QED corrections to Planck’s radiation law and photon thermodynamics

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(September 6, 2018)

Abstract

Leading corrections to Planck’s formula and photon thermodynamics arising from the pair-mediated photon-photon interaction are calculated. This interaction is attractive and causes an increase in occupation number for all modes. Possible consequences, including the role of the cosmic photon gas in structure formation, are considered. [hep-th/9308089]

PACS numbers: 12.20.Ds, 05.30.Jp, 14.80.Am, 98.80.Cq
I. INTRODUCTION

Electromagnetic radiation in thermal equilibrium is a fundamental physical system which has played a crucial, if serendipitous, role in opening up new frontiers in physics; few systems are as ubiquitous or have played as significant a role in the progress of modern physics and astronomy. Indeed it was in an attempt to describe the properties of cavity radiation that Planck stumbled upon the notion of quantization in 1900, and, 65 years later, it was while testing a microwave antenna designed for communications research that Penzias and Wilson discovered the cosmic microwave background radiation, the single most important clue to the past history of the universe. The recent discovery by Smoot and coworkers [1] of large-angle temperature fluctuations in the background radiation, which, by contrast, had long been anticipated, promises to be another milestone in the quest for the origins of the universe. The distribution law discovered by Planck accurately describes the equilibrium properties of an assembly of photons over a vast range of temperatures and scales, from terrestrial cavity radiation (from which it was deduced) to hot stellar atmospheres, and, of course, including the 2.73K cosmic background radiation. Nevertheless, there are small, high-temperature corrections to Planck’s law and photon thermodynamics which, in view of the pervasive presence of radiation systems in general and the singularly important role of the background radiation for cosmic evolution in particular, may play a significant role in an as yet undiscovered phenomenon. For this reason, as well as their intrinsic interest as high temperature modifications to a fundamental law of nature, these corrections are worthy of serious attention. This communication is devoted to the calculation of these corrections and the exploration of some of their consequences.

The Planck law basically describes a noninteracting gas of massless, spin-one bosons (not subject to number conservation), driven to equilibrium by interaction with an external system, e.g., the atoms in the walls of the cavity in the case of cavity radiation, and the charged particles present in the cosmic fluid in the case of the background radiation. The virtual nonexistence of an interaction between photons under ordinary conditions, a hallmark of
Maxwell electrodynamics and the working principle of virtually all existing telecommunication systems, is what sets the photon gas apart from other ideal gases. To highlight this difference, let us consider the fact that the mean free path for hydrogen under standard conditions is about $10^{-6}$ m, allowing hydrogen to behave like a continuous medium on scales larger than a micron. By contrast, the same quantity for a photon gas is a staggering $10^{61}$ m [cf. Eq. (19) below], implying that under ordinary conditions photons behave like a collection of independent, free-streaming particles on any physically meaningful scale.

The absence of interaction between photons and the linearity of electrodynamics are of course violated by the process of pair creation, a fact that was realized by Halpern [2] as early as in 1933. Not long before this, Delbrück [3] had suggested that quantum effects would cause photons to be scattered by an external electric field. Delbrück’s phenomenon, which was observed in the scattering of 1.33 Mev gamma rays by the Coulomb field of the lead nucleus some years later [4], is essentially the interaction of a real photon with a virtual one [5]. By contrast, the interaction of two real photons predicted by Halpern has never been detected because of its extreme weakness. The center-of-mass photon-photon cross section is equal to $0.031\alpha^2 r_e^2(\omega/m)^6$ for $\omega/m \ll 1$ and nearly equal to it almost up to the pair production threshold $\omega = m$ [6]; here $\alpha$ is the fine-structure constant, $r_e$ the classical electron radius, $\omega$ the photon energy, and $m$ the electron mass (natural units with $\hbar = c = k = 1$ will be used unless otherwise stated). For 633 nm light, this cross section is about $4 \times 10^{-64}$ cm$^2$, and it rises steeply to about $3 \times 10^{-30}$ cm$^2$ near the production threshold. These minute cross sections are the reason why phenomena involving the interaction of real photons have not been much discussed in connection with nonlinear electrodynamic effects, attention having been focused instead on the magnetic counterpart of Delbrück scattering and other effects [7]. It is reasonable to expect, however, that these tiny magnitudes will eventually succumb to high-precision experiments, or to new observations. Indeed a scheme has already been proposed for detecting the photon-photon interaction by colliding laser beam interferometry, using a modification of Michelson interferometers developed for gravity wave detection [8].

