Thickness dependence of spin Peltier effect visualized by thermal imaging technique

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II. EXPERIMENTAL CONFIGURATION AND PROCEDURE

The SPE was measured for a Pt/YIG film with a gradient of the YIG thickness, \( \nabla t_{\text{YIG}} \). \( \nabla t_{\text{YIG}} \) was introduced by obliquely polishing a single-crystalline YIG (111) film grown by LPE on a single-crystalline Gd3Fe5O12 (GGG) (111) substrate. Since we introduced a gradual thickness change to a film with the size of \( 10 \times 10 \text{ mm}^2 \), \( \nabla t_{\text{YIG}} \) is almost uniform in the measurement range of 1 mm [Fig. 1(f)]. The value of \( \nabla t_{\text{YIG}} \) along the y direction was obtained with a cross-sectional scanning electron microscope as 9.2 \( \mu \text{m/mm} \) [Fig. 1(f)]. After the polishing, a U-shaped Pt film with the thickness of 5 nm was sputtered on the surface of the YIG film. The longer lines of the U-shaped Pt were arranged parallel to the \( \nabla t_{\text{YIG}} \) direction [Fig. 1(a),(d)]. In the microscope image of the Pt/YIG/GGG sample in Fig. 1(d), the yellow (gray) area above (below) the white dot-
FIG. 1: (a) A schematic illustration of the SPE measurement using a Pt/YIG/GGG sample by means of the lock-in thermography method. A charge current, \( J_c \), is applied to the U-shaped Pt film fabricated on the YIG film with a thickness gradient, \( \nabla t_{\text{YIG}} \). (b),(c) Time \( t \) profile of an input a.c. charge current, \( J_c \), and output temperature change, \( \Delta T \), for the (b) SPE and (c) Joule heating configurations. (d) An optical microscope image of the sample. The yellow (gray) area above (below) the white dotted line corresponds to the YIG film with \( \nabla t_{\text{YIG}} \) and the magnetization, \( \mathbf{M} \), in the YIG via the interfacial exchange coupling and induces a heat current across the Pt/YIG interface via the SPE. The heat current results in a temperature change, \( \Delta T \), which satisfies the following relation of the SPE \[3,4\]:

\[
\Delta T = (\mathbf{J}_c \times \mathbf{M}) \cdot \mathbf{n},
\]

where \( \mathbf{n} \) is the normal vector of the Pt/YIG interface plane. Significantly, the temperature change induced by the SPE is localized around the Pt/YIG interface in length scale of micrometers owing to the formation of dipolar heat sources \[11\]. Therefore, the temperature change generated at each position on the YIG is not broadened within the scale of the spatial resolution of our infrared camera (\(~10 \mu m\)). We can thus obtain local information of the spatial dependent SPE reflecting \( t_{\text{YIG}} \) at each position.

The \( t_{\text{YIG}} \) dependence of our measurements is determined as 92 nm for the bare Pt/YIG/GGG sample by multiplying the \( t_{\text{YIG}} \) gradient and the spatial resolution of our infrared camera. Measurements were also performed using a black-ink-coated Pt/YIG/GGG sample to obtain the uniform infrared emission property of the sample. The surface of the sample was coated with insulating black ink with a thickness of 20 – 30 \( \mu m \), which mainly consists of SiZrO\(_4\), Cr\(_2\)O\(_3\), and iron oxide-based inorganic pigments \[16\]. Hereafter, we focus mainly on the bare Pt/YIG/GGG sample to obtain the precise thickness dependence of the SPE (note that the thick black-ink layer may reduce the spatial resolution of thermal images).

The \( t_{\text{YIG}} \) dependence of the SPE can be obtained by visualizing distribution of the temperature change by means of LIT \[23,28\]. A rectangular a.c. charge current, \( J_c \), with the amplitude \( J_0 \) (\( \Delta J_c \)), frequency \( f \), and zero (non-zero) d.c. offset \( (J_0') \) was used as an input for the SPE (Joule heating) measurement [Fig. 1(b),(c)]. By extracting the first harmonic response of the temperature change in the SPE (Joule heating) configuration, we can detect the pure SPE (Joule heating) signal free from other thermal effects \[4,6\]. Here, the SPE induced temperature change \( \Delta T_{1f} \) is defined as the component of \( \Delta T \) oscillating in the same phase as \( \mathbf{J}_c \) because the SPE exactly follows the \( \mathbf{J}_c \) oscillation in the time scale of \( 1/f \) due to the quite fast response of the \( \Delta T \) generated by the SPE \[4,20,30\]. In the LIT measurement, we obtain the first harmonic component of the infrared light emission \( \Delta I_{1f} \) caused by \( \Delta T_{1f} \). By calibrating \( \Delta I_{1f} \) to \( \Delta T_{1f} \) using an infrared emissivity, \( \epsilon \), of the sample, we obtain the temperature change induced by the SPE \[4,6\]. All measurements of the SPE were performed under a magnetic field with a magnitude of 20 mT at room temperature and atmospheric pressure, where the magnetization of YIG aligns along the field direction at 20 mT.
FIG. 2: (a) Lock-in signal of an infrared light emission $\Delta I_f$ induced by the SPE. $J_0$, $H$, and $\nabla_{\text{YIG}}$ are amplitude of the input charge current, an applied magnetic field, and the YIG-thickness gradient, respectively. (b) Lock-in amplitude of the infrared emission $A_f$ induced by the Joule heating. (c),(d) Continuous YIG-thickness $t_{\text{YIG}}$ dependence of (c) $\Delta I_f$ and the emissivity $\epsilon$ and (d) the lock-in signal of the temperature change $\Delta T_{1f}$ induced by the SPE.

