Numerical study of thermoacoustic detection of composite materials

Yan Pan¹ and Hanping Hu¹
¹Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, Anhui 230027, P. R. China

E-mail: pynash@mail.ustc.edu.cn

Abstract. In this paper, to study the applicability of thermoacoustic detection of composite materials, the distribution of thermoacoustic waves was numerically calculated when there was a nickel domain inside the silicon domain by applying a sinusoidally varying heat source on the surface of solid silicon domain. By changing the width, thickness and position of the internal nickel domain, the applicable conditions of detecting whether there is another material inside the silicon domain by the distribution of the thermoacoustic waves were studied. In addition, a method of judging the width of the inner nickel domain was given. And the corresponding error was studied.

1. Introduction
The thermoacoustic effect is the phenomenon of mutual conversion between heat and sound. According to the direction of energy conversion, the thermoacoustic effect can be divided into the following two categories [¹]: first, heat-driven acoustic oscillations, in which heat energy is converted into kinetic energy (acoustic signal); second, sound-driven heat transfer, which uses acoustic oscillations to generate heat. Thermoacoustic imaging, based on the principle of heat-driven acoustic oscillations, has become a hotspot in the field of medical imaging [², ⁴], and it is widely used in tumor detection [⁵], angiography [⁶, ⁸], mouse brain structure, functional imaging [⁹] and molecular imaging [¹⁰]. In recent years, the thermoacoustic effect has also been used to detect the thermal properties of materials and test the quality of resistance weld nugget [¹¹] and other directions. Previous studies of thermoacoustic imaging are mainly focused on the field of medicine, and few research on the application of thermoacoustic effect in the detection of composite materials. When solid material is locally heated under periodically modulated heat flow, and then due to thermal expansion and contraction, the pressure in the surrounding medium changes periodically, thereby the thermoacoustic waves emerge [¹²]. In this work, based on thermomechanical coupling equations, we calculated the distribution of thermoacoustic waves in the two-dimensional axisymmetric composite medium (nickel domain inside the silicon domain), and studied the variation of thermoacoustic distribution when the nickel domain moved up and down, and the width of the nickel domain and the thickness of the nickel domain were changed. Therefore, a method of detecting the width of the internal nickel domain was given, and the variation of the detected width error was analyzed.

2. Thermomechanical coupling equations
The equations for thermoacoustical simulation are as follows:
\[ \nabla(\nabla P) - \frac{\rho_i}{E_i} \frac{\partial^2 P_i}{\partial t^2} = -\rho_i \beta T_i \frac{\partial^3 T_i}{\partial t^2} \] (1)

\[ \nabla(K_i \nabla T_i) - \frac{K_i}{\alpha_i} \frac{\partial T_i}{\partial t} + S_i = -T_0 \beta T_i \frac{\partial P_i}{\partial t} \] (2)

Where \( K_i \) is the thermal conductivity, \( \alpha_i \) is the ambient temperature, \( \beta T_i \) is the coefficient of thermal expansion, \( \rho_i \) is the density, \( E_i \) is the elastic modulus, \( P_i \) is the pressure, \( T_i \) is the temperature, and \( S_i \) is the heat source. The subscript “i” in the above equations represents different solution domains.

3. Computational domain

The calculation area is the cylinder shown in Figure 1-(a), which is rotated by the rectangle shown in Figure 1-(b) around the left boundary and its dimensions are shown in Figure 1-(b). The outer layer of the cylinder is the air domain (this domain is for easy numerical simulation). The middle layer is silicon domain, and the inner layer is nickel domain. The heat transfer boundary condition is thermal insulation, the pressure acoustics boundary condition is impedance, which is \( 1.2[kg/m^3]*343[m/s] \). The initial value: \( T = T_0 \), \( P = 0 \). This model is two-dimensional axisymmetric, so its computational results must be two-dimensional axisymmetric too. So we only need to pay attention to the distribution of thermoacoustic waves on the cross section shown in Figure 1-(b). The sinusoidally varying heat source on the upper surface of the silicon domain is expressed as follows:

\[ S = I_0*(1+\cos(\omega*\pi*t)) \] (3)

Where \( \omega \) is the frequency of the heat source, \( I_0 \) is the power of the heat source. Since the generation of thermoacoustic waves is only related to the AC term of the heat source, the DC term is ignored. So the heat source expression becomes as follows:

\[ S = I_0*\cos(\omega*\pi*t) \] (4)

The number of grid nodes in this paper is 9595. Verified by grid independence, the number of grids satisfies the computational requirements of this paper.

