Revisiting Analytical Models of N-Type Symmetric Double-Gate MOSFETs

Rekib Uddin Ahmed and Prabir Saha

Abstract—Nowadays, the endlessly increasing demand for faster and complex integrated circuits (IC) has been fuelled by the scaling of metal-oxide-semiconductor field-effect-transistors (MOSFET) to smaller dimensions. The continued scaling of MOSFETs approaches its physical limits due to short-channel effects (SCE). Double-gate (DG) MOSFET is one of the promising alternatives as it offers better immunity towards SCEs and can be scaled to the shortest channel length. In future, ICs can be designed using DG-CMOS technology for which mathematical models depicting the electrical characteristics of the DG MOSFETs are foremost needed. In this paper, a review on n-type symmetric DG MOSFET models has been presented based on the analyses of electrostatic potential distribution, threshold voltage, and drain-current models. Mathematical derivations of the device models are described elaborately, and numerical simulations are also carried out to validate the replicability of models.

Index Terms—Analytical modeling, drain-current, n-type DG MOSFETs, potential distribution, review, threshold voltage.

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I. INTRODUCTION

For more than five decades, the semiconductor industries have been successful in providing continuous system performance improvement because of the invention of MOSFETs. Prior to this, bulky vacuum tubes were used for systems, but reliability and heat dissipations were major issues [1]. Therefore, researchers tried to realize the vacuum tube in solid-state for which the surface of semiconductors was studied thoroughly. Lilienfeld first reported the idea of enhancing the surface conductance of a semiconductor by application of electric field in 1930, but it was not successful because of the presence of large densities of surface states [2]-[3]. The first MOSFET was fabricated in 1960 by Kahng and Atalla [4] on a silicon substrate using an oxide layer (SiO₂) as the gate insulator. Circuits based on single polarity MOSFETs (either p- or n-type) suffered from large static power dissipation, thereby limited the level of integration in a chip. The breakthrough in the level of integration came in 1963 with the invention of complementary metal-oxide-semiconductor (CMOS) [5]. In CMOS technology, both the n- and p-type MOSFETs were constructed side by side on the same substrate, and are connected in series between the supply terminals, so that there is negligible static power dissipation.

The prediction proclaimed by Moore’s law has been achieved through scaling of MOSFETs. One of the most important parameters of a MOSFET is its channel length (L), defined as the distance between the source and drain. For a given technology, there is a minimum value of L below which the gate starts to lose control of the drain current (I_{ds}). This is because of the physical limits imposed by non-scalability of silicon energy band-gap (E_g), built-in potential (V_{bn}), short-channel effects (SCEs), and thermal voltage (V_T) [6]. Conventionally, MOSFETs were scaled with a scaling factor s, (s \approx 0.7). Scaling by this factor reduces L to L \times s, oxide thickness (t_{ox}) to t_{ox} \times s, while it increases doping concentration (N_{si}) to N_{si}/s [7]. But this technique cannot be continued in the sub-micron regime, because increasing N_{si} gives rise to mobility degradation of carriers and random dopant fluctuation (RDF) [7,8]. Mobility degradation occurs due to large vertical fields induced by high doping [9]. RDF is a form of process variation due to variation in the implanted dopants which alters the transistor’s properties, especially threshold voltage (V_{th}) [10]. So it is utmost important to restore the gate control of the channel without increasing doping concentration of the body. This requirement has led to creating multi-gate (MG) MOSFETs in which body of the device is undoped (or lightly doped). Fig.1 shows some examples of MG MOSFETs where the gate is wrapped around the body from either two or three or four sides.

On decreasing the L, depletion region created by the source and drain encroaches horizontally in the channel, thereby reduces the effective channel length [11]. As the drain-to-source voltage (V_{ds}) increases the depletion region becomes wider. As a result, the channel electrostatics is not only controlled by the gate but also influenced by L and V_{ds}. The observable effects arising due to loss of channel electrostatics controlled by the gate are termed as SCEs. The SCEs include the V_{th} roll-off due to the L reduction, and the drain-induced
barrier lowering (DIBL). These effects cause the $V_{th}$ to decrease upon increasing $V_{ds}$ and also degrades subthreshold slope (SS). Improvement of SCEs by using double-gate architecture was predicted in 1984, which put forward the concept of double-gate (DG) MOSFET [12]. The DG MOSFET is being studied as a key component for future ICs due to its numerous advantages such as excellent gate controllability and improvements in $V_{th}$ roll-off, off-state leakage current and channel length modulation (CLM) effects. The undoped body makes the device immune to RDF, leading to a consistency in the $V_{th}$ from device to device [8]. Due to the undoped body, depletion charge is negligible, which enhances the carrier mobility [13]. The channel inversion takes place throughout the thickness of the body and consequently increases the minority carriers due to which higher current is found [14]. Junction capacitance and mobility degradation are reduced due to which the switching speed of the device is improved [13]. Surface roughness scattering due to lower surface electric field is also reduced because of the undoped body [13,14].

All IC designs, digital or analog or mixed-signal, are verified through the use of circuit simulators before being reproduced in real silicon. For any circuit simulator to predict the performance of the ICs based on DG-CMOS technology, it should have accurate models to describe the behaviour of the constituting DG MOSFETs. The device model is a representation of characteristics or conditions in the device in the form of (a) an equation, (b) an equivalent circuit, and (c) a table, together with the proper reasoning and assumptions. Primary requirements to use a device in the simulators are electrostatic potential distribution ($\phi$) model, $V_{th}$ model, and $I_{ds}$ model. Several such models have been reported so far regarding the modeling of n-type DG MOSFETs [15–45]. A brief review on modeling of DG MOSFETs has been presented in [46,47] but the models for short-channel (nanoscale) regimes have not been considered.

Taur [15] developed a $\phi$ model for long-channel undoped DG MOSFETs where two transcendental equations had to be solved in order to describe the potential distribution in the channel. The need for solving the two equations was removed in the model given by Lu and Taur [16], and thus provided only one equation for potential distribution which in turns required numerical iteration method to get the solution. Hong et al. [17] had proposed the $\phi$ model for a long-channel lightly doped DG MOSFETs by considering the effects of fixed as well as mobile charge carriers. Taur [15] had also given a $V_{th}$ criterion for long-channel DG MOSFETs in which iterative method was used to calculate the $V_{th}$ which was later improved by Chen et al. [18] by proposing a new definition for $V_{th}$. Based on the models [15, 16], Taur et al. [19] had given a $I_{ds}$ model for long-channel DG MOSFETs by considering the effects of fixed as well as mobile charge carriers. Taur [15] had also given a $V_{th}$ criterion for long-channel DG MOSFETs in which iterative method was used to calculate the $V_{th}$ which was later improved by Chen et al. [18] by proposing a new definition for $V_{th}$. Based on the models [15, 16], Taur et al. [19] had given a $I_{ds}$ model for long-channel DG MOSFETs which had three different equations for subthreshold, linear, and saturation regions. Tsarmatzoglou et al. [20] presented the $\phi$ model for short-channel DG MOSFETs based on the parabolic potential approximation method [48] and also presented a semi-analytical model for subthreshold drain

![Fig. 1. Different types of MG MOSFETs](image-url)
II. MODELS FOR LONG CHANNEL DG MOSFETS

A. Electrostatic Potential Models

The electrostatic potential of a long-channel DG MOSFET $\phi(x)$ is one-dimensional (1-D), which is obtained by solving the 1-D Poisson’s equation governing the relationship between electric fields and charges. As shown in Fig. 2, $\phi(x)$ is a function of the distance $x$ from the gate towards the channel. The $\phi(x)$ models including Taur’s [15], and Lu and Taur’s [16] for $L = 1 \mu m$ have been considered for the derivation and analysis of $V_{th}$ and $I_{ds}$ models necessary for designing the complete device model for DG MOSFETs.

1) Taur’s Model [15]:

The $\phi(x)$ model for an undoped n-type DG MOSFET is derived by considering only the mobile charge density. This is a core model for $L = 1 \mu m$ regime obtained by solving the 1-D Poisson’s equation under gradual channel approximation (GCA) [50] assuming Boltzmann statistics for mobile charges. The GCA assumes that variation in lateral electric field much less than the variation in the vertical electric field (along $x$) so that the 2-D Poisson’s equation reduces to 1-D [51]. Finally, the $\phi(x)$ model is expressed as:

$$\phi(x) = \phi_0 - 2V_T \ln \left( \frac{q n_i}{2 \varepsilon_s V_T} \right)^{\frac{\phi_0}{\varepsilon_s k_b T}} e^{-\frac{\phi_0}{k_b T}}$$

where $\phi_0 = \phi(x = 0)$, $V_T$ is the thermal voltage, $n_i$ is the intrinsic charge density, and $\varepsilon_s$ is the dielectric permittivity of silicon. $\phi(x)$ is also defined as the amount of band bending or position of intrinsic potential at $x$ [51]. A similar form of solution (1) was earlier given by Hauser and Littlejohn [52]. Derivation of the model (1) is as follows.

