Hierarchical mechanical metamaterials built with scalable tristable elements for ternary logic operation and amplitude modulation

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Multistable mechanical metamaterials are artificial materials whose microarchitectures offer more than two different stable configurations. Existing multistable mechanical metamaterials mainly rely on origami/kirigami-inspired designs, snap-through instability, and microstructured soft mechanisms, with mostly bistable fundamental unit cells. Scalable, tristable structural elements that can be built up to form mechanical metamaterials with an extremely large number of programmable stable configurations remains illusive. Here, we harness the elastic tensile/compressive asymmetry of kirigami microstructures to design a class of scalable X-shaped tristable structures. Using these structure as building block elements, hierarchical mechanical metamaterials with one-dimensional (1D) cylindrical geometries, 2D square lattices, and 3D cubic/octahedral lattices are designed and demonstrated, with capabilities of torsional multistability or independent controlled multidirectional multistability. The number of stable states increases exponentially with the cell number of mechanical metamaterials. The versatile multistability and structural diversity allow demonstrative applications in mechanical ternary logic operators and amplitude modulators with unusual functionalities.

INTRODUCTION

Mechanical metamaterials (1–16) represent a type of artificial materials usually consisting of periodic microstructures whose architectures are carefully designed to offer mechanical properties that surpass those of conventional materials. These architected metamaterials mainly leverage the spatial motions, extreme deformations, multiple equilibrium states, and shape morphing of microstructures to obtain exotic properties and/or functionalities, such as zero/negative values of Poisson’s ratios (17–20), thermal expansion coefficients (21–24) and swelling ratios (25, 26), reprogrammable stiffness and/or dissipation (27–33), controlled acoustic wave propagation (34, 35), and tailorable multistability (9, 36, 37). The latter property (i.e., tailorable multistability) is of rapidly increasing interests because of promising potentials for applications in information processing (38–41), recyclable energy absorption (42, 43), and soft robotic systems (44–47). These materials, sometimes termed as “multistable mechanical metamaterials,” are designed to offer more than two stable states that can be switched reversibly among each other. This requires an elaborate manipulation of the energy landscape, and several strategies have been reported, including those that rely on origami/kirigami-inspired designs (48–54), snap-through instability (42, 55–59), microstructured soft mechanisms (36, 43, 60), and geometrical frustration (61–63). For example, the diversity and high foldability of prismatic geometries have been leveraged in the context of origami techniques to develop a class of three-dimensional (3D) multistable metamaterials with periodic arrangements of rigid plates and elastic hinges (49, 52, 53). The instability-based strategies mainly exploited elastic beams capable of snapping between two different stable configurations to create bistable building block elements that can be further extended to form multistable metamaterials (40, 42, 55, 64). Despite these important progresses, it remains challenging to design hierarchical metamaterials with thousands of stable states and precisely tailored steady-state properties. In particular, ample opportunities exist in the development of scalable, tristable structural elements that can be built up to form multistable mechanical metamaterials with an extremely high number (e.g., >10⁶) of programmable stable configurations.

This paper introduces a class of X-shaped kirigami microstructures as tristable building block elements, which can be extended, following a bottom-up scheme, to achieve hierarchical mechanical metamaterials with an exponentially increased number of stable states. Here, the tristability arises mainly from the elastic tensile/compressive asymmetry of kirigami microstructures, representing a distinct mechanism from those exploited in previous designs of multistable mechanical metamaterials (36, 37, 40, 42, 43, 55–61). Multimaterial 3D printing technologies enable the fabrication and experimental validation of the programmable multistability in mechanical metamaterials with 1D cylindrical geometries, 2D square lattices, and 3D cubic/octahedral lattices. The number of stable states in the developed 2D multistable mechanical metamaterials with \((M \times N)\) unit cells follows an exponential law of \((3^{M+N})\) due to the independently controlled multistability along diverse directions. Note that although a few tristable structures have been reported (65–67), they are not scalable and cannot serve as building block elements of multistable mechanical metamaterials. Quantitative studies of the underlying mechanics establish the relationship between the geometries of X-shaped kirigami microstructures and the resulting energy landscape. The unique tristable building block structure allows the design and demonstration of fundamental mechanical ternary logic gates, as well as complex logic operators, which are unachievable previously. Compared with binary logic gates, much more information can be processed through ternary/multivalued logic gates, which can effectively reduce the design complexity and speed up the serial
arithmetic operations (68–70). The ternary logic gates also have unique applications in fuzzy logic circuits (71), asynchronous circuits (72), etc. Furthermore, the elastic tensile/compressive asymmetry of kirigami microstructures and independently controlled tristability of hierarchical metamaterials enable the realization of amplitude modulators capable of filtering low-frequency vibration along different in-plane directions with desired functions, which can be used in noise suppression (73) and nonlinear communication (74). Different from the frequency/phase modulation (75, 76) of vibration, the amplitude modulation is more challenging to realize, because the traditional engineering materials usually show similar mechanical properties under tension and compression.

