PAIR-INSTABILITY SUPERNOVAE, GRAVITY WAVES, AND GAMMA-RAY TRANSIENTS

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ABSTRACT

Growing evidence suggests that the first generation of stars may have been quite massive (~100–300 $M_\odot$). If they retain their high mass until death, such stars will, after about 3 Myr, make pair-instability supernovae. Models for these explosions have been discussed in the literature for four decades, but very few included the effects of rotation and none employed a realistic model for neutrino trapping and transport. Both turn out to be very important, especially for those stars whose cores collapse into black holes (helium cores above about 133 $M_\odot$). We consider the complete evolution of two zero-metallicity stars of 250 and 300 $M_\odot$. Despite their large masses, we argue that the low metallicities of these stars imply negligible mass loss. Evolving the stars with no mass loss, but including angular momentum transport and rotationally induced mixing, the two stars produce helium cores of 130 and 180 $M_\odot$. Products of central helium burning (e.g., primary nitrogen) are mixed into the hydrogen envelope with dramatic effects on the radius, especially in the case of the 300 $M_\odot$ model. Explosive oxygen and silicon burning cause the 130 $M_\odot$ helium core (250 $M_\odot$ star) to explode, but explosive burning is unable to drive an explosion in the 180 $M_\odot$ helium core, and it collapses to a black hole. For this star, the calculated angular momentum in the presupernova model is sufficient to delay black hole formation, and the star initially forms an ~50 $M_\odot$ 1000 km core within which neutrinos are trapped. The calculated growth time for secular rotational instabilities in this core is shorter than the black hole formation time, and they may develop. If so, the estimated gravitational wave energy and wave amplitude are $E_{GW} \approx 10^{-3} M_\odot c^2$ and $h_\nu \approx 10^{-21}/d$(Gpc), but these estimates are very rough and depend sensitively on the non-linear nature of the instabilities. After the black hole forms, accretion continues through a disk. The mass of the disk depends on the adopted viscosity but may be quite large, up to 30 $M_\odot$ when the black hole mass is 140 $M_\odot$. The accretion rate through the disk can be as large as 1–10 $M_\odot$ s$^{-1}$. Although the disk is far too large and cool to transport energy efficiently to the rotational axis by neutrino annihilation, it has ample potential energy to produce a $10^{54}$ erg jet driven by magnetic fields. The interaction of this jet with surrounding circumstellar gas may produce an energetic gamma-ray transient, but given the probable redshift and the consequent timescale and spectrum, this model may have difficulty explaining typical gamma-ray bursts.

Subject headings: gamma rays: bursts — nuclear reactions, nucleosynthesis, abundances — stars: evolution — supernovae: general

1. INTRODUCTION

Simulations of the collapse of primordial molecular clouds suggest that the first generation of stars (Ostriker & Gnedin 1996) contained many extremely massive members, from 100 up to 1000 $M_\odot$ (e.g., Larson 1999; Bromm, Coppi, & Larson 1999; Abel, Bryan, & Norman 2000). While details of the mass function and the interaction of these stars with their environment have yet to be worked out, up to 1% of the baryonic mass of the universe might have participated in this generation of stars (T. Abel 2000, private communication).

Such massive stars ($M \gtrsim 100 M_\odot$) form large helium cores that reach carbon ignition with masses in excess of about 45 $M_\odot$. It is known that after helium burning, cores of this mass will encounter the electron-positron pair instability, collapse, and ignite oxygen and silicon burning explosively (e.g., Barkat, Rakavy, & Sack 1967; Woosley & Weaver 1982; Bond, Arnett, & Carr 1984; Carr, Bond, & Arnett 1984; Glatzel, El Eid, & Fricke 1985; Woosley 1986; A. Heger & S. E. Woosley 2001, in preparation). If explosive oxygen burning provides enough energy, it can reverse the collapse in a giant nuclear-powered explosion. As the mass of the helium core increases, so does the strength of the explosion and the mass of $^{56}$Ni synthesized. Masses of $^{56}$Ni ejecta over 40 $M_\odot$ and explosion energies approaching $10^{53}$ ergs are possible, with light curves brighter than $10^{44}$ ergs s$^{-1}$ for several months.

However, going to still more massive stars (over ~260 $M_\odot$ on the main sequence), a new phenomenon occurs as a sufficiently large fraction of the center of the star becomes so hot that the photodisintegration instability is encountered before explosive burning reverses the implosion (Bond et al. 1984) . This uses up all the energy released by previous burning stages and, instead of producing an explosion, accelerates the collapse. A massive black hole is born inside the star. Although this general picture has been known for some time and many explosions have been calculated, few detailed models exist for the collapse (though see Woosley, Wilson, & Mayle 1986; Stringfellow & Woosley 1988). As Bond et al. (1984) also realized, these stars are likely to be rotating rapidly, and this, as we shall see, can drastically affect the outcome of the collapse.

In this paper we consider the evolution of two stars whose masses straddle the limit between explosion and collapse: 250 and 300 $M_\odot$. We follow their entire life and death using a combination of stellar evolution and core collapse codes. Since angular momentum is an important consideration, we assume that the initial stars are rigidly rotating (with a ratio of surface centrifugal force to gravity of 20%, comparable to what is seen in O stars today) and follow the
transport of angular momentum through all burning stages (but ignoring magnetic fields). Collapse, explosive nuclear burning, and neutrino transport are all followed in a parameter-free way.

Concentrating on the collapsing stars, we find that Bond et al. (1984) correctly predicted the qualitative effects of the collapse but, in important cases (e.g., neutrino luminosity, gravitational wave emission), overestimated by orders of magnitude the amount of energy that can escape the collapse. The evolution of these stars is presented in §2 and the collapse in §3. In §4 we discuss the gravitational wave emission expected from the collapse of these massive stars.

