Progenitor Mass Distribution for Core-collapse Supernova Remnants in M31 and M33

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Abstract

Using the star formation histories (SFHs) near 94 supernova remnants (SNRs), we infer the progenitor mass distribution for core-collapse supernovae. We use Bayesian inference and model each SFH with multiple bursts of star formation (SF), one of which is assumed to be associated with the SNR. Assuming single-star evolution, the minimum mass of CCSNe is $M_\text{min} = 7.3^{+0.02}_{-0.16} M_\odot$, the slope of the progenitor mass distribution is $\alpha = -2.96^{+0.45}_{-0.25}$, and the maximum mass is greater than $M_\text{max} > 59 M_\odot$ with a 68% confidence. While these results are consistent with previous inferences, they also provide tighter constraints. The progenitor distribution is somewhat steeper than a Salpeter initial mass function ($\alpha = -2.35$). This suggests that either SNR catalogs are biased against the youngest SF regions, or the most massive stars do not explode as easily as lower mass stars. If SNR catalogs are biased, it will most likely affect the slope but not the minimum mass. The uncertainties are dominated by three primary sources of uncertainty, the SFH resolution, the number of SF bursts, and the uncertainty on SF rate in each age bin.

We address the first two of these uncertainties, with an emphasis on multiple bursts. The third will be addressed in future work.

Keywords: galaxies: individual (M31, M33) – ISM: supernova remnants – methods: statistical – stars: evolution – supernovae: general

1. Introduction

One fundamental prediction of stellar evolution theory is that the zero-age main-sequence mass ($M_{\text{ZAMS}}$) of a star determines its fate (Nomoto 1987; Woosley et al. 2002; Heger et al. 2003). In particular, theory predicts that single stars above $\sim 8 M_\odot$ eventually collapse (Woosley et al. 2002), but it is not clear if every core collapse leads to explosion. Recent investigations suggest that lower mass stars may explode more easily than higher mass stars (Radice et al. 2017), with the latter being more likely to collapse directly into a black hole (Ugliano et al. 2012; Bruenn et al. 2013; Burrows et al. 2016). However, the ease of explosion may not be monotonic with mass (Woosley & Heger 2015; Sukhbold et al. 2016). For example, Sukhbold et al. (2016) studied the explodability of progenitor stars from 9 to 120 $M_\odot$. They found that there is no clear threshold of unsuccessful SN explosion, but that stars less massive than $\sim 15 M_\odot$ tend to explode. On the other hand, the region above 15 $M_\odot$ shows both successful explosions and failed SNe. While the more massive stars are more likely to fail, there is not a monotonic trend. Instead, there appear to be islands of SN production. These “islands” of SN production complicate the mapping between progenitor mass and explosion outcome. Moreover, the final core structure of progenitors may be chaotic, further breaking the simple mapping of progenitor mass to outcome mapping (Sukhbold et al. 2018). To constrain these basic predictions of stellar evolution, it is important to observationally constrain the progenitor mass distribution for SNe.

Broadly, there are two methods for constraining the progenitor masses of CCSNe. One is to analyze images of the progenitor star taken before the SN (White & Malin 1987; Smartt et al. 2002a, 2002b, 2004, 2009; Van Dyk et al. 2003a, 2003b, 2011, 2012a, 2012b; Li et al. 2005, 2006, 2007; Maund et al. 2005, 2011, 2014a, 2014b; Hendry et al. 2006; Gal-Yam et al. 2007; Gal-Yam & Leonard 2009; Smartt 2009; Smith et al. 2011; Fraser et al. 2012, 2014). This technique has the advantage that it directly probes the progenitor star, allowing the identification of the type of star that exploded, and to infer its $M_{\text{ZAMS}}$, one compares the color and magnitude of the precursor to stellar evolution tracks.

Moreover, even when precursor images exist inferring the luminosity and mass of the progenitor requires interpreting a star magnitude and color during the last, most uncertain stages of stellar evolution, for which the star may not be in hydrostatic equilibrium (Quataert & Shiode 2012; Fuller 2017). In addition, these stars’ dusty winds may be obscuring their luminosity and consequently lowering mass estimates (Van Dyk et al. 2012a; Beasor & Davies 2016, 2017).

While the direct-imaging method constrains the type of star that exploded, there are potential limitations to the technique. For one, serendipitous precursor images are rare. As a result, only about 30 SNe have directly imaged progenitors and 38 upper limits (Van Dyk 2017). One of the largest analyses of precursor images (Smartt 2015) finds that all SN II-P detections were red super giants (RSGs), as expected. They also infer that the minimum $M_{\text{ZAMS}}$ for explosion is $7.3^{+2.9}_{-1.4} M_\odot$, observed for SN2003gd (Smartt et al. 2004, 2009). Surprisingly, even though RSGs are observed with masses up to 25–30 $M_\odot$, there are no SN II-P progenitors more massive than $\sim 17 M_\odot$ (Smartt 2015). Recently, however, Davies & Beasor (2018) applied different bolometric corrections that are more appropriate for late-stage RSGs and found a higher upper mass limit of <7 $M_\odot$ (95% confidence). Therefore, another technique with different systematics and limitations is needed to increase the number of progenitors when pre-images are not available and to validate the direct-imaging method.
The second alternative technique is to age-date the surrounding stellar populations in the vicinity of the SN explosion, and from this age, infer a progenitor mass (Walborn et al. 1993; Barth et al. 1996; Van Dyk et al. 1999; Panagia et al. 2000; Maíz-Apellániz et al. 2004; Wang et al. 2005; Crockett et al. 2008; Gogarten et al. 2009; Vinkó et al. 2009; Murphy et al. 2011, 2018; Williams et al. 2014b, 2018; Maund 2017, 2018). The age-dating technique mostly depends on well-understood properties of main-sequence and early post-main-sequence phases, and thus is relatively insensitive to details of late-stage stellar evolution.

Because this age-dating technique does not require rare precursor imaging, age-dating expands the number of progenitor estimates to many more CCSNe (Williams et al. 2014b, 2018; Maund 2017, 2018). The age-dating technique mostly depends on well-understood properties of main-sequence and early post-main-sequence phases, and thus is relatively insensitive to details of late-stage stellar evolution.

Because this age-dating technique does not require rare precursor imaging, age-dating expands the number of progenitor estimates to many more CCSNe (Williams et al. 2014b, 2018; Maund 2017, 2018) and to hundreds of supernova remnants (SNRs; Jennings et al. 2012, 2014). For example, Williams et al. (2018) used this technique to age-date 25 historic SNe. Maund (2017) used a similar age-dating technique to resolve stellar populations around 12 Type II-P SNe with identified progenitors, and Maund (2018) inferred the ages around the sites of 23 stripped-envelope SNe. Jennings et al. (2014) age-dated 115 SNRs, demonstrating a way to increase the number of progenitor masses by at least a factor of 10.

