Capacity Analysis for Cooperative Vehicular Networks with Different Fading Channels

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Abstract. The cooperative vehicular networks (CVNs) enable vehicles to obtain information of remote infrastructures through cooperative vehicles. Nevertheless, the tradeoff between the cooperative vehicular ratio and transmission capacity of CVN is a challenge. In this paper, an analytical framework is developed to study the impacts of cooperation, interference and channel fading on the system capacity. Moreover, based on the proposed analytical framework, a closed-form expression is derived to reveal the relationship between transmission capacity and other network parameters, such as outage probability, vehicular density and adjacent infrastructures distance. Numerical results show that an optimal cooperative vehicular ratio leads to the maximum capacity. The results can also provide a preferable guidance for the design of CVNs.

1. Introduction

Capacity is one of the most fundamental performance metrics in vehicular networks, which provides guidance for system configuration[1]. In the cooperative vehicular networks (CVNs), some vehicles termed Vehicles-of-Interests (Vols) obtain the required files from infrastructure and relay vehicles. However, an increasing number of cooperative vehicles result in increased interference and reduction of system capacity. Therefore, it is important to study the problem of balancing system capacity and cooperative vehicle ratio under different fading channels.

In the current studies [2]–[4], there have been a lot of researches on vehicular network performance, however the cooperative communication in the case of using multiple relay methods has not been thoroughly studied. A new cooperative communication strategy is proposed in [5], which leveraged vehicle-to-vehicle (V2V), vehicle-to-infrastructure (V2I) communication, and mobility to boost capacity of vehicular networks. However, the disk model used in it ignored the fading and interference in the communication process which may not fit in the real scenario.

In this paper, the capacity of the CVN network is analyzed synthetically based on the highway scenario, considering different fading in different communication scenes. In addition, a closed-form expression of transmission capacity is derived by establishing an analysis framework, and the approximate optimal cooperative vehicular ratio is obtained by a newton iteration method. Through closed form and simulation of transmission capacity, the relationship between system capacity and outage probability, vehicle density and distance between adjacent infrastructures is studied, and the trade-off between cooperative vehicle ratio and system capacity in CVN is solved.

The rest of this paper is organized as follows. The Section 2 introduces the system model and communication strategy. The calculation of V2V and V2I transmission capacity is elaborated in
Section 3. Section 4 provides simulations to verify the results of theoretical analysis and make more discussions. Finally, we conclude this paper in Section 5.

2. System model

2.1. Network model

As shown in Figure 1, consider a bi-directional highway with length $M$ and an equally spaced infrastructure. The infrastructures installed at both ends of the road and no other facilities are set between the neighboring infrastructures. Assume that all infrastructures are connected to the same central cloud. The central server fully knows the data transmission process of the system and the information required by each VoI. To simplify the analysis, the width of the road is ignored and multiple lanes in the same direction are considered as one. In Figure 1, we define an area that contains a V2V and V2I communication as a single segment with a length of $L$, which is equal to the distance of adjacent infrastructures, hence the number of sections on the whole highway is $n = [M / L]$, $M \gg L$, where $[x]$ denotes the smallest integer not less than $x$. In addition, the model analysis of highway is extensible, for example, it can be used for 2D grid model, and the capacity of the 2D network can be calculated by adding the capacity of each separate road.

As for traffic model, we consider that the distribution of vehicles in each segment follows Poisson process with density $\lambda$. For analytical tractability, we assume that all vehicles drive at the same speed denoted $v$ in this paper. Note that transmission capacity analysis proposed in this paper is irrelevant to speed, which is shown in section 3. To simplify the analysis, we assume that the vehicle is either a VoI with a download request or a relay vehicle which only delivers data. Let $\mu$ denote the ratio of VoIs, and the ratio of relay vehicles is $1 - \mu$.

![Figure 1. Highway cooperative vehicular networks](image)

2.2. Communication model

During the V2I and V2V communication, assume that all infrastructure points and vehicles have the same radio range, denoted by $r_i$ and $r_v$, respectively. Similarly, $W_i$ and $W_v$ are defined as transmission rates of infrastructures and vehicles, respectively. To study the effect of fading on V2V communication, Nakagami-m distribution is employed to reflect the small-scale fading of vehicular channels which has been proven to fit well to the vehicular channel measurements [6]. Consequently, the probability density function (PDF) of received signal envelope from a relay vehicle is:

$$f(x;m,\omega) = \frac{2m^m}{\Gamma(m)\omega^m}x^{2m-1}\exp(-\frac{m}{\omega})x^2,$$

