The effects of gluon depletion on $J/\psi$ suppression in $pA$ and $AA$ collisions

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Abstract

The enhanced suppression of $J/\psi$ production at large $x_F$ in $pA$ collisions is studied in the framework of gluon depletion at large $x_1$. The nonperturbative process that modifies the gluon distribution as the gluons propagate in nuclear matter is described by an evolution equation with a kernel to be determined by phenomenology. With nuclear shadowing and anti-shadowing taken into account, the effect on the gluon distribution is shown to be a depletion in excess of 40% at $x_1 \approx 0.8$ for $A > 100$. There is a small amount of enhancement of the gluon distribution at small $x_1$, but it does not lead to any contradiction with the existing data on $J/\psi$ suppression in the central region. Extensions to $\psi'$ suppression and $AB$ collisions are also investigated in the framework of gluon redistribution.

1 Introduction

In an earlier paper [1] we presented the phenomenological evidence for the depletion of high-momentum gluons as a projectile proton traverses a target nucleus in $pA$ collisions. Here we present a more complete discussion of the subject with more technical details and with extension to nucleus-nucleus collisions.

Our approach is unconventional in that we do not make the usual assumption that the parton distributions in a proton remain unaltered as the proton propagates through a nucleus, even if a hard subprocess occurs deep in the nucleus. That assumption amounts to factorization, a property that has been proven for $pp$ collision, but clearly must fail for $pA$ collision if $A$ is infinitely large. For realistic nuclear sizes our alternative assumption that the parton distribution can be modified led to $J/\psi$ suppression [2] that cannot be distinguished from the effects of the usual mechanisms [3, 4]. What is different now is the appearance of new data on $J/\psi$ suppression in $pA$ collisions at large $x_F$ [5]. Since those data cannot be explained in terms of hadronic absorption of the produced $c\bar{c}$ state [3, 7, 8], we find more
direct support for the idea of gluon depletion before the hard subprocess that produces the $c\bar{c}$ state \[1\].

The data of the Fermilab E866 experiment \[5\] on the $J/\psi$ suppression in $pA$ collisions at 800 GeVc are expressed as the ratio

$$R(x_F, A) = \sigma_A(x_F) / A \sigma_N(x_F) = A^{\alpha(x_F) - 1},$$

where $\sigma_{N,A}$ is the cross section for $J/\psi$ production by a proton on a nucleon ($N$) or on a nucleus ($A$). An analytical formula for $\alpha(x_F)$ is given in \[5\]

$$\alpha(x_F) = 0.96 \left(1 - 0.0519x_F - 0.338x_F^2\right),$$

for $-0.1 < x_F < 0.9$. Equation (2) differs significantly from the one given in their preprint \[9\], on the basis of which the analysis in \[1\] was done. Here we perform a reanalysis based on the new parameterization in Eq. (2).

In our view there are three sources that can contribute to the $x_F$ dependence, and we write them in a product form

$$R(x_F, A) = G(x_F, A) N(x_F, A) H(x_F, A),$$

where $H(x_F, A)$ represents the hadronic absorption of the $c\bar{c}$ state, $N(x_F, A)$ nuclear shadowing, and $G(x_F, A)$ the gluon depletion effect, defined as

$$G(x_F(x_1), A) = g(x_1, A) / g(x_1, 0),$$

$x_1$ being the momentum fraction of the gluon in the projectile proton, and $g(x_1, A)$ being the effective gluon distributions of that projectile in a nucleus $A$ at the point of $c\bar{c}$ production, averaged over the penetration depth at which the hard process occurs.

By $H(x_F, A)$ we mean the absorption factor that operates between the production of the $c\bar{c}$ state and the detection of the final $J/\psi$ due to any mechanism, including the interaction with comovers. Since no good arguments have been advanced to show that $H(x_F, A)$ can have a significant dependence on $x_F$ \[8, 11, 12\], we shall assume in the following that $H(x_F, A)$ is independent of $x_F$. This is not a serious limitation in our formalism. If a reliable $x_F$ dependence is found at a later date, we can easily incorporate it in our analysis. For now we adopt the usual Gerschal-H"ufner form.

$$H(A) = \exp [-\sigma \rho L(A)]$$

where $\sigma$ is the absorption cross section, $\rho$ the nuclear density, and $L(A)$ the mean path length in $A$ that a $c\bar{c}$ state propagates.

In the next section the nuclear shadowing factor $N(x_F, A)$ will be discussed. We shall find a simple formula that can represent the change in the gluon distribution in the target nucleus due to shadowing and anti-shadowing. Such a formula facilitates the analysis, and offers a simple parametrization that is convenient to use, independent of the particular problem that we apply it to here.

The determination of the depletion factor $G(x_F(x_1), A)$ is the main theme of this paper. In Sec. 3 we shall go beyond a review of the content of Ref. \[1\], not only because the data has
changed from those in the preprint [4], resulting in numerical differences, but also because we shall improve on [1] in some technical details and include some new material.

