Joint quantum state tomography of an entangled qubit–resonator hybrid

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Abstract. The integration of superconducting qubits and resonators in one circuit offers a promising solution for quantum information processing (QIP), which also realizes the on-chip analogue of cavity quantum electrodynamics (QED), known as circuit QED. In most prototype circuit designs, qubits are active processing elements and resonators are peripherals. As resonators typically have better coherence performance and more accessible energy levels, it is proposed that the entangled qubit–resonator hybrid can be used as a processing element. To achieve such a goal, an accurate measurement of the hybrid is first necessary. Here we demonstrate a joint quantum state tomography (QST) technique to fully characterize an entangled qubit–resonator hybrid. We benchmarked our QST technique by generating and accurately characterizing multiple states, e.g. \( |gN\rangle + |e(N-1)\rangle \) where \( |g\rangle \) and \( |e\rangle \) are the ground and excited states of the qubit and \( |0\rangle, \ldots, |N\rangle \) are Fock states of the resonator. We further provided a numerical method to improve the QST efficiency and measured the decoherence dynamics of the bipartite hybrid, witnessing dissipation coming from both the qubit and the \( N \)-photon Fock state. As such, the joint QST presents an important step toward actively using the qubit–resonator element for QIP in hybrid quantum devices and for studying circuit QED.
1. Introduction

Superconducting quantum circuits based on Josephson junctions [1] are a promising candidate for realizing quantum computation and simulation [2–4]. In recent years, this field has witnessed tremendous progress, featuring the significantly improved device performance in both coherence and controllability. Coherence time approaches the range of 10–100 $\mu$s in three-dimensional (3D) resonator cavity-protected qubits [5, 6] and, more recently, in two-dimensional resonator cavity-protected qubits where circuit integration is possible [7, 8]. Qubit gate fidelities are high around 90% for a variety of qubits [9–11]. This figure of merit has been further improved to 99.8% [12], approaching the desired error threshold for fault-tolerant quantum computation. With the introduction of parametric amplifiers based on Josephson junctions, fast quantum non-demolition measurement and realtime feedback control become possible [13–15]. Quantum processors featuring resonator-based elements such as quantum bus and memory have been realized for preliminary demonstration of quantum algorithms, providing the hope to build a quantum version of the von Neumann computing machine at the intermediate scale [16–19].

Superconducting resonators have played a significant role in the above-mentioned progress. The strong couplings achieved between qubit and resonator also founded an important field, circuit quantum electrodynamics (QED), where atom–light interaction can be studied on-chip using artificial atoms and quantized microwave photons [20, 21]. Circuit QED has gained significant advances due to the flexibility in circuit design. Recent experiments have demonstrated the deterministic control of non-classical states in single and two coupled resonator cavities [22–24]. Superpositions of microwave photons at different colors have been realized [25] using frequency-tunable resonator cavities [26]. The dramatic enhancement of the 3D resonator’s coherence has allowed the experimental observation of the single-photon Kerr effect when the single-photon interaction strength exceeds the photon decay rate [27].

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Not limited to intracavity photons, propagating microwave photons and their entanglement have also been demonstrated on-chip [28, 29].

In most prototype circuit designs, qubits are the active processing elements and resonators passively serve as bus communicator, storage memory or part of the readout circuit. With better coherence performance and many accessible energy levels, the resonator may also be included in the active processing unit. Experimentally it has been shown that the refocusing and dynamical decoupling techniques can improve the coherence of an entangled state formed between a flux qubit and a microscopic two-level system (TLS) [30]. Here the TLS can be replaced by a resonator with better parameter control, thus providing the possibility of encoding a logical qubit using the two states \(|g\rangle\) and \(|e\rangle\), where \((|g\rangle, |e\rangle)\) are the ground and excited states of the qubit and \(|0\rangle, |1\rangle, \ldots, |N\rangle\) are Fock states of the resonator. Such a proposal is also suggested using a hybrid spin-photon qubit, which couples a frequency-tunable resonator to spin ensembles [31, 32]. In the device, both superconducting circuits and spin ensembles play active roles; the states of the qubit are defined by where the excitation quantum is stored [32].

Before introducing such a qubit–resonator hybrid as a basic processing unit [33], first we need to be able to measure it correctly. So far, most quantum state tomography (QST) techniques have been limited to either solely the qubit or the resonator in superconducting circuits; here, we demonstrate the simultaneous readout of the qubit–resonator hybrid to map out its complex density matrix. We used a second qubit to detect the resonator state. Such a technique would allow us to gain more useful information as compared to just probing the qubit or resonator alone when studying the circuit QED system. It would also shed light on how to efficiently operate a hybrid quantum device [34], bridging e.g. superconducting qubits and spin ensembles, for quantum information processing (QIP).

We benchmarked our QST technique by deterministically generating and accurately detecting various entangled states, e.g. \(|g N\rangle + |e (N - 1)\rangle\) for \(N\) up to 4. As such, we can access more energy levels in both the qubit and the resonator, a key to more powerful QIP. For the Bell-type state \(|g 1\rangle + |e 0\rangle\), we measured an improved fidelity value as compared to that of the two-qubit Bell state, which can be used for violating Bell’s inequality [35]. Furthermore, we provided a numerical method to improve the detection efficiency and measured the decoherence dynamics of the entangled system, witnessing dissipation coming from both the qubit and the \(N\)-photon Fock state by examining the time evolutions of the density matrix elements. Although demonstrated in a hybrid composed of a phase qubit and a half-wavelength coplanar waveguide resonator, we note that the joint QST technique is general and can be applied to other hybrid quantum systems using different types of qubits and/or resonator cavities.

The paper is organized in the following way. In section 2, we describe the general theory of joint quantum state tomography (QST), explaining it for the qubit, the resonator and the coupled qubit–resonator hybrid. We also discuss the theory for optimizing the detection efficiency. In section 3, we introduce our experimental setups, including sample characterization and the joint QST sequences. In section 4, we show the results of various experimentally generated and detected entangled states of the qubit–resonator hybrid, which indicate the validity of our method. We also show data describing the decoherence dynamics of the hybrid. Section 5 is the conclusion.

