Lattice QCD and the Computational Frontier

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Abstract

The search for new physics requires a joint experimental and theoretical effort. Lattice QCD is already an essential tool for obtaining precise model-free theoretical predictions of the hadronic processes underlying many key experimental searches, such as those involving heavy flavor physics, the anomalous magnetic moment of the muon, nucleon-neutrino scattering, and rare, second-order electroweak processes. As experimental measurements become more precise over the next decade, lattice QCD will play an increasing role in providing the needed matching theoretical precision. Achieving the needed precision requires simulations with lattices with substantially increased resolution. As we push to finer lattice spacing we encounter an array of new challenges. They include algorithmic and software-engineering challenges, challenges in computer technology and design, and challenges in maintaining the necessary human resources. In this white paper we describe those challenges and discuss ways they are being dealt with. Overcoming them is key to supporting the community effort required to deliver the needed theoretical support for experiments in the coming decade.
1 Introduction

The USQCD lattice collaboration wrote a computing white paper in 2019 [1]. This Snowmass white paper is intended to complement and add to the 2019 document by anticipating longer-term technology trends and the algorithmic and software development required.

The precise theoretical prediction of hadronic properties is a central part of searches for new physics over the coming decades. Lattice gauge theory is a systematically improvable theoretical tool for numerical evaluation of the Euclidean Feynman-path integral for Quantum Chromodynamics (QCD) in its nonlinear regime where it describes the hadrons as bound states of quarks. Various discretization approaches are taken, which respect or break continuum flavour and chiral symmetries to differing degrees. The lattice-gauge-theory simulation effort takes place worldwide and directly supports high-energy physics experiment by providing calculations of the properties of hadrons that are in many cases vital to the interpretation of experimental results and to the comparison of our observations with standard model predictions, thus enabling searches for new physics. A comprehensive review of important lattice predictions for a range of important quantities is carried out regularly by the Flavour Lattice Averaging Group [2].

There are significant challenges to meeting the precision goals of the next decade and beyond. These include the development of new algorithms and software that enable substantially greater numbers of degrees of freedom to be simulated on emerging supercomputers over the next 5 to 10 years. This work is required in order to reduce theoretical uncertainties by enabling calculations with larger physical lattice volumes, inclusion of both QCD and QED effects, finer lattices, and larger statistical samples. Finer lattices also give access to substantially higher energy scales.

Lattice QCD is already playing an important role in determining the hadronic contributions to the anomalous magnetic moment of the muon, discussed in a Snowmass white paper [3], by the international Muon $g-2$ Theory Initiative white paper [4] and in a white paper by the USQCD collaboration in 2019 [5]. These calculations will be critical to the interpretation of results from the Fermilab Muon $g-2$ Experiment [6], which is currently in excellent agreement with results from the earlier Brookhaven experiment [7].

Understanding the neutrino-nucleus interaction is critical to the analysis of the Deep Underground Neutrino Experiment. The interaction between an intermediate W or Z boson with a quark confined inside a neutron or proton within the nucleus is theoretically complex. This was discussed by USQCD in a 2019 white paper [8].

A robust quark-flavor physics program has also been discussed in a USQCD white paper [5]. These phenomena are typically highly suppressed in the standard model and therefore also offer promising avenues for the discovery of new physics. Contributed Snowmass white papers include improving the understanding of anomalies in B physics [9], emerging from the the Large Hadron Collider and studied further at Belle II, as well as reconciling CP violation observed in kaon experiments with the standard model and exploring rare Kaon decays [10].

Simulation at finer lattice spacing and larger volumes introduces new challenges. In this white paper we analyze the array of new challenges facing lattice QCD in the next ten years. Meeting them requires new algorithmic research, novel computer hardware design beyond the exascale, improved software engineering, and attention to maintaining human resources. If these challenges are met, lattice QCD will deliver the theoretical precision needed for interpreting experimental measurements in the foreseeable future. We begin with algorithmic challenges in Sec. 2 and provide a detailed discussion of the arithmetic intensity and hierarchical bandwidth requirements intrinsic to the various fermion formulations in use. We also provide an illustrative estimate of computational costs for problems that extend beyond the exascale. We follow in Sec. 3 with a discussion of the challenges in designing computer systems that address the computational needs. We go into some detail reviewing new memory technology and interconnect design. That review suggests further directions for algorithmic research. We discuss software-engineering challenges in Sec. 4 offering suggestions for DOE influence in improved software tools. Finally, we discuss the human-resource
challenges in Sec. 5.

2 Lattice QCD and Algorithms

In this section we introduce, briefly, the standard method for simulating QCD on a lattice and then turn to algorithmic challenges, computational requirements, and a discussion of estimated problem sizes and simulation costs in the coming years.

Lattice gauge theory formulates the Feynman path integral for QCD as a statistical mechanical sampling of the related Euclidean space path integral. The sampling is performed by Markov chain Monte Carlo (MCMC) sampling, using forms of the hybrid Monte Carlo (HMC) algorithm\[11–16\]. The partition function of QCD is sampled through the introduction of auxiliary momentum (\(\pi\)) and pseudofermion (\(\phi\)) integrals,

\[
Z = \int d\pi \int d\phi \int dU \ e^{-\pi^2/2}e^{-S_G[U]}e^{-\phi^*(D^\dagger D)^{-1}\phi}.
\]  

The sampling is weighted with the action probability functional, in which \(S_G\) is the gauge action and the sparse matrix \(D\) represents the lattice Dirac operator. The pseudofermion term provides a stochastic estimate of the fermion determinant which describes the effect of quark loops in the partition function. It involves the inverse of the gauge-covariant Dirac operator and requires linear-solver algorithms to evaluate. The momentum integral serves only to move the gluon fields (\(U\)) efficiently around their group manifold at constant energy in the HMC sampling algorithm. As computers become more capable, we are able to simulate at smaller lattice spacings with higher precision, which increases the range of length scales covered.

Present algorithms for both MCMC sampling and Dirac solvers display growing limitations as substantially greater ranges of energy scales are included in our problem, an algorithmic challenge called critical slowing down. Also for the computation of correlation functions the inversion of the matrix \(D\) is typically the computationally most expensive component. While a greater degree of trivial parallelism can be utilized for these quark correlation function calculations, a large number of inversions per gauge configuration is needed to achieve sub-percent precision for complicated observables.

When QCD is naively discretized, additional unphysical degrees of freedom appear associated with zero modes arising at the corners of the Brillouin zone in momentum space. Two techniques are commonly employed to control this problem: in the staggered fermion approach\[17\] the path integral weighting is adjusted to account for the additional degrees of freedom (“tastes”) and their contributions to the valence sector are handled theoretically, for example using staggered chiral perturbation theory. A second approach introduces a “Wilson term” which adds a large mass to the extra degrees of freedom proportional to the inverse lattice spacing, decoupling them from the theory in the continuum limit. Unfortunately the Wilson term breaks the chiral symmetry, which is the symmetry of QCD under the interchange of left and right-handed quark fields, and which is vital to the correct description of the pions that encapsulate the majority of the low-energy degrees of freedom in the system. The domain-wall-fermion approach\[18\] reintroduces the chiral symmetry in a controlled way by placing the chiral modes on opposite sides of a fictional, finite fifth dimension, at the expense of additional computational cost. Both the staggered and domain wall discretizations are employed extensively by USQCD in High Energy Physics (HEP) calculations.

