Host Star Dependence of Small Planet Mass–Radius Distributions

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Abstract

The planet formation environment around M dwarf stars is different than around G dwarf stars. The longer hot protostellar phase, activity levels and lower protoplanetary disk mass of M dwarfs all may leave imprints on the composition distribution of planets. We use hierarchical Bayesian modeling conditioned on the sample of transiting planets with radial velocity mass measurements to explore small planet mass–radius distributions that depend on host star mass. We find that the current mass–radius data set is consistent with no host star mass dependence. These models are then applied to the Kepler planet radius distribution to calculate the mass distribution of close-orbiting planets and how it varies with host star mass. We find that the average heavy element mass per star at short orbits is higher for M dwarfs compared to FGK dwarfs, in agreement with previous studies. This work will facilitate comparisons between microlensing planet surveys and Kepler, and will provide an analysis framework that can readily be updated as more M dwarf planets are discovered by ongoing and future surveys such as K2 and the Transiting Exoplanet Survey Satellite.

Key words: methods: statistical – planets and satellites: composition – stars: low-mass

1. Introduction

The Kepler survey (Borucki et al. 2011) has discovered thousands of transiting exoplanets, leading to the robust characterization of the radius distribution of small planets (Burke et al. 2015; Fulton et al. 2017). Subsequent radial velocity follow-up of transiting planets from Kepler and other surveys, as well as transit timing variations, have constrained planet masses and allowed exploration of the composition distribution of planets, often in the form of a mass–radius relationship. Constraining the composition distribution of planets provides insights into different formation pathways and the prevalence of Earth-like rocky planets.

A mass–radius relationship (hereafter M–R relation) is a key ingredient necessary for the comparison of different exoplanet populations. While Kepler and, in the near future, the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015) are able to characterize the radii of planets, WFIRST, a future microlensing survey, will only be able to characterize masses. If we wish to combine transit and microlensing surveys to constrain planet occurrence rates, an M–R relation is essential for translating masses into radii and vice versa. For example, Suzuki et al. (2016) showed that the break in the mass-ratio function of microlensing planets from the MOA-II survey occurs at a higher mass than the peak in the Kepler mass distribution by converting Kepler radii to masses with an M–R relation. However, WFIRST is much more sensitive than Kepler to planets around M dwarf hosts, due to the innate characteristics of the microlensing method. Given these disparate stellar distributions, if planet formation differs around M dwarfs compared to FGK dwarfs, this may manifest itself in the M–R relation. In this paper, we consider an M–R relation that depends on host star mass.

1.1. Planet Formation around M Dwarfs

Important differences during the protostellar phase between M and FGK dwarfs could impact planet formation. Before reaching the main sequence, stars undergo a period of contraction whereby the luminosity declines by several orders of magnitude. For low-mass stars, this pre-main sequence phase lasts longer and the luminosity difference is more pronounced, leading to an initially distant snowline that migrates far inward while the star is contracting (Kennedy & Kenyon 2008). This has important consequences for planet composition. Raymond et al. (2007) and Lissauer (2007) found that planets formed in situ around low-mass stars may be deficient in volatiles, as water-bearing planetesimals would not be scattered as often from beyond the snowline. However, if the surface density of water content in planetesimals is much higher than in our own solar system, volatile delivery to planets inside the snowline will increase (Ciesla et al. 2015). Alternatively, if the disk is long-lived, planets that form beyond the snowline will have more time to migrate inward, leading to volatile-rich planets at short orbits (Alibert & Benz 2017).

Another characteristic feature of M dwarfs is their intense activity at young stellar ages, during which flares can output bursts of XUV radiation and relativistic particles. For planets formed in situ, this high-energy radiation could provide enough heating to evaporate the atmosphere and any water content on the surface (Scalo et al. 2007). Alternatively, for gaseous planets that form outside the snowline and migrate inward, XUV radiation from the star may be enough to fully evaporate their envelopes, resulting in “habitable evaporated cores” (Luger et al. 2015). This process is more likely to occur for planets less massive than $2 M_{\oplus}$, which could result in a population of volatile-rich rocky planets. If the mechanisms described can efficiently erode planet atmospheres, we expect close-in planets around M dwarfs to experience more mass loss and to be more dense than their counterparts around FGK stars.

Kepler has shown that the radius distribution of planets around M dwarfs differs from that around FGK dwarfs, with higher occurrence rates of small planets around the former (Dressing & Charbonneau 2015; Gaidos et al. 2016). Similarly, radial velocity surveys have shown differences in the mass distribution, the most notable being a comparative lack of giant planets around M dwarfs (Bonfils et al. 2013). Occurrence rate studies with Kepler have also suggested a trend of increasing planetary heavy element mass in short orbits for decreasing...
stellar mass, seemingly at odds with the protoplanetary disk mass scaling with stellar mass (Mulders et al. 2015b). However, it is not yet clear as to how the composition distribution differs, or how this may be impacted by the stellar environment. If planets around M dwarfs had lower volatile content or experience more envelope mass loss, then we would expect planets to be more dense around low-mass stars. However, this could be counteracted by a higher surface density of water content in the protoplanetary disk or longer disk lifetimes. While differences in the stellar environment exist for planets around different types of stars, there is currently no predictive model for how these differences may imprint themselves in the observed population of planets. Answering this question motivates characterizing more planets around M dwarf stars.

1.2. M–R Relations

Empirically derived planet M–R relations have traditionally been cast as simple power laws. Lissauer et al. (2011) found $M/M_\oplus = (R/R_\oplus)^{1.2}$ by fitting a power law to Earth and Saturn. Utilizing a sample of 22 planet pairs with transit timing variation (TTV) measured masses, Wu & Lithwick (2013) found a linear relation with $M/M_\oplus = 3(R/R_\oplus)$. Weiss et al. (2013) introduced incident flux dependence into the M–R relation, using a sample of 35 planets with mass and radius measurements to find $(R/R_\oplus) = 1.78(M/M_\oplus)^{0.52} (F/\text{erg s}^{-1} \text{cm}^{-2})^{-0.03}$ for $M < 150 M_\oplus$. Weiss & Marcy (2014) used a sample of 42 Kepler planets with radial velocity (RV)-measured masses to fit a broken power law with a linear density relation $\rho_p = 2.43 + 3.39(R/R_\oplus) \text{ g cm}^{-3}$ for $R/R_\oplus < 1.5$ and $M/M_\oplus = 2.69(R/R_\oplus)^{0.93}$ for $1.5 < R/R_\oplus < 4.0$.

More recently, Wolfgang et al. (2016) used hierarchical Bayesian modeling to derive a more statistically robust, probabilistic M–R relation. They included intrinsic scatter in planet composition by modeling the M–R relation as a power law with dispersion that is normally distributed. They found $M/M_\oplus = 2.7(R/R_\oplus)^{1.3}$ for $R/R_\oplus < 4.0$ with an intrinsic scatter of $1.9 M/M_\oplus$, constrained to physically plausible densities, as the best-fit relation. The benefit of the Bayesian approach is that the uncertainties in the power-law parameters and intrinsic scatter are fully quantified. Chen & Kipping (2017) used a similar approach to fit broken probabilistic power laws to a much wider range of planet (and stellar) masses, including the transition points as additional parameters in the model. In this paper, we extend the approach of Wolfgang et al. (2016) to include dependence on host star mass.

This paper is organized as follows. In Section 2, we outline the steps taken to obtain our planet sample, detail our model, and describe how we fit it to the data. The results of our model fitting are described in Section 3, along with applications to planet heavy element mass and planet mass distributions. We discuss model selection, limitations, and future extensions of this work in Section 4 and conclude in Section 5.

