Reproving the basic principles of fuzzy sets for the decisions

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Abstract. Principles in the logic and the sets form the basis of many societal solutions that are present with physical as well as non-physical phenomena. Non-physical phenomena, for example present language variables that require the presence of the fuzzy logic and the fuzzy sets. Although specific verification of the rules used remains the same, the systematic verification for the fuzzy sets requires a strategy, including when implementing it into a computer program. The final problem has the answer to the change in the operator implemented in the program.

1. Introduction
Decisions rationally come from choices that can be considered [1]. The consideration of several choices depends on the measurement. The measurement comes from the ability of the calculation. Then the calculation is revealed from the applicable rules [2, 3]. Therefore, decisions based on ordinary concepts and logic easily implemented by programs [4]. However, when it comes to numbers with varying decimal numbers as in fuzzy sets, it must reveal its calculation strategy. This paper reveals a strategy of carrying out calculations through a computer program. But, the guideline used it to reprove the basic principles of the decision either in the fuzzy logic or the fuzzy sets.

2. A background
Systematically, based on the logic a statement $p$ that represents an event in a context is true (T) or false (F). For each statement $p$, there is a negation of $p$, that is $\overline{p}$. A pair of statements, $p$ and $q$, there is a compound statement with a logical connective (as an operator) of: disjunction $p \lor q$ for $p$ or $q$, conjunction $p \land q$ for $p$ and $q$, implication $p \Rightarrow q$ for if $p$ then $q$, or equivalence $p \iff q$ for $p \land q$ if and only if $q \land p$. Two or more statements with the connection form logical rules and then $p, q, \ldots$ as the propositions [5], see Table 1. On the other hand, a basic concept, well-defined, is a set conceptually denoted as $A$, which can be expressed as $A = \{a_i | i = 1, \ldots, n\}$, where $a_i$ are the members of the set $A$, i.e. $a_i \in A$. For example, $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is a set of integers, obviously the following numbers 5, 7, 11 is in $Z$ and also it is logically true value (T). However, it is logically possible that a set has no members. The set recognized as an empty set and denoted by $\emptyset$ or $\{}$. Then in general, each set becomes a subset of the universe of discourse or it denoted by $\mathcal{U}$, so for all sets: $A_j \subseteq \mathcal{U}$ for $j = 1, \ldots, m$ [6]. Therefore, for each set $A$ there is a complement and it denoted by $\overline{A}$, for example $\frac{1}{2} \notin Z$. Let there are $A_1 = \{a, a, b, c, d\}$, $A_2 = \{a, b, c, d\}$ and $A_3 = \{a, c, d, b\}$, and
then those sets are same or $A_1 = A_2 = A_3$. While two operators, a union $\cup$ or an intersection $\cap$, deal with a pair of sets. In general, the rules that apply to sets are similar to logic [7], as summarized in Table 2.

Philosophically, all the rules that apply in logic are also valid in a set. Suppose $A$ and $B$ are two sets, $A \cap B \neq \emptyset$ if $A \cap B = \{x | x \in A \wedge x \in B\}$. In other words, $A \cap B \neq \emptyset$ if $x \in A$ then there is one $x \in B$. It is quantified briefly as follows $x \in A, \exists x \in B \Rightarrow A \cap B \neq \emptyset$, where $\exists$ is a quantifier for there is, $\forall$ is a quantifier for for all, and $!$ means that at least one [8].

Unlike logic, a set has a size that is several members in a set denoted by $| \cdot |$. The size of the set $A$ is $|A|$. Therefore, $|\emptyset| = 0$ and $|Z| = \infty$, then $|U| = \infty$. In short $0 \leq | \cdot | \leq \infty$. Thus, philosophically it is possible to underlie a systematics other than the natural event model. Namely, the likelihood of an event occurring and true become a fact [9]. Each coin, which consists of the head and the tail will form a set, that is $C = \{H, T\}$. It is a set of two sides of a coin. An event of chanting of the coin will appear above is one of the two sides, and is expressed as an opportunity of the head or the tail, each of them denoted as $p(H)$ or $p(T)$, where each value is $1/2$. In other words, the weight of the appearance event for the head or the tail on a logically correct coin is the number of head or tail divided by the size of the set [10]. Likewise, the probability of each side of the dice appearing when chanted is $1/6$ where the set of dice eyes is $D = \{1, 2, 3, 4, 5, 6\}$ [11]. The probability of an event depends on the inclusion of that event in

