New Einstein-Hilbert type action for space-time and matter
-Nonlinear-supersymmetric general relativity theory-

Kazunari Shima
Saitama Institute of Technology
E-mail: shima@sit.ac.jp

We can perform the geometric argument of general relativity principle on (unstable) Riemann space-time just inspired by nonlinear representation of supersymmetry(NLSUSY), whose tangent space is specified by Grassmann degrees of freedom $\psi$ for SL(2,C) besides the ordinary Minkowski one $x^a$ for SO(1,3) and obtain straightforwardly new Einstein-Hilbert(EH)-type action with global NLSUSY invariance (NLSUSYGR) equipped with the cosmological term. Due to the NLSUSY nature of space-time NLSUSYGR would breaks down(Big Collapse) spontaneously to ordinary E-H action of graviton, NLSUSY action of Nambu-Goldstone fermion $\psi$ and their gravitational interaction. Simultaneously the universal attractive gravitational force would constitute the NG fermion-composites corresponding to the eigenstates of liner-SUSY(LSUSY) super-Poincare space-time symmetry, which gives a new paradigm for the unification of space-time and matter. By linearizing NLSUSY we show that the standard model(SM) of the low energy particle physics can emerge in the true vacuum of NLSUSYGR as the NG fermion-composite massless eigenstates of LSUSY super-Poincare algebra of space-time symmetry, which can be understood as the ignition of the Big Bang and continues naturally to the standard Big Bang model of the universe. NLSUSYGR can bridge naturally the cosmology and the low energy particle physics and provides new insights into unsolved problems of cosmology, SM and mysterious relations between them, e.g. the space-time dimension four, the origin of SUSY breaking, the dark energy and dark matter, the dark energy density~(neutrino mass)$^4$, the tiny neutrino mass, the three-generations structure of quarks and leptons, the rapid expansion of space-time, the magnitude of bare gauge coupling constant, etc..
Three-generations structure

Supersymmetry (SUSY) related naturally to space-time symmetry is promising for the unification of general relativity and the low energy SM in one single irreducible representation of the symmetry group. We have found by group theoretical arguments that among all $SO(N)$ super-Poincaré (sP) groups the $SO(10)$ sP group decomposed as $N = 10 = 5 + \bar{5}^*$ under $SO(10) \supset SU(5)$ may be a unique and minimal group which accommodates all observed particles including graviton in a single irreducible representation of $N$ linear($L$) SUSY. In this case 10 supercharges $Q^I$, $(I = 1, 2, \ldots, 10)$ are embedded as follows:

$$10_{SO(10)} = 5_{SU(5)} + \bar{5}^*_{SU(5)}, \quad 5_{SU(5)} = \left[ \begin{array}{c} 3^c, 1^{ew}, (\frac{\xi}{3}, \frac{\xi}{3}, \frac{\xi}{3}) : Q_a (a = 1, 2, 3) \end{array} \right] + \left[ \begin{array}{c} 1^c, 2^{ew}, (-e, 0) : Q_m (m = 4, 5) \end{array} \right],$$

i.e., $5_{SU(5)GUT}$ represents $[Q_a{:}d\text{-type}, Q_m{:}Lepton\text{-type}]$ supercharges. The massless helicity state $|h\rangle$ of gravity multiplet of $SO(10)$ sP with CPT conjugate are specified by the helicity $h = (2 - \frac{n}{2})$ and the dimension $d_{[n]} = \frac{10!}{n! (10 - n)!}$ for $n = 0, 1, \ldots, 10$ as tabulated below. To see low energy massive states we assume a maximal $SU(3) \times SU(2) \times U(1)$ invariant superHiggs-like mechanism among helicity states, i.e., all redundant high helicity states for SM become massive by absorbing lower helicity states (and decoupled) in $SM$ invariant way. The results are interesting: Spin $\frac{1}{2}$ state survivours after superHiggs-like mechanism are shown in the table (tentatively as Dirac particles).

