Modeling the Dynamics of Supraglacial Rivers and Distributed Meltwater Flow With the Subaerial Drainage System (SaDS) Model

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Abstract  Meltwater produced at the surface of glaciers and ice sheets has important implications for basal sliding rates and therefore ice flow velocities. In order to determine the role of supraglacial water in ice dynamics and predict future changes, we first need to understand and be able to accurately predict moulin input rates. To this end, we present the Subaerial Drainage System (SaDS) model. SaDS is a dynamic model that couples supraglacial runoff in the bare-ice ablation zone in a distributed sheet with flow in discrete channels. Flow in the distributed sheet drives melt through potential energy dissipation, allowing a channel network to form naturally with no prior assumptions about channel locations. We apply the model to a synthetic ice sheet margin and carry out a suite of sensitivity tests. Modeled moulin inputs show expected behaviors including large diurnal variability, multi-hour lags following peak surface melt, and demonstrate complex and diverse seasonal dynamics. The sensitivity tests illustrate the range of possible model behaviors and constrain the parameter values for which the model predicts physically realistic moulin inputs. We also apply the model to a ~20 × 27 km\(^2\) catchment on the southwestern Greenland Ice Sheet using RACMO melt forcing and previously mapped moulin locations. Modeled supraglacial lake and stream locations match those mapped from Landsat 8 images, and moulin inputs show varied daily and seasonal dynamics. These results demonstrate that the model is a promising tool to provide moulin inputs for subglacial and ice dynamic studies.

Plain Language Summary  Throughout the summer, the surface of glaciers is covered by a network of rivers and lakes. These rivers often drain through moulins (vertical shafts from the surface to the base of the ice that let water drain through the glacier). The rate of water flow through these moulins can change the speed at which glaciers flow since water can pool underneath the glacier, allowing the glacier to slide against the bedrock. This is important because changes to glacier flow speeds can impact the rate of sea level rise. In this study we introduce a new tool, called the Subaerial Drainage System (SaDS) model, that simulates flow through glacial surface rivers and lakes in order to calculate water input into moulins. We show that SaDS more accurately represents changes in flow into moulins caused by rivers drying up or expanding than previous simulations. SaDS is a significant step forward for making realistic predictions of moulin inputs, which will be key for determining how the world’s ice sheets and glaciers change as the climate continues to warm.

1. Introduction

Large volumes of meltwater are generated each summer at the surface of melting glaciers and ice sheets (Chu, 2014; Shepherd et al., 2020). This meltwater flows across the glacier surface and can access the subglacial environment through moulins and crevasses (McGrath et al., 2011). Subglacial water determines basal effective pressure and therefore sliding rates of the overlying ice. As a result, surface meltwater volumes can exert an important control on ice dynamics (e.g., Bartholomew et al., 2011; Joughin et al., 2013; Zwally et al., 2002). However, the exact relationship between surface melt rates and glacier flow velocities is complex (Sole et al., 2013; Sundal et al., 2011). While it is clear that the seasonal pattern of glacier velocity is driven by surface meltwater (Chu, 2014; Iken & Bindschadler, 1986; Nienow et al., 2005; Stevens et al., 2021), the specific impact that increasing melt rates has on ice velocity is not easily generalized across glacier types and regions (King et al., 2020). Even for apparently similar glaciers, their response to changes in surface melt rates can be different (Chu, 2014; King et al., 2020; Moon et al., 2014). In order to determine the impacts of surface melt on the dynamics of different glacial systems, we first need an accurate assessment of the routing of meltwater across the surface and the discharge rate into moulins.
In the ablation zone, meltwater flow consists of slow flow across the bare ice surface along with fast flow through discrete supraglacial channels (Fountain & Walder, 1998; Pitcock & Smith, 2019). Meltwater can often be stored in supraglacial lakes, features that are particularly common on the Greenland Ice Sheet (Chu, 2014). Supraglacial flow is more complicated within the accumulation zone, where meltwater can be stored seasonally or perennially in firn aquifers that buffer runoff (Christianson et al., 2015; Forster et al., 2014; Koenig et al., 2014), and where percolation and refreezing within firn is important, including the formation of ice slabs (Culberg et al., 2021; MacFerrin et al., 2019). Given the availability of physical models of these accumulation zone processes, here we focus on the bare-ice supraglacial drainage system within the ablation zone.

The supraglacial drainage network is not static throughout the melt season. Supraglacial streams in the ablation zone grow and shrink according to the balance between melt due to frictional heat dissipation along the wetted perimeter, melt due to solar radiation on the banks, and ablation of the adjacent ice surface reducing the channel cross section (St Germain & Moorman, 2019). Since frictional melt is proportional to the flow rate in the channel (Koziol et al., 2017; Raymond & Nolan, 2000), this balance means that small streams with low flow rates will melt out throughout the melt season, while larger streams with high flow rates will persist throughout the melt season and often perennially reform in the same location (St Germain & Moorman, 2019; Yang et al., 2018). Under certain conditions, streams can incise deeply enough for the channel opening to pinch closed due to ice deformation, forming an englacial conduit (Jarosch & Gudmundsson, 2012). These seasonal changes in the channel network lead to a temporal pattern in river discharge and moulin inputs that changes throughout the melt season (St Germain & Moorman, 2019; Yang et al., 2018), and therefore have the potential to impact subglacial effective pressure and basal sliding (Moon et al., 2014; Sole et al., 2011; Sundal et al., 2011).

A variety of models exist to represent the supraglacial drainage system. The most common approach is to precompute flow paths from a digital elevation model using a flow routing algorithm and travel times using velocity parameterizations (e.g., Arnold et al., 1998; Banwell et al., 2012; Clason et al., 2015, 2012; Smith et al., 2017; Yang et al., 2018). However, these models often assume the ice surface and drainage basins are static so that the flow paths do not change throughout the melt season, and usually represent distributed sheet flow only.

Alternatively, “transient” or “dynamic” models can apply conservation laws to mathematically compute supraglacial runoff, usually driven by the conservation of mass of liquid water, combined with flow parameterizations. These models can be formulated for flow in a distributed sheet (Flowers & Clarke, 2002) or for flow in supraglacial channels (also called spillway models) (Kingslake et al., 2015; Raymond & Nolan, 2000). Since transient/dynamic models are usually formulated as partial differential equations, they are usually more complex and computationally expensive than flow routing. However, their transient formulation allows them to more accurately represent the dynamics of supraglacial flow. These models focus on a single mode of flow, with the exception of Koziol et al. (2017), who combined distributed flow routing with supraglacial lake drainage through spillways.

Recent models have started to include both sheet and channel flow. Smith et al. (2017) empirically determined synthetic unit hydrographs (SUH) that account for both modes of flow, although the hydrograph is derived using synthetic, non-physically based basis functions, limiting its transferability. Yang et al. (2018) accounted for both types of flow in the empirical Rescaled Width Function (RWF) model. The RWF model relies on a parameterization that increases the interfluve distance to account for seasonal dynamics. However, it is not clear how transferable this parameterization is to other catchments or glaciers. Finally, Gleason et al. (2021) used the terrestrial Hillslope River Routing model (HRR; Beighley et al., 2009), a dynamic model that represents distributed hillslope and channelized flow to calculate supraglacial runoff in the Rio Behar catchment. However, the HRR model does not account for seasonal changes in the supraglacial river network.

Here, we introduce a new supraglacial flow model. The Subaerial Drainage System (SaDS) model is a physically based, dynamic model that represents coupled flow in both a distributed sheet and through supraglacial channels in order to calculate moulin inputs in the bare-ice ablation zone. SaDS improves on existing models by automatically developing a network of connected lakes and rivers with no prior knowledge of the location of drainage features. The model's dynamic formulation lets the drainage system evolve throughout the melt season, resulting in moulin inputs with strong seasonal dynamics.
et al., 2018). We then examine application of the model to internally drained catchments on the Greenland Ice Sheet in Section 4 prior to discussing the outputs from both synthetic and applied experiments in Section 5.

2. Model Description

SaDS largely follows the framework of the subglacial hydrology Glacier Drainage System model (GlaDS) (Werder et al., 2013). We combine slow, distributed flow across the glacier surface with discrete flow in supraglacial channels. Both systems are modeled by applying mass conservation with turbulent flow parameterizations, and with mass exchange allowed between the systems. However, the details of the model differ from GlaDS. Among the more important differences are that SaDS assumes the water pressure is hydrostatic, and that the hydraulic potential is not continuous between the distributed water sheet and in adjacent supraglacial channels. Importantly, we model the dynamic evolution of the supraglacial channel system. Channels grow as concentrated flow melts the underlying ice, and shrink as the ice surface adjacent to the channels melts and lowers. This configuration provides the flexibility required to represent seasonal changes in the partitioning between distributed and channelized runoff. Smaller streams can melt out, with the distributed sheet taking up the excess meltwater and transporting it downstream to larger supraglacial rivers.

