Production of the $h_c$ and $h_b$ and Implications for Quarkonium Spectroscopy

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Abstract. The recent observation of the $h_c$ is an important test of QCD calculations and provides constraints on models of quarkonium spectroscopy. In this contribution I discuss some of these implications and describe methods to search for the $h_c$ and $h_b$ via radiative transitions and other means.

1. Introduction
Over the years there have been numerous calculations of quarkonia spectra. On the one hand, first principles calculations starting with the QCD Lagrangian such as Lattice QCD and NRQCD provide a rigorous test of the theory while on the other hand, quark models can provide more intuitive insights into these systems and provide important phenomenological guidance towards their study [1]. In both cases it is absolutely necessary to test theory against experiment. The $P$-wave singlet quarkonium states are particularly significant as they are the first place we can really test our understanding of the spin-spin interaction between quarks where complications due to relativistic and other effects are less important than in light quark mesons. In this short writeup I will summarize some of the different predictions for the $1^1P_1 - 3^3P_{cog}$ splittings $^1$. We will see that the recent CLEO [2] and E835 [3] measurements of the $h_c$ mass provide an important test of theoretical predictions. I will also briefly describe alternative ways of searching for the $h_c$ and $h_b$ [4] via radiative transitions [5] and in $B$-meson decays (for the $h_c$) [6, 7, 8, 9].

2. $\Delta(M(1^1P_1) - M(3^3P_{cog}))$ as a Test of Quarkonium Calculations
There are numerous calculations of quarkonia properties [1]. The measurement of the singlet-triplet splitting is an important validation of lattice QCD calculations and pNRQCD calculations. It is also an important means of testing various models. For example, in the quark model the triplet-singlet splitting tests the Lorentz nature of the confining potential. The standard Lorentz vector 1-gluon-exchange at short distance with a Lorentz scalar confining potential gives a very short range spin-spin interaction. In contrast, a Lorentz vector confining potential implies a long-range interaction. Representative predictions for the $M(1^1P_1) - M(3^3P_{cog})$ splitting are summarized in Fig. 1. A more complete listing is given in Ref. [5].

$^1$ Where cog stands for the triplet $J=0, 1, 2$ centre of gravity.
In quark potential models the 1-gluon-exchange spin-spin interaction is described by:

$$H_{qq}^{hyp} = \frac{32\pi}{9} \frac{\alpha_s}{m_q m_{\bar{q}}} \vec{S}_q \cdot \vec{S}_{\bar{q}} \delta^3(\vec{r})$$  \hspace{1cm} (1)

The $\delta$-function is short range but will be smeared out by relativistic effects. The Godfrey-Isgur quark model [10] smears the $\delta$-function with a Gaussian and predicts $M(3P_{cog}) > M(1P_1)$. In contrast, McClary and Byers [11] include spin-independent relativistic corrections and find $M(3P_{cog}) < M(1P_1)$. Finally, Franzini [12] includes a Lorentz vector confining potential and finds $M(3P_{cog}) < M(1P_1)$ with a large splitting.

Pantaleone and Tye [13] calculated the splitting using perturbative QCD and also found a small splitting with $M(3P_{cog}) < M(1P_1)$ but noted that other contributions such as relativistic corrections and coupled channel effects could alter this result. Lattice QCD finds $M(3P_{cog}) > M(1P_1)$ but with large errors [14]. Ultimately LQCD will provide the definitive result but more precise results are needed.

The point of these examples is that there is a wide variation in the predictions. There is a strong need for experimental data to test these results.

### 3. Production of Singlet $P$-wave States

There are a number of ways to produce and detect the singlet $P$-wave states. The $h_c$ was recently observed in the reaction $\psi' \rightarrow \pi^0 h_c \rightarrow (\gamma \gamma)(\gamma h_c)$ by the CLEO collaboration [2] and a less convincing signal was seen in $\bar{p}p \rightarrow h_c \rightarrow \eta_c \gamma$ by E835 at FNAL [3]. It has been suggested that the singlet $P$-waves states could also be produced in the radiative cascades $n^3S_1 \rightarrow n^1S_0 + \gamma^{E1}(1^1P_1) + \gamma \gamma$ [5] and in $B$-meson decay, $B \rightarrow h_c + X$ [6, 7, 8, 9].

