Thermovortical effect and magnetobaryogenesis

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(Dated: August 24, 2021)

We introduce a new source for the production of hyperelectric vortical current, which originates from the effective thermal masses of the fermions. We show that the contributions of gauge interactions to the thermal masses in the hyperelectric vortical current cancel out due to the gauge symmetries; while, those of the Yukawa interactions do not cancel out and yield $J_v \simeq (3g/256\pi^2)T^2\omega$. We finally show that the resulting temperature-dependent vortical current leads to the generation of hypomagnetic fields and matter-antimatter asymmetries in the symmetric phase of the early Universe, in the temperature range $100\text{GeV} \leq T \leq 10\text{TeV}$, all starting from zero initial values. In particular, we show that only a small transient vorticity fluctuation about the zero background value in the plasma can lead to the generation of observed baryon asymmetry in the Universe (BAU), even in the presence of the weak sphaleron processes which try to wash out the asymmetries.

PACS numbers:

Introduction.— It is known that in the symmetric phase, $T > T_{EW}$, the $SU_{L}(2) \times U_{Y}(1)$ symmetry is restored and gauge bosons and fermions are all massless at tree level. So there are no bare masses for the fermions. However, at finite temperature the pole of fermion propagator, i.e. fermion dispersion relation, no longer remains at $k^2 = 0$, where $k^\mu$ is the fermion four-momentum [3]. Indeed, the particle acquires an effective thermal mass of order $T$ and, to a very good approximation, its dispersion relation becomes $E(k) \simeq \sqrt{k^2 + m(T)^2}$ [3]. This is due to the particle propagation in the early Universe plasma and its interaction with other particles in the background at temperature $T$. The effective thermal masses for different chiralities and families are obtained as follows [1, 2]:

\begin{align}
 m_{i,R}^2 &= g^2T^2 + \frac{h_r}{8}, \\
 m_{i,L}^2 &= g^2T^2 + 3g^2L^2 + \frac{h_l}{16}, \\
 m_{d,R}^2 &= g^2T^2 + \frac{g^2T^2}{16} + \frac{h_l}{8}, \\
 m_{u,R}^2 &= g^2T^2 + \frac{g^2T^2}{6} + \frac{h_u}{16}, \\
 m_{Q_i}^2 &= g^2T^2 + \frac{3g^2T^2}{10} + \frac{g^2T^2}{6} + \frac{h_u^2 + h_d^2}{16},
\end{align}

where, $m_{i,R}$ ($m_{i,L}$), $m_{Q_i}$, and $m_{u,R}$ ($m_{d,R}$) denote the masses of right-handed (left-handed) leptons, left-handed quarks, and up (down) right-handed quarks, respectively, and $i$ is the generation index. Furthermore, $g^r$, $g^l$, and $g$ are the $SU(3)$, $U_Y(1)$, and $SU(2)$ coupling constants, and $h_{FS}$ are the Yukawa coupling constants given as \footnote{The Yukawa Lagrangian density is given by $\mathcal{L}_Y = \sum_{ij} [h_{ij}^f \bar{l}_i L \Phi l_{j}R + h_{ij}^d \bar{Q}_i Q_{j} \Phi d_{j}R + h_{ij}^u \bar{h}_i \Phi u_{j}R] + h.c.$, where $\Phi = \sqrt{2} \Phi$. The fermion bases are chosen such that for right-handed fermions, left-handed leptons and left-handed quarks, $h_f h_f$, $h_u h_u$, and $h_d h_d$ are diagonal, respectively. In this basis, $h^u h^u$ and $h^d h^d$ have the same eigenvalues equal to $2m_f^2/v^2$, where $v = 246\text{GeV}$ is the Higgs vacuum expectation value.}

\begin{align}
 h_{t_r} &\doteq \text{diag}(2.94 \times 10^{-6}, 6.9 \times 10^{-4}, 1.03 \times 10^{-2}), \\
 h_{t_u} &\doteq \text{diag}(1.3 \times 10^{-5}, 7.3 \times 10^{-3}, 1.0), \\
 h_{t_d} &\doteq \text{diag}(2.7 \times 10^{-5}, 5.5 \times 10^{-4}, 2.4 \times 10^{-2}).
\end{align}

Upon substituting the relevant coupling constants in Eqs. (1) we obtain the effective thermal masses for the fermions as given in Table 1 of Appendix A. Although thermal masses influence various physical quantities, we shall show that in this study their effects are negligible, except that in the presence of vorticity they become a new source for producing the hyperelectric vortical current. Generation of electric currents parallel to the vorticity and magnetic fields are generally referred to as the chiral vortical effect (CVE) and the chiral magnetic effect (CME), respectively. These effects play important roles in particle physics and cosmology, particularly in the early Universe [3].

