Corrigendum: Correction for stress-induced optical path length changes in a refractometer cell at variable external pressure (2019 Metrologia 56 015001)

Guido Bartl, Stephanie Glaw, Frank Schmaljohann and René Schödel

PTB, Bundesallee 100, 38116 Braunschweig, Germany
E-mail: guido.bartl@ptb.de

Received 10 April 2019, revised 6 May 2019
Accepted for publication 13 May 2019
Published 3 June 2019

(Some figures may appear in colour only in the online journal)

Corrigendum: 3. Stress-induced optical path length changes

In the derivation of equation (10) a term is missing, which takes into account the replacement of glass material by the gas medium when the refractometer cell is compressed by the external pressure. As a consequence, instead of the change of the optical path length, i.e. the distance between two fixed points independent of the material boundaries, unintentionally only the change of the optical thickness is represented by equation (10). This can be resolved by setting up equation (4) as follows:

\[ n_{\text{air}} - 1 = \left[ \frac{l_{\text{c,out}} \cdot (1 - \frac{K}{3} \delta p_{\text{out}})}{\delta l_{\text{out}} - \delta l_{\text{in}}} + \delta l_{\text{c,bend}} (\delta p_{\text{out}}) \right] \]

\[ - \left( \delta l_{w_{\text{out}}} - \delta l_{w_{\text{in}}} \right)^{-1} \cdot \left[ A \otimes \mathcal{V} \right] \]

\[ + \left( n_{f.s.} - 1 - L_{f.s.} \right) \cdot l_{w_{\text{out}}} \frac{K}{3} \delta p_{\text{out}} \]

\[ + \left( \delta l_{w_{\text{out}}} - \delta l_{w_{\text{in}}} \right) \cdot (\delta p_{\text{out}}) \]

\[ + \left( l_{\text{c,out}} - l_{\text{c,in}} \right) \cdot \frac{K}{3} \delta p_{\text{out}} \]

\[ - 2 \cdot L_{f.s.} \cdot (l_{w_{\text{out}}} - l_{w_{\text{in}}}) \cdot \frac{K}{3} \delta p_{\text{out}} \]  

(2)

which is approximated by

\[ n_{\text{air}} - 1 \approx \left[ \frac{\left[ A \otimes \mathcal{V} \right]}{l_{\text{c,out}} \cdot (1 - \frac{K}{3} \delta p_{\text{out}})} \right] \]

\[ \left[ \mathcal{V} \otimes \mathcal{V} \right] \]

\[ - \left( \delta l_{w_{\text{out}}} - \delta l_{w_{\text{in}}} \right)^{-1} \cdot \left[ A \otimes \mathcal{V} \right] \]

\[ + \left( n_{f.s.} - 1 - L_{f.s.} \right) \cdot l_{w_{\text{out}}} \frac{K}{3} \delta p_{\text{out}} \]

\[ + \left( \delta l_{w_{\text{out}}} - \delta l_{w_{\text{in}}} \right) \cdot (\delta p_{\text{out}}) \]

\[ + \left( l_{\text{c,out}} - l_{\text{c,in}} \right) \cdot \frac{K}{3} \delta p_{\text{out}} \]  

(3)

in analogy to the previous equation (13). Note that the factor \((n_{f.s.} - 1 - L_{f.s.})\) now involves a ‘−1’.

However, a comparison with an all-FEM-based approach performed at NIST by Egan et al (see appendix A in [1]) has
Corrigendum: Appendix. Refractive index changes

Following [2] and [3] we applied the Lorentz–Lorenz relation to calculate the change of the refractive index of fused silica induced by external pressure. Unfortunately, this approach is in conflict with experimental data of Vedam et al [4], Ritland [5], Spinner et al [6] and Waxler et al [7]. These publications provide experimental evidence that the relation between the refractive index and the density of solid materials, in particular of fused silica, is not compatible with the Lorentz–Lorenz relation (i.e. with the assumption of a constant value of the polarizability). Therefore, instead of the previous equation (A.2) the relation between the relative change of the refractive index and the relative density change should be expressed by

\[
\frac{dn}{n} = a \frac{d\rho}{\rho}
\]

