A New Fast Edge Detection Algorithm Based on NAM-structured Plane Decomposition

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Abstract  The nonsymmetry and antipacking pattern representation model (NAM), inspired by the concept of the packing problem, uses a set of subpatterns to represent an original pattern. The NAM is a promising method for image representation because of its ability to focus on the interesting subsets of an image. In this paper, we develop a new method for gray-scale image representation based on NAM, called NAM-structured plane decomposition (NAMPD), in which each subpattern is associated with a rectangular region in the image. The luminance function of pixels in this region is approximated by an oblique plane model. Then, we propose a new and fast edge detection algorithm based on NAMPD. The theoretical analyses and experimental results presented in this paper show that the edge detection algorithm using NAMPD performs faster than the classical ones because it permits the execution of operations on subpatterns instead of pixels.

Keywords  nonsymmetry and antipacking pattern representation model (NAM); packing problem; image processing; edge detection

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Introduction

The current image representation methods are generally divided into three groups: traditional pixel representation, compression oriented representation, [1-4] and hierarchical representation[5-7]. The traditional pixel representation remains too much redundant data, which results in a high complexity of image processing. The compression-oriented representation decreases the redundancy but complicates some basic image processing. The hierarchical representation is a very important method for image processing, pattern recognition, and computer visualization. The fundamental of hierarchical representation is that the image is decomposed into blocks recursively, and then, each image block is subdivided into several equal parts every time. One of the most important hierarchical representation methods is the quadtree [8]. The image processing using the quadtree performs more quickly because it permits the execution of operations on image blocks instead of pixels [8-12]. However, although the quadtree has many merits, the symmetry of segmentation causes poor adaptability to image content and results in too many leaf nodes.

The nonsymmetry and antipacking pattern representation model (NAM), inspired by the concept of the packing problem, uses a set of subpatterns to represent an original pattern [13-17]. The asymmetrical-structured antipacking extracts more subpatterns with
large areas and reduces the number of sub patterns with small areas. The NAM is a promising method for image representation because of its ability to focus on the interesting subsets of an image. In this paper, we develop a new method for gray-scale image representation based on NAM, called NAM-structured plane decomposition (NAMPD), in which each subpattern is associated with a rectangular region in the image. The luminance function of pixels in this region is approximated by an oblique plane model.

The edge detection is in the forefront of image analysis. The classical edge detection algorithms [18] are based on pixel representation. In this paper, a new fast edge detection algorithm based on NAMPD is proposed. The theoretical and experimental results show that this algorithm obtained a significant improvement over the classical ones [19, 20].

In Section 1, the abstract model of the NAM is introduced. In Section 2, the fundamentals of the NAMPD are presented. A rule deduced from least squares methods is introduced to compute the parameter vector of the plane, and then, the encoding and decoding algorithms are presented. We propose a new and fast edge detection algorithm based on NAMPD in Section 3. A serial of experiments are done to evaluate our edge detection algorithm in Section 4. Finally, we discuss the subsequent work in Section 5.

1 Nonsymmetry and Antipacking pattern representation Model (NAM)

In the following, we review the abstract model of the NAM [13-17].

The NAM is an inverse problem (antipacking) of the packing problem. The idea of the NAM can be described as follows: Given an original pattern $\Gamma$ and a set $P = \{P_0, P_1, \ldots, P_n\}$ of $n$ predefined subpatterns, we form the specific sequence $\Gamma'$ of the instances of these subpatterns to represent the given pattern $\Gamma$. The procedure of the transform can be written as follows:

$$\Gamma' = T(\Gamma)$$

(1)

where $T(\cdot)$ is a forward transform or encoding function. The result of the encoding is given by

$$\Gamma' = \bigcup_{i=0}^{N-1} p_i(v_i, P_i, f_i)$$

(2)

where $N$ is the number of the instances of subpatterns in $\Gamma'$, and $p_i(0 \leq i \leq N-1)$ represents the $i$th instance of subpattern in $\Gamma'$, $v_i$ is the luminance function of $p_i$, $P_i \in P$ indicates that $p_i$ is an instance of $P_i$, and $f_i$ is an affine transform that can map the values of the shape parameters of $P_i$ into those of $p_i$.