2.1 Algorithmic Challenges

The development of numerical algorithms is an intellectual activity that spans physics, mathematics, and computer science. Many key algorithms such as Markov Chain Monte Carlo (MCMC) and Metropolis-Hastings algorithms emerged from theoretical physics. The hybrid/Hamiltonian Monte Carlo algorithm was developed in Lattice Gauge theory and is one of several work-horse algorithms used in training machine learning. Over the almost 40 year history of active lattice gauge theory calculations, the annual
improvement purely from algorithm development has been broadly similar to, and additional to, the gain from Moore’s law.

The needed simulations cover a much greater range of length scales and energy scales than those to date. This will require a new generation of algorithms tailored to multi-scale dynamics. In the absence of these algorithms both Dirac-matrix inversion and Monte-Carlo sampling are known to require increasingly large iteration counts or Monte-Carlo evolution times as the lattice spacing is reduced. We hope that these issues may be addressed in one of several ways.

Firstly, the Atiyah-Singer index theorem guarantees the cost of Dirac matrix solution becomes large as the continuum limit is taken. Krylov solvers are fundamentally polynomial approximations to the inverse \( f(x) = 1/x \). The order of the polynomial required for a given accuracy grows linearly in the ratio of the highest to lowest eigenvalue (condition number) of the squared Dirac operator. This dictates a loss of algorithmic efficiency as the lattice is made finer and the Nyquist frequency is increased. Therefore the Krylov solver takes a diverging number of iterations as the continuum limit is taken.

This difficulty can be countered in several ways. Firstly one can accurately compute a number of the lowest lying eigenvectors of the Dirac operator and handle these ‘exactly’ up to numerical tolerance. This removes these modes from a deflated Krylov solver and changes the effective condition number, thus accelerating convergence.

Alternative multigrid \([19–21]\) solvers have demonstrated order-of-magnitude gains for Wilson fermions by approximately and repeatedly handling degrees of freedom in the low lying eigenspace as a form of pre-conditioner. The US HEP program is presently focused on the domain-wall\([18]\) and staggered approaches, but corresponding gains for staggered \([22]\) and domain-wall-fermion \([23–27]\) discretizations are an open research activity. In all cases gauge invariance dictates that the coarse degrees of freedom must be discovered in a data-dependent way to treat more efficiently longer distance or collective degrees of freedom. This discovery or setup step is costly and limits the gains from multigrid in HMC. Better methods are required, and it is possible both eigenvector solution and multigrid setup could be phrased as a well posed potential machine-learning feature-recognition problem.

A second direction is the critical slowing down of MCMC algorithms. This manifests itself, among other metrics, as the freezing of topological charge in simulations with periodic gauge field boundary conditions. An underlying issue is that the HMC forces for different “wavelength” modes in the system receive vastly different forces in our fictitious Monte-Carlo time evolution. Several research activities are being undertaken to introduce a wavelength-dependent evolution speed in the Monte Carlo that accelerates the sampling of long-distance dynamics to evade critical slowing down. The first of these is the Riemannian Manifold Hybrid Monte Carlo (RMHMC) algorithm \([28]\), which is in some aspects similar to a “gauge invariant Fourier Acceleration” idea introduced in ref. \([29]\). This is being studied under Exascale Computing and SciDAC projects, with recent progress \([15]\) and also in a related gauge-fixed form \([30]\), suggesting that critical slowing down may be substantially reduced and possibly eliminated on small physical volumes.

Other algorithmic directions with different ways of attacking the same problem include applications of machine learning to configuration sampling. This possibility has been discussed in a dedicated Snowmass white paper \([31]\).

### 2.2 Lattice QCD as a computational problem

In this subsection we analyze the key characteristics of the computational problem that present challenges to computer hardware design. In particular, we focus on the arithmetic intensity of the various fermion discretizations.

From a computational perspective Lattice QCD is performed on structured Cartesian grids with a high degree of regularity and natural data parallelism. The central repeated operation is the solution of the gauge covariant Dirac equation. In a discrete system, the partial derivative is replaced by a finite difference
with spacing $a$

$$\left( \partial_\mu - igA_\mu(x) \right) \psi(x) \rightarrow \frac{U_\mu(x)\psi(x + a\hat{\mu}) - U_\mu^\dagger(x - a)\psi(x - a\hat{\mu})}{2a}. \tag{2}$$

Wilson’s covariant displacement preserves the gauge symmetry of QCD and is achieved with SU(3) matrices $U_\mu(x) = e^{i g A_\mu(x)}$ connecting lattice sites called gauge links. These $3 \times 3$ complex valued matrices represent the QCD generalization of the electromagnetic vector potential acting on color degrees of freedom.

The Dirac differential operator acts on a space-time field with (typically) 12 degrees of freedom at each coordinate site. It is represented as a sparse matrix on the structured grid where it typically has either 8 (or 16 points) entering the list of non-zero entries in each row, with a nearest-neighbour (or next-next-nearest neighbour) ‘cross’ geometry in four dimensions and with coordinate-dependent coefficient matrices multiplying each contribution. The solution is performed using iterative Krylov solvers, either in a standard algorithm such as conjugate gradient or with a multigrid preconditioner.

The Lattice QCD workflow is divided into two phases. First, a MCMC sampling phase generates an ensemble of the most likely gluon field configurations distributed according to the QCD action. The fields $U_\mu$ are modified throughout the MCMC, so that algorithmic accelerations like multigrid or eigenvector deflation require fresh setup or discovery of the most relevant degrees of freedom. The ensemble generation is serially dependent and represents a strong scaling computational problem. Ideally one would be able to use efficiently $O(10^4)$ computing nodes on $O(256^4)$ data points. On the largest scales this becomes a halo-exchange communication problem since the local data bandwidths vastly exceed those of inter-node communication.

In the second phase hadronic observables are calculated on each sampled configuration where many thousands of quark propagators are calculated and assembled into correlation functions. This both allows more scope for amortizing the setup cost of advanced algorithms like multigrid or deflation, and also has a high degree of trivial parallelism. Computer interconnect limitations are more easily avoided by exploiting this trivial parallelism. The local-memory bandwidth requirements of calculating quark propagators for hadronic correlation functions can be amortized and the computational rate can be brought closer to the cache-bandwidth limit through use of multiple right hand side solvers and some trivial-level parallelism between computing nodes.

We will consider the computational requirements of Wilson, Improved Staggered and domain wall fermion (DWF) actions and with both single right-hand-side and multiple right-hand-side Krylov solvers. Multiple right-hand-side solvers can be advantageous, where applicable, for Wilson and Staggered actions because the gauge field is a significant element of the memory traffic, but it can be applied to multiple vectors concurrently, suppressing this overhead. The same suppression factor already occurs quite naturally for five-dimensional chiral fermion actions such as DWF. When this natural gain is combined with solvers for multiple right-hand sides, the gain is amplified and both algorithmic and execution performance can be accelerated. We assume an $L^4$ local volume and either an 8-point (nearest neighbour) or 16-point (nearest neighbor + Naik) stencil. Under the assumption of weak scaling, the local floating-point, memory and cache bandwidths and MPI-network bandwidth per node suffice to describe large systems.

