Color superconductivity with determinant interaction in strange quark matter

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We investigate the effect of six fermion determinant interaction on color superconductivity as well as on chiral symmetry breaking. Coupled mass gap equations and the superconducting gap equation are derived through the minimisation of the thermodynamic potential. The effect of nonzero quark–antiquark condensates on the superconducting gap is derived. This becomes particularly relevant for the case of 2-flavor superconducting matter with unpaired strange quarks in the diquark channel.

While the effect of six fermion interaction leads to an enhancement of u-d superconductivity, due to nonvanishing strange quark–antiquark condensates, such an enhancement will be absent at higher densities for u-s or d-s superconductivity due to early (almost) vanishing of light quark antiquark condensates.

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I. INTRODUCTION

Cold and dense quark matter can be a color superconductor. At sufficiently high baryon densities, when nucleons get converted to quark matter, the resulting quark matter is expected to be in one kind or the other of the many different possible color superconducting phases at low enough temperatures [1]. The rich phase structure is essentially due to the fact that the quark–quark interaction is not only strong and attractive in many channels, but also many degrees of freedom become possible for quarks like color, flavor and spin. Studying the properties of color superconducting phases in heavy ion collision experiments seems unlikely in the present accelerators as one cannot avoid producing large entropy per baryon in heavy ion collisions and hence cannot produce the dense and cold environment that is needed to support the formation of superconducting phases. However, in the future accelerator facility planned at GSI for compressed baryonic matter experiments, one possibly can hope for observing fluctuations signifying precursory phenomena of color superconducting phase [2].

On the other hand, it is natural to expect some color superconducting phases to exist in the core of compact stars where the densities are above nuclear matter densities and temperatures are of the order of tens of KeV. However, to consider quark matter for neutron stars, color and electrical charge neutrality conditions need to be imposed for the bulk quark matter. Such an attempt has been made in Ref. [3] as well as in Ref. [4], where the lighter up and down quarks form the two flavor color superconducting (2SC) matter while the strange quarks do not participate in pairing. A model independent analysis was done in Ref. [5] that is valid in the limit \( m_s << \mu \) and \( \Delta \sim m_s^2/\mu \), where, \( \Delta \) is the pairing gap and \( \mu \) is the quark chemical potential. It has been shown, based upon the comparison of free energies that such a two flavor color superconducting phase would be absent in the core of neutron stars [6]. Within Nambu Jona-Lasinio (NJL) model in Ref. [7] it has been argued that such conclusions are consistent except for a small window in density range where superconducting phase is possible. There have also been studies to include the possibility of mixed phases [8] of superconducting matter demanding neutral matter on the average. Later, it was observed that the imposition of neutrality conditions lead to pairing of quarks with different Fermi momenta giving rise to gapless
modes \( \mathbb{R} \). Within a Nambu Jona Lasinio model, the two flavor superconducting quark matter (2SC) was shown to exhibit gapless modes (\( g_2 \)) arising due to the difference in the Fermi momenta of the pairing quarks, when charge and color neutrality conditions are imposed. Superconducting quark matter with unpaired strange quarks (2SC+) was shown to exhibit gapless superconductivity (\( g_2 \)) within a window of about 80 MeV in baryon chemical potential \( \Omega \). Temperature effects on the gapless modes were also studied for the two flavor quark matter in Refs. \( \mathbb{S} \).

A variational approach was used to study the chiral symmetry breaking as well as color superconductivity for the quark matter \( \mathbb{R} \). The calculations were carried out for the NJL model with the minimisation of the free energy density to study which condensate will exist at what density. Charge neutrality conditions were introduced through the introduction of appropriate chemical potentials. We note here that the possibility of diquark condensates along with quark–antiquark condensates has also been considered in Refs. \( \mathbb{S} \).

In the present work, we investigate the effect of six fermion determinant interaction on color superconductivity in Nambu JonaLasinio model. Since it is not possible to have a model with same symmetries as QCD with four quark operators alone, the so called t’Hooft term is added to the usual four fermion interaction which is of the form

\[
L_{\text{det}} \sim \text{det}_f \left[ \bar{\psi}(1 + \gamma_5) \psi \right] + \text{det}_f \left[ \bar{\psi}(1 - \gamma_5) \psi \right]
\]

where, determinant is in the flavor (u,d,s) space. This term respects chiral symmetry and breaks \( U(1) \) axial symmetry as in QCD.

While studying the dense quark matter, the effect of such a term in quark antiquark channel has been widely used \( \mathbb{R} \). However, for color superconductivity such a term is only considered at the four fermion level. The effect of six fermion interaction has been considered recently in Ref. \( \mathbb{U} \). This, however considers a different effective six fermion interaction than as given in Eq. \( \mathbb{U} \). We shall treat both chiral symmetry breaking as well as color superconductivity in the same footing using the determinant interaction as in Eq. \( \mathbb{U} \). With the variational ansatz considered, we show that such a term gives a nonzero contribution to the free energy and at higher densities, it is of similar or even larger magnitude than the contributions from the quark–antiquark condensates.

We organize the paper as follows. In the following subsection, we discuss the ansatz state with the quark–antiquark as well as the diquark condensates \( \mathbb{R} \). In section \( \mathbb{U} \) we consider the Nambu Jona-Lasinio model Hamiltonian with the determinant interaction and calculate its expectation value with respect to the ansatz state to compute the energy density as well as the thermodynamic potential. We minimise the thermodynamic potential to calculate all the ansatz functions and the resulting mass as well as superconducting gap equations here. In section \( \mathbb{U} \) we discuss the results of the present investigation. Finally we summarise and conclude in section \( \mathbb{V} \).

