Quantum repeater via entangled phase modulated multimode coherent states

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Abstract: We present a scheme of quantum repeater that uses entangled multimode coherent states which are obtained by electro-optic modulation of symmetric and antisymmetric Schrödinger cat states. Part of generated entangled frequency modes are sent to a symmetric beam splitter at the central node, while the remaining modes are kept locally in quantum memories. The entangled coherent states between remote quantum memories are conditionally prepared by photon counting measurements at the output channels of the beam splitter. We study how the effects of decoherence in the quantum channel affect statistics of photocounts and the fidelity of the prepared entanglement. Finally, we show how the proposed scheme can be useful for extending range of quantum key distribution with subcarrier wave encoding by exploiting quantum teleportation with the generated entanglement.

1. Introduction

The primary purpose of quantum communication [1, 2] is a transfer of quantum states between remote parties. Such transmission is of vital importance for a variety of applications such as secure transfer of classical messages using quantum key distribution (QKD) [3, 4], quantum metrology [5–7] and distributed computations [8, 9] to name few. The QKD is one of the most mature quantum technologies nowadays, but its direct communication distance is fundamentally limited by losses [10]. The future enhancement of the QKD distance without trusted nodes is promised with a creation of entanglement-assisted quantum networks that may surpass the loss bound [11].

The subcarrier wave (SCW) is one of promising encodings for quantum key distribution due to its robustness to environmental distortion of fiber line, interferometer-free scheme and multiplexing capacity. The encoding proved to be useful in point-to-point [12, 13], Plug&Play [14], continuous variables [15, 16] and twin-field QKD [17]. In SCW-QKD the information is encoded in a value of the phase of the applied sinusoidal optical phase modulation, that leads to specific multi-mode state. For a long range communication with SCW states, these states have to be compatible with the entanglement distribution protocol, namely quantum repeater [18], for teleporting the information from one end of the quantum repeater to another [19].

One way for completing this task would be a conversion of an information from SCW encoding into the one with available quantum repeater protocol [20]. However this method may suffer from non-ideal components that can reduce the fidelity and rate of the conversion. Alternatively, one can design a quantum repeater protocol that is compatible with SCW states. The operation of any quantum repeater is based on the principle of division the transmission channel into several
elementary links and creation of entanglement between the ends of each link. By swapping the entanglement between neighboring links, entanglement can be gradually transferred over significant distances to the target points of the quantum network.

Here we describe a quantum repeater protocol based on multimode phase-modulated coherent states generated by electro-optic modulation of Schrödinger cat states. For such multimode states, we demonstrate both the elementary link and the entanglement swapping procedure. In our model, the entanglement between two broadband quantum memories [21–29] is heralded by the results of photon detection at the central untrusted node. We show how the presented protocol is capable for performing quantum teleportation of the SCW states. As compared to quantum repeater schemes using controlled beam splitters (see e.g. [30]), the phase modulator is more flexible in use. We consider an isometrical decoherence model that is typical for continuous variables (CV) with energy transfer to environmental modes. We note that such an approach is used in a consideration of quantum channels in the CV quantum key distribution protocols where the environment is controlled by an eavesdropper. It links the proposed quantum repeater with measurement-device-independent CV-QKD.

The paper is organized as follows. In Section 2 we present the way how to create such distributed entangled coherent states. We start from building an elementary link, considering the single mode and multimode (from the point of view of the central node) cases. In Section 3 we analyze the potential performance of the proposed scheme by evaluating the fidelity and photon counting probability. Then we discuss the entanglement swapping techniques for creating long-distant entanglement in Section 4. In Section 5 we develop some intuition on how the quantum information from SCW state can be teleported through the repeater network with a help of the created entangled coherent states that would allow to connect the remote SCW QKD systems via the proposed repeater link. Finally, we discuss and summarize the results in Section 6.

2. Elementary link

First step in the establishing of quantum repeater is description of an elementary link, which creates entanglement between two remote sites. The optical scheme of this link is depicted in Fig. 1. Using the phase modulation method, users generate a multimode cat state and then send one of the frequency modes to the central relay via spectral filtering keeping other modes in quantum memories. The use of fast phase modulators makes it possible to actively change the modulation index and phase, which allows better control of the output states of Alice and Bob. To investigate the principal theoretical possibility of creating the elementary link, we perform calculation of the fidelity function and the probability of photocounts when an entanglement is created from the symmetric and antisymmetric states of the Schrödinger cat.

Alice and Bob prepare superpositions of the coherent states known as the Schrödinger cat states. These states were originally introduced in [31] as even and odd coherent states given by

$$|\Psi_\pm(\alpha)\rangle = \frac{1}{\sqrt{M_\pm(\alpha)}}|\alpha^{(\pm)}\rangle, \quad |\alpha^{(\pm)}\rangle \equiv |\alpha\rangle \pm |\alpha\rangle$$

(1)

where

$$M_\pm(\alpha) = \langle \alpha^{(\pm)} | \alpha^{(\pm)} \rangle = 2(1 \pm \exp(-2|\alpha|^2)),$$

(2)

and represent the symmetric (even) and antisymmetric (odd) Schrödinger cats: $|\Psi_+(\alpha)\rangle$ and $|\Psi_-(\alpha)\rangle$, respectively. Experimental techniques to generate optical Schrödinger cats include the method based on photon subtraction from the squeezed vacuum state [32–35], the protocol that uses homodyne detection and photon number states [36], reservoir engineering [37–39] and the method that involves making quadrature measurements of one of the modes of a biphoton NOON state [40].
We perform our analysis assuming that the input (non-modulated) state of the (central) carrier wave mode for the Alice’s and Bob’s modulators are generally two different Schrödinger cat states of the form: $|S_A\rangle$ and $|S_B\rangle$, given by

$$
|S_A\rangle = |\Psi_{\nu'}(\alpha)\rangle, \quad |S_B\rangle = |\Psi_{\nu'}(\beta)\rangle, \quad \nu, \nu' \in \{+, -\}.
$$

