Theoretical descriptions of isospin corrections for superallowed $\beta$ transitions

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Abstract. The isospin symmetry-breaking corrections $\delta_c$ obtained by self-consistent relativistic random phase approximation (RPA) calculations are compared with those obtained by shell model calculations as well as by isospin- and angular-momentum-projected nuclear density functional theory. It is found that the present theoretical uncertainty of the isospin symmetry-breaking corrections $\delta_c$ is of the order of $0.003$, rather than $0.0002$ as indicated by the shell model calculations. Whether the unitarity of the CKM matrix is fulfilled or not is still an open question.

1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2], which relates the quark eigenstates of the weak interaction with the quark mass eigenstates, is one of the key transformations in the Standard Model. Kobayashi and Maskawa were awarded the 2008 Nobel Prize in Physics “for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature” [3]. If the three-generation Standard Model is complete, the rotation embodied in the CKM matrix must be unitary. Therefore, measuring the matrix elements independently and verifying its unitarity condition provide a rigorous test for the Standard Model and set limits on new physics beyond.

To date, the most precise test is that using the three top-row elements $V_{ud}$, $V_{us}$ and $V_{ub}$ [4, 5], in which the leading term $|V_{ud}|^2$ contributes $\sim 95\%$ to the unitarity sum. Thus, determining the $|V_{ud}|$ value and restraining its uncertainty is a critical issue for both experimental and theoretical researches.

Four experimental methods to determine the $|V_{ud}|$ value so far include the nuclear $0^+ \rightarrow 0^+$ superallowed $\beta$ transitions [6], neutron decay [7], pion $\beta$ decay [8] and nuclear mirror transitions [9]. In particular, the first method provides the most precise determination [4, 5]. In order to determine the $|V_{ud}|$ value with these nuclear superallowed $\beta$ transitions, apart from the experimental $f_t$ values, the radiative corrections $\Delta_R^V$, $\delta_R^\prime$, $\delta_{NS}$ and the isospin symmetry-breaking corrections $\delta_c$ must also be taken into account [6]. The radiative corrections are due to the emission of real photons and the exchange of virtual photons and Z-bosons in semi-leptonic weak transitions [10, 11], while the isospin symmetry-breaking corrections are used to estimate...
the slight failure of the conserved vector current (CVC) hypothesis caused by the isospin $SU(2)$ symmetry-breaking in finite nuclei [11]. It is expected that the corrected $\mathcal{F}t$ values should be nucleus-independent once the radiative and isospin symmetry-breaking corrections are taken into account.

The isospin symmetry-breaking corrections $\delta_{c}$ characterize the slight reduction of the superallowed transition strengths $|M_F|^2$ from the ideal value $|M_0|^2$:

$$|M_F|^2 = |\langle f | T_\pm | i \rangle|^2 = |M_0|^2(1 - \delta_{c}),$$

with $M_0 = \sqrt{2}$ for $T = 1$ states having exact isospin symmetry. Shell model calculations [11] have been generally used to evaluate the $\delta_{c}$ corrections for the past several decades. However, recently it was pointed out that the significant effects of radial excitations have been neglected in the treatment used in Ref. [11] and hence, the $\delta_{c}$ corrections as well as the $|V_{ud}|$ value have been overestimated [12]. Alternatively, the self-consistent random phase approximation (RPA) based on microscopic mean field theories is another reliable approach for evaluating the superallowed transition strength $|M_F|^2$. Such calculations have been performed for several nuclei with the non-relativistic Skyrme Hartree-Fock approach in Ref. [13]. During the last decade, great efforts have been dedicated to developing the charge-exchange (Q)RPA within the relativistic framework [14, 15, 16]. In particular, a fully self-consistent charge-exchange RPA based on the relativistic Hartree-Fock (RHF) approach [17] was established [16]. A very satisfactory description of spin-isospin resonances was obtained without any readjustment of the energy functional. Adopting these self-consistent relativistic RPA approaches, the corrections $\delta_{c}$ have been systematically investigated in Refs. [18, 19]. Recently, $\delta_{c}$ corrections have also been evaluated within isospin- and angular-momentum-projected nuclear density functional theory (DFT) [20].

