Reduction of 12th Radial Force in Double Inverter Fed Permanent-Magnet Motors

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This paper proposes a control method to suppress vibrations in a double-star winding permanent-magnet synchronous motor (PMSM). The double-star winding PMSM, which has two three-phase windings, can be used to improve reliability and torque ripple cancellation. A 6th order torque ripple is canceled by providing a phase angle of 30° to the mechanical angle between the two windings. A temporal 12th order radial force vibration causes a breathing mode vibration, which uniformly deforms the stator core in a double-star winding PMSM. The breathing mode vibration is suppressed by superimposing the 6th order current. In this study, the temporal 12th order and spatial 6th order radial force, which causes the breathing mode vibration, was reduced by 86.3% by superimposing the 6th harmonic current via simulations. The radial acceleration was experimentally measured on the surface of a motor. The temporal 12th order radial acceleration on the motor surface exhibited a reduction of 66.5%.

Keywords: Double-star winding PMSM, vibration, torque ripple, radial force

1. Introduction

In recent years, as represented by "Net Zero 2050", global energy conservation has promoted electrification in the fields of automobiles, aircraft, and ships (1). This electrification has spread to hybrid vehicles (HV) and electric vehicles (EV), which have a motor instead of an internal combustion engine (ICE) to tow the vehicle. The noise from the traction source of the HV and EV is low because the motor is more silent than the ICE. Therefore, other acoustic noises, such as noises from electric power steering (EPS) are noticeable. In a permanent magnet synchronous motor (PMSM), EPS is installed in the immediate vicinity of a vehicle’s driver. The vibration from the EPS motor discomforts the driver who directly grips the steering wheel. The acoustic noise and vibration of the motor not only affect the steering, but also threaten its safety. In such cases, reducing vibration and acoustic noise is an issue that needs to be resolved.

There are two main reasons for the vibration caused by the mechanical structure of the PMSM: the first is torque ripple, which occurs mainly in the temporal 6th order electrical angular frequency and its integral multiples. The second is radial electromagnetic power (also known as radial force (2)), which occurs in an even order electrical angular frequency. The suppression of torque ripple has been investigated by methods such as optimizing the magnet arrangement and position of the flux barrier (3), giving a skew on rotor (4,5), and superimposing a compensated current applied to the PMSM (6,7). Suppressing the radial force, giving the skew on the rotor (3), and superimposing the compensated current has been proposed. In particular, to reduce the radial force, control methods for suppressing the temporal 2nd order radial force (8,9) and temporal 6th order radial force (10) have been investigated. These vibration suppression methods are intended for motor with general three-phase winding.

A double-star winding PMSM (DW-PMSM), which has two three-phase windings driven by two three-phase inverters improves the reliability. A phase angle between the two windings is the degree of freedom (11). The phase angle of 30° in the electrical angle cancels the temporal 6th order torque ripple. The power rating of each inverter is reduced by sharing the output torque with the two three-phase windings. According to several researchers, the suppression of the radial force and torque ripple can reduce the vibration and acoustic noise of a general three-phase PMSM. Few studies have described control methods that reduce the vibration and noise of DW-PMSMs. The issues of vibration reduction in the DW-PMSM are described as follows:

First, formulating a mathematical model that considers harmonic vibration and generating current reference from the two inverters is an important issue for DW-PMSMs. Coupling of flux linkage by coiling two windings around the common stator core complicates the formulating model. In addition, generating current references is an assignment because a current-fed DW-PMSM has four degrees of freedom due to two independent three-phase windings.

This paper proposes a control method and motor structure to suppress vibration in a DW-PMSM. DW-PMSM complicates the mathematical model due to the coupling effect between the two three-phase windings, which does not generate with general three-phase motors. Therefore, in this paper, the mathematical model considers the magnetic coupling between the windings. In the other hand, conventional vibration reduction methods target either radial force or torque ripple (2,10,11). These vibration reduction methods are also characterized by targeting motors with general three-phase winding. One of the proposed methods is a motor structure that realizes simultaneous suppression of 12th order radial force and 6th order torque ripple, and a control method that focuses on that structure, in this paper. First, we introduce the DW-PMSM, which has a structure that reduces vibration. Second, this study reveals a mathematical model that considers harmonic vibration.

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The mathematical model proposes current references that achieve the suppression of harmonic vibration. Simulations and experiments provide the validity of the proposed current reference based on the mathematical model and the suppression effect of vibration with the method.