| $p$ | $q$ | $\overline{p}$ | $\overline{p} \land q$ | $p \lor q$ | $p \land q$ | $\overline{p} \land q$ | $p \lor q \land r$ | $\overline{p} \lor q$ | $p \land q \lor r$ | $\overline{p} \lor q \land r$ | $p \land q \lor r$ |
|-----|-----|-------------|-----------------|--------|--------|-----------------|----------------|-------------|-----------------|----------------|----------------|
| T   | T   | F           | F               | T      | T      | F               | T              | T           | T               | T              | T              |
| F   | T   | F           | F               | T      | T      | T               | F              | T           | F               | F              | F              |
| F   | T   | T           | F               | T      | F      | F               | T              | F           | F               | F              | F              |
| F   | F   | T           | F               | T      | F      | F               | T              | T           | F               | F              | F              |

Table 1. Some of truth value for the statements.

| Laws | Propositions $p, q, r$ | For sets $A, B, C$ |
|------|----------------------|-------------------|
| Double negation | $\overline{\overline{p}}$ | $p$ | $\overline{\overline{A}}$ | $A$ |
| DeMorgan | $p \lor \overline{q}$ | $p \land \overline{q}$ | $A \cup \overline{B}$ | $A \cap \overline{B}$ |
| Commutative | $p \lor q$ | $q \lor p$ | $A \cup B$ | $B \cup A$ |
| Associative | $p \lor (q \lor r)$ | $(p \lor q) \lor r$ | $A \cup (B \cup C)$ | $(A \cup B) \cup C$ |
| $p \land (q \land r)$ | $(p \land q) \land r$ | $A \cap (B \cap C)$ | $(A \cap B) \cap C$ |
| Distributive | $p \lor (q \lor r)$ | $(p \lor q) \lor r$ | $A \cup (B \cap C)$ | $(A \cup B) \cap C$ |
| Idempotent | $p \lor p$ | $p$ | $A \cup A$ | $A$ |
| $p \land p$ | $p$ | $A \cap A$ | $A$ |
| Identity | $p \lor F$ | $p$ | $A \cup \emptyset$ | $A$ |
| $p \land T$ | $p$ | $A \cap U$ | $A$ |
| Invers | $p \lor \overline{p}$ | $T$ | $A \cup \overline{A}$ | $U$ |
| $p \land \overline{p}$ | $F$ | $A \cap \overline{A}$ | $\emptyset$ |
| Domination | $p \lor T$ | $T$ | $A \cup U$ | $U$ |
| $p \land F$ | $F$ | $A \cap \emptyset$ | $\emptyset$ |
| Absorption | $p \lor (p \land q)$ | $p$ | $A \cup (A \cap B)$ | $A$ |
| $p \land (p \lor q)$ | $p$ | $A \cap (A \cup B)$ | $A$ |
the set of possible events. The probability of an event not included in that possible event is 0. The probability of an event occurring as the only likelihood is 1. In other words, the probability of an event being in the range of 0 and 1 closed, that is $p() \in [0, 1]$ \[12\].

The head (H) or the tail (T) is the name for the side of each coin. Likewise, numbers 1 through 6 each are the naming of the sides of the dice. In the language, some words that refer to the inter-opposite meaning logically. Except for stating the truth values, namely the word "true" or "false" that has two conditions only. Many other words express different conditions for their meaning as is "strong", "normal", and "weak". Thus, not only do variables in mathematics take numerical values, but they also accept values referred to by words (not numeric), such as heat and cold, young and old, and others as linguistic variables \[13\].

With the presence of the set of characteristics above, there is a model as a mathematical way to understand the obscurity of observations when making decisions based on incorrect or non-numeric information and therefore called the fuzzy. In logical expression, the fuzzy expression produces the same expressions as a certainty for true or false. However, as the language of logic expresses, it will range from the value of truth that is completely false and the value of truth that is entirely true, namely in $[0, 1]$. Thus, the logical operator in the expression $p \land q$, $p \lor q$, and $\neg p$ transform into a fuzzy logic operator of: $p \land q \rightarrow \max(p, q)$, $p \lor q \rightarrow \min(p, q)$, and $\neg p \rightarrow (1 - p) \[14\].

