Transport-theoretic extensions of quantum field theories

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Abstract

We propose a new, transport-theoretic (tt) class of relativistic extensions of quantum field theories of fundamental interactions. Its concepts are inspired by Feynman’s atomistic idea about the physical world and by the extension of fluid dynamics to shorter distances through the Boltzmann transport equation. The extending tt Lagrangians imply the original Lagrangians as path-integralwise approximations. By constructing a tt Lagrangian that extends a general gauge-invariant Lagrangian, we show that a tt extension of the standard model is feasible. We define a tt Lagrangian in terms of tt fields of the spacetime variable and an additional, four-vector variable. We explain the fields of quantum field theories as certain covariant, local averages of tt fields. Only two tt fields may be needed for modeling fundamental interactions: (i) a four-vector one unifying all fundamental forces, and (ii) a two-component-spinor one unifying all fundamental matter particles. We comment on the new physics expected within the tt framework put forward, and point out some open questions.

PACS: 11.10.Kk, 12.10.Dm
Keywords: Extension, quantum field theory
1. Introduction and motivation

One can describe the strong, weak, and electromagnetic interactions at presently accessible energies, with required precision, by the standard model. Nowadays, however, this model is generally believed to be a low-energy approximation to a more fundamental theory [1]. Yet it is not clear to what kind of theory, and what physics is lost as a result of this approximation; e.g., Salam [2] suggested that the physics of quantum gravity is lost. Some theorists are looking for an improved theory of fundamental interactions outside the framework of quantum field theories in four-dimensional spacetime \( \mathbb{R}^{1,3} \) (QFTs), formulated in terms of fields of only four independent, continuous variables. To explain premises of the standard model, to extend it, and to include gravity, they study quantum field theories in higher-dimensional spacetimes \( \mathbb{R}^{1,n} \), \( n > 3 \) [3].

The ultraviolet divergences of realistic QFTs may be seen as a sign of inadequate treatment of physical processes at higher energies—processes determined by the experimentally unexplored, small-distance physics of fundamental interactions [1, 4]. Already sixty years ago, Heisenberg [5] proposed that a QFT can provide only an idealized, large-scale description of quantum dynamics, valid for distances larger than some fundamental length. As it is still not clear how to extend present QFTs of fundamental interactions to shorter distances, we will consider to this end a new quantum field theory in eight dimensional \( \mathbb{R}^{1,3} \times \mathbb{R}^{1,3} \).

It is nowadays customary to specify a QFT by choosing (1) a finite number of \( c \)-number fields of the spacetime variable \( x \in \mathbb{R}^{1,3} \) (fields), \( \psi_k(x) \), and (2) a classical Lagrangian density (Lagrangian) \( \mathcal{L} \) defined in terms of \( \psi_k \) and of their first-order, spacetime derivatives \( \partial_\mu \psi_k \). The QFT probability amplitudes, representing quantum dynamics, can then be expressed in terms of the Feynman path integral [1, 6]

\[
\int [d\psi_k] \exp \left( \frac{i}{\hbar} I \right),
\]

where the action

\[
I[\ldots, \psi_k, \ldots] \equiv c^{-1} \int \mathcal{L} \, d^4x.
\]

To extend a QFT, we are going to look for such an extending Lagrangian that the path integrals specified by it may be approximated by the corresponding QFT path integrals, providing some characteristic length of the extending Lagrangian tends to zero.

The Euler-Lagrange equations

\[
\frac{\partial \mathcal{L}}{\partial \psi_k} - \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi_k)} \right) = 0
\]

of present QFTs are nonlinear, coupled, covariant, partial-differential equations [1, 4, 6]. Following Bjorken and Drell [4] and Feynman [7], we may wonder once
again, some thirty years later, what kind of equations could be the Euler-Lagrange equations in theories that will extend present QFTs to shorter distances? Commenting on the “underlying unity” of nature, Feynman [7] noted that the partial-differential equation of motion “…we found for neutron diffusion is only an approximation that is good when the distance over which we are looking is large compared with the mean free path. If we looked more closely, we would see individual neutrons running around.” And then he wondered, “Could it be that the real world consists of little X-ons which can be seen only at very tiny distances? And that in our measurements we are always observing on such a large scale that we can’t see these little X-ons, and that is why we get the differential equations? …Are they [therefore] also correct only as a smoothed-out imitation of a really much more complicated microscopic world?” Presuming that Feynman was on the right track with this atomistic idea, we believe that the kinetic theory of gases suggests a way to obtain such Lagrangians for modeling fundamental interactions that extend the Lagrangians of QFTs to shorter distances.

In the kinetic theory of gases, one describes large-scale phenomena by a finite number of fields, so-called macroscopic variables such as number density, macroscopic mean velocity, pressure tensor, heat-flow vector, and kinetic-energy density [8, 9]. For such large-scale phenomena where changes in macroscopic variables over a mean collision time and over a mean free path are sufficiently small, one can predict the evolution of these fields accurately enough by the partial-differential equations of fluid dynamics. Otherwise, one must resort to a more detailed description of physical processes. To better approximate the evolution of macroscopic variables, one may use the one-particle distribution: a function of spacetime positions \( x \) and of four-momenta \( p \) of constituent particles, i.e., of eight independent, continuous variables. This classical function of \( x, p \in \mathbb{R}^{1+3} \) (tt field) determines the values of macroscopic variables through certain local averages over four-momenta \( p \). For a sufficiently rarefied gas, it is accepted that the equation of motion for such tt field is an integro-differential, Boltzmann transport equation, which is local in the spacetime variable \( x \), but not in the four-momentum variable \( p \). As it takes some account of gas “granularity”, the Boltzmann equation describes dynamics of physical processes also at vastly smaller distances than the partial-differential equations of fluid dynamics. These small-scale physical processes determine the large-scale, fluid phenomena described by macroscopic variables. The natural time of evolution of macroscopic variables, which can be adequately determined by the equations of motion of fluid dynamics, apparently bears no relation to the mean collision time, a vastly smaller, basic time-scale for the corresponding solution to the Boltzmann equation [11]. Within the transport-theoretic (tt) framework, this disparity causes no conceptual problems such as the hierarchy problem in elementary particle physics [12].

One can formally derive the equations of motion of fluid dynamics as such an asymptotic approximation to the Boltzmann equation that depends on its
initial and boundary conditions [11]. Which explains fluid dynamics as an asymptotic theory in the kinetic theory of gases. Inspired by this explanation, and by Feynman’s conjecture that the partial-differential equations of QFTs are actually describing large-scale phenomena due to microscopic motion of some hypothetical X-ons, we formulated [13] certain new, relativistic, integro-differential, transport equations: equations which offer such physically motivated, tt extensions of partial-differential equations of a QFT to shorter distances that contain them as large-distance approximations. When thinking about alternatives to the conventional QFTs, such extensions were found missing by Bjorken and Drell [4] on the analogy with the classical, partial-differential equations that are an idealization valid for distances larger than the characteristic length measuring the granularity of the medium. So we proposed [14] that for modeling quantum dynamics of fundamental interactions by the path-integral method we use, instead of QFT Lagrangians, such tt Lagrangians that imply them as asymptotic approximations, and whose Euler-Lagrange equations are integro-differential, transport equations. Here, our aims are: (1) to present the basic concepts of relativistic, tt quantum field theories (ttQFTs) specified by such tt Lagrangians, (2) to formally show that ttQFTs extending QFTs of fundamental interactions are feasible, (3) to elaborate on physical implications of ttQFTs, and (4) to point out relevant physical questions.

The paper is organized as follows: In Section 2, we specify basic properties of tt fields, and of tt Lagrangians extending QFT Lagrangians. We introduce a length $\lambda$ to classify present QFTs as asymptotic approximations to certain ttQFTs as $\lambda \to 0$. In Section 3, we give a covariant, tt Lagrangian that extends a general gauge-invariant Lagrangian. In Section 4, we give tt symmetry transformations generalizing those of QFTs. In Section 5, we point out what kind of physics might distinguish an extending ttQFT from the original QFT. We also consider the transition from a ttQFT to the corresponding QFT. Concluding remarks and summary are given in Sec. 6. Mathematical formalism of ttQFTs is considered in the Appendix, where we give simple tt Lagrangians extending the Klein-Gordon, Dirac, and QED Lagrangians. For convenience, we use the metric tensor $\eta^{\mu\nu} \equiv \text{diag}(-1,1,1,1)$, bispinors and Dirac matrices in the chiral representation, and the natural units in which $\hbar = c = 1$; $[L]$ denotes dimensions of length and time, and $[L]^{-1}$ those of energy, momentum, and mass.

2. Basic assumptions of tt extensions

In a ttQFT that extends a QFT, we will assume that one can study quantum processes modeled by this QFT using (1) tt fields of two independent, four-vector variables; (2) a tt Lagrangian defined in terms of tt fields and their spacetime derivatives; and (3) appropriate path integrals, specified by this tt Lagrangian.

2.1. Transport-theoretic fields
We will specify a tt Lagrangian in terms of real and complex, relativistic tt fields, say, $\Psi_j(x,p)$ of the spacetime variable $x = (t, \mathbf{r}) \in \mathbb{R}^{1,3}$ and of an additional, four-vector variable $p = (p^0, \mathbf{p}) \in \mathbb{R}^{1,3}$. The number and type of tt fields $\Psi_j(x,p)$ we will use depend on the physical system under consideration, the interactions we take account of, and the way we model them. Dimensions of the independent variable $x$ are $[L]$; for convenience, the dimensions of the independent variable $p$ and of tt fields $\Psi_j(x,p)$ are chosen to be $[L]^{-1}$ and $[L]$, respectively. In contrast to field theories in higher-dimensional spacetimes, in ttQFTs the additional independent variables $p^0, p^1, p^2$ and $p^3$ (which we may regard as the components of four-momenta of Feynman’s X-ons) do not make us modify our ideas about time and space. In ttQFTs, the spacetime is still the same as in QFTs, the four-dimensional space $\mathbb{R}^{1,3}$, and no compactification is necessary.

We will need Lorentz-scalar (scalar) tt fields $\Psi_0(x,p)$, left-handed–two-component-spinor (left-spinor) tt fields $\Psi_{1/2}(x,p)$, right-handed–two-component-spinor (right-spinor) tt fields $\Psi_{-1/2}(x,p)$, and four-vector tt fields $\Psi_1(x,p)$. On the analogy of the relativistic kinetic theory, we assume that under an inhomogeneous, proper, orthochronous, Lorentz transformation $x \rightarrow \Lambda x + a$, $a \in \mathbb{R}^{1,3}$, a tt field

$$\Psi_j(x,p) \rightarrow U(\Lambda, a)\Psi_j(x,p) \equiv D_j(\Lambda^{-1})\Psi_j(\Lambda x + a, \Lambda p),$$  \hspace{1cm} (2.1)

where $D_j(\Lambda), j = 0, 1/2, -1/2, 1$, are the conventional Lorentz-transformation matrices for scalars, left-spinors, right-spinors, and four-vectors, respectively. $[D_0 \equiv 1; D_{1/2} = D_{-1/2}^{-1}; D_1 \equiv \Lambda; \text{for the four-vector tt field } \Psi_1(x,p) \equiv p, U(\Lambda, a)\Psi_1 = p.]$

Under Lorentz transformations (2.1) the independent variables $x$ and $p$ do not mix; $x$ transforms as the spacetime four-vector, and $p$ transforms as a momentum four-vector. So no extension of special relativity is needed in ttQFTs.

To define free tt Lagrangians we will use the following local, bilinear mappings $[\Psi_j | \Psi_j']$ of tt fields $\Psi_j(x,p)$ and $\Psi'_j(x,p)$ into scalar fields:

$$[\Psi_0 | \Psi_0'](x) \equiv \int d^4p \Psi_0^*(x,-p)\Psi_0'(x,p),$$

$$[\Psi_{\pm 1/2} | \Psi_{\pm 1/2}'](x) \equiv \int d^4p \Psi_{\pm 1/2}^\dagger(x,-p)\sigma_{\pm}(p)\Psi_{\pm 1/2}'(x,p),$$ \hspace{1cm} (2.2)

$$[\Psi_1 | \Psi_1'](x) \equiv \int d^4p \Psi_1^*(x,-p)\cdot\Psi_1'(x,p).$$

In the above: $^*$ denotes complex conjugation; $^\dagger$ denotes the adjoint (i.e., transposed and complex-conjugate) spinor; the $2 \times 2$ matrix functions

$$\sigma_{\pm}(p) \equiv i(p \cdot p)^{-1/2}(p \cdot \mathbf{\sigma} \pm p^0 I)$$  \hspace{1cm} (2.3)

for $p \cdot p > 0$, where $I$ is the $2 \times 2$ identity matrix, and $\mathbf{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ is the vector of Pauli matrices; $a \cdot b \equiv a^\mu b_\mu$, $a, b \in \mathbb{R}^{1,3}$; for a function $F(p)$ of $p \in \mathbb{R}^{1,3}$, the
functional
\[ \int d^4p F(p) \equiv -i \lim_{R \to \infty} \int_{-iR}^{iR} d^0p \int_{p \cdot p < R^2} F(p) d^3p, \tag{2.4} \]

where \( F(p) \) is defined for imaginary values of \( p^0 \) through analytic continuation. Henceforth, we assume that tt fields \( \Psi_j(x,p) \) are such that all functionals (2.4) we will encounter exist and are invariant in the sense that \( \int d^4p F(\Lambda p) = \int d^4p F(p) \), see [13]. \( \int d^4p \) is an analog of the standard Euclidean definition of a fourfold integral through Wick’s rotation and symmetric integration.

Mappings \( [\Psi_j | \Psi'_j] \) are scalar fields in the sense that [13]
\[ [U(\Lambda, a)\Psi_j | U(\Lambda, a)\Psi'_j](x) = [\Psi_j | \Psi'_j](\Lambda x + a). \tag{2.5} \]

They do not explicitly depend on \( x \), and are strictly local in the sense that they ignore the \( x \)-dependence of tt fields:
\[ [\Psi_j | \Psi'_j](a) = [\Psi_j(a, p) | \Psi'_j(a, p)](a). \tag{2.6} \]

2.2. Lagrangian of a ttQFT

We denote by \( p \cdot \partial \) the covariant, substantial time derivative \( p^0 \frac{\partial}{\partial t} + p \cdot \nabla \), by \( \Psi(x,p) \) the tt field \( (\ldots, \Psi_j(x,p), \ldots) \) consisting of all tt fields \( \Psi_j(x,p) \) of the considered ttQFT, and by \( \mathcal{L}_{tt}(\Psi, p \cdot \partial \Psi) \) the tt Lagrangian defining this ttQFT. We write a tt Lagrangian as a sum of free and interaction parts,
\[ \mathcal{L}_{tt} = \mathcal{L}_{free} + \mathcal{L}_{int}, \tag{2.7} \]

which are real, scalar fields in the sense of (2.5); do not explicitly depend on the spacetime variable \( x \); and have dimensions of energy density, \([L]^{-4}\). Thus the tt action \( I[\Psi] \equiv \int \mathcal{L}_{tt} d^4x \) is a real, dimensionless, relativistic invariant.

