Modeling water droplet condensation and evaporation in DNS of turbulent channel flow

E Russo¹, JGM Kuerten¹,², CWM van der Geld¹ and BJ Geurts²

¹Department of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
²Faculty EEMCS, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands
E-mail: e.russo@tue.nl

Abstract. In this paper a point particle model for two-way coupling in water droplet-laden incompressible turbulent flow of air is proposed. The model is based on conservation laws and semi-empirical correlations. It has been implemented and tested in a DNS code based for turbulent channel flow with an Eulerian-Lagrangian approach. The two-way coupling is investigated in terms of the effects of mass and heat transfer on the droplets distributions along the channel wall-normal direction and by comparison of the droplet temperature statistics with respect to the case without evaporation and condensation. A remarkable conclusion is that the presence of evaporating and condensing droplets results in an increase in the non-dimensional heat transfer coefficient of the channel flow represented by the Nusselt number.

1. Introduction

Particle-laden turbulent flow remains a challenging topic for scientists due to the turbulent character of the flow. However, thanks to the rapid increase in computing power, many steps have been taken in the last fifty years to arrive at a better understanding of this topic. Currently, equations for some particle-laden flows can be solved by means of numerical computations. The numerical solver computes the solution of the equations which represent the flow and the equations for each particle. In a simple approach to the problem, when the particle diameters are small enough, the particle equations that are solved are based on point particle modeling (Maxey & Riley, 1983). This means that the particle is considered as a point and the particle mass is concentrated in the point. This approach for modeling the particle simplifies the particle equations, making the computational cost acceptable for practical application. However, even with the current computing power, the computation is limited to flows with a relatively small number of particles. The point particle modeling has been successfully used to perform both direct numerical simulations (DNS) and large eddy simulations (LES) of particle-laden flows (Kuerten, 2006).

In spite of the fact that there has been a growing interest in point particle modeling, the effect of mass and heat transfer in turbulent channel flows has hardly been addressed. Up to now, there have been few studies on the modeling of heat and mass transfer mechanisms in simple particle model approaches suitable for numerical computations (Mashayek & Pandya, 2003; Wang & Rutland, 2006). In particular, Mashayek (1997) made the first attempt in DNS of evaporating droplet dispersion in homogeneous turbulent flow with two-way coupling (modeling
heat and mass transfer between droplets and fluid). Here, we are considering instead the case of inhomogeneous turbulent channel flow with a gradient of temperature along the wall-normal direction of the channel. On the other hand, the heat and mass transfer of a single particle has been studied extensively in recent years (see Sazhin et al. (2005) for a detailed list of publications on this topic).

In this paper, a point particle model which takes into account mass and heat exchange is proposed. This model is based on conservation laws. The aim of this study is to test the validity of such a model by means of DNS, using an in-house code. DNS is computationally very expensive, since no turbulence model for the flow is used and therefore the grid spacing must be small enough to capture the smallest of the eddies in the turbulent flow. Nevertheless, when DNS is suitable, it is the best choice to test a new model, because it ensures that the only approximation used to solve the flow equations is the numerical approximation. Even so, we must be aware that a model is used for the droplets, which means that the effects due to the presence of droplets in the flow are not directly solved, but involves approximations (semi-empirical correlations) which are not strictly numerical.

Using our in-house code, several test cases were performed, in which the particles are evaporating and condensing water droplets. Correlations for a single spherical droplet immersed in a uniform flow are used to close the mathematical model. The approach can be further adapted to other droplet shapes by using other correlations. In our results, the effects of the heat and mass transfer phenomena on the flow and droplet behavior were analyzed. Droplet-droplet collisions (four-way coupling) were ignored because of the small number of droplets and small volume fraction considered. In case of solid particles in the presence of solid walls turbophoresis (the tendency for particles to migrate in the direction of decreasing turbulence level) takes place resulting in a higher density of particles in the near wall region. The influence of evaporation, condensation and heat flux through the walls will be analyzed to have an insight into their effect on the droplet distribution along the channel wall-normal direction.

2. Governing equations

In this section the equations of the gas are presented and the point particle equations considering heat and mass transfer are derived. Moreover, the numerical methods for gas and droplets are described.

