ABSTRACT

We investigate the impact on low energy flavour mixing phenomena of the renormalisation group running of parameters in supersymmetric theories. We have explicitly chosen flavour dependent supergravity couplings. The renormalisation group equations are displayed in general matrix form. We work in a scale dependent basis such that the hierarchy in the quark masses can be more easily utilised. Simple analytic solutions are provided in matrix form (for general boundary conditions) when only the third generation Yukawa couplings are retained in the renormalisation group equations. We solve the evolution equations of the KM matrix and the supersymmetry breaking soft terms in the squark sector. We suggest a general parametrisation of the soft analytic terms, more suitable for non-universal supergravity couplings. We point out that for flavour dependent soft terms, there are new contributions to the neutron electric dipole moment. Finally, we consider a model with maximal flavour mixing in order to estimate the effects due to the renormalisation scale dependence.
1. INTRODUCTION

Twenty years ago, the discovery of the charmed quark established the GIM mechanism and naturally explained the approximate flavour conservation in neutral current (FCNC) processes [1]. It was then generalised to include the third family with the introduction of the Kobayashi-Maskawa matrix. This settled the standard model framework enabling one to calculate, up to non-perturbative effects, flavour changing neutral current phenomena as radiative corrections. Consequently, the comparison of theoretical expectations and experimental data provides valuable information on possible new physics beyond the standard model. On top of the traditional set of parameters which characterise FCNC in the $K$-system, more recent studies of $B$-meson decays have considerably enlarged the FCNC phenomenology. A rich literature [2 – 10] is available about FCNC restrictions on supersymmetric extensions of the standard model. Nevertheless, both the LEP (and Tevatron) constraints on supersymmetric theories and some fresh insight on spontaneously broken supergravities from superstrings have encouraged a recent revival of this subject [11 – 14].

The basic supersymmetry induced FCNC effects are produced by the supersymmetric analogues of the standard model loop diagrams for neutral current processes, where gauge bosons are exchanged. Quarks and vector bosons are then replaced by squarks and gauginos. It is convenient to separate such graphs into two kinds: (a) Loops that generate FCNC effects in the standard model with the $W$-bosons replaced by charginos. (b) Flavour preserving loops in the standard model with gluons, photons and $Z$-bosons replaced by gluinos and neutralinos. If equal-charge quark and squark mass matrices are not diagonal in the same basis, their couplings to neutral gauginos can induce FCNC effects. Of course, the gluino exchange diagrams proportional to $\alpha_{\text{strong}}^2$ may be particularly relevant. There are several sources of flavour mixing in gaugino couplings that we now turn to discuss.

Within the general framework of supergravity [15], a theory is defined by the gauge and matter superfields, by their couplings to supergravity encoded in the Kähler potential, and by the generalised gauge and Yukawa couplings. The low-energy theory is then fixed by the values of the auxiliary fields which provide the spontaneous breaking of supersymmetry. Therefore, the parameters in the low-energy theory with broken supersymmetry are to be determined within the framework of a larger theory explaining all interactions including gravity. Superstrings [16] are the best prototypes, despite being incomplete. Nevertheless, recent studies [17 – 19, 12] of the field theoretical limit of superstrings have given hints on the general form of the relevant parts of the
Kähler potential as well as on their interpretation in terms of stringy symmetries. A conspicuous result of superstring model calculations is that the three families of quark superfields may couple to supergravity according to quite different terms in the Kähler potential. Therefore, the minimal hypothesis in phenomenological analyses, alleging that supergravity couplings are flavour independent is not quite confirmed by these explorations within superstring theories. The dilaton superfield in these theories does have universal supergravity couplings to quark superfields [8, 20]. But the moduli fields associated to the compact 6-dimensional manifold, needed to reduce the space-time dimension to four, have model dependent couplings to the quark superfields [17, 18, 12]. Thus, the Kähler potential can be different for each flavour. Moreover, gauge singlet fields have been conjectured to explain the so-called textures in the Yukawa couplings [21, 22]. They are assumed to get relatively large vacuum expectation values and could introduce non-universality corrections to the supergravity couplings of quark superfields.

At the level of the effective (renormalisable) theory, below the Planck scale, the supersymmetry breaking effects reduce to gaugino masses and the soft interactions in the scalar potential [23]. There appear hermitian \((mass)^2\) terms involving the scalar fields of each given \(SU(3) \times SU(2) \times U(1)\) quantum numbers and their complex conjugates. The corresponding sub-matrix of the scalar \((mass)^2\) matrix depend on the Kähler potential and on the supersymmetry breaking auxiliary fields [24, 15]. In particular, the universality or flavour independence hypothesis assumes equal masses for all squarks at the unification. At lower energies, this conjectured universality is broken by radiative corrections due to Yukawa interactions and acquire some calculable flavour dependence. Moreover, analytic \((mass)^2\) terms involving scalar fields of opposite \(SU(3)_{c} \times U(1)_{em}\) quantum numbers are generated after the electro-weak gauge symmetry breaking by the Higgs scalars. They are basically proportional to the Yukawa couplings in the superpotential. Again, if universality is assumed for the proportionality factors, referred to as \(A\)-parameters, at the unification scale, their equality is spoilt at lower energies by the calculable radiative corrections.

In most discussions of supersymmetric FCNC processes, universality of soft terms is assumed. Then, the most striking effects of radiative corrections are of two kinds: a) gauge corrections which are universal and, in general, tend to attenuate loop effects by an overall rise in the squark masses if gauginos are relatively heavy, and b) Yukawa corrections dominated by the top coupling, \(\lambda_t\), tend to align the down squark mass eigenstates to the up quarks [3 – 5, 7]. This reverses the pattern of gaugino couplings in comparison with the gauge boson ones. Chargino couplings to down squark and up quarks are approximately family diagonal while gluino and neutralino couplings
become proportional to the Kobayashi-Maskawa matrix. However, the expected physical effects are generally quite tiny with the present overall bounds on supersymmetric particles\(^1\). The \(b \rightarrow s\gamma\) transition is the remarkable exception which gives interesting information on supersymmetric theories \([7, 8, 10]\).

Motivated by superstrings, as well as symmetries proposed to explain the structure of Yukawa couplings, new analyses \([11 - 14]\) have been performed on FCNC transitions produced by non-universality in supergravity couplings. If the flavour dependence is large already at the supergravity or Planck scale, the quark mass eigenstates and the squark mass eigenstates are not superpartners under the action of supersymmetry. There is a unitary transformation in the family space reflecting the difference of the basis for physical quark and squark states. The gluino and neutralino couplings will be proportional to this unitary matrix. The natural supergravity basis for quarks and squarks is established by diagonalisation of the Kähler metrics (with its field-dependence classically fixed). In this supergravity basis, the squark (mass)\(^2\) matrix is dominantly diagonal but the eigenvalues may be different. In the same basis, the Yukawa couplings are expected to be non-diagonal. In order to define the quark mass eigenstates, one has to perform four unitary transformations to diagonalise the up and down Yukawa couplings in the family space at the supergravity scale. These unitary matrices tend to produce FCNC effects in neutral gaugino exchange already at tree level. But the difference of many orders of magnitude between the unification scale and the low energy scale suitable for FCNC calculations, suggests that the quantum running of the quark and squark masses and mixing matrices could affect the tree-level pattern.

In this paper, we present a detailed study of the renormalisation group equations for the Yukawa and soft-terms in the quark-squark sector, written as general matrices in family space \([4, 5]\). The phenomenological hierarchy in the Yukawa coupling eigenvalues is exploited using the obvious device of keeping the up quark Yukawa couplings diagonal through scale dependent rotations. The RGE for Yukawa couplings and soft terms are then easily solved in the approximation in which all Yukawa couplings but \(\lambda_t\) are neglected. It is straightforward to compute the corrections proportional to the other couplings \((\lambda_b^2, \lambda_c^2, \text{etc})\) to the lowest order. Although they may be relevant to some applications, these corrections are not included in this paper.

In Section 2, the RGE for the Kobayashi-Maskawa matrix is derived, then solved in the case in which only the third family Yukawa couplings, \(\lambda_t\) and \(\lambda_b\), are retained in the RGE. These results already exist in the literature \([4, 25]\), but here, the derivation is more transparent. In particular,

\(^1\) For a recent discussion, see e.g., Refs.\([10, 11]\).
the role of the Yukawa coupling hierarchy is exhibited so that the accuracy of the approximation can be readily estimated.

In Section 3, the RGE for the squark soft analytic terms are analysed. We argue that the current way [15] of expressing their proportionality to the Yukawa couplings should be changed to better implement the general boundary conditions at the unification or Planck scales. Our parametrisation is more suitable because Yukawa couplings relate left-handed and right-handed quarks whose supergravity properties may be very different. The RGE for the newly defined matrices of parameters are displayed and analytically solved, in the limit of $\lambda_t$ dominance, in full complex matrix form.

