Modeling and Performance Analysis of an Axial-Radial Combined Permanent Magnet Eddy Current Coupler

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\textbf{ABSTRACT} The structure and magnetic circuits of axial-radial combined permanent magnet eddy current couplers are more complicated than those of axial or radial permanent magnet eddy current couplers, the degree of coupling is deeper, which will affect the device performance. In this paper, a novel analytical model for an axial-radial combined permanent magnet eddy current coupler is proposed based on the equivalent magnetic circuit method. The distribution of the magnetic field, the relationship between the slip and torque, the influence of the structural parameters on the transmission performance, the 3-D correction and thermal compensation are studied. Combined with the concept of the micro-element method, the air gap magnetic density, eddy current density and torque are accurately calculated, which solves the problem that the existing model cannot analyse the axial-radial combined structure. To show the effectiveness of the proposed analytical model, the transmission performance of an axial-radial combined permanent magnet eddy current coupler prototype is investigated, and the analytical calculation results are compared with the results of the finite element method simulations and experimental tests.

\textbf{INDEX TERMS} Axial-radial combined permanent magnet eddy current coupler, equivalent magnetic circuit model, modelling and performance analysis.

\section{INTRODUCTION}

In recent years, permanent magnet eddy current couplers have been adopted in many industrial fields, such as the mining and metallurgical industry, oil and gas industry, chemical industry, and military manufacturing field [1]–[3]. With respect to the traditional transmission devices, the permanent magnet eddy current couplers transmit torque through a magnetic coupling force without rigid contact and mechanical connection [4]. Permanent magnet eddy current couplers are purely mechanical structures, without an electric drive, so there is no harmonic and other pollution to the power grid [5]. Other significant advantages of permanent magnet eddy current couplers are isolated protection, no ripple torque, and excellent tolerance to shaft misalignment [6]–[8].

As a novel transmission device, the axial-radial combined permanent magnet eddy current coupler has a unique double rotor structure [9], [10]. It mainly consists of a conductor rotor, a permanent magnet rotor and an adjustment device; the exploded view of the mechanical structure is shown in Fig. 1. The motor shaft and load shaft are connected to the conductor rotor and permanent magnet rotor, respectively [11]. The conductor rotor rotates together with the drive motor and cuts the magnetic line produced by the permanent...
magnet rotor [12]. An alternating eddy current field is generated in the conductor rotor and excites the induced magnetic field according to Faraday’s law [13]. The torque is calculated by superposing the induced magnetic field with the excitation magnetic field. It is transmitted from the power side to the load side [14], [15]. As shown in Fig. 2, the effective contact area between the magnetic field and the rotor can be adjusted by changing the thickness of the air gap to obtain a controllable and adjustable load rotation speed [16].

To study the performance of axial-radial combined permanent magnet eddy current couplers, two different methodologies are used: numerical methods and analytical methods [17]. The finite element method is a representative numerical method that can comprehensively consider the nonlinearity and magnetic flux leakage of the permanent magnet materials. Although the simulation results of the finite element method are extremely accurate, it is less flexible and quite demanding in terms of simulation time and computer hardware [18]. The parameters of an axial permanent magnet eddy current coupler were designed in [19]. The 3-D finite element method was used to confirm the rationality and effectiveness of the parameter design, and the torque transfer performance was also studied. To avoid the disadvantages of the finite element method, [20] proposed a mixed 2-D analytical modelling approach for a flux-concentration disk-type permanent magnet eddy current coupler with a double conductor rotor. And the 3-D finite element method was employed to validate the analytical results. Compared with numerical methods, the main advantage of analytical methods is represented by the rapidity and precision of the results, which is more appropriate for use in the design stage [5], [7], [18]. In [21], a 3-D steady-state analytical model was established for an axial-flux permanent magnet eddy current coupler, and the closed forms of the torque and axial force were obtained. In particular, the Maxwell equations are solved by establishing the magnetic scalar potential equations in the permanent magnets and air gap, and the magnetic field strength equations in the conductor rotor. The equivalent magnetic circuit method is the representative of the analytical method, which is based on the similarity between magnetic circuits and electric circuits. The abstract and complex magnetic field calculation problems can be transformed into specific circuit analytical problems according to Ampere’s Law and Kirchhoff’s law of magnetic circuit [22]. The equivalent magnetic circuit method can provide solutions quickly and accurately, which has been widely used in the design and optimization of electromagnetic equipment such as PM machines. Based on the equivalent circuit model, a new analytical model of an axial-flux eddy current device is proposed in literature [23], and the 3-D analytical computation method is employed to obtain the field solutions. Based on the analytical method, a new equivalent magnetic circuit model is proposed in literature [24] to calculate the parameter performance of the permanent magnet eddy current coupler. In the proposed equivalent magnetic circuit model, the inherent eddy current effect of the permanent magnet eddy current coupler is considered by introducing a branch magnetic circuit, which takes into account the magnetomotive force and the reaction magnetic flux. A complete model is given by treating the reaction flux as the magnetic flux leakage.