The reconstruction of the original pattern is an inverse process of the above encoding function. The procedure is to reconstruct the original pattern $\Gamma$ from a specific antipacking result $\Gamma''$ based on the given set $P$ of predefined subpatterns. The procedure can be described as follows:

$$\Gamma = T^{-1}(\Gamma') + E$$

(3)

where $T^{-1}(\cdot)$ is an inverse transform or decoding function, and $E$ is a set of residue patterns. The NAM is a nondistortion model from $\Gamma''$ to $\Gamma$ when $E$ is an empty set, otherwise a distortion model.

Many nondistortion models of the NAM have been investigated in literatures [13-17]. In this paper, we only discuss the distortion model of the NAM.

The concept of the nonsymmetry in the NAM means that the structure of antipacking is asymmetrical, which is relative to the symmetry of the hierarchical structure. The fundamental of hierarchical representation methods is that the image is segmented into blocks recursively, and then, each image block is subdivided into several parts, which are all of the same size. The symmetry of segmentation makes hierarchical representation methods have poor adaptability to image content. The asymmetrically structured antipacking extracts a subpattern with a large size in the remaining part of the original pattern every time so that the number of subpatterns is much smaller than that of nodes in the hierarchical structure. Therefore, the NAM has the capability of achieving a better representation efficiency that cannot be achieved by the traditional hierarchical representation methods.

2 NAM-structured plane decomposition

Before describing the NAMPD, we first introduce the plane model, the rule for computing the plane
parameter vector, a decision rule for the homogeneous of a rectangular region, and the rectangular plane subpattern.

2.1 Plane model

Given an \( M \times N \) gray-scale image, the gray value vector of this image can be given by

\[
S = [s_0, s_1, \cdots, s_{L-1}]^T
\]

where \( L = MN \), and \( s_i \) is the gray value of the \( i \)-th pixel in the image along the raster scan order. The linear least squares estimate of \( S \) and \( \hat{S} \) will be

\[
\hat{S} = R_{MN} \begin{bmatrix} a \\ b \\ c \end{bmatrix}
\]

where \( a \) is a mean parameter, \( [b, c] \) forms a plane gradient, and \( R_{MN} = (I_{MN}, X_{MN}, Y_{MN}) \) is defined as a subspace matrix for an \( M \times N \) image. The three vectors \( I, X, \) and \( Y \) are defined as follows:

\[
I_{MN} = [1, 1, \cdots, 1]^T
\]

\[
X_{MN} = [x_0, x_1, \cdots, x_{L-1}]^T
\]

\[
Y_{MN} = [y_0, y_1, \cdots, y_{L-1}]^T
\]

where \( x_i = \text{MOD}(i, N) \), \( \forall i = 0, \cdots, L-1 \)

\[
y_i = \text{DIV}(i, N) \), \( \forall i = 0, \cdots, L-1 \)

When \( M = 2 \) and \( N = 4 \), we may write

\[
R_{2x4} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}^T
\]

2.2 Plane parameter vector

By introducing a plane parameter vector \( P = [a, b, c]^T \), we can obtain \( \hat{S} \) as follows:

\[
\hat{S} = R_{MN} P
\]

The residue vector between \( S \) and \( \hat{S} \) can be defined as

\[
E = S - \hat{S}
\]

According to Eq. (12) and Eq. (13), the energy of the residue vector \( E \) is then given by

\[
EN = E^T E
\]

\[
= (S - \hat{S})^T (S - \hat{S})
\]

\[
= (S - R_{MN} P)^T (S - R_{MN} P)
\]

Minimizing Eq. (14) can be achieved by setting the gradient of the energy \( EN \) to zero as follows:

\[
\frac{\partial EN}{\partial P^T} = -R_{MN}^T S + R_{MN}^T R_{MN} P = 0
\]

