Financial networks with static and
dynamic thresholds

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Abstract. Based on the daily data of the American and Chinese stock markets, the dynamic behavior of a financial network with static and dynamic thresholds is investigated. Compared with the static threshold, the dynamic threshold suppresses the large fluctuation induced by the cross correlation of individual stock prices and leads to a stable topological structure in the dynamic evolution. Long-range time correlations are revealed for the average clustering coefficient, average degree and cross correlation of degrees. The dynamic network shows a two-peak behavior in the degree distribution.

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1. Introduction

A financial market is a complex system composed of many interacting units, which exhibit various collective behavior [1]–[6]. How to extract its structure information has attracted much attention from physicists, and remains challenging. For example, the hierarchical structure of financial markets has been investigated with the minimal spanning tree and its variants [7]–[13]. With the random matrix theory, business sectors and unusual sectors can be clarified and topology communities are also revealed [14]–[22].

The complexity theory provides a powerful tool to understand complex networks. In recent years, much progress has been achieved in fields ranging from biology to sociology [23]–[26]. Different models and theoretical approaches have been developed to investigate the formation of network topologies, the functions emerging from networks, the combined mechanism of topologies and functions, etc [27]–[31]. Most research efforts on these schemes focus on the collective properties of the system in a steady state. However, complex systems such as financial markets are essentially nonstationary in time. For each time step, the system shows a particular topology, induced by the cross correlations of individual stock prices. The topology dynamics is important for the full understanding of the network structure. However, it is rarely touched on or elaborated in detail, to the best of our knowledge.

So far, the static topological properties of financial markets from the viewpoint of complex networks have been widely investigated [32]–[37]. An ordinary way to construct a financial network is to take individual stocks as nodes, and set a threshold to create links. If the cross correlation between two stocks is larger than the threshold, then create a link. A number of contributions have adopted an artificially given threshold for static cross correlations. However, it does not extract the dynamic characteristics of stock markets [33, 36, 37]. For example, an outstanding company with a great number of correlated companies is made bankrupt by an extreme event, and all its links are then cut down. This may lead to a temporal variation of the topological structure of the market. Therefore, a real financial network should be dynamic, to capture the dynamic evolution of the topological structure. On the other hand, as is well known, the volatilities of the stock prices exhibit a fat-tailed probability distribution. Large fluctuations of the volatilities may greatly influence the network structure and network stability and therefore should be taken into account in understanding the topology dynamics.

In this paper, we examine a dynamic financial network based on the American and Chinese stock markets. For a comparative study, both static and dynamic thresholds are, respectively, adopted in the network construction. Our purpose is to study the statistical properties of the dynamic network and, more importantly, to investigate the temporal correlations of the topology time series, such as the time series of the average clustering coefficient, the average degree and the cross correlation of degrees, by applying the detrended fluctuation analysis (DFA). Special attention is focused on the dynamic effect of the thresholds on the network structure and network stability.

In section 2, we present the data analyzed and construct the dynamic network with static and dynamic thresholds. In section 3, we investigate the time-correlations of the average clustering coefficient, the average degree and the cross correlation of degrees. The degree distribution of the network is examined, and the relevant economic sectors are identified. Finally, the conclusion comes in section 4.
2. Dynamic financial network

For a comprehensive understanding, we analyze two different stock markets, the New York Stock Exchange (NYSE) and the Chinese Stock Market (CSM), representing the mature and the emerging markets, respectively. For both markets, we investigate the daily data of 259 individual stocks, with 2981 data points from the year 1997 to 2008 for the NYSE and 2633 data points from the year 1997 to 2007 for the CSM.

We define the price return

\[ R_i(t, \Delta t) = \ln P_i(t + \Delta t) - \ln P_i(t), \]

where \( P_i(t) \) is the price of stock \( i \) at time \( t \), and the time interval is set to \( \Delta t = 1 \) day in this paper. For comparison of different stocks, we normalize the price return to

\[ r_i(t) = \frac{R_i(t) - \langle R_i \rangle}{\sigma_i}, \]

where \( \sigma_i = \sqrt{\langle R_i^2 \rangle - \langle R_i \rangle^2} \) is the standard deviation of \( R_i \) and \( \langle \cdots \rangle \) is the time average over \( t \).

