Coupled dynamics characteristics analysis of the marine multi-gearbox system under multi-source excitation

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Abstract
The marine multi-gearbox system has the characteristics of multi-source excitation and multi-gearbox coupled. The interaction among gearboxes is inevitable. To reveal the coupled mechanism, a coupled dynamic model of multi-gearbox system is proposed in this paper by the generalized finite element method and the substructure method. The dynamic model of each gearbox in the multi-gearbox system is also established. Under the same working conditions, the dynamic response of single gearbox and multi-gearbox system are obtained. The results show that the adjacent gear pairs of adjacent gearboxes are more affected by the coupled effect at low rotational speed. Compared with single gearbox, the RMS curve of multi-gearbox system of meshing forces generates some new resonance peaks, which are caused by the new modes excited by the mesh frequencies of adjacent gearboxes. The dynamic response of gear pairs which are greatly affected by coupled effect are more sensitive to the change of stiffness of couplings. Reducing the stiffness of the couplings can weaken the coupled effect of the system, and vice versa. The change of housing stiffness does not influence the coupled effect of the multi-gearbox system, and the new modes and resonance peaks generated by the coupled effect have not changed significantly.

Keywords
Multi-gearbox system, multi-source excitation, coupled effect, dynamics, finite element, substructure

Introduction
A gas turbine, steam engine, diesel engine, and electric motors are the main power sources used in ships. With the increasing demand for fuel economy and reliability of ships, the multi-gearbox system has been widely used in large powered ships.¹,² The Multi-gearbox system has various configurations, which are generally composed of parallel gearbox, cross-connect gearbox, and two-speed gearbox, etc.³ The gearboxes are connected by couplings. The power of multiple engines can be transmitted to the propeller shafts respectively or simultaneously by multi-gearbox system. For multi-gearbox

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system, each gear pair is an excitation source, so it is a complex multi-source excitation system. The gearboxes inevitably have interaction. Therefore, it is of great significance to study the coupled vibration characteristics of multi-gearbox system.

The research on coupled characteristics of gear system mainly focuses on coupled degree of freedom, coupled between stages and coupled of supporting system. For the degree of freedom coupling, the gear dynamics model has undergone a pure torsion model, a bending-torsion coupled model and a bending-torsion-axial-swing coupled model (full degrees of freedom). The research found that the full degrees of freedom coupled model can more accurately analyze the vibration characteristics of the system.4,5 Regarding the coupled between stages, the coupled dynamic characteristics of multi-stage parallel shafts and multi-stage planetary gear systems have attracted the attention of many researchers over the years. Papers by Walha et al.,6 Raclot and Velex,7 and Shen et al.8 studied the coupled dynamic characteristics of the system. The research found that the full degrees of freedom coupled model can more accurately analyze the vibration characteristics of the system.9,10 Regarding the coupled of supporting system, each gear pair is an excitation source, so it is a complex multi-source excitation system. The gearboxes inevitably have interaction. Therefore, it is of great significance to study the coupled vibration characteristics of multi-gearbox system.

Dynamic model of the multi-gearbox system
Multi-gearbox system structure and model discretization

As shown in Figure 1(a), the multi-gearbox system with two-input power and two-output power is studied in this paper. It is composed of five gearboxes, in which all gears are double-helical gears. The left and right helical gear in each double-helical gear pair, bearings, and shafts are serially numbered. As shown in Figure 1(b), GB2 and GB4 adopt multi-mesh gear transmission, which can realize the power split and confluence. When
the input power of an engine on one side is less than the power required by the load torque of the propeller, the engine on the other side split part of the power through GB2 or GB4 and transmits the power through GB3 to GB2 or GB4 on the power demand side confluence to achieve power distribution.

Compared with the lumped mass method, the generalized finite element method can better consider the characteristics of the shaft and support system. Its modeling and solution efficiency are higher than the dynamic three-dimensional contact finite element method. The literature\textsuperscript{11,37} has verified its validity and accuracy by comparing with experiments and centralized mass method. For multi-gearbox system, it is necessary to ensure the accuracy of the dynamic model and to consider the convenience of modeling. The generalized finite element method is very suitable for such complex models. The model takes the effects of bearing support position and housing flexibility on the dynamic characteristics of the system into consideration. As shown in Figure 2, the multi-gearbox system is discretized into several shaft elements, meshing elements, bearing-housing elements, housing-foundation elements, and coupling elements. By establishing the
dynamic model of each element, the overall system dynamic model is obtained by assembling according to the connection relationship between the elements.

**Subsystem model**

**Meshing element.** The double-helical gear is modeled as two helical gears. The intermediate shaft is regarded as a shaft element, and it is simulated by the Timoshenko beam element. The calculation of time-varying mesh stiffness adopts the method of Chang et al. As shown in Figure 3, project the displacement from the global coordinate system to the normal line of action. The projection vector \( V \) of the \( j \)th gear pair is

\[
V = \left\{ \cos \beta_j^l \sin \gamma^l, \pm \cos \beta_j^l \cos \gamma^l, \mp \sin \beta_j^l, \\
\right.
\]
\[
\left. r_j^p \sin \beta_j^l \sin \gamma^l, \pm r_j^p \sin \beta_j^l \cos \gamma^l, \cos \beta_j^l \sin \gamma^l, \\
\right.
\]
\[
\left. \pm \cos \beta_j^l \cos \gamma^l, \pm \sin \beta_j^l, r_j^p \sin \beta_j^l \sin \gamma^l, \\
\right.
\]
\[
\left. \pm r_j^p \sin \beta_j^l \cos \gamma^l, \pm r_j^p \cos \gamma^l \right\}
\]

(1)

where \( \beta_j^l \) is the base circle helix angle, the positive value is taken when the gear is a right-hand gear, and the negative value is taken when the gear is a left-hand gear. \( \gamma^l \) is the angle between the positive \( y \)-axis and the meshing line of the end surface in the global coordinate system. It can be expressed as \( \gamma^l = \alpha^l \mp \theta^l \). The upper half of the formula corresponds to the counterclockwise rotation of the driving wheel, while the lower half corresponds to the clockwise rotation of the driving wheel.
wheel, where \( \alpha \) is the meshing angle and \( \theta \) is the mounting angle. \( r_f^d \) and \( r_f^p \) are the base circle radius of the driving wheel and the driven wheel respectively. The projection vector takes different values when the rotation direction of the driving wheel is different. When the driving wheel rotates clockwise, the projection vector takes the upper half of the symbol, and when the driving wheel rotates counterclockwise, the projection vector takes the lower half of the symbol.

