Analysis of the $X(3960)$ and related tetraquark molecular states via the QCD sum rules

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Abstract

In this work, we study the $D\bar{D}, D\bar{D}, D\bar{D}, D\bar{D}, D\bar{D}$ and $D\bar{D}$ tetraquark molecular states with the $J^{PC} = 0^{++}$ via the QCD sum rules. The prediction $M_{X(3960)} = 3956 \pm 5 \pm 10$ MeV from the LHCb collaboration and supports assigning the $X(3960)$ as the $D^+_s D^-_s$ molecular state with the $J^{PC} = 0^{++}$. We take account of our previous works on the four-quark states consisting of two color-neutral clusters and acquire the mass spectrum of the ground state hidden-charm and doubly-charm tetraquark molecular states.

1 Introduction

In 2020, the LHCb collaboration performed an amplitude analysis of the $B^+ \to D^+ D^- K^+$ decays using the LHCb data taken at the energies $\sqrt{s} = 7, 8$ and 13 TeV corresponding to a total integrated luminosity of 9 fb$^{-1}$, and they observed that it is necessary to include the $X_{c0}(3930)$ and $X_{c2}(3930)$ with the $J^{PC} = 0^{++}$ and 2$^{++}$ respectively in the $D^+ D^-$ channel and to include the $X_0(2900)$ and $X_1(2900)$ with the $J^P = 0^+$ and 1$^-$ respectively in the $D^- K^+$ channel [1, 2]. The measured Breit-Wigner masses and widths are

$$M_{X_{c0}(3930)} = 3923.8 \pm 1.5 \pm 0.4 \text{MeV}; \quad \Gamma = 17.4 \pm 5.1 \pm 0.8 \text{MeV},$$

$$M_{X_{c2}(3930)} = 3926.8 \pm 2.4 \pm 0.8 \text{MeV}; \quad \Gamma = 34.2 \pm 6.6 \pm 1.1 \text{MeV}.$$  

Recently, the LHCb collaboration announced the observation of the $X(3960)$ in the $D^+_s D^-_s$ invariant mass spectrum with the significance of 12.6$\sigma$ in the $B^+ \to D^+_s D^-_s K^+$ decays, and the assignment $J^{PC} = 0^{++}$ is favored, while the assignments $J^{PC} = 1^{--}$ and 2$^{++}$ are rejected by at least 9$\sigma$ [3]. The measured Breit-Wigner mass and width are $M = 3956 \pm 5 \pm 10$ MeV and $\Gamma = 43 \pm 13 \pm 8$ MeV, respectively.

The thresholds of the $D^+_s D^-_s$ and $D^+ D^-$ are 3938 MeV and 3739 MeV, respectively. This leads to the possible assignments of the $X(3960)$ and $X_{c0}(3930)$ as the same particle. The ratio of the branching fractions [3],

$$\frac{\Gamma(X \to D^+ D^-)}{\Gamma(X \to D^+_s D^-_s)} = 0.29 \pm 0.09 \pm 0.10 \pm 0.08,$$

implies the exotic nature of this state, as it is harder to excite an $s\bar{s}$ pair from the vacuum compared with the $u\bar{u}$ or $d\bar{d}$ pair, and the conventional charmonium states predominantly decay into the $D D$ and $D^* D^*$ states rather than into the $D^+_s D_s$ and $D^+_s D^+_s$ states. For a short and concise review of the experimental status of the $X, Y$, and $Z$ states, one can consult Ref. [4].

The $DD$ and $D_s D_s$ molecular states have been explored via several theoretical approaches before and after the discovery of the $X(3960)$, such as the lattice QCD, the...
heavy quark symmetry plus light flavor $SU(3)$ symmetry, and the $SU(4)$ symmetry with breaking effects (with the couple-channel effects) [5–12]. In those theories, the $D$ and $D_s$ are physical mesons, while in the QCD sum rules, we choose the color-singlet-color-singlet type four-quark currents, there are two color-neutral clusters, which couple potentially to the tetraquark states with two color-neutral clusters, the color-neutral clusters have the same quantum numbers as the physical mesons except for the masses, and are compact objects, just like the diquark-antidiquark type (color antitriplet-triplet type) tetraquark states. The local currents require that they have the average spatial sizes $\sqrt{\langle r^2 \rangle}$ of the same magnitudes as the conventional mesons.

