Type-2 Fuzzy Curve Model

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Abstract. The paper discusses about the formulation of type-2 fuzzy curve model. The generalization is carried out due to the existence of complex uncertainty which cannot be represented with classical type-1 fuzzy set. Hence, type-2 fuzzy set is proposed to define this type of complex uncertainty. Based on the complex uncertainty of data, fuzzy set theory type-2 with fuzzy number type-2 concept is used to represent the data with complex uncertainty. This process re-defines the data as type-2 fuzzy data which is also the result obtained by generalizing type-1 fuzzy data. Therefore, B-spline function is chosen to show the development of type-2 B-spline curve model via generalization. It is then followed by a number of processes, i.e. fuzzification, reduction and defuzzification are defined to model type-2 fuzzy B-spline curve to obtain a crisp type-2 fuzzy curve.

1. Introduction
The problem of uncertainty that arises in the modelling field often is a significant problem that caused the modeling process cannot be done. One area that has experienced uncertainty modeling is geometric modeling [1-3]. In the study of geometric modeling, the problem of uncertainty inherent in the data representation form curves and surfaces. This problem has been overcome by the use of fuzzy set theory type-1 (TSKJ1) [4] in defining uncertainty data problems. Thus, this makes the curve and surface produced a model curves and surfaces fuzzy type-1 covered by [5-8]. The uncertainty of complex problems has a higher level of uncertainty than usual uncertainty problem. This causes uncertainty TSKJ1 in defining complex problems cannot be applied. Therefore, the theory of fuzzy sets of type-2 (TSKJ2) [9] used to define the uncertainty complex problem. Whereas, for the purpose of modeling of curves and surfaces of complex uncertainty data problem, the concept of type-2 fuzzy numbers (NKJ2) used to define uncertainty complex data problems. Thus, this data is also known as type-2 fuzzy data (control) point. Finally, curve and surface generated will provide a model d type-2 fuzzy curve and surface.

This paper was written to study the fuzzy curve model type-2 focused on the defining issues of uncertainty of complex data. The formation of type-2 fuzzy curve will be discussed in the next section. Also discussed are fuzzification process, reduction process and defuzzification process for type-2 to obtain a single curve model.
2. Type-2 Fuzzy Data (Control) Points

In this section, the definitions of type-2 fuzzy data (control) points are given which are defined by using type-2 fuzzy set theory and also type-2 fuzzy number focused on type-2 fuzzy interval fuzzy number.

**Definition 1:** Let \( P_p \) crisp control points} and \( \tilde{P} = \{ \tilde{P}_i | \tilde{P}_i \) type-2 fuzzy control points\} are set of type-2 fuzzy control points (T2FCP) with \( P_i \subset X \) with \( X \) is universal set and \( \mu_p(P_i) : P \rightarrow [0, 1] \) is the membership function where can be defined as \( \mu_p(P) = 1 \) and then it can be formulated as \( \tilde{P} = \{ (P_i, \mu_p(P_i)) | P_i \in \mathbb{R} \} \). Therefore,

\[
\mu_p(P) = \begin{cases} 
0 & \text{if } P_i \notin X \\
\frac{c}{0,1} & \text{if } P_i \in X \\
1 & \text{if } P_i \in X 
\end{cases}
\]  

(1)

for each \( i \), \( \tilde{P} = \{ \tilde{P}_i, P_i, \tilde{P}_j \} \) with \( \tilde{P}_i = \{ \tilde{P}_i^+, \tilde{P}_i^- \} \) where \( \tilde{P}_i^+ \) and \( \tilde{P}_i^- \) are left-left, left, left-right T2FCP respectively and also \( \tilde{P}_j = \{ \tilde{P}_j^+, \tilde{P}_j^- \} \) where \( \tilde{P}_j^+ \) and \( \tilde{P}_j^- \) are right-left, right, right-right T2FCP respectively.

After defining T2FCP, then the next definitions are the fuzzification process (alpha-cut operation), type-reduction and defuzzification.

**Definition 2:** Let \( \tilde{P} \) are T2FCPs with \( P_i \in \tilde{P} \) where \( i = 0, 1, \ldots, n-1 \). Then, \( \tilde{P}_i^\alpha \) are the alpha-cut operation known as fuzzification process for T2FCP which can be given as equation as follows.

\[
\tilde{P}_i^\alpha = \left\{ \tilde{P}_i^+, P_i, \tilde{P}_i^- \right\} \\
= \left\{ \tilde{P}_i^+, P_i, \tilde{P}_i^- \right\}
\]  

(2)

where \( \alpha \in (0, 1] \) and \( i = 1, 2, \ldots, n \) and \( j = 0, 1, \ldots, n \).

**Definition 3:** Let \( \tilde{P}_i^\alpha \) are set of \( \tilde{P}_i \) T2FCP which has been through the fuzzification process, then the type-reduction for \( \tilde{P}_i^\alpha \) can be defined as
\[ \overline{P} = \left( \sum_{j=0}^{n} \sum_{k=0}^{m} \left( \frac{1}{3} \overline{P}_{\alpha_{j}} \right) , \overline{P}_{\alpha_{j}} \right) \]

which the \( \overline{P}_{\alpha_{j}} \) and \( \overline{P}_{\alpha_{j}} \) both are the type-reduction results for left and right \( \overline{P}_{\alpha_{j}} \) with \( i = 0,1,...,n \).