The coordinate of the meshing element can be defined as \( \mathbf{q}_j^m = \{x_j^m, y_j^m, z_j^m, \theta_{x_j^m}, \theta_{y_j^m}, \theta_{z_j^m}, x_j^f, y_j^f, z_j^f, \theta_{x_j^f}, \theta_{y_j^f}, \theta_{z_j^f}\} \). The deformation along the meshing line can be expressed as

\[
\sigma_j^m = \mathbf{V} \mathbf{q}_j^m
\]  
(2)

The motion equations for a helical gear pair can be written as

\[
\begin{align*}
 m_j^d \ddot{x}_j^d + (c_j^{\sigma} \dot{x}_j^d + k_j^{\sigma} \alpha_j^d) \cos \beta_j^d \sin \gamma_j^d &= 0 \\
m_j^d \ddot{y}_j^d + (c_j^{\sigma} \dot{y}_j^d + k_j^{\sigma} \alpha_j^d) \cos \beta_j^d \cos \gamma_j^d &= 0 \\
m_j^d \ddot{z}_j^d + (c_j^{\sigma} \dot{z}_j^d + k_j^{\sigma} \alpha_j^d) \sin \beta_j^d &= 0 \\
m_j^d \ddot{\theta}_x^d + (c_j^{\sigma} \dot{\theta}_x^d + k_j^{\sigma} \alpha_j^d) \cos \beta_j^d \sin \gamma_j^d &= 0 \\
m_j^d \ddot{\theta}_y^d + (c_j^{\sigma} \dot{\theta}_y^d + k_j^{\sigma} \alpha_j^d) \cos \beta_j^d \cos \gamma_j^d &= 0 \\
m_j^d \ddot{\theta}_z^d + (c_j^{\sigma} \dot{\theta}_z^d + k_j^{\sigma} \alpha_j^d) \sin \beta_j^d &= 0
\end{align*}
\]  
(3)

\( m_j^d \) is the mass of the driving wheel, \( m_j^f \) is the mass of the driven wheel. \( I_{x_j^d}, I_{y_j^d}, I_{z_j^d} \) are the rotational inertia of the driving wheel in the \( x, y, z \) directions. \( c_j^{\sigma} \) is the meshing damping of the gear pair, and the meshing damping ratio is 0.07.

The calculation formula of meshing damping \( c_j^{\sigma} \) for a gear pair is

\[
c_j^{\sigma} = 2Z_k \sqrt{k_i/(1/m_i^{eq} + 1/m_q^{eq})}
\]  
(4)

where \( k_i \) is the mean value of meshing stiffness; \( \xi \) is the meshing damping ratio, and its value range is generally 0.03–0.17; \( m_i^{eq}, m_q^{eq} \) are the equivalent mass of the driving wheel and the driven wheel.

The matrix form of the dynamic equation of the meshing element is shown in equation (5).

\[
M_j^m \ddot{\mathbf{q}}_m(t) + C_j^m \dot{\mathbf{q}}_m(t) + K_j^m \mathbf{q}_m(t) = 0
\]  
(5)

where \( \mathbf{q}_m(t) \) is the node displacement column vector of the driving wheel and the driven wheel of the \( j \)th meshing element, and \( M_j^m \), \( C_j^m \), \( K_j^m \) are the corresponding mass matrix, damping matrix, and stiffness matrix.

**Shaft element.** Using the Timoshenko beam element with two nodes to establish the shaft element, as shown in Figure 4. The generalized coordinates of the \( j \)th shaft element can be defined as

\[
\mathbf{q}_j^s = \{x_j, y_j, z_j, \theta_{x_j}, \theta_{y_j}, \theta_{z_j}, x_{j+1}, y_{j+1}, z_{j+1}, \theta_{x_{j+1}}, \theta_{y_{j+1}}, \theta_{z_{j+1}}\}
\]  
(6)

Where \( x_j, y_j, z_j, x_{j+1}, y_{j+1}, z_{j+1} \) is the lateral displacement of the node along each coordinate axis, and \( \theta_{x_j}, \theta_{y_j}, \theta_{z_j}, \theta_{x_{j+1}}, \theta_{y_{j+1}}, \theta_{z_{j+1}} \) is the torsion angle of the node around each coordinate axis. The calculation method of the stiffness matrix and the element mass matrix of the shaft element can refer to Stringer. 39

The matrix form of the dynamic equation of the shaft element is:

\[
M_j^s \ddot{\mathbf{q}}_s(t) + C_j^s \dot{\mathbf{q}}_s(t) + K_j^s \mathbf{q}_s(t) = 0
\]  
(7)
Where $q_{jb}$ is the node displacement column vector of the $j$th shaft element, and $M_{jb}$, $C_{jb}$, $K_{jb}$ are the $j$th shaft element mass matrix, damping matrix, and stiffness matrix. The system damping adopts the Rayleigh model.

\[
C_{jb} = a_0 M_{jb} + a_1 K_{jb}
\]

Where $a_0$ is the mass coefficient, $a_1$ is the stiffness coefficient.

**Bearing-housing element.** As shown in Figure 5 is the $j$th bearing-housing element, one end node of the bearing-housing element is at the midpoint of the bearing, and the other end node is at the housing node, that is, at the center hole of the bearing seat. The bearing stiffness matrix $K_{jb}$ can be expressed as equation (9). Most of the coupled stiffness of bearings is very small, and the calculation equation only retains the main stiffness $k_{xx}$, $k_{yy}$, $k_{zz}$, $k_{xu}$, $k_{yv}$. The mass of the housing node $M_{jh}$ is obtained by the static substructure method. The 2 nodes 12 degrees of freedom dynamic model of the bearing-housing element is shown in equation (10).

