Dissipative generalized Chaplygin gas as phantom dark energy

Norman Cruz,1∗ Samuel Lepe,2† and Francisco Peña3‡

1Departamento de Física, Facultad de Ciencia, Universidad de Santiago, Casilla 307, Santiago, Chile.
2Instituto de Física, Facultad de Ciencias Básicas y Matemáticas, Pontificia Universidad Católica de Valparaíso, Avenida Brasil 2950, Valparaíso, Chile.
3Departamento de Ciencias Físicas, Facultad de Ingeniería, Ciencias y Administración, Universidad de la Frontera, Avda. Francisco Salazar 01145, Casilla 54-D, Temuco, Chile.

The generalized Chaplygin gas, characterized by the equation of state \( p = -A/\rho^\alpha \), has been considered as a model for dark energy due to its dark-energy-like evolution at late times. When dissipative processes are taken account, within the framework of the standard Eckart theory of relativistic irreversible thermodynamics, cosmological analytical solutions are found. Using the truncated causal version of the Israel-Stewart formalism, a suitable model was constructed which cross the \( \omega = -1 \) barrier. The future-singularities encounter in both approaches are of a new type, not included in the classification presented by S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D71, 063004 (2005).

PACS numbers: 98.80.Jk, 04.20.Jb

I. INTRODUCTION

In the framework of general relativity, the acceleration in the expansion of the universe during recent cosmological times, first indicated by Supernovae observations and also supported by the astrophysical data obtained from WMAP indicates the existence of an exotic fluid with negative pressure, which constitutes about the 70 per cent of the total energy of the universe. To know the nature and the behavior of this dark energy is one of the great challenges of cosmology and, indeed, of fundamental physics.

Within the different candidates to play the role of the dark energy, the generalized Chaplygin gas (GCG), in which the pressure \( p \) and the energy density \( \rho \) are related by

\[ p = -A/\rho^\alpha, \tag{1} \]

where \( A \) and \( \alpha \) are constants, has emerged as a possible unification of dark matter and dark energy, since its cosmological evolution is similar to an initial dust like matter and a cosmological constant for late times. This idea was first proposed in [2]. Using supernova observations this model has showed to be consistent for any value of \( \alpha \) in the range \( 0 \leq \alpha < 2 \).

Nevertheless, L. Amendola et al [3] found that the latest CMB anisotropy data seems favoured GCG as a dark energy model instead of a candidate for a unified dark model. The parameter \( \alpha \) is rather severely constrained, i.e., \( 0 \leq \alpha < 0.2 \) at the 95% confidence level. Others works, such as [4] tends to confirm that adiabatic Chaplygin gas is ruled out, as unified model, unless the parameter \( \alpha \) be very close to zero. A Chaplygin quartessence model with vanishing pressure perturbation, the called Silent Chaplygin, has shown to be consistent with CMB data [5].

Since we are interested here only in the late time acceleration and the phantom behavior, we will considered a GCG as a model for dark energy. Specifically, our principal aim is to investigate the possibilities that GCG models could realize the phantom divide \( \omega = -1 \) crossing phenomenon. Previous works have indicated this possibility. For example, using an extension of the Chaplygin gas model, where \( \mathcal{A} \) was taken as a function of the scale factor, \( a \), of the form \( \mathcal{A}a^{-m} \) (\( \mathcal{A} \) is a constant and \( m > 0 \)), Meng et al [6] showed that exist some cases which can realize state parameter \( \omega = -1 \) crossing.

Other approaches have considered dissipative effects in GCG models, using the framework of the non causal Eckart theory [7]. In [8] a viscous GCG was investigated from the point of view of the dynamical analysis, assuming that there is bulk viscosity in the linear barotropic fluid and GCG. It is found that the equation of state of GCG can cross the boundary \( \omega = -1 \).

