Three-dimensional matter-wave interferometry of trapped single ion

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We report on a demonstration of three-dimensional Ramsey interferometry with a trapped $^{171}\text{Yb}^+$ ion. We applied a momentum kick to the ion in a direction diagonal to the trap axes to initiate three-dimensional motion using a mode-locked pulse laser. The interference signal was analyzed theoretically to demonstrate three-dimensional matter-wave interference. This work paves the way to realizing matter-wave interferometry using trapped ions.

The wave nature of matter offers the potential of measuring physical quantities with high precision. Matter-wave interferometers normally exploit entanglement between the internal state and the motional state of matter. The accumulated matter-wave phases for different paths are derived from the interference signal of the internal states after the closing pulse of the interferometer. Since atoms and ions provide us with exquisite control of individual quanta using electronic, magnetic and optical means, they have been widely used as wave-like matters in many sensing applications. Neutral atom systems have been extensively used and there are a large number of demonstrations of precision sensing such as in measurement of the relativistic effects in electromagnetic interactions [1–3], atomic and molecular properties [4, 5], and measurement of inertial displacement [6–11].

In contrast, a quantum sensing based on a matter-wave interference with trapped ions has so far not been demonstrated. In an ion trap, the Schrödinger cat state, which is the quantum superposition of classically distinct states, was realized for the first time for a laser-cooled ion prepared in the motional ground state [12]. Later, a scheme to create the cat state using ultrashort pulses from a mode-locked laser was realized, and spin-motion entanglement was created on a short time scale [13–16]. This has also been achieved with ions in the thermal regime [17–19]. Campbell and Hamilton recently proposed a scheme to utilize a trapped ion in such a Schrödinger cat state for a precise rotation sensing [20]. In their proposal, an ion undergoing two-dimensional circular motion is utilized to construct a Sagnac interferometer. While matter-wave interference of ions moving along one of the symmetry axes of the trap has been realized, interferometry for ions in motion along multiple symmetry axes is an important next step for the new types of sensing applications.

In this Letter, we demonstrate a matter-wave interferometry of a trapped ion in a three-dimensional motion, initiated by a momentum kick in a direction diagonal to any of the trap symmetry axes. We applied laser pulses to the ion, which puts it into a superposition state made up of the original spin state and motion, and the opposite spin state with additional momentum due to the momentum kick. This spin-motion coupling is applied along the three symmetry axes of the trap simultaneously, and half of the ion wave packet travels with a harmonic potential relative to the other half of the ion wave packet in the original spin and motion state. In the experiment, we applied second spin-motion coupling pulse after the evolution time to close the interferometer. When the evolution time of this Ramsey type interferometer is an integer multiple of the trap period, constructive interference is observed. Since trap frequencies for the three axes are different, the interference signal shows a complicated interrogation-time dependence. A slight change in the frequency of one of the trap axes causes a sensitive change in the interference signal, indicating that the interference arises from ion motion in three dimensions. We theoretically analyzed the interference signal and found good agreement between the measured data and the theory.

We trapped a single $^{171}\text{Yb}^+$ ion using a linear Paul trap. We used a mode-locked frequency-tripled Nd:YVO$_4$ laser with a pulse duration of $\tau \sim 15$ ps, a center wavelength of $2\pi/k = 355$ nm, and a repetition rate $f_{\text{rep}} = 120.47$ MHz [15, 16]. The laser drove stimulated Raman transitions between the two hyperfine ground states $|\downarrow\rangle = |F = 0, m_F = 0\rangle$ and $|\uparrow\rangle = |F = 1, m_F = 0\rangle$ to give momentum kicks to the ion. Acousto-optic modulators (AOMs) were placed in both optical paths and were used to tune the frequency difference between the two beams to match the hyperfine splitting $\omega_{\text{HF}} = n\omega_{\text{rep}} + \omega_{\text{AOM}}$, where $\omega_{\text{AOM}}$ is the frequency shift generated by the AOMs. We chose $n = 102$, and $\omega_{\text{AOM}} = 354$ MHz. Each optical path had the same length by adjusting the variable delay of the pulse train through each path so as to arrive at the ion simultaneously. All of $355$ nm beam paths were covered with a case to prevent fluctuations of the beam spot position due to air flow.

