Effects of knot characteristics on tensile breaking of a polymeric monofilament

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Abstract. The relationship between knot characteristics and breaking mechanisms was investigated for the model filament of a poly(vinylidene fluoride) fishing line. The colouring procedure adopted in this study enabled us to define the accurate breaking position within the knots. Comparison of the breaking positions of a series of torus knots suggested that the breaking position gradually shifted from inside to outside the knots with increasing crossing number. Interestingly, a corresponding effect of knot geometry was also recognized as the synchronized decrease of tensile strength of the knotted filaments. The morphological observation of the fractured tips obtained after tensile breaking revealed that both squeezing and rotation identified for each knot predominantly determined the position and strength of the knot.

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1. Introduction

It has been commonly recognized that a knotted strand always breaks just outside the entrance of the knot [1]. Breaking never occurs in the straight-line part. Thus, breaking a knotted strand is much easier than breaking an unknotted strand [2].

Even for knotted strands, the difference in knot geometry affects the ease of tensile breaking. This fact is well known among anglers who must make knots in their fishing lines. Through trial and error, anglers have sought the strongest knot. However, material science has not approached the predominant factor of knot strength, although some theoretical studies have been applied for knotted molecules [3]–[6]. The reason lies in the unknown mechanism of knot breaking. When the knotted strand is stretched, breaking occurs quickly. Even if such a breaking process is recorded with a high-speed camera, the fractured tips of the broken strand are outside the observed area as soon as the breaking occurs, due to strict tension recovery. The virtual rebinding of both tips could reproduce the original knot shape, but the entangled state of knot geometry prevents an accurate prediction of the breaking position of the knot. Furthermore, the knot becomes smaller with stretching, meaning that any given position within the knot moves continuously. Progressive knot squeezing also leads to deformation of the strand diameter, which influences the breaking mechanism of the knot.

The mechanism of knot breaking has been examined for a significantly soft strand, which reduces breaking recovery. Stasiak et al [7] used a high-speed camera to record the knot breaking of boiled spaghetti as a model strand of DNA molecules [8]. Their results indicated that breaking occurred at the entrance of the knot, so that the knot remained after breaking. This result was supported by their theoretical calculation where the highest strain was predicted around the knot entrance. In contrast, the knot appearance was not recognizable in fishing lines for the simplest trefoil knot, as discussed later in this study. This experimental fact suggested that breaking occurred within the squeezed knot.

The position of knot breaking depends on the kind of strands examined. Knot geometry also significantly affects the breaking mechanism. A complicated, highly entangled knot retains its original appearance even after tensile breaking. Therefore, knot geometry dominates the

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breaking mechanism during stretching. In this study, we examined the relationship between knot characteristics and breaking mechanisms for a polymeric monofilament used as fishing line.

2. Experimental

2.1. Material

The sample monofilament used was commercial fishing line of poly(vinylidene fluoride) (PVDF) having a diameter of 0.205 mm, kindly supplied by Kureha Chemical, Japan. Its flexibility, initial homogeneity in fibre diameter, smooth surface and circular shape of the cross-section were preferred to minimize the friction force effects on tying knots.

2.2. Measurements

Tensile tests of the knotted monofilaments were performed with a Tensilon tensile tester UTM-5T. These tests were always conducted at room temperature, with a cross-head speed of 30 mm min\(^{-1}\), corresponding to the initial strain rates of 0.1 min\(^{-1}\). The total filament length was always 300 mm. Prior to the tensile tests, 3.92 N (400 kgf) was pre-loaded to all knotted samples to tie the knots sufficiently. In the case of the tensile tests for the rotated filament without a knot, the given times of filament rotation were induced under the same pre-loading of 3.92 N before the tests. Then, the rotated filaments were tensile-broken using the above-mentioned tensile tester.