∗ncruz@lauca.usach.cl
†slepe@ucv.cl
‡fcampos@ufro.cl
It is interesting to note that the GCG itself may behave like a fluid with viscosity, in the context of the Eckart formalism. J. C. Fabris et al. [10] found an equivalence between the GCG and a dust like fluid with a special election of both parameter $m$ and $\alpha$, where $m$ is related with the viscosity coefficient throughout the relation $\xi(\rho) = \beta \rho^m$.

It is well known that the Eckart approach is a non causal theory. The full causal theory developed by Israel and Stewart [11] leads to stable behavior under a wide range of conditions and transient phenomena on the scale of the mean free path/time [12]. In this theory the character of the evolution equation is very complicated, nevertheless we were able to find solutions in the truncated case for dissipative GCG cosmological models.

In the present paper we have investigated the behavior of a flat universe filled with a viscous GCG, using first the non causal approach of Eckart and then the truncated version of the Israel-Stewart formalism. In the context of the Eckart formalism, we show analytical solutions, which presents future-singularities, which were found restricting the range of the parameters $m$ and $\alpha$.

Using a truncated version of the Israel-Stewart formalism we were able to find a suitable model that present future-singularities, but with an equation of state corresponding to quintessence. This model has a relaxation time compatible with the expansion of the universe in a regime near to the thermodynamic equilibrium.

The organization of the paper is as follows: in Section II we present the field equations for a flat FRW universe filled with a viscous GCG within the framework of the Eckart theory. Cosmological solutions are obtained solving a non linear differential equation for the Hubble parameter. Exact solutions are obtained in terms of LerchPhi function for $m$ and $\alpha$ satisfying the constraint given by $m = -(\alpha + 1/2)$. In Section III we solve the equations of the truncated version of the Israel-Stewart formalism, using an Ansatz for the Hubble parameter. In section IV we characterize the future-singularities obtained. In section V we summarize our results.

II. THE GENERALIZED CHAPLYGIN GAS AND THE ECKART THEORY

The FRW metric for an homogeneous and isotropic flat universe is given by
\[ ds^2 = -dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2), \] (2)

where $a(t)$ is the scale factor and $t$ represents the cosmic time. In the following we use the units $8\pi G = 1$. The field equations in the presence of bulk viscous stresses are
\[ \left( \frac{\dot{a}}{a} \right)^2 = H^2 = \frac{\rho}{3}, \] (3)
\[ \frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{6} (\rho + 3(P + \Pi)), \] (4)

where $\Pi$ is the bulk dissipative pressure. The conservation equation is given by
\[ \dot{\rho} + 3H(\rho + p + \Pi) = 0. \] (5)

Assuming that the dark component is GCG, the equation of state can be rewritten in the form
\[ p(\rho) = -M^4(1+\alpha) \rho^{-\alpha}, \] (6)

where $M$ has dimension of mass and $\alpha$ is a constant. The original Chaplygin gas correspond to the election $\alpha = 1$. We note that $\alpha = -1$ yields the state equation corresponding to a cosmological constant.

If we put the Eq. (6) in the barotropic form $p(\rho) = \omega \rho$, the parameter $\omega$ take the following form in terms of the Hubble parameter
\[ \omega (H, \alpha) = -\left( \frac{M^4}{3H^2} \right)^{1+\alpha}. \] (7)

From Eqs. (9) to (14) is direct to obtain a single evolution equation for $H$:
\[ 2\dot{H} + 3 (1 + \omega(H, \alpha)) H^2 = -\Pi. \] (8)

In the first order thermodynamic theory of Eckart [8] $\Pi$ is given by
\[ \Pi = -3H \xi, \] (9)
and, in order to obtain solutions of the equation (8), we will assume that the viscosity has a power-law dependence upon the density

\[ \xi = \beta \rho^m, \quad \beta \geq 0. \]  

(10)

where \( \beta \) and \( m \) are constant parameters. Using Eq. (7), (10) and (9) in Eq. (8), we obtain the non linear equation leading the evolution of the Hubble parameter

\[ 2\dot{H} + 3 \left\{ H^2 - \left( \frac{M^4}{3} \right)^{1+\alpha} H^{-2\alpha} + 3^m \beta H^{2m+1} \right\} = 0. \]  

(11)

Despite the complexity of this equation it is possible to obtain analytic solutions for special cases in which the parameters \( \alpha, \beta \) and \( m \) are constrained. In the following subsections these solutions are explicitly showed.

