Spin squeezing and entanglement in superposition of nonlinear spin coherent states

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Abstract
We study squeezing and entanglement in superposition of a class of nonlinear spin coherent states. We analyze these properties as a function of the coherence and superposition parameters, number of qubits and the Hamiltonians involved, by presenting analytical and numerical results. We specifically observe that a subclass of these states, the even non-linear cat states are maximally entangled and squeezed, while another related subclass, the odd non-linear cat states, are entangled but not squeezed at all.

1. Introduction
The spin coherent states were introduced by Radcliff [1], in similarity to the bosonic coherent states and since, their generation, application and properties have been studied extensively [1–5]. Similarly, spin squeezed states, with all their variations; have been the focus of intensive investigations due to their relevance in interferometers, precision spectroscopy [3, 6] and entanglement studies [7–19]. The state is said to be spin squeezed if the uncertainty of one of the component in a direction perpendicular to the mean spin, is smaller than the standard quantum limit [1]. The spin squeezed states may be generated by application of nonlinear Hamiltonians to appropriate states [4, 12–20] or by superposition of spin states [18–20]. Ordinary coherent states are neither squeezed nor entangled; however, the production of squeezed and entangled states by application of non-linear Hamiltonians to spin coherent states [4, 8, 14, 21], and through their superposition, for example cat states, have also been considered [18–20]. Several classes of nonlinear coherent states have also been introduced and their properties and applications have been studied [22–25]. Entangled and squeezed spin coherent states have many applications in quantum information, and in the final analysis, they are based on the correlations in the system, though different kinds are involved; therefore, one may expect some kind of interrelationship between them. In this article we intend to study squeezing and entanglement in a general superposition of nonlinear spin coherent states. We organize the rest of the paper as follows. In section 2 a class of nonlinear spin coherent states (NSCS) and their superpositions (SNSCS) are considered. Squeezing properties of (SNSCS) are analyzed in section 3. The squeezing properties of the (NSCS) have been reported recently [21]. Entanglement properties of NSCS and SNSCS are studied in sections 4 and 5 respectively. Finally section 6 is devoted to discussion and conclusions.

2. Nonlinear spin coherent states and their superposition
We consider a system of $N$ spin $\frac{1}{2}$ in a coherent state that is described by [1]

$$|\eta\rangle = (1 + |\eta|^2)^{-j} \sum_{m=0}^{2j} \binom{2j}{m} \eta^m |j, -j + m\rangle$$

(1)
Where, \( \eta = e^{-i\xi} \tan \left( \frac{\theta}{2} \right) \) and \( j = \frac{N}{2} \). The nonlinear spin coherent state \( |\eta\rangle_{nl} \) is defined by

\[
|\eta\rangle_{nl} = f(\hat{N}) \eta(N) |\eta\rangle
\]

(2)

Where, \( f(\hat{N}) \) is a nonlinear function of the operator \( \hat{N} = \hat{J} + j. \) The following nonlinear spin coherent state \( |\eta, t\rangle \) satisfies equation (2) [21]

\[
|\eta, t\rangle = e^{-itF(\hat{N})} |\eta\rangle
\]

(3)

This implies that the nonlinear Hamiltonian \( F(\hat{N}) \) has converted the coherent state \( |\eta\rangle \) to the nonlinear one \( |\eta, t\rangle \). By definition

\[
f(\hat{N}) = e^{it[\hat{N} + 1] - F(\hat{N})}
\]

And substituting for \( |\eta, t\rangle \) in equation (2) we obtain

\[
f(\hat{N}) \eta(N) |\eta\rangle_{nl} = \eta(2j - N) |\eta\rangle_{nl}
\]

(4)

Where \( |\eta\rangle_{nl} = |\eta, t\rangle \). Now, we choose \( F(\hat{N}) = \hat{N}^2 \) and use equation (3) to write

\[
|\eta, t\rangle = (1 + |\eta|^2)^{-j} \sum_{m=0}^{2j} (\frac{2j}{m})^{\frac{1}{2}} \eta^m e^{-it(j+m-j)} |j, -j + m\rangle
\]

(5)

Squeezing properties of these states have been studied previously [21]. We now consider a general superposition of these states as follows

\[
|S\rangle \equiv |\eta, \gamma, t\rangle = \frac{1}{N} \{ |\eta, t\rangle + se^{it} |\eta, t\rangle \}
\]

(6)

where, \( \gamma \) and \( s \) are real parameters and the normalization constant \( N \) is given by

\[
N = \left\{ 1 + s^2 + 2s \cos \gamma \left( 1 - |\eta|^2 \right) \left( 1 + |\eta|^2 \right)^{\frac{1}{2}} \right\}^{-\frac{1}{2}}
\]

(7)

In the following sections the squeezing and entanglement properties of these states will be considered.

