Quark description of the Nambu-Goldstone bosons
in the color-flavor locked phase

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Abstract

We investigate the color-singlet order parameters and the quark description of the Nambu-Goldstone (NG) bosons in the color-flavor locked (CFL) phase. We put emphasis on the NG boson (phason) called “H” associated with the $U_B(1)$ symmetry breaking. We qualitatively argue the nature of H as the second sound in the hydrodynamic regime. We articulate, based on a diquark picture, how the structural change of the condensates and the associated NG bosons occurs continuously from hadronic to CFL quark matter if the quark-hadron continuity is realized. We sharpen the qualitative difference between the flavor octet pions and the singlet phason. We propose a conjecture that superfluid H matter undergoes a crossover to a superconductor with tightly-bound diquarks, and then a crossover to superconducting matter with diquarks dissociated.

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I. INTRODUCTION

Quantum Chromodynamics (QCD) describing the dynamics of quarks and gluons has the rich phase structure depending on the temperature and the baryon density. In particular, the region at low temperature and high density is the arena of a number of possibilities competing. Our knowledge in condensed matter physics provides us with useful information on dense QCD matter. For example, we know that the attractive force between electrons mediated by phonons brings about superconductivity of metals, and that a fermionic $^3\text{He}$ liquid has the superfluid (B, A, and A$_1$) phases originating from pair condensation of $^3\text{He}$ atoms [1, 2]. It is natural to pursue similar possibilities for nucleons [3] and quarks [4].

Actually, the pairing force in the NN interaction leads to a finite $^1S_0$ gap in nuclear matter. The superfluid properties of neutron matter and neutron-rich nuclei have been paid much attention in both nuclear and neutron star physics. When the baryon density is large enough, a triplet-state $^3P_2$ superfluidity of neutron matter is possibly favored [3, 5]. At extremely high density, quarks are expected to participate in the dynamics directly [6] and eventually the perturbative QCD calculation at weak coupling is feasible owing to asymptotic freedom. As a matter of fact, the one gluon exchange interaction between quarks is attractive in itself in the color anti-triplet channel, which results in the color-superconducting phase [7, 8].

In the case of three massless flavors, the color-flavor locked (CFL) phase [9] is well established at sufficiently high density. The CFL phase preserving a symmetry under the color-flavor rotation is similar to the B phase of $^3\text{He}$ where a symmetry remain when the orbit and spin rotations are locked together. Interestingly enough, it has been discussed that the spontaneous breaking of global symmetries in the CFL phase has the same pattern as that in hadronic matter, as briefly reviewed below. It leads us to a conjecture that superfluid (hyper-nuclear) hadronic matter can be smoothly connected to CFL quark matter (i.e., quark-hadron continuity [10]). If this happens, the physical content of the Nambu-Goldstone (NG) bosons must continuously change from one to the other phase.

In order to investigate the low energy dynamics governed by the NG bosons in the CFL phase, the non-linear representation of the chiral effective Lagrangian has been studied [11, 12, 13, 14]. There are, on the other hand, some attempts to describe the NG bosons in terms of quarks directly [15, 16]. An apparent problem thus is that the effective Lagrangian approach suggests the (pionic) NG bosons consisting of four quarks, i.e., two quarks and
two quark-holes. Due to this structure, as discussed in \cite{11,12,13}, the inverse mass ordering of the NG bosons occurs, which leads to the possible meson condensation in the presence of finite quark masses or an electron chemical potential. The fact that the NG bosons are made of four quarks has not been fully taken into account in quark based calculations. \cite{16} Hence, it is important to formulate the quark description of the NG bosons in the CFL phase clearly.

The purpose of this paper is to give plain expressions for the NG bosons in the CFL phase. Such a description enables us to have a clear speculation on the quark-hadron continuity from the point of view of the quark content of the condensates and the associated NG bosons. We would emphasize that the structural change of the quark content can be intuitively understood, even though it is hard to imagine how the correspondence is realized, as originally conjectured in \cite{10}, between the color-flavor octet quarks and the flavor octet baryons, or between the massive gauge bosons and the vector mesons.