A. The ansatz for the ground state

We shall use the notations and conventions of Ref. \( \mathbb{G} \) and recapitulate briefly the construction of the variational ansatz for the ground state. We take it as a squeezed coherent state involving quark–antiquark as well as diquark condensates as given by \( \mathbb{G} \).

\[
|\Omega\rangle = \mathcal{U}_d|\text{vac}\rangle = \mathcal{U}_d \mathcal{U}_Q|0\rangle.
\]

Here, \( \mathcal{U}_Q \) and \( \mathcal{U}_d \) are unitary operators creating quark–antiquark and diquark pairs respectively. Explicitly, the operator, \( \mathcal{U}_Q \) is given as

\[
\mathcal{U}_Q = \exp \left( \int q_{0i}(k)^\dagger (\sigma \cdot k)h_i(k)q_{0i}(k)dk - h.c. \right).
\]

In the above, \( q_{0} \) \( (\bar{q}_{0}) \) is the two component particle annihilation (antiparticle creation) operator of the four component quark field operator \( \psi \). The superscript ‘0’ refers to the fact that these operators correspond to “free” Dirac fields including, in general, a current quark mass. The quark–antiquark condensate function \( h_i(k) \) is a real function of \( |k| \) which describes vacuum realignment for chiral symmetry breaking for quarks of a given flavor \( i \). We shall take the condensate function \( h_i(k) \) to be the same for u and d quarks and \( h_3(k) \) as the chiral condensate function for the s-quark. Clearly, a nontrivial \( h_i(k) \) shall break the chiral symmetry. Summation over three colors and three flavors is understood in the exponent of \( \mathcal{U}_Q \) in Eq. \( \mathbb{G} \). Similarly, the unitary operator \( \mathcal{U}_d \) describing diquark condensates is given as

\[
\mathcal{U}_d = \exp(B_d \dagger - B_d)
\]
where, $B^I_d$ is the pair creation operator as given by

$$B^I_d = \int \left[ \bar{q}^a_r(k)\gamma^I r f(k)q^b_r(-k)\gamma_I \epsilon_{ij}^3 \epsilon_{ij}^3 + \bar{q}^a_r(k)r f_1(k)q^b_r(-k)\gamma_I \epsilon_{ij}^3 \epsilon_{ij}^3 \right] dk.$$  \hspace{1cm} (5)

In the above, $i, j$ are flavor indices, $a, b$ are the color indices and $r (= \pm 1/2)$ is the spin index. The operators $q(k)$ are related to $q^0(k)$ through the transformation $q(k) = UQq^0(k)U_Q^{-1}$. As noted earlier we shall have the quarks of colors red and green ($a=1,2$) and flavors $u,d$ ($i=1,2$) taking part in diquark condensation. The blue quarks ($a=3$) do not take part in diquark condensation. Note that we have assumed the ‘condensate functions’ $f(k)$ to be independent of flavor color indices. We give a post-facto justification for this to be that the function depends upon the average energy and average chemical potential of the quarks that condense and is the same for red up and green down or green up and red down quarks, when color isospin is unbroken.

Finally, to include the effects of temperature and density we next write down the state at finite temperature and density $|\Omega(\beta, \mu)\rangle$ taking a thermal Bogoliubov transformation over the state $|\Omega\rangle$ using thermofield dynamics (TFD) as described in Refs. [13, 19]. We then have,

$$|\Omega(\beta, \mu)\rangle = U_{\beta,\mu}|\Omega\rangle = U_{\beta,\mu}U_Q|0\rangle.$$  \hspace{1cm} (6)

where $U_{\beta,\mu}$ is given as

$$U_{\beta,\mu} = e^{B^I(\beta,\mu)-B(\beta,\mu)},$$  \hspace{1cm} (7)

with

$$B^I(\beta,\mu) = \int \left[ q^a_I(k)\theta_-(k,\beta,\mu)\bar{q}^b_I(k) + \bar{q}^a_I(k)\theta_+(k,\beta,\mu)\bar{q}^b_I(k) \right] dk.$$  \hspace{1cm} (8)

In Eq. (8) the ansatz functions $\theta_\pm(k,\beta,\mu)$ will be related to quark and antiquark distributions, and, the underlined operators are the operators in the extended Hilbert space associated with thermal doubling in TFD method. In Eq. (8) we have suppressed the color and flavor indices on the quarks as well as the functions $\theta(k,\beta,\mu)$.

All the functions in the ansatz in Eq. (8) are to be obtained by minimising the thermodynamic potential. We shall carry out this minimisation in the next section.

II. EVALUATION OF THERMODYNAMIC POTENTIAL AND GAP EQUATIONS

As discussed earlier we shall consider here 3-flavor Nambu Jona Lasinio model including the determinant interaction given by equation (1). The Hamiltonian is then given as

$$H = \psi^\dagger \left( -i\alpha \cdot \nabla + \gamma^0 \hat{m}\psi \right) - \sum_{A=0}^{8} \left[ (\bar{\psi}\gamma^A \psi)^2 - (\bar{\psi}\gamma^5 \lambda^A \psi)^2 \right]$$

$$- G_D (\bar{\psi}\gamma^5 \epsilon^{bc} \psi^C)(\bar{\psi}\gamma^5 \epsilon^{bc} \psi^C) + K \left[ det_f(\bar{\psi}(1 + \gamma_5)\psi) + det_f(\bar{\psi}(1 - \gamma_5)\psi) \right]$$  \hspace{1cm} (9)

where $\psi^{i,a}$ denotes a quark field with color ‘a’, $(a = r, g, b)$ and flavor ‘i’, $(i = u, d, s)$ indices. The matrix of current quark masses is given by $\hat{m} = \text{diag}(m_u, m_d, m_s)$ in the flavor space. As noted earlier, we shall assume isospin symmetry with $m_u = m_d$. In Eq. (9), $\lambda^A$, $A = 1, \cdots , 8$ denote the Gellman matrices acting in the flavor space and $\lambda^0 = \sqrt{\frac{2}{3}} \mathbf{1}_f$, $\mathbf{1}_f$ as the unit matrix in the flavor space. The four point interaction term $\sim G_s$ is symmetric in $SU_V(3) \times SU_A(3) \times U_V(1) \times U_A(1)$. In contrast, the determinant term $\sim K$ which for the case of three flavors generates a six point interaction which breaks $U_A(1)$ symmetry. If the diquark and the mass terms are neglected, the overall symmetry is $SU_V(3) \times SU_A(3) \times U_V(1)$. This spontaneously breaks to $SU_V(3) \times U_V(1)$ implying the conservation of the baryon number and the flavor number. The current quark mass term introduces additional explicit breaking of chiral symmetry and the axial flavor current is not completely conserved.

