How heavy can the Fermions in Split Susy be?  
A study on Gravitino and Extradimensional LSP.

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Abstract

In recently introduced Split Susy theories, in which the scale of Susy breaking is very high, the requirement that the relic abundance of the Lightest SuperPartner (LSP) provides the Dark Matter of the Universe leads to the prediction of fermionic superpartners around the weak scale. This is no longer obviously the case if the LSP is a hidden sector field, such as a Gravitino or an other hidden sector fermion, so, it is interesting to study this scenario. We consider the case in which the Next-Lightest SuperPartner (NLSP) freezes out with its thermal relic abundance, and then it decays to the LSP. We use the constraints from BBN and CMB, together with the requirement of attaining Gauge Coupling Unification and that the LSP abundance provides the Dark Matter of the Universe, to infer the allowed superpartner spectrum. As very good news for a possible detection of Split Susy at LHC, we find that if the Gravitino is the LSP, than the only allowed NLSP has to be very purely photino like. In this case, a photino from 700 GeV to 5 TeV is allowed, which is difficult to test at LHC. We also study the case where the LSP is given by a light fermion in the hidden sector which is naturally present in Susy breaking in Extra Dimensions. We find that, in this case, a generic NLSP is allowed to be in the range 1-20 TeV, while a Bino NLSP can be as light as tens of GeV.

1 Introduction

Two are the main reasons which lead to the introduction of Low Energy Supersymmetry for the physics beyond the Standard Model: a solution of the hierarchy problem, and gauge coupling unification.

The problem of the cosmological constant is usually neglected in the general treatment of beyond the Standard Model physics, justifying this with the assumption that its solution must come from a quantum theory of gravity. However, recently [1], in the light of the landscape picture developed by

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a new understanding of string theory, it has been noted that, if the cosmological constant problem is solved just by a choice of a particular vacua with the right amount of cosmological constant, the statistical weight of such a fine tuning may dominate the fine tuning necessary to keep the Higgs light. Therefore, it is in this sense reasonable to expect that the vacuum which solves the cosmological constant problem solves also the hierarchy problem.

As a consequence of this, the necessity of having Susy at low energy disappears, and Susy can be broken at much higher scales ($10^6 – 10^9$ GeV).

However, there is another important prediction of Low Energy Susy which we do not want to give up, and this is gauge coupling unification. Nevertheless, gauge coupling unification with the same precision as with the usual Minimal Supersymmetric Standard Model (MSSM) can be achieved also in the case in which Susy is broken at high scales. An example of this is the theories called Split Susy [1, 2] where there is an hierarchy between the scalar supersymmetric partners of Standard Model (SM) particles (squarks, sleptons, and so on) and the fermionic superpartners of SM particles (Gaugino, Higgsino), according to which, the scalars can be very heavy at an intermediate scale of the order of $10^9$ GeV, while the fermions can be around the weak scale. The existence for this hierarchy can be justified by requiring that the chiral symmetry protects the mass of the fermions partners.

While the chiral symmetry justifies the existence of light fermions, it can not fix the mass of the fermionic partners precisely at the weak scale. As a consequence, this theory tends to make improbable the possibility of findingSusy at LHC, because in principle there could be no particles at precisely 1 TeV. In this paper, for Split Susy, we do a study at one-loop level of the range of masses allowed by gauge coupling unification, finding that these can vary in a range that approximately goes up to 20 TeV. A possible way out from this depressing scenario comes from realizing that cosmological observations indicate the existence of Dark Matter (DM) in the universe. The standard paradigm is that the Dark Matter should be constituted by stable weakly interacting particles which are thermal relics from the initial times of the universe. The Lightest Supersymmetric Partner (LSP) in the case of conserved R-parity is stable, and, if it is weakly interacting, such as the Neutralino, it provides a perfect candidate for the DM. In particular, an actual calculation shows that in order for the LSP to provide all the DM of the universe, its mass should be very close to the TeV scale. This is the very good news for LHC we were looking for. Just to stress this result, it is the requirement the the DM is given by weakly interacting LSP that forces the fermions in Split Susy to be close to the weak scale, and accessible at LHC.

In three recent papers [2, 3, 4], the predictions for DM in Split Susy were investigated, and revealed some regions in which the Neutralino can be as light as $\sim 200$ GeV (Bino-Higgsino), and some others instead where it is around a 1 TeV (Pure Higgsino) or even 2 TeV (Pure Wino). As we had anticipated, all these scales are very close to one TeV, even though only the Bino-Higgsino region is very good for detection at LHC.

Since the Dark Matter Observation is really the constraint that tells us if this kind of theories will be observable or not at LHC, it is worth to explore all the possibilities for DM in Split Susy. In particular, a possible and well motivated case which had been not considered in the literature, is the case in which the LSP is a very weakly interacting fermion in a hidden sector.

In this paper, we will explore this possibility in the case in which the LSP is either the Gravitino, or a light weakly interacting fermion in the hidden sector which naturally appears in Extra
Dimensional Susy breaking models of Split Susy [1 5].

We will find that, if the Gravitino is the LSP, than all possible candidates for the NLSP are excluded by the combination of imposing gauge coupling unification and the constraint on hadronic decays coming from BBN. Just the requirement of having the Gravitino to provide all the Dark Matter of the universe and to still have gauge coupling unification would have allowed weakly interacting fermionic superpartners as heavy as 5 TeV, with very bad consequences on the detectability of Split Susy at LHC. This means that these constraints play a very big role. The only exception to this result occurs if the NLSP is very photino like, avoiding in this way the stringent constraints on hadronic decays coming from BBN. However, as we will see, already a small barionic decay branching ratio of $10^{-3}$ is enough to rule out also this possibility.

For the Extradimensional LSP, we will instead find a wide range of possibilities, with NLSP allowed to span from 30 GeV to 20 TeV.

The paper is organized as follows. In section 2, we study the constraints on the spectrum coming from the requirement of obtaining gauge coupling unification. In section 3, we briefly review the relic abundance of Dark Matter in the case the LSP is an hidden sector particle. In section 4, we discuss the cosmological constraints coming from BBN and CMB. In section 5, we show the results for Gravitino LSP. In section 6, we do the same for a dark sector LSP arising in extra dimensional implementation of Split Susy. In section 7, we draw our conclusions.

## 2 Gauge Coupling Unification

Gauge coupling unification is a necessary requirement in Split Susy theories. Here we investigate at one loop level how heavy can be the fermionic supersymmetric partner for which gauge coupling unification is allowed. We will consider the Bino, Wino, and Higgsino as degenerate at a scale $M_2$, while we will put the Gluinos at a different scale $M_3$.

Before actually beginning the computation, it is interesting to make an observation about the lower bound on the mass of the fermionic superpartners. Since the Bino is gauge singlet, it has no effect on one-loop gauge coupling unification. In Split Susy, with the scalar superpartners very heavy, the Bino is very weakly interacting, its only relevant vertex being the one with the light Higgs and the Higgsino. This means that, while for the other supersymmetric partners LEP gives a lower bound of $\sim 50$-100 GeV [11], for the Bino in Split Susy there is basically no lower limit.

Going back to the computation of gauge coupling unification, we perform the study at 1-loop level. The renormalization group equations for the gauge couplings are given by:

$$\Lambda \frac{dg_i}{d\Lambda} = \frac{1}{(4\pi)^2} b_i(\Lambda) g_i^3$$

where $b_i(\Lambda)$ depends of the scale, keeping track of the different particle content of the theory according to the different scales, and $i = 1, 2, 3$ represent respectively $\sqrt{5/3}g', g, g_s$. We introduce two different scales for the Neutralinos, $M_2$, and for the Gluinos $M_3$, and for us $M_3 > M_2$.

