Isotope effect on gyro-fluid edge turbulence and zonal flows

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Abstract

The role of ion polarisation and finite Larmor radius on the isotope effect on turbulent tokamak edge transport and flows is investigated by means of local electromagnetic multispecies gyro-fluid computations. Transport is found to be reduced with the effective plasma mass for protium, deuterium and tritium mixtures. This isotope effect is found for both cold and warm ion models, but significant influence of finite Larmor radius and polarisation effects are identified. Sheared flow reduction of transport through self generated turbulent zonal flows and geodesic acoustic modes in the present model (not including neoclassical flows) is found to play only a minor role on regulating isotopically improved confinement.

Keywords: isotope effect, edge turbulence, zonal flows, particle transport

1. Introduction

The isotope effect in tokamak plasmas refers to experimentally measured improved confinement properties with increasing ion mass $m_i$, as observed from hydrogen (H) to deuterium (D) and tritium (T) plasmas [1–3]. While most present tokamak experiments for practical reasons run with deuterium, actual fusion plasmas like in ITER will consist of the D-T plasma fuel mixed with the Helium ash.

Whether specific theory based transport scaling models could, for certain underlying instabilities e.g. driven by trapped electron modes (TEM), roughly account for the experimental scalings has been controversial [4–6]. In any case the observed improvement in confinement is apparently inconsistent with primitive mixing length approximations of the turbulent gyro-Bohm cross-field diffusivities $\chi \sim \rho_i \sim \sqrt{m_i}$, where $\rho_i \sim \sqrt{m_i}$ is the ion gyro-radius.

A region of particular interest for transport in tokamaks is the plasma edge around the last closed magnetic flux surface. Turbulence and transport in the edge pedestal region crucially determine the low-to-high confinement transition, the pedestal width, and further transport of particles and heat across the scrape-off layer onto plasma facing components.

The isotope effect on computations of three-dimensional tokamak edge turbulence has been studied initially with a drift-Alfven fluid model, where the turbulent particle and heat transport have been found to be only weakly dependent on ion mass [7]. This was attributed to two competing mechanisms: a higher ion mass leads to more adiabatic electron dynamics [8] and consequently a weaker radial transport (for a collisional coupling coefficient $C \sim 1/m_i$). This effect more or less cancels the basic gyro-Bohm transport which is proportional to $\sqrt{m_i}$. The sensitivity was found to be larger for collisional electrostatic drift wave transport compared to electromagnetic drift-Alfven transport [7].

Initial computations on the isotope effect on ion temperature gradient (ITG) core turbulence revealed a trend for favourable isotope scaling for the ion thermal diffusivity, which had partly been attributed to the ITG growth rate decreasing with isotope mass [9, 10].

Recent experiments [11], theoretical work [12] and gyro-kinetic computations of ITG and TEM turbulence [13, 14] have investigated the isotope effect on turbulent and/or neoclassical zonal flows, and on geodesic acoustic modes (GAMs) [15]. Increasing isotope mass has been found to lead to stronger zonal flows and GAMs, which in turn reduce the radial turbulent particle transport magnitude.
In this work we specifically focus on the influences of ion polarisation and ion temperature through finite Larmor radius (FLR) effects on drift-Alfvén edge turbulence, zonal flows and GAMs by means of three-dimensional isothermal gyro-fluid multi-species simulations for various isotopic compositions.

We introduce the model and numerical code employed in section 2. The mass dependence of our model is discussed in section 3. Results form numerical simulations are presented in section 4, where we start with a general observation of the isotope effect which is then studied more systematically by considering various parts of the model independently. Conclusions and an outlook are presented in section 5.

2. Local isothermal gyro-fluid model and code

In the local (delta-f) isothermal multi-species gyro-fluid model, the normalised equations for the fluctuating gyro-center densities \( n_s \) are [16]
\[
\frac{dn_s}{dt} = -g_s \frac{\partial \phi}{\partial y} - \nabla \mu_{\parallel} + K(\phi_s + \tau n_s),
\]
where the index \( s \) denotes the species with \( s \in \{ e, i, j \} \) for the main plasma components (electrons with index \( e \) and main ions with index \( i \)) plus one or more additional ion species with index \( j \) (for simulations of mixtures of e.g. \( i = \) deuterium and \( j = \) tritium).