In Section 4, the RGE for the hermitian sector of the $(mass)^2$ matrices are analytically solved within the same approximation.

The calculation of the gluino loop contribution to the electric dipole moment [26] of the neutron is revisited in Section 5, in view of possible effects of non-universality in the soft terms. We notice that contributions proportional to $m_b$ or $m_t$ which are usually neglected, could overcome current contributions proportional to $m_d$ or $m_u$. In order to estimate their relative strength, the upper limits [8,10,11] on the flavour mixing components of the scalar $(mass)^2$ matrices are used.

In Section 6, we explore the effect of the RGE evolution on FCNC effects in the presence of flavour dependent supergravity couplings. In the absence of a realistic model, we define an ad hoc one, where the mixing matrices relating the basis that diagonalise the Kählerian metrics and the Yukawa couplings, respectively, are maximal. With simple assumptions on the Kähler potential and the superpotential, we find that in some extreme cases the existent upper bounds on FCNC generating parameters can be saturated.

2. QUARK MIXING MATRICES

Let us begin with the RGE for Yukawas and KM matrices. We define the unitary matrices $U_R$, $U_L$, $V_R$ and $V_L$ by the diagonalisation of the up and down Yukawa couplings:

$$U_R^\dagger \lambda U_L = \begin{pmatrix} \lambda_u \\ \lambda_c \\ \lambda_t \end{pmatrix}$$

$$V_R^\dagger \lambda_D V_L = \begin{pmatrix} \lambda_d \\ \lambda_s \\ \lambda_b \end{pmatrix}$$

(2.1)

where all the parameters have their dependence on $t = \ln (\Lambda / \Lambda_0) / (4\pi)^2$, defined by the well-known
RGE\(^2\):
\[
\begin{align*}
\frac{d\lambda_U}{dt} &= \lambda_U \left[ 3 \left( \lambda_U^{\dagger} \lambda_U + \text{Tr} \lambda_U^{\dagger} \lambda_U \right) + \lambda_D^\dagger \lambda_D - 2 C^u_{\alpha} g^2_{\alpha} \right] \\
\frac{d\lambda_D}{dt} &= \lambda_D \left[ 3 \left( \lambda_D^{\dagger} \lambda_D + \text{Tr} \lambda_D^{\dagger} \lambda_D \right) + \lambda_U^{\dagger} \lambda_U + \text{Tr} \lambda_U^{\dagger} \lambda_U - 2 C^d_{\alpha} g^2_{\alpha} \right] \\
2 C^u_{\alpha} g^2_{\alpha} &= \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{13}{9} g_1^2 \\
2 C^d_{\alpha} g^2_{\alpha} &= \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{2}{9} g_1^2
\end{align*}
\tag{2.2}
\]

It is convenient to write \(\lambda_U\) and \(\lambda_D\) in terms of the hermitian matrices
\[
Y_U = \lambda_U^{\dagger} \lambda_U , \quad Y_D = \lambda_D^{\dagger} \lambda_D \tag{2.3}
\]
diagonalised by the left-handed unitary transformations \(U_L\) and \(V_L\) only, and \(\lambda_U \lambda_U^{\dagger}, \lambda_D \lambda_D^{\dagger}\) diagonalised by \(U_R\) and \(V_R\). We then work with (2.3) only, the right-handed counterpart being analogous. The RGE can then be cast in the form:
\[
\begin{align*}
\frac{dY_U}{dt} &= \{3Y_U + Y_D, Y_U\} + 2 \Delta_U Y_U \\
\Delta_U &= 3 \text{Tr} Y_U - 2 C^u_{\alpha} g^2_{\alpha} \\
\frac{dY_D}{dt} &= \{3Y_D + Y_U, Y_D\} + 2 \Delta_D Y_D \\
\Delta_D &= 3 \text{Tr} Y_D + \text{Tr} Y_L - 2 C^d_{\alpha} g^2_{\alpha}
\end{align*}
\tag{2.4}
\]

Now diagonalise \(Y_U\) and \(Y_D\):
\[
\begin{align*}
\hat{Y}_U &= U_L^\dagger Y_U U_L = \begin{pmatrix} \lambda_u^2 \\ \lambda_c^2 \\ \lambda_t^2 \end{pmatrix} \\
\hat{Y}_D &= V_L^\dagger Y_D V_L = \begin{pmatrix} \lambda_d^2 \\ \lambda_s^2 \\ \lambda_b^2 \end{pmatrix}
\end{align*}
\tag{2.5}
\]

and define the KM matrix (an analogous one may be defined for the right-handed states)
\[
K_L = V_L^\dagger U_L \tag{2.6}
\]

Then the RG evolution for the eigenvalues in \(\hat{Y}_U\) follows from (2.2):
\[
\frac{d\hat{Y}_U}{dt} = \left[ 6 \hat{Y}_U + 2 \Delta_U + 2 \left( K_L^\dagger \hat{Y}_D K_L \right)_{(d)} \right] Y_U \tag{2.7}
\]

\(^2\) See, e.g., Ref.[4].
where we denote the separation of any matrix into diagonal and non-diagonal parts, $M = M_{(d)} + M_{(nd)}$. Consider, e.g., $U_L^\dagger \frac{dU_L}{dt} U_L$, which is an element of $SU(3)/H$ where $H$ is the $SU(3)$ Cartan algebra. The off-diagonal counterpart of (2.7) coming from (2.2) reads:

$$
\left[ U_L^\dagger \frac{dU_L}{dt}, Y_D \right] = \left\{ \left( K_L^\dagger Y_D K_L \right)_{(nd)}, \hat{Y}_U \right\}
$$

(2.8a)

Notice that $K_L^\dagger Y_D K_L$ is just $Y_D$ in the basis where $Y_U$ and SU(2) weak are diagonal (which will play a major rôle in the next sections) at each $t$. Analogously,

$$
\left[ V_L^\dagger \frac{dV_L}{dt}, \hat{Y}_D \right] = \left\{ \left( K_L^\dagger Y_U K_L \right)_{(nd)}, \hat{Y}_U \right\}
$$

(2.8b)

Finally,

$$
\frac{dK_L}{dt} = K_L U_L^\dagger \frac{dU_L}{dt} - V_L^\dagger \frac{dV_L}{dt} K_L
$$

(2.9)

At this point we can take advantage of the strong hierarchy of the (mass)$^2$ eigenvalues for up-quarks on the one hand ($m_t^2 \gg m_c^2 \gg m_u^2$), and for down-quarks on the other hand ($m_b^2 \gg m_s^2 \gg m_d^2$). Taking the components of (2.8)

$$
\begin{align*}
\left( U_L^\dagger \frac{dU_L}{dt} \right)_{ab} &= \left( \frac{m_a^2 + m_b^2}{m_a^2 - m_b^2} \right) \left( K_L^\dagger Y_D K_L \right)_{ab} \\
(a, b = u, c, t; \ a \neq b) \\
\left( V_L^\dagger \frac{dV_L}{dt} \right)_{ab} &= \left( \frac{m_a^2 + m_b^2}{m_a^2 - m_b^2} \right) \left( K_L^\dagger Y_D K_L \right)_{ab} \\
(a, b = d, s, b; \ a \neq b)
\end{align*}
$$

(2.10)

one obtains the evolution of $U_L$ and $V_L$ from the (mass)$^2$ hierarchy:

$$
\begin{align*}
\left( U_L^\dagger \frac{dU_L}{dt} \right)_{ab} &= \xi_{ab} \left( K_L^\dagger Y_D K_L \right)_{ab} \\
\left( V_L^\dagger \frac{dV_L}{dt} \right)_{ab} &= \xi_{ab} \left( K_L^\dagger Y_U K_L \right)_{ab}
\end{align*}
$$

(2.11)

$$
\xi_{ab} = \begin{cases} 
1 & (m_a < m_b) \\
0 & (a = b) \\
-1 & (m_a > m_b)
\end{cases}
$$

Next order approximations in $(m_s^2/m_t^2)$ and $(m_c^2/m_t^2)$ are easily obtained (but more difficult to integrate), though useless in physical applications. From (2.9) and (2.11), we derive the RGE for the elements of the evolving KM matrix. We fully use the hierarchy among masses of equal-charge.
quarks and get the following results\textsuperscript{3, 4}:
\[
\begin{align*}
\frac{d}{dt} \ln K_{bt} &= \zeta + (\lambda_t^2 + \lambda_b^2) \\
\frac{d}{dt} \ln K_{bc} &= \zeta + \lambda_b^2 |K_{bu}|^2 \\
\frac{d}{dt} \ln K_{st} &= \zeta + \lambda_t^2 |K_{dt}|^2 \\
\frac{d}{dt} \ln K_{bu} &= \zeta + \lambda_b^2 |K_{bc}|^2 \\
\frac{d}{dt} \ln K_{dt} &= \zeta + \lambda_t^2 |K_{st}|^2 \\
\zeta &= - (\lambda_t^2 + \lambda_b^2) |K_{bt}|^2
\end{align*}
\] (2.12)