3. Learning approach

To reaffirm the relationship between the logic and the set as learning is that an element of any set $A$ has only two possibilities, which are included and not included in the set $A$. Two possibilities are represented as logical values (F) and (T). Based on binary numbers the logical values are rewritten with 0 and 1. However, the interpretation of logic variables as implication of the relationship between logic and sets, for example in computer programming using Python, is expressed as follows: pure logic, if $a==2$; logic and set, if $a in b$; or logic variable, if $a==b$ \[15\]. Thus, in simplifying some of the above laws, for example, it can be re-expressed by involving variables $x$ and $y$ where logically the meaning of universal set $\mathcal{U}$ shift to 1 (one) and so the meaning of empty set $\emptyset$ shift to 0 (null). So, in the language logic "negation $x$" be

$$\neg x = 1 - x,$$ \[1\]

in language logic may be written as "$x$ and $y$", where logically it is $x \land y$, and its simplification becomes

$$x \land y = xy.$$ \[2\]

Meanwhile, "$x$ or $y$" becomes $x \lor y$, or

$$\frac{(x \lor y)}{(x \land y)} = \frac{(\neg \land \neg y)}{(\neg \land y)} \quad \text{DeMorgan Law}$$
$$\frac{x \lor y) = 1 - (\neg \land y)}{x \land y) = 1 - (\neg \land y)} \quad \text{Double Negation Law}$$

Equation (1)

$$1 = 1 - (1 - x) \land (1 - y) \quad \text{Eq. (1)}$$
$$1 = 1 - ((1 - x) \land (1 - y)) \quad \text{Eq. (1)}$$
$$1 = 1 - ((1 - x)(1 - y)) \quad \text{Eq. (2)}$$

Equation (3)

$$= 1 - (1 - y - x + xy)$$
$$= 1 - 1 + y + x - xy$$
$$= y + x - xy$$

By learning that if $a \in A$ then the element is worth 1, conversely if $a \notin A$ then the element is 0. The approach to value states that the degree of membership in the A set is denoted by $\mu_A(x)$. If $x$ is included in the set $A$ then $\mu_A(x) = 1$ and if vice versa then $\mu_A(x) = 0$, or

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$ \[4\]
where $\mu_A$ is a characteristic function of the set $A$. Meanwhile, the degree of membership for each element in the fuzzy set is in an interval of $[0, 1]$ [16].

**Definition 1. Fuzzy Set:**
A fuzzy set $A$ in $\mathcal{U}$ is expressed with a membership function

$$\mu_A(x) : \mathcal{U} \rightarrow [0, 1]$$ (5)

$\forall x \in \mathcal{U}$ and $\mu_A(x) \in [0, 1]$.

The definition confirms that there is a membership degree $[0, 1]$ and a domain that is a subset of $\mathcal{U}$. In Cartesian coordinate, the membership degree on $y$-axis, while the domain on $x$-axis. Therefore, a membership function is a function that transforms the domain in $\mathcal{U}$ into $[0, 1]$.

Thus,

$$\mu_\mathcal{U}(x) = 1,$$ (6)

and

$$\mu_\emptyset(x) = 0.$$ (7)

Defining is an approach to express clearly the basis of operations in the fuzzy logic and the fuzzy set and learning of the laws that apply in the fuzzy logic and the fuzzy set. The basic operations in a fuzzy set as following

(i) Complement:
Suppose that $A$ is a fuzzy set in the universal set $\mathcal{U}$, the complement of a fuzzy set $A$ is $\overline{A}$ expressed as

$$\mu_{\overline{A}}(x) = 1 - \mu_A(x).$$ (8)

(ii) Intersection:
Assuming that $A$ and $B$ are fuzzy sets in the universal set $\mathcal{U}$, the basic operation for the intersection between $A$ and $B$ is $(A \cap B)$ and expressed it as

$$\mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x) = \min(\mu_A(x), \mu_B(x)).$$ (9)

(iii) Union:
Assuming that $A$ and $B$ are fuzzy sets in the universal set $\mathcal{U}$, the basic operation for the union of $A$ and $B$ is $(A \cup B)$ and expressed it as

$$\mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x) = \max(\mu_A(x), \mu_B(x)).$$ (10)

