Analysis of Longitudinal Force of Beam and Rail of Continuously Welded Rails (CWR) on Railway Bridge

Zhiping Zeng\textsuperscript{1,2}, Ji Hu\textsuperscript{1}, Qiang Zeng\textsuperscript{1}, Zhibin Huang\textsuperscript{1,3*}, Huatuo Yin\textsuperscript{4} and Hui Huang\textsuperscript{4}

1 School of Civil Engineering, Central South University, Changsha, Hunan 410075, China
2 MOE Key Laboratory of Engineering Structure of Heavy Haul Railway, Central South University, Changsha, Hunan 410075, China
3 Southeast Coastal Railway (Fujian) Co., Ltd, Fuzhou, Fujian, 350001, China
4 Guangzhou Metro Design & Research Institute Co., Ltd., Guangzhou, Guangdong 510010, China
Email: 1146401456@qq.com

Abstract. To study the longitudinal force of CWR on viaduct, a track-bridge-pier finite element model is established. Taking a multi-span simply supported beam with a maximum span of 32.7m of an elevated CWR as an example, the additional expansion and contraction forces, displacement between rail and beam and the force of pier are calculated, and whether the rail stress meets the requirements when setting constant resistance fasteners is checked. The results show that: (1) For the left and right lines, the maximum additional expansion forces of single strand rail are both 211.13kN, and the maximum relative displacements between beam and rail are both 6.572mm. (2) The maximum value of the additional expansion and contraction forces and the relative displacement between beam and rail of the same line occur at the same position. The left line is at ZFZ29 pier and the right line is at ZFS31 pier. (3) The maximum force of pier in this section is 500.80kN, and the pier numbers are ZFZ27 and ZFS29. (4) The rail stress is less than the allowable stress of 352MPa, and the rail strength meets the requirements.

1. Introduction
Under the influence of train load, temperature change and other factors, the rail and beam will have longitudinal interaction, produce relative displacement and additional internal force [1-3].

The researches on Continuously Welded Rails (CWR) on bridge mainly study the longitudinal interaction mechanism between track and bridge and the structural design of track and bridge to ensure that the track and bridge structure meet the requirements of strength and stability under temperature and train load [4-5].

In this paper, taking the CWR on a viaduct as an example, the spatial structure model is established by using the finite element software ANSYS [6-8], and the longitudinal mechanical properties are calculated and analyzed, which could provide a reference basis for the design of laying CWR on the bridge.
2. Track-Bridge-Pier Finite Element Calculation Model

2.1. Model Assumptions and Treatment
The finite element model of beam-rail interaction under temperature change is established. There is no regulator in the design scheme, and the longitudinal force of CWR is calculated according to the concrete simply supported beam. In order to eliminate the influence of boundary conditions, 180 m subgrade sections are set at both ends. The rail and bridge are simulated by beam element respectively, the vertical characteristic of the fastener is simulated by linear spring element, and the longitudinal characteristic of the fastener is simulated by nonlinear spring. The spring action direction is to prevent the displacement of rail relative to sleeper.

2.2. Model Parameters

2.2.1. Bridge Parameters. The finite element model of track-bridge structure is established based on ANSYS. The ballastless track structure is adopted. The bridge is a simply supported beam bridge, and the bearings are arranged in the form of ‘fixed-movable’, the basic parameters of the bridge are shown in table 1.

| line | Pier number | Span(m) | Stiffness(kN/cm) | Bearing       |
|------|-------------|---------|-----------------|---------------|
| Left line | ZFZ9  | 24.7 | 26149          | Fixed- Movable |
|       | ZFZ10 | 24.7 | 2178           | Fixed- Movable |
|       | ZFZ11 | 24.7 | 1918           | Fixed- Movable |
|       | ZFZ12 | 24.7 | 2056           | Fixed- Movable |
|       | ZFZ13 | 24.7 | 2178           | Fixed- Movable |
|       | ZFZ14 | /    | 28149          | /             |
|       | ZFZ27 | 32.7 | 28149          | Fixed- Movable |
|       | ZFZ28 | 32.7 | 1667           | Fixed- Movable |
|       | ZFZ29 | /    | 28149          | /             |
| Right line | ZFS9  | 24.7 | 28149          | Fixed- Movable |
|       | ZFS10 | 24.7 | 2878           | Fixed- Movable |
|       | ZFS11 | 24.7 | 2358           | Fixed- Movable |
|       | ZFS12 | 24.7 | 2167           | Fixed- Movable |
|       | ZFS13 | 24.7 | 5649           | Fixed- Movable |
|       | ZFS14 | /    | 28149          | /             |
|       | ZFS29 | 32.7 | 28149          | Fixed- Movable |
|       | ZFS30 | 32.7 | 1667           | Fixed- Movable |
|       | ZFS31 | /    | 28149          | /             |

