The refinement of dynamic deformation equation for AISI1215 steel using the Zener-Hollomon parameter

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Abstract. The flow stress of a metal being deformed depends on temperature, strain rate, and strain. The Johnson-Cook model describes these dependencies in a mathematical form with six material parameters, of which five have to be derived from forming experiments. However, the coupled interaction between parameters on flow stress has not been considered, and the solution of equation coefficients is difficult. This paper presents the development of a less complicated relationship between flow stress and forming parameters based on the Zener-Holloman parameter. In this paper, AISI1215 steel was chosen as experimental material and was deformed in the range of the temperatures of 750–850 °C and strain rate of 0.01–30 s⁻¹ to resolve and verify the presented model. The investigation results show that the coefficients could be solved easily in the present equation through simple hot simulation experiments without complex and disputed mathematics. The experimental results were agreement with the modeling predictions. Here the dynamic deformation behavior was only considered before dynamic recrystallization.

1. Introduction
The flow stress of materials that undergo dynamic deformation due to high temperature and strain rate is affected by strain, strain rate, and deformation temperature [1]. Dynamic deformation behavior is an essential consideration in process simulations to determine the hot forming applications of materials; this parameter is usually described using constitutive models [2–4], such as Johnson–Cook (J–C).

Dynamic recrystallization (DRX) is promoted when the strain exceeds the critical strain ($\varepsilon_c$). Thus, the flow stress of materials should be divided into two stages of modeling: with or without DRX dependent on $\varepsilon_c$ [5]. In the first stage, the flow stress of materials without DRX is an empirical and simple multiplication of strain, strain rate, and deformation temperature, such as in the J–C model [6]. The empirical equation has been improved from the J–C model [7–8]. However, the interaction between the strain rate and deformation temperature is not considered in these models. Therefore, calculating the coefficients in these equations is difficult and usually requires a disputed mathematical method [9]. Momeni [10] proposed a physically based model for the dynamic deformation behavior of materials, but this model is complicated and difficult to solve.

Equations with only a few parameters are easy to solve. For hot deformation processing, Sellar [11] combined strain rate and deformation temperature into the Zener–Hollomon (Z) parameter, i.e., strain rate–corrected temperature. In other words, the effect of strain rate and deformation temperature on flow stress can be described by the Z parameter. In the present work, a constitute model defined by the Z parameter was developed for the dynamic deformation behavior before DRX. The effect of DRX on the flow stress of materials during dynamic deformation should be considered in future works.
2. Hot compression experiments and results
Commercial AISI 1215 steel with a chemical composition of 0.07% C, 0.06% Si, 0.8% Mn, 0.05% P, and 0.28% S was utilized as the experimental material. The samples for thermal simulation examination were cut to 8 mm in diameter and 12 mm in length from the bar. Hot compression tests were conducted on Gleeble-1500 thermo-mechanical simulator at different temperatures of 700 °C, 750 °C, 800 °C, and 850 °C at four strain rates of 0.1, 1, 10, and 30 s\(^{-1}\) because of the maximum rate limit of the facility. The specimens were first reheated to 1200 °C with 80 °C/s (15 s), held for 5 min, and then cooled to the deformation temperature at a cooling rate of 5 °C/s. The typical stress-strain curves obtained from hot compression tests are shown in figure 1.

As shown in figure 1, the flow stress increased as the deformation temperature was decreased from 850 °C to 700 °C for a given strain rate of 1 s\(^{-1}\) and decreased as the strain rate increased from 30 s\(^{-1}\) to 0.1 s\(^{-1}\) for a given deformation temperature of 700 °C. The hardening mechanism of strain rate was generally attributed to the tangled dislocation structures that hinder the dislocation movement at high strain rates. The softening mechanism of temperature was attributed to the decrease in P–N force of dislocation movement and dynamic recovery when no DRX occurred. When the strain exceeds the critical strain for DRX, recrystallization softening will overcome work hardening at the given strain rate and deformation temperature.

