Analysis of students’ mathematical reasoning

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Abstract. The reasoning is one of the mathematical abilities that have very complex implications. This complexity causes reasoning including abilities that are not easily attainable by students. Similarly, studies dealing with reason are quite diverse, primarily concerned with the quality of mathematical reasoning. The objective of this study was to determine the quality of mathematical reasoning based perspective Lithner. Lithner looked at how the environment affects the mathematical reasoning. In this regard, Lithner made two perspectives, namely imitative reasoning and creative reasoning. Imitative reasoning can be memorized and algorithmic reasoning. The Result study shows that although the students generally still have problems in reasoning. Students tend to be on imitative reasoning which means that students tend to use a routine procedure when dealing with reasoning. It is also shown that the traditional approach still dominates on the situation of students' daily learning.

1. Introduction

Reasoning is a very important aspect of mathematical ability in teaching and learning of mathematics. The National Council of Teachers Mathematics \[1\] revealed that mathematics is reasoning that any activities in mathematics will not be detached from reasoning. Consequently, reasoning becomes a much-needed basic capability to improve general mathematical abilities. Program for International Student Assessment \[2\] mentioned the reasoning as one of the fundamental mathematical capabilities continually would be a strategic issue in the future.

Reasoning, in fact, always be a major concern of a mathematician. Its role in solving mathematical problems \[3\] but the implications of reasoning is a crucial issue that always attends in daily learning. Some researchers have revealed the low level of mathematical reasoning achievement in junior high school \[4\], even Arwinie \[5\] reported that from the 40 students of the junior high school who he studied, only one being able to achieve with good reasoning. While Wardhani and Rumiati \[6\] said that the lack of imagination and creativity into the causes of students are not able to use reasoning skills well. Finding Wardhani and Rumiati \[6\] provides an overview of the condition explicitly reality problems faced junior high school students at the time associated with reasoning. It also provides illumination that issues regarding the reasoning are a complex issue, not only just a basic mathematical skill but the ability that requires students to think comprehensively. On the other hand, Rizta, Zulkardi and Hartono \[7\] next give a more detailed analysis of the obstacles students in reasoning, including the ability of the analyzing, generalizing, integrating, giving a reason, and the solving non-routine problems. They argued that the mathematical reasoning abilities of the students in the fourth such capabilities only reached an average of 13.33\% for the category of very high and 14.82\% for the higher categories. In other words, only 28.15\% of the students achieving good categories.
Bjuland [8] revealed that the complexity of reasoning regarding the process of the five mathematical processes; namely: sense-making, conjecturing, convincing, reflecting, and generalizing. Sense-making is the ability to build a schema problems and represent knowledge. Sense-making is the adaptation and association process of new information acquired with prior knowledge. This process occurs when the mathematical situation is understood and then try to be communicated in mathematical symbols or languages. Conjecturing has the sense to predict theoretical activities and conclusions based on incomplete facts. Products from the conjecturing are done through the completion strategy of arguing and communicating mathematical activity. The completion strategy is then implemented through an activity called convincing. While reflecting are activities undertaken to re-evaluate the three processes earlier. These five abilities are in principle a series of capability processes in relation to the quality of mathematical reasoning.

As Bludjan pointed out, the activity of arguing and communicating mathematically became the core of the quality of mathematical reasoning. Basically, these two activities are closely related to the activity of expressing ideas or ideas in justifying the claims submitted. The idea or the idea could arise from the process of meaning that comes from prior experience and prior knowledge [8]. Lithner [9] has a similar opinion to Bludjan by classifying two types of reasoning based on their quality [10], namely: imitative reasoning and creative reasoning. The imitative reasoning is solely reasoning based on the experience before and effort that are original. Activities copying example of textbooks or remembering Algorithm certain mathematical reasoning fall into the category of this kind. Reasoning Imitative is superficial, meaning reasoning based solely on the surface only, based on mathematical characteristics deeply. The Imitative reasoning consists of two categories, namely: memorized reasoning and algorithmic reasoning. Both are distinguished from strategies used in solving mathematical reasoning. The characteristics of memorized reasoning, first, the strategy built on complete answer remembered by rote (memory); second, strategy built by writing each step that has been memorized before. While the characteristics of algorithmic reasoning, first, the strategy built by remembering an algorithm that guarantees that the solution can be achieved correctly, second, implementing a strategy consisting of nontrivial transformation or actions follow a set of rules. Although algorithmic reasoning includes imitative reasoning, but in certain cases (e.g. when solving non-routine problems), algorithmic reasoning is very reliable, even by maths. The creative reasoning is reasoning that is based on creativity, the ability to produce work that is original and meaningful [9]. Lithner [9] described four characteristics of reasoning creatively, (1) novelty, refers to the new fact, that is the order of reasoning created or recreated, (2) flexibility, means the ability to use approaches and different adaptations from the specific problems, (3) plausibility, means that there are arguments in favour of the chosen strategy and explain why the conclusion gathering of the true or plausible, (4) a sound mathematical foundation, meaning that the argument is built based on a deep mathematical characteristics.