In all cases the radiative decay $h_{c,b} \rightarrow \eta_{c,b} + \gamma$ results in a clean final state. To estimate the BR requires knowing all important partial decay widths. The $E1$ width for the $h_c$ is given by
\[ \Gamma[h_c(1^P_1) \rightarrow \eta_c(1^S_0) + \gamma] = \frac{4}{9} \alpha e_Q^2 \omega^3 |(1^S_0|r|1^P_1)|^2 = 354 \text{ keV} \] (2)

where \( \alpha = 1/137.036 \) is the fine-structure constant, \( e_Q \) is the quark charge in units of \( |e| \) (2/3 for \( Q = c \) and \(-1/3 \) for \( Q = b \)), and \( \omega \) is the photon’s energy. The overlap integrals were obtained using the wavefunctions of Ref. [10]. The strong widths are estimated to be [5, 18, 19]

\[ \Gamma[h_c(1^P_1) \rightarrow \text{hadrons}] = \frac{5}{2n_f} \times \Gamma[\chi_{c1}(3^P_1) \rightarrow \text{hadrons}] = 533 \text{ keV} \] (3)

\[ \Gamma[h_c(1^P_1) \rightarrow gg + \gamma] = \frac{36}{5} \frac{e_Q^2}{\alpha_s} \Gamma[h_c(1^P_1) \rightarrow gg] = 52 \text{ keV} \] (4)

where \( n_f \) is the number of light quark flavours in the final state which we will take to be 3. For \( b \bar{b} \) we combined the theoretical estimates for the radiative transitions \( \chi_{b1}(3^P_1) \rightarrow \Upsilon(3^S_1)\gamma \) with the measured BR’s [20] to estimate the \( \chi_{b1} \) hadronic width [5]. For the \( h_c \), \( \mathcal{B}[h_c \rightarrow \eta_c + \gamma] = 37.7\% \) and for the \( h_b \), \( \mathcal{B}[h_b \rightarrow \eta_b + \gamma] = 41.4\% \).

It was pointed out long ago that a promising way to produce the \( h_c \) (and \( h_b \)) is via the decay \( \psi'(2S) \rightarrow h_c + \pi^0 \) (and \( \Upsilon(3S) \rightarrow h_b\pi^0 \) [21, 22, 23, 24]. Estimates for the branching ratio are \( \mathcal{B}[\psi' \rightarrow h_c + \pi^0] = 0.1 - 0.3\% \) [21, 22, 23, 24]. Combining this result with the predicted BR for \( \mathcal{B}(h_c \rightarrow \eta_c + \gamma) \) gives \( \mathcal{B}(\psi' \rightarrow h_c\pi^0) \times \mathcal{B}(h_c \rightarrow \eta_c\gamma) \approx 3.8 \times 10^{-4} \) which would yield roughly 400 events for \( 10^6 \) \( \psi' \)'s produced. Likewise, we obtain \( \mathcal{B}(\Upsilon(3S) \rightarrow h_b\pi^0) \times \mathcal{B}(h_b \rightarrow \eta_b\gamma) \approx 4 \times 10^{-4} \) which also yields \( \approx 400 \) events for \( 10^6 \) \( \Upsilon(3S) \)'s. Kuang and Yan [25] have also considered the related spin-flip transition \( \Upsilon(3S) \rightarrow h_b(1^P_1) + \pi\pi \) which may provide an additional path to the \( h_b \).