In the fermion sector, just three generations of quark and lepton states survive as shown in the table. In the bosonic sector, gauge fields of SM in vector states and one Higgs field of SM in

| $|h|$ | 3 | 5/2 | 2 | $\bar{5}$/2 | 1 | $\bar{5}$/2 | 0 |
|-----|---|-----|----|---------|---|---------|---|
| $d_{[n]}$ | 1_{[10]} | 10_{[9]} | 45_{[8]} | 120_{[7]} | 210_{[6]} | 252_{[5]} | 210_{[4]} |

| $SU(3)$ | $Q_e$ | $SU(2) \otimes U(1)$ |
|--------|------|----------------|
| 1 | 0 | $(\nu_e \quad \nu_\mu \quad \nu_\tau)$ |
|    | -1 | $(\nu_e \quad \nu_\mu \quad \nu_\tau)$ |
|    | -2 | $(\nu_e \quad \nu_\mu \quad \nu_\tau)$ |
| 3 | 5/3 | $(u \quad c \quad t)$ |
|    | 2/3 | $(d \quad s \quad b)$ |
|    | -1/3 | $(h \quad f \quad m)$ |
|    | -4/3 | $(r \quad i \quad n)$ |
| 6 | 4/3 | $(P \quad X)$ |
|    | 1/3 | $(Q \quad Y)$ |
|    | -2/3 | $(R \quad Z)$ |
| 8 | 0 | $(N_1 \quad N_2)$ |
|    | -1 | $(E_1 \quad E_2)$ |
the scalar states survive. Besides those observed states, one color-singlet neutral vector state and one double-charge color-singlet spin $\frac{1}{2}$ state are survived, which can be tested experimentally. We will show in the next section that no-go theorem for constructing non-trivial $SO(N > 1)$ SUGRA can be circumvented by adopting the nonliner (NL) representation of SUSY, i.e. by introducing the degeneracy of space-time through NLSUSY degrees of freedom.

**Nonlinear-Supersymmetric General Relativity Theory (NLSUSYGR)**

For simplicity we discuss $N = 1$ without the loss of the generality.

The fundamental action nonlinear supersymmetric general relativity theory (NLSUSYGR) has been constructed by extending the geometric arguments of Einstein general relativity (EGR) on Riemann space-time to new space-time inspired by NLSUSY, where tangent space-time is specified not only by the Minkowski coordinate $x_\alpha$ for $SO(1,3)$ but also by the Grassmann coordinate $\psi_\alpha$ for $SL(2,C)$ related to NLSUSY. They are coordinates of the coset space $\frac{superalgebra}{GL(4,R)}$ and can be interpreted as NG fermions associated with the spontaneous breaking of $GL(4,R)$ down to $GL(4,R)$. The NLSUSYGR action, is given by

$$L_{\text{NLSUSYGR}}(w) = -\frac{c^4}{16\pi G} |w| \{\Omega(w) + \Lambda\},$$

$$|w| = \det w^a_{\mu} = \det\{e^a_{\mu} + t^a_{\mu}(\psi)\},$$

$$t^a_{\mu}(\psi) = \frac{\kappa^2}{2\ell} (\bar{\psi} \gamma^\mu \psi - \partial_\mu \bar{\psi} \gamma^\mu \psi),$$

where $G$ is the Newton gravitational constant, $\Lambda$ is a (small) cosmological term and $\kappa$ is an arbitrary constant of NLSUSY with the dimension (mass)$^{-2}$. $w^a_{\mu}(x) = e^a_{\mu} + t^a_{\mu}(\psi)$ and $w^{\mu}_{\alpha} = e^{\mu}_{\alpha} - t^{\mu}_{\alpha} + t^{\mu}_{\rho} \theta^{\rho}_{\alpha} - t^{\mu}_{\sigma} \theta^{\sigma}_{\rho} \rho^{\rho}_{\alpha} + t^{\mu}_{\mu} \varepsilon^{\mu}_{\sigma} \theta^{\sigma}_{\rho} \rho^{\rho}_{\alpha}$ which terminate at $O(\ell^4)$ for $N = 1$ are the invertible unified vierbeins of new space-time. $e^a_{\mu}$ is the ordinary vierbein of EGR for the local $SO(1,3)$ and $t^a_{\mu}(\psi)$ is the mimic vierbein analogue (actually the stress-energy-momentum tensor) of NG fermion $\psi(x)$ for the local $SL(2,C)$. (We call $\psi(x)$ superon as the hypothetical fundamental spin $\frac{1}{2}$ particle carrying the supercharge of the supercurrent of the global NLSUSY.) $\Omega(w)$ is the unified scalar curvature of new space-time computed in terms of the unified vierbein $w^a_{\mu}(x)$. Interestingly Grassmann degrees of freedom induce the imaginary part of the unified vierbein $w^a_{\mu}(x)$, which represents straightforwardly the fermionic matter contribution. Note that $e^a_{\mu}$ and $t^a_{\mu}(\psi)$ contribute equally to the curvature of space-time, which may be regarded as the Mach’s principle in ultimate space-time. (The second index of mimic vierbein $t$, e.g. $\mu$ of $t^a_{\mu}$, means the derivative $\partial_{\mu}$.) $s_{\mu\nu} = w^a_{\mu} \eta_{ab} w^b_{\nu}$ and $\bar{s}^a_{\mu}(x) \equiv w^{\mu}_{\alpha}(x) w^{\alpha}_{\mu}(x)$ are unified metric tensors of new spacetime. $L_{\text{NLSUSYGR}}(w)$ (2) is invariant under the following NLSUSY transformations:

$$\delta^{\text{NL}} \psi = \frac{1}{\kappa^2} \zeta + i \kappa^2 (\bar{\xi} \gamma^\rho \psi) \partial_\rho \psi,$$

$$\delta^{\text{NL}} e^a_{\mu} = i \kappa^2 (\bar{\xi} \gamma^\rho \psi) \partial_\rho e^a_{\mu},$$

where $\zeta$ is a constant spinor parameter and $\partial_{[\rho} e^{a_{\mu]}} = \partial_{\rho} e^a_{\mu} - \partial_{\mu} e^a_{\rho}$, which close on $GL(4,R)$, i.e. new NLSUSY (3) is the square-root of $GL(4,R)$;

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi = \Xi^\mu \partial_\mu \psi,$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] e^a_{\mu} = \Xi^\rho \partial_\rho e^a_{\mu} + e^a_{\rho} \partial_\mu \Xi^\rho,$$

where $\Xi^\mu = 2i \kappa (\bar{\xi}_1 \gamma^\rho \xi_2) - \bar{\xi}_1 \xi_2 e^{a}_{\mu}(\partial_\rho e^{a}_{\sigma})$, and induce the following $GL(4,R)$ transformations on the unified vierbein $w^a_{\mu}$ and the metric tensor $s_{\mu\nu}$

$$\delta_{\zeta} w^a_{\mu} = \bar{\xi} \gamma^\nu \partial_\nu w^a_{\mu} + \partial_\mu \bar{\xi} \gamma^\nu w^a_{\nu},$$

$$\delta_{\zeta} s_{\mu\nu} = \bar{\xi} \gamma^k \partial_ks_{\mu\nu} + \partial_\mu \bar{\xi} \gamma^k s_{k\nu} + \partial_\nu \bar{\xi} \gamma^k s_{\mu k},$$

(5)
where $\xi^\rho = i\kappa^2 (\tilde{\xi}^\rho \psi)$.

NLSUSY GR action (1) possesses promising large symmetries isomorphic to $SO(N)\ (SO(10))$ SP group; namely, $L_{NLSUSYGR}(w)$ is invariant under spacetime symmetries: [new NLSUSY] $\otimes$ [local GL$(4, \mathbb{R})] \otimes$ [local Lorentz] and under internal symmetries: [global $SO(N)$] $\otimes$ [local $U(1)^N$] in case of $N$ superons $\psi^i, i = 1, 2, \cdots, N$. Note that the no-go theorem is overcome (circumvented) in a sense that the nontrivial $N(N > 8)$-extended SUSY with gravity has been constructed in the NLSUSY invariant way.

**Big Collapse (BC) of ultimate space-time (NLSUSYGR)**

New (empty) space-time described by NLSUSYGR action $L_{NLSUSYGR}(w)$ is unstable due to NL-SUSY structure of tangent space-time and collapses (called Big Collapse) spontaneously to ordinary Riemann space-time with the cosmological term and fermionic matter superon (called superon-graviton model (SGM)). $L_{SGM}(e, \psi)$ can be recasted formally as the following familiar form

$$L_{NLSUSYGR}(w) = L_{SGM}(e, \psi) = -\frac{c^4}{16\pi G} |w| \{ R(e) + \Lambda + \tilde{T}(e, \psi) \}, \quad (6)$$