(similar to [8]) with \(a = 0.226 \pm 0.010\) being a fit parameter which is determined by linear regression of the experimental results from [4] for fused silica. Consequently, the experimentally-based factor \(\mathcal{L}\) must read

\[
\mathcal{L} = a \cdot n.
\]

Conclusion

Considering the corrections described above, the partially FEM-based approach from [9] yields the updated figure 6 which shows the dependence between the resulting corrections and the external gas pressure affecting the cell windows of our particular cell geometry. However, as mentioned above, the all-FEM-based approach from Egan et al [1] provides more reliable results and yields the pressure-dependent correction shown in figure 7.

Compared to the previously published correction in [9] its magnitude is decreased so that, for instance, at 1000 hPa...
the contribution to the air refractivity is of the order of 2.4 nm/420 mm ≈ 6 × 10⁻⁹ which corresponds to a relative effect of approximately 2 × 10⁻⁵ at standard conditions.

Acknowledgments

We appreciate a thorough and critical discussion of our previously published results from [9] with J Stone and P Egan from NIST, USA. This interaction has lead to the revealing of the corrections shown in the present corrigendum and the FEM data from P Egan enabled us to improve the accuracy of the correction.

ORCID iDs

René Schödel https://orcid.org/0000-0002-7597-9036

References

[1] Egan P F, Stone J A, Scherschligt J K and Harvey A H 2019 Measured relationship between thermodynamic pressure and refractivity for six candidate gases in laser barometry J. Vac. Sci. Technol. A 37 031603
[2] Birch K P, Downs M J and Ferriss D H 1988 Optical path length changes induced in cell windows and solid etalons by evacuation J. Phys. E: Sci. Instrum. 21 690
[3] Bönsch G and Potulski E 1998 Measurement of the refractive index of air and comparison with modified Edlén’s formulae Metrologia 35 133
[4] Vedam K, Schmidt E D D and Roy R 1966 Nonlinear variation of refractive index of vitreous silica with pressure to 7 kbars J. Am. Ceram. Soc. 49 531–35
[5] Ritland H N 1955 Relation between refractive index and density of a glass at constant temperature J. Am. Ceram. Soc. 38 86–8
[6] Spinner S and Waxler R M 1966 Relation between refractive index and density of glasses resulting from annealing compared with corresponding relation resulting from compression Appl. Opt. 5 1887–9
[7] Waxler R M and Weir C E 1965 Effect of hydrostatic pressure on the refractive indices of some solids J. Res. Nat. Bur. Stand. 69A 325–33
[8] Shelton D P 1992 Lens induced by stress in optical windows for high-pressure cells Rev. Sci. Instrum. 63 3978–82
[9] Bartl G, Glaw S, Schmaljohann F and Schödel R 2019 Correction for stress-induced optical path length changes in a refractometer cell at variable external pressure Metrologia 56 015001
Correlation for stress-induced optical path length changes in a refractometer cell at variable external pressure

Guido Bartl, Stephanie Glaw, Frank Schmaljohann and René Schödel

PTB, Bundesallee 100, 38116 Braunschweig, Germany

E-mail: guido.bartl@ptb.de

Received 11 October 2018, revised 6 November 2018
Accepted for publication 8 November 2018
Published 30 November 2018

Abstract
In the field of interferometry for high-precision length measurements the influence of the refractive index of air can either be eliminated by completely measuring in a vacuum or by the application of a refractometer cell which enables the precise in situ determination of the refractivity. In the latter case, systematic effects can occur in connection with the operation of the refractometer cell due to mechanical stress induced at different pressures. Operation at a pressure range beyond the usual atmospheric pressure variation is needed when, for instance, the compressibility of a material measure is investigated or the pressure dependence of the refractive index of gases is aimed at. In this work an approach is shown for the correction of pressure-induced effects for the type of interference refractometer being used at PTB when starting from a reference state in vacuum. The required correction of the refractivity’s value was found to be of the order of $2 \times 10^{-8}$ for a refractometer cell of 420 mm length which corresponds to a relative contribution of $7 \times 10^{-5}$ when determining the air refractivity $n-1$ at standard laboratory conditions. For instance, in the application of high-precision length measurements (up to 1000 mm) the effect amounts to a length correction of several nanometers.