Unitarity is easily checked. The RGE for remaining elements of the KM matrix are more involved, so we take advantage of the strong hierarchy in the non-diagonal elements to write them as follows:
\[
\begin{align*}
\frac{d}{dt} \ln K_{du} &\simeq \lambda_s^2 |K_{su}|^2 + \lambda_b^2 |K_{bu}|^2 + \lambda_c^2 |K_{dc}|^2 + \lambda_t^2 |K_{dt}|^2 \\
\frac{d}{dt} \ln K_{sc} &\simeq \lambda_s^2 |K_{su}|^2 + \lambda_b^2 |K_{bc}|^2 + \lambda_c^2 |K_{dc}|^2 + \lambda_t^2 |K_{st}|^2 \\
\frac{d}{dt} \ln K_{dc} &\simeq \lambda_s^2 |K_{su}|^2 + \lambda_b^2 |K_{bc}|^2 + \lambda_c^2 |K_{dc}|^2 + \lambda_t^2 |K_{dt}|^2 \\
\frac{d}{dt} \ln K_{su} &\simeq \lambda_s^2 |K_{su}|^2 + \lambda_b^2 |K_{bc}|^2 + \lambda_c^2 |K_{dc}|^2 + \lambda_t^2 |K_{bu}|^2
\end{align*}
\] (2.13)

These results have been obtained by a different method in Ref.\textsuperscript{25}. It is notice-worthy that the present approach clearly shows that the relevant approximation is given by (2.11). The RG evolution of the KM matrix elements in (2.13) is negligible compared to those in (2.12). The latter are easily integrated if one neglects the small matrix elements in the r.h.s. In practice, it is enough to put $|K_{bt}|^2 \simeq 1$ to get
\[
\begin{align*}
\frac{K_{bu} (\Lambda)}{K_{bu} (\Lambda_0)} &= \frac{K_{bc} (\Lambda)}{K_{bc} (\Lambda_0)} = \frac{K_{st} (\Lambda)}{K_{st} (\Lambda_0)} \\
&= \frac{K_{dt} (\Lambda)}{K_{dt} (\Lambda_0)} = e^{I_t + I_b} \\
I_t &= \int_t^0 dt \lambda_t^2 (t) \quad I_b = \int_t^0 dt \lambda_b^2 (t)
\end{align*}
\] (2.14)

\textsuperscript{3} Notice that in spite of their real aspect, Eqs.(2.12) and (2.13) are RGE for complex quantities.
\textsuperscript{4} For simplicity the index L is omitted.
\textsuperscript{5} And, previously, in Ref.\textsuperscript{4} for $\lambda_t^2 \gg \lambda_b^2$. 

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while all other KM matrix elements have a negligible $t$-dependence. Assume $\lambda_t^2 \gg \lambda_b^2$, then, from the RGE for $\lambda_t^2$, one gets

$$
e^{-I_t} = \left(1 - \frac{\lambda_t^2(t)}{\lambda_{\text{crit}}^2(t)}\right)^{1/12}
\frac{1}{\lambda_{\text{crit}}^2(t)} = 12 \int_t^0 dt \prod_{\alpha} \left(\frac{g_{\alpha}^2(t)}{g_{\alpha}^2(0)}\right)^{2C_{\alpha}/b_{\alpha}}$$

(2.15)

where $\lambda_{\text{crit}}^2(t)$ is the $\lambda_t^2$ value corresponding to a Landau pole at $t_0$. Hence, the RG evolution of the KM matrix can be dismissed in most problems, or taken into account by simple approximations as in (2.14).

It goes without saying that everything can be repeated ipsis litteris for the unitary transformations $U_R, V_R, K_R$ on the right-handed quarks.

3. ANALYTIC SOFT TERMS

In this Section, we establish the RGE for the so-called soft analytic scalar interactions. Those relevant for the phenomenology of flavour mixing are:

$$H_1 U^c \eta_U Q + H_2 D^c \eta_D Q + \text{h.c.},$$

(3.1)

where isospin indices are omitted, $\eta_U$ and $\eta_D$ are matrices in the family indices. The matrix elements of $\eta_U$ and $\eta_D$ are proportional to the supersymmetry breaking scale, hence to $m_{3/2}$. As discussed in the introduction (see also (3.5) below), it is simple to assume them to be also proportional to the corresponding matrix elements of the Yukawa couplings, $\lambda_U$ and $\lambda_D$, respectively. The current assumption of “universality”, i.e., flavour independence of the soft terms is usually stated by setting [15] $\eta_U = A\lambda_U$, $\eta_D = A\lambda_D$ where $A$ is $0\left(m_{3/2}\right)$. Since flavour independence is broken in the RGE by the effect of Yukawa interactions, this is usually taken into account by replacing $A$ by a diagonal matrix and neglecting the scale dependence of the KM matrix. In more specific calculations (e.g., in that of the electric dipole moment of the neutron in models where $A$ is assumed to be universal) the matrix character of the soft terms have to be more carefully manipulated [27]. Nevertheless, the soft couplings $\eta_U$ and $\eta_D$ are not expected to be in general exactly flavour independent in supergravity theories. In such circumstances, one should choose some parametrisation of $\eta_U$ and $\eta_D$ which simply embodies the assumed dependence on $\lambda_U$ and $\lambda_D$ and still preserves the generality of supergravity breaking terms. Obviously, it is always possible to
define a matrix $A$ as $\lambda^{-1}\eta$ (or $\eta\lambda^{-1}$), but since these soft terms and the Yukawa couplings link quarks with different $q$-numbers ($L$ and $R$ states, respectively) let us scrutinise the RGE for $\eta^U$,

$$
\frac{d\eta^U}{dt} = \eta^U \left[ 5\lambda^U \lambda^U + \lambda^{D\dagger} \lambda^{D} + 3 \text{Tr} (\lambda^{U\dagger} \lambda^U) - 2C^u_\alpha g^2_\alpha \right] + 2\lambda^U \left[ 2\lambda^{U\dagger} \eta^U + 3 \text{Tr} (\lambda^{U\dagger} \eta^U) + \lambda^{D\dagger} \eta^{D} - 2C^u_\alpha g^2_\alpha M_\alpha \right]
$$

(3.2)

and its analogous for $\eta^D$, replacing $U \leftrightarrow D$ (and adding the leptonic couplings $\text{Tr} \lambda^{L\dagger} \lambda^{L}$). Actually, the choice $\eta^U = \lambda^U A^U$ and $\eta^D = \lambda^D A^D$ is quite a satisfactory definition leading, e.g., to the following equation for the matrix $A^U$

$$
\frac{dA^U}{dt} = 5A^U Y_U + Y_U A^U + Y_D A^D + 6 \text{Tr} (A^U Y_U) - 4C^u_\alpha g^2_\alpha M_\alpha
$$

(3.3)

This choice has been used in the literature in connection with FCNC effects.

However, in the more general context of flavour dependent supergravity couplings, the choice in (3.3) is not the best in view of the structure of the boundary conditions at the unification scale $\Lambda_0$, not far from the Planck mass. Indeed, consider an effective supergravity theory [15] defined by some Kahler potential $K(X, \ldots, H_1, H_2, U_i, Q_i, D_1 \ldots)$ ($i = 1, 2, 3$ is the family index) and a superpotential:

$$
W = H_1 U^c \lambda^U(X) Q + H_2 D^c \lambda^D(X) Q + \ldots
$$

(3.4)

where we have defined the chiral field $X$ as the goldstino (i.e., supersymmetry breaking) direction, such that the auxiliary field $F_X \sim 0 (m_{3/2})$ (in units $M_{\text{Planck}} = 1$). Then, the trilinear coupling matrix $\eta^U$ will take the form [15, 24]:

$$
\eta^U_{ij} = \left\{ \left( \frac{\partial}{\partial X} + \frac{1}{2} \frac{\partial K}{\partial X} \right) \lambda_{ij}^U(X) + \Gamma^H_{XH} \lambda_{ij}^U(X) + \Gamma^k_{X_i} \lambda_{kj}^U(X) + \Gamma^k_{X_j} \lambda_{ik}^U(X) \right\} F^X
$$

$$
\Gamma^H_{XH} = K^{H\ast H} \frac{\partial}{\partial X} K_{H\ast H}
$$

$$
\Gamma^k_{X_i} = K^{k\ast i} \frac{\partial}{\partial X} K_{\ast i}
$$

$$
\Gamma^k_{X_j} = K^{k\ast j} \frac{\partial}{\partial X} K_{\ast j}
$$

(3.5)

where $K^{H\ast H}$, $K^{k\ast i}$ and $K^{k\ast j}$ denote the effective Kahler metrics for $H_1$, $U$ and $Q$, respectively. This formula clearly shows how flavour mixing can be introduced in the $\eta$’s, through either the “Kahler connexions” $\Gamma, \bar{\Gamma}$ or a non-trivial dependence of the Yukawa couplings on the goldstino scalar partner, $X$.

