Thermophysical analysis of three-dimensional magnetohydrodynamic flow of a tangent hyperbolic nanofluid

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Abstract
In this article, we use the Buongiorno model to study the three-dimensional tangent hyperbolic nanofluid flow over a stretched sheet. The study investigates the impact of velocity slip on the flow and heat transfer features in a tangent hyperbolic nanofluid. For a better fit with experimental observations, we assume that the nanoparticle mass flux at the boundary is zero as opposed to a prescribed concentration at the surface. Using appropriate transformations, we reduce the partial differential equations that describe the momentum, energy, and concentration transport to ordinary differential equations. Numerical solutions of the equations are obtained using the spectral method. Graphical illustration of the physical influence of various parameters on the flow features, the skin friction coefficient, and the local Nusselt number is given. The results indicate that particle Brownian motion has a negligible impact on the rate of heat transfer. The impact of the modified Weissenberg number, a measure of the ratio of elastic to viscous forces, causes a reduction in the fluid velocity. The results are shown to be good agreement with those in related studies in the literature.

KEYWORDS
hyperbolic nanofluid, MHD flow, spectral method, stretching sheet

1 INTRODUCTION

The study of a viscous incompressible fluid flow past a stretching sheet arises in many applications in science and engineering. Some of the important applications may be found in the processing of polymers and plastics, in solar receivers, and in solar ponds. The effect of viscosity is significant near a solid surface. Previous investigations indicate that when the free fluid motion exceeds the stretching rate, the velocity boundary layer thickness increases. Additionally, the friction force cannot be ignored when dealing with fluid flows on a solid interface. The significant role of friction between solid and liquid interfaces in mechanical devices has been highlighted in the study by Yu et al.1 When evaluating the physical properties of boundary layer fluid flows, the velocity slip is an important aspect that must be considered. Beavers and Joseph2 pioneered the study and analysis of the velocity slip condition and suggested that the condition is suitable for
macroscale systems but not for micro and nanoscales. The impact of fluid velocity slip on an unsteady flow was investigated by Hosseini et al. They used the rational Bernstein collocation method as the method of solution. The effect of slip on the boundary layer flow over a non-linearly shrinking sheet was studied by Ghosh et al where they obtained dual solutions of the flow equations. A mathematical model for the flow and heat transfer in an unsteady non-Newtonian fluid was presented by Hamid et al. In the study, they considered the impact of fluid buoyancy, the velocity slip as well as Dufour and Soret effects on the fluid properties. They concluded that an increase in the slip parameter coupled with a reduction in the magnetic field increases the fluid velocity.

Magnetohydrodynamics (MHDs) has many useful applications, for example, in heat pumps, generators, plasma studies, cooling reactors, and flow meters, Su et al. Waqas et al studied the flow of a MHD micropolar fluid over a non-linearly stretching surface. Their study included an investigation of the effects of mixed convection, viscous dissipation, Joule heating, and a convective boundary condition. MHD boundary layer flow along a stretching cylinder was studied by Mukhopadhyay. He analyzed the flow and thermal characteristics of an axisymmetric boundary layer flow and considered the effects of velocity slip and a uniform magnetic field. Additional studies relating to MHD boundary layer flows may be found in References 9-12.

There are limitations to the degree of heat conductivity in conventional fluids. Lately, the use of fluids containing suspended nanosized metallic particles, known as nanofluids has become more prevalent. Several studies have reported exponential growth in research into the effectiveness of these fluids. Nanofluids are created by the suspension of nanoparticles in certain base fluids such as water, kerosene oil, ethylene glycol, or engine oil. The nanoparticles serve as agents for improving the thermophysical features of the base fluid to achieve higher heat transfer rates. The primary interest in many studies is to gain a fundamental understanding of the mechanisms that enhance heat transfer rates in nanofluids. The important mechanisms identified in the literature include particle Brownian motion, particle migration, and thermophoresis. Buongiorno suggested these two mechanisms as the most significant in enhancing the thermal conductivity of nanofluids. The Buongiorno model has been adopted in many nanofluid heat transfer studies which include those in References 19, 20. Some significant research efforts have been made to assess the degree of heat transfer rate intensification in nanofluids. This has included, for example, experimenting with different nanoparticle concentrations, the size of the nanoparticles, and the choice of the base fluid. The shape of the nanoparticles also plays an important role in the properties of nanofluids. Research into nanofluid heat transfer processes has shown that in forced convection processes, higher heat transfer rates are achieved when the concentration of the nanoparticles is high. Nanofluids are used in large-scale manufacturing industries, transportation, and engineering processes, for example, in solar water heating, diesel combustion, microdevices, and fuel cells. The MHD stagnation point flow of a nanofluid through a stretched sheet with convective heating was studied by Ibrahim and Haq while Jamaludin et al investigated the MHD and mixed convection effects in a nanofluid flow with slip conditions. The equations were solved using the MATLAB bvp4c method.

