Experimental demonstration of continuous quantum error correction

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The storage and processing of quantum information are susceptible to external noise, resulting in computational errors that are inherently continuous[1]. A powerful method to suppress these effects is to use quantum error correction[2–4]. Typically, quantum error correction is executed in discrete rounds where errors are digitized and detected by projective multi-qubit parity measurements[5, 6]. These stabilizer measurements are traditionally realized with entangling gates and projective measurement on ancillary qubits to complete a round of error correction. However, their gate structure makes them vulnerable to errors occurring at specific times in the code and errors on the ancilla qubits. Here we use direct parity measurements to implement a continuous quantum bit-flip correction code in a resource-efficient manner, eliminating entangling gates, ancilla qubits, and their associated errors. The continuous measurements are monitored by an FPGA controller that actively corrects errors as they are detected. Using this method, we achieve an average bit-flip detection efficiency of up to 91%. Furthermore, we use the protocol to increase the relaxation time of the protected logical qubit by a factor of 2.7 over the relaxation times of the bare comprising qubits. Our results showcase resource-efficient stabilizer measurements in a multi-qubit architecture and demonstrate how continuous error correction codes can address challenges in realizing a fault-tolerant system.

A successful quantum error correction (QEC) code decreases logical errors by redundantly encoding information and detecting errors in a more complex physical system. Such a system includes both the qubits encoding the logical quantum information and the overhead resources to perform stabilizer measurements. In a fault-tolerant QEC code, the benefit from error correction needs to outweigh the cost of extra errors associated with this overhead. In the past decade, discrete QEC has been realized in various physical systems such as ion traps[7–9], defects in diamonds[10], and superconducting circuits[11–17]. The stabilizer measurements in these realizations are a dominant source of error [17] because they are indirect and require extra resources, including ancillas and entangling gates.

We demonstrate an alternative form of QEC known as continuous QEC in which continuous stabilizer measurements eliminate the cycles of discrete error correction as well as the need for ancilla qubits and entangling gates[18–20]. Continuous measurements have previously been used to study the dynamics of wavefunction collapse and, with the addition of classical feedback, to stabilize qubit trajectories and correct for errors in single qubit dynamics[21–23]. In systems of two or more qubits, direct measurements of parity can be used to prepare entangled states through measurement[24–29]. Here, we use two direct continuous parity measurements and associated filtering[30] to correct bit-flip errors while maintaining logical coherence. Errors are detected on a rolling basis, with the measurement rate as the primary limitation to how quickly errors are detected.

We realize our code in a planar superconducting architecture using three transmons as the bare qubits. As depicted in Fig. 1, we implement the ZZ parity measurements using two pairs of qubits coupled to joint readout resonators[29, 31]. Each resonator is coupled to its associated qubits with the same dispersive coupling $\chi_i$ with $i$ indexing the resonator, thereby making the resonator reflection response when the associated qubit pair is in $|01\rangle$ identical to the response when the pair is in $|10\rangle$. For each resonator, we set the probe frequency to be at the center of this shared odd parity resonance. To approximately implement a full parity measurement, we make the line-width $\kappa_i$ (636 kHz, 810 kHz) of each resonator smaller than its respective dispersive shift $\chi_i$ (2.02 MHz, 2.34 MHz). When the qubit pair is in either $|00\rangle$ or $|11\rangle$, the resonance frequency is sufficiently detuned from the odd parity probe tone to keep the cavity population low and the reflected phase responses for the two even states nearly identical. After reflecting a parity tone off a cavity, the signal is amplified by a Josephson Parametric Amplifier[32] in phase-sensitive mode aligned with the informational quadrature.

We implement the three qubit repetition code using two ZZ parity measurements as stabilizers: $Z_0Z_1$ and $Z_1Z_2$, with $Z_j$ being the Pauli $Z$ operator on qubit
j. The codespace can be any of the four subspaces with definite stabilizer values, so we choose the subspace with positive parity values (+1, +1) for simplicity. This choice of codespace is spanned by the logical code states $|0_L\rangle = |000\rangle$ and $|1_L\rangle = |111\rangle$. The three remaining possible stabilizer values identify error subspaces in which a qubit has a single bit-flip ($X$) error relative to the codespace. A change in parity heralds that the logical state has moved to a different subspace with a different logical state encoding.

