Multi-objective bi-level photovoltaic access planning based on copula theory and included angle cosine method

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Abstract. A multi-objective stochastic bi-level programming method for PV access planning is proposed by cooperating both the Copula theory and included angle cosine method. In order to consider the effect of dependent uncertain irradiance on distribution network operations, the scenarios for correlated uncertain irradiance are obtained by combining the Copula theory and Latin hypercube sampling. Then, to coordinate the distribution network loss and voltage violation, included angle cosine method is applied to determine the weight of different objective functions in the programming model. In addition, the optimal power flow model of the radial distribution network is approximated by second-order cone programming losslessly. The case study verifies the feasibility of the proposed model.

1. Introduction

Solar energy has been widely considered as one of the most promising renewable resources and has experienced rapid development in recent decades. The proliferation of photovoltaic units (PV) variously benefits distribution networks, mainly involving voltage profiles improvement and losses reduction [1]. Hence, there are intrinsic motivations for full exploitation of PV. In light of the advent of PV, the techniques for planning, designing and operating distribution networks have changed profoundly.

Research on PV access planning mainly focuses on the effect of the uncertain power of PVs to distribution system operations. Reference [2] analyzed the three-phase unbalanced degree due to the PV power output. A K-means Clustering Analysis method was proposed to produce scenarios of PV output for choosing the PV locations in [3]. PV access planning is optimized for improving the grid reinforcement in [4]. However, the correlation between the PV power output of different buses in a distribution network is ignored [2-4]. Considering the correlation of uncertainties in engineering applications, the copula theory [5] is a powerful tool to model the dependent uncertainties [6], which provides the information about the degree of dependence between the PVs, as well as the dependence structure. It can also handle linear and non-linear, flexible, dependence relationships. In addition, it is also unconstrained to marginal distribution (MD) type of PV power output.

Besides, because the distribution network loss and voltage violation are two different factors considered in the planning, multi-objective programming method is usually used in such problems. The methodology for solving the multi-objective programming can be divided into two categories:
Pareto-based heuristic algorithm [7] and the weighted sum approach [8]. The former is computationally expensive and may not achieve the best solution. The latter converts the multi-objective optimization into a single-objective optimization by weighting the different objectives. Because the objective weighted sum approaches are performed with a large given data, it is not suitable for the model whose solution is unknown. Included angle cosine method [9] is one of the subjective weighted sum approaches and applied widely due to its accuracy.

In this paper, a multi-objective stochastic bi-level programming model for PV access planning is proposed. Copula theory and included angle cosine method are employed to transform the stochastic model into a single-objective determined programming model. The feasibility of the proposed method is verified in the IEEE 33-bus distribution network.

2. Problem formulation

2.1. Upper-level optimization model

The upper-level optimization model aims to find the optimal allocation of the total PV capacity at different buses in a distribution network.

The objective function and constraints of the upper-level are the following.

\[
\min C^T X
\]
\[\text{s.t. } V^T X = S\]
\[\underline{x}_i \leq x_i \leq \bar{x}_i, i = 1, \ldots, n\]

(1)

Where \(X\) is the vector whose elements are \(x_i\) denoting the PV capacity at the \(i\)-th bus, and \(S\) represents the total PV capacity to be allocated in the PV access planning. In addition, \(\underline{x}_i\) and \(\bar{x}_i\) denote the PV capacity limits at the \(i\)-th bus. For the sake of brevity, the cost vector of PV array at different buses is denoted by \(C\).

2.2. Lower-level optimization model

The lower level optimization model aims to coordinate the distribution network loss and voltage violations considering the dependent uncertain power output of PVs at different buses, which is a multi-objective stochastic bi-level optimization problem.

The objective function is the following.

\[
\min \left\{ E\left( P_{\text{loss}} \right) + \alpha E\left( \sum_{i=1}^{n} (V_i - V_{\text{ref},i})^2 \right) \right\}
\]

\[
(2)
\]

Where \(E\) denotes the expected value due to the uncertain irradiance, \(P_{\text{loss}}\) is the distribution network loss, \(V_i\) and \(V_{\text{ref},i}\) are bus voltage magnitude and voltage reference value at the \(i\)-th bus, respectively. \(\alpha\) and \(\beta\) are the weight coefficients of the two objective functions to be determined.