Because the bearing mass is smaller than other parts, the bearing mass $M_{jb}$ is generally ignored in the calculation. In equation (10), the structure of the damping matrix $C_{jb}$ is the same as the stiffness matrix $K_{jb}$, which is calculated by using the Rayleigh model of equation (6).

\[
K_{jb} = \begin{bmatrix}
k_{xx} & k_{xy} & k_{xz} & k_{xu} & k_{xy} & k_{xv} \\
k_{yx} & k_{yy} & k_{yz} & k_{yv} & k_{yx} & k_{yv} \\
k_{zx} & k_{zy} & k_{zz} & k_{zu} & k_{zy} & k_{zu} \\
k_{ux} & k_{uy} & k_{uz} & k_{ux} & k_{uy} & k_{uz} \\
k_{vx} & k_{vy} & k_{vz} & k_{vx} & k_{vy} & k_{vz} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
M_{jb} & 0 \\
0 & M_{jh}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_{jb}(t) \\
\ddot{q}_{jh}(t)
\end{bmatrix} + \begin{bmatrix}
-C_{jb} & C_{jb} \\
-C_{jh} & C_{jh}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_{jb}(t) \\
\dot{q}_{jh}(t)
\end{bmatrix} + \begin{bmatrix}
K_{jb} & -K_{jb} \\
-K_{jh} & K_{jh}
\end{bmatrix}
\begin{bmatrix}
q_{jb}(t) \\
q_{jh}(t)
\end{bmatrix} = 0
\]

$\begin{bmatrix}
M_{jb} & 0 \\
0 & M_{jh}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_{jb}(t) \\
\ddot{q}_{jh}(t)
\end{bmatrix} + \begin{bmatrix}
-C_{jb} & C_{jb} \\
-C_{jh} & C_{jh}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_{jb}(t) \\
\dot{q}_{jh}(t)
\end{bmatrix} + \begin{bmatrix}
K_{jb} & -K_{jb} \\
-K_{jh} & K_{jh}
\end{bmatrix}
\begin{bmatrix}
q_{jb}(t) \\
q_{jh}(t)
\end{bmatrix} = 0
$
where \( \mathbf{q}_h^j \) is the displacement column vector of the housing node of the \( j \)th housing-foundation element, the dimension is \([n \times 1]\), and \( n \) is the number of bearing nodes multiplied by 6.

**Coupling element.** The multi-gearbox system uses a diaphragm coupling. The structure of the diaphragm coupling is shown in Figure 7, which is mainly composed of three parts: the driving end, the driven end, and the coupling diaphragm. The driving end and driven end can be considered as two shaft nodes, each node has six degrees of freedom, the stiffness between the two nodes is the radial stiffness of the coupling diaphragm \( k_{cxx}, k_{cyy} \), axial stiffness \( k_{czz} \), bending stiffness \( k_{u_{xx}}, k_{u_{yy}} \), torsional stiffness \( k_{u_{zz}} \). Among them, \( k_{cxx} = k_{cyy} \), \( k_{u_{xx}} = k_{u_{yy}} \).

The stiffness matrix of the coupling element can be expressed as equation (12), where \( k_{c11} = k_{c12} = k_{c21} = k_{c22} \). The expression form of sub-matrix \( k_{c11} \) is shown in equation (13).

\[
\mathbf{K}_c = \begin{bmatrix}
k_{c11} & -k_{c12} \\
-k_{c21} & k_{c22}
k\end{bmatrix}
\quad (12)
\]

\[
\mathbf{k}_{c11} =
\begin{bmatrix}
k_{c_{xx}} & k_{c_{yy}} \\
k_{c_{yy}} & k_{c_{zz}}
k_{u_{xx}} & k_{u_{yy}} \\
k_{u_{yy}} & k_{u_{zz}}
k\end{bmatrix}
\quad (13)
\]

The motion equation for the \( j \)th coupling element can be expressed as equation (14).

\[
\mathbf{M}_c \ddot{\mathbf{q}}_c^j(t) + \mathbf{C}_c \dot{\mathbf{q}}_c^j(t) + \mathbf{K}_c \mathbf{q}_c^j(t) = 0
\quad (14)
\]

Where \( \mathbf{q}_c^j \) is the node displacement column vector of the \( j \)th coupling element, and \( \mathbf{M}_c, \mathbf{C}_c, \mathbf{K}_c \) are the corresponding mass matrix, damping matrix, and stiffness matrix.

**Overall system dynamics model**

For the multi-gearbox dynamic model considering the housing, the overall stiffness matrix composition is shown in Figure 8. When the block in the figure is composed of two kinds of shadows, the submatrix is the superposition of the corresponding element submatrices. If the housing is rigid, the rows and columns where the housing is free can be crossed out. There are different types of elements in the system, so it is
necessary to assemble the element matrix into the whole matrix in turn.

Assemble the mass, stiffness, and damping matrices of each element divided above according to the physical connection relationship, and establish a linear time-varying multi-gearbox coupled dynamic model considering the time-varying mesh stiffness. The system is discretized into 236 nodes, each with 6 degrees of freedom. The motion equation for the system is

$$ M(t) \ddot{X} + C(t) \dot{X} + K(t)X = F(t) $$

Where $M(t)$ is the overall mass matrix of the system, $C(t)$ is the overall damping matrix of the system, $K(t)$ is the overall stiffness matrix of the system, $X(t)$ is the column vector of displacement of all nodes, and $F(t)$ is the column vector of system external load. Because the mesh element considers the time-varying mesh stiffness of gear pairs, the stiffness matrix of the system is time-dependent.

To compare and analyze with the coupled system, the dynamic model of each gearbox in the multi-gearbox system is established by using the above method, and these models are not repeated in this paper.

