Studying the natural gas market under demand uncertainty using a heterogeneous distributed computing environment

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Abstract. In the paper, we propose a new model of the natural gas market simulation. It takes into account the random nature of the natural gas demand that depends, for example, on temperature fluctuations during the heating period. The natural gas supplier strategy is optimized with the reservation of the necessary amount of fuel to compensate for demand fluctuations. To implement simulation modeling, we developed a scalable application with the DISCOMP framework. We apply DISCOMP to develop and use scalable applications in a heterogeneous distributed computing environment. The application significantly increases the number of model runs and improves the simulation accuracy in comparison with the Mathcad computations on a personal computer.

1. Introduction

Nowadays, there is a lot of work to research the natural gas market. For example, the models GSAM [1] and NEMS [2] are used to study and forecast the North American natural gas market. The models TIGER [3], GASTALE [4], GASMOD [5], and GaMMES [6] analyse the European natural gas market. The models RWGTM [7] and WGM [8] are developed for the study of the global natural gas market. These models are based on different approaches and assumptions. The models GASTALE and WGM utilize the searching for Cournot equilibrium of the natural gas market. The models TIGER, GASMOD, and GSAM study the interconnections between the natural gas production, distribution and selling on the market. The model MAGELAN [9] optimizes the natural gas supply for the long term perspective, including investments into the natural gas production facilities and infrastructure.

However, the high accuracy of modelling results using such models and considering different market uncertainties can only be achieved through high-performance computing (HPC).

We study the relevant problem of regulation of seasonal and random fluctuations of the natural gas consumption under market conditions applying a new simulation model. The optimal working gas value of the underground natural gas storage (UNGS) depends on the demand. Also, the income of the current time period and its mathematical expectation during the subsequent periods affect the UNGS working gas. The mathematical expectation of income is calculated on the base of multiple solutions of the income maximization problem. The different random demands are simulated with the Monte Carlo method. The supplier possibility to change the size of the minimum UNGS working gas value depends on the market price of natural gas. The analysis of the mathematical expectation of the provider’s income defines the optimal value of the UNGS working gas.

We developed the simulation model using the specialized DISCOMP framework. It supports the development and use of scalable applications (distributed applied software packages) in a distributed
computing environment [10]. The simulation model execution in such environment enables us to significantly increase the number of model runs. In addition, the accuracy of the simulation modeling results in comparison with the aforementioned models executed on personal computers is improved.

2. The natural gas demand under uncertainty

There are summer and winter periods of the gas industry operation. Let us denote the summer and winter performance indices as 1 and 2 respectively. The natural gas demand at the $i$-th period is specified as

$$q_i = a_i - b p_i, \quad i = 1, 2,$$

where:

- $q_i$ is the volume of natural gas consumed during the period $i$ (billions of m$^3$),
- $p_i$ is the gas price at the $i$-th period,
- $b$ is the given positive factor of a demand function,
- $a_i$ is a random variable.

The values of $a_i$ are generated according to the truncated normal distribution within intervals $[q_i, \bar{a}_i]$ . $i = 1, 2$. The natural gas production in the summer and winter are denoted as $d_1$ and $d_2$ (billions of m$^3$) respectively.

Smoothing fluctuations of the natural gas demand are carried out by means of UNGS. Usually, the natural gas injection into UNGS is performed in the summer season. The natural gas deliverability from UNGS is performed during the winter season. The working gas value of UNGS at the end of the $i$-th period is a linear function of the price $\hat{p}_i$ which regulates the injection rate:

$$s_i = \begin{cases} \hat{s}, & \hat{p}_i \in [0, \frac{v - \hat{s}}{c}], \\ v - \hat{c}\hat{p}_i, & \hat{p}_i \in (\frac{v - \hat{s}}{c}, v]. \end{cases}$$

where:

- $v$ is the working gas capacity of UNGS in billions of m$^3$,
- $\hat{s}$ is the base gas value of UNGS in billions of m$^3$,
- $\hat{c}$ is the given positive value that characterizes the tendency of a manufacturer to keep the natural gas in UNGS.