Tangent hyperbolic nanofluid models constitute a recent development in heat and mass transfer fluids in boundary layer flows. The model takes into account the shear thinning and thickening behavior of the fluid, Hayat et al. The boundary layer flow of a MHD tangent hyperbolic nanofluid over a stretching sheet was studied by Khan et al. The application of the generalized Fourier heat law to the flow of tangent hyperbolic liquids over a sheet stretching nonlinearly was studied by Waqas et al. Recent studies on MHD tangent hyperbolic flows can be found in References 30-33.

The equations that model the transport processes are generally nonlinear and have been solved in the literature using a variety of numerical or semianalytical methods. Some recent studies on nanofluid used the homotopy analysis method to solve the conservation equations. Spectral collocation methods, such as the spectral relaxation, spectral local linearization, spectral quasilinearization, successive linearization, spectral perturbation, piecewise successive linearization, and the spectral homotopy analysis methods are recently developed numerical techniques. These methods promise rapid convergence rates with good accuracy. The methods have been used to solve many boundary value problems.

In this study, Buongiorno’s model presented in Reference 36 is extended to study of flow and heat transfer characteristics of a three-dimensional (3D) tangent hyperbolic nanofluid. We use the spectral local linearization method to solve the reduced conservation equations. We investigate the impact of the fluid velocity slip and free nanoparticle movement at the boundary.

2 | MATHEMATICAL FORMULATIONS

Consider the steady incompressible flow of a tangent hyperbolic nanofluid over a permeable stretching sheet (see Figure 1). The model is 3D with the z-coordinate orthogonal to the sheet which is stretched in the xy-plane. The mass
flux velocity is \( w = w_0 \), where \( w_0 > 0 \) indicates suction and \( w_0 < 0 \) is for injection. The equations of mass, momentum, energy, and nanoparticle concentration for the flow are\(^{36,38-40}\):

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu(1-n) \frac{\partial^2 u}{\partial z^2} + \sqrt{2} \Gamma \nu \frac{\partial u}{\partial z} \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u, \tag{2}
\]

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu(1-n) \frac{\partial^2 v}{\partial z^2} + \sqrt{2} \Gamma \nu \frac{\partial v}{\partial z} \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho} v, \tag{3}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} + \beta \left[ D_B \frac{\partial c}{\partial z} \frac{\partial T}{\partial z} + D_T \frac{\partial^2 T}{\partial z^2} \right]^2, \tag{4}
\]

\[
u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z} = D_B \frac{\partial^2 c}{\partial z^2} + \frac{D_T}{T_{\infty}} \frac{\partial^2 T}{\partial z^2}. \tag{5}
\]

The associated boundary conditions are

\[
u = \nu_w(x) = ax + N_1 \alpha \frac{\partial u}{\partial z}, \quad v = \nu_w(y) = by + N_2 \alpha \frac{\partial v}{\partial z}, \]

\[w = w_0, \quad T = T_w, \quad D_B \frac{\partial c}{\partial z} + D_T \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \]

\[u \to 0, \quad v \to 0, \quad T \to T_{\infty}, \quad c \to c_{\infty} \text{ as } z \to \infty, \tag{6}
\]

