A Consistent Calculation of Heavy Meson Decay Constants and Transition Wave Functions in the Complete HQEFT

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Abstract

Within the complete heavy quark effective field theory (HQEFT), the QCD sum rule approach is used to evaluate the decay constants including $1/m_Q$ corrections and the Isgur-Wise function and other additional important wave functions concerned at $1/m_Q$ for the heavy-light mesons. The number of unknown wave functions or form factors in HQEFT is shown to be much less than the one in the usual heavy quark effective theory (HQET). The values of wave functions at zero recoil are found to be consistent with the ones extracted from the interesting relations (which are resulted from the HQEFT) between the hadron masses and wave functions at zero recoil. The results for the decay constants are consistent with the ones from full QCD sum rule and Lattice calculations. The $1/m_Q$ corrections to the scaling law $f_M \sim F/\sqrt{m_M}$ are found to be small in HQEFT, which demonstrates again the validity of $1/m_Q$ expansion in HQEFT. It is also shown that the residual momentum $v \cdot k$ of heavy quark within heavy-light hadrons does be around the binding energy $\bar{\Lambda}$ of the heavy hadrons, which turns out to be in agreement with the expected one in the HQEFT. Therefore such a calculation provides a consistent check on the HQEFT and shows that the HQEFT is more reliable than the usual HQET for describing a slightly off-mass shell heavy quark within hadron as the usual HQET seems to lead to the breakdown of $1/m_Q$ expansion in evaluating the meson decay constants. It is emphasized that the introduction of the ‘dressed heavy quark’ mass is useful for the heavy-light mesons ($Qq$) with $m_Q >> \Lambda >> m_q$, while for heavy-heavy bound states ($\psi_1 \psi_2$) with masses $m_1$, $m_2 >> \Lambda$, like bottom-charm hadrons or similarly for muonium in QED, one needs to treat both particles as heavy effective particles via $1/m_1$ and $1/m_2$ expansions and redefine the effective bound states and modified ‘dressed heavy quark’ masses within the HQEFT.

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I. INTRODUCTION

It has been seen that the heavy quark effective field theory (HQEFT) with keeping both quark and antiquark fields can provide a consistent description on both exclusive and inclusive decays of heavy hadrons. The extracted values of $|V_{cb}|$ from both exclusive and inclusive decays have shown a good agreement. The lifetime differences among bottom mesons and hadrons can also be well understood within the new framework of HQEFT. Especially, it has been noticed that at zero recoil, $1/m_Q$ corrections in both exclusive and inclusive decays are automatically absent without imposing the equation of motion $iv \cdot DQ_v = 0$ when the physical observables are presented in terms of heavy hadron masses, this is the main point that differs from the usual heavy quark effective theory (HQET) or the usual heavy quark expansion. In the usual framework, $1/m_Q$ corrections in the inclusive decays are absent only when the inclusive decay rate is presented in terms of heavy quark mass ($m_Q$) rather than the heavy hadron mass ($m_H$), the situation seems to be conflict with the case in the exclusive decays where the normalization is given in term of heavy hadron mass. Such an inconsistency in the usual HQET may be the main reason that leads to the difficulty for understanding the lifetime differences among the bottom hadrons. These observations indicate that the contributions and effects from antiquark fields should play a significant role for understanding hadronic structures and can become important for certain physical observables.

Our basic point of considerations is based on the physical picture that a heavy hadron ($Qq$) containing a single heavy quark ($Q$) and satisfying the condition among the heavy quark mass $m_Q$, the light quark mass $m_q$ and the binding energy $\bar{\Lambda}$ may be regarded as a ‘dressed heavy quark’ with an off-mass shell by an amount of binding energy $\bar{\Lambda}$. In this case, the usual quark-hadron duality should be extended to a ‘dressed-heavy quark’ - ‘heavy-light hadron’ duality. Thus a more reliable heavy quark expansion for heavy-light hadron systems should be carried out in terms of the “dressed heavy quark” mass defined as

$$\hat{m}_Q \equiv m_Q + \bar{\Lambda} = m_H - O(1/m_Q) = \lim_{m_Q \to \infty} m_H$$

with $m_H$ the heavy hadron mass. Before proceeding, we would like to point out that this picture cannot naively be applied to the hadrons with

$$m_Q, m_q >> \bar{\Lambda}$$

This is because such hadrons must be treated as heavy-heavy hadrons, like $B_c$ meson system. Similarly, the muonium ($\mu e$) must also be treated as heavy-heavy bound state system in QED. In general for heavy-heavy system ($\psi_1 \psi_2$) with masses $m_1, m_2 >> \bar{\Lambda}$, one should make expansion for both heavy particles in the bound state and redefine the effective fields and effective bound states.

It has been shown that the new framework of HQEFT with keeping the antiquark fields enables us to describe such an off-mass shell heavy quark within hadrons as the $1/m_Q$...
corrections to the transition matrix elements automatically vanish at zero recoil. Of particular, the number of transition form factors involved in the HQEFT is much less than the one in the usual HQET. In addition, the new framework of HQEFT also results in some interesting relations among meson masses and transition formfactors at zero recoil, which enables us to extract the important transition form factors from the known heavy meson masses. It is those special features that allow us to check self-consistently the validity of the new framework of HQEFT when applying it to the heavy-light hadron systems. This can simply be carried out by comparing the values of transition formfactors and residual momentum of heavy quark in hadron, which are extracted from the relations between the meson masses and formfactors, with those obtained from directly evaluating the corresponding hadronic matrix elements by using some reliable approaches. In this paper, we are going to adopt QCD sum rule approach for a practical calculation. Our paper is organized as follows. In Sec. II, we briefly review the heavy quark expansion and present a general description on form factors concerned up to order $1/m_Q^2$, their corresponding hadronic matrix elements in heavy meson weak decays and weak transitions between two heavy mesons are explicitly defined. In Sec. III, we apply the QCD sum rules to two point Green functions of heavy quark currents within the framework of HQEFT, and calculate three form factors $F$, $G_1$ and $G_2$ as well as two composite factors $g_1$ and $g_2$ concerned in the decay and coupling constants of heavy pseudoscalar and vector mesons. It also enables us to extract a reasonable value of the binding energy $\bar{\Lambda}$. In Sec. IV, we investigate three point Green functions of heavy quark operators and apply the QCD sum rule to evaluate the Isgur-Wise function and some additional wave functions appearing at the order of $1/m_Q$. It is also shown that the residual momentum of the heavy quark within the hadron is truly around the binding energy, i.e., $iv \cdot D \sim v \cdot k \approx \bar{\Lambda}$ is found to be a good approximation in simplifying the evaluation of the hadronic matrix elements. In Sec. V, we add the two-loop perturbative contributions to the two-point collerator for the purpose of seeing the importance of QCD radiative corrections. A brief summary with some remarks is presented in Sec. VI.

II. HADRONIC MATRIX ELEMENTS IN HQEFT

It has been shown that by decomposing the original heavy quark field in QCD into effective quark field and antiquark field and integrating out both the antiquark field and the small components of quark field, we arrive at an effective theory for only the large components of quark field $Q_v^+$, its effective Lagrangian has the following form

$$L_{\text{eff}}^{(++)} = L_{\text{eff}}^{(0)} + L_{\text{eff}}^{(1/m_Q)} = \bar{Q}_v^+ i\not{D}\parallel Q_v^+ + \frac{1}{m_Q} \bar{Q}_v^+ (i\not{D}_\perp) Q_v^+ + O\left(\frac{1}{m_Q^2}\right),$$

(2.1)

where $Q_v^+$ is the heavy quark effective field in this new framework of HQEFT, its momentum is $k = P_Q - m_Q v$ with $P_Q$ the momentum of the original heavy quark field in QCD. The operators $\not{D}_\parallel$ and $\not{D}_\perp$ are defined as

$$ \not{D}_\parallel \equiv \not{\varphi} (v \cdot D), \quad \not{D}_\perp \equiv \not{\varphi} (v \cdot D) $$

(2.2)
with $v^\mu$ an arbitrary four-vector satisfying $v^2 = 1$. In general, a mass dimension parameter $\Lambda$ may appear in the Lagrangian depending on the redefinition of heavy quark effective field theory, here we take $\Lambda = 0$ for the convenience in comparison with the convention in the usual heavy quark effective theory (HQET).

