Thermal Fluctuations And Correlations Among Hairs Of Stable Quantum ADS
Kerr-Newman Black Hole

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We have already derived the Criteria for thermal stability of charged rotating quantum black holes, for horizon areas that are large relative to the Planck area. The derivation is done by using results of loop quantum gravity and equilibrium statistical mechanics of the Grand Canonical ensemble.

It is also shown that in four dimensional spacetime, quantum ADS Kerr-Newman Black hole is thermally stable within certain range of its’ parameters. In this paper, the expectation values of fluctuations and correlations among horizon area, charge and angular momentum of stable quantum ADS black hole are calculated within the range of stability. Interestingly, it is found that leading order fluctuations of charge and angular momentum, in large horizon area limit, are independent of the values of charge and angular momentum at equilibrium.

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I. INTRODUCTION

Semiclassical analysis shows that nonextremal, asymptotically flat black holes are thermally unstable due to decay under Hawking radiation, with negative specific heat \[1\]. This motivated the study of thermal stability of black holes, from a perspective that relies on a definite proposal for quantum spacetime (like Loop Quantum Gravity, [2, 3]). A consistent understanding of quantum black hole entropy has been obtained through Loop Quantum Gravity [4, 5], where not only has the Bekenstein-Hawking area law been retrieved for macroscopic (astrophysical) black holes, but a whole slew of corrections to it, due to quantum spacetime fluctuations have been derived as well [6]-[11], with the leading correction being logarithmic in area with the coefficient \(-\frac{3}{2}\).

Classically a black hole, in general relativity, is characterized by its’ mass (\(M\)), charge (\(Q\)) and angular momentum (\(J\)). Intuitively, therefore, we expect that thermal behaviour of black holes will depend on all of these parameters. The simplest case of vanishing charge and angular momentum has been investigated longer than a decade ago [12] - [14] and that has been generalized, via the idea of thermal holography [15], [16], and the saddle point approximation to evaluate the canonical partition function corresponding to the horizon, retaining Gaussian thermal fluctuations. This body of work has been generalized recently [17] for charged rotating black holes. There it is shown that anti-de Sitter(ADS) Kerr-Newman black hole (for a certain range of its’ parameters) is thermally stable. In fact the conditions for thermal stability of a macroscopic quantum black hole with arbitrary number of hairs in arbitrary spacetime dimension has already been derived too [18].

In this paper, using previous knowledge [17], thermal fluctuations and correlations among all the hairs i.e. charge, horizon area and angular momentum are calculated. These are calculated in the limit of large horizon area.

The paper is organized as follows: In section 2, the idea of thermal holography, alongwith the concept of (holographic) mass associated with horizon of a black hole is briefly reviewed along with detail discussion of quantum black hole algebra and quantum geometry. This section also contains a short revision of grand canonical partition function of charged rotating black hole (ADS Kerr-Newman Black Hole) and condition for its’ thermal stability. In the next section, detailed calculation of thermal fluctuations are done for ADS Kerr-Newman black hole. Last section contains a brief summary and outlook.

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II. THERMAL HOLOGRAPHY

In this section, we briefly review some part of our earlier work, essential for this paper and hence some overlapping with that [17] is inevitable.

A. Mass Associated With horizon

Black holes at equilibrium are represented by isolated horizons, which are internal boundaries of spacetime. Hamiltonian evolution of this spacetime gives the first law associated with isolated horizon (b) and is given as,

\[ \delta E^t_h = \frac{k^t}{8\pi} \delta A_h + \Phi^t \delta Q_h + \Omega^t \delta J_h \]

where, \( E^t_h \) is the energy function associated with the horizon, \( k^t \), \( \Phi^t \) and \( \Omega^t \) are respectively the surface gravity, electric potential and angular velocity of the horizon; \( Q_h \), \( A_h \) and \( J_h \) are respectively the charge, area and angular momentum of the horizon. The label \( 't' \) denotes the particular time evolution field \( (t^\mu) \) associated with the spatial hypersurface chosen. \( E^t_h \) is assumed here to be a function of \( A_h, Q_h \) and \( J_h \).