For either multiple right-hand sides (multiRHS) or for DWF, we take the fifth dimension size $L_s \equiv N_{\text{rhs}}$. A cache-reuse factor $\times N_{\text{stencil}}$ for fermion fields is possible, but only if the cache is of sufficient capacity. We can count the words accessed per 4d site by each node, and the surface volume that must be communicated in a massively parallel simulation with this local volume. Here $N_d$ is the number of dimensions (excluding the fifth dimension for DWF), and $N_s$ and $N_c$ are the number of spin and color degrees of freedom, respectively. These comprise

- Fermion: $N_{\text{stencil}} \times (N_s \in \{1, 4\}) \times (N_c = 3) \times (N_{\text{rhs}} \in \{1, 16\})$ complex,
- Gauge: $2N_d \times N_c^2$ complex.

Similarly we can count the floating point operations as proportional to the number of points in the stencil $N_{\text{stencil}}$ and the number of fermion spin degrees of freedom per point after (any) spin projection $N_{\text{rhs}}$. The
cost of spin projection, and the cost of SU(3) matrix multiplication is

- SU(3) MatVec: $66 \times N_{hs} \times N_{\text{stencil}}$.

We tabulate the arithmetic intensity and surface to volume ratio of the different kernels in Table 1.

| Action            | Fermion Vol | Surface | $N_s$ | $N_{hs}$ | $N_{rhs}$ | Flops  | Bytes  | Bytes/Flops |
|-------------------|-------------|---------|-------|----------|-----------|--------|--------|-------------|
| Staggered         | $L^4$       | $3 \times 8 \times L^4$ | 1     | 1        | 1         | 1146   | 1560   | 1.36        |
| Wilson            | $L^4$       | $8 \times L^3$   | 4     | 2        | 1         | 1320   | 1440   | 1.09        |
| DWF               | $L^4 \times N$ | $8 \times L^3$ | 4     | 2        | $L_s$     | $L_s \times 1320$ | $N_{rhs} \times 864$ | 0.65        |
| Wilson-RHS        | $L^4$       | $8 \times L^3$   | 4     | 2        | $N_{rhs}$ | $N_{rhs} \times 1320$ | $N_{rhs} \times 864$ | 0.65        |
| Staggered-RHS     | $L^4$       | $3 \times 8 \times L^3$ | 1     | 1        | $N_{rhs}$ | $N_{rhs} \times 1146$ | $N_{rhs} \times 408$ | 0.36        |

Table 1: Arithmetic intensity is the ratio of the floating point instructions executed to the bytes accessed for each of the cache, memory and interconnect subsystems. The arithmetic intensities (single precision) and surface to volume ratio for different forms of the QCD action and solver. Here there is significant possible cache reuse on the data due to the stencil nature, however this is only possible with an idealized large-capacity cache. Note that for Wilson and staggered fermions the arithmetic intensity can be greatly improved for valence analysis by increasing the number of right-hand sides as indicated by the last two lines of the table.

Since for multiRHS Wilson or for DWF $\sim \frac{1}{L}$ of the fermion data comes from off node, scaling the fine operator requires a bidirectional interconnect bandwidth,

$$B_{\text{network}} \sim \frac{2N_{hs}B_{\text{memory}}}{N_s} \times R_{\text{eff}}$$

(3)

where $R_{\text{eff}}$ is the effective reuse factor obtained for the stencil in caches (assuming that the memory bandwidth $B_{\text{memory}}$ is saturated). The product $B_{\text{memory}} \times R_{\text{eff}}$ represents the actual data throughput from the entire memory system, including all levels of cache, and this actual cache access rate can be empirically measured using the arithmetic intensity and a measured single node performance in flop/s via,

$$B_{\text{cache}} = R_{\text{eff}}B_{\text{memory}} = (\text{flops/second}) \times (\text{Bytes/flop})$$

It is now common for performance counters and profiling to measure $B_{\text{memory}}$ and sometimes $B_{\text{cache}}$ in isolation, giving several ways to determine the cache reuse rate $R_{\text{eff}}$ and check that memory bandwidth is saturated. The network bandwidth should be sufficient to perform all transfers in the same amount of time as the computation, and can be overlapped with interior terms in the computation that represent the majority of the calculation. Equation (3) reflects a factor of two (send + receive bidirectional bandwidth), and spinor projection compression (Wilson, DWF) and a geometrical $\frac{1}{L}$ for a nearest neighbour halo depth of size one. This argument and the above expressions are easy to generalise to multiRHS HISQ fermions, which acquire a $3/L$ factor due to the larger stencil.

As a concrete example, a four-GPU computer node delivering a measured 10 TFlop/s performance on single node DWF action with a $32^4$ local volume obtains an L2 cache throughput of

$$B_{\text{cache}} = 10000 \times 0.65 = 6500\text{GB/s}.$$  

This implies a per-node aggregate interconnect throughput requirement of 200 GB/s if communication and computation are exactly overlapped.\(^1\)

\(^1\)Our model is robust and numerically matches detailed profiling on nodes that actually perform at 10 TF/s for our working code (four Nvidia A100 four Mellanox HDR 200 Gbit/s cards), delivering 180 GB/s of bidirectional bandwidth to our application, and overlapping communication with computation almost exactly.
With a large cache capacity one projects that for standard Krylov algorithms, the interconnect requirement is proportional to the local cache bandwidth. The coefficient is $\frac{1}{L}$ and typically $O\left(\frac{1}{16}\right)$ which can lead to exceedingly high interconnect requirements, that are comparable to local high-speed memory bandwidths.

Algorithmic research in advanced MCMC sampling algorithms that both avoid critical slowing down and improve the performance on realistic computer networks is required to deliver many of the physics results that support the experimental program discussed above.

2.3 Sample simulation parameters and costs

In addition to the above common physics goals, we discuss some specific ambitions and computational costs for simulations with chiral (or domain wall) fermions. The ambitions for improved staggered-fermion simulations are very similar.

Present chiral fermion lattice calculations use simulation volumes up to $96^3 \times 192$ and use the most powerful supercomputers presently available to the Department of Energy. Improved staggered-fermion simulation volumes are currently as large as $144^3 \times 288$ and are expected to grow to $192^3 \times 384$.

Our physics goals require calculations with ensembles of gauge fields with physical volumes large enough to ensure that finite-volume effects are under control. Such simulations require increased lattice sizes.

A number of specific simulations, Table 2.3, have been proposed with estimated costs in a Snowmass white paper [10], and the same methodology can be used to estimate the requirements of the ideal ensemble for flavor physics.

The final entry is associated with physics in the B-meson system indicated in Snowmass white paper [9]. A $256^3 \times 512$ lattice at a lattice spacing $a = 0.04$ fm ($a^{-1} \sim 5$ GeV) would allow us to simulate up/down, strange, charm, and bottom quarks at their physical mass in a 10 fm box with $m_\pi L = 7$.