We define an instantaneous equal-time cross correlation between two stocks by

\[ G_{ij}(t) = r_i(t)r_j(t). \]

Our financial network is constructed in the following way: take individual stocks as nodes and set a threshold \( \zeta \) to create links. At each time step, if the cross correlation \( G_{ij}(t) > \zeta \), then add a link between stocks \( i \) and \( j \); otherwise, cut the link. Due to the dynamic evolution of \( G_{ij}(t) \), the connections between stocks vary with time, regardless of the static or the dynamic threshold. This results in a dynamic topology of the financial network. To understand the robustness and stability of the network structure, introducing a proper threshold is very important. We first consider a static threshold \( \zeta \propto Q_s \),

\[ Q_s = \frac{2}{N(N - 1)T} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{t=1}^{T} G_{ij}(t), \]

where \( T \) is the total time interval and \( N \) is the total stock number. Since \( Q_s \) is the average cross correlation over the total time and all stocks, the static threshold is reasonable in the sense of providing a uniform standard for the cross correlations of all stocks. As shown in figures 1(a) and (c), \( Q_s \) takes rather small values, 0.17 for the NYSE and 0.37 for the CSM. However, the cross correlation \( G_{ij}(t) \) is defined by the individual price returns and fluctuates according to the price dynamics. The static threshold may suffer from the large fluctuation of the cross correlation \( G_{ij}(t) \). Hence, we introduce a dynamic threshold \( \zeta \propto Q_d(t) \),

\[ Q_d(t) = \frac{2}{N(N - 1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} G_{ij}(t). \]

\( Q_d(t) \) takes only the average over all stocks in a single time step and therefore fluctuates synchronously with the cross correlations \( G_{ij}(t) \). As shown in figures 1(b) and (d), large values of \( Q_d(t) \) are observed for both the NYSE and CSM, in addition to the small values for most time steps. Therefore, the dynamic threshold may suppress the large fluctuation induced by \( G_{ij}(t) \) and accordingly creates a stable network structure. In this paper, we consider the static threshold from \( \zeta = 0.25 Q_s \) to \( 4 Q_s \), and the dynamic threshold from \( \zeta = 0.25 Q_d \) to \( 6 Q_d \).
Figure 1. $Q_s$ and $Q_d(t)$ for the NYSE and CSM are shown in (a)–(d).

In figure 2, the average clustering coefficient $C(t)$ is displayed for $\zeta = Q_s$ and $\zeta = Q_d(t)$. The average clustering coefficient $C(t)$ is defined by

$$C(t) = \frac{1}{N} \sum_{i=1}^{N} c_i(t),$$

where $c_i(t)$ is the clustering coefficient of node $i$ [28], denoting the ratio of the triangle-connection number of node $i$ to the maximum possible triangle-connection number of the node. It is observed in figure 2 that the average clustering coefficient $C(t)$ of $\zeta = Q_d(t)$ is apparently more stable than that of $\zeta = Q_s$ for both the NYSE and CSM. The common characteristic shared by $\zeta = Q_s$ and $\zeta = Q_d(t)$ is that their average clustering coefficients are large. Especially for the dynamic threshold $\zeta = Q_d(t)$, the average clustering coefficient $C(t)$ keeps its values around 0.88 and 0.85, respectively, for the NYSE and CSM at a highly clustering level, implying a close relationship of stocks in the financial markets.

3. Topology dynamics

The topological properties of a complex network are usually described by the average clustering coefficient, the average degree and the cross correlation of degrees. We investigate the topology dynamics by computing the time correlations. The autocorrelation function is widely adopted to
Figure 2. The average clustering coefficients $C(t)$ corresponding to $Q_s$ and $Q_d(t)$ for the NYSE and CSM are displayed in (a)–(d). The time-averaging values of $C(t)$ are 0.78, 0.88, 0.68 and 0.85, respectively.

measure the time correlation. However, it shows large fluctuations for nonstationary time series. Therefore, we apply the DFA method [38, 39].