The third term in Eq. (9) describes a scalar diquark interaction in the color antitriplet and flavor antitriplet channel. We have not included, in the present investigation, a pseudoscalar term with the same coupling which leads to Goldstone mode condensation. Such a form of Lagrangian can arise e.g. by Fiertz transformation of a four point
Current current interaction having quantum numbers of single-gluon exchange \[^{20}\]. In that case the diquark coupling \(G_D\) is related to the scalar coupling as \(G_D = 0.75G_s\).

Using the variational ansatz state in Eq. (6) one can calculate the expectation values of various operators \[^{3}\]. We evaluate the expectation values

\[
\langle \Omega(\beta, \mu) | \tilde{\psi}_\alpha(k) \tilde{\psi}_\beta(k') | \Omega(\beta, \mu) \rangle = \delta^{ij} \delta^{ab} A_{\pm \alpha \beta}^{ia}(k, \beta, \mu) \delta(k - k'),
\]

(10)

and,

\[
\langle \Omega(\beta, \mu) | \tilde{\psi}_\alpha(k) | \tilde{\psi}_\beta(k') | \Omega(\beta, \mu) \rangle = \delta^{ij} \delta^{ab} A_{\pm \alpha \beta}^{ia,jb}(k, \beta, \mu) \delta(k - k'),
\]

(11)

where,

\[
A_{\pm}^{ia}(k, \beta, \mu) = \frac{1}{2} \left[ 1 \pm \left( F_1^{ia}(k) - F_1^{ia}(k) \right) \right] \left( \gamma^0 \cos \phi(k) + \alpha \cdot \hat{k} \sin \phi(k) \right) \left( 1 - F_1^{ia}(k) - F_1^{ia}(k) \right).
\]

(12)

In the above, \(\tilde{\psi}(k)\) is the Fourier transform of \(\psi(x)\) \[^{4}\]. The effect of the diquark condensates and their temperature and/or density dependences are encoded in the functions \(F_1^{ia}(k)\) and \(F_1^{ia}(k)\) given as

\[
F_1^{ia}(k) = \sin^2 \theta^{ia}(k) + \sin^2 f_{\theta}^{ia,jb}(k) \left(1 - \delta^{a3}\right),
\]

(13)

and,

\[
F_1^{ia}(k) = \sin^2 \theta_{\tau}^{ia}(k) + \sin^2 f_{\tau}^{ia,jb}(k) \left(1 - \delta^{a3}\right).
\]

(14)

We have defined \(\cos 2\theta^{ia,jb} = 1 - \sin^2 \theta^{ia} - \sin^2 \theta^{jb} \) with \(i \neq j\) and \(a \neq b\). The \(\delta^{a3}\) term indicates that the third color does not take part in diquark condensation. Further, we have introduced the notation for the quark-antiquark condensate functions as a 'shift' from their vacuum values as \(\phi_\tau(k) = \phi_\tau^0(k) - 2h_\tau(k)\), with \(\cot \phi_\tau^0(k) = m_\tau/\sqrt{(k^2 + m_\tau^2)}\).

We also have for diquark operators,

\[
\langle \Omega(\beta, \mu) | \tilde{\psi}_\alpha(\mathbf{x}) \tilde{\psi}_\beta(0) | \Omega(\beta, \mu) \rangle = - \frac{1}{(2\pi)^3} \int e^{i\mathbf{k} \cdot \mathbf{x}} \mathcal{P}_{\pm \gamma \alpha}^{ia,jb}(k, \beta, \mu) d\mathbf{k},
\]

(15)

where,

\[
\mathcal{P}_{\pm \gamma \alpha}^{ia,jb}(k, \beta, \mu) = \frac{\epsilon^{ij} \epsilon^{ab}}{4} \left[ S_{\pm \gamma \alpha}^{ia,jb}(k) \cos \left( \frac{\phi_\tau - \phi_j}{2} \right) + \left( \gamma^0 \cos \left( \frac{\phi_\tau + \phi_j}{2} \right) - \alpha \cdot \hat{k} \sin \left( \frac{\phi_\tau + \phi_j}{2} \right) \right) A_{\pm \gamma \alpha}^{ia,jb}(k) \right] \gamma_5 C,
\]

(16)

and,

\[
\mathcal{P}_{\pm \gamma \alpha}^{ia,jb}(k, \beta, \mu) = \frac{\epsilon^{ij} \epsilon^{ab} C \gamma_5}{4} \left[ S_{\pm \gamma \alpha}^{ia,jb}(k) \cos \left( \frac{\phi_\tau - \phi_j}{2} \right) + \left( \gamma^0 \cos \left( \frac{\phi_\tau + \phi_j}{2} \right) - \alpha \cdot \hat{k} \sin \left( \frac{\phi_\tau + \phi_j}{2} \right) \right) A_{\pm \gamma \alpha}^{ia,jb}(k) \right].
\]

(17)

Here, \(C = i\gamma^2\gamma^0\) is the charge conjugation matrix (we use the notation of Bjorken and Drell) and the functions \(S(k)\) and \(A(k)\) are given as,

\[
S^{ij,ab}(k) = \sin 2f(k) \cos 2\theta^{ia,jb}(k, \beta, \mu) + \sin 2f_1(k) \cos 2\theta^{ia,jb}(k, \beta, \mu),
\]

(18)

and,

\[
A^{ij,ab}(k) = \sin 2f(k) \cos 2\theta^{ia,jb}(k, \beta, \mu) - \sin 2f_1(k) \cos 2\theta^{ia,jb}(k, \beta, \mu),
\]

(19)
These expressions are used to calculate thermal expectation value of the Hamiltonian and compute the thermodynamic potential.

