The double domain structure of pair contact process with diffusion

Sungchul Kwon and Yup Kim
Department of Physics and Research Institute of Basic Sciences, Kyung Hee University, Seoul 130-701, Korea

(Received: March 23, 2022)

I. INTRODUCTION

The concept of universality makes possible to understand and classify the various and complicate critical behavior of a number of equilibrium models. For nonequilibrium critical phenomena, extensive studies during past decades revealed that various nonequilibrium systems also exhibit universal behavior characterized by several features. Hence, it has been an important issue to identify nonequilibrium universality classes by finding common physical features.

Among nonequilibrium critical phenomena, absorbing phase transitions (APT’s) from fluctuating active states into absorbing states in which the system is trapped forever have been a field of growing interest during last decades. Recent theoretical and numerical studies show that APT’s exhibit universality and it can be classified according to conservation laws, dimensionality of systems and symmetries of absorbing states. However only a few universality classes have been identified so far. Directed percolation (DP), parity conserving (PC) and directed percolation (DP), parity conserving (PC) are well studied classes among others. DP includes systems with no special attributes except the time reversal symmetry, so that most systems studied so far belong to this class.

As a research direction to search for further unknown universality classes, coupled systems have been studied recently. A coupled system is a multi-species system in which each species is coupled to the others in certain ways such as bidirectional and unidirectional coupling in linear or quadratic ways. However the coupled systems do not always exhibit new critical behavior. For bidirectionally coupled systems, the critical behavior depends on the manner of the coupling. For instance, quadratically coupled DP systems still belong to DP class despite their complex behavior. However, linearly coupled systems belonging to DP or PC class exhibit mean-field or non-trivial critical behavior. Linear and unidirectionally coupled systems exhibit new critical behavior at a particular point where all sub-systems are critical.

Among single species systems, pair contact process with diffusion (PCPD) can be regarded as a two species system. PCPD has been extensively studied during last years due to its nontrivial critical behavior. PCPD is a stochastic reaction-diffusion model, which evolves by the competition of two processes, fission (2A → 3A) and annihilation (A → ∅). In addition, each particle performs isotropic diffusion. Without diffusion, the model is so-called pair contact process (PCP) belonging to DP class. Since the reactions involve pairs, diffusing solitary particles are not engaged in the binary reactions. However when two solitary particles form a pair, the reactions take place. On the other hand, solitary particles are created from pairs by diffusion. Hence, PCPD can be regarded as a bidirectionally coupled two species system in which the order of the coupling is linear in the direction from pairs to solitary particles and quadratic in the opposite direction. This observation leads to the cyclically coupled DP and pair annihilation which exhibits the similar type of critical behavior to that of PCPD.

In this paper, we investigate the domain structure of PCPD. When a spreading domain is formed from localized activities, the quadratic coupling from solitary particles to pairs allows the point-free region in which only solitary particles exist. We call the point-free region so-called uncoupled region. On the other hand, the linear coupling from pairs to solitary particles results in the so-called coupled region in which pairs and solitary par-
PCPD has three sectors in configuration spaces according to the existence of pairs (P) and solitary particles (S). The one is the configurations in which both pairs and solitary particles are present (PS-ensemble). We call configurations with at least one pair (two solitary particles) P-ensemble (S-ensemble). In P-ensemble (S-ensemble), solitary particles (pairs) may be present or not. The P-ensemble is the reactive subspace of Ref. 31. A conventional ensemble includes configurations with at least two particles which can be either two solitary particles or one pair. We call the conventional ensemble All-ensemble. Since solitary particles are effectively linearly coupled to pairs, the existence of pairs implies the existence of solitary particles. Hence, PS-ensemble should coincide with P-ensemble asymptotically. However, solitary particles transform into pairs by collisions, the coupling in this direction is quadratic. Hence, the existence of solitary particles does not always guarantee the presence of pairs due to the long life time of solitary particles. So S-ensemble coincide with All-ensemble. As a result, there are two distinct ensembles in PCPD.