II. NAMBU-GOLDSTONE BOSONS

We shall limit our discussion to the chiral limit with three massless flavors since we are interested in the ideal realization of the NG bosons at first. Although a finite $m_s$ might change the phase structure qualitatively \cite{17}, the chiral limit would be a well-defined starting point and an optimal underpinning to consider more complicated situations.

In the superfluid phase of diquarks the relevant degrees of freedom are given by diquarks rather than quarks because quarks are all gapped. \textit{This is a dynamical assumption meaning that we are working in the CFL phase}. The statement that the NG bosons in the CFL phase consist of four quarks is generic not depending on the assumption. The picture, as we argue below, that the four-quark boson is regarded as a diquark-dihole state is derived from the assumption. In other words we specify our problem by taking a diquark model picture.

We define the left- and right-handed diquark fields as follows;

\begin{equation}
\phi_{L \alpha}^* = \epsilon_{ijk} \epsilon_{\alpha \beta \gamma} q_L^C \bar{q}_{Lj}^{\beta} q_{Lk}^{\gamma}, \quad \phi_{R \alpha}^* = \epsilon_{ijk} \epsilon_{\alpha \beta \gamma} q_R^C \bar{q}_{Rj}^{\beta} q_{Rk}^{\gamma},
\end{equation}

where the complex conjugation (*) is attached in order that $\phi_{L,R \alpha}$ transforms as a triplet under the color and flavor rotations as in \cite{11}. The Latin and Greek indices represent flavor and color, respectively. It is assumed that dominant diquarks are anti-symmetric in
spin, color, and therefore flavor. Since the condensate has positive parity, which is favored energetically, we have $\langle \phi_{L,i}^{\alpha} \rangle = -\langle \phi_{R,i}^{\alpha} \rangle \propto \delta_{i\alpha} \Delta$ in the CFL phase if we choose a certain gauge. The left-handed diquark condensate breaks the symmetries as $SU_{C}(3) \times SU_{L}(3) \times U_{L}(1) \rightarrow SU_{C+L}(3) \times Z_{L}(2)$ ($\phi_{L}$ is invariant under $q_{L} \rightarrow -q_{L}$). The right-handed diquark condensate breaks the symmetries likewise $[9]$. The residual symmetry is in a sense similar to the custodial symmetry in the Higgs-Kibble model $[18]$. After all, the chiral symmetry breaking takes place in the CFL phase as

$$SU_{C}(3) \times SU_{L}(3) \times SU_{R}(3) \times U_{V}(1) \times U_{A}(1) \rightarrow SU_{C+L+R}(3) \times Z_{L}(2) \times Z_{R}(2), \quad (2)$$

as argued first in $[9]$. We should remark that the $U_{A}(1)$ symmetry is explicitly broken due to the axial anomaly and thus the residual discrete symmetry is only $Z(2)$ in fact. Nevertheless we leave both $Z_{L,R}(2)$ symmetries for later convenience. In the subsequent paragraphs we will clarify the nature of the NG bosons associated with $[2]$.

Let us first consider about the pions in the CFL phase, that is, the mesonic excitations forming an octet of $SU_{C+L+R}(3)$. As stated above, a vectorial symmetry is preserved when the color rotation is locked with both the left- and right-handed flavor (opposite) rotations. Only the axial part of chiral symmetry (i.e., coset space $SU_{L}(3) \times SU_{R}(3)/SU_{L+R}(3)$) is broken actually and the NG bosons appear corresponding to the axial fluctuations. The important point is that the pions should be colorless because the local color symmetry is never broken spontaneously $[19]$. In other words, gapless fluctuations with non-trivial color charge are to be absorbed in the longitudinal polarization of the gauge fields (Anderson-Higgs mechanism). As the resulting NG bosons must be all colorless, it is rather preferable to begin with color-singlet order parameters characterizing the symmetry breaking pattern $[2]$.