Keywords: refractive index, refractivity, interference refractometer, refractometry, length measurement

(Some figures may appear in colour only in the online journal)

1. Introduction

The determination of the refractive index of air by interferometric techniques began in the 19th century as stated in the introductory review in [1]. Still nowadays a common technique for the determination of the refractive index of air is the application of interference refractometers which allow the precise measurement of the refractivity of a gas by the comparison of the optical path lengths through the gaseous medium and a (vacuum) reference path. More than 30 years ago widespread investigations came up to determine the accuracy of such refractometers in the field of interferometric dimensional measurements [2] and to compare the results with values from an empirical formula [3] (which has been followed by modifications since then [4–6]). Depending on the design of the refractometer cell these paths may appear very different and there is a variety of realisations (e.g. [6–10]). Some consist of a single evacuated light path which is compared to an outer path through a free gas, while others combine a path through a vacuum and a gas filled cavity side by side. Depending on the design principle the operation mode differs from type to type and, hence, the occurring systematic effects also do. For instance pressure changes of the gas under examination can either mechanically affect the own gas cavity (internal change) or the whole refractometer cell (external change). While differences caused by geometric and stress-induced
optical path length changes have been taken into account in former measurements at PTB for pressure changes inside the cavity [6], the stress-induced path length changes resulting from variations in the environment of the refractometer cell must be handled differently. On the one hand, the influence of the geometric difference can be determined by a simple in-place measurement with the refractometer cell being in a reference state with vacuum inside and around the cell, so that a detected path difference can be used as a correction for this influence [6, 11, 12]. On the other hand, stress-induced influences require detailed consideration. Hence, this work deals with the necessary corrections for systematic effects on the measurement result, which occur for a refractometer cell exposed to external pressure changes. The focus is on the derivation of a theoretical model equation enabling the identification of these effects. The application of the model requires knowledge of a quantity which is determined for practical purposes by the finite-element method (FEM). Eventually the validity of the FEM approach is supported by means of experimental data.

2. Optical paths in the refractometer cell

The refractometer cells used at PTB consist of a fused silica tube with polished end faces (figure 1). The open ends of the tube are sealed by fused silica plates which are wrung onto the end faces so that the molecular forces keep the bodies in tight contact (similar to the wringing of gauge blocks). In the middle of the tube there is a junction to a vacuum pump for the high vacuum evacuation of the inner cavity. For the application during measurements the complete device is placed in a pressure tight chamber which can be operated between atmospheric and high vacuum conditions [12].

The cell depicted in figure 1 is 420 mm long, has an inner diameter of 10 mm and an outer diameter of 19 mm. The sealing windows have lateral dimensions of 20 mm by 50 mm and a thickness of 5 mm. In order to deflect light reflexions from the windows away from the optical axis, they are slightly inclined to the tube’s axis and not perfectly parallel to each other. Moreover, the windows are wedge-shaped along the long edge in reverse direction to each other. As a consequence, the optical paths through the centre of and along the tube differ by the various geometric conditions.

For the following theoretical model the geometric paths are defined by

\[ l_{\text{out}} = l_{\text{c,out}} + l_{w1,\text{out}} + l_{w2,\text{out}} = l_{\text{c,out}} + l_{w,\text{out}} \]  

(1)

and

\[ l_{\text{in}} = l_{\text{c,in}} + l_{w1,\text{in}} + l_{w2,\text{in}} = l_{\text{c,in}} + l_{w,\text{in}} \]  

(2)

as sketched in figure 2, where \( l_{\text{c,\text{in}}} \) is the reference path through vacuum and \( l_{\text{c,\text{out}}} \) is the path through a gas at a specific pressure. As described in [12] the use as an interference refractometer yields the measured optical path length difference between the path through air and the reference path through vacuum, which is defined by the symbol \([A \odot V]\) in the expression

\[ [A \odot V] = N \cdot \frac{\lambda}{2} \]  

