2γ-decays of scalar mesons (σ(600), f₀(980) and a₀(980)) in the Nambu-Jona-Lasinio model

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The two-photon decay widths of scalar mesons σ(600), f₀(980) and a₀(980) are calculated in framework of the local Nambu-Jona-Lasinio model. The contributions of the quark loops (Hartree-Fock approximation) and the meson loops (next 1/Nc-approximation where Nc is the number of colors) are taken into account. These contributions, as we show, are the values of the same order of magnitude. For the f₀ decay the K-loop contribution turns out to play the dominant role. The results are in satisfactory agreement with modern experimental data.

I. INTRODUCTION

In recent papers [1, 2], the radiative decay \( \phi \to f₀γ \) and \( f₀(α₀) \to ρ(ω)γ \) widths within the local Nambu-Jona-Lasinio (NJL) model [3, 4, 5, 6, 7, 8, 9, 10, 11] have been calculated. In these works, we took into account not only the quark loop contributions but also the meson loop contributions, moreover, in the decays of the \( f₀(980) \) meson the kaon loop contribution is the dominant one. It is worth noticing that the situation here is similar to the one that takes place in the case of \( φ \to f₀γ, \, f₀ \to ργ, \, f₀ \to ωγ \) decays. Let us note that due to the explicit gauge invariant form of the amplitude the relevant loop integrals do not contain ultraviolet divergences. Thus, the explicit dependence of these amplitudes of the external momenta was obtained [12].

In this paper, we will consider the two-photon decays of the scalar mesons σ(600), f₀(980) and a₀(980). These decays was considered in a recent paper [13] where a rather rough \( q^2 \) approximation for quark loop integrals evaluation was used. In our case we consider the full integral corresponding to the quark loop and thus, we can obtain a complete dependence of the amplitude of external momenta.

In the case of the quark loop we consider only a real part of the relevant loop integral. This prescription permits us to take into account the condition of the "naive" quark confinement. Some theoretical arguments supporting this procedure can be found in [14]. As for the meson loops, both the real and the imaginary parts were taken into account.

The structure of our paper is the following. In Section II, the NJL quark-meson Lagrangian, corresponding parameters and the coupling constants of our model are defined. In Section III, the methods of quark and meson loop calculation are given.

In Section IV, the contributions of quark and meson loops to the amplitudes and the widths of two-photon decays of the scalar meson are presented.

In Section V we discuss the results obtained.

II. LAGRANGIAN OF THE NJL MODEL

The Lagrangian of interaction of mesons and quarks within the NJL model has the form [3]:

\[
\mathcal{L} = \bar{q} \left[ i \gamma \partial - M + eQ \hat{A} + g_{\sigma_\alpha} \lambda_\alpha \sigma_\alpha + g_{\sigma_\gamma} \lambda_\gamma \sigma_\gamma + g_u \lambda_u a_0 + \right. \\
\left. + i \gamma_5 g_π \left( \lambda_{\pi^+} \pi^+ + \lambda_{\pi^-} \pi^- \right) + i \gamma_5 g_K \left( \lambda_K^{+} K^+ + \lambda_K^{-} K^- \right) \right] q,
\]

where \( \bar{q} = (\bar{u}, \bar{d}, \bar{s}) \), and \( u, d, s \) are the quark fields, \( M = \text{diag} (m_u, m_d, m_s) \) with \( m_u = m_d = 263 \text{ MeV}, m_s = 406 \text{ MeV} \) - constituent quark mass matrix; \( Q = \text{diag} (2/3, -1/3, -1/3) \) is the quark electric charge matrix, \( e \) is the elementary electric charge (\( e^2/4\pi = α = 1/137 \)), \( \lambda_\alpha = \left( \sqrt{2} λ_\alpha + λ_8 \right) / \sqrt{3}, \lambda_\gamma = \left( -λ_0 + \sqrt{2} λ_8 \right) / \sqrt{3}, \lambda_{\pi^\pm} = \left( λ_1 \pm i λ_3 \right) / \sqrt{2}, \lambda_K^{\pm} = \left( λ_4 \pm i λ_5 \right) / \sqrt{2} \) where \( λ_i \) are the well-known Gell-Mann matrices and \( λ_0 = \sqrt{2/3} \text{ diag} (1, 1, 1) \). Scalar isoscalar
mesons $f_0$, $\sigma$ are the mixed states

\[
\begin{align*}
  f_0 &= \sigma_u \sin \alpha + \sigma_s \cos \alpha, \\
  \sigma &= \sigma_u \cos \alpha - \sigma_s \sin \alpha,
\end{align*}
\]

(2)

with the mixing angle $\alpha = 11.3^\circ$.

