Can electron beams be really focused by bent crystals?

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Abstract

The short period atomic plane modulation is suggested to be applied to modify the electron channeling potential in order to make it possible to considerably increase electron channeling efficiency by the crystal structure brake ensured by either a plane crystal cut or amorphous layer beneath the crystal entrance surface. The achieved channeling efficiency of 60-70 % can considerably facilitate the process of high energy electron beam focusing by specially cut bent crystals. Possible parameters of one TeV electron beam focusing region are estimated using simulations.

Key words: electron, channeling, high energy, linear collider, beam focusing

1 Introduction

The possibility to deflect channeling particles by bent crystals is known since 1976 [1]. Its applications for both positively charged particle beam extraction and collimation have been widely demonstrated and possess perspectives to be...
used at the LCH and FCC [2-4]. Another promising application of bent crystals is beam focusing [5-8]. Since the effective field strength of crystal planes exceeds one kilotesla [9], bent crystals are able to focus high energy beams within a centimeter focal length. The high efficiency of all the bent crystal applications to positively charged particle beam manipulation originates from both the high channeling stability and capture probability, the latter of which can be additionally increased by a crystal structure break [10, 11].

However besides positron beam sharp focusing, future $e^+e^-$ colliders will also need the same of negatively charged electron ones. Meanwhile, experiments demonstrate [12, 13] that electron beam deflection efficiency remains relatively small even for thin, moderately bent crystals. Besides the strong electron scattering by nuclei, the poor electron channeling efficiency originates also from the unsuitable electron planar potential coordinate dependence, which also makes the method [10, 11] of channeling efficiency increase practically inapplicable.

Developing the advantages of the electron dechanneling rate fall at TeV energies [14], we suggest in this Letter to modify the electron planar potential by a short period small amplitude atomic plane modulation [15, 16] in order to further decrease the electron dechanneling rate as well as to increase, both directly and by the method [10, 11], the probability of electron capture into the stable channeling motion. We expose justified parameters of the focusing region to clarify the perspectives of bent crystal application at both the ILC and CLIC as well as to provide a benchmark for possible focusing scheme improvements.
Figure 1. Averaged potential of (111) (top) and (110) (bottom) Si planes, modulated with the amplitudes, indicated on the right, and averaged over the modulation period.

2 Channeling efficiency increase by both short period plane modulation and crystal structure break

A salient feature of the electron average potential (i. e. of the dependence of electron potential energy in the averaged field of crystal planes on the transverse coordinate $x$, measured along the normal to the latter) are sharp dips (one for (110) and two for (111) transverse lattice period) in the region of high nuclear density - see Fig. 1, resulting both in the small acceptance of the low transverse energy states and enhanced nuclear scattering, immediately depopulating the latter [14]. In its turn the region of the highest transverse energies of channeled electrons both readily shrinks at a moderate crystal bending and is intensively depopulated by the strong transverse energy fluctuations $\delta \varepsilon_\perp = \varepsilon_x \theta_s$, induced by the electron nuclear scattering [14, 17] at a random angle $\theta_s$ and enhanced by the large electron channeling oscillation velocity $v_x \simeq \theta_{ch}$ within the high nuclear density region, where $\theta_{ch}$ is critical.
Figure 2. Angular distributions of 855 MeV electrons after passing through a 30 μm Si crystal at the incidence on (111) (top) and (110) (bottom) crystal planes with zero angle and 20 μrad divergence.

channeling angle.

All these circumstances explain the low, about 20% in Fig. 2, efficiency of channeled 855 MeV electron deflection, observed in the MAMI experiment [12] in a 30 μm Si (111) crystal, well reproduced by the simulation method [18, 12]. Fig. 2 also demonstrates the better deflection efficiency of (111) planes in the case of electrons. Note also that only (111) channeling plane orientation is available in ultrathin crystals [12]. Besides the mentioned above direct factors favoring the fast electron dechanneling, a large dispersion of the channeling electron oscillation period makes unfeasible the method of channeling efficiency increase by a crystal structure break [10, 11]. Owing to this rea-
soning one can readily assume that both the fraction value and stability of
electron channeling can be increased by smearing out the potential dips to
about a half of the transverse lattice period.