where $R(e)$ is the Ricci scalar curvature of ordinary EH action and $\tilde{T}(e, \psi)$ represents the kinetic term and the gravitational interaction of superons. Remarkably the first term should reduces to NLSUSY action in Riemann-flat $e^\mu_a(x) \rightarrow \delta^\mu_a$ space-time, i.e. the arbitrary constant $\kappa$ of NLSUSY is fixed to

$$\kappa^{-2} = \frac{c^4}{8\pi G} \Lambda. \quad (7)$$

Note that Big Collapse induces the rapid expansion of space-time due to the Pauli principle for fermion superon; $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = [g_{\mu\nu}(e) + h_{\mu\nu}(e, \psi)] dx^\mu dx^\nu$ and simultaneously constitutes all possible gravitational composite-eigenstates of space-time symметry. SGM action, whose gravitational evolution ignites Big Bang of the present observed universe. NLSUSY scenario predicts the dimension of space-time is four, for space-time supersymmetry for $SO(1, D - 1)$ and $SL(d, C)$ requires

$$\frac{D(D - 1)}{2} = 2(d^2 - 1), \quad (8)$$

which holds only for $D = 4, d = 2$.

**Evolution of NLSUSYGR/SGM**

NLSUSYGR(SGM) with $\Lambda > 0$ evolves toward the true vacuum. The gravity is the universal attractive force and creates all possible gravitational composites of superons, which is the all possible products of supercharges and corresponds to (massless) helicity-eigenstates of $SO(10)$ linear(L) SUSY $\mathfrak{sp}$ algebra of asymptotic space-time symmetry. (Note that the leading term of supercharge is the superon field.) This means that all component fields of USUSY supermultiplet are expressed as such composites of superons (called SUSY compositeness) as USUSY transformation of USUSY supermultiplet are reproduced under the NLSUSY transformations of the constituent superons. Simultaneously the equivalence of NLSUSY action and the USUSY action holds (called NL/L SUSY relation) in the sence that USUSY action reduces to NLSUSY action when SUSY compositeness is inserted in USUSY component fields. To see the low energy (vacuum) behavior of $N = 2$ SGM (NLSUSYGR) we consider SGM in asymptotic Riemann-flat space-time, where $N = 2$ SGM reduces to essentially $N = 2$ NLSUSY action. We will show the equivalence of $N = 2$
NLSUSY action to $N = 2$ LSUSY QED action (called NL/L SUSY relation), i.e.
\[
L_{\text{NL/SUSYGR}} (w^\mu) = L_{\text{SGM}} (e^\mu, \psi) + L_{\text{NL/SUSY}} (\psi) + [\text{surface terms}] = f_\xi L_{\text{LSUSY}} (\nu^i, D, \cdots)
\]
where $f_\xi$ is the function of vacuum values $\xi^i$ of auxiliary fields. NL/L SUSY relation is shown explicitly by substituting the following SUSY compositeness into the LSUSY QED theory. For example, the SUSY compositeness for the **minimal gauge** vector supermultiplet ($\nu^i, \lambda^i, A, \phi, D$) are
\[
\nu^i = -i/2 \xi \kappa e^{ij} \bar{\psi}^j \gamma^i \psi^j |w|, \quad \lambda^i = \xi \left[ \psi^i |w| - i/2 \kappa^2 \partial_a \{ \gamma^a \bar{\psi}^i \gamma^i (1 - i \kappa^2 \bar{\psi}^k \partial \gamma^k) \} \right],
\]
\[
A = 1/2 \xi \kappa \bar{\psi}^i |w|, \phi = \frac{1}{2} \xi \kappa e^{ij} \bar{\gamma}^j \gamma^i |w|, D = \frac{\xi}{\kappa} |w| - i/8 \xi \kappa^3 \partial_a (\bar{\psi}^i \gamma^i \bar{\psi}^j \nu^j). \quad \cdots
\]
and similar results for the scalar supermultiplet. NL/L SUSY relation shows the equivalence (relation) of two theories irrespective of the renormalizability. For the **non-minimal most general** gauge vector and scalar supermultiplet NL/L SUSY relation $f_\xi = 1$ predicts the magnitude of the bare gauge coupling constant.
\[
f(\xi, \bar{\xi}, \xi_c) = \bar{\xi}^2 - (\xi^i)^2 e^{-4\xi c} = 1, \quad i.e., \quad e = \frac{\ln(\frac{\xi^i}{\bar{\xi}} - 1)}{4\xi_c},
\]
Broken LSUSY(QED) gauge theory is encoded in the vacuum of NLSUSY theory as composites of NG fermion. And SM may be an low energy effective theory of SGM.