RESULTS
Design concepts and demonstration of multistable mechanical metamaterials with hierarchical constructions

Figure 1A presents a schematic illustration of the hierarchical design for a multistable mechanical metamaterial consisting of 2D periodic octagonal cells extended in a square array (4 × 4 in this example). Detailed illustrations of the design of the octagonal cell are in fig. S1 and note S1. The octagonal cell can be regarded as a combination of two X-shaped structures, because the X-shaped structure shows the same mechanical responses with that of the structure in the purple frame (Fig. 1A). Such an X-shaped structure serves as a tristable building block structure. Here, the tristability mainly arises from the kirigami microstructures that offer distinct tensile and compressive moduli under uniaxial loadings, such that the X-shaped building block structure can possibly reach an equilibrium state, when the connecting bar (blue) is moved along the x axis. PolyJet multimaterial 3D printing (fig. S2) allows precise fabrication of the designed multistable mechanical metamaterials (see Materials and Methods for details). Here, the hinges, connecting bars, and substrate are all made of a hard polymer (elastic modulus $E \approx 450$ MPa; VeroWhite, Stratasys), while the kirigami microstructures are made of a soft elastomer (elastic modulus $E \approx 0.5$ MPa; TangoBlackPlus, Stratasys). Quantitative mechanics modeling of the kirigami microstructures based on finite element analyses (FEAs; see Materials and Methods for details) shows a bending-dominated deformation mechanism under uniaxial stretching (Fig. 1, B and C, and fig. S3), resulting in a much lower tensile modulus ($E_t$) than the compressive modulus ($E_c$). Experimental measurements of deformed configurations and stress-strain curve show excellent agreement with FEA results, verifying the contrasting tensile/compressive moduli, as evidenced by the modulus ratio of $E_c/E_t (\approx 101)$. Owing to such kirigami designs, the X-shaped building block structure can offer two additional stable states with the connecting bar moved leftward/rightward (Fig. 1D). Both the computed and measured load-displacement curves for rightward motion of the connecting bar have an additional minimum point (excluding the load-free state), and the corresponding force is negative (i.e., antiparallel with the x axis), pushing the bar further rightward to another equilibrium state (Fig. 1E). The calculated strain energy curve shows three minimum points (I, II, and III), which confirms the tristability of the X-shaped building block structure (Fig. 1E). Note that the modulus ratio ($E_c/E_t$) of the kirigami microstructure should be sufficiently high to ensure the tristability of the X-shaped building block structure (fig. S4).

The tristability of the X-shaped building block structure enables a large number of stable configurations in the hierarchical mechanical metamaterials. Figure 1F presents all of the nine stable states of the octagonal cell. To decouple the deformations along the x and y directions, we separate the two connecting bars along the out-of-plane direction (i.e., the z direction) in the central region of the octagonal cell (movie S1). This design allows independent control of the stable states along the horizontal and vertical directions through translational motions of the connecting bars (movie S1). Linear array of the octagonal cell along the x and y directions gives rise to a rapidly increased number of stable states. For example, the mechanical metamaterial with a 2 × 2 array of octagonal cells has 3^2 (i.e., 81) stable configurations (fig. S5), because the three stable locations of all four connecting bars can be individually addressed. In general, the number of stable states in mechanical metamaterials with $M \times N$ array increases exponentially (3^M × N) with the total number of connecting bars. This indicates 6561 (i.e., 3^8) stable configurations for the mechanical metamaterial with a square array of 4 × 4 octagonal cells. Figure 1G and movie S2 present five different stable states of the fabricated mechanical metamaterial, suggesting the capability of reshaping each octagonal cell into one of the nine possible configurations in Fig. 1F. The geometric reconfiguration can be easily implemented through translational motions of connecting bars. Specifically, pushing all the four horizontal bars (along the x axis) rightward in the as-fabricated configuration (① in Fig. 1G) can reshape the metamaterial into configuration ②. On the basis of configuration ①, pushing all the four vertical bars (along the y axis) downward and the bottom horizontal bar back to the initial configuration leads to the formation of configuration ①. Figure S6 provides 81 stable configurations of this mechanical metamaterial by moving only four connecting bars while fixing the positions of two middle horizontal/vertical bars.

