Modeling and Prediction of Human Driver Behavior: A Survey

Kyle Brown, Katherine Driggs-Campbell, and Mykel J. Kochenderfer

Abstract—We present a review and taxonomy of 200 models from the literature on driver behavior modeling. We begin by introducing a mathematical formulation based on the partially observable stochastic game, which serves as a common framework for comparing and contrasting different driver models. Our taxonomy is constructed around the core modeling tasks of state estimation, intention estimation, state estimation, and motion prediction, and also discusses the auxiliary tasks of risk estimation, anomaly detection, behavior imitation and microscopic traffic simulation. Existing driver models are categorized based on the specific tasks they address and key attributes of their approach.

1 INTRODUCTION

Automated vehicles generally need to operate in close proximity to humans. This task is challenging because human behavior can be difficult to predict. The cognitive processes that govern human decision-making are inherently unobservable. Skills, preferences, and driving “style” vary widely among drivers. Moreover, complex interactions between drivers are commonplace in the traffic ecosystem. The task of modeling human driver behavior, though challenging, must be addressed to enable safe and efficient automated driving systems.

Due to its importance, the body of existing driver modeling literature is large. There are a wide variety of problem formulations, model assumptions, and evaluation criteria. Various reviews of existing driver behavior models have been published recently. Notable examples include the 2011 review of tactical behavior prediction models by Doshi and Trivedi [1], the 2014 review of motion prediction and risk estimation models by Lefèvre, Vasquez, and Laugier [2] and the 2016 review of human factors both in and around automated vehicles by Ohn-Bar and Trivedi [3]. Earlier reviews include the 1985 critical review by Michon [4], the 1994 survey of cognitive driver models by Ranney [5], and the 1999 review of car-following models by Brackstone and McDonald [6]. Each survey focuses on different subsets of driver behavior modeling tasks. Others touch relatively lightly on driver modeling in the context of a broader discussion [7–10].

This survey provides a broad, up-to-date review of the state of the art. We introduce a common mathematical framework and taxonomy for understanding existing driver models and drawing meaningful connections between them. Models are examined from the perspective of highly automated driving. We categorize each model according to the specific tasks that it addresses and key attributes of its approach. We review a total of 200 papers.

2 MATHEMATICAL FORMULATION

We begin by presenting a general mathematical framework for describing the microscopic dynamics of traffic—dynamics that arise from the behavior of multiple interacting, decision-making agents operating in a complex, partially observable environment. This framework is the discrete-time multi-agent partially observable stochastic game (POSG) [11]. The POSG formulation is introduced below and summarized in Figure 1 and Table 1.

Consider an arbitrary traffic scene consisting of $n$ agents. Let $x_i^{(t)}$ and $b_i^{(t)}$ represent the physical state and the internal state, respectively, of agent $i$ at time $t$. The physical state describes attributes such as the agent’s position, orientation, and velocity, whereas the internal state encompasses attributes such as the agent’s navigational goals, behavioral traits, and “mental model” of the surrounding environment.

Let $z_i^{(t)} \sim G_i(x_i^{(t)}, \ldots, x_n^{(t)})$ represent the potentially non-deterministic process by which agent $i$ observes the surrounding environment at time $t$. We call $G_i$ the observation function, and $z_i^{(t)}$ the observation. In general, the physical environment is only partially observed (i.e., $z_i^{(t)}$ is lossy). As agent $i$ processes the information in each new observation, its internal state evolves over time. We describe this process by $b_i^{(t+1)} \sim H_i(b_i^{(t)}, z_i^{(t)})$, where $H_i$ is the internal state update function.

At each time step, agent $i$ selects a control action $u_i^{(t)}$ according to $u_i^{(t)} \sim \pi_i(b_i^{(t)})$, where $\pi_i$ is called the policy function and its argument reflects the fact that the decision originates from the agent’s internal state. Agent $i$’s physical state evolves over time according to $x_i^{(t+1)} \sim F_i(x_i^{(t)}, u_i^{(t)})$, where $F_i$ is a discrete-time stochastic state-transition function.
model is a collection of assumptions about the human observation function. In the context of the POSG formulation, a multi-agent driving scenario, (e.g., change lanes, overtake the lead vehicle)—that a driver is trying to do, [318], trait estimation, driver is trying to do, [318], trait estimation, and motion prediction. Most models in the literature address one or more of these core modeling tasks, particularly the latter three. Closely related to these are auxiliary tasks such as risk estimation, anomaly detection, and behavior imitation. Another auxiliary modeling task, commonly used for testing and validation of automated driving software, is microscopic traffic simulation.

State estimation is the process by which the robot extracts a coherent estimate of the physical environment state—including the current physical states $x_{1:n}$ of the surrounding vehicles—from raw sensor information. This estimate can be deterministic, or it can reflect uncertainty via parametric (e.g., Gaussian) or non-parametric (e.g., particle) probability distributions. State estimation is the gateway to effective reasoning about the robot’s environment, in the sense that all other modeling tasks are predicated on the information that it infers.

Of course, the robot must reason not only about the physical states of other drivers, but also about their internal states. Intention estimation is the process by which the robot infers what a driver might intend to do in the immediate future. Intention estimation usually involves computing a probability distribution over a finite set of possible behavior modes—often corresponding to navigational goals (e.g., change lanes, overtake the lead vehicle)—that a driver might execute in the current situation.

Whereas intention estimation reasons about what a driver is trying to do, trait estimation reasons about factors that affect how the driver will do it. Broadly speaking, traits encompass skills, preferences, and “style,” as well as properties like fatigue, distractedness, etc. In the POSG framework, trait estimation may be interpreted as the process of inferring the “parameters” of the driver’s policy function $\pi$ on the basis of observed driving behavior.

3.1 Driver Modeling Tasks

Specific driver modeling tasks that might be addressed in a robot’s planning and control stack include state estimation, intention estimation, trait estimation, and motion prediction. Most models in the literature address one or more of these core modeling tasks, particularly the latter three. Closely related to these are auxiliary tasks such as risk estimation, anomaly detection, and behavior imitation. Another auxiliary modeling task, commonly used for testing and validation of automated driving software, is microscopic traffic simulation.

State estimation is the process by which the robot extracts a coherent estimate of the physical environment state—including the current physical states $x_{1:n}$ of the surrounding vehicles—from raw sensor information. This estimate can be deterministic, or it can reflect uncertainty via parametric (e.g., Gaussian) or non-parametric (e.g., particle) probability distributions. State estimation is the gateway to effective reasoning about the robot’s environment, in the sense that all other modeling tasks are predicated on the information that it infers.

Of course, the robot must reason not only about the physical states of other drivers, but also about their internal states. Intention estimation is the process by which the robot infers what a driver might intend to do in the immediate future. Intention estimation usually involves computing a probability distribution over a finite set of possible behavior modes—often corresponding to navigational goals (e.g., change lanes, overtake the lead vehicle)—that a driver might execute in the current situation.

Whereas intention estimation reasons about what a driver is trying to do, trait estimation reasons about factors that affect how the driver will do it. Broadly speaking, traits encompass skills, preferences, and “style,” as well as properties like fatigue, distractedness, etc. In the POSG framework, trait estimation may be interpreted as the process of inferring the “parameters” of the driver’s policy function $\pi$ on the basis of observed driving behavior.

3 Drivers Behavior Models

In the context of the POSG formulation, a driver behavior model is a collection of assumptions about the human observation function $G$, internal-state update function $H$ and policy function $\pi$. The model approximates the inherently unobservable processes occurring as a human driver observes, thinks, and acts. The implications of a given driver model’s assumptions depend on its applications.

3.1 Drivers Behavior Models

In the context of the POSG formulation, a driver behavior model is a collection of assumptions about the human observation function $G$, internal-state update function $H$, and policy function $\pi$. The model approximates the inherently unobservable processes occurring as a human driver observes, thinks, and acts. The implications of a given driver model’s assumptions depend on its applications.

TABLE 1

| Notation associated with the Partially Observable Stochastic Game (POSG) formulation of a general multi-agent driving scenario. Subscripts denote the agent index, and superscripts denote the time step.
|---|
| POSG Notation |
| $x_i^{(t)} \in X_i$ | physical state (of agent $i$ at time $t$) |
| $b_i^{(t)} \in B_i$ | internal state |
| $u_i^{(t)} \in U_i$ | control action |
| $z_i^{(t)} \in Z_i$ | observation |
| $x_i^{(t+1)} \sim F_i(x_i^{(t)}, u_i^{(t)})$ | state transition function |
| $z_i^{(t)} \sim G_i(x_i^{(t)}, \ldots, x_n^{(t)})$ | observation function |
| $b_i^{(t+1)} \sim H_i(b_i^{(t)}, z_i^{(t)})$ | internal state update function |
| $u_i^{(t)} \sim \pi_i(b_i^{(t)})$ | policy function |

Fig. 1: The evolution of an n-agent Partially Observable Stochastic Game (POSG) visualized as a graphical model. Each layer (into the page) of the graphical model corresponds to a different agent. Time increases from left to right. Edges represent the direction of information flow.

TABLE 1

Notation associated with the Partially Observable Stochastic Game (POSG) formulation of a general multi-agent driving scenario. Subscripts denote the agent index, and superscripts denote the time step.

| POSG Notation |
|---|
| $x_i^{(t)} \in X_i$ | physical state (of agent $i$ at time $t$) |
| $b_i^{(t)} \in B_i$ | internal state |
| $u_i^{(t)} \in U_i$ | control action |
| $z_i^{(t)} \in Z_i$ | observation |
| $x_i^{(t+1)} \sim F_i(x_i^{(t)}, u_i^{(t)})$ | state transition function |
| $z_i^{(t)} \sim G_i(x_i^{(t)}, \ldots, x_n^{(t)})$ | observation function |
| $b_i^{(t+1)} \sim H_i(b_i^{(t)}, z_i^{(t)})$ | internal state update function |
| $u_i^{(t)} \sim \pi_i(b_i^{(t)})$ | policy function |

3 Drivers Behavior Models

In the context of the POSG formulation, a driver behavior model is a collection of assumptions about the human observation function $G$, internal-state update function $H$, and policy function $\pi$. The model approximates the inherently unobservable processes occurring as a human driver observes, thinks, and acts. The implications of a given driver model’s assumptions depend on its applications.