After black hole formation and following the collapse of roughly another 100 $M_\odot$, the outer layers of the helium core have sufficient angular momentum to hang up in a disk roughly another 100 the outer layers of the helium emission expected from the collapse of these massive stars.

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After black hole formation and following the collapse of roughly another 100 $M_\odot$, the outer layers of the helium core have sufficient angular momentum to hang up in a disk that accretes onto the $\sim 140 M_\odot$ black hole at a rate in excess of 1 $M_\odot$ s$^{-1}$. The characteristics of this accreting black hole are very similar to the class of black hole accretion disk systems that are thought to power gamma-ray bursts (GRBs), and it is possible that the disks formed by the collapse of these very massive stars may produce a new class of collapsar-like (MacFadyen & Woosley 1999; MacFadyen, Woosley, & Heger 2001) gamma-ray transients (GRTs). These GRTs may be the most directly observable consequence of the collapse of massive Population III stars (§5).

2. PRECOLLAPSE EVOLUTION OF 250 AND 300 $M_\odot$ STARS

2.1. Population III Stars

It has long been thought that the first generation of stars after the big bang might have been characterized by an initial mass function skewed to more massive members (e.g., Silk 1983; Carr & Rees 1984). Recently, Abel et al. (2000) have studied the formation and fragmentation of primordial, zero-metallicity molecular clouds using a three-dimensional code with adaptive mesh. Dark matter dynamics, hydrodynamics, and the relevant chemical and radiative processes were followed down to a scale of 0.5 pc. They concluded that the typical mass of the first-generation stars is $\sim 100 M_\odot$.

Based upon the amount of energy released by hydrogen fusion, the need to reionize a significant fraction of the universe prior to redshift 5 using the light of such stars, and a crude estimate of the ionization efficiency of a given ultraviolet photon, one estimates that roughly 0.01%–1% of the baryonic mass of the universe may have been incorporated into such stars. For a $10^{11} M_\odot$ galaxy this corresponds to $\sim 10^9$–$10^7$ stars, an estimate that probably exceeds the total number of merging double neutron star binaries or black hole binaries in the Milky Way, $10^3$–$10^5$ per 10 Gyr (Fryer, Woosley, & Hartmann 1999b). There is thus ample theoretical basis for assuming the formation of an appreciable number of primordial stars having mass up to several hundred $M_\odot$.

2.2. Mass Loss

Because of their short lifetime, there are no nearby examples of these zero-metallicity, high-mass stars. Unfortunately, this makes determining the mass loss of these stars from observations very difficult. Today, mass loss in such massive stars is dominated by radiative processes (Appenzeller 1986; Figer et al. 1999), and the stars probably lose most of their mass before dying. For zero metallicity, however, as would have characterized these first stars, radiative mass loss was negligible (Kudritzki 1999).

Still, one must consider mass loss driven by nuclear pulsations. The critical mass for the onset of the nuclear pulsational instability (Schwarzschild & Härn 1959) is very uncertain (90–420 $M_\odot$ for stars of solar metallicity; Appenzeller 1986), and the resulting mass-loss rate even more so. For very low metallicity, both the temperature dependence of hydrogen burning and the gravitational potential at the surface of the star are different. Appenzeller's (1970) estimate that a 130 $M_\odot$ star would lose $\sim 30 M_\odot$ during its main-sequence lifetime is certainly an overestimate for Population III stars. Baraffe et al. (2001) find the mass loss from nonrotating pulsational instabilities to be much lower. For stars of 200 and 300 $M_\odot$, they find that the total mass ejected from pulsational instabilities during hydrogen burning is less than a few percent of the total mass. However, there may be mass loss toward the end of central helium burning for main-sequence star masses above $\sim 300 M_\odot$ (Baraffe et al. 2001). The effect of this mass loss is, as yet, unknown, but it is unlikely to affect the helium core of a 300 $M_\odot$ star and hence will not affect the results of the stars modeled in this paper.

2.3. Presupernova Evolution of 250 and 300 $M_\odot$, $Z = 0$ Stars

We thus considered the evolution of two massive, rotating, Population III stars of 250 and 300 $M_\odot$ evolved at constant mass. By "Population III" we mean that the initial composition is 76% H and 24% He with no initial abundance of anything heavier. The evolution of these stars was followed from central hydrogen ignition until core collapse using a one-dimensional hydrodynamic stellar evolution code, KEPLER (Weaver, Zimmerman, & Woosley 1978). Stellar rotation (constant along shells) was included as described in Heger, Langer, & Woosley (2000). That is, mixing and redistribution of angular momentum induced by convection, shear, Eddington-Sweet circulation, etc., were followed, but the centrifugal force terms were not included in the stellar structure equations. Semi-convection was treated as in Woosley & Weaver (1995), but no overshoot mixing was assumed. So long as the ratio of centrifugal force to gravity remains small, as it did during the epoch followed by KEPLER, this should be an accurate approximation.

For the initial angular momentum of the rotating zero-age main-sequence (ZAMS) stars, we assumed rigid rotation with a velocity 20% Keplerian at the equator, comparable to the typical value of observed O stars.

More details of the evolution of these stars and many other Population III models will be given elsewhere (A. Heger & S. E. Woosley 2001, in preparation). Here we note only those characteristics relevant to forming black holes and producing jets and gravity waves. After central helium depletion ($T = 5 \times 10^8$ K), the masses of the helium cores in the 250 and 300 $M_\odot$ models were 130 and 180 $M_\odot$. The hydrogen envelopes were also appreciably enriched in helium and the products of helium burning with $X = 16.0\%$, $Y = 68.0\%$, and $Z = 16.0\%$ for the 250 $M_\odot$ model and $X = 19.4\%$, $Y = 72.2\%$, and $Z = 8.4\%$ (mostly CNO) for the 300 $M_\odot$ model. The luminosities of the two stars were $3.8 \times 10^{40}$ and $5 \times 10^{40}$ erg s$^{-1}$, respectively, and the central carbon abundance was 6.0% and 4.8% by mass.

