"POWER-INFORMATION-SOCIETY" MODEL

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Summary. The paper deals with the construction and primary analysis of "Power-Information-Society" model, which is a combination of the model of information warfare and the "Power-Society" model. The constructed model has the form of a system from parabolic and two differential equations. The model is studied numerically. Some sociological interpretation is given for the results of the mathematical analysis.

1 INTRODUCTION

The complex of issues related to information warfare, information dissemination in a population, Internet security and its influence on the population attracts increasing attention among researchers of various specialties - both sociologists (see, for example, [1-3]) and mathematicians. The goal of the latter, in the most general terms, is to study these processes on the basis of mathematical modeling methods [4-9]. However, information processes are usually considered in isolation from other processes occurring in a society, while they can be influenced by factors of a very different nature.

On the other hand, other social processes, such as a distribution of power in an hierarchy taking into account the influence of a civil society, can also be influenced by information warfare.

In this paper, two models are combined: the model of information warfare [10] and the "Power-Society" model [4, 11].

2 "POWER-SOCIETY" MODEL

The "Power-Society" model [4, 11] considers interaction between an empowered hierarchy and a civil society. Let the number of authorities be sufficiently large, and the empowered hierarchy is a "continuous environment". The coordinate $x$ characterizes the instance's place in the hierarchy: the larger $x$ is, the lower the instance is. We will assume that $x \in [0;1]$. Let $p(x,t)$ be the level of power of the instance $x$ at the moment $t$. The rate of change of the function $p(x,t)$ is determined by the following factors.

1) The difference between power flows received from the nearest neighbors in hierarchy or given to them. It is assumed here that the greater the difference between values of the current...
power in instances is, the greater the rate of change \( p(x,t) \) is. This factor is described by the term 
\[
\frac{\partial}{\partial x} \left[ \kappa(p, \frac{\partial p}{\partial x}, x, t) \frac{\partial p}{\partial x} \right],
\]
where \( \kappa(p, \frac{\partial p}{\partial x}, x, t) \) is a positive function.

2) The amount of power flows received by an authority from the distant levels of hierarchy or given to them. It is assumed here that the greater the difference between the values of current power in instances is, the greater the rate of change \( p(x,t) \) is. This factor is described by the term

\[
\int_0^1 \chi[p(x'), p(x), x', x] [p(x', t) - p(x, t)] dx'.
\]

3) The reaction of the civil society, described by a function \( F(x, p(x,t)) \).

In this paper we consider a simplified case of the "Power-society" system, which also takes into account the following assumptions:

1) a function \( \kappa \) corresponding to the mechanism of power transfer by command is constant. Let us denote by \( \varepsilon \) this constant function, and we assume that it is sufficiently small.

2) there is no mechanism for transmitting commands over one's head;

3) reaction of the population is a function of deviation from the ideal level of power. By analogy with [12], we consider the case of the existence of two stable levels of power \( \phi_1(x) \) \( \phi_2(x) \), and each of them is optimal. Owing to the smoothness of the function \( F(p, x) \), the degenerate stationary equation \( F(p, x) = 0 \) has a root \( \phi_2(x) \). So, \( F(p, x) = -k_1(x)(p - \phi_1(x))(p - \phi_2(x))(p - \phi(x)). \)

Given the absence of power flows across the boundaries of the hierarchy, we obtain the final equation:

\[
\frac{\partial p}{\partial t} = \frac{\partial}{\partial x} \left[ \kappa(p, \frac{\partial p}{\partial x}, x, t) \frac{\partial p}{\partial x} \right] - k_1(x)(p - \phi_1(x))(p - \phi_2(x))(p - \phi(x)),
\]

(1)

\[
\left. \frac{\partial p}{\partial x} \right|_{x=0} = \left. \frac{\partial p}{\partial x} \right|_{x=1} = 0.
\]

(2)

3 THE MODEL OF INFORMATION WARFARE

The model of information warfare assumes that there are two sources of information in a population. They are antagonistic in the sense that an individual that has become a spreader of one "party" is at the same time insusceptible to the information from the other party. Let \( N_0 \) be the size of the society. In the framework of the model under consideration (see [10]), it is assumed that an individual who is not covered by information can receive it either from the media or through interpersonal communication. Let \( X(t) \) be the number of spreaders from the first party and \( Y(t) \) be the number of spreaders from the second party. Parameters \( a_i, i=1,2 \), and \( \beta_i, i=1,2 \), characterize the intensity of the dissemination through the media and interpersonal communication for particular party, respectively. Note that the intensity of
information dissemination through interpersonal communication is also proportional to the number of individuals already covered by this source of information. It is assumed also that the rate of information dissemination (that is, the number of individuals involved per unit of time) is composed of the information dissemination rates of each of the above methods. The rate is also proportional to the number of uncovered individuals, that is, \( N_0 - X(t) - Y(t) \). It is assumed here that any of still uncovered individuals are always able to receive the piece of information being disseminated and to perceive it.

