Spinor moving frame, M0–brane covariant BRST quantization and intrinsic complexity of the pure spinor approach

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To exhibit the possible origin of the inner complexity of the Berkovits’s pure spinor approach, we consider the covariant BRST quantization of the D=11 massless superparticle (M0–brane) in its spinor moving frame or twistor-like Lorentz harmonics formulation. The presence of additional twistor-like variables (spinor harmonics) allows us to separate covariantly the first and the second class constraints. After taking into account the second class constraints by means of Dirac brackets and after further reducing the first class constraints algebra, the dynamical system is described by the cohomology of a simple BRST charge $Q^{\text{susy}}$ associated to the $d = 1$, $n = 16$ supersymmetry algebra. The calculation of the cohomology of this $Q^{\text{susy}}$ requires a regularization which implies the complexification of the bosonic ghost associated to the $\kappa$–symmetry and further leads to a complex (non-Hermitian) BRST charge $\tilde{Q}^{\text{susy}}$ which is essentially the ‘pure spinor’ BRST charge $Q^B$ by Berkovits, but with a composite pure spinor.
1 Introduction

Recently a serious breakthrough in covariant description of quantum superstring theory has been reached in the framework of the Berkovits pure spinor approach [1]: a technique for loop calculations was developed [2] and the first results were given in [2, 3]. On the other hand, the pure spinor superstring was introduced as -and still remains- a set of prescriptions for quantum superstring calculations, rather than a quantization of the Green-Schwarz superstring. In particular, the measure defining the functional integration over the pure spinor ghosts was guessed [2] and checked on consistency [2] rather than derived. Despite a certain progress in relating the pure spinor superstring [1] to the original Green-Schwarz formulation [4], and also [5] to the superembedding approach [8, 9], the origin and geometrical meaning of the pure spinor formalism is far from being clear. Possible modifications of pure spinor formalism are also considered [6, 7]. In particular, an additional non-minimal sector appeared to be needed to further proceed with loop calculations [7]. A deeper understanding of how the pure spinor BRST operator, and other ingredients of the pure spinor approach, appear on the way of a straightforward covariant quantization of a classical action might, in particular, provide a resource of possible non-minimal variables and give new suggestions in further development of loop calculations.

In this context, the Lorentz harmonic approach [10]–[21], in the frame of which a significant progress toward a covariant superstring quantization had already been made in late eighties [11, 12], looks particularly interesting. Although no counterpart of the recent progress in loop calculations [2, 3] has been ever reached in the Lorentz harmonics framework, its relation with the superembedding approach [8, 9], clear group-theoretical and geometrical meaning [10, 13, 14, 15] and twistor-likeness [14, 16, 17, 18, 21] suggest it as a promising starting point of the search for the origin and geometrical meaning of the pure spinor formalism and its non-minimal modifications. We also hope that the further development of twistor-like Lorentz harmonic approach, in the pragmatic spirit which characterizes the pure spinor approach of [1, 2, 3], might lead to a convenient and transparent way of the covariant quantum description of superstring. A natural first stage in such a program is to study the covariant quantization of superparticle, and in particular, of the $D=11$ massless superparticle [22, 23] or $M0$–brane, also less studied in comparison with $D=10$ and $D=4$ superparticle models.

A supertwistor covariant quantization of the massless $D=11$ superparticle has been recently considered in [24]. It starts from twistor-like Lorentz harmonics formulation of the $M0$–brane [25] leads to the linearized $D=11$ supergravity multiplet in the superparticle quantum state spectrum (in agreement with the light–cone results of [23]) and exhibits a possible origin of the hidden $SO(16)$ symmetry of the $D = 11$ supergravity [27].

In this letter we report the results of the study of the BRST quantization of the $D=11$ massless superparticle in its twistor-like formulation [25, 24]. We find a simple reduced BRST charge describing this model and show that the calculation of its cohomology requires regularization which is made by complexification of the bosonic ghost for the $\kappa$–symmetry. Then the superparticle spectrum is described by cohomology of a complex BRST charge calculated at vanishing bosonic ghost. We discuss the relation of this complex BRST charge with the pure spinor BRST operator by Berkovits. This allows us to explain the intrinsic complexity of the pure spinor BRST charge. We also present the similar complex Lorentz harmonic BRST charge for superstring, which is essentially the Berkovits BRST operator but with composite pure spinors constructed from harmonics and the complexified bosonic ghosts. Derivation of this BRST operator by covariant quantization of superstring in its spinor moving frame formulation [16, 17] is an interesting problem for future study.

1 The form of the pure spinor ghost measure appeared in [2] as a result of a series of very elegant but indirect arguments involving the picture changing operator characteristic of the RNS string.

2 See [14] for $D=4$, [21] for $D=10$ and [16, 17, 18, 26] for the twistor-like Lorentz harmonic or spinor moving frame formulations of superstrings, standard and Dirichlet super-p-branes.
2 M0-brane in spinor moving frame formulation

The Brink-Schwarz superparticle action can be written in first order form as $S_{BS}^{1} = \int_{W^{1}}(P_{m}^{m} - \frac{1}{2}d\tau e P_{m}P^{m})$. Here $P_{m}(\tau)$ is the auxiliary momentum variable, $e(\tau)$ is the worldline einbein and

$$
\Pi^{m} := dx^{m} - id\theta^{m}\theta := d\tau\Pi^{m}, \qquad \Pi^{m} := \partial_{\tau}x^{m}(\tau) - i\partial_{\tau}\theta^{m}(\tau)\Gamma_{a\beta}^{m}\theta^{a}(\tau) \tag{2.1}
$$

is the pull-back of the bosonic supervielbein of flat superspace (Volkov-Akulov one-form) to the superparticle worldline. The above formulae are valued in any dimensions. The action of $D=11$ massless superparticle [22] is singled out by the $m = 0, 1, \ldots, 9, \# (\# \equiv 10)$ and $\alpha = 1, \ldots, 32$.

The einbein $e(\tau)$ plays the rôlé of Lagrange multiplier and produces the mass shell constraint $P_{m}P^{m} = 0$. Since this is algebraic, if its general solution is known, one may substitute it for $P_{m}$ in $S_{BS}^{1}$ and to obtain a classically equivalent formulation of the $D$- (here 11-) dimensional Brink-Schwarz superparticle. The moving frame or twistor-like Lorentz harmonics formulation of [25] [24] (see [14] for $D=4$ and [21] for $D=10$) can be obtained just in this way.

It is easy to solve the constraint $P_{m}P^{m} = 0$ in a non-covariant manner: in a special Lorentz frame a solution with positive energy reads e.g. $\beta_{(a)} = \frac{p_{a}}{2}(1, \ldots, -1) = \frac{p_{a}}{2}(\delta_{(a)}^{0} - \delta^{#}_{(a)})$. The solution in an arbitrary frame follows from this by making a Lorentz transformation,

$$
P_{m} := U_{m}^{(a)}\beta_{(a)} = \frac{p_{a}}{2}(u_{(a)}^{0} - u_{(a)}^{#}), \quad U_{m}^{(a)} := (u_{(a)}^{0}, u_{(a)}^{i}, u_{(a)}^{#}) \in SO(1, D - 1). \tag{2.2}
$$

Since $P_{m} = P_{m}(\tau)$ is dynamical variable in the superparticle action, the same is true for the Lorentz group matrix $U$ when it is used to express $P_{m}$ through Eq. (2.2), $U_{m}^{(a)} = U_{m}^{(a)}(\tau)$. Such moving frame variables [16] [17] are called Lorentz harmonics [14] [15] (light-cone harmonics in [10]).