The parallel velocities \( u_{\parallel s} \) and the vector potential \( A_{\parallel s} \) evolve according to [16]
\[
\frac{dA_{\parallel s}}{dt} + \varepsilon_s \frac{du_{\parallel s}}{dt} = -\nabla(\phi_s + \tau n_s) + 2\varepsilon_s \tau_s K(u_{\parallel s}) + \beta_s \tau_s \frac{\partial A_{\parallel s}}{\partial y} = C\mu_{\parallel s},
\]
and are closed by the gyro-fluid polarisation equation
\[
\sum_s a_s \left[ \Gamma_n \left( \frac{\n_0}{\n_s} - 1 \right) \phi_s \right] = 0,
\]
and Ampere’s law
\[
-\nabla^2 A_{\parallel s} = J_0 = \sum_s a_s u_{\parallel s}.
\]
The gyro-screened electrostatic potential acting on the ions is given by
\[
\phi_s = \Gamma_s (\rho_s^2 \kappa_s^2)^{\frac{1}{2}} \phi_k,
\]
where \( \phi_k \) are the Fourier coefficients of the electrostatic potential. The gyro-average operators \( \Gamma(b) \) and \( \Gamma(b) = \Gamma_0^{1/2}(b) \) correspond to multiplication of Fourier coefficients by \( k_0(b)e^{-b} \) and \( k_0(b)2e^{-b/2} \), respectively, where \( k_0 \) is the modified Bessel function of zeroth order and \( b = \rho_s^2 \kappa_s^2 \). We here use approximate Padé forms with \( \Gamma_0(b) \approx (1 + b)^{-1} \) and \( \Gamma_1(b) \approx (1 + b/2)^{-1} \) [17].

The perpendicular \( E \times B \) adveotive and the parallel derivative operators for species \( s \) are given by
\[
\frac{d_s}{dt} = \frac{\partial}{\partial t} + \{ \phi_s \},
\]
\[
\nabla_{\parallel} = \frac{\partial}{\partial x} - \beta \{ A_{\parallel s} \},
\]
where we have introduced the Poisson bracket as
\[
\{ f, g \} = \left( \frac{\partial f}{\partial x} - \frac{\partial g}{\partial y} \right) dt.
\]
For the magnetic field geometry we employ a radially local three-dimensional flux-tube model with locally perpendicular flux-tube coordinates \( \{ x, y, z \} \), where \( x \) is a (radial) flux-surface label, \( y \) is a (perpendicular) field-line label and \( z \) is the position along the magnetic field-line. In circular toroidal geometry with major radius \( R \), the curvature operator is given by
\[
K = \omega_B \left( \sin z \frac{\partial}{\partial x} + \cos z \frac{\partial}{\partial y} \right),
\]
where \( \omega_B = 2L_c/R \), and the perpendicular Laplacian is given by
\[
\nabla_{\perp}^2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right).
\]
Flux surface shaping effects in more general tokamak or stellarator geometry [18, 19] are here neglected for simplicity. Spatial scales are normalised by the drift scale \( \rho_0 = \sqrt{T_m m_e / e B} \), where \( T_m \) is a reference electron temperature, \( B \) is the reference magnetic field strength and \( m_0 \) is a reference ion mass, for which we use the mass of deuterium \( m_d = m_0 \). The temporal scale is normalised by \( c_0 / L_c \), where \( c_0 = \sqrt{T_e / m_0} \), and \( L_c \) is the generalized profile gradient scale length.

The main species dependent parameters are
\[
a_s = \frac{Z_s n_0}{n_s}, \quad \tau_s = \frac{T_i}{T_e}, \quad \mu_s = \frac{m_s}{Z_s m_0}, \quad \rho_s^2 = \mu_s \tau_s \rho_0^2, \quad \epsilon_s = \frac{\mu_s (q R)^2}{L_c},
\]
setting the relative concentrations, temperatures, mass ratios and FLR scales of the respective species. \( Z_s \) is the charge state of the species \( s \) with mass \( m_s \) and temperature \( T_e \).

The local gradient scale lengths are fixed for all species, \( n_s = \left[ \partial \ln n_s / \partial x \right] = 1 / L_{ns} \), where \( n_s(x) \) is the respective background density, satisfy the electrically neutral equilibrium condition \( \sum_s a_s n_s = 0 \). In this local flux-tube model the background density gradient is stationary and constitutes a fixed drive (in contrast to ‘global’ flux-driven models with consistently evolving profiles, compare for example [20]). The present gyrofluid model is based on \( \delta - f \) gyrokinetics, which means that only small fluctuations of the densities and potentials are evolved on a static background. This is well valid for mid-pedestal regions in the closed-field line edge plasma, whereas computations in the scrape-off-layer region rather would require use of a full-\( f \) gyrofluid model [21–23].