where \( u, v, \) and \( w \) are the components of velocity along the direction of \( x-, y-, \) and \( z- \) axes, respectively. It is further assumed that \( \nu_w(x) = ax \) and \( \nu_w(y) = by, \) where \( a > 0, \) and \( b \) is a constant such that \( (b > 0) \) corresponds to a stretching sheet and \( (b < 0) \) corresponds to a shrinking sheet. The boundary condition \( D_B, \) Brownian motion \( \frac{\partial c}{\partial z} + \frac{D_T}{T_{\infty}} \frac{\partial T}{\partial z} = 0 \) at \( z = 0 \) physically signifies that the volume fraction of nanoparticle at the surface is controlled passively but not actively. This condition ensures a random nanoparticle movement at the boundary. Here \( T \) is the temperature of the nanofluid, \( C \) is the nanoparticle volume fraction, \( T_w \) is the temperature of the constant surface, \( T_{\infty} \) and \( C_{\infty} \) are the constant temperature and nanoparticle volume fraction of the ambient fluid, respectively, \( \alpha \) is the nanofluid thermal diffusivity, \( \nu \) is the kinematic viscosity of the fluid, \( N_1 \) and \( N_2 \) are dimensional slip coefficients, \( D_T \) is the thermophoretic diffusion coefficient, \( D_B \) is the Brownian diffusion coefficient and \( \beta = \frac{(\rho c)_p}{(\rho c)_f} \) where \( (\rho c)_p \) is the nanoparticle material’s effective heat capacity, and finally \( (\rho c)_f \) is the
heat capacity of the fluid. The integer \( n \) is the power-law index, \( \Gamma \) is the Williamson parameter, \( \sigma \) is the fluid electrical conductivity, \( B_0 \) is the magnetic induction parameter, and \( \rho \) is the fluid density.

Using the similarity transformations,

\[
\begin{align*}
    u &= ax f'(\eta), v = ay g'(\eta), w = -\sqrt{a}\alpha[f(\eta) + g(\eta)], \\
    \eta &= \frac{T - T_\infty}{T_w - T_\infty}, \\
    \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \\
    \phi(\eta) &= \frac{C - C_\infty}{C_\infty},
\end{align*}
\]

(7)

where \( f \) and \( g \) denotes the function of nondimensional stream and the primes denote the differentiation with respect to \( \eta \).

Equations (2) to (6) takes the form

\[
\begin{align*}
    ((1 - n) + nWey f''') Pr f''' - f''^2 + (f + g) f''' - Mf' &= 0, \\
    ((1 - n) + nWey g''') Pr g''' - g''^2 + (f + g) g''' - Mg' &= 0, \\
    \theta'' + (f + g) \theta' + Nb \theta' \phi' + Nt \theta'\phi^2 &= 0, \\
    \phi'' + Le(f + g) \phi' + \frac{Nt}{Nb} \theta'\phi'' &= 0.
\end{align*}
\]

(8)  (9)  (10)  (11)

The nondimensional boundary conditions are,

\[
\begin{align*}
    f'(0) &= 1 + A f''(0), g'(0) = \lambda + B g''(0), f(0) = S, \\
    g(0) &= 0, \theta(0) = 1, \phi'(0) + \frac{Nt}{Nb} \theta'(0) = 0, \\
    f'(\infty) &= 0, g'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0,
\end{align*}
\]

(12)

where \( Wey_x \) and \( Wey_y \) are the modified Weissenberg numbers, \( M \) is the magnetic field parameter, \( A \) and \( B \) are the slip parameters (dimensionless), \( \lambda \) represents the parameter of constant stretching, where \( \lambda > 0 \) signifies a stretching sheet while \( \lambda < 0 \) is indicative of a shrinking sheet. In this study \( S \) is one of the significant parameters representing the constant mass flux, where \( S > 0 \) corresponds to suction while \( S < 0 \) corresponds to injection and \( S = 0 \) represents an impermeable plate, \( Pr \) is the Prandtl number, \( Le \) is the Lewis number, \( Nb \) and \( Nt \) are the parameters representing the Brownian motion and the thermophoresis, respectively. These parameters are defined as

\[
\begin{align*}
    Wey_x = \sqrt{2\Gamma} \sqrt{\frac{a}{\alpha}} ax, Wey_y = \sqrt{2\Gamma} \sqrt{\frac{a}{\alpha}} ay, M = \frac{\sigma \alpha}{\rho a}, \lambda = \frac{\beta}{\alpha}, S = -\frac{w_0}{\sqrt{a}}, Pr = \frac{\nu}{\alpha}, Le = \frac{a}{D_B}, Nb = \frac{\beta D_B (C_a - C_\infty)}{\alpha}, Nt = \frac{\beta D_B (T_\infty - T_w)}{T_w a}, \lambda = \sqrt{\alpha a N_1}, \quad \text{and} \quad B = \sqrt{\alpha a N_2}.
\end{align*}
\]