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See, e.g., Refs. [4, 8].
Therefore, we favour a different decomposition of the soft terms, as follows,

\[
\begin{align*}
\eta^U &= B_R^U \lambda_U + \lambda_U B_L^U \\
\eta^D &= B_R^D \lambda_D + \lambda_D B_L^D
\end{align*}
\]

(3.6)

whose advantages become clear comparing with (3.5). This decomposition is defined up to terms commuting with \( \lambda_U, \) (\( \lambda_D, \) resp.) which can be arbitrarily transferred from \( B_R \)'s to \( B_L \)'s, or vice-versa. In our expressions there below, we shall equally share the family independent parts between \( B_R \) and \( B_L \). The RGE for the \( B_R \)'s and \( B_L \)'s immediately follow from (3.2). However, it is more convenient to treat these RGE in such a way as to easily utilise the hierarchy of Yukawa couplings, in particular, the large value of the top mass. The obvious idea is to keep \( \lambda_U(\Lambda) \) diagonal at any scale \( \Lambda \) by the transformations \( U_R, U_L \) defined in (2.1). This amounts to choosing a rotating basis because of (2.8a) and its analogue for \( U_R \). Let us denote the diagonal Yukawa coupling matrix by \( \hat{\lambda}_U(\Lambda) = U_R^\dagger(\Lambda)\lambda_U(\Lambda)U_L(\Lambda) \), and any matrix in flavour space in the corresponding basis by

\[
\hat{M}(\Lambda) = U^\dagger(\Lambda)M(\Lambda)U(\Lambda)
\]

(3.7)

where \( U = U_R \) or \( U_L \), according as \( M \) acts on \( R \) or \( L \) states, respectively. Hence, one derives the RGE for \( \hat{M}(\Lambda) \) in the rotating basis as

\[
\frac{d\hat{M}}{dt} = U^\dagger(\Lambda) \frac{dM}{dt} U(\Lambda) + \left[ \hat{M}, U^\dagger \frac{dU}{dt} \right]
\]

(3.8)

where the last term takes into account the rotating basis. From the discussion of the previous Section, one knows that the effect of this last term is quite small in the \( L \)-sector, where the KM matrix in (2.11) is known. On the contrary, the corresponding KM matrix for the \( R \)-sector is unknown and so are the angular velocities defined by the analogue of (2.11).

Let us now write down the RGE for the \( B' \)'s in (3.6). Unless otherwise stated, all matrices are written in the rotating basis (diagonal \( \lambda_U \)). (To simplify the notation we omit the “\(^\dagger\)” symbol on these matrices). From (3.6), (2.2) and (3.2) one then derives the following RGE equations in the (real) diagonal \( \lambda_U \) basis:

\[
\begin{align*}
\frac{dB_R^U}{dt} &= 2B_R^U \lambda_U^2 + 4\lambda_U^2 B_R^U + 6 \text{ Tr} \left( \lambda_U^2 B_R^U \right) - 2C_\alpha g_a^2 M_\alpha \mathbb{I} + \left[ B_R^U, B_R^U \frac{dU_R}{dt} \right] \\
\frac{dB_L^U}{dt} &= 5B_L^U \lambda_U^2 + \lambda_U^2 B_L^U + 6 \text{ Tr} \left( \lambda_U^2 B_L^U \right) - 2C_\alpha g_a^2 M_\alpha \mathbb{I} + \left[ B_L^U, \lambda_D^\dagger \lambda_D \right] + 2\lambda_D^\dagger B_R^D \lambda_D + 2\lambda_D^\dagger B_L^D
\end{align*}
\]

(3.9)
\[ \frac{d B_R^D}{dt} = 2 B_R^D \lambda_D \lambda_D + 4 \lambda_D^2 B_R + \left[ 6 \text{ Tr} \left( \lambda_D^2 B_R^D \right) - 2 C^d \alpha g^2 M_\alpha \right] B_R^D + \left[ B_R^D, U_R^\dagger \frac{d U_R}{dt} \right] \]

\[ \frac{d B_L^D}{dt} = 5 B_L^D \lambda_U \lambda_D + \lambda_U^\dagger \lambda_D B_L^D + \left[ 6 \text{ Tr} \left( \lambda_U^\dagger \lambda_D B_L^D \right) 2 C^d \alpha g^2 M_\alpha \right] B_L^D + \left[ B_L^D, \lambda_U^\dagger \right] + 2 \lambda_D^2 B_L^U \alpha + 2 \lambda_D^2 B_L^U \alpha + \left[ B_L^D, B_L^U \alpha \right] \]

\[ \text{where } M_\alpha \text{ are the gaugino masses and the terms proportional to the identity matrix } \mathbb{1} \text{ have been equitably distributed between } B_L \text{ and } B_R. \text{ The last term in each equation corresponds to the action of an element of the SU(3) algebra whose expression is given by (2.11). Notice that } U_L^\dagger (d U_L / dt) \text{ is known in terms of the KM matrix but } U_R^\dagger (d U_R / dt) \text{ is unknown.} \]

Let us now illustrate some of the feature of (3.9) by a simple, yet possibly realistic, case where the RGE can be solved analytically. This is so if all Yukawa couplings can be neglected but the top one, \( \lambda_t \). This approximation which kills many interesting terms in the RGE above, allows one to compare with previous work\(^7\) in the literature where family mixing is neglected. Indeed, the solutions of (3.9) in this approximation are as follows:

\[ (B_L^U + B_R^U)_{ii} (t) = (B_L^U + B_R^U)_{ii} (0) - 4 \sum_\alpha C^d \alpha g^2 \xi M_0 \]

\[ - \frac{1}{2} \left( 1 + \delta_{ii} \right) \left[ (B_L^U + B_R^U)_{33} (0) + \xi M_0 \right] \]

\[ \lambda_R^{ii} (t) = e^{-\alpha_R \xi M_0} (B_L^U)_{ii} (0) \]

\[ \lambda_L^{ii} (t) = e^{-\alpha_L \xi M_0} (B_L^U)_{ii} (0) \]

\[ \alpha_R^{23} = \alpha_R^{13} = 2 \quad \alpha_L^{23} = \alpha_L^{13} = 5 \]

\[ \alpha_R^{32} = \alpha_R^{31} = 4 \quad \alpha_L^{32} = \alpha_L^{31} = 4 \]

\[ \alpha_L^{12} = \alpha_L^{21} = \alpha_L^{21} = \alpha_L^{21} = 0 \]

\[ (B_R^D)_{ij} (t) = (B_R^D)_{ij} (0) - 2 \delta_{ij} \sum_\alpha C^d \alpha g^2 \xi M_0 \]

\[ (i, j = 1, 2, 3) \]

\[ (B_L^D)_{ii} (t) = (B_L^D)_{ii} (0) - 2 \sum_\alpha C^d \alpha g^2 t M_0 - \frac{1}{6} \rho \delta_{ii} \left( (B_R^U + B_L^U)_{33} (0) + \xi M_0 \right) \]

\[ (B_L^D)_{12} (t) = (B_L^D)_{12} (0) \]

\[ (B_L^D)_{i3} (t) = e^{-\xi M_0} (B_R^D)_{i3} (0) \quad (i = 1, 2) \]

\[ (B_L^D)_{3i} (t) = e^{\xi M_0} [(B_L^D)_{3i} (0) - 2 I (B_R^U)_{3i} (0)] \]

\[ \text{\(^7\) Analytic solutions without family mixing in the } \lambda_t\text{-dominance approximation can be found in [4] and [28] with universal supergravity couplings.} \]
where \( I_t \) and \( e^{-I_t} \) are defined in (2.15), \( M_0 \) is the gaugino mass at the unification scale \( (t=0) \), and \( \rho \) and \( \xi \) are defined by:

\[
\rho = \frac{\lambda^2(t)}{\lambda^2_{\text{crit}}(t)} = 1 - e^{-12I_t} \quad (3.11)
\]

\[
\xi = -12\lambda^2_{\text{crit}}t - 1.
\]

Notice that \( e^{I_t} \simeq 1 \), so that the sizeable effect in the \( B^D_{\ell}, B^D_R \) matrix elements are the gauge contributions in the diagonal and the top trilinear term in \( (B^D_{\ell})_{33} \).

The consistently complex character of all the relations in this section is notice-worthy.