A. Solutions with \( m = -(\alpha + 1/2) \)

If in Eq. (11) we choose the exponents of \( H \) to be equal, i.e., \(-2\alpha = 2m + 1\) we obtain

\[ 2\dot{H} + 3H^2 - \beta(\alpha)H^{-2\alpha} = 0, \]  

(12)

where \( \beta(\alpha) \) is defined by

\[ \beta(\alpha) \equiv 3^{1/2-\alpha} \left[ \beta + \frac{1}{\sqrt{3}} M^4(1+\alpha) \right]. \]  

(13)

Integrating Eq. (12) we obtain an implicit relation for \( H \) as a function of the cosmic time \( t \)

\[ t = \frac{H^{-1}}{3(1+\alpha)} \operatorname{LerchPhi} \left( \frac{\beta(\alpha)}{3} H^{-2(1+\alpha)}, 1, \frac{1}{2(1+\alpha)} \right), \]  

(14)

where the \( \operatorname{LerchPhi} \) function is defined as follows

\[ \operatorname{LerchPhi}(z, a, \nu) = \sum_{n=0}^{\infty} \frac{z^n}{(n+\nu)^a}. \]  

(15)

It was mentioned in [10] that a GCG (without viscosity) is equivalent to a dust with viscosity, if the parameters \( \alpha \) and \( m \) satisfy the constraint \( m = -(\alpha + 1/2) \). In our case this constraint was imposed in order to obtain analytic solutions of a GCG with viscosity.

B. Solutions with future-singularities in the non causal approach

It is easy to show graphically that above solutions, for positive values of \( \alpha \), represents universes with future-singularities. Before to show an analytical solution of Eq. (11), let us discuss some interesting results related to a pure GCG without dissipation. From Eqs. (9) and (11), we obtain the evolution of the energy density \( \rho \) in terms of the scale factor \( a \), or equivalently in terms of the redshift \( z = a_0/a - 1 \). The expression for \( \rho(z) \) is given by (for \( \alpha \neq -1 \))

\[ \rho = M^4 \left[ 1 + g (1 + z)^{3(1+\alpha)} \right]^{1/(1+\alpha)}, \]  

(16)

where \( g \) is a parameter that can be fitted by observational data.

Using the above equation in (11) we can obtain an expression for \( \omega \) in terms of the redshift

\[ \omega(z, \alpha) = -\left( 1 + g (1 + z)^{3(1+\alpha)} \right)^{-1}. \]  

(17)

From Eq. (17) is direct to see that for negative \( z \) (future cosmic time), we can obtain \( \omega = -1 \). The phantom case can be obtained if \( g < 0 \). A. Sen and R. J. Scherrer [13] explore the possible scenarios when the usual constraint on the parameters \( \alpha \) and \( g \) are relaxed. They consider the possibility of take \( g < 0 \) and \( \alpha < -1 \). Two cases of phantom
dark energy are obtained. One of them is a late phantom GCG model which gives \( \omega < -1 \) with \( \omega \) decreasing with time. A future singularity occurs at finite value of the cosmic time.

