A Fish Biology Chaotic System and its Circuit Design

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Abstract. Applications of dynamical systems in chaos theory arising in several areas are investigated and circuitual implementations of dynamical systems are studied by many researchers. In this work, we consider the fish-biology dynamical system introduced by Foutayeni and Khaladi (2016). By taking a different set of parameter values, we observe chaos in the Foutayeni-Khaladi fish-biology model and derive a new fish-biology chaotic system. Properties of the new fish-biology chaotic system are analyzed by means of phase portraits, Lyapunov exponents, and Kaplan-Yorke dimension. An electronic circuit realization is shown to validate the chaotic behavior of the new 3-D fish-biology chaotic system. The circuit experimental results of the 3-D fish-biology chaotic system show agreement with numerical simulations.

1. Introduction

Chaotic systems are nonlinear dynamical systems with at least one positive Lyapunov exponent [1-2]. The chaotic systems are characterized by “butterfly effect”, i.e. they are highly sensitive to initial conditions. Chaotic systems generate trajectories with highly complex properties and hence chaotic systems are an important area of study in science and engineering.

Chaos has applications in science and engineering such as planets systems [3], biology [4], ships in wave [5], beam system [6], radars [7], oscillators [8], robotics [9], machine [10], environmental [11], circuits [12-14], etc. In these applications, the scientists have used various existing chaotic systems and also developed new chaotic models. Such new chaotic systems and models have been implemented using MATLAB, FPGA, electronic circuits and also real circuit designs.

In 2016, Foutayeni and Khaladi [15] presented a three-dimensional dynamical systems model with delay that describes the dynamics of three fish populations with one predator and two preys. In this work, we work on the 3-D fish-biology system derived by Foutayeni and Khaladi [15] to obtain a new 3-D fish-biology chaotic system. We achieve this by considering a different set of parameters for the fish-biology dynamical system reported in [15].

In Section 2, the main results of the 3-D fish-biology chaotic system are discussed in detail and phase portraits are presented.
In Section 3, a circuit implementation of the fish-biology chaotic system is shown to facilitate practical feasibility of the theoretical model. Electronic circuit realization is an important topic for practical implementation of chaotic systems [16]. Section 4 concludes with a summary.

This paper is organized as follows. Section 2 describes the new chaotic system, its phase plots and equilibrium points. Section 3 describes the dynamic analysis of the new chaotic system. Section 4 depicts an electronic circuit realization of the new chaotic system. Section 5 draws the main conclusions.

2. A new fish biology chaotic system

First, we consider the fish-biology dynamical system studied by Foutayeni and Khaladi [15]. This dynamical system is given by the three-dimensional space equations:

\[
\begin{align*}
\dot{x} &= r_1 \left(1 - \frac{x}{K}\right) - c_{12} xy - a_{13} xz \\
\dot{y} &= r_2 \left(1 - \frac{y}{K}\right) - c_{23} yz \\
\dot{z} &= (\delta_1 x + \delta_2 y) z - \mu z - \theta_1 x - \theta_2 yz
\end{align*}
\]

The fish-biology system (1) modelled by Foutayeni and Khaladi describes the population biology model of three fish populations consisting of a predator and 2 preys.

In (1), \(x(t), y(t)\) and \(z(t)\) represent the population density of the fishes at any time \(t > 0\), where \(x(t), y(t)\) denote two preys with growth rates \(r_1, r_2\) and \(z(t)\) denotes a predator with decay rate \(\mu\) among the 3 fish populations. Here, \(K\) stands for carrying limit (threshold value) in the environment. Also, \(c_{ij}\) represents the competition coefficient between the fish populations \(i\) and \(j\). We suppose that \(a_{ij}\) is the predation rate coefficient. We also suppose that \(\delta_1\) and \(\delta_2\) are the maximum predator conversion rates.

The first fish population \(x(t)\) competes with the second fish population \(y(t)\) and it is a prey of the third fish population \(z(t)\). The second fish population \(y(t)\) competes with the first fish population \(x(t)\) and it is a prey of the third fish population \(z(t)\). The third fish population \(z(t)\) is the predator of the first and second fish populations, viz. \(x(t)\) and \(y(t)\).

Foutayeni and Khaladi [15] found that the system (1) has seven equilibrium points including the origin and showed that the origin is an unstable saddle point for the parameter values

\[
\begin{align*}
 r_1 &= 0.41, \quad r_2 = 0.21, \quad c_{12} = 0.001, \quad c_{23} = 0.002, \quad a_{13} = 0.02, \quad a_{23} = 0.01 \\
 \delta_1 &= 0.07, \quad \delta_2 = 0.08, \quad \theta_1 = 0.06, \quad \theta_2 = 0.07, \quad K = 100, \quad \mu = 0.1
\end{align*}
\]

In this paper, we show that the fish-biology system (1) displays chaotic behaviour for a different set of parameter values, viz.

