Thermal conductivity of gasholders during gas storage

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Abstract. A dynamic problem of gasholders during gas storage has been considered. Each layer of gasholders has its own physical properties, and the heat transfer coefficient depends on the coordinate system. The solution of a dynamic problem is framed using a finite integral transform technique.

1. Introduction

To eliminate the daily non-uniformity of gas consumption, vessels are built near cities for storing the entire excess gas in the night hours to return it to the gas distribution network of the city. Gasholders are used for this purpose. Gasholders are the large-capacity vessels designated for gas storage [1]. Vessels might be bowl shaped (vertical and horizontal) and spherical [2]. Gasholders may be low-pressure and high-pressure vessels [3]. Depending on the ground location, there are terrestrial and underground gasholders [1].

Gases obtain peculiar properties, which significantly affect their storage arrangement. The paramount ones are their flammability and explosiveness [2]. Gas storage is effected at low temperatures. Constant low temperature inside the gasholder is maintained with a chiller. Gasholders are heat insulated to reduce any heat influx through their walls, lid and bottom. Because of the temperature increase inside the gasholder during storage, gas evaporation occurs. Besides, gases have large coefficients of volumetric expansion, i.e. their volume sufficiently expands with temperature increase. Some amount of water is dispersed in gases with its solubility increase with the temperature growth [4]. Consequently, temperature plays a significant role in gas storage.

Therefore, it is necessary to calculate heat distribution of the products stored in the gasholder. On the basis of the laws of heat conduction, we can define the non-uniform temperature distribution in each layer of a gasholder, given that the layers are made of solid materials and have their own physical properties. Thermal conductivity of one-component spherical constructions is considered in detail in papers [5] – [9]. In problem solutions of these papers, the heat conduction coefficient is a constant value. Should non-uniform layers of gasholders be considered, their heat conduction coefficient generally depends on coordinates.

2. Methods

In case the temperature pattern is spherically symmetrical, and the heat conduction coefficient is $r$ - coordinate dependant, equation has the form:
\[
c_j \rho_j \frac{\partial T_j(r,t)}{\partial t} = \lambda_j(r) \frac{\partial^2 T_j(r,t)}{\partial r^2} + \left( \frac{\partial \lambda_j(r)}{\partial r} + \frac{2\lambda_j(r)}{r} \right) \frac{\partial T_j(r,t)}{\partial r} + Q_j(r,t),
\]

(1)

Here \( T_j(r,t) \) is the temperature field of \( j \)-layer of the sphere; \( t \) is the time; \( c_j \) is the specific thermal capacity of \( j \)-layer of the sphere; \( \rho_j \) is the density of \( j \)-layer of the sphere; \( \lambda_j(r) \) is the thermal conductivity coefficient of \( j \)-layer of the sphere; and \( Q_j(r,t) \) is the internal heat source.

Let us present equation (1) in the following form:

\[
c_j \rho_j \frac{\partial T_j(r,t)}{\partial t} = \lambda_j(r) \frac{\partial^2 T_j(r,t)}{\partial r^2} + \left( \frac{\partial \lambda_j(r)}{\partial r} + \frac{2\lambda_j(r)}{r} \right) \frac{\partial T_j(r,t)}{\partial r} + Q_j(r,t),
\]

having introduced the indicators \( A_j(r) = \lambda_j(r); B_j(r) = \frac{\partial \lambda_j(r)}{\partial r} + \frac{2\lambda_j(r)}{r}, \) we get:

\[
\frac{c_j \rho_j}{A_j(r)} \frac{\partial T_j(r,t)}{\partial t} = \frac{\partial^2 T_j(r,t)}{\partial r^2} + \frac{B_j(r) \partial T_j(r,t)}{A_j(r)} + \frac{1}{A_j(r)} Q_j(r,t)
\]

(2)

To solve equation (2), let us set the initial and boundary conditions. On the outer surface of the spherical vessel, we set the temperature of the environment and the law of convective heat exchange between the body surface and the environment; we set the conditions of ideal thermal contact at the interface of \((j-1)\) and \(j\)-layers, and the heat flux density in the vessel centre [8]:

\[
T_j(r,0) = f_j(r),
\]

(3)

\[
\lambda_j(r) \frac{\partial T_j(r,t)}{\partial r} \bigg|_{r=R_j} = q_c,
\]

(4)

\[
T_{j-1}(r,t) \bigg|_{r=R_{j-1}} = T_j(r,t) \bigg|_{r=R_{j-1}},
\]

