Shot-noise governed Coulomb blockade in a single Josephson junction

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We have investigated the influence of shot noise on the IV-curves of a single mesoscopic Josephson junction. We find that the blockade of the Cooper pair current is strongly suppressed in the presence of shot noise due to tunnelling in an adjacent SIN junction. Our experimental findings can be accounted for by an extension of the phase correlation theory. Shot-noise effect in a resistive environment $R$ can be characterized by the effective noise temperature $T_N = eI/R/2k_B$, which means that a Josephson junction can easily detect shot noise from a current well below 1 pA.

The role of quasiparticle tunnelling on the decoherence phenomena in Josephson junctions has become of great importance now when schemes for quantum computation are under development [1]. In the absence of dissipation from environmental modes, shot noise provides the ultimate dephasing mechanism at low temperatures. We have investigated the influence of external shot noise on a single Josephson junction in a strongly resistive environment, in which Coulomb blockade (CB) of Cooper pair current [2-3] takes place owing to the delocalization of the phase variable [4].

The Coulomb blockade of Cooper pairs is very sensitive to fluctuations. Inherently, it is influenced by Johnson-Nyquist noise, which is predicted to result in a power-law-like increase of conductance both as a function of temperature and voltage [4]. The exponent of the power law, $2\rho - 2$, is governed by the parameter $\rho = R/R_Q$ where $R$ describes the dissipative ohmic environment and $R_Q = h/4e^2$. Hence, in the case of large exponents $2\rho - 2 \gg 1$, there is a high resolution against tiny changes in temperature, or alternatively, a high sensitivity to any external noise sources.

In this Letter we report first investigations of “noise spectroscopy” using the Coulomb blockade of Cooper pairs as a sensitive detector. We have induced shot noise by a separately biased superconductor-insulator-normal metal (SIN) tunnel junction. The quasiparticle current is found to strongly reduce the CB of Cooper pairs: the influence can be resolved down to currents of a few pA in our experiments. Our findings can be accounted for by including the effect of shot noise into the phase-phase correlation functions. To our knowledge, the present work is the first one to study quantitatively the effect of nonlinear dissipative elements on the Cooper pair tunnelling, as well as the first attempt to extend the phase-fluctuation theory [4] to account for an independent shot-noise source.

When the supercurrent channel is blocked off, the current is carried by incoherent tunnelling of Cooper pairs. Using $P(E)$-theory [4], the zero-bias conductance of the junction can be expressed via the real and the imaginary part of $J(t) = \langle \varphi(t) - \varphi(0) \rangle = J_R(t) + i J_I(t)$:

$$G_0 = \frac{dI}{dV}_{V \to 0} = -2e^2E_0^2 \int_{-\infty}^{\infty} t dt e^{J_R(t)} \sin J_I(t), \quad (1)$$

where the real part, $J_R(t) = J_T(t) + J_N(t)$, contains the contributions $J_T$ from the equilibrium Johnson-Nyquist noise and $J_N$ from the shot noise. Without the shot noise

$$J(t) = J_T + i J_I = 2 \int_{-\infty}^{\infty} \frac{d\omega}{\omega} Re Z(\omega) e^{-i\omega t} - 1 \frac{1}{1 - e^{-\beta\omega}}. \quad (2)$$

For ohmic environment, $Z(\omega) = (1/R + i\omega C_T)^{-1}$ where $C_T$ is the junction capacitance. The imaginary part $J_I(t) = -\pi \rho (1 - \sqrt{\beta^2 + \gamma^2})$ sign does not depend on temperature and in the low-temperature limit $\hbar = h/k_B T \gg \tau$,

$$J_T(t) = 2\rho \left( \frac{1}{2} \left[ e^{-t/\tau} \pi t \right] + e^{t/\tau} \pi t \right) - \ln \left( \frac{\hbar}{\pi \tau} \frac{e^{\pi t}}{\hbar} - \gamma \right), \quad (3)$$

where $\tau = R C_T$. The temperature dependence is important only at long times $t \gg \hbar / \tau$, where temperature determines the phase diffusion, and $J_N(t) \approx -2\pi \rho t / \hbar = -2\pi \rho k_B T / h$. Now we consider the contribution $J_N(t)$ of the shot noise. From the current–current spectral density $S_I = 2eI$ one can find the voltage-voltage spectral density:

$$S_V = |Z(\omega)|^2 S_I = \frac{2eIR^2}{1 + (\tau \omega)^2}. \quad (4)$$

Then using the Josephson relation $\hbar \partial \varphi / \partial t = 2eV$ one can obtain the phase-phase spectral density $S_\varphi = (4e^2 / \hbar^2 \omega^2) S_V$ and finally the contribution of the shot noise to $J_R$ is

$$J_N(t) = \frac{\pi I R^2}{e R_Q^2} \int_{-\infty}^{\infty} \frac{d\omega}{\omega^2} \cos \omega t - 1 \frac{1}{1 + (\tau \omega)^2}$$

$$= -\frac{\pi^2 I \tau R^2}{2e} \left( e^{-|t|/\tau} + \frac{|t|}{\tau} - 1 \right). \quad (5)$$

