Singularities of ultra–relativistic expanding quark–gluon plasma within Quark–gluon transport theory

A. Ya. Berdnikov, Ya. A. Berdnikov *, A. N. Ivanov, V. A. Ivanova, V. F. Kosmach, V. M. Samsonov †, N. I. Troitskaya

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Department of Nuclear Physics, State Technical University of St. Petersburg, 195251 St. Petersburg, Russian Federation

Abstract

We follow Quark–gluon transport theory and analyse singularities of the ultra–relativistic and spherical expanding quark–gluon plasma. Within the linearized QCD oscillations and instabilities of the ultra–relativistic and spherical symmetric expanding quark–gluon plasma near global thermodynamical equilibrium are investigated in dependence on a chemical potential $\mu(T)$ of non–strange light quarks and antiquarks, a strange quark mass $m_s$, a temperature $T$ and a hydrodynamical velocity $u$. We calculate the chromoelectric permeability tensor for the quark–gluon plasma at rest and ultra–relativistic moving. We show that the contribution of chemical potential of light quarks and antiquarks can be neglected to the chromoelectric permeability. We show that the account for the non–zero mass of strange quarks and antiquarks diminishes the value of the plasma frequency. We show that the plasma frequency of the ultra–relativistic and spherical symmetric expanding quark–gluon plasma is enhanced by a Lorentz factor compared with the plasma frequency of the quark–gluon plasma at rest. We find that the ultra–relativistic and spherical symmetric expanding quark–gluon plasma behaves like a collisionless thermalized plasma.

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*E–mail: berdnikov@twonet.stu.neva.ru
†E–mail: samsonov@hep486.pnpi.spb.ru, St. Petersburg Institute for Nuclear Research, Gatchina, Russian Federation
1 Introduction

One of the most meaningful and challenging problems of the modern high–energy physics is theoretical and experimental investigation of the quark–gluon plasma (QGP) [1]. There is a believe that the QGP phase of the quark–gluon system can be realized in ultra–relativistic heavy–ion collisions [1]. In this paper we study the problem of theoretical description of singularities of the QGP, such as oscillations and instabilities, produced in the intermediate state of ultra–relativistic heavy–ion collisions in the center of mass frame of colliding ions. In our approach to the description of the QGP we follow the quark–gluon transport (QGT) theory outlined in [2–4] and based on the relativistic kinetic theory nicely expounded in the book [5].

The main objects of the QGT approach are one–particle distribution functions \( f_q(x,p) \), \( \bar{f}_q(x,p) \) and \( f_g(x,p) \) defining probabilities to find a quark \( q = u, d \) or \( s \), an antiquark \( \bar{q} = \bar{u}, \bar{d} \) or \( \bar{s} \) and a gluon \( g \) at a space–time point \( x = x^\mu = (t, \vec{x}) \) with a 4–momentum \( p = p^\mu = (E, \vec{p}) \), respectively [2–4]. The distribution functions of quarks, \( f_q(x,p) \), and antiquarks, \( \bar{f}_q(x,p) \), are hermitian \( N_C \times N_C \) matrices, whereas the gluon distribution function \( f_g(x,p) \) is a hermitian \( (N_C^2 - 1) \times (N_C^2 - 1) \) matrix in colour space of a \( SU(N_C) \) colour gauge group. These distribution functions obey the transport equations [2,3,6]

\[
\begin{align*}
    p^\mu D_\mu f_q(x,p) + \frac{1}{2} g_s p^\mu \frac{\partial}{\partial p_\nu} \{G_{\mu\nu}(x), f_q(x,p)\} &= C_u[f_u, f_d, f_s, \bar{f}_d, \bar{f}_s, \bar{f}_q, f_g], \ldots, \\
    p^\mu D_\mu \bar{f}_q(x,p) - \frac{1}{2} g_s p^\mu \frac{\partial}{\partial p_\nu} \{G_{\mu\nu}(x), \bar{f}_q(x,p)\} &= \bar{C}_u[f_u, f_d, f_s, \bar{f}_d, \bar{f}_s, \bar{f}_q, f_g], \ldots, \\
    p^\mu D_\mu f_g(x,p) + \frac{1}{2} g_s p^\mu \frac{\partial}{\partial p_\nu} \{G_{\mu\nu}(x), f_g(x,p)\} &= C_g[f_u, f_d, f_s, \bar{f}_d, \bar{f}_s, \bar{f}_q, f_g],
\end{align*}
\]

(1.1)

where \( g_s \) is a quark–gluon coupling constant, the symbol \( \{ \ldots, \ldots \} \) denotes anticommutators, \( D_\mu \) and \( \bar{D}_\mu \) are the covariant derivatives in the fundamental and adjoint representations, respectively, which act as

\[
\begin{align*}
    D_\mu &= \partial_\mu - i g_s [A_\mu(x), \ldots], \\
    \bar{D}_\mu &= \bar{\partial}_\mu - i g_s [\bar{A}_\mu(x), \ldots].
\end{align*}
\]

(1.2)

The gluon field potentials \( A_\mu(x) \) and \( \bar{A}_\mu(x) \) are determined by

\[
    A_\mu(x) = A^a_\mu t^a_C, \quad \bar{A}_\mu(x) = \bar{A}^a_\mu(x) T^a_C.
\]

(1.3)

The matrices \( t^a_C \) and \( T^a_C \) \((a = 1, 2, \ldots, N_C^2 - 1)\) are the generators of the \( SU(N_C) \) gauge group in the fundamental, \( t^a_C = (\lambda^a_C)_{ij}/2 \) \# with indices \( i (j) \) running over \( 1, 2, \ldots, N_C \), and adjoint, \( (T^a_C)_{bc} = -i f^{abc} \) \# with indices \( a (b, c) \) running over \( 1, 2, \ldots, N_C^2 - 1 \), representations, where \( f^{abc} \) are the structure constants of the \( SU(N_C) \) gauge group defined by

\[
    [X^a_C, X^b_C] = i f^{abc} X^c_C,
\]

(1.4)

where \( X^a_C \) is either \( t^a_C \) or \( T^a_C \). The matrices \( t^a_C \) and \( T^a_C \) are normalized by the conditions

\[
\begin{align*}
    \text{tr}_C \{t^a_C t^b_C\} = \delta^{ab}/2 \quad \text{and} \quad \text{tr}_C \{T^a_C T^b_C\} = N_C \delta^{ab}. \quad \text{Then}, \quad G_{\mu\nu}(x) \quad \text{and} \quad \bar{G}_{\mu\nu}(x) \quad \text{are the gluon}
\end{align*}
\]

