We studied the energy dependence of the 2D skew scattering from strong potential, for which the Born approximation is not applicable. Since the skew scattering cross section is zero both at low and at high energies, it exhibits a maximum as a function of energy of incident electron. We found analytically the shape of the maximum for an exactly solvable model of circular-barrier potential. Within a rescaling factor, this shape is universal for strong potentials. If the repulsive potential has an attractive core, the discrete levels of the core become quasilocal due to degeneracy with continuum. For energy of incident electron close to the quasilocal state with zero angular momentum, the enhancement of the net cross section is accompanied by resonant enhancement of the skew scattering. By contrast, near the resonance with quasilocal states having momenta \( \pm 1 \), the skew scattering cross section is an odd function of energy deviation from the resonance, and passes through zero, i.e., it exhibits a sign reversal. In the latter case, in the presence of the Fermi sea, the Kondo resonance manifests itself in strong temperature dependence of the skew scattering.

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I. INTRODUCTION

The phenomenon of skew scattering has its origin in a spin-orbit part \( \propto \hat{\sigma} \cdot [k \times \nabla V(\rho)] \) of the scattering potential, \( V(\rho) \). Due to this part, the scattering amplitude between the states with momenta \( k \) and \( k' \) acquires a contribution, \( f^a \propto \hat{\sigma} \cdot [k \times k'] \), which is asymmetric with respect to the scattering angle, \( \theta \), between \( k \) and \( k' \). Therefore, in two dimensions, differential cross section acquires an asymmetric contribution

\[
\frac{d\sigma^a}{d\theta} = 2\text{Re} [(f^s)^* f^a] \propto \sigma \sin \theta
\]

(1)

where \( f^s \) is the symmetric part of the scattering amplitude, i.e., the scattering amplitude in absence of spin-orbit potential, and \( \sigma = \pm 1 \) is the spin projection on the normal to the 2D plane.

Skew scattering, introduced almost 80 years ago, had recently attracted a lot of interest, since it is a key ingredient of the anomalous Hall effect\(^{3,4,5,6,7,8}\), as well as of the spin-Hall effect\(^{9,10,11,12,13,14,15,16,17}\). Detailed theoretical calculations of \( \sigma^a \) were carried out for atomic systems\(^{18}\). Complexity of these calculations stems from the fact that \( \sigma^a \equiv 0 \) in the lowest Born approximation for \( f^a \). The results of the second Born approximation for \( f^a \), yielding a finite \( \sigma^a \), are presented in Ref. 18 for the model of the Thomas-Fermi screening of the charge of a nucleus.

Calculations of Ref. 18 were recently utilized in Ref. 12 to estimate the magnitude of skew scattering by a donor impurity in 2D electron gas. Concerning the energy dependence of the skew-scattering, \( \sigma^a(E) \), in calculation of the anomalous Hall and spin-Hall effects, it should be taken at \( E = E_F \), where \( E_F \) is the Fermi energy. Thus, the skew scattering is sensitive to the electron density via \( E_F \). Then the question about the explicit form of \( \sigma^a(E) \) in two dimensions arises. Nontriviality of this dependence stems from the fact that at low energy \( f^a(E) \), is dominated by the angular momentum, \( l = 0 \). As a result, the skew-scattering cross section, \( \sigma^a(E) \), turns to zero at \( E \to 0 \). Since \( \sigma^a(E) \) turns to zero at large \( E \) as well, we conclude that it should pass through a maximum at a certain finite energy.

Study of the energy dependence of the skew scattering is the focus of the present paper. We demonstrate that this dependence exhibits especially rich behavior for a “strong” scattering potential with characteristic magnitude, \( V_0 \), and characteristic radius, \( b \), satisfying the condition \( V_0 \gg \hbar^2/mb^2 \), where \( m \) is the electron mass. For such potentials, the Born approximation does not apply at low energies. Then the calculation of the skew scattering requires the knowledge of scattering phases in the absence of spin-orbit coupling, while the spin-orbit coupling should still be treated perturbatively.

Even more interesting energy dependence of the skew scattering emerges in the case when the scattering po-
Concerning the resonances, around energies $V_I$. Kondo regime. Concluding remarks are presented in Sec. IV. In Sec. V. we consider the skew scattering in the initial. Resonant skew scattering is studied in detail in Sec. VI. 

The paper is organized as follows. In Sec. II we derive a general expression for skew-scattering part of the 2D transport scattering cross section. This expression is linear in spin-orbit coupling and contains all the scattering phases in the absence of spin-orbit coupling. In Sec. III we extract the energy dependence of the skew scattering in different domains for a model circular-barrier potential. Resonant skew scattering is studied in detail in Sec. IV. In Sec. V. we consider the skew scattering in the Kondo regime. Concluding remarks are presented in Sec. VI.

