Rare Decay of K Meson

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Abstract

We investigate the rare decay processes of the K mesons, $K_{L,S} \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in LR model with lepton family number being well conserved. In these processes, scalar operators $(\bar{s}d)(\bar{\nu}_\tau \nu_\tau)$, which are derived from box diagram in LR model, play an important role due to an enhancement factor $M_K/m_s$ in the matrix element $<\pi|\bar{s}d|K>$. It is emphasized that the $K_L$ decay process through the scalar operator is not the CP violating mode, so the $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ remains non-zero even in the CP conserved limit. We present the pion energy spectrum for these processes and discuss the effects of LR model.

1 Introduction

Decay of the neutral K meson taught us the violation of CP symmetry. Our understanding about the CP violation is based on one complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix $[1]$, however, our knowledge of the CKM phase is poor today. The projects of the B-factory are starting at KEK and SLAC, where CKM sector of SM will be tested by using e.g. unitarity triangle in the B meson system. The precise determination of the CKM parameters will be one of the most important progresses to understand the nature, physics of violated symmetry.

Experiments in the K meson system have entered new period. That is the observation of the rare process, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and the search for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Recently, the signature of decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has been observed by E787 Collaboration $[2]$ and the reported branching ratio is $4.2^{+9.7}_{-3.5} \times 10^{-10}$, which is consistent with the expectation value in SM. Additional data are expected as well as the improvement of the experimental data in the near future $[3]$. This situation forces us the detail study of the rare decays of the K mesons. The decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$

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is one of the most promising processes since it is a CP violating mode in SM. This mode is theoretically clean to extract the CKM parameter $\eta$ [4].

In this paper, we investigate rare decays of the K mesons in Left-Right (LR) model [3] introducing right handed neutrinos. However, in the model, the neutrino masses are zero in the tree level and lepton flavor is well conserved [3]. (Also see analyses in the other models [7]). There appear scalar and tensor operators $\bar{s} [1, \sigma^\mu \nu] d (\bar{\nu} [1, \sigma_{\mu \nu}] \nu)$ from the LR box diagrams, in which left and right handed gauge bosons $W_L$ and $W_R$ are exchanged. The scalar operators have an enhancement factor $M_K/m_s$ in the matrix element $< \pi | \bar{s} d| K >$ ($M_K$ and $m_s$ is K meson and strange quark mass respectively). Thus the scalar operator may have large contribution to the rare decays of K mesons, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_{L,S} \rightarrow \pi^0 \nu \bar{\nu}$. An important point is that the CP property of the scalar interaction is different from the V-A interaction $(\bar{s}d)_{V-A}(\bar{\nu}_l \nu_l)_{V-A}$ in SM. The decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ through the scalar operator is not CP violating one, so we have non-zero branching ratio $B(K_L \rightarrow \pi^0 \nu \bar{\nu}_l)$ even in the CP conserved limit ($\eta \rightarrow 0$). Thus, it is interesting to estimate how large the effect of the scalar operator in the pion energy spectrum.

This paper is organized as follows. In section 2, we first discuss the rare decays of the K meson rather generally with an effective Lagrangian including the scalar interaction. The scalar interaction in LR model is briefly discussed. In section 3, we show the pion energy spectrum in LR model, section 5 is devoted to discussion and summary.

## 2 Rare Decay Through the Scalar Operator

Rare decays, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_{L,S} \rightarrow \pi^0 \nu \bar{\nu}$, are flavor changing neutral current (FCNC) processes which are loop-induced one in SM and short-distance dominated one, so theoretically very clean modes [4]. The matrix elements involved in these decays are related to experimentally well known leading decay $K^+ \rightarrow \pi^0 e^+ \nu$ using isospin symmetry, and corrections to this relations have been studied [8]. In this section, we start our discussion from following effective Lagrangian:

$$
\mathcal{L}_{\text{eff}} = -\frac{4kG_F}{\sqrt{2}} \sum_{l=e,\mu, \tau} \left[ C_{SM}^l (\bar{s}_L \gamma^\mu d_L) (\bar{\nu}_{L,l} \gamma_\mu \nu_{L,l}) + C_{LR}^l (\bar{s}_L d_R) (\bar{\nu}_{L,l} \nu_{R,l}) \right] + \text{h.c.},
$$

