The flux-dependent amplitude of broadband noise variability in X-ray binaries and active galaxies

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\textbf{ABSTRACT}
Standard shot-noise models, which seek to explain the broadband noise variability that characterises the X-ray lightcurves of X-ray binaries and active galaxies, predict that the power spectrum of the X-ray lightcurve is stationary (i.e. constant amplitude and shape) on short time-scales. We show that the broadband noise power spectra of the black hole candidate Cyg X-1 and the accreting millisecond pulsar SAX J1808.4-3658 are intrinsically non-stationary, in that RMS variability scales linearly with flux. Flux-selected power spectra confirm that this effect is due to changes in power-spectral amplitude and not shape. The lightcurves of three Seyfert galaxies are also consistent with a linear relationship between RMS variability and flux, suggesting that it is an intrinsic feature of the broadband noise variability in compact accreting systems over more than 6 decades of central object mass. The RMS variability responds to flux variations on all measured time-scales, raising fundamental difficulties for shot-noise models which seek to explain this result by invoking variations in the shot parameters. We suggest that models should be explored where the longest time-scale variations are fundamental and precede the variations on shorter time-scales. Possible models which can explain the linear RMS-flux relation include the fractal break-up of large coronal flares, or the propagation of fluctuations in mass accretion rate through the accretion disk. The linear relationship between RMS variability and flux in Cyg X-1 and SAX J1808.4-3658 is offset on the flux axis, suggesting the presence of a second, constant-flux component to the lightcurve which contributes \(\sim 25\%\) of the total flux.

\textbf{Key words:} X-rays: stars – stars: individual: Cyg X-1 – stars: individual: SAX J1808.4-3658 – galaxies: active – galaxies: Seyfert

\section{INTRODUCTION}
The broadband noise variability which characterises the X-ray lightcurves of both X-ray binary systems (XRBs) and active galactic nuclei (AGN) is commonly modelled as a shot-noise process. The simplest shot-noise models, where the lightcurve is constructed from a stochastic series of independent overlapping shots with a single characteristic decay time-scale (e.g. Terrell 1972), produce a red-noise type power spectrum at high frequencies (\(P(\nu) \propto \nu^{-\alpha}\), where \(\alpha = 2\)), which flattens to \(\alpha = 0\) at low frequencies (i.e. the variability becomes white-noise on long time-scales). This simple form is qualitatively similar to the power-spectral shapes of XRBs, which flatten below \(\sim 0.1\) Hz (e.g. van der Klis 1995) and AGN, which flatten on much longer time-scales (< \(10^{-6}\) Hz, M{\textsuperscript{c}}Hardy 1988, Edelson & Nandra 1999), but fails to reproduce the typically observed red-noise slopes, \(\alpha \sim 1.0\). More complex shot-noise models, which invoke a broad distribution of decay time-scales, can reproduce the correct high-frequency shapes (Lehto 1989; Lochner, Swank & Szymkowiak 1991), including the additional high-frequency break which is seen above \(\sim 1\) Hz in some XRB power spectra (e.g. Cyg X-1, Belloni & Hasinger 1990). Physical interpretations of shots typically invoke magnetic flares in the X-ray emitting corona which is likely to be common to both XRBs and AGN (e.g. Poutanen & Fabian 1999).

It is well-known that the power spectra of XRBs are non-stationary (i.e. have a variable shape and amplitude) on long time-scales (> days, e.g. Belloni & Hasinger 1990). There is some evidence for non-stationarity of AGN power spectra on much longer time-scales (Uttley et al. 1999). The shot-
noise models described above can account for non-stationary power-spectra by invoking changes in the parameters of the shot-noise process on long time-scales, but predict that on short time-scales the power spectrum should be stationary. Because individual power-spectra measured on very short time-scales (seconds) are noisy, this prediction is difficult to test. However, we can test whether power-spectral shape and/or amplitude varies systematically with short time-scale variations in X-ray flux by binning the power-spectra of small segments of observed XRB and AGN lightcurves according to their flux.

The relationship between long-time-scale variations in flux and power-spectral amplitude has been studied in a few XRBs, e.g. a somewhat complex correlation between RMS variability and flux has been noted during the decay of the hard X-ray transient GRO J0422+32 (Denis et al. 1994). Surprisingly, the possibility of systematic short-time-scale variations of power-spectral shape and amplitude with flux has not previously been investigated in detail. The standard practice of normalising power spectra by squared mean flux (and RMS variability by mean flux), for comparison between different objects and instruments, causes information regarding the flux to be lost. In this letter, we show that this practice has helped to keep hidden a startling fact about the X-ray variability of XRBs and AGN: that the RMS variability of their broadband noise component scales with flux variations on time-scales as short as seconds, while the power-spectral shape remains constant. This discovery poses particular problems for standard shot-noise models, as we shall discuss.