Let us first concentrate on the contribution of the determinant term to the energy expectation value. When expanded the determinant term will have six terms, each involving three pairs of quarks of different flavors. These are to be ‘contracted’ among themselves in all possible manner, while taking expectation values. Further, while considering quark–antiquark condensates, it is clear that the contractions of the same color will be dominant over that with different colors by a factor $N_c$, where $N_c$ is the number of colors. With the present case of two flavor superconductivity, this leads to the fact that out of the six terms in the expansion of the determinant, only two terms that are proportional to the strange quark–antiquark condensate $(\bar{s}s)$ will be dominant over the rest. These latter four terms are suppressed at least by a factor $N_c$. The dominant two terms are the ones involving contraction of strange quark–antiquarks having the same color. This simplification arises because we are considering only u-d superconductivity here. For the rest of our calculations, we take contributions of these two terms only. Explicitly these two terms are given as

$$V_{\text{det}} = +K\langle \text{det}_f[\bar{\psi}(1 + \gamma_5)\psi]\rangle + \text{det}_f[\bar{\psi}(1 - \gamma_5)\psi]\rangle$$

$$= -8KI_s^{(3)}\left(I_D^2 + 2I_s^{(1)}I_s^{(2)}\right).$$

Here,

$$I_s^{(i)} = -\frac{1}{2}\langle \bar{\psi}^i\psi^i\rangle = \sum_{a=1}^{3} \int \frac{dk}{(2\pi)^3} \cos \phi^i (1 - F^{ia} - F_1^{ia})$$

is proportional to the quark–antiquark condensate for $i$-th flavor, and,

$$I_D = \frac{1}{4}\langle \bar{\psi}e^{ia}\gamma^5\epsilon^{ij}\epsilon^{j}_{ab}\psi\rangle$$

$$= \frac{1}{(2\pi)^3} \int d\mathbf{k} \cos \left(\frac{\phi^i - \phi^j}{2}\right) \left[\sin 2f_1(\mathbf{k})(1 - \sin^2 \theta_- - \sin^2 \theta^2) + \sin 2f_1(\mathbf{k})(1 - \sin^2 \theta^1 - \sin^2 \theta^2_+ - \sin^2 \theta^2_+)\right]$$

is the diquark condensate. It is then straightforward to calculate the expectation value of the Hamiltonian. This can be written as

$$\epsilon = \langle H \rangle = T + V_S + V_D + V_{\text{det}},$$

the various terms arising from the kinetic, the scalar, the diquark and the determinant interaction terms of the Hamiltonian respectively. Explicitly, the kinetic energy part in Eq. (20) is given as

$$T = \langle \Omega(\beta, \mu)|\bar{\psi}(-i\alpha \cdot \nabla + \gamma^0 m_i)\psi|\Omega(\beta, \mu)\rangle$$

$$= -\frac{2}{(2\pi)^3} \sum_{i=1}^{3} \sum_{a=1}^{3} \int d\mathbf{k} (|\mathbf{k}| \sin \phi^i + m_i \cos \phi_i) (1 - F^{ia} - F_1^{ia}),$$

where, $F^{ia}$ and $F_1^{ia}$ are given by Eq.s [15] and Eq. [14].

The contribution from the scalar interaction term in Eq. (19) turns out to be

$$V_S = -G_s\langle \Omega(\beta, \mu)|\sum_{A=0}^{8} \left[\bar{\psi}\lambda^A\psi\right]^2 - (\bar{\psi}\gamma^5\lambda^A\psi)^2\rangle |\Omega(\beta, \mu)\rangle = -8G_s \sum_{i=1,3} I_s^2,$$

where, $I_s^i$ is given in Eq. (21).
Similarly, the contribution from the diquark interaction from Eq. (23) to the energy density is given as

\[ V_D = -G_D\langle \Omega(\beta, \mu) \rangle (\bar{\psi} \gamma^5 \gamma^\mu \gamma^\nu \psi) \langle \Omega(\beta, \mu) \rangle = -16G_D I_D \]  

with \( I_D \) defined in Eq. (22), and the contribution from the determinant term is as given in Eq. (20).

To calculate the thermodynamic potential we shall have to specify the chemical potentials relevant for the system. Here we shall be interested in the form of quark matter that might be present in compact stars older than few minutes so that chemical equilibration under weak interaction is there. The relevant chemical potentials in this case are the baryon chemical potential \( \mu_B \) and the chemical potential \( \mu_E \) associated with electromagnetic charge, and the two color electrostatic chemical potentials \( \mu_3 \) and \( \mu_8 \). The chemical potential is a matrix that is diagonal in color and flavor space, and is given by

\[ \mu_{ij,ab} = (\mu \delta_{ij} + Q_{ij} \mu_E) \delta_{ab} + (Q_{3ab} \mu_3 + Q_{8ab} \mu_8) \delta_{ij}. \]  

(27)

Demanding the color superconducting ground state to be invariant under SU(2) \(_c\) gauge group, we can choose the chemical potential \( \mu_3 \) to be zero.

The total thermodynamic potential, including the contribution from the electrons, is then given by

\[ \Omega = T + V_S + V_D - \langle \mu N \rangle - \frac{1}{\beta} S + \Omega_e \]  

(28)

where, we have denoted

\[ \langle \mu N \rangle = \langle \psi^{ia} \bar{\psi}^{iab} \psi^{jb} \rangle = 2 \sum_{i,a} \mu^{ia} I_v^{ia}. \]  

(29)

In the above, \( \mu^{ia} \) is the chemical potential for the quark of flavor \( i \) and color \( a \), which can be expressed in terms of the chemical potentials \( \mu \), \( \mu_E \) and \( \mu_8 \) using Eq. (27). Further

\[ I_v^{ia} = \frac{1}{(2\pi)^3} \int dK (F^{\alpha a} - F_0^{\alpha a}) \]  

(30)

is proportional to the number density of quarks of a given color and flavor. The thermodynamic potential for electrons is given as

\[ \Omega_e = -\frac{\mu_E^2}{12\pi^2} \left( 1 + 2\pi^2 \frac{T^2}{\mu_E^2} \right). \]  

(31)

Here, the electron mass is assumed to be zero which suffices for the system we are considering.

