Acoustic Metal

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Metal reflects electromagnetic waves because of the large conductivity that is responsible for dissipation. During which the waves undergo a 180° phase change that is independent of the frequency. There is no counterpart material for acoustic waves. Here we show that by using an array of acoustic resonators with a designed high-density dissipative component, an “acoustic metal” can be realised that strongly couples with sound over a wide frequency range not otherwise attainable by conventional means. In particular, we show the acoustic Faraday cage effect that when used as a ring covering an air duct, 99% of the noise can be blocked without impeding the airflow. We further delineate the underlying volume requirement for an acoustic metal based on the constraint of the causality principle. Our findings complement the missing properties of acoustic materials and pave the way to the strong wave-material couplings that are critical for the applications as high-performance audio devices.

I. INTRODUCTION

Large dissipation coefficient does not always lead to effective energy absorption. The best example is metal, which has a large conductivity but reflects the electromagnetic (EM) waves strongly. Such reflections are noted to be nearly 180° out of phase with the incident wave and independent of the frequency. While in recent years resonance-based acoustic metamaterials have achieved exceptional material properties, including zero and negative parameter, chiral micropolar behaviour, nonreciprocity by a magnetic-like circular flow, and exotic absorption, there is still no “acoustic metal” that reflects sound in the same way as metal reflects EM waves. This is due to the very high contrast between the solid and air impedance that prevents the effective coupling and damping in broadband. Recently, by a designed integration of multiple acoustic resonators, broadband near-perfect absorption was achieved. In this work, we push the dissipative component of the resonator array an order of magnitude higher so that the structure becomes an acoustic metal that reflects sound waves instead of absorbing them. In analogy to the metallic mesh that shields the microwaves from leaking out of the microwave oven or the Faraday cage for shielding the EM waves, a circular ring of such acoustic metal, lining the inner wall of a short air duct (~4 cm), is shown to block almost all the low-frequency noise within an octave. Such blocking effect evidences the strong couplings with sound.

In what follows, Section II presents a “phase diagram” to position the acoustic metal in relation to other acoustic materials as characterised by their impedance. In Section III we describe the design strategy for the acoustic metal, followed by the presentation of experimental results in Section IV. The underlying volume requirement for an acoustic metal, in relation to its effective frequency range, is described in Section V. We conclude with a short recapitulation in Section VI.

II. ACOUSTIC METAL AND THE SOFT BOUNDARY

By denoting the specific acoustic impedance of the air by \( Z_0 \), the displacement velocity on sample’s surface equals to the superposition of the particle velocities of the incident and reflected waves, \( v = p_i/Z_0 - p_r/Z_0 \). Here, \( p_i \) and \( p_r \) are the respective incident and reflected sound wave pressure modulations and the negative sign is due to their opposite propagating directions. Conservation of energy dictates that \( |p_r| = |p_i - v Z_0| \leq |p_i| \) which corresponds to the dimensionless \( \tilde{v} = v Z_0/p_i \) inside a circle of radius 1 on the complex plane \( \tilde{v} = \tilde{v}' + i \tilde{v}'' \),

\[ |\tilde{v} - 1| \leq 1, \]

as shown in Figure I. This circle can serve as a “phase-diagram” to represent the distinct motions of the various sample materials in response to the incident sound wave. At the origin is the motionless hard boundary. Along the circular edge, coloured by green, the total pressure \( p = p_i + p_r \)
\[ 2p_i - vZ_0 = (2 - \bar{\nu})p_i, \] 
always differs from \( v \) by a phase of \( \pi/2 \), hence lossless. At the far right of the circle, \( p \) approaches zero for the condition of soft boundary, at which point \( \bar{\nu} = 2 \) reaches its maximum magnitude implying the maximum coupling with sound. From the circumference of the circle the absorption coefficient

\[ A = 1 - |p_r/p_i|^2 = 1 - |\bar{\nu} - 1|^2 \quad (1) \]

monotonously increases (indicated by the colour gradient) as a function of decreasing radial distance to the centre and reaches the maximum absorption \( A_{\text{max}} \) at \( \bar{\nu} = 1 \) (i.e., the impedance-matching condition). Most of the solid materials are around the origin while the porous acoustic materials such as foam and wool extend to the centre of the circle along the real axis (the grey region). Since a metal is characterised by its high conductivity for dissipation, \( \sigma \), hence analogly we introduce the acoustic conductivity and permittivity as \( \nu = (\sigma - i\omega\varepsilon)\nu \). Under the incident sound with pressure modulation \( p_i \), \( p = 2p_i - vZ_0 \)