where $x \geq 0, \omega \geq 0, m \geq 1/2$, $\omega$ is the mean power and $m$ is shape factor indicating the severity of fading, $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function.
A unicast transmission scenario is considered in V2I communication. In fact, the calculation of downlink capacity in the unicast scene is the same as in multicast scene, since the infrastructure is the only transmitter. In the multicast scenario, different channels are allocated to different receivers to avoid interference. Due to the high-speed mobility of the vehicle, the received signal fluctuates, Rayleigh fading is used in V2I scene to characterize the fading environment. Note that the Rayleigh fading is actually a special case of the Nakagami-m distribution when $m = 1$. Hence the received power $S$ received by the vehicles also follows a gamma distribution $Ga(m, \omega)$. 

2.3. Communication strategy in CVN

The communication strategy will be introduced in this subsection. Some VoIs on each street want to download large files which are split into multiple pieces in different infrastructures. The communication process in whole system is divided into two stages: V2I and V2V. In the first phase, VoIs and relay vehicles both get different required data from infrastructures. In the second phase, the VoIs continue to receive data from relay by exploiting the mobility of vehicles and V2V communications when they move outside the coverage of infrastructure.

During V2V region, the strategy in [7] is adopted in this paper to maximize the number of vehicle pairs which include a VoI and its exclusive relay vehicle. As shown in figure 1, we set the $+x$ direction to the right. Choose the leftmost relay vehicle which has at least one VoI as the first transmitter and the leftmost VoI in its range be the first receiver. In order from left to the right, we choose the next transmitter as the nearest relay vehicle to the current transmitter which at least one VoI can be found within its coverage. Repeat the above process until reach the rightmost boundary of V2V region.

3. Theoretical analysis

As mentioned before, considering the fading and interference experienced by the received signal in the communication process, the SINR model is used to determine the connection probability between the receiver and the transmitter. When the received SINR exceeds a certain threshold $\eta$, the transmission is assumed to be successfully completed. Therefore, the outage probability $P_{out}$ can be defined as:

$$P_{out} = \Pr\left\{ \frac{S}{I + N_0} \leq \eta \right\} = \sum_{K = 0}^{\infty} \Pr\left\{ \frac{S}{I} \leq \eta | N_i = K \right\} \Pr\{N_i = K\},$$

where $I$ represents the power of interference and $N_0$ is the noise power at the receiver. $N_i$ denotes the number of interference relay vehicles. The transmission capacity of the system can be defined as:

$$C = W\lambda(1 - P_{out}),$$

where $W$ is the expected transmission rate and $\lambda$ is the density of successful simultaneous transmissions. According to formula (3), we can see that the transmission capacity of the system in this paper is relevant to the connectivity probability and the density of receivers. Obviously, the density of VoIs is related to its spatial distribution, and the connectivity probability is related to the SINR of the system. As a result, it is reasonable to assume that the vehicle is driving at the same speed and the analysis can be used in other velocity models.

3.1. V2V region capacity analysis

Assume that there are $N_p$ relay vehicles communicate with VoIs in V2V region. Using the analysis above, the capacity is:

$$C_{v2v} = W_p\lambda(1 - P_{out}) = W_p(1 - P_{out}) E[N_p]/(L - 2r_f),$$

where $(L - 2r_f)$ is the length of V2V region.