The decay constants of a heavy pseudoscalar meson $P$ and a heavy vector meson $V$ are defined by

$$<0|\bar{q}_\gamma \gamma_5 Q|P(v)> = i f_P m_P v^\mu,$$

$$<0|\bar{q}_\gamma Q|V(\epsilon, v)> = f_V m_V \epsilon^\mu$$

with $Q$ the original quark field in QCD. Here $\epsilon^\mu$ is the polarization vector of a vector meson. $|P(v)>$ and $|V(\epsilon, v)>$ are the pseudoscalar and vector meson states in QCD.

In the new framework of HQEFT, current operators composed by one heavy quark and one light (or heavy) quark can be expanded in terms of operators in the effective theory $\bar{q} \Gamma Q \rightarrow \bar{q} \Gamma Q^+_v + \frac{1}{2m_Q} \bar{q} \frac{i}{iD_\perp} (iD_\perp)^2 Q^+_v + O(1/m_Q^2),$

$$\bar{Q} \Gamma Q \rightarrow \bar{Q}^+_v \Gamma Q^+_v + \frac{1}{2m_Q} \bar{Q}^+_v \frac{i}{iD_\perp} (iD_\perp)^2 Q^+_v + \frac{1}{2m_Q} Q^+_v (-i \frac{i}{iD_\perp})^2 \frac{1}{m_Q} \Gamma Q^+_v + O(1/m_Q^2).$$

Correspondingly, it is useful to introduce an effective heavy hadron state $|H_v>$ for exhibiting a manifest heavy quark spin-flavor symmetry. Such an effective hadron state in HQEFT is related to the hadron state $|H>$ in QCD via

$$\frac{1}{\sqrt{m_{H^c} m_H}} <H^c|\bar{Q} \Gamma Q|H> = \frac{1}{\sqrt{\tilde{\Lambda}_H \tilde{\Lambda}_H}} <H^c|\bar{Q}^+_v \Gamma Q^+_v|H_v> = 2 \tilde{\Lambda} v^\mu$$

with

$$\tilde{\Lambda}_H^{(v)} \equiv m_{H^{(v)}} - m_{Q^{(v)}}.$$ (2.6)

The renormalization condition for $|H_v>$ is given by

$$<H_v|\bar{Q}^+_v \gamma^\mu Q^+_v|H_v> = 2 \tilde{\Lambda} v^\mu$$

with

$$\tilde{\Lambda} = \tilde{\Lambda}_H - O(1/m_Q) = \lim_{m_Q \rightarrow \infty} \tilde{\Lambda}_H$$

being a heavy flavor independent binding energy that reflects the effects of the light degrees of freedom in the heavy hadron. Obviously, the above normalization condition preserves spin-flavor symmetry. We would like to address again that above renormalization is only applicable for the heavy-light hadrons with $m_Q >> \tilde{\Lambda} >> m_q$. For heavy-heavy bound state system, i.e. $m_1, m_2 >> \tilde{\Lambda}$, such as bottom-charm meson $B_{c}$ and muonium ($\mu e$), one needs to redefine the effective bound states by considering $1/m_Q$ expansion for both heavy particles.
In the heavy quark expansion, $1/m_Q$ order corrections to the hadronic matrix elements arise from not only the current expansion (2.4) but also the effective Lagrangian, i.e., insertion of $L_{eff}^{(1/m_Q)}$ into the matrix elements. By including all these corrections, we arrive at the following results for the hadronic matrix elements of the currents in (2.4)

$$\left. \frac{\hat{A}_M}{m_M} \right|_M < 0|q\bar{q}Q|M> \rightarrow <0|q\bar{q}Q^+_v|M_v> - \frac{1}{2m_Q} <0|q\bar{q}\Gamma\left(\frac{1}{i\not{P}_\perp}\right)^2Q^+_v|M_v> + O(1/m^2_Q),$$

$$\left. \frac{\hat{A}_M\hat{A}_M}{m_M^2m_M} \right|_M < M'|\bar{Q}'\Gamma Q|M> \rightarrow <M'_v|\bar{Q}'_v\Gamma^+Q_v^+|M_v> - \frac{1}{2m_Q} <M'_v|\bar{Q}'_v\Gamma\left(\frac{1}{i\not{P}_\perp}\right)^2Q_v^+|M_v>$$

$$- \frac{1}{2m_Q} <M'_v|\bar{Q}'_v\left(-i\not{P}_\perp\right)^2\frac{1}{-i\not{P}_\perp}\Gamma Q_v^+|M_v> + O(1/m^2_Q).$$

(2.8)

The relevant hadronic matrix elements may be parameterized as follows

$$<0|q\bar{q}Q^+_v|M_v> = \frac{F}{2}Tr[\Gamma M],$$

$$<0|q\bar{q}D\left(\frac{1}{i\not{v}D}\right)^2Q^+_v|M_v> = -FG_1Tr[\Gamma M],$$

$$<0|q\bar{q}\left(\frac{1}{i\not{v}D}\right)^2\sigma_{\alpha\beta}F^{\alpha\beta}Q^+_v|M_v> = 2FG_2Tr[i\sigma_{\alpha\beta}\Gamma P+\frac{i}{2}\sigma^{\alpha\beta}M] = -2FG_2d_MT Tr[\Gamma M],$$

$$<M'_v|\bar{Q}'_v\Gamma Q_v^+|M_v> = -\xi(y)Tr[\bar{N}\Gamma M],$$

$$<M'_v|\bar{Q}'_v\Gamma\left(\frac{1}{i\not{v}D}\right)^2Q_v^+|M_v> = -\kappa_1(y)\frac{1}{\Lambda}Tr[\bar{N}\Gamma M],$$

$$<M'_v|\bar{Q}'_v\Gamma\left(-i\not{v}D\right)^2\sigma_{\alpha\beta}F^{\alpha\beta}Q_v^+|M_v> = \frac{1}{\Lambda}Tr[\kappa_{\alpha\beta}(v,v')\bar{N}\Gamma M P+\frac{i}{2}\sigma^{\alpha\beta}M].$$

(2.9)

where $y = v \cdot v'$. $M(v)$ is the spin wave function

$$\mathcal{M}(v) = \sqrt{\Lambda}P_+ \begin{cases} -\gamma^5, & \text{for pseudoscalar meson } P \\ \not{\phi}, & \text{for vector meson } V \end{cases}$$

(2.10)

and

$$d_M = \begin{cases} 3, & \text{for pseudoscalar meson } P \\ -1, & \text{for vector meson } V \end{cases}$$

(2.11)

where $F$, $G_1$ and $G_2$ are constants, and $\xi(y)$ and $\kappa_1(y)$ the Lorentz scalar functions. Actually, $\xi(y)$ is the well-known Isgur-Wise functions. The Lorentz tensor $\kappa_{\alpha\beta}(v,v')$ can be decomposed into

$$\kappa_{\alpha\beta}(v,v') = i\kappa_2(y)\sigma_{\alpha\beta} + \kappa_3(y)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha)$$

(2.12)

with $\kappa_2(y)$ and $\kappa_3(y)$ being the Lorentz scalar functions.

Combining (2.3), (2.7) and (2.9) we have

$$f_M = \sqrt{\frac{\Lambda}{\Lambda_Mm_M}}F\{1 + \frac{1}{m_M}(G_1+2d_MG_2)\}.$$  

(2.13)
Thus the ratio between the vector and pseudoscalar meson constants is given by

$$\frac{f_{V}m_{V}^{1/2}}{f_{P}m_{P}^{1/2}} = \left(\frac{\bar{\Lambda}}{\Lambda}\right)^{1/2}(1 - \frac{8}{m_{Q}}G_{2}). \quad (2.14)$$

As is known, the normalization of the Isgur-Wise function at zero recoil point is given by $\xi(1) = 1$. The additional wave functions $\kappa_{1}(y)$, $\kappa_{2}(y)$ and $\kappa_{3}(y)$ characterize the next-to-leading order symmetry-breaking corrections to $\xi$. From (2.9) and (2.12), it is easily seen that the hadronic matrix element at zero recoil is irrelevant to $\kappa_{3}(1)$. While $\kappa_{1}(1)$ and $\kappa_{2}(1)$ are found to be related to the meson masses $[2,3]\bar{\Lambda}_{M} = m_{M} - m_{Q} = \bar{\Lambda} - \left(\frac{1}{m_{Q}} - \frac{\bar{\Lambda}}{2m_{Q}^{2}}\right)(\kappa_{1}(1) + d_{M}\kappa_{2}(1)). \quad (2.15)$