2. Theory of joint quantum state tomography

QST is to reconstruct an unknown quantum state entirely based on experimental data. The state of a quantum system can be fully represented by a density matrix. An intricacy of quantum
mechanics is related to the off-diagonal entries of the density matrix, which reflect phase coherence of a quantum state. As a single local measurement is typically not sufficient to determine the density matrix, in particular its off-diagonal entries, we repetitively generate the same quantum state and perform various local measurements [38]. Reconstruction of the density matrix requires a complete set of local measurements. For a linear system such as a resonator, it typically takes much more local measurements due to its linearity and large Hilbert space; the density matrix is computed after knowing all of the measurement results. Here we give relevant formulae that are applicable to our experiment.

2.1. Qubit tomography

The Hamiltonian of a superconducting qubit is

\[
\frac{H(t)}{\hbar} = \Delta(t)\sigma_z + \frac{\Omega(t)}{2}(\sigma_+ + \sigma_-),
\]

where \(h\Delta\) is the qubit energy-level spacing and \(\Omega\) describes the amplitude and phase of the microwave signals applied to the qubit. For a phase qubit, the probabilities of \(|g\rangle\) (\(P_g\)) and \(|e\rangle\) (\(P_e\)), i.e. the diagonals (\(\rho_{|g\rangle}|g\rangle\) and \(\rho_{|e\rangle}|e\rangle\)) of the qubit density matrix \(\rho\), can be measured using a superconducting quantum interference device (SQUID) integrated with it.

To fully reconstruct the qubit density matrix \(\rho\), we need to carry out a complete set of local measurements. This is done by applying three unitary rotations \(U_I\), \(U_X\) and \(U^Y\) to the qubit before readout. \(U_I\) is a null operation, \(U_X\) is a \(\frac{\pi}{2}\) rotation around the \(x\)-axis in the Bloch sphere and \(U^Y\) is a \(\frac{\pi}{2}\) rotation around the \(y\)-axis. Their matrix expressions in the qubit basis are

\[
U_I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U_X = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}, \quad U^Y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.
\]

After rotation \(U_i\), \(i \in \{I, X, Y\}\), \(\rho\) changes to \(\rho'^i\), with the resulting new diagonals given by

\[
\rho'_{mm} = \langle m|U^i \rho(U^i)^\dagger|m\rangle = \sum_{m'm''} U^{i*}_{mm'} U^{i}_{m'm''} \rho_{m'm''},
\]

which can be directly measured by SQUID (\(m, m', m'' \in \{g, e\}\)). The off-diagonals of \(\rho\) (\(\rho_{|g\rangle|e\rangle}\) and \(\rho_{|e\rangle|g\rangle}\)) are related to diagonals of \(\rho'^i\); therefore, \(\rho\) can be reconstructed from a set of over-constrained equations following equation (3) via the least-squares optimization.

2.2. Resonator tomography

Due to its linearity, it is difficult to prepare and measure the non-classical states in a resonator via only classical means. Here we use a coupled phase qubit to control and detect the complex resonator states. The Hamiltonian of a qubit, \(Q\), and a resonator, \(R\), in the resonator rotating frame is

\[
\frac{H(t)}{\hbar} = \Delta(t)\sigma_z + \left(\frac{g}{2}\sigma_+ a + \frac{\Omega_Q(t)}{2}\sigma_+ + \frac{\Omega_R(t)}{2}a^\dagger\right) + \text{h.c.},
\]

where the \(a^\dagger\) and \(a\) (\(\sigma_+\) and \(\sigma_-\)) are the resonator (qubit) raising and lowering operators respectively, \(\Delta(t) = \omega_Q(t) - \omega_R\) is the \(Q-R\) detuning, \(g/2\pi\) is the \(Q-R\) interaction strength, \(\Omega_R\) (\(\Omega_Q\)) describes the amplitude and phase of the microwave signals applied to resonator \(R\) (qubit \(Q\)) and h.c. is the Hermitian conjugate of the terms in parentheses.

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In the simplest case of qubit \( Q \) in its \(|g\rangle\) state, zero \( Q-R \) detuning (\( \Delta = 0 \)) and no microwave signals (\( \Omega_R = \Omega_Q = 0 \)), only the \( Q-R \) interaction terms in equation (4) remain and the qubit \(|e\rangle\)-state probability will evolve in the following way:

\[
P_e(t) = \frac{1}{2} \left( 1 - \sum_{n=0}^{\infty} \rho_{nn} \cos(\sqrt{ng}t) \right),
\]

where \( \rho_{nn} \) is the \( n \)th diagonal term of the resonator density matrix \( \rho \), expressed in the Fock-state basis. Measuring the time evolution of \( P_e(t) \), we can obtain \( \rho_{nn} \) by the least-squares fit.

Similar to the qubit QST, to map out the resonator density matrix \( \rho \), we apply unitary operations to effectively relocate the off-diagonals of \( \rho \) to diagonals of the resulting new matrix for measurement [22, 39]. The microwave signals applied to the resonator can coherently displace the resonator state, described by the displacement operator \( D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) \), where \( \alpha = \frac{1}{2} \int \Omega_R(t) \, dt \). \( D(\alpha) \) can be expanded in the resonator Fock-state basis as

\[
\langle n | D(\alpha) | n' \rangle = e^{-|\alpha|^2/2} \sqrt{n'!} \sum_{k=0}^{\min(n, n')} \frac{\alpha^{n-k} (-\alpha^*)^{n-k}}{k!(n-k)!(n'-k)!}.
\]

The resonator density matrix \( \rho \) changes to a new matrix \( \rho'' \), with the resulting new diagonals given by

\[
\rho''_{nn} = \langle n | D(\alpha) \rho D(\alpha)^\dagger | n \rangle = \sum_{n'n''} D(\alpha)_{nn'} D(\alpha)_{n'n''}^* \rho_{n'n''}.
\]

which can be measured by the coupling qubit according to equation (5). Experimentally, there are negligible photon populations beyond certain energy level \(|M\rangle\); therefore, we take into account only the energy levels \(|0\rangle, \ldots, |M\rangle\) when calculating equation (7). Choosing a set of \( \alpha \) and measuring the resulting new matrix diagonals, \( \rho''_{nn} \), we can infer the original density matrix \( \rho \) from equation (7).

Another method to fully characterize the resonator is the parity-measurement-based Wigner tomography, where measurement of the coupling qubit after the qubit–resonator dispersive interaction is used to determine the Wigner function, \( W(\alpha) \), and thus the density matrix \( \rho \). \( W(\alpha) = (2/\pi) \text{Tr} (D(-\alpha) \rho D(\alpha) P) \), where \( P \) is the parity operator [22]. In our experiment, \( W(\alpha) \) can be calculated using the measured \( \rho''_{nn} \) through \( W(\alpha) = \sum_n (-1)^n \rho''_{nn} \). We note that the parity measurement relies on accurately tuning the dispersive interaction between the qubit and resonator, which is more difficult to implement in our case due to the limited qubit dephasing time, \( T_2 \). Our method is less sensitive to dephasing noise and we directly obtain \( \rho \) from the measured \( \rho''_{nn} \) without calculating \( W(\alpha) \).

