Optical Model Potential Using CINDA Library for Neutron Induced Cross Section Reactions for Spherical Uranium-235,238 Isotopes

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ABSTRACT. The calculation are based mainly on the nuclear optical model potential and relevant parameters are collected and selected from References Input Parameter Library (RIPL) which is being developed under the international project coordinated by the International Atomic Energy Agency (IAEA). The analyzing of a complete energy range has done starting from threshold energy for each reaction. The cross sections are reproduced in fine steps of incident neutron energy with 0.01MeV intervals with their corresponding errors. The recommended cross sections for available experimental data taken from CINDA library have been calculated for all the considered neutron induced reactions for spherical U-235 and U-238 isotopes. The calculated results are analyzed and compared with the experimental data. The optimized optical potential model parameters give a very good agreement with the experimental data over the energy range 0.001-20MeV for neutron induced cross section reactions (n,f), (n,tot), (n,el), (n,inl), (n,2n), (n,3n), and (n,γ) for spherical U-235 and U-238 target elements.

1. INTRODUCTION

The excitation functions for induced neutron nuclear reactions (n,f), (n,tot), (n,el), (n,inl), (n,2n), (n,3n), and (n,γ) measured for U-235 and U-238 with the aid of CINDA libraries. The cross sections for these reactions have been evaluated for the exact estimation of the cross sections among different authors. This paper describes the standard optical model potential analyses of the spherical U-235 and U-238 target elements. The References Input Parameter Library (RIPL) used for input parameters. These data are used in the real and imaginary part of optical model potential-special emphasis is placed in this study on the isotope dependence of the optical model potential.

2. RECOMMENDED CROSS SECTION

The available measured data from CINDA library for the cross section (n,f), (n,tot) and (n,el) reactions for U-235 and for the cross sections (n,f), (n,tot), (n,2n) and (n,3n) reactions for U-238 respectively. Those data have been plotted interpolated and recalculated in different fine steps and for different energy ranges of incident neutron by using Matlab-8.0 in order to calculate the recommended cross section with a minimum $\chi^2$ for U-235 neutron induced reactions is equal to 0.001046 and a minimum $\chi^2$ for U-238 neutron induced reaction is equal to 0.0006. The interpolation for the nearest data for each energy interval as a function of cross sections and their corresponding errors have been done using Matlab-8.0. The sets of experimental cross sections with their corresponding errors data are collected for different authors with different energy ranges. They re-arranged according to different energy intervals for different energy ranges. The normalization for the statistical distribution of cross sections errors to the corresponding cross section values for each author has been done. The interpolated values are calculated to obtain the recommended cross section which is based on the weighted average calculation according to the following expressions [1]:
\[
\sigma_{w.d.} = \frac{\sum_{i=1}^{n} (\Delta \sigma_i)^2}{\sum_{i=1}^{n} 1} \quad (1)
\]

The standard deviation error is:
\[
S.D. = \frac{1}{\sqrt{\sum_{i=1}^{N} (\Delta \sigma_i)^2}} \quad (2)
\]

Where \( \sigma_i \) is the cross section value, \( \Delta \sigma_i \) is the corresponding error for each cross section value.

Figs. 1 to 4 illustrate the recommended cross sections for the above mentioned reactions as calculated in the present work compared with CINDA library. The results are in good agreement with the measured data [2-13].

3. THEORETICAL BASIS OF OPTICAL MODEL POTENTIAL

In the frame of the optical model, all the interactions between the nucleons of the projectile and the nucleons of the target are replaced by an average and central interaction \( V(r) \) between the projectile and the target in their ground states. The nuclear optical model used to describe the interaction between two nuclei is inspired by the optical phenomenon. The nuclear medium diffracts one part of the incident wave which models the incident particle and another part of the wave is refracted [14]. As the nucleon-nucleon interaction is a short range interaction, the potential \( V_r \times f_j (r, r_j, a_i) \), which is approximately the sum of nucleon-nucleon interactions, has the same behavior. The nucleons in the core of the nucleus undergo only the interaction with their closest neighbors. Due to this saturation of the nuclear forces, \( V_r \times f_j (r, r_j, a_i) \) is uniform inside the nucleus and then decreases exponentially in the surface region [15]. The present evaluations are based mainly on the calculations the optical model potential. A standard form of the optical model potential and relevant parameters used in the present work contains volume, surface, and spin-orbit parts, each having real and imaginary components. This Potential can be written as follows [16, 17, 18]:

\[
V(r,E) = -V_r \times f_j (r, r_j, a_i) + i \left[ 4\pi a_g \times W_d \times \left( \frac{df_j (r, r_j, a_i)}{dr} \right) - W_v \times \exp(-X^2) - W_s \times f_j (r, r_j, a_i) \right] + \frac{\lambda_j^2}{r} \times \left[ V_{so} \times \left( \frac{df_{so} (r, r_{so}, a_{so})}{dr} \right) + iW_{so} \times \left( \frac{df_{so} (r, r_{so}, a_{so})}{dr} \right) \right] \times \ell \cdot s
\]

In Eq 3, \( V_r \) and \( W_v \) are the real and imaginary volume potential well depths, \( W_s \) is the well depth for the surface derivative term, \( W_g \) is the well depth for the global nucleon-nucleon optical potential, \( V_{so} \) and \( W_{so} \) are the real and imaginary well depths for the spin-orbit potential, and \( \lambda_j^2 \) is the pion Compton wavelength squared (\( \approx 2 \)). The quantity \( \ell \cdot s \) is the scalar product of the orbital and intrinsic angular momentum operators and is given by [16]:

\[
\ell \cdot s = \ell \quad \text{for } j = \ell + \frac{1}{2}
\]

\[
\ell \cdot s = -(\ell + 1) \quad \text{for } j = \ell - \frac{1}{2}
\]

The \( f_j (r, r_j, a_i) \) are radial-dependent form factors. The real potential, imaginary potential and form factors are defined below [16]:
a. Real Potential

\( V_{r}, V_{so} \) are the depths of real potential in (MeV).

Since

\[
V_{i} = V_{i0} + V_{i1} \times E + V_{i2} \times E^2 + (V_{i3} + V_{i4} \times E) \times (N - Z) / A \quad \text{with } i = r, so
\]

(6)

Where \( V_{i0}, V_{i1}, V_{i2}, V_{i3}, V_{i4} \) and \( V_{so0}, V_{so1}, V_{so2}, V_{so3}, V_{so4} \) are the depth parameters of real potential in (MeV) taken from (RIPL). \( Z, N, \) and \( A \) are the numbers of protons, neutrons and nucleons in the target nuclide respectively. \( E \) is the energy of incident particle.

b. Imaginary Potential

\( W_{d}, W_{r}, W_{g}, W_{so} \) are the depths of imaginary potential in (MeV).

Since

\[
W_{i} = W_{i0} + W_{i1} \times E + W_{i2} \times E^2 + (W_{i3} + W_{i4} \times E) \times (N - Z) / A \quad i = d, r, g, so
\]

(7)

Where \( W_{d}(1.2647 \text{ fm}), W_{r}, W_{g}, W_{so}(1.2647 \text{ fm}) \); and \( W_{so}(W_{so0}, W_{so1}, W_{so2}, W_{so3}, W_{so4}) \) are the depth parameters of imaginary potential in (MeV) taken from (RIPL).

c. Form Factor

Wood – Saxon form factors is permitted for \( f_{i}(r, r_{i}, a_{i}) \) terms in Eq 3, is as follows:

\[
f_{i}(r, r_{i}, a_{i}) = \frac{1}{1 + \exp(X_{i})} \quad \text{with } i = r, d, v, so \quad \text{ (Wood–Saxon form factor )}
\]

(8)

Where \( X_{i} = (r - R_{i}) / a_{i} \) with \( i = r, d, g, \) and \( v, so \)

(9)

\( r \) is the radial distance in (fm). The nuclear radius \( R_{i} \) is given by:

\[
R_{i} = (r_{i0} + r_{i1} \times E) \times A^{3/2} + C_{i}
\]

(10)

And the form used for the diffuseness, \( a_{i} \), is given by:

\[
a_{i} = a_{i0} + a_{i1} \times E
\]

(11)

Where:

\[
\begin{align*}
& r_{r}(r_{r0} r_{r1} C_{r} a_{r0} a_{r1}); r_{d}(r_{d0} r_{d1} C_{d} a_{d0} a_{d1}); r_{g}(r_{g0} r_{g1} C_{g} a_{g0} a_{g1}) \\
& r_{so}(r_{so0} r_{so1} C_{so} a_{so0} a_{so1})
\end{align*}
\]

are the geometry parameters of real potential in (fm) taken from (RIPL). The optical potential program has been built in the present work using Matlab-8.0, to calculate the real and imaginary optical potential as a function of radial distance and the energy of induced neutron for spherical U-235 and U-238 target elements.