4. SCALAR MASS MATRICES

The RGE for the scalar (mass)\(^2\) soft terms are available in the literature in full matrix form \([4,8]\). However the universality hypothesis has been used in the solution \([28]\). The one-loop RGE for the relevant masses are,

\[
\frac{dm^2_Q}{dt} = \left\{ \lambda_U^\dagger \lambda_U + \lambda_D^\dagger \lambda_D, m^2_Q \right\} + 2 \left( m^2_{H_1} \lambda_U^\dagger \lambda_U + m^2_{H_2} \lambda_U^\dagger \lambda_D + \lambda_U^\dagger m^2_D \lambda_U + \lambda_D^\dagger m^2_U \lambda_D \right.
\]

\[
+ \eta_U^\dagger \eta_U + \eta_D^\dagger \eta_D \right) - 8C^Q_{\alpha} g^2_{\alpha} M^2_{\alpha}
\]

\[
\frac{dm^2_U}{dt} = 2 \left\{ \lambda_U^\dagger m_U, m_U \right\} + 4 \left( m^2_{H_1} \lambda_U^\dagger \lambda_U + \lambda_U m^2_Q \lambda_U^\dagger + \eta_U^\dagger \eta_U \right) - 8C^U_{\alpha} g^2_{\alpha} M^2_{\alpha} + ... \quad (4.1)
\]

\[
\frac{dm^2_{H_i}}{dt} = 6 \text{Tr} \left( m^2_{H_i} \lambda_U^\dagger \lambda_U + m^2_Q \lambda_U^\dagger \lambda_U + m^2_U \lambda_U^\dagger + \eta_U^\dagger \eta_U \right) - 8C^H_{\alpha} g^2_{\alpha} M^2_{\alpha} + ...
\]

where \( \text{Tr} \) is the trace on flavour indices. The RGE for \( m^2_D \) and \( m^2_{H_2} \) are obtained from those of \( m^2_U \) and \( m^2_{H_1} \), respectively by the exchanges \( U \leftrightarrow D, H_1 \leftrightarrow H_2 \) and inclusion of the leptonic terms. Because of non-universality, there is the additional term \( Y_i S \) in the RGE, where,

\[
S(t) = \frac{g^2_i(t)}{g^2_i(0)} \text{Tr} \left( Y m^2 \right) \quad (4.2)
\]

The last term in each RGE takes into account the change of the basis at each energy if, for instance, \( \lambda_U \) is kept diagonal. Let us then display the solutions of the above RGE under the assumption that all Yukawa couplings but \( \lambda^2_U \) are neglected, consistently with (3.10). In the basis in which \( \lambda_U \) is diagonal, we obtain for those hermitian matrices:

\[
(m^2_Q)_{33}(t) = (m^2_Q)_{33}(0) + \frac{g^2_i}{3} S(0) - \frac{\rho}{6} K - \gamma^Q t M^2_0
\]

\[
(m^2_U)_{33}(t) = (m^2_U)_{33}(0) - \frac{4}{3} \gamma_U^Q S(0) - \frac{\rho}{3} K - \gamma^U t M^2_0
\]
\[ m_D^2(t) = m_D^2(0) + \left( \frac{2g_1^2}{3} t S(0) - \gamma^D t M_0^2 \right) \mathbb{1} \]

\[ (m_Q^2)_{ij}(t) = (m_Q^2)_{ij}(0) + \frac{g_1^2}{3} t S(0) \delta_{ij} + (e^{-2I_t} - 1) (B_L^{U*})_{3i} (B_L^U)_{3j}(0) - \gamma^Q t M_0^2 \delta_{ij} \]

\[ (m_U^2)_{ij}(t) = (m_U^2)_{ij}(0) - \frac{4}{3} g_2^2 t S(0) \delta_{ij} + (e^{-4I_t} - 1) (B_R^{U*})_{i3} (B_R^U)^*_{j3}(0) - \gamma^U t M_0^2 \delta_{ij} \]

\[ (m_Q^2)_{i3}(t) = e^{-I_t} (m_Q^2)_{i3}(0) - \frac{\rho}{6} A_0 (B_L^{U*})_{3i}(0) \]

\[ (m_U^2)_{i3}(t) = e^{-2I_t} (m_U^2)_{i3}(0) - \frac{\rho}{3} A_0 (B_R^{U*})_{i3}(0) \]  \hspace{1cm} (4.3)

where \( i, j = 1, 2 \). We have defined the following quantities:

\[ A_0 = (B_R^U + B_L^U)_{33}(0) + \xi M_0 \]

\[ K = (m_Q^2 + m_U^2 + m_{H_2} \mathbb{1})_{33}(0) + (1 - \rho) |A_0|^2 - \xi^2 M_0^2 \]

\[ - 8 \sum_{\alpha} C_{\alpha}^u g_\alpha^2 (\xi + 1) t M_0^2 \]

\[ \gamma^Q = \frac{32}{3} g_3^2 \left( 1 + 3 g_3^2 t \right) + 6 g_2^2 \left( 1 - g_2^2 t \right) + \frac{2}{9} g_1^2 \left( 1 - 11 g_1^2 t \right) \]

\[ \gamma^U = \frac{32}{3} g_3^2 \left( 1 + 3 g_3^2 t \right) + \frac{32}{9} g_1^2 \left( 1 - 11 g_1^2 t \right) \]

\[ \gamma^D = \frac{32}{3} g_3^2 \left( 1 + 3 g_3^2 t \right) + \frac{8}{9} g_1^2 \left( 1 - 11 g_1^2 t \right) \]  \hspace{1cm} (4.4)

As already discussed in the literature [3, 4, 8], starting from universal soft terms, the flavour dependence due to the RG running only appears in the diagonal matrix elements. In the general case instead, the flavour dependence of the (mass)² terms gets contributions from the \( B_L^U, B_R^U \) matrix elements. Also in this Section, the complex nature of the various parameters is explicitly respected.

Let us estimate the impact of the running of the soft terms from the unification scale down to the SU(2) × U(1) symmetry breaking scale. Assuming \( h_t^2 \simeq 1 \) at \( \Lambda \sim 0 \left( M_Z \right) \), one gets the numerical values:

\[ t = -.2 \quad \rho = .8 \quad \xi = 2 \]

\[ e^{-I_t} = .87 \quad I_t = .13 \]

\[ 2 \sum C_{\alpha}^u g_\alpha^2 t = 2 \sum C_{\alpha}^d g_\alpha^2 t = -1.9 \]

\[ \gamma^Q t = 6.7 \quad \gamma^U t = \gamma^D t = 6.3 \]  \hspace{1cm} (4.5)

The substitution of these numbers in (3.11), (4.3) and (4.4) gives the following relations,

\[ (B_L^U + B_R^U)_{ii}(t) = (B_L^U + B_R^U)_{ii}(0) + 4 M_0 - .4 (1 + \delta_{3j}) A_0 \]
\[
(B_R^D + B_L^D)_{ii}(t) = (B_L^D + B_R^D)_{ii}(0) + 4M_0 - .13\delta_{i3}A_0 \quad (i = 1, 2, 3)
\]
\[
(B_L^U)_{i3}(t) = .5 (B_L^U)_{i3}(0) \quad (i = 1, 2)
\]
\[
(m_Q^2)_{33}(t) = (m_Q^2)_{33}(0) - .4\tilde{m}_0^2 - .025A_0^2 + 5.7M_0^2
\]
\[
(m_Q^2)_{ij}(t) = (m_Q^2)_{ij}(0) + 6.7M_0^2\delta_{ij} - .25 (B_L^U)^*_{3i} (B_L^U)_{3j}(0)
\]
\[
(m_Q^2)_{i3}(t) = .87 (m_Q^2)_{i3}(0) - .13A_0 (B_L^U)^*_{3i}(0) \quad (i, j = 1, 2)
\]
\[
(m_Q^2)_{33}(t) = (m_Q^2)_{33}(0) - .8\tilde{m}_0^2 - .05A_0^2 + 4.3M_0^2
\]
\[
(m_U^2)_{ij}(t) = (m_U^2)_{ij}(0) + 6.3M_0^2\delta_{ij} - .4 (B_R^U)^*_{i3} (B_R^U)_{j3}
\]
\[
(m_U^2)_{i3}(t) = .87 (m_U^2)_{i3}(0) - .13A_0 (B_R^U)_{i3}(0)
\]

where,
\[
m_0^2 = \frac{1}{3} (m_Q^2 + m_U^2 + m_H^2)_{33}(0)
\]
\[
A_0 = (B_L^U + B_R^U)_{33}(0) + 2M_0
\]

and \(M_0\) is the mass of the gauginos at the unification scale, assumed to be all equal.

Since FCNC effects from the non-universality of \(m_U^2, m_D^2, m_Q^2\) are expected to be relatively small, it is convenient to expand the contributions to the possible FCNC processes in powers of the following matrices:
\[
(\delta^U_R)_{ij} = \frac{m_U^2}{\tilde{m}_U^2} \delta_{ij} - \delta_{ij}
\]
\[
(\delta^U_L)_{ij} = \frac{m_Q^2}{\tilde{m}_Q^2} \delta_{ij} - \delta_{ij}
\]

(4.7)

where
\[
\tilde{m}_Q^2 = \frac{1}{3} \text{Tr} m_Q^2(0) + \gamma^Q |t| M_0^2 + \frac{g_t^2}{3} t S(0)
\]
\[
\tilde{m}_U^2 = \frac{1}{3} \text{Tr} m_U^2(0) + \gamma^U |t| M_0^2 - \frac{4}{3} g_t^2 t S(0)
\]

(i.e., only the gauge contribution in RGE are included in the definition of these “average” masses).

