Superconducting Junctions with Ferromagnetic, Antiferromagnetic or Charge-Density-Wave Interlayers

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Abstract. Spectra and spin structures of Andreev interface states and the Josephson current are investigated theoretically in junctions between clean superconductors (SC) with ordered interlayers. The Josephson current through the ferromagnet-insulator-ferromagnet interlayer can exhibit a nonmonotonic dependence on the misorientation angle. The characteristic behavior takes place if the \( \pi \) state is the equilibrium state of the junction in the particular case of parallel magnetizations. We find a novel channel of quasiparticle reflection (\( Q \) reflection) from the simplest two-sublattice antiferromagnet (AF) on a bipartite lattice. As a combined effect of Andreev and \( Q \) reflections, Andreev states arise at the AF/SC interface. When the \( Q \) reflection dominates the specular one, Andreev bound states have almost zero energy on AF/\( s \)-wave SC interfaces, whereas they lie near the edge of the continuous spectrum for AF/d-wave SC boundaries. For an \( s \)-wave SC/AF/\( s \)-wave SC junction, the bound states are found to split and carry the supercurrent. Our analytical results are based on a novel quasiclassical approach, which applies to interfaces involving itinerant antiferromagnets. Similar effects can take place on interfaces of superconductors with charge density wave materials (CDW), including the possible d-density wave state (DDW) of the cuprates.

Superconducting heterostructures involving ferro- and/or antiferromagnets manifest unusual properties associated with spin effects, and are of both fundamental interest and important for technological applications. Superconductor-ferromagnet-superconductor (SC/F/SC) junctions are known to display \( 0 - \pi \) transitions with varying the temperature or the interlayer width. We have demonstrated theoretically that the \( 0 - \pi \) transition can show up also at fixed temperature and interlayer width in superconducting junctions with a three-layer FIF interface, as a function of the misorientation angle between the magnetizations of two F layers separated by the insulating barrier [1]. The dependence of the Josephson current on the misorientation angle \( \phi \) becomes especially simple in the tunneling limit, when the critical current takes the form

\[
J_c(T, \phi) = J_c^{(p)}(T) \cos^2(\phi/2) + J_c^{(a)}(T) \sin^2(\phi/2)
\]

Here \( J_c^{(p)}(T) \) and \( J_c^{(a)}(T) \) are critical currents in tunnel junctions with parallel and antiparallel orientations of the exchange fields in the three-layer interface. The \( 0 - \pi \) transition can take place with varying \( \phi \), if \( J_c^{(p)}(T) \) and \( J_c^{(a)}(T) \) have opposite signs. This is the case when the junction with parallel magnetizations is in the \( \pi \)-state, since always \( J_c^{(a)}(T) > 0 \). The transition results in a nonmonotonic dependence of \( |J(T, \phi)| \) on \( \phi \). This effect can be used for switching the junction from the zero state to the \( \pi \) state by varying the misorientation angle.

The angle is changed in the FIF interlayer with applied magnetic field, if it is larger than the coercive force in one of the F layers and less than in the other layer.

