Exploring dark matter, neutrino mass and \( R_{K^{(*)},\phi} \) anomalies in 
\( L_\mu - L_\tau \) model

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Abstract

We investigate Majorana dark matter in a new variant of \( U(1)_{L_\mu - L_\tau} \) gauge extension of Standard Model, where the scalar sector is enriched with an inert doublet and a \((\bar{3}, 1, 1/3)\) scalar leptoquark. We compute the WIMP-nucleon cross section in leptoquark portal and the relic density mediated by inert doublet components, leptoquark and the new \( Z' \) boson. We constrain the parameter space consistent with PLANCK limit on relic density, PICO-60 and LUX bounds on spin-dependent direct detection cross section. Furthermore, we constrain the new couplings from the present experimental data on \( \text{Br}(\tau \to \mu \nu_\tau \bar{\nu}_\mu) \), \( \text{Br}(B \to X_s \gamma) \), \( \text{Br}(B \to K\tau\tau) \), \( R_K \) and \( B_s - \bar{B}_s \) mixing, which occur at one-loop level in the presence of \( Z' \) and leptoquark. Using the allowed parameter space, we estimate the form factor independent \( P'_{4,5} \) observables and the lepton non-universality parameters \( R_{K^{(*)}} \) and \( R_\phi \). We also briefly discuss about the neutrino mass generation at one-loop level.

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I. INTRODUCTION

Though the experimental measured values of various physical observables are in excellent agreement with the Standard Model (SM) predictions, there are many open unsolved problems like the matter-antimatter asymmetry, hierarchy problem and the dark matter (DM) content of the universe etc., which make ourselves believe that there is something beyond the SM. In this regard, the study of rare semileptonic $B$ decay processes provide an ideal testing ground to critically test the SM and to look for possible extension of it. Although, so far we have not observed any clear indication of new physics (NP) in the $B$ sector, there are several physical observables associated with flavor changing neutral current (FCNC) $b \to s l^+ l^-$ processes which have $(2 - 4)\sigma$ [1-6] discrepancies. Especially, the observation of $3\sigma$ anomaly in the $P'_5$ angular observables [4] and the decay rate [5] of $B \to K^*\mu^+\mu^-$ processes have attracted a lot of attention in recent times. The decay rate of $B_s \to \phi\mu^+\mu^-$ has also $3\sigma$ deviation compared to its SM prediction [3]. Furthermore, the LHCb Collaboration has observed the violation of lepton universality in $B^+ \to K^+ l^+ l^-$ process in the low $q^2 \in [1, 6]$ GeV$^2$ region [2]

$$R_{K}^{\text{Expt}} = \frac{\text{Br}(B^+ \to K^+ \mu^+\mu^-)}{\text{Br}(B^+ \to K^+ e^+ e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036,$$

which has a $2.6\sigma$ deviation from the corresponding SM result [7]

$$R_{K}^{\text{SM}} = 1.0003 \pm 0.0001.$$

In addition, an analogous lepton non-universality (LNU) parameter ($R_{K^*}$) has also been observed in $B^0 \to K^{*0} l^+ l^-$ processes [1]

$$R_{K^*}^{\text{Expt}} = \frac{\text{Br}(B^0 \to K^{*0} \mu^+\mu^-)}{\text{Br}(B^0 \to K^{*0} e^+ e^-)} = 0.66^{+0.11}_{-0.07} \pm 0.03, \quad q^2 \in [0.045, 1.1] \text{ GeV}^2,$n

$$= 0.69^{+0.11}_{-0.07} \pm 0.05, \quad q^2 \in [1.1, 6] \text{ GeV}^2,$$

which correspond to the deviation of $2.2\sigma$ and $2.4\sigma$ from their SM predictions [8]

$$R_{K^*}^{\text{SM}}|_{q^2 \in [0.045, 1.1] \text{ GeV}^2} = 0.92 \pm 0.02, \quad R_{K^*}^{\text{SM}}|_{q^2 \in [1.1, 6] \text{ GeV}^2} = 1.00 \pm 0.01.$$

To resolve the above $b \to s l^+ l^-$ anomalies, we extend the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ with a local $U(1)_{L\mu-L\tau}$ symmetry. The anomaly free $L_\mu - L_\tau$ gauge extensions [9] [10] are captivating with minimal new particles and parameters, rich in phenomenological
perspective. The model is quite simple in structure, suitable to study the phenomenology of DM, neutrino and also the flavor anomalies. It is well explored in dark matter context in literature [11–14], in the gauge and scalar portals. The approach of adding color triplet particles to shed light on the flavor sector thereby connecting with dark sector is interesting. Leptoquarks (LQ) are not only advantageous in addressing the flavor anomalies, but also act as a mediator between the visible and dark sector. Few works were already done with this motivation [15–18].

Leptoquarks are hypothetical color triplet gauge particles, with either spin-0 (scalar) or spin-1 (vector), which connect the quark and lepton sectors and thus, carry both baryon and lepton numbers simultaneously. They can arise from various extended standard model scenarios [19–30], which treat quarks and leptons on equal footing, such as the grand unified theories (GUTs) [19–22], color SU(4) Pati-Salam model [23–27], extended technicolor model [28, 29] and the composite models of quark and lepton [30]. In this article, we study a new version of $U(1)_{L_{\mu}-L_{\tau}}$ gauge extension of SM with a $(\bar{3}, 1, 1/3)$ scalar LQ (SLQ) and an inert doublet, to study the phenomenology of dark matter, neutrino mass generation and compute the flavor observables on a single platform. The SLQ mediates the annihilation channels contributing to relic density and also plays a crucial role in direct searches as well, providing a spin-dependent WIMP-nucleon cross section which is quite sensitive to the recent and ongoing direct detection experiments such as PICO-60 and LUX. The $Z'$ gauge boson of extended $U(1)$ symmetry and the SLQ also play an important role in settling the known issues of flavor sector. In this regard, we would like to investigate whether the observed anomalies in the rare leptonic/semileptonic decay processes mediated by $b \rightarrow s l^+l^-$ transitions, can be explained in the present framework. We analyze the implications of the model on both the DM and flavor sectors, in particular on $B \rightarrow V l^+l^- (V = K^*, \phi)$ decay modes. In literature [31–51], there were many attempts being made to explain the observed anomalies of rare $B$ decays in the scalar leptoquark model.

The paper is structured as follows. We describe the particle content, relevant Lagrangian and interaction terms, pattern of symmetry breaking in section-II. We derive the mass eigenstates of the new fermions and the scalar spectrum in section-III. We then provide a detailed study of DM phenomenology in prospects of relic density and direct detection observables in section-IV. Mechanism of generating light neutrino mass at one-loop level is illustrated in section-V. Section-VI contains the additional constraint on the new parameters
obtained from the existing anomalies of the flavor sector, like \( \text{Br}(\tau \to \mu \nu_\tau \nu_\mu) \), \( \text{Br}(B \to X_s \gamma) \), \( \text{Br}(B \to K \tau \tau) \), \( R_K \) and \( B_s - \bar{B}_s \) mixing. We then investigate the impact of additional \( U(1)_{L_\mu - L_\tau} \) gauge symmetry on the \( R_K \), \( R_\phi \) LNU parameters and optimized \( P^*_{4,5} \) observables in section-VII. We summarize our findings in Section-VIII.

