Metastable shock waves in the region of their ambiguous representation

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Abstract. The behavior of perturbed shocks belonging to Hugoniot segments with ambiguous representation of the shock-wave discontinuity is considered. The segments are adjacent to the region of the shock wave instability \( L > 1 + 2M \) from the side of greater and lower pressure. Here, \( L \) is the D'yakov parameter and \( M \) is the post-shock Mach number. The model equation of state is used. For the constructed Hugoniot curve, the ambiguity sections are overlapped by the shock wave neutral stability regions. The three-dimensional simulations with varying post-shock parameters and perturbation intensity are performed. It is shown that shock waves considered are metastable: for any intensity of the shock wave, there are intervals of the perturbation threshold values inside of which the shock is neutrally stable or unstable. Neutral stability is characterized by the weak damping of secondary waves occurring as result of the perturbation.

1. Introduction
Previously [1–4], we have shown that the linear theory of the stability of a planar shock wave in a medium with an arbitrary equation of state [5–7] makes it possible to establish the presence of regions of anomalous behavior of shock waves on the shock adiabat, but as a whole it does not correctly determine their boundaries and the character of the appearing anomalies (a detailed review of the works on the considered problem is given in [3, 4]). In particular, in the region of the ambiguous representation of the shock wave, arising when one of the linear conditions of its instability is fulfilled and overlapping the corresponding region of the shock adiabat \([8, 9]\), it either decays with the formation of the composite compression wave (the condition \( L < -1 \); here and below \( L \) is the D'yakov parameter \([5, 6]\)) or shows a metastable behavior (the condition \( L > 1 + 2M \), where \( M \) is the post-shock Mach number in the shock attached frame). The metastable behavior manifests itself in the following way: for every shock wave intensity there are intervals of the perturbation threshold values inside of which the shock is stable (the perturbed shock-wave recovers its initial parameters), or unstable. In the latter case the transition to non-stationary mode characterized by presence of switching secondary waves is observed. In the switching secondary waves the post-shock parameters changes between the two parts of the Hugoniot separated from each other by the instability segment \( L > 1 + 2M \). Also it is possible that for metastable shock wave the condition of neutral stability in the framework of linear stability analysis is fulfilled. Neutral stability is characterized by the weak damping of secondary waves occurring as result of the forced perturbation. The model equation of state \([10]\) or its modification \([3]\) can be used to study the behavior of perturbed shocks belonging to the Hugoniot segments with ambiguous representation of the shock wave discontinuity adjacent to
the region of the shock wave instability \( L > 1 + 2M \) from the side of greater and lower pressure. For the constructed Hugoniot curve, the ambiguity sections are overlapped by the shock wave neutral stability regions. The rest of the paper is organized in the following way. In the next section the problem formulation is given. Then the three-dimensional simulations with varying post-shock parameters are performed.

2. Problem formulation

We use equations, which express conservation of mass, momentum and energy for the viscous and heat conducting medium:

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial (\mathbf{F}^\alpha (\mathbf{U}) + \mathbf{D}^\alpha)}{\partial x} + \frac{\partial (\mathbf{F}^\beta (\mathbf{U}) + \mathbf{D}^\beta)}{\partial y} + \frac{\partial (\mathbf{F}^\gamma (\mathbf{U}) + \mathbf{D}^\gamma)}{\partial z} = 0,
\]

where \( \mathbf{U} = (\rho, m_x, m_y, m_z, \rho E)^T \) is the vector of conservative variables; \( \mathbf{F}^\alpha \) and \( \mathbf{D}^\alpha \) are inviscid and viscous fluxes in the direction of the \( \alpha \)-axis of the Cartesian coordinates \((x, y, z)\). The former are given by

\[
\mathbf{F}^\alpha (\mathbf{U}) = (\rho v_\alpha, m_x v_\alpha + p \delta^{\alpha x}, m_y v_\alpha + p \delta^{\alpha y}, m_z v_\alpha + p \delta^{\alpha z}, \rho v_\alpha H)^T.
\]

The following notations are used: \( \rho \) is the density, \( E = e + 0.5v^2 \) denotes the total specific energy, \( H = e + 0.5v^2 + p/\rho \) is the total enthalpy, \( p \) is the pressure, \( \delta^{\alpha \beta} \) is the Kronecker symbol, \( v_\alpha \) are the components of the velocity vector, \( m_\alpha \) are the components of the momentum density vector. The viscous part of fluxes is given by

\[
\mathbf{D}^\alpha = (0, -\tau_{\alpha x}, -\tau_{\alpha y}, -\tau_{\alpha z}, q_\alpha)^T,
\]

where the tensor of the viscous stresses and the heat flux \( q \) were determined by expressions

\[
q_\alpha = -\lambda \nabla_\alpha h
\]

and

\[
\tau_{\alpha \beta} = \mu \left( \nabla_\alpha v_\beta + \nabla_\beta v_\alpha - (2/3) \delta^{\alpha \beta} p \right) \nabla_\kappa v_\kappa,
\]

where \( \nabla \) is the gradient operator. The expression for \( q \) has a model character given by the caloric form of the equation of state used in the calculation. The system is closed by the model equation of state

\[
e(V, p) = (1 - \exp(-p^2) + 10^{-3} p V) (4 - \exp(-(V - 4)^2)).
\]

