\( \tilde{U}(12) \) A New Symmetry 
Possibly Realizing in Hadron Spectroscopy

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Starting from the multi-local Klein-Gordon equations with Lorentz-scalar squared-mass operator we give a covariant quark representation of the general composite mesons and baryons with definite Lorentz transformation property. The phenomenologically observed hadron mass spectra is pointed out to satisfy possibly the approximate symmetry under the \( \tilde{U}(4) \) transformation group concerning the spinor freedom of light constituent quarks, including the chiral transformation as a subgroup. This symmetry predicts the existence of new type of chiral mesons and baryons out of the conventional framework in non-relativistic quark model: For light \( q\bar{q} \) systems, the scalar \( \sigma \)- and axial-vector \( a_1 \)-nonets are predicted to exist as relativistic \( S \)-wave states besides the ordinary \( P \)-wave state mesons. Two “exotic” \( J^{PC} = 1^{-+} \) mesons are predicted to exist as relativistic \( P \)-wave states, which are possibly assigned as \( \pi_1(1400) \) and \( \pi_1(1600) \). For light quark baryons the extra \( 56 \) with positive parity and the extra \( 70 \) with negative parity of the static \( SU(6) \) are predicted to exist as the ground state chiral particles.

§1. Introduction

(The present status of level-classification of hadrons) There exist the two contrasting, non-relativistic and relativistic, viewpoints of level-classification. The former is based on the non-relativistic quark model (NRQM) with the approximate \( LS \)-symmetry and gives a theoretical base to the PDG level-classification. The latter is embodied typically in the NJL model with the approximate chiral symmetry. It is widely accepted that \( \pi \) meson nonet has the property as a Nambu-Goldstone boson in the case of spontaneous breaking of chiral symmetry.

|                | Non-Relativistic | Relativistic |
|----------------|------------------|-------------|
| Model          | NRQM             | NJL model   |
| Approx. Symm.  | \( LS \)-Symm.   | Chiral Symm.|
| Evidence       | Bases for PDG    | \( \pi \) nonet as NG boson |

Table I. Two Contrasting Viewpoints of Level Classification

Owing to the recent progress, both theoretical and experimental, the existence of light \( \sigma \)-meson as chiral partner of \( \pi(140) \) seems to be established\(^1\) especially through the analysis of various \( \pi\pi \)-production processes. This gives further a strong support to the relativistic viewpoint.

Thus, the hadron spectroscopy is now confronting with a serious problem, existence of the seemingly contradictory two viewpoints, Non-relativistic and Extremely
Relativistic ones.

(The purpose of this letter) is, unifying these two viewpoints, to propose a covariant level-classification scheme of light-through-heavy quark (and possibly of gluonic) hadrons. We shall point out a possibility that an approximate symmetry of \( \tilde{U}(12) \supset \tilde{U}(4) \otimes U(3)_F \), here \( \tilde{U}(4) \) (we denote the homogeneous Lorentz group, \( \mathcal{L}_4 \), as \( \tilde{U}(4) \)) \( (U(3)_F) \) being a pseudo-unitary Lorentz (unitary) group concerning Dirac spinor (flavor) of light quarks, is realized in nature in the world of hadron spectroscopy.

Here the \( \tilde{U}(12) \) symmetry is mathematically the same as the one that appeared in 1965 to generalize covariantly the static \( SU(6)_{SF} \) symmetry \( (SU(6)_{SF} \supset SU(2)_S \otimes U(3)_F) \) in NRQM. However, in the case of \( \tilde{U}(12)_{SF} \) at that time only the boosted Pauli-spinors are taken as physical components out of the fundamental representations of \( \tilde{U}(4) \). Now in the present scheme of the \( \tilde{U}(12) \) symmetry all general Dirac spinors are, inside of hadrons, to be treated as physical.

(History of symmetry and level classification) First we shall review briefly a history of the hadron-level classification since the birth of hadron physics. It is based on a representation of some symmetry, which is deeply connected to the composite picture of hadrons.

In 1933 Yukawa\(^2\) predicted existence of \( \pi \)-meson, “symbolic particle of Strong Interaction and Hadron Physics.”