The coupling constants from the Lagrangian are defined in the following way:

\[
\begin{align*}
  g_{\sigma_u} &= (4I^A(m_u,m_u))^{-1/2} = 2.43, \\
  g_{\sigma_s} &= (4I^A(m_s,m_s))^{-1/2} = 2.99, \\
  g_{\pi} &= \frac{m_u}{F_{\pi}} = 2.84, \\
  g_{K} &= \frac{m_u + m_s}{2F_{K}} = 3.01,
\end{align*}
\]

where we use the Goldberger-Treiman relation for $g_{\pi}$ and $g_{K}$ constants, $F_{\pi} = 92.5$ MeV and $F_{K} = 1.2 \ F_{\pi}$, and $I^A(m,m)$ is the logarithmically divergent integral which has the form:

\[
I(m,m) = \frac{N_c}{(2\pi)^2} \int d^4k \frac{\theta(k^2 - k^2)}{(k^2 - m_u^2)(k^2 - m_s^2)(k^2 - m_q^2)} \left( \ln \frac{\Lambda^2}{m^2 + 1} - \frac{\Lambda^2}{\Lambda^2 + m^2} \right), \quad N_c = 3.
\]

This integral is written in the Euclidean space. The cut-off parameter $\Lambda = 1.27$ GeV is taken from [1, 2].

### III. QUARK AND MESON LOOP INTEGRALS

The amplitudes of the $2\gamma$ decay can be expressed in terms of the quark and meson loop integrals. The quark loop contribution to the amplitude is given by two triangle type Feynman diagrams:

\[
T_{\mu\nu}^q = -\frac{\alpha}{4\pi} \int d^4k \gamma_\nu \left( \frac{\gamma_\mu}{(k^2 - m_q^2)} - \frac{1}{(k^2 - m_q^2)} \right) + \left( (q_1, \mu) \leftrightarrow (q_2, \nu) \right).
\]

(3)

Applying the Feynman procedure of joining of the denominators

\[
\int \frac{1}{(k^2 - m_q^2)} \frac{1}{(k^2 - m_q^2)} = \int_0^1 dx \int_0^1 dy \frac{1}{(k - q_x y)^2 - (m_q^2 + q_y^2)^2},
\]

(4)

we obtain for $T_{\mu\nu}^q$:

\[
T_{\mu\nu}^q = \frac{\alpha}{\pi} (g_{\mu\nu}(q_1q_2) - q_1\nu q_2\mu) T^q,
\]

(5)

\[
T^q = 2m_q \int_0^1 dx \int_0^1 dy \frac{1 - 4y^2x\bar{x}}{m_q^2 - M_3^2y^2x\bar{x}},
\]

(6)

For meson loops an additional Feynman diagram with two photon-two meson vertex contributes as well. To restore the general gauge invariant form of the amplitude, we can nevertheless consider only two triangle type Feynman diagrams:

\[
\Delta T_{\mu\nu}^M = \frac{\alpha}{4\pi} \int d^4k \frac{(k + q_1)_\mu(k - q_2)_\nu}{(k^2 - M^2)(k^2 - M^2)(k^2 - M^2)} + \left( (q_1, \mu) \leftrightarrow (q_2, \nu) \right).
\]

(7)

Extracting the term $\sim q_1\nu q_2\mu$ and adding the relevant term $\sim g_{\mu\nu}$ we obtain:

\[
T_{\mu\nu}^M = \frac{\alpha}{\pi} (g_{\mu\nu}(q_1q_2) - q_1\nu q_2\mu) T^M,
\]

(8)

\[
T^M = \frac{\alpha}{\pi}(g_{\mu\nu}(q_1q_2) - q_1\nu q_2\mu) M^2,
\]

(9)
with
\[ T^M = 2 \int_0^1 dx \int_0^1 dy y^2 x \frac{y^2 x \bar{x}}{M^2 - M^2_z y^2 x \bar{x}}. \] (10)

Standard evaluation of these integrals leads to
\[ T^q = -\frac{1}{m_q} F(z_S^q), \] (11)
\[ T^M = -\frac{z_S^M}{4M^2} \Phi(z_S^M), \] (12)
where \( z_S^q = 4m_q^2/M_S^2 \), \( z_S^M = 4M^2/M_S^2 \).
\[ F(z) = \text{Re} \left[ 1 + (1 - z) \Phi(z) \right], \]
\[ \Phi(z) = z\phi(z) - 1, \]
\[ \phi(z) = \begin{cases} \frac{\pi^2}{2} - \ln \frac{1 + \sqrt{1 - z}}{2} + i\pi \ln \frac{1 + \sqrt{1 - z}}{1 - \sqrt{1 - z}}, & z < 1, \\
\arctan \frac{1}{\sqrt{z - 1}}, & z > 1. \end{cases} \] (13)

We remind that for the quark loop contribution the imaginary part of the function \( \Phi(z) \) must be omitted, and for the meson loop contribution both the real and possible imaginary parts are relevant.

