Inflation and Bounce from Classical and Loop Quantum Cosmology Imperfect Fluids

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We investigate how various inflationary and bouncing cosmologies can be realized by imperfect fluids with a generalized equation of state, in the context of both classical and loop quantum cosmology. With regards to the inflationary cosmologies, we study the intermediate inflation scenario, the $R^2$ inflation scenario and two constant-roll inflation scenarios and with regards to the bouncing cosmologies we study the matter bounce scenario, the singular bounce and the super bounce scenario. Within the context of the classical cosmology, we calculate the spectral index of the power spectrum of primordial curvature perturbations, the scalar-to-tensor ratio and the running of the spectral index and we compare the resulting picture with the Planck data. As we demonstrate, partial compatibility with the observational data is achieved in the imperfect fluid description, however none of the above scenarios is in full agreement with data. This result shows that although it is possible to realize various cosmological scenarios using different theoretical frameworks, it is not guaranteed that all the theoretical descriptions are viable.

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I. INTRODUCTION

Admittedly, one of the most fascinating observations the last ten years was the direct detection of gravitational waves, but as some of the most influential minds in gravitational waves admit, the gravitational waves were expected to be directly observed since their indirect observation back in the early 90’s. What was not expected at all in astrophysics and cosmology was the observation of late-time acceleration in the late 90’s [1]. This observation had set the stage for new research towards the consistent modelling of this late-time acceleration era of our Universe. It is conceivable that this late-time acceleration era cannot be solely expressed by a perfect matter fluid, and therefore this allows the use of more general cosmological fluids to be potential candidates for the consistent description of the late-time era, but also for other evolution eras of our Universe, like for example the early-time acceleration era. Also it is known that the effective equation of state of our Universe is believed to change at late-times, taking values very close to $-1$, with $w = -1$ being known as the phantom divide line. Obviously, such cosmic evolution needs quite general forms of dark fluids in order to consistently describe it. Nowadays, several cosmological frameworks make use of imperfect fluids [2,6], which in some cases are also viscous, and this description is an alternative to modified gravity cosmological frameworks [10–14]. Particularly, it is known that imperfect fluids can even describe a phantom evolution without the need for a phantom scalar to drive the evolution [2,4]. Also, it has been shown in Refs. [7,9] that early-time acceleration can also be described by imperfect fluids. Moreover in a recent work [15] it was demonstrated how singular inflationary cosmology can be realized by imperfect fluids.

The purpose of this work is to investigate how several inflationary and bouncing scenarios can be realized by imperfect fluids. We shall use two different theoretical frameworks, namely classical cosmology and Loop Quantum Cosmology (LQC) [16–20] (see [18,19] where the derivation of the Hamiltonian in LQC was firstly derived to yield the modified Friedmann equation, and also see [20] for a recent derivation of the effective Hamiltonian in LQC, which was derived by demanding repulsive gravity, as in Loop Quantum Gravity). In both cases we shall investigate which imperfect fluid can realize various inflationary and bouncing cosmology scenarios. The inflationary cosmology [21,29] and bouncing cosmology [30–45] are two alternative scenarios for our Universe evolution. In the case of inflation, the Universe starts from an initial singularity and accelerates at early times, while in the case of the bouncing cosmology, the Universe initially contracts until it reaches a minimum radius, and then it expands again. With regards to inflation, we shall be interested in four different inflationary scenarios, namely the intermediate inflation [46–49], the Starobinsky inflation [50,53], and two constant-roll inflation scenarios [62]. With regards to bouncing cosmologies, we shall be interested in realizing several well studied bouncing cosmologies, and particularly the matter bounce scenario [39,45], the superbounce scenario [54,55] and the singular bounce [56,58].

As we already mentioned we shall use two theoretical frameworks, that of classical cosmology and that of LQC. After presenting the reconstruction methods for realizing the various cosmologies with imperfect fluids, we proceed to the realization of the cosmologies by using the reconstruction methods. In the case of classical cosmology, we will calculate the power spectrum of primordial curvature perturbations, the scalar-to-tensor ratio and the running of the spectral index for all the aforementioned cosmologies, and we compare the results to the recent Planck data [59]. The main outcome of our work is that, although the cosmological scenarios we study in this paper are viable in
other modified gravity frameworks, these are not necessarily viable in all the alternative modified gravity descriptions. As we will demonstrate, in some cases the resulting imperfect fluid cosmologies are not compatible at all with the observational data, and in some other cases, there is partial compatibility.