The axial-radial combined permanent magnet eddy current coupler has a special rotor structure. Therefore, the structure and magnetic circuits are more complicated than those of axial or radial permanent magnet eddy current couplers. And the degree of coupling is deeper, which will affect the device performance. Most of the existing research has studied a permanent magnet eddy current coupler with a single structure, and there are few studies on the influence of equipment parameters on the performance. The main contribution of this paper is to present a new analytical model of an axial-radial combined permanent magnet eddy current coupler based on the equivalent magnetic circuit model. First, the axial and radial equivalent magnetic circuit models are established, and the air gap magnetic density model, eddy current loss and torque model are studied. Although the conductor is implemented by a solid conductor, the approach allows one to determine the equivalent reluctance as a function of the design parameters. Then, the coupled magnetic circuit model is studied, the 3-D correction and thermal compensation are applied. Finally, the transmission performance of an axial-radial combined permanent magnet eddy current coupler prototype is investigated to show the effectiveness of the proposed analytical model. And the analytical calculation results are compared with the finite element method simulations and experimental tests.

II. EQUIVALENT MAGNETIC CIRCUIT MODELLING AND MODELLING ASSUMPTION

A. EQUIVALENT MAGNETIC CIRCUIT MODELLING

The axial-radial combined permanent magnet eddy current coupler has both an axial magnetic circuit and a radial magnetic circuit. To analyse the magnetic circuit structure, the axial and radial magnetic circuits are simplified as shown in Fig. 3.

B. MODELLING ASSUMPTION

During the operation of the axial-radial combined permanent magnet eddy current coupler, the axial main magnetic flux plays a major role in the main magnetic flux. Therefore, the axial torque transmission ability of the coupler is stronger,
but the change of the axial air gap will cause a rapid change of the transfer rate. The ratio of the radial main magnetic flux to the main magnetic flux is small, but with the change of the axial air gap, the torque transmitted by radial structure is stable. Considering the complexity of eddy current distribution in conductor rotor, the following assumptions are put forward in the calculation of eddy current loss and transfer torque by equivalent magnetic circuit method:

1) The magnetic flux leakage of permanent magnet is ignored when the air gap is very small.
2) The magnetic field of permanent magnet is evenly distributed in the air gap.
3) The effective flux area of the conductor rotor cut by the permanent magnet is taken as the calculated area of each pole of the conductor rotor.
4) The permeability and conductivity of the conductor rotor are constant.
5) Ignore the saturation of the cover and flange.

III. EQUIVALENT MAGNETIC CIRCUIT MODELLING OF AN AXIAL-RADIAL COMBINED PERMANENT MAGNET EDDY CURRENT COUPLER

A. AXIAL MAGNETIC CIRCUIT MODEL

Based on the symmetry of the axial magnetic field, the axial structure of the axial-radial combined permanent magnet eddy current coupler can be converted into a 2-D axial structure for analysis, which is shown in Fig. 4. It is assumed that \( t_{s1} = t_{s2} \) for the convenience of calculation.

1) AIR GAP MAGNETIC DENSITY MODEL

The equivalent magnetic circuit shown in Fig. 5 can be obtained from the 2-D axial model shown in Fig. 4 based on the equivalent magnetic circuit method to simplify the magnetic circuit analysis. It is possible to calculate the magnetic flux density on the air gap and the conductor rotor while the eddy current effect and the magnetic field coupling are neglected.

According to the reluctance and the equivalent magnetomotive force formula, the parameters of the equivalent magnetic circuit shown in Fig. 5 can be calculated as

\[
R_m = \frac{t_p}{\mu_0 \mu_r \omega_1 h_p} \\
R_g = \frac{(t_s + t_c)}{\mu_0 \omega_1 h_p} 
\]

where \( R_m \), \( \mu_r \), and \( h_p \) are the reluctance, the relative permeability, and the radial length of the permanent magnet, respectively. \( R_g \) is the total reluctance of the air gap and the conductor rotor. \( \mu_0 \) is the vacuum permeability.

To improve the accuracy of modelling, the reluctance of the back iron is divided into two parts based on different regions of the adjacent permanent magnets or the air gap which corresponds to \( R_{s1} \) and \( R_{s2} \) shown in Fig. 6, respectively

\[
R_{s1} = \frac{\omega_2}{\mu_0 \mu_r h_p t_s} \\
R_{s2} = \frac{\omega_1}{\mu_0 \mu_s (h_p t_{s1} + \omega_1 h_p)}
\]

where \( \mu_s \) is the relative permeability of the back iron.

