Collective Mode in a $d_{x^2-y^2} + id_{xy}$ Superconductor

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We consider a superconducting state with a mixed symmetry order parameter components, e.g. $d + i s$ or $d + id'$ with $d' = d_{xy}$. We argue for the existence of the new orbital magnetization mode which corresponds to the oscillations of relative phase $\phi$ between two components around an equilibrium value of $\phi = \frac{\pi}{2}$. It is similar to the so called “clapping” mode in superfluid $^3He - A$. We estimate the frequency of this mode $\omega_0(B, T)$ depending on the field and temperature for the specific case of magnetic field induced $d' = d_{xy}$ state. We find that this mode is tunable with an applied magnetic field with $\omega_0(B, T) \propto B \Delta_0$, where $\Delta_0$ is the magnitude of the d-wave order parameter.

We also estimate the velocity $s(B, T)$ of this mode. Pacs Numbers: 74.20.-z, 74.25.Nf

The order parameter in high-Tc superconductors is widely believed\cite{1,2} to be of a $d_{x^2-y^2}$ symmetry. However more careful consideration indicates that the symmetry of the state in high-Tc might be lower in a number of cases. The symmetry allows the secondary components to be generated whenever there is a perturbing field. These secondary components are considers as: $d + is$ or $d + id'$ generation has been addressed in recent literature for the case of inhomogeneity due to wall scattering\cite{3,4} or due to vortex texture\cite{5,6}. Similarly, the generated $id'$ component near magnetic impurity and spontaneous violation of time reversal symmetry with global $d + id'$ has also been discussed\cite{7,8}. Another possibility for a global $d + id'$ phase has been pointed out\cite{9,10} where the external magnetic field applied to two-dimensional $d$-wave superconductor generates $id'$.

Direct experimental determination of the second superconducting component nevertheless remains a substantial challenge, see e.g.\cite{11}. The smallness of induced component and the large superconducting response of the underlying $d$ component makes the detection of the secondary component difficult.

We present here an excitation which constitutes a direct test of the induced component of the order parameter. This excitation is uniquely tied to the existence of the secondary out of phase component of the order parameter. Consider the most general situation of a complex order parameter which can be generally written as $\Delta_0 + i \Delta_1$ where $\Delta_0$ is the original $d_{x^2-y^2}$ component and $\Delta_1$ is the induced $s$ or $d_{xy}$ component which is orthogonal to the initial $d_{x^2-y^2}$ state. Define the global (Josephson) phase of the order parameter $\nu$ and the relative phase $\phi$ as:

$$
\Delta = |\Delta_0| \exp(i\phi(r))|\Delta_1| \exp(i\nu(r)) \tag{1}
$$

If the order parameter $\Delta$ is to be defined at the Fermi surface, then the functions $\Delta_i, i=0,1$ implicitly contain the angular dependence. The global phase $\nu(r)$ can be position dependent and even singular as is the case for the vortex configuration. We will focus on the relative phase $\phi$. Its dynamics by definition is related to the appearance of the secondary component $\Delta_1$. While the equilibrium value of the relative phase in most cases is argued to be $\pm \frac{\pi}{2}$ resulting in a violation of the time reversal symmetry, $\phi$ may oscillate around this value and represent a new collective mode that can be used to detect the induced component. Similar relative phase mode has been considered before in context of two band superconductors\cite{23} and heavy fermions\cite{24} and in\cite{25}, where the collective modes of d-wave superconductor in zero field were considered.

This collective mode is akin to a spin wave in the orbital moment of the superconducting ground state. The dispersion of this mode has a gap similar to the Larmour frequency in case of spins. As has been shown in\cite{24}, the relative phase oscillations also can be viewed as an internal Josephson effect, similar to the clapping mode in $^3He - A$\cite{26} or to the relative phase dynamics in a 2-band superconductor\cite{27}. In the sense of a magnetic excitation, that can respond to a time dependent magnetic field in a resonant manner, this mode is also comparable to the longitudinal NMR in $^3He - A$\cite{28}. The details of the dynamics are clearly model dependent.

