Neutrino emission from neutron star crusts

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Abstract. Neutrino production is the dominant cooling process in neutron stars. After the rapid cooling during the protoneutron star stage, the neutron star crust is formed. Neutrinos continue to be produced in the crust, which can escape from the surface. This neutrino production is an important process to the final cooling phase of the star. They are produced in the crystalline lattice formed by nuclei permeated by an electronic gas. Quantum oscillations of the electron density with respect to the lattice of nuclei generates plasmons, which decay in a pair of neutrinos. Many works have studied the plasma of the nucleus in the star, however, without incorporating the effect of the lattice in the crust of the neutron star. The objective of this work is to include the lattice in the calculation of the plasmon decay rate using quantum field theory at finite temperature. It is not common to find studies on the crystalline lattice of the star using quantum field theory and the calculations generally used for neutrino emissivity consider a homogeneous and isotropic medium, which can not be directly applied in the context of neutron star crust.

1. Introduction
Neutron stars are the most fascinating stars in the universe. In fact, these small stars have an estimated radius of \( R \approx 10 \text{ Km} \), measured masses of \( M \approx 1.5 \, M_\odot \) and average densities of \( \rho \approx 7 \times 10^{14} \, \text{g} \cdot \text{cm}^{-3} \), which is often greater than the standard nuclear density, \( \rho_0 \approx 2.8 \times 10^{14} \, \text{g} \cdot \text{cm}^{-3} \). The crust is a small portion of the neutron star, approximately \( 0.3 - 0.5 \text{ Km} \) (see [2]), however, process which occur is the crust are important for the understanding of the thermal relaxation and cooling of the star. The crust has a variety of structures containing free electrons, neutrons, screened nuclei, being partially ionized atoms or fully ionized nuclei. Nuclei are structured in a crystal lattice (Coulomb crystal), permeated by an electron gas in a valence band of the coulomb crystal.

Our objective in this work is to include the effects of the lattice on the plasmon decay in a pair of neutrino and antineutrino. There are some papers and books [1] with calculations in the case of a free electron plasma. However, the presence of the crystalline lattice in the plasma may change the plasmon decay rate due to the Coulomb interaction of this lattice, which can be introduced as an external field.

2. Methods
We observe that the highest stability structure of a one-component-plasma (OCP) crystalline lattice is reached by a body-centered cubic (bcc) or face-centered-cubic (fcc) lattice [1]. We adopt the bcc configuration for the neutron star’s crust. This lattice forms a Coulomb crystal, which the ion coupling parameter has the lower limit \( \Gamma > 172 \) [1] and depends on temperature.
$T$, atomic number $Z$ and the radius of Wigner-Seitz $a_i$, which depends of the ion density. The ion coupling parameter reads

$$\Gamma = \frac{Z^2 e^2}{a_i k_b T}. \quad (1)$$

This calculation is based on a crystalline lattice immersed in an homogeneous and isotropic gas of electrons. However in our case the local energy density inside the cells begin to have an important influence for the neutrino pair production mechanism.

A more realistic charge distribution in the plasma should be considered in the quantum field calculation of the neutrino production in the crust. In the case of a classical and non relativistic approach, we first calculate the electric potential $V_{ext}(x, y, z)$ generated by the lattice bcc, which is given by

$$V_{ext}(x, y, z) = \frac{Q}{4\pi \epsilon_0} \sum_{n,m,p} \left\{ \frac{1}{\sqrt{(a n - x)^2 + (a m - y)^2 + (a p - z)^2}} + \frac{1}{\sqrt{[(a (n - \frac{1}{2}) - x)]^2 + [(a (m - \frac{1}{2}) - y)]^2 + [(a (p - \frac{1}{2}) - z)]^2}} \right\} \Theta(r - R). \quad (2)$$

Performing the Fourier transform of $V_{ext}(r)$ and considering non-stationary oscillations in this lattice we have

$$V_{ext}(r, t) = \sum_q V(q) e^{i(q.r - \omega t)}, \quad (3)$$

where $\omega$ is the optical longitudinal vibration [6] originating in the instantaneous displacement of the bcc lattice. At the high frequency limit of lattice vibrations, the ions are unable to keep pace with rapid field variations, so they remain virtually stationary.

