Prospects for Neutron Star Equation of State Constraints using “Recycled” Millisecond Pulsars

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Abstract. “Recycled” millisecond pulsars are a variety of rapidly-spinning neutron stars that typically show thermal X-ray radiation due to the heated surface of their magnetic polar caps. Detailed numerical modeling of the rotation-induced thermal X-ray pulsations observed from recycled millisecond pulsars, including all relevant relativistic and stellar atmospheric effects, has been identified as a promising approach towards an astrophysical determination of the true neutron star mass-radius relation, and by extension the state of cold matter at densities exceeding those of atomic nuclei. Herein, I review the basic model and methodology commonly used to extract information regarding neutron star structure from the pulsed X-ray radiation observed from millisecond pulsars. I also summarize the results of past X-ray observations of these objects and the prospects for precision neutron star mass-radius measurements with the upcoming Neutron Star Interior Composition Explorer (NICER) X-ray timing mission.

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1 Introduction

The state of cold matter at densities exceeding those of atomic nuclei remains one of the principal outstanding problems in modern physics. Neutron stars provide the only known setting in the Universe where these physical conditions occur naturally. Thermal X-ray radiation from the physical surface of a neutron star can serve as a powerful tool for probing the poorly understood behavior of the matter in the ultra-dense stellar interior (see, e.g., [29] for a comprehensive review). This is possible because of the unique mapping between the pressure and density of neutron star matter and the stellar radius and mass. For neutron stars with thermal radiation confined to a small fraction of the surface, realistic modeling of the rotation-induced flux variations of the observed “hot spot” radiation can, in principle, yield the true $M \sim R$ relation [35] [11] [10]. Such emission geometry is observed in several varieties of neutron stars including rotation-powered “recycled” millisecond pulsars (MSPs). Recycled MSPs are a population of old neutron stars, characterized by rapid rotation rates (hundreds of Hertz), exceptional rotational stability and low magnetic fields ($\sim 10^8$–$10^9$ G). It is commonly accepted that these neutron stars are a product of low-mass X-ray binaries [11], acquiring their rapid spin rates via accretion of matter and angular momentum. At the end of their spin-up phase, they are reactivated (i.e., “recycled”) as rotation-powered, radio-loud pulsars, meaning that the observer radiation is generated at the expense of the rotational kinetic energy of the neutron star.

Over the past $\sim 15$ years, extensive studies with Chandra and XMM-Newton have shown that many of these neutron stars are seen as X-ray sources due to their hot ($\sim 10^6$ K) polar caps [52] [10] [12] [18]. The inferred emitting areas indicate that this radiation is localized in regions on the stellar surface that are much smaller than the total surface area, but comparable in radius to what is expected for pulsar magnetic polar caps, $R_{pc} = (2\pi R/cP)^{1/2}$, where $R$ is the stellar radius and $P$ its spin period. This finding is consistent with pulsar electrodynamics models, which predict heating of the polar caps by a backflow of energetic particles along the open magnetic field lines [23]. Sophisticated modeling of the X-ray spectra and pulse profiles (i.e. waveforms) of MSPs can offer a probe of the mass and radius of the star. This approach, originally proposed by [35] in the context of “recycled” MSPs, can serve as a valuable probe of key NS properties that are inaccessible by other observational means.

Below, I provide an overview of the standard approach used for modeling the surface emission from neutron stars in general, and rapidly spinning objects such as MSPs, in particular. I summarize the application of this method in practice to existing data obtained with XMM-Newton. I conclude by describing the forthcoming Neutron Star Interior Composition Explorer X-ray timing mission and the precise measurement of neutron star structure it is expected to achieve.
In the vicinity of a neutron star, gravity greatly affects the photons as they propagate from the surface to a distant observer. For many practical purposes, the commonly used spherical Schwarzschild + Doppler formalism provides a sufficiently accurate description of the space-time near the stellar surface. It should be noted, however, that for objects spinning at rates greater than \( \sim 300 \) Hz, the rapid rotation results in appreciable oblateness of the neutron star although the external space-time is still well represented by the Schwarzschild metric [14] [51]. In such instances it is necessary to consider an oblate spheroid. For MSPs with spin frequencies up to \( \sim 800 \) Hz, the effect on the stellar oblateness on the observed hot spot flux modulations can be as high as 5–30%, while the effect on the space-time quadrupole is 1-5% [10]. In this regime, the Hartle-Thorne metric [24] provides an accurate approximation of the space-time [8]. For the fastest spinning neutron stars (beyond \( \sim 1000 \) Hz), higher order space-time multipoles are non-negligible, which necessitates numerically solving the field equations for a given neutron star equation of state [15] [47].