\#Here \( \lambda^a_C \) are Gell–Mann’s matrices of the \( SU(N_C) \) group.
mean–field strength tensors in the fundamental and adjoint representations

\[ G_{\mu\nu}(x) = \partial_\mu A_\mu(x) - \partial_\nu A_\mu(x) - i g [A_\mu(x), A_\nu(x)], \]
\[ G_{\mu\nu}(x) = \partial_\mu A_\mu(x) - \partial_\nu A_\mu(x) - i g_s [A_\mu(x), A_\nu(x)]. \]  

The gluon mean–field is induced by the current of quarks, antiquarks and gluons [3]

\[ D_\mu G^{\mu\nu}(x) = \partial_\mu G^{\mu\nu}(x) - i g_s [A_\mu(x), G^{\mu\nu}(x)] = j^{\nu}(x), \]  

where the current \( j^{\nu}(x) \) is determined by quark, antiquark and gluon distribution functions

\[ j^{\nu}(x) = -\frac{1}{2} g_s \int \frac{d^3p}{(2\pi)^3 E_p^\nu} p_\mu \left\{ \sum_{q=u,d,s} g_q f_q(x,p) - \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} g_{\bar{q}} \bar{f}_{\bar{q}}(x,p) \right\} \]
\[ -\frac{1}{N_C} \text{tr} C \left[ \sum_{q=u,d,s} g_q f_q(x,p) - \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} g_{\bar{q}} \bar{f}_{\bar{q}}(x,p) \right] + 2 g_s i t^a_C f^{abc} f_q(x,p)^{bc} \],

where \( g_q, g_{\bar{q}} \) and \( g_g \) are spin degeneracies of quarks, antiquarks and gluons. The transport equations (1.4) supplemented by equations (1.6) and (1.7) represent a complete set of relativistic kinetic equations for a non–equilibrium quark–gluon system in Vlasov’s approach [5].

Equations (1.1), (1.6) and (1.7) should be considered together with colour independent quantities [3] which are (i) the baryon number current \( b^\nu(x) \)

\[ b^\nu(x) = \int \frac{d^3p}{(2\pi)^3 E_p^\nu} p_\mu \text{tr} C \left[ \sum_{q=u,d,s} g_q f_q(x,p) - \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} g_{\bar{q}} \bar{f}_{\bar{q}}(x,p) \right] \]  

and (ii) the energy–momentum tensor \( t^{\mu\nu}(x) \)

\[ t^{\mu\nu}(x) = \int \frac{d^3p}{(2\pi)^3 E_p} p_\mu p_\nu \text{tr} C \left[ \sum_{q=u,d,s} g_q f_q(x,p) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} g_{\bar{q}} \bar{f}_{\bar{q}}(x,p) + g_g f_g(x,p) \right]. \]  

The r.h.s. of (1.1) is defined by the collision terms \( C_u, C_d, C_s, C_{\bar{u}}, C_{\bar{d}}, C_{\bar{s}} \) and \( C_g \). These collision terms vanish in the collisionless limit, when the gluon mean–field effects dominate for the evolution of the QGP. Such a dominance appears when the characteristic frequency of variations of a gluon mean–field becomes much greater than a frequency of parton collisions [3].

As has been emphasized by Mrówczyński in Ref.[3] the collision terms of the QGP kinetic equations (1.1) have not been jet derived, but they can be taken in the form of the collision terms of the Waldmann–Snider kinetic equations [3,7] describing a non–equilibrium system of spinning particles [5]. Recall, that the Waldmann–Snider kinetic equations play an important role in the kinetic theory of multicomponent atomic gases [5,8].

The collision terms in the relaxation time approximation can be approximated by [3]

\[ C_u[f_u, f_d, f_s, \bar{f}_u, \bar{f}_d, \bar{f}_s, f_g] = \nu_u p_\mu u^\mu(x) [f_u^{eq}(x,p) - f_u(x,p)], \]
\[ C_d[f_u, f_d, f_s, \bar{f}_u, \bar{f}_d, \bar{f}_s, f_g] = \nu_d p_\mu d^\mu(x) [f_d^{eq}(x,p) - f_d(x,p)], \]
\[ C_s[f_u, f_d, f_s, \bar{f}_u, \bar{f}_d, \bar{f}_s, f_g] = \nu_s p_\mu s^\mu(x) [f_s^{eq}(x,p) - f_s(x,p)], \]
\[ C_{\bar{u}}[f_u, f_d, f_s, \bar{f}_u, \bar{f}_d, \bar{f}_s, f_g] = \nu_{\bar{u}} p_\mu u^\mu(x) [\bar{f}_{\bar{u}}^{eq}(x,p) - \bar{f}_{\bar{u}}(x,p)], \]
\[ C_{\bar{d}}[f_u, f_d, f_s, \bar{f}_u, \bar{f}_d, \bar{f}_s, f_g] = \nu_{\bar{d}} p_\mu d^\mu(x) [\bar{f}_{\bar{d}}^{eq}(x,p) - \bar{f}_{\bar{d}}(x,p)], \]
determined by the QGP can be described in terms of the chromoelectric permeability tensor and for the QGP near global thermodynamical equilibrium oscillations and instabilities allow to analyse singular properties of the QGP such as oscillations and instabilities. The calculation of $\epsilon$ is achievable only for the plasma constituents [10].

Neglecting the contribution of statistics, i.e. unities with respect to exponentials in Eqs. (1.11) and (1.12), distribution functions (1.11) and (1.12) reduce themselves to the form introduced by Jüttner [9].

As has been pointed out in Refs. [2,3] such a transport approach to the QGP allows to analyse singular properties of the QGP such as oscillations and instabilities. The main aim of this paper is to analyse oscillations and instabilities of the QGP in dependence on a chemical potential of massless quarks, and the Bose–Einstein distribution function for gluons

$$f^{eq}_g(x,p)_{ab} = \delta_{ab} n_g(p,T) = \delta_{ab} \frac{1}{e^{u^\mu p_\mu / T} - 1}.$$  (1.12)

Neglecting the contribution of statistics, i.e. unities with respect to exponentials in Eqs. (1.11) and (1.12), distribution functions (1.11) and (1.12) reduce themselves to the form introduced by Jüttner [9].

