Abstract

High luminosity muon-muon colliders would provide a powerful new probe of Higgs boson physics through $s$-channel resonance production. We discuss the prospects for detection of Higgs bosons and precision measurements of their masses and widths at such a machine.
The feasibility of constructing high luminosity muon-muon colliders is currently under investigation \cite{1,2} and a first overview of the phenomenology has been given \cite{3}. The fact that the muon is 200 times more massive than the electron makes such colliders very attractive for both practical and theoretical reasons:

(i) synchrotron radiation does not limit their circular acceleration and multi-TeV energies can be realized;

(ii) the beam energy resolution is not limited by beamstrahlung smearing;

(iii) the $s$-channel production of Higgs boson resonances ($\mu^+\mu^- \rightarrow h$) would make possible precision studies of the Higgs sector.

If electroweak symmetry breaking is realized via a scalar field Higgs sector, then one of the primary goals of future colliders must be to completely delineate the Higgs spectrum and measure the Higgs masses, widths and couplings. In this Letter we present a quantitative study of the merits of $s$-channel Higgs production at a $\mu^+\mu^-$ collider with excellent beam energy resolution.

Two specific muon collider schemes are under consideration. A high energy machine with 4 TeV center-of-mass energy ($\sqrt{s}$) and luminosity of order $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ \cite{4} would have an energy reach appropriate for pair production of heavy supersymmetric particles \cite{5} or, in the absence of Higgs bosons, the study of strong scattering of longitudinally polarized $W$ bosons \cite{3,5,6}. A lower energy machine, hereafter called the First Muon Collider (FMC), could have c.m. energy around 0.5 TeV with a luminosity of order $2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ \cite{4} for unpolarized beams. It is the latter machine that may be most directly relevant to the $s$-channel Higgs process. The most costly component of a muon collider is the muon source (decays of pions produced by proton collisions) and the muon storage rings would comprise a modest fraction of the overall cost \cite{7}. In order that full luminosity be maintained at all c.m. energies where Higgs bosons are either observed or expected, it is thus envisioned that multiple storage rings could eventually be constructed with c.m. energies spanning the
desired range.

For s-channel studies of narrow resonances, the energy resolution is an important consideration. A Gaussian shape for the energy spectrum of each beam is expected to be a good approximation, with an rms deviation most naturally in the range $R = 0.04\%$ to $0.08\%$ [8]. By additional cooling or chromaticity corrections, this can either be decreased to $R = 0.01\%$ or increased to $R = 1\%$, respectively. The corresponding rms error $\sigma$ in $\sqrt{s}$ is given by

$$\sigma = (0.04 \text{ GeV}) \left( \frac{R}{0.06\%} \right) \left( \frac{\sqrt{s}}{100 \text{ GeV}} \right). \quad (1)$$

The critical issue is how this resolution compares to the calculated total widths of Higgs bosons. Widths for the Standard Model Higgs $h_{SM}$ and the three neutral Higgs bosons $h^0$, $H^0$, $A^0$ of the Minimal Supersymmetric Standard Model (MSSM) are illustrated in Fig. [9]; for the MSSM Higgs bosons, results at $\tan \beta = 5$ and 20 are shown. For $R \lesssim 0.06\%$, the energy resolution in Eq. (1) can be smaller than the Higgs widths in many cases; the total luminosity required for Higgs discovery by energy scanning is also minimized by employing the smallest possible $R$.

The $s$-channel Higgs resonance cross section is

$$\sigma_h = \frac{4\pi \Gamma(h \rightarrow \mu\mu) \Gamma(h \rightarrow X)}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2}, \quad (2)$$

where $h$ denotes a generic Higgs boson which decays to a final state $X$. The effective cross section $\overline{\sigma}_h$ is obtained by convoluting with the Gaussian distribution in $\sqrt{s}$:

$$\overline{\sigma}_h = \int \sigma_h(s') \exp \left[ -\frac{(-\sqrt{s'} - \sqrt{s})^2}{2\sigma^2} \right] \frac{1}{\sqrt{2\pi}\sigma} d\sqrt{s'} \quad (3)$$

For $\sigma \gg \Gamma_h$, $\overline{\sigma}_h$ at $\sqrt{s} = m_h$ is given by

$$\overline{\sigma}_h = \frac{\pi \Gamma_h}{2\sqrt{2\pi}\sigma} \sigma_h(\sqrt{s} = m_h) \quad (4)$$

and for $\Gamma_h \gg \sigma$

$$\overline{\sigma}_h = \sigma_h(\sqrt{s} = m_h). \quad (5)$$
Since the backgrounds vary slowly over the expected energy resolution interval \( \sigma_B = \sigma_B \).