The simplest choice is to form a singlet order parameter from a triplet $\phi_{L,R,i}^{\alpha}$ and an anti-triplet $\phi_{L,R,i}^{\alpha}$ in color space. Then we have $6 \times 6 = 36$ combinations (left- and right-handed triplet and anti-triplet) for the flavor indices. The condition that it should have positive parity and break chiral symmetry reduces those into 17 combinations as follows;

$$\tilde{\sigma} = \phi_{L,i}^{\alpha} \phi_{R,i}^{\alpha} + \phi_{R,i}^{\alpha} \phi_{L,i}^{\alpha}, \quad (3)$$

$$C_{V}^{a} = \phi_{L,i}^{*} \left( \frac{\lambda^{a}}{2} \right)_{ij} \phi_{L,j}^{\alpha} + \phi_{R,i}^{*} \left( \frac{\lambda^{a}}{2} \right)_{ij} \phi_{R,j}^{\alpha}, \quad (4)$$

$$C_{A}^{a} = \phi_{L,i}^{*} \left( \frac{\lambda^{a}}{2} \right)_{ij} \phi_{R,j}^{\alpha} + \phi_{R,i}^{*} \left( \frac{\lambda^{a}}{2} \right)_{ij} \phi_{L,j}^{\alpha}, \quad (5)$$
where $\lambda^a/2 \ (a = 1, \ldots, 8)$ form an algebra of su(3) in flavor space. $\tilde{\sigma}$ is a singlet under the flavor SU$_V$(3) transformation, but neither $C^a_V$ nor $C^a_A$ are. The essential point is that only $\tilde{\sigma}$ serves as a proper order parameter for the CFL phase. This can be seen algebraically from $\text{tr}\lambda^a = 0$. Otherwise ($\langle C^a_{V,A} \rangle \neq 0$), no vectorial subgroup of chiral symmetry would survive. One might wonder where the color and flavor are locked in the order parameter \(3\) as they should be. The answer is that the choice of $\tilde{\sigma}$ already reflects the fact that the system is in the CFL phase. This is because the only way to preserve the chiral SU$_V$(3) symmetry with a finite diquark condensate is the color-flavor locking and thus the presence of the unbroken SU$_V$(3) symmetry signifies the CFL phase (see also the argument in \[20\]).

In the construction of the chiral effective Lagrangian, the choice of the flavor-singlet currents $(X^\dagger \partial X$ and $Y^\dagger \partial Y$ in \[11\]) corresponds to the choice of \(3\) in the present formulation.

As we stated before, the four-quark nature is generically derived from the symmetry breaking pattern \(2\) and the color-singletness of the order parameter. The diquark description like \(3\) is, however, based on the diquark model picture. Actually the four-quark order parameter can be given by other expressions once some Fierz transformation is applied to \(3\). In principle one cannot distinguish the CFL phase from the hadronic phase with exotic chiral symmetry breaking \[21\] only from the symmetry breaking pattern.

Apparently $\tilde{\sigma}$ is a counterpart of the ordinary sigma meson, $\sigma \sim \bar{q}_R q_L + \bar{q}_L q_R$, which is regarded as a four-quark object as discussed in \[16\]. We must be cautious about this statement, however. It does not mean that $\tilde{\sigma}$ is a bound state of two quarks and two holes. We should rather understand that one diquark (dihole) only supplies the color charge as a background compensating for the other dihole (diquark). Also it should be noted here that $\tilde{\sigma}$ has an essential difference from $\sigma$. Under either $Z_L(2)$ or $Z_R(2)$ transformation, $\tilde{\sigma}$ is invariant but $\sigma$ is not. It should be noted that $\langle \sigma \rangle$ breaks the $U_A(1)$ symmetry up to $Z(2)$ that amounts to only a part of unbroken $U_V(1)$. Therefore $Z_L(2) \times Z_R(2)$ in the CFL phase is a larger symmetry than $\langle \sigma \rangle$ leaves. This difference is responsible for the inverse mass ordering of the NG bosons \[11\] and, in general, the attribute of higher dimensionful order

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1 For convenience, we will simply call a state excited by the operator $q$ a hole. It is actually mixed with an anti-quark whose contribution is negligible at high density and low temperature.