The overall appearances and series of cross-sections of the knotted fibres before and after breaking were observed with an Olympus BH2-UMA optical microscope. For cross-section analysis, the sample fibre was embedded in epoxy resin; a Reichert UltraCut S microtome equipped with a glass knife was used to cut the assembly perpendicular to the fibre axis every 25 µm. These cut surfaces were sketched, and the long axis of the fibre cross-section was estimated from them.

The fractured tips were observed with a scanning electron microscope using a JEOL JEM-25D operated at 5 kV. The sample surface was coated with 8 nm Au with the use of a JEOL ion spatter JFC-1000.

3. Results and discussions

3.1. Knot geometries

Various knots were introduced in this study to investigate the relationship between knot geometry and breaking characteristics. These knots were compared in terms of universality. Knot theory enables the appropriate ordering of knot geometry using the crossing number. This theory defines the topological properties even for complicated knots [9, 10].

Knot theory assumes a closed loop without the ends of the strands. The virtual cutting of these theoretical closed loops provides several types of geometries of the open knots [11]. In this study, we adopted symmetrical sets of open knots, which are listed in figure 1. The crossing number (i.e. the number of the strand crossing) is the most important topological factor to distinguish various knots. The simplest trefoil knot has a crossing number of 3 (figure 1). This trefoil knot is defined as a ‘31’ knot, according to Alexander’s categorization [12, 13].
Figure 1. Comparisons of knot appearances of torus series (a) before tightening, (b) loosely tightened and (c) tightly squeezed. The internal region (red arrow) was rounded by the ring part (blue arrow) from the outside with knot squeezing.

subscript of 1 means the torus knot series is prepared by one-side entering the strand through its ring part. For the torus series, the crossing number increases by two because the strand passes through twice within the knot, giving odd values of crossing number. Therefore, the torus knot after $3_1$ is $5_1$. A comparison of knot appearances depicted in the top and middle rows of figure 1 reveals that the ring part (blue arrow) rounds to the internal strands (red arrow), which twist within the knot and they gradually squeeze with stretching. The increase of crossing number signifies the twisting and rounding of the strand. Therefore, the fully squeezed appearance is similar to that of a green caterpillar (bottom row of figure 1).

3.2. Knot strength

Tensile strengths of these knots were compared. If the examined strand has a distribution of tensile strength in an unknotted state, the effect of knots on the tensile strength cannot be accurately identified. Therefore, we selected the PVDF monofilament as the standard strand. The circular cross-section of this filament minimized the friction on knot squeezing and also enabled us to estimate the deformation within the knot, as discussed later.

Figure 2 depicts tensile strength as a function of crossing number for the torus series of knots. Introducing a knot reduces the tensile strength, compared to that of the unknotted straight line. As the crossing number increased, the tensile strength gradually decreased. Beyond a crossing number of 9, a constant level of 500 MPa was obtained. Therefore, at least for the torus series of
knots, the complexity of the knot provided lower breaking strength and a constant value beyond the critical crossing number.

3.3. Breaking position

The breaking position of the knotted filament was also investigated. As described in the introduction, it has been thought that knots broke at the entrance of the knot, independent of knot geometry [1]. However, our experimental result for the knotted PVDF monofilament revealed that knot appearance was not retained on both fractured tips. Thus, the origin of such a difference between predicted and obtained results should be clarified.

One possible explanation is that the knotted tip disentangles just after breaking at the knot entrance, which nominally gives an unknotted appearance of the fractured tips. In order to confirm this possible scenario, the position just outside of the entrance of the \(3_1\)-knotted filament was cut by a blade when the filament was stretched by loading 90% of the tensile strength. The filament broke immediately, but the knot appearance remained at either fractured tip even when the same tests were attempted several times. These results suggest that the \(3_1\) knot never disentangles even if tensile breaking occurs at the entrance of the knot in the case of the PVDF monofilament adopted in this study. Thus, tensile breaking occurs within the \(3_1\) knot.