### 2.3. Joint tomography for a qubit–resonator hybrid

As an auxiliary qubit is needed to measure the resonator, two qubits, \( Q_1 \) and \( Q_2 \), and one resonator \( R \) are needed for a qubit–resonator hybrid. The hybrid is \( Q_1-R \) and \( Q_2 \) is for the resonator readout. In the resonator rotating frame, the Hamiltonian of the system is

\[
\frac{H(t)}{\hbar} = \Delta_1(t) \sigma_+^{(1)} \sigma_-^{(1)} + \Delta_2(t) \sigma_+^{(2)} \sigma_-^{(2)} + \left( \frac{g_1}{2} \sigma_+^{(1)} a + \frac{g_2}{2} \sigma_+^{(2)} a \right) + \frac{\Omega_R(t)}{2} a^\dagger + \frac{\Omega_1(t)}{2} \sigma_+^{(1)} + \frac{\Omega_2(t)}{2} \sigma_+^{(2)} + \text{h.c.},
\]

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where $\sigma_x^{(i)}$ ($\sigma_z^{(i)}$) is the raising (lowering) operator of $Q_i$ ($i = 1$ or 2), $a^\dagger$ ($a$) is the raising (lowering) operator of $R$, $\Delta_i(t)$ is the $Q_i$–$R$ detuning, $g_i/2\pi$ is the $Q_i$–$R$ interaction strength, $\Omega_R$ ($\Omega_1$) describes the amplitude and phase of the microwave signals applied to the resonator $R$ (qubit $Q_1$) respectively and h.c. is the Hermitian conjugate of the terms in parentheses.

The density matrix $\rho$ of the $Q_1$–$R$ hybrid is defined in the basis $|Q_1\rangle \otimes |R\rangle$, i.e. $|g0\rangle$, $|g1\rangle$, $|gn\rangle$, $|e0\rangle$, $|e1\rangle$, $|en\rangle$, ... Diagonals of $\rho$ can be measured with the help of $Q_2$. For measurement, $Q_2$, originally in its $|g\rangle$, is tuned into resonance with $R$ while $Q_1$ is tuned off resonance. After an interaction time $t$ and no microwave signals during the period, a simultaneous readout of $Q_1$ and $Q_2$ yields the four probabilities, $P_{gg}$, $P_{ge}$, $P_{eg}$ and $P_{ee}$. Ignoring decoherence, the four probabilities versus the interaction time $t$ are given by

$$
\begin{align*}
P_{gg}(t) &= \frac{1}{2} \left( P_g^{(1)} + \sum_{n=0}^{\infty} \rho_{|gn\rangle|gn\rangle} \cos(\sqrt{n}g_2 t) \right), \\
P_{ge}(t) &= \frac{1}{2} \left( P_g^{(1)} - \sum_{n=0}^{\infty} \rho_{|gn\rangle|gn\rangle} \cos(\sqrt{n}g_2 t) \right), \\
P_{eg}(t) &= \frac{1}{2} \left( P_e^{(1)} + \sum_{n=0}^{\infty} \rho_{|en\rangle|en\rangle} \cos(\sqrt{n}e_2 t) \right), \\
P_{ee}(t) &= \frac{1}{2} \left( P_e^{(1)} - \sum_{n=0}^{\infty} \rho_{|en\rangle|en\rangle} \cos(\sqrt{n}e_2 t) \right),
\end{align*}
$$

(9)

where $P_g^{(1)}$ and $P_e^{(1)}$ are the occupation probabilities of $Q_1$ in its initial state. By measuring the time evolution of $P_{gg}(t)$, $P_{ge}(t)$, $P_{eg}(t)$ and $P_{ee}(t)$, the diagonals of the density matrix can be obtained through the least-squares fit: $\rho_{mn,mm}$, $m \in \{g, e\}$ and $n$ is the Fock-state index. (For clarity in matrix manipulations, we remove ‘$|$’ and ‘‘$\rangle$’ here and below in this section.)

To do QST of the $Q_1$–$R$ hybrid, microwave signals are applied to $Q_1$ and $R$ as described previously, noted by unitary rotations $U^i$ for $i \in \{I, X, Y\}$ and $D(\alpha)$ respectively. The combined action can be expressed in the joint system basis as

$$
(mn|U^i \otimes D(\alpha)|m'n') = U^i_{mn} D(\alpha)_{nn'}.
$$

(10)

After the action, the density matrix $\rho$ changes to a new matrix $\rho^{i,\alpha}$. The resulting diagonals of $\rho^{i,\alpha}$ relate to $\rho$ in the following way:

$$
\rho^{i,\alpha}_{mn,mm} = (mn|U^i \otimes D(\alpha))\rho(U^i \otimes D(\alpha))^\dagger |mn\rangle = \sum_{m'n'm'n'} U^i_{mn,mm'} D(\alpha)_{nn'} D(\alpha)_{nn'}^* \rho_{m'n',m'n'},
$$

(11)

which can be measured following equation (9). By applying a set of actions as defined in equation (10) and measuring the resulting new matrix diagonals, we can infer the original density matrix $\rho$ of the $Q_1$–$R$ via the least-squares fit following equation (11).

### 2.4. How to choose $\alpha$ for efficient resonator tomography

As discussed previously, for the qubit QST, we perform three unitary rotation operations $U^i$ before the qubit readout. The information extracted from all three measurements are sufficient
to determine the qubit density matrix. For the resonator QST and the joint QST, we choose a set of \( \alpha \) and perform the displacement operations \( D(\alpha) \) before the resonator readout using the auxiliary qubit. As a resonator possesses a much larger Hilbert space (here we only consider energy levels up to \(|M\rangle\), where \(|M\rangle\) is the highest energy level being populated), we need a certain number of \( \alpha \), such that we could infer the original density matrix with enough confidence. But choosing more than enough \( \alpha \) would take up a lot of experimental and computational resources. The problem is: how can we minimize the number of \( \alpha \) without affecting our confidence when inferring the original density matrix?