II. NEW \( L_\mu - L_\tau \) MODEL WITH A SCALAR LEPTOQUARK

We study the well known anomaly free \( U(1)_{L_\mu - L_\tau} \) extension of SM with three neutral fermions \( N_e, N_\mu, N_\tau \), with \( L_\mu - L_\tau \) charges 0, 1 and \(-1\) respectively. A scalar singlet \( \phi_2 \), charged \(+2\) under the new \( U(1) \) is added to spontaneously break the local \( U(1)_{L_\mu - L_\tau} \) gauge symmetry. We also introduce an inert doublet \((\eta)\) and a scalar leptoquark \( S_1(3, 1, 1/3) \) with \( L_\mu - L_\tau \) charges 0 and \(-1\) to the scalar content of the model. We impose an additional \( Z_2 \) symmetry under which all the new fermions, \( \eta \) and the leptoquark are odd and rest are even. The particle content and their corresponding charges are displayed in Table. I.

The Lagrangian of the present model can be written as

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} Z'_{\mu \nu} Z'_{\mu \nu} - g_{\mu \nu} \bar{N}_L \gamma^\mu \gamma_5 N_L Z'_{\mu} - g_{\mu \nu} \bar{N}_R \gamma^\mu \gamma_5 N_R Z'_{\mu} + g_{\mu \nu} \tau_L \gamma^\mu \tau_L Z'_{\mu} + g_{\mu \nu} \tau_R \gamma^\mu \tau_R Z'_{\mu}
+ \bar{N}_e i \tau \eta N_e + \bar{N}_\mu (i \tau \phi - g_{\mu \tau} Z'_{\mu} \eta) N_\mu + \bar{N}_\tau (i \phi + g_{\mu \tau} Z'_{\mu} \eta) N_\tau - \frac{f_\mu}{2} \left( \Re \{\bar{N}_\mu N_\mu \phi_2^\dagger \} + \text{h.c.} \right)
- \frac{f_\tau}{2} \left( \Re \{\bar{N}_\mu N_\mu \phi_2^\dagger \} + \text{h.c.} \right) - \frac{1}{2} M_{ee} \bar{N}_e N_e - \frac{1}{2} M_{\mu \tau} \Re \{\bar{N}_\mu N_\mu \} - \sum_{q=d,s,b} \left( y_{qR} \bar{d}^{\dagger}_q \tau_1^a S_1 N_\mu + \text{h.c.} \right)
+ \left| (i \partial_\mu - \frac{g}{2} \tau^a \cdot W_\mu - \frac{g'}{2} B_\mu) \eta \right|^2 + \left| (i \partial_\mu - 2 g_{\mu \tau} Z'_{\mu}) \phi_2 \right|^2 + \left| (i \partial_\mu - \frac{g'}{3} B_\mu + g_{\mu \tau} Z'_{\mu}) S_1 \right|^2
- V(H, \eta, \phi_2, S_1),
\]

where the scalar potential \( V \) is

\[
V(H, \eta, \phi_2, S_1) = \mu_H^2 H^d H + \lambda_H (H^d H)^2 + \mu_\eta (\eta^\dagger \eta) + \lambda_H \eta (H^d H) \eta^\dagger + \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_H \eta (H^d H) (\eta^\dagger \eta) + \lambda \eta^\dagger \eta (H^d H) (\eta^\dagger \eta)
+ \lambda_H \eta^\dagger \eta (H^d H) \eta^\dagger \eta + \frac{\lambda_H \eta^\dagger \eta}{2} \left[ (H^d H)^2 + \text{h.c.} \right] + \mu_2 (\phi_2^\dagger \phi_2^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \mu_S (S_1^\dagger S_1) + \lambda_S (S_1^\dagger S_1)^2
+ \lambda_H \eta (\phi_2^\dagger \phi_2) + \lambda_H S_1 (S_1^\dagger S_1) \right] (H^d H) + \lambda_S (\phi_2^\dagger \phi_2) (S_1^\dagger S_1) + \lambda_S (\phi_2^\dagger \phi_2) (S_1^\dagger S_1)
+ \lambda_S (S_1^\dagger S_1) (\eta^\dagger \eta).
\]

The gauge symmetry \( SU(2)_L \times U(1)_Y \times U(1)_{L_\mu - L_\tau} \) is spontaneously broken to \( SU(2)_L \times U(1)_Y \) by assigning a VEV \( v_2 \) to the complex singlet \( \phi_2 \). Then the SM Higgs doublet breaks the
SM gauge group to low energy theory by obtaining a VEV $v$. The new neutral gauge boson $Z'$ associated with the $U(1)$ extension absorbs the massless pseudoscalar in $\phi_2$ to become massive. The neutral components of the fields $H$ and $\phi_2$ can be written in terms of real scalars and pseudoscalars as

\begin{align}
H^0 &= \frac{1}{\sqrt{2}}(v + h) + \frac{i}{\sqrt{2}}A^0, \\
\phi_2 &= \frac{1}{\sqrt{2}}(v_2 + h_2) + \frac{i}{\sqrt{2}}A_2. \tag{7}
\end{align}

The inert doublet is denoted by $\eta = \left(\begin{array}{c} \eta^+ \\ \eta^0 \end{array}\right)$, with $\eta^0 = \frac{\eta_e + \eta_\mu}{\sqrt{2}}$. The masses of its charged
and neural components are given by

\[
M^2_{\eta_c} = \mu^2_{\eta_c} + \frac{\lambda H v^2}{2} + \frac{\lambda_{\eta_c}^2}{2} v_{2}^2, \\
M^2_{\eta_e} = \mu^2_{\eta_e} + \frac{\lambda_{\eta_e}^2}{2} v_{2}^2 + \left( \lambda\eta + \lambda_{\eta_e} \eta + \lambda_{\eta_e}'' \eta \right) \frac{v^2}{2}, \\
M^2_{\eta_o} = \mu^2_{\eta_o} + \frac{\lambda_{\eta_o}^2}{2} v_{2}^2 + \left( \lambda\eta + \lambda_{\eta_o} - \lambda_{\eta_o}'' \eta \right) \frac{v^2}{2}. \tag{8}
\]

The masses obtained by the colored scalar and the gauge boson $Z'$ are

\[
M^2_{S_1} = 2\mu^2_S + \lambda_{HS} v^2 + \lambda_{S_2} v_{2}^2, \\
M_{Z'} = 2v_{2} g_{\mu\tau}. \tag{9}
\]

In the whole discussion of the results, we consider the benchmark values for the masses of the scalar spectrum as $(M_{S_1}, M_{\eta_c}, M_{\eta_e, o}) = (1.2, 2, 1.5)$ TeV.

