A Low-Complexity Near-Maximum-Likelihood Parallel Detection Algorithm with Imperfect Channel State Information

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Abstract. The existing parallel detectors can approach the performance of the maximum likelihood detector (MLD). But, the required computational complexity is very huge for QAM with large constellation size. In this letter, a novel parallel detector, which focuses on the process of channel gain matrix instead of received signals, is proposed. The complex analysis shows that the new parallel detector’s computational complexity at least reduces an order than typical parallel detectors’ and its detection performance still keep near optimal even with imperfect channel state information.

1. Introduction

In MIMO communication systems, the maximum likelihood (ML) detection algorithm is optimal. Unfortunately, its complexity grows exponentially as, where M is the number of points in the signal constellation and NT is the numbers of transmitting antennas. Therefore, it is hard to be realized in the practical MIMO communication systems with large numbers of antennas [1].

In order to achieve the near optimal detection performance and keep a lower computational complexity compared with MLD, many parallel detection algorithms are proposed in [2]-[7]. The classical parallel detectors include Noise predictive partial Decision feedback (NP-PDF) detector [5][6] and generalized parallel interference cancellation (GPIC) detector [7]. Because these parallel detectors perform the parallel detection process on the received signals, it is hard for them to present a satisfactory trade-off between performance and complexity [8]-[9].

In this letter, in order to solve above problem, a novel parallel detector, which is called lattice reduction aided linear parallel detector (LRALPD), is proposed. In LRALPD, the main process is performed on channel gain matrix. The received signal over the sub-channel with largest noise impact is first detected using MLD. The new channel gain matrix composed of the remained sub-channels is transformed into more orthogonal matrix. Then the rest received signals are detected with the new channel matrix through linear parallel detector. The complexity analysis show that the required computational complexity of LRALPD can be significantly reduced especially for 64QAM MIMO system. The bit error ratio (BER) performance simulation shows that LRALPD can approach the performance of MLD for 64QAM MIMO systems with imperfect CSI when the variance of CSI estimation error is 0.01[10]. In this letter, superscript †, T and H denote the pseudoinvers, transpose and conjugate transpose of matrix respectively. Denotes the 2-norm of vectors.
2. System Model

2.1. MIMO System Model
Consider a MIMO wireless communication system that employs $N_t$ transmitting antennas and $N_r$ receiving antennas ($N_r \geq N_t$) can be expressed as

$$y = Hx + n = \sum_{i=1}^{N_t} h_i x_i + n$$  \hspace{1cm} (1)

where $y = [y_1, y_2, \cdots, y_{N_r}]^T \in \mathbb{C}^{N_r}$, $x = [x_1, x_2, \cdots, x_{N_t}]^T \in \mathbb{C}^{N_t}$, $n = [n_1, n_2, \cdots, n_{N_r}]^T \in \mathbb{C}^{N_r}$, $H \in \mathbb{C}^{N_r \times N_t}$ and $h_i$ is the $i$-th column of $H$. The elements of $H$ are independent and identically distributed (i.i.d.) with zero-mean unit-variance complex Gaussian distribution for a flat fading channel. The elements of $n$ is complex additive white Gaussian noise with variance $\sigma_n^2$. Suppose that $E\{nn^H\} = \sigma_n^2 I_{N_r}$ and $E\{xx^H\} = I_{N_t}$. We assume a flat fading environment, where $H$ is constant over a frame and changes independently from the current frame to the next.

2.2. Imperfect CSI Model
Assume that $H_0$ represents the perfect CSI. The imperfect CSI due to channel estimation is modeled as $N_r \times N_t$ additive error matrix $\Delta H$, and the entries of $\Delta H$ are also assumed to be i.i.d. with zero-mean $\sigma_\Delta^2$-variance. The channel matrix at the receiver denoted by $H$ can be expressed as

$$H = H_0 + \Delta H$$  \hspace{1cm} (2)

2.3. Complex-valued Lattices
A complex-valued lattice of rank $m$ is defined as [7]

$$L = \{a \mid a = \sum_{i=1}^{N_r} z_i b_i, \ z_i \in \mathbb{Z}_j\}$$  \hspace{1cm} (3)

where $b_i \in \mathbb{C}^*$, and $b_i$ is a complex vector. $\mathbb{Z}_j = \mathbb{Z} + j\mathbb{Z}$ denotes the set of complex integers.