From (3) and (4), the total reluctance of the back iron is

\[
R_s = 2\omega_2 (h_p h_s + \omega_1 h_p) + 4\omega_1 h_p t_s/\mu_0 \mu_s h_p t_{s1} (h_p h_s + \omega_1 h_p)
\]

The reluctance \( R_1 \) corresponding to the magnetic flux leakage of the permanent magnet through the back iron can be obtained by the leakage permeance as (6). For the convenience of calculation, it is assumed that the direction of the magnetic flux leakage is perpendicular to the permanent magnet, and the depth of the magnetic flux leakage through
the back iron is \( t_{s1}/2 \).

\[
P_1 = \mu_0 h_p \ln \left( 1 + \frac{t_{s1}}{2(t_p + t_g)} \right)
\]  

(6)

Similarly, the permeance corresponding to the magnetic flux leakage from the permanent magnet to the adjacent permanent magnet can be obtained as

\[
P_{mm} = \mu_0 h_p \ln \left( 1 + \frac{t_g}{\omega_2} \right)
\]  

(7)

Based on the relationship between the permeance and the reluctance, the reluctance of the two parts of the magnetic circuit of (6) and (7) can be obtained as

\[
R_1 = \ln \left( 1 + \frac{t_{s1}}{2(t_p + t_g)} \right) / \mu_0 h_p
\]  

(8)

\[
R_{mm} = \ln \left( 1 + \frac{t_g}{\omega_2} \right) / \mu_0 h_p
\]  

(9)

According to Kirchhoff’s law, the air gap flux without considering the eddy current effect and magnet field coupling can be expressed as

\[
\Phi_{ga1} = \Phi_{ra1}pC / \left\{ t_pC + \ln a \ast \left( \frac{1}{2} b \left[ c(t_c + t_g) + \omega_1(\omega_2 t_{s1} + \omega_1 \omega_2 + 2 \omega_1 t_{s1}) \right] \right) \right\}
\]  

(10)

where the coefficients \( a, b, \) and \( c \) are given by

\[
a = 2 + [\omega_2 t_{s1} + 2 t_g(t_p + t_g)] / [2 \omega_2(t_p + t_g)]
\]

\[
b = \mu_0 h_p^2 \mu_r
\]

\[
c = 4 t_{s1} \mu_s (t_{s1} + \omega_1)
\]  

(11)

The air gap magnetic density is divided into two categories when the eddy current effect is not considered: one is the axial air gap magnetic density in the gap of the permanent magnets whose value is approximately zero; the other is the air gap magnetic density of the permanent magnet magnetic density shown in (12)

\[
B_{ga1}' = \Phi_{ra1}pC (h_p \omega_1)^{-1} / \left\{ t_pC + \ln a \ast \left( \frac{1}{2} b \left[ c(t_c + t_g) + \omega_1(\omega_2 t_{s1} + \omega_1 \omega_2 + 2 \omega_1 t_{s1}) \right] \right) \right\}
\]  

(12)

According to Faraday’s law, the conductor rotor will induce the eddy current due to the alternating magnetic field and excite the induced magnetic field. Therefore, the air gap magnetic density considering eddy current effect should be composed of (12) and the air gap magnetic density of eddy current induced magnetic field \( B_{cal1} \). Combining Ampere’s law, the relationship is given by (13).

\[
\oint_c H dl = \int_{r_1}^{r_2} \int_0^{2\pi} r_c \delta B_{gal1} \omega dy dx
\]  

(13)

where \( r_c \) and \( \delta \) are the average radius and the conductivity of the axial conductor rotor, \( \omega \) is the relative speed of two rotors. And the differential equation can be obtained as

\[
\frac{dB_{cal1}}{dx} = \frac{\mu_0 \delta \omega t_c}{2(t_c + t_m + t_g)} B_{cal1} = \frac{\mu_0 \delta \omega t_c}{2(t_c + t_m + t_g)} B_{gal1}
\]  

(14)

To simplify the calculation, only the conductor rotor facing the permanent magnet is considered, and the part between the permanent magnets is ignored. The magnetic density generated by the induced current can be obtained in the light of the boundary conditions as

\[
B_{cal1} = \frac{\Phi_{gal1}}{h_p \omega_1} \left[ \frac{\cosh \left( \frac{\mu_0 \delta \omega t_c}{4(t_c + t_m + t_g)} \right)}{\cosh \left( \frac{\mu_0 \delta \omega t_c}{4(t_c + t_m + t_g)} \right)} \right] e^{\frac{\mu_0 \delta \omega t_c}{4(t_c + t_m + t_g)}} - B_{ga1}'
\]  

