3D CFD TRANSIENT NUMERICAL SIMULATION OF SUPERFLUID HELIUM

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Abstract. Numerical simulations of superfluid helium are necessary to design the next generation of superconducting accelerator magnets at CERN. Previous studies have presented the thermodynamic equations implemented in the Fluent CFD software to model the thermal behavior of superfluid helium. Momentum and energy equations have been modified in the solver to model a simplified two-fluid model. In this model, the thermo-mechanical effect term and the Gorter-Mellink mutual friction term are the dominant terms in the momentum equation for the superfluid component. This assumption is valid for most of superfluid applications. Transient thermal and dynamic behavior of superfluid helium has been studied in this paper. The equivalent thermal conductivity in the energy equation is represented by the Gorter-Mellink term and both the theoretical and the Sato formulation of this term have been compared to unsteady helium superfluid experiments. The main difference between these two formulations is the coefficient to the power of the temperature gradient between the hot and the cold part in the equivalent thermal conductivity. The results of these unsteady simulations have been compared with two experiments. The first one is a Van Sciver experiment on a 10 m long, and 9 mm diameter tube at saturation conditions and the other, realized in our laboratory, is a 150×50×10 mm rectangular channel filled with pressurized superfluid helium. Both studies have been performed with a heating source that starts delivering power at the beginning of the experiment and many temperature sensors measure the transient thermal behavior of the superfluid helium along the length of the channel.

1. Introduction
Superfluid helium, thanks to its high equivalent thermal conductivity ($10^3$ to $10^4$ W/(m.K)) is used as a refrigerant to cool down the next generation of superconducting magnet designed at CERN at low temperature (1.6 K - 2.1 K). These superconducting magnets must be able to reach high magnetic field (around 8 Tesla) and cooling liquids flow in confined space (accelerator magnet). Simulations of the transient thermodynamic behavior of superfluid helium are fundamental to design cryogenic system avoiding any quench of the magnet or, at least, helping to control their propagation. Previous papers use Kitamura [1] simplified model to simulate the thermal behavior of the superfluid helium [2, 3] in CFD commercial softwares. Those papers mostly focus on the steady state regime and the equivalent thermal conductivity in the heat equation is represented by the theoretical formulation of the Gorter-Mellink term [4] where coefficients are extracted from the Hepak database [5].

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This paper focuses on the simulation of the transient thermal behavior of superfluid using the Gorter-Mellink term coefficient extracted from the Hepak data and from Sato [6]. Those simulations model the superfluid thermal and dynamic behavior using the CFD software Fluent® [7]. Both simulations have been compared with Van Sciver's transient experiment [8] on a long tube filled with saturated superfluid helium. They have also been compared with an experiment performed in our laboratory on a 150x50x10 mm rectangular channel filled with pressurized superfluid helium. This experiment has been instrumented with several pressure and temperature sensors to compare with the simulations.

2. Numerical model

Superfluid helium is viewed as a mixture of two components according to the theory of Landau. This is called the “two-fluid” model where the inviscid superfluid component and the dissipative normal fluid component flow in opposite directions [9, 10]. The simplified “two-fluid” model has been implemented in Fluent® software based on an element-based finite volume method (FVM). In this model, the thermomechanical effect term and the Gorter-Mellink mutual friction term are the dominant terms in the momentum equation for the superfluid component. This simplification changes the transport equation for the superfluid component from a hyperbolic to a parabolic equation, thus the second-sound effect is not modeled. These equations are discretized in three-dimensional space with a hexahedral mesh. The additional components (in red in the equations 4 and 5) are implemented as source terms in Navier-Stokes equations using the C language. Then the conservative and momentum equations for superfluid component lead to the following relationships:

\[ \rho = \rho_s + \rho_n \]  
\[ \rho \mathbf{u} = \rho_s \mathbf{u}_s + \rho_n \mathbf{u}_n \]  
\[ \frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{u}) = 0 \]  
\[ \rho \frac{\partial \mathbf{u}}{\partial t} = -\rho (\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla p - \nabla \left[ \frac{\rho_s \rho_n}{\rho} \left( \frac{s}{A \rho_n \sqrt{T}} \right)^{3/2} \nabla T \right] \]  
\[ + \eta \left[ \nabla^2 \mathbf{u} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}) - \frac{\rho_s^3}{A \rho_n \rho_s \sqrt{T}} \right] \left[ \nabla^2 (\nabla T) + \frac{1}{3} \nabla (\nabla \cdot \mathbf{T}) \right] + \rho \mathbf{g} \]  

And the energy equation is written as:

\[ \rho \frac{\partial}{\partial t} (C_r T) = -\rho C_p (\mathbf{u} \cdot \nabla T) - \nabla \cdot \left( \frac{f(T,p)}{\sqrt{T}} \right)^{1/n} \nabla T \]  

Two different equivalent thermal conductivity have been implemented in the energy equation. One is used by Van Sciver [8] where the coefficient m and the function f(T,p) in the Gorter-Mellink term are directly extracted from the Hepak database. In that case, the coefficient m is equal to 3. The other, where the m coefficient is equal to 3.4, has been first defined by Bon Mardion [11, 12] and used by Sato [6] in recent works. The f(T,p) correlations have been extracted from Sato’s paper in this second case.