The large-scale magnetic fields [6, 7] and the baryon asymmetry [8, 10] in the Universe are two important puzzles in particle physics and cosmology, the origin and evolution of which have been investigated through various models [11–17]. In the symmetric phase of the early Universe before the electroweak phase transition (EWPT), these two seemingly unrelated problems are intertwined via the Abelian anomalous effects including the Abelian anomaly, $\nabla_{\mu} j^\mu \sim \tilde{E}_Y, \tilde{B}_Y$, and the CME [13, 18, 21, 22–26]. The evolution of the magnetic fields is also con-

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nected to that of the fluid vorticity through the CVE. Generically, in the absence of chirality, the CVE is inactive when all particles of the vortical electroweak plasma have the same temperature. The main result of this work is that when thermal masses of all particles are taken into account, the CVE becomes active in the presence of vorticity and leads to the creation of a sufficiently strong hypermagnetic field, which then produces the particle asymmetries due to the Abelian anomaly, even when the weak sphalerons are present. As we shall show, the contributions of thermal masses to the CVE which originate from the gauge interactions cancel out due to the gauge invariance, while those of the Yukawa processes do not cancel out and result in the aforementioned effects.

CVE. — The chiral vortical effect, generically induced by the rotation of chiral matter, refers to the generation of an electric current parallel to the vorticity field. The hyperelectric chiral vortical current for one fermion species with two different handedness is obtained as

\[ J_{\mu, r}^c = Q r \xi_{c,r}(T, m_r, \mu_r) \omega^\mu, \tag{3} \]

where \( r = \pm \) represents the chirality, \( Q_r = -g Y_c / 2 \), \( Y_c \) is the hypercharge of the chiral fermion, \( \omega^\mu = (\epsilon^{\mu
u\rho\sigma} / R(t))^u_\mu \nabla_\nu u_\sigma \) is the vorticity four-vector, with the totally anti-symmetric four-dimensional Levi-Civita symbol specified by \( \epsilon^{0123} = -\epsilon^{0123} = 1 \), \( u^\mu = \gamma (1, \dot{v} / R(t)) \) is the four-velocity of the plasma normalized such that \( u^\mu u_\mu = 1 \), \( \gamma \) is the Lorentz factor, \( R(t) \) is the scale factor, and \( \nabla \) is the covariant derivative with respect to the Friedmann-Robertson-Walker (FRW) metric

\[ ds^2 = dt^2 - R^2(t) \delta_{ij} dx^i dx^j. \]

Furthermore, \( \xi_{c,r} \) is given by

\[ \xi_{c,r}(T, m_r, \mu_r) = r \frac{\beta^2}{8\pi^2} \int_0^\infty dk \frac{2k^2 + m_r^2}{E_{k,r}} \tilde{F}_+(E_{k,r}, \mu_r), \tag{4} \]

where \( \tilde{F}_+(E_{k,r}, \mu_r) = f_{FD}(E_{k,r} - \mu_r) + f_{FD}(E_{k,r} + \mu_r) \),

\[ E_{k,r} = \sqrt{k^2 + m_r^2}, \quad f_{FD}(x) = \frac{1}{e^{x+1}} \]

is the Fermi-Dirac distribution function, \( \beta = 1 / T \), and \( m_r \) and \( \mu_r \) are the mass and chemical potential of the chiral fermion, respectively.

In the symmetric phase of the early Universe plasma at high temperatures, \( \mu_r / T < 1 \). Using this fact, and also assuming \( m_r / T \ll 1 \), the expression for \( \xi_{c,r} \) given by Eq. (4) reduces to

\[ \xi_{c,r}(T, m_r, \mu_r) = r \left( \frac{T^2}{24} - \frac{m_r^2}{16\pi^2} + \frac{\mu_r^2}{8\pi^2} \right). \tag{5} \]

For a massless fermion this reduces to the form

\[ \xi_{c,r}(T, 0, \mu_r) = r \left( \frac{T^2}{24} + \frac{\mu_r^2}{8\pi^2} \right). \tag{5'} \]

Using Eqs. (4-5) for all chiral fermions of the electroweak plasma, the hyperelectric chiral vortical coefficient \( c_r \), defined by \( J_{\mu, r}^c = c_r \omega^\mu \), can be obtained as follows

\[ c_r(t) = \sum_{i=1}^{n_G} \left[ \frac{g_i'}{48} \left( -Y_{1i} T_{1i}^2 + Y_{1L} T_{1L}^2 N_w - Y_{dR} T_{dR}^2 N_c - Y_{uR} m_{uR}^2 N_c + Y Q T_{Q_i}^2 N_c N_w \right) \right. \]

\[ - \frac{g_i'}{32\pi^2} \left( -Y_{1i} m_{1i}^2 + Y_{1L} m_{1L}^2 N_w \right. \]

\[ - Y_{dR} m_{dR}^2 N_c - Y_{uR} m_{uR}^2 N_c + Y Q m_{Q_i}^2 N_c N_w \]

\[ + \frac{g_i'}{16\pi^2} \left( -Y_{1i} \mu_{1i}^2 + Y_{1L} \mu_{1L}^2 N_w - Y_{dR} \mu_{dR}^2 N_c - Y_{uR} \mu_{uR}^2 N_c + Y Q \mu_{Q_i}^2 N_c N_w \right) \right]. \tag{6} \]

