Coherence non-activating measurement

Xueyuan Hu

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Abstract
We define the coherence non-activating measurement as the positive operator-valued measurement which gives the same result whether or not the coherence in a quantum state is destroyed. A connection is built between the coherence activating ability of a measurement and its ability to steer quantum states when coherence non-activating measurements are allowed. Then, we study the quantum discord based on coherence non-activating measurement and its behavior under local incoherent operations. Our results contribute to the study of resource non-activating condition, which is a complement to the well-studied resource non-generating condition.

Keywords Quantum measurement · Quantum coherence · Resource theory · Quantum correlation

1 Introduction
Quantum measurement, one of the fundamental elements in quantum theory, is an indispensable procedure in all the quantum information protocols. It underlies the phenomena such as the uncertainty principle [1,2] and the quantum steering [3,4] and provides the priority in tasks like quantum state discrimination [5]. Moreover, the reference basis of a quantum measurement plays an important role for the notion of non-classicality [6,7]. With the development of the quantum resource theory, quantifiers of quantumness for measurements are proposed [8,9], and remarkably, discrimination tasks are designed to give an operational characterization [5]. In this article, we ask two essential questions concerning the resource theory of quantum measurement: (Q1) what is the minimal constraint on free measurements (i.e., measurements which are not able to reveal quantum resource contained in any state to a

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Xueyuan Hu
xyhu@sdu.edu.cn

1 School of Information Science and Engineering, Shandong University, Qingdao 266237, China
classical observer), and (Q2) how to judge whether a given measurement is free in a device-independent way.

The resource theory of quantum coherence [10–12] has attracted much recent interest because it has many promising aspects, such as the analytical expressions for various measures of coherence in states [13], the clear characterization of different sets of free operations [14–18], and most importantly, the existence of a natural coherence destroying map [19,20]. Therefore, it is of interest to study (Q1) and (Q2) in the regime of quantum coherence. If a measurement does not activate coherence, or in other words, gives the same result whether or not the quantum coherence in states is destroyed, then the measurement should not be considered “quantum.” Hence, this non-activating condition is natural for a free measurement, analogous to the non-generating condition on free operations.

The definition of quantum discord [21] relies on quantum measurement. It quantifies the minimum fraction of correlation that cannot be detected by the local positive operator value measurements (POVMs). Instead of the whole set of POVMs, some works focus on the discord defined on some specific measurement. For example, for measurement-induced disturbance [22] or diagonal discord [23], the measurement is a projection to the eigenvector of marginal density matrix. The so-called classical discord is based on classical local measurements [24]. When the measurement is a projection to a fixed basis, the corresponding discord is called the basis-dependent discord [16,25], which is closely related to quantum coherence.

In this article, we define the coherence non-activating measurement as the POVMs which gives the same result whether or not the coherence in a quantum state is destroyed. We derive a device-independent criterion to witness whether a measurement can activate the coherence. Further we investigate the quantum discord based on incoherent measurement (QDI), which is similar to the traditional discord, but the local measurement is limited to the set of incoherent measurements. Interestingly, the discord based on incoherent measurements equals the basis-dependent discord. The behavior of QDI under local incoherent operations are also explicitly studied.

2 Coherence non-activating measurement

A positive operator-valued measure (POVM) is associated with a set of positive operators \{M_j\} satisfying \(\sum_j M_j = 1\). Instead of the measurement outcomes \(j\), one cares about the probability distribution of the outcomes

\[ p_j = \text{tr}(\rho M_j), \]

where \(\rho\) is the state we put into the detection. If we get the same measurement result even if the coherence in \(\rho\) is destroyed, we say that the measurement \(\{M_j\}\) is coherence non-activating; namely, it is not able to detect the resource contained in \(\rho\).

In the resource theory of quantum coherence, a reference basis \(\{|k\rangle\}\) is fixed. A state is incoherent if its density matrix is diagonal on the reference basis. Let \(\mathcal{I}\) be the set of incoherent states. A coherence destroying map \(\lambda\) satisfies two conditions [19]:

1. \(\lambda(\rho) \in \mathcal{I}, \forall \rho\).

Based on the coherence destroying map, we review two conditions on quantum operations [19]. The coherence non-generating condition on a quantum operation $\mathcal{E}$ reads

$$\mathcal{E} \circ \lambda = \lambda \circ \mathcal{E} \circ \lambda.$$  \tag{2}

If it is satisfied, then $\mathcal{E}$ cannot generate coherence in any incoherent state. Notably, the non-generating condition is the minimal constraint on free operations. A dual form of non-generating condition is the coherence non-activating condition

$$\lambda \circ \mathcal{E} = \lambda \circ \mathcal{E} \circ \lambda.$$  \tag{3}

A quantum channel satisfying the coherence non-activating condition cannot make use of any coherence in the input state to affect of incoherent part of the output state. This condition is natural for free measurements, in analog to the non-generating condition on free operations. Notice that there are coherence non-activating operations which are not coherence non-generating.

A coherence non-activating measurement is then defined as follows.

**Definition 1** (Coherence non-activating measurement) A quantum measurement $\{M_j\}$ is said to be coherence non-activating, if it satisfies

$$\text{tr}(\rho M_j) = \text{tr}(\lambda(\rho) M_j), \quad \forall \rho, j.$$  \tag{4}

By definition, the coherence destroying map is not unique [19] and the definition of coherence non-activating measurement relies heavily on the choice of the resource destroying map. However, if we require the coherence destroying map to be a completely-positive and trace-preserving (CPTP) map, then it is unique and given by the completely dephasing map $\Delta(\cdot) = \sum_k |k\rangle\langle k| \mathcal{O} |k\rangle\langle k|$ [20]. As proved in Ref. [8], when $\lambda = \Delta$, a measurement $\{M_j\}$ is coherence non-activating if and only if it is an incoherent measurement (IM) whose POVM elements $M_j$ are diagonal on the incoherent basis,

$$M_j = \Delta(M_j), \quad \forall j.$$  \tag{5}

In Appendix A, we extend this result and prove that for a general coherence destroying map $\lambda$, Eq. (5) is still necessary for a coherence non-activating measurement. It means that the most natural choice of coherence-destroying map (i.e., the completely dephasing map) imposes the minimal constraints on the coherence non-activating measurement, and any set of coherence non-activating measurement is subset to IM. Hence, we will focus on the incoherence measurements.

