Dynamics Analysis of CBR Reducer with Tooth Modification

XiaoXiao Sun\(^1\), Liang Han\(^1,^a\)

\(^1\)School of Mechanical Engineering, Southeast University, Nanjing, P.R. China

\(^a\)melhan@seu.edu.cn

Abstract. China Bearing Reducer (CBR) is a one stage new type cycloid drive reducer which has large transmission ratio, high payload, high torsional stiffness, high tilting stiffness, compact size. In this paper, the cycloid gear with tooth modification of CBR reducer was investigated to analyse the contact force and stress. Firstly, the mechanical design of CBR reducer was introduced. Then, the profile equation of cycloid gear was derived by inverted gear train method. And the modified profile equation was derived by adding a small increment into the profile equation. Secondly, the dynamic model was built by multi-dynamics theory. Thirdly, the number of simultaneous contact teeth, contact force, stress with respect to modification value were investigated by dynamics software. The results show that the larger is the clearance between the cycloid teeth and pin, the larger are the contact force and stress.

1 Introduction

The cycloid drive has the advantages of large reducer ratio, high torque, high precision, high stiffness and compact structure. It has been widely used in precision reducer, such as RV reducer. In this paper, the CBR reducer is introduced and the dynamics characteristics are analysed.

Many researches had been done by scholars on cycloid-pin gear reducer. Yang and Blanche [1,2] discussed the formulas of cycloid drive and investigated the effect of machining tolerances on backlash and torque ripple. Then they presented an analytical and computer-aided analysis and synthesis of cycloid drives. Teruaki Hidaka et al. [3] proposed a method by using equivalent dynamic model with equivalent error to analyze rotational transmission error. Subsequently, the effects of machining and assembly errors of elements on the rotational transmission error were investigated [4]. And the mutual effects of error of the elements on rotational transmission error were analyzed [5]. Xin Li et al. [6] proposed a double crank ring-plate-type cycloid drive which does not need output unit and places no limit on the size of the tumbler bearing, and working principles, advantages and design issues were discussed. Ta-Shi Lai [7] presented a cycloid drive with roller drive, and the equation of geometric design and meshing is derived. Joong-Ho Shin and Soon-Man Kwo [8] proposed an approach to derive cycloid profile by means of the principle of the instant velocity center. The epicycloid and hypocycloid were both discussed. Anh Duc Pham et al. [9] presented an efficient FE analysis procedure for the hysteresis characteristics of a cycloid reducer using a nonlinear spring with a dead zone. Zhong-Yi Ren et al. [10] proposed a new tooth modification method of cycloid disc by adjusting the position of 5 key points.

The aim of the paper is to investigate the contact force of cycloidal gear reducer. It will be expected to analyze the load sharing among the multiple contact tooth pairs, the distribution and variation of the contact stress. Three different modification values for force analysis of the CBR reducer are presented in the paper.
2 Design and profile equation of cycloid gear

2.1 Mechanical design
CBR reducer is a new type of cycloid gear reducer, which adopts a symmetrical transmission structure. As shown in Fig. 1, it is mainly composed of bearing end cover, outputting flange, cross roller bearing, crank bearing, disc connector, turning bearing, crank shaft, pins, cycloid gear, case, outputting bearing and input support. Compared with RV reducer, it has the advantages of simple structure, small number of parts, simple assembly process and low manufacturing cost.

Because of its compact structure, it can achieve the installation dimension of the harmonic drive reducer. So, it can be used in robot joints with space restricted instead of harmonic reducer, and its payload, torsional stiffness and instantaneous impact resistance are several times higher than that of the harmonic drive reducer.

At the same time, in order to improve the transmission accuracy, we designed a disc connector between the output flange and the cycloid gear. Through the disc connector, the differential motion produced by the cycloid gear and the pins is transferred to the output flange. As shown in Fig. 2, there are two pairs of rounded rectangular holes on the disc connector, which are connected with the boss of the cycloid gear and the boss of the output flange respectively. In order to improve the transmission efficiency, rollers are placed between rounded rectangular hole and boss of cycloid gear and output flange to avoid sliding friction.

The parameters of CBR25 are shown in Table 1, we will analyse this type in later section.