In our scheme it is possible to obtain phantom evolution in the special case, which is straightforward to integrate, when \( m = 1/2 \). The bulk viscosity coefficient \( \xi \) takes the form

\[
\xi = \beta \rho^{1/2} = \sqrt{3} \beta H.
\]

Introducing Eq. (18) in Eq. (8), we obtain

\[
2 \dot{H} + 3 \left(1 - \sqrt{3} \beta\right) H^2 = -3 \omega (H) H^2,
\]

which with the election \( \beta = 1/\sqrt{3} \), the above equation has the solution

\[
H(t) = H_0 \left[1 + \frac{3}{2} (2\alpha + 1) \left(\frac{M^4}{3H_0^4}\right)^{1+\alpha} (t - t_0)\right]^{1/(2\alpha+1)}.
\]

(20)

Assuming that \( 2\alpha + 1 = -\delta < 0 \), the solution (20) can be rewritten as

\[
H(t) = H_0 \left[1 - \frac{(t - t_0)}{t_s}\right]^{-1/\delta},
\]

(21)

obtaining, from Eq. (18), the explicit expression for the energy density,

\[
\rho(t) = 3H_0^2 \left[1 - \frac{(t - t_0)}{t_s}\right]^{-2/\delta},
\]

(22)

which blows up at the time \( t_s \), given by

\[
t_s = \frac{2}{3\delta} \left(\frac{3H_0^2}{M^4}\right)^{(1-\delta)/2}.
\]

(23)

The solution given in Eq. (22) implies that \( \dot{H} + H^2 > 0 \), i.e., the acceleration is positive. An expression for \( a(t) \) can be easily obtained integrating Eq. (22)

\[
a(t) = a_0 \exp \left\{ \frac{\delta}{\delta + 1} H_0 t_s \left[1 - \frac{(t - t_0)}{t_s}\right]^{-(1+\delta)/\delta} - 1 \right\},
\]

(24)

which at \( t - t_0 \to t_s \) blows up, representing a solution with a future-singularity. In brief, a GCG with bulk viscosity contains big rip solutions if \( \alpha < -1/2 \) and if the bulk viscosity coefficient takes the specific form \( \xi = \frac{1}{\sqrt{3}} \rho^{1/2} \).

It is interesting to compare the above results with those obtained in [13], where the model with a future singularity occurs when \( \gamma < 0 \) and \( \alpha < -1 \). Nevertheless, it is easy to show from Eq. (17) that if \( \gamma < 0 \) it is possible to find, in the past or in the future evolution of the universe (depending on the value of \( \gamma \)) that the fluid behaves as a stiff matter. This result is not physically plausible as we know from the cosmological observations that ruled out stiff matter, which would be of the importance only in the very early universe (see, for example [14]). In our case, a CGC fluid with bulk viscosity, the solution with future singularity is obtained for \( \gamma > 0 \) and \( \alpha < -1/2 \).

Note the exponential behavior of the scale factor with time. In the case where dark energy component obey the state equation \( p = (\gamma - 1)\rho \) and \( \xi \sim \rho^{1/2} \) there are also big rip solutions but with a power-law expansion for the scale factor of the form

\[
a(t) = a_0 \left(1 - \frac{t - t_0}{t_s}\right)^{2(\gamma - \sqrt{3} \beta)}/(\gamma - \sqrt{3} \beta),
\]

(25)

where \( a_0 = a(t = t_s) \). This solution was found by Barrow [17] and if the following constrains on the parameters \( \beta \) and \( \gamma \) are imposed [10]

\[
\sqrt{3} \beta > \gamma,
\]

(26)

the scale factor blows up to infinity at a finite time \( t_s > t_0 \), given by

\[
t_s = \frac{2}{3(\sqrt{3} \beta - \gamma)H_0^{-1}}.
\]

(27)
III. A TRUNCATED VERSION OF THE ISRAEL-STEWART FORMALISM

In this approach we consider an imperfect fluid whose dissipative bulk pressure, $\Pi$, obeys the causal transport Eq. \[ 17 \]

$$ \Pi + \tau \dot{\Pi} = -3H \xi, $$

where $\tau$ is the bulk relaxation time and is the crucial thermodynamic parameter in the Israel-Stewart formalism that ensures causality.