To be specific we will focus on the field induced $\Delta_1 = id_{xy}$ secondary component in the bulk. In this case quasiparticle spectrum is fully gapped in the field with the minimal gap vanishing at zero field. The $d + id'$ state breaks time reversal symmetry and has a finite magnetic moment $M_z$ perpendicular to the superconducting plane\cite{29,30}. The mode discussed above in this case will
correspond to the *longitudinal* oscillations of the condensate magnetic moment around its equilibrium value. The frequency of this mode will contain information about the details of the coupling between components and will provide insight into the overall order parameter structure. Below we will focus on a 2d superconductor at the fields $H \approx H_{c2}(T)$ and therefore we will ignore small $(O(H-H_{c2}))$ difference between induction $\mathbf{B}$ and applied field $\mathbf{H}||z$.

We find the resonance frequency for the *relative* phase oscillations of $\phi$. It turns out to be a gapped propagating mode with:

$$\omega^2(B, k) = \omega_0^2(B, T) + s_{ij}(B, T)k_i k_j,$$

$$\omega_0(B) \simeq \frac{\eta B}{N(0)} \Delta_0(B, T) \tag{2}$$

$$s_{ij} = \delta_{ij} s^2 \tag{3}$$

Here $s = (a + b B^2) \Delta^2_0$ is the mode velocity, $a, b$ are some constants and $\eta$ is a constant, a measure of the strength of the interaction, which we discuss in more detail below: $N(0)$ is the Density of States (DOS) at the Fermi level; both $i, j$ refer to in-plane coordinates $x, y$. This mode is a longitudinal magnetization oscillation $\delta M_z(t, r) \propto \delta M_z \exp[\omega(B, k)t - ikr]$ and is tunable by the *external field*. There are, in effect, two consequences arising from a non-zero $\eta$ which may be used to estimate its magnitude. In the presence of a secondary order parameter $\Delta_1 = id_{xy}$, the temperature dependent upper critical field $H_{c2}(T)$ has an additional contribution which is quadratic in $(1-T/T_c)$. This results in an upward curvature with a scaling field which depends on $\eta$. Thus the upward curvature in $H_{c2}(T)$ and the resonance frequency should be related in that they are both derivable form the same energy scale.

![FIG. 1. Schematic representation of the longitudinal magnetization oscillations caused by the oscillation of the relative phase angle between $\Delta_0$ and $\Delta_1$ with an equilibrium value $\pi/2$. The frequency of this mode is linear in the B field and is also linear in the magnitude of the $\Delta_0$. This mode is therefore tunable by the field and by temperature. We estimate $\omega_0(B, T) \simeq \Delta_1(B, T)$ and is in the Kelvin range.](image)

We now will prove the existence of the “clapping mode” for $d + id'$ state. The free energy has the standard form:

$$F = F_0 + F_1 + \frac{B^2}{8\pi} \tag{4}$$

$$F_0 = \frac{\alpha_0}{2} |\Delta_0|^2 + \frac{b_0}{4} |\Delta_0|^4 + \frac{K_{ij}}{2} |D_1 \Delta_0 D_1 \Delta_0^*| \tag{5}$$

$$F_1 = \frac{\alpha_1}{2} |\Delta_1|^2 + \frac{K'_{ij}}{2} |D_1 \Delta_1 D_1 \Delta_1^*| \tag{6}$$