We apply the model equations on a two-dimensional, unstructured triangular mesh (Figure 1 and Figure S1 in Supporting Information S1), where sheet flow occurs across the elements, and channel flow occurs along the edges of the triangular elements. We first describe the sheet and channel model equations as applied to each individual element and edge in the domain, and then explain how the equations are combined to represent the complete domain. Additional details of the numerical implementation are provided in Text S1 of Supporting Information S1. Model variables and their symbols are described in Table 1, and model parameters with their symbols are described in Table 2. The model’s computational requirements are shown in Table S2 of Supporting Information S1.

| Table 1 |
| --- |
| Model Variables Symbols, Descriptions, and Units |

| Variable | Description | Units |
| --- | --- | --- |
| $h_s$ | Distributed sheet water depth | m |
| $h_c$ | Channel water depth | m |
| $H_c$ | Channel incision depth | m |
| $\Sigma$ | Water-filled channel cross section | m² |
| $A_e$ | Incised cross section | m² |
| $z_s$ | Distributed sheet surface elevation | m |
| $\phi_s$ | Distributed sheet hydraulic potential | Pa |
| $\phi_c$ | Channel hydraulic potential | Pa |
| $q_s$ | Distributed sheet width-averaged flow | m²·s⁻¹ |
| $q_c$ | Channel flow | m³·s⁻¹ |
| $\Xi_s$ | Potential energy dissipation in distributed sheet | W·m⁻² |
| $\Xi_c$ | Potential energy dissipation in channels | W·m⁻² |
| $w_c$ | Channel width | m |
| $f_e$ | Exchange fraction | – |

| Table 2 |
| --- |
| Parameters (Top Group) and Physical Constants (Bottom Group) |

| Parameter | Description | Units | Default value |
| --- | --- | --- | --- |
| $\alpha_s$ | Distributed sheet flow exponent | – | 5/4 |
| $\beta_s$ | Distributed sheet flow exponent | – | 3/2 |
| $\alpha_c$ | Channel flow exponent | – | 5/3 |
| $\beta_c$ | Channel flow exponent | – | 3/2 |
| $k_s$ | Distributed sheet hydraulic conductivity | m²⁻α_s·s⁻¹ | 0.5 |
| $k_c$ | Channel hydraulic conductivity | m²⁻α_c·s⁻¹ | 10 |
| $r$ | Channel width-to-depth ratio | – | 3 |
| $\zeta$ | Exchange ratio | – | 0.2 |
| $m_s$ | Distributed sheet melt rate | m w.e. s⁻¹ | – |
| $m_c$ | Channel melt rate | m w.e. s⁻¹ | 0 |
| $\Delta t$ | Timestep | s | 240 |
| $\rho_w$ | Density of water | kg·m⁻³ | 1,000 |
| $\rho_i$ | Density of ice | kg·m⁻³ | 850 |
| $L$ | Latent heat of fusion | J·kg⁻¹ | 3.34 × 10⁵ |
| $g$ | Gravitational acceleration | m·s⁻² | 9.81 |
2.1. Sheet Model

Slow, distributed flow along the glacier surface is represented as a two-dimensional continuous water sheet in order to develop a physics-based model. We define $h_s$ to be the water sheet thickness, and $z_s$ to be the ice surface elevation (Figure S2 in Supporting Information S1). We assume the water pressure to be equal to the hydrostatic potential (Flowers & Clarke, 2002), and so the hydraulic potential is

$$\phi_s = \rho_w g z_s + \rho_w g h_s.$$  

(1)

Here, $\rho_w$ is the density of water and $g$ is gravitational acceleration. Following Flowers and Clarke (2002) and Werder et al. (2013), the sheet model is based on conservation of mass. The evolution equation for the water depth is

$$\frac{\partial h_s}{\partial t} + \nabla \cdot q_s = m_s + \frac{\Xi_s}{\rho_w L},$$  

(2)

where $L$ is the latent heat of fusion, $q_s$ is the width-averaged mass flux, $m_s$ is the surface melt source term, and $\Xi_s$ represents melt by frictional potential energy dissipation.

The width averaged mass flux is calculated using a turbulent flow parameterization,

$$q_s = -k_s h_s^{\alpha_s} \left| \nabla \phi_s \right| \frac{\rho_w g}{\rho_w g} h_s^{\beta_s - 2} \left| \nabla \phi_s \right| \frac{\rho_w g}{\rho_w g}.$$  

(3)

The flow exponents $\alpha_s > 1$ and $\beta_s > 1$ control the turbulence parameterization. For instance, $\alpha_s = 1$ and $\beta_s = 2$ would represent laminar flow. We take $\alpha_s = 5/4$ and $\beta_s = 3/2$ to represent fully turbulent flow (Werder et al., 2013). This parameterization is agnostic about whether flow occurs over impermeable ice or within a weathering crust. By adjusting the conductivity ($k_s$), the parameterization can be tuned to match the specific domain of interest. Since the flow velocity can be calculated as $v_s = |q_s|/h_s$, the conductivity can be tuned to match in-situ or assumed flow velocities. Alternatively, SaDS can be used to calculate flow velocities if the conductivity is known, and these can be compared to known values from direct measurement or other models, or used to drive flow routing models to compare modeled moulin inputs.

The potential energy dissipation term $\Xi_s$ represents the energy dissipated by friction between the water and the underlying ice, so that

$$\Xi_s = |q_s| \cdot \nabla \phi_s.$$  

(4)

The direct inclusion of $\Xi_s$ in Equation 2 assumes that the ice is at the melting point and so all of the energy dissipated by friction is used immediately to melt the underlying ice. For applications where this is not true, this energy should first be used to warm the surface ice layer, and any remaining energy should be used as a melt.
source term in Equation 2. For example, this could be achieved by adding Ξ to the net surface energy balance in a coupled surface energy balance and subsurface heat conduction model (e.g., Hill et al., 2021).

The ice surface elevation is evolved to account for surface lowering due to surface melt and melt by potential energy dissipation. The time evolution of the ice surface is therefore calculated as

\[
\frac{dz_s}{dt} = -\frac{\Xi_s}{\rho_i L} - m_s \rho_w \rho_i,
\]

(5)

where \(m_s\) is the surface melt rate and \(\rho_i\) is the density of the surface layer of ice. We assume a low ice density of 850 kg m\(^{-3}\) since this represents the density of the weathered surface layer, consistent with densities commonly used to convert geodetic volume changes to mass changes (Huss, 2012; Zemp et al., 2010). This weathered ice density is most appropriate for the distributed water sheet, however we have used a constant ice density including for the beds of supraglacial channels. Using a higher density along streams would cause channels to close faster and would slightly lower the equilibrium channel cross section. This would be most important for small channels that are on the cusp of melting out, although it is unclear whether the ice along these shallow channels is weathered or not. Therefore, we continue to use a constant ice density of 850 kg m\(^{-3}\).

### 2.2. Channel Model

The supraglacial channel model is based on the same principles as the sheet model, namely mass conservation with a turbulent flow parameterization. As in the sheet model, flow in channels melts the underlying ice. Additionally, channels are allowed to grow and shrink in both width and depth due to the balance between melt-out due to surface melt and down-cutting due to frictional melt.

As with the sheet model, the channel model begins with the hydraulic potential. Assuming hydrostatic potential we have

\[
\phi_c = \rho_w g z_c + \rho_w g (h_c - H_c),
\]

(6)

where \(z_c\) is the elevation of the lip of the channel, \(h_c\) is the depth of water in the channel, and \(H_c\) is the incision depth (the distance from the base of the channel to the lip) (Figure S2 in Supporting Information S1).

We apply conservation of mass along the channel length to describe the time evolution of the water-filled cross-sectional area in the channel. We include a mass source term from melt by potential energy dissipation, \(\Xi_c\), and a prescribed melt term at the base of channels \(m_c\). This prescribed source term could represent additional thermal erosion from the draining of lakes that have been warmed by solar radiation, for example. We set this additional melt to zero in the current work, consistent with most spillway models (e.g., Kingslake et al., 2015), but we have included it in the model description as it may be important for future applications. Therefore, the time evolution equation for the channel water-filled area \(\sigma\) is

\[
\frac{d\sigma}{dt} + \frac{\partial \phi_c}{\partial s} = w_c m_c + \frac{\Xi_c}{\rho_w L},
\]

(7)

where \(q_c\) is the volume flux in the channel, and \(w_c\) is the width of the channel at the level of the water surface.