In order to obtain an equation for $\Pi$ in terms of Hubble parameter $H$, we derive Eq. \[ 8 \] introducing then Eq. \[ 28 \]. This yields

$$ \Pi(H) = \tau \left( 2\dot{H} + 3\omega(H, \alpha)H^2 + 6(1 + \omega(H, \alpha))H\dot{H} \right) - 3\xi H. $$

The parameter $\tau$ is related to the energy density throughout the equation

$$ \tau = \xi = \beta \rho m^{-1}, $$

where $\xi$ is given by Eq. \[ 10 \]. In \[ 16 \] big rip solutions were found within the framework of the full causal theory of Israel-Stewart, for universes filled a dark component obeying the state equation $p = (\gamma - 1)\rho$. For this case, using the Ansatz $H = A(t_s - t)^{-1}$, where $A \equiv H_0t_s$ and $t_s$ is the time when the future-singularity occurs, we were able to find the exact expression for $A$.

A. Solutions with future-singularities in the causal approach

In the following we will generalize the expression for the Ansatz thinking in the more complex structure of the equation of state corresponding to a GCG. Our election is

$$ H = A\Delta^{-q}, $$

where $A = H_0(t_s - t_0)^q$, $\Delta \equiv t_s - t$ and $q$ is a positive constant, which is determined by consistence of the equations. Using the above Ansatz and Eqs. \[ 13 \], \[ 17 \] and \[ 10 \], in Eq. \[ 29 \], we obtain

$$ \Pi(\Delta) = -3m \beta \Delta^{-2q(m+1/2)} \left( a(m) - b(m, q)\Delta^{q-1} - c(m, q)\Delta^{2(q-1)} - ad(m, q; \alpha)\Delta^{2(q+2\alpha-1)} \right), $$

where the coefficients $a(m)$ and $b(m, q)$ are given by the following expressions

$$ a(m) = 3A^{2(m+1/2)}, \quad b(m, q) = 2qA^{2m}, $$

and $c(m, q)$ and $d(m, q; \alpha)$ by

$$ c(m, q) = \frac{2}{3}q(q+1)A^{2(m-1/2)}, \quad d(m, q; \alpha) = 2q \left( \frac{M^4}{3} \right)^{1+\alpha} A^{2(m-1-\alpha)}. $$

Condition $\Pi < 0$. Since from thermodynamics arguments (as required from the second law), $\Pi < 0$, we will explore the possible scenarios which are obtained depending on the values of the parameters $q, m,$ and $\alpha$.

Case $q = 1$. Let us investigate the special case with $q = 1$, where the factors with the time dependence $\Delta^{q-1}$ in Eq. \[ 29 \], becomes constant. In this situation we obtain constraint for $A$ which are independent of the parameter $m$. In brief, we have the following possibilities in order to ensure $\Pi < 0$:

$$ A > \frac{1 + \sqrt{5}}{3}, \quad \alpha = 0, $$

and

$$ A \geq \frac{1 + \sqrt{5}}{3}, \quad \alpha < 0. $$
In the first case it is straightforward to see that the time \( t_s \) has a lower limit given by

\[
    t_s > \frac{1 + \sqrt{5}}{3} H_0^{-1}. \tag{37}
\]

**Case \( q > 1 \).** Choosing for simplicity \( \alpha = 0 \), the sign of Eq. \( \text{[32]} \) becomes dependent of the sign of the factor \( a(m) - b(m, q)\Delta^q - c(m, q)\Delta^{2(q-1)} \). Since the powers of the \( \Delta \) functions are both positives, they are decreasing functions of the cosmic time. This implies that if we demand the above factor to be positive at \( t = 0 \) it will positive for the future evolution. Solving the constraint

\[
    \left( a(m) - b(m, q)\Delta^q - c(m, q)\Delta^{2(q-1)} \right) |_{t=0} > 0, \tag{38}
\]

and since \( a(m), b(m, q), c(m, q) \) are functions of \( A = H_0(t_s - t_0)^q \), at \( t = 0 \) Eq. \( \text{[38]} \) gives a constraint for \( t_s \) in the form of \( \mathcal{P}(t_s) > 0 \), where \( \mathcal{P}(t_s) \) is a polynomial of the form

\[
    \beta_1 t_s^q - \beta_2 t_s^{q-1} - \beta_3 t_s^{-q}, \tag{39}
\]

which can be solved for \( t_s \).