Hereinafter, temporal Nth order represents the temporal Nth order of the electrical angular frequency. Temporal and spatial denote the time and spatial orders, respectively. On the other hand, physical quantities that do not have spatial order, such as current and torque, omit the term.

2. Target PMSM

2.1 Motor Specifications

Fig. 1 shows the motor winding arrangement and structure employed in this study. The motor has two three-phase windings with a phase angle of 30° in the mechanical angle between each winding and is a concentrated PMSM, which generates a radial force and torque ripple larger than the distributed PMSM. When the phase difference is 30° in the mechanical angle, the pole pairs should be odd because the 6th order torque ripple cancels out according to Eq. (1) when the amplitudes of the 6th order torque ripple generated by each winding are the same.

\[
T_{6th}(\sin(6\alpha t) + T_{6th}\sin(6(\alpha t + P_n\alpha_m))) = 0
\]

where \(T_{6th}\) is the amplitude of the 6th order torque ripple, \(P_n\) is the number of pole pairs, and \(\alpha_m\) is the winding phase angle in the mechanical angle. When \(\alpha_m = \pi/6\) and \(P_n = 2\) m an odd number, the 6th order components cancel each other. The PMSM in this study has 5-pole and 12-slot pairs (10P/2S) motor. Therefore, the 6th order torque ripple is canceled to prevent its generation in this motor. The windings of the motor in this study have a phase angle, but the parameters of each winding are the same.

2.2 Control Scheme and Decoupling

A transformation matrix, which transforms the three-phase reference frame to the d-q reference frame, is defined as

\[
T_p(\delta) = \frac{1}{\sqrt{3}} \begin{bmatrix}
\cos(\delta) & \cos(\delta - 2\pi/3) & \cos(\delta + 2\pi/3) \\
-\sin(\delta) & -\sin(\delta - 2\pi/3) & -\sin(\delta + 2\pi/3)
\end{bmatrix}
\]

and the matrix corresponds to 6 phases,

\[
T_1(\theta) = \begin{bmatrix}
T_p(\theta) & O_{23} & T_p(\theta + \alpha)
\end{bmatrix}
\]

where \(O_{23}\) is a 2 \times 3 null matrix, and \(\theta\) is the electric angle of the rotor.

The DW-PMSM causes interference between the two winding sets. The inductance matrix on the d-q reference frame of the motor is

\[
L_1 = T_1(\theta) \cdot L(\theta) \cdot T_1^T(\theta) = \begin{bmatrix} L_{dq} & M_{dq} \\ M_{dq} & L_{dq} \end{bmatrix}
\]

with

\[
L_{dq} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \quad M_{dq} = \begin{bmatrix} M_d & 0 \\ 0 & M_q \end{bmatrix}
\]

where matrix \(L(\theta)\) is the phase inductance, and the components of matrix \(L_{dq}\) and \(M_{dq}\) are the d-q inductances.

The inductance matrix \(L_1\) has magnetic interference, which does not appear with a general three-phase PMSM because Eq. (3) is not a diagonal matrix. Therefore, controlling the current requires magnetic decoupling, which makes the control complex. The decoupled inductance matrix is defined by the transformation matrix (Eq. (5): \(T_{dq}\)) as

\[
L_{dq} = \begin{bmatrix} L_{D1} & 0 & 0 \\ 0 & L_{Q1} & 0 \\ 0 & 0 & L_{Q2} \end{bmatrix}
\]

where the inductances are defined as

\[
L_{D1} = L_d + M_d \\
L_{Q1} = L_q + M_q \\
L_{D2} = L_d - M_d \\
L_{Q2} = L_d - M_d
\]

and \(\alpha\) is the phase angle of the electrical angle between the two windings. This transformation defines new D-Q reference frames, which are indicated by \([D1, Q1, D2, Q2]\) frames. The flux linkage of the PMSM is expressed as

\[
\begin{bmatrix} \psi_{D1} \\ \psi_{Q1} \\ \psi_{D2} \\ \psi_{Q2} \end{bmatrix} = L_{dq} \cdot \begin{bmatrix} i_{D1} \\ i_{Q1} \\ i_{D2} \\ i_{Q2} \end{bmatrix} + \begin{bmatrix} \sqrt{3}\psi_{PM} \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

where, \(\psi_{D1}\) to \(\psi_{Q2}\) are the flux linkages of the reference frames corresponding to the subscripts, \(i_{D1}\) to \(i_{Q2}\) are the current, and \(\psi_{PM}\) is the flux linkage of the permanent magnets (PMs).

