A statistical method for pitch measurement and evaluation

Xiaoyu Cai¹,², Junjie Wu³, Yuan Li², Xuedian Zhang¹, Lihua Lei² and Jiasi Wei²,³,*

¹School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, 516 Jungong Rd., Shanghai, China.
²Shanghai Institute of Measurement and Testing Technology, 1500 Zhangheng Rd., Shanghai, China
³*Corresponding author e-mail: caixiaoyu@simt.com.cn

Abstract. Nano grating is one of the common standards in micro-nano scale dimensional measurement and calibration. Errors terming from the measurement system, measurement procedure, data process and the object (specimen) affect the grating evaluation result. Either more precise measurement systems or optimization in methodology is underdeveloped to realize more accurate measurement. In this study, a measuring procedure, using scanning probe microscopy to measure 1D grating, is put forward firstly, mainly to reduce cosine error. Then a measuring statistical model is established based on the statistical characteristics of measurement system, terming from both the machine and the standard. By measuring the grating area for enough times, the mathematical equations are fitted and the statistical characteristic variables, i.e. the mathematical expectation and variance of both the measurement machine error and the grating are to be solved. Practically, the grating is measured in roughly the same area for several times using a nano measurement machine, with unknown error. The grating pitch as well as the errors of the machine stage were evaluated.

1. Introduction

Nano gratings, including 1D and 2D gratings, are common references in nano-dimensional metrology, whose pitches characterize lateral accuracy and reliability. In grating measurement, errors from not only specimen and measurement instrument as well as measurement procedure and data process. On specimen, the surface roughness, the uniformity of the grating pattern should bring errors to the pitch while the influences of sampling interval, surface quality and noises in sampling data should be decreased by data processing [1, 2]. On instrument and measurement procedure, the influences of shape and tilt angle of probe tip should be avoided mechanically. Particularly, a metrological scanning probe microscopy is used to scan a certain area on the grating surface, line by line forming a micro-image of the grating. In order to reduce the cosine error, scan axis is configured to be perpendicular to the grating lines, or the line for evaluating is manually cut off as close as to the perpendicular angle.

2. Measurement Method

In order to reduce the cosine error, angle between the scan axis which is ideally perpendicular to the grating lines and the x- or y-piezo axis of the scanner of the measurement machine is calculated by pre-experiments of the grating. However, this approach will involve coupling error of the x- and the y-
driver besides of single drive positioning error. In this study, a novel approach is proposed to scan the grating and the cosine error is eliminated by mathematical method.

2.1. The measurement model

For typical measurement, the model composes of the probe, the motion stage, working as scanner, and the grating sample to be measured. The motion stage has independent motion vectors of X-axis, Y-axis and Z-axis, whose errors are independent.

The scanner’s coordinate system x-y-z sets its X-axis, Y-axis and Z-axis in line with the motion stage (figure 1-a), while the sample’s coordinate system x’-y’-z’ sets its X-axis along with the width direction of the grating and Y-axis along with the ideal length direction of the grating (figure 1-b).

![Figure 1. a. Scanner’s coordinate system, b. Sample’s coordinate system.](image)

There are four hypotheses in this model. (1)There is usually an unknown angle $\theta$ between the x and x’, which causes cosine error during measurement, as the same circumstance in practice. (2) The angle between z and z’ is small enough to ignore. (3) The origins of both coordinates are coincident, i.e. the scanning starting point. Based on the above hypotheses, the simplified coordinate system is illustrated in figure 2. (4) There’s no gross error in measurements. And the pitches measured accord with a same distribution, meanwhile the errors of each scanner also, respectively, accord with a same distribution. Moreover, the uncertainty from the pitch and the scanner is independent from each other.

