Abstract
Granada oscillation code (GraCo) is a software constructed to compute adiabatic and non-adiabatic oscillation eigenfunctions and eigenvalues. The adiabatic version gives the standard numerical resolution, and also the Richardson extrapolation, different sets of eigenfunctions, different outer mechanical boundary conditions or different integration variables. The non-adiabatic version can include the atmosphere-pulsation interaction. The code has been used for intensive studies of δ Scuti, γ Doradus, β Ceph., SdO and SdB stars. The non adiabatic observables “phase-lag” (the phase between the effective temperature variations and the radial displacement) and $\delta T_{\text{eff}}$ (relative surface temperature variation) can help to the modal identification. These quantities together with the energy balance (“growth rate”) provide useful additional information to the adiabatic resolution (eigenfrequencies and eigenfunctions).

Keywords
Stars · Stellar oscillations · Numerical resolution

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1 Introduction
GraCo (Moya et al., 2003) is a software developed to solve non-radial adiabatic and non-adiabatic oscillation equations. It is written in fortran95 language. It can be used for models all over the HR diagram. GraCo is able to work with three different sources of equilibrium models: CESAM (Morel, 1997), Granada Code (Claret, 1999) and JMSTAR (Lawlor and MacDonald, 2006). The numerical technique used is the so called Henyey relaxation method as it is described in Unno et al. (1989) (Section 18.2). The simple representation of the system of differential equations in terms of second-order centred differences is adopted for the numerical resolution.

2 Adiabatic case
The adiabatic system of differential equations is described in Unno et al. (1989) (p. 161). The code has the possibility of choosing between two sets of eigenfunctions. Both sets include the radial displacement ($\xi_r$) and the Eulerian perturbation of the gravitational potential ($\phi'$). The sets differ in the use of the Lagrangian or the Eulerian variation of the pressure, and the addition or not of a function of the radial displacement ($U \xi_r/r$, with $U = d \ln m/d \ln r$, $m$ the mass and $r$ the radius), to the derivative of the Eulerian perturbation of the gravitational potential ($d\phi'/dr$) (Vorontsov et al., 1976).

As boundary conditions GraCo uses those prescribed in Unno et al. (1989) (pp 162 ff). The solutions must satisfy regularity conditions at the innermost mesh-point. As a first surface condition the continuity of $\phi'$ and its first derivative are imposed. As second surface condition, the mechanical one, the program offers two possibilities: 1) The Lagrangian variation of the pressure vanishes ($\delta p = 0$), or 2) makes use of the isothermal reflective wave boundary condition (see Unno et al. (1989), pp 163 ff).

Another degree of freedom of GraCo is the variable of integration. The program can solve the system of differential equations as a function of the logarithm of the radius ($\ln r$) or the ratio between the radius and the pressure ($r/P$). The first one largely weights the inner regions and the second the outer ones. Depending on the physics to be tested, the user can choose the most convenient variable.

In order to solve the eigenvalue problem, the outer boundary condition for the gravitational potential is removed. In this case, for each trial eigenfrequency we have a unique solution. But not all the solutions obtained with every trial eigenfrequency fulfill the removed boundary condition. The spectra of the system is the set of eigenfrequencies and eigenfunctions fulfilling this outer boundary condition.

As the system of differential equations has been replaced by a system of centred difference equations of second order, the truncation error in the eigenfrequencies and the eigenfunctions are of the order $N^{-2}$, with $N$ the number of mesh points of the equilibrium model. To obtain more accurate
eigenfrequencies, the code can make use of the so called Richardson extrapolation [Shibahashi et al., 1981], a combination of the values obtained with $N$ and $N/2$ mesh points cancelling the leading order error.

Finally, GraCo can compute the first order rotational splitting through the Ledoux coefficient [Ledoux, 1951]. The eigenfrequencies for radial modes can be obtained in two ways: 1) Using the LAWE second order differential equation, or 2) setting $\ell = 0$ in the standard non-radial system of equations.

Fig. 1 shows the adiabatic eigenfrequency differences between all the possible aforementioned options, only radial modes are depicted. The equilibrium model used is the last of the step1 of Task 2 [Moya et al., 2007] (this volume), with 4042 mesh points, $1.5M_\odot$ and $X_e = 0.4$. As reference we have calculated eigenfrequencies with the following options: $X = (\ell = 0, \text{no Richardson}, p', \delta p = 0, \ln \tau)$. For each comparison we have changed one single degree of freedom, remaining the rest unchanged. We show the differences obtained in the range [200,2500] $\mu$Hz, that is, from the fundamental radial mode to a frequency slightly larger than the cutoff frequency (around 2250 $\mu$Hz). Top panel presents the differences obtained when two outer mechanical boundary conditions are used. Reference line is $\delta P = 0$, and the comparison is with the use of the isothermal reflective wave outer boundary condition. The differences for large frequencies are of the order of units of $\mu$Hz. These differences are similar to those obtained by other codes (J.C. Suárez, private communication), but a larger study of these differences is still needed. The use of higher order integration procedures, as the Richardson extrapolation, do not change significantly this differences. In bottom panel, the rest of the comparisons are depicted. The Richardson extrapolation gives differences of the order of tenths of $\mu$Hz, the use of $r/P$ as integration variable provides small differences always lower than 0.008 $\mu$Hz. The LAWE differential equations and the use of the Lagrangian variation of pressure as eigenfunction provide similar differences smaller than 0.05 $\mu$Hz, but its profile is not constant. For a comprehensive study of these differences see Moya et al. (2007) (this volume).