$$F_{int} = \frac{\eta}{2} |D_x, D_y| \Delta_0 \Delta_1^* + h.c. \tag{7}$$

where $\Delta_0, \Delta_1$ are $d$ and $d'$ components of the order parameter, $\alpha_0 = \alpha_0(T/T_0 - 1)$, $T_0$ is the ordering temperature for $\Delta_0$. The corresponding $a_0 > 0 = N_0$ is always positive, as are $b_0 > 0$ and $K_{ij}, K'_{ij} > 0$. $D_i = \partial_i - i \frac{2e}{c} A_i$, with $\mathbf{B} = \nabla \times \mathbf{A}$. For simplicity we take $K_{ij} = K \delta_{ij}$ and similarly for $K'$ term. Because we are near $H_{c2}$, the flux expulsion is nearly non-existent and $B$ and $H$ are nearly equal. The interaction term in this form has been proposed earlier [20][22]. One can see that the GL free energy is indeed a scalar. The interaction term, using $|D_x, D_y| = ieB$ can be written as

$$F_{int} = ieB \Delta_0 \Delta_1^* + h.c. \tag{8}$$

which corresponds to a coupling of the magnetic field with an *intrinsic* orbital moment along $z$-axis: $< M_z > = \frac{ie}{2} \Delta_0 \Delta_1^* + h.c.$.

Up to now the work on secondary component has been focused on the equilibrium solution for a particular model for induced component, say in $d+is$ or $d+id'$ state. Here we will assume that the amplitude for the secondary $\Delta_1$ has been developed and we will look at the relative phase $\phi$ oscillations only. It is convenient to introduce the relative phases of each component:

$$\Delta_0 = |\Delta_0| \exp(i \phi_0), \quad \Delta_1 = |\Delta_1| \exp(i \phi_1) \tag{9}$$

which are related to the introduced above $\nu = \phi_0, \phi = \phi_1 - \phi_0$. Derivation of the Eq.(3) then proceeds as follows. We use the Josephson relations for each component:

$$\frac{\partial F}{\partial N_i} = \mu_i, \quad \frac{\partial F}{\partial \phi_i}, i = 0, 1 \tag{10}$$

Where $\mu_i$ are chemical potentials for particles in $\Delta_0, \Delta_1$ condensate and similarly, $N_i$ are conjugated number of particles. In the equilibrium, where $\mu_0 = \mu_1 = \mu$, we find for a relative phase motion:

$$\dot{\phi} = -i \frac{\partial \mu_0}{\partial N_0} + \frac{\partial \mu_1}{\partial N_1} - 2 \frac{\partial \mu_0}{\partial N_1} \frac{\partial F}{\partial \phi} \tag{11}$$
where in the above we have taken into account the fact that although \( \mu_0 = \mu_1 \) their derivatives are different. The term in the brackets is on the order of \( N(0) \): 
\[
\frac{\partial \mu_0}{\partial N_0} + \frac{\partial \mu_1}{\partial N_1} - 2 \frac{\partial \mu_0}{\partial N_1} = \rho^{-1} \approx N(0). 
\]
For the general values of the relative phase \( \phi \) the terms in free energy \( F \) that contribute to the derivative in Eq.(11) are \( F_{int}, \) Eq.(3) and terms with gradients \( K_{ij}, K_{ij}' \). 

\[
F_{int} = \eta B_z ||\Delta_0|| \Delta_1 \sin \phi \tag{12}
\]
with minimum \( F \) reached at \( \phi = -\pi/2 \text{sgn}(B_z) \). In general, there is an additional term proportional to the squares of the two (the bare and the induced) order parameters. However it contributes little new to the physics of the problem. Its effects are largely quantitative. We stress here that the phase \( \phi_0 \) of \( \Delta_0 \) is not constant in the external field in the mixed state. Nevertheless the \( F_{int} \) in Eq.(12) depends on the relative phase only.

\[
\text{FIG. } 2. \text{ The } F_{int} \text{ profile as a function of the relative phase angle } \phi \text{ between } \Delta_0 \text{ and } \Delta_1 \text{ is shown. Although the minimum is reached at } \phi = \frac{\pi}{2}, \text{ the finite stiffness for } \phi \text{ oscillations leads to the finite frequency mode at } \omega_0(B, T). \text{ We have ignored the dependence on the sign of } B \text{ in the figure.}
\]

We find:

\[
\dot{\phi} = \frac{1}{\rho} \eta ||\Delta_0|| \Delta_1 B \cos \phi - s^2 \nabla^2 \phi \tag{13}
\]

Here \( s \) is given by Eq.(3). Minimizing the free energy Eq.(4) with respect to the magnitude from this equation, Eq.(3) follows immediately. The velocity \( s \) is field dependent and also can be used in experiments to detect the presence of \( id' \) component.

