Phase-controlled phonon laser

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Abstract

A phase-controlled ultralow-threshold phonon laser is proposed by using tunable optical amplifiers in coupled-cavity-optomechanical system. The multiplicative behavior of the individual enhancements, by engineering the phases and strengths of external parametric driving, makes it possible to achieve the strong-coupling regime of optomechanics, where the switching among radiation-pressure, parametric amplification, and three-mode optomechanical couplings can be realized and ultralow-threshold phonon lasing is observable. This opens up novel prospects for applications in, e.g. quantum acoustics, nonlinear phonon devices, and ultrasensitive motion sensing.

1. Introduction

As a promising platform to study fascinating macroscopic quantum phenomena [1], cavity optomechanics [2, 3] has received tremendous attentions in recent years. All kinds of optomechanical couplings and applications [4–9] have been opened up due to remarkable experimental advances in e.g., mechanical ground-state cooling [10, 11], optomechanical non-reciprocity [12–14], optomechanically induced transparency [15, 16], non-classical state preparation [17, 18], coherent state transfer between light and sound [19, 20], and various phonon-mediated hybrid devices [21, 22]. To extend more applications, on the one hand, the unique regime of single-photon quantum optomechanics [23, 24], however, is still pursued in current experimental efforts; on the other hand, we need to realize convenient tuning, especially the switching between different optomechanical couplings.

To realize single-photon coupling, many theoretical schemes have been proposed based on, for examples, undriven two-cavity set-ups [25], optomechanical arrays [26], Josephson effect [27], and the transient scheme [28]. Very recently, the parametric drive has been used to enhance the nonlinear coupling [29–34] in some systems. By exploiting coupled cavities, many applications have been studied, such as single-photon generation [35], steady-state entanglement [36], thermal phonon squeezing [37], and phonon laser [38]. In the study of phononic devices [39–41], the phonon laser [42–45] plays a key role in integrating coherent phonon sources, detectors, and waveguides [46]. Phonon lasing have been demonstrated in the electromechanical resonator [47], the nanomechanical resonator [48], the vertical cavity structure [49], and the compound microcavity system [50], and some schemes are also proposed to realize phonon laser in the quantum-dot system [51, 52]. In particular, the ultralow-threshold phonon laser [53–55] still gives rise to the broad interest and remains largely unexplored.

In this paper, we present a scheme for both enhancing optomechanical couplings into the single-photon strong-coupling regime and realizing the switching between different optomechanical interactions using optical parametric amplifiers (OPAs). The key idea is to put two OPAs into both the auxiliary cavity and the
optomechanical system, which leads to the squeezing of transformational optical modes. Due to the squeezing, we can obtain exponentially enhanced radiation-pressure, parametric amplification, and three-mode optomechanical couplings, which are controlled by the phase difference from the two OPAs. As one of applications, we study a phase-controlled ultralow-threshold phonon laser in detail. In addition, we consider the noise of the squeezed modes, which can be suppressed greatly via dissipative squeezing or an additional optical mode. With current experimentally accessible parameters, our scheme should be feasible to study quantum optomechanics.

The paper is organized as follows. In section 2, we first propose an optomechanical model with two coupled cavities containing driven nonlinear optical media for OPAs, and develop the Hamiltonian description of the optomechanical system. Then, by engineering the phases and strengths of external parametric driving, we realize the exponentially enhanced optomechanical interactions and the switching among radiation-pressure, parametric amplification, and three-mode optomechanical couplings in section 3. As one of the applications, in section 4, we study the ultralow-threshold phonon lasing in detail. Based on parameters realized with existing technologies, the physical systems are discussed in section 5, and then we summarize in section 6.

2. Model

We consider an optomechanical system with two coupled cavities, and each cavity contains a driven nonlinear optical medium for OPA, as shown in figure 1(a), which can be described by the Hamiltonian ($\hbar = 1$)

$$H = H_e + H_m + J(a_d a_d^\dagger + a_d^\dagger a_d),$$

with

$$H_e = \sum_j \omega_j a_j^\dagger a_j + \Lambda_j (a_j^\dagger a_j^\dagger - e^{i\Phi_j} a_j^\dagger a_j + \text{h.c.})$$

$$H_m = \omega_m b^\dagger b - g_0 (a_d^\dagger a_d (b^\dagger + b))$$

describing the optical modes containing two different OPAs, and describing the optomechanical system associated with the 2nd cavity. Here, $a_d$ and $b$ are the annihilation operators for the $j$th ($j = 1, 2$) cavity mode with frequency $\omega_d$ and the mechanical mode with frequency $\omega_m$, respectively, $J$ is the photon-hopping interaction strength between two cavities, and $g_0$ is the radiation-pressure optomechanical coupling strength.

