Pure quantum freezing of the 5\textsuperscript{th} dimension

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Abstract

It is shown that super thin and super long gravitational flux tube solutions in the 5D Kaluza-Klein gravity have the Planck scale regions ($\approx l_{Pl}$) where the metric signature changes from $\{+,-,-,-,-\}$ to $\{-,-,-,-,\}$. Such a change occurs too rapidly according to one of the paradigms of quantum gravity which holds that the Planck length is the minimal length in the nature and consequently the physical quantities cannot change very quickly over this length scale. To avoid such a dynamic it is hypothesized that a pure quantum freezing of the dynamics of the 5\textsuperscript{th} dimension takes place. As a continuation of the flux tube metric in the longitudinal direction the Reissner-Nordström metric is proposed. As a consequence of such a construction one can avoid the appearance of a point-like singularity in the extremal Reissner-Nordström solution.

1 Introduction

One of the intriguing problems of Kaluza-Klein gravity is how the observed 4D universe can be connected with the hypothesized 5 (or multi) dimensional Kaluza-Klein gravity? In this paper we would like to show that on a super thin and super long gravitational flux tube there takes place a pure quantum freezing of the 5\textsuperscript{th} dimension. This mechanism is based on one of paradigms of quantum gravity: the Planck length is the minimal length in the nature. It is easy to understand that this statement leads to the conclusion that no physical quantity can vary on a macroscopical scale over the extent of such a microscopical length as the Planck length. We will use this statement for the metric signature. For example, the following dynamic is impossible: having a metric signature near $r < r_H - l_{Pl}$ as $\{+,-,-,-,-\}$ and near $r > r_H + l_{Pl}$ as $\{-,-,-,-,\}$ (with $r_H$ is some constant). According to the above statement in the region $|r - r_H| < l_{Pl}$ there should be some quantum gravitational effects which will prevent such a dynamic. In this paper we do not give any microscopical description how this happens, but we consider only the consequences of such a prohibition. Such an approach is similar to the macroscopical thermodynamical investigation of a physical process when we are not interested in the microscopical description of the process.

2 Short description of super thin and super long gravitational flux tubes

The 5D super thin and super long gravitational flux tube metric is

$$ds^2 = \frac{dt^2}{\Delta(r)} - \Delta(r)e^{2\psi(r)} \left[ d\chi + \omega(r)dt + Q \cos \theta d\varphi \right]^2 - dr^2 - a(r)(d\theta^2 + \sin^2 \theta d\varphi^2),$$

(1)

where $\chi$ is the 5\textsuperscript{th}, extra coordinate; $r, \theta, \varphi$ are 3D spherical-polar coordinates; $r \in \{-\infty, +\infty\}$ is the longitudinal coordinate; $Q$ is the magnetic charge.

The metric (1) gives us the following components of the electromagnetic potential $A_\mu$

$$A_\chi = \omega(r) \quad \text{and} \quad A_\varphi = Q \cos \theta$$

(2)

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and the Maxwell tensor

\[ F_{\tau t} = \omega'(r) \quad \text{and} \quad F_{\theta \varphi} = -Q \sin \theta. \]  

(3)

This means that we have radial Kaluza-Klein electric \( E_r \propto F_{\tau r} \) and magnetic \( H_r \propto F_{\theta \varphi} \) fields.

Substituting this ansatz into the 5D Einstein vacuum equations

\[ R_{AB} - \frac{1}{2} \eta_{AB} R = 0 \]  

(4)

\( A, B = 0, 1, 2, 3, 5 \) and \( \eta_{AB} \) is the metric signature, gives us

\[ \frac{\Delta''}{\Delta} - \frac{\Delta'^2}{\Delta^2} + \frac{\Delta \alpha'}{\Delta} - \frac{\Delta' \psi'}{\Delta} + \frac{q^2}{a^2 \Delta^2} e^{-4\psi} = 0, \]  