In the effective theory below $M_2$, we have the SM, which implies:

$$b^{SM} = \left(\frac{41}{10}, -\frac{19}{7}, -7\right)$$
Between $M_2$ and $M_3$:
\[
 b^{\text{split1}} = \left( \frac{9}{2}, -\frac{7}{6}, -7 \right)
\]

Between $M_3$ and $\tilde{m}$, which is the scale of the scalars:
\[
 b^{\text{split2}} = \left( \frac{9}{2}, -\frac{7}{6}, -5 \right)
\]

and finally, above $\tilde{m}$ we have the SSM:
\[
 b^{\text{ssm}} = \left( \frac{33}{5}, 1, -3 \right)
\]

The way we proceed is as follows: we compute the unification scale $M_{\text{GUT}}$ and $\alpha_{\text{GUT}}$ as deduced by the unification of the SU(2) and U(1) couplings. Starting from this, we deduce the value of $\alpha_s$ at the weak scale $M_Z$, and we impose it to be within the $2\sigma$ experimental result $\alpha_s(M_Z) = 0.119 \pm 0.003$. We use the experimental data: $\sin^2(\theta_W(M_Z)) = 0.23150 \pm 0.00016$ and $\alpha^{-1}(M_Z) = 128.936 \pm 0.0049$.[12]

A further constraint comes from Proton decay $p \rightarrow \pi^0 e^+$, which has lifetime:
\[
 \tau(p \rightarrow \pi^0 e^+) = \frac{8 f^2 \pi M^4_{\text{GUT}}}{\pi m_p \alpha_{\text{GUT}}^2 ((1 + D + F) A \alpha_N)^2} = \left( \frac{M_{\text{GUT}}}{10^{16}\text{GeV}} \right)^4 \left( \frac{1}{35} \right)^2 \left( \frac{0.015 \text{GeV}^3}{\alpha_N} \right) 1.3 \times 10^{35}\text{yr}
\]

where we have taken the chiral Lagrangian factor $(1 + D + F)$ and the operator renormalization $A$ to be $(1 + D + F)A \simeq 20$. For the Hadronic matrix element $\alpha_N$, we take the lattice result[13] $\alpha_N = 0.015\text{GeV}^3$. From the Super-Kamiokande limit[14], $\tau(p \rightarrow \pi^0 e^+) > 5.3 \times 10^{33}\text{yr}$, we get:

\[
 M_{\text{GUT}} > \left( \frac{\alpha_N}{0.015\text{GeV}^3} \right)^{1/2} \left( \frac{\alpha_{\text{GUT}}}{1/35} \right)^{1/2} 4 \times 10^{15}\text{GeV}
\]

An important point regards the mass thresholds of the theory. In fact, the spectrum of the theory will depend strongly on the initial condition for the masses at the supersymmetric scale $\tilde{m}$. As we will see, in particular, the Gluino mass $M_3$ has a very important role for determining the allowed mass range for the Next-Lightest Supersymmetric Particle (NLSP), which is what we are trying to determine. In the light of this, we will consider $M_2$ as a free parameter, with the only constraint of being smaller than $\tilde{m}$. $M_3$ will be then a function of $M_2$ and $\tilde{m}$, and its actual value will depend on the kind of initial conditions we require. In order to cover the larger fraction of parameter space as possible, we will consider two distinct and well motivated initial conditions. First, we will require gaugino mass unification at $\tilde{m}$. This initial condition is the best motivated in the approach of Split Susy, where unification plays a fundamental role. Secondarily, we will require anomaly mediated gaugino mass initial conditions at the scale $\tilde{m}$. This second kind of initial conditions will give results quite different from those of Gaugino mass unification, and, even if in this case the Gravitino can not be the NLSP, the field $\psi_X$, which will be a candidate LSP from extradimensions that we will introduce in the next sections, could be still the LSP.
2.1 Gaugino Mass Unification

Here we study the case in which we apply gaugino mass unification at the scale $\tilde{m}$.

In [2], a 2-loop study of the renormalization group equations for the Gaugino mass starting from this initial condition was done, and it was found that, according to $\tilde{m}$ and $M_2$, the ratio between $M_3$ and $M_2$ can vary in a range $\sim 3 - 8$. We shall use their result for $M_3$, as the value of $M_3$ will have influence on the results, tending to increase the upper limit on the fermions’ mass.

At one loop level, we can obtain analytical results. After integration of eq.(11), we get the following expressions:

$$M_{GUT} = e^{8\pi^2 \frac{1}{g_2^2(M_Z)} - \frac{1}{g_1^2(M_Z)} M_Z^{b_{1s} - b_{2s}} M_2^{(b_{1p} - b_{1s}) - (b_{2p} - b_{2s})} M_3^{(b_{1p} - b_{1s}) - (b_{2p} - b_{2s})} \left(\frac{1}{\tilde{m}} \right)^{\frac{1}{g_1^2 - g_2^2}}$$

(8)

$$\frac{1}{g_2^2(M_Z)} = \frac{1}{g_2^2(M_Z)} - \frac{1}{8\pi^2} \ln \left( M_Z^{-b_{2s}} M_2^{b_{2p} + b_{2s}} \right)$$

(9)

$$\frac{1}{g_2^2(M_Z)} = \frac{1}{g_2^2(M_{GUT})} + \frac{1}{8\pi^2} \ln \left( M_Z^{-b_{3m}} M_2^{b_{3p} + b_{3m}} \right)$$

(10)

It turns out that two loops effect are important to determine the predicted value of $\alpha_s(M_Z)$. Since our main purpose is to have a rough idea of the maximum scale for the fermionic masses allowed by Gauge Coupling Unification, we proceed in the following way. In [2], 2-loop gauge coupling unification was studied for $M_2 = 300$ GeV and 1 TeV. Since the main effect of the 2-loop contribution is to raise the predicted value of $\alpha_s(M_Z)$, we translate our predicted value of $\alpha_s(M_Z)$ to match the result in [2] for the correspondent values of $M_2$. Having set in this way the predicted scale for $\alpha_s(M_Z)$, we check what is the upper limit on fermion masses in order to reach gauge coupling unification. The amount of translation we have to do is: 0.008.

In fig.1 we plot the prediction for $\alpha_s(M_Z)$ for $M_2 = 300$ GeV, 1 TeV, and 5 TeV. We see that for 5 TeV, unification becomes impossible. And so, 5 TeV is the upper limit on fermionic superpartner allowed from gauge coupling unification. Note that the role of the small difference between $M_3$ and $M_2$ is to raise this limit.

In fig.2 and fig.3 we plot the predictions for $\alpha_{GUT}(M_{GUT})$ and for $M_{GUT}$, for the same range of masses. We see that unification is reached in the perturbative regime, with unification scale large enough to avoid proton decay limits. Note, however, that for $M_2 = 5$ TeV, the limit is close to a possible detection.

Finally, note that with this Gaugino mass initial conditions, the Wino can not be the NLSP if the Gravitino is the LSP, as shown in [2].

5
Figure 1: In the case of gaugino mass unification at scale \( \tilde{m} \), we plot the unification prediction for \( \alpha_s(M_Z) \). The results for \( M_2 = 300 \text{ GeV}, 1 \text{ TeV} \) and \( 5 \text{ TeV} \) are shown. The horizontal lines represent the experimental bounds.

Figure 2: In the case of Gaugino mass unification at scale \( \tilde{m} \), we plot the prediction for \( \alpha_s(M_{\text{GUT}}) \). The results for \( M_2 = 300 \text{ GeV}, 1 \text{ TeV} \) and \( 5 \text{ TeV} \) are shown.

As we will see later, a particular interesting case for the LSP in the hidden sector is given by a Bino NLSP. For this case, we need to do a more accurate computation, splitting the mass of the Gauginos, from that of the Higgsinos, and taking the Wino mass roughly two times larger than the Bino mass, as inferred from [2] for gaugino mass unification initial conditions. In fig.4 we show what is the allowed region for the mass of the Bino and the ratio of the Hissino mass and Bino mass, such that gauge coupling unification is attained with a mass for the scalars, \( \tilde{m} \), in the range \( 10^5 \text{ GeV} - 10^{18} \text{ GeV} \). Raising the Higgsino mass with respect to the Bino mass has the effect of lowering
the maximum mass for the fermionic superpartners. This is due to the fact that, raising the Higgsino mass, the unification value for the U(1) and SU(2) couplings is reduced, so that the prediction for $\alpha_s(M_Z)$ is lowered.