(15)

Then the axial magnetic density of the axial-radial combined permanent magnet eddy current coupler can be expressed as

\[
B_{gal1} = \frac{\Phi_{gal1}}{h_p \omega_1} \left[ \frac{\cosh \left( \frac{\mu_0 \delta \omega t_c}{4(t_c + t_m + t_g)} \right)}{\cosh \left( \frac{\mu_0 \delta \omega t_c}{4(t_c + t_m + t_g)} \right)} \right] e^{\frac{\mu_0 \delta \omega t_c}{4(t_c + t_m + t_g)}}
\]  

(16)

2) EDDY CURRENT LOSS AND TORQUE MODEL

The conductor rotor will cut the magnetic lines of the permanent magnet rotor and generate eddy current on the surface layer when it is driven by the prime mover. The eddy current is alternating, the eddy current loss is also located on the conductor rotor. Therefore, the eddy current loss could be analysed from the conductor rotor as shown in Fig. 7.

Taking one element \( dS^* \) on the conductor rotor, the concept of the micro-element method shows that when \( dS^* \) is small enough, the segment conductor can be approximated as a piece of cubic copper strip. Taking a small piece of \( dr_*^c \) on the copper strip, the calculation formulas (17) and (18) of the induced electromotive force \( d\varepsilon \) and the resistance \( R_{cu}^* \) on the \( dr_*^c \) section are established by

\[
d\varepsilon = (v \times B_{gal1}) \ dr_*^c 
\]  

(17)

\[
R_{cu}^* = dr_*^c / \left\{ \left[ \delta \varepsilon (r_c - r_*^c) \right] dS^* \right\}
\]  

(18)

By integrating (17) and (18), the induced electromotive force \( E \) and resistance \( R_{cu}^* \) on the whole conductor rotor can
be obtained

\[ E = B_{gal} \omega r_c I_c \]
\[ R_{eu} = \ln \left[ \pi \left( r_c + r'^{c} \right) \right] \frac{t_c}{\delta \nu \left( r_c - r'^{c} \right)} \]

The eddy current \( I_c \), eddy current loss \( P_{loss1} \) and torque \( T_{out} \) on the conductor rotor can be further obtained by (19) and (20)

\[ I_c = \frac{N B_{gal} \omega r_c \delta \nu (r_c - r'^{c})}{t_c \ln \left[ \pi (r_c + r'^{c}) \right]} \]
\[ P_{loss1} = \frac{\delta \nu (r_c - r'^{c}) \left( N B_{gal} \omega r_c \right)^2}{t_c \ln \left[ \pi (r_c + r'^{c}) \right]} \]
\[ T_{out} = \frac{\delta \nu \delta \omega (r_c - r'^{c}) \left( N B_{gal} \omega r_c \right)^2}{t_c \ln \left[ \pi (r_c + r'^{c}) \right]} \]

where \( N \) is the logarithm of permanent magnets.

**B. RADIAL MAGNETIC CIRCUIT MODEL**

As shown in in Fig. 8, the radial structure of the axial-radial combined permanent magnet eddy current coupler can be converted into a 2-D radial structure for analysis, and then the 3-D correction is applied.

1) AIR GAP MAGNETIC DENSITY MODEL

According to the equivalent magnetic circuit method, the radial equivalent magnetic circuit shown in Fig. 9 can be obtained from the radial magnetic circuit of the axial-radial combined permanent magnet eddy current coupler shown in Fig. 8.

Fig. 5 and Fig. 9 illustrate that the axial and radial equivalent magnetic circuit configurations are identical, but it is still difficult to calculate the relevant parameters. According to the formula of magnetoresistance, the reluctance of the permanent magnet and the total reluctance of the air gap and the conductor rotor are

\[ R'_{m} = N t'_p / \mu_0 \mu_r 2 \pi r'_p h'_{p} \]
\[ R'_g = N \left( t'_g + t'_p \right) / \mu_0 \alpha 2 \pi r'_p h'_{p} \]

where \( \alpha \) is the polar arc coefficient, \( h'_p \) and \( r'_p \) are the axial length and radius of the permanent magnet, respectively, and \( R'_{g} = R'_{g1} + R'_{g2} \) is the inner diameter of the radial permanent magnet back iron.

