Comb Type Pilot Based Channel Estimation using Dft Based Interpolation for Spatial Modulated OFDM Systems

Anetha Mary Soman, R Nakkeeran, Shiniu Mathew John

Abstract: An integration of Spatial Modulation with Orthogonal Frequency Division Multiplexing (SM OFDM) is a recently evolved transmission technique. In practical scenarios, channel estimation is significant for detecting transmitted data coherently. Impulse response based interpolation technique that provides channel frequency response estimate with reduction in noise error is proposed for comb type pilot based channel estimation of SM OFDM system along with 1D interpolation techniques under frequency selective channel. This scheme focuses on carrying out smoothing and estimation in time domain and transforming output back to the frequency domain. BER performance is investigated for Rayleigh channel employing COST 207 project model on two test urban environments (Typical and Bad) for 4 and 16 QAM SM OFDM systems. Results show that the Least Square estimator with DFT interpolation performs finer compared to all one dimensional interpolation methods with less computational complexity by employing FFT algorithms.

Keywords: Multiple Input Multiple output (MIMO), Multicarrier modulation, Spatial Modulation, Channel Estimation, Interpolation.

I. INTRODUCTION

High data rates, spectral efficiency, low complexity and flexibility are some of the requirements of upcoming wireless communication systems. MIMO technology has emerged as a favored aspirant in the account of these measures [1]. OFDM, a multicarrier modulation technique, transforms a frequency selective channel to several parallel flat sub channels. Combining OFDM with MIMO technique increases spectral efficiency and improve link reliability [2-5]. Spatial modulation (SM), a space modulation technology is a new low complexity data throughput enhancing technique introduced by Mesleh. R and Haas. H in the year 2006 [6] and remains as a unique single stream MIMO transmission technique. The index of one of the available antennas being active avoids Inter Channel Interference (ICI). SM-OFDM system model, an alternative MIMO technique is analyzed in this paper [7].

Based on SM mapper, multiple information bits are mapped into single information symbol and a corresponding antenna number. Selected transmit antenna involves in transmission and other antennas remain silent for corresponding sub channel and time instant. At the receiver side, for each subcarrier, estimation of symbol and active antenna number takes place and original information bits are demapped. The transmitted bits depends on a constellation diagram and the number of antennas used for transmission. [8]. Ninety percentage complexity reductions are obtained by integrating SM in OFDM.

Perfect channel state information is always impractical and hence estimation of channel is of vital importance. Estimation of channel is an integral part of receiver design. It is a challenging issue in wireless systems. The channel response varies over time because of the movement of transmitters, receivers and scattering objects. Therefore, the estimation of channel is important for SMOFDM systems. For single carrier SM systems, estimation of the channel has been considered in [9] [10]. Pilot based channel estimate where pilots are added along with the symbols carrying information [11] is commonly used to obtain channel state information (CSI) at the receiver. Iterative channel estimation based detection systems are proposed in [12]. Estimation of channel for coherent SM OFDM systems relies on pilot sequence adapted and channel characteristics. Channel estimation based on pilot tones is one among favorable methods for frequency selective channels [13]. Few subcarriers are used in the pilot mode for the initial estimation process. Estimation techniques are based on frequency and time domain techniques. Initially to find the channel estimate on pilot tones, least squares (LS) algorithm is applied in the frequency domain. The frequency response for the known pilot subcarrier is estimated as an equation.

$$\hat{H}_{LS} = \text{diag}(X_1)^{-1} Y_1 + \text{diag}(X_1)^{-1} W_1$$

(1)

$Y_1$ is the received samples, $H$ is channel matrix, $W_i$ is the noise component at each subcarrier for $n^{th}$ OFDM symbol and $X_1$ is the input data. Using interpolation, the estimates on all subcarriers can be found. The interpolation process is denoted as

$$\hat{H} = Q\hat{H}_{LS}$$

(2)

Where $Q$ represents the interpolation matrix. The aim of the channel estimation technique is to acquire $Q$ with less computation and to attain greater accuracy for a given system.

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This paper discusses channel estimation using Discrete Fourier Transform (DFT) in addition to one dimensional interpolation methods. The DFT method can come up with acceptable results by lessening noise on estimated channel coefficients [14]. A classical comb type pilot based frame format used for MIMO is not appropriate for SM-OFDM systems. Since a single antenna at transmit side is active at a time and all other antennas remain silent for each subcarrier,
a new frame structure is put forward for \( N_t \times N_r \) SM OFDM systems [15]. To trace frequency selectivity of the channel, pilot tones are placed precisely in the frequency domain. Interpolation methods are then used to obtain the frequency response of channel at data symbols.

