The effect of $K^+$ potential on the nuclear equation of state for the $K^+$ production in heavy ion collisions by using a quantum molecular dynamics model

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Abstract. This work investigates and focuses on the effect of $K^+$ potential on the nuclear equation of state (soft and hard EoS) for $K^+$ production in heavy ion-collision by using the quantum molecular dynamics model (QMD). Production of a cross-section of $K^+$ mesons in $^{197}$Au + $^{197}$Au collision at incident energy 1.1 (polar angle 56$^\circ$) and 1.46 (polar angle 40$^\circ$ and 56$^\circ$) A GeV respectively, as well as studying impact parameter between 4.14 and 12.41 fm. The production cross-section as a function of the laboratory momentum ($p_{lab}$) is computed and compared with KaoS experiments. The calculations performed with and without Brown-Rho parameters ($K^+N$) as well as the soft and hard equation of state. The results displayed the theoretical calculations with the soft EoS are similar to the hard EoS. The theoretical calculations with soft and hard EoS while including $K^+N$ potential tend to be consistent with the KaoS experiments, this indicates that the production of the cross section for $K^+$ mesons as a function of the laboratory momentum is one of sensitive probes to extract information on the $K^+N$ potential at high densities.

1. Introduction

Relativistic heavy-ion collisions at intermediate energy provide a unique possibility to approach the nuclear matter with a density of 2-3 $\rho_0$. $\rho_0= 0.16$ fm$^3$ which presents the nuclear-saturated density. It is important to examine the theories and experiments in heavy-ions collisions that exhibit the behaviour of a nuclear equation of state (EoS) at high temperatures and density. Furthermore, the results of heavy-ion collisions are not only interesting in the fields of nuclear and particle physics, but also in understanding other fields of physics, such as the properties of the core of the compact star, the evolution of the early universe and the stability of neutron star [1-3].

According to the previous research [4-5], the central $^{197}\text{Au} + ^{197}\text{Au}$ collisions at the incident energy in which the density of nuclear matter is about 2–3 times that of a normal nuclear matter density were studied. The reports revealed that the production of strange mesons below the
production thresholds of these particles in free $NN$ collisions is used as a sensitive probe to test the behaviour of nuclear matter at high densities. The rest mass of $K^\pm$ is 0.454 GeV. Additionally, the threshold of $K^+$ in $NN$ collisions (in the laboratory system) is defined by the associate production of $NN \rightarrow K^+\Lambda N$, which is 1.58 GeV. Similarly, it is 2.5 GeV for the $K^-$ production via a pair creation $NN \rightarrow NNK^-K^+$. Another research studied the density reached in the reaction zone [6-10]. The results that were presented showed that the stiffness of nuclear matter depended on the density that was reached in the reaction zone, this is due to the specific production mechanism of their rather long mean free path ($\approx$ 5 fm at normal nuclear density). Therefore, $K^+$ mesons are ideal probes for exploring the high-density phase of a heavy-ion reaction, they are also useful in studying the stiffness of the EoS. Calculations by Zheng et al. [11] illustrated that the FOPI data on the kaon flow [12] is the best explanation in using kaon potential ($K^+ N \approx$ 30 MeV) given by Brown-Rho (BR) Parameterization [13]. The $K^+ N$ the in-medium potential is repulsive, while $K^- N$ the in-medium potential is strongly attractive. The collected motion of the $K^+$ is investigated in order to deduce the strength of the $K^+$ potential by measuring the $K^+$ cross-section in heavy-ion collision.

In this paper, we presented the cross-section production of $K^+$ mesons at incident energy 1.1 and 1.46 A GeV as well as impact parameter from 4.14 to 12.41 fm by utilizing the quantum molecular dynamics (QMD) model. The theoretical results of the cross-section production of $K^+$ mesons with and without the $K^+$ the kaon potential by using the nuclear equation of state which was computed and compared with to KaoS experiments [14].