We define the size of each domain as follows (See Fig. 1). The size of the whole domain at time $t$ ($R(t)$) is defined as the distance between the leftmost and the rightmost particle. When both pairs and solitary particles exist simultaneously, we can define the size of the coupled region ($R_C(t)$) and the size of the uncoupled region ($R_U(t)$). Since solitary particles are linearly coupled to pairs, we define the size of the coupled region ($R_C(t)$) as the spreading distance of pairs ($R_p$) defined as the distance between the leftmost and the rightmost pair. Hence, we have three different lengths, $R$, $R_p$ and $R_U$ in PCPD. To take into account the three length scales at the same time, one should use PS-ensemble in which only two lengths, $R_p$ and $R_U$ are the fundamental length scales of PCPD due to $R = R_p + R_U$.

At $p_c = 0.133 \ 519(3)$, we perform defect Monte Carlo simulations with a pair on one dimensional empty lattice. We run simulations up to $t = 10^7$ time steps using $3.6 \times 10^6$ independent runs. We measure the squared
sizes, $R^2$, $R_p^2$ and $R_U^2$ for surviving PS-ensemble. The PCPD has two absorbing states, vacuum and states with one diffusing solitary particle. Hence, we stop the simulations when the total number of particles $N$ is less than two. At the criticality, the squared sizes scale as $R^2 \sim t^{2/Z}$, $R_p^2 \sim t^{2/Z_p}$ and $R_U^2 \sim t^{2/Z_U}$. Fig. 2 shows the scaling plots of the squared sizes, $R^2/t^{2/Z}$. We obtain the best scaling plot with $Z = 1.663(5)$, $Z_p = 1.61(1)$ and $Z_U = 1.768(8)$ respectively. The errors of our estimates should be larger due to the error of $p_c$. Within the numerical errors at the criticality, our estimate of $Z$ agree with the previous studies [27], especially $Z = 1.70(5)$ [10]. Since $Z_U > Z_p$, the total spreading distance $R(= R_p + R_U)$ should scale as $R \sim t^{1/Z_p}$, and $R_U$ plays the role of the correction to the scaling as in unidirectionally coupled systems. Hence, we conclude $Z = Z_p = 1.61(1)$ which is the smallest value among the estimates of previous studies [27]. For reference, we also measure $R^2$ using All-ensemble. The difference of All-ensemble from PS-ensemble or P-ensemble is that All-ensemble includes configurations without pairs. Since solitary particles spread diffusively, it is expected that $R$ averaged over All-ensemble scales differently from that of PS-ensemble. From the scaling plot of $R^2/t^{2/Z}$, we estimate $Z = 1.676(3)$ for All-ensemble which is larger than that of PS-ensemble (not shown). The slow spreading of the whole domain in All-ensemble results from the diffusive motions of solitary particles in configuration without pairs.

Since $R$ scales as $R = R_p(1 + R_U/R_p)$, the correction to the $R$ is $Q = R_U/R_p$ which decays as $Q \sim t^{-\phi}$ with $\phi = (Z_U - Z_p)/Z_p Z_U$. To see how the correction decays slowly in time, we plot $Q$ in Fig. 3. When $Q \ll 1$, the correction is negligible. However, as shown in the inset, $Q$ is still comparable to one even at $t = 10^7$. We obtain the best scaling plot of $Qt^\phi$ with $\phi = 0.054(4)$. As $Q$ decays with very small $\phi$, it is practically impossible to reach the asymptotic scaling regime of $R \sim R_p$. As a result, one should take the double domain structure into account for the more precise measurement of the dynamic exponent $Z$.