Apparently, all of the incoherent measurements are compatible, because they can be generated from incoherent projective measurement $\{|k\rangle\langle k|\}$ as $M_j = \sum_k m_{jk} |k\rangle\langle k|$. Conversely, if a measurement $\mathcal{S} = \{S_j\}$ is compatible with the incoherent projective measurement $\{|k\rangle\langle k|\}$, then $\mathcal{S} \in \text{IM}$. The reason is as follows. From the definition
of compatibility [4], $S$ and $|k\rangle\langle k|$ are compatible, if and only if a measurement $G = \{G_\lambda\}$ exists such that both $S$ and $|k\rangle\langle k|$ can be generated from $G$, i.e.,

$$|k\rangle\langle k| = \sum_\lambda p(k|\lambda, 0)G_\lambda,$$

(6)

$$S_j = \sum_\lambda p(j|\lambda, 1)G_\lambda,$$

(7)

where $p(k|\lambda, 0)$ and $p(k|\lambda, 1)$ are conditional probabilities. Because $p(k|\lambda, 0)$ are positive and each incoherent projector $|k\rangle\langle k|$ is of rank 1, Eq. (6) implies that every $G_\lambda$ is proportional to some incoherent projector. Substituting the form of $G_\lambda$ to Eq. (7), we obtain that each $S_j$ is diagonal on the incoherent basis, so $S \in IM$. Therefore, the condition $M \in IM$ is necessary and sufficient for the compatibility of $M$ and $|k\rangle\langle k|$. Recalling that when Alice and Bob previously share a maximally entangled state, Alice can steer Bob’s state if and only if she can implement incompatible measurements [4], we arrive at the following proposition.

**Proposition 1** Suppose that Alice and Bob previously share a maximally entangled state, and that Alice can implement incoherent measurements for free. Alice can steer Bob’s state if and only if she can implement a measurement $M \notin IM$.

When the state shared between Alice and Bob is not maximally entangled, then the incompatibility of Alice’s measurements is necessary but may not be sufficient for steering. Hence, Bob’s is convinced that Alice can implement measurements other than incoherent measurement as long as his state is steered.

A consequence of this proposition is that one can employ steering inequalities as criteria for incoherent measurements. Inspired by the quantum steering inequality proposed in Ref. [26], we derive an inequality for witnessing coherent measurement. The relationship between Eq. (8) and the steering inequality in Ref. [26] is analyzed in Appendix B.

**Theorem 1** Let $M = \{M_\alpha\}_{\alpha=0}^{n-1}$ be a quantum measurement on $d$-dimensional systems. If an orthonormal basis $\{|\varphi_\alpha\rangle\}_{\alpha=0}^{d-1}$ exists such that the inequality

$$\sum_{\alpha=0}^{d-1} \langle \varphi_\alpha | M_\alpha | \varphi_\alpha \rangle > \sum_{i=0}^{d-1} \max_\alpha | \langle \varphi_\alpha | i \rangle |^2$$

(8)

holds, then $M \notin IM$.

**Proof** Notice that one can always set $n \geq d$. If $n < d$, we can construct an equivalent measurement $M'$, with $M'_\alpha = M_\alpha$ for $\alpha < n$ and $M'_\alpha = 0$ for $n \leq \alpha < d$.

If $M \in IM$, then we have $M_\alpha = \sum_{i=0}^{d-1} m_{\alpha i} |i\rangle\langle i|$. Because $n \geq d$, $M_\alpha \geq 0$ and $\sum_{\alpha=0}^{n-1} M_\alpha = I$, we have $\sum_{\alpha=0}^{d-1} m_{\alpha i} \leq \sum_{\alpha=0}^{n-1} m_{\alpha i} = 1$, and consequently,

$$\sum_{\alpha=0}^{d-1} \langle \varphi_\alpha | M_\alpha | \varphi_\alpha \rangle = \sum_{i=0}^{d-1} \sum_{\alpha=0}^{d-1} m_{\alpha i} | \langle \varphi_\alpha | i \rangle |^2$$

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\[\begin{align*}
&= \sum_{i=0}^{d-1} \sum_{\alpha=0}^{d-1} m_{\alpha i} \left|\braket{\varphi_\alpha|i}\right|^2 \\
&\leq \sum_{i=0}^{d-1} \max_{\alpha} \left|\braket{\varphi_\alpha|i}\right|^2.
\end{align*}\]

(9)

Therefore, if the above inequality is violated for some orthonormal basis \(\{|\varphi_\alpha\rangle\}_{\alpha=0}^{d-1}\), the quantum measurement \(M\) is not an incoherent measurement. It completes the proof. \(\square\)

From this theorem, Alice can convince Bob that she can implement a measurement \(M \not\in \text{IM}\) in the following way. Alice and Bob previously share a maximally entangled state \(|\Phi\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |ii\rangle\). Alice performs the measurement \(M\) and tells Bob her results. Then, Bob performs a projective measurement \(\{|\varphi_\alpha\rangle\}_{\alpha=0}^{d-1}\) and checks whether their results coincide. The probability that their results coincide reads

\[\frac{1}{d} \sum_{\alpha=0}^{d-1} \langle \varphi_\alpha | M_\alpha | \varphi_\alpha \rangle.\]

If it is larger than \(\frac{1}{d} \sum_{i=0}^{d-1} \max_{\alpha} \left|\braket{\varphi_\alpha|i}\right|^2\), Bob is convinced that Alice’s measurement \(M\) is coherence activating.