The 1-D Poisson's equation for the silicon region considering only mobile charge density is expressed as:

$$\frac{d^2 \phi(x)}{dx^2} = \frac{q}{\varepsilon_s} n_i e^{-\frac{\phi_0}{k_b T}}$$

where $q$ is the elementary charge, $k_b$ is the Boltzmann constant, and $T$ is the temperature. By interpreting in terms of $d\phi$ and integrating both sides, (2) can be rewritten as:

$$\int_{0}^{\phi(x)} \frac{d\phi}{x} d\left( \frac{d\phi}{dx} \right) = \int_{\phi_0}^{\phi(x)} \frac{q}{\varepsilon_s} n_i e^{-\frac{\phi_0}{k_b T}} d\phi.$$

On solving (3):

$$\left( \frac{d\phi}{dx} \right)^2 = \frac{2k_b T n_i}{\varepsilon_s} e^{-\frac{\phi_0}{k_b T}} - e^{-\frac{\phi_0}{k_b T}}.$$

Integrating both sides of (4):

$$\int_{\phi_0}^{\phi(x)} \frac{d\phi}{\sqrt{e^{-\frac{\phi_0}{k_b T}} - e^{-\frac{\phi_0}{k_b T}}}} = \sqrt{\frac{2k_b T n_i}{\varepsilon_s}} \int_{0}^{x} d\phi.$$

Considering $e^{-\frac{\phi(x)}{k_b T}} - e^{-\frac{\phi_0}{k_b T}} = t$ will imply:

$$e^{-\frac{\phi_0}{k_b T}} = t + e^{-\frac{\phi_0}{k_b T}}.$$

Differentiating (6) with respect to $\phi$:

$$\frac{d}{d\phi} e^{-\frac{\phi_0}{k_b T}} = \frac{dt}{d\phi} = \frac{q}{k_b T} e^{-\frac{\phi_0}{k_b T}}.$$

Rearranging the terms of (7):

$$d\phi = \frac{k_b T}{q} e^{-\frac{\phi_0}{k_b T}} dt = \frac{k_b T}{q} \left( \frac{dt}{1 + e^{-\frac{\phi_0}{k_b T}}} \right).$$

Substituting (8) and (6) in (5) will yield:

$$\int_{0}^{t} \frac{dt}{\sqrt{T \left( t + e^{-\frac{\phi_0}{k_b T}} \right)}} = \sqrt{\frac{2k_b T n_i}{\varepsilon_s k_b T}} \int_{0}^{x} dx.$$

Considering $\sqrt{T} = x$ in (9) and substituting $dt = 2xdx$ in (9) will imply:

$$\phi(x) = \phi_0 - 2V_T \ln \left( \frac{q n_i}{2 \varepsilon_s V_T} \right)^{\frac{\phi_0}{\varepsilon_s k_b T}} e^{-\frac{\phi_0}{k_b T}}$$
\[ f \left( \frac{2dx}{z^2 + e^{\phi_0}} \right) = \sqrt{\frac{2q^2n_i}{\varepsilon_{si}k_B T} x}. \]  

(10)

\[ \frac{2}{q} \frac{\tan^{-1} \left( \frac{x}{\phi_0} e^{2\phi_0/k_B T} \right)}{e^{2\phi_0/k_B T}} = \sqrt{\frac{2q^2n_i}{\varepsilon_{si}k_B T} x}. \]  

(11)

\[ z = e^{2\phi_0/k_B T} \tan \left( \sqrt{\frac{2q^2n_i}{\varepsilon_{si}k_B T} e^{2\phi_0/k_B T} x} \right). \]  

(12)

\[ t = e^{\phi_0/k_B T} \tan^2 \left( \sqrt{\frac{2q^2n_i}{\varepsilon_{si}k_B T} e^{2\phi_0/k_B T} x} \right). \]  

(13)

\[ \frac{q\phi(x)}{e^{k_B T}} = \frac{q\phi_0}{e^{k_B T}} e^{2\phi_0/k_B T} \tan^2 \left( \sqrt{\frac{2q^2n_i}{\varepsilon_{si}k_B T} e^{2\phi_0/k_B T} x} \right). \]  

(14)

\[ \frac{q\phi(x) - \phi_0}{e^{k_B T}} = 2 \ln \left( \tan \left( \sqrt{\frac{2q^2n_i}{\varepsilon_{si}k_B T} e^{2\phi_0/k_B T} x} \right) \right). \]  

(15)

Rearranging terms of (15):

\[ q(\phi(x) - \phi_0) = 2q \ln \left( \tan \left( \sqrt{\frac{2q^2n_i}{\varepsilon_{si}k_B T} e^{2\phi_0/k_B T} x} \right) \right). \]  

(16)

The surface potential at \( x = t_{si}/2 \) in (15) is expressed as:

\[ \phi_s \equiv \phi \left( x = \frac{t_{si}}{2} \right) = \frac{2k_B T}{q} \ln \left( \frac{t_{si}}{2} \sin \left( \beta \frac{t_{si}}{2} \right) \right). \]  

(24)

Equation (23) is the \( \phi(x) \) model given by Lu and Taur which has been taken by many research groups [22–23], [33–34] to model the short-channel DG MOSFET characteristics. Applying boundary condition at silicon-oxide interface:

\[ E_{ox} \frac{d^2 \phi(x)}{dx^2} + q \frac{d\phi(x)}{dx} = \frac{E_s - E_{ox}}{t_{ox}}. \]  

(25)

where \( V_g \) is the applied gate voltage and \( \Delta \chi_{ms} \) is the workfunction difference between the gates and the silicon as shown in Fig. 3. In case of undoped body \( \Delta \chi_{ms} = 0 \) for mid-gap metal gate, \( -E_g/2q \) for n⁺ polysilicon, and \( E_g/2q \) for p⁺ polysilicon. Differentiating (23) with respect to \( x \):

\[ \frac{d\phi}{dx} = \frac{2V_g}{t_{si}} \sqrt{\frac{2\phi_0}{\varepsilon_{si}k_B T}} \sin \left( \frac{\beta t_{si}}{2} \right) \tan \left( \frac{\beta t_{si}}{2} \right). \]  

(26)
\[ V_F = \frac{3}{2} \chi_m \phi_F - \ln \left[ \frac{1}{2} \left( \frac{\xi_{\text{eff}} b p^T}{q^4 n_i} \right) \right] \]

(27)

\[ \ln \beta = \ln \cos \beta + 2\tau \beta \tan \beta, \]

with \( r = \frac{\xi_{\text{eff}} b s}{\xi_{\text{eff}} s} \). The value of \( \beta \) has to be calculated from (27) using numerical iterations like Newton-Raphson method [54]. Numerical iteration and algorithms increase the computation time. Fast and efficient method has to be adopted to make the model suitable for circuit simulation. Yu et al. [25] developed a computation method which eliminated the need for numerical iterations.

3) Hong et al. [17]:

The \( \phi(x) \) models of DG MOSFETs developed in [15, 16] are valid for the undoped silicon body. The work has been extended by Hong et al. [17] by proposed the \( \phi(x) \) model for the lightly doped silicon body with spatially varying doping profiles. The \( \phi(x) \) model derived through solving the 1-D Poisson’s equation considering both the fixed and mobile charge density.

\[ \frac{d^2 \phi(x)}{dx^2} = \frac{q n_i^2}{\epsilon_{\text{si}}} e^{-\frac{\phi(x)}{k_B T}} + q N_{\text{si}}(x) \epsilon_{\text{si}}, \]

(28)

where \( N_{\text{si}}(x) \) is the spatially varying doping distribution in the silicon body (can be continuous or discrete). Consideration of fixed and mobile charge density in a lightly-doped silicon body is required from the accuracy point of view [31,32]. Because, the effect of mobile charge density cannot be neglected in the above subthreshold regime [33] and its inclusion in Poisson’s equation enhances the model accuracy [55]. Substituting \( q N_{\text{si}}(x) = \frac{d^2 g(x)}{dx^2} \), (28) can be written as:

\[ \phi(x) = \frac{k_B T}{q} Z(x) + \phi_F + g(x). \]

(29)

Differentiating (29) twice with respect to \( x \):

\[ \frac{d^2 \phi(x)}{dx^2} = \frac{k_B T}{q} \frac{d^2 Z(x)}{dx^2} + \frac{d^2 g(x)}{dx^2}. \]

(30)

Substituting (30) in (28) will yield:

\[ \frac{k_B T}{q} \frac{d^2 Z(x)}{dx^2} + \frac{d^2 g(x)}{dx^2} = \frac{q n_i^2}{\epsilon_{\text{si}}} e^{-\frac{\phi(x)}{k_B T}} \left[ Z(x)+g(x) \right] + q N_{\text{si}}(x) \epsilon_{\text{si}}, \]

which on solving will yield:

\[ \frac{d^2 Z(x)}{dx^2} = \frac{q n_i^2}{k_B T \epsilon_{\text{si}}} e^{\frac{\phi(x)}{k_B T}} e^{-\frac{\phi(x)}{k_B T}}. \]

(31)

