Research Article
Vertical Dynamic Response of the 2-DOF Maglev System considering Suspension Nonlinearity

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In order to study the dynamic characteristics of the nonlinear system of the maglev train, a vibration model of the two-degree-of-freedom nonlinear suspension system of the maglev train is established in this paper. Based on the linearized model of the suspension control module, the suspension system is controlled by the state feedback method. Compared with the results of the Runge–Kutta method, the accuracy of the settlement results is verified. This paper analyzes the influence of system parameters and feedback control parameters on the system. The results show that the linear stiffness mainly affects the left-right migration of the resonance peak and the difference between the maximum and minimum of the resonance region of the suspension frame; the nonlinear stiffness mainly affects the slope of the resonance region of the car body and the suspension frame; displacement feedback control parameters can reduce the amplitude of the system, velocity feedback control parameters can make the state change of the suspension frame more smoothly, and acceleration feedback control parameters make the car body and the suspension frame have a strong coupling effect. According to the Hurwitz criterion of Hopf bifurcation, this paper deduces the conditions that the control parameters should satisfy when the equilibrium point is stable, and the instability produces periodic vibration under PID control. Through numerical simulation, it is found that the system has complex dynamic behavior, the results show that the system works under some conditions, and there are multiple stable and unstable limit cycles simultaneously. Therefore, the system will appear alternately between multiperiod and chaotic motion, which will affect the stability of the system.

1. Introduction

Maglev train has the advantages of good safety, low noise, strong climbing ability, high running speed, smooth, and comfortable [1–5]. There have been great development in recent years, and commercial routes using EMS technology already exist in Japan, China, and Korea [6]. The development and continuation of the maglev railway as an ordinary wheel-rail railway is expected to replace the traditional low-speed wheel-rail transportation system [7]. Maglev train is a kind of emerging railway technology, and many key structures have strong nonlinear characteristics, such as suspension control, metal rubber, and air springs. It is necessary to analyze and calculate it. The EMS type maglev train uses electromagnetic suction to overcome the weight of the train, and the magnitude of the electromagnetic force is proportional to the square of the current and inversely proportional to the square of the suspension gap. Therefore, the mathematical model of the maglev system has open-loop instability [8, 9]. When the maglev train is running, it is necessary to maintain a certain suspension gap, and only good stability can ensure the safety and comfort of the train running.

Due to the nonlinear dynamic characteristics of the maglev train suspension system, many scholars have proposed and improved different suspension control schemes. Wang et al. [10] established a system model in which a joint structure replaces a single magnetic levitation unit, and a
control strategy based on the linear-quadratic optimal control method is proposed. In the case of external disturbances, it has a good control performance. Li et al. [11] considered inherent nonlinearity, module coupling, coupling, and misalignment of sensors and actuators, and a feedback linear controller based on the mathematical model of the maglev module is proposed, which reduces the dynamic coupling between the two points. At present, the suspension control model of the maglev train mostly adopts the single electromagnet suspension system model, and the linearized model is applied to the classical control theory—PID or PD controller for control. In the suspension control of the maglev train, it is divided into voltage [12, 13] and current [14–18] as the control target.

The main methods for nonlinear vibration analysis are the perturbation method, multiscale method, incremental harmonic balance method [19–21], and average method [22]. Wang et al. [18] established a simplified coupling model of the single suspension point and the elastic beam, and the Hopf bifurcation phenomenon caused by time delay and the influence of control parameters on forced vibration in periodic solutions of subharmonic resonance is discussed by using the multiscale method. Wu and Zeng [19] established a three-degree-of-freedom model of the maglev train body, maglev frame, and electromagnet, and the numerical solution is in good agreement with the incremental harmonic balance method. Zhang et al. [22] studied the nonlinear vehicle suspension system with a single degree of freedom using the average method. By comparing our understanding of different approaches, the IHB method can solve both the weak nonlinear system and the strong nonlinear system, which is a more comprehensive solution method, so this paper uses this method to study suspension control and nonlinear suspension.