Figure 3: In the case of Gaugino mass unification at scale $\tilde{m}$, we plot the prediction for $M_{\text{GUT}}$. The results for $M_2 = 300$ GeV, 1 TeV and 5 TeV are shown, together with the lower bound on $M_{\text{GUT}}$ from Proton decay.

Figure 4: Shaded is the allowed region for the Bino mass and the ratio of the Higgsino mass and the Bino mass, in order to obtain Gauge Coupling Unification with a value of the scalar mass $\tilde{m}$ in the range $10^5$ GeV-$10^{18}$ GeV. We take $M_2 \approx 2M_1$ as inferred from gaugino mass unification at the GUT scale \[2\].
2.2 Gaugino Mass Condition from Anomaly Mediation

Of the possible initial conditions for the Gaugino mass which can have some influence on the upper bound on fermions mass, there is one which is particularly natural, and which is coming from Anomaly Mediated Susy breaking, and according to which the initial conditions for the gaugino masses are:

\[ M_i = \frac{\beta_i g_i}{g_i} m_{3/2} \sim \frac{c_i g_i^2}{16\pi^2} m_{3/2} \]  \hspace{1cm} (11)

where \( \beta_i \) is the beta-function for the gauge coupling, and \( c_i \) is an order one number. These initial conditions are not relevant for the Gravitino LSP, as in this case the Neutralinos are lighter than the Gravitinos; but they can be relevant in the case the LSP is given by a fermion in the hidden sector, as we will study later. Further, the study of this case is interesting on its own, as it gives an upper bound on the fermionic superpartners which is higher with respect to the one coming from gaugino mass unification initial conditions.

The study parallels very much what done in the former section, with the only difference being the fact that in this case, as computed in [2], the mass hierarchy between the Gluinos and the Gauginos is higher (a factor \( \sim 10 - 20 \) instead of \( \sim 3 - 8 \)). This has the effect of raising the allowed mass for the fermions. We do the same amount of translation as before for the predicted \( \alpha_s(M_Z) \). The result is shown in fig.5 and gives, as upper limit, \( M_2 = 18 \) TeV.

![Graph showing \( \alpha_s(M_Z) \) vs. Log(\( \tilde{m}/\text{GeV} \))](image_url)

Figure 5: In the case of Gaugino mass condition from anomaly mediation at scale \( \tilde{m} \), we plot the unification prediction for \( \alpha_s(M_Z) \). The results for \( M_2 = 300 \) GeV, 1 TeV and 18 TeV are shown. The horizontal lines represent the experimental bounds.

In fig.6 and fig.7, we plot the predictions for \( \alpha_{\text{GUT}} \) and \( M_{\text{GUT}} \) for the same range of masses, and we see that unification is reached in the perturbative regime, and that the unification scale is large enough to avoid proton decay limits, but it is getting very close to the experimental bound for large values of the mass \( \tilde{m} \).
Figure 6: In the case of Gaugino mass from Anomaly Mediation, we plot the prediction for $\alpha_{GUT}$. The results for $M_2 = 300$ GeV, 1 TeV and 18 TeV are shown.

Figure 7: In the case of Gaugino mass from Anomaly Mediation, we plot the prediction for $M_{GUT}$. The results for $M_2 = 300$ GeV, 1 TeV and 18 TeV are shown, together with the lower bound on $M_{GUT}$ from Proton decay.

As we can see, in the case of Gaugino Mass from Anomaly Mediation, the upper limit on fermion mass is raised to 18 TeV. This last one can be interpreted as a sort of maximum allowed mass for fermionic superpartners.

It is important to note that, as pointed out in [2], in this case the Bino can not be the NLSP.
3 Hidden sector LSP and Dark Matter Abundance

An hidden sector LSP which is very weakly interacting can well be the DM from the astrophysical and cosmological point of view. Its present abundance can be given by two different sources: it can be a thermal relic, if in the past the temperature was so high that hidden sector particles were in equilibrium with the thermal bath, or it can be present in the universe just as the result of the decay of the other supersymmetric particles.

We concentrate in the case in which the thermal relic abundance is negligible, which is generically the case for not too large reheating temperatures, and the abundance is given by the decaying of the other supersymmetric particles into the LSP. A discussion on the consequences of a thermal relic abundance of Gravitinos is discussed in [6].

In our case, the relic abundance of the heavier particles is what determines the final abundance of the LSP, and so it is the fundamental quantity to analyze. In the very early universe, the typical time scale of the cosmic evolution $H^{-1}$ is much larger than the time scale of interaction of a weakly interacting particle, and so a weakly interacting particle is in thermal equilibrium. Therefore, its abundance is given by the one of a thermal distribution. As the temperature of the universe drops down, the interaction rate is not able anymore to keep the particle in thermal equilibrium, and so the particle decouples from the thermal bath, and its density begins to dilute, ignoring the rest of the thermal bath. We say in this case that the particle species freezes out.

In the case of weakly interacting particles around the TeV scale, the freeze out temperature is around decades of GeV, and so they are non relativistic at the moment of freezing out. In this case, the relic abundance of these particles is given by the following formula [7, 8, 9]:

$$\Omega_{NLSP}h^2 \approx 0.1 \left( \frac{10^{-9}\text{GeV}^{-2}}{<\sigma v>} \right) \left( \frac{15}{\sqrt{g_*}} \right) \left( \frac{10^{19}\text{GeV}}{M_{pl}} \right) \left( \frac{x_f}{30} \right) \left( \frac{h^2}{0.5} \right)$$

(12)

where $<\sigma v>$ is the thermally averaged cross section at the time of freeze out, $x_f = \frac{m_{NLSP}}{T_f}$ where $T_f$ is the freeze out temperature, $g_*$ is the effective number of degrees of freedom at freeze out, and $h$ is the Hubble constant measured in units of 100Km/(sec Mpc).

It is immediate to see that, for weakly interacting particle at 1 TeV, the resulting $\Omega$ is of order unity, and this has led to the claim that the Dark Matter bounds some supersymmetric partners to be at TeV scale. In this paper, we shall check this claim for an LSP in the hidden sector.

Once the weakly interacting particles are freezed out, they will rapidly decay to the NLSP, which, being lighter, will be in general still in thermal equilibrium. So, it will be the NLSP the only one to have a relevant relic abundance, determined by the freeze out mechanism, and so it will be the NLSP that, through its decay, will generate the present abundance of the LSP.

In Split Susy, the NLSP can either be the lightest Neutralino, or the lightest Chargino. The Neutralino is a mixed state of the interaction eigenstates Bino, Wino, and neutral Higgsinos, and is the lightest eigenstate of the following matrix [3]:

$$
\begin{pmatrix}
M_1 & 0 & -\kappa_2 v \\
0 & M_2 & -\kappa_1 v \\
-\kappa_1 v & -\kappa_2 v & 0 & -\mu \\
\mu & \mu & 0
\end{pmatrix}
$$

(13)
which differs from the usual Neutralino matrix in low energy Susy for the Yukawa coupling, which have their Susy value at the Susy breaking scale $\tilde{m}$, but then run differently from that scale to the weak scale.

The Chargino is a mixed eigenstate of charged Higgsino and charged Wino, and is the lightest eigenstate of the following matrix:

\[
\begin{pmatrix}
M_2 \\
\frac{k_w}{2} \\
\mu
\end{pmatrix}
\]

The actual and precise computation of the thermally averaged cross section of the NLSP at freeze out, which determines the $\Omega_{NLSP}$, is very complicated, because there are many channels to take care of, which depend on the abundance of the particles involved, and on the mixing of states, creating a very complicated system of differential equations. A software called DarkSusy has been developed to reliably compute the relic abundance \[10\], and, in a couple of recent papers \[2, 3\], it has been used to compute the relic abundance of the Neutralino NLSP in Split Susy. In this kind of theories, in particular, this computation is a bit simplified, since the absence of the scalar superpartners makes many channels inefficient. Nevertheless, in most cases, the computation is still too complicated to be done analytically.

