Continual Reinforcement Learning with Multi-Timescale Replay

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Abstract

In this paper, we propose a multi-timescale replay (MTR) buffer for improving continual learning in RL agents faced with environments that are changing continuously over time at timescales that are unknown to the agent. The basic MTR buffer comprises a cascade of sub-buffers that accumulate experiences at different timescales, enabling the agent to improve the trade-off between adaptation to new data and retention of old knowledge. We also combine the MTR framework with invariant risk minimization [Arjovsky et al., 2019] with the idea of encouraging the agent to learn a policy that is robust across the various environments it encounters over time. The MTR methods are evaluated in three different continual learning settings on two continuous control tasks and, in many cases, show improvement over the baselines.

1 Introduction

Artificially intelligent agents that are deployed in the real world have to be able to learn from data streaming in from a nonstationary distribution and incrementally build on their knowledge over time, while operating with limited computational resources; this is the challenge of continual learning [Ring, 1993]. Artificial neural networks (ANNs), however, have long been known to suffer from the problem of catastrophic forgetting [McCloskey and Cohen, 1989], whereby, in a context where the data distribution is changing over time, training on new data can result in abrupt erasure of previously acquired knowledge. This precludes them from being able to learn continually.

Typically, the testing of methods for mitigating catastrophic forgetting has been conducted in the context of training on a number of distinct tasks in sequence. A consequence of this format for evaluation is that many continual learning techniques make use of the boundaries between tasks in order to consolidate knowledge during training [Ruvolo and Eaton, 2013; Kirkpatrick et al., 2017; Zenke et al., 2017]. In the real world, however, changes to the data distribution may happen gradually and at unpredictable timescales, in which case many of the existing techniques are simply not applicable, prompting the community to pose task-agnostic and task-free continual learning as desiderata for our agents [clw, 2016, 2018].

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Reinforcement learning [Sutton and Barto, 1998] is a paradigm that naturally poses these challenges, where the changes to the data distribution can occur unpredictably during the training of a single task and can arise from multiple sources, e.g.: (i) correlations between successive states of the environment, (ii) changes to the agent’s policy as it learns, and (iii) changes to the dynamics of the agent’s environment. For these reasons, catastrophic forgetting can be an issue in the context in deep reinforcement learning, where the agents are neural network-based. One commonly used method to tackle (i) is experience replay (ER) [Lin, 1992, Mnih et al., 2015], whereby the agent’s most recent experiences are stored in a first-in-first-out (FIFO) buffer, which is then sampled from at random during training. By shuffling the experiences in the buffer, the data are then identically and independently distributed (i.i.d.) at training time, which prevents forgetting over the (short) timescale of the buffer since the distribution over this period is now stationary.

1.1 Experience replay for continual reinforcement learning

The community has naturally investigated whether ER can be used to mitigate forgetting over the longer timescales that are typically associated with continual learning, particularly because it does not necessarily require prior knowledge of the changes to the data distribution. One key observation that has been made in both a sequential multi-task setting [Isele and Cosgun, 2018, Rolnick et al., 2019], as well as in a single task setting [de Bruin et al., 2015, 2016, Zhang and Sutton, 2017, Wang and Ross, 2019], is that it is important to maintain a balance between the storage of new and old experiences in the buffer. By focusing just on recent experiences, the agent can easily forget what to do when it revisits states it has not seen in a while, resulting in catastrophic forgetting and instability; by retaining too many old experiences, on the other hand, the agent might focus too much on replaying states that are not relevant to its current policy, resulting in a sluggish and/or noisy improvement in its performance.

In this paper, we propose a multi-timescale replay (MTR) buffer to improve continual reinforcement learning, which consists of a cascade of interacting sub-buffers, each of which accumulates experiences at a different timescale. It was designed with the following three motivating factors:

- Several of the previously mentioned replay methods use just two timescales of memory in order to strike a balance between new and old experiences [Isele and Cosgun, 2018, Rolnick et al., 2019, Zhang and Sutton, 2017]. For example, one method in [Isele and Cosgun, 2018] combines a small FIFO buffer with a reservoir buffer that maintains a uniform distribution over the agent’s entire history of experiences [Vitter, 1985] - this means that the composition of the replay database will adjust to short term changes in the distribution (with the FIFO buffer) and to long term changes (with the reservoir buffer), but it will not be as sensitive to medium term changes. Our method, on the other hand, maintains several sub-buffers that store experiences at a range of timescales, meaning that it can adjust well in scenarios where the rate of change of the distribution is unknown and can vary over time.

