On the flux jumps in the flux creep regime of type - II superconductors

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Abstract

The spatial and temporal evolution of small perturbations of the temperature and electromagnetic field for the superconducting slab placed in a parallel magnetic field in the regime of thermally activated flux creep is studied. On the basis of theoretical analysis of thermal diffusion and Maxwell’s equations we find the onset field $B_j$ of the flux-jump instability field and its dependence on the external magnetic-field sweep rate $\dot{B}_e$.

Key words: thermal and electromagnetic perturbations, critical state, flux creep.

Introduction

As we know that the conventional II type superconductor are characterized by a high value of the critical current and upper critical field, and is therefore widely used for various technological applications. One of the main topic at the technical application of superconductors is their stability with respect to flux jumps [1-3]. The flux jumps or avalanches are associated with a sudden puncture of magnetic flux into the volume of the sample, and in turn, an increase of temperature and the decrease of critical current density. The jump phenomena have been observed in conventional hard superconductors [1-6], as well as in high-temperature superconductors, recently [7, 8].

The critical state stability against flux jumps in hard and composite superconductors has been discussed in a number of theoretical and experimental papers [1-6]. The general concept of the thermomagnetic instabilities in type-II superconductors was developed in literature [4, 5]. The dynamics of small thermal and electromagnetic perturbations, whose development leads to the flux jump, have been investigated theoretically in detail by Mints and Rakhmanov [5]. The authors have found the stability criterion for the flux jumps in the framework of adiabatic and dynamic approximations in the viscous flux flow regime of type-II superconductors. Conventionally, thermomagnetic instabilities were interpreted in terms of thermal runaway triggered by local energy dissipation in the sample [5]. According to this theory, any local instability causes a small temperature rise, the critical current is decreased and magnetic flux moves much easily under the Lorentz force. The additional flux movement dissipates more energy further increasing temperature. This positive feedback loop may lead to a flux jumps in the superconductor sample.

Theoretical investigations of small thermal and electromagnetic perturbations in a various regimes with a various current-voltage characteristics is one of key problems of electrodynamics of superconductors. Moreover, there is a special interest to this problem in the flux creep regime of type-II superconductors, since all superconducting devices used for a large scale applications operate under this regime. In our previous work, the dynamics of small thermal and electromagnetic perturbations has been studied in the flux flow regime, where voltage current-current characteristics of hard superconductor is described by linear dependence of $j(E)$ at sufficiently large values of electric field $E$ [9]. In the region of weak electric fields the current-voltage characteristics $j(E)$ of superconductors is highly nonlinear due to thermally activated dissipative flux motion. In the flux creep regime a differential conductivity $\sigma$ strongly depends on the electric field $E$. The nonlinear conductivity $\sigma(E)$ significantly affects the dynamics of thermal and electromagnetic processes in superconductors. In particular, it results in the dependence of the flux-jump field $B_j$ on variation of external parameters, in particular on the magnetic field sweep rate $\dot{B}_e$.

A theoretical analyze of the flux jumping in the flux creep regime, where the current-voltage characteristics of a sample is a nonlinear have been carried out recently by Mints [10] and by Mints and Brandt [11]. However, a careful study the dynamics of the thermal and electromagnetic perturbations in the regime weak electric field with nonlinear current-voltage characteristics associated with flux creep is still lacking.

Objectives

We report the results of theoretical simulations of the spatial and temporal evolution of small perturbations of the temperature and electromagnetic field for the superconducting slab placed in a parallel magnetic field in the regime of thermally activated flux creep. On the basis of theoretical analysis of thermal diffusion and Maxwell equations we find the onset field $B_j$ of the flux-jump instability field and its dependence on the external magnetic-field sweep rate $\dot{B}_e$. It is assumed that the magnetic diffusion is slower than the thermal diffusion.