2.1. Gas

The gas is air, which is the carrier phase of the multiphase channel flow. The gas is considered in an Eulerian way. The gas is assumed to behave as an incompressible fluid, thus the continuity equation is:

$$\nabla \cdot \mathbf{u} = 0$$  \hspace{1cm} (1)

The mathematical model of the air consists of the Navier-Stokes equations in rotational form:

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \times \mathbf{u} + \nabla P = \nu \Delta \mathbf{u} + \mathbf{F} + \mathcal{L}_u$$  \hspace{1cm} (2)

which are the same equations as in Kuerten (2006), with the additional coupling term:

$$\mathcal{L}_u = - \sum_{i=1}^{N} \frac{d(m_i \mathbf{v}_i)}{dt} \delta (\mathbf{x} - \mathbf{x}_i) = - \sum_{i=1}^{N} m_i \frac{d\mathbf{v}_i}{dt} \delta (\mathbf{x} - \mathbf{x}_i) - \sum_{i=1}^{N} \mathbf{v}_i \frac{dm_i}{dt} \delta (\mathbf{x} - \mathbf{x}_i)$$  \hspace{1cm} (3)

In (2) $\omega = \nabla \times \mathbf{u}$ is the vorticity, $P = p/\rho_f + \frac{1}{2} \mathbf{u}^2$, $\nu$ is the fluid kinematic viscosity, $p$ the fluctuating part of the pressure, $\rho_f$ is the fluid density, and $\mathbf{F}$ is the driving force, chosen
constant in time and space. In (3) \(m_i\) and \(\mathbf{v}_i\) are the mass and velocity of droplet \(i\) respectively. The coupling term can be split into two contributions which represent the momentum transfer from the droplet to the air due to the drag and due to evaporation and condensation respectively. The delta-function \(\delta(x - x_i)\), where \(x_i\) is the position of droplet \(i\), indicates that the coupling terms are only present at the locations of the droplets.

The temperature equation is obtained by applying the conservation of the total enthalpy over a control volume \(\Delta V\) and taking into account the contributions due to the presence of droplets in this volume. Thus, the total enthalpy in \(\Delta V\) can be written as the sum of the enthalpy of the fluid and of the droplets within \(\Delta V\):

\[
\int_{\Delta V} \left[ \left( \rho_a c_{p,a} + \rho_v c_{p,v} \right) T_g + \rho_v \ell_0 \right] \, dV + \sum_{i=1}^{N} \rho_i c_{p,l} T_i \, V_i \tag{4}
\]

where \(\rho_a\) and \(c_{p,a}\) are the air mass density and specific heat capacity respectively, \(\rho_v\) and \(c_{p,v}\) the vapor mass density and specific heat capacity, \(\ell_0\) the latent heat of water at \(T = 0^\circ\mathrm{C}\), \(N\) the number of droplets within \(\Delta V\), \(T_i\) the temperature of droplet \(i\), \(c_{p,l}\) the specific heat capacity of water, and \(V_i\) the volume of droplet \(i\). By applying the first law of thermodynamics to (4) and dividing by \(\Delta V\), the following evolution equation for the gas temperature is obtained:

\[
\left( \rho_a c_{p,a} + \rho_v c_{p,v} \right) \left( \frac{\partial T_g}{\partial t} + \nabla \cdot (\mathbf{u} T_g) \right) = \nabla \cdot (k_g \nabla T_g) + \mathcal{L}_{T1} + \mathcal{L}_{T2} + \mathcal{L}_{T3} \tag{5}
\]

with:

\[
\mathcal{L}_{T1} = D \left( c_{p,v} - c_{p,a} \right) \nabla \rho_v \cdot \nabla T_g \tag{6}
\]

\[
\mathcal{L}_{T2} = - \sum_{i=1}^{N} c_{p,a} T_g \frac{dm_i}{dt} \delta(x - x_i) \tag{7}
\]

\[
\mathcal{L}_{T3} = - \sum_{i=1}^{N} h_m A_i \left( T_{g_i} - T_i \right) \delta(x - x_i). \tag{8}
\]

\(\mathcal{L}_{T1}, \mathcal{L}_{T2},\) and \(\mathcal{L}_{T3}\) are the coupling terms representing the diffusion of energy due to the diffusion of water vapor, the gain (loss) of energy due to condensation (evaporation) of the droplets, and the heat exchange between fluid and droplets, respectively. Equation (5) is obtained from equation (4) assuming the gas incompressible\(^1\), according to (1), and assuming \(c_{p,a}, c_{p,v}\), and \(c_{p,l}\) constant in time and space. In (8) \(h_m\) the heat transfer coefficient, \(A_i\) the surface area of the \(i\)-droplet, and \(T_{g_i}\) the temperature of the gas at the location of droplet \(i\).

