“Atmospheric Neutrino” and “Proton Decay” Data Exclude Some New Dark Matter Scenarios

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Abstract

Models in which the "dark" halo particles have mutual and potentially also appreciable nuclear interactions have been considered by various authors. In this note we briefly point out strategies for a most sensitive search for these particles. We show that a particular matter/anti-matter symmetric variant due to Farrar et al. is excluded by combining bounds on proton decay from various experiments and from super-Kamiokande and atmospheric neutrino measurements at super-Kamiokande.

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Introduction

In the following we note that existing data strongly limit dark matter scenarios involving “MIMMPs”: Moderately Interacting, Moderately Massive \((m \sim m(\text{Nucleon}))\) Particles.[1] While some of our arguments are general, we present them in the context of a specific scenario due to G. Farrar et al. [2] which motivated this work. The MIMMPs there are an extreme variant of R. Jaffe’s[3] hexa-quark \(H = uuddss\). Jaffe’s original bag model calculations suggested that \(m(H) < 2 \cdot m(\Lambda)\) so that \(H \to \Lambda + \Lambda\) decay is forbidden. Experiments searching for a weakly decaying \(H\) have not found it to date. Farrar et al. postulate a more tightly bound \(H\):

\[
m(H) < 2 \cdot m(\text{Nucleon}) \sim 1860\text{MeV.}
\]

(1)

explaining the failure of the above searches and making the stable \(H\) a cold dark matter candidate. Such a scenario is viable only if \(H\) is very small:

\[
r(H) \sim 0.1 - 0.2\text{Fermi.}
\]

(2)

The residual interactions between the color neutral extremely compact \(H\) and nucleons/nuclei are then very small ensuring[2] that:

i) \(H\) particles do not bind to nuclei;

ii) elastic \(H\)-nucleon cross sections are smaller than normal hadronic cross sections;

iii) the mismatch in scales strongly suppresses \(H\) production in accelerators.[4]

i) explains why \(H\)-nuclei composites did not manifest in anomalous isotopes. Small nuclear cross sections:

\[
\sigma(H - N) = F(1) \cdot 10^{-26} \text{cm}^2 \quad F(1) < 10^{-3}
\]

(3)

prevent \(H\) particles from manifesting in the X-ray balloon experiment[5].

Finally, iii) explains the absence of \(H\) in accelerator missing mass searches.

If the \(H\) particles make up the galactic halo with local density \(\sim 0.4 \text{GeV/cm}^3\), then their local flux is:

\[
\Phi(H(\text{local})) = v(H) \cdot n(H) \sim 6 \cdot 10^6/(\text{cm}^2 \text{ sec}).
\]

(4)
In the next section (II) we will briefly comment on how the detection of such a MIMMP signal can be readily achieved with minimal modifications of existing experimental set-ups looking for the more conventional halo WIMPs (Weakly Interacting Massive Particles).

Going beyond the above $H$ dark matter scenario Farrar et al. envision[6] a matter/anti-matter symmetric universe in which the ordinary baryon excess is matched by an $\bar{H}$. Excess baryonic number in the dark matter segregated there during the QCD phase transition. At first sight it seems to be trivially excluded as $\bar{H}$’s impinging on the earth and the sun would annihilate with ordinary baryons. Also, $H-\bar{H}$ annihilate in the halo and more so at earlier, denser cosmic epochs. The mismatch between energy/distance scales of quarks in $\bar{H}$ and in the nucleon can strongly suppress annihilation:

$$\Sigma(\text{annihilation})(\bar{H} - N) = F(2) \cdot 10^{-25} \text{cm}^2/\beta. \quad (5)$$

where $10^{-25} \text{ cm}^2$ is a normal hadronic annihilation cross section, and the $1/\beta$ factor is appropriate for exothermic processes. Following Farrar we make the drastic assumption:

$$F(2) \sim 10^{-16} \quad (6)$$

We show in Sec. IV that this and the tuning of $F(1)$ from Eq. (3) can avoid the difficulties with terrestrial $\bar{H}$ annihilations. Still we find in Sec. IV that the $H-\bar{H}$ scenario violates bounds on “solar” and on ”atmospheric neutrinos”, as too many neutrinos with energies in the 30-50 MeV range and with $\sim 240$ MeV are generated by annihilations of $\bar{H}$’s in the sun.

