Proving Regulatory Compliance: 
A Computational Complexity Analysis of Elementary Variants

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**Abstract.** Organisations model their processes using so-called business process models, to allow for verification of their correctness with respect to regulatory requirements and business rules. Automated methods for checking compliance, however, have to deal with the high complexity of the requirements as well as the significant size and quantity of process models in an organisation, which may prevent process models from being checked efficiently and timely. This paper provides a computational complexity analysis of the problem of proving regulatory compliance of process models. We investigate the computational complexity of each variant of the problem resulting from a combination of three binary properties associated to the regulatory framework, determining the regulatory requirements that a process model needs to follow to be compliant. These binary properties are whether the framework contains one or multiple obligations, whether the obligations are global or conditional, and whether only literals or formulae can be used to describe the obligations. For each variant of the problem we study the computational complexity of proving full compliance, partial compliance, and non-compliance. This analysis allows to under-
stand the specific features of the problem leading to intractability issues, thus potentially guiding future research towards developing feasible solutions for the problem in practical settings.

**Keywords:** Business Process Models, Regulatory Compliance, Computational Complexity

1. **Introduction**

Showing compliance with the given regulatory requirements is a critical step that organisations need to perform to prove their trustworthiness. These regulatory requirements consist of rules governing the procedures and processes carried out in the organisations operating in the governed environment. Furthermore, organisations commonly develop so-called business rules on top of the regulatory requirements, which specify additional constraints to their business processes to meet organisational goals (e.g. a customer has to be older than 18 in order to be eligible for a loan). However, as organisations grow, so do their processes and business rule collections. As a consequence, the number of regulations and rules that the organisation needs to comply with is generally large [1], and non-compliance may lead to significant fines and law suits [2].

In order to ensure continuous compliance to each of those rules, processes are modelled with so-called business process models, to allow for verification of their correctness with respect to those rules using automated compliance checking techniques. The problem of proving regulatory compliance of process models can be generally understood as checking whether the possible ways of executing a process model comply with the given regulatory requirements. Process models are represented as a collection of tasks, whose in turn represent the atomic actions composing the executions of the model, and a coordinating structure determining the possible execution orders of the tasks in the model. The regulatory requirements, on the other hand, are typically formulated in some form of temporal logic or deontic logic [3]. Different degrees of compliance can be identified for a process model. It is considered to be fully compliant when all of its executions satisfy the regulatory requirements, partially compliant when there exists at least one satisfactory execution, and non compliant when it does not contain any satisfactory execution.

Business processes can be prevented from being checked efficiently and timely, as the total amount of possible executions of a process model may be very large, and the requirements may be complex to be checked. The computational complexity of proving regulatory compliance depends on a number of different features of the problem. These features can be grouped into a set of different variants, each of which belonging to a specific computational complexity class.

In this paper, we provide a structured overview of a selection of compliance variants of proving regulatory compliance of process models, and provide a detailed computational complexity analysis for each of these selected variants [4].

The remainder of the paper is structured as follows: Section 2 introduces the concepts around business processes and business process models, together with the regulatory framework they are subject to. Subsequently, Section 3 defines the problem of proving regulatory compliance of structured

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[1] This paper extends and completes the existing work of Colombo Tosatto et al. [4] related to the computational complexity of these problem’s variants.
business process models with respect to a regulatory framework, and the elementary variants of the compliance problem. Sections 4 and 5 analyse and discuss the computational complexity of the various variants of the problem with respect to verifying full and partial compliance respectively. Finally, Section 6 concludes the paper, discussing the implications of the results and outlining possible further directions of research.

2. Preliminaries

2.1. Acyclic Structured Process Models

We first introduce process models, representing a collection of executions that an organisation uses to achieve its business objectives. We limit our computational complexity analysis to problems dealing with acyclic structured process models. These models are similar to the structured workflows defined by Kiepuszewski et al. [5], and can be preliminarily defined as the models whose components are properly nested. That is, a sub-component of such a process model, which we refer to as process block (Definition 2.1), always has at most one parent component and a set of children, equivalent to a tree structure.

Additionally, we consider only acyclic process models. These models do not contain loops, meaning that every task is limited to be executed at most once in each of the model’s executions. This limitation has the advantage that the number of possible executions of a process model are guaranteed to be finite.

2.1.1. Semantics

A structured process model is recursively composed of so-called process blocks. These process blocks can be either single tasks, or a collection of process blocks organised within a structure describing the possible execution orders of its components. One of such structures is for instance a sequence, describing that the process blocks contained have to be executed in the specified order. The possible structures that constitutes these process blocks are detailed in Definition 2.1.

Definition 2.1. (Process Block)

A process block $B$ is a directed graph: the nodes are called elements and the directed edges are called arcs. The set of elements of a process block are identified by the function $V(B)$ and the set of arcs by the function $E(B)$. The set of elements is composed of tasks and coordinators. There are 4 types of coordinators: and_split, and_join, xor_split and xor_join. Each process block $B$ has two distinguished nodes called the initial and final element. The initial element has no incoming arc from other elements in $B$ and is denoted by $b(B)$. Similarly the final element has no outgoing arcs to other elements in $B$ and is denoted by $f(B)$.

A directed graph composing a process block is defined inductively as follows:

- A single task constitutes a process block. The task is both initial and final element of the block.
- Let $B_1, \ldots, B_n$ be distinct process blocks with $n > 1$: 

Definition 2.2. (Annotated process)

Let $P$ be a structured process and $T$ be the set of tasks contained in $P$. An annotated process is a pair: $(P,\text{ann})$, where $\text{ann}$ is a function associating a consistent set of literals to each task in $T$: $\text{ann} : T \mapsto 2^\mathcal{L}$. The function $\text{ann}$ is constrained to the consistent literals sets in $2^\mathcal{L}$.

Example 2.3. (Payment Task Annotation)

Considering a process model describing the possible procedures to handle transactions in a store, such model could contain a task representing accepting payment from a customer for the purchased items. This accepting payment task would be associated with a set of literals representing the effects of the task.

For the sake of this example, we can assume that such annotation contains two literals: $\alpha$ and $\beta$, where $\alpha$ represents the effect that the purchased goods have been paid for, and $\beta$ represents the effect of issuing a receipt for the processed payment to the customer.

Annotations allow to keep track of the events holding at the various stages of an execution, allowing the regulatory requirements to be evaluated. Another benefit of using annotated tasks is that this allows the option of retracting propositions from the process state and replace them with their negation, representing that the execution of some tasks can negate the effect of previously executed tasks. Definition 2.4 details the semantics of how a process state is updated by the execution of a task.
update between the states of a process during its execution is inspired by the AGM\cite{2} belief revision operator \cite{6}.

**Definition 2.4. (State update)**
Given two consistent sets of literals $L_1$ and $L_2$, representing the process state and the annotation of a task being executed, the update of $L_1$ with $L_2$, denoted by $L_1 \oplus L_2$ is a set of literals defined as follows:

$$L_1 \oplus L_2 = L_1 \setminus \{\neg l \mid l \in L_2\} \cup L_2$$

### 2.1.3. Process execution

Depending on the process blocks composing a process model, its tasks can be executed in various orders. We use the term *execution* to refer to these ordered sequences of tasks, following the allowed execution ordering by the process blocks of the process model. An execution of a process always start with the start task and conclude with the end task. Definition\cite{2} recursively defines the allowed executions of a process model according to its process blocks.

**Definition 2.5. (Execution)**
Given a structured process model identified by a process block $B$, the set of its executions $\Sigma(B) = \{\epsilon \mid \epsilon$ is a sequence and is an execution of $B\}$. The function $\Sigma(B)$ is defined as follows:

1. If $B$ is a task $t$, then $\Sigma(B) = \{(t)\}$. The result is a set containing a single sequence composed by the single task $t$.

2. if $B$ is a composite block with sub-blocks $B_1, \ldots, B_n$:
   
   (a) If $B = \text{SEQ}(B_1, \ldots, B_n)$, then $\Sigma(B) = \{\epsilon_1 + \epsilon \cdots + \epsilon \mid \epsilon_i \in \Sigma(B_i)\}$ and $+\epsilon$ the operator concatenating two executions.

   (b) If $B = \text{XOR}(B_1, \ldots, B_n)$, then $\Sigma(B) = \Sigma(B_1) \cup \cdots \cup \Sigma(B_n)$

   (c) If $B = \text{AND}(B_1, \ldots, B_n)$, then $\Sigma(B) = \{\epsilon_1 \times \epsilon_2 \cdots \times \epsilon \mid \epsilon_i \in \Sigma(B_i)\}$ where $\times_\epsilon$ is the binary commutative interleaving operator having the following behaviour:

   i. $\epsilon_1 \times_\epsilon \epsilon_2 = \{\epsilon \mid \epsilon$ is a sequence representing a combinatorial combination of $\epsilon_1$ and $\epsilon_2$ that preserves the relatives orders of the tasks from $\epsilon_1$ and $\epsilon_2\}$

We use the term *trace* to refer to an execution of a process along with the associated process states that hold after the execution of each task. We show in Definition\cite{2} how these process states are progressively constructed using the annotations of the executed tasks.