The present discussion deals with photon interactions in equilibrium distributions, the
opposite extreme to highly-ordered, far-from-equilibrium distributions characteristic of laser light. While at low (i.e., $\ll m$) temperatures a photon gas is basically pure, at sufficiently high temperatures a significant contamination of electron-positron (and possibly other) pairs may be present, making it inappropriate to treat the gas as a single-component system. To ensure that the photon gas is effectively free of contamination, we shall restrict our treatment to temperatures for which the photon-photon collision rate dominates the collision rate of the photons with the pair-produced electrons or positrons so that the latter play no significant role in the thermodynamics of the photon gas. We shall see below that the condition $T \leq 1.5 \times 10^8$ K, where $T$ is the temperature of the photon gas, is sufficient for this purpose.

Now for this range of temperatures, the parameter $T/m$ is no larger than $2.5 \times 10^{-2}$ and can be treated as small. Since, for dimensional reasons, the dependence of the corrections we are seeking on $T$ must be through the combination $T/m$, we can achieve a considerable simplification in the calculations by limiting them to the leading order in this parameter, with essentially no loss in accuracy. Our objective, therefore, is to derive the thermodynamics of a photon gas (i.e., the charge-zero, equilibrium state of QED) up to and including the leading correction in the parameters $\alpha$ and $T/m$.

The leading contribution to photon-photon scattering is of second order in $\alpha$ and originates in the box diagram in which four external photon lines are attached to a closed electron loop [9]. In leading order, the matrix element for this diagram is directly related to the interaction energy of a pair of photons, so that the corrections in question will be of relative order $\alpha^2(T/m)^4$, the power of 4 resulting from the four electron lines of the box diagram in the limit $T/m \to 0$. Obviously, these corrections will only be manifested under extreme conditions, requiring very high temperatures or very large spatial dimensions. Such conditions may be achievable in controlled experiments, or may have prevailed during the evolution of the universe subsequent to the annihilation of positrons. For example, the elastic properties of the cosmic photon gas may have played a role in the early stages of density perturbation growth and structure formation. As will be shown below, the cosmic background radiation has in the past been under conditions such that the photon-photon
collision rate exceeded the expansion rate. This fact implies the possibility of independently propagating sound waves or supporting growing density perturbations for the photon gas. Accordingly, the results derived below will be explored in some detail with a view to their cosmological implications, although there do not appear to be any observable consequences at this time.

II. PHOTON-PHOTON INTERACTION ENERGY

The thermodynamics of a photon gas can be conveniently derived in the canonical ensemble where the density matrix and the partition function are given by

\[ \hat{\rho}(\beta, V) = Z^{-1}(\beta, V) \exp[-\beta \hat{H}(V)], \]

\[ Z(\beta, V) = \text{tr}\{\exp[-\beta \hat{H}(V)]\}. \]

Here \( V \) is the volume and \( \beta = T^{-1} \). To the accuracy sought here, the Hamiltonian may be written as \( \hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} \), where \( \hat{H}_0 \) describes noninteracting photons and \( \hat{H}_{\text{int}} \) is the effective interaction Hamiltonian resulting from the box diagram. Recall that our objective is to calculate \( \hat{\rho} \) in the limit \( T/m \to 0 \). Since the amplitude resulting from the box diagram is convergent, this limit can be implemented by taking the static limit, \( m \to \infty \), of \( \hat{H}_{\text{int}} \) in Eqs. (1) and (2), a step that considerably simplifies the calculation. One simply calculates the amplitude for the box diagram in the static limit, then converts this into \( \hat{H}_{\text{int}} \), in much the same way that the Born amplitude is converted into an interaction potential when dealing with potential scattering.

