RESPONSE OF THE COST OF EQUITY TO LEVERAGE: AN ALTERNATIVE PERSPECTIVE

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ABSTRACT

In this paper we examine the change in a corporation’s cost of equity as the corporation increases leverage. Standard textbook treatments present the well-known Modigliani-Miller hypothesis that the cost of leverage increases linearly with increases in the debt-to-equity ratio in keeping with a constant cost of capital for the firm. Less frequently, textbooks present the Modigliani-Miller argument that, if the cost of debt rises with high levels of leverage, the cost of equity will increase at a decreasing rate or even decline in order to keep the overall cost of capital constant. Standard textbook presentations continue with additional discussions concerning tax effects and bankruptcy costs but without mention of the cost of equity. These presentations leave the impression that the cost of equity remains as presented in the Modigliani-Miller framework. In this paper we present theoretical and empirical arguments in support of our claim that the cost of equity increases slowly with moderate increases in debt but increases dramatically as leverage increases sufficiently to cause equity investors to fear bankruptcy.

INTRODUCTION

What Should Students Know About the Cost of Equity?

Two fundamental relationships form the core knowledge base that students should possess about the cost of equity, both from a corporate finance perspective and an investment perspective. First, because debt has a senior payment claim both in the case of normal business operations and in the case of bankruptcy, equity has more risk and the cost of equity is greater than the cost of debt. This holds regardless if the firm uses very little leverage or a great deal of leverage. Second, because debt is cheaper than equity, increasing leverage may increase per unit payments to equity, but at the cost of increasing variability in these payments. This increase in financial risk causes equity owners to demand a higher percentage return. It is the path of
this increased required return to equity that we argue is inadequately addressed by textbooks and provides the focus of this paper. Because textbook discussions of the relationship between the cost of equity and leverage typically depend on the theory developed by Modigliani and Miller, we continue with a discussion of their arguments.

**Modigliani-Miller Model**

Modigliani and Miller’s (1958) Nobel Prize winning paper on capital structure still underpins textbook discussions of capital structure. At the core of their argument is the proposal that a firm’s value is determined solely by the cash flow and the risk of the cash flow created by the firm’s assets. Thus, under perfect market conditions, the value of the firm, and hence its overall cost of capital, is not influenced by the decision to finance the firm by debt or by equity. This result is referred to as the irrelevance proposal, or as Modigliani-Miller Proposition I (MM I), and is supported by the argument that investors can achieve whatever level of leverage they desire in their investment in a firm by borrowing on their own to supplement their personal equity investment (the “homemade leverage” argument).

Because using debt financing replaces the higher cost of equity with lower costing debt, in order for the cost of capital to remain constant, the cost of equity must increase with leverage and must do so at a prescribed rate as shown in equation (1) below. This relationship is referred to as Modigliani-Miller Proposition II (MM II).

\[
RR_{\text{equity}} = \text{Unlevered Required Return of Equity} + D/E \times (\text{ROA} - \text{Cost of Debt})
\]

Where: \(RR_{\text{equity}}\) is the required return on equity for a given level of leverage, \(D/E\) is the debt-to-equity ratio (measures leverage), and \(\text{ROA}\) is the expected return on assets.

The cost of debt is assumed to be constant across levels of leverage.

Thus, the required return to equity is a constantly increasing linear function of leverage as measured by the D/E ratio. A constantly increasing cost of equity is consistent with constantly increasing financial risk borne by equity holders as the total amount of fixed payments to debt holders increases with leverage. The rationale that this increase in cost can be measured by equation (1) above results because
only this increase will maintain a constant overall cost of capital. To support the validity of MM II, it is argued that, if the markets incorrectly price equity at any level inconsistent with the cost of equity as determined by equation (1), arbitrage profits will be available and market transactions will move the stock price to the correct level.

Modigliani and Miller consider the relaxation of several key perfect market assumptions, including a constant cost of debt with increasing leverage and the absence of corporate taxes. They find their results robust to the relaxation of these assumptions. These results, although widely accepted in the academic community, conflict with the observed behavior of financial managers who clearly believe that the financing decision and the level of leverage used matters. Furthermore, keeping the overall cost of capital constant in the face of a rising cost of debt required Modigliani and Miller to argue that the rate of increase in the cost of equity slowed, or even reversed, as leverage grew high, a seemingly illogical behavior on the part of equity investors. Modigliani and Miller (1963) revisit the tax issue and their revised results lead to the equally untenable position that the optimum capital structure is 100% debt.

**Textbook Treatment**

Textbook discussions of the cost of equity, as included in descriptions of capital structure, provide a fairly standard representation of the effect of leverage on the cost of equity. We find this standard representation lacking in two important aspects. We next discuss the standard presentations and then identify the two aspects of these presentations that we suggest are inconsistent with rational investor behavior, short changing the student. Textbook presentations generally begin with the Modigliani-Miller assumptions that capital markets are efficient and there is no tax on corporate earnings. They emphasize that the value of the firm is determined by the income produced by the firm’s assets and that this asset value does not change with the method of financing. Thus, the total value of debt and equity remains constant across levels of leverage. These presentations generally identify this relationship as the Modigliani-Miller irrelevance proposition, or MM I.

Discussion of the cost of equity follows from this proposition. Textbook presentations may or may not emphasize that the cost of equity is higher than the cost of debt due to lower risk for debt, but do indicate a higher cost of equity relative to the cost of debt. These presentations continue with a discussion of the impact of leverage on the cost of equity. The discussion shows that leverage creates a higher
EPS for the stockholder, but at the cost of higher financial risk. The precise increase in the cost of equity is shown as a mathematical identity whereby the value of the firm is held constant in accordance with the basic proposition that the value of the firm is solely determined by the value of the firm’s assets. In this case, the cost of equity is determined by MM II (equation (1)) and the overall cost of capital is determined by the cost of debt and cost of equity weighted by their respective book values. A graphic presentation of the cost of equity, cost of debt and cost of capital as leverage increases is normally provided. Generally, the graphic presentation is made in debt-to-equity space revealing a linear relationship between leverage and the cost of equity.

Most textbooks explicitly recognize that the cost of debt does increase with increasing levels of leverage due to the fear of default. However, an explicit connection between the increasing cost of debt and the optimum capital structure is often absent. Indeed, in some cases, the argument made is that an increase in the cost of debt is consistent with a constant overall cost of capital. For example in their undergraduate textbook, *Fundamentals of Corporate Finance*, Brealey, Myers and Marcus (2013) state that the overall cost of capital remains constant despite an increasing cost of debt: “Essentially because holders of risky debt begin to bear part of the firm’s operating risk. As the firm borrows more, more of the risk is transferred from stockholders to bondholders” (p. 454). Implicit in the statement is the assumption that stockholders remain essentially unscathed by bankruptcy.

