Many-body theory interpretation of deep inelastic scattering

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ABSTRACT: We analyze data on deep inelastic scattering of electrons from the proton using ideas from standard many-body theory involving bound constituents subject to interactions. This leads us to expect, at large three-momentum transfer $q$, scaling in terms of the variable $\tilde{y} = \nu - |q|$. The response at constant $|q|$ scales well in this variable. Interaction effects are manifestly displayed in this approach. They are illustrated in two examples.

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The cross section for deep inelastic scattering (DIS) of electrons on unpolarized protons (and, mutatis mutandis, neutrons) is usually expressed as [1]:

$$\frac{d^2\sigma}{d\Omega d\nu} = \sigma_M \left[ 2W_1(|q|, \nu) \tan^2 \frac{\theta}{2} + W_2(|q|, \nu) \right],$$

(1)

where $\sigma_M$ is the Mott cross section, $\theta$ the scattering angle and $\nu$ the energy transfer. In this brief note we discuss only $W_1$.

$W_1$ is generally considered as a function of $Q^2 = |q|^2 - \nu^2$ and Bjorken $x = Q^2/2m\nu$ for scattering by protons initially at rest in laboratory frame. The advantages of using the Lorentz scalar variables $Q^2$ and $x$ are discussed in standard texts [2]. The data show that $W_1(Q^2, x)$ obeys Bjorken scaling at large values of $Q^2$; it depends primarily on $x$. The weak dependence of $W_1$ on $Q^2$ is well understood via the perturbative QCD theory developed by Gribov, Lipatov, Altarelli and Parisi (GLAP) [2]. The DIS data are usually interpreted by going to the infinite momentum frame where $x$ is identified as the fraction of the momentum carried by the quark responsible for the deep inelastic scattering. This interpretation has been very helpful in understanding DIS and in interpreting high energy reactions.
In many-body theory it is natural to study the response of a system in its rest frame at fixed values of the momentum transfer $q$ as a function of the energy transfer $\nu$. The response to a scalar probe, for example, is viewed as the distribution of the strength of the state $\sum_i e^{i q \cdot r_i} |0\rangle$, created by the probe, among the eigenstates of the system belonging to momentum $q$. Fig. 1 shows the various domains of the response of a proton in the $|q|, \nu$ plane. The thick line $|q| = \nu$ separates the spacelike response above the line, and the timelike below the line. The thin line shows the $e - p$ elastic scattering kinematical condition:

$$\nu_{el} = \sqrt{\left(|q|^2 + m^2\right)} - m. \quad (2)$$

At large values of $|q|$ the $\nu_{el} = |q| - m$ up to terms of order $m^2/2|q|$. There can not be any response above the line $\nu_{el}(|q|)$ since none of the target states can have energy less than $\sqrt{\left(|q|^2 + m^2\right)}$. The dashed lines show parabolae: $|q|^2 - \nu^2 = Q^2$ for $Q^2 = 5$ and 10 GeV$^2$. They intersect the $\nu_{el}(|q|)$ curve at
$x = 1$ and approach the $\nu = |q|$ line at $\nu \to \infty$ or equivalently as $x \to 0$. In most of the literature the authors have considered the variation of $W_1$ along these parabolae which do not enter the timelike region. Here we study the variation of $W_1$ along the dash-dot lines, which have constant $|q|$ and enter the timelike region.

In fig.2 we show the proton $W^p_1(q, \nu)$, obtained from the MRS(A) fit of ref.3 to $e^-p$ and other cross sections, at several values of $|q|$ as a function of $\nu - |q| \equiv \tilde{y}$. The data show that at large values of $q$ the $W^p_1(q, \nu)$ depends primarily on $\tilde{y}$. This scaling has a simple interpretation in the many-body theory, related to the well known $y$-scaling [4], as will be discussed below. Since $\nu - |q| = -m\xi$, where $\xi$ is the Nachtmann [5, 6] scaling variable, it is closely related to Bjorken scaling as well. The Nachtmann variable, which results from operator product expansion studies, coincides with $x$ at large $Q^2$ and is generally used to extend the applicability of Bjorken scaling to lower $Q^2$. The small scaling violations seen in fig.2 originate from the gluonic radiative corrections as in the standard approach based on GLAP evolution equations [4]. Contrary to the case of $x$-scaling, in $\tilde{y}$ scaling $Q^2$ has a large
variation, from zero at $|q| = \nu (\tilde{y} = 0)$ to $\sim 2m|q|$ at $\nu = |q| - m (\tilde{y} = m)$, which does not appear to spoil the quality of scaling.