As a photon climbs out of the deep gravitational potential, its energy is diminished by 1 + \( z_g = (1 - R_S/R)^{-1/2} \), where \( R_S = 2GM/c^2 \). Additionally, the trajectory of a photon emitted at an angle \( \theta > 0 \) relative to the local radial direction is deflected, resulting in an angle \( \psi > \theta \) measured at infinity. In Schwarzschild geometry, the relation between these two quantities is expressed by the elliptical integral [36]:

\[
\psi = \int_R^\infty \frac{dr}{r^2} \left[ 1 - \frac{1}{r^2} \left( 1 - \frac{R_S}{r} \right) \right]^{-1/2}
\]

where

\[
b = \frac{R}{\sqrt{1 - R_S/R}} \sin \theta
\]

is the impact parameter of a light ray originating from radius \( R \) (at the neutron star surface) that is emitted at an angle \( \theta \). As the use of the ray-tracing integral is computationally demanding, for many applications it is more convenient to use the greatly simplified approximate formula [5] [37]

\[
\cos \psi \approx \frac{\cos \theta - R_S/R}{1 - R_S/R}
\]

which can be used for \( R > 2R_S \), where it achieves fractional errors of only a few percent at the largest values of \( \theta \). However, if high accuracy is desired, the exact expression needs to be used. Due to the deflection of the photon paths by the immense gravitational field, a larger fraction of the stellar surface is visible to an observer at any instance. In the weak-field regime, the visibility condition is simply \( \cos \psi = \cos \theta > 0 \). In contrast, in the presence of strong gravity regions on the far side of the neutron star relative to the observer surface are viewable up to an angle \( \cos \psi_c \), corresponding to the maximum impact parameter \( b_{\text{max}} = R/\sqrt{1 - R_S/R} \equiv R^\infty \), the so-called radius at infinity.

For MSPs, the rapid motion of the neutron star surface induces a substantial Doppler effect, parameterized
Fig. 2. The emergent intensity of a non-magnetic neutron star hydrogen atmosphere with an effective temperature of $2.1 \times 10^6$ Kelvin and a surface gravity of $2.4 \times 10^{14}$ cm s$^{-2}$ (corresponding to a neutron star with mass $1.4 M_\odot$ and intrinsic radius 12 km). The different spectra correspond to increasing emission angle with respect to the surface normal (from top to bottom, respectively) in logarithmic steps of 0.3 in $\cos \theta$. Note the shift in the peak of the spectrum towards lower energies in with increasing angle, in addition to the overall decline in intensity. This energy-dependent limb-darkening effect arises due to a neutron star with mass $1.4 M_\odot$ and a surface gravity of $2 \times 10^5$. 

Photons emitted from the back side of the compact object as seen by the observer, in addition to following a curved trajectory, have to travel an additional distance compared to a photon emitted radially from the near side. The time lag of the photon as recorded by an observer at infinity is given by the elliptical integral \[ \Delta t(b) = \frac{1}{c} \int_R^\infty \frac{dr}{1 - R_S/R} \left\{ \left[ 1 - \frac{b^2}{r^2} \left( 1 - \frac{R_S}{r} \right) \right]^{-1/2} - 1 \right\} \] (7)

This time delay translates into a phase lag ($\Delta \phi$) of a photon \[ \Delta \phi = \frac{2\pi}{T} \Delta t \] (8) which produces the measured rotational phase $\phi_{\text{obs}} = \phi + \Delta \phi$ [51]. For $R/R_S = 2.5$, the largest value of $\Delta t$, obtained for light rays with maximum impact parameter $b_{\text{max}} = (1 + z_\odot)R$, is $\approx 60 \mu$s. These propagation time differences amount to a few percent of the rotation period of a typical MSP so they need to be taken into account when considering high-quality data.