It is easy to check that the leading order contributions to the hadronic matrix elements in the HQEFT, which are characterized by the decay constant $F$ and the Isgur-Wise function $\xi$ defined in (2.9), are the same as the ones in the usual HQET. Nevertheless, to the next-to-leading order, differences occur between the two frameworks. In the usual HQET, the transition matrix elements between two heavy mesons are parameterized by six functions denoted as $\xi_{i} \text{ and } \chi_{i} (i = 1, 2, 3)$. Here $\xi_{1}$, $\xi_{2}$ and $\xi_{3}$ arise from the current expansion, and $\chi_{1}$, $\chi_{2}$ and $\chi_{3}$ from the insertion of the effective Lagrangian into the hadronic matrix elements. All these six quantities are functions of the recoil parameter $y = v \cdot v'$. In addition, there are two more parameters $\lambda_{1}$ and $\lambda_{2}$ which appear in the $1/m_{Q}$ order corrections to heavy meson masses. Unlike, in the new framework of HQEFT, it has been noticed that the evaluation of the hadronic matrix elements is greatly simplified when the antiquark contributions are included $[1,3]$. As a consequence, all transition matrix elements concerned at $1/m_{Q}$ order can be characterized by only three wave functions $\kappa_{1}(y)$, $\kappa_{2}(y)$ and $\kappa_{3}(y)$. Furthermore, the $1/m_{Q}$ order corrections to meson masses were found to be naturally related to their values at zero recoil, i.e., $\kappa_{1}(1)$ and $\kappa_{2}(1)$. In other words, instead of the six wave functions $\xi_{i}(y)$ and $\chi_{i}(y) (i=1,2,3)$ and two parameters $\lambda_{1}$ and $\lambda_{2}$ in the usual HQET, we only need to evaluate three wave functions $\kappa_{i} (i = 1, 2, 3)$ in the new framework of HQEFT up to the order $1/m_{Q}$. At zero recoil, only two parameters $\kappa_{1}(1)$ and $\kappa_{2}(1)$ are relevant. Similar comments hold for the corrections to meson decay constants. In the HQEFT one encounters only two constants $G_{1}$ and $G_{2}$ at $1/m_{Q}$ order.

Within the new framework of HQEFT, two important parameters $\kappa_{1}(1)$ and $\kappa_{2}(1)$ have been extracted from meson mass spectrum in refs. $[2]$. To have an independent check for the HQEFT, it would be very useful to evaluate these two wave functions directly by a field-theoretical method within the framework of HQEFT. In the following sections, we shall present an QCD sum rule study for these form factors.

**III. QCD SUM RULE CALCULATION OF $F$, $G_{1}$ AND $G_{2}$**

QCD sum rule approach has widely been used to calculate hadronic matrix elements in QCD and has also been applied to effective theories of QCD. It turns out to be a powerful analytic approach to estimate non-perturbative effects. The basic idea of QCD sum rule formalism is to study the analytic properties of correlation functions, and to treat the bound
state problems in QCD from quark-hadron duality considerations. So that one could start at short distance physics and moves to large distance physics where confinement effects become important and resonances emerge as a reflection of confinement. Here we shall briefly outline the main steps of sum rule treatments. In general, the fixed point gauge for the background fields is used in calculating Feynman diagrams.

In order to evaluate the parameters $F$, $G_1$ and $G_2$ involved in the meson decay constants, we consider the following two-point correlator

$$\Pi(\omega) = i \int d^4x e^{iP \cdot x} \langle 0 | (\bar{q} \Gamma_M Q)_x (0), (\bar{Q} \Gamma_M q)_0 | 0 \rangle,$$  \hspace{1cm} (3.1)

where $\Gamma_M$ has appropriate Lorentz structure so that the two currents in (3.1) interpolate the heavy meson of interest. It is convenient to choose

$$\Gamma_M = \begin{cases} -i\gamma^5, & \text{pseudoscalar meson} \ P \\ \gamma_\mu - v_\mu, & \text{vector meson} \ V \end{cases}$$  \hspace{1cm} (3.2)

The total external momentum in (3.1) is $P = m_Q v + k$ with the momentum $k$ in (3.1) being the residual momentum of the heavy quark. The correlator $\Pi$ is an analytic function of $2v \cdot k + k^2/m_Q$ with discontinuities for its positive values. Here $k^\mu = k_T^\mu + k_L^\mu$ and $k_T^\mu = (v \cdot k)v^\mu$ being the transverse and longitudinal part of $k$ separately. Particularly, under the definition $\omega \equiv 2v \cdot k + k^2/m_Q$, one has

$$2v \cdot k + \frac{k^2}{m_Q} = \omega + \frac{\omega^2}{4m_Q} + O(1/m_Q^2).$$  \hspace{1cm} (3.3)

In eq.(3.1) we have represented $\Pi$ as an analytic function of the variable $\omega$.

Phenomenologically, the two-point function $\Pi(\omega)$ can be written as the sum of three parts: a pole contribution from the ground state mesons associated with the heavy-light currents; a dispersion integral over a physical spectral function; and subtraction terms, namely,

$$\Pi_{\text{phen}}(\omega) = -\left(\sum_{\text{pole}}\right) \frac{\langle 0 | \bar{q} \Gamma_M Q | M \rangle \langle M | \bar{Q} \Gamma_M q | 0 \rangle}{P^2 - m_M^2 + i\epsilon}$$

$$+ \int_{\omega_c}^{\infty} d\nu \frac{\rho_{\text{phys}}(\nu)}{\nu - \omega - i\epsilon} + \text{subtractions},$$  \hspace{1cm} (3.4)

where the two matrix elements should be expanded by using eq.(2.8). ($\sum_{\text{pole}}$) means summation over polarization for vector mesons.

On the other hand, using the Feynman rules of the HQEFT, the correlation function in HQEFT is evaluated perturbatively in the deep Euclidean region ($\omega \ll \Lambda$) via

$$\Pi(\omega) = i \int d^4x e^{iP \cdot x} \left\{ \langle 0 | (\bar{q} \Gamma_M Q^+_v)_x (0), (\bar{Q}^+_v \Gamma_M q)_0 | 0 \rangleight.$$  

$$+ \frac{1}{2m_Q} \langle 0 | (\bar{q} \Gamma_M \frac{1}{i\not{D}_{\perp}}(i\not{D}_{\parallel})^2 Q^+_v)_x (0), (\bar{Q}^+_v \Gamma_M q)_0 | 0 \rangle$$  

$$+ \frac{1}{2m_Q} \langle 0 | (\bar{q} \Gamma_M Q^+_v(x), (\bar{Q}^+_v \Gamma_M q)_0 | 0 \rangle$$  

$$+ O(1/m_Q^2) \right\}$$  \hspace{1cm} (3.5)
In lower Euclidean region, the non-perturbative contributions become important. In QCD sum rule analysis, the two-point correlator receives contributions not only from the pure perturbative ones but also the ones from condensates, which characterizes non-vanishing vacuum expectation values of local operators in the operator product expansion (OPE). It is thought that by including these condensates, the QCD confinement effects may be accounted for at the transition from perturbative region to non-perturbative region. Writing the perturbative contributions as the form of an integral over a theoretic spectral function, the theoretical result for the two-point correlator (3.1) becomes

\[ \Pi_{\text{theor}}(\omega) = \int d\nu \frac{\rho_{\text{pert}}(\nu)}{(\nu - \omega - i\epsilon)} + \Pi_{NP} + \text{subtractions.} \]  

(3.6)

A basic assumption in QCD sum rule is the quark-hadron duality. Due to this duality one can model the contributions of higher resonance states by the perturbative continuum starting at a threshold energy \( \omega_c \). In other words, we assume \( \rho_{\text{phys}} = \rho_{\text{pert}} \). Equating the phenomenological side and the theoretical side, up to the order of \( 1/m_Q \) one arrives at

\[ 2Tr[\Gamma_M P_+ \Gamma_M]F^2 \frac{F^2}{4} \left[ 1 + \frac{2}{m_Q} (G_1 + 2d_m G_2) \right] \frac{\Lambda}{2\Lambda m_Q} \frac{m}{m_Q} \frac{1}{\omega - \omega_M + i\epsilon} (1 - \frac{\omega - \omega_M}{4m_Q}) \]

\[ = \int_0^{\omega_c} d\nu \frac{\rho_{\text{pert}}(\nu)}{(\nu - \omega - i\epsilon)} + \Pi_{NP} + \text{subtractions} \]  

(3.7)

with \( \omega_M \equiv 2\Lambda \). In deriving (3.7) we have used (2.9) as well as the relations

\[ Tr[\Gamma_M M(v)]Tr[M(v)\Gamma_M] = -2\Lambda Tr[\Gamma_M P_+ \Gamma_M] \]  

