Review of Progress in Magnetorotational Instability

Himawan Winarto

Department of Physics, Princeton University, Princeton, New Jersey 08544, USA
Email: hwinarto@princeton.edu

Abstract. I will describe the current numerical and experimental approaches for magnetorotational instability which is believed to be the driving mechanism of angular momentum transport in magnetized accretion disk. In the recent computational study, the incorporation of fully anisotropic pressure tensor on hybrid-MHD simulation shows interesting properties in the kinetic regime with low collisionality. The phenomenon can also be investigated further by the laboratory MRI experiments, for example through the use of liquid Galinstan alloy (GaInSn). Unlike observations of astrophysical plasma, laboratory astrophysical experiments can be used to investigate MRI over broad range of parameters with direct diagnostics.

1. Introduction

The angular momentum transport problem in accretion disk has been puzzling physicist for decades. Accretion disks itself is an amalgamate of matters in plasma state that can be usually found in the surrounding of massive astrophysical bodies. Most accretion disks plasma satisfy the Magnetohydrodynamics(MHD) criteria. From the observations, we know that particles in the system are accreting or falling towards the central bodies, defying the angular momentum conservation. This accretion process can be very efficient to convert the rest mass of the particles into outgoing radiation. In some system, the conversion rate has been theoretically predicted to be as high as 40%.[1] Due its ubiquitous and fundamental nature, this problem attracts have attracted many studies in the recent years.

Starting in the late twentieth century, there were various attempt to explain the angular momentum transport mechanism. The widely accepted model of turbulent angular momentum transport is the $-\nu$ viscosity model by Shakura and Sunyaev in 1973.[2] In this model, the angular momentum transport is modeled due to the viscosity that depends on the local sound speed $c_s$ and scale height of the system $H$ that can be written as,

\[ \nu = \alpha c_s H. \] (1)

Here $\alpha$ is the dimensionless parameter that can be retrieved through observations. This model is successful as it matches the profile of various light emission observations from accretion disks.

There are two main events where $\alpha$ values can be estimated. The first event is the light curves from dwarf novae outburst which transition time can be retrieved from the viscosity model to be $t_{visc} \sim \frac{R^2}{\nu}$, where $R$ is the radial scale associated with the system. Observations from various sources estimated the value of $\alpha$ to be in range of 0.1 to 0.3.[3-6] The second large scale event is the soft x-ray transients(STXs) emission from massive astrophysical bodies. Due to large gravitational potential of black hole or neutron stars, the emitted light can undergo self-irradiation process.[7] Observing the
spectrum of secondary light emission can give the value of $\alpha$ in the range of 0.2 to 0.4.[8] Some super-massive black holes, for example the Sagittarius A* in the center of our galaxy, have accretion disks that exhibit properties of collisionless plasma regime.[9]

While $\alpha$-viscosity model works well, it does not explain the fundamental process on how the angular momentum can be transported. Balbus and Hawley proposed Magneto-rotational instability (MRI) as the driving mechanism of angular momentum transport in their seminal paper in 1991.[10] MRI is a form of interchange instability in the weakly magnetized plasma with differential rotation. The magnetic field in the system can be modeled as a weak spring constant that threads two particles. From classical derivation, we can see that when one particle is pulled away from the center of rotation, the other one will transfer its angular momentum and fall closer to the center. By assuming the initial angular velocity of $v_\phi = r_\Omega(r)$ and doing perturbation solution on ideal MHD equations, we obtain the MRI instability can exist when the rotation profile satisfies

$$\frac{\partial \alpha^2}{\partial \ln r} \leq 0.$$  

(2)

For the regular Keplerian motion with the value of $q \equiv -\frac{d \ln \Omega}{d \ln r} = \frac{3}{2}$, it can be shown that the system is always unstable to MRI.

![Figure 1. Classical model of MRI treating the magnetic field as a weak spring connecting two masses orbiting a central body.](image)

2. Computational Approach

Starting from the first paper by Balbus and Hawley, myriad computational studies on MRI have been done to simulate the angular momentum transport. While these simulations have been successful to predict the linear growth phase of MRI, the simulation on late non-linear stage gives varying results of $\alpha$ values. Traditionally, the simulation is done by providing constant magnetic field in longitudinal direction or commonly known as finite flux simulation. However, such strong directed magnetic field is hard to find in such system, most of the recent computational utilize zero net flux approach. While it is not very physical, the net flux simulation tends to give higher $\alpha$ results than the zero-net flux simulation.[11] However, the net flux simulations tend to have convergence problem for late non-linear stage which is important for MRI study.

