Quantum Critical Metals:
Dynamical Planckian Scaling and Loss of Quasiparticles

Haoyu Hu\textsuperscript{1,2}, Lei Chen\textsuperscript{1}, Qimiao Si\textsuperscript{1}

\textsuperscript{1}Department of Physics and Astronomy, Rice Center for Quantum Materials, Rice University, Houston, TX 77005, USA

\textsuperscript{2}Donostia International Physics Center, P. Manuel de Lardizabal 4, 20018 Donostia-San Sebastian, Spain

Strange metals develop near quantum critical points (QCPs) in a variety of strongly correlated systems. Some of the important issues include how the quantum critical state loses quasiparticles, how it drives superconductivity, and to what extent the strange-metal physics in different classes of correlated systems is interconnected. In this review, we confront some of these issues from the vantage point of heavy fermion metals. We will describe the notion of Kondo destruction and how it leads to a transformation of the Fermi surface from “large” to “small” when the system is tuned across the QCP, a loss of quasiparticles everywhere on the Fermi surface when it is perched at the QCP, and a dynamical Planckian scaling in various physical properties including charge responses. We close with a brief discussion about the connections between the strange-metal physics in heavy fermion metals and its counterparts in the cuprates and other correlated materials.
I. INTRODUCTION

Large classes of quantum materials host strongly correlated electrons \[1,2\]. Many of them feature unconventional superconductivity. One connection among the strongly correlated systems is illustrated in Fig. 1(a). The superconducting transition temperature \((T_c)\) is several percent of the effective Fermi temperature \((T_0)\) across several material classes of strongly correlated superconductors, with each temperature scale spanning about three decades. A \(T_c/T_0\) ratio of a few percent qualifies these systems as “high-\(T_c\)” superconductors, given that this ratio is about two orders of magnitude smaller in conventional (Bardeen-Cooper-Schrieffer \[3\]) superconductors. Another connection lies in their normal states at temperatures above the superconducting transition temperature \((T > T_c)\), which are often strange metals with a non-Fermi liquid \((T–linear)\) form in the temperature dependence of the electrical resistivity and, accompanying it, a slew of other exotic properties.

The link between the strange-metal normal state and unconventional superconductivity in heavy fermion systems is particularly striking. Indeed, heavy fermion metals represent a prototype setting in which quantum critical metallicity has been elucidated \[9\], in part because \(T_c\) being relatively small in absolute magnitude opens up a large window of temperature over which the strange-metal properties can be explored. These systems often possess antiferromagnetic (AF) correlations. Heavy fermion superconductors per se is a venerable topic, and have now grown into a material family of about 50 members. By contrast, the strange-metal behavior and its association with quantum criticality have only been the focus during the relatively recent past.

It is natural for quantum criticality to nucleate unusual properties \[10\]. Indeed, as a system is tuned towards its quantum critical regime at a given low (but nonzero) temperature, the entropy is expected to be maximized \[11, 12\]. The behavior has been demonstrated in \(\text{CeCu}_{6-x}\text{Au}_x\) across multiple tuning parameters, as illustrated in Figs. 1(b,c) \[5\]. In this sense, quantum critical systems are particularly soft and are prone to the formation of unusual excitations and novel phases.

That strange metals develop via quantum criticality is clearly demonstrated in heavy fermion metals. We illustrate the point in \(\text{YbRh}_2\text{Si}_2\) and \(\text{CeRhIn}_5\), via their respective phase diagrams [Fig. 1(d,e)]. Each exhibits an AF order at ambient conditions. In \(\text{YbRh}_2\text{Si}_2\), a magnetic field applied perpendicular to its tetragonal plane of about 0.7 T (or one applied
FIG. 1: a Superconducting transition temperature ($T_c$) vs. the effective Fermi temperature ($T_0$, extracted from entropy and other means) for various superconductors from different material families (adapted from Ref. [4], courtesy J. D. Thompson).
FIG. 1: (cont’d) b Illustration of the evolution in the entropy near a QCP, based on measurements of the uniaxial Grüneisen ratios in CeCu$_{6-x}$Au$_x$ at $x_c = 0.1$ (from Ref. [5]). Panel b shows the parameter space of the uniaxial stress along the $a, b, c$ axis. The red arrow marks the direction with the steepest slope of entropy $\nabla S$. Away from the QCP, one can tune the system in directions that are perpendicular to the $\nabla S$ direction (the red plane), along which the Grüneisen ratio is not singular. c Illustration of the entropy maximization near the QCP, in the parameter space of the shear stress $\sigma_{x,y}$.

Temperature-control parameter phase diagrams of d YbRh$_2$Si$_2$ under a magnetic field [6, 7] and e CeRhIn$_5$ under pressure [8]. The colors represent the temperature exponent $\gamma$ of the resistivity ($\rho$), determined by a logarithmic derivative of $\Delta \rho(T) \equiv \rho(T) - \rho(T = 0)$ with respect to log $T$, signifying a $T^\gamma$ dependence.

within the plane of about 66 mT) tunes the system to its QCP [6], where a $T$-linear resistivity [13] occurs over more than three decades in temperature [14]. In CeRhIn$_5$, a quantum critical “fan” develops near a pressure of 2.3 GPa [15, 16] with a nearly-$T$-linear resistivity [8].

