Investigation of the optical properties of a spherical distributed Bragg reflector

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Abstract. The dependence of the optical properties of a spherical distributed Bragg reflector (SDBR) on the radius of the SDBR core is studied. The SDBR reflection spectra were calculated by the method of spherical wave transfer matrices for different polarizations and different angular modal numbers of spherical waves. It is shown that if the core radius is much larger than the Bragg wavelength in the core substance, the width and spectral position of the stop-band are the same for both polarizations (TM and TE) of spherical waves and do not depend on the angular modal number of the spherical wave. If the core radius is less than several Bragg wavelengths in the core substance, a narrow dip appears in the reflection spectrum in the region of the stop-band for one polarization and some values of the angular modal number of spherical waves. It is shown that the reason for the appearance of this dip is the occurrence of a resonance in the SDBR quarter-wave layer closest to the core.

1. Introduction

A spherical distributed Bragg reflector (SDBR) consists of periodically alternating quarter-wave layers with high and low refractive index that surround a transparent dielectric sphere (core) [1]. Due to Bragg diffraction in the reflection spectra of the SDBR there are bands with a reflection coefficient close to unity — stop-bands. In consequence of spherical symmetry, the stop-bands for the radial directions of propagation of light spectrally overlap and form an omnidirectional stop-band.

Due to the omnidirectional stop-band, SDBRs can be used to control spontaneous emission [2], to create low-threshold lasers [3] and single-photon light sources [4]. However, the detailed dependence of the optical properties of the SDBR on the parameters of the SDBR structure is still not investigated. This paper is aimed at studying the dependence of the omnidirectional SDBR stop-band on the radius of the SDBR core at different polarizations and different values of the angular modal number of a spherical wave.

2. Calculation of the reflection spectra of the SDBR

The structure under study consists of a central core with a radius of $R_{co}$ and a refractive index of $n_{co}$ and a SDBR (figure 1). The SDBR consists of two pairs of quarter-wave layers with refractive indices $n_1$ and $n_2$ (figure 1). For the calculation, refractive index values were used, corresponding to the refractive indices of SiO$_2$ and Si in the near infrared region: $n_1 = 1.45$, $n_2 = 3.46$, $n_{co} = 3.46$. The thickness of each SDBR layer is equal to $\lambda_B/4n$, where $\lambda_B = 1.53 \mu$m is the Bragg wavelength.

To calculate the reflection spectra of the SDBR, we used the decomposition of the electromagnetic wave field in the basis of vector spherical harmonics and the method of spherical wave transfer matrices [5].
The reflection coefficient of the SDBR was determined from the transfer matrix of the SDBR [6, 7]. Spherical electromagnetic waves have two independent polarizations: TM (the magnetic field has only a tangential component to the boundaries of the SDBR layers) and TE (the electric field has only a tangential component to the boundaries of the SDBR layers). Spherical waves are characterized by angular modal numbers \( l \) (positive integer, \( l = 1, 2, 3, \ldots \)) and \( m \) (integer, \( -l \leq m \leq l \)) [5]. The SDBR reflection coefficient depends on the polarization and the angular modal number \( l \) of the spherical wave and does not depend on \( m \).

The spherical wave propagates simultaneously in all directions, therefore the omnidirectional stop-band for spherical waves is the area of overlapping of all partial stop-bands for spherical waves of all polarizations with all angular modal numbers.

Figure 2 (a, b, c) shows the calculated reflection spectra of the SDBR (figure 1), for the core radius \( R_{co} = 100 \lambda_{Br} / n_{co} \) and for some values of the angular modal number \( l = 1, 10, 20 \). The reflection band with a reflection coefficient close to 1 (stop-band) is present on all spectra in the region near \( \omega / \omega_{Br} = 1 \) \((\omega_{Br} = 2\pi c / \lambda_{Br} - \) is the Bragg frequency, \( c \) is the speed of light in vacuum). The shape of the reflection band, its width and spectral position practically coincide for both TM and TE polarizations and for different values of the angular modal number \( l \). The overlap area of these bands is shown in figure 2 (a) with a horizontal two-sided arrow. Additional calculations showed that if the core radius is much larger than the Bragg wavelength in the core substance, the width and spectral position of the SDBR stop-band do not depend on the core radius, polarization and angular modal number of the spherical wave in the studied range of values \( 1 \leq l \leq 20 \). Accordingly, the width of the stop-band determines the width of the omnidirectional stop-band. The spectra for the TM and TE polarizations approximately coincide with each other in the range of parameters that we studied \((1 \leq l \leq 20, \ 100 \lambda_{Br} / n_{co} \leq R_{co} \leq 1000 \lambda_{Br} / n_{co})\). The explanation of the reasons for this coincidence requires a separate large detailed study, which lies beyond the scope of this article.

