Seesaw model with hidden $SU(2)_H \times U(1)_X$ gauge symmetry

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Abstract

We propose a seesaw model with a hidden gauge symmetry $SU(2)_H \times U(1)_X$ where two types of standard model singlet fermions in realizing a seesaw mechanism are organized into $SU(2)_H$ doublet. Then we formulate scalar and gauge sector, neutrino mass matrix and lepton flavor violations. In our gauge sector, $Z-Z'$ mixing appears after spontaneous symmetry breaking and we investigate constraint from $\rho$-parameter. In addition we discuss $Z'$ production at the large hadron collider via $Z-Z'$ mixing, where $Z'$ tends to dominantly decay into heavy neutrinos.

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I. INTRODUCTION

Generation of non-zero neutrino masses is one of the important issues which require an extension of the standard model (SM). Moreover, we expect smallness of the neutrino mass indicates a hint of structure of new physics beyond the SM. Actually, many mechanisms to generate neutrino masses are discussed such as canonical seesaw [1–4], inverse seesaw [9, 10], linear seesaw mechanisms [7, 8, 10], and so on. A linear seesaw mechanism (as well as inverse seesaw) is one of the interesting scenarios to realize tiny neutrino masses in which two types of SM singlet fermions are introduced; they are often denoted by $N_R^c$ and $S_L$. In many cases, the introduction of these singlets are simply assumed in ad hoc way. Even if we extend a gauge group such as left-right symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [9, 10], only one type of singlet can be embedded in the right-handed lepton doublet. Thus, a new hidden $SU(2)$ gauge symmetry is one of the promising candidates to unify $N_R^c$ and $S_L$ in one doublet. In order to have $SU(2)_H$ gauge anomaly cancellations for right-handed new fermions, even number of them is only allowed [12]. This is also one of the unique natures of the $SU(2)_H$ gauge symmetry model, and we could obtain a specific feature such as prediction of one massless neutrino in the minimal scenario as we will discuss in the main text.

In this letter, we discuss a seesaw model with a hidden gauge symmetry of $SU(2)_H \times U(1)_X$ in which extra neutral fermions are introduced as $SU(2)_H$ doublet giving two types of SM singlet fermions after spontaneous symmetry breaking. Introducing an $SU(2)_L \times SU(2)_H$ bi-doublet boson in our scalar sector, we can obtain Yukawa coupling among $SU(2)_L$ and $SU(2)_H$ lepton doublets which can realize the linear seesaw mechanism or Type-I seesaw like mechanism depending on parameter region. Then we formulate neutrino mass matrix and lepton flavor violation (LFV) induced by the same Yukawa coupling generating the neutrino mass. In addition, we discuss $Z$-$Z'$ mixing in our gauge sector, taking into account the constraint from $\rho$-parameter. Finally, we also consider collider physics in our model, focusing on $Z'$ production via the $Z$-$Z'$ mixing. In our scenario, $Z'$ tends to dominantly decay into heavy neutrinos when it is kinematically allowed, and its branching ratio shows clear difference from $Z'$ in other neutrino models with extra $U(1)$ such as $U(1)_{B-L}$ type as its $Z'$ should decay into SM fermions [13, 16].

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1 Another approach applying $SU(2)_L$ triplet fermion with hidden $U(1)$ symmetry can be referred to ref. [11].
This letter is organized as follows. In Sec. II, we introduce our model, and formulate scalar sector, neutral gauge sector, neutrino mass matrix, and lepton flavor violations. Then, we discuss collider phenomenologies focusing on $Z'$ boson which dominantly decays into heavy neutrinos. Finally we devote the summary of our results and the discussion in Sec.III.

II. MODEL SETUP

In this section, we formulate our model in which we introduce hidden $SU(2)_H \times U(1)_X$ gauge symmetry. In scalar sector, we introduce new scalar fields $H_2$, $\Phi$, $\Delta$ and $\varphi$ which are doublet, doublet, real triplet and singlet under $SU(2)_H$ with $U(1)_X \times U(1)_Y$ charges $(x, 0), (0, 1/2), (0, 0)$ and $(x, 0)$, and only $\Phi$ is $SU(2)_L$ doublet while the others are singlet. Also SM-like Higgs doublet is denoted as $H_1$. In our scenario, all these scalar fields develop vacuum expectation values (VEVs) inducing spontaneous symmetry breaking. The scalar fields are written by their components as follows:

$$H_1 = \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}} (v_1 + h_1^0 + i\eta_{h1}) \end{pmatrix}, \quad H_2 = \begin{pmatrix} \varphi \\ \frac{1}{\sqrt{2}} (v_2 + h_2^0 + i\eta_{h2}) \end{pmatrix}, \quad \Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} \delta_3 \\ \delta^* - \delta_3 \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} (\kappa_1 + \phi_1^0 + i\eta_{\phi_1}) \\ \phi_2^+ \end{pmatrix}, \quad \varphi = \frac{1}{\sqrt{2}} (v_\varphi + \varphi_R + i\varphi_I),$$

where $v_{1,2}$ and $\kappa_{1,2}$ are VEVs for corresponding fields. The VEV of triplet is given by $\langle \delta_3 \rangle = v_\Delta/\sqrt{2}$ derived from scalar potential shown below. In addition, $SU(2)_H$ doublet fermions $\Sigma^a_R$ are introduced which is taken as right-handed and SM gauge singlet. We write

|           | $L_L^a$ | $e_R^a$ | $\Sigma^a_R$ | $\Phi$ | $H_2$ | $H_1$ | $\Delta$ | $\varphi$ |
|-----------|---------|---------|--------------|--------|-------|-------|----------|----------|
| $SU(2)_H$| 1       | 1       | 2            | 2      | 2     | 1     | 3        | 1        |
| $SU(2)_L$| 2       | 1       | 1            | 2      | 1     | 2     | 1        | 1        |
| $U(1)_Y$ | $-\frac{1}{2}$ | -1     | 0            | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 0        |
| $U(1)_X$ | 0       | 0       | 0            | $x$    | 0     | 0     | $x$      |          |

