Gribov Noise on the Lattice Axial Current Renormalisation Constant

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October 23, 2018
Abstract

We study the influence of Gribov copies, in the Landau gauge, on the lattice renormalisation constant $Z_A$ of the axial current, obtained from a Ward identity on quark state correlation functions, with the Clover action, in quenched $SU(3)$ gauge theory. A comparison between the gauge invariant determination of $Z_A$ and the gauge dependent one is discussed. On a $16^3 \times 32$ lattice at $\beta = 6.0$ and with $K = 0.1425$, the values, on a sample of 36 configurations, are: $Z_A = 1.08(5)$ (gauge dependent calculation) and $Z_A = 1.06(2)$ (gauge independent calculation). We find that the residual gauge freedom associated to Gribov copies induces observable effects, which, at the level of numerical accuracy of our simulation, are included in the statistical uncertainty inherent in a Monte Carlo simulation. Doubling the statistics suggests that the fluctuation due to the lattice Gribov ambiguity scales down at least as fast as a pure statistical error.
1 Introduction

In continuum non Abelian field theories, most popular choices of fixing the gauge (e.g. Landau, Coulomb) suffer from the Gribov ambiguity [1]. It is now well established that this problem also affects the lattice formulation of these theories [2]-[4]. This problem has been neglected for a long time because, in principle, the computation of gauge invariant operators in compact lattice theories does not require gauge fixing. Fixing the gauge is, however, necessary in several cases. Monopole studies in SU(2) pure gauge theory have been done in the unitary gauge and the effect of the Gribov ambiguity on the number of SU(2) monopoles has been investigated [5, 6]. The authors conclude that, in their case, the Gribov noise does not exceed the statistical uncertainty. In SU(3) gauge theory, gauge fixing is essential in the the computation of gauge dependent quantities, such as gluon and quark propagators. There are now several studies of lattice propagators. The gluon propagator has been calculated in [7]-[9] with the aim of studying the mechanism through which the gluon may become massive at long distances. More recent attempts have investigated its behaviour as a function of momentum [10, 11]. Analogous studies have also been performed on the quark propagator (see, for example [9]). In practice, there are also cases in which it is convenient to implement a gauge dependent procedure for the computation of gauge invariant quantities [12]-[14]. For example, smeared fermionic interpolating operators are widely being used in lattice QCD spectroscopy and phenomenology, in order to optimise the overlap of the lower-lying physical state with the operator. The point-splitged smeared operators are gauge dependent, and therefore the gauge must be fixed before they are calculated. In particular, the calculation of the decay constant of the $B$ meson in the static approximation, in which the $b$-quark has infinite mass, requires the computation of the two point correlation function of the axial current. The isolation of the lightest state at large times is not possible if local (gauge invariant) operators are used. A nice way out consists in smearing the bilocal operator over a small cube and extracting $f_B$ by forming suitable ratios of smeared and local correlation functions [13]. This is an explicitly gauge dependent procedure which is most naturally carried out in the Coulomb gauge. In ref.[14] the smeared - smeared correlation functions on a few individual configurations were computed. Two Gribov copies were produced per configuration. The Gribov noise on individual configurations was found to
vary from 1% to 50% depending on the time-slice, which implies that it may still be a considerable effect after averaging over configurations. However, it was not possible to estimate its effect beyond individual configurations. The reason is that in such a study other sources of error dominate, such as the systematic error arising from fitting the exponential decay of the correlation function with time. Thus the isolation of the Gribov noise is difficult.

In this paper we study a different physical quantity, namely the renormalisation constant $Z_A$ of the lattice axial current. A knowledge of these renormalisation constants is necessary for matching the matrix elements computed using lattice simulations to those required in a definite continuum renormalisation scheme. Provided that the lattice spacing is sufficiently small it is possible to calculate these renormalisation constants in perturbation theory. For a more reliable determination of these constants it has been suggested to impose the chiral Ward identities of QCD non-perturbatively \cite{17, 18}. Here we focus our attention on the determination of the rôle of the Gribov ambiguity in the calculation of $Z_A$, obtained from quark state correlation functions.