In fact, we usually choose the color antitriplet-triplet type, singlet-singlet type and octet-octet type four-quark currents to interpolate the tetraquark states, as they couple potentially to the tetraquark states with the same quantum numbers, such as the valence quarks, $J^{PC}$, etc. The currents with the same quantum numbers could mix with each other under re-normalization, we have to introduce the mixing matrixes to diagonalize those currents. If we choose the diagonal currents, they do not mix with each other and are expected to couple potentially to the physical tetraquark states, which maybe have several Fock components, such as the color antitriplet-triplet type, singlet-singlet type, and octet-octet type components; furthermore, the physical masses are invariant under re-normalization. However, we cannot acquire the mixing matrixes without calculating the anomalous dimension matrixes. It is better to choose the current having the same substructure as the main Fock component. But which component is the main Fock component? At the present time, we usually choose the criterion that if we acquire good QCD sum rules to reproduce the mass (and width) of the tetraquark state, then we reach the conclusion tentatively that its main Fock component has the same structure as the interpolating current. In the QCD sum rules, we refer the color singlet-singlet type tetraquark states as the molecular states.

In our earlier works, we have studied the mass spectroscoopy of the hidden-charm and doubly-charm tetraquark (molecular) states with the QCD sum rules in a comprehensive way, and make possible assignments of the $X$, $Y$, $Z$, and $T$ states, such as the $X(3860)$, $X(3872)$, $T_{cc}(3875)$, $X(3915)$, $X(3940)$, $X(4160)$, $Z_{cc}(3900)$, $Z_{cs}(3985/4000)$, $Z_{c}(4020)$, $Z_{c}(4050)$, $Z_{c}(4055)$, $Z_{c}(4100)$, $Z_{c}(4200)$, $Z_{c}(4250)$, $Y(4260)$, $Y(4360)$, $Z_{c}(4430)$, $Z_{c}(4600)$, and $Y(4660)$ [13–18]. For more literatures on the QCD sum rules for the exotic states, one can consult the review [19]. In the present work, we extend our previous works on the molecular states to study the $D\bar{D}$, $D\bar{D}$, $D_s\bar{D}$, $D\bar{D}$, $D\bar{D}$, and $D_s\bar{D}$ tetraquark molecular states with the QCD sum rules and make possible assignment of the $X(3960)$ in the picture of molecular states. Furthermore, we compare our predictions to the existing works, where the $DD$, $DD$, and $D_s\bar{D}$ molecular states have been partly studied [20–22]. If the molecular states (more precisely, color-singlet-color-singlet type tetraquark states) really exist, they can decay into their constituents, through the Okubo-Zweig-Iizuka super-allowed fall-apart mechanism, saving possible in the phase-space, regardless of weak bound states or higher resonances.

The following of the article: we obtain the QCD sum rules for the hidden-charm and doubly-charm molecular states in Section 2; we offer the numerical results and discussions in Section 3; conclusion is saved for Section 4.

