TESTING COSMOLOGICAL MODELS WITH A Lyα FOREST STATISTIC: THE HIGH END OF THE OPTICAL DEPTH DISTRIBUTION

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ABSTRACT

We pay particular attention to the high end of the Lyα optical depth distribution of a quasar spectrum. Based on the flux distribution (Miralda-Escudé et al.), a simple yet seemingly cosmological model—differentiating statistical, Δα—the cumulative probability of a quasar spectrum with Lyα optical depth greater than a high-value τα—is emphasized. It is shown that two different models—the cold dark matter model with a cosmological constant and the mixed hot and cold dark matter model, both normalized to COBE and local galaxy cluster abundance—yield quite different values of Δα: 0.13 for the former versus 0.058 for the latter, for τα = 3.0 at z = 3. Moreover, it is argued that Δα may be fairly robust to compute theoretically because it does not seem to depend sensitively on small variations of simulation parameters such as the radiation field, cooling, the feedback process, radioactive transfer, the resolution, and the simulation volume within the plausible ranges of the concerned quantities. Furthermore, it is illustrated that Δα can be obtained sufficiently accurately from currently available observed quasar spectra for τα ~ 3.0–4.0, when observational noise is properly taken into account. We anticipate that analyses of observations of quasar Lyα absorption spectra over a range of redshift may be able to constrain the redshift evolution of the amplitude of the density fluctuations on small-to-intermediate scales, therefore providing an independent constraint on Ω0, Ωa, and Δv.

Subject heading: cosmology; theory—hydrodynamics—intergalactic medium—large-scale structure of universe—quasars: absorption lines

1. INTRODUCTION

Any acceptable theory for growth of structure has to pass the tests imposed by observations in our local (z = 0) universe. Among those, the most stringent is provided by observations of clusters of galaxies (Bahcall & Cen 1992, 1993; Oukbir & Blanchard 1992; White, Efstathiou, & Frenk 1993; Viana & Liddle 1995; Bond & Myers 1996; Eke, Cole, & Frenk 1996; Pen 1996), simply because they probe the tail of a Gaussian (or alike) distribution, which depends extremely strongly on some otherwise fairly stable quantities such as the rms of density fluctuation amplitude. In addition, a model has to match COBE observations of the universe at the epoch of recombination (Smoot et al. 1992). The combination of these two relatively fixed points defines the shape and amplitude of the assumed power spectrum, thus limiting significantly the allowable parameter space for the Gaussian family of the cold dark matter cosmological models; one is essentially left undecided on how to adjust the following three parameters: n, Ω0, or Ωb, where n is the primordial power spectrum index, Ω0 is the current cosmological constant, and Ωb is the density parameter of the hot dark matter. Critical differentiators are likely to come from areas that are as far removed as possible from both the COBE epoch (on large scales) and our local vantage point (on scales of ~8 h⁻¹ Mpc).

In the intervening redshifts, among those accessible to current observations, the Lyα forest observed in spectra of high-redshift quasars (e.g., Carswell et al. 1991; Rauch et al. 1992; Petitjean et al. 1993; Schneider et al. 1993; Cristiani et al. 1995; Hu et al. 1995; Tytler et al. 1995; Lanzetta et al. 1995; Bahcall et al. 1996) provides possibly the single richest set of information about the structure of the universe at low-to-moderate redshift. Furthermore, each line of sight to a distant quasar samples indiscriminately the distribution of neutral hydrogen gas (hence total gas) over a wide redshift range (z ~ 0–5) in a random fashion (i.e., a quasar and the foreground-absorbing material are unrelated); thus, the Lyα forest constitutes perhaps the fairest sample of the cosmic structure in its observed redshift range.

