Influence of technological load on the stability of synchronous anti-phase motion of the jaws of a vibratory crusher

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Abstract. Vibratory jaw crushers are based on the effect of self-synchronization of jaws and body, when its elements vibrate with frequencies that are in a rational relationship due to the influence of links already existing in the system. The technological load acting on the jaws of a vibratory crusher during its operation can have a significant effect on which operating mode of the crusher (synchronous antiphase or synchronous inphase) is implemented. This paper investigates the influence of the proposed crushing model on the dynamics of a vibratory jaw crusher. The model takes into account the elastoplastic properties of the rock being crushed and its one-way connection with the jaws. The destruction of the material, its movement along the crushing chamber and the change in the properties of the processed rock as it is crushing has been implemented. Conclusions are made about the influence of the processed rock on the implementation of the crusher operating modes

1. Introduction

Vibratory jaw crushers (VJC) are common machines for processing and crushing rock. The main feature that sets them apart from other machines for crushing material is the use of self-synchronization effect in design. This effect consists in the fact that the elements of the system begin to vibrate with the same or multiple frequencies due to the connections already existing in the system. This makes it possible to exclude from the design of the machine rigid and unreliable kinematic connections that ensure the mutual movement of the crusher jaws [1]. However, practice has shown that, depending on the physical and mechanical properties of the treated rock and the conditions of impact with the jaws, the type of self-synchronization (phase of mutual oscillations) of vibration exciters can change, which affects the efficiency of the crushing process. The working mode is the synchronous antiphase mode of the crusher vibrations.

There are studies in which the force interaction between the VJC jaws and the processed material is modeled by introducing an equivalent dissipation coefficient into a dynamic system [2-3]. In these works, the conditions for the stability of the in-phase and anti-phase modes of operation of the VJC were obtained, but many properties of the crushed rock itself and the effects occurring in the crushing chamber (movement of the rock along it, changes in the properties of the material and its chipping during processing) were not taken into account. The article [4] proposes a model of material accounting in the VJC, based on the laws of crushing and distribution of the processed rock in the crushing chamber. In this case, the movement of the jaw is set by the kinematic functions of time, i.e. the rock being crushed does not affect this movement. Another approach that allows to simulate
crushing of a single piece and crushing “under the blockage” is the use of the discrete element method (DEM) [5, 6]. This approach allows us to consider the crushing of each piece in the rock flow, however, the movement of the jaw is similarly set in a kinematic manner and does not experience the opposite effect of the processed material. Thus, the interaction between the jaws of the VJC and the rock on which they act is not fully understood. The influence of this interaction on the stability of the operating modes of the VSC is the subject of this article. The further development of the model created by the authors of [7] and its influence on the movement of the air pressure motor, taking into account the imperfection of electric motors, is proposed.

2. Vibratory jaw crusher model
The VJC (figure 1) consists of an absolutely rigid body with mass \( m \) and a moment of inertia \( J \) relative to its centre of mass \( O \). At points \( D_1, D_2 \), the body is suspended on a package of soft springs with transverse and longitudinal stiffnesses and linear-viscous damping coefficients \( k_1, k_2, b_1, b_2 \). The position of the body is defined by offsets of the center of mass \( x \) and \( y \), as well as by rotation relative to the center of mass \( \phi \). At points \( A_1 \) and \( A_2 \), at the distance of \( 2a \) from each other on elastic torsion bars, jaws with centres of mass \( C_1 \) and \( C_2 \) are symmetrically suspended. Torsion bars have the same linear characteristics with torsional stiffness \( k_3 \) and linear-viscous damping coefficient \( b_3 \). The jaws have the same masses \( \mu \) and moments of inertia \( J_0 \) relative to their centres of mass. Jaw length \( 2l \). The position of the jaws is defined by the relative (relative to the body) angles \( \phi_1 \) and \( \phi_2 \). At points \( B_1 \) and \( B_2 \) at an angle \( \beta \) to the surfaces of the jaws at a distance \( b \), two unbalanced vibrators are attached to the jaws. The mass of each unbalance is \( m_e \) and is concentrated at points \( E_1 \) and \( E_2 \), the eccentricity of both unbalances is \( e \). The position of the unbalances is set by the angles \( \theta_1 \) and \( \theta_2 \) relative to the horizontal. Unbalances rotation speed \( \omega_1 \) and \( \omega_2 \). The characteristics of asynchronous motors driving the unbalances: critical moment \( M_c \), critical slip \( s_c \) and coefficient of friction in their bearings \( f \).