Determining the value of $\mu_{A \cap B}(x)$ or the value $\mu_{A \cup B}(x)$ depend on the result (truth) of comparison logic operator: $\mu_A(x) < \mu_B(x)$ or $\mu_A(x) > \mu_B(x)$. Thus, even though according to the laws in Table 2, $\mu_{\overline{A \cap B}}(x)$ becomes $\mu_{\overline{A}}(x)$ and $\mu_{\overline{A \cup B}}(x)$ adjusted to $\mu_{\overline{A}}(x)$, but logically and systematically the determination must be clearly express it or perhaps structurally through algebra. It shows that the need for some interpretations of the laws of the fuzzy logic and sets for learning to understand the principles of the fuzzy logic and the fuzzy sets.

4. Evidence
First is about the proof of all the laws in Table 2 for fuzzy logic and fuzzy sets. Second is about reproving some that logically needs proof systematically, and then show treatment of numbers in $[0, 1]$ for programming in Python as a refinement of the proof of those laws.
### Table 3. The laws of the fuzzy logic and the fuzzy sets.

| Laws            | Propositions \( p, q, r \) | For fuzzy sets \( A, B, C \) |
|-----------------|-------------------------------|-----------------------------|
| Double negation | \( 1 - (1 - p) \) ⇔ \( p \)  | \( \mu_A(x) = \mu_A(x) \)  |
| DeMorgan        | \( 1 - \max(p, q) \) ⇔ \( \min(1 - p, 1 - q) \) | \( \mu_{(A \cup B)}(x) = \mu_{(A \cup B)}(x) \) |
|                 | \( 1 - \min(p, q) \) ⇔ \( \max(1 - p, 1 - q) \) | \( \mu_{(A \cap B)}(x) = \mu_{(A \cap B)}(x) \) |
| Commutative     | \( \max(p, q) \) ⇔ \( \max(q, p) \) | \( \mu_{(A \cup B)}(x) = \mu_{(B \cup A)}(x) \) |
|                 | \( \min(p, q) \) ⇔ \( \min(q, p) \) | \( \mu_{(A \cap B)}(x) = \mu_{(B \cap A)}(x) \) |
| Associative     | \( \max(p, \max(q, r)) \) ⇔ \( \max(\max(p, q), \max(p, r)) \) | \( \mu_{(A \cup (B \cup C))}(x) = \mu_{(A \cup (B \cup C))}(x) \) |
|                 | \( \min(p, \min(q, r)) \) ⇔ \( \min(\min(p, q), \min(p, r)) \) | \( \mu_{(A \cap (B \cap C))}(x) = \mu_{(A \cap (B \cap C))}(x) \) |
| Distributive    | \( \max(p, \min(q, r)) \) ⇔ \( \min(\max(p, q), \max(p, r)) \) | \( \mu_{(A \cup (B \cup C))}(x) = \mu_{(A \cup (B \cup C))}(x) \) |
|                 | \( \min(p, \max(q, r)) \) ⇔ \( \max(\min(p, q), \min(p, r)) \) | \( \mu_{(A \cap (B \cap C))}(x) = \mu_{(A \cap (B \cap C))}(x) \) |
| Idempoten       | \( \max(p, p) \) ⇔ \( p \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
|                 | \( \min(p, p) \) ⇔ \( p \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
| Identity        | \( \max(p, 0) \) ⇔ \( p \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
|                 | \( \min(p, 1) \) ⇔ \( p \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
| Invers          | \( \max(p, 1 - p) \) ⇔ \( 1 \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
|                 | \( \min(p, 1 - p) \) ⇔ \( 0 \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
| Domination      | \( \max(p, 1) \) ⇔ \( 1 \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
|                 | \( \min(p, 0) \) ⇔ \( 0 \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
| Absorption      | \( \max(p, \min(p, q)) \) ⇔ \( p \) | \( \mu_{A}(x) = \mu_{A}(x) \) |
|                 | \( \min(p, \max(p, q)) \) ⇔ \( p \) | \( \mu_{A}(x) = \mu_{A}(x) \) |

### 4.1. Propositions and their proof

The first evidence is classically derived as follows [17].