Moreover, by computing the resolution for Eq. (15), we obtain the least squares estimate \( \hat{P} \) of the plane parameter vector as follows:

\[
\hat{P} = (R_{MN}^T R_{MN})^{-1} R_{MN}^T S
\]

2.3 The homogeneous of a rectangular region

It is clear that a region with less detail can be antipacked by subpatterns with large size. However, a region with more detail might require more subpatterns with small size to achieve an acceptable distortion. Before discussing the decision rule for the homogeneous of a rectangular region, we first give some relative definitions. Let \( D = (x, y, h, w) \) be a rectangular region in a gray-scale image, where \( (x, y) \) is the start point (the top left corner) of the rectangular region \( D \), \( h \) \( \text{MOD}(h, 2) = 0 \) is the height of \( D \), and \( w \) \( \text{MOD}(w, 2) = 0 \) is the width of \( D \).

Definition 1 The initial residue energy of the rectangular region \( D \) is defined as

\[
E_I = \sum_{i=0}^{N/4-1} (S_i - R_{2x2} \hat{P})^T (S_i - R_{2x2} \hat{P})
\]

where \( S_i \) is the gray value vector of the \( i \)-th 2-by-2 region in \( D \), and \( \hat{P} \) is the least squares estimate of the plane parameter vector in \( S_i \).

Definition 2 The united residue energy of the rectangular region \( D \) is defined as

\[
E_U = (S_D - R_{Nyw} \hat{P})^T (S_D - R_{Nyw} \hat{P})
\]

where \( S_D \) is the gray value vector of \( D \), and \( \hat{P} \) is the least squares estimate of the plane parameter vector in \( S_D \).

Our major concern now is to establish a decision rule for the homogeneous of the rectangular region \( D \), which requires a hypothesis test of the following form:

Hypothesis \( H_0 \) The rectangular region \( D \) is homogeneous, that is, it can be represented by one subpattern.

Hypothesis \( H_1 \) The rectangular region \( D \) is heterogeneous, that is, it cannot be represented by one subpattern.

In the case of the hypothesis \( H_0 \), antipacking a
subpattern of size $h$-by-$w$ will not cause perceptual distortion, and the following rule
\[ E_i > \tau E_U \] (19)
holds, where $\tau(0 < \tau < 1)$ is a scale factor. However, in the case of the hypothesis $H_1$, antipacking a subpattern of size $h$-by-$w$ will cause perceptual distortion, and the following rule
\[ E_i \leq \tau E_U \] (20)
holds. From rule (19) and rule (20), the decision rule for the homogeneous of the rectangular region $D$ is obtained as follows:
\[ E_i > \tau E_U \]
\[ E_i < \tau E_U \] (21)

2.4 Rectangular plane subpattern

The rectangular plane subpattern is a plane model, which is picked up from a rectangular region of the original gray-scale image $I$ to simulate the gray value vector of this region. The symbol for a rectangular plane subpattern is $\hat{p}(D, \hat{P})$, where $D$ is a rectangular region, and $\hat{P}$ is the least squares estimate of the plane parameter vector.

2.5 Encoding and decoding algorithms

NAMPD is a new representation method based on NAM, which is defined with the following characteristics:

(1) The original pattern $\Gamma$ is restricted to a gray-scale image $I$.
(2) There is only one subpattern, i.e., the rectangular plane subpattern in the set $P$.
(3) The affine transform $f_i$ is restricted to any combination of translation and scaling.
(4) The rectangular region of each subpattern in the image $I$ must be homogeneous.

The result of the encoding is given by following equation:
\[ \hat{p}(D, \hat{P}) = \bigcup_{i=0}^{N} p_i(D, \hat{P}) \] (22)
where $N$ is the number of the instances of rectangular plane subpatterns in $\Gamma'$, and $p_i(D, \hat{P})$ ($0 \leq i \leq N - 1$) represents the $i^{th}$ instance of rectangular plane subpattern in $\Gamma'$.

The following sections describe the encoding and decoding algorithms in detail.

2.5.1 Encoding algorithm

The main idea of the encoding algorithm for the NAMPD is antipacking a series of rectangular plane subpatterns, in which the height and the width are both even, from the gray-scale image $I$ and then storing them in a queue $Q$ of subpatterns.