An interesting characteristic of these two stars was their
production of primary nitrogen. As a consequence of both the initial CNO deficiency and the helium-rich nature of their envelopes (resulting from rotationally induced mixing), the hydrogen-burning shell was characterized by a relatively shallow entropy gradient. Thus, the envelope remained compact and rapidly rotating with only a small entropy barrier separating it from the helium core. During the growth of the helium-burning convective core, traces of carbon and oxygen were mixed into the hydrogen-burning shell by meridional circulation. Creation of a CNO mass fraction as small as \( \gtrsim 10^{-8} \) was sufficient to increase significantly the nuclear energy generation rate and make the hydrogen-burning shell convective. More significantly, during late stages of helium burning, the shear between the convective hydrogen shell and the core became large enough to lead to a significant dredge-up of the helium core, i.e., helium and large amounts of helium-burning products (carbon, oxygen, and neon) were mixed into the envelope. Because of the short time remaining in the life of the star at this point, only part of the carbon and oxygen was processed into nitrogen. For the 250 \( M_\odot \) star this major mixing event happened at a central helium mass fraction of \( \sim 25\% \), while in the 300 \( M_\odot \) star it occurred at the end of central helium burning. In the 250 \( M_\odot \) star the mass fraction of primary nitrogen in the envelope was 7.75\% (i.e., a total of 9.48 \( M_\odot \)), while carbon and oxygen had mass fractions of 0.26\% and 7.97\%, respectively. Since in the 300 \( M_\odot \) star the dredge-up occurred only toward the end of central helium burning, the material mixed into the envelope showed a clear signature of elements produced at the end of hot helium burning. For the 300 \( M_\odot \) star the mass fractions were 1.28\% for nitrogen (i.e., a total of 1.56 \( M_\odot \)), 0.0533\% for carbon, 5.68\% for oxygen, 0.964\% for neon, and 0.395\% for magnesium.

The increased energy generation afforded by the (suddenly) larger abundance of CNO led to the formation of red supergiants in both stars. The radius of the 250 \( M_\odot \) model expanded from \( 2.3 \times 10^{12} \) to \( \sim 10^{14} \) cm by the time it encountered the electron-positron pair instability; the 300 \( M_\odot \) model expanded from \( 4 \times 10^{12} \) to \( \sim 1.5 \times 10^{14} \) cm. The mixing events occurred \( \sim 10^4 \) yr before core collapse in the 250 \( M_\odot \) model and \( \sim 10^6 \) yr before collapse in the 300 \( M_\odot \) model. Note that all these estimates of mixing, nitrogen nucleosynthesis, and timescales are sensitive to the highly uncertain physics of convection and rotational instabilities.

The new large radius and high metal content in this supergiant phase can cause significant mass loss and may remove the envelope, particularly if the star is in a binary. Once the helium core is revealed, unless a small amount of hydrogen remains, an even more pulsationally unstable situation may be created. Appenzeller (1986) summarizes evidence that bare helium cores above \( \sim 16 M_\odot \) may be unstable to nuclear-driven pulsations. However, especially for the 300 \( M_\odot \) model, the time remaining until the star died was very short. If the star can avoid losing an additional 40 \( M_\odot \) of helium (i.e., \( \dot{M} \lesssim 0.004 \, M_\odot \, \text{yr}^{-1} \)), a black hole will form.

3. COLLAPSE AND BLACK HOLE FORMATION

Following helium depletion, both the 250 and 300 \( M_\odot \) stars encountered the electron-positron pair instability. The 250 \( M_\odot \) star experienced a very deep bounce, penetrating into the pair-unstable region, then violently rebounding because of the excess energy created by explosive oxygen and silicon burning. The peak temperature and density during the bounce were \( 6.37 \times 10^9 \) K and \( 1.08 \times 10^7 \) g cm\(^{-3} \) (Fig. 1). So much of the core was heated to temperatures in excess of \( \sim 5 \times 10^9 \) K that 43.0 \( M_\odot \) of \( ^{56}\text{Ni} \) was synthesized. We estimate that this is very nearly the maximum mass that can explode by nuclear burning alone and so represents a nearly maximal mass of \( ^{56}\text{Ni} \) and explosion energy, \( 9 \times 10^{52} \) ergs of kinetic energy at infinity, almost 100 times that of an ordinary supernova. Heger et al. (2000) calculated a peak luminosity for this model of \( \sim 10^{44} \) ergs s\(^{-1} \) (\( M_{\text{bol}} = -21 \)) lasting for about 150 days. Roughly the same peak luminosity and duration characterize the event with and without the hydrogen envelope, but in the case of the helium core the peak was delayed about 100 days and completely powered by \( ^{56}\text{Co} \) decay. Since the kinetic energy of the explosion was so high, the interaction with circumstellar matter may also have been quite brilliant. These are truly “hypernovae” (Woosley & Weaver 1982), but no strongly relativistic matter was ejected and such explosions will not produce a GRT.

The 300 \( M_\odot \) model (180 \( M_\odot \) helium core), on the other hand, was so tightly bound and gained so much kinetic energy prior to oxygen burning that even the fusion of the entire core to silicon and iron was unable to reverse its infall. It made a massive black hole. At the latest time reliably calculated with KEPLER, the central temperature had reached \( 10^{10} \) K and the nuclei in the center of the star had photodisintegrated to 72\% \( \alpha \)-particles and 28\% nucleons (\( \varepsilon_c = 0.50 \)). Unlike what is seen in ordinary supernova models, even though temperature-dependent partition func-

![Fig. 1.—Internal structure of the 250 \( M_\odot \) star at maximum central density as a function of mass coordinate. Panel (a) gives the mass fractions of the dominant chemical species. Note that “iron” denotes the sum of all isotopes of the iron group elements. Panel (b) gives temperature, density, and velocity. Inward movement (\( v_{\text{infall}} \)) is drawn as a dashed line and outward movement (\( v_{\text{exp}} \)) as a dotted line. The (logarithmic) density scale is 3.25 times that of the temperature scale.](image-url)
tions were included in the calculation, the higher entropy of these models results in complete photodisintegration of heavy elements. The density at $10^{10}$ K was $5.5 \times 10^7$ g cm$^{-3}$, the total kinetic energy of infall $5.8 \times 10^{52}$ ergs, and the net binding energy (internal plus potential) $-7.9 \times 10^{52}$ ergs. Figure 2 shows the composition, temperature and density structure, and distribution of specific angular momentum at this time. In subsequent discussions we define this to be the $t = 0$ collapse model.

