Numerical production of /s/-like sound as Helmholtz resonance in the turbulence-induced pressure field inside a two-dimensional oral front cavity

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Abstract: A numerical production of /s/-like sound is dealt with by means of computational fluid dynamics in a two-dimensional model of oral front cavity. The basic hypothesis is that the target sound is the outcome of the lowest or first-mode resonance to the pressure field inside the front cavity, whose resonance frequency is in the range 5–8 kHz. The model domain is composed of a small semi-closed area representing the front cavity, the upstream half-space, the downstream half-space and two channels that connect the upstream/downstream half-spaces to the cavity. Computation was carried out with the method of direct numerical simulation on the two-dimensional Navier-Stokes equations. The pressure wave emitted into the downstream half-space has a continuous spectrum with a single spectral peak around the assumed resonance frequency. The spatial/temporal characteristics of the turbulence and pressure fields are studied in relation to the so-called quadrupole field.

Keywords: Sibilant, Air turbulence, Navier-Stokes equations, Helmholtz resonance, Two-dimensional model, Direct numerical simulation

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1. INTRODUCTION

The primary cause of fricative consonants is attributed to air turbulence where a constriction at some section in the vocal tract is thought to play an essential role for the generation of turbulence [1–3]. A number of studies, such as [4–37] among others, have been done since 1950s on the spectral natures of different fricatives and the associated shapes of vocal tract, and the results have been used to build synthesis models in the framework of source-filter theory.

Among these preceding studies, an extensive work by Toda [37] on /s/, /ʃ/ and others revealed that the vital portion of vocal tract for each specific category of fricative changes considerably across the subjects of different genders and languages, in different utterance postures (seated vs. stretched), and for different sound sequences (sustained vs. words with different vowels); and the sounds so spoken show a large diversity in their spectral profiles. Here, the shapes of vocal tract were identified with the midsaggital and coronal sections.

There have been several results with the approach of fluid dynamics. Howe and McGowan [38] obtained a theoretical solution for /s/. Adachi [39], Grandchamp et al. [40], van Hirtum et al. [41] searched for a numerical solution for /s/ with the use of LES (large-eddy simulation) with three-dimensional non-structured elements, and recently Yoshinaga et al. [42,43] reported a result on the production of sibilant fricatives /s/ and /ʃ/ experimentally and numerically using a simplified three-dimensional vocal tract model.

It has been generally considered, as stated in Catford [2], that the agent to bring forth /s/ is the turbulence caused by impingement on the teeth, a kind of wake turbulence as contrasted to the channel turbulence that is caused directly by air jet. The model domains in [38] and [43], as well as in other model experiments such as Shadle [16], were set up based on this assumption. However, it is worth noting an argument posed by Meyer-Eppler [5] that the upper and lower incisors were not essential to pronounce a perfect German /s/ since the lower incisors had been found not to contribute to pronouncing it effectively, and the patients whose upper incisors had been extracted could restore the faculty to pronounce a normal /s/ after a short lisping stage.

On the basis of the preceding investigations, the present paper studies the basic mechanism of turbulence-induced pressure field generated inside the front cavity and the resultant pressure wave emitted into the free-field in the
Our concern is the sole role of front cavity under no acoustical coupling with other parts of vocal tract. Hence the model domain will be composed of three parts: the air supply, the front cavity and the air discharge. The problem will be dealt with in a two-dimensional setting by taking the following two points into consideration. The governing equations will be solved with the method of direct numerical simulation (DNS).

First, since the power spectra of natural /s/ were reported to have several spectral peaks at frequencies below 10 kHz where the most dominant one lies in the range \(5\sim8\) kHz or thereabouts, we shall take a hypothesis that it arises from the lowest or the first-mode resonance of sound inside the front cavity coupled with its connecting inflow and outflow channels. Hence, our model domain will assume its first natural frequency, \(f_1\), to lie in the above mentioned frequency band. Since \(f_1\) is associated with a domain composed of a small semi-closed area and its connecting narrow channels, it must depend essentially on the total extent of domain rather than the representative length, that is, the resonance is in the mode of Helmholtz resonator.

Second, by taking account of Meyer-Eppler’s argument, the channel at the outlet side of mouth cavity will be configured to be straight. This differs from an L-like channel assumed in [38] and [43], which forces the airflow to bend at right angles.

Admittedly, the two-dimensional modeling differs from the three-dimensional one because of (i) the nature of turbulent flow-field itself, typically known as a longer lifespan of vortexes in the former than the latter, and (ii) the structure of inflow constriction in the model geometry. Regarding the latter, the two-dimensional inflow constriction, when embedded in the three-dimensional space, represents a narrow slit of uniform width extending in the coronal direction, whereas the coronal cross-section of the real constriction is a small, oval hole or the like; accordingly, the real midsagittal section of inflow constriction is too wide to adopt for the two-dimensional counterpart. Also, the real midsagittal section of front cavity is too large for the two-dimensional model for a similar reason.

2. PROBLEM SETTING

2.1. Two-dimensional Model Domain

Figure 1 shows a schematic view of our model domain where CAV is the front cavity, UHS and DHS are upstream and downstream half-spaces, and UCHN and DCHN are narrow channels, UHS and UCHN together take the role of air supply where UHS is an air tank of infinitely large capacity kept at a constant pressure rise relative to the standard atmosphere, \(p_s\), from which the air is supplied to CAV through UCHN that constricts suitably to regulate the airflow. Inside CAV the supplied air is expected to become turbulent. Then, it is discharged into DHS whose pressure is kept at \(p_s\), through DCHN that works as the incisors gap and nearby area. UHS and DHS are defined in relative to offsets \(U_0\) and \(D_0\) at which their origins are set so that \(U_0 = (u_0, u_0)\) and \(D_0 = (d_0, d_0)\), respectively, are the midpoints of the exit of UCHN and DCHN. Hereafter the set \{UCHN, CAV, DCHN\} will be referred to as the U-C-D construct, or U-C-D for short.

Toda [37] reported several MRI images and many sketches of vocal tract at midsaggital section for /s/ (and others), and several graphs of area-functions; the former show different shapes and sizes of front cavity, from slender to fat, with or without a notch at bottom side, etc., and the latter enable us to get their approximate dimension. It is considered that the Helmholtz resonance is the main reason for allowing such a diversity in the shape and size for /s/. See Sect. 2.3.2.

By referring to these evidences, three model domains, to be made in small size for reduction of computational load, will be configured so as to work as a Helmholtz resonator with \(f_1\) in the desired range, driven by turbulence. The first one, labeled \(D_1\), will have a rectangular CAV so that \(f_1\) takes about 10 kHz. It will work as a pilot model for the two tasks: (i) to determine a reasonable mesh discretization to numerically reproduce the pressure wave that contains high-frequency components up to 10 kHz, and (ii) to get a general understanding on the temporal/spatial behavior of the underlying turbulence field. UCHN will take a funnel-like shape for smooth airflow from UHS, and DCHN will also take a similar shape for the same reason. The second and third ones, labeled \(D_2\) and \(D_3\), will be those which are supposed to produce /s/-like sounds in two contrastive shapes of CAV. UCHN and DCHN will be similar to those of \(D_1\). In what follows, \(D\) stands for the generic name of \(D_1, D_2\) and \(D_3\).
Since there is no a priori knowledge about the spatial behavior of turbulence field in CAV, discretizing CAV into uniform square elements is the most desirable to obtain an accurate numerical solution. Hence, CAV will be composed of rectangles whose side lengths satisfy a simple integral ratio. The boundaries of UCHN and DCHN will be configured only with line segments and simple curves.

