OZI violation in low energy $\omega$ and $\phi$ production in the $pp$ system in a quark-gluon model $^{*\dagger}$

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Abstract

We investigate OZI violation in near-threshold $\omega$ and $\phi$ production in the $pp$-system. Assuming ideal $\omega/\phi$ mixing (corrections are estimated), the energy dependence of the ratio $R_{\omega/\phi}$ is analyzed in a perturbative quark-gluon exchange model up to the third order in the strong coupling constant $\alpha_s$ with the proton represented as a quark - scalar diquark system. We give a very natural explanation of the violation of the OZI rule in $\omega/\phi$ production and its energy dependence near the production thresholds.

PACS: 12.30-x, 12.40-y, 13.60.Le, 21.45+v, 24.85+p

Keywords: Meson production, quark-gluon model, OZI rule

$^{*}$supported in part by the Forschungszentrum FZ Jülich (COSY)
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The OZI rule, formulated by Okubo, Zweig and Iizuka [1] - [3] provides a direct link to the quark-structure of hadrons in QCD: it states that meson production via disconnected quark lines is suppressed relative to connect $q\bar{q}$ excitations. One interesting example for this conjecture is the production of $\omega$ and $\phi$ mesons. Assuming ideal SU(3) mixing of octet-singlet representation, then

$$|\omega\rangle = \cos \theta_v |\eta_8\rangle + \sin \theta_v |\eta_1\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$|\phi\rangle = -\sin \theta_v |\eta_8\rangle + \cos \theta_v |\eta_1\rangle = s\bar{s},$$

i. e. the $\phi$-meson is a pure $s\bar{s}$ state and its excitation is OZI forbidden. Deviations from ideal mixing [4] yield the ratio [5]

$$R_{\text{OZI}} = \frac{\sigma(i \rightarrow f\phi)}{\sigma(i \rightarrow f\omega)} = \tan^2(\Delta \theta_v = 3.7^0) = 4.2 \cdot 10^{-3}.$$  

$R_{\phi/\omega}$ has been investigated extensively for various systems, such as in $\pi N$ [6, 7], $\gamma N$ [8, 9], (radiative decays of) vector mesons [10, 11], $NN$ annihilation [12, 13] and $NN$ systems [14, 15]. In this note we focus on near-threshold $\phi, \omega$ production in pp collisions, where recently data on the total $\omega, \phi$ cross sections and on the $\phi/\omega$ ratio have become available [16]-[20].

Currently, most investigations on near threshold $pp \rightarrow pp\omega/\phi$ production are based on meson exchange models both for $\omega$ [13] -[23] and $\phi$ production [24] - [27], with or without the inclusion of baryon resonances [28, 29]. As the $\omega$ and the $\phi$ meson carry the same quantum numbers ($J^\pi = 1^-, T = 0$), the leading contributions (without baryon resonances) involve the ($\omega, \phi$)$\rho\pi$ and ($\omega, \phi$)pp coupling (Fig. 1). Now $g_{\omega\rho\pi}$ and $g_{\phi\rho\pi}$ can be extracted, via vector meson dominance, from their decay into the $\gamma\pi$ channel [11], while $g_{\omega\text{pp}}$ is controlled from modern meson exchange models for the NN interaction [30]. Thus a comparison of $\phi/\omega$ production as a function of the excess energy $Q = \sqrt{s} - (2M_p + m_{\phi,\omega})$ should provide detailed information on $g_{\phi\text{pp}}$, which involves a typical uncertainty up to one order of magnitude in the literature [28]. Exploring this uncertainty, meson exchange models give qualitatively the right trend: an increasing OZI suppression of $\phi$-production for decreasing $Q$ (in comparing with the data, the $\phi/\omega$ ratio is normalized to the experimental ratio at the DISTO energy [16], as the poor knowledge of the pp initial state interaction at the relevant energies prohibits a quantitative normalization of the $\phi, \omega$ cross sections at momentum transfers of typically 1 GeV/c).