### II. GENERAL FORMALISM

As a result of the spin-orbit term 

$$\hat{H}_{so} = \lambda \rho \frac{1}{\rho} \frac{dV(\rho)}{d\rho} \hat{z}$$  

in the Hamiltonian, where $\lambda$ is the spin-orbit constant and $L_z = -i\hbar \partial / \partial \theta$ is the z-component of the orbital angular momentum, we have $\delta_{l,\sigma} \neq \delta_{l,-\sigma}$, where $\delta_{l,\sigma}$ are the scattering phases in the channel with orbital momentum $l$.

Characteristics of the skew scattering, relevant for transport \([21]\) is 

$$I^a(E) = \frac{\sigma}{\pi} \int_0^{2\pi} d\theta \left( \frac{d\sigma^a}{d\theta} \right) \sin \theta,$$  

where $d\sigma^a/d\theta$ is the asymmetric part of the differential scattering cross section 

$$\frac{d\sigma^a}{d\theta} = \frac{d\sigma^s}{d\theta} + \frac{d\sigma^a}{d\theta}.$$  

While $\sigma$ is explicitly present in Eq. \([22]\), $I^a(E)$ is, in fact, $\sigma$-independent. Expression for $d\sigma^a/d\theta$ in terms of scattering phases reads 

$$\frac{d\sigma^a}{d\theta} = \frac{i}{2\pi \hbar} \sum_{l,l'} (e^{2i\delta_{l,\sigma}} - 1)(e^{-2i\delta_{l',\sigma}} - 1) \sin[(l - l')\theta].$$  

Eq. \([23]\) yields for $I^a(E)$ the following general relation: 

$$I^a(E) = \frac{2\sigma}{\pi \hbar} \sum_{l = l + 1} \frac{i \text{sign}(l - l')}{(\cot \delta_{l,\sigma} - i)(\cot \delta_{l',\sigma} + i)}$$  

$$= \frac{\sigma}{2 \pi \hbar} \sum_{l} \left\{ \sin[2(\delta_{l,\sigma} - \delta_{l+1,\sigma})] - \sin[2\delta_{l,\sigma}] + \sin[2\delta_{l+1,\sigma}] \right\}.$$  

Obviously, the two last terms of the second line of Eq. \([24]\) have vanishing contribution, and we will omit them later on.

It is convenient to separate the spin-dependent and spin-independent parts of scattering phases 

$$\delta_{l,\sigma}(E) = \delta^s_{l}(E) + \sigma \delta^a_{l}(E).$$  

Spin-orbit correction, $\delta^s_{l}(E)$, can be expressed in terms of radial eigenfunctions of the continuous spectrum, $\chi_l(\rho, E)$, in the absence $\hat{H}_{so}$ as follows 

$$\delta^s_{l}(E) = \frac{2m\lambda}{\hbar^2} \int_0^\infty d\rho \frac{1}{\rho} \frac{dV(\rho)}{d\rho} \chi^2_l(\rho, E).$$  

Large-$\rho$ behavior of properly normalized $\chi_l(\rho, E)$ is 

$$\chi_l(\rho, E) = \sqrt{\frac{\pi \rho}{2}} \left[ \cos \delta^s_{l}(k) J_{|l|}(k \rho) - \sin \delta^s_{l}(k) N_{|l|}(k \rho) \right].$$
Substitution of Eq. (7) into Eq. (6) and expansion over \( \delta_i \) yields
\[
I^a(E) = \frac{2}{\pi k} \sum_i (\delta_i^l - \delta_{i+1}^l) \cos 2(\delta_i^l - \delta_{i+1}^l). \tag{10}
\]

Via simple transformation of the sum in Eq. (10) and insertion of the expression for \( \delta_i^l \), Eq. (9), we find
\[
I^a(E) = \frac{4m\lambda}{\pi k h^2} \sum_{l=-\infty}^{\infty} \int_{0}^{\infty} \frac{d\rho}{\rho} dV(\rho) \left[ \chi_l^2(\rho, E) - (l + 1) \chi_{l+1}^2(\rho, E) \right] [2(\delta_i^0 - \delta_{i+1}^0)]. \tag{11}
\]

In Eq. (11), it is also convenient to make use of the identity
\[
\cos 2(\delta_i^0 - \delta_{i+1}^0) = 1 - 2\left( \frac{\tan(\delta_i^0) - \tan(\delta_{i+1}^0)}{1 + \tan^2(\delta_i^0)} \right)^2. \tag{12}
\]

The advantage of using this identity is that the first term does not contribute to the sum in Eq. (11), and the expression for \( I^a(E) \) acquires the form
\[
I^a(E) = \frac{16m\lambda}{\pi k h^2} \sum_{l=-\infty}^{\infty} \int_{0}^{\infty} \frac{d\rho}{\rho} dV(\rho) \left[ \chi_l^2(\rho, E) - (l + 1) \chi_{l+1}^2(\rho, E) \right] \left( \frac{\tan(\delta_i^0) - \tan(\delta_{i+1}^0)}{1 + \tan^2(\delta_i^0)} \right)^2. \tag{13}
\]

The form Eq. (13) clearly illustrates that, for a weak scattering potential, skew scattering is \( \propto V^3 \). Indeed, when the scattering phases are small, the last fraction in Eq. (13) can be replaced by \( (\delta_i - \delta_{i+1})^2 \propto V^2 \), while for \( \chi_l(\rho) \) one can use the free radial wave functions \( \chi_l(\rho, E) = (\pi \rho/2)^{1/2} J_l(k \rho) \). Another power of \( V \) comes from \( \frac{dV(\rho)}{d\rho} \). Eq. (13) is also convenient for the analysis of the energy dependence of the skew scattering from a strong potential. We perform this study in the next Section.