2.2. Water vapor

The water vapor is also treated in an Eulerian way. The total water mass rate of change over the volume \(\Delta V\) is the sum of the rate of change due to diffusion, convection, and evaporation and condensation of the droplets within \(\Delta V\):

\[
\int_{\Delta V} \left[ \frac{\partial \rho_v}{\partial t} + \nabla \cdot (\rho_v \mathbf{u}) - \nabla \cdot (D \nabla \rho_v) \right] \, dV = - \sum_{i=1}^{N} \frac{dm_i}{dt}. \tag{9}
\]

By dividing (9) by \(\Delta V\) the following convection-diffusion equation is obtained:

\(^1\) The gas mass density, which is the sum of the mass densities of air and vapor is constant in time and space: \(\rho_g = \rho_a(x,t) + \rho_v(x,t) = \text{const}\). However, \(\rho_a(x,t)\) and \(\rho_v(x,t)\) are function of position \(x\) and time \(t\).
\[
\frac{\partial \rho_v}{\partial t} + \nabla \cdot (\rho_v \mathbf{u}) = \nabla \cdot (D \nabla \rho_v) + \mathcal{L}_v
\]  

(10)

where the term:

\[
\mathcal{L}_v = - \sum_{i=1}^{N} \frac{dm_i}{dt} \delta (\mathbf{x} - \mathbf{x}_i)
\]

(11)

is the coupling term which represents the mass transfer between the liquid phase and the water vapor by condensation or evaporation.

2.3. Droplets

The droplets (dispersed phase) are considered in a Lagrangian way. For each water droplet, the following equation of motion (Maxey & Riley, 1983) is solved:

\[
\frac{d \mathbf{v}_i}{dt} = \frac{\mathbf{u}(\mathbf{x}_i, t) - \mathbf{v}_i}{\tau_d} (1 + 0.15Re_p^{0.687})
\]

(12)

where \( \mathbf{v}_i \) is the velocity of droplet \( i \), \( \mathbf{u}(\mathbf{x}_i, t) \) the velocity of the gas at the location of the droplet, \( Re_p \) the Reynolds number of the droplet, \( \tau_d = \frac{\rho_d d^2}{15 \mu_g} \) the droplet relaxation time with \( d \) the droplet diameter, \( \rho_d \) the droplet mass density, and \( \mu_g \) the dynamic viscosity of the gas. Since droplets are small and heavy compared to the gas phase, we can assume that the drag force is the only relevant force between the two phases. Moreover, we do not consider gravity. The Lagrangian droplet tracking is performed by integrating the trajectory equation:

\[
\frac{d \mathbf{x}_i(t)}{dt} = \mathbf{v}_i
\]

(13)

In order to derive an equation for the droplet temperature which takes into account the heat and mass transfer to and from the gas, we can apply the first law of thermodynamics to droplet \( i \). This yields:

\[
\frac{d}{dt} (m_i c_{p,i} T_i) = h_v \frac{dm_i}{dt} + h_m A_i (T_{g_i} - T_i)
\]

(14)

where the variation of the internal energy of the i-droplet is balanced by condensation or evaporation (1\textsuperscript{st} term on the right-hand side) and the heat transfer due to convection and conduction (2\textsuperscript{nd} term on the right-hand side); kinetic and potential energy are ignored. Defining the enthalpy of the water as \( h_l = c_l T \), and the droplet volume by \( V_i \), equation (14) can be written as:

\[
\rho_l c_{p,l} V_i \frac{dT_i}{dt} = (h_v - h_l) \frac{dm_i}{dt} + h_m A_i (T_{g_i} - T_i)
\]

(15)

Equation (15) is closed using correlations for \( \frac{dm_i}{dt} \) and \( h_m \). Assuming that the gas mass density is constant in time and space and applying the conservation of mass around a droplet, the following expression for the total mass flux of vapor \( N_{\rho,v} \) (mass per second per unit area) is obtained:

\[
N_{\rho,v} = \frac{D \rho_g}{\delta} \ln \left( \frac{1 - x_{v,\delta}}{1 - x_{v,0}} \right)
\]