2. Methods

2.1. Data

In order to constrain the M–R relation, we need a sample of planets with well-characterized mass and radius measurements and uncertainties. For homogeneity, we choose to use the sample of transiting planets with RV mass measurements. There is a comparable number of transiting planets with TTV mass measurements, but those planets have been shown to have systematically lower densities than planets with RV mass measurements (Jontof-Hutter et al. 2014; Weiss & Marcy 2014), likely due to different observational biases in the two techniques (Steffen 2016; Mills & Mazeh 2017). We use only planets with radii below $8 R_\oplus$, as we are most interested in the M–R relation of small planets (rocky planets and mini-Neptunes/super-Earths). There is evidence for a transition in the M–R relation from Neptunian to Jovian planets at around $\sim 11 R_\oplus$ (Chen & Kipping 2017), so adopting a cutoff of $8 R_\oplus$ is a conservative measure to ensure we are safely outside of the giant planet regime. Physically, as a planet becomes more massive, the core becomes more dense and degeneracy pressure plays an increasingly important role, causing the M–R relation to flatten at around a Jupiter radius (Chabrier & Baraffe 2007).

The planet catalog for this work was downloaded from the NASA Exoplanet Archive1 on 2017 June 13. Our first cuts are made to exclude planets with $R > 8 R_\oplus$, circumbinary planets, and those without either transit or RV measurements. As our model incorporates host star uncertainties, we require that a planet has both a well-defined RV semi-amplitude measurement $(K)$ and a planet radius to host star radius ratio $(r = R_p/R_\star)$ measurement, with uncertainties. While transit depth may be seen as a more fundamental parameter derived from a transit light curve, stellar limb-darkening causes the transit depth to deviate from the simple $\delta = \left(\frac{R_\oplus}{R_\star}\right)^2$ relation (Mandel & Agol 2002), with the limb-darkening law and coefficients varying from star to star. To avoid having to treat limb-darkening for each individual star, we use the radius ratio instead of the transit depth as the primary transit parameter. For each planet, we verify the source paper for the RV measurement to ensure that each value reported in the Exoplanet Archive is correct. We remove any planets without reported $K$ or $r$ values, or those with only upper limits. In the case of planets with transit depth measurements but no $r$ (HD 219134 b and c), we calculate $r$ from the transit depth assuming no limb darkening. For those planets without reported eccentricity or inclination measurements, we assume $e = 0$ and $i = 90^\circ$.

The final planet catalog includes 88 planets. The distribution of host star mass for these planets is shown in Figure 1. Given that we are most interested in how the M–R relation differs around stars of low mass, it is unfortunate that we only have six planets orbiting stars with $M < 0.7 M_\odot$; the majority of planets in the sample orbit G-type stars. This is largely because most RV follow-up campaigns target GK dwarfs as solar analogues. However, there are many ongoing and future RV observing programs that are specifically designed to characterize planets around M dwarfs, e.g., MAROON-X (Seifahrt et al. 2016), SPIRou (Moutou et al. 2015), CARMENES (Quirrenbach et al. 2014), and ESPRESSO (González Hernández et al. 2017) as a follow-up to transit observations, e.g., TESS, MEarth (Berta et al. 2012), ExTra (Bonfils et al. 2015), SPECULOOS (Burdanov et al. 2017), and APACHE (Christelle et al. 2013), so this number is expected to increase in the near future (Kains et al. 2016). Despite the current lack of planets with well-characterized radii and masses around M dwarfs, our analysis will provide a framework that can be revisited when the sample size increases drastically in the near future. A host star mass-dependent M–R relation is also necessary to study trends in exoplanet populations with host star mass and to allow

1 https://exoplanetarchive.ipac.caltech.edu/
comparison between Kepler planets and those discovered with the microlensing method.

2.2. Model

Since we are investigating the dependence of the $M$–$R$ relation on host star mass, we choose to start with the same framework as Wolfgang et al. (2016) in order to isolate this dependence. At a given radius, a planet’s mass is drawn from a Gaussian distribution where the mean of the Gaussian distribution is a power law:

$$\frac{M}{M_\odot} \sim \text{Normal}\left(\mu = C\left(\frac{R}{R_\odot}\right)^\gamma, \sigma = \sigma_M\right)$$

(1)

and the standard deviation parameterizes the intrinsic scatter. The fact that the mass is drawn from a distribution makes this a probabilistic, rather than deterministic, relation.

In Wolfgang et al. (2016), $C$, $\gamma$, and $\sigma_M$ are the three parameters to be fit to the data. In our case, we expand each of these to include host star mass dependence:

$$\gamma = \gamma_0 + \ln\left(\frac{M_*}{M_\odot}\right)\gamma_s$$

(2)

$$C = C_0 + \ln\left(\frac{M_*}{M_\odot}\right)C_s$$

(3)

$$\sigma_M = \sqrt{\sigma_0^2 + \ln\left(\frac{M_*}{M_\odot}\right)\sigma_s}$$

(4)

where we have introduced three new parameters that parameterize the host star dependence: $\gamma_s$, $C_s$, and $\sigma_s$. In the case where these three parameters are all zero, we obtain the original $M$–$R$ relation of Wolfgang et al. (2016), which is independent of host star mass. In Equations (2)–(4), $M_*$ is the mass of the planet’s host star such that, at a solar mass $M_\odot$, the host star mass-dependent terms drop out, and the $M$–$R$ relation is given solely by $\gamma_0$, $C_0$, and $\sigma_0$. Parameterizing the $M$–$R$ relation in this manner allows both the mean planet mass and intrinsic scatter to smoothly vary as a function of host star mass. This specific choice of parameterization is chosen for simplicity, similar to the choice of parameterizing the $M$–$R$ relation as a power law. Our aim is to allow the $M$–$R$ relation to change flexibly with host star mass, while still minimizing the number of parameters.

Wolfgang et al. (2016) considered an additional model, where the intrinsic scatter in mass is allowed to vary as a function of planet radius. While this may be physically reasonable, they find that this parameter is consistent with zero and there is no strong evidence for requiring its inclusion. For this reason and for the sake of simplicity, we do not include any dependence of the intrinsic scatter on radius. However, following Wolfgang et al. (2016), we do include a maximum density constraint, based on the mass of a planet composed of a pure iron core. At a given planet radius, the maximum physically plausible planet mass is given by:

$$\log_{10}(M_{\text{pureFe}}) = -b + \sqrt{b^2 - 4a(c - R)} \frac{2a}{}$$

(5)

where $a = 0.0975$, $b = 0.4938$, and $c = 0.7932$ (Fortney et al. 2007). Imposing this constraint has the effect of truncating the normal distribution of masses at a given radius. This is more constraining for small planet radii and severely limits the range of masses that these small planets can have. At around 1.5 $R_\odot$, this maximum mass limit no longer constrains the $M$–$R$ relation significantly. In this way, although we have not included a parameter that allows the intrinsic scatter to vary with radius, the range of masses is being constrained at small radii by this maximum mass limit.

A graphical representation of our model is shown in Figure 2. As input for our model, we take the following for each planet: observed radial velocity semi-amplitude $K_{\text{obs}}$ with uncertainty $\sigma_{K_{\text{obs}}}$, observed planet radius to host star radius ratio $r_{\text{obs}}$ with uncertainty $\sigma_{r_{\text{obs}}}$, period $P$, eccentricity $e$, and inclination $i$. We also take as input a list of observed host star masses $M_{\text{obs}}$ with uncertainty $\sigma_{M_{\text{obs}}}$, as well as observed stellar radii $R_{\text{obs}}$ with uncertainty $\sigma_{R_{\text{obs}}}$. We only include each host star once, such that multiple planets around the same star will be drawing from the same host star properties during each step of the fitting. If asymmetric error bars are reported for a given measurement, we use the average of the two as the uncertainty.