### III. CIRCULAR-BARRIER POTENTIAL

It is a textbook knowledge\(^{20}\) that spin-independent scattering cross section from a strong potential, \( V \gg h^2/(m_0 b)^2 \), has different forms in three energy intervals:

(i) low-energy scattering, \( E \ll \frac{h^2}{m_0 b^2} \), in which only zero orbital momentum, \( l = 0 \), contributes to the cross section;

(ii) intermediate energies (semiclassical regime), \( \frac{h^2}{m_0 b^2} \ll E \ll V \left[ V/\left( \frac{h^2}{m_0 b^2} \right) \right] \), where the scattering cross section is determined by high \( l \gg 1 \), and finally,

(iii) high-energy scattering, \( E \gg V \left[ V/\left( \frac{h^2}{m_0 b^2} \right) \right] \), which corresponds to the Born approximation.

As we will see below, the skew scattering exhibits different universal behaviors in the above three domains. We will see that, within the interval (ii), in contrast to the spin-independent scattering, the skew scattering has an additional scale at \( E \sim V_0 \), where it passes through the maximum value. Note that, for strong potential, the energy \( E \sim V_0 \) is intermediate between \( h^2/mb^2 \) and \( mV_0^2b^2/h^2 \), which are the boundaries of the interval (ii).

We will perform calculations for a model of circular-barrier potential \( V(\rho) = V_0 \theta(b - \rho) \). In principle, this model allows to incorporate spin-orbit interaction nonperturbatively. However, we will use the fact that this interaction is weak and evaluate Eq. (13) expanded with respect to the coupling parameter, \( \lambda \).

Within the model \( V(\rho) = V_0 \theta(b - \rho) \), the phases \( \delta_i^0 \) are found from matching at \( \rho = b \) the expressions for \( \chi'(\rho)/\chi(\rho) \) inside and outside the well. This yields
\[
\tan(\delta_i^0(k)) = \frac{J'_l(k b) I_l(b v) - \frac{\nu}{\nu b} J'_l(k b) J_l(b v)}{N'_l(k b) I_l(b v) - \frac{\nu}{\nu b} J'_l(k b) N_l(b v)}, \tag{14}
\]
where
\[
\nu = \sqrt{v_0 - k^2}, \quad v_0 = 2mV_0/h^2. \tag{15}
\]

It is convenient to rewrite Eq. (14) in a different form using the recurrence relations for \( J_l(z) \) and \( N_l(z) \)
\[
\tan(\delta_i^0(k)) = \left( \frac{1}{k b} - \frac{J'_l(k b)}{J_{l+1}(k b)} \right) J_l(k b) - \frac{\nu}{\nu b} \left( \frac{1}{k b} - \frac{J'_l(k b)}{J_{l+1}(k b)} \right) N_l(k b), \tag{16}
\]

Also, within a model of a circular well, we have \( \frac{dJ_l}{d\rho} \propto \delta(\rho - b) \), so that the general expression Eq. (11) assumes the form
\[
I^a(E) = \frac{4\lambda V_0}{k} \sum_{l=1}^{\infty} \left( \frac{\tan(\delta_i^0) - \tan(\delta_{i+1}^0)}{1 + \tan^2(\delta_i^0)} \right)^2 \frac{l(l - 1)}{Z_l - Z_{l+1}}. \tag{17}
\]

Here we have introduced a notation
\[
Z_l = \frac{1 + \tan^2(\delta_i^0(k))}{\left[ J_l(k b) - \tan(\delta_i^0(k))N_l(k b) \right]^2}, \tag{18}
\]
so that the spin-orbit corrections to the scattering phases are given by
\[
\delta_i^0 = \frac{\pi \lambda V_0}{2Z_l}. \tag{19}
\]