Considering the limited engine power makes it possible to study the stability of synchronous modes.

![Figure 1. Vibratory jaw crusher scheme.](image)

The case of uniform supply of crushed material to the crushing chamber and uniform distribution of the processed rock along the height of this chamber is simulated. Thus, the chamber is completely filled with rock at any given time. To describe the mechanical properties of the rock and its movement along the crushing chamber, the following phenomenological model is used: the entire crushing chamber is divided into three zones (figure 1), to each of which the element \( El_{i0} \) (figure 2) is added at
a distance \( l_{(i)} \) from torsion bars. Each element is an ideally elastic-plastic body and consists of a linear elastic element with stiffness \( k_{e(i)} \) and a dry friction element with the ultimate force \( F_{\text{lim}(i)} \). During crushing, these elements do not move vertically. The cycle of movement of the jaws consists of two phases - the compression phase, when the jaws approach each other and the reverse phase of the jaws movement, when they move away from each other. During the compression phase, the elastic element is compressed until the ultimate force of the dry friction element is reached. After that, until the end of compression, the element's response stops increasing. After the elastic element has straightened to an unstressed state during the reverse movement phase, before the start of a new compression phase, the element does not exert forces on the jaws. Each such element can only withstand a certain number of cycles of interaction with the jaw, after which it collapses. The rock is loaded in the low-cycle fatigue mode. During compression, a complex stress-strain state is realized in the pieces, in the most weakened places of the piece, where pores and tiny cracks are present, damage begins to accumulate, which then develop into macro-cracks and the piece collapses. To simulate the accumulation of damage, each element is assigned a function of damage \( \chi_{(i)} \), which at the initial moment has a value of 0, and at the moment of destruction turns into 1. Accumulation of damage proceeds in accordance with the hypothesis of linear damage accumulation, the kinetic equation is as follows

\[
\frac{d\chi_{(i)}}{dt} = \frac{1}{t_{cr} F_{(i)}(t)}
\]

Here, \( t_{cr} \) is the time of rock destruction under loading by a symmetric sinusoidal load with an amplitude \( F \). In this model, the simplest approximation \( F = C \left(t_{cr}\right)^n \) is used, where \( C \) and \( n \) are the experimental constants of the material; they are selected so that each element collapses in 10 vibrations. During the oscillations, function (1) is integrated over time. As soon as the damage of all elements reaches \( \chi_{(i)} = 1 \), the element is destroyed.

During the reverse phase of the jaws movement immediately following the fracture, there is no technological load. This reflects the movement of rock down the crushing chamber. Then the whole cycle is repeated.

Consider the behavior of a part of an element consisting of a series-connected spring and a dry friction damper when compressed by a force \( F_{(i)} \) in the compression phase. Relative displacement \( \Delta u_{(i)} \) of points touching of the element and jaws.

\[
\Delta u_{(i)} = \begin{cases} 
F_{(i)}/k_{e(i)}, & F_{(i)} < F_{\text{lim}(i)} \\
\infty, & F_{(i)} = F_{\text{lim}(i)}
\end{cases}
\]

The dependence of the compressive force \( F_{(i)} \) on the displacement \( \Delta u_{(i)} \) is written in the form:

Figure 2. Scheme of the element that reproduces the interaction of the rock and jaw.
For subsequent numerical integration, it is expedient to replace the piecewise linear function (3) with a smooth function. As an approximation of the piecewise linear function (3), we will use a smooth function of the hyperbolic tangent.