3.1. Collapse without Rotation

Beyond this point the effects of neutrino trapping were no longer negligible. The Kepler model was mapped into a one-dimensional Lagrangian code (Herant et al. 1994; Fryer et al. 1999a), which included the effects of general relativity (assuming spherical symmetry and without back-reaction from gravitational wave emission) and neutrino transport. It is useful to compare the ensuing collapse of this $300 M_\odot$ star to that of an ordinary $15 M_\odot$ supernova progenitor. Just as in ordinary supernovae, as the core of the $300 M_\odot$ star contracted, the rate of electron capture increased. The removal of electron degeneracy pressure and cooling via neutrino emission helped to create a runaway collapse. However, there were several important differences between the structures of the 300 and 15 $M_\odot$ stars. As noted previously, the entropy in the more massive core was larger and thus favored the more complete photodisintegration of heavy elements and $x$-particles. General relativity also played a more significant role. The collapse of a $15 M_\odot$ core halted when its central density exceeded a few times $10^{14}$ g cm$^{-3}$, i.e., when nuclear forces and neutron degeneracy pressure became important. However, the core of a $300 M_\odot$ star was so large that it collapsed into a black hole before nuclear forces could affect the collapse, less than 1 s after the code link at $T_c = 10^{10}$ K.

Figure 3 shows the velocity, electron fraction ($Y_e$), temperature, and density profiles of the $300 M_\odot$ core at times 0, 500, 800, and 930 ms after the code link. Before the initial collapse, the entropy of the core was $\sim 10 k_B$ per nucleon (compared to $\sim 1 k_B$ per nucleon for most core collapse supernovae). Hence, for a given central collapse density, the $300 M_\odot$ star was hotter and neutrino emission more efficient. Neutrinos from electron capture initially streamed out from the core, quickly lowering its lepton number and electron degeneracy pressure. However, because the core of a $300 M_\odot$ star was so much bigger than a $15 M_\odot$ supernova progenitor, electron neutrinos started to be trapped at a lower central density (Bond et al. 1984). At first, neutrino absorption on nucleons dominated the electron neutrino opacity, but as the temperature rose, scattering on electron-positron pairs quickly became the most important opacity source. Although $\mu$ and $\tau$ neutrinos escaped the collapsing star more easily, trapping the electron neutrinos halted the deleptonization in the core. But at this point, the collapse to a black hole was already inevitable. The large population of electron neutrinos in the core actually caused the central electron fraction to rise, as the core tried to reach an equilibrium between the electron/antineutrino neutrinos and the electron/proton fractions in the core (see Fig. 3).

In our simulations, we followed the collapse until some region of the star fell within its last stable orbit (for the nonrotating star, this is $6GM_{\text{enclosed}}/c^2$). We used the simplifying assumption that the core “formed a black hole” at this point. A rigorous determination of the first trapped surface would require a multidimensional general relativistic calculation that is beyond the scope of this paper. The exact point in the core where the initial black hole formed is also very sensitive to neutrino transport, especially for the $\mu$ and $\tau$ neutrinos that dominate the neutrino cooling. By this definition, the black hole first formed at $\sim 20 M_\odot$, but the entire inner $35 M_\odot$ of the core was very close to its last stable orbit and relatively small changes in the $\mu$ and $\tau$ transport could have given an initial black hole mass anywhere in the range of $15-35 M_\odot$. No matter where the black hole initially formed, though, without rotation, most of the helium core would quickly become part of the black hole.

The bulk of the potential energy released during the collapse was dragged into the black hole. Without rotation to drive a dynamo, it is unlikely that a strong magnetic field
could have emerged during the collapse to tap into any of the potential energy. Likewise, a nonrotating collapse will not emit much energy in gravitational waves. Bond et al. (1984) argued that much of the potential energy could be released in a neutrino fireball. However, their analytic calculation overestimated the temperature at the neutrinosphere, which drastically overestimated the total neutrino luminosity. In our simulations, most of the neutrinos were trapped in the flow and only 1% of the gravitational potential energy was released in neutrinos, most in $\mu$ and $\tau$ neutrinos (Fig. 4). The collapse of the nonrotating 300 $M_\odot$ star proceeded without so much as a whimper.

### 3.2. Collapse with Rotation

However, these stars are rotating, and this changes the results. As with the nonrotating model, the rotating collapse was simulated by first mapping the KEPLER output into the one-dimensional neutrino transport code. The effect of rotation was included by adding a centrifugal term, $a_{\text{cent}} = j^2/r^3$, to the force equation and conserving angular momentum locally, $j \equiv j(m, t) = j(m)$, for each mass $m$. We mapped $j$ from the KEPLER output (Fig. 2) by setting $j(r) = r^2 \Omega(r)$ and added the equatorial force to the entire star. This approximation obviously overestimates the effect of angular momentum everywhere except in the equatorial plane, but it allowed us to follow the first 1.5 s of the star’s collapse quickly and gave approximately the correct composition and mass distribution. Recall that in the nonrotating simulation, a black hole formed 1 s after mapping from the KEPLER output into our one-dimensional neutrino transport code. In the rotating simulation, centrifugal forces were extremely important (Fig. 5) in slowing the collapse. After 1.5 s, the central density of the rotating core was only $5 \times 10^{10} \text{ g cm}^{-3}$.