Based on this assumption, a boundary-fitted mesh will be applied to discretize \( \mathcal{D} \); it defines a set of ‘structured’ finite elements as a generalization of finite difference meshes to fit curved boundaries.

### 2.2. The Initial-boundary Value Problem

Under the assumption of omitting the equation of energy, the governing equations are the Navier-Stokes equations for the unknowns \( \rho, p \) and \( \mathbf{u} = (u, v) \) in \( \mathcal{D} \):

\[
\begin{aligned}
\frac{\partial p}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} &= 0 \\
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} &= \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \\
\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho uv)}{\partial x} + \frac{\partial (\rho vv + p)}{\partial y} &= \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}
\end{aligned}
\]

(1)

where \( \rho \) is the density, \( u \) and \( v \) are the \( x \) and \( y \) components of flow velocity, and \( p \) is the pressure representing the deviation from \( p_s \). The \( \tau_{xx}, \tau_{xy} \), etc. are the stress tensor components for viscosity with Lamé’s coefficients \( \mu \) and \( \lambda \). Under the assumption that \( p \) is small, the constitutive law

\[
\frac{\partial p}{\partial \rho} = c^2 \quad (c \text{ is the sound velocity})
\]

(2)

holds, which makes the system of equations closed. The physical constants of air assume the following values at \( p_s = 1 \text{ atm} \): density \( \rho_s = 1.29 \text{ kg/m}^3 \), \( c = 340 \text{ m/s} \), \( \mu = 18.2 \times 10^{-6} \text{ Pa·s} \) and \( \lambda = -(2/3)\mu \).

The unknowns are assumed to satisfy the following boundary and initial conditions, where \( \partial \mathcal{D} \) is the boundary of \( \mathcal{D} \), \( \partial p/\partial n \) is the normal derivative of \( p \) to \( \partial \mathcal{D} \), and \( p_0 \) \( (p_0 > 0) \) is a constant pressure rise from \( p_s \).

**Boundary condition.**

\[
\frac{\partial p}{\partial n} = 0, \quad u = v = 0 \quad \text{on} \quad \partial \mathcal{D}
\]

(3)

**Initial condition.**

\[
\begin{cases}
    p = p_0, \quad u = v = 0 \quad \text{in UHS} \\
    p = 0, \quad u = v = 0 \quad \text{otherwise}.
\end{cases}
\]

(4)

The quantity \( p_0 \) stands for the so-called intra-oral pressure (IOP). It is the only parameter that determines the behavior of unknowns for a given \( \mathcal{D} \).

We shall be concerned with both the temporal and spatial aspects of \( p, u \) and \( q \). In order to discriminate the two aspects, the following convention will be taken. When it is meant to be the spatial distribution, a suffix ‘-field’ will be put such as \( p\text{-field}, u\text{-field} \) and \( q\text{-field} \); alternatively, when it is meant to be the temporal behavior, a suffix ‘-wav’ will be put such as \( p\text{-wav} \), which is a synonym of waveform \( p \), etc.

### 2.3. Resonance of Turbulence-induced Pressure in a Semi-closed Domain

#### 2.3.1. Lighthill’s quadrupole

The behavior of turbulence-induced pressure field can be best understood in relation to Lighthill’s quadrupole term [45]. By eliminating the first time-derivative of momentum in the three-dimensional Navier-Stokes equations under the omission of viscosity terms, an inhomogeneous wave equation of \( \rho \) with a right-hand side term, \( q \), as defined by Eq. (6) below, is derived. When expressed in \( p \) in place of \( \rho \), the equation is

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = q
\]

(5)

with

\[
q \overset{\text{def}}{=} \sum_{ij} \frac{\partial^2 \rho u_i u_j}{\partial x_i \partial x_j}
\]

(6)

where \( \Delta \) is the Laplacian operator and \( q \) is the quadrupole term. In the right-hand side of Eq. (6), \( \rho \) was replaced with \( \rho_0 \) in [45]. For the two-dimensional problem, Eqs. (5) and (6) are read in terms of \( x_1 = x, x_2 = y, u_1 = u \) and \( u_2 = v \) for \( i = 1,2 \) and \( j = 1,2 \).

Although \( p \) and \( u_i \) are coupled mutually, it is considered that the reaction of \( p \) to \( u_i \) is smaller than the reverse one. Hence with negligence of the former, Eq. (5) is regarded as an inhomogeneous wave equation in \( p \) with an ‘external source function’ \( q \). This view immediately leads to the following when applied to the resonance of \( p \) in CAV: (i) the resonance would take place at the natural frequencies, \( f_i \) \( (i = 1,2,\ldots) \), of the homogeneous wave equation in \( p \):

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0 \quad \text{in} \quad \mathcal{D}
\]

subject to Eq. (3) for \( p \)

(7)

and (ii) the actual resonance at a particular \( f_i \) takes place only when the spectrum of \( q \) covers \( f_i \) with sufficient intensity over a certain sub-domain of CAV. Note that such a sub-domain changes dynamically in shape and location in the turbulent \( u\text{-field} \).

#### 2.3.2. U-C-D construct as a Helmholtz resonator

Let \( \Omega \) be a finite domain in the three-dimensional space. Obviously, the smallest or first-mode resonance frequency, \( f_1 \), of the wave equation of \( p \) in \( \Omega \) subject to \( \partial p/\partial n = 0 \) on \( \partial \Omega \) takes on a trivial 0 Hz with which a constant-valued modal shape function is associated, though this mode is in fact not oscillatory.

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The so-called Helmholtz resonator can be viewed as dealing with the first-mode resonance resulting from a small opening on \( \partial \Omega \) by which the definition domain of problem is extended to the free-field so as to make \( f_1 > 0 \), yet the associated modal shape function remain constant-valued approximately over \( \Omega \) except near the opening. In this context of modification on \( \Omega \), there may be a multiple opening in arbitrary shape; let \( \Omega' \) denote the derived domain. Then, \( \Omega' \) and the first-mode resonance, respectively, may also be called the Helmholtz resonator and the Helmholtz resonance. It must be noted that the similarity transformation of \( \Omega' \) with a scaling factor \( \alpha \) changes \( f_i \) (\( i = 1, 2, \cdots \)) to \( f_i / \alpha \).

