Inflaton as a pseudo-Nambu-Goldstone boson

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Abstract

In the realistic model of cosmic inflation the inflaton potential should be flat and stable under quantum corrections. It is natural to imagine that there is some symmetry behind and an idea of the inflaton as a Nambu-Goldstone boson of spontaneous breaking of some symmetry has been examined. We give a general formulation of this idea using the non-linear realization of Nambu-Goldstone boson in low-energy effective theory with some explicit symmetry breaking to generate non-trivial potential. The potential is naturally a simple function, typically the mass term of inflaton, and the scenario of “warm inflation” should necessarily be applied under the present observational constraints. We investigate the generation mechanism of necessary thermal dissipation term in inflaton field equation for “warm inflation” without large thermal corrections to inflaton potential. A simple numerical analysis is given to investigate the viability of this scenario.
1 Introduction

The slow-roll inflation by the potential energy by almost flat potential as a function of inflaton field is considered to be most probable. In many candidates of the models of inflation potential (see [1], for example, for the status of various models) the common problem is the stability under quantum corrections by the fields which necessarily interact with inflaton for the reheating. In general the potential of scalar field with some interactions get large quantum corrections, which courses the naturalness problem in many places in particle physics and cosmology. The ideas of “natural inflation” [2, 3] and its improvements [4, 5], “warm little inflaton” [6, 7] and “minimal warm inflation” [8, 9, 10, 11] are proposed as a solution of this problem by assuming the inflaton as a pseudo-Nambu-Goldstone boson. Since the exact Nambu-Goldstone boson can not have potential for symmetry reason, the potential of pseudo-Nambu-Goldstone boson due to certain explicit soft symmetry breaking effect is not suffered by large quantum corrections.

On the other hand, because of the symmetry reason, the form of such a potential is restricted to be a simple one, which is not acceptable in normal inflation scenario by too large tensor-to-scalar ratio as a necessary result of large field inflation. In the scenario of warm inflation [12, 13] with radiation in almost thermal equilibrium even under the accelerated expansion of the universe, such a simple inflaton potential is acceptable. Since the additional friction by thermal dissipation term in the inflaton field equation due to the interaction with radiation keeps the value of the inflaton field small as well as realizing enough number of e-folds in slow-roll inflation.

In this letter we give a general arguments of warm inflation with an inflaton as pseudo-Nambu-Goldstone boson. In the next section we introduce a general formulation of Nambu-Goldstone field in low-energy effective theory with non-linear realization [14, 15]. In section 3 we discuss some explicit soft symmetry breaking to produce inflaton potential and thermal dissipation term in inflation field equation. In section 4 we solve the fundamental equations of warm inflation in slow-roll limit and give a simple numerical analysis to investigate the viability of this scenario. In the last section we conclude.

2 Effective theory with non-linear realization

The dynamics of Nambu-Goldstone boson at energies much lower than the energy scale of corresponding spontaneous symmetry breaking is well described by low-energy effective theory with non-linear realization (see [16, 17, 18] for general formalism). In this letter we consider only a very simple model of the symmetry breaking $\text{U}(1) \rightarrow \mathbb{Z}_2$ at the energy scale $f$ as a minimal case. The non-linear Nambu-Goldstone field is

$$U = e^{i\phi/f},$$

where $\phi$ is the Nambu-Goldstone field which transforms as $\phi \rightarrow \phi + \theta f$ with $0 < \theta \leq 2\pi$ under the $\text{U}(1)$. We assume that the spontaneous symmetry breaking is effectively triggered by a vacuum expectation value of a scalar field with $\text{U}(1)$ charge 2, and the low-energy effective theory is invariant under $\mathbb{Z}_2$ transformation $U \rightarrow -U$ corresponding to $\theta = \pi$. The low-energy effective Lagrangian of the field is constructed in the notion of a derivative expansions (we neglect non-trivial background space-time metric for a while).

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} f^2 \partial^\mu U \partial_\mu U + \mathcal{O}(\partial^4).$$
In this simple model the first term gives kinetic term only and the interaction with derivative couplings are included in higher order terms. There is no potential as a function of $\phi$ without derivatives. In this letter we investigate low-energy effective theory at the lowest order.