For simplicity, we take the two parametric driving frequencies satisfying $\omega_{d1} = \omega_{d2} = \omega_d$. In the interaction picture $H_0 = \frac{\alpha_0}{2} (a_d^\dagger a_d + a_d^\dagger a_d)$, the Hamiltonian of the system can be written as

$$H_0 = \frac{\alpha_0}{2} (a_d^\dagger a_d + a_d^\dagger a_d) + J a_d^\dagger a_d (b^\dagger + b)$$

$$+ \sum_j \omega_j a_j^\dagger a_j + \Lambda_j (a_j^\dagger a_j^\dagger - e^{i\Phi_j} a_j^\dagger a_j + \text{h.c.})$$

$$+ \omega_m b^\dagger b - g_0 (a_d^\dagger a_d (b^\dagger + b))$$

$$+ (A_d a_d^\dagger a_d + \text{h.c.}) (b^\dagger + b)$$

$$+ (A_d a_d^\dagger a_d + \text{h.c.}) (b^\dagger + b)$$

Figure 1. (a) Schematic diagram of the optomechanical system with coupled cavities. Each cavity contains an OPA with driving amplitude $A_j$, frequency $\omega_j$, and phase $\Phi_j$, respectively. The photon-hopping interaction $J$ leads to the supermodes $A_j$ with the frequency $W_j$. (b) By tuning the phase difference $\Delta \Phi = \Phi_d - \Phi_0$, it is flexible to switch among the radiation-pressure $A_j^\dagger A_j (b^\dagger + b)$, parametric amplification $(A_d a_d^\dagger a_d + \text{h.c.}) (b^\dagger + b)$, and three-mode $(A_d^\dagger A_d^\dagger a_d a_d^\dagger + \text{h.c.}) (b^\dagger + b)$ optomechanical couplings.
where the detuning $\Delta_j = \omega_j - \omega_d / 2$. To diagonalize $H_s$, we define the squeezed mode operator $a_{sj}$ via the transformation [29]:

$$a_j = \cosh(r_{dj}) a_{sj} - e^{-i\phi_0} \sinh(r_{dj}) a_{sj}^\dagger,$$

where $r_{dj} = (1/4) \ln [(\Delta_j + 2\Lambda_j)/(\Delta_j - 2\Lambda_j)]$ and $\Delta_j = \omega_j - \omega_d / 2$, which requires $|\Delta_j| > |2\Lambda_j|$ to avoid the system instability.

The Hamiltonian of the system can be changed into

$$H = \sum_{j=1}^{2} \omega_{aj} a_{aj}^\dagger a_{aj} + \omega_{bj} b^\dagger b - g_{s2} a_{s2}^\dagger a_{s2}(b^\dagger + b) + \frac{1}{2} \sum_{j<k} g_{p2}(e^{-i\phi_0} a_{s2}^\dagger a_{s2}^\dagger a_{s2}^\dagger a_{s2} + h.c.) (b^\dagger + b)
+ \gamma (\lambda_1 a_{s1}^\dagger a_{s1} - \lambda_2 a_{s1} a_{s1}^\dagger + h.c.)
- F(b^\dagger + b) + C,$$

where effective mode frequency

$$\omega_j = (\Delta_j - 2\Lambda_j) \exp(2r_{dj}),$$

the dispersive optomechanical coupling strength $g_{s2} = \frac{g_0 \lambda_2}{\sqrt{\Delta_j^2 - 4\Lambda_j^2}}$, parametric OM interaction $g_{p2} = \frac{g_0 \lambda_0}{\sqrt{\Delta_j^2 - 4\Lambda_j^2}}$. The photon-hopping between cavities becomes coherent and parametric coupling with coupling strength

$$\lambda_1 = \sinh(r_{dj}) \sinh(r_{j2}) e^{i\phi_0} + \cosh(r_{dj}) \cosh(r_{j2}),$$

and

$$\lambda_2 = \cosh(r_{dj}) \sinh(r_{j2}) e^{i\phi_0} + \sinh(r_{dj}) \cosh(r_{j2}) e^{i\phi_0},$$

respectively. There is also effective additional pump to mechanical mode with $F = g_0 \sinh^2(r_{j2})$ and constant $C = \sum_{j=1}^{2} \Delta_j \sinh^2(r_{dj}) - 2\Lambda_j \cosh(r_{dj}) \sinh(r_{dj})$.

### 3. Phase-controlled optomechanical systems

For convenient discussion, we define the effective coupling ratio between the squeezing and coherent terms $f_1 \equiv (\lambda_2(\omega_{s1} - \omega_{s2}) / \lambda_0(\omega_{s1} + \omega_{s2}))$. With the rotating wave approximation (RWA) (appendix A), it is obvious that we can reserve the squeezing (coherent) term for $f_1 \gg 1$ ($f_1 \ll 1$), which can be used to realize different optomechanical interactions.

When we have $f_1 \gg 1$, the squeezing term can be used to enhance optomechanical coupling strength [31], and we can further diagonalize the two-mode squeezing terms (appendix B) via the squeezing transformation ($j = k$):

$$a_{sj} = \cosh(r) A_j - e^{-i\phi} \sinh(r) A_j^\dagger,$$

with $A_j$ is the annihilation operator for the supermode $j$ with frequency $W_{j(k=j)} = \omega_j \cosh^2(r) + \omega_k \sinh^2(r) - \frac{1}{2} \gamma \sinh(2\gamma/2)$. The effective interaction Hamiltonian can be rewritten as

$$H_{\text{int}} = -\sum_{j=1}^{2} G_j A_j^\dagger A_j (b^\dagger + b)
+ \sum_{j<k=1}^{2} (G_{jk} A_j A_k + h.c.) (b^\dagger + b)
- (G_{s2} A_s^\dagger A_s + h.c.) (b^\dagger + b),$$