3 | SKIN FRICTION COEFFICIENTS AND HEAT TRANSFER

The local skin friction coefficients \( Cf_x \), and \( Cf_y \), and the local Nusselt number \( Nu_x \) are given as

\[
\begin{align*}
    Cf_x &= \frac{\tau_{ux}}{\rho U_w^2}, \quad Cf_y = \frac{\tau_{uy}}{\rho V_w^2} \quad \text{and} \quad Nu_x = \frac{q_w}{k(T_w - T_\infty)},
\end{align*}
\]

(13)

where \( \tau_{ux} \) and \( \tau_{uy} \) are the skin friction coefficients in the \( x \) and \( y \) directions and \( q_w \) is the heat flux from the surface of the sheet. These are given as

\[
\begin{align*}
    \tau_{ux} &= \mu \left( (1 - n) \frac{\partial u}{\partial z} + \frac{n\Gamma}{\sqrt{2}} \left( \frac{\partial u}{\partial z} \right)^2 \right) \bigg|_{z=0}, \quad \tau_{uy} = \mu \left( (1 - n) \frac{\partial v}{\partial z} + \frac{n\Gamma}{\sqrt{2}} \left( \frac{\partial v}{\partial z} \right)^2 \right) \bigg|_{z=0}, \quad \text{and} \quad q_w = -k \left( \frac{\partial T}{\partial z} \right) \bigg|_{z=0}
\end{align*}
\]

(14)
Substituting Equation (9) into Equation (14) and using Equation (13), we obtain

\[
\left( \frac{Re_x}{Pr} \right)^{\frac{1}{2}} C f_x = (1 - n)f'''(0) + \frac{nWe_x f''(0)^2}{2},
\]

\[
\left( \frac{Re_y}{Pr} \right)^{\frac{1}{2}} \lambda^2 C f_y = (1 - n)g'''(0) + \frac{nWe_y g''(0)^2}{2}.
\]

and \( Nu_x = -\theta'(0) \),

(15)

where \( Re_x = \frac{n u_x}{v} \) and \( Re_y = \frac{n u_y}{v} \) are the local Reynolds numbers.

4 | METHOD OF COMPUTATIONAL SOLUTIONS

The spectral local linearization method is used to solve the nonlinear boundary value problem defined by Equations (8) to (11) with boundary conditions (12). The method combines the Gauss-Seidel method for decoupling equations and the quasilinearization technique of Bellman and Kalaba.\(^{41}\) In using the spectral local linearization method, the system of equations is decoupled, and the nonlinear components linearized. The solution of the linearized system is then approximated using the Chebyshev spectral collocation method. The method has been used by researchers to solve various mathematical models, such as in the studies.\(^{42-44}\) A detailed description of the method for systems of differential equations can be found in Reference 45.

Applying the spectral local linearization method on Equations (8) to (11) gives

\[
a_{0,r} f'''_{r+1} + a_{1,r} f''_{r+1} + a_{2,r} f'_{r+1} + a_{3,r} f_{r+1} = R_{1,r},
\]

(16)

\[
b_{0,r} g'''_{r+1} + b_{1,r} g''_{r+1} + b_{2,r} g'_{r+1} + b_{3,r} g_{r+1} = R_{2,r},
\]

(17)

\[
\theta_{r+1}'' + c_{1,r} \theta_{r+1}' = R_{3,r},
\]

(18)

\[
\phi_{r+1}'' + d_{1,r} \phi_{r+1}' = R_{4,r},
\]

(19)

where the primes denote partial derivatives with respect to \( \eta \). The boundary conditions are given by

\[
f_{r+1}(0) = S_{0} f'_{r+1}(0) = 1 + A f''_{r+1}(0), f'_{r+1}(\infty) = 0,
\]

\[
g_{r+1}(0) = 0, g'_{r+1}(0) = \lambda + B g''_{r+1}(0), g'_{r+1}(\infty) = 0,
\]

(20)

\[
\theta_{r+1}(0) = 1, \theta_{r+1}(\infty) = 0,
\]

\[
\phi_{r+1}(0) + \frac{N_{t}}{N_{b}} \phi_{r+1}'(0) = 0, \phi_{r+1}(\infty) = 0.
\]

