EVIDENCE FOR NEUTRINO BEING LIKELY A SUPERLUMINAL PARTICLE

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Abstract Experimental evidence and theoretical argument in favor the claim of neutrino being likely a superluminal particle, a tachyon, are discussed from six aspects with details.

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1. Introduction

The question of superluminal particle (tachyon) has a long history. For a broad review on superluminal motion, see Racami’s paper[1]. In this paper we will concentrate on some crucial points which seem to obstruct the general acceptance of the tachyon concept in physics community. They are:

(a) Does the principle of the limiting speed deny the existence of a tachyon?
(b) Can the puzzle about the violation of causality in superluminal motion be solved?
(c) Can we find a quantum equation for tachyon and what is its symmetry?
(d) Could a possible tachyon be the neutrino? What is the crucial evidence on this claim?
(e) There is an analytical continuation of mass in the transition from subluminal motion to superluminal one. What is its physical meaning?
(f) Does the so-called Pseudotachyon[2] exist?

We will discuss these questions in next six sections before the summary and further discussion will be made in the final.

2. The Principle of Limiting Speed and the Principle of Causality

In the theory of special relativity (SR), the kinematic relation of a particle reads:

\[ E^2 = \vec{p}^2 c^2 + m_0^2 c^4 \]  

(1)

where \( E, \vec{p} \) and \( m_0 \) are energy, momentum and rest-mass of the particle respectively. Then the velocity \( \vec{u} \) of the particle can easily be derived as:

\[ \vec{u} = \vec{p} c^2 / E \]  

(2)

\[ \vec{p} = m_0 \vec{u} \sqrt{1 - u^2 / c^2}, \quad E = \frac{m_0 c^2}{\sqrt{1 - u^2 / c^2}} \]  

(3)
Obviously, the speed \( u \) of particle cannot exceed that of light: \( u < c \). In other words, there is an upper limit \( c \) for all (subluminal) particles with nonzero rest-mass \( m_0 \).

Because of the great success of SR, the existence of a limiting speed gradually became a principle to construct new theory in physics. Sometimes, it took an equivalent form as a principle of causality — any particle, or energy, or information can only be traveling in a speed smaller than (or at most equal to) \( c \) and along the time direction from the past to the future.

By a careful reading on the original paper of Einstein in 1905, we understand that there are two relativistic postulates for the establishment of SR: the principle of relativity (A) and the principle of constancy of the speed of light (B). The B is necessary to establish the Lorentz transformation (LT) before the A can make sense in two (or more) inertial systems (\( S, S' \) etc.) quantitatively. In other words, the light-speed \( c \) exhibits itself as a constant and serves as a means to fix the relationship of coordinates between two systems. There is no need for \( c \) being an upper limit of particle velocity. The latter concept is merely a lemma of Eq.(3) which is an outcome of SR rather than a starting-point of SR.

Of course, the light-speed \( c \) displays itself as a singular point in the space-time as shown by the LT:

\[
x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}},
\]

where \( v \) is the relative velocity between two systems \( S \) and \( S' \). Note that \( v \) always has an upper limit \( c \) because we observers are composed of ordinary particles obeying Eq.(3). However, on the other hand, being the coordinates of some event, \( x \) and \( t \) can take arbitrary values on the (two-dimensional) \( x-t \) diagram.

Let’s consider a tachyon (P) with uniform velocity \( u = x/t \) in the \( S \) system. The particle’s world-line OP is a straight line delivered from the origin O and located below the line \( x = ct \) (see [3-5]). Now consider another \( S' \) system moving with a velocity \( v \) relative to \( S \). Then the velocity of tachyon in \( S' \) will be \( u' = x'/t' \). A strange phenomenon occurs as follows. In accompanying with the increase of \( v \), the \( x' \) axis rotates counterclockwise and crosses the
OP line at $v = c^2/u$. Just at this moment, the time coordinate $t'$ of tachyon would become negative suddenly:

$$t'_p < 0, \quad (u > c, \ v > c^2/u). \quad (5)$$

This was regarded as "tachyon traveling backward in time" or a "violation to the principle of causality" because the tachyon seems (in $S'$ system) to send the information into the past! Many physicists pondered the above puzzle being nonsense and believed even firmly the principle of limiting speed. So little attention has been paid to discussions on the superluminal motion except in some literatures, e.g., [1-5].

Evidently, there is no way out in the pure classical theory. A possible way out of this puzzle is to refer to the Stuekelberg-Feynman rule in quantum electrodynamics — a (negative-energy) electron traveling backward in time is equivalent to a (positive-energy) positron traveling forward in time. Then a "reinterpretation principle" was proposed by Sudarshan and Recami that a (negative-energy) tachyon traveling backward in time may be viewed as an (positive-energy) antitachyon traveling forward in time[1,6].