which describes the typical optomechanical forms including the radiation-pressure, parametric amplification, and three-mode optomechanical couplings. Here $G_j$ is the effective coupling of optomechanical systems, where

$$G_1 = g_0 \cosh(2r_{j2}) \sinh^2(r),$$

$$G_2 = g_0 \cosh(2r_{j2}) \cosh^2(r),$$

respectively.
with

\[
\frac{r}{4} = \ln \frac{\omega_1 + \omega_2 + |J'|}{\omega_1 + \omega_2 - |J'|},
\]

\[
F' = e^{i\Phi_0} \left[ \cosh (r_{t1}) \sinh (r_{t2}) + \cosh (r_{t2}) \sinh (r_{t1}) e^{i\Delta \Phi} \right],
\]

depending on the phase difference \(\Delta \Phi = \Phi_{d1} - \Phi_{d2}\). As illustrated in figure 1(b), the phase difference \(\Delta \Phi\) determines the effective optomechanical couplings. As a comparison with the previous proposals [29, 31], the coupling \(G\) is greatly enhanced as the product of enhancement from the single-mode [29] and two-mode [31] squeezing. Here \(\Phi = \arg (J')\) and the explicit expressions for the parameters \(\lambda_{1,2}, \omega_{jm}, G_{jk}, G_{m}\) can be found in appendix B. We note that in previous works, time-dependent linear coupling [36] and modulated nonlinear coupling [32, 33] were also used to enhance the optomechanical coupling in specific physical systems, with however a finite value of the maximal enhancement factor (i.e., not exponentially improved as in our scheme).

In figure 2(a), the optomechanical coupling strengths \(G_{1,2}\) are plotted with parameters provided in the caption, which demonstrate the significant enhancement by controlling the phase \(\Delta \Phi\) and show the strong-coupling regime is achievable (i.e., \(G_{1,2} \approx \kappa\)) for \(\Delta \Phi\) around the optimal \(\Delta \Phi = \pi\). Because we choose the parametric pump detuning \(\Delta_1 < 0\) and \(\Delta_2 > 0\), which lead to \(r_{d1} < 0\) and \(r_{d2} > 0\), the effective \(|J'|\) reaches its maximum when \(\Delta \Phi = \pi\) and the minimum when \(\Delta \Phi = 0\). In the figure 2(b), we plot the dependence of supermode frequencies \(|W_1|/\omega_{jm}, W_2/\omega_{jm}\) on \(\Delta \Phi\). When \(\Delta \Phi\) tends to 0 or \(2\pi\), we have the coupling strength \(G_2 \gg G_1\), while the other couplings \(G_{jk}\) and \(G_{m}\) can be ignored for \(|W_1 + W_2 \pm \omega_{jm}| \gg G_{jk}\) and \(|W_1 - W_2 \pm \omega_{jm}| \gg G_{m}\). To show the enhanced coupling strengths for different driving amplitudes \(\Lambda_1\) and \(\Delta \Phi\), we plot the equipotential lines of \(f_1, G_{1}/\omega_{jm}, G_2/\omega_{jm}\), and \(\eta = G_1/ G_2\) in the figure 2(c). The inner region surrounded by the blue line \(f_1 = 10\) means that only the squeezing term dominates and the RWA is appropriate. The red line \(G_2 = 0.1\omega_{jm} > \kappa\) and green line \(\eta = \frac{1}{20}\) show that only the second optomechanical coupling reaches the strong-coupling regime. When the parameters \(\Lambda_1\) and \(\Delta \Phi\) tend to the central area, as shown by both the pink and black lines, both \(G_1\) and \(G_2\) can reach strong-coupling regime. We can study the non-classicality of two-mode field in the region crossing from weak coupling to the strong-coupling regime [37], and photon pair can be realized by engineering the phases and strengths of external parametric driving.

With appropriate parameters, the parametric amplification coupling forms in equation (11) can also been obtained when \(\omega_{jm} \gg G_{jk}\) and \(|W_1 - W_2 \pm \omega_{jm}| \gg G_{m}\), and meanwhile the frequency matching \(|W_1 + W_2 \pm \omega_{jm}| \approx 0\) is satisfied. The detailed discussion for the parametric amplification can be found in the appendix B. Compared to previous schemes that also employ the parametric interaction [58–60], the coefficient of parametric amplification is further improved by our coupled-cavity configuration, which can be used to generate photon–phonon pairs.

When only one parametric driving field exists in the cavity, we can still realize enhanced optomechanical coupling without phase control. If the parametric driving field exists in the second cavity, it means \(\Lambda_1 = 0\)
(t_{d1} = 0), and all coupling forms are same to the above. The phase difference does not appear in the expression of effective coupling $|J' | = 2J \sinh (t_{d1})$, which means the coupling parameters cannot be tuned by the phase difference. It needs a stronger photon-hopping interaction $J$ because of no product factor $\sinh (t_{d1})$ or $\cosh (t_{d1})$ in the effective $|J' |$. We can still realize the controlled optomechanical coupling by tuning $\Lambda_2$, $\Delta_2$, and $J$.

