Photoproduction of electron-positron pairs in the presence of hyperacoustic oscillations

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March 27, 2022

Abstract

We report on the recent progress in the investigation of the influence of hyperacoustic vibrations on the coherent electron-positron pair creation by high-energy photons in crystals. In dependence of the values for the parameters, the presence of the deformation field can either enhance or reduce the cross-section. This can be used to control the parameters of the positron sources for storage rings and colliders.

PACS Nos.: 41.60.-m, 78.90.+t, 43.35.+d, 12.20.Ds

1 Introduction

The basic source to creating positrons for high-energy electron-positron colliders is the electron-positron pair creation by hard bremsstrahlung photons produced when a powerful electron beam hits an amorphous target. One possible approach to increase the positron production efficiency is to use a crystal target as a positron emitter. If the formation length exceeds the interatomic spacing, the interference effects from all atoms within this length are important and the cross-section for the pair creation can change essentially compared with the corresponding quantities for a single atom. From the point of view of controlling the parameters of various processes in a medium, it is of interest to investigate the influence of external fields, such as acoustic waves, temperature gradient etc., on the corresponding characteristics. Our considerations of diffraction radiation, transition radiation, parametric X-radiation, channelling radiation, bremsstrahlung by high-energy electrons, have shown that the external fields can essentially change the angular-frequency characteristics of the radiation intensities. In [1, 2, 3] we have investigated the influence of the hypersonic wave excited in a crystal on the process of electron-positron pair creation by high-energy photons. The case of simplest crystal with one atom in the lattice base and the sinusoidal deformation field generated by the hypersonic were considered in [1]. To have a considerable influence of the acoustic wave, high-frequency hypersonic is needed. Usually this type of waves is excited by high-frequency electromagnetic field through the piezoelectric effect in crystals with a complex lattice base. In [2] we have generalized the results of [1] for crystals with a complex base and for acoustic waves with an arbitrary profile. In particular, the numerical calculations are carried out for the quartz single crystal and for the photons of energy 100 GeV. The results of the numerical calculations on the base of the formulae given in [2] for the pair creation cross-section by the photons of energy 3.5 GeV are presented in the recent paper

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where the scheme of experimental setup is proposed for the corresponding measurements on the photon channel of the Yerevan synchrotron. The influence of acoustic waves to the bremsstrahlung by high-energy electrons in crystals is investigated in [4].

2 Cross-section for the coherent pair creation

Consider the creation of electron-positron pairs by high-energy photons in a crystal (see figure 1 for the diagram of the process). We denote by \((\omega, \mathbf{k}), (E_+, \mathbf{p}_+), \) and \((E_-, \mathbf{p}_-)\) the energies and momenta for the photon, positron, and electron respectively. When acoustic waves are excited the positions of atoms in the crystal can be written as \(\mathbf{r}_n^{(j)} = \mathbf{r}_{ne}^{(j)} + \mathbf{u}_n^{(j)}\), where \(\mathbf{r}_{ne}^{(j)}\) determines the equilibrium position of an atom in the situation without the deformation, \(\mathbf{u}_n^{(j)}\) is the displacement of the atom caused by the acoustic field. The collective index \(n\) enumerates the elementary cell and the superscript \(j\) enumerates the atoms in a given cell of a crystal. We will consider displacements of the form

\[
\mathbf{u}_n^{(j)} = u_0 f(k_s r_{ne}^{(j)}),
\]  

where \(u_0\) and \(k_s\) are the amplitude and wave vector of the acoustic wave, \(f(x)\) is an arbitrary function with the period \(2\pi\), \(\max f(x) = 1\). For a lattice with a complex cell the coordinates of the atoms can be presented in the form \(\mathbf{r}_n^{(j)} = \mathbf{R}_n + \rho^{(j)}\), where \(\mathbf{R}_n\) determines the positions of the atoms for one of primitive lattices, and \(\rho^{(j)}\) are the equilibrium positions for other atoms inside \(n\)-th elementary cell with respect to \(\mathbf{R}_n\).

\[
\text{Figure 1: The pair creation by high-energy photon with the momentum } \mathbf{k}. \text{ Vectors } \mathbf{g} \text{ and } m\mathbf{k}_s \text{ stand for the momenta transferred to the crystal and to the external field.}
\]

The cross-section for the pair creation process is presented in the form \(d\sigma = N_0 (d\sigma_n + d\sigma_c)\), where \(d\sigma_n\) and \(d\sigma_c\) are the incoherent and coherent parts of the cross-section per atom and \(N_0\) is the number of atoms in the crystal. The coherent part is determined by the formula

\[
\frac{d\sigma_c}{dE_+} = \frac{e^2 N}{\omega^2 N_0 \Delta} \sum_{m, \mathbf{g}} \frac{g_{m\parallel}^2}{g_{m\parallel}^2} \left( \frac{\omega^2}{2E_+ E_-} - 1 + \frac{2\delta}{g_{m\parallel}} - \frac{2\delta^2}{g_{m\parallel}^2} \right) |F_m(\mathbf{g}_m u_0)|^2 |S(\mathbf{g}_m, \mathbf{g})|^2,
\]

where \(e\) is the electron charge, \(N\) is the number of cells in the crystal, \(\Delta = a_1 a_2 a_3\) is the unit cell volume for an orthogonal lattice with the lattice constants \(a_1, a_2, a_3\),

\[
\mathbf{g}_m = \mathbf{g} - m\mathbf{k}_s, \quad m = 0, \pm 1, \pm 2, \ldots,
\]