A recently proposed method to determine $Z_A$ and other renormalisation constants, based on truncated quark Green functions in momentum space \cite{19} can also in principle be afflicted by Gribov fluctuations.

Since reasonably small errors are expected, in this kind of calculations, it is crucial to investigate the rôle of the Gribov noise. Moreover, the renormalisation constant $Z_A$ of the axial current is particularly well suited to the study of the Gribov fluctuations, mainly for two reasons. Firstly, $Z_A$ can be obtained from chiral Ward identities in two distinct ways: a gauge independent one, which consists in taking the matrix elements between hadronic states, and a gauge dependent one, which consists in taking the matrix elements between quark states. Hence, there is an explicitly gauge invariant estimate of $Z_A$ which is free of Gribov noise and which can be directly compared to the gauge dependent, Gribov affected, estimate. The second advantage is that $Z_A$ is obtained by solving a first degree algebraic equation for each lattice time slice, thus avoiding the usual systematic errors arising from fitting exponentially decaying signals in time.

\footnote{More recently, a high statistics study of $f_B$ in the static limit \cite{16} uses a different method for constructing ratios of smeared and local correlators which avoids fitting. This method, however, requires a large temporal extension of the lattice.}
2 Chiral Ward identities for $Z_A$

The theoretical framework for the non-perturbative evaluation of $Z_A$ for Wilson fermions, has been developed in [18]. The renormalisation constant is obtained through Ward identities generated by axial transformations. A first application of these techniques in numerical simulations using the Wilson action was attempted in [20]. The extension of these methods to the $O(a)$ improved Clover action [21] ($a$ is the lattice spacing) was presented in [22], which we follow most closely. Here we only give a brief outline of the results which are essential to our work.

In this study terms that, close to the continuum limit, are effectively of $O(a)$ are eliminated by using the Clover action [23] and rotating all quark fields of the matrix elements according to the "improved improvement" prescription of [24]:

$$\psi \to (1 - \frac{ra}{2} \gamma \cdot \vec{D})\psi \quad ; \quad \bar{\psi} \to \bar{\psi}(1 + \frac{ra}{2} \gamma \cdot \vec{D})$$ (1)

($r$ is the Wilson term parameter; in this work $r = 1$). The two fermion local operators considered in the following are the axial and vector currents and the pseudoscalar density:

$$A^f_\mu(x) \equiv \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\lambda^f}{2}\psi(x)$$

$$V^f_\mu(x) \equiv \bar{\psi}(x)\gamma_\mu\frac{\lambda^f}{2}\psi(x)$$

$$P^f_5(x) \equiv \bar{\psi}(x)\gamma_5\frac{\lambda^f}{2}\psi(x)$$ (2)

($f$ is a flavour label and the notation is generic for any quark fields $\psi$ and $\bar{\psi}$). In order to ensure that the lattice axial current $A^f_\mu$ has the correct chiral properties, it is normalised by a renormalisation constant $Z_A$ [18]; this implies that

$$2\rho = \frac{\partial \int d^3\vec{y} < A^f_4(\vec{y}, t_y)P^{+f}_5(0, 0) >}{\int d^3\vec{y} < P^{+f}_5(\vec{y}, t_y)P^{+f}_5(0, 0) >} = \frac{2m}{Z_A}$$ (3)

($m$ is the bare quark mass and $|P>$ the pseudoscalar state). As can be seen from the above equation, $\rho$ is gauge invariant.
The gauge dependent calculation of $Z_A$ relies on the following Ward identity for quark Green functions \[18, 22\]

\[
2\rho \text{Tr} \left[ \int \! d^4 x \int \! d^3 \vec{y} < u_\alpha(y) (\bar{u}(x)\gamma_5 d(x)) \bar{d}_\beta(0) > \right] = \left( \frac{1}{Z_A} - \rho r a \right) \text{Tr} \left[ \int \! d^3 \vec{y} < (\gamma_5 d(\vec{y})\bar{d}(0) + u(\vec{y})\bar{u}(0)\gamma_5) >_{\alpha,\beta} \right] \quad (4)
\]