In the above equation, \( n_G \) is the number of generations, \( N_c = 3 \) and \( N_w = 2 \) are the ranks of the non-Abelian \( SU(3) \) and \( SU(2) \) gauge groups, respectively, and the relevant hypercharges are

\[ Y_{1L} = -1, \quad Y_{1R} = -2, \quad Y_Q = \frac{1}{3}, \quad Y_{uR} = \frac{4}{3}, \quad Y_{dR} = -\frac{2}{3}. \tag{7} \]

At temperatures \( T \leq 10^{14} \text{GeV} \), the Abelian \( U_Y(1) \) gauge interactions cause all particles to be in thermal equilibrium, and therefore have the same temperature. As explained earlier, all chiral fermions acquire thermal masses, given by Eq. (4). When we substitute the fermion thermal masses and the hypercharges, given by Eqs. (1-2), into Eq. (6), two major cancellations occur due to gauge symmetry. First, the original \( T^2 \) terms which appear in the first parenthesis cancel each other. Second, all terms coming from the contributions of gauge interactions to thermal masses in the second parenthesis of Eq. (6) cancel one another exactly, as well. Therefore, only the contributions due to the Higgs coupling to thermal masses remain and Eq. (6) simplifies to

\[ c_r(t) = \sum_{i=1}^{n_G} \left[ \frac{g_i'}{32\pi^2} \left( h_{i1}^2 \frac{T^2}{8} + 3 h_{i2}^2 \frac{T^2}{8} - 3 h_{i1}^2 \frac{T^2}{8} \right) \right. \]

\[ + \frac{g_i'}{8\pi^2} \left( \mu_{1i}^2 - \mu_{1L}^2 + \mu_{dR}^2 - 2 \mu_{uR}^2 + \mu_{Q_i}^2 \right) \right]. \tag{8} \]

When the thermal masses are neglected, Eqs. (6-8) reduce to the usual forms [41], and there would be no CVE in a chirally balanced plasma. However, as we have shown, even in the absence of chirality, \( c_r \) is nonzero due to thermal masses, and hence CVE is active.

Let us neglect the tiny \( \mu_r \) terms in Eq. (8) and focus on the role of thermal masses in the CVE. Since the top quark Yukawa coupling \( h_t \) is very much larger than other Yukawa couplings, it suffices to consider only \( h_t \) and ne...
glect all other $h_{fS}$ and obtain

$$c_{v}(t) \simeq \frac{3g^2}{256\pi^2}T^2 = (0.00118)g^2T^2. \quad (9)$$

As can be seen, there exists a significant contribution to the hyperelectric vortical current which might be consequential at high temperatures.\(^5\) This is the first main result of this study. As we shall show, this new source for the CVE plays an important role in the generation of the hypermagnetic fields and the matter-antimatter asymmetries.

CME.— The chiral magnetic effect refers to the generation of an electric current parallel to the magnetic field in an imbalanced chiral plasma. The hyperelectric chiral magnetic current for one fermion species with two different handedness is

$$J_{B_{\mu}} = Q_r \xi_{B_{\mu}}(T, m_{\tau}, \mu_{\tau})B^\mu. \quad (10)$$

Here, $B^\mu = (e^{\nu\rho\sigma}/2R(t)^3)\nu_{(\rho}F_{\sigma)}$ is the hypermagnetic field four-vector, $F_{\rho\sigma} = \nabla_{\rho}A_{\sigma} - \nabla_{\sigma}A_{\rho}$ is the field strength tensor of the $U(1)_{Y}$ gauge fields, and $\xi_{B_{\mu}}(T, m_{\tau}, \mu_{\tau})$ is

$$\xi_{B_{\mu}}(T, m_{\tau}, \mu_{\tau}) = \frac{rQ_r}{4\pi^2} \int_{0}^{\infty} dk \tilde{F}_-(E_{k,r}, \mu_{\tau}), \quad (11)$$

where $\tilde{F}_-(E_{k,r}, \mu_{\tau}) = f_{FD}(E_{k,r} - \mu_{\tau}) - f_{FD}(E_{k,r} + \mu_{\tau})$. In the massless limit, for $\mu_{\tau}/T \ll 1$, $\xi_{B_{\mu}}$ reduces to the simple form $\xi_{B_{\mu}}(T, 0, 0) = \frac{rQ_r}{4\pi^2} \mu_{\tau}.$ \(^4\) \(^5\)

Before the EWPT, the terms of order $\mu_{\tau}/T$ are usually neglected in the dynamical equation for obtaining the physical quantities such as the occupation number density, energy density, pressure, entropy density, etc. However, for having non-zero CME, the Fermi-Dirac distribution function should be expanded to first order in $(\mu_{\tau}/T)$. In fact, at $\mu_{\tau} = 0$ limit, the C-symmetry enforces $\xi_{B_{\mu}} = 0$, so the interdependent CME and Abelian anomaly effects disappear. Upon expanding $\tilde{F}_-(E_{k,r}, \mu_{\tau})$ to first order in $(\mu_{\tau}/T)$ and using Eq. (11), $\xi_{B_{\mu}}$ can be obtained as

$$\xi_{B_{\mu}}(T, m_{\tau}, \mu_{\tau}) = \frac{rQ_r}{4\pi^2} \mu_{\tau} \left[1 + \delta_{\xi}(m_{\tau}/T)\right], \quad (12)$$

where $\delta_{\xi}(m_{\tau}/T)$ is given by

$$\delta_{\xi}(m_{\tau}/T) = 2\int_{0}^{\infty} dy \frac{e^{y\sqrt{(m_{\tau}/T)^2 - 1}}}{\left(e^{y\sqrt{(m_{\tau}/T)^2 - 1}} + 1\right)^2} \quad (13)$$