**Case \( 0 < q < 1 \).** For this case \( \Pi < 0 \) if \( \alpha < 0 \)

**Condition \( |\Pi| << |p| \).** We have just found the constraints on the parameters \( \alpha \), depending of which values are given for \( q \), in order to obtain a negative dissipative pressure. Note that these constraints are independent of \( m \).

Constraints on \( m \) appears when we consider that the Israel-Stewart theory is derived under the assumption that the thermodynamical state of the fluid is close to equilibrium, which means that the nonequilibrium bulk viscous pressure should be small when compared to the local equilibrium pressure that is \( |\Pi| << |p| \). Using Eqs. \( \text{[32]}, \text{[4]}, \text{[7]} \) the above condition takes the form

\[
    \frac{|\Pi(\Delta)|}{|p(\Delta)|} = 3^{m-1}\beta \left( \frac{M^4}{3} \right)^{-(1+\alpha)} A^{2\alpha} \Delta^{-2q(m+1/2+\alpha)} \times \left( a(m) - b(m, q)\Delta^q - c(m, q)\Delta^{2(q-1)} - \alpha d(m, q; \alpha)_\Delta^{q(3+2\alpha)} \right) < < 1. \tag{40}
\]

It is straightforward to check out that both conditions, \( \Pi < 0 \) and \( |\Pi| << |p| \) are satisfied in the following cases:

**Case \( q = 1 \)

\[
    \alpha = 0, \quad m + \frac{1}{2} < 0, \quad \tag{41}
\]

and

\[
    A = \frac{1 + \sqrt{5}}{3}, \quad \alpha < 0 \quad m < \frac{1}{2}. \tag{42}
\]

**Case \( q > 1 \)

\[
    \alpha = 0, \quad m + \frac{1}{2} < 0, \quad \tag{43}
\]

We finally found the value of the parameter \( \beta \), constraining the relaxation time \( \tau \) to be lower than the Hubble time, i.e., \( \tau < H^{-1} \). The expression for \( \tau \) is given by

\[
    \tau = \frac{\xi}{\rho} = 3^{m-1}\beta A^{2(m-1/2)} \Delta^{-2q(m-1/2)} H^{-1}. \tag{44}
\]

Since the relation \( \tau < H^{-1} \) must be satisfied for all times, at \( t = 0 \) we obtain and upper limit for the parameter \( \beta \), which can be evaluated from

\[
    \left( 3^{m-1}\beta A^{2(m-1/2)} \Delta^{-2q(m-1/2)} \right) |_{t=0} < 1, \tag{45}
\]

in terms of \( t_s \). In order to satisfy \( \tau < H^{-1} \) for all times we need \( m < 1/2 \). If this condition holds, \( \tau \) is a decreasing function of the cosmic time and becomes equal to zero at \( t_s \). We can take, for example, \( \alpha = 0, m < -1/2 \) (from the
condition \( m < -1/2 \). In other words, with \( m < -1/2 \) (and \( q = 1 \) and \( \alpha = 0 \)), it is possible to have a realistic models with \( \Pi < 0 \), \(|\Pi| \ll p \) and \( \tau < H^{-1} \) for all times during the cosmic evolution.