2 QCD sum rules for the hidden-charm and doubly-charm molecular states

Firstly, let us write down the two-point correlation functions $\Pi(p^2)$,

$$\Pi(p^2) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle,$$

where the currents $J(x) = J_{DD}(x)$, $J_{D\bar{D}}(x)$, $J_{D_s\bar{D}}(x)$, $J_{DD}(x)$, $J_{D_s\bar{D}}(x)$, $J_{D_s\bar{D}}(x)$,

$$J_{DD}(x) = \bar{q}(x)i\gamma_5 c(x) \bar{c}(x)i\gamma_5 q(x),$$

$$J_{D\bar{D}}(x) = \bar{q}(x)i\gamma_5 c(x) \bar{c}(x)i\gamma_5 s(x),$$

$$J_{D_s\bar{D}}(x) = \bar{s}(x)i\gamma_5 c(x) \bar{c}(x)i\gamma_5 s(x),$$

$$J_{DD}(x) = \bar{q}(x)i\gamma_5 c(x) \bar{c}(x)i\gamma_5 c(x),$$

$$J_{D\bar{D}}(x) = \bar{q}(x)i\gamma_5 c(x) \bar{c}(x)i\gamma_5 s(x),$$

$$J_{D_s\bar{D}}(x) = \bar{s}(x)i\gamma_5 c(x) \bar{c}(x)i\gamma_5 c(x),$$

with $q = u, d$. There are two color-singlet clusters in the currents $J(x)$, each has the same quantum numbers as the corresponding charmed mesons with the quantum numbers $J^{PC} = 0^{++}$. The two color-neutral clusters are in relative $S$-wave, thus the currents $J(x)$ have the quantum numbers $J^{PC} = 0^{++}$. We choose the local four-quark currents, the couplings to the two-meson scattering states are neglectful [23, 24], we isolate the ground state tetraquark molecule contributions to get the hadronic representation,

$$\Pi(p^2) = \frac{J^{2}_{X/T}}{M_{X/T}^2 - p^2} + \cdots,$$
where the pole residues $\hat{\lambda}_{X/T}$ are defined by

$$
(0|\langle 0|X/T(p)\rangle = \hat{\lambda}_{X/T}.
$$

(7)

There are two heavy quark propagators and two light quark propagators after contracting the quark fields in the correlation functions $\Pi(p^2)$ using the Wick's theorem. If each heavy quark line emits a gluon and each light quark line contributes a quark-antiquark pair, we acquire the quark-gluon operators $GG\bar{q}q\bar{q}$, which have dimension 10. Therefore, we should calculate the vacuum condensates at least up to dimension 10 to assess the convergence of the operator product expansion. We assume vacuum saturation and take account of the vacuum condensates $\langle \bar{q}q \rangle$, $\langle \bar{q}Gq \bar{q}G \rangle$, $\langle \bar{q}q \rangle^2$, $\langle \bar{q}G \rangle$, $\langle Gq \rangle$, $\langle Gq \bar{q} \rangle^2$, and $\langle Gq \rangle^2$, where $q=u$, $d$ or $s$. For the SU(3) mass-breaking effects, we set $m_u = m_d = 0$ and consider the terms proportional to $m_s$.

After accomplishing the operator product expansion, we obtain the QCD representation and then reach the QCD spectral densities through dispersion relation, finally we match the hadrons with the QCD sides, and apply Borel transform on the variable $P^2 = -p^2$ to get the QCD sum rules,

$$
\hat{\lambda}_{X/T} \exp\left(-\frac{M^2_{X/T}}{t^2}\right) = \int_0^\infty f d\rho_{\text{QCD}}(s) \exp\left(-\frac{s}{t^2}\right).
$$

Detailed expressions of the spectral densities $\rho_{\text{QCD}}(s)$ can be obtained by contacting the corresponding author via e-mail.

Then, we obtain the masses of the scalar molecular states through a fraction,

$$
M^2_{X/T} = -\frac{\int_{4m_s^2}^0 ds \frac{d}{ds} \rho_{\text{QCD}}(s) \exp\left(-\frac{s}{t^2}\right)}{\int_{4m_s^2}^\infty ds \rho_{\text{QCD}}(s) \exp\left(-\frac{s}{t^2}\right)} \mid_{t = \frac{1}{\tau^2}}.
$$

(9)

### 3 Numerical results and discussions

For sake of numerical calculation, we choose the standard values of vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$, $\langle \bar{g}\sigma Gq \rangle = m_G^2 \langle \bar{q}q \rangle$, $\langle \bar{g}\sigma G \rangle = m_G^2 \langle \bar{s}s \rangle$, $m_G^2 = (0.8 \pm 0.1) \text{ GeV}^2$, and $\langle \bar{G}G \rangle = 0.33 \text{ GeV}^4$ at the energy scale $\mu = 1 \text{ GeV}$ [25–28].