(5)

\[ \frac{\alpha''}{a} + \frac{\alpha' \psi'}{a} - \frac{2}{a} \frac{Q^2}{a^2 \Delta^2} e^{2\psi} = 0, \]  

(6)

\[ \psi'' + \psi'^2 + \frac{\alpha' \psi'}{a} - \frac{2}{a^2} \frac{Q^2}{a^2 \Delta^2} e^{2\psi} = 0, \]  

(7)

\[ \frac{\Delta'^2}{\Delta^2} + \frac{\alpha'^2}{a^2} - 2 \frac{\Delta' \psi'}{\Delta} - \frac{4}{a} + 4 \frac{\alpha' \psi'}{a} + \frac{q^2}{a^2 \Delta^2} e^{-4\psi} + \frac{Q^2}{a^2 \Delta^2} e^{2\psi} = 0. \]  

(8)

\( q \) is the electric charge. These equations are derived after substituting the expression (9) for the electric field in the initial Einstein’s equations. The 5D (\( \chi t \))-Einstein’s equation (4D Maxwell equation) is taken as having the following solution

\[ \omega' = \frac{q}{a\Delta} e^{-3\psi}. \]  

(9)

For the determination of the physical sense of the constant \( q \) let us write the (\( \chi t \))-Einstein’s equation in the following way:

\[ \left( \omega' \Delta^2 e^{3\psi} 4\pi a \right)' = 0. \]  

(10)

The 5D Kaluza-Klein gravity after the dimensional reduction indicates that the Maxwell tensor is

\[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]  

(11)

That allows us to write the electric field as \( E_r = \omega' \). Eq. (10), with the electric field defined by (11), can be compared with the Maxwell’s equations in a continuous medium

\[ \text{div} \vec{D} = 0 \]  

(12)

where \( \vec{D} = \epsilon \vec{E} \) is an electric displacement and \( \epsilon \) is a dielectric permeability. Comparing Eq. (12) with Eq. (10) we see that the magnitude \( q/a = \omega' \Delta^2 e^{3\psi} \) is like to the electric displacement and the dielectric permeability is \( \epsilon = \Delta^2 e^{3\psi} \). This means that \( q \) can be taken as the Kaluza-Klein electric charge because the flux of the electric field is \( \Phi = 4\pi a \vec{D} = 4aq \).

As the electric \( q \) and magnetic \( Q \) charges are varied it was found [1] that the solutions to the metric in Eq. (5)-(8) evolve in the following way:

1. \( 0 \leq Q < q \). The solution is a regular gravitational flux tube. The solution is filled with both electric and magnetic fields. The longitudinal distance between the \( \pm r_H \) surfaces increases, and the cross-sectional size does not increase as rapidly as \( r \to r_H \) with \( q \to Q \). The values \( r = \pm r_H \) are defined in the following way \( \Delta(\pm r_H) = 0 \).

2. \( Q = q \). In this case the solution is an infinite flux tube filled with constant electric and magnetic fields. The cross-sectional size of this solution is constant (\( a = \text{const.} \)).

3. \( 0 \leq q < Q \). In this case we have a singular gravitational flux tube located between two (\( + \)) and (\( - \)) electric and magnetic charges at \( \pm r_{\text{sing}} \). At \( r = \pm r_{\text{sing}} \) this solution has real singularities at the location of the charges.

We focus on the case when \( q \approx Q \) but \( q > Q \). In this case there is a region \( |r| \leq r_H \) where the solution is similar to a tube filled with almost equal electric and magnetic fields. The length \( L = 2r_H \) of the throat of the flux tube (with \( |r| < r_H \)) depends on the relation \( \delta = 1 - Q/q \), i.e. \( L \to \infty \) but \( \delta > 0 \). A numerical analysis [2] shows that the spatial cross section of the tube \( (t, \chi, r = \text{const}) \) does not change significantly, i.e. \( a(r_H) \approx 2a(0) \). The cross section of the tube at the center \( a(0) \) is arbitrary and we choose it as \( a(0) \approx l_{Pl} \). This gives a super thin and super long gravitational flux tube: \( L \to \infty, a(0) \approx l_{Pl} \).
The function \( \Delta(x) e^{2\psi(x)} \).