These simulation goals clearly demonstrate a need for computers at least 10x more capable than the coming Exaflop computers. Since the performance is required to be delivered on a real-code performance basis, and efficiency will not be 100%, more than an order of magnitude improvement, perhaps, from both algorithms and computing are required.

| Lattice volume | $a^{-1}$ GeV | Exaflop hours |
|----------------|-------------|---------------|
| $32^3 \times 64$ | 1.4         | 1.5           |
| $40^3 \times 96$ | 1.7         | 3.5           |
| $48^3 \times 64$ | 2.1         | 7.5           |
| $48^3 \times 96$ | 1.8         | 7.54          |
| $64^3 \times 128$ | 2.4       | 25            |
| $96^3 \times 192$ | 2.8      | 120           |
| $64^3 \times 256$ | 2.4       | 50            |
| $96^3 \times 384$ | 2.8       | 250           |
| $128^3 \times 512$ | 5.0      | 1500          |

Table 2: Proposed lattice volumes and cost estimates in sustained Exaflop hours, scaled from current simulations on Cori (NERSC) and Summit (ORNL). Volumes and estimates are proposed in Snowmass white paper [10], while the final entry uses the same methodology to estimate the cost of the most expensive proposed B-physics capable simulation.
3 Computational landscape and challenges

We have demonstrated above that there is a deep need for future supercomputing that reaches far beyond the Exascale over the next decade. Since Moore’s law is hitting scaling limits, it is important to consider whether prospects for greater computing power are realistic.

In this section we discuss the opportunities presented by some new technologies that give realistic prospects for improving system-level design. In particular we will consider new memory and packaging technologies that may enable better recruitment of expensive logic silicon for useful computation. Tree-dimensional memory and advanced packaging offer the opportunity to make connections to memory in numbers governed by microscopic rather than macroscopic constraints. In this transition there is “plenty of room at the bottom” for sustained gains to be made in the more efficient use of transistors as a response to slowing of Moore’s law for transistor density.

DOE HEP computation is likely to remain composed of both “high-throughput computing” (HTC or capability) and “high-performance computing” (HPC or capacity). HTC exploits the trivial parallelism of independent processing nodes and independent Unix processes, and achieves high aggregate throughput by running many work items in parallel. High-performance computing has an enormous investment in message-passing-based interconnects. Almost all HPC software is written to the Message Passing Interface standard (MPI) and this appears a good long term investment because it supports the asynchronous decoupling of long latency operations, such as remote data access, from local processor execution.

Within each Unix process, the elements of the computation performed by both HPC and HTC may or may not be amenable to acceleration. There are a variety of acceleration options, the most frequent of which are Graphics Processor Unit (GPU) accelerators (typically from vendors such as Nvidia, AMD or Intel).

The principal technical difference between a HPC capable system and a HTC computer is the performance of the interconnect and potentially the degree of vendor specificity of the software environment; for example whether the computing nodes are on a closed network among other issues.

While exotic hardware can seem attractive, it is worth remembering that at the time of writing, the fastest computer in the world is the RIKEN Fugaku system, which is based on a standard ARM multi-core CPU architecture that is commonly used in mobile phones, combined with an integrated network and high bandwidth memory. It has vector-computing acceleration, but does not make large compromises in either software or special-purpose directions.

Some simple physics explains much of recent and presumably future computer architecture trends. This enables us to attempt some degree of informed forecasting, perhaps even over a ten-year timescale, as we discuss in the following subsections.

3.1 Wire-delay considerations

Wire delay is the amount of time for a signal to propagate down a wire, and is proportional to the “discharge time constant” $RC$, where $R$ is the resistance and $C$ the capacitance. We can model a wire as an isolated semi-infinite rod of metal of volume $L \times \pi r^2$ and look at the scaling of resistance and capacitance as all dimensions are “shrunk” by the same scale factor. This gives rise to the discharge time constant,

$$RC \sim 2\rho \epsilon L^2 \frac{r^2}{\log(r_0/r)} \sim \text{const} \times \frac{L^2}{r^2},$$

where $\rho$ is the metal resistivity, $\epsilon$, the dielectric permittivity and $r, r_0 \ll L$ are length scales small enough that the potential remains well approximated by the long rod approximation.

This is profound, because, to a great degree (ignoring logarithmic terms and finite size effects), wire delay depends only on the geometry or aspect ratio of the wire. Shrinking a process by the same scale factor in all dimensions does not reduce wire delay. Wire delay places an irreducible floor below which faster transistors make no difference.
It is interesting to note that, historically, process advances announced in the press have included materials changes designed solely to move this wire delay floor out of the way of transistor scaling: “copper interconnect” (180 nm) and “low-k“ dielectric (100 nm) improved $\rho$ and $\epsilon$ respectively changing the RC time constant. The balance of a timed circuit becomes worse as transistors become faster without changes to the materials that control wire delay.

The immediate corollary of the aspect ratio dominance of wire delay is the following:

**Multiple processing units with broad wire long-haul buses is the only possible high speed wiring strategy for a planar 10 billion transistor device.**

There must be a low number of long-range “broad” wires (bus/interconnect) since they consume area, and a high number of short-range “thin” wires giving densely connected regions (cores).

One might note here that the mean wire length *could* be reduced with devices containing multiple logic layers of transistors, but this would require *radical* changes to silicon manufacturing and cooling technology to obtain truly three-dimensional high logic, whether by chip-stacking or single-silicon-chip lithography. One might hope that, in future, current thermal issues can be solved and that silicon cubes can add a radical new dimension to digital computing.

### 3.2 Hierarchical memory and multiple address spaces

One important application of the previous simple analysis of wire delay arises when we consider the connection of a calculation device to memory chips. Long copper traces across a printed circuit board fall into precisely the worst case corner for wire delay. Further, given the speed of light and GHz-scale clock rates, distances of order 30 cm ($= 1$ light-nanosecond) set the length scale at which transmission-line behaviour sets in. These considerations and, at high speed and with long wires, also, transmission line dynamics, impose constraints on memory technologies.

Data motion is often the single greatest performance and power bottleneck to be addressed in computation. A nuanced approach to analysing algorithms will distinguish data references by their most likely origin (e.g. cache, local memory, remote node). For a bandwidth analysis, the ratio of the floating point instructions executed to the bytes accessed for each of the cache, memory and interconnect subsystems is called the *arithmetic intensity* of the algorithm and is a key classification to understand if future computers are engineered to support efficient execution of the algorithm.

Since the cost of floating point units has decreased exponentially, while the cost of macroscopic copper traces has remained (relatively) static, it is often the case that execution is memory limited. Many scientific algorithms can be categorised as either bandwidth or FPU bound, for example, using the Berkeley roofline model [32], and as discussed above, the single-node performance of QCD codes are typically bandwidth bound with the L2-cache bandwidth being dominant for multiple right-hand sides, while either the network bandwidth and/or memory is the rate-limiting step in gauge ensemble generation.

In order to substantially address memory limitations it is necessary to change both the aspect ratio of the wires involved, and vastly increase the number of wires that can carry data concurrently. Fortunately the computing industry has already developed technology directions responding to this imperative with the 3d “High Bandwidth Memory” (HBM) technology. It is becoming a widespread solution. These chips contain memory layers with large numbers of vertical metal rods (through silicon vias or TSV’s) used as high-bit-count buses skewering the device, Figure 1. The ability to stack such chips vertically, with solder connections between TSV’s, gives a favorable aspect ratio, low wire delay and power. Furthermore, the miniaturization enables a high bit-lane count connection that gives a massive data rate for reading memory pages; it is exciting that after a long period of conservatism in memory design, there is no clearly defined barrier to further increases in the number of bit lanes and bandwidth.