Inclusion of fermions in an U(1) symmetric way reveals the non-trivial nature of Nambu-Goldstone boson. Introduce a left-handed fermion $\psi_L$ with U(1) charge 1 and a right-handed fermion $\psi_R$ with U(1) charge 0, then the simplest effective Lagrangian is

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} f^2 \partial^\mu U^\dag \partial_\mu U + \bar{\psi} i \gamma^\mu \psi - \mu \bar{U} \psi_L \psi_R - \mu U^\dag \bar{\psi}_R \psi_L,$$

(3)

where $\mu$ is a real parameter with dimension one [6]. The mass of the fermion becomes clear once we redefine the field as $U^\dag \psi_L \rightarrow \psi_L$ (analogous to “constituent quark field”). The effective Lagrangian becomes in this new field

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} f^2 \partial^\mu U^\dag \partial_\mu U + \bar{\psi} i \gamma^\mu \psi - \mu \bar{\psi} \psi - \frac{1}{f} \partial_\mu \phi \bar{\psi}_L \gamma^\mu \psi_L.$$

(4)

We see that the derivative coupling of the Nambu-Goldstone field $\phi$ with fermion U(1) current, and fermion one-loop quantum corrections do not give potential terms of $\phi$ without derivatives. The field equation of $\phi$ contains the divergence of fermion U(1) current, which can be rewritten as a pseudo-scalar density by using fermion field equation, and

$$\Box \phi + \frac{\mu}{f} \bar{\psi} \gamma_5 \psi = 0.$$

(5)

Even if we consider a thermal bath of the fermion, no effect (thermal mass, dissipation term, for example) emerges in the equation of $\phi$, because the thermal average of the parity-odd pseudo-scalar density vanishes. Here, and in this letter, we apply the Hartree approximation to evaluate thermal effects for a simple and intuitive understanding [19].

In this way there are no quantum corrections to the potential and no thermal effects for Nambu-Goldstone field. We need to introduce some explicit soft breaking of U(1) symmetry whose energy scale is much less than the energy scale of spontaneous symmetry breaking $f$.

### 3 Inflaton potential and dissipation term

Consider a spurious scalar field $\Sigma$ with U(1) charge $-2$. Then, the following potential term is arrowed.

$$V_{\text{eff}} = -\frac{1}{8} B \Sigma U \Sigma + \text{h.c.},$$

(6)

where $B \simeq f^3$ is a real quantity which are determined by the dynamics of spontaneous U(1) symmetry breaking. This form is analogous to the explicit chiral symmetry breaking term by current quark masses in the low-energy effective Lagrangian for pions which are Nambu-Goldstone bosons under the spontaneous chiral symmetry breaking by QCD dynamics. The real vacuum expectation value of the spurious field, $\langle \Sigma \rangle = v_\Sigma \ll f$, gives the following $\mathbb{Z}_2$ symmetric potential

$$V_{\text{eff}} = -\frac{1}{4} M^4 \cos(2\phi/f) = -\frac{1}{4} M^4 + \frac{1}{2} \frac{M^4}{f^2} \phi^2 - \frac{1}{6} \frac{M^4}{f^4} \phi^4 + \mathcal{O}(\phi^6),$$

(7)
where $M^4 \equiv Bv_\Sigma \simeq f^3v_\Sigma$. Since this potential should be considered in the range $\phi \ll f$ as that in the low-energy effective theory, the dominant term is the mass term of $\phi$ with the mass $m^2_\phi \simeq f v_\Sigma$. Here, we simply neglect the constant term, which belongs to the cosmological constant (or dark energy) problem beyond the scope of this work.

The quartic coupling in eq. (7) gives quantum corrections to the potential, but all the correction is higher order in the expansion of $M^4/f^4 \simeq v_\Sigma/f$ [14,15] (we need to use dimensional regularization not to break U(1) symmetry). This expansion is equivalent to the derivative expansion in eq. (2) with $\partial^2/f^2 \sim M^4/f^4$, so that the mass term is the same order of the kinetic term. Therefore, the inflaton potential under $\phi \ll f$

$$V \simeq \frac{1}{2}m^2_\phi \phi^2$$

(8)

is stable under the quantum corrections as long as we stay in the lowest order of the expansion in $M^4/f^4 \simeq v_\Sigma/f$.