In Sec. 4 we shall extend the result from the study of the \( pA \) problem to \( AB \) collisions. We shall show how the enhanced depletion at large \( x_F \) does not affect the \( J/\psi \) suppression at small \( x_F \), which is where the existing data for \( AB \) collisions were collected. We shall have nothing to add on the subject of enhanced suppression observed in \( Pb-Pb \) collisions beyond what we have advanced in Ref. [2].

\section{Nuclear Shadowing and Anti-shadowing}

The nuclear shadowing and anti-shadowing problem has been studied phenomenologically by Eskola et. al. [10, 11]. Instead of focusing on the physics of the origin of the problem in QCD, they analyzed the deep inelastic scattering data of nuclear targets. On the basis of DGLAP evolution [12] they can determine the parton distribution at any \( Q^2 > 2.25 \text{ GeV}^2 \). The results are given in terms of numerical parametrizations (called EKS98 [11]) of the ratio

\[ N_i^A(x, Q^2) = f_i/A(x, Q^2)/f_i(x, Q^2), \tag{6} \]

where \( f_i \) is the parton distribution of flavor \( i \) for the free proton and \( f_i/A \) is that for a proton in a nucleus \( A \).

For the purpose of \( J/\psi \) production we are interested in Eq. (6) for only the gluons. We consider only the dominant subprocess \( g_1(x_1) + g_2(x_2) \rightarrow c + \bar{c} \), where \( x_1 \) is the momentum fraction of the projectile gluon whose distribution is \( g_1(x_1) \), and \( x_2 \) is that of the target gluon whose distribution is \( g_2(x_2) \). For the \( c\bar{c} \) state produced with momentum fraction \( x_F = x_1 - x_2 \), the usual kinematical relations are

\[ x_{1,2} = [(4\tau + x_F)^{1/2} \pm x_F]/2, \quad x_1 x_2 = \tau \equiv M_{J/\psi}^2/s, \tag{7} \]

where it is assumed that the \( c\bar{c} \) state that turns to \( J/\psi \) is produced near threshold. Thus the virtuality of the subprocess \( g + g \rightarrow c + \bar{c} \) is given by the value of \( M_{J/\psi}^2 \), or slightly higher. We shall take \( Q^2 = 10 \text{ GeV}^2 \), a value that is chosen in EKS98 [11] to give explicit values of \( N_i^A(x, Q^2) \). For simplicity we shall label the gluon distribution at \( Q^2 = 10 \text{ GeV}^2 \) by \( N(x_2, A) \).

The numerical output of EKS98 for \( N(x_2, A) \) is shown by the points in Fig. 1 for \( A = 50, 100, \) and \( 200 \). We exhibit only the values for \( x_2 \) in the range \( 0.01 \leq x_2 \leq 0.12 \), since that is the range relevant for the production of \( J/\psi \) at 800 GeV/c for \( 0 < x_F < 0.8 \). For the purpose of convenience in using those values of \( N(x_2, A) \) in analytic manipulation and computation, we propose a simple formula that contains the shadowing and anti-shadowing effects. Since the cross-over of the two effects occurs at \( x_2 = 0.02 \) where \( N(x_2, A) = 1 \) for all \( A \), it is sensible to use an auxiliary variable \( \xi \), defined by

\[ \xi = 3.912 + \ln x_2 \tag{8} \]

which vanishes at \( x_2 = 0.02 \). Moreover, we notice that \( \ln N(x_2, A) \) depends linearly on \( \ln A \) to a good approximation, so it suggests a power-law behavior

\[ N(x_2, A) = A^{\beta(x_2)}. \tag{9} \]
The exponent \( \beta(x_2) \) can be determined by fitting the data for \( A = 100 \). In terms of \( \xi \) we find a good fit with the parametrization

\[
\beta(\xi) = \xi(0.0284 + 0.0008\xi - 0.0041\xi^2),
\]

the result of which is shown in Fig. 2, where the points are for \( \ln N(x_2, A) / \ln A \) obtained from EKS98 at \( A = 100 \). This convenient formula for \( \beta(\xi) \) can then be used in conjunction with Eq. (9) to determine \( N(x_2, A) \) for other values of \( A \). The curves in Fig. 1 exhibit the good agreement between the data and our parameterization for \( A = 50, 100, \) and 200.

In the following we shall simply use Eq. (9) as a summary of the effects of nuclear shadowing and anti-shadowing for problems in \( pA \) collisions where the relevant range of \( x_2 \) is in the interval \( 0.01 \leq x_2 \leq 0.12 \).

3 Evolution of Gluon Distribution in a Nucleus

We now may regard \( R(x_F, A) \) and \( N(x_2, A) \) as known phenomenologically. Thus, from Eq. (3) we may write

\[
G(x_F, A)H(A) = A^{\alpha(x_F) - \beta(x_2(x_F))^{-1}}.
\]

Although the form of \( H(A) \) is given by Eq. (5), we do not know the value of \( \sigma \) in the present circumstance where we allow the possibility of gluon depletion. Hence, we treat \( H(A) \) temporarily as unknown, along with the depletion factor \( G(x_F, A) \). However, the \( x_F \) dependence is completely known from the RHS of Eq. (11).