Bao et al. [23] established a 2-DOF nonlinear dynamics model, by fitting the experimental data of the nonlinear components. It is found that the stiffness expression of the two-system suspension is a unary quartic polynomial, and good fitting results are obtained under different preloads. Zheng et al. [24] used polynomials to describe the nonlinear stiffness of two-system suspension. The influence of nonlinear characteristics on system stability is studied, and it is found that the nonlinear spring system is chaotic. Han and Jing [25] through theoretical analysis and formula derivation of air spring, the obtained formula shows that the elastic restoring force of the air spring is a quartic polynomial with one variable about the spring shape. The above literature shows the feasibility of using polynomial to represent the maglev train two-system suspension model with air spring. The maglev system model in this paper adopts cubic polynomial to represent the two-system nonlinear suspension, and the dynamic behavior of the system is analyzed theoretically.

To sum up, the existing literature mostly focuses on the suspension control of the single iron suspension model, and there are few maglev system models with two-system nonlinear suspension. To solve this problem, the nonlinear suspension model equation of the maglev system with two degrees of freedom is established in this paper, and after applying sinusoidal excitation to the suspension frame, the system response of the suspension frame and the vehicle body is studied. Taking velocity feedback control parameters as an example, the condition of Hopf bifurcation in the system is obtained. Finally, the bifurcation and chaos phenomena of the maglev system are analyzed by numerical simulation, and it is found that the system is in the alternating state of multiperiod and chaos in some working conditions.

2. Model

2.1. Vehicle-Rail Coupling Model. The nonlinear suspension vibration model of the maglev train with two degrees of freedom is shown in Figure 1.

In Figure 1, $m_1$ and $m_2$ are the mass of the suspension frame and the body, respectively, $z_1$ and $z_2$ are the displacement of the suspension frame and the body, respectively, $h_0$ is the expected value of suspension clearance, the relation between the restoring force and displacement of the nonlinear spring is $F_k = k_1(z_2 - z_1)^3 + k_3(z_2 - z_1)^5$, $k_1$ and $k_3$ are linear and nonlinear stiffness coefficients of the suspension frame, respectively, $c$ is the linear damping coefficient of the suspension frame, and $F_0$ and $\omega$ represent the amplitude and frequency of the excitation force, respectively. The relationship between electromagnetic attraction $F$ [26, 27] and suspension clearance is...
\[ F(i, c) = \frac{\mu_0 N^2 S}{4} \left[ \frac{i(t)}{c(t)} \right]^2. \] (1)

\( \mu_0 \) is the air permeability, \( N \) is the number of turns of electromagnet winding, \( S \) is the magnetic pole area of the electromagnet, \( i(t) \) is the electromagnet current at time \( t \), and \( c(t) = c_0 - z_1 \). It can be seen from equation (1) that the function relation between electromagnetic attraction and suspension clearance is inverse square and nonlinear, and it is proved that the maglev train suspension system is unstable and needs to be controlled.

The Taylor series expansion of equation (1) at the equilibrium point \( (c_0, i_0, h_0) \) is obtained as
\[
F(i, c) = F(i_0, c_0) + \Delta F = F + F_1(i_0, c_0) \Delta i(t) + F_c(i_0, c_0) \Delta c(t) + a(\Delta i, \Delta c).
\] (2)

By ignoring higher-order terms, the linearized form of equilibrium neighborhood can be obtained as
\[
F(i, c) = F(i_0, c_0) + k_i \Delta i(t) - k_c \Delta c(t).
\] (3)

In the previous formula, \( c_0 \) and \( i_0 \) are the suspension gap and current at the equilibrium point, respectively, and \( F(i_0, c_0) \) is the electromagnetic force generated by the electromagnet at the equilibrium point, and maglev train gravity, and other large reverse. The coefficients \( k_i \) and \( k_c \) are, respectively, written as follows:
\[
k_i = F_i(i_0, c_0) = \left( \frac{\partial F}{\partial i} \right)_{i=i_0,c=c_0}
\] (4a)
\[
k_c = F_c(i_0, c_0) = \left( \frac{\partial F}{\partial c} \right)_{i=i_0,c=c_0}
\] (4b)

Using the state feedback method to control the current, with displacement-speed-acceleration as feedback variables, the control current is expressed as [14–18]
\[
\Delta i(t) = k_p \Delta c(t) + k_c \Delta \dot{z}(t) + k_k \Delta \ddot{z}(t).
\] (5)

The performance of the state feedback closed-loop system is determined by \( k_p, k_c, \) and \( k_k \).