The channel melt source term \(\Xi_c\) is calculated as

\[
\Xi_c = \left| q_c \frac{\partial \phi_c}{\partial s} \right|
\]

(8)

This term represents potential energy dissipation in the channels, which is applied as thermal energy to melt the ice along the entire channel perimeter. This term is analogous to the dissipation term in the sheet model (Equation 4). Here we have simplified the model by applying heat dissipation along the entire channel perimeter, rather than the wetted perimeter (the portion of the channel in contact with the water and excluding the portion of the channel walls exposed to the air) only. To apply heat dissipation along the wetted perimeter would cause the channel cross-sections to change over time and we would need to model this evolution by discretizing the channel perimeter.
Next, we model the evolution of the total incised channel cross-section, \( A_c \), as the balance between the net melt rate and melt due to energy dissipation,

\[
\frac{\partial A_c}{\partial t} = \frac{\Xi}{\rho_i L} + w_c \frac{\rho_w}{\rho_i} (m_c - m_i).
\]  

(9)

The elevation of the channel lip is evolved according to the rate of surface lowering in the adjacent ice sheet surface due to surface melt,

\[
\frac{dz_c}{dt} = -m_i \frac{\rho_w}{\rho_i}.
\]

(10)

We calculate volume flux in the channels using a turbulent flow parameterization,

\[
q_c = -k_c w_c h_c^\alpha \left[ \frac{1}{\rho_w g} \frac{\partial \phi_c}{\partial s} \right]^\beta - 2 \frac{1}{\rho_w g} \frac{\partial \phi_c}{\partial s}.
\]

(11)

In this equation, \( \alpha > 1 \) and \( \beta > 1 \) have the same role as \( \alpha_s \) and \( \beta_s \) in Equation 3. We take \( \alpha = 5/3 \) and \( \beta = 3/2 \), representing fully turbulent Darcy flow (Leeson et al., 2012; Raymond & Nolan, 2000; Werder et al., 2013).

To derive the equations for the evolution of the channel water thickness and incision depth from Equations 7 and 9, we assume that channels have a rectangular cross section with a constant aspect ratio. This means that \( w_c = r H_c \), where \( r \) is the constant width-to-depth ratio. Under this assumption, we derive the final equations for the evolution of the water depth and incision depth:

\[
\frac{w_c \partial h_c}{\partial t} + \frac{\partial q_c}{\partial s} = w_c \frac{\Xi}{\rho_w L} - h_c \frac{\partial w_c}{\partial t},
\]

(12)

and,

\[
\frac{dH_c}{dt} = \frac{1}{2} \frac{\rho_w}{\rho_i} (m_c - m_i) + \frac{1}{2w_c \rho_i L} \frac{\Xi}{\rho_w L}.
\]

(13)

From these two equations, combined with Equation 10, we calculate the evolution of the hydraulic potential:

\[
\frac{d\phi_c}{dt} = \rho_w g \left( \frac{dz_c}{dt} + \frac{\partial h_c}{\partial t} - \frac{dH_c}{dt} \right).
\]

(14)

### 2.3. Mass Exchange

We wish to represent the fact that flow begins at high elevations of supraglacial catchments as a distributed sheet and becomes channelized at lower elevations once sufficient flow accumulates, without assuming prior knowledge of the channel network. We model this process by allowing mass to be transferred from the distributed sheet into the channel system. We treat every interior edge in the triangular mesh equally as a possible location for a supraglacial stream and let the channel network evolve naturally according to the balance between heat dissipation and surface melt.

Mass exchange is regulated by the hydraulic potential difference between the sheet and the channels. For each element, the model calculates the projection of the sheet flow vector on the outward unit normal to each edge,

\[
q_e = q_s \cdot n.
\]

If \( q_e < 0 \), the flow in the sheet is directed inwards along the edge, and so mass is not transferred to the edge. If \( q_e > 0 \) flow is directed outwards along the edge, and we calculate the sheet potential on the element edge. We let a fraction \( f_e \) \((0 \leq f_e \leq 1)\) of the mass crossing the edge be taken up by the channel so that the total mass transferred from the element to the edge is

\[
m_e = f_e q_e.
\]

(15)
where \( l \) is the length of the edge. Dividing by the surface area of the channel, the rate of change of channel water depth due to mass exchange is

\[
\left( \frac{d h_c}{d t} \right)_{\text{exchange}} = \frac{m_e}{w_i l}.
\]  

(16)

We add this term to Equation 12, and remove this amount of mass from the element.

Any physical parameterization of the exchange fraction should satisfy two conditions. First, the channel should be hydrologically disconnected from the sheet when the water in the channel is below the sheet surface. That is, the flow from the adjacent sheet elements should freely flow into the channel. Second, the channel should overflow toward the downhill neighboring element when the water in the channel is above the water level of the neighboring sheet.

We parameterize the exchange fraction, \( f_e \), as a function of the channel incision depth \( (H_c) \), water depth in the channel \( (h_c) \), and the sheet water depth interpolated to the channel edge \( h_{sc} \). Ideally, we would transfer all the mass crossing an edge into the channel until it is exactly full, at which point zero mass would be transferred. However, such a sharp transition is too numerically unstable. Instead, the exchange fraction is equal to 1 until the water depth in the channel reaches a threshold ratio, \( \zeta \), from the top of the channel. For instance, if \( \zeta = 0.1 \), the exchange fraction is 1 until the water depth reaches the top 10% of the channel wall. We also enforce that the exchange fraction reaches 0 when the water surface height in the channel is greater than or equal to the interpolated sheet water height. This means that the systems are decoupled, so the channel system is free to evacuate the excess water and the sheet system continues routing water from high to low potential. The exchange fraction is linearly interpolated when the channel water thickness is between these extremes. Mathematically, this parameterization is:

\[
f_e = \begin{cases} 
1, & h_c \leq (1 - \zeta) H_c \\
1 - \frac{h_c - (1 - \zeta) H_c}{\zeta H_c + h_{sc}}, & (1 - \zeta) H_c < h_c < H_c + h_{sc} \\
0, & h_c \geq H_c + h_{sc}.
\end{cases}
\]  

(17)

2.4. Summary of Model Equations

The five fundamental model equations are the evolution equations for \( h_i \) (Equation 2), \( z_i \) (Equation 5), \( h_c \) (Equation 12), \( H_c \) (Equation 13), and \( \phi_e \) (Equation 14):

\[
\frac{\partial h_i}{\partial t} + \nabla \cdot \mathbf{q}_i = m_s + \frac{\Xi_s}{\rho_w L} - \frac{m_e}{A},
\]  

(18)

\[
\frac{\partial z_i}{\partial t} = -\frac{\Xi_i}{\rho_i L w_i} - m_e \frac{\rho_w}{\rho_i}
\]  

(19)

\[
\frac{\partial h_c}{\partial t} + \frac{\partial q_e}{\partial s} = w_c m_s + \frac{\Xi_c}{\rho_w L} - h_c \frac{\partial w_c}{\partial t} + \frac{m_e}{l}
\]  

(20)

\[
\frac{d H_c}{d t} = \frac{1}{2} \rho_i \left( m_s - m_e \right) + \frac{1}{2 w_c} \frac{\Xi_c}{\rho_i L}
\]  

(21)

\[
\frac{d \phi_e}{d t} = \rho_w g \left( -m_e \frac{\rho_w}{\rho_i} + \frac{\partial h_c}{\partial t} + \frac{d H_c}{d t} \right)
\]  

(22)

where \( A \) is the area of a triangular element. The flux parameterizations for \( \mathbf{q}_i \) (Equation 3) and \( q_e \) (Equation 11), and the heat dissipation parameterizations for \( \Xi_s \) (Equation 4) and \( \Xi_c \) (Equation 8) complete the system.
2.5. Numerical Methods

We solve the model equations using finite volume methods on the unstructured triangular mesh domain. The methods are described briefly here, and more detail is provided in Text S1 of Supporting Information S1.

The sheet model is a relatively standard finite volume problem, which we solve using a first-order element-centered upwind method with a second-order gradient reconstruction method. We incorporate the sheet-channel coupling term directly into the calculation of the sheet flux at element boundaries to ensure mass conservation.

For the channel model, we implement a standard first-order, one-dimensional finite volume numerical method to evolve the water thickness $h_c$. This method considers each edge independently, using the nodes as source and sink terms.

The difficulty in solving the channel equations stems from the complex arrangement of edges, where any number of edges connects to each node. To calculate the hydraulic potential gradient, we calculate the channel potential at each node by solving an inverse-distance weighted linear least squares problem. Using the resulting nodal potential, it is straightforward to calculate the along-channel potential gradients in Equation 11.

The edge network is assembled from the individual edges by enforcing mass conservation at nodes. For each node, we identify the upstream edges, and the edge with the largest downhill potential gradient, called the outlet edge. The flux in all upstream edges is summed to determine the total mass flowing into the node. If the node is a moulin or a free-flux boundary node, this mass is removed from the domain, and no flux is routed to any of the downhill edges. If the node is an interior node, all the mass flowing into the node is directed out along the outlet edge. This scheme conserves mass exactly and naturally forms a dendritic network. It is similar to the scheme used in GlaDs (Werder et al., 2013).

The model is stepped forward in time using a standard explicit fourth-order Runge-Kutta method. The timestep is chosen at run-time, and is strongly dependent on the maximum melt rate and the minimum edge length. The timestep is typically between 30 and 240 s. The convergence is evaluated by considering the maximum absolute change in the state variables ($h_c$, $H_c$, $\phi_c$, $h_s$, and $\phi_s$) over each timestep.

2.6. Boundary Conditions

Boundary conditions for the sheet model are imposed on Equation 2. We impose a hybrid boundary condition that combines Neumann and free-flux boundary conditions on the domain boundary. For each boundary element, we calculate the outward normal component of flow, $q_n$.