In addition to the sizes, we also measure the number of pairs ($N_p$) and of solitary particles ($N_s$) averaged over all samples, and the survival probability of PS-ensemble ($P_{ps}$) and All-ensemble ($P_{all}$). As solitary particles are linearly coupled to pairs, $N_s$ is proportional to $N_p$. At criticality, $N_p$ scales as $N_p \sim t^\eta$. Fig. 4 shows the scaling plot $N_p/t^\eta$. We obtain the best scaling plot with $\eta = 0.275(5)$. The inset show the ratio of $N_s/N_p$, which converges to one as expected. $P_{ps}(t)$ and $P_{all}(t)$ de-

![FIG. 2: (Color online) The scaling plots of various sizes. (a) The scaling plot of $R^2$ (black) and $R_U^2$ (blue) with $Z = 1.663$ and $Z_U = 1.768$. (b) the scaling plot of $R_p^2$ with $Z_p = 1.61$.](Image)

![FIG. 3: The scaling plot of the ratio $Q = R_U/R_p$. The main plot shows $Qt^\phi$ with $\phi = 0.054$. The inset shows the double logarithmic plot of $Q(t)$.](Image)

![FIG. 4: The scaling plot of $N_p$. The main plot shows $N_p/t^\eta$ with $\eta = 0.275$. The inset shows the semilogarithmic plot of the ratio $N_s/N_p$.](Image)
of the coupled region ($R$) of the whole domain ($U$). The difference of the coupling ways results in the double domain structure of the whole domain, the coupled and the uncoupled region respectively. As a result, the size of the whole domain ($R$) is given as the sum of the size of the coupled region ($R_p$) and of the uncoupled region ($R_U$). Hence, $R_p$ and $R_U$ are the basic length scales characterizing the scaling behavior of the spreading domain in PCPD. We numerically find that $R_p$ and $R_U$ scale as $R_p \sim t^{1/Z_p}$ and $R_U \sim t^{1/Z_U}$ with $Z_U > Z_G$ at criticality. Hence, $R$ should asymptotically scale as $R \sim t^{1/2}$ with $Z = Z_p$ and $R_U$ plays the role of the correction to the scaling. However, the direct measurement of $R$ leads to the underestimate of the asymptotic value of $Z$ because the correction $Q = R_U/R_p$ decays with very small exponent. Since it is practically impossible to reach the asymptotic scaling region of $R \sim t^{1/2}$, it is important to take the domain structure into account in simulations for more precise estimate of the dynamic exponent $Z$ of PCPD.

We classify particle configurations into four ensembles, which are finally reduced to two distinct ensembles, $P$-ensemble and All-ensemble respectively. All-ensemble includes configurations without pairs, while $P$-ensemble does not. The survival probabilities of two ensembles decay with the same exponent. However, the whole domain appears to spread more slowly in All-ensemble than in $P$-ensemble due to the diffusive motions of solitary particles in configurations without pairs. Hence, in addition to the domain structure, the diffusive motions of solitary particles in All-ensemble raise another correction to the scaling of the total spreading distance $R$ which does not appear in $P$-ensemble.

As the linear-quadratic bidirectional coupling is the common feature of various PCPD studied so far, the double domain structure should appear in other PCPD variants. Among PCPD variants, we investigate the domain structure of the bosonic PCPD with soft-constraint of Ref. [10] which is known to exhibit the clear power-law decays at the criticality. For this model, we also confirm the existence of the double domain structure and the critical spreading behavior similar to that of PCPD studied in this paper. Hence, the double domain structure is a common feature of PCPD variants.

As shown in recent studies, as PCPD extremely slowly approaches to its asymptotic scaling region, it is very difficult to identify the critical behavior. The bidirectional coupling should be the main reason in itself. On the other hand, the double domain structure naturally appears due to the linear-quadratic bidirectional couplings, which enhance the long-time drift of dynamic exponent $Z$. One can overcomes the latter effect by considering the scaling behavior of sub-domains separately as in unidirectionally coupled systems.