Despite the attractiveness of above argument, we still need a rigorous proof which can only be provided by a quantum equation for the tachyon. Before we turn to the quantum realm, let’s explore the above puzzle into a full paradox[5].

From the LT (4), the addition law for velocity can easily be derived as:

$$u' = \frac{u - v}{1 - uv/c^2}. \quad (6)$$

There is a singular point for $v$ located at $c^2/u$ as long as $u > c$. Once $v > c^2/u$, instead of Eq.(5), now we see that $u'$ changes its sign from positive to negative:

$$u' < (-c), \quad (u > c^2/v, \ or \ v > c^2/u). \quad (7)$$

However, Eq.(7) still remains as a puzzle because we have the momentum $p' > 0$ as easily proved by the LT:

$$p' = \frac{p - vE/c^2}{\sqrt{1 - v^2/c^2}}, \quad E' = \frac{E - vp}{\sqrt{1 - v^2/c^2}}. \quad (8)$$
Now we write down the counterparts of Eqs. (1) and (3) for tachyon being:

\[ E^2 = p^2 c^2 - m_s^2 c^4, \]  

\[ p = \frac{m_s u}{\sqrt{u^2/c^2 - 1}}, \quad E = \frac{m_s c^2}{\sqrt{u^2/c^2 - 1}}, \]  

(9)  

(10)

where the real \( m_s \) is named the proper mass or tachyon mass.

We will derive (9) and (10) below. Now just combining (8) with (10), we find:

\[ p' = \frac{m_s(u - v)}{\sqrt{1 - v^2/c^2} \sqrt{u^2/c^2 - 1}} > m_s c > 0, \]  

(11)

\[ E' = \frac{m_s(c^2 - uv)}{\sqrt{1 - v^2/c^2} \sqrt{u^2/c^2 - 1}} < 0, \quad (u > c^2/v, \text{ or } v > c^2/u). \]  

(12)

Then new puzzle emerges: How can a particle have \( u' < 0 \) whereas its \( p' > 0 \)? What is the meaning of \( E' < 0 \)? (The energy must be positive definite in a classical theory). We call all above puzzles, Eqs. (5), (7), (11) and (12) the "superluminal paradox"[5].

Evidently, to solve the superluminal paradox, we need more than a "reinterpretation principle" as a generalization to the Stuekelberg-Feynman rule. Even the latter needs a deeper explanation.

### 3. Dirac Equation Versus the Tachyon Equation

Obviously, we have to go beyond the realm of classical physics and innovate new quantum theory for the tachyon. Then immediately, we are facing a challenge: Do we already have a quantum theory for SR? If we don’t have a reliable quantum theory for SR, i.e., a new interpretation and derivation of SR on the basis of quantum mechanics (QM), we will have no that for tachyon either. Then a controversial problem for more than 70 years arose: Do SR and QM have the common essence? Are they really consistent, compatible or even identical at the deeper level?

In the past, some physicists considered SR being a classical theory which is different
from the QM in their nature and the successful combination of SR and QM to innovate
the relativistic QM (RQM), the quantum field theory (QFT) and particle physics (PP) is
a success of some "complementarity". We don’t think so. We have been pondering on two
facts for a long time.

First, the derivation of famous formula $\lambda = h/p$ ($p = \hbar k$) by de Broglie in 1924 is
by no means a simple combination of SR with the quantum theory of early stage at his
time. What de Broglie did is to derive the "half" of quantum theory $p = \hbar k$ from another
"half" of quantum theory $E = \hbar \omega$ by using the whole theory of SR. His thinking clearly
showed that SR and quantum theory do have the common essence[7]. The second fact comes
from biology: The combination of living beings of different species cannot reproduce their
descendants. It would be interesting to mention an exotic example. The mating of a horse
and a donkey gives birth of a mule, but the latter can no longer have descendant. Now we
know that the genetic factor in inheritance is DNA. So our problem becomes the following:
What are the DNA of RQM, QFT and PP inherited from SR and QM respectively?

In searching for the answer to above question, we have been gradually accumulating ten
arguments for it[7], focusing on one relativistic principle (as the substitution of original two
postulates) — a basic symmetry in one inertial system — based on QM as follows.