3.1 Drivers Behavior Models

In the context of the POSG formulation, a driver behavior model is a collection of assumptions about the human observation function $G$, internal-state update function $H$, and policy function $\pi$. The model approximates the inherently unobservable processes occurring as a human driver observes, thinks, and acts. The implications of a given driver model’s assumptions depend on its applications.
**Motion prediction** is the process by which the robot infers the future physical states $x^{t+1}_{1:m(t)}$ of the surrounding vehicles. Motion prediction is the canonical task of driver behavior modeling in the sense that it reasons about the variables that have the most direct influence on the ego vehicle’s motion planning. Effective motion prediction generally requires state estimation, intention estimation, and trait estimation. This relationship is apparent from the POSG graphic in Figure 1 where the state variables $x^{(i)}$ and $b^{(i)}$ at time $t$ form a Markov blanket between the past and the future [13]. Intuitively, the future motion of vehicles depends on where they are, what the drivers intend to do, and how they intend to do it.

**Risk estimation** involves quantifying the risk inherent in a given scenario. Risk estimation is particularly relevant in advanced driver assistance systems (ADAS), which must decide if, when, and how to intervene. Risk estimation is fundamentally linked to motion prediction and the other core tasks, since risk is a measure of how “unsafe” one or more drivers’ future motion is expected to be. For an excellent review of risk-estimation models, see the survey by Lefèvre, Vasquez, and Laugier [2].

**Anomaly detection** involves recognizing when the behavior of one or more traffic participants defies expectations. Such information might be crucial in activating safety features that modify robot behavior to be more cautious in a given situation. Anomaly detection usually amounts to quantifying the mismatch between prediction (i.e., the output from motion prediction and the other core tasks) and observation.

**Behavior imitation** aims to make the automated driving behavior more “human-like”. Imitating humans can be desirable, for example, if the goal is to produce a familiar-feeling ride for passengers or a familiar interaction experience for other drivers.

**Microscopic traffic simulation** is related to motion prediction. For the purposes of this survey, we make the following distinction: **Prediction**—a discriminative task—deals with what will happen, whereas **simulation**—often a generative task—deals with what could happen. Put another way, prediction is a tool for forecasting the development of a given situation, whereas simulation is a tool for exploring a wide range of potential situations, often with the goal of probing the robot’s planning and control stack for weaknesses that can be addressed by system developers.

## 4 Categorization of Existing Models

The purpose of this survey is to review and classify models on the basis of the assumptions they make and the ramifications thereof. Note that we do not compare models based on their performance. Moreover, we do not address the degree to which models are evaluated and vetted experimentally. Rather, the survey is based on fundamental attributes of each proposed model as described in the publication that introduces it. We will begin by categorizing models based on the tasks they aim to address.

Table 3 (in the appendix) identifies the specific tasks addressed by each of the 200 models reviewed in this survey. Some driver models address multiple modeling tasks, while others concentrate on only one. Each row of the table corresponds to the publication in which the given model was introduced, while each column corresponds to one of the tasks described above. The final column of the table is used to identify models that are described in the context of a **behavior planning** algorithm (i.e., an algorithm for planning the actions of an automated vehicle).

In the following sections, we take a closer look at the characteristics of existing models that address one or more of the core driving modeling tasks: **state estimation**, **intention estimation**, **trait estimation**, and **motion prediction**. We devote most of our attention to the latter three.

Our first objective for each task is to highlight fundamental similarities and differences between the assumptions and methods employed by existing models. To this end, each of sections 4.1 to 4.4 introduces task-specific algorithmic axes along which models are compared and contrasted. The discussion for each core task (except state estimation) is supplemented and summarized by a **comparison** table that identifies where each surveyed model falls along the selected dimensions.

We also aim to provide a closer look at how existing models actually work. Hence, discussion of each task is also supplemented by a **keyword** table, which identifies important task-specific keywords and lists all surveyed models associated with each keyword. Keywords are broken into five categories. **Architecture** keywords (e.g., dynamic Bayesian network, support vector machine, etc.) describe specific components or methodologies that a model incorporates. **Training** keywords (e.g., expectation maximization, genetic algorithms, etc.) describe how a model’s parameters are selected. **Theory** keywords (e.g., clustering, time series analysis) allude to the theoretical underpinnings of a model. **Scope** keywords (e.g., intersection, highway merging) identify target applications for which a model is proposed, or on which it is evaluated. Finally, **Evaluation** keywords (e.g., root-mean square error, precision over recall) identify specific metrics by which a model’s performance is characterized.

### 4.1 State Estimation

Though state estimation is a crucial component within any fully automated driving software stack, it is addressed by relatively few publications in the driver modeling literature. Many models that ignore state estimation rely on post-processed datasets or simulation to provide accurate localization.

State estimation algorithms are usually implemented with an approximate recursive Bayesian filter. Examples include variants of the Kalman filter, particle filter, moving average filter, and Bayesian occupancy (histogram) filter. Some advanced state estimation models take advantage of the structure inherent in the driving environment to improve filtering accuracy. Examples include models based on dynamic Bayesian networks (DBN) and multiple model unscented Kalman filters (MM-UKF), as well as the multiperspective tracker [14]. Table 3 identifies the keywords associated with the 27 surveyed models that explicitly address state estimation. These keywords essentially correspond to the principal algorithmic components used by the respective models for addressing the state estimation task.
4.2 Intention Estimation

We begin our analysis of intention estimation models by introducing (in Table 5) a list of keywords and the specific models that are associated to them.

The second part of our analysis is a comparison of existing intention estimation models along three axes. First, we consider the assumptions made by existing models about the intention space, or the set of possible behavior modes that may exist in a driver’s internal state, b. Second, we consider the different ways in which existing models represent uncertainty in the intention hypothesis. Third, we identify several overlapping inference paradigms that characterize the way models actually perform the intention estimation task. Table 5 catalogues 107 models according to where they fall within the categories identified in each of the above areas.

4.2.1 Intention Space

Existing models vary widely in the way they represent the space of possible driver intentions. The intention space is usually defined explicitly, although it can also be learned in an unsupervised manner [15]. We identify a non-exclusive, non-comprehensive list of behavior mode categories used by models in the literature.

Route behavior modes are defined in terms of the structure of the roadway network, and may consist of a single decision (e.g., *turn right*) or a sequence of decisions (e.g., *turn right → go straight → turn right again*) that a driver may intend to execute. Lane-change intentions are a fundamental case of route intentions [4].

Configuration intentions are defined in terms of spatial relationships to other vehicles. For example, intention estimation for a merging scenario might involve reasoning about which gap between vehicles the target car intends to enter. The intention space of a car in the other lane might be whether or not to yield and allow the merging vehicle to enter. Configuration behavior modes are sometimes described as homotopies [17], which correspond to the various ways vehicles might pass ahead of or behind each other.

Some models consider modes of longitudinal driving behavior or modes of lateral driving behavior. For example, a model highway driving behavior might distinguish between the behavior modes of *car-following* and *cruising*. A possible set of lateral behavior modes might include *lane keeping, preparing to lane change, lane changing* [18].

Examples of uncommon intention spaces include intent to comply with traffic signals [19], possible emergency maneuvers [20] and intentional or unintentional maneuvers [21].

Behavior mode categories are often combined. For example, a model might reason about routes through an intersection and, additionally, whether or not a driver intends to yield to conflicting traffic along any particular route. The set of applicable behavior modes can vary depending on the specific context. While some models are tailored for a single operational context, others incorporate explicit context-dependent intention spaces with a scheme for restricting the set of applicable behavior modes based on observed features of the traffic environment.

4.2.2 Intention Hypothesis

The next consideration for comparing and contrasting models is the way they represent uncertainty in the intention hypothesis, $P(b_{i}^{(t)})$. The most common representation is a discrete probability distribution over possible Intentions. In contrast, the point estimate hypothesis ignores uncertainty and simply assigns a probability of 1 to a single (presumably the most likely) behavior mode. A few surveyed models employ a particle distribution [23], [24]. Some models compute a discrete distribution over scenarios, which corresponds to a (potentially sparse) distribution over joint intention assignments to all target vehicles [25]–[27].

4.2.3 Intention Inference Paradigms

Existing computational models estimate driver intention in different ways. We frame this discussion around several overlapping inference paradigms under which models may be grouped.

Recursive estimation algorithms operate by repeatedly updating the intention hypothesis at each time step based on the new information received. In contrast, single-shot estimators compute a new hypothesis from scratch at each inference step. The latter may operate over a history of observations, but it does not store any information between successive inference iterations.

Bayesian models are based on Bayes’ rule and the laws of conditional probability. Most of the surveyed models that fall under this umbrella are based on probabilistic graphical models. Black-box models have many non-interpretable parameters whose values are usually set by minimizing some loss function over a training dataset. Most black-box models, even if their final output is interpreted as a probability distribution, do not operate within the Bayesian framework.

Many models operate by comparing observed motion history to set of prototype trajectories or prototype policies, and deducing which prototype is the “closest” match. A prototype trajectory can be represented by a single trajectory, a set of trajectories, or a parametric or non-parametric distribution over trajectories. They can be based entirely on road geometry, extracted from a dataset of pre-recorded trajectories, or generated (e.g., with a motion planning/prediction algorithm or closed-loop control policy).

