Majorana Neutrino Mass Matrices with a Texture Zero and a Cofactor Zero under Current Experimental Texts

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Abstract

The Majorana neutrino mass textures with a texture zero and a vanishing cofactor are reconsidered in the light of current experimental results. A numerical and systematic analysis is carried out for all viable patterns. In particular, we focus on the phenomenological implication of correlations between three mixing angle (especially for $\theta_{23}$), Dirac CP-violating phase $\delta$, the effective Majorana neutrino mass $m_{ee}$. We demonstrated that the correlations between these variables play an important role in the model selection and can be measured in future long-baseline oscillation and neutrinoless double beta decay. Among the six viable patterns, it is the type-III with normal hierarchy and type-VI with inverted hierarchy that have the parameter space where the atmospheric neutrino mixing angle $\theta_{23}$ is less than maximal and the Dirac CP-violating phase covers its best-fit value.

Keywords: Majorana neutrino mass matrix, texture zero and cofactor zero, current experimental text.
By this time, various neutrino oscillation experiments have provided us with convincing evidences for massive neutrinos and leptonic flavor mixing with high degree of accuracy[1–3]. The large reactor angle $\theta_{13}(\approx 9^\circ)$ opens the door for us to explore the leptonic Dirac-CP violation and the mass hierarchy in the future long-baseline oscillation experiments. The absolute neutrino mass scale are strongly constrained by the cosmology observation[4, 5] and neutrinoless double-beta decay ($0\nu\beta\beta$) experiments (for a review, see [6]). On the other hand, it is still the main theoretical challenge to understand dynamic origin behind the observed bi-large structure of leptonic flavor mixing and the mass hierarchy spectrum. Although a full theory is still missing, several flavor symmetries have been proposed within the seesaw mechanism to reduce the number of free parameters in Yukawa sector and reveal the phenomenologically acceptable mixing pattern. These ideas include texture zeros[8], hybrid textures[9, 10], zero trace[11], zero determinant[12], vanishing minors[13, 14], two traceless submatrices[15], equal elements or cofactors[16], hybrid $M^{-1}_\nu$ textures[17].

Among these models, the textures with zero elements or zero minors are particularly appealed, which is not only because the textures can be naturally realized by introducing proper flavor symmetry, but also they are stable against the one-loop quantum corrections as the running of RGEs from seesaw scale $\Lambda$ to the electroweak scale $\mu \approx M_Z$.

In this work, we reconsider the Majorana neutrino mass textures with one texture and one vanishing minor, which have been studied in Ref.[14]. These models can be realized by $Z_{12} \times Z_2$ flavor symmetry. There are six types of textures compatible with the neutrino oscillation data. Our aim is to perform a more updated and complete analysis on these texture based on the current experimental data including the large reactor angle and the new cosmological bound on the sum of neutrino mass. In particular, we want to analyze the correlations among the Dirac CP-violating phase $\delta$, the three mixing angle, the Jarlskog invariant $J_{CP}$, the effective neutrino mass $m_{ee}$.

As is pointed out in Ref.[14], the six textures compatible with the neutrino oscil-
lation data are given by

\[
(M_\nu)^I = \begin{pmatrix}
\triangle & 0 & \times \\
0 & \times & \times \\
\times & \times & \times 
\end{pmatrix} \\
(M_\nu)^{II} = \begin{pmatrix}
\times & 0 & \times \\
\times & \times & \times \\
\times & \times & \times 
\end{pmatrix} \\
(M_\nu)^{III} = \begin{pmatrix}
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times 
\end{pmatrix} \\
(M_\nu)^{IV} = \begin{pmatrix}
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times 
\end{pmatrix} \\
(M_\nu)^{V} = \begin{pmatrix}
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times 
\end{pmatrix}
\]

where the "0" stands for the zero element and the "△" stands for the zero cofactor.