Opposite to meson-exchange models we follow the quark-based OZI arguments more explicitly and construct the contributions to $\phi/\omega$ production in a quark-gluon model. The leading terms up to third order gluon exchange as summarized in Fig. 2. Both sets of diagrams in 2 (a,b) favor $\omega$ versus $\phi$ production. For the 'mesonic' component the basic
difference stems from the diagram in fig. 2 (a). For ideal \( \phi/\omega \) mixing, only the \( \omega \) can be produced by the exchange of two gluons (without or with interchange of two quark lines), whereas \( \Phi \) production is strictly forbidden. For the 'nucleonic' component 2 (b) the difference is similarly striking: the leading 3g-exchange piece in \( \omega \)-production, i.e. colorless (Pomeron) 2g exchange, followed by \( q\bar{q} \) excitation, is again strictly forbidden for \( \phi \) production, where the direct excitation of the \( ss \) quark-anti quark in the \( \Phi \) meson without the interchange of quark lines yields the only contribution. Collecting just for a very qualitative estimate the corresponding color matrix elements (and adding thereby noncrossed and crossed gluon diagrams and leaving out common factors)

\[
R_{\text{colour}}^{1} = \frac{|M_{3g}^{\text{no ex}}|_{\phi}^{2}}{[M_{2g}^{\text{ex}} + M_{3g}^{\text{no ex}} + M_{\text{pom}}^{\text{ex}}]_{\omega}^{2}} = \left( \frac{7}{7 + 8 + 32} \right)^{2} = 2.2 \cdot 10^{-2}
\]

\( \phi \) production is suppressed by roughly 2 orders of magnitude relative to \( \omega \) production.

The main steps and approximations for a detailed calculation are readily summarized. As energy and momentum transfers \( \Delta E \sim \Delta q \sim 1 \text{ GeV} \) are far below the onset of perturbative QCD, we model the transition amplitude with effective quark and gluon degrees of freedom. The transition operator, integrated over the internal proton and meson Jacobi-coordinates (in coordinate space) is given as (\( \lambda = \phi, \omega \))

\[
M_{qq \rightarrow q\bar{q},q\bar{q}}(R) = \langle \phi_{p}(r, \rho)|\phi_{p}(r', \rho')\phi_{\lambda}(r_{\lambda})|V_{q \rightarrow q\bar{q}}(R)|V_{qq \rightarrow q\bar{q}}(R)||\phi_{p}(r, \rho)|\phi_{p}(r', \rho') \rangle
\]

as a function of the relative pp coordinate \( R \). Above the \( qq \) and \( q \rightarrow q\bar{q} \) interaction is derived from the relativistic one-gluon exchange operator \[31, 32\], where the corresponding (particle, antiparticle) Dirac spinors are expanded up to \( (\frac{m_{q}}{\omega(q)})^{2} \) in the quark mass \( m_{q} = 330 \text{ MeV} \) and the quark energy \( \omega(q) \), respectively (note that for equal momentum sharing among the quarks of the protons with \( q \sim p/3 \) (\( p \sim 1 \text{ GeV/c} \) is the proton momentum in the initial state), the expansion parameter yields \( \sim (2/5)^{2} \)). Then \[33, 35\]

\[
V_{qq \rightarrow q\bar{q}}(\Sigma_{ij}) = \frac{4\pi\alpha_{s}}{m_{g}^{2}} \frac{\lambda_{i} \lambda_{j}}{4} (V_{c} + V_{ss} + V_{LS} + V_{T})e^{-\frac{m_{q}^{2}}{4}r^{2}_{ij}}
\]

with the strong coupling constant \( \alpha_{s} \sim 2 \), the constituent gluon mass \( m_{g} \sim 800 \text{ MeV} \) \[36, 37\] and the central, spin, spin-orbit and tensor components; \( \lambda_{i,j} \) are SU(3) color matrices. Similarly we obtain for the \( q \rightarrow q\bar{q} \) excitation

\[
V_{q \rightarrow q\bar{q}}(\Sigma_{ij}) = -\frac{\alpha_{s}}{8\sqrt{\pi}} \frac{m_{q}^{3}}{m_{g}} \frac{\lambda_{i} \lambda_{j}}{4} (\sigma_{i} x \sigma_{j}) e^{-\frac{m_{q}^{2}}{4}r^{2}_{ij}} + i \frac{4}{m_{g}^{2}} e^{-\frac{m_{g}^{2}}{4}r^{2}_{ij}} \sigma_{i} \nabla_{j}
\]