Solution model of the dynamic model

The system dynamics model established in this paper has many nodes, many degrees of freedom, and a large matrix dimension, so the Fourier series method is used to solve the dynamic equations to improve the calculation efficiency. According to the idea of the constant parameter in the Fourier series method, the variable in equation (15) is expressed as the sum of the mean value and the fluctuation value, then $K(t) = K_0 + \Delta K$, $x(t) = x_0 + \Delta x$, $\dot{x}(t) = \dot{x}_0 + \Delta \dot{x}$, $\ddot{x}(t) = \Delta \ddot{x}$ and $\dddot{x}(t) = \Delta \dddot{x}$ can be obtained by derivation. Put them into equation (15) to get equation (16).

$$ M(t) \dddot{x} + C(t) \ddot{x} + K(t)x_0 + \Delta K \dot{x} + K_0 \Delta x + \Delta K \Delta x = P_0 $$

Arrange the shift term to obtain equation (17), where $\Delta x$ is an unknown quantity, and $\Delta x_0$ is used to approximately replace $\Delta x$, where $x_0$ is obtained by solving statics equation (18).

$$ M(t) \Delta \dddot{x} + C(t) \Delta \ddot{x} + K(t) \Delta x = - \Delta K (x_0 + \Delta x) 
≈ - \Delta K (x_0 + \Delta x_0) = - \Delta K \Delta x_0 = F(t) $$

$$ K_0 x_0 = P_0 $$

At this time, only the left side of the equation has unknowns, and $\Delta x, \Delta \dot{x}, \Delta \ddot{x}$, and $F(t)$ are respectively expanded to the $k$-th order Fourier series with the basic frequency of the mesh frequency:

$$ \Delta x = \Delta x_0 + \sum_{n=1}^{k} [a_n \cos(n \omega t) + b_n \sin(n \omega t)] $$

$$ \Delta \dot{x} = \sum_{n=1}^{k} [-n \omega a_n \sin(n \omega t) + n \omega b_n \cos(n \omega t)] $$

$$ \Delta \ddot{x} = \sum_{n=1}^{k} [-n^2 \omega^2 a_n \cos(n \omega t) - n^2 \omega^2 b_n \sin(n \omega t)] $$

$$ F(t) = F_{ext0} + \sum_{n=1}^{k} [c_n \cos(n \omega t) + d_n \sin(n \omega t)] $$

Substitute equations (19)–(22) into equation (17) to make the sine and cosine terms on both sides of the equation equal, and satisfy the equation (23) for any order frequency. $c_n$ and $d_n$ are known, $a_n$ and $b_n$ can be calculated. Then $a_n$ and $b_n$ of each order are substituted into equation (19) to obtain the displacement fluctuation $\Delta x$, and then obtain the dynamic displacement $x$ of the system.

$$ \begin{cases} 
- n^2 \omega^2 M + K_0 \cdot a_n + n \omega C \cdot b_n = c_n \\
- n \omega C \cdot a_n + (- n^2 \omega^2 M + K_0) b_n = d_n 
\end{cases} $$

Dynamics analysis of multi-gearbox system

As shown in Figure 9, compared with a single gearbox, the multi-gearbox system is a complex multi-source excitation system with multiple excitation forces of different frequencies. For the convenience of analysis and discussion, the time-varying mesh stiffness excitation is considered in this paper. The mesh frequency of GB1, GB2, and GB3 in the multi-gearbox system is expressed as $f_{m1}$, $f_{m2}$, and $f_{m3}$ respectively. To reveal the coupled mechanism of the multi-gearbox system, this section will analyze the natural frequency difference between the multi-gearbox system and the single gearbox, the coupled effect of multiple gearbox system, and the influence of the stiffness of couplings and housings. The model parameters in this paper are shown in Tables 1 and 2. The parameters of GB4 are the same as GB2, and the parameters of GB5 are the same as GB1. Due to the symmetrical structure of the multi-gearbox system, the dynamic responses of GB1, GB2, and GB3 are analyzed below.

Natural frequency analysis of single gearbox and multi-gearbox system

The natural frequencies of the multi-gearbox system and the single gearboxes GB1, GB2, and GB3 in the
system are calculated by equation (25). Among them, \( [M] \) is the mass matrix, \( [K] \) is the stiffness matrix, \( \omega_i \) is the \( i \)-th natural frequency, and \( \Phi_i \) is the mode shape corresponding to each natural frequency. Table 3 shows the natural frequencies of single gearbox and multi-gearbox systems. It can be seen that many new natural frequencies are derived after coupled. For example, 558.36 Hz of the 193rd order, 644.06 Hz of the 209th order and 833 Hz of the 243rd order in the natural frequency of the system do not exist in the natural frequency of the single gearbox. Part of the natural frequency of a single gearbox appears in the multi-gearbox system. The 174th order of GB1 and the 1401st order of multi-gearbox system are both 55,071.14 Hz. The natural frequencies of the 402nd order of GB2 and the 1391st order of the multi-gearbox system are both 51,396.07 Hz. The natural frequencies of the 264th order of GB3 and the 1416th order of multi-gearbox system are both 72,441.81 Hz. Symmetric mode shape, which is the repeated-root of the natural frequency of single gearbox, also appears in multi-gearbox system. In a multi-gearbox system, GB1 and GB5 are the same, and GB2 and GB4 are the same, so the natural frequency of the system has four repeated-roots, such as 481st–484th order. The multi-gearbox system has partial modes of single gearbox. Besides, the coupled effect has generated some new modes in multi-gearbox system. In the following, the influence of the new modes on the dynamic response will be analyzed in detail.

\[
\omega_i^2 [M] \Phi_i = [K] \Phi_i
\]  

(24)

**Coupled effect of multi-gearbox system**

**Gearbox 1.** Under the same working conditions, the dynamic meshing forces of multi-gearbox system and single gearbox are compared. The dynamic

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**Table 1.** Side helical gear parameters.

|          | GB1 | GB2 | GB3 |
|----------|-----|-----|-----|
| Teeth number | 106/37 | 45/150 | 150/54 | 150/125 | 75/168 |
| Module (mm)  | 5   | 7   | 7   | 7   | 8   |
| Pressure angle (°) | 20 | 20 | 20 | 20 | 20 |
| Helix angle (°) | 26.652 | 29.087 | 29.087 | 29.087 | 27.918 |
| Face width (mm) | 100 | 100 | 100 | 100 | 65 |