3. Determining the optimal demand value and injection volume into UNGS

The natural gas demand at the $i$-th period is the sum of the working gas in UNGS $s_{i-1}$ at the period beginning (value of $s_0$ is known) and the volume of the natural gas production $d_i$, $i = 1, 2$. The total natural gas demand is formed from the gas demand and the value of injection into UNGS. Hence, the equality of the demand to the proposal at the $i$-th period, $i = 1, 2$ is defined as:

$$d_i + s_{i-1} = q_i + s_i.$$ 

The natural gas sales on the market during the period $i$ are denoted as:

$$TR_i = p_i q_i, \quad i = 1, 2.$$ 

A manufacturer estimates the potential benefit from the natural gas injection into UNGS that can be represented in the following form of the fictitious income:

$$TR_i = \hat{p}_i s_i, \quad i = 1, 2.$$ 

The total income for two periods is calculated as:

$$TR = \sum_{i=1}^{2} (TR_i + TR_i).$$

Using the Monte Carlo method, the values of random variables $a_i$, $i = 1, 2$ are calculated. Let $\bar{a}_i$ be some values of these variables. The optimization of natural gas consumption and injection into UNGS values at $i$-th period, $i = 1, 2$ is formulated as the following problem of the income maximization:
The process is repeated. Next, find $c_{\text{opt}} = s_{\text{opt}}$, otherwise $c_{\text{opt}} = c_{\text{cp}}, s_{\text{opt}} = c_{\text{cp}}$. Stop.

$$
p, q_i + \hat{p}, s_i \rightarrow \text{max},$$

\text{s.t. } q_i = \alpha_i - b_i p_i,

$$
s_i = v - c \hat{p}_i,$$

$$
d_i + s_{i+1} = q_i + s_i,$$

$$
p_i \in [0, \frac{\alpha_i}{b_i}], \quad \hat{p}_i \in [\frac{v - \bar{s}}{c}, \frac{v}{c}].$$

(5)

4. The mathematical expectation of income

Let $N$ be the number of tests or sets of values of independent random variables $a_i, i = 1, 2$. The multiple solutions of the problem (1) - (5) with $\alpha_i = a_i^k, i = 1, 2$ results in $N$ income values $TR_i^k, k = 1, 2, \ldots, N$. The mathematical expectation of income is defined as:

$$
MTR = \frac{1}{N} \sum_{k=1}^{N} TR_i^k.
$$

(6)

The maximization of the $MTR$ function by the selected parameters $c$ and $\bar{s}$ from the given segments $[\underline{c}, \bar{c}]$ and $[\underline{s}, \bar{v}]$ with $\underline{s} \leq \bar{s} \leq \bar{v}$ enables us to determine the optimal working gas in UNGS.

5. Algorithm for maximizing the mathematical expectation of income

The method of coordinate descent with the golden section algorithm is chosen to solve the problem of maximizing the $MTR$ function. We propose the following algorithm of the $MTR$ function maximization that includes the following steps:

Step 1. Fix the parameter $\hat{s}$ at a certain level, for example, $\hat{s} = \bar{s}$. Then define the small positive constant $\varepsilon$ as the golden section algorithm accuracy and specify the number $N$ of tests to calculate the $MTR$ function values.

Step 2. Calculate the initial division points $c_1 = \bar{c} - \frac{2(\bar{c} - \underline{c})}{1 + \sqrt{5}}$ and $c_2 = \bar{c} + \frac{2(\bar{c} - \underline{c})}{1 + \sqrt{5}}$. Next, find the $MTR$ function values at these points as $MTR(c_1, \hat{s})$ and $MTR(c_2, \hat{s})$ respectively. If $MTR(c_1, \hat{s}) \leq MTR(c_2, \hat{s})$ then $\underline{c} = c_1, c_1 = c_2, c_2 = \bar{c}$, and $c_2 = \bar{c} - \frac{2(\bar{c} - \underline{c})}{1 + \sqrt{5}}$. The process is repeated until $|\bar{s} - \underline{s}| < \varepsilon$. Calculate $c_{\text{opt}} = \frac{\bar{c} + \underline{c}}{2}$ and go to Step 3.

