New Members in the $0^+ (0^{++})$ Family

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Abstract

Recent BES data on $J/\psi \to \phi \pi \pi$ indicate that there is a possible new spin-0 state ($f_0(1790)$) with a mass of $m = 1790^{+40}_{-30}$ MeV/c$^2$. Assuming it to be an iso-singlet $0^+(0^{++})$, we propose a new mixing scheme to describe this and the other three known iso-singlet $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ states by adding iso-singlet hybrid states to the usual basis of two iso-singlet quarkonia and one glueball. Since there are two iso-singlet hybrid states, $(u\bar{u} + d\bar{d})g/\sqrt{2}$ and $s\bar{s}g$, this new basis implies existence of another iso-singlet state $X$. Using known data, we estimate the ranges of the mixing parameters. We find two sets of solutions with X mass predicted to be about 1820 MeV and 1760 MeV, respectively. We also study implications on the decay properties of these new states.
The BES collaboration has recently obtained evidence for a new broad state in the spectrum of $\pi\pi$ in $J/\psi \rightarrow \phi\pi\pi$ decay. Their results indicate that it is a $0^+$ state with mass and width given by $m = 1790^{+40}_{-30}\text{MeV}/c^2$ and $\Gamma = 270^{+60}_{-30}\text{MeV}/c^2$. The observed branching ratio for $\mathcal{B}(J/\psi \rightarrow \phi f_0(1790)) \cdot \mathcal{B}(f_0(1790) \rightarrow \pi\pi)$ is determined to be $(6.2 \pm 1.4) \times 10^{-4}$ [1]. This resonant state is named as $f_0(1790)$.

In the energy range of 1 to 2 GeV, three $0^+(0^{++})$ states: $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ have been experimentally confirmed [2]. The possible new state $f_0(1790)$ may be a new member of the $0^+(0^{++})$ family, even though its isospin and G-parity have not well determined yet. Close et al. suggested that the three resonant states ($f_0(1370)$, $f_0(1500)$ and $f_0(1710)$) may be mixtures of a scalar glueball $G$, an iso-singlet quarkonium $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ and an $S = s\bar{s}$ [3]. Several other authors have also discussed the mixing mechanism of the three resonant states and related phenomenology [4, 5, 6]. If the possible new $f_0(1790)$ state is another member of the $0^+(0^{++})$ family, it may also mix with the three known states. To describe this possible new state, it is necessary to enlarge the previously used basis in terms of constituent quarks and gluons. A natural way of achieving this is to introduce the $q\bar{q}g$ hybrid states, even though other possibilities exist [7]. We therefore propose that the basis in terms of constituent quarks and gluons for a unified description of states in the $0^+(0^{++})$ family is composed of the glueball state $G$, quarkonia $N = (u\bar{u} + d\bar{d})/\sqrt{2}$, $S = s\bar{s}$ and the two new hybrid states: $(u\bar{u} + d\bar{d})g/\sqrt{2}$ and $(s\bar{s})g$. With this picture, we predict existence of a yet to be discovered new physical state $X$. Since the hybrid states have very different structure compared with the usual glueball and quarkonia, the new states will have some distinctive new signatures in some decays, especially for the doubly OZI suppressed decay processes. In this paper we study implications of this mixing mechanism.

Below 1 GeV there are also other $0^{++}$ states, such as $f_0(600)$ and $f_0(980)$ [2]. These states have masses much lower than that of other members of the $0^+(0^{++})$ family mentioned above, therefore $f_0(600)$ and $f_0(980)$ can hardly mix with the other heavier states. We will not discuss them in this paper.

If the parameters are known, one can diagonalize the mass matrix and obtain the eigen-masses and mixing parameters of the physical states. The mixing parameters are, however, completely governed by non-perturbative effects which cannot be reliably evaluated at present. Therefore, we will use the experimental data, as much as possible, as inputs to obtain the mixing parameters.
We now study possible structures for the mixing. The effective Hamiltonian $H$ for the system cannot be calculated from QCD yet because of complicated non-perturbative effects. With certain simplifications, the form of the mass matrix for the $G$, $N$ and $S$ states has been suggested by Close et al. and some other authors [5, 6], where $G$ can strongly couple to both quarkonia $N$ and $S$, but the element $\langle N|H|S \rangle$ is obviously OZI suppressed and can therefore be neglected at the lowest order approximation. Since this coupling is flavor-independent, one has the relation $e = \langle G|H|\xi_S \rangle = \langle G|H|\xi_N \rangle/\sqrt{2}$. In analog, we assume that only the coupling of glueball to the hybrids is strong, thus $f = \langle G|H|S \rangle = \langle G|H|N \rangle/\sqrt{2}$ is substantial while other matrix elements can be practically set to be null. With the approximation described here, the mass matrix can be expressed as