Finally, game-theoretic models are distinguished by being “interaction-aware”. They explicitly consider possible situational outcomes in order to compute or refine an intention hypothesis. This “interaction-awareness” can be as simple as pruning intentions with a high probability of conflicting with other drivers, or it can mean computing the Nash equilibrium of an explicitly formulated game with a payoff matrix. Some models in this category are interwoven with motion prediction algorithms, so that the results of motion prediction are used to inform the output of intention estimation in an iterated manner.
4.3 Trait Estimation

As with intention estimation, our analysis of trait estimation models also begins with a table (Table 6) of keywords and models associated with each keyword.

We compare existing trait estimation models along the following three dimensions. First, we consider the trait space—the set of trait parameters whose values each model selects. Second, we discuss how existing models represent uncertainty in the trait hypothesis. Third, we examine several non-exclusive inference paradigms that describe important aspects of the operation of existing trait estimation models. Table 7 catalogues 39 models along these dimensions.

4.3.1 Trait Space

Some of the most widely known driver models are simple parametric controllers with tuneable “style” or “preference” policy parameters that represent intuitive behavioral traits of drivers. For example, the Intelligent Driver Model (IDM) has five parameters that govern longitudinal acceleration as a function of the relative distance and velocity to the lead vehicle: minimum desired gap, desired time headway, maximum feasible acceleration, preferred deceleration, maximum desired speed [25]. Some other examples of “style” parameters include “aggressiveness” [29], “distraction” level [30], and “politeness” factor [31].

Some models encode driver preferences in a parametric cost (or reward) function that drivers are assumed to be “trying to optimize.” Reward function parameters often correspond to the same intuitive notions mentioned above (e.g., preferred velocity, etc.), the important difference being that they parametrize a reward function rather than a closed-loop control policy.

A few models incorporate attention parameters to model whether (or to what extent) a driver is attentive to the driving task. In a similar vein are models that reason about physiological traits like “reaction time” [32]. Such models are underrepresented in this survey, as we focus on high-level behavior. For a survey that dives more deeply into driver modeling at the operational level, see the 2003 review by Macadam [33] as well as the 2007 survey by Plöchl and Edelmann on vehicle dynamics [12].

Finally, the non-interpretable parameters of black-box policy or reward models (e.g., neural networks, Gaussian Mixture models) can be considered an implicit representation of driver traits.

4.3.2 Trait Hypothesis

We next consider the matter of how existing trait estimation models represent uncertainty in their estimate. In almost all cases, the hypothesis \( P(\beta_i(t) \mid z^{(t)}) \) is represented by a point estimate, \( \hat{\beta}_i(t) \in B \), rather than a distribution. There are, however, a few notable exceptions: Several models employ particle distributions to represent a belief \( P(\beta_i(t)) \) over IDM parameters for individual agents [29], [34]–[36]. Sadigh, Landolfi, Shankar, et al. maintain a discrete distribution over possible trait “clusters” to which a particular driver’s reward function parameters might belong [37]. A few models employ continuous distributions, including a Gaussian distribution over IDM parameters [38] and a log-concave distribution over reward function parameters [39].

4.3.3 Trait Inference

Finally, we consider the major inference paradigms that characterize existing trait estimation models.

Trait estimation can be performed offline or online. In the offline paradigm, estimated trait parameters are computed prior to deploying the model. The selected parameter values usually remain fixed during operation, meaning that they only reflect the population of drivers whose behavior was observed in the training data. In the online trait estimation paradigm, models reason in real time about the traits of currently observed—perhaps previously unobserved—drivers. Some models combine the two paradigms by computing a prior distribution offline, then tuning it online. This tuning procedure often relies on Bayesian methods.

One simple approach to offline trait estimation is to set trait parameters heuristically. Specifying parameters manually is one way to incorporate expert domain knowledge into models.

Many trait estimation models employ some form of optimization to select driver trait parameters. Table 6 identifies a wide variety of algorithms used for optimization-based estimation.

When model parameters represent a reward function, agents are no longer necessarily modeled as simple feedback controllers. Rather, they are modeled as maximizing the expected reward (or, equivalently, minimizing the expected reward) of their actions over time. The task of inferring a reward function from observed behavior belongs to the field of inverse optimal control, also known as inverse reinforcement learning.

In some approaches, trait parameters are modeled as contextually varying, meaning that they vary based on the region of the state space (the context) or the current behavior mode. Such models essentially define adaptive controllers, where adaptation laws can be stochastic or deterministic, and can be learned from data or specified heuristically.

4.4 Motion Prediction

As we did for intention estimation and trait estimation, we catalog motion prediction models according to keywords (see Table 8).

We also compare motion prediction models based on the following attributes. First, what (if any) vehicle dynamics models does the model employ? Second, how is uncertainty modeled—both at the scene level and the agent level? Third, what is the prediction paradigm of each model? Table 9 categorizes the models based on these attributes.

4.4.1 State-Transition models

Within our POSG framework, the state transition function \( F \) defines how the physical state of a vehicle evolves over time as a driver executes control inputs. In the context of motion prediction, the choice of vehicle dynamics model can influence the degree to which predicted motion is physically realizable. Vehicle models used in the driver modeling
literature can be classified into several partially overlapping categories.

Four-wheel dynamic models propagate the vehicle state forward in time by solving for the forces generated by each of the vehicle’s tires at the tire-road interface and applying these forces to the vehicle system over some time period. These “full car” models can vary substantially in terms of the way they model tire friction, longitudinal and lateral load transfer, and many other factors that affect vehicle motion. Most driver modeling applications do not require the level of fidelity of a four-wheel dynamic vehicle model, but some require accurate vehicle modeling for maneuvers at the extremes of the vehicle performance envelope [41].

Two-wheel models lump the front wheels and rear wheels into a single wheel per axle. For this reason, they are often referred to as bicycle models. Bicycle dynamic models, similar to four-wheel models, operate by computing the forces at each of the lumped “tires” and transmitting those forces through the vehicle body. Like four-wheel dynamic models, bicycle dynamic models also vary substantially in how they account for (or ignore) effects like load transfer and nonlinear tire friction relationships. Bicycle kinematic models have the same two-wheel geometry as dynamic bicycle models, but they rely on the simplifying assumption that tires experience zero lateral slip. This no-slip assumption means that motion is computed purely from the geometry of the vehicle model—without reasoning about forces at the tire-road interface. Kinematic models are simpler than dynamic models, but the no-slip assumption can lead to significant modeling errors (especially at high speeds and large steering angles).

Single wheel or unicycle dynamic models treat the vehicle as a point mass whose state is represented by its 2D position, heading angle and velocity. The control inputs are usually acceleration and yaw rate (turn rate). It is generally assumed that the vehicle moves in the direction of its heading angle, although certain models allow for some level of lateral side-slip.

Many models in the literature assume linear state-transition dynamics. Linear models can be first order (i.e., output is position, input is velocity), second order (i.e., output is position, input is acceleration), and so forth. Longitudinal and lateral motion are usually decoupled, often with different order equations of motion (e.g., second order longitudinal dynamics with first order lateral dynamics). The modeling error induced by the assumption of linear dynamics depends on the application. At highway speeds, for example, this assumption can be quite reasonable so long as lateral acceleration remains low.

A few state-transition models employed in the literature model vehicle motion directly with some kind of parametric spline. Examples include cubic splines, quintic splines, Chebychev polynomials, and Bézier curves. In such cases, the spline itself often constitutes the state-transition function.

Discrete state-transition models (i.e., the state- and action-spaces are discrete) are sometimes employed in applications where low-level vehicle dynamics can be abstracted away.

Probabilistic state-transition models capture uncertainty over the future state as a function of the current state and action. In other words, the state-transition function is modeled as a conditional distribution.

Some state-transition models are learned, in the sense that the observed correlation between consecutive predicted states results entirely from training on large datasets. Such data-driven models come in many forms. Some incorporate an explicit transition model where the parameters are learned, whereas others simply output a full trajectory.

Table 9 categorizes existing motion prediction models based on the type of state-transition model $F$ that each uses to describe the input-output behavior of the vehicle system. Rows without any entry correspond to motion prediction models that either do not incorporate a state-transition model or simply fail to describe the specific model they use.

4.4.2 Motion Hypothesis

The motion hypothesis is generally an estimate of $x_{t:t_f}^{n}$, the future states of all agents in the scene from the next time step $t + 1$ to some prediction horizon $t_f$. Uncertainty over these variables can be modeled in many ways. In order to provide insight into the similarities and differences between existing approaches, we separate motion hypothesis uncertainty into two categories: scene-level uncertainty and agent-level uncertainty.

At the scene level, multi-scenario motion prediction models reason about the different possible scenarios that may follow from an initial traffic scene, where each scenario is usually (though not necessarily) characterized by a unique combination of predicted behavior modes for each participant in the traffic scene. Other models reason only about a single scenario, ignoring multimodal uncertainty at the scene level. Some models reason only about a partial scenario, meaning they predict the motion of only a subset of vehicles in the traffic scene, usually under a single scenario.

The motion hypothesis can also be represented as a belief tree. Belief trees generalize discrete distributions over scenarios because each individual node of the belief tree might represent a partial scenario. In most cases, the belief tree category applies to models that represent the motion hypothesis (perhaps implicitly) as part of behavior planning.

On the agent-level, deterministic motion hypothesis representations from the literature include single trajectories (i.e., a single sequence of states per target agent), sequences of bounding boxes, and splines. Note that some approaches use splines for trajectory generation, but that the final representation is still just a sequence of states rather than a spline. Probabilistic agent-level motion hypotheses (i.e., distributions over trajectories) can be represented with unimodal Gaussian distributions, Gaussian mixture distributions, and particle sets. Some approaches use probabilistic occupancy distributions, created by binning a continuous space into finite cells and modeling the probability that any one cell is occupied at a given time.

Rather than reasoning about the likelihood of future states, some models reason about reachability. Reachability analysis implies taking a worst-case mindset in terms of predicting vehicle motion. Examples of motion hypotheses

9. Occupancy distributions are called occupancy grids when the binning is rectilinear.
in this category include forward reachable sets, backward reachable sets. Some models use empirical reachable sets, which balance the robustness of reachable sets and the expressiveness of probabilistic hypotheses.