so that the dimensionless

\[ \tilde{\nu} = \frac{2Z_0}{1/(\sigma - i\omega\varepsilon) + Z_0}, \quad (2) \]

which is along the real axis in Figure 1, provided \( \omega \varepsilon \ll \sigma \) is negligible as that in a metal. Furthermore, to have the reflection, \( p_r \), 180 degrees out of phase with \( p_i \), where \( p_r = (1 - \bar{\nu})p_i = p_i(1 - \sigma Z_0)/(1 + \sigma Z_0) \), \( \sigma \) also has to meet another requirement that \( \sigma > 1/Z_0 \). Combining the above two conditions, we can delineate the section coloured in red in Figure 1 as the region for acoustic metals. This region is noted to be far beyond the scope of the conventional acoustic materials coloured by grey. In Figure 1 we show that the absorption has a peak when \( \sigma = 1/Z_0 \), with the growth of \( \sigma \) the perfect acoustic metal (PAM) limit is approaching when \( \omega \rightarrow \infty \) at the soft boundary limit of (the dimensionless \( \tilde{\nu} = 2 \)). In Figure 1 we illustrate the geometry of the acoustic metal resonator array intended to realise the acoustic counterpart to the Faraday cage effect for the EM waves.

### III. DESIGN STRATEGY

Our strategy is to design an array comprising a large number of resonators within a sub-wavelength scale so that the resonators respond in unison. While the response of each of the resonators can be delineated mathematically by the Lorentzian form, the array as a whole would respond as an acoustic metal as shown below.

Consider an array of resonators whose resonance frequencies are uniformly distributed. The oscillatory nature of the individual \( \varepsilon_n \) may tend to cancel each other while, in contrast, the individual \( \sigma_n \) are always positive and can accumulate. As a result, the overall permittivity becomes small while the conductivity can increase to a large value that is approximately constant in frequency as desired.

Mathematically, the velocity of the \( n \)th resonator is given

\[ v_n = \frac{-i\omega f_n}{\omega_n^2 - \omega^2 - i\omega\beta_n} p = (\sigma_n - i\omega\varepsilon_n) p \quad (3) \]

with \( \omega_n(f_n) \) being its resonant frequency (oscillator strength), and \( \beta_n \) the damping coefficient. Denoting its surface area by \( a_n \), only the averaged \( \bar{\nu} = \sum_n a_n v_n/\sum_n a_n \) over all the resonators can couple to the propagating waves, owing to the sub-wavelength nature of the array. Hence \( p = 2p_i - vZ_0 \) in Equation (3). Together with the definition of \( v \), we obtain an expression similar to Equation (2):

\[ \bar{\nu} = \frac{\tilde{\nu}}{p_i/Z_0} = \frac{2Z_0}{1/(\sigma - i\omega\varepsilon) + Z_0}. \quad (4) \]

In Equation (4) the averaged \( \bar{\sigma} \) and \( \bar{\varepsilon} \) of the composite are defined by

\[ \bar{\sigma} = \sum_n a_n \sigma_n = \frac{\omega^2}{\sum_n a_n} \sum_n \frac{a_n f_n \beta_n}{(\omega_n^2 - \omega^2)^2 + \omega^2 \beta_n^2}; \quad (5a) \]

\[ \bar{\varepsilon} = \sum_n a_n \varepsilon_n = \frac{1}{\sum_n a_n} \sum_n \frac{a_n f_n (\omega_n^2 - \omega^2)}{(\omega_n^2 - \omega^2)^2 + \omega^2 \beta_n^2}; \quad (5b) \]

Equation (5a) shows that indeed, the averaged \( \bar{\sigma} \) can accumulate and become large in magnitude whereas the summed terms in Equation (5b) are always opposite in sign between adjacent \( \omega_n \), hence cancelling each other.