Theories of metallic QCPs have two general types. One class of theory is based on the fluctuations of Landau’s order parameter, as described by the Hertz-Millis-Moriya approach [17, 18]. Typically, this order parameter corresponds to a spin-density-wave (SDW) order at an AF wavevector $Q$. In this case, the nonzero ordering wavevector $Q$ links narrow “hot” regions of the Fermi surface to each other. The order parameter fluctuations couple to electrons from a small portion (“hot” region) of the Fermi surface (Fig. 2). Meanwhile, the majority of the Fermi surface remains “cold” (Fig. 2): the order parameter fluctuation connects one point on the cold region of the Fermi surface to another point in the Brillouin zone where the energy level lies substantially away from the Fermi energy. Correspondingly, for the electronic states in the cold region of the Fermi surface, the quantum critical fluctuations have a minimal effect and the quasiparticles retain their integrity [19, 21]. The electrical transport will not show the strange-metal behavior given that the quasiparticles, being long-lived, will short-circuit the electrical transport.

To realize the strange-metal behavior, it is necessary to destroy the quasiparticles on the entire Fermi surface. This takes place in the second type of theory for metallic quantum criticality, which goes beyond the Landau framework [22, 24].
FIG. 2: Schematic illustration of the Fermi surface for a SDW QCP. States in a small portion of the Fermi surface (red stripes with a width of $\sim \sqrt{T}$ for three dimensions) can be scattered by the low-energy critical bosons of wave vector $Q$. These states are “hot” in that they experience strong scattering by the order parameter fluctuations. Meanwhile, the majority of the Fermi surface remains “cold” (blue region), where Landau quasiparticles are left intact. The electrical transport is dominated by the contributions from the “cold” region of the Fermi surface and, thus, will not show strange metal behavior.

In this article, we survey the beyond-Landau quantum criticality. We start by considering how quasiparticles can be (critically) destroyed. The central theme here is that, for bad metals such as heavy fermion systems, the quasiparticles are fragile to begin with and their formation takes place through a process that is non-perturbative in electron correlations and, yet, well-understood. This understanding sets the stage for confronting the central challenge, viz. how the quasiparticles are lost. For heavy fermion metals, the Kondo effect underlies the formation of heavy quasiparticles, whereas the Kondo destruction leads to their suppression. We suggest that these understandings are relevant to the loss of quasiparticles in a variety of strongly correlated systems, including the doped cuprates, the iron chalcogenides and certain organic superconductors. In addition to surveying the theoretical issues, we will describe some of the salient experimental developments [25–31].
II. QUANTUM CRITICAL METALS – HOW TO DESTROY QUASIPARTICLES

To see how the quasiparticles can be lost everywhere on the Fermi surface, we start from their formation away from the QCP.

A. Quasiparticles: the robust and the fragile

For quantum many-body systems, the physics at low energies is analyzed in terms of building blocks and their symmetry-allowed interactions [2]. Traditionally, one takes bare electrons as the building blocks and treat the electron-electron interactions order by order in perturbation theory [32]. The notion of quasiparticles survives up to the infinite orders of the perturbation series. In that sense, quasiparticles are rather robust. For a long time, the validity of Fermi liquid theory was largely unquestioned for systems in dimensions higher than one; indeed, Fermi liquid was considered to be the only fixed point of the renormalization-group (RG) flow in such dimensions [33, 34]. A quasiparticle corresponds to a sharp peak in the electron spectral function as a function of frequency for a fixed wavevector. It has the physical meaning of a dressed electron; its quantum numbers are exactly those of a bare electron or hole, namely charge $\pm e$ and spin $\frac{h}{2}$. Their Fermi statistics dictates a decay rate that goes as $(k_B T)^2$ or $E^2$ as temperature ($T$) or energy ($E$, measured from the Fermi energy) goes to zero. In the Green’s function language, the self-energy $\Sigma(k, \omega)$ retains the Fermi liquid form up to infinite orders of the perturbative expansion [32]. This turns out to ensure a nonzero value for the quasiparticle weight, $Z_k$.

Sufficiently strong electron correlations can lead to other forms of the building blocks for the low-energy physics. For example, heavy fermion systems involve local $f$-electron-derived moments and itinerant $spd$-electron bands as the starting point for the description of their low-energy properties [2, 9, 35, 37]. In that case, quasiparticles are fragile, with a weight that is exponentially small.

Consider the Kondo lattice Hamiltonian (Box 1). We start from the parameter regime when the Kondo interaction succeeds in driving the formation of a Kondo singlet, which can be pictured as a bound state between a local moment and a triplet particle-hole combination of the conduction electrons. Breaking the bound state leads to not only bare conduction electrons, but also a composite fermion formed between the local moment and a conduction
electron. The composite fermions have the same quantum numbers as bare electrons, and they hybridize with the conduction electrons to form heavy quasiparticles. These quasiparticles have a large effective mass and a small quasiparticle weight: \( Z \) is exponentially small and, in practice, is of the order \( 10^{-3} \).

When the quasiparticles are this fragile, competing interactions can readily destroy it.

**B. Quantum criticality from Kondo destruction**

The notion of Kondo destruction quantum criticality invokes fluctuations that go beyond a Landau order parameter. For Kondo lattice systems (Box 1), it captures the dynamical competition between the Kondo and RKKY interactions. The corresponding QCP is illustrated in Fig. 3(a) [9, 22], in the space of temperature and non-thermal control parameter, \( \delta = T_K^0/I \), the ratio of the bare Kondo temperature to the RKKY interaction.