Using Eqs. (10-12) for all chiral fermions, and neglecting $\delta_{\xi}(m_{\tau}/T)$ (see Appendix B), the hyperelectric chiral magnetic coefficient $c_{B_{\mu}}$, defined by $J_{B_{\mu}}^\mu = c_{B_{\mu}}B^\mu$, can be obtained as

$$c_{B_{\mu}}(t) = \frac{g^2}{4\pi^2} \sum_{i=1}^{n_{\mu}} \left[Y_{i\mu_{\tau} \mu_{i}}^2 \mu_{i} N_{w} + Y_{i\mu_{\tau} \mu_{i}}^2 \mu_{i} N_{w} + \right.$$

$$
\left. + Y_{i\mu_{\tau} \mu_{i}}^2 \mu_{i} N_{w} - Y_{Q}^2 \mu_{Q} N_{w} \right]. \quad (14)
$$

In the temperature range of our interest, the Yukawa interactions of all fermions except the electron are in thermal equilibrium. Furthermore, due to the flavor mixing in the quark sector, all up or down quarks belonging to different generations with distinct handedness have the same chemical potential. The weak sphaleron processes which try to wash out the asymmetries are in thermal equilibrium, as well. Using these assumptions with the hypercharge neutrality condition and $\mu_{B}/3 - \mu_{R} - 2\mu_{L} = 0$ conservation laws, all chemical potentials are obtained in terms of the right-handed electron chemical potential (see Ref. [20] for more details), and $c_{B_{\mu}}$ simplifies to

$$c_{B_{\mu}} = \frac{g^2}{4\pi^2} \frac{711}{481} \mu_{\tau}. \quad (15)$$

The evolution equations close to the EWPT.— Now, we shall show how the effective thermal masses can lead to the generation of the hypermagnetic fields and the matter-antimatter asymmetries in the symmetric phase, which is our second main result.

Considering the perturbative chirality flip reactions for the right-handed electron, we obtain the evolution equation of its asymmetry $\eta_{\mu_{\tau} \mu_{\tau}} = (n_{\mu_{\tau} \mu_{\tau}} - \bar{n}_{\mu_{\tau} \mu_{\tau}})/s$ as (see Refs. [18, 19, 41, 46, 47], and Appendix C)

$$\frac{d\eta_{\mu_{\tau} \mu_{\tau}}}{dt} = \frac{g^2}{4\pi^2} s \left(\tilde{E}_{Y} \tilde{B}_{Y}\right)$$

$$+ \left(\frac{\Gamma_{0}}{t_{EW}}\right) \left(\frac{1 - x}{\sqrt{x}}\right) \left(\eta_{\mu_{\tau} \mu_{\tau}} - \eta_{\mu_{\tau} \mu_{\tau}} - \frac{\eta_{0}}{2}\right), \quad (16)$$

where, $\left(\frac{\Gamma_{0}}{t_{EW}}\right) \left(\frac{1 - x}{\sqrt{x}}\right)$ is the chirality flip rate, $\Gamma_{0} = 121$, $x = (t/t_{EW}) = (T_{EW}/T)^2$ is given by the Friedmann law, $t_{EW} = (M_{pl}/2T_{EW}^2)$, $M_{pl} = (M_{pl}/1.66\sqrt{g^*})$, and $M_{pl}$ is the Plank mass. Taking the CVE and the CME into account, we obtain the anomalous Maxwell’s equations for the hypercharge-neutral plasma in the symmetric phase of the expanding Universe as (see Refs. [41, 48, 50], and Appendix C)

$$\frac{1}{R} \nabla \cdot \tilde{E}_{Y} = 0, \quad \frac{1}{R} \nabla \cdot \tilde{B}_{Y} = 0, \quad (17)$$

$$\frac{1}{R} \nabla \times \tilde{E}_{Y} + \frac{\partial \tilde{B}_{Y}}{\partial t} + 2H \tilde{B}_{Y} = 0, \quad (18)$$

\(^4\) Using Eq. (3) and the numerical values $\xi_{\nu_{e}, \nu_{\tau}}(T, m_{\tau})$ given in Table II for all particles, the exact value is $c_{\nu}(N/T) = (0.00096)g^2T^2$.