In this paper, we consider both the possibility that the NLSP is a Neutralino and a Chargino. In the case of Neutralino NLSP, we modify the Dark Susy code \[10\] and adapt it to the case of Split Susy. We consider the cases of pure Bino, pure Wino, pure Higgsino, and Photino NLSP. In the case of Chargino NLSP, we consider the case of charged Higgsino and Charged Wino as NLSP, and we estimate their abundance with the most important diagram. We will see, in fact, that in this case a more precise determination of the relic abundance is not necessary.

Once the NLSP has freezed out, it will dilute for a long time, until, at the typical time scale of 1 sec, it will decay gravitationally to the LSP, which will be stable, and will constitute today’s Dark Matter. It’s present abundance is connected to the NLSP “would be” present abundance by the simple relation:

\[
\Omega_{LSP} = \frac{m_{LSP}}{m_{NLSP}} \Omega_{NLSP}
\]

Already from this formula, we may get some important information on the masses of the particles, just comparing with the case in which the Neutralino or the Chargino is the LSP. In fact, since $\frac{m_{LSP}}{m_{NLSP}} < 1$, $\Omega_{NLSP}$ has to be greater than what it would have to have if the NLSP was the LSP, in order for the LSP to provide all the DM. The abundance of the NLSP is inversely proportional to $<\sigma v>$, and this means that we need to have a typical cross section smaller than the one we would obtain in the case of a weakly interacting particle at TeV scale. This result can be achieved in two ways: either raising the mass of the particles, since $\sigma \sim \frac{1}{m^2}$, or by choosing some particle which for some reason is very low interacting.

The direction in which the particle become very massive is not very attractive from the LHC detection point of view, but still, in Split Susy, is in principle an acceptable scenario.

The other direction instead immediately lets a new possible candidate to emerge, which could be very attractive from the LHC detection point of view. In fact, in Split Susy, a pure Bino NLSP is almost not interacting, the only annihilation channel being the one into Higgs bosons in which an Higgsino is exchanged. In this case the relic abundance has $\Omega_{NLSP} \gg 1$, and this was the reason
why a pure Bino could not be the DM in Split Susy [2, 3]. In the case of a gravitationally interacting LSP, as we are considering, this over abundance would go exactly into the right direction, and it could open a quite interesting region for detection at LHC.

4 Cosmological Constraints

Since we wish the LSP to be the Dark Matter of the universe, so, we impose its abundance to cope with WMAP data [15, 16].

In general, for low reheating temperature, only the weakly interacting particles are thermally produced, and only the NLSP will remain as a thermal relic in a relevant amount. Later on, it will decay to the LSP. This decay will give the strongest cosmological constraints.

In fact, concentrating on the Gravitino, which interacts only gravitationally, we can naively estimate its lifetime as:

\[ \Gamma \sim \frac{m_{\text{NLSP}}^3}{M_{\text{pl}}^2} \]  

where the \( m_{\text{NLSP}} \) term comes from dimensional analysis. In reality, we can easily do better. In fact, as we have Goldstone bosons associated to spontaneous symmetry breaking, the breaking of supersymmetry leads to the presence of a Goldstino, a massless spinor. Then, as usually occurs in gauge theories, the Goldstino is eaten by the massless Gravitino, which becomes a massive Gravitino with the right number of polarization. Therefore, the coupling of the longitudinal components of the Gravitino to the LSP will be determined by the usual pattern of spontaneous symmetry breaking, and in particular will be controlled by the scale of symmetry breaking. This means that the coupling constant may be amplified.

In fact, if we concentrate on the Gauginos for simplicity, we can reconstruct their coupling to the Goldstino simply by looking at the symmetry breaking term in the lagrangian in unitary gauge, then reintroducing the Goldstino performing a Susy transformation, and promoting the transformation parameter to a new field, the Goldstino. The actual coupling is then obtained after canonical normalization of the Goldstino kinetic term, which is obtained performing a Susy transformation of the mass term of the Gravitino, and remembering that in the case of Sugra, the Susy transformation of the Gravitino contains a piece proportional to the vacuum energy. In formulas, the Gaugino Susy transformation is given by:

\[ \delta \lambda = \sigma^{\mu\nu} F_{\mu\nu} \xi \]  

where \( \xi \) is the Goldstino. This implies that the mass term of the Gaugino sources the following coupling between the Gaugino and the Goldstino:

\[ \delta (m_{\lambda\lambda}) \supset m_{\lambda\lambda} \sigma^{\mu\nu} F_{\mu\nu} \xi \]  

The Goldstino kinetic term comes from the Gravitino tranformation, which is:

\[ \delta \psi_\mu = m_{\text{Pl}} \partial_\mu \xi + i f \sigma_\mu \bar{\xi} \]  

so, the Gravitino mass term produces the Goldstino kinetic term:

\[ \delta (m_{gr\psi\psi}) \supset m_{gr} f m_{\mu} \xi \sigma^{\mu} \partial_\mu \xi = f^2 \xi \sigma^{\mu} \partial_\mu \xi \]
where in the last expression we used that \( m_{gr} = \frac{f}{m_{Pl}} \). So, after canonical normalization, we get the following interaction term:

\[
L_I = \frac{m_\lambda}{f^2} \lambda \sigma^{\mu \nu} F_{\mu \nu} \xi_c
\]  

(21)

where \( \xi_c = \xi f \) is the canonically normalized Goldstino. After all this, we get an enhanced decay width like this:

\[
\Gamma \sim \frac{1}{M_{Pl}^2} \left( \frac{m_{NLSP}}{m_{gr}} \right)^2 m_{NLSP}^3
\]  

(22)

Note that this is independent on the particle species, as it must be by the equivalence principle [17].

Plugging in some number, we immediately see that, for particles around the TeV scale, without introducing a big hierarchy with the Gravitino, the time of decay is approximately \( \sim 1 \text{ sec} \), and this is right the time of Big Bang Nucleosynthesis (BBN).

This is the origin of the main cosmological bound. In fact, the typical decay of the LSP will be into the Gravitino and into its SM partner. The SM particle will be very energetic, especially with respect to a thermal bath which is of the order of 1 MeV, and so it will create showers of particles, which will destroy some of the existent nuclei, and enhance the formation of others, with the final result of altering the final abundance of the light elements [17].

There is also another quantity which comes into play, and it is what we can call the "destructive power". In fact, the alteration of the light nuclei abundance will be proportional to the product of the abundance of the decaying particle and to the energy release per decay. This information is synthesized in an upper limit on the variable \( \xi \) defined as:

\[
\xi = B \epsilon Y
\]  

(23)

where \( B \) is the branching ratio for hadronic or electromagnetic decays (it turns out that hadronic decays impose constrains a couple of order of magnitude more stringent that electromagnetic decays), \( \epsilon \) is the energy release per decay, and finally \( Y = \frac{n_s}{n_\chi} \), where \( n_s \) is the number of photons per comoving volume, and \( n_\chi \) the number of decaying particles per comoving volume. Again, it is easy to see what will be the lower limit on the upper limit on \( \xi \). For the moment, we will neglect the dependence on the branching ratio \( B \), because, clearly, one of the two branching ratios must be of order one. Then, we understand that the most dangerous particles for BBN will be those particles that decay when BBN has already produced most of the nuclei we have to see today. A particle which decays earlier than this time, will in general have its decay products diluted and thermalized with an efficiency that depends on the kind of decay product of the particle: either baryonic or electromagnetic, and it turns out that the dilution for electromagnetic decays is much more efficient. So, it is clear that the upper limit on the "destructive power" \( \xi \) will be lower for particles which decay after BBN. For these late decaying particles, we can estimate what the upper limit on \( \xi \) should be with the following argument. \( \xi \) will become dangerous if the energy release is bigger than 1 MeV, in order for the decay product to be able to destroy nuclei, and also if \( Y \) is greater than \( \frac{n_s}{n_\chi} \), which represent the number of baryons per comoving volume opportunely normalized. Plugging in the numbers, with, again, \( B \sim 1 \), we get

\[
\xi_{\text{dangerous}} \gtrsim 10^{-14} \text{GeV}
\]  

(24)

This values of \( \xi_{\text{dangerous}} \) is more or less where the limit seems to apply in numerical simulations for late decaying particles, and it is in fact independent on the particular kind of decay, as in this case there are not dilution issues [18, 19]. For early decaying particles the limits do depend on the kind of decay, and they get more and more relaxed as the decay time becomes shorter and shorter, until
there is practically no limit on particles which decay earlier than $\sim 10^{-2}$ sec. Notice however that, from the estimates above on the decay time, the particles which we will be interested in will tend to decay right in the region where these limits apply.