In 1949 Fermi and Yang\(^3\) had proposed the composite model of \( \pi \)-mesons. In 1956 Sakata\(^4\), extending the framework of F.Y. model, proposed a composite model for the new particles (mesons and baryons), taking Sakata triplet \( S(P,N,A) \) as the fundamental constituents. The strangeness of Nishijima-Gell-Mann rule\(^5\) was identified with the number of \( A \)-particle. In 1959 Ikeda, Ogawa and Ohnuki\(^6\) presented the mathematical framework of \( U(3) \) symmetry in Sakata model, after an important notice by Ogawa\(^7\) that each member of Sakata triplet has a certain equality, considered from the role of its numbers \( N_i \) \( (i = P,N,A) \) as quantum numbers, suggesting a possible certain symmetry. The similar approach was proposed by Yamaguchi, too.

The choice of Sakata triplet as a fundamental triplet was inadequate, leading to a difficulty in baryon assignment, and replaced\(^8\) with the (light) quark triplet \( q(u,d,s) \) by Gell-Mann and Zweig\(^9\) in 1964. Shortly after the appearance of quark model in 1964, Sakita\(^10\), Gürsey-Radicati\(^11\) and Pais\(^12\) independently proposed the \( SU(6)_{SF} \) theory, treating the intrinsic spin of quarks as a group-theoretical object.

The success of \( SU(6) \) theory was remarkable in that i) all the ground state mesons and baryons experimentally observed are assigned satisfactorily to its \( 35 \) and \( 56 \) representation, respectively. ii) Moreover, the famous ratio of nucleon magnetic moments \( \mu_P/\mu_N = -3/2 \), is predicted. iii) Furthermore, concerning the \( P \)-wave Yukawa interaction of mesons with baryons, the desirable \( F/D \) ratio is insisted\(^*\) to be also obtained.

However, all the above results are depending on the assignment of baryons into the corresponding symmetrical \( 56 \) representation, which seems to be, in simple intuition, contradictory with the sacred spin-statistics connection of elementary particles.

\(^*\) However, note that this interaction has the zero static limit.
In 1965 Salam, Delbourgo and Strathdee\textsuperscript{13}, and Sakita-Wali\textsuperscript{14} independently proposed the \( \tilde{U}(12) \) symmetry as a relativistic extension of the \( SU(6)_{SF} \) symmetry, and set up the Bargmann-Wigner equation to be satisfied by free hadron wave functions. Then assigning the baryons and mesons to the completely symmetrical representation \( 364 \) and \( 144 \), respectively, Salam et al. showed that all the above desirable results of the \( SU(6) \) (including the \( F/D \) ratio of Yukawa interaction) is reproduced. However, they assumed there that the “boosted multi-Pauli spinors”, which reduce to the multi-Pauli spinors at hadron rest frame, are only allowed to be physical and the effective vertex is to be (“fundamentally broken”)\textsuperscript{*}) \( \tilde{U}(12) \) symmetric. It is now well-known that a relativistic extension of the \( SU(6)_{SF} \) symmetry as a mathematical group is impossible as no-go theorem\textsuperscript{15}. 

In 1968 Roman and one of the authors (S.I.)\textsuperscript{16} derived the \( SU(6) \) results from a different physical consideration on its covariant generalization, applying a kind of the bosonization method. That is, starting from general Lorentz invariant 4 quark interactions and imposing them a “rest-condition,” to be \( SU(6) \) symmetric at the rest frame of all quarks in a channel where all quark and anti-quark numbers are separately conserved.

In order to treat the excited hadrons it is necessary to introduce the freedom of relative space-coordinates between constituent quarks. This is a straight-forward extension of the \( SU(6) \) theory from the composite picture, which was developed (first proposed by Greenberg\textsuperscript{17}, 1964) by many authors in the (so-called, simple and realistic) symmetrical quark model. They assumed there that constituent quarks, inside of hadrons, behave non-relativistically\textsuperscript{18}, leading to the approximate \( LS \)-symmetry\textsuperscript{19} in the hadron spectroscopy. So the hadron wave functions in this scheme is the tensors in the \( SU(6)_{SF} \otimes O(3)_{L} \) space.