As has been discussed in details in Refs. [2,3] such a transport approach to the QGP allows to analyse singular properties of the QGP such as oscillations and instabilities. The main aim of this paper is to analyse oscillations and instabilities of the QGP in dependence on a chemical potential of light $u$ and $d$ quarks $\mu(T)$ as a function of a temperature $T$, a mass $m_s$ of strange quark quarks and antiquarks and a hydrodynamical velocity $u^\mu$. In the electromagnetic plasma such phenomena as oscillations and instabilities can be described in terms of singularities of the electric permeability tensor $\varepsilon^{ij}(k)$, where Latin indices run over $i = 1,2,3$ and $k^\mu = (\omega, \vec{k})$ is a 4–momentum [10]. As usual the electric permeability tensor is expanded into transversal and longitudinal parts with respect to a 3–momentum $\vec{k}$ [10]

$$\varepsilon^{ij}(k) = \varepsilon_T(\omega, \vec{k}) \left( \delta^{ij} - \frac{k^i k^j}{\vec{k}^2} \right) + \varepsilon_L(\omega, \vec{k}) \frac{k^i k^j}{\vec{k}^2}. \quad (1.13)$$

The calculation of $\varepsilon_T(\omega, \vec{k})$ and $\varepsilon_L(\omega, \vec{k})$ depends on the model describing a dynamics of plasma constituents [10].

As has been pointed out in Refs. [2,3] the calculation of the chromoelectric permeability tensor within QCD is achievable only for the linearized QCD. In the linearized QCD and for the QGP near global thermodynamical equilibrium oscillations and instabilities of the QGP can be described in terms of the chromoelectric permeability tensor $\varepsilon^{ij}(k)$ determined by [3]

$$\varepsilon^{ij}(k) = \delta^{ij} - \frac{i}{\omega} \sigma^{ij}(k) - \frac{i}{\omega^2} k^\ell [\sigma^{ij\ell}(k) - \sigma^{\ell ij}(k)], \quad (1.14)$$
where $\sigma^{\alpha\beta\lambda}(k)$ is the colour conductivity tensor [3].

The paper is organized as follows. In section 2 we discuss the calculation of the colour conductivity tensor $\sigma^{\alpha\beta\lambda}(k)$. We write down the components of the colour conductivity tensor in dependence on a chemical potential of light non–strange quarks and antiquarks, a mass of strange quarks and a hydrodynamical velocity of the QGP. In section 3 we calculate the chromoelectric permeability tensor for the QGP at rest. We show that the contribution of a chemical potential of light quarks and antiquarks can be neglected, but the contribution of the non–zero mass of strange quarks and antiquarks leads to screening of strange quarks and antiquarks for the formation of the chromoelectric permeability. This entails the decrease of the plasma frequency calculated in Refs.[2,3]. The numerical estimate is carried out for $m_s = 135\text{ MeV}$ and $T = 175\text{ MeV}$. In section 4 we calculate the chromoelectric permeability tensor for the ultra–relativistic and spherical symmetric expanding QGP. We show that the chromoelectric permeability tensor retains its form calculated for the QGP at rest save the value of the plasma frequency which becomes enhanced by a twice Lorentz factor of the ultra–relativistic motion of the QGP. In the Conclusion we discuss the obtained results. We argue that our results are in agreement with those obtained by Elze and Heinz [2] and Mrówczyński [3]. Analytical expressions for the solutions of dispersion relations derived in Refs.[2,3] for singularities of the QGP defining oscillations and instabilities are fully retained. The distinctions of our results from those of Refs.[2,3] concern the magnitudes of the plasma frequency which is slightly diminished for the QGP at rest due to screening of strange quarks and antiquarks and substantially enhanced by a twice Lorentz factor for the ultra–relativistic and spherical symmetric expanding QGP.

## 2 Colour conductivity tensor of quark–gluon plasma

The colour conductivity tensor $\sigma^{\alpha\beta\lambda}(k)$, the components of which define the chromoelectric permeability $\varepsilon$ and derived in Ref.[3], can be transcribed as follows

$$\sigma^{\alpha\beta\lambda}(k) = ig_s^2 \int \frac{d^3p}{(2\pi)^3} p^\mu p^\nu \left[ \sum_{q=u,d} \frac{1}{p \cdot (k + i\nu_q u)} \frac{\partial n_q(p, T)}{\partial p_\lambda} + \frac{1}{p \cdot (k + i\nu_s u)} \frac{\partial n_s(p, T)}{\partial p_\lambda} \right. + \left. \sum_{q=\bar{u},\bar{d}} \frac{1}{p \cdot (k + i\nu_q u)} \frac{\partial n_{\bar{q}}(p, T)}{\partial p_\lambda} + \frac{1}{p \cdot (k + i\nu_s u)} \frac{\partial n_{\bar{s}}(p, T)}{\partial p_\lambda} + 2N_C \frac{\partial n_g(p, T)}{\partial p_\lambda} \right],$$

(2.1)

where we have set $g_q = g_{\bar{q}} = g_g = 2$. For the calculation of the colour conductivity tensor Eq.(2.1), we would use the probabilities of the light quarks and antiquarks defined by Eq.(1.11) and the probabilities of strange quarks and antiquarks given by

$$n_s(p, T) = n_{\bar{s}}(p, T) = \frac{1}{e^{u^\mu p_\mu / T} + 1}.$$  

(2.2)

At the rest frame when $u^\mu = (1, \vec{0})$ the probabilities of strange quarks and antiquarks read

$$n_s(\vec{p}, T) = n_{\bar{s}}(\vec{p}, T) = \frac{1}{e^{\sqrt{\vec{p}^2 + m_s^2} / T} + 1},$$

(2.3)
where $m_s = 135 \text{MeV}$ [11] is the mass of the strange quark and antiquark. The value of the current $s$–quark mass $m_s = 135 \text{MeV}$ has been successfully applied to the calculation of chiral corrections to the amplitudes of low–energy interactions, form factors and mass spectra of low–lying hadrons [12] and charmed heavy–light mesons [13]. Unlike massless antiquarks $\bar{u}$ and $\bar{d}$ for which a low–temperature suppression, $T \to 0$, is caused by a chemical potential $\mu(T)$, strange quarks and antiquarks are suppressed at $T \to 0$ by virtue of a non–zero mass $m_s$.

For the QGP, realized for ultra–relativistic heavy–ion collisions, a hydrodynamical velocity $u$ we define as

$$u^\mu = (\gamma, \gamma \vec{v}) = \left(\frac{1}{\sqrt{1 - v^2}}, \frac{\vec{v}}{\sqrt{1 - v^2}}\right), \quad (2.4)$$

where $\vec{v}$ is a 3–dimensional hydrodynamical velocity of the QGP such as $v \sim 1$. This gives the constraint $\gamma \gg 1$.

