The Chelyabinsk superbolide: a fragment of asteroid 2011 EO$_{40}$?

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ABSTRACT
Bright fireballs or bolides are caused by meteoroids entering the Earth’s atmosphere at high speed. Some have a cometary origin, a few may have originated within the Venus–Earth–Mars region as a result of massive impacts in the remote past but a relevant fraction is likely the result of the break-up of asteroids. Disrupted asteroids produce clusters of fragments or asteroid families and meteoroid streams. Linking a bolide to a certain asteroid family may help to understand its origin and pre-impact dynamical evolution. On 2013 February 15, a superbolide was observed in the skies near Chelyabinsk, Russia. Such a meteor could be the result of the decay of an asteroid and here we explore this possibility applying a multistep approach. First, we use available data and Monte Carlo optimization (validated using 2008 TC$_3$ as template) to obtain a robust solution for the pre-impact orbit of the Chelyabinsk impactor ($a = 1.62$ au, $e = 0.53$, $i = 3:82$, $\Omega = 326:41$ and $\omega = 109:44$). Then, we use this most probable orbit and numerical analysis to single out candidates for membership in, what we call, the Chelyabinsk asteroid family. Finally, we perform N-body simulations to either confirm or reject any dynamical connection between candidates and impactor. We find reliable statistical evidence on the existence of the Chelyabinsk cluster. It appears to include multiple small asteroids and two relatively large members: 2007 BD$_7$ and 2011 EO$_{40}$. The most probable parent body for the Chelyabinsk superbolide is 2011 EO$_{40}$. The orbits of these objects are quite perturbed as they experience close encounters not only with the Earth–Moon system but also with Venus, Mars and Ceres. Under such conditions, the cluster cannot be older than about 20–40 kyr.

Key words: celestial mechanics – minor planets, asteroids: general – minor planets, asteroids: individual: 2007 BD$_7$ – minor planets, asteroids: individual: 2008 TC$_3$ – minor planets, asteroids: individual: 2011 EO$_{40}$ – planets and satellites: individual: Earth.

1 INTRODUCTION
The decay of asteroids in the main belt region is one of the sources of small near-Earth asteroids or meteoroids. The shattered pieces resulting from the collisional, tidal or rotational break-up of a rubble-pile asteroid can spread along the entire orbit of the parent body on a time-scale of hundreds of years (Tóth, Vereš & Kornoš 2011). These meteoroid streams can cause meteor showers on the Earth when their paths intersect that of our home planet (e.g. Jopek & Williams 2013). Exceptionally bright meteors are popularly known as fireballs. More properly, relatively small impacting objects entering the Earth’s atmosphere at high speed and reaching an apparent magnitude of $-14$ or brighter are called bolides; if the magnitude is $-17$ or brighter they are known as superbolides (Ceplecha et al. 1999). Superbolides can produce very powerful ballistic shock waves as they move at hypersonic speeds and explosive shock waves when they fragment in the atmosphere or hit the ground to form an impact crater (e.g. Ens et al. 2012). They are also parents of meteorite showers as meteorite-dropping bolides, seeding the ground with fragments of extraterrestrial material (e.g. Foschini 2001). Although not capable of triggering global devastation, they are powerful enough to provoke a significant amount of local damage. These events are not exclusive of the Earth but have also been predicted (Dycus 1969; Adolfsson, Gustafson & Murray 1996; Christou & Beurler 1999; Bland & Smith 2000; Christou 2004, 2010; Domokos et al. 2007; Christou, Vaubaillon & Withers 2008; Christou et al. 2012 and observed (Selsis et al. 2005; Christou, Vaubaillon & Withers 2007; Hueso et al. 2010; Daubar et al. 2013) in other planets. The connection between asteroidal debris and bolides was first proposed by Halliday (1987), further explored by Williams (2002, 2004) and Jenniskens (2006) and first confirmed observationally by Trigo-Rodríguez et al. (2007). Since then, new examples of bolides associated with asteroids have been reported (Trigo-Rodríguez et al. 2009, 2010; Madiedo et al. 2013).