In the basis where the charged mass matrix is diagonal, the neutrino mass texture
\(M_\nu\) under flavor basis is given by

\[
M_\nu = V M_{\text{diag}} V^T
\]

where \(M_{\text{diag}}\) is the diagonal matrix of neutrino mass eigenvalues \(M_{\text{diag}} = \text{diag}(m_1, m_2, m_3)\). The Pontecorvo-Maki-Nakagawa-Sakata matrix\[18\]

\[
V = U \cdot P
\]

can be parameterized as

\[
V = UP = \begin{pmatrix}
c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23}
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha} & 0 \\
0 & 0 & e^{i(\beta+\delta)}
\end{pmatrix}
\]

where the abbreviation \(s_{ij} = \sin \theta_{ij}\) and \(c_{ij} = \cos \theta_{ij}\) is used. In neutrino oscillation experiments, CP violation effect is usually reflected by the Jarlskog rephasing invariant quantity \[19\] defined as

\[
J_{CP} = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \delta
\]

Following the same step in Ref.\[14\], the texture zero and the zero minor in \(M_\nu\) gives the mass ratio \(\frac{m_1}{m_2}, \frac{m_1}{m_3}\) and Majorana CP-violating phases \((\alpha, \beta)\) in terms of the \((\theta_{12}, \theta_{23}, \theta_{13}, \delta)\).i.e.

\[
\frac{m_1}{m_2}e^{-2i\beta} = \frac{-K_1L_1 - K_2L_2 + K_3L_3 \pm (K_1^2L_1^2 + (K_2L_2 - K_3L_3)^2 - 2K_1L_1(K_2L_2 + K_3L_3))^\frac{1}{2}}{2K_1L_3} e^{2i\delta}
\]

(5)
\[
\frac{m_1 e^{-2i\beta}}{m_3} = \frac{-K_1 L_1 - K_2 L_2 + K_3 L_3 \pm (K_1^2 L_1^2 + (K_2 L_2 - K_3 L_3)^2 - 2K_1 L_1 (K_2 L_2 + K_3 L_3))^{\frac{1}{2}}}{2K_1 L_2}
\]

(6)

where \(K_1 = U_{x1} U_{y1}, K_2 = U_{x2} U_{y2}, K_3 = U_{x3} U_{y3}\) and

\[
L_i = (U_{pj} U_{qj} U_{sk} - U_{tj} U_{uj} U_{vk} U_{wk}) + (j \leftrightarrow k)
\]

(7)

with \((i, j, k)\) a cyclic permutation of \((1, 2, 3)\). With the help of (5) and (6), the magnitudes of mass ratios are

\[
\rho = \left| \frac{m_1}{m_3} e^{-2i\beta} \right|, \quad \sigma = \left| \frac{m_1}{m_2} e^{-2i\alpha} \right|
\]

(8)

as well as the two Majorana CP-violating phases

\[
\alpha = -\frac{1}{2} \arg \left( \frac{m_1}{m_2} e^{-2i\alpha} \right), \quad \beta = -\frac{1}{2} \arg \left( \frac{m_1}{m_3} e^{-2i\beta} \right)
\]

(9)

The neutrino mass ratios \(\rho\) and \(\sigma\) are related to the ratios of two neutrino mass-squared ratios obtained from the solar and atmosphere oscillation experiments as

\[
R_\nu \equiv \frac{\delta m^2}{|\Delta m^2|} = \frac{2\rho^2(1 - \sigma^2)}{2\sigma^2 - \rho^2 - \rho^2\sigma^2}
\]

(10)

where \(\delta m^2 \equiv m_2^2 - m_1^2\) and \(\Delta m^2 \equiv m_3^2 - m_1^2\). For normal neutrino mass hierarchy (NH), the latest global-fit neutrino oscillation experimental data, at the 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) confidential level, is list as follows [22]

\[
\begin{align*}
\theta_{12} &= 33.6^{(+1.2,+2.2,+3.3)}_{(-1.0,-2.0,-3.0)} \\
\theta_{23} &= 38.4^{(+1.6,+3.6,+14.6)}_{(-1.2,-2.2,-3.2)} \\
\theta_{13} &= 8.9^{(0.5,+0.9,+1.3)}_{(-0.4,-0.9,-1.4)} \\
\delta m^2 &= 7.54^{(+0.26,+0.46,+0.64)}_{(-0.22,-0.39,-0.55)} \times 10^{-5} eV^2 \\
\Delta m^2 &= 2.43^{(+0.06,+0.12,+0.19)}_{(-0.10,-0.16,-0.24)} \times 10^{-3} eV^2
\end{align*}
\]

(11)