Figure 2 presents the design of multistable mechanical metamaterials with 1D cylindrical geometries and 3D cubic/octahedral lattices. Figure 2A shows the schematic illustration of the cylindrical mechanical metamaterial with torsional multistability, which consists of four layers of torsional unit cells. Each torsional unit cell is composed of a driving ring, a constraint ring, a bearing, and an X-shaped tristable building block structure (movie S3). The central region of the X-shaped structure is clamped with the constraint ring and connected to the driving ring through the bearing. This design enables two additional stable configurations by applying clockwise or counterclockwise rotations to the driving ring, as evidenced by the triple-well energy profile (fig. S7). Figure 2B and movie S4 provide experimental and computational results that highlight five different stable configurations of this torsional multistable metamaterial. To stabilize at configuration ① from the initial configuration, we rotate the driving rings along the same direction in all of the four torsional unit cells. Because the three stable configurations of each torsional unit cell can be individually controlled, this mechanical metamaterial offers 3^4 (i.e., 81) stable configurations in total (fig. S8). In this case, the number of stable states in the mechanical metamaterials also increases exponentially (3^M) with the number (M) of unit cells.

Figure 2C, fig. S9, and note S1 show two representative multistable metamaterial designs constructed by extending the octagonal cell (similar to that in Fig. 1) into 3D space, following the cubic and octahedral lattice topologies. Specifically, the architecture of cubic mechanical metamaterial can be obtained by rotating the octagonal cell with respect to the black dashed lines in the leftmost panel (Fig. 2C), followed by connection to the rigid frames (blue color) at
Fig. 1. Design concepts and demonstration of 2D multistable mechanical metamaterials with X-shaped kirigami microstructures. (A) Schematic illustration of the hierarchical construction of a 2D multistable mechanical metamaterial, including the octagonal cells, X-shaped building block structure, and kirigami microstructures. (B) Optical images and FEA results of the kirigami microstructures at undeformed, stretched, and compressed states. (C) Nominal stress-strain curve of the kirigami microstructure in (B), under both the uniaxial tension and compression. (D) Optical images and FEA results of the three different stable configurations of the 3D-printed X-shaped building block structure. (E) Dependences of the normalized force and the normalized strain energy on the horizontal displacement applied to the X-shaped tristable building block structure in (D). A denotes the cross-sectional area of the microstructure; $E_c$ and $E_t$ denote the compressive and tensile moduli, respectively; $d$ denotes the distance marked in (D). (F) Experimental demonstration of the stable configurations of an octagonal cell in the mechanical metamaterial. The red arrows indicate the directions in which the horizontal and vertical connecting bars move. The middle state where no connecting bar moves is marked by a red dashed frame. (G) Experimental demonstration of five representative stable configurations of a 3D-printed mechanical metamaterial with the same geometric parameters as that in (A). Scale bars, 1 mm (B), 5 mm (D and F), and 25 mm (G). Photo credits: Hang Zhang, Tsinghua University.
the hinges and spatial separation of the connecting bars at the central region of the cube (see movie S5 for details). Each cubic unit consists of three independently controlled bars, each of which has three stable positions, due to the tristability of the X-shaped building block structure (fig. S10). Thereby, the cubic unit has $3^3$ (i.e., 27) stable configurations (fig. S11). Figure 2D (left) and movie S6 show a few typical stable configurations of a 3D-printed mechanical metamaterial with $2 \times 2 \times 1$ cubic units. Because this metamaterial has eight individually addressable connecting bars, it can offer $3^8$ (i.e., 6561) stable configurations in principle. The octahedral unit (referred to as Case 2 in Fig. 2C) contains six independently controlled bars and can thereby render much more stable configurations than the cubic unit. Theoretically, the 3D-printed mechanical metamaterial with $2 \times 2 \times 1$ octahedral units (Fig. 2D, right) can offer $3^{20}$ (i.e., 3,486,784,401) stable configurations, which is inaccessible previously. The extreme number of stable states holds promise for applications in information processing, as demonstrated by a type of mechanical ternary logic gates and combined logic operators in a subsequent section.