The $H = \text{Halo Scenario: How Soon Can It Be Tested?}$

In a collision with a nucleus $(A, Z)$ the $H$ imparts recoil energy:

$$T(\text{recoil}) \sim m(H)V(H)^2(1 - \cos(\theta)) \cdot [m(H)/A \cdot m(N)] \sim 0.02 \text{keV}/(A/100) \quad (7)$$

with $V(H) \sim 3 \cdot 10^7 \text{cm/sec} \sim$ virial velocity $\sim 10^{-3}c$. The atomic number $A$ of detector materials is often large ($\sim 100$) yielding coherent cross section/gram enhanced by $A^2$ as compared with the cross section on Hydrogen. The $\cos(\theta)$ distribution is uniform for the pure S-wave scattering.
Low background bolometric and/or other underground devices sensitively searching WIMPs via nuclear recoil have been operating for several decades. With $T_{\text{recoil}} > a$ few keV threshold, these are sensitive to WIMPs of masses $> 50$ GeV but not to $H$ particles leaving the $H$ scenario untested.

Note, however, that the large recoil threshold in the bolometer is related to the small ($\sim 10^{-36}$ cm$^2$ cross sections) of the WIMPS that it was designed to detect. The latter require massive bolometers and long observation times. Only $E_{\text{recoil}} > E_{\text{threshold}} \sim$ few keV causes the temperature of the massive super-cooled bolometric detectors to change perceptibly. Also, it dictated the large-A nuclei used to enhance (by an $A^2$ factor) the number of interactions per gram of detector.

However, $H - N$ cross sections are far larger, by 6-8 orders of magnitude, and the $H$-flux is also larger, by an $m(X)/m(H)$ factor of order $\geq 100$. Thus small, O(10 gr), bolometric detectors with correspondingly low O(.1 keV) thresholds for recoil energy can be used. Further the composition of these can include light elements with an increase $\sim 1/A$ of the actual energy deposited (see Eq. (7)). Such minute test runs if done at sufficiently shallow locations where the $H$ signal is not suppressed would record many $H$ particles in short times!

Using Eq. (7) we find a rate of continuous energy deposition:

$$d(Q)/dt \sim 2 \cdot F(1)10^{-12} \text{cal/gram \cdot sec}$$  \hspace{1cm} (8)

in any material (the coherence enhanced $H$-nuclear scattering compensates the $1/A$ in Eq. (3) and $F(1)$ is (fudge) factor #1 from Eq. (3) above.

In passing we note that the $H$ particles penetrate to the level of any condensed matter-atomic laboratory and one may wonder if even such a tiny heating up may not effect ultra-sensitive micro-/nano-Kelvin experiments.

$H + \bar{H}$ Annihilations in the Halo

Before addressing $\bar{H}$ annihilations with baryons in earth/sun, we consider $H - \bar{H}$ annihilations in the galactic halo. Let the corresponding cross section be $\sim F(3) \cdot 10^{-25}/\beta \text{cm}^2$, 

\hspace{1cm}
with $F(3)$ being another "fudge" factor suppressing the $H - \bar{H}$ annihilation relative to standard nucleon-antinucleon hadronic annihilations. With $H - \bar{H}$ halo density $n(H) \sim n\bar{H} \sim 0.1/cm^3$ the annihilation rate $[dn/dt]/n$ is:

$$t^{-1} \sim nV\sigma(\text{ann}) \sim c \cdot F(3)10^{-26} \sim F(3) \cdot (3 \cdot 10^{-16}) \text{sec}^{-1}. \quad (9)$$

Each annihilation releasing $\sim 3.5$ GeV in pions should yield at least $3\pi_0$ or $\sim 6\gamma$s of average energy of $\sim 200$ MeV: The resulting flux from halo $H - \bar{H}$ annihilations in the galactic halo with radius $R \sim 3 \cdot 10^{22} cm$ is:

$$\Phi(200 \text{MeV } \gamma) = Rn/t \sim F(3) \cdot 10^8/cm^2 \text{ sec.} \quad (10)$$

Bounds on diffuse $\gamma$ rays, $\Phi(200 \text{MeV}) < 10^{-5}/cm^2 \text{ sec}$, imply

$$F(3) < 10^{-13}. \quad (11)$$

i.e., $H - \bar{H}$ annihilation cross sections which are 13 orders of magnitude smaller than those of nucleon-antinucleon seem unlikely.