In eq.(4) we work explicitly with up and down quark fields with spinor labels $\alpha$ and $\beta$. The trace is over colour indices. The expectation values on both sides of (4) are evaluated as functions of $t_y$. Taking the value of $2\rho$ obtained using eq.(3), $Z_A$ can then be determined. In order to enhance the signal, we add in both sides of (4) the four contributions $(\alpha,\beta)=(1,3), (3,1), (2,4)$ and $(4,2)$, which were found to give the clearest signal \[22\]. A plateau in $t_y$ is typically obtained and $Z_A$ is estimated from it. The crucial point is that both sides of eq.(4) are gauge dependent, and thus this determination of $Z_A$ is in principle sensitive to the Gribov noise.

A gauge invariant determination of $Z_A$ is obtained through the Ward identity

\[
2\rho \int \! d^4 x \int \! d^3 \vec{y} < P_3^f(x) A_\nu^g(y) V_{\rho}^h(0) > = -i \left( \frac{Z_V}{Z_A} - \rho r a \right) f^{fgh} \int \! d^3 \vec{y} < V_\nu^f(y) V_{\rho}^h(0) > \\
- i \left( \frac{1}{Z_V} - \rho r a \right) f^{fhl} \int \! d^3 \vec{y} < A_\nu^g(y) A_{\rho}^l(0) > \quad (5)
\]

In the above equation, the vector current renormalisation constant $Z_V$ is also needed. For the Clover action, $Z_V$ is calculated with the aid of the so-called conserved and improved vector current

\[
V_{\mu}^{CI}(x) = \frac{1}{4} \left[ \bar{\psi}(x)(\gamma_\mu - r)U_\mu(x)\psi(x + \hat{\mu}) + \bar{\psi}(x + \hat{\mu})(\gamma_\mu + r)U_\mu^\dagger(x)\psi(x) \right] \\
+ (x \to x - \hat{\mu}) + \frac{r}{2} \sum_\rho \partial_\rho \left( \bar{\psi}(x)\sigma_{\rho\mu}\psi(x) \right) \quad (6)
\]

Since the current is conserved, its renormalisation constant is precisely 1, and, since it is “improved”, its matrix elements have no corrections of $O(a)$. The normalisation constant $Z_V$ of the local vector current $V_\mu(x)$ is determined
through the ratio of the vacuum to vector state matrix elements of the two vector currents \[25\]:

\[
Z_V \equiv \frac{\langle 0| V_{\mu}^{CI}(0)| V_{\mu} \rangle}{\langle 0| V_{\mu}(0)| V_{\mu} \rangle}
\] (7)

(here \(| V_{\mu} >\) denotes the vector state). By calculating all correlation functions of eq.(3) and \(Z_V\) from eq.(7) and by requiring that eq.(3) holds at each \(t_y\), we can determine \(Z_A\) in an explicitly gauge invariant fashion for which the Gribov ambiguity is irrelevant.

One more comment is in place here: the terms proportional to \(\rho ra\) on the right-hand-side of eqs.(4) and (5) arise from the rotations of the fermion fields defined in eq.(1), which are inherent to Clover action improvement \[23\]. The \(\gamma \cdot D\) rotations, combined with the equations of motion, generate contact terms which, to \(O(a)\), give rise to the terms proportional to \(\rho ra\) \[22\].

3 Lattice Gauge Fixing and Gribov Copies

Gauge fixing and the generation of Gribov copies on the lattice is by now a standard procedure. Given a thermalised configuration generated by a Monte Carlo simulation, the Landau gauge is fixed by minimising the functional \[20\]

\[
F[U^g] \equiv -\frac{1}{V} \text{Re} \sum_{\mu=1}^{4} \sum_n U_{\mu}^g(n)
\] (8)