According to Newton’s second law, we get
\[
\begin{cases}
    m_1 \ddot{z}_1 - c_1 (\dot{z}_2 - \dot{z}_1) - k_1 (z_2 - z_1) - k_3 (z_2 - z_1)^3 = F \cos(\omega t) - F(i, c) + mg, \\
    m_2 \ddot{z}_2 + c_2 (\dot{z}_2 - \dot{z}_1) + k_1 (z_2 - z_1) + k_3 (z_2 - z_1)^3 = 0.
\end{cases}
\] (6)

Let \( x = z_1 \) and \( y = -z_2 \); then, equation (6) can be re-written as
\[
\begin{cases}
    (m_1 + k_k) \ddot{x} + k_1 \dot{x} \dot{y} + \left( k_k k_p - k_c \right) x - c y - k_1 y - k_3 y^3 = F \cos(\omega t), \\
    m_2 (\ddot{x} + \ddot{y}) + c_2 \dot{y} + k_1 y + k_3 y^3 = 0.
\end{cases}
\] (7)

Formula (7) is normalized as follows.

We make \( m_0 = m_1 + k_k \), \( \omega_1 = \sqrt{k_k k_p - k_c / m_0} \), \( \omega_2 = \sqrt{k_k / m_2} \), \( \mu = m_2 / m_0 \), \( \xi = c / m_2 \), \( \xi_1 = k_k / m_0 \), \( \gamma = k_3 / m_0 \), and \( f = F / m_2 \). Then, equation (7) can be converted into
\[
\begin{align*}
    \ddot{x} + \xi_1 \dot{x} + \omega_1^2 x - \mu \xi \dot{y} - \omega_1 \xi y - \mu y y^3 &= f \cos(\omega t), \\
    \ddot{y} + \xi \dot{y} + \omega_2^2 y + \mu \dot{y} y^3 &= 0.
\end{align*}
\] (8)

Let us write it in the matrix form
\[
\mathbf{M} \ddot{X} + \mathbf{C} \dot{X} + (\mathbf{K}_1 + \mathbf{K}_3) X = \mathbf{F}.
\] (9)

In the previous formula, we get
\[
\begin{array}{c}
    x \\
    y \\
    \xi \\
    \ddot{x} \\
    \ddot{y}
\end{array}
\]
2.2. Incremental Harmonic Balance Calculation. Let $\tau = \omega t$ 
($X' = dx/d\tau$, and $(X'' = d^2x/d\tau^2$); then, formula (9) can 
be written as

$$\omega^2 \mathcal{M} X^{''} + \omega \mathcal{C} X' + (\mathcal{K}_1 + \mathcal{K}_3) X = \mathcal{P}. \quad (11)$$

Let $X_0$ be the solution at a certain moment under the 
disturbance of fixed frequency $\omega_0$, then its adjacent state can 
be expressed in incremental form as

$$\omega^2 \mathcal{M} X^{''} + \omega \mathcal{C} X' + (\mathcal{K}_1 + \mathcal{K}_3) X = \mathcal{P},$$

$$\omega^2 \mathcal{M} (X_0 + \Delta X)' + \omega \mathcal{C} (X_0 + \Delta X)' + (\mathcal{K}_1 + \mathcal{K}_3) (X_0 + \Delta X) = \mathcal{P},$$

$$\omega^2 \mathcal{M} (X_0 + \Delta X)' + \omega \mathcal{C} (X_0 + \Delta X)' + \mathcal{K}_1 (X_0 + \Delta X) + \mathcal{K}_3 (X_0 + \Delta X)^3 = \mathcal{P},$$