From Eq. (19), we can express \( Z_l \) via the Bessel functions.
We start the analysis of Eq. (17) for the strong potential $V_0 \gg \frac{k^2}{2m}$ from the low-energy domain (i), $kb \ll 1$. In this domain, spin-independent scattering is dominated by a single phase

$$\delta_0^0 \approx \arctan \left[ \frac{\pi}{2 \ln(kb)} \right],$$

which follows from $z \ll 1$ behavior of $N_0(z)$. The phases with higher momenta are much smaller, namely $\delta_0^0(E) \sim (kb)^2$. On the other hand, parameters $Z_l$ grow rapidly with $l$,

$$Z_l = v_0 b^2 \Gamma^2(l) \left( \frac{2}{kb} \right)^{2l},$$

as follows from Eq. (20) (here $\Gamma(l)$ is the Gamma-function). For this reason we can replace the sum in Eq. (17) by a single term, $(\delta_0^0)^2 / Z_1$, so that

$$I^a(E) \bigg|_{kb \ll 1} = \frac{4\lambda v_0}{k} \frac{(\delta_0^0)^2}{Z_1} \left( \frac{2\pi^2 \lambda}{b} \right) \frac{kb}{\ln^2(kb)}.$$  

First we emphasize that, unlike $\sigma^s$, the low-energy skew scattering is governed by two phases, $\delta_0^0$ and $\delta_0^0$; the latter phase enters into $Z_1$, as can be seen from Eq. (17). This is a natural consequence of the form Eq. (2) of the spin-orbit Hamiltonian. We also note that, compared to $\sigma^s$, which behaves as $\sigma^s(E) \propto 1 / \ln^2(kb)$, Eq. (23) contains an additional factor $(kb) \propto E^{1/2}$, growing rapidly with $E$. Although derived for particular model of a circular-well potential, this low-$E$ result is universal within a factor for a general “strong” potential, $v_0 b^2 \gg 1$.

Next we consider the energy domain $1 \ll kb \ll \frac{v_0}{\sqrt{\nu/k}}$, which belongs to (ii). In this domain, the following simplifications are possible. Firstly, since $\nu b \approx \frac{v_0}{\sqrt{\nu/k}} \gg kb$, we can replace $I'_r(\nu b)/I(\nu b)$ in Eq. (20) by 1. Such a replacement is valid only for $l \ll \nu b$. However, the relevant range of momenta in this domain is narrower, namely $l < kb \ll \nu b$. This is because, for $l < kb$, the Bessel functions, $J_l(kb)$ and $N_l(kb)$ oscillate, while for $l > kb$ they behave as $(kb/l)^l$ and $N_l(kb) \sim (kb/l)^{-l}$. This behavior suggests that, $\delta_0^0$ falls off very rapidly once $l$ exceeds $kb$, and the sum over $l$ in Eq. (17) should be terminated at $l = kb$. Note now, that in the interval $1 < l < kb$, the ratio $l/(kb)$ in Eq. (10) can be neglected compared to $\nu/k$. This yields the following simplified expression for the phases, $\delta_0^0(E)$,

$$\tan \delta_0^0 = - \tan \left\{ kb + \arctan \left( \frac{\nu}{k} \right) - \frac{\pi l}{2} - \frac{\pi}{4} \right\}.$$  

On the other hand, neglecting $l/(kb)$ in Eq. (20) leads us to the conclusion that in the interval $1 < l < kb$ all $Z_l$ are equal to each other and are equal to

$$Z_l = \frac{\pi kb}{2} \left( 1 + \frac{\nu^2}{k^2} \right) = \frac{\pi v_0 b}{2k}.$$  

Then the combination $l/Z_l - (l - 1)/Z_{l-1}$ simplifies to $2k/(\pi v_0 b)$. Lastly, we notice that $\delta_0^0$ and $\delta_0^0$ are related as

$$\delta_0^0 - \delta_0^0 = \pi/2.$$  

This follows from Eq. (24). Using the latter relation, it is easy to see that all combinations containing phases in front of the square brackets in the sum Eq. (17) are equal to 1. Thus, the sum over $l$ in Eq. (17) is simply equal to $2k^2/(\pi v_0)$, and the result for $I^a(E)$ reads

$$I^a(E) \bigg|_{1 < kb \ll \sqrt{\nu/k}} = \frac{8\lambda}{\pi} kb.$$  

We see that the two asymptotes Eq. (23) and Eq. (27) match at $kb \sim 1$. In both limits $I^a(E)$ increases essentially as $\propto E^{1/2}$ with $E$. Curiously, the magnitude of potential, $v_0$, drops out from the asymmetric part of the scattering rate in both limits.

The growth of $I^a(E)$ with energy is terminated at $E = V_0$, i.e., at $kb \sim \nu b$, which also belongs to domain (ii). This can be seen from Eqs. (10), (20) in the following way. For $l \sim \nu b \ll kb$ we can use the asymptotic expression $I'_r(\nu b)/I(\nu b) \approx (1 + \nu^2/2)^{1/2}$ for the ratio $I'_r(\nu b)/I(\nu b)$. Then Eq. (20) yields

$$Z_l = \frac{\pi kb}{2} \left[ 1 + \left( \frac{l}{kb} - \nu b \sqrt{\nu^2 + \nu^2 b^2} \right) \right].$$  

Using the same asymptotic form in the expression Eq. (10) for phases, $\delta_0^0$ amounts to replacement of $\nu/k$ by $\nu b + \nu^2/2$ in the argument of the arctangent in Eq. (21). Still, the difference $(\delta_0^0 - \delta_0^0)$ remains $\pi/2$ with accuracy of a small parameter $1/(kb)$.