The generation of dissipation term in the field equation of inflaton for warm inflation is more complicated, because the thermal mass, which is the dominant thermal effect, must be canceled out. At least two fermions are required with some tuned parameters.

First consider the thermal correction from a single fermion field with the symmetry breaking term

$$\mathcal{L}_{\text{eff}}^{bx} = -\bar{\psi}_L \gamma_5 \psi_R + \text{h.c.}$$

(9)
in the effective Lagrangian of eq. (3), where $\varphi$ is a spurious field with U(1) charge 1. Redefine the field as $U^\dagger \psi_L \rightarrow \tilde{\psi}_L$, and introduce a real vacuum expectation value of the spurious field $\varphi$ as $m \ll f$ and $m \ll \mu$, then we have

$$\mathcal{L}_{\text{eff}}^{bx} = -m \cos(\phi/f) \bar{\psi} \psi + m \sin(\phi/f) \bar{\psi} \gamma_5 \psi,$$

(10)

which should be added to eq. (11). The fermion one-loop quantum correction by these interactions is independent from the inflaton field due to the nature of trigonometric functions or the unitarity of non-linear Nambu-Goldstone field, and it does not give corrections to the inflaton potential. The effective mass of the fermion under a background field $\phi$ is

$$m^\text{eff} = \mu + m \cos(\phi/f),$$

(11)

where we treat the pseudo-scalar density in eq. (10) as an interaction term. The field equation of the inflaton field becomes

$$\Box \phi + V' + \frac{\mu}{f} \bar{\psi} \gamma_5 \psi - \frac{m}{f} \sin(\phi/f) \bar{\psi} \psi - \frac{m}{f} \cos(\phi/f) \bar{\psi} \gamma_5 \psi = 0.$$  

(12)

The Hartree approximation assuming near thermal distribution of fermions with temperature $T \gg \mu$ gives

$$\langle \bar{\psi} \psi \rangle \simeq \frac{1}{6}m^\text{eff} T^2 + 4\mu^2 \frac{dm^\text{eff}}{d\phi} \frac{C}{\Gamma_\text{th}} \phi + \mathcal{O}((m/f)^2), \quad \langle \bar{\psi} \gamma_5 \psi \rangle = 0,$$

(13)

where $C$ is a constant of order 10 and $\Gamma_\text{th}^{-1}$ is the time to recover thermal equilibrium from the disturbance by small change of the value of $\phi$ in fermion effective mass. The effective inflaton field equation becomes

$$\Box \phi + V' - \frac{m}{f} \sin(\phi/f) \left[ \frac{1}{6}(\mu + m \cos(\phi/f)) T^2 - 4\mu^2 \frac{m}{f} \sin(\phi/f) \frac{C}{\Gamma_\text{th}} \phi \right] \simeq 0.$$  

(14)
The first term in the square bracket is the thermal mass term and the second term is the dissipation term. We see that the thermal mass is dominant and the dissipation is subleading effect as it has been pointed out in [19].

Consider two fermions of the same U(1) charges and interactions of eq.(3), which are connected with the \(Z_2\) symmetry (a combination of \(Z_2\) from U(1) breaking and \(Z_2\) of the exchange of \(\psi_1\) and \(\psi_2\)) as

\[
\left(\begin{array}{c}
\psi_{1L} \\
\psi_{2L}
\end{array}\right) \rightarrow e^{i\pi} \left(\begin{array}{cc}
0 & i \\
-i & 0
\end{array}\right) \left(\begin{array}{c}
\psi_{1L} \\
\psi_{2L}
\end{array}\right), \quad \left(\begin{array}{c}
\psi_{1R} \\
\psi_{2R}
\end{array}\right) \rightarrow \left(\begin{array}{cc}
0 & i \\
-i & 0
\end{array}\right) \left(\begin{array}{c}
\psi_{1R} \\
\psi_{2R}
\end{array}\right)
\]