For a uniformly distributed \( \omega_n \) each separated from the neighbours by a small \( \delta \omega \), \( \omega_n - \omega_{n-1} \), we wish to focus on the two parameter \( f_n \) and \( \beta_n \) that are crucial to the acoustic metal design. In the limit of \( \delta \omega \rightarrow 0 \), the summation in Equation (5a) becomes an integral that gives \( f_n \approx \bar{\sigma} \times 2\delta \omega \sum_n a_n/(\pi a_n) \). Since large \( \bar{\sigma} \) of acoustic metal requires large displacement velocity, we show in Appendix A that, in contrast to the displacement velocity of a single Lorentz resonator which is suppressed by the damping,

![Figure 2](image-url)  
**FIG. 2.** The surface displacement velocity of acoustic metal. a. The averaged dimensionless velocity of the acoustic metal, \( \bar{\nu} = \bar{\nu}' + i\bar{\nu}'' \), plotted as a function of frequency. Within the designed frequency band (region without shading), \( \bar{\nu}'' \rightarrow 0 \) (green) while \( \bar{\nu}' \) (red) is close to the target values (dashed) when \( \bar{\sigma}Z_0 = 9 \) and \( \bar{\varepsilon} = 0 \). b. On the complex plane, the magnitude of \( \bar{\nu} \) in the designed frequency band (region with no shading) are noted to be closely distributed around the PAM limit. The red arrow points to the direction of increasing frequency. In both a and b, the solid curves are theoretical predictions from Equation (4) and the open circles are retrieved from experimental data of a real sample shown in Figure 3.


the collective summation of the array resonators’ dissipative component can lead to the overall $\tilde{v}$ increases with the damping as long as $\beta_n$ does not exceed $2\delta\omega$.

In the following, we design the acoustic metal by targeting $\bar{\omega} = 9/Z_0$ over a broad frequency band from 290 to 625 Hz. If there are 27 resonators, $\delta\omega = 12.9 \times 2\pi$ Hz and $f_n Z_0 a_n/\sum_n a_n = 464.4$ Hz. Therefore, by choosing $\beta_n = 2\delta\omega$, as shown in Figure 2a by the solid curves, Equation 4 predicted that $\tilde{v}'' \gg \tilde{v}'$ within the designed frequency band (the blank region) with the value of $\tilde{v}'$ close to the target values (dashed line) when $\bar{\omega} = 9/Z_0$ and $\bar{v} = 0$. In the phase-diagram shown in Figure 2b, all the vectors of the dimensionless complex velocities between 290 and 625 Hz are noted to fall into a sector that has no shading. It is clear that they were mostly in the acoustic metal phase as delineated in Figure 1a and tightly distributed around the PAM limit.

IV. EXPERIMENTAL REALISATION

We wish to demonstrate the effect of the acoustic metals by realising an acoustic counterpart to the Faraday cage effect that blocks EM waves through a layer of metallic mesh whose pores are smaller than the wavelengths. For simplicity, we consider only one pore of a mesh in practice, designed as a ring covering the inner wall of an air duct as shown in Figure 1a. The actual duct used in the demonstration has a cross-sectional area $a = 78.5$ cm$^2$. The specific impedance for the air in the duct is given by $Z_0 = \rho c \sum_n a_n/(2a)$ (detailed in Appendix A), where $\rho = 1.2$ kg/m$^3$ is the air density and $c = 334$ m/s is the speed of airborne sound.

As shown in Figure 3a, the acoustic metal sample comprises 27 Helmholtz resonators (HRs) whose cavities share the same depth of $L = 74$ mm and each individual rectangular neck is connected to the central duct with the same length of $l = 2$ mm. As an approximation, the resonance frequency of the $n$th HR is given by $\omega_n = \epsilon c/\sqrt{a_n/(A_n L)}$ with $A_n = V_n/L$ being the effective cross-sectional area of the cavity and $V_n$ being the cavity volume. The frequency $\omega_n$ can be directly tuned by the areal ratio $a_n/A_n$ so as to be uniformly distributed in the designed frequency band of (290,625) Hz while the desired values of $f_n$ are obtainable from adjusting the specific $A_n$. The detailed values of $A_n/a_n$ and $A_n$ for our design are listed in Table 1 of Appendix A. By keeping these areas unchanged, we deformed their shapes so that the resonators can be assembled compactly. In order to attain a sufficiently large $\beta_n$, thin partitions were inserted into the necks so as to divide each into 16 parts (the tiny blue rectangles in Figure 3a), with the purpose of achieving larger air-solid interfaces, aimed to increase the acoustic dissipation.

