Manifestly N=3 Supersymmetric Euler-Heisenberg Action in Light-Cone Superspace

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Abstract

We find a manifestly N=3 supersymmetric generalization of the four-dimensional Euler-Heisenberg (four-derivative, or $F^4$) part of the Born-Infeld action in light-cone gauge, by using N=3 light-cone superspace.
1 Introduction

The Born-Infeld (BI) action in flat spacetime, \[ S_{\text{BI}} = \frac{1}{b^2} \int d^4x \left\{ 1 - \sqrt{-\det(\eta_{\mu\nu} + bF_{\mu\nu})} \right\}, \tag{1.1} \]
is the particular non-linear generalization of Maxwell theory, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \) The action (1.1) was initially introduced to regularize both the electric field and the self-energy of a point-like charge in electrodynamics \[. \] Much later, the BI action was recognized as the leading contribution to the effective action of open strings in an abelian background with constant field strength \( F \) \[, \] and as the essential part of the D3-brane action as well \[, \] with \( b = 2\pi\alpha'. \) The action (1.1) has many remarkable properties, e.g., causal propagation and electric-magnetic duality \[. \]

The BI Lagrangian can be rewritten to the form
\[ L = -\frac{1}{2} p^{\mu\nu} F_{\mu\nu} + H(P, Q), \tag{1.2} \]
where the auxiliary antisymmetric tensor \( p_{\mu\nu} \) and the BI structure function
\[ H(P, Q) = \frac{1}{b^2} \left( 1 - \sqrt{1 - 2b^2 P + b^4 Q^2} \right), \tag{1.3} \]
as well as the definitions
\[ P = \frac{1}{4} p_{\mu\nu} p^{\mu\nu}, \quad Q = \frac{i}{4} p_{\mu\nu} \tilde{p}^{\mu\nu}, \quad \tilde{p}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} p_{\rho\sigma}, \tag{1.4} \]
have been introduced. Eliminating \( p_{\mu\nu} \) from eq. (1.2) results in the equivalent Lagrangian
\[ L = \frac{1}{b^2} \left[ 1 - \sqrt{1 + \frac{b^2}{2} F^2 - \frac{b^4}{16} (F \tilde{F})^2} \right], \tag{1.5} \]
where we have defined \( F^2 = F^{\mu\nu} F_{\mu\nu}, \ \tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \) and \( F \tilde{F} = F^{\mu\nu} \tilde{F}_{\mu\nu}. \)

Supersymmetric generalizations of the BI action are of particular interest in connection to superstring theory (see ref. \[\] for a recent review). The super-BI actions describing D-branes can be naturally interpreted as the Goldstone-type actions associated with partial supersymmetry breaking, while they can still be duality invariant too. The manifestly N=1 supersymmetric generalization of the four-dimensional BI action in N=1 superspace was discovered long time ago \[\] (see also ref. \[\]), while its manifestly N=2 supersymmetric generalization in N=2 superspace was found only recently \[\] (see ref. \[\] too). To our knowledge, the higher (N=3 or N=4) manifestly

\[^2\text{We use } \eta_{\mu\nu} = \text{diag}(+,-,-,-) \text{ and } \hbar = c = 1.\]
supersymmetric generalizations of the four-dimensional bosonic BI action (1.1) are not known in any form.

Supersymmetry apparently prefers the parametrization of the BI action in terms of the Maxwell term $L_2 = -\frac{1}{4} F^2$ and the Maxwell stress-energy tensor squared $L_4$.

$$L_4 = \frac{1}{32} \left\{ (F^2)^2 + (F\tilde{F})^2 \right\} = \frac{1}{8} (F^+)^2 (F^-)^2, \quad F^\pm_{\mu\nu} = \frac{1}{2} \left( F_{\mu\nu} \pm i\tilde{F}_{\mu\nu} \right).$$ (1.6)

This term is known as the Euler-Heisenberg (EH) Lagrangian [11]. The EH action also appears as the bosonic part of the one-loop effective action in N=1 supersymmetric scalar electrodynamics with the parameter $b^{-1} = 2\sqrt{6}\pi m^2/e^2$. One easily finds that

$$L_{BI} = \frac{1}{b^2} \left\{ 1 - \sqrt{(1 - b^2 L_2)^2 - 2b^4 L_4} \right\} = L_2 + b^2 L_4 + O(F^6).$$ (1.7)