3. The Lattice Reduction Aided Linear Parallel Detector
In this section, the proposed new parallel detector is presented. It includes four parts.

3.1. The Preprocess of Channel Matrix
The original channel gain $H$ is first partition into $H_1$ and $H_2$. In LRALPD, it is different from the channel partition of the existing parallel detection algorithms that $H_1$ is composed of only one column of $H$ and $H_2$ is composed of $N_r - 1$ columns of $H$. For the detection of received signals over $H_1$, the MLD is used. Let $W$ denotes the nulling matrix of $H$, and then the received vectors can be decoded as

$$s = Wy = W \cdot Hx + W \cdot n$$  \hspace{1cm} (4)

where $W = W^H (WW^H)^{-1}$.

The pseudoinverse of a $N_r \times N_t$ matrix needs $3/2 N_t^2 N_r$ multiplications and $3/2 N_t^2 N_r - 1/2 N_t^2 - N_r N_t$ additions [6]. Let $n^* = W \cdot n$, we have

$$E[n^* (n^*)^H] = \sigma_n^2 WW^H$$  \hspace{1cm} (5)

that is
where $n'_i$ is the $i$-th entries of $n'$ and $w_i$ is the $i$-th row vector of $W$. From (4) to (6), we can conclude that the $i$-th received signal is impacted seriously by the noise if $\|w_i\|$ is max. The index of the worst sub-channel is

$$i = \arg \max_{j \in \{1, 2, \ldots, N_r\}} \|w_j\|^2$$

then $H_1 = h_i$ and $H_2$ is composed of the remained column vectors.

### 3.2. Precancellation Process

After $H_1$ is determined, ML detection method is used to generate the first group of candidate received symbols. Let $s_M = [s_1, s_2, \ldots, s_M]^T$ denotes a vector, which is transmitted over $H_1$, composed of all the symbols in the constellation of M-QAM and then we have

$$s_i = h_i \cdot s_M$$

then, the interference due to the first group of symbols can be cancelled. Subtract $s_i$ from the received signals, that is

$$Y_{\text{row}} = Y^\text{M} - s_i$$

where $Y^M$ is a $N_s \times M$ matrix composed of $M$ vector $y$, that is

$$Y^M = \begin{bmatrix} y & \ldots & y \end{bmatrix}_{M}$$

### 3.3. Linear Parallel Detection Process Based on ZF Criterion

It is well known that the smaller the condition number of $H$ is, the better the performance of a detection algorithm is. If the channel matrix $H$ that has a large condition number could be transformed into a new matrix $\hat{H}_1$ that has a smaller condition number, the performance of a detection algorithm could be improved.

The columns of $H_1$ can be regarded as a set of basis. Then lattice reduction algorithm is applied to transform the matrix $H_2$ into a new matrix $\hat{H}_2$ that is much better conditioned than $H_2$. The relationship of $\hat{H}_2$ and $H_2$ can be defined as

$$\hat{H}_2 = H_2 \cdot T$$

where $T$ is unimodular matrix and contains only complex integers. The complex LLL algorithm in (11) requires $(N_r - 1)^3 N_s \log(N_r - 1)$ complex operations (including multiplication and addition) [8].

According to (11), The nulling matrix $W_2$ can be computed as

$$W_2 = (\hat{H}_2)^T = \hat{H}_2^* \left[\hat{H}_2 (\hat{H}_2)^T\right]^T$$

then, we can obtain a matrix $\tilde{Z}$, which is composed of the received signal vectors over the $H_2$.

$$\tilde{Z} = W_2 \cdot Y_{\text{row}}$$

The signal vectors in $\tilde{Z}$ is a shifted and scaled version from original QAM constellation. So, it is needed to shift the entries back to original constellation, that is
where \( Q(\cdot) \) denotes the quantization and round operation \([9]\). Hard decision operation is performed to obtain a matrix \( S_{ZF} \) that is composed of candidate symbol vectors. The final candidate signal vectors can be generated by combining \( s_k \) and \( S_{ZF} \), that is

\[
\tilde{S} = \begin{bmatrix} s_k \\ S_{ZF} \end{bmatrix}
\]