The linear map between the measurement results \( \rho^\alpha \) and the original \( \rho \) is defined in equation (7), which depends on the choice of the set \( \{\alpha\} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\} \). To write it in a simple linear transformation form, we write \( \rho \) in a vector form and put all measurement outputs \( \rho^\alpha \) into another vector. We have a vector \( \tilde{\rho} \) for each density matrix \( \rho \)

\[
\tilde{\rho} = [\rho_{00}, \ldots, \rho_{0M}, \rho_{10}, \ldots, \rho_{1M}, \ldots, \ldots, \rho_{MM}]
\]  

(12)

and a vector \( \tilde{\rho}' \) for all measurement outputs with all \( \alpha \)

\[
\tilde{\rho}' = [\rho_{00}^{\alpha_0}, \rho_{11}^{\alpha_1}, \ldots, \rho_{MM}^{\alpha_M}, \rho_{00}^{\alpha_0}, \ldots, \rho_{MM}^{\alpha_M}].
\]  

(13)

Equation (7) can be written as

\[
\tilde{\rho}' = \mathcal{T}(\{\alpha\})\tilde{\rho},
\]

(14)

where \( \mathcal{T}(\{\alpha\}) \) is the transfer matrix including the actions of all \( \alpha \).

In order to obtain a stable numerical solution, we construct an objective function based on numerical stability. Given a small change \( \delta\tilde{\rho} \) of \( \tilde{\rho} \) and the corresponding change \( \delta\tilde{\rho}' \) of \( \tilde{\rho}' \), we derive the following equations:

\[
\mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})^\dagger\tilde{\rho} = \mathcal{T}(\{\alpha\})\tilde{\rho},
\]

(15)

\[
\mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})^\dagger(\tilde{\rho} + \delta\tilde{\rho}) = \mathcal{T}(\{\alpha\})\tilde{\rho} + \delta\tilde{\rho}',
\]

(16)

\[
\mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})\delta\tilde{\rho} = \mathcal{T}(\{\alpha\})\delta\tilde{\rho}',
\]

(17)

\[
\delta\tilde{\rho} = (\mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\}))^{-1}\mathcal{T}(\{\alpha\})\delta\tilde{\rho}',
\]

(18)

where \( \mathcal{T}(\{\alpha\})^\dagger \) is the Hermitian conjugate of \( \mathcal{T}(\{\alpha\}) \). Based on equations (15) and (18) and using Cauchy–Schwarz inequality, we obtain the following inequalities:

\[
\frac{1}{\| \tilde{\rho} \|} \leq \frac{\| \mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})^\dagger \|}{\| \mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})^\dagger \|},
\]

(19)

\[
\| \delta\tilde{\rho} \| \leq \| (\mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\}))^{-1} \| \| \mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})^\dagger \delta\tilde{\rho}' \| .
\]

(20)

We obtain the relative error inequality following equations (19) and (20) as

\[
\frac{\| \delta\tilde{\rho} \|}{\| \tilde{\rho} \|} \leq \text{cond} (\mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})) \frac{\| \mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})^\dagger \delta\tilde{\rho}' \|}{\| \mathcal{T}(\{\alpha\})\mathcal{T}(\{\alpha\})^\dagger \tilde{\rho}' \|},
\]

(21)
where the condition number of $\mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\})$ is defined as
\[ \text{cond} \left( \mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}) \right) = \| \mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}) \| \| (\mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}))^{-1} \| . \] (22)

The condition number controls the error of $\tilde{\rho}$. Taking $\| \cdot \|$ as $\| \cdot \|_2$ [40], we have
\[ \text{cond} \left( \mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}) \right) = \frac{\lambda_{\max} \left( \mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}) \right)}{\lambda_{\min} \left( \mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}) \right)} , \] (23)

where $\lambda_{\min}$ and $\lambda_{\max}$ are the smallest and largest eigenvalues of $\mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\})$, respectively.

If $\text{cond}(\mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}))$ is large, the linear system is ill conditioned. Small experimental errors of $\delta \tilde{\rho}'$ will be amplified and give large errors in $\delta \tilde{\rho}$. So we construct the optimization objective function as
\[ F(\alpha) = \min_{\alpha} \frac{\lambda_{\max} \left( \mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}) \right)}{\lambda_{\min} \left( \mathcal{T}(\{\alpha\})^\dagger \mathcal{T}(\{\alpha\}) \right)} . \] (24)

The objective function $F$ is minimized by particle swarm optimization [41] to obtain an optimal choice of $\alpha$ when the number of $\alpha$ is set.

Using particle swarm optimization, we checked two cases of $M = 4$ and 6, where $|M|$ is the highest energy level populated in the initial state of the resonator. In the actual calculation, we included more energy levels as the operation $D(\alpha)$ can potentially excite energy levels higher than $|M|$. The case of a larger $M$ certainly requires more local measurements (more $\alpha$) for inferring $\rho$. In figure 1(a), we display the optimized set of $\alpha$ (red squares) in the complex plane for the case of $M = 6$ when 13 $\alpha$ are used. It is seen that the optimized set

**Figure 1.** Choice of $\alpha$. (a) The optimized set of $\alpha$ (red squares) shown in the complex plane for 13 $\alpha$ and $M = 6$. For comparison, a circular set of $\alpha$ with a similar distribution pattern is shown (black dots). The circular set is defined as $\{|\alpha_r| \exp(i2\pi k/13), \ k = 0, 1, \ldots, 12\}$, where $|\alpha_r| = 0.8$. $F$ values of the two sets of $\alpha$ are close. (b) $F$ defined in equation (24) as a function of $|\alpha_r|$ for the circular set of $\alpha$ defined in (a). Numerical simulation results suggest that the circular set of $\alpha$ with $|\alpha_r|$ in the circled region can be used for reliably inferring $\rho$. (c) The minimized $F_{\text{min}}$ (red squares) as functions of the number of $\alpha$ for $M = 4$ and 6 as indicated. $F$ (black dots) as functions of the number of $\alpha$ for the circular set of $\alpha$ with $|\alpha_r| = 0.8$. The minimum numbers of $\alpha$ required for reliably inferring $\rho$ are 9 and 13 (indicated by arrows) for $M = 4$ and 6, respectively.
of $\alpha$ tend to align in a circle. For comparison, we also plot a circular set of $\alpha$ defined as 
$\{|\alpha_r| \exp(i2\pi k/13), k = 0, 1, \ldots, 12\}$ (black dots), where $|\alpha_r| = 0.8$. The two sets of $\alpha$ have very similar distribution patterns and their $F$ values are close. In figure 1(b), we vary $|\alpha_r|$ and plot the calculated $F$ as a function of $|\alpha_r|$ for the circular set of 13 $\alpha$. $F$ values are generally small for $|\alpha_r| \in [0.7, 1.0]$ (indicated by a circle in the figure). We numerically simulated the density matrix reconstruction process, during which we introduced artificial noise to check whether a set of $\alpha$ is reliable for inferring $\rho$. The simulation results suggest that a circular set of $\alpha$ can be good if its $F$ value is close to the minimum $F$ located by particle swarm optimization using the same number of $\alpha$. Details of the simulation and its connection with the optimization method will be discussed elsewhere.