- The MTR buffer also draws inspiration from psychological evidence that the function relating the strength of a memory to its age follows a power law [Wixted and Ebbesen, 1991]; forgetting tends to be fast soon after the memory is acquired, and then it proceeds slowly with a long tail. In the MTR buffer, as a result of the combination of multiple timescales of memory, the probability of a given experience lasting beyond a time \( t \) in the database also follows a power law; in particular, it approximates a \( \frac{1}{t^2} \) decay (Appendix A.3).

- While shuffling the data to make it i.i.d. helps to prevent forgetting, it also discards structural information that may exist in the sequential progression of the data distribution - something that is preserved to a degree in the cascade of sub-buffers in the MTR method. Invariant risk minimization (IRM) is a recently developed method that uses the assumption that the training data has been split up into different environments in order to train a model that is invariant across these environments and is thus more likely to be robust and generalise well. In a second version of the MTR model, the MTR-IRM agent, we apply the IRM principle by treating each sub-buffer as a different environment to see if it can improve continual learning by encouraging the agent to learn a policy that is invariant over time.

We test the two MTR methods in RL agents trained on continuous control tasks in a standard RL setting, as well as in settings where the environment dynamics are continuously modified over time. We find that the standard MTR agent is the best continual learner overall when compared to a
number of baselines, and that the MTR-IRM agent improves continual learning in some of the more nonstationary settings, where one would expect an invariant policy to be more beneficial.

2 Preliminaries

2.1 Soft Actor-Critic

We used the soft actor-critic (SAC) algorithm [Haarnoja et al., 2018a] for all experiments in this paper, adapting the version provided in [Hill et al., 2018]. SAC is an actor-critic RL algorithm based on the maximum entropy RL framework, which generalises the standard RL objective of maximising return by simultaneously maximising the entropy of the agent’s policy:

\[ \pi^* = \arg \max_{\pi} \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t (r(s_t, a_t) + \alpha H(\pi(\cdot|s_t))) \right] \]  

(1)

where \( s_t \) and \( a_t \) represent the state visited and action taken at time \( t \) respectively, \( r(s_t, a_t) \) is the reward at time \( t \), \( \pi \) is a stochastic policy defined as a probability distribution over actions given state, \( \pi^* \) represents the optimal policy, \( \gamma \) is the discount factor, \( H \) is the entropy and \( \alpha \) controls the balance between reward and entropy maximisation. This objective encourages the agent to find multiple ways of achieving its goal, resulting in more robust solutions. Robustness was a particularly important factor in choosing an RL algorithm for this project, since the added nonstationarity of the environment in two of the experimental settings can easily destabilise the agent’s performance easily. In initial experiments, we found that SAC was more stable that other algorithms such as DDPG [Lillicrap et al., 2015]. We used the automatic entropy regulariser used in [Haarnoja et al., 2018b], which was found to be more robust than a fixed entropy regularisation coefficient.

2.2 Invariant Risk Minimisation

Invariant risk minimisation [Arjovsky et al., 2019] is an approach that seeks to improve out-of-distribution generalisation in machine learning models by training them to learn invariant or stable predictors that avoid spurious correlations in the training data, a common problem with the framework of empirical risk minimisation [Vapnik, 2006]. While typically the training data and test data are randomly shuffled in order to ensure they are from the same distribution, IRM poses that information is actually lost this way, and it starts with the assumption that the data can be split up into a number of different environments \( e \in \mathcal{E}_{tr} \). The IRM loss function encourages the model to learn a mapping that is invariant across all the different training environments, with the hope that, if it is stable across these, then it is more likely to perform well in previously unseen environments. The IRM loss is constructed as follows:

\[ \min_{\Phi: \mathcal{X} \rightarrow \mathcal{Y}} \sum_{e \in \mathcal{E}_{tr}} R_e(\Phi) + \lambda \cdot ||\nabla_w|_{w=1,0}R_e(w \cdot \Phi)||^2 \]  

