1 Introduction

Nowadays, it is straightforward that energy efficiency is a crucial aspect of embedded systems where a huge number of small and very specialized autonomous devices interacting together through many kinds of media (wired/wireless network, bluetooth, GSM/GPRS, infrared . . . ). Moreover, we know that the uniprocessor paradigm will no longer hold in those devices. Even today, a lot of mobile phones are already equipped with several processors.

In this ongoing work, we are interested in multiprocessor energy efficient systems, where task durations are not known in advance, but are know stochastically. More precisely, we consider global scheduling algorithms for frame-based multiprocessor stochastic DVFS (Dynamic Voltage and Frequency Scaling) systems. Moreover, we consider processors with a discrete set of available frequencies.

In the past few years, a lot of work has been provided in multiprocessor energy efficient systems. Most work was done considering static partitioning strategies, meaning that a task was assigned to a specific processor, and each instance of this task runs on the same processor. First of those work where devoted to deterministic tasks (with a task duration known beforehand, or the worst-case is considered), such as [1, 8, 4, 5], and later probabilistic models were also considered [7, 6]. Only a little work has been provided about global scheduling, such as [3], but for deterministic systems, or [9], using some slack reclamation mechanism, but not really using stochastic information.

As far as we know, no work has been provided with global scheduling on stochastic tasks. We propose to work towards this direction. Notice that the frame-based model we consider in our work, where every task share the same period, is also used by many researchers, such as [8, 6, 9].

2 Model

We consider \( n \) sequential tasks \( \tau_1, \ldots, \tau_n \). Task \( \tau_i \) requires \( x \) cycles with a probability \( c_i(x) \), and its maximum number of cycles is \( w_i \) (Worst Case Execution Cycles, or WCEC). The number of cycles a task requires is not known before the end of its execution. We consider a frame-based model, where all tasks share the same deadline and period \( D \) and are synchronous. In the following \( D \) denote the frame length.

3 Global Scheduling Algorithm

In [2], we have provided techniques allowing to schedule such a task set on a single CPU. The main idea is to compute (offline) a function giving, for each task, the frequency to run the task based on the time elapsed in the current frame. This function, \( S_i(t) \), gave the frequency at which \( \tau_i \) should run if it started at time \( t \) in the current frame. Here, for the sake of clarity, we are going to consider the symmetric function of \( S \): \( \hat{S}_i(d) = S_i(D - d) \) gives the frequency for \( \tau_i \) if this task is started \( d \) units of time before the end of the frame.

In the uniprocessor case, we were able to give schedulability guarantees, as well as good energy consumption performance. We want to be able to provide both in this multiprocessor case, using a global scheduling algorithm. As far as we know, global scheduling algorithm on multiprocessor system using stochastic tasks, and a limited number of available frequencies, has not been considered so far.

The idea of our scheduling algorithm is to consider that a system with \( m \) CPU, and a frame length \( D \), is close to a system with a single CPU, but a frame length \( m \times D \), or, with a frame length \( D \), but \( m \) times faster. We then first compute a set of \( n \) \( \hat{S} \)-functions considering the same set of tasks, but a deadline \( m \times D \). A very naive approach would consist in considering that when a task ends at time \( t \), the total remaining available time before the deadline is the sum of remaining time available on each CPU, which means \( D - t \) on the current CPU, and \( D - t_p \) on the other ones, where \( t_p \) is the worst time at which the task currently running on \( \Pi_p \) will end. Then, we could use \( \hat{S}_i(d) \) to choose the frequency.

Unfortunately, this simple approach does not work, because a single task cannot use time on several CPUS simul-
However, if the number of tasks is reasonably greater than the number of CPUs, we think that in most cases, $S_i(d)$ will not require to use more than the available time on the current CPU, and somehow, will let the available time on other CPUs for future tasks. And when $S_i(d)$ requires more time than actually available, we just use a faster frequency.

Of course, we need to ensure the schedulability of the system, which cannot be guaranteed with the previous approach: for instance, at the end of a frame, we might have some slack time unusable because too short to run any of the remaining task. But as this time has been taken into account when we chose the frequency of previous tasks, we might miss the deadline if we do not take any precaution.

