Abstract: The bulk of air showers initiated by very high energy cosmic rays exhibits a longitudinal development in depth with a single well-defined shower maximum. However, a small fraction of showers has a profile that differs considerably from this average behavior. In extreme cases, such anomalous longitudinal profiles can even have two distinct shower maxima. We discuss the properties of the primary interactions that lead to such profiles. Simulations are used to estimate the rate of anomalous profiles in dependence of primary energy, mass, and characteristic features of hadronic multiparticle production at very high energies.

Keywords: CONEX, Air Showers, Longitudinal Profiles, Double Bumps, Leading Particles

1 Introduction

Cosmic ray detectors like HiRes [1], the Pierre Auger Observatory [2], or Telescope Array [3] measure the longitudinal profiles of air showers initiated by ultra-high energy cosmic ray particles. The data of these experiments offers the possibility to study hadronic interactions beyond the reach of man-made accelerators. In practice, however, the interpretation of the data is difficult because of the unknown mass composition of the cosmic particle beam.

In this paper, we present a Monte Carlo study of longitudinal profiles using the CONEX air shower simulation program [4] to study if ‘anomalous’ longitudinal shower profiles could provide a unique signature for properties of hadronic interactions. Measurements [5], simulations [6] and theoretical considerations (e.g. [7]) suggest that the longitudinal development of the electromagnetic component of ultra-high energy air showers is to a good approximation universal, i.e. that the shape of the profiles is very similar for the bulk of showers irrespective of the primary energy, particle type or hadronic interaction model. In rare cases, however, simulated profiles can significantly deviate from this universal shape and anomalous shower profiles can be found that in extreme cases show two completely separated maxima.

Examples of such anomalous shower profiles are shown in Fig. 1. The profile in the left panel is generated by a primary helium nucleus with an energy of \(10^{17}\) eV. The first interaction occurs at 19 g cm\(^{-2}\) and results in a sub-shower with a maximum at around 700 g cm\(^{-2}\). A spectator nucleon with a quarter of the primary energy penetrates deeply into the atmosphere before it interacts at 583 g cm\(^{-2}\) and creates a second sub-shower that reaches its maximum beyond 1000 g cm\(^{-2}\). Similar cases are shown in Fig. 1(b) and (c), but now for primary protons. Here, instead of a spectator nucleon, the second shower is created by the leading particle from the first interaction.

These examples suggest, that the anomalous shower profiles originate from leading or spectator particles that penetrate deeply into the atmosphere before interacting. The probability for propagating a slant depth distance greater than \(\Delta X\) without interacting is given by

\[
P(\Delta X) = e^{-\frac{\Delta X}{\lambda}},
\]

where \(\lambda\) denotes the hadronic interaction length in air. Assuming that a propagation distance of at least 300 g cm\(^{-2}\) is needed to experimentally distinguish these showers from the universal shower profile, one would expect \(P \simeq 3 \cdot 10^{-3}\) and \(P \simeq 8 \cdot 10^{-4}\) for \(10^{17}\) eV and \(10^{19}\) eV primaries using a hadronic interaction length as predicted by the SIBYLL interaction model. The actual rate of occurrence will however be smaller, because for an experimental observation of anomalous shower profiles, the inelasticity of the interaction must be in a suitable range such that both sub-showers carry a fraction of the primary energy that is large enough for a detection.