For a time series $A(t')$, we eliminate the average trend from the time series by introducing

$$B(t') = \sum_{t''=1}^{t'} [A(t'') - A_{ave}],$$

where $A_{ave}$ is the average of $A(t')$ in the total time interval [1, $T$]. Uniformly dividing [1, $T$] into windows of size $t$ and fitting $B(t')$ to a linear function $B_i(t')$ in each window, we define the DFA function as

$$F(t) = \sqrt{\frac{1}{T} \sum_{t'=1}^{T} [B(t') - B_i(t')]^2}. \quad (7)$$

In general, $F(t)$ will obey a power-law scaling behavior $F(t) \sim t^{\theta}$. The exponents $\theta > 1.0, 0.5 < \theta < 1.0$ and $0 < \theta < 0.5$ indicate unstable, long-range correlating and anti-correlating time series, respectively. $\theta = 0.5$ corresponds to the Gaussian white noise, while $\theta = 1.0$ represents the $1/f$ noise.

3.1. Average clustering coefficient

The DFA function of the average clustering coefficient $C(t)$ is shown in figure 3. For the static threshold $\xi = Q_s$, a two-stage scaling behavior is observed with a crossover phenomenon in
Figure 3. The DFA function of the average clustering coefficient $C(t)$ is plotted on a log–log scale. The inset shows the time-averaging clustering coefficient for different threshold values. Dashed lines are the power-law fits. (a) and (c) For the static thresholds for the NYSE and CSM. Circles, crosses and triangles are for $\zeta = 0.5Q_s$, $Q_s$, and $2Q_s$, respectively. (b) and (d) For the dynamic thresholds for the NYSE and CSM. Circles, crosses and triangles are for $\zeta = Q_d(t)$, $2Q_d(t)$ and $3Q_d(t)$, respectively.

between. For the NYSE, the exponent $\theta$ takes the values $\theta = 0.76$ for $t < t_c$ and $\theta = 1.04$ for $t > t_c$. For the CSM, $\theta = 0.71$ for $t < t_c$ and $\theta = 0.97$ for $t > t_c$. The crossover time $t_c \sim 25$ days. This result indicates that the average clustering coefficients are temporally correlated for $t < t_c$ and then transit to the $1/f$ noise for $t > t_c$. In contrast to that of $\zeta = Q_s$, the DFA function of $\zeta = Q_d(t)$ shows a clean power-law behavior, with the exponent $\theta = 0.59$ for the NYSE and 0.60 for the CSM, indicating long-range time correlation.

To investigate the robustness of the above results and the stability of the network structure during the dynamic evolution, we may adjust the level of the thresholds. For the static threshold, we consider two alternatives $\zeta = 0.5Q_s$ and $\zeta = 2Q_s$. As shown in figures 3(a) and (c), one does not find any power-law behavior. In other words, the network structure is rather sensitive to the specific value of the static threshold. For the dynamic threshold, we observe that the DFA function of $C(t)$ remains qualitatively the same for $\zeta \geq Q_d(t)$. In figures 3(b) and (d), for example, the results of $\zeta = 2Q_d$ and $3Q_d$ are displayed. Clean power-law behavior is detected with the exponent $\theta = 0.61$ and 0.61 for the NYSE and $\theta = 0.62$ and 0.64 for the CSM. Higher
dynamic thresholds yield stronger time correlations (i.e. larger values of $\theta$). This implies that the links created by large $G_{ij}(t)$ are rather stable.

To further understand the topological pattern, we calculate the time-averaging clustering coefficient $\overline{C}$ for different thresholds,

$$\overline{C} = \frac{1}{T} \sum_{t=1}^{T} C(t).$$

(8)

As shown in the inner panel of figures 3(a) and (c), $\overline{C}$ of the static threshold $\zeta \leq 0.5Q_s$ is close to 1, but as the threshold increases to $1.5Q_s$, $\overline{C}$ sharply drops to 0. As shown in the inset of figures 3(b) and (d), the time-averaging clustering coefficient $\overline{C}$ shows a mild decay for the dynamic threshold and presents rather high clustering even for the threshold $\zeta = 6Q_d(t)$ for both the NYSE and CSM.