If the parametric driving field exists in the first cavity, we have $\Lambda_2 = 0$ ($t_{d2} = 0$). The coupling strength becomes $G_1 = g_0 \sinh^2 (r)$ and $G_2 = g_0 \cosh^2 (r)$, and the enhanced optomechanical coupling can still be obtained, which is equivalent to the model [31]. Here we do not need the two-mode squeezing, and the OPA can be put into the auxiliary cavity, which may be easier to implement in the experiment.

4. Phase-controlled phonon laser

The last term in equation (11) provides the crucial interaction for realizing the phonon laser (with the dominated term $G_{p12}$ in this three-mode optomechanical system) [50], which cannot be realized in previous schemes [29, 31]. This interaction is a triply-resonant interaction, with the advantage that the pump and idle optical field are resonantly enhanced. When the triply-resonant frequency $W_1 - W_2 \approx \omega_m$ is matched, we have the parameter $f_1 \ll 1$. By a similar transformation (appendix C) with the equation (10), we obtain

$$G_{p12} = - \frac{g_0}{2} e^{-i\Phi} \cosh (2t_{d2}) \sin (\theta),$$

with $\Phi = \arg (J')$, $\theta = \arctan [|J' | / (\omega_{j2} - \omega_{j3})]$, and $t_{d2}' = \cosh (t_{d3}) \cosh (t_{d4}) + \sinh (t_{d3}) \sinh (t_{d4}) e^{i\Delta \Phi}$.

In figure 3(a), we plot the optomechanical couplings $|G| / \omega_m$ versus the phase difference $\Delta \Phi$. The solid line is the triply-resonant phonon lasing coupling strength $|G_{p12}|$ versus the phase difference $\Delta \Phi$. The $|G_{p12}|$ is almost same as the $|G_{p12}|$, which can reach the strong-coupling regime $|G_{p12}| \approx \kappa$, and there is no obvious change with the increasing of the phase difference. The other couplings are shown by the dashed lines, which have the similar form with the $|G_{p12}|$, which can also reach the strong-coupling regime. However, the effective interactions of optomechanical system depend on the frequency of supermodes, which are shown in the appendix C. When the frequency matches $W_1 - W_2 \approx \omega_m$, we have $|G_j | / \omega_m$. The threshold pump power $P_{th}$ is obtained, which is equivalent to the model [31]. Here we do not need the two-mode squeezing, and the OPA can be put into the auxiliary cavity, which may be easier to implement in the experiment.

$$P_{th} = \frac{|G_{p12}|^2 \Delta N \kappa}{(W_1 - W_2 - \omega_m)^2 + (\kappa/2)^2},$$

where $\Delta N = N_+ - N_- \approx N_0$, with $N_0 = A_{j1}^+ A_{j1}$ and $N_- = A_{j2}^+ A_{j2}$. The gain has a spectral bandwidth $\kappa$ and $W_1 - W_2 = \omega_m$ is corresponding to the maximum gain.
The threshold condition \( S = \gamma_m \) determines the emitted phonon number, which is shown in figure 3(c). The solid lines are stimulated emitted phonon number \( n_b[n_m] = \exp[2(S - \gamma_m)/\gamma_m] \) as a function of the density \( N_i \) for different \( \Delta \Phi \). If there is no any OPA in the cavity, the emitted phonon number \( n_b \) with the resonance \( W_1 - W_2 = \omega_m \) is shown by the dashed line in figure 3(c). Clearly, it indicates an ultralow-threshold phonon laser by tuning the phase difference \( \Delta \Phi \).

The black square points denote the threshold density \( N_i \) for \( S = \gamma_m \) in figure 3(c). We know the threshold pump power as \( P_{th} = N_i \kappa W_1 \), and we obtain

\[
P_{th} \approx \frac{\gamma_m W_1[(W_1 - W_2 - \omega_m)^2 + (\kappa/2)^2]}{|G_{p12}|^2},
\]

which is plotted as the function of phase difference \( \Delta \Phi \) in figure 3(b). There are two dips, which mean an ultralow-threshold power with the near resonance \( W_1 - W_2 \approx \omega_m \). The ultralow-threshold power \( P_{th} \) is related to the frequency difference \( W_1 - W_2 \) controlled by the strengths and phases of parametric driving terms. From the figure 3, it is noted that the threshold density \( N_i \leq 1 \) can be obtained by changing the phase difference \( \Delta \Phi \).

In other words, the phonon lasing is possible with an ultralow-threshold power, as low as single-photon level. Although \( P_{th} \) is calculated for the \( A_1 \), we can derive the actual pump photons in the physical cavity modes as \( \langle a_1^\dagger a_1 \rangle \approx \cos^2(\frac{\pi}{2}) \cosh(2R_2) \times N_i, (y = 1, 2) \). Considering the threshold density \( N_i \), above, we find that the actual power of the threshold pump can be greatly reduced with a factor \( \cosh(2R_2)/\cosh^2(2R_2) \) compared with no OPAs \( (A_{1,2} = 0) \). For the parameters shown in figure 3(b), we have the lowest threshold pump photon number as \( \langle a_1^\dagger a_1 \rangle \approx 0.2 \).