Jennings et al. (2012, 2014) preliminarily constrained the minimum mass \( M_{\text{min}} \), maximum mass \( M_{\text{max}} \), and power-law slope \( \alpha \) of the progenitor mass distribution, assuming SNRs were unbiased tracers of recent SNe. Jennings et al. (2012, 2014) were not able to infer all three parameters simultaneously, and instead employed several models to constrain the distribution using KS statistics. In a smaller initial sample, Jennings et al. (2012) found a \( M_{\text{min}} \) for CCSNe between 7.0 and 7.8 \( M_\odot \). Fixing the power-law slope to 2.35 (Salpeter IMF), Jennings et al. (2014) found a \( M_{\text{max}} \) of 35\( ^{+4.5}_{-3.0} \) for an expanded sample. If instead, they assumed no \( M_{\text{max}} \), they found a steeper power-law slope of 4.2\( ^{+0.3}_{-0.3} \). In either model, they found that either the most massive stars are not exploding at the same frequency as lower masses, or there is a bias against SNRs in the youngest regions.

Initial results from age-dating have been promising, but these preliminary analyses could be improved in two ways. First, Jennings et al. (2014) adopted one median age and uncertainty for each SNR, which is only appropriate if there is one well-defined peak for the SFH. In contrast, the data is often consistent with there being more than one burst of SF (see Figure 1). Second, Jennings et al. (2014) did not infer the \( M_{\text{min}}, M_{\text{max}}, \) and the power-law slope simultaneously. To appropriately infer these parameters, one needs to fit for all of them at the same time.

In this paper, we begin building a complete statistical inference framework that handles these previous limitations. Here, we use a Bayesian inference framework, to infer the

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Star formation history (SFH) within 50 pc of four SNRs in M31 and M33: Sometimes, (a) the SFH identifies one clear age (one burst of SF). However, often (b)–(d) there is more than one burst of SF in the last 80 Myr. Only one burst in each SNR is associated with the progenitor that exploded. The others are random unassociated bursts of SF. For some, the SFH has many young bright main-sequence stars and the SFH is quite certain (a), and for others there are very few bright main-sequence stars within 50 pc, and the SFH is quite uncertain (d).
parameters of the progenitor age distribution simultaneously, taking multiple bursts of SF into account. Instead of focusing on masses directly, we first infer the minimum age \((t_{\text{min}})\), maximum age \((t_{\text{max}})\), and slope of the age distribution \(\beta\). We then use the results of stellar evolution models (Marigo et al. 2017) to infer the progenitor mass distribution associated with this age distribution.

An outline of the paper is as follows. In Section 2, we present a Bayesian inference technique to infer the CCSN progenitor age distribution. This section also describes the assumptions and technique to transform this age distribution into a mass distribution. Section 3 presents the results. In Section 4, we discuss our results in the context of other progenitor analyses, theory, and major potential biases. We summarize our results in Section 5 and discuss future directions.

2. Methods

This section describes the methods for inferring the progenitor age and mass distribution for SNRs in M31 and M33. The primary inference is the age distribution rather than the mass distribution for several reasons. First, the fundamental result for each SNR is the age of the local stellar population. Second, to infer the progenitor mass distribution, one makes assumptions about the mapping from age to mass. The most basic mapping assumes single-star evolution. However, binary evolution can significantly affect this mapping. Therefore, we first infer the progenitor age distribution assuming single-star evolution to allow future investigations using binary evolution. Since it is a standard assumption, we then convert the progenitor age distribution into a mass distribution.

The methods are presented as follows. Section 2.1 briefly describes the selection criteria for the M31 and M33 SNR catalogs; we include brief discussions on how these selections may impose biases in the SNR catalog. Section 2.2 briefly describes the method for inferring the SFHs from Hubble Space Telescope (HST) photometry for each SNR. In Section 2.3, we describe the method for converting each SFH into an age probability density function (PDF) for each SNR. Then, in Section 2.5, we use hierarchical Bayesian inference to infer the progenitor age distribution for all SNRs, and convert this age distribution into an initial mass distribution for all SNRs. In Section 2.6, we simulate data to test the hierarchical model and to identify any biases.

2.1. SNR Catalogs

We analyze the age distribution of 94 SNRs for M31 (Lee & Lee 2014) and M33 (Long et al. 2010) that also have high quality overlapping HST imaging. 62 of the SNRs in our analysis are in M31 and the rest are in M33.

The M31 SNR candidates were selected based on their [S II]: Hα morphology, and the absence of blue stars (Lee & Lee 2014). Their primary motivation in omitting objects with blue stars was to remove H II regions from the catalog. However, this decision may bias against including SNRs associated with the youngest stellar populations. In contrast, the M33 SNR candidates were selected only based on their elevated [S II]: Hα, regardless of size or morphology (Long et al. 2010). One of the disadvantages with the M31 data is that there is very little follow-up spectroscopy, in contrast with M33. However, Lee & Lee (2014) had the benefit of earlier surveys, and they covered the entire disk of M31. It is possible that they may have included some very faint objects that may not be SNRs. Nevertheless, Lee & Lee’s (2014) catalog is the best extragalactic SNR survey available at the moment in comparison with other M31 catalogs. For example, the Magnier et al. (1995) M31 catalog did not include the [S II]:Hα criteria. This criteria is very important for identifying SNRs since elevated [S II]:Hα ratios are characteristic of shocked gas (Long et al. 2010).

The primary focus of this paper is to constrain the progenitors of CCSNe, not SN Type Ia. Even though these catalogs do not provide the type of SN that created each SNR, there are ways to reduce the SN Ia contamination in the catalogs. The SN Ia rate is about one-fourth of the overall SNe rate (Li et al. 2011), and for M33 the SN Ia fraction is expected to be less than for a galaxy like M31. CCSNe are associated with the explosion of massive stars (Smartt et al. 2009), and therefore younger stellar populations. While Type Ia SNe are associated with older stellar populations. Therefore, by eliminating any SNR with zero SF within the last 80 Myr, one can effectively remove likely Type Ia SNRs from the analysis. Jennings et al. (2014) took this approach and found that the fraction of SNRs with no SF in the last 80 Myr was consistent with the fraction of expected SN Ia in M31 and M33. If there is any SN Ia contamination in our catalogs, the fraction will be low and will not have a statistically significant impact on the distribution.

2.2. Star Formation Histories

The SFHs that we use originate from Jennings et al. (2014; for the SNRs in M33) and from Lewis et al. (2015; for the M31 SNRs). To infer the SFHs, these authors first calculate the photometry for all stars surrounding an SNR at a given distance. Jennings et al. (2014) used DOLPHOT\(^5\) to calculate the photometry of all stars within 50 pc of the SNRs in M33. Later, Lewis et al. (2015) calculated the SFH for M31 in 100 pc × 100 pc regions throughout the Panchromatic Hubble Archive Treasury (PHAT) footprint (Williams et al. 2014a). For each SNR in the Lee & Lee (2014) M31 catalog, we use the SFH from the corresponding 100 pc × 100 pc region calculated by Lewis et al. (2015). They selected stars with S/N > 4, sharpness squared < 0.15, and crowding < 1.3. These parameters ensure that the objects are high probability (high S/N), not extended sources (sharpness), and distinguishable from neighboring stars in crowded fields (crowding). For the details of deriving the SFHs, we refer the reader to those manuscripts.

