Multi-Higgs models. Perspectives for identification of wide set of models in future experiments at colliders in the SM-like situation

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Talks given at
Int. Workshop on Multi-Higgs Models.
Lisbon, Portugal 2-5/09/2014
and
Int. Conf.-Session of Sect. of Nucl. Phys. of Phys. Sc. Div. of RAS
Physics of Fundamental Interactions, Moscow, Russia 17-21/11. 2014

Abstract

Higgs mechanism of EWSB can be realized in both well known minimal model and with more complex non-minimal Higgs models.

These non-minimal models contain new Higgs bosons – neutral $h_a$ and charged $H_b^\pm$. Necessary step in the discovery of such model is observation of these additional Higgses. We discuss the potential of such researches at modern and future colliders in the light of recent LHC results, for wide set of models, including 2HDM as a simplest example.

Our conclusion is rather pessimistic. Discovery of new neutral Higgs boson at LHC is a very difficult task. Some windows to find them appear at accidentally favorable parameters of the theory. The regular way in the detection of these models is the study of processes with production of charged Higgs bosons (better, at Linear Collider).

1 Introduction

This report is based on papers prepared together with M. Krawczyk [1]–[3] and K. Kani- shev [4], preliminary results were published in [5].

We are based on the following interpretation of modern data [6].

1. Higgs boson $h$ with mass 125 GeV is discovered at LHC.
2. Its properties are close to those in minimal Standard Model (SM) [7] – in other words, SM-like situation is realized.

This situation does not ruled out extended Higgs sector. Higgs mechanism of EWSB can be realized in both well known minimal model and with more complex non-minimal Higgs sector, containing new neutral Higgs bosons $h_a$ and charged Higgs bosons $H_b^\pm$. Necessary step in the discovery of such model is observation of these additional Higgses. Such discovery is a challenge for the next stage of LHC and $e^+e^-$ LC.
1.1 Models

The non-minimal Higgs models are devised to solve various physical problems. In this report we consider from common point of view a large group of models with more or less standard description of Higgs phenomenon but with richer set of fundamental scalar fields. That are models with \( n \) fundamental weak isodoublets, \( p_2 \) complex weak isosinglets \( S_2 \) and \( p_1 \) real weak isosinglets \( S_1 \): \( nHDM + p_2(\mathcal{HS}_2M) + p_1(\mathcal{HS}_1M) \). The models of this group are under wide discussion now.

Examples:

1. **1HDM** – model with single Higgs doublet don’t allow CP violation and FCNC. We denote this model as SM (Standard Model).
2. **2HDM** with huge literature – see e.g. [8], [9]. At some values of parameters 2HDM can explain CP violation, FCNC, etc. At another values of parameters 2HDM gives Dark Matter (Inert doublet Model) (without CP violation in Higgs sector and FCNC) (see e.g. [10]). One more set of parameters realizes Higgs sector of MSSM.
3. **2HDM + 1(\mathcal{HS}_2M)** describes Higgs sector of nMSSM (see also [11]).
4. **3HDM** at suitable set of parameters describes models with dark matter and possible CP violation and FCNC ([12, 13]).
5. **nHDM** at \( n \geq 3 \), in particular \( n = 6 \) ([14]), can describe dark asymmetric matter (see e.g. [15]).

These models give \( n - 1 \) pairs of charged Higgs bosons \( H_b^\pm \) with masses \( M_b^\pm \) and \( 2n - 1 + 2p_2 + p_1 \) neutral scalar Higgs particles \( h_a \) with masses \( M_a \), having either definite or indefinite CP parity (in the latter case we have CP violation). Variants with suitable Yukawa sector allow flavour changing neutral currents (FCNC), etc.

- We don’t discuss models with alternative explanations of situation or (and) with additional mechanisms, supplemented or changed standard Higgs mechanism – little Higgs, orbifold, radion, models with Higgs triplets,... We don’t find common description of this group of models.

1.2 Basic points

The discovery of new scalars \( h_a \), \( H_b^\pm \) is the key problem of Higgs physics. A number of papers, devoted this task, study different non-minimal models with various benchmark parameters of new particles [16]. They find usually that many ”natural” approaches in these problems turn out either non-realistic or very difficult (for example, demand extremely high luminosity integral).