4.4.3 Motion Inference

We now consider how the motion hypothesis is actually generated. Existing approaches can be loosely grouped according to three key paradigms: Closed-loop forward simulation, open-loop independent trajectory prediction, and game theoretic prediction.

In the forward simulation paradigm, motion hypotheses are computed by rolling out a closed-loop control policy $\pi$ for each target vehicle. The model computes a control action for each agent at each time step based on the observations received up to and including that time step, then propagates the entire scene forward in time. This process is repeated until the prediction horizon $t_f$ is reached. Algorithm 1 is a generic version of forward simulation prediction.

Algorithm 1 Motion Prediction via Forward Simulation

\[
\begin{align*}
\text{for } t & \in t_1, \ldots, t_f - 1 \\
\text{for } i & \in 1, \ldots, n \\
& x_i(t) \leftarrow G_i(x_i(t), b_i(t-1)) \quad \triangleright \text{receive observation} \\
& b_i(t) \leftarrow H_i(x_i(t), b_i(t-1), z_i(t)) \quad \triangleright \text{update internal state} \\
& u_i(t) \leftarrow \pi_i(b_i(t)) \quad \triangleright \text{select action} \\
& x_i(t+1) \leftarrow F_i(x_i(t), u_i(t)) \quad \triangleright \text{step forward}
\end{align*}
\]

Forward simulation can be performed with a deterministic state representation or a probabilistic state representation (e.g., a Gaussian state as in Kalman prediction, a particle set state estimate). Some models reason about multimodal uncertainty on the scene-level by performing multiple (parallel) rollouts associated with different scenarios.

Because forward simulation involves closed-loop control, motion prediction algorithms within this paradigm can be described as nominally “interaction aware.” The level of “interaction-awareness” depends on $G_i$, which encodes assumptions about what agent $i$ observes at each time step, as well as $H_i$ and $\pi_i$, which extract what the agent does with that information. Through the predicted observation $z_i(t)$, the control action prediction $u_i(t)$ for a given agent can be conditioned on the concurrent predicted states $x_{i:n}(t)$ of its neighbors for each time step $t$ within the prediction window. A deeper notion of “interaction awareness” will be discussed in the context of game theoretic models.

Most motion prediction models assume that the intentions and traits of each driver remain fixed throughout the prediction window, although several models do reason about how the driver internal state $b$ might evolve during the prediction window.

The closed-loop action policy used in forward simulation-based models can take many forms. It can be deterministic or stochastic (the former is more common for motion prediction, while the latter is more common for traffic simulation). Simple examples include rule-based heuristic control laws like the Intelligent Driver Model. More sophisticated examples include closed-loop policies based on neural networks, dynamic Bayesian networks, and random forests. As mentioned previously, some models incorporate adaptive control policies in which the policy parameters vary by context.

Many models operate under the independent prediction paradigm, meaning that they predict a full trajectory independently for each agent in the scene. These approaches are “interaction-unaware” because they are open-loop; though they may account for interaction between vehicles at the current time $t$, they do not explicitly reason about interaction over the prediction window from $t + 1$ to $t_f$.

The simplest motion prediction models in this class are based on various combinations of constant velocity, constant acceleration, constant yaw rate or constant steering angle. More advanced models include spline-based trajectory optimization, deep neural networks (sequence-to-sequence encoder-decoder architectures, CNNs, RNNs), Gaussian processes, hierarchical mixtures of experts, and variational GMMs.

Because independent trajectory prediction models ignore interaction, their predictive power tends to quickly degrade as the prediction horizon extends further into the future. Neural network-based models tend to excel in this category.

Several models operate within a game-theoretic motion prediction paradigm. We do not imply that all models in this category compute the Nash equilibria of a multi-agent game. Rather, for our purposes, “game theoretic” means that the predicted future motion of some agents is somehow explicitly conditioned on the predicted future motion of other agents in the scene. In other words, agents are modeled as “looking ahead” to consider the possible ramifications of their actions. This notion of looking ahead makes game-theoretic prediction models more deeply “interaction aware” than forward simulation models based on reactive closed-loop control.

Game theoretic models vary in duration and information structure. Each player plays multiple times in a repeated game, whereas each player plays only once in a one-off game. All players select their actions at once in a simultaneous game, whereas they take turns in a sequential (Stackelberg) game \([42]–[45]\). Some reviewed models treat full trajectories as a single action, meaning that a long-horizon prediction can still be formulated as a non-repeated game.

Game theoretic models can also differ in reward structure. In a fully cooperative game, all agents are trying to achieve the same outcome. An example is joint trajectory optimization, wherein the collective motion of all vehicles in the scene is computed by minimizing some global cost function \([17]\). In a fully non-cooperative (zero-sum) game, the individual objective of each agent is diametrically opposed to all others. There exists a spectrum of semi-cooperative reward structures between these two extremes.

Some model-predictive control policy models (including those used within a forward simulation paradigm) fall into the game theoretic category because they explicitly predict the future states of their environment (including other cars) before computing a planned trajectory. Several approaches model the human driver as a “best-response” agent with full access to the planned paths of the other vehicles \([57], [46], [47]\). This formulation constitutes a Stackelberg game where the human plays second.
Some game theoretic models seek to reduce the complexity of reasoning about all pairwise interactions between vehicles by focusing only on the most “relevant” pairwise interactions [48], [49].

A few models use recursive reasoning, meaning that trajectory plans are recursively computed for each agent based on the most recent predicted plans for the agent’s neighbors [49], [50]. Some models employ “game-theoretic” training procedures (for trait estimation), and then apply the trained models in a forward simulation paradigm for prediction [51].

5 CONCLUSION
To our knowledge, this is the first survey to examine the field of driver behavior modeling along all of the specific dimensions that we have discussed. We believe that this review represents a unique contribution in its breadth, the specific details that it conveys, and the unified framework within which it examines existing models. The aim of these contributions is to provide a resource to help researchers navigate the complex landscape of driver behavior modeling research.

There is also much useful information that, for various reasons, is not presented in this survey. For example, we opted not to meticulously catalog specific model input and output features in our taxonomy. We also do not explicitly quantify the degree to which surveyed models are evaluated and vetted. Though such information would undoubtedly be useful, we chose instead to focus the survey on fundamental modeling techniques. The bottom line is that our chosen survey structure represents just one limited perspective from which to compare and categorize driver models.

Preparation of this survey has led to a few insights about the current state of driver modeling research. The behavior modeling field could benefit tremendously from adopting a “standard” modeling framework and simulation platform. Researchers could submit driver models complying with a standard interface and evaluate their own models against other submitted models on a variety of scenarios. Relevant projects include Common-Road [52], CARLA [53], Simulation of Urban MOBility (SUMO) [54], and AutomotiveDrivingModels.jl (https://github.com/sisl/AutomotiveDrivingModels.jl).

Along the same lines, it would be helpful to define a standard suite of tests and benchmarks—particularly for intention estimation and motion prediction. Tests could be attached to one or more publicly available datasets, perhaps among those catalogued by Kang, Yin, and Berger [55].

All models make major assumptions about the processes that govern human cognition and behavior, but many authors fail to clearly identify these assumptions. We recommend that authors of future publications frame their models within a stochastic game framework like the POSG framework introduced in section 2. In other words, authors should explicitly define, when relevant, the state space, action space, observation space, and internal state space, as well as the state transition function, the observation function, the policy function and the internal state transition function associated with their models. Doing so will help clarify important assumptions regarding human observation, information processing, and decision-making.

ACKNOWLEDGMENTS
This work was supported by Qualcomm and by the National Science Foundation under grant no. DGE 1656518. The authors would like to thank Ahmed Sadek, Mohammad Naghshvar, and the team at Qualcomm Corporate Research for their insightful feedback.

Kyle J. Brown Kyle J. Brown received a B.S. degree in Mechanical Engineering from Brigham Young University in 2016. He is currently pursuing a PhD degree in Mechanical Engineering at Stanford University. He is a member of the Stanford Intelligent Systems Laboratory.

Katherine Driggs-Campbell Katie Driggs-Campbell received her BSE in Electrical Engineering from Arizona State University and her MS and PhD in Electrical Engineering and Computer Science from the University of California, Berkeley. She is currently an Assistant Professor in the ECE Department at the University of Illinois at Urbana-Champaign. Her research focuses on exploring and uncovering structure in complex human-robot systems to create more intelligent, interactive autonomy. She draws from the fields of optimization, learning & AI, and control theory, applied to human-robot interaction and autonomous vehicles.

Mykel J. Kochenderfer Mykel J. Kochenderfer received B.S. and M.S. degrees in Computer Science from Stanford University in 2003 and a Ph.D. degree from the University of Edinburgh in 2006. He was a member of the technical staff at MIT Lincoln Laboratory, where he worked on airspace modeling and aircraft collision avoidance. He is now an Associate Professor of Aeronautics and Astronautics at Stanford University and the director of the Stanford Intelligent Systems Laboratory.
REFERENCES

[1] A. Doshi and M. M. Trivedi, “Tactical Driver Behavior Prediction and Intent Inference: A Review,” IEEE International Conference on Intelligent Transportation Systems (ITSC), 2011.

[2] S. Lefèvre, D. Vasquez, and C. Laugier, “A survey on motion prediction and risk assessment for intelligent vehicles,” Robomech., vol. 1, 2014.

[3] E. Ohn-Bar and M. M. Trivedi, “Looking at Humans in the Age of Self-Driving and Highly Automated Vehicles,” IEEE Transactions on Intelligent Vehicles, 2016.

[4] J. A. Michon, “A critical view of driver behavior models: what do we know, what should we do?” In Human Behavior and Traffic Safety, 1985, pp. 485–520.