A manifestly N=4 supersymmetric generalization of the BI action is known to be the formidable problem, though it is highly desirable, e.g., for an investigation of quantum properties of D3-branes and their comparison to supergravity [12, 13]. Even the $N > 2$ supersymmetrization of the EH-term $L_4$, representing the four-derivative terms $(F^4)$, is non-trivial. The additional terms with four derivatives in the N=4 BI action were determined in ref. [14] in N=1 superspace, by imposing the $SU(3)$ internal symmetry on three N=1 chiral multiplets extending an N=1 (abelian) vector multiplet to an N=4 vector multiplet, with manifest (linearly realised) N=1 off-shell supersymmetry. The manifestly N=2 supersymmetric form of the N=4 EH action was derived in ref. [15] in N=2 projective superspace, while its equations of motion can also be written in terms of on-shell N=4 superfields in harmonic superspace [16]. It is the purpose of this Letter to write down an off-shell, manifestly N=3 supersymmetric formulation of the N=4 EH action in N=3 light-cone superspace.

Our paper is organized as follows. In sect. 2 we introduce a light-cone gauge and rewrite the EH Lagrangian in terms of physical (transverse) degrees of freedom up to the relevant order. In sect. 3 we introduce N=3 light-cone superspace and deduce an N=3 supersymmetric generalization of the EH action in terms of a single N=3 light-cone superfield. The obstructions encountered in our efforts to find a similar, manifestly N=4 supersymmetric EH action in N=4 light-cone superspace are discussed in Conclusion (sect. 4).

## 2 EH action in light-cone gauge

The light-cone formulation of a gauge theory (in light-cone gauge) keeps only physical (transverse) degrees of freedom in the field theory by giving up its manifest
Lorentz invariance. The light-cone formulation is, therefore, very suitable for an off-shell formulation of N-extended supersymmetric gauge field theories with manifest supersymmetry in N-extended light-cone superspace [17, 18, 19].

We define light-cone coordinates in Minkowski spacetime as

\[ x^+ = \frac{1}{\sqrt{2}} (x^0 + x^3), \quad x^- = \frac{1}{\sqrt{2}} (x^0 - x^3), \]
\[ x = \frac{1}{\sqrt{2}} (x^1 + ix^2), \quad \bar{x} = \frac{1}{\sqrt{2}} (x^1 - ix^2), \]

and similarly for the gauge vector field, \( A_\mu \rightarrow (A^+, A^-, A, \bar{A}) \). The real coordinate \( x^+ \) is going to be considered as ‘light-cone time’. The linear transformation (2.1) of spacetime coordinates is obviously non-singular (with the Jacobian equal to \( i \)), while it does not preserve the Minkowski metric (i.e. it is not a Lorentz-transformation).

The light-cone gauge reads

\[ A^+ = 0 . \] (2.2)

In this (physical) gauge the \( A^- \) component of the gauge field \( A_\mu \) is supposed to be eliminated via its (non-dynamical) equation of motion, whereas the transverse components \( (A, \bar{A}) \) are supposed to represent the physical propagating fields.

It is easy to solve the equation of motion for \( A^- \) in the Maxwell theory, where it takes the form of a linear equation in the light-cone gauge (cf. refs. [17, 18, 19]). It becomes, however, a highly non-trivial problem in the BI or EH theory, where it takes the form of a non-linear partial differential equation. The equations of motion amount to the conservation law for the \( p^- \)-tensor,

\[ \partial^\mu p_{\mu\nu} = 0 , \] (2.3)

while the \( p_{\mu\nu} \) in the BI theory is given by

\[ p_{\mu\nu} = \frac{b^2 F_{\mu\nu} - \frac{b^4}{4} (F \tilde{F}) \tilde{F}_{\mu\nu}}{\sqrt{1 + \frac{b^2}{2} F^2 - \frac{b^4}{16} (F \tilde{F})^2}} . \] (2.4)