As well as the thermal contribution, for small $I$ the shot-noise contribution is significant only for large $t \gg \tau$.
where the shot noise modifies the phase diffusion and 
\( J_{T}(t) + J_{N}(t) = -2\pi\rho k_{B}(T + T_{N})/\hbar \), where \( T_{N} = eIR/2k_{B} \) is the noise temperature.

Using the asymptotic expressions for \( J_{T}(t) \) and \( J_{N}(t) \) at \( t \gg \tau \), one can derive an analytical expression for the conductance:

\[
G_{0} = \frac{2e^{2}E_{J}^{2}}{h^{2}} \sin(\pi \rho) e^{-2\rho \gamma E} \left( \frac{2\pi}{\hbar \beta} \right)^{2\rho - 2} \tau^{2} \times dB(a/2 + \rho, 1 - 2\rho), \tag{6}
\]

where \( \Lambda = 2\pi\rho k_{B}T_{N}/\hbar, a = 2\rho T_{N}/T, \gamma_{E} = 0.577... \) is the Euler constant and \( B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y) \) is the beta function. Using the functional relations between gamma functions, one can check that in the limit \( T_{N} \to 0 \) \((a \to 0)\), Eq. (6) coincides with the conductance

\[
G_{0} = \frac{2\pi e^{2}E_{J}^{2}}{h^{2}} e^{-2\rho \gamma E} \left( \frac{2\pi}{\hbar \beta} \right)^{2\rho - 2} \tau^{2} \Gamma(\rho)/(2\rho) \propto T^{2\rho - 2} \tag{7}
\]

calculated in Ref. [3] without the shot noise. In the opposite limit \( T_{N} \gg T \) \((a \to \infty)\) one can use the asymptotic relation \( \Gamma(z + y) \sim \Gamma(z)z^{y} \) at \( z \to \infty \), and Eq. (6) yields

\[
G_{0} = \frac{4\pi \rho e^{2}E_{J}^{2}}{h^{2}} (2\pi\rho k_{B})^{2\rho - 2} (T + T_{N})^{2\rho - 2} \tag{8}
\]

for small \( \rho \). In contrast to Eq. (7), Eq. (8) cannot be extended to large \( \rho \), where the Coulomb blockade takes place and our experiment has been done. This is because times \( t \sim \tau \) become relevant and the oscillating term \( \sin(\pi \rho) \) is not cancelled at \( a \neq 0 \). But one may expect that the effect of shot noise still can be accounted for by the expression \( G \sim (T + \gamma(\rho)T_{N})^{2\rho - 2} \), where \( \gamma(\rho) \) may be considered as a \( \rho \)-dependent Fano-factor of order 1.

Our experiments were performed using a circuit layout which is depicted in Fig. [1]. The physical structure consists of four basic elements: 1) an Al-AIO_x-Al Josephson junction (JJ) with a tunnel resistance of \( R_{JJ} = 4 - 8 \) kΩ, 2) a superconducting-normal Al-AIO_x-Cu tunnel junction (SIN) with \( R_{SIN} = 6 - 27 \) kΩ, 3) a thin film Cr resistor of \( R_{C} = 23 - 67 \) kΩ (20 µm long), located within a few µm from the Josephson junction, and 4) a similar Cr resistor \( R_{B} \) in front of the SIN junction. Altogether we investigated three samples, the parameters of which are listed in Table I.

The circuits were fabricated using electron beam lithography and four-angle evaporation. The Cr resistor (5 nm thick, 100 nm wide) was evaporated first at an angle of \(-18^\circ\), followed by the Al-island at \(-38^\circ\). After oxidation, the sample holder was rotated by 45° around the z-axis and the SQUID-loop was deposited by a second Al-evaporation at \(+38^\circ\). Finally, the SIN-junction was made in copper deposition at \(+6^\circ\).

The JJ junction was, in fact, made of two 100 × 100 nm² junctions in a SQUID geometry in order to facilitate tuning of its Josephson energy. The Josephson energy \( E_{JJ}^{\text{max}} \) at no magnetic flux was calculated from the tunnelling resistance using the Ambegaokar-Baratoff relation. The minimum Josephson coupling energy, \( E_{JJ}^\text{min} \), was obtained from the minimum of critical current \( I_{C}(\Phi) \) as a function of external flux \( \Phi \), and assuming a linear dependence between \( E_{JJ} \) and \( I_{C} \). The Coulomb energy \( E_{C} = e^{2}/2C \) was estimated from the IV-curves in the normal state: the sum of junction capacitances, \( C = C_{SIN} + C_{JJ} \), was obtained from the voltage offset at large bias voltages using the formula \( V_{\text{offset}} = \frac{e}{2C} \). The ratio \( E_{JJ}/E_{C} \) could be tuned over the range 0.33 and 4.3 (see Table I). External noise was filtered out by 1.5 MHz low-pass filters at the top of the cryostat and by 1 m of Thermocorex cable at the mixing chamber.