For each planet, we introduce the parameters $R_{\text{true}}$ and $M_{\text{true}}$, which represent the true radius and mass of that planet. The true planet radii are assumed to follow some underlying distribution given by $\alpha$ (a uniform distribution for the purposes of this paper), while the true planet masses are given by our host star mass-dependent $M$–$R$ relation. The true planet masses and radii are both restricted to be above zero, and the true planet masses are restricted to be below the iron density constraint given above. We also introduce parameters corresponding to the true values of our observables: $K_{\text{true}}$, $r_{\text{true}}$, $M_{\text{true}}$, and $R_{\text{true}}$. $K_{\text{true}}$ is calculated from the planet’s period, inclination, eccentricity, true mass, and host star mass while $r_{\text{true}}$ is calculated from the planet’s true radius and host star
radius, as shown here:

\[
K_{\text{true}} = 0.6387 \left( \frac{P}{\text{day}} \right)^{-1/3} \left( \frac{M_{\text{true}}}{M_\odot} \right) \frac{\sin i}{\sqrt{1 - e^2}}
\times \left( \frac{M_{\text{true}}}{M_\odot} \right) + 3 \times 10^{-6} \left( \frac{M_{\text{true}}}{M_\odot} \right)^{-2/3} \text{ [m s}^{-1}]\]

\[
r_{\text{true}} = 0.009168 \left( \frac{R_{\text{true}}}{R_\odot} \right) \frac{R_{\text{true}}}{R_{\text{true}, \text{obs}}}.
\]

Our full hierarchical model, including our list of priors for these parameters, is shown in Equations (7) and (8). \(U\) represents a uniform distribution, \(N\) represents a normal distribution, \(T\) represents truncation bounds, and \(\sim\) represents “is distributed as”:

\[
K_{\text{obs}} \sim N(K_{\text{true}}, \sigma_K)
\]
\[
r_{\text{obs}} \sim N(r_{\text{true}}, \sigma_r)
\]
\[
M_{\text{obs}, \text{true}} \sim N(M_{\text{true}, \text{true}}, \sigma_M)
\]
\[
R_{\text{x, obs}} \sim N(R_{\text{x, true}}, \sigma_{R_{\text{x}}})
\]
\[
M_{\text{true}} \sim N(\mu, \sigma_M) T(0, M_{\text{true}, \text{true}}).
\]

3. Results

3.1. Model Fit

Table 1 summarizes the posteriors for our host star mass-dependent \(M-R\) relation, and they are visualized in Figure 3. We find that the three parameters encoding host star mass dependence \(\{C_0, \gamma_0, \sigma_0\}\) are each consistent with zero. This can be seen in the set of 2D contour plots shown in Figure 3, where the zero points for these three parameters (shown in blue) all lie within the 1\(\sigma\) contours. Further, the median values of \(C_0, \gamma_0\), and \(\sigma_0\) all closely match those found in Wolfgang et al. (2016) and are much less sensitive to the priors than the three host star mass parameters, suggesting that the \(M-R\) relation does not have a strong dependence on host star mass.

Though the host star mass parameters are all consistent with zero, it is possible that the current mass–radius data set is insufficient to robustly reveal any true host star mass dependencies in the planet \(M-R\) distribution. Our analysis constrains the extent to which the host star mass parameters can deviate from zero; all the posterior distributions in Figure 3 are more strongly peaked than the assumed priors. The distribution of \(\gamma_0\) peaks at \(\sim 0.25\), indicating a slight preference for a steeper slope in the \(M-R\) relation toward higher host star masses, but is only discrepant from 0 at a 0.43\(\sigma\) level. The host star mass-dependent scatter, \(\sigma_0\), shows the most evidence for being discrepant from 0, with a median of \(-4.18\%\) and 75% of the posterior samples being below zero. This would suggest a

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2 http://mc-stan.org
Table 1
Summary Statistics of the Six Population Level Parameters from Our Model Posteriors

| Parameter | 15.9% Median | 84.1% |
|-----------|-------------|-------|
| $C_\gamma$ | 0.245 | 2.97 | 3.52 |
| $C_\sigma$ | -1.77 | -0.26 | 0.67 |
| $\gamma_0$ | 1.16 | 1.29 | 1.43 |
| $\gamma_\sigma$ | -0.09 | 0.22 | 0.61 |
| $\sigma_0$ | 2.59 | 3.35 | 4.31 |
| $\sigma_\sigma$ | -12.06 | -4.18 | 1.39 |

larger scatter in the $M-R$ relation toward lower host star masses. However, given that none of the three parameters rules out no host star mass dependence, we stress that this is only what the current data set suggests with this model, and may not reflect underlying trends in the planet population.

Figure 4 shows our host star mass-dependent $M-R$ relation marginalized over the posterior distribution for two different host star masses, representing M stars (in red) and F stars (in blue). As previously stated, the model prefers a shallower $M-R$ relation with higher intrinsic scatter toward lower host star mass. This difference is most apparent at higher planet radii, with the spread of planet masses for rocky planets (<1.5 $R_\oplus$) being negligibly different between M and F host stars. Two of the most dominant limiting factors for this model are the small number of transiting planets with RV-measured masses with 2.5 < $R/R_\odot$ < 8.0, as well as the small number of those planets orbiting M dwarfs. This can be seen in Figure 4, where the majority of planets fall between 1.5 $R_\oplus$ and 2.5 $R_\oplus$, and orbit solar-mass stars. These two limitations are largely due to the selection criteria of RV followup, where brighter (up to $T_{\text{eff}} \sim 6200\, \text{K}$) stars and lower radii planets are preferred, in the quest to find and characterize small, potentially rocky planets.

3.2. Incident Flux Dependence

As M dwarfs can have luminosities hundreds to thousands of times fainter than their GK counterparts, planets with the same period will have vastly different incident fluxes from their host star depending on the type of star. Overall, given similar period distributions, we expect planets around M dwarfs to have much lower incident fluxes. Incident flux has clear physical connections to planet composition. For example, there is a lack of ultra-short period planets with radius <2 $R_\oplus$, thought to arise from photoevaporation (Sanchis-Ojeda et al. 2014) due to the extreme incident fluxes of these close-in planets. Evidence for photoevaporation can also be found in Kepler radius distribution, which has been shown to have a gap at $\sim$1.8 $R_\oplus$ (Fulton et al. 2017). Therefore, there is the possibility that any differences in the $M-R$ relation due to host star mass can be attributed to different distributions of incident flux. We briefly explore incident flux dependence in the $M-R$ relation and examine its effects on the host star mass dependence.

We model incident flux dependence into the $M-R$ relation in an analogous fashion to the host star mass dependence in our primary model. We introduce three new parameters {$C_f$, $\gamma_f$, $\sigma_f$} that modify the power-law slope, normalization, and intrinsic scatter and are scaled by the natural log of the incident flux. Equations (2)–(4) then become:

$$C = C_0 + \ln \left( \frac{M_e}{M_\odot} \right) C_i + \ln \left( \frac{S}{100 \, S_\odot} \right) C_f$$

$$\gamma = \gamma_0 + \ln \left( \frac{M_e}{M_\odot} \right) \gamma_i + \ln \left( \frac{S}{100 \, S_\odot} \right) \gamma_f$$

$$\sigma_M = \sqrt{\sigma_0^2 + \ln \left( \frac{M_e}{M_\odot} \right) \sigma_i + \ln \left( \frac{S}{100 \, S_\odot} \right) \sigma_f}$$

where the incident flux $S$ for a given planet is calculated using

$$\frac{S}{S_\odot} = \left( \frac{R_e}{R_\odot} \right)^2 \left( \frac{T_{\text{eff}}}{T_{\text{eff}, \odot}} \right)^4 \left( \frac{M_e}{M_\odot} \right)^{-2/3} \left( \frac{P}{P_\odot} \right)^{-4/3}.$$  

Our incident flux-dependent model is then a nine-parameter model, where each different combination of incident flux and host star mass gives a distinct $M-R$ relation.

