Astrophysical relevance of $\gamma$ transition energies

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The relevant $\gamma$ energy range is explicitly identified where additional $\gamma$ strength has to be located for having an impact on astrophysically relevant reactions. It is shown that folding the energy dependences of the transmission coefficients and the level density leads to maximal contributions for $\gamma$ energies of $2 \leq E_\gamma \leq 4$ unless quantum selection rules allow isolated states to contribute. Under this condition, electric dipole transitions dominate. These findings allow to more accurately judge the relevance of modifications of the $\gamma$ strength for astrophysics.

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Introduction. Predictions made by nuclear theory are essential for all nucleosynthesis studies but especially for those dealing with explosive processes proceeding far from the line of nuclear stability. Reactions with highly unstable nuclei appearing in stellar explosions cannot be directly studied in the laboratory and most properties required to model the reactions cannot be measured yet. Current radioactive ion beam facilities are still limited to a region around stability and often reactions cannot be measured unless either the target or residual nucleus is long-lived or stable. Because of the low interaction energies in astrophysically relevant reactions, the statistical Hauser-Feshbach model \cite{1} can be used to predict reaction rates provided the level density in the compound nucleus is sufficiently high \cite{2}. The model requires input based on nuclear structure physics, such as nuclear masses and deformations, optical model potentials, and nuclear level densities. These are exploited to calculate transmission coefficients (average widths) which, in turn, determine the reaction cross section. The reliability of the predictions hinges on two questions: (i) Is the model applicable for a given reaction and energy, and (ii) are the relevant inputs known or reliably predicted?

In the past and present, much effort has been and still is devoted to measure and understand photon strength functions. Often discussed is the electric dipole (E1) strength which exhibits a pronounced peak at the Giant Dipole Resonance (GDR) energy. The GDR can be well described by a Lorentzian although it proved necessary to introduce an additional energy dependence in the low-energy tail (see, e.g. \cite{3,4}). Recently, a number of experimental investigations indicated additional strength confined to a small energy range in the low-energy tail (see, e.g. \cite{5,6,7,8,9}). Theory provides different possibilities to explain the additional strength, such as collective vibration of a neutron skin against an inert core composed of protons and neutrons (pygmy resonance) or other collective modes (see, e.g. \cite{10,11,12,13,14,15,16}). Accordingly, the predictions regarding location and width of this additional E1 resonance vary. The possible impact on astrophysical capture reactions is frequently quoted as motivation for investigating these phenomena. It is the aim of this paper to substantiate these claims and to outline some general considerations for the importance of altered $\gamma$ strengths in astrophysics.

Energetics. The average transmission coefficient $T = 2\pi \rho(\varepsilon_\gamma)$ is the central quantity in Hauser-Feshbach type calculations, relating an average width to the level spacing $D$. The capture cross section $\sigma$ is proportional to the transmission $T_i$ in the initial channel, to the $\gamma$ transmission $T_\gamma$, and the total transmission $T_{\mathrm{tot}}$ which comprises all energetically possible channels:

$$\sigma(E_{\mathrm{proj}}) \propto \sum_{J,\pi} T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} = T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} . \quad (1)$$

For laboratory reactions, $T_i$ describes the formation of a compound nucleus from the target ground state and particle emission back into the initial channel. In astrophysical plasmas, it additionally accounts for compound formation from thermally excited target states. The formed compound state is characterized by its excitation energy $U$, spin $J$, and parity $\pi$. The excitation energy $U = S_{\mathrm{proj}} + E_{\mathrm{proj}}$ is computed from the separation energy of the projectile in the compound nucleus $S_{\mathrm{proj}}$ and the projectile energy $E_{\mathrm{proj}}$. De-excitation of the compound state by $\gamma$ emission is described by

$$T_{\gamma}^{U,J,\pi} = \sum_{{\mu=0}}^{\mu_{\max}} T_{\gamma}^{U,J,\pi} + \int_{U_{\gamma=0}}^{U} \sum_{J',\pi',\mu'} \rho(E',J',\pi') T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} = \int_{U_{\gamma=0}}^{U} \sum_{J',\pi'} \rho(E',J',\pi') T_{\gamma}^{U,J,\pi} T_{\gamma}^{U,J,\pi} \, dE' \quad (2)$$

where the first sum includes discrete states $\mu$ and an integration over a level density $\rho = 1/D$ is performed for the energy region with many, unresolved states with energy $E'$, spin $J'$, and parity $\pi'$. The transmission coefficients on the right-hand side include $\gamma$ emission with all multipole orders allowed by the standard spin and parity selection rules. Only the lowest multipole orders are important and therefore most Hauser-Feshbach calculations only include E1 and M1 transitions, a few also E2. Here the descriptions of \cite{2,17} were used for E1 and M1. However, the following discussion focuses on E1 transitions because the results can be understood in terms of the electric dipole alone. For the discussed cases where the level density enters, E1 is dominating. Neglecting M1...
for charged projectiles because the relevant projectile energies are below 100 keV which is almost negligible compared to nucleosynthesis processes, the relevant neutron energies are in the range 0 \( \leq E_\gamma \leq S_a + E_a \). For astrophysical neutron capture is \( E_a \ll S_a \).