The reluctance of the back iron is also divided into two parts: \( R'_{g1} \) is the reluctance corresponding to the permanent magnet, and \( R'_{g2} \) is the reluctance of the air gap between the adjacent permanent magnets.

\[ R'_{g1} = 2 \pi (1 - \alpha) r'_p / N \mu_0 \mu_r h'_p \]
\[ R'_{g2} = 2 \pi \alpha r'_p / N \mu_0 \mu_r (h'_p t'_s + 2 \pi \alpha r'_p h'_p) \]

The reluctance \( R'_g \) corresponding to the magnetic flux leakage of the permanent magnet through the back iron and the reluctance \( R'_{mm} \) of the adjacent permanent magnets can be obtained through the permeance

\[ R'_g = \ln \left( 1 + \frac{t'_s}{2 \left( t'_g + t'_p \right)} \right) / \mu_0 h'_p \]
\[ R'_{mm} = \ln \left( 1 + \frac{t'_s}{2 \pi (1 - \alpha) r'_p} \right) / \mu_0 h'_p \]

According to (10), the air gap flux and the air gap flux density without considering eddy current effect and magnetic field coupling can be obtained as

\[ \Phi_{gr1} = \frac{8 \pi \Phi_{r} N \alpha t'_p t'_s \mu_s \ln (A + B)}{\left[ 2 N t'_p \left( 2 \ln A + \ln B \right) + 2 \pi \alpha \mu_r r'_p \ln (A + B) \right]} + \left[ 4 N^2 \left( t'_g + t'_p \right) t'_s \mu_s + 8 \pi^2 \alpha (1 - \alpha) r'_p^2 \right] + \left( 4 \pi \alpha r'_p \right) \]

where the coefficients \( A \) and \( B \) are respectively

\[ A = \left( 1 + \frac{t'_s}{2 \left( t'_g + t'_p \right)} \right)^{-1} \]
\[ B = \left( 1 + \frac{N t'_g}{2 \pi (1 - \alpha) r'_p} \right)^{-1} \]

The magnetic density \( B_{gr1} \) of the air gap considering eddy current effect should be composed of \( B_{gr1}' \) and the air gap magnetic density \( B_{cr1} \) of eddy current induced magnetic field.
According to Ampere’s law and calculus theory, a differential equation similar to (14) can be obtained.

\[
\frac{dB_{cr1}}{dx} = \frac{\left(\mu_0 \delta h'_{cr1} \omega t'_{cr1}\right) B_{cr1}}{2 \left(R + t'_c + t'_p + t'_g\right)} = \frac{\left(\mu_0 \delta h'_{cr1} \omega t'_{cr1}\right) B'_{gr1}}{2 \left(R + t'_c + t'_p + t'_g\right)}
\]

Then the radial magnetic density of the axial-radial combined permanent magnet eddy current coupler can be given as

\[
B_{gr1} = \frac{N \Phi_{gr1}}{2\pi r_p h_p} \left[ \cosh \left(\frac{\pi(1-\alpha) r'_p \mu_0 \delta h'_{gr1} \omega t'_{gr1}}{2(R + t'_c + t'_p + t'_g)}\right) \right] \left[ \cosh \left(\frac{\pi r'_p \mu_0 \delta h'_{gr1} \omega t'_{gr1}}{2N(R + t'_c + t'_p + t'_g)}\right) \right] e^{\frac{\pi r'_p \mu_0 \delta h'_{gr1} \omega t'_{gr1}}{2N(R + t'_c + t'_p + t'_g)}} \]

(34)

2) EDDY CURRENT LOSS AND TORQUE MODEL

It is necessary to proceed from the conductor rotor when analysing the eddy current loss of the radial structure. As shown in Fig. 10, taking one element \(dS_e\) on the conductor rotor, the thought of the micro-element method shows that when \(dS_e\) is small enough, it can be approximately a segment of cube copper strip. If a small segment of \(dh_e^*\) is taken from the copper strip, the induced electromotive force \(d\epsilon\) and the resistance \(R_{ceu}\) on \(dh_e^*\) section are established as

\[
d\epsilon = (v \times B_{gr1}) dh_e^*
\]

(35)

\[
R_{ceu} = dh_e^*/(\delta v_t dS_e)
\]

(36)

The electromotive force and the resistance of the whole copper ring can be obtained by integrating (35) and (36). Finally, the eddy current \(I_{e1}\), eddy current loss \(P_{loss1}\) and torque \(T_{out1}\) are given as

\[
I_{e1} = \frac{B_{gr1} \omega v t'_p (2r'_c - t'_c)}{\ln \left(\frac{1}{\pi(2r'_c - t'_c)}\right)}
\]

(37)

\[
P_{loss1} = \left[ \frac{t'_p \delta h'_e v \left[N B_{gr1} \omega (R' + t'_c + t'_p + t'_g)\right]}{\ln \left(\frac{1}{\pi(2r'_c - t'_c)}\right)} \right]^2
\]