The constraints of the lower level optimization model are voltage magnitude constraints and line power flow constraints, respectively.

\[V_i \leq V_i \leq \bar{V}_i, i = 1, \ldots, n\]

\[P_i \leq \bar{P}_i, i = 1, \ldots, l\]

(3)

(4)

Other constraints are the power flow equations.

\[P_i = U_i \sum_{j=1}^{n} U_j \left( G_{ij} \cos \theta_j + B_{ij} \sin \theta_j \right)\]

\[Q_i = U_i \sum_{j=1}^{n} U_j \left( G_{ij} \sin \theta_j - B_{ij} \cos \theta_j \right)\]

(5)

The active power and reactive power at each bus are determined by the PVs capacity and loads, which connects the upper-level model and the lower level model.
3. Solution methodology

By discretizing the objective function at the lower level, (2) can be reformulated as follows.

\[ \frac{1}{m} \sum_{j=1}^{m} (\alpha P_{\text{lös},j} + \beta \sum_{i=1}^{n} (V_{i,j} - V_{\text{ref},j})^2) \]  \hspace{1cm} (6)

Where \( j \) denotes the \( j \)-th scenario, so that the expected value of (2) is calculated by the mean of the \( m \) scenarios. Then, the objective function of the bi-level optimization model is formulated as follows.

\[ \min C^* X + \frac{1}{m} \sum_{j=1}^{m} (\alpha P_{\text{lös},j} + \beta \sum_{i=1}^{n} (V_{i,j} - V_{\text{ref},j})^2) \]  \hspace{1cm} (7)

The scenarios and weight in (6) are derived from the following approaches.

3.1. Copula theory

The irradiance data in a radial distribution network are correlated so that the uncertain power output of PVs at different buses are dependent. To model the dependence structure of uncertain PV power output of different buses the copula theory is applied in this paper.

According to the Sklar theorem [10], if \( F_1(w_1) \) and \( F_2(w_2) \) are marginal cumulative distribution function of the two random variables \( w_1 \) and \( w_2 \), the joint cumulative distribution function is \( F_{ij}(w_1, w_2) \), then there exists a Copula function \( C \) in the following:

\[ F_{ij}(w_1, w_2) = C(F_1(w_1), F_2(w_2)) \]  \hspace{1cm} (8)

By selecting a proper Copula function \( C \) according to the statistical data of random variables, the dependence structure of multi random variables can be modelled accurately.

In this paper, Copula function is selected by the distance to the statistical data of irradiance in the distribution network and the scenarios can be obtained with the function in MATLAB.

3.2. Latin hypercube sampling

The Latin hypercube sampling (LHS) is a type of stratified sampling technique that has been used in many research articles [11-14]. The main advantage of the LHS is a higher sampling efficiency and robustness compared to the traditional Monte Carlo technique (Although, it is noticed that some modified Monte Carlo methods may be more efficient than the traditional one [15-16]).

In this paper, LHS is used to select a proper number of valuable scenarios from those obtained with Copula function.

3.3. Included angle cosine method

Included angle cosine method uses the samples and the angle cosine of the deviation between the optimal value vector and the worse value vector to determine the weight of each objective function. The smaller is the angle, the larger is the cosine value, and the greater will be the weight. The task consists of the following steps:

Firstly, set the optimal value vector of objective functions denoted as \( S^* = (S_1^*, S_2^*, \ldots, S_m^*) \), and the worse value vector \( S_* = (S_1^*, S_2^*, \ldots, S_m^*) \), where \( S_i^* \) and \( S_{i*} \) are formulated as follows:

\[ S_i^* = \begin{cases} \max_{1 \leq j \leq n} \{a_{ij}\} & i \in I_1 \\ \min_{1 \leq j \leq n} \{a_{ij}\} & i \in I_2 \end{cases} \]  \hspace{1cm} (9)

\[ S_{i*} = \begin{cases} \min_{1 \leq j \leq n} \{a_{ij}\} & i \in I_1 \\ \max_{1 \leq j \leq n} \{a_{ij}\} & i \in I_2 \end{cases} \]  \hspace{1cm} (10)

Where, \( I_1 \) denotes the profit index and \( I_2 \) denotes the cost index. \( a_{ij} \) is the element of sample matrix, which denotes the distribution network loss or voltage violation of the \( j \)-th scenario (\( i = 1 \) or 2).
Because the value of $a_{ij}$ is not calculated before the upper-level optimization is solved, $a_{ij}$ is estimated by an expert system like those in analytic hierarchy process.