When \( \delta \) is sufficiently large, the Kondo interaction dominates and a Kondo singlet is formed in the ground state, as illustrated in Fig. 3(c). As the RKKY interaction \( I \) is increased, i.e. when the parameter \( \delta \) is tuned downward, the RKKY interaction becomes important and promotes correlations of a spin singlet between the local moments. This process is detrimental to the formation of the Kondo singlet. When it suppresses the Kondo singlet in the ground state, the composite heavy quasiparticles are lost.

Thus, both the formation and loss of quasiparticles can be considered by analyzing the fate of the Kondo singlet or, more specifically, the amplitude of the Kondo singlet in the ground state. Our strategy is to start from the Kondo side, and see whether and how the dynamical competition of the RKKY interaction brings about the suppression of this Kondo-singlet amplitude. One can in principle also work from the opposite end, by analyzing the Kondo lattice in terms of a quantum nonlinear sigma model representation; the results of such analyses [38–45] are consistent with the conclusions we present here.

Box 1 provides further details on how the dynamical competition from the RKKY interactions suppresses the Kondo singlet and, by extension, quasiparticles. The key is a new fixed point, marked red in Box 1, panel (b). Here, the Kondo-singlet amplitude vanishes in the ground state, and the weight of the Landau quasiparticle goes to zero. This fixed point is interacting (as opposed to Gaussian), where \( k_B T \) is the only energy scale.
FIG. 3: a Kondo destruction quantum criticality of a Kondo lattice, in which a Kondo destruction energy scale $E_{\text{loc}}^*$ vanishes at the QCP. Here, $T_N$, $T_{\text{FL}}$ and $T_0$ denote
FIG. 3: (cont’d) the temperatures for an AF ordering, the crossover into a Fermi liquid state and the initial onset of Kondo correlations, respectively. On the two sides of the QCP are b an AF order without the formation of a Kondo singlet and c a paramagnetic phase with a Kondo-singlet ground state. In the AF phase, the quasiparticles only involve d the conduction electrons, in contrast to the paramagnetic phase, in which e the Kondo singlets in the ground state yield composite heavy fermions in the excitation spectrum. The Fermi surface for the AF phase is small f in that it only involves the conduction electrons, while that for the paramagnetic phase is large g in that it also counts the number of local moments. Panel a and b-g are adapted from Refs. [9, 22] and Ref. [2], respectively.

C. Global phase diagram

The introduction of Kondo destruction has inspired considerations of new quantum phases in the AF Kondo-lattice systems. These phases are not only distinguished by the Landau order parameters but also by the existence or absence of the Kondo singlet in the ground state. This has led to a global phase diagram (Fig. 4) in the two-parameter space of $J_K$, the Kondo coupling, and $G$, which specifies the extent of the quantum fluctuations in the local-moment magnetism. The $G$ axis captures the tuning of dimensionality or geometrical frustration.

The stability of the AF$_S$ phase has been analyzed in terms of a quantum nonlinear sigma model representation of the Kondo lattice. Using the AF$_S$ phase as the starting point, there are three routes for quantum phase transitions to the paramagnetic heavy fermion (P$_L$) phase. Trajectory I describes a direct transition, with a Kondo destruction QCP at the border of the AF order. Trajectory II passes through an intermediate AF$_L$ phase, which corresponds to the SDW order from the heavy quasiparticles of the P$_L$ phase. A Kondo destruction transition takes place inside the AF order, while the QCP from the AF order to the paramagnetic phase is of the SDW type. Trajectory III passes through an intermediate P$_S$ phase, which could involve non-magnetic order such as a valence-bond solid or an underlying spin liquid. Generically, the Luttinger theorem of the Kondo lattice is obeyed, as can be seen from how the local-moment part and conduction electrons respond to the adiabatic insertion of an external flux. The paramagnetic heavy fermion (P$_L$) phase itself, as described earlier, represents the standard phase of a Kondo lattice.
FIG. 4: Global phase diagram of the heavy fermion systems in the parameter space of $J_K$, the Kondo coupling, and $G$, which characterizes the quantum fluctuations in the local-moment magnetism. P and AF represent the paramagnetic and antiferromagnetic phases of the system, respectively. The subscripts S and L denote the size of the Fermi surface, indicating “small” and “large” respectively. Adapted from Refs. [2, 38, 46, 53].

From the perspective of the paramagnetic heavy fermion phase, the three trajectories of quantum phase transitions delineate a variety of ways for the Landau quasiparticles to be destroyed. Since the initial advancement of the global phase diagram [38], there has been considerable effort in exploring this phase diagram, both theoretically [39, 45, 53, 56] and experimentally [45, 52]. In addition to the Hall effect and quantum oscillations measurements, which we will describe below, thermopower has been utilized to probe the Fermi surface reconstruction and elucidate the global phase diagram [57]. To illustrate the underlying physics, we will for the most part keep our discussion focused on the trajectory I of the global phase diagram, which is represented by the phase diagram, Fig. 3(a), in the
space of temperature \((T)\) and control parameter \((\delta)\).