An important contribution of the optimization method is that it can tell the minimum number of $\alpha$ needed for reliably inferring $\rho$. In figure 1(c), we show the minimized $F_{\text{min}}$ (red squares) as functions of the number of $\alpha$ for $M = 4$ and 6 as indicated. For comparison, we plot $F$ (black dots) as functions of the number of $\alpha$ when these $\alpha$ are equally separated around a circle centered at the origin in the Re($\alpha$)–Im($\alpha$) complex plane with a radius of 0.8, i.e. the circular set with $|\alpha_r| = 0.8$. Figure 1(c) suggests that for reliably inferring $\rho$, 9 $\alpha$ are required for $M = 4$ and 13 $\alpha$ for $M = 6$ (indicated by arrows). We confirmed the validity of these two numbers experimentally.

The optimization method can be flexibly tailored to target certain elements of $\rho$, in which even fewer $\alpha$ can be used if we do not need to know all the elements of $\rho$. More details of the optimization method for reliably inferring $\rho$ will be discussed elsewhere.

3. Experimental setup

3.1. Sample characterization

Figure 2(a) shows the device photo and schematics of the sample used in this experiment. Two qubits, $Q_1$ and $Q_2$, are capacitively coupled to a superconducting coplanar waveguide resonator $R$. $Q_1$ is used to entangle with resonator $R$, forming the $Q_1$–$R$ hybrid unit. $Q_2$ is used to measure resonator $R$ for joint QST.

Measurements were performed below 20 mK in a cryogen-free dilution refrigerator. Microwave pulses were generated by a custom arbitrary waveform generator, which can generate pulses with sub-nanosecond resolution in envelopes [42]. We used superconducting phase qubits, with integrated flux bias (for control) and SQUID (for readout) [43]. The repetitive rate of microwave pulses is set close to 1 kHz, allowing sufficient time for the sample to cool, such that non-equilibrium quasiparticle injections due to SQUID’s switching should be negligible [44]. Figure 2(b) displays the spectroscopy measurement data of two qubits, showing that their resonance frequencies (y-axis) can be tuned by current applied to their flux bias (x-axis). The corresponding pulse sequence is illustrated in figure 2(c), together with detailed qubit potential well changes showing how the operation is done. 1—The qubit ($Q_1$ or $Q_2$) starts in its ground state in the left-hand potential well. 2—A square pulse is applied to its flux bias to tilt the well such that the energy-level spacing changes, which corresponds to changing the qubit resonance frequency. 3—A continuous microwave with a fixed frequency is then applied to the qubit. If the microwave frequency matches the resonance frequency of the qubit, the qubit will be excited, otherwise it remains in the ground state. 4—The sequence ends with a triangle-shape measure pulse, which tilts the potential well significantly, such that the excited state will tunnel.
Figure 2. Sample characterization. (a) Device photos (top) and circuit schematic (bottom). The superconducting circuit chip (top-right), approximately 6.25 × 6.25 mm² in size, was enclosed in an aluminum box (top-left) and loaded onto the 20 mK stage of the dilution refrigerator. Circuit elements used in this experiment are as labeled. The superconducting coplanar waveguide resonator is coupled to qubits $Q_1$ and $Q_2$ using interdigitated capacitors. Each qubit is measured by the SQUID and manipulated by the flux bias integrated with it. (b) Spectroscopy of both qubits, showing the qubit $|e\rangle$-state probability $P_e$ (colorbar on the right) versus microwave frequency and flux bias. The avoided level crossings both around 6.210 GHz are due to qubit–resonator interactions, which give the resonator resonance frequency. The qubit–resonator coupling strengths are indicated. (c) Pulse sequence for spectroscopy measurement and illustration of the qubit potential well change during the pulse sequence. 1—The qubit ($Q_1$ or $Q_2$) starts in its ground state in the left-hand potential well. 2—A square pulse is applied to its flux bias to tilt the well such that the energy-level spacing (qubit resonance frequency) changes. 3—A continuous microwave on resonance with qubit drives the qubit to oscillate between its $|g\rangle$ and $|e\rangle$. 4—The sequence ends with a triangle-shape measure pulse, which tilts the potential well significantly, such that the excited state will tunnel to the right-hand well (not drawn), while the ground state remains in the left-hand well. An on-chip SQUID is used to determine whether the qubit tunnels or not.
to the right-hand well (not drawn), while the ground state remains in the left-hand well. The SQUID is then used to differentiate whether the qubit tunnels, giving a count of 1 if it tunnels (qubit in |e⟩) or 0 if no tunnel happens (qubit in |g⟩). By running the same sequence hundreds of times, we can collect the probabilities of the qubit in |g⟩ and |e⟩.

Qubit $Q_1$ ($Q_2$) has an energy lifetime $T_1 = 490\,\text{ns}$ ($590\,\text{ns}$) and a Ramsey $T_2 = 140\,\text{ns}$ ($160\,\text{ns}$) at its idle point of 6.05 GHz (5.96 GHz). Both spectroscopy data show splittings centered around $\omega_R/2\pi = 6.210\,\text{GHz}$, which is the resonance frequency of resonator $R$ as designed. Resonator $R$ has an energy lifetime $T_{1R} = 2.6\,\mu\text{s}$ and a Ramsey $T_{2R} = 3.8\,\mu\text{s}$. The $Q_1$–$R$ coupling strength is $g_1/2\pi = 34\,\text{MHz}$ and the $Q_2$–$R$ coupling strength is $g_2/2\pi = 33\,\text{MHz}$, as shown in figure 2(b).