When the centrifugal force exceeded 25% of gravity for any Lagrangian zone (Fig. 5), the (one-dimensional) collapse calculation was again halted and remapped into a two-dimensional code based upon the smooth particle hydrodynamics prescription (Herant et al. 1994; Fryer & Heger 2000). Use of the two-dimensional code was delayed until this point in order to make the problem computationally tractable. The rotational axis was aligned with the
code's axis of symmetry, and the angular momentum of each particle was given by the rotation rate of the zones in the one-dimensional calculation: \( j(r) = \Omega(r)r^2 \cos^2(\theta) \), where \( \theta \) is the angle above the plane of rotation. The net effect was to give (initially) spherical shells constant angular velocity and a total angular momentum equal to what they had in the Kepler model at \( t = 0 \).

A total of three two-dimensional simulations were performed this way, each employing a different treatment of the angular momentum. The models will be referred to by three names: (model A) no rotation (this is inconsistent with the assumed density structure at the last remapping but calculated just for comparison); (model B) the star rotated with the angular momentum distribution at the end of the one-dimensional calculation, which was the same as prescribed by the KEPLER calculation; in the continued evolution the angular momentum of each particle was conserved; and (model C) the same as model B, but with angular momentum transport between particles mediated via an \( \alpha \)-disk prescription (\( \alpha_{\text{disk}} = 0.1 \); see Fryer & Heger 2000 for details). In the remainder of this section we will discuss the collapse of the star and the resultant black hole formation under these three assumptions. A summary of results is given in Table 1.

As might be expected from our nonrotating collapse simulation, model A collapsed directly into a black hole 250 ms after the beginning of the two-dimensional simulation. As before, we assumed that the star has made a black hole when matter at any radius lay within its last stable orbit for the given distribution of matter. At that point, all particles inside this radius were removed and replaced by a perfectly absorbing inner boundary condition at that radius. This way, we were able to follow the continued accretion into the black hole (§ 5).

Those models that included the KEPLER angular momentum distribution (Fig. 2) evolved very differently from the nonrotating ones. As the star collapsed, neutrinos became trapped in the flow, maintaining the high entropy of the 300 \( \odot \) star, along with the rapid rotation (\( Y_e \sim 0.35 \)). The high entropy of the 300 \( \odot \) star, along with the rapid rotation (\( Y_e \sim 0.35 \)), enabled the collapse to proceed without forming a black hole. However, the collapsing material was still able to reach very high temperatures, causing the neutrino emission to decrease dramatically. Note that only 1% of the potential energy released in the collapse actually makes it out of the black hole.

Table 1

| Model | \( M_{\text{variable}} \) (\( \odot \)) | \( T_{\text{coll}} \) (s) | \( M_{\text{BH}} \) (\( \odot \)) | \( \dot{M}_{\text{BH}}^{\text{disk}}/\dot{M}_{\text{BH}}^{-1} \) | \( M_{\text{PrototBH}}^{-1} \) (\( \odot \)) | \( \dot{\mu}_{\text{BH}}^{\text{disk}}/\mu_{\text{BH}}^{-1} \) (s) | \( M_{\text{BHdisk}}^{\text{disk}}/M_{\text{BH}}^{-1} \) (\( \odot \)) | \( \alpha_{\text{disk}} g_{\text{disk}}^{\text{disk}}/M_{\text{BH}}^{-1} \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| A     | ...            | 1.75           | 13             | 0              | ...            | ...            | ...            | ...            |
| B     | 5–40           | 3.7            | 35             | 0.73           | 90             | 3.5            | 136            | 0.74           |
| C     | 12–35          | 2.8            | 13             | 0.55           | 70             | 3.6            | 122            | 0.72           |

\(^a\) Model A neglects rotation. Model B assumes the initial rotation of the progenitor model and conserves locally the angular momentum for the duration of the simulation. Model C uses the same initial rotation as model B but evolves the angular momentum using an \( \alpha \)-disk prescription (\( \alpha_{\text{disk}} = 0.1 \)).

\(^b\) Time elapsed after the simulation is mapped from the KEPLER output.

\(^c\) The “disk” quantities (last three columns) refer to the time, black hole mass, and black hole rotation at which the accretion rate drops below 10 \( \odot \) s\(^{-1}\).
tion, halted the collapse of the core and even produced a weak "bounce" at a central density of a few times $10^{12} \text{ g cm}^{-3}$ (Fig. 6). The structure of the resulting core was similar to the proto-neutron star cores encountered in ordinary core collapse supernovae, but here it was much larger. Usually, in lower mass supernovae, after nuclear forces and neutron degeneracy pressure halt the collapse, the core bounces, sending a shock into the star. When the bounce shock stalls, it forms a proto-neutron star core capped by the accretion shock produced as the remainder of the star falls onto the core. In the case of core collapse supernovae, the proto-neutron star has a mass of roughly $1 \, M_{\odot}$ and a radius of $\sim 400 \text{ km}$. In the present simulations of the $300 \, M_{\odot}$ star, our weak "bounce" yielded a $50 \, M_{\odot}$ "proto-black hole" with a radius of 1000 km (Fig. 7). By "proto-black hole" we mean a hot dense neutronized core that has not contracted inside its event horizon, but which would do so if it lost its internal entropy.

Material continued to accrete onto this core through a shock at a rate of $10^{-100} \text{ M}_{\odot} \text{ s}^{-1}$. By 1.1 s after the beginning of the two-dimensional simulations (2.6 s after the initial collapse in Fig. 2), the proto-black hole mass had increased to nearly $70 \, M_{\odot}$ (Fig. 8). Neutrino emission, dominated by $\mu$ and $\tau$ neutrinos (since the electron neutrinos were trapped), cooled the proto-black hole, and the radius of the accretion shock slowly shrank. From Figure 9 we see that the total neutrino luminosity during much of the simulation was well above $10^{54} \text{ ergs s}^{-1}$. Such high neutrino luminosities are suggestive of a supernova-like explosion or perhaps even a GRB (Fuller & Shi 1998).