The new ab initio modeling of Lyα clouds by following the gravitational growth of baryonic density fluctuations on small-to-intermediate scales (~100 kpc to several megaparsecs in comoving length units) produced a remarkably successful account of the many observed properties of Lyα clouds (Cen et al. 1994; Zhang, Anninos, & Norman 1995; Hernquist, Katz, & Weinberg 1996; Miralda-Escudé et al. 1996). However, at first sight it appears that three different cosmological models—a CDM + Λ model (Hubble constant H0 = 65 km s⁻¹ Mpc⁻¹, Ω0,CDM = 0.3645, Ω0,Λ = 0, α = 0.6; Cen et al. 1994), a biased critical density CDM model (H0 = 50 km s⁻¹ Mpc⁻¹, Ω0,CDM = 0.95, Ω0,b = 0.05, α = 0.7; Hernquist et al. 1996), and a high-amplitude critical density CDM model (H0 = 70 km s⁻¹ Mpc⁻¹, Ω0,CDM = 0.96, Ω0,b = 0.04; Zhang et al. 1995)—approximately fit the observations, gauged by column density distribution, Doppler width distribution, etc. While the overall agreement between models and observations is encouraging from an astrophysicist’s point of view because it implies that the physical environment produced by the hydrocodes of widely distributed photoionized gas does correspond to the real world, it seems that more sensitive tests are demanded from a cosmologist’s point of view in order to test models. In this Letter, we show that a statistic based on the Lyα flux distribution may serve as a potentially strong discriminator between cosmological models. We argue that, while the column density distribution is useful in providing information about the Lyα clouds, it requires
postobservation fitting procedures, such as line profile fitting and line de-blending, and consequently is superimposed by additional uncertainties related to the procedures themselves. Naively, it may seem that measures based on the flux (or optical depth) distribution may be redundant, given that we already have the traditional column density distribution, but we note that in complex multivariate problems (Kendall 1980) such as that of the Lyα forest, it should not be taken for granted that a one-to-one correspondence or correlation between the two exists. In other words, two different flux distributions may yield a similar column density distribution considering the many factors involved, including real space clustering, thermal broadening, the peculiar velocity effect, and line profile fitting. Therefore, it may be profitable to deal directly with the flux distribution.

2. STATISTIC: FRACTION OF SPECTRUM WITH $\tau_{\text{obs}} > \tau_0$

We use two of the current popular models—the cold dark matter model with a cosmological constant ($\Lambda$CDM) and the mixed hot and cold dark matter model (MDM)—to demonstrate the applicability of the statistic.

We begin by showing the variance of the density fluctuations [$\sigma = (\langle \delta^2 \rangle - 1)^{1/2}$, where $\delta = \delta \rho / \langle \rho \rangle$, calculated using linear theory] as a function of the comoving radius of a sphere in the $\Lambda$CDM model (solid curve) and the MDM model (dashed curve) at $z = 3$ in Figure 1. Both the $\Lambda$CDM model (Hubble constant $H_0 = 65$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_0_{\Lambda\text{CDM}} = 0.3645$, $\Lambda_0 = 0.6$, $\Omega_0 = 0.0355$, $\sigma_8 = 0.79$) and the MDM model ($H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_0_{\Lambda\text{CDM}} = 0.74$, $\Omega_0_{\text{HDM}} = 0.2$, $\Omega_0 = 0.06$, $\sigma_8 = 0.70$; Ma 1996) are normalized to both the COBE on large scales and the local galaxy cluster abundance (on scales of $\sim 8$ h$^{-1}$ Mpc comoving). Both models involve a slight tilt of the spectrum (plus a small gravitational wave contribution to the temperature fluctuations on large scales in the $\Lambda$CDM model) in order to achieve the indicated $\sigma_8$ values. We see a somewhat modest difference in the amplitude of density fluctuations in the two models. The density fluctuations are larger in the $\Lambda$CDM than in the MDM model by (26%, 33%) on scales of (0.1 h$^{-1}$ Mpc, 1.0 h$^{-1}$ Mpc), respectively.

Focusing on the high end of the optical depth distribution, the following statistic is examined for the purpose of testing cosmological models—the fraction of pixels in a quasar spectrum with Lyα optical depth greater than $\tau_0$.