The remaining paper is ordered as follows. Section II explains system model for SM OFDM Systems. Section III includes estimation of channel with description. Section IV presents impulse response based channel estimation. Section V provides performance analysis and Section VI gives the conclusion.

II. SPATIAL MODULATION OFDM SYSTEM MODEL

Let \( Z(k) \) represents \( d \times N \) matrix of binary data to be transmitted. From the matrix, \( d \) represents the entire number of bits/symbol in a subcarrier, \( N \) represents the total number of OFDM subcarriers. Based on SM mapper represented in Fig.1 the data \( Z(k) \) is mapped to a \( N_t \times N \) matrix, \( N_t \) represents the entire number of antennas used for transmission. In the matrix, entire elements of \( X_t(k) \) remain zero except at mapped location of the transmitting antenna. On each OFDM sub channel, the number of bits transmitted is represented as

\[
m = \log_2(N_t) + \log_2 M
\]

(3)

\( M \) represents the modulation degree.

![Figure 1 SM OFDM Mapper](image)

Each row vector of \( X_t(k) \) is dealt by an OFDM modulator where data symbols gets transformed from serial to parallel form. The data symbols in each row are named as subcarrier. If each OFDM symbol consists of \( N \) data symbols, then there are \( N \) number of subcarriers. To transmit known data, pilot insertion is done which aid in channel estimation at receiver. Interpolation methods are then used to obtain the obtained data to estimate the channel by pilot based equalizer.

Where \( N \) represents the number of FFT points. To prevent distortion caused by Inter Symbol Interference (ISI) in channel cyclic prefix (CP) is affixed at starting of each OFDM Symbol. CP is the copy of the end part of the particular OFDM symbol. The resultant OFDM symbol after the insertion of CP is transformed to serial form and gets passed through the channel. For multipath fading environment, the response of channel in time domain between \( i^{th} \) transmitting antenna and \( j^{th} \) receiving antenna is written as

\[
h_{ij}(n) = \sum_{k=0}^{L-1} h_{ij,k}(n - \tau_{ij,k})
\]

(5)

Where \( L \) is the number of paths, \( h_{ij,k} \) is complex time varying channel coefficients and \( \tau_{ij,k} \) is the delay of \( k^{th} \) path.

At receiver, symbols are initially transformed to parallel form. Then CP is removed from symbols. Frequency domain symbols are obtained by passing the OFDM symbols through DFT block and are used to estimate the channel by a pilot based equalizer. Further procedure processes the obtained data to estimate the transmitted OFDM symbols.
Parallel to serial conversion takes place and demapping is done such that the binary data is obtained as transmitted. The OFDM demodulator output for $k^{th}$ subcarrier is written as

$$y(k) = H(k)x(k) + w(k) k = 1,2,\ldots,N$$  \hspace{1cm} (7)

$$x_j(k) = \begin{bmatrix} 0 \ldots x_q(k) \ldots 0 \end{bmatrix}^T.$$  \hspace{1cm} (8)

Where $h_j(k)$ is the channel coefficient between transmit antenna $j$ and receive antenna $r$, $x_q(k)$ is $q^{th}$ active antenna symbol from constellation figure and $w_r(k)$ is complex valued White Gaussian Noise with variance $\sigma_w^2$ and zero mean.

Matrix form representation of signal model is rewritten as

$$[y(k)] = [H(k)x(k)] + [w(k)] k = 1,2,\ldots,N$$  \hspace{1cm} (9)

In spatial modulation the detection process necessitates the finding of the transmitting antennas index and the modulation symbol sent on it. For each OFDM subchannel, Maximum likelihood (ML) based detection is performed. The optimum detection rule is

$$\hat{x}(k) = \arg\max_{\{x_j(k)|0\leq j\leq Q\}} \left| \Re\{\sum_{k=1}^{N} h_j(k)x(k)\} \right|$$  \hspace{1cm} (10)

for $1 \leq j \leq N_t$ , $1 \leq q \leq M$. $x_q(k)$ represents $q^{th}$ active antenna symbols from constellation figure, $h_j(k)$ represents the $j^{th}$ column of $H(k)$.