The flux per unit frequency from a hot spot on a neutron star measured by a distant observer can be expressed as \[ F(\nu) = I(\nu) d\Omega \] (9)
where $I(\nu)$ is the intensity of the radiation as measured at infinity and $d\Omega$ is the apparent solid angle subtended by the hot spot on the sky. Transforming both quantities to the rest frame of the hot spot yields

\[ F'(\nu) = (1 - R_S/R)^{3/2} \eta^3 I'(\nu', \theta') \cos \theta' d\cos \theta \frac{dS'}{d\cos \psi} D^2 \] (10)

Here, the variables marked with primes are measured in the rest frame of the stellar surface [35], with $\cos \theta' = \eta \cos \theta$ and $dS \cos \theta = dS' \cos \theta'$. $I'(\nu', \theta')$ is the emergent radiation intensity, $dS'$ is the emission area and $D$ is the distance between the star and observer. The three Doppler factors are a result of the transformation of the intensity. An additional factor is obtained upon integration over a frequency interval considering that $d\nu = (1 - R_S/R)^{3/2} \eta d\nu'$. Using equation (10), the time-dependent flux observed from the rotating hot spot can be determined for a given phase $\phi(t)$ (in the range 0 to $2\pi$) using the relations between $\phi(t)$, $\theta$ and $\psi$ in equations (1) and (2) and the appropriate emission model to compute $I'(\nu', \theta')$, which is described next.

### 2.2 Neutron Star Atmosphere Emission

The commonly accepted evolutionary scenario posits that MSPs acquire their rapid spins due to accretion of matter and angular momentum in a low-mass X-ray binary system [3]. Therefore, it is natural to expect MSPs to possess a substantial atmospheric layer. Due to gravitational setting, hydrogen is expected to surface within seconds and dominate the surface emission. An optically-thick hydrogen atmosphere of thickness $\sim 1$ cm can be obtained with as little as $10^{-20} M_\odot$ of hydrogen.

Non-magnetic hydrogen atmosphere models applicable to MSPs have been developed independently by different groups over the past 20 years [51] [22] [26] [22]. They all yield virtually identical results, with differences of only...
Fig. 3. (Left) Representative model light curves for a rotating neutron star with $M = 1.4 \, M_\odot$, $R = 10 \, \text{km}$ and two point-like antipodal hot spots for representative geometric configurations (right panel). The solid curves in each plot correspond to a H atmosphere (blue) and isotropic blackbody emission (red). The dashed lines show the effects of Doppler boosting and photon travel time delays for a spin frequency of 250 Hz. (Right) Orthographic map projection of the NS surface for the four pulse profiles (the roman numerals I–IV correspond to the lightcurves from top to bottom, respectively). The dashed line is the axis connecting the two diametrically opposite hot spots while the dotted line is the direction to the observer. The hatched area corresponds to the portion of the star not visible to the observer. Due to gravitational bending of light, for a typical NS $\sim 80\%$ of the surface is visible at any given time.

$\sim 1\%$ around the peaks of the emergent spectra. The models consider a static, plane-parallel atmosphere that is in radiative equilibrium, and composed of completely ionized hydrogen. As appropriate for MSPs, the surface is assumed to be weakly magnetized ($B \ll 10^{10} \, \text{G}$), meaning that the effects of the magnetic field on the opacity and equation of state of the atmosphere can be safely ignored.

Relative to a standard Planck spectrum, the radiation from a neutron star atmosphere has peak emission that occurs at higher energies for the same effective temperature and exhibits an overall flux depression, which allows the conservation of bolometric flux [44] [54]. As a consequence, if modeled using a blackbody, a neutron star covered by an atmosphere would be measured to have a temperature much higher than its actual effective temperature, resulting in a grossly underestimated emitting area. Furthermore, the beaming pattern of the atmosphere is intrinsically non-uniform with radiation intensity declining as the angle with respect to the surface normal, resulting in the familiar limb-darkening effect (see Figure 2). In addition to a change in the total flux, there is a significant shift in the peak energy of the spectrum. For a blackbody spectrum no such variations are expected. This implies that although the emission spectrum is qualitatively similar to the case of a Planck spectrum, the observed shape and photon energy dependence of the rotation-induced pulsations of any localized emission on the surface (such as hot spots seen from MSPs) will differ greatly.