Four degrees of freedom, \(D1\) to \(Q2\), appear in the transformation matrix. Two of them have functions that are the same as the d-q reference frame in a general three-phase PMSM. Magnet torque is generated by the current in the \(Q1\) frame, and the current for flux-weakening control is fed on the \(D1\) frame. The other degrees of freedom are redundant. In general, the currents in the \(D2-Q2\) frames are controlled to 0 A.

3. 12th Order Radial Force Suppression Method

3.1 Effect of Radial Force on Vibration

The electromagnetic excitation force of a radial gap motor, which has a cylindrical rotor and is often adopted, is larger in the radial direction than in the tangential direction. The stator core of the radial gap motor has an annular shape. Deforming this annular core by a radial force generates vibration and acoustic noise in the stator core and motor case. The most important from an acoustic noise point of view are low circumferential mode numbers. The reason is that
vibrations with a small spatial order have a greater effect on acoustic noise \(^{[10]}\). The annular stator core excites the electromagnetic excitation force in the radial direction, and the force causes a bending vibration of the annulus. In particular, the temporal 6th and 12th order radial forces generate a spatial 0th order annular vibration and tend to generate acoustic noise. The spatial 0th order component of the annular vibration is distributed uniformly around the stator core and changes periodically with time. The spatial 0th order mode vibration of the annulus is referred to as the pulsating vibration mode or breathing mode vibration.

### 3.2 Derivation of Radial Force

As previously mentioned, double winding creates four degrees of freedom, including two redundant degrees of freedom for controlling the current. Superimposing the 6th harmonic current on the redundant degrees of freedom suppresses the 12th order radial force. The radial force applied to the tooth surface is expressed with the Maxwell stress equation as

\[
\tau_r(t, \theta) = \frac{b_s - b_t}{2\mu_0} dS \tag{9}
\]

where \(dS\) is a minute area on an arbitrary surface, \(b_s\) is the magnetic flux density in the radial direction in the minute area, and \(b_t\) is the tangential magnetic flux density in the minute area. Subsequently, the following two hypotheses help to formulate the equation: (i) The magnetic flux density passes uniformly over the surface of the teeth. (ii) The magnetic flux density in the tangential direction is extremely small compared to the magnetic flux density in the radial direction. Based on the abovementioned assumptions, simplifying Eq. (9) as

\[
\tau_r(t, \theta) \cong \frac{1}{2\mu_0} b_t^2 \tag{10}
\]

where \(s\) is the area of the tooth surface facing an air gap and \(\mu_0\) is the magnetic permeability of free space \((=0.4\pi \times 10^{-6}\) H/m). The flux linkage approximates the magnetic flux density as

\[
b_r \cong \psi \tag{11}
\]

\[
\tau_r(t, \theta) \cong \frac{1}{2\mu_0 N}\psi^2 \tag{12}
\]

where \(N\) is the number of turns per tooth. Furthermore, assuming constant speed rotation, the flux linkage is not a function of time \(t\) but a function of the magnetic pole position \(\theta\).

The flux linkage of the U-phase is used to approximately express the radial force applied to a U-phase tooth. Dividing the flux linkage, as shown in Eq. (13), gives the square of the flux linkage of the U-phase in Eq. (14):

\[
\psi_u = \psi_{uc} + \psi_{um} \tag{13}
\]

\[
\psi_u^2 = \psi_{uc}^2 + \psi_{um}^2 + 2\psi_{uc}\psi_{um} \tag{14}
\]

Using the current on the D-Q reference frames and D-Q transformation matrix (Eq. (6)), the flux linkage generated by the current is

\[
\psi_c = T_{DQ} \cdot L_{DQ} \cdot i_{DQ} \tag{16}
\]

\[
\psi_{uc} = \frac{1}{\sqrt{3}} (L_{D1} i_{D1} \cos(\theta) - L_{Q1} i_{Q1} \sin(\theta) - L_{D2} i_{D2} \cos(6\theta + \theta_{6th}) \sin(\theta) - L_{Q2} i_{Q2} \sin(6\theta + \theta_{6th}) \cos(\theta)) \tag{17}
\]