(15)

with \(U \rightarrow -U\). The only difference of these two fermions is the sign of the breaking term

\[
\mathcal{L}_{\text{br}}^{\text{eff}} = -\varphi \bar{\psi}_1 \psi_1 + \varphi \bar{\psi}_2 \psi_2 + \text{h.c.},
\]

(16)

which becomes

\[
\mathcal{L}_{\text{eff}}^{\text{br}} = -m \cos(\phi/f) \bar{\psi}_1 \psi_1 + m \sin(\phi/f) \bar{\psi}_1 i \gamma_5 \psi_1
\]

\[+ m \cos(\phi/f) \bar{\psi}_2 \psi_2 - m \sin(\phi/f) \bar{\psi}_2 i \gamma_5 \psi_2,
\]

(17)

in redefined left-handed fermion fields with \(\langle \varphi \rangle = m\). The largest contribution in the thermal mass in eq.(14), which is proportional to \(m\), is canceled:

\[
\Box \phi + V' - \frac{m}{f} \sin(\phi/f) \left[ \frac{1}{3} m \cos(\phi/f) T^2 - 8 \mu^2 \frac{m}{f} \sin(\phi/f) \frac{C}{\Gamma_{\text{th}}} \right] \phi \sim 0.
\]

(18)

The contributions which are proportional to the even power of \(m\) are not canceled, and the subleading thermal mass term remains. We assume for simplicity that \(\Gamma_{\text{th}}\) is the same for each fermion.

We need more fermions to cancel the remaining thermal mass term. Introduce the mirror fields \(\tilde{\psi}_1, \tilde{\psi}_2\) and \(\tilde{\varphi}\) which are copies of the corresponding fields \(\psi_1, \psi_2\) and \(\varphi\), respectively, with a new mirror \(Z_2\) symmetry \(\psi \leftrightarrow \tilde{\psi}\) and \(\varphi \leftrightarrow \tilde{\varphi}\). We assume that the physics behind the explicit breaking results the vacuum expectation values of the spurious fields as \(\langle \varphi \rangle = m\) and \(\langle \tilde{\varphi} \rangle = im\) (the mirror \(Z_2\) symmetry is also broken). Then the additional breaking terms in redefined left-handed fermion fields are

\[
\mathcal{L}_{\text{eff}}^{\text{br}} = -m \sin(\phi/f) \bar{\tilde{\psi}}_1 \tilde{\psi}_1 - m \cos(\phi/f) \bar{\tilde{\psi}}_1 i \gamma_5 \tilde{\psi}_1
\]

\[+ m \sin(\phi/f) \bar{\tilde{\psi}}_2 \tilde{\psi}_2 + m \cos(\phi/f) \bar{\tilde{\psi}}_2 i \gamma_5 \tilde{\psi}_2.
\]

(19)

which should be compared with eq.(17) showing different appearance of trigonometric functions. The cancelation of the thermal mass term in inflaton field equation happens as

\[
\Box \phi + V' + 8 \mu^2 \left( \frac{m}{f} \right)^2 \frac{C}{\Gamma_{\text{th}}} \phi \sim 0,
\]

(20)

where we assume again that \(\Gamma_{\text{th}}\) is the same for each fermion. This mechanism utilizing combinations of trigonometric functions has been proposed in [6]. In this way, even if it is possible, we need some complicated mechanism to produce dissipation term without large thermal effect to the potential. Another mechanism is proposed in [8] where the dissipation is not thermal but topological assuming that the inflaton is an axion with topological coupling to some Yang-Mills gauge field. See [20, 21] for further possibilities.
We estimate $\Gamma_{th} \simeq h^2 \mu^2 / T$ following [6] by considering the decay of the fermions in almost thermal equilibrium through the interaction

$$\mathcal{L}_{\text{int}} = -h \bar{\psi} R \Phi^\dagger \ell_L + \text{h.c.},$$

(21)

where $h$ is a coupling constant of this Yukawa coupling in which $\ell_L$ and $\Phi$ (they are also in thermal equilibrium) are possibly identified as left-handed lepton doublet and Higgs doublet in the standard model of particle physics, respectively as in [22]. Then the coefficient of the dissipation term is $\Gamma \simeq (C / h^2)(m / f)^2 T$, which does not depend on the inflaton field, where newly defined $C$ is a constant of the order of 10.