(3)

in terms of the measured integral and fractional orders of interference \( N \) and the vacuum wavelength \( \lambda \) of the used light. Apart from that, the difference can also be written in terms of the respective optical path lengths \( n_x l_y \) with \( x \) and \( y \) being placeholders for the different indices of the refractive index \( n \) and the geometric length \( l \):

\[ [A \odot V] = (n_{x,\text{out}} \cdot l_{w,\text{out}} + n_{\text{air}} \cdot l_{\text{c,\text{out}}}) - (n_{x,\text{in}} \cdot l_{w,\text{in}} + 1 \cdot l_{\text{c,\text{in}}}). \]  

(4)

The quantities marked with a tilde specify a state under the influence of external pressure and ‘f.s.’ refers to fused silica. Consequently, a similar expression can be derived for the refractometer cell being in equilibrium, i.e. inside and outside under vacuum conditions.
\[ [V \otimes V] = (n_{f,s,\text{out}} \cdot l_{w,\text{out}} + 1 \cdot l_{c,\text{out}}) - (n_{f,s} \cdot l_{w,\text{in}} + 1 \cdot l_{c,\text{in}}) \] (5)

which can serve as a reference state to determine geometric differences along the inner and outer path.

3. Stress-induced optical path length changes

When the refractometer cell is exposed to external pressure, the optical paths undergo mechanical stress in the glass material (fused silica). Therefore, using integration over the total differential of the optical path length \( d(n, l_x) = l_x \, dn_y + n_y \, dl_x \) (again with \( x \) and \( y \) being placeholders for the different indices) the expression

\[ \delta(n, l_x) = \int d(n, l_x) = l_x \, \delta n_y + n_y \, \delta l_x \] (6)

can be derived assuming that the small \( \delta \)-changes follow a linear relation, which is then equivalent to

\[ \delta(n, l_x) = n \, l_x - n_x l_x. \] (7)

The combination of the two previous equations yields

\[ \tilde{n} l_x = n l_x + l_x \, \delta n_y + n_y \, \delta l_x \] (8)

or, when \( \delta n_y = 0 \), i.e. \( n_y \) is not affected by pressure changes,

\[ n \tilde{l}_x = n_x \right (l_x + \delta l_x). \] (9)

With the equations (8) and (9), the former equation (4) can be converted to

\[ [A \otimes V] = [V \otimes V] + n_{\text{air}} \cdot l_{w,\text{out}} + n_{\text{air}} \cdot \delta l_{w,\text{out}} + n_{f,s,\text{out}} \cdot \delta l_{w,\text{out}} \]
\[ + n_{\text{air}} \cdot (l_{c,\text{out}} + \delta l_{c,\text{out}}) \]
\[ - (n_{f,s} \cdot l_{w,\text{in}} + n_{f,s} \cdot \delta l_{w,\text{in}} + l_{w,\text{in}} \cdot \delta n_{f,s}) \]
\[ - (l_{c,\text{in}} + \delta l_{c,\text{in}}). \] (10)

and by replacing \( n_{f,s,\text{out}} \cdot \delta l_{w,\text{out}} = n_{f,s} \cdot \delta l_{w,\text{out}} \) with help of equation (5) and approximating \( n_{f,s,\text{out}} \cdot \delta l_{w,\text{out}} = (n_{f,s} + \Delta n) \cdot \delta l_{w,\text{out}} \), \( \Delta n \ll n_{f,s} \) we eventually have

\[ [A \otimes V] = [V \otimes V] + n_{\text{air}} \cdot l_{w,\text{out}} - n_{f,s} \cdot l_{w,\text{in}} \]
\[ + n_{\text{air}} \cdot \delta l_{w,\text{out}} + n_{f,s} \cdot \delta l_{w,\text{in}} \]
\[ - n_{f,s} \cdot \delta l_{w,\text{in}} - l_{w,\text{in}} \cdot \delta n_{f,s} \]
\[ - \delta l_{c,\text{in}}. \] (11)