The posterior distributions for {$C_f$, $\gamma_f$, $\sigma_f$} for our standard six-parameter model compared to the nine-parameter incident-flux-dependent model are shown in Figure 5. We find that the posterior distributions for $C_f$ and $\gamma_f$ widen when incident flux dependence is introduced. We also note that, while the original model prefers shallower slopes toward lower-mass stars, this trend is reversed with incident flux dependence. With the current data set, it is difficult to disentangle the effects of incident flux and host star mass on the $M-R$ relation. Both may affect planet composition, but currently there is no empirical proof for either affecting the $M-R$ relation and no evidence to prefer one dependence over the other.

While this model does include both host star mass and incident flux dependence, this extra set of parameters is not justified by the limited data set available. In Section 4.1, we demonstrate by information criterion considerations that this model is not strongly preferred over the six-parameter model. For this reason and for simplicity, we stick to the six-parameter host star mass-dependent $M-R$ relation as the primary relation in the following sections. We have tested the conclusions of the following sections with the incident flux-dependent model and have found the conclusions to be unchanged.

3.3. Exploring Other Data Sets

3.3.1. TTV Data Set

As a consistency check, we repeat the modeling above using the TTV data set instead of the RV data set, as well as the combined RV + TTV data set. We compile the TTV data set from the NASA Exoplanet Archive, with an identical cut on planet radius of $R < 8 \, R_\oplus$ as well as a cut on planet mass of $M < 25 \, M_\oplus$ to remove planets with physically unlikely masses. For TTV planets, we replace the radial velocity semi-amplitude with the rotation semi-amplitude and mass measurements, we assume the RV measurement. The TTV data set contains 63 planets compared to 88 RV planets, but has the advantage of more planets orbiting low-mass stars (13 planets with host star masses below 0.7 $M_\odot$, compared to six planets with RV), although they come from...
fewer distinct systems. Due to overlap between the two data sets, the combined RV + TTV data set has 143 planets.

The posterior distributions of the three host star mass-dependent parameters for our standard model fitted to the RV, TTV, and RV + TTV data sets are shown in Figure 6. With the TTV data set, we find that there is no evidence for host star mass dependence, as the posteriors for the three host star mass-dependent parameters are consistent with zero. The median value of $\gamma_s$ using the TTV data set is $-0.08$, which translates to a slightly shallower power-law slope toward higher host star masses, a trend opposite to what we found with the RV data set. Rather than revealing anything insightful about the underlying population, this is likely a result of our limited sample and further evidence for no host star mass dependence evident in the current sample of planets. The median value of $\sigma_s$ is $-1.88$ using the TTV data set, in the same direction as the RV data set. With the RV+TTV data set, the posteriors tighten, but are still centered at 0.

3.3.2. Potential TESS Data Set

The lack of evidence for host star mass dependence in the $M$–$R$ relation from fitting our model to the RV, TTV, and RV + TTV data sets begs the question of how much data would be needed to demonstrate a dependence if such a dependence were to exist. The biggest hope for the near future is the TESS survey and subsequent radial velocity follow-up, given the increased number of M dwarf planets TESS expects to find compared to Kepler. Using a simulated catalog of TESS planets from Sullivan et al. (2015), we construct a hypothetical future data set of transiting planets with RV-measured masses. We assume that each planet orbiting an M dwarf with a $V$ magnitude above some cut will have an RV mass measurement.
Figure 4. Host star mass-dependent $M - R$ relation for two characteristic stellar masses: 0.42 $M_\odot$, a typical M dwarf (red) and 1.2 $M_\odot$, a typical F dwarf (blue). The shaded region corresponds to the central 68% of masses drawn at a given radius with the $M - R$ relation parameters marginalized over their posterior distributions. The narrow feature below 1.5 $M_\odot$ is due to the pure iron maximum density restriction. The colored points represent the observed masses and radii of planets in our sample along with their reported uncertainties. The color of the points represents the host star mass as given by the color bar, with redder points having lower host star mass and bluer points having higher host star mass. Triangles are upper limits in mass as reported by the original authors, although we use more complete information in our modeling. Compared to the representative $M - R$ relation for F stars, the $M - R$ relation for M dwarfs has a shallower slope and higher intrinsic scatter. However, the posterior distributions for the host star mass-dependent parameters are wide enough to be consistent with no host star mass dependence.

Figure 5. Posterior distributions from our nine-parameter model, which includes both incident flux and host star mass dependence in the $M - R$ relation. Here we show only the host star mass-dependent parameters, $\{C_s, \gamma_s, \sigma_s\}$. The posteriors from the six-parameter model with only host star mass dependence are shown in red, while those from the nine-parameter model with incident flux dependence are shown in blue, with the 1$\sigma$ and 2$\sigma$ contours shown. We find that the posteriors for $C_s$ and $\gamma_s$ widen when incident flux dependence is included, but the posterior distribution for $\sigma_s$ is slightly tighter. Additionally, the peak of the $\gamma_s$ distribution shifts from positive to negative, which corresponds to preferring a steeper slope in the $M - R$ relation for low-mass stars when incident flux dependence is included.

Figure 6. Posterior distributions from our six-parameter model fit with three different data sets: RV (red), TTV (blue), and RV + TTV (green). As in Figure 5, we show only the host star mass-dependent parameters, $\{C_s, \gamma_s, \sigma_s\}$. We find that the posterior distributions agree between the three data sets, with the TTV data also showing no evidence for host star mass dependence.

We use a cut of $V_{mag} < 14$ for potentially rocky planets ($R_p/R_*$ < 2) and a cut of $V_{mag} < 13$ for those with gaseous envelopes ($2 < R_p/R_*$ < 8) to simulate the preference of many RV follow-up programs to characterize rocky planets. We perform similar cuts on the FGK sample, using a cut of $K_s, mag < 10$ for rocky planets and $K_s, mag < 7$ for gaseous planets. We then generate RV semi-amplitudes for each planet using two different models: the first using our six-parameter model with $\{C_0 = 3.0, C_i = 0.5, \gamma_0 = 1.3, \gamma_i = -0.5, \sigma_0 = 2.0, \sigma_i = -3.0\}$ and the second using a three-parameter model with no host star mass dependence $\{C = 3.0, \gamma = 1.3, \sigma = 2.0\}$. We make a further cut on the sample to ensure each planet has $K_{expected} > 1 \text{ m s}^{-1}$ (taken from Sullivan et al. 2015), in line with the sensitivity of RV spectrographs such as MAROON-X (Seifahrt et al. 2016). Finally, we generate normally distributed fractional errors for $K$, $r$, $R_s$, and $M_s$ using values similar to those found in the current sample of small planets with mass and radius measurements. These mean fractional errors are 0.2, 0.015, 0.03, and 0.1, respectively. The true values of these parameters are perturbed with these errors to simulate our observables.

We fit both data sets to each of the two models used to generate them: the six-parameter model with host star mass dependence and the three-parameter model without. We do this for 10 realizations of each data set. We then calculate the difference in the Watanabe–Akaikes information criterion (WAIC, further discussed in Section 4.1) for each data set between the two models as a means to test whether we can successfully distinguish which is the correct model. We find that this information criterion correctly prefers the model with no host star mass dependence for the data set generated without host star mass dependence in nine of 10 realizations. Further, compared to the smaller, current RV data set, the 1$\sigma$ uncertainties for the three host star mass-
dependent parameters decrease by a factor of \(\sim 3\). For the data set generated with host star mass dependence, we similarly find that 10 out of 10 realizations correctly prefer the model with host star mass dependence by at least 2\(\sigma\). We also note that seven out of 10 realizations exclude 0 for \(C_1\) at a 1\(\sigma\) level, with 10 out of 10 and four out of 10 realizations excluding zero for \(\gamma_1\) and \(\sigma_1\), respectively.