$$\Delta_\eta = P(> \tau_0),$$

where $P(> \tau_0)$ is the cumulative distribution of Lyα optical depth. In Figure 2, we show the results of the $\Lambda$CDM model (thick, solid curve) and the MDM model (thick, dashed curve) obtained from detailed synthetic Lyα spectra (Miralda-Escudé et al. 1996). A sampling bin of 2.0 km s$^{-1}$ is used, and the spectrum is degraded by a point-spread function with a 6.0 km s$^{-1}$ FWHM. Note that noise is not added for the two thick curves in Figure 2. The box size, resolution, and physics input of the two simulations are identical (for details of the $\Lambda$CDM model, see Miralda-Escudé et al. 1996). The differences between the two model simulations are (1) different input power spectra, (2) different background models, and (3) there is one additional (hot) dark matter species in the MDM simulation. Ionization equilibrium between photoionization and recombination is assumed. We require that the average flux decrement in each model, $\langle D \rangle = 1 - \exp(-\tau_{\text{Ly}\alpha})$, be equal to the observed value, $\langle D \rangle_{\text{obs}} = 0.36$ at $z = 3$ (Press, Rybicki, & Schneider 1993). This is achieved by adjusting the following parameter: $\mu = (\Omega_0 h / 0.015 h^{-2}) / (0.7 / 10^2 s^{-1})^{1/2}$, where $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$.
$\Omega_{b}$ is the mean baryonic density, and $j_H$ is the hydrogen photoionization rate. This normalization procedure for the flux distribution is necessary in order to fix its overall amplitude, because of the large uncertainties of the observed values of $\Omega_{b}$ and $j_H$. The fitted values of $\mu$ are 1.90 and 1.47, respectively, for the $\Lambda$CDM and the MDM models (note that the value of $\mu$ for the MDM model is obtained after the temperature of the intergalactic medium in the model is raised; see below).

Although the $\Lambda$CDM model yields a temperature of the clouds consistent with observations, as indicated by the $b$-parameter distribution (Miralda-Escudé et al. 1996), it is found that the mean temperature of the intergalactic medium in the MDM model at $z = 3$ is unrealistically low, with $(T) \sim 100$ K (compared with $(T) \sim 15,000$ K in the $\Lambda$CDM). The reason is that our self-consistent treatment of the structure formation (star/galaxy formation) and the ionizing radiation field does not produce sufficient photoionization/photo-heating of the intergalactic medium because of the very small fraction of baryons that have collapsed to form stars or quasars in the MDM model by $z = 3$ (for details of our treatment of atomic processes, radiation, and galaxy formation, see Cen 1992 and Cen & Ostriker 1993). It would be meaningless to generate the Ly$\alpha$ spectrum for the MDM model using such low temperature. Instead, we uniformly raise the gas temperature in the MDM model to $2 \times 10^4$ K (but retain other properties of the gas, including density and velocity) and then generate the Ly$\alpha$ spectrum with the thermal broadening effect of the gas with the putative high temperature. To test the sensitivity of the results on the artificial temperature adjustment in the MDM model, we also compute the results by raising the temperature to $4 \times 10^4$ and $1 \times 10^4$ K, respectively. We find that at $\tau_0 = (3.0, 4.0, 5.0)$, the results are $\Delta_\tau = (0.054, 0.039, 0.029)$, $(0.058, 0.041, 0.032)$, and $(0.071, 0.051, 0.041)$ for three cases with $T = 4 \times 10^4, 2 \times 10^4$, and $1 \times 10^4$ K, respectively. One more experiment is made for the MDM model: instead of setting the temperature uniformly to $2 \times 10^4$ K, $2 \times 10^4$ K is added uniformly to the temperature of each cell, and we find that the results of $\Delta_\tau$ for the two cases are indistinguishable within the quoted digits, at all three $\tau_0$. It seems that the results depend only weakly on the temperature within the reasonable range, with the trend that higher temperatures yield lower fractions of pixels with high optical depth.

We find that $\Delta_\tau(\Lambda$CDM) = (0.12, 0.10, 0.088) and $\Delta_\tau$(MDM) = (0.058, 0.040, 0.031), at $\tau_0 = (3.0, 4.0, 5.0)$, respectively, i.e., a difference between the two models by a factor of $(2.1, 2.5, 2.8)$ at the three $\tau_0$'s. It is likely that the high end of the Ly$\alpha$ optical depth distribution in the MDM model (thick, dashed curve, Fig. 2) would have been further reduced had we run the simulation with sufficient (i.e., realistic) photoionization/photoheating, since reduced cooling and increased thermal pressure would result in less condensation of dense regions responsible for the high Ly$\alpha$ optical depth considered here. The countervailing effect is that the current MDM simulation may have overcooled the dense clouds, making them much smaller and thus much less capable of intercepting O$\alpha$ lines of sight. However, examinations of the cloud sizes and densities in the dense regions, as well as the found trend of higher temperatures yielding lower high optical depth fractions (see the preceding paragraph), indicate that this effect is unimportant for the MDM model; there do not seem to exist superdense clouds in the MDM model. In short, we anticipate that a more realistic MDM simulation, with a higher photoionization field, would produce even smaller $\tau_0$ than that of the current MDM simulation.