**Definition 2.6. (Trace)**
Given an annotated process $(B, \text{ann})$ and an execution $\epsilon = (t_1, \ldots, t_n)$ such that $\epsilon \in \Sigma(B)$, a trace $\theta$ is a finite sequence of states: $(\sigma_1, \ldots, \sigma_n)$. Each state of $\sigma_i \in \theta$ is a pair: $(t_i, L_i)$ capturing what holds after the execution of a task $t_i$, expressed by a set of literals $L_i$. A set $L_i$ is constructed as follows:

\footnote{The operator is named after the initials of the authors introducing it: Alchourrón, Gärdenfors, and Makinson.}
1. $L_0 = \emptyset$

2. $L_{i+1} = L_i \oplus \text{ann}(t_{i+1})$, for $1 \leq i < n$.

To denote the set of possible traces resulting from a process model $(B, \text{ann})$, we use $\Theta(B, \text{ann})$.

In Example 2.7, we illustrate an annotated process model, and its possible executions and traces.

**Example 2.7. (Annotated Process Model)**

Fig. 1 shows a structured process containing four tasks labelled $t_1$, $t_2$, $t_3$ and $t_4$ and their annotations. The process contains an AND block followed by a task and an XOR block nested within the AND block. The annotations indicate what has to hold after a task is executed. If $t_1$ is executed, then the literal $a$ has to hold in that state of the process. Table 1 shows the possible executions ($\Sigma(B)$) and traces ($\Theta(B, \text{ann})$) of the model.

| $\Sigma(B)$          | $\Theta(B, \text{ann})$                      |
|----------------------|-----------------------------------------------|
| (start, $t_1$, $t_3$, $t_4$, end) | ((start, $\emptyset$), ($t_1$, $\{a\}$), ($t_3$, $\{a, c, d\}$), ($t_4$, $\{-a, c, d\}$), (end, $\{-a, c, d\}$)) |
| (start, $t_2$, $t_3$, $t_4$ end) | ((start, $\emptyset$), ($t_2$, $\{b, c\}$), ($t_3$, $\{b, c, d\}$), ($t_4$, $\{-a, b, c, d\}$), (end, $\{-a, b, c, d\}$)) |
| (start, $t_3$, $t_1$, $t_4$ end) | ((start, $\emptyset$), ($t_3$, $\{c, d\}$), ($t_1$, $\{a, c, d\}$), ($t_4$, $\{-a, c, d\}$), (end, $\{-a, c, d\}$)) |
| (start, $t_3$, $t_2$, $t_4$ end) | ((start, $\emptyset$), ($t_3$, $\{c, d\}$), ($t_2$, $\{b, c, d\}$), ($t_4$, $\{-a, b, c, d\}$), (end, $\{-a, b, c, d\}$)) |

Table 1: Executions and traces of the annotated process in Fig. 1

2.1.4. **Advantages**

There are two main advantages of limiting the computational complexity analysis to this specific class of processes. The first advantage is related to the process models being acyclic such that every task in the model can be executed at most once in each execution, which leads to the process model to contain a finite number of possible executions. When showing that a problem belongs to the $\textbf{NP}$ complexity class, one needs to show that checking whether a solution is correct can be done in polynomial time. In case of proving partial compliance, for instance, only a single execution needs to be found.
that satisfies the regulatory requirements. However, as loops allow tasks to be executed multiple times in a single execution, parsing such an execution can potentially be no longer in polynomial time with respect to the size of the process model. Therefore, we opted to focus our computational complexity analysis on acyclic process models.

The second advantage is related to the process models being structured (as defined by Kiepuszewski et al. [5]), which allows to verify the soundness\(^3\) of these models in polynomial time with respect to their size.

### 2.1.5. Disadvantages

The reduced expressivity of the allowed process model is the obvious disadvantage of the applied limitations. Although the absence of cycles in these models can be circumvented by abstracting repeated tasks into a single component in the model, reducing whole cycles as single tasks inhibits the capability of checking the compliance of the inner details of these repetitions.

Similarly, limiting the study to structured process models leaves out all unstructured models that could be required to represent more edge-case real world scenarios. However, the SAP R/3 collection used by SAP to customize their R/3 ERP product [9] contains 604 process models, of which 493 are structured and behaviorally correct (i.e. sound), including only 1 model that is structured but not acyclic. As such, the restrictions on process structures assumed in this paper do not impose a severe limitation on the amount of real world scenarios supported.

### 2.2. Regulatory Framework

The executions of the process model are required to follow the regulatory requirements expressed by the regulatory framework, which is the second component of the problem. Depending on whether all, some, or none of the executions satisfy these requirements, a different compliance level is determined for the process model.

The regulatory requirements in a regulatory framework are described using obligations. These obligations are specified using a subset of Process Compliance Logic (PCL), introduced by Governatori and Rotolo [10]. In the remainder of this section, we introduce the semantics of the different properties identifying the problem’s variants, and the advantages and disadvantages of adopting these features to define a regulatory compliance problem.

#### 2.2.1. Semantics

We introduce now the semantics for the binary features describing the regulatory framework. Through their combinations these features determine the semantics of the regulatory framework and the obligations composing it.

The first binary feature we discuss concerns whether an obligation is global or local\(^4\). A global obligation is in force for the entire duration of an execution, while the in force interval of a local

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\(^3\)A process is sound, as defined by van der Aalst [7][8], if it is live and deadlock free.

\(^4\)Local obligations are also referred to as conditional obligations, as they rely on a condition to be verified for them to be applicable.
obligation is determined by its trigger and deadline conditions. The *in force interval* of an obligation determines the part of an execution in which the obligation is checked to determine whether the execution is compliant. In the case of local obligations, the *trigger* and *deadline* elements describing the obligation are checked against the process’ states of an execution to identify the in force intervals as described in Definition 2.8.

**Definition 2.8. (Local Obligation)**
A local obligation $O^o(\pi, \tau, \delta)$, where $o \in \{a, m\}$ represents whether the obligation represented by the obligation is of type achievement or maintenance. The element $\pi$ represents the requirement of the represented obligation, the element $\tau$ its trigger, and the element $\delta$ its deadline.

![Figure 2: Conditional Obligation](image)

Figure 2 illustrates an in force interval of a local obligation within a trace. This simplistic representation shows that an in force interval is identified in a trace between two points: the *trigger* and the *deadline*. These two points in a trace represent a state in the trace satisfying the trigger of the obligation, and a state satisfying the deadline.

Compliance with respect to an obligation is evaluated over the states composing a *trace*. Definition 2.9 provides an initial intuitive informal description of the semantics for both types of obligations (i.e. achievement and maintenance).

**Definition 2.9. (Achievement and Maintenance Obligations)**
An in force obligation is fulfilled depending on its type as follows:

**Achievement** If this type of obligation is in force in an interval, then the requirement specified by the regulation must be satisfied in a trace in at least one point in the interval before the deadline is satisfied. If this is the case, then the obligation in force is considered to be satisfied. Otherwise it is violated.

**Maintenance** If this type of obligation is in force in an interval, then the requirement must be satisfied *continuously* in all points of the interval until the deadline is satisfied. If this is the case, then the obligation in force is then satisfied, otherwise it is violated.

### 2.2.2. Multiple in force intervals

It is possible that a trace contains multiple points where the trigger condition of an obligation is satisfied. Figure 3 shows a trace having 2 points satisfying the trigger labelled *trigger 1* and *trigger
2, and 2 points satisfying the deadline of the obligation labelled *deadline 1* and *deadline 2*. Notice that due to the multiple points in the trace satisfying the trigger of the obligation being evaluated, the obligation has then multiple in force intervals in the execution being analysed, and each in force interval requires to be evaluated by considering the type and requirement condition of the obligation.

When an obligation has multiple in force instances within a trace, a single event is allowed to satisfy multiple instances. In other words, a state containing the required proposition to satisfy an obligation can satisfy multiple in force instances of it, as the state is not consumed by satisfying the instances. The main reason why we handle multiple instances in this way is twofold: first, obligations only verify execution states without changing the states themselves, meaning that they cannot consume events that are used to satisfy them. Second, some counting and / or labelling techniques could be adopted to tackle these multi-instance case differently, however this would increase the computational complexity burden of the problem, hence our decision to limit the current study to these simpler variants.

In general, different in force intervals of an obligation in a single trace can be checked independently for their satisfaction. This is always the case when these in force intervals are not overlapping. However, when two in force intervals overlap, the two intervals can be collapsed in a single in force interval as detailed in Lemma 2.10.