The free part of a tt Lagrangian
\[ \mathcal{L}_{free} \equiv \frac{1}{2} \sum_j \left\{ [\Psi_j | p \cdot \partial \Psi_j] + [p \cdot \partial \Psi_j | \Psi_j] \right\}, \tag{2.8} \]

where \( [\Psi_j | p \cdot \partial \Psi_j] \) and \( [p \cdot \partial \Psi_j | \Psi_j] \), \( j = 0, 1 \), are equal and real if \( \Psi_j \) is real. The Euler-Lagrange equations of \( \mathcal{L}_{free} \) are the covariant, partial-differential equations
\[ p \cdot \partial \Psi_j(x,p) = 0, \tag{2.9} \]

by (A.7). [For solutions to (2.9) see [14]. For an alternative to definition (2.8) see (A.27).] According to (2.9), all Euler-Lagrange equations of free tt Lagrangians are of the same simple form: the tt approach entails certain unification and simplification of free Lagrangians. On the analogy of the kinetic theory of gases, we
may consider equations of motion (2.9) for free tt fields as a classical description of free streaming of Feynman’s X-ons: an analog of Newton’s first law.

We assume that the interaction part $L_{\text{int}}$ of a tt Lagrangian acts strictly locally on tt fields $\Psi(x,p)$, i.e., satisfies a relation such as (2.6), and so does not depend on the spacetime derivatives $\partial \Psi$ of $\Psi$. If regarded as a part of a classical tt description of Feynman’s X-ons, the interaction part $L_{\text{int}}(\Psi)$ of a tt Lagrangian $L_{tt}$ describes strictly local interactions of infinitesimal entities: an example of the Aristotle concept of contact forces.

2.3. Extension of a QFT

To be an extension of certain QFT, a ttQFT has to respect the correspondence principle; i.e., as pointed out by Weidner [15]: “...when a more general theory is applied to a situation in which a less general theory is known to apply, the more general theory must yield the same predictions as the less general theory.” So, when calculating the ttQFT transition amplitude for a quantum process that can be adequately described already by the QFT, we should be able to replace the ttQFT path integral, specified by the tt Lagrangian $L_{tt}(\Psi, p \cdot \partial \Psi)$ in terms of functions $\Psi(x,p)$ of eight independent variables, with the corresponding QFT path integral, specified by the QFT Lagrangian $L(..., \psi_k, \partial \psi_k, ...)$ in terms of functions $\psi_k(x)$ of only four independent variables. Let us show how such a replacement of ttQFT with the QFT can be explained.

Inspired by the explanation of fluid dynamics as an approximate theory in the kinetic theory of gases [8–11], we introduce the set of asymptotic tt fields

$$\Psi_{\text{as}}(x,p) \equiv \sum_k F_k(p) \psi_k(x).$$

They are sums of appropriate products of the QFT fields $\psi_k(x)$ and of functions $F_k(p)$ of $p \in \mathbb{R}^{1,3}$ that are the same for all $\Psi_{\text{as}}(x,p)$, though they may depend on $L_{tt}$ but (to simplify) not on the initial and final states $[F_k(p)]$ will play here a role analogous to that of the Maxwellian distribution and corrections to it. We then hypothesize that the quantum process for which a ttQFT transition amplitude is adequately approximated by the QFT one is in general such that:

(a) We can specify the initial and final states of this process by the QFT fields $\psi_k(x)$.

(b) We can obtain an adequate approximation to the ttQFT path integral specified by the tt Lagrangian $L_{tt}$ and the initial and final states by integrating over all such asymptotic tt fields $\Psi_{\text{as}}(x,p)$ whose fields $\psi_k$ are constrained by the initial and final states in the same way as in the corresponding QFT path integral specified by $L$. Thus the actual variables of the ttQFT path integral over such asymptotic tt fields are the same as in the corresponding QFT path integral: the fields $\psi_k(x)$.

(c) These two path integrals are equivalent.
Whenever these three conditions are met, we will regard (i) the Lagrangian \( \mathcal{L} \) of the original QFT as a path-integralwise approximation to the tt Lagrangian \( \mathcal{L}_{tt} \), and (ii) the tt Lagrangian \( \mathcal{L}_{tt} \) as a tt extension of the QFT Lagrangian \( \mathcal{L} \) provided \( \mathcal{L}_{tt} \) with \( \Psi \) restricted to \( \Psi_{as} \) equals \( \mathcal{L} \), i.e.,

\[
\mathcal{L}_{tt}(\Psi_{as}, p \cdot \partial \Psi_{as}) = \mathcal{L}(\ldots, \psi_{k}, \partial \psi_{k}, \ldots).
\]  

(2.11)

In the classical asymptote \( \hbar \to 0 \), the formal oscillatory behaviour of the integrand of (1.1) suggests the main contribution to the Feynman path integral is due to the fields \( \psi_{k}(x) \) that satisfy the boundary conditions imposed by the initial and final states and make the action \( I \) stationary by satisfying the Euler-Lagrange equations (1.3). On the analogy of this formal asymptote in QFTs, in this paper we will formally simulate such quantum processes where the QFT is adequate by replacing the above condition (b) with the following assumptions: (1) The interaction part \( \mathcal{L}_{int} \) of the tt Lagrangian \( \mathcal{L}_{tt} \) depends on a characteristic length \( \lambda, [\lambda] = [L] \), and, in general, tends to infinity if \( \lambda \to 0 \). (This length \( \lambda \) is an analog of the singular parameter introduced in 1924 by Hilbert to formally obtain fluid dynamics as an asymptotic approximation in the kinetic theory of gases [11].) (2) The asymptotic tt fields (2.10) are the most general tt fields that satisfy the Euler-Lagrange equations of \( \mathcal{L}_{tt} \) in the leading order of \( \lambda \). (3) We are considering such transitions where \( \lambda \) is relatively so small that we can obtain adequate approximations to the ttQFT path integrals by integrating over the asymptotic tt fields \( \Psi_{as}(x,p) \) with fields \( \psi_{k}(x) \) consistent with the initial and final states. We will refer to such asymptote \( \lambda \to 0 \) as the QFT asymptote since we assumed that eventually as \( \lambda \to 0 \): (i) the tt Lagrangian \( \mathcal{L}_{tt} \) actually depends only on the QFT fields \( \psi_{k}(x) \), and (ii) tt transition amplitudes can be expressed in terms of the path integrals specified by the QFT Lagrangian \( \mathcal{L} \) (which may depend on \( \lambda \)). In Sec. 5.2, we will mention how quantum processes one models by a QFT may come about.

Henceforth, we assume that functions \( F_{k}(p) \) are such that the incorporation (2.10) of QFT fields \( \psi_{k}(x) \) into the asymptotic tt fields \( \Psi_{as}(x,p) \), the \( p \)-dependence of asymptotic tt fields \( \Psi_{as}(x,p) \), and the definition (2.11) of a tt extension of a QFT Lagrangian \( \mathcal{L} \) are relativistically invariant in the sense that

\[
U(\Lambda, a)\Psi_{as}(x, p) = \sum_{k} F_{k}(p)U(\Lambda, a)\psi_{k}(x),
\]

(2.12)

\[
\mathcal{L}_{tt}(U(\Lambda, a)\Psi_{as}, p \cdot \partial U(\Lambda, a)\Psi_{as}) = \mathcal{L}(\ldots, U(\Lambda, a)\psi_{k}, \partial U(\Lambda, a)\psi_{k}, \ldots).
\]

(2.13)

By (2.12), the asymptotic tt fields (2.10) with the Lorentz-transformed QFT fields equal the Lorentz-transformed asymptotic tt fields.

2.4. Macroscopic variables

Take a ttQFT that extends certain QFT. Motivated by the macroscopic variables in the kinetic theory of gases, we assume that for each kind of QFT fields
ψ_k(x) there is a special covariant average of any ttQFT field Ψ(x, p) over the independent variable p, namely, the tt macroscopic variable

$$\psi_k[x; \Psi] = \int d^4p \mathcal{F}_k(p)\Psi(x, p)$$

(2.14)

of Ψ(x, p), where \( \mathcal{F}_k(p) \) is such a function of \( p \in \mathbb{R}^{1,3} \) that \( \psi_k[x; \Psi] \) and \( \psi_k(x) \) are the same kind of relativistic fields, have the same dimensions, satisfy relations

$$\psi_k[x; \Psi] = \psi_k(x)$$

(2.15)

and

$$\psi_k[x; U(\Lambda, a)\Psi] = U(\Lambda, a)\psi_k[x; \Psi]$$

(2.16)

and have the same physical significance in the following sense: If a tt field Ψ(x, p) is consistent with certain initial and final states, we can take its macroscopic variables \( \psi_k[x; \Psi] \) as a QFT path between these two states. Hence, the initial and final states constrain the macroscopic variables \( \psi_k[x; \Psi] \) of ttQFT fields \( \Psi(x, p) \) in the same way as the QFT fields \( \psi_k(x) \). So we may generalize (2.15) and regard QFT fields as the macroscopic variables of the ttQFT extension and vice versa; this connection between ttQFT and QFT fields is relativistically invariant, by (2.16). By (2.15) and (2.10), an asymptotic tt field is completely described by its macroscopic variables:

$$\Psi(x, p) = \sum_k \mathcal{F}_k(p)\psi_k[x; \Psi] .$$

(2.17)

Using macroscopic variables \( \psi_k[x; \Psi] \) and the characteristic functions \( \mathcal{F}_k(p) \) appearing in (2.10), we define the macroscopic projection operator

$$\mathcal{P}_{as} \Psi \equiv \sum_k \mathcal{F}_k(p)\psi_k[x; \Psi] ;$$

(2.18)

$$\mathcal{P}_{as}^2 \Psi = \mathcal{P}_{as} \Psi, \mathcal{P}_{as} \Psi_{as} = \Psi_{as}, \text{ and } \psi_k[x; \mathcal{P}_{as} \Psi] = \psi_k[x; \Psi], \text{ by (2.10) and (2.15).}$$

If Ψ is a tt path, then \( \mathcal{P}_{as} \Psi \) is an asymptotic tt field (2.10) with fields \( \psi_k \) compatible with initial and final states. Thus any ttQFT path Ψ(x, p) consistent with certain initial and final states can be written as a sum

$$\Psi(x, p) = \Psi_{as}(x, p) + (1 - \mathcal{P}_{as})\Psi(x, p) ,$$

(2.19)

where \( \Psi_{as} \) is defined by (2.10) with \( (\ldots, \psi_k(x), \ldots) \) being a path between these two states in the original QFT. Which shows that extending a QFT by a ttQFT, we actually add tt fields \( (1 - \mathcal{P}_{as})\Psi \) to asymptotic tt fields, so to speak; so an infinite number of ttQFT paths corresponds to one QFT path. In the QFT asymptote,
the part \( (1 - \mathcal{P}_{as})\Psi \) of any ttQFT path \( \Psi \) is “neglected”. We have yet to find out how the parts \( (1 - \mathcal{P}_{as})\Psi \) of ttQFT paths \( \Psi \) are constrained by the initial and final states, and by the ttQFT Lagrangian \( \mathcal{L}_{tt} \). If the initial and final values of \( (1 - \mathcal{P}_{as})\Psi \) are uniquely determined, the extending ttQFT implies no new particles in addition to those considered in the original QFT. And if they are equal to zero, the asymptotic tt fields \( \Psi_{as} \) are ttQFT paths.

In Section 3 and in the Appendix, we are going to consider tt extensions of QFT Lagrangians. In this paper, we will not consider: (1) how to define the macroscopic variables and tt path integral appropriate to a given tt Lagrangian and certain initial and final states; (2) the quantization and derivation of Feynman’s rules in ttQFTs; (3) which specific gauge and ghost terms, and regularization and renormalization procedures, a ttQFT implies in the QFT it extends; and (4) how a tt extension modifies the divergent integrals of a QFT.

3. Transport-theoretic extensions of a QFT Lagrangian

In this Section, we are going to look into the physical, conceptual problems in constructing a tt extension of a QFT Lagrangian. To this end, we will make the following basic assumptions considered in Sec. 2: (1) A tt Lagrangian is defined in terms of tt fields. It is a sum (2.7) of free and interaction parts, which are real, scalar fields and do not depend explicitly on \( x \). (2) The free part of a tt Lagrangian is given by (2.8). (3) The interaction part of a tt Lagrangian is local and without derivative couplings. It depends on a characteristic length \( \lambda \) and, in general, tends to infinity if \( \lambda \rightarrow 0 \). (4) When calculating the ttQFT probabilities amplitudes for quantum processes that are adequately modeled already by the QFT to be extended, we may take as tt paths the asymptotic tt fields (2.10) that satisfy the tt Euler-Lagrange equations in the leading order in \( \lambda \) and whose fields \( \psi_k(x) \) are compatible with the initial and final states. (6) So computed ttQFT probability amplitudes equal the corresponding QFT ones.

Step by step to ever higher energies, it took some fifty years of intense theoretical and experimental effort to establish first the QED and then the standard model. To show that a bottom-up tt approach to fundamental interactions that takes account of these results is feasible, we are going to construct a ttQFT formally encompassing the standard model. To this end, we start with the general gauge-invariant Lagrangian [16]

\[
\mathcal{L}_{II} \equiv -\frac{1}{4} \left( \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - C_{abc} A_{b\mu} A_{c\nu} \right) \left( \partial^\mu A^\nu_a - \partial^\nu A^\mu_a - C_{ade} A^\mu_d A^\nu_c \right)
- \bar{\psi}_m \left[ \gamma^\mu \partial_\mu \psi_m + (i \gamma^\mu A_{a\mu} T^a_{mn} + m_{mn} + \phi_i \Gamma^i_{mn}) \psi_n \right]
- \frac{1}{2} (\partial^\mu \phi_i + i A^\mu_a R_{i,j}^a \phi_j)(\partial_\mu \phi_i + i A_{b\mu} R^b_{i,k} \phi_k) + \frac{1}{2} \mu^2 \phi_i \phi_i - \frac{1}{4} \mu^2 v^2 (\phi_i \phi_i)^2.
\]

In the above: (1) \( A_a(x) \) are real, four-vector, gauge fields; \( \psi_m(x) \) are complex, chiral-bispinor fields; and \( \phi_i(x) \) are real, scalar fields. (2) \( \bar{\psi}_m(x) \equiv \psi_m^\dagger(x) \beta \), with \( \psi_m^\dagger \) being the bispinor adjoint to \( \psi_m \) and \( \beta = i \gamma^0 \); and \( \gamma^\mu \) are the four \( 4 \times 4 \) Dirac...
matrices. (3) \( T_{mn}^a \) and \( R_{ik}^a \) are components of Hermitian matrices representing the Lie algebra of the gauge group of \( \mathcal{L}_{II} \) (\( iR_{ik}^a \) are real). \( T_{mn}^a, R_{ik}^a \), and the structure constants \( C_{abc} \) are proportional to one or more gauge coupling constants. (4) \( \Gamma_{mn}^i \) are components of Yukawa coupling matrices; \( m_{mn} \) are components of a bare-mass matrix; and \( \mu^2 \) and \( v^2 \neq 0 \) are real constants. (5) Here and in what follows, the repeated, Lorentz or gauge indices are summed over.