with respect to \(g\). Here \(V\) is the lattice volume and \(U_{\mu}^g(n) \equiv g(n)U_{\mu}(n)g(n + \mu)^\dagger\) is the compact \(SU(3)\) gauge field, gauge transformed by a local gauge transformation \(g(n)\). The extrema of \(F\) correspond to configurations that satisfy the gauge condition \(\partial_{\mu}A_{\mu}^g = 0\) in discretised form. The minimisation of \(F\) is obtained through iteration: each lattice site is visited and \(F\) is minimized locally. After several lattice sweeps \(F\) becomes constant, and the gauge is fixed. This is nothing else but a discretised analogue of the continuum formulation \[27\], according to which the local minima of the functional \(F[A_{\mu}^g] = -\sum_{\mu=1}^{4} \text{Tr} \int d^4x (A_{\mu}^g)^2\) with respect to \(g(x)\) correspond to configurations in the Landau gauge. Even if there is a straightforward correspondence between the lattice and continuum gauge fixing procedures, it is interesting to note that on the lattice, because of discretisation, there can be more minima than in the continuum \[28\].
The production of Gribov copies consists in generating gauge equivalent configurations, by applying random gauge transformations to our thermalised link configuration \cite{26}, \cite{2}. The gauge is then fixed and $F$ is measured. Since it is a gauge dependent quantity, a different value of $F$ for two gauge equivalent, gauge fixed configurations means that they are Gribov copies. The over-relaxation technique of \cite{29} (which consists in accelerating the gauge fixing algorithm by raising the gauge transformation $g(x)$ to a real tunable power $\omega$ at every iteration) has been implemented at fixed $\omega$. The over-relaxation itself can be used for the generation of Gribov copies, by varying the value of $\omega$, as proposed in \cite{30}. In this work we have opted for the random gauge transformation method.

4 Results

We work in the framework of the quenched approximation with the Clover action of SU(3) gauge theory. The lattice volume is $V = 16^3 \times 32$ and $\beta = 6.0$. After 3000 thermalising sweeps, 36 configurations were generated, separated by 1000 sweeps. An 8 hit Metropolis algorithm was used. For each thermalised configuration, we generated 6 Gribov copies. This was done by fixing the gauge both on the original configuration and on 5 gauge equivalent replicas, obtained by applying random gauge transformations. It is remarkable that, in our case, each random gauge transformation produced a Gribov copy. This high probability to find Gribov copies is a characteristic of large volume lattices in the confined region, as discussed in \cite{30,31}. We fix the Landau gauge, using the over-relaxation algorithm suggested in \cite{29} at fourth order in the over-relaxation parameter $\omega = 1.72$. The stopping condition we have imposed is that the relative variation $\delta F/F$, of the minimised functional $F$, between two consecutive gauge fixing sweeps be less than $10^{-8}$. This value guarantees a good quality of the gauge fixing, allows us to distinguish Gribov copies, and it is typically reached after a number of sweeps which varies between 500 and 1500. The gauge fixing was done on an IBM Risc 6000/550 equipped with 128 Mbyte of RAM memory and with a CPU working at 42 MHz; with this machine a single gauge fixing sweep takes about 40 s. The quark propagators, rotated as indicated by eq.(1) were obtained at a Wilson hopping parameter value of $K = 0.1425$, which, for the Clover action, corresponds to a pion of roughly 900 MeV.
Before passing to a detailed discussion of the Gribov noise, it is appropriate to present a comparison of the results for $Z_A$, obtained in a gauge invariant way (see eq.(5)), to those based on the gauge dependent identity of eq.(4). This calculation has been already presented in [22], on the first 18 configurations of our ensemble; here we have doubled the statistics. Moreover, we have re-gauged the first 18 configurations in order to reach the better quality of gauge fixing required for this study. In Fig.(1) we show the behaviour of the two estimates of $Z_A$, as a function of $t_y$, calculated on the same ensemble of 36 configurations. The gauge invariant values of $Z_A$ show a flat behaviour already at $t_y = 5$ and with small error bars. The new value of $Z_A$, obtained with the gauge independent technique, over 36 configurations, is $Z_A = 1.06(2)$ to be compared with the old one obtained over 18 configurations: $Z_A = 1.09(3)$ (see ref.[22]).