**Lemma 2.10. (Overlapping In Force Intervals)**

Given a trace $\theta$ and a local obligation $\mathcal{O}^\pi = \langle \pi, \tau, \delta \rangle$ having two in force intervals $i_1, i_2$ in $\epsilon$. If $i_1$ and $i_2$ overlap, then depending on the type of the obligation, the two in force interval can be collapsed as $i_{1+2} = i_1$ considering the effective in force intervals when the obligation’s type is achievement, or the in force intervals when it is maintenance.

**Proof:**

We prove the lemma by checking the two cases depending on the type of the obligation:

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5We detail the semantics concerning how traces satisfy obligations in general in Definition 3.1 and Definition 3.2, depending on the type of the obligation being evaluated (achievement or maintenance).

6Two in force intervals are considered to be overlapping if one of the intervals has its trigger point or deadline point in the other interval.

7An effective in force interval is the interval of an achievement obligation that terminates when: a) the deadline is reached without the satisfying condition, ending with a violation, of b) the satisfying condition is reached and the in force interval terminates without waiting for the deadline to appear.
Achievement When there are two overlapping effective in force intervals ($i_1$ and $i_2$) for an achievement obligation, two options are possible:

1. when $i_1$ is successfully terminated, the termination also ends $t_2$ effective in force interval, practically making $i_2$ a sub interval of $i_1$ and having both intervals satisfied. This is shown in Figure 4.

2. when $i_1$ is unsuccessfully terminated the execution can be labeled not compliant with the obligation. Whether $i_2$ is successful or not becomes irrelevant for determining the compliance state of the execution.

Maintenance From Definition 3.2 we know that a required property needs to be maintained within an in force interval. When such property needs to be maintained for the duration of two in force intervals ($i_1$ and $i_2$), and these intervals are overlapping, this is equivalent to maintain the property for the first interval, as the deadline of the first interval would also terminate the second interval.

\[ \square \]

![Figure 4: Successful Termination of 2 Overlapping In Force Instances](image)

3. Problem specification

3.1. Satisfying Obligations

Before formally defining the semantics of obligations, we introduce some syntax shorthands to avoid cluttering and improve readability in Notation 1.

**Notation 1.** We write $\sigma_l$ to denote $\sigma \in \theta$ such that $\sigma \models l$. Similarly, we write \text{contain}(l, \sigma)$ to denote a task-state pair appearing in a trace $(t, \sigma)$ such that $l \in \text{ann}(t)$.

Using these shorthands, we formally define achievement and maintenance obligations as follows:
Definition 3.1. (Comply with Achievement)
Given an achievement obligation $O^a(\pi, \tau, \delta)$ and a trace $\theta$, $\theta$ is compliant with $O^a(\pi, \tau, \delta)$ if and only if: $\forall \sigma_\tau, \exists \sigma_\pi$ contain $(\tau, \sigma_\tau)$ and $\sigma_\tau \leq \sigma_\pi$ and $\neg \exists \sigma_\delta | \sigma_\tau \preceq \sigma_\delta \prec \sigma_\pi$.
A trace not complying with an achievement obligation is considered to violate it.

Definition 3.2. (Comply with Maintenance)
Given a maintenance obligation $O^m(\pi, \tau, \delta)$ and a trace $\theta$, $\theta$ is compliant with $O^m(\pi, \tau, \delta)$ if and only if: $\forall \sigma_\tau, \exists \sigma_\delta$ contain $(\tau, \sigma_\tau)$ and $\sigma_\tau \leq \sigma_\delta$ and $\forall \sigma | \sigma_\tau \leq \sigma \leq \sigma_\delta, \pi \in \sigma$.
A trace not complying with a maintenance obligation is considered to violate it.

From Definition 3.1 and Definition 3.2, it can be observed that states satisfying the deadline of an obligation are often critical to determining whether a trace satisfies an obligation or not. For instance, when considering an obligation of type maintenance, its satisfaction cannot be determined until a state satisfying the deadline is found, while its violation can be determined as soon as a state not satisfying the requirement is encountered when the obligation is in force.

To avoid a scenario where an in force instance of an obligation being evaluated reaches the end of a trace while a state satisfying its deadline is not encountered (thus potentially leaving the in force instance in a non-evaluated state), we adopt Assumption 3.3. This assumption states that the final task of each trace of a process model always satisfies the deadlines of each obligation being evaluated. Note that the task end is always the last task in each trace of a process model.

Assumption 3.3. (Final Deadline)
Given a structured process model $(P, \text{ann})$ and an obligation $O^o(\pi, \tau, \delta)$, $\forall \theta \in \Theta(P, \text{ann})$: if $(e, \sigma)$ is the last task-state tuple in $\theta$, then $\sigma \models = \delta$.

One of the binary features determining the difficulty of the problem’s variants being studied consists of the expressivity allowed to be used by the element composing the obligations of the regulatory framework. The easier variant restricts these elements to be represented by single propositional literals, while the more difficult one would allow propositional formulae.

When the easier feature is adopted, the behaviour of the obligations being checked can be considered to collapse to checking whether tasks having such propositional literals annotated are executed, instead of checking the evolving execution’s state. As stated in Definition 2.4, when a task is executed, the propositional literals contained in its annotations become part of the execution state. When the elements of an obligation, such as the trigger, deadline, and requirement, are composed by simple propositional literals, checking the execution’s states satisfying them can be reduced to check whether a task having such literal in their annotation is executed.

3.2. Compliance Classifications

When a process model is checked for compliance the following three categorisations can be determined. A process is said to be fully compliant if every trace of the process is compliant with the regulatory framework\(^8\). A process is partially compliant if there exists at least one trace that is compliant with the regulatory framework, and not compliant if there is no trace complying with the framework.

\(^8\)Notice that by “compliant with the regulatory framework”, we refer to a trace fulfilling each in force interval along the trace itself for each obligation belonging to the regulatory framework.
Definition 3.4. (Process Compliance)
Given a process \((P, \text{ann})\) and a regulatory framework composed by a set of obligations \(\mathcal{O}\), the compliance of \((P, \text{ann})\) with respect to \(\mathcal{O}\) is determined as follows:

- **Full compliance** \((P, \text{ann}) \vdash^{F} \mathcal{O}\) if and only if \(\forall \theta \in \Theta(P, \text{ann}), \theta \vdash \mathcal{O}\).
- **Partial compliance** \((P, \text{ann}) \vdash^{P} \mathcal{O}\) if and only if \(\exists \theta \in \Theta(P, \text{ann}), \theta \vdash \mathcal{O}\).
- **Non compliance** \((P, \text{ann}) \nvdash \mathcal{O}\) if and only if \(\neg \exists \theta \in \Theta(P, \text{ann}), \theta \vdash \mathcal{O}\).

3.3. Problem variants
Given the regulatory framework defined above, the problem can be categorised by three binary properties, resulting in 8 distinct variants. These variants of the problem will be studied in the remainder of this paper and can be formulated as follows:

1/n  A regulatory framework containing a single obligation (1) or a set of obligations (n).
G/L  A regulatory framework containing only global obligations (G) or also containing local obligations (L).
-/+  A regulatory framework containing obligations limited to be described using propositional literals (-) or allowing such obligations to be described using propositional formulae (+).

When considering the binary properties defining the variants of the problem, it is easy to see that each property allows an easy and a hard option:

- It is easier to check compliance against a single obligation (1) than multiple ones (n).
- A global obligation (G) allows a single in force instance spanning for the entirety of the traces, making it the easier counterpart, as local obligations (L) may allow multiple in force instances, which also need to be computed in each possible trace of the process model.
- Obligations limiting their elements to propositions (-) instead of formulae (+) are easier to verify, as the satisfaction of their elements can be reduced to checking the execution of tasks having such elements in their annotations. Checking the satisfaction of the elements of an obligation allowing formulae always requires to consider the trace’s history, making it harder.

Considering two variants of the problem sharing two of the three binary properties, it is fair to assume that the one being defined by the harder differentiating property, is at least as computationally complex as the one defined by the easier differentiating property. For instance, the variant \(nG+\) is at least as difficult as both \(nG-\) or \(1G+\). This is illustrated graphically in Figure 5 below, where the potential increase in complexity is indicated by the direction of the arrows.
3.3.1. Advantages

The two types of obligations considered, along with the three binary features of the regulatory framework determining the variants of the problem, are expressive enough to represent the majority of obligations contained in legal documents used in real world scenarios, as discussed by Hashmi et al. [11]. Namely, when the three difficult binary features are chosen to identify a variant of the problem, these combination allow to represent problems involving multiple obligations whose expressiveness allow to capture fairly complex behaviours.

3.3.2. Disadvantages

Despite the ample expressivity provided by the features of the variants of the problem being studied, there are still some limitations. One of such limitations is for instance the impossibility of representing actual data and variables in the problem. For example representing counters using variables, and obligations over their allowed or disallowed values. Another limitation, related to the adopted semantics, is the inability of having multiple in force instances of an obligation requiring distinct events to satisfy each instance.