Here, the isospin symmetry-breaking corrections $\delta_{c}$ obtained by self-consistent relativistic RPA calculations will be compared with those obtained by shell model calculations as well as by projected DFT. Together with the most up-to-date $|V_{us}|$ and $|V_{ub}|$ values, the unitarity of the CKM matrix will also be discussed.

2. Discussion

In Table 1, the isospin symmetry-breaking corrections $\delta_{c}$ for the $0^+ \rightarrow 0^+$ superallowed transitions obtained by self-consistent RHF+RPA calculations [18] with PKO1 parametrization [17] are compared with those obtained by shell model calculations (T&H) [11] and isospin- and angular-momentum-projected nuclear density functional theory [20] with the Skyrme force SV [21]. In order to clearly see the different predictions among these three approaches, the differences of the isospin symmetry-breaking corrections $\delta_{c}$ are shown in the left panel of Fig. 1 as a function of the charge $Z$ of the daughter nucleus, where the T&H results are taken as references, while the uncertainties of $\delta_{c}$ are not taken into account.

On the one hand, it is found that the RHF+RPA results are systematically smaller than those of T&H by around 0.4% in most cases. Since it was pointed out that the corrections $\delta_{c}$ obtained by shell model calculations may be overestimated due to the neglect of radial excitations, it is worthwhile to notice this systematic discrepancy for future investigations. Meanwhile, it should also be pointed out that, in the present relativistic RPA calculations, the effects of pairing correlations and deformation are not taken into account. In addition, the missing charge symmetry breaking (CSB) and charge independence breaking (CIB) forces are expected to enlarge the corrections $\delta_{c}$ by $\sim 0.1\%$ according to the calculations in Ref. [13].

On the other hand, it is found that the projected DFT results are similar to the T&H results on average, while the individual values differ from each other by about 0.1 $\sim$ 0.2% in most cases. Nevertheless, three $pf$-shell nuclei – $^{62}$Ga, $^{68}$As, and $^{74}$Rb – should be paid more attention to. In particular, the uncertainty of the experimental $ft$ value for $^{62}$Ga has become relatively small [6], thus the underestimate (or overestimate) of the correction $\delta_{c}$ for this
Table 1. Isospin symmetry-breaking corrections $\delta_c$ for the $0^+ \rightarrow 0^+$ superallowed transitions obtained by self-consistent RHF+RPA calculations [18] with PKO1 parametrization [17] are compared with those obtained by the shell model calculations (T&H) [11] and isospin- and angular-momentum-projected nuclear density functional theory [20] with the Skyrme force SV [21]. All values are expressed in %.