**Relationship between mechanical properties and geometrical designs of kirigami microstructures and X-shaped building block structures**

Understanding of the microstructure-property relationship is essential to the hierarchical design of proposed multistable mechanical metamaterials. Here, we focus on the X-shaped building block structure and establish the connection of its key geometric parameters to the resulting energy landscape. Considering the two-level construction, the geometric parameters can be divided into two categories (one related to the kirigami microstructure and the other to the X-shaped composite). Figure 3A presents a schematic illustration of the kirigami microstructure, where the geometric parameters include the cut lengths, $l_1$ and $l_2$, the cut width $c$, as well as the overall dimensions (width $a$ and length $b$) of the periodical unit. To ensure that the compressive response is very close to that of the parent material (i.e., to reduce the strain required to result in the self-contact of the microstructures), the cut width should be as small as possible and is fixed as 200 μm in this study, considering the precision of exploited commercial 3D printer (layer thickness, ~30 μm).
dimensionless cut lengths, i.e., \( \bar{l}_1 = l_1 / a \) and \( \bar{l}_2 = l_2 / a \), represent two dominant design parameters that affect the mechanical properties of kirigami microstructures. The other two dimensionless parameters are fixed as \( \bar{b} = b / a = 0.2 \) and \( \bar{c} = c / a = 0.02 \) in this set of analyses (Fig. 3), noting that their effects on the stress-strain curve of the kirigami microstructure are illustrated in fig. S12. On the basis of the energy method and incompressible Mooney-Rivlin law, a finite-deformation theoretical model can be developed to...
predict the stress-strain curve of the kirigami microstructure (fig. S13 and note S2). Figure 3B provides the measured and computed tensile stress-strain curves of kirigami microstructures with six different groups of cut lengths ($\bar{l}_1$ and $\bar{l}_2$). An excellent linear mechanical response can be observed, up to ~80% strain for $\bar{l}_1 = 0.2$ and ~40% strain for $\bar{l}_2 = 0.4$, which can be partially attributed to the relative linear stress-strain curve of the 3D-printed elastomer material (TangoBlackPlus) (fig. S14). The theoretical and FEA results are in good agreement with the experiments, indicating the mechanics model and FEA as reliable tools to guide the design of kirigami microstructures. The tensile stress-strain curves of kirigami microstructures with larger cut lengths of $\bar{l}_2$ are shown in fig. S15, where the deviations of FEA and experimental results at large strains are attributed mainly to the fracture at the ends of cuts. Figure 3C shows the contour plot of the initial elastic modulus of kirigami microstructures in terms of cut lengths $\bar{l}_1$ and $\bar{l}_2$. The tensile elastic modulus basically increases with decreasing $\bar{l}_1$ or increasing $\bar{l}_2$ because of the increased length of bending-dominated segments. To increase the compressive modulus of kirigami microstructures, the connection region (highlighted in red, Fig. 3D) can be replaced by hard polymers (VeroWhitePlus). Figure 3D shows the stress-strain curve of the 3D-printed composite kirigami microstructure ($\bar{l}_1 = 0.8$ and $\bar{l}_2 = 0.4$) under both the uniaxial tension and compression. Compared with the homogeneous design with the same geometric parameters, the compressive elastic modulus of the composite design is increased from 0.5 to 1.2 MPa, while the tensile mechanical responses are fairly close, leading to an increased modulus ratio ($E_s/E_t$) (from 101 to 240). In this case, the simulated microstructure deformations under tension and compression also agree well with the optical images (Fig. 3E).