### III. RESULTS AND DISCUSSION

Figure 2(a) shows the $\Delta I_f$ signal from the Pt/YIG/GGG sample in the SPE configuration with $J_0 = 8$ mA and $f = 5$ Hz. The infrared signal appears only on the Pt/YIG structure but disappears on the Pt/GGG structure [compare Figs. 2(d) and 2(a)]. The sign of $\Delta I_f$ is reversed when the charge current $J_c$ is reversed. These characteristics are consistent with the symmetry of the SPE and imply that the observed infrared signal comes from the SPE.[3,4]

To focus on the $t_{\text{YIG}}$ dependence of the SPE, the $y$ dependence of the $\Delta I_f$ values is plotted in Fig. 2(c), where the $\Delta I_f$ values are averaged along the $x$ direction in the area surrounded by the dotted line in Fig. 2(a). The $\Delta I_f$ value gradually increases with small oscillation in the $t_{\text{YIG}}$ dependence. The oscillation originates from the oscillation of the infrared emissivity $\epsilon$ of the sample due to multiple reflection of the infrared light in the YIG film.[3,4] To remove the oscillation in the $t_{\text{YIG}}$ dependence of the SPE, $\epsilon$ was measured by the LIT with the Joule heating configuration with $J_c = 0.5$ mA, $f = 5$ Hz, and $T_0 = 8.0$ mA. Figure 2(b) shows an image of the amplitude of the infrared emission $A_f$ induced by the Joule heating. Since the temperature change induced by the Joule heating is uniform on the Pt film, the $A_f$ distribution on the Pt film solely depends on the $\epsilon$ distribution. The $t_{\text{YIG}}$ dependence of $\epsilon (\propto A_f)$ is plotted in Fig. 2(c), where the $A_f$ values are averaged along the $x$ direction in the same area as that for the SPE signal. We found that $\epsilon$ and $\Delta I_f$ show the similar oscillating behavior. By calibrating $\Delta I_f$ by $\epsilon$, we obtained the $t_{\text{YIG}}$ dependence of the temperature change $\Delta T_{1f}$ induced by the SPE [Fig. 2(d)]. The $\Delta T_{1f}$ value monotonically increases with increasing $t_{\text{YIG}}$.

We also measured the $t_{\text{YIG}}$ dependence of the SPE in the black-ink/Pt/YIG/GGG sample [Fig. 3]. The sample with the black ink exhibits high and uniform emissivity without the spacial oscillation of $\epsilon$ appeared in the bare Pt/YIG/GGG sample [compare gray lines in Figs. 2(c) and 3(c),(d)]. In the SPE configuration with $J_0 = 8$ mA and $f = 5$ Hz, positive and negative $\Delta I_f$ signals appear on the left and right Pt films, respectively [Fig. 3(b)]. By calibrating $\Delta I_f$ with $\epsilon$, we obtained the $t_{\text{YIG}}$ dependence of the temperature change $\Delta T_{1f}$ induced by the SPE [Fig. 3(e)]. We found that $\Delta T_{1f}$ monotonically increases with increasing $t_{\text{YIG}}$ even when the sample surface was coated with the black ink and the non-uniformity of the infrared light emissivity was removed.

The obtained $t_{\text{YIG}}$ dependence cannot be explained by a simple exponential approximation used in the previous studies on the SPE and SSE.[6,12,13,18] Based on the simple assumption that the magnon diffuses in the YIG film with a magnon diffusion length $l_m$, the simple exponential approximation has been used for the analysis of the $t_{\text{YIG}}$ dependence:

$$\Delta T \propto 1 - \exp\left(-t_{\text{YIG}}/l_m\right).$$

However, in general, this expression cannot be used for the small thickness region since the exponential function should be modulated by the boundary conditions for the spin and heat currents. In fact, when the experimental result is fitted by using Eq. (1), the fitting result shows significant discrepancy in small thickness regions $t_{\text{YIG}} < 4$ $\mu$m as shown in Fig. 3. The observed continuous $t_{\text{YIG}}$ dependence of the SPE thus requires advanced understanding of the spin-heat conversion phenomena.