4. Comparison of different heat source powers

The computational results are shown in Figure 2-(a) (b) (c) (d) when the heat source power is 0.01w/mm² 0.1w/mm² 1w/mm² and 10w/mm² respectively. By comparison, the thermoacoustic waves' amplitude is smaller and the distribution of the thermoacoustic waves is not ideal when the heat source power is 0.01w/mm² and 0.1w/mm² respectively. The thermoacoustic waves' amplitude is larger and the distribution of the thermoacoustic waves is ideal and almost constant when the heat source power is 1w/mm² and 10w/mm² respectively. The reason is that the computational model is very small, the thermoacoustic waves' wavelength is also very small in order to use it to detect, so the frequency must be large through the formula \( v = \lambda * f \). The frequency of the heat source is the same as
the thermoacoustic waves’, so the frequency of the heat source is also large. The acoustic energy converted from heat energy is divided by more thermoacoustic waves equally at the same time, so the power of the heat source must be particularly large in order to use thermoacoustic waves to detect better. So in this paper, the heat source power is all taken 1w/mm².

![Figure 2](image)

**Figure 2.** Comparison of the computational results for different heat source powers

5. **Result analysis**

We studied the distribution of thermoacoustic waves when there was not nickel domain inside the silicon domain as a comparison. The distribution of thermoacoustic waves at 5.4e-8s is shown in Figure 3-(a). The distribution of thermoacoustic waves is approximately one-dimensionally distributed along the z direction. Due to the influence of the right border, the distribution of thermoacoustic waves on the right boundary is not one-dimensionally. The thermoacoustic waves on the straight line ab at 5.4e-8s 6.7e-8s and 7.8e-8s are shown on the asterisk curve circular curve and square curve in Figure 3-(b). By this three curves, the thermoacoustic waves’ crest is basically the same and unchanged at all times.
Figure 3. The computational results when there is not a nickel domain in the silicon domain

The distribution of thermoacoustic waves at 6.7e-8s is shown in Figure 4-(a) when there is nickel domain inside the silicon domain. The thermoacoustic waves on the straight line ab and cd at 6.7e-8s are shown on the solid curve and dotted curve in Figure 4-(b). By the Figure, the pressure at the right part of the lower boundary of the silicon domain is zero, and the pressure at the left part is not zero, by which it is possible to detect that there is another object inside the silicon domain. In addition, the thermoacoustic waves on the straight line cd at 6.7e-8s are basically the same with the thermoacoustic waves at 6.7e-8s in Figure 3-(b). The wavelength of the thermoacoustic waves on straight line ab at 0.2-0.4mm is different from others, which is due to the different physical properties of the two materials.

Figure 4. The computational results when there is a nickel domain in the silicon domain

The distribution of thermoacoustic waves near the lower boundary along the r direction from 0 to 1 is shown in Figure 5. By changing the width, thickness and position of the internal nickel domain, we found that the distributions of thermoacoustic waves at the same place were similar and they all had inflection points as shown in Figure 5. So we take the position of ‘a’ point (shown in Figure 5) as the right boundary of the inner nickel domain, by which we deduce that the width of the inner nickel domain shown in Figure 4 is 0.4mm. The error is 0.1mm, and the relative error is 20%. 
Figure 5. The distribution of thermoacoustic waves near the lower boundary

6. The results when the nickel domain changes

6.1. The results when the nickel domain moves up and down

The computational results are shown in Figure 6 when the nickel domain moves up and down. The distributions of thermoacoustic waves are basically the same. It is all possible to detect that there is another object inside the silicon domain by the difference of the distribution of thermoacoustic waves near the lower boundary. The difference is that the closer the nickel domain is to the lower boundary, the more obvious the difference of the distribution of thermoacoustic waves near the lower boundary is. So it is easier to detect that there is another object inside the silicon domain when the internal nickel domain is closer to the lower boundary of the silicon domain.

Figure 6. The computational results when the nickel domain moves up and down

The detected widths of the nickel domain and the corresponding errors, according to the method shown in Figure 5, are shown in Table 1 when the nickel domain moves up and down. The distance in the Table 1 is the distance from the lower boundary of the nickel domain to the lower boundary of the silicon domain. From the Table 1, it is all possible to detect the width of the nickel domain when the nickel domain moves up and down. But the closer the nickel domain is to the lower boundary, the smaller the error and the relative error are. Therefore, when using this method, it is better to add the heat source to the outer boundary farther from the inner object.