On 2013 February 15, 03:20:33 GMT a superbolide was observed in the skies near Chelyabinsk, Russia. The event is believed to have been caused by a relatively small impacting object (17–20 m)
2 BEFORE IMPACT: A MONTE CARLO APPROACH

If the Chelyabinsk superbolide was the result of the decay of a larger asteroid and we want to identify the putative parent body or bodies, the first step is having a well-defined, statistically robust impactor orbit prior to its collision. Unfortunately, the range of orbital parameters from the solutions provided by the various authors (see Table 1) is too wide to be useful in a systematic search. The impactor came from the direction of the rising Sun and no pre-impact observations have been released yet. The well-known pre-impact orbit of the Chelyabinsk superbolide. The solution obtained in this work is also included and it matches that of Nakano. Standard deviations are provided when known. See the text for details.

Entering the Earth’s atmosphere at high speed and a shallow angle. Calculations by Adamo (2013), Borovicka et al. (Green 2013), Chodas & Chesley, Emel’yanenko et al., Lyytinen, Lyytinen & Matson, Nakano, Proud (2013), Zuluaga & Ferrin and Zuluaga, Ferrin & Geens (2013) revealed that the parent object was one of the Apollo asteroids that periodically cross the orbit of the Earth (see Table 1). In this Letter, we assume that the meteoroid responsible for the Chelyabinsk event was the result of a relatively recent asteroid break-up event and use numerical analysis to single out candidates to be the parent body or bodies. Then we perform N-body calculations to further study any possible dynamical connection between the candidates and the superbolide. Our analysis indicates that the Chelyabinsk impactor was a small member of a not-previously-identified young asteroid family. The most probable pre-impact orbit is obtained in Section 2 using Monte Carlo optimization techniques. The candidate selection procedure is described and available information on the candidate bodies is presented in Section 3. The results of our N-body calculations together with the numerical model are shown in Section 4. The proposed new asteroid family is characterized in Section 5. Results are discussed and conclusions are summarized in Section 6.

Table 1. Currently available solutions for the pre-impact orbit of the Chelyabinsk superbolide. The solution obtained in this work is also included and it matches that of Nakano. Standard deviations are provided when known. See the text for details.

| Authors            | \(a (\text{au})\) | \(e\)  | \(i (\degree)\) | \(\Omega (\degree)\) | \(\omega (\degree)\) | \(P_{\text{0.03 au}}\) | \(P_{\text{0.0000263 au}}\) | \(\beta_{\text{max}}\) |
|---------------------|------------------|--------|-----------------|----------------------|----------------------|--------------------------|-----------------------------|--------------------------|
| Adamo (2013)        | 1.60             | 0.53   | 4.07            | 326.46               | 109.36               | 0.950 8                  | 0.000 0633                  | 0.986 2                  |
| Borovicka et al.    | 1.55 ± 0.07      | 0.50 ± 0.02 | 3.6 ± 0.7    | 326.41               | 109.7 ± 1.8          | 0.889 1                  | 0.000 1696                  | 0.989 9                  |
| Chodas & Chesley1   | 1.73             | 0.57   | 4.2             | 326.41               | 109.7 ± 1.8          | 0.889 1                  | 0.000 1696                  | 0.989 9                  |
| Emel’yanenko et al.2| 1.77             | 0.58   | 4.3             | 326.41               | 109.7 ± 1.8          | 0.889 1                  | 0.000 1696                  | 0.989 9                  |
| Lyytinen3           | 1.66             | 0.52   | 4.05            | 326.43               | 109.0 ± 1.8          | 0.889 1                  | 0.000 1696                  | 0.989 9                  |
| Lyytinen & Matson4  | 1.660 079 129    | 0.524 094 58 | 4.235 664 | 326.457 642          | 114.670 125          | 0.330 2                  | 0.000 0001                  | 0.875 6                  |
| Nakano5             | 1.623 3665       | 0.531 1191 | 3.871 28     | 326.425 24          | 109.708 44          | 0.899 9                  | 0.000 1183                  | 0.985 8                  |
| Proud (2013)        | 1.47 ± 0.03      | 0.52 ± 0.05 | 4.61 ± 2.09  | 326.53 ± 0.1         | 96.58 ± 2.04        | 0.768 1                  | 0.000 0001                  | 0.508 2                  |
| Zuluaga & Ferrin6   | 1.73 ± 0.23      | 0.51 ± 0.08 | 3.45 ± 2.02  | 326.70 ± 0.79       | 120.62 ± 2.77       | 0.341 6                  | 0.000 002                   | 0.743 0                  |
| Zuluaga et al. (2013)| 1.27 ± 0.05 | 0.44 ± 0.02 | 3.0 ± 0.2    | 326.54 ± 0.08       | 95.1 ± 0.8          | 0.978 3                  | 0.000 02                   | 0.151 3                  |
| Zuluaga et al.7     | 1.368 ± 0.006    | 0.470 ± 0.010 | 4.0 ± 0.3    | 326.479 ± 0.003     | 99.6 ± 1.3          | 1.000 02                 | 0.020 3                   | 0.203 0                  |

