Capturing students’ covariational reasoning levels while solving integrals problem

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Capturing students’ covariational reasoning levels while solving integrals problem

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Abstract. Covariational reasoning has defined as the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other. Level of covariational reasoning will indicate how far students’ comprehension about varying quantities in tandem. This study investigates students’ covariational reasoning level while solving integrals problem. Thirty one undergraduate students were participated to complete written test. Interviews were conducted to reveal the students’ covariational reasoning level while solving covariational problems. The result capture that students were able to construct the relation of dependent variable that changes in tandem with the independent variable, but they only stay in maximum level 3 out of 5 level in covariational reasoning. However, students appeared to have difficulty in applying the concept of integrals. These findings suggest that learning in calculus should place increased emphasis on coordinating images of two quantities changing in tandem and solving daily life problem about the application of integrals concept.

1. Introduction
Covariational reasoning is defined as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" [1]. To indicate how far students’ comprehension about varying quantities in tandem, a students is given a level classification according to the behaviour appears while he or she solve covariational problem. Study [1] have found that there are 5 level of covariational reasoning, and behaviors list to identify the appearance of each level. Several studies [1-4] have shown that covariational reasoning is fundamental for students’ understanding calculus concepts. Calculus concepts can be categorized into 4 areas: function, limits & continuity, the derivative, and the integrals [5].

Many researchers had investigated about covariational reasoning, such as exponential relationships [6, 7], trigonometry [8], rate of change [9, 10], function [1, 9, 10], the fundamental theorem of calculus [11], graphics of functions [1, 12], and differential equations [6]. Since the definite integral concept is one of the main concepts in studying calculus and many researchers [13-16] indicated that students have consistently difficulties in understanding and implementing the concept of definite integral (with specific problems are computational error and misconception in evaluate the proposed integrals), this study investigates students’ covariational reasoning level while solving integrals problem.
2. Experimental method
This study was conducted on the A class mathematics’ education study program of Universitas Islam Negeri Sunan Ampel (UINSA), in academic year 2016 which was chosen by purposively sampling. A class consists of 2 male and 29 female. 31 voluntary students who had studied about definite integrals topic in calculus class were set up to answer three items of the proposed problems relating to an analysis of covarying two quantities.

The instrument consists of three items, and had validated by two senior lecturer of the mathematics’ department. 31 students invited to complete three written problems that involve interpreting and representing how quantities vary in tandem, which were identically designed in context but the problems have different representations (algebraic representation, graphical, and numerical representation). Five of these students (labeling with Student A, B, C, D, and E) were voluntary invited to participate in the interview sessions that concentrated on how students reason during solving covariational problems. The selection of the interview subjects was based on diversity responses on their written test. The analysis focused on students’ use of reasoning covariationally while responses to the written test, which related with five level of covariational reasoning (see table 1).

| Level of Covariational Reasoning | Behaviours |
|----------------------------------|------------|
| Level 1 Coordination             | Labelling the axes with verbal indications of coordinating the two variables (e.g., \( y \) changes with changes in \( x \)). |
| Level 2 Direction                | Constructing an increasing straight line. Verbalizing an awareness of the direction of the change in the output while considering changes in the input. |
| Level 3 Quantitative Coordination| Plotting points/constructing secant lines. Verbalizing an awareness of the variable amount of change of the output while considering changes in the input. |
| Level 4 Average Rate             | Constructing contiguous secant lines for the domain. Verbalizing an awareness of the rate of change of the output (with respect input) while considering uniform increments of the input. |
| Level 5 Instantaneous Rate        | Constructing a smooth curve with clear indications of concavity changes. Verbalizing an awareness of the instantaneous changes in the rate of change for the entire domain of the function (direction of concavities and inflection points are correct). |

Adopted from [1]

3. Results and discussion
The given problem (see figure 1) encouraged students to plot a graph of a dynamic situation with a continuously changing rate. Table 2 presents the kinds of responses that the 31 students provided on the written test. The students were able to construct the relation of dependent variable (output) that change in tandem with the independent variable (input). Here is the detail: Students made horizontal line (labeled with \( t \)) and vertical line (labeled with \( v \)) as axes in Cartesian coordinate to indicate that there are 2 quantities vary in tandem. Based on reference [1], “labelling the axes with verbal indications of coordinating the two variables” means that students in level 1 covariational reasoning. Students constructed increasing and decreasing function that is described by line up and line down in graph. According to reference [1], “constructing an increasing straight line” means that students in level 2 covariational reasoning. They verbally stated that, those line up and line down refers to direction of the change output by input. Students plotted points to coordinate amount of output by input and connecting those plotted points with secant line to illustrate the amount of change of the output by the changes in
input. It was explained in interview sessions. “Plotting points/constructing secant lines” means students in level 3 covariational reasoning [1]. All of students stand in level 3 covariational reasoning that is Quantitative Coordination, it address that they have through level 1 Coordination (awareness of the variables which change) and level 2 Direction (awareness of the direction of the change in the output while considering changes in the input). However, they have diverse graph’s shape which depends on how they calculated the amount of output by input. During the follow up interviews, the five interview subjects provided varied responses.