(3)

According to the model of electro-optic modulator [41], for the input states $|\pm \alpha\rangle_A$ and $|\pm \beta\rangle_B$, the modulated states can be described as the multimode coherent states given by

$$
|\pm \alpha\rangle_A \rightarrow |\pm \alpha\rangle_A = \otimes_{\mu=-S}^{S} |\pm \alpha_{\mu}\rangle_A, \quad |\pm \beta\rangle_B \rightarrow |\pm \beta\rangle_B = \otimes_{\mu=-S}^{S} |\pm \beta_{\mu}\rangle_B,
$$

(4)

$$
\alpha_{\mu} = U^{(A)\mu}_{\mu_0} \alpha, \quad \beta_{\mu} = U^{(B)\mu}_{\mu_0} \beta,
$$

(5)

where, in the large $S$ limit, the approximated expressions for the elements of the evolution matrix are: $U^{(A,B)}_{\mu_0} \approx e^{i\mu \phi_{A,B} I_\mu (m_{A,B})}$, where $\phi_{A,B}$ and $m_{A,B}$ are the phases and the modulation indices, respectively.

From Eq. (4) it can be readily seen that the electro-optic modulator acts like a multiport beam splitter that transforms the one-mode cat states (3) into the multimode Schrödinger cat states of the following form:

$$
|S_A\rangle \rightarrow |\Psi_A\rangle = |\Psi^{(A)}_{\nu'}(\alpha)\rangle, \quad |S_B\rangle \rightarrow |\Psi_B\rangle = |\Psi^{(B)}_{\nu'}(\beta)\rangle,
$$

(6)
where
\[
|\Psi_{\pm}^{(A)}(\alpha)\rangle = \frac{1}{\sqrt{M_{\pm}(\alpha)}}|\alpha^{(\pm)}\rangle_A, \quad |\Psi_{\pm}^{(B)}(\beta)\rangle = \frac{1}{\sqrt{M_{\pm}(\beta)}}|\beta^{(\pm)}\rangle_B, \quad (7)
\]
\[
|\alpha^{(\pm)}\rangle_A = |\alpha\rangle_A \pm |\pm\rangle_A, \quad |\beta^{(\pm)}\rangle_B = |\beta\rangle_B \pm |-\beta\rangle_B, \quad (8)
\]
\[
M_{\pm}(\alpha) = 2(1 \pm \exp(-2|\alpha|^2)), \quad |\alpha|^2 = \sum_{\mu=-S}^{S} |\alpha_{\mu}|^2. \quad (9)
\]

Thus the modulated state shared by Alice and Bob, $|\Psi_{AB}\rangle$, is the tensor product of two multimode Schrödinger cat states after the Alice’s and Bob’s modulators: $|\Psi_{AB}\rangle = |\Psi_{v,\alpha}^{(A)}(\alpha)\rangle \otimes |\Psi_{v,\beta}^{(B)}(\beta)\rangle$. This state can be written in the form
\[
|\Psi_{AB}\rangle = \frac{1}{N_{AB}} \left\{|\alpha,\beta\rangle_{AB} + v\langle \alpha, -\beta\rangle_{AB} + v|\alpha, -\beta\rangle_{AB}\right\}
\]
\[
= \frac{1}{M_{\alpha,\beta}} \left\{|\Psi_{v,v}^{(AB)}(\alpha, \beta)\rangle + v|\Psi_{v,v}^{(AB)}(\alpha, -\beta)\rangle\right\}, \quad (10)
\]
\[
M_{\pm}(\alpha, \beta) = 2 \left\{1 + \exp[-4(|\alpha|^2 + |\beta|^2)]\right\}, \quad (11)
\]

where $N_{AB} = \sqrt{M_{\alpha,\beta}M_{v,v}}$ and $|\alpha, \beta\rangle_{AB}$ represents $|\alpha\rangle_A \otimes |\beta\rangle_B$ rearranged into the tensor product of modes.

From now on we shall restrict our analysis to the important special case where $\alpha = \beta$ and, following the general approach [42, 43] to preparation of entangled states shared by Alice and Bob, we shall assume that the modes $\alpha$ that enter the modulated states, $|\Psi_{v,\alpha}^{(A)}(\alpha)\rangle \equiv |\Psi_{v,\alpha_{qm}}(\alpha_{bs})\rangle$, are divided into the two groups: the modes $\alpha_{qm}$ stored in the quantum memory and the modes $\alpha_{bs}$ put to interfere onto a symmetric $50:50$ beam splitter with the output channels $C$ and $D$. The above modes brought into interference onto the beam splitter appear to be transformed as follows
\[
\hat{\mathcal{T}}_{\alpha_{bs} \rightarrow C/D}|\pm \alpha_{bs}\rangle_A \otimes |\pm \alpha_{bs}\rangle_B = |\pm \gamma_{bs}\rangle_C \otimes |0\rangle_D, \quad \gamma_{bs} = \sqrt{2}\alpha_{bs}. \quad (12)
\]

We can now apply the transformation (12) to the state shared by Alice and Bob (see (10)). The result reads
\[
\hat{\mathcal{T}}_{\alpha_{bs} \rightarrow C/D}|\Psi_{v,\alpha}^{(A)}(\alpha_{qm}, \alpha_{bs})\rangle \otimes |\Psi_{v,\beta}^{(B)}(\alpha_{qm}, \alpha_{bs})\rangle = \frac{1}{2\sqrt{M_{\alpha,\beta}M_{v,v}}M_{\alpha,\beta}M_{v,v}} \times \sum_{\mu=0}^{\pm} \sqrt{M_{\mu}(\gamma_{bs})M_{\mu}(\alpha_{qm}, \alpha_{qm})} \left\{|\Psi_{v,v}^{(C)}(\gamma_{bs})\rangle \otimes |0\rangle_D \otimes |\Psi_{v,v}^{(AB)}(\alpha_{qm}, \alpha_{qm})\rangle + v|0\rangle_C \otimes |\mu'\rangle_{bs}^{(D)}(\gamma_{bs})\rangle \otimes |\Psi_{v,v}^{(AB)}(\alpha_{qm}, \alpha_{qm})\rangle\right\}, \quad \mu' = v\nu\mu, \quad (13)
\]

where $\gamma_{bs} \equiv \sqrt{2}\alpha_{bs}$.