Lagrangian \( \mathcal{L}_{II} \) itself does not have a \( \tau \tau \) extension as it contains the first-order derivatives also quadratically and in its interaction part, cf. (2.8). We therefore take an equivalent first-order Lagrangian, the real scalar field

\[
\mathcal{L}_I \equiv L_{ab} F_b^{\mu \nu} \left[ \partial_\nu A_{a \mu} + C_{abcd} A_{c \mu} A_{d \nu} + L_{ac} F_{c \mu \nu} \right] \\
- \bar{\psi}_m \left[ \frac{1}{2} \gamma^\mu \partial_\mu \psi_m + (i \gamma^\mu A_{a \mu} T_{mn}^a + m_{mn} + \phi_i \Gamma_{mn}^i) \psi_n \right] \\
+ \frac{1}{2} \ell_{ij} \varphi_i^T \left[ \partial_\mu \phi_i + 2i A_{a \mu} R_{ik}^a \phi_k + \ell_{ik} \varphi_k \phi_\mu \right] \\
+ \frac{1}{2} \mu^2 \phi_i \phi_i - \mu^2 v^2 \mu^2 v^{-2} b \phi_i + \mu^2 v^{-2} \ell^2 b^2, \tag{3.2}
\]

where: (1) \( A_{a, \psi_n, \phi_n} \) are the fields of \( \mathcal{L}_{II}; F_{a}(x) \) are real, antisymmetric-tensor fields; \( \varphi_i(x) \) are real, four-vector fields; and \( b(x) \) is a real, scalar field. (2) Real, dimensionless parameters \( L_{ab} \) and \( \ell_{ij} \) are elements of two invertible matrices, and \( \ell \neq 0 \) is a real, dimensionless parameter. (3) \( a\partial_\mu b \equiv a(\partial_\mu b) - (\partial_\mu a)b \). Within the framework of QFTs, Lagrangian \( \mathcal{L}_I \) is equivalent to \( \mathcal{L}_{II} \) since their Euler-Lagrange equations are equivalent [17]. Physical significance of parameters \( L_{ab}, \ell_{ij}, \) and \( \ell \) remains to be found out. If \( L_{ab} = \delta_{ab}, \ell_{ij} = \delta_{ij}, \) and \( \ell = 1 \), the Lagrangian \( \mathcal{L}_I \) is actually, up to a divergence term, a general gauge-invariant Lagrangian in the first-order formalism [18]. Hence \( \mathcal{L}_I \) is also gauge-invariant (up to a divergence term) since \( L_{ab} \) and \( \ell_{ij} \) are elements of invertible matrices.

Unlike the conventional Lagrangian \( \mathcal{L}_{II} \) of non-Abelian gauge theories, the equivalent first-order Lagrangian \( \mathcal{L}_I \) has already two basic properties of tt Lagrangians: Owing to the additional fields \( F_{a}(x) \) and \( \varphi_i(x) \), the first-order derivatives appear only linearly, and there are no derivative couplings, i.e., no interaction terms involving derivatives. Lagrangian \( \mathcal{L}_I \) also has no quartic interaction terms; to this end we introduced the scalar field \( b(x) \), which is therefore not needed if \( v^{-2} = 0 \) in \( \mathcal{L}_{II} \).

3.1. A \( \tau \tau \) extension of a non-Abelian–gauge theory

To construct a ttQFT Lagrangian that formally extends the QFT Lagrangian \( \mathcal{L}_I \), we take the tt field \( \Psi = (\Psi_0, \Psi_{1/2}, \Psi_{1}) \) with real \( \Psi_0 \) and \( \Psi_{1/2} \) but complex \( \Psi_{1/2} \). And we incorporate the fields \( \phi_i(x), \varphi_i(x), b(x), \psi_m(x), A_a(x), \) and \( F_{a}(x) \) of \( \mathcal{L}_I \) in the asymptotic tt field \( \Psi_{as} = (\Psi_{0as}, \Psi_{1/2as}, \Psi_{1as}) \) as follows:

\[
\Psi_{0as}(x,p) = f_0((p-p)\phi_i(x) + 2f_{11}(p-p)\varphi_i(x) \cdot p + f_2(p-p)b(x),
\Psi_{1/2as}(x,p) = (\sigma_-(p), I f_{1/2m}(p-p)\psi_m(x),
\Psi_{1as}(x,p) = (1 - P_1) f_{3a}(p-p) A_a(x) + 2f_{4a}(p-p)F_a(x)p. \tag{3.3}
\]
Here: \((\sigma_-(p), I)\) is a \(4 \times 2\) matrix; \((Ta)^\mu \equiv T^{\mu\nu} a_\nu\) for a second-rank tensor \(T\) and a four-vector \(a\); the longitudinal projection \(P_1 a \equiv (p-p)^{-1}(p-a)p\) for a four-vector \(a\); and \(f_{0i}(p-p), f_{1i}(p-p), f_2(p-p), f_{32m}(p-p), f_{3a}(p-p), \) and \(f_{3a}(p-p)\) are such real functions of \(p-p > 0\) that the real matrices \(C^4_{00}, C^2_{11}, C^1_{1/21/2}, C^1_{33}, C^2_{44}, C^2_{01}, \) and \(C^2_{34}\) with components

\[
C^n_{uvst} \equiv \int d^4p (p-p)^{n-1} f_{us}(p-p)f_{vt}(p-p) \tag{3.4}
\]

\((C^n_{uvst} = C^n_{vufts})\) exist and are invertible,

\[
\int d^4p f_2(p-p)f_{0i}(p-p) = 0, \quad \int d^4p p\cdot p f_2(p-p)f_{1i}(p-p) = 0, \tag{3.5}
\]

\[
\frac{1}{4} C^{3/2}_{1/21/2mn} = \delta_{mn}, \quad L_{ab} = -\frac{1}{2} C^2_{34ab}, \quad \ell_{ij} = -C^2_{01ij}. \tag{3.6}
\]

If \(L_{ab}\) and \(\ell_{ij}\) are given, relations (3.6) are constraints on \(f_{3a}, f_{4a}, f_{0i}, \) and \(f_{1i};\) otherwise, relations (3.6) determine \(L_{ab}\) and \(\ell_{ij}\).

To construct a tt extension of \(L_I\) we will use: scalar fields

\[
\phi_1[x; \Psi] \equiv C^{-1}_{00ij} \int d^4p f_{0j}(p-p)\Psi_0(x,p), \quad b[x; \Psi] \equiv (C^{-2}_{22})^{-1} \int d^4p f_2(p-p)\Psi_0(x,p); \tag{3.7}
\]

chiral-bispinor fields

\[
\psi_m[x; \Psi] \equiv C^{-1}_{1/21/2mn} \int d^4p f_{1/2m}(p-p) \left( -\frac{\sigma_+(p)}{I} \right) \Psi_{1/2}(x,p); \tag{3.8}
\]

four-vector fields

\[
\varphi_1[x; \Psi] \equiv 2C^{-2}_{11ij} \int d^4p f_{1j}(p-p)\Psi_0(x,p), \tag{3.9}
\]

\[
A_1[x; \Psi] \equiv \frac{4}{3} C^{-1}_{33ab} \int d^4p f_{3b}(p-p)(1 - P_1)\Psi_1(x,p); \tag{3.10}
\]

and antisymmetric–rank-two–four-tensor fields

\[
F_1[x; \Psi] \equiv C^{-2}_{44ab} \int d^4p f_{4b}(p-p)[\Psi_1(x,p)\otimes p - p\otimes \Psi_1(x,p)]. \tag{3.10}
\]

Here: \(C^{-n}_{uv}\) denotes the matrix inverse to \(C^n_{uv}\) (i.e., \(C^{-n}_{uvst}C^n_{uv't't} = \delta_{st}\)); central, big brackets in (3.8) are \(4 \times 2\) matrices; and \((a\otimes b)^{\alpha\beta} \equiv a^\alpha b^\beta\) is the direct product of four-vectors \(a\) and \(b\). Fields (3.7)–(3.10) have properties (2.15) and (2.16) required of macroscopic variables; so,

\[
\phi_1[x; \Psi_{as}] = \phi_1(x), \quad b[x; \Psi_{as}] = b(x), \tag{3.11}
\]

\[
\psi_m[x; \Psi_{as}] = \psi_m(x), \quad \varphi_1[x; \Psi_{as}] = \varphi_1(x), \tag{3.11}
\]

\[
A_1[x; \Psi_{as}] = A_1(x), \quad F_1[x; \Psi_{as}] = F_1(x). \tag{3.11}
\]
We will refer to the scalar and four-vector tt fields $\Psi_0(x,p)$ and $\Psi_1(x,p)$ as the bosonic tt fields, and to the left-spinor tt field $\Psi_{1/2}(x,p)$ as the fermionic tt field, since they are related to boson and fermion fields of $L_I$ through (3.7)–(3.11).

Let us now consider the tt Lagrangian

$$L_{tt} = L_{free}(\Psi, p \cdot \partial \Psi) + L_2(\Psi) + L_3(\Psi) + L_\lambda(\Psi),$$

where: (1) According to (2.8), the free tt Lagrangian

$$L_{free}(\Psi, p \cdot \partial \Psi) = \left[ \Psi_0 | p \cdot \partial \Psi_0 \right] + \frac{1}{2} \left[ \Psi_{1/2} | p \cdot \partial \Psi_{1/2} \right] + \frac{1}{2} \left[ p \cdot \partial \Psi_{1/2} | \Psi_{1/2} \right] + \left[ \Psi_1 | p \cdot \partial \Psi_1 \right].$$

For the asymptotic tt fields (3.3), $L_{free}$ equals the sum of all terms in $L_I$ that contain spacetime derivatives, by (A.18)–(A.20) and (3.6). (2) We use

$$L_2 \equiv L_{ab}F_{bj\nu}[x; \Psi]L_{ac}F^\mu_{bj}[x; \Psi] - \overline{\psi}_m[x; \Psi]m_{mn}\psi_n[x; \Psi]$$

$$+ \frac{1}{2}\ell_i\phi_{ji}(x; \Psi)\ell_j\phi_{ik}(x; \Psi) + \frac{1}{2}\lambda^2\phi_i(x; \Psi)\phi_i(x; \Psi) + \mu^2v^{-2}\ell^2b^2[x; \Psi],$$

$$L_3 \equiv C_{\alpha\beta\gamma}L_{ab}F^\mu_{bj}[x; \Psi]A_{\mu}[x; \Psi]A_{\nu}[x; \Psi]$$

$$- \overline{\psi}_m[x; \Psi]\{i\gamma^\mu A_{\mu}[x; \Psi]T^a_{mn} + \phi_i(x; \Psi)\Gamma^i_{mn}\}\psi_n[x; \Psi]$$

$$+ i\ell_i\phi_{ji}(x; \Psi)\phi_{ik}(x; \Psi)A_{\alpha}[x; \Psi]R^\alpha_{ik}\phi_k[x; \Psi] - \mu^2v^{-2}\ell^2b[x; \Psi]\phi_i(x; \Psi)\phi_i(x; \Psi).$$

Thus $L_2(\Psi_{as})$ equals the sum of all quadratic terms in $L_I$ that contain no spacetime derivatives, whereas $L_3(\Psi_{as})$ equals the sum of all cubic interaction terms, by (3.11). (3) Real, scalar field

$$L_\lambda \equiv \lambda^{-2} \sum_j \left[ \Psi_j | (1 - P_j) \Psi_j \right]$$

with parameter $\lambda > 0$, $[\lambda] = [L]$, and projections

$$P_0\Psi_0 \equiv f_{03}(p \cdot p)\phi_i(x; \Psi) + 2f_{13}(p \cdot p)\phi_i(x; \Psi)\cdot p + f_2(p \cdot p)b[x; \Psi],$$

$$P_{1/2}\Psi_{1/2} \equiv (\sigma_-, I) f_{1/2m}(p \cdot p)\psi_m[x; \Psi],$$

$$P_1\Psi_1 \equiv (1 - P_3)a(p \cdot p)A_{\alpha}[x; \Psi] + 2f_3a(p \cdot p)F_\alpha[x; \Psi]p$$

[Note that $P_1P_3 = P_3P_1 = 0$, $P_3\Psi_{jas} = \Psi_{jas}$, and $L_\lambda(\Psi_{jas}) = 0$. The tt Lagrangian $L_{tt}$ defined by (3.12)–(3.16) is a real, scalar field, by (2.8) and since we obtained its interaction part from the interaction part of $L_I$ by replacing its fields with the corresponding fields (3.7)–(3.10) [which transform under Lorentz transformations of the tt field $\Psi$ the same way as the original fields of $L_I$, by (2.16)], and then adding the real, scalar field $L_\lambda$.

If $\lambda \to 0$, either $L_\lambda(\Psi) = 0$, or $L_\lambda(\Psi)$ and the tt Lagrangian $L_{tt}$ tend to infinity. Taking into account (A.7),

$$[P_j\Psi_j | \Psi_j'] = [\Psi_j | P_j\Psi_j'], \quad P_j^2 = P_j, \quad j = 0, 1/2, 1,$$
we can conclude that (1) the Euler-Lagrange equations of $L_{\lambda}(\Psi)$ read

$$(1 - \mathcal{P}_j)\Psi_j = 0, \quad j = 0, 1/2, 1,$$  (3.18)

and (2) their most general solutions are the asymptotic tt fields (3.3). As for every asymptotic tt field (3.3) the tt Lagrangian $\mathcal{L}_{tt}$ equals the QFT Lagrangian $\mathcal{L}_I$, i.e.,

$$\mathcal{L}_{tt}(\Psi_{as}, p \cdot \partial \Psi_{as}) = \mathcal{L}_I,$$  (3.19)

we may follow the arguments presented in Sec. 2.3 and consider $\mathcal{L}_I$ as a path-integralwise approximation to $\mathcal{L}_{tt}$ in the asymptote $\lambda \to 0$. So we may presume that the tt Lagrangian $\mathcal{L}_{tt}$ is a tt extension of the QFT Lagrangian $\mathcal{L}_I$. The gauge-invariant Lagrangian $\mathcal{L}_{II}$ does not have mass terms needed to model fundamental interactions. It is customary to get rid of this deficiency by assuming that its gauge symmetry is spontaneously broken, and replace the scalar fields $\phi_i(x)$ with $\phi_i(x) + \phi_{gi}$, $\phi_{gi} \phi_{gi} = v^2$. This way $\mathcal{L}_{II}$ acquires necessary, quadratic mass terms but no linear terms [6]. If we accordingly shift the scalar fields in $\mathcal{L}_I$,

$$\phi_i(x) \to \phi_i(x) + \phi_{gi}, \quad b(x) \to b(x) + \frac{1}{2} v^2 / \ell,$$  (3.20)

then (1) in $\mathcal{L}_I$ we do not generate linear terms excepting a divergence term, and (2) $\mathcal{L}_I$ becomes equivalent to $\mathcal{L}_{II}$ whose scalar fields $\phi_i(x)$ have been replaced with the shifted fields $\phi_i(x) + \phi_{gi}$. The $x$-independent tt field

$$\Psi_{HG}(p) \equiv \left( f_{0i}(p \cdot p) \phi_{gi} + \frac{1}{2} v^2 \ell^{-1} f_2(p \cdot p), 0, 0 \right), \quad \phi_{gi} \phi_{gi} = v^2,$$  (3.21)

is a tt counterpart to the ground state $\phi_i(x) = \phi_{gi}$ and $b(x) = \frac{1}{2} v^2 / \ell$ of $\mathcal{L}_I$. Namely, the tt Lagrangian (3.12)–(3.16) with $\Psi \to \Psi + \Psi_{HG}$ has no linear terms (except for a divergence term), and for $\Psi = \Psi_{as}$, it equals $\mathcal{L}_I$ whose scalar fields have been shifted as specified by (3.20).