3. Squeezing in SNCS

We choose the squeezing parameter \( \xi^2 \), introduced by Sorensen and et al [10]

\[
\xi^2 = \frac{N \langle (\Delta J_n)^2 \rangle}{\langle J_n^2 \rangle + \langle J_n \rangle^2}
\]

(8)

where \( \hat{J}_n = n \cdot \hat{J} \) and \( n \) are orthonormal vectors in three dimensional space; the criterion of squeezing is given by

\[
\xi^2 < 1
\]

(9)

To obtain \( \xi^2 \), we need to compute the relevant expectation values in three orthogonal directions, which we chose to be the Cartesian ones. First, performing some calculations, we obtain the expectation values of the following intermediary operators

\[
\langle \hat{J}_+ \rangle = \frac{1}{N^2} \eta(2j)e^{it} \left( B^{2j-1}_{2j} + B^{-2j-1}_{2j} \right)
\]

(10)

\[
\langle \hat{J}_- \rangle = \frac{1}{N^2} \eta(2j)e^{-it} \left( D^{2j-1}_{2j} + D^{-2j-1}_{2j} \right)
\]

(11)

\[
\langle \hat{J}_x \rangle = \frac{1}{N^2} \eta(2j) \left( 1 - s^2 \right) \left( E^{2j-2}_{2j} + E^{-2j-2}_{2j} \right)
\]

(12)

\[
\langle \hat{J}_y \rangle = \frac{1}{N^2} \eta(2j) \left( 1 + s^2 \right) \left( F^{2j-2}_{2j} + F^{-2j-2}_{2j} \right)
\]

(13)
\[ \langle \hat{j}_x \rangle = \frac{2j |\eta|^2}{N^2} \left( (2j - 1)^2 + \frac{1}{A_+^2} \right) - s \cos \gamma \left( \frac{2j - 1}{A_+^2} - \frac{A_{2j-2}^2}{A_{2j}^2} \right) \] 

(14)

\[ \langle \hat{j}_y \rangle = \frac{2j |\eta|^2}{N^2} \left( (2j - 1)^2 + \frac{1}{A_+^2} \right) - s \cos \gamma \left( \frac{2j - 1}{A_+^2} - \frac{A_{2j-2}^2}{A_{2j}^2} \right) \] 

(15)

where

\[ A_\pm = (1 \pm |\eta|^2) \]
\[ B_\pm = (1 \pm e^{\pm i \theta} |\eta|^2) \]
\[ D_\pm = (1 \pm e^{\mp i \phi} |\eta|^2) \]
\[ E_\pm = (1 \pm e^{\pm i \theta} |\eta|^2) \]
\[ F_\pm = (1 \pm e^{\pm i \phi} |\eta|^2) \] 

(16)

Now, we may write down the required expectation values in terms of the intermediary operators obtained in equations (10)–(16) as follows

\[ \langle \hat{j}_x \rangle = \frac{1}{2} (\langle \hat{j}_- \rangle + \langle \hat{j}_+ \rangle) \] 

(17)

\[ \langle \hat{j}_y \rangle = \frac{1}{2} (\langle \hat{j}_- \rangle - \langle \hat{j}_+ \rangle) \] 

(18)

\[ \langle \hat{j}_x^2 \rangle = \frac{1}{4} (\langle \hat{j}_+ \rangle^2 + \langle \hat{j}_- \rangle^2 + \langle \hat{j}_+ \rangle \langle \hat{j}_- \rangle + \langle \hat{j}_- \rangle \langle \hat{j}_+ \rangle) \] 

(19)

\[ \langle \hat{j}_y^2 \rangle = \frac{1}{4} (\langle \hat{j}_+ \rangle^2 + \langle \hat{j}_- \rangle^2 - \langle \hat{j}_+ \rangle \langle \hat{j}_- \rangle - \langle \hat{j}_- \rangle \langle \hat{j}_+ \rangle) \] 

(20)

Substituting for equations (17)–(20) we obtain

\[ \langle \hat{j}_x \rangle = \frac{\eta}{N^2} \left( \frac{e^{i \theta} B_{2j-1} - e^{-i \phi} D_{2j-1}}{A_{2j}^2} (1 - s^2) + \frac{e^{i \phi} B_{2j-1} - e^{-i \theta} D_{2j-1}}{A_{2j}^2} (2is \sin \gamma) \right) \] 