(1)

where $\kappa = \alpha/(2\pi \sin^2 \Theta_W)$. The first term is given by SM contributions [4, 8] and the second and third terms are given by new contributions in our model. The scalar and tensor operators may generally appear from box diagram when one consider a model which contains the right handed charged gauge boson $W_R$. In this paper, we only discuss the contribution from the scalar operator and show the explicit form of the coefficients $C_{LR}^l$ and $C_{RL}^l$.

First we show the decay amplitude for the neutral K meson, $K_L$ and $K_S$ ($|K_{L,S} > \equiv p|K^0 > \pm q|\bar{K}^0 >$, $CP|K^0 > \equiv -|\bar{K}^0 >$). We assume neutrinos are massless, thus each term in the effective Lagrangian do not interfere in the decay process. The decay amplitude $A(K_{L,S} \rightarrow \pi^0 \nu \bar{\nu})$ are:

$$
A(K_{L,S} \rightarrow \pi^0 \nu \bar{\nu}) = -\frac{G_{FK}}{\sqrt{2}} \left( (pC_{SM}^l \mp qC_{SM}^* l) (\bar{s} d) \right) (\bar{\nu}_l \nu_l)_{V-A}
$$
\[ + \left( p(C_{LR}^I + C_{RL}^I) \pm q(C_{LR}^I + C_{RL}^I)^* \right) < \bar{s}d > (\bar{\nu}\nu) \]
\[ + \left( p(C_{LR}^I - C_{RL}^I) \mp q(C_{LR}^I - C_{RL}^I)^* \right) < \bar{s}d > (\bar{\nu}\gamma_5\nu) \]  
\text{(2)}

where \( < \mathcal{O} > \) is a short hand notation for \( < \pi^0|\mathcal{O}|K^0 > \).

The CP conserved limit corresponds to \( p = q \) with all coefficients \( C_{SM}^I, C_{LR}^I \) and \( C_{RL}^I \) being real. In this limit, the decay amplitude \( A(K_L \rightarrow \pi^0\bar{\nu}_L\nu_L) \) through the V-A interaction is zero, while \( A(K_S \rightarrow \pi^0\bar{\nu}_L\nu_L) \) is nonzero and the decays through the scalar operators \( A(K_{L,S} \rightarrow \pi^0\bar{\nu}_{R(L)}\nu_{R(L)}\) remain non-zero generally. In LR symmetric case \( (C_{LR}^I = C_{RL}^I) \), \( K_S \) decay through the scalar operators are the CP violating mode, while CP is conserved for \( K_L \) decay. Thus decays of neutral K meson in LR symmetric case are summarized as follows:

\[ \bullet K_L \text{decay} \]
\[ \{ (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} \Rightarrow \mathcal{C}P, \]
\[ (\bar{s}d)_{S}(\bar{\nu}\nu)_{S} \Rightarrow \mathcal{C}P \text{ Conserving} \]

\[ \bullet K_S \text{ decay} \]
\[ \{ (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A} \Rightarrow \mathcal{C}P \text{ Conserving}, \]
\[ (\bar{s}d)_{S}(\bar{\nu}\nu)_{S} \Rightarrow \mathcal{C}P \}

Experimentally we do not observe neutrinos, so the pion energy spectrum is obtained by summing these contributions which have different CP properties each other. The \( K_L \) decay through V-A operator is suppressed due to CP symmetry, but decays through the scalar operators are CP conserved ones and furthermore its matrix element is enhanced by use of equation of motion of:

\[ < \pi^0|\bar{s}d|K^0 > = \frac{p_{-} < \pi^0|(\bar{s}d)_{V-A}|K^0 >}{m_d - m_s} \sim \frac{M_K}{m_s} \times M_K f^\pm, \]
\text{(3)}

where \( (\bar{s}d)_{V-A} = f_+p_+^d + f_-p_0^d \) and \( p_{\pm} = P_K \pm p_s \). Thus, the contribution of the scalar interaction to the decay amplitude \( A(K_L \rightarrow \pi^0\bar{\nu}\nu) \) is sizable and dominates at CP conserved limit.