(16)

In this equation \( \delta \) is the distance normal to the droplet surface over which the gradient of vapor concentration exists, and \( x_{v,\delta} \) and \( x_{v,0} \) are the vapor mass fraction at \( \delta \) and on the surface of the droplet, respectively. With the help of the mass-transfer correlation for a sphere
\[ Sh = 2 + 0.6 \frac{Re_d^{1/2} Sc^{1/3}}{2+0.6} \] (Bird et al., 1960), the rate of change of the droplet mass can be written as:

\[
\frac{dm_i}{dt} = - \frac{m_i Sh}{3 \tau_d Sc} \ln \left( \frac{1 - x_{v,\delta}}{1 - x_{v,0}} \right)
\]

where \( Sh \) is the Sherwood number, \( Sc \) the Schmidt number. The driving quantity of (17) is the difference \( x_{v,\delta} - x_{v,0} \), where \( x_{v,0} \) can be written in terms of the fluid temperature and saturation pressure. The saturation pressure in Pascal is calculated with Antoine’s relation:

\[
p_{v,\text{sat}}(T_g) = 10^5 \exp \left( \frac{3816.44}{226.87 + T_g} - 273.15 \right)
\]

where \( T_g \) is in Kelvin. For forced convection around a sphere, the heat-transfer correlation gives \( h_m \) (Bird et al., 1960):

\[
\frac{h_m d_i}{K_g} = 2 + 0.6 \frac{Re_p^{1/2} Pr^{1/3}}{(2+0.6)}
\]

where \( K_g \) is the thermal conductivity of the gas, and \( Pr \) the Prandtl number.

### 3. Numerical method and simulations set-up

The proposed model for two-way coupling in droplet-laden turbulent flow is tested by means of DNS. The DNS code based on an Eulerian-Lagrangian approach solves the continuity (1) and Navier-Stokes equations (2) for incompressible flow, the equation for the gas temperature (5), the equation for the water vapor concentration (10), and the equations for the position (13), velocity (12), mass (17) and temperature (15) of each droplet. For the fluid, the DNS of the turbulent channel flow is performed using an extension of the in-house pseudo-spectral code as described in Kuerten (2006). Simulations are performed at Reynolds number \( Re = 150 \) (based on friction velocity \( u_\tau \) and half the channel height). The domain has a size of \( 4\pi H \) in streamwise direction and \( 2\pi H \) in spanwise direction, where \( H \) is half the channel height. In the streamwise and spanwise direction 128 Fourier modes have been used, in the wall-normal direction 129 Chebyshev collocation points. For the streamwise, wall-normal, and spanwise directions the notation \( x, y, \) and \( z \) is used, respectively. On the upper (warm) wall a uniform and constant heat flux is supplied to the channel, which is equal to the heat flux extracted from the lower (cold) wall in order to conserve the total thermal energy of the system. This implies that Neumann boundary conditions for temperature are applied.

The equations for the droplets are integrated in time with the same three-stage Runge-Kutta method as used for the nonlinear terms in the equations for the continuous phase. The fluid properties velocity, temperature and vapor mass density, are calculated at the droplet positions by tri-linear interpolation. It has been checked by comparison with fourth-order accurate interpolation that this method of interpolation is sufficiently accurate for the calculation of statistical properties. The calculation of the coupling terms between fluid and droplets requires special attention if a spectral method is adopted. Fourier transform of the delta functions gives rise to large contributions at high wave numbers. This induces numerical instabilities in case only few droplets are present in the flow. The contributions to high wave numbers are for a large part canceled if the number of droplets is of the same order of magnitude as the number of grid points. Moreover, extra smoothing of the source terms is obtained by distributing the contribution from a droplet over the eight neighboring grid points. For this distribution the same weights are used as for the tri-linear interpolation of the fluid properties to the droplet positions. The coupling terms are integrated explicitly in time, similar to the treatment of the nonlinear terms in the governing equations. Also for the droplets periodic conditions are
applied in the flow periodic directions (spanwise and streamwise), which means that when a droplet leaves the domain in one of these directions, it re-enters the domain from the other side. If a droplet approaches a wall within a distance of its radius, elastic collisions are applied without heat transfer.