In terms of the integrated luminosity \( L \), total event rates are given by \( L \sigma \); roughly \( L = 20 \text{ fb}^{-1}/\text{yr} \) is expected for the FMC. Predictions for \( \sigma_{h_{SM}} \) for inclusive SM Higgs production are given in Fig. 2 versus \( \sqrt{s} = m_{h_{SM}} \) for resolutions of \( R = 0.01\%, 0.06\% \) and 0.1\%. For comparison, the \( \mu^+\mu^- \rightarrow Z^* \rightarrow Z h_{SM} \) cross section is also shown, evaluated at the value \( \sqrt{s} = m_Z + \sqrt{2} m_{h_{SM}} \) for which it is a maximum.

**SM Higgs Boson**

The optimal strategy for SM Higgs discovery at a lepton collider is to use the \( \mu^+\mu^- \rightarrow Z h \) mode (or \( e^+e^- \rightarrow Z h \)) because no energy scan is needed. Studies of \( e^+e^- \) collider capabilities indicate that the SM Higgs can be discovered if \( m_{h_{SM}} < 0.7 \sqrt{s} \) and its mass determined to a precision [9]

\[
\Delta m_{h_{SM}} \simeq 0.4 \text{ GeV} \left( \frac{20 \text{ fb}^{-1}}{L} \right)^{1/2} \tag{6}
\]

if \( m_{h_{SM}} \lesssim 140 \text{ GeV} \). At the LHC the \( h_{SM} \rightarrow \gamma\gamma \) mode is deemed viable for \( 80 \lesssim m_{h_{SM}} \lesssim 150 \text{ GeV} \), with a 1\% mass resolution [10]. Once the \( h_{SM} \) signal is found, the measurement of its width becomes the paramount issue, and it is this task for which s-channel resonance production at a \( \mu^+\mu^- \) collider is uniquely suited.

For \( m_{h_{SM}} < 2m_W \) the dominant \( h_{SM} \)-decay channels are \( b\bar{b} \), \( WW^* \), and \( ZZ^* \), where the star denotes a virtual weak boson. The light quark backgrounds to the \( b\bar{b} \) signal can be rejected by \( b \)-tagging. For the \( WW^* \) and \( ZZ^* \) channels we employ only the mixed leptonic/hadronic modes (\( \ell\nu j \) for \( WW^* \) and \( 2\ell 2j \), \( 2\nu 2j \) for \( ZZ^* \), where \( \ell = e \) or \( \mu \) and \( j \) denotes a quark jet), and the visible purely-leptonic \( ZZ^* \) modes (4\( \ell \) and 2\( \ell 2\nu \)), taking into account the major electroweak QCD backgrounds. For all channels we assume a general signal and background identification efficiency of \( \epsilon = 50\% \), after selected acceptance cuts [11]. In the case of the \( b\bar{b} \) channel, this is to include the efficiency for tagging at least one \( b \). The signal and background channel cross sections \( \epsilon \sigma BF(X) \) at \( \sqrt{s} = m_{h_{SM}} \) for \( X = b\bar{b}, WW^* \) and \( ZZ^* \) are presented in Fig. 3 versus \( m_{h_{SM}} \) for a resolution \( R = 0.06\%; \)
\( BF(X) \) includes the Higgs decay branching ratios for the signal, and the branching ratios for the \( W, W^* \) and \( Z, Z^* \) decays in the \( ZZ^* \) and \( WW^* \) final states for both the signal and the background. Also shown for each channel is the statistical significance of the signal, \( n_\sigma = S / \sqrt{B} \), where \( S \) and \( B \) are the signal and background rates, \( L \sigma BF(X) \); an integrated luminosity of \( L = 1 \text{ fb}^{-1} \) is assumed. A detectable \( s \)-channel Higgs signal is realizable for all \( m_{h_{SM}} \) values between the current LEP I limit and \( 2m_W \) except in the region of the \( Z \) peak; a luminosity \( L \sim 10 \text{ fb}^{-1} \) at \( \sqrt{s} = m_{h_{SM}} \) is needed for \( 85 \lesssim m_{h_{SM}} \lesssim 100 \text{ GeV} \).