Student A plotted points and constructed secant lines correctly, but he did not construct contiguous secant lines then produce a smooth curve. When prompted to explain the rational for his graph (see figure 2), he said.

I remember the relationship of distance, velocity, and time. Distance is the area of velocity and time and I used definite integral to find data of changing distance in each hour. Then I constructed $s(t)$ graph in the Cartesian coordinate, time in horizontal axe and distance in vertical axe (Level 1 Coordination). Based on my graphical sketch, I consider that $s(t)$ must contain increasing function at $t=0$ until $t=100$ and decreasing function when $t=100$ until $t=160$, since the sign of distance is negative (Level 2 Direction). By using the definite integral concept, I compute the amount of change in $s(t)$ each hour (Level 3 Quantitative Coordination). Then, I find the exact value of $s(t)$ during 160 hours. But it still perform a set of points, then I made secant line to link the points.

This condition similar with [17], it caused by lacking of checking process. They did not check the amount of output by input besides the given point in problem. Meanwhile it will address in drawing contiguous secant lines to produce smooth curve.

Student B made plotting points and constructed secant lines correctly except increasing function after $t=100$ (see figure 3). Actually this case similar with student A that can’t draw contiguous secant line and produce smooth curve [17]. But she has lack in understanding negative sign in distance, then assuming that distance always positive. When prompted to explain why she constructed that type of graph, she stated as follows.

I just draw graph like what the result of my integral process. Then I see negative sign in distance, I think distance is always positive so I ignore that and continuing drawing my graph.

Student C plotted points and drew secant lines, but she did not use formula $s = v \cdot t$ correctly (see figure 4). She misconception about the relation of $s$, $v$, and $t$ as distance is the area of velocity and time. Moreover, she could not see the problem as application of definite integrals concept in daily life. This condition like in reference [15]. Besides that, she misunderstanding in interpreting negative sign in distance. When prompted to explain the rationale for her graph, she told.

When I read the problem, I remember the formula $s = v \cdot t$. So I used it to find amount of $s(t)$ in each hour. Then I make plotting points and connect each points. And about negative sign in distance, I think the distance always positive (no negative value).

Student D and E have the same case as Student C (misconception in applying formula $s = v \cdot t$), but in different perceptions while interpreting negative sign in distance. Student D used the true value of negative sign in distance (see figure 5) whereas Student E believe that negative sign in distance means no value or zero, so they made all distance in negative value become zero (see figure 6).
Table 2. Data responses of problem.

| Responses types                                      | Number of students out of 31 providing each response type | Note                                      |
|------------------------------------------------------|-----------------------------------------------------------|-------------------------------------------|
| - Plotting points and constructing secant lines       | 1                                                         | Represented by Student A in figure 2      |
| - Applying definite integrals concept accurately      | 1                                                         | Represented by Student B in figure 3      |
| - Good understanding of negative sign in distance     | 1                                                         |                                           |
| - Lack in understanding of negative sign in distance  | 1                                                         |                                           |
| - Misconception in applying formula $s = v \cdot t$   | 6                                                         | Represented by Student C in figure 4      |
| - Assuming that distance always in positive value     | 6                                                         |                                           |
| - Misconception in applying formula $s = v \cdot t$   | 13                                                        | Represented by Student D in figure 5      |
| - Assuming that negative sign in distance as no value or zero | 10                                                        | Represented by Student E in figure 6      |

Figure 1. The covariational problem.
Figure 2. Graph of student A.

Figure 3. Graph of student B.
Figure 4. Graph of student C.

Figure 5. Graph of student D.
Figure 6. Graph of student E.

4. Conclusions
The result captures that students were able to construct the relation of dependent variable that change in tandem with independent variable, but they only stand on level 3 out of 5 level in covariational reasoning. Those are: coordination, direction, quantitative coordination, average rate, and instantaneous rate. However, students appeared to have difficulty in forming image of continuously changing rate and could not accurately applying the concept of integrals. This findings suggest that learning in calculus should place increased emphasis on coordinating image of two quantities changing in tandem about instantaneous rate of change and conceptual knowledge in integral technique with daily life problem. Further studies are needed to investigate how to develop students' covariational reasoning levels in order to make deeper comprehension in organizing the relationship between changing quantities.

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