U-C-D of Fig. 1 is regarded as such an \( \Omega' \) when considered in the two-dimensional space. If the two channels would be rectangular in lengths \( l_k \times \) widths \( w_k \) (\( k = 1, 2 \)), then the following formula (which is derived by the usual mass-spring analogy) gives an approximate value of \( f_1 \):

\[
f_{11} = \frac{c}{2\pi} \sqrt{\frac{1}{S} \left( \frac{w_1}{l_1 + \epsilon_1} + \frac{w_2}{l_2 + \epsilon_2} \right)}
\]

where \( S \) is the area of CAV, and \( \epsilon_1 \) and \( \epsilon_2 \) are the so-called end corrections. It tells that (i) \( f_{11} \propto 1/\sqrt{S} \) for fixed \( w_k \) and \( l_k \), (ii) \( f_{11} \) increases as \( w_k \) increase or tends to null as \( w_k \) go to shrink for fixed \( S \) and \( l_k \), (iii) \( f_{11} \) remains unchanged if \( S \) and \( w_k \) are doubled but \( l_k \) are unchanged, etc. Sample values \( \{ S = 6 \text{mm}^2, \; w_1 = 0.1 \text{mm}, \; l_1 = 5 \text{mm}, \; w_2 = 0.5 \text{mm}, \; l_2 = 3 \text{mm} \} \) give \( f_{11} = 9.54 \text{kHz} \) if omitting \( \epsilon_1 \) and \( \epsilon_2 \).

It is expected that \( f_{11} \) approximates \( f_1 \) also for (non-rectangular) UCHN and DCHN when \( w_1 \) and \( w_2 \) of the formula are assigned the widths at the inlet and outlet of CAV; the validity will be checked numerically in Sect. 3.

Oscillation in \( \Omega' \) is in fact a damped one since it loses part of kinetic energy continuously by radiation at the opening. Then, a question arises whether the quality factor, \( Q \), decreases as the size of opening increases. In the case of cylindrical pipe of radius \( a \) to open to the free-field, it is positively answered since the reflection coefficient is a decreasing function in \( 2\pi a/\lambda \) (\( \lambda \) is the wavelength) \([46]\). (See also \([47]\).) It is very likely that the same holds for \( \Omega' \) with pipes in general shape, and also for those in the two-dimensional space such as ours.

The wall loss of a cylindrical pipe changes the characteristic impedance \([48]\). But the effect is negligible if \( a \simeq 0.5 \text{mm} \) or larger in the frequency range above a few \( \text{kHz} \).

### 2.4. Discretization

#### 2.4.1. Finite-element scheme

The governing equations are solved on the following boundary-fitted mesh whose elements are associated each with a single set of nodal values \( \{ p, u, v \} \) at its barycenter. The scheme is a finite-element version of MacCormack’s explicit one \([49]\) to which the so-called artificial viscosity of fourth-order is further added to suppress the numerical instability arising from the nonlinear terms.

CAV is discretized with uniform squares whose side length is denoted by \( h_c \). As seen below, all other parts are discretized on the base of \( h_c \). Hence it represents the fineness of the entire boundary-fitted mesh.

DCHN is discretized with quadrilaterals whose side lengths are enlarged smoothly from \( h_c \) to connect the outlet of CAV with DHS. The same is applied to UCHN to connect the inlet of CAV with UHS.

DHS is discretized with a finite number of rectangles in the way their side lengths are stretched toward the infinity starting from those lying on the boundary of DCHN. A function, \( \phi(s) \), that maps a finite interval to the half line \( (R^+) \) controls the rate of stretching. See Appendix for the method. The same is applied to UHS.

#### 2.4.2. Numerical values of \( q \) and \( A \)

The quadrupole and area velocity will be used as auxiliary quantities to understand the simulated result.

The term \( q = \partial^2 \rho u / \partial x_i \partial x_j \) at element \( e \) is evaluated with the nodal values of \( u \) at \( e \) and its eight surrounding elements. The area velocity, \( A \), is the two-dimensional counterpart of the volume velocity such that it is associated with a line segment \( \Gamma \) in \( D \) as

\[
A = \left| \int_{\Gamma} u_n ds \right|
\]

where \( u_n \) is the normal component of \( u \) to \( \Gamma \). \( \Gamma \) will be either of the inlet and outlet openings of CAV to monitor the airflow. The line integral is evaluated with the trapezoidal rule.

#### 2.4.3. Numerical stability

Under the condition that the square elements of side length \( h_c \) are the smallest ones in \( D \), the scheme is considered to be stable when the time step \( \Delta t \) satisfies

\[
(c + \sqrt{u^2 + v^2})\Delta t / h_c \leq 1 / \sqrt{2}
\]

for \( u = (u, v) \) in those elements.

The actual \( \Delta t \) is taken, in view of Eq. (10), as

\[
c\Delta t / h_c = 0.5
\]

by assuming \( \sqrt{u^2 + v^2} \leq 0.4c \) all the time. The amount of artificial viscosity is determined so as not to introduce an excess damping.

#### 2.4.4. Time-averaged quantities of \( p, A \) and \( q \)

Let \( T \) be the timespan for taking the time-average.

The pressure variable \( p \) is expected to realize an /s/-like sound. In this context, \( p \) may be called the sound pressure. However, there need two points to be made clear. First, the waveform of \( p \), or \( p \)-wave, represents the so-called instantaneous sound pressure observed at a fixed location.
Second, it contains a portion of static \( p_0 \) and certain infrasonic components. As far as the sound pressure level is concerned, these low-frequency components must be removed from \( p \).

Hence, the sound pressure level of \( p \)-wav, written as \( \tilde{p} \), denotes the root-mean-square (or rms) over \( T \) of the preprocessed \( p \)-wav with high-pass filter of cutoff frequency 100 Hz. Then, it will be represented in dB (re. 0 dB = 20 \( \mu \)Pa).

The time-averaged \( A \), written as \( \tilde{A} \), is given as

\[
\tilde{A} = \int Adt/T
\]

over \( T \). It will be represented in \( \text{cm}^2/\text{s} \).

The time-averaged \( q \), written as \( \tilde{q} \), is the rms of \( q \)-wav over \( T \). It will be represented in dB (re. 0 dB = 1 \text{atm/m}^2).  

2.4.5. Identification of \( f_i \) and \( Q \) 

Approximate values of the first and/or second natural frequencies associated with Eq. (7) can be found by spectral analysis on \( p \) solved with the initial condition:

\[
\begin{align*}
\{ u & = v = 0 \text{ in } \mathcal{D} \cr
p & = p_i \text{ in CAV, and } p = 0 \text{ otherwise} \}
\end{align*}
\]

in place of Eq. (4), where \( p_i \) is a small quantity, e.g., \( 1.0 \times 10^{-5} \text{atm} \). A timespan of 3 ms is sufficient for simulation to identify \( f_1 \) and/or \( f_2 \) of the present problem. Also the \( Q \)-factor associated with \( f_1 \) can be estimated from the decay rate of a band-pass filtered waveform of \( p \) processed with the center frequency \( f_1 \).

3. SIMULATION

3.1. Preliminaries

Since the natural sounds of /s/ spoken as a single sustained tone appear to be a stationary noise with a unique spectral profile when observed in the free-field, the simulated sound \( p \) is expected to have the same property. In this sense the spectral profile and the sound pressure level are considered to be the key criteria to evaluate the nature of \( p \).

