Nuclear mass systematics and nucleon-removal thresholds; application to $^{17}$Na.

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A survey of known threshold excitations of mirror systems suggests a means to estimate masses of nuclear systems that are uncertain or not known, as does a trend in the relative energies of isobaric ground states. Using both studies and known mirror-pair energy differences, we estimate the mass of the nucleus $^{17}$Na and its energy relative to the $p+^{16}$Ne threshold. This model-free estimate of the latter is larger than that suggested by recent structure models.

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The spectra of radioactive nuclei are most intriguing, especially of those at or just beyond a drip line. To date not even the masses of many are known. Of those for which some information does exist, details of spectra are often poorly known at best. Few if any excited states have been identified. Likewise, the spin-parities of many of the states that are known have not been or are uncertainly assigned. However, the advent of radioactive ion beams and their scattering from nuclei means that properties of quite exotic nuclei can be sought. One example is the specification of properties of the proton unstable $^{15}$F found from studies of $^{14}$O-$p$ scattering$^{[1,2]}$.

With light mass systems having charge number $\pi$ and neutron number $\nu$, there is often the possibility to link the structures of mirror systems. Usually there is a reasonably well known spectrum of a nucleus ($^{A+1}_Z X_{\nu=N+1}^{\pi}$), which may be treated as a compound of a neutron ($n$) with $^{A}_{Z}X_{\nu=N}$ to define a nuclear model with which, assuming charge invariance of the nuclear force and adjusting for Coulomb effects, the spectrum of the mirror, $^{A+1}_{Z+1}Q_{\nu=Z}$, may be predicted.

Of many past articles in which this symmetry has been used, just four$^{[3,4]}$ are cited as examples. In$^{[3]}$, a symmetry was applied with a shell model scheme, adapted to give the low lying structure of $^{15}$C (from $n+^{14}$C), and spectra of other mass-15 nuclei deduced. That included a spectrum for the proton-unstable $^{15}$F. In$^{[4]}$, the same systems were studied using the collective approach of a multichannel algebraic scattering (MCAS)$^{[5]}$ method. In that study, not only was a spectrum of resonance states of $^{15}$F predicted, but so also were low energy differential cross sections from the scattering of radioactive $^{14}$O ions from hydrogen; cross sections found to be in good agreement with experiment$^{[1,2]}$. Subsequently$^{[6]}$, resonances were found in the predicted energy region.

Recently, Timofeyuk and Descouvemont$^{[4]}$ and Fortune, Lacaze, and Sherr$^{[6]}$ used microscopic structure models to define the spectrum of $^{17}$C, treated as $n+^{16}$C. Then, using the charge symmetry argument, they ascertained a spectrum for $^{17}$Na (treated as $p+^{16}$Ne). Both studies expect there to be a set of resonances in the low excitation spectrum of the particle unstable $^{17}$Na and that the ground state of $^{17}$Na would be a broad resonance of spin-parity $\frac{3}{2}^+$. The centroid energy (width) has been predicted as 2.4 (1.36) MeV$^{[4]}$ and 2.7 (2.2) MeV$^{[6]}$. However there is very little actually known about the $^{17}$Na system. An early tabulation of nuclear masses$^{[5]}$ put the mass excess (from theory) for $^{17}$Na at 35.61, 35.81, and 35.84 MeV. Using 35.61 MeV for $^{17}$Na and for $^{16}$Ne is particle emissive.

Defining two mirror nuclei by $X = (^{\pi=Z}_Z X_{\nu=N})$ and $Y = (^{\pi=A}_Z Y_{\nu=Z})$, let the energies of the nucleon plus nucleus thresholds be

\[
Th(nX) = E(n+X) - E_{g.s.} \left[ (^{A+1}_{Z+1}X_{\nu=N+1}) \right]
\]

and

\[
Th(pY) = E(p+Y) - E_{g.s.} \left[ (^{A+1}_{Z+1}Y_{\nu=N}) \right].
\]

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\( \Delta(Th) \) denotes the difference \( Th(nX) - Th(pY) \). These energies for the mass-13 systems are

\[\begin{align*}
\text{C}^{13} - \text{N}^{13} & : Th(nC) = 4.95 \; ; \; Th(pC) = 1.94 \\
\Delta(Th) & = 3.01 \\
\text{B}^{13} - \text{O}^{13} & : Th(nB) = 4.88 \; ; \; Th(pN) = 1.51 \\
\Delta(Th) & = 3.37.
\end{align*}\]  

(1)  

(2)

The data values leading to the \( Th(nX), Th(pY) \), and \( \Delta(Th) \) were taken from the Ame2003 compilation [12], as are all that are specified and used hereafter.