As did Modigliani and Miller, textbook discussions then relax the no corporate tax assumption resulting in the conclusion that an optimum capital structure consists of 100% debt. This result relies on the implicit assumption that the cost of equity remains the same as given by MM II. Thus, the increasing cost of equity that was just sufficient to offset the use of cheaper debt when debt offered no tax advantage is now insufficient to keep the overall cost of capital constant, resulting instead in an ever decreasing cost of capital with each incremental addition of a unit of debt to that capital structure. This unrealistic result begs for resolution, which is then presented by bankruptcy costs.

Textbooks generally present a resolution to the 100% debt dilemma by identifying a tradeoff between the upside of the tax reduction and the downside of increased bankruptcy cost. When interest expense is taxable, as leverage increases, the higher total interest payments result in more tax deductions and lower taxes. The amount by which the higher interest payments lower taxes is commonly referred to as the interest expense tax shield. As leverage increases, however, the possibility of default also increases. Bankruptcy results in a firm having to pay a number of,
what can be significant, direct costs (e.g., legal, accounting, and administration fees) as well as indirect costs (e.g., loss of reputation and ability to purchase supplies/inventory on credit). The tradeoff theory suggests that an optimum capital structure exists when the present value of the tax shield equals the present value of bankruptcy costs. Significantly, textbook discussions of bankruptcy costs do not include a discussion of the cost of equity. Rather, bankruptcy costs appear to affect the debt holder. For example, Brealey, Myers and Marcus (2013) state that, in the case of bankruptcy involving highly marketable assets such as hotel properties, “The direct bankruptcy costs are restricted to items such as legal and court fees, real estate commissions, and the time the lender spends sorting things out” (p. 461). Notice, the cost is borne by the “lender.” Of course, in this situation, the stockholder bears no additional cost because this investor has lost everything. But no mention is made of the stockholder’s loss. The student is left to assume the cost of equity must remain as provided by the MM II equation, unaffected by fears of bankruptcy. Textbooks frequently discuss indirect costs of bankruptcy as well. One indirect cost often discussed is the games shareholders are assumed to play at the expense of the bondholder (an example of agency). It may seem to the student that the stockholder actually relishes the prospect of bankruptcy.

In many textbooks, the presentation of the tradeoff theory is supplemented with a further discussion of the financing decision. For example, textbooks frequently present the pecking order theory, in part to explain the lack of the utilization of the tax shield by highly profitable companies. Yet these additional discussions remain silent about the cost of equity. Again, the student is left to assume the cost of equity must remain as determined by the MM II equation.

In this paper, we argue that standard textbook explanations of capital structure are remiss for two reasons. First, we maintain that increases in the probability of bankruptcy do affect the cost of equity and find textbook presentations remiss because no consideration is given to this relationship. We argue this point in greater detail in Section II. Second, we maintain that the issue for the corporation is its market value weighted cost of equity and again find textbook presentations remiss because book values are used to determine the cost of equity. Modigliani and Miller assume the value of the corporation is derived only from income produced by its assets and all earnings are paid out as dividends. Textbook presentations emphasize the former and implicitly assume the latter, thus ignoring the possibility that market forces may price income streams differently depending on the division of the income stream. Brealey, Myers and Marcus (2013, p. 446) provide an example relying on the wisdom of Yogi Berra to make the point to students that value is unchanged by
selecting different financing options. The story proceeds as follows: Yogi is brought an after-game pizza. When asked by the delivery person whether he should slice the pizza, as usual, into four slices, Yogi asks for the pizza to be sliced into eight pieces because he is especially hungry on this night. The moral is clear. Slicing the pizza does not increase the total size of the pizza, thus implying that slicing the value of the corporation between debt and equity does not increase total asset value. We argue that the illustration misses the point that the market value of the corporation depends on the market value of the debt and equity issued by the corporation. We suggest that, just as a pizza vendor may be able to sell eight slices of a given pizza for more than four slices of the same pizza, even if the total size of the pizza remains the same, how the firm slices claims to its cash flow may make a difference as to the total value of the slices. We also argue this point in greater detail in Section II. In Section III we provide empirical evidence for our arguments. A conclusion is found in Section IV.

THEORETICAL CONSIDERATIONS

Implications of the Modigliani-Miller Model

We argue that rational investor response to moderate increases in leverage is inconsistent with key implications of Modigliani-Miller Proposition II. Careful analysis of MM II reveals several important implications of which we find no direct mention in extant literature. It is these key implications that we suggest are inconsistent with rational investor behavior.
Table 1:
Implications of Modigliani-Miller Proposition II for the Prototype Firm

| % Debt  | 0.00% | 10.00% | 20.00% | 30.00% | 40.00% | 50.00% | 60.00% | 70.00% | 80.00% | 90.00% |
|--------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| D/E    | 0.00  | 0.11   | 0.25   | 0.43   | 0.67   | 1.00   | 1.50   | 2.33   | 4.00   | 9.00   |
| Shares Out | 2.00 | 1.80   | 1.60   | 1.40   | 1.20   | 1.00   | 0.80   | 0.60   | 0.40   | 0.20   |
| Earnings | $10.00 | $10.00 | $10.00 | $10.00 | $10.00 | $10.00 | $10.00 | $10.00 | $10.00 | $10.00 |
| EAS    | $10.00 | $9.50  | $9.00  | $8.50  | $8.00  | $7.50  | $7.00  | $6.50  | $6.00  | $5.50  |
| EPS    | $5.00  | $5.28  | $5.63  | $6.07  | $6.67  | $7.50  | $8.75  | $10.83 | $15.00 | $27.50 |
| Book Equity | $100 | $90    | $80    | $70    | $60    | $50    | $40    | $30    | $20    | $10    |
| ROBE   | 10.00% | 10.56% | 11.25% | 12.14% | 13.33% | 15.00% | 17.50% | 21.67% | 30.00% | 55.00% |
| MM II RR | 10.00% | 10.56% | 11.25% | 12.14% | 13.33% | 15.00% | 17.50% | 21.67% | 30.00% | 55.00% |
| Share Price | $50 | $50    | $50    | $50    | $50    | $50    | $50    | $50    | $50    | $50    |
| St. Dev. ROBE | 5.00% | 5.56% | 6.25% | 7.14% | 8.33% | 10.00% | 12.50% | 16.67% | 25.00% | 50.00% |
| P/E    | 10.00 | 9.47   | 8.89   | 8.24   | 7.50   | 6.67   | 5.71   | 4.62   | 3.33   | 1.82   |

*“Earnings” and “EAS” are in $millions. “EAS” is earnings available to shareholders. “ROBE” is return to book equity. “St. Dev. ROBE” is the standard deviation of return on book equity. The calculation of “St. Dev. ROBE” value is described in End Note 10.*

We report these key results in Table 1 above for a “Prototype Firm” under various levels of leverage. Based on assumed structural characteristics for the firm, we allow leverage to vary and identify required market results given that MM II holds. We assume that the firm has assets of $100,000,000 with shares outstanding of 2,000,000, resulting in a market price of $50 per share for the unlevered firm. Assuming market efficiency, the average annual return on investment, ROI, is 10%. Thus, on average, the firm has operating income of $10,000,000 (0.10 * $100,000,000). Consistent with Modigliani’s and Miller’s analysis, we assume that all earnings are paid out as dividends. Given the number of shares outstanding and market price per share, the firm has unlevered earnings per share (EPS) of $5 ($10,000,000/2,000,000) and a price to earnings (P/E) ratio of 10 ($50/$5). Finally, we assume the level of business risk for the “Prototype Firm” corresponds to a
required annual return of 10% for an unlevered equity investment and a standard deviation in annual returns of 5%.