In order to fix the argument, a point in DHS located 4 cm downstream the exit of DCHN was chosen as the reference point. In what follows, \( P^* \) and \( p^* \)-wav, respectively, denote the reference point and the waveform of pressure at this point. Our interest is directed to numerical solutions satisfying \( p^* \lesssim 90 \text{dB} \).

Computation was carried out in double-precision arithmetic under OpenMP environment on Windows machine with an i7-processor (3.67 GHz). Each run of simulation was done for the first 100 ms of the phenomenon. The outputs \( \{p, u, v, q\} \) at the prespecified observation points were saved in multi-channel wave files in single-precision format after down-sampling to 192 kHz. (The sampling frequency of the original output data is 68 MHz when \( h_e = 0.01 \text{mm} \).) Similarly, the outputs of \( A \) at the inlet and outlet of CAV, denoted as \( A_0 \) and \( A_1 \), respectively, were saved in another wave file.

Postprocessing on the wave files was done for the timespan of \( T = 90 \text{ms} \) by skipping the first 10 ms (for removal of the initial transient response) to obtain the sound pressure levels, power spectra, auto- and cross-correlations, etc. Unless otherwise stated, the short-term Fourier transform was obtained by DFT with a 1,024-point rectangular window running at 75%-overlapping to take the average. The symbol \( F \) is used to denote this transform such as \( Fp \) on \( p \)-wav.

3.2. Model Domain \( \mathcal{D}_1 \)

Figure 2 shows U-C-D of \( \mathcal{D}_1 \). It occupies a total area of 22.4 mm\(^2\) of which CAV is 6.0 mm\(^2\). \( \{\Gamma_0, \Gamma_1\} \) are the inlet and outlet of CAV (see Eq. (9)), and \( \{U_0, D_0\} \) are the points to which the origins of UHS and DHS, respectively, are attached (see Fig. 1).

The first two natural frequencies \( \{f_1, f_2\} \) and \( Q \) associated with \( f_1 \) were identified as \( f_1 = 9.8 \text{kHz} \) (\( Q = 4.5 \)) and \( f_2 = 18.9 \text{kHz} \) by a preliminary run of simulation as explained in 2.4.5. For comparison, \( f_1 = 9.4 \text{kHz} \) when \( w_1 \) and \( w_2 \) assume the inlet and outlet widths of CAV.

3.2.1. Choice of \( h_e \)

Six values of \( h_e = 1/N \text{ (mm)} \) were tested:

\[
N = 40, 50, 60, 80, 100 \text{ and } 120.
\]

Figure 3 summarizes the power spectra of \( p^* \)-wav, together with the associated \( \tilde{p}^* \) and \( \tilde{A}_0 \), at \( p_0 = 0.8 \text{cmH}_2\text{O} \) for six values of \( N \).

The number of quadrilateral elements (\( M_qe \)) and the machine time for executing 100 ms of simulation (\( T_{ex} \)) are as follows:

\[
\begin{array}{c|cccccccc}
N & 40 & 50 & 60 & 80 & 100 & 120 \\
\hline
M_{qe} & 72.2 & 95.7 & 117.8 & 157.7 & 205.3 & 334.9 \\
T_{ex} & 4.3 & 8.6 & 10.7 & 17.5 & 28.5 & 58.4 \\
\end{array}
\]

(Units: \( M_{qe} \) \( 10^3 \) elements, and \( T_{ex} \) hours.)

In order to determine a reasonable lower bound of \( N \), we shall obtain the Kolgomorov scale \( N_{Kol} = Re^{3/4} \) [50]
where $Re$ is the Reynolds number estimated a posteriori. Take $\tilde{A}_0$ of $N = 120$, and regard $U = \tilde{A}_0/w_0$ as the representative flow velocity of the problem where $w_0$ is the width of $\Gamma_0$. Then $U = 8.93 \text{ m/s}$, and $Re = UL/v = 1.96 \times 10^3$ ($L = 4 \text{ mm}$ and $v = 18.2 \times 10^{-6} \text{ m/s}$), from which it follows that $N_{Kol} \sim 294$, hence $N \sim 74$ ($h_c \sim 1/74 \text{ mm}$).

On the other hand, a practical upper bound of $N$ would be 120 since the cases $N > 120$ ($h_c < 1/120 \text{ mm}$), which were not tested, would solve the equations more accurately, but require a too heavy computational load in time and memory.

Taking the above facts into consideration, it was decided to choose $N = 100$ among the three cases $N = 80$, 100 and 120, i.e.,

$$h_c = 0.01 \text{ mm} \quad (13)$$

as a compromise between accuracy and computational resource. We shall use $h_c$ of Eq. (13) throughout the simulations which follow.

3.2.2. Response to different values of $p_0$

Figure 4 shows the power spectra of $p^*$-wav for five values of $p_0 = 0.8, 0.6, 0.5, 0.4$ and $0.3 \text{ cmH}_2\text{O}$, together with their $\tilde{p}^*$ and $\tilde{A}_0$. Here, the case $p_0 = 0.8 \text{ cmH}_2\text{O}$ is identical to the case $N = 100$ of Fig. 3.

At first glance, the five spectral curves come almost parallel each other, having a prominent peak near $f_1$. These peaks are certainly the first-mode resonance. There are two less prominent humps near the origin (0 kHz) and near 20 kHz. The latter is the second-mode resonance. The former is not a resonance, however. It is caused by the high-intensity spectral components of $q$ in the low-frequency region built up near the outlet of CAV (See Sect. 3.2.4 below).

The tails on the higher-frequency sides become less steep as $p_0$ increases. This is the consequence of high-frequency components in $q$ building up more and more as $p_0$ increases.

3.2.3. Sound pressure at different locations

Figure 5 shows the observation points to acquire $p$-wav. The distances of $\{U_1, \ldots, U_k\}$ from $U_0$ are $\{2.0, 4.0, 2.83, 2.83 \text{ cm}\}$; and the distances of $\{D_1, \ldots, D_8\}$ from $D_0$ are $\{2.0, 4.0, 8.0, 16.0, 2.83, 2.83, 5.66, 5.66 \text{ cm}\}$. Here, $D_2$ is the same as $P^*$.

We shall take the case $p_0 = 0.8 \text{ cmH}_2\text{O}$. Figure 6 shows the power spectra of $p$-wav at nine points in CAV, four points in DHS, and two points in UHS, together with their sound pressure levels. Here, the spectral curves are labeled the point names in bold type where the source waveforms are acquired, such as $C_1$ is the spectral curve of $p$-wav at point $C_1$. The full range of the vertical axis of Fig. 6 is twice as wide as Fig. 4, so all the spectral curves in Fig. 6 are drawn as if the resonance are less acute than those in Fig. 4.

These curves can be classified into the following three groups according to the areas they belong to.
Group I (CAV). $C_1,\ldots, C_9$ are all decreasing as the frequency increases, keeping within 5 dB of level difference among them except in the low-frequency region near the origin where the difference between $C_6$ and $C_4$ amounts to 15 dB. There appear small humps around $f_1$, which indicate the existence of resonance.