**Table 2.** Stiffness and damp parameters.

|                      | \( k_{xx} \) | \( k_{yy} \) | \( k_{zz} \) | \( k_{ixx} \) | \( k_{iyy} \) | \( k_{i zz} \) |
|----------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Coupling stiffness (N/m) | \( 9.8 \times 10^7 \) | \( 9.8 \times 10^7 \) | \( 7.5 \times 10^5 \) | \( 4.5 \times 10^4 \) | \( 4.5 \times 10^4 \) | \( 8.8 \times 10^10 \) |
| Bearing stiffness (N/m) | \( 2 \times 10^9 \) | \( 2 \times 10^9 \) | \( 10^6 \) | \( 10^6 \) | \( 10^6 \) |
| Bearing damp          | \( 10^6 \) | \( 10^6 \) | \( 10^4 \) | \( 10^4 \) | \( 10^4 \) |

**Figure 9.** Schematic diagram of double gearbox coupled vibration.
transmission error and dynamic meshing force are calculated by equations (25) and (26). As shown in Figure 10(a) is RMS of dynamic meshing force of No.1 meshing element of multi-gearbox system and single gearbox in GB1. It can be seen that there are some new resonance peaks in the RMS curve of the dynamic meshing force of the multi-gearbox system before 415 rpm, and the corresponding resonance speeds are 35, 115, 260, and 385 rpm. After 415 rpm, the RMS curves of dynamic meshing force of multi-gearbox system and single gearbox coincide. Figure 10(b) shows the spectrogram of the dynamic meshing force of the No.1 meshing element of the multi-gearbox system at 0–1000 rpm. It is observed that in addition to the frequency components of GB1’s mesh frequency \( f_{m1} \) and \( 2f_{m1} \), the meshing frequency component \( f_{m2} \) of GB2 is also very significant before 415 rpm. 

\[
\text{DTE} = \frac{V}{C_{q_m}} \quad (25)
\]

\[
F_d = k_{m}\text{DTE} + c_{m}\text{DTE} \quad (26)
\]

The time domain and frequency domain of dynamic meshing force of No.1 meshing element in GB1 are compared between multi-gearbox system and single gearbox at 385 rpm. As shown in Figure 11(a), The fluctuation of meshing force of No.1 meshing element in multi-gearbox system is 3910 N, and that of No.1 meshing element in single gearbox is 1117 N, which increases by 2.5 times after coupled. It can be seen from Figure 11(b) and (c) that the increase of fluctuation is mainly due to the frequency component of \( f_{m2} \) appearing after coupled, which accounts for a large proportion in the spectrogram.

The mesh frequencies and the corresponding natural frequencies of the system at 35, 115, 260, and 385 rpm. After 415 rpm, the RMS curves of dynamic meshing force of multi-gearbox system and single gearbox coincide. Figure 10(b) shows the spectrogram of the dynamic meshing force of the No.1 meshing element of the multi-gearbox system at 0–1000 rpm. It is observed that in addition to the frequency components of GB1’s mesh frequency \( f_{m1} \) and \( 2f_{m1} \), the meshing frequency component \( f_{m2} \) of GB2 is also very significant before 415 rpm.

| Modes | GB1 (Hz) | Modes | GB2 (Hz) | Modes | GB3 (Hz) | Modes | Multi-gearbox system (Hz) |
|-------|---------|-------|---------|-------|---------|-------|--------------------------|
| 1     | 106.74  | 1     | 22.51   | 1     | 21.44   | 1     | 1.44                     |
| 2     | 115.79  | 2     | 28.90   | 2     | 29.53   | 2     | 2.97                     |
| 3     | 124.33  | 3     | 36.68   | 3     | 46.94   | 3     | 4.90                     |
| 4     | 141.54  | 4     | 37.37   | 4     | 47.37   | 4     | 4.97                     |
| 5     | 159.95  | 5     | 45.74   | 5     | 49.61   | 5     | 6.08                     |
| 6     | 165.22  | 6     | 51.70   | 6     | 57.79   | 6     | 16.92                    |
| 7     | 229.86  | 7     | 54.74   | 7     | 68.48   | 7     | 21.67                    |
| 8     | 234.33  | 8     | 62.07   | 8     | 91.14   | 8     | 21.98                    |
| 9     | 277.04  | 9     | 75.99   | 9     | 91.37   | 9     | 24.24                    |
| 10    | 294.26  | 10    | 93.73   | 10    | 94.71   | 10    | 29.67                    |
| 15    | 454.33  | 65    | 597.24  | 134   | 10,364.34 | 33    | 76.139                   |
| 16    | 483.41  | 143   | 2701.33 | 135   | 10,364.34 | 110   | 247.30                   |
| 18    | 505.58  | 144   | 2701.43 | 264   | 72,444.81 | 193   | 558.36                   |
| 19    | 581.12  | 402   | 51,396.07 | 209   | 644.06  |
| 20    | 708.99  |       |         | 243   | 833.22  |
| 21    | 748.09  |       |         | 360   | 1667.69 |
| 22    | 840.32  |       |         | 481   | 2701.25 |
| 34    | 1667.23 |       |         | 482   | 2701.25 |
| 94    | 10,248.47 |     |         | 483   | 2701.35 |
| 95    | 10,248.77 |    |         | 484   | 2701.35 |
| 174   | 55,071.14 |   |         | 836   | 10,248.47 |
| 181   | 837     |       |         | 838   | 10,248.77 |
| 182   | 839     |       |         | 846   | 10,364.34 |
| 183   | 847     |       |         | 1391  | 51,396.07 |
| 184   | 1401    |       |         | 1416  | 72,444.81 |
| 185   | 1416    |       |         |       |         |
lateral-rotational motions of shaft2 and shaft3. Shaft1 has no motions. Therefore, in Figure 10, resonance peaks are generated at 35, 115, 260, and 385 rpm.

Gearbox 2. As shown in Figure 13(a) is RMS of dynamic meshing force of No.3 meshing element of multi-gearbox system and single gearbox in GB2. It can be seen that there are some new resonance peaks in the RMS curve of the dynamic meshing force of the multi-gearbox system before 335 rpm, and the corresponding resonance speeds are 260 and 300 rpm. After 335 rpm, the RMS curves of dynamic meshing force of multi-gearbox system and single gearbox coincide. The RMS of the dynamic meshing force of the single gearbox No.3 meshing element has a resonance peak at 279 rpm, but there is no resonance peak in the multi-gearbox system. Figure 13(b) shows the spectrogram of the No.3 meshing element at 0–1000 rpm.

