Second-harmonic generation in focused beam fields of phased-array transducers in a nonlinear solid with a stress-free boundary

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Abstract

In nonlinear acoustic harmonic generation in solids with stress-free boundaries, such a boundary is known to destructively change the second harmonic generation, and the pulse-echo method is not practically applicable. Focused beams have often been used for fluid nonlinearity and biomechanical imaging in pulse-echo test setups. This paper considers the focused beam fields of linear phased-array transducers to ensure that pulse-echo harmonic generation can be applied to solids with stress-free boundaries. The fundamental and second-harmonic beam fields that are focused and reflected at the stress-free and rigid boundaries are calculated and their properties are investigated in terms of the received average fields. The phase difference between the two second-harmonic components after reflection from the boundary—that is, the reflected and the newly generated second harmonic—is emphasized. The phase difference is used to explain the improved and accumulated second harmonic observed in the simulation results.

Keywords: harmonic generation; stress-free boundary; phased-array transducer; beam focusing

1. Introduction

When monochromatic ultrasonic waves are transmitted in a material, the nonlinear elastic properties of the material distort the waveform and generate harmonics [1]. Nonlinear ultrasonic methods often measure these harmonics to obtain the nonlinear parameter $\beta$ defined by the amplitude ratio of the second harmonic to the fundamental, and draw conclusions about the material...
Nonlinear parameters are measured predominantly using the longitudinal wave in the transmission mode [2–4], which needs to approach both sides. Therefore, this technique may be limited in field measurement scenarios where only one surface of the component can be accessed.

The pulse-echo method, which enables the single-side access of the test component, provides a useful tool for practical applications of nonlinear ultrasonic measurement. Related theoretical and experimental studies are found in the fields of biomedical imaging [5, 6] and fluid nonlinearity [7, 8], where the rigid reflectors are considered. However, in the case of solid materials or structures, the issue with using the nonlinear harmonics reflected from the stress-free interface is that such a boundary destructively alters the nonlinear generation process, resulting in difficulty in obtaining reliable results for nonlinear parameters. Breazeale and Lester [9], and Bender et al. [10] have noted that in the pure plane-wave case, the harmonic component generated during forward propagation will theoretically diminish to zero on returning to its origin after reflection from the stress-free boundary. Although the oblique incidence reflection may not disappear when the reflected second harmonic reaches the other, the magnitude of the nonlinear wave is still very low [11]. According to theory and experimentation, when diffraction and attenuation are considered, a nonlinear wave reflected from the stress-free boundary can be detected for normal incidence. In principle, however, this method requires that the measured sample be thick enough and may reduce the accuracy for thin samples [12, 13].

The focused beam is considered to be a method of optimizing the pulse-echo nonlinear wave field with stress-free boundaries because such beams exhibit significantly different phase behaviour from plane waves [14]. According to our recent work, the phase difference between the newly generated and reflected second harmonics was found to be about \( \pi/2 \) after being reflected from the rigid boundary or the stress-free boundary [15]. Thus, the received second harmonic is greatly improved due to the reinforcing addition of these two components and is measurable in the pulse-echo configuration. Although the study was performed in fluids, where focused transducers are easy to employ, we can also use other methods that focus the beam in nonlinear ultrasonic testing of solids in the pulse-echo mode. Phased-array transducers can generate focused beams in solids and have been used to inspect microcracks [16].

Before fully utilizing the phased-array technique, it is necessary to figure out the focused-beam field generation and reflection at the stress-free boundary.

In this paper, we model the nonlinear acoustic beam fields generated by linear phased-array transducers in the pulse-echo setup. The fundamental and second-harmonic beam fields, which are focused and reflected at the stress-free and rigid boundaries, are calculated and their characteristics are examined with the received average fields. The phase difference between the two second-harmonic components after reflection from the boundary—that is, the reflected and the newly generated second harmonic—is emphasized. The phase difference is used to account for the improved and accumulated second harmonics observed in the simulation results. We also study the effect of element numbers used in the phased-array transducers.

