AC Losses of a Tri-axial Superconducting Cable with Balanced Three-phase Current Distributions

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Abstract. High Temperature Superconducting (HTS) cables have been studied because of low loss and compactness, compared with conventional copper cables. Three-phase cables are usually composed of three single-phase concentric cables. Recently, a tri-axial cable composed of three concentric phases, has been intensively developed, because it has advantages such as reduced amount of HTS tapes, low leakage fields, low heat in leak and compactness, compared with the three single-phase cables. We analyzed the three-phase balanced current distributions in the tri-axial cable as functions of winding pitches of three concentric phase layers. We formulated the general form of AC losses in the case of the transport current and the field in different phase. We analyze AC losses of a tri-axial cable with balanced three-phase current distributions.

1. Introduction
High Temperature Superconducting (HTS) cables have been intensively developed as potential cables for a metropolitan area in many countries because of extremely low loss and compactness compared with conventional copper cables [1-9]. Some of them have been designed, fabricated and tested [6-9].

Many developed HTS cables are usually composed of single-phase coaxial cables so far, and hence three-phase cable requires the three single-phase coaxial cables. Since the balanced three-phase currents have small fringe field, a tri-axial cable, which supplies three-phase currents concentrically along the same axis, has been proposed. The tri-axial cable has great advantages of large reduction of HTS tapes and cable size compared with the three single-phase cables [9]. However, there is an inherent imbalance in the three-phase currents in the tri-axial cable due to differences in radii of the three-phase current layers. Since an imbalanced three-phase current and voltage distribution should restrict transport current capacity, enhance AC loss and deteriorate the power quality [9], it is important to realize the balanced three-phase distribution. We theoretically demonstrated that the three-phase currents are balanced in the tri-axial cable by following means; the HTS layers are composed of two longitudinal sections, and all section cable pitches should be adjusted to supply the balanced three-phase currents [4].

Since an external field with different phase from that of a transport current is applied on the tri-axial cable, we formulated the AC loss in the case of the out-of-phase external field and applied it to a tri-axial cable in previous papers [5,10].
Since the unknown number of twist pitches $6 > \text{number of equations} 4$, we can add two additional equations analytically. As far as we assume the following relations, we can solve the problem.

\begin{align}
\frac{1}{\pi} \ln \frac{r_a}{r_c} + \frac{1}{\pi} \ln \frac{y l_c}{l_c} + \frac{1}{\pi} \ln \frac{1-y l_c}{l_c} = 0 \\
\frac{1}{\pi} \ln \frac{r_a}{r_c} + \frac{1}{\pi} \ln \frac{y l_c}{l_c} + \frac{1}{\pi} \ln \frac{1-y l_c}{l_c} = 0
\end{align}

In this expression, we label the layers phase-a to phase-c from the most inner layer in the tri-axial cable, $r_a$ is radius of the phase-a, $l_{ai}$ is twist pitch of phase-a of the $i$th section, and $y$ is the ratio of the $i$th section to the total length.

2.2. Analytical solutions of twist pitches for 1 layer/phase

Since the unknown number of twist pitches $6 > \text{number of equations} 4$, we can add two additional equations analytically. As far as we assume the following relations, we can solve the problem.

\begin{align}
l_{ai} = l_{a2}, \quad l_{bi} = l_{b2}
\end{align}
This condition represents that twist pitches of both longitudinal sections of phase-a and -b are the same, and hence it is easy to fabricate the cable for inner two layers. Inserting the relations (2) into the fundamental equations (1), we can obtain the following solutions.

\[
\begin{align*}
    l_{a1} &= l_{a2} = e_1 g_a(r_a, r_a, r_a), \\
    l_{b1} &= l_{b2} = -e_2 g_b(r_a, r_a, r_a), \\
    l_{a1} &= e_2 f_a(y) \cdot g_a(r_a, r_a, r_a), \\
    l_{a2} &= -e_2 f_a(y) \cdot g_a(r_a, r_a, r_a)
\end{align*}
\]

(3)

Here, \( e_1 \) and \( e_2 \) are twist direction \( \pm 1 \) (clockwise and anticlockwise), and functions \( f \) and \( g \) are as follows.

\[
\begin{align*}
    f_{a1} &= \frac{y}{1+y}, \\
    f_{a2} &= \frac{1-y}{y}
\end{align*}
\]