Ideal strong measurements of both code stabilizers project the logical state into either the original codespace or one of the error spaces, effectively converting analog errors to correctable digital errors. In contrast, measurements with a finite rate of information extraction, like the homodyne detection used in this experiment, result in the qubit state undergoing stochastic evolution such that the logical subspaces are invariant attractors[33]. The observer receives noisy voltage traces with mean values that are correlated to stabilizer eigenvalues and variances that determine the continuous measurement collapse timescales. Monitoring both parity stabilizers in this manner suppresses analog drifts away from the logical subspaces, while providing a steady stream of noisy information to help identify and correct errors that do occur.

First we experimentally investigate how to extract parity information from such noisy voltage traces. Previous work has shown that Bayesian filtering is theoretically optimal [30, 34]. Here, we implement a simpler technique with performance theoretically comparable to that of the Bayesian filter while using fewer resources on our FPGA controller[30]. We first filter the incoming voltage signals with a 1536 ns exponential filter to reduce the noise inherent from measuring our system with a finite measurement rate (0.40 MHz) and call this signal $V_i(t)$ for resonator $i$. We normalize $V_i(t)$ such that $\langle V_i(t) \rangle = -1$ corresponds to the system being in an odd parity state, and $\langle V_i(t) \rangle = 1$ corresponds the the system in an even parity state. Here we have defined expectation values as averaging over all possible noise realizations. As shown in Fig. 2a, we monitor the trajectories of $V_i$ for signatures of bit-flips using a thresholding scheme[30, 35, 36]. Supposing we prepare an even-even parity state, a bit-flip on one of the outer qubits is detected when one of the signals goes lower than a threshold $\Theta_1 = -0.50$ while the other signal stays above another threshold, $\Theta_2 = 0.72$. A flip of the central qubit is detected when both signal traces fall below a threshold $\Theta_3 = -0.39$. These thresholds are numerically chosen based on experimental trajectories to maximize detection efficiencies of flips while minimizing dark counts and misclassification errors due to noise. When a thresholding condition is met, the controller sends out a corrective $\pi$-pulse to the qubit on which the error was detected. The controller also performs a reset operation on the voltage signals in memory to reflect the updated qubit state. As shown in Fig. 2b, when a deterministic flip is applied to the $|000\rangle$ state, the system is reset back to $|000\rangle$ faster with feedback than through natural $T_1$ decay.

To characterize the code, we first check the ability of the controller to correct single bit-flips. We prepare the qubits in $|000\rangle$ and apply the parity readout tones for 16$\mu$s. After 4$\mu$s of readout to let the resonators reach
steady state, we apply a $\pi$-pulse to one of the qubits, inducing a controlled error. We record if and when the controller detects the error and sends out a correction pulse. Errors are successfully detected on $Q_0$ with 90% efficiency, $Q_1$ with 86% efficiency, and $Q_2$ with 91% efficiency. The primary source of inefficiency is $T_1$ decay bringing the qubits back to ground before detection can happen. On average, the controller corrects an error 3.1-3.4 $\mu$s after the error occurs, with the full probability density function over time shown in Fig. 3a. We also characterize a dark count rate for each flip variety by measuring the rate at which the controller detects a qubit flip after preparing in the ground state (3.4, 1.0, 4.0) ms$^{-1}$. In comparison, the thermal excitation rates for each qubit are estimated to be (1.8, 1.0, 2.0) ms$^{-1}$.

We next investigate the dominant source of logical errors while running the code: two bit flips occurring in quick succession. When two different qubits flip close together in time relative to the inverse measurement rate, the controller may incorrectly interpret the signals as an error having occurred on the unflipped qubit. The controller then flips this remaining qubit, resulting in a logical error. For continuous error correction, this effect results in a time after an error occurs we call the dead time, when a following error cannot be reliably corrected. To characterize this behavior, we prepare the system in the ground state and apply two successive bit-flips with different times between the pulses. We then check if the controller responds with the right sequence of correction pulses. In Fig. 3b, we show the controller’s interpretation of successive flips on $Q_0$ and $Q_2$ as a function of time between them. We mark the dead time at the point where the probability of a logical error crosses the probability of successfully correcting the state. Among the possible pairs and orderings of two qubit errors, the dead times vary from 1.6-2.6 $\mu$s.