$$M = \begin{pmatrix} M_{\xi_S} & 0 & e & 0 & 0 \\ 0 & M_{\xi_N} & \sqrt{2}e & 0 & 0 \\ e & \sqrt{2}e & M_G & f & \sqrt{2}f \\ 0 & 0 & f & M_S & 0 \\ 0 & 0 & \sqrt{2}f & 0 & M_N \end{pmatrix}, \quad (1)$$

where $M_{\xi_S} = \langle \xi_S|H|\xi_S \rangle$, $M_{\xi_N} = \langle \xi_N|H|\xi_N \rangle$, $M_G = \langle G|H|G \rangle$, $M_S = \langle S|H|S \rangle$ and $M_N = \langle N|H|N \rangle$ are the diagonal matrix elements of $M$.

Diagonalizing the above matrix, one obtains the mass eigenvalues and physical states in terms of the quarkonia, hybrids and glueball. We parameterize the relation between the physical states and the basis as

$$F_{\text{phys}} = UB_{\text{basis}}, \quad U = \begin{pmatrix} v_1 & w_1 & z_1 & y_1 & x_1 \\ v_2 & w_2 & z_2 & y_2 & x_2 \\ v_3 & w_3 & z_3 & y_3 & x_3 \\ v_4 & w_4 & z_4 & y_4 & x_4 \\ v_5 & w_5 & z_5 & y_5 & x_5 \end{pmatrix}, \quad (2)$$

where $F_{\text{phys}}^T = (|X\rangle, |f_0(1790)\rangle, |f_0(1710)\rangle, |f_0(1500)\rangle, |f_0(1370)\rangle)$ and $B_{\text{basis}}^T = (|\xi_S\rangle, |\xi_N\rangle, |G\rangle, |S\rangle, |N\rangle)$. Here the state $X$ is an extra $0^{++}$ state predicted in this scheme.

As $H$ is not derivable and therefore neither all the matrix elements, we need to determine them by fitting data except the scalar glueball mass $M_G$. In our later discussions we will take the lattice calculation results of [8] to constrain $M_G$ to be within the range $1.5 \sim 1.7$.
GeV. The mixing parameters $v_i$, $z_i$ and $y_i$ depend on the seven parameters $M_{\xi_2,\xi_N,G,S,N}$, $e$ and $f$. The available data which are directly related to these parameters are the four known eigenmasses of $f_0(1790, 1710, 1500, 1370)$. To completely fix all the parameters, more information is needed. To this end, we use information from the ratios of the measured branching ratios of $f_0(1790, 1710, 1500, 1370)$ to two pseudoscalar mesons listed in Table 1.

The effective Hamiltonian of scalar state decaying into two pseudoscalar mesons can be written as

\[ H_{eff}^{PP} = f_1 \text{Tr}[X_F P_F P_F] + f_2 X_G \text{Tr}[P_F P_F] 
+ f_3 X_G \text{Tr}[P_F] \text{Tr}[P_F] + f_4 \text{Tr}[X_H P_F P_F] 
+ f_5 \text{Tr}[X_H P_F] \text{Tr}[P_F] + f_6 \text{Tr}[X_H] \text{Tr}[P_F] \text{Tr}[P_F] 
+ f_7 \text{Tr}[X_H] \text{Tr}[P_F P_F] + f_8 \text{Tr}[X_H] \text{Tr}[P_F P_F] \text{Tr}[P_F]. \]  

(3)

Here $X_F$ is the flavor matrices of iso-singlet quarkonia components of $X_i$ where the subscript $i = 1,\ldots,5$ labels the five physical states. The detailed expression for $X_F$ is given as

\[ X_F = a\lambda^0 + b\lambda^8 = \begin{pmatrix} \frac{u\bar{u} - d\bar{d}}{2} & 0 & 0 \\ 0 & \frac{u\bar{u} - d\bar{d}}{2} & 0 \\ 0 & 0 & s\bar{s} \end{pmatrix} \]

\[ \begin{pmatrix} \sum_i \frac{x_i}{\sqrt{2}} X_i & 0 & 0 \\ 0 & \sum_i \frac{y_i}{\sqrt{2}} X_i & 0 \\ 0 & 0 & \sum_i y_i X_i \end{pmatrix} \]  