Substituting (2.2) for $P_{m}$ in $S_{BS}^{1}$, one arrives at the action $S_{M0} = \int_{W^{1}}\frac{1}{2}p^{++}u_{m}^{--}\Pi^{m}$ where the vector $u_{m}^{--} = u_{m}^{0} - u_{m}^{#}$ is light–like as follows from the orthogonality and normalization of the timelike $u_{m}^{0}$ and spacelike $u_{m}^{#}$ vectors which, in their turn, follow from $U \in SO(1, 10)$ in Eq. (2.2).

Moreover, the further analysis shows that the above expression for $S_{M0}$ hides the twistor–like action, a higher dimensional ($D=11$ here) generalization of the $D=4$ Ferber–Schirafuji action [28]. Indeed it can be written in the following equivalent forms [25] [21]

$$
S_{M0} := \int d\tau L = \int_{W^{1}}\frac{1}{2}p^{++}u_{m}^{--}\Pi^{m} = \int_{W^{1}}\frac{1}{32}p^{++}v_{\alpha q}^{-}\Pi^{m}\Gamma_{m}^{\alpha\beta}, \tag{2.3}
$$

$$
\alpha = 1, 2, \ldots, 32 \quad (n \text{ in general}), \quad q = 1, \ldots, 16 \quad (n/2 \text{ in general}),
$$

where the first form of the action is as above, while the second form is twistor–like (cf. [28]). Instead of two–component Weyl spinors of the Ferber super-twistor [28], the action of Eq. (2.3) includes the set of 16 bosonic 32–component Majorana spinors $v_{\alpha q}^{-}$ which satisfy the following kinematical constraints (see [16] [17] [21] [25]),

$$
\left\{ \begin{array}{ll}
2v_{q}\delta_{\alpha q}v_{a\beta} &= u_{m}^{--}\Gamma_{m}^{\alpha\beta} \quad (a), \\
v_{q}\Gamma_{m}v_{p}^{-} &= \delta_{qp}u_{m}^{--} \quad (b), \\
v_{\alpha q}^{--}C^{\alpha\beta}v_{\beta p}^{-} &= 0 \quad (c), \\
u_{m}^{--}u_{m}^{--} &= 0 \quad (d).
\end{array} \right. \tag{2.4}
$$

In [21] we presented the super-twistor quantization of the M0–brane model (2.3). Here we perform the Hamiltonian analysis of the system and consider its BRST quantization.

2.1 Vector and spinor Lorentz harmonics. Spinor moving frame

Although, in principle, one can study the dynamical system using just the kinematical constraints (2.4), it is more convenient to treat the light–like vector $u_{m}^{--}$ as an element of moving frame and the set of 16 $SO(1, 10)$ spinors $v_{\alpha q}^{-}$ as part of the corresponding spinor moving frame. These moving
frame variables are also called (vector and spinor) Lorentz harmonics (see [29] for the notion of harmonics).

The vector Lorentz harmonics \( u_{m}^{\pm \pm}, u_{m}^{i} \) [10] are defined as elements of the \( 11 \times 11 \) Lorentz group matrix, Eq. (2.2). In the lightlike basis they are given by

\[
U_{m}^{(a)} = (u_{m}^{-}, u_{m}^{+}, u_{m}^{i}) \in SO(1, 10), \quad m = 0, 1, \ldots, 9, \# \, ; \quad i = 1, \ldots, 9 , \quad (2.5)
\]

where \( u_{m}^{\pm \pm} = v_{m}^{0} \pm u_{m}^{\#}. \quad \) The three-blocks splitting (2.5) is invariant under \( SO(1, 1) \otimes SO(9) \); \( SO(1, 1) \) rotates \( u_{m}^{0} \) and \( u_{m}^{\#} \) among themselves and, hence, transforms their sum and differences, \( u_{m}^{\pm \pm} = u_{m}^{0} \pm u_{m}^{\#} \), by inverse scaling factors.

The fact that \( U \in SO(1, 10) \) implies the constraints

\[
U^{T} \eta U = \eta \quad \Leftrightarrow \begin{cases}
    u_{m}^{-} u_{m}^{-} = 0, & u_{m}^{+} u_{m}^{+} = 0, & u_{m}^{\pm} u_{m}^{i} = 0 , \\
    u_{m}^{-} u_{m}^{+} = 2 , & u_{m}^{i} u_{m}^{j} = \delta_{ij} 
\end{cases} \quad (2.6)
\]

or, equivalently, the unity decomposition

\[
\delta_{m}^{n} = \frac{1}{2} u_{m}^{++} u_{n}^{--} + \frac{1}{2} u_{m}^{-+} u_{n}^{+-} - u_{m}^{i} u_{n}^{i} \quad \Leftrightarrow \quad U \eta U^{T} = \eta . \quad (2.7)
\]

The spinor harmonics [14 [15 [19] or spinor moving frame variables [16 [17 [18] \( \nu_{aq}^{\pm} \) are the elements of the \( 32 \times 32 \) \( Spin(1, 10) \) matrix

\[
V_{a}^{(b)} = (\nu_{aq}^{-}, \nu_{aq}^{+}) \in Spin(1, 10) \quad (\alpha = 1, \ldots, 32 , \quad q = 1, \ldots, 16) . \quad (2.8)
\]

They are ‘square roots’ of the associated vector harmonics in the sense that

\[
V^{T}(a)V = \Gamma^{m} U_{m}^{(a)} , \quad V^{T} \tilde{\Gamma}_{m} V = U_{m}^{(a)} \tilde{\Gamma}_{(a)} , \quad (2.9)
\]

which express the \( Spin(1, 10) \) invariance of the Dirac matrices.

Equation in (2.4a) is just the \( (a) = (--) \) component of the first equation in (2.9) taken in the Dirac matrices realization in which \( \Gamma^{0} \) and \( \Gamma^{\#} \) are diagonal and \( \Gamma^{i} \) are off-diagonal. Eq. (2.4b) comes from the upper diagonal block of the second equation in Eq. (2.9). To complete the set of constraints defining the spinorial harmonics, we have to add the conditions expressing the invariance of the charge conjugation matrix \( C \),

\[
V C V^{T} = C \quad , \quad V^{T} C^{-1} V = C^{-1} , \quad (2.10)
\]

which give rise to the constraint (2.4b).