Compared with directly using the definition of coherence non-activating measurement, the advantages of this criteria are as follows. Firstly, it is easier to check an inequality rather than an equality in experiment. Secondly, this criterion is device-independent in the sense that Alice cannot cheat. Thirdly, one need not to reconstruct the measurement elements by using this criterion.

Generally speaking, Eq. (8) is not the necessary condition for \(M \not\in \text{IM}\), so there are situations where Eq. (8) is not satisfied for any \(\{|\varphi_\alpha\rangle\}_{\alpha=0}^{d-1}\) even though \(M\) is not incoherent. However, for projective measurements with white noise, we prove that Eq. (8) is the necessary and sufficient condition that the measurement is not incoherent. The measurement elements of a projective measurement with white noise \(\tilde{\Pi} = \{\tilde{\Pi}_\alpha\}_{\alpha=0}^{d-1}\) can be written as

\[\tilde{\Pi}_\alpha = \lambda |\varphi_\alpha\rangle \langle \varphi_\alpha | + \frac{1-\lambda}{d} I,\]

(10)

where \(0 \leq \lambda \leq 1\) and \(\{|\varphi_\alpha\rangle\}\) is an orthonormal basis other than the incoherent basis. Clearly, the measurement \(\tilde{\Pi}\) is incoherent only when \(\lambda = 0\). If \(\{|\varphi_\alpha\rangle\}\) and \(\{|i\rangle\}\) are mutually unbiased bases, we choose \(|\varphi_\alpha\rangle = |\phi_\alpha\rangle\), \(\forall \alpha\), and Eq. (8) becomes

\[d\lambda + (1 - \lambda) > 1,\]

which holds for \(\lambda \neq 0\). If \(\{|\varphi_\alpha\rangle\}\) and \(\{|i\rangle\}\) are not mutually unbiased, we choose \(\{|\varphi_\alpha\rangle\}\) to be mutually unbiased with \(\{|i\rangle\}\) but not with \(\{|\phi_\alpha\rangle\}\). Hence, the right-hand side of Eq. (8) equals 1, and the left-hand-side reads

\[\sum_{\alpha=0}^{d-1} \langle \varphi_\alpha | \tilde{\Pi}_\alpha | \varphi_\alpha \rangle = 1 + \lambda \left[ \sum_{\alpha=0}^{d-1} \left|\braket{\phi_\alpha|\varphi_\alpha}\right|^2\right] - 1.\]

(11)

Because \(\{|\varphi_\alpha\rangle\}\) and \(\{|\phi_\alpha\rangle\}\) are not mutually unbiased, we can arrange the ordering of \(\{|\varphi_\alpha\rangle\}\) such that \(\sum_{\alpha=0}^{d-1} \left|\braket{\phi_\alpha|\varphi_\alpha}\right|^2 > 1\). Hence, the left-hand-side is strictly larger than 1 if \(\lambda \neq 0\). This completes the proof.
3 Quantum discord based on incoherent measurements

Before we study quantum discord based on incoherent measurements, we briefly review the definition of traditional quantum discord. For a bipartite state $\rho_{AB}$, the total correlation between $A$ and $B$ is quantified by the mutual information $I_{A:B}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$, where $\rho_{A(B)} = \text{tr}_{B(A)}(\rho_{AB})$ is the reduced density matrix of system $A(B)$, and $S(\rho) = -\text{tr}(\rho \log_2 \rho)$ is the von Neumann entropy. The maximal amount of information that can be revealed by local POVM on $A$ is called the classical correlation $J_{B|A}(\rho_{AB}) = \max_{\{M_\mu\} \in \text{POVM}} [S(\rho_B) - \sum_\mu p_{M_\mu} S(\rho_{B|M_\mu})]$, where $p_{M_\mu} = \text{tr}(\rho_{AB} M_\mu^A \otimes I_B^B)$ is the probability to get the measurement result $\mu$ and $\rho_{B|M_\mu} = \text{tr}_A(\rho_{AB} M_\mu^A \otimes I_B^B)/p_{M_\mu}$ is the resulted state of $B$ after the measurement. The difference between total correlation and classical correlation is called quantum discord $\delta_{B|A}(\rho_{AB}) = I_{A:B}(\rho_{AB}) - J_{B|A}(\rho_{AB})$.

Now we are ready to define the incoherent correlation and quantum discord based on incoherent measurement.

**Definition 2** For a bipartite state $\rho_{AB}$, the incoherent correlation on $A$ is defined as the maximal information gain about $B$ as a result of an incoherent measurement on $A$

$$J^I_{B|A}(\rho_{AB}) := \max_{\{M_\mu\} \in \text{IM}} \left[ S(\rho_B) - \sum_\mu p_{M_\mu} S(\rho_{B|M_\mu}) \right], \quad (12)$$

where $p_{M_\mu} = \text{tr}(M_\mu^A \otimes I_B^B \rho_{AB})$ and $\rho_{B|M_\mu} = \text{tr}_A(M_\mu^A \otimes I_B^B \rho_{AB})/p_{M_\mu}$ are the probability and the resulted state of $B$ for the measurement result $\mu$. The quantum discord based on the incoherent measurement (QDI) is defined as the difference between the mutual information and the incoherent correlation

$$D^I_{B|A}(\rho_{AB}) := I_{A:B}(\rho_{AB}) - J^I_{B|A}(\rho_{AB}). \quad (13)$$

In the definition of traditional quantum discord $\delta_{B|A}(\rho_{AB})$, the optimization is taken over the whole set of POVM, and the optimal measurement need not to be projective. Here, for the discord based on incoherent measurement $D^I_{B|A}(\rho_{AB})$, the optimization is restricted to the set of incoherent measurement. Therefore, $D^I_{B|A}(\rho_{AB})$ is lower bounded by $\delta_{B|A}(\rho_{AB})$, which is nonnegative. In the following, we prove that the optimal incoherent measurement which reaches the minimization in $D^I_{B|A}(\rho_{AB})$ is just the projection to incoherent basis.