In the gauge dependent case, we have taken the average over the Gribov copies, in the way that will be discussed below. In this case, the $Z_A$ behaviour becomes flat only at $t_y = 10$ showing a large sensitivity to the contact terms of the Ward identity at small $t_y$ values. The new value of $Z_A$, obtained from the gauge dependent Ward identity, as can be seen from Fig.(1) is $Z_A = 1.08(5)$; to be compared to the value obtained from 18 configurations, $Z_A = 1.14(8)$ (see ref.[22]). As already stressed in [22], this method gives results that are compatible within the errors with the gauge independent ones. Moreover, the error of the gauge invariant calculation of $Z_A$ is always smaller than the error of its gauge dependent counterpart. This is due to the fact that the quark state correlation functions fluctuate more than the gauge invariant correlation functions, but it may also indicate the presence of another effect, which is probably the Gribov ambiguity. In the hypothetical case of two determinations of $Z_A$, affected by the same statistical error, the gauge dependent estimate should fluctuate more due to the Gribov noise. Then the amount of Gribov noise could be estimated as the difference (in quadrature) of the two errors.

Normally, in a standard simulation of gauge dependent quantities, one does not generate Gribov copies. Consequently, one measures a given quantity by taking the average and error over the ensemble of the gauge fixed configurations that have been generated. This implies a particular and arbitrary choice of Gribov copies. The error estimated, for example, by a jackknife method, is not purely statistical as it implicitly contains the uncertainty due to the particular choice of a Gribov copy.
In our case, having generated $G = 6$ copies for each of the $N = 36$ thermalised configurations, there are $G^N$ possible combinations that we may consider when forming a particular ensemble. To the best of our knowledge, the distribution of the Gribov copies of a given configuration is unknown; thus the weight to be associated to it is arbitrary. Moreover, any technique used to generate different Gribov copies selects a particular copy in a completely uncontrolled way. Hence, we assume that the particular choice of different combinations of Gribov copies when forming a statistical ensemble is arbitrary. In order to exhibit the effect of such arbitrariness, we show in Fig.(2) the behaviour of $Z_A(t_y)$ for 4 arbitrary choices of copies. The 4 different behaviours are compatible, and the same is true for the jackknife errors. It is clear, however, that the presence of Gribov copies is a visible effect; each of the six estimates of $Z_A$ shown has a slightly different profile as a function of $t_y$.

We now expose the procedure we implemented for taking into account the Gribov ambiguity. The gauge dependent traces of the two and three point correlation functions appearing in eq.(4), calculated on a single configuration $c$ and for a particular Gribov copy $g$ are denoted by:

\[ T_2(t_y; c, g) = \text{Tr} \left[ \int d^3 \vec{y} (\gamma_5 d(y) \bar{d}(0) + u(y) \bar{u}(0) \gamma_5) \right] \]

\[ T_3(t_y; c, g) = \text{Tr} \left[ \int d^4 x \int d^3 \vec{y} u(y) (\bar{u}(x) \gamma_5 d(x)) \bar{d}(0) \right] \]  \hspace{1cm} (9)

(in the above equations the Dirac indices have been implicitly averaged over, as explained in Sect.(2)). Then, for a given configuration $c$, we consider our "best estimate" of these matrix elements to be their average over the $G = 6$ Gribov copies:

\[ \bar{T}_{2,3}(t_y; c) = \frac{1}{G} \sum_{g=1}^{G} T_{2,3}(t_y; c, g) \]  \hspace{1cm} (10)

The average of the above $\bar{T}$’s over the 36 configurations will be taken as our "best estimate" $< T >$ for the gauge dependent traces. On the other hand, $\rho$, being gauge invariant, does not depend on a particular choice of Gribov copies, but only on the configuration ensemble. Then $Z_A(t_y)$ is obtained by applying eq.(3) as follows

\[ Z_A(t_y)^{-1} = 2 \rho(t_y) \frac{< \bar{T}_3(t_y) >}{< \bar{T}_2(t_y) >} + \rho(t_y) ra \]  \hspace{1cm} (11)
The error is obtained by a standard jacknife method performed on the quantities \( \langle \bar{T}_2(t_y) \rangle \), \( \langle \bar{T}_3(t_y) \rangle \) and \( \rho(t_y) \), by decimating one configuration at a time. This is the gauge dependent \( Z_A(t_y) \) result shown in Fig.(1).