Substituting \( \exp \left[ \frac{q}{k_B T} g(x) \right] / N_{\text{si}} = f(x) \) and \( \frac{q n_i^2}{k_B T \epsilon_{\text{si}}} = \xi \), (31) is re-written as:

\[ \frac{d^2 Z(x)}{dx^2} = \xi e^{Z(x)} f(x). \]

(32)

The terms \( \xi \) and \( f(x) \) in (32) contain the effect of nonlinear coupling between the mobile and fixed charge densities. Presence of the \( f(x) \) makes this modeling scheme unique from the exiting \( \phi(x) \) model [56] for the DG MOSFET. In order to derive the analytical solution for \( \phi(x) \), the (32) (in Cartesian coordinate) is transformed into the cylindrical coordinate.

\[ \frac{d^2 Z_c(t)}{dt^2} + \frac{1}{\tau} \frac{dZ_c(t)}{dt} = \xi e^{Z_c(t)} F(t), \]

(33)

where \( Z_c(t) = Z(x) - 2 \), \( \ln \tau = x \), and \( F(t) = f(\ln \tau) \). In order to solve (32), two new variables are introduced: \( \beta = \tau \frac{dZ_c}{dt} \) and \( \eta = \tau^2 F(t) e^{Z_c(t)} \). Differentiating \( \beta \) with respect to \( \tau \) will yield:

\[ \frac{d\beta}{dt} = \frac{dZ_c(t)}{dt} + \tau \frac{d^2 Z_c(t)}{dt^2}. \]

(34)

Substituting \( \frac{d^2 Z_c(t)}{dt^2} \) from (34) in (33):

\[ \frac{d\beta}{dt} = \tau^2 \xi e^{Z_c(t)} F(t). \]

(35)

Differentiating \( \eta \) with respect to \( \tau \) will yield:

\[ \frac{dn}{d\tau} = 2\tau F(t) e^{Z_c(t)} + \tau^2 F'(t) e^{Z_c(t)} + \tau^2 F(t) e^{Z_c(t)} \frac{dZ_c}{d\tau}. \]

(36)

Substituting \( \beta = \tau \frac{dZ_c}{d\tau} \) and rearranging the terms of (36):

\[ \tau e^{Z_c(t)} F(t) = \frac{dn}{d\tau} \frac{1}{2+\frac{dZ_c}{d\tau}+\beta}. \]

(37)

On substituting (37) in (35) will further transform the (33) to:

\[ d\beta + p(\tau) \beta = \xi d\eta, \]

(38)

where \( p(\tau) = 2 + \tau^2 F'(t) / F(t) \) is the spatial function related to the doping profile, i.e. whether continuous or discrete doping. Equation (38) is integrated to obtain:

\[ \frac{\beta^2}{2} + p(\tau) \beta = h = \xi \eta, \]

(39)

where \( h = -\xi \eta_0 - 2p + 2 \) is an integration constant to be determined from boundary conditions. Substituting \( \beta = \tau \frac{dZ_c}{d\tau} \) and \( \eta = \tau^2 F(t) e^{Z_c(t)} \) in (39) and using (32) will yield:

\[ \frac{d^2 Z_c(t)}{dt^2} \left[ \frac{p(\tau)-1}{\tau} \frac{dZ_c(t)}{dt} - \tau \right] \left( \frac{dZ_c(t)}{dt} \right)^2 + h \tau^2 = 0. \]

(40)

On solving (40), the general solution of 1-D Poisson’s equation can be readily obtained as:

\[ Z_c = -p \ln \tau + A - 2 \ln \left[ \cos \left( \frac{\tau}{2} \sqrt{-(p-2)^2 - 2h} \right) - \frac{B \sqrt{-(p-2)^2 - 2h}}{2h} \right]. \]

(41)

where \( A \) and \( B \) are the integration constants. Here the \( \beta(\tau) \) is approximated as:

\[ \beta(\tau) = -p + \sqrt{-(p-2)^2 - 2h} \]

(42)
Applying boundary condition at the silicon-oxide interface:
\[
e_{ox} \frac{V_g - \Delta \chi_{ms} - \phi_s}{t_{ox}} = e_{sl} \frac{d\phi}{dx}\bigg|_{x = t_{sl}/2}.
\] (47)

Substituting \( \frac{d\phi}{dx}\bigg|_{x = t_{sl}/2} \) from (4) in (47):
\[
e_{ox} \frac{V_g - \Delta \chi_{ms} - \phi_s}{t_{ox}} = \sqrt{2e_{sl}k_bTn_1 \frac{q\phi_s}{e^{\frac{q\phi_s}{k_bT}} - e^{\frac{-q\phi_s}{k_bT}}}}.
\] (48)

\( \phi_s \) is increased with increase in \( V_g \), whereas the center potential \( \phi_0 \) attains a constant value. For greater value of \( V_g \) (more than threshold), the term \( \phi_0 \) in (48) can be neglected which will imply:
\[
e_{ox} \frac{V_g - \Delta \chi_{ms} - \phi_s}{t_{ox}} = \sqrt{2e_{sl}k_bTn_1 e^{\frac{q\phi_s}{k_bT}}}.
\] (49)

Since \( \frac{\phi_0}{t_{ox}} = C_{ox} \), so (49) can be re-written as:
\[
C_{ox}(V_g - \Delta \chi_{ms} - \phi_s) = \sqrt{2e_{sl}k_bTn_1 e^{\frac{q\phi_s}{k_bT}}}
\] (50)

Substituting \( V_g - \Delta \chi_{ms} - \phi_s = \phi_s \) in (50) and on solving:
\[
\phi_s = \frac{2k_bT}{q} \ln \left[ \frac{e_{ox}V_{gt}}{\sqrt{2e_{sl}k_bTn_1}} \right] \] (51)

Since the threshold condition is given by:
\[
V_{th} = \Delta \chi_{ms} + \phi_s
\] (52)

Substituting \( \phi_s \) from (51):
\[
V_{th} = \Delta \chi_{ms} + \frac{2k_bT}{q} \ln \left[ \frac{e_{ox}V_{gt}}{\sqrt{2e_{sl}k_bTn_1}} \right].
\] (53)

The \( V_{th} \) model (53) is a transcendental equation which needs to be solved numerically. The \( \phi_s \) increases with the increase in \( V_g \), and the \( \phi_0 \) asymptotically approach a constant value: \( \phi_{0, max} = (k_bT/q) \ln[2\pi^2 e_{sl}k_bT/q^2 n_{sl}] \) with slope \( 2C_{ox} \). Volume inversion takes place in the subthreshold region and volume inversion, no band bending occurs.

2) Chen et al. [18]:

Chen et al. [18] defined the \( V_{th} \) as the required \( V_g \) at which the inversion charge sheet density \( Q_{inv} \) at minimum potential position (virtual cathode) reaches a value \( Q_{th} \) which is sufficient enough to turn on the device [33]. Fig. 4 shows the threshold condition defined for DG MOSFETs. The effective conductive path is located at \( x = t_{sl}/4 \) from the top and bottom surfaces. The \( V_{th} \) model for the long-channel DG MOSFET is:
\[
V_{th} = \Delta \chi_{ms} + \frac{2k_bT}{q} \ln \left[ \frac{e_{ox}V_{gt}}{\sqrt{2e_{sl}k_bTn_1}} \right].
\] (54)

The value of \( Q_{th} \) is determined as \( 3.2 \times 10^{10} \) cm\(^{-2}\). Similar expression (54) has been deduced by Hamid et al. [33].
C. Drain-Current Models

The $I_{ds}$ models can be broadly classified into potential based and charge based models. In the potential based models, the $I_{ds}$ is expressed through indirect function of applied $V_g$ and $V_{ds}$. Whereas, in charge based models, the $I_{ds}$ is expressed in terms of terminal charges, as an implicit function of $V_g$ and $V_{ds}$.

1) Taur et al. [19]:

The model [19] is a surface potential based model in which $I_{ds}$ is expressed in terms of applied bias. The pre-requisite for the model is electrostatic potential models [15,16]. The drain current expression is:

$$I_{ds} = \mu \frac{w}{L} \left[ \frac{2k_BT}{q} \right]^2 \left[ g_x(\beta_s) - g_x(\beta_d) \right]$$

(55)

where $g_x(\beta) = \left[ \beta \tan \beta - \frac{\beta^2}{2} + 2r_\beta \tan^2 \beta \right]$ with $\beta_s$ and $\beta_d$ are the values of $\beta$ at the source and drain ends respectively. Three different equations have been used for subthreshold, linear, and saturation regions by approximating the values of $\beta$. The $I_{ds}$ model is based on Pau-Sah's double integral, which is based on GCA [50]. The GCA is valid for most regions of MOSFET operation except beyond the pinch-off point. Charge-sheet model [57] is then introduced to obtain the implicit equations for $I_{ds}$ model. The detailed derivation is as follows.