MHD simulations commonly use the shearing-box boundary conditions which can simulate the differential rotation properties locally.[12] Typical MHD simulations gives the value of $\alpha$ in the order of $O(10^{-3})$, which is orders of magnitude smaller than the observed values.[11, 13] Another approach to the simulation is to incorporate the gravitation stratification which consider the gravitation potential of the particles in the longitudinal direction. These simulations tend to provide similar value of $\alpha$ parameter, thus they are mostly used to study the accretion disk structure.

Recent simulations started to consider the kinetic effect in the system. The first approach is to utilize the Chew-Goldberger-Low (CGL) closure which is the same as to consider fast gyrotropization
limit of particle collisions. In place of the usual adiabatic closure in ideal MHD, the CGL approximation utilize fully gyrotrropic pressure tensor which can be reduced to two scalar pressures $p_\parallel$ and $p_\perp$. The results coming from the simulations that utilize this closure gives slightly higher volume average $\alpha$ value than MHD simulation in net flux simulation.[14] Further attempt has been done by using some partial or full kinetic approach which treat just the ion or both ion and electrons using Particle-in-Cell method. The results show significant increase for the measured $\alpha$ values which is in the order of $O(10^{-1})$.[15, 16]

The promising result of collisionless simulations raise some questions on how the low collisionality factor could increase the value of $\alpha$ parameter. One of the way to investigate the behavior is through the use of fully anisotropic pressure tensor with some gyrotropization scheme. This is done through the use of $Izanagi$ code that is developed by Kota Hirabayashi.[17] The code implement gyrotropization model that force the decay of anisotropic pressure tensor to its gyrotrropic form. In fully gyrotrropic system, we can write the pressure tensor as two scalars,

$$p_\alpha = p_\perp I + (p_\parallel - p_\perp)\hat{B}\hat{B},$$

where $I$ is the identity tensor. The decay is controlled by gyrotropization parameters $v_\alpha$ and $\Omega_\alpha$ that can be adjusted in the simulation. In this closure, the MRI system is evolved with the setup described in the influential paper of Hawley et al.[11] The full set of equations can be written as,

$$\frac{\partial \rho}{\partial t} + (\rho \bar{v}) = 0,$$

$$\frac{\partial}{\partial t} (\rho \bar{v}) + \nabla \cdot \left( \rho \bar{v} \bar{v} + \frac{B^2}{2} I - \hat{B}\hat{B} \right) = 0,$$

$$\frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{v} \times \bar{B}),$$

$$S_{ijk} = c_{ijk} E_i B_k,\quad (7)$$

$$\frac{\partial}{\partial t}(\rho v_i v_j + p_{ij} + B_i B_j) + \partial_k \left( \rho v_i v_j v_k + p_{ij} v_k + p_{ik} v_j + p_{jk} v_i + S_{kij} + S_{kji} \right) = B_i v_k \partial_k B_j$$

$$+ B_j v_k \partial_k B_i - B_k v_j \partial_j B_k - B_k v_j \partial_i B_k - v_g (p_{ij} - p_{g,ij}) + \Omega_\alpha (c_{ijk} p_{jk} \bar{B}_i + c_{jik} p_{ik} \bar{B}_i). \quad (8)$$

Here we denote $\rho$ as the plasma density, $\bar{v}$ as the plasma velocity, $\bar{B}$ as the local magnetic field, and $c_{ijk}$ as the Levi-Civita asymmetric tensor.

The results of the simulations show significant increase for low gyrotropization parameters as shown in the figure 2 and 3. It is also interesting to see that the pressure anisotropy distribution in figure 4 resembles the one that can be found in the hybrid simulation.[15] The effect of low gyrotropization can be easily seen as the value of pressure contribution $\alpha_p$ is approaching the Maxwell tensor component($\alpha_M$). If the pressure is fully gyrotrropic, the value should be related to $\alpha_p = \left( \frac{p\perp - p\parallel}{B^2} \right) \alpha_M$. As shown in the picture, the fluctuations of both values are not correlated indicating the presence of non-gyrotrropic pressure tensor.