For a prescribed kirigami microstructure design (with modulus ratio $E_s/E_t$), two additional design parameters govern the energy landscape of the X-shaped building block structure, including the angle $\theta$ of the X-shaped structure and the length ratio ($L/L_0$) of the kirigami microstructure to the total length (Fig. 3F). On the basis of the above energetic model of kirigami microstructures, the principle of virtual work can be used to calculate the reaction force for a prescribed displacement, thereby allowing the prediction of load-displacement curves of the X-shaped building block structure. Figure 3 (H and I) elucidates the effects of these two parameters on mechanical responses of the X-shaped building block structure with a homogeneous kirigami design ($E_s/E_t = 101$). As the angle $\theta$ increases, the force required to trigger the switch of stable state increases (Fig. 3H), resulting in an increased energy barrier $E_{\text{barrier}}$ of the landscape (fig. S16A). Note that the effect of friction at hinges is not very evident because of the relatively small area of contact interfaces (~4%, relative to the in-plane projection area of the mechanical metamatarial). For the X-shaped building block structure with the same length ratio ($L/L_0$), the increased angle ($\theta$) also enables a distinct configuration of the higher-order stable state, as evidenced by the larger displacement of the equilibrium state (Fig. 3G). In addition, the X-shaped building block structures can survive high levels of deformations, as evidenced by the large stretchability (or compressibility) measured experimentally (fig. S17A). The excellent deformability can ensure the reliable switch of the stable state without any failure. Note that the tristability disappears for $\theta < 24°$ because of the vanishing energy barrier. Figure 3I shows that the length ratio ($L/L_0$) of the kirigami microstructure only affects the energy barrier $E_{\text{barrier}}$ and plays a negligible role on the displacement of the higher-order stable states (fig. S16B). A lower length ratio ($L/L_0$) is preferred to increase the energy barrier, thereby enhancing the stability of higher-order modes. Figure 3J and fig. S16C illustrate that the use of composite kirigami design in the X-shaped building block structure yields an enlarged energy barrier and improved tristability, in comparison to that with a homogeneous kirigami design. Specifically, the modulus ratio (i.e., 240) of the composite kirigami microstructure containing harder materials is much larger than that (i.e., 101) of the homogeneous design (Fig. 3D). For the X-shaped building block structure with $\theta = 30°$, the use of composite kirigami design enables the tristability, while the use of homogeneous kirigami design does not. For the X-shaped building block structures with $\theta = 30°$ or 35°, the use of composite kirigami design results in a larger force required to switch to high-order stable state, as compared with the case of homogeneous kirigami design (Fig. 3J). The theoretical results show reasonable agreements with the FEA and theoretical results (Fig. 3, H to J). The relatively large deviations near the peak of the load-displacement curve (e.g., in the case of $\theta = 30°$, $L/L_0 = 0.38$, and $\eta = 101$) are mainly due to the large nominal strain (i.e., >100%) of kirigami microstructure, where the theoretically calculated stress-strain curves of the kirigami microstructure show large discrepancies from the FEA and experimental results. In addition, detailed discussions on the applicability of the FEA methods are provided in note S3.

**Mechanical ternary logic gates**

The design flexibility of the X-shaped tristable building block structure facilitates the application in mechanical ternary logic operation, which is unachievable using bistable building block structures (39, 40). For example, in the mechanical system presented by Raney et al. (40), it is very challenging to combine many basic gates for complex logic operations. In comparison to previously reported works (39, 40), the ternary logic gates presented here exploit modular designs, which facilitates the extension of basic gates for complex operations. In addition, compared with the binary logic operation, the ternary logic operation can transmit a larger amount of information and exploit a reduced number of basic gates to complete the same logic operation, showing advantages for applications in fuzzy logic (71) and arithmetic and signal processing (68).

Figure 4A presents the schematic illustration of a mechanical ternary NOT gate that is composed of two modules, including an analog-to-digital converter and a digital displacement processor (fig. S18). The analog-to-digital converter is realized by the X-shaped tristable building block structure that converts the continuous displacement input into three discrete values. Specifically, the small displacement input that leads to the first-order stable state (initial configuration) of the X-shaped structure is indicated by 0, and the large negative/positive displacement inputs corresponding to two symmetric high-order stable states (deformed configurations) are indicated by −1 and 1, respectively. The digital displacement processor is realized by the mechanical conversion design proposed by Merkle (77), which can reverse the direction of the input displacement. A T-shaped groove is introduced in the substrate to limit the out-of-plane (i.e., z direction) displacements of the converter. Figure 4B and movie S7 provide the experimental demonstration of the operations of the fabricated NOT gate. The ternary logic operations of AND (A ∧ B) and OR (A ∨ B) (Fig. 4C) are more complicated than the binary operations, and the corresponding mechanical gates can be achieved by introducing strategic designs of digital displacement processor. Basically, the key is to enable the conversion of two input signals into a desired output signal, according to the truth
tables (Fig. 4C). Specifically, the output of the AND gate should depend on the minimum of two inputs, while the output of the OR gate should depend on the maximum of two inputs. For example, the digital displacement processor of the AND gate is composed of three parts (Fig. 4D and fig. S19): (i) a converter that serves to transform two displacements into a single output displacement (i.e., at the middle of the converter); (ii) latches that push the converter to move and restrict the converter from being pulled by the kirigami microstructures; and (iii) kirigami microstructures that pull the converter back when the two latches both pull the converter. Figure 4D