The equivalent conductivity is initially equal to 1000 W/(m.K) to avoid numerical instability caused by the small temperature gradients at the beginning of the calculation. In the simulations presented in
this paper, the superfluid to normal helium transition has not been modelled. Thus, the calculations stop when the temperature in one cell of the fluid domain reaches the lambda temperature.

3. Comparison with the Van Sciver’s experiment
In the literature, the only experiment including multiple sensors measuring thermal transient regime in He II has been described by Van Sciver [8]. The experiment and the results of the simulations have been presented in this section.

3.1. Experiment and numerical conditions
This experiment has been designed to measure the He II transient heat transfer in a one-dimensional system. The apparatus, shown in Fig. 8, is composed by a helical coil (10 m long and 9 mm internal diameter) with a heater on the bottom delivering 22.2 kW/m² and an He II bath on top (T_b=1.802 K). The experiment was conducted with He II at saturation conditions and 6 temperature sensors measure the transient thermal behaviour at the surface of the tube (cf. Fig. 1). Since the superfluid helium does not show important effect of gravity, our simulated model is a 3D horizontal cylinder of the same length and diameter of Van Sciver's tube with adiabatic side wall. The mesh, generated on ICEM CFD [13], counts 107 065 cells (cf. Fig. 2).

| Cell number | Wall refinement | Min cell size | Max cell size |
|-------------|----------------|--------------|--------------|
| 107 085     | 5.10⁻⁷ m      | 5.10⁻¹¹ m    | 1.10⁻³ m³   |

Figure 1. Geometry of V. Sciver’s experiment.

Figure 2. 3D Mesh of the tube.

3.2. Results
The results of the simulations with the combination of m and f(t) given by the Hepak data base and the Sato’s work are compared with the experiment for the temperature sensor located at 2 m from the heater (cf. Fig. 3). The two simulations indicate a similar overall pattern with the experimental measurement points. Despite the general tendencies, the two simulations overestimated temperature increase even if the one with m = 3.4 is closer to the experimental results of Van Sciver. The main difference between the two correlations lies in the temperature rise of the fluid near the heater. At t = 2.5 s, the temperature calculated with the Sato’s correlation at the heater is 35 mK higher than the Hepak’s one.
Figure 3. Temperature difference evolution in the tube.

The numerical simulation with the Sato’s coefficients for $t = 3.75$ s and $t = 5$ s could not be plotted because it predicts the superfluid to normal transition (Lambda point) close to the heater at around $t = 3.2$ s. The numerical code is not intended to simulate the superfluid transition. At $t = 5$ s, the temperature given by the computations with the coefficients from Hepak is close to the experiment but still overestimates it (about 20 mK) at 0.1 m to the heater. The calculations predict a temperature of 2.153 K at the wall of the heater and therefore an imminent transition. Van Sciver’s results indicate the appearance of the film boiling at around 9 s after the beginning of the experiment. The two simulations therefore provide a significantly higher temperature rise near the heater. The heater is wrapped at the surface of the copper tube. The copper part represents a thermal resistance between the heater and the superfluid helium inside the tube. Further calculations need to be performed taking into account the thermal diffusion by conduction in the copper tube which is neglected in these simulation (no solid part have been modelled).

4. Comparison with the 10-mm channel experiment
An experiment has been performed in our laboratory on a small channel placed in a pressurize superfluid helium bath. Both, steady state and transient regime have been compared to simulations using Hepak and Sato coefficients.

4.1. Experiment and numerical conditions
The experiment is a rectangular channel with dimensions of 140 mm length, 50 mm width and 10 mm height. It has a heater at one end and is open on a helium bath on the opposite end. The channel is immersed in a superfluid helium bath at 1 bar. The side are made of stainless steel and composed of two lateral wedges and two plates containing the temperature and pressure sensors as the figure 4 shows. The closed end is made of G10 fiberglass and contains a notch to insert the nickel-chromium heater ($\text{NiCr } \rho_{\text{elec}} = 1, 08\mu\Omega\cdot\text{m}$). All the parts surrounding the superfluid channel are 20 mm thick to thermally insulate it from the helium bath during a transient experiment. Superconducting Y2Ba4Cu7Oy cable are delivering power to the heater to avoid any extra heat dissipation in the bath and in the solid part of the channel. Eight temperature sensors (Lakeshore Cernox CX-1050-BC) are located on the top plate, in the center, every 15 mm started from the heater. Another temperature sensor is placed behind the heater.
To model the conditions of the experiment, a mesh of the channel respecting its dimensions and containing 73542 cells was realized on ICEM (Fig. 5). The steel walls is modeled by adiabatic walls,
the heater by a wall providing a constant heat flow. The opening on the bath is modeled by a wall with a fixed temperature at 1.8 K. The pressure of the bath is constant at 1 bar. The mesh has been refined near the heater to model the proper transient temperature gradient at the beginning of the simulation. The time step is $10^{-4}$ s at the beginning of the calculation and is increased to $10^{-2}$ s after a two hundred milliseconds.