In a theory with a local \( SO(1, 1) \otimes SO(9) \) symmetry containing only one of the two sets of 16 constrained spinors (2.5), say \( \nu_{aq}^{-} \), these can be treated as homogeneous coordinates of the \( SO(1, 10) \) coset giving the celestial sphere \( S^{9} \); specifically (see [15])

\[
\{ \nu_{aq}^{-} \} = \frac{Spin(1, 10)}{[Spin(1, 1) \otimes Spin(9)] \otimes \mathbb{K}_{9}} = S^{9} , \quad (2.11)
\]

where \( \mathbb{K}_{9} \) is the abelian subgroup of \( SO(1, 10) \) defined by

\[
\delta \nu_{aq}^{-} = 0 , \quad \delta \nu_{aq}^{+} = k_{+, q}^{i} v_{aq}^{-} , \quad i = 1, \ldots, 9 . \quad (2.12)
\]

Our superparticle model contains just \( \nu_{aq}^{-} \) and is invariant under \( SO(1, 1) \otimes Spin(9) \) transformations. Hence the harmonics sector of its configuration space parametrize \( S^{9} \) sphere.

In principle, the constraint Eqs. (2.6), as equivalent to Eq. (2.5), can be solved by expressing the vector harmonics in terms of 55 parameters \( l^{(a)}(b) = -l^{(b)}(a) \), \( U_{m}^{(a)} = U_{m}^{(a)} (l^{(b)}(c)) \),

\[
U_{m}^{(a)} = (u_{m}^{-}, u_{m}^{+, i} u_{m}^{i}) = U_{m}^{(a)} (l^{(d)}(c)) = \delta_{m}^{a} + \eta_{m}(b) l^{(b)}(a) + \mathcal{O}(l^{2}) . \quad (2.13)
\]
Furthermore, Eqs. (2.9), (2.10) imply that spinorial harmonics parametrize the double covering of the $SO(1,10)$ group element $U_{\alpha}^{(a)}(l)$ and, hence, that they also can be expressed through the same $l^{(a)(b)} = -l^{(b)(a)}$ parameters, $V_{a}^{(\beta)} = V_{a}^{(\beta)}(l)$,

$$V_{a}^{(\beta)} = (v_{a\alpha}, v_{a\dot{\alpha}}) = V_{a}^{(\beta)}(l^{(a)(b)}) = \pm \left( \delta_{a}^{\beta} + \frac{1}{4} l^{(a)(b)} \Gamma_{(a)(b)}^{(\beta)} + \mathcal{O}(l^{2}) \right). \quad (2.14)$$

The identification of the harmonics with the coordinates of $SO(1,10)/H$ corresponds, in this language, to setting to zero the $H$ coordinates in the explicit expressions (2.13), (2.14). In our case with $H = [SO(1,1) \otimes SO(9)] \otimes \mathbb{K}_{0}$ this implies $l^{0\#} = l^{i} = l^{+i} = 0$ so that the $SO(1,10)$ matrix is constructed with the use of 9 parameters $l^{-i} := l^{0} - l^{#i}$,

$$u_{a}^{-} = \delta_{a}^{-} + \delta_{a}^{+} l^{-i} + \frac{1}{2} \delta_{a}^{+} (l^{-j} l^{-j})^{-1}, \quad u_{a}^{+} = \delta_{a}^{+}, \quad u_{a}^{i} = \delta_{a}^{i} + \frac{1}{2} \delta_{a}^{+} l^{-i}. \quad (2.15)$$

$$v_{a\alpha} = \delta_{a\alpha}^{\alpha} + \frac{1}{2} l^{-i} \Gamma_{\alpha \beta}^{\alpha \beta}, \quad v_{a\dot{\alpha}} = \delta_{a\dot{\alpha}}^{\alpha}. \quad (2.16)$$

In distinction to the general Eqs. (2.13), (2.14), the above equations are not Lorentz covariant. Although the use of the explicit expressions (2.13), (2.14) (their complete form can be found in [3]) is not practical, it is useful to have in mind the mere fact of their existence which, in particular, makes transparent that the spinorial and vector harmonics carries the same degrees of freedom.

3 M0-brane Hamiltonian mechanics and the BRST charge $Q_{\text{susy}}$

3.1 Primary constraints of the D=11 massless superparticle model

The phase space $(Z^{N}, P_{N})$ of our superparticle model includes the coordinates and momenta

$$Z^{N} := (x^{a}, \theta^{a}, \rho^{++}, U_{m}^{(a)} \text{ or } V_{\beta}^{(a)}) \quad \text{or} \quad P_{N} = \frac{\partial L}{\partial \dot{Z}^{N}} := (P_{a}, \pi_{\alpha}, P_{++}^{(\rho)}, P_{m}^{(\alpha)} \text{ or } P_{(\alpha)}^{(\beta)}), \quad (3.1)$$

restricted by the kinematical constraints (2.6) or (2.9), (2.10) and also by the following primary constraints characteristic of the M0-brane in the spinor moving frame formulation (2.3)

$$\Phi_{a} := P_{a} - \frac{1}{2} P^{++} u_{a}^{-} \approx 0 \quad \iff \quad \Phi_{\alpha\beta} := \Phi_{a \alpha \beta}^{a} = P_{\alpha \beta}^{(a)} - \rho^{++} v_{\alpha \beta}^{-} v_{\alpha \beta}^{-} \approx 0 \quad (3.2)$$

$$d_{\alpha} := \pi_{\alpha} + i P_{a} \alpha \beta \Gamma_{\alpha \beta}^{a} \approx 0, \quad \pi_{\alpha} := \frac{\partial L}{\partial \dot{x}^{\alpha}}, \quad P_{m} := \frac{\partial L}{\partial \dot{z}^{m}}, \quad (3.3)$$

$$P_{++}^{(\rho)} := \frac{\partial L}{\partial \dot{\rho}^{++}} \approx 0, \quad (3.4)$$

and

$$P_{m}^{(\alpha)} := \frac{\partial L}{\partial \dot{V}_{m}^{(\alpha)}} \approx 0 \quad \text{or} \quad P_{(\alpha)}^{(\beta)} := \frac{\partial L}{\partial \dot{V}_{(\alpha)}^{(\beta)}} \approx 0, \quad (3.5)$$

Here \(\approx\) denotes weak equalities [3], the equalities which may be used only after all the Poisson brackets are calculated. This latter are defined by $[P_{a}, Z^{N}]_{PB} := -\delta_{a}^{N}$.

Since the canonical Hamiltonian $d\tau H_{0} := dZ^{N} P_{N} - d\tau L$ of the massless superparticle is zero in the weak sense, $H_{0} \approx 0$, its Hamiltonian analysis reduces to the analysis of the constraints. The presence of the harmonics in the phase space (3.1) makes possible to split covariantly the whole set of the constraints on the first and second class ones (which is not possible in the original Brink-Schwarz formulation).