We have fabricated the designed structure by the stereolithography 3D printing technology. To carry out the measurement, the sample was connected to two circular impedance tubes of the same inner diameter on the two sides as shown in Figure 3a. During the experiments, a loudspeaker continuously generates sinusoidal signals from one end. Two microphones on the front tube recorded the reflected sound, $p_r$, from the structure while another two microphones on the rear measuring the transmitted $p_t$. Acoustic foam was put at the rear end to insure there was no reflection. The results are shown in Figure 3b by open circles. Within the designed frequency band (the area without shading), $|t| = |p_t/p_i| \sim 0.1$ (red) and $|r| = |p_r/p_i| \sim 0.89$ (green) indicating that about 99% of the forward propagating energy has been blocked by the acoustic metal. The phase of reflection $\angle r \simeq \pi$ in the designed frequency band that consistent with the signature of a
metal. Based on these experimental data, we can retrieve the values of \( \vec{v} \) as shown by the open circles in Figure 2. Excellent agreement with the theoretical design target (solid curves) is seen. The relationship between \( \vec{v} \), \( r \) and \( t \) is detailed in Appendix A.

Figures 3a and e show the numerically simulated in-plane air velocity field \( \vec{v}_{||} = \vec{v}_{||}/v_{i} \) on the duct cross-section, which intersects all the HRs' openings as denoted by the dashed line in Figure 3a. The two figures demonstrate in detail how the HRs attain their metallic behaviour by blocking the sound waves in the duct. For sound frequencies in the designed frequency band, e.g., at \( \omega = 590 \times 2\pi \) Hz, all the in-phase components, i.e. real parts, of the displacement velocity (\( \vec{v}' \)) vectors in Figure 3a are pointed towards the HRs, with a "source" of outward arrows at the centre. Such a source draws the air in the ducts from both sides of the plane so that, on the transmission side, the air displacement velocity is opposite to that of the incident wave's \( v_{i} \), thereby cancelling it. Meanwhile, the out-of-phase component of the displacement velocity (\( \vec{v}'' \)) vectors in Figure 3a presents a distinct feature that \( \nabla \cdot \vec{v}'' \approx 0 \) everywhere—which means that the air expelled from the resonating HR is completely compensated by the intakes of the others with different resonance frequencies. Such in-phase and out-of-phase behaviours of the displacement velocity correspond exactly to the two equations in Equation (6).

Be noted that our simulations did not consider the airflow in the duct since the airflow speed is usually only a few percent of the sound speed (wind tunnel being the exception), therefore only has a negligible effect.

The above conclusions can be further confirmed experimentally by individually measuring the 27 HRs by blocking the 26 resonators and retrieving the relevant normalised velocity \( \vec{v}_{n} = v_{n}/v_{i} \) for the one not blocked. As illustrated by the three examples in Figure 3b, all the \( \vec{v}'_{n} > 0 \) (denoted by the red curves) confirms that the velocity is directed towards the relevant HRs in Figure 3b. In contrast, at any given frequency in the design band, all the HRs at those frequencies below the given frequency contribute positively to \( \vec{v}'_{n} \) (green curves), and those at frequencies above the given frequency contributes negatively to \( \vec{v}'_{n} \). Such behaviour corresponds exactly to that indicated by Figure 3b.

V. VOLUME REQUIREMENT

The conductivity, \( \sigma \), of an acoustic metal cannot be arbitrarily large. As proven in Appendix B it is constrained by the given volume because of the principle of causality. To be specific, for any structure targeting a constant \( \sigma \) over a broadband frequency regime, causality dictates that its volume, \( V \), must have a minimum volume requirement:

\[
V \geq \sigma Z_{0} \frac{a}{\pi^{2}} \int_{0}^{\infty} \frac{1 + \zeta}{1 - \zeta} d\lambda = V_{\text{min}},
\]

where, \( \lambda = 2\pi c/\omega \) the wavelength and \( \zeta = 2i\omega \delta \omega/\pi \times \sum_{n} 1/(\omega_{n}^{2} - \omega^{2} - i\omega \beta_{n}) \) for the structures of our design. According to Equation (6), \( V_{\text{min}} \) is proportional to \( \sigma Z_{0} \) as shown in Figure 4 by the dashed line. Therefore, a PAM of \( \sigma \to \infty \) over a finite bandwidth will need an infinitely large volume, hence impossible in practice. It is worth to mention that the result of causality constraint in Ref. 19 is a specific case of Equation (6) with \( \sigma Z_{0} = 1 \) and \( 2a = \sum_{n} a_{n} \).