TABLE I: Charge assignments of leptons and scalar fields including new field under $(SU(2)_H \times U(1)_X) \times (SU(2)_L \times U(1)_Y)$, where the upper index $a$ is the number of family that runs over 1-3 while $\alpha$ runs over 1-2 $n$ ($n$ is integer), and all of them are singlet under $SU(3)_C$. 

This letter is organized as follows. In Sec. II, we introduce our model, and formulate scalar sector, neutral gauge sector, neutrino mass matrix, and lepton flavor violations. Then, we discuss collider phenomenologies focusing on $Z'$ boson which dominantly decays into heavy neutrinos. Finally we devote the summary of our results and the discussion in Sec.III.
\[ \Sigma^{\alpha}_{R} \] with their components as

\[
\Sigma^{\alpha}_{R} = \begin{pmatrix} N^{\alpha}_{R} \\ (S^{\alpha}_{\pm}) \end{pmatrix},
\]

where both component fields are electrically neutral, and \( \alpha \) runs over 1-2n \( (n \text{ is integer}) \); we require even number of \( \Sigma_{R} \) for guaranteeing the theory to be anomaly free \[12\]. In our discussion below, however, we fix \( n \) to be 1 for simplicity: \( \alpha = 1,2 \).

The mass term of \( \Sigma_{R} \) and new Yukawa coupling are given by

\[
L = \bar{M}_{\alpha \beta}(\Sigma^{\alpha}_{R})(i\sigma_{2})(\Sigma^{\beta}_{R}) - y_{\alpha \beta} \bar{\Sigma}^{\alpha}_{R} \Delta(i\sigma_{2})(\Sigma^{\beta}_{R}) + f_{\alpha \beta} \bar{L}^{\alpha}_{R} \Phi \Sigma^{\beta}_{R} + h.c.,
\]

where \( \sigma_{2} \) is the second Pauli matrix and \( \Phi \equiv (i\sigma_{2})\Phi(i\sigma_{2}) \). Note here that \( \bar{M}_{\alpha \beta} \) should be anti-symmetric matrix due to anti-symmetric contraction of \( SU(2)_{H} \) indices in the term. It suggests that \( \bar{M} \) reduces the matrix rank by one, and we cannot formulate the active neutrino mass matrix. Thus, we introduce \( \Delta \) that leads to the second term as we will see later. The bi-doublet plays an role in inducing the Dirac mass that is also needed to construct the neutrino mass matrix. Moreover, \( H_{2} \) and \( \varphi \) play a role in breaking the gauge symmetry of \( SU(2)_{H} \times U(1)_{X} \) spontaneously and avoiding massless Goldstone boson associated with breaking of global symmetry in the scalar potential. Then scalar potential is written such as

\[
V = -m_{H_{1}}^{2}H_{1}^{\dagger}H_{1} - m_{H_{2}}^{2}H_{2}^{\dagger}H_{2} - m_{\varphi}^{2}\varphi^{\dagger}\varphi + \tilde{m}_{\Delta}^{2}Tr[\Delta^{\dagger}\Delta] + \tilde{m}_{\varphi}^{2}Tr[\Phi^{\dagger}\Phi]
+ \mu_{\Delta}(H_{2}^{\dagger}\Delta H_{2} + h.c.) + \lambda(\varphi^{*}H_{1}^{\dagger}\Phi H_{2} + h.c.) + \lambda'(\varphi H_{1}^{\dagger}\Phi H_{2} + h.c.) + \lambda_{\varphi}(\varphi^{*}\varphi)^{2}
+ \lambda_{H_{1}}(H_{1}^{\dagger}H_{1})^{2} + \lambda_{H_{2}}(H_{2}^{\dagger}H_{2})^{2} + \lambda_{\varphi}Tr[\Phi^{\dagger}\Phi] + \lambda_{\Delta}Tr[\Delta^{\dagger}\Delta]^{2} + \lambda_{\varphi}Tr[(\Delta^{\dagger}\Delta)^{2}]
+ \lambda_{H_{1}H_{2}}(H_{1}^{\dagger}H_{1})(H_{2}^{\dagger}H_{2}) + \lambda_{H_{1}\Psi}(H_{1}^{\dagger}H_{1})Tr[\Phi^{\dagger}\Phi] + \lambda_{H_{2}\varphi}(H_{2}^{\dagger}H_{2})Tr[\Phi^{\dagger}\Phi]
+ \lambda_{\Delta H_{1}}(H_{1}^{\dagger}H_{1})Tr[\Delta^{\dagger}\Delta] + \lambda_{\Delta H_{2}}(H_{2}^{\dagger}H_{2})Tr[\Delta^{\dagger}\Delta] + \lambda_{\Delta \varphi}(\varphi^{*}\varphi)Tr[\Delta^{\dagger}\Delta]
+ \lambda_{H_{1} \varphi}(H_{1}^{\dagger}H_{1})(\varphi^{*}\varphi) + \lambda_{H_{2} \varphi}(H_{2}^{\dagger}H_{2})(\varphi^{*}\varphi) + \lambda_{\Delta \varphi}Tr[\Delta^{\dagger}\Delta]Tr[\Phi^{\dagger}\Phi]
+ \lambda_{\varphi}(\varphi^{*}\varphi)Tr[\Phi^{\dagger}\Phi] + \lambda_{\Delta H_{2}} \sum_{i=1}^{3}(H_{2}^{\dagger}H_{2})Tr[\Delta^{\dagger}\sigma^{i}\Delta] + \lambda_{\Delta \varphi} \sum_{i=1}^{3}Tr[\Delta^{\dagger}\sigma^{i}\Delta](\Phi^{\dagger}\sigma_{i}\Phi),
\]