In this report we show that such conclusion does not related to an unfortunate choice variant of non-minimal model or its parameters but is general feature for all models of the first group with almost arbitrary parameters, allowing modern data – SM-like situation (sect. [2]). To show that, we use sets of Sum rules (sect. [3]) which form either extensions of known in 2HDM Sum Rules for more wide class of models with arbitrary set of parameters or new Sum Rules founded recently. In the SM-like situation Sum Rules allow describe many properties of new Higgs bosons (sect. [4]). First, we discuss in sect. [4,4] possible values of couplings of new neutrals. Next, we discuss in sect. [4.2] properties and production of new neutral Higgses.

In the sect. [4] we consider important particular case – most general 2HDM (with \( h_1 \) – discovered Higgs boson and two new neutrals \( h_{2,3} \)) in SM-like situation. First, we note that the coupling \( Zh_2h_3 \) is not small, while couplings \( Zh_1h_2 \) and \( Zh_1h_3 \) are small. Second, we consider triple Higgs vertex \( h_1h_2h_3 \) and show that it is hardly probable to see for violation of SM prediction in its observation, except exotic cases.

We summarize results in sect. [6].
1.3 Relative couplings

In the discussion we use relative couplings, defined as ratios of the couplings of each neutral Higgs boson $h^a$ for given model or in experiment with the fundamental particle $P$ to the corresponding SM couplings:

$$
\chi^a_P = \frac{g^a_P}{g^a_{SM}}, \quad \chi^{exp}_P = \frac{g^{exp}_P}{g^{exp}_{SM}} \quad (P = V(W, Z), q = (t, b, ...), \ell = (\tau, ...)). \quad (1)
$$

The neutrals $h_a$ generally have no definite parity. In the CP conserving case some of $h_a$ are scalars, another are pseudoscalars. In the CP conserving case we have

$$
\prod_a \chi^a_V = 0, \quad \prod_a |\chi^a_f| = \prod_a |\chi^a_f| \text{ for each fermion } f. \quad (2)
$$

In particular, for the CP-conserving case of $2HDM$ (where $h^3 = A$) $\chi^3_V = 0$.

The models with charged Higgs boson contain important vertex $H^\pm W^\mp h^a$. For models with single charged Higgs we consider relative couplings (generally complex)

$$
\chi^a_{H^\pm W^\mp} = \frac{g(H^\pm W^\mp h^a)}{M_W/v}. \quad (3)
$$

Below we omit attribute ”relative” almost everywhere.

2 Modern status. SM-like situation

The intensive study of recently discovered Higgs boson makes very probable that the SM-like situation is realized in Nature:

1) Single observed Higgs boson $h$ has mass $M_h \approx 125$ GeV, we denote it as $h_1$.
2) Its couplings to fundamental particles $P$ (gauge bosons $V$ and fermions $f$) are close to the SM expectations within experimental accuracy (see e.g. [7]):

$$
\varepsilon_P = |1 - |\chi^{1}_P|^2| \ll 1 \quad (P = V(W, Z), \quad f = (t, b, \tau, ...)). \quad (4)
$$

(The measuring of loop obliged decays $h \rightarrow gg$ and $h \rightarrow \gamma\gamma$ gives limitation also for relative sign of different couplings.)

The realization of SM-like situation don’t shoot the doors for realization of non-minimal Higgs models. No doubts that the SM-like situation in the non-minimal model can occur if additional Higgs bosons are very heavy and are coupled only weakly with usual matter (decoupling limit) (see e.g. [17]). It was found 15 years ago that, at finite precision of future experiments at LHC and at the planned high energy $e^+e^-$ collider even the simplest non-minimal model 2HDM with the special choice of the Yukawa interaction 2HDM-II (as in MSSM) allows several possible windows significantly differing from the decoupling limit and implementing the SM-like situation [18]. Naturally, such windows exist in other models as well.

The successful experiments reduce $\varepsilon_P$ and, consequently, the region of the allowed parameters of each non-minimal model.