Group II (DHS). Before examining the spectral profiles of $p$, let us see how $p$ propagates in both DHS and UHS. Figure 7 is a snapshot of high-pass filtered $p$-field with cutoff frequency 100 Hz, together with $p^*$-wav for the timespan of simulation (100 ms).

$D_1,\ldots, D_4$ have prominent spectral peaks near $f_1$ showing the first-mode resonance. Regarding $\tilde{p}$ at the eight points including those whose spectral curves are not shown in Fig. 6, the following holds.

First, for all these $\tilde{p}$ except at $D_4$ which is the farthest among $D_i$ ($i = 1,\ldots, 8$), the regression equation:

$$ \tilde{p} = -2.9775x + 94.967 \text{ dB} \tag{14} $$

holds with $R^2 = 0.9971$ where $x$ is the ratio of distances $D_0D_i$ to $D_0D_1$. This conforms to the theoretical law of attenuation-by-distance ($-3$ dB per distance doubled) in the two-dimensional half or whole space.

Second, $\tilde{p}$ at $D_4$ is lower by 2.06 dB than the value estimated by Eq. (14). This must be due to the effect of stretched meshes that are too coarse to cope with the high-frequency components of $p$ propagating thereinto. A large droop in the tail on the higher-frequency side of $D_4$ is the evidence as compared to $D_1$ and $D_2$. Also $D_3$ shows a similar tendency though to a lesser degree.

Group III (UHS). The tails on the higher/lower-frequency sides of $U_1$ and $U_2$ are raised considerably as compared to those of Group II. The former comes from the second-mode resonance whose occurrence is due to a long and thin geometry of UCHN that behaves almost as an open pipe, while the latter is due to the pressure wave caused by a random fluctuation of air-jet orientation occurring just downstream of the inlet of CAV, whose spectral components concentrate below 1 kHz or thereabouts. Although this effect propagates both upstream and downstream, it is much larger in the upstream side because of the location where it originates.

3.2.4. Quadrupole in CAV

Take also the case $p_0 = 0.8 \text{ cmH}_2\text{O}$. Figure 8 shows the $q$-field in gray scale (top-left) together with the underlying $u$-field in oriented line segments (top-right) in CAV at $t = 57.0$ ms. Here, the gray scale represents $|q|$ up to $1.0 \times 10^6 \text{ atm/m}^2$ at which the scale becomes saturated. (It can be seen in red and blue for positive and negative $q$ in the pdf-version of the paper.) The areas of high-intensity $|q|$ in dark gray will be referred to as the...
hotspots. The oriented line segments, shown decimated to 1/16 in density, represents the magnitude and direction of \( \mathbf{u} \).

The figure also shows the high-frequency components above 7 kHz (high-pass filtered at cutoff 7 kHz) contained in the \( q \)-field (bottom-left) and \( u \)-field (bottom-right). The hotspots appearing therein are the agent that causes resonance of \( p \) at \( f_1 \) (and higher). In what follows, we shall look into the spatial difference in the spectrum of \( q \)-wav inside CAV.

Figure 9 shows (i) the power spectra of \( q \)-wav in two frequency-dB ranges together with \( \tilde{q} \) at nine points \( C_1, \ldots, C_9 \), and (ii) \( q \)-waves at points \( C_4, C_6 \) and \( C_8 \). Here, the spectral curves are labeled the respective point names in bold type such as \( C_4 \) for the spectral curve of \( q \)-wav at point \( C_1 \), as in the case of \( p \)-wav.

In the top-left graph of (i), all nine curves are decreasing as the frequency increases toward 40 kHz within 10 dB of level difference at maximum among them and without a hump around \( f_1 \). Note that in terms of rms values, the level difference is 3.9 dB.

The top-right graph of (i) is a zoom-in view of the above by limiting to the audio-frequency range. Here, three curves labeled \( \tilde{C}_4, \tilde{C}_6 \) and \( \tilde{C}_8 \) in thick line are the smoothed ones obtained by quadratic curve-fitting on \( C_4, C_6 \) and \( C_8 \). They highlight in the frequency domain the essential nature of the respective \( q \)-waves that differ from each other in the amplitude, acuteness, and density of their randomly oscillating motion shown in the time domain.

| \( q \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) | \( C_7 \) | \( C_8 \) | \( C_9 \) |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| dB    | 98.4    | 95.3    | 97.6    | 97.5    | 96.5    | 98.5    | 98.6    | 94.7    | 96.0    |

(Units: \( \tilde{q} \) dB (re. 0 dB = 1 atm/m²))

![Figure 9](image9.png)

**Fig. 9** Properties of \( q \)-wav in CAV (\( p_0 = 0.8 \text{cmH}_2\text{O} \)).

(i) Top: Power spectra with their rms-values at all nine points; (ii) Bottom: Waveforms at three selected points.

\( \tilde{C}_4 \) slants down the least as the frequency increases, being almost flat below 6 kHz; it resembles a white noise the most. \( \tilde{C}_6 \) is lower than \( \tilde{C}_4 \) by about 5 dB. Point \( C_4 \) is at the middle-left of CAV, directly downstream the inlet of CAV, and \( C_8 \) is at the bottom-center.

In contrast to the above two, \( \tilde{C}_6 \) slants down the most, taking the largest level at low frequencies among all nine curves, about 15 dB high from that of \( \tilde{C}_4 \) near 0 Hz. \( C_6 \) is at the middle-right of CAV, which is the nearest point to the outlet of CAV. This area is where the flow becomes stagnant, which is considered to boost the intensity of \( q \) at low-frequencies due to slow-down of transport velocity of hotspots, thereby the intensity of \( p \) near 0 Hz is raised.

At this point, let \( \rho_{ij} \) be the normalized correlation function between \( q \)-waves at points \( C_i \) and \( C_j \). (It is the auto-correlation if \( i = j \), or the cross-correlation if \( i \neq j \).) Here, every point is apart from others by 0.37 mm or more. Figure 10 presents the graphs of \( \rho_{4j} \) and \( \rho_{6j} \) plotted against the delay \( \tau \) (\( \tau \geq 0 \)) for \( j = 1, \ldots, 9 \), as the most contrasting cases.

**Case** \( \rho_{4j} \): \( \rho_{44} \) decays rapidly from 1.0 as \( \tau \) increases, dropping to less than 0.02 in magnitude when \( \tau > 0.05 \text{ ms} \), and every \( \rho_{4j} \) for \( j \neq 4 \) stays within the magnitude less than 0.02 for all \( \tau \geq 0 \). This implies that \( q \)-wav at \( C_4 \) has a nature of a white-noise strongly and is almost independent of \( q \)-wav at other points.

**Case** \( \rho_{6j} \): \( \rho_{66} \) decays slowly in contrast to \( \rho_{44} \), as well as \( \rho_{6j} \) (\( j \neq 6 \)) keep a noticeable magnitude even at large values of \( \tau \), among which \( \rho_{63} \) undulates the most (\( C_3 \) is at the top-right, near the outlet of CAV).

The above result reveals that the quadrupole tends to lose the randomness both in time and space where the place is far from the air jet.