The algorithm we propose is composed of two phases, one off-line, and one on-line. The off-line one consists in performing a (virtual) static partitioning, aiming at reserving enough time in the system for each task. This phase is close to what we did in [2] with Danger Zones. The on-line phase uses both this pre-reservation to ensure the schedulability (but performing dynamic changes to this static partitioning), and the $\hat{S}$-functions, to improve the energy efficiency.

### 3.1 Virtual Static Partitioning

We first perform a “virtual static partitioning”. The aim of this partitioning is not to assign a task to a processor, but to make sure that every task can be executed. A task does not have to run on its assigned processor, but we know that some time has been reserved for this task, which allows to guarantee the schedulability.

This static partitioning can be performed in many ways, but we propose in Algorithm 1 to do it as balanced as possible, by sorting tasks according to their WCEC.

**Algorithm 1: Static partitioning**

```
Ap = 0 ∀p; // Reserved time on Πp
Tp = {} ∀p; // Tasks assigned to Πp
foreach τi descending sorted by wi do
  q = argminq Ap; // cpu with the largest not yet assigned time
  if D − Aq > \frac{w_i}{T_M} then
    Aq = Aq + \frac{w_i}{T_M}; // τi reservation
    Tp = Tp ∪ τi;
  else
    Failed!
```

After this first step of virtual static partitioning, we can see the system as in Figure 1 left part. Notice that it is not because we cannot manage to do this virtual partitioning that the system is not schedulable. But at least, if we manage to do so, then we can ensure that the system is schedulable. This virtual static partitioning can be computed offline, and used for the whole life of the system.

![Figure 1 Left: Static partitioning. Right: State of the system after having started tasks \{τ1, ..., τ7\}. Notice that reservations (dashed tasks) correspond to worst cases, while effective tasks (plain lines) are actual execution times, and change from frame to frame. Vertical axis is frequency, horizontal is time. Then areas correspond to amount of computation.](image)

### 3.2 On-line algorithm

Based on the virtual static partitioning, the main idea of the on-line part is to start a task at a frequency which allows it to end before the beginning of the “reserved” part of the frame. For instance, in Figure 1, τ1 could start on Π1 using all the space between the beginning of the frame, and the reserved space for τ5. But we will see situations where the scheduler needs to give more time for τ1. In such cases, we can also move, for instance, τ5 or τ6 on Π2, or τ12 to Π3. By doing so, and because we never let a running task using the reserved time of another (not started) task, we can guarantee that, if we were able to build a partitioning in the on-line phase, no task will never miss its deadline. Of course, as soon as a task starts, we release the reserved time for this task.

The on-line part of the algorithm is given in Algorithm 4. We first give some explanation about two procedures we need in the main algorithm.

#### 3.2.1 MoveTasksOut

This procedure (Algorithm 2) aims at moving enough tasks from CPU Πp, enough space (the quantity s in the algorithm) is available, or no task can be moved anymore. For instance, in Figure 1 at time $t = 0$, we may want to run $\tau_1$ on $\Pi_1$ at frequency $f_2$. But according to the worst case of $\tau_1$, we do not have enough time to run this task between 0, and the beginning of the reserved area of $\tau_5$. However, we can move $\tau_3$ to $\Pi_3$, and $\tau_5$ or $\tau_6$ to $\Pi_2$.

While $s$ units of time is not available, we take the largest task on $\Pi_p$, and put it on the CPU with the largest free space. This is of course a heuristic, since finding the optimal choice is probably NP-hard or at least intractable problem.