If, for instance, we now perform a measurement on the output mode $C$ to distinguish the states $|\Psi_{v,v}^{(C)}(\gamma_{bs})\rangle$ and $|\Psi_{v,v}^{(C)}(\gamma_{bs})\rangle$, the multimode state will collapse onto either $|\Psi_{v,v}^{(AB)}(\alpha_{qm}, \alpha_{qm})\rangle$ or $|\Psi_{v,v}^{(AB)}(\alpha_{qm}, \alpha_{qm})\rangle$, respectively. Thus preparation of the entangled coherent cat states is heralded by the parity of clicks of a photon-number-resolving detector placed at the output channel $C$.

Now we introduce the modified symmetric cat state
\[
|\tilde{\Psi}_{+}(\alpha)\rangle = \frac{1}{\sqrt{M_{+}(\alpha)}}\left(|\tilde{\alpha}\rangle + |\tilde{\alpha}\rangle\right), \quad |\tilde{\alpha}\rangle = |\pm\rangle \equiv |\pm\rangle - e^{-|\alpha|^2/2} |0\rangle, \quad (14)
\]
where $M_{\alpha}(\alpha) = M_+(\alpha) - 4 \exp(-|\alpha|^2)$, defined in terms of the coherent states renormalized by subtracting the vacuum contribution. Then we have a set of three orthonormal states: $|\Psi_+(\sqrt{2}\alpha_{bs})\rangle \equiv |\Phi_+(\gamma_{bs})\rangle$, $|\Psi_-(\sqrt{2}\alpha_{bs})\rangle \equiv |\Phi_-(\gamma_{bs})\rangle$, and $|0\rangle$ that can be conveniently used to render the state (13) into the form

$$
\hat{T}_{\alpha_{bs},\alpha_{bs} \rightarrow CD} |\Psi^{(A)}_{\alpha_qm, \alpha_{bs}}\rangle \otimes |\Psi^{(B)}_{\alpha_qm, \alpha_{bs}}\rangle = \sqrt{P^{(\nu \nu)}_0} |0, 0\rangle_{CD} \\
\otimes |\Psi^{(A)}_{\nu \alpha_qm} \rangle \otimes |\Psi^{(B)}_{\nu \alpha_qm} \rangle + \sum_{\mu = \pm} \sqrt{P^{(\nu \nu)}_\mu} \left\{|\Phi^{(C)}_{\mu \nu \alpha_{bs}}\rangle \otimes |0\rangle_D \otimes |\Psi^{(AB)}_{\nu \alpha_qm, \alpha_{bs}}\rangle + \nu |0\rangle_C \otimes |\Phi^{(D)}_{\mu \nu \alpha_{bs}}\rangle \otimes |\Psi^{(AB)}_{\nu \alpha_qm, -\alpha_{bs}}\rangle\right\},
$$

where $P^{(\nu \nu)}_\mu$ is the probability for the states $|\Phi^{(\nu \nu)}_{\mu \nu \alpha_{bs}}\rangle \otimes |0\rangle_D$ and $|0\rangle_C \otimes |\Phi^{(D)}_{\mu \nu \alpha_{bs}}\rangle$ to be detected at the output channels of the beam splitter, whereas $P^{(0 \nu \nu)}_0$ is the probability to detect the vacuum state $|0\rangle_C \otimes |0\rangle_D$. The expressions for $P^{(\nu \nu)}_\mu$ are given by

$$
P^{(\nu \nu)}_\mu = \text{Prob}(\mu |\nu \nu) = \frac{1}{4} M^{(\nu \nu)}_{\alpha_{bs}} M^{(\nu \nu)}_{\nu \alpha_{bs}} M^{(\nu \nu)}_{\alpha_{bs} \nu \alpha_{bs}} M^{(\nu \nu)}_{\nu \alpha_{bs} \nu \alpha_{bs}}, \quad \mu \in \{+, -\}, \quad (16a)
$$

$$
P^{(0 \nu \nu)}_0 = \text{Prob}(0 |\nu \nu) = \frac{M^{(0 \nu \nu)}_{\alpha_{bs} \nu \alpha_{bs}}}{M^{(0 \nu \nu)}_{\nu \alpha_{bs} \nu \alpha_{bs}} \exp(-|\gamma_{bs}|^2)}. \quad (16b)
$$

In the special case, where the modulated cat states are identical and $\nu' = \nu$, the relations (16) can be simplified as follows

$$
P^{(\nu \nu)}_{\nu} = P^{(\nu)}_{\nu} = \frac{\sinh(2|\alpha_{qm}|^2) \sinh(2|\alpha_{bs}|^2)}{[\exp(|\alpha|^2) + \nu \exp(-|\alpha|^2)]^2}, \quad |\alpha|^2 = |\alpha_{qm}|^2 + |\alpha_{bs}|^2, \quad (17a)
$$

$$
P^{(\nu \nu)}_{\nu} = P^{(\nu)}_{\nu} = \frac{\cosh(2|\alpha_{qm}|^2) [\cosh(2|\alpha_{bs}|^2) - 1]}{[\exp(|\alpha|^2) + \nu \exp(-|\alpha|^2)]^2}, \quad (17b)
$$

$$
P^{(\nu \nu)}_{0} = P^{(\nu)}_{0} = \frac{[\exp(|\alpha_{qm}|^2) + \nu \exp(-|\alpha_{qm}|^2)]^2}{[\exp(|\alpha|^2) + \nu \exp(-|\alpha|^2)]^2}. \quad (17c)
$$

Figure 2 shows that the antisymmetric states turn out to be more sensitive to the amount of signal sent to the beam splitter than the symmetric ones. It is noteworthy that if we do not extract the vacuum component from the symmetric cat state, then in an obvious way we can observe $P^{(+)}_+ = 1 - P^{(+)}_-$.  