(2)

where \( \Phi \) is the mapping induced by the model that maps the inputs to the outputs (and is a function of the model parameters), \( R_e \) is the loss function for environment \( e \), \( w \) is a dummy variable and \( \lambda \) is a parameter that balances the importance of the empirical loss (the first term) and the IRM loss (the second term). The goal of the IRM loss is to find a representation \( \Phi \) such that the optimal readout \( w \) is the same (i.e. the gradient of the readout is zero), no matter the environment; this way, when a new environment is encountered, it is less likely that the policy will have to change in order to suit it. In the next section, we describe how the IRM principle is applied in one version of the MTR replay database where the different environments correspond to experiences collected at different timescales.

3 Multi-Timescale Replay

The multi-timescale replay (MTR) database of size \( N \) consists of a cascade of \( n_b \) FIFO buffers, each with maximum size \( \frac{N}{n_b} \), and a separate overflow buffer (also FIFO), which has a dynamic maximum size that is equal to the difference between \( N \) and the number of experiences currently stored in the cascade (Figure 1(a)). New experiences of the form \( (s_t, a_t, r_{t+1}, s_{t+1}) \) are pushed into the first sub-buffer. When a given sub-buffer is full, the oldest experience in the buffer is pushed out, at which
point it has two possible fates: (i) with a predefined probability $\beta_m$, it gets pushed into the next sub-buffer, or (ii) with probability $1 - \beta_m$ it gets added to the overflow buffer. If the total number of experiences stored in the cascade and the overflow buffer exceeds $N$, the overflow buffer is shrunk with the oldest experiences being removed until the database has at most $N$ experiences. Once the cascade of sub-buffers is full, the size of the overflow buffer will be zero and any experience that is pushed out of any of the sub-buffers is discarded. During training, the number of experiences sampled from each sub-buffer (including the overflow buffer) is proportional to the fraction of the total number of experiences in the database contained in the sub-buffer. Figure 1(b) shows the distribution of ages of experiences in the MTR buffer, and a mathematical intuition for how the MTR buffer results in a power law distribution of memories is given in Appendix A.3.

3.1 IRM version of MTR

In the IRM version of the MTR agent (MTR-IRM), we assume that the set of experiences in each sub-buffer of the MTR cascade corresponds to data collected in a different environment. Under this assumption, we can apply the IRM principle to the policy network of the SAC agent, so each $R^e(\Phi)$ corresponds to the policy loss calculated using the data in the corresponding sub-buffer. While it would be interesting to apply the IRM to the value losses too, in this work, for simplicity we only applied it to the policy loss of the agent. The per-experience policy loss for SAC is as follows:

$$L_\pi(\phi, s) = E_{a \sim \pi_\phi}[\alpha_t \log(\pi_\phi(a|s)) - Q_{\theta_1}(s, a)]$$ (3)

where $\pi$ are the parameters of the policy network, $\pi_\phi$ is the conditional action distribution implied by the policy network, $s$ is the state, $a$ is the action sampled from $\pi_\phi$, $\alpha_t$ is the dynamic entropy coefficient and $Q_{\theta_1}$ is the Q-value function implied by one of the two Q-value networks used in SAC.

The policy loss at each iteration is calculated by averaging the per-experience loss shown above over a mini-batch of experiences chosen from the replay database. In combination with IRM, however, the overall policy loss is as evaluated as follows:

$$L_{\pi_{IRM}}(\phi) = E_{s_t \sim \mathcal{D}}[L_\pi(\phi, s_t)] + \lambda_{IRM} \sum_{i=1}^{n_b} \frac{|D_i|}{|D_{cascade}|} E_{s_t \sim \mathcal{D}_i} \left[ ||\nabla_{w_{1,2}} L_\pi(\phi, s_t, w) ||^2 \right]$$ (4)

where $|D_i|$ is the number of experiences in the $i^{th}$ sub-buffer in the cascade, $|D_{cascade}|$ is the total number of experiences stored in the cascade of sub-buffers, $w$ is a dummy variable, and $L_\pi$ is overloaded, such that:

$$L_\pi(\phi, s_t, w) = E_{a \sim \pi_\phi}[\alpha_t \log(\pi_\phi(w \cdot a|s)) - Q_{\theta_1}(s, a)]$$ (5)

The balance between the original policy loss and the extra IRM constraint is controlled by $\lambda_{IRM}$.