Many fundamental and practical problems involve interfaces with antiferromagnets. In particular, many of the properties of HTSC cuprate materials probably arise from a competition between antiferromagnetic and superconducting order, and many situations involve such natural or fabricated boundaries. We have studied interfaces between itinerant antiferromagnets and normal metals or superconductors and demonstrated that a new spin-dependent channel of quasiparticle reflection, the so-called \( Q \) reflection, occurs on the interfaces [2]. Parallel to the interface, the momentum component of low-energy normal-metal quasiparticles changes by \( Q_y \) in a \( Q \) reflection event, where \( Q \) is the wave-vector of the antiferromagnetic pattern and \( y \) is the direction parallel to the interface. Assuming comparatively small Fermi velocity mismatches and taking into account the nesting condition \( E_F(p + Q) = -E_F(p) \) in itinerant antiferromagnets, one can see within the mean-field tight-binding model on the half-filled square lattice that normal metal quasiparticles with energies less than or comparable to the antiferromagnetic gap change their momenta by \( Q \) and reverse the signs of their velocities in a \( Q \) reflection event. Consequently, such quasiparticles experience spin-dependent retroreflection at antiferromagnet-normal metal (AF/N) transparent interfaces.
Quasiparticle bound states below the AF and SC gaps at AF/SC interfaces arise as a combined effect of Andreev and Q reflections. Among a variety of subgap states, low-energy states $E_B \ll \min\{m, \Delta\}$ are of special interest since they can result in low-temperature anomalies in the Josephson critical current, as well as low-bias anomalies in the conductance. Here $m$ and $\Delta$ are the sublattice electronic magnetization and the superconducting s-wave or d-wave order parameters. In the absence of the interface potential $h$, the dispersive bound state energies at the (110) and (100) AF - s-wave superconductor (AF/sSC) interfaces can be represented as $E_{s}(k_{F}) = \pm \Delta_{s} \sqrt{R_{sp}(k_{F})}$. Here $R_{sp}(k_{F})$ is the normal state reflection coefficient for specular reflection from the interface, which occurs even in the absence of any interface potentials due to a mismatch of Fermi velocities in the AF and the sSC. Since normal-metal states are presumably identical in the left and right halfspaces, under the conditions $\Delta \ll m, t$, the mismatch in the model controls the parameter $m/t$. If the magnetic order parameter $m$ is much less than the hopping matrix element $t$, Q reflection dominates $R_{sp}(k_{F}) \ll 1$, and bound state energies almost coincide with the Fermi level $\pm \Delta_{s} \leq m$. In particular, for the (110) interface on the square lattice we find $R_{sp}(k_{F}) = \left[ 1 + \left( \frac{\sqrt{2}v_{F,\perp}(k_{F})}{am} \right)^{2} \right]^{-1}$, where $a$ is the lattice spacing and $v_{F,\perp}(k_{F})$ the normal-state Fermi velocity component along the boundary normal.

![Figure 1](image)

**FIGURE 1.** Eigenvalues for the (100) AF/sSC interface as a function of $k_{F}$: (a) $\mu = 0.0$ and $h = 0.0$ and (b) $\mu = 2.0t$. Order parameter values in the bulk: $\Delta_{s,b} = 0.4t$, $m_{g} = 0.7t$. Here, one sees explicitly the presence and dispersion of the bound state band inside the gap.

from the selfconsistent eigenvalues of the Bogoliubov-de Gennes equations. Interface bound states show up inside the main gap of the spectrum as a distinct band, which disperses with the momentum component $k_{y}$ along the interface. The two gap edges seen in Fig. 1 are associated with the superconducting (lesser) and the antiferromagnetic (larger) gaps. Interface potentials $h$ present near the interface tend to suppress the bound states resulting from Q-reflection and move their positions towards the gap edge. In the regime where $h$ is of the order of $t$, we find that the main effect of the specular reflection channel is to cause a stronger dispersion of the bound state energy. One can identify additional extrema in the wave vector dependence of the bound state energy $E(k_{y})$. A typical example is seen in Fig. 1 where $h = 2.0t$. The new stationary points in the dispersion lead to additional LDOS peaks near the interface.

Dispersive bound state energies on an antiferromagnetic - d-wave superconductor interface (AF/dSC) can be represented as $E_{b}(k_{F}) = \pm \Delta_{b}|(k_{F})| \sqrt{Q_{Q}(k_{F})}$. They lie near the edges of the continuous spectrum, when $Q$ reflection dominates. This contrasts with the case of a (110) surface of a dSC confined with an impenetrable wall, where zero-energy Andreev states are formed.

For an sSC/AF/sSC junction, due to a finite width $l$ of the AF interlayer, the low-energy bound states are split and carry the supercurrent. If no potential barriers are present on the boundaries, and $l \ll \xi_{s}$, $\Delta_{s} \ll m \ll t$, we find the following energies for interface states: $E_{b} = \pm \sqrt{D\Delta_{s}} \cos(\chi/2)$, where $\Delta_{s}$ is the order parameter phase difference, $D(k_{y}) = 4K(k_{y})(K(k_{y}) + 1)^{-2}$ is the transparency of the N/AF/N junction and $K(k_{y}) = \exp(2ml/[v_{F,\perp}(k_{y})])$. For large interlayer width, $K, D \ll 1$, there are low-energy states in the junction which result in low-temperature anomalous behavior of the critical current.

Similar effects for CDW/SC interfaces have been studied recently in [3]. Subgap Andreev states arise at CDW/dSC and DDW/sSC interfaces. At the same time there are no subgap states at CDW/sSC interfaces due to the absence of interface-induced pair-breaking processes. The interface states also do not arise at DDW/dSC interfaces since pair-breaking effects from DDW and dSC compensate each other in this case. In dSC/CDW/dSC and sSC/DDW/sSC Josephson junctions, the interface low-energy bound states are split and strongly influence the Josephson current.

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