The plasma beta parameter
\[
\beta = \frac{4\pi n_p (q R)^2}{B^2 L_c^2},
\]
controls the shear-Alfvén activity, and
\[ C = 0.51 \left( \frac{e_{Li}}{c_0} \right) \frac{m_e}{m_0} \left( \frac{qR}{L_i} \right)^2, \]

mediates the collisional parallel electron response for \( Z = 1 \) charged hydrogen isotopes.

The present computational study is based on the isothermal electromagnetic multi-species gyro-fluid model [16, 24]. According to its isothermal character, the model does not include temperature gradient driven (ITG or ETG) instabilities, which is usually regarded as an acceptable approximation for L-mode edge plasmas. Core turbulence studies or parameters close to H-mode conditions would rather require the use of a full thermal model. Thus in the present study no ITG mode specific characteristics like an instability threshold or related effects (like the formation of avalanche induced staircases) are included.

The theoretical and computational basis of plasma edge turbulence models and simulations is for example reviewed in [25]. Our local flux-tube code (‘TOEFL T3L’) employs a globally consistent flux-tube geometry [26] with a shifted metric treatment of the coordinates [27] to avoid artefacts. Global flux-tube consistency requires periodic boundary conditions in the (radial) \( x \)-direction. Gyrofluid densities are adapted at the \( x \)-boundaries to ensure zero vorticity radial boundary conditions for finite ion temperatures (\( \tau_i > 0 \)). Global flux-tube consistency requires periodic boundary conditions in the \( y \)-direction and quasi-periodicity (shifted) in \( z \)-direction [26].

We use an Arakawa–Karniadakis numerical scheme for advancing the moment equations [28–30]. The present multi-species code [20] has been cross-verified in the local cold-ion limit with the tokamak edge turbulence standard benchmark case of Falchetto et al [31], and with the results of finite Larmor radius SOL blob simulations of Madsen et al [32].

3. Ion isotope mass dependence in the multi-species gyro-fluid model

The isotope masses enter the multi-species gyro-fluid equations in several terms. The term with the advective derivative of the parallel velocity in (2) is proportional to \( \mu_i \) via \( \epsilon_F \). The different ion masses also enter via the gyro-radius \( R_i = \sqrt{\mu_i T_e} \rho_i \). This sets the magnitude of gyro-averaging operators depending on \( b = \mu_i^2 k_\perp^2 = -\mu_i \tau_i V_\perp^2 \). Explicitly, mass dependent effects in the polarisation equation (3) can be seen when expanding \( \Gamma_0 = 1 = (1 + b)^{-1} - 1 = -b(1 + b) = \mu_i \tau_i V_\perp^2 \Gamma_0 \). For cold ions the polarisation equation thus reduces to

\[ \sum_s a_s [n_s + \mu_i V_\perp^2 \phi] = 0, \quad (5) \]

or

\[ \mu_{eff} V_\perp^2 \phi = - \sum_s a_s n_s, \quad (6) \]

which depends on the effective mass \( \mu_{eff} = \sum_s a_s \mu_i \). For a single-ion plasma \( \mu_{eff} = a_e \mu_e + a_i \mu_i \approx a_i \mu_i = 1 \), when this ion mass is taken as the reference mass. In a multi-ion-species plasma the relative ion masses and concentrations determine the effective polarisation through \( \mu_{eff} \).

For illustration of the isotope effect on polarisation we construct a multi-species quasi-2D gyro-fluid model, which corresponds to the classic Hasegawa–Wakatani (HW) model [33]. Here we assume a constant magnetic field, ignore curvature and electromagnetic effects and consider cold ions. The parallel response then reduces to \( \nabla_\parallel [\mu_i (n_s - \phi)] \). The multi-species gyro-fluid density equations in this limit are

\[ \frac{d}{dt} n_s = - g \frac{\partial \phi}{\partial y} - \nabla_\parallel [\mu_i (n_s - \phi)]. \quad (7) \]

We multiply (7) each with \( a_s \) and sum up all equations:

\[ \frac{d}{dt} \sum_s a_s n_s = - \left( \sum_s a_s g \frac{\partial \phi}{\partial y} \right) - \sum_s a_s \nabla_\parallel [\mu_i (n_s - \phi)]. \]