To proceed we need some theoretical input on the possible form of \( G(x_F, A) \), or more directly the gluon distribution \( g(x_1, z) \) in the projectile, where \( z \) is the distance traversed in a nucleus. Note that we have refrained from referring to the projectile as the proton, since the possible modification of \( g(x_1, 0) \) for \( z > 0 \) implies that the incident proton loses its usual identity, in particular, the nature of its partonic content, as the projectile, now identified only as a flux of partons, propagates in the nuclear medium. How \( g(x_1, z) \) evolves in the nuclear medium is clearly a nonperturbative process that involves multiple scatterings of gluons and quarks at low virtualities. Nevertheless, for every incremental distance, \( dz \), that a gluon travels the modification that \( g(x_1, z) \) undergoes must be perturbative in that \( g(x_1, z + dz) - g(x_1, z) \) is small and is proportional to \( dz \). It is therefore reasonable to adopt an evolution equation similar in spirit to that of DGLAP [12], but with the change in resolution scale \( d\ln Q^2 \) replaced by the change in penetration depth \( dz \), so that we write

\[
\frac{d}{dz}g(x, z) = \int_0^1 \frac{dx'}{x'}g(x', z)Q(\frac{x}{x'}),
\]

where the unknown kernel \( Q(x/x') \) controls the gain and loss of the gluons in \( dz \). \( Q(y) \) cannot be determined by perturbative calculation, as the splitting functions in pQCD for \( Q^2 \) evolution. Equation (12) is similar to the nucleonic evolution equation proposed in [13], except that this is now at the parton level. In Eq. (12) the quark sector has been left out for simplicity. To be more complete one should include also the effects of the couplings of gluons with the quarks, a task that is deferred to the future. Thus what we can achieve now is the
determination of an effective kernel \( Q(y) \) that can account for the enhanced suppression of \( J/\psi \) at large \( x_F \).

To determine the \( z \) dependence of \( g(x, z) \) let us take the moments by defining

\[
g_n(z) = \int_0^1 dx \ x^{n-2} g(x, z) \tag{13}
\]

and

\[
Q_n = \int_0^1 dx \ x^{n-2} Q(y). \tag{14}
\]

Then by the convolution theorem, Eq. (12) becomes

\[
dg_n(z)/dz = g_n(z)Q_n, \tag{15}
\]

whose solution is

\[
g_n(z) = g_n(0)e^{zQ_n}. \tag{16}
\]

It is possible that Eq. (15) is valid only when \( z \) is large enough, in which case Eq. (16) should be modified to read

\[
g_n(z) = g_n(z_0)e^{(z-z_0)Q_n} \tag{17}
\]

for \( z \) greater than some positive value of \( z_0 \).

To proceed, let us substitute Eq. (4) in (11) and define

\[
J(x_1, A) \equiv g(x_1, 0)A^{a(x_F(x_1)) - \beta(x_2(x_1)) - 1} \tag{18}
\]

where the interrelationships among \( x_1, x_2 \) and \( x_F \) are specified by Eq. (7). For \( g(x_1, 0) \) we use the canonical form

\[
g(x_1, 0) = g_0(1 - x_1)^5. \tag{19}
\]

The final result is insensitive to its form and independent of \( g_0 \), which we shall set to be 1. Thus we may regard \( J(x_1, A) \) as known. Since we also have

\[
J(x_1, A) = g(x_1, A)H(A), \tag{20}
\]

its moments are, by virtue of Eqs. (3) and (16),

\[
J_n(z) = g_n(0)\exp[z(Q_n - \sigma\rho)]. \tag{21}
\]

Here and in the following we shall use \( z \) (until Sec. 5) to denote the average penetration depth (i.e., \( z \equiv \bar{z}_A \)) in \( A \) when a \( c\bar{c} \) state is produced by \( gg \) annihilation. It is then also the average length that the \( c\bar{c} \) must travel in \( A \) and be subject to hadronic absorption, i.e., \( z = L(A) \).

We can determine \( J_n(z) \) by taking the moments of \( J(x_1, A) \), as expressed in Eq. (18). However, there is a problem in evaluating \( J_n(z) = \int_0^1 dx_1 x_1^{n-2} J(x_1, z) \), since \( J(x_1, A) \) is
Table 1: Values of the coefficients $k_i$

| $A$ | $z$   | $k_0$         | $k_1$         | $k_2$         | $k_3$         | $k_4$         |
|-----|------|---------------|---------------|---------------|---------------|---------------|
| 100 | 4.177| $-0.5444$     | $-0.2063$     | $5.87 \times 10^{-3}$ | $-8.35 \times 10^{-3}$ | 0.424         |
| 200 | 5.262| $-0.6267$     | $-0.2354$     | $6.64 \times 10^{-3}$ | $-9.39 \times 10^{-3}$ | 0.486         |

ill-defined at $x_1 = 0$. According to Eq. (4), $x_2$ diverges as $x_1 \to 0$; thus limiting $x_2$ to 1 implies that $x_1$ cannot be less than $M_{j/\psi}^2/s$, a small but nonvanishing value. Furthermore, $x_F$ becomes negative at small $x_1$, and we lose any knowledge about $\alpha(x_F)$ for $x_F < -0.1$ [5]. Also, $\beta(x_2)$ is not reliably known at large $x_2$, so the RHS of Eq. (13) cannot offer accurate determination of $J(x_1, A)$ as $x_1 \to 0$. These defects can be suppressed by the factor $x_1^{n-2}$ in the integrand, if we restrict $n$ to $\geq 3$. We shall therefore determine $J_n$ only for $n \geq 3$.

