In this talk we discuss recent progress concerning precise predictions for the LHC. We give a status report of an application of the GOLEM method to deal with multi-leg one-loop amplitudes, namely the next-to-leading order QCD corrections to the process $pp \rightarrow VV + \text{jet}$, where $V$ is a weak boson $W, Z$.

The Large Hadron Collider (LHC) at CERN will probe our understanding of electroweak symmetry breaking and explore physics in the TeV region. A detailed theoretical knowledge of various kinds of Standard Model backgrounds is indispensable for these studies. Precise predictions for multi-partonic cross sections are only possible by including higher order corrections such that renormalisation and factorisation scale dependencies are tamed.

During the Les Houches 2005 workshop, the process $pp \rightarrow VV + \text{jet}$ was identified as one of the most important missing next-to-leading order (NLO) calculations. Indeed, by considering at least one additional jet in the final state, one can improve the Higgs signal significance. Then, this process represents an important background for the production of $H + \text{jet}$ and new physics. Moreover, it is an important test before approaching more complicated multi-particle processes at NLO.

The process with a charged vector boson pair has been evaluated recently by two independent groups. However, the evaluation for $ZZ + \text{jet}$ is still missing.

1 Preliminaries

The process is composed of three partonic reactions:

$$gg \rightarrow VVg, \quad qg \rightarrow VVq, \quad gq \rightarrow VVq \quad (1)$$
with \( V^2 F Z \, \hat{W} \, W^+ g \). In fact, we only have to evaluate the helicity amplitudes of the first partonic process in [1]:

\[
q(p_2; 1) + q(p_2; 2) . \, V \, (p_3; 3) + V \, (p_4; 4) + g(p_5; 5) 
\]  
(2)

Indeed, the other partonic processes are obtained by applying a momentum crossing on the partons. For massless quarks, the allowed helicities are \( 1 = 2; 5 \, 2 \, f; \, g, \, 3; \, 4 \, 2 \, f; \, 0; \, g \) which leads in general to 36 different helicity amplitudes. The amplitude can be written as:

\[
M = m^3 m^4 m^5 h^2 j^3 j^4 j^5 1 i
\]  
(3)

where we introduce the spinor string \( s^4 s^5 \), which contains the coupling of a vector boson to a quark line given by

\[
i_{\text{vertex}, V_{qq}} = e (a_{V \, f f} b_{V \, f f} s) = (g_v^+ + g_v^-) \]  
(4)

where \( s = (1 \, 5) = 2 \) and \( g_v = e(a_{V \, f f} b_{V \, f f}) \). We work in dimensional regularisation and treat \( s \) by applying the \( \Gamma \) Hooft-Veltman scheme. Therefore, the \( g_v \) coupling of the \( V \, q \) need a finite renormalisation proportional to \( a \, g_v \) (i.e., \( g_v \).)

Before turning to helicity methods we notice that Bose symmetry \( [1] \), parity, and charge conjugation combined with parity, relate different helicity amplitudes with each other [1]. Indeed, we only have to produce the following independent helicities:

\[
M = \begin{pmatrix} M^0 & M^{0+, \, 0} & M^{0+} \end{pmatrix} \]  

Then, we use these discrete transformations in order to generate the 36 helicities.

For the evaluation of the helicity amplitudes one preferably uses the spinor helicity formalism [3]. Before writing down the polarization vectors for the different helicities, we introduce two light-like auxiliary vectors \( k_3, k_4 \) in order to replace the massless momenta \( p_3, p_4 \) of the \( V \) bosons, such that \( p_3 + p_4 = k_3 + k_4 \). One needs:

\[
k_3 = \frac{1}{2} \left[ (1 + ) p_3 (1 - ) p_1 \right] \quad \text{and} \quad k_4 = \frac{1}{2} \left[ (1 + ) p_4 (1 - ) p_3 \right] 
\]  
(5)

where \( s = 1 \frac{4M^2}{m^2} \). The polarization vectors for the different helicities of the massless vector bosons can now be written now as:

\[
\begin{align*}
\eta^3_{3^3} &= \frac{1}{2} \frac{h^4 j^3 j^i i}{h^4 i} \quad (6) \\
\eta^3_{3^3} &= \frac{1}{2} \frac{h^3 j^3 j^i i}{h^3 i} \quad (7) \\
\eta^3_{3^3} &= \frac{1}{2} \frac{[(1 + )k_3^3 (1 - )k_4^3]}{[3]} \quad (8)
\end{align*}
\]