4 Experiments

4.1 Setup

The two MTR methods were evaluated in RL agents trained with SAC [Haarnoja et al., 2018a] on two different continuous control tasks (RoboschoolAnt and RoboschoolHalfCheetah) where the strength
of gravity in the environments was modified continuously throughout training in three different ways: fixed gravity, linearly increasing gravity, and gravity fluctuating in a sine wave. The idea was to see how the agents could cope with changes to the distribution that arise from different sources and at different timescales. In the fixed gravity setting, which constitutes the standard RL setup, the changes to the distribution result from correlation between successive states and changes to the agent’s policy. In the other two settings, continual learning is made more challenging because changes are also made to the dynamics of the environment. In the linear setting, the gravity is adjusted slowly over time, with no setting being repeated at any point during training; in the fluctuating setting, the changes are faster and gravity settings are revisited so that the relearning ability of the agent can be observed.

In order to evaluate the continual learning ability of the agents, their performance was recorded over the course of training (in terms of mean reward) on (i) the current task at hand, and (ii) on an evenly spaced subset of the gravity environments experienced during training (−7, −9.5, −12, −14.5 and −17 m/s²). It is worth noting that initial experiments were run in a traditional multi-task setting, where the gravity was uniformly sampled from an interval of [−17, −7] throughout training, i.e. the tasks are interleaved, in order to ensure that it is possible to learn good policies for all tasks in the same policy network (Figure A.2). Further experimental details and a table of the hyperparameters used for training are given in Appendix A.4.

4.2 Results

Across all three continual learning settings, the basic MTR agent appears to be the most consistent performer, demonstrating either the best or second best results in terms of training reward and mean evaluation reward in all tasks, indicating that recording experiences over multiple timescales can improve the tradeoff between new learning and retention of old knowledge in RL agents. The MTR-IRM agent achieved the best evaluation reward in two of the more nonstationary settings for the HalfCheetah task, but not in the Ant task, indicating that learning a policy that is invariant across time can be beneficial for generalisation and mitigating forgetting, but that it might depend on the particular task setting and the transfer potential between the different environments. Below, we discuss the results from each setting in more detail. All plots show moving averages of mean reward over three runs per agent type with standard error bars.

Fixed gravity In the fixed gravity experiments, the FIFO and MTR agents were consistently the best performers (Figure 2), with both agents achieving a similar final level of training reward in both the HalfCheetah and Ant tasks. One would expect the FIFO agent to be a relatively strong performer in this setting, since the environment dynamics are stable and so the retention of old memories is likely to be less crucial than in the other two more nonstationary settings. The fact that the basic MTR agent performs as well as the FIFO agent shows that the replay of some older experiences is not holding back the progress of the agent, but also that it does not seem to particularly help the overall performance either. The MTR-IRM agent, on the other hand, performed poorly in the fixed gravity setting, presumably because there is not enough nonstationarity to reap any generalisation benefits from learning an invariant representation for the policy, and instead the IRM constraints just slow down the pace of improvement of the agent.

Linearly increasing gravity In the linearly increasing gravity experiments, the FIFO agent performed best in terms of training reward, but was the worst performer when evaluated on the 5 different gravity settings on both tasks. This is somewhat intuitive: the FIFO agent can be expected to do well on the training task as it is only training with the most recent data, which are the most pertinent to the task at hand; on the other hand, it quickly forgets what to do in gravity settings that it experienced a long time ago (Figure A.1a)). In the HalfCheetah task, the MTR-IRM agent surpassed all other agents in the evaluation setting by the end of training, with the standard MTR agent coming second, perhaps indicating that, in a more nonstationary setting (in contrast with the fixed gravity experiments), learning a policy that is invariant over time can lead to a better overall performance in different environments. It was difficult, however, to identify any presence of forward transfer in the MTR-IRM agent in plotting the individual evaluations rewards over time (Appendix A.1).