Using \( \sum_s a_s g = 0 \), inserting (6) and neglecting parallel ion dynamics yields (\( n_{\parallel i} = 0 \))

\[ \mu_{eff} \frac{d}{dt} \nabla_\perp^2 \phi \approx \nabla_\parallel [\mu_i (n_{\parallel i} - \phi)] \approx \nabla_\parallel [\mu_i n_{\parallel i}]. \]

Defining \( D \equiv k_\perp^2/C \) as HW dissipative coupling coefficient (where \( C \) is normalised to the reference ion mass) and the electric drift vorticity \( \Omega = \nabla_\perp^2 \phi \), the reduced model equations read:

\[ \frac{d}{dt} n_e = D(\phi - n_e) - g \frac{\partial \phi}{\partial y}, \quad (8a) \]

\[ \mu_{eff} \frac{d}{dt} \Omega = D(\phi - n_e). \quad (8b) \]

The remaining gyro-fluids are passively advected while the potential \( \phi \) can be determined from \( \Omega \). The mass of the secondary ion species thus enters via the effective polarisation mass in the vorticity equation: higher effective mass reduces the perpendicular inertial response and weakens the turbulent vorticity drive maintained by parallel dissipative coupling. The perpendicular wave number spectrum is rescaled by \( k_\perp \mu_{eff} = k_\perp \mu_{eff} = k_\perp = -k_\parallel \delta \phi \), adapting to the drift scale given by the effective mass.

This can be illustrated by linear analysis of the multi-species HW equations, which results in a real drift wave frequency of \( \omega = \omega_0 \sqrt{1 + K^2} \) and an approximate growth rate (in the weakly nonadiabatic limit) \( \gamma \approx (\omega/\omega_0)^2 \) with \( \omega_0^2 = D(1 + K^2)/K^2 \). The drift wave frequency is reduced and the growth rate enhanced for increasing effective mass.

Similar to this inertial isotope mass effect on polarisation through the drift scale, we can expect gyro-screening and gyro-averaging FLR effects for warm ions through the gyro-scales.

In summary, the ion isotope mass enters into the isothermal 3d gyro-fluid equations in three ways: (i) Re-scaling of the
effective drift scale \( \rho_0 \rightarrow \sqrt{\mu_{\text{eff}} \rho_0} \) in the ion polarisation entering through the term \((I_0 - 1)\tau_\parallel \) in (3). (ii) Re-scaling of the ion gyro-scales \( \rho_i = \sqrt{\mu_{\text{eff}} \tau_\parallel \rho_0} \) in the gyro-screening and gyro-averaging FLR operators for finite ion temperatures \( \tau_\parallel > 0 \). (iii) Parallel ion response scaled by \( \epsilon_i \sim \mu_i \) in (2).

As for any (of arbitrary multiple) ion species \( |\mu_2| \ll |\mu_1| \) the adiabatic and electromagnetic response is mainly carried by the electrons, we do not expect the third effect to be of major relevance.

In the following we numerically study the relative importance of isotope polarisation and FLR effects with the complete 3d isothermal gyro-fluid model in order to identify their roles on turbulence, transport and flows.

4. Computational results

Nominal reference simulation, and roughly corresponding physical, parameters are listed in table 1. To neglect electromagnetic effects, we also perform simulations with \( \beta = 0 \). All simulations are initialised with the same turbulent bath given by the sum over multiple small-amplitude sinusoidal modes in density fluctuations. All simulations are carried deeply into the non-linearly saturated state which, depending on plasma composition, typically sets in at \( t \sim 200 \) normalized time units. Statistics are sampled over at least 8000 time units in the turbulent state. In the subsequent plots the standard deviation of fluctuations around the mean of the respective time series is used as error bars. Numerical dissipation of the form \( \nu_0 \partial_{zz}^2 - \nu \nabla_\perp^4 \) with \( \nu_0 = 0.01 \) and \( \nu = 0.02 \) is added to each advective derivative.