4. Full Compliance

We first focus on discussing the computational complexity of proving full compliance for the 8 variants of the problem.

4.1. Existing Results

Computational complexity results already exist for 5 of the 8 variants of the problem, as shown in Table 2.

Figure 5: Compliance Complexity Lattice.
| Problem’s Variant | Source | Complexity Class |
|-------------------|--------|-----------------|
| 1G-               | Colombo Tosatto et al. [12] | P |
| 1L-               | Colombo Tosatto et al. [13] | P |
| nL-               | Colombo Tosatto et al. [13] | P |
| 1L+               | Colombo Tosatto et al. [14] | coNP-complete |
| nL+               | Colombo Tosatto [15] | coNP-complete |

Table 2: Full Compliance Complexity

From the table we can see that three of the variants have been shown to allow polynomial solutions, while the two more difficult variants belong to the computational complexity class coNP-complete. In the remainder of this section we first discuss the existing results, and later we provide new computational complexity results for the not yet classified variants of the problem.

4.1.1. Existing Result: 1G-

Two algorithms [12] have been proposed to show that this variant of the problem can be solved in polynomial time. Each algorithm solves the problem for one of the obligation’s types, and we briefly outline them below.

**Achievement**

Let *erase* be a procedure that, when applied to a process block, removes the block and is recursively applied to its parent process block. This procedure stops without removing the process block when applied to a type XOR. Full compliance of a process model can be checked against a global obligation using the two following steps:

1. Apply *erase* to each task containing the required condition of the obligation in their annotation.
2. If the *erase* procedure removed the process block corresponding to the whole process model, then the process is fully compliant.

This algorithm ensures that every possible trace of the model must execute a task allowing to satisfy the required condition of the obligation.
Boundary

In this case, two necessary conditions are required to prove full compliance of a process model:

1. Every task that can be executed at the beginning of a trace must contain the required condition of the obligation in its annotations.

2. There must be no task in the process having the negation of the required condition of the obligation in its annotations.

This algorithm ensures that the required condition appears in the first process state of every trace, and is not removed by the execution of further tasks.

4.1.2. Existing Results: 1L- and nL-

The variant 1L- can be solved in polynomial time using the pattern approach. This approach verifies the complement of proving full compliance of a process model, meaning that this approach checks whether the process model contains a trace violating the given local obligation. Using this approach allows to determine that a process is not fully compliant when a violating trace is identified. However, when such a trace cannot be found in a process model, then the model can be classified as fully compliant.

The pattern approach checks the process blocks of the process model for some patterns leading to the process block to contain a violating trace for the local obligation being evaluated. These patterns are referred to as Δ-constraints in the approach, and are generated from the elements composing the local obligation, as described by Definition 4.1. A set of Δ-constraints is generated in accordance to the type of obligation and its parameters.

Definition 4.1. (Parametrising Δ-constraints)

Given an obligation $O_o(\pi, \tau, \delta)$, depending on its type $o$ it is parametrised into Δ-constraints as follows:

- $\overline{A}\Delta(\neg\pi, \tau, \delta)$ when the type is achievement. This represents searching for a trace in the model where $\neg\pi$ is the case when a task having $\tau$ annotated is executed, and $\neg\pi$ stays the case until a task having $\delta$ annotated is executed. If such a trace is found, then it corresponds to find a trace violating the given achievement obligation.

- $\overline{M}\Delta(\neg\pi, \tau, \delta)$ when the type is maintenance. This represents searching for a trace in the process where $\neg\pi$ is the case when a task having $\tau$ annotated is executed, or for a trace where $\neg\pi$ becomes the case before a task having $\delta$ annotated is executed. This corresponds to finding a trace violating the given maintenance obligation.

Classifications are used to measure whether a process block satisfies the pattern specified by one of the Δ-constraints, and keep track of how the process block may partially satisfy the pattern. Keeping track of partial pattern satisfaction is used in the approach to evaluate a Δ-constraint satisfaction from the evaluation of its sub-process blocks. We now outline the steps of the algorithm.

1. For each Δ-constraint obtained from the parameters of the local obligation.
(a) For each task having the trigger of the obligation in its annotation (trigger task), evaluate whether the pattern represented by the $\Delta$-constraint can be satisfied when considering this particular task as the one starting an in force instance of the local obligation.

i. Classify each task in the process model.

ii. Recursively classify a process block in the model by aggregating the evaluations of its sub-blocks.

iii. If one or more process blocks satisfy the pattern, then there exist a trace in the process model violating the local obligation in this particular in force instance started by the trigger task being currently considered.

2. If a violating trace is identified by the approach, then the process model can be classified as not fully compliant with the given local obligation. Otherwise, if no such trace is found, then the process model is fully compliant.

This approach allows to evaluate full compliance for a variant $1L^-$. The evaluation is polynomial as the number of possible trigger tasks is bounded by the size of the process, and the number of $\Delta$-constraints is constant. Also, notice that classifying and aggregating the classifications to evaluate process blocks can be performed in constant time.

Finally, the same pattern approach procedure can be iterated for each obligation composing a regulatory framework containing multiple obligations. This still allows to decide in polynomial time whether a process model is fully compliant also for the variant $nL^-$ of the problem.

4.1.3. Existing Result: $1L^+$

It has been proven [14] that the tautology problem can be reduced to the compliance problem variant $1L^+$. Through this reduction, this variant of the problem is shown to belong to the computational complexity class coNP-complete.

The process model in this reduction is constructed in such a way that the final state of each trace represents one of the possible interpretations of the formula from the tautology problem. In Definition 4.2, we provide a detailed explanation of the process model construction, as it is reused by several other reductions, both for existing and new computational complexity results.

**Definition 4.2. (Interpretation Process Construction)**

Given a propositional logic formula $\varphi$, a process model $P$, containing a set of traces whose aggregated collection of process states represent each possible interpretation of $\varphi$, is constructed as follows:

1. For each proposition appearing in $\varphi$ an XOR block is created containing the following branches:
   - A branch containing a task having the proposition annotated.
   - A branch containing a task having the negation of the proposition annotated.

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9The tautology problem consists of verifying whether a propositional formula is true in each of its possible interpretations.
2. Each XOR block constructed in 1. is organised within an AND block, each composing a branch of the process block.\(^{10}\)

3. The AND block constructed in 2. is included in a SEQ block, preceded by a start task and followed by an end task. Additionally, we would like to point out that the following alternative is sometimes used when the regulatory framework allows local obligations:

- The AND block constructed in 2. is included in a SEQ block, preceded by a start task and followed by a test task having a proposition \(test\) annotated. The SEQ block is then concluded by an end task.\(^{11}\)

4. The start task is annotated with the set of proposition containing all the positive variants of the propositions appearing in \(\varphi\).\(^{12}\)

Considering the process construction detailed above, we illustrate in Figure 6a the possible resulting outcomes, and we show here how it allows to provide each possible interpretation for \(\varphi\) through the states of its traces.

When one of the XOR blocks constructed in step 1 is executed, one of the tasks composing it is executed and either the positive or negative variant of a proposition is introduced in the process state. When each of the XOR blocks are executed, the resulting process state contains a positive or negative variant of each of the propositions appearing in \(\varphi\). This corresponds to a possible interpretation of the propositional formula.

Considering the possible ways as these XOR blocks can be executed, as detailed in Definition 2.5, we obtain that each possible trace corresponds to one of the possible permutations of the positive/negative value assignment for the propositions of \(\varphi\). By considering every possible trace going through each of these XOR blocks,\(^{13}\) the resulting process states after the execution of these blocks corresponds then to the collection of the possible interpretations of \(\varphi\).

When the constructed process includes a test task (Figure 6b), it can be used to specifically target these final process states by using it along with local obligations having their trigger and deadline components set to the \(test\) proposition annotated on the test task. However, when a test task is not used in the process construction, the process states leading to the final ones still represent possible interpretations for the formula \(\varphi\) thanks to the initial process state assigned through the annotation of the start task.

\(^{10}\)We would like to bring to notice that an AND block is not strictly necessary in the construction, but is used to emphasise that the execution order between the XOR blocks is not important. As a matter of fact a simple SEQ block including the list of XOR blocks would have been sufficient for the construction purpose.

\(^{11}\)The purpose of this alternative construction is to leverage the capability of local obligations to evaluate specific process states in traces. In this case, a local obligation could have the proposition \(test\) as both its trigger and deadline, leading to a situation where the obligation’s in force interval would span exactly the process state holding after the execution of the task test. As we see in the remainder of Definition 4.2, the process states obtained after executing each of the XOR blocks are the ones sufficient to cover every possible interpretation for the given propositional formula \(\varphi\).

\(^{12}\)Note that potentially this initial state annotated in the start task can be any combination of the positive/negative variants of the proposition included in \(\varphi\). However for the sake of simplicity we stick to a starting annotation only including the positive variants.