3.2 Second class constraints and Dirac brackets

Keeping in mind that, upon solving the kinematical constraints (2.6) and (2.9), (2.10), the spinorial and vectorial harmonics are expressed through the same parameter $l^{(a)(b)}$, Eqs. (2.13), (2.14), we will use the Language of vector harmonics in the analysis of the bosonic second class constraints and the spinorial harmonics to separate covariantly the fermionic first and second class constraints.
It is convenient to begin with separating the set of 121 primary constraints $P_{(a)}^m \approx 0$ (3.5) in a set of 55 constraints $d_{(a)(b)} := P_{(a)}^m U_{m(b)} - P_{(b)}^m U_{m(a)}$ and the 66 constraints $K_{(a)(b)} := P_{(a)}^m U_{m(b)} + P_{(b)}^m U_{m(a)}$ (see [17]). The 55 constraints $d_{(a)(b)}$ commute with the kinematical constraints (2.3), which we denote by $\Xi^{(a)(b)} := U_m^a U_m^b - \eta^{(a)(b)} \approx 0$, and generate the Lorentz group algebra

$$d_{(a)(b)} := P_{(a)}^m U_{m(b)} - P_{(b)}^m U_{m(a)} \approx 0 \ , \quad [\Xi^{(a)(b)} , d_{(a')(b')} ]_{PB} \approx 0 \ , \quad (3.6)$$

$$[d_{(a)(b)} , d^{(c)(d)} ]_{PB} = -4\delta_{[a]}^{[c]} \delta_{d ]}^{(b])} \ . \quad (3.7)$$

In contrast, the 66 constraints $K_{(a)(b)}$ are manifestly second class constraints as far as they are conjugate to the (also second class) 66 kinematical constraints (2.6), $[\Xi^{(a)(b)} , K_{(a')(b')} ]_{PB} \approx 4\delta_{(a)(b)}^{(a')(b')} \ . \quad (3.8)$

At this stage we can introduce Dirac brackets [30] allowing to treat the constraints (3.8) as strong equalities

$$[\ldots , \ldots ]_{DB^h} = [\ldots , \ldots ]_{PB} - \frac{1}{4}[\ldots , K_{(a)(b)} ]_{PB} [\Xi^{(a)(b)} , \ldots ]_{PB} + \frac{1}{4}[\ldots , \Xi^{(a)(b)} ]_{PB} [K_{(a)(b)} , \ldots ]_{PB} \ , \quad (3.9)$$

The further study shows the presence of the following fermionic and bosonic second class constraints, the latter split in mutually conjugate pairs

$$d^{++}_q := v^{++}_q d_a \approx 0 \ , \quad \{d^{++}_q , d^{++}_p \}_{PB} = -2i\rho^{++}\delta_{pq} \ , \quad (3.10)$$

Here $d^{++}+j = d^{+j} + d^{#j}$ is one of the element appearing in the $SO(1,1) \otimes SO(9)$ invariant splitting of the Lorentz $SO(1,10)$ generator $d_{(a)(b)}^+ = (d_0^+ , d^\pm , d^j , d^{#j})$, $v^{++}_q$ is an element of the inverse spinor moving frame matrix $V^{-1}_a^{\alpha} = (v^{++}_q , v^{--}_q) \in Spin(1,10)$ which obeys $v^{++}_q v^{++}_q = 0$ and $v^{++}_q v^{--}_q = \delta_{qp}$. In $D=11$ this is expressed through the original spinor harmonics by $v^{++}_q = \pm iC\alpha^{\beta}v^{++}_q$, which is an equivalent form of Eqs. (2.10).

Following Dirac [30], we would like to introduce the Dirac brackets allowing to treat the second class constraints as strong equalities. For our M0–brane model it is convenient to do this in two stages (starred and doubly starred brackets in [30]). On the first stage one introduces the Dirac brackets for sector of harmonic variables, i.e. for the second class constraints $K_{(a)}$, 

$$[\ldots , \ldots ]_{DB^h} = [\ldots , \ldots ]_{PB} - \frac{1}{4}[\ldots , K_{(a)(b)} ]_{PB} [\Xi^{(a)(b)} , \ldots ]_{PB} + \frac{1}{4}[\ldots , \Xi^{(a)(b)} ]_{PB} [K_{(a)(b)} , \ldots ]_{PB} \ , \quad (3.11)$$

while on the second stage one finds the Dirac brackets for all the second class constraints,

$$[\ldots , \ldots ]_{DB} = [\ldots , \ldots ]_{DB^h} +$$

$$+ [\ldots , P^{[p]}_{++} ]_{PB} [ (u^{++} P - \rho^{++}) , \ldots ]_{DB^h} - [\ldots , (u^{++} P - \rho^{++}) ]_{DB^h} [P^{[p]}_{++} , \ldots ]_{PB} -$$

$$- [\ldots , u^{[p]} P ]_{DB^h} \frac{1}{\rho^{++}} [d^{++}+j , \ldots ]_{DB^h} + [\ldots , d^{++}+j ]_{DB^h} \frac{1}{\rho^{++}} [u^{[p]} P , \ldots ]_{DB^h} -$$

$$- [\ldots , d^{++}_q ]_{DB^h} \frac{i}{2\rho^{++}} [v^{++}_q , \ldots ]_{DB^h} \ . \quad (3.12)$$

Using these Dirac brackets one can treat all the second class constraints as the strong equalities,

$$\Xi^{(a)(b)} := U^{(a)} U^{m(b)} - \eta^{(a)(b)} \approx 0 \ , \quad K_{(a)(b)} := P_{(a)}^m U_{m(b)} + P_{(b)}^m U_{m(a)} \approx 0 \ ; \quad (3.13)$$

$$d^{++}_q := v^{++}_q d_a = 0 \ ; \quad \rho^{++} = u^{++} P_a \ ; \quad P^{[p]}_{++} = 0 \ ; \quad u^{[p]} P_a = 0 \ ; \quad d^{++}+j = 0 \ . \quad (3.14)$$
3.3 First class constraints and their algebra

The remaining constraints of the M0–brane model \((2.13)\), \((d^{(a)(b)} = (d^{(0)}, d^{\pm j}, d^{ij}), d^{(0)} := \frac{1}{2}d^{0\#})\)

\[
d_q^+ := v_q^a d_a \approx 0, \quad u^{a-} \Phi_a = u^{a-} P_a =: P^{--} \approx 0, \tag{3.15}
\]

\[
d^{ij} \approx 0, \quad d^{(0)} \approx 0, \quad d^{--i} \approx 0, \tag{3.16}
\]
give rise to the first class constraints. Their Dirac bracket algebra is characterized by

\[
[d^{ij}, d^{kl}]_{DB} = 4d^{[k[i} \delta^{j]l]}, \quad [d^{ij}, d^{--k}]_{DB} = 2d^{-[i} \delta^{j]k}, \quad [d^{(0)}, d^{\pm i}]_{DB} = \pm 2d^{\pm i}, \tag{3.17}
\]

\[
[d^{--i}, d^{--j}]_{DB} = \frac{i}{2p^q d^q} d^q \gamma^{ij} d_p, \tag{3.18}
\]

\[
[d^{ij}, d_p]_{DB} = -\frac{1}{2} \gamma^{ij} d_q, \quad [d^{(0)}, d_p]_{DB} = -d^q, \quad [d^{(0)}, P^{---}]_{DB} = -2P^{--}, \tag{3.19}
\]

\[
\{d^q, d_p\}_{DB} = -2i \delta_{qp} P^{--}. \tag{3.20}
\]

