Heat transport and spin-charge separation in the normal state of high temperature superconductors

Qimiao Si
Department of Physics and Astronomy, Rice University, Houston, TX 77251-1892

Hill et al. have recently measured both the thermal and charge conductivities in the normal state of a high temperature superconductor. Based on the vanishing of the Wiedemann-Franz ratio in the extrapolated zero temperature limit, they conclude that the charge carriers in this material are not fermionic. Here I make a simple observation that the prefactor in the temperature dependence of the measured thermal conductivity is unusually large, corresponding to an extremely small energy scale $T_0 \approx 0.15$ K. I argue that $T_0$ should be interpreted as a collective scale. Based on model-independent considerations, I also argue that the experiment leads to two possibilities: 1) The charge-carrying excitations are non-fermionic. And much of the heat current is in fact carried by distinctive charge-neutral excitations; 2) The charge-carrying excitations are fermionic, but a subtle ordering transition occurs at $T_0$.

Spin-charge separation has long been proposed to describe the normal state of the high temperature superconductor, and continues to be the subject of extensive current work. It has, however, received no unambiguous experimental support, many indirect evidences notwithstanding. One natural means to probe spin-charge separation is to compare the temperature dependences of spin transport and charge transport properties. Spin injection experiments have recently been carried out in the cuprates; these experiments, however, have yet to yield quantitative information about spin transport. A less direct alternative to spin transport in this context is heat transport, if the electronic contribution can be unambiguously separated from the phononic one. While such a separation is easy to achieve in highly conductive metals, it is in general very difficult for strongly correlated metals.

Very recently, Hill et al. have extracted the electronic contribution to the thermal conductivity ($\kappa$) in the normal state of an optimally electron-doped PCCO. This is achieved at very low temperatures, where scattering is dominated by elastic processes. The normal state arises in a magnetic field applied along the $c$-axis $H \approx 13$ T, which is above the bulk upper critical field $H_{c2}$. (The thermal conductivity is field-independent at $H > H_{c2}$.) At roughly the same magnetic field, the electrical conductivity ($\sigma$) is essentially temperature-independent, up to small weak-localization-like corrections. The authors take advantage of the fact that the measured zero-field thermal conductivity $\kappa(H = 0)$ goes to zero in approximately a $T^3$ fashion, in sharp contrast to what is generally expected for a disordered d-wave superconductor. They assume that $\kappa(H = 0)$ is entirely due to phonons, and identify the electronic contribution to the normal-state thermal conductivity as $\kappa_e = \kappa(H \approx 13$ T) − $\kappa(H = 0)$. $\kappa_e$ is found to have an asymptotic low temperature form

$$\lim_{T \to 0} \kappa_e \sim T^{\alpha+1}$$

where $\alpha \approx 2.6$. This is in strong violation of the Wiedemann-Franz (WF) law: According to this law, $\alpha$ should be equal to zero reflecting the linear temperature dependence of the specific heat of the fermionic quasiparticles in a Fermi liquid. From this perspective, the experiment implies a missing entropy. The authors conclude that the carriers of the charge current are not fermionic.

Here I point out that the prefactor in the temperature dependence of the thermal conductivity should be considered to be unusually large, if the dominant contribution to $\kappa_e$ were due to non-fermionic charge-carrying excitations. To see this, we note that the measured $\kappa_e$ becomes larger than

$$\kappa_{WF} \equiv L_0 T \sigma \quad (2)$$

at a temperature $T_0 \approx 0.15 K$. (Here $L_0 \equiv \pi^2 k_B^2 / 3e^2$ is the Lorenz number of a Fermi liquid.) Namely, the experimental data can be cast in the form,

$$\lim_{T \to 0} \frac{\kappa_e}{\kappa_{WF}} \approx \left( \frac{T}{T_0} \right)^\alpha \quad (3)$$

Using $\sigma = e^2 (dn/d\mu) D_{charge}$, where $dn/d\mu$ and $D_{charge}$ are the electronic compressibility and charge diffusion constant, respectively, we can rewrite

$$\kappa_{WF} = \frac{L_0 T(e^2/m) n_{charge}}{N} \left( \frac{dn/d\mu}{D_{charge}/N_0} \right)$$

where the subscript 0 labels quantities in the absence of interactions. It follows that $\kappa_{WF}$ does not explicitly depend on the velocity and, equivalently, the bandwidth, of the charge-carrying excitations.