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1 In refs. [18] our problem was to understand potential of Photon Collider (PLC) in distinguishing different Higgs models in the case when LHC and new $e^+e^-$ colliders will show no visible deviations from SM – SM-like scenario.
3 Sum Rules

The freedom in parameters of discussed models is limited by Sum Rules (SR) for couplings, that will be key point of main part of our discussion. Some of these SR’s were known for some particular models, here we extend them for wider class of models, other SR’s are new.

- **The couplings** $\chi^a_V$ of the EW gauge bosons $V = W/Z$ to all neutral Higgs scalars $h^a$ are real due to Hermiticity of Lagrangian. In models like nHDM (with $p_i = 0$) $\chi^a_z = \chi^a_V = \chi^a_t$.

  These $\chi^a_V$ coincide with elements of rotation matrix, describing transition from neutral components of Higgs fields $\phi_j$ to the physical neutral Higgs bosons $h^a$ in the Higgs basis. The unitarity of this transformation matrix results in the Sum Rules, which are valid for all discussed models both with and without CP violation:

  \[ \sum_a (\chi^a_W)^2 = 1, \sum_a (\chi^a_Z)^2 = 1 : \text{all Higgs models with doublets and singlets} \]  

  This argumentation extend directly that given in [1] for most general 2HDM. (For particular case of 2HDM such SR’s were obtained earlier in [19, 20]). One can speak that these SR’s mean that masses of gauge bosons are given by single v.e.v. $v$.

- **The couplings** $\chi^a_f$ of each definite fermion $f$ (quark or lepton) to all neutral Higgs scalars $h^a$ are generally complex. (For CP-conserving case of $2HDM$ $\chi^h_f$, $\chi^H_f$ are real and $\chi^A_f$ is imaginary).

  These Sum Rules naturally occur in $nHDM + p_2HSn_2M + p_1HSn_1M$ at arbitrary $n$ and $p_i$ in the case when weak isosinglets don’t couple to fermions. To prove this we start with Sum Rule proven for 2HDM with definite Yukawa interaction (Model I or Model II) in [19, 18, 1]. Let us write general Yukawa term for interaction of down-type fermion $f$ to neutral components $\phi_{j,0}$ of $\phi_j$ as $\Delta L = \sum_j g_{jf}^\dagger \tilde{\psi}_j \phi_{j,0} \psi_f$. Simple reparameterization $\phi_{j,0} = N \sum_k g_{jk} \phi_{j,k}$ (where $N$ – the normalization factor) transforms this term to the form $\Delta L = g_{j,0}^\dagger \tilde{\psi}_j \phi_{j,0} \psi_f$, which coincides with that for Model I (or II) in 2HDM – $f$-selective reparameterization basis [4]. For the latter case Sum Rule has been proved in [19, 20] in the form

  \[ \sum_a (\chi^a_f)^2 = 1 : \text{models without Yukawa interaction with isoscalars } S_i \]  

  Our argumentation shows that these SR’s are valid in much more general class of models than those discussed in [19, 20].

- **The non-diagonal couplings to EW gauge bosons $H^\pm W^\mp h^a$** are generally complex. The Higgs potential is naturally invariant under rephasing transformation $\phi_i \to \phi_i e^{i\alpha_i}$, compensated by the corresponding phase rotation of some coefficients. This rephasing freedom results in the phase freedom of couplings $\chi^a_{H^\pm W^\mp} \to \chi^a_{H^\pm W^\mp} e^{i\beta}$, keeping phase difference between $\chi^a_{H^\pm W^\mp}$ for different $a$. The SR’s for these quantities were obtained first in the most general 2HDM [4]. The method of derivation of these SR’s allow to extend result for all models with single charged Higgs boson and arbitrary number of Higgs singlets, $2HDM + p_2(HS_2M) + p_1(HS_1M)$,

  \[ |\chi^a_V|^2 + |\chi^a_{H^\pm W^\mp}|^2 = 1 : \text{2HDM + isoscalars with arbitrary Yukawa sector} \]  

\footnote{Similar argumentation is valid for up-type fermion with the change $\phi_{j,0} \to \phi_{j,0}^*$.}

\footnote{Another proof of this SR is similar to that developed in [1] for 2HDM. Couplings $\chi^a_f$ can be expressed via couplings $\chi^a_V$, $\chi^a_{H^\pm W^\mp}$ and parameters of transformation of Higgs basis to the $f$-selective basis. Orthogonality of this transformation results in SR [4].}
Some relations for different Yukawa sectors in 2HDM. The Yukawa couplings for different fermions are generally independent on each other. In some widely spread models of Yukawa sector these couplings are correlated.