1. Formulation of the problem

Mathematical problem of theoretical study the dynamics of thermal and electromagnetic perturbations in a superconductor sample in the flux creep regime can be formulated on the basis of a system nonlinear diffusion-like equations for the thermal and electromagnetic field perturbations with account nonlinear relationship between the field and current in superconductor sample. Bean [1] has proposed the critical state model which is successfully used to describe magnetic properties of type II superconductors. According to this model, the distribution of the magnetic flux density $\vec{B}$ and the transport current density $\vec{j}$ inside a superconductor is given by a solution of the equation

$$\text{rot}\vec{B} = \vec{j}.$$  \hspace{1cm} (1)

When the penetrated magnetic flux changes with time, an electric field $\vec{E}$ is generated inside the sample according to Faraday’s law

$$\text{rot}\vec{E} = -\frac{d\vec{B}}{dt}.$$ \hspace{1cm} (2)

The temperature distribution in superconductor is governed by the heat conduction diffusion equation

$$\nu(T)\frac{dT}{dt} = \nabla \left[ \kappa(T) \nabla T \right] + j(E).$$ \hspace{1cm} (3)

Here $\nu = \nu(T)$ and $\kappa = \kappa(T)$ are the specific heat and thermal conductivity, respectively. The above equations should be supplemented by a current-voltage characteristics of superconductors, which has the form

$$j = j_c(T, B) + j(E).$$

In order to obtain analytical results of a set Eqs. (1)-(3), we suggest that $j_c$ is independent on magnetic field induction $B$ and use the Bean critical state model $j_c = j_c(B_0, T)$ [1], where $B_0$ is the external applied magnetic field induction. We assume that the dependence of the critical current density on temperature of the sample is linear [9]. For the sake of simplifying of the calculations, we perform our calculations on the assumption of negligibly small heating $(T - T_0 \ll T_e - T_0)$ and assume that the temperature profile is a constant within the across sample and thermal conductivity $\kappa$ and heat capacity $\nu$ are independent on the temperature profile; where $T_0$ and $T_e$ are the equilibrium

}\hspace{1cm} (4)

...
and critical temperatures of the sample, respectively, [9]. We shall study the problem in the framework of a macroscopic approach, in which all length scales are larger than the flux-line spacing; thus, the superconductor is considered as an uniform medium.

The system of differential equations (1)-(3) should be supplemented by a current-voltage curve \( j = j(E) \). In the flux creep regime the current-voltage characteristics of type II conventional superconductors is highly nonlinear due to thermally activated dissipative flux motion [12, 13]. For the logarithmic current dependence of the potential barrier \( U(j) \), proposed by [14] the dependence \( j(E) \) has the form

\[
j = j_c \left[ \frac{E}{E_b} \right]^{1/n}.
\]  

(4)

where the constant parameter \( n \) depends on the pinning regimes and can vary widely for various types of superconductors. In the case \( n = 1 \) the power-law relation (4) reduces to Ohm’s law, describing the normal or flux-flow regime [15]. For infinitely large \( n \), the equation describes the Bean critical state model \( j = j_c \) [1]. When \( 1 < n < \infty \), the equation (4) describes nonlinear flux creep. In this case the differential conductivity \( \sigma \) is determined by the following expression

\[
\sigma = \frac{d\gamma}{dE} = \frac{j_c}{nE_b}.
\]  

(5)

According to relation (5) the differential conductivity decreases with the increasing of the background electric field \( E \), and strongly depends on the external magnetic field sweep rate \( E_b \). Therefore the stability criterion also strongly depends on the differential conductivity \( \sigma \). For the typical values of \( j_c \approx 10^8 \text{A/cm}^2 \), \( E_b = 10^{-2} \text{V/cm} \) we obtain \( \sigma \approx 10^{10}/\text{V/cm} \). It follows from this estimation [11] that the differential conductivity \( \sigma \) which determines the dynamics of thermomagnetic instability for is high enough. We assume, for simplicity, that the value of \( n \) temperature and magnetic-field independent.