The flow is initialized by a statistically stationary turbulent velocity and temperature field obtained from a previous simulation without droplets and with the same boundary conditions for gas temperature. The total number of droplets is 400,000, with an initial diameter $d_i/H = 5.10 \times 10^{-3}$. This corresponds to Stokes number $St = \tau_p^+ = \frac{\tau_p u_{2i}^2}{\nu} = 25$. In this way, the initial volume fraction is so low that droplet-droplet collisions can be ignored but two-way coupling is relevant (Elghobashi, 1994). The initial spatial droplet distribution is random but homogeneous, and the initial droplet velocity and temperature are taken the same as the flow velocity and temperature at the droplet location. The initial water vapor mass density is uniform: its value is chosen in order to have the relative humidity as depicted in Figure 1. Thus, initially in the upper half of the channel evaporation takes place and in the lower half condensation. The saturation pressure changes along the wall-normal direction because of the boundary conditions: the mean temperature of the flow increases with $y$. The initial mean temperature of the fluid and the particles is illustrated in Figure 2.

In the next section results of this simulation are compared with the results of a second simulation without mass transfer but with heat transfer between the phases using the same settings. The results of this simulation are indicated in the figures with the tag ‘solid’.

4. RESULTS

In this section, results of the evaporating and condensing water droplet-laden turbulent channel flow are presented and, whenever relevant, compared with the results of the simulation considering solid droplets with heat exchange with the fluid. In Figure 1 the mean droplet temperature is shown as a function of the wall-normal coordinate at several instances of time. Averages of droplet properties are calculated by subdividing the channel geometry into 128 slabs in wall-normal direction and averaging over all droplets present in each slab. The figure includes results at the initial time, at $t^+ = 80$, which is well within the initial, transient phase in which phase change plays an important role, and at $t^+ = 2400$, in which a statistically steady temperature distribution is reached. At $t = 0$ the fluid temperature $T_{\text{fluid}}$ and the droplet temperature $T_{\text{drop}}$ are the same. In the initial phase, the droplet temperature decreases rapidly where evaporation occurs (upper half of the channel) and increases rapidly where condensation occurs (lower half of the channel). On the contrary, the solid droplets temperature and the fluid temperature which are affected only by convective heat transfer show a slower change. The evaporation (condensation) process subtracts (releases) energy from (to) the droplets because of the latent heat necessary to evaporate (condense). When the stationary state is reached, the mean temperatures of fluid and droplets are almost the same again, because the system is in equilibrium. A remarkable observation is that the mean temperature difference between the two walls is smaller in the equilibrium phase than in the initial condition or in the simulation without mass transfer. We define the Nusselt number for this flow as:

$$Nu = \left( \frac{d\overline{T_g}}{dy}_{\text{wall}} \right) \frac{\Delta \overline{T_g}}{2H}$$

where overbars denote mean values, averaged over the two homogeneous directions of the flow, $\Delta \overline{T_g}$ is the difference in gas temperature between the two walls and the derivative in the numerator is evaluated at the wall. The results show that the Nusselt number, and hence the heat transfer, increases by the presence of evaporating and condensing particles.

The relative humidity and the vapor mass density are shown as a function of the wall-normal coordinate at several values of time in Figure 2 and Figure 3, respectively. Initially, the vapor
mass density is constant. Since the temperature is not constant, the relative humidity is larger than 100% in the lower half and smaller than 100% in the upper half of the channel. This causes condensation and evaporation respectively in the two halves of the channel. As a result, the vapor mass density and relative humidity decrease in the lower half and increase in the upper half. In the equilibrium state the droplet and fluid properties fluctuate around a statistically steady value. In this state the relative humidity is close to 100% throughout the channel and the vapor mass density is lower in the cold half of the channel than in the warm half.

![Figure 1.](image-url)

**Figure 1.** Droplet temperature as a function of wall-normal coordinate for the cases with and without mass transfer between the phases for several instances of time.

The initial value of the vapor mass density is chosen in such a way that the relative humidity is equal to 100% not exactly in the middle of the channel, but at a wall-normal position slightly shifted towards the lower wall. Therefore, and because of the homogeneous random distribution of the droplets, the number of evaporating droplets is larger than the number of condensing droplets, resulting in a decrease of the average fluid temperature of the channel flow, as shown in Figure 1.