Algorithm 1 Encoding part of NAMPD

Input A gray-scale image $I$.

Output A queue $Q$ of the subpatterns extracted from the image $I$.

Step 1 Empty the queue $Q$ and set all the pixels of the image $I$ unmarked.

Step 2 Search the first unmarked point $(x, y)$ from the image $I$ along the raster scan order. If there is no unmarked point in the image $I$, go to Step 7. Else, execute the following substeps sequentially.

Substep 1 Assign 2 to the variables $h$ and MaxHeight.

Substep 2 Assign 4 to the variables $w$ and MaxArea.

Substep 3 Repeat executing $A$, $B$, and $C$ until $x + w$ than the width of the image $I$.

A. If $h \times w \leq \text{MaxArea}$, go to $C$.

B. If the rectangular region $(x, y, h, w)$ in image $I$ satisfies the decision rule (21), assign $h \times w$ to the variable MaxArea and $h$ to the variable MaxHeight. Else, terminate the execution of the loop.

C. Increase the variable $w$ by 2.

Substep 4 If $w > 2$, then assign 2 to the variable $w$ and increase the variable $h$ by two. Else, go to Step 3.

Substep 5 If $y + h \leq$ the height of the image $I$, go back to Substep 3.

Step 3 Work out the rectangular plane subpattern $p(D, \hat{P})$, where $D$ is the rectangular region $(x, y, \text{MaxHeight}, \text{MaxArea} / \text{MaxHeight})$, and $\hat{P}$ is calculated according to Eq. (16).

Step 4 Mark all pixels of the rectangular region $D$ in image $I$.

Step 5 Add the subpattern $p$ to the queue $Q$.

Step 6 Repeat Step 2 to Step 5 until there is no unmarked point in image $I$.

Step 7 Output the queue $Q$.

2.5.2 Decoding algorithm

The decoding algorithm gets rectangular plane subpatterns from $Q$ orderly and sets the gray values of
the pixels on the image $I$ according to these subpatterns.

**Algorithm 2 Decoding part of NAMPD**

**Input** A queue $Q$ of subpatterns, the height $h$ and the width $w$ of original image.

**Output** A gray-scale image $I$ restored from the queue $Q$ of subpatterns.

**Step 1** Initialize an $h$-by-$w$ gray-scale image $I$, assign 0 to the variable $i$, and assign the size of the queue $Q$ to the constant $N$.

**Step 2** Assign the $i^{th}$ instance of rectangular plane subpatterns in the queue $Q$ to the variable $p$.

**Step 3** Work out the rectangular region $D$ and the gray value vector $\hat{S}$ of the subpattern $p$ according to Eq. (5).

**Step 4** Set the gray values of the pixels of the rectangular region $D$ in image $I$ with the gray value vector $\hat{S}$.

**Step 5** Increase the variable $i$ by one.

**Step 6** Repeat Step 2 to Step 6 until the variable $i$ equals the constant $N$.

**Step 7** Output the gray-scale image $I$.

3 A new fast edge detection algorithm based on NAMPD

The edge, which is defined by a discontinuity in gray-level values, is one of the most important features of visual information since it corresponds to discontinuities in the physical, photometrical, and geometrical properties of objects. Edge detection, which captures these significant features in the image, is the initial step in image analysis. The classical edge detection algorithms\cite{17} are based on pixel representation, which extracts edge information from pixels. In this paper, the proposed edge detection algorithm based on NAMPD extracts edge information directly from subpatterns instead of pixels. Before discussing our new edge detection algorithm, we first introduce some definitions.

3.1 Fundamentals

Without loss of generality, we assume that the ideal edge cuts a subpattern through the center of it and that the ideal edge within a subpattern has no curving part. Based on the ideal edge model shown in Fig. 1, we are interested in relating edge parameters, namely, edge strength $s$ and edge orientation $\theta$ of a rectangular plane subpattern.