A convenient alternative to the above procedure is to take advantage of a well known result, known as the Heisenberg-Euler Lagrangian [10], that recasts the static limit of the box diagram amplitude as an effective interaction Lagrangian expressed in terms of the field variables. The density corresponding to this Lagrangian is

\[ \hat{L}_{\text{int}}(\hat{E}, \hat{B}) = \frac{2\alpha^2}{45m^4}[(\hat{E}^2 - \hat{B}^2)^2 + 7(\hat{E} \cdot \hat{B})^2]. \]
A simple calculation shows that with \( \hat{\mathcal{L}}_{\text{int}} \) treated to first order, the interaction Hamiltonian density is given by \( \hat{\mathcal{H}}_{\text{int}} = -\hat{\mathcal{L}}_{\text{int}} \), i.e., \( \hat{\mathcal{H}} \) is equal to \( \frac{1}{2}(\hat{\Pi}^2 + \hat{\mathcal{B}}^2) - \hat{\mathcal{L}}_{\text{int}}(\hat{\Pi}, \hat{\mathcal{B}}) \). Note that the canonical momentum density \( \hat{\Pi} \) is now equal to \( \hat{\mathcal{E}} + \partial \hat{\mathcal{L}}_{\text{int}}(\hat{\mathcal{E}}, \hat{\mathcal{B}})/\partial \hat{\mathcal{E}} \), the second term being the effect of the interaction [11].

The next step is to calculate the diagonal matrix elements of the interaction Hamiltonian \( \hat{\mathcal{H}}_{\text{int}} \) in the photon number representation (these being the only ones needed in the following calculations). It is convenient to carry out this calculation in the Coulomb gauge using the complex field \( \hat{\mathcal{G}} = \hat{\Pi} + i\hat{\mathcal{B}} \). The plane-wave expansion of \( \hat{\mathcal{G}} \) in finite-volume normalization appears as
\[
\hat{\mathcal{G}}(t, \mathbf{x}) = \sum_{\mathbf{k}, \lambda} \frac{i k}{\sqrt{2kV}} f_\lambda(\hat{\mathbf{k}}) \times \left[ \hat{a}_{\mathbf{k}, \lambda}(t) \exp(i\mathbf{k} \cdot \mathbf{x}) - \hat{a}_{\mathbf{k}, \lambda}^\dagger(t) \exp(-i\mathbf{k} \cdot \mathbf{x}) \right],
\]
where \( k = |\mathbf{k}| \) and \( \hat{\mathbf{k}} = \mathbf{k}/k \). Here the unit vectors \( \mathbf{e}_\lambda(\hat{\mathbf{k}}) \), \( \lambda = 1, 2 \), represent the two polarization directions normal to \( k \), and the complex vector \( f_\lambda(\hat{\mathbf{k}}) \) is given by \( \mathbf{e}_\lambda(\hat{\mathbf{k}}) + i\hat{\mathbf{k}} \times \mathbf{e}_\lambda(\hat{\mathbf{k}}) \). Furthermore, \( \hat{a}_{\mathbf{k}, \lambda} \) and \( \hat{a}_{\mathbf{k}, \lambda}^\dagger \) are standard destruction and creation operators subject to the commutation relations
\[
[\hat{a}_{\mathbf{k}, \lambda}(t), \hat{a}_{\mathbf{k}', \lambda'}^\dagger(t)] = \delta_{\mathbf{k}, \mathbf{k}'} \delta_{\lambda, \lambda'}.
\]
It is useful to note here that the product \( f_\lambda(\hat{\mathbf{k}}) \cdot f_{\lambda'}(\hat{\mathbf{k}}') \) vanishes for \( \mathbf{k} = \mathbf{k}' \).