These results suggest that, while variance is still a factor, for this set of model parameters we would likely be able to find evidence for host star mass dependence with a future TESS data set. However, these results are contingent upon a few factors. While we generated 10 realizations of the radial velocity measurements, each realization used the same set of radii taken from the simulated catalog of Sullivan et al. (2015). The true variance between data sets should be higher if radii were generated alongside masses. We used a simplistic treatment of the errors on both the masses and the radii, as well as the radial velocity follow-up strategy. The real catalog may look very different from the ones studied here. Further, we only tested one set of model parameters, and only one model parameterization with host star mass dependence: there are many plausible combinations of these parameters that may describe the underlying \(M-R\) relation. Despite these limitations, we present this as one method of determining whether or not TESS may help us distinguish between \(M-R\) relations with and without host star mass dependence.

### 3.4. Planet Mass Distributions

The Kepler survey has provided a wealth of potential planets, with over 4500 planet candidates discovered around a wide variety of host stars. This large sample of planet candidates has enabled many exoplanet population studies, including how exoplanet systems may differ around various types of stars. A pair of papers by Mulders et al. (2015a, 2015b) (hereafter MPA15a,b) has provided evidence for increasing heavy element mass in planetary systems in short orbits (period between 2 and 150 days) around lower-mass stars. They find that the average heavy element mass in short orbits scales roughly inversely with host star mass, from 3.6 \(M_\oplus\) around F stars to 7.3 \(M_\oplus\) around M stars. This trend is at apparent odds with the observed trend of protoplanetary disk masses increasing with host star mass, derived from millimeter-wave observations (Andrews et al. 2013).

As with any analysis of the Kepler sample involving planet masses, the results in MPA15b are heavily reliant on the assumed \(M-R\) relation. They derived their results using both the deterministic \(M-R\) relation in Weiss & Marcy (2014) as well as the probabilistic \(M-R\) relation in Wolfgang et al. (2016), and found the trend to be robust to the assumed \(M-R\) relation. However, they use the best-fit deterministic relation from Wolfgang et al. (2016) rather than using the truly probabilistic formulation. More specifically, they did not take into account the intrinsic scatter of the \(M-R\) relation or the maximum density constraint derived from a pure iron core. This would have the effect of overestimating planet masses at small radii, which could skew the observed trend given that smaller planets have higher occurrence rates around lower-mass stars. Additionally, uncertainties in the adopted \(M-R\) relation parameters are not taken into account, which would cause the errors reported to be underestimated. In this section, we seek to test the robustness of this trend, and to see to what extent it depends on the assumed planet \(M-R\) relation. We will use the posteriors from our host star mass-dependent \(M-R\) relation for this purpose. We largely follow the methodology of MPA15b in deriving planet occurrence rates, which we briefly describe below.

For the purposes of this calculation, we use the Q1-16 Kepler Objects of Interest (KOIs; Mullally et al. 2015) catalog along with the corresponding list of target stars observed in these quarters. In order to isolate the effect of using our own \(M-R\) relation posteriors, we ensure that our KOI catalog completely matches that used in MPA15b. Giant stars are removed from the stellar catalog given their position in \(\log(g)-T_{\text{eff}}\) space according to the prescription in Ciardi et al. (2011). Stellar noise during a transit is derived from the Combined Differential Photometric Precision (CDPP) metric (Christiansen et al. 2012) at 3, 6, and 12 hr timescales for each observing quarter, which were downloaded from the MAST archive. For each star, we take the median of the CDPP of all quarters at each timescale and fit a power law. The CDPP for a given star and a given transit duration is then calculated using the star’s CDPP power-law fit.

The occurrence rate \(f_{\text{occ}}\) of a given planet with radius and orbital period \(R_p, P\) in a bin of stellar effective temperature \(T_{\text{eff}}\) is defined as the inverse of the number of stars in that bin for which the planet would be detectable \(N_s\), multiplied by a factor \(f_{\text{geo}}\) to account for the geometric probability for the planet to transit:

\[
\frac{1}{f_{\text{geo}}N_s(T_{\text{eff}}, R_p, P)}.
\]

We use \(T_{\text{eff}}\) bins that correspond to M, K, G, and F stars based on the recommendation of the Exoplanet Study Analysis Group 13, with upper bin edges of 3900, 5300, 6000, and 7300 K. For each star, the signal-to-noise ratio (S/N) of the planet is calculated given the transit depth, the number of transits, and the stellar noise that the system would have with the planet orbiting that star. The detection efficiency, given that S/N, is then calculated based on the number of transits and a cumulative gamma function that is empirically derived from planet transit injection and recovery tests (Christiansen et al. 2015). For detailed calculations, see Equations (2)–(9) in MPA15a, as well as MPA15b.

In order to calculate planetary heavy element mass as a function of host star mass, we first sample from our host star mass-dependent \(M-R\) relation to obtain a set of the six parameters \(\{C_0, C_1, \gamma_0, \gamma_1, \sigma_0, \sigma_1\}\). At the same time, we resample the KOI population by bootstrapping (drawing \(N_p\) planets from the sample with replacement). Then, for each planet we sample \(N_p\) masses using the set of host star mass-dependent \(M-R\) relation parameters, as well as the planet’s radius and host star mass from normal distributions. To retrieve the heavy element mass instead of the total mass, we cap the mean of the probabilistic \(M-R\) relation at \(22 M_\oplus\) in order to replicate the measured median core masses of giant planets (Miller & Fortney 2011). Each mass is sampled from a truncated normal distribution where the mean and standard deviation are given by our \(M-R\) relation, and the truncation is due to the maximum density of a planet with a pure iron core.

For each sample of a planet’s mass, we calculate the planet’s
In order to test the robustness of this trend in heavy element mass, we would like to know to what extent the increase at lower host star mass is dependent on our assumptions. First, we test the assumption that heavy element mass is equivalent to planet mass up to a limiting mass, chosen to be 22 $M_\oplus$ in MPA15b due to the median core mass of giant planets. Recent evidence shows that the correlation between heavy element mass and total mass for giant planets roughly scales as $M_e \propto \sqrt[3]{M}$, based on thermal and structural evolution models (Thorngren et al. 2016). This correlation is fit to masses above $\sim 20 M_\oplus$. We adopt this scaling for planets with $R > 4 R_{\oplus}$, which roughly corresponds to a mass of 20 $M_\oplus$ in our model, with $M_e \propto M$ below 4 $R_{\oplus}$. To smoothly transition between these two functions, we adopt a logistic function with a transition point at 4 $R_{\oplus}$ and a scale parameter of 1. For giant planets with $R > 10 R_{\oplus}$, the $M-R$ relation changes to become approximately flat (Chen & Kipping 2017), and so we assume the heavy element mass is flat past this radius.

The second assumption we test is the period cut we apply on the planet sample. A planet with a period of 150 days has a significantly different irradiation depending on what type of star it is orbiting. By applying a cut of 2–150 days regardless of host star mass, we are applying an uneven cut in planet irradiation. To account for this, we use the location of the snowline as a proxy for planet irradiation and use the scaling found in Ida & Lin (2005), where $d_{\text{snowline}} = 2.7 a u (M_*/M_\odot)^{0.5}$. This is equivalent to a linear scaling of the period with host star mass, by Kepler’s third law. We take 2–150 days to be the period bounds for a host star with 1 $M_\odot$, and scale both the inner and outer bound by $M_*/M_\odot$. For example, for a planet with a host star mass of 0.5 $M_\odot$, it is only included in the sample if its period falls between 1 and 75 days.