**Figure 10.** RMS of dynamic meshing force and spectrogram of No.1 meshing element in GB1: (a) comparison of RMS of dynamic meshing force of No.1 meshing element between multi-gearbox system and single gearbox and (b) spectrogram of No.1 meshing element of multi-gearbox system at 0–1000 rpm.

**Figure 11.** Time and frequency domain of dynamic meshing force of No.1 meshing element in multi-gearbox system and single gearbox at 385 rpm: (a) dynamic meshing force of multi-gearbox system and single gearbox, (b) the spectrum of single gearbox, and (c) the spectrum of multi-gearbox system.
meshing element of the multi-gearbox system. The spectrogram of the No.3 meshing element only has its mesh frequency. There is no meshing frequency component of adjacent gearbox GB1 in the spectrogram.

The mesh frequencies and the corresponding natural frequencies of the system at 260, 279, and 300 rpm are shown in Table 5. The mesh frequency of a single gearbox at 279 rpm does not have a close natural frequency in a multi-gearbox system. This shows that some natural frequencies of GB2 have changed after coupled, which changes the dynamic response of the system.

Figure 14(a) shows RMS of dynamic meshing force of No.7 meshing element of multi-gearbox system and single gearbox in GB2. No.7 meshing element is adjacent to GB3. It can be seen that the curves coincide, indicating that the modes are not affected by the coupled. As shown in Figure 16(b), the meshing force spectrogram of the No.5 meshing element only has its meshing frequency and its double frequency.

**Table 4.** The mesh frequencies at 35, 115, 250, and 385 rpm and the corresponding system natural frequency.

| Speed (rpm) | $f_{m1}$ (Hz) | $f_{m2}$ (Hz) | $f_{m3}$ (Hz) | Mode | Natural frequency (Hz) |
|-------------|---------------|---------------|---------------|------|------------------------|
| 35          | 61.8333       | 75.2027       | 45.1216       | 33   | 76.13952               |
| 115         | 203.1667      | 247.0946      | 148.2568      | 110  | 247.305                |
| 260         | 459.3333      | 558.6486      | 335.1892      | 193  | 558.3609               |
| 385         | 680.1667      | 827.2297      | 496.3378      | 243  | 833.2296               |

**Table 5.** The mesh frequencies at 260, 279, and 300 rpm and the corresponding system natural frequency.

| Speed (rpm) | $f_{m1}$ (Hz) | $f_{m2}$ (Hz) | $f_{m3}$ (Hz) | Mode | Natural frequency (Hz) | Mode shape          |
|-------------|---------------|---------------|---------------|------|------------------------|---------------------|
| 260         | 459.3333      | 558.6486      | 335.1892      | 193  | 558.3609               | Shaft3 lateral-rotational motion |
| 279         | 492.9000      | 599.4730      | 359.6838      | 202  | 578.5111               |                     |
|             |               |               |               | 203  | 611.8411               |                     |
| 300         | 530.0000      | 644.5946      | 386.7568      | 209  | 644.0655               | Shaft3 lateral-rotational motion |

**Gearbox 3.** The cross-connect gearbox GB3 is less affected by the coupling effect. As shown in Figure 17(a), the RMS curve of the meshing force of the No. 9 meshing element of the multi-gearbox system and the single gearbox coincides. As shown in Figure 17(b), there is only the meshing frequency component of GB3 in the spectrogram.

It is found that the adjacent gear pairs of adjacent gearboxes are more easily affected by the coupled effect. In the multi-gearbox system, the RMS of the dynamic meshing force of the No.1 meshing element generate some new resonance peaks with larger amplitudes, which are caused by the new modes excited by the mesh frequencies of adjacent gearboxes. The frequency components of adjacent gearboxes appear in their meshing force spectrum, and the amplitude is very large. More attention should be paid to such situations in multi-source excitation systems. The No.5 meshing element in GB2 is far away from other gearboxes, so the coupled effect does not influence it.

**Influence of the stiffness of couplings**

By increasing and decreasing the stiffness of the couplings, the effect on the coupled dynamic characteristics of the system is studied. As shown in Table 6, the original stiffness of the coupling is recorded as $K_{coupling}$, which increases and decreases the stiffness of all couplings in the system by 10 times, 100 times, and 1000 times.
times, a total of seven cases. The dynamic responses of No.1 meshing element of GB1, No.3 and No.7 meshing element of GB2, and No.9 meshing element of GB3 are analyzed.

Figure 18 shows the RMS curve of the dynamic meshing force of the No.1 meshing element under different coupling stiffnesses. It is observed that the resonance peaks of 385, 260, 115, and 35 rpm due to coupled under $10^{-1}K_{\text{coupling}}$; $10^{-2}K_{\text{coupling}}$ disappear. At 385 rpm, the 253rd order natural frequency of $10^{-1}K_{\text{coupling}}$; $10^{-2}K_{\text{coupling}}$ and $10^{-2}K_{\text{coupling}}$; $10^{-3}K_{\text{coupling}}$ are 833 Hz, but shaft1 and shaft2 have no motions, so the dynamic meshing force RMS curve does not produce resonance peak. At 260, 115, and 35 rpm, shaft1 and shaft2 also have no motions, so there are no resonance peaks.

Under $10K_{\text{coupling}}$; $10^{3}K_{\text{coupling}}$, the 385 rpm resonance peak of the meshing force RMS curve of the No.1 meshing element disappeared, but a larger

Figure 12. Mode shapes of shaft2 and shaft3: (a) 243rd mode shape, (b) 193rd mode shape, (c) 110th mode shape, and (d) 33rd mode shape.
A resonance peak was produced at 280 rpm. At this time, $10^2K_{\text{coupling}}$ corresponds to the natural frequency of 599.7524 Hz at the 202nd order of the system, and the shaft2 of GB1 and shaft3 of GB2 have bending-torsional coupled motions, as shown in Figure 19. From the meshing force spectrum of No.1 meshing element under $K_{\text{coupling}}$, it is found that with the increase of stiffness of couplings, the maximum amplitude of $f_{m2}$ in the spectrum increases, and the corresponding speed changes, as shown in Figure 20.