(In practice the radial interactions are expanded as a superposition of Gaussians with different strength and width parameters). With these ingredients the 'Pomeron' exchange
is formulated from the noncrossed and crossed 2 g exchange, coupled to a color singlet \( ^{3}S_{1} - ^{1}P_{1} \) state

Similarly, the proton and the mesons are also modelled in a Gaussian basis. Thus the vector meson wave functions are represented as

\[
\phi_{\lambda}(r_{ij}) = e^{-r_{ij}^2 a_{\lambda}^2} \left[ 1/2(i)1/2(j) \right]_{\text{spin, flavour}}^{1m,00} \left( \delta_{ij} \sqrt{3} \right)_{\text{colour}}
\]

with \( a_{\lambda} = \sqrt{\frac{2}{3}} r_{rms} \) (i.e. the root mean square radius of the meson). For the proton we introduce an additional approximation to simplify the complicated 6q and 6q(q\overline{q}) many body problem: we represent the proton as a quark-scalar diquark system

\[
\phi_{p}(r, \rho)_{ijk} = \sum_{n=1}^{3} c_n e^{-r_{ij}^2 a_n^2} \left[ 1/2(i)1/2(j)1/2(k) \right]_{\text{spin, flavour}}^{0,0} \epsilon_{ijk} \sqrt{6}
\]

with the parameters \( c_n, a_n \) from resonating group calculations of the proton. Treating the scalar diquark as a boson with mass \( m_{s} \) (without antisymmetrizing its quark structure with the additional quark) dramatically simplifies antisymmetrization and the calculation of the spin-flavor-color matrix elements. We remark that the quark-scalar diquark configuration of the proton is supported from strong \( qq \) correlations in scalar diquarks with \( S = T = 0 \); opposite, the probability of axial diquarks with \( S = T = 1 \) is suppressed by more than one order of magnitude compared to scalar diquarks in the proton.

In a final step we calculate the total \( \omega, \phi \) cross sections and their ratio as a function of \( Q \)

\[
\sigma(Q) \sim \int \sum_{\text{spins}} |M_{pp \rightarrow pp\lambda}(Q)|^2 \frac{1}{2\omega_{\lambda}(k)} \delta(p'_1 + p'_2 + k) \delta(\sqrt{\sigma_{pp \rightarrow q\overline{q}q\overline{q}}(R)} - E_{p'_1} - E_{p'_2} - \omega_{\lambda}(k))dV_{ps}
\]

\((dV_{ps} \text{ denotes the integration over the 3-body phase space})\) with

\[
M_{pp \rightarrow pp\lambda}(Q) = \langle \chi_{pp}(R)|M_{qq \rightarrow q\overline{q}q\overline{q}}(R)|\chi_{pp}(R) \rangle.
\]

Neglecting the meson-proton interaction, \( \chi_{pp}(R) \) is the distorted \( pp \) wave, which is obtained for the \( pp \) final state interaction from an expansion in a scattering length-effective range parametrization. Without quantitative information on the \( pp \) initial state interaction we use

\[
|\chi_{pp}(R)\rangle = e^{ipR} \sqrt{\frac{\sigma_{pp} \ell}{\sigma_{pp}^{\text{total}}}} \sim e^{ipR} \frac{1}{2}
\]