The authors then derive the SFHs from the color–magnitude diagram (CMD) for each field using the program MATCH (Dolphin 2002, 2012, 2013). MATCH generates model CMDs that include the effects of observational errors, foreground and internal dust extinction, and distance, and then generates SFHs to maximize the likelihood of the observed CMD. The modeled magnitudes and colors are based upon stellar evolution tracks and isochrones from Marigo et al. (2017), which is an updated version of PARSEC Girardi et al. (2010). See the respective manuscripts for the extinction and distances used.

A major assumption of this technique is that the young population within ~50 pc is coeval with the progenitor. Stellar cluster studies suggest that over 90% of stars form in clusters containing more than 100 members with M > 50 M\(_\odot\) (Lada &

\(^5\) The original reference is Dolphin (2002), and updated versions are available online.
Lada 2003). Furthermore, these stars likely remain spatially correlated on physical scales up to \( \sim100 \) pc during 100 Myr. This spatial correlation continues even for low mass clusters that are not gravitationally bound (Bastian & Goodwin 2006). Therefore, by studying the stars surrounding these SNRs, we can determine the age of the star that exploded. Our previous studies (Gogarten et al. 2009; Murphy et al. 2011; Williams et al. 2014b) have confirmed that this assumption is reasonable.

In both analyses, the SFH is calculated using logarithmic spaced age bins (\( \Delta \log_{10}(t/\text{yr}) \)), and the youngest edge of the minimum age bin is \( \log_{10}(t/\text{yr}) = 6.6 \). This technique is similar to isochrone fitting, but it uses the entire CMD to infer the recent SFH. Therefore, unlike simple isochrone fitting, this technique fits for multiple ages. Figure 1, shows four examples of the SFH derived by MATCH.

Even though Jennings et al. (2014) reported SFHs for the SNRs in both M31 and M33, we only use their M33 SFHs. The SNR catalogs that they used for M31 lack homogeneous SNR identification: Magnier et al. (1995), Braun & Walterbos (1993), and Williams et al. (1995). They were mainly identified using [S II]-to-H\( \alpha \) ratios and there was no confirmation using more reliable techniques, such as radio or X-ray observations. Later, Lee & Lee (2014) published an M31 SNR catalog with many more observations to constrain SNR candidacy. This was the first full coverage catalog with a homogeneous survey using [S II]-to-H\( \alpha \). To identify SFHs for the SNRs in M31, we cross-correlate the SNR positions from Lee & Lee (2014) with the spatially resolved catalog of SFHs in the PHAT footprint (Lewis et al. 2015). The cross-correlations yields 65 SNRs with at least some SFH in the last 80 Myr. Table 1 gives the median ages and corresponding progenitor mass for each SNR in M31.

There are three primary sources of uncertainty in estimating the age for each SNR. For one, the resolution of the SFH limits the certainty for each age bin. The resolution of each age bin is \( \Delta \log_{10}(t/\text{Myr}) = 0.1 \) for M31 and \( \Delta \log_{10}(t/\text{Myr}) = 0.05 \) for M33. Second, there are often multiple bursts of SF (see Figure 1). These multiple bursts often dominate the uncertainty in estimating the age for each SNR. Third, the SF rate for each bin has an uncertainty.

The primary purpose of this paper is to develop a hierarchical Bayesian model to handle the multiple bursts. In doing so, we automatically consider the resolution of each age bin. The third source of uncertainty requires translating the SFH and its uncertainty into a probability distribution for the age. We leave this transformation for future work (Murphy et al. 2018). For now, we simply convert the best-fit SFH into a PDF for each SNR.

2.3. Age Probability Densities for Each SNR Progenitor

The first step is to convert the SFH into a progenitor age distribution function for each SNR, \( P_k(t) \), where the index \( k \) references each SNR. This PDF has units of 1/Myr. We assume that the probability density is proportional to the star formation rate (SFR), and the normalization \( M_* \) is the total amount of stars formed in the last \( T_{\text{max}} \) Myr:

\[
P_k(t) = \frac{\text{SFR}(t)}{M_*(T_{\text{max}})} \quad \text{for} \quad t < T_{\text{max}}.
\]

Single-star evolutionary models predict a \( M_{\text{min}} \) for core collapse around \( \sim8 \) \( M_\odot \) (Woosley et al. 2002), which corresponds to a \( t_{\text{max}} \) of \( \sim45 \) Myr (Marigo et al. 2017). To properly model and infer this \( t_{\text{max}} \), the PDF must include ages above this. Otherwise, the inference algorithm would just detect the artificial cutoff in the PDF. On the other hand, if \( T_{\text{max}} \) is too large, then one adds significant uncertainty in the form of SFH that is clearly too old. For this manuscript, we adopt \( T_{\text{max}} = 80 \) Myr.

The discreet version of the PDF for SFH is

\[
P_k(i) = \frac{\text{SFR}(i)}{M_*(T_{\text{max}})}.
\]

where each bin is indexed by \( i \) and the set of bins associated with each SFH is \{\( i \)\}. Given this discrete PDF, the probability of a star being associated with bin \( i \) is

\[
P_{\text{SF}}(i) = P_k(i) \cdot \Delta t_i.
\]

The best-fit SFH for SNe and SNRs often show distinct bursts of SF. In many cases, the SFH is simple, and there is one clear burst of SF (Figure 1(a)) for an SNR. However, this is not always the case. Sometimes there is more than one burst of SF (Figures 1(b)–(d)). A priori, it is unclear which burst is associated with the SNR, and this represents a significant source of uncertainty in our analysis. Therefore, to properly infer the underlying progenitor age distribution, one needs to also model the unassociated bursts of SF. In the following derivation, we consider the SF in each bin, \( i \), as independent bursts of SF.

2.4. Progenitor Age Distribution Model

The simplest model that one might consider is a power-law distribution with minimum and maximum age. This kind of model has the minimum number of parameters that one might expect for the distribution of SN progenitors. To ensure that such a model is a reasonable approximation, we stack the age distributions for all SNRs (see Figure 2). In doing so, one can note that the SF bursts seem to be drawn from two distributions. One is the power-law distribution associated with the SN progenitors. The second is a uniform distribution that probably represents random unassociated bursts.