If the measured $\kappa_e$ is dominated by the contribution of the charge-carrying excitations, we can express $\kappa_e/\kappa_{WF}$ in the following general form:

$$\frac{\kappa_e}{\kappa_{WF}} = \frac{C_0}{C_0^0} \frac{N_0}{(dn/d\mu)} \frac{D_{heat}}{D_{charge}} \quad (5)$$
where $D_{\text{heat}}$ is the entropy diffusion constant and $C_v$ the specific heat due to the charge-carrying excitations. In this case, the scattering times for both the heat and charge currents reflect the elastic scattering time of the same excitation. As a result, the ratio $D_{\text{heat}}/D_{\text{charge}}$ is expected to depend only on equilibrium interaction parameters. These interaction parameters should mostly cancel out in the product $\frac{D_{\text{heat}} N_0}{D_{\text{charge}}(dn/d\mu)}$. (In the Fermi liquid theory with s-wave scattering, this product is equal to $m/m^*$ making $\frac{\kappa_e}{\kappa_{WF}}$ exactly equal to unity.) The temperature dependence of the ratio $\frac{\kappa_e}{\kappa_{WF}}$ then describes, in a dimensionless form, the temperature dependence of the specific heat due to the charge-carrying excitations. It is then clear that the prefactor of the asymptotic low-temperature dependence of $\kappa_e$ should be considered to be anomalously large: The effective bandwidth of the charge-carrying excitations would be of order $0.15kT$. Such a small bare scale is essentially impossible.

We are then forced to associate $T_0$ with a collective scale. Unlike for a bare scale such as a bandwidth, it is entirely reasonable for a collective scale to be this small. On purely phenomenologically grounds, there are two possible interpretations as illustrated in Figs. 1a and 1b.

Distinctive charge-neutral excitations in the normal state: If the charge-carrying excitations indeed have non-fermionic statistics, such that their contribution to the specific heat has a higher than linear temperature dependence, much of the heat current is necessarily carried by some distinctive charge-neutral excitations. This conclusion is reached as follows. In this picture, the contribution to the thermal conductivity due to the charge-carrying excitations would have to be

$$\frac{\kappa_{\text{charge}}}{\kappa_{WF}} \approx \left( \frac{T}{W} \right)^\beta$$

(6)

where the exponent $\beta > 0$ reflects both the statistics and the dispersion of the charge-carrying excitations, and $W$ is the effective bandwidth. Since $W$ is expected to be much larger than $T_0$, in the measured temperature range $\kappa_{\text{charge}}$ is necessarily much smaller than the measured $\kappa_e$ as illustrated in Fig. 1a – hence our conclusion. We note that a microscopic theory for such a picture needs to produce a temperature-independent carrier concentration relevant to transport and, at the same time, a specific heat that goes as $T^{1+\beta}$.

Experimentally, the plot of $\kappa_e/T$ versus $T$ starts to deviate from the low temperature power-law form as the temperature is increased through $T_0$. Within our picture, $T_0$ would be some collective temperature scale associated with a transition that affects mostly the charge-neutral excitations. It follows that the temperature dependence of $\kappa_{\text{neutral}}$ is very different from that of $\kappa_{\text{charge}}$, reflecting the separation of electrons into the charge-carrying and charge-neutral excitations.

Measurement of the heat transport cannot tell us about the quantum number of the charge-neutral excitations. A natural candidate, of course, would be that these are spin-carrying excitations. Measurement of the spin transport would help clarify the nature of these charge-neutral excitations.

Ordering transition in the normal state: An alternative possibility is that, the charge-carrying excitations above $T_0$ are in fact fermions. A subtle ordering transition takes place around $T_0$. Again, it is natural for $T_0$ to be so small since it is a collective scale.

In this picture, the transition around $T_0$ leads to a sharp drop in the specific heat but leaves the electrical conductivity largely unaffected. In addition, the fermionic excitations are gapped out at $T << T_0$. The reason that $\kappa_e/T$ goes to zero in the asymptotic low temperature limit is presumably related to what is responsible for the vanishing $\kappa(H = 0)/T$. While it is in principle possible, this picture, in its simple form, requires some delicate balance between the quasiparticle contribution and collective contribution to the charge current such that the electrical conductivity is essentially unchanged as the temperature is lowered through $T_0$. The collective contribution is of relevance here since disorder is not very strong and pinning is expected to be weak. Whether such a balance can be achieved in specific microscopic models remains to be seen.

Experimentally, $\kappa_e/T\sigma$ at temperatures above $T_0$ is indeed close to $L_0^2$, though somewhat larger than, $L_0$. 

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**FIG. 1.** Schematics of two possible pictures. The solid lines give the ratio of the total electronic thermal conductivity, $\kappa_e$, to $\kappa_{WF}$, defined in Eq. (5). In a) the dotted line describes the contribution from charge-carrying excitations; the remaining contribution comes from charge-neutral excitations. In b), the excitations are fermionic and the low temperature behavior reflects an ordering transition around $T_0$. 

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One way to differentiate these two pictures is to study the behavior of $\kappa_e$ at temperatures high compared to $T_0$ (but still low enough so that elastic scatterings dominate). In the first picture, the mean free path of the charge-neutral excitations is in general very different from that of the charge-carrying excitations. As illustrated in Fig. 1a, we expect $\kappa_e/\kappa_{WF}$ to be very different from unity for $T >> T_0$. In the second picture, on the other hand, we expect $\kappa_e/\kappa_{WF} \approx 1$ for $T >> T_0$; see Fig. 1b. Unfortunately, the experiment in PCCO is limited to temperatures not too high compared to $T_0$. Analogous experiments in other cuprates may help clarify the situation.

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