The coefficients in Equations (16) to (19) are defined as

\[
a_{0,r} = ((1 - n) + nWe_x f'''(0)) Pr, a_{1,r} = nWe_x Pr f''' + f_r + g_r, a_{2,r} = -2f'_r - M,
\]

\[
a_{3,r} = f''(0), b_{0,r} = ((1 - n) + nWe_y g'''(0)) Pr, b_{1,r} = nWe_y Pr g''' + f_r + g_r,
\]

\[
b_{2,r} = -2g'_r - M, b_{3,r} = g''_r, c_{1,r} = f_r + g_r + N b \phi' + 2N t \theta', d_{1,r} = L e(f_r + g_r),
\]

\[
R_{1,r} = a_{0,r} f'''_{r+1} + a_{1,r} f''_{r+1} + a_{2,r} f'_{r+1} + a_{3,r} f_{r+1} - F_1,
\]

\[
R_{2,r} = b_{0,r} g'''_{r+1} + b_{1,r} g''_{r+1} + b_{2,r} g'_{r+1} + b_{3,r} g_{r+1} - F_2,
\]

\[
R_{3,r} = \theta''_{r+1} + c_{1,r} \theta'_{r+1} - F_3, R_{4,r} = \phi''_{r+1} + d_{1,r} \phi'_{r+1} - F_3.
\]
The domain $\eta \in [0, L_\infty]$ is transformed into the interval $x \in [-1, 1]$ using a suitable linear transformation. We define the Gauss-Lobatto grid points as $x_j = \cos \frac{j \pi}{N}, j = 0, 1, \ldots, N$. The unknown functions are arranged in matrix form using the Chebyshev differentiation matrix as defined in Reference 46. The derivatives are expressed as

$$\frac{df}{dx} = \sum_{k=0}^{N} D_{jk} f(x_k) = DF, \quad j = 0, 1, \ldots, N.$$ (21)

Higher order derivatives are represented as $\frac{d^n f}{dx^n} = D^n F$, where in this case $n$ is the order of the derivative, $D$ is the differentiation with respect to $x$, and $F = [f(x_0), \ldots, f(x_N)]^T$.

Applying the spectral method to the scheme in Equations (16) to (19), we obtain

$$\begin{align*}
(a_0, D^3 + a_1, D^2 + a_2, D + a_3, I) F_{r+1} &= R_{1,r}, \\
(b_0, D^3 + b_1, D^2 + b_2, D + b_3, I) G_{r+1} &= R_{2,r}, \\
(D^2 + c_1, D) \theta_{r+1} &= R_{3,r}, \\
(D^2 + d_1, D) \phi_{r+1} &= R_{4,r},
\end{align*}$$

where $I$ is an $(N+1) \times (N+1)$ identity matrix and the bold coefficients represent diagonal matrices. In matrix form:

$$\begin{align*}
AF_{r+1} &= R_{1,r}, \\
BG_{r+1} &= R_{2,r}, \\
C\theta_{r+1} &= R_{3,r}, \\
E\phi_{r+1} &= R_{4,r}
\end{align*}$$ (26)

where $A = a_0, D^3 + a_1, D^2 + a_2, D + a_3, I$, $B = b_0, D^3 + b_1, D^2 + b_2, D + b_3, I$, $C = D^2 + c_1, D$, and $E = D^2 + d_1, D$.

The approximate solutions for $f, g, \theta,$ and $\phi$ are obtained by solving Equation (26) iteratively starting from suitable initial solutions.

5 | RESULTS AND DISCUSSION

The conservation equations are solved using the method of spectral local linearization. The results of the numerical simulations are given in Tables 1, 2 and Figures 2-11. The parameters values are chosen from published studies in the literature.

The accuracy of the computed solution is verified by comparing our solutions with those given by Akbar et al.47 for certain parameter values. The results are depicted in Table 1 where the solution generally agrees with previously published results up to five significant digits of accuracy.

Table 2 shows the variation of the local Nusselt number with the thermophoresis and Brownian motion parameters. The local Nusselt number does not change significantly with the Brownian motion parameter. This may be an indication of the limitation of particle Brownian motion in impacting the degree of heat conductivity of the nanofluid. However, as the thermophoresis parameter increases, the local Nusselt number decreases. The physical reason is that an increase in thermophoresis enhances the diffusion of nanoparticles to the cold regions and this increases the thermal boundary layer thickness. This, in turn, leads to a reduction in the heat transfer rate.