We note that quantum optomechanical effects have attracted a wide range of interests for both fundamental studies and practical applications of nanomechanical devices [61–64]. For the phonon laser, as shown in figure 3, our scheme already reaches \( |G_{p12}|/\kappa = 1 > 0.1 \), which makes it possible to observe non-trivial quantum effects such as phonon anti-bunching [65], as detectable [48] via the Hanbury Brown–Twiss measurements of the output of the optomechanical cavity [66]. Therefore, such a non-classical phonon laser provides the key source for future fundamental studies and applications of quantum or nonlinear acoustics, such as vacuum Casimir–Rabi splittings [67], squeezed phonons [68, 69], and bistable phonon emission [70]. If the RWA is not satisfied, the pure effects [71] of counter-rotating terms should be observed. It is worth noting that the thermal noise of mechanical oscillators is detrimental for the coherent single phonon processes, while the negativity of the mechanical oscillator’s Wigner function can still be observed with the thermal phonon occupation \( n_{th} = 2 \) [66], which is corresponding to a 100 MHz mechanical mode in 10 mK environment.

5. Discussion

In view of rapid advances of experimental technologies, our scheme holds the promise to be realizable in a wide range of physical systems [72–76], such as whispering-gallery-mode (WGM) micro-resonators [4, 13, 77] and superconducting electromechanical systems [74, 78–83]. For the optical WGM system, the parametric amplification can be introduced to the high-Q optical mode by cavity enhanced second-order or third-order optical nonlinearities [73, 84].

For the superconducting circuits, the degenerate parametric oscillation of LC resonance configuration can be realized by the Josephson effects, and the electromechanical coupling can be realized by coupling to a drum-shaped capacitor [85]. In addition, some optomechanical couplings can be mimicked with superconducting circuits, such as quadratic optomechanics [86] and optomechanical-like coupling [87].

In particular, we can employ the second-order optical nonlinearity in WGM resonators [73] as the OPAs to fabricate the optomechanical system with two coupled cavities. We consider the nonlinearity as an OPA [77] with the frequency \( \lambda = 1550 \) nm, and the optomechanical interaction can be realized by an optical mode dispersive coupling with a symmetric radial breathing mechanical mode with the frequency \( \omega_m/2\pi = 100 \) MHz [13] and \( g_0 = 0.001 \omega_m \). The WGM resonators can couple through evanescent field [88], with the photon-hopping interaction strength \( J \) tunable by adjusting the distance between two resonators. Here, we choose a optical cavity decay rate \( \kappa = 0.1 \omega_m \), which is feasible for practical experimental system.

Based on the experimental parameters above, we plot the equipotential lines \( f_1 = 10 \) (blue line) and \( f_2 = 0 \) versus \( \Lambda_1 \) and \( \Delta \Phi \) in figure 4(a). The red line shows the equipotential line \( f_2 = 0 \) for \( J = 0.01 \omega_m \) and the area between the blue and red lines fully satisfies both the RWA and stable conditions. The unsteady area becomes larger when the photon-hopping interaction \( J \) increases, which is described by the black line with \( J = 0.02 \omega_m \). In figure 4(b), we plot the coupling \( G_2/\omega_m \) versus phase difference \( \Delta \Phi \) for the \( \Lambda_1 = 9.91 \omega_m \). With the increasing of the phase difference, the optomechanical coupling is exponentially enhanced. When the phase difference \( \Delta \Phi \) approaches the unsteady area as shown in figure 4(a), the optomechanical coupling has two maximum values,
and the area between the two peaks means that the system is unstable. The figure 4 (c) shows the smaller unsteady area with $\Lambda_1 = 9.92\omega_m$, which is corresponding to the figure 4 (a).

In the presence of a parametric drive, the noise due to the optical cavity dissipation might also be amplified. To circumvent the amplified noise, a possible strategy is to introduce a broadband single-mode or two-mode squeezed vacuum via dissipative squeezing [29–31]. This steady-state technique has recently been implemented experimentally [89–91], and recently it has been experimentally demonstrated that squeezed light can be used to cool the motion of a macroscopic mechanical object without resolved-sideband condition [92]. One can also take advantage of the tunability of the parametric drive to avoid significant perturbation of the initial photon state [30]. It is feasible to suppress the cavity noise in the experiment for realizing the optomechanical strong-coupling regime.

6. Conclusion

We present a scheme for enhancing phase-controlled optomechanical couplings into the single-photon strong-coupling regime by optical squeezing. With two OPAs in two coupled optical cavities, we obtain the squeezing of transformational optical modes, which leads to exponentially enhanced optomechanical systems. The phase difference between the two driving fields on OPAs can control the enhanced radiation-pressure, parametric amplification, and three-mode optomechanical couplings. In particular, the three-mode optomechanical coupling can be used to realize a low-threshold phonon laser, and the threshold pump power is decreased greatly with the giant enhancement of mechanical gain. With current technologies, our scheme should be experimentally realizable. This allows us to explore a number of interesting quantum optomechanics applications ranging from single-photon sources to non-classical quantum states.