![Figure 2. The simplified measurement coordinate.](image)

2.2. The measurement procedure and the evaluation

The first scan angle is 0° along the x-axis, in the scanner coordinate system. P1 is the result of the pitch evaluated from the first scan. The second scan angle is perpendicular to the first one, i.e. along the y-axis. P2 is the result of the pitch evaluated of the second scan[3]. In both scans, since only one out of the x-scanner and the y-scanner is working, while the other one is regarded as keeping still, no coupling error will be involved. Figure 3 shows the two scanning routes on the gratings and geometric relation of the pitches. Ideally, the grating pitch P can be solved in equation (1).
However, the errors term from the X-Y motion stages and that from measurement results of the sample are considered in this method, and statistically analyzed. The mathematical model of the measurement comes to the equation (2) and (3),

\[ P_1 = \frac{1}{\cos \theta} P + \delta_x \]  
\[ P_2 = \frac{1}{\sin \theta} P + \delta_y \]  

where \( \delta_x \) is the error of the measurement machine in x-axis, \( \delta_y \) is the error of the measurement machine in y-axis, \( \theta \) is the angle between the coordinates of the motion stage and the grating on the sample. The final pitch result, \( P \), is derived into the equation (4).

\[ P = \frac{1}{\sqrt{\frac{1}{(P_1 - \delta_x)^2} + \frac{1}{(P_2 - \delta_y)^2}}} \]  

Then the mathematical expectation and the variance of pitch evaluated is respectively \( E(P_1), E(P_2), D(P_1), D(P_2) \), by unbiased estimation. Derived from equation (4), the mathematical expectation of the final pitch result \( E(P) \) and the variance of the final pitch result \( D(P) \) are got from equation (5) and equation (6):

\[ E(P) = \frac{1}{\sqrt{\frac{1}{(E(P_1) - E(\delta_x))^2} + \frac{1}{(E(P_2) - E(\delta_y))^2}}} \]
\[ D(P) = \frac{1}{1 + \frac{1}{D(P_1) - D(\delta_x)}} + \frac{1}{1 + \frac{1}{D(P_2) - D(\delta_y)}} \]  

where \( E(\delta_x), E(\delta_y), D(\delta_x), D(\delta_y) \) are respectively the mathematical expectation and the variance of motion error in x-axis and error in y-axis.

Based on the hypotheses (4), \( E(P), D(P), E(\delta_x), E(\delta_y), D(\delta_x), \) and \( D(\delta_y) \) are constants to be solved in the measurement. Meanwhile, \( E(P_1), E(P_2), D(P_1), D(P_2) \) are calculated by the measurement result. By a non-linear fit of (5) and (6), all the unknown value can be solved.[4]

In this statistic evaluation method of 1D-pitch measurement, the pitch of the sample is indicated by \( E(P) \), while \( \sigma(P) \), the square root of \( D(P) \), is regarded as the measurement error. Furthermore, the errors of the x- and y-scanner are solved to be \( E(\delta_x) \) and \( E(\delta_y) \), with a variance of \( D(\delta_x) \), and \( D(\delta_y) \), which means by this method the error of the scanners are separated from the measurement error and estimated.

3. Discuss on the Scan Angle \( \theta \)

In the model, the angle \( \theta \) is not specified, yet still has impacts on the accuracy and efficiency of the measurement and evaluation. Practically, when the angle \( \theta \) is close to 0° or 90°, the measurement is difficult to carry out within the limited scannig range or time. The range of angle is discussed in this section. For a sample with ideal homogeneity, the error is only from measurement machine. Derived from equation (1), the standard deviation \( \Sigma_P \) is as equation (8),

\[ \sigma_p = \sqrt{\frac{D(P_1)}{1 + \left(\frac{P_1}{P_2}\right)^2} + \frac{D(P_2)}{1 + \left(\frac{P_1}{P_2}\right)^2}} = \sqrt{\frac{\sigma_1^2}{1 + \left(\frac{P_1}{P_2}\right)^2} + \frac{\sigma_2^2}{1 + \left(\frac{P_1}{P_2}\right)^2}} \]

where, \( \sigma_1 \) and \( \sigma_2 \) is respectively standard deviation of \( P_1 \) and \( P_2 \). Assume that \( D(P_1) \geq D(P_2) \), \( \sigma_2 = k \cdot \sigma_1 \), \( \left(\frac{P_1}{P_2}\right)^2 = a \), thus

\[ \sigma_p = \sqrt{\frac{\sigma_1^2}{1 + \left(\frac{P_1}{P_2}\right)^2} + \frac{(P_1)^6 \cdot \sigma_2^2}{(1 + \left(\frac{P_1}{P_2}\right)^2)^2}} = \sqrt{\frac{1 + k^2 a^3}{(1 + a^2)^2} \cdot \sigma_1} \]

(8)