III. MIXING IN THE FERMION AND SCALAR SECTOR

The fermion and scalar mass matrices take the form

\[
M_N = \begin{pmatrix} \frac{1}{\sqrt{2}} f_\mu v_2 & M_{\mu\tau} \\ M_{\mu\tau} & \frac{1}{\sqrt{2}} f_\tau v_2 \end{pmatrix}, \\
M_S = \begin{pmatrix} 2\lambda_H v^2 & \lambda_{H2} v v_2 \\ \lambda_{H2} v v_2 & 2\lambda_2 v_{2}^2 \end{pmatrix}. \tag{10}
\]

One can diagonalize the above mass matrices by $U^T_{\alpha(\zeta)} M_{N(S)} U_{\alpha(\zeta)} = \text{diag} [M_{N_{-\alpha}(H_{1,2})}]$, where

\[
U_\theta = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \tag{11}
\]

with $\zeta = \frac{1}{2} \tan^{-1} \left( \frac{\lambda_{H2} v v_2}{\lambda_2 v_{2}^2 - \lambda_H v^2} \right)$ and $\alpha = \frac{1}{2} \tan^{-1} \left( \frac{M_{\mu\tau}}{(f_\tau - f_\mu)(v/\sqrt{2})} \right)$.

We denote the scalar mass eigenstates as $H_1$ and $H_2$, with $H_1$ is assumed to be observed Higgs at LHC with $M_{H_1} = 125.09$ GeV and $v = 246$ GeV. The mixing parameter $\zeta$ is taken minimal to stay with LHC limits on Higgs decay width. We indicate $N_-$ and $N_+$ to be the fermion mass eigenstates, with the lightest one ($N_-$) as the probable dark matter in the present work.
IV. DARK MATTER PHENOMENOLOGY

A. Relic abundance

The model allows the dark matter ($N_-$) to have gauge and scalar mediated annihilation channels. The possible contributing diagrams are provided in Fig. 1 which are mediated by ($H_1, H_2, \eta^+, \eta^0, S_1, Z'$). Majorana DM in $H_{1,2}$ portal (upper row in Fig. 1) has already been well explored in literature [52, 53]. Here, we focus on ($Z', S_1, \eta$)-mediated channels (middle and bottom rows in Fig. 1) contributing to DM observables, which we later make connection with radiative neutrino mass as well as flavor observables. The relic abundance of dark matter is computed by

$$\Omega h^2 = \frac{2.14 \times 10^9 \text{ GeV}^{-1}}{M_{\text{pl}} g_*^{1/2}} \frac{1}{J(x_f)}. \quad (12)$$

Here the Planck mass, $M_{\text{pl}} = 1.22 \times 10^{19}$ GeV and $g_* = 106.75$ denotes the total number of effective relativistic degrees of freedom. The function $J(x_f)$ reads as

$$J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma v \rangle(x)}{x^2} dx. \quad (13)$$
FIG. 2: Behavior of relic density plotted against DM mass with $M_{H_2} = 2.2$ TeV, shown with varying $M_{Z'}$ and $g_{\mu\tau}$ (left panel) and $y_{qR}$ (right panel). Black horizontal dotted lines denote the $3\sigma$ range of PLANCK limit \[54\].

The thermally averaged annihilation cross section $\langle \sigma v \rangle$ is given by the expression

$$\langle \sigma v \rangle(x) = \frac{x}{8 M_D^2 K_2^2(x)} \int_{4 M_D^2}^\infty \hat{\sigma} \times (s - 4 M_D^2) \sqrt{s} \frac{x}{M_D} K_1 \left( \frac{x \sqrt{s}}{M_D} \right) ds,$$

where $K_1, K_2$ denote the modified Bessel functions and $x = M_D/T$, where $T$ is the temperature. The analytical expression for the freeze out parameter $x_f$ is

$$x_f = \ln \left( \frac{0.038 \ g \ M_{pl} \ M_D \langle \sigma v \rangle(x_f)}{(g_s x_f)^{1/2}} \right).$$

Here $g$ represents the number of degrees of freedom of the dark matter particle $N_-$.

As seen from the left panel of Fig. 2, the relic density with $s$-channel contribution is featured to meet the PLANCK limit \[54\] near the resonance in propagator $(H_1, H_2, Z')$, i.e., near $M_- = \frac{M_{prop}}{2}$. We restrict our discussion to the mass region (in GeV), $100 \leq M_{Z'} \leq 1000$, $80 \leq M_- \leq 1000$ and also $H_2$ is considered to be sufficiently large such that its resonance doesn’t meet the PLANCK limit below 1 TeV region of DM mass. Now, in this mass range of DM, the channels mediated by $(Z', \eta, S_1)$ drive the relic density observable, where the gauge coupling $g_{\mu\tau}$ controls the $s$-channel contribution, while $Y_{ll}$, $y_{qR}$ are relevant in $t$-channel contributions. The relevant parameters in our investigation are $(M_-, g_{\mu\tau}, M_{Z'}, Y_{ll}, y_{qR})$. The effect of these parameters on the relic abundance is made transparent in Fig. 2, where we fixed $Y_{ll} \sim 10^{-2}$, in order to explain neutrino mass at one loop level. Left panel shows the variation of relic density with varying gauge parameters $g_{\mu\tau}$ and $M_{Z'}$, right panel depicts the behaviour with varying $y_{qR}$ parameter. No significant constraint on $M_{Z'}, g_{\mu\tau}$ parameters.
is observed, however relic density has an appreciable footprint on \( M_\gamma - (y_q R)^2 \) parameter space, which will be discussed in the next section.

**B. Direct searches**

Moving to direct searches, the WIMP-nucleon cross section is insensitive to direct detection experiments as \( Z' \) couples differently to Majorana fermion (axial-vector type) and quarks (vector type) \cite{55, 56}. The \( t \)-channel scalar \((H_1, H_2)\) exchange can give spin-independent contribution, but it doesn’t help our purpose of study. In the scalar portal, one can obtain contribution from spin-dependent (SD) interaction mediated by SLQ, of the form

\[
\mathcal{L}_{\text{eff}} \simeq \frac{y_q^2 R \cos^2 \alpha}{4(M_{S_1}^2 - M_{S_1}^2)} (y_{q R}^2) N_\gamma^\alpha N_\gamma^5 N_\gamma^\mu \gamma^5 N_\gamma^\mu q. \quad (16)
\]

The corresponding cross section is given by \cite{55}

\[
\sigma_{S_1} = \frac{M_{S_1}^2 M_n^2}{\pi (M_\gamma + M_n)^2} \cos^4 \alpha \left[ y_{q R}^2 \Delta_d + y_{s R}^2 \Delta_s \right]^2 J_N (J_N + 1), \quad (17)
\]

where the angular momentum \( J_N = \frac{1}{2} \), \( M_n \approx 1 \text{ GeV} \) for nucleon. The values of quark spin functions \( \Delta_{d,s} \) are provided in \cite{55}. Now, it is obvious that it can constrain the parameters \( M_\gamma \) and \( (y_q R)^2 \). Fig. 3 left panel displays \( M_\gamma - (y_q R)^2 \) parameter space (green and red regions) remained after imposing PLANCK \cite{54} 3\( \sigma \) limit on current relic density. Here, the region shown in green turns out to be excluded by most stringent PICO-60 \cite{57} limit on SD WIMP-proton cross section, as seen from the right panel.