Extracting the 12th order component (i.e., including the 12\(\theta\) term from the square of Eq. (17) gives

\[
\psi_{uc}^2(12\theta) = \frac{1}{12} (\bar{L}_{D2}^2 \bar{L}_{Q2}^2 - \bar{L}_{Q2}^2 \bar{L}_{D2}^2) \cos(12\theta + 2\theta_{6th}) \tag{18}
\]

\[
\psi_{uc}^2(12\theta) = \psi_{um}^2 \cos(12\theta + 2\theta_{6th}) + \psi_{um}^2 \cos(12\theta + \theta_{11m}) + \psi_{um}^2 \cos(12\theta + \theta_{11m}) \tag{20}
\]

\[
\psi_{uc}^2(12\theta) = \psi_{um}^2 \cos(12\theta + 2\theta_{6th}) + \psi_{um}^2 \cos(12\theta + \theta_{11m}) + \psi_{um}^2 \cos(12\theta + \theta_{11m}) \tag{21}
\]

### 3.3 Suppressing the Radial Force

From the abovementioned equation, the radial force is simplified by dividing it into components that depend on the harmonic current amplitude \((l_{D2}, l_{Q2})\) and other components, as shown in Eq. (22) and Eq. (23), respectively. The hypothesis that the two amplitudes of harmonic currents are set equal to \(l_{D2} = l_{Q2} = l_{6th}\) entails the following formulas:

\[
F_{uc}(12\theta) = \frac{A_{\psi}}{12} (\bar{L}_{D2}^2 \bar{L}_{Q2}^2 - \bar{L}_{Q2}^2 \bar{L}_{D2}^2) \cos(12\theta + 2\theta_{6th}) \tag{22}
\]

\[
\psi_{um}^2 \cos(12\theta + 2\theta_{6th}) + \psi_{um}^2 \cos(12\theta + \theta_{11m}) + \psi_{um}^2 \cos(12\theta + \theta_{11m}) \tag{23}
\]
optional angular frequency. The transfer functions of the PI controller constitute the PIR controller, which increases the gain of the R controller, which is in parallel with the PI controller in Fig. 2, to suppress the 12th order radial force.

By canceling Eq. (23) with Eq. (22), it is possible to derive Eq. (24). Eq. (22) shows the 12th order radial force generated by the 5th harmonic current in the command value in the proposed method. The PI controller retains the steady-state error in the component 6 of the 12th order radial force, which tends to generate acoustic noise. A PIR controller solves this problem. The PI controller and a resonance controller (R controller), which is in parallel with the PI controller in Fig. 2, constitute the PIR controller, which increases the gain of the optional angular frequency. The transfer functions of the PI controller and R controller are

\[ G_{PI}(s) = K_p \left( 1 + \frac{1}{T_1 s} \right) \]  
\[ G_{R}(s) = \frac{2K_p \alpha}{\omega_n^2 + 2\omega_n \frac{\alpha}{T_1} + (\alpha^{-2})} \]

where \( K_p \) is the proportional gain, \( T_1 \) is the integration time, \( K_r \) is the R controller gain, \( \omega_n \) and \( \alpha \) are the cutoff frequency and motor synchronization speed equal to the electrical angular frequency, respectively. As shown in Fig. 3, the R controller has a high gain at a specific frequency, and the width of the high-gain area in the Bode plot and gain of the R controller are determined by the two parameters \( (\omega_n, K_r) \).

### 4. Simulation results

#### 4.1 Simulation System

Finite element method (FEM) simulation verifies the effectiveness of the harmonic current on the temporal 12th order radial force, which tends to generate acoustic noise. A simulation system is illustrated in Fig. 4. Simulation uses an ideal current source to disregard the error and delay components of the controller. An ideal constant-speed feed load is utilized for the DW-PMSM. The simulation evaluates two types of radial forces. The first type of radial force is the temporal variation in the integral value of the radial force applied to the surface of the U-phase tooth facing the air gap. Harmonic analysis using a one-dimensional FFT evaluates the force. This measurement indicates the evaluation of the member force applied to the tooth surface. The second type of radial force is the one on the cylinder facing the air gap of all teeth of the stator core. The temporal 12th order and spatial 0th order radial forces are component to evaluate because the temporal 12th order radial force causes the breathing mode vibration, which tends to generate acoustic noise. The simulation with various amplitudes of the 6th, 7th, and 8th spatial harmonics of the flux linkage affects the voltage value of the radial force applied to the surface of the U-phase tooth.

#### 4.2 Verification of Radial Force of Member Force

According to Eq. (22) and Eq. (23), changing the phase of the harmonic current varies the amplitude of the temporal 12th order radial force.
Radial vibration and torque ripple suppression method (Takumi Soeda et al.)