For these two cases (where we also include the $M_2$ scaling in the second case), along with our initial assumptions, we calculate the average heavy element mass per star in bins of heavy element mass, for each of our four host star mass bins. We then integrate this curve along the heavy element mass axis to obtain the total average heavy element mass per star. The results are shown in Table 2. We find that scaling the heavy element mass as $\sqrt[3]{M}$ decreases the total heavy element mass across all four host star mass bins. Scaling the period bounds with the location of the snowline decreases the heavy element mass for M and K stars, making the slope shallower. Despite changing our assumptions for the heavy element mass and period scaling, the trend of increasing heavy element mass toward lower host star mass is still present. Figure 8 shows the contribution of different heavy element mass regimes to the total heavy element mass per star; the integral of each curve in

| Assumptions | M | K | G | P |
|-------------|---|---|---|---|
| (1) MPA15b  | 11.2 $^{+4.9}_{-3.6}$ | 7.4 $^{+1.6}_{-1.3}$ | 7.0 $^{+1.2}_{-1.0}$ | 5.4 $^{+1.1}_{-0.9}$ |
| (2) $M_2 \propto \sqrt[3]{M_p}$ | 10.6 $^{+3.4}_{-3.1}$ | 6.7 $^{+1.2}_{-1.0}$ | 6.2 $^{+0.9}_{-0.8}$ | 4.9 $^{+0.9}_{-0.8}$ |
| (3) Snowline scaling + 2 | 8.4 $^{+2.3}_{-2.2}$ | 5.9 $^{+1.1}_{-1.0}$ | 6.1 $^{+1.0}_{-0.9}$ | 5.0 $^{+0.9}_{-0.9}$ |
| (4) Incident flux dep. + 3 | 8.4 $^{+2.4}_{-2.2}$ | 5.7 $^{+1.3}_{-1.2}$ | 5.6 $^{+1.2}_{-1.1}$ | 4.7 $^{+1.1}_{-1.0}$ |
| (5) TTV + 3 | 5.9 $^{+1.9}_{-1.6}$ | 3.9 $^{+0.9}_{-0.7}$ | 3.9 $^{+0.9}_{-0.8}$ | 3.1 $^{+0.7}_{-0.7}$ |
| (6) TTV + 4 | 7.0 $^{+2.3}_{-2.2}$ | 4.4 $^{+1.2}_{-1.1}$ | 4.4 $^{+1.2}_{-1.1}$ | 3.8 $^{+1.2}_{-1.1}$ |
| (7) RV+TTV + 3 | 7.6 $^{+3.1}_{-2.9}$ | 5.3 $^{+1.4}_{-1.2}$ | 5.2 $^{+1.4}_{-1.2}$ | 4.4 $^{+1.3}_{-1.2}$ |
| (8) RV+TTV + 4 | 7.6 $^{+3.0}_{-2.8}$ | 5.4 $^{+1.3}_{-1.0}$ | 5.5 $^{+1.0}_{-0.9}$ | 4.6 $^{+0.9}_{-0.9}$ |

These results are shown in Figure 7. For these results, $N_m = 25$ and $N_p = 500$. The data shown are the median over $N_p$ samples, with error bars again representing the 15.9 and 84.1 percentiles. For each $T_\text{eff}$ bin, we plot the corresponding host star mass by taking the median of the masses of each star in the bin, with error bars again representing the 15.9 and 84.1 percentiles. We find that our results are consistent with the findings of MPA15b. The average planetary heavy element mass increases from $5.4^{+1.1}_{-0.9} M_\oplus$ around F stars to $11.7^{+4.9}_{-3.6} M_\oplus$ around M stars. With respect to MPA15b, our error bars are roughly twice as large and our average masses slightly higher, but the overall trend is consistent.

With the assumed model of the host star mass-dependent $M-R$ relation, our results are consistent within error with neither a scaling similar to the scaling of protoplanetary disk masses with host star mass, nor a flat trend. We fit a power law to the results and find a power-law index of $-0.9$. We also fit a power law to each of the $N_p$ individual samples of the planet population and $M-R$ relation parameters. These are shown as faded lines in Figure 7 and give a rough visualization of the spread of this trend with host star mass.

We have tested several combinations of parameters to see what would be consistent with a flat trend of planetary heavy element mass with host star mass. Keeping $\{C_0, \gamma_0, \sigma_0\}$ fixed to their median values as listed in Table 1, we find that the following combination of $\{C_0 = 1, \gamma_0 = 1.6, \sigma_0 = -5\}$ is sufficient to produce a flat trend. This results in a flat $M-R$ relation for planets with $R > 1.3 R_\oplus$ orbiting M dwarf ($M \sim 0.42 M_\odot$) stars. As seen in our model posteriors in Figure 3, this set of parameters is well outside the 2$\sigma$ contours and is heavily disfavored by the current data set.
average heavy element mass per star, binned and scaled by heavy element mass. The red curve is for planets around M dwarf stars, with the green, blue, and purple curves representing K, G, and F stars respectively. The shaded region shows the 68% region, obtained through bootstrap resampling and sampling of the $M-R$ relation parameters. These results are for the third line in Table 2, where we scale the heavy element mass as $M_{\text{p}}^2$ and scale the period bounds with the location of the snowline. The integral under each curve gives the total heavy element mass reported in Figure 7. We discover around M dwarfs, as well as the limited radius to which we are limited by the small number of planets included in Figure 8. We can see that the increase in the average heavy element mass per star, binned and scaled by heavy element mass, we also calculate mass distributions for planets around M dwarfs that planet occurrence is higher for planets with $M_{\text{p}} < 20 M_{\oplus}$. We find that the increased planetary heavy element mass around M dwarfs at short periods is due to higher occurrence of planets with $M_{\text{p}} < 20 M_{\oplus}$.

Figure 8 gives the total heavy element mass shown in Figure 7. Both the snowline and heavy element mass scalings are included in Figure 8. We can see that the increase in the average heavy element mass for M dwarfs is due to planets with heavy element masses up to $\sim 16 M_{\oplus}$ contributing more mass than those around F, G, and K stars. Only at masses higher than $16 M_{\oplus}$ do planets contribute more heavy element mass for F, G, and K stars compared to M stars.

Using a similar methodology to calculating the planet heavy element mass, we also calculate mass distributions for Kepler planets around M stars as well as FGK stars, shown in Figure 9. For this calculation, we use the period range scaling with the snowline, and we bin by planet mass as well as stellar $T_{\text{eff}}$. We find that planet occurrence is higher for planets with $3 < M/M_{\oplus} < 32$ around M dwarfs than FGK dwarfs. Beyond $32 M_{\oplus}$, we are limited by the small number of planets discovered around M dwarfs, as well as the limited radius range to which we fit the $M-R$ relation. Below $3 M_{\oplus}$, the Kepler sample is thought to be incomplete for the period range under consideration (Christiansen et al. 2015).

We find an occurrence rate of $1.03^{+0.40}_{-0.24}$ planets per M dwarf with $M < 10 M_{\oplus}$ and $1 < P < 100$ day. Compared to the HARPS M dwarf sample for the same mass and period bounds, which found 0.88 planets per star (Bonfils et al. 2013), our derived result is a factor of 1.17 higher but consistent within 1σ. We find $0.30^{+0.19}_{-0.12}$ planets per M dwarf with $10 < M/M_{\oplus} < 100$ and $1 < P < 100$ day, which is a factor of 6 higher than the HARPS result of 0.05. We attribute this higher frequency of massive planets to our sampling of the masses of each planet, which has the effect of flattening the distribution. Since our masses are more uncertain than directly measured RV masses, planets with smaller radii at higher occurrence rates will have some significant posterior probability at higher masses.