(5)

\[
\lambda_{j-1}(r) \frac{\partial T_{j-1}(r,t)}{\partial r} \bigg|_{r=R_{j-1}} = \lambda_j(r) \frac{\partial T_j(r,t)}{\partial r} \bigg|_{r=R_{j-1}},
\]

(6)

\[
\lambda_m(r) \frac{\partial T_m(r,t)}{\partial r} \bigg|_{r=R_m} + \alpha_m \left[ T_m(r,t) - T_{\text{ambient}}(t) \right] \bigg|_{r=R_m} = 0
\]

(7)

Here \( \lambda_j(r) \), \( \lambda_m(r) \) are the heat transfer coefficients of \( j \)-layer and last layer of the spherical gas holder, \( (j=1,m) \); \( q_c \) is the heat flux from the internal heat source; \( \alpha_m \) is the heat transition coefficient of the \( m \)-layer of the gas holder; and \( T_{\text{ambient}}(t) \) is the temperature of the ambient air.

3. Results

The problem solution (2) - (7) will be presented as the total of a steady-state and a dynamic problem [7]:

\[
T_j(r,t) = S_j(r) + P_j(r,t),
\]

here \( S_j(r) \) is the solution of a steady-state problem; and \( P_j(r,t) \) is the solution of a dynamic problem.
The solution of a steady-state problem for a spherical and a cylinder-shaped multi-layered gas holder with non-uniform layers is considered in paper [10].

The solution of a dynamic problem will be framed using a finite integral transform technique [7], [8]. To exclude the $r$-coordinate, along which the material properties of the spherical gas holder are changing, the image transfer equation [7] will be used:

$$ U_j(\mu_j, \tau) = \int_{R_{j-1}}^{R_j} \rho_j(r) P_j(r, \tau) W_j(r, \mu_j) dr, \quad (8) $$

here $\rho_j(r)$ is the weight function; $W_j(r, \mu_j)$ is the kernel of integral transformation; and $\mu_j$ is the eigen value of the problem.

The weight function $\rho_j(r)$ is defined from equation [7]:

$$ \frac{d \rho_j(r)}{dr} - \frac{B_j(r)}{A_j(r)} \rho_j(r) = 0 \quad (9) $$

Solving equation (9), we get:

$$ \rho_j(r) = \exp \left( \int_{R_{j-1}}^{r} \frac{B_j(r)}{A_j(r)} dr \right) $$

The kernel of integral transformation $W_j(r, \mu_j)$ is the solution of the auxiliary Sturm-Liouville problem [8]:

$$ \frac{d^2 W_j(r, \mu_j)}{dr^2} + \frac{B_j(r)}{A_j(r)} \frac{d W_j(r, \mu_j)}{dr} + \mu_j^2 W_j(r, \mu_j) = 0, \quad (10) $$

$$ \lambda_j(r) \left. \frac{d W_j(r, \mu_j)}{dr} \right|_{r=R_{j-1}} = q_c, \quad (11) $$

$$ W_{j-1}(r, \mu_j) \left|_{r=R_{j-1}} = W_j(r, \mu_j) \left|_{r=R_{j-1}} \right. \right. \quad (12) $$

$$ \lambda_{j-1}(r) \left. \frac{d W_{j-1}(r, \mu_j)}{dr} \right|_{r=R_{j-1}} = \lambda_j(r) \left. \frac{d W_j(r, \mu_j)}{dr} \right|_{r=R_{j-1}} \right. \right. \quad (13) $$

$$ \lambda_n(r) \left. \frac{d W_n(r, \mu_j)}{dr} \right|_{r=R_{j-1}} + \alpha_n \left[ T_n(r, \mu_j) \right]_{r=R_{j-1}} = 0, \quad (14) $$

Solving equation (10), we get:

$$ W_j(r, \mu_j) = C_{1j} e^{\frac{r}{\lambda_j(r)} - B_j(r) - \int_{R_{j-1}}^{r} \lambda_n(r) \rho(r) dr} + C_{2j} e^{\frac{r}{\lambda_j(r)} - B_j(r) + \int_{R_{j-1}}^{r} \lambda_n(r) \rho(r) dr}, \quad (15) $$

here coefficients $C_{1j}$ and $C_{2j}$, as well as the $\mu_j$-values are determined through the boundary conditions (11)-(14).

For the image transfer, it is necessary to apply formula (8) termwise to equation (2) and initial condition [7].