![Figure 1. Exploded view of CBR Reducer.](image1)

![Figure 2. Assembly of transmission parts.](image2)

The parameters of CBR25 are shown in Table 1, we will analyse this type in later section.

| Parameter                          | Value   |
|------------------------------------|---------|
| Radius of pin position $r_p$       | 29.6    |
| Pin radius $r_{pp}$                | 0.975   |
| Teeth number of Pins $z_p$         | 50      |
| Teeth number cycloid gear $z_c$    | 49      |
| Transmission ratio $i$             | 49      |
| Eccentricity $a$                   | 0.462   |
| Short width coefficient $K_1$      | 0.7804  |
| Tooth width $w_d$                  | 6.3     |

2.2 Tooth profile with modification
The analysis method of inverted gear train coordinate systems (IGTCS) is used to establish standard profile equation. As shown in Fig. 3, an overall fixed coordinate system $S_f (X_fO_fY_f)$ and a moving coordinate system...
Sp(Xp−Oc−Yp) fixed to the pin gear are established in the center of the pin gear. And a moving coordinate system Sm(Xm−Om−Ym) is established in the center of the cycloid gear. In the initial position, the Yp axis coincides with the Yf axis, and the Yc axis is parallel to the Yf axis. In the IGTS, the cycloid gear rotates \( \phi_c \) at the angular speed \( w_c \) counter clockwise around the point \( O_c \) (that is, \( Sc \) rotates \( \phi_c \) around \( O_c \)). According to the relative motion relation, the pin gear will rotate \( \phi_p \) at the angular speed \( w_p \) counter clockwise around the point \( O_p \) (that is, \( Sp \) rotates \( \phi_p \) around \( O_p \)). So, we can get

\[
i_H^m = \frac{w_c}{w_p} = \frac{\phi_c}{\phi_p} = \frac{z_c}{z_p}
\]

(1)

Where \( i_H^m \) is the transmission ratio of the cycloid gear and the pin gear. \( zc \) is the number of teeth of the cycloidal gear and \( zp \) is the number of teeth of pins.

![Figure 3. Standard tooth profile of cycloid disc.](image)

Based on the theory of gear meshing and differential geometry, the standard cycloid profile in \( Sc \) can be expressed as

\[
\begin{align*}
x_c &= \left( R_p - R_{rp} S_p \right) \cos \left[ (1 - i_H^m) \phi_p \right] - \left( a - K_1 R_{rp} S_p \right) \cos \left( i_H^m \phi_p \right) \\
y_c &= \left( R_p - R_{rp} S_p \right) \sin \left[ (1 - i_H^m) \phi_p \right] + \left( a - K_1 R_{rp} S_p \right) \cos \left( i_H^m \phi_p \right)
\end{align*}
\]

(2)

Where, \( R_p \) is the radius of pin gear and \( R_{rp} \) is the radius of the pin, \( a \) is the eccentricity of the input crank shaft, \( K_1 \) is short width coefficient, \( K_1 = a z_p / R_p S_p = 1 + K_r^2 - 2 K_1 \cos \phi \).

To compensate for the errors caused by assembly and manufacturing, it is necessary to modify the cycloidal gear profile. Three ways of modification are commonly used: isometric modification (modifying the pin radius \( R_{rp} \)), offset modification (modifying the pin gear radius \( R_p \)) and angle rotation modification (modifying the rotation angle \( \phi \) of cycloid gear). It is often the compound use of these three methods, especially the compound uses of offset modification and isometric modification. The cycloidal gear profile with modifications can be obtained by adding a small increment of the pin center position \( \Delta R_{rp} \) to \( R_{rp} \), a small increment of the pin radius \( \Delta r_p \) to \( r_p \) and a small increment of the cycloid disc rotational angle \( \delta \) to \( \phi_c \) in (2). The equation for the modified profile can be written as (3).

\[
\begin{align*}
x_c' &= \left( R_p + \Delta R_p \right) - \left( R_{rp} + \Delta R_{rp} S_p \right) \cos \left[ (1 - i_H^m) \phi_p - \delta \right] - \\
&\frac{a}{\rho_p + \Delta r_p} \left( R_{rp} + \Delta R_{rp} \right) - z_p \left( (R_p + \Delta R_p) S_p \right) \cos \left[ i_H^m \phi_p + \delta \right]
\end{align*}
\]

\[
\begin{align*}
y_c' &= \left( R_p + \Delta R_p \right) - \left( R_{rp} + \Delta R_{rp} S_p \right) \sin \left[ (1 - i_H^m) \phi_p - \delta \right] + \\
&\frac{a}{\rho_p + \Delta r_p} \left( R_{rp} + \Delta R_{rp} \right) - z_p \left( (R_p + \Delta R_p) S_p \right) \sin \left[ i_H^m \phi_p + \delta \right]
\end{align*}
\]

(3)

3 Dynamic modelling
Tooth modification will affect the dynamic performance of cycloidal speed reducer. In this paper, neglecting the stiffness of the bearing, an eight degrees of freedom (DOF) discrete dynamic model of CBR reducer is established as shown in Fig. 4. As illustrated in Fig. 4, the structure of CBR reducer is symmetrical, so we can build the dynamic model with half of it, just four DOFs. Each component is represented by a rigid disc of radius $R$ and polar mass moment of inertia $J$. The cycloid disc mesh is represented by two sets of periodically time–varying mesh spring–damping elements with $K_m(t)$ and $C_m(t)$. The crank shaft and cycloid disc mesh is represented by two sets of spring damping elements with $K_p$ and $C_p$. The cycloid disc and disc connector mesh is represented by two sets of spring damping elements with $K_b$ and $C_b$. The contact of disc connector and carrier is represented by two sets of spring damping elements with $K_a$ and $C_a$.