Finally, for the entropy density for the quarks we have

\[ s = -\frac{2}{(2\pi)^3} \sum_{i,a} \int dK \left( \sin^2 \theta_{+}^a \ln \sin^2 \theta_{+}^a + \cos^2 \theta_{+}^a \ln \cos^2 \theta_{+}^a \\
+ \sin^2 \theta_{-}^a \ln \sin^2 \theta_{-}^a + \cos^2 \theta_{-}^a \ln \cos^2 \theta_{-}^a \right). \]  

(32)

Now the functional minimisation of the thermodynamic potential \( \Omega \) with respect to the chiral condensate function \( h_i(k) \) leads to

\[ \cot \phi_i(k) = \frac{m_i + 8G_s I_s^{(ij)} + 8K|\epsilon_{ijk}|S^{(jk)} I_s^{(kl)} + 4K I_D^2 \delta_{ij}}{|k|} \equiv \frac{M_i}{|k|}. \]  

(33)

The last term in the numerator indicates the explicit dependence of the strange quark–antiquark condensate function on the light diquark condensate \( I_D \).

Substituting this back in Eq. (21) yields the mass gap equation as

\[ M_i = m_i + \frac{8G_s}{(2\pi)^3} \int \frac{M_i}{|k|} \sum_{a=1,3} (1 - F^{\alpha a} - F_0^{\alpha a})dK + 8K|\epsilon_{ijk}|I_s^{(j)} I_s^{(k)} + 4K I_D^2 \delta_{ij}, \]  

(34)
with, \( \epsilon_i = \sqrt{k_i^2 + M_i^2} \) being the energy of the constituent quarks of \( i \)-th flavor. Note that for the two flavor superconductivity as considered here, the strange quark mass is affected explicitly by the superconducting gap given by the last term on the right hand side Eq. (22). Of course, there is implicit dependence on the gap in the second term through the functions \( F \) and \( F_1 \) (given in Eqs. 13 and 14). Further, when chiral symmetry is restored for the light quarks i.e., when the scalar condensate for the nonstrange quarks vanishes, still, the determinant term gives rise to a density dependent dynamical strange quark mass. Such a mass generation is very different from the typical mechanism of quark mass generation through quark–antiquark condensates [17].

Next, the minimisation of the thermodynamic potential \( \Omega \) with respect to the diquark and di-antiquark condensate functions \( f(k) \) and \( f_1(k) \) yields

\[
\tan 2f(k) = \frac{4(2G_D + K I_s^{(3)})}{\bar{\epsilon} - \bar{\mu}} = \frac{\Delta}{\bar{\epsilon} - \bar{\mu}} \cos \left( \frac{\phi_1 - \phi_2}{2} \right) \quad (35)
\]

and

\[
\tan 2f_1(k) = \frac{4(2G_D + K I_D^{(3)})}{\bar{\epsilon} + \bar{\mu}} = \frac{\Delta}{\bar{\epsilon} + \bar{\mu}} \cos \left( \frac{\phi_1 - \phi_2}{2} \right) \quad (36)
\]

where, we have defined the superconducting gap \( \Delta = 4(2G_D - K I_s^{(3)})I_D \), with \( I_D \) as defined in Eq. (22). Further, in the above \( \bar{\epsilon} = (\epsilon_1 + \epsilon_2)/2 \), \( \bar{\mu} = (\mu_u + \mu_d)/2 = (\mu_u - \mu_d)/2 \) is \( \mu = \mu_E/6 + \mu_s/\sqrt{3} \). It is thus seen that the diquark condensate functions depend upon the masses of the two quarks which condense through the function \( \cos [(\phi_1 - \phi_2)/2] \). The function \( \cos \phi_i = M_i/\epsilon_i \), can be different for \( u,d \) quarks, when the charge neutrality condition is imposed. Such a normalisation factor is always there when the condensing fermions have different masses as has been noted in Ref. [16] in the context of CFL phase.

Substituting the solutions for the condensate functions given in Eqs. (33), (35) and (36) in the expression for \( I_D \) in Eq. (22), we have the superconducting gap equation given by

\[
\Delta = \frac{4(2G_D + K I_s^{(3)})}{(2\pi)^3} \int \frac{d\vec{k} \Delta \cos^2 \left( \frac{\phi_1 - \phi_2}{2} \right)}{\omega_- \cos 2\theta_- + \omega_+ \cos 2\theta_+}. \quad (37)
\]

In the above, \( \omega_\pm = \sqrt{\Delta^2 \cos^2 \left( (\phi_1 - \phi_2)/2 \right) + \bar{\epsilon}_\pm^2}, \bar{\epsilon}_\pm = \bar{\epsilon} \pm \bar{\mu} \), and, \( \cos 2\theta_\pm = 1 - \sin^2 \theta_\pm^u - \sin^2 \theta_\pm^d \).

Finally, the minimisation of the thermodynamic potential with respect to the thermal functions \( \theta_\pm(k) \) gives

\[
\sin^2 \theta_\pm^a = \frac{1}{\exp(\beta \omega_\pm^a) + 1}. \quad (38)
\]