Secondly, calculate the relative deviation matrix $R = (r_{ij})_{m \times n}$ of $S^*$ and $\Delta = (\delta_{ij})_{m \times n}$ of $S^*$ according to samples, where $r_{ij}$ and $\delta_{ij}$ are denoted by the following equations.

$$r_{ij} = \frac{|S_j^*-a_{ij}|}{\max_j \{a_{ij}\} - \min_j \{a_{ij}\}}$$  \hspace{1cm} (11)$$

$$\delta_{ij} = \frac{|S_j^*-a_{ij}|}{\max_j \{a_{ij}\} - \min_j \{a_{ij}\}}$$  \hspace{1cm} (12)$$

Thirdly, calculate the weight coefficients $\omega_k$. Before that, calculate the included angle cosine between the $i$-th row vector of $R = (r_{ij})_{m \times n}$ and the responding row vector of $\Delta = (\delta_{ij})_{m \times n}$:

$$v_i = \cos(r_i, \delta_i) = \frac{\sum_{j=1}^{n} (r_{ij} \cdot \delta_{ij})}{\sqrt{\sum_{j=1}^{n} r_{ij}^2 \cdot \sum_{j=1}^{n} \delta_{ij}^2}}$$ \hspace{1cm} (13)$$

By normalizing $v_i$, weight coefficients for different objective functions are obtained as follows.

$$\omega_k = \frac{v_k}{\sum_{i=1}^{n} v_i}$$ \hspace{1cm} (14)$$

In this paper, included angle cosine method is used to determine the value of $\alpha$ and $\beta$.

3.4. Second-order cone programming for approximating the power flow model

The power flow equations in rectangular form for radial distribution networks can be approximated by second-order cone losslessly [17]. With the second-order cone-based power flow model, the multi-objective bi-level optimization model proposed in this paper is transformed into a determined convex optimization model, which can be handily solved by the off-the-shelf solver.

4. Case study

In this section, case studies are applied to the IEEE 33-bus distribution system to validate the effectiveness of the proposed model and solution methodology. The numerical experiments are conducted in MATLAB R2014a on a personal computer with an Intel Core (i7, 2.80GHz) and 8GB memory. The Cplex toolbox is invoked to solve the optimization model.

There are 9 buses in the IEEE 33-bus system to integrate PVs (seen in Figure 1), namely, 4, 8, 14, 18, 21, 24, 25, 30, 32. The maximum capacity of PVs in each bus is 6 MW, and the total capacity of all buses does not exceed 19 MW. The power factor is assumed to be 0.8 lagging for all PVs. The voltage range in the distribution network is 0.93~1.07 p.u. [3]. Solar irradiance data obtained from the Baseline Measurement System of the Solar Radiation Research Laboratory of the National Renewable Energy Laboratory (NREL) is employed [18].

![Figure 1. The structure of 33-bus distribution system.](image-url)
4.1. Scenario analysis
The case selects four typical days all the year round. For each typical day, the dependent PV power output data of different buses are used to perform the Copula theory. In details, marginal PDF of PV power output of each bus is described as Beta distribution, and the Gaussian copula function is selected to model the joint probability distribution of the PV power output of all the buses to be chosen. 5000 scenarios are generated from Copula theory with the function in MATLAB. Then, the number of scenarios is reduced to 800 by using the Latin hypercube sampling.

4.2. Weight of different object functions
The weight of the two objective functions in the lower level model is calculated by the included angle cosine method proposed in this paper. Firstly, the optimal value vector and the worse value vector of two objective functions are estimated as (9) and (10). Secondly, calculate the relative deviation matrix defined as (11) and (12). At last, calculate the weight coefficients defined as (13) and weight of the two object functions are obtained after normalizing the included angle cosine as (14). It is noticed that the weight is determined by a particular expert system, which is common in subjective weighted sum approaches. In this case, the weight of the two objective functions is (0.7, 0.3) due to that network losses are concerned more than the voltage violation.