2. Basic equations

Let us formulate a differential equations governing the dynamics of small temperature and electromagnetic field perturbation in a superconductor sample. We study the evolution of the thermal and electromagnetic penetration process in a simple geometry - superconducting semi-infinite sample \( x \geq 0 \). We assume that the external magnetic field induction \( B_0 \) is parallel to the \( z \)-axis and the magnetic field sweep rate \( B_s \) is constant. When the magnetic field with the flux density \( B_0 \) is applied in the direction of the \( z \)-axis, the transport current \( j(t) \) and the electric field \( E(x,t) \) are induced inside the slab along the \( y \)-axis. For this geometry the spatial and temporal evolution of small thermal \( \Delta T(x,t) \) and electromagnetic field \( \delta E(x,t) \) perturbations

\[
\delta T = \Theta(x) \exp[\gamma t],
\]

\[
\delta E = \epsilon(x) \exp[\gamma t],
\]

is described by Maxwell equations coupled to the thermal diffusion equation

\[
\nu \gamma \Theta = \epsilon \frac{d^2 \Theta}{dx^2} + j_c \epsilon,
\]

\[
\frac{d^2 \epsilon}{dx^2} = \frac{4\pi}{c^2} \gamma \left[ j_c \frac{E_b}{nE_b} \epsilon - \frac{j_c}{T_c - T_0} \Theta \right],
\]

(8)

where \( \gamma \) is the eigenvalue of the problem to be determined. It is clear that the rate \( \gamma \) characterizes the time development of the instability. In the case when \( \nu \gamma T \geq 0 \), small thermal and electromagnetic perturbations increase and the stability margin corresponds to the case when \( \gamma = 0 \). It should be noted that the nonlinear diffusion-type equations (8) and (9), totally determine the problem of the space-time distribution of the temperature and electromagnetic field profiles in the flux creep regime with a nonlinear current-voltage characteristics in the semi-infinite sample.

3. Dispersion relation

Let us derive the dispersion equation to determine the eigenvalue problem. As we know [5] that a nature of the flux jumps depends on the competition between diffusive and dissipative processes through the dimensionless parameter

\[
\tau = \frac{4\pi \kappa}{c^2} - \frac{D_t}{D_m},
\]

where \( D_t = \kappa/\nu \) is the thermal diffusivity and \( D_m = c^2/4\pi \sigma \) the magnetic diffusivity coefficients, respectively. Therefore the flux instability criterion is determined mainly by the relation of the magnetic \( D_m \) and thermal \( D_t \) diffusion coefficients. As we have mentioned above, the differential conductivity \( \sigma(E) \), which determines the dynamics of the instability is high in the flux creep regime and the parameter \( \tau \) is high enough, also. It is clear that this picture for the flux jumps corresponds to the limiting case \( \tau \gg 1 \). Consequently, it can be assumed that the initial rapid heating stage of a flux jump takes place on the background of a "frozen-in" magnetic flux. Therefore, under this dynamic approximation, we obtain from (9) the relation between electric field \( \epsilon(x,t) \) and temperature \( \Theta(x,t) \) perturbations in the following form

\[
\frac{j_c}{nE_b} - \frac{j_c}{T_c - T_0} \Theta = 0.
\]

(10)

We notice that the last relation between \( \epsilon(x,t) \) and temperature \( \Theta(x,t) \) has been derived in the assumption that the decrease of the critical current density \( j_c \) resulting from a temperature perturbation \( \Theta(x,t) \) compensates with increase of the resistive current density \( j_r \) resulting from an electric field perturbation \( \epsilon(x,t) \), so the total current density remains constant [5]. Upon substituting the expression (10) into the equation (8) and excluding the variable \( \epsilon(x,t) \) one can get the differential equation for the distribution of thermal perturbation, which can be conveniently presented in the following dimensionless form

\[
\frac{d^2 \Theta}{d\rho^2} - \rho \Theta = 0.
\]

(11)

Here we introduced the following dimensionless variables

\[
\rho = \frac{\gamma - z}{r}, \quad \frac{1}{r} = \left[ \frac{aL^2}{\kappa E_L} \right]^{1/3}, \quad E_L \approx B_s L, \quad a = \frac{j_c}{T_c - T_0}, \quad z = \frac{x}{L}.
\]