In 1970 one of the present authors (S.I.) proposed its relativistic generalization, the Urciton Scheme\textsuperscript{20} (applying exciton-picture\textsuperscript{\textsuperscript{**}}) of constituent quarks), where the kinematical region is extended into Boosted \( SU(6)_{SF} \otimes O(3,1) \) space (\( O(3,1) \) being Lorentz space), and accordingly the framework is manifestly covariant. This scheme gives an effective method to treat both hadron spectroscopy and reactions on common footing and has been applied actually for these three decades as the covariant oscillator quark model\textsuperscript{22}.

\section*{§2. Covariant Framework for Describing Composite Hadrons}

\textbf{(Wave Functions of Mesons and Baryons)} In order to introduce the quark-composite picture of hadrons, we set up the following wave functions(WF) for mesons and baryons, respectively.

\begin{align*}
\text{Meson} : \Phi^{AB}_{A}(x_{1},x_{2}), & \quad \text{Baryon} : \Phi_{A_{1}A_{2}A_{3}}(x_{1},x_{2},x_{3}); \quad (2.1)
\end{align*}

\textsuperscript{*}) Note that the Bargmann-Wigner equation with a definite mass itself is not \( \tilde{U}_{DS}(4) \) (accordingly \( \tilde{U}(12) \))-symmetric.

\textsuperscript{**}) Here the strange symmetric properties (now accepted to be as the color-singlet behavior) of baryon wave function is reduced to a bose-quantization of (ur-)exciton quarks\textsuperscript{21}. 

\textsuperscript{1}U(12) a new symmetry possibly realizing in hadron spectroscopy
where \( A, A_i(\alpha, a); \alpha = 1 \sim 4(a) \) denote Dirac(flavor) indices of respective quark constituents. \( B = (\beta, b) \) its conjugate ones; and \( x_1 \) etc. denote the Lorentz four-vectors of the space-time coordinates of constituents.

(\textit{Klein-Gordon equation and mass term}) We start from the Yukawa-type Klein Gordon equation as a basic wave equation

\[
\frac{\partial^2}{\partial X^2} - \mathcal{M}^2 (r, \mu, \partial/\partial r, \partial/\partial X, \partial/\partial i X) \Phi(X, r, \cdots) = 0, \quad \mathcal{M}^2 = \mathcal{M}_{\text{conf}}^2 + \delta \mathcal{M}^2, \quad (2.2)
\]

where \( X_\mu(r \text{ for mesons, } r_1, r_2 \text{ for baryons}) \) are the center of mass (relative) coordinates of hadron systems. In the squared mass operator \( \mathcal{M}^2 \) the confining-force part \( \mathcal{M}_{\text{conf}}^2 \) is assumed to be Lorentz-scalar and \( A, (B) \)-independent in the case of light-quark hadrons, leading to the mass spectra with the \( \tilde{U}(12) \) symmetry and accordingly also with the chiral symmetry. As its concrete model we apply the covariant oscillator in COQM, leading to the straightly-rising Regge trajectories. The effects due to perturbative QCD and other possible effects \( \delta \mathcal{M}^2 \) are neglected in this paper.

The WF are separated into the positive (negative)-frequency parts concerning the internal WF of relativistic composite hadrons with a definite mass and a definite total spin \( (J = L + S) \) which are tensors in the \( \tilde{U}(DS) \times O(3,1) \) space by expanding them in terms of covariant bases of complete set, being a direct product of eigenfunctions, in the respective sub-space. We choose the BW spinors and the covariant oscillator functions (with a definite metric type) as them.

(\textit{Spinor WF}) The internal WF is, concerning the spinor freedom, expanded in terms of complete set of relevant multi-spinors, Bargmann-Wigner (BW) spinors. The BW spinors are defined as multi-Dirac spinor solutions of the relevant local Klein-Gordon equation:

\[
(\frac{\partial^2}{\partial X^2} - M^2)W_{\alpha \cdots}^{\beta \cdots}(X) = 0
\]

\[
W_{\alpha \cdots}^{\beta \cdots}(X) \equiv \sum_P [e^{iP \cdot X} W_{\alpha \cdots}^{(+ \cdots \beta \cdots \cdots}(P) + e^{-iP \cdot X} W_{\alpha \cdots}^{(- \cdots \beta \cdots \cdots}(P)). \quad (2.5)
\]