And take the MS masses $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ and $m_s(m_s) = (0.959 \pm 0.005) \text{ GeV}$ from the Particle Data Group [29]. Additionally, we consider the energy-scale dependence of the quark condensates, mixed quark condensates, and MS masses [30].

$$
\langle \bar{q}q \rangle = \langle \bar{q}q \rangle(1\text{GeV}) \left[ \frac{a(1\text{GeV})}{a(\mu)} \right]^{-\frac{s}{m^2}}.
$$

$$
\langle \bar{g}\sigma Gq \rangle = \langle \bar{g}\sigma Gq \rangle(1\text{GeV}) \left[ \frac{a(1\text{GeV})}{a(\mu)} \right]^{-\frac{s}{m^2}}.
$$

$$
\langle \bar{g}\sigma G \rangle = \langle \bar{g}\sigma G \rangle(1\text{GeV}) \left[ \frac{a(1\text{GeV})}{a(\mu)} \right]^{-\frac{s}{m^2}}.
$$

$$
\langle \bar{G}G \rangle = \langle \bar{G}G \rangle(1\text{GeV}) \left[ \frac{a(1\text{GeV})}{a(\mu)} \right]^{-\frac{s}{m^2}}.
$$

$$
\begin{aligned}
\rho_{\text{QCD}}(s) &= \frac{1}{b_0} \left[ 1 - b_1 \log t + b_2 \log^2 t - \log^3 t + b_3 b_4 \log^3 t \right], \\
\tilde{\rho}_{\text{QCD}}(s) &= -\frac{1}{b_0} \left[ 1 - b_1 \log t - b_2 \log^2 t - b_3 b_4 \log^3 t \right],
\end{aligned}
$$

where $t = \log \frac{s}{\mu^2}$, $b_0 = \frac{2}{3} \log \frac{\Lambda^2}{\mu^2}$, $b_1 = \frac{1}{10} \log \frac{\Lambda^2}{\mu^2}$, $b_2 = \frac{2}{3} \log \frac{\Lambda^2}{\mu^2}$, $\Lambda = 213 \text{ MeV}$, $296 \text{ MeV}$, and $339 \text{ MeV}$ for the quark flavors $\nu_f = 4, 5$ and $3$, respectively, as the heavy quark masses play a crucial role in the QCD sum rules for the full heavy baryons, hidden(doubly)-charm(bottom) multiquark states, etc. [17, 18, 31]. It is preferable to select the quark flavor number $\nu_f = 4$ and evolve the QCD spectral densities $\rho_{\text{QCD}}(s)$ to the desired energy scales to extract the molecule masses. To avoid possible failure of the energy-scale formula due to the small $DD$ ($DD$) mass and reach the most uniform parameters, we tentatively assign the $Z_3(3900)$ ($T_{2s}(3875)$) as the hidden-charm (doubly-charm) molecule state $D^0 + D^* \bar{D} (D^* + D^* \bar{D})$ with the quantum numbers $J^{PC} = 1^{++} (J^P = 1^+)$ and choose the same energy scale $\mu = 1.3 \text{ GeV}$ (1.4 GeV) as in our previous work [17] ([18]).

For conventional heavy mesons and quarkonia, the energy gaps between the ground states and the first radial excited states are about $0.5 \sim 0.7 \text{ GeV}$ [29]. To avoid contaminations from the higher resonances and continuum states, we choose the continuum threshold parameters in this work as $\sqrt{\rho_0} = M_{X/T} + 0.4 \sim 0.7 \text{ GeV}$ and gradually adjust the continuum threshold parameters and Borel parameters via trial and error.