where \( \tilde{H}_{2} = i\sigma_{2}H_{2}^{*} \) and we take all couplings as real parameters, and \( \sigma_{i} \) (i=1,2,3) are Pauli matrices. Note that the terms associated with operator \( H_{2}^{\dagger}\Delta H_{2}, \varphi^{*}H_{1}^{\dagger}\Phi H_{2} \) and \( \varphi H_{1}^{\dagger}\Phi \tilde{H}_{2} \) play a role to prevent massless Goldstone boson from appearing. Furthermore these terms can realize small VEVs of \( \Phi \) and \( \Delta \) which are preferred for neutrino mass generation.
A. Scalar sector

Firstly we assume $\varphi$ develops a VEV in higher scale compared to other VEV scale. The VEV is derived by $\partial V/\partial v_\varphi = 0$, providing $v_\varphi \simeq \sqrt{m_\varphi^2/\lambda_\varphi}$. Then the terms in mass parameter are modified as

$$
\mathcal{V} \supset -m_X^2 H_1^\dagger H_1 - m_{H_2}^2 H_2^\dagger H_2 + m_\Delta^2 Tr[\Delta^\dagger \Delta] + m_\Phi^2 Tr[\Phi^\dagger \Phi] \\
+ \mu(H_1^\dagger \Phi H_2 + h.c.) + \mu'(H_1^\dagger \Phi H_2 + h.c.),
$$

(5)

$$
m_X^2 = m_X^2 - \lambda_{X\varphi}v_\varphi^2, \quad m_Y^2 = m_Y^2 + \lambda_{Y\varphi}v_\varphi^2, \quad \mu(\mu') = \lambda(\lambda')v_\varphi,
$$

(6)

where $X = \{H_1, H_2\}$ and $Y = \{\Phi, \Delta\}$. The VEVs of the other scalar fields are obtained by solving the conditions

$$
\frac{\partial \mathcal{V}}{\partial v_1} = \frac{\partial \mathcal{V}}{\partial v_2} = \frac{\partial \mathcal{V}}{\partial \kappa_1} = \frac{\partial \mathcal{V}}{\partial \kappa_2} = \frac{\partial \mathcal{V}}{\partial v_\Delta} = 0.
$$

(7)

In our scenario, we require relations among VEVs such that $\kappa_{1,2} \ll v_{1,2}$ to realize a seesaw mechanism as discussed below. Then VEVs are approximately given by

$$
v_1 \simeq \sqrt{\frac{4\lambda_{H_1} m_{H_1}^2}{4\lambda_{H_1} \lambda_{H_2} - \lambda_{H_1 H_2}^2}} v_2 \simeq \sqrt{\frac{4\lambda_{H_2} m_{H_2}^2 - 2\lambda_{H_1 H_2} m_{H_1}^2}{4\lambda_{H_1} \lambda_{H_2} - \lambda_{H_1 H_2}^2}},
$$

(8)

$$
\kappa_1 \simeq \frac{\sqrt{2\mu'v_1 v_2}}{2m_\Phi + \lambda_{H\Phi}v_1^2 + \lambda_{H\Phi}v_2^2}, \quad \kappa_2 \simeq \frac{\sqrt{2\mu'v_1 v_2}}{2m_\Phi + \lambda_{H\Phi}v_1^2 + \lambda_{H\Phi}v_2^2},
$$

(9)

$$
v_\Delta \simeq \frac{\mu_\Delta v_\Delta^2}{2 m_\Delta^2},
$$

(10)

where we chose $(\lambda_\Delta + \lambda'_\Delta)v_\Delta^2 \ll m_\Delta^2$ and omit contribution from quartic terms assuming it is subdominant in deriving the triplet VEV; we also ignored $\lambda'_{\Delta H_2(\Phi)}$ coupling assuming it is sufficiently small for simplicity. We thus see that $\kappa_{1,2}$ can be smaller than $v_{1,2}$ by choosing parameters $\mu$ and $\mu'$ to be small compared with other mass parameters. In our case of $\kappa_{1,2} \ll v_{1,2}$ and assuming a mixing associated with $\{\varphi, \Delta\}$ is small, CP-even scalar bosons $h_1^0$ and $h_2^0$ from $H_1$ and $H_2$ can have sizable mixing. Then squared mass matrix for $h_{1,2}^0$ is obtained as

$$
\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} h_1^0 \\ h_2^0 \end{pmatrix}^T \begin{pmatrix} 2\lambda_{H_1} v_1^2 & \lambda_{H_1 H_2} v_1 v_2 \\ \lambda_{H_1 H_2} v_1 v_2 & 2\lambda_{H_2} v_2^2 \end{pmatrix} \begin{pmatrix} h_1^0 \\ h_2^0 \end{pmatrix}.
$$