However, inside the proto-black hole scattering and absorption dominated the neutrino opacity, and beyond the

![Fig. 6](image6.png)

**Fig. 6.**—Velocity vs. radius 250 ms after mapping the simulation into the two-dimensional code (model B). Points correspond to particles at different radii and latitudes. Thermal pressure, aided by support from angular momentum, slows the collapse of the core, causing a weak "bounce." The material along the equator is supported by angular momentum and collapses much slower than the material along the poles. Hence, at any given radius, there is a range of velocities (the slowest speeds correspond to the material along the equator). Note the accretion shock developing at 2000 km.

![Fig. 7](image7.png)

**Fig. 7.**—Proto-black hole 0.5 s before black hole formation. Color denotes temperature in $10^{9}$ K, and the vectors represent the direction and magnitude of the particle velocity. The simulation was actually only half of this circle and was reflected about the $z$-axis for display purposes. It used a total of 25,000 particles though only $\sim 15,000$ are shown in the figure. At this time, the proto-black hole has a mass of roughly $78 \, M_{\odot}$ and size of $\sim 1100 \text{ km}$. Roughly 20 $M_{\odot}$ resides in the inner 100 km, and it is this inner core that first forms a black hole, but as soon as it collapses, the entire proto-black hole (which is $90 \, M_{\odot}$ at collapse) quickly accretes onto the black hole.

![Fig. 8](image8.png)

**Fig. 8.**—Density vs. enclosed mass for the collapsing core 1.75 s (red; lower curve) and 2.6 s (blue; upper curve) after reaching a central temperature of $T_{c} = 10^{10}$ K. The points correspond to particles at different radii and latitudinal angles. The "enclosed mass" is defined by the mass inside a sphere with the same radius as the particle distance from the center. Because of the range in fall velocities, at any given mass coordinate, there is a range of densities. The proto-black hole mass increases as material accretes through the accretion shock. However, neutrino cooling causes the core to contract, and the accretion shock actually moves inward. These two effects cause the density to increase dramatically.
The electron neutrinos (solid line) do not decrease significantly until the black hole expands enough to produce a cool accretion disk. The neutrino luminosity is shown in Fig. 9. Neutrinos interacting with nucleons and scattering on electrons were also included in the calculation, and these too provided totally insufficient energy to reverse the infall. However, shortly after the black hole formed, the accretion shock moved inward and angular momentum became more important. An accretion disk eventually formed, and that could be relevant to an explosion. Thus, it was important to determine just when the black hole formed. As the proto–black hole contracted, its inner core was slowly compressed down to the event horizon. As with the nonrotating model, we followed that collapse until the material was compressed down to the radius of the marginally stable circular orbit (e.g., Shapiro & Teukolsky 1983):

\[ r_{ms} = M_{BH}[3 + Z_2 - \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}] \]

where

\[ Z_1 = \frac{1}{3} \left[ 1 - \frac{a_{BH}^2}{M_{BH}^2} \right] \left[ \frac{1}{3} + \frac{a_{BH}}{M_{BH}} + \frac{1}{3} - \frac{a_{BH}}{M_{BH}} \right] \]

and \( a_{BH}/M_{BH} \) and \( M_{BH} \) are the dimensionless angular momentum and mass of the black hole. At any given radius in the proto–black hole, we calculated both \( a_{BH}/M_{BH} \) and \( M_{BH} \) and thus determined when that radius fell inside the marginally stable orbit. When this happened, we stopped the simulation and removed all of the particles within that radius. The total mass of these particles formed the initial black hole mass. In this manner, although we did not physically model the collapse to a black hole, we obtained roughly the correct time of collapse and the initial black hole mass (Table 1).

However, this simplifying procedure may obscure some very interesting physics that might have gone on in a fully relativistic, three-dimensional simulation. Our collapsing star had a large amount of angular momentum. As it collapsed, momentum conservation caused it to spin up. As long as the rotational energy remains less than \( \sim 14\% \) of the gravitational potential energy, the proto–black hole is stable against triaxial deformation. However, when \( T/|W| > 0.14 \), instabilities can occur (e.g., Shapiro & Teukolsky 1983). By plotting the energy ratios \( T/|W| \) as a function of radius for the proto–black hole 0.5 and 0 s before black hole collapse, we see that much of the rotating proto–black hole was unstable to triaxial deformations (Fig. 10). The growth time for such instabilities is (Schutz 1983)

\[ \tau \sim \frac{T/|W|}{\Omega} \left( \frac{\Omega c}{c} \right)^{-5}, \]

where \( \Omega \) is the rotational velocity at radius \( r \). Figure 10 also shows the instability growth time versus mass 0.5 and 0 s before black hole formation.
before the collapse of the proto–black hole. According to our analysis, instabilities would have had time to grow in our rotating models, possibly forming smaller clumps that would have collapsed and merged to form the central black hole (Bond et al. 1984). By comparing the growth time to the time to collapse for both our rotating models, we can estimate the range of masses in the proto–black hole that might develop these instabilities (Table 1). Equation (4) is not sufficiently accurate to prove decisively that such instabilities will develop before the proto–black hole collapses. Even if these instabilities occur, they would not affect the accretion onto the black hole significantly (§ 5) because the individual clumps will merge and form a central black hole at roughly the same time as our assumed stable model, but they would affect the emission of gravity waves (§ 4).