**3. Photodetection and decoherence**

**3.1. Probability of photo-counts**

In this section, we begin with the decoherence effects for the states transmitted through the fiber channels from Alice and Bob to the beam splitter (central) node. The decoherence processes will affect the output states of the beam splitter, $|\Phi^{(S)}_{\pm \gamma_{bs}}\rangle = |\Phi^{(S)}_{\pm R} \gamma\rangle$, where $S$ stands for the output channel of the beam splitter ($S$ is either $C$ or $D$), that enter the right hand side of Eq. (15). For simplicity, we restrict ourselves to the one-mode case with $|\Phi^{(S)}_{\pm \gamma_{bs}}\rangle = |\Phi^{(S)}_{\pm \gamma} \rangle$ and assume that the noisy channels are identical and describe them by introducing additional environmental (ancillary) mode as follows [44, 45]. An isometry representing the Stinespring dilation of the fiber channel where the signal mode is supplemented with an extra system $E$ (the channel’s environment) can be described as the following mapping

$$
|\gamma\rangle_S \rightarrow |\gamma_S\rangle_S \otimes |\gamma_E\rangle_E = |\gamma_S, \gamma_E\rangle \equiv |\gamma\rangle. \quad |\gamma|^2 = |\gamma_S|^2 + |\gamma_E|^2. \quad (18)
$$
Fig. 2. Probabilities $P_\pm$ and $P_0$ to get considered cat states at the output of the beam splitter computed as a function of $|\alpha|^2$ and $r_{bs} = |\alpha_{bs}|^2/|\alpha|^2$.

where $|\gamma_s|^2 = \eta|\gamma|^2$, $|\gamma_a|^2 = (1 - \eta)|\gamma|^2$ and $\eta$ is the channel (fiber) transmittance.

One of the methods to discriminate between the symmetric and antisymmetric cat states is to perform photocounting measurements that may distinguish the parity of the photon number registered by a photodetector. For a photon-number resolving photodetector, the probability to detect $k$ photons in the signal mode conditioned on the event that the initial state $\Psi^{(A)}_\mu \otimes \Psi^{(B)}_\nu$ is projected onto the entangled state $|\Psi^{(AB)}_\mu\rangle$

\[
\text{Prob}(k|\mu, \nu') = P(k|\mu'), \quad \mu' = \nu' \nu \mu
\]  

that expressed in terms of the probability of clicks $P(k|\mu)$ determined by the statistics of photocounts for the cat states $|\Phi_\mu(\gamma)\rangle$. According to the well-known Kelley-Kleiner formula [46, 47], this probability reads

\[
P(k|\mu) = \langle \Phi_\mu(\gamma) | \hat{\Pi}_k | \Phi_\mu(\gamma) \rangle, \quad P(k|0) = \delta_{k0},
\]  

where $\hat{\Pi}_k$ is the positive-operator-valued measure for a number-resolving detector given by

\[
\hat{\Pi}_k = \frac{(|\xi\hat{a}_s|^k)}{k!} e^{-|\xi\hat{a}_s|^2}, \quad \hat{a}_s = \hat{a}_a \hat{a}_s,
\]
where $\xi$ is the efficiency of the detector. We can now apply the algebraic identity

$$\text{Tr}_E \left| \gamma^{(+)} \right\rangle \left\langle \gamma^{(+)} \right| = \frac{1}{4} \left\{ M_+ (\gamma_e) \left| \gamma_s^{(+)} \right\rangle \left\langle \gamma_s^{(+)} \right| + M_- (\gamma_e) \left| \gamma_s^{(-)} \right\rangle \left\langle \gamma_s^{(-)} \right| \right\}$$

(22)

to deduce the following expressions of the probabilities (20)

$$P(k|+) = \frac{1}{4M_+ (\gamma)} \left\{ M_+ (\gamma_e) C_+ (k, \gamma_s) + M_- (\gamma_e) C_- (k, \gamma_s) \right\}, \quad k > 0,$$

(23)

$$P(k|-) = \frac{1}{4M_- (\gamma)} \left\{ M_- (\gamma_e) C_+ (k, \gamma_s) + M_+ (\gamma_e) C_- (k, \gamma_s) \right\}, \quad P(k|0) = 0,$$

(24)

where

$$C_{\mu} (k, \gamma_s) = \langle \gamma_s^{(\mu)} | \hat{E}_k | \gamma_s^{(\mu)} \rangle = \frac{2}{k!} \frac{\xi |\gamma_s|^{2k}}{k!} \left\{ e^{(1-\xi)|\gamma_s|^2} + (-1)^k \mu e^{-(1-\xi)|\gamma_s|^2} \right\}.$$  

(25)

In what follows, we consider the heralding outcomes described by the two different parity of clicks, $p_c$, and thus discriminate between the cases, where the number of registered photons is either odd or even. The sum of the corresponding probabilities

$$P(\text{even}|\pm, 0) = \sum_{n=0}^{\infty} P(2n|\pm, 0), \quad P(\text{odd}|\pm, 0) = \sum_{n=0}^{\infty} P(2n+1|\pm, 0),$$