![Ideal edge model](image)

**Definition 3** Given a rectangular plane subpattern $p(D, \hat{P})$, where $\hat{P} = [a, b, c]^T$ is the plane parameter vector, and the edge strength $s$ of $p$ is defined as follows:

$$s = b^2 + c^2. \quad (23)$$

**Definition 4** Given a rectangular plane subpattern $p(D, \hat{P})$, where $\hat{P} = [a, b, c]^T$ is the plane parameter vector, and the edge orientation $\theta$ of $p$ is defined as follows:

$$\theta = \begin{cases} \arctan \frac{b}{c}, & c \neq 0 \\ \frac{\pi}{2}, & c = 0 \end{cases} \quad (24)$$

3.2 Edge detection algorithm

The edge detection algorithm gets rectangular plane subpatterns from $Q$ orderly and works out the edge strength and orientation for each subpattern.

**Algorithm 3 Edge detection**

**Input** A queue $Q$ of the subpatterns.

**Output** A queue $E$ of the edge parameters.

**Step 1** Empty the queue $E$, assign 0 to the variable $i$, and assign the size of the queue $Q$ to the constant $N$.

**Step 2** Assign the $i^{th}$ instance of rectangular plane subpatterns in the queue $Q$ to the variable $p$.

**Step 3** Work out the edge strength $s$ and edge orientation $\theta$ of the subpattern $p$ according to Eq. (23) and Eq. (24), and set the edge parameter vector $e = (s, \theta)$.

**Step 4** Add the edge parameter vector $e$ to the queue $E$.

**Step 5** Increase the variable $i$ by one.

**Step 6** Repeat Step 2 to Step 5 until the variable $i$ equals $N$.

**Step 7** Output the queue $E$. 
3.3 Computational complexity analysis

Suppose that the number of subpatterns of an \(H\)-by-\(W\) gray-scale image \(I\) is \(N\) and that \(p(D, \hat{D})\) is a subpattern of the image \(I\), where \(D = (x, y, h, w)\) is a rectangular region in the image \(I\). It can be seen from Eq. (23) that the computation of edge strength for the subpattern \(p\) requires two multiplications and one addition using our proposed algorithm. The computation of edge strength for a pixel requires 25 multiplications and 24 additions using LoG operator\(^{19}\) and 18 multiplications and 16 additions using Sobel operator\(^{20}\). Table 1 summarizes the computational complexity of edge strength with different algorithms for the subpattern \(p\). It can be seen in Table 1 that the computational complexity of the algorithm based on NAMPD is much lower than that of LoG operator and Sobel operator for a subpattern. The comparison of the computational complexity of these algorithms for the image \(I\) is shown in Table 2. It should be noted that the computational complexity of our proposed edge detection algorithm is associated with the number of the subpatterns, because this algorithm extracts edge information directly from subpatterns (rectangular regions) instead of pixels. The computational complexity of edge orientation can be deduced in the same way.

### Table 1 Comparison of the computational complexity of computing edge strength using different algorithms for the subpattern \(p\)

| Algorithms | Multiplications | Additions |
|------------|-----------------|-----------|
| LoG        | \(25hw\)        | \(24hw\)  |
| Sobel      | \(18hw\)        | \(16hw\)  |
| NAMPD      | 2               | 1         |

### Table 2 Comparison of the computational complexity of computing edge strength using different algorithms for the image \(I\)

| Algorithms | Multiplications | Additions |
|------------|-----------------|-----------|
| LoG        | \(25HW\)        | \(24HW\)  |
| Sobel      | \(18HW\)        | \(16HW\)  |
| NAMPD      | \(2N\)          | \(N\)     |

4 Experimental results

To demonstrate the effectiveness of the edge detection algorithm based on NAMPDPD, we design two experiments to evaluate the experimental results of our algorithm, compared with LoG\(^{19}\) and Sobel\(^{20}\) algorithms. All experiments have been performed on a computer equipped with an Intel Pentium M 1.73GHz processor and 512MB RAM.

The first experiment has been performed using four test images, as shown in Fig. 2. All of them are gray-scale images with a size of \(512\times512\) at 8 bpp. These images have been chosen due to the consideration of their different features.