For the long channel devices, the total electron current density is the sum of the drift and diffusion current density [51,58]:

$$J_n(x,y) = qn(x,y)\mu_n E_x + qD_n \frac{dn(x,y)}{dx},$$

(56)

where $E_x = -\frac{\partial \phi(x)}{\partial x}$ is the vertical electric field in the silicon body and $D_n = \mu_n V_T$ is the electron diffusion coefficient [51]. Substituting $E_x$ and $D_n$ in (56):

$$J_n(x,y) = -qn(x,y)\mu_n \frac{\partial \phi(x)}{\partial x} - \frac{k_BT}{q} \frac{dn(x,y)}{dx},$$

(57)

where $n(x,y) = n_i e^{-\frac{\phi(x)}{k_BT}}$ is the electron density. On rearranging the terms of $n(x,y)$:

$$\frac{n(x,y)}{n_i} = e^{-\frac{\phi(x)-\phi_F}{k_BT}},$$

which on solving will yield:

$$\phi(x) - \frac{k_BT}{q} \ln \left[ \frac{n(x,y)}{n_i} \right] = \phi_F.$$  

(58)

Differentiating (58) with respect to $x$:

$$\frac{d\phi(x)}{dx} + \frac{k_BT}{q} \frac{dn(x,y)}{dx} = \frac{d\phi_F}{dy}.$$  

(59)

Substituting (59) in (57):

$$J_n(x,y) = -q\mu_n n(x,y) \frac{d\phi_F}{dy}. \quad \text{(60)}$$

The $I_{ds}$ is expressed in terms of $J_n(x,y)$ [51] as:

$$I_{ds} = \mu_n \left( \frac{2W}{L} \right) \int_0^{V_{ds}} \frac{4\epsilon Si k_BT}{q \tau_{si}} \beta \tan \beta \left[ -2V_T \left( \frac{1}{\beta} + (2r+1) \tan \beta + 2r \beta \sec^2 \beta \right) \right] d\beta.$$  

(70)

Fig. 4. Schematic showing the inversion charge sheet density at threshold condition. (Dashed lines represent the effective conductive path).
Referring to the expression (27):

$$\frac{v_g - \Delta \chi_{ms}}{2v_T} - \ln \left( \frac{2}{\tanh \left( \frac{2v_\phi k_B T}{q^2 n_i} \right)} \right) = \ln \beta - \ln \cos \beta + 2r \beta \tan \beta. \tag{68}$$

Differentiating (68) with respect to $\beta$:

$$d\phi_F = \frac{-2v_T}{\beta} \left( 1 - \frac{2r + 2r \beta \sec^2 \beta}{2r \beta} \right). \tag{69}$$

Substituting (67) and (69) in (63) will yield (shown at the bottom of the previous page):

Changing the integral limit of (70) from $\int_0^{V_{ds}} d\beta$ to $\int_{\beta_s}^{\beta_d} d\beta$:

$$I_{ds} = -\mu_n \left( \frac{2W}{L} \right) \left( \frac{2k_B T}{\epsilon q} \right)^2 \int_{\beta_s}^{\beta_d} \left[ \tan \beta + (2r + 1) \beta \tan^2 \beta + 2r \beta^2 \tan \beta \sec^2 \beta \right] d\beta \tag{71}$$

$$\int_{\beta_s}^{\beta_d} \left[ \tan \beta + (2r + 1) \beta \tan^2 \beta + 2r \beta^2 \tan \beta \sec^2 \beta \right] d\beta$$

There are three integrals to be solved in (71) which are: $\int \tan \beta d\beta$, $\int \beta \tan^2 \beta d\beta$, and $\beta^2 \tan \beta \sec^2 \beta d\beta$. Solution of the integrals are expressed as:

$$\int \tan \beta d\beta = \ln \sec \beta \tag{72}$$

$$\int \beta \tan^2 \beta d\beta = \int (\tan^2 \beta - 1) d\beta = \int \frac{\sec^2 \beta - \sec^2 \beta}{\beta} d\beta = (\ln \sec \beta - \frac{\sec^2 \beta}{2} \frac{\beta}{\beta}) \tag{73}$$

$$\int \beta^2 \tan \beta \sec^2 \beta d\beta = \beta^2 \tan \beta \sec^2 \beta d\beta - \int \left( \frac{d}{d\beta} \beta^2 \tan \beta \sec^2 \beta d\beta \right) d\beta = \frac{1}{2} \beta^2 \tan \beta - \beta \tan \beta + \ln \sec \beta + \frac{\beta^2}{2} \tag{74}$$

Substituting (72-74) in (71) will yield:

$$I_{ds} = \mu_n \left( \frac{2W}{L} \right) \left( \frac{2k_B T}{\epsilon q} \right)^2 \left[ \beta \tan \beta - \frac{\beta^2}{2} + r \beta^2 \tan^2 \beta \right]_{\beta_s}^{\beta_d}. \tag{75}$$

Equating the terms: $\ln \beta - \ln \cos \beta + 2r \beta \tan \beta = f_r(\beta)$ [from (68)] and $\beta \tan \beta - \frac{\beta^2}{2} + r \beta^2 \tan^2 \beta = g_r(\beta)$ [from (55) and (75)]. At source end $\beta = \beta_s$ and $\phi_F = 0 \ V$. So,

$$f_r(\beta_s) = \frac{V_g - \Delta \chi_{ms}}{2v_T} - \ln \left( \frac{2}{\tanh \left( \frac{2v_\phi k_B T}{q^2 n_i} \right)} \right)$$

$$= \frac{V_g - V_0}{2v_T} \tag{76}$$

where $V_0 = \Delta \chi_{ms} + 2v_T \ln \left( \frac{2}{\tanh \left( \frac{2v_\phi k_B T}{q^2 n_i} \right)} \right)$. At drain end, $\beta = \beta_d$ and $\phi_F = V_{ds}$. So,

$$f_r(\beta_d) = \frac{V_g - V_{ds}}{2v_T}. \tag{77}$$

In the linear region of operation, $f_r(\beta_s) = f_r(\beta_d) \gg 1$ which implies $\beta_s, \beta_d > \frac{\pi}{2}$. So, the term $f_r(\beta)$ in (76) and $g_r(\beta)$ in (77) are reduced to $2r \beta \tan \beta$ and $r \beta^2 \tan^2 \beta$ respectively. Therefore,

$$f_r(\beta_s) \equiv \beta_s \tan \beta_s = \left( \frac{V_g - V_0}{2v_T} \right) \frac{1}{2r} \tag{78}$$

Similarly, $f_r(\beta_d) \equiv \beta_d \tan \beta_d = \left( \frac{V_g - V_{ds}}{2v_T} \right) \frac{1}{2r} \tag{79}$

and the expression (75) reduces to:

$$I_{ds,LIN} = \mu_n \left( \frac{2W}{L} \right) \left( \frac{2k_B T}{\epsilon q} \right)^2 \left[ \beta \tan \beta - \frac{\beta^2}{2} + r \beta^2 \tan^2 \beta \right]_{\beta_s}^{\beta_d}. \tag{80}$$

On substituting (78) and (79) in (80):

$$I_{ds,LIN} = \mu_n \left( \frac{2W}{L} \right) \left( \frac{2k_B T}{\epsilon q} \right)^2 \left[ \frac{1}{4r} \left( \left( \frac{V_g - V_0}{2v_T} \right)^2 - \left( \frac{V_g - V_{ds}}{2v_T} \right)^2 \right) \right]$$

$$= \frac{\mu_n W}{L} C_{ox} \left[ (V_g - V_0)^2 - (V_g - V_{ds})^2 - V_{ds}^2 + 2(V_g - V_0)V_{ds} \right] \tag{81}$$

where $V_{th} = V_0 + \delta$, $\delta$ is the second-order effects. The $I_{ds,LIN}$ (81) is the drain current expression for the linear region. The $\delta$ is derived as follows.

Considering $\phi_F = 0$ in (25), the $\phi_s$ at the source region is expressed as: $\phi_s = -\frac{2k_B T}{q} \ln \left( \frac{\tau_s}{2 \sqrt{2 \pi}} \frac{q^2 n_i}{2} \cos(\beta) \right)$. Since the threshold condition is given by: $V_{th} = \Delta \chi_{ms} + \phi_s$ [15], the expression of the $V_{th}$ in (81) is written as:

$$V_{th} = \Delta \chi_{ms} - \frac{2k_B T}{q} \ln \left( \frac{\tau_s}{2 \sqrt{2 \pi}} \frac{q^2 n_i}{2} \right) - \frac{2k_B T}{q} \ln \cos \beta \tag{82}$$

$$= V_0 + \frac{2k_B T}{q} \ln \frac{\beta \sin \beta}{\cos \beta} \sin \beta = V_0 + \frac{2k_B T}{q} \ln \beta \tan \beta - \frac{2k_B T}{q} \sin \beta \tag{83}$$

In the strong inversion condition, the $\beta \rightarrow \frac{\pi}{2}$ which implies the term “$\ln \sin \beta$” in (82) is $\approx 0$. So,

$$V_{th} = V_0 + \frac{2k_B T}{q} \ln \beta \tan \beta = V_0 + \delta \tag{83}$$

with $\delta = \left( \frac{2k_B T}{q} \right) \ln \beta \tan \beta$. Substituting (78) in (83) will yield:

$$\delta = \frac{2k_B T}{q} \ln \left( \frac{V_g - V_0}{2v_T} \right) \frac{1}{2r}. \tag{84}$$

Equation (84) is the second-order effect $\approx 0.05 \ V$.