\[
F_{i(0)} = \begin{cases} 
    k_e(0) \Delta u_{i(0)}, & \Delta u_{i(0)} < \frac{F_{\text{lim}}(0)}{k_e(0)} \\
    F_{\text{lim}}(0), & \Delta u_{i(0)} \geq \frac{F_{\text{lim}}(0)}{k_e(0)}
\end{cases}
\]  

(3)

Here, \(\Delta u_{\text{max}(i)}\) is the maximum distance between the touch points of the element and the jaws (end of the reverse phase); \(\Delta u_{\text{min}(i)}\) – the minimum distance between the points of contact of the element and the jaws (end of the compression phase); \(H(\phi_1 - \phi_2)\) – Heaviside step function, indicates that this part of the power characteristic works only in the compression phase; \(H(\phi_1 - \phi_2)\) is the Heaviside step function, indicates that this part of the power characteristic works only during the reverse phase of the jaws; \(H(F_{\text{lim}(i)} - k_e(0)(\Delta u_{i(0)} - \Delta u_{\text{min}(i)}))\) is the Heaviside function, indicates that only straightening occurs during the reverse phase of the jaws springs, without further stretching it; \(H(1 - \chi_{i(0)})\) is the Heaviside function indicates the destruction of an element and the absence of a reaction after this destruction; \(L_{x(i)}\) is the initial length of the element.

3. Differential equations of system movement

Crusher vibrations are described by the following differential equation:

\[
\mathbf{M} \ddot{\mathbf{q}} = \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}})
\]

(5)

Here, \(\mathbf{q} = \{x, y, \phi, \phi_1, \phi_2, \theta_1, \theta_2\}^T\) is the vector of the degrees of freedom of the system; \(\mathbf{M}\) - symmetric mass matrix of the system with dimension 7x7; \(\mathbf{F}\) is the vector of the right parts, which includes elastic and dissipative forces in the package of soft springs and torsion bars, the interaction forces of the unbalanced vibrator and the jaw, the force of the electric motor, the friction force in the bearings of the unbalanced vibrator and the force of interaction of the jaws and elements.

\[
\begin{align*}
M_{11} &= m + 2(\mu + m_r); & M_{12} &= 0; & M_{13} &= 2(\mu \cos(\phi_{cw}) + m_b \cos(\beta)); & M_{14} &= \mu \cos(\phi_{cw}) + m_b \cos(\beta); \\
M_{15} &= M_{16} = -m_e \sin(\theta_1); & M_{17} &= m_e \sin(\theta_2); & M_{22} &= m + 2(\mu + m_r); & M_{23} &= 0; \\
M_{24} &= \mu \sin(\phi_{cw}) + m_b \sin(\beta); & M_{25} &= -M_{24}; & M_{26} &= m_e \cos(\theta_1); & M_{27} &= m_e \cos(\theta_2); \\
M_{33} &= J + 2J_o + 2\mu(a^2 + l^2) + 2m_e(a^2 + b^2) + 4m_e \cos(\phi_{cw}) + 4m_e \sin(\phi_{cw}); \\
M_{34} &= J_o + \mu l^2 + m_b b^2 + \mu \alpha \sin(\phi_{cw}) + m_b \sin(\beta); & M_{35} &= M_{34}; & M_{36} &= m_e(a \cos(\theta_1) + b \cos(\beta - \theta_1)); \\
M_{37} &= -m_e(a \cos(\theta_1) + b \cos(\beta - \theta_2)); & M_{44} &= J_o + \mu l^2 + m_b b^2; & M_{45} &= 0; & M_{46} &= m_e \sin(\beta - \theta_1); \\
M_{47} &= 0; & M_{55} &= J_o + \mu l^2 + m_b b^2; & M_{56} &= 0; & M_{57} &= -m_e \sin(\beta - \theta_2); & M_{66} &= m_e^2; & M_{67} &= 0; & M_{77} &= m_e^2
\end{align*}
\]

(6)