We can experimentally verify Equation (6) by checking the volumes of different samples. To do so, we fabricated another two (the schematics in Figure 4) with smaller conductivities: \( \sigma = 3/Z_{0} \) and \( \sigma = 1/Z_{0} \) (impedance-matching). If we compare the real volumes, \( V \), of all the three samples (stars) with \( V_{\text{min}} \), it is clear that they are very close to this causally optimal limit. Hence, they exhibit the linear relationship seen in Figure 4.

A useful approximation of Equation (6) (detailed in Appendix B) is given by

\[
V \simeq V_{\text{min}} \simeq \sigma Z_{0} \frac{2a}{\pi^{2}}(\lambda_{1} - \lambda_{N}),
\]

in which, \( \lambda_{1(N)} = 2\pi c/\omega_{1(N)} \) are the wavelengths at the lower and upper bounds of the designed frequency band. Take our first sample as an example, \( \sigma Z_{0} = 9 \) within the band (290,625) Hz, so that \( 2\sigma Z_{0}(\lambda_{1} - \lambda_{N})/\pi^{2} \) gives an effective length about 1.1 m and, for the duct of cross-sectional area \( a = 7.85 \times 10^{-3} \text{ m}^{2} \), \( V \simeq 8.64 \text{ L} \) that is quite close to the sample’s real volume of 7.92 L.

VI. CONCLUSION

We have realised the acoustic counterpart of EM wave’s metallic behaviour by integrating an array of HR resonators each with a large dissipative coefficient. Instead of absorbing the acoustic energy, the composite array strongly reflects sounds within an octave (from 290 to 625 Hz). A circular ring of acoustic metal lining the inner wall of an air duct blocks 99% of the noise energy within a distance 1/30 of the longest wavelength, without impeding the airflow. One can therefore
expect its application to effectively reduce noise generated by fans and heavy machinery (as shown in Appendix C and the Supplementary demo video). We further delineate the causality constraint on absorption to acoustic metals. The proposed novel acoustic material properties enrich the possibilities for new types of insertion loss panels. In particular, features such as large bandwidth, low spectral dispersion, and high susceptibility are critical for many acoustic elements such as absorbers, lenses, and audio devices.

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**Appendix A: Supplementary Note for the Design Methodology**

Figure 5a is a schematic drawing for a single HR used in our design. For the nth one, we denote the area of its opening (red) as $a_n$, If the volume of the sector-cylindrical cavity is $V_n$ and the depth along the radius is $L$, we can define its effective area $A_n = V_n/L$ (green). In our case, the length of neck is $l = 2$ mm and $L = 74$ mm, the resonance frequencies, $\omega_n$, of the 27 HRs together with the associated $a_n$ and $A_n$ are listed in Table I. Here, the values of $\omega_n$ were from the FEM simulation by COMSOL Multiphysics, in which the evanescent waves at the openings and the weak couplings between neighbouring HRs have all been included.

Unlike the Lorentz resonator whose response was always suppressed by the damping mechanism, our proposed acoustic metal can have large damping while simultaneously display large displacement velocity. Here, we will try to visualise this fact by drawing the $\tilde{v}$ of our sample in the phase-diagram, as shown by the top row in Figure 5b, when the damping coefficient $\delta$ takes values from $0.02\omega_l$ to $2.0\omega_l$. It is clear that the overall motion intensities of $\tilde{v}$ increase if $\beta < 2\delta\omega$ and only start to drop when $\beta > 2\delta\omega$. Therefore, we have chosen the value of $2\delta\omega$ for $\beta$ in the design.