|                  | PKO1 [18] | T&H [11] | SV [20] |
|------------------|-----------|----------|---------|
| $^{10}$C $\rightarrow$ $^{10}$B  | 0.082     | 0.175(18) | 0.559(56) |
| $^{14}$O $\rightarrow$ $^{14}$N  | 0.114     | 0.330(25) | 0.303(30) |
| $^{18}$Ne $\rightarrow$ $^{18}$F  | 0.270     | 0.565(39) |
| $^{22}$Mg $\rightarrow$ $^{22}$Na  |           | 0.380(22) | 0.243(24) |
| $^{26}$Si $\rightarrow$ $^{26}$Al  | 0.176     | 0.435(27) |
| $^{30}$S $\rightarrow$ $^{30}$P  | 0.497     | 0.855(28) |
| $^{34}$Ar $\rightarrow$ $^{34}$Cl  | 0.268     | 0.665(56) | 0.865(87) |
| $^{38}$Ca $\rightarrow$ $^{38}$K  | 0.313     | 0.765(71) |
| $^{42}$Ti $\rightarrow$ $^{42}$Sc  | 0.384     | 0.935(78) |
| $^{26}$Al$^m$ $\rightarrow$ $^{26}$Mg  | 0.139     | 0.310(18) | 0.494(49) |
| $^{34}$Cl $\rightarrow$ $^{34}$S  | 0.234     | 0.650(46) | 0.679(68) |
| $^{38}$K$^m$ $\rightarrow$ $^{38}$Ar  | 0.278     | 0.655(59) |
| $^{42}$Sc $\rightarrow$ $^{42}$Ca  | 0.333     | 0.665(56) | 0.767(77) |
| $^{46}$V $\rightarrow$ $^{46}$Ti  |           | 0.620(63) | 0.759(76) |
| $^{50}$Mn $\rightarrow$ $^{50}$Cr  |           | 0.655(54) | 0.740(74) |
| $^{54}$Co $\rightarrow$ $^{54}$Fe  | 0.319     | 0.770(67) | 0.671(67) |
| $^{62}$Ga $\rightarrow$ $^{62}$Zn  |           | 1.48(21)  | 0.925(93) |
| $^{66}$As $\rightarrow$ $^{66}$Ge  | 0.475     | 1.56(40)  |
| $^{70}$Br $\rightarrow$ $^{70}$Se  | 1.140     | 1.60(25)  |
| $^{74}$Rb $\rightarrow$ $^{74}$Kr  | 1.088     | 1.63(31)  | 2.06(21)  |

nucleus represents the dominant contribution to the value of chi-square per degree of freedom $\chi^2/\nu$, which is crucial to determine the confidence level of the nuclear structure-dependent corrections. Therefore, the latter corrections in $^{62}$Ga could become one of the critical cases in future theoretical investigations.

Combining the $|V_{ud}| = 0.97273(27)$ (PKO1) [18], $|V_{ud}| = 0.97425(22)$ (H&I) [6] or $|V_{ud}| = 0.97459(24)$ (SV) [20], and the most updated values of the other two top-row matrix elements $|V_{us}| = 0.2252(9)$ and $|V_{ub}| = 0.00389(44)$ [4], the sum of squared top-row elements of the CKM matrix extracted from the superallowed $\beta$ transitions is shown in the right panel of Fig. 1. These results are also compared with the most up-to-date values obtained in neutron decay [4], pion $\beta$ decay [4] and nuclear mirror transitions [9]. It is interesting to note that whether the unitarity of the CKM matrix is fulfilled or not is still an open question. Remarkably, adopting different theoretical frameworks, the theoretical uncertainty of the isospin symmetry-breaking corrections $\delta_c$ is of the order of $\sim 0.003$, rather than $\sim 0.0002$ as it was concluded from shell model calculations in Ref. [5].

3. Summary

The isospin symmetry-breaking corrections $\delta_c$ obtained by the self-consistent relativistic RPA calculations are compared with those obtained by shell model calculations as well as by isospin- and angular-momentum-projected nuclear density functional theory. It is pointed out that the
Figure 1. Left panel: Differences of the isospin symmetry-breaking corrections $\delta_c$ as a function of the charge $Z$ of the daughter nucleus. The T&H results [11] are taken as references, while the uncertainties of $\delta_c$ are not taken into account. Right panel: The sum of squared top-row elements of the CKM matrix, where the $|V_{ud}|$ values are obtained by RHF+RPA with PKO1 [18], shell model (H&T) [6], projected DFT with SV [20], as well as neutron decay [4], pion $\beta$ decay [4], and nuclear mirror transitions [9], respectively.

present theoretical uncertainty of the isospin symmetry-breaking corrections $\delta_c$ is of the order of magnitude $\sim 0.003$, rather than $\sim 0.0002$ as suggested by shell model calculations. Thus, whether the unitarity of the CKM matrix is fulfilled or not is still an open question.

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