The advent of HBM is driving a rather revolutionary development across the industry towards on-package integration of memory, where a sizable volume of memory is physically co-located with the pro-
Figure 1: A micrograph of a through-silicon via bus structure giving thousands of bit lanes connecting memory chips with using a stacked configuration to give wiring geometries favorable from an energy and delay perspective.

Figure 2: Independent of the underlying computational technology (cores, GPUs, FPGAs etc), the underlying physics of electronics is driving a hierarchical computing node memory system organization in the direction of accelerators.

cessing unit. This may be done either through 2.5D integration with memory stacks placed along side a processing unit on an interposer (i.e. a miniature circuit board), or full 3d integration where memory chips are bonded on-top of a computational processing unit using TSV’s. Recent Nvidia GPUs have used a silicon interposer and recent GPU and EPYC CPU designs from AMD are multichip modules. Intel has developed an “EMIB” multi-die package technology that enables bridging between adjacent die. Rather than using a large silicon interposer, this links together neighboring die in a multichip package much as a child might join together two 8x8 LEGO squares to make a 16x8 surface using a small 2x2 backing block.

A further significant emerging memory technology is a novel form of non-volatile phase-change memory, typically based on an amorphous/crystalline glass cell. This should enable increased memory density, and in particular is suggested to support many bit-cells vertically stacked in multiple layers, promising four times higher density than DRAM. The most exciting application lies in plans to give such devices conventional DRAM electronic interfaces. It is a reasonable assumption that once a page is open, the column accesses should be no slower than in a conventional DRAM since the electronics will be similar, while page-miss latency may be larger. Certainly, there are already standards for non-volatile memory modules, and this is
likely a disruptive technology for large memory algorithms, such as the use of Dirac eigenvectors and the assembly of multi-hadron correlation functions.

It is reasonable to assume that on a decade time frame, memories will remain hierarchical and likely become even more hierarchical than at present. Based on the above arguments there are good fundamental electronic reasons to have computational devices with the following properties:

- one hundred or more independent processing units.
- internal caching with several to many times local memory bandwidth.
- local in-package memory with several TB/s high bandwidth memory ranging to 100 GB scale or more.
- slower conventional DRAM-based system memory with greater capacity (multi TB) and a few 100’s of GB/s bandwidth.
- Non-volatile memory with even greater capacity, longer latency and potentially reduced bandwidth.

Such a memory organization is illustrated in Figure 2. In this diagram the red data transfers could either be managed by user software, or by demand-driven fetching (at the cache line or memory page level of granularity). This organization is potentially independent of the computational technology. For example, low power multi-core CPUs with high bandwidth memory could be used as accelerators for multi-core CPUs with large slow memory.

Given such hierarchical memory, there is a significant question about how this should be managed. There are two logical solutions, and several means of implementing each.

The first is the “easiest” from a computer vendor perspective: to solve the problem with application and library software. One can expose these as multiple address spaces and have the programmer or programming interface handle data placement. This is a familiar element of the (now) traditional GPU or “offload” model, which can be implemented by giving the programmer two types of pointer: ’host’ and local ’device’ pointers, and requiring the programmer to keep track carefully of what data is stored where.

Another common software solution, taken by SYCL [35], is to break arrays into distinct data-storage containers and accessors, with the process of gaining access implicitly triggering data motion.

The second and more elegant solution is to solve the problem with operating system software. This presents a single logical address space in Unix virtual memory, but moves physical pages of data to a nearby location as and when required by a computational unit. This is a generalization of the familiar Unix virtual-memory swapping mechanism and could in future be systematised to cover even more hierarchical memory systems including both volatile, non-volatile, HBM etc... Such “unified” address space solutions presently introduce some performance overhead, but greater flexibility of memory page sizes to amortize transfer overheads would surely make this a good, long-term solution that simplifies the programming challenge.

Unified Virtual Addressing, as used in CUDA, SYCL and HIP, is a good way to organize this and minimise the programmer burden. This uses existing low latency “translation look-aside buffer” (TLB) virtual memory mechanisms to search for a local copy of data. The default page in user space for HPC systems could be promoted from 4KB to 2MB (or an intermediate page size introduced) so that paging overheads are amortised on large arrays. This viewpoint would naturally extend to hierarchical systems comprising both slower non-volatile memory and faster conventional DRAM.

In the Fugaku computer, high bandwidth memory has been combined with multi-core ARM processors, using HBM as the main memory. In some cases, such as the Intel Knight’s Landing, HBM has been used in conventional CPUs as a cache. However with multi-gigabyte caches the status information (tags) are also huge and must be stored in the HBM. This makes the process of checking for the data in the cache expensive. In contrast, a virtual memory translation typically hits in the translation cache. Larger user space page sizes would enable good coverage in translation caches and good efficiency for page migration.
Page sizes as small as 4KB are almost certain to lead to low performance in user space in future, and delivered bandwidth will grow with the granularity of transfer. One might hope that an improved virtual memory system would have an option to only use larger memory pages, such as 2MB in future to amortise kernel overheads.

The development of such a coherent virtual memory direction for hierarchical memory may require incentives from the Department of Energy ASCR leadership and could perhaps lead to significant benefits to computing environments.

### 3.3 Interconnect technology

From a technological perspective, the interconnect is the most significant lagging capability in computer evolution, as illustrated in Figure 3. The ratio of memory bandwidth to local floating-point performance and the interconnect bandwidth to local floating-point performance has changed in the last twenty years from numbers of order one, to numbers of order ten (memory) and order one thousand (network). While it is true that these fat nodes are capable of larger problems, permitting a lower surface to volume ratio and, thus, a proportionally lower network demand, it is still the case that parallel algorithms must be designed to avoid the substantial cost of accessing remote data. We must invest in developing efficient algorithms that recognise that new computers have a series of weakly linked islands of high performance to realise the potential of computing ten years from now. Fortunately the separation of length scales is a fundamental element of physics and effective theories, and multi-scale algorithms are a reasonable goal for research investment.

We may also ask: is limited network bandwidth a technological necessity? While 100 Gbit/s of copper trace in a larger printed circuit board (PCB) will cost as little as pennies, the same 100 Gbit/s of bandwidth in copper cables costs just under $100 USD. A similarly performing 100 Gbit/s active optical cable over four bit lanes costs of order $1000 USD.

The arguments given in the above discussion on wire delay and transmission line behavior are not applicable to fiber-optic cables. Modern interconnect cables that exceed roughly 2 m in length are predominately fiber optic. They increase costs, as optical transceiver technology uses exotic non-silicon processes (that differ from standard logic silicon), and cables (such as QSFP56) have electrical interfaces with active optical transceivers on either end. This raises price and power substantially.

The stated purpose of very large-scale integration (VLSI) is the reduction of cost by combining multiple “parts” in a single chip. Significant industrial effort is being focused on technologies such as silicon photonics to implement fiber optic transceiver logic in a standard silicon process. Combined with vertical-cavity surface-emitting laser (VCSEL) technology there is a real possibility of a chip-to-chip light path, perhaps multiplexing laser cells with different colors, with greatly increased bandwidth removing active optical transceivers and alleviating bandwidth limitations. However the appetite and focus of industry on this direction may depend in large part upon Department of Energy requirements for future supercomputers.