First we define the corresponding various free Dirac spinors for constituent quarks and anti-quarks with hadron 4-momentum \( P_\mu \) as single-index BW spinors:

\[
\psi_{q,\alpha}(X) \equiv W_{\alpha}(X), \quad u_{q,\alpha}(P_\mu) = W_{\alpha}^{(+)}(P), \quad u_{q,\alpha}(-P_\mu) = W_{\alpha}^{(-)}(P), \quad (2.6)
\]

\[
\psi_{\bar{q},\alpha}(X) \equiv W_{\bar{q},\alpha}(X), \quad \bar{u}_{\bar{q},\alpha}(P_\mu) = W_{\bar{q},\alpha}^{(-)}(P), \quad \bar{u}_{\bar{q},\alpha}(-P_\mu) = W_{\bar{q},\alpha}^{(+)}(P). \quad (2.7)
\]

For mesons(baryons) the BW spinors are bi-Dirac (tri-Dirac) spinors.

meson : \( \psi_{N,A}^{(\pm)}(P_N, r) = \sum_W W_{\alpha}^{(\pm \beta \cdots \cdots}(P_N) M_{N,\alpha}^{(\pm \beta \cdots \cdots}(r, P_N) \quad (2.8)\)

baryon : \( \psi_{N,A_1, A_2, A_3}^{(\pm)}(P_N, r_1, r_2) = \sum_W W_{\alpha_1, \alpha_2, \alpha_3}^{(\pm \beta \cdots \cdots}(P_N) B_{N, \alpha_1, \alpha_2, \alpha_3}^{(\pm \beta \cdots \cdots}(r_1, r_2, P_N). \quad (2.9)\)
The irreducible composite hadrons are summarized in Tables II and IV, respectively, for $qq$-mesons and $qqq$-baryons. Here it is to be noted that the intrinsic spin-freedom for totality of BW spinors of $q\bar{q}$ mesons is $4 \times 4 = 16$, four times of $2 \times 2 = 4$ for boosted Pauli spinors. Correspondingly new types of BW spinors appear. For $qqq$ baryons $G(P)$ and $F(P)$, which include 1 and 2 negative energy Dirac components, respectively, appear in addition to the conventional $E(P)$ (with all positive energy Dirac components), boosted multi-Pauli spinor; and also to be noted that, although the BW equation with a definite mass itself is not $\tilde{U}_{DS}^{(4)}$ symmetric, the Klein-Gordon equation with a Lorentz-scalar mass-squared is generally invariant.

(Space-time WF) The internal WF is, concerning the relative space-time freedom, expanded in terms of the complete set of covariant oscillator eigen-functions, where (by applying a Lorentz-invariant subsidiary condition\textsuperscript{24}) to “freeze” the relative-time freedom) the general symmetry $O(3,1)$ of the original $\mathcal{M}^2_{\text{conf}}$ is reduced into the non-relativistic $O(3)$ symmetry.

Here it is noteworthy that the above choice of bases is desirable from the phenomenological facts i) that the constituent quark inside of hadrons behaves like a free Dirac-particle\textsuperscript{*}) (implied by BW-spinors) and ii) that in the global structure of hadron spectra (Regge trajectory and so on) is well described by the corresponding oscillator potential.

(Transformation rule for hadrons and chiral symmetry) By using the covariant quark representation of composite hadrons given above we can derive automatically their rule for any (relativistic) symmetry transformation from that of constituent quarks. The physical meaning of chiral transformation is clearly seen from the operations of its infinitesimal generator on the respective constituent quark spinors:

$u(P) \rightarrow u'(P) = -\gamma_5 u(P) = u(-P); \quad v(P) \rightarrow v'(P) = -\gamma_5 v(P) = v(-P)$. That is, the chiral transformation transforms the members of relevant BW-spinors with each other. Accordingly, if $\mathcal{M}^2$ operator is a Lorentz-scalar and independent of Dirac indices, the hadron mass spectra have effectively the $\tilde{U}(4)$ symmetry and accordingly, also the chiral symmetry. Here it is to be stressed that this is simply a phenomenological assumption. An intention is in this work not to treat a dynamical problem from a conventional composite picture, but is to propose a kinematical framework for describing composite hadrons covariantly. The validity of the above assumption is checked only by comparing its predictions with experimental and phenomenological facts.