As argued in [17], mass can be defined on the isolated horizon.

B. Quantum Black Hole Algebra And Quantum Geometry

Like for all quantum systems, an operator algebra of fundamental observables is required to have a proper quantum description of black holes. Classically, generic black holes are represented by four parameters \( (M, Q, J, A) \), with three of them independent. It is not possible to have a black hole with \( M = 0 \) and \( Q, J \neq 0 \). So, additional structures i.e. charge and angular momentum are fundamental observables in a quantum theory. We choose area \( (A) \) as the third fundamental observable. So, Mass\( (M) \) becomes the secondary observable i.e. \( M = M(A, Q, J) \). So, the algebraic approach of black hole quantization gives, \( Q, \hat{J}, \hat{A} \) as quantum operators of fundamental observables and \( \hat{M}(\hat{H}_b) \) as quantum operator of secondary observable. All these correspond to the isolated horizon of a black hole.

In Loop Quantum Gravity (LQG), quantum black holes are represented by spin network, collection of graphs with links and vertices [19]. Spin networks are duals of cellular decompositions of space, where a certain volume is associated to a vertex and each boundary area with certain links. So, geometry of a black hole horizon is completely determined by the intersections of the graphs with its boundary. These intersections are labelled with \( \gamma \) that \( j \) is the spin projection quantum number of the \( p \)-th link, can take on the values \( j=1,\ldots,jp \). A system of \( n \) particles each having a spin \( j_p \) with states in a single-particle tensor product Hilbert space \( \mathcal{H}_b = \mathcal{H}_{b}^{(j)} \otimes ... \otimes \mathcal{H}_{b}^{(jn)} \). Simultaneous eigenstates of the \( i \)-th component \( \hat{j}_p^i \) of the angular momentum operator \( \hat{J}_p \) is an orthonormal basis for \( \mathcal{H}_b \). These states are the spin network states. \( \hat{j}_p^i \) have eigenvalues \( m_p \) and \( j_p(j_p+1) \) respectively, where \( m_p \) is the spin projection quantum number of the \( p \)-th link, can take on the values \( (-j_p, -j_p+1, \ldots, j_p-1, j_p) \). So, spin network states can be explicitly denoted as \( |(j_p,m_p)^n,\ldots\rangle \), with \( n = p_{\text{max}} \).

Now, LQG gives the action of black hole horizon area operator \( \hat{A} \) and angular momentum operator \( \hat{J} \) respectively as [20],

\[ \hat{A} |(j_p,m_p)^n,\ldots\rangle = A |(j_p,m_p)^n,\ldots\rangle = 8\pi l_p^2 \gamma \sum_{p=1}^n m_p \sqrt{j_p(j_p+1)} |(j_p,m_p)^n,\ldots\rangle \]

Where, \( A = 8\pi l_p^2 \gamma \sum_{p=1}^n m_p \sqrt{j_p(j_p+1)} \equiv \text{area of black hole horizon} \), \( l_p \equiv \text{planck length} \) and \( \gamma \equiv \text{Immirzi parameter} \).

\[ \sum_{p=1}^n m_p \hat{j}_p^i |(j_p,m_p)^n,\ldots\rangle = \frac{l_p}{l_p^2 \gamma} \delta_1^i |(j_p,m_p)^n,\ldots\rangle = \delta_1^i \sum_{p=1}^n m_p |(j_p,m_p)^n,\ldots\rangle \]
Where, \( J = l_p^2 \gamma \sum_{p=1}^{n} m_p \) \( \equiv \) Angular momentum of the black hole.