FIG. 1: A scanning electron microscope picture of sample 1 and a schematic view of the circuit. The chrome resistor is denoted by Cr, the superconductor-normal junction by SIN, and the Josephson junction in a SQUID-loop configuration by JJ1 and JJ2. For constructional details, see text.

| \( R_{JJ}^{C}(k\Omega) \) | \( R_{SIN}^{C}(k\Omega) \) | \( R_{C}(k\Omega) \) | \( R_{B}(k\Omega) \) | \( E_{C} \) | \( E_{JJ}^{\text{min}} / E_{JJ}^{\text{max}} \) |
|---|---|---|---|---|---|
| 1 | 8.1 | 27.3 | 22.6 | 0.1 | 65 | 22 / 78 |
| 2 | 7.8 | 5.8 | 54.2 | 0.1 | 50 | 83 / 83 |
| 3 | 4.3 | 10 | 67 | 53 | 35 | 14 /150 |

TABLE I: Device parameters for our three samples numbered consecutively by the first column. The next two columns give the tunnelling resistance of the Josephson junction \( R_{JJ}^{C} \) and the SIN junction \( R_{SIN}^{C} \). \( R_{C} \) and \( R_{B} \) denote the impedances in the immediate vicinity of the SIN and Josephson junctions, respectively. The last two columns indicate the Coulomb energy, \( E_{C} \), and the minimum \( E_{JJ}^{\text{min}} \) and maximum \( E_{JJ}^{\text{max}} \) values of the Josephson energy. The energies are given in µeV.
conclude that, in the small $E_J$ limit, the conduction in the JJ is predominantly caused by inelastic Cooper pair tunnelling, and no extra leakage is present at the lowest temperatures. Under these circumstances, any change in $G_0$ can be assigned to an additional external source of noise in the circuit.

The Coulomb blockade at small voltages is most clearly using measurements of differential conductance $\frac{dI}{dV}$. Fig. 2 displays the measured $\frac{dI}{dV}$ for sample 3 at zero quasiparticle current as well as at a few values of $I_{\text{SIN}}$ ranging from 0.01 nA to 0.1 nA; the data was taken at the minimum value of $E_J = 14 \mu\text{eV}$. The bias voltage $V$ to the JJ was applied via the chrome resistor while the SIN-junction was current biased through $R_{\text{bias}} = 100 \Omega$. The minimum amplitude of the Coulomb blockade dip $G_{\text{min}} = \left[\frac{dI}{dV}\right]_{\text{min}}$ is seen to increase monotonically with $I_{\text{SIN}}$. In addition, there is a small shift by $\Delta I_{\text{JJ}} \sim -0.2\cdot I_{\text{SIN}}$ in the location of the minimum conductance with increasing $I_{\text{SIN}}$. Sample 1 yielded $\Delta I_{\text{JJ}} \sim -0.22 \cdot I_{\text{SIN}}$, but in sample 2, the large value of $E_J$ prevented any quantitative analysis.

The IV-curve ($I_{\text{SIN}}$ vs. $V_{\text{SIN}}$, see Fig. 1) of the SIN junction is illustrated in the inset of Fig. 3. The biasing was applied through JJ while the voltage was recorded via $R_C$. In the subgap region, there is only a small current, presumably due to Andreev reflection processes. At the gap edge, the current increases rapidly and the differential resistance $R_d = \frac{dI}{dV}$ drops down to 5 - 10 kΩ at currents 0.05 nA - 1 nA. This means that the resistive environment $R$ seen by the Josephson junction varies with the biasing of the SIN junction. When $I_{\text{SIN}} < 20 - 30$ pA, $R_{\text{SIN}} \gg R_C$ and the environment is purely governed by $R_C$. On the other hand, in the regime 0.05 - 1 nA, we may approximate $R^{-1} \approx R_C^{-1} + (R_B + R_d)^{-1}$. Here, we neglect all the second order terms which might give a noticeable contribution to the dissipative part of the impedance.

In addition to shot noise, the SIN junction might give a contribution via the Johnson–Nyquist noise, which is determined by the differential resistance $R_d$ at the biasing point. Since $R_d$ is not monotonous and has a minimum as a function of $I_{\text{SIN}}$, this would result in a nonmonotonous current dependence for the Coulomb blockade. None of our samples, however, showed any nonmonotonous behavior. The absence of any re-entrant type of behavior supports our observation that we are dealing with the effect of shot noise.