In Figure 21, for the No.3 meshing element, the resonance peak at 260 rpm generated by the coupled disappears under $10^{-1}K_{\text{coupling}}$. A resonance peak with a larger amplitude was produced at 275 rpm under $10K_{\text{coupling}}$. In Figure 22, for the No.7 meshing element, the resonance peak amplitude

**Table 6.** Coupling stiffness variation case.

| Case | Stiffness |
|------|-----------|
| 1    | $10^4K_{\text{coupling}}$ |
| 2    | $10^2K_{\text{coupling}}$ |
| 3    | $10K_{\text{coupling}}$ |
| 4    | $K_{\text{coupling}}$ |
| 5    | $10^{-1}K_{\text{coupling}}$ |
| 6    | $10^{-2}K_{\text{coupling}}$ |
| 7    | $10^{-3}K_{\text{coupling}}$ |

resonance peak was produced at 280 rpm. At this time, $10K_{\text{coupling}}$ corresponds to the natural frequency of 599.7524 Hz at the 202nd order of the system, and the shaft2 of GB1 and shaft3 of GB2 have bending-torsional coupled motions, as shown in Figure 19. From the meshing force spectrum of No.1 meshing element under $K_{\text{coupling}}$, it is found that with the increase of stiffness of couplings, the maximum amplitude of $f_{m2}$ in the spectrum increases, and the corresponding speed changes, as shown in Figure 20.

In Figure 21, for the No.3 meshing element, the resonance peak at 260 rpm generated by the coupled disappears under $10^{-1}K_{\text{coupling}}$. A resonance peak with a larger amplitude was produced at 275 rpm under $10K_{\text{coupling}}$. In Figure 22, for the No.7 meshing element, the resonance peak amplitude
decreases under $10^{-1} K_{\text{coupling}} \sim 10^{-3} K_{\text{coupling}}$. New resonance peaks are generated at 95 and 115 rpm under $10 K_{\text{coupling}} \sim 10^3 K_{\text{coupling}}$. Figure 23 shows the spectrogram of meshing force of No.7 meshing element under $K_{\text{coupling}} \sim 10^{-3} K_{\text{coupling}}$. It can be seen that the amplitude of $f_{m3}$ caused by coupled decreases with the decrease of stiffness of couplings. For the No.1 meshing element and No.7 meshing element, which are greatly influenced by coupled effect, they are more sensitive to the change of stiffness of couplings. When the stiffness of coupling decreases, the coupling effect weakens, on the contrary, the coupling effect increases. For GB3, which is not affected by the coupled effect, the RMS curve of meshing force under different coupling stiffness does not change significantly, as shown in Figure 24.

According to analysis and comparison, gear pairs which are greatly affected by coupled are sensitive to the change of the stiffness of couplings. Reducing the

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**Figure 15.** The spectrum of No.7 meshing element at 340 rpm for multi gearbox system and single gearbox.

**Figure 16.** RMS of dynamic meshing force and Spectrogram of No.5 meshing element in GB2: (a) comparison of RMS of dynamic meshing force of No.5 meshing element between multi-gearbox system and single gearbox and (b) spectrum diagram of No.5 meshing element of multi-gearbox system at 0–1000 rpm.

**Figure 17.** RMS of dynamic meshing force and spectrogram of No.9 meshing element in GB3: (a) comparison of RMS of dynamic meshing force of No.9 meshing element between multi-gearbox system and single gearbox and (b) spectrum diagram of No.9 meshing element of multi-gearbox system at 0–1000 rpm.
stiffness of the couplings can make the resonance peak of the RMS curve of the meshing force generated by the coupled disappear or the amplitude will be reduced, and the coupled effect will be weakened. Increasing the stiffness of the coupling will produce a resonance peak with a larger amplitude and increase the coupled effect. With the coupling stiffness increasing, new modes are generated. When the excitation frequency is close to the new mode, a large dynamic response occurs. Therefore, the influence of the stiffness change of the multi-gearbox system on the dynamic characteristics mainly affects the system modal, which in turn affects the dynamic response of the system.

**Influence of supporting stiffness**

The housing is an important supporting component of the marine multi-gearbox transmission system. To analyze the influence of the housing on the vibration response of the multi-gearbox coupling system, the dynamic meshing force and bearing reaction force in the system were investigated under four housing stiffness cases. As shown in Table 7, the four cases are without housing, $K_{\text{housing}_1}$, $10^3K_{\text{housing}}$, and $10^2K_{\text{housing}}$ respectively.

Figure 25 shows the RMS of the dynamic meshing force curve of the No.1 meshing element. It changes little under the four cases, indicating that the housing has little influence on the meshing force in the multi-gearbox transmission system. This conclusion is consistent with the influence of the housing on the system in a single gearbox. It can be seen that the resonance peaks of 385 and 260 rpm caused by the coupled of multiple gearboxes have not changed, indicating that the stiffness of the housing has little influence on the coupled effect of the multi-gearbox system. The

![Figure 18. RMS of dynamic meshing force of No.1 meshing element under different coupling stiffness: (a) RMS of meshing force of No.1 meshing element under $K_{\text{coupling}}=10^{-3}K_{\text{coupling}}$ and (b) RMS of meshing force of No.1 meshing element under $K_{\text{coupling}}=10^2K_{\text{coupling}}$.](image)

![Figure 19. Mode shapes of shaft2 and shaft3 of the 202th order at $10^4K_{\text{coupling}}$: (a) shaft 2 mode shape and (b) shaft 3 mode shape.](image)
The meshing force is not sensitive to the change of the stiffness of the housing mainly because it has little influence on the gear rotor mode in the system. It can be seen from the results that the resonance peak of the dynamic meshing force RMS curve does not change under several housing stiffnesses. It indicates that the mode of the gear-rotor system does not change with the change of the stiffness of the housing.

There are many bearings in the system. In this section, taking No.4 bearing in GB1 as an example, the
influence of housing stiffness on bearing reaction force is analyzed. Figure 26 shows the RMS curve of the bearing reaction force of No.4 under the four cases. It can be seen that the influence of the housing stiffness on the bearing reaction force is mainly in the low-speed stage, and the curve does not change significantly after 315 rpm. In Figure 26(a), the RMS curve of the bearing reaction force after coupled the housing produces a new resonance peak at 90 rpm. It indicates that the coupled housing will generate new modes of the system.