4 Dynamics and numerical evaluation

The fundamental equations of warm inflation are the following [13].

\begin{align*}
\ddot{\phi} + (3H + \Gamma) \dot{\phi} + V' & = 0, \\
3H^2 & = \frac{1}{M_p^2} \left( \frac{1}{2} \dot{\phi}^2 + V + \rho_r \right), \\
\dot{\rho}_r + 4H \rho_r & = \Gamma \dot{\phi}^2.
\end{align*}

(22)

(23)

(24)

Here, $H$ is the Hubble parameter, $\rho_r = C_R T^4$ is the energy density of radiation in almost thermal equilibrium with $C_R = g_* \pi^2 / 30$, where the effective degrees of freedom $g_*$ which includes the fermion fields in previous section, and $M_p$ is the reduced Planck mass. The model in this letter, the inflaton as a pseudo-Nambu-Goldstone boson, we have obtained inflaton potential of eq.(8), $V \simeq m_V^2 \phi^2 / 2$, and dissipation coefficient as $\Gamma \simeq C_T T$ with $C_T \equiv (C / h^2)(m / f)^2$. There are conditions which should be satisfied among dimension-full quantities so that the low-energy effective theory is applicable. The value of the inflaton field should be less than the energy scale of spontaneous $U(1)$ symmetry breaking: $\phi \ll f \lesssim M_p$. Since the mass of inflaton $m_V$ is generated by the effect of explicit $U(1)$ symmetry breaking, it should be less than $f$: $m_V \ll f$ (more precisely $v_\Sigma \ll f$). The mass of fermions $\mu$ should be less than $f$ and the energy scale of explicit symmetry breaking $m$ should be less than $\mu$ and $f$: $m \ll \mu \ll f$. The temperature should be higher than the fermion mass, but lower than the energy scale of spontaneous symmetry breaking: $\mu \ll T \ll f$.

The slow-roll approximation of the fundamental equations are

$$\dot{\phi} \simeq \frac{V'}{3H(1 + Q)}, \quad 3H^2 \simeq \frac{V}{M_p^2}, \quad 4\rho_r \simeq 3Q \dot{\phi}^2,$$

(25)

where $Q \equiv \Gamma / 3H$. These equations are satisfied when the slow-roll conditions $\epsilon \ll Q$ and $\eta \ll Q$ are satisfied, where

$$\epsilon \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \simeq 2 \left( \frac{M_p}{\phi} \right)^2, \quad \eta \equiv M_p^2 \frac{V''}{V} \simeq \epsilon$$

(26)

under the present simple potential of eq.(8). In the usual inflation without radiation, the inflaton field value must be larger than the Planck scale for slow-roll in this kind of simple
potential, and this large field inflation gives too large value of tensor-to-scalar ratio. In warm inflation the inflaton field value can be smaller than Planck scale for slow-roll as far as $Q \gg 1$. We first analytically solve the slow-roll equations and give the formulae for typical values of $\phi$, $H$, $Q$ and $T/H$ during slow-roll inflation.

The derivative in time $t$ can be replaced by the derivative in e-folds $N$ with $dt = H^{-1}dN$, and we obtain the difference of the values of field at $N = 0$ and $N = N_e$ from slow-roll equations as

$$ (\phi(N = 0))^{6/5} - (\phi(N = N_e))^{6/5} \approx \frac{6}{5} N_e \left( \frac{144 C_R^4 M_p^4 m_V^2}{C_T^4} \right)^{1/5}. \quad (27) $$

This gives typical values of inflaton field and Hubble parameter during slow-roll inflation as

$$ \phi_{\text{typ}} \equiv \left( \frac{6}{5} N_e \right)^{5/6} \left( \frac{144 C_R^4 M_p^4 m_V^2}{C_T^4} \right)^{1/6}, \quad H_{\text{typ}} \approx \frac{1}{\sqrt{6} M_p} \phi_{\text{typ}}. \quad (28) $$