By the use of the Bianchi identity, \( \partial^\mu \tilde{F}_{\mu\nu} = 0 \), we find the following equation for \( A^- \):

\[ \partial^\mu F^-_- = b^2 \left\{ -\frac{1}{2} \partial^\mu F^-_- F^2 + \frac{1}{4} \tilde{F}_{\mu\nu} \partial^\mu (F \tilde{F}) + \frac{1}{4} F_{\mu\nu} \partial^\mu F^2 \right\} \]
\[ + b^4 \left\{ -\frac{1}{16} (F \tilde{F}) \tilde{F}_{\mu\nu} \partial^\mu F^2 + \frac{1}{16} \partial^\mu F_{\mu\nu} (F \tilde{F})^2 \right\} + \frac{1}{32} \tilde{F}_{\mu\nu} \partial^\mu (F \tilde{F})^2 - \frac{1}{32} F_{\mu\nu} \partial^\mu (F \tilde{F})^2 \] . (2.5)
We use a perturbative Ansatz, in powers of the small parameter $b^2$, for a solution to eq. (2.5),

$$A^-(x) = \sum_{n=0}^{\infty} b^{2n} A_{(2n)}^-(x) .$$

As regards the leading and sub-leading terms, we find

$$A^-(0) = \partial + (\bar{\partial} A + \partial \bar{A}) ,$$

$$A^-(2) = \frac{1}{(\partial^+)^2} \left[ \frac{1}{2} \partial^\mu F_{\mu-} F^2 + \frac{i}{2} \bar{F}_{\mu-} \partial^\mu (F \bar{F}) + \frac{1}{4} F_{\mu-} \partial^\mu F^2 \right] \bigg|_{A^- = A^-(0)} ,$$

where we have used the notation $\partial^+ = \partial / \partial x^-$.

According to eq. (1.7), the EH term $L_4$ is the leading $b^2$-correction to the Maxwell term $L_2$ in the BI theory. The light-cone formulation of the BI Lagrangian in the same approximation is thus given by the terms written down on the right-hand-side of eq. (1.7) after a substitution of eq. (2.2) and the first line of eq. (2.8). After some algebra and partial integration we find

$$L[A, \bar{A}] = -\frac{1}{4} F^2 + \frac{b^2}{8} (F^+)^2 (F^-)^2$$

$$= -A \Box \bar{A} + 2b^2 \left[ (\partial \bar{A})^2 + \partial^+ \bar{A} \frac{\partial^2}{2 \partial^+} A - \partial^+ A \frac{\partial^2}{\partial^+} \bar{A} \right]^2 + O(b^4) ,$$

where we have used the notation $\partial = \partial / \partial x$ and $\bar{\partial} = \partial / \partial \bar{x}$. Eq. (2.8) can be thought of as the light-cone EH Lagrangian. Its N=3 supersymmetrization is discussed in the next sect. 3.

3 N=3 light-cone superspace action

The light-cone N=3 supersymmetry algebra reads

$$\{Q^m, \bar{Q}_n\} = -\sqrt{2} \delta^m_n P^+ , \quad m, n = 1, 2, 3 ,$$

where the supersymmetry charges $Q^m$ transform in the fundamental representation of $SU(3)$. A natural representation of the algebra (3.1) in N=3 light-cone superspace $Z = (x^\mu, \theta^m, \bar{\theta}_n)$ is given by

$$Q^m = -\frac{\partial}{\partial \theta^m} + \frac{i}{\sqrt{2}} \theta^m \partial^+ ,$$

$$\bar{Q}_n = \frac{\partial}{\partial \bar{\theta}_n} - \frac{i}{\sqrt{2}} \bar{\theta}_n \partial^+ .$$
The covariant derivatives in $N=3$ light-cone superspace are
\[ D^m = -\frac{\partial}{\partial \theta_m} - \frac{i}{\sqrt{2}}\theta^m \partial^+ , \]
\[ \bar{D}_n = \frac{\partial}{\partial \bar{\theta}^n} + \frac{i}{\sqrt{2}}\bar{\theta}^n \partial^+ . \] (3.3)

They anticommute with the supersymmetry charges (3.2) and obey the same algebra (3.1). The irreducible off-shell representations of $N=3$ light-cone supersymmetry are easily obtained by imposing the covariant chirality condition on $N=3$ light-cone superfields $\phi(Z)$,
\[ D^m \phi(Z) = 0 . \] (3.4)