(38)

\[
T_{out1} = \left[ \frac{t'_p \delta h'_e v \omega \left[N B_{gr1} (R' + t'_c + t'_p + t'_g)\right]}{\ln \left(\frac{1}{\pi(2r'_c - t'_c)}\right)} \right]^2
\]

(39)

IV. COUPLED MAGNETIC CIRCUIT MODELLING, 3-D CORRECTION AND THERMAL COMPENSATION OF AN AXIAL-RADIAL COMBINED PERMANENT MAGNET EDdy CURRENT COUPLER

A. COUPLED MAGNETIC CIRCUIT MODEL

The difference from the traditional magnetic circuit modelling method of permanent magnet eddy current coupler is that the coupling effect of axial and radial magnetic circuit should be fully considered in the modelling of the axial-radial combined permanent magnet eddy current coupler. The axial and radial hybrid magnetic circuit structure is mostly used in the design of rotating equipment such as motors. A few literatures apply it to the design of permanent magnet eddy current couplers, and the modelling method is proposed under the premise of neglecting the influence of magnetic field coupling. In fact, magnetic field coupling plays an indispensable role in the performance of axial-radial combined permanent magnet eddy current couplers.

1) AIR GAP MAGNETIC DENSITY MODEL

The model of the coupled magnetic circuit and the equivalent coupled magnetic circuit model are shown in Fig. 11 and Fig. 12, respectively.

The premise of (2) and (25) is that the permanent magnet is assumed to face the air gap, but that may not always be the case, so they are corrected to

\[
R_{ga} = \frac{\eta_1(t_c + t_g)}{\mu_0 h_p}, \quad R_{gr} = \frac{\eta_2(N t'_c + t'_g)}{2\pi \alpha r_p h_p}
\]

(40)

where the correction coefficients are

\[
\eta_1 = \frac{t_c + t_p + t_g}{h_p + t'_c + t'_p + t'_g}, \quad \eta_2 = \frac{t'_c + t'_p + t'_g}{h_p + t'_c + t'_p + t'_g}
\]
The axial and radial reluctance obtained from the permeance of the permanent magnet passing through the magnetic flux leakage of the back iron are corrected as follows, and the correction coefficients are given together.

\[
R_{1a} = \frac{t_c' + t_p' + t_g' + \omega_1}{\mu_0 h_p \left(t_c' + t_p' + t_g' + \omega_1\right)} \ln \left(1 + \frac{t_1}{2h_p}\right)
\]

\[
R_{1r} = \frac{t_c + t_p + t_g + h_p'}{\mu_0 t_p' \left(t_c + t_p + t_g + h_p'\right)} \ln \left(1 + \frac{t_1}{2t_p}\right)
\]

\[
\eta_3 = \frac{t_c' + t_p' + t_g'}{t_c' + t_p' + t_g' + \omega_1}, \quad \eta_4 = \frac{t_c + t_p + t_g + h_p'}{t_c + t_p + t_g + h_p'}
\]

The corrected air gap flux can be obtained according to Kirchhoff’s law, and the air gap flux density considering the eddy current effect can be further obtained.

\[
B_{g22} = \frac{\Phi_{g22} e^{\frac{\mu_0 \delta t_{in} v t_{in}}{2 h_p \omega_1}} \cosh \left(\frac{\mu_0 \delta t_{in} v t_{in}}{h_p \omega_1}\right)}{\cosh \left(\frac{\mu_0 \delta t_{in} v t_{in}}{h_p \omega_1}\right)}
\]

\[
B_{g22} = \frac{N \Phi_{g22} e^{\frac{\mu_0 \delta t_{in} v t_{in}}{2 h_p \omega_1}} \cosh \left(\frac{\mu_0 \delta t_{in} v t_{in}}{h_p \omega_1}\right)}{\cosh \left(\frac{\mu_0 \delta t_{in} v t_{in}}{h_p \omega_1}\right)}
\]

2) EDDY CURRENT LOSS AND TORQUE MODEL

When calculating the eddy current loss and the torque of the coupled magnetic circuit, the difference is that the eddy current effect should be considered in the air gap magnetic density

\[
P_{\text{loss3}} = \frac{N \delta v \left(r_c - t_c\right) \left(B_{g22}^{2} \omega r t_c\right)^2}{\ln \left[\pi \left(r_c + t_c\right)\right]}
\]

\[
P_{\text{loss4}} = \frac{N \delta v t_c' \left(B_{g22}^{2} \omega (r'_c + t'_c + t_p' + t_g')^2}{\ln \left[\pi \left(2r'_c - t_c\right)\right]}
\]

\[
T_{\text{out3}} = \frac{N \delta v o t_c \left(r_c - r_c'\right) \left(B_{g22}^{2} o r t_c\right)^2}{\ln \left[\pi \left(r_c + r_c'\right)\right]}
\]

\[
T_{\text{out4}} = \frac{N \delta v o t_c' \left(B_{g22}^{2} (r'_c + t'_c + t_p' + t_g')^2}{\ln \left[\pi \left(2r'_c - t_c\right)\right]}
\]

where \(P_{\text{loss3}}\) and \(T_{\text{out3}}\) are the eddy current loss and torque of the axial magnetic circuit, \(P_{\text{loss4}}\) and \(T_{\text{out4}}\) are the eddy current loss and torque of the radial magnetic circuit, respectively.