### 3.4. Determining the position \( k \)

The matrix \( \tilde{S} \) contains \( M \) candidate signal vectors. The position \( k \) of final decision result can be computed by

\[
k = \arg\min_{k=1,2, \cdots, M} \|Y_{row} - H_k S_{ZF} \|
\]

After \( k \) is determined, the final decision result can be selected from matrix \( \tilde{S} \). The final result \( \hat{s} \) can be expressed as

\[
\hat{s} = \begin{bmatrix} s_k^i \\ S_{ZF}^k \end{bmatrix}
\]

where \( s_k^i \) is the \( k \)-th element of \( s_k \) and \( S_{ZF}^k \) is the \( k \)-th column vector of matrix \( S_{ZF} \).

### 4. Complexity and Performance

#### 4.1. Complexity Analysis

The purpose of this letter is to provide a parallel detection algorithm with near-optimal detection performance, with low complexity and simpler architecture. In this section, the complexity of the proposed is summarized in Table I. The number of complex multiplications and additions required by the proposed algorithm measures the complexity.

| Multiplications | Additions |
|-----------------|-----------|
| A \( 3/2 N_t^2 N_r \) | \( 1/2( N_t -1)^2 \) \( N_r -1/2 N_t^2 - N_r (N_r +1) \) |
| B \( N_t \times M \) | \( (N_r -1) \times M \) |
| C \( 3/2(N_t-1)^2 N_r \) | \( 5/4( N_t -1)^2 (N_r +1)- N_t ( N_r +1)^2 \) |
| \( 1/2 N_t (N_r +2) M \) | \( 3/2 N_t (N_r -1)(M +2) \) |
| \( (N_t-1)(N_r-1) M \) | \( (N_t -1)( N_r -2) M \) |
| D \( (N_t-1)(N_r-1) M \) | \( 3/4( N_t -1) N_r M + (N_t +1) M \) |
| \( 1/2 M N_r \) | \( 1/2 M (N_r -1) \) |

The complexity curves for 64QAM are shown in Figure 1. For comparison, the complexity curves of NP-PDF algorithm \([4]\) \([5]\) and GPIC algorithm \([6]\) are also show in Figure 2. The performance of
GPIC algorithm is affected by the parameters $N$ and $L$, where $N$ is columns of $\mathbf{H}$, and $L = N_T - N$. The larger $N$ is, the better the performance of GPIC algorithm is. The complexity of GPIC algorithm is denoted by $G(N, L)$ Figure 2. Similarly, the performance of NP-PDF algorithm, which is denoted by NP ($N$), is affected by the parameter $N$. In Figure 2, we can see that our proposed algorithm is the lowest with 64QAM. The required complex operations are no more than $10^8$ with $N_T \leq 8$ for $N_T \leq 5$ for 64QAM.

It is noted that the complexity of LLL algorithm in fourth line of Table I is an upper bound. A large number of complex operations occur only when the original channel matrix $\mathbf{H}$ is poorly conditioned [7]. In practice, the required complex operations are less than the theoretical value.

4.2. Performance Analysis

In this subsection, the bit error rate (BER) performance of LRALPD and MLD is show through the numeric simulation. For a 64QAM MIMO system with $N_T = N_R = 4$, there is 644 combinations in all. The complexity is too high. So we only simulated a 64QAM MIMO system with $N_T = N_R = 3$. In Fig. 4, for perfect and imperfect CSI, we can observe that our proposed detection algorithm approach the performance of the MLD algorithm.

![Figure 1. Complexity of LRALPD, GPIC and NP-PDF for 64QAM.](image)

![Figure 2. BER performance of LRALPD and MLD with perfect and imperfect CSI for 64QAM.](image)

5. Conclusion

In this letter, a low complexity parallel detection algorithm with near-optimal detection is proposed. Only ZF criterion is employed in this algorithm. It is associated with lattice reduction to reduce the computational effort significantly for which the constellation size $M$ of QAM is large and approach the performance of the MLD algorithm. The simulation results show the required complex operations of proposed algorithm approach $10^3$ with $N_T = 4$. For a 64QAM MIMO system, the required complex operations are less than $10^4$ with $N_T \leq 5$. And the BER performance is still approach the MLD with perfect and imperfect CSI.

6. References

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