In order to derive the colour conductivity tensor given by Eq.(2.1) one has to calculate the deviations of the distribution functions of the QGP constituents from their equilibrium state as well as gradients of these functions are smaller compared with the equilibrium state. Then, traces of these functions vanish, i.e.

$$\text{tr}_C \{ f_q(x, p) \} = \text{tr}_C \{ \bar{f}_q(x, p) \} = \text{tr}_C \{ f_g(x, p) \} = 0.$$  

Substituting Eq.(2.5) into Eq.(1.7) we obtain

$$j^\mu(x) = -\frac{g_s}{2} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \times \left\{ \sum_{q=u,d,s} g_q \delta f_q(x, p) - \sum_{q=u,d,s} g_q \delta \bar{f}_q(x, p) + 2 g_g i t_G^a f^{abc} \delta f_g(x, p)^{bc} \right\}. \quad (2.6)$$

According to [3] the distribution functions $\delta f_q(x, p)$, $\delta \bar{f}_q(x, p)$ and $\delta f_g(x, p)$ can be obtained as solutions of the linearized Eq.(1.1) with collision terms given by Eq.(1.10). The corresponding solutions read [3]

$$\delta f_q(x, p) = -ig_s \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{p \cdot (k + i\nu_q u)} p^\beta G_{\beta\nu}(k) \frac{\partial n_q(p, T)}{\partial p^\nu},$$

$$\delta \bar{f}_q(x, p) = +ig_s \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{p \cdot (k + i\nu_q u)} p^\beta \bar{G}_{\beta\nu}(k) \frac{\partial \bar{n}_q(p, T)}{\partial p^\nu},$$

$$\delta f_g(x, p) = -ig_g \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{p \cdot (k + i\nu_g u)} p^\beta G_{\beta\nu}(k) \frac{\partial n_g(p, T)}{\partial p^\nu}. \quad (2.7)$$
where \( n_q(p, T) \), \( n_{\bar{q}}(p, T) \) and \( n_{\bar{q}}(p, T) \) are equilibrium distribution functions of quarks, antiquarks and gluons, respectively. Then, \( G_{\beta\nu}(k) \) and \( G_{\beta\nu}(k) \) are Fourier transforms of the gluon mean-field strength tensors in the fundamental and adjoint representations, respectively, such as

\[
G_{\alpha\beta}^{ab}(k) = -i f^{abc} G_{\beta\nu}^{ab}(k). 
\]

(2.8)

Substituting Eq. (2.7) in Eq. (2.6) we get

\[
j(x) = i g_s^2 \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} \int \frac{d^3p}{(2\pi)^3} \frac{p^\alpha p^\beta}{E_p} \left[ \sum_{q=u,d,s} \frac{g_q}{p \cdot (k + i\nu_q u)} \frac{\partial n_q(p, T)}{\partial p_\lambda} \right] G_{\beta\nu}(k) =
\]

\[
= \int \frac{d^4k}{(2\pi)^4} j(x) e^{-ik \cdot x} = \int \frac{d^4k}{(2\pi)^4} \sigma^{\alpha\beta\lambda}(k) G_{\beta\lambda}(k) e^{-ik \cdot x},
\]

(2.9)

where we have used the relation \( f^{abc} f^{dbc} = N_C \delta^{ad} \) and denoted \( j(x) = \sigma^{\alpha\beta\lambda}(k) G_{\beta\lambda}(k) \) [3]. The tensor \( \sigma^{\alpha\beta\lambda}(k) \) is the colour conductivity tensor determined by Eq. (2.1) [3].

The components of the colour conductivity tensor required for the calculation of the chromoelectric permeability tensor are determined by

\[
\sigma^{i0j}(k) = 2i g_s^2 \gamma \int \frac{d^3p}{(2\pi)^3} p^i \times \left\{ \left( \frac{p^j}{|\vec{p}|} - \nu^j \right) \left[ \frac{N_C}{(\omega + i\nu_\gamma |\vec{p}| - (\vec{k} + i\nu_\gamma \vec{v}) \cdot \vec{p}} e^{\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T} \right]
\]

\[
+ \frac{1}{\omega + i\nu_\gamma |\vec{p}| - (\vec{k} + i\nu_\gamma \vec{v}) \cdot \vec{p}} e^{\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T} \left( e^{\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T - 1} \right) \right\},
\]

\[
\sigma^{ij}(k) = 2i g_s^2 \gamma \int \frac{d^3p}{(2\pi)^3} p^i p^j \times \left\{ \left( \frac{p^j}{|\vec{p}|} - \nu^j \right) \frac{N_C}{(\omega + i\nu_\gamma |\vec{p}| - (\vec{k} + i\nu_\gamma \vec{v}) \cdot \vec{p}} e^{\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T} \right\},
\]

(7)
where we have set \( \nu_u = \nu_d = \nu_u = \nu_d = \nu_d = \nu_q \) and \( \nu_s = \nu_s \). This is justified by \( C, P \) and \( T \) invariance of strong interactions.

Now we can proceed to calculating the components of the colour conductivity tensor Eq. (2.10) for the QGP in different coordinate frames.

### 3 Quark–gluon plasma at rest

The components of the colour conductivity tensor for the QGP at rest \((\vec{v} = 0\) and \(\gamma = 1\)) and \(\mu(T) = m_s = 0\) have been calculated by Mrówczyński [3]. Our results should differ from his calculations by non-zero values of a chemical potential of light quarks \(\mu(T)\) and the \(s\)-quark mass \(m_s\). Setting \(\vec{v} = 0\) and \(\gamma = 1\) the components of the colour conductivity tensor given by Eq. (2.10) acquire the form

\[
\sigma^{0j}(k) = \frac{2ig^2}{T} \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{|\vec{p}|} \left\{ \frac{N_C}{(\omega + i\nu_q)|\vec{p}| - \vec{k} \cdot \vec{p}} \frac{e^{|\vec{p}|/T}}{(\omega + i\nu_q)|\vec{p}| - \vec{k} \cdot \vec{p}} \right. \\
\left. + \frac{1}{(\omega + i\nu_q)|\vec{p}| - \vec{k} \cdot \vec{p}} \frac{e^{|\vec{p}|/T}}{e^{|\vec{p}|/T}} \right\},
\]

\[
\sigma^{ij}(k) = \frac{2ig^2}{T} \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{|\vec{p}|} \left\{ \frac{N_C}{(\omega + i\nu_q)|\vec{p}| - \vec{k} \cdot \vec{p}} \frac{e^{|\vec{p}|/T}}{(\omega + i\nu_q)|\vec{p}| - \vec{k} \cdot \vec{p}} \right. \\
\left. + \frac{1}{(\omega + i\nu_q)|\vec{p}| - \vec{k} \cdot \vec{p}} \frac{e^{|\vec{p}|/T}}{e^{|\vec{p}|/T}} \right\}.
\]
The components $\sigma^{ij}(k)$ enter to the definition of the chromoelectric permeability determined by Eq. (1.14) in the antisymmetric combination $(\sigma^{ij}(k) - \sigma^{ji}(k))$. Since the tensor $\sigma^{ij}$ defined by Eq. (3.1) is fully symmetric, so it is obvious that $(\sigma^{ij}(k) - \sigma^{ji}(k)) = 0$. Thereby, a non-trivial contribution of the colour conductivity tensor to the chromoelectric permeability of the QGP at rest comes from the components $\sigma^{0j}(k)$ given by Eq. (5.1).