2 THE FLUX-DEPENDENT VARIABILITY OF X-RAY BINARIES

We obtained public archival data from Rossi X-ray Timing Explorer (RXTE) observations of Cyg X-1 (in the low state), observed 23 October 1996 (Nowak et al. 1999) and the recently discovered accreting millisecond pulsar, SAX J1808.4-3658 (Wijnands & van der Klis 1998a), observed 18 April 1998, of duration 18 ksec and 23 ksec respectively (using the PCA instrument and including only periods where all 5 Proportional Counter Units (PCUs) were switched on). Cyg X-1 is an obvious choice for study, as it is the most well-known black hole X-ray binary. The accreting millisecond pulsar SAX J1808.4-3658 is a good example of a neutron star system which displays strong broadband noise variability, so any similarity in the variations of the power spectra of these objects will provide strong evidence that the power-spectral variability is intrinsic to the process that produces the broadband noise, independent of whether the central object has a hard surface or an event horizon. Furthermore, the observations we select show no evidence for discernable low-frequency quasi-periodic oscillations (QPOs) or components other than the broadband noise which we wish to investigate here. Using PCA binned and event mode data from the observations of Cyg X-1 and SAX J1808.4-3658 respectively, we made lightcurves with a resolution of 16 ms for each source in the 2–13.1 keV energy range. The mean fluxes in this energy range are 3377 counts s$^{-1}$ and 409 counts s$^{-1}$ for Cyg X-1 and SAX J1808.4-3658 respectively.

We first investigated the dependence of RMS variability on local X-ray flux, in other words, does the RMS variability of a small section of lightcurve depend on the mean flux of that section of lightcurve? In order to answer this question, we split each lightcurve into 10 s segments and measured the power spectrum for each segment (subtracting the contribution due to Poisson noise estimated from the mean total count rate of the lightcurve segment). We applied the standard RMS-squared normalisation to the power spectrum, so that integrating the power over a given frequency range yields the contribution to lightcurve variance due to variations in that frequency range. Taking the square root of the integrated power yields the RMS variability $\sigma$. By using this method, rather than simply measuring $\sigma$ directly from the lightcurve, we can study how the amplitude of the power spectrum in a defined frequency range responds to changes in the mean flux.

Figure 1. Local flux dependence of mean $\sigma$ for Cyg X-1 and SAX J1808.4-3658. The dotted lines mark the best-fitting linear models described in the text.
in flux. We first chose to measure $\sigma$ over the frequency range 0.1–10 Hz, which incorporates the full spectrum of broadband noise from white-noise to red-noise, as can be seen from the power spectra of these observations (Nowak et al. 1999, Wijnands & van der Klis 1998b). Due to the stochastic nature of red-noise lightcurves, there is significant scatter in individual measures of $\sigma$, so we binned $\sigma$ as a function of segment flux using relatively narrow flux bins to examine the form of the $\sigma$-flux relationship over a broad range of flux. Using a minimum number of 30 measurements per bin to ensure an accurate estimate of the standard error in the mean $\sigma$, the resulting $\sigma$-flux relation for both sources is plotted in Fig. 2. The lightcurves of both Cyg X-1 and SAX J1808.4-3658 show a remarkably linear dependence of RMS variability on flux.

Visual inspection of Fig. 2 shows that the $\sigma$-flux trend in both sources does not pass through the origin, i.e. there is a constant offset in flux and possibly also $\sigma$. To test the goodness of fit of a linear model, including a constant offset from the origin, we fitted a function of the form $\sigma = k(F - C)$, where $F$ is the flux and $k$ and $C$ are constants. This linear model provides a good fit to the data, yielding $\chi^2$ values of 28.7 (for 22 degrees of freedom) for Cyg X-1 and 17.2 (20 degrees of freedom) for SAX J1808.4-3658.