A particle is always not pure in the sense of its wavefunction (WF) always containing
two contradictory fields, $\varphi(x, t)$ and $\chi(x, t)$. They evolve with space and time essentially
with opposite phase, e.g., in free motion[8]:

$$\varphi \sim \exp \left[ \frac{i}{\hbar} (\vec{p} \cdot \vec{x} - Et) \right],$$

(13)

$$\chi \sim \exp \left[ -\frac{i}{\hbar} (\vec{p} \cdot \vec{x} - Et) \right].$$

(14)

But in the WF $\psi$ of a concrete particle, $\varphi$ dominates $\chi$, so the latter has to follow the
space-time evolution of the former:

$$\psi \sim \varphi \sim \chi \sim \exp \left[ \frac{i}{\hbar} (\vec{p} \cdot \vec{x} - Et) \right], \quad (|\varphi| > |\chi|).$$

(15)
By contrast, in the WF $\psi_c$ of its antiparticle, because now $\chi_c$ dominates $\varphi_c$, so:

$$\psi_c \sim \chi_c \sim \varphi_c \sim \exp \left[ -\frac{i}{\hbar}(\vec{p} \cdot \vec{x} - Et) \right], \quad (|\chi_c| > |\varphi_c|). \quad (16)$$

Under the newly defined space-time inversion ($\vec{x} \rightarrow -\vec{x}$, $t \rightarrow -t$), the theory (equation) remains invariant but with its concrete solution transforming from a particle to its antiparticle due to:

$$\left\{ \begin{array}{l}
\varphi(\vec{x}, t) \rightarrow \varphi(-\vec{x}, -t) \rightarrow \chi(\vec{x}, t), \\
\chi(\vec{x}, t) \rightarrow \chi(-\vec{x}, -t) \rightarrow \varphi(\vec{x}, t).
\end{array} \right. \quad (17)$$

Note that the operators of momentum and energy for a particle:

$$\hat{\vec{p}} = -i\hbar \nabla, \quad \hat{E} = i\hbar \frac{\partial}{\partial t} \quad (18)$$

are now transformed into that for its antiparticle:

$$\hat{\vec{p}}_c = i\hbar \nabla, \quad \hat{E}_c = -i\hbar \frac{\partial}{\partial t}. \quad (19)$$

So the observable momentum and energy of the particle (15) or its antiparticle (16) are all the same $\vec{p}$ and $E(>0)$ respectively. Operators (18) and (19) are exactly the DNA in the RQM, QFT and PP inherited from QM and SR respectively.

Now we are in a position to establish the quantum equation for tachyon (carrying proper-mass $m_s$) in comparison with Dirac equation (carrying rest-mass $m_0$)[12]:

**Dirac equation**

$$\left\{ \begin{array}{l}
\frac{i\hbar}{m_0} \varphi = i\hbar \vec{\sigma} \cdot \nabla \varphi + m_0 c^2 \varphi \\
\frac{i\hbar}{m_0} \chi = i\hbar \vec{\sigma} \cdot \nabla \chi - m_0 c^2 \chi,
\end{array} \right. \quad \left\{ \begin{array}{l}
\frac{i\hbar}{m_0} \varphi = i\hbar \vec{\sigma} \cdot \nabla \varphi + m_s c^2 \varphi \\
\frac{i\hbar}{m_0} \chi = i\hbar \vec{\sigma} \cdot \nabla \chi - m_s c^2 \chi,
\end{array} \right. \quad (20)$$

$$\varphi = \frac{1}{\sqrt{2}}(\phi_L + \phi_R), \quad \chi = \frac{1}{\sqrt{2}}(\phi_L - \phi_R), \quad \varphi = \frac{1}{\sqrt{2}}(\phi_L + \phi_R), \quad \chi = \frac{1}{\sqrt{2}}(\phi_L - \phi_R),(21)$$

$$\left\{ \begin{array}{l}
\frac{\partial}{\partial t} \phi_L = i\hbar \vec{\sigma} \cdot \nabla \phi_L + m_0 c^2 \phi_R \\
\frac{\partial}{\partial t} \phi_R = -i\hbar \vec{\sigma} \cdot \nabla \phi_R + m_0 c^2 \phi_L,
\end{array} \right. \quad \left\{ \begin{array}{l}
\frac{\partial}{\partial t} \phi_L = i\hbar \vec{\sigma} \cdot \nabla \phi_L - m_s c^2 \phi_R \\
\frac{\partial}{\partial t} \phi_R = -i\hbar \vec{\sigma} \cdot \nabla \phi_R + m_s c^2 \phi_L,
\end{array} \right. \quad (22)$$

$$\frac{\partial}{\partial t} \rho + \nabla \cdot \vec{j} = 0, \quad \frac{\partial}{\partial t} \rho + \nabla \cdot \vec{j} = 0, \quad (23)$$
\[ \rho = \varphi^\dagger \varphi + \chi^\dagger \chi = \phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R, \quad \rho = \varphi^\dagger \chi + \chi^\dagger \varphi = \phi_L^\dagger \phi_L - \phi_R^\dagger \phi_R, \quad (24) \]

\[ \vec{j} = -c(\varphi^\dagger \vec{\sigma} \chi + \chi^\dagger \vec{\sigma} \varphi) \quad \vec{j} = -c(\varphi^\dagger \vec{\sigma} \varphi + \chi^\dagger \vec{\sigma} \chi) \quad (25) \]

Here we choose the \( \vec{\sigma} \) matrix as the minus of the original one in Dirac equation such that its counterpart — the tachyon equation can describe a left-handed neutrino together with a right-handed antineutrino.