This is sufficient to replace by 1 all combinations containing phases in front of the square brackets in the sum Eq. (17). Upon substituting Eq. (28) into Eq. (17), the summation over $l$ can be easily performed, and we obtain

$$I^a(E) = \frac{4\lambda v_0}{k} \sum_{l=1}^{kb} \left[ l/Z_l - (l - 1)/Z_{l-1} \right]$$  

$$= \frac{4\lambda v_0}{k} \frac{kb}{Z_{kb}} \left[ \frac{8\lambda v_0}{\pi} \sqrt{\nu b/k} \frac{\sqrt{\nu b/k}}{1 + (1 - \sqrt{\nu b/k})^2} \right].$$  


Now we see that $I^a(E)$ reaches a maximum at $k = (v_0/2)^{1/2}$, which corresponds to $E = V_0$. The maximal value is equal to

$$I^a(V_0/2) = \frac{4(\sqrt{2} + 1)}{\pi} \lambda \sqrt{v_0}.$$  

(30)

For smaller energies, Eq. (29) reproduces the result Eq. (27). Although Eq. (29) was derived for $E < V_0$, it remains applicable also for above-barrier scattering, $E > V_0$. It is seen that, after passing the maximum, $I^a(E)$ falls off as $E^{-1/2}$. The energy dependence of $I^a(E)$ is illustrated in Fig. 2.

We note that the position of maximum in $I^a(E)$ is model-dependent, in the sense, that for a general potential with radius, $\sim b$, and magnitude, $\sim V_0$, the position of maximum can differ from $V_0/2$ by a numerical factor $\sim 1$. However, the existence of maximum in $I^a(E)$, followed by $E^{-1/2}$ decrease, is model-independent. Within a scaling factor, Eq. (29) applies up to the boundary, $E \sim mV_0^2b^2/\hbar^2$, of the domain (iii), where the Born approximation applies. From $E \sim V_0$ to this boundary $I^a(E)$ drops by a large factor $\sim \sqrt{v_0}b^2$. In the high-energy tail (iii), the behavior of $I^a(E)$ depends strongly on the shape of the potential.

IV. $I^a(E)$ IN THE PRESENCE OF QUASILOCAL STATES

The simplest model in which the quasilocal states emerge, is the repulsive scattering potential with attractive core, as shown in Fig. 1. Quasilocal state does not affect the scattering process when the deviation of energy of the incident electron from the resonance exceeds the width, $\Gamma$, of the quasilocal state. For the model potential Fig. 1 the calculation of the width (with prefactor) is presented in the Appendix. Below we consider skew scattering near the resonance for two particular cases: 1. Resonance for zero angular momentum, $l = 0$. In this case, the phase $\delta_0(E)$ changes by $\pi$ as $E$ is swept across the resonance. As we have seen above, in the low-energy limit, $kb \ll 1$, the phases $\delta_0^\pm b$ fall off rapidly with $l$. In calculating $I^a$, the phases, $\delta_{\pm 1}(E)$, should be retained, since there is no skew scattering without them. All phases $\delta_{\pm 1}(E)$ with $|l| \geq 2$ can be neglected. 2. Resonance for angular momenta, $l = \pm 1$. Now the phases $\delta_{\pm 1}(E)$ exhibit resonant behavior. One has to retain the phase $\delta_0(E)$; all phases $\delta_{\pm 1}(E)$ with $|l| \geq 2$ can be neglected.

Cases 1 and 2 are dramatically different because $\dot{H}_{so}$ does not split the level $l = 0$, but does split levels $l = \pm 1$. For this reason, in the case 1, $\delta_{\pm 1}(E)$ can be calculated perturbatively from Eq. (8), so that for $\delta_0$ we have

$$\delta_0 = - \arctan \left[ \frac{\Gamma_0}{2(E - E_0)} \right].$$  

(31)

In the case 2, $\delta_0$ is non-resonant and is still not affected by $\dot{H}_{so}$, while both scattering phases $\delta_1$ and $\delta_{-1}$ exhibit a resonant behavior

$$\delta_{1,-1} = - \arctan \left[ \frac{\Gamma_1}{2(E - E_1 - \delta_{1,-1})} \right].$$  

(32)

Now we have to express the splitting $\delta E_{1,-1}$ through $\dot{H}_{so}$. We have

$$\delta E_{1,-1} \equiv \sigma \delta E_1 = \lambda \sigma \int_0^\infty d\rho \rho \left\{ R_0^0(\rho, E_1) \right\}^2.$$

(33)

where $R_0^0(\rho, E)$ is the radial wave function of the discrete state, i.e., the coupling to continuum is neglected. Important is that the width, $\Gamma_1$, of the state $E_1$ is unaffected by the spin-orbit interaction.