Similar expressions were obtained in \( [12] \), where the imaginary part of the quark loop contribution was taken into account.

### IV. SCALAR MESONS DECAY AMPLITUDES AND WIDTHS

The vertices of the quark-meson and quark-photon interactions were given above. The vertices of the meson-meson interaction in the framework of NJL model have the form (for details see \([3]\)):
\[ V_{\sigma \pi \pi} = V_{a_0 \pi \pi} = 0, \]
\[ V_{\sigma \rho K} = V_{a_0 \rho K} = -2(2m_u - m_s) \frac{g_K^2}{g_{\sigma \rho}}, \]
\[ V_{\sigma \pi K} = 2\sqrt{2}(2m_s - m_u) \frac{g_K^2}{g_{\sigma \pi}}, \]
\[ V_{\sigma \rho \pi} = -4m_u \frac{g_\rho^2}{g_{\sigma \rho}}. \]

The general structure of the two-photon scalar meson decay amplitudes has the form
\[ T_{S\gamma\gamma} = -\frac{\alpha g_{\sigma \pi}}{\pi m_u} (g_{\mu \nu} (q_1 q_2) - q_1^\mu q_2^\nu) a_{S\gamma\gamma}. \] (14)

The expression for the width has the form:
\[ \Gamma_{S\gamma\gamma} = \frac{M_S^2}{64\pi} \frac{\alpha^2 g_\sigma^2}{\pi^2 m_u^2} |a_{S\gamma\gamma}|^2. \] (15)

The amplitude \( a_{u_0 \gamma\gamma} \) of \( a_0 \rightarrow \gamma \gamma \) contains the contribution of \( u, d \) quarks and the \( K \)-meson intermediate states. The color-charge factor associated with \( u, d \) quarks is \( N_c \left( \frac{4}{3} - \frac{1}{3} \right) = 1 \). Thus,
\[ a_{u,d}^{u,d} = F(z_{a_0}^u). \] (16)

Taking the \( K \)-meson loop contribution we obtain:
\[ a_{a_0 \gamma\gamma} = a_{a_0 \gamma\gamma}^{u,d} + a_{a_0 \gamma\gamma}^K = \]
\[ = F(z_{a_0}^u) - \frac{m_u}{g_{\sigma \pi}} \frac{2(2m_u - m_s) g_K^2}{4g_{\sigma \pi} M_K^2} z_{a_0}^K \Phi(z_{a_0}^K) = 0.482 - 0.114 = 0.367. \] (17)
The corresponding width is
\[ \Gamma_{a_0(980) \to \gamma \gamma} = (2.25 \text{ KeV}) |a_{a_0 \gamma \gamma}|^2 = 0.29 \text{ KeV}. \]

In the case of the \( f_0 \to \gamma \gamma \) decay we also have the contribution of \( u, d \) and \( s \) quarks and the \( K \)-meson intermediate states. The color-charge factor associated with \( u, d \) quarks is \( N_c \left( \frac{1}{2} + \frac{1}{3} \right) = \frac{5}{3} \) for \( \sigma_u \)-component of \( f_0 \) and \( N_c \left( \frac{1}{2} \right) = \frac{1}{2} \) for \( \sigma_s \)-component of \( f_0 \). Taking the \( K \)-meson and the \( \pi \)-meson loop contribution we obtain
\[ a_{f_0 \to \gamma \gamma} = \frac{5}{3} F(z_{f_0}^u) \sin \alpha - \frac{\sqrt{2}}{3} F(z_{f_0}^s) \left( \frac{g_{\sigma_u} m_u}{g_{\sigma_u} m_u} \right) \cos \alpha + \]
\[ + \left( \frac{- g_K^2 m_u}{g_{\sigma_u}^2} 2(2m_u - m_s) \sin \alpha + \frac{g_K^2}{g_{\sigma_u} g_{\sigma_s}} \frac{m_u}{4M_K^2} 2\sqrt{2} (2m_s - m_u) \cos \alpha \right) \Phi(z_{f_0}^K) \Phi(z_{f_0}^K) - \]
\[ \sin \alpha \frac{m_u^2 g_{\sigma_u}^2}{M_K^2} \Phi(z_{f_0}^u) = 0.157 - 0.417 - 0.022 + 0.589 + 0.082 - 0.038 i = 0.385 - 0.038 i. \]  
(18)

For the width we have
\[ \Gamma_{f_0(980) \to \gamma \gamma} = (2.25 \text{ KeV}) |a_{f_0 \gamma \gamma}|^2 = 0.33 \text{ KeV}. \]