**Theorem 2** The discord based on incoherent measurement has the following equivalent expressions

$$D^I_{B|A}(\rho_{AB}) = \sum_{i=0}^{d-1} p_i S\left(\rho_B^i\right) + S(\rho_A) - S(\rho_{AB}) \quad (14)$$

$$D^I_{B|A}(\rho_{AB}) = I_{A:B}(\rho_{AB}) - I_{A:B}(\rho_{\bar{A}B}), \quad (15)$$

$$D^I_{B|A}(\rho_{AB}) = C_r(\rho_{AB}) - C_r(\rho_{\bar{A}B}) - C_r(\rho_A). \quad (16)$$
Here \( p_i = \text{tr} \left[ (|i\rangle_A \langle i| \otimes I_B) \rho_{AB} \right] \) and \( \rho_{B|i} = \text{tr}_A \left[ (|i\rangle_A \langle i| \otimes I_B) \rho_{AB} \right] / p_i \) are the probability and the resulted state of \( B \) after Alice implemented incoherent projective measurement and got the result \( i \). \( \rho_{AB} = \Delta_A \otimes I_B(\rho_{AB}) \), and \( C_r(\rho) = S(\Delta(\rho)) - S(\rho) \) is the relative entropy of coherence.

**Proof** We first prove the equivalence between Eqs. (13) and (14). Because the measurement \( M \) on \( A \) is coherence non-activating, the measurement element is diagonal in the incoherence basis, i.e., \( M_{\mu} = \sum_{i=0}^{d-1} m_{\mu i} |i\rangle \langle i| \), and the resulted state of \( B \) for the measurement result \( \mu \) is then written as

\[
\rho_{B|M_{\mu}} = \frac{1}{p_{M_{\mu}}} \text{tr}_A \left[ \left( \sum_{i=0}^{d-1} m_{\mu i} |i\rangle \langle i| \otimes I_B \right) \rho_{AB} \right] 
= \sum_{i=0}^{d-1} \frac{m_{\mu i} p_i}{p_{M_{\mu}}} \rho_{B|i},
\]

(17)

Notice that \( \text{tr}(\rho_{B|M_{\mu}}) = \text{tr}(\rho_{B|i}) = 1 \), and we have \( \sum_{i=0}^{d-1} \frac{m_{\mu i} p_i}{p_{M_{\mu}}} = 1 \), so \( \{ \frac{m_{\mu i} p_i}{p_{M_{\mu}}} \}_{i} \) is a probability distribution for arbitrary \( \mu \). By the concavity of Von Neumann entropy, \( S(\rho_{B|M_{\mu}}) \geq \sum_{i=0}^{d-1} \frac{m_{\mu i} p_i}{p_{M_{\mu}}} S(\rho_{B|i}) \). Hence, the following inequality holds for all incoherent measurements \( \{ M_{\mu} \} \):

\[
\sum_{\mu} p_{M_{\mu}} S(\rho_{B|M_{\mu}}) \geq \sum_{i=0}^{d-1} p_i S(\rho_{B|i}).
\]

(18)

On the other hand, \( \min_{\{ M_{\mu} \} \in \text{IM}} \sum_{\mu} p_{M_{\mu}} S(\rho_{B|M_{\mu}}) \leq \sum_{i=0}^{d-1} p_i S(\rho_{B|i}) \), because the incoherent projective measurement belongs to \( \text{IM} \). Therefore, Eqs. (13) and (14) are equivalent.

Equation (15) is equivalent to Eq. (14) because \( \rho_{AB} = \sum_{i=0}^{d-1} p_i |i\rangle \langle i| \otimes \rho_{B|i} \) and then \( I_{A:B}(\rho_{AB}) = S(\rho_B) - \sum_{i=0}^{d-1} p_i S(\rho_{B|i}) \). The equivalence between Eqs (15) and (16) is obtained directly by definition. This completes the proof. \( \square \)

Theorem 2 indicates that the quantum discord based on incoherent measurement equals the basis-dependent discord defined in Ref. [16]. As proved in Ref. [16], the basis-dependent discord vanishes not only for incoherent-quantum states, but also for coherent states which have a decomposition

\[
\rho_{AB} = \sum_{j} \rho_A^j \otimes \rho_B^j,
\]

(19)

such that all \( \rho_A^j \) are perfectly distinguishable by the incoherent projective measurement. This result is natural by using Definition 2. The incoherent correlation on \( A \) reaches the mutual information if there exists a local incoherent measurement on \( A \) which can reveal the mutual information between \( A \) and \( B \). That is to say, the bipartite state is separable with each \( \rho_A^j \) distinguishable by some incoherent measurement.
4 Behavior of QDI under local incoherent operations

Similar to the local creating property of the traditional discord [27,28], the discord based on incoherent measurement can also be created by local incoherent operations [16]. In the following, we study the behavior of $D_{B|A}(\rho_{AB})$ under local operations.

(P1) $D_{B|A}(\rho_{AB})$ does not change under local unitary on $B$ or local incoherent unitary on $A$.