[5] T. A. Ramey, “Models of Driving Behavior: A Review of Their Evolution,” Accident Analysis and Prevention, vol. 26, no. 6, pp. 733–750, 1994.

[6] M. Blackstone and M. McDonald, “Car-following: a historical review,” Transportation Research Part F: Traffic Psychology and Behaviour, pp. 181–196, 1999.

[7] K. Bengler, K. Dietmayer, B. Farber, M. Maurer, C. Stiller, and H. Winner, Three decades of driver assistance systems: Review and future perspectives, 2014.

[8] A. F. Palomo, S. Lefèvre, G. Schüldich, J. Kong, and F. Borrelli, “Automated driving: The role of forecasts and uncertainty: A control perspective,” European Journal of Control, vol. 24, pp. 14–32, 2015.

[9] W. Zhan, A. L. de Fortelle, Y. T. Chen, C. Y. Chan, and M. Tomizuka, “Probabilistic prediction from planning perspective: Problem formulation, representation simplification and evaluation metric,” in IEEE Intelligent Vehicles Symposium (IV), 2018, pp. 1150–1156.

[10] S. Riedmaier, T. Ponn, S. Member, D. Ludwig, B. Schick, and F. Dierich, “Statistical Based Safety Assessment of Automated Vehicles,” IEEE Access, vol. 8, pp. 87 456–87 477, 2020.

[11] H. W. Kuhn, “11. Extensive Games and the Problem of Information,” in Contributions to the Theory of Games, Volume II. Princeton: Princeton University Press, 1953, pp. 193–216.

[12] M. Plochl and J. Edelmann, “Driver models in autonomous driving application,” Vehicle System Dynamics, vol. 45, pp. 7–8, 2007.

[13] D. Koller and N. Friedman, Probabilistic Graphical Models: Principles and Techniques - Adaptive Computation and Machine Learning: The MIT Press, 2009.

[14] N. Deo, A. Rangesh, and M. M. Trivedi, “How would surround vehicles move? A unified Framework for Maneuver Classification and Motion Prediction,” IEEE Transactions on Intelligent Vehicles, vol. 3, no. 1, pp. 129–140, 2018.

[15] L. Guo and Y. Jia, “Modeling, Learning and Prediction of Longitudinal Behaviors of Human-Driven Vehicles by Incorporating Internal Human Decision-Making Process using Inverse Model Predictive Control,” in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2019, pp. 5279–5283.

[16] X. Ying, C. Iy, H. Wang, H. Yang, Y. Ai, D. Cao, E. Velenis, and F. Y. Wang, “Driver Lane Change Intention Inference for Intelligent Vehicles: Framework, Survey, and Challenges,” IEEE Transactions on Vehicular Technology, vol. 9545, no. c, pp. 1–1, 2019.

[17] J. Schulz, K. Hirsenkorn, J. Lochner, M. Werling, and D. Burschka, “Evaluation of collaborative maneuvers through cooperation: multi-agent planning,” in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2019, pp. 3426–3431.

[18] S. Riedmaier, T. Ponn, S. Member, D. Ludwig, B. Schick, and F. Dierich, “Statistical Based Safety Assessment of Automated Vehicles,” IEEE Access, vol. 8, pp. 87 456–87 477, 2020.

[19] H. W. Kuhn, “11. Extensive Games and the Problem of Information,” in Contributions to the Theory of Games, Volume II. Princeton: Princeton University Press, 1953, pp. 193–216.

[20] J. A. Michon, “A critical view of driver behavior models: what do we know, what should we do?” In Human Behavior and Traffic Safety, 1985, pp. 485–520.

[21] S. Tryhub and G. L. Masala, “Detection of driver’s inattention: a real-time deep learning approach,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 3993–3998.

[22] D. S. González, M. Gaarz, J. S. Dibangoye, C. Laugier, D. S. González, M. Garzón, J. S. Dibangoye, and C. Laugier, “Human-Like Decision-Making for Automated Driving in Highways,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), IEEE, 2019, pp. 2087–2094.

[23] L. Sun, W. Zhan, and M. Tomizuka, “Probabilistic Prediction of Interactive Driving Behavior via Hierarchical Inverse Reinforcement Learning,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2018.

[24] J. Hardy and M. Campbell, “Contingency planning over probabilistic obstacle predictions for autonomous road vehicles,” IEEE Transactions on Robotics, 2013.

[25] E. Galceran, A. G. Cunningham, R. M. Eustice, and E. Olson, “Multi-policy decision-making for autonomous driving via change-point-based behavior prediction: Theory and experiment,” Autonomous Robots, vol. 41, no. 6, pp. 1367–1382, 2017.
REFERENCES
REFERENCES

[101] J Hayakawa and B Dariush, “Ego-motion and Surrounding Vehicle State Estimation Using a Monocular Camera,” in IEEE International Vehicles Symposium (IV), 2019, pp. 2550–2556.

[102] A. Houenou, P. Bonnifait, V. Cherfaoui, and W. Yao, “Vehicle trajectory prediction based on motion model and maneuver recognition,” in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), IEEE, 2015, pp. 4363–4369.

[103] Y. Hu, W. Zhan, and M. Tomizuka, “Probabilistic Prediction of Vehicle Semantic Intention and Motion,” in IEEE Intelligent Vehicles Symposium (IV), 2018.

[104] Y. Hu, W. Zhan, and M. Tomizuka, “A Framework for Probabilistic Generic Traffic Scene Prediction,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), IEEE, 2016, pp. 2790–2796.

[105] Y. Hu, W. Zhan, L. Sun, and M. Tomizuka, “Multi-modal Probabilistic Prediction of Interactive Behavior via an Interpretable Model,” in IEEE Intelligent Vehicles Symposium (IV), 2019, pp. 557–563.

[106] Y. Hu, L. Sun, and M. Tomizuka, “Generic Prediction Architecture Considering both Rational and Irrational Driving Behaviors,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 3539–3546.

[107] Y. Hu, A. Nakhaei, M. Tomizuka, and K. Fujimura, “Interaction-aware Decision Making with Adaptive Strategies under Merging Scenarios,” in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2020, pp. 151–158.

[108] C. Hubmann, N. Queetschlich, J. Schäule, J. Bernhard, D. Althoff, and C. Stiller, “A POMDP Maneuver Planner For Occlusions in Urban Scenarios,” in IEEE Intelligent Vehicles Symposium (IV), 2019, pp. 2172–2179.

[109] D. Isele, “Interactive Decision Making for Autonomous Vehicles in Dense Traffic,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 2141–2147.

[110] A. Jain, H. S. Koppula, S. Soh, B. Raghavan, A. Singh, and A. Saxena, “BrainInCars: Car That Knows Before You Do with Sensory-Fusion Deep Learning Architecture,” ArXiv, 2016.

[111] J. Jiao and K. Han, “Probabilistic modeling of vehicle acceleration and state propagation with long short-term memory neural networks,” in IEEE Intelligent Vehicles Symposium (IV), 2019, pp. 2236–2242.

[112] B. C. Jügade, A. C. Victoria, and V. B. Cherfaoui, “Shared Driving Control between Human and Autonomous Driving Systems via Conflict resolution using Non-Cooperative Game Theory,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 2141–2147.

[113] N. Kaempchen, K. Weiss, M. Schaefer, and K. C. Dietmayer, “IMM object tracking for high dynamic driving maneuvers,” in IEEE Intelligent Vehicles Symposium (IV), 2004, pp. 825–830.

[114] E. Kaiser, C. Hermes, C. Wohler, H. Thümmler, and F. Kummert, “Recognition of situation classes at road intersections,” in IEEE International Conference on Robotics and Automation (ICRA), 2010.

[115] A. Kesting and M. Treiber, “Calibrating car-following models by using trajectory data methodological study,” Transportation Research Record, no. 2086, pp. 148–156, 2008.

[116] A. Kesting, M. Treiber, and D. Helbing, “Enhanced intelligent driver model to access the impact of driving strategies on traffic capacity,” in IEEE Transactions on Intelligent Transportation Systems (ITSC), vol. 11, no. 4, pp. 797–802, 2013.

[117] C. Kuefler, J. Morton, T. Wheeler, and M. Kochenderfer, “Imitating driver behavior with generative adversarial networks,” in IEEE Intelligent Vehicles Symposium (IV), 2017.

[118] P. Kumar, M. Perrollaz, S. Lefevre, and C. Laugier, “Learning-based approach for online lane change intention prediction,” in IEEE Intelligent Vehicles Symposium (IV), no. IV, pp. 797–802, 2013.

[119] C. Laugier, I. E. Paromtchik, M. Perrollaz, M. Yong, J. D. Voder, C. Tay, K. Mehnachka, and A. Négre, “Probabilistic analysis of dynamic scenario and collision risk to improve driving safety,” IEEE Intelligent Transportation Systems Magazine, 2011.

[120] N. Lee, W. Choi, P. Vernaza, C. B. Choy, P. H. S. Torr, and M. Chandraker, “DESIRE: Distant Future Prediction in Dynamic Scenes with Interacting Agents,” in IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), 2017.

[121] S. Lefèvre, C. Laugier, and J. Ibáñez-Guzmán, “Risk Assessment at Road Intersections: Comparing Intention andExpectation,” IEEE Intelligent Vehicles Symposium (IV), 2012.

[122] S. Lefèvre, C. Laugier, and J. Ibáñez-Guzmán, “Evaluating Risk at Road Intersections by Detecting Conflicting Intentions,” in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), INRIA, 2012, RR-7904.

[123] S. Lefèvre, C. Sun, R. Bajcsy, and C. Laugier, “Comparision of parametric and non-parametric approaches for vehicle speed prediction,” in American Control Conference (ACC), 2014, pp. 3494–3499.

[124] S. Lefèvre, A. Carvalho, and F. Borrelli, “Autonomous car following: A learning-based approach,” in IEEE Intelligent Vehicles Symposium (IV), 2015.