The limit in eq. (24) translates into another useful parameter:

$$\Omega_{\text{dangerous}} \gtrsim 10^{-7}$$  \hspace{1cm} (25)

for the contribution of the NLSP around the time of nucleosynthesis. An easy computation shows that, imposing $\Omega_{DM} \sim 1$ today, we get that the contribution of NLSP goes as:

$$\Omega_{NLSP} \sim 10^{-7} \frac{m_{NLSP}}{m_{gr}} \left( \frac{\text{MeV}}{T} \right)$$  \hspace{1cm} (26)

This estimates are obviously very rough, but they are useful to give an idea of the physics which is going by, and they are, at the end of the day, quite accurate. They nevertheless tell us that we are really in the region in which these limits are effective, with two possible consequences: on one hand, a big part of the parameter region might be excluded, but also, on the other hand, this tells us that a possible indirect detection through deviations from the standard picture nucleosynthesis might reveal new physics.

In two recent papers [18, 19], numerical simulation were implemented to determine the constraints on $\xi$, both for the hadronic and the electromagnetic decays, and we shall use their data. (See also [20, 21] where a similar discussion is developed.)

Cosmological constraints come also from another observable. A late decaying particle can in fact alter the thermal distribution of the photons which then will form the CMB, introducing a chemical potential in the CMB thermal distribution bigger than the one which is usually expected due to the usual cosmic evolution, or even bigger than the current experimental upper bound [22, 23]. Analytical formulas for the produced effect are given in [24, 25]. Nevertheless, it is useful to notice that the induced chemical potential $\mu$ is mostly and hugely dependent on the time of decay of the particles. In particular, we see that:

$$\mu \sim e^{-\frac{\tau_{dc}}{\tau_{NLSP}}}$$  \hspace{1cm} (27)

where $\tau_{NLSP}$ is the lifetime of the NLSP, and $\tau_{dc} \sim 10^6$ s is the time at which the double Compton scattering of the photons is no more efficient. The $\xi_{\text{dangerous}}$ for this quantity is $\xi_{\text{dangerous}} \sim 10^{-9}$ GeV. So, we conclude that basically, for $\tau_{NLSP} < \tau_{dc}$, there are no limits, while for $\tau_{NLSP} > \tau_{dc}$, the limit from nucleosynthesis is stronger. We easily see that this constraint never comes into play in our work.

From formula (22), we can already extract an idea of what will be the final result of the analysis. In fact, we can avoid the limits from nucleosynthesis by decaying early. This means that, according to (22), we need to let the ratio $m_{NLSP}/m_{LSP}$ to grow, and consequently $\Omega_{NLSP}$ has to grow as well. This leads to two directions: either a very massive LSP or a a very weakly interacting LSP. The first direction goes in agreement with one of the directions we had found in order to match the constraint from $\Omega_{DM}$, and tells us that, in general, a massive NLSP will be acceptable from the cosmological point of view. However, it will have chances to encounter the constraints coming from gauge couplings unification. The other direction is to have an NLSP whose main annihilation channel is controlled by another particle, which can be made heavy. As an example, this is the case for the Bino, whose channel is controlled by the Higgsino: so, we might have a light Bino, if the Higgsino will be heavier.
5 Gravitino LSP

In this section, we concentrate in detail on the possibility that the LSP in Split Susy is the Gravitino, and that it constitute the Dark Matter of the universe. We shall consider the mass of the Gravitino as a free parameter, and we shall try to extract information on the mass and the nature of the NLSP. However, an actual lower limit on the Gravitino mass can be expected in the case Susy is broken directly, as in that case the mass of the Gravitino should be: $m_{gr} \sim \frac{\tilde{m}^2}{M_{pl}}$, where $\tilde{m}$ is the Susy breaking scale. Since, roughly, in Split Susy $\tilde{m}$ is as light as $\sim 100$ TeV, we get the lower limit $m_{gr} \gtrsim 10^{-8}$ GeV, which, as we will see, is lower than the region we will concentrate on.

As we learnt in the former two sections, there are two fundamental quantities to be computed: the lifetime of the NLSP, and $\Omega_{NLSP}$.

As we said before, we shall consider both the Neutralino and the Chargino as LSP. The decaying amplitude of a Neutralino into Gravitino plus a Standard Model particle was computed in [26, 27, 28].

For decay into Photons:

$$\Gamma(\chi \rightarrow \gamma, gr) = \frac{|N_{11} \cos(\theta_w) + N_{12} \sin(\theta_w)|^2 m_\chi^5}{48\pi M_{pl}^2} \left(1 - \frac{m_{gr}^2}{m_\chi^2}\right)^{3/2} \left(1 + 3 \frac{m_{gr}^2}{m_\chi^2}\right)$$

where $\chi = N_{11}(-i\tilde{B}) + N_{12}(-i\tilde{W}) + N_{13}\tilde{H}_d + N_{14}\tilde{H}_u$ is the NLSP. As we see, eq. (22) reproduces the right behavior in the limit $m_{NLSP}/m_{LSP} \gg 1$. This decay will contribute only to ElectroMagnetic (EM) energy.

The leading contribution from hadronic decays comes from the decay into $Z, gr$ and $h, gr$. These decays will contribute to EM or Hadronic energy according to the branching ratios of the SM particles.

The decay width to $Z$ boson is given by:

$$\Gamma(\chi \rightarrow Z, gr) = \frac{|-N_{11} \sin(\theta_w) + N_{12} \cos(\theta_w)|^2 m_\chi^5}{48\pi M_{pl}^2} \left(1 - \frac{m_{gr}^2}{m_\chi^2}\right)^{3/2} \left(1 + 3 \frac{m_{gr}^2}{m_\chi^2}\right)$$

where

$$F(m_\chi, m_{gr}, m_z) = \left(1 - \left(\frac{m_{gr} + m_Z}{m_\chi}\right)^2\right) \left(1 - \left(\frac{m_{gr} - m_Z}{m_\chi}\right)^2\right)^{1/2}$$

$$G(m_\chi, m_{gr}, m_z) = 3 + \frac{3 m_{gr}^3}{m_\chi^3} \left(-12 + \frac{m_{gr}}{m_\chi} + \frac{m_Z^4}{m_\chi^4} - \frac{m^2_{gr} m_Z^2}{m^2_\chi} \left(3 - \frac{m_{gr}^2}{m^2_\chi}\right)\right)$$