### III. DYNAMICAL PLANCKIAN SCALING

At the QCP \([\text{c.f. Fig. 3(a)}]\), \(k_B T\) is the only energy scale, and this leads to dynamical properties in which \(\hbar \omega\) scales with \(k_B T\). The dynamical spin susceptibility at the AF wavevector \(Q\) is found \([22, 58]\) to have the following dynamical Planckian scaling form:

\[
\chi(Q, \omega) = \frac{1}{(-i\hbar \omega)^\alpha W^{-1} \left( \frac{\hbar \omega}{k_B T} \right)}. \tag{1}
\]

Here, \(W = A M(\omega/T)\), with \(A\) being a constant prefactor and

\[
M(\omega/T) = \left( \frac{T}{-i\omega} \right)^\alpha \exp \left[ \alpha \psi(1/2 - i\omega/2\pi T) \right], \tag{2}
\]

where \(\psi\) is the digamma function.

The calculated exponent \(\alpha\) is fractional, and is close to being 0.75 for the Ising anisotropic case (between 0.72 and 0.78 when different methods are used for the calculation) \([59-62]\) and about 0.71 for the case with SU(2) spin symmetry \([63]\). At a general wavevector \(q\), the dynamical spin susceptibility takes the following form:

\[
\chi(q, \omega) = \frac{1}{\theta(q) + A (-i\omega)^\alpha M(\omega/T)}. \tag{3}
\]

Here, \(\theta(q) = I_Q - I_q\), where \(I_q\) is the RKKY interaction expressed in wave vector space. The comparable critical exponents, obtained from calculations at the QCPs of the Ising-anisotropic and SU(2)-symmetric Kondo lattice models, imply the universal quantum critical behaviors of the dynamical spin susceptibility.

These theoretical results provide the understanding of the inelastic neutron scattering data measured in CeCu\(_{6-x}\)Au\(_x\) at its quantum critical concentration \(x_c = 0.1\) \([28]\) (see also Ref. \([29]\)). The experiments show not only the \(\hbar \omega/k_B T\) scaling form but also a fractional exponent \(\alpha \approx 0.75\).

The Kondo destruction QCP also predicted the temperature dependence of the NMR relaxation rate. When the hyperfine form factor does not have a strong dependence on the wavevector, the NMR relaxation rate \(1/T_1\) is determined by the local spin susceptibility, leading to \([22, 58]\):

\[
\frac{1}{T_1} \sim \text{constant}. \tag{4}
\]
By contrast, if the hyperfine coupling has a strong $q$-dependence leading to a cancellation of the contributions from the dynamical spin susceptibility near the AF wavevector (as in the well-known case of the oxygen-site NMR relaxation rate of the optimally-hole-doped cuprates \[64\, 65\]), the NMR relaxation rate has the following temperature dependence \[66\]:

$$\frac{1}{T_1} \sim T^\alpha.$$  \hspace{1cm} (5)

The results from the silicon-site NMR experiments in YbRh$_2$Si$_2$ found the NMR relaxation rate to be strongly dependent on the applied magnetic field \[67\, 68\]. When combined with the $\mu$SR results, they have allowed the extraction of the relaxation rate $1/T_1$ at the quantum critical magnetic field \[66\, 67\], and the result is consistent with the prediction of Eq. (4). Whereas the measured copper-site NMR relaxation rate in CeCu$_{6-x}$Au$_x$, at the quantum critical concentration $x_c = 0.1$, is compatible with the expectation of Eq. (5) \[66\].

Importantly, charge response, particularly the optical conductivity, has also been found to be critical \[30\]. This would have been unusual for an SDW QCP, where the singular fluctuations are in the magnetic sector. Theoretically, at the Kondo destruction QCP, the engagement of the Kondo process in the quantum criticality suggests the relevance of the single-particle and charge sectors to quantum criticality \[69\]. The corresponding responses, including the optical conductivity, obey dynamical Planckian scaling. Experimental evidence for the involvement of the charge sector in quantum criticality has also been provided in beta-YbAlB$_4$ \[70\]. Further evidence for a singular charge response has come from other theoretical studies \[71\, 73\].

Finally, as $k_B T$ is the only energy scale at the QCP, the electronic scattering rate takes the form $1/\tau \sim (k_B T)/\hbar$. With the Umklapp scattering that is generically present in quantum critical metals, this relationship leads to strange-metal behavior in the temperature dependence of the electrical resistivity.

### IV. TRANSFORMATION OF LARGE-TO-SMALL FERMI SURFACE AND LOSS OF QUASIPARTICLES

An important characteristic of the Kondo destruction quantum criticality is a transformation of a “large” to “small” Fermi surface across the QCP. This turns out to be intimately connected to a loss of quasiparticles everywhere on the Fermi surface at the QCP.
A. Large to small Fermi surface transformation across the QCP

In the paramagnetic phase, the ground state has a nonzero Kondo-singlet amplitude [Fig. 3(c)]. Correspondingly, composite fermions develop in the low-energy single-electron excitation spectrum [Fig. 3(e)], and the Fermi surface is large in the sense that it incorporates both the conduction electrons and the Kondo-destroyed composite fermions. This is characterized by a nonzero amplitude, $b^*$, for a pole \cite{74, 75} of the conduction-electron self-energy in energy space:

$$\Sigma(k, \omega) = \frac{(b^*)^2}{\omega - \varepsilon_f^*}.$$  \(\text{(6)}\)

Here the self energy is specified via the Dyson equation: $G_c(k, \omega) = [\omega - \varepsilon_k - \Sigma(k, \omega)]^{-1}$. Correspondingly, the conduction-electron Green’s function contains two poles, respectively at energies

$$E_{k}^{\pm} = \frac{1}{2} \left[ \varepsilon_k + \varepsilon_f^* \pm \sqrt{(\varepsilon_k - \varepsilon_f^*)^2 + 4(b^*)^2} \right].$$  \(\text{(7)}\)

They correspond to the heavy fermion bands. The nonzero $b^*$ specifies a Kondo resonance and leads to a large Fermi surface. The quasiparticle weight is $Z_L \propto (b^*)^2$ (Fig. 5). The damping rate has the Fermi liquid $T^2$ and $E^2$ form.