Putting aside an evident but technically too challenging possibility of a con-
siderable thermal vibration amplitude increase by an intense crystal heating,
we prefer the recently suggested [15] and already realized [16] idea of the
small amplitude short period crystal undulator. Such a Si crystal undulator
with modulated (periodically bent) (110) planes was grown by the method
of molecular beam epitaxy. The (110) planes modulation appeared [16, 19]
due to the periodic introduction of Ge atoms into the Si crystal growing in
the ⟨100⟩ direction. If the modulation (undulator) period \( \lambda_u \), is much shorter
than the period \( \lambda_{ch} \) of channeling motion (see Fig. 1 in [15]), the latter is
governed by the planar potential additionally averaged over the modulation
period \( \lambda_u \). This “second averaging” naturally smears out the unwanted sharp
potential dips over the range determined by the modulation amplitude \( a \). Fig.
1 demonstrates that the potentials of the modulated (111) and (110) planes
well enough approach harmonic ones at a considerable part of the channel
width \( d \) at modulation amplitudes \( a = 0.3 \, \text{Å} \) and \( a = 0.4 \, \text{Å} \), respectively. Fig.
2 demonstrates that the modification of both planar potentials by the modu-
lation results in the dechanneling process deceleration at \( 250 \mu\text{rad} < \theta_x < 750 \mu\text{rad} \) as well as in the 15-20% relative increase of the deflection efficiency at
\( \theta_x > 800 \mu\text{rad} \). However, since both these effects turn out to be moderate, to
reach a really decisive increase of the negatively charged particle channeling
efficiency we suggest here to apply the idea of crystal structure break introduc-
tion [10, 11] becoming feasible for electrons in crystals with the short period
plane modulation.
Figure 3. One TeV electron focusing to the most stable channeling region inside a bent modulated Si (111) crystal channel.

Figure 4. Effective electron potential of the bent modulated Si (111) planes (top) and transverse coordinate electron distribution (bottom) within one (111) transverse period $d$. 
Channeling efficiency can be increased by a slight focusing of highly parallel particle beam within each channel. Following [10, 11] such a focusing can be accomplished by a thin \((0.08\lambda_{ch} - 0.12\lambda_{ch})\) crystalline plate separated from the crystal bulk by either an empty [10] or amorphous [11] plane layer of the thickness \(0.12\lambda_{ch} - 0.17\lambda_{ch}\). Since both of the approaches [10, 11] preserve a single crystal structure both in the front and behind its break, each channel of the front thin crystal plate can serve as a lens for a one in the crystal bulk. Note also, that possible deformations accompanying both the break formation and crystal bending can be compensated by an appropriate choice of particle beam incidence direction to the planes (of about one microradian in the one TeV example below). The discussed approach works well in a semi-parabolic focusing potential, approached by the natural inter-planar potential for positively charged particles and, as suggested above, also by the planar potential of a short-period modulated crystal for negatively charged ones (see Fig. 1).

Another necessary condition of a very low incident beam divergence is for sure fulfilled for the quite small emittance beams of the future \(e^+e^-\) linear colliders (see Table 30.1 in [20]).

Fig. 3 illustrates by fifty trajectories the channeling efficiency increase of one TeV electrons in a bent Si (111) crystal. Electrons are first accelerated towards the channel center by the focusing planar field at \(z < 6\ \mu m\) (red color on the left), and then (see both Figs. 3 and 4) freely shift towards the potential minimum inside the crystal structure break region of \(6\ \mu m < z < 36\ \mu m\). The effective potential of bent (111) planes is shown both on the right in Fig. 3 and on the top in Fig. 4.
Figure 5. One TeV electron beam focusing. Bent crystal dimensions and the crystal cut region (top, left), bending radius and angle (top, right), beam focusing by a pair of crystals, focusing length, focus size and electron trajectories in the focusing $xz$ plane (bottom).

3 One TeV electron beam focusing region example

Below we will proceed from the ideas of Refs. [5-7] for positively charged beams to present a possible geometry of one TeV electron focusing region which implements additionally the idea of crystal structure break [10, 11]. It was realized recently [8] that focusing crystals with a skew back face [6, 7] will encounter technological problems at centimeter focal lengths. To avoid the latter, ”a bent plane-parallel silicon plate whose side edges are rotated at a small angle with respect to crystallographic planes” [8] was suggested. We will consider the crystal assisted focusing of a one TeV electron beam with initial transverse size $\Sigma_x = 1 \, \mu m$ as an example. Fig. 5 depicts both the focusing crystal parameters and electron trajectories in the focusing plane $xz$, being also the plane of channeling oscillations and crystal bending. It is assumed that electron focusing in the normal $yz$ plane is accomplished by the usual means. Below we present the simulation results allowing us to
demonstrate the consequences of both the short period crystal modulation and crystal structure brake as well to explain both the tight interrelation and severe limitations of the dimensions of the electron focusing region depicted in Fig. 5. Figs. 6 and 7 present, respectively, the differential and the integral electron angular distributions behind the Si (111) and Si (110) crystals, revealing that the crystal structure brake application to a crystal with the short period plane modulation makes it possible to increase the **electron deflection efficiency** up to the appropriate levels of 70 and 60 % for (111) and Si (110) planes, respectively.

Rather close values of the channeling inefficiency at the crystal entrance on the one hand and the dechanneling loss percentage inside the crystal on the other, which both have dropped down to 15% in Si (111) and 20 % in Si (110), reflect the reached optimal balance of these devastating processes. Note that electron dechanneling at such a low loss level can’t be described by a single exponential function which only could make rigorous dechanneling length introduction possible [14]. Though the rate of dechanneling process considerably falls at one TeV, it still limits the **crystal thickness** $l = 1 \, mm$ measured along the electron velocity. The same do the radiative losses, which can be simulated by the method [21-23].