(4)

and $P_F$ is the pseudoscalar octet,

\[ P_F = \begin{pmatrix} x_0 & \frac{x_0 \eta + x_{-\eta} \eta'}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} & K^+ \\
\frac{\pi^-}{\sqrt{2}} & -x_0 & \frac{\eta_1 \eta + \eta_1' \eta'}{\sqrt{2}} & K^0 \\
K^- & \bar{K}^0 & y_\eta \eta + y_{\eta'} \eta' \end{pmatrix}. \]  

(5)

In the above, $x_{\eta,\eta'}$ and $y_{\eta,\eta'}$ describe the $\eta - \eta'$s mixing, and

\[ x_\eta = y_{\eta'} = \frac{\cos \theta - \sqrt{2} \sin \theta}{\sqrt{3}}, \]
\[ x_{\eta'} = -y_\eta = \frac{\sin \theta + \sqrt{2} \cos \theta}{\sqrt{3}}, \]  

(6)
where $\theta = -19.1^\circ$ is the mixing angle of $\eta$ and $\eta'$.

The concrete expressions of $X_G$ and $X_H$ are

$$X_G = \sum_i z_i X_i,$$

(7)

$$X_H = (a\lambda^0 + b\lambda^8)g = \begin{pmatrix} \frac{u\bar{u} + dd}{2} g & 0 & 0 \\ 0 & \frac{u\bar{u} + dd}{2} g & 0 \\ 0 & 0 & s\bar{s} g \end{pmatrix} = \begin{pmatrix} \sum_i \frac{w_i}{\sqrt{2}} X_i & 0 & 0 \\ 0 & \sum_i \frac{w_i}{\sqrt{2}} X_i & 0 \\ 0 & 0 & \sum_i v_i X_i \end{pmatrix}.$$  

(8)

The $f_{6-10}$ terms in the above effective Hamiltonian describing the decay modes with two-meson final states are OZI suppressed as can be seen from Figure 1((6)-(10)). The contributions from these terms can be neglected to a good approximation. Within this approximation, 5 parameters (actually 4 parameters $\xi_i = f_{1+i}/f_1$ when considering ratios of branching ratios) are needed to describe decay modes with two pseudoscalar mesons in the final states. We obtain the decay width $\Gamma(X_i \rightarrow \pi\pi, K\bar{K}, \eta\eta, \eta\eta')$ in terms of the parameters $\xi_1 = f_2/f_1$, $\xi_2 = f_3/f_1$, $\xi_3 = f_4/f_1$ and $\xi_4 = f_5/f_1$.

Using the above mass matrix and the decay amplitudes, our task is now reduced to see if the four eigenmasses of $f_0(1370, 1500, 1710, 1790)$, and eight ratios of the branching ratios listed in Table 1 can be described by the 11 parameters (7 parameters in the mass matrix with $M_G$ in the range of 1.4 to 1.7 GeV plus the 4 parameters $\xi_i$ in the decay amplitudes) in some reasonable ranges. This by no means is a trivial task. We, however, do find parameter spaces which can give reasonable fit to experimental data. Since the mass of $X$ is not known, we consider two types of solutions for the mass $M_X$ of $X$: (a) $M_X > M_{f_0(1790)}$, and (b) $M_X < M_{f_0(1790)}$. We display the results in the following.

Case (a) $M_X > M_{f_0(1790)}$

In Table 2, we list the input parameters and the resulting eigenmasses and branching ratios obtained. Our procedure to obtain the fitted values for the input parameters is guided by obtaining numbers which are mostly consistent with the central values of the known data as the parameters being within the reasonable ranges described earlier. The mixing matrix
is given by

\[
U = \begin{pmatrix}
-0.986 & -0.107 & -0.109 & -0.065 & -0.030 \\
-0.131 & +0.972 & +0.146 & +0.121 & +0.045 \\
-0.077 & -0.173 & +0.273 & +0.937 & +0.105 \\
-0.061 & -0.099 & +0.617 & -0.284 & +0.725 \\
+0.041 & +0.063 & -0.715 & +0.147 & +0.679
\end{pmatrix}.
\]

The dominant component of \( f_0(1790) \) is \((u\bar{u} + d\bar{d})g/\sqrt{2}\), whereas \( s\bar{s}g \) is the dominant one in \( X \). The main components of \( f_0(1710) \), and \( f_0(1500, 1370) \) are S and mixtures of N and G, respectively. The mass of \( X \) is approximately 1.823 GeV.