The absence of the Kondo-singlet amplitude on the other side of the QCP [Fig. 3(b)], implies the absence of the composite fermion as a well-defined excitation [Fig. 3(d)]. Correspondingly, the Fermi surface is small [Fig. 3(f) and Fig. 5(a)] in the sense that it incorporates conduction electrons only.

The jump of the large-to-small Fermi surface across the QCP is experimentally testable \cite{22, 24, 58, 76}. Probing this jump has been the subject of intriguing experiments. Across the field-induced QCP in YbRh$_2$Si$_2$, a remarkable sequence of measurements \cite{25, 26} have identified a rapid isothermal crossover in the normal Hall coefficient. The crossover width extrapolates to zero in the $T = 0$ limit. This jump of the Hall coefficient provides evidence for a jump in the Fermi surface across the QCP. Moreover, the location of the crossover maps out a new temperature scale [Fig. 1(d)] \cite{25, 26, 77}. Separately, in CeRhIn$_5$, measurements of the de Haas-van Alphen effect have provided evidence of a sharp jump of the Fermi surface across the pressure-induced QCP \cite{27}. Additional evidence for a Fermi-surface transformation across the QCP has come from Hall effect measurements in pressurized CeRhIn$_5$ \cite{78}.
FIG. 5:  

a The quasiparticle spectral weight, $Z_S$ and $Z_L$, for the small and large Fermi surfaces respectively as a function of $\delta$, showing a large-to-small Fermi surface transformation across the Kondo destruction QCP, and the loss of quasiparticles everywhere on the Fermi surface at the QCP \[9\].

b Illustration of the quantum critical charge fluctuation at the Kondo destruction QCP between an AF state, illustrated by the bottom left box where the staggered red arrows denote the long-range magnetic order of local moments and the blue circles represent the Fermi surface of the conduction electrons,
FIG. 5: (cont’d) and a paramagnetic state, illustrated by the bottom right box where a Kondo singlet is formed between the local moments (red arrow) and the spins of conduction electrons in the form of a spin-triplet combination of particles and holes (red solid and open dots). Adapted from Ref. [30].

B. Loss of quasiparticles at the QCP

As the system approaches the Kondo destruction QCP from the side of large Fermi surface, the Kondo-singlet amplitude goes to zero. The residue of the pole in the conduction-electron self-energy $\Sigma(k, \omega)$ of Eq. (6) vanishes. Correspondingly, the quasiparticle weight on the large Fermi-surface, $Z_L$ vanishes (Fig. 5(a)). As dictated by continuity, the quasiparticle weight on the small Fermi surface, $Z_S$, vanishes as well upon approaching the QCP from the other side of the phase diagram (Fig. 5(a)).

How to probe the destruction of quasiparticles at the QCP? In heavy fermion metals, angle-resolved photoemission spectroscopy (ARPES) measurements have yet to reach adequate resolution to address the issue. Scanning tunneling spectroscopy (STS) in the heavy fermion compound YbRh$_2$Si$_2$ [79] has provided evidence that the single-particle excitations are a part of its quantum criticality [80]. Related evidence has come from the probe of Kondo-driven excitations in the quantum critical regime by time-resolved terahertz spectroscopy [81]. Recently, the current shot noise has been used as a new probe of strongly correlated metals. The observed reduction of the Fano factor provides fairly direct evidence for the loss of quasiparticles in the quantum critical regime of YbRh$_2$Si$_2$ [82, 83].

C. Implications of the singular charge response: dynamical Kondo effect and high-$T_c$ superconductivity

That charge responses are singular and obey dynamical Planckian scaling at a magnetic QCP carries a special significance. It implicates a charge-spin entanglement at the Kondo destruction QCP, in spite of a vanishing amplitude of the Kondo singlet in the ground state. In fact, it has been shown that a dynamical Kondo correlation persists in this regime: a nonzero Kondo coupling in the Hamiltonian dictates that the cross local moment-conduction electron spin correlations operate at nonzero frequencies [63]. More generally, recent work,
both theoretical and experimental, have provided evidence for amplified entanglement at the Kondo-destruction QCP.

Qualitatively, the Kondo destruction QCP features quantum fluctuations between a phase that has a Kondo singlet in the ground state and with an accompanying large Fermi surface on the one hand, and a phase that has no Kondo singlet in the ground state and with a corresponding small Fermi surface on the other hand (Fig. 5 (a,b)). Since the composite fermions carry both charge and spin, the fact that they are critically suppressed at the Kondo destruction QCP means that the charge sector is an inherent component of the quantum criticality (Fig. 5(b)).