**V. Radiative neutrino mass**

To generate light neutrino mass at one-loop level, we can write the interaction term using the inert doublet \( \eta \) as

\[
\sum_{i=e,\mu,\tau} Y_{ai} (\bar{\ell}_L)_i \bar{\eta} N_{iR}. \quad (18)
\]

The corresponding diagram is shown in Fig. 4. Assuming \( m_0^2 = (M_{\eta_e}^2 + M_{\eta_\mu}^2)/2 \) is much greater than \( M_{\eta_e}^2 - M_{\eta_\mu}^2 = \lambda_{\eta e}^2 v^2 \), the expression for the radiatively generated neutrino mass \cite{59} is given by

\[
(M_\nu)_{\alpha\beta} = \frac{\lambda_{\eta e}^2 v^2}{16\pi^2} \sum_{i=e,\mu,\tau} \frac{M_{Di}}{m_0^2 - M_{Di}^2} \left[ 1 - \frac{M_{Di}^2}{m_0^2 - M_{Di}^2} \ln \frac{m_0^2}{M_{Di}^2} \right]. \quad (19)
\]
FIG. 3: Left panel depicts the $M_- - (y_{qR})^2$ parameter space consistent up to 3σ level of PLANCK limit [54] on relic density. Right panel gives the SD WIMP-proton cross section as a function of DM mass. Dashed lines represent the recent bounds obtained from PICO-60 [57] and LUX [58]. Green (red) data points in both the panels represent PLANCK allowed and PICO excluded (PLANCK and PICO allowed).

Here $M_{Di} = (U^T M_N U)_i = \text{diag}(M_{ee}, M_-, M_+)$ and the fermion mass eigenstates $N_{Di} = U_{ij}^i N_j$. With a sample parameter space, $(Y_{ll}, \chi_H^\nu) \sim (10^{-2}, 10^{-5})$ and $(m_0, M_-, M_{ee}, M_+) \sim (1.5, 0.4, 3, 3)$ TeV, one can explain neutrino mass ($m_\nu$) near eV scale. Thus, the light neutrino mass generation can be successfully achieved in the proposed model.

FIG. 4: Radiative generation of neutrino mass.
VI. FLAVOR PHENOMENOLOGY

The general effective Hamiltonian responsible for the quark level transition \( b \rightarrow s l^+ l^- \) is given by \cite{60, 61}

\[
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1}^{6} C_i(\mu)O_i + \sum_{i=7,9,10} \left( C_i(\mu)O_i + C_i'(\mu)O'_i \right) \right],
\]

where \( G_F \) is the Fermi constant and \( V_{qq'} \) denote the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The \( C_i \)'s stand for the Wilson coefficients evaluated at the renormalized scale \( \mu = m_b \) \cite{62}, where the sum over \( i \) includes the current-current operators \((i = 1, 2)\) and the QCD-penguin operators \((i = 3, 4, 5, 6)\). The quark level operators mediating leptonic/semileptonic processes are given as

\[
\begin{align*}
O_7^{(l)} &= \frac{e}{16\pi^2} \left( \bar{s} \gamma^\mu \left( m_s P_{L(R)} + m_b P_{R(L)} \right) b \right) F^{\mu
u}, \\
O_9^{(l)} &= \frac{\alpha_{\text{em}}}{4\pi} \left( \bar{s} \gamma^\mu P_{L(R)} b \right) \left( \bar{l} \gamma^\mu l \right), \quad O_{10}^{(l)} = \frac{\alpha_{\text{em}}}{4\pi} \left( \bar{s} \gamma^\mu P_{L(R)} b \right) \left( \bar{l} \gamma^\mu \gamma^5 l \right),
\end{align*}
\]

where \( \alpha_{\text{em}} \) denotes the fine-structure constant and \( P_{L,R} = (1 \pm \gamma_5)/2 \) are the chiral operators. The primed operators are absent in the SM, but can exist in the proposed \( L_\mu - L_\tau \) model.

The previous section has discussed the available new parameter space consistent with the DM observables which are within their respective experimental limits. However, these parameters can be further constrained from the quark and lepton sectors, to be presented in the subsequent sections.

A. \( B_s - \bar{B}_s \) mixing

In this subsection, we discuss the constraint on the new parameters from the mass difference between the \( B_s \) meson mass eigenstates \((\Delta M_s)\), which characterizes the \( B_s - \bar{B}_s \) mixing phenomena. In the SM, \( B_s - \bar{B}_s \) mixing proceeds to an excellent approximation through the box diagram with internal top quark and \( W \) boson exchange. The effective Hamiltonian describing the \( \Delta B = 2 \) transition is given by \cite{63}

\[
\mathcal{H}_{\text{eff}} = \frac{G_F^2}{16\pi^2} \lambda_t^2 \frac{M_W^2}{2} S_0(x_t) \eta_B (\bar{s}b) V_{ts} (\bar{s}b) V_{ts},
\]

where \( \lambda_t = V_{tb} V_{ts}^* \), \( \eta_B \) is the QCD correction factor and \( S_0(x_t) \) is the loop function \cite{63} with \( x_t = m_t^2/M_W^2 \). Using Eqn. \( \Delta M_s^{\text{SM}} = \frac{\langle \bar{B}_s | \mathcal{H}_{\text{eff}} | B_s \rangle}{M_{B_s}} = \frac{G_F^2}{6\pi^2} \lambda_t^2 \eta_B \hat{f}_{B_s} f_{B_s} M_{B_s} S_0(x_t) \), \( \Delta M_s^{\text{SM}} = 2|M_{12}^{\text{SM}}| = \frac{\langle \bar{B}_s | \mathcal{H}_{\text{eff}} | B_s \rangle}{M_{B_s}} = \frac{G_F^2}{6\pi^2} \lambda_t^2 \eta_B \hat{f}_{B_s} f_{B_s} M_{B_s} S_0(x_t) \).
The SM predicted value of $\Delta M_s$ by using the input parameters from [64, 65] is
\[
\Delta M_s^{\text{SM}} = (17.426 \pm 1.057) \text{ ps}^{-1},
\] (24)
and the corresponding experimental value is [64]
\[
\Delta M_s^{\text{Expt}} = 17.761 \pm 0.022 \text{ ps}^{-1}.
\] (25)

Even though the theoretical prediction is in good agreement with the experimental $B_s - \bar{B}_s$ oscillation data, it does not completely rule out the possibility of new physics.

The box diagrams for $B_s - \bar{B}_s$ mixing in the presence of singlet SLQ and $N_{\pm}$ are shown in Fig. 5. The effective Hamiltonian in the presence NP is given by
\[
\mathcal{H}_{\text{eff}} = \frac{(y_s R y_{bR})^2}{128 \pi^2 M_{S_1}^2} \cos^2 \alpha \sin^2 \alpha C_{B_s}^{\text{NP}} (s\bar{b})_{V+A} (s\bar{b})_{V+A},
\] (26)
where
\[
C_{B_s}^{\text{NP}} = 4k (\chi_-, \chi_+, 1) + 2\sqrt{\chi_- \chi_+} j (\chi_-, \chi_+, 1) + \chi_+ j (\chi_+, \chi_-, 1),
\] (27)
with $\chi_{\pm} = M_{\mp}^2 / M_{S_1}^2$ and $k (\chi_+, \chi_+, 1)$, $j (\chi_+, \chi_+, 1)$ are the loop functions [15]. Using Eqn. [26], the mass difference of $B_s - \bar{B}_s$ mixing due to the exchange of $S_1$ and $N_{\pm}$ is found to be
\[
\Delta M_s^{\text{NP}} = \frac{(y_s R y_{bR})^2}{48 \pi^2 M_{S_1}^2} \cos^2 \alpha \sin^2 \alpha C_{B_s}^{\text{NP}} \eta_{B_s} f_{B_s}^2 M_{B_s},
\] (28)

FIG. 5: Box diagrams of $B_s - \bar{B}_s$ mixing with leptoquark in the loop.
Including the NP contribution arising due to the SLQ exchange, the total mass difference can be written as

$$\Delta M_s = \Delta M_s^{\text{SM}} \left[ 1 + \frac{C_{B_{i}}^{\text{NP}} \cos^2 \alpha \sin^2 \alpha}{8 G_F^2 V_{tb}^2 V_{ts}^* M_W^2 S_0(x_t)} \left( \frac{(y_{sR} y_{bR})^2}{M_{S_1}^2} \right) \right]. \quad (29)$$

Using Eqns. (24) and (25) in (29), one can put bound on $(y_{qR})^2$ and $M_-$ parameters.