In notable cases, such as the IBM BlueGene computing line, system on a chip technology has enabled integrating not just network transceivers but also a distributed router element. This enables the integration of not only a network interface but also the switch system in the node silicon, substantially reducing the added cost of the high performance interconnect from over a thousand dollars per node to dollars per node. At the time of introduction BlueGene/Q had bandwidth equivalent to eight contemporary infiniband cards but connected nodes together within a rack with glueless wiring (i.e. no intermediate chips). Optical transceivers were used on the surface of a computer rack containing 1024 nodes, suppressing the cost of optics in the system. This example emphasizes the importance of Department of Energy ASCR leadership in encouraging innovative system integration.
| Location | System | Interconnect (GB/s) per node (X+R) | Floating point performance (GF/s) per node | Memory Bandwidth (GB/s) per node | Year | System peak (PF/s) | FP / Interconnect | FP / Memory | Memory / Interconnect |
|----------|--------|-----------------------------------|---------------------------------|--------------------------------|------|-------------------|------------------|-------------|---------------------|
| LLNL     | BlueGene/L | 2.1                               | 5.6                             | 5.5                             | 2004 | 0.58              | 2.7              | 1.0         | 2.6                |
| ANL      | BlueGene/P | 5.1                               | 13.6                            | 13.6                            | 2008 | 0.56              | 2.7              | 1.0         | 2.7                |
| ANL      | BlueGene/Q | 40                                | 205                             | 42.6                            | 2012 | 20                | 5.1              | 4.8         | 1.1                |
| ORNL     | Titan    | 9.6                               | 1445                            | 250                             | 2012 | 27                | 150.5            | 5.8         | 26.0               |
| NERSC    | Edison   | 32                                | 460                             | 100                             | 2013 | 2                 | 14.4             | 4.6         | 3.1                |
| ORNL     | Cori/KNL | 32                                | 3050                            | 450                             | 2016 | 28                | 95.3             | 6.8         | 14.1               |
| ORNL     | Summit   | 50                                | 42000                           | 5400                            | 2018 | 194               | 840.0            | 7.8         | 108.0              |
| RIKEN    | Fugaku   | 70                                | 3072                            | 1024                            | 2021 | 488               | 43.9             | 3.0         | 14.6               |
| NERSC    | Perlmutter/GPU | 200                            | 38800                           | 6220                            | 2022 | 58                | 194.0            | 6.2         | 31.1               |
| ORNL     | Frontier | 200                               | 181200                          | 12800                           | 2022 | >1630             | 906.0            | 14.2        | 64.0               |

Figure 3: Evolution of high end HPC computer systems over the last two decades. The interconnect has only increased slightly in performance while larger increases have been seen in local memory bandwidth and particularly in (peak) per node performance with the introduction of accelerated computing as powerful (and expensive) nodes. These are often accompanied by powerful memory networks interior to the node. This growing ratio of both local floating point and local memory bandwidth to interconnect performance leaves interconnect a bottleneck in many current algorithms for Lattice QCD ensemble generation.

### 3.4 Processing Units

At the core of a computing system is the processing unit, defining the interface a programmer controls directly, if at least through a compiler, and which accesses the memory to perform its assigned tasks.

The traditional programming unit is the microprocessor, which is commonly called a processor “core”. They present a logical state and instruction fetching and execution. They are mature and typically highly optimised for the highest performance for (cleverly giving the illusion of) serially executing a single sequence of instructions. Since processor cores optimised in this way are large and power hungry, it has become common to improve floating point throughput and power by using short vector “single instruction multiple data” (SIMD) units to increase the density of floating point hardware. However, this design does introduce constraints and not all HEP computing can benefit from such hardware. Lattice QCD does, and it can make use of all current SIMD architectures and even longer vectors would be useful.

Modern GPUs provide powerful alternatives to CPUs and deliver excellent performance and power performance for a number of reasons. Firstly, an accelerator architecture may be more specialized than processor cores that target the best general purpose single-thread performance (and a full range of features). Secondly, by using a private memory system, more aggressive technology decisions may be made. A host CPU is retained for executing general code that is not well handled by a GPU thread, and an offload model is used where critical loops and data are marked by software for execution and placement on the accelerator device.

All GPUs at this time present a parallel multi-dimensional (1d to 3d) loop as the primitive looping construct. Similar to the Connection Machine computer, the machine model is that each instance of the parallel loop body is presented as a different virtual machine or thread. However, syntactically, the implementation is less elegant without data-parallel expressions in the high level language.

Although GPUs are fundamentally SIMD architectures, addressing modes and masked execution are cleverly used to obscure this fact and present a scalar processing model to the programmer, called “Single Instruction Multiple Thread” (SIMT). In SIMT, a single instruction fetch unit controls `multiple` logical threads, typically a number of O(32). When some threads choose yes, and other threads choose no the divergence leads to loss of parallel throughput.

Accesses to thread-private data (stack, local memory and what the programmer would think of as local variables - if not in registers) are addressed in a way that efficiently interleaves accesses to corresponding local memory locations by each thread in a physical memory array. One might imagine that electronically the “thread” index within a parallel execution group dictates the byte address within a hardware SRAM data bus. This ensures that when a group of software “threads” concurrently execute the same instruction,
they all access the matching variable on their respective stack or local memory. The accesses will be transferred as a physically contiguous data transaction even though the virtual addresses are relative to different stack pointers or in a local memory space.

Both GPUs and CPUs make use of forms of “hyperthreading” but with varying degrees of expected parallelism. CPUs are very much focused on single thread performance (i.e., time to solution for for single serial calculation) while GPUs are rather more focused on parallel performance or throughput. Modern CPU processor cores will concurrently execute instructions from between one and four independent processes or threads, each of which maintains its own instruction pointer and program state. In contrast up to 32 independent instruction pointers are common in current GPU processors. The low thread count of the CPU makes for smaller register files with lower latency to access, while GPUs use larger and multi-banked register files with potentially longer latency. In this way, a GPU is able present a greater number of independent outstanding loads to the memory system with greater throughput from the additional memory level parallelism. This parallelism is easier to find and manage in the GPU since the independent threads are trivially parallel while in the CPU direction significant engineering resource is devoted to dependency tracking.

One might imagine that conventional microprocessors could, in principle, add addressing modes that facilitate a similar SIMT model in their vector extensions and become more GPU-like to make vectorization easier. It is also possible low power CPU multi-core designs coupled with GPU style high bandwidth memory systems could provide viable alternatives for some of the HPC workload, particularly those which do not map well to the data parallel model. The very large peak performance of GPUs is relevant to dense matrix operations, but bandwidth limits the performance of many other common algorithms. The success of the Fugaku computer combining low power Fujitsu/ARM processor cores with 3D memory and an integrated network is a good example.