Here \( L = \frac{cB_s}{4\pi j_c} \) is the magnetic field penetration depth. Thus, the condition of existence of a non-trivial solutions of Eq. (11) allows to define the spectrum of eigenvalues of \( \gamma \) and the instability threshold, accordingly. The equation (11) has an exact solution in terms of Airy functions given as the following form

\[
\Theta(\rho) = c_1(\rho)Ai(\rho) + c_2(\rho)Bi(\rho),
\]

(12)

where \( Ai(\rho) \) and \( Bi(\rho) \) are the Airy functions. Here constants of integration \( c_1 \) and \( c_2 \) are determined from the thermal boundary conditions. Substituting the last solution (12) into the thermal boundary conditions (see, for example [9]) we find that \( c_2 = 0 \) and \( \Theta(\rho) = c_1(\rho)Ai(\rho) \). Applying the second boundary condition \( \Theta(1) = 0 \) we get an equation to determine the eigenvalues of the problem

\[
J_{2/3}(a_n) = J_{-2/3}(a_n),
\]

where \( a_n \) are the zeros of the Bessel function and growing with increasing \( n \); For example, for \( n = 1 \) the stability criterion is presented as

\[
\lambda_1 = \pi^2/3 \gamma_1.
\]

(13)

Using the value for the magnetic field penetration depth, we can easily obtain from (13) an expression for the threshold magnetic field \( B_j \) at which the branching instability occurs

\[
B_j = \frac{4\pi j_c}{c} \sqrt{\frac{\kappa}{\pi nE_L}}.
\]

(14)

Let us now estimate the threshold field for a typical values of parameters \( j_c \approx 10^7 \text{A/m}^2 \), \( T_c - T_0 \approx 10 \text{K} \), \( \kappa \approx 10^{-1} \text{W/K} \cdot \text{m} \), \( n=10 \). The
background electric field $E_L \approx B_L L$, induced by the magnetic-field variation $B_L \approx 10^{-2} \div 10^{-5}$ T/s is of the order of $E_L \approx 10^{-4} \div 10^{-5}$ V/m for the value of $L = 0.01$ m. We can easily estimate that the threshold field has the value $B_j \approx 1 \div 3T$.

4. Discussion

Experimentally, the background electric field is created by the sweeping rate of applied magnetic field $B_e$. As can be seen from the relation (14) the threshold field $B_j$ is decreased with the increasing of background electric field. It is noticeable that the dependence of the flux-jump field $B_j$ on the sweeping rate $B_e$ of the applied magnetic field has been verified by a numerous experiments [8, 16-18]. An intensive numerical analysis on the sweep rate dependence of the threshold field has been performed recently in [19]. Recent magnetization measurements [8, 16] have shown that the value of the threshold field $B_j$ decreases as the sweep rate $B_e$ increases. A theoretical investigations on the dependence of threshold field on the varying external magnetic field has been performed in detail recently by Mints [10]. Within the framework of the flux jump instability theory [4, 5] a rapid variation of the applied magnetic field acts as the instability-driving perturbation, and that threshold field $B_j$ should decrease with increasing the sweeping rate $B_e$ [8]. The numerical studies [19] have demonstrated that the flux jumps take place when the sweep rate $B_e$ increases up to a certain value, where the number of jumps increases with the sweep rate $B_e$. As the sweep rate further increases, these simulation results show that the flux-jump field decreases and approaches a saturation value, which is fairly close to the experimental value of about $1.2$ T/s [8]. However, as has been mentioned in [16], an experimental investigations on the dependence of the threshold field for the flux jump field $B_j$ on the external magnetic field sweep rate $B_e$ is very little. Experimentally, can be observed a complex behavior of dependence of the threshold field $B_j$ on the sweep rate $B_e$. The results of experiments of Ref. [16] demonstrated that $B_j$ is independent of the sweep rate in a defined range of temperatures. Thus, the near independence of $B_j$ on the sweeping rate remains to be explained. In some conventional superconductors both the independence of $B_j$ on the sweeping rate [3, 4] and its growth at a high sweeping rate [20] were detected. It has been suggested [4] that a nonuniform heating may be responsible for such an effect. However, theoretical understanding of the thermomagnetic instabilities at such conditions is still lacking. Note, however, that some details of the local field behavior depend indeed on the sweeping rate, as, for example, the number and amplitude of the jumps. Gerber at. al. [18] have demonstrated that at low values of the sweep rates the number of flux jumps decreases as sweep rate increases. At still high sweep rates the amplitude of flux jumps becomes independent of the sweep rate and saturates to the limit with further increasing sweep rate.