Where the flow is directed into the domain from the boundary ($q_n < 0$), we use a Neumann boundary condition to specify the flow entering the domain. In this study, the flux is set to zero because of our choices of domains. In future work, this flux can be used to represent additional flow from upstream of the modeled domain. Where the flow is directed toward the boundary ($q_n > 0$) we impose that the flux does not change across the boundary, which allows mass to freely leave the domain. This condition is imposed numerically by directly transferring the flux value from boundary element centroids to boundary edges. We do not need to impose conditions on the gradient in Equation 3 because our numerical method is able to calculate the potential gradient for boundary elements without specifying the value at the boundary.

For the channel model, we impose boundary conditions on the mass conservation Equation 12. For this, we partition the nodes in the domain into Neumann and free-flux groups. For Neumann nodes we impose a zero-flux condition. For free-flux nodes we impose that the total flux flowing toward the node leaves the domain. Together, these conditions allow the channel system to naturally form in the upper reaches and evacuate water through the domain outlet. As in the sheet model, we do not require conditions on the directional derivative at the boundary in the flow parameterization (Equation 11).

3. Synthetic Ice Sheet Margin Experiments

We test the model using the synthetic ice sheet margin geometry of the Subglacial Hydrology Model Intercomparison Experiment (SHMIP; de Fleurian et al., 2018). The ice sheet margin geometry represents a land-terminating segment of the margin, and is similar to that used in Werder et al. (2013). We present modeled moulin
hydrographs using standard parameters as well as a suite of sensitivity tests to characterize the model's spectrum of behaviors.

### 3.1. Ice Sheet Margin Domain

The SHMIP ice sheet margin geometry represents an idealized 100 km long, 20 km wide ice sheet margin (Figure 1). The surface elevation from de Fleurian et al. (2018) is given by

\[ z = 6 \left( \sqrt{x + 5000} - \sqrt{5000} \right) + 1. \]  

(23)

We solve the model equations on a triangular mesh with 1,205 nodes, 3,443 edges, and 2,239 elements. The mesh is refined by a factor of 10 at lower elevations, where the channel density is expected to be higher due to the greater melt rate. The maximum element area is \( 6.0 \times 10^6 \) m² at the upper boundary, and \( 6.0 \times 10^5 \) m² at the lower boundary. The minimum edge length is 625 m near the terminus.

Since our synthetic domain has no valleys and ridges to drive flow accumulation, flow instead accumulates along the channel paths that have the steepest downhill potential gradient. This makes it difficult to predict which nodes will be on flow paths. Therefore, we randomly place a large number of moulins throughout the domain, with a higher density of moulins at lower elevation. In this initial stage, we place moulins at up to 60% of nodes. To decide which moulins to keep, we apply the flow-routing stage of the channel model to predict the flow paths. Since removing a moulin impacts downstream moulins, we remove moulins one at a time. At each step, we remove the moulin with the lowest input, and then recalculate the flow paths. We continue this process until we are left with a prescribed number of moulins. For the baseline scenario, we keep 50 moulins. This corresponds to the number of moulins in SHMIP case B4.

### 3.2. Surface Melt Parameterization

We force the supraglacial model with the simple degree-day melt model used in the SHMIP experiment (de Fleurian et al., 2018), which we summarize here. By using this melt parameterization, we remove extra noise and variability that would be introduced by instead using melt forcing from regional climate simulations. We follow the SHMIP model exactly, except for the diurnal amplitude function.

The daily mean sea-level temperature \( T_0 \) (°C) is parameterized as

\[ T_0(t) = -16 \cos \left( \frac{2\pi t}{t_{\text{year}}} \right) - 5 + \Delta T, \]

where \( t_{\text{year}} \) is the number of seconds in one year and \( t \) is the current time in seconds. We take the offset \( \Delta T = 0 \), corresponding to the standard SHMIP case “D3” (de Fleurian et al., 2018). The air temperature is distributed across the domain assuming a constant and uniform lapse rate \( \Gamma = -0.0075 \) °C m⁻¹,

\[ T(z) = \Gamma z + T_0. \]

Melt \( m \) (m w.e. day⁻¹) is calculated using the degree-day model

\[ m(t) = \max \left\{ 0, [\Gamma z + T_0(t)]D(t)\text{DDF} \right\}, \]

where \( \text{DDF} = 0.01 \) m°C⁻¹ day⁻¹ is the degree day factor. \( D(t) \) is a diurnally varying melt factor, calculated as

\[ D(t) = 1 - A \cos \left( \frac{2\pi t}{t_{\text{day}}} \right), \]

where \( A \) is the relative amplitude and \( t_{\text{day}} \) is the length of 1 day in seconds (86,400 s). We have changed this diurnal factor from the sine function in the SHMIP model (de Fleurian et al., 2018) to a cosine function so that the melt rate reaches its minimum at midnight and its maximum at noon local solar time. Figure S3 in Supporting Information S1 shows the melt for the synthetic ice sheet margin geometry over the duration of the melt season. This degree day model is used as the source term for the distributed sheet (\( m_s \)).
3.3. Baseline Scenario

The baseline scenario uses default values as listed in Table 2, forced by the SHMIP melt model. To allow the supraglacial channel dimensions to adjust to balance the surface melt rate, we first run the model for one melt season as a spinup period. At the end of this period the channel dimensions are nearly in equilibrium with the rate of mass transfer from the elements, and so we use these dimensions as initial conditions for the second melt season. Since there is no topography to influence channel locations, we increase small channels to a minimum channel incision depth of 25 cm to give channels the potential to grow everywhere. This means that even if a channel melted out during the spinup period it is initialized with a 25 cm incision depth. This process represents the perennial nature of large supraglacial rivers (Pitcher & Smith, 2019; St Germain & Moorman, 2019), while the minimum depth of 25 cm represents the rapid channelization of supraglacial runoff when snow melts at the beginning of the melt season (Pitcher & Smith, 2019; Yang et al., 2018).

The modeled water sheet thickness, channel flow, and moulin discharge are shown in Figure 2. The sheet thickness is highly variable due to its interaction with the channel system. Large channels draw water out of the sheet, leaving localized regions with very little water in the distributed system. Regions with low channel density, in particular downstream of large moulins, have the most water in the distributed system.

The channel flow maps show evidence of seasonal dynamics. Early in the melt season (Figure 2b) there are numerous small channels with flow <5 m³s⁻¹. By peak melt (Figure 2d) there is significant flow (>50 m³s⁻¹) in the largest channels due to the higher melt rate. Late in the melt season (Figure 2f) the smaller channels have melted out, concentrating more flow into the large channels that remain compared to the early season map, even though the melt rate is the same.

The discharge in the largest moulins peaks at almost 70 m³s⁻¹. This discharge is high, but not unprecedented, as Echelmeyer and Harrison (1990) reported river flow ranging from 50 to 80 m³s⁻¹ in a large river on Jakobshavn Isbræ. The bulk of the moulins have inputs that peak between 10 and 30 m³s⁻¹, which is more in line with other measurements of large supraglacial rivers (Pitcher & Smith, 2019).

The moulin hydrographs show additional evidence of seasonal dynamics. Figure 3 highlights the discharge (teal), diurnal amplitude (yellow), and lag time (purple) for moulins representing four typical responses: (a) In Figures 3a, 3e, and 3i, the discharge and diurnal amplitude are both roughly symmetric and peak shortly after peak melt, and the lag time is nearly constant. This behavior is similar to what would be predicted by a supraglacial flow-routing model. (b) In Figures 3b, 3f, and 3j the diurnal amplitude and discharge in the highlighted case both peak before peak melt and taper off after, and the lag time relaxes to its steady state value over the first half of the melt season. However, other moulins with the same pattern of diurnal amplitude may peak before or after...
peak melt. (c) In Figures 3c, 3g, and 3k there are two separate peaks in the diurnal amplitude. Discharge picks up rapidly in the early season and peaks well after peak melt, with a rapid drop off toward the end of the melt season. In this case, the lag time shows a similar pattern to Figure 3j, but with a second small peak coinciding with the second peak in diurnal amplitude. (d) Several moulins shut off throughout the melt season as the channels that feed them melt out (Figures 3d, 3h, and 3l).

3.4. Sensitivity Tests

We ran a large suite of sensitivity tests to investigate the full range of the model's behavior and to constrain the set of parameter values where the model produces physically realistic outputs. Since many of the model parameters are poorly constrained, these tests are important to inform applications of the model to real domains. The sensitivity tests are summarized in Table 3. Each test varies a single parameter to test its effect on moulin discharges. Here, we present hydrographs for the sensitivity tests (Figure 4) and describe the results from a selection of tests in detail.

3.4.1. Parameter Sensitivity

As shown in Figure 4, the hydraulic conductivity values and the width-to-depth ratio strongly control the moulin inputs. From cases C1 and C2, we see that the sheet conductivity controls the moulin diurnal amplitude, while the channel conductivity controls the moulin discharge rates. With a lower conductivity (case C2) the larger moulins have captured more flow, while the smaller moulins are more likely to shut off.