With \( L = 30 \text{ fb}^{-1} \) devoted to an energy scan around \( \sqrt{s} = m_{h_{SM}} \), the total Higgs width could be determined with reasonable accuracy. For example, for \( m_{h_{SM}} \sim 120 \text{ GeV} \) the error \( \Delta \Gamma_{h_{SM}} \) would be of order 0.002 GeV (0.008 GeV) for \( R = 0.01\% \) (\( R = 0.06\% \)). In addition, the event rate in a given channel measures \( \Gamma(h_{SM} \rightarrow \mu^+\mu^-) \times BF(h_{SM} \rightarrow X) \). Then, using the measured branching fractions, the \( h_{SM} \rightarrow \mu\mu \) partial width can be determined, providing an important test of the Higgs coupling.

**MSSM Higgs Bosons**

The MSSM has three neutral Higgs bosons \( h^0 \) (CP-even), \( H^0 \) (CP-even), and \( A^0 \) (CP-odd). There is a theoretical upper bound on the mass of the lightest state \( h^0 \) of \( m_{h^0} \lesssim 130 \) to 150 GeV \[12,13\]. If \( m_{A^0} \gtrsim 2m_Z \) (typical of grand unified models), the couplings are approximately \[14\]

\[
\begin{align*}
\mu^+\mu^-, b\bar{b} & \quad t\bar{t} & \quad ZZ, W^+W^- \\
\tau^-\tau^+ & \quad 1 \quad -1 \quad 1 \\
\tau^-\tau^+ & \quad \tan \beta \quad -1/\tan \beta \quad 0 \\
\tau^-\tau^+ & \quad -i\gamma_5 \tan \beta \quad -i\gamma_5/\tan \beta \quad 0 \\
\end{align*}
\] (7)

\( m_{A^0} \lesssim m_Z \), then \( H^0 \) is SM-like, while \( h^0 \) has couplings like those for \( H^0 \) above.

A \( \mu^+\mu^- \) collider provides two particularly unique probes of the MSSM Higgs sector. First, the couplings of the SM-like MSSM Higgs boson deviate sufficiently from exact SM
Higgs couplings that it may well be distinguishable from the $h_{SM}$ by measurements of $\Gamma_h$ and $\Gamma(h \rightarrow \mu^+\mu^-)$ at a $\mu^+\mu^-$ collider, using the $s$-channel resonance process. For instance, in the $b\bar{b}$ channel $\Gamma_h$ and $\Gamma(h \rightarrow \mu^+\mu^-) \times BF(h \rightarrow b\bar{b})$ can be measured with sufficient accuracy so as to distinguish the $h^0$ from the $h_{SM}$ for $m_{A^0}$ values as high as 500 GeV.

The second dramatic advantage of a $\mu^+\mu^-$ collider in MSSM Higgs physics is the ability to study the non-SM-like Higgs bosons, e.g. for $m_{A^0} \gtrsim 2m_Z$ the $H^0, A^0$. An $e^+e^-$ collider can only study these states via $Z^* \rightarrow A^0 H^0$ production, which could easily be kinematically disallowed since GUT scenarios typically have $m_{A^0} \sim m_{H^0} \gtrsim 200-250$ GeV. In $s$-channel production the $H^0, A^0$ can be even more easily observable than a SM-like Higgs. This is because the partial widths $\Gamma(H^0, A^0 \rightarrow \mu^+\mu^-)$ grow rapidly with increasing $\tan \beta$, implying [see Eqs. (4) and (5)] that $\sigma_{H^0, A^0}$ will become strongly enhanced relative to SM-like values. $BF(H^0, A^0 \rightarrow b\bar{b})$ is also enhanced at large $\tan \beta$, implying an increasingly large rate in the $b\bar{b}$ final state. Thus, we concentrate here on the $b\bar{b}$ final states of $H^0, A^0$ although the modes $H^0, A^0 \rightarrow t\bar{t}, H^0 \rightarrow h^0h^0, A^0 A^0$ and $A^0 \rightarrow Z h^0$ can also be useful.

Despite the enhanced $b\bar{b}$ partial widths, the suppressed (absent) coupling of the $H^0 (A^0)$ to $WW$ and $ZZ$ means that, unlike the SM Higgs boson, the $H^0$ and $A^0$ remain relatively narrow at high mass, with widths $\Gamma_{H^0}, \Gamma_{A^0} \sim 0.1$ to 3 GeV. Since these widths are generally comparable to or broader than the expected $\sqrt{s}$ resolution for $R = 0.06\%$ and $\sqrt{s} \gtrsim 200$ GeV, measurements of these Higgs widths could be straightforward with a scan over several $\sqrt{s}$ settings, provided that the signal rates are sufficiently high. The results of a fine scan can be combined to get a coarse scan appropriate for broader widths.