It is clear from Figure 2 that the higher the $\tau_0$, the more model-differentiating power $\Delta_\tau$ possesses. However, detecting higher $\tau_0$ is difficult because of noise and telescope systematics. To see how noise affects the statistic, we generated synthetic spectra with noise added in the following way. By definition, the signal-to-noise ratio at the continuum is $S/N = N_{\text{src}}/(N_{\text{src}} + N_{\text{noise}})^{1/2}$, where $N_{\text{src}}$, $N_{\text{noise}}$, and $S/N$ are the number of source photons, the number of noise photons, and the signal-to-noise ratio, respectively, per frequency pixel at the continuum. Thus, given $S/N$ and $N_{\text{noise}}$, we can obtain $N_{\text{src}}$. To simplify the illustration (without loss of generality), we assume that $N_{\text{noise}}$ is dominated by the detector readout noise. This is a good approximation for a bright quasar only, where the number of sky photons are small (because of a shorter exposure time) compared with the readout noise of, say, a CCD detector. For example, the gain of the HIRES CCD detector on the Keck telescope is 6.1 electrons, so the number of photons due to the CCD readout noise integrated over 5 spatial pixels (for each frequency bin) is $N_{\text{noise}} = 5 \times 6.1^2 = 186$. For a $V = 16.5$ mag quasar at 5000 Å with 1 h integration time, about 4 photons from the sky per spatial pixel give a total count of sky photons per frequency pixel (integrated over the 5 spatial pixels) of only 20 photons (see, e.g., Hu et al. 1995). A frequency pixel in the simulated noise-free spectrum with flux $f$ contains $fN_{\text{src}}$ photons. When noise is added, the “observed” number of photons (subtracted by the known CCD noise) in the pixel will be $Poisson(fN_{\text{src}} + N_{\text{noise}})$, where Poisson($X$) means a Poisson-distributed random number with the mean equal to $X$. The resultant distributions $P(\tau_R)$ are also shown in Figure 2 for three continuum signal-to-noise ratios of $S/N = (50, 100, 150)$ for each model (thin, solid curves for the $\Lambda$CDM model and thin, dashed curves for the MDM model). The 6 $\sigma$ statistical error bars are also shown for the case with $S/N = 100$ (other cases have comparable error bars but are not shown, in order to maintain the readability of the plot), assuming a quasar absorption spectrum coverage of unit redshift range about $z = 3$ with a sampling bin of 2 km s$^{-1}$. The $N_{\text{CDM}}$ value of the HIRES CCD detector is adopted in the calculation. We see that three values of $S/N = (50, 100, 150)$ are able to preserve the differences between the two models up to $\tau_{\text{sys}} \sim (2.5, 3.0, 3.5)$, respectively. Another complication, telescope systematics, may cause further difficulties. Nevertheless, it seems that $\Delta_\tau$ for $\tau_0 = 3.0$ can be fairly accurately obtained with a high statistical accuracy in good Keck spectra (e.g., Womble, Sargent, & Lyons 1996 achieved a typical signal-to-noise ratio of 150 per resolution element using the HIRES spectrograph on the Keck telescope).

With a preliminary comparison of the simulation results using $\Delta_\tau$ with observations (Rauch et al. 1997, Fig. 1) at $\tau_0 = 3.0$ at $z \sim 3$, it appears that the result of the $\Lambda$CDM model matches the observed value well ($\sim 0.10$ computed vs. $\sim 0.11$ observed), while the MDM model seems to predict a value (0.04) lower than observed.