The reaction of a shock waves with initial density \( \rho_0 = 0.1821 \) and initial pressure \( p_0 = 0.1 \) to a perturbation is considered. Corresponding Hugoniot has an S-shape in the \( p-u \) variables such that the shock wave stability criterion \( L < 1 + 2M \) is violated. The flow region is considered that bounds a square-shaped section of the shock wave surface. The problem is solved in a coordinate system attached to the unperturbed shock wave. The symmetry conditions are set on the lateral surfaces of the computational domain. The initial pre-shock parameters (density \( \rho_0 \), velocity \( v_0 \) and pressure \( p_0 \)) are fixed at the inflow boundary, which is supersonic in the coordinate system under consideration. Non-reflecting boundary conditions are imposed at the outflow boundary. The initial data correspond to the shock wave with the perturbation of density in the ball-shaped region located at some distance ahead of the shock wave surface. The introduction of viscous fluxes makes it possible to somewhat suppress the influence of the numerical effects related to the motion of the shock wave along the mesh, but it does not determine the choice of any wave configuration in the region of the non-uniqueness of the solution. The calculations are carried out on meshes with up to \( 10^7 \) computational cells, and the mesh step in the coordinate corresponding to the direction of the shock wave propagation decreases in the vicinity of the shock surface to better resolve its displacements during interaction with perturbations. The mesh step at the shock-wave surface was equal to \( h/L = 0.002 \). The Reynolds number \( \text{Re} = \rho_0 v_0 L/\mu \) was chosen.
Figure 1. Transition to the oscillating mode of a metastable shock wave in a model medium under the influence of a perturbation in three-dimensional formulation \((p = 2.3)\).

from the condition of the resolution of the shock-wave structure sufficient for the calculation aims (diminishing of numerical perturbations caused by the motion of the artificial shock structure along the computational grid); the Prandtl number was taken to be unity.

To integrate the system of equations (1), the finite volume method scheme is applied with one integration point per the cell face. Reconstruction of the solution at the integration points on the faces of the computational cells is carried out using TVD (total variation diminishing) limiters.

3. Interaction of metastable shock wave with perturbations

Let us consider a metastable shock wave that belongs to the lower segment of the ambiguous representation of the shock-wave discontinuity \((p_1 = 2.3)\). This segment is adjacent to the segment of the shock wave instability \(L > 1 + 2M\) from the side of lower pressure. The perturbation has the form of ball-shaped region with decreased density being equal \(0.9 \rho_0\), where \(\rho_0\) is the primary pre-shock density. Interaction of the shock wave with the perturbation is shown in figure 1, where the shock-wave surface and density distribution in the region of compressed matter are shown. The result of the interaction is the secondary wave (i.e. wave in the post-shock region of space consistent with conditions at the perturbed shock-wave surface) spreading from the site of interaction. The wave has the following structure. Consider the local post-shock pressure distribution along the shock-wave surface. The leading part of the wave is compression shock that switches the local post-shock parameters to the upper segment of ambiguous representation of the shock-wave discontinuity. This shock is followed by the rarefaction wave. In this wave the local value of the post-shock pressure gradually decreases up to the instability boundary. Then the region of rapid decrement is observed which tends to form the rarefaction jump, which turns the post-shock parameters to the lower segment of ambiguous
Figure 2. Transition to the oscillating mode of a metastable shock wave in a model medium under the influence of a perturbation in the three-dimensional formulation ($p = 3.6$).