3 Non-adiabatic resolution

Additional information for asteroseismology is provided by GraCo with the resolution of the non-adiabatic set of differential equations described in Unno et al. (1989) (pp 261 ff). In the non-adiabatic resolution the eigenfrequencies and the eigenfunctions are no longer real. This makes it possible to obtain the so called non-adiabatic observables: 1) The “phase-lag” $(\phi^T \equiv \phi(r_e^T) - \phi(T))$ defined as the phase difference between the Lagrangian variation of the effective temperature and the radial displacement. 2) The relative variation of the Lagrangian variation of the effective temperature $(\delta T_e/T_e)$. And 3) The energy balance of each mode measured by the “growth rate”, directly related with the imaginary part of the modal eigenfrequency.

The code uses the adiabatic solutions obtained for a given mode as trial functions for the non-adiabatic relaxation procedure. The inner boundary conditions are those described in Unno et al. (1989) (p. 229). It is in the outer region where GraCo presents some complexity. The code can treat the photosphere as a boundary condition or introduce the atmosphere-pulsation interaction resolution described by Dupret et al. (2002). This interaction is described imposing the atmosphere to be in thermal equilibrium and the diffusion approximation for the radiative flux to be no longer valid. In this case two different sets of differential equations are solved, one for the stellar core and envelope and another for the atmosphere. A transition layer and outer boundary conditions for the atmosphere must be defined.

Fig. 2 shows the values of the non-adiabatic observables for a standard $\delta$ Scuti model of $1.8M_\odot$. In this figure we can see how the values in the case labeled “with” (where the atmosphere-pulsation interaction is included) are clearly different from those labeled “without” (the photosphere treated as the outer boundary layer). This illustrates the importance of the inclusion of the atmosphere-pulsation interaction for
Granada oscillation Code (GraCo)

Fig. 2 |$\delta T_{\text{eff}}/T_{\text{eff}}$| (top panel) and $\phi^l$ (degrees, bottom panel), as a function of the pulsation constant $Q$ (in days) for different modes with spherical degrees $l = 0, 1, 2, 3$. A model of a $1.8 M_\odot$ is studied with $X_e = 0.44$, a MLT parameter $\alpha = 1$ and the CEFF equation of state. Results obtained “with” (+) and “without” atmosphere (×) in the non-adiabatic treatment are compared.

4 Conclusions

GraCo is a complete software for the resolution of systems of differential equations related with the stellar oscillations. The general integration scheme used in the code is the Henyey relaxation method as it is explained in Unno et al. (1989).

In the adiabatic frame, different integration schemes can be used to obtain the eigenfrequencies and eigenfunctions: 1) Two outer mechanical boundary conditions, 2) two choices for the set of eigenfunctions, 3) two choices of the integration variables, 4) the use or not of the Richardson extrapolation and, 5) for radial modes the use of the LAWE second order differential equation or set $\ell = 0$ in the standard non-radial differential equations.

On the other hand, the main characteristic of the code is that the non-adiabatic set of differential equations can also be solved. Two different treatments of the photosphere can be used. The first considers the atmosphere as a single boundary layer, and the second describes the atmosphere-pulsation interaction. This makes it possible to obtain more accurate non-adiabatic observables (phase-lag and $\delta T_{\text{eff}}/T_{\text{eff}}$) crucial for modal identification through the multicolor photometry. On the other hand, non-adiabatic studies also allow to study the modal energy balance, giving a theoretical range of observable modes.

Finally we want to remark that the analytical expressions of the differential equations, eigenfunctions and boundary conditions, and the numerical procedure followed in GraCo can be found in Unno et al. (1989).

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The top panel shows theoretical predictions for two specific Strömgren photometric bands (\((b-y)\) and \(y\)) for a given theoretical model using three MLT \(\alpha\) parameters in the fundamental radial mode regime (pulsation constant near 0.033 days). The 3rd overtone regime (pulsation constant near 0.017 days) is shown in the bottom panel. The usual observational errors for these quantities from the ground are: \(\pm 1^\circ\) for the phase differences and \(\pm 0.01\) for the amplitude ratio.

Fig. 3

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