**Proof** Let $\rho'_{AB} = U_A^1 \otimes U_B \rho_{AB} U_A^{1\dagger} \otimes U_B^\dagger$, where $U_A^1$ and $U_B$ are arbitrary incoherent unitary on $A$ and unitary on $B$. Because an incoherent unitary $U^1$ satisfies the commutativity property $U^1 \Delta(\cdot) U^{1\dagger} = \Delta[U^1(\cdot) U^{1\dagger}]$, we have $\rho'_{AB} = \Lambda_A \otimes \mathbf{I}_B(\rho_{AB}) = U_A^1 \otimes U_B \rho_{AB} U_A^{1\dagger} \otimes U_B^\dagger$. Furthermore, local unitary does not change the mutual information, so $I(\rho'_{AB}) = I(\rho_{AB})$ and $I(\rho'_{AB}) = I(\rho_{AB})$. It follows from Eq. (15) that $D_{B|A}(\rho'_{AB}) = D_{B|A}(\rho_{AB})$. This completes the proof. □

(P2) $D_{B|A}(\rho_{AB})$ cannot be increased by local operations on $B$.

**Proof** Here, we first prove that discarding a subsystem on $B$ side does not increase the QDI defined on $A$, i.e., $D_{BB'|A}(\rho_{ABB'}) \geq D_{B|A}(\rho_{AB})$, where $\rho_{AB} = \text{tr}_{B'}(\rho_{ABB'})$. To this end, we employ Eq. (16) and obtain

$$D_{BB'|A}(\rho_{ABB'}) - D_{B|A}(\rho_{AB})$$

$$= [C_r(\rho_{ABB'}) - C_r(\rho_{ABB'}) - C_r(\rho_A)]$$

$$- [C_r(\rho_{AB}) - C_r(\rho_{AB}) - C_r(\rho_A)]$$

$$= [C_r(\rho_{ABB'}) - C_r(\rho_{AB}) - C_r(\rho_{AB})]$$

$$- [C_r(\rho_{AB}) - C_r(\rho_{AB}) - C_r(\rho_{AB})]$$

$$= I_{AB:B'}(\rho_{ABB'}) - I_{AB:B'}(\rho_{ABB'}) \geq 0. \quad (20)$$

The last inequality is because local operations cannot increase mutual entropy.

Any operation $\Lambda_B$ on $B$ can be realized by appending an ancilla $B'$, applying a local unitary $U_{BB'}$ on $B$ and $B'$, and then discarding $B'$. From Eq. (15), appending an ancilla on $B$ side does not change the QDI on $A$. Hence, we have

$$D_{B|A}(\rho_{AB}) = D_{BB'|A}(U_{BB'}(\rho_{AB} \otimes \rho_{B'}) U_{BB'}^\dagger)$$

$$\geq D_{B|A}(I_A \otimes \Lambda_B(\rho_{AB})). \quad (21)$$

This completes the proof. □

This monotonic property makes $D_{B|A}(\rho_{AB})$ a proper quantification of correlations. Actually, the traditional quantum discord also satisfies this property.

Also, (P2) is a generalization of a result in Ref. [29], which says that the remaining coherence defined as $C^T(\rho_{AB}) = C_r(\rho_{AB}) - C_r(\rho_{AB}) - C_r(\rho_{AB})$ is nonnegative.

From Eq. (16), $C^T(\rho_{AB}) \geq 0$ is equivalent to $D_{B|A}(\rho_{AB}) \geq D_{B|A}(\rho_{AB})$, which is a special case of (P2). Reference [29] points out that $C^T(\rho_{AB}) = 0$ for states.
with vanishing $D_{B|A}$ or $D_{A|B}$, but leaves it open whether other states satisfy this equation. Here, we give a positive answer to this problem. To this end, we consider the maximally entangled state $\rho_{AB}^m = |\Psi\rangle\langle\Psi| = \frac{1}{\sqrt{2}} (|+0\rangle + |+1\rangle)$ and $|\pm\rangle = \frac{1}{\sqrt{2}} ((0) \pm |1\rangle)$. Employing Eq. (15), we obtain $D_{B|A}(\rho_{AB}^m) = D_{B|A}(\rho_{AB}^m) = 1$. It means that the completely dephasing map on $B$ causes equal amount of decrease in the total correlation and the incoherent correlation.

Another related question is whether the monogamy relation holds when multipartite systems are considered, i.e., whether $D_{BB'|A}(\rho_{ABB'})$ is no less than $D_{B|A}(\rho_{AB}) + D_{B'|A}(\rho_{AB'})$ for any tripartite state $\rho_{ABB'}$. We give a negative answer to this question. Actually, for GHZ state $|\Psi\rangle = \frac{1}{\sqrt{2}} ((000) + (111))$, the monogamy relation $D_{B|A}(\rho_{AB}^{\text{GHZ}}) + D_{B'|A}(\rho_{AB}^{\text{GHZ}}) - D_{BB'|A}(\rho_{ABB}^{\text{GHZ}}) = -1 < 0$ holds, but for $|W\rangle = \frac{1}{\sqrt{3}} ((001) + (010) + |100\rangle)$, we have $D_{B|A}(\rho_{AB}^W) + D_{B'|A}(\rho_{AB'}^W) - D_{BB'|A}(\rho_{ABB}^W) = 2 - \log_2 3 > 0$. Generally, $D_{B|A}(\rho_{AB}) + D_{B'|A}(\rho_{AB'}) - D_{BB'|A}(\rho_{ABB'}) = I_{B:B'}(\rho_{ABB'}) - I_{B:B'}(\rho_{ABC})$, where $I_{A:B|C}(\rho_{ABC}) = S(\rho_{AC}) + S(\rho_{BC}) - S(\rho_{ABC}) - S(\rho_{B})$ is the conditional mutual information. Our results show that $I_{A:B|C}(\rho_{ABC})$ is not monotonic under local operations on $C$.