(26)

and the no-click probability, $P(0|\mu)$, gives unity and we have the completeness identity

$$P(0|\pm, 0) + P(\text{even}|\pm, 0) + P(\text{odd}|\pm, 0) = 1.$$  

(27)

By substituting Eqs. (23)–(25) into relations (26) we obtain the expressions for the probabilities of the detection outcomes determined on the parity of click

$$P(p_c|+) = \frac{1}{4M_+ (\gamma)} \left\{ M_+ (\gamma_e) C_+ (p_c, \gamma_s) + M_- (\gamma_e) C_- (p_c, \gamma_s) \right\}, \quad p_c \in \{\text{even}, \text{odd}\},$$

(28)

$$P(p_c|-) = \frac{1}{4M_- (\gamma)} \left\{ M_- (\gamma_e) C_+ (p_c, \gamma_s) + M_+ (\gamma_e) C_- (p_c, \gamma_s) \right\}, \quad P(p_c|0) = 0,$$

(29)

where

$$C_+ (\text{even}, \gamma_s) = 2e^{-|\gamma_s|^2} (\cosh(\xi)|\gamma_s|^2 - 1) \left\{ e^{(1-\xi)|\gamma_s|^2} \pm e^{-(1-\xi)|\gamma_s|^2} \right\},$$

(30a)

$$C_+ (\text{odd}, \gamma_s) = 2e^{-|\gamma_s|^2} \sinh(\xi)|\gamma_s|^2 \left\{ e^{(1-\xi)|\gamma_s|^2} \pm e^{-(1-\xi)|\gamma_s|^2} \right\}.$$  

(30b)

Note that this relations can be readily generalized to the case of multimode signal, where $|\gamma_s\rangle \rightarrow |\gamma_s^{(x)}\rangle \equiv |\gamma_1^{(x)}, \ldots, \gamma_N^{(x)}\rangle$ and $N_s$ is the number of signal modes registered by a broad band detector, by replacing $|\gamma_s|^2$ and $\xi|\gamma_s|^2$ with $|\gamma_s^{(x)}|^2 = \sum_{i=1}^{N_s} |\gamma_i^{(x)}|^2$ and $\sum_{i=1}^{N_s} \xi_i |\gamma_i^{(x)}|^2$.

Referring to Fig. 3, it is seen that, for the limiting case of the ideal photodetector and the lossless channel with the efficiency and the transmittance both equal to unity, $\xi = \eta = 1$, the probability of odd (even) number of clicks vanishes for the symmetric (antisymmetric) states, $P(\text{odd}|+) = 0 (P(\text{even}|-) = 0)$, and thus the cat states are perfectly distinguishable. It can also be noted that the probability turns out to be quite sensitive to the detector efficiency $\xi$.

### 3.2. Performance and entanglement

By definition the total probabilities of state discrimination can be written as follows

$$P_s^{(\mu')}(p_c) = \sum_{\mu \in \mathbb{R}} \text{Prob}(p_c|\mu, \nu') \text{Prob}(\mu|\nu') = \sum_{\mu \in \mathbb{R}} P(p_c|\mu') P^{(\mu')}_\nu.$$  

(31)
Explicit expressions for the probabilities can also be obtained via the substitution \( \xi = \xi \eta \rho_{bn} \). For example,
\[
\begin{align*}
P_s^{(-)} \text{ (odd)} &= \frac{\sinh(2 \xi x) \sinh(2(1 - \xi) x)}{4 \sinh^2(x)}, \\
P_s^{(-)} \text{ (even)} &= \frac{\sinh^2(\xi x) \cosh(2(1 - \xi) x)}{2 \sinh^2(x)}, \\
P_s^{(c)} \text{ (odd)} &= \frac{\sinh(2 \xi x) \cosh(2(1 - \xi) x)}{4 \sinh(x) \cosh(x)}, \\
P_s^{(c)} \text{ (even)} &= \frac{\sinh^2(\xi x) \sinh(2(1 - \xi) x)}{2 \sinh(x) \cosh(x)}.
\end{align*}
\]

It is noteworthy that the probabilities from Eqs. (32a) and (32b) will respectively behave in exactly the same way as the probabilities \( P_{-}^{(-)} \) and \( P_{-}^{(+)} \) from Eqs. (17a) and (17b) up to the substitution of \( \rho_{bn} \) by \( \xi \). The same will be with other probabilities. This obviously means that the surfaces from figures 4 can be easily recreated with the limitation of the parameter \( \xi \). Also, \( P_s^{(c)} \text{ (odd)} = P_s^{(-)} \text{ (odd)} = 1/4 \) at \( \xi = 1/2 \).

The corresponding fidelities for the final state is obtained via the well from the Bayesian formula
\[
F_{\mu}^{(\nu'\nu)}(p_c) = \frac{\text{Prob}(p_c|\mu, \nu') \text{Prob}(\mu|\nu')}{P_s^{(\nu'\nu)}(p_c)} = \text{Prob}(\mu|p_c, \nu').
\]

Now we can write the density matrix for the final state, taking into account decoherence and detectors’ outputs
\[
\hat{\rho}_{AB}(p_c|\nu') = \sum_{\mu=\pm} F_{\mu}^{(\nu'\nu)}(p_c)|\Psi_{\mu}^{(AB)}(\alpha_{qm}, \alpha_{qm})\rangle\langle \Psi_{\mu}^{(AB)}(\alpha_{qm}, \alpha_{qm})|.
\]

As is shown in Fig. 4, at \( \eta < 1 \), decoherence induced contributions to the total probabilities will impair distinguishability. The results indicate that the statistics of photocounts for the antisymmetric cat state at small \( |\alpha|^2 \) and \( |\alpha_{bn}|^2 \) is less sensitive to the decoherence effects as
Fig. 4. The total probability of distinguishing states, depending on the input of the detector and the output clicks, plotted against the mean photon number $|\alpha|^2$ (the fraction value $r_{bs}$ is indicated) and channel transmittance $\eta$.