Recall that we have introduced the optimum scheduling scheme in Section II. The scheme can lead to the maximum number of vehicle-pairs which is denoted by $N_p^{max}$. Similar to [5], the distance
between i-th and i+1-th relay vehicles can be formulated as 
\[ J_i, i=1,2,... \] , and \( J_0 \) is the distance between the region point and the first vehicle. Note that the distribution of \( J_i, i=1,2,... \) are independent and identically distributed (i.i.d) and the inter-vehicle distances are exponentially distributed, we regard the counting process \( N_p \) as a renewal process. According to the Elementary Delayed Renewal Theorem, the formula can be given by:
\[
\lim_{t \to \infty} E[N(t)](t)^{-1} = (\bar{X})^{-1}, \text{ where } E[N(t)] \text{ is the expectation of the number of vehicles involved in V2V communication.}
\]
In the renewal process, the time \( t \) can be approximated to \( L-2r_a \), and \( \bar{X} \) is the expectation of \( E[J_i] \). Moreover, the maximum expectation of the number of V2V communication vehicles is:
\[
\max_{1 \leq i \leq K} - 2 \left( E[L_p] \right) \left( E[J_i] \right)^{-1}, \text{ where the calculation results of } E[L_p] \text{ follows that:}
\]
\[
e^{-\mu \rho \rho_0} \rho_0 \exp\left(-\mu \rho \rho_0\right), \quad (5)
\]
More calculation details have been introduced in [7]. After getting the number of vehicles, the next step is to consider the interference in the communication process. According to the section II, the number of interference vehicles follows the Poisson distribution and can be given by:
\[
\Pr\{N_i = K\} = \lambda_i^k (K!)^{-1} \exp(-\lambda_i) = (\mu \rho)^k (K!)^{-1} \exp(-\mu \rho).
\]
To further explore the relationship among capacity, cooperative communication and parameters affecting performance, we consider the special case which interference with i.i.d. As mentioned above, Nakagami-m fading channel is used in this paper to describe V2V communication. Accordingly, the received power \( I_i (i=1,2,...L) \) is a Gamma distributed random variable with the same fading parameter \( m_i \) and average power \( \omega_i \). In addition, we can conclude that the total interference power \( I \) follows the Gamma distribution \( I \sim Ga(mL, \omega_i) \). Since the white noise power is negligible compared to the co-channel interference power, the outage probability is given by:
\[
P_{out} = 1 - \Pr\{S/1 \geq \eta\} = 1 - \Pr\{S/1 \geq \eta | N_i = K\} \Pr\{N_i = K\}
\]
\[
= 1 - \sum_{K=1}^{\infty} I_{gamma}(m_i K, \omega_i)(\mu \rho)^k (K!)^{-1} \exp(-\mu \rho), \quad (7)
\]
where \( I_{gamma}(a,b) = \Gamma(a + b) / \Gamma(a) \Gamma(b) B(a,b) \) \( e^{-x} = (m/\omega) / ((m/\omega) + (\eta m / w_i)), B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx \) is the incomplete Beta function[8]. By combining (4) and (7), the channel capacity of V2V communication can be:
\[
C_{V2V} = W_e E[N_p_{\text{max}}] (1 - P_{out}) / (L-2r_a) = W_e \sum_{i=1}^{\infty} I_{gamma}(m_i K, \omega_i)(\mu \rho)^k (K!)^{-1} \exp(-\mu \rho)(E[J_i])^{-1} \quad (8)
\]
### 3.2. V2I region capacity analysis

As mentioned above, the V2I region is unicast transmission. The interference among receivers can be ignored by assigning different channels to the receivers during communication. Therefore, the outage probability \( P_{out} \) can be defined as the successful reception probability that SINR at the receiver does not exceed the threshold value:
\[
P_{out} = 1 - \Gamma(m_i \omega_i / N_0 \eta) / \Gamma(m_i).
\]
**Proof:** In Section II, we already know the PDF of power \( S \) received by the vehicles. Taking the noise power into account, the outage probability can be numerically calculated as:
During the V2I communication, both relay vehicles and VoIs receive capacity only from infrastructures. Accordingly, the transmission capacity can be obtained as follows:

\[ C_{\text{V2I}} = C_{\text{R2I}} = W_I (1 - P_{\text{out}}). \]  

### 3.3. System capacity and optimal cooperative vehicular ratio

In this subsection, we first give the total system capacity of the entire highway. According to the "renewal theorem", each road section is independent of each other, and the performance analysis results are the same. Combined with the capacity of V2I and V2V, the total capacity of highway CVN system is:

\[ C_{\text{sys}} = n(C_{\text{V2I}} + C_{\text{V2V}}). \]  

Since the V2I capacity is independent of the value of \( \mu \), the following analysis will only consider the capacity of the V2V region. In the case where other parameters are determined, the \( \mu \) corresponding to the maximum capacity of the system is the optimal ratio \( \mu^* \). It is a feasible method to obtain the extreme point by finding the first derivative of (8) and we can obtain:

\[ C_{\text{V2V}}(\mu^*) = (W_I \sum_{L=1}^{\infty} I_{\text{C}}(mK, w)(\mu^* \rho)^K (K!E[J_i]))^{-1} \exp(-\mu^* \rho)^{-\mu^* \rho}) = 0. \]  

However, (13) is a transcendental equation, and the analytical solution of this equation is difficult to find. The Newton iteration method can be adopted here to obtain an approximate solution [9]. The iteration formula is as follows:

\[ \mu_{k+1} = \mu_k - C_{\text{V2V}}'(\mu_k) (C_{\text{V2V}}(\mu_k))^{-1}, \]  

where \( k \) is number of iterations and \( C_{\text{V2V}}(\mu_k) \) is the second derivative of \( C_{\text{V2V}} \) function. Considering that when \( \mu_{k+1} - \mu_k < 0.001 \), iterative termination and \( \mu_{k+1} = \mu^* \). To guarantee a more accurate approximate solution, let the initial value \( \mu_0 = q(1/\rho) \), where \( q \) is a positive value.