2. Theories

2.1. Kaons in dense matter

The natural framework to studying the interactions between pseudoscalar mesons and baryons at low energies is the chiral perturbation theory (ChPT). Utilizing the Chiral Lagrangian, the field equations for the $K^\pm$ mesons are derived from the Euler-Lagrange equations [15, 16]

$$\left[(\partial_\mu \pm iV_\mu)^2 + (m^*_K)^2\right] \phi_{K^\pm}(x) = 0. \quad \text{(1)}$$

Here the upper sign “+” is for the $K^+$-meson and the lower sign “−” for $K^-$. $V_\mu$ is the vector potential and $m^*_K$ is the effective mass of the kaon, which read as

$$V_\mu = \frac{3}{8f_\pi^2} j_\mu, \quad V_\mu = (V_0, V), \quad V_0 = (V_0, V),$$

$$m^*_K = \sqrt{m_K^2 - \frac{\sum_{NN} f_{\pi}^2}{f_\pi^2} \rho_s + V_\mu V^\mu}, \quad \text{(3)}$$

$m_K = 0.946$ A GeV presents the kaon mass. Due to the bosonic character, the coupling of the scalar field to the mass term is no longer linear, as, with the baryons, the quadratic provides additional contribution originating from the vector field. The effective quasi-particle mass is defined by the equation (3), which is a Lorentz scalar and is equal for both $K^+$ and $K^-$. The $K^\pm$ single-particle energy is exhibited as follows

$$\omega_{K^\pm}(k, \rho) = \sqrt{k^2 + m^2_{K^\pm}} \pm V_0, \quad \text{(4)}$$

$k^* = k \mp V$ is represented as the kaon effective momentum, $k_\mu = (k_0, k)$, $V_\mu = (V_0, V)$. The kaon vector field is introduced by minimal coupling into the Klein-Gordon with opposite signs for $K^+$ and $K^-$. $m^*_K$ is the kaon effective (Dirac) mass. The kaon and antikaon potentials $U_{K^\pm} = (k, \rho)$ are defined by
\[
U_{K^\pm} (k, \rho) = \omega_{K^\pm} (k, \rho) - \omega_0 (k),
\]

where presented by
\[
\omega_0 (k) = \sqrt{k^2 + m^2_k}.
\]

The covariant equations of motion are obtained by using the classical (test particle) limit from the relativistic transport equation for the kaons which can be derived from the equation (1). They represent analogous to the corresponding relativistic equations for baryons as these followings;
\[
\frac{dq^\mu}{d\tau} = k^*_{\mu^0} m^*_{K^+},
\]
\[
\frac{dk^{*\mu}}{d\tau} = \frac{k^*_{\mu}}{m^*_{K^+}} F^{\mu\nu} + \partial^\mu m^*_{K^+}.
\]

Here \( q^\mu = (t, k) \) serve as the coordinates in the Minkowski space and \( F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu \) is also the field strength tensor for \( K^+ \). While \( K^- \) is where the sign of the vector field changes. The equations of motion are identified, however, here \( F^{\mu\nu} \) has to be replaced by \( -F^{\mu\nu} \).

As referenced by [11], in this work, the Brown and Rho parameterization (BRP) is being used:
\[
\sum_{KN} = 450 \text{ MeV}, \quad f_\pi^2 = 0.6 f_\pi^2 \text{ for the vector field while the } \frac{f_\pi^2}{f_\pi^2} = f_\pi^2 \text{ for the scalar parts are provided by } - \sum_{KN}/f_\pi^2 \rho_s. \] Up to saturation density, the Brown and Rho potential is \( K^+ N \approx 30 \text{ MeV} \) (the \( K^+ N \) potential)

2.2. Quantum Molecular Dynamics (QMD) Model
Throughout the quantum molecular dynamics [17] framework the baryon dynamics are described where each nucleon is represented by a coherent state of this following from;
\[
\psi(r, p, t) = \frac{1}{(2\pi L)^{3/4}} \exp \left\{ -\frac{(r - r_0)^2}{4L} \right\} \exp \{ i p \cdot (r - r_0) \},
\]

here the \( r_0 \) presents the centre of the Gaussian wave pocket and \( L = 1.08 \text{ fm}^2 \) is the width of the wave pocket. Correspondingly, the density of a system with \( N \) nucleons in coordinated space is provided as
\[
\rho (r, t) = \sum_{i=1}^{N} \frac{1}{(2\pi L)^{3/2}} \exp \left\{ -\frac{(r - r_{i0})^2}{2L} \right\}.
\]