In order to define the exact breaking position, the knotted filament was marked using a colouring procedure. The knotted part was completely painted with black ink when the filament was stretched by loading 90% of the tensile strength. Such preloading prevented filament movement and deformation within the knot after the knot was coloured. The exposed surface of the filament was marked black, but the internal part of the knot remained transparent. Therefore, the fractured tips had patterned colourings, and an accurate fracture point could be estimated by comparing the patterns before and after breaking.
The breaking position of the $3_1$ knot was examined first. Figures 3(a) and (b) illustrate the set of knot appearances observed from opposite sides. The initial filament cross-section was circular, but the filament was significantly bent and squashed after knotting. Specifically, the rounding part (figure 3(b)) was compressed. The fractured tips were always sets of J- and I-shaped tips (figure 3(c)). Both of the front parts of the fractured tips were black; thus the breaking position corresponded to the part previously exposed outside when colouring. In order to correlate the alphabetic positions of A–G before and after breaking, the colouring pattern was virtually schemed (figure 3(d)). It should be noted that the real knot could not be disentangled, due to plastic deformation during knotting. Outside the knot, the filament was fully coloured. As the filament entered from entrance position A within the knot, it became transparent. From position B, the filament was again coloured, corresponding to the beginning point of the region rounding to the knot. Afterwards, it reached shoulder position C with sharp bending, and then reached centre position D. The final breaking occurred at position E. From the next position F, the filament rounded again to the knot, and went out from the knot at position G. Considering that these positions were arranged symmetrically to centre position D, it was found that fractured position E corresponded to shoulder position C.

For quantitative analysis of the filament constraints, the cross-section at each critical position denoted above was estimated. Figure 4(a) reveals the series of cross-sections for a residual fractured tip of a $3_1$ knot. For this analysis, the longer J-shaped tip was straightened and embedded in epoxy resin, and the assembly was microtomed perpendicular to the filament axis every 25 µm. The positions denoted by the letters correspond to those depicted in figure 3. At entrance position A, the cross-section exhibited a fully circular shape, but changed to a semicircular shape and then an arch-like shape at shoulder position C. Approaching centre position D, it reverted to a semicircular shape again and finally reached the actual breaking position E. The front shape of the fractured tip was distorted because breaking occurred less perpendicular to the filament axis. Figure 4(b) plots the length of the long axis of the cross-sections at each position from knot entrance A to centre position D, which corresponded to the internal region of the knot. Here, the long axis was defined as the longest line that divided the semicircular or arch-shaped cross-section into two halves having the same areas. The maximum value in the length of the long axis was obtained at shoulder position C, indicating the highest constraint within the knot. It should be noted that the cross-section area was constant throughout the whole distance from A to D.

It is easily understandable that the series of cross-sectional shapes within the knot should be reversible at symmetrical centre position D. The filament cross-section is most constrained at two corresponding positions within the knot. When the vertical tensile tests were conducted for ten $3_1$-knotted filaments, the top and bottom side combinations of J- or I-shaped fractured tips were obtained an equal number of times. Namely, breaking at shoulder positions C and E occurred five times each when the results in ten-time tensile tests were compared, indicating the equal probability of filament breaking.

In contrast, the knot appearance became complex with increasing crossing number (figure 1). The two internal filaments entering from the right and left entrances of the knots were twisted around each other with distortion, and were then rounded by the initial ring part. Even for these highly entangled knots, apparent constraint was recognized at the shoulder position, corresponding to the starting position of the rounding part.
Figure 3. Images before and after breaking of 3₁-knotted monofilament. (a) and (b) Photographs of a squeezed knot taken with reversing views. An initially loose knot was tightened and subsequently squeezed before breaking, as illustrated in this photo set. (c) Both sides of fractures obtained after the tensile test. Here, the tightly knotted filament was painted black before the test. (d) Schematic representation of the recoiled part in a knot hypothetically restored from a matching collation of fractured tips on both sides. Letters correspond with each other among these figures.
Figure 4. Position dependence of cross-section for a fractured residual tip of a prior $3_1$ knot. The letters indicate the position corresponding to those in figure 3. (a) Series of cross-section changes for the longer J-shaped tip depicted in figure 3(c). (b) Plots of the long axis of cross-sections for evaluation of the constraint distribution within a knot, including the position assignment indicated by the above corresponding marks.