CLEO recently observed the \( h_c \) in \( \psi' \rightarrow h_c + \pi^0 \) [2]. They measured \( \mathcal{B}(\psi' \rightarrow \pi^0 h_c) \times \mathcal{B}(h_c \rightarrow \gamma\eta_c) = (2 - 6) \times 10^{-4} \). This is in good agreement with the theoretical prediction and is an important validation of the \( \psi' \rightarrow h_c + \pi^0 \) calculations.

Another possibility for producing the singlet \( P \)-wave \( c\bar{c} \) and \( b\bar{b} \) states is via electromagnetic cascades [5] such as:

\[ \psi(2S) \rightarrow M_1 \eta_c(2S) + \gamma \rightarrow E_1 h_c(1P) + \gamma \rightarrow E_1 \eta_c(1S) + \gamma \gamma \gamma. \] (5)

As before, we need the BR’s to estimate the expected number of events. The M1 transition widths are given by:

\[ \Gamma[\psi'(2^3S_1) \rightarrow \eta_c'(2^1S_0) + \gamma] = \frac{4e\alpha^2}{3\alpha_s} \omega^3 |\langle f|j_0(kr/2)|i\rangle|^2 = 0.051 \text{ keV} \] (6)

where we take \( m_c = 1.628 \text{ GeV} \) and as before use the wavefunctions of Ref. [10]. Using the measured \( \psi' \) width gives \( \mathcal{B}[\psi'(2^3S_1) \rightarrow \eta_c'(2^1S_0) + \gamma] = 0.018\% \). For the next decay in the chain, an E1 transition, we estimate [5]:

\[ \Gamma[\eta_c'(2^1S_0) \rightarrow h_c(1^P_1) + \gamma] = \frac{4}{3} e_Q^2 \omega^3 |(1^P_1|r|1^S_0)|^2 = 51.3 \text{ keV}. \] (7)

and

\[ \Gamma(1^S_0 \rightarrow gg) = \frac{27\pi}{5(\pi^2 - 9)} \alpha_s \times \Gamma(3^S_1 \rightarrow ggg) = 7.4 \text{ MeV} \] (8)

where we haven’t shown the QCD corrections that were included in obtaining this result. This results in \( \mathcal{B}(\eta_c' \rightarrow h_c\gamma) = 0.69\% \). Combining this result with the \( \psi' \rightarrow \eta_c'\gamma \) BR we obtain \( \mathcal{B}(\psi' \rightarrow \eta_c'\gamma) \times \mathcal{B}(\eta_c' \rightarrow h_c\gamma) \approx 10^{-6} \) which would yield only 1 event per \( 10^6 \) \( \psi' \)'s. Similarly, we
find $\mathcal{B}(\Upsilon(3S) \to \eta'_b \gamma) \times \mathcal{B}(\eta'_b \to h_b \gamma) = 2.6 \times 10^{-7}$ resulting in only 0.3 events per $10^6 \, \Upsilon(3S)$’s. In a gross understatement, this would be quite the challenge for experimentalists.

The final possibility is to produce the $h_c$ in $B$ decay. This mode has been explored in a number of papers [6, 7, 8, 9] and is supported by the Belle observation [26] of the $\eta_c(2S)$ in $B \to \eta_c(2S)K$. Combining the estimate of $\mathcal{B}[B \to h_c + K] \approx 0.1\%$ [6, 7, 8] with the BR for $h_c \to \eta_c \gamma$ we obtain $\mathcal{B}(B \to h_c + K) \times \mathcal{B}(h_c \to \eta_c \gamma) \sim 4 \times 10^{-4}$. Belle and Babar should be able to observe this.

4. Summary
In the last decade there has been considerable theoretical progress, especially in lattice QCD. We need comparable experimental results to check these calculations. Recently the $h_c$ has been observed which provides an important reality check for theory. The $h_c$ is found to be almost degenerate with the $1^3P_{cog}(c \bar{c})$ and referring to Fig. 1 we see that the new measurements rule out a Lorentz vector confining potential, and demonstrate the need for improved models and for improved calculations using PQCD.

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