By introducing the pulse trains from opposite directions along the axial trap axis shown in the inset of Fig. 1(b), a one-dimensional momentum kick was given to the ion along the axial trap axis. Figure 1(a) depicts the position-momentum phase space in the one-dimensional
interference experiment. First, the ion was prepared in a \( |\downarrow\rangle \) state using Doppler cooling and optical pumping. Therefore, the ion motional state is thermal, and is expressed as a superposition of coherent states. For simplicity, we express the initial state as a coherent state \( |\downarrow, \alpha_z\rangle \) (step 1 in Fig. 1(a)). Next, we applied a \( \pi/2 \) pulse using a 200 ns long pulse train. This pulse train is in the strong excitation regime \[21\], and applies the same momentum kick for each coherent state contained in the thermal state. The momentum kick \( i\eta_z \) causes the ion to transfer to a superposition state \( (|\downarrow, \alpha_z\rangle + |\uparrow, \alpha_z + i\eta_z\rangle)/\sqrt{2} \) (step 2). After an interrogation time \( T_{\text{int}} \) (step 3), we applied a \( \pi/2 \) pulse again, which partially transfers the \( |\downarrow\rangle \) state to a \( |\uparrow\rangle \) state with added momentum \( i\eta_z \), and transfers the \( |\uparrow\rangle \) state to a \( |\downarrow\rangle \) state with added momentum \(-i\eta_z \) (step 4). Finally we detected the probability of the ion being in the \( |\uparrow\rangle \) state.

First, the Raman pulse was irradiated along the axial trap axis with a counter-propagating configuration. Figure 1(b) shows the transition probability from state \( |\downarrow\rangle \) to state \( |\uparrow\rangle \) as a function of the interrogation time between two \( \pi/2 \) pulses for a one-dimensional momentum kick. As can be clearly seen, constructive interference appears at integer multiples of the trap period \( (T_z) \) determined by the axial trap frequency \( \omega_z = 2\pi \times 475 \text{ kHz} \). After the second \( \pi/2 \) pulse for \( \omega_z T_{\text{int}} = 2m\pi \) \( (m \text{ is the integer in step 4 in Fig. 1(a))}, \) the transition probability reaches 0.8 when the two wave packets match, but is only around 0.5 when the wave packets did not match for \( \omega_z T_{\text{int}} \neq 2m\pi \). In Fig. 1(b), the measured transition probability for the peaks is smaller than unity, because of imperfect Rabi oscillations due to the finite temperature.

Next, under the same trap conditions, we changed the direction of the momentum kick from along the axial trap axis to along the direction diagonal to all trap axes. The measured matter-wave interference results are shown in Fig. 1(c). Peaks appear at integer multiples of the trap period, similar to the case for one-dimensional matter-wave interference in Fig. 1(b). However, peak height variations from 0.6 to 0.9 are observed in this configuration, in contrast to the results shown in Fig. 1(b); this is a clear indication of three-dimensional interference.

In order to clarify this complicated motion, we theoretically investigated three-dimensional matter-wave in-
The probability $P$ of detecting the ion in the $|\uparrow\rangle$ state after the interference process can be expressed as:

$$P = \frac{1}{2} + \frac{1}{2} P_x P_y P_z,$$

where $P_j (j = x, y, z)$ describes the wave function overlap along the $j$ axes:

$$P_j = e^{-2\eta_j (1 - \cos(\omega_j T_{\text{int}}))} \cos(\eta_j^2 \sin(\omega_j T_{\text{int}})).$$

Here $\eta_j$ is the mean phonon number, $\eta_j = k_j \sqrt{\hbar/2m\omega_j}$ is the Lamb-Dicke parameter, and $\omega_j$ is the trap frequency, in the $j$ direction.

So as to investigate the result of three dimensional interference in detail, we observed the interference with a longer interrogation time. Figure 2(a) displays the results for interference initiated by the diagonal momentum kick. The red solid line shows the results of fitting using Eq. (1). We used $\bar{n}$ determined based on the ion temperature (0.94 mK) obtained from the fitting results for the one-dimensional experiment. Three trap frequencies $\omega_x, \omega_y$, and $\omega_z$, the amplitude of the peaks, and the offset were taken as free parameters. From the fitting results, the trap frequencies were obtained to be $(\omega_x, \omega_y, \omega_z) = 2\pi \times (1.35, 1.23, 0.492)$ MHz, respectively. These are consistent with the measurement results using sideband spectroscopy. In order to take into account uncertainties in the intensity of the $\pi/2$ pulse, we introduced the amplitude and the offset of the interference as free parameters. The complex interference pattern shows good agreement with the theoretical predictions.