Actually, to author’s knowledge, the above tachyon equation was found earlier by Chodos et al. [9], Ciborowski and Rembielinski [10] and Chang [11] by different approaches but in four-component form. However, we prefer to use the two-component spinor (\( \varphi, \chi \) or \( \phi_L, \phi_R \)), showing the symmetries clearly [12].

First, both Dirac equation and tachyon equation respect the basic symmetry, i.e., the invariance of space-time inversion (17) explicitly. (There is merely an exchange of \( \varphi \) and \( \chi \) in the mass term.) A simple substitution of plane WF to (20) yields the desired kinematic relation (9) immediately.

The tachyon velocity \( u \) is identified to the group velocity \( u_g \) of wave:

\[ u = u_g = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{\vec{p}c^2}{E} > c, \quad (26) \]

and Eq.(10) follows accordingly.

Then next question arises: Although it is important to find both Dirac equation and tachyon equation obeying the same basic symmetry, but what is their distinction?

4. What is the Convincing Evidence for Neutrino Being Likely a Tachyon?

The discussion on tachyon is by no means an academic problem as that before 1960s. Since 1970s, a series of experiments like the beta decay of tritium show that the neutrino might have nonzero mass \( m_\nu \), which is defined by Eq.(1):

\[ E^2 = p^2c^2 + m_\nu^2c^4. \quad (27) \]
Because of great difficulty in the measurement of neutrino’s energy $E$, especially its momentum $p$, the accuracy of experiments has been quite low. However, in 1996, it was reported that the global averaged value of $m^2$ for electron-neutrino seemed to be negative[13]:

$$m^2(\nu_e) = -27 \pm 20(eV^2). \quad (28)$$

Later, the experimental technique was improved to control better the energy loss of beta particle in the source and nine data in 1991-1995 were excluded because they were judged as unreliable, resulting in new average in 2000 as[13]:

$$m^2(\nu_e) = -2.5 \pm 3.3(eV^2). \quad (29)$$

Similar situation occurs for the muon-neutrino[13]:

$$m^2(\nu_\mu) = -0.016 \pm 0.023(MeV^2). \quad (30)$$

The tau-neutrino was just discovered in FermiLab in 2000, no experimental data on its $m^2(\nu_\tau)$ is reported yet.

To author’s knowledge, the majority of physics community don’t pay enough attention to the minus sign in Eqs.(28)-(30), which are regarded meaningless in statistics (so the present data are still compatible with $m^2 = 0$). Alternatively, some physicists believe that even if neutrino does have some tiny mass, it must be the rest-mass of a Dirac particle (i.e., $m^2 > 0$). Only a few papers like [9], [6], [10], [14] and [11] etc. were considering that the experimental data of negative $m^2$, though far from accurate, do strongly hint the neutrino being a tachyon as discussed in Eqs.(9) and (10) with a real proper mass $m_s$ ($m^2 = -m^2_s$).

We hope that the experimental accuracy could be raised in the near future so that whether the neutrino is a tachyon or not will be verified unambiguously. However, in our point of view, the convincing evidence for the neutrino being a tachyon is already lying in the fact of parity violation[12, 15, 16].

Since the historical discovery of parity violation by Lee-Yang[17], and Wu et al.[18], all decay processes in weak interaction where neutrinos participate in are explained by the
two-component neutrino theory[19], which implies that there are only left-handed neutrino and right-handed antineutrino in nature whereas no right-handed neutrino or left-handed antineutrino exists. What renders this kind of permanent longitudinal polarization of neutrino possible is its mass being regarded as zero at that time. Would the neutrino have a nonzero rest-mass, no matter how tiny it is, the neutrino, say a $\nu_L$, would be a Dirac particle with velocity $u < c$ in a system $S$. Then an another observer in $S'$ system moving with respect to $S$ with a velocity $v > u$ would see the $\nu_L$ as a $\nu_R$! There would be no permanent longitudinal polarization at all. We would still need a four-component neutrino theory with parity conservation. This obviously contradicts the experimental facts of parity violation.

Some physicists think that even the neutrino has some rest-mass, the experimental facts of parity violation could still be accounted for by the V-A theory in the standard model which is endowed with the property of parity violation. We don’t think so. Would particles in weak interaction process be all Dirac particles, there would be no parity violation phenomenon at all. In particular, one even cannot discriminate a static fermion being left-handed or right-handed polarized[20]. This is because the solution to the Dirac equation describing a plane wave along $z$ axis reads:

$$\psi \sim \phi \sim \chi \sim \phi_L \sim \phi_R \sim \exp \left[ \frac{i}{\hbar} (p z - E t) \right],$$  \hspace{1cm} (31)

which will give $\phi_L = \phi_R$ if $p = 0$.