(3.8)

and

\[ \frac{P^2 - m^2}{m_Q} = (\omega - \omega_M)[1 + \frac{\omega + \omega_M}{4m_Q} + O(1/m_Q^2)]. \]  

(3.9)

The relevant Feynman diagrams are plotted in Fig.1. Fig.1(a) is the lowest order perturbative diagram. For the non-perturbative effects, it is sufficient in the present case to consider only the contributions of the quark condensate, the gluon condensate and the mixed quark-gluon condensate, which have values \((\alpha_s \equiv g_s^2/4\pi)\)

\[ < \bar{q}q > \approx -(230) \text{ MeV}^3; \]

\[ i < \bar{q} \sigma_{\alpha\beta} F^{\alpha\beta} q > \approx -0.8 < \bar{q}q >; \]

\[ \alpha_s < FF > \approx \alpha_s < F_{\alpha\beta} F_a^{\alpha\beta} > \approx 0.04 \text{ GeV}^4. \]  

(3.10)

We only keep terms up to order \( \alpha_s \) for the condensates. Neglecting the light quark mass, we obtain the following results at the renormalization scale \( \mu \sim 2\Lambda \)

\[ F^2 \left[ 1 + \frac{2}{m_Q} (G_1 + 2d_m G_2) - \frac{1}{m_Q} (\omega_M^{(1)} + d_M \omega_M^{(2)}) + \frac{\Lambda}{m_Q} \frac{1}{\omega - \omega_M + i\epsilon} (1 - \frac{\omega - \omega_M}{2m_Q}) \right] = \]

\[ \frac{3}{8\pi^2} \int_0^{\omega_c} d\nu \left[ \frac{1}{4m_Q} + \frac{1}{\nu - \omega - i\epsilon} (\nu^2 - \frac{3\nu^3}{4m_Q}) + < \bar{q}q > \left[ \frac{1}{\omega - \omega_M + i\epsilon} + \frac{1}{\nu - \omega - i\epsilon} (\frac{1}{4m_Q} + \frac{\omega - \omega_M}{2m_Q}) \right] \right] \]

\[ + \frac{\alpha}{\pi} < FF > \left[ \frac{1}{24\omega^2} - \frac{15 - 4d_M}{96m_Q\omega} \right] + \text{subtractions.} \]  

(3.11)
In order to improve the convergence and suppress the importances of higher-resonance states, we apply the Borel operator

$$\hat{B}_T^{(\omega)} \equiv T \lim_{n \rightarrow \infty} \frac{\omega^n}{\Gamma(n)} \left( -\frac{d}{d\omega} \right)^n$$

(3.12)

to both sides of (3.11) with $T = \frac{\omega}{\omega_n}$ being held fixed. In the dispersion integral, this Borel transformation yields an exponential damping factor which effectively suppresses high-resonance contributions. On the non-perturbative terms, Borel transformation enhances the importance of low dimension condensates. Furthermore, the subtraction terms in (3.11) may also be eliminated by this Borel transformation.

The resulting Borel transformed sum rule reads

$$F^2 \left[ 1 + \frac{2}{m_Q} (G_1 + 2d_MG_2) - \frac{1}{2m_Q \Lambda} (\omega_M^{(1)} + d_M\omega_M^{(2)}) + \frac{\Lambda}{m_Q} - \frac{\bar{\omega}_M}{2m_Q} e^{-\omega_M/T} \right] =$$

$$\frac{3}{8\pi^2} \int_0^{\omega_c} d\nu e^{-\nu/T} \left( \nu^2 - \frac{3\nu^3}{4m_Q} \right)$$

$$- \langle \bar{q}q \rangle \left( 1 + \frac{4\alpha_s}{3\pi} \right) - i \langle \bar{q}\sigma_{\alpha\beta}F_{\alpha\beta}q \rangle \left[ \frac{1}{4T^2} + \frac{1}{m_Q T} \left( \frac{3}{8} - \frac{d_M}{12} \right) \right]$$

$$+ \frac{\alpha_s}{\pi} \left[ \frac{1}{T^2} - \frac{163 + 147d_M}{576m_T} \right] - \frac{\alpha_s}{\pi} <FF> \left( \frac{1}{24T} + \frac{15 - 4d_M}{96m_Q} \right).$$

(3.13)

Besides the $1/m_Q$ corrections shown explicitly in (3.13), the pole energy $\omega_M$ and the threshold energy $\omega_c$ also receive $1/m_Q$ corrections. We may write them as

$$\omega_M \equiv 2\bar{\Lambda}_M = \omega_M^{(0)} + \frac{1}{m_Q} (\omega_M^{(1)} + d_M\omega_M^{(2)}),$$

$$\omega_c = \omega_0 + \frac{1}{m_Q} (\omega_1 + d_M\omega_2).$$

(3.14)

With these formulae, eqs.(2.13) and (2.14) can be rewritten as

$$f_M = \frac{F}{\sqrt{m_M}} \left\{ 1 + \frac{1}{m_Q} (g_1 + 2d_Mg_2) \right\}$$

(3.15)

and

$$\frac{f_V m_V^{1/2}}{f_P m_P^{1/2}} = 1 - \frac{8}{m_Q} g_2$$

(3.16)

with $g_1$ and $g_2$ being two composite factors defined as

$$g_1 \equiv G_1 - \frac{\omega_M^{(1)}}{4\bar{\Lambda}},$$

$$g_2 \equiv G_2 - \frac{\omega_M^{(2)}}{8\bar{\Lambda}}.$$
and the usual HQET in the limit $m_Q \to \infty$. We shall not repeat the analysis for the leading order sum rule analysis, our numerical results are presented in Fig.4, where we have used $\alpha_s(2\bar{\Lambda}) \simeq 0.34$. From the stability of the curves, we are led to the following solutions for the parameters

$$\omega_0 = 1.8 \pm 0.3 \text{ GeV},$$
$$\frac{\omega_M^{(0)}}{2} = \bar{\Lambda} = 0.53 \pm 0.08 \text{ GeV},$$
$$F = 0.30 \pm 0.06 \text{ GeV}^{3/2}. \quad (3.18)$$

We now proceed to the next-to-leading order sum rule analysis in (3.13). Putting (3.14) into (3.13), and expanding (3.13) in $1/m_Q$, we obtain, at $1/m_Q$ order, two sum rule formulae which are relevant to the spin-symmetry conserving and violating corrections, respectively. These two sorts of corrections are easy to be distinguished because the latter is proportional to $d_M$. To find out the solutions, it is useful to first evaluate the quantities $\omega_1$ and $\omega_2$ by requiring optimal stability of $\omega_M^{(1)}$ and $\omega_M^{(2)}$ with respect to the Borel parameter $T$ in the allowed sum rule windows. Using the central values of $\omega_0$, $\bar{\Lambda}$ and $F$ in (3.18), we then obtain for $\omega_i$, $\omega_M^{(i)}$, $G_i$ and $g_i$ the numerical results plotted in Figs.5-10. When the Borel parameter $T$ takes the reliable values $T = 1 \pm 0.2$ GeV, one can read off the following solutions

$$\omega_1 = 1.5 \pm 0.2 \text{ GeV}^2,$$
$$\omega_M^{(1)} = 0.86 \pm 0.10 \text{ GeV}^2,$$
$$G_1 = 0.95 \pm 0.15 \text{ GeV},$$
$$g_1 = 0.54 \pm 0.12 \text{ GeV},$$
$$\omega_2 = -0.15 \pm 0.05 \text{ GeV}^2,$$
$$\omega_M^{(2)} = -0.16 \pm 0.03 \text{ GeV}^2,$$
$$G_2 = -0.09 \pm 0.03 \text{ GeV},$$
$$g_2 = -0.06 \pm 0.02 \text{ GeV}. \quad (3.19)$$

In the usual HQET [8] two parameters $G_1$ and $G_2$ were defined for the $1/m_Q$ corrections to decay constants. The sum rule calculation in [8] yielded an unexpectedly large value for $|G_1|$: $G_1 \approx -4\bar{\Lambda} \approx -2.0$GeV, which leads to the breakdown of $1/m_Q$ expansion for decay constants. In the new framework of HQEFT, however, as can be seen from eqs.(3.15) and (3.16), $1/m_Q$ corrections to the physical decay constants $f_M$ and the ratio are actually characterized by the composite factors $g_i$. Though $G_1$ in eq.(3.15) is large, $|G_1|/m_Q$ remains small enough so that the $1/m_Q$ expansion in the new framework of HQEFT appears to be more reliable.