$$\omega^3 \mathcal{M} (X_0 + \Delta X)' + \omega \mathcal{C} (X_0 + \Delta X)' + \mathcal{K}_1 (X_0 + \Delta X) + \mathcal{K}_3 X_0^3 + 3 \mathcal{K}_3 X_0^2 \Delta X = \mathcal{P},$$

$$\omega^2 \mathcal{M} \Delta X^{''} + \omega \mathcal{C} \Delta X' + (\mathcal{K}_1 + \mathcal{K}_3) \Delta X = \mathcal{P} - \left[ \omega^2 \mathcal{M} X_0^{''} + \omega \mathcal{C} X_0' + (\mathcal{K}_1 + \mathcal{K}_3) X_0 \right]. \quad (13)$$

Fourier expansion of exact solution $X_0$ and incremental 
$\Delta X$ is as follows:

$$\begin{cases} x_{j0} = a_{j0} + \sum_{i=1}^{N} (a_{j,k}\cos k\tau + b_{j,k}\sin k\tau), \\
\Delta x_{j} = \Delta a_{j0} + \sum_{i=1}^{N} (\Delta a_{j,k}\cos k\tau + \Delta b_{j,k}\sin k\tau). \quad (14) \end{cases}$$

We define

$$A_j = [a_{j0}, a_{j1}, a_{j2}, \ldots, a_{jN}, b_{j0}, b_{j1}, b_{j2}, \ldots, b_{jN}]^T,$$

$$\Delta A_j = [\Delta a_{j0}, \Delta a_{j1}, \Delta a_{j2}, \ldots, \Delta a_{jN}, \Delta b_{j0}, \Delta b_{j1}, \Delta b_{j2}, \ldots, \Delta b_{jN}]^T, \quad \text{(15)}$$

We make 
$$(S = \text{diag} [C_1, C_2]), \quad (A = [A_1, A_2]^T), \quad \left(\Delta A = [\Delta A_1, \Delta A_2]^T\right), \quad (X_0 = S A_0), \quad \text{and} \quad (\Delta X_0 = S \Delta A).$$

Applying the Galerkin averaging process to integrate equation (13), taking $\tau$ as the integration variable and $[0,2\pi]$ as the integration interval, we can obtain

$$\int_{0}^{2\pi} (\Delta x)^T [\omega^2 \mathcal{M} \Delta X'' + \omega \mathcal{C} \Delta X' + (\mathcal{K}_1 + 3 \mathcal{K}_3) \Delta X] d\tau$$

$$= \frac{1}{2\pi} \int_{0}^{2\pi} (\Delta x)^T \left[ \mathcal{P} - \left[ \omega^2 \mathcal{M} X_0^{''} + \omega \mathcal{C} X_0' + (\mathcal{K}_1 + \mathcal{K}_3) X_0 \right] \right] d\tau.$$ 

We tidy up

$$\left\{ \frac{1}{2\pi} \int_{0}^{2\pi} S^T \left[ \omega^2 \mathcal{M} S'' + \omega \mathcal{C} S' + (\mathcal{K}_1 + 3 \mathcal{K}_3) S \right] d\tau \right\} \Delta A$$

$$= \frac{1}{2\pi} \int_{0}^{2\pi} S^T \left[ \mathcal{P} - \left[ \omega^2 \mathcal{M} S'' + \omega \mathcal{C} S' + (\mathcal{K}_1 + \mathcal{K}_3) S \right] A \right] d\tau.$$ 

We get the linearized equation for $X_0$ and increment $\Delta X$: 

$$K_{mc} \Delta A = R. \quad (18)$$

Among them,

$$K_{mc} = \frac{1}{2\pi} \int_{0}^{2\pi} S^T \left[ \omega^2 \mathcal{M} S'' + \omega \mathcal{C} S' + (\mathcal{K}_1 + 3 \mathcal{K}_3) S \right] d\tau,$$

$$R = \frac{1}{2\pi} \int_{0}^{2\pi} S^T \left[ \mathcal{P} - \left[ \omega^2 \mathcal{M} S'' + \omega \mathcal{C} S' + (\mathcal{K}_1 + \mathcal{K}_3) S \right] A \right] d\tau.$$ 