3.2. Average degree

The average degree $K(t)$ is defined as

$$K(t) = \frac{1}{N} \sum_{i=1}^{N} k_i(t),$$

(9)

where $k_i(t)$ is the degree of node $i$, denoting the number of links directly connected with it [40]. For the network with both the static and the dynamic threshold, there is no weight and no direction in the network, and at most one link between any two nodes. Therefore, the degree of a node represents the number of its adjacent nodes in the network. For $\zeta = Q_s$, $K(t)$ exhibits large fluctuations between 0 and $N - 1$, as shown in figures 4(a) and (c). $K(t) = N - 1$ indicates that every node directly connects to all other nodes in the network, whereas $K(t) = 0$ corresponds to a set of isolated nodes. The fluctuation of the CSM is obviously stronger, and $K(t)$ often comes rather close to 0 and $N - 1$. For $\zeta = Q_d(t)$, $K(t)$ is almost bounded by two envelope curves, as shown in figures 4(b) and (d). The upper envelope is around $K(t) \sim 130–140$, indicating that one node is adjacent to about half of all other nodes on average, whereas the lower envelope is around $K(t) \sim 70–80$, indicating that one node and about one quarter of all other nodes are adjacent on average. The upper envelope implies a high–low symmetric distribution of the cross correlations $G_{ij}(t)$, with one half of the data points below the average and the other half of the data points above the average. The upper envelope sounds reasonable, for the larger $G_{ij}(t)$ are usually well above the average. However, the lower envelope suggests a high–low asymmetric distribution of the cross correlations $G_{ij}(t)$. Why there is such a lower envelope remains to be understood.

We then compute the DFA function of the average degree $K(t)$. Similar to that of the average clustering coefficient, unstable behavior is detected for the static threshold, whereas a stable network structure is observed for the dynamic threshold, as shown in figure 5. For the static threshold, the DFA function for the NYSE shows power-law behavior with the exponent $\theta = 0.61$ for $\zeta = 0.5Q_s$, two-stage scaling for $\zeta = Q_s$, and no power-law behavior for $\zeta = 2Q_s$. The DFA function for the CSM shows power-law behavior with the exponent $\theta = 0.65$ and 0.71 for $\zeta = 0.5Q_s$ and $Q_s$, and no power-law behavior for $\zeta = 2Q_s$. These results can also be
Figure 4. The average degree $K(t)$ is displayed. (a) and (c) For the static threshold $\zeta = Q_s$ for the NYSE and CSM. (b) and (d) For the dynamic threshold $\zeta = Q_d(t)$ for the NYSE and CSM.

understood from the time-averaging degree

$$K = \frac{1}{T} \sum_{t=1}^{T} K(t). \quad (10)$$

As shown in the insets of figures 5(a) and (c), the network has a large number of links for lower static thresholds; however, the number of links sharply falls off to nearly zero as the threshold increases up to $1.5Q_s$.

For the dynamic threshold, robust power-law behavior is observed. The exponent $\theta$ of $\zeta = Q_d(t)$ is measured to be 0.58 for the NYSE and 0.56 for the CSM, respectively, somewhat close to $\theta = 1/2$ of the Gaussian white noises. But as $\zeta$ increases, the network structure stabilizes, with the exponent $\theta = 0.60$ for the NYSE and 0.61 for the CSM. The time-averaging degree $K$ is also found to decay slowly as the dynamic threshold increases, and the network shows significant correlation for a high threshold as $6Q_d(t)$, as shown in the insets of figures 5(b) and (d).