2.2.2. Track Structure Parameters
(1) Standard 60kg/m rail is adopted, and the elastic modulus of rail is 2.06×10^5 Mpa, the cross-sectional area of a single rail is 77.45cm^2; The linear expansion coefficient is taken as 1.18×10^-5/℃.
(2) SFC type constant resistance fastener is adopted, the fastener spacing is 0.6m, the longitudinal resistance of fastener is taken as 9kN for each group, and the slip value is 2mm.
(3) The elastic modulus of concrete bridge is 35.5GPa, and the linear expansion coefficient of concrete beam is 1×10^-5 /℃ calculation.
(4) According to Code for Design of Railway Continuous Welded Rail (TB 10015-2012) [9], the maximum rail temperature in this area is 61.7 °C and the minimum rail temperature is -1.7 °C. The daily temperature difference of concrete beam is calculated as 30 °C, the locking rail temperature is selected as 30 ± 5 °C, and the maximum temperature change range is 36.7 °C.

2.2.3. Model Establishment. The overall beam and rail interaction models are shown in figure 2.

![Figure 2. Finite element model of expansion force and temperature force of beam rail interaction.](image)

3. Calculation and Analysis

3.1. Calculation of Additional Expansion and Contraction Force of Rail, Relative Displacement Between Beam and Rail And The Force of Pier

When the concrete bridge temperature rises 30 °C, the additional expansion and contraction forces of the rails and relative displacements between the beam and rail are shown in figure 3 to 10. The force of each pier is shown in table 2.

![Figure 3. Additional expansion and contraction force of left rail (ZFZ9-ZFZ14).](image)

![Figure 4. Relative displacement of left track between the beam and rail (ZFZ9-ZFZ14).](image)
According to figure 2 to 9, for the left line, the maximum additional expansion force on the single strand rail is 211.13kN, which occurs at the pier numbered ZFZ29; The maximum relative displacement between beam and rail also occurs at this position, which is 6.572mm. For the right line, the maximum additional expansion force on the single strand rail is 211.13kN, which occurs at the pier.
numbered ZFS31; The maximum relative displacement between beam and rail also occurs at this position, which is 6.572mm. Therefore, for the left and right lines, the values of the maximum additional expansion force of single strand rail and the values of the maximum relative displacement between beam and rail are the same. The maximum value of the additional expansion and contraction forces and the relative displacement between beam and rail for each line occur at the same position.

As is shown in table 2, the maximum force of pier in the section is 500.80kN, and the pier numbers are ZFZ27 and ZFS29.

Table 2. Longitudinal force of pier (kN / rail).

| Pier number | Force  | Pier number | Force  |
|-------------|--------|-------------|--------|
| ZFZ9        | 165.08 | ZFS9        | 164.12 |
| ZFZ10       | 34.50  | ZFS10       | 36.14  |
| ZFZ11       | 22.10  | ZFS11       | 21.00  |
| ZFZ12       | 30.76  | ZFS12       | 26.86  |
| ZFZ13       | 63.52  | ZFS13       | 78.10  |
| ZFZ14       | 0      | ZFS14       | 0      |
| ZFZ27       | 250.40 | ZFS27       | 250.40 |
| ZFZ28       | 95.10  | ZFS28       | 95.10  |
| ZFZ29       | 0      | ZFS29       | 0      |

Therefore, in the design, construction and maintenance of CWR on the bridge, more attention should be paid to the bridge-track structure at these piers, ZFZ29, ZFS31, ZFZ27 and ZFS29.