When taking the typical values for the quark masses $m_b = 4.8 \pm 0.10$GeV and $m_c = 1.35 \pm 0.10$GeV, we obtain from (3.18) and (3.19) the following values for the decay constants of bottom and charm meson without including QCD corrections caused by the running energy scale from $\mu \simeq m_b$ to $\mu \simeq 2\bar{\Lambda}$

$$f_B(2\bar{\Lambda}) = 0.135 \pm 0.035 \text{ GeV}, \quad f_B^*(2\bar{\Lambda}) = 0.147 \pm 0.034 \text{ GeV},$$
$$f_D(2\bar{\Lambda}) = 0.246 \pm 0.097 \text{ GeV}, \quad f_D^*(2\bar{\Lambda}) = 0.308 \pm 0.091 \text{ GeV}. \quad (3.20)$$
Note that all the quantities $\omega_M^{(0)}$, $\omega_0$, $F$, $\omega_M^{(i)}$, $\omega_i$, $G_i$ ($i = 1, 2$) and $g_i$ obtained by QCD sum rules are scale dependent. And the results in (3.18) and (3.19) are corresponding to the values with the renormalization scale at $\mu \simeq 2\bar{\Lambda} \approx 1\text{GeV}$. So do the decay constants in (3.20).

From the results given in (3.18) and (3.19), we arrive at the following relations from eq. (3.16)

\[
\frac{f_B\cdot m_B^{1/2}}{f_B m_B^{1/2}} \approx 1.10 \pm 0.03, \quad (3.21)
\]

\[
\frac{f_D\cdot m_D^{1/2}}{f_D m_D^{1/2}} \approx 1.31 \pm 0.07. \quad (3.22)
\]

These two ratios agrees well with the lattice calculations [10] which lead to 1.12 ± 0.05 and 1.34 ± 0.07, respectively.

The QCD corrections may be considered via the renormalization of the current $J = \bar{q}\Gamma Q$. In renormalization-group-improved perturbation theory, up to the next-to-leading order the values of the decay and coupling constants at energy scale $\mu = m_Q$ are given by

\[
f_M(m_Q) \approx \sqrt{\frac{\bar{\Lambda}}{m_M\Lambda_M}} F(m_Q)[1 + d_M\frac{\alpha_s(m_Q)}{6\pi}][1 + \frac{1}{m_Q}(G_1 + 2d_MG_2)]
\]

\[
= \frac{F(m_Q)}{\sqrt{m_M}}[1 + d_M\frac{\alpha_s(m_Q)}{6\pi}][1 + \frac{1}{m_Q}(g_1 + 2d_Mg_2)], \quad (3.23)
\]

\[
F(m_Q) \approx \left[\frac{\alpha_s(2\bar{\Lambda})}{\alpha_s(m_Q)}\right]^{6/\beta} \left(1 - 0.894\frac{\alpha_s(m_Q) - \alpha_s(2\bar{\Lambda})}{\pi} - \frac{\alpha_s(m_Q)}{2\pi}\right) F(2\bar{\Lambda}) \quad (3.24)
\]

with $\beta = 33 - 2n_F$, $M = B, D, B^*, D^*$ and $m_Q = m_b, m_c$. From the results of $f_M(2\bar{\Lambda})$ given in (3.20), we then have

\[
f_B(m_b) = 0.159 \pm 0.042 \text{ GeV}, \quad f_{B^*}(m_b) = 0.166 \pm 0.038 \text{ GeV},
\]

\[
f_D(m_c) = 0.251 \pm 0.099 \text{ GeV}, \quad f_{D^*}(m_c) = 0.293 \pm 0.086 \text{ GeV}. \quad (3.25)
\]

which is consistent with the experimental upper limit $f_B(m_B) < 200 \text{ MeV}$ and $f_D(m_D) < 290\text{ MeV}$ and also some theoretical upper bounds [12]. The results also agree with the lattice calculations [13–17] and with full QCD calculations [18].

The ratio in eq. (2.14) is now modified to be

\[
\frac{f_V m_V^{1/2}}{f_P m_P^{1/2}} = \frac{(\bar{\Lambda}_P/\bar{\Lambda}_V)^{1/2}(1 - \frac{2\alpha_s(m_Q)}{3\pi})(1 - \frac{8}{m_Q}G_2)}{(1 - \frac{2\alpha_s(m_Q)}{3\pi})(1 - \frac{8}{m_Q}g_2)}, \quad (3.26)
\]

which yields

\[
\frac{f_B\cdot m_B^{1/2}}{f_B m_B^{1/2}} \approx 1.05 \pm 0.03, \quad (3.27)
\]

\[
\frac{f_D\cdot m_D^{1/2}}{f_D m_D^{1/2}} \approx 1.22 \pm 0.07. \quad (3.28)
\]
IV. QCD SUM RULE EVALUATION ON WAVE FUNCTIONS $\xi$ AND $\kappa_1$

It is easily seen that to the leading order of heavy quark expansion, $m_Q \to \infty$, there is no difference between the HQEFT and the usual HQET, thus the procedure of calculating Isgur-Wise function $\xi(y)$ is the same as in the usual HQET. The main task in this section is to evaluate the two additional wave functions $\kappa_1(y)$ and $\kappa_2(y)$ involved at $1/m_Q$ order. For completeness, we also briefly outline the calculation of the Isgur-Wise functions $\xi(y)$ in HQEFT.

To evaluate the wave functions, we need to consider three point Green functions of the relevant operators. For the Isgur-Wise function $\xi(y)$, it relates the following three-point correlation function at leading order of $1/m_Q$

$$\Xi(\omega, \omega') = \int d^4x d^4y e^{i(P' \cdot x - P \cdot y)} <0|\bar{q}\Gamma^+ M |Q_j> <0|\bar{Q} \Gamma M q|0>$$

$$= \int d^4x d^4y e^{i(k' \cdot x - k \cdot y)} <0|\bar{q}\Gamma^+ M q|0> <0|\bar{Q} \Gamma M q|0> + O(1/m_Q),$$

(4.1)

where the Dirac structure $\Gamma$ of the heavy-heavy current can in principle be arbitrary. For the present case, we take $\Gamma = \gamma^\mu$. As $\Xi(\omega, \omega')$ is an analytic function in $\omega = 2v \cdot k + O(1/m_Q)$ and $\omega' = 2v' \cdot k' + O(1/m_Q)$ with discontinuities on the positive real axis, one can write the phenomenological representation as

$$\Xi_{\text{phen}} = -\sum_{\text{pole}} \frac{<0|\bar{q}\Gamma^+ M q|0>}{\Lambda_M \Lambda_M'(\omega_M - \omega - i\epsilon)(\omega'_M - \omega' - i\epsilon)} m_M m_{M'} + \int d\nu d\nu' \frac{\rho_{\text{phys}}(\nu, \nu')}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \text{subtractions.}$$

(4.2)

The first term in (4.2) is a double-pole contribution and the second represents the higher resonance contributions in the form of a double dispersion integral over physical intermediate states. Parameterizing the three matrix elements in (4.2) by using (2.3) and noticing the following relation

$$Tr[\Gamma_M M(v')] Tr[\hat{\mathcal{M}}(v') \Gamma M(v)] Tr[\hat{\mathcal{M}}(v) \Gamma_M] = 4\Lambda^2 Tr[\Gamma_M P_+ \Gamma P_+ \Gamma_M],$$

(4.3)

the double-pole term becomes

$$\Xi_{\text{pole}} = \frac{Tr[\Gamma_M P_+ \Gamma P_+ \Gamma_M]}{(\omega_M - \omega - i\epsilon)(\omega'_M - \omega' - i\epsilon)} \frac{\Lambda^2}{\Lambda_M \Lambda_M'} \frac{m_M m_{M'}}{m_Q^2} F^2 \xi.$$ (4.4)

Theoretically, the correlation function (4.1) may be written as

$$\Xi_{\text{theor}} = \int d\nu d\nu' \frac{\rho_{\text{pert}}(\nu, \nu')}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \Xi_{NP} + \text{subtractions}$$ (4.5)

which can be calculated perturbatively in deep Euclidean region $(\omega, \omega' \ll \Lambda)$. Note that there are two momentum variables for the correlator (4.1), a double Borel transformation $\hat{B}_\nu(\omega') \hat{B}_\nu(\omega)$ should be applied to both sides of the sum rule. Because of the heavy quark
symmetry, \( \omega \) and \( \omega' \) are symmetric in (4.1), and thus it is natural and convenient to choose \( \tau = \tau' = 2T \). It is useful to define \( \omega_{\pm} = \frac{1}{2}(\omega \pm \omega') \), and integrate first the spectral function over \( \omega_- \) at the region \(-\nu_+ < \nu_- < \nu_+\). Finally the quark-hadron duality allows us to write

\[
\tilde{\Xi}_{\text{phen}} = 2 \int_0^{\omega_0(y)} d\nu_+ e^{-\nu_+/T} \tilde{\rho}_{\text{pert}}(\nu_+) + \Xi_{NP};
\]

where \( \tilde{\Xi} \) denotes the result obtained by applying double Borel operators to \( \Xi \), and

\[
\tilde{\rho}_{\text{pert}}(\nu_+) = \int_{-\nu_+}^{\nu_+} d\nu_- \rho_{\text{pert}}(\nu_+, \nu_-).
\]