Although the qualitative picture we have described is the same for both rotating models (B and C), the actual formation time of the black hole differs considerably. This is because in model C angular momentum was transported out of the core using the $\alpha$-disk prescription. This lowered the spin of the black hole ($a_{\text{BH}}$), which increased the radius of the marginally stable orbit ($r_{\text{ms}}$) and ultimately caused a black hole to form more quickly. By choosing a high disk viscosity ($\alpha_{\text{disk}} = 0.1$), our model C estimates the extreme effect of viscosity, and the true answer probably lies between models B and C. Fortunately, these differences do not affect our quantitative estimates of gravitational wave emission or GRT energies (§§ 4 and 5).

4. GRAVITATIONAL WAVES

Even if the core does not break into several pieces, since $T/|W|$ exceeds the secular instability criterion ($\approx 0.14$), the star might have still developed a barlike configuration. Gravitational wave emission for rotating bodies in the context of supernova collapse has been studied in great detail (for reviews see Zwerger & Müller 1997; Rampp, Müller, & Ruffert 1998). We will use here the expressions derived by Zwerger & Müller (1997) for the quadrupole wave amplitude, the gravitational wave field ($h_+$), and the total gravitational energy emitted (eqs. [20]–[22] of Zwerger & Müller 1997). In all of our simulations, $|\mathcal{E}_{52}|$ peaked near $1.5 \times 10^{-21} \, \text{cm}$, nearly 4 orders of magnitude greater than most of the simulations of core collapse supernovae by Zwerger & Müller (1997). The amplitude of the corresponding waveform was also high: $h_+ = 1.3 \times 10^{-21} / d(\text{Gpc})$. Recall that 300 $M_\odot$ stars are only likely to form at very high redshifts ($z \approx 15$), corresponding to distances beyond 7.5 Gpc for $H_0 \approx 60 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$. For most of these stars, $h_+ \approx 10^{-22}$. The total energy emitted in gravitational waves during the first few seconds of collapse was $\sim 10^{-3} c^2 M_\odot$. During this time, a 100 $M_\odot$ black hole formed and roughly $10^{-2}$ of its rest mass energy was converted into gravitational waves. Like the neutrino emission, our calculation of the energy lost to gravitational waves is much smaller than that predicted by Bond et al. (1984). However, should the core of the proto–black hole break into pieces, the gravitational wave emission could be much larger.

Unfortunately, the development of secular instabilities requires the ability to model the proto–black hole over many orbital periods in three dimensions. While we can follow the collapse all the way to black hole formation in two dimensions, the development of instabilities (and a more accurate estimate of gravitational wave emission) awaits future three-dimensional simulations. We would be happy to provide any of our models to researchers who would like to attempt this problem.

5. BLACK HOLE ACCRETION AND GAMMA-RAY TRANSIENTS

Once the black hole formed, the rest of the dense inner core quickly collapsed inside, but in the rotating models, material along the equator was slowed by centrifugal force. If there is sufficient angular momentum to form an accretion disk, this material will be slowed and pile up outside the hole. The disk that forms would be similar to those studied by Popham et al. (1999). Such black hole accretion disk systems are believed to power gamma-ray bursts, through either neutrino annihilation or magnetic field–powered jets. To assess whether the collapse of 300 $M_\odot$ stars might produce some sort of gamma-ray transient, we must further examine the properties of the disk and black hole.

We followed the accretion into the black hole by placing an absorptive inner boundary in the core at the marginally stable orbit. As particles moved inside the marginally stable orbit ($r_{\text{ms}}$), their mass and angular momentum were added to the black hole. The boundary condition corresponding to the last stable orbit expanded as the black hole gained mass. For the nonrotating model, almost all of the potential energy of the accreting material was carried into the black hole (recall Fig. 4). Even in the rotating models, much of the star collapsed onto the black hole before a stable accretion disk could form. When angular momentum was conserved locally (model B, Fig. 11), a stable disk formed only after roughly 140 $M_\odot$ of the star has accreted (Fig. 11).

To produce a jet, energy generated in the disk and transported, by either neutrino emission or magnetohydrodynamic processes to matter above and below the black hole along the rotational axis, must generate sufficient pres-

![Fig. 11.—Accretion disk 6.5 s after black hole formation assuming local angular momentum conservation (model B). Velocities are color coded in units of 1000 km s$^{-1}$. At this time, the accretion rate through the pole is less than 0.1 $M_\odot$ s$^{-1}$ and the disk mass is roughly 30 $M_\odot$. However, only 5200 particles remain in the simulation (4600 are shown in the figure), and it is difficult to follow the accretion through the disk. The color represents radial velocity, showing that a large part of the disk is now stable.](image)
sure and momentum to reverse the implosion of material accreting along the poles (MacFadyen & Woosley 1999). The evolution of the polar accretion and disk accretion rates and the efficiency for this transport thus dictate if and when an explosion is likely to occur. Figure 12 shows the total accretion rates for our three simulations along with an analytic estimate assuming that the mass falls in at the free-fall time:

\[ t_{\text{ff}} = \frac{\pi}{2} \sqrt{\frac{r_0^3}{GM_{\text{enclosed}}}}. \]  

(5)

This analytic estimate does not take into account pressure forces or angular momentum, both of which affect the accretion rate. The accretion rate in the nonrotating model shows a large dip in the accretion rate over the mass range ~40–80 \( M_\odot \). This occurs because nuclear burning injects energy into the collapsing material and slows its infall. In the rotating models, the entire proto–black hole burns into iron elements before it collapses into a black hole. Hence, in the rotating models nuclear burning does not affect the infall after black hole formation and there is no dip in the accretion rate. The accretion rate decreases at lower black hole masses in the rotating models because some of the material has enough angular momentum to support itself in a disk.