\(^5\) It is worth mentioning that although after the EWPT fermions acquire bare masses, the contributions of these masses to the electric chiral vortical current cancel out, since the right- and left-handed fermions have the same masses and electric charges.
\[
\frac{1}{R} \nabla \times \vec{B}_Y - \left( \frac{\partial \vec{E}_Y}{\partial t} + 2H \vec{E}_Y \right) = \vec{J}
\]
\[
= \vec{J}_{\text{Ohm}} + \vec{J}_{\text{cv}} + \vec{J}_{\text{cm}},
\]
\[
\vec{J}_{\text{Ohm}} = \sigma \left( \vec{E}_Y + \vec{v} \times \vec{B}_Y \right),
\]
\[
\vec{J}_{\text{cv}} = c_v \vec{\omega},
\]
\[
\vec{J}_{\text{cm}} = c_B \vec{B}_Y,
\]

where \( H = \dot{R}/R \) is the Hubble parameter. In the following, we choose a simple monochromatic Chern-Simons configuration for the hypermagnetic field \( \vec{B}_Y = (1/R)\nabla \times \vec{A}_Y \), and the velocity field \( \vec{v} = (1/R)\nabla \times \vec{S} \) [19, 41]. To do this, we choose \( \vec{A}_Y = \gamma(t) (\sin kz, \cos kz, 0) \), and \( \vec{S} = \tau(t) (\sin kz, \cos kz, 0) \), for their corresponding vector potentials [51, 53].

Using the aforementioned configurations, the evolution equation for the velocity field reduces to \( \frac{\partial \vec{v}}{\partial t} = -\nu k^2 \vec{v} \) [41], where the kinematic viscosity \( \nu \approx 1/(5\alpha T^2) \) [54, 55].

Neglecting the displacement current in the lab frame and using the aforementioned configurations, the hyperelectric field and the evolution equation for the hypermagnetic field amplitude are obtained, as follows

\[
\vec{E}_y = \frac{k'}{\sigma} \vec{B}_y - \frac{c_v}{\sigma} k' \vec{v} - \frac{c_B}{\sigma} \vec{B}_y,
\]
\[
\frac{dB_Y(t)}{dt} = \left[ -\frac{1}{t} - \frac{k'^2}{\sigma} + \frac{c_B k'}{\sigma} \right] B_Y(t) + \frac{c_v}{\sigma} k'^2 \nu(t),
\]

where \( k' = k/R = kT \). Using \( \mu_f = (6s/T^2)\eta_f \) with Eq. [24], we obtain \( \langle \vec{E}_Y, \vec{B}_Y \rangle \), which appears in the Abelian anomaly equations, as

\[
\langle \vec{E}_Y, \vec{B}_Y \rangle = \frac{B_Y^2(t)}{100} \left[ \frac{k'}{T} - \frac{6 s g'^2}{4 \pi^2 T^2} \left( \frac{711}{481} \eta_c R \right) \right] \]
\[
- \frac{3 g'}{256 \pi^2} \frac{k' T}{100} \nu(t) B_Y(t).
\]

In Eq. (25), the last term is due to the CVE; while, the second term in the first bracket is due to the CME. For an illustrative example we consider the velocity as two successive Gaussian pulses with opposite profiles as

\[
v(x) = \frac{v_0}{b\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2b^2}} - \frac{v_0}{b\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2b^2}},
\]

where \( b, v_0, \) and \( x_{0,1} \) denote the width, the amplitude and the occurrence times of the fluctuations, respectively.

We now set the initial values of the hypermagnetic field amplitude and all matter-antimatter asymmetries to zero, i.e. \( B_Y^{(0)} = 0, \eta_{cR}^{(0)} = 0 \), and solve the set of evolution equations with the initial conditions \( b = 2 \times 10^{-4}, x_{0,1} = 5.5 \times 10^{-4}, x_{0,2} = x_{0,1} + b = 7.5 \times 10^{-4} \), and three different values for \( v_0 \), and present the results in Fig. 11. As can be seen, upon the occurrence of vorticity fluctuations, the thermal mass induced CVE leads to the generation of strong hypermagnetic fields which then produce the matter-antimatter asymmetries via the Abelian anomaly, all starting from zero initial values, in spite of the fact that the weak sphaleron processes are also taken into account. Moreover, we have found that in this study, the generation of the hypermagnetic fields and the matter asymmetries is mainly due to the CVE not the CME. We have also found that by increasing the amplitude of velocity fluctuations, the maximum and the final values of the hypermagnetic field amplitude, as well as the matter-antimatter asymmetries increase.

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6 In fact, the occurrence of any fluctuation in a plasma in a quasi-equilibrium state would normally trigger a restoring response originating from dissipative effects, such as viscous effects. Here, for simplicity we assume that the combined results of the original fluctuations and the ensuing dissipative effects have the Gaussian profiles as given by Eq. (20) [54].
Conclusion. — In this study, first we have proposed a new source for the CVE originating from the temperature-dependent effective thermal masses of the fermions in the symmetric phase. We have shown that the contributions of the gauge interactions to the thermal masses in the hyperelectric chiral vortical current cancel out due to the gauge symmetry, while those of the Yukawa processes yield a nonzero expression proportional to $T^2$. Second, we have shown that this new term can produce a hyperelectric chiral vortical current, in the presence of transient vorticity fluctuation, leading to the generation of hypermagnetic fields and matter-antimatter asymmetries from zero initial values, more importantly, when the weak sphalerons are also taken into account. This outcome has not been observed in any of the previous studies. Here we have shown that the CVE, which was considered inefficient in AMHD equations, has great impact on the generation of hypermagnetic fields and matter-antimatter asymmetries in the early Universe.