Adding the PDFs for each SNR provides a reasonable approximation for the overall progenitor age distribution. This stacked distribution is a simple sum of the individual PDFs

\[
\text{Stacked}(i) = \sum_k P_k(i).
\]

Figure 2 shows the stacked age distribution for all 94 SNRs, and suggests a model for the age distribution. There are two clear components. There is an underlying uniform distribution, \( P_0(\hat{\beta}) \), at all ages (black dashed line), which we presume is associated with the random unassociated bursts of SF. We model this uniform distribution between 0 Myr and \( T_{\text{max}} = 80 \) Myr as \( P_0(\hat{\beta}) = 1/T_{\text{max}} \). In addition, there is a power-law distribution with minimum and maximum ages. These minimum and maximum ages appear to be around \( \sim8 \) Myr and \( \sim50 \) Myr. Therefore, we model the distribution of true burst ages, \( \hat{\beta} \), as a simple power-law distribution

\[
P_{\text{SF}}(\hat{\beta}) \propto \hat{\beta}^\beta \cdot \Pi(\hat{\beta}, t_{\text{min}}, t_{\text{max}}),
\]

where \( \Pi(\hat{\beta}, t_{\text{min}}, t_{\text{max}}) \) is the unit boxcar function and is equal to one between \( t_{\text{min}} \) and \( t_{\text{max}} \), and zero outside of this range. The model parameters are the \( t_{\text{min}}, t_{\text{max}} \), and slope of the distribution (\( \beta \)), which we collectively represent as an array (\( \theta \)). We
### Table 1

**Ages and Progenitor Initial Masses for SNRs in M31**

| SNR ID | R.A. (J2000.0) (Degree) | Decl. (J2000.0) (Degree) | Progenitor Age (Myr) | \( \sigma_+ \) (Myr) | \( \sigma_- \) (Myr) | \( M_{\text{SNR}} \) (M\(_{\odot}\)) | \( \sigma_+ \) (M\(_{\odot}\)) | \( \sigma_- \) (M\(_{\odot}\)) |
|--------|-------------------------|--------------------------|----------------------|----------------|----------------|-----------------|----------------|----------------|
| 8      | 01.0589018              | +40.620914               | 45.0                 | 0.1            | 21.9           | 7.7             | 3.1            | 0.0            |
| 10     | 01.1026564              | +40.721081               | 22.2                 | 3.3            | 3.6            | 11.0            | 1.2            | 0.8            |
| 13     | 01.1393518              | +40.726681               | 7.9                  | 8.8            | 0.0            | 23.3            | 0.1            | 10.3           |
| 14     | 01.1402712              | +40.546425               | 22.8                 | 1.8            | 3.1            | 10.8            | 0.9            | 0.5            |
| 16     | 01.1652269              | +40.580177               | 18.6                 | 1.4            | 1.6            | 12.2            | 0.7            | 0.5            |
| 32     | 01.3097161              | +41.247833               | 45.0                 | 0.1            | 7.8            | 7.7             | 0.7            | 0.0            |
| 34     | 01.3986397              | +41.115501               | 42.6                 | 0.0            | 9.4            | 7.9             | 0.9            | 0.0            |
| 37     | 01.5428724              | +40.86359                | 45.0                 | 0.0            | 4.3            | 7.7             | 0.4            | 0.0            |
| 42     | 01.6060371              | +40.873493               | 35.7                 | 1.0            | 2.9            | 8.6             | 0.3            | 0.1            |
| 45     | 01.6318827              | +41.101646               | 14.2                 | 4.5            | 0.4            | 14.4            | 0.3            | 2.3            |
| 47     | 01.6862478              | +40.909912               | 28.4                 | 0.7            | 2.3            | 9.6             | 0.5            | 0.1            |
| 51     | 01.7038184              | +40.996078               | 12.1                 | 17.0           | 0.2            | 16.2            | 0.2            | 6.7            |
| 54     | 01.7659559              | +41.604427               | 45.0                 | 0.7            | 3.2            | 7.7             | 0.3            | 0.1            |
| 59     | 01.7866192              | +41.05183                | 22.7                 | 1.5            | 2.0            | 10.9            | 0.6            | 0.4            |
| 60     | 01.7957678              | +41.627312               | 23.0                 | 0.5            | 3.0            | 10.8            | 0.9            | 0.1            |
| 62     | 01.8016556              | +40.909092               | 17.9                 | 0.7            | 1.1            | 12.4            | 0.5            | 0.3            |
| 63     | 01.8666568              | +41.022324               | 22.8                 | 1.2            | 2.4            | 10.8            | 0.7            | 0.3            |
| 51     | 01.7038184              | +40.996078               | 12.1                 | 17.0           | 0.2            | 16.2            | 0.2            | 6.7            |
| 54     | 01.7659559              | +41.604427               | 45.0                 | 0.7            | 3.2            | 7.7             | 0.3            | 0.1            |
| 59     | 01.7866192              | +41.05183                | 22.7                 | 1.5            | 2.0            | 10.9            | 0.6            | 0.4            |
| 60     | 01.7957678              | +41.627312               | 23.0                 | 0.5            | 3.0            | 10.8            | 0.9            | 0.1            |
| 62     | 01.8016556              | +40.909092               | 17.9                 | 0.7            | 1.1            | 12.4            | 0.5            | 0.3            |
| 63     | 01.8666568              | +41.022324               | 22.8                 | 1.2            | 2.4            | 10.8            | 0.7            | 0.3            |
| 51     | 01.7038184              | +40.996078               | 12.1                 | 17.0           | 0.2            | 16.2            | 0.2            | 6.7            |
| 54     | 01.7659559              | +41.604427               | 45.0                 | 0.7            | 3.2            | 7.7             | 0.3            | 0.1            |
| 59     | 01.7866192              | +41.05183                | 22.7                 | 1.5            | 2.0            | 10.9            | 0.6            | 0.4            |
| 60     | 01.7957678              | +41.627312               | 23.0                 | 0.5            | 3.0            | 10.8            | 0.9            | 0.1            |
| 62     | 01.8016556              | +40.909092               | 17.9                 | 0.7            | 1.1            | 12.4            | 0.5            | 0.3            |
| 63     | 01.8666568              | +41.022324               | 22.8                 | 1.2            | 2.4            | 10.8            | 0.7            | 0.3            |

**Note.** These SNRs are from the Lee & Lee (2014) catalog. The SFHs used to derive these ages and masses are from Lewis et al. (2015). Column (1) gives the SNR ID. Columns (2) and (3) give the position of the SNR. Column (4) is the median age from the SFH. The uncertainties in the SFH allow for a range of median ages. Columns (5) and (6) give the 68th percentiles on the median age. Columns (7), (8), and (9) give the corresponding mass and uncertainties.
propose that the true age of the SF burst is either drawn from $q(\hat{t} | \theta_p)$ or $P_{tu}(\hat{t})$, and the true age, $\hat{t}$, for each burst represents a latent, or nuisance, parameter in a hierarchical Bayesian model. Formally, one would carry these nuisance parameters throughout the derivation, making the derivation cumbersome until the very end. If one assumes that the bins are not correlated, then one may marginalize each bin to find the likelihood of drawing from $q(\hat{t} | \theta_p)$ or $P_{tu}(\hat{t})$. The likelihood of bin $i$ having a burst from the uniform distribution is

$$
P_{u}(i) = \int_{i}^{i+1} P_{u}(\hat{t}) \, d\hat{t} = \frac{\Delta t_i}{t_{\text{max}}},$$

(6)

The likelihood of bin $i$ having a burst from the power-law component is

$$
P_{p}(i|\theta) = \int_{i}^{i+1} P_{p}(\hat{t}|\theta) \, d\hat{t} = \frac{\tilde{t}_{i+1}^{\beta+1} - \tilde{t}_{i}^{\beta+1}}{t_{\text{max}} - t_{\text{min}}},$$

(7)

$\tilde{t}_{i+\frac{1}{2}}$ and $\tilde{t}_{i-\frac{1}{2}}$ are the right and left sides of the bin unless the bin straddles either $t_{\text{min}}$ or $t_{\text{max}}$, the parameters for $P_{p}(i|\theta)$. If the bin straddles $t_{\text{min}}$, then the left side is $t_{\text{min}}$. If the bin straddles $t_{\text{max}}$, then the right side is $t_{\text{max}}$.