The droplet size probability distribution function is plotted at three positions in the channel in the initial phase ($t^+ = 80$ in Figure 4) and in the equilibrium phase ($t^+ = 2400$ in Figure 5). The three positions are in the middle of the channel and close to both walls. In order to compute these pdf’s, the wall-normal direction was divided into 64 bins and for every bin the number of
droplets within a certain size range was counted. The size is normalized with the initial droplet size. At \( t^+ = 80 \) in Figure 4 the distributions are still very sharp (note the horizontal axis scale), but clearly show the effect of phase change. Near the cold wall the mean droplet size is higher than near the warm wall. In the equilibrium phase (Figure 5) the distribution functions have become much broader and, although the difference between the warm and cold wall is still visible, they overlap for the three different positions. The broadening of the distribution functions is caused by the combined effects of evaporation and condensation and droplet migration. At the relatively large Stokes number of the droplets the migration of droplets, e.g. by turbophoresis, is significant. The effect of the initial condition, which its small preference for evaporation, is still visible in the equilibrium size distributions. The average size of all droplets is slightly lower than the initial droplet size.

**Figure 2.** Relative humidity as a function of wall-normal coordinate for the case with mass transfer between the phases for several instance of time.

**Figure 3.** Vapor mass density as a function of wall-normal coordinate for the case with mass transfer between the phases for several instance of time.

**Figure 4.** Droplet size probability distribution function at three positions in the channel in the initial phase \( t^+ = 80 \).

**Figure 5.** Droplet size probability distribution function at three positions in the channel in the equilibrium phase \( t^+ = 2400 \).
5. DISCUSSION AND CONCLUSIONS

In this paper a model for water droplet-laden turbulent flow including heat and mass transfer by convection and phase change is proposed. The model was implemented in a pseudo-spectral in-house DNS code for turbulent channel flow at relatively small Reynolds number, where the droplets are modeled in a Lagrangian way. The effects of phase change have been studied by comparison with results of a simulation in which only heat transfer between the phases is considered. The results show that the initial non-equilibrium state leads to evaporation and condensation in an initial phase. After a longer time an equilibrium state is reached in which the relative humidity is close to 100% over the total height of the channel. In the equilibrium state the heat transfer between the two walls, quantified by the Nusselt number, is larger than in case the droplets are absent or replaced by solid particles with the same physical properties. Apart from mean temperature, vapor mass density and relative humidity, also the droplet size distribution functions are analyzed at various wall-normal coordinates. The combined effect of phase change and migration of droplets leads to a broadening of the size distribution functions. Obviously, the mean droplet diameter is smaller near the warm wall than near the cold wall.

6. ACKNOWLEDGMENTS

This research is supported by the Dutch Technology Foundation STW, applied-science division of NWO (Netherlands Organisation for Scientific Research), and the Technology Program of the Ministry of Economic Affairs of the Netherlands. Part of this work was sponsored by the Stichting Nationale Computerfaciliteiten (National Computing Facilities Foundation, NCF) for the use of supercomputer facilities, with financial support from the Nederlandse Organisatie voor Wetenschappelijk Onderzoek, NWO.

References

BIRD, R. B, STEWART, W. E. & LIGHTFOOT, E.N. 1960 Transport Phenomena.. John Wiley and Sons.

ELGHOBASHI, S. 1994 On predicting particle-laden turbulent flows. Applied scientific research 52, 309–329.

KUERTEN, J. G. M. 2006 Subgrid modeling in particle-laden channel flow. J. Phys. Fluid 18, 025108–1–13.

MASHAYEK, F. 1997 Direct numerical simulations of evaporating droplet dispersion in forced low mach number turbulence. Int. J. Heat Mass Transfer 41, 2601–2617.

MASHAYEK, F. & PANDYA, R. V. R. 2003 Analytical description of particle/droplet-laden turbulent flows. Progress in Energy and Combustion science 29, 329–378.

MAXEY, M. R. & RILEY, J. J. 1983 Equation of motion for a small rigid sphere in a nonuniform flow. J. Phys. Fluid. 26, 886.

SAZHIN, S. S., ABDELGHAFFAR, W. A., KRUTITSKII, P.A., SAZHINA, E. M. & HEIKAL, M. R. 2005 New approaches to numerical modelling of droplet transient heating and evaporation. Int. J. Heat Mass Transfer 48, 4215–4228.

WANG, Y. & RUTLAND, C. J. 2006 Direct numerical simulation of turbulent flow with evaporating droplets at high temperature. Heat Mass Transfer 42, 1103–1110.