Since the tensor $\sigma^{0j}(k)$ is symmetric, $\sigma^{0j}(k) = \sigma^{j0}(k)$, the general expression of $\sigma^{0j}(k)$ can written as

$$\sigma^{0j}(k) = i A(\omega, \vec{k}) \left( \delta^{ij} - \frac{k^i k^j}{k^2} \right) + i B(\omega, \vec{k}) \frac{k^i k^j}{k^2}. \quad (3.2)$$

The coefficient functions $A(\omega, \vec{k})$ and $B(\omega, \vec{k})$ are defined by the integrals

$$A(\omega, \vec{k}) = -\frac{1}{2} B(\omega, \vec{k}) + \frac{g_s^2}{T} \int \frac{d^3p}{(2\pi)^3} \left\{ - \frac{N_C}{(\omega + i \nu_q) - \vec{k} \cdot \vec{n}} \frac{e^{|\vec{p}|/T}}{(e^{|\vec{p}|/T} - 1)^2} + \frac{1}{(\omega + i \nu_q) - \vec{k} \cdot \vec{n}} \frac{e^{(|\vec{p}| + \mu(T))/T}}{(e^{(|\vec{p}| + \mu(T))/T} + 1)^2} \right\},$$

$$B(\omega, \vec{k}) = \frac{2g_s^2}{T} \frac{1}{\vec{k}^2} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{N_C}{(\omega + \nu_q) - \vec{k} \cdot \vec{n}} \frac{(\vec{n} \cdot \vec{k})^2}{(e^{(\vec{n} \cdot \vec{k})^2/(\omega + \nu_q) - \vec{k} \cdot \vec{n}})} \frac{e^{|\vec{p}|/T}}{(e^{|\vec{p}|/T} - 1)^2} + \frac{(\vec{n} \cdot \vec{k})^2}{(\omega + \nu_q) - \vec{k} \cdot \vec{n}} \frac{e^{(\vec{n} \cdot \vec{k})^2}}{(e^{(\vec{n} \cdot \vec{k})^2} + 1)^2} \right\}, \quad (3.3)$$

where $\vec{n} = \vec{p}/|\vec{p}|$.

In terms of the coefficient functions $A(\omega, \vec{k})$ and $B(\omega, \vec{k})$ the transversal $\varepsilon_T(\omega, \vec{k})$ and longitudinal $\varepsilon_L(\omega, \vec{k})$ parts of the chromoelectric permeability tensor $\varepsilon^{ij}(\omega, \vec{k})$ are given by

$$\varepsilon_T(\omega, \vec{k}) = 1 + \frac{1}{\omega} A(\omega, \vec{k}),$$

$$\varepsilon_L(\omega, \vec{k}) = 1 + \frac{1}{\omega} B(\omega, \vec{k}). \quad (3.4)$$
A direct calculation of the integrals over $\vec{p}$ in Eq. (3.3) yields

$$A(\omega, \vec{k}) = \frac{g_s^2 N_c}{12} \frac{T^2}{|\vec{k}|} \left\{ \frac{2}{\omega + i \nu_g} + \left( 1 - \frac{(\omega + i \nu_g)^2}{k^2} \right) \ln \left( \frac{\omega + i \nu_g + |\vec{k}|}{\omega + i \nu_g - |\vec{k}|} \right) \right\}$$

$$+ \frac{1}{N_c} \left( 1 + \frac{3}{\pi^2} \frac{\mu^2(T)}{T^2} \right) \left[ \frac{2}{\omega + i \nu_g} + \left( 1 - \frac{(\omega + i \nu_g)^2}{k^2} \right) \ln \left( \frac{\omega + i \nu_g + |\vec{k}|}{\omega + i \nu_g - |\vec{k}|} \right) \right]$$

$$+ \frac{1}{N_c} \frac{6}{\pi^2} \frac{1}{T^2} \int_0^\infty \frac{dp}{e^{\sqrt{p^2 + m_s^2/T}} + 1} \left[ \left( \omega + i \nu_s \right) \sqrt{p^2 + m_s^2} + |\vec{k}| \right] + \left( \omega + i \nu_s \right) \sqrt{p^2 + m_s^2} - |\vec{k}| \right]$$

$$B(\omega, \vec{k}) = - \frac{g_s^2 N_c}{6} \frac{T^2}{|\vec{k}|} \left\{ \frac{2}{\omega + i \nu_g} - \frac{(\omega + i \nu_g)^2}{k^2} \ln \left( \frac{\omega + i \nu_g + |\vec{k}|}{\omega + i \nu_g - |\vec{k}|} \right) \right\}$$

$$+ \frac{1}{N_c} \left( 1 + \frac{3}{\pi^2} \frac{\mu^2(T)}{T^2} \right) \left[ \frac{2}{\omega + i \nu_g} - \frac{(\omega + i \nu_g)^2}{k^2} \ln \left( \frac{\omega + i \nu_g + |\vec{k}|}{\omega + i \nu_g - |\vec{k}|} \right) \right]$$

$$+ \frac{1}{N_c} \frac{3}{\pi^2} \frac{1}{T^2} \int_0^\infty \frac{dp}{e^{\sqrt{p^2 + m_s^2/T}} + 1} \frac{d}{dp} \left[ \frac{2}{\omega + i \nu_s} \frac{p^2 + m_s^2}{k^2} - \frac{(\omega + i \nu_s)^2}{k^2} \right]$$

For a numerical estimate of the contribution of a chemical potential to the chromoelectric permeability tensor we would use the chemical potential calculated in [14]. A temperature
we set equal to a freeze–out temperature $T = T_f$, a typical value of which amounts to $T_f = 175\,\text{MeV}$ for ultra–relativistic heavy–ion collisions. At $T = T_f = 175\,\text{MeV}$ one can find that $\mu(T)/T \approx 0.29$. This means that the contribution of a chemical potential of light quarks is insignificant for the formation of the chromoelectric permeability of the QGP, and without loss of generality one can set $\mu(T) = 0$.