The formulas indicate the occurrence of strengthening or weakening of the radial force depending on the phase. The force applied to the surface of the U-phase tooth facing the air gap of the DW-PMSM, which outputs constant torque and is given constant speed, evaluates the radial force. Let the radial force in the depth direction be uniformly distributed because it is based on a two-dimensional simulation. The force applied to the tooth was evaluated by integrating the nodal force applied to the surface of the tooth, as shown in Fig. 5 (a).

4.3 Verification of Breathing Mode Vibration

Temporal 12th order radial force Fr(12th) evaluated by integrating the nodal force applied to the surface of the tooth, as shown in Fig. 5 (a).

The DW-PMSM is given an ideal speed of $N = 1200$ rpm from outside and the motor is fed current $I_{Q1} = 10$ A for torque output by the ideal current sources. Simulations were performed on the applied harmonic currents in various phases and amplitudes. From Fig. 6 obtained by the simulations, it is confirmed that the stress applied to the member is strengthened and weakened depending on the current phase.

4.3 Verification of Breathing Mode Vibration

The simulation verifies the effect of force reduction, which has a large effect on acoustic noise. Because the temporal 12th order radial force is excited by adjacent teeth with the same phase, it gives the annular vibration to the core. Therefore, it is necessary to observe the 0th order spatial component. To obtain the 12th order temporal and 0th order spatial components, the temporal and spatial data were obtained by harmonic analysis using a two-dimensional FFT. Similar to measuring the member force, let the radial force in the depth direction be uniformly distributed because it is also based on a two-dimensional simulation.

Variables in the radial force corresponding to the harmonic current amplitude and phase were simulated with three patterns of drive conditions. Three patterns were selected: a condition that does not generate torque ($I_{Q1} = 0$ A) and conditions that produce two types of torque ($I_{Q1} = 10$, 20 A). The simulations evaluated the radial forces for 6 harmonic current amplitudes and 12 harmonic current phases under each operating condition.

The evaluation of radial force is performed using harmonic analysis by temporal and spatial two-dimensional fast Fourier Transform (FFT) lines.

Fig. 5 Measured line of radial force. The line is on the line of the teeth (tooth) facing the air gap.

Fig. 6 Member force amplitude (temporal 12th order) for various harmonic current amplitude ($I_{Q1}$) and phase ($\theta_{Q1}$).

Fig. 7 Variation in temporal 12th order and spatial 0th order radial force for various harmonic current amplitude ($I_{Q1}$) and phase ($\theta_{Q1}$).

Fig. 8 Effect of reducing temporal 12th order breathing mode vibration by superimposing temporal 6th order harmonic current.
transform (FFT) because the radial force varies temporally and spatially. The temporal 12th order radial force deforms teeth in the spatial 0th order ring mode. The simulation measures the radial force on the circle of the teeth surface facing the air gap, as shown in Fig. 5 (b), including the slot area.

The simulation results are presented in Fig. 7. The simulation results confirm that the 6th order harmonic current generates the temporal 12th order and spatial 0th order radial forces from the simulation results. Furthermore, the phase of the radial force amplitude varies by changing the phase of the harmonic current. The temporal 12th order and spatial 0th order radial forces are suppressed by canceling the radial force due to the magnetic flux of the PM and the current for torque output and radial force due to the harmonic current.

Fig. 8 shows the detailed data of the point where the spatial 0th order radial force is suppressed from the results obtained in Fig. 7. The horizontal axis shows the temporal order and the vertical axis shows the spatial order. The color scale shows the amplitude of the radial force for each order. The temporal 12th order component of the spatial 0th order radial force, which is particularly large, is suppressed by superimposing the 6th order harmonic current. The temporal 12th order and spatial 0th order radial force vary from $5.96 \times 10^{-3}$ N to $8.13 \times 10^{-3}$ N by superimposing the 6th order harmonic current on the $D$-$Q$ reference frames, and 86.3% reduction is confirmed.