It is physically obvious that both area and charge should be invariant under SO(3) rotations and that the area should also be U(1) gauge invariant. Since the angular momentum is a measure for rotation (\( SO(3) \) Group) and the charge is the generator of the U(1) global gauge group. These give,

\[
|\hat{A}, \hat{J}⟩ = |\hat{A}, \hat{Q}⟩ = |\hat{Q}, \hat{J}⟩ = 0
\]

(4)

Since \( \hat{M}(\hat{H}_b) \) is a quantum operator of secondary observable \( (M(A, J, Q)) \), Equ no. (4) can be extended as,

\[
|\hat{A}, \hat{J}⟩ = |\hat{A}, \hat{Q}⟩ = |\hat{A}, \hat{M}⟩ = |\hat{Q}, \hat{J}⟩ = |\hat{M}, \hat{Q}⟩ = |\hat{J}, \hat{M}⟩ = 0
\]

(5)

The generic quantum black hole horizon (boundary) state is denoted as, \( |j_p, m_p, q⟩ \), where \( eq \) is the eigenvalue of the charge operator \( \hat{Q} \) with \( q \) is an integer no. and \( e \) is the fundamental \( U(1) \) charge.

Equ no. (5) implies that \( |j_p, m_p, q⟩ \) is a simultaneous eigenstate of \( \hat{A}, \hat{J}, \hat{Q}, \hat{M} \) with eigenvalues as follows,

\[
\hat{A}|j_p, m_p, q⟩ = A|j_p, m_p, q⟩, \quad l_p \sqrt{\gamma} \hat{Q}|j_p, m_p, q⟩ = Q|j_p, m_p, q⟩
\]

\[
l_p^2 \gamma |j_p, m_p, q⟩ = J|j_p, m_p, q⟩, \quad l_p \sqrt{\gamma} \hat{M}|j_p, m_p, q⟩ = M|j_p, m_p, q⟩
\]

(6)

Where, \( Q = l_p \sqrt{\gamma} eq \equiv \) charge of the black hole, \( M \equiv \) mass of the black hole and rest are as before.

The Hilbert space of a generic quantum spacetime is given as, \( \mathcal{H} = \mathcal{H}_b \otimes \mathcal{H}_v \), where \( b(v) \) denotes the boundary (bulk) space. A generic quantum state is thus given as

\[
|\Psi⟩ = \sum_{b,v} C_{b,v}|\chi_b⟩ \otimes |\psi_v⟩
\]

(7)

Now, the full Hamiltonian operator \( (\hat{H}) \), operating on \( \mathcal{H} \) is given by

\[
\hat{H}|\Psi⟩ = (\hat{H}_b \otimes I_v + I_b \otimes \hat{H}_v)|\Psi⟩
\]

(8)

where, respectively, \( I_b(I_v) \) are identity operators on \( \mathcal{H}_b(\mathcal{H}_v) \) and \( \hat{H}_b(\hat{H}_v) \) are the Hamiltonian operators on \( \mathcal{H}_b(\mathcal{H}_v) \).

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\]

(10)

where, respectively, \( I_b(I_v) \) are identity operators on \( \mathcal{H}_b(\mathcal{H}_v) \) and \( \hat{H}_b(\hat{H}_v) \) are the Hamiltonian operators on \( \mathcal{H}_b(\mathcal{H}_v) \).

The first class constraints are realized on Hilbert space as annihilation constraints on physical states. The bulk Hamiltonian operator thus annihilates bulk physical states

\[
\hat{H}_v|\psi_v⟩ = 0
\]

(11)

Any generic quantum bulk Hilbert space is invariant under local \( U(1) \) gauge transformations and local spacetime rotations (the latter, as part of local Lorentz invariance). Since \( \hat{Q}_v, \hat{J}_v \) are the generators of \( U(1) \) gauge transformation and local spacetime rotation for bulk spacetime respectively, they individually annihilates bulk states i.e. \( \hat{Q}_v|\psi_v⟩ = 0, \quad \hat{J}_v|\psi_v⟩ = 0. \)

So, for generic bulk states

\[
[\hat{H}_v - \Phi \hat{Q}_v - \Omega \hat{J}_v]|\psi_v⟩ = 0
\]