To discuss the $t_{\text{YIG}}$ dependence of the SPE, some phenomenological theories are available. Here, we focus on two theories referenced in Refs. [17] and [30]. Following these theories, we calculate the $t_{\text{YIG}}$ dependence of the SPE in the Pt/YIG/GGG system. The calculation was carried out by considering the following three processes: (i) spin current is generated by the SHE in Pt; (ii) the spin current is injected into YIG due to the interfacial exchange interaction; (iii) The spin current carries heat from Pt to YIG across the interface, and the resultant temperature distribution is calculated by solving diffusion equations for the spin and heat currents in the Pt/YIG system. The former two processes require the same calculation in the two theories, but the third process gives rise to different temperature changes in the different two theories due to the different heat-generation conditions. In the theory in Ref. [17], the authors assumed that the magnons in YIG locally fol-
FIG. 3: (a) Infrared light image of a black-ink/Pt/YIG/GGG sample. $J_c$ and $H$ are a charge current applied to the Pt film and magnetic field, respectively. (b) Lock-in signal of an infrared light emission $\Delta I_{1f}$ induced by the SPE. $J_0$ and $\nabla T_{yig}$ are amplitude of the input charge current and a YIG-thickness gradient, respectively. (c) Lock-in amplitude of an infrared light emission $A_I$ induced by the Joule heating. $\Delta J_c$ and $J_0$ denote the amplitude and a d.c. offset of the input charge current, respectively. (d),(e) Continuous YIG-thickness $t_{yig}$ dependence of (d) the $\Delta I_{1f}$ and the emissivity $\epsilon$ and (e) the lock-in signal of the temperature change $\Delta T_{1f}$ induced by the SPE.

FIG. 4: Experimental results of the $t_{yig}$ dependence of the SPE for the (a) Pt/YIG/GGG and (b) black-ink/Pt/YIG/GGG samples and fitting curves using Eqs. (1)-(3).

The Bose-Einstein distribution with $\mu_m$ and $T_m$ [Fig. 5(a)]. The values of these parameters diffuse in the YIG following the Boltzmann and continuity equations, approximated into two diffusion equations: $\nabla^2 \mu_m = \mu_m / l_m^2$ and $\nabla^2 T_m = (T_m - T_p) / l_{mp}^2$, where $T_p$ is the phonon temperature in YIG, and $l_m$ and $l_{mp}$ are the magnon-spin diffusion length and the magnon-phonon thermalization length, respectively [17]. The spin and heat currents carried by magnons are defined as $j_m \propto \nabla \mu_m$ and $j_{Q,m} \propto \nabla T_m$, respectively. These formalisms mean that $j_m$ and $j_{Q,m}$ are characterized by different diffusion lengths in YIG [Fig. 5(a)]. According to Ref. [17], $l_m$ is much longer than $l_{mp}$ due to the difference in scattering time scale of the magnon-conserving and magnon-non-conserving scattering processes. Under this assumption, heat current does not come to the YIG/GGG interface due to the short length scale of $l_{mp}$, and the temperature distribution generated by the SPE is not affected by $t_{yig}$ in the length scale of $l_m$, although the magnitude of $\Delta T$ is determined by spin current at the Pt/YIG interface.
Then, by solving the above diffusion equations, we obtain the $\Delta T$ dependence of $\Delta T$ (see Appendix section for more details):

$$\Delta T \propto j_{\text{int}} \propto \frac{1}{C \coth (t_{\text{YIG}} / l_m) - C + 1}.$$  \hspace{1cm} (2)

where $C$ is a $t_{\text{YIG}}$-independent constant used as a fitting parameter in our analysis. On the other hand, in Ref. [31], the authors proposed a theory based on non-equilibrium thermodynamics, where magnetization and heat currents are induced by a non-equilibrium component of the magnetic field $H^+$. $H^*$ follows the diffusion equation $\nabla^2 H^* = H^* / l_M^2$, where $l_M$ is a diffusion length [Fig. 5(b)]. The magnetization and heat currents are induced by the gradient of $H^*$ and the local temperature $T$. The authors assumed that the heat current carried by magnons $j_{\text{M}} = \epsilon_M j_{\text{M}}$, where $\epsilon_M$ is the absolute thermomagnetic power coefficient [31]. Under the formalisms, the $t_{\text{YIG}}$ dependence of the SPE is obtained by solving the above diffusion equation and a heat diffusion equation (see Appendix section for more details):

$$\Delta T \propto \frac{\cosh (t_{\text{YIG}} / l_m) - 1}{D \sinh (t_{\text{YIG}} / l_m) + (1 - D) \cosh (t_{\text{YIG}} / l_m)},$$  \hspace{1cm} (3)

where $D$ is a $t_{\text{YIG}}$-independent constant used as a fitting parameter in our analysis.