In Fig. (1) the profile of the function \( \Delta e^{2\psi} \) (which is equal to \( G_{55} \)) is presented [2]. We see that near the values \( r = \pm r_H \) this function changes drastically from the value \( \approx +1 \) for \( -r_H + l_0 \lesssim r \lesssim r_H - l_0 \) to \( \approx -1 \) for \( r \gtrsim r_H + l_0 \) and \( r \lesssim -r_H - l_0 \). The exact solution with \( q = Q \) is

\[
a = a(0) = \frac{Q_0^2}{2} = \text{const},
\]

\[
e^{2\psi} = \frac{1}{\Delta} = \cosh \frac{r}{\sqrt{a(0)}},
\]

\[
\omega = \sqrt{2} \sinh \frac{r}{\sqrt{a(0)}},
\]

\[
G_{55} = \Delta e^{2\psi} = 1
\]

(13) (14) (15) (16)

Here we have parallel electric \( E \) and magnetic \( H \) fields with equal electric \( q_0 \) and magnetic \( Q_0 \) charges \( q_0 = Q_0 = \sqrt{2a(0)} \). For the throat the solution with \( q \approx Q \) but \( q > Q \), all the equalities of (13)-(16) are changed to approximate equalities

\[
a \approx \frac{Q_0^2}{2} = \text{const},
\]

\[
e^{2\psi} \approx \frac{1}{\Delta} = \cosh \frac{r}{\sqrt{a(0)}},
\]

\[
\omega \approx \sqrt{2} \sinh \frac{r}{\sqrt{a(0)}},
\]

\[
G_{55} = \Delta e^{2\psi} \approx 1
\]

(17) (18) (19) (20)

Such an approximation is valid only for \( |r| \lesssim r_H - l_0 \).

Now we would like to estimate the length \( l_0 \) of the region where the change of the function \( \Delta e^{2\psi} \) occurs

\[
\Delta e^{2\psi} \bigg|_{r \approx r_H + l_0} - \Delta e^{2\psi} \bigg|_{r \approx r_H - l_0} \approx 2.
\]

(21)

For this estimation we use Eq. (6). On the throat this equation is approximately

\[
-\frac{2}{a} + \frac{Q^2}{a^2} \Delta e^{2\psi} \approx 0.
\]

(22)

We can estimate \( l_0 \) by solving Einstein’s equations (5)-(8) near \( r = +r_H \) (for \( r = -r_H \) the analysis is the same) and define \( r = r_H - l_0 \) where the last two terms in Eq. (6) will have the same order

\[
\left( \frac{2}{a} \right) \bigg|_{r=r_H-l_0} \approx \left( \frac{Q^2}{a^2} \Delta e^{2\psi} \right) \bigg|_{r=r_H-l_0}.
\]

(23)
For the solution close to \( r = \pm r_H \) we try the following form

\[
\Delta(r) = \Delta_1 (r_H - r) + \Delta_2 (r_H - r)^2 + \cdots .
\]  

(24)

Substitution into Eq. (5) gives us the following solution

\[
\Delta_1 = \frac{q e^{-2\psi_H}}{a_H}.
\]  

(25)

After the substitution into Eq. (23) we have

\[
l_0 \approx \sqrt{a(0)} = l_{Pl}
\]  

(26)

here we took into account that the numerical analysis of [2] shows that \( a_H \approx 2a(0) \). Thus the change of the macroscopical dimensionless function \( \Delta e^{2\psi} \) as in Eq. (21) occurs in the Planck length. The metric (1) near \( |r| \approx r_H - l_{Pl} \) is approximately

\[
ds^2 \approx e^{2\psi_H} dt^2 - dr^2 - a(0) (d\theta^2 + \sin \theta d\varphi^2) - (d\chi + \omega dt + Q \cos \theta d\varphi)^2
\]  