This dynamical Kondo effect has important implications for how unconventional superconductivity develops out of the strange-metal normal state. The Kondo destruction quantum criticality is robust in that a large entropy – amounting to a significant portion of Rln 2 per f-site –is encoded in the quantum fluctuation spectrum. The primary degrees of freedom that is involved in this amplified quantum fluctuations are spin in nature. Indeed, a recent calculation using the cluster version of the EDMFT approach has found large intersite spin-singlet correlations in this quantum critical fluid. Through the dynamical Kondo effect, such amplified quantum fluctuations strongly influence the charge sector. In turn, the singlet spin correlations lead to pronounced spin-singlet pairing correlations. The calculations show that this process drives unconventional superconductivity with high-$T_c$: the transition temperature reaches a few percent of the effective Fermi temperature.

V. IMPLICATIONS AND BROADER CONTEXTS

A. Delocalization-localization transition in other correlated systems

We have emphasized how Kondo destruction corresponds to a delocalization-localization transition of the $f$-electrons across the QCP. Such an effect also appears in more complex $f$-electron systems, which involve entwined local degrees of freedom of both spins and orbitals. Localization-delocalization transition of this kind in a metallic environment is emerging as a unifying theme across the correlated material classes.

In the hole doped cuprates, strange-metal behavior is well established. Hall effect measurements in YBa$_2$Cu$_3$O$_y$ (YBCO), when combined with the results in underdoped...
La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) and overdoped Tl$_2$Ba$_2$CuO$_{6+\delta}$ have implicated a transition between phases with carrier concentrations $p$ and $1+p$ near the optimal doping [Fig. 6(a)] [87]. This

![Graph a](image1.png)

![Graph b](image2.png)

![Graph c](image3.png)

![Graph d](image4.png)

**FIG. 6:** a Doping ($p$) dependence of the Hall number ($n_H$) in the hole-doped cuprates [87]. The red and blue lines correspond to $n_H = p$ and $n_H = 1+p$, respectively. b Effective mass $m^*$ enhancement near the optimally doped YBCO under a high magnetic field, with the blue curves denoting $T_c$ at different magnetic fields [88]. c Phase diagram of FeTe$_{1-x}$Se$_x$, with $T_N$, $T_s$ and $T_c$ respectively representing the temperatures for AF, structural and superconducting transitions. The red dots and black diamonds correspond to the temperatures for the maximum ($R_H$ max) and zero ($R_H = 0$) of Hall resistivity. The orbital-selective Mott transition temperature ($T_{OSMT}$) is defined as the temperature at which the photoemission spectral weight of the $d_{xy}$ orbital vanishes. The gradual suppression of $T_{OMST}$ with decreasing $x$ provides evidence for an orbital selective Mott quantum phase transition. d Schematic illustration of a large-to-small Fermi surface transformation as $x$ is decreased in FeTe$_{1-x}$Se$_x$, induced by the de-hybridization of the $d_{xy}$ orbital associated with the OSMT [89].
is accompanied by the observation of mass enhancement in YBCO near optimal doping [Fig. 6(b)] [88]. While this remains an issue of active discussions [99, 100], the notion that the Fermi surface undergoes a small-to-large transformation as a function of hole doping has also been reported based on angle-dependent magnetoresistance (ADMR) measurements in La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$ [101]. Moreover, recent inelastic measurements have implicated a QCP near the optimal doping in LSCO [102]. All these provide evidence for the relevance of a QCP involving an electronic localization-delocalization to the physics of optimally hole-doped cuprates.

A localization-delocalization transition has also been evidenced in other correlated electron material classes. An orbital-selective Mott transition, with a large-to-small Fermi surface transformation, has been shown in FeTe$_{1-x}$Se$_x$ by ARPES measurements [Fig. 6(c,d)] [89]. In a doped Mott insulator of organic charge-transfer salts, some preliminary evidence for a rapid Fermi surface change in a doped Mott insulator has also emerged from Hall [103] and thermoelectric [104] measurements, In moiré systems, related properties are also being uncovered [105–107]. Intriguingly, $T_c/T_0$ of the moiré systems is also on the order of a few percent [105]. Finally, kagome and related metals with frustrated lattices, with active flat bands, have recently emerged as a platform for strange metal behavior [108–110]. A Kondo lattice description [111, 112] allows for strange metallicity in terms of localization-delocalization of electrons in compact molecular orbitals [113]; this approach leads to a phase diagram of temperature and control parameter that has now been supported by an experimentally determined temperature-pressure phase diagram in a kagome metal [114].

B. $t – J$ models with random exchange interactions

In treatment of a Hubbard-Heisenberg $t – J – U$ model with random $J$ interactions, the model can be studied by dynamical equations that are similar to the EDMFT equations [115–117]. The local dynamics are also studied by a Bose-Fermi Anderson model, and properties similar to what we have summarized in this article for the Kondo lattice systems have recently been discussed [115, 117]. In that sense, the physics of the Kondo lattice provides a perspective to view the doped Mott insulator problem as appearing in the cuprates. On the other hand, there is a recognition that the cuprate strange metals need to be addressed starting from the correlations effects in a disorder-free environment [118]. The issue of
treating the cuprate strange metal physics primarily via disordered or clean models of strong electron correlations remains a subject of investigation.