We now proceed to calculate the diagonal matrix elements of \( (\hat{\mathcal{G}}^2)^2 \) and \( \hat{\mathcal{G}}^2(\hat{\mathcal{G}}^\dagger)^2 \). Let \( |n\rangle \) represent the state with \( n(\mathbf{k}, \lambda) \) photons of momentum \( \mathbf{k} \) and polarization \( \mathbf{e}_\lambda(\hat{\mathbf{k}}) \). Then, following a lengthy but essentially straightforward calculation, we find
\[
\langle n|(\hat{\mathcal{G}}^2)^2(\hat{\mathcal{G}}^\dagger)^2|n\rangle = \frac{2}{V^2} \sum_{1,2} k_1 k_2 |f_1 \cdot f_2|^2 n_1 n_2,
\]
where the sum runs over all possible values of \( (\mathbf{k}_1, k_1) \) and \( (\mathbf{k}_2, k_2) \), and the abbreviations \( f_i = f_{\lambda_i}(\hat{\mathbf{k}}_i) \) and \( n_i = n(\mathbf{k}_i, \lambda_i) \) have been introduced. The corresponding matrix element for \( (\hat{\mathcal{G}}^2)^2 \) is obtained from this formula by replacing \( |f_1 \cdot f_2|^2 \) with \( (f_1 \cdot f_2)^2 \). Note that in these sums all terms with \( \mathbf{k}_1 = \mathbf{k}_2 \) vanish as a result of the vanishing property of \( f_1 \cdot f_2 \) mentioned above.
The photon-photon interaction energy can now be written down using the above information; the result is

$$\langle n|\hat{H}_{\text{int}}|n\rangle = -\frac{\alpha^2}{45Vm^4}\sum_{1,2}n_1, n_2 k_1 k_2(4R^2 + 7I^2),$$

(4)

where $R$ and $I$ are the real and imaginary parts of the quantity $f_1 \cdot f_2$. Note that $\langle n|\hat{H}_{\text{int}}|n\rangle$ represents a sum of negative two-body interaction terms; i.e., photons attract photons. Furthermore, both $R$ and $I$ vanish for $\hat{k}_1 = \hat{k}_2$ as a result of the vanishing of $f_1 \cdot f_2$ mentioned above; i.e., parallel photons don’t interact. The latter property can be traced to the fact that the center-of-mass energy of a pair of parallel photons vanishes, implying the same for the corresponding cross section.

III. PHOTON THERMODYNAMICS

To first order in the interaction, the density matrix of Eq. (1) can be represented as

$$\hat{\rho} = Z^{-1}\int_0^1 d\xi \exp(-\xi\beta\hat{H}_0)(1 - \beta\hat{H}_{\text{int}})\exp[(\xi - 1)\beta\hat{H}_0],$$

(5)

from which the partition function can be calculated using the result given in Eq. (4);

$$Z = \sum_{\{n\}}(1 - \beta\langle n|\hat{H}_{\text{int}}|n\rangle)\exp(-\beta\langle n|\hat{H}_0|n\rangle),$$

(6)

where $\langle n|\hat{H}_0|n\rangle$ is the energy of an assembly of noninteracting photons, given by the familiar expression $\sum n(k, \lambda)k$. The symbol $\{n\}$ in Eq. (6) indicates an unrestricted summation (since photon number is not fixed) over $n(k, \lambda)$ for each possible value of $(k, \lambda)$.