In the saturation region of operation, $\beta_s \approx \frac{\pi}{2}$ and $\beta_d \ll 1$. So, the terms $f_r(\beta_s)$ and $f_r(\beta_d)$ are reduced to $2r \beta \tan \beta$ and

$$V_{th} = V_0 + \frac{2k_B T}{q} \ln \beta \tan \beta = V_0 + \delta \tag{83}$$

with $\delta = \left( \frac{2k_B T}{q} \right) \ln \beta \tan \beta$. Substituting (78) in (83) will yield:

$$\delta = \frac{2k_B T}{q} \ln \left( \frac{V_g - V_0}{2v_T} \right) \frac{1}{2r}. \tag{84}$$

Equation (84) is the second-order effect $\approx 0.05 \ V$.

In the saturation region of operation, $\beta_s \approx \frac{\pi}{2}$ and $\beta_d \ll 1$. So, the terms $f_r(\beta_s)$ and $f_r(\beta_d)$ are reduced to $2r \beta \tan \beta$ and
\[ f_r(\beta_s) \equiv \gamma \beta_s \tan \beta_s = \left( \frac{V_g - V_a}{4V_T} \right) \] 

and \[ f_r(\beta_d) \equiv \beta_d = e^{\left( \frac{V_{g}-V_{a}-V_{ds}}{2V_T} \right)} \] 

The expression (75) reduces to:

\[ I_{ds,SAT} = \mu_n \frac{2W}{L} \frac{2e^2}{\xi} (2V_T^2) \left[ r \beta_s^2 \tan^2 \beta_s - \frac{\beta_d^2}{2} \right] \] 

Substituting (85) and (86) in (87) will yield:

\[ I_{ds,SAT} = \mu_n \frac{W}{L} C_{ox} \left( V_g - V_a \right)^2 - \frac{8kT^2 e^2}{q^2} \frac{V_g - V_a - V_{ds}}{V_T} \] 

Equation (88) is the drain current expression for the saturation region \( I_{ds,SAT} \).

In subthreshold region of operation, \( \beta_s, \beta_d \ll 1 \). So the terms \( f_r(\beta) \) and \( g_r(\beta) \) are reduced to \( \ln \beta \) and \( (\beta/2) \) respectively. On solving (76) for \( f_r(\beta) = \ln \beta \) will yield:

\[ \ln \beta_s = \frac{V_g - V_a}{2V_T} = \ln \left[ \frac{2}{\xi} \left( \frac{2e^2}{q^2 \nu_n} \right) \right] \] 

which implies:

\[ \beta_s = \frac{2}{\xi} \sqrt{\frac{2e^2}{q^2 \nu_n}} \frac{V_g - V_{xs,m}}{2V_T} \] 

Similarly,

\[ \beta_d = \frac{2}{\xi} \sqrt{\frac{2e^2}{q^2 \nu_n}} \frac{V_g - V_{xs,m} - V_{ds}}{2V_T} \] 

Online substituting (89) and (90) in (91) will finally yield the \( I_{ds,SUB} \) model for the subthreshold region.

\[ I_{ds,SUB} = \mu_n \frac{W}{L} k_b T n_i \tau_{sl} e^{\frac{V_g - V_{xs,m}}{V_T}} \left( 1 - e^{-\frac{V_{ds}}{V_T}} \right) \] 

Combining the \( I_{ds,LIN} \) (81), \( I_{ds,SAT} \) (88), and \( I_{ds,SUB} \) (92) for the different regions namely linear, saturation, and subthreshold, respectively, the complete \( I_{ds} \) model is written as:

\[ I_{ds} = \begin{cases} 
\mu_n \frac{W}{L} V_{qs} n_i \tau_{sl} e^{\frac{V_g - V_{xs,m}}{V_T}} \left( 1 - e^{-\frac{V_{ds}}{V_T}} \right) & \\
2\mu_n C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) e^{\frac{V_g - V_{a}}{V_T}} & \\
\mu C_{ox} \frac{W}{L} \left[ V_{gs} - V_{th} - 8r V_T^2 e^{\frac{V_{g}-V_{a}+V_{ds}}{V_T}} \right] & 
\end{cases} \] 

Equation (93) is the long channel core \( I_{ds} \) model for DG MOSFETs which has been subsequently augmented with various physical effects like SCE, quantum mechanical effect, and low and high field transport in order to develop \( I_{ds} \) models [22, 23] for short-channel DG MOSFETs. Fig. 5 shows the characteristics obtained from (93) for an undoped DG MOSFET with a mid-gap metal gate, in comparison with the characteristics obtained through solving the \( \beta \) from (27) by the method given by Yu et al. [25] (a) output characteristics, (b) transfer characteristics.
31] took into account the Lombardi CVT mobility model [61] to depict the drain current characteristics.

**III. MODELS FOR SHORT CHANNEL DG MOSFETS**

**A. Electrostatic Potential Models**

In the short-channel devices, due to SCEs the electric fields from the source and drain encroach horizontally into the channel and thus introduce a second dimension (y) [Fig. 6] to the channel electrostatics \( \phi(x,y) \) [11]. The modeling for nanoscale (short-channel) DG MOSFETs solves 2-D Poisson’s equation in order to derive the analytical \( \phi(x,y) \) model. The approach adopted in [33–35], derived the \( \phi(x,y) \) using superposition method where the 2-D Poisson’s equation split into 1-D Poisson and 2-D Laplace equation. The reported papers [20],[32],[36–38] adopted parabolic potential approximation, where \( \phi(x,y) \) is obtained using a parabolic function in terms of x and y. This section describes the \( \phi(x,y) \) model given by Tsormpatzoglou et al. [20] in order to address the modeling scheme for short-channel (L = 30 nm) DG MOSFET.

1) Tsormpatzoglou et al. [20]:

Tsormpatzoglou et al. [20] presented an analytical expression (94) of the \( \phi(x,y) \) along the channel of lightly-doped symmetrical DG MOSFET in weak inversion:

\[
\phi(x,y) = \frac{1}{e^{2x-1}} \left[ (V_{bl} + V_{ds} - A_y) \left( e^{\frac{L+y}{A_y}} - e^{\frac{L-y}{A_y}} \right) + (V_{bl} - A_y) \left( e^{\frac{2L-y}{A_y}} - e^{\frac{y}{A_y}} \right) + A_x \left( e^{\frac{2L}{A_x}} - 1 \right) \right],
\]

with \( A_x = V_g - \Delta X_{ms} - qN_{sib} \sigma_{sib} + \sigma_{sib}(1 - x) \). In case of a lightly-doped body, \( \Delta X_{ms} = -V_T \ln(N_{sib}/N_t) \) for mid-gap metal gates [Fig. 3], \( V_{bl} = V_T \ln(N_{sib}/N_t) \), is the built-in potential, and \( N_{sib} \) is the doping concentration of source and drain. \( A_x = \)

![Fig. 6. The cross-sectional view of a short-channel n-type symmetric DG MOSFET along with the geometrical coordinates.](image)

**Fig. 7.** Transfer characteristics (in semi-logarithmic scale) in the subthreshold region of short-channel DG MOSFET obtained from solving (97) through the numerical method.

\[
\frac{\sqrt{\frac{\varepsilon_{sib} \varepsilon_{ox} x}{\varepsilon_{ox}}} \left( 1 + \frac{\varepsilon_{ox}^2}{\varepsilon_{sib} \varepsilon_{ox}} \right)}{\varepsilon_{sib} \varepsilon_{ox} x} \text{is the natural channel length proposed by Yan et al. [9] which is described more accurately as a function channel depth in short-channel devices. The 2-D extra potential } \Delta \phi(x,y) \text{ induced in the channel due to SCEs is described by:}
\]

\[
\Delta \phi(x,y) = \frac{1}{2} \left[ (V_{bl} + V_{ds} - A_y) \left( e^{\frac{L+y}{A_y}} - e^{\frac{L-y}{A_y}} \right) + (V_{bl} - A_y) \left( e^{\frac{2L-y}{A_y}} - e^{\frac{y}{A_y}} \right) \right].
\]

Based on the 2-D extra potential induced in the channel due to SCEs, a semi-analytical expression for the subthreshold drain current is derived. In the subthreshold condition, the diffusion current dominates due to weak inversion [62]. For weak inversion, the drain current in the subthreshold condition of a long channel device can be expressed as:

\[
I_{ds,long} = \frac{W}{L} V_T \mu_n Q_{is} \left( 1 - \frac{V_{ds}}{V_F} \right),
\]

where \( Q_{is} = \frac{q N_{sib}^2}{N_{sib}} t_s e^{\phi_s} \) is the inversion charge sheet density at the source end, and \( \phi_s = A_{x=0} \) is the surface potential for long channel device. The drain current expression for a short-channel device is obtained by dividing the long channel case by correction factor (CF).

\[
I_{ds,short} = \frac{W}{L} V_T \mu_n Q_{is} \left( 1 - \frac{V_{ds}}{V_F} \right),
\]

where CF = \( \frac{1}{L} \int_{0}^{L} \frac{1}{t_s} \int_{x_i}^{x_f} e^{-\frac{\phi(x,y)}{V_F}} dx \) dy. Here, the CF has to be calculated numerically and hence the model is not applicable if developing a compact model for DG MOSFETs. However, the same can be used to properly design new DG MOSFETS.
because it is rather a semi-analytical model of SCE. Also, equation (97) allows extrapolation of various DG device specifications. The subthreshold drain current characteristics of DG MOSFET shown in Fig. 7 with $W = 1 \mu m$, $t_{si} = 5 \text{ nm}$, $t_{ox} = 1 \text{ nm}$, and $\mu_n = 500 \text{ cm}^2/\text{Vs}$ for different values of $L$ at $V_{ds} = 0.02 \text{ V}$ have been implemented using MATLAB. In addition, Simpson’s one-third method [54] has been employed to evaluate the $CF$.