Figure 5 shows the experimental results of the $t_{\text{YIG}}$ dependence of the SPE and fitting curves based on Eqs. 2 and 3. We found that Eq. 2 shows the best agreement with the experimental result and $l_m$ is estimated to be 3.9 $\mu$m and 4.0 $\mu$m for the bare and black-ink-coated samples, respectively. In contrast, Eq. 3 cannot explain the experimental result in most thickness range $t_{\text{YIG}} < 4 \mu$m and gives a shorter magnon diffusion length of 0.6 $\mu$m for both samples. Since the essential difference between the theories in Refs. [31] and [17] is whether the length scales of the spin and heat currents are same or not, these fitting results show that the spin and heat currents carried by magnons in YIG have different length scales, $l_m$ and $l_{\text{mp}}$. Significantly, $l_m \gg l_{\text{mp}}$ is necessary for deriving Eq. 2. If $l_m \sim l_{\text{mp}}$, the theory in Ref. [17] gives $t_{\text{YIG}}$ dependence written not in Eq. 2 but in Eq. 3, which cannot reproduce the experimental result. On the other hand, the theory in Ref. [17] can derive Eq. 2 by assuming that magnetization current flows much longer than that of magnetic heat current. Therefore, we conclude that spin current carried by magnons can flow much longer than that of heat current carried by magnons in YIG.

In the recent study on the SSE in Ref. [19], the authors reported non-monotonical increase of the SSE signal with $t_{\text{YIG}}$. Since the SSE signal takes a local maximum at $t_{\text{YIG}} \sim l_{\text{mp}}$, they estimated $l_{\text{mp}}$ as 250 nm from the maximum point. However, in our Pt/YIG sample, the SSE signal monotonical increases with increasing $t_{\text{YIG}}$. These results suggest that $l_{\text{mp}}$ is shorter than the $t_{\text{YIG}}$ resolution of 60 nm for our YIG sample. The conclusion is consistent with the theoretical expectation $l_{\text{mp}} \sim 1$ nm [17].

IV. CONTINUOUS $t_{\text{YIG}}$ DEPENDENCE OF THE SSE

To check the reciprocity between the SPE and SSE, we also measured the $t_{\text{YIG}}$ dependence of the SSE by using a Pt/YIG/GGG sample with a $t_{\text{YIG}}$ gradient. The YIG film used in the SSE measurement was not the same as that in the SPE measurements but it was obtained from the same YIG/GGG substrate. By obliquely polishing the surface of the YIG/GGG substrate, we obtained the YIG film with the $t_{\text{YIG}}$ gradient of 12.2 $\mu$m/mm [Fig. 6(c)]. After the polishing, a Pt film with the thickness of 50 nm was sputtered on the surface of the YIG film [Figs. 6(a) and (d)]. Because magnetic properties of the YIG film was similar to those of the YIG film used in the SPE measurements, the reciprocity between the SPE and SSE can be checked by using the Pt/YIG/GGG sample.

To obtain the $t_{\text{YIG}}$ dependence of the SSE, the SSE voltage was measured in the Pt/YIG/GGG sample by means of a micro-focused laser heating method [33, 35]. As shown in Fig. 4(a), a laser with the wavelength of 1.3 $\mu$m and the diameter of the laser spot of 5.2 $\mu$m was focused on the sample surface to generate a spin current across the Pt/YIG interface. The spin current is converted into a charge current via the inverse spin Hall effect.
FIG. 6: (a) Schematic illustration of a SSE measurement using a laser heating method. We used a Pt/YIG/GGG sample with a YIG-thickness gradient $\nabla t_{\text{YIG}}$. $H$ denotes an applied magnetic field. (b) Time $t$ dependence of a temperature change of the sample $\Delta T$ and an output SSE voltage $V_{\text{SSE}}$ induced by periodic irradiation of the laser light with frequency $f$. (c) $t_{\text{YIG}}$ profile and cross-sectional image of the sample obtained by scanning electron microscope. (d) Infrared light image of the sample. (e) The SSE voltage $V_{\text{SSE}}$ image induced by the laser heating. (f) Continuous YIG-thickness $t_{\text{YIG}}$ dependence of $V_{\text{SSE}}$ and comparison with that of the temperature change $\Delta T_{\text{SPE}}$ induced by the SPE.