The 16 fermionic and 1 bosonic first class constraints in \((3.15)\) describe the irreducible \(\kappa\)–symmetry, \(d_q^+ := v_q^a d_a\), and its superpartner \((b\text{–symmetry}), P^{--}\); these generate the \(d = 1, N = 16\) supersymmetry algebra \((3.20)\). The irreducibility of the \(\kappa\)–symmetry in the spinor moving frame formulation \((\text{in contrast with the standard one} \[32\])\) is due to the presence of the spinorial harmonics (see \([16, 21]\)). The remaining first class constraints \((3.16)\) are originally related to the generators of \([SO(1, 1) \otimes SO(9)] \otimes K_9\) subgroup of the Lorentz group \(SO(1, 10)\) \((\text{see} \ (3.7)\ \text{with} (a)(b) \neq ++i, i++\)). However, when passing to Dirac brackets, the deformation in its \([K_9, K_9]\) part appears: Eq. \((3.18)\) acquires the nonvanishing r.h.s. proportional to the product of two fermionic first class constraints \((\text{which implies moving outside the Lie algebra, to the enveloping algebra})\) \([\ref{footnote}])

One may guess that the complete BRST charge \(Q\) for the algebra of the first class constraints \((3.20)\) is quite complicated and its use is not too practical. Following the pragmatic spirit of the pure spinor approach \([1]\) we might take care of the generators of \([SO(1, 1) \otimes SO(9)] \otimes K_9\) subgroup of the Lorentz group \(SO(1, 10)\) \((\text{see} \ (3.7)\ \text{with} (a)(b) \neq ++i, i++\)). However, when passing to Dirac brackets, the deformation in its \([K_9, K_9]\) part appears: Eq. \((3.18)\) acquires the nonvanishing r.h.s. proportional to the product of two fermionic first class constraints \((\text{which implies moving outside the Lie algebra, to the enveloping algebra})\) \([\ref{footnote}])

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3.4 BRST charge for a nonlinear (sub)algebra, \(Q',\) and its reduction to \(Q^{susy}\)

The BRST charge \(Q'\) of the nonlinear sub(super)algebra \((3.18), (3.20)\) of the nonlinear superalgebra of the M0–brane first class constraints must solve the master equations

\[
\{Q', Q\}_{DB} = 0 \tag{3.21}
\]

with 'initial conditions' \(Q'\big|_{p_r^{[\lambda]} = 0, \pi_+^{[c] = 0, \pi_-^{[c] = 0}} = 0 = \lambda_q^+ d_q^+ + c^{++} P^{--} + c^{++j} d^{--j}\), where \(\lambda_q^+\) is the bosonic ghost for the fermionic \(\kappa\)–symmetry, \(c^{++\lambda}\) and \(c^{++j}\) are the fermionic ghosts for the bosonic \(b\text{–symmetry} and deformed \(K_9\) symmetry transformations, and \(P_r^{[\lambda]}\), \(\pi_+^{[c]}\), and \(\pi_-^{[c]}\) are the (bosonic and fermionic) ghost momenta conjugate to \(\lambda_q^+\), \(c^{++}\) and \(c^{++j}\), respectively: \([\lambda_q^+, P_r^{[\lambda]}]_{DB} = \delta_{qp}, \{c^{++}, \pi_+^{[c]}\}_{DB} = -1, \{c^{++j}, \pi_-^{[c]}\}_{DB} = -\delta_{ij}\). The straightforward calculations show that \(Q'\) does not contain the ghost momentum \(\pi_+^{[c]}\), and can be presented as a sum

\[
Q' = Q^{susy} + c^{++j} d^{--j} \tag{3.22}
\]

\footnote{This is actually a counterpart of the well known phenomenon of the non–commutativity of the bosonic coordinate of the d=4 superparticle which appears in standard formulation \([34]\) \((\text{see also} \ [35])\). The appearance of a nonlinear algebra of constraints was also observed for the D=4 null–superstring and null–supermembrane cases \([33]\).}
of the much simpler BRST charge

\[
Q^{\text{susy}} = \lambda_q^+ \, d_q^+ + c^{++} \, P^{--} - i\lambda_q^+ \lambda_q^+ \pi_{++}^c , \quad \{ Q^{\text{susy}} , Q^{\text{susy}} \}_{DB} = 0 ,
\]

(3.23)

and of the product \(c^{++}d^{-j}\) of the \(c^{++}\) ghost fields and the deformed \(K_3\) generator modified by additional ghost contributions,

\[
\tilde{d}^{--i} = d^{--i} + \frac{i}{2p^+} c^{++} j\delta_q p^- \partial \gamma_{qp} t_p \left[ \lambda_q^+ \lambda_q^+ \pi_{++}^c \right] - \frac{i}{4(p^+)^2} c^{++} j c^{++} d_{[P q^-]} \left[ \lambda_q^+ \lambda_q^+ \pi_{++}^c \right] .
\]

(3.24)

The BRST charge \(Q^{\text{susy}}\) corresponds to the \(d = 1, N = 16\) supersymmetry algebra

\[
\{ d^{-} , \, d_{p}^{-} \}_{DB} = -2i P^{--} \, , \quad [ P^{--} , \, d_{p}^{-} ]_{DB} = 0 \, , \quad [ P^{--} , \, P^{--} ]_{DB} \equiv 0 .
\]

(3.25)

of the \(\kappa\)- and \(b\)-symmetry generators (3.25). Its ‘nilpotency’ \(\{ Q^{\text{susy}} , Q^{\text{susy}} \}_{DB} = 0\) guaranties the consistency of the reduction of the \(Q^{\text{susy}}\)-cohomology problem to the \(Q^{\text{susy}}\)-cohomology. As such a reduction is very much in the pragmatic spirit of the pure spinor approach [1, 2], we are going to use it in this letter and to study the cohomology of (3.23).

4 Cohomology of \(Q^{\text{susy}}\) and non-Hermitian \(\tilde{Q}^{\text{susy}}\) charge

4.1 Quantum M0–brane BRST charge \(Q^{\text{susy}}\) and its cohomology problem

It is practical, omitting the overall \(\pm i\) factor, to write the quantum BRST charge (3.23) as

\[
Q^{\text{susy}} = \lambda_q^+ \, D_q^- + i c^{++} \partial_{++} - \lambda_q^+ \lambda_q^+ \frac{\partial}{\partial \varepsilon^{++}} , \quad \{ Q^{\text{susy}} , Q^{\text{susy}} \} = 0 ,
\]

(4.1)

where the quantum operators \(D_q^-\) and \(\partial_{++}\), associated with \(d_q^-\) and \(P_{++}\), obey (cf. (3.20))

\[
\{ D_{p}^- , D_q^- \} = 2i \delta_{qp} \partial_{++} \, , \quad [ \partial_{++} , D_{p}^- ] = 0 ,
\]

(4.2)

which can be identified with \(d = 1, n = 16\) supersymmetry algebra (or with its dual which is given by the algebra of the flat superspace covariant derivatives). It is convenient to use a realization of \(\partial_{++}\), \(D_q^-\) as differential operators on the \(d=1, n=16\) superspace \(W^{(1|16)}\) of coordinates \((x^{++}, \theta_q^+)\),

\[
D_q^- = \partial_{q+} + i \theta_q^+ \partial_{q+} \, , \quad \partial_{++} := \frac{\partial}{\partial x^{++}} , \quad \partial_{q+} := \frac{\partial}{\partial \theta_q^+} .
\]

(4.3)

These variables have straightforward counterparts in the so–called covariant light cone basis, \(\theta_q^+ = \theta^a v_{a q}^+\) and \(x^{++} = x^m u_m^{++}\) (see [10, 19]).