3.5 Machine Learning and non von Neumann hardware acceleration

Fixed-function acceleration is possible in the area of machine learning (ML), or more generally problems involving dense matrix multiplication, with tensor cores in GPUs and other hardware specific to large matrix multiplication. These are commonly on 64-bit and 32-bit floating point arithmetic, or even on 16-bit floating point arithmetic. Mixed precision functional units are common. Fixed function dense matrix hardware such as tensor cores have proven difficult to use because the 3x3 complex gauge link matrices do not map naturally to such hardware.

Reduced precision formats using non-IEEE mantissa and significand have been found to be useful in Machine Learning including non-IEEE 16-bit \[36, 37\] and 8-bit arithmetic \[38\]. A joint patent for a hardware implementation for the modified 16 bit floating point format was submitted by one of the authors (PB) with Intel in 2018 \[36\] wherein it was demonstrated to provide better support for training in machine-learning. Lattice QCD can also make use of reduced-precision arithmetic, and gains 64-bit precision accuracy using 32-bit or even 16-bit precision floating point arithmetic in a preconditioner or mixed-precision solver \[39\].

Machine learning can broadly be decomposed into “training” and “inference” phases. Training often involves scientist supervision for parameter tuning with time to solution being critical. Distributed machine-learning training parallelizes the training process. The use of small “mini-batches” in stochastic gradient descent leaves the problem very sensitive to Amdahl’s law slowdown. In the massively distributed limit \[40\], the machine-wide reduction of a gradient vector calculated across many randomly selected training samples in a minibatch is a significant performance bottleneck \[41\]. Here the extreme interconnect requirements of QCD and distributed machine learning align \[42\].

Inference with a previously trained network is typically performed on many independent events of data, and is in many cases a trivially parallel or HTC problem. However, when inference is used in either hard or soft real-time situations (as occurs in HEP experiment) the deadlines impose challenging performance
Accelerators from GraphCore and Cerebras are good examples of ML-optimised accelerators in addition to more standard GPUs by Nvidia, AMD and Intel. Reconfigurable computing hardware such as Field Programmable Gate Arrays (FPGAs) or spatial accelerators are powerful options for certain algorithms. They may be accessed via a dataflow model where a hardware circuit is configured in a non von Neumann approach to stream data from memory, calculate and store results in memory. They may be particular effective in eliminating software latency in real time inference environments.

### 3.6 Architectural diversity

The overview above shows that the computing landscape at this time displays a wonderful proliferation of computer architectures. It opens vibrant and healthy opportunities for innovation and competition and for the introduction of new and competitive ideas. However, it also risks introducing special-purpose or less-general features than with previous systems. The memory spaces are often fragmented with different types of pointer and location of data. There are multiple instruction-set architectures in use, each with different programming interfaces or models. A single program frequently has different segments that target different types of instruction sets within a single computer node. Indeed, some of the computing models available (such as FPGAs or spatial architectures) are not von Neumann computers. Others have dedicated fixed-function acceleration of matrix-matrix multiplication for machine learning.

This diversity presents a significant challenge to the scientific programmer with either legacy code or the need to use more than one system.

### 4 Software challenges

Text-book computer engineering \[43\] suggests that code optimizations should expose an admixture of two forms of locality: spatial-data-reference locality, and temporal-data-reference locality. Buses are used that transfer many contiguous words of data at a time in many levels of a computer memory system. For example, when a DRAM page is opened (row access), very many consecutive bit cells are read and page hits (column access) incur much less overhead than would occur with a new random access, which would give rise to different latencies. Further, multiple layers of caches are used; the access bandwidth and latency characteristics become poorer the further one travels from the processor core while the storage capacity correspondingly increases. The transfers between levels in the memory system are performed in aligned chunks called cache lines, with sizes $\in \{32, 64, 128, 256, 512\}$ bytes depending on the level and computer architecture. For example, modern Intel chips typically use a 64 B cache line in all layers of the cache hierarchy, while the IBM BlueGene/Q system made use of a 64 B L1 line size and a 128 B L2 line size.

Spatial reference locality arises because memory systems are fundamentally granular. If one accesses one word of a cache line, the entire cache line is transferred over the (rate limiting) buses in the system. In order to maximise performance, algorithms and codes should ensure that they make use of all the data in cache lines that are transferred, by laying out data to give spatial locality of reference. Architectures use cache lines based on the observation that many algorithms (such as linear algebra) have such access patterns. Temporal locality of reference arises when data referenced is likely to be referenced again soon. Thus caches store recently accessed data. Both of these hardware optimizations are relatively easy to exploit by writing software that loops or block in the right order.

Additional considerations apply when working with architectures that implement short vector instructions (single instruction, multiple data or SIMD). Particularly when the processor core is complex, these instructions represent one of the cheapest ways to to enhance peak floating point performance, as it vastly cheaper to build hardware to execute a single arithmetic instruction or load on a contiguous block of N elements of data than to generally schedule N independent and decoupled instructions. In order to exploit these instructions, code optimisations should expose what one might call spatial operation locality.
Although there are obvious applications in array and matrix processing, even matrix transposition shows that it is surprisingly hard to exploit SIMD in general because you must arrange to have same operation applied to consecutive elements of data.

### 4.1 Programming models

There are a number of different programming models one must consider in the present computing landscape. The following can all in principle be combined with MPI message passing.

Present LQCD ECP codes support:

- traditional CPU core, SIMD floating point instructions, and OpenMP threading,
- CUDA-based GPU programming,
- HIP-based GPU programming,
- SYCL-based GPU and FPGA programming.

Expected to be supported by future LQCD software:

- OpenMP 5.0 offload GPU programming,
- C++ parallel STL offload programming.

The challenge of writing high-performance and portable code is three fold:

- syntactical differences,
- semantic differences, and
- data placement.

Firstly, the syntax for offloading loops depends on the underlying software environment. Of the Department of Energy open-science pre-exascale and exascale systems (Summit, Perlmutter, Frontier and Aurora), all are accelerated with various architectures and there are three distinct native vendor programming models: Nvidia/CUDA, AMD/HIP and Intel/SYCL. One approach may be to identify a suite of abstractions that are both compact and adequate to write portable and high performance software with a single abstract interface that can itself target all of the above. Techniques introduced by the RAJA [44] and Kokkos [45] libraries use C++ device-lambda function objects to capture sequences of offloaded code. Within Lattice QCD, a similar effort has been undertaken in the ECP project on the Grid software library [46–48] and on the QUDA software library [39], which have both been ported to execute under CUDA, HIP and SYCL.

Alternately, one could rely on a single standard interface that is used to provide portability. OpenMP 5.0 offload is potentially such an interface, and may in fact be the appropriate portability layer in the long term. OpenMP has already been transformative in making multithreaded programming accessible to the average scientific programmer. However, there are other standards-based alternatives: SYCL is in principle cross-platform portable, but whether the performance of either SYCL or OpenMP reproduces vendor-native compilers giving performance portability remains to be seen. Similarly, C++ parallel STL offers a more standard C++ interface to offload and parallelism, but this is even newer than either SYCL or OpenMP offload. Parallel STL presently has support from only one GPU vendor and, although a standard, the roadmap to de facto portability remains unclear.