### III. CHANNEL ESTIMATION

Channel estimation is defined as specifying a mathematically modeled channel. The design benchmark of estimators is to reduce mean square error (MSE) and complexity in computation. Estimation algorithms are used to find channel impulse response (CIR) or channel frequency response (CFR) in the time domain and frequency domain. This paper considers comb type pilot based channel estimation for SM OFDM systems. As an initial step to the estimation process, pilot insertion should be taken care of. Too much pilot insertion cause spectrum overhead. So the pilot spacing should be as small as possible. This makes certain time varying, frequency selective channel to be in good track. Frame structure uses comb type pilot insertion. Pilot tones are set at the periodically located subcarriers of every OFDM symbol. To estimate the channel on frequency axis, interpolation is used.

Let $T_f$ represents period of pilot tones in frequency. To keep trace of channel characteristics, pilot tones must be accommodated as regularly as coherence bandwidth is. Then pilot tone period should satisfy the inequality

$$T_f \leq \frac{1}{\sigma_{max}}$$  \hspace{1cm} (11)

Where $\sigma_{max}$ represents a maximum delay spread.

To estimate the channel accurately, Least Square (LS) Algorithm is used to find known pilot’s frequency response inserted in the transmitted frame. Interpolation is then done to estimate the frequency response of the channel for each subcarrier. For an SM-OFDM system, channel state information is required to make out modulated symbols and transmit antenna indices. Compared with the classical comb type frame format, a new one is designed for SM OFDM systems as single transmit antenna gets activated and the remaining ones transmits null for each subcarrier. Fig.2 represents the frame format. In the figure, pilot tones are placed periodically over subcarriers and for the respective OFDM symbol number, each transmit antenna pass on pilots.
An LS based channel estimator performs LS estimation at pilot positions using the retrieved and known pilot symbols. Interpolation is then performed to obtain channel frequency response at entire positions. Fig.3 shows pilot aided estimator.

For all OFDM symbols, signals received on pilot subcarrier $k_p$ are represented as

$$y_i(k_p) = \rho h_i(k_p) + w_i(k_p), i = 1, 2, ..., P$$  \hspace{1cm} (10)

Where $\rho$ represents the pilot tone, $P$ represents the entire number of transmit antennas.

Channel frequency response (CFR) coefficients obtained by LS estimation are represented as

$$h_i^*(k_p) = y_i(k_p) / \rho$$  \hspace{1cm} (11)

Using the estimates found in pilot positions, estimate at data positions is acquired by applying different interpolation technique.

Rewriting LS estimated CFR by substituting values of $y_i(k_p)$

$$h_i^*(k_p) = h_i(k_p) + v_i(k_p) \text{where } v_i(k_p) = \frac{w_i(k_p)}{\rho}$$  \hspace{1cm} (13)

Some of the prominent interpolation techniques considered in this paper are linear interpolation, spline interpolation and low-pass interpolation and are comparatively considered by Coleri et.al [16]. In [17] Impulse response based channel estimation methods have been studied where the scheme does time domain estimation and transforms the result to frequency domain. Literature [18-20] details the Spline interpolation technique and [21] details the Low pass interpolation technique.
IV. IMPULSE RESPONSE BASED CHANNEL ESTIMATION

The DFT method is considered as an impulse response based channel estimation method which provides better accuracy. It enhances performance of LS estimation by removing the noise effect in time domain outside the extreme channel delay length. The channel frequency response coefficients obtained are first transformed using IDFT. A filter is then applied in the time domain. It is assumed that extreme channel delay is within the OFDM symbol CP. These are then applied with DFT to return to frequency domain.

Mathematically, if \( \hat{H}(k) \) represent estimate of channel gain at \( k^{th} \) subcarrier acquired by LS estimation, then IDFT of the channel estimate \( H'(k)_{k=0}^{N-1} \) is

\[
\text{IDFT} \left\{ \hat{H}(k) \right\} = h(n) + w(n) \Delta h'(n) n = 0,1,\ldots,N-1
\]

(14)

Where \( w(n) \) represent the noise component in time domain. Disregarding coefficients of \( h'(n) \) which contain only noise by defining the coefficients for the maximum channel delay length \( L \), \( \hat{h}_{DFT}(n) \) is written as

\[
\hat{h}_{DFT}(n) = \begin{cases} h(n) + w(n) = 0,1,\ldots,L-1 \\ 0,\ldots,o.w \end{cases}
\]

(15)

Finally, transforming remaining \( L \) elements to frequency domain we get

\[
\hat{H}_{DFT}[k] = DFT \left\{ \hat{h}_{DFT}(n) \right\}
\]

(16)

(L should be known in advance)

The block diagram of channel estimator based on DFT is shown in Fig.4.