The typical value of $Q$ is given in a complicated form as

$$ Q_{\text{typ}} \approx \sqrt{\frac{2}{3}} \left( \frac{144^2 C_R^{12}}{(4\sqrt{6})^5 C_T^9} \left( \frac{M_p}{f} \right)^{18} \left( \frac{f}{m_V} \right)^6 \left( \frac{f}{\phi_{\text{typ}}} \right)^{12} \right)^{1/15}, \quad (29) $$

and this gives the typical value of $T/H$ as

$$ \frac{T_{\text{typ}}}{H_{\text{typ}}} = \frac{3}{C_T} Q_{\text{typ}}. \quad (30) $$

Roughly speaking both $\phi_{\text{typ}} \ll f$ and $Q_{\text{typ}} \gg 1$ requires

$$ \frac{m_V}{M_p} \ll C_T^2, \quad (31) $$

namely a flat potential.

If the value of $Q_{\text{typ}}$ is large enough, the typical value of $T/H$ is large and the contribution of thermal fluctuations to the scalar (or curvature) perturbations dominates over the contribution of quantum fluctuations as it can be seen in the formula of the power spectrum [23, 24, 25, 26]

$$ P_R \simeq \left( \frac{Q}{2\sqrt{2\pi} M_p} \right)^2 \frac{1}{\epsilon} \left[ 1 + \sqrt{3\pi Q} \frac{T}{H} \right] G(Q), \quad (32) $$

where we assume $Q \gg 1$ and no inflaton particles in thermal equilibrium and

$$ G(Q) = 1 + 0.335 Q^{1.364} + 0.0185 Q^{2.315} \quad (33) $$

which is given in [6, 7] for the case of $\Gamma \propto T$. All the quantities in this formula of power spectrum are evaluated at the time of $k/a(t)H(t) = 1$, and it is a function of wave number $k$ in this sense. The spectral index of this power spectrum is approximately given by

$$ n_s = 1 - \frac{6}{5} \frac{\epsilon}{Q} + \frac{d\ln G}{d\ln Q} \cdot \frac{4}{5} \frac{\epsilon}{Q} \quad (34) $$
in the present particular form of the inflaton potential and dissipation term.

A method as a first attempt to evaluate whether this scenario can be realistic or not is to choose a set of concrete numerical values of parameters and calculate the values of important quantities. For simplicity we assume that U(1) symmetry is spontaneously broken at Planck scale, namely \( f \simeq M_p \simeq 2.4 \times 10^{18} \) GeV, which could be understood as accepting a view point that the quantum gravity should not allow a global symmetry. We choose some natural values of dimensionless parameters as \( N_e = 60, \ g_s = 100, \ C = 10 \) and \( h = 2.5 \times 10^{-9} \). This very small coupling is required for enough light fermions to be in thermal equilibrium (see the definition of \( C_T \) under \( f \simeq M_p \)). A possible problem is the thermalization with slow-rate production and decay of fermions. The value of \( mV \) is chosen to be small enough as

\[
\frac{m_V}{M_p} = C_T^2 \times 10^{-8}
\]

according to eq. (31), which makes \( \phi_{\text{typ}} \) and \( Q_{\text{typ}} \) independent from \( m \). We obtain

\[
\frac{\phi_{\text{typ}}}{f} \simeq 0.3, \quad Q_{\text{typ}} \simeq 10^3,
\]

which indicate that the low-energy effective theory is applicable and the slow-roll condition is satisfied. The resultant value \( \epsilon_{\text{typ}} \simeq 20 \) is large, but \( \epsilon_{\text{typ}}/Q_{\text{typ}} \simeq 0.01 \) is small. We choose \( m = M_p \times 10^{-10} \simeq 2 \times 10^8 \) GeV so that the typical value of Hubble parameter during inflation is small enough, \( H < 10^9 \) GeV, which is the condition for stability of the electroweak Higgs potential in the standard model of particle physics [27]. Then we obtain