3.2. Cubic interactions as the origin of quadratic ones

We now give an example of a tt Lagrangian, say, $\mathcal{L}_C$ that extends $\mathcal{L}_I$ as specified in Sec. 2.3, and has only cubic interaction terms. Introducing real functions $f(p \cdot p)$ and $f_g(p \cdot p)$ with dimensions $[L]^4$ and $[L]^2$, and a positive dimensionless constant $a_g \equiv [pf \mid pf_g]$, we define:

$$\mathcal{L}_C(\Psi, p \cdot \partial \Psi) \equiv \mathcal{L}_{free} + a_g^{-1} [pf \mid \Psi_1] \mathcal{L}_2 + \mathcal{L}_3 + a_g^{-1} [pf \mid \Psi_1] \mathcal{L}_\lambda,$$  (3.22)

where $\mathcal{L}_{free}, \mathcal{L}_2, \mathcal{L}_3,$ and $\mathcal{L}_\lambda$ are as defined in (3.13)–(3.16) with $\mathcal{P}_1$ replaced with $\mathcal{P}_1 + \mathcal{P}_1$. As the variables of the path integral specified by $\mathcal{L}_C(\Psi, p \cdot \partial \Psi)$ we take tt fields $\Psi$ of the form

$$\Psi = \Psi_t + \Psi_g,$$  (3.23)
where \( \Psi_t \equiv (\Psi_0, \Psi_{\lambda/2}, (1 - \mathcal{P}_I)\Psi_1) \) and \( \Psi_g \equiv (0, 0, pf_g) \); we note that
\[
\mathcal{L}_C(\Psi_t + \Psi_g, p \cdot \partial (\Psi_t + \Psi_g)) = \mathcal{L}_{tt}(\Psi_t, p \cdot \partial \Psi_t),
\] (3.24)
where \( \mathcal{L}_{tt} \) is the tt Lagrangian (3.12)–(3.16). If \( \lambda \to 0 \) and \( \mathcal{L}_\lambda(\Psi) \neq 0 \), then \( \mathcal{L}_C \) tends to infinity. The most general solutions to the Euler-Lagrange equations of \( \mathcal{L}_\lambda \) in (3.22) are \( \Psi_{as} + \Psi_g \), which are allowed by (3.23). As \( \mathcal{L}_C(\Psi_{as} + \Psi_g, p \cdot \partial (\Psi_{as} + \Psi_g)) = \mathcal{L}_I \), by (3.24), \( \mathcal{P}_I \mathcal{P}_l = 0 \), and (3.19), we may indeed consider the tt Lagrangian \( \mathcal{L}_C \) a tt extension of the first-order Lagrangian \( \mathcal{L}_I \).

If we replace \( \Psi \) in \( \mathcal{L}_C(\Psi, p \cdot \partial \Psi) \) with \( \Psi_t + \Psi_g \), we generate no linear terms, by (3.24). So we may see \( \Psi_g(p) \) as a ground state of a kind, though there are certain distinctions between \( \Psi_g(p) \) and the tt ground state \( \Psi_{HG}(p) \): (1) Ground state \( \Psi_g \) is not determined by the condition that the Lagrangian \( \mathcal{L}_C \) gets no linear terms if we replace \( \Psi \) with \( \Psi_t + \Psi_g \). (2) The ground state \( \Psi_g \) and the integration variable \( \Psi_t \) in path integrals specified by \( \mathcal{L}_C(\Psi_t + \Psi_g, p \cdot \partial (\Psi_t + \Psi_g)) \) do not interfere, by (3.24). Ground state \( \Psi_g \) is an additional, \( x \)-independent, permanent, Lorentz-invariant feature of the universe: a classical component of the relativistic vacuum extending throughout the universe.

If we regard the tt Lagrangian \( \mathcal{L}_C \) as the basic description of fundamental interactions and the ground state \( \Psi_g(p) \) as a historical accident, then \( \mathcal{L}_C \) should not depend on constant \( a_g \) determined by \( \Psi_g \) (for a given \( f \)). This is the case if: (1) We rescale the units of fields \( A^a_\mu \) and \( \phi_i \) by replacing them in (3.1) with \( a_g^{-1/2} A^a_\mu \) and \( a_g^{-1/2} \phi_i \). (2) We replace \( L_{ab} \) and \( \ell_{ij} \) in (3.2) with \( a_g^{1/2} L_{ab} \) and \( a_g^{1/2} \ell_{ij} \). (3) We assume that \( C_{abc}, T^a_{mn}, R^b_{ij}, \Gamma^i_{mn}, m_{mn}, \mu^2, \nu^2 \), and \( \lambda \) are proportional to \( a_g^{1/2}, a_g^{1/2}, a_g^{1/2}, a_g^{1/2}, a_g^{1/2}, a_g \), and \( a_g^{-1/2} \), respectively. In such a case, \( \lambda \to 0 \) if \( a_g \to \infty \), and we may explain the QFT asymptote \( \lambda \to 0 \) as describing transitions where the interaction between the tt ground state \( \Psi_g \) and tt field \( \Psi_t \) suppresses projections \( (1 - \mathcal{P}_j) \) of its components so much that they are negligible. Masses in \( \mathcal{L}_{II} \) with spontaneously broken symmetry are then proportional to \( a_g \). So we may interpret the mass terms in \( \mathcal{L}_{II} \) with spontaneously broken symmetry as due to interaction with the longitudinal, tt ground state \( \Psi_g \).

3.3. Unresolved questions about tt extensions

It is not clear which physical principles and properties, in addition to those mentioned at the beginning of this section and to unitarity of the S-matrix, further restrict realistic tt extensions of \( \mathcal{L}_I \). The gauge invariance of \( \mathcal{L}_{II} \) ensures renormalizability, also when it is spontaneously broken, and severely constrains the values of \( \mathcal{L}_{II} \) parameters \( C_{abc}, T^a_{mn}, R^b_{ij}, \Gamma^i_{mn}, \) and \( m_{mn} \). We do not know whether there is a tt counterpart to the concept of QFT gauge invariance (e.g., a symmetry and/or high-energy unitarity constraints [16]) that similarly restricts acceptable tt extensions of \( \mathcal{L}_I \). Any restriction would be of great help since there is an infinite variety of possible tt extensions of \( \mathcal{L}_I \), cf. (A.22)–(A.31). When formulating a tt extension, we have, e.g., the following options:

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(a) The domain of the independent variable \( p \). It may be the whole \( \mathbb{R}^{1,3} \) or only a Lorentz-invariant subspace of \( \mathbb{R}^{1,3} \).

(b) The kind of functions of \( x \) and \( p \) that \( \Psi(x, p) \) are, cf. Sec. 5.1 and [13].

(c) The number and type of \( \Psi_j(x, p) \) used. We can use, instead of the left-spinor \( \Psi_{1/2}(x, p) \), a right-spinor or a bispinor \( \Psi \) field: \( \Psi_{-1/2}(x, p) \) or \( (\Psi_{-1/2}(x, p), \Psi_{1/2}(x, p)) \). We can also use one scalar, one two-component-spinor, and one, possibly transversal, four-vector \( \Psi \) field for each Higgs-scalar field \( \phi_i \), Dirac-spinor field \( \psi_m \), and four-vector gauge field \( A_a \), respectively. On the other hand, we can construct a \( \Psi \) extension of \( \mathcal{L}_I \) analogous to (3.12)–(3.16) without using the scalar \( \Psi_0 \), by incorporating the fields \( \phi_i(x), \phi_i(x), \) and \( b(x) \) of \( \mathcal{L}_I \) into the longitudinal component of the asymptotic, four-vector \( \Psi \) field \( \Psi_{1as} \) instead of into the scalar \( \Psi_{0as} \) [e.g., by adding \( \Psi_{0as} \) to \( \Psi_{1as} \) in (3.3)]. Alternatively, the \( \Psi \) Lagrangian \( \mathcal{L}_I \) of (3.24) is a \( \Psi \) extension of \( \mathcal{L}_I \) that needs fewer \( \Psi \) fields than \( \mathcal{L}_I \), namely \( \Psi_0 \), \( \Psi_{1/2} \), and \( (1 - P_l) \Psi_1 \) instead of \( \Psi_0 \), \( \Psi_{1/2} \), \( (1 - P_l) \Psi_1 \), and \( P_l \Psi_1 \). If only one four-vector \( \Psi \) field and one two-component-spinor \( \Psi \) field suffice for modeling fundamental interactions, then no approximate \( \Psi \)QFT can do with fewer \( \Psi \) fields, in contrast to QFTs where, e.g., QED uses fewer \( \Psi \) fields than the standard model.

(d) The functions \( f_{us}(p \cdot p) \) that shape the \( p \)-dependence of the asymptotic \( \Psi \) fields (3.3). Here they are restricted only by the weak requirements (3.4)–(3.6), but may not be constant, and may also depend on \( \lambda \). They are not necessarily distinct, though those having the same first subscript are linearly independent, by (3.4); e.g., we could choose \( f_{3a}(y) \) proportional to \( \sqrt{y} f_{4a}(y) \) and/or \( f_{0i}(y) \) proportional to \( \sqrt{y} f_{1i}(y) \). We have yet to find out how to choose them so that the fields (3.7)–(3.10) are macroscopic variables. Were the asymptotic \( \Psi \) fields \( \Psi_{jas}(x, p) \) the initial parts of certain expansions of \( \Psi \) fields \( \Psi_j(x, p) \) in terms of orthogonal functions of variable \( p \), there would be additional requirements on \( f_{us}(p \cdot p) \). Which might also imply restrictions on the number and internal symmetries of QFT fields incorporated in \( \Psi_{jas}(x, p) \), cf. (5.1), (5.3), and [19].

(e) The dependence on \( \lambda \) and \( \Psi(x, p) \). We can add to the \( \Psi \) Lagrangians \( \mathcal{L}_I \) and \( \mathcal{L}_C \) any term that vanishes if \( \lambda \to 0 \) or is finite as \( \lambda \to 0 \) and zero for \( \Psi_{jas}(x, p) \).

(f) A property of \( \mathcal{L}_I \) is either explicitly displayed by its \( \Psi \) extension, or it emerges only in the QFT asymptote. E.g., it is not clear whether a \( \Psi \) Lagrangian that extends \( \mathcal{L}_I \) should have a \( \Psi \) counterpart to the gauge invariance and/or to the ground state of \( \mathcal{L}_I \), cf. (3.21).

If there is a basic \( \Psi \)QFT of fundamental interactions, better than any QFT, it is of interest to know the answers to questions raised in the above alternatives, and to the following ones:

(a) How many free parameters has the basic \( \Psi \) Lagrangian in comparison with the standard model, and how are they related?
(b) Is the basic tt Lagrangian bilinear in fermionic tt fields? cf. [20].

(c) Does the interaction part of the basic tt Lagrangian contain: (1) only cubic terms, due to the emission (creation) or absorption (annihilation) of bosonic tt fields by fermionic or bosonic tt fields; (2) only quartic terms, due to the self-scattering of bosonic tt fields or to the scattering of fermionic tt fields on bosonic ones, but no quartic terms describing self-scattering of fermionic tt fields; (3) only cubic and quartic terms? Comparing tt Euler-Lagrange equations with the Boltzmann equation [8–10], we would expect that the interaction part of the basic tt Lagrangian contains only cubic terms. However, noting that the Boltzmann equation models collisions of two particles, we would expect only quartic terms.

(d) Does a solution $\Psi$ to the basic tt Euler-Lagrange equations evolve toward an $x$-independent, Lorentz-invariant tt field, say, $\Psi_{eq}(p; \Psi)$ as the time $t \to \infty$? If so: Is $P_{as}\Psi_{eq}(p; \Psi) \neq 0$ and/or $(1-P_{as})\Psi_{eq}(p; \Psi) \neq 0$? Is $L(\Psi_{eq}(p; \Psi), p \cdot \partial \Psi_{eq}(p; \Psi)) = 0$? Is $\Psi_{eq}(p; \Psi)$ related to a ground state that endows particles with mass?

(e) How does it come about that we can adequately describe present experiments by QFTs? What physical processes make it possible to replace the basic ttQFT with some QFT? Can they be formally simulated by limiting to zero some parameters of the basic ttQFT? Under what conditions are $x$-independent terms absent from (2.10) and functions $F_k(p)$ independent of the initial and final states?

(f) What are the implications for the QFT Lagrangian that approximates the basic tt Lagrangian in the QFT asymptote? E.g., what fields are present in the asymptotic tt fields $\Psi_{as}(x, p)$? cf. (A.8), and how are possible tt and QFT stationary points related? cf. (3.21).

(g) How are the quadratic terms with no spacetime derivatives generated in Lagrangians of realistic QFTs? e.g., in $L_I$ with possibly shifted scalar fields (3.20). In the kinetic theory of gases, the linear terms of a tt Lagrangian would be determined by the independent sources, whereas the bilinear ones would describe free streaming or interactions with an underlying host medium not described by tt fields. So it seems an appealing physical proposition to assume, without any reference to possible symmetries of the basic tt Lagrangian, that it has no linear terms, and no quadratic terms in addition to those, (2.8), that describe free streaming. If so, there is a question about physical processes that result in a realistic QFT Lagrangian with no linear terms and having also quadratic terms with no derivatives. We see the following three possibilities: First, quadratic terms with no derivatives in this QFT Lagrangian could be generated within the QFT framework from a QFT Lagrangian that has no such terms and is implied in the QFT asymptote by the basic tt Lagrangian with no linear and quadratic-interaction terms. Second, various non-exclusive tt processes could provide the basic tt Lagrangian with counterparts to the quadratic terms with no derivatives of the QFT Lagrangian in question: (1) An $x$-independent, Lorentz-invariant, tt ground state that (i) does not belong to the region of integration of ttQFT path integral, (ii) represents such a permanent classical relativistic vacuum as
\( \Psi_g \) does in (3.24), and (iii) whose origin is not known. (2) A \( \text{tt} \) analogue to the QFT process of dynamical symmetry breaking [6] that can be described in the low-energy approximation by an effective \( \text{ttQFT} \), the \( \text{tt} \) Lagrangian of which has the required quadratic terms. (3) A \( \text{tt} \) phase transition analogous to the QFT process of spontaneous symmetry breaking: An \( x \)-independent, Lorentz-invariant stationary point of the basic \( \text{tt} \) Lagrangian is added here to the original \( \text{tt} \) fields, which generates an equivalent \( \text{tt} \) Lagrangian that has quadratic interaction terms but no linear ones, cf. (3.21). And third, were the functions \( F_k(p) \) in (2.10), which shape the \( p \)-dependence of the asymptotic \( \text{tt} \) fields \( \Psi_{\text{as}}(x, p) \), actually certain averages of rapidly varying functions of \( x \), then we would have to take account of correlations when evaluating the free \( \text{tt} \) Lagrangian (2.8) for asymptotic \( \text{tt} \) fields \( \Psi_{\text{as}}(x, p) \). As a consequence, the resulting QFT Lagrangian would acquire from the free \( \text{tt} \) Lagrangian also quadratic terms with no derivatives. This case would be of special interest since mass terms would appear only in the QFT asymptote.

4. Symmetry transformations

4.1. Spacetime translations

The \( \text{tt} \) Lagrangian (2.7) is assumed to be invariant under spacetime translations \( x \rightarrow x + a \) in the sense of (2.5). [Translations \( p \rightarrow p + a \) have no physical significance in \( \text{tt} \) approach; even the free \( \text{tt} \) Lagrangian (2.8) is not invariant under them.] So Noether’s theorem for translations implies that the energy-momentum tensor

\[
T^{\mu\nu}(x) = \sum_j \Re\left[\Psi_j|p^\mu \partial^\nu \Psi_j\right] - \mathcal{L}_{\text{tt}}(\Psi, p \cdot \partial \Psi)\eta^{\mu\nu},
\]

where \( \Re \) stands for real part of, satisfies the continuity equation

\[
\partial_\mu T^{\mu\nu}(x) = 0
\]

provided \( \text{tt} \) fields \( \Psi_j(x, p) \) satisfy the \( \text{tt} \) Euler-Lagrange equations [1, 21]. The energy-momentum tensor of the \( \text{tt} \) ground state \( \Psi_g(p) \) is a relativistic invariant \( T^{\mu\nu}(x) \equiv 0 \), by (3.24).