The load vector components are given by expressions (7) and (8):
\[ F_1 = -k_1 x - b_1 \dot{x} + m_e \cos(\theta_1) \dot{\theta}_1^2 - m_e \cos(\theta_1) \dot{\theta}_2^2; \]
\[ F_2 = -2k_2 y - b_2 \dot{y} + m_e \sin(\theta_1) \dot{\theta}_1^2 + m_e \sin(\theta_2) \dot{\theta}_2^2; \]
\[ F_3 = -2a^2 k_2 \phi - b_2 \phi + m_e (a \sin(\theta_1) + b \cos(\beta - \theta_1)) \dot{\theta}_1^2 - m_e (a \sin(\theta_2) + b \cos(\beta - \theta_2)) \dot{\theta}_2^2 + \sum_{i=1}^{3} F_{i1} I_{f(i)} \left( \cos(\phi_{cm} + \phi + \phi_1) - \cos(\phi_{cm} + \phi + \phi_2) \right); \]
\[ F_4 = -k_3 \phi_1 - b_3 \phi_1 + m_e b \cos(\beta - \theta_1) \dot{\theta}_1^2 + \sum_{i=1}^{3} F_{i1} I_{f(i)} \cos(\phi_{cm} + \phi + \phi_1); \]
\[ F_5 = -k_3 \phi_2 - b_3 \phi_2 - m_e b \cos(\beta - \theta_2) \dot{\theta}_2^2 - \sum_{i=1}^{3} F_{i1} I_{f(i)} \cos(\phi_{cm} + \phi + \phi_2); \]
\[ F_6 = 2M_{11} \left( (\omega_1 - \dot{\theta}_1) \cdot (\omega_1 \cdot s_{11})^{-1} + \omega_1 \cdot s_{11} \cdot (\omega_1 - \dot{\theta}_1) \right)^{-1} - m_e f_1 \cdot \dot{\theta}_1^2 \text{sign}(\dot{\theta}_1) - m_e g \cos(\theta_1); \]
\[ F_7 = 2M_{12} \left( (\omega_2 - \dot{\theta}_2) \cdot (\omega_2 \cdot s_{12})^{-1} + \omega_2 \cdot s_{12} \cdot (\omega_2 - \dot{\theta}_2) \right)^{-1} - m_e f_2 \cdot \dot{\theta}_2^2 \text{sign}(\dot{\theta}_2) - m_e g \cos(\theta_2). \]

4. Results and discussions

This system was solved numerically for different speeds of rotation of unbalances \( \omega=\omega_1=\omega_2 \) with the following parameters: \( m=4738 \) kg; \( J=2575 \) kg m\(^2\); \( \mu=3253 \) kg m\(^2\); \( m_1=172 \) kg; \( l=0.8 \) m; \( b=1.01 \) m; \( e=0.1 \) m; \( a=0.5 \) m; \( k_1=620000 \) N m\(^{-1}\); \( k_2=620000 \) N m\(^{-1}\); \( k_3=45.4 \times 10^5 \) N m; \( \varphi_1=15^\circ \); \( b_1=380 \) N s m\(^{-1}\); \( b_2=380 \) N s m\(^{-1}\); \( b_3=11198 \) N m s rad\(^{-1}\); \( \beta=27^\circ \); \( M_1=600 \) N m\(^{-1}\); \( s_1=0.12 \); \( f=0.001 \); \( F_{lim(1)}=25600 \) N; \( F_{lim(2)}=32000 \) N; \( F_{lim(3)}=38400 \) N; \( \kappa_{e1}=\kappa_{e2}=\kappa_{e3}=530000 \) N m\(^{-1}\).