The energy range of the emitted photons are in the range 0 \( \leq E_\gamma \leq S_a + E_a \). Thus, the possible photon energies \( E_\gamma \) can only be in the range 0 \( \leq E_\gamma \leq S_a + E_a \). For astrophysical neutron capture is \( E_a \ll S_a \).

 totally changes the results only by a few percent. This is in agreement with experiment 18.

From the above it follows that the energies of the emitted photons are in the range 0 \( \leq E_\gamma \leq S_{proj} + E_{proj} \). A sketch of the situation is given in Fig. 1. In astrophysical nucleosynthesis processes, the relevant neutron energies are below 100 keV which is almost negligible compared to neutron separation energies of several MeV, even for very neutron-rich, unstable nuclei. The situation is different for charged projectiles because the relevant projectile energies are shifted to 5–10 MeV, depending on the charges of target and projectile 10.

The transmission coefficient \( T^L \) for \( \gamma \) emission with multipolarity \( L \) is related to the (downward) strength function \( f \) by \( T^L_\gamma = 2 \pi E_\gamma^{2L+1} f(E_\gamma) \). It is to be noted that only for the strength function \( f \) defined in this way there is a direct connection to the Hauser-Feshbach transmission coefficients. There are many approaches to derive \( f \), each leading to a basic energy dependence of the \( E_1 \) transmission given by a Lorentzian

\[
T(E_\gamma) \propto \frac{\Gamma_{GDR} E_\gamma^4}{(E_\gamma^2 - E_{GDR}^2)^2 + \Gamma_{GDR}^2 E_\gamma^2}
\]

(3)

around the Giant Dipole Resonance (GDR) at energy \( E_{GDR} \) with a width \( \Gamma_{GDR} \), but differ in details describing the strength at very low energy 6.

Although transitions with the largest \( \gamma \) energies (close to \( U \)) are the strongest due to the \( E_\gamma^4 \) dependence (Eq. 3), they receive less weight in the integrand appearing in Eq. 2 because the nuclear level density \( \rho \) decreases with increasing \( E_\gamma \) with \( \rho(E_\gamma) \propto \exp(2 \sqrt{a(U - E_\gamma)}) \) (with \( a \) being the usual level density parameter). Thus, there will be a competition of few strong transitions with many weak ones and only a closer inspection of the integrand reveals the relative importance. As an example, the relative contribution to the integrand of different \( E_\gamma \) is shown in Fig. 2 for two nuclei. The computation was performed adopting the descriptions of 3, 12 for \( T_{E1} \) (and \( T_{M1} \)) and \( \rho \), assuming that \( E_{proj} = 60 \) keV and \( \mu_{\max} = 0 \). It can clearly be seen that the largest contribution stems from the energy range about mid-way between the ground state and the formed compound state.

FIG. 1: Sketch of the relevant energies in a compound capture reaction: target \( A \) captures a projectile \( a \) by formation of a compound nucleus \( C \). The excitation energy \( U \) of the compound nucleus depends on the projectile energy \( E_a \) and the separation energy \( S_a \) of the projectile in the compound nucleus. The nucleus \( C \) is deexcited via \( \gamma \) emission to discrete states or “average states” given by a level density (grey area). Thus, the possible photon energies \( E_\gamma \) can only be in the range 0 \( \leq E_\gamma \leq S_a + E_a \). For astrophysical neutron capture is \( E_a \ll S_a \).

FIG. 2: The integrand of Eq. 2 in the two compound nuclei \( ^{124,132}\text{Sn} \) when capturing 60 keV neutrons (renormalized to the same maximal value to show the similar shapes).
This conclusion holds even when employing different descriptions of low-energy GDR strength as given in [1]. The resulting changes in the location of the maxima are on a level of $10\%$ or smaller.

When the level density is low, the smooth averaged integral is fractionated into contributions of single, isolated states which are accounted for in the first sum of Eq. (2) including the contribution of the transition to the ground state. For light or closed-shell nuclei, the sum can extend high in excitation energy. Depending on their spin and parity, these isolated states can carry a considerable fraction of the total strength when they can be reached by a single E1 transition. Even in the absence of such a fractionation, the contribution of the transitions to the ground state may be significant in nuclei with an inherent low level density where the application of the statistical model is doubtful, anyway.

Implications for astrophysics. It has been shown that a change in the low-energy tail of the GDR strength, such as one caused by a pygmy resonance, can lead to a significant change in the neutron capture cross sections for neutron-rich nuclei [20, 21]. Although experiments have indicated additional E1 strength at low excitation energy [3, 4, 8, 9], the origin of this strength and a prediction of its properties for more neutron-rich nuclei remains controversial. Nevertheless, using the above arguments the energy can be derived at which the additional strength has to be located to have an impact on astrophysically...
relevant reactions. As above, the descriptions of $^3$ Sn and $^2$ Sn were adopted for $T_{E1}$ and $\rho$ and information for ground and excited states was taken from \cite{23} and \cite{24}.