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**Fig. 4. Design and experimental demonstration of the ternary mechanical logic gates.** (A) Schematic illustration of the NOT gate based on the X-shaped tristable structure. (B) Three operation states of the NOT gate. (C) Truth table of the AND and OR logic operations. (D and E) Six representative operation states of the AND and OR gates. (F) Integration of the three basic logic gates to realize a complex logic operation \((A \lor B) \land \neg C\), with six representative states provided herein. (G) Six representative states of a complex logic operation, \((\neg(A \lor B)) \land (C \land D)\), based on the multistable mechanical metamaterials with a square array of \(2 \times 2\) octagonal cells. Scale bars, 10 mm. Photo credits: Hang Zhang, Tsinghua University.
and movie S7 show the experimental demonstration of the fabricated AND gate for all possible operations. Following a similar strategy, the ternary OR gate (fig. S20) can be designed, and the digital displacement processor is also composed of three parts: (i) a converter with two rectangular cavities; (ii) T-shaped bars that pull the converter to move and restrict the converter from being pushed by the springs; and (iii) springs that push the converter back when the two T-shaped bars both push the converter. The logic operations are demonstrated in Fig. 4E. The flexibility of the modular design facilitates complex logic operations based on the above basic gates. Figure 4F and movie S8 provide an example that realizes the operation, \((A \lor B) \land \neg \neg (C)\), for three inputs (A, B, and C). As shown in fig. S21A, the digital displacement processor follows from a combination of digital displacement processors of three basic gates. Another example of the operation, \(\neg (A \land B)\), is shown in figs. S21B and S22 and movie S8. The quasi-static time domain diagram of these basic and complex logic operations is shown in fig. S23.

The basic logic gates (e.g., AND gate) allows demonstration of a simple voting device with two different working modes (fig. S24). Specifically, a displacement sensor is used to detect the output displacement of the AND gate. After calibration, the sensor could generate an output voltage (~24 V) to turn on the light-emitting diode (LED), if it senses an object within a preset range of distance. For the voting device, each voter has three choices, including "Support," "Abstain," and "Oppose," corresponding to the normalized output "1," "0," and "-1," respectively. In the first voting mode (fig. S24C), the LED light is turned on, only when both voters choose "Support." By changing the preset range of distance that can be sensed by the displacement sensor, the device can be switched to a different voting mode (fig. S24D), in which the LED light is turned on, as long as one voter chooses "Oppose."

The large number of stable states enabled by the multistable mechanical metamaterials allows for complex ternary operations of multiple (e.g., >3) inputs. Figure 4G presents a logic operator based on a mechanical metamaterial (with 2 × 2 array of octagonal cells) that serves as the analog-to-digital converter. Integrated with a specially designed digital displacement processor, this device can realize a complex target operation \((\neg \neg (A \lor B) \land (C \land D))\) for four different inputs (Fig. 4G and figs. S25A and S26). As shown in fig. S25A, the analog-to-digital converter consists of a 2 × 2 array of octagonal cells. The digital displacement processor of the OR gate is adopted in the x direction, and the digital displacement processor of the NOT gate is used to convert the displacement in the x direction to the y direction. Another two digital displacement processors of the AND gate are used to combine the inputs into a single output. This type of logic operator can also enable the parallel processing of the inputs along different directions, leading to two independent displacement outputs (figs. S25B and S27).