Although the code is designed to correct bit-flip errors, the code will also protect the logical computational basis states against qubit decay, extending the $T_1$ lifetimes of the logical system beyond that of the bare qubits. As opposed to a bit-flip, a qubit decaying loses any coherent phase of the logical state, and the system will be corrected to a mixed state with the same probability distribution in the computational basis as the initial state. For example, the state $\frac{1}{\sqrt{2}}(|0_L\rangle + |1_L\rangle)$ undergoing a qubit decay and correction will be restored as the density matrix $\frac{1}{2}(|0_L\rangle \langle 0_L| + |1_L\rangle \langle 1_L|)$. In the long time limit of active feedback, the system will reach a steady state described by a mixed density matrix with the majority of population (87-99.6%) in the selected codespace. The $T_1$ of a codespace is defined by the exponential time constant at which population of computational basis states in the codespace approach this steady state. The different codespaces of different parities have different $T_1$ decay times, with the longest decay time of 66 $\mu$s associated with the odd-odd subspace, as shown in Fig. 3c. The shortest lifetime, 32 $\mu$s, is associated with the even-even subspace, since the higher energy level in this codespace has three bare excitations and the lower energy has no excitations. In comparison, the bare $T_1$ values of the bare qubits range from 20-24 $\mu$s, making the logical qubit excited life 2.7 times longer than that of a bare qubit.

Although phase errors are not protected against by this code, an ideal implementation of a bit-flip code should not increase their occurrence rate. However, with our physical realization of continuous correction, we induce extra dephasing in the logical subspace through three primary channels: continuous dephasing due to the measurement tone; dephasing when going from an odd parity subspace to an even parity subspace; and dephasing related to static $ZZ$ interactions intrinsic to the chip design.
The coherence of the logical state is expected to contract rings down to the steady state for the even subspace. In the odd parity state, the resonator frequency shifts and compared to when the qubits have even parity. If one of a pair of qubits switches from an odd parity state to a steady state with a larger number of photons as compared to when the qubits have even parity. The parity measurement distinguishability is calibrated such that the resonator's associated coherent state. From the steady state dephasing rates and the resonator parameters, we independently estimate the photon number. Since \( \bar{n} \) is one of the four possible logical encodings. With the measurement tone on, but without feedback, we apply a pulse on one (or none) of the qubits, taking the state to a different (or the same) codespace, \( L \). We then tomographically reconstruct the magnitude of the coherence in the new codespace, \( |\rho_{01}^L| \), as shown in Fig. 4b. The coherences are normalized to the \( |\rho_{01}^L| \) generated by same experiment with the measurement tones off. The system demonstrates significantly less coherence when one of the parities changes from odd to even than vice versa, with reasonable agreement to the expected dephasing based on measured photon number. Since \( \bar{n} \) scales inversely with \( \kappa \) for a fixed measurement rate, a larger kappa would reduce this effect.

The third source of excess dephasing is related to static ZZ interactions among the qubits and the uncertainty in timing between when a bit-flip error occurs and when the correction pulse is applied. Performing a Ramsey sequence on \( Q_j \), where \( Q_j \) is either in the ground or excited state, we measure the coefficients of the system’s intrinsic ZZ Hamiltonian, \( H_{ZZ} = \frac{1}{2} \sum_{i < j} (Z_i Z_j) \). Since the three qubits are in a line topology, with the joint readout resonators also acting as couplers, there is significant coupling between \( Q_0 \) and \( Q_1 \) (\( \beta_{01} = 0.49 \text{ MHz} \)) and between \( Q_1 \) and \( Q_2 \) (\( \beta_{12} = 1.05 \text{ MHz} \)) while there is almost no coupling between \( Q_0 \) and \( Q_2 \) (\( \beta_{02} < 2 \text{ kHz} \)). Due to this

The first source of excess dephasing is measurement-induced dephasing, where the dephasing rate is proportional to the distinguishability of different qubit eigenstates under the measurement. Distinguishability is measured as \( D_{m,n} = |\alpha_{m} - \alpha_{n}|^2 \) where \( |m\) and \( |n\) are different basis states of the two qubits coupled to resonator \( i \), and \( \alpha_{i} \) is the resonator’s associated coherent state. By tuning the qubit frequencies, the dispersive shifts of the system are calibrated such that \( D_{01,10} \) are close to zero. The parity measurement distinguishability \( D_{01,11} \approx D_{01,00} \) determines the measurement-induced dephasing rate of the code. Due to finite \( \chi/\kappa \), the even subspaces are not perfectly indistinguishable, with the theoretical distinguishability ratio \( D_{00,01}/D_{00,11} \approx 4(\chi/\kappa)^2 \). We use this formula to calculate distinguishability ratios of 40 and 33 for resonator 0 and 1 respectively. We plot the measured distinguishability of various state pairs in Fig. 4a, and find agreement with these predicted values as well as low distinguishability between eigenstates of odd parity. This distinguishability could be lowered even further by increasing the ratio \( \chi/\kappa \).