2 Analysis of Air Shower Simulations

To estimate the frequency of detectable anomalous air showers we generated more than \(10^4\) showers for each combination of three different hadronic interaction models (SIBYLL2.1 [8], QGSJETII [9] and EPOS1.99 [10]), seven primary energies between \(E = 10^{17}\) and \(10^{20}\) eV and three primary masses (proton, helium, and iron).
Anomalous profiles are identified by comparing the \( \chi^2 \) value obtained from a fit of the simulated profile with a Gaisser-Hillas function [11], \( f_{\text{GH}} \), with the \( \chi^2 \) that results from the sum of two Gaisser-Hillas functions:

\[
\chi^2_{\text{double}} = \sum_{i=0}^{n} \frac{(N_i - f_{\text{GH}}(X_i) - f_{\text{GH}2}(X_i))^2}{V_i},
\]

with

\[
f_{\text{GH}} = f_{\text{GH}} \left( N_{\text{max},1}, X_{\text{max},1}, \lambda_1, X_0^1 \right)
\]

\[
f_{\text{GH}2} = f_{\text{GH}} \left( N_{\text{max},2}, \Delta X_{\text{max}}, \lambda_2, X_0^2 \right),
\]

where \( N_i \) denotes the simulated shower size at slant depth \( X_i \) and \( n \) is the number of generated data points. The normalisation of the functions is given by \( N_{\text{max},i} \) and \( X_{\text{max},1} \) is the depth of maximum of the first sub-shower. Following [12], the shape parameters \( \lambda_i \) and \( X_0^i \) are constrained to average values corresponding to the 'normal' universal profiles to assure a better stability of the fit. Finally, \( \Delta X_{\text{max}} \) denotes the difference between the shower maxima of the first and second sub-shower.

For an anomalous profile that originates from a deeply penetrating leading particle, we expect that the fit with two Gaisser-Hillas functions will lead to parameters that are related to the properties of the two sub-showers. The inelasticity \( \kappa \) of the first interaction can be estimated from the normalisations of the two Gaisser-Hillas functions,

\[
\kappa \approx 1 - \frac{N_{\text{max},2}}{N_{\text{max},1} + N_{\text{max},2}},
\]

and the fitted distance between the two shower maxima is related to the slant depth distance between the first and second interaction

\[
\Delta X \approx \Delta X_{\text{max}}.
\]

This relations assumes that the shower development from the interaction point is a mere translation, that is the same for both showers. This should be a good approximation, since the average slant depth distance between the interaction point and the shower maximum depends only logarithmically on energy.

In order to obtain a meaningful \( \chi^2 \) in Eq. (2), a choice must be made for \( V_i \). CONEX generates 'exact' shower sizes without statistical fluctuations. A Poissonian ansatz, \( V_i = N_i \), would lead to an increasing sensitivity of the \( \chi^2 \) to deviations from a universal profile with energy. For this study however, we prefer to have a constant sensitivity at each energy and therefore use \( V_i = kN_i/E \), where \( k \) is chosen such that \( \sqrt{\sum V_i/\sum N_i} = 0.01 \). With this re-normalisation, equal \( \chi^2 \) probabilities for all energies are assured that enable us to study the 'intrinsic' evolution of the fraction of anomalous profiles with energy. Furthermore, profile points with \( \sqrt{V_i/N_i} > 0.3 \) are excluded from the fit to avoid the influence of the long tail of deeply penetrating muonic component of the shower size profile that can not be well described by a four parameter Gaisser-Hillas function; Fluorescence detectors can not observe this tail above the night-sky background.

For the analysis of the generated showers, each profile point is fluctuated by \( \sqrt{V_i} \) using Gaussian random numbers and the \( \Delta \chi^2 \) between single- and double-Gaisser-Hillas fit is calculated. Then we define a shower as 'anomalous' if

- two Gaisser-Hillas functions significantly improve the goodness of fit (\( \Delta \chi^2 > 25 \)),
- the shower maxima are clearly separated (\( \Delta X_{\text{max}} > 300 \text{ g cm}^{-2} \)),
- both fitted \( X_{\text{max}} \) were found within the profile range,
- both fitted sub-showers have more than 20% of the primary energy.

3 Results

Using the criteria from the last section to select and define anomalous shower profiles, their fraction of the total simulated sample can be determined. As can be seen in Fig. 2, no anomalous showers are predicted for iron nuclei, which is not surprising because spectator nucleons can carry less than 2% (1/56) of the primary energy. Heavier fragment
nuclei could have a larger energy, but due to their large cross section they have a small mean free path length in the atmosphere.