In the linear gravity setting for the Ant task, the FIFO, MTR and MTR-IRM agents were equally strong on the training task (Figure 3b)), and the MTR and reservoir agents were joint best with regards to the mean evaluation reward (Figure 3d)). The MTR-IRM agent does not show the same benefits as in the HalfCheetah setting; this could be because it is more difficult to learn an invariant
Figure 2: Fixed gravity setting ($-9.81 m/s^2$). Training reward for (a) HalfCheetah and (b) Ant.

Policy across the different gravity settings across this task, with less potential for transfer between policies for the various environments. The transferability of policies across different environments and the effects of the order in which they are encountered are important aspects for future investigation.

Figure 3: Linearly increasing gravity setting. (Top) Training reward for (a) HalfCheetah and (b) Ant. (Bottom) Mean evaluation reward for (c) HalfCheetah and (d) Ant.

**Fluctuating gravity** In the fluctuating gravity setting, the performances of the various agents were less differentiated than in the linearly increasing gravity setting, perhaps because the timescale of changes to the distribution were shorter and the agents had the opportunity to revisit gravity.
environments (Figure 4). In the HalfCheetah task, the MTR-IRM agent was the best performer in terms of final training and evaluation rewards, though by a very small margin. In the Ant task, the best performer was the standard MTR agent, which reached a higher and more stable evaluation reward than any of the other agents. Once again, as in the linearly increasing gravity setting, the MTR-IRM agent struggled comparatively on the Ant task.

An interesting observation with regards to the agents’ ability to relearn can be made by comparing the individual evaluation rewards of the FIFO and MTR-IRM agents in the fluctuating gravity setting. The fluctuating performance on each of the different gravity evaluation settings can be observed very clearly in the results of the FIFO agent (Figure 5(a)), where the ups and downs in performance reflect the fluctuations of the gravity setting being trained on. While in the MTR-IRM agent, these fluctuations in performance can also be observed, the dips in performance on gravity settings that have not been experienced in a while become significantly shallower as training progresses, providing evidence that the agent is consolidating its knowledge over time (Figure 5(b)).

![Figure 4: Fluctuating gravity setting. (Top) Training reward for (a) HalfCheetah and (b) Ant. (Bottom) Mean evaluation reward for (c) HalfCheetah and (d) Ant.](image)

5 Related Work

Many existing approaches for mitigating catastrophic forgetting in neural networks use buffers for storing past data or experiences, and are collectively known as *replay-based methods* [Robins, 1995, Lopez-Paz et al., 2017, Chaudhry et al., 2019]. Here we briefly elaborate on a selection of works that investigate or make use of multiple timescales in the replay database, either in continual learning or in the standard RL setting. In [de Bruin et al., 2015], it is shown that retaining older experiences as well as the most recent ones can improve the performance of deep RL agents on the pendulum swing-up task, particularly for smaller replay databases. In [Zhang and Sutton, 2017], it is shown that *combined experience replay*, which trains agents on a combination of the most recent experiences...
(a) FIFO
(b) MTR-IRM

Figure 5: Individual Evaluation rewards for fluctuating gravity HalfCheetah with (a) FIFO buffer and (b) MTR-IRM buffer.

as they come in and those in the replay buffer, enables faster learning than training on just one or the other, particularly when the replay database is very large. In [Isele and Cosgun, 2018], various experience selection methods are evaluated for deep RL, and it was noted that for each method, a small FIFO buffer was maintained in parallel in order to ensure that the agent did not overfit to the more long-term memory buffer and had a chance to train on all experiences. In [Rolnick et al., 2019], it is shown that a 50/50 split of fresh experiences and experiences sampled from a reservoir buffer provides a good balance between mitigating forgetting and reaching a high overall level of performance on different tasks in a deep RL setting. As discussed in the Introduction, these methods employ two different timescales in the distribution of experiences used for training, while the MTR methods use multiple timescales, which makes them sensitive to changes to the data distribution that occur at a range of different speeds or frequencies.