The Grassmann odd \(c^{++}\) variable, \(c^{++} c^{++} = 0\), and the bosonic variables \(\lambda_q^+\) in (4.1) are the ghosts corresponding to the bosonic and 16 fermionic first class constraints represented by the differential operators \(\partial_{++}\) and \(D_q^-\). Their ghost numbers are 1, and this fixes the ghost number of the BRST charge to be also one,

\[
gh_{\#}(\lambda_q^+) = 1 , \quad gh_{\#}(c^{++}) = 1 , \quad gh_{\#}(Q^{\text{susy}}) = 1 .
\]

(4.4)

A non-trivial BRST cohomology is determined by the set of wavefunctions \(\Phi\) of certain ghost numbers \(g := gh_{\#}(\Phi)\) which are BRST-closed, \(Q^{\text{susy}} \Phi = 0\), but not BRST-exact, \(\Phi \neq Q^{\text{susy}}(\ldots)\). Moreover, such functions are defined modulo the BRST transformations \(i.e.\) modulo BRST-exact wavefunctions \(Q^{\text{susy}}\chi\), where \(\chi\) is an arbitrary function of the same configuration space variables of the ghost number \(gh_{\#}(\chi) = gh_{\#}(\Phi) - 1\) and the Grassmann parity opposite to the one of \(\Phi\),

\[
Q^{\text{susy}} \Phi = 0 , \quad \Phi \sim \Phi' = \Phi + Q^{\text{susy}} \chi , \quad gh_{\#}(\chi) = gh_{\#}(\Phi) - 1 .
\]

(4.5)
4.2 The nontrivial cohomology of $Q^{\text{susy}}$ is located at $\lambda_q^+\lambda_q^+ = 0$

Decomposing the wave function $\Phi = \Phi(c^+, \lambda_q^+; x^+, \theta_q^+, \ldots)$ in power series of the Grassmann odd ghost $c^+$, $\Phi = \Phi_0 + c^+\Phi_{+-}$, one finds that $Q^{\text{susy}}\Phi = 0$ for the superfield $\Phi$ implies

$$\lambda_q^+ D_q^- \Phi_0 = \lambda_q^+ \lambda_q^+ \Psi_{++} \quad (a), \quad \lambda_q^+ D_q^- \Psi_{++} = i\partial_{++} \Phi_0 \quad (b).$$

(4.6)

Using a similar decomposition for the $\chi$ superfield in (1.5), $\chi = \chi_0 + c^+K_{++}$, one finds

$$\Phi \mapsto \Phi' = \Phi + Q^{\text{susy}}\chi \Rightarrow \begin{cases} \Phi_0 \mapsto \Phi' = \Phi_0 + \lambda_q^+ D_q^- \chi_0 - \lambda_q^+ \lambda_q^+ K_{++} \quad (a), \\ \Psi_{++} \mapsto \Psi'_{++} = \Psi_{++} + i\partial_{++} \chi_0 + \lambda_q^+ D_q^- K_{++} \quad (b) \end{cases}$$

(4.7)

for the BRST transformations. Using Eqs. (4.6), (4.7) we can show that, if one assumes that the spinorial bosonic ghost $\lambda_q^+$ is non-zero, or, equivalently, that $\lambda_q^+\lambda_q^+ \neq 0$, the BRST cohomology of $Q^{\text{susy}}$ is necessarily trivial: all the BRST–closed states are BRST-exact.

Thus, if $Q^{\text{susy}}$ has a non-trivial cohomology, it must have a representation by wavefunctions with support on $\lambda_q^+\lambda_q^+ \neq 0$. In other words, the closed non-exact wavefunctions representing the non-trivial $Q^{\text{susy}}$–cohomology must be of the form $\Phi \propto \delta(\lambda_q^+\lambda_q^+)$ plus a possible $Q^{\text{susy}}$–trivial contribution.

4.3 Cohomology at vanishing bosonic ghost and complex BRST operator $\tilde{Q}^{\text{susy}}$

Thus the non-trivial cohomology of $Q^{\text{susy}}$, if exists, must allow a representation by wavefunctions of the form $\Phi = \delta(\lambda_q^+\lambda_q^+) \Phi^{++}$, where $\Phi^{++} = \Phi^{++} + c^+\Psi^0$ has ghost number two units more than $\Phi$. $\gamma_0 := gh\#(\Phi^{++}) = gh\#(\Phi^0) + 2$. But there is a difficulty with finding such wavefunctions: since the bosonic ghosts $\lambda_q^+$ are real, $\lambda_q^+\lambda_q^+ = 0$ implies $\lambda_q^+ = 0$. Then, since $Q^{\text{susy}}$ includes $\lambda_q^+$ in an essential manner, we need in a regularization allowing us to consider, at the intermediate stages, a nonvanishing $\lambda_q^+$ which nevertheless obeys $\lambda_q^+\lambda_q^+ = 0$.

This is possible if we consider $\lambda_q^+$ to be complex (cf. with the pure spinors by Berkovits [1])

$$\lambda_q^+ \mapsto \tilde{\lambda}_q^+ \neq (\lambda_q^+)^* \Rightarrow \tilde{\lambda}_q^+\lambda_q^+ = 0 \quad \text{with} \quad \tilde{\lambda}_q^+ \neq 0 \quad \text{is possible}.$$  (4.8)

The ‘regularized’ BRST charge, $Q^{\text{susy}}_{\text{reg}} := Q^{\text{susy}}|_{\lambda_q^+ \rightarrow \tilde{\lambda}_q^+}$, is thus non-Hermitian. It contains the complex ghost $\tilde{\lambda}_q^+$ rather than the real $\lambda_q^+$ in (1.1), but does not contain $(\tilde{\lambda}_q^+)^*$, and acts on the space of wavefunctions holomorphic in $\tilde{\lambda}_q^+$. Since the discussion of the previous section is not affected by above complexification $\lambda_q^+ \mapsto \tilde{\lambda}_q^+$, we conclude that the non-trivial cohomology states of the complexified BRST charge can be described by wavefunctions of the form

$$\Phi = \delta(\tilde{\lambda}_q^+\tilde{\lambda}_q^+) \Phi^{++}(\tilde{\lambda}_q^+, c^{++}; x^{++}, \theta_q^+, \ldots).$$  (4.9)