The first column in Table 1 represents results, as described above, for the unlevered firm given assumptions about the firm’s operating characteristics and assuming MM II holds. In each successive column, we increase the debt financing by 10%. In keeping with MM II, we assume the cost of debt remains constant, which we set on an *ex cathedra* basis at 5%. We assume the additional debt is used to retire shares of equity. Thus, in column 2 of Table 1, 10% of the firm’s assets are financed by debt and 90% are financed by equity. As shown in the second row, the debt-to-equity ratio (D/E) is 0.11 (0.10/0.90).

The third row, labeled “Shares Out,” reports the number of shares outstanding in millions of shares. The unlevered firm, as described above, has 2,000,000 shares outstanding. As reported in Table 1, when the firm is financed with 10% leverage, there now are 1,800,000 shares outstanding. Financing 10% of the firm with debt requires issuing $10,000,000 in debt, which is used to buy back outstanding shares. At $50 a share, a total of 200,000 ($10,000,000/$50) shares are repurchased, leaving 1,800,000 (2,000,000 – 200,000) remaining shares outstanding. Each subsequent 10% increase in debt financing results in the same number of shares being repurchased. Thus, at 20% there are 1,600,000 shares outstanding, and so on.

The fourth, fifth and sixth rows report earnings. Row four, labeled “Earnings,” reports the total earnings for the firm. We report average annual earnings in $millions which, according to assumptions made above, is 10% of the assets invested, or $10,000,000. The next row, labeled “EAS,” reports the earnings available to shareholders in $millions, which is the $10,000,000 earned by the firm minus the debt payment given the debt outstanding and the 5% interest paid on debt. As shown in Table 1, when the firm’s assets are financed 10% by debt, EAS is $9,500,000 ($10,000,000 – (0.05 * $10,000,000)). Increasing debt financing from 0% to 10% results in a $500,000 decrease in EAS, and the EAS continues to decrease by $500,000 for each additional 10% increase in debt financing given the fixed 5% interest charge. The sixth row reports earnings per share (EPS). This value is calculated by dividing EAS by the number of shares outstanding. As reported in Table 1, EPS is $5.28 ($9,500,000/1,800,000) when the firm’s assets are financed 10% by debt and $6.07 (48,500,000/1,400,000) when the firm’s assets are financed 30% by debt. EPS increases monotonically with an increase in debt as the average ROI of 10% is greater than the constant cost of debt of 5%.

The next row reports “Book Equity” in $millions. Given that the value of the firm’s assets does not change and, consistent with Modigliani-Miller Proposition II,
the value of the firm does not change with leverage, book equity plus the amount of debt issued must always equal $100,000,000. Therefore, book equity for the unlevered firm is $100,000,000 and with each debt issue of 10% of firm value, book equity declines by $10,000,000. Because we have calculated both earnings available to shareholders and book equity, it is an easy matter to calculate return on book equity, ROBE.\textsuperscript{7} For the unlevered firm ROBE = ROI = 10%. As with EPS, ROBE increases monotonically with increases in leverage as the average ROI of 10% is greater than the constant cost of debt of 5%; meaning the equity holders pocket the excess of ROI over the constant cost of debt. When 10% of the firm is financed by debt, the average ROBE is 10.56% ($9.5/$90) and, when 30% of the firm is financed by debt, the average ROBE is 12.14% ($8.5/$70).

In the next row we report the required return to equity as determined by MM II using the well-known Modigliani-Miller equation (equation (1)). For example, this relationship provides a required return for equity of 10.56% when the firm’s assets are financed by 10% debt and 12.14% when the firm’s assets are financed by 30% debt as shown by equations (2) and (3) below.

\[
RR_{\text{equity 10\% leverage}} = 10\% + .111 \times (10\% - 5\%) = 10.56\% \tag{2}
\]

\[
RR_{\text{equity 30\% leverage}} = 10\% + .429 \times (10\% - 5\%) = 12.14\% \tag{3}
\]

Thus, the required return to equity, according to MM II, is equal to the average ROBE. Indeed, if this were not the case, MM I would not hold either. The equality, however, emphasizes the mechanical nature of MM II and the lack of market behavior content in the hypothesis. MM II is intended to indicate the required return for investors and, as such, the goal of MM II should be to calculate ROME, return on market equity. There is no analysis to indicate why investors would demand exactly the return that is being observed for book equity under various levels of leverage.

The equality between ROBE and ROME, as required by MM II, results in a constant market price for equity shares regardless of the level of leverage. As shown in Row 10 of Table 1, the market share price, P, is a constant $50 across all levels of leverage. We know of no documentation of this unusual result, but it is easily illustrated from the data in Table 1. Recall that all earnings are paid out as dividends, so the ROME is simply EPS divided by the share price, P. Thus ROME= EPS/P; and
P = EPS/ROME. As shown below in equation (4) for each level of leverage the share price is $50.

\[
\text{EPS/ROME} = \frac{5}{0.10} = \frac{5.28}{0.1056} = \frac{5.63}{0.1125} = \frac{6.07}{0.1214} = \frac{6.67}{0.1333} \\
= \frac{7.50}{0.15} = \frac{8.75}{0.1750} = \frac{10.83}{0.2167} = \frac{15}{0.30} = \frac{27.50}{0.55}
\]

The constant share price accompanied by the ever increasing EPS across leverage levels leads to a monotonically decreasing price-to-earnings (P/E). And this ratio decreases dramatically. We find both the constant share price and the dramatic decrease in the P/E ratio to be suspect. In the next section we develop our arguments relative to this position.

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**Why MM II Should Not Hold Across Moderate Levels of Leverage**

As argued below, we find the supposition that the share price for a firm, with a given level of business risk, is constant across all levels of leverage to be untenable. Because a constant share price across levels of leverage is required by MM II, our position argues that MM II does not hold.