(21)

\[ \langle \hat{j}_y \rangle = \frac{\eta}{N^2} \left( \frac{e^{i \theta} B_{2j-1} - e^{-i \phi} D_{2j-1}}{A_{2j}^2} (1 - s^2) + \frac{e^{i \phi} B_{2j-1} - e^{-i \theta} D_{2j-1}}{A_{2j}^2} (2is \sin \gamma) \right) \] 

(22)

\[ \langle \hat{j}_x^2 \rangle = \frac{j(1 + s^2)}{2N^2} \left( \frac{2 |\eta|^2 (2j - 1)}{A_{2j}^2} + 1 \right) + \frac{j(2j - 1)}{2N^2} \times \left( (1 + s^2) \frac{\eta^2 e^{2i \theta} E_{2j-2}^2 - \eta^2 e^{2i \phi} F_{2j-2}^2}{A_{2j}^2} \right) + 2s \cos \gamma \left( \frac{\eta^2 e^{2i \theta} E_{2j-2}^2 - \eta^2 e^{2i \phi} F_{2j-2}^2}{A_{2j}^2} - |\eta|^2 A_{2j-2}^2 \right) \] 

(23)

\[ \langle \hat{j}_y^2 \rangle = \frac{-j(1 + s^2)}{2N^2} \left( \frac{2(2j - 1)}{A_{2j}^2} + \frac{1 + |\eta|^2}{A_+^2} \right) + \frac{j(2j - 1)}{2N^2} \times \left( (1 + s^2) \frac{\eta^2 e^{2i \theta} E_{2j-2}^2 + \eta^2 e^{2i \phi} F_{2j-2}^2}{A_{2j}^2} \right) + 2s \cos \gamma \left( \frac{\eta^2 e^{2i \theta} E_{2j-2}^2 + \eta^2 e^{2i \phi} F_{2j-2}^2}{A_{2j}^2} + |\eta|^2 A_{2j-2}^2 \right) \] 

(24)

Similarly we have

\[ \langle \hat{j}_z \rangle = \frac{1}{N^2} \left\{ -j(1 + s^2) + \frac{2j(1 + s^2)}{A_+^2} - 2sj A_{2j}^2 \cos \gamma + 4sj A_{2j}^2 \cos \gamma \right\} \] 

(25)

\[ \langle \hat{j}_z^2 \rangle = \frac{j}{N^2} \left\{ (1 + s^2) \left( j - 2(2j - 1) \frac{|\eta|^2}{A_+^2} \right) + 2j A_+ \frac{A_{2j}^2}{A_+^2} \cos \gamma + 4s(2j - 1) A_{2j}^2 \cos \gamma \right\} \] 

(26)

Now, we have all the required quantities at hand to compute the squeezing parameters in the x and y directions

\[ \xi_x^2 = \frac{2j(\langle \hat{j}_y \rangle^2 - \langle \hat{j}_x \rangle^2)}{\langle \hat{j}_x \rangle^2 + \langle \hat{j}_y \rangle^2} \] 

(27)
We have depicted $x^2$ and $y^2$ as a function of time in figure 1; we note that squeezing in the x and y directions are in opposite phases, implying alternative squeezing in perpendicular directions.

Figure 2 presents $x^2$ as a function of time and $\eta$. It demonstrates that in an interval of $\eta$, squeezing never happens in time. Moreover, above this interval, the squeezing amplitude is an increasing function of coherence parameter $\eta$ and approaches 1 while squeezing vanishes gradually. This phenomenon is best observed in figure 3, where the squeezing parameter for three values of $\eta$ is depicted.

In figure 4 we have presented the time evolution of the squeezing parameter for $\eta$ above the death region and three values of $j$; it implies that the higher $j$ values produce deeper squeezing. Moreover, the period of oscillations increases as $j$ is increased.

We have presented plots of $x^2$ as a function of time in figures 5 and 6 for several values of the parameters $\gamma$ and $s$, respectively. It is deduced from figure 5 that we have a squeezing death interval here again for a range of $\gamma$ values, specifically at $\gamma = \pi$, which corresponds to an odd nonlinear cat state. Moreover, an oscillatory behavior with an amplitude of maximum is observed at $\gamma = 0$ which corresponds to a nonlinear even cat state. Figure 6 implies that the maximum of squeezing depth occurs at $s = 1$ and a deviation from this optimal value decreases the depth of squeezing.