The one loop perturbative diagrams and lowest order nonperturbative diagrams proportional to quark condensate and mixed quark-gluon condensate are listed in Fig.2. In this section the gluon condensate can be safely neglected since its contribution is tiny. Calculation of those Feynman diagrams in Fig.2 gives

\[
F^2 \xi e^{-\omega_M/T} = \frac{3}{2\pi^2} \int_0^{\omega_0(y)} d\omega_+ e^{-\omega_+/T} \frac{\omega_+^2}{(y+1)^2}
- \langle \bar{q}q \rangle - i \langle \bar{q}\sigma_{\alpha\beta}F^{\alpha\beta}q \rangle > \frac{1+y}{8T^2}. \tag{4.8}
\]

The continuum threshold energy in (4.8) is in general a function of the recoil variable \( y \). One may employ different models for reasonable choice of this function. It is seen that if the \( 1/m_Q \) order terms and order \( \alpha_s \) terms in (3.13) are neglected, (4.8) reduces to (3.13) at the zero recoil point. This implies that we may use the same values of \( T \) and \( \omega_M \) as those in (3.13) for evaluating the wave functions \( \xi, \kappa_1 \) and \( \kappa_2 \). The values of \( T \) and \( \omega_M \) can be read from Fig.4, Fig.5 and Fig.6. Note that the threshold energy satisfies the normalization \( \omega_0(1) = \omega_0 \). Our numerical results are plotted in Fig.7, where we have used the values of \( \omega_0, \omega_M(0) \) and \( F \) obtained in the previous section. To be consistent, as the QCD radiative corrections are not considered in eq.(4.8), we have used the values given in eq.(3.18) where the results were obtained without QCD corrections. For comparison, we have used two simple models considered in [4]:

\[
\omega_0(y) = \omega_0(1) \begin{cases} 1, & \text{model 1;} \\ \frac{y+1}{2y}, & \text{model 2.} \end{cases} \tag{4.9}
\]

We now turn to the calculations for the wave functions \( \kappa_i(y) \) defined in (2.9). For that, one may consider the following three-point correlation function at \( 1/m_Q \) order:

\[
\mathcal{K}(\omega, \omega') = \int d^4x d^4y e^{i(P' \cdot x - P' \cdot y)} < 0|T\{ (\bar{q}\Gamma_{M'}\bar{Q})(x),
-\frac{1}{2m_Q} (\bar{Q}\Gamma_{iv} \cdot D) (iD_{\perp})^2 Q \}(0), (\bar{Q}\Gamma_{M}q)(y) \}|0 >
= \int d^4x d^4y e^{i(k' \cdot x - k \cdot y)} < 0|T\{ (\bar{q}\Gamma_{M'}Q'^{++}_{v})(x),
-\frac{1}{2m_Q} (Q'^{++}_{v}\Gamma_{iv} \cdot D) (iD_{\perp})^2 Q^{++}_{v} \}(0), (\bar{Q}\Gamma_{M}q)(y) \}|0 > + O(1/m_Q^2) \tag{4.10}
\]
Saturating this three-point Green function with hadron states, one gets the double-pole contribution

$$\mathcal{K}_{\text{pole}}(\omega, \omega') = -\left(\sum_{\text{pole}}\frac{m_M m_{M'}}{m_Q^2 A_M A_{M'}(\omega_M - \omega - i\epsilon)(\omega_M' - \omega' - i\epsilon)}\right) \times \langle 0|q\bar{\Gamma}_M Q^+_v|M_{\nu'}\rangle < M_{\nu'}\langle \bar{Q}^+_v\Gamma - \frac{1}{2m_Q} P^+_{\perp} (i\nabla_{\perp})^2 Q^+_v| M_{\nu} \rangle < M_{\nu} |\bar{Q}^+_v \Gamma q|0 \rangle . \quad (4.11)$$

The heavy-heavy current in (4.10) contains both spin-symmetry conserving operator

$$\frac{1}{2m_Q} \bar{Q}^+_v \Gamma - \frac{1}{i\nu D} (D_{\perp})^2 Q^+_v$$

and spin-symmetry violating operator

$$\frac{1}{2m_Q} \bar{Q}^+_v \Gamma - \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} Q^+_v .$$

The hadronic matrix elements of the latter are parameterized by the wave functions $\kappa_2(\nu)$ and $\kappa_3(\nu)$. In this note, we may consider only the Feynman diagrams shown in Fig.3, namely we will neglect radiative corrections as a first approximation. In such a treatment the resulting contributions from the spin-symmetry violating operator may proportional to the mixed quark-gluon condensate. Noticing that in the fixed point gauge

$$\langle 0| : \bar{q}(x) A_\mu(z) q(0) : |0 \rangle = \frac{1}{96} \sigma_{\mu\nu} \langle \bar{q} \sigma_{\alpha\beta} F^{\alpha\beta} q \rangle + \text{higher dimensional condensates to be neglected} , \quad (4.12)$$

it is readily seen from (2.12) that there would be no contributions to $\kappa_3(\nu)$ at the order we are considering. So in this approximation we have $\kappa_3(\nu) = 0$, namely $\kappa_3(\nu)$ only receives contributions from higher order and higher dimensional condensates which are expected to be small. For this reason, we may rewrite (4.11) as

$$\mathcal{K}_{\text{pole}}(\omega, \omega') = -\frac{1}{(\omega_M - \omega - i\epsilon)(\omega_M' - \omega' - i\epsilon)} \frac{\Lambda^2}{\Lambda_M \Lambda_{M'}} \frac{m_M m_{M'}}{m_Q^2} F^2 \frac{1}{2m_Q \Lambda} (\kappa_1 + d_M \kappa_2) , \quad (4.13)$$

where we have used (2.9) and (4.3) as well as the formula $P_\perp \sigma_{\mu\nu} \mathcal{M}(\nu) \sigma^{\mu\nu} = 2d_M \mathcal{M}(\nu)$. In evaluating the three-point Green function of (4.10), one meets a non-local operator, to be convenient of calculating $\mathcal{K}$, one may choose the axial gauge $\nu \cdot A = 0$ . Note that in this gauge the diagrams with gluons attached to a heavy quark line are absent.

Adopting the same strategy for evaluating the Isgur-Wise function $\xi(\nu)$, we arrive at the following double Borel transformed sum rule

$$-F^2 = \frac{1}{2m_Q \Lambda} (\kappa_1 + d_M \kappa_2) e^{-\omega_M/T} = \frac{1}{2\pi^2} \int_0^{\omega_c} d\omega_+ e^{-\omega_+/T} \frac{(2y + 1) \omega_+^3}{m_Q (y + 1)^3}$$

$$+ i \langle \bar{q} \sigma_{\alpha\beta} F^{\alpha\beta} q \rangle \left( \frac{3}{16m_Q T} - \frac{d_M}{24m_Q T} \right) . \quad (4.14)$$

By separately considering the spin-conserving and spin-breaking corrections to the limit case $m_Q \to \infty$ (or equivalently considering those terms with and without $d_M$), and taking the central values in (3.18), we obtain numerical results which are plotted in Fig.8. The curves in Fig.8 are corresponding to the results at $T = 1.0$GeV which is chosen by considering the stability region of the curves exhibited in Fig.4. We also present in Fig.9 and Fig.10 the values of $\kappa_1(1)$ and $\kappa_2(1)$ as functions of the Borel parameter $T$. It is seen from those two figures that $\kappa_1(1)$ and $\kappa_2(1)$ are really stable at the region around $T = 1.0$GeV. The stable
regions in Fig. 9 and Fig. 10 are consistent each other and also consistent with the ones in Fig.4-Fig.6. With these considerations and analyses, we may present the final result for \( \kappa_1(1) \) as

\[
\kappa_1 \equiv \kappa_1(1) = -0.50 \pm 0.18 \text{GeV}^2
\]  

which agrees with the one extracted from the heavy meson masses [2], where \( \kappa_1(1) \) could range from \(-0.8 \text{GeV}^2\) to \(-0.25 \text{GeV}^2\) with a favorable value \( \kappa_1(1) \approx -0.61 \text{GeV}^2 \).