At this time, abstraction and support for multiple programming models is likely the safest option for portable efficiency on near term computers. For projects that wish to reuse interfaces to acceleration, both Kokkos and RAJA provide good options as Department of Energy ASCR projects.
Secondly, the somewhat larger challenge is to write a single program that captures the differing semantics between SIMD and SIMT execution models. The optimal data layout changes with the parallelism model. Both SIMD and SIMT use underlying vector architectures and a partial “struct-of-array” transformation is needed in data arrays in memory. However they semantically differ in the behavior of local variables within functions. In a GPU, each “lane” of the underlying SIMD executes a different logical instance of the same function, and thus processes scalar items, while in a CPU local variables remain (short) vector data types. Optimal software cannot be invariant when the architecture is changed, and rather to target both efficiently it is necessary to design a programming style that transforms covariantly with the architecture. Some QCD implementations use compiler intrinsics to obtain performance, however software could be substantially simplified if (perhaps working with the Kokkos team) we could successfully advocate that C++ “std::simd” support vectors of complex number datatypes and standardise permutation and vector interleaving patterns.

Thirdly, placing data and managing data motion should be simple and even transparent. Marking up every loop with ‘copy-in’ and ‘copy-out’ pragmas is laborious, for example. As advocated above, unified virtual memory and a systematic approach to increasingly hierarchical memory by the computing industry would give a significant and long term benefit to scientific programming.

The Lattice QCD recipe for building portable code on accelerated systems could be summarised as:

- abstract offload primitives and device function, attributes,
- abstract memory allocation primitives,
- abstract communication patterns such as halo exchange, shifting, reductions and sub-block reductions.

Unless relying on unified virtual memory or similar:

- Software managed device cache for host memory regions,
- Distinguish accessors (views) of lattice objects from the storage container.

To obtain portable performance:

- Abstraction capturing SIMT and SIMD models in a single interface,
- Provide Stencil differential-operator-assistance primitives: neighbour indexing, halo-exchange patterns.

This has been achieved in at least two major code bases for the HIP, SYCL and CUDA APIs. One of these also gives good performance on several CPU SIMD architectures. Newer interfaces like OpenMP 5.0 offload and ISO Standard C++ (including parallel implementations of the C++ Standard Template Library, also known as Parallel STL or pSTL) will be adopted as and when enabled both by the maturity of the respective standards and of compiler implementations. In particular, compilers are only now starting to adopt OpenMP-5.0 features and ISO C++ parallelism is implemented in several compilers for CPU architectures and even by one GPU vendor. However, the C++ standard does not currently feature the concept of different memory address spaces and hence GPU implementations rely on automatic data movement and are currently limited to parallelism on one type of device (either the host CPU or GPU) via the STL at a time, requiring other frameworks in a hybrid application.

One hopes that this might consolidate the proliferation of programming interfaces as the lack of standardization imposes a very significant software development burden on the science community with duplication of effort for multiple systems. This software development underpins the entire community effort and requires support to make the Snowmass science goals feasible.
5 Ecosystem Challenges

For this ambitious physics program to be viable, a significant computational challenge exists, in order to enable a series of calculations on ensembles from $a^{-1} = 3 - 5$ GeV, and lattice volumes from $96^3 \times 192$ to $256^3 \times 512$. The challenges exist on multiple fronts: intellectual in developing algorithms that evade critical slowing down, software engineering to develop well-performing and portable code on an evolving range of supercomputers and programming models, and technical to remain engaged with the DOE HPC community as systems are planned and developed.

USQCD has had senior-staff, post-doctoral researchers, and post-graduate students funded under the Exascale Computing Project (ECP) and the SciDAC program. The ECP project includes algorithmic programs in “Critical Slowing Down”, and “multilevel solvers” [1, 25, 49]. It has also funded the development of high performance software portable to Exascale hardware [39, 46, 48]. The community activity is dependent on the existence of bespoke software environments that enable efficient simulation on rapidly evolving supercomputers, which requires significant expertise to develop and sustain. It is important that these gains continue to be realised, with continued funding throughout the Snowmass period, or an opportunity comparable to the available gains in computer performance will be lost.

It is important that flexible high-performance software is developed for a diverse range of architectures that tracks the DOE computing program. The life cycle of scientific code is at least 10 years, and the health of a community depends on large code bases (up to 200,000 lines of code) which do not have a secure model for development and support which places investments at risk.

A worrying trend is also identified in a recent preprint [50], which notes that innovation in the design of computing devices is now much more heavily influenced by the needs of mobile-phone companies and cloud data centers and their associated services than by Governments due to the enormous financial power wielded by the leading corporations compared to levels of investment which Governments can bring to bear. The paper also notes “Talent is following the money and the opportunities, which are increasingly in a small number of very large companies or creative startups”. Such a trend may continue to exacerbate retention challenges in the future.

5.1 Lab supported software development

Just as the large experiments require talented permanent staff at the Labs to engineer experiments and manage sophisticated long-term software systems, cost- and people-effective lattice QCD desperately requires that career paths be created to retain some of the most talented experts in software and algorithms.

The permanent staff will address HPC architectures as they emerge under the Computational Frontier. We aim to develop performance-portable high-level data-parallel code, with a write once and run anywhere approach that many domain scientists can modify effectively. This effort must track the evolution of computing. A key element of managing the science program is the early engagement with DOE HPC laboratory sites during the development, and years prior to installation, of major new facilities. The lead time for porting to new architectures lies in the region of multiple years, and early engagement with emerging architectures is required to ensure timely scientific exploitation.

5.2 Joint Lab-University Tenure Track Appointments

We believe the DOE should seek to foster the continued development of intellectual leaders in computational quantum field theory. This is now done very well at the Labs, with a large fraction of the leading personnel located there. The health of the field requires a similar cohort of individuals at the best universities, reflecting the intellectual vigor and potential of this area to contribute to DOE scientific goals. The creation of such positions can be stimulated by DOE-funded joint, five year, tenure-track appointments.

Theoretical particle physics is one of the last area of physics to recognize the importance of computation in forefront research and continued effort is urgently required to overcome this historical bias, and create
a vibrant pool of skilled young faculty, and around them their PhD students and research groups.

6 Conclusions

We have discussed the importance of Lattice QCD to key elements of the future experimental physics program. Theoretical results for hadronic parameters are critical inputs in searches for new physics: for example, the DUNE and Fermilab $g - 2$ experiments, in addition to continuing searches in flavor physics.

Lattice QCD is uniquely dependent on high-performance computing. High performance computing will likely continue to evolve rapidly, bringing both a massive potential for science, but also posing significant challenges to exploitation and delivery of this potential. This evolution must be coupled with continued innovation in developing multi-scale-capable algorithms to solve the challenge of critical slowing down in Markov-Chain Monte-Carlo and in multi-scale linear solvers for all fermion discretizations.

DOE support to develop the software and algorithmic environment in which computational quantum field theory can be performed is important to enable the rich physics plans of the community. The DOE has an opportunity to foster the development of a cohort of talented LQCD scientists doing this work.

Due to the nature of leadership-class computing facilities, continued access to institutional-cluster computing for USQCD is a substantial benefit to productivity, allowing the long tail of many small projects with modest computer requirements to be carried out using easy-to-program architectures with a low barrier to access.

Scientific productivity would be enhanced if the DOE used its influence on world supercomputing to standardize programming interfaces and ensure simple memory models that hide underlying hardware complexity.

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