A solution to eq. (3.4) in components is just given by an arbitrary complex function $\phi(x^+, x^- + \frac{i}{\sqrt{2}}\theta^m \bar{\theta}_m, x, \bar{x}; \theta^n) \equiv \phi(y; \theta)$. Its expansion in the chiral superspace reads
\[ \phi(y; \theta) = \frac{1}{\partial^+} A(y) + \frac{i}{\partial^+} \theta^m \bar{\chi}_m(y) + \frac{i}{2} \theta^m \theta^n \varepsilon_{mnp} C^p(y) + \frac{1}{3!} \varepsilon_{mnp} \theta^m \theta^n \theta^p \psi(y) . \] (3.5)

The light-cone $N=3$ supersymmetry transformation laws for the components are
\[ \delta A = i\varepsilon^n \bar{\chi}_n , \]
\[ \delta \bar{\chi}_m = \sqrt{2}\varepsilon_m \partial^+ A + \varepsilon_{mnp} \varepsilon^n \partial^+ C^p , \]
\[ \delta C^p = -i\sqrt{2}\varepsilon^{pq} \varepsilon_q \bar{\chi}_r - i\varepsilon^p \psi , \]
\[ \delta \psi = -\sqrt{2}\varepsilon_n \partial^+ C^n , \] (3.6)

where $(\varepsilon^n, \bar{\varepsilon}_m)$ are the infinitesimal anticommuting parameters.

All our field components have canonical dimensions. The complex field $A$ can be identified with the physical (translational) vector field components, the spinors $\bar{\chi}_m$ in the fundamental representation $3$ of $SU(3)$ with a triplet of photinos, the singlet spinor $\psi$ with extra photino, and the complex triplet $C^m$ with Higgs fields in $3$ of $SU(3)$. The physical content thus coincides with that of the $N=4$ supersymmetric abelian vector multiplet having a single photon field, photinos in the fundamental representation $4$ of $SU(4)$ and Higgs fields in real $6$ of $SU(4)$, after their decomposition with respect to the $SU(3)$ subgroup of the internal symmetry $SU(4)$. This is the manifestation of the well-known fact that $N=3$ and $N=4$ supersymmetric vector multiplets are physically equivalent.

It is now straightforward (though very tedious) to find the $N=3$ supersymmetric generalization of the bosonic EH light-cone action (2.8) in $N=3$ light-cone superspace,
\[ S = \int d^4x d^3\theta d^3\bar{\theta} \mathcal{L}(\phi, \bar{\phi}) = -\int d^4x (D)^3(\bar{D})^3 \mathcal{L}(\phi, \bar{\phi}) , \] (3.7)
where \((D)^3 = \varepsilon_{mnp} D^m D^n D^p\) and similarly for \((\bar{D})^3\). After some trials and errors, we find

\[
36(-i\sqrt{2})^3 L(\phi, \bar{\phi}) = -\phi \frac{\partial}{\partial \phi} \phi + 2b^2 \left\{ \frac{1}{\partial^+ \partial^-} \left( \partial \partial^+ \phi \bar{\partial} \partial^+ \phi \right) (\partial \partial^+ \bar{\phi})^2 \right. \\
+ \frac{1}{\partial^+ \partial^-} \left( \partial^+ \phi \partial^+ \phi \right) \partial^+ \bar{\partial} \partial^+ \phi + \frac{1}{2 \partial^+ \partial^-} (\partial \partial^+ \phi) (\partial \partial^+ \bar{\phi}) \bar{\partial} \phi \\
+ \frac{1}{4 \partial^+ \partial^-} \left( \partial^+ \phi \partial^- \phi \right) \partial^+ \bar{\partial} \partial^- \phi - \frac{1}{\partial^+ \partial^-} \left( \partial \partial^+ \phi \partial^+ \phi \right) (\partial \partial^+ \bar{\phi})^2 \\
- \frac{1}{2 \partial^+ \partial^-} \left( \partial^+ \phi \partial^+ \bar{\partial} \partial^+ \phi \right) \partial^+ \bar{\partial} \partial^+ \bar{\phi} - \frac{1}{2 \partial^+ \partial^-} (\partial \partial^+ \phi) \partial^+ \bar{\partial} \partial^+ \bar{\phi} \\
+ \frac{1}{2 \partial^+ \partial^-} \left( \partial \partial^+ \phi \partial^+ \bar{\partial} \partial^+ \phi \right) \partial^+ \bar{\partial} \partial^+ \bar{\phi} + \frac{1}{2 \partial^+ \partial^-} (\partial \partial^+ \phi) \partial^+ \bar{\partial} \partial^+ \bar{\phi} \right\} .
\]