4.2. Internal symmetries

We will say that a \( \text{tt} \) Lagrangian \( \mathcal{L}_{\text{tt}}(\Psi, p \cdot \partial \Psi) \) is invariant under infinitesimal, internal-symmetry transformations

\[
\Psi_j(x, p) \rightarrow \Psi_j(x, p; \epsilon) \equiv \Psi_j(x, p) + i\epsilon \sum_k t_{jk} \Psi_k(x, p)
\]

if

\[
\frac{\partial}{\partial \epsilon} \mathcal{L}_{\text{tt}}(\Psi(x, p; \epsilon), p \cdot \partial \Psi(x, p; \epsilon))\bigg|_{\epsilon=0} = 0,
\]

18
where $\epsilon$ is a parameter and $t_{jk}$ a mapping of the field $\Psi_k$ into the field $\Psi_j$. If so, the associated current

$$j(x) = \sum_{jk} \Re[\Psi_j | i p t_{jk} \Psi_k]$$

(4.5)

satisfies the continuity equation

$$\partial \cdot j(x) = 0$$

(4.6)

provided $\Psi_j$ satisfy the Euler-Lagrange equations, by (2.7) and (2.8) [1, 6, 21]. Infinitesimal transformations (4.3) appear the same in all inertial frames if, and only if $U(\Lambda, a)t_{jk} = t_{jk}U(\Lambda, a)$; in such a case, the current $j(x)$ transforms as a four-vector field under the replacement of $\Psi(x, p)$ with $U(\Lambda, a)\Psi(x, p)$. If $t_{jk}$ are such complex constants that $t^*_{kj} = t_{jk}$, then the current (4.5) is invariant under infinitesimal transformations (4.3), i.e., $\partial j(x)/\partial \epsilon = 0$ at $\epsilon = 0$.

For a global phase change of the field $\Psi_j(x, p)$,

$$\Psi_j(x, p) \rightarrow e^{i\varphi} \Psi_j(x, p), \quad \varphi \in \mathbb{R},$$

(4.7)

the associated current $j(x) = [\Psi_j | i p \Psi_j]$ is a real, four-vector field, by (A.4) and (A.6). [This current vanishes for real $\Psi_j(x, p)$, $j = 0, 1$, by (A.4)–(A.6): a real bosonic field effects no such current.] Both tt Lagrangians (3.12)–(3.16) and (3.22)–(3.23) exhibit invariance under global phase changes of the fermionic tt field $\Psi_{1/2}(x, p)$, which is analogous to the invariance of Lagrangians $L_{II}$ and $L_I$ under joint global phase changes of fermion fields $\psi_m$.

Lagrangian $L_I$ is invariant under the infinitesimal, global, gauge transformations

$\begin{align*}
A_a &\rightarrow A_a + \epsilon_a C_{abc} A_c, & L_{ab} F_b &\rightarrow L_{ab} F_b + \epsilon_b C_{abc} L_{cd} F_d, \\
\psi_m &\rightarrow \psi_m - i \epsilon_a T_{mn} \psi_n, & \ell b &\rightarrow \ell b, \\
\phi_j &\rightarrow \phi_j - i \epsilon_a R^a_{jk} \phi_k, & \ell_{ij} \varphi_j &\rightarrow \ell_{ij} \varphi_j - i \epsilon_a R^a_{jk} \ell_{kl} \varphi_l,
\end{align*}$

(4.8)

where $\epsilon_a$ are real parameters [6]. As the tt extension (3.12)–(3.16) of $L_I$ does not exhibit an analogous symmetry, let us point out such a tt extension of $L_I$ that is invariant under infinitesimal, global–internal-symmetry transformations analogous to (4.8). To this end, we take the following tt fields: real, scalar tt fields $\Psi_0(x, p)$ and $\Psi_{0j}(x, p)$; complex, left-spinor tt fields $\Psi_{1/2m}(x, p)$; real, four-vector tt fields $\Psi_{1a}(x, p)$; and associated asymptotic tt fields

$\begin{align*}
\Psi_{0as} &= f_0(p \cdot p) b(x), & \Psi_{0jas} &= f_0(p \cdot p) \phi_j(x) + 2 f_1(p \cdot p) p \cdot \varphi_j(x), \\
\Psi_{1/2mas} &= f_{1/2}(p \cdot p) (\sigma_-(p), I) \psi_m(x), & \Psi_{1aas} &= f_2(p \cdot p) A_a(x) + 2 f_3(p \cdot p) F_a(x) p. 
\end{align*}$

(4.9)
We then construct a tt extension of $\mathcal{L}_I$ on the analogy of the tt extension (3.12)–(3.16). Such tt extension of $\mathcal{L}_I$ is invariant under infinitesimal, global–internal-symmetry transformations

\[
\begin{align*}
\Psi_{1a} &\rightarrow \Psi_{1a} + \epsilon_b C_{abc} \Psi_{1c}, \\
\Psi_{1/2m} &\rightarrow \Psi_{1/2m} - i \epsilon_a T_{mn}^a \Psi_{1/2n}, \\
\Psi_{0j} &\rightarrow \Psi_{0j} - i \epsilon_a R_{jk}^a \Psi_{0k}, \\
\Psi_0 &\rightarrow \Psi_0;
\end{align*}
\]  

(4.10)

and the associated conserved currents (4.5) are $j_a(x) \equiv -\Re[\Psi_{1/2m} | i p T_{mn}^a \Psi_{1/2n}]$. Hence, there are tt extensions of $\mathcal{L}_I$ that exhibit an analog to the global–non-Abelian-gauge invariance, though they need more tt fields than tt Lagrangians (3.12)–(3.16) and (3.22)–(3.23), which exhibit less symmetry [see (A.28)–(A.34) about the local gauge invariance]. So in ttQFTs, more symmetry requires more tt fields $\Psi_j$, since the free tt Lagrangian (2.8) is not invariant under operations on macroscopic variables of its tt fields $\Psi_j$; a simpler tt model exhibits less symmetry! This makes one wonder whether ttQFTs of fundamental interactions exhibit global or local, non-Abelian–gauge symmetries only in the QFT approximation [22]; especially as one has to add terms to Lagrangians of QFTs to remove the gauge invariance, thereby allowing construction of propagators.

4.3. $\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, and chiral transformations

For scalar, four-vector, left-spinor, and right-spinor tt fields we define the charge conjugation transformation $\mathcal{C}$ as follows:

\[
\mathcal{C} \Psi_0 \equiv \Psi_0^*(x, p), \quad \mathcal{C} \Psi_1 \equiv -\Psi_1^*(x, p), \\
\mathcal{C} \Psi_{\pm 1/2} \equiv \mp \sigma_\mp(p) \sigma^2 \Psi_{\mp 1/2}(x, p).
\]  

(4.11)

Note that $U(\Lambda, a) \mathcal{C} = \mathcal{C} U(\Lambda, a)$, $\mathcal{C}^2 = 1$, and $[\mathcal{C} \Psi_j | \mathcal{C} \Psi'_j] = (-1)^{2j} [\Psi_j | \Psi'_j]^*$. Hence, $[\Psi_j | \Psi_j] = 4 \langle (1 + \mathcal{C}) \Psi_j | (1 + \mathcal{C}) \Psi_j \rangle + (-1)^{2j} \frac{4}{2} \langle (1 - \mathcal{C}) \Psi_j | (1 - \mathcal{C}) \Psi_j \rangle$, i.e., $(1 + \mathcal{C}) \Psi_j$ and $(1 - \mathcal{C}) \Psi_j$ do not interfere. In particular, real and imaginary parts of $\Psi_0$ and of $\Psi_1$ do not interfere.

For tt fields we define the space inversion transformation $\mathcal{P}$ as follows:

\[
\mathcal{P} \Psi_0 \equiv \Psi_0(\mathcal{P}_1 x, \mathcal{P}_1 p), \quad \mathcal{P} \Psi_1 \equiv (\mathcal{P}_1 \Psi_1)(\mathcal{P}_1 x, \mathcal{P}_1 p), \\
\mathcal{P} \Psi_{\pm 1/2} \equiv \sigma_\mp(p) \Psi_{\mp 1/2}(\mathcal{P}_1 x, \mathcal{P}_1 p),
\]  

(4.12)

where $\mathcal{P}_1 x \equiv (x^0, -x)$ is the space inversion of a four-vector $x$. Note that $\mathcal{P}^2 = 1$, $[\mathcal{P} \Psi_j | \mathcal{P} \Psi'_j] = \mathcal{P} [\Psi_j | \Psi'_j]$, and $U(\Lambda(g, \beta), a) \mathcal{P} = \mathcal{P} U(\Lambda(g, -\beta), \mathcal{P}_1 a)$ with $g$ and $\beta$ being the rotation and boost parameters of a proper orthochronous Lorentz transformation $\Lambda$.

For a four-vector $x$, its time-inversion $\mathcal{T}_1(x^0, x) \equiv (-x^0, x)$. For tt fields we define the time-reversal transformation $\mathcal{T}$ as follows:

\[
\mathcal{T} \Psi_0 \equiv \Psi_0^*(\mathcal{T}_1 x, \mathcal{T}_1 p), \quad \mathcal{T} \Psi_1 \equiv - (\mathcal{T}_1 \Psi_1^*)(\mathcal{T}_1 x, \mathcal{T}_1 p), \\
\mathcal{T} \Psi_{\pm 1/2} \equiv -i \alpha^2 \Psi_{\pm 1/2}^*(\mathcal{T}_1 x, \mathcal{T}_1 p).
\]  

(4.13)
Note that \( U(\Lambda(g, \beta), a) \mathcal{T} = \mathcal{T} U(\Lambda(g, -\beta), T_1 a) \), \( \mathcal{T}^2 \Psi_j = (-1)^{2j} \Psi_j \), and 

\[
[\mathcal{T} \Psi_j \mid \mathcal{T} \Psi'_j] = \mathcal{T} [\Psi_j \mid \Psi'_j].
\]

We may regard the above \( \mathcal{T} \) symmetry transformations \( C, P, \) and \( \mathcal{T} \) as extensions of the QFT symmetry transformations \( C, P, \) and \( \mathcal{T} \) with certain phase factors, because under \( C, P, \) and \( \mathcal{T} \) transformations of \( \mathcal{T} \) fields: (1) the asymptotic fields (3.3) satisfy relations analogous to (2.12), and (2) the fields (3.7)–(3.10) transform as the corresponding QFT fields; e.g., \( \psi_m[x; C \Psi] = \gamma^2 \psi_m^*[x; \Psi] \), \( \psi_m[x; P \Psi] = -\gamma^0 \psi_m[P_1 x; \Psi] \), and \( \psi_m[x; \mathcal{T} \Psi] = \gamma^3 \gamma^1 \psi_m^*[T_1 x; \Psi] \), by (3.8) and (4.11)–(4.13). [This is so only when the functions \( f_{us}(p \cdot p) \) in (3.3) are real-valued.] If a scalar or a four-vector \( \mathcal{T} \) field, \( \Psi_0(x, p) \) or \( \Psi_1(x, p) \), does not depend on the additional variable \( p \), then the \( \mathcal{T} \) symmetry transformations \( P, \mathcal{T}, \) and \( C \) have the same effect as the original QFT ones.

Any product of \( \mathcal{T} \) transformations \( C, P, \) and \( \mathcal{T} \) is equivalent to the strong-reflection \( \mathcal{T} \) transformation; i.e., for \( j = 0, \pm 1/2, 1 \), we have

\[
CPT \Psi_j = (-1)^{2j} PCT \Psi_j = PT C \Psi_j = \mathcal{T} PC \Psi_j = (-1)^{2j} \mathcal{T} CP \Psi_j.
\]

Under the combined \( C, P, \mathcal{T}, \) and \( x \to -x \) transformations of \( \mathcal{T} \) fields \( \Psi_j \), taken in any order, the terms of the \( \mathcal{T} \) Lagrangian (3.12)–(3.16) that depend bilinearly on the fermionic \( \mathcal{T} \) field \( \Psi_{1/2}(x, p) \) change sign, whereas the terms depending only on the bosonic \( \mathcal{T} \) fields \( \Psi_0(x, p) \) and \( \Psi_1(x, p) \) remain the same—just as in the case of \( \mathcal{L}_I \) or \( \mathcal{L}_{II} \). On the analogy of the CPT theorem of QFTs, we expect that also the theoretical results of realistic \( \mathcal{T} \)QFTs are invariant under combined \( C, P, \) and \( \mathcal{T} \) transformations, i.e., we expect that fundamental interactions exhibit the CPT invariance also in \( \mathcal{T} \)QFTs.

We define the \textit{chiral transformation} of spinor \( \mathcal{T} \) fields as

\[
\Psi_{\pm 1/2}(x, p) \to \mp \Psi_{\pm 1/2}(x, -p),
\]

It induces the QFT chiral transformation \( \psi_{m}[x; \Psi_{\pm 1/2}] \to \gamma^5 \psi_{m}[x; \Psi_{\pm 1/2}] \) in the asymptotic \( \mathcal{T} \) fields (3.3) and (A.8), and in the chiral-bispinor fields (3.8) and (A.13). Free \( \mathcal{T} \) Lagrangian (2.8) is invariant under \( \mathcal{T} \) chiral transformation (4.15).

5. Comments on physics of \( \mathcal{T} \)QFTs

In the \( \mathcal{T} \) approach to quantum dynamics, the first-order form \( \mathcal{L}_I \) of a general gauge-invariant Lagrangian represents physical processes in the QFT asymptote, though it is equivalent to the second-order form \( \mathcal{L}_{II} \). So due to the form (2.8) of the free \( \mathcal{T} \) Lagrangian, the \( \mathcal{T} \) approach suggests that the first-order formalism of QFTs is a more direct description of fundamental interactions than the conventional, second-order one. Which (1) may explain claims that the first-order formalism simplifies calculations in QFTs [23], (2) concurs with Schwinger’s
statement that it is “natural” [23], and (3) gives support to Greiner’s conjecture that “the Good Lord wrote the field equations in linearized form” [24], since the Euler-Lagrange equations of a first-order-form QFT Lagrangian are themselves first-order partial-differential equations.

According to ttQFTs, all spacetime derivatives in Lagrangians of QFTs, including those that appear in derivative couplings, come from the free part (2.8) of tt Lagrangians. Thus the tt approach offers a simple, unified physical explanation of the origin of all spacetime derivatives in QFT Lagrangians: the free streaming of Feynman’s X-ons in between strictly local collisions, which is described by the substantial time derivative $p \cdot \partial$.

In QFTs one can derive the appropriate path-integral formalism from the canonical formalism, and vice versa [1]. But within the tt framework this may not be so outside the QFT asymptote. If nature is nonlocal or granular in the small, Bjorken and Drell [4] expect that the canonical formalism applies only in the sense of a correspondence principle for large distances.