5. Experimental results

5.1 Experimental Setup This section addresses the experimental validation of the vibration suppression method. The radial force is not measured directly because of the force dimension. Instead, the level of vibration was measured using accelerometers. An accelerometer is attached to the case surface using an adhesive to measure the radial acceleration. Table 2 lists the parameters and experimental conditions of the motor used in the experiment. Fig. 9 and Fig. 10 shows the experimental setup. For the experiment, the motor has the same structure as the model in Fig. 1, which is employed in the simulation. A torque meter mechanically connects the DW-PMSM, which is a verified motor, to a servo motor, which is a mechanical load. A data logger (DL950: Yokogawa) stores the measured torque and radial acceleration, and harmonic analysis evaluates the torque and radial acceleration by FFT. The servo
motor gives the DW-PMSM a constant speed rotation, and the DW-PMSM outputs a constant torque. A constant speed rotation of 1000 rpm, and the $Q$ axis current, which feeds to torque, is set to a constant torque condition of 0.45 Nm with a constant current of 10 A. For simplicity, the current reference of $I_{Q1}$ is set to 0 A to invalidate the reluctance torque and flux-weakening control.

### 5.2 Verification of the Radial Acceleration

Fig. 11 shows the results of measuring the 12th order radial acceleration by changing the phase of the 6th order harmonic current on the $D_2$-$Q_2$ reference frame. The current reference is based on Eq. (15). The broken line in Fig. 11 shows the acceleration value when the 6th order harmonic current is not superimposed. Similar to the simulation, it is confirmed that the radial acceleration increases or decreases due to the superposition of the 6th order harmonic current on the $D_2$-$Q_2$ reference frame.

Fig. 12 shows the phase currents (U-phase and X-phase) and currents on the $D_2$-$Q_2$ reference frame under conditions with harmonic current and without harmonic current, respectively. The currents on the $D$-$Q$ reference frames are internal variables recorded using the digital-to-analog converter (DAC) of a microcontroller. Fig. 13 shows the results of the harmonic analysis of the radial acceleration and torque under each condition in Fig. 12. Under the condition of $I_{Q1} = 10$ A, which outputs a constant torque, the servo motor, which is a mechanical load, feeds a rotation of 1000 rpm. The temporal 12th order radial acceleration without the 6th order harmonic current is 3.16 $\times 10^{-2}$ mm/s$^2$; when the 6th order harmonic current is applied, it is 1.06 $\times 10^{-2}$ mm/s$^2$. This outcome has a reduction effect of 66.5%. In this motor, the temporal 12th order radial force causes the vibration of the spatial 0th order mode. Because acoustic noise increases as the spatial order decreases, the reduction of the 12th order radial acceleration contributes significantly to the reduction of acoustic noise. In addition, the 6th order torque ripple is reduced by the structure. The double-winding structure with a winding phase angle of 30° in the mechanical angle cancels the 6th torque. The 0th order component of torque $T_0$ is 0.460 Nm under the driving condition $I_{Q1} = 10$ A. On the other hand, the 6th order torque ripple is 1.30 $\times 10^{-2}$ Nm (without harmonic current) and 4.88 $\times 10^{-3}$ Nm (with harmonic current), which are 2.83% (without harmonic current) and 10.6% (with harmonic current), respectively, of the 0th order torque. It is considered that the imbalance of the given harmonic currents increases the 6th order torque ripple. The phenomenon of imbalance of the given harmonic current seems to have occurred due to the error in comparison with the current reference value on $Q_2$ frames, which has a smaller inductance of the four degrees of freedom on the $D$-$Q$ reference frames.

### 6. Conclusion

This paper proposes a suppression method for both the 12th order radial force and 6th order torque ripple with a DW-PMSM. The DW-PMSM was targeted to improve the reliability and suppress torque ripple. FEM analysis and measurement of experimental radial acceleration were used to evaluate the effect of the method.

The simulation compared the temporal 12th order and spatial 0th order radial force with and without harmonic current. The temporal 12th order and spatial 0th order radial force varied from 5.96 $\times 10^{-2}$ N to 8.13 $\times 10^{-2}$ N by superimposing the 6th order harmonic current on $D$-$Q$ reference frames. The simulation confirmed the 86.3% reduction effect.

The experiment measured the acceleration in the radial direction on the surface of the motor case. The 12th order radial acceleration without the 6th order harmonic current was 3.16 $\times 10^{-2}$ mm/s$^2$; when the 6th order harmonic current was applied, it was 1.06 $\times 10^{-2}$ mm/s$^2$, which resulted in a reduction effect of 66.5%.

The winding structure, in which the two three-phase windings of the motor had a phase angle of 30° in the mechanical angle, suppressed the 6th order torque ripple. When the number of pole pairs was odd, similar to the motor employed in this study, the windings canceled the 6th order torque ripple because the torque ripple output by the two three-phase windings was out of phase.

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Radial vibration and torque ripple suppression method (Takumi Soeda et al.)

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