On the other hand, in the two-component neutrino theory, the Dirac equation with $m_0 = 0$ reduces into two uncoupled Weyl equations. Only one of them:

$$i \hbar \frac{\partial}{\partial t} \phi_L = i \hbar \vec{\sigma} \cdot \nabla \phi_L$$  \hspace{1cm} (32)

is adopted whereas the other one:

$$i \hbar \frac{\partial}{\partial t} \phi_R = -i \hbar \vec{\sigma} \cdot \nabla \phi_R$$  \hspace{1cm} (33)

is discarded. Eq.(32) precisely describes the massless $\nu_L$ and $\bar{\nu}_R$. 

Note that the WF of a $\bar{\nu}_R$ moving along $z$ axis reads:

$$\phi_L \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp \left[ -\frac{i}{\hbar} (pz - Et) \right], \quad (E > 0, \ p > 0). \tag{34}$$

It is a "negative-energy" neutrino, but actually a positive-energy antineutrino with momentum $p$ and energy $E > 0$ if we use the correct operators for antiparticle, Eq.(19). Meanwhile, the spin angular-momentum operator also reads:

$$\tilde{\sigma}_c = -\tilde{\sigma}, \tag{35}$$

as can be proved by the Heisenberg motion equation. So Eq.(34) exactly describes a right-handed antineutrino without any resort to so-called "hole-theory". It was emphasized by Konopinski and Mahnaud[21], Schwinger[22] even earlier by Stueckelberg and Feynman in different form and to different extent, (see below).

Now we turn to the tachyon equation (20)-(22) with nonzero proper mass $m_s$. In contrast to Dirac equation, it is not invariant under the following space-inversion transformation ($\vec{x} \rightarrow -\vec{x}, \ t \rightarrow t$):

$$\begin{align*}
&\phi_L(\vec{x}, t) \rightarrow \phi_L(-\vec{x}, t) \rightarrow \phi_R(\vec{x}, t) \\
&\phi_R(\vec{x}, t) \rightarrow \phi_R(-\vec{x}, t) \rightarrow \phi_L(\vec{x}, t),
\end{align*} \tag{36}$$

because of the opposite signs in its mass term. This is a violation to parity symmetry and must be reflected in its solutions. Indeed, consider a plane wave like

$$\psi \sim \varphi \sim \chi \sim \phi_L \sim \phi_R \sim \exp \left[ \frac{i}{\hbar} (pz - Et) \right], \tag{37}$$

which gives:

$$\chi = \frac{cp - m_s c^2}{E} \varphi, \tag{38}$$

$$\phi_R = \frac{m_s c^2}{cp + E} \phi_L. \tag{39}$$

We see that for $E > 0$, a particle always have $|\varphi| > |\chi|$ and $|\phi_L| > |\phi_R|$, i.e., it is a left-handed neutrino $\nu_L$. But for $E < 0$, an antiparticle always have $|\chi_c| > |\varphi_c|$ and $|\phi_R| > |\phi_L|$, i.e., it is a right-handed antineutrino $\bar{\nu}_R$ with moment $p_e = -p$ and energy $E_e = -E > 0$. Moreover, the solution of $\nu_R$ or $\bar{\nu}_L$ is definitely excluded. The above feature can also be seen
from the expression for the ”charge density”, Eq.(24), being not positive definite: While the normalization for a particle $\nu_L$, $\int \rho \, d\vec{x} = 1$ implies the helicity $H = \langle \vec{\sigma} \cdot \vec{p} \rangle / |\vec{p}| = -1$, it will equal to -1 for an antiparticle $\bar{\nu}_R$ and implies $H = 1$.

In summary, only a neutrino with intrinsic left-right asymmetry can account for the parity violation in weak interactions. And only a neutrino with $m_{\nu} = 0$ or $m_{\nu}^2 < 0$ can be such an initiator of parity violation. The reason why we believe $m_{\nu}^2 < 0$ rather than $m_{\nu} = 0$ is not only due to the present experimental data tending to $m_{\nu}^2 < 0$, but largely depending on a theoretical confidence that ”a particle is always not pure and there is no exception to this rule”. So a neutrino should be a superposition of two fields $\phi_L$ and $\phi_R$, rather than one field. Just look at all known particles. Even the massless photon, despite its special character, is no exception. A left-handed and right-handed photon can be viewed as a particle and its antiparticle respectively. But a linearly polarized light-beam is a superposition of them.