The width-to-depth ratio regulates the minimum flow required to prevent channels from melting out. Narrower channels sustain channels with lower flow, which means that no moulins shut off in case W1. This leads to a lower peak input into the largest moulins. In the case of wider channels (W2), more moulins shut off and more flow is concentrated in the largest channels. There is also more exaggerated asymmetry in moulin discharges in case W2.
**Table 3**

*Deviations From the Default Parameters (Table 2) for the Sensitivity Tests*

| Case name         | Case code | Varied parameter | Value          |
|-------------------|-----------|------------------|----------------|
| Conductivity      | C1        | $k_s$            | $0.5 \text{ m}^{0.5} \text{s}^{-1}$ |
| Conductivity      | C2        | $k_c$            | $5 \text{ m}^{1/3} \text{s}^{-1}$ |
| Exchange fraction | E1        | $\zeta$         | 0.1            |
| Width-to-depth ratio | W1       | $r$             | 2.0            |
| Width-to-depth ratio | W2       | $r$             | 5.0            |
| Diurnal melt      | D1        | $D(t)$          | $D(t) = \max(0, -\pi \cos(2\pi t/\text{day}))$ |
| Seasonal melt rate| S1        | $\Delta T$      | $-2^\circ \text{C}$ |
| Seasonal melt rate| S2        | $\Delta T$      | $2^\circ \text{C}$ |
| Seasonal melt rate| S3        | $\Delta T$      | $-2^\circ \text{C}$ |
| Seasonal melt rate| S4        | $\Delta T$      | $2^\circ \text{C}$ |
| Resolution        | R1        | Number of nodes | 582            |
| Resolution        | R2        | Number of nodes | 2,451          |
| Timestep          | T1        | $dt$            | 90 s           |

*This $D(t)$ function maintains the same average melt rate and has no melt for 12 hr overnight. Cases S3 and S4 are initialized using the channel dimensions after the spinup year of the baseline case.*

**Figure 4.** Hydrographs for the sensitivity tests described in Table 3. (a) Hydrograph for the baseline case for comparison. This is the same data presented in Figure 2. (b–l) Hydrographs for the sensitivity tests indicated in the top right corner of each panel. Note the different y-axis scales for (i, j, k, and l).
In case T1, reducing the timestep from 180 to 90 s changes moulin inputs by only $\sim 1\%$ (Figure S4 in Supporting Information S1). Therefore, we are confident that our default timestep sufficiently resolves the system's dynamics.

For case E1, we decreased the channel exchange ratio $\zeta$ to 10%, which more faithfully represents the true physics but is less numerically stable. This only changes moulin inputs by $\sim 4\%$, so we believe the default value of 20% is a good choice for this scenario.

### 3.4.2. Forcing Sensitivity

We included five tests that vary the surface melt rates used to force the model. For case D1 we changed the diurnal melt forcing so that there is no melt overnight for 12 hr while maintaining the daily averaged melt rate. The main difference in this case is that the moulin inputs have a larger diurnal amplitude, although the seasonal dynamics are similar.

Cases S1 and S2 modified the melt parameterization by using temperature offsets of $\Delta T = -2^\circ C$ and $\Delta T = 2^\circ C$, respectively. The results from these cases show similar seasonal patterns of moulin inputs as the baseline, with flow rates scaled according to changes in the melt forcing.

The final sensitivity tests, S3 and S4, have the most interesting dynamics. Cases S3 and S4 are designed to mimic years with abnormally low and high melt, respectively. We change the sea level temperature in the melt model by setting $\Delta T = -2^\circ C$ for case S3 and $\Delta T = 2^\circ C$ for case S4, but initialize the model with channel dimensions from the baseline case. This means that the channels are initially out of balance with the melt intensity.

Case S3 is interesting in that it yields moulin inputs that are nearly symmetric. For the most part, moulin inputs peak soon (a few days to a week) after peak melt, and do not show dramatic differences between the early and late melt season. These moulin inputs are therefore similar to what would be predicted by flow routing models that do not represent the seasonal evolution of supraglacial channels.

In contrast, case S4 provides very dynamic moulin discharge curves (Figure 5). As in Figure 3, the moulins in Figure 5 were chosen to show four representative responses. In panels (a and e), the discharge is nearly symmetric, whereas the diurnal amplitude is nearly constant through the middle of the melt season. In panels (b and f) the discharge peaks early, and the diurnal amplitude has two distinct peaks. In panels (c and g) the discharge peaks late, and the diurnal amplitude has an even more prominent double peak. Panels (d and h) show a moulin with early peaks in both diurnal amplitude and discharge, caused by the upstream channels that feed the moulin melting out throughout the first half of the melt season. In each case, the lag time follows a similar pattern as in the baseline case (Figure 3).

### 3.4.3. Domain Sensitivity

The final tests investigate the sensitivity of modeled moulin inputs to changes in the physical domain, including the number of moulins and the mesh resolution.

To test the effect of mesh resolution on the modeled drainage system, the model is applied to a coarser mesh with 582 nodes (case R1) and a finer mesh with 2,451 nodes (case R2), compared to the default mesh with 1,205 nodes (Figure 6, Table S1 in Supporting Information S1). The meshes for cases R1 and R2 are refined by the same elevation-dependent factor as the default mesh.

As expected, we needed to use a shorter timestep (90 s) for case R2 than for the baseline case (180 s) to ensure similar numerical convergence. Interestingly, case R1 required the same timestep as the baseline case. The model runtime for these cases were 11.4 hr for the baseline case, 4.9 hr for case R1, and 46.5 hr for case R2 on a single CPU on Compute Canada's Graham cluster (Intel E5-2683 v4 Broadwell @ 2.1 GHz) with 8 GB of RAM.
The sheet thickness maps suggest that the maximum water sheet thickness decreases with the number of nodes in the mesh (Figures 6a, 6b, and 6c). This can be explained by the fact that the finer meshes have a higher density of possible channel locations, and so finer meshes allow a channel network to form that more evenly takes up mass from the distributed sheet (Figures 6d, 6e, and 6f). This finding does not suggest that a more extensive network of channels is formed; instead, it indicates a more even distribution of mass across the basin.

Figure 5. Moulin discharge (a–d), diurnal amplitude (e–h), and lag time (i–l) from case S4 for four select moulins.

Figure 6. Modeled drainage system for the baseline case (a, d, and g), case R1 with a coarse mesh (b, e, and h), and case R2 with a fine mesh (c, f, and i). The panels show (a–c) water sheet thickness and (d–e) channel flow at time $t = 182.5$ days, and (g–i) moulin inputs.
active channels forms, but that the channels that do form can more uniformly fill the domain than with a coarser mesh.

Due to the nature of the unstructured triangular meshes, each mesh contains unique node locations and moulins are placed at different nodes. This means it is difficult to directly compare moulin hydrographs, and instead we compare the results from a more qualitative perspective. The moulin hydrographs show that the difference between the baseline case and case R2 is small in terms of moulin inputs, while case R1 has significantly lower flow rates (Figures 6g, 6h, and 6i). The baseline and R2 have similar maximum input rates (approximately 70 m$^3$s$^{-1}$) and show qualitatively similar seasonal patterns. On the other hand, case R1 has significantly lower maximum input rates (approximately 45 m$^3$s$^{-1}$), but shows similar seasonal patterns.

When applied to a real setting, moulin locations would be known (e.g., from satellite imagery), but other parameters such as the conductivities, the width-to-depth ratio, and the exchange ratio would have to be tested or calibrated, and the number of the nodes in the mesh would need to be adjusted to properly resolve flow across rough topography.

4. Internally Drained Catchments on the Greenland Ice Sheet

To test our model's ability to calculate moulin inputs in a real setting, we apply the model to a ∼27 × 20 km$^2$ region of the southwestern Greenland Ice Sheet (Figure 7). We have chosen this domain so that we can compare our modeled network development with the existing map of supraglacial rivers, lakes, moulins, and catchments by created by Yang and Smith (2016) from a 19 August 2013 Landsat 8 panchromatic image. Our domain includes a subset of eight internally drained catchments, corresponding to the region shown in Figure 2 of Yang and Smith (2016), and we run the model for the 2013 melt season to coincide with their map.

4.1. Input Data

We run SaDS using a mesh with 1,984 nodes, 5,751 edges, 3,768 elements, and a maximum element area threshold of 1.5 × 10$^5$ m$^2$. The mesh is refined by a factor of 4 within a 2 km radius of nine supraglacial lake locations. Edge lengths range from 180.9 to 768.1 m with a median of 419.5 m. This minimum edge length within lakes.

![Figure 7. Overview of the Greenland model domain. (a) Hydrology features mapped by Yang and Smith (2016), with 32 m resolution ArcticDEM (Porter et al., 2018) shown as the background. The inset shows the location of the study area within Greenland in red. (b) Detailed view of the model domain and mapped rivers shown in panel (a).](image-url)
is sufficient for the current work, however if we were explicitly studying supraglacial lake dynamics, we would refine the mesh further to ensure we accurately capture the volume and surface area of each lake. The model is run with default parameters as in Table 2, with the exception of a 30 s timestep. The short timestep is necessary to ensure numerical stability with the small edge lengths compared to the synthetic experiments and to resolve supraglacial lake dynamics.