Independent of theoretical considerations, such small annihilation cross sections can lead to excessive freeze-out relic $\bar{H} + H$ densities—proportional to $1/\sigma(\text{ann})(H - \bar{H})$ (annihilations of $\bar{H}$ and ordinary baryons are even more severely limited by direct experiments). Let us assume that we start with roughly an equal number of ordinary baryon/anti-baryons—which seems natural if we have all along a charge symmetric universe with no quark-antiquark excess of $H - \bar{H}$—then the number density of the surviving $H$ and $\bar{H}$ exceeds that of the surviving baryons by:

$$n(\bar{H}) \sim n(\text{anti - proton}) \cdot F(3)^{-1} > 10^{13} \cdot n(\text{anti - protons}) \quad (12)$$

Using then the estimated ratio of surviving anti-proton and background photon densities [7]: $n(\text{anti - proton})/n(\gamma) \sim 10^{-18}$, this yields a cosmological $H - \bar{H}$ mass density or $\sim .1$ GeV/cm$^3$ exceeding by $\sim 10^5$ the expected cold dark matter density.

To address this issue one needs a comprehensive model to provide some of the underpinning of the $H - \bar{H}$ scenario which is coming soon[8]. This model should, in particular, provide a mechanism for preferring $\bar{H}$ formation over that of $H$—and for explaining the
present $n_B/n_\gamma$ ratio. We will therefore not view the above as a fatal flow of the $H - \bar{H}$ scenario, and proceed with several present-day observational bounds which jointly\cite{8} directly exclude the $H - \bar{H}$ scenario in a manner which is practically independent of particle physics and cosmology.

Terrestrial $\bar{H}$ Annihilations

When encountering ordinary baryons, $\bar{H}$ can annihilate in several ways:

$$\bar{H} + p \rightarrow \bar{\Xi} + n \text{ pions } \quad (a)$$
$$\bar{H} + p \rightarrow K(+)K(0) + \Lambda/\bar{\Sigma} + n \text{ pions } \quad (b)$$
$$\bar{H} + p \rightarrow K(+)K(0) + K(+)K(0) + \bar{p} + n \text{ pions } \quad (c)$$

\begin{align}
\tag{13}
\end{align}

The anti-baryons emerging from the primary annihilations annihilate shortly thereafter:

$$\bar{\Xi} + p \rightarrow K(+)K(0) + K(+)K(0) + n \text{ pions } \quad (a)$$
$$\Lambda/\bar{\Sigma} + p \rightarrow K(+)K(0) + n \text{ pions } \quad (b)$$
$$\bar{p} + p \rightarrow n \text{ pions } \quad (c)$$

\begin{align}
\tag{14}
\end{align}

Such events release $\sim 2.5$ GeV energy and could be detected in underground detectors like super-Kamiokande (SK) which established a remarkable proton decay bound:

$$t(\text{prot.dec}) > 10^{40}\text{ sec.}$$

\begin{align}
\tag{15}
\end{align}

The nucleons in the SK detector are annihilated by the incoming $\bar{H}$'s at a rate:

$$(dn/dt)/n = \Phi(H) \cdot \sigma(\text{ann})(\bar{H} - N) = 3F(2) \cdot f(d) \cdot 10^{-17}\text{ sec}^{-1}$$

\begin{align}
\tag{16}
\end{align}

where we used Eq. (5) and $\beta \sim 10^{-3}$. If the same bound on the proton decay rate applies also to the rate of such annihilation, we need $F(2) < 3 \cdot 10^{-24}$, a bound $10^8$ times smaller than the value of Eq. (8) suggested by Farrar, making:

$$\sigma_{\bar{H}N(\text{ann})} < 10^{-50}\text{ cm}^2 << \sigma(\text{Weak})$$

\begin{align}
\tag{17}
\end{align}
which is most unlikely. However, the above is highly oversimplified and Eq. (13) is not warranted! The point is that the numbers of $\bar{H}$ deep underground are strongly attenuated by elastic nuclear collisions.

As mentioned above, the collision of particles moving with velocity $\beta \sim 10^{-3}$ with $A \sim 20$ crust nuclei are elastic, isotropic, coherently enhanced:

$$\sigma(H - A) \sim A^2 \sigma(H - N),$$

with small $(2/A)$ fraction of the energy lost to nuclear recoil. These features and the small escape velocity from earth $\sim V(\text{escape}) \sim 11 \text{ km/sec} \sim 1/30 V(H)$ with the typical virial velocity of $\sim 300 \text{ km/sec}$, cause most of the “light” infalling $H - \bar{H}$ particles to “reflect” from the earth after a few collisions and no ambient $H - \bar{H}$ population builds up. The fraction of the $H - \bar{H}$ particles penetrating to a depth $d$ is $exp(-d/l(mfp))$ with the mean-free path given by:

$$l(mfp) = [n(A) \cdot \sigma(H - A)]^{-1} \sim 3 \text{ cm}/F(2).$$

where we used Eqs. (3),(4) with $n(A) \sim N(\text{Avogadro}) \cdot \rho/A$, $\rho \sim 2.7 A \sim 20$. The maximal $F(1) = 10^{-3}$ or $\sigma(H - N) = 10^{-2} \text{ mb}$ yields a minimal $l(mfp) \sim 30$ meters. Thus the $\sim 2$ km depth of SK is 60 mean-free paths completely extinguishing the $H - \bar{H}$ flux and no bound on $F(2)$ ensues! Note, however, that a mere factor of 10 decrease of $F(1)$ to avoid stricter putative direct bounds on $H$ energy deposition leaves SK “exposed” to only $\sim 1/500$ decreased $\bar{H}$ flux! But we do have—albeit weaker—bounds of $\sim 10^{25}$ years on the proton lifetime from experiments at shallow locations, say, $10^{25}$ years at depths of $\sim 30$ meters where there is no $\bar{H}$ flux suppression. We therefore have to maintain the original bound of Eqs. (7),(8) above:

$$F(2) < 10^{-16}.\quad (20)$$

$\bar{H}$ Annihilations in the Sun: General Features of the Resulting Neutrinos

The fate of $H - \bar{H}$’s falling on the sun is very different than in the the earth. Light Hydrogen and Helium dominate with a solar surface mass ratio of $\sim 3:1$. Equation (14) then implies that the infalling $H$’s are equally likely to collide with Hydrogen or with Helium—with half or twice their mass, respectively. Further, the escape velocity from the sun, $\sim 600$
km/sec, is twice the average virial velocity of the MIMPs. Hence, the first collision in the sun occurs at:

\[ V(\text{col})^2 = V(\text{escape})^2 + V(\text{virial})^2 \sim 5V(\text{virial})^2 \]  \hspace{1cm} (21)

Thus if \( H - \bar{H} \) loses just 20% of its energy in the first collision, it gets bound. From Eq. (4) (or more precise versions) we find that for \( A = 1 \) or 4 this always happens except for forward scattering. After such forward scatterings, \( H - \bar{H} \)’s are prone to suffer more collisions. Also getting deeper into the sun they are less likely to escape. We find that only \( \sim 2\% \) of the infalling \( H - \bar{H} \)’s reflect and 98% stay bound. Gravitational focusing also enhances the flux at the solar surface by by \([V(\text{col})/V(\text{virial})]^2 \sim 5\) making a flux of captured \( H - \bar{H} \) (= 1/2 flux of \( H + \bar{H} \))

\[ \Phi(\text{captured } \bar{H} \text{ at solar surface}) \sim 1.5 \cdot 10^7/(\text{cm})^2 \text{ sec.} \]  \hspace{1cm} (22)

Our argument is then based on the following simple steps:

(a) Unless the \( \bar{H} \)-nucleon annihilation cross sections are suppressed by 31(!) orders of magnitude relative to those of \( N - \bar{N} \), all the captured \( \bar{H} \)’s eventually annihilate.

(b) The annihilation of each \( \bar{H} \) eventually yields on average 3.5 positively charged pions and \( \sim 1.3 \) positive kaons.

(c) All the \( \pi^+ \)’s decay: \( \pi^+ \rightarrow \mu^+ + \nu(\mu) \), and with the subsequent \( \mu \) decays lead to three neutrinos per decaying pion or \( O(10) \) neutrinos (from \( \pi \) decay) per \( \bar{H} \) annihilation. Also \( \sim 70\% \) of the \( K^+ \) decay via \( K^+ \rightarrow \mu^+ + \nu(\mu) \) leading to \( \sim \) one primary neutrino from the above decay per annihilation.

This implies outgoing/incoming neutrino fluxes at the solar/earth’s surface of

\[ \Phi|_{\text{Sun}}(\pi/\mu\text{decay } \nu^{'s}) \sim 1.5 \cdot 10^8/(\text{cm}^2 \text{ sec}) \rightarrow 4 \cdot 10^3|_{\text{Earth}} \]  \hspace{1cm} (23)

\[ \phi|_{\text{Sun}}(K^+\text{decay } \nu^{'s}) \sim 1.5 \cdot 10^7/(\text{cm}^2 \text{ sec}) \rightarrow 4 \cdot 10^2|_{\text{Earth}} \]  \hspace{1cm} (24)

where the later terrestrial flux was reduced by the ratio \([R(\text{sun})/Au]^2 \sim 3 \cdot 10^{-5}\).