By definition, these matrices are defined in the diagonal \(\lambda_U\) basis. Instead, for the corresponding \(\delta_D\) and \(\delta_U\), it is more convenient to go to the diagonal \(\lambda_D\) basis through KM rotations, \(K_L, K_R\).

Hence, we define the following matrices:
\[
(\tilde{\delta}_R^D)_{ij} = \frac{K_R m_D^2 K_R^\dagger}{\tilde{m}_D^2} \delta_{ij} - \delta_{ij}
\]
\[
(\tilde{\delta}_L^D)_{ij} = \frac{K_L m_Q^2 K_L^\dagger}{\tilde{m}_Q^2} \delta_{ij} - \delta_{ij}
\]

(4.8)
where the “∼” symbol indicates the diagonal $\lambda_D$ basis for the matrices. Unfortunately, $\tilde{\delta}_D^R$ depends on the unknown $K_R$ matrix. The matrices defined in (4.6) are those usually compared to experimental data on the various FCNC processes. Thus, there are several upper bounds in the literature obtained by considering a variety of physical transitions [8, 10, 11]. However these upper bounds correspond to different assumptions and approximations, and also depend on the overall squark and gaugino masses.

5. AN EXAMPLE: THE NEUTRON E.D.M.

The neutron electric dipole moment is a precious parameter in fundamental physics as it is strongly suppressed in the Standard Model, on the contrary, it is sensitive to new sources of CP violations. Among the possible physical phases in supersymmetric theories, some are more relevant for the discussion of the neutron e.d.m., namely, the relative phases between the matrix elements in the soft analytic scalar couplings and the gaugino masses [26, 27]. We choose the usual conventions and redefine the fields so that gaugino masses are real. In this Section, we choose to work in the basis where the down-quark mass matrix is diagonal and real. With respect to the definition in the previous Sections, we have to perform KM rotations to define, e.g.,

$$\tilde{m}_Q^2 = K_L m_Q^2 K_L^\dagger$$
$$\tilde{m}_D^2 = K_R m_D^2 K_R^\dagger$$
$$\tilde{\eta}_D = K_R \eta_D K_R^\dagger$$

Since the calculation of the gluino-squark contribution to the e.d.m. $D_n$ of the neutron has been already presented many times in the literature, we just quote the simple final expression for the e.d.m. of the $d$-quark:

$$D_d = -\frac{8}{9} e \frac{\alpha_3}{4\pi} M_3^3 V_2 \text{Im} \Gamma_{dd}$$
$$\Gamma_{dd} = \int \frac{k^2 dk^2}{[k^2 + M_3^2]} \left( \frac{1}{k^2 + \tilde{m}_{D_R}^2} \tilde{\eta}_D \frac{1}{k^2 + \tilde{m}_{D_L}^2} \right)_{dd}$$

where $M_3$ is the gluino mass, only the lowest order in the analytic trilinear coupling is retained, the phase in the “μ-term” is neglected for simplicity, and the matrices $\tilde{m}_{D_R}^2$ and $\tilde{m}_{D_L}^2$ are defined as:

$$\tilde{m}_{D_R}^2 = \tilde{m}_D^2 + \left( \tilde{\lambda}_D V_2 \right)^2 - \frac{M_Z^2}{3} \sin^2 \theta_W \cos^2 \beta$$
$$\tilde{m}_{D_L}^2 = \tilde{m}_Q^2 + \left( \tilde{\lambda}_D V_2 \right)^2 - M_Z^2 \left( \frac{1}{2} - \frac{\sin^2 \theta_W}{3} \right) \cos^2 \beta$$
\[ \tan \beta = \frac{V_1}{V_2}, \quad V_i = \langle H_i \rangle. \]

(All matrices are defined in the diagonal \( \lambda_D \) basis).

In order to extract the dominant contributions in the presence of flavour dependent supergravity couplings, let us consider a simple approximation. We consider a Taylor expansion of the propagators \((k^2 - \tilde{m}^2_{D_R})^{-1}\) and \((k^2 - \tilde{m}^2_{D_L})^{-1}\) around some mean value \(\bar{m}^2\), an average over the squark and gluino masses, and then we integrate on \(k^2\). By keeping only the dominant term of each kind, using the notation introduced in (4.6), one ends up with the following expression:

\[ \text{Im } \Gamma_{dd} \simeq \frac{1}{\bar{m}^6} \left\{ \frac{\lambda_d}{12} \text{Im} \left( \mathcal{B}^D_R + \mathcal{B}^D_L \right)_{dd} - \frac{\lambda_i}{20} \text{Im} \left[ \left( \mathcal{D}^D_R \right)_{di} \left( \mathcal{B}^D_L \right)_{id} + \left( \mathcal{B}^D_R \right)_{di} \left( \mathcal{D}^D_L \right)_{id} \right] - \frac{\lambda_i}{30} \text{Im} \left[ \left( \mathcal{D}^D_R \right)_{dj} \left( \mathcal{D}^D_L \right)_{ji} + \left( \mathcal{D}^D_R \right)_{dj} \left( \mathcal{B}^D_L \right)_{id} \right] \right\} \]  

(5.4)

where \(i = b, s\); \(j = b, s, d\). Because of the Yukawa coupling hierarchy, all terms in (5.4) can be equally relevant. Indeed, let us consider the upper bounds obtained in Refs.[8,10,11] for the squark mass differences, assuming \(\bar{m} \sim 0(1 \text{ TeV})\). They are as follows:

\[
\begin{align*}
\left( \mathcal{D}^D_L \right)_{sd} &< .1 \\
\left( \mathcal{D}^D_L \right)_{ds} &< .1 \\
\left( \mathcal{D}^D_L \right)_{bd} &< .3 \\
\left( \mathcal{D}^D_L \right)_{db} &< .3 \\
\left( \mathcal{D}^D_R \right)_{ds} &< 4 \times 10^{-5} \\
\left( \mathcal{D}^D_R \right)_{db} &< 5 \times 10^{-3}
\end{align*}
\]

(5.5)

Combining these values with the quark mass ratios, we get the following estimates for the relative magnitude of the various terms appearing in (5.4):\(^8\)

\[
\begin{align*}
\frac{12}{20} \frac{m_s}{m_d} \left( \mathcal{D}^D_L \right)_{ds} &\sim 0(1) (0(0.07)) \\
\frac{12}{20} \frac{m_b}{m_d} \left( \mathcal{D}^D_L \right)_{db} &\sim 0(70) (0(20)) \\
\frac{12}{30} \frac{m_s}{m_d} \left( \mathcal{D}^D_R \right)_{ds} &\sim 0 (3 \times 10^{-4}) \\
\frac{12}{30} \frac{m_b}{m_d} \left( \mathcal{D}^D_R \right)_{db} &\sim 0(1)
\end{align*}
\]

(5.6)

Though the coefficients of \(\left( \mathcal{B}^D_L \right)_{bd}, \left( \mathcal{B}^D_R \right)_{db}\) are allowed to be rather large, upper bounds are also available for these matrix elements. A rough upper bound is obtained by assuming

\(^8\) The estimates in brackets are obtained from the bounds on the products \(\mathcal{D}^D_L \mathcal{D}^D_R\) and the assumption \(\mathcal{D}^D_L \sim \mathcal{D}^D_R\).
\[
\left( B_{L(R)}^\nu \right)_{dd} \sim 0 (\bar{m}) , \text{ where } \bar{m} \text{ is the average mass in (5.4) and then imposing bounds on } (\eta)_{bd}, (\eta)_{db} \text{ obtained from studies of supersymmetric effects in } K \text{ and } B \text{ physics [8, 10, 11]. However, once translated in terms of the parameters in (5.4), these bounds just require } \left| \left( B_{L(R)}^\nu \right)_{db} \right| \sim 0 \left( \left| \left( \tilde{B}_{L,R}^\nu \right)_{dd} \right| \right) .
\]

Thus, terms proportional to \( m_b \) can even dominate the first, more familiar, term.