3. Dynamic deformation equation described by Z parameter for AISI 1215 steel

3.1. Modeling of dynamic deformation behavior described by Z
The flow stress of materials during deformation at high temperature and strain rate can be expressed as the function of strain, strain rate, and deformation temperature:

\[
\sigma = f(\varepsilon, \dot{\varepsilon}, T)
\]  \hspace{1cm} (1)

where \(\varepsilon\), \(\dot{\varepsilon}\) and \(T\) are the strain, strain rate, and deformation temperature, respectively.

The Z parameter can be expressed as:

\[
Z = \varepsilon \exp\left(\frac{Q}{RT}\right)
\]  \hspace{1cm} (2)

where \(\dot{\varepsilon}\) is the strain rate, \(T\) is the deformation temperature, \(Q\) is the deformation activation energy, and \(R\) is the gas constant of 8.314 J mol\(^{-1}\)K\(^{-1}\).

Combining equations (1) and (2), we can express the flow stress of the materials during deformation at high temperature and strain rate as a function of strain and Z as:
The basic Z parameter, where the effect of strain rate and deformation temperature on flow stress can be ignored, is defined as $Z_0$ (293 K, 0.001 s$^{-1}$) in this study. The flow stress of materials during dynamic deformation can be described as:

$$\sigma = f(\varepsilon, Z)$$

When the Z parameter is not high enough (i.e., $Z = Z_0$), the flow stress of the materials can only be the function of strain, called quasi-static deformation, which can be expressed as [12]:

$$\sigma = \sigma_0 + Ke^n$$

where $\sigma_0$ is the yield stress, $K$ is the strength component, and $n$ is the work-hardening exponent.

The existing model of the relationship between flow stress and Z in this work was not found in the references. It was established from the experimental database by section 2. The preliminary calculation showed that the strain range was 0.02–0.4 to avoid the effect of DRX on the flow stress. All of the experimental strain rate and deformation temperature databases were calculated as Z parameters with deformation-activated energy derived using the method described in the following chapter. The typical empirical relationship of the flow stress as a function of the Z parameter is shown in figure 2 at the given strains of 0.08 and 0.24.

![Figure 2](image_url)

**Figure 2.** The relationship between flow stress and $Ln(Z/Z_0)$ at strains of (a) 0.08 and (b) 0.24, experiment and fitted flow stress-strain curves of quasi-static deformation conditions (c).

As shown in figure 2(a) and (b), the flow stress was well fitted linearly with $Ln(Z/Z_0)$. Therefore, the function of flow stress with $(Z/Z_0)$ can be defined through a linear relationship as:

$$\sigma = A(\varepsilon) + B(\varepsilon)\ln(Z/Z_0)$$

where $A(\varepsilon)$ represents the basic flow stress at $Z_0$, i.e., without the influence of strain rate and deformation temperature. It was the only function of the strain, which should be the same as equation (5). $B(\varepsilon)\ln(Z/Z_0)$ consists of the combined contribution of strain and Z on flow stress. The flow stress of the material could not be zero even when the strain was equal to zero because of the existence of internal stress, such as P–N force of dislocation [13]. The function of $B(\varepsilon)$ can be expressed as:

$$B(\varepsilon) = C + D(\varepsilon)$$

where $C$ is a material constant depending on the internal stress and $D(\varepsilon)$ is a function of flow stress and strain at a given Z parameter.

Combining equations (5–7), we can express the modeling of dynamic deformation behavior before
DRX as:

$$
\sigma = \sigma_0 + K \varepsilon^n + C \ln(Z / Z_0) + D(\varepsilon) \ln(Z / Z_0)
$$

(8)

From the physical principle, $\sigma_0$, $K$, and $n$ represent the contribution of work hardening of materials on flow stress, $C$ represents the effect of $Z$ (deformation temperature and strain rate) on flow stress, and $D(\varepsilon)$ describes the combined effect of strain and $Z$ on the flow stress.

3.2. The solution of the proposed constitutive modeling parameters

A dynamic deformation modeling described by the $Z$ parameter was promoted, as shown in equation (8). Then, the parameters of $\sigma_0$, $K$, $n$, $C$, and $Z_0$, and the function of $D(\varepsilon)$ in the equation were derived. $\sigma_0$ (197.6 MPa), $K$ (539.8), and $n$ (0.39) were fitted into equation (5) with the flow stress curve under quasi-static deformation conditions ($Z_0$). The fitted curve with open squares is shown in figure 2(c). The fitted curve was close to the flow stress curve of $Z_0$ (solid black line).