4. RESULTS AND DISCUSSION

A limited number of parameters for spherical potential are included for incident neutron particles. The energy dependence of the neutron potential based on the Uranium isotopes (Z=92, A=235, 238) is E=0.001-20MeV for spherical U-235, U-238 nuclei. Which are included in the present calculations to cover the same energy range for the same target charge and mass. The optical model potential (OMP) parameters are including in the (RIPL) optical file coordinated research project with Beijing Library. The parameters for optical model potential used in this work are tabulated in table 1 for spherical U-235 and 238 target element respectively. The global potentials are calculated for systematics utilization of nuclear radial distance \( r=1 \) to 20fm as well as real and imaginary potential. This model represents the scattering in terms of a complex potential \( V(r, E) \), see Eq 1, where the functions \( V \) and \( W \) are selected to give the potential its proper radial dependence. The real part, \( V \), is responsible for the elastic scattering it describes the ordinary nuclear interaction between target and projectile and may therefore be very similar to a shell model potential. The imaginary part, \( W \), is responsible for the absorption.

The usual optical potential for Uranium-235(spherical nucleus) has a real optical depth \( V_{ro} \) of the order of 48.8679MeV with radius of the real depth \( r_{ro} = 1.2647 \text{ fm} \) with spin orbit potential \( V_{so} = 6.2000 \text{ MeV} \) with radius \( r_{so} = 1.2647 \text{ fm} \), and imaginary optical depth \( W_{do} = \)
6.3882MeV and $W_{vo} = 0.2911\text{MeV}$ with radius $r_{d0} = 1.3501\text{fm}$ and $r_{s0} = 1.3501\text{fm}$ with energy range $(0.001-20\text{MeV})$. The radial distance is to be at most of the order of the $r = 1.20\text{fm}$ and the energy of incident neutrons have been taken at maximum cross section. All parameters used in this work for optical model potential have been taken from (Beijing) library [19]. Table 1 tabulated the optical model parameters for Uranium-235, 238 as spherical nucleus respectively. For spherical structure the real optical depth $V_{ro}$ is of the order of $49.7237\text{MeV}$ with radius of the real depth $r_{ro} = 1.2684\text{fm}$ with spin potential $V_{so} = 6.2000\text{MeV}$ with radius $r_{so} = 1.2684\text{fm}$ and imaginary optical depth $W_{do}$ is $6.5060\text{MeV}$ radius is $r_{d0} = 1.3059\text{fm}$; $W_{vo}$ is $1.4504\text{MeV}$ radius is $r_{v0} = 1.3059\text{fm}$, while deformed structure the real optical depth $V_{ro}$ is $47.5000\text{MeV}$; $V_{so}$ is $7.5000\text{MeV}$ with radius $r_{ro} = r_{so} = 1.2400\text{fm}$ and the imaginary optical depth $W_{do}$ is $2.700\text{MeV}$ with radius $r_{d0} = 1.2600\text{fm}$.

Fig. 5 shows the optical model potential for Uranium target element ($U; A = 235, 238$) induced by neutron, in which absorption $W$ is relatively weaker than elastic scattering $V$. The absorptive part, $W$, at low energies must have a very different form. It is clear from these figures that for spherical structure both absorption and scattering depth parameters for Uranium-238 are greater than that for Uranium-235, since the well depth depend on the mass number of the target element. Because of the exclusion principle, the tightly bound nucleons in the nuclear interior cannot participate in absorb the relatively low energy carried by the incident particle. The optical potential is thus often has the proper shape of being large only near the surface. At higher energy, where the inner nucleons can also participate in absorption, $W$, may look more like $V$. A spin – orbit term is also included in this optical potential. It is also peaked near the surface, because the spin density of the inner nucleons vanishes. A Wood – Saxon form factor is also included. The calculation using the optical model potential, as described in this work, does not deal with where the absorbed particles actually go; they simply disappear from the elastic channel.