§3. Level structure of mesons and baryons

(Assignment of mesons and baryons into $\tilde{U}_{SF}(12) \otimes O(3,1)$ scheme) We assign the light-quark ground state mesons and baryons to the representations $(12 \times 12^*) = 144$ and $(12 \times 12 \times 12)_{\text{Symm}} = 364$, respectively.

In the extended version (old version) of COQM the confining force is assumed

\textsuperscript{*}) The BW-spinors with total hadron momentum $P_\mu$ and $M$ are easily shown to be equivalent to the product of free Dirac spinors of the respective constituent “exciton-quarks” with momentum $p_{N,\mu}^{(i)} \equiv \kappa^{(i)} P_{N,\mu}$ and mass $m_{N}^{(i)} \equiv \kappa^{(i)} M_{N}$ \textsuperscript{(}\textit{\sum} $i \kappa^{(i)} = 1$\textsuperscript{)}. \textsuperscript{24}}
to be Lorentz-scalar (“boosted-spin” independent) and the mass spectra have the \(\tilde{U}(12)_{\text{SF}}\) symmetry (boosted \(SU(6)_{\text{SF}}\) symmetry). Thus, the WF of hadrons in the new level-classification scheme become the tensors in the \(O(3,1)_{\text{Lorentz}} \otimes \tilde{U}(4)_{\text{D.S.}} \otimes SU(3)_{\text{F}}-\text{space}(\), being extended from the ones in the \(O(3) \otimes SU(2)_{\text{P.S.}} \otimes SU(3)_{\text{F}}\) space of NRQM). The numbers of freedom of spin-flavor WF in NRQM are \(6 \times 6^* = 36\) for mesons and \((6 \times 6 \times 6)_{\text{Symm.}} = 56\) for baryons: These numbers in COQM become \(12 \times 12^* = 144\) for mesons and \((12 \times 12 \times 12)_{\text{Symm.}} = 364 = 182\) (for baryons) for anti-baryons.

Inclusion of heavy quarks is straightforward: The WF of general \(q\) and/or \(Q\) hadrons become tensors in \(O(3,1) \otimes [\tilde{U}(4)_{\text{P.S.}} \otimes SU(3)_{\text{F}}]_q \otimes [SU(2)_{\text{P.S.}} \otimes U(1)_{\text{F}}]_Q\). (Level structure of ground state mesons)\(^{25}\) In Table II we have summarized the properties of ground state mesons in the light and/or heavy quark systems. It is remarkable that there appear new multiplets of the scalar and axial-vector mesons in the \(q\bar{q}\) and \(Q\bar{Q}\) systems and that in the \(q\bar{q}\) systems the two sets (Normal and Extra) of pseudo-scalar and of vector meson nonets exist. The \(\pi\) nonet (\(\rho\) nonet) is assigned to the \(P^{(N)}_s\) \((V^{(N)}_\mu)\) state. We call the new type of particles in the extended COQM (which have never appeared in NRQM) as “chiralons”.

| mass     | Approx. Symm. | Spin WF               | \(SU(3)\) | Meson Type          |
|----------|---------------|-----------------------|-----------|---------------------|
| \(QQ\)   | \(m_Q + m_Q\) | \(L\)S symm.          | \(u_Q(P)\bar{v}^Q(P)\) | \(1\) | \(P_s, V_\mu\) |
| \(q\bar{q}\ | \(m_q + m_{\bar{q}}\) | \(q\)-Chiral Symm.    | \(u_q(P)\bar{v}^q(P)\) | \(3\) | \(P_s, V_\mu\) |
| \(Q\bar{q}\ | \(m_Q + m_{\bar{q}}\) | \(Q\)-Heavy Q. Symm.  | \(u_Q(-P)\bar{v}^Q(P)\) | \(3\) | \(S, A_\mu\) |
| \(q\bar{q}\ | \(m_q + m_{\bar{q}}\) | \(q\)-Chiral Symm.    | \(u_q(P)\bar{v}^q(P)\) | \(3\) | \(S, A_\mu\) |
| \(q\bar{q}\ | \(m_q + m_{\bar{q}}\) | Chiral Symm.          | \(\frac{1}{\sqrt{2}}(u(P)\bar{v}(P) \pm u(-P)\bar{v}(-P))\) | \(9\) | \(P_s^{(N,E)}, V^{(N,E)}_\mu\) |
|          |               |                       | \(\frac{1}{\sqrt{2}}(u(P)\bar{v}(-P) \pm u(-P)\bar{v}(P))\) | \(9\) | \(S^{(N,E)}, A^{(N,E)}_\mu\) |