C. Quantum critical point vs. quantum critical phase

One of the topical issues in the realm of quantum criticality concerns the possibility of a quantum critical phase. The global phase diagram for quantum critical heavy fermion metals, described earlier (Sec. II C and Fig. 4), delineates the close relationship between the two possibilities. Here, a quantum critical phase can develop in the regime $P_S$, where quantum fluctuations prevent the system from acquiring a long-range order. The global phase diagram suggests that both the quantum critical phase and the (beyond-Landau) Kondo destruction quantum critical points are descendents of the same “parent”, namely the strong dynamical competition between the Kondo and RKKY interactions. In a heavy fermion compound with geometrical frustration (due to a distorted kagome lattice), CePdAl, a quantum critical phase has been implicated in its pressure-magnetic field phase diagram [49]. Evidence for a quantum critical phase has also come from thermoelectric measurements in an organic charge-transfer salt [104].

In the cuprates, we have already discussed evidence for the relevance of a quantum critical point [87, 88, 101, 102]. In the LSCO family, the evidence for a quantum critical point developing near optimal superconductivity includes the observation of a peak in the specific heat (and, correspondingly, the maximization of entropy), and the presence of low-energy collective spin fluctuations with an energy scale comparable to temperature [102]. On the other hand, experimental observations have revealed that both the linear-$T$ behavior in the resistivity for LSCO [119, 120] and a quadrature scaling in the magnetoresistance for Tl$_2$Ba$_2$CuO$_{6+\delta}$ and Bi$_2$Sr$_2$CuO$_{6+\delta}$ [120] occur at doping levels beyond $p^*$, raising the possibility of a quantum critical phase. Regardless of whether the physics is driven by a quantum critical point or a quantum critical phase, the phenomenology suggests that both collective (spin) fluctuations and the electron localization-delocalization transition are involved in the low-energy physics in the strange metal regime. This bears similarity to the phenomena observed in heavy-fermion systems.
D. Loss of quasiparticles under symmetry constraints

We close by pointing to an exemplary new setting for a loss of quasiparticles. Recent developments have highlighted how strong correlations may drive correlated metallic topology \textsuperscript{[121–123]}. This line of development has pushed the frontier about the interplay between strong correlations and topology \textsuperscript{[124, 125]}. Ref. \textsuperscript{[126]} shows that, for general interacting electron systems, the set of all eigenvectors of the single-electron Green’s function form a representation space of the space group. As such, the space group symmetry constrains the single-particle excitations even when there are no Landau quasiparticles. This allows for the realization of non-Fermi liquid topology in a quantum critical phase, where the quasiparticles are destroyed \textsuperscript{[126]}. Such non-Fermi liquid topology is also shown near a quantum critical point \textsuperscript{[127]}. Conversely, for heavy fermion systems, this raises the prospect that lattice symmetry may further enrich their global phase diagram.

VI. SUMMARY AND OUTLOOK

We have highlighted the theme that quasiparticles are fragile to begin with in strongly correlated metals such as heavy fermion systems and that, in the Kondo destruction quantum criticality, the quasiparticles are lost at the delocalization-localization transition of the \textit{f}-electrons. This theme unveils a hidden Mott transition in an unlikely setting, namely between two metallic phases. As such, the unusualness of the properties here rivals what happens in the case of the standard Mott transition. By certain measure, it is even more striking because, with both sides of the transition being metallic, the Coulomb interactions are screened and it is more natural to have the quantum phase transition to be continuous. The loss of quasiparticles at the Kondo destruction QCP is accompanied not only by such spectacular feature as dynamical Planckian scaling in the spin and charge dynamics but also in a sudden transformation between large and small Fermi surfaces across the QCP.

These salient properties allow for connecting the strange metallicity of heavy fermion metals with that of a variety of strongly correlated systems. The strange metallicity in the cuprates and organic systems naturally develop in the backdrop of the parent Mott insulator phase. In the iron-based superconductors, recent experiments have provided evidence for the proximate orbital-selective Mott phase. Finally, in moiré and frustrated lattice systems,
where strange metal behavior has also been observed, correlated insulating phases may well be considered as the result of electron localization. It appears to be no coincidence that the strange metal behavior develops in all these strongly correlated material classes and that superconductivity emerges with a high transition temperature. By extension, it seems likely that, in most if not all of these systems, strange metallicity is underlined by the loss of quasiparticles on the entire Fermi surface. Exploring the issues in diverse settings and from varied perspectives promises to deepen the understanding about quantum critical metals and to uncover new connections that the strange metal physics of heavy fermion metals may have with those of a broad range of other correlated material classes.
Consider the Kondo lattice Hamiltonian:

\[ H_{KL} = \sum_k \varepsilon_k c_k^\dagger c_k + \sum_{ij} I_{ij} S_i \cdot S_j + \sum_i J_K S_i \cdot \vec{c}_i^\dagger \vec{\sigma} \cdot \vec{c}_i. \]  

(8)

The involved building blocks are the \( f \)-electrons in the form of local moments, \( S_i \), and a band of \( spd \) conduction electrons, \( c_{k\sigma} \) with an energy dispersion \( \varepsilon_k \). At each site \( i \), an AF Kondo interaction \( J_K \) couples the spin of the local moment and that of the conduction electrons, \( s_{c,i} = (1/2)c_i^\dagger \vec{\sigma} c_i \), where \( \vec{\sigma} \) denote the three Pauli matrices. Across the sites, the local moments are coupled to each other via an RKKY interaction \( I_{ij} \).