effect in Pt [22] and was detected as an electrical voltage. While this method enabled the measurement of the local SSE voltage near the laser spot, the spatial resolution was larger than the laser spot size because the temperature gradient is broadened in the sample owing to the heat diffusion. To avoid the reduction in spatial resolution, we adopted a lock-in technique in the laser SSE measurement, where the laser intensity was modulated in a periodic square waveform with frequency $f = 5$ kHz and we measured the thermal voltage $V_{\text{ff}}$ oscillating with the same frequency as that of the input laser [Fig. 6(b)]. This lock-in technique realized high spatial resolution for the SSE measurement because the heat diffusion cannot follow the oscillation of the laser intensity. Here, we defined the SSE voltage $V_{\text{SSE}}$ as $[V_{\text{ff}}(+50 \text{ mT}) - V_{\text{ff}}(-50 \text{ mT})]/2$ to remove magnetic-field-independent experimental artifacts. By scanning the position of the laser spot on the sample, we visualized the spatial distribution of the SSE voltage with high spatial resolution of $5.2 \mu m$.

Figure 6(e) shows the experimental results for the SSE measurement. In response to the laser heating, the clear voltage signal appeared in the Pt film. The $t_{\text{YIG}}$ dependence of the SSE signal is plotted in Fig. 6(f), where the $V_{\text{SSE}}$ values were averaged along the $x$ direction in the area surrounded by the dotted line in Fig. 6(e). The SSE signal monotonically increases with increasing $t_{\text{YIG}}$. Significantly, the $t_{\text{YIG}}$ dependence of the SSE demonstrated the same behavior as the SPE [Fig. 6(f)]. This result supports the reciprocity between the SPE and SSE and strengthens our conclusion in the SPE measurements.

V. CONCLUSION

In conclusion, we revealed the length scale of the spin and heat transport by magnons in YIG by measuring the $t_{\text{YIG}}$ dependence of the SPE in the Pt/YIG sample. This measurement was realized by using the YIG film with the $t_{\text{YIG}}$ gradient and the LIT method, which allows us to obtain the continuous $t_{\text{YIG}}$ dependence of the SPE in the single Pt/YIG sample. We found that the experimental result is well reproduced by assuming that the spin current flows much longer than that of the heat current carried by magnons in YIG. We also measured the $t_{\text{YIG}}$ dependence of the SSE and found that the SPE and SSE demonstrate the same behavior in the $t_{\text{YIG}}$ dependence. This understanding gives crucial information to understand the physical origin of the spin-heat conversion phenomena.

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APPENDIX: THEORETICAL DESCRIPTION OF $t_{YIG}$ DEPENDENCE OF SPE

To analyze the $t_{YIG}$ dependence of the SPE obtained from the experiments in the bare and black-ink-coated Pt/YIG/GGG samples, we introduced two phenomenological theories for the SPE, which can quantitatively derive its $t_{YIG}$ dependence. Here, we considered a junction comprising a Pt film with a thickness of $t_{Pt}$, a YIG film with a thickness of $t_{YIG}$, and a GGG film with a thickness of $t_{GGG}$ as illustrated in Figs. 4 and 7.

A. The $t_{YIG}$ dependence of the SPE based on Ref. [17]

In Ref. [17] the authors formulated the SSE and SPE based on the Boltzmann theory for magnons in a ferromagnetic insulator. In this formalism, the nonequilibrium distribution of the magnons (spins) in YIG (Pt) is described by the Bose-Einstein distribution with the magnon chemical-potential $\mu_m$ and magnon temperature $T_m$ (the spin accumulation $\mu_s$ and electron temperature $T_e$) [Fig. 5(a)]. The authors derived that the spin and heat currents in YIG (Pt) are driven by the gradients of $\mu_m$ and $T_m$ ($\mu_s$ and $T_e$), respectively, and constructed theories for the SSE and SPE.

In the YIG film, the linear-response relations for the magnon spin current $j_m$ and heat current carried by magnons $j_{Q,m}$ are derived from the Boltzmann equation for the magnons [17]:

$$
\left( \frac{2e}{\hbar} j_m \right) = - \left( \frac{\sigma_m}{k_B} L/T_p \right) \left( \nabla z \mu_m \frac{\nabla z T_m}{\nabla z T_m} \right),
$$

(4)

where $e$, $\hbar$, $\sigma_m$, $\kappa_m$, $L$, and $T_p$ are the electron charge, Planck constant, magnon spin conductivity, magnonic heat conductivity, bulk spin Seebeck coefficient, and phonon temperature, respectively. By combining these equations with the continuity equation for $j_m$ and $j_{Q,m}$, the diffusion equations for $\mu_m$ and $T_m$ can be derived [17]:

$$
\left( \frac{2e}{\hbar} j_s \right) = - \left( \frac{\sigma_{SH}}{\sigma_s} \frac{\mu_s}{T_e} \right) \left( \nabla z \mu_s \frac{\nabla z T_e}{\nabla z T_e} \right),
$$