[125] S. Lefèvre, J. Ibáñez-Guzmán, and C. Laugier, “Context-based Estimation of Driver Intent at Road Intersections,” Cites, 2011.

[126] D. Lenz, F. Diehl, M. T. Le, and A. Knoll, “Deep neural networks for Markovian interactive scene prediction in highway scenarios,” in IEEE Intelligent Vehicles Symposium (IV), IEEE, 2018, pp. 689–692.

[127] S. Levine and V. Koltun, “Continuous Inverse Optimal Control with Locally Optimal Examples,” in International Conference on Machine Learning (ICML), 2012.

[128] I. Li, D. W. Ouyler, M. Zhang, Y. Yildiz, I. Kolmanovsky, and A. R. Girard, Game Theoretic Modeling of Driver and Vehicle Interactions for Verification and Validation of Autonomous Vehicle Control Systems, 2017.

[129] Z. Li, C. Gong, C. Lu, J. Gong, J. Lu, Y. Xu, and F. Hu, “Transferable Driver Behavior Learning via Distribution Adaption in the Lane Change Scenario,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 3999–4005.

[130] T Li, J. Wu, and C. Chan, “Evolutionary Learning in Decision Making for Tangential Lane Changing Scenarios,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 1826–1831.

[131] X. Li, X. Ying, and M. C. Chuah, “GRIP: Graph-based Interaction-aware Trajectory Prediction,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 3960–3966.

[132] J Li, H. Ma, W. Zhan, and M. Tomizuka, “Coordination and Trajectory Prediction for Vehicle Interactions via Bayesian Generative Modeling,” in IEEE Intelligent Vehicles Symposium (IV), 2019, pp. 2496–2503.

[133] Z. Li, C. Gong, C. Lu, J. Gong, J. Lu, and F. Hu, “Transferable Driver Behavior Learning via Distribution Adaption in the Lane Change Scenario,” in IEEE Intelligent Vehicles Symposium (IV), 2019, pp. 193–200.

[134] M. Liebner, M. Baumann, F. Klanner, and C. Stiller, “Driver Intent Inference at Urban Intersections using the Intelligent Driver Model,” IEEE Intelligent Vehicles Symposium (IV), 2012.

[135] X. Lin, J. Zhang, J. Shang, Y. Wang, H. Yu, and X. Zhang, “Decision Making through Oclcluded Intersections for Autonomous Driving,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 2449–2455.

[136] A. Liu and D. Salvucci, “Modeling and Prediction of Human Driver Behavior,” International Conference on Human-Computer Interaction, 2001.

[137] J Liu, Y. Luo, H. Xiao, Z. Xue, H. Huang, and Z. Zhong, “An Integrated Approach to Probabilistic Vehicle Trajectory Prediction via Driver Characteristic and Intention Estimation,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 3526–3532.

[138] W. Luo, B. Yang, and F. Damerow, “If You Must, Please Provide Real-Time End-To-End 3D Detection, Tracking and Motion Forecasting with a Single Convolutional Net,” in IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), 2018.

[139] H. Ma, J. Li, W. Zhan, and M. Tomizuka, “Wasserstein Generative Learning with Kinematic Constraints for Probabilistic Interactive Driving Behavior Prediction,” in IEEE Intelligent Vehicles Symposium (IV), 2019, pp. 2477–2483.
APPENDIX A

[205] E. Ward and J. Folkesson, “Multi-classification of Driver Intentions in Yielding Scenarios,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2015.

[206] T. A. Wheeler, P. Robbel, and M. J. Kochenderfer, “Analysis of Microscopic Behavior Models for Probabilistic Modeling of Driver Behavior,” Tech. Rep., 2016.

[207] J. Wiest, M. Hoffken, U. Kreßel, and K. Dietmayer, “Probabilistic trajectory prediction with Gaussian mixture models,” IEEE Intelligent Vehicles Symposium (IV), pp. 141–146, 2012.

[208] J. Wiest, F. Kunz, U. Kreßel, and K. Dietmayer, “Incorporating categorical information for enhanced probabilistic trajectory prediction,” in International Conference on Machine Learning and Applications (ICMLA), 2013.

[209] C. Wissing, T. Nattermann, K. H. Glander, and T. Bertram, “Probabilistic time-to-lane-change prediction on highways,” IEEE Intelligent Vehicles Symposium (IV), no. IV, pp. 1452–1457, 2017.

[210] H. Woo, Y. Ji, H. Kono, Y. Tamura, Y. Kuroda, T. Sugano, Y. Yamamoto, A. Yamashita, and H. Asama, “Lane-Change Detection Based on Vehicle-Trajectory Prediction,” IEEE Robotics and Automation Letters, vol. 2, no. 2, pp. 1109–1116, 2017.

[211] L. Wörle, J. von Schleinitz, M. Graf, and A. Eichberger, “Driver Detection from Objective Criteria Describing the Driving Style of Race Car Drivers,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2019, pp. 1198–1203.

[212] S. Worrall and E. Nebot, “A probabilistic method for detecting impending vehicle interactions,” in IEEE International Conference on Robotics and Automation (ICRA), 2008, pp. 1767–1791.

[213] Z Yan, K Yang, Z Wang, B Yang, T Kaizuka, and K Nakano, “Time to lane change and completion prediction based on Gated Recurrent Unit Network,” in IEEE Intelligent Vehicles Symposium (IV), 2019, pp. 102–107.

[214] F. Yan, M. Eilers, L. Weber, and M. Baumann, “Investigating Initial Driver Intention on Overtaking on Rural Roads,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), IEEE, 2019.

[215] S. Yoon and D. Kum, “The multilayer perceptron approach to lateral motion prediction of surrounding vehicles for autonomous vehicles,” IEEE Intelligent Vehicles Symposium (IV), no. IV, pp. 1307–1312, 2016.

[216] W. Yuan, Z. Li, and C. Wang, “Lane-change prediction method for adaptive cruise control system with hidden Markov model,” Advances in Mechanical Engineering, vol. 10, no. 9, p. 168714031880293, 2018.

[217] W. Zhan, L. Sun, Y. Hu, J. Li, and M. Tomizuka, “Towards a Fatality-Aware Benchmark of Probabilistic Reaction Prediction in Highly Interactive Driving Scenarios,” in IEEE International Conference on Intelligent Transportation Systems (ITSC), 2018, pp. 3274–3280.

[218] J. Zhang and B. Roessler, “Situation Analysis and Adaptive Risk Assessment for Intersection Safety Systems in Advanced Assisted Driving,” in Autonome Mobile Systeme 2009, R. Dillmann, J. Beyrer, C. Stiller, J. M. Zöllner, and T. Girdele, Eds., Berlin, Heidelberg: Springer Berlin Heidelberg, 2009, pp. 249–258.
| Ref | Tasks Addressed | Ref | Tasks Addressed | Ref | Tasks Addressed |
|-----|----------------|-----|----------------|-----|----------------|
| 54  | – – – – – – – TE | 63  | – – – – – – – MP | 153 | – – – – – – – TE |
| 57  | SE IE – – – – MP | 66  | – – – – – – – IM | 156 | – – – – – – – IM |
| 58  | – – – – – – – TE | 67  | – – – – – – – MP | 157 | – – – – – – – BP |
| 59  | – – – – – – – MP | 68  | – – – – – – – TS | 158 | – – – – – – – BP |
| 60  | – – – – – – – TS | 69  | – – – – – – – BP | 159 | – – – – – – – TS |
| 61  | – – – – – – – BP | 70  | – – – – – – – BP | 160 | – – – – – – – BP |
| 62  | – – – – – – – BP | 71  | – – – – – – – TS | 161 | – – – – – – – TS |
| 63  | – – – – – – – TS | 72  | – – – – – – – BP | 162 | – – – – – – – BP |

**Tasks** addressed by each model: State Estimation (SE), Intention Estimation (IE), Trait Estimation (TE), Motion Prediction (MP), Anomaly Detection (AD), Risk Estimation (RE), Traffic Simulation (TS), Behavior Imitation (Im). Behavior Planning (BP) denotes that the model is introduced in the context of a behavior planning framework.
**TABLE 3**