We obtain the Borel windows, continuum threshold parameters and pole contributions, which are shown clearly in Table 1. The table clearly indicates that the pole contributions are approximately $(40 - 60)$%. Meanwhile, the central valued are larger than $50\%$, and the expression for the pole contributions is

$$
\begin{aligned}
pole &= \frac{\int_{4m_s^2}^\infty ds \rho_{\text{QCD}}(s) \exp\left(-\frac{s}{t^2}\right)}{\int_{4m_s^2}^\infty ds \rho_{\text{QCD}}(s) \exp\left(-\frac{s}{t^2}\right)},
\end{aligned}
$$

(11)
Table 1 The Borel parameters, continuum threshold parameters, pole contributions, masses, and pole residues of the molecular states

| $X/T$ | $f^P/J^P$ | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole | $M_{X/T}$(GeV) | $\lambda_{X/T}(10^{-2}$GeV$^3$) |
|-------|------------|----------------|-----------------|------|----------------|-----------------------------|
| $D\bar{D}$ | $0^{++}$ | $2.7 - 3.1$ | $4.30 \pm 0.10$ | $(40 - 63)$% | $3.74 \pm 0.09$ | $1.61 \pm 0.23$ |
| $D\bar{D}_s$ | $0^{++}$ | $2.8 - 3.2$ | $4.40 \pm 0.10$ | $(40 - 63)$% | $3.88 \pm 0.10$ | $1.98 \pm 0.30$ |
| $D_s\bar{D}_s$ | $0^{++}$ | $2.9 - 3.3$ | $4.50 \pm 0.10$ | $(41 - 62)$% | $3.98 \pm 0.10$ | $2.36 \pm 0.45$ |
| $DD$ | $0^+$ | $\bar{s}b - s\bar{u}$ | $4.30 \pm 0.10$ | $(40 - 64)$% | $3.75 \pm 0.09$ | $1.48 \pm 0.23$ |
| $DD_s$ | $0^+$ | $2.7 - 3.1$ | $4.40 \pm 0.10$ | $(41 - 64)$% | $3.85 \pm 0.09$ | $1.69 \pm 0.26$ |
| $D_sD_s$ | $0^+$ | $2.8 - 3.2$ | $4.50 \pm 0.10$ | $(41 - 63)$% | $3.95 \pm 0.09$ | $2.00 \pm 0.32$ |

The definition of the relative contributions $D(n)$ is

$$D(n) = \frac{\int_{\sqrt{s_0}}^{s_{0n}} ds \rho_{QCD,n}(s) \exp \left(-\frac{s}{T^2}\right)}{\int_{\sqrt{s_0}}^{s_{max}} ds \rho_{QCD}(s) \exp \left(-\frac{s}{T^2}\right)},$$

where the $\rho_{QCD,n}(s)$ are the spectral densities involving the vacuum condensates of dimension $n$. The contributions $|D(10)| \ll 1\%$, and satisfy the criterion of convergence of the operator product expansion. In Fig.1, we plot the absolute values of the contributions of the vacuum condensates in the case of the central values of the input parameters. The contributions of the $|D(3)|$, both with $s$-quark and without $s$-quark, dominate the operator product expansion, the remaining contributions $|D(4)|$, $|D(5)|$, $|D(7)|$, $|D(8)|$ are all $\leq 4\%$, and $|D(10)| \ll 1\%$.