Crystal **bending radius** $R = 20 \, cm$ amounts to about a dozen of the minimal bending radii $R_{min} = \varepsilon/e/E_{max}$, where $E_{max}$ is the maximal planar field. Note that the practically used radii indeed usually exceed much the value $R_{min}$, demonstrating that the later is not a completely optimal characteristic. One could introduce instead an alternative one of $R_{V_0} = \varepsilon d/V_0$, corresponding to the equal to $V_0$ centripetal potential variation at the period $d$ of planar potential, where $V_0$ is the amplitude of the latter. Since $R_{V_0} \sim 10 \, m$ for both
Figure 6. Electron angular distribution in the focusing plane for unmodified Si (111) (left) and (110) planes (no mod., no cut), the same for the planes, modulated with a short period and 0.3 Å and 0.4 Å amplitudes (mod., no cut) and for the latter with a crystal structure brake extending from $z = 10 \mu m$ to $z = 40 \mu m$ (mod., cut).

Figure 7. Integral distributions corresponding to the differential ones from Fig. 6. of the considered crystal planes, one should realize that the choice of $R \approx 2R_{V_0}$ is sufficiently grounded and can be additionally adjusted through a thorough optimization only. Knowing now both the crystal thickness and bending radius one can readily find the crystal bending angle $\varphi = l/R = 50 \mu rad$.

Let us discuss the choice of focal length of $f = 1 cm$. Since channeled electron beam inevitably acquires an angular divergence determined by the channeling angle $\theta_{ch}$ (of about 5 $\mu rad$ at 1 TeV), one should surely limit the focal length from above reducing thus the focus size $\sigma_x = f\theta_{ch}$ in the focusing plane. One the other hand, since the focus size has to be, at least, an order less than the beam size $\Sigma_x \simeq f\varphi$ before focusing, the focal length has been
Figure 8. Electron angular distribution in the plane $yz$ normal to the focusing plane for the Si (111) orientation.

also limited from below, resulting in total in the compromise choice of $f = 1 \text{ cm}$.

The latter value determines the focus sizes in both $xz$, $\sigma_x = f\theta_{ch} \sim 50 \text{ nm}$, and $yz$, $\sigma_y = f\theta_{ys} 15 \text{ nm}$, planes, where $\theta_{ys} \approx 1.53 \mu\text{rad}$ is the root mean square scattering angle in the $yz$ plane extracted from the simulated angular distribution depicted in Fig. 8.

The positron beam focusing region parameters can also be readily estimated. Since well channeled positrons scatter on atomic electrons only, their dechanneling length more then two orders exceeds that of electrons [14]. In addition, due to the more suitable averaged potential coordinate dependence, positrons are both better captured into channeling and liable to the direct application of the method [10, 11] of channeling efficiency increase. The most pronounced consequence of these advantages is a reduction of the positron multiple scattering angle in the $yz$ plane at least by an order of value. The focus size in the same plane can be accordingly reduced to about one nanometer.

Since channeling induces the positron beam divergence of about the channeling angle, as in the electron case, the positron beam focal size in the focusing $xz$
plane can’t be reduced so pronouncingly. Nevertheless, provided a small enough incident beam divergence \([20]\), the method \([10, 11]\) of crystal structure break can be applied to optimize bending radius, percentage and transverse energy spread of channeled positrons in order to diminish the focal size down, say, to 20 nm by means of both focal length and angular divergence reduction. Note that, alternatively, initial transverse size of the focused positron beam can be increased by an order or more at the cost of some focus size increase in both \(xz\) and \(yz\) planes.

4 Conclusions

Both the short period atomic plane modulation and crystal structure brake have been suggested to be applied to increase the efficiency of electron deflection by bent crystals. It has been demonstrated through simulations that the deflected electron percentage of 60-70 \%, which justify enough the crystal application for beam focusing, is achievable for 1 TeV electrons. The estimates of focus sizes for both 1 TeV electron and positron beams in the focusing \(xz\) and normal \(yz\) planes, evaluated for 1 mm focusing Si crystal bent with 20 m radius, are summarized in the Table.

The latter reads that the intrinsic channeling angular spread makes it hardly possible to reach a 10 nm focus size in the focusing plane both for electrons and positrons. In its turn nuclear scattering limits from below the possible focal size in the normal plane by 10 nm for electrons, while the less intense scattering of channeled positrons makes the same of 1 nm achievable. Since the critical bending radius, dechanneling length and inverse value of multiple scattering angle are proportional or nearly proportional to the particle energy,
Table 1

Estimates of the $e^\pm$ beam focus sizes in both focusing $xz$ and normal to it $yz$ planes.

|                | 1 TeV $e^-$ | 1 TeV $e^+$ |
|----------------|-------------|-------------|
| $\sigma_x [\text{nm}]$ | 50          | 20-50       |
| $\sigma_y [\text{nm}]$ | 15          | 1           |
| channeling efficiency, % | 50-70       | 70-90       |

one can widely adapt the above estimates to various other beam energies.

The author expects that the above findings will help the experts to estimate the perspectives of ”crystal focusing” application at future electron-positron colliders.
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