To this end, we observe a high average clustering coefficient and high average degree for the network with the dynamic threshold. Due to rather plentiful links, the network is different from a small-world network. Moreover, we also compare the network structure with an Erdos–Renyi (E–R) random network. Following the method reported in [41], we theoretically
Figure 5. The DFA function of the average degree $K(t)$ is plotted on a log–log scale. The insets show the time-averaging degrees for different threshold values. Dashed lines are the power-law fits. (a) and (c) For the static thresholds for the NYSE and CSM. Circles, crosses and triangles are for $\zeta = 0.5Q_s$, $Q_s$ and $2Q_s$, respectively. (b) and (d) For the dynamic thresholds for the NYSE and CSM. Circles, crosses and triangles are for $\zeta = Q_d(t)$, $2Q_d(t)$ and $3Q_d(t)$, respectively.

calculate the clustering coefficient of the E–R random network

$$C = \frac{\overline{K}}{N-1},$$

where $\overline{K}$ is the average degree and $N = 259$ is the total number of nodes in the network. Given $\overline{K} = 108$ and 107, respectively, the same as that of the NYSE and CSM, the clustering coefficients of the E–R random network are estimated to be 0.42 and 0.41. Numerical simulation of the E–R random network is also performed [24]. Starting with $N$ nodes, we connect every pair of nodes with probability $p$. Setting $N = 259$ and $p = 0.42$, the corresponding average degree and clustering coefficient are then computed to be 108 and 0.42, respectively, consistent with the theoretical calculations. Therefore, the clustering coefficients of the financial markets are also different from those obtained from the E–R random network.

Why is the dynamic threshold crucial in the analysis of the network structure of financial markets? One important reason is that the volatilities fluctuate strongly in the dynamic evolution, especially on the crash days [42, 43]. It induces large temporal fluctuations of the cross correlations of price returns, as is characterized by $Q_d(t)$ in figure 1. Thus the static threshold
leads to dramatic changes in the topological structure of the network. However, the dynamic threshold proportional to $Q_d(t)$ suppresses such kinds of fluctuations and results in a stable topological structure of the network.

We take two typical periods of 30 trading days as examples to study the behavior on the extreme market and the relatively stable market days. For the NYSE, we investigate the time periods from 11 September 2008 to 22 October 2008, around the financial crash in September 2008, and from 30 December 2004 to 10 February 2005 with less volatile days. For the static threshold $\zeta = Q_s$, the time-averaging degree is 166 and 56, respectively, far away from the time-averaging degree $\overline{K} = 74$ in the total time interval. The time-average clustering coefficient is 0.95 and 0.73, also significantly deviating from the time-averaging clustering coefficient $\overline{C} = 0.78$ in the total time interval. Evidently, the topology of the crash days is quite different from that of the stable market days for the static threshold. For the dynamic threshold $\zeta = Q_d(t)$, however, the time-averaging degree is 102 and 109, both around $\overline{K} = 108$. The time-averaging clustering coefficient is 0.83 and 0.89, also close to $\overline{C} = 0.88$. It indicates that the dynamic threshold stabilizes the network structure. For the CSM, the market strongly fluctuates from 29 August 2007 to 16 October 2007, while it is relatively stable from 3 July 2003 to 13 August 2003. Similarly, for the static threshold $\zeta = Q_s$, the time-averaging degree is 87 and 27, and the time-averaging clustering coefficient is 0.77 and 0.51, significantly deviating from

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**Figure 6.** The degree distribution $P(k)$ is displayed. (a) and (c) For the static thresholds for the NYSE and CSM. Circles and crosses are for $\zeta = 0.5Q_s$ and $Q_s$, respectively. (b) and (d) For the dynamic thresholds for the NYSE and CSM. Circles, crosses and triangles are for $\zeta = Q_d(t), 2Q_d(t)$ and $3Q_d(t)$, respectively.
Figure 7. The cross correlation $r(t)$ of degrees is displayed. (a) and (c) For the static threshold $\xi = Q_s$ for the NYSE and CSM. (b) and (d) For the dynamic threshold $\xi = Q_d(t)$ for the NYSE and CSM. The time-averaging values of $r(t)$ are 0.00, 0.36, $-0.20$ and 0.22, respectively, in (a)–(d).

$\overline{K} = 68$ and $\overline{C} = 0.68$, respectively. For the dynamic threshold $\xi = Q_d(t)$, however, the time-averaging degree is 103 and 111, and the time-averaging clustering coefficient is 0.83 and 0.87, respectively, close to $\overline{K} = 107$ and $\overline{C} = 0.85$.