For the inverted neutrino mass hierarchy (NH), the differences compared with the NH are so slight that we shall use the same values given above. It is noted that the global analysis tends to give a \(\theta_{23}\) less than 45° at 2\(\sigma\) and 1\(\sigma\) level. The Majorana nature of neutrino can be determined if any signal of neutrinoless double decay is observed,
implying the violation of leptonic number violation. The decay ratio is related to the effective of neutrino $m_{ee}$, which is written as

$$m_{ee} = |m_1c_{12}^2c_{13}^2 + m_2s_{12}^2c_{13}^2e^{2i\alpha} + m_3s_{13}^2e^{2i\beta}| \quad (12)$$

Although a $3\sigma$ result of $m_{ee} = (0.11 - 0.56) \text{ eV}$ is reported by the Heidelberg-Moscow Collaboration\[23\], this result is criticized in Ref \[24\] and shall be checked by the forthcoming experiment. It is believed that that the next generation $0\nu\beta\beta$ experiments, with the sensitivity of $m_{ee}$ being up to $0.01 \text{ eV}$, will open the window to not only the absolute neutrino mass scale but also the Majorana-type CP violation. Besides the $0\nu\beta\beta$ experiments, a more severe constraint was set from the recent cosmology observation. Recently, an upper bound on the sum of neutrino mass $\sum m_i < 0.23 \text{ eV}$ is reported\[5\] by Plank Collaboration combined with the WMAP, high-resolution CMB and BAO experiments.

In the numerical analysis, We randomly vary the three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) in their $3\sigma$ range. Up to now, no bound was set on Dirac CP-violating phase $\delta$ at $3\sigma$ level, so we vary it randomly in the range of $[0, 2\pi]$. Using Eq. \(10\), the mass-squared difference ratio $R_\nu$ is determined. Then the input parameters is empirically acceptable when the $R_\nu$ falls inside the the $3\sigma$ range of experimental data, otherwise they are excluded. Finally, we get the value of neutrino mass and Majorana CP-violating $\alpha$ and $\beta$ though Eq. \(8\), \(9\). Since we have already obtained the absolute neutrino mass $m_{1,2,3}$ and ($\alpha, \beta$), the further constraint from cosmology should be considered. In this work, we set the upper bound on the sum of neutrino mass $\sum m_i$ less than $0.23 \text{ eV}$.

In Fig.\[1\]-\[10\] we demonstrate the correlations for all six pattern. The main results and the discussion are summarized as follows:

1) The Type-I and Type-II patterns are phenomenological acceptable only for inverted mass hierarchy, as mentioned in Ref.\[14\]. The type-I pattern are related to the type-II pattern by the $\mu - \tau$ symmetry\[25\], leading to the similar allowed resign for both patterns. One can see from the Fig.\[1\] and Fig.\[2\] that in the light of a large $\theta_{13}$ the Dirac CP-violating phase $\delta$ can only acceptable at around $\pi/2 (3\pi/2)$. The strong bound on $\delta$ is unusual and can be verified or excluded in future experiments.
The value of $\theta_{23}$ and $\theta_{12}$ are fully covered in $3\sigma$ level range. The $m_{ee}$ for both type-I and type-II pattern lie in the range of $0.044\text{eV} < m_{ee} < 0.05\text{eV}$, which is in the scope of accuracy of $0\nu\beta\beta$ decay experiment near the future. For the maximal CP-violating $\delta = \pi/2$ or $\delta = 3\pi/2$, the $|J_{CP}| \geq 3\%$ is achieved, which is accessible to the future long-baseline neutrino oscillation experiments.

2) The allowed region of type-III patterns are shown in Fig.(3) for inverted hierarchy and Fig.(4) for normal hierarchy. For the IH pattern, $\theta_{23}$ lies below the maximality. Interestingly, the $\theta_{12} > 34^\circ$ and $\theta_{23} < 40^\circ$ are satisfied when the Dirac CP-violating phase $\delta$ falls into the region of $0^\circ \sim 50^\circ$ ($310^\circ \sim 360^\circ$), leading to $0.015\text{eV} < m_{ee} < 0.030\text{eV}$ and $|J_{CP}| < 0.03$. However, $\delta$ is also highly constrained at around $\pi/2$ ($3\pi/2$), where the full $3\sigma$ range of mixing angle are covered and $0.050\text{eV} < m_{ee} < 0.090\text{eV}$. The Jarlskog invariant $J_{CP}$ is close to its maximum, i.e. $|J_{CP}| \geq 3\%$, which is promising to be explored in the next-generation long-baseline neutrino oscillation experiment. On the other hand, the Fig.(4) illustrates more complicated correlations for the NH pattern. There are unconstrained parameter space for $\delta$, $\theta_{12}$, $\theta_{13}$ and $J_{CP}$. However, we get a strongly constrained $\theta_{23} > \pi/4$ if $\delta$ lie in the range of $120^\circ \sim 240^\circ$ and $m_{ee} < 0.005\text{eV}$, rendering it very challenging to be detected in future $0\nu\beta\beta$ experiments.