The time evolution of the \( N \)-body distribution has been determined by the motion of the centroid of the Gaussian \( \{ \hat{r}_{i0}, \hat{p}_{i0} \} \), which is propagated by the Poisson brackets,
\[
\dot{\hat{r}}_{i0} = \{ \hat{p}_{i0}, H \}
\]
and
\[
\dot{\hat{p}}_{i0} = \{ \hat{r}_{i0}, H \}
\]
where the nuclear Hamiltonian \( H \) can be read as below;
\[
H = \sum_{i=1}^{N} \sqrt{\hat{p}_{i0}^2 + m_i^2} + \sum_{i<j}^{N} \left( U_{ij}^{Str} + U_{ij}^{Coul} \right).
\]
In this instance, $U_{ij}^{Str}$ and $U_{ij}^{Coul}$ pose as the nuclear mean-field and the Coulomb interaction, respectively.

The strength of the nuclear compression is normally quoted in terms of the incompressibility by value constant $K$ (compressibility)[18], which the compressibility defined as

$$K = 9\rho^2 \frac{\partial^2}{\partial \rho^2} \left( \frac{E}{A} \right),$$

(14)
normally in order to the classify of the binding energy per nucleon ($E/A$) as a density function the Skyrme parameterizations ($U$) are used in the equation (15). Therefore, two different nuclear equations of state are commonly used. A hard-nuclear equation of state (Hard EoS) and a soft-nuclear equation of state (Soft EoS) which have a compressibility of $K = 380$ MeV and $K = 200$ MeV, respectively [17]. The Skyrme parameterizations are described by

$$U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^{\gamma}.$$

(15)

The relation of the $\alpha$, $\beta$ and $\gamma$ is shown in table 1. The $\rho_0$ presents the nuclear density, which is measured in units of the saturation density ($\rho_0 \approx 0.16$ fm$^3$).

**Table 1.** Parameters in the equations (14) and (15) are used for the soft and hard nuclear equation of state (EoS) [17].

| $K$ (MeV) | $\alpha$ | $\beta$ | $\gamma$ | EoS       |
|-----------|---------|--------|---------|-----------|
| 200       | -356    | 303    | 7/6     | Soft      |
| 380       | -124    | 70.5   | 2       | Hard      |

2.3. Cross section of the $K^+$ production in the small volume of the spherical momentum space

According to the figure 1, the collective flow of the $K^+$ from heavy ion collisions can be explained by a physical quantity such as the cross section of the $K^+$ production in the small volume of the spherical momentum space. Mathematically, the small volume of the spherical momentum space is defined as follows

$$dp \sin \theta d\theta d\phi,$$

(16)

where $d\phi = 3.6 \times (\pi/180) = 2\pi/100$ (the steradian unit) and substitute (implemented/introduced) $d\phi$ into the equation (16) thus,

$$dp \sin \theta d\theta d\phi = \frac{dp2\pi \sin \theta d\theta}{100},$$

(17)

and the $d\Omega$ is defined as

$$d\Omega = \frac{2\pi \sin \theta d\theta}{100},$$

(18)

so, from the equation (17) and (18) are given

$$dpd\Omega = \frac{dp2\pi \sin \theta d\theta}{100}.$$

(19)

The cross section of the $K^+$ production ($\sigma$) on the transverse plane transverse plane which is defined as

$$\sigma = P_{prob} \int_{\theta_{max}}^{\theta_{min}} 2\pi \theta db,$$

(20)
therefore, the cross section of the $K^+$ production ($\sigma$) on the transverse plane is

$$\sigma = P_{prob} \pi b_{max}^2,$$

where $P_{prob}$ represents the probability of the $K$ meson which is found after heavy-ion collisions. $b_{max}$ is the maximum impact parameter in nucleus–nucleus collisions. Consequently, the production cross-section of $K^+$ in relation to the small volume of the spherical momentum space, which can be read as

$$\frac{d^2 \sigma}{dp d\Omega} = \frac{P_{prob} \pi b_{max}^2}{dp 2\pi \sin \theta d\theta} \left( \text{barn} \right. \left. \frac{}{(\text{GeV}/c \cdot \text{sr})} \right).$$

\textbf{Figure 1.} The cross-section of the $K^+$ production in the spherical momentum space which the small volume.