(12)
C. Grand Canonical Partition Function

Consider the black hole immersed in a heat bath, at some (inverse) temperature \( \beta \), with which it can exchange energy, charge and angular momentum. The grand canonical partition function of the black hole is given as,

\[
Z_G = Tr(\exp(-\beta \hat{H} + \beta \Phi \hat{Q} + \beta \Omega \hat{J}))
\]  

(13)

where the trace is taken over all states. This definition, together with eqn.s [9] and [12], yields

\[
Z_G = \sum_b |C_b|^2 \langle \psi_b | \exp(-\beta \hat{H} + \beta \Phi \hat{Q} + \beta \Omega \hat{J}) | \chi_b \rangle 
\]

(14)

assuming that the bulk states are normalized. The partition function thus turns out to be completely determined by the boundary states \( Z_{G_b} \), i.e.,

\[
Z = Z_{G_b} = Tr_b \exp(-\beta \hat{H} + \beta \Phi \hat{Q} + \beta \Omega \hat{J}) 
\]

(15)

where \( g(k, l, m) \) is the degeneracy corresponding to energy \( E(A_k, Q_l, J_m) \) and \( k, l, m \) are the quantum numbers corresponding to eigenvalues of area, charge and angular momentum respectively. The application of the Poisson resummation formula [12] gives

\[
Z_G = \int dx dy dz g(A(x), Q(y), J(z)) \exp(-\beta(E(A, Q, J) - \Phi Q - \Omega J)) 
\]

(16)

where \( x, y, z \) are respectively the continuum limit of \( k, l, m \) respectively. A change of variables gives,

\[
Z_G = \int dA dQ dJ \exp[S(A) - \beta(E(A, Q, J) - \Phi Q - \Omega J)] 
\]

(17)

where, following [21], the microcanonical entropy \( S(A) \) of the horizon is defined by \( \exp S(A) \equiv \frac{\partial (\beta(A), Q(y), J(z))}{\partial (A, Q, J)} \).

D. Saddle Point Approximation and Stability Criteria

The equilibrium configuration of black hole is given by the saddle point \( (\bar{A}, \bar{Q}, \bar{J}) \) in the three dimensional space of integration over area, charge and angular momentum with fluctuations \( a = (A - \bar{A}), q = (Q - \bar{Q}), j = (J - \bar{J}) \) around the saddle point. Taylor expanding eqn [17] about the saddle point, yields

\[
Z_G = \exp[S(\bar{A}) - \beta M(\bar{A}, \bar{Q}, \bar{J}) + \beta \Phi \bar{Q} + \beta \Omega \bar{J}] 
\]

\[
\times \int da dq dj \exp\left\{-\frac{\beta}{2}(M_{AA} - \frac{S_{AA}}{\beta})a^2 + (M_{QQ})q^2 + (2M_{AQ})aq + \left((M_{JJ})j^2 + (2M_{AJ})aj + (2M_{QJ})aq\right)\right\} 
\]

(18)

where \( M_{JJ} = \frac{\partial^2 M}{\partial a^2}((\bar{A}, \bar{Q}, \bar{J})) \) etc. as described in [17].

Convergence of the integral [18] implies that the Hessian matrix \( (H) \) has to be positive definite, where

\[
H = \begin{pmatrix}
\beta_{M_{AA}}(\bar{A}, \bar{Q}, \bar{J}) & \beta_{M_{AQ}}(\bar{A}, \bar{Q}, \bar{J}) & \beta_{M_{AJ}}(\bar{A}, \bar{Q}, \bar{J}) \\
\beta_{M_{AQ}}(\bar{A}, \bar{Q}, \bar{J}) & \beta_{M_{QQ}}(\bar{A}, \bar{Q}, \bar{J}) & \beta_{M_{QQ}}(\bar{A}, \bar{Q}, \bar{J}) \\
\beta_{M_{AJ}}(\bar{A}, \bar{Q}, \bar{J}) & \beta_{M_{QQ}}(\bar{A}, \bar{Q}, \bar{J}) & \beta_{M_{QQ}}(\bar{A}, \bar{Q}, \bar{J}) 
\end{pmatrix}
\]

(19)
The necessary and sufficient conditions for a real symmetric square matrix to be positive definite is:

determinants all principal square submatrices, and the determinant of the full matrix, are positive. This condition leads to the 'stability criteria' that are described in [17]. Of course, (inverse) temperature $\beta$ is assumed to be positive for a stable configuration.