This work 1.623 75 ± 0.000 14 | 0.532 79 ± 0.000 11 | 3.817 ± 0.005 | 326.409 0 ± 0.000 7 | 109.44 ± 0.03 | 1.020 44 | 0.999 7

\[ \beta = \frac{e^2}{d} \left[ \left( \frac{d_{\text{Earth}}}{d_{\text{AU}}} \right)^2 + \left( \frac{\sigma_{\beta}}{\sigma_{d}} \right)^2 \right], \]

where \(d\) is the MOID of the test orbit in au, \(d_{\text{Earth}} = 0\) au is the minimum possible MOID, \(\sigma_{\beta}\) is assumed to be the radius of the Earth in au, \(f\) is Earth’s true anomaly used in the computation, \(f'\) is Earth’s true anomaly at the collision time and \(\sigma_{f'}\) is half the angle subtended by the Earth from the Sun (0.00488). If \(\beta > 0.368\), a collision is
solutions compiled in Table 1. If we calculate the probability of obtaining an MOID under 0.05 au ($P_{0.050}$) and 0.000 042 63 au ($P_{0.00004263}$) at the impact time (see above) and the highest $\beta$ rank for the various candidate orbits (see Table 1, last three columns), all of them are statistically less robust than the one obtained here. In these calculations, the errors associated with the orbital elements as provided by the respective authors (see Table 1) have been used when known; if unknown, the errors in Zuluaga et al. (2013) have been used. From now on, we will assume that the orbit followed by the Chelyabinsk impactor prior to its collision was the averaged one.