As there are uncertainties in the theory, we deduce $T_N$ by equating $G_0(I_{\text{SIN}})$ with $G_0(T + T_N)$. The results are displayed in Fig. 3 for samples 1 and 3. Both samples show a nearly linear increase in $T_N$ with growing $I_{\text{SIN}}$. A comparison with the formula $T_N = eIR/2k_B$, yields the Fano-factor $\gamma = 3$ and 0.5 for the samples 1 and 3, respectively. The inset of Fig. 3 displays the $I_{\text{SIN}}$-dependence of $G_0$ for sample 3. A fit using Eq. 8 with a “high-current” environment of $R^{-1} = R_C^{-1} + R_B$ yields $\gamma \sim 1$.

The measured Fano-factors, at least qualitatively, agree with our theoretical scenario, which up to now has not yet provided numerical values for the Fano factor at large $\rho$. But there are additional, neglected physical processes, which also can account for the deviation of the measured Fano-factors from 1. Especially, there is an uncertainty in the subgap regime of the SIN junction: the conduction should be caused only by Andreev processes. This would indicate that the Fano factor should be 2 for $I_{\text{SIN}}$ (tunnelling of 2e charges). At larger cur-

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2}
\caption{a) Zero bias conductance $G_0 = \frac{dI}{dV}|_{I_{\text{JJ}}=0}$ for the JJ + $R_C$ section of sample 3 as a function of temperature $T$ at $E_J = 14 \mu\text{eV}$. The solid curve illustrates Eq. 8 using the ohmic environment of $R = R_C$. b) $G_0$ as a function of Josephson energy $E_J$ at $T = 82$ mK. The solid curve illustrates the quadratic dependence obtained from Eq. 9.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure3}
\caption{Differential conductance $\frac{dI}{dV}$ vs. current $I_{\text{JJ}}$ for the JJ + $R_C$ section of sample 3 with a small current bias (via $100 \Omega$ resistor) in the SIN junction: $I_{\text{SIN}} = 0$ (□), 0.01 (○), 0.02 (▲), 0.05 (△), 0.1 nA (◇). The inset shows the $I_{\text{SIN}}$ vs. $V_{\text{SIN}}$ for the SIN-junction biased via JJ ($R_C$ was employed as the voltage lead). $T = 82$ mK.}
\end{figure}
rents, the dynamic impedance of the SIN junction is expected to lower the exponent \( \rho \). Last but not least, there is the question of correlations between quasiparticle and Cooper pair tunnelling.

Our model assumes additivity of thermal and shot noise and, therefore, cannot account for the shift of the noise and, therefore, cannot account for the shift of the Cooper pair tunnelling.

Using the experimental value of \( \gamma \) and the shift \( \Delta I_{JJ} \) for sample 3, we get an upper limit \( \kappa < 0.31 \) (at maximum correlation \( \lambda = -1 \)). We expect this limit to hold for sample 1 as well: then, by taking \( \kappa = 0.31 \), we obtain \( \lambda = -0.43 \). Thus, it appears that the correlations between \( I_{JJ} \) and \( I_{SIN} \) depend on the base resistor and that an increase in \( R_B \) tends to suppress uncorrelated tunnelling current in the SIN junction.

According to our experimental analysis, a single Josephson junction provides a good candidate for a noise detector in general. Its main virtue is the high sensitivity which comes from the large detector band width: \( \sim 1/RC \). Experimentally, changes by 1 mK in \( T_N \) can be resolved clearly in Fig. 3. This corresponds to a quasiparticle current of \( I_{qp} = 3 \) pA, which equals the sensitivity in high-resolution noise experiments of Ref. [10]. An enhancement by a factor of ten in the noise sensitivity is obtainable by improving the stability of \( \Delta T \) measurement to the level of 1% and working at \( T = 20 \) mK. This would allow, for example, detailed studies of the back-action noise of quantum amplifiers such as superconducting SETs. A larger dynamic range can be obtained by operating the device at lower \( T \).

In summary, our measurements of \( G \) vs. \( I \) curves for solitary, resistively confined small Josephson junctions show that the Cooper pair blockade can be suppressed strongly by shot noise from a near-by SIN tunnel junction. Using the framework of the phase correlation theory, we have presented a theoretical analysis of the shot-noise effect, which can be characterized by an effective noise temperature of the system \( T_N \). This approach yields a good agreement with our experimental results. We expect that other sources of noise (1/f-noise, as an example), produce similar effects. Consequently, CB of Cooper pairs can be employed as a sensitive noise detector with a resolution of 0.1 mK in \( T_N \).

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