Electron–Electron Scattering in Quantum Wires
and it’s Possible Suppression due to Spin Effects

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A microscopic picture of electron-electron pair scattering in single mode quantum
wires is introduced which includes electron spin. A new source of ‘excess’ noise for hot
carriers is presented. We show that zero magnetic field ‘spin’ splitting in quantum
wires can lead to a dramatic ‘spin’-subband dependence of electron–electron scat-
tering, including the possibility of strong suppression. As a consequence extremely
long electron coherence lengths and new spin-related phenomena are predicted. Since
electron bands in III-V semiconductor quantum wires are in general spin-split in zero
applied magnetic field, these new transport effects are of general importance.

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We show that electron-electron pair scattering in quantum wires is fundamentally different from two-dimensional (2D) or three-dimensional (3D) systems and that it is essential to include the electron spin in the analysis. We show that ‘spin’ splitting of the electron bands causes ‘spin’-subband dependence of electron pair scattering rates, and may cause a dramatic reduction of electron-electron scattering for hot electrons in one of the two spin subbands. We show that electron pair scattering can cause fluctuations of electron spin, energy and wave number, and therefore is expected to contribute ‘excess’ noise to electrical current. We expect that spin related effects in quantum wire transport, as demonstrated in the present work, will become important in mesoscopic transport experiments.

Electron-electron pair scattering is for many conditions the strongest scattering mechanism, limiting the electron life-time and the phase coherence length. More generally, electron-electron interactions cause or contribute to such diverse phenomena as superconductivity, Wigner crystallization, magnetic ordering and heavy Fermion effects. In two dimensional (2D) and three dimensional (3D) systems (but not in 1D) electron pair scattering contributes to thermalization of hot electrons. Although it does not contribute directly to diffusive transport, it enters the collision integral of the Boltzmann equation and thus in 3D and 2D (not in 1D) contributes to establish diffusive transport. The general properties of electron pair scattering in 3D have been investigated extensively [1], but detailed quantitative information for experimental semiconductor structures has only become available recently in 2D [2, 3, 4], and for 1D [5]. The weak localization regime, where impurity scattering dominates, has been intensively investigated, but little is known about the properties of electron-electron scattering in high mobility quantum wires. We assume in the present work, that electrons in a quantum wire form an ordinary Landau liquid as supported by Ref. [6] and that disorder effects are negligible. The present work concerns single mode quantum wires, with a single (or a few) transverse modes per spin orientation, i.e. wires with widths of the order of 100Å.

First, we show that electron-electron pair scattering is a phase breaking scattering process even in a ‘single-mode’ quantum wire. Fig. [7] demonstrates such a process: a ‘spin-up’
electron at \((p, \uparrow)\) scatters with a ‘spin-down’ electron at \((k, \downarrow)\), resulting in a hole at \((k, \downarrow)\), an electron at \((k, \uparrow)\) and an electron at \((p, \downarrow)\). Electron pair scattering in a single mode quantum wire can only occur for pairs of electrons in opposite ‘spin’-subbands, while it is forbidden for pairs of electrons in the same ‘spin’-subband due to energy and momentum conservation and the Pauli principle. Similarly, it was shown in Ref. [4], that in a 2DEG scattering for pairs of electrons in the same ‘spin’-subband is typically 50% weaker than for opposite ‘spin’-subbands, although not totally forbidden as in 1D.

Fig. 1b shows the resulting picture for a non-equilibrium electron propagating in a quantum wire. Electron pair scattering flips electrons between the two different ‘spin’-subbands. Therefore, pair scattering leads to fluctuations of electron spin. In general, the two ‘spin’-subbands in a quantum wire will be split in energy, and the spin states will be mixed. As Fig. 1b demonstrates, pair scattering in the presence of ‘spin’ subband splitting causes fluctuations of electron energy and wave number in addition to the spin fluctuations of a propagating electron, and therefore should be experimentally important in quantum wire devices as a new contribution to current dependent ‘excess’ noise.