For the estimate of the contribution of the non–zero value of the $s$–quark mass we would notice that the main contribution to the integral over $p$ comes from the region $p \leq T$. Due to this the $\omega$ and $|\vec{k}|$ dependence of the chromoelectric permeability induced by strange quarks and antiquarks relative to the contribution of gluons and light quarks and antiquarks enters with the factor
\begin{equation}
\frac{1}{N_C + 1} \frac{\alpha}{\pi^2} \left[ F(e^{-m_s/T}) + \frac{m_s}{T} \elln\left(1 + e^{-m_s/T}\right) \right],
\end{equation}
where $F(x)$ is Spence’s function [15]
\begin{equation}
F(x) = \int_0^1 \frac{dt}{t} \elln(1 + xt).
\end{equation}

At $N_C = 3$, $m_s = 135\,\text{MeV}$ and $T = 175\,\text{MeV}$ the factor Eq.(3.6) is less than 0.11. This implies the possibility to neglect the contribution of massive strange quarks and antiquarks to the chromoelectric permeability at the freeze–out temperature.

Thus, according to our estimate the coefficient functions Eq.(3.5) read now
\begin{align}
A(\omega, \vec{k}) &= \frac{g_s^2 N_C}{6} \frac{T^2}{|\vec{k}|} \left\{ \left[ \omega + i \nu_q \right] + \frac{1}{2} \left( \frac{1 - (\omega + i \nu_q)^2}{\vec{k}^2} \right) \elln \left( \frac{\omega + i \nu_q + |\vec{k}|}{\omega + i \nu_q - |\vec{k}|} \right) \right. \\
&\phantom{=} + \left[ \frac{\omega + i \nu_q}{|\vec{k}|} \right] + \frac{1}{2} \left( \frac{1 - (\omega + i \nu_q)^2}{\vec{k}^2} \right) \elln \left( \frac{\omega + i \nu_q + |\vec{k}|}{\omega + i \nu_q - |\vec{k}|} \right) \right\}; \\
B(\omega, \vec{k}) &= -\frac{g_s^2 N_C}{3} \frac{T^2}{|\vec{k}|} \left\{ \left[ \omega + i \nu_q \right] - \frac{1}{2} \left( \frac{1 - (\omega + i \nu_q)^2}{\vec{k}^2} \right) \elln \left( \frac{\omega + i \nu_q + |\vec{k}|}{\omega + i \nu_q - |\vec{k}|} \right) \right. \\
&\phantom{=} + \left[ \frac{\omega + i \nu_q}{|\vec{k}|} \right] - \frac{1}{2} \left( \frac{1 - (\omega + i \nu_q)^2}{\vec{k}^2} \right) \elln \left( \frac{\omega + i \nu_q + |\vec{k}|}{\omega + i \nu_q - |\vec{k}|} \right) \right\}.
\end{align}

In order to compare our results with those calculated in [2,3] we should set $\nu_q = \nu_q = \nu$. This gives
\begin{align}
A(\omega, \vec{k}) &= \frac{g_s^2(N_C + 1)}{6} \frac{T^2}{|\vec{k}|} \left\{ \left[ \omega + i \nu \right] + \frac{1}{2} \left( \frac{1 - (\omega + i \nu)^2}{\vec{k}^2} \right) \elln \left( \frac{\omega + i \nu + |\vec{k}|}{\omega + i \nu - |\vec{k}|} \right) \right. \\
B(\omega, \vec{k}) &= -\frac{g_s^2(N_C + 1)}{3} \frac{T^2}{|\vec{k}|} \left\{ \left[ \omega + i \nu \right] - \frac{1}{2} \left( \frac{1 - (\omega + i \nu)^2}{\vec{k}^2} \right) \elln \left( \frac{\omega + i \nu + |\vec{k}|}{\omega + i \nu - |\vec{k}|} \right) \right. \\
&\phantom{=} + \left[ \frac{\omega + i \nu}{|\vec{k}|} \right] - \frac{1}{2} \left( \frac{1 - (\omega + i \nu)^2}{\vec{k}^2} \right) \elln \left( \frac{\omega + i \nu + |\vec{k}|}{\omega + i \nu - |\vec{k}|} \right) \right\}.
\end{align}

The transversal and longitudinal components of the chromoelectric permeability tensor
we obtain in the form
\[
\varepsilon_T(\omega, \vec{k}) = 1 + \frac{g_s^2(N_C + 1)}{12\omega} \frac{T^2}{|\vec{k}|} \left[ \frac{\omega + \nu}{|\vec{k}|} + \frac{1}{2} \left( \frac{1 - (\omega + \nu)^2}{\nu^2} \right) \elln\left( \frac{\omega + \nu + |\vec{k}|}{\omega + \nu - |\vec{k}|} \right) \right],
\]
\[
\varepsilon_L(\omega, \vec{k}) = 1 - \frac{g_s^2(N_C + 1)}{6\omega} \frac{T^2}{|\vec{k}|} \left[ \frac{\omega + \nu}{|\vec{k}|} - \frac{1}{2} \left( \frac{1 - (\omega + \nu)^2}{\nu^2} \right) \elln\left( \frac{\omega + \nu + |\vec{k}|}{\omega + \nu - |\vec{k}|} \right) \right].
\] (3.10)

Following [2,3] and introducing the plasma frequency
\[
\omega_0^2 = \frac{1}{3} g_s^2 (N_C + 1) T^2
\] (3.11)
we obtain
\[
\varepsilon_T(\omega, \vec{k}) = 1 + \frac{\omega_0^2}{\omega} \frac{\nu}{2|\vec{k}|} \left[ \frac{\omega + \nu}{|\vec{k}|} + \frac{1}{2} \left( \frac{1 - (\omega + \nu)^2}{\nu^2} \right) \elln\left( \frac{\omega + \nu + |\vec{k}|}{\omega + \nu - |\vec{k}|} \right) \right],
\]
\[
\varepsilon_L(\omega, \vec{k}) = 1 - \frac{\omega_0^2}{\omega} \frac{\nu}{|\vec{k}|} \left[ \frac{\omega + \nu}{|\vec{k}|} - \frac{1}{2} \left( \frac{1 - (\omega + \nu)^2}{\nu^2} \right) \elln\left( \frac{\omega + \nu + |\vec{k}|}{\omega + \nu - |\vec{k}|} \right) \right].
\] (3.12)

These expressions agree well with those calculated in [2,3]. The plasma frequency (3.11) is diminished with respect to the expression given in [3]: \( \omega_0^2 = g_s^2 T^2 (2N_C + 3)/6 \). The former is due to the neglect of the contribution of massive strange quarks and antiquarks the validity of which has been demonstrated above.