### Table 1. Numerical and physical parameters.

| \( \beta \) | \( \bar{s} \) | \( \omega_B \) | \( \tau_\parallel \) | \( C \) | \( (qR/L_\parallel)^2 \) |
|------|------|------|------|------|------------------|
| 1    | 2    | 0.01 | 1    | 5    | 18,500           |

| \( n_e (\text{cm}^{-3}) \) | \( T_e (\text{eV}) \) | \( B (\text{T}) \) | \( R (\text{cm}) \) | \( L_\parallel (\text{cm}) \) | \( q \) |
|------|------|------|------|------------------|------|
| 3 \times 10^{13} | 70   | 2.5  | 165  | 4.25            | 3.5  |

### Table 2. Values of effective mass with corresponding plasma compositions.

| \( \mu_{\text{eff}} \) | \( 0.6 \) | \( 0.9 \) | \( 1.1 \) | \( 1.4 \) |
|---------------------|--------|--------|--------|--------|
| \( H : D \)         | 80 : 20| 20 : 80|        |        |
| \( D : T \)         |        |        | 80 : 20| 20 : 80|

4.1. Transport scaling with effective mass

We consider two ion species with mass ratios \( \mu_i, \mu_j \) and concentrations \( a_i, a_j \) and compare results of turbulence computations with respect to the effective mass \( \mu_{\text{eff}} = a_i \mu_i + a_j \mu_j \) both for cold ions \( (\tau_i = \tau_j = 0) \) and warm ions \( (\tau_i = \tau_j = 1) \).

The range of \( \mu_{\text{eff}} = 0.6 \) to \( \mu_{\text{eff}} = 1.4 \) includes protium–tritium–deuteron mixtures ranging from almost pure protium to almost pure tritium, see table 2. In figure 1 the average radial particle flux \( (\Gamma) = (n_e v_i) \), averaged both spatially over the simulation domain and temporally, is shown as a function of the effective mass \( \mu_{\text{eff}} \) in a plasma with two ion species. Normalisation is with respect to the reference ion mass, in our case deuterium. The transport clearly decreases with increasing effective mass for both, cold and warm ions. Increased transport levels for warm ions relative to the cold ion case are caused by the additional ion diamagnetic contribution to the curvature coupling term \( K(\tau \mu_i) \) in (1). Reduction of transport with effective mass is stronger for warm ions, which points to a pronounced role of the isotope mass on gyro-screening through FLR effects. Transport reduction for heavier plasmas shows little dependence on finite \( \beta \). We now test that the isotope mass effect found above is actually robust with variation of the collisionality parameter \( C \) from 1 to 7. Results are shown in figure 2. Plasma compositions considered are close to pure cases, i.e. \( H \) denotes a 99% protium plasma with 1% deuterium. \( D \) is pure deuterium and \( T \) denotes 99% tritium with 1% deuterium. It can be seen that transport levels for each species increase with larger collisionality \( C \), which is consistent with standard benchmark cases [31]. The isotope effect is present throughout this range of collisionalities, which indicates that the collisional response, which mainly governs the parallel electron dynamics for the present range
of parameters is not the only and major mechanism acting on transport reduction with respect to isotope mass.

4.2. Mass effects on zonal flows and GAMs

We next examine the role of zonal flows on the reduction of turbulent transport with increasing effective mass. In figure 3 (top) the ratio between zonal flow (ZF) energy $U = \sum \int d\phi \langle \phi \rangle /2$, where $\langle \cdot \rangle = \int d\phi$ is the flux-surface average, and total turbulent kinetic energy $E = \sum \int d\phi d\phi d\phi /2$ as a measure for the zonal flow strength is plotted as a function of the effective mass. While the ZF strength is generally larger for the warm ion case, both cold and warm ion cases show little dependence of the ZF strength on the effective mass: for $\tau = 0$ the ZF energy ratio is nearly unaffected, and for $\tau = 1$ it increases by approximately 5% when varying from $\mu_{\text{eff}} = 0.6$ to 1.4.

Another mechanism by which ZFs may reduce radial transport is by poloidal flow shear. Here, the shearing rate $\omega$ is defined as the inverse auto-correlation time of $\int \phi /2$, averaged over six distinct radial positions $x_p$. This is shown in figure 3 (bottom), again for cold and warm ions. While the shearing rates on average are about 40% stronger for the warm ion case compared to the cold ion case, they are again nearly unchanged (within statistical uncertainties) for variations of the effective mass.

We have conducted analyses of a number of further statistical quantities as a function of the effective mass. Characteristic time scales $\tau$ can be deduced from auto-correlation functions, where state variables are sampled at each time step to produce time-series at $x_p$. Figure 4 shows, both for density (top) and potential (bottom) auto-correlation times, the general trend of longer time scales as the effective plasma mass increases. This trend is in approximate agreement with the shift of the drift wave frequency $\omega \sim 1/(1 + \mu_{\text{eff}})$ for the wave number scale $\partial_t = 1$, and thus $\tau \sim 1 + \mu_{\text{eff}}$; the time scale increases by around 50% when varying the effective mass from $\mu_{\text{eff}} = 0.6$ to 1.4.