The MATLAB code to obtain the subthreshold drain current characteristics [Fig. 7]

```matlab
W=1000*10^(-9); % Channel width
tsi=5*10^(-9); % Body thickness
tox=1*10^(-9); % Gate oxide thickness
Eo=8.85*10^(-12); % Permittivity of free space
Eox=3.9*Eo; % Dielectric permittivity of oxide
Esi=11.68*Eo; % ___________ of silicon
Nsi=10^21; % Body doping density
Nsd=5*10^25; % Doping density
ni=1.45*10^16; % Intrinsic charge concentration
L=[10 15 20 30 50]*10^(-9); % Different// channel lengths

considered
u=500*10^(-4); % Mobility of electrons
K=1.38*10^(-23); % Boltzmann constant
T=300; % Room temperature
q=1.6*10^(-19); % Elementary charge
VT=(K*T)/q; % Thermal voltage = 26mV

Vfb=VT*log(Nsi*Nsd/(ni^2)); % Built-in potential

Vgf(Vgs-Vfb); % Drain to source voltage
Cox=Eox/tox; % Oxide capacitance

%------------------- Simpson’s 1/3rd method begins--------

for l=1:length(L)
for k=1:length(L(l))
for j=1:length(y)
xtterm(i)=(1*(Eox*x(i))==(Esi*tox))-((Eox*(x(1)-2))/((Esi*tox)*tsi));
lambda(i)=sqrt((Esi*tox*(2)*Eox)*xterm(i));
delphi(i)=(Vb+Vd-Vg(k))*exp((L1)*y(j))/lambda(i)+1)*exp((L1)*y(j))/lambda(i)+Vb-Vg(k))*exp((L1)*y(j))/lambda(i)+exp((L1)*y(j));
fi=exp(-(delphi(i)/VT));
end
end
end
```
\[ -B \left[ V_{bi} - V_T \ln \left( \frac{Q_{th} \eta_{tff}}{n_{eff}} \right) \right] \frac{2}{3} \left[ V_{bi} + V_{ds} - C (2V_{bi} + V_{Th}) \right], \]

where \( A = \left( \frac{\eta_{tff} \eta_{tff} + \eta_{eff} \eta_{eff}}{\eta_{eff} \eta_{eff}} \right) \), \( B = \frac{2}{3} \left( \frac{\eta_{eff} \eta_{eff}}{\eta_{eff} \eta_{eff}} \right) \), and \( C = \frac{2}{3} \left( \frac{\eta_{eff} \eta_{eff}}{\eta_{eff} \eta_{eff}} \right) \).

\( \lambda \) is the natural channel length along the effective conductive path = \( \frac{\varepsilon_{ox} \epsilon_{ox} \eta_{ox}}{2 \varepsilon_{ox}} \left( 1 + \frac{\varepsilon_{ox} \eta_{ox}}{\varepsilon_{eff} \eta_{eff}} \right) \). For long channel device, \( A = 1 \), and the parameter \( B \) and \( C \) tend to zero and thus, the \( V_{th} \) expression reduces to that of a long-channel DG MOSFET: \( V_{th} = \Delta X_{ms} + V_T \ln \left( \frac{Q_{th} \eta_{tff}}{n_{eff}} \right) \) as given by Chen et al. [18]. The \( Q_{th} \) for long channel DG MOSFET has been determined to be about \( 3.2 \times 10^{-9} \text{ cm}^2 \). Whereas, for a short-channel device, the \( Q_{th} \) is dependent upon the \( L, t_{ox}, t_{sl}, \) and \( V_{ds} \) by the relationship:

\[
Q_{th} = 10^{11} \left[ 1 - (5 + V_{ds}) \frac{L}{2L} \right] \text{ cm}^2.
\]

**C. Drain-Current Models**

1) Tsompartzoglou et al. [22]:

In this model, instead of the numerical approach, an analytical approach is adopted. Various effects like SCEs, series resistance, and CLM are included. Two different equations for subthreshold \( I_{ds, SUB} \) and strong inversion \( I_{ds, SI} \) have been combined through interpolation method. The detailed derivation of \( I_{ds, SI} \) is as follows.

The \( \phi \) model in [16] has been utilized to model the \( I_{ds, SI} \), and the model derivation starts from the expression (27), which will imply:

\[
\ln (\beta \cos \beta + 2r \beta \tan \beta) = \frac{V_{g} - \Delta X_{ms} - \phi_{F}}{2V_{T}} - \ln \left[ \frac{2}{t_{sl}} \frac{2x_{e}e_{ox}}{q^{2}n_{i}} \right], \tag{100}
\]

\[
\ln (\beta \sin \beta \sin \beta + 2r \beta \tan \beta) = \frac{V_{g} - \Delta X_{ms} - \phi_{F}}{2V_{T}} - \ln \left[ \frac{2}{t_{sl}} \frac{2x_{e}e_{ox}}{q^{2}n_{i}} \right], \tag{101}
\]

\[
\ln \beta \tan \beta - \ln \sin \beta + 2r \beta \tan \beta = \frac{V_{g} - \Delta X_{ms} - \phi_{F}}{2V_{T}} - \ln \left[ \frac{2}{t_{sl}} \frac{2x_{e}e_{ox}}{q^{2}n_{i}} \right], \tag{102}
\]

Replacing the term \( \beta \tan \beta \) in the \( Q_{th}(Y) \) expression (67) by \( q_{i} \) (normalized charge density) and substituting in (63) will yield the \( I_{ds, SI} \) expression as:

\[
I_{ds, SI} = \mu_{n} \left( \frac{2W}{L} \right) J_{0} \frac{\xi_{ox} e_{ox} e_{ox}}{q^{2}t_{sl}} q_{i} d\phi_{F} . \tag{103}
\]

In strong inversion, \( \beta \to (\pi/2) \), implies that (102) reduces to:

\[
\ln (\beta \tan \beta + 2r \beta \tan \beta) = \frac{V_{g} - \Delta X_{ms} - \phi_{F}}{2V_{T}} - \ln \left[ \frac{2}{t_{sl}} \frac{2x_{e}e_{ox}}{q^{2}n_{i}} \right]. \tag{104}
\]

Substituting \( q_{i} \) in (104):

\[
\ln q_{i} + 2r q_{i} = \frac{V_{g} - \Delta X_{ms} - \phi_{F}}{2V_{T}} - \ln \left[ \frac{2}{t_{sl}} \frac{2x_{e}e_{ox}}{q^{2}n_{i}} \right]. \tag{105}
\]

Differentiating (105) with respect to \( q_{i} \) will yield: \( d\phi_{F} = -2V_{T}(2r + (1/q_{i}^{2})) d q_{i} \). On substituting the \( d\phi_{F} \) in (103):

\[
I_{ds, SI} = -\mu_{n} \left( \frac{2W}{L} \right) J_{0} \xi_{ox} e_{ox} e_{ox} q_{i} dV_{T} \left( 2r + \frac{1}{q_{i}^{2}} \right) d q_{i} = \mu_{n} \left( \frac{2W}{L} \right) \left( \frac{2e_{ox}}{t_{sl}} \right) \frac{2q_{i}^{2}t_{sl}}{q^{2}t_{sl}} \left( 2r + \frac{1}{q_{i}^{2}} \right) d q_{i} = \mu_{n} \left( \frac{2W}{L} \right) \left( \frac{2e_{ox}}{t_{sl}} \right) \frac{2q_{i}^{2}t_{sl}}{q^{2}t_{sl}} \left( 2r + \frac{1}{q_{i}^{2}} \right) d q_{i} + \xi_{ox} e_{ox} e_{ox} q_{i}^{2} \left( q_{i}^{2} - q_{i}^{2} \right). \tag{106}
\]

where \( q_{is}, q_{id} \) are the values of \( q_{i} \) at source (\( \phi = 0 \)) and drain (\( \phi = V_{ds} \)) ends respectively. The expression for \( q_{i} \) can be derived from (105) as:

\[
\ln \left[ \frac{2q_{i}}{t_{sl}} \frac{2x_{e}e_{ox}}{q^{2}n_{i}} \right] = q \left( V_{g} - \Delta X_{ms} - \phi_{F} \right) - \frac{2x_{e}e_{ox}}{q^{2}t_{sl}} q_{i}. \tag{107}
\]