In the model I we have
\[ \chi^a_u = \chi^d_a = \chi^\ell_a. \quad (8a) \]

For this model the \( f \)-preferable bases coincide for all fermions, \( \beta_t = \beta_d \), \( \xi_t = \xi_d \).

In the model II the pattern relations among Yukawa couplings for different fermions take place \[ (\chi^u_a + \chi^d_a) \chi^V_a = 1 - \chi^u_a \chi^d_a. \quad (8b) \]

For this model the \( u \)-preferable bases coincide for all up-quarks, and the \( d \)-preferable basis coincide for all down quarks, \( \beta_b = \pi/2 - \beta_t \equiv \beta, \xi_b = \xi, \xi_t = 0. \)

4 Consequences from SR’s in the SM-like situation

4.1 Couplings

1. In view of (5), couplings of neutrals \( h_a \) to gauge bosons \( \chi^H_a \) are small,
\[ |\chi^H_a|^2 \ll \varepsilon_V \ll 1. \quad (9) \]

2. In view of (7), (5), the absolute values of non-diagonal couplings to EW gauge bosons \( \chi^{W \pm H\mp}_a \) are close to their maximal value while similar coupling for the observed Higgs \( \chi^{H}_a \) is small:
\[ a) \quad |\chi^{W \pm H\mp}_a|^2 \approx 1; \quad b) \quad |\chi^{W \pm H\mp}_a|^2 \sim \varepsilon_V \ll 1. \quad (10) \]

(Note in particular, that the calculations of \( H^- \rightarrow W^- h_1 \) decay at LHC in [21] are made without taking part limitation (10)). It makes numerical estimates there to be non-realistic.)

3. The SR’s for couplings to given fermions \( f \) (6) can be written as \( \sum_{a \geq 2} (\chi^a_f)^2 \approx 0 \). We will write here about the most important case \( f = t \). Since couplings \( \chi^a_t \) are generally complex, this SR can be saturated by different ways, we discuss simplest limiting cases
\[ a) \quad |\chi^a_t| < 1 \quad \text{for all} \quad h_a, \quad \text{\quad (11a)} \]
\[ b) \quad (bI) \quad \chi^a_t \approx \chi^b_0 > 1; \quad (bII) \quad \chi^a_t \approx 1(\chi^b_0) > 1, \quad \text{\quad (11b)} \]
\[ c) \quad |\chi^a_{t2}| \approx |\chi^a_{t1}| > 1, \quad \chi^a_{t2} \approx i\chi^a_{t1} \quad \text{for some} \quad h_{a_1} \text{ \ and \ } h_{a_2}. \quad \text{\quad (11c)} \]

The case (11a) provides no new interesting opportunities in the discovery of \( h_a \).

Interesting opportunities provide the case when some couplings \( \chi^a_t \) are large in their absolute value with various organizations of Yukawa sector.

The eq. (11b) describes two limiting opportunity in the organization of Yukawa sector. The case (bII) corresponds Yukawa sector similar to Model I in 2HDM. The case (bII) corresponds Yukawa sector similar to Model II in 2HDM – with pattern relation like (8).

One particular opportunity in saturation of (6) is described by eq. (11c). In this case the absolute values of couplings \( \chi^a_{i1} \) are large for two neutrals \( h_a \) only. For one of this neutrals \( Re(\chi^a_{i1}) > Im(\chi^a_{i1}) \), for another neutral imaginary part dominates. (This variant can be realized in CP conserved 2HDM \( h_{a1} = H, h_{a2} = A \)).

The opportunities (11b) and (11c) can coexist or not coexist.