As shown in figure 5, we also find an increase in the maximum of the cross-correlation between density and potential fluctuations for warm ions, but not for the cold ion case. Values stem from averages of correlation times and cross-correlations over two time series at two distinct locations on the outboard midplane.

Typical values of the cross-correlation coefficient computed for the warm ion case are approximately a third weaker than in the cold ion case, but increase with effective mass as opposed to the cold ion case. From these trends we can infer that, first, the general reduction of cross-correlation for warm ions is a result of the $\tau$ dependence in the diamagnetic curvature coupling, which facilitates a trend towards a more interchange-like phase shift distribution and thus weaker cross-correlation. Second, the increase of the cross correlation with effective mass for the warm ions then can be attributed to warm ion dynamics, which by gyro-averaging counteract the phase shift.

Figure 6 shows correlation lengths $L_y$ in the perpendicular $y$-direction of the density, electric potential, and vorticity fields. For all cases we find that $L_y$ increases by approximately 20% when varying the effective mass from 0.6 to 1.4. As the scaling is similar for both cold and warm ions, we infer that this trend could be based on the drift-scaling (which is independent of $\tau$ in contrast to direct FLR effects) by the effective mass acting on the polarisation.

On the other hand, an increase of perpendicular correlation lengths could also be attributed to stronger zonal flows. Here,
the results allow no unambiguous conclusion: while the relative ZF strength and the shearing rate both only show weak dependence on the effective mass for warm ions and not at all for cold ions, the perpendicular (‘poloidal’) correlation length increases for both cold and warm ions by around 20%.

In the context of the present (isothermal) gyro-fluid model, turbulence driven zonal flows thus only play a rather minor role as cause for the overall isotope effect on turbulent transport reduction.

Regarding the influence of isotope mass on GAMs, we have found no dependence of the GAM intensity (quantified by the spectral power of $\phi$ at the GAM frequency) in our isothermal gyro-fluid simulations for the reference parameters.

We observe that in regions of strong zonal flow shear close to the radial computational boundaries (with Dirichlet conditions on the zonal potential) the GAM amplitude is reduced, whereas the ZF activity is generally weaker in the center of the radial computational domain and the GAMs are more pronounced.

### 4.3. Mass effects through finite Larmor radius and polarisation

In order to separate the mass effects acting through FLR terms or the drift scaling of polarisation, we set up a number of computations with artificially modified equations.

First, we study the effect of mass on transport through polarisation by modifying the standard cold ion case. Set

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**Figure 4.** Auto-correlation times $\tau_c$ for density (top) and potential (bottom) fluctuations as a function of the effective ion mass $\mu_{\text{eff}}$. Cold ion cases ($\tau = 0$) are depicted with green downward symbols, warm ion cases ($\tau = 1$) with blue upward symbols. Simulation parameters are listed in table 1.

**Figure 5.** Maximum cross-correlation between density and potential fluctuations for cold ($\tau = 0$) and warm ($\tau = 1$) ion cases as functions of the effective mass: for finite Larmor radius (i.e. $\tau = 1$) a slight increase in the cross-correlation is observed with increasing ion mass. Simulation parameters are listed in table 1.

**Figure 6.** Poloidal auto-correlation lengths $L_y$ for fluctuations of density (blue squares), potential (green circles) and vorticity (red diamonds) as functions of the effective ion mass. Top: cold ion case ($\tau = 0$); bottom: warm ion cases with FLR effects ($\tau = 1$). For all cases the poloidal correlation lengths increase similarly with larger effective mass. Simulation parameters are listed in table 1.
one of simulations is run with the usual effective mass defined by $\mu_{\text{eff}} = a_i \mu_{i} + a_j \mu_{j}$ given by separate mass ratios $\mu_{i}$ and $\mu_{j}$. Set two is run with artificially setting $\mu_{i} = \mu_{j}$ in the effective mass entering in the polarisation. We perform both sets of runs with ion concentrations $a_{s1} : a_{s0} = 80 : 20$ and $a_{d1} : a_{d0} = 20 : 80$, denoted by ‘light’ and ‘heavy’, respectively. The resulting transport rates are given in figure 7 (data points are shifted slightly for better visibility). We find that the modified mass dependence in the vorticity term of the polarisation equation effectively annihilates the isotope effect, confirming the above assumptions. This now is not surprising any more, since for cold ions this term is the only mass-dependent term of relevance in the equations.