Using the model ingredients described above, for MSPs, synthetic pulse profiles can be generated by considering emission from two hot spots (corresponding to the two dipole magnetic polar caps) as a function of the neutron star rotational phase given a geometric configuration, temperature, emission area, and input neutron star mass and radius [54]. As seen Figure 3, the morphology of the flux modulations are determined in large part by the geometric configuration of the hot spot, neutron star spin axis and observer system. The choice of surface emission model is also a crucial factor as apparent from the substantially larger amplitude of the H atmosphere pulsations compared to a blackbody for the same assumed parameters.

It is evident from equations (2) and (10) that the measured radiation at infinity is highly sensitive to the choice of the neutron star compactness, i.e. the mass-to-radius ratio ($M/R$). The impact of $M/R$ on the rotation-induced X-ray modulations of a neutron star with two diametrically opposite hot spots (corresponding to the two magnetic poles of the star) covered with a non-magnetic neutron star hydrogen atmosphere. A small increase in $M/R$ results in a pronounced decrease in the amplitude of the pulsations (Figure 4), as a direct result of the strong dependence of the magnitude of the bending of light effect on $M/R$. Thus, as shown by [35] and [53], modeling of the pulsations of MSPs may can provide constraints on $M$ and $R$. 
Fig. 4. Synthetic hydrogen atmosphere light curves for different stellar radii for a 1.4 $M_\odot$ neutron star. The lines correspond to stellar radii of 9 km (dot-dashed), 12 km (solid), and 16 km (dashed). The angles $\alpha$ and $\zeta$ for each panel are assumed to have the same values as in Figure 2. Note the dramatic change in the amplitude of the rotation-induced flux variations as a function of stellar radius, caused by amplified bending of light effect for more compact stars. Two neutron star spin cycles are shown for clarity. Adapted from [11].

3 Observational Results

Rotation-powered MSPs were identified as pulsed X-ray sources by Becker and Trümper [4] in data from the ROSAT all-sky survey. The potential utility of recycled MSPs as powerful probes of the neutron star equation of state was first pointed out by Pavlov and Zavlin [35] [53], who used ROSAT data of the nearest known MSP, PSR J0437–4715 [27], to demonstrate that a model of polar cap thermal emission from a neutron star hydrogen atmosphere provides a good description of the X-ray pulse profiles of this MSP, as well as to place crude limits on the mass-radius relation.

Prompted by this promising result, deep XMM-Newton European Photon Imaging Camera (EPIC) pn observations of nearby MSPs were conducted [10] [8] [7]. These efforts confirmed that a non-magnetic hydrogen atmosphere can indeed reproduce the resulting energy-dependent X-ray pulse profiles of the two closest known MSPs, PSRs J0437–4715 and J0030+0451 (see Figures 4 and 5). In contrast, the large-amplitude pulsations are found to be incompatible with a model that considers an isotropically-emitting Planck spectrum. Furthermore, this modeling has already produced interesting constraints on the allowed neutron star equation of state (Figure 6). For PSR J0437–4715 (Figure 5), assuming 1.76 $M_\odot$ (the current measurement from radio timing [50]) the stellar radius is constrained to be $R > 11.1$ km (at 3$\sigma$ confidence; Bogdanov 2013), while for PSR J0030+0451 (Figure 6) the best constraint is $R > 10.4$ (at 99.9% confidence) assuming 1.4 $M_\odot$ [8]. These limits are already inconsistent with certain quark star and kaon condensate equations of state, illustrating that this method represents a beneficial approach to probing the neutron star EoS.