On rearranging the terms of (107):

\[
q_{i} e_{ox} e_{ox} q_{i} = \frac{t_{sl}}{2} \frac{q^{2}n_{i}}{2x_{e}e_{ox}} e_{2x_{e}e_{ox}}. \tag{108}
\]

Multiplying on both sides of (108) by \( (2e_{ox}/t_{sl}) \):

\[
2x_{e}e_{ox} q_{i} e_{ox} e_{ox} q_{i} = \frac{t_{sl}}{2} \frac{q^{2}n_{i}}{2x_{e}e_{ox}} e_{2x_{e}e_{ox}}. \tag{109}
\]

Since, \( \omega = \frac{n_{i}}{q} = \frac{LambertW(x)}{x} \), so (109) can be transformed using the LambertW function:

\[
2x_{e}e_{ox} q_{i} e_{ox} e_{ox} q_{i} = \frac{t_{sl}}{2} \frac{q^{2}n_{i}}{2x_{e}e_{ox}} e_{2x_{e}e_{ox}}. \tag{110}
\]

The LambertW(\( x \)) function in current expression was first introduced by Ortiz-Conde et al. [66]. When the channel is lightly doped, i.e. \( n = (n_{i}^{2}/N_{a}) \) and to incorporate threshold voltage roll-off effect, \( \Delta V_{th} \) is introduced in (110):

\[
q_{i} = \frac{2e_{ox}}{t_{sl}^{2}} \frac{\xi_{ox} e_{ox} e_{ox} q_{i}}{2x_{e}e_{ox}} \left( q_{i}^{2} - q_{i}^{2} \right). \tag{111}
\]

A compact \( I_{ds} \) model is obtained by combining the \( I_{ds, SI} \) and \( I_{ds, SUB} \) through interpolation function.

\[
I_{ds} = \frac{I_{ds, SI} \times I_{ds, SUB}}{I_{m, SI} + I_{m, SUB}^{m}} \tag{112}
\]
where \( I_{ds,\text{SUB}} = \mu_n \left( \frac{2W}{L} \right) \left( \frac{q}{e} \right) \left( \frac{k_BT}{q} \right)^2 \left[ 1 + e^{\frac{V_{dd,\text{SUB}}}{V_T}} \right] \)
and \( m = 1.9 - \sqrt{1.2V_{ds}} \) is a parameter that prevents the discontinuity in current characteristics at the transition from subthreshold to above-threshold region.

2) Papathanasiou et al. [23]:

This model is an improvement over the \( I_{ds} \) model given by Tsormpatzoglou et al. [22]. Papathanasiou et al. [23] provided only one equation for \( I_{ds} \) which is valid in all region of operation whereas in [22], two equations were combined through interolation function. The detailed derivation of \( I_{ds} \) model is as follows.

In the subthreshold regime \( (V_g < V_{th}) \), \( q_1^2 \) term in (106) can be approximated as zero, i.e. \( (q_1^2 \approx 0) \) which implies: \( q_1 \rightarrow \exp\left[q(V_g - \Delta X_{ms} + \Delta V_{th} - \Phi_f)/2k_BT\right] \). So, the expression (106) reduces to:

\[
I_{ds,SI} = \mu_n \left( \frac{2W}{L} \right) \left( \frac{q}{e} \right) \left( \frac{k_BT}{q} \right)^2 \left[ \frac{q_1}{2} - \frac{q_{id}}{2} \right].
\]

The \( I_{ds,\text{SUB}} \) can be approximated as [22]:

\[
I_{ds,\text{SUB}} = \mu_n \left( \frac{2W}{L} \right) \left( \frac{q}{e} \right) \left( \frac{k_BT}{q} \right)^2 e^{0.8[q_{\text{SUB}} - q_{id,\text{SUB}}]},
\]

where \( q_{\text{SUB}} = \exp\left[q(V_g - V_{th} + \Delta V_{th} - \Phi_f)/\eta k_BT\right] \) and \( \eta = (SS/V_g)\ln 10 \). On dividing the (114) by (113) will yield:

\[
\frac{I_{ds,\text{SUB}}}{I_{ds,SI}} = e^{0.8} \frac{e^{\frac{q(V_g - V_{th} - \Phi_f)}{\eta k_BT}}}{4 \frac{q(V_g - V_{th} - \Phi_f)}{\eta k_BT}},
\]

where \( V_{ge} = V_g + \Delta V_{th} \) and \( \eta_f = \frac{2-\eta}{\eta} \).

In the paper [23], (115) is expressed as:

\[
\frac{I_{ds,\text{SUB}}}{I_{ds,SI}} = \frac{e^{\frac{q(V_g - V_{th} - \Phi_f)}{\eta k_BT}}}{4 \frac{q(V_g - V_{th} - \Phi_f)}{\eta k_BT}},
\]

In this model, only one equation has to be used for both the subthreshold and strong inversion regime. So, it is decided to investigate the possibility of altering the \( z \) parameter of LambertW\((z)\) in \( q_i \), to accommodate for the change in slope of the exponent, at the point where the DG MOSFET is entering the subthreshold mode of operation. Considering:

\[
q_i = \text{LambertW}\left( \frac{q_{fs}}{\left( e^\frac{q_{fs}}{\eta k_BT} \right)} \right)
\]

In addition, to model the \( I_{ds} \), a flag \( isSI \) is used, which is 1 when the device is in strong inversion and = 0 when the device is in weak inversion. The \( isSI \) function can be implemented by using “tanh” function [41–43], which is expressed as:

\[
isSI = \frac{1}{2} + \frac{\tanh\left[\left|V_g - V_{th} + \Delta V_{th}\right|\right]}{2}
\]

Finally the \( I_{ds} \) model is expressed as:

\[
I_{ds,SI} = \mu_n \left( \frac{2W}{L} \right) \left( \frac{q}{e} \right) \left( \frac{k_BT}{q} \right)^2 \left[ \frac{q_{is} - \frac{q_{id}}{2}}{2} \right] + isSI \times \left( r \left( \frac{q_{fs}}{2} - \frac{q_{id}}{2} \right) \right).
\]

The complete \( I_{ds} \) model (118) is incorporated with various effects like surface roughness scattering, velocity saturation, series resistance between drain and source, and CLM (shown at the bottom of the page), where \( \theta \) is the mobility attenuation factor due to surface roughness scattering, \( V_{sat} \) is the high-field electron drift-velocity saturation, \( R_{sd} \) is the equivalent resistance between the source and drain, and \( F_{CLM} \) is the CLM factor. For channel electric field of \( E_g = 10^5 \text{Vcm}^{-1} \) and higher, \( V_{sat} \) in the channel reaches a value about \( V_{sat} = 10^5 \text{Vcm}^{-1} \) [67].

The empirical relationship of \( F_{CLM} \) is:

\[
F_{CLM} = 1 + \left( \frac{A}{\eta_f} \right)^4 \frac{q_{fs}}{\eta f_{\text{eff}} \left( V_{gs,\text{eff}} \right)},
\]

where A = 1.2 - \sqrt{A/L}. In order to avoid a discontinuity at \( V_g = V_{th} \) and \( V_{ds} = V_g - V_{th} \), the smoothing functions: \( V_{ge} = 2V_{th} + \left( V_g - 2V_{th}\right) \tanh\left( V_g/V_{th}\right) \)2 and \( V_{dep} = V_{ds} tanh\left( 1.5V_{ds}/V_{dep}\right)2 \) are introduced.

The \( I_{ds} \) models [22,23] are charge based compact model since the \( I_{ds} \) is expressed in terms of charge densities at source and drain ends. The short-channel models [20–23] have been integrated through Verilog-A code (given in Appendix A) in order to implement a DG MOSFET whose parameters are specified as: \( L = 30 \text{nm, } W = 50 \text{nm, } t_{ox} = 12 \text{nm, } t_{ep} = 1 \text{nm, } N_{sl} = 10^{15} \text{cm}^{-3}, N_{sd} = 10^{20} \text{cm}^{-3}, \) and \( \mu_n = 500 \text{cm}^2/Vs. \) The LambertW function has been coded using the algorithm given by Morris et al. [68]. Fig. 8 shows the \( I_{ds} \) characteristics observed in Spectre simulator for \( V_g \) sweep from 0 to 1.2 V at \( V_{ds} = 1 \text{V}. \) Fig. 8(b) ensures symmetry of the device when the

\[
I_{ds} = \frac{\mu_n}{1 + 8\left( V_g - V_{th}\right)} \left( 8 \frac{V_{gs}}{V_{sat}} \right) \left( \frac{2W}{L} \right) \left( \frac{q}{e} \right) \left( \frac{k_BT}{q} \right)^2 \left[ \frac{q_{is} - \frac{q_{id}}{2}}{2} \right] + isSI \times \left( \frac{e^{\frac{q_{fs}}{\eta k_BT}} - 1}{\eta f_{\text{eff}} \left( V_{gs,\text{eff}} \right)} \right) \times F_{CLM}
\]
polarity of \( V_{ds} \) is reversed. The transfer and output characteristics in Fig. 8 (c–d) are in close agreement with the simulation results in [23]. Fig. 9 shows the correlation between \( I_{ds} \) models (112) and (118); it is observed that the two different equations lead to similar results for the same DG MOSFET structure.