The decay amplitude for the charged K meson \( A(K^+ \rightarrow \pi^+\bar{\nu}\nu) \) is obtained in the same way:

\[ A(K^+ \rightarrow \pi^+\bar{\nu}\nu) = - \frac{G_F}{\sqrt{2}} \kappa (C_{SM} < (\bar{s}d)_V > (\bar{\nu}\nu)_{V-A} + (C_{LR} + C_{RL}) < \bar{s}d > (\bar{\nu}\gamma_5\nu) + (C_{LR} - C_{RL}) < \bar{s}d > (\bar{\nu}\gamma_5\nu) ), \]
\text{(4)}

where \( < \mathcal{O} > = < \pi^+|\mathcal{O}|K^+ > \).

Now we show explicit form of the coefficient function \( C_{LR}^I \) in LR model. [3]. We choose LR symmetric case, \( C_{LR}^I = C_{RL}^I \). There are two types of diagrams, penguin and box, which contribute to the process we are interested in. However, only the box diagrams produce the scalar operator \( (\bar{s}d)_S(\bar{\nu}\nu)_{S} \) in the effective Lagrangian. Thus we do not consider the contributions from penguin diagrams in this paper. There are tensor operators \( (\bar{s}\sigma^{\mu\nu}d)(\bar{\nu}\sigma_{\mu\nu}\nu) \) from box diagram in LR model, which will be discussed in the other place [3].

We calculate box diagrams, in which left handed \( W_L \) boson (\( W \) in SM) and right handed gauge boson \( W_R \) are exchanged [Fig.1]. There are corresponding charged Higgs diagrams due to the gauge invariance. The internal upper fermion lines correspond to the ordinary and singlet quarks, the lower correspond to SM and singlet leptons.
Figure 1: Box diagrams which contribute the effective Lagrangian for the process $K \rightarrow \pi \nu \bar{\nu}$. (a) is a contribution from $W_L$ and $W_R$. (b) and (c) are gauge boson and unphysical Higgs contributions. (d) is a contribution from unphysical Higgs $\chi_L$ and $\chi_R$.

The coefficients in the effective Lagrangian are:

$$C^d_{LR} = C^d_{RL} = \cos \theta^d_L \cos \theta^d_R \sum_{q=u,c,t} (V^*_{qs} V^{qd}) \cos \theta^q_L \cos \theta^q_R \times \beta \sqrt{x_q y_l} [d(x_q, y_l) - d(X_q, y_l) - d(x_q, Y_l) + d(X_q, Y_l)],$$

where $\theta^q_{L/R}$ is a mixing angle between singlet left/right handed quark and corresponding doublet quark, $V^{ij}$ corresponds to $3 \times 3$ CKM matrix element. $x_q, X_q, y_l, Y_l$ and $\beta$ are dimensionless parameters defined by:

$$x_q = \frac{m^2_q}{M_W^2}, \quad X_q = \frac{M_q^2}{M_W^2}, \quad y_l = \frac{m^2_l}{M_W^2}, \quad Y_l = \frac{M_l^2}{M_W^2}, \quad \beta = \frac{M_R^2}{M_W^2}, \quad (5)$$

where $m_{q/l}$ and $M_{q/l}$ is a mass of the ordinary quark/lepton and mass of the heavy singlet quark/lepton. The function $d(x, y)$ is defined by:

$$d(x, y) = \frac{4 + \beta x y}{4} \left[ \frac{x \ln x}{(1 - x)(1 - \beta x)(1 - \beta y)} + \frac{\beta \ln \beta}{(1 - \beta x)(1 - \beta y)(1 - \beta)} \right] - \frac{1 + \beta}{4} \left[ \frac{x^2 \ln x}{(1 - x)(1 - \beta x)(1 - \beta y)} + \frac{\ln \beta}{(1 - \beta x)(1 - \beta y)(1 - \beta)} \right], \quad (6)$$