The simple age distribution model, which is quite apparent in the combined stacked distribution (see Figure 2) is not so clearly apparent for the individual galaxies. Figure 3 shows the stacked distributions for the individual galaxies, M31 (left panel) and M33 (right panel). There are likely too few SNRs to clearly define the same model. Therefore, this manuscript will focus on inferring the progenitor age distribution parameters for both galaxies.

One could model the aggregate stacked distribution in the Bayesian inference (Figure 2). However, there is more constraint in modeling each individual SNR. If one models only the aggregate, then one throws away additional constraints from modeling the likelihood of each individual SNR. For example, if one SNR has a very young age, then the likelihood of this one SNR will constrain the $t_{\text{min}}$ to be quite small. Therefore, we choose to model the likelihood for each individual SNR.

2.5. Hierarchical Bayesian Inference

To self-consistently infer all three parameters, we use Bayes’ theorem to compute the joint probability $P(\theta|\text{Data})$ of our model parameters, $\theta$, given the observations. The posterior distribution for each parameter is then the integral of the joint distribution over all other model parameters.

Bayes’ theorem states that

$$
P(\theta|\text{Data}) = \frac{\mathcal{L}(\text{Data}|\theta) \, P(\theta)}{P(\text{Data})}. $$

(8)

The posterior distribution $P(\theta|\text{Data})$ relates the probability of model parameters to the probability of observing the data, $\mathcal{L}(\text{Data}|\theta)$ (also known as likelihood), and the prior distributions $P(\theta)$. The prior distribution represents any prior knowledge one has about the parameters. $P(\text{Data})$ is the normalization.

In Bayesian inference, the primary task is to develop a likelihood model for observing the data given the model parameters, $\mathcal{L}(\text{Data}|\theta)$. In this particular case, the data consists of SFHs for each SNR. If there are $N_{\text{SNR}}$ then the complete likelihood is the product of the likelihoods for each individual SNR:

![Figure 2. Age distribution for all 94 SNRs in M31 and M33 galaxies. For each SNR, we construct a probability distribution from the SFH; see Equation (1), and we add those distributions to get an age distribution for all SNRs in our sample. This stacked distribution, clearly shows two distributions. We model this as a power-law distribution with a $t_{\text{min}}$, a $t_{\text{max}}$, and a power-law slope (red dashed line). There is also a contaminating distribution, a random uniform distribution that likely represents the random unassociated bursts of star formation (black dashed line).](image-url)
Each SFH is composed of a set of bins \( \{ i \} \), which suggests a more specific definition for the likelihood for each SNR: \( \mathcal{L}_i(\text{SFH}|\theta) = \mathcal{L}_k(i|\theta) \).

Now, we may derive the hierarchical likelihood model for observing one burst that is drawn from \( P_p(t|\theta) \) and an arbitrary number of random bursts drawn from \( P_u(t) \). For illustration purposes, consider the case for which there are only two bursts, but it is not known which is drawn from \( P_p(t|\theta) \) (see Figure 4). In the following model, one and only one burst is associated with the explosion; the other is a random unassociated burst due to random uncorrelated star formation. One burst is labeled with 1, the other with 2. We represent the parameters of the power-law distribution by an array of parameters, \( \theta = (\text{min}, \text{max}, \beta) \). Our goal is to infer the posterior distribution for these parameters.

When there are two bursts, and it is not clear which burst is drawn from \( P_p(t|\theta) \) or \( P_u(t) \), then there are two hypotheses. Hypothesis one \( (H_1) \) states that burst 1 is drawn from \( P_p(t|\theta) \) and burst 2 is drawn from \( P_u(t) \). Hypothesis two \( (H_2) \) states that burst 2 is drawn from \( P_p(t|\theta) \) and burst 1 is drawn from \( P_u(t) \). Since there is no a priori information on which hypothesis is correct, \( H_j \) represents a latent parameter of the hierarchical model. Note that \( j \) loops over each burst (or bin) just like \( i \), with one important difference. Hypothesis \( H_j \) represents the hypothesis when the SF in bin \( j \) is assumed to be associated with the SN progenitor.

Now, we may derive a likelihood for each SFH, \( \mathcal{L}_k(\text{data}|\theta) \). But this likelihood depends upon the latent parameters, so one must first define the joint probability for the observed bursts and the latent parameters, \( \mathcal{L}_k(i|\theta, H) \). Using the conditional probability theorem, the joint probability is

\[
\mathcal{L}_k(i|\theta, H) = \mathcal{L}_k(i|H, \theta) \cdot P(H).
\]  

Then to obtain the likelihood of just the observed bursts, one marginalizes over the latent parameter \( H \):

\[
\mathcal{L}_k(i|\theta) = \sum_{H=1}^{N} \mathcal{L}_k(i|H, \theta),
\]

where \( N \) is the number of bins.

Next, we construct the hierarchical likelihood for each hypothesis. For hypothesis \( H_j \), the expansion of the likelihood using the conditional probability theorem, Equation (10), becomes

\[
\mathcal{L}_k(i, H_j|\theta) = P_p(j|\theta) \prod_{k=j} \cdot P_U(k) \cdot P(H_j).
\]
where \( P(H_j) \) is the probability of the hypothesis. For hypothesis \( H_j \), we set this to the probability of a star being associated with the SF in bin \( j \):

\[
P(H_j) = P_{\text{SF}}(j).
\]

Substituting these definitions, Equations (13) and (12), into Equation (11) leads to the final form of the likelihood for SNR \( k \), marginalized over all latent parameters:

\[
L_k(\{i\} | \theta) = \sum_{j=1}^{N} P_{\text{SF}}(j) \cdot P_{\text{U}}(j) \prod_{k \neq j} P_{\text{U}}(k). 
\]

Equation (14) represents a general likelihood, but one may further simplify this equation and reduce the computational time for the calculation by reducing the number of calculations within the MCMC runs. The product series in Equation (14) is essentially only a function of \( j \), \( f(j) = \prod_{k \neq j} P_{\text{U}}(k) \). Since it is only a function of \( j \), this series may be calculated before the MCMC runs. In fact, with a little clever algebra, the likelihood reduces even further. If one multiplies and divides the right-hand side of Equation (14) by \( P_A(j) \), then the product series includes \( k = j \) and the series is over all \( k \) now, i.e.,

\[
C = \prod_{k \neq j} P_{\text{U}}(k).
\]

In other words, the product series is now a constant and may be factored out of the summation. Equation (14) then becomes

\[
L_k(\{i\} | \theta) = C \cdot \sum_{j=1}^{N} \frac{P_{\text{SF}}(j)}{P_{\text{U}}(j)} \cdot P_{\text{U}}(j)/\theta. 
\]

Using the definitions of \( P_{\text{SF}}(j) \) in Equation (3) and \( P_{\text{U}}(j) \) in Equation (6), the ratio \( P_{\text{SF}}(j)/P_{\text{U}}(j) \) becomes \( P_{\text{SF}}(j) \cdot T_{\text{max}} \). The final reduced likelihood is

\[
L_k(\{i\} | \theta) = C \cdot T_{\text{max}} \sum_{j=1}^{N} P_{\text{SF}}(j) \cdot P_{\text{U}}(j)/\theta, 
\]

and the product \( C \cdot T_{\text{max}} \) is simply a constant and may be calculated before the MCMC runs.