2 When superfluidity occurs, the $U_V(1)$ symmetry is also broken up to $Z(2)$ in addition to $U_A(1) \to Z(2)$ due to the chiral condensate. The residual symmetry in this case is not $Z(2) \times Z(2)$ but only $Z(2)$ and certainly smaller than $Z_L(2) \times Z_R(2)$ realized in the CFL phase.
parameters as discussed in [21].

Now that the order parameter is given by (3), the NG bosons can be read from the operator identity to define the spontaneous symmetry breaking as formulated first in [22]. Let $Q^a_A$ be generators of the axial part of chiral symmetry, i.e.,

$$Q^a_A = \int dx j^a_A \mu = \int dx \bar{q} \gamma_5 \gamma_0 (\lambda^a/2) q.$$

By using the anti-commutation relation,

$$\{ \lambda^a/2, \lambda^b/2 \} = \delta^{ab} + d^{abc} \lambda^c/2,$$

we can readily show

$$[iQ^a_A, \tilde{\pi}^b] = \frac{i}{3} \delta^{ab} \tilde{\sigma} + id^{abc} C^c \tag{6}$$

with

$$\tilde{\pi}^a = \phi^a_{Li\alpha} \left( \frac{\lambda^a}{2} \right)_{ij} \phi_{Rj\alpha} - \phi^a_{Ri\alpha} \left( \frac{\lambda^a}{2} \right)_{ij} \phi_{Lj\alpha}. \tag{7}$$

In general, as explicitly seen in the Umezawa-Kamefuchi–Lehmann-Källén representation, we can prove that $\tilde{\pi}^a$ couples to a massless state if the expectation value of the right-hand-side of (6) is non-vanishing [22]. As a result, it follows that $\tilde{\pi}^a \to Z^{1/2} \chi^a + \cdots$ and $j^a_{A\mu} \to (f_{\pi} v^2 \partial_\mu \chi^a) + \cdots$ asymptotically, where $v$ is the velocity, $\chi^a$ is the (asymptotic) NG boson operator, and $f_{\pi} Z^{1/2} = \langle \sigma/3 \rangle$. This non-perturbative relation may be made use of to determine a non-perturbative value of $f_{\pi}$. [c.f. $f_\pi = \langle \sigma \rangle$ in the mean-field analysis of the linear sigma model.] The four-quark operators, $\tilde{\pi}^a$, are thus the interpolating fields of the pions in the CFL phase. Equivalently, by using the Jacobi identity, one can intuitively regard the pions as fluctuations around the order parameter, i.e., $\tilde{\pi}^a \sim [iQ^a_A, \tilde{\sigma}]$.

A similar description in terms of four-quark states in terms of diquarks (diquark–dihole or diquark–anti-diquark) has been argued in diquark models [23]. In contrast to the normal hadronic phase, however, we would emphasize that the four-quark nature in the CFL phase is rigid and there is no mixing with the quark–hole nor quark–anti-quark component. This is because of the unbroken $Z_L(2) \times Z_R(2)$ symmetry in the CFL phase. The “parity” under either $Z_L(2)$ or $Z_R(2)$ transformation becomes a good quantum number. The quark–hole or quark–anti-quark object has odd “parity”, while the diquark–dihole or diquark–anti-diquark object has even “parity”. Therefore, the dominant Fock-state of the pionic excitation in the CFL phase is considered as a four-quark state, apart from the instanton effect.

Next let us consider about the phason, that is, the NG boson in connection with the superfluidity. We can construct color-singlet order parameters with a non-vanishing baryon number. The simplest choice is to form a color-singlet from three triplets. In this case the flavor indices are more complicated than in the pion case, namely, $6 \times 6 \times 6 = 216$
combinations. They can be reduced to 10 combinations by parity and anti-symmetric nature
in color;

\[ H = H_L - H_R = \epsilon_{ijk} \epsilon_{\alpha \beta \gamma} (\phi_{Li\alpha} \phi_{Lj\beta} \phi_{Lk\gamma} - \phi_{Ri\alpha} \phi_{Rj\beta} \phi_{Rk\gamma}), \]  

(8)

\[ \tilde{H}_1 = \epsilon_{ijk} \epsilon_{\alpha \beta \gamma} (\phi_{Li\alpha} \phi_{Lj\beta} \phi_{Rk\gamma} - \phi_{Ri\alpha} \phi_{Rj\beta} \phi_{Lk\gamma}), \]  