Aimed at an expression to determine the gas refractive index \( n_{\text{air}} \) based on the measured optical path differences \([A \otimes V] \) and \([V \otimes V] \) the additional quantities in the previous equation have to be collected in correction terms. \( \delta l_{w,\text{out}} \) is simply the geometric length change of the window thickness due to external pressure in the outer region of the window. As the mechanical stress is quasi-isotropic in this region, one can define \( \delta l_{w,\text{out}} = -l_{w,\text{out}} \kappa \delta p_{\text{out}} \) with \( \kappa \) being the compressibility of the glass material and \( \delta p_{\text{out}} \) the pressure difference to the reference state (vacuum). The length change of the tube on the central axis of the refractometer cell

\[ \delta l_{c,\text{in}} = -l_{c,\text{in}} \frac{\kappa}{3} \delta p_{\text{out}} + (n_{\text{air}} - 1) \cdot \delta l_{c,\text{bend}} \delta p_{\text{out}} \]

takes account of the geometric length change of the tube due to external pressure and for the path replacement which comes from the external pressure bending the window into the tube by the amount \( \delta l_{c,\text{bend}} \delta p_{\text{out}} \). As illustrated in figure 3, a small fraction of vacuum path inside the tube is replaced by air beyond the window. In the outer region, the path along the tube is shortened by \( \delta l_{c,\text{out}} = -l_{c,\text{out}} \frac{\kappa}{3} \delta p_{\text{out}} \) due to the compression of the tube’s length. The bending of the windows does not have a significant effect on the external path length, as the occurring tilting angles are too small. \( \delta l_{w,\text{in}} = \delta l_{w,\text{in}}(\delta p_{\text{out}}) \) is the geometric length change of the window thickness on the central axis of the tube, but cannot be defined analogously to the previous terms, because the influence of the external pressure is not isotropic in this region. Therefore, it has to be determined by different means as described in section 4.2. The definition of the two remaining quantities specifying the changes of the refractive index of the glass material,

\[ \delta n_{f,s} = \mathcal{L}_{f,s} \left ( -\frac{1}{l_{w,\text{in}}} \delta l_{w,\text{in}}(\delta p_{\text{out}}) + 2 \cdot \frac{\kappa}{3} \delta p_{\text{out}} \right ) \]

and

\[ \delta n_{t,s,\text{out}} = \mathcal{L}_{f,s} \cdot \kappa \cdot \delta p_{\text{out}}, \]

is derived in the appendix.

With these definitions inserted into equation (11) it can be converted to an expression to determine the gas refractive index \( n_{\text{air}} \) or, here, the refractivity \( n_{\text{air}} - 1 \):
\[ n_{\text{air}} - 1 = \frac{[A \otimes V]}{L_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}})} - \frac{[V \otimes V]}{L_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}})} - \frac{(n_{\text{t.s.}} - L_{\text{t.s.}}) \cdot l_{w,in} \frac{\kappa}{\lambda} \delta p_{\text{out}} + (n_{\text{t.s.}} - L_{\text{t.s.}}) \cdot \delta l_{w,in}(\delta p_{\text{out}})}{L_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}})} + \frac{(n_{\text{t.s.}} - L_{\text{t.s.}}) \cdot \delta l_{w,in}(\delta p_{\text{out}})}{L_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}})} \]  

The fourth term can be neglected, because the contained contributions in the numerator are very small compared to the other terms. Moreover, on account of \( l_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}}) \approx \delta l_{\text{bend}}(\delta p_{\text{out}}) \) the contribution of \( \delta l_{\text{bend}}(\delta p_{\text{out}}) \) can also be omitted, so that eventually the equation reads:

\[ n_{\text{air}} - 1 \approx \frac{[A \otimes V]}{L_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}})} - \frac{[V \otimes V]}{L_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}})} - \frac{(n_{\text{t.s.}} - L_{\text{t.s.}}) \cdot l_{w,out} \frac{\kappa}{\lambda} \delta p_{\text{out}}}{L_{\text{out}} \cdot (1 - \frac{\kappa}{\lambda} \delta p_{\text{out}})} \]  