3. Results and discussion

Figure 2 displays the production cross-section for $K^+$ mesons as a function of the laboratory momentum in $^{197}$Au+$^{197}$Au collisions for the impact parameter from 4.14 to 12.41 fm at incident energy 1.1 A GeV with the polar angle 56° by using the quantum molecular dynamics model. The full circle symbols represent the KaoS data [14]. First, the green line represents the results calculated by using the soft EoS with the $K^+ N$ potential. Second, the black line shows the results calculated by using the hard EoS with the $K^+ N$ potential. Third, the orange line represents the results calculated by using the soft EoS without the $K^+ N$ potential. Next, the blue line serves to show the results calculated by using the hard EoS without the $K^+ N$ potential. From this figure, it can be seen that the result of theoretical calculation with soft EOS is similar to that of a hard EoS. Furthermore, calculated results without the $K^+ N$ potential, the production of a cross section for the $K^+$ mesons are too narrow with too high small-momentum part and too low large-momentum part. After taking into consideration the $K^+ N$ potential, which is positive potential [16] for the production cross section for $K^+$ mesons, the production cross section for $K^+$ mesons is being pushed away from nucleons, and lead to a good fit for the experimental data. Therefore, the results calculated with the soft and hard EoS when including the $K^+ N$ potential tend to be consistent with the KaoS data. In order to properly compare the theoretical results with the experimental data feather, we calculate the root mean square errors (RMSE) for each value given by utilizing the soft & hard EoS as well as with and without $K^+ N$ potential. The results are displayed in table 2. What can be assessed from this table, is that the result calculated by using soft and hard EoS with the $K^+ N$ potential has the smallest RMSE. This implies that this result is the best one for describing the KaoS data, which the KaoS experiments in these energies are confirmed by the QMD model for the first time.
Figure 2. Illustrates the production cross-section for $K^+$ mesons as a function of the laboratory momentum from $^{197}\text{Au} + ^{197}\text{Au}$ collisions at incident energy 1.1 A GeV is represented with the polar angle at 56°. The full circle symbols show the KaoS data [14]. The green line displays the results calculated by utilizing the soft EoS with the $K^+N$ potential. While the black line in the figure presents the results which are calculated using the hard EoS with the $K^+N$ potential. The orange line shows the results calculated using the soft EoS without the $K^+N$ potential. Finally, the blue line serves to show the results calculated using the hard EoS without the $K^+N$ potential.

Figure 3 represents the production cross-section for $K^+$ mesons as a function of the laboratory momentum from $^{197}\text{Au} + ^{197}\text{Au}$ collisions at incident energy 1.46 A GeV with the polar angles 40° and 56° by applying the quantum molecular dynamics model. The full solid circle symbols represent the KaoS data [14]. The dash lines are the results calculated by using the soft EoS with the $K^+N$ potential. The solid lines are the results calculated by using the hard EoS with the $K^+N$ potential. Following that, the results calculated with the soft and hard EoS which include the $K^+N$ potential tend to be consistent with the KaoS data in figure 2 are used for the confirmation of theoretical calculations. The beam energy is enhanced by the different polar angles. From this figure it is clear that the theoretical results calculated by using the soft & hard EoS and with the $K^+N$ potential tend to be consistent with the KaoS data. This indicates that the $K^+N$ potential should be taken into account in theoretical simulations of the $K^+$ production in heavy-ion collisions in order to reasonably describe the experimental data.
Figure 3. Illustrates the production cross-section for $K^+$ mesons as a function of the laboratory momentum from $^{197}$Au + $^{197}$Au collisions at incident energy 1.46 A GeV with the polar angles 40° and 56° respectively. The full solid circle symbols represent the KaoS data [14]. The dash lines are the results calculated by using the soft EoS with the $K^+$N potential. Solid serve as a representation of results calculated by using the hard EoS with the $K^+$N potential.