In [Wang and Ross, 2019], it is shown that prioritising the replay of recent experiences in the buffer improves the performance of deep RL agents, using a continuum of priorities across time. In this paper, a FIFO buffer is used, so the data is only stored at the most recent timescale, but the probability of an experience being chosen for replay decays exponentially with its age. Other non-replay-based approaches that use multiple timescales of memory to mitigate catastrophic forgetting in deep RL include [Kaplanis et al., 2018], which consolidates the individual parameters at multiple timescales, and [Kaplanis et al., 2019], which consolidates the agent’s policy at multiple timescales, using a cascade of hidden policy networks.

6 Conclusion

In this paper, we investigated whether a replay buffer set up to record experiences at multiple timescales could help in a continual reinforcement learning setting where the timing, timescale and nature of changes to the incoming data distribution are unknown to the agent. One of the versions of the multi-timescale replay database was combined with the invariant risk minimisation principle [Arjovsky et al., 2019] in order to try and learn a policy that is invariant across time, with the idea that it might lead to a more robust policy that is more resistant to catastrophic forgetting. We tested numerous agents on two different continuous control tasks in three different continual learning settings and found that the basic MTR agent was the most consistent performer overall. The MTR-IRM agent was the best continual learner in two of the more nonstationary settings in the HalfCheetah task, but was relatively poor on the Ant task, indicating that the utility of the IRM principle may depend on specific aspects of the tasks at hand and the transferability between the policies in different environments.

Future Work One important avenue for future work would be to evaluate the MTR model in a broader range of training settings, for example to vary the timescales at which the environment is adjusted (e.g. gravity fluctuations at different frequencies). Furthermore, it would be useful to evaluate the sensitivity of the MTR method’s performance to its hyperparameters ($\beta_{\text{mtr}}$ and $n_b$).
Finally, it is worth noting that, in its current incarnation, the MTR method does not select which memories to retain for longer in an intelligent way - it is simply determined with a coin toss. In this light, it would be interesting to explore ways of prioritising the retention of certain memories from one sub-buffer to the next, for example by the temporal difference error, which is used in [Schaul et al., 2015] to prioritise the replay of memories in the buffer.

7 Code

The code for this project is available at https://github.com/ChristosKap/multi_timescale_replay.

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A Appendix

A.1 Linear Gravity Individual Evaluations

In Figure A.1(a), we can see that the cycles of learning and forgetting are quite clear with the FIFO agent. In all other agents, where older experiences were maintained for longer in the buffer, the forgetting process is slower. This does not seem to be qualitatively different for the MTR-IRM agent - it just seems to be able to reach a good balance between achieving a high performance in the various settings, while forgetting slowly. In particular, it is hard to identify whether there has been much forward transfer to gravity settings that have yet to be trained on, which one might hope for by learning an invariant policy: at the beginning of training, the extra IRM constraints seem to inhibit the progress on all settings (as compared to the standard IRM agent), but in the latter stages the performance on a number of the later settings improves drastically.

![Figure A.1: Individual Evaluation rewards for linearly increasing gravity HalfCheetah. Mean and standard error bars over three runs.](image)

A.2 Multi-task (Random gravity) Experiments

A.3 Power Law Forgetting

Several studies have shown that memory performance in humans declines with a power law function of time \[ y = at^{−b} \] for some \( a, b \in \mathbb{R}^+ \) [Kahana and Adler, 2017]. Here we provide a mathematical intuition for how the MTR buffer approximates a power law forgetting function of the form \( 1/t \), without giving a formal proof. If we assume the cascade is full, then the probability of an experience being pushed into the \( k^{th} \) sub-buffer is \( \beta_{mtr}^{k−1} \), since, for this to happen, one must be pushed from the 1\( \text{st} \) to the 2\( \text{nd} \) with probability \( \beta_{mtr} \), and another from the 2\( \text{nd} \) to the 3\( \text{rd} \) with the same probability, and so on. So, in expectation, \( \frac{N_{new}}{n_{mtr}} \cdot \frac{1}{\beta_{mtr}^{k−1}} \) new experiences must be added to the database for an experience to move from the beginning to the end of the \( k^{th} \) sub-buffer. Thus, if an experience reaches the end of the \( k^{th} \) buffer, then the expected number of time steps that
Figure A.2: Multitask setting: (a) training performance and (b) evaluation performance with uniformly random gravity between \(-7\) and \(-17\) m/s\(^2\) with a FIFO buffer. This experiment shows that the agent has the capacity to represent good policies for all evaluation settings if trained in a non-sequential setting.