Note that the standard perturbative estimates, used here and in other papers, become invalid if this coupling is enormously large, at \( |\chi^a_{i1}| > 2\pi \) we come into the region of the strong interaction in Higgs sector, mediated by \( t \)-quarks.
4.2 Properties and production of neutral Higgses $h_a$

The common feature of all considered models, essential in the data analysis, is the fact that masses $M_a$ at $a > 1$, $M_b^3$ can be treated as independent input free parameters, supplemented by some couplings of neutrals to vector bosons. (The limitations for the masses can follow only from some additional hypotheses, implemented in model.) Some triple and quartic couplings are also independent parameters of theory (see 2HDM where this complete set contains masses of Higgs bosons $M_a$, $M_t$, two of three couplings $\chi^a_V$, and couplings $g(H^+H^-h_a)$, $g(H^+H^-H^-H^-)$ – see [4] in the simplest case of 2HDM). The Yukawa couplings form additional set of input parameters.

For definiteness, we assume\(^5\) $M_a > 150$ GeV and $|\chi^a_V| < 40$ for $f \neq t$. To make many statements more transparent, we will compare discussed quantities with those for would be SM Higgs boson having the same mass $H^{(wb)}_\text{SM}(M)$. We will speak that some quantity is small if it is smaller than the same quantity for the $H^{(wb)}_\text{SM}(M_a)$.

Some conclusions below about total width and observability can be changed by effects of moderately strong interaction in Higgs sector with large triple Higgs vertices like $h_a h_t h_t$, etc. They should be considered separately. Here we neglect this opportunity.

- **Effects from coupling $h_a$ to gauge bosons.** In the Standard Model, the main contribution to the width of the Higgs boson with masses larger than 150 GeV comes from the decays $h \to W^+W^-$ and $h \to ZZ$. These decays and processes like $W$ fusion provide the main signal for detection of the CP even Higgs boson. Production of the CP even Higgs boson through a gauge vertex was until recently assumed to ensure the best signal/background ratio and the least inaccuracy in the measurement of its parameters both at the LHC and at the ILC. According to eq. (2),

(i) $\Gamma_a \ll \Gamma^{(wb)}_\text{SM}(M_a)$.

(ii) The decay $h_a \to W^+W^- / ZZ$ is suppressed, observation of $h_a$ via this decay is hardly improbably.

(iii) The search for new neutral Higgs bosons in the $W$ fusion at the LHC, $e^+e^- \to Zh_a$ and $e^+e^- \to \nu\overline{\nu}h_a$ at the ILC, and $e\gamma \to \nu W^- h_a$ at the PLC (photon collider) cannot be successful \(^2\), their cross sections are roughly one order of value lower then those calculated for $H^{(wb)}_\text{SM}(M_a)$.

- **The interaction $h_a \bar{t}t$** can in principle compensate decreasing of partial widths and production cross sections, obliged by small coupling to gauge bosons.

◊ **In the case** \(^{11a}\) such compensation is absent, $h_a$ is very narrow resonance with small partial widths, small cross section of gluon fusion, observation of this particle looks very difficult task. Besides, the associative production $gg \to t\overline{t}h_a$ is suppressed as compare to SM case.

◊ **The cases** \(^{11b}\).

The two gluon width of Higgs boson $\Gamma_a^{gg}$ is saturated by contribution of $t$-quark loop. Therefore, this width is enhanced as compare would be SM case by a factor $|\chi^a_V|^2$, just as cross section of gluon fusion. Next, at $M_a > 350$ GeV the decay $h_a \to \bar{t}t$ become dominant. Besides, the associative production $gg \to t\overline{t}h_a$ is enlarged as compare to SM case.

At $M_a < 350$ GeV the cross section of $h_a$ production via gluon fusion

$$\sigma(gg \to h_a) = |\chi^a_V|^2 \sigma^{(wb)}_\text{SM}(gg \to h_a).$$

Let us consider process $gg \to h_a \to \bar{b}b$.

For Yukawa sector similar to Model I (variant (bl)) we have $\chi^a_b \approx \chi^a_h$. At $|\chi^a_V| \approx 6$ it gives total width $\Gamma(h_a) \sim 300 \div 400$ MeV for $M_a \approx 300$ GeV (contributions of $\bar{b}b$ and $gg$

\(^5\)The opportunity that some neutrals are lighter than 125 GeV cannot be excluded, this opportunity is constrained strong by modern data.
The cross section \( \sigma(gg \rightarrow h_a \rightarrow \bar{b}b) \approx |\chi^a_t|^4 \sigma_{SM}^{(wb)}(gg \rightarrow h_a \rightarrow \bar{b}b) \). In this case one can hope to see \( h_a \) as the high narrow peak in the production of \( \bar{b}b \) pairs at LHC.