2. Theory

We will try to model the pulse-echo nonlinear wave fields generated and received by a phased-array transducer, as shown in Fig. 1. In the forward propagation direction, the array transducer will radiate fundamental wave \( v_{p1} \) at the frequency \( f \), and there will be second-harmonic wave \( v_{p2} \) at the frequency \( 2f \) generated due to material nonlinearity. In the back-propagation direction after reflection from the boundary, \( v_{r1} \) denotes the reflected fundamental wave when the wave \( v_{p1} \) hits the boundary. \( v_{r21} \) is the second-harmonic wave newly generated by the reflected \( v_{r1} \) and \( v_{r22} \) is the reflected second-harmonic wave when \( v_{r1} \) hits the boundary. The total reflected wave of the second harmonic, \( v_{r2} \), will be obtained by adding \( v_{r21} \) and \( v_{r22} \). Both reflected fundamental and second-harmonic waves will be received by the array transducer. Note that in this model, the wave fields are described using particle velocity.

Nonlinear harmonic waves are produced due to material nonlinearity when the finite-amplitude wave radiating from a transducer propagates in solids. The related acoustic fields for a single-element circular transducer have been calculated before [17, 18], with the transducer modelled as a circular piston radiator mounted on a rigid baffle. In this section, we will briefly present the mathematical equations to calculate the received acoustic fields when a linear phased-array transducer composed of rectangular elements radiates the finite-amplitude longitudinal wave.
2.1 Propagation sound fields

The quasilinear solutions for the KZK-like equation yields a system of equations for \( v_{p1} \) and \( v_{p2} \). The Green’s function approach can then be used to solve these partial differential equations. Solutions of these equations are now obtained by integrating over the product of the Green’s function and appropriate source function to sum up the contributions from all source points. We thus obtain the nonlinear longitudinal wave fields in the sample in the forward propagation direction using the Rayleigh–Sommerfeld integral (RSI) method as follows:

\[
\begin{align*}
\nu_{p1}(x_1,y_1,z_1) &= -2i\kappa \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_1(x',y',0) \\
G_1(x,y,z|x',y',z) &= \int_{-\infty}^{z} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_1^2(x',y',z') \\
\nu_{p2}(x_1,y_1,z_1) &= \frac{2\beta v}{c} \int_{0}^{z} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_1^2(x',y',z') \nonumber \\
G_2(x,y,z|x',y',z') &= \int_{z}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_1^2(x',y',z')
\end{align*}
\]

where \( v \) is the acoustic particle velocity, \( c \) is the sound velocity of fundamental wave, and \( \beta = -\left(3 + \frac{\omega}{\rho c^2}\right) \), in which \( \rho \) is the density and \( c_{111} \) is the third-order elastic constant, is defined as the nonlinear parameter of solids [19]. The effect of attenuation on the nonlinear wave fields is neglected in this study. For a rectangular piston element with sides of lengths \( 2a_1, 2a_2 \) in the \( (x,y) \) directions, respectively, the source velocity field on the transducer face is given by

\[
\begin{align*}
\nu_1(x',y',z' = 0) &= v_0(x',y'), \quad |x/a_1| < 1, |y/a_2| < 1 \\
\nu_2(x',y',z' = 0) &= 0, \quad \text{otherwise}
\end{align*}
\]

Note that Equation (1) provides an exact solution to calculate the fundamental wave field, and represents the element radiation as a superposition of spherical waves radiating from point sources distributed on the plane \( z' = 0 \). When the second-harmonic wave field is calculated from Equation (2), the Green’s function used in this integral includes the contribution of the element \( dV = dx'dy'dz' \) of the virtual source formed by the primary field. When the initial sound sources are given, these equations can be used to obtain the propagation fundamental and second-harmonic wave fields.