The bosonic part of this action is given by

\[
\mathcal{L}_{\text{bos.}} = -A \frac{\partial}{\partial A} + 2b^2 \left| (\partial A)^2 + \partial^+ A \frac{\partial}{\partial A} A - \partial^+ A \frac{\partial^2}{\partial A^2} A \right|^2 \\
- \frac{1}{2} C^p \frac{\partial}{\partial A} C_p - 2b^2 \left\{ \frac{2}{\partial^+ \partial^-} \left( \partial \partial^+ C^m \bar{\partial} \partial^+ A \right) \left( \partial \partial^+ \bar{C}_m \partial \partial^+ \bar{A} \right) \\
+ \frac{1}{2 \partial^+ \partial^-} \left( \partial^+ \partial^+ C^p \partial^2 \bar{A} + \partial^2 \partial^+ C^p \partial^2 \bar{A} \right) \left( \partial^+ \bar{C}_p \partial^2 \bar{A} + \partial^2 \bar{C}_p \partial^2 \bar{A} \right) \\
+ \frac{1}{8 \partial^+ \partial^-} \left( \partial^+ \partial^+ C^p \frac{\partial}{\partial A} \right) \left( \partial^+ \bar{C}_p \frac{\partial}{\partial A} \right) \left( \partial^+ \bar{C}_p \frac{\partial}{\partial A} \right) + \frac{1}{4} C^p \left( 2 \partial \partial^+ C_p \partial \partial^+ \bar{A} \frac{\partial}{\partial A} + \bar{C}_p \partial \partial^+ \bar{A} \partial \partial^+ \bar{A} \right) \right\} \\
- \frac{1}{\partial^+ \partial^-} \left( \partial^+ \partial^+ C^p \partial^2 \bar{A} + \partial^2 \partial^+ C^p \partial^2 \bar{A} \right) \partial \partial^+ \bar{C}_p \partial \partial^+ \bar{A} \\
- \frac{1}{4 \partial^+ \partial^-} \left( \partial^+ \partial^+ C^p \frac{\partial}{\partial A} \right) \left( \partial^+ \bar{C}_p \frac{\partial}{\partial A} \right) \left( \partial^+ \bar{C}_p \frac{\partial}{\partial A} \right) + \text{h.c.} \right\} .
\]

One of the obvious features of both eqs. (3.8) and (3.9) is the apparent presence of higher derivatives, as may have been expected from the experience with the manifestly N=2 supersymmetric generalization of the BI action in the covariant N=2 superspace \([6]\). The expected correspondence to the component D3-brane effective action having non-linearly realized extended supersymmetry and no higher derivatives implies the existence of a field redefinition that would eliminate the higher-derivative terms in our action and make its N=3 supersymmetry to be non-linearly realised (i.e non-manifest) \([3]\). We also note the absence of quartic \((C^4)\) scalar terms and the on-shell \((\frac{\partial}{\partial A} = \frac{\partial}{\partial C} = 0)\) invariance of our action under constant shifts, \(C_p(x) \rightarrow C_p(x) + c_p\), which are supposed to be related to the possible interpretation of the \(C_p\) fields as the Goldstone scalars associated with spontaneously broken translations in the full N=3 BI action.
4 Conclusion