Step 3. Fix the parameter $c$ value at the level $c_{\text{opt}}$ and calculate the initial division points $s_1 = \bar{s} - \frac{2(\bar{s} - \underline{s})}{1 + \sqrt{5}}$ and $s_2 = \bar{s} + \frac{2(\bar{s} - \underline{s})}{1 + \sqrt{5}}$. Next, find the $MTR$ function values at these points as $MTR(c_{\text{opt}}, s_1)$ and $MTR(c_{\text{opt}}, s_2)$ respectively. If $MTR(c_{\text{opt}}, s_1) \leq MTR(c_{\text{opt}}, s_2)$ then $\bar{s} = s_1, s_1 = s_2, s_2 = \bar{s}$, and $s_1 = \bar{s} - \frac{2(\bar{s} - \underline{s})}{1 + \sqrt{5}}$. The process is repeated until $|\bar{s} - \underline{s}| < \varepsilon$. Calculate $s_{\text{opt}} = \frac{\bar{s} - \underline{s}}{2}$ and go to Step 4.

Step 4. Calculate the $MTR$ function value at $(c_{\text{opt}}, s_{\text{opt}})$. Stop the algorithm.

Thus, the algorithm above determines the $MTR$ function value that is close to the maximum in depend on the given accuracy $\varepsilon$ and the number $N$ of tests.

6. Case study
We consider the following functions for the natural gas demand: \( q_1 = a_1 - 0.2p_1 \) and \( q_2 = a_2 - 0.2p_2 \). It is assumed that the random variables \( a_1 \) and \( a_2 \) belong to the segments \([10, 12]\), \([30, 32]\) respectively. Figure 1 shows the histograms of the random variables frequencies.

![Histograms of random variables frequencies](image)

**Figure 1.** Histograms of the random variables frequencies \( f_1 \) (a) and \( f_2 \) (b).

The value of natural gas production was assumed to be equal to 5 billions of \( m^3 \) (\( d_1 = 5 \)) for the summer and 5 billions of \( m^3 \) (\( d_2 = 6 \)) for the winter. The working gas capacity \( v \) is equal to 3 billions of \( m^3 \). The base gas volume is equal to 1 billions of \( m^3 \) (\( s_0 = 1 \)).

To calculate each value of the \( MTR \) function, we model \( N \) (\( N = 1000 \) and \( N = 10000 \)) sets of the random variables \( a_1 \) and \( a_2 \). These variables are distributed over the segments \([10, 12]\) and \([30, 32]\) respectively in accord to the histograms above. Parameters \( c \) and \( s \) are selected from the segments \([0, 1]\) and \([1, 3]\) respectively. The accuracy \( \varepsilon = 0.01 \) is applied to stop the golden section algorithm. The results of maximizing the \( MTR \) function with the proposed algorithm are shown in table 1.

| The number of tests, \( N \) | The number of iterations | \( MTR \) | \( c_{opt} \) | \( s_{opt} \) |
|-------------------------------|--------------------------|--------|--------------|-------------|
| 1000                          | 22                       | 1013.7201 | 0.0942       | 2.2995      |
| 10000                         | 22                       | 1024.5720 | 0.0435       | 2.0615      |
| 100000                        | 19                       | 1020.0700 | 0.0441       | 1.4506      |
| 1000000                       | 28                       | 1014.7900 | 0.0660       | 2.2755      |

The simulation modeling results for the tests with \( N = 1000 \) and \( N = 10000 \) are obtained on a personal computer using the Mathcad system [11]. Other results for the tests with \( N = 100000 \) and \( N = 1000000 \) are calculated in the heterogeneous distributed computing environment. It includes resources of the HPC-clusters of the public access supercomputer center “Irkutsk Supercomputer Center of SB RAS” [12].

To run the model with various tests we developed a scalable application. The application generates a large number of independent jobs for the heterogeneous distributed computing environment. Each job runs the model with one test. After the execution of all jobs, the application provides the collection of simulation results.

We apply the DISCOMP framework to develop and run the application. The model execution is managed by DISCOMP. Independence of jobs provides high computational scalability. DISCOMP provides the speedup and efficiency of distributed computing on heterogeneous resources close to the linear speedup and efficiency equal to 1 correspondingly [13].