**Amplitude modulation of the low-frequency vibration**

Figure 5A presents the design of a bidirectional amplitude modulator with developed multistable mechanical metamaterial. Here, the kirigami microstructures connected directly to the input ports serve to weaken the transmission of associated forces for displacement loading, while the mechanical metamaterial combines the transmitted force with the tristable units to achieve a regulated displacement output. Figure 5 (B to D) shows three representative operational cases of the device for the filtering of low-frequency (i.e., ~0.017 Hz) vibration. For low vibration amplitudes (e.g., 15 mm), the amplitude modulator filters the triangular wave into a truncated triangular wave (Fig. 5B and movie S9), because the small tensile modulus of the kirigami microstructure almost blocks the transmission of the negative input displacement (Fig. 5B, bottom). As a result, the negative input displacement is suppressed tremendously (attenuation rate, ~95%), while the positive input displacement is transmitted with a relative high fidelity (attenuation rate, ~30%). For medium levels of vibration amplitude (e.g., 25 mm), the modulator filters the triangular wave into a step wave (Fig. 5C and movie S9). In this case, the positive input displacement triggers the switch of the octagonal cells into a high-order stable state, while the force associated with the negative input is not large enough to recover the octagonal cell to the original state. Here, we use the critical vibration amplitude \(A_{\text{cr}}\) to denote the displacement at which the switch of stable states occurs during vibration. The critical vibration amplitude depends primarily on the displacement of the first zero-crossing point of the load-displacement curve that corresponds to the peak point of the energy-displacement curve (see note S4 for details). In the specific example shown in Fig. 5C, the critical vibration amplitude \(A_{\text{cr}}\) to trigger this mode switch of stable state is ~20 mm (note S4 and fig. S28A). For high levels of vibration amplitude (e.g., 55 mm), the large positive input displacement moves the output port along the positive direction further, resulting in sharp conical bulges in the curves of the output displacement (fig. S28B). With a further increase in the vibration amplitude (e.g., to 60 mm), the structure can be pulled from the high-order stable state back to the initial state. On the basis of the above critical vibration amplitude, for an asymmetric vibration with the maximum positive and negative input displacements fixed at 10 and 30 mm, respectively, the triangle wave can be filtered into an output close to a square wave (Fig. 5D and movie S9). The ratios of rise time (~7.0 s) and fall time (~1.5 s) to the vibration period are ~1.17 and 2.5%, respectively, representing a fast mode switch. Here, the amplitude of the square wave depends only on the displacement of the equilibrium state of the octagonal cell. The bottom panels (V, VI, and VII) correspond to the deformed configurations marked in the curves. Because of the symmetry of the octagonal cell, the mechanical responses (fig. S29) for the vibration along a vertical direction (i.e., y direction) are very close to that along a horizontal direction (i.e., x direction) in Fig. 5 (B to D), suggesting that the modulator here can work well in both in-plane directions. In addition, the amplitude modulator shows a good durability, as evidenced by the cyclic testing, where the displacement output is quite stable after 6000 cycles, for the vibration (triangular wave form) with an amplitude of ~15 mm and a frequency of ~0.028 Hz (fig. S30). For vibration with a higher frequency (e.g., 0.05 and 0.1 Hz), the modulator can achieve a similar filtering function (fig. S28, C to E). Here, the overall size (~70 mm) of the octagonal cell and the loading rate (up to 2 mm/s) of the testing equipment based on the stepping motor set a practical limit to the frequency that can be achieved in experiments. Theoretically, the damping of the kirigami microstructure could also affect the frequency limit of the system, because the 3D-printed rubber material (i.e., TangoBlackPlus) used in the current work shows a certain level of viscoelastic behavior at room temperature. This could be addressed by using a lower-damping material (with a reduced viscoelasticity) in the fabrication. Because the developed amplitude modulator mainly leverages the multistability of the octagonal cell and the different tensile/compressive stiffness of the kirigami microstructure, similar filtering function can be expected for inputs with other wave forms (e.g., sinusoidal waves).
Typically, for robots in harsh environment (e.g., high radiation and strong magnetic fields), electronic devices would not work properly, and mechanical devices could be a good alternative. The amplitude modulator can be used under these circumstances to filter different input signals into desired output signals. For example, the modulator could serve as a displacement detector to sense whether the input displacement exceeds a given threshold or a damping platform that reduces substantially the influence of the input displacement on the output. The modulation of vibration amplitude demonstrated here can be potentially used in noise suppression (73) and nonlinear communication (74).