Thermodynamic quantities can now be calculated as ensemble averages using Eqs. (5) and (6). For example, the mean energy $U(\beta, V)$, defined by $tr(H\hat{\rho})$, can be calculated by applying the formula $-(\partial/\partial\beta)\ln Z$. To find $U$, we first substitute (4) in (6) and perform the sum over $\{n\}$ to get

$$Z = \left[1 - \beta\sum_{1,2} g(1,2)n_0(k_1)n_0(k_2)\right]Z_0,$$

(7)
where $g(1, 2)$ is the quantity multiplying the photon numbers in Eq. (4), $\bar{n}_0(k)$ is the mean occupation number for noninteracting photons as given by the Planck formula $[\exp(\beta k) - 1]^{-1}$, and $Z_0$ is the associated partition function $\prod[1 - \exp(-\beta k)]^{-2}$. The product in the last expression runs over all possible values of $k$.

To complete the calculation of $U$, we find it convenient to replace $g(1, 2)$ in (7) by its average, $\bar{g}(k_1, k_2)$, over the two polarization directions as well as the orientations of the momenta (a permissible operation since $\bar{n}_0$ only depends on $k$); the result is $-22\alpha^2 k_1 k_2/135V m^4$. Using this result in (7) and carrying out the remaining sums (in the large-volume, continuum limit), we find

$$Z = \left[1 + \frac{22\alpha^2 \beta V u_0^2}{135 m^4}\right] Z_0,$$

where $u_0$ is the mean energy density of noninteracting photons, equal to $\pi^2/15\beta^4$.

The mean energy density $u = U/V$ can now be calculated from (8); the result is

$$u = \left[1 + \frac{154\pi^2 \alpha^2}{2025} (T/T_e)^4\right] u_0,$$

where $T_e = m$ is equal to 5.9 GK. As expected, the post-Planckian correction in Eq. (9) is very small at terrestrially occurring temperatures, except possibly those achievable in experiments on thermonuclear research. The sign of the correction is also noteworthy. Although the interaction is attractive, at a given temperature the interacting photon gas has a higher energy density than the noninteracting one. To elucidate this result, let us consider the relation

$$u = u_0 + V^{-1}(1 - T\partial/\partial T)\bar{H}_{\text{int}},$$

which is valid to first order in the correction. Here $\bar{H}_{\text{int}}$ is the mean value of the interaction Hamiltonian $tr(\hat{H}_{\text{int}}\hat{\rho})$ in leading order. A straightforward calculation now yields

$$\bar{H}_{\text{int}} = -\frac{22\pi^2 \alpha^2 V}{2025} (T/T_e)^4 u_0.$$

Equation (10) shows that if, as is the case here, $\bar{H}_{\text{int}}$ depends on the temperature more strongly than linear, then $u$ will exceed $u_0$ in case the interaction is attractive. We must
therefore conclude that the negative photon-photon interaction energy is in the present instance more than compensated by an increase in the average number of photons in each mode, resulting in a net increase in the internal energy.

To verify the increase in photon occupation number just predicted, we proceed to calculate the ensemble average of the photon number operator, \( tr[\hat{n}(k, \lambda)\hat{\rho}] \). To the leading order we find, after some algebra,

\[
\bar{n}(k) = \left\{ 1 + [1 + \bar{n}_0(k)] \frac{44\pi^2\alpha^2k}{2025m} (T/T_e)^3 \right\} \bar{n}_0(k).
\]

(12)

This formula gives the leading correction to Planck’s radiation law arising from vacuum polarization. As predicted above, the correction is an increase in occupation number for all modes. Note that the increase is more pronounced for higher energy modes, reflecting the increase with energy of the photon-photon scattering cross section.