Results presented in Table 1 are based on assumptions about the firm size, number of shares outstanding for the unlevered firm and the level of business risk that associates with the given ROI (10%) and cost of debt (5%) for our “Prototype Firm.” These assumptions imply a share price of $50, but MM II requires a constant share price across all levels of leverage regardless of the exact price indicated by basic assumptions. Thus, the MM II result requires that investors are indifferent between the risk-return tradeoff existing at each level of leverage. To present the risk-return tradeoffs, we may compare expected return and standard deviation of return at every level of leverage. As reported in Table 1, for each level of leverage, we calculated a return to book equity that is identical to the market return required by MM II. We assumed a standard deviation of return of 5% for the unlevered firm, which we use to calculate standard deviation for every level of leverage. Financial risk is clearly evident from our calculations. Each increase in leverage results in an increase in variability of return and a dramatic increase in return for higher levels of leverage.

Note that the constant share price of MM II still allows investors to be risk averse. At the various levels of leverage, investors are paying the same for one share. This share does have higher risk, as measured by the standard deviation of returns,
for every level of risk, but the investor is being awarded higher payments. So investors are adhering to the risk-return tradeoff maxim. Our argument with MM II is not that the risk-return tradeoff is ignored, but that the risk-return tradeoff is forced to adhere to a particular indifference curve that does not have theoretical support and one which we find unreasonable. In the rest of this section, we present arguments suggesting that the cost of equity increases more moderately with leverage than dictated by MM I and MM II.

The data presented in Table 1 is annual data. We present the expected annual returns and the standard deviation in annual returns consistent with MM II. These calculations show that MM II requires that investors are indifferent between accepting an expected annual return of 10% with a standard deviation of 5% for the unlevered firm or accepting an expected annual return of 15% with a standard deviation of 10% for the levered firm with 50% debt or accepting an expected annual return of 55% with a standard deviation of 50% for the levered firm with 90% debt. Without a theoretical basis for this relationship we are suspect that it would actually hold. Certainly the reward to risk ratio varies greatly across the options ranging from 2.00 for the unlevered position to 1.10 for the 90% debt position. One may argue that the constant share price implication is supported in that it requires book equity and market equity values to be the same. However, casual empiricism shows that book equity and market equity values are seldom the same. So, indeed, the required equality between book and market values argues against the validity of MM II. Thus, the Modigliani-Miller arbitrage process must be relied upon to support investors’ indifference across the observed risk-return tradeoffs, but, as noted above, this process is suspect.

We next address the question of investor indifference and submit that investor reaction to changes in the risk-return tradeoff caused by the use of leverage is open to question. So, how should we describe the graphic presentation of the firm’s cost of equity to our students? We postulate that the cost of equity increases more slowly with moderate levels of leverage than envisioned by MM II. We do not, of course, argue against the proposition that stockholders are risk averse. Nor do we argue against the proposition that the use of leverage increases financial risk. Instead, what we argue is that stockholder reaction to an increase in financial risk is exaggerated by MM II for at least two reasons. First, casual observations of market behavior suggest a focus on earnings rather than variability in earnings. Second, conventional means of measuring return and risk tend to exaggerate the impact of leverage.

According to MM II and as stated above, investors in our “Prototype Firm” are indifferent between the firm being unlevered, receiving an average annual return
of 10% with a standard deviation of 5%, or the firm being financed with 50% debt that would provide an average annual return of 15% with a standard deviation of 10%. It is our experience that, when we show students these choices, the students are not indifferent; they choose the higher return. Of course, what is important is how actual market investors act. Observation of market behavior and investment analyst pronouncements suggests to us that the students are in tune with market behavior.

We posit that investors, although typically risk averse, are usually much more attuned to the return portion of the risk-return tradeoff than the risk portion. We posit that investors are more concerned about EPS than the variability of EPS. We argue that observable market behavior supports our position. A widely used tool by investment analysts is the P/E ratio. Application of this tool encourages investors to increase the price that they pay for a stock when EPS increases. Indeed, there are numerous studies that indicate that following such a strategy “beats the market.” (See, for example, Basu (1983)) There is no strategy that encourages investors to buy a stock when the return variability falls. Further, the P/E strategy is not modified by a metric measuring variability in earnings.

Investors eagerly await earnings announcements. Market pundits look for the “earnings season” to indicate market direction. These announcements are awaited to see if earnings are changing, not if the associated earnings variability is changing. Such announcements move the market, and often the announcements are made after market close to dampen their impact. We do not rely here on casual observance alone, although the uniformity of market reaction may make such observations sufficient. Numerous empirical studies show the long-term impact of positive and negative earnings announcements. Studies of standardized unexpected earnings (SUE) announcements show that unexpectedly high (low) earnings announcements lead to positive (negative) price drifts following the announcements. (See, for example, Rendleman, et. al. (1982)) The variable SUE is standardized by variability in earnings, not because investors are focused on variability in earnings, but rather to determine if the change in earnings is truly unusual. There is no measure that seeks to explain price changes by a change in the variability of earnings. Under SUE, if a firm announces a remarkably high quarterly EPS, a signal would be given to buy. If variability were the focus, this unusual EPS would be a sell signal as variability in earnings would be increased. Empirical evidence shows that it is the buy signal that is followed!

This general focus on earnings is further illustrated by what is reported in the financial press and on financial internet sites. Investors know these outlets report firms’ EPSs and highlight their P/E ratios. Yet these outlets do not typically report
variability in earnings, which is the measure of financial risk. Investors may find reports of debt ratios, which indicate leverage and financial risk, but these ratios are not routinely responsible for investment decisions. Changes in earnings are. Because investors focus on earnings, especially EPS, the result of a moderate increase in leverage will tend to be positive, thus increasing the value of the firm. The required payment to equity is not enough to keep the value of the firm constant as suggested by MM II and the cost of equity rises more slowly than suggested by MM II.

Our second rationale supporting the hypothesis that stockholder reaction to an increase in financial risk is exaggerated by MM II is that conventional means of measuring return and risk tend to exaggerate the impact of leverage. Textbook examples showing the creation of financial risk using leverage invariably illustrate the risk with a one-year time horizon. This is common and generally accepted practice, but, in this case as in others, the choice of this time horizon may distort reality. We illustrate our concern first with an example that instructors frequently use to advise students as young investors. The instructor could present students with a risk-return tradeoff, comparing U. S. government debt, say, for example, Treasury bills, with large-firm stocks.

To illustrate, according to Ibbotson (2014), from 1926 through 2013, the arithmetic average annual return to Treasury bills is 3.5% and the standard deviation in annual returns is 3.1%. In contrast, for large-firm equities, the 1926 through 2013 arithmetic average annual return is 12.1% and the standard deviation in annual returns is 20.2%. The instructor could use this data to illustrate the tradeoff between risk and return and may find students who prefer the Treasury bill over large-firm stocks when using a one-year investment horizon. And such a choice may be reasonable for an investment with a one-year horizon. But surely an instructor would counsel students against such a choice for retirement investment. A typical student would have 40 plus years until retirement. Regardless of the extent of the student’s risk aversion, the student should not select Treasury bills. There is no historic 40-year period where large-firm equities have a lower return than Treasury bills. Over this time horizon, large-firm equities are no longer riskier than Treasury bills. Using a one-year comparison provides a bias against the correct long-term investment choice.