(8)

where $\sigma_s$ and $\sigma_{SH}$ are the electrical conductivity and spin Hall conductivity, respectively. $\sigma_s'$ is defined as $\sigma_s [1 - (\sigma_{SH}/\sigma_s)^2]$. Here, we considered the relaxation-time approximation for the spin current: $\nabla z (\frac{2e}{\hbar} j_s) = \frac{\mu_s}{\tau_s}$, where $\tau_s$ is the relaxation time. From these equations, the diffusion equation for $\mu_s$ can be derived:

$$
\nabla z^2 \mu_s = \frac{\mu_s}{l_{mp}^2},
$$

(9)

where the spin diffusion length $l_{mp}$ is defined as $\sqrt{\frac{\sigma_s' \tau_s}{2}}$.

By using the above equations for the spin and heat currents, we calculated the temperature change $\Delta T_{\text{SPE}}$ induced by the SPE. In this formalism, because $l_{mp} \ll l_m$, the heat source generated in YIG by the SPE is confined in much shorter length scale than the magnon diffusion length of micrometers. Significantly, under this condition, a YIG film with a thickness of $t_{YIG}$, $1 \ll l_m$ and $l_{mp}$, is a spin-heat relaxation length, and magnon heat current carried by magnons $j_Q,m$ is much longer than $j_m$.

Because Eq. (4) leads to $j_m \propto \nabla z \mu_m$ and $j_{Q,m} \propto \nabla z T_m$, these diffusion equations indicate that $j_m$ and $j_{Q,m}$ diffuse in the length scales of $l_m$ and $l_{mp}$, respectively. The authors claimed that $l_m$ is much longer than $l_{mp}$ owing to difference in time scale of the magnon-conserving and magnon-non-conserving scattering processes [17].

In the Pt film, the spin current $j_s$ perpendicular to the Pt/YIG interface and the charge current $j_c$ flowing in the Pt film satisfy the following equation [17]:

$$
2e \hbar j_s = - \frac{\sigma_{SH}}{\sigma_s} j_c - \frac{\sigma_s'}{2} \nabla z \mu_s,
$$

(8)

where $\sigma_s$ and $\sigma_{SH}$ are the electrical conductivity and spin Hall conductivity, respectively. $\sigma_s'$ is defined as $\sigma_s [1 - (\sigma_{SH}/\sigma_s)^2]$. Here, we considered the relaxation-time approximation for the spin current: $\nabla z (\frac{2e}{\hbar} j_s) = \frac{\mu_s}{\tau_s}$, where $\tau_s$ is the relaxation time. From these equations, the diffusion equation for $\mu_s$ can be derived:

$$
\nabla z^2 \mu_s = \frac{\mu_s}{l_{mp}^2},
$$

(9)

where the spin diffusion length $l_{mp}$ is defined as $\sqrt{\frac{\sigma_s' \tau_s}{2}}$.
assumption, the heat source distribution induced by the SPE is not affected by \( t_{\text{YIG}} \) when \( t_{\text{YIG}} \gg l_{\text{mp}} \). In addition, the temperature distribution is also not affected by \( t_{\text{YIG}} \) when \( t_{\text{YIG}} \gg l_{\text{mp}} \), because the temperature change in the Pt/YIG sample appears at the Pt/YIG interface due to the interfacial heat resistance as shown in Fig. 7(a) [31]. The SPE is only affected by the amplitude of the heat sources near the Pt/YIG interface, which is proportional to the spin current at the Pt/YIG interface \( j_{\text{int}} \): \( \Delta T_{\text{SPE}} \propto j_{\text{int}} \). To obtain \( j_{\text{int}} \), we need to solve the diffusion equations [Eqs. (6, 9)]. As the boundary conditions for the diffusion equations, we assumed that the spin current does not flow at \( z = t_{\text{Pt}} \) and \( t_{\text{YIG}} \), \( j_s \) and \( j_m \) are connected continuously at the Pt/YIG interface: \( j_s(t_{\text{Pt}}) = 0 \), \( j_m(-t_{\text{YIG}}) = 0 \), \( j_s(+0) = j_m(-0) \), and \( j_{\text{int}} = g_{\text{SH}}[\mu_s(+0) - \mu_m(-0)] \) [17]. Under these boundary conditions, \( j_{\text{int}} \) is calculated by the following equation:

\[
\begin{align*}
\left( \frac{2e}{h} j_{\text{int}} \right) \left/ \left( \frac{\sigma_{\text{SH}} j_c}{\sigma_e} \right) \right. &= \frac{2 \tanh \left( \frac{\mu_0 l_{\text{Pt}}}{2e} \right)}{\frac{\sigma_e'}{\sigma_m} - \coth \left( \frac{\mu_0 l_{\text{Pt}}}{2e} \right) - \tanh \left( \frac{\mu_0 l_{\text{Pt}}}{2e} \right) - \frac{\sigma_e'}{\sigma_m} \coth \left( \frac{\mu_0 l_{\text{Pt}}}{2e} \right),}
\end{align*}
\]