Now we observe that, as the BRST charge $Q^{\text{susy}}$ does not contain any derivative with respect to the bosonic ghost $\lambda_q^+$, its regularization acts on the $\Phi^{++}$ part of the function $\Phi$ in (4.9) only,

$$Q^{\text{susy}}_{\lambda_q^+ \rightarrow \tilde{\lambda}_q^+} \delta(\tilde{\lambda}_q^+\tilde{\lambda}_q^+) \Phi^{++}(\tilde{\lambda}_q^+, c^{++}; \ldots) = \delta(\tilde{\lambda}_q^+\tilde{\lambda}_q^+) Q^{\text{susy}}\Phi^{++}(\tilde{\lambda}_q^+, c^{++}; \ldots).$$  (4.10)

where we introduced the non-Hermitian BRST charge $\tilde{Q}^{\text{susy}} = Q^{\text{susy}}|_{\lambda_q^+ \rightarrow \tilde{\lambda}_q^+}$: $\tilde{\lambda}_q^+\tilde{\lambda}_q^+ = 0$,

$$\tilde{Q}^{\text{susy}} = \tilde{\lambda}_q^+ D_q^- + ic^{++}\partial_{++},$$

$$\tilde{\lambda}_q^+\tilde{\lambda}_q^+ = 0.$$  (4.11)

which is nilpotent, $(\tilde{Q}^{\text{susy}})^2 = 0$, and can be used to reformulate the regularized cohomology problem. Note that, once we have concluded that the cohomology of $Q^{\text{susy}}$ can be described by wavefunctions of the form (4.9), we can reduce the nontrivial cohomology search to the set of such
functions, restricting as well the arbitrary superfields $\chi$ of the BRST transformations (4.7) to have the form $\chi = \delta(\lambda^+_q \lambda^+_q)\chi^{++}$.

Then the regularized cohomology problem for the complexified BRST operator ($Q^{susy}$ of (4.1)) now depending on the complexified bosonic ghost $\lambda^+_q$), reduces to the search for a $\lambda^+_q = 0$ ‘value’ of the cohomology of the operator $\tilde{Q}^{susy}$ in Eq. (4.11),

$$\tilde{Q}^{susy} \Phi^{++} = 0 \ , \quad \Phi^{++} \sim \Phi^{++'} = \Phi^{++} + \tilde{Q}^{susy} \chi^{++} \ . \quad (4.12)$$

This problem (4.12) can be reformulated in terms of components $\Phi_0^{++}$ and $\Psi^{(0)}$ of the wavefunction superfield $\Phi^{++} = \Phi_0^{++} + c^{++}\Psi^{(0)}$ giving rise to the following equations

$$\tilde{\lambda}^+_q D^-_q \Phi_0^{++} = 0 \ , \quad \tilde{\lambda}^+_q D^-_q \Psi^{(0)} = i\partial_{++} \Phi_0^{++} \ . \quad (4.13)$$

$$\Phi_0^{++} \sim \Phi_0^{++'} = \Phi_0^{++} + \tilde{\lambda}^+_q D^-_q \chi^{++}_o \ , \quad \Psi^{(0)} \sim \Psi^{(0)'} = \Psi^{(0)} + i\partial_{++} \chi^{++}_o + \tilde{\lambda}^+_q D^-_q K^{(0)} \ . \quad (4.14)$$

To obtain the cohomology of $Q^{susy}$, we have to set $\tilde{\lambda}^+_q = 0$ at the end to remove the regularization; thus we are really interested in the wavefunctions for $\lambda^+_q = 0$:

$$\Phi_0^{++}|_{\lambda^+_q=0} = \Phi_0^{++}(0, x^{++}, \theta^+_q ; \ldots) \ , \quad \Psi^{(0)}|_{\lambda^+_q=0} = \Psi^{(0)}(0, x^{++}, \theta^+_q ; \ldots) \ .$$

The further study shows that nontrivial ‘superfield’ cohomology problem of Eq. (4.12) can appear only due to non-triviality of the a (pure-spinor like) cohomology problem for the leading component $\Phi_0^{++}$ of the $\Phi^{++}$ superfield (see Eqs. (4.13), (4.14)),

$$\tilde{\lambda}^+_q D^-_q \Phi_0^{++} = 0 \ , \quad \Phi_0^{++} \mapsto \Phi_0^{++'} = \Phi_0^{++} + \tilde{\lambda}^+_q D^-_q \chi^{++}_o \ . \quad (4.15)$$

Moreover, we have found that, in its turn, the non-triviality of the reduced BRST cohomology (4.13) (($\tilde{\lambda}^+_q D^-_q$)–cohomology) requires the vanishing ghost number of the wavefunction $\Phi_0^{++}$ ($g_0 := gh_{\#}(\Phi_0^{++}) = 0$) and, in this case, is described by the kernel $D^-_q \Phi_0^{++} = 0$ of the $\kappa$–symmetry generator,

$$g_0 := gh_{\#}(\Phi_0^{++}) = 0 \ , \quad \tilde{\lambda}^+_q D^-_q \Phi_0^{++} = 0 \quad \Rightarrow \quad D^-_q \Phi_0^{++} = 0 \ . \quad (4.16)$$

With the realization (4.3), one finds that the general solution of this equation is a function independent on both $\theta^+_q$ and $x^{++}$,

$$g_0 := gh_{\#}(\Phi_0^{++}) = 0 \ , \quad \Phi_0^{++} \neq \Phi_0^{++}(x^{++}, \theta^+_q) \quad \left( \frac{\partial}{\partial x^{++}} \Phi_0^{++} = 0 \ , \quad \frac{\partial}{\partial \theta^+_q} \Phi_0^{++} = 0 \right) \ . \quad (4.17)$$

Thus the nontrivial cohomology of the BRST charge $Q^{susy}$ (4.11) is described by the cohomology of $\tilde{Q}^{susy}$ (4.11) in the sector with (vanishing bosonic ghost and) vanishing ghost number $g_0 := gh_{\#}(\Phi^{++}) = 0$ (or $g := gh_{\#}(\Phi) = -2$ for $\Phi$ in (4.9)), which in turn is described by the wavefunctions dependent on the ‘physical variables’ only. This actually reduces the problem to the quantization of the physical degrees of freedom, i.e. to a counterpart of the twistor quantization of (24) which shows that the quantum state spectrum is described by the linearized D=11 supergravity multiplet.

5 Relation with the Berkovits pure spinor BRST charge

Thus we have shown that the BRST quantization of the M0–brane in the spinor moving frame formulation (2.3) leads to the cohomology problem for complex BRST charge (4.11) (the cohomology at vanishing bosonic ghost gives the M0–brane quantum state spectrum). Now we turn to the question of relation of our complex $\tilde{Q}^{susy}$, Eq. (4.11), with also complex pure spinor BRST operator by Berkovits. This latter BRST charge has the form [1]

$$Q^B = \Lambda^\alpha d_\alpha \ , \quad \Lambda_{\alpha} \Lambda = 0 \ , \quad \Lambda^\alpha \neq (\Lambda^\alpha)^* \ , \quad (5.18)$$
where \( d_\alpha \) is the fermionic constraint of Eq. (3.3) and \( \Lambda^\alpha \) is the complex pure spinor satisfying the constraints \( \Lambda^\alpha_\alpha \Lambda = 0 \) which guarantees the nilpotency \((Q^B)^2 = 0\) of the BRST charge \((Q^B)\).