Since, we are considering quantum theory of gravity, we have to consider the effect of quantum spacetime fluctuations on microcanonical entropy of isolated horizons. It has been shown that the microcanonical entropy for macroscopic isolated horizons ($S$) has the form

$$S = S_{BH} - \frac{3}{2} \log S_{BH} + O(S_{BH}^{-1}) \quad (20)$$

$$S_{BH} = \frac{A}{4A_P}, \ A_P \rightarrow \text{Planck area}. \quad (21)$$

In reference [7], the formula (20) was derived for non-rotating black holes in four dimensional spacetime. This is based on a three dimensional SU(2) Chern-Simons theory. Where as consideration of U(1) theory gives, $-\log(S_{BH})$. Although the coefficients mismatch with each other, both are logarithmic corrections and subdominating for large horizon area. The detail study of isolated horizon for rotating black holes [23] shows that many properties of rotating black hole are like that of non-rotating ones. This hints towards the possibility of similar correction for microcanonical entropy of rotating black holes as well. The approach, of view work Hawking radiation from a black hole as quantum tunneling of particles through the event horizon, shows [24] that microcanonical entropy of Kerr-Newman black hole has form similar to that of [20]. In the reference [25], it is extensively shown that for various types of black holes in various space time dimension with various charges and angular momentums, the corrections of black hole entropy are mostly logarithmic and hence subdominating for large black hole area. So, the exact form for correction of microcanonical entropy really does not alter any calculation of the rest of the paper as we will only bother about the leading order values in large area limit ($A >> A_P$).

III. THERMAL FLUCTUATION AND CORRELATION AMONG HAIRS OF ADS KERR-NEWMAN BLACK HOLE

The expectation value of fluctuation of any quantity is the standard deviation of that quantity. It is a statistical measure of deviation for any distribution. The knowledge of probability theory and the last expression of grand canonical partition function [18] together give the standard deviation of charge ($Q$) as,

$$(\Delta Q)^2 = \frac{\int dq \; dq \; dq \; dQ \; [ (M_{AA} - \frac{S_{AA}}{\beta} + (M_{QQ}q^2 + (2M_{AQ}a)q + (M_{JJ})j^2 + (2M_{AJ}aj + (2M_{QJ})jq)] }{\int dq \; dq \; dq \; dQ \; [ (M_{AA} - \frac{S_{AA}}{\beta} + (M_{QQ}q^2 + (2M_{AQ}a)q + (M_{JJ})j^2 + (2M_{AJ}aj + (2M_{QJ})jq)] } \quad (22)$$

where, $\Delta Q$ is the standard deviation of black hole charge. Similarly, $\Delta A$ and $\Delta J$ are defined for horizon area and angular momentum of the black hole.

The correlation function between charge ($Q$) and angular momentum ($J$) is denoted as $\Delta Q J$ and is defined as,

$$\Delta Q J = \frac{\int dq \; dq \; dq \; dQ \; [ (M_{AA} - \frac{S_{AA}}{\beta} + (M_{QQ}q^2 + (2M_{AQ}a)q + (M_{JJ})j^2 + (2M_{AJ}aj + (2M_{QJ})jq)] }{\int dq \; dq \; dq \; dQ \; [ (M_{AA} - \frac{S_{AA}}{\beta} + (M_{QQ}q^2 + (2M_{AQ}a)q + (M_{JJ})j^2 + (2M_{AJ}aj + (2M_{QJ})jq)] } \quad (23)$$

Similarly, $\Delta QA$ and $\Delta JA$ are defined for the black hole.