Figure 2 shows the effect of the magnetic parameter $M$ on the velocity profiles. Here, we note that both the velocities $f'(\eta)$ and $g'(\eta)$ are reduced by an increase in the magnetic field parameter. This is because there exists an electromagnetic interplay linking the magnetic field with electrically conducting fluid. This interaction produces a retarding Lorentz force which reduces the speed of the fluid in the boundary layer. Hence, the presence of the magnetic field parameter has a
Table 1: Comparison of skin friction coefficient for different values of $M$ when $A = 0$, $Pr = 1$, $n = 0$, $S = 0$, $B = 0$, $\lambda = 0$, and $W_{ex} = W_{ey} = 0$

| $M$  | Akbar et al 47 | Present Results |
|------|----------------|-----------------|
| 0    | $-1.00000$    | $-1.00001$      |
| 0.25 | $-1.11803$    | $-1.11803$      |
| 1    | $-1.41421$    | $-1.41421$      |
| 5    | $-2.44949$    | $-2.44949$      |
| 10   | $-3.31662$    | $-3.31663$      |
| 50   | $-7.14142$    | $-7.14143$      |
| 100  | $-10.0499$    | $-10.0499$      |
| 500  | $-22.383$     | $-22.383$       |
| 1000 | $-31.6386$    | $-31.6386$      |

Table 2: Variations of the local Nusselt number $|\theta'(0)|$ with the thermophoresis and Brownian motion parameters when $A = 1$, $Pr = 6.2$, $S = 2.5$, $B = -2.5$, $\lambda = -0.1$, $n = W_{ex} = M = W_{ey} = 0$, and $Le = 5$

| $Nt$ | $Nb = 0.1$ | $Nb = 0.2$ | $Nb = 0.3$ | $Nb = 0.4$ | $Nb = 0.5$ | $Nb = 0.6$ | $Nb = 0.7$ | $Nb = 0.8$ |
|------|------------|------------|------------|------------|------------|------------|------------|------------|
| 0.1  | 2.614527   | 2.614527   | 2.614527   | 2.614527   | 2.614527   | 2.614527   | 2.614527   | 2.614527   |
| 0.2  | 2.490704   | 2.490704   | 2.490659   | 2.490703   | 2.490703   | 2.490703   | 2.490703   | 2.490703   |
| 0.3  | 2.377171   | 2.377171   | 2.377171   | 2.377171   | 2.377171   | 2.377171   | 2.377171   | 2.377171   |
| 0.5  | 2.176251   | 2.176251   | 2.176251   | 2.176251   | 2.176251   | 2.176251   | 2.176251   | 2.176251   |
| 0.7  | 2.004064   | 2.004063   | 2.004063   | 2.004063   | 2.004063   | 2.004063   | 2.004063   | 2.004063   |
| 0.9  | 1.855008   | 1.855008   | 1.855008   | 1.855008   | 1.855008   | 1.855008   | 1.855008   | 1.855008   |

Figure 2: Effect of the magnetic field parameter on the velocity profiles

Significant influence on the flow dynamics. In addition, it can be observed that for any fixed values of $M$, $f'(\eta)$ has a higher value as compared with $g'(\eta)$.

The influence of the magnetic field parameter $M$ on temperature $\theta(\eta)$ and concentration profiles $\phi(\eta)$ is shown in Figure 3. The important outcomes as observed from the figure is that both the temperature and concentration profiles increase when the magnetic field parameter is enhanced. This is because an increase in the electromagnetic force retards the fluid motion. This, in turn, leads to a thickening of the temperature and concentration boundary layer profiles.

Figures 4-7 show the impact of varying the Weissenberg number on the velocity, temperature, and nanoparticle concentration profiles. It is observed that an increase in the Weissenberg number (parallel to the axes) causes a reduction in the velocity (parallel to the axes). This is expected because the Weissenberg number is a ratio of the shear rate to the relaxation time. Consequently, increasing the Weissenberg number increases the relaxation time and so higher...
resistance is experienced and hence the velocity profiles decrease. This can be seen in Figures 4 and 5. On the other hand, Figures 6 and 7 show that the temperature and nanoparticle concentration profiles increase with the Weissenberg number.

Figure 8 shows the effect of the Prandtl number on the velocity profiles with other parameters fixed. It can be seen that both velocity profiles increase near the boundary layer with increasing Prandtl numbers. Further, it can be noted that $f'(\eta)$ has a higher value than $g'(\eta)$ for any fixed values of $\eta$ and $Pr$.