We seek a model-free scheme to estimate ground state energies of nuclei as yet unmeasured or which are poorly established. The isospin of the core nucleus will be used as a label for these energies. For example, the values for the mirror pair, \( \text{C}^{13} - \text{N}^{13} \), are formed from the single, isospin \( T = 0 \), core nucleus, \( \text{C}^{13} \), while those for the mirror pair \( \text{B}^{13} - \text{O}^{13} \) have mirror core nuclei with \( T = 1 \), \( \text{B}^{13} \) and \( \text{N}^{13} \) respectively.

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\end{align*}\]  

The associated \( \Delta(Th) \) are plotted in Fig. 2 in which the curves are theoretical results for \( T = 0 \) core nuclei \( (N = Z = \frac{A}{2}) \) with a proton. They were found from

\[\begin{align*}
\Delta(Th) = \alpha Zhc & = \frac{197.3269602}{137.035999679} Z \frac{1}{R} \\
R & = c_1 A^\frac{1}{2} + c_2 A^{-\frac{3}{2}} + r_p.
\end{align*}\]  

Ref. [13] specifies \( c_1 = 0.94 \) and \( c_2 = 2.81 \) fm. Then by taking the proton radius to be \( r_p = 0.5 \) fm, the result displayed by the dashed curve in Fig. 2 was found. The second result displayed by the solid curve was found using a radius defined without any proton radius correction \( (r_p = 0) \) and making a nonlinear curve fit to the \( T=0 \) data set to determine the coefficients. That resulted in values of \( c_1 = 1.07585 \) fm and of \( c_2 = 1.95514 \) fm.

FIG. 1: (color online) Excitation energies of particle-emission thresholds, \( Th(nX) \) (filled squares) and \( Th(pY) \) (filled circles). The connecting lines are simply to guide the eye.

Using the Ame2003 mass tabulation, the excitation energies of nucleon emission thresholds in pairs of mirror systems were evaluated and the results are displayed in Fig. 1 for all nuclei with known mass \( (A) \) and with isospin \( (T) \) less than 3. The excitation energies are mostly positive with an odd-even staggering with the (core) mass. The zero value is emphasised by the dashed lines in each panel. Negative values mean that the compound nucleus lies beyond the appropriate nucleon drip line. Mostly these are proton emissive systems. The non-zero isospin results have mirror core nuclei \( (X, Y \text{ as denoted above}) \). These excitations also show odd-even mass staggering and, with the \( Th(pY) \) particularly, involve many more compound systems lying beyond the proton drip line.

The comparisons of the light mass results \( (A \leq 20) \) are shown on larger scale in Fig. 3. The two theoretical results are very good representations of the \( T = 0 \) data set (filled circles) and they are good representations of all the data save for one stand-out data point, associated
with the mirror pair case of $^{19}\text{N}(^{18}\text{N} + n) - ^{19}\text{Mg}(^{18}\text{Na} + p)$. Of these four nuclei, there is some information on $^{18,19}\text{N}$ but there is little or nothing known about the nuclei $^{19}\text{Mg}$ and $^{18}\text{Na}$. The latter two are very exotic both being particle emissive. With all other systems, the difference between excitation values of mirror system threshold energies show a gradual trend from ~1 to ~5 MeV over the range of light masses (to $A = 20$). Though not universal, that difference tends to increase with isospin. The mirror nuclei $^{17}\text{C}$ and $^{17}\text{Na}$ are a mirror pair of current interest $^{4, 7}$ and one might expect the difference in the $n+^{16}\text{C}$ and $p+^{16}\text{Ne}$ threshold energies relative to the ground states of $^{17}\text{C}$ and $^{17}\text{Na}$ respectively, to be ~4 MeV. Given that $^{17}\text{Na}$ lies beyond the proton drip-line, and that the $n+^{16}\text{C}$ threshold lies below the $^{17}\text{C}$ ground state by 0.728 MeV, this suggests that the ground state of $^{17}\text{Na}$ would be ~3.3 MeV above the $p+^{16}\text{Ne}$ threshold; an MeV larger than the recently anticipated $^{4}$ value ($2.4 \text{ MeV}$).