(with \( \pm p \) being the CM momentum of the protons in the initial state; the \( pp \) cross sections are taken from ref; other estimates from the literature yield a qualitatively similar result.)
The results of our calculation are presented in Figs. 3 to 5. Figs. 3, 4 show the $Q$-dependence of the $\omega$ and the $\phi$ cross section. Opposite to $\phi$-production, where only one data point is published together with still preliminary data from ANKE \cite{17}, a more detailed comparison is possible for the $\omega$ meson. Within the given parameters ($\alpha_s = 1.7, m_s = 600$ MeV, $m_g = 800$ MeV) we reproduce qualitatively the experimental $Q$-dependence. There remains still a significant parameter sensitivity, especially on final state $pp$ interactions and the $q$-diquark structure of the proton. For $\phi$-production, which is calculated with the parameters fixed from $\omega$ production, the $Q$-dependence is similar, though the relative strength of $\sigma_{pp\rightarrow pp\phi}(Q)/\sigma_{pp\rightarrow pp\phi}(Q_0 = 83$ MeV) with $Q < Q_0$ decreases relative to $\omega$ production.

The most interesting quantity is the $\phi/\omega$ ratio in Fig. 5, as here we expect a reduced sensitivity to several details of the parametrization (i.e. less influence from initial and final state interactions or from the modelling of $p, \omega$ and $\phi$). Then $R_{\phi/\omega}$ decreases with decreasing $Q$ as shown from the ANKE data, however, there is still a significant variation for different parametrizations within currently accepted limits. Derivations from ideal $\phi/\omega$ mixing enhance $R_{\phi/\omega}$ by less than 10%.

The conclusions to be drawn are evident. From the experiment side more detailed information on the $R_{\phi/\omega}(Q)$ dependence is necessary, to restrict the various model parameters (here information on $R_{\phi/\omega}$ at $Q$ very close to threshold would be very interesting, as different models predict a significant variation when approaching the $\phi$ and $\omega$ thresholds); in addition, angular distributions for both vector mesons, beyond the existing data are urgently needed (for example, the $\omega$ and $\phi$ angular distribution at $Q \sim 90$ MeV are fairly isotropic \cite{16} - \cite{18}, in striking contrast to the $\omega$-data at $Q = 173$ MeV \cite{19}, which exhibit a very strong non isotropic structure).

From theory, both meson-exchange and quark-gluon models should be tested consistently on existing and future $\omega, \phi$ data (also to explore the dominant reaction mechanisms at much larger excess energies). In addition to still missing ingredients (such as the vector-meson-proton final state interaction, relativistic corrections to the production operator and its extension to gluon exchange of next order, or an improvement of $pp$ initial state interactions \cite{18} \cite{50}) most urgent seems a more refined modelling of the $q$-diquark structure of the proton (bridging the extremes from a point-like diquark to its resolution as two uncorrelated quarks \cite{51} together with the incorporation of genuine relativistic corrections, such as the Lorentz-quenching of the protons in the initial state \cite{52} \cite{53}; and a more systematic extension of the formalism to explore the role of intermediate baryon resonances both for the $\omega$ and $\phi$ channel \cite{28} \cite{29}. Progress along those lines would pave the way to other, even more subtle questions, such as the $s\bar{s}$ content of the proton and
its influence on its spin structure ([54] [55]).

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Figure 1: Leading contributions to near-threshold $pp \rightarrow pp\omega, \phi$ production in meson-exchange models. (a) 'mesonic' and (b) 'nucleonic' contribution.
Figure 2: As in Fig. 1 however, in a quark-gluon exchange model. (a) 'mesonic' 2-gluon exchange in $\omega$ and (b) 'nucleonic' Pomeron-gluon and 3-gluon exchange in $\omega, \phi$ production (for ideal $\omega, \phi$ mixing the 'pomeron' component is absent in $\phi$-production).
Figure 3: Energy dependence of the $pp \rightarrow pp\omega$ cross section. Compared are different parametrizations (see legend) with data for $\omega$ production [18] - [20].
Figure 4: As Fig. 3, however for the $pp \rightarrow pp\phi$ cross section, compared with data from refs. (16 17).
Figure 5: Q-dependence of $R_{\phi/\omega}$ for different parametizations, compared to COSY-TOF, ANKE and DISTO data ([16] - [19]). $R_{OZI} = 4.2 \cdot 10^{-3}$ shows the OZI prediction for nonideal $\omega/\phi$ mixing (see text).