3. DISCUSSION AND CONCLUSIONS

This study illustrates that high-quality quasar Ly$\alpha$ absorption spectra are potentially useful to discriminate between cosmological models. It is demonstrated that the $\Delta_\tau$ statistic—the cumulative probability of a spectrum with Ly$\alpha$ optical depth greater than a high-value $\tau_0 (\sim 3.0-5.0)$—serves as an
amplifier of the differences between models. We show that a modest difference (\(\approx 25\% - 30\%\)) in the mean amplitudes translates into a large difference in the two \(\Delta_a\)'s (by a factor larger than 2.0 for \(a_0 > 3.0\)) between the \(\Lambda CDM\) model and the MDM model at \(z = 3\). Moreover, the value of \(\Delta_a\) is at the level of 0.01–0.1, i.e., the relevant regions with \(\tau_{\alpha,0} > \tau_0\) cover a sizable portion of a quasar spectrum. Therefore, one is not dealing with small-number statistics, ensuring that \(\Delta_a\) is a potentially accurately determinable statistic statistically. The statistic is also simple in that it does not involve complicated procedures, such as line profile fitting, and hence can be applied to the observed flux distribution directly.

In addition to the need for a high signal-to-noise ratio \((S/N \approx 100\) for \(a_0 > 3.0\)), it is essential that telescope systematics be understood and scattered noise photons be minimized, which may push one to focus on the brightest quasars at this time. Furthermore, it is required that the spectroscopic resolution be sufficiently high so that the Ly\(\alpha\) optical depth can be extracted reliably from the observed flux. The latter requirement is equivalent to having a FWHM less than the Doppler width, which seems readily satisfied with current observations. Finally, we note that a stable normalization procedure for the overall flux distribution is necessary because of large uncertainties in \(\Omega_{\alpha,0}\) and \(a_0\). We adopt \(\langle D_{\alpha,0}\rangle\), as a normalization parameter in this work. The primary difficulty in fixing \(\langle D_{\alpha,0}\rangle\) is to determine the continuum, which is fairly obscured by heavy absorption at high redshift (compare, e.g., Zuo & Lu 1993 and Press et al. 1993 to see the situation); the situation is better at lower redshift.

It is equally essential to determine the robustness of the prediction of a theoretical model for the proposed statistic. This may only be made definitive by performing many simulations with varying input parameters, including the ionization radiation field, the baryonic density, the feedback processes, the simulation resolution, and the simulation volume (box size). It is expected, however, that all these effects may not change the results significantly. Let us take an example to illustrate this conjecture. A Ly\(\alpha\) cloud with a column density of \(N = 1 \times 10^{21} \text{ cm}^{-2}\) and a Doppler width \(b = 25 \text{ km s}^{-1}\) has a central Ly\(\alpha\) optical depth of \(\approx 3.0\). Since these clouds are not cooling efficiently (the cooling time is longer than the Hubble time), cooling effects are likely to be small, implying that changing the radiation field or the baryonic density would have a small dynamic effect within plausible ranges. For the same reason, these clouds are not effective in forming stars, therefore the feedback effect may be small (which, if anything, may be the result of nearby star-forming regions, which may be much rarer). These clouds are also found to be relatively large and well resolved in our current simulations, but small compared with the simulation box size (Cen & Simcoe 1997), so an increase of simulation resolution or box size would not affect the statistic substantially. Lastly, we note that, since the relevant regions are optically thin to Lyman limit photons, self-shielding effects are likely to be unimportant. So it appears that \(\Delta_a\) is a relatively easy statistic to determine theoretically.

We anticipate that the evolution of different models may be quite different, because of the dependence of the growth of density fluctuations on \(\Omega_{\alpha}, \Omega_{DMDM}\), and \(\lambda_0\) (Peebles 1980), possibly coupled with other nonlinear, redshift-dependent thermal/dynamical effects. The redshift evolution of \(\Delta_a\), computable with both observations and simulations of different models, may be potentially revealing. It is conceivable that we can constrain the amplitude of density fluctuations on small- to-intermediate scales as a function of redshift using observations of Ly\(\alpha\) clouds, and thus we can possibly constrain \(\Omega_{\alpha}, \Omega_{DMDM}\), and \(\lambda_0\).

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