Disturbance Rejection Control Based on Linear Quadratic for Nonminimum-phase Hypersonic Flight Vehicle System

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Abstract. This paper proposed a disturbance rejection control method based on linear quadratic (LQ) for nonminimum-phase discrete-time systems with unmatched signals. By introducing a new cost function that considers the influences of disturbance on the input signal, we apply the novel control method to hypersonic flight vehicle (HFV) system to solve the problem of turbulence compensation. A two-component control input is generated through systematic derivation of finite-time LQ optimal control law, improving the stability, output regulation capability, and robustness of the HFV system. Comparison analysis under different control schemes in simulation study shows the effectiveness of the proposed method.

Introduction

Hypersonic flight vehicle (HFV) is a near-space vehicle with a flying speed exceeding five times Mach and a flying altitude between 20 and 100 km. Compared with traditional general aircrafts, HFVs feature many advantages, such as fast flight speed, good lateral maneuverability, and high target strike accuracy [1]. However, from the point of view of control, HFVs are a nonlinear system with fast time-varying, strong coupling, and non-minimum phase. Complicated external disturbances and uncertainties in the aircraft mode increase the complexity of design for flight control system.

Turbulence is an important factor affecting HFV performance. With in-depth research and exploration, researchers have achieved breakthroughs in modeling turbulence [2]; thus, the current control system not only presents flexible maneuverability but also a certain robustness and adaptability [3]. In [4], a feedforward control technology is adopted for gust disturbance design of aircraft. The existing turbulence disturbance rejection methods are mostly directed to single-input and single-output aircraft systems. Therefore, the development of new disturbance rejection technology bears importance in aircraft engineering.

Linear quadratic (LQ) control has been continuously developed in recent years. The main methods include the adaptive LQ disturbance rejection method and LQ optimal control disturbance rejection method [5, 6]. The optimal control system and optimal control method of discrete linear systems are introduced in [7] and provide a theoretical basis for practical engineering modeling. Inspired by control method designed in [6], this paper raises a novel control method based on LQ for nonminimum-phase systems and provides a solution for compensating the effects of disturbance.

The reminder of this paper is arranged as follows. First, the linear system model and control objective are given. Secondly, we discuss the development of the nominal LQ control method, and make comparison between LQ control separation design and traditional LQ control design. Thirdly, numerical simulations are conducted to illustrate the validity of the proposed method. Finally, the study conclusions are drawn.
**Problem Formulation**

The common longitudinal dynamics of HFV continuous system is proposed by NASA Langley Research Center in [5] as follows:

\[
\dot{h} = V \sin(\theta - \alpha). 
\]  

(1)

\[
\dot{V} = \frac{T \cos \alpha - D}{m} - g \sin(\theta - \alpha). 
\]  

(2)

\[
\dot{\alpha} = \frac{-T \sin \alpha - L}{mV} + q + \frac{g \cos(\theta - \alpha)}{V}. 
\]  

(3)

\[
\dot{\theta} = q. 
\]  

(4)

\[
\dot{q} = \frac{M}{I_{yy}}. 
\]  

(5)

where \( h \) refers to the altitude; \( V \) denotes the velocity; \( \alpha \) represents the angle of attack; \( \theta \) stands for the pitch angle; \( q \) corresponds to the pitch rate; \( T, L, D \) specify the thrust, lift and drag, respectively; \( M \) indicates the pitch moment of \( y \)-axis; \( I_{yy} \) is the moment of inertia.

In practical application, turbulence is usually introduced into the equation of state. Under the condition of turbulence disturbance, the linear model of HFV system [5] can be generalized into the following equation:

\[
\begin{cases}
    x(k+1) = Ax(k) + Bu(k) + B_d d(k) \\
    y(k) = Cx(k)
\end{cases}
\]  

(6)

where state variable \( x = [V, \alpha, q, \theta, h]^T \), input signal \( u = [\delta, \eta]^T \), \( \delta \) denotes the attitude angle, and \( \eta \) is the time derivative of the elastic coordinates. The system matrices \( A \) and \( B \) are decided by flight conditions, such as flight altitude and Mach number, and \( C \) is a proper output matrix. \( d(k) = [V_w, \alpha_w]^T \) corresponds to disturbance signal, \( V_w \) denotes the velocity of turbulence wind, \( \alpha_w \) represents the angle of attack produced by turbulent wind, and \( B_d \) is turbulence disturbance matrix.