\(\mathbf{g}\) is the reciprocal lattice vector, \(\mathbf{g}_{m\parallel}\) and \(\mathbf{g}_{m\perp}\) are the parallel and perpendicular components of the vector \(\mathbf{g}_m\) with respect to the direction of the photon momentum \(\mathbf{k}\), \(\delta = 1/l_c\) is the minimum
longitudinal momentum transfer, and \( l_c = 2E_1E_\perp/(m_e^2\omega) \) is the formation length for the pair creation process. In \( \text{(2)} \) the summation goes under the constraint \( g_{m||} \geq \delta \). The function \( F_m(x) \) is the Fourier-transform of the function \( e^{ixf(t)} \):

\[
F_m(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ixf(t) - imt} dt, \tag{4}
\]

and \( S(g_m, g) = \sum_j u^{(j)}_g e^{i\varphi^{(j)}_g} e^{-\frac{1}{2}g_m^2 u^{(j)2}_g} \) is the factor determined by the structure of the crystal lattice base, \( u^{(j)2}_g \) is the temperature dependent mean-squared amplitude of the thermal vibrations of the \( j \)-th atom. The corresponding momentum conservation is written in the form

\[
k = p_+ + p_- + g - mk_s, \tag{5}\]

where \(-mk_s\) stands for the momentum transfer to the external field. Formula \( \text{(2)} \) differs from the formula in an undeformed crystal by the replacement \( g \rightarrow g_m \), and by an additional summation over \( m \) with weights \( |F_m(g_m u_0)|^2 \). This corresponds to the presence of an additional one-dimensional superlattice with the period \( \lambda_s = 2\pi/k_s \) and the reciprocal lattice vector \( mk_s \). As the main contribution into the cross-section comes from the terms with \( g_{m||} \sim \delta \), the influence of the deformation field may be considerable if \( |mk_s|| \geq \delta \). Combining this with the estimate that the main contribution comes from the terms for which \( |mk_s u_0| \lesssim |gu_0| \), or equivalently \( |m| \lesssim \lambda_s/a \), we find the condition: \( u_0/\lambda_s \gtrsim a/4\pi^2 l_c \). At high energies one has \( a/l_c \ll 1 \) and this condition can be consistent with the condition \( u_0/\lambda_s \ll 1 \).

If the photon moves in a non-oriented crystal, in formula \( \text{(2)} \) the summation over \( g \) can be replaced by the integration and the pair creation cross-section coincides with that in an amorphous medium. The role of coherence effects in the pair creation cross-section is essential when the photon enters into the crystal at small angle \( \theta \) with respect to a crystallographic axis (axis \( z \) in our consideration). In this case the main contribution into the coherent part of the cross-section comes from the crystallographic planes, parallel to the chosen axis which correspond to the summands with \( g_z = 0 \). The behavior of this cross-section as a function on the positron energy essentially depends on the angle \( \alpha \) between the projection of the photon momentum on the plane \((x, y)\) and \( y\)-axis (see figure 2). If the photon moves far from the corresponding crystallographic planes, the summation over the perpendicular components of the reciprocal lattice vector can be replaced by the integration: \( \sum_{g_x, g_y} \rightarrow \left[a_1 a_2/(2\pi)^2\right] \int dg_x dg_y \). When the photon enters into the crystal near a crystallographic plane (\( \alpha \) is small), two cases have to

![Figure 2: Geometry of the problem in the case of an orthogonal lattice with the axes \((x, y, z)\)](image-url)
be distinguished. For the first one $\theta \sim a_2/2\pi l_c$, the summation over $g_z$ can be replaced by integration. Under the assumption $u_0 \perp a_1$, the corresponding formula is further simplified to the form

$$\frac{d\sigma_c}{dE_+} \approx \frac{e^2N\omega^{-2}}{2\pi N_0a_2a_3} \sum_{m,g_y} \frac{|F_m(g_yu_0)|^2}{g_m^2} \left( \frac{\omega^2}{2E_+E_-} - 1 + \frac{2\delta}{g_m} - \frac{2\delta^2}{g_m^2} \right) \int dg_x \frac{1}{S(g_m,g)}, \quad (6)$$