\(^{13}\)For simplicity we are currently considering a fixed execution order of such blocks, avoiding all the possible orders allowed by the encompassing AND block.
This reduction uses a construction with a test task as described in Definition 4.2. The regulatory framework contains an achievement obligation having the propositional formula as its requirement, and set to be always evaluated only on the final state of a trace of the model.

Therefore, proving that the process model is fully compliant, corresponds that every possible interpretation of a propositional formula satisfies it, which is sufficient to conclude that the propositional formula is a tautology.

4.1.4. Existing Result: nL+

Knowing from existing analysis [15] that 1L+ is in coNP-complete, we can conclude that the more complex variant is also at least in the same or a higher computational complexity class, following the reasoning in Section 3.3. Since the variant nL+ allows more obligations than the single one allowed in the counterpart 1L+, the tautology problem can also be reduced to this more complex variant.

While this shows that the problem is at least in coNP-complete, we show that it is indeed in this computational complexity class by considering its complementary problem. The complement of the problem of checking full compliance means checking whether a process model is not fully compliant. This corresponds to checking whether there exists a trace of the model violating the regulatory framework. Such a counter-example can be still verified in polynomial time, as it would correspond to a trace whose sequential process states can be linearly parsed while checking the existence of a violation of a triggered in force instance of the obligations composing the regulatory framework. Moreover, the complementary problem closely resembles the problem of proving partial compliance, requiring to find a trace of the process model satisfying some given properties. As we are showing in Section 5, the computational complexity of this variant is in NP-complete, which allows to conclude that the
variant when checking full compliance is in coNP-complete.

4.2. New Result: Reducing Tautology to 1G+

Having introduced and discussed some of the existing computational complexity results, we now propose a reduction showing that the tautology problem can also be reduced to the variant 1G+ of the problem. A similar reduction [12] has already been used, showing that some of the harder variants of the problem of proving full compliance are in general in co-NP-complete.

Reduction 1. Given a propositional formula \( \varphi \), the reduction to 1G+ is done as follows:

BPM The business process model is constructed as an Interpretation Process without a test task as described in Definition 4.2. The result is that each trace of such a business process model represents an interpretation of the propositions composing \( \varphi \), and according to such interpretation the truth value of \( \varphi \) can be uniquely determined.

RF The regulatory framework is composed by the following global maintenance obligation: \( O^m(\varphi) \).

Proof:

1. From the construction of the business process model and Definition 4.2, it follows that each possible trace contains in its last state a possible interpretation for the proposition composing \( \varphi \).

2. From the construction of the business process model and Definition 4.2, it follows that every possible interpretation of the literals included in \( \varphi \) is considered in at least a state of a trace of the business process model.

3. From the construction of the business process model and Definition 2.4 and 4.2, given the initial annotation of the task start, the states evolving through the trace of the model represent the interpretations of the propositions composing \( \varphi \) that are still possible.

4. From 1., 2., 3., and the construction of the regulation, it follows that if none of the states in every trace of the business process model falsifies the condition of the maintenance obligation constituting the regulatory framework. This means that \( \varphi \) satisfies each of its possible interpretations, hence \( \varphi \) is a tautology.

4.3. Summary of Results

The computational complexity result concerning the variants of the problem of proving full compliance of a process model are summarised in Figure 7. The arrows in the illustration represent the relations between the variants. An arrow starting from one of the variants and pointing to another
represents that the second problem has 2 features in common with the first, and another that is more difficult. Therefore, the direction of the arrows also indicates the hardness of the variants, in terms as most as difficult as by following the directions of the arrows.

Notice that Figure 7 contains also results for the variants $nG$- and $nG+$, and were not provided in the earlier parts of this section. These results are derived using the relations between the variants of the problem shown earlier in Figure 5. The follow relationship holds regarding the computational complexity: $1G- \leq nG- \leq nL-$. Therefore, by knowing that both $1G-$ and $nL-$ are in $P$, it follows that also $nG-$ is in $P$. Similarly, we know that with respect to computational complexity it holds that $1G+ \leq nG+ \leq nL+$, and as the extremes belong to the same computational complexity class, then also $nG+$ is in $coNP$-complete (shortened to $coNP$-c in Figure 7).

### 4.4. Discussion on the Computational Complexity

From the computational complexity results, it can be observed that the variants of the problem allowing propositional formulae in the regulations composing the regulatory framework are significantly harder than the variants limiting their regulations to only use propositional literals. Therefore, the computational complexity of the problem of proving full compliance of structured business process models is strongly correlated to the expressivity allowed in the elements composing the regulations.

Proving full compliance for the variants of the problem limiting the expressivity of their regulations to propositional literals can be done in polynomial time with respect to the size of the problem, as it is sufficient to find a trace violating one of the regulations composing the regulatory framework. Looking for such violations can be done independently for each regulation, hence the number of regulations composing the regulatory framework does not bring these variants of the problem to a higher complexity class. Moreover, violations can be looked for in each of the in force intervals of the obligations independently, allowing to prove full compliance of variants using local obligations efficiently.
5. Partial Compliance

This section focuses on the problem of proving partial compliance for the 8 variants of the problem. Table 3 shows that 7 computational results already exist.

| Problem’s Variant | Source                          | Complexity Class  |
|-------------------|--------------------------------|-------------------|
| 1G-               | Colombo Tosatto et al. [12]     | P                 |
| nG-               | Colombo Tosatto et al. [4]      | NP-complete       |
| 1G+               | Colombo Tosatto et al. [4]      | NP-complete       |
| nG+               | Colombo Tosatto et al. [4]      | NP-complete       |
| nL-               | Colombo Tosatto et al. [14]     | NP-complete       |
| 1L+               | Colombo Tosatto et al. [4]      | NP-complete       |
| nL+               | Colombo Tosatto et al. [14]     | NP-complete       |

Table 3: Partial Compliance Complexity

5.1. Existing Results

We first discuss the existing results, and subsequently provide a new computational complexity classification for the variant not yet classified.

5.1.1. Existing Result: 1G-

Two algorithms [12] have been proposed to prove partial compliance in polynomial time for this problem’s variant. Each algorithm solves the variant of the problem for a specific obligation’s type, and we outline the two algorithms below.

Achievement In this case it is sufficient to verify whether the process model contains a task having in its annotations the required condition of the obligation composing the regulatory framework. This is a sufficient and necessary condition to ensure that exists a trace of the model containing it, which means that such a trace would contain a state satisfying the requirement of the obligation, hence satisfying it and allowing to label the process model partially compliant.
Maintenance  Let erase be a procedure that, when applied to a process block, removes the block from the process and is recursively applied to its parent process block. This procedure stops without removing the process block when applied to a type XOR.

When the obligation is of type maintenance, the algorithm verifies whether the process is partially compliant in 3 steps:

1. Apply erase on every task that must be executed at the beginning of a trace and does not contain in its annotations the required condition of the obligation.
2. Apply erase on each task containing in their annotation the negation of required condition of the obligation.
3. If the process block associated to the process model is not removed by the erase procedure, then the process is partially compliant.

This algorithm ensures that there exists a trace whose first process state contains the required condition, and no further executed task alters the required state.

5.1.2. Existing Result: nG-

The 3-SAT problem has been shown to be reducible to the variant nG- of the problem [4], meaning that this variant of the problem is also in NP-complete. The reduction constructs an Interpretation Process model, representing the possible interpretations of the proposition in the formula, as described in Definition 4.2.

For each clause, a single achievement obligation is created. Its required condition is set to a literal labelled as the clause, and a count-as rule is created for each of the literal in the disjunction of the clause and having the literal as the body and the clause label as the head of the count-as rule. When a state satisfies one of the count-as rules, it satisfies the achievement obligation having the related literal representing the clause in its condition. Finally, having a trace satisfying each of the obligations corresponds to satisfying each of the clause, hence the 3CNF formula.

5.2. Existing Results: 1G+ and 1L+

The satisfiability problem, which is an NP-complete problem consisting of checking whether a propositional formula allows an interpretation evaluating it to true, can be reduced to both variants [4]. For both variants, the reductions construct an Interpretation Process model without a test task, as described by Definition 4.2 considering the propositional formula being evaluated.

For both variants, a single achievement obligation is constructed having the formula as its required condition. For the variant 1L+, the obligation can use its trigger and deadline in such a way that the

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14 The problem of verifying whether a list of conjuncted logical clauses, where each clause is formed by up to 3 literals in disjunction, allows an interpretation satisfying it.

15 Consider a count as rule: $\alpha \Rightarrow \beta$, where $\alpha$ and $\beta$ are literals. If $\alpha$ is true in the process’ state, then the process’ state is considered to contain $\beta$. 
evaluation occurs only in the last process state. For the variant \(1G^+\), on the other hand, the evaluation of the obligation can be performed over partial interpretations contained in the states of the traces. However, irrespective of whether a full interpretation satisfies the obligation of the variant \(1L^+\), or a partial interpretation satisfies the obligation of the variant \(1G^+\), the formula is satisfiable.