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{channel_experiment_geometry.png}
\caption{Geometry of the channel experiment.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{channel_experiment_mesh.png}
\caption{3D Mesh of the fluid part of the channel experiment.}
\end{figure}

4.2. \textit{Results in the steady state regime}

The temperature difference between the temperature sensors and the superfluid helium bath given by the simulations and our experiment are shown in figure. 6 and 7 when steady-state is reached. According to the simulations, this regime is reached after less than 2 seconds. The error bars on the temperature measured during the experiment have been added to the figures.

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{temperature_difference_10kW.png}
\caption{Temperature difference in the channel for in the steady state regime 10 kW/m\textsuperscript{2}.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{temperature_difference_15kW.png}
\caption{Temperature difference in the channel for in the steady state regime 15 kW/m\textsuperscript{2}.}
\end{figure}
Figure 7. Temperature difference in the channel for in the steady state regime 20 and 25 kW/m²

In general, simulations based with the coefficients from Hepak give lower temperature increases than the one based on the Sato’s correlation. These differences increase for higher heat fluxes at the surface of the heater (5 mK for 15 kW/m², 20 mK for 20 kW/m² and 60 mK for 25kW/m²). Actually, the simulations with the Sato’s correlation provide a lower heat dissipation due to a lower equivalent thermal conductivity. Experimental measurements show an almost quasi linear decrease of the temperature from the heater to the open helium bath. For 10 kW/m², both simulations with the Hepak and Sato’s coefficients show very close results to the channel experiment (less than 3 mK). Nevertheless, the temperature rise is only 10 mK, which is too low compared with the sensibility of the thermal sensors (about 5 mK) and therefore do not allow any conclusions on the accuracy of these simulations. The experimental measurements fit very well with the simulation using Sato’s coefficients for 20 kW/m² and 25kW/m² (less than 5 mK). For 15 kW/m², the measured values lie between the two calculations.

In steady state, it can thus be asserted that the calculations using Sato’s coefficients for the equivalent thermal conductivity with m = 3.4 accurately predict the thermal behavior of helium (less than 5 mK) for large heat fluxes. The simulation using the Hepak data base with m = 3 underestimate the rise in temperature in the channel.

4.3. Results in the transient regime
The profiles of temperature difference at t = 200, 400, 700 and 1000 ms in the channel are shown in Fig 8 for 15, 20, 25 kW/m² on the heater. For 10 kW/m², the measurement errors was too high to compare the results to the simulations.
As for the steady-state regime, both simulations have similar patterns moving progressively from a hyperbolic profile to a linear profile over time. This trend follows the experimental results for the three heat fluxes delivered by the heater. At $t = 200$ ms, the two simulations differ close to the heater and overestimate the temperature in the channel as in Van Sciver’s experiment. At $t = 400$ ms, the temperature difference between the two simulations is increasing. The experimental results are in good agreement with the simulation using Hepak data base (less than 10 mK) contrary to the results obtained in the steady state regime. At $t = 700$ and 1000 ms, the experimental measurements are close to the simulation using Sato’s coefficient and the one using Hepak data base underestimate the rise in temperature. In the transient regime, calculations with Hepak data base better simulate the heat transfer during the first moments. On the contrary, calculations with Sato’s coefficients show better predictions when the experiment approach the steady state regime.

Further simulations have to be performed modelling the solid part of the channel to estimate the influence of the thermal dissipation of the heat in the solid element. This phenomenon could be responsible of the overestimated results of the Sato simulations during transient regime.
5. Conclusion
This paper has presented transient superfluid simulations using the simplified “two-fluid” model with coefficient extracted from both Hepak database and Sato paper to calculate the energy equation in Fluent® solver. These two type of simulations have been compared with Van Sciver’s superfluid transient experiment in a long tube. The results show good agreement with the experiment except near the heater where the superfluid to normal transition is predicted at least two times sooner. Moreover, these simulations are also compared with a transient superfluid experiment in a small channel filled with pressurized superfluid helium, performed in our laboratory. Sato simulations better fit the steady state regime in this case while Hepak simulations are close to experimental thermal behavior during the first moments. Some works are still in progress to add solid part into the simulations to model the thermal dissipation of heat in the metallic part surrounding the superfluid helium.

6. References
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