### 3 Candidate Selection

Now that the most probable orbit of the Chelyabinsk impactor has been established, the next step is finding candidates for the parent body of the impactor as we assumed that it was the result of a relatively recent asteroid break-up event. Groups of rocky fragments resulting from the disruption of an asteroid are called asteroid families. Trigo-Rodríguez et al. (2007) used the D-criterion of Southworth & Hawkins (1963), $D_{SH}$, to investigate the connection between the asteroid 2002 NY$_{10}$ and the FN300806 bolide. Alternatives to $D_{SH}$ are the D-criterion of Lindblad & Southworth (1971), $D_{LS}$, which is based on the previous one and the $D_{R}$ from Valsecchi, Jopek & Froeschl (1999). In order to investigate a possible association between the meteoroid responsible for the Chelyabinsk superbolide as characterized by the orbital solution in Table 1 and any known asteroid, we carried out a search among all the objects currently catalogued by the JPL Small-Body Database$^8$ using the three D-criteria. The lowest values of the various $D$s are found for 2011 EO$_{40}$ ($D_{SH} = 0.12$, $D_{LS} = 0.011$, $D_{R} = 0.0084$), followed by 2011 GP$_{28}$ (1.0, 0.015, 0.011) and 2010 DU$_{1}$ (1.0, 0.020, 0.04), 2008 FH (1.0, 0.040, 0.049) and 2007 BD$_{7}$ (0.93, 0.047, 0.041) among those with relatively well-known orbits. The best candidate, 2011 EO$_{40}$, was discovered on 2011 March 10 by Kowalski et al. (2011), has an absolute magnitude, $H$, of 21.7 (size of 150–330 m if $G = 0.15$ and the albedo range is 0.04–0.20) and it has been observed 20 times with an arc length of 34 d. The orbital elements of this Apollo asteroid are $a = 1.6539 \pm 0.0004$ au, $e = 0.54039 \pm 0.0001$, $i = 3.3638 \pm 0.0008$, $\Omega = 50.310 \pm 0.011$ and $\omega = 17.055 \pm 0.013$. Apollo asteroid 2007 BD$_{7}$ was discovered on 2007 January 23, has $H = 21.1$ and its orbit is based on 185 observations with a data-arc span of 14 d. Its orbital elements are $a = 1.5624 \pm 0.0012$ au, $e = 0.4980 \pm 0.0005$, $i = 4.849 \pm 0.004$, $\Omega = 343.627 \pm 0.008$ and $\omega = 219.875 \pm 0.007$. The other candidates have shorter arcs and larger $H$: 2008 FH (12 d, $H = 24.4$, $a = 1.582 \pm 0.012$ au, $e = 0.504 \pm 0.005$, $i = 3.45 \pm 0.03$, $\Omega = 5.207 \pm 0.013$ and $\omega = 264.09 \pm 0.04$), 2010 DU$_{1}$ (4 d, $H = 26.6$, $a = 1.687 \pm 0.006$ au, $e = 0.539 \pm 0.002$, $i = 3.704 \pm 0.012$, $\Omega = 147.832 \pm 0.002$ and $\omega = 74.250 \pm 0.010$) or 2011 GP$_{28}$ (1 d, $H = 29.4$, $a = 1.591 \pm 0.004$ au, $e = 0.5199 \pm 0.0015$, $i = 4.048 \pm 0.009$, $\Omega = 16.397 \pm 0.003$ and $\omega = 252.19 \pm 0.02$). All these objects are classified as Apollo asteroids, near-Earth objects (NEOs) and potentially hazardous asteroids (PHAs). Unfortunately, even the best-known orbits are based on rather short arcs. In the following, we perform N-body calculations to further study any possible dynamical connection between some of the candidates and the superbolide.
Figure 2. Results of the numerical integrations of the orbits backwards in time for 2011 EO\textsubscript{40}, 2007 BD\textsubscript{7}, 2008 FH and 2010 DU\textsubscript{1} and one representative superbolide control orbit (see the text for details). The evolution of perihelia (A), eccentricities (B), inclinations (C) and the various D-criteria ($D_{SH}$-green, $D_{LS}$-blue, $D_{R}$-red) are shown. The pink curve is always associated with the asteroid. The black curve shows the evolution of the control orbit of the meteoroid.

4 DYNAMICAL EVOLUTION

The orbital evolution of meteoroid orbits following the osculating elements in Table 1 (averaged orbit), those of the four most promising candidate objects pointed out above and several others, was computed for 0.25 Myr backwards in time using the Hermite integration scheme described by Makino (1991) and implemented by Aarseth (2003). The standard version of this serial code is publicly available from the IoA website.\textsuperscript{10} Results from this N-body code have been discussed in de la Fuente Marcos & de la Fuente Marcos (2012). Our direct integrations include the perturbations by the eight major planets, the Moon, the barycentre of the Pluto–Charon system and the three largest asteroids. For accurate initial positions and velocities, we used the heliocentric ecliptic Keplerian elements provided by the Jet Propulsion Laboratory online Solar system data service\textsuperscript{11} (Giorgini et al. 1996) and based on the DE405 planetary orbital ephemerides (Standish 1998) referred to the barycentre of the Solar system. In addition to the calculations completed using the nominal orbital elements pointed out above, we have performed 50 control simulations for each object with sets of orbital elements obtained from the nominal ones within the accepted uncertainties (3\textsigma). Meteoroid orbits have been treated similarly. Fig. 2 summarizes the results of our backwards integrations for the parent candidate asteroids. The orbital evolution of 2011 EO\textsubscript{40} matches well that of the Chelyabinsk impactor. Giving the uncertainties in the initial conditions (orbital elements) for both candidates and impactor, the agreement is good and suggests that these bodies were formed in a single (or a sequence of) break-up event(s) 20–40 kyr ago. The orbits of these objects are strongly perturbed as they experience periodic close encounters not only with the Earth–Moon system but also with Mars, Ceres and, in some cases, Venus. The objects studied here are part of a genetic family not a dynamical one, like the NEO family recently identified by de la Fuente Marcos & de la Fuente Marcos (2013).