The second source of excess dephasing occurs when a pair of qubits switches from an odd parity state to an even parity state. When two qubits coupled to one of the resonators have odd parity, the resonator is resonantly driven by the measurement tone and thus reaches a steady state with a larger number of photons as compared to when the qubits have even parity. If one of these qubits undergoes a bit-flip while the system is in an odd parity state, the resonator frequency shifts and the system undergoes excess dephasing as the resonator rings down to the steady state for the even subspace. The coherence of the logical state is expected to contract by a factor of \( e^{-\bar{n}} \), with \( \bar{n} \) being the steady state photon number of a resonator when its qubits are in an odd parity state. The system, when the qubits are in the odd state. To measure this effect, we prepare a 3-qubit logical encoding of an X-eigenstate, \( |+X_L⟩ = \frac{1}{\sqrt{2}}(|0_L⟩ + |1_L⟩) \), where \( L' \) is one of the four possible logical encodings. With the measurement tone on, but without feedback, we apply a pulse on one (or none) of the qubits, taking the state to a different (or the same) codespace, \( L \). We then tomographically reconstruct the magnitude of the coherence in the new codespace, \( |\rho_{01}^L| \), as shown in Fig. 4b. The coherences are normalized to the \( |\rho_{01}^L| \) generated by same experiment with the measurement tones off. The system demonstrates significantly less coherence when one of the parities changes from odd to even than vice versa, with reasonable agreement to the expected dephasing based on measured photon number. Since \( \bar{n} \) scales inversely with \( \kappa \) for a fixed measurement rate, a larger kappa would reduce this effect.
coupling, the definite parity subspaces have different energy splittings: In the rotating frame of the qubits, the odd-odd, odd-even, even-odd, and even-even subspaces have logical energy splittings of 0, $\beta_{12}$, $\beta_{01}$, and $\beta_{01} + \beta_{12}$ respectively. When a bit-flip occurs, the system jumps to an error space and precesses at the frequency of that error space until being corrected by the controller. Since the time from the error flip to the correction pulse is generally unknown, the state can be considered to have picked up a random unknown relative phase. The net dephasing $\zeta_{zz}$ can be calculated by averaging the potential phases over the probability distribution of time, $T$, it takes to correct an error: $e^{i\phi_0} = \langle e^{iT\Delta\beta} \rangle_T$ with $\Delta\beta$ being the energy difference between codespace and error space. Using the distributions in Fig. 3a and known $\Delta\beta$, we compute $\zeta_{zz}$ to be from 2.5 to 5.7 depending on the codespace and the qubit flipped. Although we don’t observe this dephasing directly, we perform an experiment to capture this effect. For each of the codespaces, we prepare a $|+X_L\rangle$ state in the odd-odd codespace and induce a bit-flip error while the feedback controller is active. After 6 $\mu$s, we perform tomography on all three qubits and note the time at which the correction pulse occurred. We then reconstruct the logical coherence element $\rho^{L}_{01}$ of the density matrix conditional on time it took the controller to apply the correction pulse. As shown in Fig. 4c, we observe oscillations with frequency corresponding to the effects of $ZZ$ coupling. This source of dephasing is not intrinsic to the protocol, and can be mitigated by reducing the $ZZ$ coupling between the qubits.

Our experiment extends the capabilities of continuous measurements, demonstrating active feedback on multiparticle multipartite measurement operators. We use continuous quantum error correction to detect bit flips and extend the relaxation time of a logical state. Furthermore, the protocol is implemented in a planar geometry and compatible with existing superconducting qubit architectures so can in principle be combined with other error correction methods. Future improvements could be made by reducing spurious decoherence effects through novel implementations of continuous parity measurements or optimizing coupling parameters. Specifically, increasing $\chi/\kappa$ and increasing $\kappa$ will reduce dephasing for a given measurement rate. Furthermore, lowering the static $ZZ$ coupling will reduce the observed dephasing from an error occurring at an indeterminate time. Additional feedback could be used to reduce the effects of measurement induced dephasing. By incorporating more qubits and continuous $XX$ measurements, this scheme could be extended to stabilize fully protected logical states.