(9)

\[ \tilde{H}_8^a = \epsilon_{\alpha \beta \gamma} \left\{ \epsilon_{ijk} \phi_{Li\alpha} \phi_{Lj\beta} \left( \frac{\lambda^a}{2} \right)_i \phi_{Rk\gamma} - \epsilon_{ijk} \phi_{Ri\alpha} \phi_{Rj\beta} \left( \frac{\lambda^a}{2} \right)_i \phi_{Lk\gamma} \right\}. \]  

(10)

In our notation the above expression of \( H \) gives positive parity since \( \phi_{L,R} \rightarrow -\phi_{R,L} \) under the parity transformation. In the CFL phase both \( H \) and \( \tilde{H}_1 \) are \( SU(3) \)-singlets and can take a finite expectation value, while \( \langle \tilde{H}_8^a \rangle = 0 \). A non-vanishing expectation value of \( H \) would break only the \( U(1) \) and \( U_A(1) \) symmetries up to \( Z_L(2) \times Z_R(2) \). On the other hand, \( \tilde{H}_1 \) is variant also under the axial part of chiral symmetry as well as the \( U(1) \) symmetries. A natural interpretation on \( \tilde{H}_1 \) is a composite state made of \( \tilde{\sigma} \) and \( \tilde{H} \). Therefore we will give little thought about \( \tilde{H}_1 \) hereafter though \( \langle \tilde{H}_1 \rangle \) could be qualified as an order parameter. We would mention that one can directly prove that \( (\text{Tr} X^\dagger \partial X)^2 + (\text{Tr} Y^\dagger \partial Y)^2 \) in [11] originates from \( (\partial H_L)^2 + (\partial H_R)^2 \) in the present formulation.

The interpolating field of the NG boson can be deduced in the same way as in the pion case. The generator of the \( U_V(1) \) rotation is given by \( Q_V = \int dx j_{\mu \rightarrow 0} = \int dx \bar{q}_0 q \). Then, as in the familiar Goldstone model [24], the phase of the order parameter is the NG boson (phason) [25]. We can easily show

\[ [iQ_V, H] = -6iH. \]  

(11)

Interestingly enough, the \( H \) field defined in [8] has the same quark content as the H-dibaryon [26] as already pointed out in [10, 11, 27]. Let us clarify physical interpretation on the \( H \) field. The \( H \) field has two degrees of freedom. In the \( U_V(1) \) broken phase the vacuum state is described by the coherent state of \( H \), which is the superposition of many \( H \) and \( H^* \) states. [\( H \) is a complex scalar field and \( H^* \) is its conjugate.] If we choose the real part, \( (H + H^*)/2 \), as a condensate, the phason, \( \chi_H \), is given by the imaginary part, \( \text{i}(H - H^*)/2 \). The relation between the real and imaginary parts of \( H \) under the \( U_V(1) \) transformation can be understood in analogy with the \( U_A(1) \) rotation for \( \sigma \sim \bar{q}_L q_R + \bar{q}_R q_L \) and \( \eta_0 \sim \bar{q}_L q_R - \bar{q}_R q_L \). It would be legitimate to regard both parts of \( H \) as particles just like \( \sigma \) and \( \eta_0 \) (which eventually becomes \( \eta' \) due to mixing with \( \eta_8 \) when \( m_u > m_s \sim m_d \)), though there is no clear distinction for the \( H \) field.
The velocity of $\chi_H$ has been calculated at zero temperature \cite{11} and the zero temperature value, $v^2 = 1/3$, has been prevailing in model studies \cite{14}. When it comes to the velocity of $\chi_H$ at $T \neq 0$, it is important to distinguish the collisionless regime from the hydrodynamic one. Let $\omega$ and $\tau$ be the frequency of $\chi_H$ and the characteristic collision time which is roughly of order $1/T$. The zero temperature estimate is expected to work in the collisionless regime, $\omega \tau \gg 1$, meaning $H$ with energies much higher than $T$. The collisionless regime shrinks with increasing temperature. Then, the hydrodynamic description becomes more relevant in the hydrodynamic regime, $\omega \tau \ll 1$.