3.2. Joint tomography sequence and the crosstalk-free tomography for phase qubit

We have discussed the theory of joint QST in section 2. The joint QST pulse sequence is illustrated in figure 3(a). After the state preparation in $Q_1$–$R$, we apply a single qubit rotation pulse $U^\dagger$ (20 ns long) on $Q_1$ and a resonator displacement pulse $D(\omega)$ (30 ns long) on $R$, shown as sinusoidals. We then bring $Q_2$ on resonance with it for an interaction time $t$, which varies from 0 to 50 ns, followed immediately by triangle measure pulses on both $Q_1$ and $Q_2$. The resulting two-qubit occupation probabilities $P_{gg}$, $P_{ge}$, $P_{eg}$ and $P_{ee}$ are used to analyze the prepared state in the $Q_1$–$R$ hybrid.

We note that to avoid measurement crosstalk, the measurement pulse on $Q_1$ must be applied after the $Q_2$–$R$ interaction is done, since a measure on $Q_1$ could accidentally populate resonator $R$. Energy relaxation can happen during the wait period due to the limited $Q_1$ lifetime. Therefore, using the joint QST sequence shown in figure 3(a), we need to correct for this decayed ratio.

It is possible to avoid the extra wait in $Q_1$ by performing the crosstalk-free joint QST sequence as shown in figure 3(b). The crosstalk-free QST sequence is divided into two steps. In the first step, the measurement pulse of $Q_1$ is applied right after the rotation pulse $U^\dagger$ and we only save the probabilities $P_{gg}^{\text{mea}}(t)$ and $P_{ge}^{\text{mea}}(t)$. Probabilities $P_{eg}^{\text{mea}}(t)$ and $P_{ee}^{\text{mea}}(t)$ are not saved because measurement of $Q_1$ in |e⟩ disturbs $Q_2$’s measurement due to crosstalk. In the second step, we only apply a measurement pulse on $Q_2$ and get the probabilities $P_{gg}^{\text{mea}}(t)$, $P_{eg}^{\text{mea}}(t)$ for $Q_2$ ($Q_1$ is not measured). At last, we have the four probabilities $P_{gg}(t)$, $P_{ge}(t)$, $P_{eg}(t)$ and $P_{ee}(t)$ through the relations below.

\begin{align}
P_{gg}(t) &= P_{gg}^{\text{mea}}(t), \\
P_{ge}(t) &= P_{ge}^{\text{mea}}(t), \\
P_{eg}(t) &= P_{eg}^{\text{mea}}(t) - P_{gg}^{\text{mea}}(t), \\
P_{ee}(t) &= P_{ee}^{\text{mea}}(t) - P_{ge}^{\text{mea}}(t).
\end{align}

(25)

In figure 3(c), we show an example of the measured data using the crosstalk-free QST method. Figure 3(d) shows the calculated occupation probabilities of the $Q_1$–$R$ hybrid using the data in figure 3(c).

Both regular and crosstalk-free methods yield correct and similar tomography results within experimental errors. The crosstalk-free method takes a longer time but it avoids using the extra energy-decay correction factor $e^{-t_w/T_1}$, where $t_w = 50\,\text{ns}$ and $T_1$ is $Q_1$’s lifetime.
Figure 3. Illustrations of the joint QST. (a) The regular joint QST pulse sequence (on the right of the vertical dashed line). After the state preparation in $Q_1$–$R$, we apply a single qubit rotation pulse $U^i$ (20 ns long) on $Q_1$ and a resonator displacement pulse $D(\alpha)$ (30 ns long) on $R$ (sinusoidal). We then bring $Q_2$ on resonance with $R$ for an interaction time $t$ using the square pulse, followed immediately by triangle measure pulses on both $Q_1$ and $Q_2$. $t$ varies from 0 to 50 ns. Delaying $Q_1$’s measure pulse is to avoid crosstalk. (b) The crosstalk-free joint QST pulse sequence (on the right of the vertical dashed lines). The measurement consists of two steps. In step 1 (left panel), we measure $Q_1$ immediately after $U^i$ pulses and measure $Q_2$ after its swap with resonator $R$, obtaining four probabilities $P_{gg}(t)$, $P_{ge}(t)$, $P_{eg}(t)$ and $P_{ee}(t)$. We only keep $P_{ge}(t)$ and $P_{eg}(t)$ as $Q_1$ in $|e\rangle$ affects readings of $Q_2$ due to crosstalk. In step 2, we only apply a triangle measurement pulse on $Q_2$ and get $Q_2$’s probabilities $P_{gg}(t)$ and $P_{ee}(t)$. The crosstalk-free probabilities $P_{gg}(t)$, $P_{ge}(t)$, $P_{eg}(t)$ and $P_{ee}(t)$ are given according to equation (25) for QST analysis. (c) Experimental data (dots) and fitted curves (lines) of the measured probabilities, using the crosstalk-free method for the prepared state $|g1\rangle + |e0\rangle$ of $Q_1$–$R$ while $U^i$ and $D(\alpha)$ are null operations. The upper panel shows the probabilities $P_{gg}(t)$ (red) and $P_{ge}(t)$ (blue) in step 1. The lower panel shows the probabilities $P_{eg}(t)$ (red) and $P_{ee}(t)$ (blue) in step 2. We then use equation (25) to obtain the four probabilities for QST analysis. (d) Populations inferred from data in c. The most significant occupations are in $|g1\rangle$ and $|e0\rangle$, as expected.

$T_1$ could change over time due to a slow drift of $Q_1$’s operation point, and therefore the correction factor may slightly vary. Here we mainly report the tomography results using the crosstalk-free method.
4. Experimental results

4.1. Density matrices of various states via joint tomography

We synthesized some representative entangled states of the $Q_1$–$R$ hybrid and then used the joint QST technique to map out their density matrix. We first generated the Bell-type state $|g1⟩ + |e0⟩$. To begin with, we applied a $\pi$-pulse to excite $Q_1$ to $|e⟩$, followed by tuning $Q_1$’s frequency into resonance with resonator $R$ for an interaction period, during which just half of the energy quanta entered the resonator. After the state preparation, we immediately performed the joint QST sequences as shown in figure 3. Both the regular and the crosstalk-free method reveal similar results after data correction (we only show the crosstalk-free results below). The mapped-out density matrix is shown in figure 4(a) (bars with colors), with fidelity 93.8%, entanglement of formation (EOF) [36] 87.3% and negativity [37] 87.9%. All values reported here are after readout corrections. These close-to-unit values prove that qubit and resonator are highly entangled.