The significant increase in the low gyrotropization case can be explained by the reduced rate of reconnection in low gyrotropization computational model. This has been shown briefly in the $Izanagi$ test cases.[17] The decrease in reconnection rate will create particle with strong perpendicular pressure that could lead to the increase in pressure contribution of alpha parameter.
Figure 2. Evolution of $\alpha$ parameters in the anisotropic case with low collisionality of $\nu_g = \Omega_c = 10^3$.

Figure 3. Evolution of $\alpha$ parameters in the ideal MHD case using adiabatic fluid closure.

Figure 4. Pressure anisotropy as a function of plasma parallel beta after 24 gyroperiods showing the firehose and mirror instability limit.

3. Experimental Approach

While most of MRI studies are done through computational simulation, some recent studies try to observe MRI through laboratory experiments. These experiments utilize either actual plasma or liquid metals that is spun using differential rotation profile. The main advantage of experimental approach of MRI when compared to the astrophysical observation is the capability to measure various parameters of the system with very high accuracy using direct diagnostic techniques. Compared to the pure computer simulation approach, laboratory experiments results could be considered more realistic and comprehensive. However, laboratory experiments suffer from the strong influence of physical boundary conditions that requires further modeling through computer simulations.

There is one experiment dedicated to investigate MRI using plasma: Plasma Couette Experiment(PCX) in University of Wisconsin Madison. PCX experiment utilizes multipole confinement. The main advantage of using plasma when compared to liquid metals are the high magnetic Prandtl number and magnetic Reynolds number that can easily be controlled through the manipulation of plasma parameters. The major drawbacks are coming from the confinement which is considerably harder to do for plasma. Recent result from the experiments are still limited to the characterization of flows and major hydrodynamics instability associated with the system.[18]

Liquid metal, especially liquid sodium, is prominently used for investigating MRI in laboratory setting. Compared to plasma experiment, sodium experiments are easier to handle. It is also easier to vary the rotational profile and boundary conditions which is crucial in the investigation of MRI effect. Most of the experiment utilize Taylor-Couette flow profile and produced varying results that mostly caused by hydrodynamics instability.[19-21] Some new experiments using liquid sodium are proposed
to utilize more exotic geometry with promising preliminary results which are yet to be verified.[22] More prominently, signs of MRI can also be retrieved from the Riga dynamo experiments which utilize liquid sodium.[23]

Another alternative to sodium as liquid metal is the liquid Galinstan alloy (GaInSn) which is used in Princeton MRI experiment. Compared to sodium, galinstan is less reactive and can be handled easier. However, as the density is higher, it will require considerable mechanical torque to rotate the system. In Princeton MRI experiment, the liquid galinstan is spun in cylindrical Taylor-Couette profile with boundary conditions on the top and bottom could be varied between conducting and isolating. Furthermore, the flow profile could be altered through the end caps that are spun using different speed from the inner and outer cylinders. The experimental results are extensively compared with the computer simulation using SFEMaNS code for verification.[24] Recent results includes the characterization of the flow profile, Ekman circulation, and Shercliff layer instability.[25] Further characterization of the Shercliff layer instability and high speed runs to sweep the parameter space of the experiment are scheduled in the next few months.

Figure 5. Cross section of Princeton MRI experiment device.

Figure 6. Parameter space of Princeton MRI experiment. Black dots are indicating previous runs while red dots are indicating proposed runs.

4. Conclusion

Magnetorotational instability has been widely believed as the main driving mechanism of angular momentum transport in accretion disk. It is best to characterize the angular momentum transport using $\alpha$-viscosity model. The direct observations from astrophysical systems gives the estimate value of $\alpha$ to be in the order of $O(10^{-1})$. However, most MHD simulation model of MRI provides value in the order of $O(10^{-3})$ which is significantly smaller than the observations. Recent simulation results in PIC and hybrid-MHD frameworks showed an increase to the $\alpha$ value that can match the observation. This stimulated study about the effect of collisionality in the form of pressure tensor gyrotropization. Preliminary result shows that the $\alpha$ parameter is increased as the system become less collisional. Experimental approach to MRI is very promising to prove the existence of angular momentum transport through MRI. Laboratory experiment could provide more details as various parameters of the system can be directly measured. Main challenges come from the characterization of physical boundary conditions that could be the source of various hydrodynamics instabilities.
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