Light Quark Masses and the CP violation parameter $\epsilon'/\epsilon$

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We present estimates of light quarks masses using lattice data. Our main results are based on a global analysis of all the published data for Wilson and Staggered fermions, both in the quenched approximation and with $n_f = 2$ dynamical flavors. The Wilson and Staggered results agree after extrapolation to the continuum limit for both the $n_f = 0, 2$ theories. Our best estimates, in the $\overline{MS}$ scheme at scale 2 GeV, are $\overline{m} = 3.2(4)\text{ MeV}$ and $m_s = 90(20)\text{ MeV}$ in the quenched approximation, and $\overline{m} \sim 2.7\text{ MeV}$ and $m_s \sim 70\text{ MeV}$ for the $n_f = 2$ theory. These estimates are significantly smaller than phenomenological estimates based on sum rules, but maintain the ratios predicted by chiral perturbation theory ($\chi$PT). Along with the new estimates of 4-fermion operators, lower quark masses have a significant impact on the extraction of $\epsilon'/\epsilon$ from the Standard Model.

1. LIGHT QUARK MASSES

The masses of light quarks $m_u$, $m_d$, and $m_s$ are three of the least well known parameters of the Standard Model. These quark masses have to be inferred from the masses of low lying hadrons. $\chi$PT relates the masses of pseudoscalar mesons to $m_u$, $m_d$, and $m_s$, however, the presence of the unknown scale $\mu$ in $\mathcal{L}_{\chi PT}$ implies that only ratios of quark masses can be determined. For example $2m_s/(m_u + m_d) \equiv m_s/\overline{m} = 25$ at lowest order, and 31 at next order [13]. Latest estimates using QCD sum rules give $m_u + m_d = 12(1)\text{ MeV}$ [3]. However, as discussed in [4], a reanalysis of existing data do not show any significant deviation from linearity. One thus has to use $M_V$ in order to extract $m_s$. Details of our analysis and of the global data used are given in [5].

For Wilson fermions the lattice quark mass, defined at scale $q^*$, is taken to be $m_L(q^*) = (1/2\kappa - 1/2\kappa_c)$. For staggered fermions $m_L(q^*) = m_0$, the input mass. The $\overline{MS}$ mass at scale $\mu$ is $m_{\overline{MS}}(\mu) = Z_m(\mu)a m_L(\mu)$, where $Z_m$ is the mass renormalization constant relating the lattice and the continuum regularization schemes at scale $\mu$, and $\lambda = g^2/16\pi^2$. In calculating $Z_m$, a la Lepage-Mackenzie, we use $\alpha_{\overline{MS}}$ for the lattice coupling, use “horizontal” matching, i.e. $\mu = q^* = 1/a$, and do tadpole subtraction. We find that the results are insensitive to the choice of $q^*$ in the range $0.86/a - \pi/a$ and to whether or not tadpole subtraction is done. Once $m_{\overline{MS}}(\mu)$ has been calculated, its value at any other scale $Q$ is given by the two loop running. We quote all results at $Q = 2\text{ GeV}$.

We extrapolate the lattice masses to $a = 0$ using the lowest order corrections (Wilson are $O(a)$ and Staggered are $O(a^2)$). In the quenched fits...
Table 1
Summary of results in MeV in $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV. The label $W(0)$ stands for Wilson with $n_f = 0$. An additional uncertainty of $\sim 10\%$ due to the uncertainty in the lattice scale $a$ is suppressed.

| Label | $\overline{\text{MS}}$ (2 GeV) | $m_s(M_K)$ | $m_s(M_{\phi})$ | $m_s(M_{K^*})$ |
|-------|-------------------------------|------------|----------------|----------------|
| $W(0)$ | 3.3(4) | 83(10) | 96(10) | 76(20) |
| $S(0)$ | 3.1(1) | 78(3) | 96(2) | 87(2) |
| $W(2)$ | 2.5(3) | 63(8) | 77(10) | 78(22) |
| $S(2)$ | 2.9(3) | 73(8) | 66(6) | 59(6) |

we omit points at the stronger couplings ($a > 0.5$ GeV$^{-1}$) because we use only the leading correction in the extrapolation to $a = 0$, and because the perturbative matching becomes less reliable as $\beta$ is decreased. The bottom line is that we find that the leading corrections give a good fit to the data, and in the $a = 0$ limit the two different fermion formulations give consistent results.

Our final results are summarized in Table 1.

The global data for $\overline{\text{m}}$ and the extrapolations to $a = 0$ for Wilson are shown in Fig. 1. Using the average of quenched estimates given in Table 1 we get $\overline{\text{m}}(\overline{\text{MS}}, 2 \text{ GeV}) = 3.2(4)(3) \text{ MeV}$ (quenched), where the first error estimate is the larger of the two extrapolation errors, and the second is that due to the uncertainty in the scale $a$.