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Appendix A. Validity of the RWA

In the interaction picture $H_I = \sum_{j=1}^2 \omega_j a_j^\dagger a_j + \omega_m b^\dagger b$, the interaction Hamiltonian of the system can be described by

$$H_I(t) = -g_2 a_1^\dagger a_2 (b^\dagger e^{i\omega_1 t} + b e^{-i\omega_2 t}) + g_{p2} (e^{-i\omega_1 t} a_2^\dagger a_2^\dagger + h.c.) (b^\dagger e^{i\omega_2 t} + b e^{-i\omega_1 t}) + f (\lambda_1 a_1^\dagger a_2^\dagger e^{i(\omega_1 - \omega_2) t} - \lambda_1 a_1^\dagger a_2^\dagger e^{-i(\omega_1 + \omega_2) t} + h.c.) - F (b^\dagger e^{i\omega_1 t} + b e^{-i\omega_2 t}).$$

By the effective Hamiltonian theory [93], we obtain the $U_I(t) = -i \int H_I(t') dt'$, which can be written as

$$U_I(t) = -\frac{g_2}{\omega_m} a_1^\dagger a_2 (b^\dagger e^{i\omega_1 t} + b e^{-i\omega_2 t}) - \frac{g_{p2} a_2^\dagger b^\dagger}{2\omega_2 - \omega_m} e^{i(\omega_2 - \omega_1) t - i\phi_{12}} + \frac{-i\lambda_1}{\omega_1} a_1^\dagger a_2^\dagger e^{i(\omega_1 - \omega_2) t} + h.c.) - \frac{F}{\omega_m} (b^\dagger e^{i\omega_1 t} + b e^{-i\omega_2 t}).$$

We can obtain the effective Hamiltonian as $H_{\text{eff}}(t) = \frac{H_I(t) + (\frac{dH_I(t)}{dt}, U_I(t)}{2}$. We just concern the optomechanical couplings from the terms $\lambda_1 a_1^\dagger a_2 + h.c.$ and $\lambda_2 a_1^\dagger a_2 + h.c.$ in the effective Hamiltonian.

When $f_1 \gg 1$, we have $|\omega_{11} - \omega_{12}|, |\omega_{12}| \gg |\omega_{1m}|, |\omega_{11} + \omega_{12}|$. The effective parameters have the following relation

$$\left| \frac{J_1 g_{p2}}{\omega_{12} - \omega_{11}} \right| + \left| \frac{J_2 g_{p2}}{2\omega_{12} \pm 2\omega_m} \right| \ll \left| \frac{J_3 g_{p2}}{\omega_{12} + \omega_{11}} \right| + \left| \frac{J_4 g_{p2}}{\omega_{1m}} \right| .$$

It is obvious that the effective optomechanical interactions from the term $\lambda_1 a_1^\dagger a_2^\dagger + h.c.$ and $\lambda_2 a_1^\dagger a_2 + h.c.$ are extremely weak, and we can eliminate the term $\lambda_1 a_1^\dagger a_2^\dagger + h.c.$ under RWA. For the parameter $f_1 \ll 1$, all of the optomechanical interactions are the resonant couplings. Other terms can be safely ignored when there is a frequency resonance. Under the RWA, we can neglect the effective optomechanical interactions from the term $\lambda_2 a_1^\dagger a_2 + h.c.$ under the resonant condition. Within the scope of the parameters we choose, our RWA is very reasonable, and the effective Hamiltonian can describe the optomechanical system exactly.

Appendix B. Phase-controlled OM interaction with $f_1 \gg 1$

Under the RWA, we can eliminate the term $\lambda_1 a_1^\dagger a_2^\dagger + h.c.$ when we have $f_1 \gg 1$, which means that the effective interaction from squeezing terms is much larger. It is obvious that only the squeezing terms can be reserved to enhance optomechanical coupling strength [31]. We can diagonalize the two-mode squeezing via the squeezing transformation [31]

$$a_j = \cosh(r) A_j - e^{-i\varphi} \sinh(r) A_k^\dagger (j \neq k),$$

where

$$r = \frac{1}{4} \ln[(\omega_{11} + \omega_{12} + |J'|)/(|\omega_{11} + \omega_{12} - |J'|)],$$

in which $J' = 2J_2$ and $\Phi = \arg(J')$. To avoid the system instability, we need

$$f_2 \equiv |\omega_{11} + \omega_{12} - |J'| > 0.$$

This leads to the following Hamiltonian

$$H = \omega_m b^\dagger b + \sum_{j=1}^2 W_j A_j^\dagger A_j - G_j A_j^\dagger A_j (b^\dagger + b) + \sum_{j,k=1} \left( G_{jk} A_j A_k + h.c. \right) (b^\dagger + b) - \left( G_{jk} A_j A_k + h.c. \right) (b^\dagger + b) + \left( F' - F \right) (b^\dagger + b) + C + C',$$

where the coefficients

$$W_j = \omega_{1j} \cosh^2(r) + \omega_{1j} \sinh^2(r) - |J'| \sinh(2r)/2,$$

8
neglected in the optomechanical system. We notice that the strong-coupling regime when frequency matching which means that we have the supermode frequencies $G_{r1}$ and $G_{r2}$ versus phase difference $\Delta \Phi$. (d) The frequency ($W_1 + W_2$) versus phase difference $\Delta \Phi$. The red square points show the resonant condition $W_1 + W_2 = \omega_m$. The parameters are $\Delta_1 = -400\omega_m$, $\Delta_2 = 400\omega_m$, $\Lambda_1 = 198.305\omega_m$, $\Lambda_2 = 198\omega_m$, $g_0 = 0.005\omega_m$, $J = 0.3\omega_m$, and $\kappa = 0.05\omega_m$.