In both of our rotating simulations, a disk formed in a wedge extending up to 35°–40° above the equator. When we included angular momentum transport (model C), the angular momentum in the outer layers of the helium core increased (as these layers gained angular momentum from the core). Thus, the disk in model C was initially larger, both in extent and mass, than in the case in which angular momentum was not transported (model B). Although material in the disk fell into the black hole much slower than along the poles, the density in the disk quickly became so much higher than in the polar regions that it dominated the accretion onto the black hole (beyond a black hole mass of 120 \( M_\odot \), disk accretion makes up greater than 70% of the total accretion). That is, by the time the total accretion rate dropped below 10 \( M_\odot \) s\(^{-1}\), the accretion along the polar region (the entire region beyond the 40° disk) was accreting less than 3 \( M_\odot \) s\(^{-1}\), and this rate was dropping rapidly.

Unfortunately, our limited resolution made it difficult for us to follow the accretion rate when it fell below a few \( M_\odot \) s\(^{-1}\). In model B (where the angular momentum is locally conserved), the accretion rate in the disk dropped as abruptly as the rate in the poles as the disk became entirely supported by centrifugal forces. However, where we included angular momentum transport (model C), we expect that the disk would have continued to accrete at a rate of ~1–10 \( M_\odot \) s\(^{-1}\) (for an \( \alpha_{\text{disk}} = 0.1 \)) until the entire ~40 \( M_\odot \) disk had accreted onto the black hole.

It is unlikely that any jet would have formed before the polar regions cleared and the accretion rate along the poles dropped below some critical rate. This critical rate depends upon the mechanism used to power the explosion. For the neutrino-driven mechanism, the accretion rate in the polar region must drop below 1–10 \( M_\odot \) s\(^{-1}\) so that the infalling material is optically thin to neutrinos. When the total accretion rate fell below 10 \( M_\odot \) s\(^{-1}\), corresponding to an accretion rate in the 50° polar region of 2–3 \( M_\odot \) s\(^{-1}\), the conditions around the black hole could drive vigorous outflow. At this time, we can compare the properties of our black hole accretion disk systems with those models produced by Popham et al. (1999). The black hole mass and spin for both our rotating models are listed in Table 1. The total disk mass available for rapid accretion is roughly 170 \( M_\odot - M_{\text{BHdisk}} \), where \( M_{\text{BH}} \) is the mass after disk formation (Table 1).

Popham et al. (1999) considered black hole masses between 3 and 10 \( M_\odot \). The efficiency at which neutrino annihilation converted the gravitational potential energy released into fireball energy dropped dramatically as the black hole mass increased. As the event horizon increases, (1) the density and temperature in the disk decreases, decreasing the total neutrino flux and neutrino energy; and (2) the volume of the annihilation region increases, reducing the neutrino density. From Popham et al. (1999) we find that the neutrino annihilation conversion efficiency goes as \( M_{\text{BH}}^{-n} \), where \( n \) is roughly 2–3. So even if the disk accreted at a rate of 10 \( M_\odot \) s\(^{-1}\), the energy deposited by neutrino annihilation probably would not be high enough to drive an explosion. Note that in our simulations, the accretion rate through the disk in all of our simulations was less than 10 \( M_\odot \) s\(^{-1}\) when the polar region has cleared.

Jets driven by magnetic fields might still produce a strong explosion, though. It is difficult to make any quantitative predictions about magnetic field–driven jets simply because the exact mechanisms for energizing and collimating the jet are not well understood. However, by the time the total accretion rate dropped below 10 \( M_\odot \) s\(^{-1}\), a well-defined disk had formed, as is required for most GRB central engines driven by magnetic fields. The total potential energy
available to drive an explosion was \( \epsilon_{\text{spin}} \epsilon_{\text{MHD}} M_{\text{disk}} c^2 \), where the potential efficiency of the accretion disk (\( \epsilon_{\text{spin}} \)) is \( \sim 0.11 \) for \( a/M = 0.74 \). Assuming that the efficiency of our magnetic field–driven explosion (\( \epsilon_{\text{MHD}} \)) is 0.1, the yields from models B and C are \( 6 \times 10^{53} \) and \( 10^{54} \) ergs, respectively. Beamed into 1% of the sky, these jets would have inferred isotropic explosion energies of nearly \( 10^{56} \) ergs and could produce GRTs of comparable energy. Such large gamma-ray luminosities would be easily at redshift 10 and beyond and are roughly 1–2 orders of magnitude more energetic than ordinary collapsars.

The duration of the jet can be crudely estimated from the \( x \)-disk model (Shakura & Sunyaev 1973), \( \tau_{\text{acc}} \sim r^2/(x M c^2) \), with \( x \) as the disk viscosity parameter, \( \Omega_k \) the Keplerian angular velocity, and \( r \) and \( H \) the radius and thickness of the disk, respectively. For \( r \sim 3H \sim 5000 \) km (Fig. 11) and \( x \sim 0.1 \), the timescale is very roughly 10 s, but there is considerable uncertainty in both \( x \) and the other parameters of this equation. That this timescale is approximately equal to the duration of many common GRBs is interesting but may be coincidental. We expect stars that have masses at death of over 300 \( M_\odot \) to exist only at large redshifts where the metallicity is very low (Baraffe et al. 2001). Unless the jet wanders a lot or the disk is prematurely disrupted by the explosion, the burst duration, roughly 10 s times \((1+z)\), will be too long to explain the most common GRBs. Depending too on the uncertain physics of the jet interacting with the circumstellar medium, the redshifted spectrum might also be a lot softer than more “local” GRBs. Further work to clarify these uncertainties would be worthwhile. The large energy potentially available from the model leaves room for considerable inefficiency in the GRB production.

At a minimum, our work suggests the possibility of other forms of GRTs with timescales of perhaps minutes and a hard X-ray spectrum. Such an energetic jet would likely disrupt the star in a gigantic “hypernova” explosion visible perhaps almost to the edge of the universe. If the star retained its hydrogen envelope (i.e., did not lose it to a binary companion), disk-fed accretion might continue for about a day. This could make an even longer fainter transient of some sort and may be an important source of X-ray photons in the early universe.

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