The best fitting model parameters were $k = 0.326 \pm 0.017$, $C = 850 \pm 130$ count s$^{-1}$ for Cyg X-1 and $k = 0.305 \pm 0.026$, $C = 81 \pm 30$ count s$^{-1}$ for SAX J1808.4-3658 (uncertainties are at the 90% confidence level for 2 interesting parameters). Note that the gradient of the $\sigma$-$F$ trend $k$, is equivalent to the fractional RMS variability of the variable-$\sigma$ component of the lightcurve. The constant $C$ represents a second component to the lightcurve, which does not follow the linear $\sigma$-$F$ trend but may still contribute to the total value of $\sigma$. It is not possible to disentangle the mean flux level of this second, constant-$\sigma$ component and the contribution it makes to the total $\sigma$, however the value of $C$ represents the flux of this component in the limit where it does not vary.

Changes in RMS variability measured over a given frequency range may be associated with changes in power spectral amplitude and/or shape. For example, Belloni & Hasinger (1990) show that the fractional RMS variability of Cygnus X-1 is well correlated with changes in the power spectrum, but note that these variations occur on long time-scales and show no relationship to X-ray flux. It seems unlikely that the strict linear relationship we have found between RMS variability and flux could be caused by changes in power spectral shape over the entire 0.1–10 Hz frequency range, because the power spectrum over this range is not a simple power law. Instead, the simplest explanation is that only the power-spectral amplitude changes. We can confirm this interpretation of the $\sigma$-flux relation by plotting power spectra which are binned according to lightcurve flux. Fig. 2 shows flux-binned power spectra of Cygnus X-1, obtained by averaging the power spectra of lightcurve segments of 32 s duration, according to whether the segment flux occupes the first or fourth quartile of the overall distribution of fluxes. The resulting power spectra correspond to fluxes of 3092 count s$^{-1}$ and 3679 count s$^{-1}$ for the first and fourth flux quartiles respectively, and show that the amplitude of the broadband noise variability responds to flux in the same way across the entire power spectrum. There does not appear to be any systematic change in power spectral shape. We obtain a similar result for SAX J1808.4-3658.

3 THE FLUX-DEPENDENT VARIABILITY OF AGN

Given the strong similarities between the power-spectral shapes of AGN and XRBs, which suggest that the same variability processes are at work, independent of the mass of the central object, we might expect AGN to show the same flux-dependent X-ray variability as XRBs. To test this possibility, we obtained quasi-continuous lightcurves of ~days duration for three Seyfert galaxies, NGC 4051, NGC 5506 and MCG-6-30-15, observed in Dec 1996, Jun 1997 and Aug 1997 respectively and available in the RXTE public archive. Because PCUs 3 and 4 were switched off for much of each observation, we only use data from PCUs 0, 1 and 2. We make the lightcurves from Standard 2 data, in the 2–10 keV energy band, using data from only the top layer of each PCU in order to minimise the contribution of instrumental background which adds to the Poisson noise level, applying standard selection criteria (e.g. as described in Uttley 1999) to extract good time intervals, and estimating background lightcurves using the L7 model. The observations are of total useful exposures ~ 75 ks, 90 ks and 340 ks for NGC 4051, NGC 5506 and MCG-6-30-15 respectively. We made power spectra from continuous lightcurve segments of length ~ 2500 s (corresponding to periods between Earth occultations of the source), and measured the Poisson-noise-subtracted integrated power in the $5 \times 10^{-4}$–$5 \times 10^{-3}$ Hz frequency range.

Due to the faint nature and lower variability (relative to XRBs) of AGN on the time-scales sampled, we average the integrated power into two flux bins, corresponding to mean segment fluxes lying below or above the mean flux of the entire observation. The resulting integrated powers (including standard errors) are shown in Table 4. Note that we do not directly determine the RMS variability, $\sigma$ for each segment, because statistical fluctuations in the true Poisson noise level, combined with the relatively small amount of high-

Figure 2. Flux-dependent power spectra of Cyg X-1, corresponding to the first and fourth flux quartiles (see text for details).
frequency power seen in AGN lightcurves, sometimes lead to negative integrated powers after subtraction of the noise estimate. Segments with negative integrated power cannot be used to determine \(\sigma\), but can be used to determine the mean \(\sigma^2\) and its error (note that the fractional RMS can be estimated from the mean \(\sigma^2\) and is also shown in Table 1).

These data clearly show that the variance of the X-ray lightcurves of all three AGN is dependent on X-ray flux, and furthermore the relationship between the RMS variability and the mean segment flux seems to be linear, such that the fractional RMS variability remains approximately constant, despite significant flux changes (e.g. \(\sim\)factor 3 for NGC 4051). There is not sufficient data to confirm the existence of a constant flux component to the AGN lightcurves, of the same relative strength as that seen in the XRBs, although we note that the existence of such a component is not ruled out.