On the contrary, if we block all the other 26 resonators and leave only one open (e.g., the 14th one), there is only one term left in the summation in Equation (5a) and the relevant responded velocity has a simple Lorentz-form (as shown in Figure 5c).

$$\dot{v}_{14} = \frac{-72\omega_l\omega_l/(\pi \rho c a_n)}{\omega^2 - \omega^2 - i\omega(\beta + 18\delta\omega/\pi)/\pi} p_{14}. \quad (A1)$$

The bottom row of Figure 5b shows that, in contrast to what happened in our acoustic metal, its magnitude continuously shrinks as the growth of $\beta$.

We also wish to show the role of the mode-density in the acoustic metal by a series of numerical examples. Let’s start from 20 resonators uniformly distributed in an octave, ($\omega_1, 2\omega_1$). Their $f_n = 2\omega_1 n/(19\pi \rho c a_n)$ and $\beta = 2/19 \times \omega_1$ so that meet the requirement of a good broadband absorber. Figure 5c shows its complex $\tilde{v}$ in the phase-diagram together with the reflected energy ratio $|r|^2$ and phase $\angle r$ under normal incident sounds. Nearly perfect absorption, $|r|^2 \sim 0$, is seen in the design band but $\angle r$ is away from $\pi$. Now, if we keep increasing the number of resonators, $N$, within the same frequency range, $\tilde{v}$ starts to move along the small red arrows in the phase-diagram and continue to converge towards the PAM limit, as illustrated in Figures 5d and e. Eventually, $|r|^2 \rightarrow 1$ and $\angle r \rightarrow \pi$.

The above phenomenon recall for us an optical analogy in the drying of a drop of ink. As shown in Figure 5f, during the process of its drying, a drop of red ink on glass starts to acquire a metallic greenish reflection, sometimes called “sheen”. The reason is that the red ink absorbs out the greens of transmitted light. So if the ink is very concentrated, the number of dye particles per unit volume will be large and can therefore exhibit a strong surface reflection for the frequencies of green light—similar as what we have seen in the above simulations for sounds. That means, the mechanism in this paper is not limited to acoustics as concluded by Richard Feynman as “if any material gets to be a very good absorber at any frequency, the waves are strongly reflected at the surface and very little gets inside to be absorbed.”

Finally, let’s evaluate $Z_0$ of a narrow duct for the array of objects on the sidewalks. By a “narrow” duct, we mean one whose width is small comparing to the wavelengths. So that only the waves with uniform front can travel inside. For the array also small along the wave direction, the air region in front of it is in sub-wavelength hence relatively incompressible. That means the net volume of air in and out of it is zero. Mathematically, $\sum n a_n v_n + 2a v_{rad} = 0$ with $v_{rad}$ being the particle displacement velocity of the pair of symmetric radiative sound waves pointing outward. For a given objects’ motions of $v_n$ that cause a sound pressure in the front air by $p_{rad}$,
Therefore, the transmitted and reflected sounds are given by

\[ t \rightarrow \infty \]

where, \( t \rightarrow \infty \) denotes the scattering process. Illustration should also follow the principle of causality hence constrained in a similar way as shown in Ref. [20]. That means,

\[ r(\delta t) = \int_{-\infty}^{\infty} r(\delta t) t(t-\delta t) d\delta t. \] (B1)

with \( r(\delta t) = 0 \) when \( \delta t < 0 \). Therefore, the reflection coefficient as a function of frequency,

\[ r(\omega) = \int_{-\infty}^{\infty} r(\delta t) e^{i\omega \delta t} d\delta t, \] (B2)

can be analytically continued into the complex frequency and still be analytic in the upper half-plane. In terms of the wavelength \( \lambda = 2\pi c/\omega \), that means \( r(\lambda) \) has no poles in the lower half-plane of complex \( \lambda \) but may have zeros. Similar as in Ref. [20], we consider the logarithmic function \( \ln r \) which is analytic except in the positions, \( \lambda_n \), and for \( r = 0 \). Such logarithmic singularities can be removed by multiplying the factor \( (\lambda - \lambda_n) / (\lambda - \lambda_n') \) with unity amplitude, where, * denotes the
complex conjugation. Then for
\[
\tilde{r}(\lambda) = r(\lambda) \prod_n \frac{\lambda - \lambda_n^*}{\lambda - \lambda_n},
\] (B3)

In \( \tilde{r}(\lambda) \) is analytic with no singularities in the lower half-plane.