Time derivative image is:
\[
\frac{dU_j(\mu_j,t)}{dt} = \int_{r_{j-1}}^{r_j} \rho_j(r) \left( \frac{c_j \rho_j}{A_j(r)} \frac{\partial P_j(r,t)}{\partial t} \right) W_j(r,\mu_j) dr. \tag{16}
\]

Coordinate derivative image is as follows. Let us twice integrate partially:

\[
\int_{r_{j-1}}^{r_j} \rho_j(r) \left( \frac{\partial^2 T_j(r,t)}{\partial r^2} + \frac{B_j(r)}{A_j(r)} \frac{\partial T_j(r,t)}{\partial r} \right) W_j(r,\mu_j) dr =
= \rho_j(r) \frac{\partial T_j(r,t)}{\partial r} W_j(r,\mu_j) \bigg|_{r_{j-1}}^{r_j} - \int_{r_{j-1}}^{r_j} \rho_j(r) \frac{\partial T_j(r,t)}{\partial r} dW_j(r,\mu_j) dr =
= \rho_j(r) \frac{\partial T_j(r,t)}{\partial r} W_j(r,\mu_j) \bigg|_{r_{j-1}}^{r_j} - \rho_j(r) \frac{dW_j(r,\mu_j)}{dr} T_j(r,t) \bigg|_{r_{j-1}}^{r_j} + \mu_j^2 \int_{r_{j-1}}^{r_j} \rho_j(r) T_j(r,t) W_j(r,\mu_j) dr
\]

With account for (2), and after the substitution by

\[
F_j = \rho_j(r) \frac{\partial T_j(r,t)}{\partial r} W_j(r,\mu_j) \bigg|_{r_{j-1}}^{r_j} - \rho_j(r) \frac{dW_j(r,\mu_j)}{dr} T_j(r,t) \bigg|_{r_{j-1}}^{r_j},
\]

we get:

\[
\int_{r_{j-1}}^{r_j} \rho_j(r) \left( \frac{\partial^2 T_j(r,t)}{\partial r^2} + \frac{B_j(r)}{A_j(r)} \frac{\partial T_j(r,t)}{\partial r} \right) W_j(r,\mu_j) dr = F_j + \mu_j^2 U_j(\mu_j,t)
\tag{17}
\]

The values \( \frac{\partial T_j(r,t)}{\partial r} \bigg|_{r_{j-1}}^{r_j} \) and \( \frac{\partial T_j(r,t)}{\partial r} \bigg|_{r_{j-1}}^{r_j} \) are defined through the boundary conditions (4)-(7).

Images of the source function are:

\[
G_j(\mu_j,t) = \int_{r_{j-1}}^{r_j} \rho_j(r) \frac{Q_j(r,t)}{A_j(r)} W_j(r,\mu_j) dr.
\tag{18}
\]

In this way, we have the following equation in the images:

\[
\frac{dU_j(\mu_j,t)}{dt} - \mu_j^2 U_j(\mu_j,t) = F_j + G_j(\mu_j,t)
\tag{19}
\]

Equation (19) is the linear non-uniform first-order differential equation. To start with, we will solve the homogeneous equation:

\[
\frac{dU_j(\mu_j,t)}{dt} - \mu_j^2 U_j(\mu_j,t) = 0,
\]

\[
U_j(\mu_j,t) = C_j e^{\mu_j t}
\tag{20}
\]

Supposing that \( C_j \) is the function of \( \mu_j, t \), we will substitute it into the original equation (16):

\[
\frac{d}{dt} C_j(\mu_j,t) = \left( F_j + G_j(\mu_j,t) \right) e^{\mu_j t},
\]

\[
\frac{d}{dt} C_j(\mu_j,t)
\]
\[ C_j = C_j(\mu_j,0) + \int_0^j (F_j + G_j(\mu_j,t))e^{\mu_j t} dt, \]  

Let us substitute (21) into (20):

\[ U_j(\mu_j,t) = e^{\mu_j t} \left[ U_j(\mu_j,0) + \int_0^j (F_j + G_j(\mu_j,t))e^{\mu_j t} dt \right] \]  

The opposite transfer is effected via formula [7],[8]:

\[ P_j(r,t) = \sum_{n \neq j} \frac{U_j(\mu_j,t)}{W_n} W_j(r,\mu_j), \]

here \( \|W_n\|^2 \) is the squared norm of the function \( W_n \) equal to

\[ \|W_n\|^2 = \int_{R_{nj}} \rho_n(r)r^2(r,\mu_j)dr. \]

4. Conclusions

In this way, the finite integral transformation rule was used to obtain the procedure of solving a dynamic problem of thermal conductivity for the \( j \)-level in multi-layer gasholders, given that each of the layers has its own physical properties, and heat transfer coefficient is \( r \)-coordinate dependant.

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