Figure 9 shows the influence of the Prandtl number on the temperature and nanoparticle concentration profiles. As the Prandtl number increases, the temperature and nanoparticle concentration profiles both decrease at a certain point.
**Figure 6** Effect of the Weissenberg number on the temperature and concentration profiles

**Figure 7** Effect of the Weissenberg number on the temperature and concentration profiles

**Figure 8** Effect of the Prandtl number on the velocity profiles
Physically, this is explained by the fact that an increase in Prandtl number means a reduced thermal conductivity and so reduces the thermal and concentration boundary layer thicknesses.

Figure 10 demonstrates the impact of the thermophoresis parameter on the temperature and concentration profiles. It can be seen that both profiles increase near the boundary with increasing values of $Nt$. The temperature profile $\theta(\eta)$ increases because as $Nt$ increases, transfer of fluid from a hot region to cold regions increases and this, in turn, increases the temperature of the fluid. Also, it can be noted that close to the boundary, $\theta(\eta)$ takes a higher value than $\phi(\eta)$ for any fixed values of $\eta$, and $Nt$. The concentration profiles near the boundary have a much significant increase when $Nt$ is increasing.
Figure 11 shows the effect of the Brownian motion parameter on the concentration profiles when other parameters are fixed. The concentration profiles decrease when the particle Brownian motion is increased. The explanation is that an increase in particle Brownian motion causes an irregular movement and more vigorous mixing of the fluid, and consequently, a reduction in the nanoparticle concentration profiles.

6 | CONCLUSIONS

In this study, we investigated the dynamics of 3D tangent hyperbolic nanofluid flow over a permeable stretching sheet adopting the Buongiorno’s model. It was further assumed that the flow field was subjected to an applied magnetic field that has an impact on the flow evolution.

The flow equations have been solved using the spectral local linearization method. This method was shown to give accurate solutions. From the analysis, we found, among other results that:

- For fixed values of the thermophoresis parameter, there are no changes in the local Nusselt number with increasing values of the Brownian motion parameter. However, when the Brownian motion parameter is fixed the local Nusselt number is noted to decrease when the thermophoresis parameter is increasing.
- The velocity profiles at a point decrease near the boundary with increasing values of the magnetic field parameter when other parameters are fixed.
- The velocity profiles increase when the Prandtl number is increasing and the other model parameters are fixed.
- The temperature and concentration profiles increase when the thermophoresis parameter increases.
- The concentration profiles have a significant increase near the boundary when the thermophoresis parameter is varied.

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this article.

AUTHOR CONTRIBUTIONS

Ibukun Oyelakin: Formal analysis; methodology; validation; writing-original draft. P.C. Lalramneihamwii: Formal analysis; investigation; writing-original draft. Sabyasachi Mondal: Conceptualization; formal analysis; investigation; methodology; supervision; validation; writing-review and editing. Samir Nandy: Conceptualization; formal analysis; writing-review and editing. Precious Sibanda: Conceptualization; supervision; validation; writing-review and editing.

NOMENCLATURE

- $C_{f_x}$, $C_{f_y}$: skin friction coefficients
- $Nu_c$: Nusselt number
- $Re_x$, $Re_y$: Reynolds number
- $We_x$, $We_y$: modified Weissenberg numbers
- $A$, $B$: dimensionless slip parameters
- $C$: nanofluid volume fraction
- $C_\infty$: ambient nanofluid volume fraction
- $D_B$: Brownian motion diffusion coefficient
- $D_T$: thermophoresis diffusion coefficient
- $f$, $g$: nondimensional stream functions
- $Le$: Lewis number
- $M$: magnetic field parameter
- $n$: tangent hyperbolic fluid exponent
- $Nb$: Brownian motion parameter
\( Nt \) \text{ thermophoresis parameter}  \\
\( Pr \) \text{ Prandtl number}  \\
\( q_w \) \text{ surface heat flux}  \\
\( S \) \text{ suction/injection parameter}  \\
\( T \) \text{ fluid temperature}  \\
\( T_{\infty} \) \text{ ambient temperature}  \\
\( T_w \) \text{ surface temperature}  \\
u, v, w \quad \text{velocity components along the x, y, z axes, respectively}  \\
\( \beta \) \text{ ratio of the effective heat capacity of the nanoparticle material to the heat capacity of the fluid}  \\
\( \theta, \phi \) \text{ temperature and concentration functions}  \\
\( \tau_{wx}, \tau_{wy} \) \text{ surface drag coefficients} \\

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