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**Author contributions** E.F., M.S.B., and W.P.L. conceived the experiment. W.P.L. and E.F. designed the chip. W.P.L fabricated the chip, constructed the experimental setup, performed measurements, and analysed data with assistance from M.S.B. J.D. and A.N.J. provided theoretical support. W.P.L. wrote the manuscript with feedback from all authors. All work was carried out under the supervision of I.S.

**Competing interests** The authors declare that they have no competing financial interests.
**METHODS**

**Design and fabrication** The microwave properties of the chip were simulated in Ansys high-frequency electromagnetic-field simulator (HFSS), and dispersive couplings were simulated using the energy participation method with the python package pyEPR[42]. Resonators, transmission lines, and qubit capacitors were defined by reactive ion etching of 200 nm of sputtered niobium on a silicon wafer. Al-AlOx-Al Josephson junctions were added using the bridge-free “Manhattan style” method[43]. The junctions were then galvanically connected to the capacitor paddles through a bandaid process[44]. The middle qubit is fixed frequency, and the outer two qubits are tunable with a tuning range of 260 MHz and 220 MHz. Wire bonds join ground planes across the resonators and bus lines.

**Measurement setup** A wiring diagram of our experimental setup is show in Supplementary Information Figure 1. The Josephson Parametric Amplifiers (JPAs) are fabricated with a single step using Dolan bridge Josephson junctions. They are flux pumped at twice their resonance frequency, providing narrow-band, phase-sensitive amplification. The signals are further amplified by two cryogenic HEMT amplifiers, model LNF4.8. In the output chain for resonator 0, we include a TWPA between the JPA and the HEMT to operate that JPA at a lower gain. Infrared filters on input lines are made with an Ecocarb dielectric. The outer qubits are flux tuned with off-chip coils. The FPGA board provides full control of the qubits and readout of the resonators. An external arbitrary waveform generator creates the cavity tones and JPA drives, as well as triggering the FPGA. The JPA modulation tone is split with one branch phase shifted before both go into an IQ mixer for single sideband modulation.

**FPGA Logic** The FPGA board we used for the feedback is an Innovative Integration X6-1000M board. We programmed a custom pulse generation core to drive qubit pulses and to demodulate and filter incoming readout signals. A control unit parses instructions loaded in an instruction register. These instructions may include 1) putting a specified number of pulse commands into a queue to await pulse timing; 2) resetting a pulse timer keeping track of time within a sequence while incrementing a trigger counter; and 3) resetting the pulse timer, the trigger counter, and the instruction pointer. When a pulse instruction enters the timing queue, it waits until a specified time and is then sent to one of three different possible locations. The first possible location is a pulse library where the instruction points to a complex pulse envelope of a given duration, which is then modulated by one of three CORDIC sine/cosine generators and sent to the correct DAC. These pulses are sent down one of three qubit control lines. The second possible location is to one of the CORDIC sine/cosine generators, where the instruction will increment the phase of the generator by a specified argument, thus implementing Z rotations in the qubit frame. The third location is a demodulation core, which, similarly to the qubit pulse block, retrieves a complex waveform from memory for a specified duration. This waveform is then multiplied against the complex incoming readout signals and low-pass filtered with a 32 ns exponential filter to generate the signal $V_i^{DC}$ for feedback as well as to readout projective measurements.

When the feedback control unit is active, it takes $V_i^{DC}$, applies a secondary 1536 ns exponential filter/accumulator to further reduce the noise, and then continuously checks these traces ($V_i$) against the threshold conditions for an error to have been detected. When an error is detected, the controller injects instructions for a corrective $\pi$-pulse into the pulse generation unit. Any voltage $V_i$ which went across a threshold is then inverted as to not trip further corrective pulses. After a delay such that the corrective pulse has taken effect on chip, the $V_i^{DC}$ is inverted before being accumulated into $V_i$. In conjunction with the previous inversion of $V_i$, this effectively resets the feedback controller while avoiding interpreting the corrective pulse as another error.