### B. $B \to Kl^+l^-$ process

The rare semileptonic $B \to Kl^+l^-$ process is mediated via $b \to sl^+l^-$ quark level transitions. In the current framework, the $b \to sl^+l^-$ transitions can occur via the $Z'$ exchanging one-loop penguin diagrams shown in Fig. 6.

![Penguin diagram of $b \to sll$ processes, where $l = \mu, \tau$ with leptoquark in the loop.](image)

The matrix elements of the various hadronic currents between the initial $B$ meson and $K$ meson in the final state are related to the form factors $f_{+0}$ as follows [7, 66]

$$\langle K (p_K) | \bar{s} \gamma^\mu b | B (p_B) \rangle = f_+ (q^2) (p_B + p_K)^\mu + \left[ f_0 (q^2) - f_+ (q^2) \right] \frac{M_B^2 - M_K^2}{q^2} q^\mu, \quad (30)$$

where $p_B$ ($p_K$) and $M_B$ ($M_K$) denote the 4-momenta and mass of the $B$ ($K$) meson and $q^2$ is the momentum transfer. By using Eqn. (30), the transition amplitude of $B \to K \mu^+\mu^-$ process is given by

$$\mathcal{M} = \frac{1}{25 \pi^2} \frac{y_{bR} y_{sR} g_{\mu \nu}^L}{M_{Z'}^2} \mathcal{V}_{sb}(\chi-, \chi_+) \left[ \bar{u}(p_B) \gamma^\mu (1 + \gamma_5) u(p_K) \right] \left[ \bar{v}(p_2) \gamma_\mu u(p_1) \right], \quad (31)$$
where $p_1$ and $p_2$ are the four momenta of charged leptons and $\mathcal{V}_{sb}(\chi_-, \chi_+)$ is the loop function \cite{15, 67}. Now comparing this amplitude (31) with the amplitude obtained from the effective Hamiltonian \cite{20}, we obtain a new Wilson coefficient associated with the right-handed semileptonic electroweak penguin operator $\mathcal{O}_9$ as

$$C_9^{\text{NP}} = \frac{\sqrt{2}}{2^{4+1}p_4 \alpha_{\text{em}}} V_{tb}V_{ts}^* \frac{g_{bR}sRg_{a}^2}{M_{Z'}^2} \mathcal{V}_{sb}(\chi_-, \chi_+).$$

The differential branching ratio of $B \to Kll$ process with respect to $q^2$ is given by

$$\frac{d\text{Br}}{dq^2} = \tau_B \frac{G_F^2 \alpha_{\text{em}}^2 |V_{tb}V_{ts}^*|^2}{2^8 \pi^2 M_B^3} \sqrt{\lambda(M_B^2, M_K^2, q^2)} \beta_l f_+(a_l(q^2) + \frac{c_l(q^2)}{3}),$$

where

$$a_l(q^2) = q^2 |F_P|^2 + \frac{\lambda(M_B^2, M_K^2, q^2)}{4} (|F_A|^2 + |F_V|^2)$$

$$+ 2m_l(M_B^2 - M_K^2 + q^2) \text{Re}(F_P F_A^*) + 4m_l^2 M_B^2 |F_A|^2,$$

$$c_l(q^2) = - \frac{\lambda(M_B^2, M_K^2, q^2)}{4} \beta_l^2 (|F_A|^2 + |F_V|^2),$$

with

$$F_V = \frac{2m_b}{M_B} C_T^{\text{eff}} + C_9^{\text{eff}} + C_9^{\text{NP}}, \quad F_A = C_{10},$$

$$F_P = m_l C_{10} \left[ \frac{M_B^2 - M_K^2}{q^2} \left( \frac{f_0(q^2)}{f_+(q^2)} - 1 \right) - 1 \right],$$

and

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca), \quad \beta_l = \sqrt{1 - 4m_l^2/q^2}.$$

For numerical estimation, we have used the lifetime and masses of particles from \cite{64} and the form factors are taken from \cite{68}. The upper limit on the branching ratio of $B^+ \to K^+\tau^+\tau^-$ process is \cite{64}

$$\left| \text{Br}(B^+ \to K^+\tau^+\tau^-) \right|_{\text{Expt}} < 2.5 \times 10^{-3},$$

while its predicted value in the SM is

$$\left| \text{Br}(B^+ \to K^+\tau^+\tau^-) \right|_{\text{SM}} = (1.486 \pm 0.12) \times 10^{-7}.$$

Since $Z'$ doesn’t couple to electron, the branching ratio of $B^+ \to K^+e^+e^-$ process is considered to be SM like. The anomalies of $b \to sll$ decay modes can put constraint on all the four parameters, i.e., $(y_{qR})^2$, $g_{\mu\tau}$, $M_{Z'}$ and $M_-$.
C. $B \to X_s\gamma$ process

The $B \to X_s\gamma$ process involves $b \to s\gamma$ quark level transition, the experimental limit on the corresponding branching ratio is given by [69]

$$\text{Br}(B \to X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}}^{\text{Expt}} = (3.32 \pm 0.16) \times 10^{-4}. \quad (39)$$

Fig. 7 represents the one loop penguin diagram of $b \to s\gamma$ process mediated by SLQ and $N_{\pm}$.