Figure 8. Average heavy element mass per star, binned and scaled by heavy element mass. The red curve is for planets around M dwarf stars, with the green, blue, and purple curves representing K, G, and F stars respectively. The shaded region shows the 68% region, obtained through bootstrap resampling and sampling of the $M-R$ relation parameters. These results are for the third line in Table 2, where we scale the heavy element mass as $M_{\text{p}}^2$ and scale the period bounds with the location of the snowline. The integral under each curve gives the total heavy element mass reported in Figure 7. We discover around M dwarfs, as well as the limited radius to which we are limited by the small number of planets included in Figure 8. We can see that the increase in the average heavy element mass per star, binned and scaled by heavy element mass, we also calculate mass distributions for planets around M dwarfs that planet occurrence is higher for planets with $M_{\text{p}} < 20 M_{\oplus}$.

Figure 9. Mass distributions for Kepler planets with short-period orbits around M dwarfs (red) and FGK dwarfs (blue). Distributions were obtained by sampling from the probabilistic host star mass-dependent $M-R$ relation posteriors. Error bars correspond to the central 68% of occurrence rates drawn at a given mass bin. The gray shaded regions indicate incompleteness of the sample below $3 M_{\oplus}$, the Kepler sample is incomplete, and above $32 M_{\oplus}$, the $M-R$ relation we fit has to be extrapolated. The mass distribution for planets around M dwarfs peaks at a lower mass and has higher occurrence for less massive planets than the distribution for planets around FGK dwarfs.

3.5. Minimum-mass Extrasolar Nebula

The minimum-mass solar nebula (MMSN) is an estimate of how much mass must have been in the solar protoplanetary disk to form the planets that had they formed in situ. Kuchner (2004) first showed how one could extend the concept of the MMSN to exoplanets, using a sample of RV planets in order to calculate the minimum-mass extrasolar nebula (MMEN). More recently, Chiang & Laughlin (2013) calculated the surface density profile of the MMEN using the Kepler sample of transiting planets. They found a similar power-law slope compared to the MMSN ($-1.6$ and $-1.5$, respectively) and an overdensity compared to the MMSN of about a factor of five. However, their primary result has several limitations. The $M-R$ relation they used is that derived in Lissauer et al. (2011): a power-law fitted to six solar system planets, which has since been greatly improved upon by utilizing the sample of transiting exoplanets with mass measurements. The host star mass for each individual Kepler planet is not taken into account, either for the calculation for orbital radius or for the $M-R$ relation. Finally, the occurrence rate of each individual planet is not factored in. Here, we redo the MMEN calculation, making several improvements over the initial work.

We use the Kepler occurrence rates, planet heavy element mass, and host star mass samples as described in Section 3.3 for the following calculation. For each sample, the solid surface density of the planet is given by

$$\Sigma_{\text{solid}} = \frac{M_{\text{p}}}{2\pi a^2}. \quad (14)$$

In Figure 10, we plot a 2D weighted histogram of the MMEN surface density of solids and semimajor axis for M, K, G, and F dwarfs separately, where the weights are the occurrence rates of the KOIs. We find that the surface density of solids at short orbits is higher for M dwarfs than it is for FGK dwarfs. Between F, G, and K dwarfs there is little difference, mirroring Figure 8. Compared to Chiang & Laughlin (2013), we find that the surface density profile of the MMEN for FGK dwarfs exhibits a shallower slope with a
power-law index ranging from −0.9 to −1.2, whereas the power-law index for M dwarfs, −1.8, more closely matches their result of −1.6. We find a similar normalization for the surface density of solids, which is several factors higher than that of the MMSN. While the MMEN suggests a higher surface density of solids at short orbits for disks around M dwarfs, migration likely plays a pivotal role in this trend and thus the MMEN may not reflect the true initial protoplanetary disk surface density profile (Mulders et al. 2015b). This is supported by recent work from Raymond & Cossou (2014), which showed that the surface density profiles of multiple planet systems have a wide range of power-law slopes.

4. Discussion

4.1. Model Selection

In order to test whether we are justified in adding host star mass dependence to the $M-R$ relation, we would like to estimate the predictive accuracy of models with and without host star mass dependence. Cross-validation is a robust method of evaluating the predictive accuracy of a model, but requires multiple model fits and is computationally expensive. Luckily, several alternatives exist which approximate cross-validation and are simpler to compute. For the purposes of this paper, we choose the WAIC to be our predictive measure of choice (Watanabe 2012). Similar to other information criteria, the WAIC relies on computing the log predictive density for the data sample used to fit the model, and applying a bias correction to estimate the log predictive density for a hypothetical new data sample. Unlike alternatives such as the Akaike information criterion (AIC) and deviance information criterion (DIC), the WAIC is fully Bayesian in the sense that it averages over the posterior distributions, rather than relying on point estimates (Gelman et al. 2014). Given that the posteriors for our six parameters are not all normally distributed ($\sigma_s$ in particular), this makes the WAIC an appealing choice.

We compute the WAIC for seven different models: our six-parameter host star mass-dependent $M-R$ relation, a three-parameter model with no host star mass dependence, three four-parameter models that correspond to adding one of $C_s$, $\gamma_s$, and $\sigma_s$ to the three-parameter model, our six-parameter incident flux-dependent model, and our nine-parameter model with both incident flux and host star mass dependence. We then calculate the difference in the WAIC between the six-parameter model and each other model, along with the error in the difference. We repeat this for the TTV and RV + TTV data sets discussed in Section 3.3. Our results are shown in Table 3. Lower values for a given model indicate better predicted out-of-sample fits compared to the six-parameter model. We find that generally, models with fewer parameters have a higher predicted out-of-sample fit compared to our standard six-parameter model. Our results for the incident flux dependent model vary between data sets, with incident flux dependence being most strongly favored over host star mass dependence for the combined RV + TTV data set, but slightly disfavored for the TTV-only data set. For the RV and TTV data sets, the nine-parameter model with both host star mass and incident flux dependence is disfavored, whereas for the combined RV + TTV data set, the nine-parameter model is slightly favored but with a large error in the difference. Given that the error of these estimates is often comparable to the calculated differences, nothing definitive can be said about which model should be preferred. This strengthens our conclusion that there is no evidence in the current $M-R$ data set for host star mass dependence in the $M-R$ relation.