We derive a digital elevation model (DEM) for this triangular mesh from 32 m resolution ArcticDEM mosaic tiles (Porter et al., 2018). To do this, we smooth the ArcticDEM data using a square two-dimensional moving average filter with an edge length of 45 pixels (1.44 km). The elevation of each element centroid is calculated as the average value of the smoothed ArcticDEM pixels that lie within the triangular element.

We use the seven moulin locations mapped within the subdomain by Yang and Smith (2016) to place moulins in our triangular mesh domain. We shift moulins by up to 1,290 m (median 556.3 m) to coincide with local depressions in the DEM and our modeled flow paths. This shift is not surprising since it is only slightly larger than the median edge length, and we are using an elevation model that is derived from imagery from DigitalGlobe WorldView-1, WorldView-2, and WorldView-3 satellite imagery from 2011–2017 (Porter et al., 2018), whereas the Landsat image used to map the moulin locations by Yang and Smith (2016) is from 19 August 2013. Since the acquisition times do not align exactly, it is possible that the elevation model differs from the topography at the time of the image acquisition, and so our flow routing should be expected to differ slightly.

The model is forced with RACMO2.3p2 surface melt data at 5.5 km horizontal resolution and 3 hr temporal resolution (Noël et al., 2019). We linearly interpolate this raw data spatially and temporally to calculate melt rates for each element in our triangular mesh. The RACMO data indicates the melt season for our domain spans 5 June to 15 August 2013 (Figure S6 in Supporting Information S1). To ensure the model fully captures the onset of melt and has time to drain at the end of the season, we run the model from 5 June to 18 August 2013. As for the synthetic case, we run the model for one year as a spinup period and present results for the second year. This spinup year is critical to allow channels to adjust to their equilibrium dimensions. Since in this case (unlike with the synthetic setup) we have topography to guide the location of supraglacial channels, we use the channel dimensions from the end of the spinup year directly as initial conditions in the second year.

4.2. Results

The modeled outputs are shown in Figure 8, where we have used an arbitrary threshold water depth of 30 cm to plot supraglacial river locations. The modeled drainage system closely matches the drainage system mapped by Yang and Smith (2016). SaDS predicts an extensive system of supraglacial channels that is controlled by topography: rivers form in valleys, while distributed flow is dominant along topographic ridges (Figures 8a and 8d). Several supraglacial lakes are formed by the model, shown by the deep (>1 m) water in the sheet and in the channels (Figures 8d and 8f). These lakes form in topographic depressions and are fed by both modes of flow.

As with the synthetic case, SaDS predicts dynamic and variable moulin inputs (Figure 8b). Moulin inputs (Figure 8b) have slightly higher diurnal amplitude in the first few weeks of the melt season than in the late season. We also see that moulins have distinct behaviors from one another, in particular in their relaxation time after the cessation of melt. Near 3 July, melt paused for a few days, allowing the system to empty in the middle of the melt season.

The total mass in both systems has similar dynamics to the moulin inputs (Figure 8c). The channel mass has slightly lower diurnal amplitude in the late melt season than at the beginning. We also see that the channels very quickly take up mass at the beginning of the melt season compared to the distributed system. However, when melt resumes after the mid-season pause around July 3, the systems both take a similar amount of time to reach their saturation values. The nonzero equilibrium value in both systems is due to water storage in lakes.

As a test of the impact of initial conditions following spinup, we also ran the model with all channel locations incised by at least 25 cm at the beginning of the second melt season, as we did for the non-topographically driven synthetic tests. The outputs (Figure S7 in Supporting Information S1) show extremely high diurnal amplitude in the first few weeks of the melt season with moulin inputs nearly halting overnight. If these moulin inputs are accurate, there are significant implications for subglacial hydrology. The high peak inputs throughout the day would induce large spikes in subglacial pressure and lead to a rapid early season speedup of the ice sheet. The lack of
Figure 8. (a) Comparison between the modeled drainage system (black lines) and the drainage system mapped by Yang and Smith (2016, legend elements indicated by YS16). The background is colored by the surface elevation. (b) Discharge for the moulins shown in (a) with colors corresponding to the marker color in (a). (c) Total mass in the distributed water sheet (left axis, green) and the channel system (right axis, black). The vertical dashed lines in (b and c) indicated the time of the maps shown in (d, e, and f). (d) Water thickness in supraglacial channels. (e) Channel volume flux. (f) Water sheet thickness (note the logarithmic color scale).
water inputs overnight could reverse the growth of subglacial channels during the day, extending the duration of the acceleration event and thereby enhancing mass discharge for marine terminating glaciers and mass transport to warmer, lower elevations for land terminating regions.

5. Discussion

5.1. Synthetic Ice Sheet Margin

The modeled supraglacial drainage systems behaves largely as expected. In the SHMIP-inspired synthetic ice sheet margin tests, most of the mass is transported through the channel network, while the distributed sheet acts to control the diurnal amplitude and lag time of moulin inputs.

In the baseline case, modeled moulin discharge lags behind melt forcing by ~5–7 hr, indicating efficient drainage (Smith et al., 2017). On a seasonal timescale, moulin discharge peaks a few days to a week after melt peaks. This long seasonal lag is due to the dynamics of the supraglacial drainage system. As melt intensity increases above the seasonal mean, channels grow in response to higher meltwater supply. Since this process takes several days, the channel network reaches its highest capacity after melt peaks, delaying peak moulin inputs.

However, several moulins show more complex dynamics and so are exceptions to this behavior (Figures 3 and 5). This behavior is a direct result of our model's physics-based treatment of supraglacial channel incision and melt out. For moulins that have high inputs, which are primarily at low elevations, we predict moulin inputs similar to what would be predicted by a routing model (e.g., Banwell et al., 2012). This pattern occurs when the moulin is fed by large, perennial channels that consistently take up water from the distributed sheet. When a moulin is fed by smaller channels that may become overwhelmed by the mass inputs from the sheet, or by channels that melt out, the moulin inputs resemble those of Yang et al. (2018). The inputs to these moulins have their maximum diurnal amplitude early in the season, and their discharge may peak before or after peak melt. The “double peak” diurnal amplitude response in Figures 3c and 3g occurs for the same reasons, but in this case the channel network is either able to evacuate the excess mass, or grows sufficiently by heat dissipation to take up mass from the sheet again, causing the diurnal amplitude to increase toward the second peak (Figure S8 in Supporting Information S1). Finally, moulin inputs may decrease throughout the melt season as the channels feeding the moulin melt out. This only occurs for moulins with low input rates in regions with low surface slopes.

For some moulins, the lag time between peak melt and peak moulin inputs decreases throughout the melt season (Figures 3j and 3k). This pattern indicates that flow becomes increasingly concentrated in large supraglacial channels that efficiently transport meltwater throughout the catchment as the melt season progresses.

Beyond these seasonal dynamics, the model predicts long-term evolution of the drainage system. Large channels incise by ~2 m a⁻¹, and flow in the distributed sheet melts the ice sheet surface by up to 15 cm due to heat dissipation (Figure S9 in Supporting Information S1). These channel incision rates compare well to direct observations of 2.5 m a⁻¹ in the Canadian High Arctic (St Germain & Moorman, 2019) and 3.3 cm·day⁻¹ on the Greenland ice sheet (this is equivalent to 2 m a⁻¹ assuming a 60 day melt season) (McGrath et al., 2011). Surface melt by sheet water flow is not usually considered in the energy balance or water budget of glacier surfaces. However, Bash and Moorman (2020) suggested that thin sheet flow could be an additional source of surface melt on Fountain Glacier, in the Canadian High Arctic, as they found that errors in their high resolution melt models were correlated with the density of water features.

5.2. Sensitivity Tests

We carried out a large suite of sensitivity tests (Figure 4, Table 3). From these tests, we can characterize how sensitive the model is to each parameter, as well as begin to constrain the range of values that parameters may take.

From the sensitivity test hydrographs (Figure 4), it is clear that the model is most sensitive to the channel conductivity and the width-to-depth ratio. The channel conductivity controls how much energy is available to melt the underlying ice, so reducing the conductivity causes more small channels to melt out. This acts to concentrate the flow into a few larger channels, which is why there is such inequity in moulin inputs in case C2. The width-to-depth ratio acts in a similar way. Channels with a larger width-to-depth ratio will melt out more quickly since surface melt reduces the channel cross section much more than for channels with a smaller width-to-depth ratio.
(Equation 9). On the other hand, in case W1, the channels are narrow enough that no moulins shut off, and the distribution of flow between moulins is more uniform. These parameters could be tuned for regions of interest by mapping changes in river networks over a melt season, allowing more accurate model projections.