It is known that machines of this class have two characteristic modal frequencies - \( \omega_{sym} \), corresponding to a symmetric natural shape (when the jaws vibrate in antiphase) and \( \omega_{asym} \), corresponding to a skew-symmetric natural shape of the system (when the jaws vibrate in phase). For the case of absence of technological load, these frequencies set three possible section by the frequency of rotation of the unbalances \( \omega=\omega_1=\omega_2 \), depending on which one or another mode of operation of the machine is stable. If \( \omega<\omega_{asym} \), then the synchronous antiphase mode is stable, if \( \omega_{sym}<\omega<\omega_{asym} \), then the synchronous in-phase mode is realized when \( \omega>\omega_{asym} \), then the synchronous antiphase regime becomes stable again. For this crusher \( \omega_{sym}=52 \) rad s\(^{-1}\), \( \omega_{asym}=92 \) rad s\(^{-1}\).

Figure 3 shows a graph of changes in the phase difference of the unbalance rotation \( \Delta \theta = \theta_1 - \theta_2 \) depending on the unbalance rotation speed. For vibrating crushers, the phase of oscillation of the jaw correlates with the phase of rotation of its driving unbalance, therefore the value of \( \Delta \theta \) shows which mode of movement of the crusher has been established. For the used directions of \( \theta_1, \theta_2 \), the synchronous antiphase regime corresponds to \( \Delta \theta=0 \), and the synchronous in-phase regime corresponds to \( \Delta \theta=\pi \). It can be seen in this figure that the use of this model shifts the boundaries of the stability segments of oscillation modes towards decreasing frequencies by about 10 rad s\(^{-1}\). It can be noted that in the interresonance mode, corresponding to the in-phase movement of the jaws, comminution is impossible, which is displayed by a strong spread of the frequency difference in the middle of the graph.

The plots in figure 4 ((a), (b)) oscillations of the jaws \( \phi_1 \) and \( \phi_2 \) for two frequencies of external excitation \( \omega=22 \) rad s\(^{-1}\) (a) and \( \omega=100 \) rad s\(^{-1}\) (b). I.e. demonstrate the behavior of crushing organs in pre-resonance and over-resonance modes. It can be seen that, for these frequency ranges, antiphase oscillations of the VJC are realized, corresponding to the operating modes of crushing. On these graphs, a different influence of the model is noticeable - the vibrations of the jaws are no longer harmonic, they are complex. Oscillatory motion is superimposed on a non-zero mean value of the angle of rotation, which indicates that the jaws are moving apart relative to each other. This phenomenon has been confirmed experimentally [1]. The change in the amplitude of vibrations is
associated with taking into account the destruction of the rock. At the moment when the element collapses, the resistance force falls on the jaws, and the amplitude of the oscillations increases. On the next compression cycle, the element begins to resist again, which reflects the supply of new material to the crushing chamber, and the vibration amplitude begins to decrease.

![Graph](image)

**Figure 3.** Plot of changes in the phase difference of unbalances

![Graph](image)

**Figure 4.** (a) Plot of the jaw vibrations for the case $\omega = 22 \text{ rad} \cdot \text{s}^{-1}$; (b) Plot of the jaw vibration for the case $\omega = 100 \text{ rad} \cdot \text{s}^{-1}$

5. **Conclusions**

This article discusses the issues of modeling the behavior of the VJC and the treated rock. The VJC is modeled as an absolutely rigid body with 3 degrees of freedom and two absolutely rigid jaws, each of them has 2 degrees of freedom. The system is in motion with two unbalanced vibrators of limited power, fixed on the jaws. The rock being crushed is presented in the form of a phenomenological model: the entire crushing chamber is divided into 3 zones, in each zone there is an elastoplastic element. This model simulates the elastic and plastic properties of the crushed rock, the packing of the
rock to the bottom of the crushing chamber, its destruction and chipping, and the change in these properties during the movement of the rock through the crushing chamber. The strength properties of the rock are taken into account using the hypothesis of linear damage accumulation. Damage to the elements accumulates with each interaction with the jaws. The results show the influence of such models of rock crushing on the movement of the VJC. Accounting for the material in this way leads to a change in the synchronous rotation frequency of the stability unbalances of the synchronous in-phase mode and synchronous anti-phase mode of the crusher operation. Also, due to the interaction with the cultivated rock, the nature of the machine vibrations changes.

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