The decay width to $h$ boson is given by:

$$\Gamma(\chi \rightarrow h, gr) = \frac{|-N_{13} \sin(\beta) + N_{14} \cos(\beta)|^2 m_\chi^5}{48\pi M_{pl}^2} \frac{m_\chi^2}{m^2_{gr}} F(m_\chi, m_{gr}, m_z)$$

$$\left(1 - \frac{m_{gr}^2}{m_\chi^2}\right)^2 \left(1 + \frac{m_{gr}^2}{m_\chi^2}\right)^4 - \frac{m_h^2}{m_{gr}^2} H(m_\chi, m_{gr}, m_h)$$
where $h = -H_0^0 \sin(\beta) + H_0^0 \cos(\beta)$ is the fine tuned light Higgs, and

$$H(m_\chi, m_{gr}, m_h) = 3 + 4 \frac{m_{gr}}{m_\chi} + 2 \frac{m_{gr}^2}{m_\chi^2} + 4 \frac{m_{gr}^3}{m_\chi^3} + 3 \frac{m_{gr}^4}{m_\chi^4} + \frac{m_h^2}{m_\chi^2} \left( 3 + 2 \frac{m_{gr}}{m_\chi} + 3 \frac{m_{gr}^2}{m_\chi^2} \right)$$ (33)

We further use the following branching ratios and energy release parameters:

$$B_{EM}^\chi \sim 1$$ (34)

$$\epsilon_{EM}^\chi = \frac{m_\chi^2 - m_{gr}^2}{2m_\chi}$$ (35)

$$B_{Rad}^\chi \sim \frac{\Gamma(\chi \to Z, gr) B_{had}^Z + \Gamma(\chi \to h, gr) B_{had}^h + \Gamma(\chi \to q, \bar{q}, gr)}{\Gamma(\chi \to \gamma, gr) + \Gamma(\chi \to h, gr) + \Gamma(\chi \to Z, gr)}$$ (36)

$$\epsilon_{had}^\chi = \frac{m_\chi^2 - m_{gr}^2 - m_{Z,h}^2}{2m_\chi}$$ (37)

where $\epsilon_{i}^\chi$ is the energy release per decay in the EM and in the Hadronic channel, and $B_{i}^\chi$ is the branching ratio in the EM and Hadronic channel. We use $B_{Rad}^h \sim 0.9, B_{Rad}^Z \sim 0.7$. Since it will not play an important role, we just estimate the channel $\Gamma(\chi \to q, \bar{q}, gr)$, and do not perform a complete computation. This channel provides the hadronic decays when $m_{NLSP} - m_{gr}$ is less than the $m_Z$ or $m_h$. The leading diagram in this case is given by the tree level diagram in which there is a virtual $Z$ boson or a virtual Higgs that decays into quarks.

5.1 Neutral Higgsino, Neutral Wino, and Chargino NLSP

An Higgsino NLSP will be naturally much interacting in Split Susy, quite independently of the other partners mass. In fact, there are gauge interaction and Yukawa coupling to the other particles. While the coupling to the Z vanishes for $\mu \gg m_Z$, in that case neutral Higgsino and Charged Higgsino are almost degenerate, and so the interaction with the Higgs become relevant. This means that the annihilation rate will never be very weak, and so $\Omega_{NLSP}$ will be large only for large $\mu$. An analytical computation is too complicated for our necessities, even with the simplifications of Split Susy, so, we modify the DarkSusy code [10] to adapt it to the Split Susy case, and we obtain the following relic abundance:

$$\Omega_{H^0 h^2} = 0.09 \left( \frac{\mu}{\text{TeV}} \right)^2$$ (38)

In order to avoid nucleosynthesis constraints, we need to decay early. This can be achieved either raising the hierarchy between Higgsino and Gravitino, or raising the mass of the Higgsino. Since $\Omega_{LSP} = \frac{m_{LSP}}{m_{NLSP}} \Omega_{NLSP}$, we can not grow too much with the hierarchy, and so we are forced to raise the mass of the Higgsino.

This is exactly one of the two directions to go in the parameter space we had outlined in the first sections, and it is the one which is less favourable for detection at LHC.

The results of an actual computation are shown in fig. where we plot the allowed region for the Higgsino NLSP, in the plane $m_{gr}, \delta m = m_{NLSP} - m_{gr}$. The quantity $\delta m$ well represents the available energy for decay, and, obviously, can not be negative. The Hadronic and the Electromagnetic constraints we use come from the numerical simulations done in [15, 19]. There, constraints are
given as upper limit on the quantity $\xi = B_{\text{EM,Had}} \epsilon_{\text{EM,Had}} Y$ as a function of the time of decay. We then apply this limit to our NLSPs computing both the time of decay and the quantity $\xi$ with the formulas given in the former section.

As we had anticipated, the cosmologically allowed region is given by:

$$m_{\tilde{g}} \leq 4 \times 10^2 \text{GeV} \quad (39)$$

$$m_{\tilde{H}} \geq 20 \text{TeV} \quad (40)$$

This mass range is not allowed by gauge coupling unification, as, for Gravitino LSP, we have to use the upper bound on NLSP coming from gaugino mass unification initial conditions. So we conclude that the Higgsino NLSP is excluded. This is a very nice example of how much we can constraint physics combining particle physics data, and cosmological observations.

Finally, note how the hadronic constraints raise the limit on the Higgisino mass of approximately one order of magnitude.

In the case of the Neutral Wino NLSP, and Chargino NLSP, there are basically no relevant differences with respect to the case of the Neutral Higgsino, the main reason being the fact that the many annihilations channels lead to an high mass for the NLSP, exactly in parallel to the case of the Higgsino. We avoid showing explicitly the results, and simply say that, for all of them, the cosmologically allowed parameter space is very similar in shape and values to the one for Higgsino, with just this slight correction on the numerical values:

$$m_{\tilde{g}} \leq 5 \times 10^2 \text{GeV} \quad (41)$$

$$m_{\tilde{W}^0} \geq 30 \text{TeV} \quad (42)$$

for the Wino case. Notice that the mass is a little higher than in the Higgsino case, as the Wino is naturally more interacting. In fact, its relic abundance is given by (again, using a on porpuse modified version of Dark Susy [10]):

$$\Omega_{\tilde{W}^0} h^2 = 0.02 \left( \frac{M_2}{\text{TeV}} \right)^2 \quad (43)$$

For the Chargino, the mass limit is even higher:

$$m_{\tilde{g}} \leq 10^3 \text{GeV} \quad (44)$$

$$m_{\tilde{W}^+} \geq 40 \text{TeV} \quad (45)$$

All of these regions are excluded by the requirement of gauge coupling unification.

### 5.2 Photino NLSP

As we saw in the former section, hadronic constraints pushed the mass of the NLSP different from a Bino one above the 10 TeV scale, with the resulting conflict with gauge coupling unification. Anyway, just looking at the electromagnetic constraints in fig[8], one can see that a particle that will
not decay hadronically at the time of Nucleosynthesis will be allowed to be one order of magnitude lighter. This makes the photino, $\tilde{A} = \cos(\theta_W)\tilde{B} + \sin(\theta_W)\tilde{W}_3$, a natural candidate to be a NLSP with Gravitino LSP.

Computing the relic abundance of a Photino is rather complicated, even in Split Susy. The reason is that co-hannilation channels with the charged Winos makes a lot of diagrams allowed. In order to estimate the Photino relic abundance, we then observe that, because of the fact that the Bino is very weakly interacting in Split Susy, Photino annihilation channels will be dominated by the contribution of the channels allowed by the Wino component. We then quite reliably estimate the Photino relic abundance starting from the formula for the relic abundance of a pure Wino particle we found before:

$$\Omega_{Wino NLSP} h^2 = 0.02 \left( \frac{M_2}{\text{TeV}} \right)^2$$

(46)

and consider that the Photino has a Wino component equal to $\sin(\theta_W)$ So, for the Photino case we will have:

$$\Omega_{Photino NLSP} \approx \frac{0.02}{\sin(\theta_W)^4} \left( \frac{M_{\tilde{A}}}{\text{TeV}} \right)^2 \approx 0.37 \left( \frac{M_{\tilde{A}}}{\text{TeV}} \right)^2$$

(47)

We then obtain the allowed region shown in fig.[9]. The graph is very similar to the Higgsino case, with the difference that the Electromagnetic constraints are less stringent than the Hadronic ones. This allows to have the following region:

$$700 \text{ GeV} \leq M_{\tilde{A}} \leq 5 \text{ TeV}$$

(48)

the lightest part of which might be reachable at LHC. However, already if we allow an hadronic branching ratio of $10^{-3}$, we see that the Photino NLSP becomes excluded. So, we conclude that a Photino NLSP is in principle allowed, but only if we fine tune it to be extremily close to a pure state of Photino.