**Low energy particle physics of NLSUSYGR/SGM**

NL/L SUSY relation (equivalence) gives:
$L_{N=2\text{SGM}} \rightarrow L_{N=2\text{NL/SUSY}} + [\text{surface terms}] = L_{N=2\text{SUSYQED}}$ in Riemann-flat space-time. The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\text{SUSYQED}}$,
\[
V(A, \phi, B^i, D) = -\frac{1}{2} D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2} e |B^i|^2 \right\} D + \frac{e^2}{2} (A^2 + \phi^2) |B^i|^2,
\]
which has mass spectra:
\[
m_A^2 = m_{\chi^i}^2 = 4 f^2 k^2 = \frac{4 \xi f}{\kappa}, \quad m_{B^i}^2 = m_{\bar{B}^i}^2 = m_{\chi}^2 = m_{\nu}^2 = e^2 k^2 = \frac{\xi e^2}{\kappa f}, \quad m_\nu = m_\phi = 0,
\]
which produces **mass hierarchy** by the factor $\xi$. The vacuum describes qualitatively lepton-Higgs-U(1) sector analogue of the SM: one massive charged Dirac fermion ($\psi_D = \chi + i v$), one massive neutral Dirac fermion ($\lambda_D (\lambda_D^0 \sim \lambda^1 - i \lambda^2)$), one massless vector (a photon) ($\nu_0$), one charged scalar ($B^1 + i B^2$), one neutral complex scalar ($A + i \phi$), which are composites of superons.

Revisiting SM and GUT in the SQM (superon-quintet composite model) picture may give new insight into the unsolved problems of the SM. One simple assignment of observed particles is:
\[
(e, \nu_e): \delta^{ab} Q_a Q^*_b Q_m, \quad (\mu, \nu_\mu): \delta^{ab} Q_a Q^*_b e^{im} Q_l Q_m Q_n, \quad (\tau, \nu_\tau): \epsilon^{abc} Q_a Q_c e^{im} Q^*_b Q^*_c Q_m, \quad (\pi, \nu_\pi): \epsilon^{abc} Q_a Q_b Q_c e^{im} Q^*_d Q^*_e Q_f Q_m, \quad (u, d): \delta^{abc} Q_a Q_b Q_c, \quad (c, s): \epsilon^{im} Q_l Q_m \epsilon^{abc} Q_a Q^*_b Q^*_c Q^*_m, \quad (t, b): \epsilon^{abc} Q_a Q_b Q_c Q_d Q^*_m, \quad \text{Gauge Boson: } Q_a Q^*_b, \quad \text{Higgs Boson: } \delta^{ab} Q_a Q^*_b Q_l Q_m, \quad \cdots
\]
and SQM diagram, e.g., for $\beta$-decay:
**Cosmological implications of NLSUSYGR/SGM**

The variation of SGM action $L_{SGM}(e, \psi)$ with respect to $e^\mu_\nu$ yields

Einstein equation equipping with matter and cosmological term:

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4} (\bar{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G}).$$

where $\bar{T}_{\mu\nu}(e, \psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction. Note that the cosmological term $-\frac{c^4 \Lambda}{8\pi G}$ can be interpreted as the negative energy density of space-time, i.e. the dark energy density $\rho_D$.

Big collapse (BC) may induce 3 dimensional expansion of space-time by Pauli principle:

$$ds^2 = s_{\mu\nu}(x)dx^\mu dx^\nu = \left( g_{\mu\nu}(e) + h_{\mu\nu}(e, \psi) \right)dx^\mu dx^\nu.$$  

BC produces composite (massless) eigenstates of SO(N) sP algebra due to the universal attractive force of graviton, which is the ignition of the Big Bang (BB) SM scenario. As shown in the toy model, the vacuum of the composite SGM scenario may explain naturally observed mysterious (numerical) relations: dark energy density $\rho_D \sim O(\kappa^{-2}) \sim m_\nu^4 \sim (10^{-12} GeV)^4 \sim g_{\nu}^2$, provided $\lambda_D^0$ is identified with neutrino and $f_\xi \sim O(1)$.

**References**

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