#### 3.2.2 MoveTaskIn

This procedure (Algorithm 3) aims at trying to move a task $\tau_i$ assigned to some CPU $\Pi_q$ to the CPU $\Pi_p$. The main idea
Algorithm 2: MoveTasksOut

Data: processor Π_p, current time t, space to free s
// Move out tasks from Π_p until s units of time are free from t.
1 while D − t − A_p ≥ 0 do
2   \( \tau_i = \text{next task in } T_p \) (sorted by decreasing \( w_i \));
3   \( \text{if No such } \tau_i \text{ then} \)
4     break;
5   \( q = \arg \max_{r \neq p} \{D − A_r − t_r\} \); // CPU with the maximal amount of available space
6   \( \text{if } D − A_q − t_q > \frac{w_q}{f_q} \) then
7       \( \text{// Enough place to move } \tau_i \text{ on } \Pi_q \)
8       \( T_p = T_p \setminus \tau_i; \quad A_p = \frac{w_i}{f_i}; \)
9       \( T_q = T_q \cup \tau_i; \quad A_q = \frac{w_i}{f_i}; \)

is that we first move out as many tasks as needed from \( \Pi_p \) (line 1), until we have enough space to import \( \tau_i \) (lines 2 to 6). If we have not managed to get enough space, \texttt{false} is returned (line 8). However, this algorithm is a heuristic, and is not always able to find a solution, even whether such a solution exists.

For instance (see Figure 1 right part), at the end of \( \tau_7 \), we would like to start \( \tau_8 \) on \( \Pi_1 \). But neither \( \tau_6 \) nor \( \tau_7 \) can be moved on another CPU, so our algorithm fails in finding a solution. However, a smarter algorithm could find out that by swapping \( \tau_8 \) and \( \tau_9 \), \( \tau_8 \) would be able to start on \( \Pi_1 \). Notice that giving a solution in any solvable case is probably also an NP-hard or at least intractable problem.

The procedure we give here is quite naive, and not very efficient, but we let a better algorithm for further research. The naiveness of this algorithm does not affect the schedulability at all: it just makes the system to be forced more often to accept tasks order changes, which might degrade the energy efficiency (S-functions are computed according to the given order), and the user satisfaction, if its preferences are often not respected.

Algorithm 3: MoveTasksIn

Data: processor Π_p, task \( \tau_i \)
Result: \texttt{true} if \( \tau_i \) can be moved on \( \Pi_p \), \texttt{false} otherwise
// Move enough tasks from \( \Pi_p \) to let \( \tau_i \) running
1 MoveTasksOut(\( \Pi_p \), t, \( \frac{w_q}{f_q} \));
2 \( \text{if } D − t − A_p ≥ \frac{w_i}{f_i} \) then
3   \( \text{let } q \text{ be such as } \tau_i \in \Pi_q; \)
4       \( \text{// Move } \tau_i \text{ from } \Pi_q \text{ to } \Pi_p \)
5       \( T_q = T_q \setminus \tau_i; \quad A_q = \frac{w_i}{f_i}; \)
6       \( T_p = T_p \cup \tau_i; \quad A_p = \frac{w_i}{f_i}; \)
7 return true;
8 else
9 return false;

3.2.3 Main algorithm

Here are the main steps of the procedure given in Algorithm 4 which is called each time a CPU (say \( \Pi_q \)) is available, at time \( t \), with \( \tau_i \) the next task to start. This procedure will always start at task at a speed guarantying deadlines, but not necessarily \( \tau_i \).

- \( \text{line } 1 \) We first evaluate \( d \), the remaining time we have for \( \tau_1, \ldots, \tau_n \); if \( t_q \) is the worst time where \( \Pi_q \) is going to be available (the time of the last start, plus the worst case execution time of the current task at the chosen frequency), we have:

\[
    d = (D - t) + \sum_{q \neq p} (D - t_q) = PD - \left( t + \sum_{q \neq p} t_q \right).
\]

- \( \text{line } 2 \) Let \( f = \hat{S}_i(d) \), the frequency chosen for \( \tau_i \) in the single CPU model with \( d \) units of time before the deadline. We are going to check if we can use this frequency (we assume this frequency to be a “good” one from the energy consumption point of view).

- \( \text{line } 3 \) If \( \tau_i \) was not assigned to \( \Pi_p \), we first try to move it to \( \Pi_p \) (Algorithm 3). If we have enough space on \( \Pi_p \), the situation is easy. Otherwise, we need to move some tasks out from \( \Pi_p \), in order to create enough space.