The pattern of $O(a)$ corrections in the unquenched data ($n_f = 2$) is not clear and we only consider data for $\beta \geq 5.4$. The strongest statement we can make is qualitative; at any given value of the lattice spacing, the $n_f = 2$ data lies below the quenched result. Taking the existing data at face value, we find that the average of the Wilson and staggered values are the same for the choices $\beta \geq 5.4$, $\beta \geq 5.5$, or $\beta \geq 5.6$. We therefore take this average $\overline{\text{m}}(2 \text{ GeV}) \approx 2.7 \text{ MeV}$ ($n_f = 2$ flavors), as the current estimate. To obtain a value in the physical case of $n_f = 3$, the best we can do is to assume a behavior linear in $n_f$. In which case extrapolating the $n_f = 0$ and $2$ data gives $\overline{\text{m}}(2 \text{ GeV}) \approx 2.5 \text{ MeV}$ ($n_f = 3$ flavors).

We stress that this extrapolation in $n_f$ is extremely preliminary.

We determine $m_s$ using the three different mass-ratios, $M_{K^*}/M_{\pi}^2$, $M_{K^*}/M_{\rho}$, and $M_{\phi}/M_{\rho}$. Using a linear fit to the pseudo-scalar data constrains $m_s(M_K) = 25 \overline{\text{m}}$. Using the vector mesons $M_{K^*}^2$ and $M_{\phi}$ gives independent estimates. The quenched data and extrapolation to $a = 0$ of $m_s(M_{\phi})$ are shown in Fig. 2. The average of Wilson and staggered values are $m_s(M_{\phi}) = 96(10) \text{ MeV}$ and $m_s(M_{K^*}) = 82(20) \text{ MeV}$ where the errors are taken to be the larger of Wilson/staggered data. From these we get our final estimate

$m_s = 90(15) \text{ MeV}$ (quenched).

The $n_f = 2$ data shows a pattern similar to that for $\overline{\text{m}}$. Therefore, we again take the average of values quoted in Table 1 to get

$m_s = 70(11) \text{ MeV}$ ($n_f = 2$).
The error estimate reflects the spread in the data. Qualitatively, the data show three consistent patterns. First, agreement between Wilson and Staggered values. Second, for a given value of $a$ the $n_f = 2$ results are smaller than those in the quenched approximation. Lastly, the ratio $\bar{m}/m_s(M_\phi)$ is in good agreement with the next-to-leading-order predictions of chiral perturbation theory for both the $n_f = 0$ and 2 estimates. It is obvious that more lattice data are needed to resolve the behavior of the unquenched results. However, the surprise of this analysis is that both the quenched and $n_f = 2$ values are small and lie at the very bottom of the range predicted by phenomenological analyses.

2. CP VIOLATION and $\epsilon'/\epsilon$

A detailed analysis of 4-fermion matrix elements with quenched Wilson fermions at $\beta = 6.0$ is presented in [6]. The methodology, based on the expansion of the matrix elements in powers of the quark mass and momentum, is discussed in [7]. Our estimates in the NDR scheme at $\mu = 2$ GeV are

\[
\begin{align*}
B_K &= 0.68(4), \\
B_D &= 0.78(1), \\
B_7^{3/2} &= 0.58(2), \\
B_8^{3/2} &= 0.81(3).
\end{align*}
\]  

(7)

The errors quoted are statistical. The major remaining sources of errors in these estimates are lattice discretization and quenching.

To exhibit the dependence of the Standard Model (SM) prediction of $\epsilon'/\epsilon$ on the light quark masses and the $B$ parameters we write

\[
\epsilon'/\epsilon = A \left( c_0 + c_6 B_6^{1/2} M_r + c_8 B_8^{3/2} M_r \right),
\]

(8)

where $M_r = (158 \text{ MeV}/(m_s + m_d))^2$. For reasonable choices of SM parameters Buras et al. estimate $A = 1.3 \times 10^{-4}$, $c_0 = -1.3$, $c_6 = 7.9$, $c_8 = -4.3$ [8]. Thus, to a good approximation $\epsilon'/\epsilon \propto M_r$; and increases as $B_8^{3/2}$ decreases. As a result, our estimates of $\bar{m}, m_s, B_8^{3/2}$ increase $\epsilon'/\epsilon$ by roughly a factor of three compared to previous analysis, i.e. from $3.6 \times 10^{-4}$ to $\sim 10.4 \times 10^{-4}$.

This revised estimate lies in between the Fermilab E731 (7.4(5.9) $\times 10^{-4}$) and CERN NA31 (23(7) $\times 10^{-4}$) measurements and provides a scenario in which direct CP violation can be explained within the Standard Model.

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