### 3.3. Model Domains \( \mathcal{D}_2 \) and \( \mathcal{D}_3 \)

(i) \( \mathcal{D}_2 \) with different values of \( p_0 \)

\( \mathcal{D}_2 \) (Fig. 11) is configured as a modification on \( \mathcal{D}_1 \) such that CAV is added with a substructure of area 5.0 mm² at the bottom, DCHN is extended by 1.5 mm with a ‘neck’ at the outlet of CAV, and the length of UCHN is doubled. Thus the U-C-D occupies an area of 35.1 mm² in which
CAV is 11.0 mm$^2$. The natural frequencies $\{f_1, f_2\}$ were identified as $[6.1, 9.2]$ kHz; and $Q = 8.8$ for $f_1$. ($f_{HI} = 5.9$ kHz when $w_1$ and $w_2$ assume the inlet and outlet widths.)

Figure 12 shows the power spectra of $p^*$-wav, together with the associated $\tilde{p}^*$ and $\tilde{A}_0$, at $p_0 = 0.6$, 0.4 and 0.3 cmH$_2$O. The first-mode resonance appears around $f_1$. The second-mode one can be seen around $f_2$ for $p_0 = 0.4$ and 0.3 cmH$_2$O.

As a supplementary result, $f_1$ increases from 5.5 kHz to 6.3 kHz and $Q$ decreases 11.6 to 7.8, when the width of DCHN widens from 0.55 mm to 0.85 mm at neck and from 4.4 mm to 6.8 mm at exit.

(ii) $D_3$ with different lengths of DCHN

In comparison with $D_2$, $D_3$ (Fig. 13) is configured so that CAV is a slender one with DCHN having a variable neck length $d$ mm that takes on the three values:

![Fig. 13 U-C-D of $D_3$.](image)

$\tilde{p}^*$ 92.4 87.2 81.9
$\tilde{A}_0$ 11.7 9.4 8.0

(Units: $p_0$ cmH$_2$O, $\tilde{p}^*$ dB, and $\tilde{A}_0$ cm$^2$/s)

$\begin{array}{c|ccc}
  \text{d} & 0.0 & 2.0 & 4.0 \\
  \tilde{p}^* & 89.3 & 90.4 & 91.4 \\
  \tilde{A}_0 & 11.4 & 11.6 & 11.5 \\
\end{array}$

(Units: $\tilde{p}^*$ dB, and $\tilde{A}_0$ cm$^2$/s)

Fig. 14 Power spectra of $p^*$-wav together with the associated $\tilde{p}^*$ and $\tilde{A}_0$ for three values of $d$ ($p_0 = 0.6$ cmH$_2$O).

$d = 0.0, 2.0$ and 4.0 (15)

and the same UCHN as that of $D_1$. Thus the U-C-D occupies an area of $26.6 + 0.75d$ mm$^2$.

The natural frequencies $\{f_1, f_2\}$ for $d$ of Eq. (15), respectively, were identified as $[8.1, 19.0]$ kHz, $[6.4, 18.4]$ kHz, and $[5.4, 18.1]$ kHz; and $Q$ was $[4.9, 8.4, 11.7]$ for the respective values of $f_1$. ($f_{HI} = [7.5$ kHz, $6.2$ kHz, $5.5$ kHz] when $w_1$ and $w_2$ assume the inlet and outlet widths.) In comparison with $D_2$, every $f_2$ of $D_3$ is about twice as high as that of $D_2$. This results largely from the length of UCHN, 10 mm in $D_3$ vs. 20 mm in $D_2$.

Figure 14 shows the power spectra of $p^*$-wav for $d$ of Eq. (15), together with the associated $\tilde{p}^*$ and $\tilde{A}_0$, at $p_0 = 0.6$ cmH$_2$O. As $d$ increases, $f_1$ decreases and the peak level at $f_1$ rises with increasing sharpness of resonance. Also, increase of $d$ results in decrease of $f_2$ but to a lesser degree than $f_1$.

4. DISCUSSION AND CONCLUSIONS

The proposed models have been shown to produce /s/-like sounds in terms of power spectrum. The following discusses the spatial and temporal behavior of $q$-field in the case of $D_1$ in more detail. (The cases of $D_2$ and $D_3$ parallel more or less the case of $D_1$.)

In Fig. 8, the air jet collapses at a very early stage of development. It is caused by a strong entrainment occurring in the small, semi-closed domain of CAV; accordingly, the resultant turbulent-flow prevails throughout CAV. The $q$-field shown as the top-left image is characterized by a patchy pattern sprinkled with hotspots. An animated sequence of such snapshots (sampled at every 0.01 ms) exhibits that (I) the hotspots are born continuously around the collapsing area of air jet, as well as born almost continuously where a portion of turbulent flow happens to
collide violently with the boundary wall or within itself, (II) once born, they start to deform and decay while moving around, and (III) eventually they disappear either by dying out inside or exiting from CAV. An animated sequence of high-pass filtered $q$-field (cutoff at 7 kHz, the bottom-left image shows a snapshot) reveals a finer and more dynamic structure of hotspots that essentially contribute to the resonance at $f_1$.

The three $q$-waves at the bottom part of Fig. 9 are the temporal records of hotspots passing randomly one after another over the respective locations. It is apparent that the faster in speed or the smaller in size the hotspots, the shorter the period of spike-like swings in the positive and negative directions, which results in higher-frequency components in the spectrum of $q$-wave, hence in the spectrum of $p$-wav.

Regarding the quality of tone, each $p^*$-wav of $D_2$ and $D_3$ sounds like /s/ to some degree but it is a little rough by author’s subjective judgment. The reason for the roughness may be attributed to an insufficient number of hotspots produced with the two-dimensional models.

We shall return to the argument posed by Meyer-Eppler [5] on the recovery of /s/ after extraction of upper incisors. The phenomenon can be interpreted on the basis of Helmholtz resonance as follows, under the assumption that the $q$-field inside the mouth cavity is essential. At the moment the upper incisors are extracted, the condition of Helmholtz resonance for a proper /s/ is lost since $Q$ is lowered and $f_1$ is raised by an enlarged opening at the exit of mouth (see Sect. 2.3.2). After a short lisping stage, the patient acquires consciously or unconsciously an articulatory configuration suitable for a proper resonance by narrowing the opening and/or enlarging the volume of front cavity to raise $Q$ once lowered and to lower $f_1$ once raised.

Our numerical result with the straight channel at the exit of mouth is opposed to the common theory [2]. However, it does not mean that the teeth has no effect. Under the existence of teeth, the hotspots from the resultant turbulence would contribute to the production of /s/ in corporation with those explained in (I)-(III) above.

At last, we note the working range of $p_0$. Either of $D_2$ and $D_3$ produced /s/-like sounds at $p^* \approx 90$ dB with $p_0 = 0.6$ cmH$_2$O. However, Hixon [11] reported that the introral pressure (IOP) for /s/ (averaged over nine speakers) took on 4.34, 7.20 and 10.45 cmH$_2$O at three speech effort levels. This suggests that the two-dimensional configuration brings about a lower working range of IOP than the three-dimensional one. The shape of inflow constriction mentioned in Sect. 1 must most probably be the reason for the difference.