In our previous papers [2-4], we have argued that \( <iv \cdot D> \) is of order the binding energy \( \bar{\Lambda} \). For simplifying the analyses, we have actually made a heavy quark expansion at point \( <iv \cdot D> = \bar{\Lambda} \) for inclusive decays of heavy hadrons [3,4]. To check the validity of this approximation, we may replace the non-local operator \( iv \cdot D \) in (2.9) with \( \bar{\Lambda} \) and evaluate the resulting local matrix element, then a comparison between two results should allow one to test the goodness of the approximation. By doing this, we may reparameterize the matrix elements as

\[
< M'_{v'} | \bar{Q}^\alpha_P + v' \Gamma P + \bar{\Lambda} (iD / \perp)_Q + v | M_v > = - K_1(y) \frac{1}{\Lambda} Tr[\bar{M}\bar{\Gamma}M],
\]

\[
< M'_{v'} | \bar{Q}^\alpha_P + v' \Gamma P + \bar{\Lambda} (iD / \perp)_Q + v | M_v > = - K_1(y) \frac{1}{\Lambda} Tr[\bar{M}\bar{\Gamma}M],
\]

where \( K_{\alpha\beta} \) may be decomposed in a similar way as \( \kappa_{\alpha\beta} \). Applying the sum rule approach once more to evaluate the parameter \( K_1 \) and following the same strategy as before, we yield the following sum rule formula for \( K_1 \),

\[
\frac{1}{2} F^2 K_1 e^{-\omega_M/T} = - \frac{1}{8\pi^2} \int_0^{\omega} e^{-\omega_+ / T} \frac{1 + 2y}{(1 + y)^3} \omega_+^4 - i < \bar{q} \sigma_{\alpha\beta} F^{\alpha\beta} q > \frac{3}{16},
\]

where the corresponding Feynman diagrams are the same as those in Fig.3 with the box now representing the new operator \(- \frac{1}{2m_Q} \bar{Q}^\alpha_P \Gamma P + (iD / \perp)^2 \bar{Q}^\alpha_P \). The numerical results of (4.17) are shown in Fig.11 and Fig.12, it is found that

\[
K_1(1) \approx -0.40 \text{GeV}^2
\]  

which is slightly lower than \( \kappa_1(1) \). Comparing the two numerical results of \( K_1(1) \) and \( \kappa_1(1) \), we see from (2.9) and (4.16) that the simple replacement

\[
iv \cdot D \sim v \cdot k \approx \bar{\Lambda}
\]  

is actually a reliable approximation for the operator \( iv \cdot D \).

The above demonstration supports the analyses in [2,4]. We are confirmed to believe that the HQEFT is more reliable to describe the off-mass shell heavy quark within heavy hadrons and can provide a consistent understanding on both exclusive and inclusion decays of heavy hadrons.

We may return to comment on the wave function \( \kappa_2(y) \). Up to the order considered above, the sum rule formula (4.14) leads to the value \( \kappa_2(1) \approx 0.015 \text{GeV}^2 \) which is much smaller than the one extracted from the meson masses [3] (where \( \kappa_2(1) \approx 0.056 \text{GeV}^2 \)). One of possible reasons for such a discrepancy may be seen from (4.14) in which only one mixed condensate term contributes to \( \kappa_2 \). That unique term is relevant to the spin-symmetry
breaking term and arises from the diagram Fig.3(c). As the operator parameterized by \( \kappa_2 \) in (2.9) contains gluon fields, the lowest order perturbative contributions should arise from the two-loop diagram in Fig.11. In order to improve the determination for \( \kappa_2 \), one should therefore calculate at least two-loop perturbative diagrams as well as nonperturbative diagrams up to order \( \alpha_s \) in a consistent way. Since all these diagrams are at least at the order of \( \alpha_s \), the more precise value of \( \kappa_2 \) should not be too large. We should not present calculations of the additional diagrams for \( \kappa_2 \) in this paper.

In fact there is another way to estimate the values of \( \kappa_1(1) \) and \( \kappa_2(1) \) directly from the results obtained through two-point correlator functions in Sec.III. This is again because the remarkable relation between the heavy-light hadron mass and wave functions at zero recoil resulted from the complete HQEFT. It can be seen from Eqs.(2.15) and (3.14) that the form factors \( \kappa_1(1) \) and \( \kappa_2(1) \) are simply given by

\[
\kappa_1(1) = -\frac{\omega_1^{(1)}}{2}, \quad \kappa_2(1) = -\frac{\omega_2^{(1)}}{2}.
\] (4.20)

With these relations, we obtain from the values of \( \omega_M^{(i)} \) in eq.(3.19)

\[
\kappa_1(1) \approx -0.43\text{GeV}^2, \quad \kappa_2(1) \approx 0.08\text{GeV}^2,
\] (4.21)

which are consistent with the results yielded above (eq.(4.15)) and also those obtained in Ref. [2] for \( \kappa_1(1) \) and \( \kappa_2(1) \).

V. CORRECTIONS FROM TWO-LOOP PERTURBATIVE QCD

In order to take a look at the magnitude of the effects of QCD radiative corrections, as in Ref. [8], one can now include the two-loop perturbative contributions. Their effects can be simply taken into account by replacing the perturbative contributions in the sum rule (3.11) with the following ones

\[
\Pi_{\text{pert}}(\omega) = \frac{3}{8\pi^2} \int_0^{\omega_c} d\nu \left( \frac{1}{4m_Q} + \frac{1}{\nu - \omega - i\epsilon} \right) \nu^2 \left( 1 + \frac{2\alpha_s}{\pi} \ln \frac{2\Lambda}{\nu} + \frac{13}{6} + \frac{2\pi^2}{9} - \frac{3\nu^4}{4m_Q} \right). \] (5.1)

In comparison with the results from leading QCD corrections, we have plotted the modified results with two-loop QCD corrections in Figs. (14-22). As a consequence, instead of eqs.(3.18) and (3.19), we arrive at the following modified results

\[
\omega_0 = 1.8 \pm 0.3 \text{ GeV}, \\
\frac{\omega_M^{(0)}}{2} = \overline{\Lambda} = 0.56 \pm 0.08 \text{ GeV}, \\
F = 0.38 \pm 0.06 \text{ GeV}^{3/2}.
\] (5.2)

and

\[
\omega_1 = 1.0 \pm 0.2 \text{ GeV}^2, \\
\omega_M^{(1)} = 0.65 \pm 0.10 \text{ GeV}^2,
\]
\begin{align*}
G_1 &= 0.75 \pm 0.15 \text{ GeV}, \\
g_1 &= 0.46 \pm 0.12 \text{ GeV}, \\
\omega_2 &= -0.15 \pm 0.05 \text{ GeV}^2, \\
\omega_2^{(2)} &= -0.14 \pm 0.03 \text{ GeV}^2, \\
G_2 &= -0.09 \pm 0.03 \text{ GeV}, \\
g_2 &= -0.06 \pm 0.02 \text{ GeV}. \\
\end{align*}

Correspondingly, instead of eq. (5.25), we have
\begin{align*}
f_B(m_b) &= 0.196 \pm 0.044 \text{ GeV}, & f_B^*(m_b) &= 0.206 \pm 0.039 \text{ GeV}, \\
f_D(m_c) &= 0.298 \pm 0.109 \text{ GeV}, & f_D^*(m_c) &= 0.354 \pm 0.090 \text{ GeV}. \\
\end{align*}

Comparing these values with those obtained in Sec. III, we see that the QCD radiative corrections may enlarge \( F \) by about 25\%. It implies that the radiative corrections may be significant for a more accurate determination for some physical quantities. The large values seems to be consistent with the ones from the recent calculations by Lattice QCD approach [15–17].

With those values in eq. (5.2), we yield from the sum rules in eqs. (4.14) and (4.17)
\begin{align*}
\kappa_1(1) &= -0.34 \text{ GeV}^2; \\
K_1(1) &= -0.26 \text{ GeV}^2. \\
\end{align*}

which are lower than the ones without two-loop perturbative QCD corrections. While we would like to point out that the modified results for \( \kappa_1(1) \) and \( K_1(1) \) (eqs. (5.5) and (5.6)) may not be regarded to be more reliable than the ones given in eqs. (4.15) and (4.18) since eqs. (4.14) and (4.17) contain only the leading order contributions in perturbation theory. For a complete and consistent evaluation, one should also include the next-to-leading order corrections to eqs. (4.14) and (4.17). But comparing the two values in eqs. (5.5) and (5.6) we see that the approximation \( i v \cdot D \sim v \cdot k \simeq \Lambda \) still holds, though both the values of \( \kappa_1(1) \) and \( K_1(1) \) are now smaller than those presented in Sec. IV because the input parameter \( F \) is enlarged by the two-loop perturbative corrections.