(11)

The above squared mass matrix can be diagonalized, applying an orthogonal matrix that gives mass eigenvalues

$$
m_{h_{1,2}}^2 = \lambda_{H_1} v_1^2 + \lambda_{H_2} v_2^2 \pm \sqrt{(\lambda_{H_1} v_1^2 - \lambda_{H_2} v_2^2)^2 + \lambda_{H_1 H_2} v_1^2 v_2^2},
$$

(12)
and the corresponding mass eigenstates $h$ and $H$ are obtained as

$$
\begin{pmatrix}
  h \\
  H
\end{pmatrix} = \begin{pmatrix}
  \cos \alpha & \sin \alpha \\
  -\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
  h_1^0 \\
  h_2^0
\end{pmatrix},
\quad \tan 2\alpha = \frac{\lambda_{H_1 H_2} v_1 v_2}{\lambda_{H_1} v_1^2 - \lambda_{H_2} v_2^2},
$$

where $\alpha$ is the mixing angle and $h$ is identified as the SM-like Higgs boson.

The mass eigenvalues for components of bi-doublet $\Phi$ are given by

$$
m_{\phi^0_1}^2 = m_{\phi^0_2}^2 = m_{\phi_1^+}^2 = m_{\phi_{0,1,2}}^2 = m_{\phi_1^0}^2 + \frac{1}{2}\lambda_{H_1} v_1^2 + \frac{1}{2}\lambda_{H_2} v_2^2 + \lambda_\phi (3\kappa_1^2 + \kappa_2^2),
$$

where corresponding components $\{\phi^0_{1,2}, \phi^+_1, \eta_{0,1,2}\}$ can be approximately identified with mass eigenstates for small $\kappa_{1,2}$. In addition, the mass eigenvalues are almost degenerated in our case. Scalar bosons from $\Delta$ are neutral scalar bosons and it would interact with SM particle via neutral fermion mixing and Higgs mixing. In this paper we just assume $\Delta$ is heavy whose mass is dominantly given by $m_\Delta$.

### B. Gauge sector

Here we analyze mass terms for gauge fields. The mass terms are obtained after spontaneous breaking of $SU(2)_H \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry via kinetic terms of scalar fields:

$$
L_K = (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2) + Tr[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + (D_\mu \varphi)^\dagger (D^\mu \varphi),
$$

$$
D_\mu \Phi = \partial_\mu \Phi - ig_2 W^{\mu i}_\mu \sigma^i_2 \Phi + ig_H \sigma^i_2 \Phi W^{\mu i}_H - ig_1 \frac{1}{2} B_\mu \Phi,
$$

$$
D_\mu H_1 = \partial_\mu H_1 - ig_2 W^{\mu i}_\mu \sigma^i_2 H_2 - ig_1 \frac{1}{2} B_\mu H_1,
$$

$$
D_\mu H_2 = \partial_\mu H_2 - ig_H \sigma^i_2 W^{\mu i}_H H_2 - i g_X X_\mu H_2,
$$

$$
D_\mu \Delta = \partial_\mu \Delta - ig_H \left[ \frac{\sigma^i_2}{2} W^{\mu i}_H, \Delta \right],
$$

$$
D_\mu \varphi = \partial_\mu \varphi - i g_X X_\mu \varphi,
$$

where $\sigma^i$ denotes the Pauli matrix, $W^{\mu i}_{H_\mu}$ and $X_\mu$ are $SU(2)_H$ and $U(1)_X$ gauge fields, and $g_1, g_2, g_H$ and $g_X$ are respectively gauge couplings for $U(1)_Y$, $SU(2)_L$, $SU(2)_H$ and $U(1)_X$. 


Then the mass terms for gauge fields are given by

\[
L_M = \frac{1}{8} [(g_1^2 + g_2^2)(v_1^2 + \kappa^2)\tilde{Z}_\mu \tilde{Z}^\mu + 2g_2^2(v_1^2 + \kappa^2)W_\mu^+ W^{-\mu} \\
+ g_H^2(v_2^2 + \kappa^2 + v_2^2)(W_{1\mu}^1 W_{H\mu}^1 + W_{2\mu}^2 W_{H\mu}^2) + v_2^2(g_H W_{3\mu}^3 - 4xg_X X_\mu)^2 \\
+ 2g_H \sqrt{g_1^2 + g_2^2} \tilde{Z}_\mu (\Delta \kappa^2 W_{H}^3 W_{H}^3 + 2\kappa_1 \kappa_2 W_{H}^3 W_{H}^3)], \tag{23}
\]

where we define

\[
\kappa^2 = \kappa_1^2 + \kappa_2^2, \quad \Delta \kappa^2 = \kappa_1^2 - \kappa_2^2, \quad \tilde{Z}_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_1 B_\mu - g_2 W_{3\mu}^3). \tag{24}
\]

For \(W^\pm\) boson, the mass is given by

\[
m_W = \frac{1}{2} g_2 \sqrt{v_1^2 + \kappa^2}, \tag{25}
\]

where \(\sqrt{v_1^2 + \kappa^2} = v \simeq 246\) GeV is required. In our following analysis we take \(\kappa_1 \sim \kappa_2\) so that the mass term associated with \(\Delta \kappa^2\) is negligibly small compared to other mass terms.

