HHV\rangle + |VVH\rangle)\), of course, the latter can be transformed into the former by performing a bit flip on the third photon qubit. In this step, we choose the three photons in mode 1, 2 and 3 to be in \((|HHH\rangle + |VVV\rangle)\). Subsequently, we exploit three HWPs for the \(-45^0\) rotation of polarization. The HWP is governed by the transformation matrix

\[
U_{HWP}(\delta) = \begin{pmatrix} \cos 2\delta & -\sin 2\delta \\ \sin 2\delta & \cos 2\delta \end{pmatrix},
\]

where \(\delta\) is the rotation of polarization and the single-photon polarization basis states are denoted as

\[
|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]

At last, the two photons in mode 3 and 4 interact with the coherent state in the EPR entangler 3. The whole system will evolve into

\[
|\psi'\rangle_T = (|HHHH\rangle + |VVHH\rangle + |HVVV\rangle + |VHVV\rangle)|\alpha\rangle
+ (|HHHV\rangle + |VVHV\rangle)|e^{i\theta}\alpha\rangle
+ (|HVVH\rangle + |VHVV\rangle)|e^{-i\theta}\alpha\rangle.
\]
When performing the QND measurement in the entangler 3, we can obtain the $|\chi\rangle$ state or $e^{i\phi(x)}(|HHHV\rangle + |VVHV\rangle)$. For the latter, the phase factor $e^{\pm i\phi(x)}$ could be eliminated through a dynamic phase shifter combined with classical feed forward process. Thus we can combine the states $|HHHV\rangle$ and $|VVHV\rangle$ together, the two combined states could be transformed to four-photon polarized-entangled $|\chi\rangle$ state through performing $\sigma_x$ operation on photon 4. Since the $|\chi\rangle$ state and the $|\text{cluster}\rangle$ state is equivalent under a local unitary transformation, we can continue to perform two HWPs for the $-45^\circ$ rotation of polarization R(-45) on mode 1, and 2 respectively, and then we will obtain the cluster state. As a matter of fact, the process of generating the cluster state can not be so complex, it can be omitted some steps if only we choose another three-photon maximally entangled state when we make three photons in mode 1, 2 and 3 entangled via the entangler 1 and 2. The setup schematic is shown in Fig. 3.

Fig. 3. Schematic diagram of generating four-photon polarized-entangled state $|\text{cluster}\rangle$. It is composed of three EPR entanglers described above and a half wave plate (HWP) for the $-45^\circ$ rotation of polarization on mode 3.

As described above, at first the two photons in mode 1 and 2 interact in the entangler 1, then the two photons in mode 2 and 3 pass through the entangler 2, the three photons are entangled. Here, we choose these photons to be in the maximally entangled state $|HHV\rangle + |VVH\rangle$. Afterward, we rotate the horizontal and vertical polarizations of the photon in mode 3 by utilizing $R(-45)$, the whole system evolves into

$$|\psi\rangle_{123} = |HHV\rangle + |VVH\rangle$$

Next, the photons in mode 3 and 4 interact with cross-Kerr nonlinearity in the entangler 3. The action of cross-Kerr nonlinearity makes the four photons in

$$|\psi''\rangle_T = (|HHHH\rangle + |HHHV\rangle + |VVHH\rangle - |VVVV\rangle)|\alpha\rangle + (|HHHV\rangle + |VVHV\rangle)|e^{i\phi}\alpha\rangle + (|HHHV\rangle - |VVVV\rangle)|e^{-i\phi}\alpha\rangle.$$  

It is obvious that the four-photon polarized state together with coherent state $|\alpha\rangle$ is the cluster state. So we have conditioned on an X homodyne measurement result either the state $|\text{cluster}\rangle$ or $e^{i\phi(x)}(|HHHV\rangle + |VVHV\rangle) + e^{-i\phi(x)}(|HHHV\rangle - |VVHV\rangle)$. A simple phase shift achieved via classical feed forward and then a bit flip on the fourth polarization photon transforms the second state into the first.

**IV. CONCLUSION**

In summary, a simple experimental protocol is presented to generate four-photon polarized-entangled states based on EPR entanglers. This scheme has its distinct advantages: it uses only the basic tools in quantum optical laboratories and can be implemented with the EPR entanglers, which mainly involves a weak cross-Kerr nonlinearity between signal modes and the probe coherent state followed by QND measurement. This makes us sure it is feasible in the current experimental technology. Moreover, it is possible to preparing the four-photon state $|\chi\rangle$ or $|\text{cluster}\rangle$ with high success probability since it primarily depends on three EPR entanglers which can make photons entangled and detected near deterministically. Thus, our scheme can be used in QIP.
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