Case (b) \( M_X < M_{f_0(1790)} \)

There are some differences for case (b) from case (a). The input parameters, the resulting eigenmasses and branching ratios obtained are also listed in Table 2. The mixing pattern is given by

\[
U = \begin{pmatrix}
-0.168 & +0.945 & +0.179 & +0.210 & +0.060 \\
-0.978 & -0.125 & -0.129 & -0.099 & -0.039 \\
-0.095 & -0.274 & +0.248 & +0.919 & +0.096 \\
-0.067 & -0.111 & +0.614 & -0.282 & +0.725 \\
+0.044 & +0.068 & -0.716 & +0.147 & +0.678
\end{pmatrix}.
\]

In this case, the dominant component of \( f_0(1790) \) is \( s\bar{s}g \), whereas the \((u\bar{u} + d\bar{d})g/\sqrt{2}\) is the dominant one in \( X \). The main components of \( f_0(1710) \), and \( f_0(1500, 1370) \) still are, respectively, S and mixtures of N and G. The mass of \( X \) is approximately 1.76 GeV in this case. We note that the ratio \( B(f_0(1790) \rightarrow \pi\pi)/B(f_0(1790) \rightarrow K\bar{K}) \) in this case is below the central value of the data \( B(f_0(1790) \rightarrow \pi\pi)/B(f_0(1790) \rightarrow K\bar{K}) = 3.88 \). However, due to large error associated with the data, this case cannot be ruled out at present. This may provide a crucial criteria to distinguish the two cases.

There are many possible decay modes for the new state \( f_0(1790) \) and \( X \). We will consider several two-body decay modes of these states. They are \( f_0(1790)(X) \) decays into two pseudoscalar-mesons, two vector-mesons and two-photons.

\[ f_0(1790)(X) \rightarrow PP' \]

The two-pseudoscalar meson decays have been given in eq.(3) previously. We have used several of these decay modes involving \( f_0(1370, 1500, 1710, 1790) \) to fix the parameters. Using
the mixing parameters determined, the decay modes for \( f_0(1790) \) and \( X \) can be predicted. We obtain the results for cases (a) and (b) in the following.

For case (a), we have

\[
B(f_0(1790) \to \pi\pi) : B(f_0(1790) \to K\bar{K}) \\
: B(f_0(1790) \to \eta\eta) : B(f_0(1790) \to \eta\eta')
\]

\[= 23 : 10 : 5 : 2,\]

\[
B(X \to \pi\pi) : B(X \to K\bar{K}) : B(X \to \eta\eta) : B(X \to \eta\eta')
\]

\[= 5 : 43 : 6 : 4,\]

whereas for case (b), we have

\[
B(f_0(1790) \to \pi\pi) : B(f_0(1790) \to K\bar{K}) \\
: B(f_0(1790) \to \eta\eta) : B(f_0(1790) \to \eta\eta')
\]

\[= 13 : 31 : 6 : 0.3,\]

\[
B(X \to \pi\pi) : B(X \to K\bar{K}) : B(X \to \eta\eta) : B(X \to \eta\eta')
\]

\[= 10 : 15 : 5 : 0.4.\]

Using the above obtained ratios of \( \Gamma(f_0(1790)(X) \to PP')/\Gamma(f_0(1710) \to K\bar{K}) \) and combining the measured value of \( \Gamma(f_0(1710) \to K\bar{K}) \), we obtain the corresponding values for \( f_0(1790)(X) \to PP' \) in our Table 3. Since the total width of \( f_0(1790) \) is measured at BES, one can obtain branching ratios of \( f_0(1790) \to PP' \), by contraries, for \( X \), one can only have the partial widths.