In this form the contributions in the four terms can be identified as

(i) the actual difference measurement \([A \otimes V]\) between the vacuum and the air path,
(ii) a measured correction \([V \otimes V]\) for geometric differences (e.g. tilted and wedged window profiles), which is a reference measurement as already applied in [6, 11, 12],
(iii) a correction \((n_{\text{t.s.}} - L_{\text{t.s.}}) \cdot l_{w,out} \frac{\kappa}{\lambda} \delta p_{\text{out}}\) for the quasi-isotropic compression of the glass material in the outer region of the window
and
(iv) a correction \((n_{\text{t.s.}} - L_{\text{t.s.}}) \cdot \delta l_{w,in}(\delta p_{\text{out}})\) for the deformation of the window in front of the tube’s aperture.

4. Practical results and validation

In order to obtain values which can be used as a correction for real measurements information about the parameters of equation (13) is required. While the value of the compressibility \( \kappa \) is known as a material parameter of fused silica and the pressure is anyway determined during the measurement, the resulting change of the window thickness \( \delta l_{w,in}(\delta p_{\text{out}}) \) has to be derived by different manners. As reported by other authors (e.g. [10, 13]), FEM-based simulations are appropriate in this context. Therefore, the refractometer cell in use has been modelled with the software Creo Elements/Direct Modeling 19.0 and changes at different pressure conditions have been calculated by use of MSC Nastran 2008 r1. To save computing time and to set suitable boundary conditions, it is common to divide the model at the symmetry planes, which is why only an eighth of the original geometry was needed as shown in figure 4. Subsequently, the acting pressures were applied to the outer or rather the inner and outer surfaces. Then a linear static analysis was carried out and the displacements of the relevant geometry points along the components’ axes were extracted. With these data the deformation of the structure (figure 5), such as the window curvature \( \delta l_{\text{bend}} \) and the change of its thickness \( \delta l_{w,in} \), for instance, was calculated. The calculations are based on material parameters for amorphous fused silica (Young’s modulus: 73 GPa, Poisson’s ratio: 0.16) with conservatively estimated uncertainties of 10% to cover the variation in the data from literature. To estimate the uncertainty of the FEM results the simulations are performed repeatedly with the extreme values of the input data.

4.1. Validation

In order to validate the FEM approach one can refer to the measured data reported in [6] combined with the previous FEM model, but matching the dimensions of the respective...
refractometer cell (10 mm window thickness, 18 mm inner diameter). Moreover, it has to be taken into account that the reference state of the refractometer cell was atmospheric pressure inside and outside the cell. Nevertheless, the described measurements yielded the optical path length change through one of the cell windows while it was exposed to pressure changes inside the tube which was evacuated, so that the results are appropriate for the validation purpose. By combining equations (6) and (A.4) and considering that the pressure change outside the refractometer cell is zero in this case, one gets the relation

\[ \delta OPL = n_{f.s.} \cdot \delta l_{w, in} - L_{f.s.} \cdot \delta l_{w, in} \] (14)

between the measured optical path length change \( \delta OPL \) and the geometric length change \( \delta l_{w, in} \) of the window thickness. On the one hand, the data of figure 3 in [6] shows that the resulting path length change is approximately \( \delta OPL = 8.2 \text{ nm} \pm 0.9 \text{ nm} \). The uncertainty of about 10% considers the variation in the repeated evacuation cycles, the approximated linear approach of the refractive index change and the estimate of the actual atmospheric pressure during the measurement. On the other hand, the FEM simulations yield \( \delta l_{w, in} = 8.7 \text{ nm} \pm 0.5 \text{ nm} \) at a pressure difference of 1000 hPa; i.e. the window becomes thicker when the cavity is evacuated. Then, with \( n_{f.s.} = 1.458 \) and by applying equation (14), one gets \( \delta OPL_{\text{FEM}} = 7.3 \text{ nm} \pm 0.5 \text{ nm} \). As the values of \( \delta OPL_{\text{FEM}} \) and \( \delta OPL \) are in mutual agreement within their uncertainties, the validity of the approach from section 3 proves to be trustworthy.