Our main results are given by eqs. (2.8), (3.8) and (3.9). Our initial motivation was to construct an N=4 supersymmetric generalization of the EH action in the light-cone gauge. The N=4 light-cone supersymmetry algebra is given by eq. (3.1), where the indices now take four values. Equations (3.2), (3.3) and (3.4) are still valid in N=4 light-cone superspace, where they have to supplemented by an extra (generalized reality) condition [17],

$$D^m D^n \bar{\phi} = \frac{1}{2} \varepsilon^{mnpq} \bar{D}^p \bar{D}^q \phi,$$

or, equivalently,

$$\phi = \frac{1}{48 \partial^2 + 2 \varepsilon^{mnpq} \bar{D}_m \bar{D}_n \bar{D}_p \bar{D}_q \phi}.$$ (4.1)

The restricted chiral N=4 light-cone superfield $\phi$ is equivalent to the chiral N=3 superfield in eq. (3.5). Our efforts to construct an N=4 generalization of eq. (2.8) along the similar lines (sect. 3) unexpectedly failed, while eq. (4.1) was the main obstruction. We conclude that even a manifestly N=4 supersymmetric generalization of the EH action in the light-cone gauge seems to be highly non-trivial, if any, not to mention an even more ambitious (manifest) N=4 supersymmetrization of the BI action.

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References

[1] M. Born, Proc. Roy. Soc. A143 (1934) 410;  
M. Born and L. Infeld, Proc. Roy. Soc. A144 (1934) 425;  
M. Born, Ann. Inst. Poincaré 7 (1939) 155

[2] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. 163B (1985) 123;  
A. Abouelsaood, C. G. Callan, C. R. Nappi and S. A. Yost, Nucl. Phys. B280 (1987) 599

[3] C. G. Callan, and J. Maldacena, Nucl. Phys. B513 (1998) 198;  
G. W. Gibbons, Nucl. Phys. B514 (1998) 603;  
R. G. Leigh, Mod. Phys. Lett. A4 (1989) 2767
[4] J. Plebanski. \textit{Lectures on Non-Linear Electrodynamics}, Nordita (1968);
S. Deser, J. McCarthy and O. Sarioglu, Class. and Quantum Grav. \textbf{16} (1999) 841

[5] M. K. Gaillard and B. Zumino, Nucl. Phys. \textbf{B193} (1981) 221;
G. W. Gibbons and D. A. Rasheed, Nucl. Phys. \textbf{B454} (1995) 185

[6] A. A. Tseytlin, \textit{Born-Infeld Action, Supersymmetry and String Theory}, in “Yuri Gol’fand Memorial Volume”, edited by M. Shifman, World Scientific, 2000; \texttt{hep-th/9908105}

[7] S. Cecotti and S. Ferrara, Phys. Lett. \textbf{187B} (1987) 335

[8] S. Deser and R. Puzalowski, J. Phys. \textbf{A13} (1980) 2501

[9] S. V. Ketov, Mod. Phys. Lett. \textbf{A14} (1999) 501; Nucl. Phys. \textbf{B553} (1999) 250

[10] S. M. Kuzenko and S. Theisen, \textit{Supersymmetric Duality Rotations}, München preprint; \texttt{hep-th/0001068}

[11] W. Heisenberg and H. Euler, Z. Phys. \textbf{98} (1936) 714

[12] M. Roček and A. A. Tseytlin, Phys. Rev. \textbf{D59} (1999) 106001

[13] A. De Giovanni, A. Santambrogio and D. Zanon, \textit{$(\alpha')^4$ Corrections to the \textit{N}=2 Supersymmetric Born-Infeld Action}, Milano and Leuven preprint, \texttt{hep-th/9907214}

[14] H. Liu and A. A. Tseytlin, JHEP \textbf{9910} (1999) 003

[15] F. Gonzalez-Rey and M. Roček, Phys. Lett. \textbf{434B} (1998) 303

[16] G. G. Harwell and P. S. Howe, Class. and Quantum Grav. \textbf{12} (1995) 1823;
P. S. Howe and P. C. West, Phys. Lett. \textbf{389B} (1996) 273

[17] L. Brink, O. Lindgren and B. E. W. Nilsson, Phys. Lett. \textbf{123B} (1983) 323 and
Nucl. Phys. \textbf{B212} (1983) 401

[18] S. Mandelstam, Nucl. Phys. \textbf{B213} (1983) 149

[19] S. V. Ketov, Theor. Math. Phys. \textbf{63} (1985) 470.