Let us show the behaviour of one of these models, taking for simplicity, \( q = 1 \), \( \alpha = 0 \) and \( m = -1 \). In this model the solution for the Hubble parameter is given by

\[
H = A\Delta^{-1}. \tag{46}
\]

From Eq. (52) we obtain the expression for the dissipative pressure

\[
\Pi(\Delta) = -\beta \frac{(A-A_+)(A-A_-)}{A^3} \Delta, \tag{47}
\]

where \( A_\pm = 1/3(1 \pm \sqrt{5}) \). As we saw above, \( A \) must satisfy the inequality given in (35) in order to ensure \( \Pi < 0 \).

The expression for pressure of the GCG in terms of used Ansatz is

\[
p(\Delta) = -3 \left( \frac{M^4}{3} \right)^{(1+\alpha)} A^{-2\alpha} \Delta^{2q\alpha}, \tag{48}
\]

which gives for the particular values that we are taking

\[
p(\Delta) = -M^4. \tag{49}
\]

The negative pressure of the GCG will be constant during the entire cosmic evolution. The energy density can be obtained from Eqs. (3) and (46), yielding

\[
\rho(\Delta) = 3A^2\Delta^{-2}, \tag{50}
\]

which becomes infinity at \( t_s \). The upper limit that the parameter \( \beta \) could take can be evaluated from the conditions \(|\Pi|/|p| \ll 1 \) and \( \tau < H^{-1} \). Evaluating \(|\Pi|/|p| \ll 1 \) for our model we obtain that

\[
\frac{|\Pi(\Delta)|}{|p(\Delta)|} = \frac{\beta (A-A_+)(A-A_-)}{M^4 A^3} \Delta \ll 1, \tag{51}
\]

which implies the following upper limit for \( \beta \) when equation is evaluated at \( t = 0 \)

\[
\beta \ll \frac{3A^3}{(A-A_+)(A-A_-)} \frac{M^4}{3H_0^2 t_s} (H_0 t_s)^{-1}. \tag{52}
\]

The constraint \( \tau < H^{-1} \) evaluated at \( t = 0 \) gives the other restriction on \( \beta \)

\[
\beta < 9H_0^3. \tag{53}
\]

If we choose \( \beta \) constrained by Eq. (53), Eq. (52) can be easily satisfied. This means that with a enough low viscosity, the dark energy, modeled by a GCG, leads to a superaccelerated universe.

\section*{IV. CHARACTERIZATION OF THE FUTURE-SINGULARITIES}

We will discuss briefly the conditions which presents the future-singularities founded in the above sections. In the two solutions found in section II, for \( \alpha > 0 \), the future-singularities have the following characterization:

- For \( t \to t_s \), \( a \to \infty \), \( \rho \to \infty \), and \( |p| \to 0 \)

The model explicitly calculated for the special values \( q = 1 \), \( \alpha = 0 \) and \( m = -1 \), in section III, gives a solution which presents a future-singularity characterized by

- For \( t \to t_s \), \( a \to \infty \), \( \rho \to \infty \), and \( |p| \to \text{constant} \)

In the classification realized in (18) exist four types of singularities and none of them behaves like those founded in this work. In this sense, cosmological models filled with a dissipative GCG, in the framework of the truncated Israel-Stewart formalism, present new types of future-singularities.
V. CONCLUSION

In the present paper we have found cosmological solutions for a GCG with bulk viscous stresses, in the context of both thermodynamic formalism of Eckart and Israel-Stewart. We have assumed that the viscosity has a power-law dependence upon the density, i.e., $\xi = \beta \rho^m$, where $\beta$ and $m$ are constant parameters ($\beta \geq 0$). Following the Eckart approach and in order to obtain solutions, we have derived the non linear differential equation which leads the evolution of the Hubble parameter. Analytical solutions are found constraining the parameters $\alpha$ and $m$ by the relation $m = - (\alpha + 1/2)$ and taking particular values of $\beta$. Solutions with future-singularities can be obtained taking the specific value $m = 1/2$. In this case, the scale factor grows exponentially, differing from the power law expansion of the scale factor when the universe is filled with a fluid obeying a barotropic equation of state and viscosity. As it was discussed in [13], phantom scenarios with big rip are obtained if $\alpha < -1$ and $g < 0$. Nevertheless, those obtained from models with a GCG and bulk viscosity avoid the fact that the cosmic fluid could behave like stiff matter in the past or future evolution of the universe.