5. The Full Solution to Superluminal Paradox

Now it is easy to solve the superluminal paradox, Eqs.(5), (7), (11) and (12). Evidently, since the observer in $S'$ system will see the particle (with velocity $u > c^2/v$ in $S$ system) as a negative-energy ($E' < 0$) particle, he should look it as an antiparticle with positive-energy $E_c = -E' > 0$. Meanwhile, its momentum should be $P_c = -p' < 0$ by the formula (19), in conformity with $u' < 0$ in Eq.(7). The mysterious time-reversal Eq.(5) is no more than a false appearance of sign change in the phase of WF, which, of course, cannot be reflected suitably in the ”classical” $x - t$ diagram. So all puzzles disappear now. There is no paradox at all.

However, we have to check carefully if the above ”reinterpretation rule” is really allowed by the equation? Indeed, it does work for a neutrino described by Eq.(22) in $S'$ system,
which is invariant under the ”pure time-inversion“ \((\vec{x}' \rightarrow \vec{x}', t' \rightarrow -t')\) as far as:

\[
\begin{align*}
\phi_L(\vec{x}', t') & \rightarrow \phi_L(\vec{x}', -t') \rightarrow \phi_R(\vec{x}', t') \\
\phi_R(\vec{x}', t') & \rightarrow \phi_R(\vec{x}', -t') \rightarrow \phi_L(\vec{x}', t'),
\end{align*}
\]

with a concrete solution \((p' > 0, E' < 0)\):

\[
\phi_L \sim \phi_R \sim \exp \left[ \frac{i}{\hbar} (p' x' - E' t) \right],
\]

being transformed into:

\[
\phi_R \sim \phi_L \sim \exp \left[ -\frac{i}{\hbar} (p_c x' - E_c t) \right],
\]

which is just an right-handed antineutrino with \(p_c = -p < 0, E_c = -E > 0\). Q.E.D.

Note that while neutrino equation (22) allows such kind of a ”pure time-inversion“, the Dirac equation cannot. On the other hand, both (all) equations should be invariant under a full space-time inversion \((x \rightarrow -x, t \rightarrow -t)\) while a concrete particle transforming into its antiparticle with the same momentum and (positive) energy accordingly. This is the rigorous statement of stuekelberg-Feynman rule.

Now the momentum of antineutrino in Eq.(41) is negative: \(p_c < 0\), opposite to the direction of original \(p(> 0)\). So the implication is amazing: when the \(S'\) observer chases the neutrino (with velocity \(u\) in \(S\) system) and increases his velocity \(v\) exceeding a critical value \(c^2/u\), the \(\nu_L\) will suddenly transform into a \(\bar{\nu}_R\) moving toward him[5, 6].

6. Why a Dirac Equation with imaginary mass doesn’t work?

At first sight, the difference between Eqs.(1)-(3) and Eqs.(9)-(10) amounts simply to an analytical continuation of particle mass: \(m_0 \rightarrow im_s\) (with both \(m_0\) and \(m_s\) being real). Such kind of unsuccessful attempt could be found in the literature, e.g., [23].

Look at Dirac equation versus Eq.(20). In terms of four-component form, they are:

\[
 i\hbar \frac{\partial}{\partial t} \psi = i\hbar \vec{\alpha} \cdot \nabla \psi + \beta m_0 c^2 \psi, \tag{43}
\]
\[ i\hbar \frac{\partial}{\partial t} \psi = i\hbar \vec{\alpha} \cdot \nabla \psi + \beta_s m_s c^2 \psi, \quad (44) \]

\[ \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}. \quad (45) \]

While \( \beta \) is hermitian, \( \beta_s \) is antihermitian but it works. On the other hand, a direct analytical continuation \( m_0 \rightarrow im_s \) leads to Eq.(43) with

\[ \beta_s \rightarrow \beta_s' = \begin{pmatrix} iI & 0 \\ 0 & -iI \end{pmatrix}, \quad (46) \]

which is also antihermitian. Why it is wrong?

As pointed out in [15], usually in QM, we endow an imaginary part of mass \( m \) to WF for desctoring an unstable particle:

\[ m \rightarrow m - i\Gamma/2, \quad \Gamma = \hbar/\tau, \quad |\psi|^2 \sim e^{-t/\tau}, \quad (47) \]

which implies the unitarity of WF being destroyed. So it is no surprise to see that the violation of hermitian property in Eqs.(43) with (45) allows a decoupled solution at \( p \rightarrow 0 \) like:

\[ \phi' \sim e^{-i(im_s)t} \sim e^{m_s t} \quad (48) \]
\[ \chi' \sim e^{-i(-im_s)t} \sim e^{-m_s t} \quad (49) \]

which implies a violation of unitarity too.