If both neutrals, mentioned in (11c) are not very heavy, \( M_{a1}, M_{a2} < 350 \text{ GeV} \), one can hope to observe 2 relatively narrow peaks in \( \bar{b}b \) production, typically well separated from each other.

For Yukawa sector similar to Model II (variant (bII)) the eq. (8b) results in \( \chi^a_t \approx 1/\chi^a_t \). In this case the \( gg \) partial width can be even larger than the \( \bar{b}b \) one. The cross section \( \sigma(gg \rightarrow h_a \rightarrow \bar{b}b) \approx \sigma_{SM}^{(wb)}(gg \rightarrow h_a \rightarrow \bar{b}b) \), and it is difficult to hope for observation of signal of this process in comparison with background signal \( gg \rightarrow \bar{b}b \).

At \( M_a > 350 \text{ GeV} \) contribution of \( h_a \rightarrow t\bar{t} \) decay is enlarged so that one can hope to see \( h_a \) in \( t\bar{t} \) mode.

If both neutrals, mentioned in (11c) are not heavy, \( M_{a1} > 350 \text{ GeV}, M_{a2} > 350 \text{ GeV} \), one can hope to observe either two separated enhancements in \( t\bar{t} \) production or even one enhancement (at \( |M_{a1} - M_{a2}| \leq \Gamma_{a1} + \Gamma_{a2} \)).

- **Two photon width.** The widths \( h_a \rightarrow \gamma \gamma, h_a \rightarrow Z\gamma \) are described by loop integrals with \( W \)-loop, which contribution is controlled by coupling \( \chi^a_t \), \( t \)-loop, which contribution is controlled by coupling \( \chi^a_t \), and \( H^+ \) loop, which contribution is controlled by coupling \( g(H^+H^-h_a) \). The value of the latter contribution into decay amplitude of Higgs boson \( h_a \) is given by factor

\[
\nu_a = \frac{v g(H^+H^-h_a)}{M^2_a} \tag{12}
\]

The knowledge of all masses and couplings \( \chi^a_t \) don’t limit values of coupling \( g(H^+H^-h_a) \) even in 2HDM [1].

As it was found for the simple SM-like situation \( (\chi^a_t \approx 1) \) for 2HDM in [18], at \( \nu_1 \approx 1 \) contribution of this loop into \( \Gamma(h_1 \rightarrow \gamma \gamma) \) reduces mentioned width for about 10\% , that is within modern inaccuracy of data. One can also realized SM-like situation with \( \chi^a_t \approx -1 \). In this case two photon width can be enhanced by factor \( 2 \div 2.5 \) vs SM value [18], what contradicts modern data. However variation of \( \nu_1 \) can reduce this enhancement (see modern studies in [22]).

- One should note that many results obtained in recent studies can be treated as examples of mentioned general picture for separate set of parameters and specific models [16]. Our discussion shows that almost negative results of many such studies have common origin.

### 4.3 Using of charged Higgses, etc.

Here we limit ourself by the group of models with single charged Higgs boson \( H^\pm \) (models 2HDM + \( p_2(HS_2M) + p_1(HS_1M) \)). The discovery of these charged Higgs bosons and the study of their decays should be discussed separately. Below we assume that the mass \( M^\pm \) is not extremely large and these particles have good signature.