(P3) $J_{B|A}(\rho_{AB})$ cannot be increased by coherence non-activating operations. If a quantum operation $\Lambda$ satisfies the coherence non-activating condition $\Delta \circ \Lambda = \Delta \circ \Lambda \circ \Lambda$, then

$$J_{B|A}(\rho_{AB}) \geq J_{B|A}(\Lambda_A \otimes I_B(\rho_{AB})).$$

Here, the coherence non-activating condition was introduced in Ref. [19] as a dual form of the non-generating condition. It can be interpreted as follows. A coherence non-activating channel does not make use of the coherence in any input state if one looks at the incoherent part of the output state. The proof of Eq. (22) goes as follows.

**Proof** We first show that a measurement $M = \{M_\mu\}_\mu$ on party $A$ of $\rho_{AB}' = \Lambda \otimes I_B(\rho_{AB})$ is equivalent to the measurement $M' = \{A^*(M_\mu)\}_\mu$ on party $A$ of $\rho_{AB}$, where $A^*(\cdot) = \sum \lambda K^\dagger \lambda K_\lambda$ and $K_\lambda$ are the Kaus operators of $\Lambda$. Here $M'$ is a quantum measurement because its elements $A^*(M_\mu)$ are positive and satisfy $\sum \mu A^*(M_\mu) = I$. The measurement $M$ on party $A$ of $\rho_{AB}'$ gives the probability

$$p'(M_\mu) = \text{tr} \left( (M_\mu \otimes I_B) \Lambda \otimes I_B(\rho_{AB}) \right) = \text{tr} \left( A^*(M_\mu) \otimes I_B(\rho_{AB}) \right) = p(M_\mu),$$

which is just the probability of the measurement $M'$ on party $A$ of $\rho_{AB}$. Similarly, the resulted states of $B$ read $\rho_{B|M_\mu}' = \text{tr}_A \left( (M_\mu \otimes I_B) \Lambda \otimes I_B(\rho_{AB}) \right) = \text{tr}_A \left( A^*(M_\mu) \otimes I_B(\rho_{AB}) \right) = \rho_{B|M_\mu}'$.

If $\Lambda$ is coherence non-activating, then $A^*$ preserves the incoherence of measurement, because $A^* \circ A^* = A^* \circ A^* \circ A^*$ and $A^* = \Delta$. From Theorem 2, the optimal measurement which reaches the maximum in the definition of incoherent correlation
is the projective measurement \(|j\rangle\langle j|\). Hence, the incoherent correlation in \(\rho_{AB}'\) reads

\[ J_{B|A}(\rho_{AB}') = S(\rho_B) - \sum_j p'_j S(\rho_{B|j}), \]

\[ = S(\rho_B) - \sum_j p_{M'_j} S(\rho_{B|M'_j}) \tag{24} \]

where \(\rho_B = tr_A(\rho_{AB}') = tr_A(\rho_{AB}), M'_j = A^*\langle j|j\rangle, p'_j = tr(|j\rangle\langle j|\rho_{AB}) = tr(A^*\langle j|A\langle j\rangle \rho_{AB}) = p_{M'_j}\), and \(\rho_{B|j}' = tr_A(|j\rangle\langle j|\rho_{AB}') = tr(A^*\langle j|A\langle j\rangle \rho_{AB}) = \rho_{B|j}'\).

Because \(\{M'_j\}\) is an incoherent measurement, but may not be the one that reaches the maximum in the definition of \(J_{B|A}(\rho_{AB})\), we have \(J_{B|A}(\rho_{AB}') \leq J_{B|A}(\rho_{AB})\). This completes the proof.

(P4) \(D_{B|A}(\rho_{AB})\) can be created by local incoherent operations on \(A\). Specifically, the ability of some incoherent operations to generate QDI can be activated by a parallel identity channel.

In the special case where \(A\) is a qubit, if a state has vanishing QDI, then it is either an incoherent-quantum state \(\rho^{iq} = p|0\rangle\langle 0| \otimes \rho_B^0 + (1 - p)|1\rangle\langle 1| \otimes \rho_B^1\) or a product state; hence, QDI cannot be created by maximal incoherent operations (MIO). (Here MIO is defined as the set of CPTP maps which do not generate coherence from any incoherence state \([14,30]\).) Generally, states with vanishing QDI are in the form of Eq. (19), where \(\rho_A'\) can be coherent. In Ref. [16], an example is given to show that QDI can be created by local incoherent operations on \(A\). Also, they prove that QDI is a monotone under genuine incoherent operations (GIO). (Here GIO is defined as the set of CPTP maps which map any incoherent state to itself [17].) Nevertheless, there are other coherence non-generating operations which cannot create QDI. For example, the qutrit channel with Kraus operators as \(K_0 = \frac{1}{\sqrt{2}}(|-0\rangle\langle 0| + |1\rangle\langle 1|)\), \(K_1 = \frac{1}{\sqrt{2}}(|+0\rangle\langle 0| + |0\rangle\langle 1|)\), and \(K_2 = |2\rangle\langle 2|\), where \(|\pm 0\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)\) is in MIO but not in GIO. This channel cannot create QDI from a qutrit-qudit state in the form of Eq. (19).