One might consider that $\chi_H$ is not a propagating (particle) mode but a diffusive mode in the hydrodynamic regime, for it looks like a density fluctuation. This is not the case in the superfluid phase since the number density is no longer conserved. Actually the order parameter field and the entropy density are mixed to lead to a propagating mode (second sound) in a superfluid \cite{28, 29}. In a dilute superfluid system the nature of hydrodynamic modes have been investigated \cite{20, 30, 31}. Then in the hydrodynamic regime it has been actually shown that the phason mainly corresponds to the second sound in the hydrodynamic regime. This consequence makes sense; the second sound only exists in the superfluid (symmetry-broken) phase, while the first sound is present in both the superfluid and normal phases. As to the QCD context, due to the asymptotic freedom at high density, we can expect that the analyses in a dilute system are relevant to our problem. Once the $\chi_H$ propagation is identified as the second sound, the velocity of $\chi_H$ in the hydrodynamic regime would be quite different from the zero temperature value.

In general a superfluid at finite temperature has a non-vanishing normal component whose density is $\rho_n$ as well as a superfluid one whose density is $\rho_s$. The two-fluid dynamics gives the general expression for the second sound speed as

$$c_2 = \sqrt{\frac{\rho_s s^2 T}{\rho_n \rho c_v}}, \quad (12)$$

where $\rho = \rho_n + \rho_s$. The entropy density and the specific heat are denoted by $s$ and $c_v$ respectively. The velocity, which is proportional to $\sqrt{\rho_s}$, approaches zero as the system goes closer to the critical temperature. This observation is analogous to the pion velocity vanishing at the critical temperature \cite{32} though the physics is different.

Whether we should consider in the collisionless or hydrodynamic regime depends on the energy of H and the temperature. The energy and temperature dependence of the velocity
of $\chi_H$ will be of importance, for example, in $\chi_H$-involved processes in the proto-neutron star at temperatures of tens MeV $^{14}$.

III. STRUCTURAL CHANGE

Now we shall consider how the quark-hadron continuity can be viewed from the quark content of the color-singlet condensates and the NG bosons. We will find it important to recognize that the meaning of continuity is twofold; one is with respect to chiral symmetry and the other is related to the confinement-Higgs crossover $^{33}$. Their physical meanings are distinct.

For the moment we will adopt a working definition; “hadronic matter” is used for the phase with $\langle \sigma \rangle \neq 0$, and “CFL phase” for the phase with $\langle \sigma \rangle = 0$. This is in accordance with the prevailing convention in the chiral Lagrangian approach $^{11, 27}$. In hadronic matter pions are mainly composed of a quark ($q$) and an anti-quark ($\bar{q}$) in the constituent quark picture.

As discussed some time ago $^{34}$ there can be a significant mixture of $\bar{q}^2q^2$ even in the hadronic phase and we have not only $\langle \sigma \rangle \neq 0$ but also $\langle \bar{\sigma} \rangle \neq 0$. In the CFL phase, i.e., in the phase having the $Z_L(2) \times Z_R(2)$ symmetry (apart from the axial anomaly), $\langle \sigma \rangle$ becomes zero and any $\bar{q}q$ component in the pions vanishes. The quark-hadron continuity is realized by a non-vanishing common ingredient of the pions, $\tilde{\pi}^a$, made of a diquark and a dihole. The instanton effect, of course, breaks the $U_A(1)$ symmetry explicitly and blurs a sharp phase separation. The important point is, however, that we can have a definite limit where hadronic matter is distinguished from the CFL phase with respect to the pions $^3$.

In contrast, there remain more or less controversial points in the discussion about superfluidity. This is because hyper-nuclear matter has not been completely understood. It is expected at least that hyper-nuclear matter has a finite gap leading to a superfluid phase where $U_V(1)$ is broken into $Z(2)$ $^4$. There are 8 baryons forming an octet in the $SU_V(3)$

$^3$ The limit where $Z_L(2) \times Z_R(2)$ is present does not corresponds to the absence of instantons. Otherwise, not only $U_A(1)$ but also chiral symmetry is not broken. This limit should be understood in the same sense as the effective restoration of the $U_A(1)$ symmetry discussed in $^{35}$.