![Feynman diagram of $b \to s\gamma$ processes in the presence of scalar leptoquark.](image)

Including the NP contribution, the total branching ratio of $B \to X_s\gamma$ is given by

$$\text{Br}(B \to X_s\gamma) = \text{Br}(B \to X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} \left(1 + \frac{C_{7}^{\text{NP}}}{C_{7}^{\text{SM}}}\right)^2, \quad (40)$$

where the predicted SM branching ratio is [70]

$$\text{Br}(B \to X_s\gamma)|_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} = (3.36 \pm 0.23) \times 10^{-4}. \quad (41)$$

The new $C_{7}^{\text{NP}}$ Wilson coefficient obtained from Fig. 7 is given by

$$C_{7}^{\text{NP}} = -\frac{\sqrt{2}/3}{8G_FV_{tb}V_{ts}^*} \frac{y_R y_{sR}}{M_{S_1}^2} (J_1(\chi_-) \cos^2 \alpha + J_1(\chi_+) \sin^2 \alpha), \quad (42)$$

where $J_1(\chi_{\pm})$ are the loop functions [15]. Using Eqns. (39, 41, 42) in (40), the parameters $(y_{qR})^2$ and $M_-$ can be constrained.
D. \( \tau \to \mu \nu_\tau \bar{\nu}_\mu \) process

In the presence of \( Z' \) boson, the \( \tau \to \mu \nu_\tau \bar{\nu}_\mu \) process can occur via box diagram as shown in Fig. 8. There are four possible one-loop box diagrams with the \( Z' \) connected to the lepton legs. The total branching ratio of this process is given by [71]

\[
\frac{\text{Br}(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{\text{SM}} = (17.29 \pm 0.032)\%.
\]

(43)

where the branching ratio in the SM is given by [71]

\[
\frac{\text{Br}(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{\text{SM}} = (17.39 \pm 0.04)\%.
\]

(44)

Now comparing the theoretical result with the experimental measured value [64]

\[
\frac{\text{Br}(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{\text{Expt}} = (17.39 \pm 0.04)\%,
\]

(45)

one can put bounds on \( M_{Z'} - g_{\mu \tau} \) parameter space.

| Parameters | DM-I  | DM-II | DM+Flavor |
|------------|-------|-------|-----------|
| \( M_- \) [GeV] | \( 103 - 560 \) | \( 561 - 988 \) | \( 103 - 560 \) |
| \( (y_{qR})^2 \) | \( 0 - 3.51 \) | \( 1.94 - 2.56 \) | \( 0 - 1.26 \) |

TABLE II: Predicted allowed range of parameters \( M_- \) and \( (y_{qR})^2 \). Here DM-I and DM-II represent two regions in Fig. 3 consistent with only DM observables, DM+Flavor denotes the region favored by both the dark matter and flavor studies.

Now correlating the theoretical predictions of \( R_K \), \( \text{Br}(B^+ \to K^+\tau^+\tau^-) \) and \( \text{Br}(\tau \to \mu \nu_\tau \bar{\nu}_\mu) \) with the corresponding 3\( \sigma \) experimental data, we compute the \( M_{Z'} - g_{\mu \tau} \) allowed
FIG. 9: Left panel projects the constraint on \( g_{\mu\tau} \) and \( M_{Z'} \) obtained from \( R_K \), \( \text{Br}(B \to K\tau\tau) \) and \( \text{Br}(\tau \to \mu\nu_\tau\bar{\nu}_\mu) \) observables. In the right panel, blue data points denote the allowed parameter space obtained from \( R_K \), \( B_s - \bar{B}_s \) mixing, \( \text{Br}(B \to K\tau\tau) \), \( \text{Br}(B \to X_s\gamma) \) experimental data, which are also consistent with PLANCK \[51\] and PICO-60 limit \[57\]. Here, green (red) data points denote PICO-60 and flavor excluded (PICO-60 allowed and flavor excluded) region.

Since \( Z' \) does not couple to quarks, these gauge parameters couldn’t be constrained from \( b \to s\gamma \) decay modes and \( B_s - \bar{B}_s \) oscillation data. The constraint on \( M_- - (y_{qR})^2 \) parameter space is obtained from \( R_K \), \( \text{Br}(B^+ \to K^+\tau^+\tau^-) \), \( \text{Br}(B \to X_s\gamma) \) and \( B_s - \bar{B}_s \) mixing results. In addition, the branching ratio of rare semileptonic \( B \to K\nu_l\bar{\nu}_l \) process can play a vital role in restricting these parameters. Though the proposed model can allow \( b \to s\nu_l\bar{\nu}_l \) decay modes, but the contributions of \( \mu \) and \( \tau \) leptons cancel with each other in the leading order of NP due to their equal and opposite \( L_\mu - L_\tau \) charges. Since there is no \( Z'\mu\tau \) coupling, the neutral and charged lepton flavor violating decay processes like \( B \to K^{(*)}\mu^+\tau^\pm, \tau^- \to \mu^-\gamma, \tau \to \mu\mu\mu \) do not play any role. In this analysis, we consider that the \( y_{qR} \) coupling is perturbative, i.e., \( |y_{qR}| \lesssim \sqrt{\frac{1}{\kappa}} \). Left (right) panel in Fig. 9 denotes the parameter space in the plane of \( M_{Z'} - g_{\mu\tau} \) \( (M_- - (y_{qR})^2) \) consistent with DM and flavor studies. From left panel, one can obtain the lower limit on the ratio \( M_{Z'}/g_{\mu\tau} \) around 4615 GeV, which is far more stringent than the lower limit imposed by neutrino trident production \[72\,73\], i.e., 540 GeV. It is also noted that the allowed region favored by the \((g - 2)_\mu\) anomaly is completely excluded by the constraint from the neutrino trident production \[71\]. In the right panel of Fig. 9, we redisplay \( M_- - (y_{qR})^2 \) parameter space of Fig. 3 after a combined analysis made by imposing the DM and flavor experimental limits, with the surviving region shown in blue color. In Table II, we report the allowed region of
the parameters $M_-$ and $(g_R)^2$ which are consistent with only DM studies (DM-I,II), both DM and flavor sectors (DM+Flavor).

VII. IMPLICATION ON $B(s) \to K^*(\phi)\mu^+\mu^-$ PROCESSES

The constrained parameter space discussed in the previous section can have an impact on the observables of $B \to Vl^+l^-$ process, where $V = K^*, \phi$ are the vector mesons. The $B \to V$ hadronic matrix elements of the local quark bilinear operators can be parametrized as [74, 75]

$$\langle V(k) | \bar{s}_\mu (1 - \gamma_5) b | B(p) \rangle = \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu\rho} q^\beta \frac{2V(q^2)}{M_B + M_V} - i \epsilon^*_\mu (M_B + M_V) A_1(q^2)$$

$$+ i (\epsilon^* \cdot q)(2p - q)_\mu \frac{A_2(q^2)}{M_B + M_V} + i \frac{2M_V}{q^2} (\epsilon^* \cdot q) [A_3(q^2) - A_0(q^2)] q_\mu \ ,$$

(46)

where

$$A_3(s) = \frac{(M_B + M_V)}{2M_V} A_1(s) - \frac{(M_B - M_V)}{2M_V} A_2(s) \ ,$$

(47)

$q^2$ is the momentum transfer between the $B$ and $V$ mesons, i.e., $q_\mu = p_\mu - k_\mu$ and $\epsilon_\mu$ is the polarization vector of the $V$ meson. The full angular differential decay distribution in terms of $q^2, \theta_l, \theta_V$ and $\phi$ variables is given as [76–78]

$$\frac{d^4 \Gamma}{dq^2 \ d \cos \theta_l \ d \cos \theta_V \ d \phi} = \frac{9}{32 \pi} J(q^2, \theta_l, \theta_V, \phi) \ ,$$

(48)

where

$$J(q^2, \theta_l, \theta_V, \phi) = J_1^s \sin^2 \theta_V + J_1^c \cos^2 \theta_V + (J_2^s \sin^2 \theta_V + J_2^c \cos^2 \theta_V) \cos 2\theta_l$$

$$+ J_3 \sin^2 \theta_V \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_V \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_V \sin \theta_l \cos \phi$$

$$+ (J_6^s \sin^2 \theta_V + J_6^c \cos^2 \theta_V) \cos \theta_l + J_7 \sin 2\theta_V \sin \theta_l \sin \phi$$

$$+ J_8 \sin 2\theta_V \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_V \sin^2 \theta_l \sin 2\phi \ ,$$