**State Estimation keywords** and the associated references.

| Algorithm Keyword                              | References |
|-----------------------------------------------|------------|
| Bayesian Occupancy Filter                     | 133, 147, 204 |
| Dynamic Bayesian Network                      | 77, 108, 208 |
| Extended Kalman Filter                        | 117          |
| Kalman Filter                                 | 69, 106, 121, 205, 210, 216 |
| Markov State Space Model                      | 122          |
| Moving Average Filter                         | 125          |
| Multi-Perspective Tracker                     | 123          |
| Multiple Model Unscented Kalman Filter        | 124          |
| Particle Filter                               | 97, 132, 185 |
| Algorithm Keyword | References |
|-------------------|------------|
| Adaptive Cruise Control Policy | 60, 233 |
| Bayesian Changepoint Estimation | 60, 233 |
| Bayesian Filter | 60, 73, 120, 135, 146, 155, 160, 218 |
| Bayesian Network | 60, 120, 135, 146, 155, 160, 218 |
| Conditional Probability Table | 60, 135, 141, 135, 146, 155, 160, 161, 215 |
| Conditional Variational AutoEncoder | 60, 135, 141, 135, 146, 155, 160, 161, 215 |
| Context Aware Scene Representation | 60, 141, 135, 146, 155, 160, 161, 215 |
| Context Dependent | 60, 141, 135, 146, 155, 160, 161, 215 |
| Convolutional Neural Network | 60, 135, 146, 155, 160, 218 |
| Convolutional Social Pooling | 60, 135, 146, 155, 160, 218 |
| Counterfactual Reasoning | 60, 135, 146, 155, 160, 218 |
| Coupled Hidden Markov Model | 60, 135, 146, 155, 160, 218 |
| Dirichlet Process | 60, 135, 146, 155, 160, 218 |
| Dynamic Bayesian Network | 60, 135, 146, 155, 160, 218 |
| Dynamic Time Warping | 60, 135, 146, 155, 160, 218 |
| Gated Recurrent Unit Network | 60, 135, 146, 155, 160, 218 |
| Gaussian Mixture Model | 60, 135, 146, 155, 160, 218 |
| Gaussian Process | 60, 135, 146, 155, 160, 218 |
| Gaussian Radial Basis Kernel Function | 60, 135, 146, 155, 160, 218 |
| Gibbs Sampling | 60, 135, 146, 155, 160, 218 |
| Hidden Markov Model | 60, 135, 146, 155, 160, 218 |
| Importance Weighting | 60, 135, 146, 155, 160, 218 |
| Indicator Functions | 60, 135, 146, 155, 160, 218 |
| Intelligent Driver Model | 60, 135, 146, 155, 160, 218 |
| Interacting Multiple-Model Kalman Filter | 60, 135, 146, 155, 160, 218 |
| K-Nearest Neighbors | 60, 135, 146, 155, 160, 218 |
| Kalman Filter | 60, 135, 146, 155, 160, 218 |
| Least Common Subsequence | 60, 135, 146, 155, 160, 218 |
| Long Short-Term Memory Network | 60, 135, 146, 155, 160, 218 |
| Marginal Composition | 60, 135, 146, 155, 160, 218 |
| Marginal Probability Distribution | 60, 135, 146, 155, 160, 218 |
| Markov State Space Model | 60, 135, 146, 155, 160, 218 |
| Mind Tracking | 60, 135, 146, 155, 160, 218 |
| Minimizing Overall Braking Induced by Lane Changes | 60, 135, 146, 155, 160, 218 |
| Mixture Density Network | 60, 135, 146, 155, 160, 218 |
| Mixture of Experts | 60, 135, 146, 155, 160, 218 |
| Multi-Layer Perceptron | 60, 135, 146, 155, 160, 218 |
| Multiple Model Unscented Kalman Filter | 60, 135, 146, 155, 160, 218 |
| Neural Network | 60, 135, 146, 155, 160, 218 |
| Pairwise Probability Coupling | 60, 135, 146, 155, 160, 218 |
| Particle Filter | 60, 135, 146, 155, 160, 218 |
| Piecewise Auto-Regressive Model | 60, 135, 146, 155, 160, 218 |
| Polynomial Classifier | 60, 135, 146, 155, 160, 218 |
| Probabilistic Graphical Model | 60, 135, 146, 155, 160, 218 |
| Quadratic Discriminant Analysis | 60, 135, 146, 155, 160, 218 |
| Quantile Regression Forest | 60, 135, 146, 155, 160, 218 |
| Random Forest | 60, 135, 146, 155, 160, 218 |
| Recurrent Neural Network | 60, 135, 146, 155, 160, 218 |
| Relevance Vector Machine | 60, 135, 146, 155, 160, 218 |
| Rule-Based | 60, 135, 146, 155, 160, 218 |
| Single Layer Perceptron | 60, 135, 146, 155, 160, 218 |
| Stochastic Switched Autoregressive | 60, 135, 146, 155, 160, 218 |
| Exogenous Model | 60, 135, 146, 155, 160, 218 |
| Support Vector Machine | 60, 135, 146, 155, 160, 218 |
| Two-player game | 60, 135, 146, 155, 160, 218 |

### Theory

| Algorithm Keyword | References |
|-------------------|------------|
| Clustering | 29, 77, 172 |
| Dempster Schafer Theory | 172 |
| Distribution Adaptation | 146 |
| Domain Adaptation | 146 |
| Game Theory | 146 |
| Interaction Detection | 146 |
| Inverse Reinforcement Learning | 146 |
| Level K Reasoning | 111 |
| Naive Bayes | 8 |
| Nash Equilibrium | 106 |
| Partially Observable Markov Decision Process | 111 |
| Reinforcement Learning | 111 |
| Signal Detection Theory | 111 |
| Time-Series Analysis | 111 |
| Trajectory Similarity | 111 |
| Transfer Learning | 111 |
| Tree-Search Planning | 111 |

### Scope

| Algorithm Keyword | References |
|-------------------|------------|
| Advanced Driver Assistance Systems | 9 |
| Assistive Braking | 21 |
| Assistive Steering | 21 |
| Car Following | 21 |
| Collision Mitigation | 21 |
| Highway Driving | 21 |
| Lane Changing | 21 |
| Lane Keeping | 21 |
| Merging at Intersection | 21 |
| Overtaking | 21 |
| Traffic Weaving | 21 |
| Unsignalized Intersections | 21 |
| Urban Driving | 21 |
| Vehicle to Vehicle communication | 21 |
| Yielding at Intersection | 21 |

### Evaluation

| Algorithm Keyword | References |
|-------------------|------------|
| Area Under the ROC Curve | 16, 169, 166, 168 |
| Balanced Accuracy | 62, 129 |
| Balanced Precision | 62, 129 |
| Brier Metric | 129 |
| Classification Accuracy | 129 |
| Classification Precision | 129 |
| Confusion Matrix | 129 |
| F1 Score | 129 |
| False Negative Rate | 129 |
| False Positive Rate | 129 |
| Max Time to Detection | 129 |
| Mean Lateral Offset before Detection | 129 |
| Mean Time to Detection | 129 |
| Median Time to Detection | 129 |
| Min Time to Detection | 129 |
| Negative Log Likelihood | 129 |
| Precision over Recall | 129 |
| Receiver Operating Characteristic curve | 129 |
| Root Mean Square Error | 129 |
| Standard Deviation of Time to Detection | 129 |
| True Positive Rate | 129 |
### Table 5

| Ref | Intention Space | Hypoth. | Paradigm |
|-----|----------------|----------|----------|
| 150 | - - - - - - - - | Re - Ba - - - | - - - - - - |
| 151 | - - - - - - - - | S - - - | - - - - - - |
| 152 | Ro - - - - - - - | Re SS - BB - | - - - - - - |
| 153 | Ro - - - - - - - | Re SS - | - - - - - - |
| 154 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 155 | Ro - - - - - - - | Re SS - | - - - - - - |
| 156 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 157 | Ro - - - - - - - | Re SS - | - - - - - - |
| 158 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 159 | Ro - - - - - - - | Re SS - | - - - - - - |
| 160 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 161 | Ro - - - - - - - | Re SS - | - - - - - - |
| 162 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 163 | Ro - - - - - - - | Re SS - | - - - - - - |
| 164 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 165 | Ro - - - - - - - | Re SS - | - - - - - - |
| 166 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 167 | Ro - - - - - - - | Re SS - | - - - - - - |
| 168 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 169 | Ro - - - - - - - | Re SS - | - - - - - - |
| 170 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 171 | Ro - - - - - - - | Re SS - | - - - - - - |
| 172 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 173 | Ro - - - - - - - | Re SS - | - - - - - - |
| 174 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 175 | Ro - - - - - - - | Re SS - | - - - - - - |
| 176 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 177 | Ro - - - - - - - | Re SS - | - - - - - - |
| 178 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 179 | Ro - - - - - - - | Re SS - | - - - - - - |
| 180 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 181 | Ro - - - - - - - | Re SS - | - - - - - - |
| 182 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 183 | Ro - - - - - - - | Re SS - | - - - - - - |
| 184 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 185 | Ro - - - - - - - | Re SS - | - - - - - - |
| 186 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 187 | Ro - - - - - - - | Re SS - | - - - - - - |
| 188 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 189 | Ro - - - - - - - | Re SS - | - - - - - - |
| 190 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 191 | Ro - - - - - - - | Re SS - | - - - - - - |
| 192 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 193 | Ro - - - - - - - | Re SS - | - - - - - - |
| 194 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 195 | Ro - - - - - - - | Re SS - | - - - - - - |
| 196 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 197 | Ro - - - - - - - | Re SS - | - - - - - - |
| 198 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 199 | Ro - - - - - - - | Re SS - | - - - - - - |
| 200 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 201 | Ro - - - - - - - | Re SS - | - - - - - - |
| 202 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 203 | Ro - - - - - - - | Re SS - | - - - - - - |
| 204 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 205 | Ro - - - - - - - | Re SS - | - - - - - - |
| 206 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 207 | Ro - - - - - - - | Re SS - | - - - - - - |
| 208 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 209 | Ro - - - - - - - | Re SS - | - - - - - - |
| 210 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 211 | Ro - - - - - - - | Re SS - | - - - - - - |
| 212 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 213 | Ro - - - - - - - | Re SS - | - - - - - - |
| 214 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 215 | Ro - - - - - - - | Re SS - | - - - - - - |
| 216 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 217 | Ro - - - - - - - | Re SS - | - - - - - - |
| 218 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 219 | Ro - - - - - - - | Re SS - | - - - - - - |
| 220 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 221 | Ro - - - - - - - | Re SS - | - - - - - - |
| 222 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 223 | Ro - - - - - - - | Re SS - | - - - - - - |
| 224 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 225 | Ro - - - - - - - | Re SS - | - - - - - - |
| 226 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 227 | Ro - - - - - - - | Re SS - | - - - - - - |
| 228 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 229 | Ro - - - - - - - | Re SS - | - - - - - - |
| 230 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 231 | Ro - - - - - - - | Re SS - | - - - - - - |
| 232 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 233 | Ro - - - - - - - | Re SS - | - - - - - - |
| 234 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 235 | Ro - - - - - - - | Re SS - | - - - - - - |
| 236 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 237 | Ro - - - - - - - | Re SS - | - - - - - - |
| 238 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 239 | Ro - - - - - - - | Re SS - | - - - - - - |
| 240 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 241 | Ro - - - - - - - | Re SS - | - - - - - - |
| 242 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 243 | Ro - - - - - - - | Re SS - | - - - - - - |
| 244 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 245 | Ro - - - - - - - | Re SS - | - - - - - - |
| 246 | Ro - - - - - - - | Re SS - SS - | - - - - - - |
| 247 | Ro - - - - - - - | Re SS - | - - - - - - |
| 248 | Ro - - - - - - - | Re SS - SS - | - - - - - - |