Other quantities of interest are calculated in a similar manner. For example, the pressure for the photon gas is found to be

\[
p = \left[ 1 + \frac{22\pi^2\alpha^2}{675} (T/T_e)^4 \right] p_0,
\]

(13)

where \( p(p_0) \) is the pressure for the interacting (noninteracting) case. Likewise, the speed of sound, given by \( dp/du \) in the present case, is found with the help of Eqs. (9) and (13). The result is

\[
v_s = \left[ 1 - \frac{88\pi^2\alpha^2}{2025} (T/T_e)^4 \right] \frac{c}{\sqrt{3}},
\]

(14)

where \( c \) is the speed of light in \textit{vacuum}. The speed of light in the photon gas, on the other hand, is found by first calculating the interaction energy \( \omega_{\text{int}} \) of an “external” photon of momentum \( k \) with the photon gas using Eq. (4). The result is

\[
\omega_{\text{int}} = -\frac{44\pi^2\alpha^2}{2025} (T/T_e)^4 k.
\]

(15)

The speed of light in the photon gas, \( v_\gamma \), is then found from the formula \( 1 + d\omega_{\text{int}}/dk)c; \)

\[
v_\gamma = \left[ 1 - \frac{44\pi^2\alpha^2}{2025} (T/T_e)^4 \right] c.
\]

(16)
This result implies that a photon gas in equilibrium acts as a linear, isotropic refractive medium for the propagation of electromagnetic waves, its index of refraction being equal to \( c/v_\gamma \).

As already stated, these post-Planckian corrections are very small, even at high temperatures. For example, at a temperature of \( 10^{-1}T_e \), which is of the order of the highest temperatures presently achievable in thermonuclear research, the fractional correction to Planck’s formula is of the order of \( 10^{-9} \) (one must also remember the contaminations arising from the presence of a plasma and the complications caused by the short-lived nature of the temperature peak in an actual experiment). Furthermore, as discussed earlier, the above results should be supplemented by the condition that requires a photon to collide much more frequently with another photon than with the equilibrium population of pair-produced electrons or positrons. To find the inequality that enforces this condition, we must first calculate the photon-photon collision frequency.

Using the standard result for the number of collisions in a gas, we find for the collision frequency of a given photon,

\[
\nu_{\gamma\gamma}(k_1) = V^{-1}\sum_{2} \bar{n}_0(k_2)|\hat{k}_1 - \hat{k}_2|\sigma(k_1, k_2),
\]

where \( \sigma(k_1, k_2) \) is the scattering cross section for a pair of photons of momenta \( k_1 \) and \( k_2 \). Transforming this expression to the center-of-mass system and using the value of \( \sigma \) given earlier, we find

\[
\nu_{\gamma\gamma}(k) = \frac{2224\pi^3\alpha^2}{455625}r^2\omega^3 (T/T_e)^6
\]

(18)

for the collision frequency of a photon of momentum \( k \) with the other photons.

The mean collision frequency can now be found by averaging \( \nu_{\gamma\gamma}(k) \). The result, stated as a mean free path, is given by

\[
\lambda_{\gamma\gamma}[m] = 1.8 \times 10^{-5} \left( \frac{T[K]}{5.9 \times 10^9} \right)^{-9}.
\]

(19)

This equation gives the mean free path (expressed in meters) in terms of the temperature (expressed in Kelvins). Now the condition that ensures a pure photon gas requires that
\[ \lambda_{\gamma\gamma} \ll \lambda_{\gamma e}, \text{ where } \lambda_{\gamma e} \text{ is the (partial) mean free path for the collisions of the photons with pair-produced electrons or positrons.} \]

A calculation involving the equilibrium concentration of electron-positron pairs and the Thomson cross section shows that for \( T/T_e \ll 1 \), the ratio \( \lambda_{\gamma\gamma}/\lambda_{\gamma e} \) is given by \( A(T_e/T)^{15/2}\exp(-T_e/T) \), where \( A \simeq 5 \times 10^3 \). For \( T = 1.5 \times 10^8 \) K, which is the upper limit to the range of temperatures considered in this paper, this ratio is less than 4\%, so that the purity condition is well satisfied for the designated range of temperatures. Thus for \( T \leq 1.5 \times 10^8 \) K, we have in Eqs. (8)-(16) the desired post-Planckian corrections to the thermodynamics of an assembly of photons in equilibrium.