We also do not discuss \(W_{3\mu}^3-X_\mu\) mixing, since it does not couple with SM sector directly and focus on \(W_{1\mu}^1-\tilde{Z}_\mu\) sector. Then \(W_{1\mu}^1\) mainly mixes with \(\tilde{Z}_\mu\) and corresponding mass matrix is given by

\[
L_M \supset \frac{1}{2} \left(\begin{array}{c}
\tilde{Z}_\mu \\
W_{1\mu}^1
\end{array}\right)^T \left(\begin{array}{cc}
M_{\tilde{Z}}^2 & \delta M^2 \\
\delta M^2 & M_X^2
\end{array}\right) \left(\begin{array}{c}
\tilde{Z}_\mu \\
W_{1\mu}^1
\end{array}\right), \tag{26}
\]

\[
M_{\tilde{Z}}^2 = \frac{1}{4}(g_1^2 + g_2^2)(v_1^2 + \kappa^2), \quad M_X^2 = \frac{1}{4}g_H(v_2^2 + \kappa^2), \quad \delta M^2 = \frac{1}{2} g_H \sqrt{g_1^2 + g_2^2 \kappa_1 \kappa_2}. \tag{27}
\]

Diagonalizing the mass matrix, we obtain mass eigenvalues

\[
m_{Z,\bar{Z}}^2 = \frac{M_{\tilde{Z}}^2 + M_X^2}{2} \pm \frac{\sqrt{(M_{\tilde{Z}}^2 - M_X^2)^2 + 4\delta M^4}}{2}, \tag{28}
\]

and mass eigenstates are given by

\[
\begin{pmatrix}
Z_\mu \\
Z_{\bar{Z}}^\mu
\end{pmatrix} = \begin{pmatrix}
\cos \theta_{ZZ'}, & \sin \theta_{ZZ'} \\
-\sin \theta_{ZZ'}, & \cos \theta_{ZZ'}
\end{pmatrix} \begin{pmatrix}
\tilde{Z}_\mu \\
W_{1\mu}^1
\end{pmatrix}, \tag{29}
\]

\[
\sin 2\theta_{ZZ'} = \frac{2\delta M^2}{m_{\tilde{Z}}^2 - m_{Z'}^2}. \tag{30}
\]

Here we consider the limit of \(M_{\tilde{Z}}^2, \delta M^2 \ll M_X^2\) and mass eigenvalues are approximately

\[
m_{\tilde{Z}}^2 \simeq M_{\tilde{Z}}^2 - \frac{\delta M^4}{M_X^2}, \quad m_{Z'}^2 \simeq M_X^2 + \frac{\delta M^4}{M_X^2}, \tag{31}
\]
where $m_Z$ is identified as the SM Z boson mass. Thus $\rho$-parameter in the model is shifted from 1 and given as

$$\rho \equiv \frac{m_W}{m_Z \cos \theta_W} = \frac{M_{\tilde{Z}}^2}{m_Z^2} \simeq 1 + \frac{\delta M^4}{m_Z^2 m_{Z'}^2},$$

(32)

where we used Eqs. (25) and (27) to obtain relation between $m_W$ and $M_{\tilde{Z}}$. Then we obtain allowed parameter region on $\{\delta M^2, m_{Z'}\}$ space from observed $\rho$-parameter $\rho = 1.004^{+0.0003}_{-0.0004}$ [17] with $2\sigma$ error. In the left plot of Fig. 1 we indicate the upper limit of $\sqrt{\delta M^2}$ as a function of $m_{Z'}$, while corresponding upper limit of $\theta_{ZZ'}$ is given in the right plot. We thus find that the VEVs in bi-doublet scalar are required not to be large, assuming gauge coupling $g_H$ is $\mathcal{O}(0.1)$ to $\mathcal{O}(1)$.
C. Neutral fermion mass

Here we consider neutral fermion masses including active neutrino masses. Firstly mass term for $\Sigma^\alpha_R$ can be written in component form:

$$\tilde{M}_{\alpha \beta}(\bar{\Sigma}^\alpha_R)_{\alpha}(i\sigma^2)(\Sigma^\beta_c)_{\beta} + y_{\alpha \beta}\Sigma^\alpha_R(\Delta)(i\sigma^2)(\Sigma^\beta_R)$$

$$= \tilde{M}_{\alpha \beta}[(\bar{N}^\alpha_R)(S^\beta_L) + (\bar{S}^\alpha_c)(N^\beta_R)] + \frac{y_{\alpha \beta} v}{2} [(\bar{N}^\alpha_R)(S^\beta_L) + (\bar{S}^\alpha_c)(N^\beta_R)]$$

$$\equiv M_{\alpha \beta}(\bar{N}^\alpha_R)(S^\beta_L), \quad (33)$$

where $M_{\alpha \beta}$ is general $2 \times 2$ mass matrix. Note that we don’t have diagonal term of $M_{\alpha \beta}$ without VEV of $\Delta$. After $\Phi$ developing VEV, we obtain mass terms from the Lagrangian in Eq. (3) such that

$$L \supset f_{a \beta} \kappa_1 \sqrt{2} \bar{\nu}^c_L(S^\beta_L) - f_{a \beta} \kappa_2 \sqrt{2} \bar{\nu}^c_L N^\beta_R + h.c., \quad (34)$$

where $a = 1-3$ and $\beta = 1, 2$. The mass matrix for neutral fermion is then obtained as

$$L_{mass} = \begin{pmatrix} 0 & M_{\kappa_2}^* & M_{\kappa_1}^* \\ M_{\kappa_2} & 0 & M \\ M_{\kappa_1} & M^T & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ N^c_R \\ S_L \end{pmatrix}, \quad (35)$$

where $(M_{\kappa_{1,2}})_{a \beta} = f_{a \beta} \kappa_{1,2}/\sqrt{2}$ and $M$ is given by Eq. (33). Here we assume $M_{\kappa_{1,2}} \ll M$ in our scenario and following situations can be considered depending on relative size of $\kappa_1$ and $\kappa_2$:

- $\kappa_1 \simeq \kappa_2$ and we obtain mass matrix similar to type-I seesaw mechanism.