$^4$ As stated in $^{36}$, the $U_B(1)$ symmetry generated by the baryon charge is different from $U_V(1)$ discussed here. Since we always deal with color-singlet objects, however, the $U_V(1)$ rotation is equivalent with three (six, nine, etc) times $U_B(1)$ rotation, apart from the discrete symmetries. In this sense, though it is not
symmetry. The superfluid condensate is an SU\(_V(3)\) singlet, which can be constructed by two baryon octets as

\[ \Delta_{NN} \sim \langle (\Sigma^0)^2 + (\Lambda^0)^2 + \Sigma^+ \Sigma^- + \Sigma^- \Sigma^+ + p \Xi^- + \Xi^- p + n \Xi^0 + \Xi^0 n \rangle. \]  

(13)

Moreover we can also expect \( \langle H \rangle \neq 0 \) (\( H \) is given by (8), i.e., a three diquark-like object) because it is not prohibited by the symmetry. As a matter of fact, the H-dibaryon state could be an admixture of two-nucleon-like and three-diquark-like configurations. This can be typically seen in the different treatments, namely, the resonating group method (RGM) and the MIT bag model. Unlike the pion case, we have no definite limit where we can give a rigorous statement about the structural change of the superfluid component. We can only draw a qualitative picture in an intuitive picture. In the phase where the \( Z_L(2) \times Z_R(2) \) symmetry is manifested, any nucleon made of 3 quarks cannot be present, though a nucleon pair can be. Thus it is likely that the nucleon-like condensate, \( \Delta_{NN} \), becomes less significant in the CFL phase. It means that the overlap of the wave-function in the condensate becomes larger for \( \langle H \rangle \) (probably deconfined three-diquark condensate) then. This can be interpreted as a crossover from a confined phase to a Higgs phase. [Actually the major difference between the RGM and the MIT bag model comes from the way to impose the confinement condition. The RGM is stricter in confinement than the MIT bag model.] We would note that a quite similar picture (continuity between hyper-nuclear matter and partially deconfined quark matter) has already been proposed some time ago in the context of H-matter in the neutron star. In contrast to the pion case, the important message here is that the continuity with respect to the phason is inherently indistinguishable. We summarized our point of view in Table I and Fig. I schematically.

In principle the color-superconductivity can be a different phenomenon from the chiral phase transition, though we have considered chiral symmetry to characterize the phase. The situation is even similar to the relation between the deconfinement and chiral phase transitions at finite temperature. There, the deconfinement transition is a crossover in the presence of dynamical quarks in the fundamental representation, while the chiral phase transition is well-defined in the chiral limit. In the present case, the liberation of color degrees of freedom corresponds to a crossover from hadronic matter to the color-superconducting

\[ \text{rigorously correct, we will use a sloppy notation and sometimes write } U_V(1) \text{ and } Z_V(2) \text{ to mean } U_B(1) \text{ and } Z_B(2) \text{ respectively.} \]
phase where colored diquarks play an essential role. From the point of view of the NG bosons this can be seen as dissociation of the hadrons into constituent diquarks. As far as \( H \) is concerned, we can say in the following way; the hadronic phase has a Bose-Einstein condensate (BEC) of the color-singlet H-dibaryon, while the dissociated colored diquarks lead to a superconducting state at higher baryon density, and yet they compensate for their color charge to be a color-singlet in the CFL phase. In this sense, the attractive force between diquarks controls the state of matter. If the interaction is strong enough, the state is BEC-like, and otherwise, it is BCS-like. This BEC-BCS crossover looks quite different
from that demonstrated in the $^3$He superfluid [40], discussed in the system at finite isospin chemical potential [41], and investigated in the color-superconductivity [42]. In the present case, the BEC-BCS crossover is seen, not in the Cooper pair itself, but in the combination of two (for the pion) or three (for the phason) Cooper pairs. This is actually a crossover from the BEC of H to the BEC of diquarks.