Many-body theory views the deep inelastic response as follows. The probe creates states $|i(q + k); R(-k)\rangle$ by hitting a bound constituent $i$ of the system with initial momentum $k$ and the residual system in the state $R$ with momentum $-k$. In the plane wave impulse approximation (PWIA) the final state interaction (FSI) between the struck constituent $i$ and the residual system $R$ are neglected. In this approximation the energy of the state $|i(q + k); R(-k)\rangle$ is:

$$E(i(q + k); R(-k)) = |q| + k\parallel + E(R(-k)) + \text{terms of order } \frac{1}{|q|}, \quad (3)$$

where $k\parallel$ is the projection of $k$ in the direction of $q$. At large $|q|$ the terms of order $1/|q|$ can be neglected, and the response due to the excitation of the state $|i(q + k); R(-k)\rangle$ occurs at energy transfer:

$$\nu = E(i(k + q); R(-k)) - m = |q| + k\parallel + E(R(-k)) - m. \quad (4)$$

Since the $\nu - |q|$ is independent of $q$ at large $|q|$, the response depends only on $\tilde{y}$ in the PWIA; therefore it scales.

The fact that the observed response of the proton, as seen in fig.2, scales with $\tilde{y}$ does not necessarily imply that PWIA is valid. In general the effects of the FSI on the response may not be negligible. Treatments of the FSI for very different cases of inclusive scattering of a probe from a composite system [7, 8, 9] have shown that the main effect of FSI results in a folding of the PWIA response:

$$W_1(|q|, \nu) = \int d\nu' W_{1,\text{PWIA}}(|q|, \nu')f(|q|, \nu, \nu'). \quad (5)$$

The scaling of $W_1(|q|, \nu)$ with $\tilde{y}$ can occur when the folding function representing the effect of the final state interactions becomes independent of $|q|$. For example, in the Glauber approximation, the folding function for the quasi-free scattering of electrons by nuclei becomes independent of $|q|$ at $|q| > 2$ GeV, as has been recently discussed by Benhar [8]. Weinstein and Negele [10] have shown that an analogous $y$-scaling in hard sphere Bose gas occurs even though the FSI effects are strong.
By boosting the state $|\mathcal{R}(-k), i(k)\rangle$ to large velocity, ignoring interaction between $i$ and $\mathcal{R}$, it can be shown that

$$\xi = \frac{|\mathbf{q}| - \nu}{m} = 1 - \frac{E(\mathcal{R}(-k)) + k_\parallel}{m}$$

(6)

has the usual meaning of the fraction of the momentum carried by the struck particle $i$.

The complete response is obtained by summing over all the final states. Therefore:

$$W_1(q, \nu) = \sum_i \int d^3 k \, d e \, \sigma_1(q, \nu, k, e) P_i(k, e) \delta(\tilde{y} - k_\parallel - e),$$

(7)

where the spectral function $P_i(k, e)$ is given by:

$$P_i(k, e) = \sum_\mathcal{R} |\langle \mathcal{R}(-k), i(k)|0\rangle|^2 \delta(m - E(\mathcal{R}(-k)) - e).$$

(8)

The $k, e$ can be considered as the initial momentum and energy of the struck particle, which is treated as bound, and therefore not on the mass shell. The $\sigma_1$ is the transverse cross-section for lepton scattering by a bound spin 1/2 point particle $i$, divided by $\sigma_M$. In the physical spacelike ($\tilde{y} < 0$) region, $\sigma_1 = q_i^2$ in the large $q$ limit, neglecting the mass $m_i$ of $i$. Here $q_i$ is the charge of $i$. This gives:

$$W_1(q, \nu) = W_1(\tilde{y}) = \sum_i q_i^2 \int d^3 k \, d e \, P_i(k, e) \delta(\tilde{y} - k_\parallel - e).$$

(9)

This Eq. provides the relation between the familiar $F_1(\xi) = mW_1(\xi)$ structure function and the $P_i(k, e)$ spectral function. The $\delta$-function implies that $\tilde{y} = k_\parallel + e$, closely resembling the scaling variable $y$ used in quasi-elastic electron-nucleus scattering, where it is associated with the parallel momentum of the struck nucleon.

This simple picture of the response will be modified by the color confining interactions. The mass of the nucleon contains confinement interaction contribution, while it is omitted in the energy of the struck quark, $|\mathbf{q}| + k_\parallel$. It therefore must be included in the energy $E(\mathcal{R})$ of the residual system. We expect that the confinement energy does not change significantly in the time duration of the DIS, and its main influence is via the wave functions $|0\rangle$ and $|\mathcal{R}\rangle$. However, it could also contribute to the FSI folding function (Eq.(5)).
An interesting feature of fig.2 concerns the width of the response, which amounts to only few hundred MeV independent of the value of $|q|$. This implies that deep inelastic scattering has an intrinsic energy scale of few hundred MeV. The main part of the transferred energy, of order $|q|$, goes into the kinetic energy of the struck constituent, and does not play any interesting role in the dynamics of the target system. Therefore changes in the energy $E(\mathcal{R}(-k))$ of the residual system of order 100 MeV have significant effect on the $W_1(\bar{y})$, or equivalently on $F_1(\xi)$. In the following we discuss two observed effects of $E(\mathcal{R}(-k))$ on the response.