Figure 2 (a) shows the calculated reflection spectrum at normal incidence of light for a planar distributed Bragg reflector (PDBR) with the same layer parameters (the thickness and the refractive index) as in the SDBR. The calculation was carried out using the plane-wave transfer-matrix method [8]. Due to the normal incidence of light, the spectra for the TM and TE polarizations coincide and therefore the polarization for the PDBR spectrum is not indicated in figure 2 (a). The shapes of the reflection bands in the region of the stop-band coincide for the SDBR and the PDBR. Thus, the width and spectral position of the omnidirectional stop-band for the SDBR coincide with the width and spectral position of the stop-band for normal incidence of light on the PDBR [9] with the same layer parameters.
**Figure 2.** The calculated reflection spectra of the SDBR and PDBR. The frequency normalized to the Bragg frequency $\omega_{Br}$ is plotted on the abscissa axis. The core radius and the angular modal number of the spherical wave: $R_{co} = 100 \lambda_{Br}/n_{co}$, $l = 1$ (a), $l = 10$ (b), $l = 20$ (c) and $R_{co} = 0.5 \lambda_{Br}/n_{co}$, $l = 1$ (d), $l = 2$ (e), $l = 4$ (f). The spectra for TM polarization are shown by dotted lines, the spectra for TE polarization are shown by solid lines. The reflection spectrum of the PDBR with the same layer parameters as in the SDBR, at normal incidence of light, is shown with crosses (a). Horizontal double-sided arrows (a, d) show the overlap area of the stop-bands for two different polarizations (TM and TE). Vertical arrows (e, f) show dips in the stop-band.
If the core radius is less than several Bragg wavelengths in the core substance, then the reflection spectra of the SDBR differ significantly for different polarizations and different angular modal numbers. Figure 2 (d, e, f) presents the calculated reflection spectra of the SDBR (figure 1) for the core radius \( R_{co} = 0.5\lambda_{Br}/n_{co} \) and for \( l = 1, 2, 4 \). For \( l = 1 \) (figure 2 (d)), the stop-band for TM polarization is noticeably narrower than that for TE polarization. Accordingly, the width of the overlap area of these stop-bands (shown by a horizontal two-sided arrow) is less than the width of the omnidirectional stop-band for \( R_{co} = 100\lambda_{Br}/n_{co} \) (figure 2 (a)).

For \( l = 2 \) (figure 2 (e)), the reflection spectrum for TM polarization has a deep dip at \( \omega / \omega_{Br} = 1.17 \) (marked by a vertical arrow) in the stop-band for TE polarization.

For \( l = 4 \) (figure 2 (f)), there is no dip in the reflection spectrum for TM polarization, but there is a narrow dip in the reflection spectrum for TE polarization (indicated by a vertical arrow) at the frequency \( \omega = 0.94\omega_{Br} \) which is close to the Bragg frequency.

Thus, when the radius of the core is less than several Bragg wavelengths in the substance of the core, deep dips appear in the stop-band of the SDBR reflection spectra, for some polarizations and angular modal numbers of spherical waves. In order to find out the reason of the appearance of these dips, the reflection spectra were calculated for all SDBR boundaries, for those cases when the dip occurs: \( R_{co} = 0.5\lambda_{Br}/n_{co} \), TM polarization, \( l = 2 \) (figure 3 (a)) and TE polarization, \( l = 4 \) (figure 3 (b)). Each \( i \)-th reflection spectrum is the reflection spectrum from the SDBR part, which includes all SDBR boundaries with numbers greater than or equal to 1, \( i \), \( i + 1 \), \( i + 2 \), ..., 5; the boundary numbers are shown in figure 1). The light wave falls on this boundary \( i \) from the inside. It can be seen that the dips at \( \omega / \omega_{Br} = 1.17 \) and \( \omega / \omega_{Br} = 0.94 \) (marked by vertical arrows) exist only for the 1st boundary, but are absent for all subsequent boundaries. Consequently, these dips appear due to the interference of light waves in the quarter-wave layer of the SDBR (between the boundaries 1 and 2 in figure 1), closest to the core.

Further analysis showed that the round-trip phase shift in this quarter-wave layer at the dip frequency (the sum of the phases of reflection from both boundaries of the quarter-wave layer plus the increase in the wave phase when the wave passes from the boundary 1 to the boundary 2 and back) is equal to an integer multiples of \( 2\pi \). Thus, the appearance of the dip is due to the occurrence of a resonance in the first quarter-wave layer of the SDBR, the closest to the core. Similar resonances occur in PDBRs at angles of incidence approaching 90 degrees [10].

![Figure 3](image.png)

**Figure 3.** The calculated reflection spectra of the SDBR with the core radius \( R_{co} = 0.5\lambda_{Br}/n_{co} \): TM polarization, \( l = 2 \) (a), TE polarization, \( l = 4 \) (b). Numerals 1 - 5 denote the number of the spherical boundary of the SDBR (figure 1) for which the spectrum is calculated. Vertical arrows mark the dips in the stop-band when light is reflected from the boundary 1.
The existence of these dips in the stop-band must be considered when designing devices based on SDBR. Their presence will reduce the quality factor and change the frequencies of some resonant modes in the SDBR core lying in the spectral region of the dip.

3. Conclusion
The dependence of the SDBR stop-band on the SDBR core radius has been studied for different polarizations and different angular modal numbers of a spherical wave. The reflection spectra of the SDBR are calculated using the decomposition of the electromagnetic wave field in the basis of vector spherical harmonics and the method of spherical wave transfer matrices. It was found that if the core radius is much larger than the Bragg wavelength in the core material, the width and spectral position of the stop-band are the same for both polarizations (TM and TE) and different angular modal numbers of spherical waves. In this case, the width and spectral position of the omnidirectional stop-band for SDBR coincide with the width and spectral position of the stop-band at normal light incidence for the PDBR with the same layer parameters. If the core radius is less than several Bragg wavelengths in the core substance, the width and position of the stop-band differ for different polarizations and different angular modal numbers of spherical waves. For one polarization and for some values of the angular modal number of a spherical wave, a narrow dip appears in the reflection spectrum of the SDBR in the region of the stop-band. It is shown that the cause of this dip is the occurrence of a resonance in the quarter-wave layer of the SDBR, closest to the core.

Acknowledgments
The work was financed in the framework of the state budget agreement (0040-2019-0012).

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