A similar, if less extreme, bias exists when comparing equity investment between firms with similar business risk but different levels of leverage. To illustrate the impact of time horizon in this case, we compare returns and risk for our hypothetical firm between the unlevered position and the same firm with moderate leverage levels of 10%, 30% and 50%. Data in Table 1 indicates the following risk-
return tradeoffs: the unlevered firm has an average return of 10.00% and a standard deviation of 5.00%; the firm financed with 10% debt has an average return of 10.56% and a standard deviation of 5.56%; the firm financed with 30% debt has an average return of 12.14% and a standard deviation of 7.14%; the firm financed with 50% debt has an average return of 15.00% and a standard deviation of 10.00%. MM II requires that investors as a whole are indifferent between these four choices. Certainly individual investors might prefer any one of the four choices for a one-year investment. But what would the impact of a longer term time horizon have on an investor’s choice between these four levels of leverage?

To gain insight into this choice, we simulate 1,000 annual revenue flows to the firm with an average annual return of 10% and a 5% standard deviation. Then we calculate the annual returns to the unlevered firm and the percentage return for the four levels of leverage listed above. We then use this data to calculate overlapping three-year returns and overlapping ten-year returns for each of the four leverage levels. We calculate the average one-year, three-year and ten-year return and the standard deviations for these return series. We report these results in Table 2.

**Table 2:**
Average Returns and Return Standard Deviations—Various Time Horizons and Various Levels of Leverage

| Time horizon/leverage | Unlevered  | 10% debt | 30% debt | 50% debt |
|-----------------------|-----------|----------|----------|----------|
| One-year              | 10.03%    | 10.59%   | 12.19%   | 15.06%   |
|                       | (5.10%)   | (5.65%)  | (7.27%)  | (10.19%) |
| Three-year            | 33.25%    | 35.30%   | 41.26%   | 52.44%   |
|                       | (10.70%)  | (12.04%) | (15.86%) | (23.28%) |
| Ten-year              | 160.58%   | 173.91%  | 216.28%  | 307.68%  |
|                       | (38.02%)  | (44.53%) | (65.12%) | (114.90%)|

Each cell reports the average return and the standard deviation (shown in parentheses) for a given level of leverage and a given time horizon. The values result from the same simulation of 1,000 annual returns. The returns for the levered positions are based on a constant 5% cost of debt, consistent with our earlier examples.

We report these key results in Table 1 above for a “Prototype Firm” under various levels of leverage. Based on assumed structural characteristics for the firm, we allow leverage to vary and identify required market results given that MM II holds. We assume that the firm has assets of $100,000,000 with shares outstanding of 2,000,000 resulting in a market price of $50 per share, assuming market efficiency, for the unlevered firm. Finally, we assume the level of business
risk for the “Prototype Firm” corresponds to a required annual return of 10% for an unlevered equity investment of $10,000,000 (0.10 * $100,000,000). Consistent with Modigliani’s and Miller’s analysis, we assume that all earnings are paid out as dividends. Given the number of shares outstanding and market price per share, the firm has unlevered earnings per share (EPS) of $5 ($10,000,000/2,000,000) and a price to earnings (P/E) ratio of 10 ($50/$5).

To provide this different perspective, we use the same simulated 1,000 annual returns to assets and the corresponding one-year, three-year and ten-year returns for the unlevered position and each of the three levered positions, the average of which we reported in Table 2, but this time compare the levered and unlevered positions across each observation of overlapping 3-year and 10-year returns. On a period by period basis, we compare the returns and identify the percent of the periods when the unlevered position outperformed each of the three levered positions. The results are presented in Table 3.

| Time horizon/leverage | 10% debt | 30% debt | 50% debt |
|-----------------------|----------|----------|----------|
| One-year              | 14.90%   | 14.90%   | 14.90%   |
| Three-year            | 3.91%    | 4.01%    | 4.31%    |
| Ten-year              | 1.01%    | 1.01%    | 1.01%    |

In our example, consistent with Modigliani’s and Miller’s analysis, the firm pays a constant 5% for debt. Thus, any year in which ROI is less than 5% all levered positions will underperform the unlevered position. In our simulation, this occurs for 14.90% of the years. An investor with a one-year time horizon faces a reasonable probability that the unlevered position would outperform a levered position. But, as the time horizon for holding the investment increases, the likelihood that the investor would ever be better off with the unlevered position diminishes. For investors with a three-year holding period, the number of periods in which the unlevered position provides a better return is around 4% with the probability increasing as leverage increases. Finally, if the investor holds the position for 10 years, there is only a 1% chance that the unlevered position outperforms any of the levered positions. For
an investor with a ten-year investment horizon, financial risk virtually disappears. If we assume two firms with the exact same return on assets and the exact same business risk, and further assume that one firm has no debt while the other has a moderate level of debt, an investor with a ten-year time horizon should pay more for the levered firm. The additional financial risk from leverage will impact an investor with a one-year time horizon, but not this investor.

In summary, we maintain that moderate levels of leverage increase the value of the firm because equity investors are less concerned with financial risk than assumed by MM II. We argued this on the basis that investors in general focus more on returns than risk. We also argued that financial risk resulting from leverage causes little increases in risk for the long-term investor and that this investor will pay more for shares of a levered firm than for shares of an unlevered firm identical in all other characteristics. But will this investor influence market price?

Some market participants do not care at all about financial risk. For example, high-frequency traders, who account for a significant percent of market volume, are not concerned about financial risk caused by leverage. Instead, high frequency traders care about the immediate direction of the market. Likewise, momentum traders are completely unconcerned about financial risk; they buy and sell stocks assuming that recent price trends will continue. Indeed, all market timers including technicians are blithely unaware of any impact from financial risk. MM II only holds if investors are fundamentalists who weigh risk against return. And, for these investors, as we have argued, their time horizons must be short enough so that financial risk provides a sufficiently large probability that leverage will reduce return so that the investors are unwilling to pay for an increase in EPS. We submit that these conditions are so restrictive as to make MM II improbable and that the increase in EPS created by leverage will increase firm value.

Why MM II Should Not Hold At Extreme Levels of Leverage

How do equity holders react to extreme levels of leverage? We suggest above that equity holders should react by demanding payment for bankruptcy risk in addition to payment for financial risk. It is accepted that, at high levels of leverage, debtors react by demanding higher payments because of bankruptcy risk. In other words, lenders demand higher interest payments because they fear they will not receive full payment. Modigliani and Miller (1958) recognize this possibility yet suggest that, at high levels of leverage, equity investors might react by decreasing their required return below that suggested by MM II. This reduction in the expected
cost of equity is required in order to offset the increasing cost of debt so that the overall cost of capital remains constant. But why should equity holders be willing to accept a less than “fair” payment for financial risk in the face of increasing default risk? Do equity holders somehow benefit from increasing risk of bankruptcy? Or why is it that equity holders do not fear a high increased bankruptcy risk associated with high levels of debt? The finance discipline has not responded in a constructive manner to this dilemma!