The control objective is to design a LQ control input \( u(k) \) for the linear system (6). The linear system may be nonminimum-phase and have unmatched signals. Based on the above statements, two important assumptions are presented as follows.

**Assumption 1.** Control system (6) is controllable and observable.

**Assumption 2.** If system matrices \( B \) and \( B_d \) satisfy \( B_d \neq B \beta, \forall \beta \in R^{M \times p} \), then the system features an unmatched input \( u(k) \) and disturbance \( d(k) \).

Then, the disturbance compensation design for the above two conditions is carried out below.

**Nominal LQ Control Design**

**Nominal LQ Control Separation Design**

In order to realize the compensation of disturbance and adjust the output of system, we describe a novel cost function as follows:
\[ J_N = y^T (N) P y(N) + \sum_{k=0}^{N-1} (y^T(k) Q y(k) + u^T(k) R u(k)). \]  

where \( P, Q, R \) are selected as proper matrices, one input signal \( u_i(k) \) is for system stability and output regulation.

On the basis of (7), the finite-time LQ optimal control law is deduced. First, several functions are introduced:

\[ F_i(y(k), u_i(k)) = y^T(k) Q y(k) + u^T_i(k) R u_i(k). \]  

\[ J_{N-i} = y^T (N) P y(N) + \sum_{k=i}^{N-1} F_i(y(k), y_i(k)). \]  

Minimizing \( J_{N-i} \) (from \( i = N - 1 \) to \( i = 0 \)) is necessary in (9) to reduce \( J_N \) in (7).

If control system (6) satisfies the conditions of Assumptions 1, 2, and 3, then the control input signals are respectively designed as follows:

\[ u(k) = u_i(k) + u_s(k), \quad k = 0,1,\ldots, N-1. \]  

\[ u_i(k) = -(B^T P(k+1) B + R)^{-1} B^T P(k+1) A x(k). \]  

\[ u_s(k) = u_s(k) = -(B^T P(k+1) B)^{-1} P(k+1) B_s d(k). \]  

where matrix \( P(k) \) is the positive definite solution of the \textit{Riccati} iteration equation:

\[ P(k) = A^T P(k+1) A + Q_q - A^T P(k+1) B (B^T P(k+1) B + R)^{-1} B^T P(k+1) A. \]  

This novel control signal minimizes (7) and ensures the value of \( u_i(k), \ y(k) \) small enough. The turbulence disturbance causes no effect on the system output signal \( y(N) \) at the final point.

\textbf{Comparative Analysis}

This subsection provides detailed contrasts between the novel LQ control separation method and traditional LQ control method. First, the traditional control method is introduced.

The cost function in traditional LQ control is described in [4] as follows:

\[ J_N = y^T (N) P y(N) + \sum_{k=0}^{N-1} (y^T(k) Q y(k) + u^T(k) R u(k)). \]  

Similar derivations with the cost function (14) are still adopted on the basis of the optimal development of the traditional finite-time LQ control design. Control input signals are designed as follows:

\[ u(k) = u_i(k) + u_s(k), \quad k = 0,1,\ldots, N-1. \]  

\[ u_i(k) = -(B^T P(k+1) B + R)^{-1} B^T P(k+1) A x(k). \]  

\[ u_s(k) = -(B^T P(k+1) B + R)^{-1} B^T P(k+1) B d(k). \]  

where the boundary conditions satisfy \( P(N) = Q_p = C^T P C, \ Q_q = C^T Q C. \)