(27)

near \( |r| \approx r_H + l_{Pl} \) the metric is approximately

\[
ds^2 \approx -e^{2\psi_H} dt^2 - dr^2 - a (d\theta^2 + \sin \theta d\varphi^2) + (d\chi + \omega dt + Q \cos \theta d\varphi)^2
\]  

(28)

here we took into account that numerical calculations of [2] show that \( \psi \approx \psi_H = \text{const} \) near \( |r| \gtrsim r_H \). We see that within the Planck length the metric signature changes from \( \{+,-,-,-,\} \) to \( \{-,-,-,-,\} \). Simultaneously it is necessary to mention that the metric (1) is non-singular near \( |r| = r_H \) and approximately [2]

\[
ds^2 \approx g_H dt^2 - e^{\psi_H} dt (d\chi + Q \cos \theta d\varphi) - dr^2 - a(r_H) (d\theta^2 + \sin^2 \theta d\varphi^2)
\]  

(29)

where \( g_H \) is some constant.

If we write the metric (1) in 5-bein formalism

\[
ds^2 = \omega^A \omega^B \eta_{AB}, \quad \omega^A = e^A \mu dx^\mu, \quad x^\mu = t, r, \theta, \varphi, \chi
\]  

(30)

then we see that

\[
\eta_{AB} = \{+1,-1,-1,-1,-1\} \quad \text{by} \quad |r| \lesssim r_H - l_{Pl}
\]  

(31)

\[
\eta_{AB} = \{-1,-1,-1,-1,+1\} \quad \text{by} \quad |r| \gtrsim r_H + l_{Pl}
\]  

(32)

It is necessary to note that for the mechanism presented here the change of the sign of two components \( \eta_{00} \) and \( \eta_{55} \) is very important. The reason is that the \( G_{55} \) metric component can be made dimensional in the following way

\[
G_{55} d\chi^2 = (l_0^2 G_{55}) \left( \frac{d\chi}{l_0} \right)^2
\]  

(33)

where \( l_0 \approx l_{Pl} \) is the characteristic length of the 5th dimension. In this case the quantity \( \sqrt{l_{Pl} G_{55}} \) changes

\[
\sqrt{l_{Pl} G_{55}}|_{r=r_H+l_{Pl}} - \sqrt{l_{Pl} G_{55}}|_{r=r_H-l_{Pl}} \approx l_{Pl}
\]  

(34)

by a change of the radial coordinate

\[
\Delta r \approx l_{Pl}.
\]  

(35)

Such a variation of \( G_{55} \) is possible but simultaneously the dimensionless quantity \( \eta_{00}(e_i^0)^2 \) changes

\[
\eta_{00}(e_i^0)^2|_{r=r_H+l_{Pl}} - \eta_{00}(e_i^0)^2|_{r=r_H-l_{Pl}} \approx 2 e^{2\psi_H} \gg 1
\]  

(36)

within a Planck length.

One of the basic paradigms of quantum gravity is that the Planck length is the minimal length in nature and consequently no physical fields or quantities can change in the course of the Planck length. Thus one can conclude that such a classical dynamic of the metric signature is impossible. There is only one way to avoid such dynamical behavior of this quantity: it is necessary to forbid any dynamic of the \( G_{55} \) field variable so that its last value is conserved, or in other words one has a pure quantum freezing of
the dynamic of 5th dimension. Mathematically this means that $G_{55}$ becomes a non-dynamical quantity and must not vary.