![Test images](image)

Fig. 2 Test images

These images represented by the NAMPD with the Peak Signal-to-Noise Ratio (PSNR) of 34.2 dB are shown in Fig. 3. Table 3 shows the comparison of experimental results, where \(N_{\text{NAMPD}}, ET_{\text{LoG}}, ET_{\text{Sobel}}, ET_{\text{NAMPD}}, \eta_{\text{NL}}, \text{ and } \eta_{\text{NS}}\) are the number of subpatterns, the execution time with LoG, the execution time with Sobel, the execution time with NAMPD, the speedup ratio of NAMPD to LoG, and the speedup ratio of NAMPD to Sobel. In Table 3, it is clear that our proposed algorithm performs from 18.7 to 20.2 times faster than LoG and from 17.6 to 19.0 times faster...
than Sobel, and it is also clear that $\eta_{NL}$ and $\eta_{NS}$ are both in reverse proportion to $N_{NAMPD}$. The results of edge detection are shown in Fig. 4, Fig. 5, and Fig. 6.

Table 3  Comparison of the performance among LoG, Sobel, and NAMPD

| Image | $N_{NAMPD}$ | $ET_{LoG}$ (ms) | $ET_{Sobel}$ (ms) | $ET_{NAMPD}$ (ms) | $\eta_{NL}$ | $\eta_{NS}$ |
|-------|-------------|-----------------|------------------|-------------------|------------|------------|
| F-16  | 17 143      | 318.23          | 298.83           | 17.01             | 18.7       | 17.6       |
| Orchids | 17 146      | 319.54          | 299.08           | 17.03             | 18.8       | 17.6       |
| Peppers | 16 911      | 319.23          | 300.89           | 16.86             | 18.9       | 17.8       |
| Lena  | 15 846      | 319.47          | 300.79           | 15.84             | 20.2       | 19.0       |

Fig. 4  Edge detection based on LoG

The second experiment has been performed to evaluate the accuracy of the edge detection algorithm based on NAMPDD. Two test images consisting of a $64 \times 64$ pixel array with a vertical oriented edge are shown in Fig. 7. A figure of merit $R$ is defined to evaluate the accuracy of edge detection [21].

$$R = \frac{1}{I_N} \sum_{i=1}^{I_N} \frac{1}{1 + \alpha d_i^2}$$  (25)

Fig. 5  Edge detection based on Sobel algorithm

where $I_N = \max\{I_I, I_A\}$, and $I_I$ and $I_A$ represent the number of ideal and actual edge points, $\alpha$ is a scaling factor, and $d_i$ is the separation distance of the $i$th actual edge point normal to a line of ideal edge points. The scaling factor may be adjusted to where the penalize edge is localized but offset from the true position. Table 4 shows the comparison of experimental results, where $R_{LoG}$, $R_{Sobel}$, and $R_{NAMPD}$ are the figure of merit of LoG, Sobel, and NAMPD, respectively. In Table 4, it can be seen that our proposed algorithm provides a performance comparable to LoG and Sobel algorithms.

Table 4  Comparison of the figure of merit among LoG, Sobel and NAMPD algorithms

| Image | $R_{LoG}$ | $R_{Sobel}$ | $R_{NAMPD}$ |
|-------|-----------|-------------|-------------|
| Edge 1 | 0.821     | 0.868       | 0.861       |
| Edge 2 | 0.852     | 0.938       | 0.935       |

5  Conclusion

In this paper, we have developed a new method for gray-scale image representation based on NAM, namely, NAMPD. Our proposed representation method can use a set of rectangular plane subpatterns to represent a gray-scale image. Also, a new edge
detection algorithm based on NAMPD was presented. The basic idea of our edge detection algorithm is to extract edge information directly from subpatterns instead of pixels. The theoretical analyses and experimental results show that the proposed edge detection algorithm performs much faster than the classical ones.

However, the research content about NAMPD is just a pioneer work. Currently, we are working to extend it to more image analysis, image classification, and pattern recognition algorithms.

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