3. Stability Analysis

We define 
$$(x = (x_1, x_2, x_3, x_4)^T = (x, \dot{x}, y, \dot{y})^T),$$

and the dynamic equation of the system can be written as the 
equation of the state as follows:

$$\begin{cases} x = x_2, \\
\dot{x} = f \cos (\omega t) - \xi x_2 - \omega_1^2 x_1 + \mu \omega_2^2 (y_1 - x_1) + \mu \xi (y_2 - x_2) + \mu y (y_2 - x_2)^3, \\
y = x_4, \\
\dot{y} = -\omega_2^2 (y_1 - x_1) - \xi (y_2 - x_2) - y (y_2 - x_2)^3. \quad (20) \end{cases}$$
At the equilibrium point, \((x_0 = (0, 0, 0, 0))\) linearizes to (21), and we get
\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -\xi_1 x_2 - \omega_1^2 x_1 + \mu \omega_1^2 (y_1 - x_1) + \mu \xi_1 (y_2 - x_2), \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -\omega_2^2 (y_1 - x_1) - \xi (y_2 - x_2).
\end{align*}
\] (21)

We write equation (21) as \((\dot{x} = Ax)\), where
\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\omega_1^2 - \mu \omega_2^2 - \xi_1 - \mu \xi_2 - \mu \xi_1 & \mu \omega_2^2 + \mu \xi_2 & \xi_1 & \mu \xi_2 \\
0 & 0 & 0 & 1 \\
\omega_2^2 & \xi & -\omega_2^2 & \xi
\end{bmatrix}
\]
The characteristic polynomial is
\[
J(\lambda) = \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4.
\] (22)
Among them, \((a_1 = -\xi + \xi_1 + \mu \xi), (a_2 = \omega_1^2 + \omega_2^2 - 2 \mu \xi_2 + \mu \xi_1), (a_3 = -\xi \omega_1^2 + \xi \omega_2^2 - 2 \xi \mu \omega_2^2),\) and \((a_4 = \omega_1^2 \omega_2^2)\).
The stability interval of control parameters can be obtained as
\[
k_p \geq 7494.8, \\
k_v \geq 0, \\
k_u \geq 0.
\] (23)

4. Simulation Analysis of Stability and Bifurcation

4.1. Parameter Setting and Solution Results. The parameters used in the calculation and analysis in this paper are set as follows [28]: \((m_1 = 80000kg), (m_2 = 39000kg), (k_1 = 2.05 \times 10^5N/m), (k_3 = 6 \times 10^5N/m^2), (c = 100000N \cdot s/m), (R = 0.61\Omega), (\mu_0 = 4\pi \times 10^{-7}), (z_0 = 0.01m), (N = 290), \) and \((S = 0.311m^2)\).

4.2. Influence of Track Irregularity Amplitude. After the system is stable, a period of roughness amplitude excitation is applied to the suspension to detect the stability of the system. The roughness amplitude excitation is shown in Figure 2, and the dynamic response of the vehicle under the roughness amplitude is shown in Figure 3. The feedback control parameters are \((k_p = 7800), (k_v = 50), \) and \((k_u = 5)\).

4.3. Influence of the Force Shock. When the system suddenly accelerates or brakes, the excitation is not periodic. Therefore, a shock force excitation of \(1000kN\) is applied to the suspension. The dynamic response of the system is shown in Figure 4, where feedback control parameters are the same as in Section 4.2.

4.4. Solution Result Comparison. Using the Runge–Kutta numerical method to solve equation (5), compared with the approximate analytical solution of the system calculated by the IHB method. The calculation results are shown in Figure 5. It can be seen that the calculation results of the two methods have high consistency.

4.5. Influence of System Parameters on System Motion Characteristics. Figure 6 shows that the resonance peak of the amplitude-frequency curves of the vehicle body and suspension gradually shift to the right with the increase of stiffness \(k_1\). The increase of \(k_1\) makes the amplitude of the car body tend to increase slightly, the impact on the vibration response of the suspension frame is obvious, with the increase of \(k_1\), and the difference between the maximum value and the minimum value of the suspension frame increases with a relatively obvious trend. When the excitation frequency of the external excitation sweeps this frequency range, then the vibration of the suspension frame may be aggravated, which is not conducive to the stability of the suspension frame.