- \( \text{line } 4 \) If we cannot manage to make enough space, then we are not able to start \( \tau_i \) right now. We try then the same procedure for \( \tau_{i+1} \), but we need to left-shift \( \hat{S}_i(d) \). This is not required from the schedulability point of view (we ensure the schedulability by controlling the available time), but we guess it will improve the energy consumption. For the same reason, we will need to right-shift functions of the same amount when \( \tau_i \) starts, because we have one task less to run after \( \tau_i \). (This improvement is not yet implemented in the given algorithm. It requires to be done carefully, because we might have several swapped tasks).

- \( \text{line } 5 \) If we succeeded, we try to move as many tasks as possible from \( \Pi_p \) to other CPUs (Algorithm 2), until we have enough space to start \( \tau_i \) at \( f \), or no task can be moved anymore. We then start \( \tau_i \) either at \( f \), or at the smallest frequency allowing to run \( \tau_i \) in the space we manage to free (line 11). As \( \tau_i \) was assigned to \( \Pi_p \) (possibly after some changes), we are at least sure that we can start \( \tau_i \) at \( f_{\Pi_p} \).

Notice that when StartTask is invoked, it is always possible to run a job, and therefore, we will never consider \( \tau_{n+1} \) in Algorithm 4, line 5. Because of space limitation, we will not give the proof here.

4 Work-in-progress

Here are a few points we want to look deeper, allowing to improve the energy consumption, or the number of systems we are able to schedule.
Algorithm 4: StartTask

Data: Time $t$, processor $\Pi_p$, task $\tau_i$

1. $d = P \times D - (t + \sum_{q \neq p} t_q);$ // Available time on the system
2. $f = \hat{S}(d);$ // Freq. we want to run $\tau_i$
3. if $\tau_i \notin T_p$ then
   // $\tau_i$ is not on $\Pi_p$, we try to move it in
   if not MoveTaskIn($\Pi_p, \tau_i$) then
   StartTask($t, \Pi_p, \tau_{i+1}$);
   return;
   // We have now $\tau_i \in T_p$
4. $A_p = \frac{w_i}{f_i t_i}$; // Release $\tau_i$ reservation
5. $T_p = T_p \setminus \tau_i$;
6. // Try to remove enough tasks (if needed) from $\Pi_p$ to allow $\tau_i$ to run at the desired speed $f$
7. MoveTasksOut($\Pi_p, t, \frac{D}{f}$);
8. if $D - t - A_p \leq \frac{D}{f}$ then
   // Not enough time to run $\tau_i$ at freq $f$
   $f = \frac{w_i}{D - A_p - t}$
   $t_{p+} = \frac{D}{f}$; // Worst end time for $\tau_i$
9. Start $\tau_i$ at $f$;

- At the end of a frame, assuming we can verify that after the task we start, we won’t run tasks anymore on this CPU, we can try to run tasks using the CPU until $D$. For instance, if we start a task on $\Pi_p$ at a speed which lets a free space $[t_p, D]$ too small to run any of the remaining tasks, then we should try to stretch the task to use $\Pi_p$ up to $D$.
- If we accept to change the frequency during the execution of tasks, we can use the continuous model to obtain a frequency $f$, and use two frequencies $[f]_f$ and $[\hat{f}]_f$ to “emulate” this $f$, where $[f]_f$ (resp. $[\hat{f}]_f$) stands for the smallest frequency above (resp. largest below) $f$.
- Several steps require to solve NP-hard problems by using some heuristics: Static partitioning (Algorithm 1), MoveTaskIn (Algorithm 2), and MoveTasksOut (Algorithm 3). The efficiency of the first one improves the number of systems we can accept to schedule, the second one, the number of tasks we will need to swap (not run in the right order), and the third one, how close we can stay from the uniprocessor algorithm. We may try to improve those three algorithms.
- In order to reduce leakage or static energy consumption, we could turn off CPU if they are not needed anymore before the end of the frame.

Of course, we also — and mainly — need to validate our model and show its efficiency by the way of simulations, using realistic environment and workloads.

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