(d) The above neutrino fluxes would have been detected in underground neutrino telescopes and in the large water Cherenkov counter of SK in particular. The fact that no anomalous signal has been seen there can then be used to exclude the \( H + \bar{H} \) scenario.
The impact of these extra neutrinos very strongly depends on their energy.

(e) Decays in flight of pions (of either charge) and decays at rest of positive kaons, \( K^+ \rightarrow \mu^+ + \nu(\mu) \), yield neutrinos of energies around 300 and 240 MeV, respectively. The expected (and measured at SK!) flux of neutrinos (of either the muon or electron types) is \( \sim 1/cm^2 \) sec at these energies[9]: \( \sim 4000 \) times smaller than even the lower of the new fluxes, namely, the flux of \( K^+ \) decay neutrinos in Eq. (24) above.

(f) Since this is the key point of our argument it may be helpful to rephrase it using PDG data[10] only. Let us assume a flux of \( \sim 400 \) neutrinos/cm\(^2\) sec originating from \( K^+ \) decays. These neutrinos are initially 100\% \( \nu(\mu) \) but (vacuum) oscillate enroute into \( \nu(\tau) \) and \( \nu(e) \) so that the flux arriving at earth consists of \( \sim 45\% \), 35\%, 20\% \( \tau \), \( \mu \) and \( e \) neutrinos, respectively. The weighted charged and neutral current cross sections of this neutrino mix is \( \sim 10^{-39} \) cm\(^2\). During a period of about three years these should produce in the 20 Kilotonne fiducial volume of SK 400,000 (!) events—all within the same energy and direction (namely, the solar direction) beans. This exceeds by about two orders of magnitude the totality of “atmospheric neutrino” events at SK—at all energies and from all directions.

The analysis in the following section indicates that if the stringent upper bound \( (F(2) < 10^{-16}) \) holds then the \( \bar{H} \)'s are likely to annihilate in inner, denser layers \( (\rho > 3 gr/cm^3) \) of the sun.[11] The mean-free path of pions with several hundred MeV energy there is \( \sim 50 \) times shorter than the mean distance for decay. The pions will therefore multiply scatter losing their energy—and apart from a small fraction of the positive pions which get absorbed by Helium via the \( \pi^+ + He \rightarrow \text{nucleons and/or nuclei} + \text{no pions} \)—all \( \pi^+ \) decay essentially at rest. The “Michel” spectra of these neutrinos are well known and relatively low—either sharp lines at 30 MeV or distribution with \( E(\nu) < 53 \) MeV in the stopped \( \mu \) decays. For each \( \pi^+ \) decay we have a \( \nu(\mu) \) from the primary decay and a \( \nu(\bar{\mu}) \) and \( \nu(e) \) from the decay of the \( \mu^+ \). The initial ratios of the number of neutrinos of various flavors \( \bar{\nu}(e) : \nu(e) : \nu(\mu) : \bar{\nu}(\mu) : \nu(\tau) : \bar{\nu}(\tau) = 0 : 1 : 1 : 1 : 0 : 0 \) then get modified by vacuum neutrino mixing which is maximal in the \( \nu(\tau) \) sector and large in the \( \nu(e) \) sector.

The energies of these neutrinos extend way beyond the highest Hep neutrinos expected in the standard solar models[14]. The higher energies enhance by 2-3 the cross sections on electrons and also makes the resulting Cherenkov cones align better in the direction of the
sun. Still, because of the relatively small number of neutrinos involved, it is not clear that such a signal would have been seen already at SK.

We note however that because of the composition—i.e., the inclusion of a $\bar{\nu}(e)$, and even more so due to the very high energy of these neutrinos—we can have scatterings on protons and on nuclei which can directly yield relativistic, charged positrons like $\bar{\nu}(e) + p \rightarrow e^+ n$ and analog reactions on the protons in the Oxygen, or indirectly by highly exciting (also in $\mu$ and $\tau$ neutrino neutral current interactions) the Oxygen to high nuclear levels which de-excite via $\beta/\gamma$ cascades[13]. The cross sections for all these interactions involving nuclear targets are $\sim 100 - 10^3$ times larger than those on electrons, and despite the loss of the directionality from the sun, should have been observed.

Since this analysis is rather involved and has not been done to date we will show in the next section that the stopped $K^+$ decay neutrinos are indeed there and, as indicated in (e) above, suffice to conclusively exclude the $\bar{H}$ scenario.