### 3.3. Degree distribution

The degree distribution function $P(k)$ describes the heterogeneous properties of nodes. To obtain better statistics, we take the degrees of all time steps as an ensemble. Figure 6 shows the degree distribution of the static and dynamic thresholds for the NYSE and CSM. For both thresholds, it is observed that the degree distribution is different not only from the Poisson distribution, but also from the power-law distribution of a scale-free network. For the static threshold, the degree distribution is rather sensitive to the threshold value, and the form of the degree distribution changes dramatically. For the dynamic threshold, two-peak behavior is observed with one peak around $k = 0$ and the other peak around $k = 140–150$ for both the NYSE and CSM. The peak at $k = 0$ simply tells us that there is a large number of isolated nodes in the network. The peak at $k = 140–150$ indicates that one node is adjacent to about one half of all other nodes in the network, i.e. the cross correlation distribution is high–low symmetric for the dynamic threshold.
Figure 8. The DFA function of the cross correlation of degrees is plotted on a log–log scale. Dashed lines are the power-law fits. (a) and (c) For the static thresholds for the NYSE and CSM. Circles and triangles are for $\zeta = 0.5 Q_s$ and $Q_s$, respectively. (b) and (d) For the dynamic thresholds for the NYSE and CSM. Circles, crosses and triangles are for $\zeta = Q_d(t)$, $2 Q_d(t)$ and $3 Q_d(t)$, respectively.

The two-peak structure of the degree distribution for the dynamic threshold explains the upper envelope of the average degree $K(t)$ in figure 4. Moreover, the features of the degree distribution are rather robust for different dynamic threshold values. It indicates that the large cross correlations are much bigger than the average cross correlation, so that a certain increase of $\zeta$ does not alter the network topology.

3.4. Cross correlation of degrees

The so-called assortative or disassortative mixing on the degrees refers to the cross correlation of degrees [44, 45]. ‘Assortative mixing’ means that high-degree nodes tend to directly connect with high-degree nodes, while ‘disassortative mixing’ indicates that high-degree nodes prefer to directly connect with low-degree nodes. The cross correlation of degrees is defined as

$$r(t) = \frac{M^{-1} \sum_{\alpha} j_{\alpha} k_{\alpha} - [M^{-1} \sum_{\alpha} \frac{1}{2} (j_{\alpha} + k_{\alpha})]^2}{M^{-1} \sum_{\alpha} \frac{1}{2} (j_{\alpha}^2 + k_{\alpha}^2) - [M^{-1} \sum_{\alpha} \frac{1}{2} (j_{\alpha} + k_{\alpha})]^2},$$

(12)

where $j_{\alpha}$ and $k_{\alpha}$ are the degrees of the nodes at both ends of the $\alpha_{ab}$ link, with $\alpha = 1, \ldots, M$. At a certain time, $r > 0$, $r = 0$ and $r < 0$ represent assortative mixing, no assortative mixing and disassortative mixing, respectively.

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Figure 9. The absolute values of the components $u_i$ of stock $i$ for the first four largest eigenvalues of the cross correlation matrix $F$ are displayed for the NYSE. Stocks are arranged according to economic sectors separated by dashed lines. A: basic materials; B: conglomerates; C: consumer goods; D: finance; E: healthcare; F: industrial goods; G: services; H: technology; I: utilities. (a) For the static threshold $\zeta = Q_s$ and (b) for the dynamic threshold $\zeta = Q_d(t)$.

In figure 7, $r(t)$ of $\zeta = Q_s$ and $Q_d(t)$ is shown for the NYSE and CSM. $r(t)$ fluctuates between the interval $[-1, 1]$, flipping between assortative mixing and disassortative mixing during the time evolution. Defining the time-averaging cross correlation of the degrees,

$$\bar{r} = \frac{1}{T} \sum_{t=1}^{T} r(t),$$

we obtain $\bar{r} = 0.00$ and $-0.20$ of the static threshold for the NYSE and CSM and $\bar{r} = 0.36$ and 0.22 of the dynamic threshold for the NYSE and CSM. In other words, the cross correlation of the static threshold shows no assortative mixing or disassortative mixing, whereas that of the dynamic threshold exhibits assortative mixing.