With the likelihood for each SFH defined, one may now construct the posterior distribution for \( \theta \). To find the likelihood for all data, calculate the product of all likelihoods: insert Equation (14) into Equation (16) into Equation (9). Then the posterior distribution for \( \theta \) is proportional to this likelihood times the priors for \( \theta \):

\[
P(\theta | \text{Data}) \propto L(\text{Data} | \theta) \cdot P(\theta),
\]

where \( P(\theta) = P(t_{\text{min}}) \cdot P(t_{\text{max}}) \cdot P(\beta) \).

The priors, \( P(\theta) \), for all model parameters are uniform with additional conditions specified in Table 2. Therefore, the model in Equation (17) has only three unknown parameters \( t_{\text{min}}, t_{\text{max}} \), and \( \beta \) embedded in \( L(\text{Data} | \theta) \). To infer the posterior distribution, \( P(\theta | \text{Data}) \), we use the Markov Chain Monte Carlo (MCMC) sampler emcee, a python implementation (Foreman-Mackey et al. 2013) of the affine invariant ensemble sampler by Goodman & Weare (2010). Typically, we use 10 walkers, 10,000 steps each, and burn 5000 of those. Generally, the acceptance fraction for an inference run is typically around \( \alpha = 0.6 \).

### 2.6. Testing the Hierarchical Model Using Simulated Data

To test the above hierarchical model, we produce simulated normalized SFHs for \( N_{\text{SNR}} \). For each test run, we set the number of SNRs to either 30, 100, and 300 SNRs. For each SNR, one SF event is drawn from \( P_{\text{SF}}(\tilde{i}) \) and \( N_{\text{random}} \) bursts from \( P_{\text{U}}(\tilde{i}) \). However, we draw first \( N_{\text{random}} \) from the Poisson distribution with a mean of \( \lambda = 1 \). We then map these bursts into an SFH with the same resolution that we use in MATCH runs \( \Delta (\log t) = 0.05 \). For this simple test, the probability of each burst is evenly split, \( \tilde{P}_{\text{SF}}(i) = 1/N_{\text{random}} \).

To adequately test the method and to identify any biases, we run the test 10 times with known parameters of \( t_{\text{min}} = 10 \) Myr, \( t_{\text{max}} = 50 \) Myr, and \( \beta = 0.46 \). Figure 5 show the results when \( N_{\text{SNR}} = 30, 100, \) and 300. The horizontal maroon lines show the true parameter values. The solid gray line shows the median value of the marginalized posterior distributions and the gray band shows the 68% confidence interval. With only \( N_{\text{SNR}} = 30 \) the resulting marginalized distributions are quite broad, providing little very constraint on the parameters. Note that there are only 62 SNRs for M31 and 32 for M33. Since there are so few SNRs in each sample, we emphasize the results from the combined data set.

To calculate the bias, we calculate the mean median value in each case, subtract this mean from the true value and report the uncertainty of the median. Neither \( t_{\text{min}} \) nor \( \beta \) show any discernible systematic. For \( t_{\text{max}} \) the potential systematics are \( \pm 0.9 \) for \( N_{\text{SNR}} = 30, 0.7 \pm 0.4 \) for \( N_{\text{SNR}} = 100, \) and \( 0.4 \pm 0.3 \) for \( N_{\text{SNR}} = 300 \). We find that the bias is on the order of the uncertainty in measuring the bias, and it gets smaller as the uncertainty on the measurement increases (or the number of SNRs increases). Therefore, we suggest that there is no discernible bias. If there is one, then it is significantly smaller than the bin width at 50 Myr (\( \Delta t = 5.8 \) Myr).

### 3. Results

Figures 6 and 7 show SNR results of the MCMC sampler when applying our model to the ages of both M31 and M33 SNRs. Figure 6 represents the primary inference: the posterior for the \( t_{\text{min}}, t_{\text{max}}, \) and power-law slope \( \beta \) for the age distribution. Then, to obtain Figure 7, we use an age-to-\( M_{\text{ZAMS}} \) mapping to recast this as posterior distributions for the \( M_{\text{min}}, M_{\text{max}}, \) and slope \( \alpha \) in the mass distribution.

The marginalized values for the age distribution parameters are as follows. The \( t_{\text{min}} \) is \( t_{\text{min}} \leq 4.2 \) Myr, the \( t_{\text{max}} \) is \( t_{\text{max}} = 50.3^{\pm 1.1} \) Myr, and the power-law slope for the age distribution is \( \beta = 0.38^{\pm 0.18} \). We use a one-sided 68% confidence interval to calculate the upper limit on \( t_{\text{min}} \), and we calculate the narrowest 68% confidence interval for \( t_{\text{max}} \) and \( \beta \). This upper bound on \( t_{\text{min}} \) is roughly consistent with the upper end of the youngest age bin in MATCH (4.47 Myr for M33 and 5.01 Myr for M31). This age is an upper bound because all stars more massive than about \( \sim 60 \) \( M_{\odot} \) start to have Eddington factors near one (ratio of photon force compared to...
When the Eddington factor approaches one, \( L \propto M \), and the lifetime, \( t_{\text{lifetime}} \sim M c^2 / L \), becomes a constant for these stars. Hence, all stars with masses \( \gtrsim 60 M_\odot \) have lifetimes \( \sim 4 \) Myr.

The consequence for age-dating is that it is impossible to distinguish the ages of SF bursts that are younger than \( \sim 4 \) Myr; MATCH places all of these young bursts into the youngest age bin, which ranges from 3.98 to 4.46 Myr for the M33 resolution and 3.98 to 5.01 Myr for the M31 resolution. The SFR in this youngest bin actually represents the SF between 0 and \( \sim 5 \) Myr, so we redefine the youngest age bin to include all ages below \( \sim 5 \) Myr. Practically, we cannot move the left side of this bin to zero, because we are inferring the parameters of a power-law age distribution. To avoid the singularity imposed by this assumption, we set the left side of the youngest bin to 0.5 Myr.

Even though single-star evolution most naturally predicts the progenitor mass distribution, our primary inference is on the progenitor age distribution. For one, the fundamental data for each SNR is the SFH. Second, the clear mapping between age and mass is only valid for a restricted set of single-star evolutionary models. Binary evolution may significantly complicate this mapping. Therefore, to accommodate future binary analyses, we first infer the progenitor age distribution. In this manuscript, we consider the most straightforward case, single-star evolution.