In conclusion, we claim that the numerical evidence presented herein helps us better understand the underlying mechanism of the production of /s/ by highlighting the role of channel turbulence in terms of hotspots. The remaining issue is to locate the vital area of turbulence in terms of $q$-field that produces high-frequency components to cover $f_1$, with and without the teeth, in the three-dimensional configuration.

It is worth to mention the production of /ʃ/-like sound in relation to the similarity transformation on the domain. If $D_2$ or $D_3$ is dilated with $\alpha = 1.5 \sim 2$, then $f'_1 = f_1/\alpha$ comes in a typical frequency range for the prominent spectral peak of /ʃ/ to stand, so that the dilated domain would become a model domain for production of /ʃ/-like sound. In fact, the tone quality of the resultant sounds from such dilated domains resembles a kind of /ʃ/ by author’s subjective judgment. Such a dilation expands the area of the concerned domain by a factor of $\alpha^2$, or the volume by a factor of $\alpha^3$ in case of the three-dimensional space. Interestingly, the time-stretched waveform $\hat{p}(t)$, $\hat{p}(t) = p^*-\text{wav}(t/\alpha)$ derived from $p^*-\text{wav}(t)$ of the original $D_2$ or $D_3$ gives a similar auditory impression.

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REFERENCES

[1] G. Fant, Acoustic Theory of Speech Production (Mouton, The Hague, 1960), Chap. 2.6, pp. 169–204.
[2] J. C. Catford, Fundamental Problems in Phonetics (Edinburgh University Press, Edinburgh, 1977), pp. 60–61 and pp. 154–155.
[3] K. N. Stevens, Acoustic Phonetics (MIT Press, 1998), Sect. 2.2, pp. 100–117, and Sect. 8.1, pp. 379–485.
[4] W. Meyer-Eppler, “Untersuchung zur Schallstruktur der stimmhaften und stimmlosen Geräusche,” Z. Phon., 7, 89–104 (1953).
[5] W. Meyer-Eppler, “Zum Erzeugungsmechanismus der Geräusche,” Z. Phon., 7, 196–212 (1953).
[6] G. Hughes and M. Halle, “Spectral properties of fricative consonants,” J. Acoust. Soc. Am., 28, 303–310 (1956).
[7] J. M. Heinz, Sound Generation by Turbulent Flow in an Acoustic Resonator (M.S. Thesis, Dept. Elec. Eng. Comp. Sci., MIT, 1958).
[8] P. Streivens, “Spectra of fricative noise in human speech,” Lang. Speech, 3, 32–49 (1960).
[9] J. M. Heinz and K. N. Stevens, “On the properties of voiceless fricative consonants,” J. Acoust. Soc. Am., 33, 589–596 (1961).
[10] W. Jassem, “The formants of fricative consonants,” Lang. Speech, 8, 1–16 (1965).
[11] T. J. Hixon, “Turbulent noise sources for speech,” Folia Phoniatr., 18, 168–182 (1966).
[12] K. N. Stevens, “Airflow and turbulence noise for fricative and stop consonants; static considerations,” J. Acoust. Soc. Am., 50, 1180–1192 (1971).
[13] J. L. Planagan, Speech Analysis Synthesis and Perception
T. Taguti: Numerical Production of /s/-Like Sound

(Springer Verlag, New York, 1972).

[14] J. L. Flanagan, K. Ishizaka and K. L. Shipley, “Synthesis of speech from a dynamic model of the vocal cords and vocal tract,” Bell Syst. Tech. J., 54, 485–506 (1975).

[15] J. L. Flanagan and K. Ishizaka, “Automatic generation of voiceless excitation in a vocal cord-vocal tract speech synthesizer,” IEEE Trans. Acoust. Speech Signal Process., 24, 163–170 (1976).

[16] C. H. Shadle, The acoustics of fricative consonants (Research Laboratory of Electronics Technical Report 506, Massachusetts Institute of Technology, MA, 1985).

[17] L. M. P. Pastel, Turbulent noise sources in vocal tract models (M. S. Dissertation, Massachusetts Institute of Technology, 1987).

[18] S. J. Behrens and S. E. Blumenstein, “Acoustic characteristics of English voiceless fricatives: A descriptive analysis,” J. Phon., 16, 295–298 (1988).

[19] S. J. Behrens and S. E. Blumenstein, “On the role of the amplitude of the fricative noise in the perception of place of articulation in voiceless fricative consonants,” J. Acoust. Soc. Am., 84, 861–867 (1988).

[20] P. Badin, “Acoustics of voiceless fricatives: Production theory and data,” Speech Transmission Laboratory Quarterly Progress and Status Report, 30, Roy. Inst. Tech., Stockholm, 33–55 (1989).

[21] S. G. Fletcher, Palatometric specifications of stop, affricate and sibilant sounds,” J. Speech Hear. Res., 32, 736–748 (1989).

[22] C. H. Shadle, “Articulatory-acoustic relationships in fricative consonants,” in Speech Production and Speech Processing, W. J. Hardcastle and A. Marchal, Eds. (Kluwer Academic Publishers, Dordrecht, 1990), pp. 187–209.

[23] S. G. Fletcher and D. G. Newman, “[s] and [j] as a function of lingualpalatal contact place and sibilant groove width,” J. Acoust. Soc. Am., 89, 850–858 (1991).

[24] C. H. Shadle, “The effect of geometry on source mechanisms of fricative consonants,” J. Phon., 19, 409–424 (1991).

[25] D. H. Whalen, “Perception of the English /s/-/z/ distinction relies on fricative noises and transitions, not on brief spectral slice,” J. Acoust. Soc. Am., 90, 1776–1785 (1991).

[26] A. Boothroyd and L. Medwetsky, “Spectral distribution of [s] and the frequency response of hearing aids,” J. Acoust. Soc. Am., 90, 1776–1785 (1991).

[27] K. N. Stevens, S. E. Blumstein, L. Glicksman, M. Burton and A. Boothroyd and L. Medwetsky, “Spectral distribution of /s/ from a limited set of articulatory parameters,” J. Acoust. Soc. Am., 91, 2979–3000 (1992).

[28] P. Perrier, L.-J. Boe and R. Sock, “Vocal tract area function estimation from midsagittal dimensions with CT scans and a vocal tract cast: modeling the transition with two sets of coefficients,” J. Speech Lang. Hear. Res., 35, 53–67 (1992).

[29] I. R. Titze, “Phonation threshold pressure: A missing link in glottal aerodynamics,” J. Acoust. Soc. Am., 91, 2926–2935 (1992).

[30] N. Nguyen, P. Hoole and A. Marchal, “Regenerating the spectral shapes of [s] and [j] from a limited set of articulatory parameters,” J. Acoust. Soc. Am., 96, 33–39 (1994).

[31] L. F. Wilde, Analysis and synthesis of fricative consonants (PhD Thesis, Dept. Electr. Eng. Comp. Sci., MIT, 1995).