The board’s I/O comprises the PCIe slot for exchanging data with the computer and the ADC/DACs on the analog front-end. The FPGA can stream from multiple sources to the computer along 4 data pipelines. The primary sources are $V_i^{DC}$ and a list of timestamped pulse commands. The timing of any corrective pulses can be obtained from this second source. Further data sources include raw ADC voltages, raw DAC voltages, and $V_i$, which are only used as diagnostics. On the analog front-end, there are two ADCs running at 1 GSa/s which take in the IF readout signals from the I and Q ports of an IQ mixer, treating the two ADC inputs as the real and imaginary parts of a complex signal. To drive the three qubit lines, there is one DAC running at 1 GSa/s and, due to board constraints, two DACs running at 500 MSa/s.

**Optimizing Filter Parameters** To optimize threshold values, we prepare the ground state and then flip either one or none of the qubits while taking parity traces ($V_i^{DC}$). In post processing, we filter the traces with the same exponential filter as on the FPGA to recreate $V_i$, and classify the resultant traces according to whether or not they pass the different thresholds registering as a qubit flip. We thus get a confusion matrix $P_{ij} = P(i|j)$, the probability of classifying a trace as a flip on $i$ given a preparation flip $j$, where $i, j \in \{\text{None},0,1,2\}$. The thresholds were chosen to minimize $\sum_{ij} (P_{ij} - \delta_{ij})^2$. 
We construct partial Pauli expectation values such as \( \langle X^+Y^-I \rangle \), with \( P^+, P^- \) being the plus and minus projectors for a particular Pauli \( P \) such that \( P = P^+ - P^- \). We then apply readout correction on these probabilities to mitigate the effects of readout infidelity. From this corrected data taken over many tomographic sequences, we can reconstruct full Pauli expectation values such as \( \langle X Y T \rangle \). When reconstructing logical coherences, we only measure in the \( X \) and \( Y \) bases. When reconstructing populations, we only measure in the \( Z \) basis.

### Ramsey heralding

Qubits 0 and 2 demonstrate a strong temporal bistability in qubit frequency, with a splitting of about 80 kHz and a typical switching time on the order of 1–10 s. When taking data to reconstruct logical coherences, we include five extra sequences in our AWG sequence table, each consisting of five repeated restless Ramsey measurements with free precession times of 6 μs. With a typical initial sequence length of 64 and a repetition rate of 100 μs, the qubit’s frequency state is sampled every 7 ms, allowing us to herald data runs to only include data from runs when the qubits have a particular frequency.

### Steady state dephasing

Here we derive relative dephasing rates for two qubits in a dispersive parity measurement using a classical analysis of the resonator steady states. The measurement dephasing rate is proportional to the distinguishability of resonator responses when the coupled qubits are in different eigenstates[45]. We set the probe frequency on resonance with the cavity when qubits are in the single-excitation subspace and assume that \( \chi \gg \kappa \). We also assume the external cavity coupling is much larger than the internal cavity loss, so the cavity responds with the following scattering parameter:

\[
S(f_0 := \chi (Z_0 + Z_1)) = \frac{-2f_0 + i\kappa}{-2f_0 - i\kappa} \tag{S.1}
\]
Odd parity states are perfectly indistinguishable. The distinguishability between states of opposite parity is

\[ D_{01,00} = |S(0) - S(2\chi)|^2 = \left| (-1) - \frac{-4\chi + i\kappa}{-4\chi - i\kappa} \right|^2 \approx 4 \tag{S.2} \]

With \( z = 4\chi + i\kappa \), the distinguishability between the two even parity states is:

\[ D_{11,00} = |S(-2\chi) - S(2\chi)|^2 = \left| \frac{4\chi + i\kappa}{4\chi - i\kappa} - \frac{-4\chi + i\kappa}{-4\chi - i\kappa} \right|^2 \]

\[ = \left| \frac{z - \bar{z}}{z + \bar{z}} \right|^2 = \left| \frac{(z + \bar{z})(z - \bar{z})}{|z|^2} \right|^2 \]

\[ = \left( \frac{8\chi(2\kappa)}{16\chi^2 + \kappa^2} \right)^2 \approx \left( \frac{\kappa}{\chi} \right)^2 \tag{S.3} \]

From these equations, we get the following relative dephasing (\( \Gamma \)) and measurement rates (\( \Gamma^m \)) between states of different parity and states of even parity:

\[ \frac{\Gamma_{01,00}}{\Gamma_{11,00}} = \frac{\Gamma_{01,00}^m}{\Gamma_{11,00}^m} \approx \frac{4\chi^2}{\kappa^2} \tag{S.4} \]