**Proposition 1.** Double complement law:

Let \( A \) be a fuzzy set in the universal set \( U \), then \( \forall x \in U \) as domain applies that

\[
\mu_{(\overline{A})}(x) = \mu_A(x) \tag{11}
\]

**Proof.** By implementing Eq. (8) we have

\[
\mu_{(\overline{A})}(x) = 1 - \mu_A(x) = 1 - (1 - \mu_A(x)) = 1 - 1 + \mu_A(x) = \mu_A(x)
\]

Eq. (11) is proven.

**Proposition 2.** DeMorgan laws:

Let \( A \) and \( B \) be the fuzzy sets in the universal set \( U \), then \( \forall x \in U \) as domain applies that

\[
\mu_{(A \cup B)}(x) = \mu_{(\overline{A \cap B})}(x) \tag{12}
\]

and

\[
\mu_{(A \cap B)}(x) = \mu_{(\overline{A \cup B})}(x) \tag{13}
\]

**Proof.** Implementing Eqs. (8), (9) and (10) produce evidence that \( \mu_{(A \cup B)}(x) = 1 - \mu_{A \cup B}(x) = \mu_{(\overline{A \cap B})}(x) \)
1 − max(μ_A(x), μ_B(x)) = min(1 − μ_A(x), 1 − μ_B(x)) = min(μ_{A^c}(x), μ_{B^c}(x)) = μ_{(A^c∩B^c)}(x),
and
μ_{(A∪B)^c}(x) = 1 − μ_A∪B(x) = 1 − min(μ_A(x), μ_B(x)) = max(1 − μ_A(x), 1 − μ_B(x)) = max(μ_{A^c}(x), μ_{B^c}(x)) = μ_{(A^c∩B^c)}(x). So, Eq. (12) and Eq. (13) are proven.

**Proposition 3.** Commutative laws:
Let A and B are the fuzzy sets in the universal set U, then ∀x ∈ U as domain applies that

\[
\mu_{(A∪B)}(x) = \mu_{(B∪A)}(x)
\]  

(14)

and

\[
\mu_{(A∩B)}(x) = \mu_{(B∩A)}(x)
\]  

(15)

**Proof.** By using Eqs. (9) and (10), μ_{(A∪B)}(x) be max(μ_A(x), μ_B(x)) = max(μ_B(x), μ_A(x)) = μ_{(B∪A)}(x) and μ_{(A∩B)}(x) be min(μ_A(x), μ_B(x)) = min(μ_B(x), μ_A(x)) = μ_{(B∩A)}(x). It proves Eq. (14) and Eq. (15).

**Proposition 4.** Associative laws:
Let A, B and C are the fuzzy sets in the universal set U, then ∀x ∈ U as domain applies that

\[
\mu_{(A∪(B∪C))}(x) = \mu_{((A∪B)∪C)}(x)
\]  

(16)

and

\[
\mu_{(A∩(B∩C))}(x) = \mu_{((A∩B)∩C)}(x)
\]  

(17)

**Proof.** By using Eqs. (9) and (10), the description μ_{(A∪(B∪C))}(x) = max(μ_A(x), μ_{(B∪C)}(x)) = max(μ_A(x), max(μ_B(x), μ_C(x))) = max(μ_A(x), μ_B(x), μ_C(x)) = max(max(μ_A(x), μ_B(x), μ_C(x)), μ_C(x)) = max(μ_{A∪B}(x), μ_C(x)) in the description μ_{((A∪B)∩C)}(x) = min(μ_A(x), min(μ_B(x), μ_C(x))) = min(μ_A(x), μ_B(x), μ_C(x)) = min(min(μ_A(x), μ_B(x)), μ_C(x)) = min(μ_{(A∪B)}(x), μ_C(x)) = μ_{((A∪B)∩C)}(x) prove Eq. (16) and Eq. (17).