\textit{\bar{H} Annihilations in the Sun and the Resulting Stopped $K^+$ Neutrinos}

To firm up the estimated flux of O(400/cm$^2$ sec. neutrinos of $\sim 240$ MeV energies from $K^+$ decays at rest, the various stages of evolution of the captured $\bar{H}$’s need to be studied more clearly. Let us first delineate these stages:

i) The captured MIMPs directionally diffuse (under gravity) towards the center getting to a radius $r \sim 0.42 R($sun$)$ where the density is $\sim 3 \text{ gr/cm}^3[14]$, in about 1/3 year.

ii) The strong upper bound on $\bar{H} - N$ annihilation cross sections $F(2) < 10^{-16}$ concluded in Sec. II above implies that only a few percent of the $\bar{H}$’s annihilate during stage i), but rather during the astronomical time spent in the denser core in stage ii).

iii) Each $H$ annihilation yields directly—or via subsequent annihilations of the produced anti-hyperons—one $K^+$ on average. Elastic collisions slow the kaons to kinetic energies of $\sim 20$ MeV energy (at which the kaons cannot charge exchange), \textit{before} the $K^+$’s charge exchange into the short-lived $K^0$’s.
We next fill in some details pertinent to these stages using the standard solar model of Ref. [14] for solar parameters when necessary.

i) We use a simple Drude model to estimate the inward, gravity-directed diffusion in stage i). The density along most of the way to \( r \sim 0.42 \text{ R(sun)} \) is less than 3 gr, i.e., about 1/10 of the earth’s crust density and a factor 10 coherence enhancement in scattering off the heavier earth’s elements is missing. Hence, the minimal mean-free path found in Sec. II above for \( H/\bar{H} \) elastic scattering in earth of 30 meters suggests a minimal \( L(\text{mfp}) \sim 3 \text{ km} \) during stage i) in the sun. The value appropriate for the ensuing discussion is actually the transport mfp which, due to the forward-biased angular distributions, is \( \sim 3 \) times larger, i.e., \( l(\text{transport}) > 10 \text{ km} \). The average temperature in the region of interest is \( T \sim 2 \cdot 10^6 \) Kelvin [14] yields an average (thermal) velocity of \( \bar{H} \sim 150 \text{ km/sec} \) and the average time between collisions is \( \Delta(t) = l(\text{mfp})/v(\text{thermal}) \sim 0.07 \text{ sec} \). The average gravitational acceleration in the region of interest is \( g(\text{sun}) \sim 1 \text{ km/(sec)}^2 \) [14] yields then a radial drift velocity of \( g(\text{sun}) \cdot \Delta(t)/2 \sim 35 \text{ meters/sec} \) causing an inward migration of \( \sim 0.6 \text{ R(sun)} \sim 4 \cdot 10^8 \text{ meters in about 1/3 year.} \)

ii) In a region of nucleon density \( \rho \), \( \bar{H} \) particles annihilate at a rate:

\[
 t(\bar{H} - n(\text{ann}))^{-1} = n(\text{nucl}) \cdot \beta \cdot c \cdot \sigma(\text{ann}) = \rho \cdot F(2) \cdot 2 \cdot 10^8(\text{sec})^{-1}, \tag{25}
\]

where we used Eq. (8) for the \( \bar{H} \)-nucleon annihilation cross section. Thus, for the average solar density of \( \sim 1 \text{ gr/cm}^3 \) and \( F(2) \sim 10^{-16} \), i.e., the \( \bar{H} \)'s annihilate in times far shorter than the solar lifetime. This in turn ensures that a steady state with all captured \( \bar{H} \)'s annihilation is achieved.

We next turn to the more detailed question as to where are these \( \bar{H} \) annihilations likely to occur. As we will see below, the local density around the annihilation point is important if the \( \bar{H} \)'s annihilate predominantly into anti-cascades as in Eq. (13a) above rather than into anti-lambda/Sigma’s or anti-protons as in Eqs. (13b) and (13c).

With \( F(2) \sim 10^{-16} \) most \( \bar{H} \)'s will \emph{not} annihilate in the outer dilute shells, but rather migrate first to the fairly dense (\( \rho \geq 3 \text{ gr/cm}^2 \)) shells at radius \( r < 0.42 \text{ R(sun)} \). Indeed the average density in \( R(\text{sun}) > r > 0.42 \text{ R(sun)} \) is \( \sim 3 \text{ gr/cm}^2 \) and Eq. (25) yields \( t(\bar{H} - n(\text{ann})) \sim 1/2 \text{ year exceeding the 1/3 year migration time to this radius of 0.42} \).
R(sun) estimated in the previous section.