Now we evaluate Eq. (33) for the cases 1 and 2. Upon neglecting $\delta_1$ with $|l| \geq 2$, we obtain from Eq. (33) a simplified form of $I^a(E)$

$$I^a(E) = \frac{\sigma}{\pi k} \left[ \sin(2\delta_{1,-1}) + \sin(2(\delta_0 - \delta_{1,-1})) \right]$$

(34)

$$+ \sin(2(\delta_{1,-1} - \delta_0) - \sin(2\delta_{1,-1}))$$

$$= \frac{4}{\pi k} \sin(\delta_1 - \delta_{-1}) \sin(\delta_0 - \delta_1 - \delta_{-1}) \sin\delta_0.$$  

(35)

Since $I^a$ is independent of $\sigma$, in the last identity we had dropped the subindex, $\sigma$, in the scattering phases. In the case 1, the phases $\delta_1$ and $\delta_{-1}$ are small, and Eq. (34) assumes the form

$$I^a(E) = \frac{4}{\pi k} (\delta_1 - \delta_{-1}) \sin^2 \delta_0$$

(35)

$$\approx \frac{1}{2\pi} (\delta_1 - \delta_{-1}) \frac{\Gamma_0^2}{(E - E_0)^2 + \Gamma_0^2/4}.$$
The difference \((\delta_1 - \delta_{-1})\) is proportional to spin-orbit constant, \(\lambda\), and can be evaluated from Eqs. (19) and (22), namely,

\[
\delta_1 - \delta_{-1} = \frac{\pi^3 \lambda}{4b^2} (kb)^2.
\]

Eq. (35) indicates that in the case 1 the energy dependence of the skew scattering is the same as the energy dependence of \(\sigma^s(E)\), i.e., it exhibits a resonant enhancement near \(E = E_0\).

\[
\delta I^s(E) = 2\pi \left[ \sin(2(\delta_1 - \delta_2) + \sin(2(\delta_0 - \delta_1) + \sin(2(\delta_{-1} - \delta_0) + \sin(2(\delta_{-2} - \delta_{-1})) \right] = \frac{2}{\pi k} \left[ \sin(\delta_1 - \delta_{-1} - \delta_2 + \delta_{-2}) - \sin(\delta_1 - \delta_{-1}) \right] \cos(\delta_1 + \delta_{-1} - 2\delta_0) = (\delta_{-1} - \delta_{-2}) \left[ \cos(2(\delta_1 - \delta_{-1}) + \cos(2(\delta_{-1} - \delta_0) \right].
\]

In the second identity we used the fact that \(\delta_2 + \delta_{-2} \approx 2\delta_0\), which follows from Eq. (21); in the third identity we replaced \(\sin(\delta_2 - \delta_{-2})\), which is nonzero only due to spin-orbit-induced corrections, by \((\delta_{-1} - \delta_{-2}) \approx 4\lambda k / b\). We note now, that the nonresonant parts of differences \((\delta_1 - \delta_0)\) and \((\delta_{-1} - \delta_{-0})\) are \(\pi/2\) and \(-\pi/2\), respectively [see Eq. (26)], while the resonant parts of \(\delta_1\) and \(\delta_{-1}\) are given by Eq. (32). This leads us to the final result

\[
\delta I^s(E) \approx \frac{4\lambda}{\pi b} \left\{ \frac{(E - E_1 - \delta E_0)^2 - \Gamma_1^2/4}{(E - E_1 - \delta E_0)^2 + \Gamma_1^2/4} \right\}.
\]

It is seen from Eq. (39) that the characteristic magnitude of the resonant skew scattering contribution is \(\delta I^s(E) \sim \lambda / b\). On the other hand, the background value, \(I^s(E) \sim \lambda k\) [see Eq. (27)], is much larger. This is because the background value is the sum of \(kb \gg 1\) contributions. Although \(\delta I^s(E)\) constitutes a small correction, it has a lively energy dependence. This dependence is illustrated in Fig. 4 for different values of dimensionless ratio \(\delta E_1 / \Gamma_1\). We see that, as \(\delta E_1 / \Gamma_1\) increases, the structure in \(\delta I^s(E)\) crosses over from one minimum to two minima.
behavior of Eq. (37). However, deep in the Kondo regime, the second term in the numerator, which is upon decreasing temperature, the curve, is small and we return to Eq. (35). Namely, the resonant condition, $I_a$ experiences a strong enhancement when the temperature, $T$, is lower than the Kondo temperature, $T_K$. This prominent many-body effect stems from the Hubbard repulsion of two electrons in the quasilocal level.

Spin-orbit interaction does not lift the degeneracy of the ground state, and thus, does not affect $\sigma^a(E_F)$. On the other hand, since the skew scattering is essentially due to interference of $l = 0$ and $l = \pm 1$ scattering channels, one should expect the Kondo enhancement of $\sigma^a$ alongside with enhancement of $\sigma^s$.