**Proposition 5.** Distributive laws:
Let A, B and C are the fuzzy sets in the universal set U, then ∀x ∈ U as domain applies that

\[
\mu_{(A∪(B∩C))}(x) = \mu_{(A∪B)∩(A∪C)}(x)
\]  

(18)

and

\[
\mu_{(A∩(B∪C))}(x) = \mu_{(A∩B)∪(A∩C)}(x)
\]  

(19)

**Proof.** By using Eqs. (9) and (10), the following statements prove Eq. (18) and Eq. (19). That is μ_{(A∪(B∩C))}(x) = max(μ_A(x), μ_{(B∩C)}(x)) = max(μ_A(x), min(μ_B(x), μ_C(x))) = min(max(μ_A(x), μ_B(x)), max(μ_A(x), μ_C(x))) = min(μ_{(A∪B)}(x), μ_{A∪C}(x)) = μ_{((A∪B)∩(A∪C))}(x) and μ_{(A∩(B∪C))}(x) = min(μ_A(x), μ_{(B∪C)}(x)) = min(μ_A(x), max(μ_B(x), μ_C(x))) = max(min(μ_A(x), μ_B(x)), min(μ_A(x), μ_C(x))) = max(μ_{(A∩B)}(x), μ_{A∩C}(x)) = μ_{((A∩B)∪(A∩C))}(x).

**Proposition 6.** Idempotent laws:
Let A is a fuzzy set in the universal set U, then ∀x ∈ U as domain applies that

\[
\mu_{(A∪A)}(x) = \mu_A(x)
\]  

(20)
and
\[ \mu_{(A \cap A)}(x) = \mu_A(x) \] (21)

**Proof.** By involving Eq. (9) and Eq. (10) we obtain \( \mu_{(A \cup A)}(x) = \max(\mu_A(x), \mu_A(x)) = \mu_A(x) \) and \( \mu_{(A \cap A)}(x) = \min(\mu_A(x), \mu_A(x)) = \mu_A(x) \). Thus, two formulas prove Eq. (20) and Eq. (21).

**Proposition 7.** Identity laws:

Let \( A \) is a fuzzy set in the universal set \( U \), then \( \forall x \in U \) as domain applies that
\[ \mu_{(A \cup \emptyset)}(x) = \mu_A(x) \] (22)

and
\[ \mu_{(A \cap \emptyset)}(x) = \mu_A(x) \] (23)

**Proof.** By involving the help of Eq. (9) and Eq. (10), the following lines
\[ \mu_{(A \cup \emptyset)}(x) = \max(\mu_A(x), \mu_{\emptyset}(x)) = \max(\mu_A(x), 0), \quad \text{Eq. (7)} \]
\[ = \mu_A(x), \quad \mu_A(x) \geq 0 \]

and
\[ \mu_{(A \cap \emptyset)}(x) = \min(\mu_A(x), \mu_{\emptyset}(x)) = \min(\mu_A(x), 1), \quad \text{Eq. (6)} \]
\[ = \mu_A(x), \quad \mu_A(x) \leq 1 \]

prove Eq. (22) and Eq. (23).

**Proposition 8.** Invers laws:

Let \( A \) is a fuzzy set in the universal set \( U \), then \( \forall x \in U \) as domain applies that
\[ \mu_{(A \cup \overline{A})}(x) = \mu_U(x) \] (24)

and
\[ \mu_{(A \cap \emptyset)}(x) = \mu_{\emptyset}(x) \] (25)

**Proof.** Simply, \( A \cup \overline{A} = U \) in Table 2 proves that the Eq. (24) applies \( \forall x \in U \). Similarly, \( A \cup \emptyset = \emptyset \) proves Eq. (25).

**Proposition 9.** Domination laws:

Let \( A \) is a fuzzy set in the universal set \( U \), then \( \forall x \in U \) as domain applies that
\[ \mu_{(A \cup U)}(x) = \mu_U(x) \] (26)

and
\[ \mu_{(A \cap \emptyset)}(x) = \mu_{\emptyset}(x) \] (27)

**Proof.** Similarly, two simple proofs come from Table 2, that is \( A \cup U = U \) and \( A \cap \emptyset = \emptyset \) proves Eq. (26) and Eq. (27), respectively.
Proposition 10. Absorption laws:
Let \( A \) is a fuzzy set in the universal set \( U \), then \( \forall x \in U \) as domain applies that
\[
\mu_{(A \cup (A \cap B))}(x) = \mu_A(x)
\] (28) and
\[
\mu_{(A \cap (A \cup B))}(x) = \mu_A(x)
\] (29)

Proof. By assuming that \( A \cap B = \emptyset \) and \( A \cup B = U \), simply \( \mu_{(A \cup (A \cap B))}(x) = \mu_{(A \cap B)}(x) = \mu_A(x) \) and \( \mu_{(A \cap (A \cup B))}(x) = \mu_{(A \cup B)}(x) = \mu_A(x) \). It proves Eq. (28) and Eq. (29).