**DISCUSSION**

This paper reports the design, fabrication, and characterization of a class of hierarchical mechanical metamaterials with an exponentially increased number of stable states. Starting from the programmable X-shaped tristable building block structure, hierarchical mechanical metamaterials with 1D cylindrical geometries, 2D square lattices, and 3D cubic/octahedral lattices are designed and demonstrated, with capabilities of torsional multistability or independent controlled multidirectional multistability. Validated mechanics modeling serves as the basis of the design strategy and sheds light on the underlying relationship between the microstructural geometries and the resulting energy landscape. The design flexibility and ample stable states allow demonstrative applications in mechanical ternary logic gates, including three basic gates (i.e., NOT, AND, and OR gates) and their combined logic operations. We also harness the structural diversity and independently controlled tristability to realize a type of amplitude modulator that can filter low-frequency vibration of different amplitudes into different wave forms (e.g., truncated triangular wave, step wave, and square wave). These mechanical devices hold promising potentials for uses in the motion/configuration control of soft actuators and robotics. Compared with traditional electrical devices, these mechanical devices show advantages in energy saving (77) and resistance to...
corrosion (78) in a harsh environment (e.g., high radiation and strong magnetic fields).

MATERIALS AND METHODS
Fabrication and mechanical testing of multistable mechanical metamaterials
All the samples in this study were prepared by multimaterial PolyJet 3D printing (Objet350, Stratasys). Two different materials (VeroWhitePlus and TangoBlackPlus) available through multimaterial 3D printers (Objet350, Stratasys) were adopted, and both were cured by ultraviolet lamps at room temperature. TangoBlackPlus is a soft elastomer (~0.5 MPa), mainly composed of exo-1,7,7-trimethylbicyclo[2.2.1]hept-2-yl acrylate and photoinitiators. VeroWhitePlus is a harder polymer (~680 MPa), mainly composed of exo-1,7,7-trimethylbicyclo[2.2.1]hept-2-yl acrylate, tricyclooctane dimethanol diacrylate, titanium dioxide, and photoinitiators. A digital camera (760D, Canon) recorded the deformation process of the mechanical metamaterials, as well as the operations of the ternary logic operators and the amplitude modulator. The load-displacement curves of the steady-state unit were obtained by a commercial mechanical testing machine. A customized machine enabled the loading and unloading of the amplitude modulator with a constant loading rate of preset magnitude.

Finite element analyses
The commercial software ABAQUS (SIMULIA) was used to perform the FEAs. We adopted an implicit solver to calculate the deformations of the multistable mechanical metamaterials, the tensile/compressive stress-strain curves of the kirigami microstructures, and the load-displacement curves of the multistable mechanical metamaterials, through heterogeneous architected materials and the operations of the ternary logic operators and the amplitude modulator. The load-displacement curves of the steady-state unit were obtained by a commercial mechanical testing machine. A customized machine enabled the loading and unloading of the amplitude modulator with a constant loading rate of preset magnitude.

SUPPLEMENTARY MATERIALS
Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/7/9/eabf1966/DC1

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Acknowledgments
Funding: Y.Z. acknowledges support from the National Natural Science Foundation of China (grant nos. 11722217 and 11921002), the Tsinghua National Laboratory for Information Science and Technology, the Tsinghua University Initiative Scientific Research Program (grant no. 2019Z08QCX10), and a grant from the Institute for Guo Qiang, Tsinghua University (grant no. 2019GQG01012). Author contributions: Y.Z. designed and supervised the research. Y.Z. and H.Z. led the structural designs, mechanics modeling, and experimental work, with assistance from J.W. Y.Z., H.Z., and D.F. wrote the manuscript and designed the figures. All authors commented on the paper. Competing interests: The authors declare that they have no competing interests.

Data and materials availability: All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplementary Materials. Additional data related to this paper may be requested from the corresponding authors.

Submitted 13 October 2020
Accepted 11 January 2021
Published 24 February 2021
10.1126/sciadv.abbf166

Citation: H. Zhang, J. Wu, D. Fang, Y. Zhang. Hierarchical mechanical metamaterials built with scalable tristable elements for ternary logic operation and amplitude modulation. Sci. Adv. 7, eabbf166 (2021).
Hierarchical mechanical metamaterials built with scalable tristable elements for ternary logic operation and amplitude modulation
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Sci Adv 7 (9), eabf1966.
DOI: 10.1126/sciadv.abf1966

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