4.2. Results for the correction

Consequently, in the next step the required correction terms (iii) and (iv) from equation (12) can be determined for the refractometer cell described in section 2. As there are two windows which are passed by the light on the way through the cell, the values of \( l_{w, out} \) and \( \delta l_{w, in} \) are considered as the sum of both windows, respectively. From the FEM simulations one obtains \( \delta l_{w, in} = 2 \cdot (-0.7 \text{ nm} \pm 0.2 \text{ nm}) \) at 1000 hPa external pressure which means that the thickness of the windows becomes smaller on the refractometer cell’s central axis compared to the vacuum reference state. Because the pressure range between vacuum and atmospheric conditions only provokes linear deformations in the material—in contrary to [14] where pressures of several atmospheres are investigated—the result for 1000 hPa can be scaled accordingly to smaller pressures. Then, with \( \kappa = 2.78 \times 10^{-11} \text{ Pa}^{-1} \) and \( l_{w, out} = 2 \cdot (5 \text{ mm} \pm 0.2 \text{ mm}) \), the corrections (iii) and (iv) from equation (12) can be calculated depending on the actual pressure difference. Figure 6 shows the dependence between the respective contributions and the external gas pressure affecting the cell windows.

As it turns out, the total correction ‘(iii) + (iv)’ is negligibly small at low pressures, i.e. somewhat below 50 hPa. Therefore, in [12]—where pressures below 13 hPa were dealt with—the consideration of this contribution could be omitted and only the correction (ii) was required. While the contribution of (iv) is comparatively small, the dominating part of (iii) shows the relevance of the compression of the window material. Therefore, towards higher pressures the correction becomes significant and needs to be taken into account in determining the gas refractivity. For instance, at 1000 hPa the contribution to the refractivity’s value is of the order of 7.3 nm/420 mm \( \approx 2 \times 10^{-8} \) which corresponds to a relative contribution of \( 7 \times 10^{-5} \) to the refractivity of air at standard conditions.

5. Conclusion

In the previous method of operation of the refractometer cells at PTB, the reference state used for the measurement of geometric corrections was atmospheric air pressure. This is convenient for typical measurements in the field of calibration of length standards (e.g. gauge blocks) under atmospheric conditions. But for highly precise measurements of the compressibility of a material the pressure in the environment of the sample under test and, hence, the refractometer cell is varied beyond the range of usual atmospheric pressure variations. Moreover, starting from a reference state in vacuum is reasonable to obtain small measurement uncertainties. Due to the design of the refractometer cell the derived correction is then to be used for avoiding systematic deviations of the measurement results. The derived...
Theoretical model is kept simple, could be validated by experimental data and the applicability only requires the following constraints: high homogeneity of the cell’s material and a pressure range in accordance with pure-linear material deformations. Then, for a refractometer cell with a geometry as shown in section 2, the calculation of the correction yields pressure-induced optical path length changes in the size of up to a few nanometers which affect the measurement of the air refractivity by a relative contribution on the order of up to $7 \times 10^{-5}$.

Appendix. Refractive index changes

The mechanical stress on the refractometer cell due to the external pressure induces changes of the material’s density $\rho$ and, therefore, of its refractive index $n$ as can be deduced from the Lorentz–Lorenz relation [15] written in the form

$$\rho = \frac{1}{R} \frac{n^2 - 1}{n^2 + 2}$$  \hspace{1cm} (A.1)

with $R$ being the specific refraction of the material. Following the approach from [13] via the total differential $d\rho = \frac{\partial \rho}{\partial n} dn$ one can derive the expression

$$\frac{d\rho}{\rho} = \frac{1}{\mathcal{L}} \cdot dn \quad \text{with} \quad \mathcal{L} = \frac{(n^2 - 1)(n^2 + 2)}{6n}.$$  \hspace{1cm} (A.2)