**Intention Estimation models** classified according to **Intention Space**: Routes through road network (Ro), Joint Configurations (Co), Trajectory Homotopies (Ho), Longitudinal modes (Lo), Lateral modes (La), Emergency Maneuvers (EM), Intentional vs. Unintentional (IU), Unspecified (?), Context Dependent (CD); **Hypothesis Representation**: Point Estimate (S), Discrete Distribution (D), Discrete Distribution over scenarios (DS); **Estimation Paradigm**: Recursive (Re), Single-Shot (SS), Bayesian (Ba), Black Box (BB), Trajectory Similarity (TS), Policy Similarity (PS), Game Theoretic (GT).
## TABLE 6
**Trait Estimation keywords and the associated references.**

| Algorithm Keyword | References |
|-------------------|------------|
| Trait Estimation: |            |
| Control policy parameters (Pa), Reward model parameters (Re), Non-interpretable control policy parameters (NP), Physiological trait parameters (Ph), Attention parameters (At) |            |
| Hypothesis Representation: |            |
| Point Estimate (S), Particle set (P) |            |
| Paradigm: Online (On), Offline (Off), Heuristic (H), Optimization (Op), Bayesian (B), Inverse Reinforcement Learning (IRL), Contextually Varying (CV) |            |

## TABLE 7
**Trait Estimation models** classified according to: **Trait Space**: Control policy parameters (Pa), Reward model parameters (Re), Non-interpretable control policy parameters (NP), Physiological trait parameters (Ph), Attention parameters (At) **Hypothesis Representation**: Point Estimate (S), Continuous Distribution (C), Particle set (P) **Paradigm**: Online (On), Offline (Off), Heuristic (H), Optimization (Op), Bayesian (B), Inverse Reinforcement Learning (IRL), Contextually Varying (CV).
| Algorithm Keyword                                                                 | References |
|---------------------------------------------------------------------------------|------------|
| Adaptive                                                                         | [20], [21]|
| Adaptive Cruise Control Policy                                                   | [20], [21]|
| Bezier Curves                                                                   | [20], [21]|
| Conditional Expectation                                                          | [20], [21]|
| Conditional Variational AutoEncoder                                              | [20], [21]|
| Constant Acceleration                                                            | [20], [21]|
| Constant Steering Angle                                                          | [20], [21]|
| Constant Turn Rate Constant Tangential Acceleration                              | [20], [21]|
| Dynamic Forest                                                                   | [20], [21]|
| Empirical Reachable Set                                                          | [20], [21]|
| Extended Kalman Filter                                                           | [20], [21]|
| Foresighted Driver Model                                                         | [20], [21]|
| Forward Reachable Set                                                            | [20], [21]|
| Gated Recurrent Unit Network                                                     | [20], [21]|
| Gaussian Mixture Model                                                           | [20], [21]|
| Gaussian Process                                                                 | [20], [21]|
| Gaussian Radial Basis Kernel Function                                            | [20], [21]|
| Generative Adversarial Network                                                   | [20], [21]|
| Gibbs Sampling                                                                   | [20], [21]|
| Gippy Car Following Model                                                        | [20], [21]|
| Graph Neural Network                                                             | [20], [21]|
| Hidden Markov Model                                                              | [20], [21]|
| Hierarchical Mixture of Experts                                                  | [20], [21]|
| Intelligent Driver Model                                                         | [20], [21]|
| Interaction Graph                                                                | [20], [21]|
| Iterative Semi-Network Form Game                                                | [20], [21]|
| Kalman Prediction                                                                | [20], [21]|
| Linear Gaussian                                                                  | [20], [21]|
| Long Short-Term Memory Network                                                   | [20], [21]|
| Markov Chain                                                                     | [20], [21]|
| Minimizing Overall Braking Induced by Lane Changes                               | [20], [21]|
| MITSIM Driver Model                                                              | [20], [21]|
| Mixture of Experts                                                               | [20], [21]|
| Monte Carlo Simulation                                                           | [20], [21]|
| Monte Carlo Tree Search                                                          | [20], [21]|
| Multi-Fidelity                                                                   | [20], [21]|
| Multi-Layer Perceptron                                                          | [20], [21]|
| Neural Network                                                                   | [20], [21]|
| Optimum Velocity Model                                                           | [20], [21]|
| Particle Filter                                                                  | [20], [21]|
| Perfect Information Game                                                         | [20], [21]|
| Piecewise Auto-Regressive Model                                                  | [20], [21]|
| Piecewise Uniform Distribution                                                   | [20], [21]|
| Potential Field                                                                  | [20], [21]|
| Proportional Derivative Feedback Control                                         | [20], [21]|
| Quantile Regression Forest                                                       | [20], [21]|
| Random Forest                                                                    | [20], [21]|
| Recurrent Neural Network                                                         | [20], [21]|
| Relational Recurrent Neural Network                                              | [20], [21]|
| Rule-Based                                                                       | [20], [21]|
| Simultaneous Game                                                                | [20], [21]|
| Splines                                                                          | [20], [21]|
| Stackelberg Game                                                                 | [20], [21]|
| Static Gaussian                                                                  | [20], [21]|
| SUMO Model                                                                       | [20], [21]|
| Switching                                                                        | [20], [21]|
| Tabular policy                                                                   | [20], [21]|
| Time-Delay Neural Network                                                        | [20], [21]|
| Two-player game                                                                  | [20], [21]|
| Variational                                                                     | [20], [21]|
| Variational Autoencoder                                                          | [20], [21]|
| Wasserstein Auto Encoder                                                         | [20], [21]|

| Algorithm Keyword                                                                 | References |
|---------------------------------------------------------------------------------|------------|
| Architecture                                                                    | [20], [21]|
| Motion Prediction keywords                                                       | [20], [21]|
| and the associated references.                                                  | [20], [21]|

TABLE 8

Motion Prediction keywords and the associated references.
### TABLE 9

Motion prediction models classified according to: Vehicle dynamics model: (Veh. Model:) Four Wheel (4W), Bicycle Dynamic (BD), Bicycle Kinematic (BK), Unicycle (U), Linear (L), Spline (S), Discrete(Dc), Probabilistic (P), Learned (L). Scene-level uncertainty modeling: Multi-Scenario (M), Single-Scenario (S), Partial Scenario (P), Belief Tree (Tr). Agent-level uncertainty modeling: Gaussian (G), Gaussian Mixture (GM), Particle Set (P), Single deterministic (S), Discrete Occupancy Distribution (O), Bounding Box (BB), Spline (Sp), Reachable Set (R), Backward Reachable Set (bR). **Prediction paradigm:** Forward Simulation (FS), Independent Prediction (IP), Game Theoretic (GT).

| Ref | Veh. Model | Scene | Agent | Paradigm |
|-----|------------|-------|-------|----------|
| 56  | G          |       |       | FS       |
| 57  | G          |       |       | IP       |
| 58  | G          |       |       | GT       |
| 59  | G          |       |       | X        |
| 60  | G          |       |       | Dc       |
| 61  | G          |       |       | S        |
| 62  | G          |       |       | Tr       |
| 63  | G          |       |       | GT       |
| 64  | G          |       |       | P        |
| 65  | G          |       |       | IP       |
| 66  | G          |       |       | GT       |
| 67  | G          |       |       | X        |
| 68  | G          |       |       | IP       |
| 69  | G          |       |       | GT       |
| 70  | G          |       |       | S        |
| 71  | G          |       |       | IP       |
| 72  | G          |       |       | GT       |
| 73  | G          |       |       | X        |
| 74  | G          |       |       | IP       |
| 75  | G          |       |       | GT       |
| 76  | G          |       |       | X        |
| 77  | G          |       |       | IP       |
| 78  | G          |       |       | GT       |
| 79  | G          |       |       | X        |
| 80  | G          |       |       | IP       |
| 81  | G          |       |       | GT       |
| 82  | G          |       |       | X        |
| 83  | G          |       |       | IP       |
| 84  | G          |       |       | GT       |
| 85  | G          |       |       | X        |
| 86  | G          |       |       | IP       |
| 87  | G          |       |       | GT       |
| 88  | G          |       |       | X        |
| 89  | G          |       |       | IP       |
| 90  | G          |       |       | GT       |
| 91  | G          |       |       | X        |
| 92  | G          |       |       | IP       |
| 93  | G          |       |       | GT       |
| 94  | G          |       |       | X        |
| 95  | G          |       |       | IP       |
| 96  | G          |       |       | GT       |
| 97  | G          |       |       | X        |
| 98  | G          |       |       | IP       |
| 99  | G          |       |       | GT       |
| 100 | G          |       |       | X        |
| 101 | G          |       |       | IP       |
| 102 | G          |       |       | GT       |
| 103 | G          |       |       | X        |
| 104 | G          |       |       | IP       |
| 105 | G          |       |       | GT       |
| 106 | G          |       |       | X        |
| 107 | G          |       |       | IP       |
| 108 | G          |       |       | GT       |
| 109 | G          |       |       | X        |
| 110 | G          |       |       | IP       |
| 111 | G          |       |       | GT       |
| 112 | G          |       |       | X        |
| 113 | G          |       |       | IP       |
| 114 | G          |       |       | GT       |
| 115 | G          |       |       | X        |
| 116 | G          |       |       | IP       |
| 117 | G          |       |       | GT       |
| 118 | G          |       |       | X        |
| 119 | G          |       |       | IP       |
| 120 | G          |       |       | GT       |
| 121 | G          |       |       | X        |
| 122 | G          |       |       | IP       |
| 123 | G          |       |       | GT       |

**Prediction paradigm:** Forward Simulation (FS), Independent Prediction (IP), Game Theoretic (GT).