In bulk III-V semiconductors, bands are spin split at zero applied magnetic field in all directions except [100] due to the lack of inversion symmetry (see [7]). Spin splitting has terms proportional to \(k\) and \(k^3\), typical bulk values are shown in the insert of Fig. 2. The equivalent magnetic fields which would have to be applied externally to produce a similar splitting at a Fermi energy around 20\(meV\) are quite large. In quantum wells and quantum wires, terms in addition to the bulk terms are expected [8] [9]. Spin-splittings for 2D systems have recently been measured [10] [11] [12]. For the rest of this work, we will show results taking the conduction band structure equal to that of bulk GaAs. We keep in mind that the precise value of the splitting and the spin mixing will vary for different types of quantum wires, although there are always two ‘spin’ subbands in a ‘single’ mode wire. We will not discuss sample dependent details further in the present Letter, and we will simply label the two subbands as ‘spin-up’ and ‘spin-down’.

The essence of our results can be explained with Fig. 2. We consider a pair scattering
process, where an electron in the ‘spin-up’ subband at \((\mathbf{p}, \uparrow)\) scatters with a ‘spin-down’ electron at \((\mathbf{k}, \downarrow)\). Once \(\mathbf{k}\) and \(\mathbf{p}\) are selected, the final states \((\mathbf{k} - \mathbf{q}, \downarrow)\) and \((\mathbf{p} + \mathbf{q}, \uparrow)\) are determined by energy and momentum conservation. (In the absence of spin splitting, or when the subbands are parallel: \(\mathbf{k} - \mathbf{q} = \mathbf{p}\)). The probability for this process is given by the product of the square of the Coulomb matrix element multiplied by the thermal factor \(f_{\mathbf{k}, \sigma}(1 - f_{\mathbf{k} - \mathbf{q}, \sigma'})(1 - f_{\mathbf{p} + \mathbf{q}, \sigma})\), where \(f_{\mathbf{k}, \sigma}\) are Fermi-Dirac occupation factors. Clearly, spin-splitting strongly reduces the thermal occupation probability factor for this scattering process. As a consequence, forward \((\mathbf{k} \text{ near } +\mathbf{k}_F)\) scattering is strongly suppressed for one particular spin orientation (here ‘spin-up’), while there is a small increase for the other spin orientation (here labelled ‘spin-down’). The strong ‘spin’ subband dependence of the scattering probability relies on the strong \(k\)-dependence of the spin-splitting (bulk terms are proportional to \(k\) and \(k^3\)). It can be easily seen that ‘spin’ subband dependent scattering rates are not expected for \(k\)-independent splittings. Furthermore, for scattering processes with \(k \approx -\mathbf{k}_F\) and \(q \approx -2\mathbf{k}_F\) pair scattering rates are almost independent of the subband. These facts weaken the ‘spin’ subband dependence of the total pair scattering rates, but detailed calculations outlined below show, that in many circumstances strong ‘spin’ subband dependence prevails.

To confirm this surprising result quantitatively, we calculate the scattering rates. The total scattering rate for an electron at wavevector \(\mathbf{p}, \sigma\) is expressed as:

\[
\frac{1}{\tau_{ee}}|_{\mathbf{p}, \sigma} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}, \mathbf{q}} f_{\mathbf{k}, \sigma}(1 - f_{\mathbf{k} - \mathbf{q}, \sigma'})(1 - f_{\mathbf{p} + \mathbf{q}, \sigma}) \frac{\langle k - q, \sigma'; p + q, \sigma | V | k, \sigma'; p, \sigma \rangle^2}{\epsilon(q, (E_{p, \sigma} - E_{p + q, \sigma}) / \hbar)} \times \delta(E_{p + q, \sigma} + E_{k - q, \sigma'} - E_{p, \sigma} - E_{k, \sigma'})
\]