Thus, the basic model of information warfare [8] has the following form:

\[
\frac{dX}{dt} = (\alpha_1 + \beta_1 X)(N_0 - X - Y); \\
\frac{dY}{dt} = (\alpha_2 + \beta_2 Y)(N_0 - X - Y). 
\]

### 4 "POWER-INFORMATION-SOCIETY" MODEL

Let us now combine the two models described above. Let \( X \) be the number of spreaders of an empowered party, \( Y \) be the number of opposition spreaders. Suppose that the greater the superiority (or defeat) in the information warfare is, the greater the rate of increasing (or decreasing) the amount of power is; that is, the term \( s(X - Y) \) is added to the right-hand side of equation (1).

Let further the intensity of information dissemination through the media for each of the opponents depends on the total amount of power \( P = \int_0^1 p(x,t) \, dx \), that is \( \alpha_1 = \alpha_1(P), \alpha_2 = \alpha_2(P) \), where \( \alpha_1(P) \) is an increasing function, \( \alpha_2(P) \) is a decreasing one.

Thus, the "Power-Information-Society" model is as follows:

\[
\frac{\partial p}{\partial t} = \varepsilon^2 \frac{\partial^2 p}{\partial x^2} - k_1(x)(p - \phi_1(x))(p - \phi_2(x))(p - \phi_3(x)) + s(X - Y) 
\]

\[
\frac{\partial p}{\partial x} \bigg|_{x=0} = \frac{\partial p}{\partial x} \bigg|_{x=1} = 0
\]

\[
\frac{dX}{dt} = \left[ \alpha_1(P) + \beta_1 X \right](N - X - Y) 
\]

\[
\frac{dY}{dt} = \left[ \alpha_2(P) + \beta_2 Y \right](N - X - Y)
\]

We assume that \( \varepsilon << 1 \), so the stationary authority profiles are the roots of the polynomial \( -k_1(x)(p - \phi_1(x))(p - \phi_2(x))(p - \phi_3(x)) + s(X - Y) \) provided that \( X \) and \( Y \) are the established numbers of spreaders. Depending on the parameters of the system, the polynomial has from 1 to 3 roots, let us denote them \( \psi_1(x), \psi_2(x) \) and \( \psi_3(x) \). If \( \psi_1(x) < \psi_2(x) < \psi_3(x) \) then \( \psi_1(x) \) and \( \psi_3(x) \) are stable, \( \psi_2(x) \) is an unstable one. At the same time \( \psi_1(x) \) is a participatory authority profile [12] (in this case the society is more...
democratic, and the total amount of power is less), $\psi_3(x)$ is a profile of a strong hand [12]. Functions $\varphi_1(x)$ and $\varphi_2(x)$ have the meaning of an optimal levels of power in the absence of influence of information warfare on the distribution of power in the hierarchy.

5 NUMERICAL EXPERIMENTS AND SOME SCENARIOS OF WARFARE

One of the standard ways to analyze mathematical models is to solve them numerically [13-15]. So in this section we provide a series of computational experiments.

Let us take the following initial conditions to carry out a numerical experiment:

$$X(0) = Y(0) = 0 \quad (8)$$

**Scenario 1.**

The first scenario characterizes the case when the information warfare has little influence on amount of power, i.e. $s << 1$. In this case, the polynomial

$$-k_1(x)(p-\phi_1(x))(p-\phi_2(x))(p-\phi_3(x)) + s(X-Y)$$

always has three roots, and the final authority profile depends on initial conditions: if initially there are high level of power, then authority profile is "strong hand", and if initially there is low level of power, then authority profile is participatory.

Let us take the following parameters: $\varepsilon = 0.05$; $b_1 = 0.096$; $b_2 = 0.1$; $N = 100$; $s = 0.01$; $\alpha_1(P) = 0.01(10 + P)$; $\alpha_2(P) = 0.01(10 - P)$; $\phi_1(x) = 2 - 1.5x$; $\phi_2(x) = 6 - 3.75x$; $\phi_3(x) = 8 - 5.25x$.

Let us take the function $p(x,0) = 7 - 5x$ as the initial level of power to illustrate the first case (strong hand authority profile).

The results of modeling for the number of spreaders are below (Figure 1).