- $\kappa_2 \ll \kappa_1$ and we obtain linear seesaw like hierarchy for the components in the mass matrix.

In our analysis, we take $\kappa_1 \simeq \kappa_2 \simeq \kappa/\sqrt{2}$ for simplicity and define $(M_{NS})_{a \beta} = f_{a \beta} \kappa/2$. In fact it is more natural case since there is no reason to have $\lambda \ll \lambda'$ for generation hierarchy.

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\footnote{\(U(1)_X\) forbids $H_2^*(i\sigma_2)\Delta H_2$ that leads to the non-vanishing components of (22) and (33) in the neutral fermion mass matrix, since it develops nonzero VEV of $\Delta$. Therefore, our model would spoil without $U(1)_X$.}
between $\kappa_1$ and $\kappa_2$. As a result, mass matrix for neutral fermions can be obtained as

$$L_{\text{mass}} = \begin{pmatrix} \bar{\nu}_L^c & \bar{N}_R^c & \bar{S}_L^c \\ \end{pmatrix}^T \begin{pmatrix} 0 & M_{NS}^* & M_{NS}^c \\ M_{NS}^T & 0 & M \\ M_{NS}^T & M^T & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \\ S_c^L \end{pmatrix}. \quad (36)$$

Applying seesaw approximation with $M_{NS} \ll M$, we then obtain active neutrino mass such that

$$-m_\nu \approx M_{NS}^*[M^{-1} + (M^T)^{-1}]M_{NS}^T = M_{NS}^*R^{-1}(R^T)^{-1}M_{NS}^T.$$ \quad (37)

Note here that $M_S$ is uniquely decomposed by a lower unit triangular matrix $R$, since $M_S$ is the symmetric matrix \cite{18}. Then $M_{NS}$ is rewritten in terms of experimental values as

$$M_{NS}^* = iU^T \sqrt{D_\nu O} R, \quad (38)$$

where $O$ is an arbitrary three by two matrix with $O^T O = 1_{2 \times 2}$ and $OO^T = \text{diag}(0, 1, 1)$, $m_\nu \equiv U^T D_\nu U$, $D_\nu$ is mass eigenvalues of neutrinos, and $U$ is the unitary matrix to diagonalize the neutrino mass matrix. Note here that in our scenario, we predict one massless neutrino in which we assume minimal number of $SU(2)_H$ doublet chiral fermion for anomaly cancellation. Next, we have to consider the constraint from non-unitarity, and this can be evaluated by $|\epsilon| \equiv \delta_N \delta_N^T$ \cite{19, 20};

$$|\epsilon| \approx \begin{pmatrix} 0.006 \pm 0.0063 & < 1.29 \times 10^{-5} & < 8.76 \times 10^{-3} \\ < 1.29 \times 10^{-5} & 0.005 \pm 0.0063 & < 1.05 \times 10^{-2} \\ < 8.76 \times 10^{-3} & < 1.05 \times 10^{-2} & 0.005 \pm 0.0063 \end{pmatrix}, \quad (39)$$

where $\delta_N \equiv M_{NS}^* M^{-1}$ and $\delta_N \ll 1$ is expected. Note that condition $M_{NS} \ll M$ can be easily achieved by taking VEV of bi-doublet to be small which is also motivated by $\rho$-parameter constraint discussed above. Rough estimation leads to $|\epsilon| \approx |D_\nu/M_{NS}|^2$, and this should conservatively satisfy $|\epsilon| \lesssim 10^{-5}$. Therefore, we find

$$32 \text{ eV} \lesssim M_{NS}, \quad (40)$$

where we fix to be $D_\nu \sim 0.1$ eV. In fact, required order of $M_{NS}$ is roughly $M_{NS} \sim \sqrt{D_\nu M} \sim 10^{-4}$ GeV for $M \sim 100$ GeV which satisfies the condition above. Heavier fermions are also
diagonalized by the unitary matrix and their mass eigenvalues are degenerately given by $M_{N_1,2} \approx \frac{M + M_T}{2}$ and their eigenstates are found to be

$$
\begin{pmatrix}
N_R^c \\
\bar{S}_L
\end{pmatrix} \approx \begin{pmatrix}
\frac{1}{\sqrt{2}} & -i/\sqrt{2} \\
\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
N_1 \\
N_2
\end{pmatrix}_L,
$$

where index for generation is omitted here.