3) Taur et al. [24]:

This model is an improvement over the \( I_{ds} \) model [19] by considering the effect of lateral electric field on mobile charge density which was earlier ignored due to the assumption given by the GCA model. This model augments the GCA to produce finite output conductance in the saturation region. Addition to this, the conventional definition of pinch-off and CLM effects in the saturation region has been reinterpreted. Fig. 10 shows the comparison TCAD simulation results with the \( I_{ds} \) model [19] considering the parameters: \( L = 100 \) nm, \( W = 1 \) \( \mu \)m, \( t_{sl} = 4 \) nm, \( t_{ox} = 2 \) nm, \( \varepsilon_{sl} = \varepsilon_{ox} \approx 11.8 \varepsilon_0 \), \( \mu_n = 200 \) cm/(Vs), \( N_{sD} = 10^{21} \) cm\(^{-3} \), and \( V_0 = 0.33 \) V. The TCAD simulation results [24] in Fig. 10 show that there is no pinch-off point in the channel as depicted by GCA model. The failure of the GCA model in bulk MOSFETs was previously also demonstrated in [69] through the TCAD simulation. The pinch-off point is interpreted as the condition in the channel at which there is a sign change in the vertical electric field \( E \) occurs or \( E = 0 \), which has also been suggested earlier in [70] (for the bulk MOSFETs only). The CLM in saturation region is interpreted as the movement of the point at which the oxide electric field becomes zero in the source side. The complete \( I_{ds} \) model equation is expressed as:

\[
\frac{I_{ds}}{\mu_n W} = \frac{4\varepsilon_{sl}}{t_{sl}} \left( \frac{2k_B T}{q} \right)^2 \left[ \beta \tan \beta - \frac{\beta^2}{2} + r \beta^2 \tan^2 \beta \right] \left[ \frac{V_g}{V_0} - (V_g - V_0 - V) \right]^{\beta_d} \tag{120}
\]

where \( E_0 \) is the lateral electric field at the source can be calculated numerically from the relation:
\[ E_0 = \frac{I_{ds}}{2\mu_n W C_{ox}(V_g - V_0)} \]  
(121)

The \( I_{ds} \) model (121) results are consistence with the TCAD simulation results.

IV. CONCLUSION

A comprehensive review based on the fundamental issues related to electrostatic potential, threshold voltage, and drain current formulations of analytic models for symmetric n-type DG MOSFETs for long as well as short channel have been presented in this paper. Equations for respective models have been analysed, and related derivations have been carried out for the further application of the models. Moreover, the correlation between the models carried out by various researchers has also been surveyed and discussed. This review provides an insight for understanding the mathematical models and also offers knowledge for modeling and designing the increasingly important DG MOSFETS. This work can be of interest to researchers working in these MOSFETS.

APPENDIX

A. Verilog-A Implementation of Short-Channel DG MOSFET (n-Type)

```verilog
// VerilogA for nDGMOS
`include "constants.vams"
`include "disciplines.vams"
module nDGMOS(Vgs,Vdd, Vss);
input Vgs;
inout Vdd, Vss;

// Technological Parameters
electrical Vgs, Vdd, Vss;
parameter real Eo=8.85e-12;
parameter real K=1.38e-23;
parameter real T=300;
parameter real q=1.6e-19;
parameter real ts1=12e-9;
parameter real tox=1e-9;
parameter real Nsi=1e21;
parameter real Nsd=1e26;
parameter real ni=1.45e16;
parameter real L=30e-9;
parameter real W=50e-9;
parameter real u=500e-4;
parameter real VT=0.0259;

// Model Parameters
real Vg,Vd,Esx,Esi,lambda,Vfb,Vth,Vthlong,delVth,r
  ,fixed,power,n,nd,A,Vge,Vgeff,Vx,Vdeff,FCLM,num1
  ,den1,qis,num2,den2,qid,isSI,x1,x2,SS;
```
Threshold voltage calculation [\(V_{th}\) model(98)]:

```plaintext
// Threshold voltage calculation [\(V_{th}\) model(98)]:

analog function real threshold;
input 1;
real
Eox,Esi,l,Vfb,Vbi,Vds,lambda,Qth,Q,den,k1,k2,k3;
begin
Eox=3.9*Eo;
Esi=11.68*Eo;
// Built-in potential:
Vbi=VT*ln(Nsd*Nsi/pow(ni,2));
Vds=0.02;
// Flat-band voltage:
Vfb=-VT*ln(Nsi/ni);
// Natural channel length:
lambda=sqrt(((Esi*tox*tsi)/(2*Eox))*(1+(Eox*tsi)/(4*Esi*tox))-(Eox*tsi)/(16*Esi*tox)));
// The \(Q_{th}\) (38):
Qth=1e15*pow((1-(5+Vds)*(lambda/(2*l))),2);
Q=(Qth*Nsi)/(pow(ni,2)*tsi);
den=exp(l/lambda)-1;
k1=(exp(4*l/lambda)-2*exp(2*l/lambda)+1)/pow(den,4);
k2=(2*exp(l/(2*lambda))*(1+exp(1/lambda)))/pow(den,2);
k3=(2*exp(3*l/lambda)-4*exp(2*l/lambda)+2*exp(1/lambda))/pow(den,4);
threshold=Vfb+k1*VT*ln(Q)-k2*sqrt((Vbi-VT*ln(Q))*(Vbi+Vds-2*VT*ln(Q)))-k3*(2*Vbi+Vds);
end
endfunction
```

Subthreshold slope calculation [20]:

```plaintext
// Subthreshold slope calculation [20]:

analog function real subthreshold;
input 1;
real Eox,Esi,l,lambda, alpha;
begin
Eox=3.9*Eo;
Esi=11.68*Eo;
lambda=sqrt(((Esi*tox*tsi)/(2*Eox))*(1+(Eox*tsi)/(4*Esi*tox))-(Eox*tsi)/(16*Esi*tox)));
Vth=threshold(L);
Vthlong=threshold(100e-9);
delVth=Vth-Vthlong;
r=(Esi/tox)/(Eox/tsi);
\(q=\sqrt{((Esi*tox*tsi)/(2*Eox))*(1+(Eox*tsi)/(4*Esi*tox))-(Eox*tsi)/(16*Esi*tox))}
vw=VT*ln(Nsi/ni);
lambda=sqrt(((Esi*tox*tsi)/(2*Eox))*(1+(Eox*tsi)/(4*Esi*tox))-(Eox*tsi)/(16*Esi*tox)));
Vth=threshold(L);
Vthlong=threshold(100e-9);
delVth=Vth-Vthlong;
r=(Esi/tox)/(Eox/tsi);
q=\sqrt{((Esi*tox*tsi)/(2*Eox))*(1+(Eox*tsi)/(4*Esi*tox))-(Eox*tsi)/(16*Esi*tox))}
vw=VT*ln(Nsi/ni);
lambda=sqrt(((Esi*tox*tsi)/(2*Eox))*(1+(Eox*tsi)/(4*Esi*tox))-(Eox*tsi)/(16*Esi*tox)));
Vth=threshold(L);
Vthlong=threshold(100e-9);
delVth=Vth-Vthlong;
```
V_{\text{eff}} = V_x \cdot \tanh\left(\text{pow}(1.5 \cdot V_x / V_{\text{eff}}), 2\right);
FCLM = \text{pow}(1.0 / \lambda_{\text{v,d}}, 2) \cdot \text{V_{\text{eff}}}/(V_{\text{eff}} - V_{\text{th}});
\text{num1} = \text{exp}((V_g + \text{delVth-Vfb-Vs})/(2 \cdot V_T)) \cdot \text{exp}((V_g + \text{delVth-Vth-Vs})/(2 \cdot V_T)) \cdot \text{num2} \cdot \text{den2} / (2 \cdot V_T);
x_1 = \text{fixed num1} / \text{den1};

// Normalized charge density \( q_{\text{is}} \) and \( q_{\text{id}} \) [23]:
\[ q_{\text{is}} = \text{q}_i \cdot \text{lambertw}(x_1); \]
\[ q_{\text{id}} = \text{q}_i \cdot \text{lambertw}(x_2); \]

// The isSI:
\[ \text{isSI} = (\text{tanh}(5 \cdot (V_g + \text{delVth-Vth}))/2) + 0.5; \]

// The I_{\text{is}} model (118):
\[ I(V_{\text{dd}}, V_{\text{ss}}) = \left( (u^2 \cdot W/L) \cdot (2 \cdot E_{\text{s}}/t_{\text{si}}) \cdot \text{pow}(2 \cdot V_T) \cdot 2 \cdot (q_{\text{is}}/(2 \cdot r)) \cdot (q_{\text{id}}/(2 \cdot r)) \cdot \text{isSI} \cdot r \cdot \text{pow}(q_{\text{is}}/(2 \cdot r), 2) \right) \cdot \text{pow}(q_{\text{id}}/(2 \cdot r), 2)) \cdot \text{FCLM}; \]

endmodule

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