For convenience, let us define

$$K_n(z) \equiv \ln\left[ J_n(z)/g_n(0) \right] = z(Q_n - \sigma \rho). \tag{22}$$

Using Eqs. (2), (10) and (19) in (18), we can calculate $J_n(x)$ and therefore $K_n(z)$ for $n \geq 3$. As mentioned earlier, Eq. (2) is different from a previous form of $\alpha(x_F)$ given in [1] and used in [4]. The results are shown as discrete points in Fig. 3 for $A = 100$ and 200. The corresponding values of $z$ are halves of the average total path lengths of the nuclei, i.e., $z = 3R_A/4 = 0.9A^{1/3}$ fm and are therefore $z_1 = 4.177$ and $z_2 = 5.262$ fm, respectively.

To extract the information contained in those points in Fig. 3 we need an analytical representation of $K_n$. We choose to fit $K_n$ by the following formula, different from the one used in [4],

$$K_n = \sum_{i=0}^{3} k_i n^i + k_4 n^{1/2}. \tag{23}$$

The results of our fits are shown by the smooth curves in Fig. 3. The corresponding parameters are given in Table 1. Note that the fits allow us to extrapolate smoothly to $n = 2$, where we could not calculate $J_2$.

Before we determine $Q_n$ from $K_n$ in Eq. (22), we need to verify the $z$ dependence. It should first be recognized that the experimental parameterization of the $A$ dependence, such as in Eq. (4), is not compatible with the theoretical expectation, such as in Eq. (5) and (21), except in a certain range of $A$. Since a power-law $A^n$, expressed as $e^{\rho \ln A}$, is approximately $e^{\rho z}$, where $z \propto A^{1/3}$, only when $\ln A$ is approximately $A^{1/3}$, the correspondence can only be for $60 < A < 240$. With that understanding, let us nevertheless calculate $J(x_1, z)$ for all $z < 6$ using $A = (z/0.9)^3$ in Eq. (13), take the moments, and then determine $K_n(z)$ through the first half of Eq. (22). The result is shown as points in Fig. 4 for eleven values of $z$ between 0.9 and 5.9, corresponding to $A$ from 1 to 282, and for four representative values of $n$, viz., 3, 8, 13 and 20. The straight lines are linear fits of the last six points for each value of $n$. Evidently, the $z$ dependence of $K_n(z)$ is very nearly linear for $3.4 < z < 5.6$, which corresponds to $54 < A < 240$. Thus our theoretical formalism is consistent with the
experimental data in the region where $lnA \approx A^{1/3}$. At $z = 0.9$, or $A = 1$, all points converge to $K_n = 0$, as they should. We cannot reliably apply our formalism to the collision problems where $A < 50$. Fig. 4 also suggests that even when $A$ is large, say $> 100$, the gluon evolution equation (12) may not be valid at small $z$, here used in the sense of penetration depth within the large nucleus, not the average depth. In the following we shall limit our consideration to only the linear portion of Fig. 4. In that region the second half of Eq. (22) should be treated as differentially correct, i.e.,

$$\Delta K_n(z)/\Delta z = Q_n - \rho \sigma$$

(24)

where $\Delta K_n(z) = K_n(z + \Delta z) - K_n(z)$.

Since the values of $k_i$ in Table 1 are determined in the linear region, we can use them to obtain $Q_n$. If we write

$$Q_n = \sum_{i=0}^{3} q_i n^i + q_4 n^{1/2},$$

(25)

then we have from Eqs. (23) and (24)

$$q_0 = \Delta k_0 / \Delta z + \rho \sigma, \quad q_i = \Delta k_i / \Delta z \quad (1 \leq i \leq 4).$$

(26)

For the two $z$ values in Table 1, we get (with $\Delta z = 1.086 \text{ fm}$)

$$q_0 = -0.0758 + \rho \sigma, \quad q_1 = -0.0268,$$

$$q_2 = 7.13 \times 10^{-4}, \quad q_3 = -9.6 \times 10^{-6}, \quad q_4 = 0.0568$$

(27)

in units of $\text{fm}^{-1}$.