The coefficient $C^d_{RL}$ ($l = e, \mu$) for electron and muon is negligibly small and only $C^\tau_{LR}$ contributes to the process significantly. The dependence on the internal quark is very similar to the one in the SM. For the top sector, coefficients are large by the top quark mass, but suppressed by the CKM factor $V^{ts*} V^{td} \sim \lambda^5$ compared to the one for the charm sector. The coefficient for the up quark sector is negligible. All the coefficients of the scalar interactions are suppressed by the $\beta$, so the V-A interaction in SM dominates when $M_R$ becomes large.
3 Pion Energy Spectrum

In this section, we present the pion energy spectrum by using the coefficient in section 2. In the process $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, contribution from the scalar interactions are tiny compared to the one in SM. The pion energy spectrum for the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is:

$$\frac{dB}{dx_{\pi}} \sim \sqrt{x_{\pi}^2 - 4\delta^2} \left[ (x_{\pi}^2 - 4\delta^2)|C^l_{SM}|^2 + 3\hat{t} \left( \frac{M_K}{m_d - m_s} \right)^2 (1 - \delta^2 + \xi \hat{t})^2 |C^l_{LR}|^2 \right]$$  \hspace{1cm} (7)

where $x_{\pi}$ is a normalized pion energy defined by $x_{\pi} = 2E_{\pi}/M_K$, and $\delta = m_{\pi}/M_K$, $\xi = f_{K^+\pi^0}/f_{K^+\pi^0}$ and $\hat{t} = (1 + \delta^2 - x_{\pi})$ are defined.

The process $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is more sensitive to probe the scalar interactions as discussed in the previous section. The energy spectrum is given by:

$$\frac{dB}{dx_{\pi}} \sim \sqrt{x_{\pi}^2 - 4\delta^2} \left[ (x_{\pi}^2 - 4\delta^2)|pC^l_{SM} \pm qC^l_{SM}|^2 + 3\hat{t} \left( \frac{M_K}{m_d - m_s} \right)^2 (1 - \delta^2 + \xi \hat{t})^2 |pC^l_{LR} \pm qC^l_{RL}|^2 \right]$$  \hspace{1cm} (8)

The first term is the SM contribution from V-A operator $(\bar{s}d)_{V-A}$, and the second is the contribution from the scalar operator which has a enhancement factor $M_K^2/(m_d - m_s)^2$ in the matrix element. Form factors are related to the one of experimentally well known leading decay mode $K^+ \rightarrow \pi^0 e^+ \nu$.

We show the pion energy spectrum $\frac{dB}{dx_{\pi}}$, in which we summed over neutrino flavors and normalized by a factor $10^{-10}$, in Fig.2.

Figure 2: The solid line is a pion energy spectrum in LR model, doted is the SM prediction, where we take the right handed gauge boson mass $M_R = 500 GeV$ and $\rho = \eta = 0.25$. LR contribution is large in the low energy region $x_{\pi} \sim 2\delta$, while in the high energy region the SM contribution dominates.
4 Conclusion

We have discussed rare decays of K mesons, $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_{L,S} \to \pi^0 \nu \bar{\nu}$ including the contributions from scalar operators in the effective Lagrangian, which is produced from LR box diagram. For the decay $K^+ \to \pi^+ \nu \bar{\nu}$, contribution from LR model is small compared to the one of SM. For the decay $K_L \to \pi^0 \nu \bar{\nu}$, there is significant contribution from the scalar operator, especially in the low energy region of the pion energy spectrum, which amount to about 30% enhancement to total branching ratio, with the parameters $\rho, \eta = 0.25$ and right handed gauge boson mass $M_R = 500$ [GeV]. Thus, measuring the decay $K_L \to \pi^0 \nu \bar{\nu}$ precisely is very important to probe the effect from new physics.

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