Therefore, according to the Cauchy’s theorem, the integral along the contour in Figure 6b is zero,
\[
\int_C \ln \tilde{r} d\lambda = 0.
\] (B4)

Because \(|\tilde{r}| = |r|\) and \(|r(\lambda)| = |r(-\lambda)|\), the real part of Equation (B4) gives
\[
-2 \int_0^\infty \ln |r| d\lambda = \Re \left[ \int_{C_\infty} \left( \ln r + \sum_n \ln \frac{\lambda - \lambda_n^*}{\lambda - \lambda_n} \right) d\lambda \right],
\] (B5)

here \( C_\infty \) is an lower half-circle with radius tends to infinity.

In the long wavelength limit of \(|\lambda| \to \infty\),
\[
\lim_{|\lambda| \to \infty} \ln \frac{\lambda - \lambda_n^*}{\lambda - \lambda_n} = 2i \Im(\lambda_n)/\lambda
\] (B6)

so that
\[
\int_{C_\infty} \ln \frac{\lambda - \lambda_n^*}{\lambda - \lambda_n} d\lambda = 2\pi \Im(\lambda_n),
\] (B7)

Where, \( \int_{C_\infty} d\lambda = \int_0^\infty \pi i \lambda d\gamma \) with \( \lambda = |\lambda|e^{i\gamma} \). Meanwhile, when \( \lambda \to \infty \), the composite is uniformly compressed and expanded under the external pressure \( p_i + p_r \). According to Hooke’s law, \( p_i + p_r = \kappa_0 \Delta V/V = i\kappa_0 \delta \sum_n a_n/(\omega V) \) with \( \kappa_0 \) the static bulk modulus of the composite and \( V \) their total volume. On the other hand, since \( \bar{v} = (p_i - p_r)/Z_0 = \sigma(p_i - p_r) \), the reflection coefficient can be solved as
\[
r = \frac{p_r}{p_i} = \frac{\kappa_0 \delta \sum_n a_n + i\omega V}{\kappa_0 \delta \sum_n a_n - i\omega V} = 1 + \frac{i}{\kappa_0 \delta} \frac{2V}{\kappa_0 \delta \sum \lambda a_n} + \cdots
\] (B8)

here \( \kappa = \rho c^2 \) denotes the air bulk modulus. Hence, \( \ln r \approx \frac{i\kappa}{\kappa_0 \times 4\pi V/(\sigma \rho c \sum \lambda a_n)} \) at the long wavelength limit and \( \int_{C_\infty} \ln r d\lambda = \frac{\kappa \sigma c}{\kappa_0} \times 4\pi V/(\sigma \rho c \sum \lambda a_n) \). Therefore,
\[
-\int_0^\infty \ln |r| d\lambda = \frac{\kappa}{\kappa_0 \sigma c \rho \sum_n a_n} \frac{2\pi^2 V}{\lambda} + \pi \sum_n \Im(\lambda_n)
\] (B9)

since \( \Im(\lambda_n) \leq 0 \).

For our acoustic metal, according to Equation (4),
\[
\bar{v} = 2 \left[ 1 + \sum_n \frac{a_n}{\omega Z_0} \left( \sum_n \frac{a_n f_n}{\omega^2 - \omega^2 - i\omega \beta_n} \right) \right]^{-1}
\] (B10)

in the current case, \( Z_0 = 1/\bar{\sigma} \), and
\[
r = 1 - \bar{v} = \frac{1 + \zeta}{1 - \zeta}
\] (B11)

with
\[
\zeta = \frac{i\omega}{\sigma} \sum_n a_n \frac{1}{\omega^2 - \omega^2 - i\omega \beta_n}.
\] (B12)

Because \( f_n = 2\sigma \delta \omega \sum_n a_n/(\pi a_n) \),
\[
\zeta = \frac{i 2\omega \delta \omega}{\pi} \sum_n \omega_n^2 - \omega^2 - i\omega \beta_n
\] (B13)

irrelevant to the target \( \bar{\sigma} \). Also, if the composite was rigid cavities filled with air as what in the main text, \( \kappa_0 = \kappa \), Equation (B9) becomes
\[
V \geq \sigma \rho c \sum_n a_n / 2\pi^2 \int_0^\infty \ln \left| 1 + \frac{\zeta}{1 - \zeta} \right| d\lambda.
\] (B14)

That gives a lower limit for the volume proportional to \( \bar{\sigma} \).