In the case of the $5_1$ knot, the filament broke at the transparent part (i.e. the internal part within the knot) (figure 5(b)). The colouring pattern obtained when virtually unknotted should have a symmetrical arrangement of each position within the $5_1$ knot, as for the $3_1$ knot depicted in figure 3(d). Therefore, the length of the internal part hidden within the knot (i.e. the length of the transparent part) could be estimated from the pattern of the J-shaped fractured tip ($X$). In contrast, the length of the transparent part for the I-shaped fractured tip ($Y$) corresponded to the length from the knot entrance to the breaking position. As explained above, the $3_1$ knot broke
Figure 5. Comparison of (a) tightened and (b) fractured tips for various torus knots. These knots were painted black under loading of 90% of breaking strength. (c) Length of internal part (X) and that from a knot entrance to a breaking position (Y) as a function of crossing number for various torus knots.
Figure 6. Oblateness of various torus knots at the centre position. The sketches of observed cross-sections are also included. Oblateness was defined as the ratio of the difference between the lengths of the long and short axes to the length of the long axis. The long axis was determined to be the same as that in figure 4. The short axis was defined as the perpendicular bisector of the long axis.

at the shoulder position, which was included in the painted part (figure 3). Therefore, the length from the knot entrance to the breaking position could be represented by the sum of the painted and transparent parts of the I-shaped tip in the case of the $3_1$ knot. Ten trials were performed for the series of torus knots having different crossing numbers. The obtained data of $X$ and $Y$ are summarized in figure 5(c). Some experiment errors did occur, but the apparent features were recognizable. First, the whole length of the transparent part ($X$) continuously increased with increasing the crossing number because the internal part within the knot increases, due to the narrower shape like that of a green caterpillar. In contrast, the length from the knot entrance to the breaking position ($Y$) decreased with increasing crossing number up to the $7_1$ knot. Thus, the breaking position gradually shifted from the shoulder to the entrance of the knot. However, a constant value of $Y$ was obtained beyond the $9_1$ knot. Such trends agreed with that of the tensile breaking of this series of torus knots, suggesting a strong relationship between the breaking strength and the position of the knots.

Why does the breaking position shift toward the knot entrance? As discussed above, the $3_1$ knot broke at the shoulder position, where the highest constraint was observed from the cross-section analysis. Therefore, it was expected that the increasing the crossing number would induce filament deformation of the internal part, where actual breaking occurs for the other knots. However, the experiment result was just the opposite. Figure 6 reveals the changes in oblateness, which were calculated from the filament cross-section obtained at the centre position within the knots. It was found that the constraint at the centre position decreased with increasing crossing number. This result meant that the squeezed deformation within the knot became less pronounced.

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for the knots with larger crossing numbers. The internal filament within the knot broke before full squeezing for these knots.

3.4. Breaking mechanism of knots

Why do the knots break at the internal filament for larger crossing numbers? A reason other than deformation of the internal part should exist. The fractured morphologies of the resultant tips gave us important information about this phenomenon. Figure 7 compares the scanning electron micrographs of the fractured tips of the series of torus knots. For the $3_1$ and $5_1$ knots, the filaments were squashed at the front of the fractured tip (figure 7(a)). In contrast, a different type of fracture morphology was observed for knots having larger crossing numbers (figure 7(b)). In the latter case, the fractured tip was less squeezed but exhibited characteristic patterned morphology. Particularly beyond the $11_1$ knot, only fractured tips having similar characteristic patterns were obtained. One feature of such a fractured tip is radial clefts running from a certain position on the filament surface. This morphology spreads in a fan area. The other fractured area exhibits a smooth surface with granular unevenness. In the cases of $7_1$ and $9_1$ knots, both types of fracture morphology were observed. The squeezed type (a) was predominant for the $7_1$ knot; in contrast, the fan type (b) occurred for the $9_1$ knot when the results of ten breaking tests were compared. Therefore, it was assumed that the fracture morphology transformed from the squeezed type (a) into the fan type (b) with increasing crossing number.