Other mass formulae have been proposed in the past $^{10, 14}$. The first $^{10}$ gave a formula for the mass excess of nuclei from which a value of 35.61 MeV was suggested for $^{17}\text{Na}$. Using the same formula to estimate the mass excess of $^{14}\text{Ne}$, they suggest that the $p+^{16}\text{Ne}$ threshold would lie 3.65 MeV below, i.e. $T\text{h}(p^{16}\text{Ne}) = 3.65 \text{ MeV}$. They also note the two proton-emission threshold would be 3.42 MeV below the $^{17}\text{Na}$ ground state. In $^{14}$ a formula for isobaric mass multiplet energies for $A < 40$ was given, but only for multiplets with $T \leq 2$. That isospin limit does not include the sextuplet of nuclei of which $^{17}\text{C}$ and $^{17}\text{Na}$ are members. However, in their Fig. 5 they did show a plot of all isovector Coulomb energies. That plot is very similar to the one for $\Delta(T\text{h})$ given in Fig. 2. Their isovector Coulomb energies for masses $A = 17$ and $A = 40$ are ~4 and ~7 MeV respectively; values close to those we find for $\Delta(T\text{h})$.

The mass of the exotic nucleus, $^{17}\text{Na}$, can also be estimated from another energy systematics. The energy differences of ground states of light mass isobars from one of their most stable is well approximated by $^{14}$

$$\epsilon = \epsilon(Z, A) = E(Z, A) - E(Z_s, A)$$

$$E(Z, A) = M_{Z,A} - Z M_{H} - (A - Z) M_{n}$$

$$- 0.6 Z(Z - 1) A^\frac{2}{3}. \quad (5)$$

Here, $Z_s$ is the charge number of the base nucleus, $M_n$ is the mass of the neutron, $M_H$ is the mass energy of the hydrogen atom. The units are MeV and the last term is the approximation for the Coulomb energy.

We have used Eq. (5) to estimate ground state energies of nuclei above one of the most stable of the isobars for masses 6 to 20 to compare against the tabulations given in the TUNL compilation $^{11, 15}$. Values of those estimates are shown in Table I. Therein even and odd mass results are shown in the left and right most set of columns respectively. The nucleus identified in the columns headed by ‘base’ are those used with the charge numbers $Z_s$ in each group. With the even-mass sets, the base entry lies on the line containing an energy that will be compared with one immediately below in what follows. The base energy for the odd-mass sets lies on the line about which pairs of results will be compared. Almost all of these results agree with the listed values $^{11, 15}$ to better than a percent. Those that do not are $^7\text{B}$ (10.44 vs 10.13), $^{11}\text{N}$ (12.13 vs 11.26), and $^{13}\text{N}$ (+0.06 vs -0.06). There is a close pairing of nuclei according to their isobaric spin, as has been noted before $^{14}$.

### Table I

| A  | $\Delta(T\text{h})$ (MeV) |
|----|-------------------|
| 6  | 0.5               |
| 7  | 1.5               |
| 8  | 2.0               |
| 9  | 2.5               |
| 10 | 3.0               |
| 11 | 3.5               |
| 12 | 4.0               |
| 13 | 4.5               |
| 14 | 5.0               |
| 15 | 5.5               |
| 16 | 6.0               |
| 17 | 6.5               |
| 18 | 7.0               |
| 19 | 7.5               |
| 20 | 8.0               |

![FIG. 3: (color online) The Coulomb shifts $\Delta(Th)$ for the light mass core nuclei. Notation is as used with Fig. 2.](image_url)

![FIG. 4: (color online) The energy differences between mirror ground state energies calculated using Eq. (5).](image_url)

For these light-mass nuclei, the energy differences between ground states of isotopic spin pairs, $D_T(=


TABLE I: Ground state gap energies (in MeV) of light mass isobars determined using Eq. (5).