**Acknowledgments**

This work was supported by Grant No. R01-2004-000-10148-0 from the Basic Research Program of KOSEF.
[1] H. E. Stanley, Rev. Mod. Phys. 71, S358 (1999).
[2] J. Marro and R. Dickman, Nonequilibrium phase transitions in lattice models (Cambridge University Press, Cambridge, 1999).
[3] H. Hinrichsen, Adv. Phys. 49, 815 (2000).
[4] G. Ódor, Rev. Mod. Phys. 76, 663 (2004).
[5] P. Grassberger and A. de la Torre, Ann. Phys. (NY) 122, 373 (1979).
[6] I. Jensen, J. Phys. A 32, 5233 (1999).
[7] I. Jensen, Phys. Rev. E, 50, 3623 (1994).
[8] S. Kwon and H. Park, Phys. Rev. E 52, 5955 (1995).
[9] J. L. Cardy and U. Täuber, Phys. Rev. Lett. 77, 4780 (1996).
[10] J. Kochelkoren and H. Chaté, Phys. Rev. Lett. 90, 125710 (2003).
[11] P. Grassberger, F. Krause, and T. von der Twer, J. Phys. A 17, L105 (1984); P. Grassberger, J. Phys. A 22, L1103 (1989); M. H. Kim and H. Park, Phys. Rev. Lett. 73, 2579(1994); H. Hinrichsen, Phys. Rev. E 55, 219 (1997).
[12] W. Hwang, S. Kwon, H. Park, and H. Park, Phys. Rev. E 57, 6438 (1998).
[13] O. Al Hammal, H. Chaté, I. Dornic, and M. A. Munoz, Phys. Rev. Lett. 94, 230601 (2005).
[14] H. K. Janssen, Phys. Rev. Lett. 78, 2890 (1997).
[15] S. Kwon, J. Lee and H. Park, Phys. Rev. Lett. 85, 1682 (2000).
[16] S. Kwon and H. Park, J. Korean Phys. Soc. 38, 490 (2001); G. Ódor, Phys. Rev. E 63, 021113 (2001).; S. Kwon and H. Park, Phys. Rev. E 69, 066125 (2004).
[17] H. Hinrichsen, Physica A, 291, 275 (2001).
[18] J. Noh and H. Park, Phys. Rev. Lett. 94, 145702 (2005).
[19] U. Alon, M. R. Evans, H. Hinrichsen and D. Mukamel, Phys. Rev. Lett. 76, 2746 (1996).
[20] D. Richardson, Proc. Cambridge Philos. Soc. 74, 515 (1973); N. Goldenfeld, J. Phys. A 17, 2807 (1984); J. M. Kim and J. J. Kosterlitz, Phys. Rev. Lett. 62, 2571 (1989); N. Rajewsky, D. E. Wolf, and J. Kertész, Physica A 164, 81 (1990); A. Toom, J. Stat. Phys. 74, 91 (1994).
[21] J. M. López and H. J. Jensen, Phys. Rev. Lett. 81, 1734 (1998). T. Sams et al, Phys. Rev. Lett. 79, 313 (1997).
[22] U. C. Tauber, M. J. Howard and H. Hinrichsen, Phys. Rev. Lett. 80, 2165 (1998).
[23] Y. Y. Goldschmidt, H. Hinrichsen, M. J. Howard and U. C. Tauber, Phys. Rev. E 59, 6381 (1999).
[24] H. Hinrichsen and G. Ódor, Phys. Rev. Lett. 82, 1205 (1999); Phys. Rev. E 60, 3842 (1999).
[25] S. Kwon and G. M. Schütz, Physica A 341, 136 (2004).
[26] S. Kwon and G. M. Schütz, Phys. Rev. E 71, 046122 (2005); S. Kwon and Y. Kim, ibid 72, 066122 (2005).
[27] M. Henkel and H. Hinrichsen, J. Phys. A 37, R117 (2004).
[28] I. Jenssen, Phy. Rev. Lett. 70,1465 (1993); I. Jenssen and R. Dickman, Phys. Rev. E 48, 1710 (1993).
[29] H. Hinrichsen, Physica A 291, 285 (2001).
[30] S.-C. Park and H. Park, Phys. Rev. E 73, 025105(R) (2005).
[31] R. Dickman and M. A. F. de Menezes, Phys. Rev. E 66, 045101(R) (2002).