Since the absorption cross section $\sigma$ is unknown when gluon depletion is not negligible, $q_0$ is not fixed by Eq. (27). Whatever the dynamics of gluon depletion is, we require that the total gluon momentum does not increase with $z$. Since the gluon momentum is $\int dx g(x, z) = g_2(z)$, that requirement implies in conjunction with Eq. (17) that $Q_2 \leq 0$. Choosing the upper bound $Q_2 = 0$ leads to the condition, on account of Eqs. (25) and (27),

$$q_0 = -(2q_1 + 4q_2 + 8q_3 + \sqrt{2}q_4) = -0.0295.$$  

(28)

With all parameters in Eq. (25) now determined, the $n$ dependence of $Q_n$ can be exhibited, as shown in Fig. 5. Evidently, $Q_n$ is smoothly varying and the use of the polynomials in Eqs. (23) and (25), which are different from those used in [1], is justified.

Although it is more direct to proceed immediately to the use of Eq. (17) to the determination of $g_n(z)$ and therefore $G(x_1, z)$, which is our goal, there is some advantage in an attempt to find $Q(y)$ at this point, while we are on the subject of $Q_n$. The easiest way to do that is to put $Q_n$ in form

$$Q_n = c_0 + \sum_j c_j/(n + j - 1),$$

(29)

used in [1], so that it can imply directly

$$Q(y) = c_0 \delta(1 - y) + \sum_j c_j y^j,$$

(30)
To translate from Eq. (25) and (29), we fit the values of \( Q_n \) at the 19 integer points, \( 2 \leq n \leq 20 \), determined by (24), by use of the formula (29) with a suitable number of terms in the sum. It turns out that a good fit can be achieved with three terms: \( j = 3, 4 \) and 5. The result is

\[
c_0 = -0.1988, \quad c_3 = 6.205, \quad c_4 = -23.316, \quad c_5 = 19.866 \ .
\] (31)

We show in Fig. 6 both the discrete values of \( Q_n \) at integral \( n \) and the fitted curve using Eqs. (29) and (31). The corresponding function \( Q(y) \), calculated using Eqs. (30) and (31), is shown in Fig. 7. It is evident that the \( c_0 \) and \( c_4 \) terms correspond to gluon depletion, while the \( c_3 \) terms correspond to gluon regeneration.

To determine \( G(x_1, z) \) as defined in Eq. (4), we have Eq. (17) that specifies the evolution in \( z \) in the linear region (see Fig. 4) from \( z_0 \). At this point we have no formalism to extrapolate in the nonlinear region from \( z_0 \) down to 0, and it is in reference to \( g(x_1, 0) \) that \( G(x_1, z) \) is defined. However, in view of the fact that the hadronic absorption term \( H(A) \) is known empirically to be an exponential, as in Eq. (9), i.e., \( \ln H(z) \) being linear in \( z \) for all \( z \), one may regard the nonlinear portion of Fig. 4 to be due primarily to the mismatch between \( \ln A \) and \( A^{1/3} \) at low \( A \). Then we adopt the approximation that Eq. (14) is adequate for relating \( g_n(z) \) to \( g_n(0) \). With \( Q_n \) being known from Eqs. (25), (27) and (28), and \( g_n(0) = B(n-1, 6) \) which is the beta function, we can calculate \( g_n(z) \). The result is shown in Fig. 8 by the full (open) circles for \( A = 100 \) (200). It is natural to fit the resultant \( g_n(z) \) by a linear combination of beta functions in the form

\[
g_n(z) = \sum_{i=1}^{3} a_i(z) B(n - 1, 5 + i) \ .
\] (32)

The coefficients \( a_i(z) \) are determined by fitting \( g_n(z)/g_n(0) \) in order to reduce the range of variation. For the two values \( z_1 \) and \( z_2 \), corresponding to \( A = 100 \) and 200, we obtain

\[
a_1(z_1) = 0.3526, \quad a_2(z_1) = 1.44, \quad a_3(z_1) = -0.78, \\
a_1(z_2) = 0.2362, \quad a_2(z_2) = 1.655, \quad a_3(z_2) = -0.869.
\] (33)

The curves in Fig. 8 are generated using Eqs. (32) and (33). Evidently, the fits are good.

The inverse transform of the moments in Eq. (32) is

\[
g(x_1, z) = \sum_{i=1}^{3} a_i(z)(1 - x_1)^{4+i} \ ,
\] (34)

whose implication for

\[
G(x_1, z) = g(x_1, z)/g(x_1, 0) \\
= a_1(z) + a_2(z)(1 - x_1) + a_3(z)(1 - x_1)^2
\] (35)

can readily be calculated using Eq. (33). The results for \( z_1 \) and \( z_2 \) are shown in Fig. 9. Clearly, there is significant depletion of gluons at large \( x_1 \), roughly 40% at \( x_1 \approx 0.8 \). There is a small amount of regeneration at small \( x_1 \), i.e., at around \( x_1 \approx 0.2 \). Although the enhancement at small \( x_1 \), is at the 2 to 3% level, in terms of the number of gluons in a small \( dx \) interval it is not insignificant compared to the depletion at large \( x_1 \), because \( g(x_1, 0) \) is strongly damped at large \( x_1 \). That is how the condition \( Q_2 = 0 \) is satisfied.
4 Suppression of $J/\psi$ and $\psi'$