Finally, from the proportional relation between \( \Delta T_{\text{SPE}} \) and \( j_{\text{int}} \), we obtained the temperature change generated by the SPE:

\[
\Delta T_{\text{SPE}} \propto \frac{1}{C \coth (t_{\text{YIG}}/l_{\text{mp}}) - C + 1}.
\]

where \( C \) is a \( t_{\text{YIG}} \)-independent constant.

B. The \( t_{\text{YIG}} \) dependence of the SPE based on Ref. [31]

In Ref. [31], the authors formulated the SSE and SPE based on non-equilibrium thermodynamics. In this formalism, a non-equilibrium parameter in materials was described by a non-equilibrium component of the magnetic field defined as \( H^* = H - H_{\text{eq}} \), where \( H \) and \( H_{\text{eq}} \) are the magnetic field and equilibrium magnetic field, respectively. The system is also characterized by the position-dependent temperature \( T \). The authors derived that magnetization and heat currents are driven by the gradients of \( H^* \) and \( T \) and constructed theories for the SSE and SPE.

In the YIG film, the magnetization current \( j_M \) and heat current \( j_q \) are described by the following equation [31]:

\[
\begin{align*}
\left( \frac{j_M}{j_q} \right) &= \left( \frac{\sigma_M}{\sigma_{\text{SH}}} T \kappa + \epsilon_{\text{M}}^2 \frac{\sigma_M}{\sigma_{\text{SH}}} T \right) \left( \frac{\mu_0}{\sigma_e} \nabla_j H^* - \nabla_j T \right),
\end{align*}
\]

where \( \sigma_M, \epsilon_{\text{M}}, \) and \( \kappa \) are the spin conductivity, absolute thermomagnetic power coefficient, and thermal conductivity, respectively. Notably, the heat current is divided into the non-magnetic heat current \( j_{q,\text{NM}} = -\kappa \nabla_j T \) and the magnetic heat current \( j_{q,\text{M}}: j_q = j_{q,\text{NM}} + j_{q,\text{M}} \). The authors assumed that \( j_{q,\text{M}} \) is proportional to \( j_M \) with the proportionality factor of \( \epsilon_{\text{M}} T \). In the case of the SPE, because the total heat generated by the SPE is zero, the continuity equation can be used: \( \nabla j_q = 0 \). In addition, because the system far from the Pt/YIG interface is a nearly equilibrium state, it can be assumed that the heat current vanishes at the end of the YIG \( j_q(-t_{\text{YIG}}) = 0 \).

Under these conditions, the following simple equations can be derived:

\[
\begin{align*}
\nabla_j T &= j_M / (\hat{\epsilon}_M \hat{\sigma}_M),
\hat{j}_M &= \hat{\sigma}_M \hat{\mu}_0 \nabla_j H^*,
\end{align*}
\]

where \( \hat{\epsilon}_M = \epsilon_M \kappa \), \( \hat{\sigma}_M = \sigma_M \kappa \), and \( \kappa_M = \epsilon_M^2 \sigma_M T \). Here, we considered the relaxation-time approximation for the magnetization current: \( \nabla_j j_M = H^*/\tau_{\text{YIG}} \), where \( \tau_{\text{YIG}} \) is the relaxation time for YIG. From these equations, the diffusion equation for \( H^* \) can be derived:

\[
\nabla_j H^* = H^*/\tau_{\text{YIG}},
\]

where \( \hat{\sigma}_M \) is the relaxation time. In the Pt film, the magnetization current \( j_M \) perpendicular to the Pt/YIG interface and the charge current \( j_c \) flowing in the Pt film satisfy the following equation [31]:

\[
\nabla_j H^* = H^*/l_{\text{Pt}}^2,
\]