Performing similar modifications in order to focus on specific FLR effects only is a bit more tedious for the warm ion case, since both gyro-averaging operators are mass dependent. We recall the warm ion gyro-screening term, $(\Gamma_{0,s} - 1)\tau_s = -\mu_s k_s^2(1 + \mu_s \tau_s k_s^2)$, occurring in the polarisation equation with the vorticity; and the gyro-averaging operator, $\Gamma_{1,i} = 1/(1 + \mu_i \tau_i k_i^2)$, present in the averaging of density as part of the polarisation equation and acting on the potential in curvature and parallel terms. The latter is most significantly as part of the $\mathbf{E} \times \mathbf{B}$ advection.

We introduce three modifications of the model to artificially enhance and study the relative importance of each of these FLR effects.

- Case 1 denotes the original model.
- In case 2 we set $\Gamma_{0,i} = \Gamma_{0,j}$ in the ‘vorticity’ term without modifying the FLR operators elsewhere.
- Case 3 leaves $\Gamma_{0,i}$ unaltered, but sets $\Gamma_{1,i} = \Gamma_{1,j}$ on the gyro-fluid densities in the polarisation equation (and also in the ‘vorticity-free’ boundary condition). Other occurrences of $\Gamma_{1,i}$ are left unchanged.
- Finally, case 4 considers the impact of the gyro-averaged potential in the advective terms by setting $\Gamma_{1,i} = \Gamma_{1,j}$ there without modifying it in the polarisation equation and vorticity boundary conditions.

Results for the particle transport for all cases are given in figure 8, with data points shifted slightly for improved visibility where necessary. The stronger the separation of the results between the ‘light’ and ‘heavy’ cases deviates from the standard case 1 in figure 8, the more pronounced is the mass effect in the three different modified cases 2–4, which highlight the relative importance in the FLR terms.

It can be seen that both mass-dependent FLR terms in the polarisation equation contribute significantly to the isotope effect. The main contribution is apparently caused by gyro-averaging on the gyro-fluid densities, whereas modification of gyro-screened advective terms actually leads to enhancement of the isotope effect.

4.4. Mass effects on ion inertia

We filter out inertia mass effects by introducing artificial electron–ion mass ratios for single-ion (D) computations,
and the results shown in the previous sections are not artificially modified by box size effects.

The box size variations have taken into account possible effects of the vortex and zonal flow scales which fit into the computational domain.

Further computational or numerical artefacts could occur through the radial (Dirichlet) boundary conditions and through insufficient spatial resolution. We have also analysed the dependence of such computational errors on the isotope mass by means of the energy theorem. These results have confirmed that the employed resolution and treatment of boundary conditions do not significantly modify the overall isotope effects found in this work.

5. Discussion and conclusions

We have discussed ion isotope mass effects on isothermal electromagnetic gyro-fluid tokamak edge turbulence and zonal flows. The simulations show reduced transport amplitudes in heavier plasmas in terms of the reduced radial particle flux. We emphasize that this reduction of the radial particle flux with increasing mass is contrary to gyro-Bohm scaling, which postulates an increase of the radial particle flux for heavier plasmas.

Deviation from gyro-Bohm scaling due to zonal flow activity has also been investigated in ITG turbulence. Close to turbulence threshold, the deviation is most pronounced when the system enters the so-called stair-case regime, driven by avalanches in the density [34, 35]. It should be noted that similarities to the E × B stair-case regime do also appear in local gradient-driven drift-Altén simulations as radially distinct, although much less sharp, zonal flow and density peaks.

However, the emergence of shear layers is not that pronounced as in global flux-driven ITG turbulence, since a turbulence threshold does not exist in our case and hence avalanches are absent. Mass dependence in linear instability ITG growth-rates and consequent radial decorrelation of turbulent eddies due to zonal flow shear has been shown to improve confinement [14].

In the constant temperature, drift-Altén edge turbulence simulations presented here, zonal flow activity is found to be slightly enhanced with the isotope mass, but not to an extent to be able to (fully) explain the observed transport reduction. In the present model (isothermal, without mass dependent ion–ion collisions) we do not find any significant influence of isotope mass on GAM amplitudes.

The isotope effect is stronger for warm ions (τ = 1) but persists also for cold ion cases (τ = 0), which suggests that not FLR effects are the main mechanism for transport reduction, but ion inertia effects in the polarisation mediated by the mass-dependent drift scale. This is also supported by artificially adapting the electron mass relative to the ion mass to enhance pure ion inertia effects.