With the positive directions of the alternating rotational displacements $\theta_s$, $\theta_c$, $\theta_d$, $\theta_{ca}$ and the constant external torques $T_{in}$ and $T_o$ are defined in Fig. 4, the equations of motion of the cycloidal speed reducer are written as:

$$
\begin{align*}
J_d\ddot{\theta}_o + \left[K_p(\theta_o - \theta'_o) + C_p(\theta_o - \theta'_o)\right] \cdot \alpha &= T_{in} \\
J_c\ddot{\theta}_c + \sum_{i=1}^{n} \left[K_m(\theta_{ci} - \theta_o) + C_m(\theta_{ci} - \theta_o)\right] \cdot L_i &= \\
\left[K_p(\theta_o - \theta'_o) + C_p(\theta_o - \theta'_o)\right] \cdot R_1 \ + \\
\left[K_p(\theta_o - \theta'_o) + C_p(\theta_o - \theta'_o)\right] \cdot R_2
\end{align*}
$$

(4)

where, $\theta_o - \theta'_o$ is the torsional angle of crank shaft, $\theta_o - \theta'_o$ is the relative displacement of cycloid disc and crank shaft, and $\theta_o - \theta'_o$ is the torsional angle of carrier. Subscript $i$ represents the $i$-th cycloid disc meshing tooth pair. $n$ is the total number of cycloid disc meshing tooth pair. $L_i$ is the arm of cycloidal tooth force as shown in Fig. 3.

4 Results and discussions

![Figure 5. Movement curve of input shaft.](image-url)
4.1 Conditions of the Dynamic Simulation
In order to analyze the influence of the modification on the dynamic performance of the CBR reducer, we use the modules of motion and simulation of SolidWorks to analyze the contact force and stress of the CBR reducer in this paper. To study the influence of the clearance between the teeth on the performance, a single isometric modification method is adopted because the normal gap between the teeth generated by isometric modification is equal. The number of simultaneous contact teeth, contact force and stress are analyzed under modification values of 10 \( \mu m \), 20 \( \mu m \) and 30 \( \mu m \), respectively. The rotational speed of input shaft is 60 rpm (360 deg/s). The given movement curve for the input crank shaft is as shown in Fig. 5. The acceleration phase occurs between 0 and 1 s, and the velocity is constant after 1 s. The gravitational acceleration is 9.8 \( m/s^2 \). The parts are made of alloy steel, and the relevant mechanical properties are as listed in Table 2.

| Mechanical properties | Values (Unit: \( \mu m^2 \)) |
|-----------------------|-----------------------------|
| Elastic modulus       | 210,000                     |
| Shear modulus         | 79,000                      |
| Tensile strength      | 210,000                     |
| Yield strength        | 79,000                      |

4.2 Contact force of each tooth
The forces acting on each tooth of the cycloidal gear are different when input crank shaft rotates at different degree. In order to express the force size of each tooth at different degree vividly, the force vector is used to express the direction and magnitude of each tooth. As shown in Fig.6, the force is expressed when the input crank shaft rotate at 0 deg, 72 deg, 144 deg, 216 deg, 288 deg and 360 deg with the load torque of 100 N (in SolidWorks motion software). It can be seen from the diagram that the force acting lines of each tooth are all along the common normal line of the meshing line, pointing to the same point which is the internal tangential point of pitch circle the pins and the cycloid disc.

4.3 Number of simultaneous contact teeth influenced by tooth modification
All the cycloid gear teeth are in contact with the pin teeth in the theoretical profile without modification. After the modification, the cycloid profile and the pins are not satisfied with the conjugate condition. Only one cycloid tooth is contact with pin after a small angle of rotation of cycloid gear. With the increase of load torque, the contact deformation between cycloid teeth and pin teeth is increased, and the number of contact teeth increases gradually. The number of contact teeth corresponding to different modification values are shown in the Fig.7. It can be seen from the diagram that under the same load torque, the larger the modification value is, the less the contact tooth number is. This is because the increase of the modification value causes the increase of the gap between the teeth, and the same size of the load torque cannot make the cycloid tooth produce large contact deformation to offset the gap.
4.4 Contact force influenced by tooth modification
Not considering the deformation, only one pair tooth contact with tooth modification, so the cycloid gear will produce enormous contact force. The actual situation is that the cycloid gear and the pins will have deformation. According to the size of the deformation and the Hertz contact theory, there will be different contact forces between the cycloidal teeth and the pin teeth. The first contact tooth has the maximum force, the last contact tooth has the minimum force, and the uncontacted teeth have no forces. The forces of different modification are shown in Fig. 8 with the load torque of 100 N.