To further evaluate the integral in Equation (B14), we assume that the system’s averaged acoustic conductivity ideally takes the value of \( \bar{\sigma} \) within the frequency range \( (\omega_1, \omega_N) \) and being zero elsewhere. According to the Kramers-Kronig relationship,
\[
\bar{\sigma} - i\omega \bar{\varepsilon} = -i \frac{2\bar{\sigma}}{\pi} \left[ \tanh^{-1} \left( \frac{\omega_1}{\omega} \right) - \tanh^{-1} \left( \frac{\omega_N}{\omega} \right) \right].
\] (B15)

The substitution of Equation (B15) into Equation (B11) gives
\[
r = \frac{i\pi/2 - \tanh^{-1}(\lambda/\lambda_1) + \tanh^{-1}(\lambda/\lambda_N)}{i\pi/2 + \tanh^{-1}(\lambda/\lambda_1) - \tanh^{-1}(\lambda/\lambda_N)}
\] (B16)

with \( \lambda_{1(N)} = 2\pi c/\omega_{1(N)} \). Its substitution into Equation (B14) gives
\[
V \geq \sigma \rho c \sum_n a_n / \pi^2 (\lambda_1 - \lambda_N).
\] (B17)

The above equation can provide an estimate for the volume of an acoustic metal based only on the knowledge of the target conductivity and bandwidth. Usually, this estimate will be a little bit higher than the volume’s actual lower limit, since Equation (5a) cannot so perfectly match the target in practice.

Appendix C: Applications in Ventilation Noise Reduction

The understanding about the causality constraint provides us with a new perspective for the application of noise control. According to Equation (B17), the better blocking efficiency in an air duct, that corresponds to larger values of \( \bar{\sigma} \), requires larger volume of \( V \). However, in practice, there is usually limited volume available for the noise treatments. In this case, the spectrum design becomes crucial, with which one can design the reflection ability given by a volume to concentrate in the frequencies where the noise energy is most intensive, so as to
narrow the bandwidth in Equation (B17) for a larger $\bar{\sigma}$ of better noise reduction effect. This strategy is particularly useful for the noise having a characteristic spectrum, such as from the fan and machine.

To demonstrate the advantage of our spectrum design, we have compared our composite liner with a traditional one which comprises an acrylic cavity filled by acoustic foam, as shown by the inset photo in Figure 7. The volume of the cavity is the same as our sample and connected to the air duct by the same necks. As shown in the right column of Figure 7, such a liner exhibited even transmission loss (TL) about 5 (dB) for all the measuring frequencies. In contrary, the TL of our liner was maximised within the design band of (290, 625) Hz and presented the advantages of 10 ~ 20 dB.

In acoustics, acoustic foam has been generally considered as a high dissipative material. However, as mentioned at the beginning of this paper, the associated damping is not large enough for being an acoustic metal. That is evidenced by the absorption data as shown in Figure 7. The absorption coefficient $A = 1 - |r|^2 - |t|^2 \sim 0.5$ indicated that the relevant effective acoustic conductance was only $\bar{\sigma}Z_0 \approx 1$. Which is far below the $\bar{\sigma}Z_0 = 9$ of our sample.

To show the sound blocking effect of acoustic metal, we have made a demo video in the Supplementary Material in which, a Bluetooth speaker, playing an audio recording of broadband wind noise, was placed at the bottom of the air duct with the HRs’ openings closed by a metallic ring. The loud noise can be heard at the top port. A Decibel meter near the top port recorded a sound pressure level (SPL) at 95 dB. After removing the metallic ring to expose the HRs to the noises, the emitted SPL suddenly drops to 70 dB. This contrast evidenced a very significant 25 dB of the TL for the specific wind noise.

FIG. 7. The comparison of absorption coefficient ($A$) and the transmission loss (TL) between our composite liner (red) and the traditional one of acoustic foam (green). All the data are from the experiments. The inset is the photo of the two samples. From the two plots it is clear that whereas acoustic foam is mostly an absorptive material, our acoustic metal displays superior transmission loss with low dissipation.

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