As in the QFT asymptote $\lambda$ is presumed to be, in effect, infinitesimal, we expect the physics beyond QFTs to be characterized also by a very large energy scale $\lambda^{-1}$. As the Planck length is absent from present QFTs, vastly smaller than any characteristic length of processes described by them, and gives the distance and energy at which present QFTs are expected to break down, it is possible that $\lambda \equiv \sqrt{\hbar G/c^3}$. If so, the Planck length is a fundamental unit of length in ttQFTs. However, it is not clear how to take account of gravitational forces within the tt framework. E.g., we do not know: (1) Is gravity inherent to tt approach? since there are such local averages of bosonic tt fields $\Psi_j(x, p), j = 0, 1,$ that transform under Lorentz transformations of $\Psi_j(x, p)$ as symmetric-tensor fields [e.g., (A.15)]. (2) Should the asymptotic tt fields contain the gravitational, symmetric-tensor field? i.e., can gravity be modeled by a QFT or does it qualitatively modify QFTs? cf. [25], (A.8), (A.18), and (A.20).

Certain theoretic considerations imply the existence of causal, superluminal influences that cannot be directly observed, though they are an essential component of quantum phenomena. Without them it is hard to explain, e.g., experimental violations of Bell’s inequalities [26] and recent experiments in quantum optics [27], which imply that distant events can causally influence each other faster than any light signal could have travelled between them. Present QFTs give adequate, probabilistic descriptions of these experiments; yet they give no clue about the physical processes that could effect such causal, superluminal influences. But ttQFTs may give some, since their Euler-Lagrange equations may be regarded as describing the classical transport of Feynman’s X-ons whose “speeds” $c|p/p^0|$ are not bounded. In the QFT asymptote, the additional variable $p \in \mathbb{R}^{1,3}$ is averaged out, but we expect the physics beyond QFTs to be characterized also by these infinite “speeds”. We were able to give examples of tt Euler-Lagrange equations that exhibit causal, superluminal influences, even though they do not propagate
changes in their macroscopic variables away from their sources faster than light [26].

Physical significance of Feynman’s X-ons is open to discussion. The concept of pointlike entities whose four-momenta $p$ comprise the whole $\mathbb{R}^{1,3}$ opens a whole new vista in quantum metaphysics [21] that we did not consider. Pursuing Feynman’s atomistic idea, we pretended that the partial-differential equations of QFTs model the macroscopic motion of hypothetical X-ons. On the analogy of the kinetic theory of gases, we then surmised that the evolution of such macroscopic motion can be better described by integro-differential, transport equations that asymptotically imply partial-differential equations of QFTs [13]. Hence, as an exploratory step we proposed that QFT Lagrangians be replaced by appropriate tt Lagrangians, and Feynman’s path integrals defined accordingly [14]. Which is the whole significance of X-ons in this paper; here, we may interpret the independent variable $p$ just as an additional, continuous index of “fields” $\Psi_j(x, p)$. In this connection, the following Polyakov’s remarks [28] are worth noticing: “Elementary particles existing in nature resemble very much excitations of some complicated medium (ether). We do not know the detailed structure of the ether but we have learned a lot about effective Lagrangians for its low energy excitations. It is as if we knew nothing about the molecular structure of some liquid but did know the Navier-Stokes equation and could thus predict many exciting things. Clearly, there are lots of different possibilities at the molecular level leading to the same low energy picture.”

5.1. QFTs with an infinite number of fields

On the analogy with the polynomial expansions and the method of moments in particle transport theory [8–10], we expect that any tt field $\Psi(x, p)$ can be completely described by an infinite number of relativistic fields that are certain averages over the variable $p \in \mathbb{R}^{1,3}$, cf. (2.17). Hence, in tt approach, we expect the Lagrangians of QFTs modeling more accurate or higher-energy experiments to be customized and contain not only more, specific terms, but also appropriate additional fields. Which is in line with our experiences with the standard model. For instance, considering measured values of the muon anomalous magnetic moment, Kinoshita and Marciano [29] pointed out that pure QED suffices to explain only the first four significant digits, using to this end 16 field-components altogether; whereas to obtain a complete explanation of the first five significant digits, one needs QED and QCD with additional 104 field-components. Moreover, Brodsky [30] pointed out that at higher energies beyond standard QCD, it is plausible that new fields with higher color representation will be needed, and it is conceivable that the present quark and gauge fields are themselves composite at short distances. Well, as we pointed out above, infinitely many fields are actually needed for a complete modeling of fundamental interactions in tt approach; but their physical significance is not clear. So we do not know: how they manifest themselves, what kind of matter or forces they represent, how many of them rep-
resent physical particles, how many fields of the standard model are among them, whether the asymptotic tt fields contain only a finite number of them, whether they are useful only in certain combinations, and what kind of constraints they are subject to, cf. photons, gauge fields, Faddeev-Popov ghost fields, and quarks.

Let us consider a ttQFT whose tt fields are sums

$$\Psi_s(x,p) \equiv \sum_l F_l(p) \psi_l(x)$$

(5.1)

of appropriate products of certain given functions $F_l(p)$ of $p \in \mathbb{R}^{1,3}$ and of relativistic fields $\psi_l(x)$ that specify the $x$-dependence of a particular tt field $\Psi_s(x,p)$. If we require that the substantial derivative $p \cdot \partial$ of such a sum is also a sum of the same kind, these sums cannot be finite as the presence of certain $F_l(p)$ implies the presence of $F_l(p)p^\mu$, $F_l(p)p^\mu p^\nu$, $\ldots$; which suggests how to construct the smallest set of tt fields $\Psi_s(x,p)$ that contains given asymptotic tt fields. On the analogy with the asymptotic tt fields $\Psi_{as}(x,p)$ and their macroscopic variables, we assume:

(1) There are such mappings $\psi_l[x;\Psi_s]$ of tt fields $\Psi_s(x,p)$ into relativistic fields that

$$\psi_l[x;\Psi_s] = \psi_l(x).$$

(5.2)

(2) For every inhomogeneous Lorentz transformation we have

$$U(\Lambda,a)\Psi_s(x,p) = \sum_l F_l(p)U(\Lambda,a)\psi_l(x),$$

(5.3)

so that $\psi_l[x;U(\Lambda,a)\Psi_s] = U(\Lambda,a)\psi_l[x;\Psi_s]$. So the tt Lagrangian $L_{tt}(\Psi_s,p \cdot \partial \Psi_s)$ of this ttQFT depends solely on an infinite number of relativistic fields $\psi_l(x)$ and their first-order derivatives $\partial \psi_l$, say, $L_{tt}(\Psi_s,p \cdot \partial \Psi_s) = L_\infty(\ldots, \psi_l, \partial \psi_l, \ldots)$, and transforms as a scalar field under Lorentz transformations $\psi_l \to U(\Lambda,a)\psi_l$. Thus the ttQFT in question is actually a QFT with an infinite number of fields $\psi_l(x)$. The main characteristic of such a QFT is that its Lagrangian $L_\infty$ contains derivatives $\partial \psi_l$ of its fields $\psi_l(x)$ only linearly and solely in its standard free part (2.8), which couples them bilinearly with various fields $\psi_l$, cf. (A.18)–(A.20). On the analogy of (2.11), we may regard the Lagrangian $L_\infty$ as a tt extension of a certain QFT Lagrangian $L$ if $L_\infty$ equals $L$ when we put all but a finite number of $\psi_l$ equal to zero. If the fields $\psi_l(x)$ of $L_\infty$ that are common to $L_\infty$ and $L$ are also the macroscopic variables of tt field $\Psi_s(x,p)$, they are always constrained by the initial and final states just like the same fields of $L$. How the values of the remaining fields $\psi_l(x)$ are constrained at the initial and final instants is an open question. It is possible that at least some of them do not depend on the initial and final states, e.g., they might be free or equal to zero.

5.2. Transition from a ttQFT to the QFT it extends

In the kinetic theory of gases, physical processes described by the Boltzmann transport equations exhibit asymmetry with respect to the direction of time
A perturbation of a classical gas in equilibrium presumably disappears in three stages with widely disparate time scales: (1) A microscopic phase, during which the $p$-dependence of $\Phi$ field rapidly simplifies as $\Phi$ field tends toward an asymptotic $\Phi$ field with characteristic $p$-dependence, and a smooth, slowly varying $x$-dependence. The dynamics of microscopic phase is as yet beyond direct laboratory measurements. (2) A subsequent macroscopic phase, which is characterized by smooth and slowly varying macroscopic variables. These fields, whose evolution can now be adequately modeled by the partial-differential equations of fluid dynamics, determine the $x$-dependence of the asymptotic $\Phi$ field. (3) As time $t \to \infty$, $\Phi$ field very slowly approaches an $x$-independent, Lorentz-invariant, equilibrium $\Phi$ field. The preceding applies also to a perturbation of a macroscopic phase. This classical behaviour makes us propose the hypothesis that the quantum dynamics of fundamental interactions exhibits an analogous temporal behaviour with an “arrow of time” in the following sense: For a given initial state at $t = 0$, there is (i) some QFT that is extended by the basic ttQFT of fundamental interactions, and (ii) a Lorentz-invariant subset of states, say, the asymptotic states such that: (1) The ttQFT probability for a transition from the initial state to any final state at instant $t_f$ but an asymptotic one is negligible when $t_f$ is large enough, i.e., after a while, the ttQFT probability for finding by measurements any non-asymptotic state becomes negligible. (2) The ttQFT probability for a transition from an initial asymptotic state to any non-asymptotic state is negligible. So physical processes are practically irreversible in the sense that non-asymptotic initial states eventually result in asymptotic final states, but the reverse is not to be expected. (3) As far as the present-day experiments are concerned, the ttQFT probability amplitude for a transition between two asymptotic states is adequately approximated by the QFT one. Thus points (1)–(3) offer an explanation how phenomena that we can model by QFTs eventually emerge. (4) When QFT is applicable and transition amplitude is not negligible, the predominant contribution to the QFT path integral comes from such paths that do not vary appreciably over time and space intervals comparable to $\lambda$. (5) There is a particular, $x$-independent final state, the equilibrium state, such that the ttQFT probability for transition from the initial state to any final state at instant $t_f$ but to the equilibrium state becomes negligible as $t_f \to \infty$. Thus, as $t \to \infty$ all radiation and matter disappear.

Our physical picture of how transition amplitudes evolve is entirely based on our experience and insights gained from such quantum processes where QFTs apply, which use a finite number of fields. So we cannot expect this picture to be of much help in comprehending the evolution of transition amplitudes where only some ttQFT can provide an adequate description, using in fact an infinite number of fields. However, it seems reasonable to assume that the probability amplitudes for transitions between initial and final states of several particles can be modeled by some QFT so long as these particles stay sufficiently apart in the
meantime. We think that such initial, “in” states of scattering experiments in nuclear or elementary particle physics that result in ultrarelativistic multiparticle interactions do not belong to the asymptotic states (though the final, “out” states do). If so, we might obtain information about the hypothetical tt properties of fundamental interactions by considering in detail such multiparticle interactions.

If one extrapolates backward various relativistic cosmological models, they imply under quite general conditions that sometime in the past the whole content of the universe was located within a small region, which then expanded outward. On the analogy of the transition from transport, integro-differential equations to the hydrodynamic, partial-differential equations, we therefore expect those concepts (particles and forces; fields; and dynamic laws with their constants, symmetries, and conservation laws) that are used to model the present universe to be of limited use for modeling the very early universe (or for that matter a white dwarf, a neutron star, or a black hole). The very early, in variable $x$ extremely localized universe was probably dominated by the microscopic phenomena characterized by unlimited “speeds” $c|\mathbf{p}/p^0|$, and its initial expansion faster than the subsequent one dominated by macroscopic movements whose speeds are limited by the speed of light.

A transition from a ttQFT to the QFT it extends is mathematically highly singular since it involves transition from an infinite number of fields to a finite number of them. So the results of a ttQFT have in general an essential singularity at $\lambda = 0$, i.e., at most an asymptotic expansion in powers of $\lambda$. And a QFT result is in general not the first term of a convergent expansion in powers of $\lambda$ of the corresponding ttQFT result.

6. Concluding remarks

In this paper we have put forward a new class of relativistic, quantum field theories in $\mathbb{IR}^{1.3} \times \mathbb{IR}^{1.3}$, ttQFTs, which may extend the present QFTs to higher energies without necessarily implying any new particles whatsoever. They are based on the Feynman path-integral approach to quantum dynamics, without changing in any way the basic conceptual framework of QFTs; in particular, the spacetime $\mathbb{IR}^{1.3}$ and quantum statics [21] remain unchanged. We have patterned the proposed ttQFTs on the kinetic theory of rarefied gases [8, 10, 11], which gives a more detailed and accurate description of physical processes than fluid dynamics, which is suitable only for modeling large-scale phenomena. Therefore, we hope the proposed tt approach to quantum dynamics will also enable us to find out more about the small-scale physics of fundamental interactions that underlies phenomena so far described by QFTs. This tt approach is, to our knowledge, the first-ever attempt to apply Feynman’s idea about granularity of the microscopic world [7] to quantum dynamics of fundamental interactions. His idea is an expression of the ancient atomistic concept, well proven in physics of fluids and solids [12], which we took into account in a continuous manner on the analogy with the
Boltzmann transport equation.

In Section 2, we specified the following basic assumptions of an extension of a QFT by a ttQFT: (1) Quantum processes considered can be described by the initial and final states of the QFT. (2) When calculating transition amplitudes, we replace the QFT path integrals, whose paths are fields of the spacetime variable $x$, with path integrals whose paths are tt fields of two independent, real, four-vector variables $x$ and $p$. (3) Lagrangian of the ttQFT is a sum of a free Lagrangian, whose Euler-Lagrange equations are the partial-differential equations of free streaming, and of an interaction Lagrangian that is local in $x$-variable and contains no derivatives. (4) The interaction ttQFT Lagrangian depends on a positive parameter $\lambda$. In general, it tends to infinity if $\lambda \to 0$, but not for the asymptotic tt fields that are: (i) the most general solutions to the ttQFT Euler-Lagrange equations up to the leading order of $\lambda$, and (ii) sums of QFT fields multiplied by certain functions of variable $p$. (5) The QFT description of a quantum process can be obtained from the extending ttQFT in the asymptote $\lambda \to 0$, where ttQFT transition amplitudes equal the QFT ones. As the ttQFT extending the QFT implies it when $\lambda$ becomes exceedingly small, ttQFT may be regarded as a generalization in the sense of Weinberg [20]. (6) In the asymptote $\lambda \to 0$, the asymptotic tt fields whose QFT fields are consistent with the initial and final states may be taken as the domains of integration of ttQFT path integrals. (7) Covariant averages of tt fields exist that have the same physical significance as the QFT fields.

In Section 3, we constructed a tt Lagrangian whose path integrals are formally equivalent in the asymptote $\lambda \to 0$ to the path integrals specified by a general, first-order, gauge-invariant Lagrangian. Which demonstrates that extensions of QFTs are feasible within the tt framework; and we can regard the standard model as an asymptotic approximation analogous to the fluid dynamics in the kinetic theory of gases. We do not know how many tt fields are actually needed for modeling fundamental interactions. Only two may be needed, one for fermions and one for bosons. If so, the different fermion, bispinor fields of the standard model are only different manifestations, covariant averages, of the same two-component–spinor tt field. As there are infinitely many possible tt extensions of a QFT Lagrangian, in Sec. 3.3 we pointed out certain available options. A crucial unresolved question is what physical principles characterize physically relevant tt extensions, and, possibly, even uniquely determine them up to a few adjustable parameters.

In Section 4, we considered tt symmetry transformations that induce in asymptotic tt fields the QFT transformations of QFT fields. However, it is not clear which symmetries of a QFT Lagrangian should be required of an extending tt Lagrangian: tt symmetry principles obeyed by fundamental interactions have yet to be identified.