Various \( \omega_\pm^a \)’s \( (i, a \equiv \text{flavor, color}) \) are explicitly given as

\[
\omega_\pm^{11} = \omega_\pm^{12} = \bar{\omega}_\pm, \quad \delta_\epsilon \pm \delta_\mu \equiv \omega_\pm^u \quad (39)
\]

\[
\omega_\pm^{21} = \omega_\pm^{22} = \bar{\omega}_\pm - \delta_\epsilon \pm \delta_\mu \equiv \omega_\pm^d \quad (40)
\]

for the quarks participating in condensation, and,

\[
\omega_\pm^a = \epsilon_i \pm \mu_i^a \quad (41)
\]

for the blue quarks which do not participate in superconductivity. Here \( \delta_\epsilon (= (\epsilon_1 - \epsilon_2)/2) \) is half the energy difference of the two quarks which condense, and, \( \delta_\mu = (\mu_{11} - \mu_{22})/2 = \mu_E/2 \), is half the difference of the chemical potentials of the two condensing quarks. Further \( \bar{\omega}_\pm \) have been already defined after Eq. (37). Note that when the charge neutrality conditions are not imposed, all the four quasiparticles taking part in diquark condensation will have the same energy \( \bar{\omega}_- \). It is clear from the dispersion relations given in Eq. (11) that it is possible to have zero modes, i.e., \( \omega_\pm^a = 0 \) depending upon the values of \( \delta_\epsilon \) and \( \delta_\mu \). So, although we shall have nonzero order parameter \( \Delta \), there will be fermionic zero modes or the gapless superconducting phase [21, 22].
Now using these dispersion relations, the mass gap equation Eq. (34) and the superconducting gap equation Eq. (37), the thermodynamic potential (Eq. (28)) becomes

$$\Omega = \Omega_g + \Omega_e. \quad (42)$$

In the above, $$\Omega_g$$ is the contribution from the quarks and is given as

$$\Omega_g = \frac{8}{(2\pi)^3} \int dk \left[ \sqrt{k^2 + m^2} - \frac{1}{2}(\bar{\omega}_- + \omega_+) \right]$$

$$- \frac{4}{(2\pi)^3} \sum_{m=u,d} \int dk \left[ \log(1 + \exp(-\beta\omega_m^-)) + \log(1 + \exp(-\beta\omega_m^+)) \right]$$

$$+ \frac{4}{(2\pi)^3} \int dk \left[ \sqrt{k^2 + m^2} \right]$$

$$- \frac{2}{\beta(2\pi)^3} \sum_{i=1,2} \int dk \left[ \log(1 + \exp(-\beta(\epsilon_i - \mu_3^3))) + \log(1 + \exp(-\beta(\epsilon_i + \mu_3^3))) \right]$$

$$+ \frac{6}{(2\pi)^3} \int dk \left[ \sqrt{k^2 + m^2} - \sqrt{k^2 + M_s^2} \right]$$

$$- \frac{2}{\beta(2\pi)^3} \sum_{a=1,3} \int dk \left[ \log(1 + \exp(-\beta(\epsilon_3 - \mu_3^a))) + \log(1 + \exp(-\beta(\epsilon_3 + \mu_3^a))) \right]$$

$$+ 16G_D I_D^3 + 8G_s \sum_{i=1,3} I_s^{(i)} + 32K I_s^{(2)} I_s^{(3)} + 16KI_s^{(3)} I_D^3, \quad (43)$$

where, $$\omega_{u,d}$$ are given in equations (30) and (31). The contribution of the electron $$\Omega_e$$ to the total thermodynamic potential is already given in Eq. (34). The first two lines in Eq. (43) correspond to the contributions from the quarks taking part in the condensation while the third and fourth lines correspond to the contribution from the two light quarks with the blue color. The next two lines are the contributions essentially from the strange quarks. The last line corresponds to the terms which mix up the flavors and also the diquark and quark–antiquark condensates. In fact, the last two terms in the expression of thermodynamic potential are the contributions arising due to the determinant interaction.

Thus the thermodynamic potential is a function of four parameters: the three mass gaps and a superconducting gap. These are calculated through minimisation of the thermodynamic potential, subjected to the conditions of electric and color charge neutrality. The electric and color charge neutrality constraints are given as

$$Q_E = \frac{2}{3} \rho^1 - \frac{1}{3} \rho^2 - \frac{1}{3} \rho^3 - \rho_e = 0, \quad (44)$$

and,

$$Q_S = \frac{1}{\sqrt{3}} \sum_i (\rho^{i1} + \rho^{i2} - 2\rho^{i3}) = 0, \quad (45)$$

respectively. In the above $$\rho^{ia} = \langle \psi^{ia\dagger} \psi^{ia} \rangle = 2I_{v}^{ia} (i, a \text{ not summed})$$ and $$I_{v}^{ia}$$ is as given in Eq. (32). Further, $$\rho^i = \sum_{a=1,3} \rho^{ia}$$. The thermodynamic potentials (Eqs. (28) and (33)), the charge neutrality conditions (Eqs. (44) and (45)), the mass gap equation (Eq. (34)) and the superconducting gap equation (Eq. (37)) constitute the basis of the numerical calculations that we shall discuss in the next section.

III. RESULTS AND DISCUSSIONS

For numerical calculations we have taken the values of the parameters of the NJL model as follows. The coupling constants $$G_s, G_D$$ have the dimensions of [Mass]$^{-2}$ while the six fermion coupling has a dimension [Mass]$^{-5}$. To regularise the divergent integrals we use a sharp cut-off, $$\Lambda$$ in 3-momentum space. Thus we have all together six parameters, namely the current quark masses for the nonstrange and strange quarks, $$m_q$$ and $$m_s$$, the three couplings
FIG. 1: Gap parameters when charge neutrality conditions are not imposed. Fig.1-a shows the gaps at zero temperature as a function of quark chemical potential. Fig. 1-b shows the same when the determinant interaction is not taken into account. Both the figures correspond to nonzero value for the current quark masses $m_u=5.5 \text{ MeV}=m_d$. Solid curve refers to masses of $u$ and $d$ quarks, the dotted curve refers to mass of strange quarks and the dashed curve corresponds to the superconducting gap.

$G_s, G_D, K$ and the three-momentum cutoff $\Lambda$. For simplicity, we shall also take $G_D$ to be $0.75G_s$, as may be expected from Fierzing a current current interaction. For the rest of the parameters we choose $\Lambda = 0.6023 \text{ GeV}$, $G_s\Lambda^2 = 1.835$, $K\Lambda^5 = 12.36$, $m_q = 5.5 \text{ MeV}$ and $m_s = 0.1407 \text{ GeV}$ as has been used in Ref. [23]. After choosing $m_q = 5.5 \text{ MeV}$, the remaining four parameters are fixed by fitting to the pion decay constant and the masses of pion, kaon and $\eta'$. With this set of parameters the mass of $\eta'$ is underestimated by about six percent. With this parametrization, the constituent masses of the light quarks turn out to be $M_1 = 0.368 \text{ GeV}$ for u-d quarks, while the same for strange quark comes out as $M_s = 0.549 \text{ GeV}$, at zero temperature and zero density.