Quantitatively, this enhancement can be found in the following way. Kondo effect modifies Eq. (35). Namely, the scattering phase, $\delta_0$, should be replaced by the temperature-dependent Kondo phase, $\delta_0(T)$. The low-$T$ and high-$T$ asymptotes of $\delta_0(T)$ are the following

$$\sin^2 \delta_0 = \begin{cases} 1 - \frac{\pi^2}{16} \left( \frac{T}{T_K} \right)^2, & T \ll T_K, \\ \frac{3\pi^2}{16} \left( \frac{1}{\ln^2(T/T_K)} \right), & T \gg T_K, \end{cases} \quad (40)$$

Eq. (40) describes how $I^a$ drops with increasing temperature from its “unitary” value $I^a = \pi^2 \lambda k_F$, where $k_F$ is the Fermi momentum, to the background value, given by Eq. (24) with $k \rightarrow k_F$.

A very lively behavior of the skew scattering emerges when the quasilocal level, $E_1$, is close to the Fermi level, while $E_0$ is well below the Fermi level. Then the Kondo resonance develops in $l = 0$ channel at $T < T_K$, while the phases $\delta_{\pm 1}(E_F)$ are strongly sensitive to the deviation $(E_1 - E_F)$.

A particularly interesting issue is what happens to the sign reversal of $I^a$, found in the previous Section. In this case, both $\delta_0$ and $\delta_{\pm 1}$ are resonant; $\sin \delta_0$ is defined by Kondo resonance Eq. (40), whereas $\delta_{\pm 1}$ are given by Eq. (32). Upon setting $E = E_F$ in Eq. (32) and substituting it into Eq. (40), we obtain

$$I^a(E_1, E_F, T) \approx \frac{4\delta E_1 \Gamma_1}{\pi k} \sin^2 \delta_0(T) \frac{\Gamma_1(E_F - E_1) \cot \delta_0(T) + [(E_F - E_1 - \delta E_1)(E_F - E_1 + \delta E_1) - \Gamma_1^2/4]}{[(E_F - E_1 - \delta E_1)^2 + \Gamma_1^2/4][(E_F - E_1 + \delta E_1)^2 + \Gamma_1^2/4]} \quad (41)$$

At “high” temperature, $\delta_0$ is small and we return to Eq. (37). However, deep in the Kondo regime, the behavior of $I^a(E_F)$ changes drastically. As it is seen from Eq. (41), the dominant contribution to $I^a$ comes from the second term in the numerator, which is even function of $(E_F - E_1)$, unlike the first term, which is odd. Therefore, upon decreasing temperature, the curve, $I^a(E_F)$, evolves from asymmetric to symmetric.

For $\delta_0 = \pi/2$, the shape of $I^a$ versus $E_F$, which is proportional to the electron density, is shown in Fig. 5. We see that symmetric shape undergoes a strong transformation as the splitting $\delta E_1/\Gamma_1$ increases. For each value of splitting there are two points of the sign change positioned symmetrically with respect to the point $E_F = E_1$.

V. MANY-BODY EFFECTS IN THE RESONANT SKEW SCATTERING

As it was mentioned in the Introduction, the quantity relevant for transport is $I^a(E)$ at $E = E_F$. Then the resonance condition, $E_F = E_0$, can be satisfied either if electron concentration or the potential of the attractive core can be controlled. On the other hand, even away from resonance, at $E_F > E_0$, the symmetric part of scattering cross section, $\sigma^s(E_F)$, experiences a strong enhancement when the temperature, $T$, is lower than the Kondo temperature, $T_K$. This prominent many-body effect stems from the Hubbard repulsion of two electrons in the quasilocal level.

VI. CONCLUDING REMARKS

The Lorentzian shape of $I^a(E)$, described by Eq. (35), suggests that it should be accompanied by the Fano features, since $I^a(E)$ must assume its nonresonant value away from $E = E_0$. The issue of Fano resonance near $E = E_1$ is more delicate. As can be seen from Eq. (37) at the Fano-resonance condition $\delta_1 \approx \delta_{-1} \approx \delta_0/2$ the
factor $\sin(\delta_1 - \delta_{-1})$ turns to zero. This observation can be interpreted as “cancellation” of Fano resonances due to interference in $l = 1$ and $l = -1$ channels. Similar “cancellation” was pointed out in Refs. 23,24. In these papers the photo-current, caused by infrared excitation of electron either from impurity into the conduction band23 or between two Zeeman subbands24 of $InSb$ in a strong magnetic field, was studied experimentally. In fact, there is a general similarity between the sine reversal of the skew scattering with energy and the sign reversal of photocurrent as a function of magnetic field, observed in Refs. 23,24, the underlying reason being the interference of a non-resonant and two split resonant channels.