Thus the analysis of the classical dynamic of the metric signature for the super thin and super long gravitational flux tube shows that there is a region where this quantity changes too quickly from the quantum gravity viewpoint. This leads to the fact that some pure quantum gravity effects have to happen in order to avoid such a variation of the metric signature. We do not consider the mechanism of these effects but the author’s point of view is that such mechanism can not be based on any field-theoretical consideration. This pure quantum freezing of the extra dimension is similar to a trigger which has only two states: in one state the dynamic of $G_{55}$ is switched on, and in another is switched off. Such a quantum dynamic, which can be realized only in the Planck region, is a non-differentiable dynamic. Examples of such a hypothesized non-differentiable dynamic are the above mentioned freezing of the extra dimensions, the change of the metric signature, or other phenomena [3], [4].

The next question arising in this context is the dynamic of residual degrees of freedom. Since $G_{55} = \text{const}$ we have Kaluza-Klein gravity in its initial interpretation where 5D Kaluza-Klein theory with $G_{55} = \text{const}$ is equivalent to 4D electrogravity. It lead to an idea that the spacetime with $(r \gtrsim r_H + l_{Pl})$ (the same for $r \lesssim -r_H - l_{Pl}$) will be the Reissner-Nordström solution with the corresponding electric and magnetic fields.

3 Joining of electric and magnetic fields

In this section we would like to discuss the problem of joining the electric and magnetic fields of the flux tube and the Reissner-Nordström solutions. For this we will compare the fluxes of electric and magnetic fields on the throat and the Reissner-Nordström spacetime.

3.1 Electric field

At first we consider the electric field. The Maxwell equations for the 4D case are

$$\frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^\alpha} \left( \sqrt{\gamma} D^\alpha \right) = 0$$

(37)

where $\sqrt{\gamma}$ is the determinant of the 3D spatial metric and $D^\alpha$ is an analog of the electric displacement in the electrodynamics of a medium and

$$D^\alpha = -\sqrt{g_{00}} F^{0\alpha}.$$  

(38)

For the Reissner-Nordström solution the metric is

$$ds^2 = \left( 1 - \frac{r_g}{R} + \frac{r_g Q}{R^2} \right) (4) dt^2 + \frac{dR^2}{1 - \frac{r_g}{R} + \frac{r_g Q}{R^2}} - R^2 (d\theta^2 + \cos^2 \theta d\phi^2),$$

$$r_g = \frac{2GM}{c^2}, \quad r_g Q = \sqrt{\frac{G}{c^2}} (q^2 + Q^2).$$

(39)

Therefore Eq. (37) is

$$\frac{d}{dR} \left( R^2 F^{(4)}_{tR} \right) = 0, \quad F_{tR} = \frac{d\phi}{dR}$$

(40)

here $\phi$ is the scalar potential. This equation shows that the flux $\Phi_e$ of the electric field $E_R = \phi'$ is conserved

$$\Phi_e = 4\pi R^2 \phi' = 4\pi (4) q = \text{const}.$$  

(41)

The corresponding Maxwell equation on the throat is

$$\frac{d}{dr} \left[ a (\omega' \Delta^2 e^{3\psi}) \right] = 0.$$  

(42)

Here we have also a conserved flux of an analog of electric displacement

$$\Phi_e = 4\pi a D^5 = 4\pi (5) q = \text{const}.$$  

(43)
\[ D^5 = \omega' \Delta^2 e^{3\psi} = \omega' e^{-\psi} G_{55}. \]  

(44)

The simplest assumption about two electric fields on the throat and Reissner-Nordström spacetimes is to join the fluxes (41) and (43)

\[ \Phi^5_e = \Phi^4_e \]  

(45)

here \( \Phi \) have to be calculated at the ends of throat where the freezing of the 5\(^{th} \) coordinate happens, i.e. near \( r \approx r_H - l_{Pl} \) and \( r \approx -r_H + l_{Pl} \). There \( G_{55} \approx 1 \) and

\[ \phi' \approx \omega' e^{-\psi_H} \]  

(46)

\[ \psi_H \approx \psi \bigg|_{r = r_H - l_{Pl}} \]  