It is also interesting to notice that the relation \( \kappa_1(1) = -\omega_M^{(1)}/2 \simeq -0.37 \text{ GeV}^2 \) is still satisfied well by looking at the result given in eq. (5.3) and the value of \( \omega_M^{(1)} \) in eq. (7.3). In conclusion, all the numerical results in this paper turn out to be consistent.

VI. CONCLUSIONS AND REMARKS

We have present, within the complete HQEFT, a consistent evaluation for the decay constants and wave functions of heavy-light hadron systems up to order of \( 1/m_Q \). It has been seen that the QCD Lagrangian for heavy quarks do neet to be transformed into a new heavy quark effective Lagrangian with including contributions of both particle fields and antiparticle fields, and the currents containing heavy quark can be consistently expanded in powers of \( 1/m_Q \). Though the leading operator in the \( 1/m_Q \) expansion is (must be) the same as the one in the usual HQET, the operators at order \( 1/m_Q \) begin to be different from those in the usual HQET.
In the complete HQEFT, corrections arising from both the current expansion and the insertion of Lagrangian into the heavy-light meson anihilation matrix elements are characterized by two factors $G_1$ and $G_2$. Similarly, corrections arising from both sources to the heavy meson transition matrix elements are characterized by three functions $\kappa_1(y)$, $\kappa_2(y)$ and $\kappa_3(y)$. Furthermore, the values of $\kappa_1$ and $\kappa_2$ at the zero recoil point also characterize the $1/m_Q$ order corrections to the meson masses. These makes the HQEFT be much elegant than the usual HQET.

The leading order contributions and the $1/m_Q$ corrections to heavy meson decay constants and heavy meson transition matrix elements have been investigated consistently by using QCD sum rule approach within the framework of HQEFT of QCD. For heavy-light meson decays, we have calculated the form factor $F$ at leading order ($m_Q \to \infty$), and the form factors $G_1$ and $G_2$ concerned at the $1/m_Q$ order as well as the binding energy $\bar{\Lambda}$. Particularly, we have found that the $1/m_Q$ corrections to the heavy-light meson decay constants are actually determined by two composite form factors $g_1$ and $g_2$. These two composited form factors were found to be much smaller than the heavy quark masses, which implies that the scaling law of the decay constants are only slightly breakdown. This observation shows that the $1/m_Q$ expansion in the complete HQEFT works well, which is unlike the usual HQET which may lead to, as shown in [8], the breakdown of the $1/m_Q$ expansion in evaluating the decay constants. Our results for the heavy-light meson decay constants have also shown a good agreement with the known experimental results and upper limits.

We have also calculated the Isgur-Wise function and $1/m_Q$ order spin-symmetry conserving form factor $\kappa_1$ as functions of the recoil value. It have been found that $\kappa_1(1) \approx -0.50 \pm 0.18 \text{GeV}^2$, which agrees with the value extracted from the interesting relations between meson mass and wave functions at zero recoil. We have also illustrated how the simple replacement $iv \cdot D \sim v \cdot k \approx \bar{\Lambda}$ holds. This shows that the residual momentum of heavy quark within heavy-light hadron does be around the binding energy which had been seen [3] to be the main point to understand the puzzle of the bottom hadron lifetime differences.

The spin symmetry breaking factor $\kappa_2$ has been discussed. The only diagram at leading order yields a $\kappa_2$ value which is much smaller than that obtained in [3]. Thought its value has not yet been evaluated accuratly, it implies that $\kappa_2$ must be small as it characterizes the spin symmetry breaking effects of heavy-light mesons. A further calculation of $\kappa_2$ including higher order contributions remains an interesting subject. $\kappa_3(y)$ was also found to be small since it receives contributions only from higher order radiative diagrams and higher dimensional condensates.

An interesting feature resulting from the HQEFT is that the values of $\kappa_1(1)$ and $\kappa_2(1)$ can also be simply obtained, due to the interesting relation between heavy-light meson mass and wave functions at zero recoil, from two-point Green’s function in evaluating the heavy-light meson decay constants via sum rule approach. It is remarkable that the resulting values of $\kappa_1(1)$ and $\kappa_2(1)$ in this way do agree with the results obtained from the analysis of the three-point Green’s function. Finally, we have shown that the higher order radiative corrections could be nontrival for a more accurate calculation of heavy-light meson decays. But the relations $iv \cdot D \sim iv \cdot k \approx \bar{\Lambda}$ and $\kappa_1(1) = -\omega^{(1)}_M/2$, $\kappa_2(1) = -\omega^{(2)}_M/2$ hold even when higher radiative corrections are included.

In summary, the complete HQEFT works well for describing the slightly off-mass shell heavy quark within heavy hadrons. In this paper, we have further checked the consistent
of the HQEFT in applying to the heavy-light hadron systems with $m_Q >> \bar{\Lambda} >> m_q$. For heavy-heavy bound state systems, such as bottom-charm system like $B_c$, and muonium ($\mu e$) system in QED, one needs to make $1/M$ expansion for both heavy particles and to redefine the effective fields and bound states when applying for the complete heavy particle effective field theory with keeping the antiparticle contributions [1].

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Fig.1. Feynman diagrams contributing to heavy meson decay. The thick lines are heavy quarks; the light lines are light quarks; the curves are gluon fields; the black dots represent condensates; and the external dashed lines are the heavy-light currents considered in (3.1).
Fig. 2. The lowest order Feynman diagrams contributing to $\xi$. The wave lines represent the heavy-heavy current $\bar{Q}_i' \Gamma Q_j^+$ in (4.1).

Fig. 3. The lowest order Feynman diagrams contributing to $\kappa_1$ and $\kappa_2$. The box at the up of each diagram represent the $1/m_Q$ order heavy-heavy current $-\frac{1}{2m_Q} \bar{Q}_i' \Gamma \frac{P_\perp}{m_Q} (i\partial_\perp)^2 Q_j^+$ in (4.10).
Fig. 4. Leading order sum rule result for heavy meson decay.

Fig. 5a
Fig. 5 Sum rule result for $1/m_Q$ order spin-symmetry conserving corrections to heavy meson decay.

Fig. 5b

Fig. 5c
Fig. 6c

Fig. 6. Sum rule result for $1/m_Q$ order spin-symmetry breaking corrections to heavy meson decay.

Fig. 7. Sum rule result for Isgur-Wise function $\xi(y)$. 
Fig. 8. Sum rule result for $1/m_Q$ order transition form factor $\kappa_1$ and $\kappa_2$.

Fig. 9. $\kappa_1(1)$ as a function of Borel parameter $T$. 
Fig. 10. $\kappa_2(1)$ as a function of Borel parameter $T$.

Fig. 11. Sum rule result for $K_1$. 
Fig. 12. $K_1(1)$ as a function of Borel parameter $T$.

Fig. 13. The lowest order perturbative diagram contributing to $\kappa_2$. 
Fig. 14. Leading order sum rule result for heavy meson decay when two-loop perturbative contributions are considered.

Fig. 15a
Fig. 5 Sum rule result for $1/m_Q$ order spin-symmetry conserving corrections to heavy meson decay when two-loop perturbative contributions are considered.
Fig. 16a

Fig. 16b
Fig. 6. Sum rule result for $1/m_Q$ order spin-symmetry breaking corrections to heavy meson decay when two-loop perturbative contributions are considered.

Fig. 16c

Fig. 17. Sum rule result for Isgur-Wise function $\xi(y)$ when the value of $F$ in eq.(5.2) is used as an input parameter.
Fig.18. Sum rule result for $1/m_Q$ order transition form factor $\kappa_1$ and $\kappa_2$ when the value of $F$ in eq.(5.2) is used as an input parameter.

Fig.19. $\kappa_1(1)$ as a function of Borel parameter $T$ when the value of $F$ in eq.(5.2) is used as an input parameter.
Fig. 20. $\kappa_2(1)$ as a function of Borel parameter $T$ when the value of $F$ in eq. (5.2) is used as an input parameter.

Fig. 21. Sum rule result for $K_1$ when the value of $F$ in eq. (5.2) is used as an input parameter.
Fig.22. $K_1(1)$ as a function of Borel parameter $T$ when the value of $F$ in eq.(5.2) is used as an input parameter.