B. 3-D CORRECTION MODEL

The eddy current loss and the torque of the coupled magnetic circuit are shown in (47). In order to improve the accuracy of the analytical model, they are corrected in 3-D.

\[
\begin{align*}
& P_{\text{loss}} = P_{\text{loss1}} + P_{\text{loss2}} - P_{\text{loss3}} - P_{\text{loss4}} \\
& T_{\text{out}} = T_{\text{out1}} + T_{\text{out2}} - T_{\text{out3}} - T_{\text{out4}}
\end{align*}
\]

TABLE 1. Simulation parameters of the eddy current coupler.

| Quantity | Symbol | Value |
|----------|--------|-------|
| number of axial pole-pairs | \(p_a\) | 6 |
| number of radial pole-pairs | \(p_r\) | 12 |
| thickness of axial air gap | \(g_a\) | 2mm |
| thickness of radial air gap | \(g_r\) | 2mm |
| ratio of permanent magnets | \(a\) | 0.75 |
| width of copper plate | \(t_c\) | 115mm |
| thickness of copper plate | \(t_r\) | 5mm |
| magnetic pole arc coefficient | \(\delta\) | 2 |
| thickness of axial magnetic pole | \(t_s\) | 20mm |
| thickness of radial magnetic pole | \(t_t\) | 20mm |
| conductivity of copper | \(\delta_0\) | 57MS/m |

According to the boundary conditions, the correction coefficient of the axial-radial combined permanent magnet eddy current coupler can be obtained by using the 3-D correction method

\[
k_s = 1 - \frac{\tan \lambda_m}{\lambda_m \left(1 + \tan \lambda_m \tanh \lambda_m\right)}
\]

where the parameters are

\[
\lambda_m = \frac{N \pi (L_1 + L_2)}{2 \pi r_p + N (\omega_1 + \omega_2)}, \quad \lambda_{cm} = \frac{N \pi (H_1 + H_2)}{2 \pi r_p + N (\omega_1 + \omega_2)}
\]

The corrected torque model is carried out as

\[
\begin{align*}
& P_{\text{loss3D}} = k_s P_{\text{loss}} \\
& T_{\text{out3D}} = k_s T_{\text{out}}
\end{align*}
\]

C. THERMAL COMPENSATION

The magnetic-thermal coupling relationship is complicated for the axial-radial combined permanent magnet eddy current coupler. The model established in this paper does not consider the influence of thermal field. Subsequent papers will test the prototype to study the effect of temperature on the accuracy of the model. Therefore, the influence of temperature rise on the conductivity of the conductor rotor is mainly considered in this paper. In general, the relationship between temperature and conductivity of the conductor material satisfies:

\[
\delta(T) = \frac{\delta_0}{1 + \kappa T}
\]

where \(\delta_0\) is the conductivity of the conductor rotor at 20°C, for copper and aluminium, \(\kappa = 0.004\).

V. MODEL VERIFICATION

The finite element method is used to verify the validity of the analytical model and analyse the performance of the electromagnetic field and eddy current field. The relationship between the structural parameters and performance is studied, and the selection range of the optimal parameters can be narrowed.

The main simulation parameters are shown in Table 1. And the materials of the permanent magnet, the floppy plate, the spacer plate, and the conductor rotor are selected
as NdFeB35, steel10, aluminum, and copper, respectively. To confirm the validity of the analytical model, the experiments are carried out on the test bench shown in Fig. 13. Two 15 kW dc motors are used as the prime mover and the generator load. Two speed sensors have been placed on the dc motors to measure the speeds on both sides of the axial-radial combined permanent magnet eddy current coupler. A torque sensor is placed between the axial-radial combined permanent magnet eddy current coupler and load to measure the torque of dc motor.

A. STUDY ON MAGNETIC FIELD DISTRIBUTION
The effective magnetic density distribution at the average radius of analytical model and finite element method under 5% slip is extracted as shown in Fig. 14. The trend of theoretical analysis is basically consistent with that of finite element method. The error between the average value of finite element method and the value of theoretical analysis is always below 6%.