where \(\langle k - q, \sigma'; p + q, \sigma | V | k, \sigma'; p, \sigma \rangle = e^2 F_{ijkl}^{1D}(q \times w)/(L\epsilon_o\epsilon_r)\) is the 1D Coulomb interaction matrix element. \(F_{ijkl}^{1D}(q \times w)\) is the 1D Coulomb Formfactor consisting of a four-dimensional integral involving the wave functions and the Bessel function \(K_0\), which we determine by numerical integration assuming a wire with a square cross section. The dielectric function \(\epsilon(q, (E_{p, \sigma} - E_{p + q, \sigma}) / \hbar)\) takes account of dynamic screening. For the present calculation we integrate the finite temperature Ehrenreich expression for the polarizability.
numerically for the two spin-split conduction bands. We assume that the quantum wire electron band structure is described by the bulk k.p dispersion. The integrals are calculated numerically using adaptive multipoint Gauss-Kronrod integration.

The details of an experimental quantum wire will affect the band dispersion, spin composition of the bands, the dielectric function and the matrix elements. The essential point of the present letter is the prediction of a large difference in the electron scattering rates for the two ‘spin’-subband. The effects discussed in the present letter are a consequence of the band splitting, the Pauli principle, Fermi occupation factors, and energy and momentum conservation for electron pair scattering. They are expected for many variations of the band structure, spin mixing and details of the wave functions in different types of wires.

Fig. 3 compares the excess energy and spin-subband dependence of differential pair scattering rates in a GaAs quantum wire for electrons in the ‘spin–up’ and ‘spin–down’ subbands for a wire assumed to have the conduction band structure of GaAs along [110]. The carrier concentration is $1.6 \times 10^6 \text{cm}^{-1}$, temperature $T = 1.4K$, and we assume a square wire of width 100Å and infinite confinement potential. Due to the exponential character of the Fermi population factors, the forward pair scattering rates for the ‘spin-up’ subband are many orders of magnitude lower compared to the ‘spin-down’ subband. As expected, Fig. 3b shows that the ‘spin’ subband dependence does not occur for electrons scattering with partners at $k \approx -k_F$. Figures 3a and b clearly show, that for electrons with excess energies more than 1meV, the total scattering rate is substantially larger for a hot electron in the ‘spin-down’ subband. We have investigated many combinations of ‘spin’ splitting strength, temperature and excess energy, and details will be published separately. Constructing quantum wires with specific ‘spin’-splitting, carrier concentration, and choosing particular temperature and excess energy will allow to tune the spin-dependence of the electron pair scattering rates. Further it can be seen from Fig. 3, that the total scattering rates also show some spin dependence for equilibrium electrons near the Fermi level ($\Delta = 0$), although the ‘spin’-subband dependence is not strong.

Fig. 4 shows the total electron pair scattering rates calculated by numerically integrat-
ing Eq. (1) over \(-\infty < k < +\infty\), and the corresponding scattering lengths. The ‘spin’ dependence of the forward scattering \((k \approx +k_F)\) causes a strong ‘spin’ subband dependence of the total rates. For Millikelvin temperatures scattering lengths in excess of millimeters are predicted for one of the two ‘spin’-subbands, while at Helium temperatures lengths around \(20\mu m\) are predicted. To observe such long scattering lengths, other competing scattering mechanisms have to be sufficiently weak. Impurity scattering and interface roughness scattering can be reduced by improvements in fabrication techniques. In Ref. \[5\] it was estimated, that remote ionized impurity scattering can also be reduced sufficiently. The dashed line in Fig. 4 shows the strength of acoustic phonon scattering in 2D from Ref. \[13\], comparable data for 1D are not yet available. Stronger spin splitting due to choice of a different material, or in-built electric fields, may lead to stronger suppression of scattering and longer coherence lengths. Fig. 4 also shows that the ‘spin’-subband dependence of the scattering rates disappears above a temperature larger than the typical splitting energy of here \(0.85meV\), indicated in Fig. 2, although this temperature may be much increased for materials with higher spin splitting.