2.2 Reflection sound fields

For application of the nonlinear wave equation to the reflected beam, the reflected beam is considered to propagate in the + z direction even in the back-propagation direction. To obtain the solution at the reflected field locations \( z < z_0 \), where \( z_0 \) is the thickness of the sample, we employ the coordinate transformation \( z \to (2z_0 - z) \). In the region \( z \geq z_0 \), the reflected beam fields can be described as

\[
\begin{align*}
\nu_{r1}(x_1,y_1,z_1) &= -2ik \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} R_1 \nu_{p1}(x',y',z = z_0) dx'dy' \\
G_1(x,y,z|x',y',z_0) &= dx'dy', \\
\nu_{r2}(x_1,y_1,z_1) &= \nu_{r21}(x_1,y_1,z_1) + \nu_{r22}(x_1,y_1,z_1) \\
&= -4ik \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} R_2 \nu_{p2}(x',y',z = z_0) dx'dy' + \\
&\frac{2\beta v}{c} \int_{z_0}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_1^2(x',y',z') \\
G_2(x,y,z|x',y',z') &= dx'dy'dz'
\end{align*}
\]

It can be found that the reflected fields are determined by the fundamental solutions \( \nu_{pn}, n = 1,2 \) at the interface, multiplied by the appropriate reflection coefficients \( R_n, n = 1,2 \). \( R_1 = -1 \) can be assumed for the stress-free boundary with normal incidence of the longitudinal wave. It is noted that the solution to the second harmonic of the reflected beam consists of two separate contributions. Equation (5) represents the reflected component of the second harmonic generated in the forward propagation direction. The second term is newly generated in the back-propagation direction by the reflected fundamental wave. When a wave is reflected at a large planar interface, the stationary phase method can be used to simplify
these expressions [20]. These reflected beams can be represented using the initial source as follows:

$$\begin{align*}
v_{r1}(x_1, y_1, z_1) &= R_1 \times -2ik \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_0(x', y', z = 0) G_1(x, y, z|x', y', 0) \, dx' \, dy', \\
v_{r2}(x_1, y_1, z_1) &= v_{r21}(x_1, y_1, z_1) + v_{r22}(x_1, y_1, z_1) \\
&= R_2 \times 2\beta k^2 \int_{z_0}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_{p1}^2(x', y', z') G_2(x, y, z|x', y', z') \, dx' \, dy' \, dz' + \\
&= R_2^2 \times 2\beta k^2 \int_{z_0}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} v_{p1}^2(x', y', z') G_2(x, y, z|x', y', z') \, dx' \, dy' \, dz'
\end{align*}$$

(6)

(7)

This simplification can improve the efficiency and accuracy of numerical calculations, as errors can be ignored in the discretization of reflecting boundaries.

3. Phased-array beam fields

3.1 Time delay

A linear phased-array transducer that emits a longitudinal wave is composed of a single-element source, and the radiation beam can be focused and steered by applying an appropriate delay set, called a delay law, to the elements of the array transducer [21].

In calculating a delay law for focusing and steering a linear array, a general method that does not depend on the number of elements will be used here to obtain the necessary delay times [22]. Consider an array of N elements radiating into a solid to produce a sound beam with a steering angle of \(\theta\) and a focusing distance of \(F\), as shown in Fig. 2. The focusing-time delays can be calculated as follows:

$$\Delta t_m = \frac{F}{c} \left\{ \left[ 1 + \left( \frac{Nd}{F} \right)^2 + \frac{2Nd}{F} \sin \theta \right]^{1/2} \right.\
- \left. \left[ 1 + \left( \frac{(m-N)d}{F} \right)^2 + \frac{2(m-N)d}{F} \sin \theta \right]^{1/2} \right\}$$

(8)

where \(\Delta t_m\) is the required time delay for element \(m = 0, 1, \ldots, N-1\), \(N = (N-1)/2\) and \(d\) is the centre-to-centre spacing between elements. Note that every calculated time has a positive value, which is a time delay.