4.5. Mass-adapted box and grid size test

Since a species Larmor radius $\rho_s^2 = \mu_s^2 \tau_b \rho_0^2 \propto \mu_{\text{eff}}$ we adapt the perpendicular simulation box size to yield the same number of gyro-radii per drift plane. This effectively increases the perpendicular resolution for $\mu_{\text{eff}} < 1$ and decreases it for $\mu_{\text{eff}} > 1$. Figure 10 shows results for $\mu_{\text{eff}} = 0.6$ and $\mu_{\text{eff}} = 1.4$ (data points are shifted where necessary for better visibility). These runs feature drift plane dimensions of $[38 \times 154] \rho_0$ and $[90 \times 358] \rho_0$ respectively. For all runs the number of nodes is $n_x \times n_y = 64 \times 256$.

Figure 10. Average radial particle flux (Γ) as a function of effective mass for cold (green) and warm (blue) ion cases. Here the simulation domain has been adapted according to effective plasma mass to resolve the same number of gyro radii: the isotope effect is still clearly visible, so that artificial box size effects on the numerical results can be excluded.

For cold ions, both the adapted electron mass model and the reference model yield comparable transport scaling, as expected.

For warm ions, we find that the relative inertia is only of rather minor importance compared to resolving the true gyro-radii. The isotope effect solely based on plasma inertia with identical Larmor radii is far weaker than the fully resolved case, indicating that the underlying mechanism is maintained from both inertia and FLR effects.

We consider cases with $\mu_{\text{eff}} = 0.6$ and $\mu_{\text{eff}} = 1.4$ resulting from $H : D = 80 : 20$ and $H : D = 20 : 80$, respectively. In figure 9 data points are shifted slightly around these values for better visibility.

Since a species Larmor radius $\rho_s^2 = \mu_s^2 \tau_b \rho_0^2 \propto \mu_{\text{eff}}$, we adapt the perpendicular simulation box size to yield the same number of gyro-radii per drift plane. This effectively increases the perpendicular resolution for $\mu_{\text{eff}} < 1$ and decreases it for $\mu_{\text{eff}} > 1$. Figure 10 shows results for $\mu_{\text{eff}} = 0.6$ and $\mu_{\text{eff}} = 1.4$ (data points are shifted where necessary for better visibility). These runs feature drift plane dimensions of $[38 \times 154] \rho_0$ and $[90 \times 358] \rho_0$ respectively. For all runs the number of nodes is $n_x \times n_y = 64 \times 256$.

Resulting transport levels in figure 10 can be compared with the reference results labelled ‘norm $\tau = 0$’ and ‘norm $\tau = 1$’ in figure 9, which are obtained from the standard resolution $(96 \times 256)/\rho_0 \times 2\pi$ on $64 \times 256 \times 16$ nodes.

The mass-adapted resolution yields slightly higher transport levels for the light plasma and slightly lower transport levels for the heavy plasma (compared to the standard resolution). However, the isotope effect is still clearly visible. Hence these results shown in the previous sections are not artificially modified by box size effects.

In the present model (isothermal, without mass dependent ion–ion collisions) we do not find any significant influence of isotope mass on GAM amplitudes.

The isotope effect is stronger for warm ions ($\tau = 1$) but persists also for cold ion cases ($\tau = 0$), which suggests that not FLR effects are the main mechanism for transport reduction, but ion inertia effects in the polarisation mediated by the mass-dependent drift scale. This is also supported by artificially adapting the electron mass relative to the ion mass to enhance pure ion inertia effects.
In the presented simulations with finite collisionality, electromagnetic flutter effects show no significant dependence on isotopes. However we note that in our present gyro-fluid model implementation the parallel magnetic vector potential is not subject to gyro-averaging operators, and thus electromagnetic FLR effects are neglected [16].

While the isotope effect is thus clearly present in the local isothermal gyro-fluid model, we still find a weaker influence on transport reduction and zonal flow enhancement compared to experimental results. This discrepancy could be caused by the neglect of thermal fluctuations and ion temperature gradient (ITG) and trapped electron mode (TEM) turbulence in the present model. Isotope mass effects on zonal flow damping via ion–ion collisionality and on neoclassical flows can be studied with full thermal models. In the future our delta-f gyro-fluid model is going to be extended to 6 moments including temperature fluctuations and a full-f approach, including consistent coupling of edge turbulence with scrape-off layer (SOL) fluctuations and filamentary transport.

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