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below, we display the numerical values of \( \xi_{\psi,i}(T, m_i, 0) \), denoted by \( \xi_{\psi,r,N} \), obtained by solving the integrals in Eq. (4) numerically, and the numerical values of \( \xi_{\psi,i}(T, m_i, 0) \) obtained by using its small mass approximation given by Eq. (5). For the latter two, we use the numerical values of thermal masses. It is interesting that although \( m_i/T \) is not much less than one, the values of the last two columns are within 0.2\%, indicating the validity of small mass approximation as expressed in Eq. (5).

### Table I: The effective thermal mass \( m_i \), the numerical value of \( \xi_{\psi,i}(T, m_i, 0) \), and the value of \( \xi_{\psi,i}(T, m_i, 0) = r \left[ \frac{3}{25} - \frac{8}{m_i^2} \right] = 1 \right) ^ \frac{1}{m_i} \right) ^ T^2 \) are obtained for all chiral fermions.

#### Particle

| \( \frac{m_i}{T} \) | \( \xi_{\psi,r,N}(T, m_i, 0) \) | \( \xi_{\psi,i}(T, m_i, 0) \) |
|---|---|---|
| \( c_R, \overline{c}_R, \tau_R \) | 0.1220 | 0.0415 |
| \( c_L, \mu_L, \tau_L \) | 0.2027 | 0.0414 |
| \( q_{R,L}, \overline{q}_{R,L}, \overline{q}_{R} \) | 0.4817 | 0.0402 |
| \( q_{R,L}, \overline{q}_{R,L}, \overline{q}_{R} \) | 0.5178 | 0.0396 |
| \( q_{R,L}, \overline{q}_{R} \) | 0.5750 | 0.0395 |
| \( q_{R} \) | 0.5975 | 0.0394 |

### II. APPENDIX B

In this appendix, we show that the effects of thermal masses on the hyperelectric chiral magnetic coefficient \( c_B \), through \( \delta \xi(m_i/T) \) as defined by Eqs. (11), are small. Moreover, we show analogous results for four other physical quantities, as follows. The occupation number density \( n_i \), the energy density \( \rho_i \), the pressure \( p_i \), and the asymmetry number density \( \Delta n_i = (n_i - \overline{n}_i) \), for fermion...
species ‘i’ are given by
\[ n_i = \frac{3}{4\pi^2} \zeta(3) T^3 [1 + \delta_n(m_i/T)], \]
\[ \rho_i = \frac{7 \pi^2}{8 \cdot 30} T^4 [1 + \delta_\rho(m_i/T)], \]
\[ p_i = \frac{7 \pi^2}{8 \cdot 90} T^4 [1 + \delta_p(m_i/T)], \]
\[ \Delta n_i = \mu_i T^2 [1 + \delta_{\Delta n}(m_i/T)], \]
where
\[ \delta_n(m_i/T) = \frac{2}{3\zeta(3)} \left[ \int_{m_i/T}^\infty f_{FD}(x) \sqrt{x^2 - \frac{m_i^2}{T^2}} dx - \int_0^\infty f_{FD}(x)x^2 dx \right], \]
\[ \delta_\rho(m_i/T) = \frac{120}{7\pi^4} \left[ \int_{m_i/T}^\infty f_{FD}(x) \sqrt{x^2 - \frac{m_i^2}{T^2}} dx - \int_0^\infty f_{FD}(x)x^3 dx \right], \]
\[ \delta_p(m_i/T) = \frac{120}{7\pi^4} \left[ \int_{m_i/T}^\infty f_{FD}(x)(x^2 - \frac{m_i^2}{T^2})^{3/2} dx - \int_0^\infty f_{FD}(x)x^3 dx \right], \]
\[ \delta_{\Delta n}(m_i/T) = \frac{6}{\pi^2} \left[ \int_{m_i/T}^\infty \frac{x e^x}{(e^x + 1)^2} \sqrt{x^2 - \frac{m_i^2}{T^2}} dx - \int_0^\infty \frac{x^2 e^x}{(e^x + 1)^2} dx \right]. \]
The numerical values of \( \delta_n(m_i/T), \delta_\rho(m_i/T), \delta_p(m_i/T) \) \( \delta_\rho(m_i/T), \) \( \) and \( \delta_{\Delta n}(m_i/T) \) are given in Table I. The entropy density is related to the pressure and energy density through the relation \( s_i = (\rho_i + p_i)/T \). The results show that the thermal masses influence the thermodynamics of the early Universe plasma by slightly reducing the asymmetries, the pressure and energy, entropy, and number densities. Moreover, the expansion rate of the Universe and as a result the generated baryon asymmetry will be affected, which is beyond the scope of this work and will be investigated in a separate study. In this paper, we have neglected all mentioned corrections, due to the thermal masses, to the CME coefficient, pressure, occupation number densities, asymmetries, energy and entropy densities (see Table I).