4 Ultra–relativistic and spherical symmetric expanding QGP

According to our estimates carried out in the preceding section, for the analysis of the chromoelectric permeability of a moving QGP we would use the chromoelectric conductivity tensor with components
\[
\sigma^{ij}(k) = 2i g_s^2 \gamma T \int \frac{d^3p}{(2\pi)^3} p^i \left( \frac{p^j}{|\vec{p}|} - \nu^j \right) \frac{1}{(\omega + i\nu \gamma |\vec{p}| - (\bar{k} + i\nu \gamma \vec{v}) \cdot \vec{p})} \times \left[ N_C \left( e^\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T \right)^2 + 2 e^\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T \right] ;
\]
\[
\sigma^{ij}(k) = 2i g_s^2 \gamma T \int \frac{d^3p}{(2\pi)^3} p^i \left( \frac{p^j}{|\vec{p}|} - \nu^j \right) \frac{1}{|\vec{p}| (\omega + i\nu \gamma |\vec{p}| - (\bar{k} + i\nu \gamma \vec{v}) \cdot \vec{p})} \times \left[ N_C \left( e^\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T \right)^2 + 2 e^\gamma (|\vec{p}| - \vec{v} \cdot \vec{p})/T \right].
\] (4.1)

For ultra–relativistic heavy–ion collisions in the center of mass frame one can consider the case of an ultra–relativistic and spherical symmetric expanding QGP.
For the spherical symmetric expanding QGP a hydrodynamical velocity \( \vec{v} \) should be directed along a momentum \( \vec{\rho} \), i.e. \( \vec{v} = v \vec{n} \), where \( \vec{n} = \vec{\rho}/|\vec{\rho}| \). In this case the components of the colour conductivity tensor Eq. (4.1) read

\[
\sigma^{\iota j}(k) = \frac{2i g_s^2}{T^\gamma} \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{|\vec{p}|} \left( \frac{1}{\omega + \frac{i \nu}{\gamma}} \right) |\vec{p}| - \vec{k} \cdot \vec{p} \left[ \frac{N_C e^{1/|T^\gamma|}}{e^{1/|T^\gamma|} - 1} + 2 e^{1/|T^\gamma|} \right],
\]

\[
\sigma^{\iota j}(k) = \frac{2i g_s^2}{T^\gamma} \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{p^2} \left( \frac{1}{\omega + \frac{i \nu}{\gamma}} \right) |\vec{p}| - \vec{k} \cdot \vec{p} \left[ \frac{N_C e^{1/|T^\gamma|}}{e^{1/|T^\gamma|} - 1} + 2 e^{1/|T^\gamma|} \right],
\]

where we have denoted

\[
\gamma = \sqrt{\frac{1 + v}{1 - v}}.
\]

In the ultra–relativistic limit \( v \approx 1 \) one can set \( \gamma \approx 2 \gamma \). The tensor \( \sigma^{\iota j}(k) \) is symmetric with respect to indices \( \ell \) and \( j \), \( \sigma^{\iota j}(k) = \sigma^{ji}(k) \), and, therefore, does not contribute to the chromoelectric permeability tensor \( \varepsilon^{ij}(k) \).

The tensor \( \sigma^{\iota j}(k) \) can be represented by the integral

\[
\sigma^{\iota j}(k) = i \frac{1}{3} (N_C + 1) g_s^2 T^2 \gamma^2 \int \frac{d\Omega}{4\pi} \left( \frac{n^i n^j}{\omega + \frac{i \nu}{\gamma} - \vec{k} \cdot \vec{n}} - 1 \right).
\]

The coefficient functions \( A(\omega, \vec{k}) \) and \( B(\omega, \vec{k}) \) read

\[
A(\omega, \vec{k}) = \gamma^2 \frac{g_s^2 (N_C + 1)}{6} \frac{T^2}{|\vec{k}|} \left[ \omega + \frac{i \nu}{\gamma} \right] + \frac{1}{2} \left( 1 - \frac{(\omega + \frac{i \nu}{\gamma})^2}{\omega - |\vec{k}| + \nu/\gamma} \right) \ell_n \left( \omega + |\vec{k}| + \nu/\gamma \right),
\]

\[
B(\omega, \vec{k}) = -\gamma^2 \frac{g_s^2 (N_C + 1)}{3} \frac{T^2}{|\vec{k}|} \left[ \omega + \frac{i \nu}{\gamma} \right] - \frac{1}{2} \left( \omega + \frac{i \nu}{\gamma} \right)^2 \ell_n \left( \frac{\omega + |\vec{k}| + \nu/\gamma}{\omega - |\vec{k}| + \nu/\gamma} \right).
\]

The transversal and longitudinal components of the chromoelectric permeability tensor we obtain in the form

\[
\varepsilon_T(\omega, \vec{k}) = 1 + \gamma^2 \frac{g_s^2 (N_C + 1)}{6} \frac{T^2}{|\vec{k}|} \left[ \omega + \frac{i \nu}{\gamma} \right] + \frac{1}{2} \left( 1 - \frac{(\omega + \frac{i \nu}{\gamma})^2}{\omega - |\vec{k}| + \nu/\gamma} \right) \ell_n \left( \omega + |\vec{k}| + \nu/\gamma \right),
\]

\[
\varepsilon_L(\omega, \vec{k}) = 1 - \gamma^2 \frac{g_s^2 (N_C + 1)}{3} \frac{T^2}{|\vec{k}|} \left[ \omega + \frac{i \nu}{\gamma} \right] - \frac{1}{2} \left( \omega + \frac{i \nu}{\gamma} \right)^2 \ell_n \left( \frac{\omega + |\vec{k}| + \nu/\gamma}{\omega - |\vec{k}| + \nu/\gamma} \right).
\]
In the ultra–relativistic limit, when $\gamma \simeq 2\gamma$ and $\gamma \gg 1$, we can neglect the contribution of the term $\nu/\bar{\gamma}$ and get

$$\varepsilon_T(\omega, \vec{k}) = 1 + \bar{\gamma}^2 \frac{g_s^2(N_C + 1)}{6\omega} \frac{T^2}{k^2} \left[ 1 - \frac{1}{2} \left( \frac{\omega}{|\vec{k}|} - \frac{|\vec{k}|}{\omega} \right) \ln \left( \frac{\omega + |\vec{k}|}{\omega - |\vec{k}|} \right) \right],$$

$$\varepsilon_L(\omega, \vec{k}) = 1 - \bar{\gamma}^2 \frac{g_s^2(N_C + 1)}{3\omega} \frac{T^2}{k^2} \left[ 1 - \frac{1}{2} \frac{\omega}{|\vec{k}|} \ln \left( \frac{\omega + |\vec{k}|}{\omega - |\vec{k}|} \right) \right].$$