The delay of the time-domain signal is equivalent to multiplying the frequency-domain signal by a phase term that is linear in frequency and proportional to its delay. If \(F(\omega)\) is the Fourier transform (FT) of the time-domain signal \(f(t)\), then the FT of the time-shifted signal \(f(t - \Delta t)\) can be obtained as \(\exp(i\omega \Delta t)F(\omega)\), where \(\Delta t\) is the delay time.

3.2 Received average fields and phase angles

When an element of the array transducer works as the receiver, the received average wave beams should be taken into account, and these values can be obtained with the following integral:

$$v_n(z) = \frac{1}{S_R} \int_{S_R} v_n(x, y, z) \, dS_R$$

(9)

where \(n = 1, 2\) denotes the fundamental and second-harmonic waves, and \(S_R\) is the area of the receiver element.

The received average field for the nth harmonic can be summed with appropriate reception delay times as

$$\hat{v}_n(z) = \sum_{m=1}^{N} v_{n,m}(z)e^{i\theta \Delta t_m}$$

(10)

where \(v_{n,m}(z)\) is the received signal at the mth element, \(\Delta t_m\) is the time delay applied to it and \(N\) denotes the total number of elements included in the summation.

The phases of the received average sound fields will be analysed in order to better understand the behaviours of the reflected sound beams. These phases are calculated by the function \(\phi_n(z) = \text{ATAN2}(\text{Im}(v_n(z)), \text{Re}(v_n(z)))\), \(n = 1, 2\). Note that the obtained phase angle is in the range \(-\pi\) to \(\pi\), but for the purposes of analysis, there will be 2\(\pi\) radians inserted whenever there is a jump of more than \(\pi\) radians to make them smoothly.
4. Simulation results and discussion

The propagating and reflected fundamental and second-harmonic wave fields were calculated numerically, and the received average results were then obtained. In the numerical simulations, the propagation medium was selected as Al-6061, with the sound speed $c = 6430 \text{ m/s}$, the nonlinear parameter $\beta = 5$ and the thickness $z_0 = 50 \text{ mm}$. The attenuations were neglected in this study. Each element radiated a fundamental wave of 5 MHz in frequency and $u_0 = 10^{-8} \text{ m}$ as a source displacement. The acoustic fields were expressed in terms of displacement with the use of relation $v_n = n\omega u_n, n = 1, 2$ between displacement and particle velocity, where $\omega$ is the angular frequency.

Two types of phased-array transducer were used in the simulation. The first was composed of four elements, where the element size was $e \times w = 3.2 \text{ mm} \times 10 \text{ mm}$ and the element gap was $g = 0.2 \text{ mm}$. The second was composed of 64 elements, where the element size was $e \times w = 0.5 \text{ mm} \times 10 \text{ mm}$ and the element gap was $g = 0.1 \text{ mm}$. The focal length was $F = 50 \text{ mm}$ and the steering angle was $\theta = 0^\circ$ for both transducers.

In the simulation, the initial phase of the drive signal for each element of the phased-array transducer was applied in accordance with the transmission-time delay law to focus the radiation beam at a specific focal distance. When calculating the received acoustic fields, the reception-time delay law was applied in the same way.

4.1 Four-element transducer results

When the four-element phased-array transducer was used, it was difficult to obtain a tightly focused sound beam. However, a weakly focused beam could be obtained, and the received average displacement of the fundamental wave increased after reflection from the boundary. In addition, depending on the boundary conditions, the phases of the reflected and newly generated second harmonics changed before and after reflection. Thus, the received second harmonic had the potential to exhibit a behaviour different from that of a plane wave or a single-element transducer.