We have dealt with \( D_d \), the e.d.m. of the \( d \)-quark as some comparison of the different contributions can be performed. However, the importance of terms proportional to heavy quark Yukawa couplings is even more obvious if one calculates \( D_u \), the e.d.m. of the \( u \)-quark. In this case we work directly in the diagonal \( \lambda_U \) basis used in the previous Sections. The analogue of (5.2) reads

\[
D_u = + \frac{16}{9} e^{\frac{\alpha_3}{4\pi}} M_3^3 V_1 \Im \Gamma_{uu}
\]

\[
\Gamma_{uu} = \int \frac{k^2 dk^2}{[k^2 + M_3^2]^3} \left( \frac{1}{k^2 + m_{U_R}^2 V_1} \frac{1}{k^2 + m_{U_L}^2} \right)_{uu}
\]

\[
m_{U_R}^2 = m_U^2 + (\lambda_U V_1)^2 + \frac{2M_\pi^2}{3} \sin^2 \theta_W \cos 2\beta
\]

\[
m_{U_L}^2 = m_Q^2 + (\lambda_U V_1)^2 + M_Z^2 \left( \frac{1}{2} - \frac{2\sin^2 \theta_W}{3} \cos 2\beta \right)
\]

Then the same approximations as in (5.4) (but, in this case, they could be poorer) yield,

\[
V_1 \Im \Gamma_{uu} \simeq \frac{1}{\bar{m}^6} \left\{ \frac{m_u}{12} \Im \left( B_R^U + B_L^U \right)_{uu} - \frac{m_t}{20} \Im \left[ (\delta_R^U)_{ii} (B_L^U)_{ii} + (B_R^U)_{ii} (\delta_L^U)_{ii} \right] 
\right. - \left. \frac{m_i}{30} \Im \left[ (\delta_R^U)_{ii} (\delta_L^U)_{ji} (B_L^U)_{ji} + (\delta_R^U)_{ij} (\delta_L^U)_{ii} (B_R^U)_{ji} \right] \right\}
\]

Estimates like those in (5.6) are only possible for (\( uc \)) or (\( cu \)) matrix elements. Using results from [8, 10, 11] we obtain the very poor constraints,

\[
\frac{12}{20m_u} \left( \delta_{L(R)}^U \right)_{uc} \sim 0(14) \quad (0(6))
\]

\[
\frac{12m_c}{30m_u} \left( \delta_R^U \right)_{uc} \left( \delta_L^U \right)_{cu} \sim 0(2)
\]

The largest contributions are certainly those proportional to \( m_t \), the top mass. Let us scrutinise the RGE of the relevant soft parameters in the approximation of (3.10). First, we notice that \( (B_R^U)_{ut} \) and \( (B_L^U)_{tu} \) are only slightly renormalised even for \( \lambda_t \) near the critical (“fixed-point”) value (while the transposed matrix elements get more reduced). Instead the \( (B_L^U + B_R^U) \) (33) diagonal element can be strongly reduced in the large \( \lambda_t \) limit so that non-diagonal \( (B_L^U) \) and \( (B_R^U) \) can dominate. This is even enhanced by the large contributions to \( (\delta_R^U) \), \( (\delta_L^U) \) diagonal terms for large \( \lambda_t^2 \).

Therefore, the contributions to \( D_d \) and, especially, \( D_u \) from terms proportional to \( m_b \) and \( m_t \), respectively, could be larger than those obtained by neglecting the squark mass splitting in
the propagator, which are proportional to light quark Yukawas. However, the whole discussion remains merely academic in the absence of a specific model for supergravity parameters, including the phases in the soft terms. The minimalist assumption is to require universal supergravity boundary conditions and the KM phase as the only source of CP violation. This has been shown to yield extremely small contributions to the quark e.d.m. [27]. Another rough estimate [26] consists in setting \((B_R + B_L) \sim M_3 \sim \bar{m}\) to obtain for the phase \(\phi\) of \((B_R + B_L)\), \(\sin \phi < (700 \text{ MeV/}\bar{m})^2\), from the experimental limit on \(D_N\). These two extreme cases clearly show that the effects could be important compared to the Standard Model prediction where \(D_N\) is very strongly suppressed.

6. MAXIMUM FLAVOUR MIXING IN SUPERGRAVITY

In this Section we aim at studying a model with maximal mixing at the supergravity scale, in order to get some quantitative insight about the effects at low energies. A natural model for flavour mixing would constrain both the Kahler potential and the superpotential (Yukawa couplings) in terms of symmetries. We do not know a realistic model of this sort and so we forget the naturalness requirement. Thus, we discuss a model designed to stress FCNC effects where the flavour dependence is introduced by hand in the (field dependent) supergravity metrics. For simplicity, the Yukawa couplings are assumed to be left-right symmetric. The \(U\) and \(V\) matrices at the unification scale \(\Lambda_{\text{GUT}}\) are defined from the Yukawa couplings at \(\Lambda_{\text{GUT}}\) by (2.1) and the KM matrix is given by the relation

\[
V = U K^\dagger. \tag{6.1}
\]

Let us consider the real matrix

\[
U = \frac{1}{\sqrt{6}} \begin{pmatrix}
\sqrt{3} & 1 & \sqrt{2} \\
-\sqrt{3} & 1 & \sqrt{2} \\
0 & -2 & \sqrt{2}
\end{pmatrix} \tag{6.2}
\]

(we neglect all phases in this Section). From (6.1) one gets the following model for \(\lambda_U (\Lambda_{\text{GUT}})\):

\[
\begin{align*}
\lambda_U &= \frac{\lambda_t}{3} \begin{pmatrix}
1 + \varepsilon_u + \varepsilon_c & 1 - \varepsilon_u + \varepsilon_c & 1 - 2\varepsilon_c \\
1 - \varepsilon_u + \varepsilon_c & 1 + \varepsilon_u + \varepsilon_c & 1 - 2\varepsilon_c \\
1 - 2\varepsilon_c & 1 - 2\varepsilon_c & 1 + 4\varepsilon_c
\end{pmatrix} \\
\varepsilon_u &= \frac{3\lambda_u}{2 \lambda_t} \\
\varepsilon_c &= \frac{\lambda_c}{2 \lambda_t} \tag{6.3}
\end{align*}
\]
This matrix is (almost) “democratic”, in the sense that all matrix elements are almost equal. In this sense the flavour mixing is maximal. Then for the sake of our qualitative discussion, we define $V$ as in (6.1) with $K$ phenomenologically given.

The supergravity basis is defined by the diagonalization of the Kahler matrix. In this basis, an honest assumption is to set all soft terms diagonal at $\Lambda_{\text{GUT}}$ (however, this is not necessarily so, as discussed in Ref.[6]). Thus, let us define

\[
\begin{align*}
(B^U_R)_{ab} &= \left( \frac{A^U}{2} + b^U_{Ra} \right) \delta_{ab} \\
(B^U_L)_{ij} &= \left( \frac{A^U}{2} + b^U_{Li} \right) \delta_{ij} \\
(m_Q^2)_{ij} &= (\bar{m}_Q^2 + \mu_Q^2) \delta_{ij} \\
(m_U^2)_{ab} &= (\bar{m}_U^2 + \mu_U^2) \delta_{ab} \\
(m_D^2)_{ab} &= (\bar{m}_D^2 + \mu_D^2) \delta_{ab}
\end{align*}
\]

\[\quad \quad (i, j, a, b = 1, 2, 3) \quad (6.4)\]

By performing the $U$ rotation, the soft terms are transformed into the more convenient diagonal $\lambda_U$ basis. In this basis $B^U_R$ and $B^U_L$ take the form:

\[
B^U_R = \left( \frac{A^U}{2} + \sum_a b^U_{Ra} \right) \mathbb{1} + \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{3} \Delta b^U_R & \Delta' b^U_R & \sqrt{2} \Delta' b^U_R \\
\Delta' b^U_R & -\sqrt{3} \Delta b^U_R & \Delta b^U_R \\
\sqrt{2} \Delta' b^U_R & \Delta b^U_R & 0
\end{pmatrix}
\]

\[
\begin{align*}
\Delta b^U_R &= b^U_{R1} + b^U_{R2} - 2 b^U_{R3} \\
\Delta' b^U_R &= b^U_{R1} - b^U_{R2}
\end{align*} \quad (6.5)
\]

and the analogous expression for $B^U_L$ with $R \rightarrow L$. The corresponding relations for $B^D_R$, $B^D_L$ are analogous with $U \rightarrow D$. The basis where $\lambda_D$ is diagonal and thus they are transformed by a KM rotation into the diagonal $\lambda_U$ basis.