VII. ACKNOWLEDGEMENT

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APPENDIX A

In this Appendix we derive an analytical expression for the width of a 2D quasilocal state. The Schrödinger equation for the radial part of the wave function, $R_l(\rho, E)$, in a azimuthally-symmetric potential, $V(\rho)$, reads

$$R''_l + \frac{1}{\rho} R'_l + \left(\frac{2m}{\hbar^2} [E_l - V(\rho)] - \frac{l^2}{\rho^2}\right) R_l = 0. \quad (A1)$$

Upon substitution

$$R_l(\rho, E) = \frac{\chi_l(\rho, E)}{\sqrt{\rho}}, \quad (A2)$$

Eq. (A1) acquires a Hermitian form

$$\chi''_l + \left(\frac{2m}{\hbar^2} [E_l - V(\rho)] - \frac{l^2 - 1/4}{\rho^2}\right) \chi_l = 0. \quad (A3)$$

Consider now the potential, depicted in Fig. 1 with $V(\rho) = 0$ for $\rho > b$. To calculate the width, $\Gamma_l$, we consider an auxiliary potential, $\tilde{V}(\rho)$, which coincides with $V(\rho)$ for $\rho < b$ and is a constant $V_0$ for $\rho > b$. Localized state, $E = E_l$ in this potential is stationary. Denote with $\tilde{\chi}_l(\rho)$ the corresponding radial wave function, so that

$$\tilde{\chi}'_l + \left(\frac{2m}{\hbar^2} [\tilde{E}_l - \tilde{V}(\rho)] - \frac{l^2 - 1/4}{\rho^2}\right) \tilde{\chi}_l = 0. \quad (A4)$$

Upon multiplying Eq. (A3) by $\tilde{\chi}_l$ and Eq. (A4) by $\chi_l$, subtracting and integrating from $a$ to $b$ we obtain the relation

$$\left[\tilde{\chi}_l \tilde{\chi}'_l - \chi_l \chi'_l\right]_a^b = \frac{2m}{\hbar^2} (\tilde{E}_l - E_l) \int_a^b d\rho \chi_l \tilde{\chi}_l. \quad (A5)$$

The fact that the difference $(\tilde{E}_l - E_l)$ is much smaller than $E_l$ allows us to keep in the left-hand side only the contribution from $\rho = b$. Then it is convenient to rewrite Eq. (A5) as

$$\left[\chi'_l(b) - \tilde{\chi}'_l(b)\right] = \frac{2m}{\hbar^2} \frac{(\tilde{E}_l - E_l)}{\chi_l(b) \tilde{\chi}_l(b)} \int_a^b d\rho \chi_l(\rho) \tilde{\chi}_l(\rho). \quad (A6)$$

To find the imaginary part of $\tilde{E}_l$ we use the continuity of the logarithmic derivatives of $\chi_l$ at $\rho = b$

$$\frac{\chi'_l(b)}{\chi_l(b)} = \frac{1}{2b} + k H_l^+(kb) H_l^+(kb), \quad (A7)$$

where $H_l^+(k\rho)$ is the Hankel function corresponding to outgoing wave at $\rho \to \infty$. Since $\tilde{\chi}_l(\rho)$ is real, the second term in the left-hand side of Eq. (A7) does not contribute to $3m\tilde{E}_l$. The imaginary part of $E_l$ originates from logarithmic derivative of the Hankel function in Eq. (A7). Final expression for $\Gamma_l = 3m\tilde{E}_l$ emerges upon setting $\chi_l(\rho) = \tilde{\chi}_l(\rho) = \sqrt{\rho} K_l(\nu \rho)$, where $K_l$ is the Macdonald function, which is the solution of Eq. (A1) in the barrier region, and extending the upper limit of integration in Eq. (A6) to infinity. Both steps are justified if $3m\tilde{E}_l \ll \tilde{E}_l$. Then we obtain

$$\Gamma_l(E) = \frac{\hbar^2}{\pi m} \frac{1}{J_l^2(kb) + N_l^2(kb)} \frac{K_l^2(\nu b)}{\int_a^F d\rho K_l^2(\nu \rho)}. \quad (A8)$$

FIG. 5: (Color online) The shape of $I'$ in the Kondo regime, $T \ll T_K$, and resonance with quasilocal level, $E = E_1$, is plotted from Eq. (41) versus dimensionless deviation $x = 2(E_1 - E_F)/\Gamma_1$ of $E_1$ from the Fermi level for three values of the dimensionless spin-orbit splitting, $\delta E_1/\Gamma_1$, of the level $E = E_1$. 

\[ \]
Further simplification is achieved when the core is wide enough, so that $\nu a = [2m(V_0 - E_l)a^2]^{1/2}/\hbar$ is large. Then we can use the large-$\rho$ asymptote of the Macdonald function in the integrand in Eq. (A8), which yields

$$
\Gamma_l(E) = \frac{2\hbar^2}{\pi mb} \cdot \frac{\nu e^{-2\nu(b-a)}}{J_l^2(kb) + N_l^2(kb)}.
$$

In the opposite limit, $\nu a \ll 1$, the exponent in the width, $\Gamma_l$, is $\exp(-2\nu b)$, while the dependence on $a$ enters into the prefactor.

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