The reason why Eq.(43) is correct can be seen from a simple but new derivation of Eq.(22) (in Weyl representation) from Dirac equation via a nonhermitian unitary transformation[15]:

\[ \psi' = \begin{pmatrix} \phi'_L \\ \phi'_R \end{pmatrix} \rightarrow U_s \psi' = \begin{pmatrix} iI & 0 \\ 0 & I \end{pmatrix} \psi' = \psi = \begin{pmatrix} \phi_L \\ \phi_R \end{pmatrix}. \quad (50) \]

Thus we see that the extra \( i \) coming from the analytical continuation of mass is now absorbed into the definition of \( \phi_L = i\phi'_L \) (while \( \phi_R = \phi'_R \)). The latter implies an extra phase-difference between \( \phi_L \) and \( \phi_R \) (in comparison with \( \phi'_L \) and \( \phi'_R \)) which ensures two solutions (for a same momentum) \( \nu_L \) and \( \bar{\nu}_R \) being stabilized whereas other two \( \nu_R \) and \( \bar{\nu}_L \) strictly forbidden. Moreover, the solution like (47) is definitely excluded. In short, the violation of hermitian
property for an equation (usually leading to the violation of unitarity) is now displayed in a stable but strange realization of parity violation. We see once again the subtlety of phase in QM.

7. Does So-called Pseudotachyon Exist?

In Ref.[2], a so-called "Pseudotachyon" was proposed that it behaves like tachyons in the momentum space but like subluminal particle in the ordinary space. How can one believe in such strange property which would lead to decoupling of particle from wave? Let’s study this puzzle carefully.

First, the kinematic relation (9) in momentum space leads straightforwardly to the group velocity of a tachyon as shown by Eq.(26).

Second, in the Heisenberg picture, one can use the motion equation for the position operator $x$ to find the velocity operator for Dirac equation being:

\[
\dot{x}_i = \frac{1}{i\hbar}[x_i, H] = c\alpha_i, \quad (i = 1, 2, 3)
\]

where $H = c\vec{\alpha} \cdot \vec{p} + \beta m_0 c^2$. The motion equation for $\alpha_i$ yields further the solution of $x$ being:

\[
x = x_0 + c^2 p H^{-1} t + \frac{1}{2} i\hbar \dot{x} H^{-1} e^{-2iHt/\hbar},
\]

where $c^2 p H^{-1}$ is just the classical velocity shown in (26). The last term was named the "zitterbewegung", which is complex and thus unobservable. Hence the position operator $x$ ceases to be an observable in RQM[24]. But anyway, the velocity or position of a Dirac particle does make sense in the average meaning. The above evaluation remains valid for a tachyon because it does not alter after $\beta$ being replaced by $\beta_s$ in (44).

Third, we turn to the Schrödinger picture and calculate the average velocity of a Dirac particle:

\[
\bar{v} = c \int \psi^\dagger \alpha_3 \psi d\vec{x} = c^2 p / E < c
\]
where a planewave along $z$ axis is used and the WF is normalized in a volume $V$ such that

$$\int \psi^\dagger \psi d\vec{x} = 1 \quad (54)$$

In [2], the same formula was used to calculate the average velocity of a tachyon, yielding the strange but wrong result:

$$\bar{v} = E/p < c \quad (55)$$

What’s wrong? While Eqs.(52) and (51) are correct for Dirac equation, we should return back to its starting point as shown in Eqs.(24) and (25). The tachyon equation has its own $\rho$ and $\vec{j}$ which must be derived from the continuity equation and are quite different from that of Dirac equation. Indeed, using $\int \rho d\vec{x} = 1$ to find the correct normalization constant for a plane-wave of tachyon along $z$ axis and evaluating $j_3$, we arrive at:

$$\bar{v} = c^2 p/E = u > c \quad (56)$$

as expected. In short, the correct expressions for tachyon should be Eqs.(52) and (51) with an extra matrix $\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$ inserted.

As three different approaches — the group velocity, the Heisenberg motion equation and the average velocity in Schrödinger picture — are all focusing on one outcome that a tachyon does travel in a superluminal speed $u = c^2 p/E > c$, we have more confidence than before.

8. Summary and Discussion

(a) Numerous experimental tests have been supporting the validity of SR, which stands even more firm than ever before. However, it is possible to construct a superluminal theory compatible with SR. Indeed, both subluminal and superluminal quantum theories are respecting the same basic symmetry, i.e., the invariance under the (newly defined) space-time inversion, Eq.(17).

(b) According to our present understanding, no boson but fermion could be superluminal
as long as the parity symmetry, i.e., the invariance under the space inversion, Eq.(36), is violated to maximum. The present experimental data, especially that of parity violation, do strongly hint that the neutrino might be such a superluminal particle, i.e., a tachyon with permanent longitudinal polarization.

(c) Interesting enough, the Lorentz transformation (LT) and the addition law for velocity are valid for both subluminal and superluminal phenomena as long as the relative velocity $v$ of observers in two inertial systems is constrained: $|v| < c$. All of our discussion can be meaningful only if it is based on SR, LT and the invariance of light-speed $c$.

