Assessment of BSL $k$-$\omega$ Turbulence Model for Predicting Flow Characteristics in a Solitary Wave Motion

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Abstract. A deep understanding of boundary layer characteristics under wave motion, especially bottom shear stress on the seabed, is critical in sediment transport calculation and modeling. In this present study, boundary layer characteristics under solitary wave motion over the smooth bed are investigated through the Baseline (BSL) $k$-$\omega$ model. It will also consider the result of a laboratory experiment performed in a closed conduit generation system. The conclusion said that the BSL $k$-$\omega$ model can replicate well on horizontal and vertical velocity distribution at Reynolds number ($R_e$) = 2.25 x 10$^5$. In addition, a turbulent spike which was occurred during decelerating phases of oscillatory motion can be predicted well also by the BSL $k$-$\omega$ model at Reynolds number ($R_e$) = 6.06 x 10$^5$.

Keywords: Solitary wave, flow characteristics, BSL $k$-$\omega$ turbulence model

1. Introduction
Understanding the water wave boundary layer is essential in the calculation, analysis, and modeling of coastal sediment transport. A solitary wave has thin crests but no trough has often been used as a rough approximation for the surface profile of ocean water waves generated in shoaling water (Figure 1). Many studies have investigated non-linear wave boundary with laboratory experiments and also numerical analysis [2][3][4][5]. Despite this, an investigation into the properties of solitary wave boundary layers is still limited.

The impact of viscous damping under solitary waves was firstly introduced theoretically in 1948 [6]. Then, this theory has been used and converted into temporal variation form to study the characteristic of cnoidal wave boundary layers at high Ursel numbers [7]. Some laboratory experiments methods such as using a particle image velocimetry (PIV) and supported by analytical analysis have been done also to observe the behavior of the laminar solitary wave boundary layer [8][9][10]. Next, direct numerical
Simulations (DNS) and Reynolds Averaged Navier-Stokes equations (RANS) have been used to predict solitary wave behavior [11][12].

Figure 1. (a) A solitary wave approaching the beach of Molokai, Hawaii [1]; (b) A sketch of solitary wave exact solution.

The laboratory experiment on solitary wave motion has been conducted to scrutinize transition to turbulent solitary boundary layers, but the maximum Reynolds number was still at 1.8 x 10^6 [13, 14, and 15]. Among the experimental studies, facilities used could be categorized into two classifications: a wave flume or wave tank with the free surface [9][10][16][17][18][19][20] and a closed conduit-oscillating water tunnel [13][14][15]. Based on the result of an experiment, a wave flume fall in the laminar flow regime and still cannot inspect in transitional and or turbulent flow regime. In contrast to a wave flume, a U-Shaped oscillating and closed conduit water tunnel enables achieved higher Reynolds number. The present study discusses transitional to the turbulence of solitary wave motion behavior through a numerical experiment using the BSL k-ω turbulence model by combining with previous experimental studies [14][15]. The BSL k-ω turbulence model was chosen because it can examine the oscillatory wave boundary layer well [21]. Once it can accurately predict behavior in transition to a turbulent flow regime, this turbulence model will be helpful for further research in a fully developed turbulent flow.
2. Materials and Methods

2.1. Experimental condition

The following equation shows the exact solution for free stream velocity in a solitary wave motion:

\[ U = U_c \text{sech}^2(\alpha t) \]

(1)

\[ \alpha = \frac{3H}{4h^3} \sqrt{g(h + H)} \]

(2)

where \( U_c \) is the maximum velocity at the crest of the wave, \( t \) is the time, \( c \) is the wave celerity, \( h \) is the water depth, \( H \) is the wave height and \( g \) is the gravitational acceleration.

Experimental data used to develop the BSL \( k-\omega \) model was obtained from a laboratory experiment using a closed conduit generation system [14][15]. Instantaneous velocities were measured at 17-22 points vertically using Laser Doppler Velocimetry (LDV) and at 10ms intervals over 50 continuous solitary motion cycles. The experiment conditions related to maximum velocity under wave crest (\( U_c \)) and Reynolds number (\( R_e \)) are tabulated in Table 1, while a recorded signal of horizontal velocity profiles at 2 (two) measuring points are displayed in Figures 2 and Figure 3 below.