The resolution sensitivity tests (R1 and R2) demonstrate that the modeled moulins inputs and drainage system behavior are similar between the default mesh and the fine resolution mesh (Figures 6g and 6h). The coarse mesh predicts lower moulins inputs (Figures 6g and 6i) and an erratic channel network, where large rivers (with flux $>10 \text{ m}^3\text{s}^{-1}$) are represented by only two or three edges (Figure 6e). These findings suggest that the coarse resolution mesh used in case R1 is insufficient to resolve the supraglacial drainage system. The increase in resolution for case R2 does not appear to be worth the trade-off in model runtime, since the behavior is qualitatively similar to the baseline case, yet the runtime was four times higher. For these reasons, we believe the baseline mesh with a median edge length of 1,232 m is the best choice for this simple synthetic domain.

The lower peak discharge in case M1 compared to the high peak discharge in the baseline run (nearly 70 m$^3$s$^{-1}$) suggests that the moulins density in case M1 may be more appropriate than the density in the baseline case. This problem is unique to this synthetic domain since we cannot use satellite imagery or direct observations to place moulins. This finding is interesting as 100 moulins was the highest number considered by the SHMIP experiment, but here we have found this is the most appropriate number for the supraglacial drainage system.

We also see that changes in the total melt volume do not significantly change the drainage system's behavior as long as the system is in balance with the melt rate. Cases S1 and S2, where the model is run for two years to reach equilibrium, show very similar responses, only scaled in proportion to the mean melt rates. On the other hand, cases S3 and S4, where the channel network is initially not in equilibrium, show complex responses. This likely has important implications for extreme melt years on the Greenland Ice Sheet. While it is beyond the scope of this study, it is an interesting question how these complex moulins inputs would impact the development of the subglacial drainage system in extreme melt years.

The set of default parameters in Table 2 is informed by previous modeling studies and direct measurements (Gleason et al., 2016; Pitcher & Smith, 2019; Werder et al., 2013), and in the case of the exchange ratio $\zeta$, to preserve numerical stability. While the conductivities have been measured and reported, it is less clear how to apply the width-to-depth ratio. Measured width-to-depth ratios have been reported as 3.4 to 12 (Knighton, 1985), 4.4 to 9.5 (Yang et al., 2016), and 6.8 to 17.3 (Gleason et al., 2016), however, these ratios are usually taken to be the ratio of the water depth to the wetted width. We have purposely used the ratio of total incision depth of the channel to the total width in our model because we believe adjusting the channel width at the fast timescale of changes in the water depth is nonphysical and would result in numerical instabilities. Measurements of this width-to-incision depth ratio are less common. For example, St Germain and Moorman (2019) reported that deeply incised rivers on Fountain Glacier may have an equal ratio of total incised depth to width. Therefore, we use a ratio of 3 as a midpoint between these cases, although this parameter remains poorly constrained.

5.3. Application to the Greenland Ice Sheet

We applied the model to a small domain on the Greenland Ice Sheet composed of several connected internally drained catchments. The modeled drainage system qualitatively agrees well with maps created from satellite imagery (Yang & Smith, 2016), and the system shows similar dynamics to the synthetic ice sheet margin tests. This Greenland drainage test demonstrates that the model is able to predict realistic drainage systems when forced with regional climate model data, DEM-derived surface topography, and known moulins locations. These results are promising for future application of the model to different catchments and for utilizing its outputs to force subglacial hydrology models.

We note that the map we have used to validate the model (Yang & Smith, 2016) was created from a Landsat 8 image from 19 August 2013. This date is outside the melt season predicted by the RACMO melt data, and is five days later than the model outputs we have compared it to (14 August 2013). Since the satellite-derived map was created from the panchromatic image, it is possible that some of the larger rivers were still active at the time of the mapping, or that the dry remnants of the supraglacial drainage system were mapped. It is also possible that the RACMO data incorrectly predicts the timing of the end of the melt season, so that the entire drainage system was still active.
The most significant difference compared to the synthetic case is perhaps the presence of supraglacial lakes. SaDS implicitly calculates filling and overtopping of lakes without requiring DEM sinks to be filled and without pre-calculating lake basins. Since we do not fill sinks in the DEM, water in the sheet and channel systems naturally accumulates in depressions until the lake overflows. The water stored in the lake elements spills over into downhill elements, and the edges spill over into downstream channels. The channels along the lake drainage path take up flow from the elements, effectively creating a lake spillway. This has the same effect as an explicit treatment of supraglacial lakes (e.g., Banwell et al., 2012; Kingslake et al., 2015), but happens automatically as a result of the model's structure. In this study, we have not simulated rapid or complete drainage of supraglacial lakes, only their continuous and slow spilling over by overfilling. Rapid drainage could be included by introducing a moulin within the lake basin that only activates during a specified known drainage period or when the lake reaches a specific water depth.

5.4. Assumptions and Simplifications in the Model Formulation

In developing our mathematical model of the drainage system, we have intentionally used relatively simple parameterizations and reduced the model to a few key equations rather than modeling every detail of the relevant physical mechanisms in order to keep the model transparent and numerically tractable. It is worthwhile to discuss some of the assumptions in more detail.

The model assumes that heat dissipation in the channels melts ice along the entire channel perimeter, rather than just the wetted perimeter (Equation 9). This assumption does not capture the details of the physics at play, but it does capture the end result, which is that channel cross section increases as flow in the channel melts the underlying ice. Any energy lost by melting the channel walls above the water surface could be at least partially compensated for by increasing the channel conductivity or reducing the width-to-depth ratio, both of which make it easier for channels to expand.

We acknowledge that our channel geometry parameterization is simplified. The assumption of rectangular channels with a uniform aspect ratio lets us formulate the channel model equations in a straightforward way with a single parameter (the width-to-depth ratio) controlling the geometry of the channel cross-section, instead of requiring additional equations to evolve the channel perimeter (e.g., Jarosch & Gudmundsson, 2012). Other geometries may work, for example, a triangular cross section, but they would be at least as simplified as our choice. The important behavior here is simply that channels should grow in both width and depth, which our parameterization captures. The impacts of a linear relationship between channel incision depth and width is an interesting question, and nonlinear relationships should be investigated in future work. As part of this assumption, we neglect solar radiation in computing channel melt rates. Radiative melt along channels would modify the channel cross-sectional shape in a complex way (St Germain & Moorman, 2019) that cannot easily be captured in the current framework. Radiative melt could be parameterized through the additional melt term in Equations 12 and 13 in future applications. Within the current width-to-depth ratio framework, this melt would need to be applied uniformly along the entire channel perimeter, which is clearly a coarse simplification. For this preliminary work, we therefore neglect solar radiation-derived melt.

We have not directly addressed all the details of the onset of surface melt in our channel network. Supraglacial channels typically become filled with snow throughout the winter which must be melted before the bare-ice channel walls are exposed. This snow can sometimes plug the channel, forcing the water to be diverted around the snow-plug (St Germain & Moorman, 2019). These processes are beyond the current resolution capability of the model, but would likely have interesting consequences for moulin inputs at the beginning of the melt season.

More generally, while the model does account for rivers, distributed flow, and lakes, we have excluded some aspects of the supraglacial drainage system. We leave it as future work to include the effect of crevasses interrupting and capturing flow (e.g., Clason et al., 2012) and transport, refreezing and/or storage within snow and firn (e.g., Meyer & Hewitt, 2017). Until these mechanisms are incorporated, it is important to carefully choose domains where they are expected to be unimportant.

In the synthetic test, we intentionally used a coarser mesh at higher elevations in order to increase the computational efficiency, which inhibits our ability to resolve a potentially dense network of small channels at high elevations. Similarly, in our application to the Greenland Ice Sheet catchment, we are unable to quantitatively compare classic watershed properties such as stream order, stream density, and topology due to the limited resolution of...
Despite this difficulty, the locations of main channels and the extent of branching tributaries agrees with the Landsat-derived map (Yang & Smith, 2016), and in both the Greenland Ice Sheet and synthetic test we focused on large channels that are responsible for the majority of the meltwater flow. Where smaller-scale catchment processes are of interest, the mesh resolution can be adjusted accordingly.

We used two different initialization procedures to represent the perennial nature of supraglacial channels: (a) applying the channel dimensions from the end of the spinup period, and (b) implementing a minimum incision depth of 25 cm across the domain. For the Greenland case (Figure 8 and Figure S6 in Supporting Information S1), the initialization has a significant impact on moulin inputs in the first few weeks with much stronger diurnal swings in the minimum incision depth case due to extremely fast flow through the over-developed channel network before small channels melt out. The model is capable of representing either behavior, and since there are limited measurements of supraglacial river discharge over the entire melt season, and particularly when melt begins in the spring, we cannot be certain which of these is more realistic.

Finally, the model does not currently have the capability to directly assimilate observed channel networks as an initial condition. Given the disparity of scales between satellite-derived drainage system maps (<15 m resolution) and our mesh for the Greenland test (median edge length >400 m), this would involve extensively modifying the mesh or implementing a new method to embed the one-dimensional channels within the two dimensional drainage system. This is an interesting question for future work. It is worth noting that, in the current configuration, the drainage network may be considered as an output of the model (as in Figure 8). This opens the possibility of using SaDS as a tool to create drainage system maps in addition to predicting moulin inputs. In this way, SaDS could be used in combination with existing methods to derive drainage systems from satellite imagery (e.g., Yang & Smith, 2016) to obtain two independent realizations of the supraglacial drainage system. This advantage is unique to the current framework, as initializing the model with an observed network would remove the independence between these methods.