### 5.3 Bino NLSP

Bino NLSP is a very good candidate for avoiding all the cosmological constraints. In Split Susy, a Bino NLSP is almost not interacting. For a pure Bino, the only interaction which determines its relic abundance is the annihilation to Higgs bosons through the exchange of an Higgsino. Since this cross section is naturally very small, by eq.[12], $\Omega_{NLSP}$ is very big. This means that, in order to create the right amount of DM (see eq.[15]), we need to make the Gravitino very light. And this is exactly what we need to do in order to avoid the nucleosynthesis bounds. We conclude then that, of the two directions to solve the DM and the nucleosynthesis problems that we outlined in the former sections, a Bino NLSP would naturally pick up the one which is the most favorable for LHC detection. However, as we saw in the section on Gauge Coupling Unification (see fig.[4]), the Higgsino can not be much heavier than the Bino. This implies that a Bino like NLSP will have to have some Higgsino component unless it is very heavy and the off diagonal terms in the mass matrix are unimportant. As a consequence, new annihilation channels opens up for the Bino NLSP through its Higgsino component. This has the effect of diminishing the relic abundance of a Bino NLSP with respect to the naive thinking we would have done if we neglected the mixing. As a result, the cosmological constraints begin to play an important role in the region of the spectrum we are interested in, and, at the end, considering the upper limit from gauge coupling unification, exclude a Bino NLSP.
In order to compute the Bino NLSP relic abundance, we again modify the Dark Susy code\textsuperscript{10}. The results are shown in fig.\textsuperscript{10}. As anticipated, we see that the relic abundance strongly depends on the ratio between the Bino and the Higgsino masses $M_1, \mu$. The relic abundance of the Bino NLSP becomes large enough to avoid the cosmological constants only for so large values of the ratio between $\mu$ and $M_1$ which are not allowed by Gauge Coupling Unification. We conclude, then, that a Bino NLSP is not allowed.

6 Extradimensional LSP

When we consider generic possibilities to break Susy, we can have, further than the Gravitino, other fermions in a hidden sector which are kept light by an R symmetry. The implications of these fermions as being the LSP can be quite different with respect to the case of Gravitino LSP, as we will see in this section. Here, we concentrate on Susy breaking in Extra Dimensions, where, as it was shown in \textsuperscript{11}, it is very generic to expect a light fermion in the hidden sector.

In Susy breaking in Extra Dimension, one can break Susy with a radion field, which gets a VEV. Its fermionic component, the Goldstino, is then eaten by the Gravitino which becomes massive. Even though at tree level there is no potential, one sees that at one loop the Casimir Energy makes the radius unstable. One can compensate for this introducing some Bulk Hypermultiplets, finding that, however, the cosmological constant is negative. Then, in order to cancel this, one finds that he has to introduce another source of symmetry breaking, a chiral superfield $X$ localized on the brane (see for example \textsuperscript{1}). This represents a sort of minimal set to break Susy in Extra Dimensions. If one protects the $X$ field interactions with a U(1) charge, than one finds that the interactions with the SM particles are all suppressed by the 5 dimensional Plank Mass, of the form:

$$\int d^4 \theta \frac{1}{M_5^2} X^\dagger X Q \dagger Q$$

This induces the following mass spectrum \textsuperscript{11}:

$$m_{gr} \sim \frac{\pi M_5^3}{M_4^2}; m_S \sim \frac{\pi M_5^3}{M_4^2}; m_i, \mu, m_{\psi X} \sim \frac{\pi M_5^3}{M_4^2}$$

where $M_4^2 \sim rM_5^3$ are the 4 and 5 dimensional Plank constants, and where $M_i$ are the gaugino masses.

It is quite natural to use the extradimension to lower the higher dimensional Plank mass to the GUT scale, a la Horawa-Witten \textsuperscript{29}, $M_5 \sim M_{GUT} \sim 3 \times 10^{16}$ GeV. We have this range of scales \textsuperscript{11}:

$$m_{gr} \sim 10^{13}\text{GeV}; m_S \sim 10^9\text{GeV}; m_{\text{radion}} \sim 10^7\text{GeV}; M, \mu, m_{\psi X} \sim 100\text{GeV}$$

We notice that we have just reached the typical spectrum of Split Susy, in a very natural way: we break Susy in Extra Dimension, stabilize the moduli, and we introduce a further Susy breaking source to compensate for the cosmological constant. We further notice that there is no much room to move the higher dimensional Plank mass $M_5$ away from the Horawa-Witten value. In fact, the fermion mass scales as $(\frac{M_4}{M_5})^8$, so, a slight change of $M_5$ makes the fermions of Split Susy generically either too heavy, making them excluded by gauge coupling unification, or too light, making them conflict with collider bounds.
Concerning the study of the LSP, we notice that the fermionic component of the $X$ field we have to introduce in order to cancel the cosmological constant is naturally light, of the order of the mass of the Gauginos. So, it is worth to investigate the case in which this fermion is the LSP, and how this case differs from the case in which the LSP is the Gravitino.

Concerning the DM abundance, nothing changes with respect to the case of the gravitino LSP, so, we can keep the former results.

Next step it is to evaluate the decay time, to check if the nucleosynthesis and CMB constraints play a role.

To be concrete, let us concentrate on the Higgsino NLSP. When the Higgsino is heavier than the Higgs, the leading contribution to the decay of the Higgsino will come from the tree level diagram mediated by the operator\[1\]:

$$\int d^2\theta \frac{m^2 X}{M_5^2} H_u H_d$$

The decay time is then given by:

$$\tau = \frac{128}{\pi} \left( \frac{M_4}{\pi M_5} \right)^8 \left( 1 + \frac{m_{\psi X}^2}{m_H^2} - \frac{m_h^2}{m_H^2} \right)^{-1} \left( \frac{(m_H^2 + m_{\psi X}^2 - m_h^2)^2}{4m_H^2} - m_{\psi X}^2 \right)^{-1/2}$$

In the limit of $m_H \gg m_{\psi X}, m_h$ this expression simplifies to:

$$\tau \sim \frac{128}{\pi^9 m_H} \left( \frac{M_4}{M_5} \right)^8$$

Estimating with the number we just used before, we get:

$$\tau \sim 10^{-14} \left( \frac{\text{TeV}}{m_H} \right) \text{sec}$$

This time is so long before nucleosynthesis, that all the BBN constraints we found in the former case for the Gravitino now disappear. Clearly, this statement is not affected if we vary $M_5$ in the very small window allowed by the restrictions on the fermionic superpartners’ spectrum. Basically, in this mass regime, the only constraint which will apply will be the one from $\Omega_{DM}$. As we can see from the formula for the higgsino relic abundance, the region where Higgsino is lighter than Higgs is not relevant, and is excluded by the constraint on Dark Matter abundance.

So, we conclude that nucleosynthesis and CMB constraints do not apply in the case in which the LSP is the field $\psi_X$, and the Higgsino is the NLSP, the only constraints which applies are the one coming from $\Omega_{DM}$ and the one coming from gauge coupling unification. In fig\[1\] we show the allowed region. While the full region is quite large, and covers a rather large phase space, there is a region where the Higgsino is rather light, $\sim 2$ TeV, and the mass of the field $\psi_X$ is constrained quite precisely to be around 2 TeV. The region is bounded from above by the limit on gauge coupling unification at around 18 TeV, as in this case we must allow also for anomaly mediated initial conditions for Gauginos mass at the intermediate scale. This region is not extremely attractive for LHC.
For gaugino NLSP, the situation is very similar, as the decay of the gauginos to the field $\psi_X$ is mediated by the same kind of operator as for the Higgsino case \[1\]:

$$\int d^2\theta \frac{m^2 X}{M_5^2} WW$$

(56)

where $W$ is the gaugino vector supermultiplet. Clearly, again in this case, the decay time will be way before the time of BBN. In this cases, the curves that delimitate the allowed region are practically identical to the one of the Higgsino, with the only difference that the region where the NLSP is the lightest and it is practically degenerate with the $\psi_X$, correspond to an higher mass of $\sim 3$ TeV, more difficult to see at LHC.