For this case, we map the progenitor age distribution to a progenitor mass distribution. To do so, we make a few necessary assumptions. We assume single-star evolution, solar metallicity of \( Z = 0.019 \), and the stellar evolution models of Marigo et al. (2017). To convert \( t_{\text{min}} \) and \( t_{\text{max}} \) to its counterpart in mass space \( (M_{\text{max}} \text{ and } M_{\text{min}}) \), we use the results of stellar evolution models (solid, black curve in Figure 8). However, mapping the slope in age \( (\beta) \) to a slope in mass \( (\alpha) \) using the stellar evolution model curve is less trivial, and instead we use a log-linear fit. For a simple power-law age-to-mass mapping, the transformation would be analytic and simple. Unfortunately, the slope for the age-to-mass mapping from stellar evolution is not a single power-law slope. Therefore, to determine the most appropriate power-law approximation, we fit a power-law to the age-to-mass mapping curve and use this to transform each \( \beta \) in the posterior distribution to an \( \alpha \):

\[
m \propto t^{-0.7} = 21.9 \, M_\odot \cdot \left( \frac{t}{10 \text{ Myr}} \right)^{-0.70}.
\]

Formally, the power-law index of the age-to-mass map changes slightly from \( -1 \) to \( -0.6 \) in the mass range that we consider, but for the purposes of our simple mapping we use a log-linear fit that produces a slope of \( -0.7 \) (see Figure 8).

Finally, under the assumption that all CCSNe result from the explosion of single stars, the posterior distributions for the mass distribution parameters are in Figure 7. The marginalized

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*Figure 5.* Marginalized parameters from simulated tests of the Bayesian hierarchical model. From left to right, the number of SNRs in each column is 30, 100, and 300. We run the test 10 times to compare the true parameter values (maroon solid line) with the median value of the marginalized posterior distributions (gray solid line). If there is a bias in any of the parameters, it is smaller or on the order of the statistical uncertainties in every case. These tests also show that the progenitor age distributions are very poorly constrained for low SNR numbers.
of, and the slope is $59 \max 50.3 \max 0.5$ vs. death time from stellar evolution models.

Therefore, the detection limit at which MATCH can derive a star formation rate. Therefore, our minimum age is $t_{\text{min}} < 4.2$ Myr, the maximum age is $t_{\text{max}} = 50.3 \max 0.5$ Myr, and the power-law slope is $\beta = 0.38 \pm 0.32$. The $t_{\text{min}}$ is consistent with the minimum age for which MATCH can derive a star formation rate. Therefore, our $t_{\text{min}}$ is actually an upper limit on the minimum age.

Figure 6. Posterior distributions for the model parameters. We report the one-side 68% confidence interval for the upper limit on $t_{\text{min}}$, and the lower limit on $M_{\text{max}}$. For the rest of the parameters, we report the mode and the narrowest 68% confidence interval for the uncertainty. Given the model shown in Figure 4, the minimum age is $t_{\text{min}} < 4.2$ Myr, the maximum age is $t_{\text{max}} = 50.3 \max 0.5$ Myr, and the power-law slope is $\beta = 0.38 \pm 0.32$. The $t_{\text{min}}$ is consistent with the minimum age for which MATCH can derive a star formation rate. Therefore, our $t_{\text{min}}$ is actually an upper limit on the minimum age.

Figure 7. Posterior distribution for progenitor mass distribution parameters. To find $M_{\text{min}}$, $M_{\text{max}}$, and the power-law slope in between, we use the results of Figure 6 and an age-to-mass mapping. The $M_{\text{max}}$ is $M_{\text{min}} = 7.33 \pm 0.02 M_\odot$, the $M_{\text{max}}$ is $M_{\text{max}} > 59 M_\odot$, and the slope is $\alpha = -2.96 \pm 0.25$, somewhat steeper than the Salpeter initial mass function ($\alpha = 2.35$). The $M_{\text{max}}$ is consistent with the detection limit at which MATCH can derive a star formation rate. Therefore, the $M_{\text{max}}$ is actually a lower limit on the $M_{\text{max}}$.

parameters for all SNRs are $M_{\text{min}} = 7.33 \pm 0.02 M_\odot$, the $M_{\text{max}}$ is greater than $M_{\text{max}} > 59 M_\odot$, and the slope of the progenitor distribution is $\alpha = -2.96 \pm 0.25$.

Figure 8. $M_{\text{JAMS}}$ vs. death time from stellar evolution models (black curve; Marigo et al. 2017). We use this curve to map the progenitor age distribution parameters into the progenitor mass distribution parameters. Specifically, we interpolate using the black curve to map $t_{\text{min}}$ to $M_{\text{max}}$ and $t_{\text{max}}$ to $M_{\text{min}}$ (see Section 3). Mapping the power-law slope in age to a power-slope in mass is less trivial. Therefore, we fit a log-linear line (green line) to the stellar evolution model (black curve) and use this fitting formula to map $\beta$ to $\alpha$.

Figure 7 emphasizes the results for both M31 and M33. Table 3 reports the results for both and for each galaxy separately. As the simulated inferences suggested, the low number of SNRs in each galaxy provides very loose constraints on the parameters. Hence, we emphasize the results from the combined data set.

4. Discussion

In general, the inferred $M_{\text{min}}$, $M_{\text{max}}$, and slope $\alpha$ are consistent with previous estimates (Smartt et al. 2004, 2009; Jennings et al. 2014). The primary difference being that the uncertainties of this manuscript are more constrained.

4.1. Minimum Mass

In general, stellar evolution theory predicts that stars above about 7–11 $M_\odot$ experience core collapse (Woosley et al. 2002). This lower mass limit depends on a variety of factors, such as the model selected and the chemical composition of the star, e.g., helium abundance, metallicity, convection, and convective overshoot parameter (Woosley et al. 2002). For example, Iben & Renzini (1983) reported that a variation in chemical composition leads to a variation in the $M_{\text{min}}$ of $M_{\text{min}} = 8–9 M_\odot$. In addition to this, the overshoot parameter can reduce the minimum value significantly. Eldridge & Tout (2004) found that extra mixing, in the form of convective overshooting, moves the $M_{\text{min}}$ to lower masses for SN. For example, in Bressan et al. (1993) they reported a value of 5–6 $M_\odot$ when assuming overshoot mixing by half of a pressure scale height for metallicity and abundance of Z = 0.02 and Y = 0.28.

Recent observational constraints on progenitor masses suggest that the minimum mass is near the theoretical prediction. Based on the observations of 20 type II-P SNe, the minimum value estimated for CCSNe is $7.33 \pm 0.02$ (Smartt et al. 2009). Using simple KS tests on a similar sample of SNRs as in this manuscript, Jennings et al. (2012) inferred a $M_{\text{min}}$ of 7.3 $M_\odot$. These are all consistent with our more precise $M_{\text{min}}$ determination of $7.33 \pm 0.02 M_\odot$.
are difficult to model, so these results could provide new insight into late-stage evolution of massive stars. However, we need to take binarity effects and biases into account when modeling our data. The fact that we are getting a \( M_{\text{min}} \) that is on the low side of possible predictions may be a sign that binarity plays a significant role in which stars explode. For example, in a binary, the primary star could explode giving a kick to the secondary star that then explodes in an older region. Mergers are another way in which a massive star could explode in a relatively old region. It is expected that nearly 24% of massive stars merge and form rejuvenated stars (Sana et al. 2012). The resulting merged star is more massive, but it will have an age that is more consistent with a lower mass star. This could cause associations between CCSNe and stellar populations that are otherwise too old to have single stars that would explode. Zapartas et al. (2017) predict that 15\(^{+9}_{-8}\)% of CCSNe will be late explosions.