The \( D = 11 \) pure spinor \( \Lambda^\alpha \) in general carries 46 (23 complex) degrees of freedom. A specific 39 parametric solution \( \tilde{\Lambda} \) can be found using spinorial harmonics \( v_\alpha q^- \), Eq. (2.11). It is given by

\[
\tilde{\Lambda}_\alpha = \tilde{\lambda}_q^+ v_\alpha q^- , \quad \tilde{\lambda}_q^+ \tilde{\lambda}_q^- = 0 \quad \Rightarrow \quad \tilde{\Lambda}_\alpha \tilde{\Lambda} = 0 .
\]  

(5.19)

Indeed, as harmonics obey \( v_\alpha \Gamma_q v^-_p = \delta_{pq} v^-_q \), Eq. (2.4a), \( \tilde{\Lambda}_\alpha \tilde{\Lambda} = \tilde{\lambda}_q^+ \tilde{\lambda}_q^- \) which vanishes due to the condition \( \tilde{\lambda}_q^+ \tilde{\lambda}_q^- = 0 \) imposed on the complex 16 component \( SO(9) \) spinor \( \tilde{\lambda}_q^+ \). This latter may be identified with the complex zero norm spinor entering the complex charge \( Q^\text{susy} \), Eq. (4.1).

Furthermore, as far as the \( \kappa \)-symmetry generator \( D^-_q \) is basically \( v_q^- d_\alpha \), one finds that our complex \( Q^\text{susy} \) of Eq. (4.11) is essentially (up to the simple \( c \) term) just the Berkovits BRST operator (5.18), but with a particular pure spinor \( \tilde{\Lambda}^\alpha \) (5.19) instead of a generic pure spinor \( \Lambda^\alpha \),

\[
\tilde{Q}^\text{susy} = Q^B|_{\Lambda^\alpha = \tilde{\lambda}_q^+ v_q^-} + ic^{++} \partial^{++} ,
\]  

(5.20)

Thus a counterpart (5.20) of the Berkovits BRST charge (5.18) appears when calculating the cohomologies of the regularized version of the BRST charge (4.1) which is obtained directly by quantizing the \( D = 11 \) superparticle in the framework of its twistor–like Lorentz harmonics formulation (2.3). In our BRST operator \( \tilde{Q}^\text{susy} \) (5.20), the zero norm complexified \( \kappa \)-symmetry ghost \( \tilde{\lambda}_q^- \) carries 30 of the 39 degrees of freedom of the composite the \( D = 11 \) pure spinor \( \| \). The remaining 9 degrees of freedom in this pure spinor correspond to the \( S^9 \) sphere of the light–like eleven–dimensional momentum modulo its energy, parametrized by the spinorial harmonics, Eq. (2.11).

Although one may notice the difference in degrees of freedom (46 versus 39), it is not obvious that all the degrees of freedom in a pure spinor are equally important in the case of \( D = 11 \) superparticle. Moreover, this mismatch disappears in the ‘stringy’ \( D = 10 \) case (see below).

In conclusion, let us stress once more that, of all the cohomologies of the complex Berkovits–like BRST charge \( Q^\text{susy} \), only their values at vanishing bosonic ghost, \( \tilde{\lambda}_q^+ = 0 \), describe the cohomologies of the M0–brane BRST charge \( Q^\text{susy} \) and, hence, the superparticle spectrum. The \( Q^\text{susy} \) cohomologies for \( \tilde{\lambda}_q^- \neq 0 \) (corresponding to nonzero ghost number of the wavefunctions) are reacher and are related with the spinorial cohomologies of [36].

6 Conclusion and outlook

The main conclusion of our present study of the M0–brane case is that the twistor–like Lorentz harmonic approach [16, 21, 24], originated in [10, 11, 12], is able to produce a simple and practical BRST charge. This makes interesting the similar investigation of the \( D = 10 \) Green–Schwarz superstring case. For instance, for the IIB superstring the Berkovits BRST charge [1] looks like

\[
\tilde{Q}^B_{\text{IIB}} = \int \Lambda^\alpha d_\alpha^1 + \int \Lambda^\alpha d_\alpha^2 , \quad \Lambda^\alpha \sigma_{\alpha \beta} \Lambda^\beta = 0 = \Lambda^\alpha \sigma_{\alpha \beta} \Lambda^\beta^2
\]  

(6.21)

with two complex pure spinors \( \Lambda^\alpha^1 \) and \( \Lambda^\alpha^2 \) multiplying respectively the left– and right–handed stringy counterparts of the superparticle fermionic constraints (3.3). By analogy with our study of M0–brane (see (5.19)), one may expect that the BRST quantization of the spinor moving frame formulation [16, 17] of the Green–Schwarz superstring would lead, after some reduction and on the way of regularization of the ‘honest’ (‘true’) hermitian BRST charge, to the cohomology problem for the complex charge of the form (6.21) but with composite pure spinors

\[
\tilde{\Lambda}^1 = \tilde{\lambda}_p^+ v_p^- , \quad \tilde{\Lambda}^2 = \tilde{\lambda}_p^- v_p^+ , \quad \tilde{\lambda}_p^+ \tilde{\lambda}_p^- = 0 = \tilde{\lambda}_p^- \tilde{\lambda}_p^+ .
\]  

(6.22)
Here $\lambda^\pm_p$ are two complex 8 component $SO(8)$ spinors and the stringy harmonics $v^\pm_p$ are the homogeneous coordinates of the non-compact 16-dimensional coset

$$\{V_{(j)}^\alpha\} = \{(v^{-\alpha}_p, v^{+\alpha}_p)\} = \frac{Spin(1,9)}{SO(1,1) \otimes SO(8)}, \quad (6.23)$$

characteristic for the spinor moving frame formulation of the (super)string [16, 17] and describing the spontaneous breaking of the spacetime Lorentz symmetry by the string model.

It is important that, in distinction to M0–brane case, the $D=10$ solution (6.22) of the pure spinor constraints in (6.21) carries the same number of degrees of freedom ($44 = 2 \times 8 + 2 \times 14$) that the pair of Berkovits pure spinors $\Lambda^{\alpha_1}, \Lambda^{\alpha_2}$ ($22 + 22$). Hence it provides the general solution of the $D=10$ pure spinor constraints in terms of harmonics (6.23) and two complex $SO(8)$ spinors of zero square so that its substitution for the generic pure spinor of [1] should not produce any anomaly or other problem related to the counting of degrees of freedom.

Further development of the present approach is related to the covariant BRST quantization of superstring in spinor moving frame formulation [16, 17] and to understanding whether/how a cohomology problem for the complex BRST charge (6.21) appears on this way.

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