Textbook presentations recognize a tradeoff between the present value of the tax benefits of debt and the present value of bankruptcy costs that occur with increasing levels of debt. These bankruptcy costs are direct costs, such as legal fees, and indirect costs, such as the ability to purchase supplies/inventory on credit, both of which are very different from “fear of bankruptcy” additional payments demanded by bond holders and lenders. What a paradox that bond holders receive payment for bankruptcy risk but not stock holders! Moreover, bond holders receive payment before equity holders, and, when bankruptcy occurs, equity holders may lose their entire investment before bond holders suffer any loss. Indeed, this difference in priority alone justifies the higher payment to equity relative to debt at any level of leverage. Is it not logical then, that when the probability of bankruptcy is high, because of a combination of business risk and leverage, that equity holders will demand a very high return? This should be observed in the market by investors lowering equity prices when bankruptcy looms in order to receive a very high return if, somehow, the firm overcomes the business risk from operations and the financial risk from leverage.

**EMPIRICAL EVIDENCE**

*Moderate Increases in Leverage*

We have argued that, with moderate increases in leverage, the cost of equity grows more slowly than required by MM II and that, for very highly levered firms, the cost of equity rises precipitously. Our arguments, of course, run counter to widely held dogma and will likely be received with some reservation. We point out, however, that there are implications of MM II, such as the constant share price across leverage, which seem quite suspect. Still it behooves us to look for empirical evidence in support of MM II.

Possible sources of empirical evidence to support MM II could consist of: 1) evidence of arbitrage activity to establish MM II equilibrium; 2) evidence of increases
in the cost of equity as leverage increases among similar firms; 3) evidence of equal share price, scaled for asset size and shares outstanding, as leverage increases among similar firms; 4) evidence of constantly decreasing P/E ratios as leverage increases among similar firms. The last three possibilities are consistent with relationships reported in Table 1.

With regard to Possibility 1, we simply note that we know of no report of arbitrage activity to support MM II equilibrium. Possibility 2 presents timing difficulties and measurement issues as it is not obvious how the reported changes in debt ratios match reported returns to equity associated with changing debt ratios. Possibility 3 reduces the timing issues but presents the difficulty of scaling as price will depend on decisions concerning the number of shares outstanding relative to a given value of book equity. Possibility 4 also reduces the timing issue and has the advantage of removing the difficult scaling problem. So, we conduct an empirical test of possibility 4.

Our sample first consists of the firms comprising the Dow Jones Industrial Average (DJIA) as of January 1, 2009. We then eliminate the four financial firms (American Express Company; Bank of America; JPMorgan Chase & Co.; The Travelers Companies, Inc.) because the nature of their financial structure has the potential to bias our results. Finally, we further adjust our sample by adding Nike, Inc., which was added to the DJIA on September 20, 2013. Thus our final sample includes twenty-seven firms from the DJIA.

We gather data for these firms for a five-year period: 2009 through 2013. This sample should provide a general similarity in firm business risk allowing an opportunity to measure the impact of changes in the debt ratio. We then calculate the debt ratio at the end of each fiscal year. (A description of the procedure used to determine the debt ratios is found in Appendix 1.) There is a considerable range of debt ratios across our twenty-seven sample firms, but in all cases the ratios may be considered to represent moderate to low levels of leverage. Table 4 reports the firms with the two highest and the two lowest debt ratios among the twenty-seven firms for each of the five sample years and reports the value of the debt ratios. Debt ratios for all twenty-seven firms for each year are available from the authors.
Table 4: Extreme Sample Observations of Debt Ratios by Years

| Year | Highest Ratios | Next Highest Ratios | Next Lowest Ratios | Lowest Ratios |
|------|---------------|---------------------|--------------------|--------------|
|      | Ratio         | Firm                | Ratio             | Firm         |
| 2009 | 35.00%        | McDonald's          | 4.18%             | Exxon        |
|      |               | DuPont              |                    |              |
| 2010 | 35.98%        | McDonald's          | 4.11%             | Intel        |
|      |               | Coca-Cola           |                    |              |
| 2011 | 37.89%        | McDonald's          | 4.85%             | Nike         |
|      |               | Coca-Cola           |                    |              |
| 2012 | 38.52%        | McDonald's          | 3.47%             | Nike         |
|      |               | Coca-Cola           |                    |              |
| 2013 | 41.17%        | Coca-Cola           | 6.55%             | General Electric |
|      |               | McDonald's          |                    |              |

We also gather the P/E ratio for each firm in our sample for each of the five years in our sample period. (A description of the procedure used to determine the P/E ratios is also found in Appendix 1.)

We conduct ordinary least square regressions of the debt ratio against the P/E ratio. We conduct separate regressions for each of the five sample years. As described above, according to MM II, increases in financial risk should increase the required cost of equity in a prescribed manner, leaving share price unchanged but constantly decreasing the P/E ratio. Support for MM II would be found if the changes in the debt ratio explain a significant part of the variation in the P/E ratios and if the slope coefficient for the debt ratio is significantly negative showing a decrease in the P/E ratio across firms as leverage increases.

Our results, shown in Table 5, provide no evidence to support the MM II position. As shown in Table 2, according to MM II, the P/E ratio should show a constant decrease with increases in the debt ratio. Our results indicate that there is no relationship between the debt ratio and the P/E ratio for our sample firms. In four of the five years, the adjusted r-squared is less than zero. In three of the five
years the slope coefficient, inconsistent with MM II, is positive. In no case is the relationship with the debt ratio and the P/E ratio significant at the 0.10 level. The relationship is strongest for 2011, but in this year the relationship is contrary to the MM II hypothesis. Thus, comparing the debt and P/E ratios of our sample DJIA firms provides no evidence in support of MM II. Rather, the evidence is consistent with the view that with moderate levels of leverage the cost of equity rises slowly.15

Table 5:
Annual Regressions of P/E Ratios Against Debt Ratios for Sample DIJA Firms

| Year | 2009  | 2010  | 2011  | 2012  | 2013  |
|------|-------|-------|-------|-------|-------|
| Adjusted R-squared | -4.13% | -3.36% | 5.31% | -2.20% | -4.15% |
| Y-intercept | 14.79 | 15.24 | 9.71 | 13.42 | 16.56 |
| Slope coefficient | -0.013 | 0.198 | 0.285 | 0.361 | -0.064 |
| t-statistic | -0.09 | 0.39 | 1.57 | 0.68 | -0.05 |

We conduct ordinary least square regressions for each of the five sample years, regressing debt ratios against P/E ratios. We omit any observation with a negative P/E ratio resulting in a sample size of 26 for years 2009, 2012, and 2013. There are no omissions for 2010 and 2011 resulting in a sample size of 27.