Figure 22. RMS of dynamic meshing force of No.7 meshing element under different coupling stiffness: (a) RMS of meshing force of No.7 meshing element under $K_{\text{coupling}} = 10^{-3} K_{\text{coupling}}$ and (b) RMS of meshing force of No.7 meshing element under $K_{\text{coupling}} = 10^3 K_{\text{coupling}}$.

Figure 23. The meshing force spectrogram of No.7 meshing element under $K_{\text{coupling}} = 10^{-3} K_{\text{coupling}}$. 
As shown in Figure 26(b) and (c), the RMS curves of the bearing support reaction forces of without housing and $10^3K_{\text{housing}}$ completely coincide, and the 90 rpm resonance peak generated by the coupled housing disappears under $10^3K_{\text{housing}}$. This means that when the housing stiffness is very rigid, the influence of the housing can be ignored in the prediction of the system’s dynamic characteristics. The resonance peak before 200 rpm changes significantly when the housing stiffness is reduced, As shown in Figure 26(d). The bearing reaction force is more sensitive to the change of the housing stiffness mainly because the bearing is installed on the housing. In the dynamic model, the bearing-housing element is coupled with the housing-foundation element, and the change of the housing stiffness will change the support stiffness, which in turn affects the bearing reaction force.

**Table 7.** Cases of different housing stiffness.

| Case | Stiffness       | Case | Stiffness       |
|------|-----------------|------|-----------------|
| 1    | Without housing | 2    | $K_{\text{housing}}$ |
| 3    | $10^3K_{\text{housing}}$ | 4    | $10^{-3}K_{\text{housing}}$ |

**Conclusion**

To study the coupled dynamics characteristics of the marine multi-gearbox system, a multi-gearbox coupled model was established by using the finite element theory and substructure technique. In addition, three corresponding dynamic models of single gearboxes are established for comparison. In this paper, the natural frequency and the RMS of meshing force of multi-gearbox system and single gearbox are analyzed. And the dynamic response of the multi-gearbox system under different stiffness of coupling and housing. The main conclusion are summarized as follows:

1. It is found that the adjacent gear pairs of adjacent gearboxes are more easily affected by the coupled effect. The coupled of multiple gearboxes will make the system generate new modes. The RMS curve of meshing force produces some new resonance peaks, which are caused by the new modes excited by the mesh frequencies of adjacent gearboxes. At this time, the mesh frequency components of adjacent gearboxes account for a large proportion of the response spectrum.

2. Reducing the stiffness of the coupling can make the resonance peak of the RMS curve of the meshing force generated by the coupled
disappear or the amplitude will be reduced, and the coupled effect will be weakened. Increasing the stiffness of the coupling will produce a resonance peak with a larger amplitude and increase the coupled effect. Gear pairs which are greatly affected by coupled are sensitive to the change of the stiffness of couplings.

3. The influence of the housing on the dynamic response of the system is mainly concentrated in the low-speed stage. Housing has a more significant influence on the bearing support force. When the housing is very rigid, the model can ignore it and reduce the calculation amount. On the contrary, it should be considered in the model.

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Appendix

Notation

| Symbol | Description |
|--------|-------------|
| $\beta^j$ | The base circle helix angle |
| $\gamma^j$ | The angle between the positive y-axis and the meshing line of the end surface in the global coordinate system |
| $\alpha$ | The meshing angle |
| $\theta$ | The mounting angle |
| $r_p^j$ | The base circle radius of the driving wheel |
| $r_s^j$ | The base circle radius of the driven wheel |
| $k_m^j$ | The mean value of meshing stiffness |
| $c_m^j$ | The meshing damping |
| $m_p^j$ | The mass of the driving wheel |
| $m_s^j$ | The mass of the driven wheel |
| $I_{xp}^j$, $I_{yp}^j$ | The rotational inertia of the driving wheel in the x, y, z directions |
| $I_{zp}^j$, $I_{yp}^j$ | The rotational inertia of the driven wheel in the x, y, z directions |
| $k_{xx}$ | The bearing stiffness in the x direction |
| $k_{yy}$ | The bearing stiffness in the y direction |
| $k_{zz}$ | The bearing stiffness in the z direction |
| $k_{\theta x \theta x}$ | The bearing stiffness in the $\theta_x$ direction |
| $k_{\theta y \theta y}$ | The bearing stiffness in the $\theta_y$ direction |
| $q_m^j$ | The node displacement column vector of the $j$th meshing element |
| $q_s^j$ | The displacement column vector of the shaft node of the $j$th shaft element |
| $q_b^j$ | The displacement column vector of the bearing node of the $j$th bearing-housing element |
| $q_h^j$ | The displacement column vector of the housing node of the $j$th bearing-housing element |
| $q_c^j$ | The node displacement column vector of the $j$th coupling element |
| $C_m^j$ | The damping matrix of the $j$th meshing element |
| $C_s^j$ | The damping matrix of the $j$th shaft element |
| $C_b^j$ | The damping matrix of the $j$th bearing-housing element |
| $C_h^j$ | The super-element equivalent damping matrix |
| $M_m^j$ | The mass matrix of the $j$th meshing element |
| $M_s^j$ | The mass matrix of the $j$th shaft element |
| $M_h^j$ | The super-element equivalent mass matrix of housing |
| $M_c^j$ | The mass matrix of the $j$th coupling |
| $K_m^j$ | The stiffness matrix of the $j$th meshing element |
| $K_s^j$ | The stiffness matrix of the $j$th shaft element |
| $K_b^j$ | The stiffness matrix of the $j$th bearing-housing element |
| $K_h^j$ | The super-element equivalent stiffness matrix of housing |
| $K_c^j$ | The stiffness matrix of the $j$th coupling |
| $M_{global}$ | The overall mass matrix of the system |
| $C_{global}$ | The overall damping matrix of the system |
| $K_{global}$ | The overall stiffness matrix of the system |
| $P_0$ | The column vector of system external load |
| $x(t)$ | The column vector of displacement of all nodes |