Having determined $G(x_1, A)$ in the previous section, we can now return to the problem of charmonium suppression, including $\psi'$. The Fermilab E866 experiment [5] gives data for both $J/\psi$ and $\psi'$ in the form of $\alpha(x_F)$. In using Eq. (3) to calculate $R(x_F, A)$, and then $\alpha(x_F)$, we have Eqs. (35), (9) and (5) for $G(x_1, A)$, $N(x_2, A)$ and $H(A)$, respectively. For both $J/\psi$ and $\psi'$ we use $\sigma = 6.5$ mb in $H(A)$. The results on $\alpha(x_F)$ are shown as curves in Fig. 10 for $J/\psi$ and Fig. 11 for $\psi'$. The two curves in each figure are for $A = 100$ and 200; they are sufficiently close to each other to be almost independent of $A$, thereby affirming our gluon evolution model for that range of $A$.

The most significant part of what we have learned from this work on the $pA$ data is that it is hard to reproduce the strong damping of the measured $\alpha(x_F)$ at large $x_F$ without a substantial amount of gluon depletion at large $x_1$, as seen in Fig. 9. Furthermore, if there is significant modification of the parton distribution as the penetration depth increases, then it is hard to justify the notion that a proton can traverse a large part of the target nucleus without any changes, or that it can be wounded upon the first collision with a nucleon and then remaining unchanged thereafter.

So far out attention has been given to the $x_F > 0$ region only where data exist. However, the $x_F < 0$ region has interesting physics also, and the data can be obtained either at RHIC, or at a fixed target experiment with proton being the target for a heavy-ion beam. At large negative $x_F$, $x_1$ would be small where Fig. 9 shows a very small enhancement, not depletion. On the other hand, $x_2$ would be large, where $N(x_2, A)$ would exhibit not only anti-shadowing, but also the EMC and Fermi motion effects. As an illustration of the $x_F$ dependence, we have performed the calculation for $J/\psi$ suppression in $pW$ collision at three energies and for all $x_F$, negative as well as positive. The absorption cross section is set at $\sigma = 8$ mb, a value found in the next section. The values of $N(x_2, A)$ for the full $x_2$ range are obtained from EKS98 [11]. The results are shown in Fig. 12. We see that the negative $x_F$ region shows very little dependence on energy and reflects mainly the property of $N(x_2, A)$.

Gluon depletion of the type discussed in this paper has no effect on the suppression of $J/\psi$ in the $x_F < 0$ region. However, there is another type of gluon depletion, discussed in the second paper in Ref. [2], called nonlinear depletion, that can influence the survival probability in the $x_F < 0$ region. When $x_2$ is large, the fast gluons in the rear part of the nucleus can catch up and interact with the slow gluons released by the $pN$ collisions in the front part of the nucleus, leading to a depletion of gluon at large $x_2$. Although this involves the interaction between gluons in different nucleons in the nucleus, it is not nuclear shadowing, since the conventional nuclear shadowing at low $x_2$ does not require the invasion of an external proton to initiate the cascading interactions of gluons among the broken nucleons. We have no prediction on the nature of the effect of this type of depletion. If the experimental data on the production rate differ significantly from the curves in Fig. 12 in the $x_F < 0$ region, then there will be strong motivation to consider this unusual type of gluon depletion.
5 Nucleus-nucleus Collisions

Having investigated the $pA$ collision problem above, and finding the necessity to consider the depletion of gluons in the projectile before the production of $c\bar{c}$ states in the $x_F > 0$ region, it is natural to ask what the implication would be for the $J/\psi$ suppression problem in $AB$ collisions. Even without any detailed calculations it is straightforward to infer that there will be enhanced suppression at large $x_F$. However, the currently available data on $J/\psi$ suppression in nuclear collisions are from CERN-SPS, and are limited to the central rapidity region. It is therefore our burden to show that the gluon depletion mechanism discussed in Sec. 3 is consistent with the existing nuclear collision data at small $x_F$.

In Sec. 3 we have used $z$ mainly as the average penetration depth in the target nucleus where $c\bar{c}$ is produced, although in the evolution equation (12) $z$ is the actual path length. Since in this section we need to average the production point over all impact parameters in the $AB$ collisions, we now restore $z$ to be actual path length of a gluon in a nucleus. Thus we allow it to vary from 1 to 12 fm. We use Eq. (16) to determine $g_z$ and the nuclear shadowing functions $F_A$ and $F_B$ are shown by the formula (32). The resulting values of $a_i(z)$ are shown by the points in Fig. 13, they are in turn fitted by quadratic equations of the form

$$a_i(z) = b_{i0} + b_{i1}z + b_{i2}z^2 \ .$$

The result is shown by the curves in Fig. 13. The corresponding coefficients are

$$b_{i0} = 0.974, \quad b_{i1} = -0.175, \quad b_{i2} = 0.0068,$$

$$b_{i0} = 0.0866, \quad b_{i1} = 0.402, \quad b_{i2} = -0.0202,$$

$$b_{i0} = -0.066, \quad b_{i1} = -0.226, \quad b_{i2} = 0.0142. \quad (37)$$

Using Eqs. (36) and (37) in (34), we can evaluate $g(x_1, z)$ at all $x_1$ and $z$.