D. Yukawa interactions and lepton flavor violation

The Yukawa interactions including SM charged leptons are obtained from third term of Eq. (3) such that

$$
f_{a\beta} \bar{L} \Phi \Sigma R + h.c. \supset f_{a\beta} \left[ \bar{\ell}_L^a N_R^\beta \phi_1 \right] + h.c.
$$

$$
\approx \frac{f_{a\beta}}{\sqrt{2}} \left[ \bar{\ell}_L^a P_R (N_1^\beta - i N_2^\beta) \phi_1 - \bar{\ell}_L^a P_R (N_1^\beta + i N_2^\beta) \phi_1 \right] + h.c.
$$

where $\{\ell^1, \ell^2, \ell^3\} = \{e, \mu, \tau\}$ and we omit interactions containing only neutral fermions. Then the formula of lepton flavor violations (LFVs), $\ell_a \rightarrow \ell_b \gamma$, is given by [22, 23]

$$
\text{BR}(\ell_a \rightarrow \ell_b \gamma) \approx \frac{48\pi^3 \alpha_{em} C_{ab}}{G_F^2} \left| a_{Rab}(N_1^k, \phi_1) + a_{Rab}(N_2^k, \phi_1^-) + a_{Rab}(N_1^k, \phi_2^-) + a_{Rab}(N_2^k, \phi_2^-) \right|^2,
$$

where $C_{21} \approx 1$, $C_{31} \approx 0.1784$, $C_{32} \approx 0.1736$, $G_F \approx 1.17 \times 10^{-5}$ GeV$^{-2}$, and

$$
a_{Rab}(\rho, \sigma) \approx \frac{1}{2(4\pi)^2} \sum_{k=1}^2 f_{bk} f_{ka} \int_0^1 dx \int_0^{1-x} dy \frac{xy}{(x^2 - x)m_{\rho}^2 + xm_\sigma^2 + (1 - x)m_\sigma^2}.
$$

Experimental upper bounds for these LFV processes are respectively given by BR($\mu \rightarrow e\gamma$) $\lesssim 4.2 \times 10^{-13}$, BR($\tau \rightarrow e\gamma$) $\lesssim 3.3 \times 10^{-8}$, and BR($\tau \rightarrow \mu\gamma$) $\lesssim 4.4 \times 10^{-8}$ [24, 25]. We find that the LFV constraints can be easily avoided. For example, taking $m_{\phi_{1,2}} = 1000$ GeV and $m_{N_{1,2}} = 400$ GeV, current $\mu \rightarrow e\gamma$ constraint of $BR(\mu \rightarrow e\gamma) < 4.2 \times 10^{-3}$ require Yukawa couplings to satisfy $\sum_{k=1}^2 f_{bk} f_{ka} \lesssim 0.1$.

E. Collider physics

Here we discuss $Z'$ production at the LHC. In our model, $Z'$ can be produced via $Z - Z'$ mixing where interaction among $Z'$ and the SM fermions is obtained as:

$$
\mathcal{L} \supset g_2 \sin \theta_{Z'Z} Z'_{\mu} J_{Z}^\mu,
$$

where

$$
\theta_{Z'Z} \approx \theta_{ZZ} \frac{M_{Z'}}{M_Z} \frac{m_{N_1}^2}{M_Z^2} \approx \theta_{ZZ} \frac{m_{N_1}^2}{M_Z^2} \frac{M_{Z'}}{M_Z}.
$$

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where $J^\mu_Z$ is the neutral current in the SM. Then the $Z'$ production cross section via Drell-Yang process is proportional to suppression factor of $\sin^2 \theta_{ZZ'}$. Here we estimate $Z'$ production cross section using CalcHEP \cite{26} by use of the CTEQ6 parton distribution functions (PDFs) \cite{27}, implementing relevant interactions. In Fig. 2, we show $Z'$ production cross section at the LHC 14 TeV as a function of $m_{Z'}$ for several values of $\theta_{ZZ'}$. We thus find that $\theta_{ZZ'} \sim 10^{-4}$ is preferred to obtain the cross section which could be tested at the LHC. Note also that $\kappa$ cannot be too small to obtain sizable $\theta_{ZZ'}$ value. From Eq. (31), we estimate

$$\theta_{ZZ'} \sim 0.18 g_H \frac{\kappa^2}{m_{Z'}^2} \sim 3 \times 10^{-4} g_H \left( \frac{\kappa}{20 \text{ GeV}} \right)^2 \left( \frac{500 \text{ GeV}}{m_{Z'}} \right)^2,$$

where we used $m_Z^2 \ll m_{Z'}^2$. Thus we should require $\kappa \gtrsim 10 \text{ GeV}$ to obtain $\theta_{ZZ'} \gtrsim 10^{-4}$ assuming $g_H$ is $\mathcal{O}(1)$ value. In that case Yukawa coupling $f_{a\beta}$ is $\sim \mathcal{O}(10^{-5})$ to realize $M_{NS} \sim 10^{-4} \text{ GeV}$ for neutrino mass generation.

In our model, $Z'$ dominantly decays into extra neutral fermions $N_1, 2$, if the decay process is kinematically allowed where $Z'\bar{N}_i N_i$ terms are obtained as:

$$\mathcal{L} \supset \bar{N}_i \gamma_5 \tilde{N}_i D \mu \gamma^\mu \Sigma_R^\alpha \Sigma_R^\alpha \sim \frac{1}{2} \bar{N}_i \gamma_5 \gamma_\mu (\bar{N}_1 \gamma_\mu \gamma_5 N_2 - \bar{N}_2 \gamma_\mu \gamma_5 N_1),$$

where we have applied Eq. (29). Note that $Z'$ can also decay into scalar bosons from

FIG. 2: $Z'$ production cross section at the LHC 14 TeV as a function of $m_{Z'}$ for several values of $\theta_{ZZ'}$.
bi-doublet but we assume these scalar bosons are heavy and the decay modes are kinematically forbidden. Branching ratios (BRs) of $Z' \to f_SM(f_SM$ denotes a SM fermion) are suppressed by small $\sin \theta_{ZZ'}$, and we have $BR(Z' \to f_SM)/BR(Z' \to N_i N_j) \propto \sin^2 \theta_{ZZ'} g_H^2/g_2^2$. Thus one finds $BR(Z' \to f_SM) \ll BR(Z' \to N_i N_j)$, if gauge coupling $g_H$ is not too small. Then collider constraints from $pp \to Z' \to f_SM$ are not significant in our model, requiring $g_H \gg \sin \theta_{ZZ'}/g_2$.