(4)

\[
\begin{align*}
    g_a &= 2\pi r_a \left( \frac{1}{2u_3} \ln u_3 \left[ \ln u_3 + \sqrt{\left( \ln u_3 \right)^2 + \left(2u_3 \ln u_3\right)^2} \right]^{1/2} \right), \\
    g_b &= 2\pi r_a \left( \frac{1}{2\ln u_3} \left[ -\ln u_3 + \sqrt{\left( \ln u_3 \right)^2 + \left(2\ln u_3\right)^2} \right]^{1/2} \right), \\
    g_c &= 2\pi r_a u_2 \left[ \ln u_3 + 2\ln u_3 + \sqrt{\left( \ln u_3 \right)^2 + \left(2\ln u_3\right)^2} \right]^{1/2}
\end{align*}
\]

(5)

The twist pitches of phase-a and phase-b are depending on the cable radii only and not depending on the section length ratio \( y \). Since the twist pitches of phase-c, on the other hand, are depending on \( y \) and moreover described in the form of multiply of \( f \) as a function of \( y \) only and \( g \) as a function of radii of the cable, we can treat functions \( f \) and \( g \) independently to study the pitch characteristics.

2.3. Twist pitches for 1 layer/phase

Since the twist pitches are composed of functions \( g \) or \( f \cdot g \), we can obtain \((g/2\pi r_a)\) as a function of \( u_3 \) with parameters \( u_3 = 1.1 \) or \( 1.2 \), according to the equation (5), as shown in Fig.2. It is found that all twist pitches are not strongly depending on \( u_3 \) within region of \( 1.1 \) to \( 1.2 \), and decrease with increasing \( u_3 \). The twist pitch of phase-a \( l_a \) is always larger than that of phase-b \( l_b \). Since the function \( f_{a1} \) is inversely proportional to \( f_{a2} \) and varies from 1/3 to 3 according to \( y \) from 0.1 to 0.9, the twist pitch of phase-c \( (l_c/2\pi r_a) \) varies from 4.2 to 0.47. However, the twist pitch \( l_c \) is the shortest as far as the both section lengths are around the same order, namely \( y \approx 0.5 \).

3. Analysis of AC loss for 1 layer /phase tri-axial cable

Since the cable is concentrically composed of superconducting layers wound with HTS tapes, magnetic fields are generated along both the axial direction and the azimuthal direction of the layers, and thereby both fields directly in parallel with the tape wide surface [5]. This allows us to use a

![Figure 2](image-url)
superconducting slab model to analyze the hysteresis loss which is the dominant contribution to the AC losses, as shown in Fig. 3. In this formulation we use Bean model for the critical state model.

3.1. General formulation of AC loss

Since the amplitudes and phases of the both surface fields on the slab are different in the tri-axial cable, we define the phase of the left surface field as a reference phase. Then the both surface fields are described in general, as follows.

\[ B_s = B_m \sin \omega t, \quad B'_s = B'_m \sin(\omega t - \phi_0) \]

\[ b_m = B_m / B_p, \quad b'_m = B'_m / B_p, \quad B_p = \mu_0 J_c d \]  \tag{6}

where, \( \omega \) is angular frequency, \( \phi_0 \) is the phase difference, \( B_s \) and \( B'_s \) are the left and right surface fields on the slab, and \( b_m \) and \( b'_m \) are fields normalized by a penetration field \( B_p \), \( j_c \) is the critical current density, \( d \) is a half thickness of the slab, \( \mu_0 \) is permeability in free space.

As far as the external field is not large enough to reach the center of the slab (\( b_m + b'_m < 2 \)), the fields from the both surfaces do not interact with each other and changes independently in the slab. We can obtain the following equation.

\[ Q = \frac{B_p^2}{3\mu_0} \left( b_m^3 + b'_m^3 \right) \quad [\text{J/m}^3/\text{cycle}] \]  \tag{7}

When the external field is so large enough to reach the center of the slab (\( b_m + b'_m \geq 2 \)), the fields from the both surfaces interact with each other, and hence loss mechanism is different. The time behaviors of the field distributions in the slab are shown in Fig. 3. The AC loss per unit volume during a cycle is obtained as following equation according to the previous paper [5].

\[ Q = \frac{B_p^2}{24\mu_0} \left[ b_m^3 \left( 5 - 3\cos 2\omega t \right) + b'_m^3 \left( 5 - 3\cos 2(\omega t - \phi_0) \right) + b_m^2 b'_m \left( -4 \cos \phi_0 + 3\sin (\omega t_1 + \phi_0) - \sin (2\omega t_1 + \phi_0) \right) \right. \]

\[ \left. + 2b_m^2 \left( -3 - \cos 2\phi_0 + 3\sin (\omega t_1 - 2\phi_0) - \sin (2\omega t_1 - 2\phi_0) \right) \right] + b'_m^2 \left( 6 + 6\cos 2\omega t_1 + b_m^2 \left( 6 + 6\cos 2(\omega t_1 - \phi_0) \right) \right) \]  \tag{8}

\[ \text{Figure 3 Time behaviours of magnetic field distributions in a superconducting slab after the minimum fields in the case of out-of-phase external fields.} \]
Where, the time $t_1$ is given as follows.