### 4 DISCUSSION

The relation between flux and the amplitude of broadband noise variability which we have presented here, suggests that the lightcurves of XRBs (and possibly AGN) are made from at least two components. One component shows a striking linear dependence of RMS variability on flux, while the other component may contribute a constant RMS to the lightcurve or, more simply, may not vary at all. We can crudely estimate the spectral shape of this second, possibly constant component by determining the RMS-flux relation for lightcurves in four energy bands and measuring the value of the intercept on the flux axis (\(C\)) for each band. In Fig. 3 we plot the value of \(C\) in each energy band expressed as a fraction of the mean flux in that band. The errors on the relative flux are determined from errors in \(\sigma\) estimated from fitting a linear model to the RMS vs. flux data for that energy band. Note however that because the fluxes in each energy band are strongly correlated with one another (e.g. Maccarone, Coppi & Poutanen 2000), the relative uncertainty between energy bands is likely to be significantly smaller than indicated by the error bars we determine here. With this caveat in mind, Fig. 3 suggests that, if \(C\) represents the flux of a constant component to the lightcurve, its spectral shape is very similar to that of the total spectrum and hence the variable component.

This result seems strange if we naively expect that any constant components to the lightcurve should have a different origin and therefore a different spectrum to the variable component. We note however, that provided that the temperature of the corona is maintained, it will continue to Comptonise seed photons regardless of whether it is dynamically variable (e.g. flaring) or ‘quiet’ and non-variable. Therefore the constant component to the X-ray lightcurve may be associated with quiet regions of the corona or patches of the corona above inactive regions of the accretion disk (if variability is driven by variations in seed photon number).

The \(\sim 25\%\) of total flux which the constant component contributes could indicate a \(\sim 25\%\) covering fraction of quiet coronal regions (this may represent a time-averaged covering fraction, since coronal regions might switch from being quiet to being variable). An alternative possibility is that the constant component is due to Thomson scattering of the primary continuum photons in extended, hot gas with radius \(R > 10\) light-seconds to smear out any variability, temperature \(T \sim 10\) keV so that no absorption edges are observed and optical depth \(\tau \sim 0.25\). However, a plasma of this extent should be a strong source of thermal bremsstrahlung emission. Assuming a spherical geometry and uniform density, the bremsstrahlung luminosity from the extended plasma region would scale linearly with its radius so that such a region would contribute \(> 10\%\) of the X-ray luminosity of the system in bremsstrahlung emission and is ruled out by observations.

The variable component of the lightcurve shows a linear dependence of RMS variability on flux, which is not consistent with the expectations of simple shot-noise models, which predict that the power spectrum is stationary. At first glance, the obvious explanation is that the parameters of the shot-noise process are changing on short time-scales. For example, if the average amplitude of shots varies on short time-scales, the flux and RMS variability will vary in proportion

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**Table 1.** Flux-related changes in the variance of NGC 4051, NGC 5506 and MCG-6-30-15.

|            | \(\mu_{\text{tot}}\) | \(\mu\) | \(n\) | \(\sigma^2\) | \(\sigma_{\text{frac}}\) | \(\mu\) | \(n\) | \(\sigma^2\) | \(\sigma_{\text{frac}}\) |
|------------|----------------------|--------|------|-------------|-------------------|--------|------|-------------|-------------------|
| NGC 4051   | 3.4                  | 1.9    | 17   | 0.047 ± 0.024 | 11.4 %            | 5.3    | 16   | 0.32 ± 0.07 | 10.7 %            |
| NGC 5506   | 28.1                 | 25.5   | 20   | 0.21 ± 0.05  | 1.8 %             | 31.2   | 19   | 0.41 ± 0.11 | 2.1 %             |
| MCG-6-30-15| 12.2                 | 10.3   | 73   | 0.28 ± 0.05  | 5.1 %             | 14.1   | 70   | 0.56 ± 0.07 | 5.3 %             |

Mean data are shown for low and high flux segments (corresponding to segment fluxes below and above the mean for the entire observation, given by \(\mu_{\text{tot}}\)). \(\mu\) is the mean flux \((2–10\) keV, count s\(^{-1}\)) for each flux bin, \(n\) is the number of segments in the flux bin, \(\sigma^2\) is the mean variance \((\text{count}^2\) s\(^{-2}\)) and \(\sigma_{\text{frac}}\) is the fractional RMS variability \((\sigma_{\text{frac}} = (\sigma^2/\mu^2)^{\frac{1}{2}})\).