3) We present the scatter plots of type-IV in Figure.(5) for IN case and Figure.(6) for NH case. For the inverted hierarchy, the Dirac CP-violating phase $\delta$ is limited to two regions, i.e. $120^\circ \sim 240^\circ$ and highly constrained points $\delta = \pi/2$ ($3\pi/2$). The $\theta_{23}$ are all above maximal and thus phenomenologically ruled out at $2\sigma$ level. The range of $m_{ee}$ is the same to that of type-III. This is not a coincidence but because the type-IV and type-III are related by the $\mu-\tau$ symmetry. Thus the $\theta_{23}^{IV}$ and $\delta^{IV}$ of type-IV are respectively equal to the $\frac{\pi}{4} - \theta_{23}^{III}$ and $\pi - \delta^{III}$ of type-III pattern. The same situation also appears in NH case.

4) The allowed region of type-V pattern are illustrated in Fig.(7) and Fig.(8). One can see from the figure that for the IH case, the Dirac CP-violating phase $\delta$ are phenomenologically acceptable in the range of $0^\circ \sim 80^\circ$ ($280^\circ \sim 360^\circ$) and $90^\circ \sim 95^\circ$ ($265^\circ \sim 270^\circ$). Although covering the whole $3\sigma$ data, the $\theta_{23}$ are excluded at $2\sigma$.
level when the $\delta$ lies in the range $50^\circ \sim 80^\circ (280^\circ \sim 310^\circ)$. We also get $0.015\text{eV}< m_{ee} < 0.040\text{eV}$ for $0^\circ < \delta < 80^\circ (280^\circ < \delta < 360^\circ)$ and $0.045\text{eV}< m_{ee} < 0.085\text{eV}$ for $90^\circ < \delta < 95^\circ (265^\circ \sim 270^\circ)$. No strong bound on $J_{CP}$ are obtained. For the NH case, we have $\delta$ constrained in the range $0^\circ \sim 40^\circ (320^\circ \sim 360^\circ)$ with $\theta_{12} > 34^\circ$, $|J_{CP}| < 0.02$ and $90^\circ \sim 100^\circ (260^\circ \sim 270^\circ)$ with $|J_{CP}| \geq 0.03$. For both $\delta$ allowed region, the $\theta_{23}$ is below the maximality and $0.005\text{eV}< m_{ee} < 0.080\text{eV}$.

5) In Fig.(9) and Fig.(10), we present the allowed region for type-VI pattern. As the previous cases, the type-VI pattern relates with the type-V pattern via $\mu - \tau$ symmetry and therefore we have $\delta$ restricted to the range of $85^\circ \sim 90^\circ (100^\circ \sim 180^\circ)$ and $180^\circ \sim 260^\circ (270^\circ \sim 275^\circ)$. The effective mass of $0\nu\beta\beta$ decay is the same as the ones of type-V, i.e. $0.015\text{eV}< m_{ee} < 0.040\text{eV}$. Just like the IH case, the $\theta_{23}$ and $\delta$ of type-VI are qual to the $\pi/4 - \theta_{23}$ and $\delta + \pi$ of type-V. In this sense, although the $\delta$ covers the $180^\circ$ which is at around the best-fit value of CP-violating phase from the neutrino oscillation experiments\cite{22}, the $\theta_{23}$ is up the maximality and thus ruled out by $2\sigma$ results.