The cross section for $\mu^+\mu^- \rightarrow A^0 \rightarrow b\bar{b}$ production with $\tan \beta = 2, 5$ and 20 (including an approximate cut and $b$-tagging efficiency of 50%) is shown versus $m_{A^0}$ in Fig. 4 for beam resolution $R = 0.06\%$. Also shown is the significance of the $b\bar{b}$ signal for delivered luminosity $L = 0.1$ fb$^{-1}$ at $\sqrt{s} = m_{A^0}$. Discovery of the $A^0$ and $H^0$ will require an energy scan if $Z^* \rightarrow H^0 + A^0$ is kinematically forbidden; a luminosity of 20 fb$^{-1}$ would allow a scan over 200 GeV at intervals of 1 GeV with $L = 0.1$ fb$^{-1}$ per point. The $b\bar{b}$ mode would yield a $5\sigma$ signal at $\sqrt{s} = m_{A^0}$ for $\tan \beta \gtrsim 2$ for $m_{A^0} \lesssim 2m_t$ and for $\tan \beta \gtrsim 5$ for all $m_{A^0}$.
For $m_{A^0} \gtrsim m_Z$ ($m_{A^0} \lesssim m_Z$), the $H^0$ ($h^0$) has very similar couplings to those of the $A^0$ and would also be observable in the $b\bar{b}$ mode down to similar $\tan\beta$ values. Discovery of both the $H^0$ and $A^0$ MSSM Higgs bosons would be possible over a large part of the $m_{A^0} \gtrsim m_Z$ MSSM parameter space.

In summary, $\mu^+\mu^-$ colliders offer significant new opportunities for probing the Higgs sector. The $s$-channel resonance production process is especially valuable for precision Higgs mass measurements, Higgs width measurements, and the search for Higgs bosons with negligible $hZZ$ couplings, such as the $H^0$, $A^0$ Higgs bosons of the MSSM. The techniques discussed here in the SM and MSSM theories are generally applicable to searches for any Higgs boson or other scalar particle that couples to $\mu^+\mu^-$. 

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FIGURES

1. Total width versus mass of the SM and MSSM Higgs bosons, for $\tan \beta = 5$ and 20 in the MSSM.

2. Cross sections versus $m_{h_{SM}}$ for inclusive SM Higgs production: (i) the $s$-channel $\sigma_h$ [Eq. (3)] for $\mu^+\mu^- \to h_{SM}$ with $R = 0.01\%$, 0.06\% and 0.1\% and (ii) $\sigma(\mu^+\mu^- \to Zh_{SM})$ at $\sqrt{s} = m_Z + \sqrt{2}m_{h_{SM}}$. 
3. The (a) $h_{SM}$ signal and (b) background cross sections, $\epsilon\sigma_{BF}(X)$, for $X = b\bar{b}$, and useful $WW^*$ and $ZZ^*$ final states (including a channel-isolation efficiency of $\epsilon = 0.5$) versus $m_{h_{SM}}$ for SM Higgs $s$-channel production with resolution $R = 0.06\%$. Also shown: (c) the statistical significance $S/\sqrt{B}$ for the three channels for $L = 1\, \text{fb}^{-1}$.

4. (a) The effective $b\bar{b}$-channel cross section, $\epsilon\sigma_{A^0}BF(A^0 \rightarrow b\bar{b})$, for $s$-channel production of the MSSM Higgs boson $A^0$ versus $\sqrt{s} = m_{A^0}$, for $\tan\beta = 2, 5$ and 20, beam resolution $R = 0.06\%$ and channel isolation efficiency $\epsilon = 0.5$; and (b) corresponding statistical significance of the $A^0 \rightarrow b\bar{b}$ signal for $L = 0.1\, \text{fb}^{-1}$ delivered at $\sqrt{s} = m_{A^0}$. 
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FIG. 4. (a) The effective $b\bar{b}$-channel cross section, $c\sigma_{A^0}BF(A^0 \rightarrow b\bar{b})$, for $s$-channel production of the MSSM Higgs boson $A^0$ versus $\sqrt{s} = m_{A^0}$, for $\tan \beta = 2, 5$ and 20, beam resolution $R = 0.06\%$ and channel isolation efficiency $\epsilon = 0.5$; and (b) corresponding statistical significance of the $A^0 \rightarrow b\bar{b}$ signal for $L = 0.1\text{ fb}^{-1}$ delivered at $\sqrt{s} = m_{A^0}$. 

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