(47)

here we have used the conditions

\[ \frac{\partial}{\partial r} \left[ r^2 \left( \frac{Q(r)}{r^2} \right) \right] = 0 \]  

(52)

where \( r \) is the radial coordinate on the throat and the tails. It allows us to introduce the flux of magnetic field for the throat and the tails which will be equal

\[ \Phi^5_m = 4\pi R^2 \frac{Q}{R^2} = 4\pi a \frac{Q}{a} = \Phi^5_m. \]  

(53)

This permits us to introduce the magnetic charge

\[ Q = Q. \]  

(54)

### 3.2 Magnetic field

Now we will repeat a similar calculation for the magnetic field. The Maxwell equation for the magnetic field having the information about the flux of magnetic field is the same for the throat and the tails (Reissner-Nordström spacetimes)

\[ \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial x^\alpha} \left[ \sqrt{\gamma} \left( \frac{-1}{2\sqrt{\gamma}} \epsilon^{\alpha\beta\delta} F_{\beta\delta} \right) \right] = 0 \]  

(50)

here the magnetic field \( B^\alpha \) can be introduced

\[ B^\alpha = \frac{-1}{2\sqrt{\gamma}} \epsilon^{\alpha\beta\delta} F_{\beta\delta}. \]  

(51)

More concretely

\[ \frac{d}{dr} \left[ r^2 \left( \frac{Q}{r^2} \right) \right] = 0 \]  

(52)

### 3.3 Joining the metric

After the quantum freezing of the 5\(^{th} \) dimension the 5D metric (1) will be

\[ ds^2 = A(r) dt^2 - B(r) dr^2 - a(r)(d\theta^2 + \sin^2 \theta d\phi^2) - [d\chi + \omega(r) dt + Q \cos \theta d\phi]^2. \]  

(55)

The interpretation of this metric in the initial Kaluza-Klein gravity tells us that we have the 4D metric

\[ ds^2 = A(r) dt^2 - B(r) dr^2 - a(r)(d\theta^2 + \sin^2 \theta d\phi^2) \]  

(56)

and the electromagnetic potential

\[ A_{\mu} = \{ \omega, 0, 0, Q \cos \theta \}. \]  

(57)
In the previous subsections we have joined the electric and magnetic fields. Now we have to consider the metric components. We will interpret the metric (56) as a Reissner-Nordström metric. After freezing of the 5th coordinate the corresponding equations become 5D Einstein’s equations but without the (55) Einstein equation, i.e. we have the ordinary 4D electro-gravity. Taking this into account the metric (55) will be the Reissner-Nordström metric (39). We will join the $g_{\theta\theta}$ and $g_{t\mu}$ components of the metrics (1) and (56). One can note that joining the $g_{rr}$ components of these metrics is not necessary as they measure the distance on the transversal direction to the surface of joining. We do not join (as is normally done) the first derivatives of the metric components since on the 5D throat and the tails there are the different sets of equations: on the throat one has the 5D Einstein’s equations but on the tails one has the equations of 4D electro-gravity \(^1\).

Joining of $g_{\theta\theta}$ gives

$$g^{(4)}_{rr}(r_0) = a l_{Pl}^{ \pm r_H \mp r_{Pl}} \Rightarrow r_0^2 = a(0) = l_{Pl}. \quad (58)$$

For the $g_{tt}$ components

$$\left(1 - \frac{r_g}{r_0} + \frac{r_a^2 q^2}{r_0^2}\right)^{(4)} dt^2 = e^{2\psi} \left(\frac{r_g}{l_{Pl}}\right)^{(5)} dt^2 \quad (59)$$

here $t^{(4)}$ and $t^{(5)}$ are the time coordinates on the throat and the tails correspondingly. One can rewrite this relation as

$$\frac{dt^{(5)}}{dt^{(4)}} = e^{-\psi} \sqrt{1 - \frac{r_g}{l_{Pl}} + \frac{r_a^2 q^2}{l_{Pl}^2}}. \quad (60)$$

Only with such a relation between the time coordinates on the throat and the tails will time pass equally on the hypersurface of the junction.