**Extreme Leverage**

We have argued that, when extreme levels of leverage cause bondholders to demand higher returns because of bankruptcy fears, equity holders must also be increasing the required return in excess of what otherwise would be demanded as payment for financial risk. As equity holders will lose before bondholders, they must logically demand payment for bankruptcy risk if bondholders are demanding payment for bankruptcy risk. Providing empirical evidence of this relationship is fraught with difficulties in separating the price impact of bankruptcy fears from other factors. And, we would not expect to find evidence of new equity issues when bankruptcy risk is high because of the expected high cost of such an issue. We, therefore, seek to find anecdotal evidence of the behavior of equity holders when bankruptcy fear has been high.

One may argue that, in recent market experience, the 2008 financial crisis would provide a time when bankruptcy fear was high for selected firms in financial distress. We use a year-end financial article, Kiviat (2008), to identify firms with
big loses during 2008. These were firms where stockholders feared bankruptcy. According to Modigliani and Miller (1958) and Brealey, Myers and Marcus (2013), when lenders begin to worry about bankruptcy because of high leverage, they demand a higher return, but stockholders reduce the rate at which their required return would normally increase to compensate for financial risk. We have argued that stockholders should fear bankruptcy more than bondholders. When bankruptcy looms, therefore, stockholders should demand a higher return than otherwise required to compensate for financial risk. Although this higher required return will not be observable from new stock issues, the higher return will be observed when new stockholders purchase outstanding shares at prices sufficiently low so as to guarantee high returns if the firm survives.

To see if our expectations are met, we determine the return to the firms identified in Kiviat (2008) from their low point in 2008 through the end of our sample period December 2013. (We add Ford Motor Company to our sample because of the publicity surrounding Ford’s possible bankruptcy during the fall of 2008.) If our argument is correct, the shares of firms that suffered large loses in 2008 would be sold at distress prices reflecting the new investors’ high required returns. If the firms survived, the new investors should enjoy high actual returns reflecting their high required return engendered by the fear of bankruptcy. On the other hand, for some of the firms the investors’ fears should be realized. These firms should experience bankruptcy and a return of -100%. To determine actual returns, from the 2008 nadir to the end of 2013, we search Yahoo Finance for the lowest adjusted close in 2008 and compare that return to the adjusted close reported on Yahoo Finance for December 31, 2013. Results reported in Table 6 indicate that this was exactly what happened. Surviving firms experienced very high returns indeed and some firms experienced a return of -100%. We submit this result as anecdotal evidence that when high levels of leverage engender fear of bankruptcy, the cost of equity climbs steeply in opposition to the supposition of Modigliani and Miller.
Table 6:
Returns to Extreme Losers of 2008

| Company          | 2008 Lowest Adjusted Close | Dec 31 2013 Adjusted Close | 2008 to 2013 Gain |
|------------------|-----------------------------|-----------------------------|-------------------|
| AIG              | $22.10                      | $50.58                      | 129%              |
| Ambac            | $0.76                       | $24.56                      | 3,132%            |
| Borders          | $0.37                       | $0.00                       | -100%             |
| Crocs            | $0.94                       | $15.92                      | 1,594%            |
| Fannie Mae       | $0.30                       | $3.01                       | 3,078%            |
| Ford             | $1.17                       | $14.95                      | 1,178%            |
| Freddie Mac      | $0.23                       | $7.31                       | 3,078%            |
| MF Global        | $1.73                       | $0.00                       | -100%             |
| Pier 1 Imports   | $0.31                       | $22.73                      | 7,232%            |
| Ruby Tuesday     | $1.08                       | $6.93                       | 542%              |

CONCLUSIONS

We have argued that textbook presentations of the response of stockholder required return to equity is deficit in two important aspects. In both cases, we argue, these deficits associate with an uncritical acceptance of the Modigliani and Miller (1958) propositions. Specifically, we argue that: 1) at moderate levels of leverage, the required return to equity rises less than postulated by Modigliani and Miller because equity investors’ reaction to leverage emphasizes the higher expected EPS rather than the additional financial risk; and 2) at high levels of leverage, when debt holders demand compensation for bankruptcy risk, stockholders will also demand compensation for bankruptcy risk. The latter argument is in contrast to the Modigliani-Miller argument that, when bankruptcy risk is high, stockholders will reduce the payment required for financial risk. If one accepts our arguments, then the Modigliani-Miller Irrelevance Proposition does not hold. We support our theoretical arguments by two empirical studies. Examination of the relationship between debt and P/E ratios among sample DJIA securities over a five-year period fails to find an inverse relationship between these ratios as required by the Modigliani-Miller propositions. Further, we show that during the 2008 market crisis for firms that appeared to be in danger of bankruptcy, new stockholders demanded a very high
expected return as evidenced by the low prices they paid for the stocks and the subsequent high actual returns if the firms avoided bankruptcy.

Given these results, instructors could discuss optimum capital structure using the sketch below (Figure 1). The instructor can explain that the cost of equity, \( k_e \), is more than the after-tax cost of debt, \( k_d \), at every level of leverage for two reasons. First, payment to debt holders is more certain. The instructor can emphasize the certain payment to the bondholder required by the bond indenture relative to the uncertain payment to the shareholder. Second, the payment to debt is tax deductible. To illustrate this point, the instructor may wish to provide a simple income statement example. The instructor can show how both of these benefits result in a higher expected EPS for the stock holder. The instructor could then use a simple WACC example to show how the overall cost of capital would fall using debt. The instructor would need to take care, however, to emphasize the downside of using more debt to the stockholders. A careful example showing multi-year returns or multiple scenarios for a single year could introduce the topic of financial risk. This provides the instructor with another chance to emphasize the risk-reward tradeoff showing an increase in \( k_e \). As we have argued, however, we feel that the instructor should show a declining overall cost of capital, because of both the tax effect and the relatively slow increase in \( k_e \) at moderate levels of leverage. Finally, the instructor can emphasize, the increase in both \( k_e \) and \( k_d \) as both the bond holders and stockholders begin to fear bankruptcy. We feel that this discussion should emphasize the loss of invested wealth rather than the emphasis on legal costs of bankruptcy that textbooks tend to emphasize.
Of course, these are not the only topics that an instructor may wish to cover when discussing a corporation’s decision between issuing debt or equity. Among other possible topics an instructor could include are indirect costs of bankruptcy, effect of a debt issue on financial control, need to preserve financial slack, pecking order considerations in issuing debt versus equity. On a purely theoretical level, the instructor may wish to discuss the topics of homemade leverage and the Modigliani-Miller arbitrage action in support of MM II.