In Table 1, we display the predicted masses and pole residues of the scalar tetraquark molecular states without strange, with strange and with hidden-strange. The prediction $M_{D_sD_s} = 3.98 \pm 0.10$ GeV is in very good agreement with the experimental value $M = 3956 \pm 5 \pm 10$ from the LHCb collaboration and supports assigning the $X(3960)$ to be the $D_s^0\bar{D}_s^-$ molecular state; the observations of the $D\bar{D}, DD, D\bar{D}_s, DD_s$, and $D_sD_s$ molecular states would shed light on the nature of the $X(3960)$. The predicted mass $M_{D\bar{D}} = 3.74 \pm 0.09$ GeV lies nearby the $\psi(3770)$, which decays dominantly to the $DD$ pair and serves as a $D$ factory; however, the $\psi(3770)$ has the quantum numbers $J^{PC} = 1^{--}$ rather than $0^{++}$; there is no overlap between the $\psi(3770)$ and $X(3740)$. On the other hand, the $X_{c0}(1P)$ has the mass $3414.71 \pm 0.30$ MeV and also has no overlap with the $X(3740)$; the observation of the $X(3740)$ plays an important role in establishing the lowest tetraquark states. The predicted mass $M_{D_s\bar{D}_s} = 3.88 \pm 0.10$ GeV is much larger than the value $3.74 \pm 0.13$ GeV from Ref.[20] due to the different schemes in treating the QCD sum rules, such as choosing the heavy quark masses and pole contributions.

In Ref. [21], Zhang and Huang carry out the operator product expansion only up to the vacuum condensates of dimension 6 and neglect the contributions of the higher dimensional condensates. They determine

![Fig. 1](image-url)
the Borel windows in their scheme of the input parameters, such as the heavy quark masses and vacuum condensates, and acquire the predicted mass $M_{D_0 D_s} = 3.91 \pm 0.10 \text{ GeV}$. As the higher dimensional vacuum condensates play a great important role in obtaining the flat Borel platforms, taking into account those contributions would lead to better QCD sum rules. In Ref.[22], Albuquerque et al. take account of the vacuum condensates up to dimension 8 and partly radiative $\mathcal{O}(\alpha_s)$ corrections for the perturbative terms, and then choose the energy scale of the QCD spectral density $\mu = 4.5 \text{ GeV}$ and the continuum threshold parameter $\sqrt{s_0} = 6.7 \text{ GeV} + 2m_c(\mu)$ to extract the mass $M_{D_0 D_s} = 4.169 \pm 0.048 \text{ GeV}$. Such large energy scale $\mu$ and continuum threshold parameter $s_0$ should weaken the predictive ability.

In Tables 2 and 3, we also present the predictions of the molecular states involving the color-neutral clusters having the same quantum numbers as the $D^*$ and $D_s^*$ mesons for completeness [17, 18]; all those predictions should be compared to the experimental data in the future to examine the nature of the exotic states. In Tables 1, 2, and 3, we take account of all uncertainties of the input parameters and acquire the uncertainties $\pm 0.10 \text{ GeV}$ and $\pm 0.09 \text{ GeV}$ by rounding a number up or down based on the existing decimals. In fact, the difference between the uncertainties $|\delta|$ is less than 0.01 GeV.

In Fig.2, we plot the masses of the $DD$, $DD_0$, $D_s D_s$, $DD_s$, $DD_0$, and $D_s D_s$ molecular states according to the variations of the Borel parameters. Regardless of whether it contains $s$ quark or not, we have standardized the widths of the Borel windows to 0.4 GeV$^2$ by intercepting the pole contributions $(40 - 60)\%$. Flat platforms emerge in the Borel windows, and the uncertainties stemming from the Borel parameters can be safely ignored; the predictions are reliable for those molecule masses.

### Table 2

The possible assignments of the ground state hidden-charm molecular states, the isospin limit is implied. The molecular states with the constituents $D^*$ and $D_s^*$ are taken from Ref.[17]