The calculations that have provided the basis for the notion of Kondo destruction is an extended dynamical mean field theory (EDMFT) \cite{ref128,ref130}; for a recent review, see Ref. \cite{ref131}. This approach treats the dynamical interplay between the Kondo and RKKY interactions of a Kondo lattice [panel (a)]. The EDMFT approach corresponds to a non-perturbative summation of an infinite series of skeleton diagrams. They are generated by an effective action functional and are systematic and conserving.

In this approach, the fate of the Kondo-singlet amplitude is characterized by the nature of local correlation functions. The latter are determined from a Bose-Fermi Kondo/Anderson model:

\[ H_{BFK} = \sum_k E_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_p \omega_p \phi_p^\dagger \phi_p \\
+ J_K S \cdot \frac{c_0^\dagger \vec{\sigma} c_0}{2} + g : S : + \sum_p (\phi_p^\dagger + \phi_{-p}^\dagger) + h_{loc} S^z. \]  

(9)

Here the dispersion \( E_k \) and \( \omega_p \) are associated with a fermionic and a bosonic bath. Their couplings to the local moment have the strength of \( J_K \) and \( g \), respectively. Self-consistency equations are expressed in terms of the local correlators of the Bose-Fermi Kondo model.

The Kondo destruction is seen from the RG flow of the Bose-Fermi Kondo model. The RG is analyzed at one loop from an \( \epsilon \)-expansion, first carried out in the model with Ising anisotropy \cite{ref128} and subsequently extended to the model with SU(2) spin symmetry \cite{ref132,ref133}. For the SU(2) and \( xy \)-spin symmetry cases, the RG analysis has been carried out to two and higher loops \cite{ref134,ref135}. Here, \( \epsilon \) describes the power-law spectrum of the bosonic bath:

\[ \rho_b(\omega) \equiv \sum_p \delta(\omega - w_p) \propto |\omega|^{1-\epsilon} \quad \text{for } |\omega| < \Lambda. \]  

(10)
Panel (b) illustrates the flow that is associated with the RG beta-functions in the \( \epsilon \)-
expansion, for a positive \( \epsilon \). In the absence of the bosonic Kondo coupling (i.e. when
\( g = 0 \)), any nonzero \( J_K \) flows away from the decoupled fixed point and towards the Kondo
fixed point \[136\]. The bosonic coupling \( g \) leads to two new fixed points and a separatrix
in the \( J_K - g \) plane. The critical (red) fixed point controls the physics on the separatrix,
corresponding to a critical destruction of the Kondo phase. On the right of the separatrix,
the system flows to a Kondo-destroyed (green) fixed point where \( J_K \) vanishes altogether.
The nature of the critical (red) Kondo destruction fixed point is to be contrasted with
that of the Kondo fixed point. There are three key distinctions. First, the Kondo fixed
point describes a phase with a nonzero amplitude of the Kondo singlet in the ground state,
whereas, in the phase associated with the Kondo destruction fixed point, the Kondo-singlet
amplitude vanishes in the ground state.

Second, and concomitantly, the distinction appears in the low-lying excitations. For the
Kondo destruction (red) fixed point, the vanishing of the Kondo-singlet amplitude in the
ground state implies that the weight of any Landau quasiparticle goes to zero. This
can also be explicitly seen from the finite-size spectrum of the many-body excitations as
determined by the numerical-renormalization-group (NRG) approach: the spectrum can
no longer be fit in terms of a combination of any quasiparticles \[137\].

Third, the Kondo destruction fixed point is critical and interacting (as opposed to being Gaussian). Thus, \( k_B T \) is the only energy scale. Accordingly, singular responses such
as the local spin and charge susceptibilities have a dynamical Planckian \( (\hbar \omega / k_B T) \) scal-
ing. This has been seen both in the Kondo destruction fixed point of the Bose-Fermi
Kondo/Anderson model as determined by a dynamical-large-\( N \) (where the index \( N \) ap-
ppears in the spin channel) approach \[69, 71\] and in the SU(2) case \[69, 137\].

We note on the extra fixed points that exist beyond the \( \epsilon \)-expansion. It turns out that, for
small \( \epsilon \), the SU(2) Bose-Fermi Kondo/Anderson model has more fixed points beyond those
that are accessed by the \( \epsilon \)-expansion method. Panel (c) shows the RG flow diagram when
\( \epsilon \) is sufficiently small (or \( s = 1 - \epsilon \) is sufficiently large) \[138\]. As \( \epsilon \) further increases, the
fixed points are pair-wise annihilated, and this has recently been understood analytically
based on a \( 1/S \) expansion (where \( S \) is the spin size) \[139\].
Schematic illustration of the Kondo lattice model, which contains the Kondo coupling $J_K$ between local moments and conduction electrons, hopping parameters $t_{ij}$ between the conduction electrons, and RKKY interactions $I_{ij}$ between the local moments. Adapted from Ref. [22]. RG flow of the BFKM from studies based on an $\epsilon$-expansion [134] and continuous time quantum Monte Carlo method [138] and a large-$S$ expansion approach [139] at $1 > s > s^*$. The RG flow diagram illustrates two categories of stable fixed points: one associated with an infinite $J_K$, representing the Kondo phases, and the other characterized by $J_K = 0$, indicating the local moment phases. An unstable fixed point corresponds to a Kondo-destruction quantum critical point.
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