Table 2. The root mean square errors [19] (RMSE) for the calculated results the production cross-section for $K^+$ mesons as a function of the laboratory momentum from $^{197}$Au + $^{197}$Au collisions at incident energy 1.1 A GeV.

| Energy          | RMSE |
|-----------------|------|
|                 | SWO  | HWO  | SW   | HW   |
| 1.1 A GeV (56°) | 0.0160 | 0.0132 | 0.0061 | 0.0052 |

4. Conclusion

We use the quantum molecular dynamics model to simulate the production of $K^+$ at incident energy 1.1 and 1.46 A GeV and the impact parameter between 4.14 and 12.41 fm, in order to analyze the production of a cross-section for $K^+$ mesons as a function of the laboratory momentum from $^{197}$Au + $^{197}$Au collisions in polar angles 0.56° (1.1 A GeV) and 40°, 56° (1.46 A GeV) respectively, to compare calculated results with the KaoS data. We observed that the production of a cross-section for $K^+$ mesons as a function of the laboratory momentum from...
$^{197}$Au+$^{197}$Au collisions after taking in to account the $K^+N$ potential and using the soft & hard equation of state show that the theoretical results are in a good agreement with the experimental data which indicates that one should include the $K^+N$ potential as well as usage of the soft & hard nuclear equation of state in the theoretical simulations in order to reasonably describe the production characteristics of a cross-section for $K^+$ mesons as a function of the laboratory momentum in $^{197}$Au+$^{197}$Au collisions. In other words, the production cross-section for $K^+$ mesons as a function of the laboratory momentum is one of the more sensitive probes to extract information on the $K^+N$ potential at high densities.

Acknowledgments

This work is financially supported by Science Classrooms in University Affiliated School Project (SCiUS). Thanks also go to Demonstration School University of Phayao and Department of Physics, School of Science, University of Phayao for supporting facilities. Acknowledgments are also to Miss Nipawan Narueprempree and Prof. Yu Ming Zheng for helpful comments and corrections.

References

[1] Akmal A, Pandharipande V R and Ravenhall D G 1998 Phys. Rev. C 58 1804
[2] Brown G E and Bethe H A 1994 Astrophys. J. 423 659
[3] Li G Q, Lee C H and Brown G E 1997 Phys. Rev. Lett. 79 5214
[4] Hartnack C, Jaenicke J, Sehn L, Stöcker H and Aichie J 1994 Nucl. Phys. A 580 643
[5] Fuchs C 2006 Prog. Part. Nucl. Phys. 56 1
[6] Ko C M 1984 Phys. Lett. B 138 361
[7] Aichie J and Ko C M 1985 Phys. Lett 55 2661
[8] Strum et al (KaoS Collaboration) 2001 Phys Rev. Lett. 86 39
[9] Fuchs C, Faessler A, Zabrodin E and Zheng Y M 2001 Phys. Rev. Lett. 86 1974
[10] Förster A et al (KaoS Collaboration) 2003 Phys Rev. Lett. 91 152301
[11] Zheng Y M, Fuchs C, Faessler A, Yan Y P and Kobdaj C 2004 Phys. Rev. C 69 034907
[12] Herrmann H 1999 Prog. Part. Nucl. Phys. 42 187
[13] Brown G E and Rho M 1996 Nucl. Phys. A 569 503
[14] Förster A et al (KaoS Collaboration) 2007 Phys. Rev. C 75 024906
[15] Li G Q and Ko C M 2001 Nucl. Phys. A 594 460
[16] Srisawad P et al 2018 J. Phys.: Conf. Ser. 1144 012102
[17] Aichelin J 1991 Phys. Revp. 202 233
[18] Hartnack C et al 1998 Eur. Phys. J. A 1 151
[19] Chai T and Draxer R R 2014 Geosci. Model Dev. 7 1250