**Figure 3.** Energy-dependent flux of possible constant components to the lightcurves of Cyg X-1 and SAX J1808.4-3658, relative to the mean flux in each band.
to the shot amplitude. Hence the linear relation between RMS variability and flux reflects the linear relationship of both variables to a single underlying varying parameter, the shot amplitude. However, this model cannot simply explain why the RMS variability is dependent on flux on all measured time-scales, a fact which becomes apparent if we plot the RMS-flux relation for Cygnus X-1 using 1 s time bins, determining \( \sigma \) in the 2–20 Hz frequency range (see Fig. 4). The fact that the linear relationship between \( \sigma \) and flux extends to a greater range of fluxes than are obtained by determining the relation using 10 s segments, implies that the linear RMS-flux relation applies to flux variations on time-scales as low as 1 s (which are averaged out by the 10 s segments used earlier). Therefore, if shot parameter variations are the cause of the RMS-flux relation, they must occur on as broad a range of time-scales as the variations in the lightcurve itself (at least down to 1 s), and we are left with the circular problem of trying to explain the broadband noise in X-ray lightcurves with a model which itself requires a broadband noise component (to describe the variation in shot parameters) in order to be consistent with the data.

The simplest solution may be to discard the standard shot-noise models altogether. Instead, the linear dependence of RMS variability on flux may be more simply explained if the lightcurves are instead made ‘from the top down’, with the primary cause of variability being large, long-time-scale variations on which the shorter-time-scale variations are later superimposed. For example, large-scale energy releases in the corona (e.g. through magnetic reconnection) might further sub-divide into a fractal structure, where the energy emitted by each sub-unit is proportional to the energy of its parent unit, but the time-scales for energy emission and the number of sub-units remain independent of total energy content (perhaps related to characteristic time-scales in the corona or disk).

Interestingly, a linear dependence of RMS variability on flux is a natural outcome of the mechanism for producing red-noise variability proposed by Lyubarskii (1997), where fluctuations in mass accretion rate at different disk radii are propagated through the disk to produce variations on all time-scales in the inner disk (and associated corona). In this model, fractional variations in mass accretion rate are produced on time-scales greater than the viscous time-scale, so that longer time-scale variations are produced in the outer disk and shorter time-scale variations in accretion rate are superimposed on them as they propagate inwards. Thus, a linear RMS-flux relation will be produced if the fractional amplitude of accretion rate variations is independent of the actual accretion rate (as suggested by the model, which assumes that changes in accretion rate are caused by fractional fluctuations in the disk viscosity parameter). Either of these suggested models might, in principle, explain the RMS-flux relation we see, but we leave a detailed comparison of their predictions with observations for a future work.

5 CONCLUSIONS

We have shown that the amplitude of the broadband noise variability in the lightcurves of the black hole candidate Cyg X-1 and the millisecond X-ray pulsar SAX J1808.4–3658 is dependent on flux, in that the RMS variability for a given segment of the lightcurve scales (on average) linearly with the segment mean flux. The linear relation between flux and RMS variability has a positive offset on the flux axis, suggesting the existence of a second, constant-flux component to the lightcurve which contributes \( \sim 25\% \) of the total flux. The shape of the power spectrum (at least on short time-scales) remains independent of flux. The X-ray lightcurves of AGN are also consistent with the same linear scaling of RMS variability with flux, suggesting that this behaviour is an intrinsic feature of the broadband noise, which is characteristic of the lightcurves of compact accreting systems across at least 6 decades of central object mass.

The spectrum of the constant component to the lightcurve is similar to the total spectrum, suggesting that it may correspond to quiet, non-variable regions in the X-ray emitting corona. The unusual behaviour of the variable component of the lightcurve, is inherently difficult to explain using standard shot-noise models. An alternative, ‘top down’ approach is more suitable, where the longest-time-scale variations precede the smallest. Possibilities include large coronal flaring regions which break down into a fractal structure of smaller sub-units, or models where variability is caused by variations in mass accretion rate which propagate through the disk, so that shorter time-scale variations from the inner parts of the disk are superimposed on longer time-scale variations from further out.

If the linear RMS-flux relation is indeed common to the broadband noise variability of all compact accreting systems, it should provide a useful tool for separating out components of lightcurves which are not related to the broadband noise component. It will also prove interesting to see how other features of XRB power spectra (such as QPOs) scale with flux.

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