To study the memory effect, we again compute the DFA function of $r(t)$. As shown in figure 8, the DFA function of $\zeta = Q_s$ shows power-law behavior with the exponents $\theta = 0.73$ for the NYSE and 0.77 for the CSM. For $\zeta = 0.5Q_s$, no power-law behavior is observed. It further confirms the unstable network structure for the static threshold. For the dynamic threshold, the
Figure 10. Similar to figure 9, but with different static and dynamic thresholds. (a) For the static threshold $\zeta = 1.1 Q_s$ and (b) for the dynamic threshold $\zeta = 6 Q_d(t)$.

DFA function of $r(t)$ shows nearly the same power-law behavior for $\zeta = Q_d(t)$, $2 Q_d(t)$ and $3 Q_d(t)$, with the exponents $\theta = 0.60$ for the NYSE and 0.63 for the CSM.

3.5. Economic sector

The community or sector structure may specify different intercorrelated subsets of networks [14–18]. Economic sectors have also been identified by the cross correlations of individual stock price-returns with the random matrix theory [19, 20, 46], clustering effect [7] and topological properties of graphs [11]. To further understand the topological structure of the financial networks, we may investigate the dynamic effect of economic sectors with the random matrix theory, e.g. one may analyze the eigenvalues and eigenvectors of the cross correlation matrix of price returns. With a similar procedure, we first introduce the normalized individual degrees $\tilde{k}_i(t) = (k_i - \langle k_i \rangle)/\sigma_{k_i}$, with $\sigma_{k_i}$ being the standard deviation of $k_i$, and $\langle \cdots \rangle$ being the time average over $t$. We then construct the cross correlation matrix $F$ of individual degrees $\tilde{k}_i(t)$, whose elements are

$$F_{ij} = \frac{1}{T} \sum_{t=1}^{T} \tilde{k}_i(t) \tilde{k}_j(t),$$

(14)
and compute its eigenvalues and eigenvectors. As shown in figure 9, we observe economic sectors for both the static threshold $\zeta = Q_s$ and the dynamic threshold $\zeta = Q_d(t)$ for the network of the NYSE. In the figure, the absolute values of the components $u_i$ of stock $i$ are displayed for the first four largest eigenvalues. For the eigenvector of the largest eigenvalue $\lambda_0$, the stocks show a relatively uniform contribution. This is the so-called ‘market mode’ [47, 48], in which all the stocks are affected by a common market mechanism with the same bias. The second largest eigenvalue $\lambda_1$ is dominated by the stocks of the utilities, whereas the third largest eigenvalue $\lambda_2$ by the stocks of the basic materials. The fourth largest eigenvalue $\lambda_3$ does not show significant peaks and therefore one cannot identify the specific sector. For the CSM, the dynamic effect of the standard economic sectors is weak, and we need a careful analysis to reveal its unusual sectors as reported in [20].

Here we should emphasize that the sector structure of the network with the dynamic threshold is rather robust from $\zeta = Q_d(t)$ to $6Q_d(t)$, while that with the static threshold is very sensitive to the specific value of the threshold. As shown in figure 10, the composition of stocks for the first four large eigenvalues remains statistically unchanged for the large dynamic threshold $\zeta = 6Q_d(t)$. For the static threshold, however, one can hardly identify the economic sector when increasing the static threshold even up to $\zeta = 1.1Q_s$. As the static threshold is up to $\zeta = 1.5Q_s$, the network becomes almost a set of isolated nodes.

4. Conclusion

We investigate the topology dynamics of a financial network by a comparative study with static and dynamic thresholds, based on the daily data of the American and Chinese stock markets. For both stock markets, the dynamic threshold properly suppresses the large fluctuation induced by the cross correlations of individual stock prices and creates a rather robust and stable network structure during the dynamic evolution, in comparison to the static threshold. Long-range time correlations are revealed for the average clustering coefficient, the average degree and the cross correlation of degrees.

The average clustering coefficient and average degree for both static and dynamic thresholds are large, indicating the strong interactions between stocks in financial markets. A two-peak behavior is observed in the degree distribution for the dynamic threshold, very different from the power-law behavior of a scale-free network.

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