Similarly occurs for the Bino NLSP, with the only difference that the region allowed by the Dark Matter constraint is a bit different with respect to the case of Higgsino and Wino. We obtain the allowed region shown in fig.12 where we see that the spectrum is very light, with Bino and Higgsino starting at tens of GeV, and gluinos at 200 GeV, with $\psi_X$ in the range $10^1 - 10^3$ GeV. This is a very good region for LHC. Notice that the upper limit on the Bino mass is again 5 TeV, as in the case the Bino is the NLSP, we can not have anomaly mediated initial conditions for Gaugino mass at the intermediate scale.

7 Conclusions

In Split Susy, the only two motivations to expect new physics at the TeV scale are given by the requirement that gauge coupling unification is achieved, and, mostly, that the stable LSP makes up the Dark Matter of the Universe. This is true in the standard scenario where the LSP is a neutralino. Here we have investigated the other main alternative for the LSP, that is that the LSP is constituted by a hidden sector particle. A natural candidate for this is the Gravitino, which here we studied quite in detail. Nevertheless, it is true that among the different possibilities we have in order to break Susy, one can expect the appearance in the spectrum of another light fermion protected by $R$ symmetry. Here, as an example, we study the case of a light fermion arising in Susy breaking in Extra Dimension.

The requirement to obtain gauge coupling unification limits the masses for the fermions to be less than 5 TeV or 18 TeV, according to the different initial conditions for the Gaugino masses at the the intermediate scale $\tilde{m}$.

In this range of masses, we have seen how constraints from Nucleosynthesis put strong limits on the allowed region. In fact, there are two competing effects: in order to avoid Nucleosynthesis constraints, the NLSP must decay to the LSP early, and this is achieved creating a big hierarchy between the NLSP and the LSP. On the other hand, this hierarchy tends to diminish the produced $\Omega_{LSP}$, and in order to compensate for it, the NLSP tends to be heavy. This goes against the constraints from gauge coupling unification. This explains why a large fraction of the parameter space is excluded.

The details depend on the particular LSP and NLSP.
7.1 Gravitino LSP

Gravitino LSP forces us to consider Gaugino Mass Unification at the intermediate scale as initial condition. This implies that we have to live with the more restrictive upper limit on fermionic masses from gauge coupling unification: 5 TeV.

At the same time, the typical decay time of an NLSP to the Gravitino is at around 1 sec, and this goes exactly into the region where constraints from Nucleosynthesis on ElectroMagnetic and Hadronic decays apply.

The final result is that only if the NLSP is very pure Photino like, then the Gravitino can be the LSP, with a Photino between 700 GeV and 5 TeV. If the NLSP is different by this case, then the Gravitino can not be the LSP. The reason is that a very Photino like NLSP can avoid the constraints on BBN on hadronic decays, which are much more stringent than the ones coming from electromagnetic decays, and so it can be light enough to avoid the upper limit on its mass coming from gauge coupling unification.

This is very good news for the detectability of Split Susy at LHC. In fact, if the Gravitino was the LSP, than the NLSP could have been much heavier than around 1 TeV, making detection very difficult. In this paper we show that this possibility is almost excluded.

7.2 ExtraDimensional LSP

Following the general consideration that in breaking Susy we might expect to have some fermion other than the Gravitino in the hidden sector which is kept light by an R symmetry, we have studied also the possibility that the fermionic component of a chiral field which naturally arises in Extra Dimensional Susy breaking is the LSP. In this case the time of decay is so early that no Nucleosynthesis bounds apply, so, the only constraints applying are those from Dark Matter and gauge coupling unification.

Concerning gauge coupling unification, in this case we must consider the possibility that the Gaugino Mass initial conditions are also those from anomaly mediation. This implies that we have to give the upper limit of 18 TeV to Fermions’ mass.

Having said this, the lower bound on the mass is given by the DM constraint. In fact, it is clear that \( \Omega_{NLSP} \) must be greater than \( \Omega_{LSP} \). As a consequence, the Mass of the NLSP has to be greater than the one found in [2] for the case in which these particles where the LSP. As a consequence, Charginos and Neutralinos NLSP are in general allowed, but they are restricted to be heavier than 1 TeV. It is quite interesting that in this cases, the LSP is restricted to be in the range 100-1000 GeV.

Again, there is an exception: the Bino. In this case, the LSP and NLSP are much lighter than in the case of the others Neutralinos NLSP, with an NLSP as light as a few decades of GeV.

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Figure 8: Constraints for the Higgsino NLSP, Gravitino LSP. There is no allowed region. The long dashed contour delimitates from above the excluded region by the hadronic constraints from BBN, the dash-dot-dot contour represents the same for EM constraints from BBN, the dash-dot lines represent the region within which $\Omega_{DM}$ is within the experimental limits; finally, we show the short dashed contours where the Higgsino decays to Gravitino at 1 sec, $10^5$ sec, and $10^{10}$ sec, and the solid line where the Higgsino is 5 TeV heavy, which represents the upper bound for Gauge Coupling Unification. For Neutral Wino and Chargino NLSP, the result is very similar.
Figure 9: Shaded is the allowed region for the Photino NLSP, Gravitino LSP. The long dashed contour delimitates from the left the region excluded by CMB, the dash-dot-dot contour delimitates from above the region excluded by the EM constraints from BBN in the case $B_h \sim 0$, the dashed-dot-dot-dot countor represents the same for $B_h \sim 10^{-3}$. The region within the dash-dot lines represents the region where $\Omega_{DM}$ is within the experimental limits; finally, we show the short dashed contours which represent where the Photino decays to Gravitino at 1 sec, $10^5$ sec, and $10^{10}$ sec, and the solid contour where the Photino is 5 TeV heavy, which represents the upper limit for Gauge Couplig Unification. We see that already for $B_h \sim 10^{-3}$ a Photino NLSP is excluded.
Figure 10: Constraints for Bino NLSP. The long dashed contour delimitates from the left the excluded region by the hadronic constraint from BBN, the dash-dot-dot contour represents the same for EM constraints from BBN, the dash-dot lines represents the ratio between the Higgsino mass and the Bino mass necessary for $\Omega_{LSP}$ to be equal to observed DM amount; the solid line represents the upper limit from GUT, while the dash-dot-dot-dot line represents the lower limit from LEP; we also show in short dashed the contours where the Bino decays to Gravitino at 1 sec, $10^5$ sec, and $10^{10}$ sec, and in dotted some characteristic contours for the Bino mass. CMB constraint plays no role here. We take $M_2 \simeq 2M_1$, as inferred from gaugino mass unification at the GUT scale \cite{footnote}, and we see that no allowed region is present.
Figure 11: Shaded is the allowed region for the Neutral Higgsino NLSP, ψ_X LSP. Since there are no constraints from CMB and BBN, the only constraints come from Ω_{DM}, which delimitates the region within the dash-dot lines, and Gauge Coupling Unification, which set the upper bound of 18 TeV with the solid line. For Neutral Wino and Chargino NLSP the result is very similar.
Figure 12: Shaded is the allowed region for the Bino NLSP, $\psi_X$ LSP. The dash-dot lines represent the ratio between the Higgsino mass and the Bino mass in order for $\Omega_{LSP}$ to be equal the observed amount of DM. The dotted lines are some characteristic countors for the Bino mass. The solid line is the upper limit from GUT, while the dash-dot-dot-dot line is the lower limit from LEP. We take $M_2 \simeq 2M_1$. 