In Section 5, we touched on physics beyond QFTs that the tt approach sug-
gests on the analogy of the kinetic theory of gases. We pointed out such ttQFTs that are actually QFTs with (i) an infinite number of fields, (ii) massless, first-order, free Lagrangians, and (iii) no derivative couplings. We discussed a transition from a ttQFT to a QFT.

In this paper, we have presented the basic premises of a new, tt approach to the extension of QFTs to higher energies, and indicated possibilities and open questions. One has yet to determine realistic, internally consistent ttQFTs of fundamental interactions, find out their underlying principles and physical contents (e.g., how they describe classical systems and inferred superluminal influences), and work out typical, observable, qualitative and quantitative consequences (in particular, experimentally testable corrections to QFTs). Preliminary identification and study of the simplest, relevant, model ttQFTs is probably necessary to gain an understanding of realistic ttQFTs: how they work, how they modify dynamic concepts of QFTs, what kind of physics beyond QFTs they can describe, how rigid and logically isolated they are in the sense of Weinberg [12], what approximations and mathematical methods make sense and when, and what methods and results of contemporary QFTs in higher dimensions are applicable to ttQFTs.

Acknowledgements

We thank M. Poljšak and A. Verbovšek for many useful discussions.
Appendix A

In this Appendix we have collected relations and examples of interest in constructing tt extensions of Lagrangians of QFTs.

A.1 Properties of $\int d^4p$, $\sigma_{\pm}(p)$, and $[\Psi_j | \Psi'_j]$  

By (2.4), $\int d^4p F(p) = \int d^4p F(-p)$ and $(\int d^4p F(p))^* = \int d^4p F^*(p)$: if $F(p)$ is a real-valued function of real $p$, then $\int d^4p F(p) \in \mathbb{R}$. As the region of integration of the fourfold integral $\int d^4p$ is symmetric, $\int d^4p F(p) = 0$ if $F(p)$ is odd function of any component of $p$. Introducing spherical variables so that  

$$p^0 = i r^{1/2} \cos \vartheta, \quad p = r^{1/2} \sin \vartheta \sin \varphi, \sin \vartheta \cos \varphi, \cos \vartheta, \quad (A.1)$$

we can infer that  

$$\int d^4p f(p\cdot p) = \pi^2 \int_0^\infty r f(r) dr,$$

$$\int d^4p f(p\cdot p) p^\alpha p^\beta = \frac{1}{4} \eta^{\alpha\beta} \int d^4p p\cdot p f(p\cdot p), \quad (A.2)$$  

$$\int d^4p f(p\cdot p) p^\alpha p^\beta p^\gamma p^\delta = \frac{1}{2^{\alpha+2}} [\eta^{\alpha\beta} \eta^{\gamma\delta} + \eta^{\alpha\gamma} \eta^{\beta\delta} + \eta^{\alpha\delta} \eta^{\beta\gamma}] \int d^4p (p\cdot p)^2 f(p\cdot p).$$

The reason we defined by (2.4) the invariant, linear functional $\int d^4p$ on functions $F(p)$ of $p \in \mathbb{R}^{1,3}$ in terms of $F(ip^0, p)$ is that the fourfold integrals over $\mathbb{R}^{1,3}$ of the functions of $p$ in (A.2) do not exist.

We use the following properties of $\sigma_{\pm}(p)$:

$$\sigma_{\pm}^\dagger(p) = \sigma_{\pm}(-p), \quad \sigma_{\pm}^*(p) \sigma^2 = \sigma^2 \sigma_{\pm}(p),$$

$$D_{\gamma/2}(\Lambda) \sigma_{\pm}(\Lambda^{-1}p) = \sigma_{\pm}(p) D_{-\gamma/2}(\Lambda), \quad (A.3)$$

$$\sigma_+(p) \sigma_-(p) = \sigma_-(p) \sigma_+(p) = -I, \quad \begin{pmatrix} 0 & \sigma_+(p) \\ -\sigma_-(p) & 0 \end{pmatrix} = (p\cdot p)^{-1/2} \gamma^\mu p_\mu.$$  

Hence, under transformations $\Psi_{\pm/2} \rightarrow U(\Lambda, a) \Psi_{\pm/2}$ the products $\sigma_+(p) \Psi_{\pm/2}$ transform as the right-spinor and left-spinor, tt fields $\Psi_{\mp/2}$ do, cf. (2.1).

For all tt fields $\Psi_j$ and $\Psi'_j$, $j = 0, \pm 1/2, 1$, and a complex constant $z$,

$$[\Psi_j | z \Psi'_j] = z [\Psi_j | \Psi'_j], \quad [\Psi_j | \Psi'_j] = [\Psi_j | \Psi'_j]^*, \quad (A.4)$$

$$[\Psi_j(-p) | \Psi'_j(-p)] = (-1)^{2j} [\Psi_j | \Psi'_j], \quad (A.5)$$

$$[\Psi_j | i(a\cdot p) \Psi'_j] = [i(a\cdot p) \Psi_j | \Psi'_j] \quad \forall \ a \in \mathbb{R}^{1,3}, \quad (A.6)$$

$$[\Psi_j | \Psi'_j] = 0 \quad \forall \ \Psi_j' \Longrightarrow \Psi_j = 0, \quad (A.7)$$

by (2.2)–(2.4) and (A.3). Hence, $[\Psi_j | \Psi_j]$ is a real, but possibly negative, scalar field, and $[\Psi_j | ip \Psi_j]$ is a real, four-vector field.
A.2 Asymptotic tt fields

We take scalar fields \( \phi_i(x) \), chiral-bispinor fields \( \psi_m(x) \), four-vector fields \( \varphi_i(x) \) and \( A_a(x) \), symmetric–traceless–second-rank–four-tensor fields \( M_a(x) \), and antisymmetric–second-rank–four-tensor fields \( F_a(x) \). And we incorporate them in the following asymptotic tt fields that satisfy (2.12):

\[
\Psi_{0\text{as}}(x,p) \equiv f_{0i}(p \cdot p)\phi_i(x) + 2f_{1i}(p \cdot p)\varphi_i(x) \cdot p + 6f_{2a}(p \cdot p)p \cdot M_a(x)p,
\]

\[
\Psi_{\pm 1/2\text{as}}(x,p) \equiv [\sigma_-(p)]^{1/2 \pm 1/2}(I, \mp \sigma_+(p))f_{1/2m}(p \cdot p)\psi_m(x),
\]

\[
\Psi_{1\text{as}}(x,p) \equiv (1 - \mathcal{P}_l)f_{3a}(p \cdot p)A_a(x) + \mathcal{P}_l f_{7i}(p \cdot p)\varphi_i(x)
\]

\[
+ [2f_{4a}(p \cdot p)M_a(x) + 2f_{5a}(p \cdot p)F_a(x) + f_{6i}(p \cdot p)\phi_i(x)p]
\]

(Note, bispinor fields \( \psi_m \) are incorporated in spinor tt fields \( \Psi_{\pm 1/2\text{as}} \)). Here: the repeated indices are summed over; \( (Ta)^\alpha \equiv T^{a\beta}a_\beta \) for a second-rank–four-tensor \( T \) and a four-vector \( a \); \( (I, \mp \sigma_+(p)) \) is a \( 2 \times 4 \) matrix, with \( \sigma_+(p) \) given by (2.3); \( \mathcal{P}_l a \equiv (p \cdot p)^{-1}(p \cdot a) p \) for a four-vector \( a \) \{ \( \mathcal{P}_l = 1 - \mathcal{P}_l \) is a 4-matrix, \( \mathcal{P}_l \mathcal{P}_l = 0 \) \}; \( \eta \) is the metric tensor; and \( f_{0i}(y), f_{1i}(y), f_{2a}(y), f_{3a}(y), f_{4a}(y), f_{5a}(y), f_{6i}(y), f_{7i}(y) \) are some complex functions of \( y \in \mathbb{R}^4 \); functions \( f_{ui} \) with the same first subscript are assumed to be linearly independent.

A second-rank four-tensor \( T \) can be written as a sum

\[
T = F + M + \phi \eta,
\]

where \( F \equiv \frac{1}{2}(T - T^t) \) is an antisymmetric tensor; \( M \equiv \frac{1}{2}(T + T^t) - \frac{1}{4}(TrT)\eta \) a symmetric–traceless tensor; and \( \phi \equiv \frac{1}{4}TrT \) a scalar \( (T^t = TrT = T_{\mu\nu}\eta^{\mu\nu} \) are the transpose and trace of \( T \), respectively). Hence, we may incorporate tensor fields \( T_a(x) \) into the four-vector, asymptotic tt field \( \Psi_{1\text{as}}(x,p) \), either piecewise, using their parts \( F_a(x), M_a(x), \phi_a(x) \), or as a whole, choosing \( 2f_{4a} = 2f_{5a} = f_{6a} \) in (A.8). By (A.8), we can incorporate scalar, four-vector, and symmetric–second-rank–four-tensor fields into a scalar or a four-vector asymptotic tt field. However, we cannot incorporate an antisymmetric-tensor field \( F(x) \) into a scalar, asymptotic tt field \( \Psi_{0\text{as}}(x,p) \), nor a scalar field and/or a symmetric–second-rank–four-tensor field into a transversal, four-vector tt field \( \Psi_{1\text{as}}(x,p) \) such that \( \mathcal{P}_l \Psi_{1\text{as}} = 0 \).

In general, we can incorporate a field that is a direct product of a totally symmetric, rank-\( m \)-four-tensor field and of a rank-\( n \)-four-tensor (or chiral bispinor) field into an asymptotic, rank-\( n \)-four-tensor (or two-component-spinor) tt field.

For the asymptotic tt fields (A.8),

\[
\begin{align*}
[\Psi_{0\text{as}} | \Psi_{0\text{as}}] & = C^1_{00ij} \phi_i^* \phi_j - C^2_{11ij} \varphi_i^* \cdot \varphi_j + 3C^3_{22ab} \text{Tr}(M^a_1 M_b), \\
[\Psi_{\pm 1/2\text{as}} | \Psi_{\pm 1/2\text{as}}] & = C^1_{1/21/2mn} \overline{\psi}_m \psi_n, \\
[\Psi_{1\text{as}} | \Psi_{1\text{as}}] & = \begin{cases} 
\frac{5}{4} C^1_{33ab} A_a^* \cdot A_b - C^2_{44ab} \text{Tr}(M^a_1 M_b) - C^2_{55ab} \text{Tr}(F^a_1 F_b) \\
- C^2_{66ij} \phi_i^* \phi_j + \frac{1}{4} C^1_{77ij} \varphi_i^* \cdot \varphi_j ,
\end{cases}
\end{align*}
\]

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by (2.2)–(2.4) and (A.2), where $T^\dagger$ is the tensor adjoint to $T$, and complex constants

$$C_{uvij}^n \equiv \int d^4 p (p \cdot p)^{n-1} f_{ui}^*(p \cdot p) f_{vj}^*(p \cdot p), \quad n \geq 1. \quad (A.11)$$

Let us regard $C_{uvij}^n$ as elements of Hermitian matrices $C_{uv}^n$, assume that they are invertible, and denote their inverses by $C_{uv}^{-n}$. We can choose the functions $f_{0i}$, $f_{1i}$, $f_{2a}$, $f_{2b}$, $f_{3a}$, $f_{4a}$, $f_{5a}$, $f_{6a}$, and $f_{7i}$ in such a way that $C_{00}^1, C_{11}^2, C_{12}^3, C_{13}^4, C_{14}^5, C_{25}^6, C_{26}^7, C_{37}^8$ are proportional to identity matrices (as they are if only one function of each sort is present, since $C_{uv}^n$ are then positive numbers); in such a case, there is no interference in (A.10) between the components of asymptotic tt fields $\Psi_{jas}(x, p)$.

The asymptotic tt fields (A.8) suggest the following fields as potential macroscopic variables: scalar fields

$$\phi_i[x; \Psi_0] \equiv C_{00ij}^{-1} \int d^4 p f_{0j}^*(p \cdot p) \Psi_0(x, p),$$

$$\phi_i[x; \Psi_1] \equiv C_{66ij}^{-2} \int d^4 p f_{6j}^*(p \cdot p) p \cdot \Psi_1(x, p); \quad (A.12)$$

chiral-bispinor fields

$$\psi_m[x; \Psi_{\pm 1/2}] \equiv C_{1/21/2 mn}^{-1} \int d^4 p f_{1mn}^*(p \cdot p) \left( \mp I \frac{\mp I}{-\sigma_-(p)} \right) [\sigma_+(p)]^{1/2 \pm 1/2} \Psi_{\pm 1/2}(x, p), \quad (A.13)$$

where central, big brackets are $4 \times 2$ matrices; four-vector fields

$$\varphi_i[x; \Psi_0] \equiv 2C_{11ij}^{-2} \int d^4 p f_{1j}^*(p \cdot p) \Psi_0(x, p),$$

$$A_a[x; \Psi_1] \equiv \frac{4}{3} C_{33ab}^{-1} \int d^4 p f_{3b}^*(p \cdot p) (1 - \mathcal{P}_l) \Psi_1(x, p),$$

$$\varphi_i[x; \Psi_1] \equiv 4C_{77ij}^{-1} \int d^4 p f_{7j}^*(p \cdot p) \mathcal{P}_l \Psi_1(x, p); \quad (A.14)$$

symmetric–traceless–second-rank–four-tensor fields

$$M_a[x; \Psi_0] \equiv 2C_{22ab}^{-3} \int d^4 p f_{2b}^*(p \cdot p) \Psi_0(x, p) [p \otimes p - \frac{1}{3} p \cdot p \eta],$$

$$M_a[x; \Psi_1] \equiv C_{44ab}^{-2} \int d^4 p f_{4b}^*(p \cdot p) [\Psi_1(x, p) \otimes p + p \otimes \Psi_1(x, p) - \frac{1}{2} p \cdot \Psi_1(x, p) \eta]; \quad (A.15)$$

and antisymmetric–second-rank–four-tensor fields

$$F_a[x; \Psi_1] \equiv C_{55ab}^{-2} \int d^4 p f_{5b}^*(p \cdot p) [\Psi_1(x, p) \otimes p - p \otimes \Psi_1(x, p)]; \quad (A.16)$$
They satisfy (2.15) and (2.16); e.g., $M_a[x; U(λ, a)Ψ_0] = λ^{-1}M_a[Ax + a; Ψ_0]$. For the asymptotic $tt$ fields (A.8), by (2.2)–(2.4) and (A.2), the currents corresponding to the global phase invariance (4.7) are:

$$[Ψ_{0as} | ipΨ_{0as}] = −ℜ\{iφ_1^*(C^2_{10 ij}φ_j + 2C^3_{12 ia}M_a)\} ;$$

$$[Ψ_{±1/2as} | ip^μΨ_{±1/2as}] = ±1/4C^3_{12/12/1/2} m^μψ_m μψ_n ;$$

(A.17)

$$[Ψ_{1as} | ipΨ_{1as}] = ℜ\{iA^a_1(C_{35 ab}F_b + 2/3C_{34 ab}M_b) + iφ_1^*\{1/3C_{71 iia}M_a + 1/2C^2_{76 iij}φ_j)\} .$$

The free $tt$ Lagrangians (2.8) were defined so that:

$$L_{free}(Ψ_{0as}, p·∂Ψ_{0as}) = ℜ\{(1/2C^2_{01 ij}φ_1^*νμ + C^3_{21 a j}M^*_{a μν})∂_νφ_jμ\} ,$$

(A.18)

$$L_{free}(Ψ_{±1/2as}, p·∂Ψ_{±1/2as}) = ±1/8C^3_{12/12/12} m^μψ_m μψ_n ;$$

(A.19)

$$L_{free}(Ψ_{1as}, p·∂Ψ_{1as}) = −ℜ\{(1/4C^2_{53 ab}F_{a νμ} + 1/3C^2_{43 ab}M^*_{a νμ})∂_νA_{μμ} + (1/4C^2_{37 a j}M^*_{a μμ} + 1/4C^2_{67 i j}φ_1^*νμ)∂_νφ_jμ\} ,$$

(A.20)

by (2.8) and (A.17). If $f_{11}(p·p) ≡ 0$, then $L_{free}(Ψ_{0as}, p·∂Ψ_{0as}) = 0$; if $f_{3a}(p·p) ≡ f_{11}(p·p) ≡ 0$, then $L_{free}(Ψ_{1as}, p·∂Ψ_{1as}) = 0$. As we assumed in Sec. 2.2 that the interaction part of a $tt$ Lagrangian is strictly local and contains no derivative couplings, results (A.18)–(A.20) tell us which combinations of fields and their derivatives may appear in such a QFT Lagrangian that has a $tt$ extension (2.11) with asymptotic $tt$ fields (A.8). Note that addition of $x$-independent terms to asymptotic $tt$ fields (A.8) would add only divergences to (A.18)–(A.20).