The polarization vector \( \eta^4_{4^4} \) is obtained with the relabeling \( 3 \rightarrow 4 \). The two helicity states of the gluon are given as usual by:

\[
\begin{align*}
\eta^5_s &= \frac{1}{2} \frac{h^2 j^5 j^i i}{h^5 i} \quad \text{and} \quad \omega^5_s = \frac{1}{2} \frac{h^5 j^5 j^i i}{[5]} 
\end{align*}
\]  
(9)

\(^a\)used only for the case \( V = 2 \).

\(^b\)See for example [3].
where j is a reference vector to be chosen in a convenient way. If $j = 2$, a convenient choice for $M^{34}$ is $j = 2$ and for $M^{34+}$ $j = 1$. In this way the spinor expression from
the gluon can be attached to the spin chain.

By multiplying $M^{34+}$ with $h^5 j_2 i h^5 j_1 j_2 i$ and $M^{34}$ with $h^1 j_3 j_4 i h^1 j_3 j_4 i$, we are now able to close the spinor string to a trace:

$$M^{34+} = \frac{m^{34} 4}{\pi^2} \text{tr}(\bar{\psi}_2 3 4 5 \psi_1 5 \psi_3 \psi_1)$$

$$M^{34} = \frac{m^{34} 4}{\pi^2} \text{tr}(\bar{\psi}_2 3 4 5 \psi_3 \psi_2 \psi_5 5)$$

In this representation it is easy to extract a global spinorial phase for each helicity amplitude.

2 Reduction of tensor integrals

The method used to reduce and to evaluate the one-loop tensor integrals is the General One-Loop Evaluator for Matrix elements (GOLEM). This formalism is able to proceed from a Feynman diagrammatic representation of a given scattering amplitude to a computer code which provides a numerically stable and accurate answer for the desired cross section.

First we generate automatically the Feynman diagram's analytical expressions (FeynArts, QGRAPH), and sort the matrix element denoted A by helicity and colour properties. The mapping of the diagrammatic input onto a Lorentz tensorial basis can be accomplished with algebraic manipulation programs (FORM). The matrix element is then expressed in terms of some integral basis $I_B$ to be discussed below:

$$A(\bar{p}_1; j; \cdots) = \sum_{B I G} C_{B I G} (s_{jk}; \cdots) I_B T_I(\bar{p}_1; j; \cdots)$$

where the coefficients $C_{B I G}$, depending on $M$ and dressed variables $s_{ij} = (p_i + p_j)^2$, have to be supplied using algebraic programs (Maple, Mathematica). The integral basis of our reduction algorithm only contains the scalar integrals $I_1^2, I_3^8, I_3^{2+}, I_4^{n+2}$, For evaluating these functions, we use algebraic/numerical algorithm implemented in the flexible Fortran 90 code GOLEM90.

3 Results

With our method based on the GOLEM library, we have obtained the virtual order $O^2$ corrections for all helicity amplitudes of both processes. Using spinor helicity methods we have obtained analytical formulas for the coefficients of all basis scalar integrals. In order to check the correctness of our results, we have evaluated the virtual part of both processes twice using independent calculations and obtained full agreement. As an illustration we show the contribution of the virtual correction to some typical distributions. Only the contributions which are related to finite basis integrals are plotted. For the full result the real emission corrections remain to be included.

4 Tuned comparison of pp! $W + W$ jet

A tuned comparison of the NLO QCD corrections to the $W + W$ jet production at the LHC has been done with two other groups. We compared the integrated LO cross section and for a fixed phase-space point the interference term between tree-level and one-loop virtual amplitude for all channels. The results obtained by the different groups using different calculational techniques agree within 6 to 9 digits. The comparison of full NLO cross sections is still in progress.
Figure 1: The finite virtual NLO contribution to the helicity component $+ + +$ of the partonic process $qq'\rightarrow ZZg$. The invariant mass of the $Z$ pair is shown on the left and the $p_T$ distribution on the right. We use the cut $p_T^{jet}>20$ GeV and a parton and beam pipe separation cut of $|\phi_j|>150^\circ$.

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