These expressions can be transcribed in terms of the ultra–relativistic, spherical symmetric expanding plasma frequency

$$\bar{\omega}_0^2 = \frac{1}{3} \bar{\gamma}^2 g_s^2(N_C + 1) T^2 = \bar{\gamma}^2 \omega_0^2. \quad (4.8)$$

In terms of the plasma frequency $\bar{\omega}_0^2$ the transversal and longitudinal components of the chromoelectric permeability tensor read

$$\varepsilon_T(\omega, \vec{k}) = 1 + \frac{\bar{\omega}_0^2}{2k^2} \left[ 1 - \frac{1}{2} \left( \frac{\omega}{|\vec{k}|} - \frac{|\vec{k}|}{\omega} \right) \ln \left( \frac{\omega + |\vec{k}|}{\omega - |\vec{k}|} \right) \right],$$

$$\varepsilon_L(\omega, \vec{k}) = 1 - \frac{\bar{\omega}_0^2}{k^2} \left[ 1 - \frac{1}{2} \frac{\omega}{|\vec{k}|} \ln \left( \frac{\omega + |\vec{k}|}{\omega - |\vec{k}|} \right) \right]. \quad (4.9)$$

These expressions are in analytical agreement with those obtained in Refs.[2,3] for the QGP at rest.

It is obvious that the expressions Eq.(4.9) for the transversal and longitudinal components of the chromoelectric permeability tensor should be retained even if the inverse relaxation times $\nu_q$ and $\nu_g$ are not equal, i.e. $\nu_q \neq \nu_g$. In fact, for $\bar{\gamma} \gg 1$ the contributions of the terms $\nu_q/\bar{\gamma}$ and $\nu_g/\bar{\gamma}$ are much less than unity and can be neglected as well as $\nu/\bar{\gamma}$. Thus, the ultra–relativistic and spherical symmetric expanding QGP behaves like a collisionless thermalized plasma.

### 5 Conclusion

Within Quark–gluon transport theory outlined in Refs.[2,3] we have investigated the contributions of a chemical potential of light quarks and antiquarks $\mu(T)$ and a non–zero value of a mass $m_s$ of strange quarks and antiquarks to the chromoelectric permeability of the QGP at rest and the ultra–relativistic and spherical symmetric expanding.

For the QGP at rest we have shown that the contribution of a chemical potential of light (massless) $u$ and $d$ quarks and antiquarks does not affect the formation of the chromoelectric permeability of the QGP and can be neglected. In turn, the account for a non–zero value of a strange quark mass $m_s$ has led to the chromoelectric permeability which might be produced in the QGP constituted with light $u$ and $d$ quarks, $\bar{u}$ and $\bar{d}$ antiquarks and gluons only. The numerical estimate, carried out for $m_s = 135$ MeV and $T = 175$ MeV, a typical freeze–out temperature of the QGP produced in ultra–relativistic heavy–ion collisions, has shown almost complete screening of massive strange quarks.
quarks and antiquarks for the formation of the chromoelectric permeability of the QGP. Such a decoupling of massive strange quarks and antiquarks from the quark–gluon system can be expressed in terms of a plasma frequency \( \omega_0^2 \) decreased from the value \( \omega_0^2 = g_s^2 T^2 (2N_C + N_F)/3 \) calculated in Refs. [2,3] for massless quarks and antiquarks at \( N_F = 3 \), the number of quark flavours, to the value \( \omega_0^2 = g_s^2 T^2 (2N_C + N_F - 1)/3 \) obtained in this paper. This means that massive strange quarks and antiquarks are not material for oscillations and instabilities which can be induced in the QGP [2,3] at the freeze–out temperature \( T = 175 \text{ MeV} \).

The extension of the results obtained for the QGP at rest to the case of the ultra–relativistic and spherical symmetric expanding QGP, which can be produced in ultra–relativistic heavy–ion collisions in the center of mass frame of colliding ions, has shown that the ultra–relativistic and spherical symmetric expanding QGP behaves like a collisionless thermalized plasma. We have found that in the ultra–relativistic and spherical symmetric expanding QGP the plasma frequency \( \omega_0 \) becomes enhanced by a factor \( \bar{\gamma} \simeq 2 \gamma \), i.e. \( \bar{\omega}_0 \simeq 2\gamma \omega_0 \), where \( \gamma = 1/\sqrt{1 - v^2} \) is a Lorentz factor of the ultra–relativistically moving QGP with a hydrodynamical 3–velocity \( v \sim 1 \), relative to the plasma frequency of the QGP at rest. Since for the ultra–relativistic QGP \( \gamma \gg 1 \), the plasma frequency of the ultra–relativistic and spherical symmetric expanding QGP is much greater than the plasma frequency of the QGP at rest \( \bar{\omega}_0 \gg \omega_0 \).

We would like to accentuate that the expressions for the chromoelectric permeability of the ultra–relativistic and spherical symmetric expanding QGP as well as for the QGP at rest, calculated in our paper, agree with the chromoelectric permeability tensor calculated in Refs.[2,3] for the QGP constituted with massless \( u, d, s \) quarks, antiquarks and gluons. The results are in analytical agreement up to the replacement of the plasma frequency. This is \( \omega_0^2 = g_s^2 T^2 (2N_C + N_F)/3 \rightarrow \omega_0^2 = g_s^2 T^2 (2N_C + N_F - 1)/3 \) for the plasma at rest and \( \omega_0^2 = g_s^2 T^2 (2N_C + N_F)/3 \rightarrow \bar{\omega}_0^2 = 4\gamma^2 g_s^2 T^2 (2N_C + N_F - 1)/3 \) for the ultra–relativistic plasma. This means that all dispersion relations derived in Ref.[2,3] are valid in our case. However, due to the enhancement of the plasma frequency \( \bar{\omega}_0 \gg \omega_0 \), caused by a relativistic motion of the QGP, all singularities of the QGP at rest obtained in Ref.[2,3], should be shifted to the region of very high frequencies in the case of the ultra–relativistic and spherical symmetric expanding QGP.
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