Their low inferred surface magnetic fields ($\sim 10^8$–$10^9$ G), small emitting areas ($\lesssim 3$ km radius), extraordinary rotational stability, and dominant and steady non-transient surface emission make MSPs fairly “clean” laboratories for studies of fundamental neutron star physics. As such, they can provide constraints on NS structure via thermal pulse shape modeling that are complementary to those derived from other approaches (e.g., using thermonuclear bursts from X-ray binaries [34] [46] [48] [39] and spectroscopy of quiescent X-ray binaries [19] [21] [25]) and thus warrant extensive studies at X-ray energies.

One advantage of using recycled MSPs is the availability of (or the possibility of obtaining) highly precise distance measurement from very long baseline interferometry (VLBI) or high-precision radio pulsar timing. The most notable example is PSR J0437–4715 for which the parallax distance has been measured to within an unprecedented $\pm 0.8\%$ (156.3 $\pm$ 1.3 parsecs) [16]. This greatly diminishes the uncertainty introduced in the emitting area, which (as seen from equation 10) is strongly covariant with the distance between the neutron star and the observer.

Perhaps more importantly, binary MSPs can offer particularly stringent constraints on the equation of state via an independent high-precision mass measurement from ra-
dio pulse timing combined with a $M/R$ measurement from X-ray observations. There is growing observational evidence that MSPs are systematically more massive than the canonical $1.4 \, M_\odot$, as expected for neutron stars spun-up by accretion; this group includes the two most massive neutron stars known, PSRs J1614–2230 and J0348+0432 with $\approx 2 \, M_\odot$ [17] [2]. This places them in an interesting region of the $M−R$ plane, away from the locus of model tracks around $R = 10 \, \text{km}$ and $M = 1.4 \, M_\odot$ (see Figure 7).

Moreover, binary MSPs permit an independent determination of the observer’s viewing angle ($\zeta$) of the neutron star since its value is expected to coincide with the measurable orbital inclination due to the expected alignment of the spin and orbital angular momentum during the accreting spin-up phase. When combined with additional geometric constraints, e.g., from modeling of Fermi LAT $\gamma$-ray pulsations [19], this reduces the number of free model parameters even further, thereby providing much more refined bounds on the mass-radius relation.

4 The Neutron Star Interior Composition Explorer

Existing X-ray data of MSPs are not of sufficient quality to provide meaningful constraints on neutron star structure. Nevertheless, the have served to demonstrate that MSPs can serve as astrophysical laboratories for studying ultra-dense matter. At present, further improvements in neutron star mass-radius measurements of MSPs are hindered by the design limitations of existing X-ray observatories. Their potential utility makes MSPs obvious targets for future X-ray observatories aimed at obtaining new and refining existing measurements of the $M−R$ relation [32].

Indeed, this has served as one of the principal science drivers for the Neutron Star Interior Composition Explorer (NICER) X-ray timing instrument, currently scheduled for launch in late 2016. NICER is an approved NASA Explorer Mission of Opportunity that will be deployed as an attached experiment on the International Space Station. Its scientific payload is a non-imaging X-ray timing instrument that is composed of an array of 56 X-ray concentrator and silicon drift detector pairs sensitive in the 0.2–12 keV band. See Gendreau et al. [20] for a more detailed overview of the design and expected performance characteristics of NICER. Its unique combination of large effective area (nearly 2000 cm$^{-2}$ at 1.5 keV), relatively low background, and high precision timing capabilities ($\sim 100$ nanoseconds absolute time resolution) is specifically tailored for effective studies of the thermal X-ray pulsations from MSPs.

Within the nominal 18-month science mission, NICER will target the nearest recycled MSPs, PSRs J0437–4715, J0030+0451, J2124–3358 and perhaps others, in very long exposures ($\sim 1–1.5$ Megaseconds), which will produce $\sim 1 \times 10^6$ source photons for each target. This number of counts is sufficient to arrive at a $\sim 5\%$ uncertainty in the measurement of the neutron star radius [41]. As demonstrated by [33], measuring the mass-radius relation of several neutron stars to better than 10% would enable very strong limits on the allowed equation of state at extreme densities [33]. Therefore, observations of MSPs with NICER hold the promise to produce a strong empirical constraint of the long sought-after pressure-density relation of cold supranuclear matter, which would have profound implications for astrophysics and nuclear physics alike.
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