Figure 2, a and Figure 2, b present the results of calculating the \( MTR \) function values obtained with the parameter \( c \) variation in the interval \([0, 1]\) and \( s = 1 \).
The greatest value of the $MTR$ function reaches at $c = 0.05$. Figure 3, a and Figure 3, b show the results of calculating the $MTR$ function values with $c = 0.05$ and $s$ belonging to the segment $[1, v]$.

Figure 3. Dependence of the $MTR$ function on the parameter $s$ for $N=1000$ (a) and $N=10000$ (b).

Figure 3, a and Figure 3, b demonstrate that the parameter $s$ growth causes the increase of the mathematical expectation of income. Since $s = 1.4$, it stabilizes at 1022.

The experimental study (Figure 2 and Figure 3) has shown that the extra number $N$ of runs improves the calculations accuracy. At the same time, the graph of the $MTR$ function is stabilized.

7. Conclusions
In the paper, we propose the model of the natural gas market regulation under demand uncertainty.

The fluctuations of natural gas demand occur largely because of the stochastic and seasonal temperature variations during the heating period. The random demand values are calculated with the Monte Carlo method. The mathematical expectation of income is calculated on the base of multiple computations of the income maximization problem for different random demand values. The analysis of the income mathematical expectation enables us to determine the working gas in UNGS. A natural gas supplier can change the working gas value in UNGS depending on the gas market price.

In addition, we propose the algorithm for maximizing the mathematical expectation of income. It is based on the coordinate descent and golden section methods.

The frequency histograms of the observed random values were used in the case study. The computation results showed that the number of simulation runs significantly increases the calculations accuracy.
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References
[1] Gabriel S A, Manik J and Vikas S 2003 Computational experience with a large-scale, multi-period, spatial equilibrium model of the North American natural gas system. *Netw. Spat. Econ.* 3(2) 97
[2] Gabriel S A, Kydes A S and Whitman P 2001 The national energy modeling system: a large-scale energy-economic equilibrium model. *Oper. Res.* 49(1) 14
[3] Lochner S 2011 Identification of congestion and valuation of transport infrastructures in the European natural gas market *Energy* 36(5) 2483
[4] Boots M G, Rijkers F A and Hobbs B F 2004 Trading in the downstream European gas market: a successive oligopoly approach *Energy J.* 25(3) 73
[5] Holz F, Von Hirschhausen C and Kemfert C 2008 A strategic model of European gas supply (GASMOD) *Energy Econ.* 30(3) 766
[6] Abada I, Gabriel S, Briat V and Massol O 2013 A generalized Nash–Cournot model for the northwestern European natural gas markets with a fuel substitution demand function: The GaMMES model *Netw. Spat. Econ.* 13(1) 1
[7] Hartley P R and Medlock III K B 2009 Potential futures for Russian natural gas exports *Energy J.* 30(Special Issue) 73
[8] Gabriel S A, Kiet S and Zhuang J 2005 A mixed complementarity-based equilibrium model of natural gas markets *Oper. Res.* 53(5) 799
[9] Lochner S and D Bothe D 2009 The development of natural gas supply costs to Europe, the United States and Japan in a globalizing gas market–model-based analysis until 2030 *Energy Policy* 37(4) 1518
[10] Feoktistov A, Kostromin R, Sidorov I and Gorsky S 2018 Development of Distributed Subject-Oriented Applications for Cloud Computing through the Integration of Conceptual and Modular Programming *Proc. 41st Int. Convention on information and communication technology, electronics and microelectronics* ed K Skala (Riejka: IEEE) pp 256–261
[11] Parulekar S. J. 2006 Numerical Problem Solving Using MathCad *Chem. Eng. Ed.* 40(1) 14
[12] Information retrieved from http://hpc.icc.ru
[13] Feoktistov A, Sidorov I, Tchernykh A, Edeleve A, Zorkalzev V, Gorsky S, Kostromin R, Bychkov I, Avetisyan A 2018 Multi-Agent Approach for Dynamic Elasticity of Virtual Machines Provisioning in Heterogeneous Distributed Computing Environment *Proc. Int. Conf. on high performance computing and simulation* ed W W Smari (Orleans: IEEE) pp 909–916