Using the causal but reduced version of the Israel-Stewart formalism we have found a suitable physically solution from the point of view the thermodynamics requirements expressed in the conditions $\tau < H^{-1}$ and $|\Pi/p| << 1$. We have explicitly showed a case in which is possible to cross the barrier $w = -1$.

The future-singularities encounter in our models, in the framework of the non causal and causal formalism, are of a new type, not included in the classification presented in [18].

VI. ACKNOWLEDGEMENTS

NC and SL acknowledge the hospitality of the Physics Department of Universidad de La Frontera where part of this work was done. SL acknowledges the hospitality of the Physics Department of Universidad de Santiago de Chile. We acknowledge the partial support to this research by CONICYT through grant Nº 1040229 (NC and SL). It also was supported from DIUFRO Nº 120618, of Dirección de Investigación y Desarrollo, Universidad de La Frontera (FP) and DI-PUCV, Grants 123.784/06 and 123.105/05 (SL), Pontificia Universidad Católica de Valparaíso.

[1] S. Perlmutter et al, Astrophys. J. 517 (1999) 565; P.M. Garnavich et al, Astrophys. J. 493 (1998) L53; A.G. Riess et al, Astron. J. 116 (1998) 1009; D.N. Spergel et al, Astrophys. J. Suppl. 148 (2003) 175; A.G. Riess, Astrophys. J. 607 (2004) 665.
[2] M.C. Bento, O. Bertolami and A.A. Sen, Phys. Rev. D66 (2002)043507.
[3] M. Makler, S. Q. de Oliveira and I. Waga, Phy. Lett. B555 2003 1; J. C. Fabris, S. V. B. Gonçalves and P. E. de Souza, astro-ph/0207430 Y.Gong and C. K. Duan, Class. Quant. Grav. 21 (2004) 3655; Mon. Not. Roy. Astron. Soc. 253 (2004) 847; Y. Gong, 0503 (2005) 007.
[4] L. Amendola, F. Finelli, C. Burigana and D. Carturan, JCAP 0307:005,2003
[5] D. Carturan and F. Finelli, Phys. Rev. D68, 103501 (2003); H. Sandvik, M. Tegmark, M. Zaldarriaga and I. Waga, Phys. Rev. D69, 123524 (2004).
[6] L. Amendola, I. Waga and F. Finelli, JCAP 0511: 009, 2005
[7] X. Meng, M. Hu and J. Ren, CAP 0511: 009, 2005
[8] C. Eckart, Phys. Rev. 58, 919 (1940).
[9] Xiang-Hua Zhai, You-Dong Xu and Xin-Zho Li, astro-ph/0511814
[10] J. C. Fabris, S. V. B. Gonçalves and R. de Sá Ribeiro, astro-ph/0503362.
[11] W. Israel, Ann.Phys. 100, 310 (1976); W. Israel and J. M. Stewart, Phys. Lett. A58, 231 (1976).
[12] W. A. Hiscock and J. Salmonson, Phys. Rev. D43, 3249 (1991); R. Martens, Class. Quantum Grav.12, 1455 (1995).
[13] A. A. Sen and R. J. Scherrer, Phys. Rev. D72, 063511 (2005).
[14] T. Banks and W. Fischler, hep-th/0111142 [hep-th/0310288]
[15] J. D Barrow, Phys. Lett. B 180, 335-339 (1987); J. D Barrow, Nucl. Phys. B 310, 743 (1988).
[16] M. Cataldo, N. Cruz and S. Lepe, Phys. Lett. B 619, (2005)5-10.
[17] R. Maartens, astro-ph/9609119
[18] S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D71, 063004 (2005).