Fig. 3 shows the two-dimensional beam profile of the four-element transducer for the fundamental and second harmonics with the stress-free ($R = -1$) and rigid ($R = 1$) boundary conditions. The boundary condition did not affect the calculated results of the fundamental wave (Fig. 3a).

The four-element phased-array transducer provided lightly focused motion for fundamental and secondary harmonics. Since the number of elements used was small, the fundamental wave seemed to focus at a shorter distance than the specific focal length of 0.05 m. However, this focusing behaviour was improved for the second-harmonic waves, as shown in Figs 3b and c.

The beam field of the second harmonic varied greatly depending on the boundary conditions used. In other words, the second-harmonic wave was concentrated mostly near the reflection boundary in the stress-free boundary case. However, strong wave fields for the rigid boundary case were also formed in the back-propagation domain. This difference affected the average displacement of the second harmonic received after its reflection from the boundary. The overall behaviour of the average displacements received for the fundamental and second harmonics is explained and discussed in greater detail below.

Fig. 4 shows the distribution of the received average displacement of the fundamental and second-harmonic waves for the four-element phased-array transducer.
Note that the received average displacement of the fundamental wave gradually increases in Fig. 4a for both boundary conditions. This behaviour compares well with the pulse-echo behaviour of a single-element transducer, where the received average displacement decreases [12]. This is due to the beam focusing of the phased-array transducer. Fig. 4b shows the received average displacement of the second harmonic. The figure includes all the reflected and newly generated second harmonics, and the total second harmonic received after its reflection from the rigid ($R = 1$) and stress-free ($R = -1$) boundaries.

In the back-propagation direction from the rigid boundary ($R = 1$), the total second-harmonic displacement became much larger than in the case of the stress-free boundary ($R = -1$). The rigid boundary ($R = 1$) provided a relatively strong second-harmonic accumulation, but the stress-free boundary ($R = -1$) provided a weakly cumulative behaviour. This difference can be explained through the phase analysis of two second-harmonic components received after the reflection boundary.

To better understand the above behaviour, we analysed the phase of the received average displacement, as shown in Fig. 5. This phase was calculated using the following function: $\phi_n(z) = \text{ATAN2} \left( \text{Im} \left( \tilde{u}_n(z) \right), \text{Re} \left( \tilde{u}_n(z) \right) \right), n = 1, 2$.

Fig. 5 shows the phase analysis results for the fundamental and second-harmonic waves. The phase of the reflected fundamental wave was continuous at the rigid boundary ($R = 1$). A phase difference of $\pi$ was observed before and after reflection at the stress-free boundary ($R = -1$). This means, as expected, the inverse reflection of the incoming fundamental wave at the stress-free boundary. Fig. 5b shows the phase analysis results for the second harmonic. Note that the phase of the reflected second harmonic was continuous at the rigid boundary ($R = 1$). It was found that the phase difference between the newly generated and reflected second harmonics was about $\pi/3$ for the rigid boundary ($R = 1$). On the other hand, the phase difference between these two components was about $4\pi/3$ for the stress-free boundary ($R = -1$). Thus, in both boundary conditions, the resulting second-harmonic wave was partially reinforced and a weakly accumulating total second-harmonic amplitude was obtained in the back-propagation direction after reflection (Fig. 5b).

Because of the large phase difference between the two second-harmonic components reflected from the rigid boundary ($R = 1$), their summed second-harmonic amplitude became much higher than in the stress-free boundary ($R = -1$) case, as shown in Fig. 5b.

### 4.2 Sixty-four-element transducer results

Fig. 6 shows the two-dimensional beam profile of the 64-element phased-array transducer for the fundamental and second harmonics with the stress-free ($R = -1$) and the rigid ($R = 1$) boundary conditions. The boundary condition did not affect the calculated results of the fundamental wave (Fig. 6a).

The 64-element phased-array transducer provided tightly focused motion for the fundamental and second harmonics. Since the number of elements used was large, the fundamental wave appeared to focus at the specified focal length
of 0.05 m. The focusing of the second harmonic also occurred at the correct position, as shown in Figs 6b and c, regardless of the boundary conditions used.