Figures 2(b–e) show phase-space diagrams for the ion for the timings shown in Fig. 2(a). At time (b), $T_{\text{int}}$ is an integer multiple of the trap period along the $y$ axis ($T_y$), but not along the $x$ and $z$ axes ($T_x, T_z$). Constructive interference is not observed because the wave packets are in different positions along the $x$ and $z$ axes in the harmonic potential.

When $T_{\text{int}}$ is an integer multiple of $T_y$ (at time (c)), constructive interference is observed even though $T_{\text{int}}$ is not an integer multiple of $T_x$ and $T_z$. The difference between times (b) and (c) arises from the fact that the

![Graph and diagrams](image-url)
trajectory along the z axis is bigger than that along the x and y axes, due to the difference in the strength of the momentum kick and the trap confinement strength [23]. The peak at time (d) is slightly larger than that at time (c). This can be naturally understood because $T_{\text{int}}$ is an integer multiple of $T_y$ and $T_z$. At time (e) when $\pi_{\text{int}}$ is the same as all three trap periods, the strongest peak appears due to perfect wave packet overlap.

![Graph](image)

**FIG. 3.** Axial trap frequency dependence of three-dimensional interference. The axial trap frequency is shifted by changing the voltage applied to the ion trap. Blue points represent an average of 900 or 600 measurements, and red solid lines are theoretical fits using Eq. (1). (a) $\omega_z = 2\pi \times 0.492$ MHz. (b) $\omega_z = 2\pi \times 0.484$ MHz. (c) $\omega_z = 2\pi \times 0.474$ MHz.

To further investigate the three-dimensional interference, $\omega_z$ values of $2\pi \times 492$ kHz, $2\pi \times 484$ kHz, and $2\pi \times 474$ kHz were used, as shown in Fig. 3. The radial trap $\omega_x$ and $\omega_y$ values through this experiment were $2\pi \times 1.34$ MHz and $2\pi \times 1.23$ MHz, respectively. The red, green, and purple diamonds at the top of each figure show times when integer multiples of $T_x$, $T_y$ and $T_z$, respectively, are obtained. Please pay attention to the peaks around 8.2 µs and 10.2 µs (indicated with pale purple shading). The peak near 8.2 µs decreases as $\omega_z$ increases. At $\omega_z = 2\pi \times 492$ kHz, a large interference signal is observed because the timing of the 4th period of $T_z$, the 11th period of $T_x$, and the 10th period of $T_y$, are matching. By changing $\omega_z$ to $2\pi \times 474$ kHz, the timing of the 4th period of $T_z$ does not match, and the interference signal strength is reduced. In contrast, the peak near 10.2 µs increases as $\omega_z$ increases because the timing of the 5th period of $T_z$, the 14th period of $T_x$, and the 13th period of $T_y$ match at $\omega_z = 2\pi \times 474$ kHz.

![Graph](image)

**FIG. 4.** Measured interference fringes for long interrogation times. Blue points represent an average of 300 measurements, and red solid lines are theoretical fits using Eq. (1) with an attenuation term $e^{-\gamma t}$.

We next investigated the coherence time for three-dimensional ion motion. Figure 4 shows the three-dimensional interference for various ranges of $T_{\text{int}}$. In order to take into account the coherence time, we added an attenuation factor $e^{-\gamma t}$ into the second term of Eq. (1). Each red solid curve in Fig. 4 is a best fit to the experimental results for each interrogation time for a fixed ion temperature and attenuation coefficient $\gamma$. The results indicate that the decoherence time is 6.2 ms. Even for large $T_{\text{int}}$ values up to 600 µs, the interference signal is well reproduced by the theoretical curves, indicating that the coherence of ion motion in all directions decays over the same time scale.

In summary, we have demonstrated three-dimensional matter-wave interference for a trapped single $^{171}$Yb$^+$ ion. We applied a momentum kick to the ion in the direction diagonal to the trap axes to induce three-dimensional ion motion. We investigated the interference signal as a function of interrogation time between two $\pi/2$ pulses. We theoretically predicted the interference pattern and found good agreement with the experimental result. The results of the present study open the possibility of applying a trapped ion in a multi-dimensional motion for a sensing application such as a rotation sensing [20]. Since the
dynamics of the ion during the interrogation time reflects the imperfections of matter-wave interferometers [24], the full understanding of the free time evolution of an ion is an important ingredient for improving the sensitivity of the matter-wave interferometers.

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