Table 1. The detected widths and the errors when the nickel domain moves up and down

| Distance (mm) | Detected width (mm) | Error (mm) | Relative error |
|---------------|---------------------|------------|---------------|
| 0.40          | 0.335               | 0.165      | 33%           |
| 0.35          | 0.350               | 0.150      | 30%           |
6.2. The results when the width of the nickel domain changes

The computational results are shown in Figure 7 when the width of nickel domain changes. By the first two figures, it is all possible to detect that there is another object inside the silicon domain. But, when the nickel domain is very small shown in Figure 7-(c), the difference of the distribution of thermoacoustic waves near the lower boundary is not obvious, it is difficult to detect that there is another object inside the silicon domain. The reason is that the thermoacoustic waves pass faster in the silicon domain, the waves pass to the lower surface earlier. Then the thermoacoustic waves near the lower surface pass to the left as shown in Figure 7-(d).

| Width (mm) | Pressure (Pa) | Error (%) |
|-----------|---------------|-----------|
| 0.30      | 0.365         | 135       | 27%   |
| 0.25      | 0.375         | 125       | 25%   |
| 0.20      | 0.400         | 100       | 20%   |
| 0.15      | 0.415         | 085       | 17%   |
| 0.10      | 0.430         | 070       | 14%   |
| 0.05      | 0.445         | 055       | 11%   |
| 0.00      | 0.475         | 025       | 5%    |

Figure 7. The computational results when the width of nickel domain changes

The detected widths of the nickel domain and the corresponding errors, according to the method shown in Figure 5, are shown in Table 2 when the width of nickel domain changes. From the Table 2, when the width of nickel domain is less than 0.15mm, the width cannot be detected by this method. When the width is between 0.15mm and 0.16 mm, the inflection point is not obvious, it is difficult to detect the width. When width is more than 0.16mm, the inflection point is obvious, it is possible to
detect the width. And as the width becomes larger, the errors are basically the same, they are 0.1mm, and the relative error decreases. So such a detection method is not applicable when the internal object is very small.

| Table 2. The detected widths and the errors when the width of nickel domain changes |
|---------------------------------------------------------------|
| Width (mm) | Detected width (mm) | Error (mm) | Relative error |
|-------------|----------------------|------------|----------------|
| 0-0.15      | Can’t detect         |            |                |
| 0.15-0.16   | Not obvious          | 0.105      | 52.5%          |
| 0.20        | 0.095                | 0.100      | 33.3%          |
| 0.30        | 0.200                | 0.100      | 25.5%          |
| 0.40        | 0.300                | 0.100      | 20.0%          |
| 0.50        | 0.400                | 0.100      | 16.7%          |
| 0.60        | 0.500                | 0.100      | 24.3%          |
| 0.70        | 0.600                | 0.110      | 13.7%          |
| 0.80        | 0.690                |            |                |

6.3. The results when the thickness of the nickel domain changes

The computational results are shown in Figure 8 when the thickness of the nickel domain changes. (The distance from the lower boundary of the nickel domain to the lower boundary of the silicon domain does not change). From the Figure 8, it is all possible to detect that there is another object inside the silicon domain regardless of whether the thickness increases or decreases. The difference is that the difference of the distribution of thermoacoustic waves near the lower boundary of the silicon domain lasts longer when the thickness increases.

| Table 3. The detected widths and the errors when the thickness of the nickel domain changes |
|---------------------------------------------------------------|
| Thickness (mm) | Detected width (mm) | Error (mm) |
|----------------|----------------------|------------|
| 0.10           | 0.396                | 0.104      |
| 0.15           | 0.398                | 0.102      |
| 0.20           | 0.400                | 0.100      |
| 0.25           | 0.400                | 0.100      |
| 0.30           | 0.395                | 0.105      |
| 0.35           | 0.395                | 0.105      |
7. Conclusions
When there is nickel domain inside the silicon domain, it is possible to detect that there is another object inside the silicon domain by the difference of the distribution of thermoacoustic waves near the lower boundary of the silicon domain. Furthermore the width of the nickel domain can also be detected. When the nickel domain is closer to the lower boundary of the silicon domain, it is easier to detect that there is another object inside the silicon domain, and the error of the detected width is smaller. When the width of nickel domain is less than 0.15mm (15% of total length), it is difficult to detect that there is another object inside the silicon domain, the width cannot be detected too. When the width becomes larger, the error of the detected width is basically the same. The error is only related to the distance from the lower boundary of the nickel domain to the lower boundary of the silicon domain.

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