\[
H_{\text{typ}} \simeq 8 \times 10^5 \text{GeV}, \quad T_{\text{typ}} \simeq 2 \times 10^{11} \text{GeV},
\]

which give large \( T/H \simeq 3 \times 10^5 \) and small \( T/f \simeq 9 \times 10^{-8} \) as they should be. The fermion mass \( \mu \), which does not explicitly appear in physical quantities, should take some value satisfying \( m < \mu < T_{\text{typ}} \) for thermal equilibrium fermions. The mass of inflaton is \( m_V \simeq 6 \times 10^6 \) GeV, which corresponds to \( \langle \Sigma \rangle = v_\Sigma \simeq 0.02 \) MeV.

These theoretically consistent values of parameters, which properly realize the hierarchy of scales, give the approximate values of the amplitude and spectral index of scalar perturbations as \( A_s \simeq 2 \times 10^{-9} \) and \( n_s \simeq 1 \), which are acceptable in comparison with those obtained by observations, \( A_S = (2.10 \pm 0.03) \times 10^{-9} \) and \( n_s = 0.965 \pm 0.004 \) by [28], for example, within this simple analysis for order estimates. We observe, however, that it could be difficult to obtain the value \( n_s < 1 \) in this scenario. In low-energy effective theory with \( \phi \ll f \ll M_p \), the formulae of slow-roll parameters in eq. (26) indicate large slow-roll parameters, and a large value of \( Q \) is necessary for slow-roll inflation \( (\epsilon/Q \ll 1 \) and \( \eta/Q \ll 1) \) with enough number of e-folds. On the other hand, in the approximate formula of spectral index, eq. (31), the third term dominates over the second term for \( Q \gtrsim 10 \) in the right-hand side, and \( n_s \) can not be less than one, which accords with the analyses of chaotic warm inflation in [6, 25]. Since we can not exclude this scenario by this simple analysis of this work, further precise and detailed analyses are necessary. It could be possible that this scenario, a general scenario of inflaton as a pseudo-Nambu-Goldstone boson in low-energy effective theory, would be hard to reconcile with observation in the end.
5 Conclusions

We have examined the possibility that the inflaton is a pseudo-Nambu-Goldstone boson. Starting with an introduction of low-energy effective theory of Nambu-Goldstone boson in non-linear realization and showing no quantum corrections to the potential and no thermal effects, the way to generate a non-trivial potential and thermal effects through the explicit breaking of the symmetry has been reviewed. Though we could choose more general pattern of spontaneous breaking of the symmetry group $G$ to the group $H$ with Nambu-Goldstone fields living in the space $G/H$, as it has been investigated in [5], we chose U(1) symmetry breaking as the simplest case, which is enough to understand essential facts. The result of this choice gives a simple model which is similar to the model in [6], but the form of fermion interactions and inflaton potential is more restricted for symmetry reason. We have solved the slow-roll equations taking care of the hierarchy of scales in this model, and shown that reasonable value of $A_s$ and $n_s$ in the power spectrum of scalar perturbations can be obtained with some natural values of parameters within our simple analysis for order estimates. A simple observation, however, would indicate that the value $n_s < 1$ could be difficult to be realized because of large value of $Q$ is required in low-energy effective theory for slow-roll inflation with enough number of e-folds. Further precise and detailed analyses are necessary to clarify the viability of a general scenario of inflaton as a pseudo-Nambu-Goldstone boson in low energies effective theory.

In the model building point of view, the dynamics of spontaneous U(1) symmetry breaking and the origin of explicit breaking (vacuum expectation values of spurious fields) should be investigated. In the phenomenological point of view the effect of the derivative coupling of inflaton to fermion U(1) current (the last term in eq.(4)), which pretends a chemical potential term under the time dependence of inflaton, should be investigated. We need to use thermal field theory beyond Hartree approximation for this investigation as well as to obtain more precise form of the dissipation term and also to clarify the role of parity-odd pseudo-scalar densities.

An interesting possibility that this inflaton becomes cold dark matter, which has been proposed in [22], is worth to be examined within the restriction by symmetry. We need to investigate the end of inflation and reheating process probably by numerical simulations.

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