| $T_{cc}$ | $J^P$ | $M_f (\text{GeV})$ | Assignments |
|---------|-------|---------------------|--------------|
| $DD$    | $0^+$ | $3.75 \pm 0.09$     |              |
| $DD_0$  | $0^+$ | $3.85 \pm 0.09$     |              |
| $D_s D_s$ | $0^+$ | $3.95 \pm 0.09$     |              |
| $D_s D_s$ | $0^+$ | $4.04 \pm 0.11$     |              |
| $D_s^* D^*$ | $0^+$ | $4.12 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $0^+$ | $4.22 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $1^+$ | $3.88 \pm 0.11$     | ?? $T_{cc}(3875)$ |
| $D_s^* D_s^*$ | $1^+$ | $3.97 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $1^+$ | $3.99 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $1^+$ | $4.00 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $1^+$ | $4.08 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $1^+$ | $4.19 \pm 0.09$     |              |
| $D_s^* D_s^*$ | $2^+$ | $4.02 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $2^+$ | $4.10 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $2^+$ | $4.20 \pm 0.10$     |              |

### Table 3

The possible assignments of the ground state doubly-charm molecular states, the isospin limit is implied. The molecular states with the constituents $D^*$ and $D_s^*$ are taken from Ref.[18]

| $T_{cc}$ | $J^P$ | $M_f (\text{GeV})$ | Assignments |
|---------|-------|---------------------|--------------|
| $DD$    | $0^+$ | $3.75 \pm 0.09$     |              |
| $DD_0$  | $0^+$ | $3.85 \pm 0.09$     |              |
| $D_s D_s$ | $0^+$ | $3.95 \pm 0.09$     |              |
| $D_s D_s$ | $0^+$ | $4.04 \pm 0.11$     |              |
| $D_s^* D^*$ | $0^+$ | $4.12 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $0^+$ | $4.22 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $1^+$ | $3.88 \pm 0.11$     | ?? $T_{cc}(3875)$ |
| $D_s^* D_s^*$ | $1^+$ | $3.97 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $1^+$ | $3.99 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $1^+$ | $4.00 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $1^+$ | $4.08 \pm 0.10$     |              |
| $D_s^* D_s^*$ | $1^+$ | $4.19 \pm 0.09$     |              |
| $D_s^* D_s^*$ | $2^+$ | $4.02 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $2^+$ | $4.10 \pm 0.11$     |              |
| $D_s^* D_s^*$ | $2^+$ | $4.20 \pm 0.10$     |              |

### 4 Conclusion

In this work, we construct the color-singlet-color-singlet type local four-quark currents to study the $DD$, $DD_0$, $D_s D_s$, $DD_s$, $DD_0$, and $D_s D_s$ tetraquark molecular states with the QCD sum rules. We carry out the operator product expansion up to the vacuum condensates of dimension 10 and acquire the molecule masses after detailed analysis. The prediction $M_{D_0 D_s} = 3.98 \pm 0.10 \text{ GeV}$ is in very good agreement with the experimental value $M = 3956 \pm 5 \pm 10$ from the LHCb collaboration and supports assigning the $X(3960)$ to be the $D_s^+ D_s^-$ molecular state; the predictions for the $DD$, $DD_s$, $DD_0$, and $D_s D_s$ molecular states can be confronted to the
Fig. 2 The masses with variations of the Borel parameters, where the $A$, $B$, $C$, $D$, $E$, and $F$ denote the $D\bar{D}, D\bar{D}_s, D\bar{D}_s, D\bar{D}, D\bar{D}_s$, and $D\bar{D}_s$ molecular states, respectively.
experimental data in the future to examine the assignment of the $X(3960)$. Furthermore, we take account of our previous works on the molecular states involving the color-neutral clusters having the same quantum numbers as the $D^*$ and $D_s^*$ mesons and acquire the mass spectrum of the ground state hidden-charm and doubly-charm tetraquark molecular states.

Acknowledgements
This work is supported by National Natural Science Foundation, Grant Number 12175068.

Authors’ contributions
All authors contributed to the study conception and design. All authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Funding
This research was supported by the National Natural Science Foundation of China through Grant No.12175068.

Availability of data and materials
The data are available via contacting the corresponding author upon request.

Declarations

Competing interests
The author declare that they have no competing interests.

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Received: 3 October 2022   Accepted: 18 November 2022
Published online: 05 December 2022

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