B. THE RELATIONSHIP BETWEEN SLIP AND TORQUE
The influence of different slip (0~100%) on torque is studied in this paper. The torque curve shown in Fig. 15 indicates that the peak point of torque is under 23% slip. The torque increases rapidly with the increase of slip in the range of 0~23%. During this period, the eddy current intensity of the conductor rotor is increased, which also increases the magnetic coupling force, the intensity of the induced magnetic field, and the ability of transmitting torque. After 23% slip, the intensity of the induced magnetic field is strong enough to weaken the intensity of the exciting magnetic field, the torque decreases rapidly and then inclines to be stable.

C. THE INFLUENCE OF STRUCTURAL PARAMETERS ON TRANSMISSION PERFORMANCE
1) THE INFLUENCE OF POLE PAIRS ON THE AIR GAP MAGNETIC FLUX DENSITY
The influence of axial and radial pole pairs at the average radius on the air gap magnetic density is studied. As shown in Fig. 16, when the thickness of the air gap is fixed, the slip is a constant, and the other parameters are almost unchanged, the pole pairs will determine the magnitude of the air gap magnetic induction directly. With the increase of pole pairs, the magnetomotive force increases, causing the air gap magnetic density to increase rapidly and reach a peak point at a certain moment. In addition, the reluctance and magnetic flux leakage are also enhanced. If the number of pole pairs continues to increase, the increased magnetic potential will be completely consumed by the reluctance and the magnetic flux leakage, so the air gap magnetic density tends to be saturated.

2) THE INFLUENCE OF THE THICKNESS OF THE CONDUCTOR ROTOR ON TRANSMISSION PERFORMANCE
The magnetic field induced on the conductor rotor will enhance the air gap magnetic density as shown in Fig. 17 (a). It can be found that the optimal thicknesses of the axial and radial conductor rotors are 5-10 mm and 4-8 mm, respectively. However, the thickness of the conductor rotor cannot be selected by the air gap magnetic density, because an
3) THE INFLUENCE OF THE THICKNESS OF THE CONDUCTOR ROTOR ON EDDY CURRENT DENSITY
Theoretically, the eddy current density will increase as the thickness of conductor rotor increases when the other parameters are fixed. If the thickness of the conductor rotor reaches the optimal value and continues to increase, the eddy current density will not increase, or even decrease.

The simulation of eddy current density is performed on the axial-radial combined permanent magnet eddy current coupler to verify the above conclusions. As shown in Fig. 18 and Fig. 19, with the thickness of the axial conductor rotor increases from 7 mm to 9 mm under 5% slip, the eddy current density also increases. However, when the conductor rotor thickness reaches 10 mm, the eddy current density decreases, and the radial conductor rotor eddy current density is similar to the axial condition.

4) THE INFLUENCE OF THE THICKNESS OF THE CONDUCTOR ROTOR ON THERMAL FIELD
The eddy current effect is generated in the conductor rotor. The resistance and eddy current increase together with the thickness of the conductor rotor, which increases the torque and temperature. Therefore, the maximum temperature of the device must exist in the conductor rotor. If the thickness of the copper rotor continues to increase after reaching the skin depth, the resistance will increase and the temperature will continue to rise, but the torque will not increase, or even decrease due to the thermal effect of eddy current loss. Fig. 20 and Fig. 21 (a) show the temperature distribution of the axial and radial conductor rotors under the rated load. It is shown that the variation of temperature with the thickness of
The axial and radial conductor rotors are consistent with the above conclusions.

The thickness of the conductor rotor will also affect the temperature rise of the permanent magnet. If the temperature is high excessively, the permanent magnet will be demagnetized permanently. The temperature distribution of the permanent magnet and the influence of the conductor rotor thickness on permanent magnets (PMS) temperature are shown in Fig. 22 and Fig. 21 (b). The figures demonstrate that the maximum temperature of axial permanent magnet is close to the central region of axial conductor rotor, the maximum temperature of radial permanent magnet is at the center outside the radial permanent magnet.

VI. CONCLUSION

A new analytical model has been developed to predict the air gap magnetic density, eddy current loss and torque of an axial-radial combined permanent magnet eddy current coupler. The validity and accuracy of the proposed model are in good agreement with the results of the finite element method and experimental tests. The proposed analytical model has less flexibility and a fast computation. The distribution of the magnetic field, the relationship between the slip and torque, the influence of the structural parameters on the transmission performance, the 3-D correction and thermal compensation for the coupled magnetic circuit model are also studied, which provides guidance for the optimal design and ways to promote the performance. Further research on the transient model and control strategies of motors, such as permanent magnet synchronous motors and induction motors, will be carried out on axial-radial combined permanent magnet eddy current couplers in the future.

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