Fig. 2 represents authority profile at different moments of time and the roots of the polynomial

$$-k_1(x)(p-\phi_1(x))(p-\phi_2(x))(p-\phi_3(x)) + s(X-Y).$$

**Scenario 2.**

We take the function $p(x,0) = 5 - 5x$ as initial conditions to illustrate the second case (the establishment of the participatory authority profile). The results of the simulation are shown in Fig. 3 and Fig. 4.
Fig. 1. Red line: the number of followers of the empowered party \( X(t) \), blue line: the number of followers of the opposition \( Y(t) \)

Figure 2. Red line: the function \( p(x,t) \), blue lines: the roots of the polynomial \(-k(x)(p-\phi(x))(p-\phi_2(x))(p-\phi_3(x)) + s(X-Y)\)
Figure 3. Red line: the number of followers of the empowered party $X(t)$, blue line: the number of followers the opposition $Y(t)$.

Figure 4. Red line: the function $p(x,t)$, blue lines: the roots of the polynomial $-k_1(x)(p - \phi_1(x))(p - \phi_2(x))(p - \phi_3(x)) + s(X - Y)$. 
Scenario 3.

If information warfare strongly influences the level of power, two roots of the polynomial 
\(-k_1(x)(p-\phi_1(x))(p-\phi_2(x))(p-\phi_3(x)) + s(X-Y)\) degenerate at some time, as a result, only one stable stationary distribution of power remains for any initial conditions. If the empowered party wins in information warfare, the remaining root corresponds to strong hand authority profile, if the opposition wins, then the participatory authority profile is observed.

Let us consider the case when empowered party wins (Fig. 5). Let us take the following parameters and the initial level of power: 
\[ \varepsilon = 0.05; b_1 = 0.09; b_2 = 0.1; \ N = 100; \ s = 2; \]
\[ \alpha_1(P) = 0.02(10 + P); \ \alpha_2(P) = 0.02(10 - P); \ \phi_1(x) = 2 - 1.5x; \ \phi_2(x) = 6 - 3.75x; \]
\[ \phi_3(x) = 8 - 5.25x, \ p(x,0) = 4 - 4x. \] The simulation results are shown in Fig. 5,6.

![Graph](image)

**Fig. 5.** Red line: the number of followers of the empowered party \(X(t)\), blue line: the number of followers of the opposition \(Y(t)\)

Note that in this case at some points (for example, \(t = 0.7\)), some (not the highest) instance has the maximum level of powers throughout the hierarchy. This is the case of the confederative authority profile [11].

Scenario 4.

Next consider the case when the opposition wins (Fig. 7). Let us take the following parameters and the initial authority profile: 
\[ \varepsilon = 0.05; b_1 = 0.053; b_2 = 0.1; \ N = 100; \ s = 1; \]
\[ \alpha_1(P) = 0.02(4 + P); \ \alpha_2(P) = 0.02(4 - P); \ \phi_1(x) = 2 - 1.5x; \ \phi_2(x) = 6 - 3.75x; \]
\[ \phi_3(x) = 8 - 5.25x, \ p(x,0) = 4 - 4x. \] The simulation results are shown in Fig. 7,8.
Fig. 6. Red line: the function \( p(x,t) \), blue lines: the roots of the polynomial
\[
-k_1(x) \left( p - \phi_1(x) \right) \left( p - \phi_2(x) \right) \left( p - \phi_3(x) \right) + s \left( X - Y \right)
\]

Fig. 7. Red line: the number of followers of the empowered party \( X(t) \), blue line: the number of followers of the opposition \( Y(t) \)
Fig. 8. Red line: the function $p(x,t)$, blue lines: the roots of the polynomial $-k_1(x)(p-\phi_1(x))(p-\phi_2(x))(p-\phi_3(x))+s(X-Y)$

A confederative authority profile is also possible in this case if the opposition has a rather weak propaganda, and the number of supporters of the empowered party is greater at initial time. The stronger this advantage is (the farther the point of intersection of the graphs of the functions $X(t)$ and $Y(t)$ from the origin is), the stronger the authority profile is attracted to
the strong hand authority profile at those moments when the advantage of the empowered party in information warfare still exists.

6 CONCLUSIONS

The "Power-Information-Society" model was constructed, combining the model of information warfare and the "Power-Society" model. Various scenarios were considered. It is established that depending on initial conditions both participatory and strong hand authority profile can exist in the case when information warfare has little effect on the level of power. In the case of strong influence of information warfare, regardless of the initial conditions, only one stationary authority profile is possible: it is the strong hand authority profile when the empower party wins and participatory level of power when the opposition wins. At the same time, confederative authority profile (some (not the highest) instance has the maximum powers throughout the hierarchy) is possible at some moments.

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