Table II. Level structure of ground-state mesons

(Level structure of mesons in general)\(^{25}\) The global mass spectra of the ground and excited state mesons are given by

\[
M_N^2 = M_0^2 + N\Omega = m_N^{(1)} + m_N^{(2)}. \tag{3.1}
\]

Their quantum numbers are given in Table III. Here it is to be noted that some chiralons have the “exotic” quantum numbers from the conventional NRQM viewpoint.

| \((q\bar{q})\) | \(P\) | \(C\) | \(N\) | \((q\bar{Q} \text{ or } q\bar{q})\) | \(P\) | \(N\) |
|---------------|------|------|------|-----------------|------|------|
| \(P^{(N)}_s\)\(|V^{(N)}_\mu\) | \(\otimes\) \(|L, N\) | \((-1)^{L+1}\) | \((-1)^{L+S}\) | \(\text{all}\) | \(P_s, V_\mu\) \(\otimes\) \(|L, N\) | \((-1)^{L+1}\) | \(\text{all}\) |
| \(P^{(E)}_s\)\(|V^{(E)}_\mu\) \(|L, N\) | \((-1)^{L+1}\) | \((-1)^{L+S}\) | \(0, 1\) | \(S, A_\mu\) \(\otimes\) \(|L, N\) | \((-1)^{L}\) | \(0, 1\) |
| \(S^{(N)}_s\)\(|A^{(N)}_\mu\) \(|L, N\) | \((-1)^L\) | \((-1)^L\) | \(0, 1\) | \(q\bar{Q}\) \(|L, N\) | \((-1)^{L+1}\) | \(N\) |
| \(S^{(E)}_s\)\(|A^{(E)}_\mu\) \(|L, N\) | \((-1)^L\) | \((-1)^{L+1}\) | \(0, 1\) | \(P_s, V_\mu\) \(\otimes\) \(|L, N\) | \((-1)^{L+1}\) | \(\text{all}\) |

Table III. Level structure of Mesons in general: We are able to infer\(^{26}\) that the chiral symmetry concerning the light quarks is valid (still effective) for the ground (first excited) state of \(n\bar{n}\) and \(n\bar{Q}\) meson systems, while the symmetry will prove invalid from the \(N\)-th \((N \geq 2)\) excited hadrons.

(Level Structure of Baryons) The baryon WF in Eq. (2.1) should be full-symmetric
(except for the color freedom) under exchange of constituent quarks: The full-symmetric total WF in the extended scheme is obtained, in the following three ways, as a product of the sub-space WF with respective symmetric properties:

\[
|\rho F\sigma_S = |\rho_S| F\sigma_S ; \quad \frac{1}{\sqrt{2}}(|\rho\alpha_\alpha| F\sigma_{\alpha} + |\rho\beta| F\sigma_{\beta}) ; \quad |F_A\rho\sigma_A ;
\]

where \(\rho \otimes \sigma = \gamma\) is the conventional two, \(\rho\) and \(\sigma\) spin, 2 by 2 matrix representation of the 4 by 4 Dirac matrix. \(|S\rangle, |\alpha(\beta)\rangle\) and \(|A\rangle\) mean the full-symmetric, \(\alpha(\beta)\)-type partial symmetric and full anti-symmetric subspace WF, respectively. The intrinsic parity operation is given by \(\hat{P} = \Pi_{i=1}^{3}\gamma_{(i)}\), that is, the parity of \((E^+, G^+, F^+)\) BW spinors are \((+, -, +)\) and those of \((E^-, G^-, F^-)\) BW spinors are \((- , +, -)\).