Figure 7 shows that the highest point of the resonance peak of the amplitude-frequency curves of the vehicle body and suspension gradually shifts to the right with the increase of stiffness \(k_3\). Comparing Figures 6(a) and 7(a), it can be found that, with an increase in \(k_1\), the frequency corresponding to the formant peak of the peak of the car body formant is increased, and increasing \(k_3\) gradually increases the
Figure 3: Roughness amplitude excited dynamic response. (a) Car body. (b) Suspension.

Figure 4: Dynamic response of the system under impact excitation. (a) Car body. (b) Suspension.

Figure 5: Comparison of numerical and analytical solutions of the amplitude-frequency curve. (a) Car body. (b) Suspension.
Comparing Figures 6(b) and 7(b), it is found that, $k_3$ makes the difference between the maximum value and the minimum value of the resonance peak decrease with the increase of $k_3$, and the amplitude-frequency response curve becomes smoother as the frequency changes, which is helpful for the stability of suspension.

4.6. Influence of Feedback Control Parameters on System Motion Characteristics. Figures 8–10 show the response results of the car body and the suspension frame under different feedback control parameters. I$_p$, $k_3$, $k_1$, and $k_2$ are the feedback gains of the car body and suspension frame, respectively. With the gradual increase of $k_p$, the vibration amplitude of the car body and suspension frame has a relatively obvious decrease, the dynamic response of $ky$ to the suspension frame is more obvious, and the maximum and minimum values of the formant are reduced and makes the curve drop at the formant smoother. The increase of $ka$ increases the peak value of the resonance area of the car body and the suspension frame. However, as can be seen from Figure 10(b), the increase of $ka$ makes the suspension amplitude decrease the fastest when the excitation frequency $\omega$ is greater than 5.

4.7. Hopf Bifurcation

**Theorem 1** (see [29, 30]). We set the coefficient algebraic equation as

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots + a_{n-1}\lambda + a_n = 0. \tag{24}$$
The necessary and sufficient conditions for having a pair of pure imaginary eigenvalues and the remaining $n-2$ eigenvalues have negative real parts which are

$$a_i > 0 (i = 1, 2 \ldots n), \Delta_i > 0 (i = n - 3, n - 5 \ldots), \Delta_{n-1} = 0.$$  \hspace{1cm} (25)

From the coefficients of equation (25), the following Hurwitz determinant can be constructed:

$$\Delta_i = \begin{vmatrix} a_1 & 1 & 0 & 0 & \ldots & 0 \\ a_3 & a_2 & a_1 & 1 & \ldots & 0 \\ a_5 & a_4 & a_3 & a_2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{2i-1} & a_{2i-2} & a_{2i-3} & a_{2i-4} & \ldots & a_i \\ \end{vmatrix}$$  \hspace{1cm} (26)

where if $i > n$, then $a_i = 0$.

Applying Theorem 1 to equation (23), the condition for the existence of a pair of pure imaginary eigenvalues and the remaining eigenvalues with negative real parts is

$$a_i > 0 (i = 1, 2, 3, 4, 5),$$

$$\Delta_3 = \begin{vmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \\ \end{vmatrix},$$

$$= -a_4a_1^2 + a_1a_2a_3 - a_3^2 = 0.$$  \hspace{1cm} (27)

So, the critical control parameter of system (21) is

$$k_p = 7494.7,$$

$$k_v = 0,$$  \hspace{1cm} (28)

$$k_a = 0.$$
Figure 10: Effects of different acceleration control parameters on the system. (a) Car body. (b) Suspension. (c) Car body partial magnification.

Figure 11: Continued.
Figure 11: Hopf bifurcation of the car body. (a) $k_y = -0.1$, $x_0 = 0.01$. (b) $k_y = 0$, $x_0 = 0.01$. (c) $k_y = -0.1$, $x_0 = 0.01$.