A.3 Examples of $tt$ extensions

According to (2.10)–(2.11), a minimum requirement that a $tt$ Lagrangian $L_{tt}$ is a $tt$ extension of a QFT Lagrangian $L$ defined in terms of certain fields is that (1) these fields are among the fields that define the asymptotic $tt$ fields $Ψ_{as}(x, p)$, and (2) the asymptotic $tt$ Lagrangian $L_{tt}(Ψ_{as}, p·∂Ψ_{as})$ equals $L$. To get an idea how we can meet these two conditions, we give a few examples. We will not investigate alternatives to definition (3.15) of the part $L_λ(Ψ)$ of $tt$ Lagrangian that determines the asymptotic $tt$ fields $Ψ_{as}$.

To consider $tt$ extensions of the Lagrangian for the Dirac equation,

$$L_D = −1/2\overline{ψ}γ^μ∂_μψ - m\overline{ψ}ψ ,$$

(A.21)

we take a left-spinor $tt$ field $Ψ_{1/2}(x, p)$ and an asymptotic $tt$ field $Ψ_{1/2as}(x, p) = (σ_−(p), I)f_{1/2}(p·p)ψ(x)$ with a real function $f_{1/2}(p·p)$. The $tt$ Lagrangians

$$L'_D = 4C^{−3/2}_{1/2/2/2} ℜ\{[Ψ_{1/2} | p·∂Ψ_{1/2}] − mC^{−1}_{1/2/2/2}[Ψ_{1/2} | Ψ_{1/2}]\} \ (A.22)$$

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and
\[ \mathcal{L}_D'' = 4C_{1/2}^{-3/2} \Re[\psi_{1/2} | p \cdot \partial \psi_{1/2}] - m \overline{\psi}[x; \psi_{1/2}] \psi[x; \psi_{1/2}] \]  
(A.23)

are real, scalar fields and equal \( \mathcal{L}_D \) for \( \Psi = \Psi_{as} \), by (A.8), (A.19), (A.10), (2.15), and (3.11). If \( C_{1/2}^{-1} = 4 \) and \( C_{1/2}^{-1} = m > 0 \), then \( \mathcal{L}_D = \mathcal{L}_{free} - [\psi_{1/2} | \psi_{1/2}] \). There is no direct extension of the Lagrangian \( \mathcal{L}_{KG} \equiv -|\partial \phi|^2 - m^2 \phi^2 \) for the Klein-Gordon equation of a real, massive, scalar field, by (A.18). The Lagrangian
\[ \mathcal{L}_E \equiv -\varphi^\mu \partial_\mu \phi + \varphi^\mu \varphi_\mu - m^2 \phi^2 \]  
(A.24)
can be regarded as an equivalent to \( \mathcal{L}_{KG} \), because its Euler-Lagrange equations are equivalent to the Klein-Gordon equation and condition \( \varphi = \partial \phi \). For a real, scalar field \( \psi_0(x, p) \) with
\[
\psi_{0as}(x, p) = f_1(p \cdot p) [m(p \cdot p)^{1/2} \phi(x) + 2 \varphi(x) \cdot p],
\]
we can check that \( \psi \) Lagrangians
\[ \mathcal{L}'_E \equiv 2m^{-1} C_{11}^{-5/2} [\psi_0 | p \cdot \partial \psi_0] - C_{11}^{-2} [\psi_0 | \psi_0] \]  
(A.25)
and
\[ \mathcal{L}''_E \equiv 2m^{-1} C_{11}^{-5/2} [\psi_0 | p \cdot \partial \psi_0] + \varphi[x; \psi_0] \cdot \varphi[x; \psi_0] - m^2 \phi^2[x; \psi_0] \]  
(A.26)
equal \( \mathcal{L}_E \) for \( \psi_0 = \psi_{0as} \). If \( C_{11}^2 = \frac{1}{2} m \), then \( \mathcal{L}'_E = \mathcal{L}_{free} - [\psi_0 | \psi_0] \). We can generalize the free \( \psi \) Lagrangian \( [\psi_1 | p \cdot \partial \psi_1] \) of a real, four-vector field \( \psi_1(x, p) \) as the following sum of two real, scalar fields:
\[ s_l [\partial_1 \psi_1 | p \cdot \partial_1 \psi_1] + s_t [(1 - \partial_l) \psi_1 | p \cdot \partial (1 - \partial_l) \psi_1] , \]  
(A.27)
where \( s_l \) and \( s_t \) are two real parameters; for \( s_l = s_t = 1 \), this generalization equals \( [\psi_1 | p \cdot \partial \psi_1] \). Generalization (A.27) with \( s_l = 0 \) will enable us to construct a local-gauge-invariant \( \psi \) Lagrangian that equals for certain asymptotic \( \psi \) fields the first-order QED Lagrangian
\[ \mathcal{L}_{QE} = F_{\mu \nu} (\partial_\nu A_\mu + F_{\mu \nu}) - \overline{\psi} \left[ (\frac{1}{2} \gamma^\mu \partial_\mu + ie \gamma^\mu A_\mu + m) \right] \psi . \]  
(A.28)
To this end, we choose a complex, left-spinor \( \psi_{1/2}(x, p) \) and a real, four-vector \( \psi \) field \( \psi_1(x, p) \); the asymptotic \( \psi \) fields
\[
\psi_{3/2as}(x, p) = f_{3/2}(p \cdot p) (-\sigma_-(p), I) \psi(x), \\
\psi_{1as}(x, p) = f_3(p \cdot p) A(x) + 2f_5(p \cdot p) F(x) p ;
\]  
(A.29)
and fields (A.13), (A.16), and
\[ A[x; \psi_1] = C_{33}^{-1} \int d^4 p f_3(p \cdot p) \psi_1(x, p) . \]  
(A.30)
Take the tt Lagrangian

\[
\mathcal{L}'_{QE} \equiv 2C^{-2}_{53}[(1 - \mathcal{P}_l)\Psi_1 \mid p \cdot \partial (1 - \mathcal{P}_l)\Psi_1] + \text{Tr}(F^\dagger [x; \Psi_1]F[x; \Psi_1])
\]
\[
+ 4C^{-3/2}_{1/2} \mathcal{R}[\Psi_{1/2} \mid [p \cdot \partial - eG(p \cdot p)\Psi_1] \Psi_{1/2}]
\]
\[
- mC^{-1}_{1/2} \Psi_{1/2} \Psi_{1/2},
\]

where \(G(p \cdot p)\) is a real-valued function such that

\[
C_{1/2}^{3/2} = \pi^2 \int_0^\infty y^{3/2} f_{1/2}^2 (-y) f_3 (-y) G(-y) \, dy,
\]
\[
4C_{1/2}^{13} = \pi^2 \int_0^\infty y f_3 (-y) G^{-1} (-y) \, dy.
\]

For the asymptotic tt fields (A.29) the tt Lagrangian \(\mathcal{L}'_{QE}\) equals \(\mathcal{L}_{QE}\). And \(\mathcal{L}'_{QE}\) is invariant under the following tt local-gauge transformations:

\[
\Psi_{1/2}(x, p) \to e^{ie\alpha(x)} \Psi_{1/2}(x, p),
\]
\[
\Psi_1(x, p) \to \Psi_1(x, p) - G^{-1}(p \cdot p)\mathcal{P}_l \partial \alpha(x).
\]

Under tt local-gauge transformations (A.33), the fields (A.13), (A.16), and (A.30) and fields in (A.29) transform as under the local-gauge transformations of QED,

\[
\psi[x; \Psi_{1/2}] \to e^{ie\alpha(x)} \psi[x; \Psi_{1/2}],
\]
\[
A[x; \Psi_1] \to A[x; \Psi_1] - \partial \alpha(x), \quad F[x; \Psi_1] \to F[x; \Psi_1].
\]

We do not know how to construct such a gauge-invariant part \(\mathcal{L}_\lambda\) of a tt Lagrangian that in general tends to infinity if \(\lambda \to 0\) but for the asymptotic tt fields (A.29) that are the most general solution to its Euler-Lagrange equations, cf. (3.15). We could do without a term such as \(\mathcal{L}_\lambda\) in a tt Lagrangian extending \(\mathcal{L}_{QE}\), were there a mechanism inherent to \(\mathcal{L}'_{QE}\) that would select the asymptotic tt fields (A.29) as the domain of the tt path integral. If so, \(\mathcal{L}'_{QE}\) would be a tt extension of \(\mathcal{L}_{QE}\) that is invariant under tt counterpart (A.33) to the local-gauge transformations (A.34) of QED.
References

[1] See, e.g., S. Weinberg, The Quantum Theory of Fields (Cambridge University Press, Cambridge 1995) Vol. I, Secs. 12, 1.3, 9.1–9.6, 7.2, 7.3, and 9; and the references therein.

[2] A. Salam, in The Physicist’s Conception of Nature, ed. J. Mehra (D. Reidel, Dordrecht, 1973) p. 430;
C. J. Isham, A. Salam and J. Strathdee, Phys. Rev. D3 (1971) 1805; D5 (1972) 2548.

[3] See, e.g., D. Bailin, Contemp. Phys. 30 (1989) 237;
D. J. Gross, Nucl. Phys. B (Proc. Suppl.) 15 (1990) 43;
G. G. Ross, Contemp. Phys. 34 (1993) 79;
S. Weinberg, in Twentieth Century Physics, ed. L. M. Brown, A. Pais and B. Pipard (IOP Publishing and AIP Publishing, New York, 1995) Vol. III, p. 2033.

[4] J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields (McGraw-Hill, New York, 1965) Secs. 11.2 and 11.3;
J. Schwinger, in The Physicist’s Conception of Nature, ed. J. Mehra (D. Reidel, Dordrecht, 1973) p. 413.

[5] W. Heisenberg, Ann. Phys. (Leipzig) 32(1938) 20.

[6] See, e.g., T. P. Cheng and L. F. Li, Gauge Theory of Elementary Particle Physics, (Claredon Press, Oxford, 1992) Secs. 1.2, 1.1, 5.3, 13.1, 5.1, and 8.1.

[7] R. P. Feynman, R. B. Leighton, and M. Sands, The Feynman Lectures on Physics (Addison-Wesley, Reading, Mass., 1965) Vol. II, Sec. 12.7.

[8] See, e.g., R. L. Liboff, Kinetic Theory (Prentice-Hall, Englewood Cliffs, N. J., 1990) Ch. 3 and Sec. 2.4.

[9] See, e.g., M. M. R. Williams, Mathematical Methods in Particle Transport Theory, (Butterworths, London, 1971) Sec. 2.7 and Ch. 11.

[10] See, e.g., S. R. de Groot, W. A. van Leeuwen and Ch. G. van Weert, Relativistic Kinetic Theory (North-Holland, Amsterdam, 1980) Secs. I.2, VI.1 and VII.

[11] See, e.g., H. Grad, in Application of Nonlinear Partial Differential Equations in Mathematical Physics, Proc. Symp. Appl. Math. XVII (Am. Math. Soc., Providence, R. I., 1965) p. 154, and references therein.

[12] See, e.g., S. Weinberg, Dreams of a Final Theory (Pantheon Books, New York 1992).

[13] M. Ribarić and L. Šušteršič, Transp. Theory Stat. Phys. 24(1995) 1.

[14] M. Ribarić and L. Šušteršič, Int. J. Theor. Phys. 34(1995) 571.

[15] R. T. Weidner, in The New Encyclopaedia Britannica (Encyclopaedia Britannica, Chicago, 1986) 15th edition, Vol. 25, p.845.
[16] See, e.g., S. Weinberg, Phys. Rev. D7(1973) 1068, Sec. II; Rev. Mod. Phys. 46 (1974) 255, Secs. I and III.

[17] J. Schwinger, Phys. Rev. 91(1953) 713; R. L. Arnowitt and S. I. Fickler, Phys. Rev. 127(1962) 1821.

[18] See, e.g., L. D. Faddeev and A. A. Slavnov, Gauge Fields: Introduction to Quantum Theory (Benjamin Cummings, Reading, Mass., 1980) 2nd edition, Sec. 3.2.

[19] For an example see M. Ribarič and L. Šušteršič, Transp. Theory Stat. Phys. 16 (1987) 1041, Secs. 4.5 and 5.1.

[20] S. Weinberg, Ann. Phys. (N.Y.) 194 (1989) 336.

[21] For an introduction see, e.g., A. Sudbery, Quantum Mechanics and Particles of Nature (Cambridge University Press, Cambridge 1988) Chs. 7, 5, and 2.

[22] See, e.g., C. D. Froggatt and H. B. Nielsen, Origin of Symmetries, (World Scientific, Singapore, 1991), and references therein.

[23] See, e.g., J. Bernstein, Rev. Mod. Phys. 46(1974) 7, footnote 32; D. G. C. McKeon, Can. J. Phys. 72 (1994) 601.

[24] W. Greiner, Quantum Mechanics, an Introduction, (Springer Verlag, Berlin, 1989) Sec. 13.2.

[25] For some related comments see K. Gottfried and V. F. Weisskopf, Concepts of Particle Physics (Claredon Press, Oxford 1984) Vol. I, Sec. 13c; R. P. Feynman, F. B. Moringio, and W. G. Wagner, Feynman Lectures on Gravitation, ed. B. Hatfield (Addison-Wesley, Reading, Mass., 1995) Secs 1.4 and 1.5; I. Percival, Phys. World 10 (1997) (3) 43, and references therein.

[26] See, e.g., M. Ribarič and L. Šušteršič, Fizika B 3, (1994) 93; Found. Phys. Lett. 7 (1994) 531; and references therein.

[27] See, e.g., R. Y. Chiao, P. G. Kwiat, and A. E. Steinberg, Sci. Am. 269 (1993) (2), 38; R. Y. Chiao, J. Boyce, and J. C. Garrison, in Fundamental Problems in Quantum Theory, ed. D. G. Greenberger and A. Zeilinger (The New York Academy of Sciences, New York, 1995) p. 400, and references therein.

[28] A. M. Polyakov, Gauge Fields and Strings (Harwood Academic Publishers, Chur, 1987) Sec. 1.2.

[29] T. Kinoshita and W. J. Marciano, in Quantum Electrodynamics, ed. T. Kinoshita (World Scientific, Singapore, 1990) p. 419.

[30] S. J. Brodsky, Preprint SLAC-PUB-95-6781, [hep-ph/9503391].