**APPENDIX 1**

**Procedure For Determining Debt Ratios**

A. Fiscal year end (FYE) financial statements are obtained from annual reports, 10-Ks, or both. All dollars are in millions. For companies whose FYE is in January or February, data is from 2010 to 2014. For all other companies, data is from 2009 to 2013.
B. Total assets included cash equivalents, marketable securities, etc.; assets for sale; investments; and financing related assets (e.g., notes receivables). If the company has a separate financing subsidiary (Caterpillar, General Electric), the subsidiaries “total assets” were excluded.

C. Total liabilities included short-term notes payable, current portion of long-term debt due, long-term debt, and short-term and long-term capital leases if itemized on the balance sheet. Capital leases are assumed not material if not itemized on the balance sheet. If the company has a separate financing subsidiary (Caterpillar, General Electric), the subsidiaries financing line items were excluded, similar to how “total assets” were adjusted.

Procedure For Determining Price/Earnings Ratios

A. Stock Prices at Fiscal Year End (FYE) 2009 through 2013 were downloaded from http://finance.yahoo.com/. Because Home Depot and Walmart have January FYEs, the prices for those companies are from 2010 through 2014. The specific values are the adjusted close prices closest to but not past the FYE date or month as given the FYE financials. For example, if a company has a December 31st FYE, the December 31st 2013, December 31st 2012, December 30th 2011, December 31st 2010, and December 31st 2009 adjusted close prices are used.

B. Annual Earnings Per Share (EPS)

Companies report “basic” EPS and “diluted” EPS in their FYE financials. The “basic” EPS value is used for determining the P/E ratios. Further, “basic” EPS can consist of both EPS from continuing operations and EPS from other activities (e.g., discontinued operations). Some companies break out the “basic” EPS into these two components, some do not. For those companies that did not, the EPS from continuing operations was not readily apparent and the EPS used is the net (total) “basic” EPS.

END NOTES

1. Lander and Pettengill (2012) show that the Modigliani-Miller arbitrage process, and thus both MM I and MM II, are inconsistent with an increase in the cost of debt with increasing leverage.

2. Book values do not change when all earnings are paid out as dividends. Also textbooks do not mention market values implicitly making the assumption that market values equal book values.
3. An exception to this presentation is provided by Brigham and Daves (2013) where the graphic presentation of the cost of equity is shown in debt ratio space. Lander and Pettengill (2010) argue that presentation of the cost of equity in the standard debt-to-equity space provides students with an inappropriate impression that the increase in the cost of equity is moderate at high levels of leverage.

4. We choose to exemplify textbook examples using the Brealey, Myers and Marcus undergraduate textbook because of the high regard with which this textbook is held and the special rigor of their undergraduate text. We examine other textbooks listed in the reference to ensure consistency with our comments.

5. The interest expense deduction is not available in all countries. Estonia, for example, has no corporate tax.

6. Miller is quoted using this example to summarize their theory quickly in Tanous (1997).

7. The normal calculation of return on equity, ROE, is calculated as earnings available to shareholders divided by book equity value. We take the liberty of labeling this value ROBE because of the importance of the difference in return to book value and return to market value for our analysis.

8. The precise assumption made concerning the level of business risk as measured by the standard deviation of returns and the required return for that business risk do not affect the resulting conclusions. If, for example, we had assumed an average annual return of 8% with a standard deviation in annual returns of 4%, we still would have reached the conclusion that share prices would be constant across all levels of leverage if MM II were to hold absolutely. The exact price would depend on the ratio of total assets to shares outstanding for the unlevered firm. A firm with $200,000,000 in assets and 8,000,000 shares outstanding would dictate a constant price of $25.

9. Investors are indifferent because they equally value, in present value terms, each risk-return tradeoff. Consider two assets. The first has a one year effective life, is expected to generate $105 at the end of year 1, and has a 5% expected rate of return. In present value terms, this asset is worth $100. The second also has a one year effective life, but is expected to generate $110 at the end of year 1, and has a 10% expected rate of return. In present value terms, this asset also is worth $100. We would say investors are indifferent, in present value terms to the two assets.
10. The expected return to the firm with any given level of leverage may be determined as follows: \( \text{ROBE} = \frac{(\text{EBI} - I)}{(K \times \text{TA})} \), where ROBE is return to book equity, EBI is expected earnings before interest, I is the constant interest payment for the level of leverage, K is \( 1 - \text{the percent of debt} \) (in decimal format) and TA is total assets. Expected return to the unlevered firm is assumed to be normally distributed with a mean of 10% and a standard deviation of 5%. The variance of expected returns is determined as \( \frac{1}{K^2} \times \text{Var}(\text{EBI}/\text{TA}) \). For the unlevered firm, the variance of expected returns is \( \frac{1}{1^2} \times (25\%) = 25\% \). When the firm is 10% debt financed, the variance of expected returns is \( \frac{1}{0.9^2} \times (25\%) = 30.86\% \). The standard deviation reported in Table 1 is the square root of this value.

11. Instructors may find the calculation of standard deviation a more effective way of illustrating financial risk than the traditional one year’s return with three states of nature: good, ordinary and bad.

12. For an alternative view see Pastor and Stambaugh (2012).

13. We ran the simulation 10 times with each simulation including 1,000 revenue observations. From these 10 simulations we selected for presentation the simulation that provided results closest to the assumed firm structure. This selection does not bias our comparison between leverage levels, but merely allows our comparisons to be made with the assurance that our comparisons are made with data that are consistent with the annual returns calculated in Table 1.

14. The 1% is as high as it is because of an extreme outlier in an annual return. In one year, the simulation showed an annual return of less than -37%. This annual return was included in ten of the ten-year overlapping return periods. In each one of these ten-year periods, but in no other ten-year period, the unlevered firm outperforms. This extreme value was more than 8 standard deviations below the mean; the highest annual return was just three standard deviations above the mean. We suspect that if the simulation was run again there would be zero periods in which the unlevered position outperforms over a ten-year period. In the interest of academic integrity, we report results from the original simulation.

15. We also test for a relation between leverage and the cost of equity. We find the debt ratios for our sample firms as described in Appendix 1. The five-year returns to equity from the 12/13/08 to 12/31/13 adjusted close prices (Yahoo Finance). There should be a positive relationship between the debt ratio and the cost of equity if MM II holds. We run two regressions. First we regress the 2009 debt ratio against the five-year return for each firm in our sample with the following results: \( R^2 = 0.61\% \), adjusted \( R^2 = -3.36\% \); the sample regression line
of the 2009 debt ratio against the five-year return to equity is \( Y = 30.03\% + 0.16 \times X \) with \( t = 0.39 \) for the slope coefficient. We also regressed the change in the debt ratio over the five-year period against the five-year return for each firm with the following results: \( R^2 = 0.90\% \), adjusted \( R^2 = -3.06\% \); the sample regression line of the 2009 to 2013 change in debt ratio against the five-year return to equity is \( Y = 34.10\% - 0.30 \times X \) with \( t = 0.47 \) for the slope coefficient. Contrary to the very basis of MM II, we find no evidence of increased required return with regard to moderate increases in leverage.

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