The expression [18] and (22) together give,

$$\langle (\Delta Q)^2 \rangle = -\frac{2}{\beta} \cdot \frac{1}{Z_G} \cdot \frac{\partial Z_G}{\partial M_{QQ}} \quad (24)$$

Similarly, $\langle (\Delta A)^2 \rangle, \langle (\Delta J)^2 \rangle, \langle (\Delta QA) \rangle, \langle (\Delta JA) \rangle, \langle (\Delta Q J) \rangle$ are defined by taking partial derivatives with respect to $(M_{AA} - \frac{S_{AA}}{\beta}), M_{JJ}, M_{QA}, M_{JA}$ and $M_{QJ}$ respectively i.e.

$$\langle (\Delta A)^2 \rangle = -\frac{2}{\beta} \cdot \frac{1}{Z_G} \cdot \frac{\partial Z_G}{\partial (M_{AA} - \frac{S_{AA}}{\beta})} \quad (25)$$

5
\[(\Delta J)^2 = -\frac{2}{\beta} \cdot \frac{1}{Z_G} \cdot \frac{\partial Z_G}{\partial M_{JJ}}\]  \hspace{1cm} (26)

\[\Delta QA = -\frac{1}{\beta} \cdot \frac{1}{Z_G} \cdot \frac{\partial Z_G}{\partial M_{QA}}\]  \hspace{1cm} (27)

\[\Delta JA = -\frac{1}{\beta} \cdot \frac{1}{Z_G} \cdot \frac{\partial Z_G}{\partial M_{JA}}\]  \hspace{1cm} (28)

\[\Delta QJ = -\frac{1}{\beta} \cdot \frac{1}{Z_G} \cdot \frac{\partial Z_G}{\partial M_{QJ}}\]  \hspace{1cm} (29)

Equation no. (18), (19) and (24) together give,

\[(\Delta Q)^2 = \frac{2}{|H|} \cdot \left( (\beta M_{AA} - S_{AA}) \cdot \beta M_{JJ} - (\beta M_{AJ})^2 \right)\]  \hspace{1cm} (30)

where, \(|H|\) is the determinant of the hessian matrix\((H)\).

Equation no. (18), (19) and (25) together give,

\[(\Delta A)^2 = \frac{2}{|H|} \cdot \left( (\beta M_{QQ} M_{JJ} - (M_{JQ})^2) \right)\]  \hspace{1cm} (31)

Equation no. (18), (19) and (26) together give,

\[\Delta QA = \frac{2}{|H|} \cdot \left( \beta^2 (M_{AQ} M_{JJ} - M_{JQ} M_{AJ}) \right)\]  \hspace{1cm} (32)

Equation no. (18), (19) and (27) together give,

\[\Delta JA = \frac{2}{|H|} \cdot \left( \beta^2 (M_{AJ} M_{QQ} - M_{JQ} M_{AQ}) \right)\]  \hspace{1cm} (33)

Equation no. (18), (19) and (28) together give,

\[\Delta QJ = \frac{2}{|H|} \cdot \left( \beta M_{JQ} (\beta M_{AA} - S_{AA}) - \beta^2 M_{AQ} M_{AJ} \right)\]  \hspace{1cm} (34)

The AdS Kerr-Newman black hole is given in BoyerLindquist coordinates as

\[ds^2 = -\Delta_r \rho^2 (dt - \frac{a \sin^2 \theta}{\Sigma} d\phi)^2 + \Delta_\theta \rho^2 \left( \frac{r^2 + a^2}{\Sigma} d\phi - adt \right)^2 + \rho^2 \Delta_\theta d\theta^2\]  \hspace{1cm} (36)

where, \(\Sigma = 1 - \frac{\rho^2}{r^2}, \quad \Delta_r = (r^2 + a^2)(1 + \frac{\rho^2}{r^2}) - 2Mr + Q^2, \quad \Delta_\theta = 1 - \frac{a^2 \cos^2 \theta}{r^2}, \quad \rho^2 = r^2 + a^2 \cos^2 \theta, \quad a = \frac{J}{M}.