When \( a = \frac{1}{k} \cdot \sigma_p \) could reach the minimum, i.e.,

\[ \sigma_{p_{\min}} = \frac{\sigma_1 \sigma_2}{\sigma_1 + \sigma_2} \leq \sqrt{k} \sigma_1 = \min (\sigma_1, \sigma_2) \]

(9)

and on that occasion,

\[ \theta = \arctan \left( \frac{1}{\sqrt{k}} \right) = \arctan \left( \frac{\sigma_1}{\sigma_2} \right) \]

(10)
Table 1 illustrates how the range of $\sigma_p$, in form of the multiple of $\sigma_1$, depends on the angle $\theta$, which can be referred in this method practice when loading the sample.

| range of $\theta$ | range of $\sigma_p$ ($\times \sigma_1$) |
|-------------------|----------------------------------------|
| [30°, 60°]        | [0.5, 0.6615]                          |
| [15°, 75°]        | [0.5, 0.9014]                          |

4. Experiments and Discussion

The measurement of $P_1$ and $P_2$ needs to be carried out for several times to get enough raw data. Before each measurement, $\theta$ is suggested to be readjust to another acute angle just by rotating the sample roughly.

A 1D grating sample with the calibrated pitch 3.005 $\mu$m is measured by a nano measurement machine (NMM) with an AFM probe. The sample is fabricated with semiconductor processing. The machine has a typical stage with x-y-z isolated motion, which is measured by interferometers.

The experiments were carried out at least four times to cover the demonstrated fan-like pattern as figure 4, so-called fan-scan. The fan-scan is designed to further eliminate the hysteresis of the motion stages. When loading the sample on the sample stage, the angle $\theta$ was managed to be within the range of [30°, 60°] just by hand roughly. Thanks to the homogeneity of the sample, the scanning start point O was also unspecified. The scanning direction is respectively along x, y, -x and -y, as the arrows illustrated in figure 4. After one set of fan scan, the sample was turned by a small angle by hand and another set of fan-scan started. By each fan-scan, a pair of $P_1$ and $P_2$ was calculated. At least four times of fan-scan is needed in this method.

Figure 4. Fan-like scanning.

Figure 5. Non-linear fit result.
In order to get a more accurate and illustrative result, extra experiment data was contributing to the solution. A non-linear curve was fitted as figure 5, as well as the solutions which were shown in table 2. Due to the high accurate performance of the NMM and the low ambient noise, the result showed sub-nanoscale errors and an accurate pitch, compared with the calibrated data.

In conclusion, the method is not only for the sample calibration, but also reference to instrument calibration and evaluation when using 1D grating transferring standard. In the future work, the appropriated time of the fan-scan experiment set needs to be figure out for time-efficiency as well as result confidence. More experiments need to be carried out in different instruments to find out the adaptability of the method.

| Table 2. Result of the pitch measurement. |
|-----------------------------------------|
| $E(P)$ | $E(\delta_x)$ | $E(\delta_y)$ | $P_{\text{cal}}$ |
| 3.0054$\mu$m | 0.0006$\mu$m | 0.0005$\mu$m | 3.005$\mu$m |

5. Acknowledgments

This work was financially supported by National Key Research and Development Project “Basic Technology and Key Components on Manufacturing” (2019YFB2004900), Scientific Research Programs of Shanghai Municipal Bureau of Quality and Technical Supervision (2018-06) and Shanghai sailing program (19YF1441700).

References

[1] Dai, Gaoliang, Ludger Koenders, Frank Pohlenz, et al. "Accurate and traceable calibration of one-dimensional gratings." Measurement Science and Technology 16.6(2005):1241-1249.

[2] Huang Q, Gonda S, Misumi I, et al. Research on pitch analysis methods for calibration of one-dimensional grating standard based on nanometrological AFM[J]. The International Society for Optical Engineering 2006:628007.

[3] Lei LH, Liu Y, Chen X, et al. Fast and Accurate Calibration of 1D and 2D Gratings. Advanced Materials Research 317–319(2011):2196–2203

[4] HP Wang, L Zhao, DS Wang, et al. Uniform design experimental data based on MATLAB multi-nonlinear minimum square-by-multiplier fitting [J]. Chemical Engineering & Equipment, 09 (2010):29-32