[32] C. H. Shadle and C. Scully, “The articulatory-acoustic-aerodynamic analysis of [s] in VCV sequences,” J. Phon., 23, 53–66 (1995).

[33] T. Cho, S.-A. Jun and P. Ladedefoged, “Acoustic and aerodynamic correlates of Korean stops and fricatives,” J. Phon., 30, 193–228 (2002).

[34] S. L. Nissen and R. A. Fox, “Acoustic and spectral characteristics of young children’s fricative productions: a developmental perspective,” J. Acoust. Soc. Am., 118, 2570–2578 (2005).

[35] S. Narayanan and A. Alwan, “Noise source models for fricative consonants,” IEEE Trans. Speech Audio Process., 8, 328–344 (2000).

[36] M. Tabain, “Variability in fricative production and spectra: implications for the hyper- hypo- and quantal theories of speech production,” Lang. Speech, 44, 57–94 (2001).

[37] M. Toda, Étude articulatoire et acoustique des fongatives sibilants (Thèse de doctorat de phonétique, Université de Paris III, 2009).

[38] M. S. Howe and R. S. McGowan, “Aeroacoustics of [s],” Proc. R. Soc. Lond. A, 561, 1005–1029 (2005).

[39] S. Adachi, Y. Tanabe and K. Honda, “Numerical simulation of fricative sound sources,” Proc. Conf. Turbulences, Zentrum für Allgemeine Sprachwissenschaft, October (2005).

[40] X. Grandchamp, A. Van Hirtum, X. Pelorson, K. Nozaki and S. Shimojo, “Towards sibilant /s/ modelling: preliminary computational results,” Paper 2074, Acoustics 08 Paris (2008).

[41] A. Van Hirtum and K. Nozaki, “Numerical simulation of airflow through simplified vocal tract geometries relevant to speech production: preliminary results,” Proc. V Eur. Conf. Comp. Fluid Dynamics ECCOMAS CFD 2010, Lisbon (2010).

[42] T. Yoshinaga, K. Nozaki and S. Wada, “Study on production mechanism of sibilant [s] using simplified vocal tract model,” Proc. Internoise 2015, pp. 95–102 (2015).

[43] T. Yoshinaga, K. Nozaki and S. Wada, “Experimental and numerical investigation of the sound generation mechanisms of sibilant fricatives using a simplified vocal tract model,” Phys. Fluids, 30, 035104 (2018).

[44] T. Taguti,”Production of sibilant [s] by means of computational fluid dynamics,” Proc. Autumn Meet. Acoust. Soc. Jpn., Paper 1-1-1 (2015) (in Japanese).

[45] M. J. Lighthill, “On sound generated aerodynamically I. General theory,” Proc. R. Soc. Lond. A, 211, 568–587 (1952).

[46] H. Levine and J. Schwinger, “On the radiation of sound from an unflanged circular pipe,” Phys. Rev., 73, 383–406 (1948).

[47] L. L. Beranek, Acoustics (McGraw-Hill, New York, 1954), Chap. 5.

[48] A. H. Benade, “On the propagation of sound in a cylindrical conduit,” J. Acoust. Soc. Am., 44, 616–623 (1968).

[49] R. W. MacCormack, “A numerical method for solving the equations of compressible viscous flow,” AIAA J., 20, 1275–1281 (1982).

[50] P. A. Davidson, Turbulence: An Introduction for Scientists and Engineers, 2nd ed. (Oxford University Press, 2015), Chap. 7.

APPENDIX: DISCRETIZATION OF DHS

Let \( \phi(s) \) be a smooth function in \( s, s \in I = [0, l] \), that satisfies (i) \( \phi(0) = 0 \), (ii) \( \phi'(0) = 1 \), (iii) \( \phi''(s) \geq 0 \), and (iv) \( \phi(s) \to \infty \) as \( s \to l \), and let \( s_i \) be such that \( s_i = \frac{lh_0}{l} \) (\( i = 0, 1, \ldots, m \)) where \( h_0 = l/m \).

By use of \( \phi(s) \), DHS is discretized into rectangles with \( m \) vertical lines \( L_i \) (\( i = 0, 1, \ldots, m \)) and \( 2m + 1 \) horizontal lines \( H_j \) (\( j = 0, \pm 1, \ldots, \pm (m-1) \)):

\[
V_i = \{ (x, y) \mid x = \phi(s_i), -\infty < y < \infty \} \\
H_j = \{ (x, y) \mid 0 \leq x < \infty, y = \text{sgn}(j)\phi(s_{j}^{\pm}) \}.
\]

Introduce two points \( a \) and \( b \), \( (0 < a < b < l) \), to construct \( \phi(s) \) with three piecewise functions as

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\[
\phi(s) = \begin{cases} 
\phi_a(s) & \text{if } 0 \leq s \leq a \\
\phi_b(s) & \text{if } a < s \leq b \\
\phi_l(s) & \text{if } b < s < l
\end{cases}
\]
where
\[
\begin{align*}
\phi_a(s) &= s \\
\phi_b(s) &= s + (B - 1) \int_a^s \frac{(s - a)/(b - a)}{ds} \\
\phi_l(s) &= \phi_b(b) + B \int_b^l \frac{ds}{1 - ((s - b)/(l - b))^2}
\end{align*}
\]

Put \(\Delta V_i = V_{i+1} - V_i\). Then it holds, for small \(h_0\),
\[
\begin{align*}
\Delta V_i &= h_0 \text{ if } s_i < a \\
\Delta V_i &\approx h_0 \phi'(s_i) \text{ if } s_i \geq a.
\end{align*}
\]
The same property holds for \(\Delta H_j = |H_{j+1} - H_j|\).

The following is a guideline for selecting concrete values of \(\{h_0, m, a, b, l, B\}\) to discretize DHS. Here, \(E\) denotes the line segment representing the exit opening of DCHN. The theoretical \(\phi(l - 0) = \infty\) shall be replaced with \(\phi(l) = \phi_\infty\) where \(\phi_\infty\) is a large number, e.g. \(10^{20}\), for numerical computation.
1. Interface with DCHN. Put \(a = (4/3)Y\) (\(Y\) is the half length of \(\Gamma\)), and \(h_0 = h_e\) (\(h_e\) is the length of element sides lying on \(E\)).
2. Stretch control at \(s = b\). Let \(\lambda\) be the wavelength of plane pressure wave oscillating at 5 kHz. Put \(B = (\lambda/20)/h_0\). Then, determine \(b\) so as to satisfy \(\phi(b) = 2\lambda\) and \(\phi'(b) = B\).
3. Size of interval \(I\). Set \(m = [(b + 2a)/h_0]\) ([\(\xi\)] is the integral part of \(\xi\)), and \(l = mh_0\).

**Note 1.** The discretized elements at the positive side of \(V_{m-1}\), as well as at the positive side of \(H_{m-1}\) and at the negative side of \(H_{m+1}\), are all very large rectangles because of \(\phi(l) = \phi_\infty\); they are called the farthest rectangles with (two or four) sides that are very large.

**Note 2.** The unknowns \(p\) and \(u\) on the farthest rectangles are subject to the boundary condition Eq. (3).