Have passed since that experience was added to the first buffer is given by:

\[
\hat{t}_k = \mathbb{E}[t|\text{end of } k^{th} \text{ buffer}] = \sum_{i=1}^{k} \frac{N}{n_b} \cdot \frac{1}{\beta_{mtr}^{i-1}}
\]

\[
= \frac{N}{n_b} \cdot \beta_{mtr} \cdot \frac{1}{1 - \beta_{mtr}} \cdot \left( \frac{1}{\beta_{mtr}^k} - 1 \right)
\]

If we approximate the distribution \(\mathbb{P}(t|\text{end of } k^{th} \text{ buffer})\) with a delta function at its mean, \(\hat{t}_k\), and we note that the probability of an experience making it into \((k + 1)^{th}\) buffer at all is \(\beta_{mtr}^k\), then, by rearranging Equation 7, we can say that the probability of an experience lasting more than \(\hat{t}_k\) time steps in the database is given by:

\[
\mathbb{P}(\text{experience lifetime} > \hat{t}_k) = \frac{1}{\hat{t}_k \left( \frac{n_b}{N} \cdot \frac{1 - \beta_{mtr}}{\beta_{mtr}} \right) + 1}
\]

In other words, the probability of an experience having been retained after \(t\) time steps is roughly proportional to \(\frac{1}{t}\).

The expected number of experiences requires to fill up the MTR cascade (such that the size of the overflow buffer goes to zero) is calculated as follows:

\[
\sum_{i=1}^{n_b} \frac{N}{n_b} \cdot \frac{1}{\beta_{mtr}^{i-1}}
\]

which for \(N = 1e6\), \(n_b = 20\) and \(\beta_{mtr} = 0.85\), evaluates to 7 million experiences.

### A.4 Experimental Details

**Gravity settings and baselines** The gravity changes in each of the different settings are shown in Figure A.3. The fixed and linear gravity experiments were run for 5 million time steps, but the fluctuating gravity was run for 12 million steps, with 3 full cycles of 4 million steps. The value of the gravity setting was appended to the state vector of the agent so that there was no ambiguity about what environment the agent was in at each time step.

The MTR and MTR-IRM methods were compared with FIFO, reservoir and half-reservoir-half-FIFO baselines. In the last baseline, new experiences are pushed either into a FIFO buffer or a reservoir buffer (both of equal size) with equal probability, and sampled from . The maximum size of each database used is 1 million experiences and was chosen such that, in every experimental setting, the agent is unable to store the entire history of its experiences.
Hyperparameters  Below is a table of the relevant hyperparameters used in our experiments.

Table 1: Hyperparameters

| PARAMETER                                      | VALUE                                      |
|------------------------------------------------|---------------------------------------------|
| # HIDDEN LAYERS (ALL NETWORKS)                 | 2                                           |
| # UNITS PER HIDDEN LAYER                       | 256                                         |
| LEARNING RATE                                  | 0.0003                                      |
| OPTIMISER                                      | ADAM                                        |
| ADAM β₁                                         | 0.9                                         |
| ADAM β₂                                         | 0.999                                       |
| REPLAY DATABASE SIZE (ALL BUFFERS)             | 1E6                                         |
| # MTR SUB-BUFFERS $n_b$                        | 20                                          |
| $\beta_{mtr}$                                   | 0.85                                        |
| HIDDEN NEURON TYPE                              | ReLU                                        |
| TARGET NETWORK $\tau$                           | 0.005                                       |
| TARGET UPDATE FREQUENCY / TIME STEPS           | 1                                           |
| BATCH SIZE                                      | 256                                         |
| # TRAINING TIME STEPS                          | 5E6 (FIXED), 5E6 (LINEAR), 1.2E7 (FLUCTUATING) |
| TRAINING FREQUENCY / TIME STEPS                | 1                                           |
| GRAVITY ADJUSTMENT FREQUENCY / TIME STEPS      | 1000                                        |
| EVALUATION FREQUENCY / EPISODES                | 100                                         |
| # EPISODES PER EVALUATION                      | 1                                           |
|IRM POLICY COEFFICIENT                          | 0.1                                         |