where \( l_{\text{Pt}}^2 = \sqrt{\sigma_M \hat{\mu}_0 t_{\text{Pt}}} \).
FIG. 7: Numerical simulations of the temperature change, $\Delta T$, induced by dipolar heat sources in a one-dimensional model of a Pt/YIG/GGG junction. The bottom of the GGG substrate and top of the Pt film are in contact with the heat bath and air, respectively. As a simple model for the SPE, we set a dipolar heat source, $\pm Q$, near the Pt/YIG interface. The positive (negative) heat source $+Q$ ($-Q$) is uniformly placed in Pt (an area with a thickness, $t_Q$, in YIG). $t_{Pt} = 5 \, \mu m$, $t_{YIG} = 100 \, \mu m$, and $t_{GGG} = 500 \, \mu m$ are the thicknesses of Pt, YIG, and GGG, respectively. We set interfacial heat resistances to the Pt/YIG and YIG/GGG interfaces. The details of calculation conditions are mentioned in Refs. [4] and [6]. $\Delta T_{SPE}$ ($\Delta T_{YIG}$) is defined as the temperature change between the top of Pt and bottom of GGG (the top and bottom of YIG). (a) Spatial profile of $\Delta T$ when $t_Q$ is set to 5 nm, which corresponds to the physical picture derived from Ref. [17]. (b) Spatial profile of $\Delta T$ when $t_Q$ is set to 500 nm, which corresponds to the physical picture derived from Ref. [31].

By using the above equations for the magnetization and heat currents, we calculated the temperature change $\Delta T_{SPE}$ induced by the SPE. In this formalism, because $j_{q,M}$ is proportional to $j_M$, the heat source generated in YIG by the SPE is broadened in the length scale of the magnon diffusion length of micrometers. In contrast, the heat source generated in Pt is confined in the thin Pt film with the thickness $t_{Pt}$ of 5 nm in our experiment. In this condition, the temperature change in the Pt/YIG sample practically appears in YIG [Fig. 7(b)]. Therefore, we assumed that $\Delta T_{SPE} \simeq \Delta T_{YIG}$, where $\Delta T_{YIG}$ is the temperature change generated in the YIG film [see a numerical calculation result shown in Fig. 7(b)]. Then, we can calculate $\Delta T_{SPE}$ by the following equation:

$$\Delta T_{SPE} \simeq \Delta T_{YIG} = \int_{-t_{YIG}}^{0} \nabla_z T(z) dz = \frac{1}{\epsilon_M \sigma_M} \int_{-t_{YIG}}^{0} j_M(z) dz,
$$

where Eq. (13) was used. To obtain $j_M$, we need to solve the diffusion equations [Eqs. (15), (17)]. As the boundary conditions for the diffusion equations, we assumed that the magnetization current does not flow zero at $z = t_{Pt}$ and $-t_{YIG}$, and $H^*$ and $j_M$ are continuous at the Pt/YIG interface: $j_M(t_{Pt}) = 0$, $j_M(-t_{YIG}) = 0$, $H^*(+0) = H^*(-0)$, $j_M(+0) = j_M(-0)$. Under these boundary conditions, $j_M$ is calculated as the following equation [31]:

$$j_M(z) = -j_0 \left[ \sinh(z/t_{YIG}) \coth(t_{YIG}/\tilde{t}_{YIG}) - \cosh(z/t_{YIG}) \right],
$$

$$j_0 = \frac{j_{MS} \cosh(t_{Pt}/t'_{Pt}) - 1}{j_{MS} \cosh(t_{Pt}/t'_{Pt}) + r_{12} \sinh(t_{Pt}/t'_{Pt}) \coth(t_{YIG}/\tilde{t}_{YIG})},
$$

$$j_{MS} = -\theta_{SH} \left( \frac{\mu_B}{e} \right) j_e,
$$

$$j_{MS} = -\theta_{SH} \left( \frac{\mu_B}{e} \right) j_e,
$$

(18)
where \( r_{12} = \langle \hat{l}_{PM}\sigma_{PM} \rangle / \langle \hat{l}_{FI}\sigma_{FI} \rangle \). Finally, by substituting Eq. [19] in Eq. [18], we obtain the temperature change

\[
\Delta T_{FI} = \frac{[\cosh (t_{PM}/\hat{l}_{PM}) - 1] \left[ \cosh \left( \frac{t_{YIG}}{\hat{l}_{YIG}} \right) - 1 \right]}{\cosh (t_{PM}/\hat{l}_{PM}) \sinh \left( \frac{t_{YIG}}{\hat{l}_{YIG}} \right) + r_{12} \sinh (t_{PM}/\hat{l}_{PM}) \cosh \left( \frac{t_{YIG}}{\hat{l}_{YIG}} \right)} \left[ \cosh \left( \frac{t_{YIG}}{\hat{l}_{YIG}} \right) - 1 \right],
\]

\[
\propto \frac{1}{D \sinh \left( \frac{t_{YIG}}{\hat{l}_{YIG}} \right) + (1 - D) \cosh \left( \frac{t_{YIG}}{\hat{l}_{YIG}} \right)},
\]

where \( D \) is a \( t_{YIG} \)-independent constant. In the main text, we replaced the symbol \( \hat{l}_{YIG} \) with \( l_m \) for simplicity.

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