Next, comparisons are made as follows.
Influence of Turbulence Disturbance. At time $k = N - 1$, optimal control laws (15), (16), and (17) are applied to system (6), thus resulting in the following:

$$y_{\text{trad}}(N) = C(A - (B^TQ_pB + R)^{-1}B^TQ_pA)x(N - 1) + (I - CB(B^TQ_pB + R)^{-1}B^TC^T)CB_d d(N - 1).$$  \hspace{1cm} (18)

where the effect of turbulence disturbance has not been completely eliminated. At time $k = N - 1$, optimal control law (10), (11), and (12) are used to system (6), resulting in the following:

$$y_{\text{new}}(N) = C(A - (B^TQ_pB + R)^{-1}B^TQ_pA)x(N - 1).$$  \hspace{1cm} (19)

in which the influence of turbulence disturbance to the system output has been completely eliminated.

Afterward, an assumption is presented as follows:

$$y_{\text{trad}}(N) - y_{\text{new}}(N) = (I - CB(B^TQ_pB + R)^{-1}B^TC^T)CB_d d(N - 1).$$  \hspace{1cm} (20)

where $(I - (B^TQ_pB + R)^{-1}CB^TC^T) > 0$. Therefore, $|y_{\text{trad}}(N)| > |y_{\text{new}}(N)|$.

The above derivations prove that the novel control method based on LQ is simpler but more efficient in solving the problem of system stability, turbulence rejection, and output tracking than the existing method.

Numerical Simulation

In this section, the novel control separation method based on LQ is used to HFV system with turbulence disturbance, and the simulation results are drawn to prove the validity of the developed control scheme.

A continuous-time longitudinal dynamics model of HFV is shown in [5], and the sampling time $T = 0.1$ s is selected to achieve a corresponding discrete-time model as (6). The system parameter matrices $A, B$ and $B_d$ are shown in [5]. The flight conditions are chosen as 8 Mach number and a flight altitude of 85700 ft.

The output matrix is set as $C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$, and longitudinal turbulence disturbance is set as $V_w = 5$ ft/s, $\alpha_w = \sin(0.4k) + 2\sin(0.2\pi k) + 0.1$ rad/s. The initial state is selected as $x_0 = [0.1, 0.1, 0.1, 0.1, 0.1]^T$, and $N = 200$.

**Case 1: Traditional LQ Design with Different Weight Matrices**

Comparison is made between traditional LQ control methods (14) with different weight matrices $P, Q, R$. Figure 1(a) shows the optimal control signal and output signal with weight matrices $P = Q = R = 2I, I = [1, 0; 0, 1]^T$. The response curves in Figure 1(a) show that disturbance influences the system, and the output $y(k)$ fluctuates relatively at around 0. Hence, the matrices are regulated as $P = Q = 10I, R = I$. Figure 1(b) shows the system responses. The system output $y(k)$ is improved with limited effects, and the effect of turbulence disturbance signal cannot be eliminated completely. Also, finding a set of matrices presents difficulty in practical engineering.
Case 2: New LQ Design with and Without Disturbance Compensation

This case compares a system with no disturbance signal $u_d(k)$ with a system that uses the new LQ control separation. For convenience of analysis, weight matrices are set as $P = Q = R = 2I$. As shown in Figure 2(a), the system features no turbulence compensation signal $u_d(k)$, that is, $u(k) = u_i(k)$. Disturbance significantly influences the system, for which the safety and stability of HFV performance cannot be guaranteed. However, as depicted in Figure 2(b), with the new LQ control separation, the system output $y(k)$ is approaching 0, thus ensuring system stability.

The validity of the adopted LQ control methods is explained through the above cases. Case 1 discusses the effect of different weight matrices to system output, whereas Case 2 presents the advantages of the new LQ control separation method.

Summary

This paper proposed a control separation method through LQ, which has solved the problem of disturbance rejection for nonminimum-phase discrete systems with unmatched signals. By designing a novel cost function and dividing the input signal into two parts, the new method has achieved improved system stabilization and good regulation of output signal. The proposed control scheme features strong capabilities to realize the desired disturbance compensation performances. The effectiveness of the control method has been verified by simulation.

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