**Figure 1.** Processes on getting \( \overline{P}_{\alpha_{j}} \) and \( \overline{P}_{\alpha_{j}} \).

**Definition 4:** Let \( \overline{P}_{\alpha_{j}} \) are T2FCP which had been defined by type-reduction process. Then, \( \overline{P}_{\alpha_{j}} \) is called defuzzification of type-reduction fuzzified T2FCP, \( \overline{P}_{\alpha_{j}} \) if for each \( \overline{P}_{\alpha_{j}} \in \overline{P} \),

\[ \overline{P} = \left\{ \overline{P}_{\alpha_{j}} \mid i = 0,1,...,n \right\} \]

with every \( \overline{P}_{\alpha_{j}} = \frac{1}{3} \sum_{i=0}^{n} \left( \frac{1}{3} \overline{P}_{\alpha_{j}} \right) \).

3. **Type 2 Fuzzy B-spline Curve Modelling**

After defining uncertainty complex data by using the type-2 fuzzy set theory, then these type-2 fuzzy data points can be used in modelling those data through the curves and surfaces function in geometric modelling field. In this research and among the curves and surfaces function, B-spline curve function is selected as the fundamental of modelling the type-2 fuzzy data which can be extended to surface forms.

Therefore, the type-2 fuzzy B-spline curve, which is defined by using interpolation method can be given through the Definition 5 as follows.
**Definition 5:** Let \( \tilde{D}_u \in \mathbb{R}^m \) are set of type-2 fuzzy data points (T2FDPs) with \( 0 \leq u \leq n + 1 \). Then, the type-2 fuzzy interpolation B-spline curve (T2FIBsC) can be defined as follows:

\[
\tilde{B}_{si}(s_u) = \sum_{i=0}^{n} \tilde{P}_i N_{i,k}(s_u) = \tilde{D}_u
\]

where \( s_u \) are crisp parameter set with gives type-2 fuzzy curve which is interpolated T2FDPs, \( \tilde{D}_u \) and \( \tilde{P}_i \) are T2FCPs which constructed type-2 fuzzy control polygon from T2FDPs and \( N_{i,k} \) is basis B-spline function.

Based on the Equation 5, \( \tilde{P}_i N_{i,k}(s_u) = \left\{ \tilde{P}_i^{<} N_{i,k}(s_u), P_i N_{i,k}(s_u), \tilde{P}_i^{>} N_{i,k}(s_u) \right\} \) where \( P_i N_{i,k}(s_u) \), \( \tilde{P}_i^{<} N_{i,k}(s_u) \) and \( \tilde{P}_i^{>} N_{i,k}(s_u) \) are crisp B-spline curve, left and right type-2 fuzzy B-spline curve with

\[
\tilde{P}_i^{<} N_{i,k}(s_u) = \left\{ \tilde{P}_i^{<} N_{i,k}(s_u), \tilde{P}_i^{<} N_{i,k}(s_u), \tilde{P}_i^{<} N_{i,k}(s_u) \right\}
\]

and

\[
\tilde{P}_i^{>} N_{i,k}(s_u) = \left\{ \tilde{P}_i^{<} N_{i,k}(s_u), \tilde{P}_i^{<} N_{i,k}(s_u), \tilde{P}_i^{<} N_{i,k}(s_u) \right\}
\]

Therefore, for example illustration of T2FIBsC, this model applied for verification offline handwriting signature. This verification process includes the process of fuzzified, fuzzification, type-reduction and defuzzification processes. The instance of this verification offline handwriting signature can be given as Figure 2 as follows.
Figure 2. T2FIBsC which modelled (a) fuzzified, (b) fuzzification, (c) reduction and (d) defuzzification of offline handwriting signature.

Figure 2 indicates that the implementation of T2FIBsC in verification offline handwriting signature based on the existing complex uncertainty data. The complex uncertainty data of offline handwriting signature existed based on the error of human style writing offline signature which cannot be exactly the same shape. Thus, for security purpose this verification needed by using this T2FIBsC where this model has the type-2 fuzzy set theory to handle and define the complex uncertainty data.

4. Discussion
Based on the developmental model of T2FIBsC, this model shows that the complex uncertainty data can be modelled by interpolation B-spline curve function after the complex uncertainty data are defined by T2FST especially type-2 fuzzy number. The contribution of this model it can model the data which had uncertainty properties instead of the perfect data. Therefore, it can give best fit data through curve which is different when the perfect data are consider to be modelled and the others data which are vague will be rejected. Then, the results will give different result which is the modelling exact data and uncertain data are more accurate and reasonable than the modelling only used the exacts data.

5. Conclusion
As conclusion, the developed model can be used for both uncertain data such as uncertainty data and complex uncertainty data. The uncertainty data is defined by type-1 fuzzy set theory which can be derived from T2FST when T2FST is being reduced to type-1 fuzzy set theory because the type-1 fuzzy set is in the left and right footprint of T2FST.

For further studies of this model, it can be extended with complicated functions such as rational B-spline and also it can perform in surface forms since the surface functions are defined based on curve functions.

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