In Table.1, we present some generic predictions of all viable textures at two value of $\delta$. i.e. $\delta \simeq \pi/2$ where the Dirac CP symmetry is maximal violated and the best-fit point $\delta \simeq \pi$. One can see from the Table that the value of $\theta_{12}$, $\theta_{23}$ and $m_{ee}$ indicates important phenomenological implication for the model selection. We are particular interested in the type-III with the normal hierarchy and the type-VI with the inverted hierarchy. In both cases, two important feature emerges: (i) there leaves a space for $\theta_{23} < \pi/4$ indicated by the experimental data at $2\sigma$ level. (ii) the Dirac CP-violating phase $\delta$ is covered to its best-fit value: $1.09\pi\ [22]$. Despite (i) and (ii) are not fully established yet, they are noteworthy in the model building\cite{26} and deserve to be examined in future experiments.

In conclusion, the neutrino mass matrix with zero element and zero cofactor are stable against the running of GREs and can be realized by introducing discrete flavor symmetries with scalar singlets\cite{14}. In this work, we carry out a numerical and comprehensive analysis of the viable textures with the current experimental data. We study the correlation between the Dirac CP-violating phase $\delta$, three mixing angle and
the $0\nu\beta\beta$ effective mass $m_{ee}$. We examine the predictive powers of these correlations in the future experiments and demonstrate that they are essential in model selection. We present some notable predictions for all the survived textures at $\delta \simeq \pi/2$ and $\delta \simeq \pi$. Interestingly, the type-III(NH) and type-VI(IH) are found to be phenomenologically interesting by the fact that $\theta_{23}$ are possible located below $\pi/4$ and the Dirac CP-violating phase $\delta$ covers its best-fit value $1.09\pi$. We except that a cooperation between theoretical study from the flavor symmetry point view and a phenomenology study with updated experimental data will help us reveal the structure of neutrino mass texture.

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Figure 1: The plots for pattern type-I (IH).

Figure 2: The plots for pattern type-II (IH).
Figure 3: The plots for pattern type-III (IH).

Figure 4: The plots for pattern type-III (NH).
Figure 5: The plots for pattern type-IV (IH).

Figure 6: The plots for pattern type-IV (NH).
Figure 7: The plots for pattern type-V (IH).

Figure 8: The plots for pattern type-V (NH).
Figure 9: The plots for pattern type-VI (IH).

Figure 10: The plots for pattern type-VI (NH).
Table I: Some generic predictions for the viable textures at $\delta \approx \pi/2$ and $\delta \approx \pi$.

| Texture   | $\delta \approx \pi/2$                                      | $\delta \approx \pi$                                      |
|-----------|-----------------------------------------------------------|-----------------------------------------------------------|
| type-I(IH)| $0.044\text{eV} < m_{ee} < 0.050\text{eV}$              | not allowed                                               |
| type-II(IH)| $0.044\text{eV} < m_{ee} < 0.050\text{eV}$             | not allowed                                               |
| type-III(IH)| $\theta_{23} < \pi/4$, $0.050\text{eV} < m_{ee} < 0.090\text{eV}$ | not allowed                                               |
| type-III(NH)| $0.002\text{eV} < m_{ee} < 0.030\text{eV}$     | $\theta_{23} < \pi/4$, $m_{ee} < 0.007\text{eV}$       |
| type-IV(IH)| $\theta_{23} > 47^\circ$, $0.050\text{eV} < m_{ee} < 0.090\text{eV}$ | $\theta_{12} > 34^\circ$, $\theta_{23} > 48^\circ$, $0.015\text{eV} < m_{ee} < 0.030\text{eV}$ |
| type-IV(NH)| $0.002\text{eV} < m_{ee} < 0.030\text{eV}$     | $\theta_{23} > \pi/4$, $0.007\text{eV} < m_{ee} < 0.014\text{eV}$ |
| type-V(IH)| $0.040\text{eV} < m_{ee} < 0.080\text{eV}$               | not allowed                                               |
| type-V(NH)| $38^\circ < \theta_{23} < 43^\circ$, $0.025\text{eV} < m_{ee} < 0.075\text{eV}$ | not allowed                                               |
| type-VI(IH)| $\theta_{23} > 42^\circ$, $0.050\text{eV} < m_{ee} < 0.080\text{eV}$ | $\theta_{23} > 40^\circ$, $0.010\text{eV} < m_{ee} < 0.035\text{eV}$ |
| type-VI(NH)| $\theta_{23} > 47^\circ$, $0.030\text{eV} < m_{ee} < 0.075\text{eV}$ | $\theta_{12} > 34^\circ$, $\theta_{23} > 49^\circ$, $0.010\text{eV} < m_{ee} < 0.025\text{eV}$ |