Study of the quality of reasoning becomes a very important in order to see the influence of the learning environment on improving mathematical reasoning abilities. This study is also a key reference in this paper, where an overview of the quality of Lithner perspective will provide valuable information for teachers in creating an approach and a pedagogical framework aimed to improve mathematical reasoning abilities.

2. Methods

The research approach in this study is a qualitative research. The goal of the research is to uncover naturally on students' mathematical reasoning based Lithner perspective. The research was conducted in the second year of the academic year 2016/2017 in one junior high school in Tangerang city. The research sample was set at 33 students of class IX. Research begins with the provision of mathematical reasoning tests. The test results in the form of student’s answer then analyzed and categorized using the criteria of reasoning, namely: Memorized Reasoning (MR), Algorithmic Reasoning (AR), and Creative Reasoning (CR). The completeness of student answers is not taken into account in this analysis; but if the student's answers do not appear at all, giving wrong answers or beyond the proposed problem, then the students' reasoning is deemed to appear (NA). Details of such a criteria can be seen in Table 1.
Table 1. Criteria for Grouping Mathematical Reasoning

| Code | Explanation |
|------|-------------|
| NA   | Answer wrong /blank/ not suitable |
| MR   | Explain the answers without giving a supportive reason, mentioning and defining supporting terms against the answers given |
| AR   | Outlining the answers in detail, give reasons for the answers given |
| CR   | Answer contains components fluency, flexibility, originality, and elaboration |

The next stage is to conduct an interview. The purpose of this interview is to confirm the student's answers in order to verify the grouping of mathematical reasoning done in the previous stage. Interviewees are determined on the basis of theoretical sampling representing high, middle, and low-ability students. The results of these interviews it is possible to change the grouping when grouping different interpretation of the results by means of students who are actually in answer to the problems posed.

3. Result and Discussion

The study results show data on the quality of mathematical reasoning, as seen in the table below.

Table 2. Data on The Quality of Mathematical Reasoning

| Question | Quality of Reasoning |
|----------|----------------------|
|          | NA   | MR   | AR   | CR   |
| 1        | 26   | 0    | 1    | 8    |
| 2        | 0    | 19   | 16   | 0    |
| 3        | 32   | 0    | 0    | 3    |
| 4        | 32   | 0    | 1    | 2    |
| 5        | 28   | 4    | 3    | 0    |

In Table 2, the number of students in the NA category (except number 2) is dominant compared to other categories. This shows that in general, students still experience constraints when dealing with mathematical reasoning. In the question 1, students experiencing difficulties in making a conjecture that meets the conditions of figure 1 and 2. Since students are unusual with this kind of problem, most students forget to use the answer option to create conjectures that satisfy both conditions. One student can determine the answer correctly by utilizing the choice of answers so categorized as algorithmic reasoning. Eight other students make conjectures directly by giving reasons for unfulfilled conditions if the load is worth 8 grams. Although these eight students do not explain for other unfulfilled conditions, at least raising the idea of conjecture is an original thought that is not easy for other students. Because of this thinking, the eight students are categorized as creative reasoning.

![Figure 1](image1.png)  ![Figure 2](image2.png)

**Figure 1.** Student Answer to Question 1, Category CR  **Figure 2.** Student Answer to Question 1, Category AR
Excerpts of the following interviews provide reinforcement for CR category on question 1.

Teacher : Can you explain the purpose of this answer? (Shown answer Student 20 on question 1)
Student 20 : At that time, I took 8 grams, lost the same that 20 grams
Teacher : So...
Student 20 : Because picture 1a is not the same? So I make sure it is 7 grams.
Teacher : Why not 6 grams or 5 grams?
Student 20 : It is too small, sir?
Teacher : Of which student 20 believe that 6 grams or 5 grams do not satisfy?
Student 20 : (Somewhat thinking) Hmm ... Yeah well yes ... But, figure 2 unfilled, must be heavy to the right. (Pointing to the load image in figure 2)

On question 2, students commonly can determine the extent of the field with the right, but wrong in drawing conclusions. In other words, students cannot utilize preliminary calculations for proper inference. Nevertheless, MR and AR category remains can be identified by looking at the pattern of the students' answers on the use of formula for the area of a rectangle.

A total of 19 students were supposedly using the rectangular area formula as a routine procedure when dealing with rectangular objects. In addition, there are 16 students allegedly using the rectangular square formula as a step to be taken to find out the field capacity of known size in the matter. In figure 3, the students immediately calculate the area of a rectangle using a mathematical formula that has been previously known. The way students do such a settlement is categorized as MR. Meanwhile, in figure 4, students use the rectangular formula as the idea (source illumination) in determining the field capacity according to rectangular matter. The way students do such settlement is categorized as AR.

In addition, there are also 3 students first draw a rectangle shape as the initial idea to understand the issues in question explicitly.