III. APPENDIX C

Anomalous magnetohydrodynamics (AMHD).— In the presence of the CME and the CVE, in the Eckart frame, the energy momentum tensor acquires extra contributions as [57, 58]
\[ T^{\mu\nu}(CME) = c_v [u^{\mu} B^{\nu} + u^{\nu} B^{\mu}], \]
and
\[ T^{\mu\nu}(CVE) = \frac{1}{2} \Delta n_{5,t} [u^{\mu} \omega^{\nu} + u^{\nu} \omega^{\mu}], \]
where \( \Delta n_{5,t} \) is the total axial asymmetry density given by
\[ \Delta n_{5,t} = \sum_{i=1}^{n_f} [\Delta n_{i,R} - N_w \Delta n_{i,L} + N_c \Delta n_{i,R} - N_w N_c \Delta n_{Q_i}]. \]
Therefore, the energy-momentum tensor \( T^{\mu\nu} \) and the total hyperelectric current \( J^\mu \) in the presence of the CME and the CVE for the early Universe plasma are given as follows:
\[ T^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} - pg^{\mu\nu} + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} - F^{\nu\sigma} F_{\mu\sigma} \]
\[ + \pi^{\mu\nu} + c_v [u^{\mu} B^{\nu} + u^{\nu} B^{\mu}] + \Delta n_{5,t} [u^{\mu} \omega^{\nu} + u^{\nu} \omega^{\mu}], \]
\[ J^\mu = \rho_3 u^\mu + J^\mu_{\text{cm}} + J^\mu_{\text{cv}} + \nu^\mu, \]
\[ J^\mu_{\text{cm}} = c_B B^\mu, \]
\[ J^\mu_{\text{cv}} = c_v \omega^\mu, \]
where \( \rho = 3p = g^* \pi^2 T^4/30 \) are the pressure and the energy density, \( g^* = 106.75 \) is the effective number of relativistic degrees of freedom, \( \rho_3 \) is the hyperelectric charge density, and \( u^\mu \) and \( T^{\mu\nu} \) are the hyperelectric dissipative current and viscous stress tensor, respectively [32]. The equations of AMHD consisting of energy-momentum conservation, Maxwell’s equations and the anomaly relations, may be expressed covariantly as [32, 57, 58, 60]
\[ \nabla_\mu T^{\mu\nu} = 0, \tag{34} \]
\[ \nabla_\mu F^{\mu\nu} = J^\nu, \tag{35} \]
\[ \nabla_\mu \tilde{F}^{\mu\nu} = 0, \tag{36} \]
\[ \nabla_{\mu,j}^\mu = C_r E_{\mu} B^\mu, \tag{37} \]
where \( \tilde{F}^{\mu\nu} = \frac{1}{2} F_{\mu\rho} u^\rho F^{\nu\sigma} \) is the dual field strength tensor, \( E^\mu = E^\mu_{\text{el}} u^\nu \) is the hyperelectric field four-vector, \( C_r = r \left( \frac{1}{2} \right)^2 \) is the chiral anomaly coefficient, \( j^\mu_{\text{el}} \) is the corresponding fermionic chiral matter current given by
\[ j^\mu_i = (n_i - \bar{n}_i) \omega^\mu + \xi_{B,i}(T, m_r, \mu_r) B^\mu + \xi_{\omega,i}(T, m_r, \mu_r) \omega^\mu + V_i^\mu, \tag{37} \]
and $V^\mu_\tau$ is the chiral particle dissipative current \cite{32, 57, 59, 60}. Hereafter, we focus on small velocity approximation, $v \ll 1$. Then the expressions for the four-vectors become

$$B^\mu = \gamma \left( \vec{v} \cdot \vec{B}_Y, \right) \approx \left( \vec{v} \cdot \vec{B}_Y, \right),$$

$$\omega^\mu = \gamma \left( \vec{v} \cdot \vec{a}, \right) \approx \left( \vec{v} \cdot \vec{a}, \right),$$

$$a^\mu = \gamma \left( \vec{v} \cdot \vec{a}, \right) \approx \left( \vec{v} \cdot \vec{a}, \right),$$

$$E^\mu = \gamma \left( \vec{v} \cdot \vec{E}_Y, \right) \approx \left( \vec{v} \cdot \vec{E}_Y, \right),$$

where $a^\mu = \Omega^\mu_{\nu} u^\nu$ is the acceleration four-vector, $\Omega_{\mu\nu} = \nabla_\mu u_\nu - \nabla_\nu u_\mu$ is the vorticity tensor, $a^\mu = R^\mu_{\nu\rho}$ is the acceleration three-vector.\footnote{In the derivative expansion of the hydrodynamics, $\partial_t \sim \vec{\nabla} \cdot \vec{v}$, so $\vec{v} \cdot \vec{E}_Y \approx v^2 B_Y$, and we have ignored the terms of $O(v^2)$ \cite{32, 23}.}