$$\sin(\omega t_1 - \phi_n) = \left(2 - b_m\right)/b_m'$$  \hspace{1cm} (9)

3.2. AC loss of 1 layer/phase

Since the fields in the tri-axial cable have both axial (z-directional) and azimuthal (θ-directional) components, we should take account of both field components for the AC loss. The self field of the cable is divided into two directional fields, namely azimuthal and axial field. The axial transport current flows in the outer region of the layer, while the azimuthal current flows in the inner region of the layer according to the magnetic field distribution [5].

The main parameters of the tri-axial cable are listed in Table 1. We can analytically obtain the all twist pitches satisfying the balanced three-phase distribution by using equation (3), and thereby we can calculate all the magnetic field components on the all layers and obtain the AC losses of all layers. Figure 4 shows the AC loss as a function of the section length ratio $y$ averaged with the entire cable length, because each section of the cable generates different AC loss due to different pitches.

It is found that the AC loss has a minimum loss about 1.3 W/m at $y \approx 0.45$, gradually becomes large apart from the minimum point, and rapidly increases both above 0.85 and below 0.15. In general, since the two cable section lengths are not so different, the AC loss has the same order around 1.5 W/m.

4. Discussion

The magnetic fields generating the AC loss is composed of azimuthal and axial fields. Since the axial fields are generated by the axial transport currents which are the same currents flowing on the same layers, the azimuthal AC losses of both section due to the azimuthal fields are the same as long as the transport currents are the same. Therefore, the average AC losses are depending on the axial AC losses due to the axial fields, because the axial fields are largely changed by twist pitch and twist direction.

In order to investigate the difference between the axial AC losses of the cable at $y = 0.2$ and 0.45, we breakdown the axial AC losses into that of all phases and sections as shown in Fig.5. In this figure, Total-1 means the total loss of the 1$^{st}$ section. It is found that the total axial AC loss of 1$^{st}$ section at $y=0.2$ is the largest and hence the total loss of both sections at $y=0.2$ is larger than that at $y=0.45$. The

| Table 1. Main parameters of tri-axial cable |
|--------------------------------------------|
| Radius of phase-a, -b, -c | 30, 35, 40 [mm] |
| HTS tape critical current | 250 [A] |
| HTS cross section | 4×0.15 [mm] |
| Critical current of each layer | 6 [kA] |
| Operation current | 5 [kA] |

Figure 4 Average AC loss as a function of $y$. 

5
Total-1 loss is larger than the Total-2 at y=0.2, while the Total-2 is larger than Total-1 at y=0.45.

This reason stems from the twist pitches and twist directions. Table 2 shows the twist pitches of all phases and the sections at y=0.45 and 0.2. The twist pitches of both sections at phase-a and b are the same as pointed at the previous section of the balanced three-phase distributions, while the twist pitches of both section at phase-c are depending on the section length ratio y. Since the twist pitch $l_{c1}$ at y=0.2 is the smallest, the axial magnetic field is strongest and causes large AC losses not only at phase-c but also at other phases.

Although the twist pitch $l_{c1}$ at y=0.45 is smaller than $l_{c2}$, the AC loss of 2nd section is a little bit larger than that of 1st section. Since the twist directions of 1st section of phase-c and b are different, the addition of both axial field vectors is larger than that of the same direction.

Table 2. Twist pitches [m] of all phases and sections at y=0.45 and 0.2

|       | $l_{a1}$ | $l_{a2}$ | $l_{b1}$ | $l_{b2}$ | $l_{c1}$ | $l_{c2}$ |
|-------|----------|----------|----------|----------|----------|----------|
| y=0.45| 0.429    | 0.429    | -0.310   | -0.310   | -0.259   | 0.317    |
| y=0.2 | 0.429    | 0.429    | -0.310   | -0.310   | -0.143   | 0.573    |

5. Conclusion

We analyzed the three-phase balanced distributions of 1 layer/phase tri-axial cable analytically by introducing the same twist pitch of both sections at phase-a and b, and then investigate the AC losses of the tri-axial cable as a function of the section length ratio y. The AC loss has a minimum value at y ≈0.45 and becomes large at both large and small y. This is explained that the AC loss depends on the twist pitches.

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