Since the migration time is proportional to $F(1)$ and the annihilation time is inversely proportional to $F(2)$, further reduction of either factor strengthens the above conclusion. We note that if the $H/\bar{H}$ lived much longer so as to achieve true thermal equilibrium than having the average $H/He$ mass, it would then sink to $r = 1/4 \ R(\text{sun})$ wherein half the solar mass is contained and the local density is $\rho \sim 20 \text{ gr/cm}^3$. We thus assume that most $\bar{H}$ annihilations occur at densities $\rho > 3 \text{ gr/cm}^2$.

iii) The “primary” $\bar{H}$ annihilation reactions Eqs. (13a), (13b), (13c) above yield 0, 1 and 2 Kaons per annihilation, respectively (“Kaon” = $K^+$ or $K^0$), and as most annihilations occur on protons and not on neutrons $\sim 60\%$ of these are $K^+$.

If all antihyperons also annihilate before decaying, then reactions 14 would supply the missing 2 and 1 kaons in case (a) and (b). The $K^+$ particles emerge from all annihilation reactions with kinetic energies $\sim 200 \text{ MeV}$ on average, so that $\beta/\gamma \sim 1$, and decay after traveling on average $l(\text{decay}) \sim 300 \text{ cm}$. The branching into the $\nu(\mu) + \mu^+$ is $68\%$ and the neutrino energies for decay at rest are $240 \text{ MeV}$. At energies of $200 \text{ MeV}$ a $\sigma(\text{el}) \sim 12 \text{ mb}$ cross section for elastic $K^+n$ scattering can be read off from the Particle Data Group[10]. We estimate that the cross section for charge exchange (CEX), namely, $\sigma(K^+n \to K^0p) < 3 \text{ mb}$. Since $K^0$ has a $50\% K(S)$ component which quickly decays into final states without neutrinos, the CEX reaction can potentially quench the $K$ decay neutrino signal. We will argue next that this is not the case as only a small fraction of the $K^+$’s charge exchange before decaying is very small. The argument is as follows:

Kaons with $T < 200 \text{ MeV}$ lose on average about $35\%$ of their kinetic energies in each quasielastic collision with the $H/He$. After about five elastic collisions the initial $T \sim 200 \text{ MeV}$ kaons degrade to $T < 30 \text{ MeV}$. Once at this energy the kaons cannot break the tightly bound Helium nuclei so as to charge exchange on the constituent neutrons. The 4:1 ratio of $\sigma(\ell)/\sigma(\text{CEX})$ and the fact that only $1/8$ of the nucleons encountered are neutrons (on which CEX can happen) implies that the mean-free path (mfp) for $K$-nucleon elastic scattering is $\sim 30$ times shorter than the mfp for CEX. Hence, in most cases, energy degradation to below the “threshold” for CEX happens prior to CEX; thus, independently of the local solar density the CEX and loss of $K^+$ can be neglected.
If all antihyperons produced in the primary $\bar{H}$ annihilations eventually annihilate themselves as well (and we argue that most indeed do), then $\bar{s}$ number conservation guarantees that there will be at least two Kaons per each primary $\bar{H}$ annihilation. With the charge bias due to the excess of proton over neutron targets slightly favoring $K^+$ over $K^0$ production and the fact that $\sim 20\%$ of all $p-\bar{p}$ annihilations with no net $\bar{s}$ excess yields in $\sim K\bar{K}$ pairs, we expect $\sim 1.3$ $K^+$ per annihilation. This was indeed the starting point of our estimated flux of neutrinos from $K^+$ decays in Eq. (24) above.

We next show that antihyperons with annihilation cross sections of approximately

$$\sigma(\text{ann}) \sim 100 \text{ mb}/\beta$$  \hspace{1cm} (26)

do annihilate prior to decaying. (The annihilation cross section in Eq. (26) is inferred from Fig. 37-19 in the PDG[10] as $\sigma(\text{tot})\bar{p}-p-\sigma(\text{el})\bar{p}-p$ at p(Lab) = GeV/c corresponding to $\beta = 1$, and where Coulomb enhancements are minimal. Being a normal hadronic cross section we do not have any longer the freedom of choosing its value!)

This can be an important issue. The semi-leptonic branching decay of the cascade is very small and the neutrinos from such decays have energies lower than the $\sim 230$ MeV in $K^+$ decays.