4.2. Systematical proof
Logically, Proposition 1 to Proposition 10 requires systematic evidence to reveal the truth of fuzzy logic that can be applied to prove some laws in fuzzy sets. As proven able something, from all propositions, Proposition 8 was taken as an example in this case and algebraically is proof by involving Eq.(2) and Eq. (3).

From the left side of the Eq. (24),
\[
\mu_{(A \cup \overline{A})}(x) = \mu_A(x) \lor \mu_{\overline{A}}(x)
\] leaves two options: If \( \mu_A(x) < 1 - \mu_A(x) \), then \( \max(\mu_A(x), 1 - \mu_A(x)) = 1 - \mu_A(x) \neq \mu_U(x) \); If \( \mu_A(x) > 1 - \mu_A(x) \), then \( \max(\mu_A(x), 1 - \mu_A(x)) = \mu_A(x) \neq \mu_U(x) \). Therefore, by involving Eq. (3)
\[
\mu_{(A \cup \overline{A})}(x) = \mu_A(x) \lor \mu_{\overline{A}}(x), \quad \text{Eq. (10)}
\] leaves two options: If \( \mu_A(x) < 1 - \mu_A(x) \), then \( \max(\mu_A(x), 1 - \mu_A(x)) = 1 - \mu_A(x) \neq \mu_U(x) \); If \( \mu_A(x) > 1 - \mu_A(x) \), then \( \max(\mu_A(x), 1 - \mu_A(x)) = \mu_A(x) \neq \mu_U(x) \). Therefore, by involving Eq. (3)
\[
\mu_{(A \cup \overline{A})}(x) = \mu_A(x) \lor \mu_{\overline{A}}(x), \quad \text{Eq. (10)}
\]
end up with choices that can’t be finish statement, which are: If \( \mu_A(x) < 1 - \mu_A(x) \), then \( \min(\mu_A(x), 1 - \mu_A(x)) = \mu_A(x) \neq \mu_U(x) \); If \( \mu_A(x) > 1 - \mu_A(x) \), then \( \min(\mu_A(x), 1 - \mu_A(x)) = 1 - \mu_A(x) \neq \mu_U(x) \). Therefore, Eq. (25) be
\[
\mu_{(A \cup \overline{A})}(x) = \mu_A(x) \lor \mu_{\overline{A}}(x), \quad \text{Eq. (10)}
\]
4.3. Program

Suppose two or more computers alternately or together provide input to one computer that controls the process machine. Each computer processes the information through fuzzy computing which involves several laws of the fuzzy logic and the fuzzy sets and the computational results are in $[0, 1]$. Suppose one of the computers $A$ sends $\mu_A(x) = 0.21$, computer $B$ sends $\mu_B(x) = 0.212$, and computer $C$ sends a response of $\mu_C(x) = 0.210$, where input $x$ is feedback from the machine controller computer and makes the decision to use the best calculation, as simulated through the following Python program:

```python
a = 0.21
b = 0.212
c = 0.210
if a==b:
    print "a==b"
if a<=b:
    print "a<=b"
if a==c:
    print "a==c"
if min(a,c)==min(b,c):
    print "min ok"
if max(a,c)==max(b,c):
    print "max ok"
if max(a,c)<=max(b,c):
    print "max <= ok"
```

with the following results: Decision from line 6 is $a<=b$; The decision for line 8 is $a==c$; Decision line 10 is $\text{min ok}$; Decision line 14 is $\text{max}<=\text{ok}$. Whereas the other decision lines do not have outcomes. It shows that logically all propositions in a fuzzy set must have an agreement on the number of digits. In other words, decisions involve the concept of a comparison of $<=$ or $=>$ instead of $==$.

5. Conclusion

Reproving the basic principles, the laws for the fuzzy logic and the fuzzy sets, not only provides an understanding of implementation but also to develop a strategy for implementing it through programming, namely by involving two comparison operators in the smaller and larger sides to avoid mismatch in the calculation. Furthermore, some other evidence relating to the membership of the fuzzy sets becomes the basis of further studies.

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