5. CONCLUSION

We have evaluated the neutron induced nuclear cross section data of spherical Uranium-235, and 238 isotopes for considerable energy range. The recommended cross sections are in good agreement with experimental data. The reliability in this work is to estimate the global optical parameters chosen for the energy range $0.001-20\text{MeV}$ from Beijing library for spherical U-235 and 238 target elements of neutron induced reactions. The results confirm that the global optical potential parameters are appropriate for these calculations. Hence, the optical model potential is successful in accounting for neutron induced reactions and leads to an understanding of the nucleon-nucleon interactions.

Fig. 1. The recommended cross section of the U-235(n,f) reaction (Left side) and U-235(n,tot) reaction (Right side) compared with CINDA library. Data in Left side: Data1: Ref. [2] Belloni et al. (2011). Data2: Present Work (PW). Data in right side: Data1: Ref. [3] Kegel et al. (1997). Data2: Present Work (PW).
Fig. 2. The recommended cross section of the U-235(n,el)U-235 reaction (Left side) and U-238(n,f) reaction (Right side) compared with CINDA library. Data in left side: Data1: Ref. [4] Smith and Guenther (1982). Data2: Present Work (PW). Data in right side Data1: Ref. [5] Meadows et al. (1989). Data2: Ref. [6] Shcherbakov et al. (2001). Data3: Ref. [7] Nolte et al. (2007). Data4: Present work (PW).

Fig. 3. The recommended cross section of the U-238(n,tot)U-238 reaction (Left side) and U-238(n,2n) reaction (Right side) compared with CINDA library. Data in left side: Data1: Ref. [8] Poenitz et al. (1981). Data2: Ref. [9] Meister et al. (2001). Data3: Present work (PW). Data in right side: Data1: Ref. [10] Frehaut et al. (1980). Data2: Ref. [11] Raics et al. (1990). Data3: Ref. [12] Konno et al. (1993). Data4: Present work (PW).

Fig. 4. The recommended cross section of the U-238(n,3n) reaction compared with CINDA library. Data1: Ref. [13] Zhou (1978). Data2: Ref. [10] Frehaut et al. (1980). Data3: Present Work (PW).
Fig. 5. The Optical Model Potential of neutron induced reaction on spherical $^{235}$U; $^{238}$U as a function of radial distance. Parameters are taken from Beijing Library.

No.1 Related to Data1 in the left side of fig. 1.

No.1 Related to Data1 in the right side of fig. 1.

No.1 Related to Data1 in the left side of fig. 2.

No.1 & 2 related to Data1 & Data 2 in the right side of fig. 2.

No.1 Related to Data1 in the left side of fig. 3.

No.1, 2 & 3 related to Data1, Data 2 & Data 3 in the right side of fig. 3.

No.1& 2 related to Data1& Data 2 in the left side of fig. 4.
Table 1 Parameters for optical model potential used for spherical Uranium-235 Uranium-238 from Beijing Library [19]. a is for U-235; b is for U-238.

| \( V \) | \( V_r \) | \( V_s \) | \( V_{so} \) | \( V_{s0} \) | \( V_{so0} \) |
|---|---|---|---|---|---|
| \( V_r \) | 0.0000<sup>a</sup> | -24.0000<sup>a</sup> | 0.0235<sup>a</sup> | -4.4650<sup>a</sup> | 48.8679<sup>a</sup> |
| \( V_s \) | -12.0000<sup>b</sup> | 0.1283<sup>b</sup> | 6.3882<sup>a</sup> | 0.6020<sup>b</sup> | 1.2684<sup>b</sup> |
| \( V_{so} \) | 0.0000<sup>a</sup> | 0.0000<sup>a</sup> | 0.0000<sup>a</sup> | 0.0000<sup>a</sup> | 6.2000<sup>a</sup> |

Geometry parameters of real potential in (fm) Depth parameters of imaginary optical in (MeV)

| \( W \) | \( W_d \) | \( W_{d1} \) | \( W_{d0} \) | \( a_0 \) | \( r_0 \) |
|---|---|---|---|---|---|
| \( W_{d} \) | 0.1283<sup>b</sup> | 6.3882<sup>a</sup> | 0.6020<sup>b</sup> | 1.2684<sup>b</sup> |
| \( W_{d1} \) | 0.1521<sup>b</sup> | 6.5060<sup>b</sup> | |
| \( W_{d0} \) | |

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