If the $\bar{H}$ annihilations produce in the first step Eq. (13) only anti-cascades and if the anti-cascade decayed before annihilating and if also the anti-Sigma/anti-Lambda hyperons that it “cascaded” into also decay before they in turn annihilate, then the number of $K^+$ generated will be suppressed to $\sim 10\%$ of our original estimate. We still have $\sim .1$ $K^+$ per $\bar{H}$ originating from (the inevitable!) annihilations of the stable anti-protons. The very strong exclusion of the $\bar{H}$ scenario indicated in item (f) of the previous section may well be enough. Still it is instructive to show that most anti-hyperons do annihilate. The mean-free path for annihilations is:

$$l(\text{mfp})(\text{Hyp})(\text{ann}) \sim 1/(\rho \cdot N(\text{Av}) \cdot \sigma(\text{ann})) \sim 16 \text{ cm}/\rho$$  \hspace{1cm} (27)

is for $\rho \sim 1-3$ gr/cm$^2$ comparable to or shorter than the sum of decay mean-free paths (for $\beta/\gamma \sim 1$) 5 and 8.7 cm for the charged and neutral anti-cascades and 7.9 cm for the anti-lambdas into which the anti-cascades decay. We have argued in i) above that captured $\bar{H}$’s annihilate mainly after sinking to solar shells with densities $> 3$ gr/cm$^2$. Thus, even if
reaction 14(b),(c) were inoperative, the flux of $\sim 230$ MeV muon neutrino originating from the sun is roughly the same as in Eq. (24) above.

The anti-hyperons would fail to annihilate before decaying if the arguments in i) notwithstanding most $\bar{H}$ annihilations occur in regions of density $< \sim 0.1$ gr/cm$^3$. However, in this case about 50% of the $\pi^-/\pi^+$'s decay in flight. The $\sim 20$ times as many neutrinos from $\pi^-$ and $\mu^-$ decays (or the $\mu\nu^-$ capture) will no longer have the low energy ($\sim 30$-50) MeV of neutrinos emerging from stopped pion decays. The SK new “atmospheric neutrino” signal will then be even stronger.

We should point out that there would be no conflict with the SK observations if the $\bar{H} - n$ annihilation rate in the sun was smaller than $10^{-20}$ sec$^{-1}$. For hubble residence time the $\bar{H}$'s would settle in regions of density $\sim 20$ gr/cm at $r \sim R$(sun)/4 where $\sim 1/2$ the solar mass. However, to ensure such small annihilation rates we need unacceptably small $\bar{H} - N$ annihilation cross sections (smaller than weak interaction cross sections)

$$\sigma(\bar{H} - \text{nucleon}) < 10 - 56 \text{ cm}^2 \quad (F(2) < 10^{-31}).$$

Summary

The $H$ and $H - \bar{H}$ scenarios are unlikely to arise in QCD. No credible QCD calculation suggests the very strong binding of Eq. (1) The quark density within the small ($r \sim 0.1$-0.2 Fermi) $H$ is $\sim 100$ times that in a nucleon, and the “weak” interactions in asymptotically free QCD cannot bind quarks with large “uncertainty” kinetic energies $\sim$ GeV/quark. Imposing suppression factors $F(2), F(3) < 10^{-16}$ on $\bar{H}$ annihilations is also rather extreme.

The $H + \bar{H}$ scenario is—even allowing for all the above—directly excluded by existing experimental bounds and measurements; namely, the nucleon decay bounds and measurements of atmospheric neutrinos in the super-Kamiokande underground water Cherenkov detector. This experiment made new discoveries and rules out many wrong theories/speculations.

We believe also that the more conservative, $H$-only scenario can be directly ruled out by simple experiments, though here a small scale bolometric experiment at shallow depth and/or analysis of some existing data that we are unaware of may be required.
Very little of the above is truly original; the idea of using the sun as a gigantic elementary physics laboratory has been admirably pursued by J. Bahcall. Also the more specific idea of using underground neutrino telescope to look for annihilations of much heavier halo particles—the more conventional SUSY neutralinos—has been suggested before[15],[16].

Finally the present $H$ and/or $H + \bar{H}$ scenario were not conceived by the present authors, and considering their “conservative” attitude to QCD, could not have been in any event. Still combining these themes and experimental data to restrict even extreme variants of new models is useful.

Acknowledgments

We would like to thank Tom Shutt and G. Farrar for helpful discussions which initiated this work. It is very satisfying that Farrar and collaborators recently concluded that the $H + \bar{H}$ scenario is not viable—though not necessarily for the same reasons presented here[8].

This work started in the fall of 2002 while I was visiting Princeton and discussed “indirect” dark matter detection with Tom Shutt, David Spergel and Paul Steinhardt.

[Note added: Just before submitting our paper, we received the paper in Ref. [8] of G. R. Farrar and G. Zaharijas. This paper not only presents the $H + \bar{H}$ scenario, but also addresses most of the points in our paper—arriving, however, at different, much more optimistic conclusions.]

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