Anomalous shower profiles of helium and proton primaries occur with a very similar frequency and the fractions are of the same order of magnitude as estimated with the simple analytic model, Eq. 1. The fraction decreases with energy as it would be expected due to the rise of the cross sections with energy in the simulations. In case of SIBYLL and EPOS, the fraction rises slightly again at the highest energies. The reason for this behaviour could be the Landau-Pomeranchuk-Migdal (LPM) effect [13] on photons produced by the decay of a leading π⁰. The LPM effect significantly increases the mean free path of electromagnetic particles above $10^{18} - 10^{19}$ eV, and in the case of SIBYLL and EPOS a leading π⁰ can carry more than 50% of the primary energy. In the case of QGSJETII no leading π⁰ are produced. Further studies are needed to confirm this hypothesis.

The correlation between the fitted $\Delta X_{\text{max}}$ and the slant depth difference $\Delta X = X_n - X_{\text{first}}$ of the two most relevant interaction points is shown in Fig. 3. Here $X_{\text{first}}$ is the interaction depth of the primary particle in the atmosphere and $X_n$ the interaction depth of the $n$th leading particle¹, where interactions are only considered if they have a minimal inelasticity of $\kappa > 0.15$. As can be seen, the distance between the shower maxima is a good estimator for $\Delta X$ for the majority of the showers. In addition to the events populating the diagonal, there is a cluster of events at small $\Delta X$ but large $\Delta X_{\text{max}}$. These events are due to deeply penetrating sub-showers created not by leading particles, so $X_n$ and thus also $\Delta X$ are not the correct quantities.

As a further test of the sensitivity of the anomalous profile fraction to the hadronic interaction length, the response of the event fraction to a decrease of the hadronic cross sections in the simulation is studied. For this purpose all cross section above $10^{15}$ eV are rescaled logarithmically during CONEX simulations by a factor $f(E)$,

$$f(E) = 1 + (f_{19} - 1) \frac{\ln (E/10^{15} \text{ eV})}{\ln(10^{19} \text{ eV}/10^{15} \text{ eV})},$$

where $E$ is the energy and $f_{19}$ is the rescaling factor of the cross section at $10^{19}$ eV [14]. The resulting fractions for diminished cross sections are shown in Fig. 4. As can be seen, decreasing the cross-section of SIBYLL to 80% means a 50% and 100% higher fraction at $10^{18}$ eV and $10^{19}$ eV respectively and a reduction to 60% increases the rate even further.

4 Conclusions

In this paper we presented a simulation study on anomalous longitudinal shower profiles. These showers exhibit in extreme cases two distinct shower maxima and originate from deeply penetrating spectator nucleons or leading particles. It was shown that their rate is expected to be largest at low energies and for light primary masses. It is however worthwhile noting that the absolute rate estimates given in Fig. 2 are only indicative and depend on the selection criteria as well as on the precision of the profiles, $V_i$.

1. For $n = 1$ this refers to the leading particle of the primary interaction, for $n = 2$ to the leading particle of the interaction of the leading particle of the primary interaction, etc..

![Figure 2: Fraction of anomalous shower profiles for different primaries and interaction models.](image1)

![Figure 3: Correlation between $\Delta X$ and $\Delta X_{\text{max}}$ for protons (SIBYLL).](image2)
Figure 4: Anomalous profile fraction for a modified cross section in SIBYLL.

estimate of the rate of anomalous showers detectable by current detectors requires to study detailed detector simulations, which is beyond the scope of this work.

The experimental detection of such showers would be an unambiguous proof of the presence of light nuclei in the cosmic particle beam. Moreover, if it is possible to detect enough of these events, they could provide a novel tool to study the cross section and inelasticity of hadronic interactions at ultra-high energies.

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