Here we assume that the mass of $Z'$ satisfies $2m_{N_{1,2}} < m_{Z'} < 2m_{N_{1,2}^2}$, so that $Z'$ decays into $N_{1(2)} N_{1(2)}^1$ pair; $N_i^1$ denote the lighter mass eigenstate and we omit the upper index in the following. Then $N_{1,2}$ decays as $N_{1,2} \to \ell^\pm W^\mp, Z \nu, h \nu$ via light-heavy neutrino mixing. In Table. II, we show $\sigma(pp \to Z' \to N_{1,2} N_{1,2} \to W^\pm W^\mp \ell^\mp \ell'^\mp)$ at the LHC 14 TeV for some benchmark values of $(m_{Z'}, \sin \theta_{ZZ'})$

| $(m_{Z'}, \sin \theta_{ZZ'})$ | $(400 \text{ GeV}, 10^{-3})$ | $(800 \text{ GeV}, 10^{-3})$ | $(400 \text{ GeV}, 10^{-4})$ |
|-----------------------------|-----------------|-----------------|-----------------|
| $\sigma BR$                | 0.22 fb         | 0.018 fb        | 0.0022 fb       |

**TABLE II:** $\sigma(pp \to Z' \to N_{1,2} N_{1,2} \to W^\pm W^\mp \ell^\mp \ell'^\mp)$ at the LHC 14 TeV for some benchmark values of $(m_{Z'}, \sin \theta_{ZZ'})$

where these processes provide multi-lepton and jets via decay of W and Z bosons. In fact, cross section of these SM processes are larger than our signal cross section and we need relevant selection and kinematical cuts to suppress number of background events. More detailed analysis including detector simulation and cut analysis is beyond the scope of this paper. Also if $Z'$ is heavier and/or $\sin \theta_{ZZ'}$ is smaller, the cross section becomes much smaller but it could be accessible at the high-luminosity (HL) LHC with integrated luminosity of 3000 fb$^{-1}$. Note that we can distinguish our $Z'$ from other $Z'$ such as that from $U(1)_{B-L}$, since $Z' \to f_SM f_SM$ mode is expected to be absent in our case.

Before closing this section, we discuss the width of heavy neutrino $N_{1,2}$ in our scenario.
Parametrizing mixing between active neutrino and heavy neutrino by $\theta_{\nu N}$, we estimate the decay width for $N_{1,2} \rightarrow W^{\pm} \ell^{\mp}$ by

$$\Gamma_{N \rightarrow W^{\pm} \ell^{\mp}} \simeq \frac{g_3^2 \theta_{\nu N}^2}{64\pi} \frac{(m_N^2 - m_W^2)^2}{m_N^3} \left( 2 + \frac{m_N^2}{m_W^2} \right),$$

where $m_N$ indicate heavy neutrino mass. Taking $m_N = 150$ GeV we obtain $(\Gamma_{N \rightarrow W^{\pm} \ell^{\mp}})^{-1} \sim 1.8 \times 10^{-16} \theta_{\nu N}^{-2} \pi$ for decay length. Thus heavy neutrino decays before reaching detector for $\theta_{\nu N} \sim M_{NS}/m_N \sim 10^{-6}$.

III. SUMMARY AND DISCUSSIONS

In this paper we have proposed a seesaw model based on a hidden gauge symmetry $SU(2)_H \times U(1)_H$ in which two types of singlet fermions to realize a seesaw mechanism are unified into a doublet of hidden $SU(2)_H$. Then a Yukawa interaction among the $SU(2)_H$ doublet fermion and the SM lepton doublet is realized by introducing bi-doublet scalar filed under $SU(2)_L \times SU(2)_H$.

Then we have formulated scalar sector and gauge sector of our model taking into account $\rho$-parameter constraint from $Z$-$Z'$ mixing. The neutral fermion mass matrix has been analyzed in which active neutrino mass is derived via Type-I like seesaw mechanism. In our scenario, we predict one massless neutrino in which we assume minimal number of $SU(2)_H$ doublet chiral fermion for anomaly cancellation. We have also taken into account constraints from non-unitarity and LFV, and found the constraints can be avoided easily.

Finally we have discussed collider physics, focusing on $Z'$ production via $Z$-$Z'$ mixing. Our $Z'$ can dominantly decay into heavy neutrinos $N_{1,2}$ and a SM fermion pair decay mode tends to be absent due to suppression by small $Z$-$Z'$ mixing effect. Then cross section of $\sim 0.22$ fb can be obtained for $pp \rightarrow Z' \rightarrow \bar{N}_{1,2}N_{1,2} \rightarrow W^{\pm}W^{\pm} \ell^{\mp} \ell^{\mp}$ with $(m_Z, \sin \theta_{ZZ'}) = (400 \text{ GeV}, 10^{-3})$ which would be tested by future LHC experiments. More parameter region can be tested at the HL-LHC.

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