\( f_0(1790)(X) \to VV' \)

Now let us turn to the case of decays with two vector mesons \( VV' \) in the final state. The effective Hamiltonian is similar to that for the pseudoscalar-meson case in eq.(3). One just replace the \( PFPF \) by \( \partial^\mu V^\nu \partial_\nu V_\mu \) at appropriate places with \( V \) being the vector nonet.

Since the measurements on such \( VV' \) channels, even for the confirmed resonant states \( f_0(1370), f_0(1500) \) and \( f_0(1710) \) are absent, it is not possible to make a definite evaluation on their branching ratios yet, and even the ratios among the branching ratios. But these decay modes should occur with substantial branching ratios. We will come back to this later.

\( f_0(1790)(X) \to \gamma\gamma \)
We now consider the two-photon decay modes. In the spirit of Ref.\[11\] that the decay amplitude is proportional to the electric charge coupling of the two photon at the quark level, ignoring mass-dependent effects, we obtain the following.

For case (a)

\[
\Gamma(X \to \gamma\gamma) : \Gamma(f_0(1790) \to \gamma\gamma) : \Gamma(f_0(1710) \to \gamma\gamma) : \Gamma(f_0(1500) \to \gamma\gamma) : \Gamma(f_0(1370) \to \gamma\gamma) = 0.06 : 0.16 : 3.42 : 10.39 : 12.98. \quad (9)
\]

For case (b)

\[
\Gamma(X \to \gamma\gamma) : \Gamma(f_0(1790) \to \gamma\gamma) : \Gamma(f_0(1710) \to \gamma\gamma) : \Gamma(f_0(1500) \to \gamma\gamma) : \Gamma(f_0(1370) \to \gamma\gamma) = 0.36 : 0.11 : 3.17 : 10.41 : 12.94. \quad (10)
\]

We have proposed a mixing scheme of isosinglet $0^+(0^{++})$ states: quarkonia, glueball and hybrid state $q\bar{q}g$ to accommodate a possible new state $f_0(1790)$ and other known states in the $0^+(0^{++})$ family with masses in the range between 1 to 2 GeV. Using known experimental data, we have been able to obtain information on the mixing parameters. The related phenomenology indicates that the parameters obtained from this fitting are within reasonable ranges. If the $f_0(1790)$ state is confirmed, this scheme predicts the existence of a new particle $X$. This can be tested further with BES data.

The decay channels of $f_0(1790)$ and $X$ to two pseudoscalar mesons can provide important information about the mixing mechanism and distinguish cases (a) and (b). For case (a) all possible decay modes have large branching ratios which may be measured by improved experiments, whereas for case (b), the decays of $f_0(1790)$ to $\eta\eta$, $\eta\eta'$, and $X$ to $\eta\eta'$ have substantially smaller branching ratios. Particularly the ratio for $r = B(f_0(1790) \to \pi\pi)/B(f_0(1790) \to K\bar{K})$ is a very important criterion to test which case is more realistic since for case (a) this ratio is about 2 and for case (b) it is about 0.4. This can be easily understood by noticing that the main component of $f_0(1790)$ in case (a) is $(u\bar{u} + d\bar{d})g/\sqrt{2}$ which has a much larger probability to transit into $\pi\pi$ compared with case (b) where the main component of $f_0(1790)$ is $s\bar{s}g$. Therefore the central value ($r = 3.88$) of the present data favors case (a) over case (b) although a definitive conclusion cannot be drawn at this stage due to larger experimental errors.
Another interesting fact to note is that \( f_0(1710) \) is observed in the \( K\bar{K} \) spectrum in \( J/\psi \rightarrow \omega K\bar{K} \) and \( J/\psi \rightarrow \phi K\bar{K} \) decays, but not in \( J/\psi \rightarrow \phi \pi\pi \). There has been some theoretical effort to explain this observation \[16\]. In our picture this is also very natural since the main component of \( f_0(1710) \) is \( s\bar{s} \) which does not directly transit to \( \pi\pi \), so that \( f_0(1710) \rightarrow \pi\pi \) would be much suppressed compared with \( f_0(1710) \rightarrow K\bar{K} \).

Obviously, by contraries, the radiative decay modes to \( \gamma\gamma \) would be very difficult to measure via \( J/\psi \rightarrow \gamma f_0(1790)(X) \rightarrow \gamma\gamma\gamma \), because the final states with only three photons are hard to be reconstructed. We hope that our experimental colleagues can figure out some ways to make the difficult measurements.