The beam field of the second harmonic did not depend heavily on the boundary conditions used, and showed almost the same behaviour. In other words, the second harmonic was concentrated mostly near the reflection boundary in all of the boundary conditions used. Similar focusing behaviour produced similar behaviour in the distribution of the received average displacement, as shown in Fig. 6b. Another point to note is that the transverse beam width of the second harmonic
Fig. 8. Phase analysis results for the 64-element transducer: (a) fundamental wave, and (b) second-harmonic wave was much narrower than that of the fundamental wave. This may be the main reason that the focused second harmonic is used preferentially in biomedical applications [7].

Fig. 7 shows the distribution of the received average displacement of the fundamental and second harmonics for the 64-element phased-array transducer. Note that the received average displacement of the fundamental wave sharply increases in Fig. 7a. This behaviour compares well with the behaviour of the four-element transducer, where the received average displacement gradually increased. This is due to the tight beam focusing of the 64-element phased-array transducer. Fig. 7b shows the received average displacement of the second harmonic. The figure includes all the reflected and newly generated second harmonics, and the total second harmonic received after its reflection from the rigid and stress-free boundaries. Both boundary conditions provided strongly cumulative second-harmonic displacements and their total second-harmonic displacements are almost the same in the back-propagation direction from each boundary. This kind of behaviour can be explained by the phase analysis of the two second-harmonic components received after the reflection boundary. In addition, it was observed that the received average displacements of both fundamental and second harmonics attained maximum values in the vicinity of $z = 2F$.

Fig. 8 shows the phase analysis results for the fundamental and second harmonics. The phase analysis results for the fundamental wave are presented in Fig. 8a, and show a behaviour similar to that of the four-element phased-array transducer shown in Fig. 4a, where the phase of the reflected fundamental wave was continuous at the stress-free boundary ($R = 1$) and a phase difference of $\pi$ was observed after reflection from the stress-free boundary ($R = -1$).

Fig. 8b shows the phase analysis results for the second harmonic. Note that the phase of the reflected second harmonic was continuous at the rigid boundary ($R = 1$). It was found that the phase difference between the newly generated and reflected second harmonics was about $\pi/2$ regardless of the boundary conditions used ($R = 1$ or $R = -1$). Thus, in both boundary conditions, the resulting second harmonic was partially reinforced and a strong accumulation of the total second-harmonic displacement was obtained in the back-propagation direction after reflection (Fig. 8b). Since the phase difference between the reflected and newly generated second-harmonic components was the same for both boundary conditions, their summed second-harmonic amplitudes were almost the same, as shown in Fig. 8b. In addition, the total second-harmonic generation was greatly improved, and its magnitude was at least one order higher than that of the four-element transducer (Fig. 8b). This was due to the tight focusing of the 64-element transducer. All of these trends in the 64-element transducer in solids are similar to those in a spherically focused transducer in fluids [15].

Conclusions

This paper used focused beams of ultrasonic phased-array transducers to investigate the non-linear harmonic generation of pulse-echo modes in solids with stress-free surfaces. A phased-array transducer consisting of 4 and 64 elements was considered with two different boundary conditions—stress-free and rigid boundaries—for com-
comparison purposes. The average displacement of the received fundamental and second harmonics was calculated when the focused radiation beam at the reflection boundary propagated back to the transducer. The phase difference between the two second-harmonic components after reflection from the boundary—that is, the reflected and newly generated second harmonics—described well the second-harmonic behavior observed in the simulation results. The focused beam of the 64-element phased-array probe produced a greatly improved and strongly accumulative second-harmonic generation. This is very similar to the sound-field characteristics of a second harmonic generated by a spherically focusing single-element transducer and can be used effectively in future for the pulse-echo nonlinearity testing of solid samples.

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