![Figure 4 DFT based channel estimator](image)

V. PERFORMANCE ANALYSIS

This section quantifies the bit error rate (BER) performance of 4×4 SM OFDM system under frequency selective channels. System parameters employed for the simulation are specified in Table 1

| Parameters                  | Value                  |
|-----------------------------|------------------------|
| Carrier frequency           | 2GHz                   |
| FFT size                    | 32                     |
| SNR                         | 30dB                   |
| Number of subcarriers       | 256                    |
| Guard Interval              | 4                      |
| Antenna Configuration       | 4×4                    |
| Cyclic prefix               | 32                     |
| Channel property            | TU (Typical Urban) and BU (Bad Urban) channel model based on the COST 207 Project |
| Modulation Scheme           | M-QAM(4,16)            |
| Pilot ratio                 | 1:8                    |
| Bandwidth                   | 1MHz                   |
| Channel length (L)          | 12                     |

Table 1 System Parameters
The following assumptions were made in the simulation

a) Normalized Total transmit power
b) Uncorrelated Data symbols
c) Optimum receiver (ML)
d) 4 OFDM symbols per frame
e) Statistically independent Multipath channels for the different pathways
f) Perfect Time and frequency synchronization
g) Guard interval greater than maximum delay spread.

h) Simulations for different signal to noise ratio(Es/N0)
i) COST 207 project models on two test urban environments (Typical and Bad)

Table 2 shows the Power Delay Profile of two test urban environments (Typical Urban and Bad Urban)[22]

| Urban Environment (Typical) | Urban Environment (Bad) |
|-----------------------------|-------------------------|
| Delay [us] | Fractional power | Doppler category | Delay [us] | Fractional power | Doppler category |
| 0.0 | 0.092 | CLASS | 0.0 | 0.033 | CLASS |
| 0.1 | 0.115 | CLASS | 0.1 | 0.089 | CLASS |
| 0.3 | 0.231 | CLASS | 0.3 | 0.141 | CLASS |
| 0.5 | 0.127 | CLASS | 0.7 | 0.194 | GAUS1 |
| 0.8 | 0.115 | GAUS1 | 1.6 | 0.114 | GAUS1 |
| 1.1 | 0.074 | GAUS1 | 2.2 | 0.052 | GAUS2 |
| 1.3 | 0.046 | GAUS1 | 3.1 | 0.035 | GAUS2 |
| 1.7 | 0.074 | GAUS1 | 5.0 | 0.140 | GAUS2 |
| 2.3 | 0.051 | GAUS2 | 6.0 | 0.136 | GAUS2 |
| 3.1 | 0.032 | GAUS2 | 7.2 | 0.041 | GAUS2 |
| 3.2 | 0.018 | GAUS2 | 8.1 | 0.019 | GAUS2 |
| 5.0 | 0.025 | GAUS2 | 10.0 | 0.006 | GAUS2 |

The BER performance under Typical urban channel model for a 4 QAM SM OFDM system is shown in Fig.5. Low pass estimated channel with DFT came near to that of perfect channel within 2.3 SNR for BER of $10^{-6}$. Error floor happens as SNR increases for linear interpolation. Spline interpolation and low pass interpolation performance remain almost the same. Out of the three interpolation techniques specified, low pass interpolation with DFT performs better when compared to Spline and linear interpolation. In all the three cases, DFT based interpolation performs better with a reduction in noise error.

The BER performance remains almost the same as that of a Typical Urban model and the same conclusion can be drawn.

![Figure 5 BER performance of 4 QAM SM-OFDM under Typical Urban channel model](image)

![Figure 6 BER performance of 4 QAM SM-OFDM under Bad Urban channel model](image)

The BER performance under Bad Urban channel model for a 4 QAM SM OFDM system is shown in Fig.6. The constellation size increases, error floor happens to linear interpolation and low pass interpolation with DFT outperforms when compared to other interpolation techniques. Low pass and Spline with DFT remains robust for frequency selective channels.
VI. CONCLUSION

This paper addressed the channel estimation process of the SM OFDM system. Various details of employing interpolation techniques in the channel estimation process are mainly focused. Results show that the Least Square estimator with DFT based interpolation performs finest of all one dimensional interpolation methods with less computational complexity by employing FFT algorithms. Out of the 1D interpolation techniques considered, Low pass interpolation with DFT outperforms when compared to others. The performance differences among the interpolation techniques vary according to the coherence bandwidth of the channel. It works effectively when channel delays are integer multiples of sampling time. Incorporation of DFT algorithm enhances BER performance by eliminating the noise effect outside the maximum channel delay length.

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Figure 7 BER performance of 16 QAM SM OFDM under Typical Urban channel model

Figure 8 BER performance of 16 QAM SM OFDM under Bad Urban channel model
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