The parameter C and strain-related function ($D(\varepsilon)$) built up the slope item ($B(\varepsilon)$) together following equation (6) and (7). Therefore, these parameters can be obtained if the line fitted between $\ln(Z / Z_0)$ and flow stress is completed at a given strain. The deformation activation energy $Q$ must be calculated first to get the $Z$ parameter. $Q$ is related to material composition and deformation parameters. The effects of deformation temperature and strain rate on the quasi-static deformation conditions ($Z_0$) can be ignored; thus, the value of $Q$ under $Z_0$, named as $Q_{Z_0}$, can be calculated by the composition of AISI 1215 steel, and the output was 185 kJ/mol [14]. Under non-quasi-static deformation conditions, $Q$ can be solved using multivariate linear regression with the peak flow stress ($\sigma_p$) of the compression test based on the three functions describing the relationship among flow stress, strain rate, and deformation temperature, divided by different stress levels as follows[15-16]:

$$
\varepsilon = A_1 \sigma_p^m \exp\left(-\frac{Q}{RT}\right)
$$

(9)

$$
\dot{\varepsilon} = A_2 \exp(\beta \sigma_p) \exp\left(-\frac{Q}{RT}\right)
$$

(10)

$$
\varepsilon \exp(Q / RT) = A[\sinh(\alpha \sigma_p)]^n
$$

(11)

where $A_1$, $m$, $A_2$, $\beta$, $A$, $\alpha$, and $n$ are the material constants.

In general, the power law of equation (9) is usually preferable for low stress at high deformation temperature or low strain rate, whereas the exponential function of equation (10) is suitable for high stress at low deformation temperature or high strain rate. However, the hyperbolic sine function of equation (11) can be used for a wide range of stress. The material constants $m$, $\beta$, and $\alpha$ are related to $\alpha = \beta / m$. Through the partial differentiation of equation (9) and (10) at constant deformation temperatures, we can use the following equations for low [figure 3(a)] and high [figure 3(b)] stress levels to obtain $m$ and $\beta$. The effect of temperature on parameters $m$, $\beta$ was small, and the average values of $m$, $\beta$, $\alpha$ were 3.36, 0.0238, and 0.0071, respectively.

Taking the natural logarithms of both sides of equation (11), we can obtain the following formula:

$$
\ln[\sinh(\alpha \sigma_p)] = \frac{\ln \varepsilon}{n} + \frac{Q}{nRT} - \frac{\ln A}{n}
$$

(12)
The relationship between $\ln(\varepsilon)$ and $\sigma_p$ at 973 and 1023 K.

The relationship between $\ln(\varepsilon)$ and $\ln(\sigma_p)$ at 1073 and 1123 K.

The relationship between $\ln(\varepsilon)$ and $\ln[\sinh(\alpha\sigma_p)]$ at different deformation temperatures.

When the deformation temperature in equation (12) is constant, the linear function can describe the relationship between $\ln[\sinh(\alpha\sigma_p)]$ and $\ln \varepsilon$ well, and the slope yields $\frac{1}{n}$, as shown in figure 3(c). The average value of $n=5.52$ was determined by the calculation.

The functional relationship between $\ln[\sinh(\alpha\sigma)]$ and $1/T$ can be described by liner function well if strain rate was fixed in equation (12), differentiating the equation (12) tended to:

$$Q = Rn \left[ \frac{\partial \ln[\sinh(\alpha\sigma)]}{\partial (1/T)} \right]_\varepsilon.$$

(13)
Figure 4. Evaluating the value of \( Q \) by plotting \( \ln(\sinh(\alpha \sigma_p)) \) vs. \( 1/T \) with different strain rates (a); and at strain rate 10 s\(^{-1}\) with different strains (b).