The behavior of the metastable shock wave, which belongs to the upper segment of ambiguous representation of the shock-wave discontinuity ($p_1 = 3.6$), is shown in figure 2. The perturbation has the form of ball-shaped region with decreased density being equal $1.1\rho_0$, where $\rho_0$ is the primary pre-shock density. One can see the following picture of the wave phenomena. The leading secondary wave is a rarefaction wave, which tends to form the rarefaction shock in the interval of pressure corresponding to unstable segment of the Hugoniot. This rarefaction wave is followed by the compression shock with transition from the lower branch of the Hugoniot to the upper one. After the compression shock there is a rarefaction wave switching the local post-shock parameters back to the lower segment of ambiguous representation of the shock-wave discontinuity. Comparing solutions for perturbed shocks belonging to the lower and upper instability segments we conclude that the limiting behavior is non-stationary mode in the both cases regardless of the different composition of the secondary wave formed just after the interaction. Another type of solution is shown in figure 3, where the perturbation of the same form as in figure 2 is applied to the shock wave with the post-shock pressure $p_1 = 2.1$. One can see that the shock-wave surface recovers its initial plane form. However, the damping is weak, as indicated by the entropy waves in the post-shock region. The weak damping of the secondary waves can be explained as follows. For the constructed Hugoniot curve, the ambiguity
sections are overlapped by the shock wave neutral stability region. From theory and numerical experiments (see [4]) it follows that neutrally stable shock waves are characterized by weak damping of perturbations which means that the dependence of the perturbation amplitude on time obeys another law than the analogous dependence for stable shock waves. The second feature of neutrally stable shock waves is that the energy flux of secondary waves is directed from the shock wave surface. In practice the consequence of these features is complicated flow pattern behind the shock wave caused by interactions of secondary (transverse) waves. Presence of the neutral stability region was found in phase diagram of real gases and metals [11–13].

Let us consider the time dependence of the front-pressure maximum for the perturbed shock waves with post-shock states at several points distributed in the lower Hugoniot segment, which correspond to ambiguous representation of the shock-wave discontinuity. The parameters of the initial perturbation are the same in all calculations. The results of calculations are shown in figure 4. The dependencies show two types of the front pressure perturbations behavior. The first one is a weak damping which is characteristic of neutrally stable shock wave, the second one correspond to transition to the non-stationary mode of the shock wave propagation. In the latter case the front pressure oscillations increase in time and go to the constant amplitude. It should be noted that weak damping of the neutrally stable shock wave perturbations is the result of non-linear analysis, while linear shock wave stability theory predict emission of non-damping acoustic waves. This conclusion of the linear theory initially obtained for shock waves considered as discontinuity is recently established for shock waves with finite relaxation structure [14].
Figure 4. Pressure pulsations at the shock-wave fronts for the oscillating regime of a metastable shock wave ($p_1 = 2.3$ and $2.5$) and for shock waves under conditions of neutral stability ($p_1 = 2.0$ and $2.1$).

4. Conclusion
The calculations with a model equation of state revealed two types of behavior of a metastable shock wave. In the first case weak damping of front pressure perturbations is observed. This can be explained by the fulfillment of the neutral stability condition. In the second case transition to the non-stationary (oscillatory) mode of the shock wave propagation with finite amplitude of the front pressure oscillations is observed. This is valid both for the shock waves which correspond to the lower segment of ambiguous shock wave discontinuity representation and for the shock waves corresponding to the upper one. In the cases of the lower and the upper segments the primary switching waves (before interaction with lateral boundaries) have different structure being a sequence of wave elements: compression and rarefaction waves.

The waves observed on the shock-wave surface in the region of its ambiguous representation, are explained by “switching” of the local post-shock parameters between the allowed wave configurations. In this respect one can track the analogy with the cellular structure of the detonation waves in the limiting case, when the transverse waves initiate switching of the local post-shock parameters between the values corresponding to the “equilibrium” shock adiabat and...
the shock adiabat built under the condition of freezing of chemical reactions. Unlike detonation waves, in this case “switching” occurs between different sections of the equilibrium shock adiabat.

References
[1] Konyukhov A V, Likhachev A P, Oparin A M, Anisimov S I and Fortov V E 2004 J. Exp. Theor. Phys. 98 811–9
[2] Konyukhov A V, Likhachev A P, Fortov V E, Oparin A M and Anisimov S I 2007 J. Exp. Theor. Phys. 104 670–3
[3] Konyukhov A V, Likhachev A P, Fortov V E, Anisimov S I and Oparin A M 2009 JETP Lett. 90 25–31
[4] Konyukhov A V, Likhachev A P, Fortov V E, Khishchenko K V, Anisimov S I, Oparin A M and Lomonosov I V 2009 JETP Lett. 90 18–24
[5] Dyakov S P 1954 Zh. Eksp. Teor. Fiz. 27 288–95
[6] Kontorovich V M 1957 Sov. Phys. JETP 6 1179–80
[7] Erpenbeck J J 1962 Phys. Fluids 5 1181–7
[8] Kuznetsov N M 1985 Sov. Phys. JETP 61 275–84
[9] Kuznetsov N M 1989 Phys. Usp. 32 993–1012
[10] Ni A L, Sugak S G and Fortov V E 1986 High Temp. 24 435–40
[11] Lomonosov A V, Fortov V E, Khishchenko K V and Levashov P R 2000 AIP Conf. Proc. 505 85–8
[12] Bates J W and Montgomery D C 2000 Phys. Rev. Lett. 84 1180–3
[13] Wetta N, Pain J C and Heuze O 2018 Phys. Rev. E 98 033205
[14] Kulikovskii A G 2018 Dokl. Phys. 63 334–6