We will now comment on the significance and on experimental predictions. We have introduced a microscopic picture for electron pair scattering in single mode quantum wires and calculated scattering rates. Such work is essential to understand microscopic details of transport, or other details such as spin-relaxation, which has recently attracted attention in 2D \[14\]. We also demonstrated a new source of ‘excess’ (i.e. current induced) noise. The predicted ‘spin’-subband dependence of electron pair scattering rates leads to the prediction of a range of novel ‘spin’-subband dependent transport properties. The present work demonstrates that electron spin can have even more dramatic effects in quantum wires than in 2D. Investigations of mesoscopic transport have progressed to the point where very detailed electronic spectroscopy of quantum dots coupled to quantum wire electron wave guides can be performed (see e.g. Ref. \[15\]). ‘Spin’ subband dependence may allow high resolution experiments of magnetic sublevels in quantum dots, and it may lead to electron spin polarization effects in hot electron transport.
In summary, we have introduced a microscopic picture of electron pair scattering for quantum wires, which includes spin. We demonstrated it to be a source of phase breaking and ‘excess’ noise. We show that ‘spin’ splitting leads to unequal forward and total pair scattering rates for electrons in the ‘spin–up’ and ‘spin–down’ subbands of a quantum wire. We predict the possibility of strong reduction of pair scattering for one of the two ‘spin’-subbands, and very long ‘spin’ subband dependent coherence lengths. Several other spin-related effects may arise as a consequence of ‘spin’-subband dependent electron pair scattering rates.

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FIGURES

FIG. 1. Microscopic picture of typical electron-electron pair scattering process in a single mode quantum wire. (a) ‘Spin-up’ electron at \((p, \uparrow)\) scatters with ‘spin-down’ electron at \((k, \downarrow)\). Pair scattering of electrons in the same ‘spin’ subband is forbidden in 1D. (b) Electron pair scattering in a quantum wire causes fluctuations of the electron spin. If the bands are ‘spin’-split, electron energy and wave vector fluctuate as well, giving rise to a new source of ‘excess’ noise for a current of hot carriers.

FIG. 2. Schematic diagram of electron–electron pair scattering process in a quantum wire with spin splitting. An electron \((p, \uparrow)\) is scattered by electron \((k, \downarrow)\). Diagram shows typical spin–splitting of the conduction band expected in a quantum wire along GaAs [110] near the Fermi energy in zero applied magnetic field. Note that spin composition of the bands is mixed and dependent on the details of the wire. For temperatures low compared to the energy separation of states \((p + q, \uparrow)\) and \((k, \downarrow)\) (here approximately 0.85 meV as indicated), the population factors entering the scattering probability will lead to a dramatic suppression of forward pair scattering for electrons in the ‘spin-up’ subband and to an enhancement of the scattering rate for the ‘spin-down’ subband. Insert shows typical values for spin splitting in the bulk.

FIG. 3. Differential scattering rates for electrons in the ‘spin–up’ and ‘spin–down’ subbands of a quantum wire with ‘spin’ splitting. (a) As a consequence of the different Fermi population factors electron–electron scattering with partners near \(+k_F\) is substantially lower for one particular spin orientation (here spin up), while scattering for the opposite spin orientation (here spin down) is enhanced. (Numerical anomalies at \(q = 0\), where no dephasing takes place, are eliminated from the figure). (b) For scattering with partners near \(-k_F\) the scattering rates are essentially independent of the ‘spin’ subband.
FIG. 4. Total electron–electron pair scattering rates determined by numerical integration of Eqn. (1) over $-\infty < k < +\infty$. Results are shown for a quantum wire, with conduction subband dispersions assumed to be those of bulk GaAs oriented along the [100] and [110] crystal orientations. Due to spin splitting the total scattering rates are strongly suppressed for one of the two ‘spin’ subbands (here ‘spin–up’), while they are increased for the opposite ‘spin’ subband. Dashed line shows acoustic phonon scattering for a 2DEG from Ref. 13.