It is, however, relevant here to comment regarding the choice of the parameters. There have been different sets of parameters by other groups also [25, 26, 27] for the three flavor NJL model. Although the same principle as above is used e.g. in Ref. [25], the resulting parameter sets are not identical -- in particular, the dimensionless coupling $K\Lambda^5$ differs by as large as 30 percent as compared to the value used here. This discrepancy lies on different treatment of the $\eta'$ meson. Since NJL model does not confine, and because of the large mass of the $\eta'$ meson ($m_{\eta'} = 958 \text{ MeV}$), it lies above the threshold for $q\bar{q}$ decay with an unphysical imaginary part for the corresponding polarization diagram. This is an unavoidable feature of NJL model and leaves uncertainty which is reflected in the difference in the parameter sets by different groups. Within this limitation regarding the parameters of the model, however, we proceed with the above parameter set which has already been used in the study of the phase diagram of dense matter in Ref. [17] as well as in the context of equation of state for neutron star matter in Ref. [24].

Let us begin with the discussions of results when the charge neutrality conditions are not imposed. At zero temperature, the behaviour of the gap parameters as functions of quark chemical potential are displayed in Fig.1-a. We may point out that these solutions for the gaps correspond to the solutions for which the thermodynamic potential is minimised. In fact, in general, for certain values of the chemical potential, there can be several solutions of the gap equations particularly near the critical chemical potential. We have chosen the ones which minimise the thermodynamic potential.

At low chemical potentials $\mu_q < \mu_1 \sim 350 \text{ MeV}$, the diquark gap vanishes and the masses of the quarks stay at their vacuum values. The entire region below $\mu_q = \mu_1$ corresponds to vacuum solution and has zero baryon number. At $\mu_q = \mu_1$ a first order phase transition takes place and the system is a two flavor color superconductor. The diquark gap jumps from zero to about 95 MeV at this point. At the same point, the masses of $u$ and $d$ quarks drop from their vacuum values of 370 MeV to 50 MeV. The baryon number density also jumps from zero to 0.42 $\text{fm}^{-3}$ at this
chemical potential. Because of the flavor mixing six Fermi interaction, this cross over transition for the light quarks is reflected also as dropping of the strange quark mass at this chemical potential from its vacuum value of 549 MeV to about 470 MeV. Although this transition is not a first order transition due to the nonzero current quark masses, there exist metastable phases. The masses of the light quarks in these metastable phases are the nontrivial solutions of the mass gap equations, but, have higher thermodynamic potential as compared to the solutions corresponding to stable phases which are shown in Fig.1-a. With the increase in $\mu_q$, the superconducting gap increases until it reaches a maximum at $\mu_q \sim 475$ MeV. Beyond this point, the quark chemical potential is increased, the mass of strange quark drops again. Due to the determinant interaction this leads to a drop in superconducting gap as may be obvious from Eq. (37). For comparison we have plotted in Fig. 1-b the masses and the superconducting gap for the case where the determinant coupling is zero, while all other couplings remain unchanged. In this case, it may be noted that the (nearly) vanishing of light quark masses does not have any effect on the strange quark mass. The strange quark mass starts to drop only when quark chemical potential is larger than the vacuum value of the mass of the strange quark as only then the finite density contributions become nonzero in the mass gap equation for the strange quark. In contrast, with nonzero coupling $K$, because of the flavor mixing, the dropping of strange quark mass starts for quark chemical potentials smaller than the vacuum mass of the strange quark as shown in Fig. 1-a. Since the charge neutrality condition depends very sensitively on the strange quark mass, determinant coupling therefore can have important consequences in deciding the phase structure.

We would also like to note that, to evaluate the thermodynamic potential, we have to solve self consistently the three mass gap equations (Eq. (34)) and the superconducting gap equation (Eq. (37)). These equations are all coupled. However, some simplification occurs for higher densities if we take the current quark masses to be zero. This leads to solving only two coupled gap equations - the strange quark mass gap equation and the superconducting gap equation for densities when chiral symmetry is restored for the light u,d quarks. Although the nature of chiral symmetry transition changes from a cross over to a first order transition, the values of the constituent masses of the quarks do not change very much. Henceforth we shall limit our discussions to this case when current quark masses for the light quarks are taken to be zero. In Fig.2 we show the results of such a calculation for zero temperature when the charge neutrality conditions are not imposed. The general behaviour of the graphs are similar to those in Fig. 1. The important difference is that, the chiral transition is a sharp first order transition at zero temperature. The constituent quark masses at zero densities for up (down) and strange quarks are 354 MeV and 546 MeV as compared to 368 MeV and 549 MeV respectively, when current quark masses of u,d quarks are taken to be nonzero. The critical chemical potential turns out to be 335 MeV for the case of nonzero K. We also note that as the quark chemical potential increases beyond 480 MeV, the superconducting gap starts decreasing as the strange quark–antiquark condensate decreases so that the effective diquark coupling also decreases, as may be clear from Eq. (37).

There have been calculations where the effect of the determinant interaction is retained for mass gap calculations, but the effect of such an interaction is not included in the superconducting gap equation (37). We plot in Fig. 3 the superconducting gap including the effect of determinant interaction for the u-d superconductivity gap for the case when the charge neutrality conditions are not imposed. As may be seen, including the effect of strange quark–antiquark condensate through the determinant interaction leads to an enhancement of the superconducting gap and this can be as large as a twenty five percent for chemical potentials of about 400 MeV. This is because the ‘effective’ coupling for superconductivity increases from $G_D$ to $G_D - K \langle \bar{s}s \rangle /4$. At higher densities however, this effect diminishes as the magnitude of the quark–antiquark condensate itself decreases. Such effects can play an important role in deciding which phase can occur at what density as we come down in the density. In particular, such kind of enhancement will be there for u-d superconductivity, but will be absent for u-s or d-s superconducting gaps since the light quark–antiquark condensate vanishes much earlier.