In the SM-like situation [1] Sum Rules (7) shows that the coupling \( H^\pm W^\mp h_1 \) is weak while the couplings \( H^\pm W^\mp h_a \) with \( a \geq 2 \) are close to possible maximal value (10). It gives following results:

- The partial width \( \Gamma(H^+ \rightarrow W^+h_1) \) is small, while at \( M^\pm > M_W + M_a \) the partial width \( \Gamma(H^+ \rightarrow W^+h^a) \) is relatively large \( (a \geq 2) \).
- The production of Higgs boson \( h_1 \) in association with \( H^\pm W^\mp \) or in decay \( H^+ \rightarrow W^+h_1 \) is hardly observable.

\[ \text{Even small coupling } h_a \text{ to } W/Z \text{ increases its total width (at } |\chi^a_t|^2 \approx 0.1 \text{ - up to } 1 \div 1.4 \text{ GeV).} \]

\[ \text{Note that some authors obtain too optimistic results due to insufficient consideration of limitations given by SM-like situation for parameters of model.} \]
The search for Higgs bosons $h_a$ can be successful in the following channels:
- $q_1 q_2 \to H^+ h_a$, $q \bar{q} \to W^+ H^+ h_a$ at LHC,
- $\gamma \gamma \to \nu H^- h_a$, $e^+ e^- \to H^+ W^+ h_a$ at $e^+ e^-$ collider,
- $\gamma \gamma \to H^\pm W^\mp h_a$ at PLC.

Certainly, $e^+ e^-$ collider and PLC have advantages due to much better background conditions.

## 5 Some effects in the general 2HDM

Most general 2HDM is described by potential with 14 parameters. It contains 2 fundamental fields $\phi_1$, $\phi_2$. The unitary transformation from these fields to the fields $\phi'_1 = a_{11} \phi_1 + a_{12} \phi_2$, $\phi'_2 = a_{21} \phi_1 + a_{22} \phi_2$ with corresponding transformation of parameters describe the same physical reality. This transformation is described by 3 parameters. Therefore, total number of physical parameters of model is 11 (see, e.g. [1]).

These 11 parameters can be chosen as the observable independent free parameters: $v = 246$ GeV – v.e.v. of Higgs field, masses of 3 neutral Higgs bosons $h_a$ with $a = 1, 2, 3$ $M_a$ and mass of charged Higgs bosons $M^\pm$, two of three couplings $\chi^a V$, couplings $g(H^+ H^- h_a)$ and $g(H^+ H^- H^+ H^-)$ (limitation of freedom – couplings $\chi^a V$ are limited by relation (5)) [1].

If subsequent observation will support SM-like situation for $V$ and $t$ with reasonable accuracy, even modern data on Higgs two photon width will give also limitation for coupling $g(H^+ H^- h_1)$.

### 5.1 Couplings $Z h_a h_b$

As it was shown in [1], we have in 2HDM

$$\chi_{Z}^{(ab)} = g(Z h_a h_b) = - \varepsilon_{abc} \chi_c^V .$$  \hspace{1cm} (13)

In the SM-like situation [1] it means that among couplings $Z h_a h_b$ only the coupling $Z h_2 h_3$ is not small. Therefore the search for Higgs bosons $h_a$, $h_b$ can be successful in the processes $q \bar{q} \to h_2 h_3$ at LHC; $e^+ e^- \to h_2 h_3$ at $e^+ e^-$ collider.

The cross section of similar processes with production $h_1 h_2$ or $h_1 h_3$ are negligibly small.

### 5.2 Triple Higgs production [23]

Many authors hope to see for triple Higgs vertex $g(h_1 h_1 h_1)$, having in mind two goals.

◊ These observations should verify SM in one more point.

◊ If other Higgs bosons are very heavy, observation of relatively light $h_1 h_1$ pair could give us information about new particles and interaction before discovery of mentioned heavy Higgs bosons.

The accuracy of corresponding experiments cannot be high enough since in each case corresponding experiments deal with interference of two production channels with identical initial state. For example at LHC, main part of amplitude of gluon fusion $gg \to h_1 h_1$ is a sum $A(x_1)^2 + B \chi_1^1 \chi_{111}$ (see below for notation), the term with $A$ corresponds independent production of Higgses from $t$-loop and the term with $B$ – production of Higgses via $h_1 h_1 h_1$ vertex. In addition, in 2HDM it is difficult to expect large deviations of $g(h_1 h_1 h_1)$

\footnote{For example, for 100 TeV hadron collider with total luminosity 3/ab one can hope to reach accuracy in the extraction of this vertex 40\% [20].}
from its SM value. For the CP conserved case and with moderate values of parameters such conclusion was obtained in \[24\].