As a final topic on squeezing in this section we study the effect of the structure of nonlinear Hamiltonian on the squeezing properties. We have plotted the squeezing parameter for three powers of $\hat{N}$, that is $F(\hat{N}) = \hat{N}^2$, $\hat{N}^3$ and $\hat{N}^4$, as a function of time, in figure 7. We note that while the amplitude of the oscillations (the maximum squeezing depth) is independent of the power of the operator $\hat{N}$; its period is a decreasing function of the latter.
4. Entanglement in NSCH

In this section we study entanglement in NSCH. We use concurrence as the measure of entanglement as expressed by [11, 26]

**Figure 3.** $\mathcal{C}_2$ as a function of $t$ for $\gamma = 0$, $s = 1$ and $j = 1$; $\eta = 2.5$ (dashed-dotted line), $\eta = 4$ (dotted line) and $\eta = 6$ (solid line).

**Figure 4.** $\mathcal{C}_2$ as a function of $t$ for $\gamma = 0$, $s = 1$ and $\eta = 3$; $j = 1$ (dashed-dotted line), $j = 2$ (solid line) and $j = 3$ (dotted line).

**Figure 5.** $\mathcal{C}_2$ as a function of $t$ and $\gamma$ for $\eta = 2$, $s = 1$ and $j = 1$. 

4. Entanglement in NSCH

In this section we study entanglement in NSCH. We use concurrence as the measure of entanglement as expressed by [11, 26]
Where, \( \lambda_i \) are the eigenvalues of matrix

\[
R = r_{ij} \left( \hat{\sigma}_x \otimes \hat{\sigma}_y \right) \rho_{ij}^{\text{RS}} \left( \hat{\sigma}_y \otimes \hat{\sigma}_y \right)
\]

and \( \rho_{ij} \) is the reduced density matrix. We consider a two-qubit system \( (j = 1) \) first. The density matrix corresponding to NSCH, equation (5), is given by

\[
\rho = \frac{1}{1 + |\eta|^2} \begin{pmatrix}
|\eta|^4 & |\eta|^2 \eta e^{-3i\gamma} & |\eta|^2 \eta e^{-3i\gamma} & \eta^2 e^{-4i\gamma} \\
|\eta|^2 \eta^* e^{3i\gamma} & |\eta|^2 & |\eta|^2 & \eta e^{-i\gamma} \\
|\eta|^2 \eta^* e^{3i\gamma} & |\eta|^2 & |\eta|^2 & \eta e^{-i\gamma} \\
\eta^2 e^{4i\gamma} & \eta^* e^{i\gamma} & \eta^* e^{i\gamma} & 1
\end{pmatrix}
\]

Performing the required intermediate calculations, we obtain the concurrence function for the NSCH as follows.

\[
C = \max \left\{ 0, \frac{4 |\eta|^2 |\sin \gamma|}{(1 + |\eta|^2)^2} \right\}
\]

We have plotted \( C \) as function of \( t \) for three real values of \( \eta \) in figure 8. It is observed that although \( C \) is an oscillating function of time, but entanglement is sustained in NSCH except at some isolated points. Moreover, we note that \( C \) is a decreasing function of \( \eta \).
5. Entanglement in SNSCS

We now consider entanglement in SNSCS as expressed by equation (6). Performing some straightforward calculations, the non-zero matrix elements of the density matrix, for the SNSCS of the two-qubit system are obtained as follows

\[
\rho_{11} = \frac{|\eta|^4(1 + se^{-i\gamma})(1 + se^{i\gamma})}{(1 + s^2)(1 + |\eta|^2)^2 + 2s(-1 + |\eta|^2)^2 \cos \gamma} \tag{33}
\]

\[
\rho_{12} = \rho_{13} = \rho_{21} = \rho_{31} = \frac{|\eta|^2\eta e^{-3i\gamma}(1 - se^{-i\gamma})(1 + se^{i\gamma})}{(1 + s^2)(1 + |\eta|^2)^2 + 2s(-1 + |\eta|^2)^2 \cos \gamma} \tag{34}
\]

\[
\rho_{14} = \rho_{24} = \rho_{34} = \frac{|\eta|^2 \eta^2 e^{-4i\gamma}(1 + se^{-i\gamma})(1 + se^{i\gamma})}{(1 + s^2)(1 + |\eta|^2)^2 + 2s(-1 + |\eta|^2)^2 \cos \gamma} \tag{35}
\]

\[
\rho_{22} = \rho_{23} = \rho_{32} = \rho_{33} = \frac{|\eta|^4(1 - se^{-i\gamma})(1 - se^{i\gamma})}{(1 + s^2)(1 + |\eta|^2)^2 + 2s(-1 + |\eta|^2)^2 \cos \gamma} \tag{36}
\]