The origin of the fan-type fracture is considered here. As indicated in figure 8, similar morphology was observed when the unknotted filament was rotated and broken by tensile force. Breaking strength gradually decreased with the increasing number of rotations. This phenomenon was coincident with that of the crossing number presented in figure 2. Beyond the critical number, strength achieved a constant value, also similar to that in figure 2. This result meant that a higher crossing number induced the significant rotation of the internal filament within the knots, resulting in breaking before full squeezing.

Such a breaking mechanism of the knot having a higher crossing number is proposed in figure 9. The internal filament was rotated but the outside one was not; thus, the filament began to rotate abruptly as soon as it entered the knot. Stretching such a filament induces a difference of rotation effect at the boundary of the filament rotation (a). The surface region of the filament could be exposed to a greater rotation moment than the core part; thus, the filament would start to break at a certain point on the filament surface (b). The initial clefts would then run in radial directions from this point. If the physical property of the filament was homogeneous within the cross-section, such clefts would run at the same velocity. Therefore, the cleft area would expand like a fan (c). Simultaneously, the residual cross-section of the surviving filament gradually reduced; thus, the residual cross-section would be broken at a certain point, due to the torsional force (d). Such rotated fracturing provided the characteristic morphology with granular unevenness in an area other than the former radial clefts. Indeed, a similar granular morphology could be observed when this filament was broken by a tensile load in liquid nitrogen at $-150^\circ\text{C}$. This result indicated that the torsional deformation in (d) occurred instantaneously at room temperature. The balance between bending deformation and torsion within a given knot determines both breaking position and strength of the knot, depending on the crossing number applied.
| Knot | Description | Figure | Note |
|------|-------------|--------|------|
| $3_1$ | Squeezed type | (a) | N/A |
| $5_1$ | Rotated type | (b) | N/A |
| $7_1$ | | | |
| $9_1$ | | | |
| $11_1$ | Squeezed type | | N/A |
| $13_1$ | Rotated type | | N/A |

**Figure 7.** Tensile fractured morphologies of various torus knots. (a) Squeezed type. (b) Rotated type. Red and blue dotted rectangular areas of a fractured surface of a $13_1$ knot are enlarged in (c) and (d).
In contrast, Pieranski et al [7] reported that the strength of torus knots increases with increasing crossing number for a nylon monofilament. This means that the material used is one of the predominant factors to determine knot breaking behaviour. This discrepancy between our PVDF monofilament and their nylon one might be also attributed to the above-mentioned balance of different deformation modes. The higher resistance for torsion and/or ease of bending could decrease the breaking strength of the knot with the lower crossing number. However, further work is required to evaluate the true effect of the filament material used. We expect that theoretical analysis is another approach to understanding the knot breaking mechanism.

*Figure 8.* Tensile strength of rotated filaments (a) as a function of the rotation number and their fracture morphologies (b) with the corresponding rotation numbers.
4. Conclusions

When the torus series of knots were made in a PVDF monofilament, the tensile strength gradually decreased with increasing crossing number. The two reasons for this phenomenon were squeezing and rotation of the filament. At lower crossing numbers, the knot was significantly squeezed due to the lower rotation of the filament. Therefore, breaking occurred at the shoulder position within the knot, where the highest bending was obtained. When the crossing number was increased, the enhanced filament rotation induced torsion within the knot, resulting in breaking at one of the entrances of the knot before squeezing into the entire knot. These effects of filament squeezing...
and rotation led to a breaking-position shift from inside to outside the knot with increasing crossing number. These results suggest that both squeezing and rotation, which are characteristic for a given knot, dominate the breaking position of the knotted strands.

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