4.2. Limitations

4.2.1. Data Set

The limited number of transiting planets with RV mass measurements and their uneven occupation of parameter space is perhaps the most significant factor limiting this work. A total of 48% of planets in our sample have a host star mass between 0.9 and 1.1 $M_\odot$, and only six planets have a host star mass below 0.7 $M_\odot$. Given our model parameterization, where host star dependence scales as $\ln M_\ast$, the difference between the coefficient of the host star mass-dependent parameters is $\sim$0.2 between 0.9 and 1.1 $M_\odot$, but $\sim$0.6 between 0.5 and 0.9 $M_\odot$. Under this scaling, the $M-R$ relation is most significantly different for planets around M dwarf stars, but we only have six such planets. Furthermore, 58% of planets in our sample have radii between 1.5 and 4.0 $R_\oplus$. More planets with radii between 4.0 and 8.0 $R_\oplus$ would further constrain the slope of the power law and allow a more substantive investigation into whether the

![Figure 10. Two-dimensional weighted histogram of the MMEN for different types of stars, using the Kepler sample. Samples are drawn for each KOI using our host star mass-dependent $M-R$ relation posteriors, and weighted by the occurrence rate of the KOI. The solid black line indicates the cumulative center of each orbital radius bin, and the dashed and dot-dashed lines show the results from the Chiang & Youdin (2010) MMSN and the Chiang & Laughlin (2013) MMEN. We find that the slope of the surface density profile of the MMEN is steeper for M dwarfs, resulting in more mass at short orbits around them.](image)

| Table 3 | Difference in Watanabe–Akaike Information Criterion (WAIC) Compared to the Standard Six-parameter Host Star Mass-dependent Model, Along with the Error in the Difference |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Model   | RV                          | TTV                         | RV+TTV                                |
| No $M_\ast$ dependence | −14.8 ± 4.6 | −8.4 ± 2.4 | −17.3 ± 4.6 |
| $C_s$    | −14.8 ± 2.9 | −5.2 ± 2.1 | −17.0 ± 2.7 |
| $\gamma_s$ | −11.7 ± 2.1 | −7.4 ± 2.2 | −11.9 ± 2.6 |
| $\sigma_s$ | −7.5 ± 4.0 | −2.8 ± 1.6 | −9.4 ± 3.7 |
| $C_s$, $\gamma_s$, $\sigma_s$ | −5.9 ± 3.8 | 2.5 ± 2.2  | −8.0 ± 3.3 |
| $C_s$, $C_\ell$, $\gamma_s$, $\sigma_s$, $\sigma_f$ | 4.3 ± 3.2 | 40.1 ± 6.3 | −1.7 ± 4.7 |

Note. Results are shown for three data sets and six different models. Negative numbers favor the model in question over the six-parameter model. We find that models with fewer parameters are favored.
scatter of the \( M-R \) relation changes with radius. These problems are twofold: first, there is the detection of transiting planets within our parameters of interest and, second, the RV follow-up of these planets. Both issues need to be addressed, although mass–radius studies are more sensitive to the number of planets subjected to RV follow-up given the wealth of data provided by \textit{Kepler}.

Fortunately, there are several surveys and experiments scheduled to become operational in the near future that will alleviate these issues. \textit{TESS}, scheduled to launch in 2018, is an all-sky NASA-sponsored mission designed to monitor \( \sim200,000 \) of the brightest nearby stars. \textit{TESS} is expected to find \( \sim400 \) planets with \( R < 2 \, R_{\oplus} \) hosted by M dwarfs (Sullivan et al. 2015), compared to \( \sim130 \) found by \textit{Kepler}. On the RV follow-up side, several high-resolution spectrographs (e.g., MAROON-X, SPIRou, ESPRESSO) are under development and planned to coincide with \textit{TESS}. There are also efforts currently underway to search for planets around nearby M dwarfs (e.g., \textit{CARMENES}, \textit{MEarth}, \textit{IR RV} spectrograph). Since \textit{TESS} is scheduled to observe the brightest nearby M dwarfs, RV follow-up of planets around M dwarfs in the era of \textit{TESS} will be more feasible. As shown in Section 3.3, a future data set of planets from \textit{TESS} with radial velocity mass measurements may be able to distinguish between \( M-R \) relations with and without host star mass dependence.

4.2.2. Physical Basis

Given that we are empirically fitting the \( M-R \) relation, there is no shortage of parametrizations we could have considered. Much like the decision to characterize the \( M-R \) relation as a power law, the decision to scale the \( M-R \) relation parameters by the natural log of host star mass was based on simplicity and intuitive understanding, rather than any physical basis. There is no reason to think that this scaling should be physically preferred over a power law scaling, for instance. Our motivation for this paper was to allow for host star mass dependence and see how much information is in the current data set. For this reason, we do not consider alternative parameterizations.

Ultimately, \( M-R \) relations should move away from strictly empirical relations and toward a physically motivated distribution. One possible step in this direction is to use a mixture model. Results from the \textit{Kepler} survey show a gap in the radius distribution of small planets at short orbital periods (Fulton et al. 2017). This bimodal distribution is thought to arise from two separate planet populations: those with significant H/He envelopes and those without. The gap is consistent with evidence that planets below \( \sim1.6 \, M_{\oplus} \) having densities consistent with a purely rocky composition (Weiss & Marcy 2014; Dressing et al. 2015; Rogers 2015). Modeling these two planet populations separately using a mixture model would give each planet a probability of falling into either population, depending on its radius. This would be an improvement over current efforts that model a break in the power law around \( 1.6 \, M_{\oplus} \), as it would account for the overlap between these two populations.

4.2.3. Mass Conditioned on Radius

In this paper we parameterize the \( M-R \) relation in terms of mass as a function of radius, in order to directly apply the relation to the \textit{Kepler} sample of planets, which generally lack mass measurements. Framing the \( M-R \) relation as \( M(R) \) allows mass estimates of \textit{Kepler} planets to be readily obtained. Furthermore, the radius measurements for the planets in our sample are generally much more precise than the mass measurements. However, one could argue that mass is the more fundamental physical quantity and the relation should be cast as radius as a function of mass. An \( R(M) \) relation would also be applicable to the sample of microlensing planets, which have mass constraints but no radius measurements. If desired, one could easily obtain radius as a function of mass by switching mass and radius in Equation (1) and fitting for a new set of parameters. Ultimately, a combined joint mass–radius distribution would solve this issue, which would allow one to obtain either relation.

4.2.4. Selection Effects

In using the sample of transiting planets with RV mass measurements, we are subject to a host of poorly characterized selection effects. For example, the choice to follow up a \textit{Kepler} planet is not completely transparent and, while guidelines exist, it is often ultimately a human decision. Planets orbiting M dwarfs are typically not favored to be chosen for RV follow-up, due to their intrinsic faintness and the desire to characterize planets around solar analogs. Furthermore, upper limits when a planet is not detected are not always published, which could lead to a bias toward more massive planets as they are more likely to be detected. The heterogeneity of the data set also poses a problem: some planets in the sample were discovered by both RV and transit methods independently, and we do not restrict our catalog to one specific survey or program such as \textit{Kepler}. Finding a way to model these selection effects is a difficult task, but necessary to remove any potential biases.

4.2.5. Incident Flux Dependence

In Section 3.2 we explored adding incident flux dependence to the \( M-R \) relation. We found that, with the current data set, we are unable to distinguish between the need for host star mass dependence and the need for incident flux dependence. This is in agreement with previous work by Weiss et al. (2013), who found weak dependence of planet radius on incident flux for small planets. Despite this, incident flux is a key parameter to take into account when constraining planet composition distributions. The incident flux on a planet can affect whether or not a planet retains its atmosphere, and its thermal evolution, particularly for those in close-in orbits (Scalo et al. 2007). An increased sample of planets with mass and radius measurements will warrant revisiting this incident flux dependence. Additionally, the sample of transiting planets with TTV mass measurements will provide another avenue to explore this dependence, given that TTV techniques prefer planets on longer periods than RV methods (Mills & Mazeh 2017).

5. Conclusion

We have modeled host star mass dependence in the planet \( M-R \) relation by introducing three new parameters to the probabilistic \( M-R \) relation first established in Wolfgang et al. (2016). We fit the model to the current sample of transiting planets with RV-measured masses and find that the host star mass-dependent parameters are consistent with zero and there is no strong evidence for host star mass dependence in the \( M-R \) relation. We have tested the observed trend in Mulders et al. (2015b) of increasing planetary heavy element mass toward
lower-mass stars and have found this trend to be robust against many of their assumptions. This trend also manifests itself in the MMEN, with the surface density pro
many of their assumptions. This trend also manifests itself in lower-mass stars and have found this trend to be robust against
upcoming surveys such as TESS and subsequent RV follow-up.

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