### 4.2. Maximum Mass and Slope of the Distribution

At the upper end of the progenitor age distribution, there are very few observational constraints. Smartt et al. (2009) suggested that the upper mass for SN IIP progenitors is \( \sim 17 \ M_{\odot} \), which is significantly lower than the observed masses of RSGs, the progenitors of SN IIP. However, their sample only included 18 detections and 27 upper limits on flux. More recently, Davies & Beasor (2018) suggested that a more accurate application of bolometric corrections brings the upper limit mass up, more in line with the RSG observations. Even so, this upper limit for SN IIP progenitors does not need to be the upper limit for explosions in general.

More massive stars are expected to lose much of their mass and explode as other types of SNe. Jennings et al. (2014) used KS statistics to constrain the upper end of the progenitor mass distribution for 115 SNRs in M31 and M33. The KS statistic does not allow for a self-consistent inference of all distribution parameters. Therefore, they first estimated the \( M_{\text{min}} \), then estimated the slope assuming no \( M_{\text{max}} \), and found a slope of \( \alpha = 4.2^{+0.3}_{-0.3} \). They also considered a second scenario; they set the slope to Salpeter (\( \alpha = 2.35 \)) and inferred a \( M_{\text{max}} \) of \( M_{\text{max}} = 35^{+3}_{-4} \). In either scenario, they concluded that either the most massive stars are not exploding at the same rate as lower mass stars or there is a bias against observing SNRs in the youngest SF regions.

With a more sophisticated Bayesian analysis, in which we infer all parameters simultaneously, we find that the \( M_{\text{max}} \) is above 59 \( M_{\odot} \), and the slope is \( \alpha = -2.96^{+0.45}_{-0.25} \). The lower limit on the \( M_{\text{max}} \) is consistent with our detection limit. Therefore, within our detection limits, we find no discernible \( M_{\text{max}} \). The slope is somewhat steeper than Salpeter, similar to Jennings et al. (2014), but not quite as steep. This significant difference demonstrates the advantage of inferring all parameters of the progenitor distributions simultaneously.

If there is a bias associated with the SNR catalog it might affect the \( M_{\text{max}} \) and/or the slope of the distribution, making it steeper or more positive, but the bias is very unlikely to affect the \( M_{\text{min}} \). Observable SNRs are very likely biased to certain regions of the ISM and may be biased to certain ages (Elwood et al. 2017; Sarbadhikary et al. 2017). We do not account for this bias, but instead report the progenitor age distribution and mass distribution as observed. However, as we incorporate the uncertainties in the SFH (future work), the distribution of minimum ages will likely incorporate even older ages. With regard to the \( M_{\text{min}} \), Figure 2 shows an abrupt drop at around 50 Myr. It is very unlikely that a general environmental bias would mimic such a drop in the progenitor distribution.

To improve the accuracy of these constraints, we identify several assumptions of our analysis that need verification or improvement. There are three dominant sources of uncertainty in the analysis: SFH resolution, multiple bursts of SF, and the uncertainty in the SFR. This current paper addresses the first two, but does not address the uncertainty in the SFR. In the future, we plan to develop a hierarchical likelihood model that includes the distribution of SFR for each age bin. For example, see Murphy et. al (2018). Furthermore, the nonlinear conversion from an age distribution, \( P(t_{\text{age}}) \), to a mass distribution, \( P(M) \), imposes a non-uniform prior on the mass distribution parameters. Explicitly, \( P(M) = P(t_{\text{age}}) \cdot dt_{\text{age}}/dM \), and Equation (18) implies that \( dt_{\text{age}}/dM \) is roughly \( t_{\text{age}} \propto M^{-2.4} \). When converting from \( P(t_{\text{age}}) \) to \( P(M) \), \( dt/dM \) acts like a prior. In this particular case, \( dt/\text{d}M \) may impose a bias toward low masses for \( M_{\text{min}} \) and \( M_{\text{max}} \).

### 5. Conclusion

Using a Bayesian hierarchical model, we infer the progenitor age distribution from the SFHs near 94 SNRs. Of these SFHs, 32 are for SNRs in M33 and 62 are for SNRs in M31. The SFHs for the 32 SNRs in M33 were previously published in Jennings et al. (2014). The remaining 62 SNRs in M31 are new and result from correlating the SNR locations from Lee & Lee (2014) with the resolved SFH map of M31 from Lewis et al. (2015). Technically, there are 71 SNRs from Lee & Lee (2014) that are in the PHAT footprint, but 8 of these had no SFH within the last 60 Myr. These 8 (or 11%) are most likely SN Ia SNR candidates, which we omit from our catalog. From the remaining 94 SNRs, we infer a \( t_{\text{min}} \) for CC of \( t_{\text{min}} < 4.2 \) Myr, a \( t_{\text{max}} = 50.3^{+0.5}_{-0.2} \) Myr, and power-law slope in between of \( \beta = 0.38^{+0.18}_{-0.32} \). Assuming single-star evolution, this age distribution corresponds to a progenitor mass distribution with a \( M_{\text{min}} \) of \( M_{\text{min}} = 7.33^{+0.02}_{-0.16} \ M_{\odot} \), a slope of \( \alpha = -2.96^{+0.45}_{-0.25} \), and a \( M_{\text{max}} \) of \( M_{\text{max}} < 59 \ M_{\odot} \) (see Figure 7).
The $M_{\text{min}}$ is consistent with the estimates from direct-imaging surveys. Since there are far more local SNRs than local SNe, the precision is much higher. Within our detection limits, we find no evidence for an upper mass. However, we do infer a progenitor mass distribution that is somewhat steeper than the Salpeter initial mass function. Either SNR catalogs are significantly biased against finding SNRs in the youngest SF regions, or the most massive stars are not exploding as often as the lowest masses.

Another major assumption of this work is assuming single-star evolution in transforming the progenitor age distribution to a mass distribution (Marigo et al. 2017). Under different assumptions, the parameters may systematically shift. For example, Zapartas et al. (2017) suggest that mass transfer and mergers in binary evolution could increase $t_{\text{max}}$ for CCSNe. This issue has also been explored by Maund (2017, 2018) in which they argue that even though their derived ages from resolved stellar populations are consistent with those derived from measurements of Hα for nearby H II regions using single-star stellar population synthesis models, it is possible that they could potentially be in disagreement with binary population synthesis models. Given the single-star assumption of this manuscript, these binary effects would lead to a lower $M_{\text{min}}$ inference. The Bayesian framework developed here easily allows for other models, including the delayed distributions caused by binary evolution. In fact, the Bayesian evidence will provide a means to estimate whether single-star or binary models best represent the progenitor age distribution.

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