The symmetry properties of ground state light-quark baryon WF and their level structures thus determined are summarized in Table IV.

| \(W^{(+)}\) | spin-flavor wave function | \(B^{D}\) | static | \(SU(6)\) |
|-----------------|-----------------|-----------|---------|---------|
| \(E^{(+)}:\) | \(|\rho_S| F\sigma_S = |\rho_S| F\sigma_S\) | \(\Delta_{3/2}^{\Omega}\) | \(10 \times 4 = 40\) | \(\mathbf{56}\) |
| | | \(\rho_S\frac{1}{\sqrt{2}((F)_{\alpha\alpha}|\sigma_{\alpha} + |F)_{\beta\beta}|\sigma_{\beta})\) | \(N_{1/2}^{\Omega}\) | \(8 \times 2 = 16\) | | |
| \(G^{(+)}:\) | \(\frac{1}{\sqrt{2}((F)_{\alpha\alpha}|\sigma_{\alpha} + |F)_{\beta\beta}|\sigma_{\beta})\) \(\frac{1}{\sqrt{2}((F)_{\alpha\alpha}|\sigma_{\alpha} + |F)_{\beta\beta}|\sigma_{\beta})\) | \(\Delta_{3/2}^{\Omega}\) | \(10 \times 4 = 20\) | | |
| | \(\frac{1}{\sqrt{2}((F)_{\alpha\alpha}|\sigma_{\alpha} + |F)_{\beta\beta}|\sigma_{\beta})\) | \(N_{3/2}^{\Omega}\) | \(8 \times 4 = 32\) | | |
| | \(\frac{1}{\sqrt{2}((F)_{\alpha\alpha}|\sigma_{\alpha} + |F)_{\beta\beta}|\sigma_{\beta})\) | \(N_{1/2}^{\Omega}\) | \(8 \times 2 = 16\) | | |
| | \(\frac{1}{\sqrt{2}((F)_{\alpha\alpha}|\sigma_{\alpha} + |F)_{\beta\beta}|\sigma_{\beta})\) | \(A_{1/2}^{\Omega}\) | \(1 \times 2 = 2\) | \(70\) | |
| \(F^{(+)}:\) | \(|\rho_S| F\sigma_S = |\rho_S| F\sigma_S\) | \(\Delta_{3/2}^{\Omega}\) | \(10 \times 4 = 40\) | | |
| | | \(\rho_S\frac{1}{\sqrt{2}((F)_{\alpha\alpha}|\sigma_{\alpha} + |F)_{\beta\beta}|\sigma_{\beta})\) | \(N_{3/2}^{\Omega}\) | \(8 \times 2 = 16\) | \(\mathbf{56}'\) | |

Table IV. Level structure of ground-state qqQ-baryon: \(12H_3 / 2 = 364 / 2 = 182\).

Here it is remarkable that there appear chiralons in the ground states. That is, the extra positive parity \(\mathbf{56}'\)-multiplet of the static \(SU(6)\) and the extra negative parity \(\mathbf{70}\)-multiplet of the \(SU(6)\) in the low mass region. It is also to be noted that the chiralons in the first excited states are expected to exist. The above consideration on the light-quark baryons are extended directly to the general light and/or heavy quark baryon systems: The chiralons are expected to exist also in the \(qqQ\) and \(qQQ\)-baryons, while no chiralons in the \(QQQ\) system.

§4. Experimental Candidates for Chiral Particles

In our level-classification scheme a series of new type of multiplets of the particles, chiralons, are predicted to exist in the ground and the first excited states of \(qq\) and \(qQ\) or \(Q\bar{q}\) meson systems and of \(qqQ\), \(qQQ\) and \(qQQ\)-baryon systems. Presently we can give only a few experimental candidates or indications for them: \(\langle qq\text{-mesons} \rangle\)

One of the most important candidates is the scalar \(\sigma\) nonet to be assigned as \(S^{(N)}(1S_0)\) : \(|\sigma(600)\rangle, |\kappa(900)\rangle, |\alpha_0(980)\rangle, |f_0(980)\rangle\). The existence of \(\sigma(600)\) seems to be established through the analyses of, especially, \(\pi\pi\)-production processes. A firm experimental evidence \(27\) for \(\kappa(800\text{–}900)\) through the decay process \(28\) \(D^+ \rightarrow K^-\pi^+\pi^+\) was reported recently at the conference, Hadron2001.

In our scheme respective two sets of \(P_s\)- and of \(V_{\mu}\)-nonets, to be assigned as \(P_s^{(N,E)}(1S_0)\)
and \( V_\mu^{(N,E)}(3S_1) \), are to exist: Out of the five vector mesons (stressed\(^{29}\)) as problems with vector mesons, \([\rho'(1450), \rho'(1700), \omega'(1420), \omega'(1600), \phi(1690)]\), the lower mass \( \rho'(1450) \) and \( \omega'(1420) \), and the \( \phi(1690) \) are naturally able to be assigned as the members of \( V_\mu^{(E)} \)-nonets.