with an effective structure factor determined by the integral on the right and $g_m \parallel \approx -mk_z + \theta g_y \geq \delta$. In the second case we assume that $\delta \sim 2\pi\theta/\alpha/a_1$. Now the main contribution comes from the terms with $g_y = 0$ and the formula for the cross-section takes the form

$$\frac{d\sigma_c}{dE_+} \approx \frac{e^2N}{\omega^2N_0A} \sum_{m,g_z} \frac{g_z^2}{g_m^2} \left( \frac{\omega^2}{2E_+E_-} - 1 + \frac{2\delta}{g_m} - \frac{2\delta^2}{g_m^2} \right) |F_m(g_mu_0)|^2 |S(g_m,g)|^2, \quad (7)$$

where $g_m \parallel \approx -mk_z + \psi g_x$, $\psi = \alpha \theta$, and the summation goes under the condition $g_m \parallel \geq \delta$.

The numerical calculations for the cross-section are carried out in the case of SiO$_2$ single crystal with the Moliere parametrization of the atomic potentials and for the deformation field generated by the transversal sinusoidal acoustic wave of $S$ type with frequency 5 GHz. To deal with an orthogonal lattice, we choose as an elementary cell the cell including 6 atoms of silicon and 12 atoms of oxygen (Shrauf elementary cell). For this choice the $y$ and $z$ axes of the orthogonal coordinate system ($x, y, z$) coincide with the standard $Y$ and $Z$ axes of the quartz crystal, whereas the angle between the axes $x$ and $X$ is equal to $\pi/6$. The vector of the amplitude of the displacement is directed along $x$-direction, $u_0 = (u_0, 0, 0)$, and the velocity is $4.687 \cdot 10^5$ cm/sec. The vector determining the direction of the hypersound propagation lies in the plane $YZ$ and has the angle with the axis $Z$ equal to 0.295 rad. The numerical calculations for various values of the parameters in the problem show that, in dependence of the values for the parameters, the presence of the deformation field can either enhance or reduce the cross-section. This can be used to control the parameters of the positron sources for storage rings and colliders.

As an illustration of the enhancement in figure 4 (left panel) we have depicted the quantity $d\sigma_c/dE_+$ evaluated by formula (6) as a function of the ratio $E_+^\omega$ for $u_0 = 0$ (dashed curve) and $2\pi u_0/a_2 = 6.75$ (full curve). The deformation is induced by the transversal acoustic wave of the $S$ type with frequency 5 MHz and $\theta = 0.0042$ rad, $\omega = 50$ GeV. As the cross-section is symmetric under the replacement $E_+^\omega \rightarrow 1 - E_+^\omega$, we have plotted the graphs for the region $0 \leq E_+^\omega \leq 0.5$ only. In figure 5 (right panel) the cross-section evaluated by formula (6) is presented as a function of $2\pi u_0/a_2$ for the positron energy corresponding to $E_+^\omega = 0.5$. The values of the other parameters are the same as those for the left panel. Note that for the chosen values of the parameters one has $\lambda_s \approx 9.4 \times 10^{-4}$ cm, whereas $l_c \approx 1.9 \times 10^{-6}$ cm for the energies $E_+ = E_- = 25$ Gev and, hence, $\lambda_s \gg l_c$.

In figure 6 (left panel) we have presented the cross-section evaluated by formula (7) as a function of the ratio $E_+^\omega$ for $u_0 = 0$ (dashed curve) and $2\pi u_0/a_1 = 2.75$ (full curve) in the case $\psi = 0.0022$. The values for the other parameters are the same as in figure 5. In figure 7 (right panel) we have plotted the cross-section evaluated by formula (7) as a function of $2\pi u_0/a_1$ for the positron energy corresponding to $E_+^\omega = 0.5$ and for $\psi = 0.001$. The values for the other parameters are the same as for the left panel.
Figure 3: Coherent part of the cross-section, $10^{-3}(m_\gamma^2\omega/e^6)d\sigma_c/dE_+$, in the quartz single crystal for the sinusoidal transversal acoustic wave of the $S$ type with frequency 5 GHz, evaluated by formula (6), as a function of $E_+/\omega$ (left panel) for $u_0 = 0$ (dashed curve), $2\pi u_0/a_2 = 6.75$ (full curve) and as a function of $2\pi u_0/a_2$ (right panel) for the positron energy corresponding to $E_+/\omega = 0.5$. The values for the other parameters are as follows: $\theta = 0.0042$ rad, $\omega = 50$ GeV.

Figure 4: Pair creation cross-section, $10^{-3}(m_\gamma^2\omega/e^6)d\sigma_c/dE_+$, evaluated by formula (7), as a function of $E_+/\omega$ (left panel) for $u_0 = 0$ (dashed curve), $2\pi u_0/a_2 = 2.75$ (full curve) and as a function of $2\pi u_0/a_2$ (right panel) for the positron energy corresponding to $E_+/\omega = 0.5$. The values for the other parameters are as follows: $\psi = 0.0022$, $\omega = 50$ GeV.
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