In the case of the \( \sigma \to \gamma \gamma \) decay we have
\[ a_{\sigma \to \gamma \gamma} = \frac{5}{3} F(z_{\sigma}^u) \cos \alpha + \frac{\sqrt{2}}{3} F(z_{\sigma}^s) \left( \frac{g_{\sigma_u} m_u}{g_{\sigma_u} m_u} \right) \sin \alpha - \]
\[ \frac{g_K^2}{g_{\sigma_s}^2} \frac{m_u}{M_K^2} 2(2m_u - m_s) \cos \alpha + \frac{g_K^2}{g_{\sigma_u} g_{\sigma_s}} \frac{m_u}{4M_K^2} 2\sqrt{2} (2m_s - m_u) \sin \alpha \right) \Phi(z_{\sigma}^K) \Phi(z_{\sigma}^K) - \]
\[ \cos \alpha \frac{m_u^2 g_{\sigma_u}^2}{M_K^2} \Phi(z_{\sigma}^u) = 1.89 + 0.057 - 0.041 - 0.043 + 0.92 - 0.08 i = 2.78 - 0.98 i. \]  
(19)

The corresponding width is
\[ \Gamma_{\sigma(600) \to \gamma \gamma} = (0.51 \text{ KeV}) |a_{\sigma \gamma \gamma}|^2 = 4.3 \text{ KeV}. \]  
(20)

The experimental value of the mass and the width of the \( \sigma \) meson is not well established. We present the width of \( \sigma \) for two other masses: \( M_\sigma = 450 \text{ MeV} \) and \( M_\sigma = 550 \text{ MeV} \). They are
\[ \Gamma_{\sigma(450) \to \gamma \gamma} = 2.18 \text{ KeV}, \]
\[ \Gamma_{\sigma(550) \to \gamma \gamma} = 3.53 \text{ KeV}. \]

The comparison of our results with the experimental data and some other model predictions is given in Table[1]

V. CONCLUSION

The calculations of the radiative decays in the NJL model show an important role of both the quarks and meson loops. Moreover, for the \( f_0 \) meson decay the kaon loop turns out to provide the dominant contribution. It is worth noticing that the situation here is similar to the one that takes place in the case of \( \phi \to f_0 \gamma \) [1], \( f_0 \to \rho(\omega) \gamma \) [2] decays. This fact permits one to understand the success of such models as the model of a kaon molecule [11] as well as the four-quark model [21, 22, 30]. The NJL model used here allows us to take into account both the quark-antiquark state, which manifests itself in the form of quark loops, and the hidden four-quark state, which shows up as meson loops. Let us emphasize that in the framework of the standard NJL model we can describe the \( 2 \gamma \) decays without any additional parameters.

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TABLE I: The table of two-gamma decays of the scalar mesons $\sigma(600)$, $f_0(980)$ and $a_0(980)$.

| $\Gamma (a_0 \rightarrow \gamma \gamma)$ (exp.), KeV | $\Gamma (a_0 \rightarrow \gamma \gamma)$ (theor.), KeV |
|-----------------------------------|-----------------------------------|
| $0.30 \pm 0.10$ [$^{17}$] | 0.29 (This paper) |
| $\Gamma (f_0 \rightarrow \gamma \gamma)$ (exp.), KeV | $\Gamma (f_0 \rightarrow \gamma \gamma)$ (theor.), KeV |
| $0.20^{+0.095+0.147}_{-0.083-0.117}$ [$^{18}$] | 0.33 (This paper) |
| $0.28^{+0.09}_{-0.13}$ [$^{19}$] | 0.21 $\pm$ 0.26 [$^{20}$] |
| $0.42 \pm 0.06 \pm 0.18$ [$^{21}$] | 0.22 [$^{22}$] |
| $0.29 \pm 0.07 \pm 0.12$ [$^{23}$] | 0.33 [$^{24}$] |
| $0.31 \pm 0.14 \pm 0.09$ [$^{25}$] | 0.31 [$^{26}$] |
| $0.63 \pm 0.14$ [$^{27}$] | 0.28$^{+0.09}_{-0.15}$ [$^{28}$] |
| $\Gamma (\sigma \rightarrow \gamma \gamma)$ (exp.), KeV | $\Gamma (\sigma \rightarrow \gamma \gamma)$ (theor.), KeV |
| 4.1 $\pm$ 0.3 [$^{32}$] | 4.3 (This paper) |
| 3.8 $\pm$ 1.5 [$^{19}$] | |
| 5.4 $\pm$ 2.3 [$^{27}$] | |
| 10 $\pm$ 6 [$^{33}$] | |

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