\[
\rho_{24} = \rho_{34} = \rho_{42} = \rho_{43} = \frac{|\eta|^2 \eta e^{-2i\gamma}(1 - se^{-i\gamma})(1 + se^{i\gamma})}{(1 + s^2)(1 + |\eta|^2)^2 + 2s(-1 + |\eta|^2)^2 \cos \gamma} \tag{37}
\]

\[
\rho_{44} = \frac{(1 + se^{-i\gamma})(1 + se^{i\gamma})}{(1 + s^2)(1 + |\eta|^2)^2 + 2s(-1 + |\eta|^2)^2 \cos \gamma} \tag{38}
\]

We are now prepared to calculate the concurrence for this state [26]. The result is too complicated to produce its analytic expression here. We have presented the results in figures 9 to 13. The concurrence \(C\) is a periodic function of time in all the cases. We have plotted \(C\) as a function of time for several values of \(\eta\) in figure 9. It demonstrates that larger \(\eta\) corresponds to smaller values of the concurrence.

To see the role of \(j\) we have depicted the \(C\) as a function of time for 3 value of \(j\) in figure 10. It implies that the period is independent of \(j\), while its amplitude is a decreasing function of the latter.

The behavior of concurrence as a function of time and the superposition parameter \(s\) may be examined in figure 11. It is an oscillating function of \(s\); its period is independent of the latter while the maximum amplitude is obtained at \(s = 1\) and it is diminished as \(s\) deviates from this value in any direction.

Figure 12 presents a plot of \(C\) as a function of time and \(\gamma\). The former is an oscillating function of the latter as we expect. It is observed that at \(\gamma = \pi\), which along with \(s = 1\), corresponds to a nonlinear odd cat state, the maximum of entanglement is achieved and the constant value of 1 is obtained for concurrence. Moreover, deviations from these \(\gamma\) values, moves back the concurrence to its oscillatory behavior.

Finally, we wish study how the nonlinear Hamiltonians affect the measure of entanglement. We have plotted the latter for three powers of \(\hat{N}\), that is \(F(\hat{N}) = \hat{N}^2, \hat{N}^3\) and \(\hat{N}^4\) as a function of time, in figure 13. We note that while its period is diminished as the power of the \(\hat{N}\) increases, the amplitudes of the oscillations remain unchanged.
Figure 9. $C$ as a function of $t$ for $\gamma = \frac{n}{2}$, $s = 1$ and $j = 1$; $\eta = 2$ (dashed-dotted line), $\eta = 3$ (dotted line) and $\eta = 4$ (solid line).

Figure 10. $C$ as a function of $t$ for $\gamma = \frac{n}{2}$, $s = 1$ and $\eta = 4$; $j = 1$ (dashed-dotted line), $j = \frac{3}{2}$ (dotted line) and $j = 2$ (solid line).

Figure 11. $C$ as a function of $t$ and $s$ for $\gamma = 0$, $\eta = 1$ and $j = 1$. 
6. Conclusions

We have studied squeezing and entanglement in a class of general superposition of nonlinear coherent states. Two subclasses of the latter; that is nonlinear even and odd cat states have also been pinpointed.

The basic results regarding squeezing in an N-qubit SNSCS state are as follows:

(a) The State is alternatively squeezed in x and y directions.

(b) Even nonlinear cat states, corresponding to superposition coefficient 1, are maximally squeezed, while nonlinear odd ones, corresponding to $-1$, are not squeezed at all.

(c) Larger $j$ corresponds to smaller squeezing parameter, that is deeper spin squeezing.

(d) The squeezing vanishes at large real values of $\eta$; that is the spin squeezing parameter goes to unity as the latter goes to very large values.

(e) The period of squeezing oscillations is a decreasing function of the power $n$ for the Hamiltonian $F(\hat{N}) = \hat{N}^n$, while the period remains constant in time.

The basic results regarding entanglement are as follows:

(f) The entanglement in NSCS is an oscillating function of time, but it remains entangled continuously except at some isolated points.

(g) The entanglement measure in SNSCS is reduced as the number of the qubits increases.
(h) The entanglement measure in SNSCS is a decreasing function of the real coherence parameter $\eta$.

(i) The period of entanglement oscillations is independent of $j$, while its amplitude is a decreasing function of the latter.

(j) Nonlinear even and odd cat states corresponding to the superposition coefficients $\pm 1$, display the maximum of entanglement.

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