5.3. Existing Result: \(nG^+\)

By exploiting the complexity relations between the variants of the problem shown in Figure 5, we can derive that \(nL^+\) is at least as difficult as \(nG^+\), leading to the following computational complexity relations: \(nG^- \leq nG^+ \leq nL^+\) and \(1G^+ \leq nG^+ \leq nL^+\). Given that the computational complexity for both variants \(1G^+\) and \(nL^+\) is NP-complete, it follows from the computational complexity relation \(1G^+ \leq nG^+ \leq nL^+\) that the computational complexity of \(nG^+\) is also in NP-complete.

5.4. Existing Results: \(nL^-\) and \(nL^+\)

The *hamiltonian path problem* can be reduced to these two variants of the problem of proving regulatory compliance [14]. The reduction constructs a process model containing a task for each of the nodes in the graph being checked for a hamiltonian path. These tasks are organised within a single AND block, allowing traces corresponding to the possible orders in which these nodes can be considered. Additionally, the regulatory framework is composed by a set of obligations for each of the nodes that produce a violation when two tasks appear in sequence, and the corresponding nodes in the graph are not connected. Given this construction, the existence of a trace compliant with each obligation correspond to a sequence of the nodes which can be traveled in the graph through the edges, and without node repetitions. Finally, as the obligations constructed in the obligations can be represented using literals, this reduction is allowed for both variants.

5.5. New Result: \(1L^-\) is in P

We now show that the variant \(1L^-\) belongs to the computational complexity class \(P\), disproving Conjecture 1 as originally proposed in [4].

**Conjecture 1.** (\(1L^-\) is in NP)

We currently have no information about the computational complexity of \(1L^-\). That is, we cannot infer its membership to a computational complexity class in a similar way as for \(nG^+\), as in this case the simpler variant \(1G^-\) is in \(P\).

While it seems that moving from \(G\) to \(L\) does not increase the complexity of the problem as much as when moving from \(-\) to \(+\), or from \(1\) to \(n\), we believe that such movement should be still be capable of bringing the computational complexity of the problem’s variant into NP.

We back our conjecture using the intuition that the move towards conditional obligations allows multiple instances of the same obligation to be in force over a single trace. As a result, even for the variant \(1L^-\), multiple instances would be required to be verified for every trace. This resembles the variant \(nG^-\), where multiple obligations are required to be verified over a trace, and is in NP.
To show that the variant $1L$- is solvable in polynomial time, we propose a solution using an adapted version of the pattern approach \[13\]. This adapted version aims at checking whether the process model is partially compliant instead of checking for non-compliance, as in the version outlined earlier in Section 4.1.2.

In its original version, the pattern approach produces a set of $\Delta$-constraints using the parameters of the local obligation $O^o\langle \pi, \tau, \delta \rangle$\[16\], as shown in Definition 4.1. The parametrisation of the $\Delta$-constraints can be adapted to allow for checking for the existence of a compliant trace instead of a violating one. The re-parametrisation can be formally defined as follows:

**Definition 5.1. (Parametrisation of achievement into a $\Delta$-constraint)**
A local achievement obligation $O^a\langle \pi, \tau, \delta \rangle$ can be parametrised into $\check{A}\Delta(\pi, \tau, \delta)$, which checks for a trace where $\pi$ holds when a task having $\tau$ annotated is executed, or a trace where $\pi$ becomes the case before a task having $\delta$ annotated is executed. Such a trace, if it exists, satisfies the given local achievement obligation. To avoid confusion we refer to this mapping as $\check{A}\Delta(\pi, \tau, \delta)$.

**Definition 5.2. (Parametrisation of maintenance into a $\Delta$-constraints)**
A local maintenance obligation $O^m\langle \pi, \tau, \delta \rangle$ can be parametrised into $\check{M}\Delta(\pi, \tau, \delta)$, which checks for a trace where $\pi$ holds when a task having $\tau$ annotated is executed, and it is not retracted before a task having $\delta$ annotated is executed. Such a trace, if it exists, satisfies the given local maintenance obligation. To avoid confusion we refer to this mapping as $\check{M}\Delta(\pi, \tau, \delta)$.

The two adapted parametrisation $\check{A}\Delta(\pi, \tau, \delta)$ and $\check{M}\Delta(\pi, \tau, \delta)$ can be verified using the pattern approach as outlined in Section 4.1.2. With the adapted parametrisation, satisfying the new constraints represents that there exists a trace in the process model satisfying the in force instance of the local obligation for the specific trigger task being considered during the evaluation. Thus, when the analysis for a trigger task cannot satisfy any of the patterns, we can label the trigger task as not executable, as not being able to satisfy any pattern means that there is no trace in the model satisfying the in force instance of the local obligation for this trigger task.

After executing the patterns approach, and assigning the not executable labels, we can determine whether the process model is partially compliant as follows:

- Apply an erase procedure\[17\] to the tasks labelled not executable. The effect of this procedure is removing executions from the model containing these not executable tasks.

- If the process block associated to the model is not removed by the erase procedure, then there exists a trace of the process model executing only trigger tasks whose in force intervals are satisfiable. For Theorem 5.3, this is a sufficient condition to determine that the process model contains a trace satisfying the obligation, hence the process model can be said to be partially compliant.

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*16* Recall that $\pi$ represents the requirement of the obligation, $\tau$ its condition, $\delta$ its deadline, and $o$ its type.

*17* Let erase be a procedure that when applied to a process block, it removes the block from the process and is recursively applied to its parent process block. This procedure stops without removing the process block when applied to a type XOR.
**Theorem 5.3. (Instance Independence)**

Given a process model $P$, and an obligation $O^o = \mathcal{O}(\pi, \tau, \delta)$. Let $T$ be the set of tasks in $P$ containing $\tau$ in their annotation. If for each task in $T$, there exists a trace in $P$ satisfying the in force instance of $O$ there originating from that task, then there exists a trace in $P$ compliant with $O$.

Before proceeding with proving the correctness of Theorem 5.3, we first introduce the notion of restricted compliance.

**Definition 5.4. (Restricted Compliance)**

Given a process model $P$, and an obligation $O^o = \mathcal{O}(\pi, \tau, \delta)$, let $T = \{t_1, \ldots, t_n\}$ be a set of tasks in $P$ such that each of those tasks contain the trigger $\tau$ of $O$ in its annotation. A trace $\theta$ of $P$ is restricted compliant with $O$ with respect to $T$ (denoted $\theta \models_{T} O$), iff $\forall t_i \in T \cap \theta$, $\theta$ satisfies the in force instance of $O$ originating from $t_i$.

From Definition 5.4, a trace restricted compliant with respect to a task not included in the trace is considered to comply with it, which follows directly from Definition 3.1 and Definition 3.2. Note that the borderline case where a trace does not contain any of the tasks specified in the set, is considered to be restricted compliant with them. We use the concept of restricted compliance in the proof of Theorem 5.3, Corollary 5.6, and Lemma 5.5.

**Proof:**

We prove the correctness of Theorem 5.3 by induction:

**Base Case** ($n = 2$): given two tasks $t_1, t_2$ in a process model $P$ satisfying the trigger of the obligation $O$, there exists a trace $\theta$ of $P$ such that $\theta \models_{\{t_1, t_2\}} O$. The correctness of the base case follows directly from Lemma 5.5.

**Induction Step** ($n > 2$): given a set of tasks $t_1, \ldots, t_n$ in a process model $P$ satisfying the trigger of the obligation $O$, there exists a trace $\theta$ of $P$ such that $\theta \models_{\{t_1, \ldots, t_n\}} O$. Considering now the case $n + 1$ where the task $t_{n+1}$ is considered, Corollary 5.6 ensures the existence of a trace $\theta'$ of $P$ such that $\theta' \models_{\{t_1, \ldots, t_n, t_{n+1}\}} O$.

Since both the base case and the inductive step have been proved as true, by mathematical induction, Theorem 5.3 is then correct.

**Lemma 5.5. (Pair Independence)**

Given a process model $P$ and an obligation $O^o = \mathcal{O}(\pi, \tau, \delta)$, for every pair of tasks $t_1, t_2$ in $P$, where the annotations of $t_1, t_2$ satisfy the trigger $\tau$ of $O$, if there exists a trace in $P$ satisfying the in force instance of $O$ activated in $t_1$, then there exists a trace $\theta$ in $P$ such that $\theta \models_{\{t_1, t_2\}} O$.

We prove the correctness of the lemma for each of the two types of obligations: achievement and maintenance.

**Proof:**

**Achievement**

1. From the premise and Definition 3.1 describing the semantics of the obligation, we have that there are two in force instances of $O$, one starting in $t_1$ and one starting in $t_2$.  