Fig. 5. The fidelity, meaning the distribution over the components of the density matrix of the final conditional state.

compared to the symmetric states. Figure 5 shows the fidelities for the final state. Given that these are reciprocals of probabilities, it is more advantageous for the final state to send a smaller fraction to the beam splitter.

Presented results shows that applicability of Schrödinger cat states for quantum repeater protocols is challenging. In addition to the fact that the preparation of such states is a non-trivial task, their fragility plays a significant role when propagating in an optical fiber.

4. Entanglement swapping

In this section we shall discuss the entanglement swapping protocol. Following the general principles presented in [42], we can establish entanglement between remote points of two elementary links ($A_1 - B_1$ and $A_2 - B_2$) via entanglement swapping. As is shown in Fig. 6, for
this purpose, we can use one of the other modes of phase modulated light generated at sites $B_1$ and $B_2$ and send them onto another 50:50 beam splitter.

![Fig. 6. Entanglement swapping scheme. $A_1$, $A_2$, $B_1$, $B_2$ are user links.](image)

For simplicity, we denote the total probabilities found in the previous section as follows

$$P_s^{(v', v_1)}(p_c)|_{A_1B_1} \equiv p_1, \quad P_s^{(v', v_2)}(p_c)|_{A_2B_2} \equiv p_2.$$  \hfill (35)

Then the probability distribution for numbers of attempts is

$$P(n_1, n_2) = p_1 q_1^{n_1 - 1} p_2 q_2^{n_2 - 1},$$  \hfill (36)

where $q_i = 1 - p_i$.

Now we can introduce a convenient probability value from the difference in the number of attempts

$$\text{Prob}(|n_1 - n_2| = k) = \frac{p_1 p_2 (q_1^k + q_2^k)}{2(1 - q_1 q_2)} (2 - \delta k 0)$$  \hfill (37)

and calculate the following expectation values

$$\langle n_w \rangle \equiv \langle |n_1 - n_2| \rangle = \frac{p_2^2 q_1^2 + p_1^2 q_2^2}{p_1 p_2 (1 - q_1 q_2)}, \quad \langle n_i \rangle \equiv \langle n_1 + n_2 \rangle = \frac{p_1 + p_2}{p_1 p_2},$$  \hfill (38)

$$\langle n_{\text{max}} \rangle \equiv \langle \max(n_1, n_2) \rangle = \frac{\langle n_i \rangle + \langle n_i \rangle}{2}, \quad \langle n_{\text{min}} \rangle \equiv \langle \min(n_1, n_2) \rangle = \frac{\langle n_i \rangle - \langle n_i \rangle}{2}.$$  \hfill (39)

As in the work [48], we can estimate the preparation time and waiting time as follows

$$T_{\text{prep}} = \frac{L \langle n_{\text{max}} \rangle}{c}, \quad T_{\text{wait}} = \frac{L \langle n_w \rangle}{c}.$$  \hfill (40)

For successful operation of the scheme, it is necessary that the waiting time does not exceed the storage time for the state inside the quantum memory, in other words, $T_{\text{wait}} \leq T_{\text{store}}$. Figure 7 shows that the order of storage time in the worst case is $10^{-3}$. In the case of small losses, one can count on a more realistic order of magnitude $10^{-6} \div 10^{-5}$ s. The order of magnitude generally corresponds to those from work [48].

Essentially, taking the density matrices from two elementary links

$$\hat{\rho}_{A_1B_1}(p_c|v'v_1) \equiv \hat{\rho}_1 = \sum_{\mu_1 = \pm} F_{\mu_1}^{(1)} |\psi^{(A_1B_1)}(\alpha_{qm}, \alpha_{qm})\rangle \langle \psi^{(A_1B_1)}(\alpha_{qm}, \alpha_{qm})|,$$  \hfill (41a)

$$\hat{\rho}_{A_2B_2}(p_c|v'v_2) \equiv \hat{\rho}_2 = \sum_{\mu_2 = \pm} F_{\mu_2}^{(2)} |\psi^{(A_2B_2)}(\alpha_{qm}, \alpha_{qm})\rangle \langle \psi^{(A_2B_2)}(\alpha_{qm}, \alpha_{qm})|.$$  \hfill (41b)
one can obtain the general state by their tensor product

$$\hat{\rho}_1 \otimes \hat{\rho}_2 \rightarrow \hat{\rho}_{12} = \sum_{\mu=\pm} F^{(12)}_\mu |\Psi^{(A_1 A_2)}_\mu (\alpha_{qm}, \alpha_{qm})\rangle \langle \Psi^{(A_1 A_2)}_\mu (\alpha_{qm}, \alpha_{qm})|.$$  \hspace{1cm} (42)

The corresponding probabilities and fidelity values of the new state will then be expressed as

$$P^{(12)}_s (p_c) = \sum_{\mu_1, \mu_2 = \pm} \text{Prob}(p_c | \mu_1 \mu_2) \text{Prob}(\mu_1 | \mu_1 \mu_2) \text{Prob}(\mu_1, \mu_2)$$

$$= \sum_{\mu_1, \mu_2 = \pm} P(p_c | \mu_1 \mu_2 \mu) P^{(\mu_1 \mu_2)}_\mu F^{(1)}_{\mu_1} F^{(2)}_{\mu_2}.$$ \hspace{1cm} (43)

$$F^{(12)}_\mu = \frac{1}{F^{(12)}_s} \sum_{\mu_1, \mu_2 = \pm} P(p_c | \mu_1 \mu_2 \mu) P^{(\mu_1 \mu_2)}_\mu F^{(1)}_{\mu_1} F^{(2)}_{\mu_2}.$$ \hspace{1cm} (44)

We note that subcarrier modes remaining at sites $A_1$ and $A_2$ can be used for further entanglement swapping with other elementary links.