It is necessary to mention that the analysis presented in this section is very simple and, for example, does not allow us to determine the mass $m$ for the Reissner-Nordström solution. A more exact calculation might be possible using quantum field-theoretical language.

4 Conclusions and discussion

In this paper we have shown that the super thin and super long gravitational flux tube solutions in 5D Kaluza-Klein gravity has two regions where the classical description can not be applied. Some metric components change too quickly: the metric signature changes from $\{+,-,-,-,-\}$ to $\{-,-,-,-,\}$ and $\Delta G_{55} \approx 2$ within a Planck length. To avoid such a variation the dynamical quantity $G_{55}$ must be frozen. Then the initial flux tube metric can be extended on the right and left ends to the Reissner-Nordström spacetimes \(^2\). At the junctions there occurs a pure quantum freezing of the dynamic of $G_{55}$ metric component. Such an object looks like two extremal Reissner-Nordström spacetimes $(r_g^2/4 < r_a^2 q^2)$ connected with a super thin and super long flux tube filled with the electric and magnetic fields. Let us note that the point of view presented here is based only on an idea which does not depend on the details whatever theory of quantum gravity one considers.

Such a model allows us to resolve successfully one of the hardest problems of general relativity: the avoidance of singularities. Our construction shows that for an extremal Reissner-Nordström solution (at least for $q > Q$) the gravitational field becomes so strong that the dynamic on the extra dimension becomes excited and a flux tube from one singularity to another one appears. The force lines do not converge in a pointlike singularity but leave our universe to another one (or to a remote part of our universe) through the flux tube and there appear near another almost singularity.

There is another argument for the pure quantum freezing mechanism presented here. We see that near $|r| \lesssim r_H - l_{Pl}$ the metric component $G_{55} \approx 1$ to a large accuracy. This means that with the same accuracy we have a dynamical freezing of the 5th coordinate but near the surface where the change of metric signature occurs the quantity $G_{55}$ becomes dynamical.

Let us note that such mechanism works only for some special extremal Reissner-Nordström solutions having electric and magnetic charges with the relation $q > Q$. The question arises: is it possible to extend this result about avoidance of a singularity to other Reissner-Nordström solutions? In this connection we have to emphasize that the analysis carried out here is very simple and a more careful analysis with

\(^1\)Let us remember that according to the initial interpretation of the Kaluza-Klein gravity the 4D electro-gravity spacetime can be considered as 5D spacetime with frozen 5th dimension ($G_{55} = \text{const}$)

\(^2\)Let us note that additionally one can freeze 4D metric $g_{\mu\nu}$ and we will have only Maxwell electrodynamic on the flat space which is similar to the AdS/CFT correspondence idea.
the quantization of corresponding field quantities can give more exact results. In this connection one can mention the results of Ref. [5] where it is shown that the super thin and super long flux tube solutions without freezing of the 5th dimension have interesting properties on the tails: the magnetic fields decreases faster then the electric field and there is a rotation connected with the magnetic field on the throat. One can hypothesis that these properties will remain the same with the freezing of the 5th coordinate (maybe in some weaker form).

The construction presented here (two Reissner-Nordström spacetimes connected by a flux tube) can be considered as a model of a spacetime with a frozen 5th dimension but with piecewise sections where the 5th dimension is unfrozen. Similar ideas about spacetime regions with compactified and uncompactified phases was considered by Guendelman in Ref. [6]. The construction presented in [6] is also based on a flux tube (i.e. the Levi-Civita - Bertotti -Robison solution [7]-[9]) inserted between two 4D black holes. Similarly in Ref. [10] a model of the classical electron with a Levi-Civita -Bertotti -Robison flux tube between two Reissner-Nordström black holes is considered.

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