2. If these two instances are overlapping, then from Lemma\[2,10\] we know that both are satisfied as they are independently satisfied. Therefore, in this proof we only consider the \textit{not overlapping} case.\[18\]

3. Considering only the literals relevant for obligation $O$, we have 4 possible states worth considering: $(\pi, \delta); (\pi, \neg\delta); (\neg\pi, \delta); (\neg\pi, \neg\delta)$. Notice that we do not need to keep track of the literal $\tau$, as in this case it is only relevant in the annotation of the task known to be triggering the second instance of $O$.

4. The possible states after the first in force instance of $O$ among the 4 considered in (3) are the ones resulting from satisfying the instance, which are: $(\pi, \neg\delta)$ when only the required event occurs, and $(\pi, \delta)$ when both the required event and the deadline occur at the same time.

\begin{itemize}
\item $(\neg\pi, \delta)$: This cannot be true at the end of the first in force instance of $O$, because the in force instance must be satisfied and $\neg\pi$ with $\delta$ means that the in force interval terminated at the deadline without finding the required event, meaning that the instance has not been fulfilled.
\item $(\neg\pi, \neg\delta)$: This state cannot be true at the end of the first instance, as either one of the two events are required to terminate the in force interval in either a satisfying or failing way.
\end{itemize}

5. From the premise we know that the second in force instance of $O$ allows a trace to satisfy it. Given the possible state $(\pi, \neg\delta)$ resulting from the first instance, identified in (4), the second in force instance would be satisfied. Therefore we consider the possible cases which would always allow the second in force instance to be falsified:

(a) The first case we consider requires to execute a task having $\neg\pi$ annotated before the execution of $t_2$. This would allow the second in force instance not to be immediately achieved. Additionally, no task with $\pi$ annotated must be executed within the in force instance before its termination.

\begin{itemize}
\item Considering the case where a task having $\neg\pi$ annotated is executed before $t_2$, if this would be unavoidable, and no tasks having $\pi$ annotated can be executed after $t_2$, then the second in force instance would not allow to be satisfied by any trace of the model, contradicting the premise.
\end{itemize}

The only case allowing a task having $\neg\pi$ annotated being unavoidable between the execution of the first in force instance and the second, is that $t_1$ is contained in an XOR block. However this case would allow a trace avoiding $t_1$, therefore avoiding the task having $\neg\pi$ annotated. Moreover from the premise we know that one of such traces avoiding $t_1$ must satisfy the second in force instance.

Finally we can conclude that in this case both instances can be considered satisfied by a single trace, as a trace avoiding to activate an in force instance of an obligation by avoiding its trigger task, it is considered to be restricted compliant with it.

\[18\]Remember that an in force instance for an achievement obligation is considered the interval between the trigger and the state where the obligation is achieved, or between the trigger and the state where the deadline is achieved, failing the obligation.
(b) The second case we consider requires to execute a task having $\neg \pi$ annotated before the execution of $t_2$, and requires the execution of a task having $\delta$ annotated before the execution of $t_2$. This would allow to violate the second in force instance of $O$ as soon as $t_2$ is executed.

- If the tasks having respectively $\neg \pi$ and $\delta$ annotated would be unavoidable after the execution of $t_1$, then in the case when also $t_1$ would be unavoidable in every trace, we could conclude that no trace would satisfy the in force instance of $O$ in $t_2$. However this contradicts the premise that such a trace exists.

Therefore, we can assume that $t_1$ is avoidable, meaning that it is in an XOR block. From this it follows that there exists a trace avoiding $t_1$, and therefore avoiding the two tasks having respectively $\neg \pi$ and $\delta$ annotated that would have been unavoidable if $t_1$ would have been executed. Moreover, from the premise we know that one of such traces avoiding $t_1$ must satisfy the second in force instance.

Finally, we can conclude that in this case there exists a trace restricted compliant with the two trigger tasks.

6. From the premise we know that the second in force instance of $O$ allows a trace to satisfy it. Given the possible state $(\pi, \delta)$ resulting from the first instance, identified in (4), the second in force instance would be satisfied. Therefore, we consider the possible cases which would always allow the second in force instance to be falsified:

(a) The case allowing the second in force instance to not be satisfiable would require to have an unavoidable task executable between $t_1$ and $t_2$ that have $\neg \pi$ annotated. However, such case can be reduced to the case discussed in (5).(b), which means that there would be a trace avoiding the first in force instance and satisfying the second.

Similarly, consider the second possibility where also a task having $\neg \delta$ must be executed between $t_1$ and $t_2$, and after $t_2$ no executable tasks can achieve $\pi$. This case can be reduced to the one discussed in (5).(a), which means that there would be a trace avoiding the first in force instance and satisfying the second. As such, for this case there exists a trace restricted compliant with both trigger tasks.

7. Having shown that every possible scenario falsifying the theorem also falsifies its premises, we can then conclude that the theorem is correct for the case where the obligation is of the achievement type.

\[ \square \]

**Proof:**

**Maintenance**

1. From the premise and Definition [3.2] describing the semantics of the obligation, we have that there are two in force instances of $O$, one starting in $t_1$ and one starting in $t_2$.

2. If these two instances are overlapping, then from Lemma [2.10] we know that both are satisfied as they are independently satisfied. Therefore, in this proof we only consider the *not overlapping* case.
3. Considering only the literals relevant for obligation $O$, we have 4 possible states worth considering: $(\pi, \delta); (\pi, \neg\delta); (\neg\pi, \delta); (\neg\pi, \neg\delta)$. Notice that we do not need to keep track of the literal $\tau$, as in this case it is only relevant in the annotation of the task known to be triggering the second instance of $O$.

4. The only possible state after the first in force instance of $O$ among the 4 considered in (3) is $(\pi, \delta)$, as from the premise the instance must be satisfied.

- $(\pi, \neg\delta)$ and $(\neg\pi, \neg\delta)$: These states cannot be true at the end of the first in force instance of $O$, because $\delta$ is required to terminate the instance.

- $(\neg\pi, \delta)$: This cannot be true at the end of the first in force instance of $O$, because the in force instance must be satisfied and $\neg\pi$ would prevent that according to Definition 3.2 describing the semantics of the obligation.

5. From the premise we know that the second in force instance of $O$ allows a trace to satisfy it. Given the state from (4) the second in force instance would be satisfied. Therefore we consider the possible cases which would always allow the second in force instance to be falsified:

(a) The first case we consider is where a $\neg\pi$ must always be executed within the second in force instance after $t_2$ and before any $\delta$ is added to the process state concluding the instance. This allows to falsify the instance in each of its traces.

- This cannot be the case as it would falsify the premise that the model contains at least a trace satisfying the second in force instance of the obligation.

(b) The second case allowing only traces falsifying the second in force instance is one where a task having $\neg\pi$ annotated must be executed before the second in force instance is activated, and no other task with $\pi$ annotated can be executed before the activation.

- If this case would always be required to be executed in the model, then it would go against the premise that the second in force instance allows a trace satisfying it.

- Therefore this case must be avoidable. A first possible way is having the task having $\neg\pi$ annotated being in an XOR block. However this would make such task avoidable, meaning that there would be a trace satisfying the second in force instance.

- The second and last possible way is that the task $t_1$ activating the first in force instance is within an XOR block with the task having $\neg\pi$ annotated.

  - This would allow to have the task with $\neg\pi$ annotated being unavoidable after the execution of $t_1$ and the activation of the first in force instance.

  - However this structure would allow a trace avoiding both $t_1$ and the task having $\neg\pi$ annotated. Such trace be restricted compliant with both trigger tasks.

6. Having shown that every possible scenario falsifying the theorem also falsifies its premises, we can then conclude that the theorem is correct for the case where the obligation is of the maintenance type. 

☐
Corollary 5.6. (Transitive Independence)
Given a process model \( P \), and an obligation \( O : O^\rho(\pi, \tau, \delta) \), let \( T : t_1, \ldots, t_n \) be a set of tasks in \( P \) such that each of those tasks contain the trigger \( \tau \) of \( O \) in its annotation. Let \( \Theta \) be a non empty set of traces of \( P \) such that \( \forall \theta \in \Theta, \theta \models_T O \).

Given a task \( t_{n+1} \) containing the trigger \( \tau \) of \( O \) in its annotation, and not already contained in \( T \), if there exists a not empty set \( \Theta' \) containing some traces of \( P \) such that \( \forall \theta' \in \Theta', \theta' \models_{t_{n+1}} O \), then there must exist a set \( \Theta'' \) containing some traces of \( P \) such that \( \forall \theta'' \in \Theta'', \epsilon'' \models_{T \cup t_{n+1}} O \).

Proof:
We prove the correctness of Theorem 5.6 by contradiction. Let us assume that the set \( \Theta'' \) is empty. We identify the properties that must hold for \( \Theta'' \) to be empty by analysing the following complementary subsets of \( \Theta \):

1. Let \( \Theta^0 \) be a subset of \( \Theta \) containing the traces of \( P \) not containing any of the tasks in \( T \).
   