In Fig.1 we have demonstrated the dependence of the threshold field $B_j$ on the external magnetic field sweep rate. As can be seen, the value of $B_j$ decreases as the sweep rate increases. As the sweep rate increases the value of $B_j$ decreases and it tends to saturate at high sweep rates [8]. Magnetic field dependence of the critical current density only slows down the decrease of the field $B_j$ with increasing external magnetic field sweep rate [10, 11]. We note that for the case Kim-Anderson model [12], the absolute value of the exponent in the power formula decreases from $1/2$ to $1/3$, so $B_j \sim B_e^{-1/3}$ [10].

Let us qualitatively estimate the temperature dependence of the first flux jump field $B_j(T)$. In order to compare the last formula with experimentally determined first flux jump field, one must know the temperature dependencies of the critical current density and the specific heat of the sample. As has been mentioned in literature [5, 16] a quantitative estimation of $j_c(T)$ from the available experimental and theoretical models is difficult because of the uncertainty in the values of the critical current $j_c$ at a given field and temperature. To determine $j_c(T)$, different approaches have been taken. Empirically, the critical current density $j_c(T)$ can be presented in the form

$$j_c = j_0 (1 - t^m)^n,$$

where $1 < m < 2$, $t = T/T_c$. The different exponents $n=1$ and $2$ refer to the most common cases discussed in the literature, where the critical current exhibits a linear and a quadratic dependence on $T/T_c$. There is experimental evidence [21], which indicates that the temperature dependence of the critical current density approximately linear at low temperatures. However, at higher temperatures, where flux creep effects are dominant, the temperature dependence of the critical current density can be presented as

$$j_c(T) = j_c(0) (1 - t^2)^2.$$

commonly accepted in literature [6]. Assuming that the thermal conductivity is a linear function of temperature, we can easily obtain an expression for the temperature dependence of the threshold field $B_j(T)$ (Fig 2.)

$$B_j(T) \approx \sqrt{t (1 - t^2)^2}.$$

Fig.2. The temperature dependence of the flux jump field

Conclusion

We have performed a theoretical study of dynamics of small thermal and electromagnetic perturbations in type-II superconductors in the flux creep regime with a nonlinear current-voltage characteristics. For this purpose, the space-time evolution of temperature and electric field was calculated using the heat diffusion equation, coupled with
Maxwell’s equations and material law, assuming that the applied magnetic field is directed parallel to the surface of the sample. We found the threshold field \( B_j \) for the occurrence of thermomagnetic instability assuming that the heat flux diffusion is considerably faster than the magnetic flux diffusion in superconductor. The obtained stability criterion for the thermomagnetic flux jumps demonstrates the extremely high sensitivity of the threshold field \( B_j \) on the values of the critical current density \( j_c \), thermal conductivity \( \kappa \), and external magnetic field sweep rate \( \dot{B}_e \). Thus, the stability condition (14) for the thermomagnetic flux jumps directly reflects the magnetic sweep rate \( \dot{B}_e \) dependence on the threshold field \( B_j \). It follows from the criterion that the value of the threshold field \( B_j \) is inversely proportional to the square root of the magnetic-field sweeping rate \( \dot{B}_e \). Therefore, with the increase of sweeping rate \( \dot{B}_e \) the threshold field \( B_j \) decreases. Finally, we have discussed the temperature dependence of the threshold field, briefly.

**Acknowledgements**

This study was supported by the NATO Reintegration Fellowship Grant and Volkswagen Foundation Grant. Part of the computational work herein was carried on in the Condensed Matter Physics at the Abdus Salam International Centre for Theoretical Physics.

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