4.5 Stress of cycloid gear influenced by tooth modification
The contact stress between teeth can be calculated by finite element method (FEM). SolidWorks simulation is the FEM module, and it can connect seamlessly with the motion module. The constraints and the force of the analysis result of the motion module can be imported into the simulation module for transient dynamic analysis. Here, the contact stress of cycloid gear is the focus. We only mesh the cycloid gear and calculate its stress. The meshing structure of cycloid gear is shown in Fig. 9, where the number of elements are 32007 and the number of nodes are 65321. In Fig. 10, the stress cloud of cycloid gear is shown in the same load torque with different modification values. We can see that the stress with no modification is smallest, the stress with 30 μm modification is largest.
5 Conclusion
This article presents the design and dynamics simulation of CBR reducer. CBR is a one-stage reducer with a compact structure and a wide range of installation size. The modified profile equation of cycloid gear was derived and the multibody dynamic equation was built. Different modification values were used to build CAD model for dynamics analysis to investigate the number of simultaneous contact teeth, contact force, contact stress with respect to modification value. The simulation results showed that the large gap between the teeth will cause large force and stress. So a suitable gap can better balance the contact force and thermal expansion of CBR reducer.

Acknowledgment
This project is supported by Science and Technology Program of Jiangsu Province (Grant No. SBE2015000030), the Fundamental Research Funds for the Central Universities and the Research Innovation Program for College Graduates of Jiangsu Province (Grant No. KYLX16_0187). These supports are gracefully acknowledged.

References
[1] J. G. Blanche and D. C. H. Yang, “Cycloid drives with machining tolerances,” J. Mech. Transm. Autom. Des., vol. 111, no. 3, pp. 337–344, (1989).
[2] D. C. H. Yang and J. G. Blanche, “Design and application guidelines for cycloid drives with machining tolerances,” Mech. Mach. Theory, vol. 25, no. 5, pp. 487–501, (1990).
[3] T. Hidaka, H. Y. Wang, T. Ishida, K. Matsumoto, and M. Hashimoto, “Rotational transmission error of k-h-v planetary gears with cycloid gear : 1st report, analytical method of the rotational transmission error,” Trans. Jpn. Soc. Mech. Eng. Ser.C, vol. 60, no. 570, pp. 645–653, (1994).
[4] T. Ishida, H. Wang, T. Hidaka, and M. Hashimoto, “Rotational transmission error of k-h-v-type planetary gears with cycloid gears. 2nd report, effects of manufacturing and assembly errors on rotational transmission error.” Trans. Jpn. Soc. Mech. Eng. Ser. C, vol. 60, no. 578, pp. 3510–3517, (1994).

[5] H. Wang, T. Ishida, T. Hidaka, and M. Hashimoto, “Rotational transmission error of k-h-v-type planetary gears with cycloid gear : 3rd report, mutual effects of errors of the elements on the rotational transmission error,” Trans. Jpn. Soc. Mech. Eng. Ser. C, vol. 60, no. 578, pp. 3518–3525, (1994).

[6] X. Li, W. He, L. Li, and L. C. Schmidt, “A new cycloid drive with high-load capacity and high efficiency,” J. Mech. Des., vol. 126, no. 4, pp. 683–686, (2004).

[7] T. S. Lai, “Geometric design of roller drives with cylindrical meshing elements,” Mech. Mach. Theory, vol. 40, no. 1, pp. 55–67, (2005).

[8] J. H. Shin and S. M. Kwon, “On the lobe profile design in a cycloid reducer using instant velocity center,” Mech. Mach. Theory, vol. 41, no. 5, pp. 596–616, (2006).

[9] A. D. Pham, T. L. Tran, and H. J. Ahn, “Hysteresis curve analysis of a cycloid reducer using non-linear spring with a dead zone,” Int. J. Precis. Eng. Manuf., vol. 18, no. 3, pp. 375–380, (2017).

[10] Z. Y. Ren, S. M. Mao, W. C. Guo, and Z. Guo, “Tooth modification and dynamic performance of the cycloidal drive,” Mech. Syst. Signal Process., vol. 85, pp. 857–866, (2017).

[11] K. S. Lin, K. Y. Chan, and J. J. Lee, “Kinematic error analysis and tolerance allocation of cycloidal gear reducers,” Mech. Mach. Theory, vol. 124, pp. 73–91, (2018).