Table 1. Experiment condition.

| Type of experiment | \( U_c \) (cm/s) | Reynolds number (\( R_e \)) |
|--------------------|----------------|---------------------------|
| Flow velocity      | 42.9           | 2.25 x 10^5               |
| Flow velocity      | 78.5           | 6.06 x 10^5               |

![Figure 2](image2.png)

**Figure 2.** The horizontal velocity distribution at measurement point \( z = 0.031 \) cm from the bottom of closed conduit solitary motion generation system at \( R_e = 2.25 \times 10^5 \).
Figure 3. The horizontal velocity distribution at measurement point \((z) = 0.032\) cm from the bottom of closed conduit solitary motion generation system at \(Re = 6.06 \times 10^5\)

2.2. Turbulence model description

The equation of motion within the boundary layer for a 1-D incompressible unsteady flow can be written as:

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial z}
\]

(3)

where \(t\) is the time, \(p\) is the pressure, \(\tau\) is the shear stress and \(x\) is the horizontal coordinate. At the axis of symmetry or outside boundary layer \(u = U\), therefore

\[
\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{\rho} \frac{\partial \tau}{\partial z}
\]

(4)

For turbulent flow,

\[
\frac{\tau}{\rho} = \nu \frac{\partial u}{\partial z} - uu'
\]

(5)

The Reynolds stress \(-\rho uu'\) may be expressed as \(-\rho uu' = \rho \nu \left( \frac{\partial u}{\partial z} \right)\), where \(\nu\) is the eddy viscosity. And Eq. (5) becomes

\[
\frac{\tau}{\rho} = (\nu + \nu_t) \frac{\partial u}{\partial z}
\]

(6)

The basic idea of the BSL \(k-\omega\) model is to retain the robust and accurate formulation of a \(k-\omega\) model in the near-wall region [22] and to take advantage of the free stream independence of the \(k-\epsilon\) model in the outer part of the boundary layer [23]. The BSL model gives results similar to the \(k-\omega\) model in the inner boundary layer but gradually changes to a \(k-\epsilon\) model towards the outer boundary layer and the free stream velocity. A detailed description of the turbulence model, governing equations, numerical technique, and boundary condition can be obtained from earlier publications [24].
3. Result and discussions
Since the time-variation of measured free stream velocity shown in Figure 4(a) and Figure 7(a) slightly deviate from the exact solution, the measured $U(t)$ was used as an input in the BSL $k-\omega$ model. Furthermore, computation had been carried out for single wave cycles or single wave input, it was considering the definition of a solitary wave.

![Graph](image1)

**Figure 4.** (a) Free stream velocity; (b) vertical velocity distribution during the accelerating phases and (c) vertical velocity distribution during the decelerating phases at $Re = 2.25 \times 10^5$. 

Figure 5. Comparison horizontal velocity distribution between BSL \( k-\omega \) model results and experiment data at several depth of measurement points at \( z = 0.031 \) cm, \( z = 0.050 \) cm, \( z = 0.071 \) cm and \( z = 0.101 \) cm.

Figure 5 illustrates a comparison of measured data from a closed conduit generation system of horizontal velocity profiles at various measurement points with BSL \( k-\omega \) model simulation result when \( R_e = 2.25 \times 10^5 \). \( z \) (cm) is the cross-stream distance from the bed level and \( u \) (cm/s) is stream-wise velocity. It can be seen that the experimental data and the BSL \( k-\omega \) model result seem to be in good agreement, particularly during the acceleration phase.
Figure 6. Comparison horizontal velocity distribution between BSL \( k-\omega \) model results and experiment data at several depth of measurement points at \( z = 0.032 \) cm, \( z = 0.053 \) cm, \( z = 0.084 \) cm and \( z = 0.101 \) cm.

Comparison of measured data obtained from closed conduit generation system of horizontal velocity profiles at several measurement points and the BSL \( k-\omega \) model output results are displayed in Figure 6. It can be observed that there is a good agreement between the experimental data and the result of the BSL \( k-\omega \) model, particularly during the acceleration phase. However, the BSL \( k-\omega \) model fails to predict well, especially during the decelerating at measurement point \( z = 0.101 \) cm. Although the BSL \( k-\omega \) model is unsuccessful in generating turbulence fluctuation at certain measurement points as mentioned before at some measurement depths closed to the bottom of conduit, at \( z = 0.032 \) cm, \( z =
0.053 cm, \( z = 0.084 \) cm, the BSL \( k-\omega \) model can predict and generate well on turbulence fluctuation during the decelerating phase.

Figure 7 recapitulates the distribution of vertical velocity during the accelerating and decelerating phases of solitary motion. It can be observed that there is a good agreement between experimental data and the BSL \( k-\omega \) model.
obtained from the closed conduit generation system and the results of the BSL $k$-$\omega$ model during the accelerating phases as displayed in Figure 7(b). However, during the decelerating phases, the BSL $k$-$\omega$ model seems unable to predict accurately due to turbulence generation (Figure 7(c)).

4. Conclusions

The BSL $k$-$\omega$ turbulence model has been used to conduct numerical studies to determine velocity profiles' transitional to turbulent behavior under solitary wave motion. This turbulence model could reasonably be considered able to predict horizontal velocity distribution during the accelerating phase. Moreover, this model still gave a good prediction compared with experimental data during the decelerating phase, especially in several measurement points near the closed conduit generation system bottom. However, the BSL $k$-$\omega$ model failed to replicate turbulence spike at velocity profile when measuring depths were getting higher from the closed conduit bed. Further numerical development is required to improve the lack of the BSL $k$-$\omega$ model in predicting turbulence fluctuation, particularly in a fully developed turbulent flow regime.

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