Similar expressions also result for the $m^2$ parameters, which can be written as:

\[
\begin{align*}
m_I^2 &= \left( \bar{m}_I^2 + \frac{1}{3} \sum_i \mu_I^2 \right) + \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{3} \Delta \mu_I^2 & \Delta' \mu_I^2 & \sqrt{2} \Delta' \mu_I^2 \\
\Delta' \mu_I^2 & -\sqrt{3} \Delta \mu_I^2 & \Delta \mu_I^2 \\
\sqrt{2} \Delta' \mu_I^2 & \Delta \mu_I^2 & 0
\end{pmatrix}
\end{align*}
\]

\[\quad \quad (I = Q, U, D) \quad (6.6)\]
In order to proceed and fix some parameters in view of a quantitative analysis, we consider a simple Ansatz for the quark quadratic terms in the Kahler potential

\[ K(T, \Phi^A, ... ) = \sum_A \frac{\Phi^A \Phi^{A*}}{(\text{Re } T)^{-n_A}} + ... \]  

(6.7)

where \( \Phi^A = Q^i, U^a, D^a \). The moduli fields introduced in the effective supergravity by the compactification of six dimensions in string theory, are represented here by the overall modulus, denoted \( T \). The exponents \( n_A \) are the modular weights that specify the behaviour of each superfield \( \Phi_A \) under modular transformations of the compact manifold. We shall assume them to be negative integer numbers [17, 18]. Then, from the general supergravity expressions:

\[ m_{AA*}^2 = (F^M F^{N*}) \left( Z_{MN*} Z_{AA*} - R_{MN*AA*} \right) \]  

(6.8)

\[ \eta_{ABC} = F^M D_M \lambda_{ABC} \]

\[ D_M = Z^{-1} \left( \partial_M + \frac{1}{2} \partial_M K \right) Z \]

\[ Z_{AB*} = \partial_A \partial_{B*} K \]  

(6.9)

one gets the soft scalar couplings and masses in terms of the supersymmetry breaking auxiliary fields \( F^M \). Here we consider the simplest case in which these auxiliary fields are just \( F^S \) and \( F^T \), corresponding to the dilaton sector and to the overall modulus sector in the effective supergravity theory obtained from the orbifold compactification of superstrings. Accordingly, we introduce the Goldstino angle \( \theta \) [12], such that \( \tan \theta = F^S / F^T \), and neglect possible phases. Then the gaugino masses at \( \Lambda_{GUT} \) are all equal and given by \( M_0^2 = |F^S|^2 \), while for the other parameters one obtains [12]

\[ m_A^2 = \frac{M_0^2}{3} \left( 1 + (1 + n_A) \cot^2 \theta \right) \]

\[ (B_R^U)_{ab} = -M_0 \left( \frac{1}{2} + \frac{1}{\sqrt{3}} \left( \frac{3}{2} + \frac{n_{H_1}}{2} + \frac{n_a U}{2} \right) \cot \theta \right) \delta_{ab} \]  

(6.10)

with analogous expressions for \( B_L^U, B_R^D, B_L^D \).

These expressions can be substituted in (4.6) to allow a numerical investigation. The integers \( n_i \) (modular weights) are free parameters. We choose to fix their values \( n_1 = -1, n_2 = -2, n_3 = -3 \) for the first, second and third families. Modular weights of Higgs fields are chosen to be -2. These choices are compatible with known results from superstring compactification where \( n_i \) is -2 for non oscillated twisted fields of orbifold models. Anyway, this particular choice is only meant to
shift the degeneracy of $B$’s and $m^2$’s at the unification scale in a way which is qualitatively consistent with our expectations from supergravity theory. We impose that masses of squarks at the compactification scale are all positive. This entails that:

$$\tan^2 \theta > -(n_m + 1) \quad (6.11)$$

where $n_m$ is the lowest modular weight. In our case $\tan^2 \theta > 2$. As a matter of fact, this is not strictly necessary since a negative squark mass$^2$ at the unification scale will be turned positive (mostly) by strong interaction radiative corrections, cf. (4.6). But if one assumes similar modular weights for the lepton sector, then (6.11) ensures that sneutrino and selectron masses are consistent with experimental bounds.

![Fig.1: The left mass insertion $\delta_{db}^L$ as function of $\theta$ (rad), the full line is the renormalised curve (see text).](image)

From (6.5), (6.6), (6.10) and (4.6), one obtains the predictions for the matrices (4.7), (4.8) in this particular model. It is then possible to vary $\theta$ and plot $\delta_{D,i,j}^L$ as functions of $\theta$ (see figs. 1, 2 and 3 where $\theta$ is measured in radians). As a rule, effects of renormalisation are quite severe. On each plot we have depicted two curves, one without renormalisation but for the gluino contribution $7M_0^2$ to the average squark mass$^2$, compared to the renormalised one. As already discussed in the
introduction, the net effect of renormalisation is to reduce the value of mass insertions as the squark masses get a large common contribution proportional to the gluino mass \([12, 13, 14]\). Otherwise, the effect of renormalisation is small for the \((ds)\) insertion whereas it is large for the other two.

Notice that the chosen range for \(\theta\) is such that mass insertions are within the phenomenological bounds discussed in \([8, 10, 11]\).

\[ t \]

Fig. 2 The mass insertion \(\delta_{sb}^L\) as function of \(\theta\) (rad),

the full line is the renormalised curve (see text).

Because of the recent measurements of the \(b \rightarrow s\gamma\) transition, let us consider the helicity flipping analogue of \((4.7), (4.8)\). They are given by the flavour mixing elements in the analytic sector of the mass\(^2\) matrix. Let us define,

\[
\delta_{LR}^D = \frac{K(B_D^D m_D + m_DB_L^D) K_f}{m^2}.
\]  

\((6.12)\)

and use \((6.5), (6.6)\) and \((4.6)\) to obtain the following numerical expressions:

\[
\left(\delta_{LR}^D\right)_{sd} = -0.05 \cot \theta + 0.9 \frac{m_s}{M_0}
\]

\[
\left(\delta_{LR}^D\right)_{db} = -0.04 \cot \theta \frac{m_b}{M_0}
\]

\[
\left(\delta_{LR}^D\right)_{sb} = -0.06 \cot \theta + 0.015 \frac{m_b}{M_0}.
\]  

\((6.13)\)
which are all below the experimental bounds.

\begin{center}
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{The mass insertion $\delta_{\text{dis}}^L$ as function of $\theta$ (rad), the full line is the renormalised curve (see text).}
\end{figure}
\end{center}

Since we are interested in renormalisation effects, we have concentrated on the quark sector. Indeed, quantum corrections are much smaller in the leptonic sector where the large QCD and top couplings do not contribute (quantum effects can be bigger in the large $\tan \beta$ scenario). However the experimental FCNC restrictions are much stronger because of the limits on $\mu \rightarrow e\gamma$ and similar processes. Let us generalise the mixing matrix $U$ in (6.2) to the leptons, choose the lepton modular weights to be those assigned to quarks in the “same family”. Then we can estimate the analogues of (4.8) and (6.12) in the lepton sector. By imposing the experimental bounds for $\mu \rightarrow e\gamma$ on the resulting expressions for $(\delta_L)_{e\mu},(\delta_R)_{e\mu}$ and $(\delta_{LR})_{e\mu}$, one gets a strong constraint on the Goldstino angle, $\tan^2 \theta > 10$. With this restriction on $\theta$, the values of $(\delta_L^D)_{ij}$ in the figures lie well below their phenomenological limits.

In summary, we have considered a simple model to estimate FCNC effects including the RGE running, with maximal family mixing at the unification scale. The flavour dependent supergravity couplings is due to the moduli fields, represented in our example by a single superfield, $T$. Thus FCNC effects are proportional to $\cot \theta$ (or $\cot^2 \theta$), which measures the supersymmetry break-
ing component along the flavour dependent direction. The resulting FCNC effects are small for two reasons: (i) The universal gluino contribution to the squark masses is very large, a model-independent feature. (ii) The contributions of the moduli sector to the scalar mass $^2$ are proportional to $(n_i + 1) \cot^2 \theta$ which are basically non-positive if the modular weights $n_i \leq -1$ as usually found in orbifold models: this fact sets a physical upper limit on the supersymmetry breaking component along the moduli direction. Expressed as a bound on $\cot \theta$ it restricts the FCNC effects to the relatively small values in the figures, even in the maximal mixing model discussed here.

This negative contribution to the squark mass $^2$ may be considered as an artefact of the model, or more generally, of flavour dependence induced by the moduli couplings. Even if this is an interesting natural mechanism to generate non-universal supergravity couplings, it is certainly possible to contemplate other possibilities. However, unless quarks and leptons exhibit very different patterns, there is another general constraint on FCNC effects. Indeed, whatever mechanism generates flavour dependence, the mixing effects in the lepton sector are severely restricted by experiments. Hence supersymmetry breaking auxiliary fields with flavour dependent couplings are bounded, as in the example here above, and quark flavour changing effects are expected to be well below the experimental limit. But, a priori, quarks and leptons could couple to supergravity in quite different ways.

7. CONCLUSION

Within the supergravity framework, most of the unitary transformations needed to diagonalise the quark masses acquire a physical meaning. The RGE are then needed to make the connection with low energy flavour mixing phenomena. The systematic study presented here, shows how the quark mass hierarchies helps simplifying results. This is so even if the tree-level mixing is large. The effects of the running down to low energies of the squark mass matrices is sizeable in many respects, and the factorisation of the A-terms into $B_L$ and $B_R$ is a useful tool. Nevertheless, some uncertainties remain due to the dependence on the unknown analogue of the CKM matrix for right-handed quarks.

Approximate discrete symmetries could be invoked to justify that the large flavour mixing in the Yukawa couplings would disfavour non-universality in the Kahler potential. An elegant analysis [22] shows how ad hoc broken continuous symmetries could naturally control non-universality patterns in the squark masses and prevent FCNC problems. It would be relevant to study how natural these symmetries are in the context of supergravity.
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