These three students allegedly only used the rectangular images to illustrate problems in the matter, then using the illustrations to solve the problem. An excerpt of the interview below provides clarity regarding the students in answering the question.
Teacher : Try to look at your test! Do I want to ask, why did you write the formula for the rectangular area here? (Pointing to answer Student 24 in question 2)

Student 2 : If it is wrong, this is a rock concert in the square-shaped field.

Teacher : Are you sure!

Student 20 : Yes, because it is a rectangle, but I count area of its, sir?

Teacher : How about Student 18?

Student 18 : I use the rectangular formula as an aid to know the capacity of the concert audience.

Teacher : Oh, I see... How about Student? Why draw a rectangle first?

Student 5 : The same as Student 20. Because there is a long and wide, I multiply

Excerpts of the interview above provide a clearer picture of how students in answering question 2. Since student 5 using the image merely as a routine procedure in resolving problems related to the rectangle, then student 5 is categorized as MR. So, for question 2, there are 22 people categorized as MR, the rest as many as 13 people are categorized as AR.

For question 3, students are required to solve problems with non-routine procedures. Students can actually take advantage of the ratio of the inner square area to the outer square area, but because the student's calculation focuses on the same right-angled triangle on all four square corners, it makes it difficult for students to calculate the area of the right triangle on the square in it. Only 3 students are capable of performing non-routine procedures so CR is categorized.

In question 4, students also experienced difficulties in utilizing the formula for the area of the circle. Most students guessed that a small pizza is cheaper, because of the smaller diameter stuck up with great pizza. A student tries to make use of the circle formula, but he does not relate to other conditions (in this case the price for each pizza), so the final answer becomes imprecise. Meanwhile, 2 students were able to use the exact circle formula. Although initially using the circle formula, but the creativity of the settlement appears from the division operation with the price of each pizza which is then multiplied by 1 zed.

Figure 6. Student Answer to Question 4, Category AR

Figure 7. Student Answer to Question 4, Category CR

In question 5, the student is only able to solve the problem for the D option. There are 4 students who answer the problem by using the formula around the rectangle as a routine procedure. These four students are categorized as MR. In addition, there are 3 students who take advantage circumference of a circle, by way of sum up the line segments into others forming the circumference. These three students are categorized as AR.
By taking a close look the various responses of the students from the five questions above, it appears that the use of imitation reasoning still dominates if compared to creative reasoning. It is not certainly independent of the students' learning environment of everyday life, where the traditional approach is still to be excellent teachers in presenting the study of mathematics. As a result, students stop at the computational results but do not arrive at the actual solution. Lithner [9] has ever revealed four-step solution regarding reasoning; (1) a task-solving situation is entered, in which students are exposed to problematic situations to determine how to solve the problem, (2) strategy choice, try to choose strategies that are believed to be able to solve the difficulties encountered, (3) strategy implementation, running a strategy, if the chosen strategy does not fit, go back to the previous step, (4) conclusion, any solution must come to the conclusion, that the conditions in which the answer to the problems was obtained.

4. Conclusion
Based on the founded study and the results, the conclusion that can be delivered from this research is as follows. The student still runs into constraints when dealing with the reasoning in general. The quality of students' mathematical reasoning is still dominated by imitative reasoning, where the faced situation problems of the student is fixated on the application of routine in daily lessons.

References
[1] National Council of Teachers Mathematics 2000 *Principles and standards for school mathematics* (Reston V A; National Council of Teachers of Mathematics)
[2] Programme for International Student Assessment 2015 *Draft mathematics framework* (http://www.oecd.org/pisa/pisaproducts/pisa2015draftframeworks.htm)
[3] Rohana 2015 The enhancement of students’ teacher mathematical reasoning ability through reflective learning *Journal of Education and Practice* 6 108
[4] Pratiwi W I 2014 Penerapan metode penemuan terbimbing untuk meningkatkan kemampuan pemahaman dan penalaran matematis serta self-confidence siswa SMP *Tesis* (Bandung: Universitas Pendidikan Indonesia)
[5] Arwinie N 2014 Meningkatkan kemampuan penalaran dan komunikasi matematis serta self-concept siswa MTs melalui pembelajaran berbasis masalah *Tesis* (Bandung: Universitas Pendidikan Indonesia)
[6] Wardhani S and Rumiati 2011 *Instrumen penilaian hasil belajar matematika SMP: Belajar dari PISA dan TIMSS* (Yogyakarta: P4TK Matematika)
[7] Rizta A, Zulkardi and Hartono Y 2013 Pengembangan soal penalaran model TIMSS matematika SMP *Jurnal Penelitian dan Evaluasi Pendidikan* 17 230
[8] Rosita C D 2014 Kemampuan penalaran dan komunikasi matematis: Apa, mengapa, dan bagaimana ditingkatkan pada mahasiswa *Jurnal Euclid* 1 33
[9] Lithner J 2008 A research framework for creative and imitative reasoning *Educational Studies in Mathematics* 67 255
[10] Bergqvist T 2005 How students verify conjectures: Teachers’ expectations *Journal of..."
Mathematics Teacher Education 8 171