The first, studied by Close and Thomas [11], concerns the difference between the responses due to valence u and d quarks in the proton. Let $V_u(\bar{y})$ and $V_d(\bar{y})$ be the contributions of valence u and d quarks to the $W_1^p(\bar{y})$. When the lepton strikes the valence d quark, the remaining two valence u quarks are left in the residual state $\mathcal{R}_1$ with spin 1. In contrast, when a valence u quark is struck, the residual u-d pair is in states $\mathcal{R}_0$ with spin 0 with probability 0.75, and $\mathcal{R}_1$ with probability 0.25. Therefore $\chi_1(\bar{y}) \equiv 9V_u(\bar{y})$ is the response due to final states $\mathcal{R}_1$ (normalized to unit particle charge), while $\chi_0(\bar{y}) \equiv 1.5(V_u(\bar{y}) - 2V_d(\bar{y})$ is that for $\mathcal{R}_0$. The

$$E(\mathcal{R}_1(-k)) - E(\mathcal{R}_0(-k)) \sim \frac{2}{3}(m_{\Delta} - m),$$

in perturbation theory, and therefore we expect $\chi_1(\bar{y})$ to be shifted by $\sim 0.2$ GeV from $\chi_0(\bar{y})$. Fig.3 shows that these responses obtained from the MRS(A) parton distributions at $|q| = 10$ GeV are indeed shifted by $\sim 0.1$ GeV from each other, at $\bar{y} < -0.2$. In the PWIA, this shift should be independent of $\bar{y}$, provided the color magnetic interaction can be treated perturbatively. The fact that the shift is only $\sim 0.1$ GeV indicates that it has nonperturbative contributions. Differences in FSI can also have an influence.

The second example concerns the modification of the deep inelastic response by nuclear effects [12] as first observed by the EMC collaboration [13]. The EMC ratio $R_A(x)$ of the cross section per nucleon, for nucleus with mass number $A$ to that for the deuteron, does not show any $Q^2$ dependence within the experimental errors. The observed $R_A(x)$ has been extrapolated using Local Density Approximation to obtain the ratio $R_{NM}(x)$ for uniform nuclear matter [14]. In fig.4 we show the $W_1^d(\bar{y})$ for the deuteron calculated from the MRS(A) fits, and the $W_1^{NM}(\bar{y})$ at $|q| = 10$ GeV. As we see from this figure, the nuclear matter response is quite similar to that of the deuteron. It
is a bit broader, due to the Fermi motion of nucleons in matter, but mainly it is shifted towards higher $\nu$ due to nuclear binding. Fig. 4 also shows the response of noninteracting nucleons distributed according to the momentum distribution of nucleons in nuclear matter, also calculated using realistic interactions [15]. The observed response is shifted relative to the Fermi motion broadened response by $\sim 40-60$ MeV, which is comparable to the average nucleon removal energy of $\sim 62$ MeV in nuclear matter [16]. For these reasons, the conventional nuclear physics approach is quite successful in describing the EMC ratio for nuclear matter at $x > 0.4$ [17].

In the parton model the struck particle is assumed to be on mass-shell before and after the interaction with the electron [18]. In this case

$$\nu = \sqrt{m_i^2 + (k + q)^2} - \sqrt{m_i^2 + k^2} \leq |q|,$$

(11)

and all of the response is at negative $\tilde{y}$, in the spacelike region. The same is valid at the leading twist-two order of the operator product expansion [8].

In many body theory, however, a timelike response occurs either due to initial state interactions, which can make $E(\mathcal{R}(-k))$ large enough to give a
positive right hand side of eq. (4), or because of FSI. The initial energy of the struck constituent is identified with $e = m - E(R(-k))$ and not with the on shell energy $\sqrt{m_i^2 + |k|^2}$ used in eq. (11). This timelike response contributes to various sum rules. For example, the Coulomb sum in quasi-elastic electron nucleus scattering is defined as the integral of the longitudinal response over both space and time like regions. The longitudinal response of deuterium has been calculated with realistic forces [19]; it extends into the timelike region, and that region has to be included to fulfill the Coulomb sum.

In fact, the shifts in $W_1(\tilde{y})$ illustrated in figs. 3 and 4 will move part of the response into the timelike region, barring FSI effects, and thus lead to a violation of sum rules involving $W_1(\tilde{y} < 0)$.

In conclusion, we obtain new insights in the deep inelastic response of nucleons by applying standard many-body theory and relate the scaling function to the nucleon spectral function in the lab frame. The natural scaling variable of many body theory, $\tilde{y}$, equals $-m\xi$ of the conventional approach to DIS. While $\tilde{y}$ scaling is derived assuming bound constituents which are
subject to initial and final state interaction, $\xi$- or $x$-scaling is obtained assuming free constituents without FSI. The occurrence of scaling thus cannot automatically be taken as evidence for scattering from free constituents.

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