The generalized Smarr formula for the AdS Kerr-Newman Black Hole is given as \[26\]

\[M^2 = \frac{A}{16\pi} + \frac{\pi}{A} (4J^2 + Q^4) + \frac{Q^2}{2} + \frac{J^2}{r^2} + \frac{A}{8\pi l^2} (Q^2 + \frac{A}{4\pi} + \frac{A^2}{32\pi^2 l^2})\]  \hspace{1cm} (37)

where the cosmological constant \((\Lambda)\) is defined in terms of a cosmic length parameter as \(\Lambda = -1/l^2\).

As before, our interest is in astrophysical (macroscopic) charged, rotating black holes whose horizon area exceeds by far the Planck area. It has shown \[18\] that, ADS kerr-newman black holes are stable if \(\frac{A}{l^2} \gg \frac{Q^2}{A}, \frac{J}{A}\). So, we can approximate \[26\] as follows

\[M \approx \frac{A^{3/2}}{16\pi^{3/2} l^2} + \frac{A^{1/2}}{4\pi^{1/2}} + \frac{\pi^{1/2} Q^2}{A^{1/2}} + \frac{8\pi^{3/2} J^2}{A^{3/2}}\]  \hspace{1cm} (38)
Equation no. (19), (30) and (38) together give,

\[
(\Delta Q)^2 \approx \frac{3A_p A}{8\pi^2 l^2}
\]  \hspace{1cm} (39)

Equation no. (19), (31) and (38) together give,

\[
(\Delta A)^2 \approx 16A_p A
\]  \hspace{1cm} (40)

Equation no. (19), (32) and (38) together give,

\[
(\Delta J)^2 \approx \frac{3A_p A^2}{64\pi^3 l^2}
\]  \hspace{1cm} (41)

Equation no. (19), (33) and (38) together give,

\[
\Delta QA \approx -8A_p Q
\]  \hspace{1cm} (42)

Equation no. (19), (34) and (38) together give,

\[
\Delta JA \approx -24A_p J
\]  \hspace{1cm} (43)

Equation no. (19), (35) and (38) together give,

\[
\Delta QJ \approx -\frac{12A_p J Q}{A}
\]  \hspace{1cm} (44)

Of course, last six expressions are only the leading order terms in large horizon area limit.

It is extremely interesting to note that ((\Delta J)^2) and ((\Delta Q)^2) are independent of J and Q respectively. It means for a large black hole there are finite amount of fluctuations of charge and angular momentum even for a almost neutral, nonrotating ADS black hole.

The measure of fluctuations of the above six fluctuations are given as,

1. Measure of Area fluctuation

\[
\frac{\Delta A}{A} \approx 4\sqrt{\frac{A_p}{A}}
\]  \hspace{1cm} (45)

2. Measure of Charge fluctuation

\[
\frac{\Delta Q}{Q} \approx \sqrt{\frac{3}{8\pi^2}} \cdot \sqrt{\frac{A_p A}{Q l}}
\]  \hspace{1cm} (46)

3. Measure of Angular Momentum fluctuation

\[
\frac{\Delta J}{J} \approx \sqrt{\frac{3}{64\pi^3}} \cdot \sqrt{\frac{A_p A^2}{J^2 l^2}}
\]  \hspace{1cm} (47)

4. Measure of Charge - Area correlation

\[
\sqrt{\frac{|\Delta QA|}{QA}} \approx \sqrt{\frac{8A_p}{A}}
\]  \hspace{1cm} (48)

5. Measure of Charge - Angular Momentum correlation

\[
\sqrt{\frac{|\Delta QJ|}{QJ}} \approx \sqrt{\frac{12A_p}{A}}
\]  \hspace{1cm} (49)
6. Measure of Area - Angular Momentum correlation

\[ \sqrt{\frac{\Delta AJ}{AJ}} \approx \sqrt{\frac{24A_P}{A}} \]  