We can estimate the magnitude of the energy gap up to a \( O(1) \) prefactors:

\[
\omega_0(B, T) \simeq ||\Delta_1(B, T)|| \simeq \frac{\eta B}{N(0)} \Delta_0(B, T) \tag{14}
\]
as one can easily see from minimization of the total \( F \) with respect to \( \Delta_1 \). For estimate for the coupling constant \( \eta \) and magnitude of \( \Delta_1 \) see \[22\]. We estimate thus

\[
\frac{\omega_0(B, T)}{\Delta_0(B, T)} \simeq \left( \frac{\rho^2}{\xi^2} \right) \left( \frac{H}{Hc2} \right) \tag{15}
\]

This result turns out to be very similar to the “clapping mode” in \(^3\text{He} - A\), where frequency was found to scale with the gap in the whole temperature range as well \[28\]. We also find asymptotics in \( H \):

\[
\omega_0 \sim \begin{cases} 
\frac{H}{H_{c2}} & H \leq H_{c2} \\
\frac{1}{\sqrt{H_{c2}} - H} & H \to H_{c2}
\end{cases} \tag{16}
\]

and we note immediately that the mode frequency \( \omega_0 \ll \Delta_0 \). Therefore this mode will be sharp. The damping coming from the low-lying quasiparticles in the nodes of \( d \)-wave gap will not affect this mode because the phase space for the decay will be small.

In above estimates we assumed that the field induced component \( id' \) will be present even outside the GL region for which the above formulas have been derived. We assumed that once the field induced component is present in GL region it will persist to a lower temperatures. Realizing the limitations of estimates made beyond GL regime we present Eq.(16) as an order of magnitude estimate only. From Eq.(15) it follows that the gap \( \omega_0 \) can be made in the range of \( 0.1 - 1K(2-20GHz) \) in the field \( H = 1 - 10T \) similar to the \( \Delta_1 \) estimates \[22\].

The longitudinal oscillations of magnetization \( M_z \) will also lead to the resonance at \( \omega_0(B, T) \) in the AC susceptibility of the superconducting state. Experimentally the proposed clapping mode can be observed in NMR in the field. Another possible method of detecting this resonance is the ultrasound attenuation (taken at lower fields), used previously in heavy fermions to investigate the excitations of the order parameter. For any of the resonance techniques used to search for the “clapping mode” resonance at \( \omega_0(B, T) \) Eq.(15) the vortex cores contribution would be the biggest source of background. We note however that \( \omega_0(B, T) \) can still be measured if the resonance is sharp enough.

In conclusion, we considered the relative phase oscillation mode that can be used to detect the secondary component in the time reversal violating superconducting state, such as \( d + id' \) or \( d + is \). Two components of the order parameter are characterized by respective amplitudes and phases. The relative phase between two components can oscillate around its equilibrium value \( \phi = \pm \frac{\pi}{2} \). This mode is very similar to the “clapping mode” in superfluid \(^3\text{He} - A\). For a specific model we choose the case of field induced \( id' \) component. We show how the relative phase mode frequency is governed by external magnetic field and \( d \)-wave gap magnitude \( \omega_0(B, T) \propto B \Delta_0 \) and is therefore tunable by external magnetic field. The scale for \( \omega_0 \) and mode velocity \( s \) is set by the magnitude of the induced gap \( \Delta_1 \) and we estimated the frequency for different fields, Eq.(13). Among other probes this mode could be experimentally detected by investigating the ac magnetic susceptibility and possibly by ultrasound attenuation in the in the mixed state as a function of applied field \( H \).
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