So far in our discussion we have implicitly assumed the existence of compact (tightly bound) diquarks. In the weak coupling regime diquarks become spatially spreading, which would cause another (and more familiar) crossover from the BEC of diquarks to the BCS. This type of the BEC-BCS crossover is well-known and have been intensely studied in condensed matter physics. Thus, as the baryon density increases, the system should undergo the two-step crossover; BEC of colorless H $\rightarrow$ color-superconductivity, and then BEC of colored diquarks $\rightarrow$ BCS. This scenario is sketched in Fig. 1.

IV. CONCLUSIONS AND AN OUTLOOK

In this paper, we discussed the quark description of the NG bosons in the CFL phase. We emphasized the four-quark nature of the pions stemming from the residual $Z(2)$ symmetry and the color-singletness. We wrote down the plain expressions for the interpolating fields of the NG bosons motivated by the diquark picture. Then we discussed the H particle, that is the NG boson associated with the $U_B(1)$ symmetry breaking, in the hydrodynamic regime. We point out the velocity of the phason is given by the second sound speed which vanishes at the critical temperature. Finally we proposed a conjecture about the quark-hadron continuity. If the quark-hadron continuity is realized, the NG boson must change its nature continuously from one to the other phase. Based on the constituent diquark picture and the quark-hadron continuity hypothesis, we drew the two-step crossover scenario; BEC of H $\rightarrow$ BEC of diquarks, and then BEC of diquarks $\rightarrow$ BCS. This scenario can be confirmed by developing the H matter description discussed in [38] or the tightly-bound diquark picture as discussed in [23].

Although the diquark model leading to too light H dibaryons may not be realistic in the vacuum, we can expect that it is a proper description at high baryon density where the color-superconductivity occurs. A three-body problem in terms of diquarks at finite density would be the next step to go further into quantitative investigations. As shown in Fig. 1 in
the diquark interaction through the 't Hooft term is affected by the chiral condensate. This may cause an entanglement between the chiral condensate and the diquark, in other words, between the pions and the phason. It would be intriguing to study how far the phase transition with respect to the pions can bring about the structural change of the phason.

In this work the explicit breaking of the $U_A(1)$ symmetry has been regarded as an external perturbation smearing a sharp distinction based on chiral symmetry. It would be a challenging problem to study its effect not only on the pions but also on the superfluid structure. Since it is known that a strong three-body repulsion induced by the instanton effect makes the H-dibaryon weakly bound or unbound theoretically [43], the instanton-induced interaction will affect the content of the superfluid component.

Our speculation implies that diquarks would become important for the hadronic phase if it is close to CFL quark matter at high baryon density. This must be a robust consequence even in the presence of finite $m_\pi$ as long as there is no first order phase transition. The nature of diquarks in the hadronic phase deserves further investigation not only in the vacuum [44] but rather at high baryon density. This is partially because the importance of diquarks would be seen once the chiral condensate is vanishingly small. A possible suggestion is that, near the chiral phase transition at high temperature where the chiral condensate melts, the diquark correlation can be relevant to thermodynamic quantities, as has been already pointed out [45].

In order to make our argument applicable, for instance, to neutron star physics, it is necessary to take account of finite $m_\pi$ and the neutrality conditions. The CFL phase is robust as long as $m_\pi^2/\mu < 2\Delta_{\text{CFL}}$ [46]. As discussed in [17], if $m_\pi$ is large enough to suppress the hyperon number density, the transition from ordinary nuclear matter to the CFL phase is discontinuous. Detailed calculations, however, suggest the presence of H matter in the neutron star [38], though further analyses are needed to reveal the actual properties of H matter in the cores of compact stellar objects. Since the diquark picture tends to give rise to tightly-bound H-dibaryon, we may well think that the calculation with diquarks taken into account would result in the existence of H matter and our discussion here is not altered qualitatively. If our scenario is realized, then it would be quite interesting to see how the nature of H (or $\chi_H$) affects the internal structure of the vortices in a superfluid along the lines of [47].

We believe that our pictorial understanding could shed light on the non-perturbative
region between hadronic and quark matter and cast novel and challenging problems.

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