Neutron capture on very neutron-rich nuclei can be relevant in $r$ process nucleosynthesis \cite{20, 25}, on proton-rich nuclei in the $p$ process \cite{26, 27}. Figs. 4-7 show the maximally contributing $\gamma$ energies for sequences of Sn, Pb, and Ru isotopes, respectively. The weighting by $T_i$ is considered according to Eq. 1. The neutron separation energy is decreasing in an isotopic chain with increasing neutron number but at the same time the maximal level density is also decreasing due to the lower excitation energies $U$ encountered. In almost all shown cases the energy of the maximal E1 contribution is in the range $2 \leq E_\gamma \leq 4$ MeV. Jumps can be found for nuclei where the selection rules allow the compound-nucleus ground state to be reached by a combination of s-wave neutrons and E1 $\gamma$s. In this case the relevant $E_\gamma$, sensitive to modifications of the strength $f$, is a sharply defined energy and equal to $S_n + E_{proj} \approx S_n$.

Experimentally, additional E1 strength was found in $^{132}$Sn and attributed to a pygmy resonance \cite{7}. However, it is located several MeV above $S_n$, in accordance with theoretical predictions. Thus, it does not affect the neutron capture cross section. Comparing to an available prediction of the pygmy resonance energy within an isotopic chain, it can be seen that it is predicted to always lie well above $S_n$ \cite{14, 15, 16}. If this is confirmed, it would mean that the pygmy resonance does not play a role in astrophysical neutron capture unless it is sufficiently wide to bring some additional strength below $S_n$. To have a wide and strong pygmy resonance may prove difficult, however, without violating the E1 sum rule (Thomas-Reiche-Kuhn sum rule). On a side note it should be remembered that the Hauser-Feshbach model cannot be applied when the level density is too low at the compound formation energy \cite{17}. This will occur in nuclei with $S_n$ of only a few MeV. In the absence of
resonances direct capture will dominate neutron capture on nuclei close to the neutron dripline, which does not excite collective modes in the $\gamma$ emission to the continuum and is not sensitive to pygmy effects. Furthermore, most $r$ process models predict the $r$ process path to be characterized by an $(n,\gamma)-\gamma(n,n)$ equilibrium in which individual cross sections do not play a role\textsuperscript{28}. In these models, neutron captures only have a moderate effect during the relatively fast freeze-out at the end of the $r$ process. Those captures will also occur closer to stability as compared to the $r$ process path.

Energetically, the situation is different for capture of charged particles. Due to the Coulomb barrier, the astrophysically relevant interaction energies are shifted to higher energies. Reactions with charged particles occur at the line of stability and on the proton-rich side of the nuclear chart in $p\alpha$, $rp$\textsuperscript{29}, and $vp$ process\textsuperscript{30} nucleosynthesis. In the most extreme case, non-equilibrium proton captures involve protons of $5-7\text{MeV}$ maximum energy. For $\alpha$ particles, the maximum energies are around $10-12\text{MeV}$. These energies depend on the charge of the target nucleus\textsuperscript{19}. Since they are comparable to or exceed the proton or $\alpha$ separation energies, respectively, the sensitive $E_\gamma$ will be at or above the projectile separation energy in the compound nucleus. Otherwise, similar rules apply as for neutron capture. However, a widely accepted explanation of the pygmy resonance is that it is caused by a collective motion of a neutron skin against a proton-neutron core. This is not likely to occur in neutron-deficient nuclei but other ways to generate additional strength beyond the GDR may still be found.

Figs.\textsuperscript{31-33} show the relevant $\gamma$ energies in comparison to the separation energies for proton and $\alpha$ capture on Sn and Sm isotopes. It is interesting to note that the energies again remain in the range of about $2 \leq E_\gamma \leq 4$, even for $\alpha$ captures with negative $Q$ values.

**Summary.** The relevant $\gamma$ energy range was explicitly identified where additional $\gamma$ strength has to be located for having an impact on astrophysically relevant reactions. It was shown that folding the energy dependences of the transmission coefficients and the level density leads to maximal contributions for $\gamma$ energies in the range of about $2 \leq E_\gamma \leq 4\text{MeV}$. The distributions show a full width at half maximum of about $2\text{MeV}$. Quantum selection rules allow isolated states to contribute only in special cases, mainly for neutron capture around closed shells or at low neutron separation energy when also the application of the statistical model becomes problematic. This can be seen in Figs.\textsuperscript{34,35} and\textsuperscript{7} for neutron capture on $^{208-211}\text{Pb}$, as well as on $^{100,124,126,128-130}\text{Sn}$ and most isotopes beyond $^{130}\text{Sn}$ where the neutron separation energy remains low. These findings allow to more accurately judge the astrophysical relevance of modifications of the $\gamma$ strength, either found experimentally or derived theoretically. The importance of experimentally obtaining spectroscopic information for nuclei with low inherent level densities far off stability is also evident.

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