The cross section for the production of $J/\psi$ in $AB$ collisions can be calculated in the standard way. We shall just write it down as follows (see, e.g., Refs [2, 3]):

$$\sigma_{J/\psi} = \int d^2b \int dz_A dz_B \rho_A(s, z_A) \rho_B(s - \bar{b}, z_B) \rho_B(s - \bar{b}, z_B)$$

$$\cdot \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} g_A(x_1, L_B - z_B) g_B(x_2, L_A - z_A) N(x_1, A) N(x_2, B)$$

$$\cdot e^{-\sigma_0[\rho_A(L_A + z_A) + \rho_B(L_B + z_B)]} \delta_{gg\rightarrow c\bar{c}}(x_1, x_2) \ , \quad (38)$$

where we have included the $z_{A,B}$ dependences in the gluon distributions $g_{A,B}(x_{1,2}, L_{B,A} - z_{B,A})$ and the nuclear shadowing functions $N(x_1, A)$ and $N(x_2, B)$. $L_{A,B}$ are the path length through $A(B)$ at the distances $s$ and $s - \bar{b}$, respectively, from the centers of the nuclei, i.e.

$$L_A = (R_A^2 - s^2)^{1/2} \ , \quad L_B = (R_B^2 - (s - \bar{b})^2)^{1/2} \ . \quad (39)$$

The gluon distribution $g_A(x_1, L_B - z_B)$ is given by [see Eq. (33)]

$$g_A(x_1, L_B - z_B) = \sum_{i=1}^{3} a_i(L_B - z_B)(1 - x_1)^{4+i} \ , \quad (40)$$
and similarly, for $g_B(x_2, z_A)$, the coefficients $a_i(z)$ in Eq. (34) are replaced by $L_A - z_A$, which is the distance that a parton in $B$ travels in $A$ before the production of $c \bar{c}$ at $z_A$. The distance that a $c \bar{c}$ state travels in $A$ is $L_A + z_A$, and $z_A$ is integrated from $-L_A$ to $+L_A$.

For the energies at CERN-SPS, $\sqrt{s} \approx 18 - 20$ GeV, and for $x_F \approx 0$, the hard cross section

$\hat{\sigma}_{gg \rightarrow c \bar{c}}(x_1, x_2)$ restricts the gluon momentum fractions to $x_1 \approx x_2 \approx M_{J/\psi}/\sqrt{s} \approx 0.16$.

The survival probability is

$$S^{AB}_{J/\psi} = \frac{\sigma^{AB}_{J/\psi}}{\sigma^{AB(0)}_{J/\psi}} = N_{AB}^{-1} \int d^2b d^2s \int_{-L_A}^{L_A} dz_A \int_{-L_B}^{L_B} dz_B W(b, s, z_A, z_B)$$

(41)

where

$$W(b, s, z_A, z_B) = G_A(x_1, L_B - z_B) G_B(x_2, L_A - z_A) N(x_1, A) N(x_2, B) \cdot e^{-\sigma_a[\rho_A(L_A+z_A)+\rho_B(L_B+z_B)]},$$

(42)

and $N_{AB}$ is the same integral in Eq. (11) but with $W(b, s, z_A, z_B)$ replaced by 1. $G_A(x_1, L_B - z_B)$ is as given in Eq. (33) except that $z$ is replaced by $L_B - z_B$. Because of both the gluon enhancement at $x_{1,2} \approx 0.16$ and the anti-shadowing, the absorption cross section $\sigma_a$ now has to be somewhat larger than before [2]. An overall agreement with all the $pA$ and $AB$ collision data, except the $Pb-Pb$ case, can be achieved with the use of one value of $\sigma_a = 8$ mb. The result is given in Table 2 for the various $AB$ cases. Fig. 14 shows how those values compare with the experimental data [14] by the straightline segments that connect those successive points. It is evident that apart from the $Pb-Pb$ case the agreement with the data is satisfactory. Thus we can conclude that the gluon depletion mechanism used to treat the $pA$ problem leads to no disagreement with the $AB$ collisions—except that the $Pb-Pb$ case remains as an anomaly.

Finally, we compute the $x_F$ dependence of the $J/\psi$ suppression factor for just one nuclear-collision case as an example, which we take to be $Pb-Pb$. We consider two energies: $E_{lab} = 160$ GeV and $\sqrt{s} = 60$ GeV for RHIC. Except for the kinematics in $\hat{\sigma}_{gg \rightarrow c \bar{c}}$ that affects the values of $x_1$ and $x_2$, the cross section and survival factor can be calculated as before. The results are shown in Fig. 15, which exhibits a substantial degree of suppression at large $x_F$. Any data at large $x_F$ would put considerable constraint on the models that attempt to explain the anomalous suppression at small $x_F$.

| $AB$   | $pp$ | $pC$ | $pAl$ | $pW$ | $pU$ | $OCu$ | $OU$ | $SU$ | $PbPb$ |
|--------|------|------|-------|------|------|-------|------|------|--------|
| $S$    | 1    | 0.85 | 0.82  | 0.70 | 0.68 | 0.67  | 0.58 | 0.56 | 0.49   |
6 Conclusion