Figure 12: Continued.
Taking the $k_y$ bifurcation parameter as an example, the motion characteristics of the system change as shown in Figures 11 and 12. As can be seen from Figures 11(a) and 12(a), when the speed feedback control parameter $k_d < 0$, the system is unstable, the amplitudes of suspension and the car body do not converge. Figures 11(b) and 12(b) show that,
Figure 14: Phase diagram and Poincare cross section of the system when $k_p = 7620$.

Figure 15: Phase diagram and Poincare cross section of the system when $k_p = 7600$.

Figure 16: Phase diagram and Poincare cross section of the system when $k_p = 7600.18$. 
when the speed feedback control parameter reaches the critical value $k_d = 0$, Hopf bifurcation occurs in the system and periodic vibration occurs; Figures 11 and 12(c) show that, when the speed feedback control parameter $k_d > 0$, the maglev system tends to be stable, and the amplitude of suspension and the car body tends to zero. It means that the suspension gap of the suspension frame converges to the desired suspension gap of 0.01 m.

4.8. Analysis of Bifurcation and Chaotic Motion. In order to analyze the influence of feedback control parameters on the dynamic response of the system, with $k_p$ as the variable parameter, using the numerical solution method to solve equation (8), the motion characteristic map of the system under parameter excitation is obtained, as shown in Figure 13. The main parameters of the system are as follows: $F = 11000 \text{kN}, k_y = -0.9, k_a = 0.5$, and $\omega = 3.4$.

With the change of the feedback control parameter $k_p$, the system will produce complex dynamic behavior. To this end, we study the phase plane and Poincare section of the system.

When $k_p = 7620$, the system is a single-cycle motion, as shown in Figure 14. When $k_p = 7600$, the system is multiperiod motion, as shown in Figure 15. When $k_p = 7600.18$, the system is in chaotic motion, as shown in Figure 16.

5. Conclusion

In this paper, a two-degree-of-freedom nonlinear suspension dynamic model of a maglev train is established. According to the Hurwitz stability criterion, the boundary conditions for the stability of the suspension system are obtained, under sinusoidal excitation applied by suspension. The nonlinear response characteristics of the system are analyzed using the incremental harmonic balance method, and the following conclusions are drawn:

1. The linear stiffness of the nonlinear suspension is gradually increased, the formants of the body and suspension will shift to the right, the amplitude of the car body formant increases slightly, and the maximum and minimum values at the resonance peak of the suspension frame tend to increase, which will cause the suspension frame to vibrate more violently under the external excitation in this frequency range.

2. The nonlinear stiffness of the nonlinear suspension is gradually increased and will increase the slope of the body resonance peak. In the process of external excitation from small to large, the amplitude of the car body will have a huge drop in a short period of time. At this time, it has poor riding comfort; the difference between the maximum value and the minimum value at the resonance peak of the suspension frame decreases with the increase of nonlinear stiffness and reduces the vibration intensity of the suspension frame. It is helpful to the vibration stability of the suspension frame.

3. If the PID controller is used, sinusoidal excitation is applied to the suspension frame. $k_p$ can effectively weaken the amplitude of the resonance peak of the car body and suspension and has a large positive effect on the stability of the suspension system. As $k_p$ increases, the amplitude of the resonance peak of the car body is only slightly reduced. The amplitude-frequency curve of the suspension frame has better smoothness, weakens the curve bulge at the maximum value of the formant, and makes the vibration response change of the maglev system smoother. Moreover, $k_a$ has a certain reduction effect on the overall vibration amplitude of the suspension frame. The increase of $k_a$, the peaks of the resonance area of the car body, and the suspension frame are increased. In addition, it can be found that the increase of $k_a$ can effectively suppress the vibration of the suspension frame when the excitation frequency $\omega$ is greater than 5.

4. This paper also analyzes the bifurcation and chaotic motion of the system, the bifurcation diagram of the system is obtained, from the phase diagram and Poincare cross section diagram. It can be found that, as the system parameters change, there will be an alternation of multiperiod motion and chaotic motion, and it is not conducive to the stability of the suspension system.

Data Availability

The data are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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