We note that in previous experiments, the excitation in resonator $R$ for state $|g1⟩ + |e0⟩$ could be transferred to $Q_2$, resulting in the Bell state of two qubits $|ge⟩ + |eg⟩$, which was used to violate Bell’s inequality [35]. We have also performed a similar operation and characterized the resulting two-qubit Bell state, with its density matrix shown in black frames in figure 4(a). The apparent loss of bar height (black frames) compared to $|g1⟩ + |e0⟩$ (bars with color) is consistent with the energy relaxation and dephasing processes that occurred during the extra operation time for $|ge⟩ + |eg⟩$. For the two-qubit Bell state, the fidelity, EOF and negativity values are 90.8, 82.2 and 82.3%, respectively. The fact that we can detect a much better Bell-type state in the qubit–resonator hybrid suggests that this system may be useful for quantum computation and other applications relying on highly entangled quantum state.

To demonstrate the bosonic nature of the resonator, we have created more general states $|gN⟩ + |e(N−1)⟩$, with total $N$ photons in the system. The generation sequence is shown in figure 4(e). We first create the $N−1$ Fock state in resonator $R$ using $Q_1$ [22, 45], followed by injecting another photon into $Q_1$ and then swapping half of it into $R$, with a swap time depending on $N$. The measured density matrices for $N = 2$ and 3 are shown in figures 4(b) and (c).

To demonstrate the accuracy of our joint QST technique, we synthesized and detected $(|g⟩ + |e⟩) \otimes |\tilde{a}⟩$, where $|\tilde{a}⟩$ represents the coherent state in resonator $R$. $|\tilde{a}⟩$ was generated by applying a microwave pulse directly to the resonator with a calibrated displacement $\alpha = 0.8$. During the same time, we excited $Q_1$ to $|g⟩ + |e⟩$ by applying a $\frac{\pi}{8}$-pulse on $Q_1$ (see the pulse sequence in figure 4(f)). The joint QST result of $(|g⟩ + |e⟩) \otimes |\tilde{a}⟩$ is shown in figure 4(d), with a fidelity of 91.1%. EOF and negativity are all zero, showing that the state is not entangled at all. The fact that all elements, including those very small ones of the density matrix, show correct values indicates the accuracy of the QST technique.

4.2. Decoherence dynamics of the qubit–resonator hybrid

Using only 13 $\alpha$ after optimization as compared to 60 $\alpha$ in a previous experiment [46] when detecting resonator $R$, we can perform the joint QST relatively fast. Therefore, we were able to ‘film’ the decoherence dynamics of the qubit–resonator hybrid [46]. We have done so for the series of entangled states $|gN⟩ + |e(N−1)⟩$. In figure 5(a), we show the representative snapshots of the generated states $|g1⟩ + |e0⟩$ during its free decay at delay = 0, 90, 180 and 270 ns. Time evolutions of the three major matrix elements ($\rho_{|g1⟩,|e0⟩}$, $\rho_{|g1⟩,|g1⟩}$ and $\rho_{|e0⟩,|e0⟩}$) are
Figure 4. Benchmark the joint QST. Representative $Q_1-R$ density matrices (bars with colors) probed using the crosstalk-free method for (a) $|g1\rangle + |e0\rangle$ with fidelity 93.8%, EOF 87.3% and negativity 87.9%, (b) $|g2\rangle + |e1\rangle$ with fidelity 80.1% and negativity 74.6%, (c) $|g3\rangle + |e2\rangle$ with fidelity 72.2% and negativity 68.9% and (d) $|e\rangle + |g\rangle$ with fidelity 91.1%. All values are after readout corrections. EOF and negativity values are for entanglement verification, both reaching 100% for maximum entanglement and zero for no entanglement. (e), (f) The generation pulse sequences for $|gN\rangle + |e(N-1)\rangle$ and $|g\rangle + |e\rangle \otimes |\tilde{\alpha}\rangle$, respectively. Pulses are as indicated. The square pulses on $Q_1$ in (e) are to swap photons into resonator $R$, with the first $N-1$ squares swapping one photon into $R$ each and the final one swapping half a photon into $R$ for entanglement. Joint QST sequences on the right side of the vertical dashed lines are omitted for clarity (see figure 3). In (a) we also display the $Q_1-Q_2$ Bell-state density matrix (black frames) generated by an extra $Q_2-R$ iSWAP following the generation of $|g1\rangle + |e0\rangle$ in $Q_1-R$. The $Q_1-Q_2$ Bell state has the fidelity 90.8%, EOF 82.2% and negativity 82.3%. The apparent loss compared to state $|g1\rangle + |e0\rangle$ in $Q_1-R$ (bars with colors in (a)) is due to energy relaxation and dephasing that occurred during the extra $Q_2-R$ iSWAP.

shown in figures 5(b) and (c). Time evolutions of the major matrix elements ($\rho_{gN}(gN)$ and $\rho_{g(N-1)}^{(e(N-1))}$) for $N = 2, 3$ and 4 are shown as well. For clarity, we only display the absolute values of the matrix elements.

In our experiment, $Q_1$’s $T_1 = 490$ ns and $T_2 = 140$ ns, while the resonator’s $T_{1R} = 2.6 \mu$s and $T_{2R} = 3.8 \mu$s. Dephasing is dominated by qubit, since the off-diagonals (black dots) completely disappear in less than 200 ns as shown in figure 5(b). However, as $N$ increases, there are slight curvature changes in the data, as guided by black lines in figure 5(b). This trend
Figure 5. Decoherence dynamics of the entangled states $|g N⟩ + |e(N - 1)⟩$ in the $Q_1 - R$ hybrid. (a) Snapshots of the density matrices of the initially created $|g1⟩ + |e0⟩$ during free decay at different delay times as indicated. (b) Major off-diagonal elements $ρ_{|g N⟩|e(N - 1)⟩}$ versus delay for different $N$ as indicated. For $N = 1$, the corresponding matrix element is marked by a black oval shown in the left-most panel in (a). Lines are guides for the eye. (c) Major diagonal elements $ρ_{|g N⟩⟨g N|}$ (red dots) and $ρ_{|e(N - 1)⟩⟨e(N - 1)|}$ (blue dots) versus delay for different $N$ as indicated. For $N = 1$, the corresponding matrix elements are marked by ovals with the same colors in the left-most panel in (a). Lines are simulations using known qubit and resonator lifetimes.

may be associated with the unique property of the bipartite hybrid, where both Markovian noise related to the resonator [46] and $1/f$ noise related to the qubit [43] contribute to dephasing. The details of the dephasing dynamics will be presented elsewhere.