| Base | A | Z | $\epsilon(Z,A)$ | Base | A | Z | $\epsilon(Z,A)$ |
|------|---|---|-----------------|------|---|---|-----------------|
| Li   | 6 | 1 | 28.19          | Li   | 7 | 2 | 11.66          |
| Li   | 6 | 2 | 4.05           | 7Li  | 7 | 4 | -0.24          |
| Li   | 6 | 4 | 3.09           | 7Li  | 7 | 5 | 10.13          |
| Be   | 8 | 2 | 28.09          | Be   | 9 | 2 | 30.91          |
| Be   | 8 | 3 | 17.02          | Be   | 9 | 3 | 14.55          |
| Be   | 8 | 5 | 16.36          | Be   | 9 | 5 | -0.46          |
| Be   | 8 | 6 | 26.32          | Be   | 9 | 6 | 13.93          |
| B    | 10 | 2 | 39.42          | B    | 11 | 3 | 34.34          |
| B    | 10 | 3 | 23.33          | B    | 11 | 4 | 12.88          |
| B    | 10 | 4 | 2.00           | B    | 11 | 6 | 0.07           |
| B    | 10 | 6 | 1.64           | B    | 11 | 7 | 11.26          |
| C    | 12 | 4 | 28.23          | C    | 13 | 5 | 15.21          |
| C    | 12 | 5 | 15.21          | C    | 13 | 7 | -0.06          |
| C    | 12 | 7 | 14.97          | C    | 13 | 8 | 14.92          |
| C    | 12 | 8 | 26.80          | C    | 13 | 8 | 14.92          |
| N    | 14 | 5 | 24.72          | N    | 15 | 5 | 32.66          |
| N    | 14 | 6 | 2.36           | N    | 15 | 6 | 11.91          |
| N    | 14 | 8 | 2.40           | N    | 15 | 8 | 0.13           |
| N    | 15 | 6 | 10.94          | N    | 15 | 8 | 10.94          |
| O    | 16 | 6 | 23.06          | O    | 17 | 6 | 26.35          |
| O    | 16 | 7 | 12.97          | O    | 17 | 7 | 11.16          |
| O    | 16 | 9 | 12.39          | O    | 17 | 9 | -0.19          |
| O    | 16 | 10| 22.20          | O    | 17 | 10| 10.90          |
| Ne   | 18 | 6 | 31.32          | Ne   | 19 | 6 | 41.00          |
| Ne   | 18 | 7 | 17.54          | Ne   | 19 | 7 | 22.53          |
| Ne   | 18 | 8 | 1.23           | Ne   | 19 | 8 | 5.64           |
| Ne   | 18 | 10| 1.10           | Ne   | 19 | 10| -0.03          |
| Ne   | 18 | 11| 7.43           | Ne   | 19 | 11| 7.43           |

$\epsilon(Z,A) - \epsilon(A-Z,A)$, are plotted in Fig. 1. Of note is that for the three separate isospin values, the trend is for the energy differences of the pairs to decrease as mass increases. We surmise that this will be the case for pairs with higher isospin values, and for the $T = 2.5$ pair of $^{17}\text{C}$ and $^{17}\text{Na}$ of particular interest herein. With that surmise, $\epsilon(11,17)$ (for $^{17}\text{Na}$) would be $\sim 25.5 \pm 0.5$ MeV above the $^{17}\text{O}$ ground state.

By inverting Eq. (5), we can find an atomic mass-energy for exotic nuclei, which for mass-17 systems are listed in Table II. The values of $\epsilon(Z,A)$ for the five known nuclei were taken from the TUNL tabulation [11, 12] and the atomic mass-energies for the neutron, $^1\text{H}$, and all mass-16 nuclei required were taken from the mass tabulation of Ame2003 [12]. All evaluated masses of the known mass-17 nuclei agree with the tabulated ones [12] to better than 1 part in a million so that the derived nucleon-core nucleus thresholds ($Th(nX)$ and $Th(pY)$) also agree with tabulated values [11, 12].

TABLE II: Mass-17 system properties deduced from inversion of Eq. (5). The base system defining $E(Z,A)$ is $^{17}\text{O}$. All energies are in units of MeV and masses are in units of $u$.

| Nucleus | $\epsilon(Z,A)$ | $M_{Z,A}$ | $Th(nX)$ | $Th(pY)$ |
|---------|-----------------|-----------|-----------|-----------|
| C       | 26.35           | 17.02259  | 0.73      | 23.33     |
| N       | 11.16           | 17.00845  | 5.88      | 13.11     |
| O       | 0.00            | 16.99914  | 4.14      | 13.78     |
| F       | -0.19           | 17.00209  | 16.8      | 0.60      |
| Ne      | 10.92           | 17.01778  | 15.6      | 1.49      |
| Na      | 25.5            | 17.03752  | 26.8      | -3.66     |

Consequently, assuming a gap energy for $^{17}\text{Na}$ above the base line of $^{17}\text{O}$ to be $25.5 \pm 0.5$, the atomic mass-energy of $M_{17\text{Na}}$ would be $17.03752 \pm 0.00054u$; $3.66 \pm 0.5$ MeV above the proton-$^{16}\text{Ne}$ threshold.

In summary, structure models [4, 6] have suggested that the centroid of the resonant ground state of $^{17}\text{Na}$ relative to the $p^{+}{^{16}\text{Ne}}$ threshold is 2.4 and 2.7 MeV. A previous mass formula suggested a value of 3.65 MeV. Considering available particle emission data, and a theoretical formulation of the same, we assess it to be $3.66 \pm 0.5$ MeV.