5 THE CHELYABINSK ASTEROID FAMILY

So far, our numerical results are somewhat consistent with 2011 EO\textsubscript{40} and other minor bodies being members of a young asteroid family but, can we identify additional members tracing a putative Chelyabinsk asteroid complex? and, what is more critical, can we reasonably conclude that they could be the result of a break-up event? An analysis based on the various D-criteria shows about 20 candidates to be part of the proposed Chelyabinsk asteroid family. Unfortunately, most of them have $H > 25$ and very short arcs (a few days) so the actual characterization of the family is rather speculative although only objects with $D_{R} < 0.05$ have been tentatively selected. With this restriction, the orbital parameters of the proposed family and their spreads are $a = 1.66 \pm 0.08$ au, $e = 0.54 \pm 0.02$, $i = 3.7 \pm 1.3$, $\Omega = 162^\circ \pm 114^\circ$ and $\omega = 173^\circ \pm 96^\circ$. In order to check if these numbers are compatible with a gentle break-up event, we start a simulation with 100 test particles moving in orbits similar to that of 2011 EO\textsubscript{40} but with negligible spread in their orbital elements at aphelion. This is equivalent to having a smoothly disintegrating rubble-pile asteroid in which the relative velocities of the resulting fragments are basically zero. Fig. 3 shows the standard deviation

\textsuperscript{10} http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm

\textsuperscript{11} http://ssd.jpl.nasa.gov/?planet_pos

Figure 3. Time evolution of the dispersion of the orbital elements of a set of particles resulting from the smooth break-up of a rubble-pile asteroid as described in the text.
of the various elements of the test particles as a function of the time. Although this calculation gives no obvious constraint on the age of the family, the long-term values of their standard deviations match well the values obtained above even if we do not consider non-gravitational effects that could be important for small objects.

6 DISCUSSION AND CONCLUSIONS

Although the numerical and statistical evidence in favour of a Chelyabinsk asteroid family or complex is quite encouraging, the ultimate proof of a truly genetic relationship between all these objects requires spectroscopy or, much better, sample-return (e.g. Barucci et al. 2012). The analysis of the Chelyabinsk meteorites shows that they are chondrite breccias (Bischoff et al. 2013) so the parent meteoroid may be of the S class. Our calculations did not include the Yarkovsky effect (see e.g. Bottke et al. 2006) which may have a non-negligible role on the medium, long-term evolution of objects as small as the ones studied here. Proper modelling of the Yarkovsky force requires knowledge on the physical properties of the objects involved (for example, rotation rate, albedo, bulk density, surface conductivity, emissivity) which is not the case for the objects discussed here. Detailed observations during future encounters with the Earth should be able to provide that information. On the short term, the Yarkovsky force mainly affects $a$ and $e$ but within the dispersion range found here. Its effects are negligible if the objects are tumbling or in chaotic rotation. The non-inclusion of this effect has no major impact on the assessment completed.

In this Letter, we have obtained a statistically robust solution for the pre-impact orbit of the Chelyabinsk superbolide. Assuming that such a meteoroid could be the result of the decay of an asteroid, we have singled out some candidates for membership in a putative Chelyabinsk asteroid family or complex and tested, using

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