We now discuss the energy relaxation process as indicated by time evolutions of diagonals $ρ_{|g N⟩⟨g N|}$ (red dots) and $ρ_{|e(N - 1)⟩⟨e(N - 1)|}$ (blue dots) in figure 5(c). For $ρ_{|g N⟩⟨g N|}$, only the event of $|N⟩ → |N - 1⟩$ in the resonator contributes to its decay; therefore, $ρ_{|g N⟩⟨g N|}$ should decay at a rate similar to the $N$-photon Fock state. $ρ_{|g N⟩⟨g N|}$’s decay time is $T_{1R}/N$ [47]. For $ρ_{|e(N - 1)⟩⟨e(N - 1)|}$, both events of $|e⟩ → |g⟩$ in the qubit and $|N - 1⟩ → |N - 2⟩$ in the resonator contribute to its decay; therefore, $ρ_{|e(N - 1)⟩⟨e(N - 1)|}$ should decay at a rate adding the two dissipation channels. $ρ_{|e(N - 1)⟩⟨e(N - 1)|}$’s decay time is $[1/T_1 + 1/(T_{1R}/(N - 1))]^{-1}$. In figure 5(c), we plot the simulation results (red and blue lines) using known qubit and resonator lifetimes. The apparent agreement between the experimental data (dots) and simulations (lines) suggest that both the qubit and the $N$-photon Fock state dissipation channels exist and that the overall observed dissipation behavior agrees with theory.
5. Conclusion

In conclusion, we have demonstrated a practical joint QST technique to accurately detect the complex entangled states in a qubit–resonator hybrid, which allows us to access more energy levels in both the qubit and resonator. We benchmarked the joint QST technique using various states, including $|gN⟩ + |e(N − 1)⟩$ for $N = 1–4$. We further provided a numerical optimization method for QST and used it to efficiently measure the decoherence dynamics of the hybrid, verifying the effects of both dissipation channels associated with the qubit and the $N$-photon Fock state in the resonator. The technique we demonstrated here is necessary for using the qubit–resonator system as an active processing unit for QIP in hybrid quantum devices, where the qubit can be superconducting Josephson junctions or other quantum systems, such as spin ensembles. The capability of detecting both qubit and resonator simultaneously allows more information to be gained when studying the atom–photon interaction in circuit QED. It also provides a natural method to study the complex decoherence dynamics of a hybrid, where different types of noise contribute to decoherence.

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References

[1] Yu Y, Han S, Chu X, Chu S-I and Wang Z 2002 Science 296 889
[2] You J Q and Nori F 2005 Phys. Today 58 42
[3] Clarke J and Wilhelm F K 2008 Nature 453 1031
[4] You J Q and Nori F 2011 Nature 474 589
[5] Paik H et al 2011 Phys. Rev. Lett. 107 240501
[6] Rigetti C et al 2012 Phys. Rev. B 86 100506
[7] Megrant A et al 2012 Appl. Phys. Lett. 100 113510
[8] Barends R et al 2013 Phys. Rev. Lett. 111 080502
[9] Lucero E et al 2008 Phys. Rev. Lett. 100 247001
[10] Chow J M et al 2009 Phys. Rev. Lett. 102 090502
[11] Chow J M et al 2012 Phys. Rev. Lett. 109 060501
[12] Gustavsson S et al 2013 Phys. Rev. Lett. 110 040502
[13] Riste D, Bulkink C C, Lehnert K W and DiCarlo L 2012 Phys. Rev. Lett. 109 240502
[14] Vijay R et al 2012 Nature 490 77
[15] Devoret M H and Schoelkopf R J 2013 Science 339 1169
[16] Sillanpaa M A, Park J I and Simmonds R W 2007 Nature 449 438
[17] Niskanen A O et al 2007 Science 316 723
[18] DiCarlo L et al 2009 Nature 460 240
[19] Mariantoni M et al 2011 Science 334 61
[20] Wallraff A et al 2004 Nature 431 162

New Journal of Physics 15 (2013) 125027 (http://www.njp.org/)
[21] Astafiev O et al 2007 Nature 449 588
[22] Hofheinz M et al 2009 Nature 459 546
[23] Merkel S T and Wilhelm F W 2010 New J. Phys. 12 093036
[24] Wang H et al 2011 Phys. Rev. Lett. 106 060401
[25] Zakka-Bajjani E et al 2011 Nature Phys. 7 599
[26] Lähteenmäki P, Paraoanu G S, Hassel J and Hakonen P J 2013 Proc. Natl Acad. Sci. USA 110 4234
[27] Kirchmair G et al 2013 Nature 495 205
[28] Mallet F 2011 Phys. Rev. Lett. 106 220502
[29] Eichler C et al 2011 Phys. Rev. Lett. 107 113601
[30] Gustavsson S et al 2012 Phys. Rev. Lett. 109 010502
[31] Kubo Y et al 2011 Phys. Rev. Lett. 107 220501
[32] Carretta S, Chiesa A, Troiani F, Gerace D, Amoretti G and Santini P 2013 Phys. Rev. Lett. 111 110501
[33] Mischuck B and Molmer K 2013 Phys. Rev. A 87 022341
[34] Xiang Z L, Ashhab S, You Y J and Nori F 2013 Rev. Mod. Phys. 85 623
[35] Ansmann M et al 2009 Nature 461 504
[36] Audenaert K, Verstraete F and DeMoor B 2001 Phys. Rev. A 64 052304
[37] Peres A et al 1996 Phys. Rev. Lett. 77 1413
[38] Liu Y X, Wei L F and Nori F 2005 Phys. Rev. B 72 014547
[39] Shalibo Y et al 2013 Phys. Rev. Lett. 110 100404
[40] Golub G H and Van Loan C F 1996 Matrix Computations 3rd edn (Baltimore, MD: John Hopkins University Press) p 56
[41] Kennedy J and Eberhart R 1995 Proc. IEEE Int. Conf. Neural Networks vol 4 p 1942
[42] Chen Y et al 2012 Appl. Phys. Lett. 101 182601
[43] Sank D et al 2012 Phys. Rev. Lett. 109 067001
[44] Wenner J et al 2013 Phys. Rev. Lett. 110 150502
[45] Wang Z L et al 2013 Appl. Phys. Lett. 102 163503
[46] Wang H et al 2009 Phys. Rev. Lett. 103 200404
[47] Wang H et al 2008 Phys. Rev. Lett. 101 240401