\[ W_2 = \omega_2 \cosh^2(r) + \omega_1 \sinh^2(r) - |J'| \sinh(2r)/2, \]  

\[ G_1 = g_0 \cosh(2r_{12}) \sinh^2(r), \]  

\[ G_2 = g_0 \cosh(2r_{23}) \cosh^2(r), \]  

\[ G_{12} = \frac{g_0}{2} e^{i\Phi} \cosh(2r_{12}) \sinh(2r), \]  

\[ G_{11} = \frac{g_0}{2} e^{i(2\Phi - \Phi_0)} \sinh(2r_{12}) \sinh^2(r), \]  

\[ G_{22} = \frac{g_0}{2} e^{i\Phi} \sinh(2r_{12}) \cosh^2(r), \]  

\[ G_{p12} = \frac{g_0}{2} e^{i(\Phi_0 - \Phi)} \sinh(2r_{12}) \sinh(2r), \]  

\[ F' = g_0 \cosh(2r_{12}) \sinh^2(r), \]  

\[ C' = (\omega_2 + \omega_1) \sinh^2(r) - |J'| \sinh(r) \cosh(r). \]  

We notice that $F' = F$ and $C + C'$ are only the displaced term and a constant, respectively, which can be neglected in the optomechanical system.

We have discussed the radiation-pressure optomechanical coupling in the main text. With the appropriate parameters, the parametric amplification coupling forms can also been obtained.

To obtain the effective Hamiltonian, we notice that there are the following conditions: (a) $f_1 \gg 1$ (RWA); (b) $|\Delta_1| > |2\Lambda_1|$ and $f_2 > 0$ (stable conditions). Naturally, the system parameters are chosen to satisfy $|\Delta_1| > |2\Lambda_1|$. Equi potential lines $f_1 = 10$ (blue line) and $f_2 = 0$ (red line) versus $\Lambda_1$ and $\Delta \Phi$ are plotted in figure B1(a), and the area between the blue and red lines fully satisfies the above conditions. In figure B1(b), we plot the coupling $G_2/\omega_m$ (red-dashed line) and $G_{12}/\omega_m$ (blue-solid line) versus phase difference $\Delta \Phi$, and we can reach the strong-coupling regime when $\Delta \Phi = \pi$, however, which is much smaller than the mechanical frequency $\omega_m$.

The supermode frequencies $|W_1|/\omega_m$ (red-solid line) and $W_2/\omega_m$ (blue-dashed line) are shown in figure B1(c), which means that we have $\omega_m \gg G_{12}$, $|W_{12} \pm \omega_m| \gg |G_{12}|$ and $|W_{12} \pm \omega_m| \gg |G_{12}|$. We can find that the frequency matching (red square points) $|W_1 + W_2 - \omega_m| \approx 0$ in figure B1(d), which leads that only the term $G_{12}$ can be reserved. It is obvious that we can also obtain the parametric amplification coupling forms when $|2W_j \pm \omega_m| \approx 0$ with appropriate parameters.
Appendix C. Phase-controlled OM interaction with $f_i \ll 1$

When $f_i \ll 1$, we can neglect the term $\lambda_2 a_1 a_2 + \text{h.c.}$, and the Hamiltonian of the system can be written as

$$H = \sum_{j=1}^{2} \omega_j a_j^\dagger a_j + \omega_m b^\dagger b - g_{a_2} a_2^\dagger a_2 (b^\dagger + b)$$

$$+ g_{p_2} (e^{-i\phi} e^{\lambda_2} a_2^\dagger + \text{h.c.}) (b^\dagger + b)$$

$$+ J (\lambda_2 a_2 a_2^\dagger + \text{h.c.}) - F (b^\dagger + b) + C. \quad (36)$$

To diagonalize the interaction term $\lambda_1$, we introduce the transformation

$$a_1 = \cos \left( \frac{\theta}{2} \right) A_1 + e^{-i\phi} \sin \left( \frac{\theta}{2} \right) A_2,$$

$$a_2 = \cos \left( \frac{\theta}{2} \right) A_2 - e^{-i\phi} \sin \left( \frac{\theta}{2} \right) A_1, \quad (37)$$

where $\theta = \arctan \left( \frac{\omega_1}{\omega_2} \right)$, in which $J' = 2J\lambda_1$ and $\Phi = \arg(J')$.

The Hamiltonian can be written as

$$H = \omega_m b^\dagger b + \sum_{j=1}^{2} W_j A_j^\dagger A_j - G_j A_j^\dagger A_j (b^\dagger + b)$$

$$+ \sum_{j \neq k=1}^{2} (G_{jk} A_j A_k + \text{h.c.}) (b^\dagger + b)$$

$$+ (G_{p_2} A_2^\dagger A_2 + \text{h.c.}) (b^\dagger + b) - F (b^\dagger + b) + C, \quad (39)$$

where the coefficients

$$W_1 = \omega_{a_1} \cos^2 \left( \frac{\theta}{2} \right) + \omega_{a_2} \sin^2 \left( \frac{\theta}{2} \right) - |J'| \sin \left( \theta \right)/2, \quad (40)$$