\[
\begin{align*}
 r_1 &= 0.41, \quad r_2 = 0.21, \quad c_{12} = 0.001, \quad c_{23} = 0.001, \quad a_{13} = 0.02, \quad a_{23} = 0.01 \\
 \delta_1 &= 0.1, \quad \delta_2 = 0.1, \quad \theta_1 = 0.02, \quad \theta_2 = 0.02, \quad K = 1000, \quad \mu = 0.1
\end{align*}
\]

For numerical calculations, we take the initial conditions for the fish-biology system (1) as

\(x(0) = x_0 = 0.1, \quad y(0) = y_0 = 0.2, \quad z(0) = z_0 = 0.3\) (4)

Lyapunov exponents of the fish-biology system (1) are determined using Wolf’s algorithm [17] in MATLAB for the parameter values (3) and the initial conditions (4) as follows:

\[L_1 = 0.02648, \quad L_2 = 0, \quad L_3 = -0.03916\]

The presence of a positive Lyapunov exponent \(L_1\) confirms that the fish-biology system (1) is chaotic for the choice of parameters given in (3). The Kaplan-Yorke dimension of the fish-biology chaotic system (1) is obtained as

\[D_{KY} = 2 + \frac{L_1 + L_2}{|L_1|} = 2.6762\]
Since $L_1 + L_2 + L_3 < 0$, it is evident that the fish-biology chaotic system (1) is dissipative. Thus, the system orbits of the fish-biology chaotic system (1) are ultimately confined into a strange chaotic attractor.

Figures 1 (a)-(d) show the phase portraits of the strange attractor of the fish-biology chaotic system (1) for the parameter values given in (3) and initial conditions (4).

The time-evolution of the Lyapunov exponents of the system (1) is depicted in Figure 2.
3. Circuit implementation of the fish-biology chaotic system
In this section, fish-biology system (1) was designed as an electronic circuit (Fig. 4). The circuit design of the chaotic system was implemented in MultiSIM. The circuit employs simple electronic elements such as resistors, capacitors, multipliers and operational amplifiers. In this study, a linear scaling is considered as follows:

\[
\begin{align*}
\dot{x} &= r_x \left( 1 - \frac{x}{50} + 50c_1_{xy} - 10a_{2}x + z \right) \\
\dot{y} &= r_y \left( 1 - \frac{y}{50} + 50c_2_{xy} - 10a_{3}x + z \right) \\
\dot{z} &= 4x y + 4y z - \mu z
\end{align*}
\] (7)

For the circuital implementation, we express the fish-biology chaotic system (7) as follows.

\[
\begin{align*}
\dot{x} &= \frac{1}{C_1 R_1} x - \frac{1}{C_1 R_2} y - \frac{1}{C_1 R_3} y - \frac{1}{C_1 R_4} z \\
\dot{y} &= \frac{1}{C_2 R_1} x - \frac{1}{C_2 R_2} y - \frac{1}{C_2 R_3} y - \frac{1}{C_2 R_4} z \\
\dot{z} &= \frac{1}{C_3 R_1} x + \frac{1}{C_3 R_2} y + \frac{1}{C_3 R_3} y + \frac{1}{C_3 R_4} z
\end{align*}
\] (8)

Here, \(x, y, z\) are the voltages across the capacitors \(C_1, C_2, \text{ and } C_3\), respectively. We choose the values of the circuital elements as follows:

\[
\begin{align*}
R_1 &= 975.609 \, \Omega, R_2 = 19.51219 \, M \Omega, R_3 &= 8 \, M \Omega, R_4 = 2 \, M \Omega \\
R_5 &= 1.90476 \, M \Omega, R_6 = 38.09534 \, M \Omega, R_7 = R_8 = 4 \, M \Omega \\
R_9 &= R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = 100 \, K \Omega \\
C_1 &= C_2 = C_3 = 1 \, nF
\end{align*}
\] (9)

The supplies of all active devices are ±15 Volt. Oscilloscope results of the circuit are represented in Figs. 4. The similarity between the numerical simulations (Fig. 1) and the oscilloscope results (Figs. 4) shows the feasibility of the theoretical system (1).

4. Conclusions
In this work, by taking a different set of parameter values, we observed chaos in the Foutayeni-Khaladi fish-biology model (2016) and derived a new fish-biology chaotic system. The dynamical properties of the fish-biology chaotic system are reported by means of Lyapunov exponent spectrum and Kaplan-Yorke dimension. The obtained results confirm the complex dynamical behaviors. Finally, electronic circuit design of the fish-biology chaotic system has been implemented and validated using the MultiSIM software to verify the numerical simulations results. The output results of MultiSIM show good qualitative agreement with the MATLAB simulations of the new fish-biology chaotic system.
Figure 3. Circuit design for the fish-biology chaotic system (1)

Figure 4. The phase portraits of fish-biology chaotic system (1) observed on the oscilloscope in different planes (a) x-y plane, (b) y-z plane, and (c) x-z plane
5. References

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