3.2. Checking and Calculation of Rail Strength

Each rail shall have sufficient strength to ensure that it will not fail under the joint action of dynamic bending stress, temperature stress, expansion additional stress, train starting / braking stress and other additional stresses. At this time, the sum of various stresses borne by the rail shall not exceed the specified allowable value $[\sigma]$, and the calculation formula is shown in formula (1):

$$\sigma_d + \sigma_t + \sigma_z + \sigma_f \leq [\sigma]$$

(1)

Where:

- $\sigma_d$ —— Maximum dynamic bending tensile stress of rail (MPa);
- $\sigma_t$ —— Temperature stress (MPa);
- $\sigma_z$ —— Maximum additional stress on rail (MPa);
- $\sigma_f$ —— Traction (braking) stress on rail (MPa);
- $[\sigma]$ —— the allowable stress of rail (MPa), according to Code for Design of Railway Continuous Welded Rail (TB 10015-2012) [9]. The yield strength of U71Mn rail is 457MPa, and the safety factor $K=1.3$, so $[\sigma]=352$MPa.

The checking calculation of rail strength is shown in table 3.

Table 3. Statistics of rail stress (MPa).

| indexes     | Dynamic bending stress | Temperature stress | Additional expansion stress | Additional braking stress | Total stress | Allowable stress |
|-------------|------------------------|--------------------|-----------------------------|--------------------------|--------------|-----------------|
| Tension     | 119.99                 | 91.02              | 27.26                       | 11.80                    | 250.07       | 352             |
| compression | 154.63                 | 91.02              | 27.26                       | 11.80                    | 284.71       | 352             |
It can be seen from the table that the sum of tensile stress and compressive stress borne by the rail is 250.07MPa and 284.71MPa, both less than the allowable stress 352MPa, and the rail strength meets the requirements.

4. Conclusion
Taking the CWR on the multi-span simply supported beam bridge as an example, the following conclusions can be drawn after analyzing the longitudinal mechanical properties of the beam-rail interaction model:

(1) For the left line, the maximum additional expansion force on the single strand rail is 211.13kN, which occurs at the pier numbered ZFZ29; The maximum relative displacement between beam and rail also occurs at this position, which is 6.572mm. For the right line, the maximum additional expansion force on the single strand rail is 211.13kN, which occurs at the pier numbered ZFS31; The maximum relative displacement between beam and rail also occurs at this position, which is 6.572mm.

(2) For the left and right lines, the values of the maximum additional expansion force of single strand rail and the values of the maximum relative displacement between beam and rail are the same.

(3) The maximum value of the additional expansion and contraction forces and the relative displacement between beam and rail for each line occur at the same position, and both are located at the pier.

(4) The maximum stress of pier in the section is 500.80kN, and the pier numbers are ZFZ27 and ZFS29.

(5) The total tensile stress borne by the rail is 250.07MPa, and the total compressive stress borne by the rail is 284.71MPa, both less than the allowable stress 352MPa, so the rail strength meets the requirements.

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References
[1] Lu Y R. 2004 Research and Application of CWR [M] Beijing: China Railway Press.
[2] Wei F, Gao L, Zhao L, et al. 2014 Research about mechanical characteristics of position restriction structure on beam end of CRTSI slab ballastless track on long-span bridge[C] ICRE, Beijing 191-200.
[3] Wang P. 2001 Behavior of FRP Jacketed Concrete Columns Under Eccentric Loading [J] Journal of Composts for Construction 5 (3) 146-152.
[4] Bae Y, Hamad M. 2019 Quantification of Ballasted Track-bridge Interaction Behavior Due to the Temperature Variation through Field Measurements [J] NDT and E International 103 (12) 84-97.
[5] Anna M R, Andrzej N. 2018 Live Load Spectra for Railway Bridges in USA [J] Konferencja Naukowa 15 (18) 3-7.
[6] Xuan N. 2014 A Finite Element Model of Vehicle-Cable Stayed Bridge Interaction Considering Braking and Acceleration: The 2014 World Congress on Advances in Civil, Environmental, and Materials Research, Busan Korea[C].
[7] Xie K Z, Zhao W G, Cai X P, Wang P, Zhao J, Giovanni L. 2020 Interaction between Track and Long-Span Cable-Stayed Bridge: Recommendations for Calculation [J] Mathematical Problems in Engineering 2020.
[8] Dai G L, Ge H, Liu W S, Chen Y F. 2017 Interaction analysis of Continuous Slab Track (CST) on long-span continuous high-speed rail bridges [J] Structural Engineering and Mechanics 63 (6).

[9] Ministry of Railways of the People's Republic of China. Code for Design of Railway Continuous Welded Rail: TB 10015-2012 [S] Beijing China Railway Publishing House 2013. (in Chinese)