Here, we observe that the ability of a quantum channel to create QDI can be activated by a parallel identity channel. Precisely, although a channel \(\Lambda_A\) cannot create QDI in any state \(\rho_{AB}\) with vanishing \(D_{B|A}\), it is possible that \(\Lambda_A \otimes \mathbb{I}_{A'}\) can create \(D_{B|AA'}\). As an example, we consider the initial state \(\rho_{AA'B} = \frac{1}{2}|000\rangle\langle 000| + \frac{1}{2}|\Psi^+\rangle\langle \Psi^+| \otimes |1\rangle\langle 1|\), where \(|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)\). Here the incoherent bases of qubits \(A, A'\) and \(B\) are all chosen to be \(|0\rangle, |1\rangle\). Because the two states \(|00\rangle\) and \(|\Psi^+\rangle\) of \(AA'\) can be distinguished by incoherent measurements, \(\rho_{AA'B}\) has zero QDI on \(AA'\). When a depolarizing channel \(\Lambda_{2}^{\text{dep}}(\rho) = p\rho + (1 - p)\frac{1}{2}\) operates on the qubit \(A\), the three-qubit state becomes

\[ \rho_{AA'B}' = \frac{1}{2} \left[ p|00\rangle\langle 00| + (1 - p)\frac{I_2}{2} \otimes |0\rangle\langle 0| \right] \otimes |0\rangle\langle 0| \]

\[ + \frac{1}{2} \left[ p|\Psi^+\rangle\langle \Psi^+| + (1 - p)\frac{I_2}{2} \otimes \frac{I_2}{2} \right] \otimes |1\rangle\langle 1|. \]
Because $|\Psi^+\rangle$ cannot be distinguished from $|10\rangle$ by incoherent measurement, the output state $\rho_{AA'B}^{\text{dep}}$ is not in the form of Eq. (19), and hence, $D_{B|A|A'}(\rho_{AA'B}^{\text{dep}}) > 0$ for $0 < p < 1$. Therefore, although a qubit depolarizing channel $\Lambda_2^{\text{dep}}$ (which is in MIO) cannot create QDI, the tensor product channel $\Lambda_2^{\text{dep}} \otimes I$ has the ability to create QDI.

Now we define the completely QDI non-generating channel as follows. A quantum channel $\Lambda$ is completely QDI non-generating if $\Lambda \otimes I$ does not have the ability to create QDI.

**Proposition 2** A quantum channel is completely QDI non-generating if and only if it is a composition of GIO and incoherent unitary operations.

**Proof** For the “if” part, because incoherent unitary does not change QDI, we only need to prove that GIO are completely QDI non-generating. If $\Lambda \in \text{GIO}$, then $\Lambda(|i\rangle\langle i|) = |i\rangle\langle i|$ by definition, so we have $\Lambda \otimes I(|ij\rangle\langle ij|) = \Lambda(|i\rangle\langle i|) \otimes |j\rangle\langle j| = |ij\rangle\langle ij|$, which means that $\Lambda \otimes I$ is also in GIO. Therefore, $\Lambda \otimes I$ does not have the ability to create QDI, and then $\Lambda$ is completely QDI non-generating.

For the “only if” part, let us consider the following tripartite state $\rho_{AA'B} = \frac{1}{d} \sum_{j=0}^{d-1} |j\rangle_A \langle j| \otimes |\phi_j\rangle_{A'} \langle \phi_j| \otimes |j\rangle_B \langle j|$, where $\{|j\rangle_A\}_{j=0}^{d-1}$ is the incoherent basis of $A$, $\{|\phi_j\rangle_{A'}\}_{j=0}^{d-1}$ are linearly independent states of $A'$ which cannot be distinguished by incoherent measurement. By definition, $D_{B|A|A'}(\rho_{AA'B}) = 0$. If $\Lambda$ is completely QDI non-generating, then $\Lambda(\rho_{AA'B}) = \frac{1}{d} \sum_{j=0}^{d-1} \Lambda(|j\rangle_A \langle j|) \otimes |\phi_j\rangle_{A'} \langle \phi_j| \otimes |j\rangle_B \langle j|$ has vanishing QDI on $AA'$. It means that $\Lambda(|j\rangle_A \langle j|) \otimes |\phi_j\rangle_{A'} \langle \phi_j|$ can be perfectly distinguished from each other by incoherent measurements. Because these states are product states and $|\phi_j\rangle_{A'}$ are indistinguishable by incoherent measurement, the $d$ states $\Lambda(|j\rangle_A \langle j|)$ can be distinguished by incoherent measurement. It follows that $\{\Lambda(|j\rangle_A \langle j|)\}$ is also the incoherent basis, i.e., there exist an incoherent unitary $U_I$ such that $\Lambda(|j\rangle_A \langle j|) = U_I|j\rangle \langle j| U_I^\dagger$, $\forall j$. Hence, $U_I^\dagger \Lambda(\cdot) U_I$ is GIO, and $\Lambda$ is a composition of GIO and incoherent unitary operations. This completes the proof.

**5 Conclusions**

The coherence non-activating measurement, as well as the quantum discord based on it, has been explicitly studied. If a POVM gives the same result when the coherence in a quantum state is destroyed, then its measurement elements should be diagonal in the incoherent basis. In order to witness that a POVM is not an incoherent measurement, we derive an inequality Eq. (8) and show that it is tight when projective measurement with white noise is considered.

When the set of POVMs in the definition of classical correlation and quantum discord is restricted to the coherence non-activating measurement, we obtain the incoherent correlation and the quantum discord based on incoherent correlation (QDI). Differing from the traditional quantum discord, where the optimal POVM may not be a projection, the optimal incoherent measurement in the definition of QDI is always the projection to the incoherent basis.
The incoherent correlation and QDI defined on A do not change under unitary on B or incoherent unitary on A and do not increase under any local operations on B. The monogamy relation for QDI does not hold in general. The incoherent correlation is monotonically decreasing under coherence non-activating quantum operations. QDI defined on A can be created by local coherence non-generating operations on A. Interestingly, the ability of some operations to create QDI can be activated by a parallel identity operation. We define the completely QDI non-generating channels as the quantum operations which cannot create QDI even if a parallel identity operation is employed, and prove that a quantum channel is completely QDI non-generating if and only if it is a composition of GIO and incoherent unitary operations.

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Compliance with ethical standards

Conflicts of interest The authors declare that they have no conflict of interest.