We use the Lindhard-Mermin response function for a degenerate gas in which the ions are fully ionized, where the limit $\tau \rightarrow \infty$ gives the non-degenerate case of Lindhard dielectric function. For well-defined elementary excitations in the collisionless limit, $\omega \tau \gg 1$, when the lifetime of these excitations is long compared to the period of oscillations. we write the dielectric function $\epsilon(q, \omega)$ in the Lindhard-Mermin:

$$\epsilon(q, \omega) = 1 + \left( \frac{4\pi e^2}{q^2} \right) \frac{\delta n(q, \omega)}{U_{sc}(q)}, \quad (4)$$

where

$$\delta n(q, \omega) = \frac{B(q, \omega + i/\tau) U_{sc}(q)}{1 - \frac{1}{1 - i\omega \tau} \left( 1 - \frac{B(q, \omega + i/\tau)}{B(q, 0)} \right)}. \quad (5)$$

The dielectric function in this case is used to generate the screening potential (6) and find the plasma frequency $\omega_{pl}$, where the frequency depends on an effective electron mass of $m_e(r)$. When we work with a free electron plasma we can calculate the effective mass using $m_e^* = m_e \gamma_r$. However, in this case where at each position of the cell we have spatial dependent induced potentials $V_{ind}(r)$, which leads us to a mass dependent on the position in the cell. This method of calculating the effective mass can be done via [5]. The induced and screening potentials depends on the external potential and the dielectric function of gas, which are given as, respectively,
\[ V_{sc}(q, \omega) = \frac{V_{ext}(q, \omega)}{\epsilon(q, \omega)}, \]  

and

\[ V_{\text{ind}}(r, t) = V_{sc}(r, t) - V_{\text{ext}}(r, t). \]  

Writting the dielectric function by Lindhard’s theory, we can find the plasmon dispersion relation to the quantum and non-relativistic case. From the dispersion relation we can calculate the decay rate of plasmons in pairs of neutrinos and antineutrinos. Using the linear response theory of quantum field theory at finite temperature, we can obtain how an electric field modifies an electron gas and write the periodic electrical potential generated by the crystalline lattice immersed in an electron gas. In this lattice configuration it is possible to observe phenomena such as Friedel’s oscillations, which can also be seen in quantum mechanics, generating a non-homogeneity in the density of electrons on this surface from neutron stars.

The electric potential of the lattice enters as an external source to influence the Dirac fields of the electrons in that plasma, thus modifying the Dirac equation of motion. The linear response can be written as

\[ \delta\langle A_\mu(x_i, t) \rangle = i \int_{t_0}^{t} dt \text{ Tr } \{ \hat{\rho}(H_{\text{ext}}(t)), A_\mu(x, t) \}. \]  

(8)

The dispersion relation of collective excitations is necessary for calculating the decay rate. To find the dispersion relation it is necessary to obtain the field response (see [3]) that in this work is given by

\[ \delta\langle A_\mu(x_i, t) \rangle = \int dt \int d^3x' \partial_\alpha F^{\alpha\nu}_{\text{ext}} D^{R}_{\nu\mu}, \]  

where \( \delta\langle A_\mu(x_i, t) \rangle \) describes the field response and the dispersion relation can be derived from this equation. The matrix \( F^{\alpha\mu}_{\text{ext}} \) represents the external field generated from the lattice acting on the plasma of electrons and \( D^{R}_{\nu\mu} \) is the real propagator of the photons, which reads

\[ D^{R}_{\nu\mu} = \frac{1}{G - k^2} P^T_{\nu\mu} + \frac{1}{F - k^2} P^L_{\nu\mu} + \frac{\rho}{k^2} \frac{k_\nu k_\mu}{k^2}, \]  

where \( P^T \) transverse projector and the \( P^L \) longitudinal projector. The oscillations of the charged particles produce quasi-particles with two modes of transverse oscillations and one longitudinal mode. Transverse oscillation modes are called phonons, which are quasi-acoustic vibration particles and the longitudinal modes are called plasmons, which are the quasi-particles representing the quantum of density of the charge oscillation in that medium. As is well known in the literature, such plasmons may decay into neutrinos. There is an important quantity to be calculated which is the emissivity, whose flux represents the energy density loss by the star at a given time. Emissivity can be calculated by making use of the polarization tensor \( \Pi_{\mu\nu}(K) \) (see [4]) for which we can obtain the decay rate and thus write the emissivity \( Q \):

\[ Q = \sum_\nu \int \frac{d^3k}{(2\pi)^3} [2n_B(\omega_\nu(k))\omega_\nu(k)\Gamma_\nu(k) + n_B(\omega_\nu(k))\omega_\nu(k)\Gamma_\nu(k)] \]  

(11)

in equation above \( \Gamma \) represent this case decay rate and \( n_B \) the Bose-Einstein distribution.
3. Results to be obtained
The determination of neutrino pair production to the neutrino luminosity of the crust is still in progress and as a first step we determine the electric field configuration inside a lattice cell. The energy density and the electron screening of the ions in the lattice is also determined. Friedel oscillations of electron density are considered and will have an important effect on the energy density inside the lattice cell.

Some developments of this work may be useful for future works with thermal relaxation of the neutron star crust, and applications in cores of white dwarfs, superconductivity in the crystalline lattice of their crusts.

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