We want to stress that the values of \( \langle \bar{T}_2 \rangle \) and \( \langle \bar{T}_3 \rangle \) fluctuate more than their ratio \( \langle \bar{T}_3 \rangle / \langle \bar{T}_2 \rangle \). The latter quantity is gauge invariant, according to eq.(11). For example, at \( t_y = 10 \), \( \delta \langle \bar{T}_2 \rangle / \langle \bar{T}_2 \rangle = 5.6\% \), \( \delta \langle \bar{T}_3 \rangle / \langle \bar{T}_3 \rangle = 5.1\% \) and \( \frac{\delta (\bar{T}_3/\bar{T}_2)}{\langle \bar{T}_3/\bar{T}_2 \rangle} = 2.3\% \). The strong reduction of the relative error indicates a great sensitivity of \( \bar{T}_{2,3} \) to the Gribov fluctuation, as opposed to a relative stability of the gauge invariant quantities.

In order to estimate the uncertainty arising from a particular choice of copies, out of the \( 6^{36} \) possible ones, we have applied a procedure which takes this arbitrariness into account. We have chosen randomly \( 10^4 \) combinations of copies of our ensemble and have calculated \( Z_A(t_y) \) for each one of these combinations at fixed \( t_y \). The histogram of the values obtained for \( Z_A(t_y) \) is well fitted by a Gaussian, the r.m.s. width of which is taken as an estimate of the fluctuation. In Fig.(3 a) we compare the jacknife error of our ”best estimate” to the width of the Gaussian. We see that for all \( t_y \) the width is smaller than the jacknife error. This implies that the fluctuations induced by the particular choice of Gribov copy when forming the ensemble are small and do not overcome the statistical uncertainty.

As it is also important to understand how the above behaviour scales with increasing the number of configurations, we have performed the same analysis for the first 18 configurations (half of our ensemble). The result is shown in Fig.(3 b). Comparing Fig.(3 a) to Fig.(3 b), we note that both errors scale at least as \( \sqrt{2} \). Thus, within our moderate statistics, we find that the error, even if affected by the Gribov ambiguity, decreases with increasing configuration number.

In conclusion, our investigation, even within its limitations, shows that, on the lattice, the residual gauge dependence associated with the Gribov copies does not generate an overwhelming fluctuation of the \( Z_A \) measurements performed by the gauge dependent method. Nevertheless the arbitrariness associated to a particular choice of Gribov copies is a visible effect which manifests itself, especially in the behaviour of \( Z_A \) as a function of \( t_y \). The jacknife error is greater in the gauge dependent determination than in the gauge independent one. However, in the former case, even though the error is afflicted by the Gribov uncertainty, it is still decreasing with increasing
statistics. This implies that the uncertainty arising from the Gribov ambiguity may be a secondary effect. Analogous studies on different physical quantities and renormalisation constants could further support this belief.

5 Acknowledgements

We warmly thank G. Martinelli, C.T. Sachrajda and M. Testa for many discussions and participation throughout this work. Useful discussions with V.N. Gribov and A.J. van der Sijs are also gratefully acknowledged. We also thank the IBM - Semea for providing us with 32 Mbytes of memory.

6 Figure Captions

FIGURE 1: Comparison of the gauge independent calculation of $Z_A(t_y)$ (diamonds) to the gauge dependent one (crosses). The errors are jacknife. The crosses have been slightly displaced to help the eye.

FIGURE 2: The gauge dependent calculation of $Z_A(t_y)$ for 4 arbitrarily chosen Gribov copies. The errors are jacknife.

FIGURE 3: Comparison of the jacknife error (crosses) to the r.m.s. Gaussian width (diamonds) due to the Gribov ambiguity (see text). The crosses have been slightly displaced to help the eye. Case (a) is with 36 configurations; case (b) with the first 18 configurations

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