(d) The superluminal paradox is over. All puzzles stemming from the classical concept disappear in the reasonable quantum theory. Indeed, the solution to superluminal paradox poses a very severe and interesting test on the validity of Eq.(17) which in turn is based on the new concept about the symmetry between a particle and its antiparticle (including Eq.(19)). Now we have a sound basis for the "Stueckelberg-Feynman-Sudarshan-Recami reinterpretation rule".

(e) Five topics on the research of neutrino were discussed in Ref.[16]. The puzzle of neutrino oscillation seems more interesting. Either Eq.(1) or Eq.(9) yields a simple relation between the group velocity $u_g = \frac{d\omega}{dk}$ and the phase velocity $u_p = \frac{\omega}{k}$ as:

$$u_g u_p = c^2.$$

Hence, if neutrinos ($\nu_e$, $\nu_\mu$ and $\nu_\tau$) are really tachyons with $u_g > c$ but $u_p < c$, they would be difficult to form coherent superposing state during their motion. So the probability of flavor mixing, i.e., the oscillation among them would be strongly suppressed. We guess that it might be one of the reasons why the present fitting value for the rest-mass of neutrino from the experimental data of oscillation is so tiny in comparison with the data shown in Eqs.(28)-(30).

(f) In the development from nonrelativistic QM to RQM, the position and velocity operators cease to be hermitian, i.e., they do not correspond to direct observables but only make sense in the average meaning. Besides, as is well known, the Klein-Gordon equation has
its Hamiltonian being nonhermitian[25] and its ”probability density” ($\rho$) being not positive definite[8,24]. Now we see similar situation occurred in the tachyon equation too. All above features in RQM are intimately linked with the antiparticle’s degree of freedom — a particle is always composed of two contradictory fields, not one field.

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References

[1] E. Recami, *Foundation of Phys.* **31**, 1119 (2001).
[2] G. Salesi, *Int. J. Mod. Phys.* A **12**, 5103 (1997).
[3] O.M. Bilaniuk, V.K. Deshpande and E.C.G. Sudarshan, *Am. J. Phys.* **30**, 718 (1962).
[4] O.M. Bilaniuk et al. *Phys. Today* **22**, May 43; Dec. 47 (1969).
[5] G-j Ni, *Superluminal paradox and neutrino*, preprint, hep-ph/0203060.
[6] E. Giannetto, G.D. Maccarrone, R. Mignani, E. Racami, *Phys. Lett.* **B178**, 115 (1986).
[7] G-j Ni, *Ten arguments for the essence of special relativity*, Proceedings of the 23rd Workshop on High-energy Physics and Field Theory, (Protvino, Russia, June, 2000) p.275-292.
[8] G-j Ni, H. Guan, W-m Zhou, J. Yan, *Chin. Phys. Lett.* **17**, 393 (2000).
[9] A. Chodos, A.I. Hauser and V.A. Kostelecky, *Phys. Lett.* **B150**, 431 (1985).
[10] J. Ciborowski and J. Rembielinski, *Europ. Phys. J.* C**8**, 157 (1999).
[11] T. Chang and G-j Ni, *An explanation on negative mass-square of neutrinos*, accepted by Fizika B, hep-ph/0009291.
[12] G-j Ni and T. Chang, *Is neutrino a superluminal particle?* preprint, [hep-ph/0103051].

[13] Review of Particle Physics, *Phys. Rev.* D54, 280 (1996); *Europ. Phys. J.* C15, 350 (2000).

[14] T. Chang, *Beyond relativity*, preprint (2000).

[15] G-j Ni, *J. Shaanxi Normal Univ.* (Natural Sci. Ed.) 29, (3) 1 (2001).

[16] G-j Ni, *WULI (physics)*, April (4) 255 (2002).

[17] T.D. Lee and C.N. Yang, *Phys.Rev.* 104, 254 (1956).

[18] C.S. Wu et al. *Phys.Rev.* 105, 1413 (1957).

[19] T.D. Lee and C.N. Yang, *Phys.Rev.* 105, 1671 (1957).

[20] L.H. Ryder, *Quantum Field Theory* (Cambridge, Cambridge Univ. Press, 1996).

[21] E.J. Konopinski and H.M. Mahmaud, *Phys.Rev.* 92, 1045 (1953).

[22] J. Schwinger, *Proc. Nat. Acad. Sc. US*, 44, 223 (1958).

[23] J. Bandukwala and D. Shay, *Phys.Rev.* D9, 889 (1974).

[24] G-j Ni and S-q Chen, *Advanced Quantum Mechanics* (Press of Fudan University, 2000); The English version will be published by Rinton Press in 2002.

[25] H. Feshbach and F. Villars, *Rev. Mod. Phys.* 30, 24 (1958).