### 5. Teleportation

Once an entanglement is established between the remote parties, it can be used for teleporting a phase information from the SCW state. We consider a case of Alice and Bob sharing a single frequency entangled coherent state for developing some intuition on how the teleportation can be implemented. The shared state is

$$|\Psi^{A B}_s\rangle = K_\pm \cdot (|\alpha_A\rangle |\alpha_B\rangle \pm |\alpha_A\rangle |\alpha_B\rangle),$$ \hspace{1cm} (45)

with $K_\pm = 1/\sqrt{2 (1 \pm e^{-4|\alpha|^2})}$ being a normalization constant. Charlie at the Alice’s site holds a SCW state with an amplitude $\alpha$ and the phase $\phi_c$ of SCW state that is targeted for teleportation:

$$|\Psi^\text{SCW}\rangle = \otimes_{\mu=-S}^S |\mu(m_C)\rangle e^{i\mu \phi_c} |\alpha\rangle,$$ \hspace{1cm} (46)
where we assume that the phase coherence between Alice and Charlie is maintained. The overall wavefunction of Alice, Bob and Charlie (4) is

$$|\Psi\rangle = |\Psi^e_{AB}\rangle \otimes |\Psi_{SCW}\rangle,$$  

where the down indices (A,B) indicate the presence of the state to Alice and Bob respectively.

$$\phi_c \quad \begin{array}{c|c|c|c|c} & D_1 \text{clicks} & & D_2 \text{clicks} \\ \hline \phi_a & \langle\alpha| & \langle\alpha| & \langle-\alpha| & \langle-\alpha| \\ \phi_a + \pi & \langle-\alpha| & \langle-\alpha| & \langle\alpha| & \langle\alpha| \\ \phi_a + \pi/2 & \langle\mp i\alpha| & \langle\pm i\alpha| & \langle\pm i\alpha| & \langle\mp i\alpha| \\ \phi_a + 3\pi/2 & \langle\pm i\alpha| & \langle\mp i\alpha| & \langle\mp i\alpha| & \langle\pm i\alpha| \end{array}$$

Table 1. Truth table for heralded state at Bob’s site \(|\Psi_B\rangle\) after performing teleportation with entangled coherent state \(|\Psi^e_{AB}\rangle\) depending on a click of a particular detector and value of Charlie’s SCW phase with respect to Alice’s SCW phase \(\phi_a\).

The scheme of the teleportation from Charlie to Bob is depicted in Fig. 8a. The mapping of information from Charlie’s SCW state to Bob’s state may be implemented in the following steps:

1. Alice applies phase modulation with the same modulation frequency, index \(m_C\) as the Charlie’s SCW state but with a fixed phase \(\phi_a\).

2. The Charlie’s and Alice’s states are interfered on 50-50 beam splitter.
3. The presence of a single photon at particular sidebands at both channels of the beam splitter are monitored by a single photon detectors, namely detector $D_1$ and detector $D_2$.

4. After a click from any of the detectors and announcement of the detector by Alice, Bob locally shifts phase of his state with the value of the shift depending on which detector clicked. Hence the Charle’s SCW phase is transfer to a phase of Bob’s single mode coherent state.

Below we describe these steps in details. The application of the phase modulation by Alice to the shared entangled state (47) transforms it into

$$|\Psi_{AB}^\pm\rangle \rightarrow K_\pm \left( \otimes_{\mu=-S}^S |J_\mu(m_C)e^{i\mu\phi_\alpha}(A,B)\rangle|\alpha_B\rangle \pm \right.$$

$$\otimes_{\mu=-S}^S |J_\mu(m_C)e^{-i\mu\phi_\alpha}(A,B)\rangle|-\alpha_B\rangle \right) \otimes |\Psi_{SCW}\rangle. \quad (48)$$

The 50-50 beam splitter transforms the Alice’s and Charlie’s spatial modes into the two modes $\hat{d}_1$ and $\hat{d}_2$:

$$\hat{d}_{1,\mu} = \frac{\hat{d}_\mu + \hat{c}_\mu}{\sqrt{2}}, \quad (49)$$

$$\hat{d}_{2,\mu} = \frac{\hat{d}_\mu - \hat{c}_\mu}{\sqrt{2}}, \quad (50)$$

where $\hat{d}_\mu, \hat{c}_\mu, \hat{d}_{1,\mu}, \hat{d}_{2,\mu}$ are annihilation operators for Alice, Charlie $\mu$-th sidebands and $\mu$-th sideband of modes being monitored by detectors $D_1$ and $D_2$, respectively.

The state after the beam splitter is:

$$|\Psi\rangle \rightarrow K_\pm \left( \otimes_{\mu=-S}^S |J_\mu(m_C)\frac{\alpha(e^{i\mu\phi_\alpha} + e^{-i\mu\phi_\alpha})}{\sqrt{2}}\rangle_{(D_1,\mu)} |J_\mu(m_C)\frac{\alpha(e^{i\mu\phi_\alpha} - e^{-i\mu\phi_\alpha})}{\sqrt{2}}\rangle_{(D_2,\mu)} |\alpha_B\rangle \pm \right.$$

$$\otimes_{\mu=-S}^S |J_\mu(m_C)\frac{-\alpha(e^{i\mu\phi_\alpha} - e^{-i\mu\phi_\alpha})}{\sqrt{2}}\rangle_{(D_1,\mu)} |J_\mu(m_C)\frac{-\alpha(e^{i\mu\phi_\alpha} + e^{-i\mu\phi_\alpha})}{\sqrt{2}}\rangle_{(D_2,\mu)} |-\alpha_B\rangle \right). \quad (51)$$