According to \[4\], in the most general 2HDM mentioned triple coupling is expressed via measurable quantities (factor $M_1^2/v$ is SM result, quantity $\chi_{111}^{hh}$ is relative coupling)

$$g(h_1 h_1 h_1) = \frac{M_1^2}{v} \chi_{111}^{hh}$$

$$\chi_{111}^{hh} = \chi_1^V \left\{ 1 + (1 - (\chi_1^V)^2) \left( (1 + 2(\chi_1^V)^2) + \sum_{b=1}^2 2M_b^2(\chi_b^V)^2 + \frac{(v^2 \Lambda_3 - 2M_2^2)}{M_1^2} + \frac{v^2 Re(\Lambda_7^* \chi_1^W H^\pm)}{M_1^2 \chi_1^V} \right) \right\} ;$$

$$v \Lambda_3 = \sum_a \chi_a^V g(H^+ H^- h_a), \quad v \Lambda_7 = \sum_a \chi_a^V g(H^+ H^- h_a).$$

Here real $\Lambda_3$ and complex $\Lambda_7$ are parameters of Higgs potential in Higgs basis, expressed via measurable quantities.

In SM-like situation \[4\] with $\varepsilon_V = |1 - \chi_1^V| \ll 1$ it is easy to estimate

$$\chi_{111}^{hh} \approx \left\{ 1 - \varepsilon_V/2 + \varepsilon_V^2 \left[ \frac{v g(H^+ H^- h_1) - M_2^2}{M_1^2} + 3 \right] + O(\varepsilon_V^3) \right\}$$

$$= \left\{ 1 - \varepsilon_V/2 + \varepsilon_V^2 \left[ \frac{M_2^2}{M_1^2} (\nu_1 - 1) + 3 \right] + O(\varepsilon_V^3) \right\} .$$

(15)

In the second line result is rewritten via quantity $\nu_1 \[12\].

One can see that at moderate values of parameters $\chi_{111}^{hh}$ is close to 1, and it is difficult to expect sizable effect.

For $\varepsilon_V \approx 0.1$ the sizable deviation from SM value can appear at $M_1 > 1$ TeV and high enough value of parameter $|\nu_1 - 1| \sim 1$ or larger.

In this respect one should note that the radiative corrections to discussed triple Higgs vertex can be not small \[25\]. However, the same corrections shift value of Higgs mass $M_1$ so that at $\chi_1^V = 1$ (strong SM-like case) the corrected value of $g(h_1 h_1 h_1)$, expressed via corrected value of $M_1$ don’t differ practically from that in SM \[27\].

- Special opportunity appears in the SM situation at $M_2 > 250$ GeV if $|\chi_1^2| > 1$. In this case Higgs boson $h_2$ is relatively narrow and the cross section of gluon fusion $gg \to h_2$ can be larger than that for would be SM Higgs boson with mass $M_2$. In this case process $gg \to h_2 \to h_1 h_1$ can be seen as resonant production of $h_1 h_1$ pair. It allow in principle to discover $h_2$ at LHC. Similar opportunity is absent at $e^+ e^-$ colliders.

### 6 Summary

We assume here that the SM-like situation is realized.

In this case for wide class of Higgs models many discussed now ways for observation of additional neutral scalars (new Higgses) are difficult (or impossible).

The production of $h_a$ together with charged Higgses look most perspective approach.

One can consider opportunity to find scalars $h_a$ with mass $M_a < 350$ GeV as a relatively narrow peaks in $b\bar{b}$ and (or) $h_1 h_1$ channels.

Results of $h_1 h_1$ production may differ significantly from the predictions of the SM only at very large values of $M_a$ and (or) vertex $g(H^+ H^- h_1)$.

We hope that similar picture is realized in many other models.
Acknowledgment

I am thankful my co-authors in some of presented studies K. Kanishev and M. Krawczyk. The discussions with F. Boudjema, I. Ivanov, P. Osland, M. Vysotsky were useful. This work was supported in part by grants RFBR 15-02-05868, NSh-3802.2012.2, Program of Dept. of Phys. Sc. RAS and SB RAS "Studies of Higgs boson and exotic particles at LHC” and Polish Ministry of Science and Higher Education Grant N202 230337.

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