Using Eqs. \cite{45, 48} the temporal and spatial components of $j^\mu_\tau = \left( j^0_\tau, j^\tau_i / R \right)$ become

$$j^0_\tau = (n_\tau - \bar{n}_\tau) + \xi_{e,\tau}(T, m_\tau, \mu_\tau) \vec{v} \cdot \vec{B}_Y + \xi_{e,\tau}(T, m_\tau, \mu_\tau) \vec{v} \cdot \vec{a},$$

$$\vec{j}_\tau = (n_\tau - \bar{n}_\tau) \vec{v} + \xi_{e,\tau}(T, m_\tau, \mu_\tau) \vec{B}_Y + \xi_{e,\tau}(T, m_\tau, \mu_\tau) \vec{a} + \sigma^\tau \vec{E}_Y,$$

where $\sigma^\tau$ is the relevant chiral conductivity. Since $\mu_\tau / T \ll 1$, we have considered only the Ohmic effect in the dissipative current, $V^\mu_\tau = \sigma^\tau \left[ E^\mu + T (u^\mu u^\nu - g^\mu\nu) \nabla_\nu (\mu_\tau / T) \right]$, and ignored the second part \cite{64}.

High temperature of the early Universe plasma and low-velocity limit in this work, imply that $j^0_\tau \approx (n_\tau - \bar{n}_\tau)$, to a very good approximation \cite{59, 60}. The anomaly equations, given in Eq. \cite{23}, can be written out as

$$\partial_t j^0_\tau + \frac{1}{R} \vec{\nabla} \cdot \vec{j}_\tau + 3H j^0_\tau = C_t E_\mu B^\mu;$$

Upon taking the spatial average of Eq. \cite{40}, the boundary term vanishes and we obtain

$$\partial_t (n_\tau - \bar{n}_\tau) + 3H (n_\tau - \bar{n}_\tau) = C_t (\vec{E}_Y \cdot \vec{B}_Y),$$

Using the relation $\dot{s} / s = -3H$, the anomaly equation reduces to the simplified form

$$\partial_t \left( \frac{n_\tau - \bar{n}_\tau}{s} \right) = \frac{C_t}{s} (\vec{E}_Y \cdot \vec{B}_Y),$$

where $\frac{n_\tau - \bar{n}_\tau}{s}$ is the asymmetry $\eta_\tau$, and $s = 2\pi^2 g^* T^3 / 45$ is the entropy density.

High temperature of the early Universe plasma and low-velocity limit imply that not only $\rho_{total} \approx \rho_{el}$ but also the new terms, $T^{\mu\nu}(CME)$ and $T^{\mu\nu}(CVE)$, are small compared to the other terms in the total energy-momentum tensor. Neglecting these terms in the energy-momentum conservation equations, we obtain \cite{41}\footnote{The rhs. of Eq. \cite{23}, $\vec{E}_Y \cdot \vec{J}$, is usually neglected.}

$$\partial_t \rho + \vec{\nabla} \cdot \left[ \rho (\rho + p) \frac{\vec{v}}{R} \right] + 3H (\rho + p) = \vec{E}_Y \cdot \vec{J},$$

$$\partial_t \rho + \frac{1}{R} \vec{\nabla} \cdot \left[ (\rho + p) \frac{\vec{v}}{R} + 3H (\rho + p) \right] \frac{\vec{v}}{R} + \partial_\tau \rho + H (\rho + p) \frac{\vec{v}}{R}$$

$$+ (\rho + p) \partial_\tau \vec{v} + (\rho + p) \frac{\vec{v} \cdot \vec{v}}{R} \frac{\vec{v}}{R} + \frac{\vec{v} \cdot \vec{v}}{R} \frac{\vec{v}}{R}$$

$$= \rho_{total} + \left( \vec{J} \times \vec{B}_Y \right),$$

where the total hyperelectric current and charge densities are given by

$$\vec{J} = \rho_{el} \vec{v} + c_B \vec{B}_Y + c_v \vec{a} + \sigma \vec{E}_Y,$$

$$\rho_{total} \approx \rho_{el} = \sum_{i=1}^{n_{e,\tau}} \left[ Y_{i,R} j^0_{i,R} + N_w Y_{i,L} j^0_{i,L} + N_c \left( Y_{d,R} j^0_{d,R} + Y_{u,R} j^0_{u,R} + Y_Q N_w j^0_{Q,R} \right) \right].$$
Using $\eta = \frac{W}{e}$, the above total charge neutrality condition reduces to the following form [20]

$$6\eta_Q - \eta_{L_1} - \eta_{R_1} - 2\eta_{L_2} - 2\eta_{L_3} + 13\eta_0 = 0. \quad (46)$$

As a result, for the incompressible and totally hypercharge neutral plasma in the presence of the helical hypermagnetic field, the momentum equation reduces to the simple form of $\frac{d\vec{v}}{dt} = -\nu \vec{k}^2 \vec{v} \quad [41]$, where the kinematic viscosity $\nu \simeq 1/(5\alpha Y T) \quad [54, 55]$. 