**Remark 1:** If initial value is close enough to the extreme point of (13), we hold that the solution converges. Finding the lower bound \( \mu_{\text{low}}^* \) of \( \mu^* \) is a suitable choice. Since \( K \) has a negative correlation with \( I_{\text{C}}(mK, w) \) and gradually decreases to 0, the cumulative function will reach the maximum value. As a result, when \( \forall K \in [K^*, \infty) I_{\text{C}}(mK, w) \equiv 0 \), the \( \mu^* \) corresponding to the \( K^* \) is \( \mu_{\text{low}}^* \).

\[ \mu_{\text{low}}^* = (1/\rho) \ln(I_{\text{C}}(mK, w)/\Delta L + 1) \]  

and we can set \( \mu_0 = \mu_{\text{low}}^* \) [8].

### 4. Simulation results

In this section, we test the results of theoretical analysis. Based on realistic highway scenarios and typical parameter settings in the DSRC protocol [10], we set the length of highway is 50km, and the other simulation parameters are set as follows: \( r_I = 500 \), \( r_r = 200 \), \( W_r = 2 \), \( W_I = 20 \), \( \alpha = 4 \), \( \eta = 3 \), \( m_I = 0.5 \), \( \omega_I = 0.2 \), \( m_r = 1 \), \( \omega_r = 1 \). 

Figure 2 reveals the relationship between the proportion of cooperative vehicles and the transmission capacity in a segment. From the figure, we can see that the capacity increases first and then decreases, which is consistent with our analysis. When the proportion of VoIs increases, although...
the amount of information transmitted increases, the interference also reduces the performance and ultimately the capacity. In addition, we use the Newton interpolation method described above to calculate the optimal ratio $\mu^*$ for each Poisson density. It can be seen from the figure that the capacity corresponding to the optimal value is basically the maximum value of the system, and the simulation is consistent with the analysis result. As a result, reasonable control of the ratio of VoIs can maximize the capacity of the cooperative vehicular networks. It is also explicitly that with the increase of $\rho$, the transmission capacity gets improved, which is in line with the expectations of the paper.

![Figure 2. Relationship between $\mu$ and capacity](image1)

![Figure 3. Relationship between $L$ and capacity](image2)

![Figure 4. Comparison of transmission capacity between SINR model and disk model](image3)

Figure 2 demonstrates the relationship between the system capacity and the distance of adjacent infrastructures. It can be seen that the overall capacity declines as the distance between infrastructures increases. This phenomenon can be explained by the fact that the number of grids become less when increasing the distance of infrastructures. At the same time, it can be observed that there is a slight increase in the small range. It is due to the fact that there is a positive correlation between $L$ and $E[t_i]$ in the case of the fixed number of grids. The jump in the function reflects the sudden change in the number of grids. Based on this conclusion, the deployment of the base station can be optimized in areas with different vehicle proportions. In addition, when the Poisson distribution density is 30, the optimal ratio $\mu^*$ is approximately 0.466, $\mu = 0.5$ is closest to $\mu^*$ at these three densities, and the capacity of the star curve is highest in the three curves. Therefore, the impact of cooperative ratio is consistent with the analysis in figure 2.
Figure. 4 compares the change of capacity between the model in this paper (labeled SINR model) and the model in literature [9] (labeled unit disk model). In [9], the communication between nodes adopted unit disk model, and the analysis of vehicle connection probability is relatively simple. Therefore, under the same communication strategy, its transmission capacity is slightly larger than the capacity calculated in this paper, and gradually rises to be the maximum value and eventually remains stable. The above phenomenon is consistent with the previous analysis. There are obviously fading interference and other factors in the real communication scene, so the analysis of capacity in this paper is more in line with the real situation.

5. Conclusion
In this paper, we analyze system capacity in cooperative vehicular networks. Specifically, different fading channels are considered in V2I and V2V, and the closed expression of system transmission capacity is obtained. In addition, a tradeoff problem between the cooperative ratio and transmission capacity is presented for CVNs. The newton iteration method is formulated to figure the optimal cooperative vehicular ratio which can achieve maximum capacity. The simulation results further verify the relationship between the system capacity and various network parameters, including vehicle density and inter-infrastructure distance. Moreover, our research results will help to design the high-performance CVN system.

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