We next extend our discussion to the case when the charge neutrality conditions ($Q_E=0=Q_S$) are imposed. We compute the thermodynamic potential numerically as follows. For given values of the quark chemical potential, $\mu_q$, and the electric and color charge potentials, $\mu_E$ and $\mu_8$, the coupled mass gap equations and the superconducting gap equations are solved self consistently. The values of $\mu_E$ and $\mu_8$ are varied so that the charge neutrality conditions (Eq. (34) and Eq. (35)) are satisfied. The resulting solutions are then used in Eq. (37) to compute the thermodynamic potential. In doing so, we also check for existence of multiple solutions of the gap equation and if they exist, we choose the solution which has the least value for the thermodynamic potential. In Fig.4 we show the dependence of the superconducting gap on quark chemical potential when charge neutrality condition is imposed. The superconducting gap starts becoming nonzero for $\mu_q$ greater than 350 MeV. At this point the chiral symmetry for the light quarks is restored. This has also its effect on the strange quark mass which drops from its vacuum value of about 546 MeV to 470 MeV. The superconducting gap increases smoothly with $\mu_q$ until $\mu_q$ attains a value of about 430 MeV. At this point the gap jumps from 80 MeV to 106 MeV and then increases slowly up to a maximum value of about 170.5 MeV at $\mu_q = 450$ MeV. Beyond this value of $\mu_q$, the magnitude of the strange quark–antiquark condensate decreases.
leading to a drop of superconducting gap through the determinant interaction.

As the quark chemical potential increases, strange quarks help in maintaining the charge neutrality conditions. We might observe here that the strange quark mass starts decreasing already at $\mu_q = 425$ MeV, when charge neutrality conditions are imposed. This may be compared with $\mu_q \simeq 480$ MeV, when charge neutrality conditions are not imposed (see Fig. 2a). As we have already mentioned, there could be multiple solutions of the gap equation and whichever has the least free energy is the stable solution. It could so happen that a solution of the mass gap equation which
is metastable when charge neutrality conditions are not imposed, can be the only solution when charge neutrality conditions are imposed.

The pairing between the quarks with charge neutrality conditions correspond to stressed pairing i.e. the pairing of the quarks of different species which differ in their Fermi momenta [28]. This gives rise to possibility of a gapless superconducting phase, the QCD analogue of Sarma phase [29]. In the present case, between the quark chemical potentials $\mu_q \simeq 350$ MeV and $\mu_q \simeq 425$ MeV, the system is in the gapless phase. The number densities of $u$ and $d$ quarks participating in the condensation are plotted in Fig. 5. As the quark chemical potential is increased beyond
425 MeV the solution for the gap jumps to a higher value of about 106 MeV almost similar to the case when charge neutrality conditions are not imposed. This corresponds to the usual BCS solution. In this case, the number densities of u quarks and d quarks participating in the condensation are the same. The charge neutrality conditions however are maintained by the blue colored u,d quarks, the electron as well as the strange quarks. One essential effect of including strange quarks is that the electron density starts decreasing for chemical potentials greater than the strange quark mass and the strange quarks carrying the negative charge maintain electric charge neutrality condition. This has the effect that the lowest excitation energy $\omega_2 = \bar{\omega} + \delta_\mu$ in the BCS pairing case becomes large due to both the large value of the gap as well as due to the small magnitude of the electron chemical potential.

It may be worthwhile to mention here that the gapless modes in superconductivity were known theoretically in the context of superconducting matter with finite momentum [21] as well as in condensed matter systems with magnetic impurities [22]. Recently it has been investigated for cold fermionic atoms [30, 31]. Gapless modes in the context of quark matter has been first proposed in Ref. [22] for color flavor locked matter. However, this corresponded to a metastable phase. Gapless modes in neutral quark matter were first emphasized for the two flavor color superconductivity in Ref. [4] and for the 2SC+ s quark matter in Ref. [7]. Stable gapless modes for color flavor locked matter were first proposed in Ref. [32] and have been confirmed in Ref. [15] in a more general structure for the gap. The temperature dependence of the CFL gapless modes has been studied in Ref. [33]. We might mention here that the effects of nonzero strange quark mass and the charge neutrality condition on superconducting gaps have also been studied within a Ginzberg Landau approach in Ref. [34].

**IV. SUMMARY**

We have analysed here the effect of flavor mixing t’Hooft six fermion interaction both for chiral symmetry breaking as well as two flavor superconductivity in NJL model. The method is a variational one with an explicit construct for the trial state including quark–antiquark as well as diquark condensates. The determinant interaction affects explicitly both the u-d superconducting gap as well as the mass gap for the strange quarks. For the strange quarks, a density dependent mass arises even when the quark–antiquark condensates vanish. Such type of generation of dynamical mass is entirely distinct from the typical mechanism of spontaneous chiral symmetry breaking through quark–antiquark condensates [17]. Further, the u-d superconducting gap gets enhanced as inclusion of such a term effectively increases the diquark coupling by an amount proportional to the strange quark–antiquark condensate.

We have focussed our attention here to the two flavor superconducting phase, with unpaired strange quarks. The variational method adopted can be directly generalised to include color flavor locked phase and one can then make a free energy comparison regarding the possibility of which phase would be thermodynamically favorable at what density. In case of CFL phase, such an enhancement of superconducting u-s (d-s) condensates will not be there as the corresponding $\langle \bar{u}u \rangle$ will vanish much earlier than $\langle \bar{s}s \rangle$ condensates, when the density is increased. Since strange quark mass is a sensitive parameter in maintaining the charge neutrality condition, this will be important while comparing free energies of different phases of charge neutral quark matter.

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