6. Conclusions

We have presented the Subaerial Drainage System Model, SaDS. The model combines surface runoff through both a distributed water sheet and within a network of supraglacial channels to calculate moulin input rates. Its unique formulation allows both systems to be modeled dynamically, with mass exchange terms coupling the systems. The model does not rely on pre-determined maps of the supraglacial drainage system. Instead, a network of lakes and rivers emerges naturally depending on the balance between channel erosion and surface melt.

The model was tested using the Subglacial Hydrology Model Intercomparison Project (SHMIP) (de Fleurian et al., 2018) synthetic ice sheet margin geometry. Modeled moulin inputs show a wide range of realistic, dynamic behavior. Supraglacial channels with low flow rates melt out, leading to reduced diurnal amplitude in moulin inputs, while large rivers are found to be perennial features, and are responsible for the majority of the runoff transport.

We ran a large suite of sensitivity tests to characterize the range of behaviors the model is capable of reproducing and to constrain possible parameter values. The sensitivity tests suggest that the interannual variability in surface melt is more important for the seasonal patterns of moulin inputs than the absolute melt rate. This finding likely has interesting implications for the development of the subglacial drainage system, and therefore basal sliding, in extreme melt years.

The model was successfully applied to a ∼20 × 27 km² region of southwest Greenland. The modeled drainage system includes several supraglacial lakes and an extensive network of supraglacial rivers that agrees well with a satellite-derived map of the drainage system (Yang & Smith, 2016). These results suggest the model should be transferable to other similar catchments and in different years.

We have made several simplifying assumptions in formulating the model to ensure it is transparent and numerically feasible. Perhaps most important is our assumption that channels have rectangular cross-sections with uniform ratios of total incision depth to channel width. When this ratio is measured in the field, it is usually reported as the ratio of the water depth, not the total incision depth, to the wetted channel width. Therefore, to constrain our
model further, we require more field measurements of the total incision depth, width, and cross-sectional shape of incised supraglacial channels across a wide range of discharges.

Compared to existing models, we predict more dynamic and variable moulin inputs. Our model encompasses the behavior of existing models (e.g., Banwell et al., 2012; Yang et al., 2018), as well as behaviors not predicted by existing models. We do this without relying on prior knowledge of supraglacial river locations or lake basins. The trade-off, however, is complexity. Our model both is mathematically and numerically complex compared to most other models. This makes it best suited for applications where the specific dynamics of moulin inputs are important. For this type of application, SaDS is a significant step forward.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

Model outputs and analysis code are available at https://doi.org/10.5281/zenodo.4923858. The model is available by contacting the lead author. The supraglacial drainage system map used to validate the model outputs is available as Supporting Information to Yang and Smith (2016). ArcticDEM data are freely available from the Polar Geospatial Center (https://www.pgc.umn.edu/data/arcticdem/). The RACMO surface melt data set is available by contacting the Institute for Marine and Atmospheric research Utrecht University (https://www.projects.science.uu.nl/iceclimate/models/racmo.php).

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References

Arnold, N., Richards, K., Willis, I., & Sharp, M. (1998). Initial results from a distributed, physically based model of glacier hydrology. Hydrological Processes, 12(2), 191–219. https://doi.org/10.1002/(SICI)1099-1085(199802)12:2<191::AID-HYP571>3.0.CO;2-C

Banwell, A. F., Arnold, N. S., Willis, I. C., Tedesco, M., & Ahlstrom, A. P. (2012). Modeling supraglacial water routing and lake filling on the Greenland Ice Sheet. Journal of Geophysical Research: Earth Surface, 117(F4). https://doi.org/10.1029/2012JF002393

Bartholomew, I., Nienow, P., Sole, A., Mair, D., Cowton, T., Palmer, S., & Wadham, J. (2012). Modelling the delivery of supraglacial meltwater to the ice/bed interface: Application to southwest Devon Ice Cap, Nunavut, Canada. Journal of Glaciology, 58(208), 361–374. https://doi.org/10.3189/2012JoG11J142

Beighley, R. E., Eggert, K. G., Dunne, T., He, Y., Gummadri, V., & Verdin, K. L. (2009). Simulating hydrologic and hydraulic processes throughout the Amazon River Basin. Hydrological Processes, 23(8), 1221–1235. https://doi.org/10.1002/hyp.7252

Christiansen, K., Kohler, J., Alley, R. B., Nuth, C., & van Pelt, W. J. J. (2015). Dynamic perennial firn aquifer on an Arctic glacier. Geophysical Research Letters, 42(5), 1418–1426. https://doi.org/10.1002/2014GL062806

Chu, V. W. (2014). Greenland ice sheet hydrology: A review. In Physical Geography: Earth and Environment, 38(1), 19–54. https://doi.org/10.1177/0309133313507075

Clason, C., Mair, D. W., Burgess, D. O., & Nienow, P. W. (2012). Modelling the delivery of supraglacial meltwater to the ice/bed interface: Application to southwest Devon Ice Cap, Nunavut, Canada. Journal of Glaciology, 58(208), 361–374. https://doi.org/10.3189/2012JoG11J142

Clason, C., Mair, D. W., Nienow, P. W., Bartholomew, I. D., Sole, A., Palmer, S., & Schwanghart, W. (2015). Modelling the transfer of supraglacial meltwater to the ice/bed interface: Application to southwest Devon Ice Cap, Nunavut, Canada. Journal of Glaciology, 61(228), 897–916. https://doi.org/10.1017/jog.2018.78

de Fleurian, B., Werder, M. A., Beyer, S., Brinkerhoff, D. J., Delaney, I., Dow, C. F., et al. (2018). SHMIP the subglacial hydrology model intercomparison project. Journal of Glaciology, 64(248), 897–916. https://doi.org/10.1017/jog.2018.78

Eichelmeyer, K., & Harrison, W. D. (1990). Jakobshavn Isbrae, West Greenland: Seasonal variations in velocity - Or lack thereof. Journal of Glaciology, 36(122), 82–88. https://doi.org/10.3189/S0022143000005591

Flowers, G. E., & Clarke, G. K. C. (2002). A multicomponent coupled model of glacier hydrology I. Theory and synthetic examples. Journal of Geophysical Research: Solid Earth, 107(B11), 8281–8299. https://doi.org/10.1029/2001JB001122

Forster, R. R., Box, J. E., Van Den Broeke, M. R., Miège, C., Burgess, E. W., Van Angelen, J. H., et al. (2014). Extensive liquid meltwater storage in firm within the Greenland Ice Sheet. Nature Geoscience, 7(4), 95–98. https://doi.org/10.1038/ngeo2043

Fountain, A. G., & Walder, J. S. (1998). Water flow through temperate glaciers. Reviews of Geophysics, 36(3), 299–328. https://doi.org/10.1029/97RG03579

Gleason, C. J., Smith, L. C., Chu, V. W., Legleiter, C. J., Pitcher, L. H., Overstreet, B. T., et al. (2016). Characterizing supraglacial meltwater channel hydraulics on the Greenland Ice Sheet from in situ observations. Earth Surface Processes and Landforms, 41(4), 2111–2122. https://doi.org/10.1002/esp.3977

Gleason, C. J., Yang, K., Feng, D., Smith, L. C., Liu, K., Pitcher, L. H., et al. (2021). Hourly surface meltwater routing for a Greenlandic supraglacial catchment across hillslopes and through a dense topological channel network. The Cryosphere, 15(5), 2315–2331. https://doi.org/10.5194/tc-15-2315-2021

Hill, T., Dow, C. F., Bash, E. A., & Copland, L. (2021). Application of an improved surface energy balance model to two large valley glaciers in the St. Elias Mountains, Yukon. Journal of Glaciology, 67(262), 297–312. https://doi.org/10.1017/jog.2020.106
Yang, K., Smith, L. C., Karlstrom, L., Cooper, M. G., Tedesco, M., van As, D., et al. (2018). A new surface meltwater routing model for use on the Greenland Ice Sheet. *The Cryosphere*, 12(12), 3791–3811. https://doi.org/10.5194/tc-12-3791-2018

Zemp, M., Jansson, P., Holmlund, P., Gätter-Roer, I., Koblet, T., Thee, P., & Haeberli, W. (2010). Reanalysis of multi-temporal aerial images of Storglaciären, Sweden (1959–99) – Part 2: Comparison of glaciological and volumetric mass balances. *The Cryosphere*, 4(3), 345–357. https://doi.org/10.5194/tc-4-345-2010

Zwally, H. J., Abdalati, W., Herring, T., Larson, K., Saba, J., & Steffen, K. (2002). Surface melt-induced acceleration of Greenland Ice Sheet flow. *Science*, 297(5579), 218–222. https://doi.org/10.1126/science.1072708