Thereby, the value of \( Q \) could be decided with the slopes of \( \ln(\sinh(\alpha \sigma)) \) vs. \( 1/T \) with different strain rate, multiplied by \( R_n \) according to equation (13). The output was shown in figure 4(a) where the linear slope increased with the increased in strain rate from 0.1 s\(^{-1}\) to 30 s\(^{-1}\). The trend is consistent with other studies [14-16]. As the value of \( n \) (5.52) have calculated, the fitting results of \( Q \) could be computed for different strain rate. The results of figure 4(b) indicated that the evolution of \( Q \) was closely linked to the strain rate from 0.1 to 30 s\(^{-1}\), which could be described by a linear function as follows:

\[
Q = 0.09 \varepsilon + 1.83 \tag{14}
\]

After obtaining the value of \( Q \), the linear effect of \( \ln(Z/Z_0) \) on flow stress was investigated, and the mean value of \( R^2 \) was about 0.91. The scatter data of \( A(\varepsilon) \) and \( B(\varepsilon) \) could be plotted in figure 5. As seen in figure 5, the trends of scatter data of variable vs. function for \( A(\varepsilon) \) and \( B(\varepsilon) \) was similar. The modus of equation (5) was adopted to fit the functional relationship of \( B(\varepsilon) \). According to the data in figure 5, the fitted equation of \( B(\varepsilon) \) was obtained as flows:

\[
B(\varepsilon) = C + D(\varepsilon) = 4.93 + 7.8 \times \varepsilon^{0.63} (R^2 : 0.96) \tag{15}
\]

The fitted curves shown in figure 5 indicated the fitted curves closely agreed with the scatter data for \( A(\varepsilon) \) and \( B(\varepsilon) \).

Figure 5. The scatter data and fitted curve of \( \varepsilon - A(\varepsilon) \) and \( \varepsilon - B(\varepsilon) \).

Finally, the dynamic deformation behavior of AISI 1215 steel could be described by the constitutive equation in the following:
\[ \sigma = 197.6 + 539.8 \times \varepsilon^{0.39} + (4.93 + 7.8 \times \varepsilon^{0.64}) \ln(ZZ_0) \]  

(16)

\[ Z = \dot{\varepsilon}_Z \exp(Q_{Z0}/RT_Z) = \dot{\varepsilon}_Z \exp \left\{ 100000[0.09 \dot{\varepsilon}_Z + 1.83]/RT_Z \right\} \]  

(17)

\[ Z_0 = \dot{\varepsilon}_Z \exp(Q_{Z0}/RT_Z) \]  

(18)

where \( \dot{\varepsilon}_{Z_0} \) was 0.001 s\(^{-1} \), \( T_{Z_0} \) was 298 K, \( Q_{Z0} \) was 185 kJ/Mol.

3.3. Comparison of calculation and experiments

Though equations from (16) to (18), the flow stress could be computed at given deformation conditions for experimental steel. The comparison between the hot compression tests experimental data and the calculation value by the proposed model were shown in figure 6.

In figure 6, it can be seen that the calculation results are in good agreement with the most experimental results. However, the new constitutive model can not predict the whole flow stress curve after DRX. As can be seen in figure 6, the flow stress curves of 750 °C*0.1 s\(^{-1} \), 800 °C*0.1 s\(^{-1} \), and 850 °C*0.1 s\(^{-1} \) occurred DRX in evidence for the flow stress curves had a single peak. The value of \( \varepsilon_c \) calculated by the model of Jonas [3] was about 0.18, 0.17 and 0.15 for these three deformation conditions respectively. It could be seen that the fitted data marked as black solid deviated the experimental data shown as white open gradually from the point of \( \varepsilon_c \) in figure 6, and this could be accepted for the behavior of DRX had not been considered in the present model.

![Figure 6. Comparison of the experimental and calculated flow stress.](image)

4. Conclusions

A new constitutive model was proposed in the present work by applying the Zener-Hollomon parameter and considering the effect of strain rate on the activation energy \( Q_{def} \). The new model coupled the interaction effect between three deformation parameters (strain, strain rate and temperature) on flow...
stress to characterize the hot deformation behavior of materials before DRX. The model could predict the behavior of flow stress well before the DRX, and the model parameters could be derived from limited experiments. The calculation using the proposed model shown a good agreement with the hot compression experimental data of AISI 1215 steel well before DRX.

Acknowledgment
This work was supported by a grant from the Key Research and Development Foundation of Anhui Province of China (1704a0902049).

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