(49)

$\theta_l$ is the angle between $l^-$ and $B$ in the dilepton frame, $\theta_V$ is defined as the angle between $K^-$ and $B$ in the $K^-\pi^+$ ($K^-K^+$) frame, the angle between the normals of the $K^-\pi^+$ ($K^-K^+$) and the dilepton planes is given by $\phi$. The complete expression for $J(q^2, \theta_l, \theta_V, \phi)$ as a function of transversity amplitudes can be found in the Ref. [79]. The transversity amplitudes
written in terms of the form factors and Wilson coefficients are as follows [79]

\[ A_{\perp L,R} = N\sqrt{2\lambda} \left[ \left( C_{9\text{eff}} + C_{9\text{NP}} \right) \mp C_{10} \right] \frac{V(q^2)}{M_B + M_V} + \frac{2m_b}{q^2} C_7 T_1(q^2), \]
\[ A_{\parallel L,R} = -N\sqrt{2(2M_B^2 - M_V^2)} \left[ \left( C_{9\text{eff}} + C_{9\text{NP}} \right) \mp C_{10} \right] \frac{A_1(q^2)}{M_B - M_V} + \frac{2m_b}{q^2} C_7 T_2(q^2), \]
\[ A_{0 L,R} = -\frac{N}{2M_V\sqrt{s}} \left[ \left( C_{9\text{eff}} + C_{9\text{NP}} \right) \mp C_{10} \right] \]
\[ \times \left( (M_B^2 - M_V^2 - q^2)(M_B + M_V) A_1(q^2) - \lambda \frac{A_2(q^2)}{M_B + M_V} \right) + 2m_B C_7 \left( (M_B^2 + 3M_V^2 - q^2) T_2(q^2) - \frac{\lambda}{M_B^2 - M_V^2} \right) \],
\[ A_t = 2N\sqrt{\frac{\lambda}{q^2}} C_{10} A_0(q^2), \quad (50) \]

where

\[ N = V_{tb}V_{ts}^* \left[ \frac{G_F^2 \alpha_{em}^2}{3 \cdot 2^{10} \pi^5 M_B^3 q^2 \beta_i \sqrt{\lambda}} \right]^{1/2}, \quad \lambda = \lambda(M_V^2, M_B^2, q^2). \quad (51) \]

The dilepton invariant mass spectrum for \( B \to Vl^+l^- \) decay after integration over all angles [76] is given by

\[ \frac{d\Gamma}{dq^2} = \frac{3}{4} \left( J_1 - \frac{J_2}{3} \right), \quad (52) \]

where \( J_i = 2J_{i^e}^e + J_{i^e}^e \). The most interesting observables in these decay modes are the lepton non-universality parameter defined as

\[ R_V = \frac{\text{Br}(B \to V\mu^+\mu^-)}{\text{Br}(B \to Ve^+e^-)}, \quad (53) \]

the form factor independent (FFI) observables [80]

\[ P'_4 = \frac{J_4}{\sqrt{-J_2^e J_2^c}}, \quad P'_5 = \frac{J_5}{2\sqrt{-J_2^e J_2^c}}. \quad (54) \]

After getting familiar with the different observables and the allowed values of the new parameters, we now proceed for numerical analysis in the full dilepton mass region i.e., \( 4m^2_l \leq q^2 \leq (M_B - M_V)^2 \), leaving the regions around \( q^2 \sim m^2_{J/\psi} \) and \( m^2_{\psi'} \). The cuts are employed to remove the dominant charmonium resonance \((c\bar{c}) = J/\psi, \psi'\) backgrounds from \( B \to V (c\bar{c}) \to VL^+l^- \). In Fig. 10, we show the behaviour of \( R_{K^*} \) (left panel) and \( R_\phi \) (right panel) with respect to \( q^2 \) in the full kinematically accessible physical region. In these
FIG. 10: The $q^2$ variation of $R_{K^*}$ (left panel) and $R_\phi$ (right panel) LNU parameters in the $L_\mu - L_\tau$ model. Here the blue dashed lines represent the SM prediction, the cyan (magenta) bands stand for the NP contribution from the dark matter studies i.e., DM-I (DM-II). Orange bands are due to the contribution from both the flavor and DM sectors (DM+Flavor). The experimental data points (with $2\sigma$ error bars) in the blue dashed lines stand for the SM contribution, the orange bands are due to the allowed region of parameters shown in Table II, favored by both DM and flavor (DM+Flavor) and cyan (magenta) bands for only DM case i.e., DM-I (DM-II). The bin-wise experimental values of $R_{K^*}$ are shown in black. From the left panel of Fig. 10, it can be seen that the measured value of $R_{K^*}$ in the $q^2 \in [0.045, 1.1]$ GeV$^2$ region can be accommodated within $2\sigma$ (DM-I), the $q^2 \in [1.1, 6]$ GeV$^2$ bin result can be explained within $1\sigma$ (DM-I) and $2\sigma$ (DM-II and DM+Flavor). Though there is no experimental evidence for $R_\phi$ parameter, the additional NP contribution arising from the allowed parameter space of all cases (DM-I,II and DM+Flavor) provide significant deviation from the SM prediction, implying the presence of lepton universality violation in the $B_s \to \phi \mu^+ \mu^-$ process. In Table III, we present our predicted values of $R_{K^*}$ and $R_\phi$ for different bins. The $q^2$ variation of famous optimized observables $-P_4'$ (top-left panel) and $P_5'$ (top-right panel) of $B \to K^* \mu^+ \mu^-$ process are depicted in Fig. 11. The bottom panel of this figure describes analogous plots for $B_s \to \phi \mu^+ \mu^-$ process in both the high and low recoil limit. It should be noted that $P_4^{LHCB} = -P_4'$. In the low $q^2$ region, our predictions on $-P_4'$ observable of $B \to K^* \mu^+ \mu^-$ process is in very good agreement with the LHCB data. For $B \to K^* \mu^+ \mu^-$ decay mode, we are able to explain the $P_5'$ observable within $1\sigma$ of the experimental limit in the full $q^2$ region (excluding the intermediate resonance regions). We notice profound deviation between the
FIG. 11: Top panel represents the variation of $P_4'$ (left panel) and $P_5'$ (right panel) observables of $B \to K^* \mu^+ \mu^-$ process with respect to $q^2$. The behaviour of $P_4'$ (left panel) and $P_5'$ (right panel) for $B_s \to \phi \mu^+ \mu^-$ are shown in the bottom panel. The bin-wise experimental data points with error bars are shown in black [4]. Note that $P_{4,5}'|_{LHCb} = -P_{4,5}'$.

results of SM and the presented $L_{\mu} - L_{\tau}$ model on the $P_{4,5}'$ observables for $B_s \to \phi \mu^+ \mu^-$ decay modes. The numerical values of all these observables are given in Table [II]. We found that our results on the angular observables of $B \to Vll$ process, obtained from DM-I parameter space are almost consistent with the corresponding measured experimental data.