For sake of simplicity in the limit of a low amplitude $\alpha \ll 1$, we truncate the energetical Hilbert space up to a single photon. For an ideal single photon detector that measures particular sideband with index $\mu$ at $D_1$ ($D_2$) channel, a detection of a single photon would project the state into

$$|\Psi\rangle \rightarrow \frac{\hat{P}_{D_1(D_2),\mu} |\Psi\rangle}{\langle \Psi | \hat{P}_j |\Psi\rangle^2}. \quad (52)$$

where

$$\hat{P}_{D_1(D_2),\mu} = |0_{S..1_{\mu}..0_{-S}}D_1(D_2)|0_{S..0_{\mu}..0_{-S}}D_1(D_2)\rangle |0_{S..1_{\mu}..0_{-S}}D_1(D_2)|0_{S..0_{\mu}..0_{-S}}D_1(D_2)\rangle^\dagger. \quad (53)$$

Assuming that Alice’s phase being fixed to zero $\phi_\alpha = 0$, the Charlie’s SCW phase is mapped into the phase of Bob’s coherent state as it is summarized in Table 1, where we used $\frac{1}{\sqrt{2}}(|\alpha\rangle_B \pm i|\alpha\rangle_B) \approx |\mp i\alpha\rangle_B$ with corresponding fidelity shown in Fig. 8c.

For example, if $|\Psi_{AB}^+\rangle$ is used and $D_1$ detects a photon on the sideband +1, the Bob’s states is projected as

$$|\Psi_B\rangle \approx \begin{cases} |\alpha\rangle, & \phi_c = 0, \\
|\alpha\rangle, & \phi_c = \pi, \\
|\alpha\rangle, & \phi_c = \pi/2, \\
|\alpha\rangle, & \phi_c = 3\pi/2. \end{cases} \quad (54)$$
The teleported state carries an exact reproduction of the Charlie’s phase, if this phase coincides with the Alice’s one up to π phase shift. If the Charlie’s phase that is different from the Alice’s one by an odd multiple of π/2, the teleported state at low photon number can be well approximated to the coherent state with a value of Charlie’s SCW phase up to π-shift. Fidelity of more than 99% is possible at amplitude smaller than 0.25 as it is shown in Fig. 8b. The same mapping occurs, up to a local π phase shift by Bob, for a detection of a single photon by $D_2$ detector at +1 sideband.

It is remarkable, that the value of the Charlie’s SCW phase at the Bob’s teleported state is flipped in one basis and not in the another, e.g. we get $|\alpha\rangle_B$ for $\phi_c = 0$, but $|-\alpha\rangle_B$ for $\phi_c = \pi/2$ instead of $|\alpha\rangle_B$. In some sense, it is similar to conventional discrete variable qubit teleportation [49] using linear optics and photon detectors that is possible only for half of the cases due to impossibility to distinguish all four Bell states [50]. Here the teleportation of a single bit from two sets of bases is done with the rate-limited fidelity in one basis that comes from a nature of entangled coherent states sharing only a single entanglement bit of information [51]. However for an application in QKD, in particular BB84 protocol, the proposed teleportation procedure is sufficient, as during the sifting procedure after a measurement of the Bob’s state the basis-dependent phase shift can be compensated. The use of multiple detectors monitoring different sidebands will proportionally increase the rate.

6. Discussion and conclusions

In this work, we have proposed to use phase modulated multimode Schrödinger cat states for producing entanglement between remote parties (Alice and Bob) and entanglement swapping. This approach to quantum repeaters is based on multimode coherent states generated by an electro-optic modulator. For instance, as is illustrated in Fig. 9, the modes of a multimode entangled state may, in principle, be distributed and stored in quantum memories located at different sites. Since these modes differ in frequency, the DWDM demultiplexer can be employed to send them to different remote locations of a star topology network.

Assuming multimode quantum memory [28,52], we have studied how decoherence will affect the performance of the proposed scheme and have analyzed the statistics of photocounts for a photon-number-resolving detector and its segmented detector implementation. In general, our calculations have shown that the cat states are highly susceptible to decoherence, and the required storage time for entanglement generation grows significantly, regardless of the type of states. In
particular, we can talk not only about the use of the multimode memory, whose efficiency, based on the presented works, remains rather low. However, for example, in [29] demonstrates efficient multimode memory specifically in the context of repeaters. It should be noted here that such memory can be very limited in terms of storage time, and therefore it is necessary to look for alternative approaches. For instance, the system presented in our work can work without the multimode quantum memory. In quantum memory, users can leave only the carrier mode at the central frequency, distributing the sidebands over the repeater nodes. Then, after extracting this mode from the memory, re-modulation can be carried out, after which the carrier mode can be stored again. Due to the high efficiency of the electro-optic modulator, such a process can be cyclical.

We have also found that using the antisymmetric cat states is preferable to the symmetric ones. Here we note that at small amplitudes, the dominant component of the initial symmetric cat states is determined by the vacuum state, which does not create entanglement, but in the article, we cut out the vacuum component. The antisymmetric state has no such need. There are also additional difficulties with the suggested scheme related to the complexity of the cat states preparation and the need for multimode quantum memory to implement the described repeater protocol. Nevertheless, our analysis suggests the feasibility of the proposed scheme for antisymmetric cat states and relatively short distances to the central node. We think that approaches to quantum repeaters using phase modulation to produce and control multimode states may lead to promising methods for the creation and distribution of entanglement. The produced entanglement may be used for connecting remote SCW QKD networks by means of quantum teleportation.

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Disclosures

The authors declare no conflicts of interest.

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