4.3. PV access planning results
By solving the multi-objective bi-level optimization proposed in this paper, PV capacity of each bus with different weighting results are obtained. The results are shown in Figure 2, where PV capacity of bus 24 and 25 are always much larger than those of other buses, in the contrast, the PV capacity of bus 14 is the smallest when the weight result is (0.3, 0.7) and that of bus 30 is the smallest when the weight result is (0.7, 0.3). The results arise from the fact that the weighted mean value of voltage violation and distribution network loss of the whole distribution system increase much less when PV capacity of bus 24 and 25 increase (the mean network losses of the two cases are 0.2 and 0.12, respectively, and the mean of voltage violations are 0.08 and 0.1, respectively). However, PV capacity of bus 14 will give rise to more voltage violation while the PV capacity of bus 14 will increase much less distribution network loss of the whole distribution system.

![Figure 2. PV Capacity at each bus.](image)

Furthermore, when the PV capacity of the distribution networks increases, the voltage magnitude of the buses will decrease generally, so the voltage violations in the objective function are caused mainly by that voltage magnitude decreases which also helps to reduce the network losses. The weight, (0.7, 0.3), can correspond to the above fact that focusing on the network losses is more worthwhile.
5. Conclusions
The proposed multi-objective bi-level PV access planning model is a powerful tool to evaluate the PV capacity considering the effect of dependent uncertain PV output power on the distribution network operations by cooperating the Copula theory and included angle cosine method. From the above case studies, the integrated PV capacity is much larger at the buses where fewer network losses and voltage violations will be added to the distribution system when the PV capacity increases. The proposed method can also apply to other stochastic programming problem with different objective functions considering the dependent uncertainties.

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References
[1] D Remon, A M Cantarellas, J M Mauricio, P Rodriguez 2017 Power system stability analysis under increasing penetration of photovoltaic power plants with synchronous power controllers[J] IET Renewable Power Generation 11 733-741
[2] He Ting, et al 2017 Optimal planning of distributed photovoltaic considering three-phase unbalanced degree[C] Industrial Electronics Society, IECON 2017-43rd Annual Conference of the IEEE IEEE
[3] Lu, Siqi, Xiaorong Wang, Junyong Wu 2018 [C] IOP Conference Series: Earth and Environmental Science 108(5) IOP Publishing
[4] Bibin, Huang, Li Qionghui, Gao Fei 2017 [J] Energy Procedia 105 427-432
[5] Nelsen R B 2007 An introduction to copulas[M] Springer Science & Business Media
[6] G Papaefthymiou, D Kurowicka 2009 Using Copulas for Modeling Stochastic Dependence in Power System Uncertainty Analysis[J] IEEE Transactions on Power Systems 24 40-49
[7] C -J Ye, M -X Huang 2015 Multi-objective optimal power flow considering transient stability based on parallel NSGA-II[J] IEEE Transactions on Power Systems 30 857-866
[8] Marler, R Timothy, Jasbir S Arora 2010 The weighted sum method for multi-objective optimization: new insights[J] Structural and multidisciplinary optimization 41(6) 853-862
[9] Wang H, HE Y, HOU Z, et al 2009 Chaotic local adding-weight linear forecasting algorithm based on included angle cosine[J] High Voltage Engineering 6 044
[10] Rüschendorf L 2009 On the distributional transform, Sklar's theorem, and the empirical copula process[J] Journal of Statistical Planning and Inference 139(11) 3921-3927
[11] Schmidt R, Voigt M, Mailach R 2019 Latin Hypercube Sampling-Based Monte Carlo Simulation: Extension of the Sample Size and Correlation Control[M]//Uncertainty Management for Robust Industrial Design in Aeronautics Springer, Cham 279-289
[12] Vikram V, Roy H 2017 Hierarchical Latin Hypercube Sampling[J] Journal of the American Statistical Association
[13] Bayraksan G 2018 [J] Operations Research Letters 46(2) 173-178
[14] J C Helton, F J Davis 2003 [J] Reliability Engineering & System Safety 81 23-69
[15] A Bernardini, P Maffezzoni, L Daniel, A Sarti 2018 [J] IEEE Transactions on Circuits and Systems I: Regular Papers 65(4) 1363-1376
[16] Guido Carpinelli, Pierluigi Caramia, Pietro Varilone 2015 [J] Renewable Energy 76 283-295, ISSN 0960-1481
[17] L Gan, N Li, U Topcu, S H Low 2012 Exact convex relaxation of optimal power flow in tree networks arXiv preprint arXiv 1208 4076
[18] Solar radiation research laboratory of NREL [Online] Available: https://midcdmz nrel.gov/srrl rsp2/