Appendix

A Coherence non-activating measurement for general resource destroying map

In this subsection, we will prove that for a general coherence destroying map \( \lambda \), Eq. (5) is a necessary condition for coherence non-activating measurements.

Now \( \lambda \) is not required to be a CPTP map. It is defined as a map which satisfies the following two conditions:

1. \( \lambda(\rho) \in \mathcal{I}, \forall \rho. \)
2. \( \lambda(\rho_i) = \rho_i, \forall \rho_i \in \mathcal{I}. \)

Hence, for a given coherence destroying map \( \lambda \), each incoherent state \( \rho_i \) defines a family \( \text{Fami}(\rho_i) \), which is the set of states mapped to \( \rho_i \) by \( \lambda \). Namely,

\[
\text{Fami}(\rho_i) = \{ \rho | \lambda(\rho) = \rho_i \}\tag{25}
\]

The families of states have the following properties:

1. There is only one incoherent state in a single family.
2. Every state belongs to a family.
3. Any two families do not overlap.

We will give the proof for the qubit case, and it can be generalized directly to the high-dimensional case. The elements of a qubit measurement \( M \) can be expressed as

\[
M_j = \sum_{l=0}^{3} m^{(j)}_l \sigma_l, \tag{26}
\]
where $\sigma_0$ is the two-dimensional identity matrix, and $\sigma_1, \sigma_2, \sigma_3$ are Pauli matrices. Here we label $m_j = (m^{(j)}_1, m^{(j)}_2, m^{(j)}_3)$ and require that $m_j \neq 0$ (because otherwise $M_j$ is proportional to the identity matrix). Let $\rho_0$ be an incoherent state whose Bloch vector is $r_0$, and $\rho$ be a state in the family of $\rho_0$, $\rho \in \text{Fami}(\rho_0)$. If a qubit measurement $M = \{M_j\}$ is coherence non-activating, then from Eq. (4),

$$\langle r - r_0 \rangle \cdot m_j = 0, \ \forall \rho \in \text{Fami}(\rho_0), \quad (27)$$

where $r$ is the Bloch vector of $\rho$. It means that the family $\text{Fami}(\rho_0)$ locates on a plane perpendicular to the vector $m_j$ and crossing with $z$-axis at $r_0$ in the Bloch presentation. Because the vector $m_j$ is fixed by the measurement element $M_j$, families of other incoherent states should locate on planes which are parallel to $\text{Fami}(\rho_0)$.

Suppose $M_j$ is not diagonal, or equivalently $m_j$ is not on $z$-axis. States which are above $\text{Fami}(|0\rangle\langle 0|)$ or below $\text{Fami}(|1\rangle\langle 1|)$ do not belong to any family. This is in contrast to Property (2) of the families. Therefore, each measurement element $M_j$ of a coherence non-activating measurement is diagonal on the incoherent basis.

### B Relation between Eq. (8) and the steering inequality

Suppose Alice and Bob previously share a maximally entangled bipartite state $\Phi = |\Phi\rangle\langle\Phi|$ with $|\Phi\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle$. Alice can implement two sets of quantum measurements $M_1 = \{|i\rangle\langle i|\}_{i=0}^{d-1}$ and $M_2 = \{|M_\alpha\rangle\rangle_{\alpha=0}^{n-1}$, where $n \geq d$. In order to check whether his state is steered by Alice, Bob implements the projective measurements $F_1 = \{|\varphi^i\rangle\langle \varphi^i|\}_{i=0}^{d-1}$ and $F_2 = \{|\varphi^\alpha\rangle\langle \varphi^\alpha|\}_{\alpha=0}^{d-1}$ accordingly. The steering functional reads

$$S(\{M_1, M_2\}, \{F_1, F_2\}, \Phi) = \sum_{i=0}^{d-1} \text{tr}(|i\rangle\langle i| \otimes |\varphi^i\rangle\langle \varphi^i|) + \sum_{\alpha=0}^{d-1} \text{tr}(M_\alpha \otimes |\varphi^\alpha\rangle\langle \varphi^\alpha|)$$

$$= \frac{1}{d} \sum_{i=0}^{d-1} |\langle i|\varphi^i\rangle|^2 + \frac{1}{d} \sum_{\alpha=0}^{d-1} |\varphi^\alpha\rangle\langle \varphi^\alpha| M_\alpha |\varphi^\alpha\rangle^\dagger. \quad (28)$$

According to Theorem 1 of Ref. [26], if Bob’s state is not steered by Alice, then the following inequality holds

$$S(\{M_1, M_2\}, \{F_1, F_2\}, \Phi) \leq 1 + \max_{i, \alpha} |\langle \varphi^i| \varphi^\alpha\rangle|, \quad (29)$$

for all choices of the projective measurements $F_1$ and $F_2$. Because Alice’s measurement $M_1$ is a projective measurement, we choose $F_1 = \{|i\rangle\langle i|\}_{i=0}^{d-1}$, without loss of
generality. Then, the above inequality is equivalent to

$$\sum_{\alpha=0}^{d-1} \langle \varphi_\alpha | M_\alpha | \varphi_\alpha \rangle \leq d \max_{i, \alpha} \left| \langle i | \varphi_\alpha \rangle \right|, \forall F_2.$$  \hspace{1cm} (30)

From Proposition 1, the violation of this inequality indicates that $M \notin \text{IM}$. Because $\max_{i, \alpha} \left| \langle i | \varphi_\alpha \rangle \right| \geq \frac{1}{d} \sum_{i=0}^{d-1} \max_\alpha |\langle \varphi_\alpha | i \rangle|^2$, the violation of Eq. (30) is sufficient for Eq. (8), which is in turn sufficient for $M \notin \text{IM}$. Therefore, Eq. (8) is a tighter criterion for witnessing coherent measurement.

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