$$W_2 = \omega_{a_2} \cos^2 \left( \frac{\theta}{2} \right) + \omega_{a_1} \sin^2 \left( \frac{\theta}{2} \right) + |J'| \sin \left( \theta \right)/2, \quad (41)$$

$$G_1 = g_0 \cosh \left( 2r_{t_2} \right) \sin^2 \left( \frac{\theta}{2} \right), \quad (42)$$

$$G_2 = g_0 \cosh \left( 2r_{t_2} \right) \cos^2 \left( \frac{\theta}{2} \right), \quad (43)$$

$$G_{12} = -\frac{g_0}{2} e^{i(\phi_{t_2} + \Phi)} \sinh \left( 2r_{t_2} \right) \sin \left( \theta \right), \quad (44)$$

$$G_{11} = \frac{g_0}{2} e^{i(2\phi + \Phi_{t_2})} \sinh \left( 2r_{t_2} \right) \sin^2 \left( \frac{\theta}{2} \right), \quad (45)$$

$$G_{22} = \frac{g_0}{2} e^{i\Phi_{t_2}} \sinh \left( 2r_{t_2} \right) \cos^2 \left( \frac{\theta}{2} \right), \quad (46)$$

$$G_{p_2} = \frac{g_0}{2} e^{-i\phi} \cosh \left( 2r_{t_2} \right) \sin \left( \theta \right). \quad (47)$$

We know that the phonon laser can be realized by the Hamiltonian

$$H = \omega_m b^\dagger b + \sum_{j=1}^{2} W_j A_j^\dagger A_j + G_{p_2} A_2^\dagger A_2 b + \text{h.c.}, \quad (48)$$

when we have $\omega_m \gg G_{1j} |W_j + W_k \pm \omega_m| \gg |G_{jk}|$, and $|W_i - W_k \pm \omega_m| \approx 0$.

In figure C1(a), we plot the equipotential lines $f_i = 0.1$ (blue line) versus $\Lambda_1$ and $\Delta \Phi$, and the area between the blue and red lines fully satisfies $f_i \ll 1$. The frequencies $W_1/\omega_m$ (red-dashed line), $W_2/\omega_m$ (blue-dashed line), and $(W_i - W_j) / \omega_m$ (blue-solid line) are shown in figure C1(b). We find the frequency matching (red square points) $|W_1 - W_2 - \omega_m| \approx 0$, which leads that only the term $G_{p_2}$ can be reserved. While the RWA is satisfied, we can realize the phase-controlled phonon laser. In figure C1(b), the frequency $|2W_2 - \omega_m| \approx 0$ can also be matched, which is denoted by the green dots. It is obvious that we can obtain the parametric amplification coupling form $G_{z_2}$ only by tuning the phase difference.
In the interaction picture $H_0 = \omega_m b^\dagger b + \sum_{j=1}^2 W_j A_j^\dagger A_j$, the Hamiltonian can be rewritten as

$$H = -2 \sum_{j=1}^2 G_j A_j^\dagger A_j (b^\dagger e^{i \omega_j t} + b e^{-i \omega_j t})$$

$$+ \sum_{j<k}^2 (G_{jk} A_j^\dagger A_k b^\dagger e^{i (\omega_j - \omega_k) t} + \text{h.c.})$$

$$+ (G_{pjk} A_j^\dagger A_k b e^{i (\omega_j - \omega_k) t} + \text{h.c.}),$$

(49)

where we have neglected the displaced term and the constant. In the RWA, the terms $A_j A_k b$ and $A_j A_k b^\dagger$ can be also ignored for $W_1 > W_2 > 0$ with the selected parameters. Considering the optical cavity decay rate and mechanical dissipation, we can obtain the Heisenberg equation of motion for the mechanical mode

$$\frac{db}{dt} = -2 \sum_{j<k}^2 i G_{jk} A_j^\dagger A_k e^{i (\omega_j - \omega_k) t}$$

$$+ \sum_{j=1}^2 i G_j A_j^\dagger A_j e^{i \omega_j t} - \sum_{j<k}^2 i G_{jk} A_j^\dagger A_k e^{i (\omega_j + \omega_k - W_j - W_k) t}.$$  

(50)

If the effective optical cavity decay rate exceeds the mechanical dissipation rate, we can obtain the mechanical gain [50]

$$\mathcal{G} \approx \frac{|G_{pjk}| \Delta \kappa}{(W_j - W_k - \omega_m)^2 + (\kappa/2)^2} - 2 \sum_{j=1}^2 \frac{|G_j|^2 \Delta \kappa}{(\omega_m)^2 + (\kappa/2)^2}$$

$$- \sum_{j<k}^2 \frac{|G_{jk}|^2 \sqrt{N_j N_k}}{(W_j + W_k - \omega_m)^2 + (\kappa/2)^2},$$

(51)

where $N_j = \langle A_j^\dagger A_j \rangle$ and $N_k = \langle A_k^\dagger A_k \rangle$. From the figures C1 and 3, we notice that $\omega_m \gg G_j$, $|W_j + W_k \pm \omega_m| \gg |G_{jk}|$ when we have $W_1 - W_2 - \omega_m \approx 0$, which means that the other couplings almost have no influence on the mechanical gain and the threshold pump power.

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