**IV. IMPLICATIONS FOR THE COSMIC PHOTON GAS**

At this point the question of the meaning of \( V \) in the above derivations must be considered more carefully since the naive model of an enclosure with impenetrable walls is clearly not tenable at the range of temperatures where post-Planckian corrections become significant. Indeed it is necessary to examine the role of whatever mechanism serves to confine the photon gas to ensure that it does not appreciably affect the thermodynamics, just as was done with pair-produced electrons and positrons. As an example, consider the cosmic photon gas, where one might impose the requirement that the scale of inhomogeneities in the cosmic gravitational field, which is of the order of the cosmic scale parameter \( R \), be much larger than the photon mean free path. This requirement basically translates into the inequality \( \lambda_{\gamma\gamma} \ll V^{1/3}, \) where \( V \sim R^3 \), a condition that can be interpreted to mean that photon-photon collisions must be far more frequent than photon-graviton collisions [12]. Using Eq. (19), we can rewrite the last condition in terms of the temperature and arrive at the requirements

\[ V^{1/3}[m] \gg 4.0 \times 10^9 \left[ \frac{T[K]}{1.5 \times 10^8} \right]^{-9}, \quad T \leq 1.5 \times 10^8 \text{ K.} \]

Observe that these restrictions on temperature and volume also guarantee that on scales larger than \( \lambda_{\gamma\gamma} \) the photon gas is in the hydrodynamic regime (in contrast to the independent-particle regime prevalent under ordinary conditions), and essentially behaves like an elastic
continuum. For example, such a gas can support or propagate density perturbations of spatial dimensions $d$, provided $V^{1/3} \gg d \gg \lambda_{\gamma\gamma}$. It is clear, however, that the necessary conditions for such phenomena are extreme, suggesting an examination of the early stages of the universe.

According to the standard model [13], about one hour after the big bang the contents of the universe had cooled to a temperature of about $1.5 \times 10^8$ K, and the spatio-temporal scales of the universe, which for the present discussion may be taken to be the Hubble distance and time, were of the order of $10^{12}$ m and $10^3$ s respectively. The requirements expressed in Eq. (20) were thus satisfied. Disregarding for a moment the rest of the contents of the universe at this time, we can see that the photon gas behaves hydrodynamically on scales of the order of $d$, provided $10^9$ m $\ll d \ll 10^{12}$ m. For example, density perturbations characterized by wavelengths and frequencies of the order of $10^{10}$ m and $10^{-2}$ s$^{-1}$, respectively, satisfy these conditions and could have been supported by the cosmic photon gas. Needless to say, the free electrons present in the cosmic fluid at this time (due to the baryon asymmetry of the universe) cannot be disregarded since they play a major role by virtue of their frequent collisions with the photons. A simple calculation shows that at $T \simeq 1.5 \times 10^8$ K, the ratio $\nu_{\gamma e}/\nu_{\gamma\gamma}$ is about $10^4$, and it rapidly increases according to $(T/T_c)^{-6}$ as the universe cools. Indeed by the time the free electrons disappear due to recombination, the photon gas is essentially noninteracting, as can be seen by considering Eq. (19). Although at the time of $T \sim 10^8$ K the cosmic photon gas was capable of independently supporting density perturbations and possibly playing a distinct role in structure formation, its dynamics was largely dominated by the free electrons, thereby obviating any interesting effects associated with the photons.

It is amusing in this connection to recall that the phenomenon of sonoluminescence, or the creation of light from sound, was observed some sixty years ago. The results of this paper suggest that the opposite phenomenon, which may be named the luminosonic effect, may also have occurred, albeit early in the life of the universe, and with x-rays as “light” and infrasonic waves as “sound.”
ACKNOWLEDGMENTS

This work was supported in part by a research award from the California State University, Sacramento.
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