   (a) If \( \Theta^0 \) is not empty, then for \( \Theta'' \) to be empty, the task \( t_{n+1} \) must be included in each of the traces of \( P \). If this would not be the case, then there would be a trace not containing any task in \( T \) and also not containing \( t_{n+1} \), which would make this trace restricted compliant with \( O \) with respect to \( T \cup t_{n+1} \), which in turn would mean that \( \Theta'' \) is not empty (I).
      
      i. For the case where some traces of \( P \) contain exclusively \( t_{n+1} \) and none of the other tasks in \( T \), such traces must not satisfy the in force instance of \( O \), otherwise \( \Theta'' \) would not be empty. The case where some traces of \( P \) contain \( t_{n+1} \) and some of the other tasks in \( T \) is analysed in 2.(a) of this proof.
   
   (b) If \( \Theta^0 \) is empty, then there cannot be a trace avoiding every task in \( T \cup t_{n+1} \), and as analysed in 1.(a).i of this proof, the traces of \( P \) containing exclusively \( t_{n+1} \) must not be restricted compliant with respect to \( t_{n+1} \), and traces also containing some of the other tasks in \( T \) is analysed in 2.(a) of this proof.

2. Let \( \Theta^n \) be a subset of \( \Theta \) containing the traces of \( P \) that contain a non empty subset of the tasks in \( T \) and \( \forall \theta \in \Theta^n \), it is still true that \( \theta \models_T O \).
   
   (a) If \( \Theta^n \) is not empty, then \( \forall \theta \in \Theta^n \): \( \epsilon \) must include \( t_{n+1} \), otherwise such a trace in \( \Theta^n \) not including \( t_{n+1} \) would be restricted compliant with \( O \) with respect to \( T \cup t_{n+1} \), and each of these traces containing \( t_{n+1} \) must also not be restricted compliant with \( O \) with respect to \( T \cup t_{n+1} \) (\( \theta \not\models_{T \cup t_{n+1}} O \)). We analyse the requirements for this to be the case below in two distinct cases:
      
      • When \( t_{n+1} \) is executed in a trace before every \( t \in T \), for such case to fail, either a task being executed after the in force instance originating from \( t_{n+1} \) leads to violations in the following in force instances, or a task executing before or during the in force instance originating in \( t_{n+1} \) leads to violating it. The first option cannot be the case, as in that scenario the trace would not belong to \( \Theta^n \) as it would not be restricted compliant with \( O \) with respect to \( T \). Meaning that the only possible case is the second option (II).
• When $t_{n+1}$ is executed in a trace after one or more $t \in T$, for such case to fail a task executed before or within the in force instance originating from $t_{n+1}$ must lead to its violation. These cases are: a task containing the negation of the requirement being executed within the interval, or before it and not retracted in case of a maintenance obligation, and a task containing the negation of the requirement being executed before the instance and the lack of a task containing the requirement being executed within the instance (III).

(b) If $\Theta^n$ is empty, then it means that $\Theta^0$ must not be empty for $\Theta$ to contain a trace restricted compliant with $O$ with respect to $T$. Hence the same analysis as the one shown in 1.(a) in this proof applies here.

Following from the analysis above, we can see that (II) leads to the cases analysed in Lemma 5.5 which concluded that the in force instances originating from the second task could not be satisfied by the traces of the model, in this case leading to $\Theta'$ to be empty.

Similarly, when considering (I) and (III) together, they also imply that the requirements for $\Theta''$ to be empty, also require $\Theta'$ to be empty, which again contradicts the premises of the theorem.

\[\square\]

5.6. Discussion on the Computational Complexity

The computational complexity results for the variants of problem when partial compliance is being checked are summarised in Figure 8.

![Figure 8: Partial Compliance Complexity Lattice.](image)

From a first glance at the results, it is noticeable that only 2 of the variants are solvable in $P$ when partial compliance is being checked, while 4 of the variants are solvable in $P$ when full compliance is being checked. Similarly for checking full compliance, also for the problem of checking partial compliance using obligations whose elements are allowed to be represented using formulae is again an element driving the computational complexity of the problem in the $NP$ space. Additionally, from
Figure 8 we can also observe that having multiple obligations instead of a single one seems to be sufficient to drive the computational complexity in the NP space.

5.6.1. Relations with Proving Non Compliance

Having provided the computational complexity results for the variants of the problem of proving both full and partial compliance of process models, we can extrapolate the computational complexities of proving non compliance of the same variants. Proving non compliance of a process model is the complement of proving its partial compliance, as it requires to verify that none of the traces comply with the given regulations. Figure 9 illustrates this graphically by showing that the set of partially compliant processes are disjoint from the non compliant ones. Moreover the illustration also shows that fully compliant process models are also partially compliant, as they always contain at least a compliant trace.19

![Compliance relations](image)

Figure 9: Compliance relations.

6. Conclusion

In this paper, we have collected existing computational complexity results for some versions of the problem of proving regulatory compliance of process models, and provided some new computational complexity results.

6.1. Complexity of Proving Full Compliance

Considering the computational complexity results for proving full compliance, shown in Figure 7, it can be observed that the variants are evenly split between the P and coNP-complete complexity classes. In particular, it can be observed that the variants of the problem allowing formulae to represent the elements of the obligations (represented by a + in the variants acronyms) fall into the computational complexity class coNP-complete while the other variants, limiting the expressivity of the elements composing the obligations to propositional literals, are in P.

19Considering how we define process models, it cannot be the case that a model contains no traces, hence for this particular problem we can say that a for all implies an exists, which is not always the case in classical logic.
Allowing formulae to represent the elements composing the obligations being checked has the obvious advantage that such obligations are more expressive, but at a computational complexity cost. When obligations are restricted to have their elements being represented by propositional literals, it is possible to determine exactly which tasks’ executions would satisfy each of the elements composing such obligations. This knowledge allows to determine whether the process model being studied is compliant by analysing its structure directly, avoiding the burden to have to study its traces explicitly. Having to explicitly study the traces of a model has the disadvantage that the space required to be studied increases by a combinatorial factor, as a process model composed by a number of tasks can potentially allow a number of traces that is combinatorial with respect to the number of tasks. This is the primary reason why these problems are not solvable in polynomial time when formulae are allowed in the elements of obligations.

6.2. Complexity of Proving Partial Compliance

When we consider Figure 8 containing the computational complexity results for proving partial compliance of process models, it can be observed that 6 out of 8 of the variants belong to the computational complexity class \( \text{NP}\)-complete, and only 2 of the variants are in \( \text{P}\).

In the same way as for the computational complexity results for checking full compliance, when formulae are allowed in the elements of the obligations, then the problem is not solvable efficiently anymore, and in this case falls under the computational complexity class \( \text{NP}\)-complete. In addition, when proving partial compliance of process models, it can be noticed from the computational complexity results that moving from a regulatory framework containing one obligation to one containing many (represented by an \( n \) in the variants acronyms), it is a sufficient condition to drive the computational complexity of the problem to the \( \text{NP}\)-complete space.

When a regulatory framework is composed of multiple obligations, a trace of a process model is required to satisfy each obligation in the framework in order for the model to be considered partially compliant. Unfortunately, this requirement negates the capability of verifying compliance by directly analysing the structure of the process model, requiring in turn to analyse the individual traces to identify one satisfying each obligation. The difference between the problem of proving full compliance and proving partial compliance, is that the former can be solved by solving its complement, i.e. looking for a trace including a violation for one of the obligations in the regulatory framework. Unfortunately, this is not feasible when checking for partial compliance, as checking its complement would require to prove that every trace contains at least a violation, leading to potentially having to check every trace of the model. This is also the very reason why when checking full compliance of process models, some variants can actually be solved efficiently while their partial compliance checking counterparts cannot.

6.3. Complexity of Proving Non Compliance

We have shown that proving non compliance is the complement of proving partial compliance, graphically illustrated in Figure 9. This results in 2 variants that are solvable efficiently, while the remaining 6 variants (the one including multiple obligations, or the ones adopting formulae in the elements of the obligations) are in the \( \text{coNP}\)-complete computational complexity class.
6.4. Discussion on the Variants

We analysed the computational complexity of some variants of the problem of proving regulatory compliance of process models. These variants are determined by a combination of three binary features, where each feature has an easy and a difficult option.

We opted to study the computational complexity of these variants as the considered features allow a fair degree of expressivity when the more complex binary features are selected, meaning that the more difficult variants of the problem studied can represent a fair amount of real world problems.

While even when considering the more complex variants, some additional features may be desirable to capture more convoluted real world problems, we opted to restrict our study as these variants allow to identify a clear division between the problems that can be efficiently solved and the ones that cannot. Therefore, while more features could have been included in the analysis to provide a broader coverage or real world scenarios, the current selection allowed a more focused analysis capable of identifying this computational complexity border capable of separating easier problems from the intractable ones.

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