With the present data, we are not able to make detailed predictions for the decay modes with two vector mesons in the final state. However, from the diagrams shown in Fig.1, we can expect that some decay modes may be measured and help us to gain more information about the properties of the new states \( f_0(1790) \) and \( X \). The radiative decays such as \( J/\psi \rightarrow \gamma f_0(1790) \rightarrow \gamma VV' \) and \( J/\psi \rightarrow \gamma X \rightarrow \gamma VV' \) are promising channels to study \( f_0(1790) \) and \( X \) states. A particularly interesting channel is \( J/\psi \rightarrow \gamma \phi \omega \) since this is a doubly OZI suppressed processes if there is not a hybrid intermediate state. With a hybrid state \( X \), the process \( J/\psi \rightarrow \gamma X \rightarrow \gamma \phi \omega \) may have a large branching ratio. This may happen for case (a) since \( X \) has a mass about 1820 MeV, but not possible for case (b) since in this case the \( X \) mass of 1760 MeV is below the threshold. We strongly urge our experimental colleagues to carry out precise measurements to test the mechanism proposed here.

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Note Added When we were ready to submit this paper, we saw a paper on the archive by B. A. Li (hep-ph/0602072) who cited that the BES collaboration reported the observation of a state \( X(1810) \) in the spectrum of \( \omega\phi \) in \( J/\psi \rightarrow \gamma \omega\phi \). This newly observed state fits our prediction of case (a) well. B. A. Li proposed the \( X(1810) \) to be a four-quark state which is different from our hybrid state description. After submitting the paper on the archive we also became aware of the papers by Vijande et al. who discussed \( f_0(1790) \) in the framework of mixing of a chiral nonet tetraquarks with conventional \( q\bar{q} \) states. We
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| Parameter                        | (a) Fitted | (b) Fitted | Parameter                        | (a) Fitted | (b) Fitted |
|----------------------------------|------------|------------|----------------------------------|------------|------------|
| $M_{H_S}$(GeV)                   | 1.82       | 1.79       | $e$(GeV)                         | 0.03       | 0.03       |
| $M_{H_N}$(GeV)                   | 1.78       | 1.75       | $f$(GeV)                         | 0.08       | 0.08       |
| $M_G$(GeV)                       | 1.43       | 1.43       | $\xi_1$                         | 1.27       | 1.46       |
| $M_S$(GeV)                       | 1.69       | 1.69       | $\xi_2$                         | 0.41       | 0.40       |
| $M_N$(GeV)                       | 1.42       | 1.42       | $\xi_3$                         | 0.80       | 0.10       |
| $M_X$(GeV)                       | 1.823      | 1.758      | $\xi_4$                         | 0.10       | 0.10       |
| $M_{f_0(1790)}$(GeV)            | 1.786      | 1.794      |                                 |            |            |
| $M_{f_0(1710)}$(GeV)            | 1.713      | 1.712      |                                 |            |            |
| $M_{f_0(1500)}$(GeV)            | 1.516      | 1.516      |                                 |            |            |
| $M_{f_0(1370)}$(GeV)            | 1.301      | 1.301      |                                 |            |            |

TABLE II: The values for the parameters in the mass matrix and the $PP$ decay amplitudes.
|                                | (a)  | (b)  |
|--------------------------------|------|------|
| $BR(f_0(1790) \to \pi\pi)$    | 23.0%| 1.3% |
| $BR(f_0(1790) \to K\bar{K})$  | 10.3%| 3.1% |
| $BR(f_0(1790) \to \eta\eta)$  |  4.5%|  0.6%|
| $BR(f_0(1790) \to \eta\eta')$ |  2.3%|  0.03%|
| $\Gamma(X \to \pi\pi)$ MeV    |  5.2 | 10.2 |
| $\Gamma(X \to K\bar{K})$ MeV  | 44.8 | 14.8 |
| $\Gamma(X \to \eta\eta)$ MeV  |  6.8 |  5.4 |
| $\Gamma(X \to \eta\eta')$ MeV |  4.6 |  0.4 |

TABLE III: The branching ratios of $f_0(1790) \to PP'$ and the widths of $X \to PP'$.

FIG. 1: The diagrams correspond respectively to terms in eq. (3). The last five terms are OZI suppressed ones.