In our attempt to understand the enhanced suppression of $J/\psi$ at large $x_F$, we have found that the depletion of gluons at large $x_1$ in the projectile is the most natural explanation for the effect. We have proposed an evolution equation for the gluon distribution as the gluons propagate in a nuclear medium. The depletion at high $x_1$ contributes to a mild growth of the gluon distribution at small $x_1$. However, that growth does not lead to any contradiction with the existing data on $J/\psi$ suppression at mid-rapidity. Indeed, we have gone further to show where to find informative clues on the dynamics of suppression at large (positive and negative) $x_F$ in both $pA$ and $AB$ collisions.

What we have done here is only a modest first step towards understanding parton evolution in nuclear matter. While concentrating on the gluons, we have ignored the influence of the quark sector, a subject to be investigated at a later point. The depletion of quarks at large $x$ reveals itself in the suppression of dilepton and leading meson production at large $x_F$, the experimental evidences for which exist, though in subtle ways. Because of the conservation of Fermion number, the degradation of the quark distribution at large $x$ cannot be substantial. Nevertheless, the influence on the gluon distribution at small $x$ may not be negligible.

An important implication of this work is that in $pA$ or $AB$ collisions the concept of a nucleon propagating through nuclear matter as an identifiable, fixed entity needs modification. The usual notion that in nuclear collisions the total transverse energy $E_T$ is proportional to the number of nucleon-nucleon collisions would seem to have difficulty in reconciling with the insistence that the nucleons remain unaltered, if each inelastic collision of the nucleons contributes a fraction of their energies to $E_T$. The wounded nucleon model \cite{15} makes a crude approximation of what goes downstream as an average quantity that is different from the incident nucleon, but ignores the way it changes as it propagates. Our evolution equation indicates that the parton flux changes continuously and may emerge with a profile that cannot be identified with that of a free nucleon in any sensible comparison. The revelation made by this understanding will undoubtedly affect many aspects of high-energy nuclear collisions.

Acknowledgment

We are grateful to Kari Eskola for providing us with the Fortran codes for EKS98. This work was supported, in part, by the U.S.-Slovakia Science and Technology Program, the U. S. National Science Foundation under Grant No. INT-9319091 and by the U. S. Department of Energy under Grant No. DE-FG03-96ER40972.
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Figure Captions

Fig. 1 Nuclear shadowing and anti-shadowing factor taken from EKS98 [1] for $Q^2 = 10$ GeV$^2$. The curves are fits by the simple formula in Eq. (9).

Fig. 2 The $\beta(\xi)$ function used to fit EKS98 data points at $A = 100$.

Fig. 3 $K_n$ calculated from the moments at discrete $n$ and fitted by the formula in Eq. (23).

Fig. 4 The $z$ dependences of $K_n(z)$. The lines are straight-line fits in the large $z$ region.

Fig. 5 $Q_n$ as calculated from Eq. (23).

Fig. 6 The points of $Q_n$ are determined from Eq. (23) or Fig. 5 at integer $n$ values; the curve is a fit using Eq. (29).

Fig. 7 The kernal $Q(y)$ as calculated from Eq. (30).

Fig. 8 $g_n$ for two values of $A$.

Fig. 9 The ratio of gluon distributions, $G(x_1, A)$, for two values of $A$.

Fig. 10 $\alpha(x_F)$ for $J/\psi$ production. The data points are from Ref. [5]; the curves represent our result.

Fig. 11 $\alpha(x_F)$ for $\psi'$ production. The data points are from Ref. [5]; the curves represent our result.

Fig. 12 The survival probability for $J/\psi$ production in $pW$ collisions at different energies for the entire range of $x_F$.

Fig. 13 The $z$ dependences of the coefficients $a_i(z)$.

Fig. 14 The survival probability in $AB$ collisions. The data points are from Ref. [14]; the line is composed of straightline sections connecting the calculated points listed in Table 2.

Fig. 15 The survival probability for $J/\psi$ in $Pb-Pb$ collision for all $x_F$ at two energies.
The graph shows the function $G(x_1, A)$ for two different values of $A$: $A=100$ and $A=200$. The function decreases as $x_1$ increases, indicating a negative relationship between $x_1$ and $G(x_1, A)$ for both values of $A$. The dashed line represents $A=100$, and the solid line represents $A=200$. The curve for $A=100$ is slightly higher than the curve for $A=200$ at lower values of $x_1$, while they converge as $x_1$ approaches 1.
\[ \alpha(x_F) \]

- \( \psi' \)
- \( A=100 \)
- \( A=200 \)
\( E_{\text{lab}} = 160 \text{ GeV} \)

\( A = B = ^{207}\text{Pb} \)

\[ \left( \frac{d\sigma_{AB}}{dx_F} / \frac{d\sigma_{pp}}{dx_F} \right) \]

\( s^{1/2} = 60 \text{ GeV} \)