E. Dynamic Dephasing

When the resonator is not at steady state, one can have significantly increased dephasing rates after a parity flip. Here we will consider the effect of a bit flip error taking an odd parity qubit state to an even parity state while the parity measurement is on. In this case, the measurement tone is on resonance with the cavity and the cavity field will initially be in a steady state \( \alpha_0 \). When the qubit parity is flipped from odd to even, the cavity evolves as two copies, one for each even parity basis state (\( \alpha_{00} \) and \( \alpha_{11} \)). As a simplifying approximation, we assume the measurement tone is turned off at the moment the parity changes as to capture just the transient dynamics. There are two equivalent methods\cite{45} to calculate the net dephasing \( \zeta \). The first can be obtained by integrating the rate at which information leaves the cavity, \( \Gamma^m = \frac{2}{\kappa} |\alpha_{00} - \alpha_{11}|^2 \). The second can be obtained by integrating the rate at which the cavity dephases the qubit, \( \dot{\phi} = 4\chi \text{Im}[\alpha_{00} \alpha_{11}^*] \), with \( 4\chi \) being the frequency difference between the |00\> resonance and the |11\> resonance. Here we use the second method to simplify the calculation. We work in the rotating frame of the odd-parity resonance and define \( k \equiv \kappa / 2 - 2i\chi \) to get two cavity equations, one associated with each basis state:

\[ \dot{\alpha}_{00} = \left( 2\chi i - \frac{\kappa}{2} \right) \alpha_{00} \]

\[ \dot{\alpha}_{11} = \left( -2\chi i - \frac{\kappa}{2} \right) \alpha_{11} \tag{S.5} \]

\[ \alpha_{00}(t) = \alpha_{00} e^{-kt} \]

\[ \alpha_{11}(t) = \alpha_{00} e^{-kt} \tag{S.6} \]

\[ \zeta = \int_0^\infty 4\chi \text{Im}[\alpha_{00} \alpha_{11}^*] \, dt = 4\chi \text{Im} \left[ \int_0^\infty \alpha_{00} \alpha_{11}^* \, dt \right] \]

\[ = |\alpha_0|^2 4\chi \text{Im} \left[ \int_0^\infty e^{-2kt} \, dt \right] \]

\[ = |\alpha_0|^2 4\chi \text{Im} \left[ \frac{1}{2k} \right] = |\alpha_0|^2 4\chi \text{Im} \left[ \frac{2\kappa}{|2k|^2} \right] \]

\[ = |\alpha_0|^2 \frac{16\chi^2}{\kappa^2 + 16\chi^2} \approx |\alpha_0|^2 \tag{S.7} \]

Therefore, the magnitude of the final coherence between |00\> and |11\>, |\( \rho_{00,11}^f \)|, will be dephased from the initial coherence between |01\> and |10\>, |\( \rho_{01,10}^i \)|:

\[ |\rho_{00,11}^f| = e^{-\zeta} |\rho_{01,10}^i| = e^{-|\alpha_0|^2} |\rho_{01,10}^i| \tag{S.8} \]
Extended Data Figure 1. Cryogenic wiring diagram. The Josephson parametric amplifiers (JPAs) operate in reflection, and additionally have off chip coils not shown. The JPAs also provide narrow-band gain, so when the readout chains are combined at room temperature, the combined noise at each cavity frequency is dominated by the noise amplified by that cavity’s JPA. Each superconducting coil has its leads connected by a small piece of copper wire on the sample box, forming a low frequency (<1 Hz) RL filter with the coil. The room temperature wiring is also shown, but with linear elements (attenuators, amplifiers, filters, isolators) removed.
Extended Data Figure 2. Extended data plot for Fig. 3b. Pairs of qubits are flipped with varying time between flips. When different qubits are flipped, the red region represents the controller detecting a flip on the third qubit, resulting in a logical error. When the same qubit is flipped twice, the red region represents the probability of the controller not detecting a flip (which is not a logical error). Blue and purple regions represent the probability of a single flip being detected. The orange region represents the probability of the controller detecting some other sequence of flips. Dotted lines represent the dead time, when the red probability matches the green probability.
Extended Data Figure 3. Extended data plot for Fig. 3c. Effective $T_1$ for all four definite parity subspaces are measured and fit to an exponential. Blue traces are taken with feedback off. Orange traces are taken with feedback on. The black dotted line represents the lifetime of a bare qubit (24 µs).