Equation Nos. (45), (48), (49), (50) imply respectively that Measure of Area fluctuation \( (\Delta A) \), Charge - Area correlation \( \left( \sqrt{\frac{\Delta QA}{Q^2}} \right) \), Charge - Angular Momentum correlation \( \left( \sqrt{\frac{\Delta QJ}{Q^2J}} \right) \) and Area - Angular Momentum correlation \( \left( \sqrt{\frac{\Delta AJ}{AJ}} \right) \) are vanishly small for large black holes \( (A >> A_P) \). These results imply that we are dealing not only around equilibrium point but also it is a stable equilibrium point. The surprising fact is that these measures of correlations are independent of charge \( (Q) \), angular momentum \( (J) \) of the black hole.

The expression no.(38) implies that for large black hole area \( (A) \),

\[ \frac{A^{3/2}}{16\pi^{3/2}l^2} > \frac{\pi^{1/2}Q^2}{A^{1/2}}, \quad \frac{A^{3/2}}{16\pi^{3/2}l^2} > \frac{8\pi^{3/2}J^2}{A^{3/2}} \]  

Equation nos. (46), (47) and (51) together give,

\[ \frac{\Delta Q}{Q} > \sqrt{\frac{6A_P}{A}} \]  

\[ \frac{\Delta J}{J} > \frac{1}{64\pi^3} \sqrt{\frac{3A_P}{2A}} \]  

Last two expressions imply that measure of charge and angular momentum fluctuations are vanishly small for large black holes \( (A >> A_P) \). Although last two expressions of (52) and (53) are the lower bounds, but still they are independent of charge \( (Q) \) and angular momentum \( (J) \) of the black hole and eventually are zero in large black hole limit.

IV. SUMMARY AND DISCUSSION

We reiterate that our analysis is quite independent of specific classical spacetime geometries, relying as it does on quantum aspects of spacetime. The construction of the partition function used standard formulations of equilibrium statistical mechanics augmented by results from canonical Quantum Gravity, with extra inputs regarding the behaviour of the microcanonical entropy as a function of area beyond the Bekenstein-Hawking area law, as for instance derived from Loop Quantum Gravity \[7\]. We use classical metric only as an input which gives the dependence of mass of black hole \( (M) \) on its’ charge \( (Q) \), area \( (A) \) and angular momentum \( (J) \).

In large horizon area limit, it turns out that for a quantum ADS black hole, leading order fluctuations of charge \( (\Delta Q)^2 \) and angular momentum \( (\Delta J)^2 \) are independent of its’ charge \( (Q) \) and angular momentum \( (J) \). This implies even a black hole with vanishingly small charge \( (Q) \) and angular momentum \( (J) \) can have finite fluctuations in respective quantities. Our analysis can be trivially extended for black holes with any number of hairs in any space time dimension \[18\]. The \( S_{AA} \) term is present everywhere in the calculation. The non vanishing contribution of this term is pure artifact of quantum fluctuation of spacetime. Thermal fluctuations are present along with this as we are considering black hole to be immersed in a extended thermal bath. So, thermal fluctuations and correlations, that we have calculated, take care of quantum fluctuation of spacetime within it automatically. This is extremely interesting in its’ own merit. We choose this example of quantum ADS black hole as AdS/CFT correspondence tells that string theory on AdS space is dual to a conformal field theory (CFT) on the boundary of that AdS space \[27\] \[28\]. It has also been shown using the AdS/CFT correspondence that the asymptotically AdS black hole is dual to a strongly coupled gauge theory at finite temperature \[29\] - \[32\]. It is possible to study the strongly correlated condensed-matter physics using the AdS/CFT correspondence. Holographic model of superconductors has also been constructed from...
blackhole solutions using the AdS/CFT correspondence [33]. Hence our results on quantum ADS black hole may have some imprints on possible applications for the strongly correlated condensed-matters systems.

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