Out of the three established \( \eta, [\eta(1295), \eta(1420), \eta(1460)] \) at least one extra, plausibly \( \eta(1295) \) with the lowest mass, may belong to \( F_s^{(E)}(1S_0) \) nonet.

Recently the existence of two “exotic” particles \( \pi_1(1400) \) and \( \pi_1(1600) \) with \( J^{PC} = 1^{-+} \) and \( I = 1 \), observed\(^{30}\) in the \( \eta \pi, \rho \pi \) and other channels, is attracting strong interests among us. These exotic particles with a mass around 1.5GeV may be naturally assigned as the first excited states \( S^{(E)}(1P_1) \) and \( A_\mu^{(E)}(3P_1) \) of the chiralons. \( (g\bar{Q} \text{ or } Q\bar{q}-\text{mesons}) \) Recently we have shown at the conference some indications for existence of the two chiralons in \( D- \) and \( B-\text{meson systems} \)\(^{31} \) and \( ^{32} \) obtained through analyses of the \( \Upsilon(4S) \) or \( Z^0 \) decay processes, respectively.

\[
\begin{align*}
D_1^+ &= A_\mu(3S_1), \quad J^P = 1^+ \quad \text{in} \quad D_1^+ \to D^* + \pi, \\
B_0^0 &= S(1S_0), \quad J^P = 0^+ \quad \text{in} \quad B_0^0 \to B + \pi.
\end{align*}
\]

\( (q\bar{q}-\text{baryons}) \) The two facts have been a longstanding problem that the Roper resonance \( N(1440)_{1/2^+} \) is too light to be assigned as radial excitation of \( N(939) \) and that \( A(1405)_{1/2^-} \) is too light as the \( L = 1 \) excited state of \( A(1116) \). In our new scheme they are reasonably assigned to the members of ground state chiralons with \([SU(6), SU(3), J^P]\), respectively, as

\[ N(1440)_{1/2^+} = F(56', 8, 1/2^+), \quad A(1405)_{1/2^-} = G(70, 1, 1/2^-). \]

The particle \( \Delta(1600)_{3/2^+} \) which is lighter than \( \Delta(1620)_{1/2^-} \) may also belong to the extra \( 56' \) of the ground state chiralons. This situation is shown in Table V.

| \( SU(6) \) | \( SU(3), J^P \) | \( SU(3), J^P \) |
|---|---|---|
| 56 | 8, 1/2 | N(939), A(1116), \( \Sigma(1192), \Sigma(1318) \) |
| 56' | 8, 1/2 | N(1440), \( \Sigma(1660) \) |
| 70 | 8, 1/2 | N(1535) |
| 1, 1/2 | \( A(1405) \) |

Table V. Assignment of \( q\bar{q}-\text{baryons} \): The baryons in the boxes are candidates of chiralons.

§5. Concluding Remarks

We have presented an attempt for Level-classification scheme unifying the seemingly contradictory two viewpoints, Non-relativistic one with \( LS \)-symmetry and Relativistic one with Chiral symmetry \( \text{.} \) As results, We have predicted the existence of New Chiral Particles in the lower mass regions “Chiralons”, which had never been appeared in NRQM. We have several good candidates for chiralons, for example, \( \sigma \)-nonet \( \{ \sigma(600), \kappa(800), a_0(980), f_0(980) \} \) as “Relativistic” \( S \)-wave states of \( q\bar{q} \); \( \pi_1(1400) \) and \( \pi_1(1600) \) with \( J^{PC} = 1^{-+} \) as “Relativistic” \( P \)-wave states of \( q\bar{q} \); Roper resonance \( N(1440)_{1/2^+} \) and \( SU(3) \) singlet \( A(1405)_{1/2^-} \) as “Relativistic” \( S \)-wave states of \( q\bar{q} \).

Further search for chiralons, both experimental and theoretical, is urgently required for developing hadron physics.
$\hat{U}(12)$ a new symmetry possibly realizing in hadron spectroscopy

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