VIII. SUMMARY AND CONCLUSION

Summarizing the article, we have studied Majorana dark matter in a new version of $U(1)_{L_{\mu} - L_{\tau}}$ gauge extension of the standard model. The model is free from triangle gauge anomalies with the inclusion of three neutral fermions with $L_{\mu} - L_{\tau}$ charges 0, 1 and $-1$. A scalar singlet, charged +2 under the new $U(1)$ is added to spontaneously break the $L_{\mu} - L_{\tau}$ gauge symmetry, thereby giving masses to the new fermions and the neutral boson $Z'$ associated with gauge extension. In addition, the scalar sector is enriched with an inert doublet and a $(\bar{3}, 1, 1/3)$ scalar leptoquark to obtain the neutrino mass at one-loop level and
address the flavor anomalies respectively. All the new fermions, leptoquark and inert doublet are assigned with charge $-1$ under $Z_2$ symmetry. Choosing the lightest mass eigenstate of the new fermion spectrum as dark matter, we made a thorough study of Majorana dark matter in relic density and direct detection perspective. The channels contributing to relic density are mediated by the scalar leptoquark, $Z'$ and inert doublet components. As $Z'$-mediated cross section is insensitive to direct detection experiments in Majorana dark matter case, only leptoquark portal channels contribute to spin-dependent WIMP-nucleon cross section. Imposing PLANCK limit on relic density and well known PICO-60, LUX bounds on spin-dependent cross section, we have constrained the new parameters of the model. We have also showed the mechanism of generating light neutrino mass radiatively using the inert doublet.

We have further restricted the new parameters from quark and lepton sectors i.e., by comparing the theoretical predictions of $\text{Br}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu)$, $\text{Br}(B \rightarrow X_s \gamma)$, $\text{Br}(B^+ \rightarrow K^+ \tau^+ \tau^-)$, $\text{BR}(K^+ \rightarrow \mu^+ \nu_e l^-$

| Observables | Values for SM | Values for DM-I | Values for DM-II | Values for DM+Flavor |
|-------------|--------------|----------------|-----------------|---------------------|
| $R_{K^+}|q^2_{[0.045,1.1]} \text{ GeV}^2$ | 0.949 | 0.825 - 0.949 | 0.884 - 0.89 | 0.9 - 0.949 |
| $B$ | 0.993 | 0.732 - 0.993 | 0.852 - 0.865 | 0.887 - 0.993 |
| $\downarrow$ | 0.998 | 0.793 - 0.998 | 0.882 - 0.893 | 0.91 - 0.998 |
| $K^*$ | $P_5^|q^2_{[1,6]} \text{ GeV}^2$ | $-0.057 \pm 0.004$ | $-0.074 \rightarrow -0.057$ | $-0.064 \rightarrow -0.063$ | $-0.063 \rightarrow -0.057$ |
| $l^+$ | $P_5^|q^2_{[1,6]} \text{ GeV}^2$ | $-0.805 \pm 0.064$ | $-1.144 \rightarrow -0.805$ | $-0.942 \rightarrow -0.926$ | $-0.921 \rightarrow -0.805$ |
| $l^+$ | $P_4^|q^2_{[1,6]} \text{ GeV}^2$ | $0.398 \pm 0.024$ | $0.025 - 0.398$ | $0.242 - 0.26$ | $0.288 \rightarrow 0.398$ |
| $P_4^|q^2_{[1,6]} \text{ GeV}^2$ | $0.852 \pm 0.068$ | $0.662 - 0.852$ | $0.78 - 0.789$ | $0.8 - 0.852$ |

| Observables | Values for SM | Values for DM-I | Values for DM-II | Values for DM+Flavor |
|-------------|--------------|----------------|-----------------|---------------------|
| $R_\phi|q^2_{[0.045,1.1]} \text{ GeV}^2$ | 0.9499 | 0.794 - 0.9499 | 0.868 - 0.876 | 0.89 - 0.9499 |
| $B_s$ | 0.994 | 0.712 - 0.994 | 0.843 - 0.858 | 0.881 - 0.994 |
| $\downarrow$ | 0.998 | 0.776 - 0.998 | 0.874 - 0.886 | 0.9 - 0.998 |
| $\phi$ | $P_5^|q^2_{[1,6]} \text{ GeV}^2$ | $-0.049 \pm 0.004$ | $-0.064 \rightarrow -0.049$ | $-0.055 \rightarrow -0.054$ | $-0.053 \rightarrow -0.049$ |
| $l^+$ | $P_5^|q^2_{[1,6]} \text{ GeV}^2$ | $-0.743 \pm 0.059$ | $-1.07 \rightarrow -0.743$ | $-0.875 \rightarrow -0.86$ | $-0.837 \rightarrow -0.743$ |
| $l^-$ | $P_4^|q^2_{[1,6]} \text{ GeV}^2$ | $0.421 \pm 0.036$ | $4.91 \times 10^{-3} - 0.421$ | $0.266 - 0.284$ | $0.311 \rightarrow 0.421$ |
| $P_4^|q^2_{[1,6]} \text{ GeV}^2$ | $0.872 \pm 0.07$ | $0.687 - 0.872$ | $0.8 - 0.812$ | $0.825 - 0.872$ |

TABLE III: Predicted numerical values of LNU parameters ($R_V$) and $P_{4,5}$ observables of $B \rightarrow Vl$, $V = K^*$, $\phi$ processes in the high and low recoil limits.
$R_K$ and $B_s - \bar{B}_s$ mixing with their corresponding 3σ experimental data. The neutral and charged lepton flavor violating decay processes are absent due to zero $Z'\tau\mu$ coupling. And also the vanishing $Z'q\bar{q}$ coupling restricts the involvement of $Z'$ in $B_s - \bar{B}_s$ mixing, $b \to s\gamma$ processes at one-loop level. We have then investigated the implication on $P'_{4,5}$, $R_{K^*}$ and $R_\phi$ observables of $B(s) \to K^*(\phi)l^+l^-$ decay modes in the full kinematically allowed $q^2$ region for two cases i.e., dark matter and flavor allowed, only dark matter allowed parameter space. We found that the $R_{K^*}$ observable obtained from the parameter space consistent with only dark matter ($M_- \leq 560$ GeV) is within its 1σ, only dark matter ($M_- > 560$ GeV) and both dark matter and flavor is within 2σ experimental limit. In the presence of new physics, the violation of lepton universality is observed in $B_s \to \phi\mu^+\mu^-$ process, thus, can be probed in LHCb experiment. We noticed that the proposed $L_\mu - L_\tau$ model is also able to explain the LHCb experimental data of the famous optimized $P'_{4,5}$ observables of $B \to K^*l^+l^-$ process in the high recoil limit. We also perceived that the form factor independent observables for $B_s \to \phi\mu^+\mu^-$ decay modes have sizeable deviation from the standard model. We observed that the parameter region satisfying only dark matter observables for $M_- \leq 560$ GeV have a good impact on the flavor anomalies. To conclude, we have made a comprehensive study of Majorana dark matter, neutrino mass generation and flavor anomalies in a $U(1)_{L_\mu - L_\tau}$ gauge extended model. This simple framework survives all the current experimental limits on dark matter and flavor observables, can be probed in upcoming high luminosity experiments.

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