While in [13] the pressure was changed inside the refractometer cell, in the present case the derivation has to deal with external pressure. Therefore, the relative differential volume change of a cylindrical volume segment $V = \pi \cdot (D_{\text{tube}}/2)^2 \cdot l_{\text{w,in}}$ of the window in front of the tube with an inner diameter $D_{\text{tube}}$ is

$$\frac{dV}{V} = \frac{dl_{\text{w,in}}}{l_{\text{w,in}}} + 2 \cdot \frac{dD_{\text{tube}}}{D_{\text{tube}}}.$$  \hspace{1cm} (A.3)

Then, because of $-\frac{dV}{V} = \frac{d\rho}{\rho} = \frac{1}{\mathcal{L}} \cdot dn$ and $dD_{\text{tube}} = -\frac{2}{l_{\text{w,in}}} \cdot D_{\text{tube}} \cdot d\rho_{\text{out}}$ one gets after carrying out the integration

$$\delta n = \mathcal{L} \cdot \left( -\frac{1}{l_{\text{w,in}}} \delta l_{\text{w,in}} + 2 \cdot \frac{\kappa}{3} \cdot \delta \rho_{\text{out}} \right)$$  \hspace{1cm} (A.4)

and, in the special case of $dl_{\text{w,in}} = -\frac{2}{3} l_{\text{w,in}} \cdot d\rho_{\text{out}}$, \hspace{1cm}

$$\delta n = \mathcal{L} \cdot \kappa \cdot \delta \rho_{\text{out}}.$$  \hspace{1cm} (A.5)

References

[1] Barrell H and Sears J E 1939 The refraction and dispersion of air and dispersion of air for the visible spectrum Phil. Trans. R. Soc. A 238 1–64
[2] Schellekens P, Wilkening G, Reinboth F, Downs M J, Birch K P and Sprockel J 1986 Measurements of the refractive index of air using interference refractometers Metrologia 22 279
[3] Edlén B 1966 The refractive index of air Metrologia 2 71
[4] Birch K P and Downs M J 1993 An updated Edlén equation for the refractive index of air Metrologia 30 155
[5] Ciddor P E 1996 Refractive index of air: new equations for the visible and near infrared Appl. Opt. 35 1566–73
[6] Bönsch G and Potulski E 1998 Measurement of the refractive index of air and comparison with modified Edlén’s formulae Metrologia 35 133
[7] Birch K P and Downs M J 1988 The results of a comparison between calculated and measured values of the refractive index of air J. Phys. E: Sci. Instrum. 21 694
[8] Andersson M, Eliasson L and Pendell R L R 1987 Compressible Fabry–Perot refractometer Appl. Opt. 26 4835–40
[9] Birch K P, Reinboth F, Ward R E and Wilkening G 1993 The effect of variations in the refractive index of industrial air upon the uncertainty of precision length measurement Metrologia 30 7
[10] Egan P F, Stone J A, Rickor J E, Hendricks J H and Strouse G F 2017 Cell-based refractometer for pascal realization Opt. Lett. 42 2944–7
[11] Schödel R and Bönsch G 2001 Precise interferometric measurements at single crystal silicon yielding thermal expansion coefficients from 12 °C to 28 °C and compressibility Proc. SPIE 4401 54–62
[12] Schödel R, Walkov A, Voigt M and Bartl G 2018 Measurement of the refractive index of air in a low-pressure regime and the applicability of traditional empirical formulae Meas. Sci. Technol. 29 064002
[13] Birch K P, Downs M J and Ferriss D H 1988 Optical path length changes induced in cell windows and solid etalons by evacuation J. Phys. E: Sci. Instrum. 21 690
[14] Shelton D P 1992 Lens induced by stress in optical windows Appl. Opt. 31 1550–60
[15] Born M and Wolf E 1999 Principles of Optics: Electromagnetic Theory of Propagation, Interference, and Diffraction of Light 7th edn (Cambridge: Cambridge University Press) ch 2.3

ORCID iDs

René Schödel @ https://orcid.org/0000-0002-7597-9036