We need to note that the perturbation aspects in LQC are not transparent enough and assume that there are no non-trivial quantum gravitational modifications arising due to presence of inhomogeneities. As it was shown in [63], a consistent Hamiltonian framework does not allow this assumption to be true. The perturbations issues that may arise in the context of the present work, are possibly more related to some early works in LQC [64], so any calculation of the primordial power spectrum should be addressed as we commented above.

The main results of this work are the following: firstly, in our attempt to realize the intermediate inflation scenario, the Starobinsky inflation scenario and two constant-roll inflation scenarios, by using imperfect fluids in the context of classical cosmology, only the Starobinsky model and one of the two constant-roll scenarios resulted to a spectral index of primordial curvature perturbations compatible with the Planck 2015 observational data. However in all cases the scalar-to-tensor ratio was incompatible with observations. Secondly, we performed the same analysis for bouncing cosmologies, and specifically for the matter bounce scenario, the superbounce and the singular bounce scenario. As we demonstrated, only the singular bounce yielded a spectral index compatible with the latest Planck data, but in this case too, the scalar-to-tensor ratio was not compatible with the data. In all cases, we found what is the imperfect fluid description in the context of LQC, however, no perturbation analysis was performed for the reasons we discussed earlier.

This paper is organized as follows: In section II we describe in some detail the reconstruction methods for imperfect fluid cosmology realization, in the cases of classical and LQC cosmology. In section III we present the various inflationary and bouncing cosmologies and we study how it is possible to realize these by classical and LQC imperfect fluids. Finally, the conclusions follow in the end of the paper.

II. COSMOLOGY WITH CLASSICAL AND LQC IMPERFECT FLUIDS: GENERAL FORMALISM

In this section we shall briefly present the theoretical framework of imperfect fluid cosmology, both in the classical and the LQC cases. We assume that the geometric background is a Friedmann-Robertson-Walker metric of the form,

$$\text{d}s^2 = -\text{d}t^2 + a^2(t) \sum \text{d}x_i^2,$$

where $a(t)$ denotes as usual the scale factor. The LQC case is an extension of the classical description, in which case the classical equations of motion are modified by using the LQC effective Hamiltonian, which in effect modifies the Friedmann equations. In this section we describe in some detail how the LQC imperfect fluid framework is constructed, and also we demonstrate that it can lead to the classical imperfect fluid theory, if the classical limit is taken.

A. Classical Imperfect Fluids

We start of our analysis with the classical viscous fluid case, and for a detailed analysis the reader is referred to [2, 4–9]. It is conceivable that the imperfect fluid cosmology is a limiting case of the viscous fluid case, and we show this explicitly now. Consider a viscous fluid with effective energy density $\rho$ and effective pressure $p$, which are related to each other as follows,

$$p = F(\rho) - B(a(t), H, \dot{H}, ...),$$

where the function $B(a(t), H, \dot{H}, ...)$ stands for the bulk viscosity, and as it can be seen it is a function of the scale factor, the Hubble rate and of the higher derivatives of the Hubble rate. The function $F(\rho)$ is the homogeneous part of the effective equation of state (EoS) given in Eq. (2), and the inhomogeneity in the EoS is introduced by the bulk viscosity, which needs to be positive in order for the entropy change to have a positive sign during irreversible processes [9]. The FRW equations for the metric (1) are,

$$\rho = \frac{3}{\kappa^2} H^2, \quad p = -\frac{1}{\kappa^2} \left( 3H^2 + 2\dot{H} \right),$$

The energy momentum tensor corresponding to the viscous fluid with EoS (2) is given below,

$$T_{\mu\nu} = \rho u_\mu u_\nu + \left( F(\rho) + B(H, \dot{H}, ...) \right) \left[ g_{\mu\nu} + u_\mu u_\nu \right],$$

(4)
where \( u_\mu \) is the comoving four velocity, and its specific form for the metric (1) is \( u_\mu = (1, 0, 0, 0) \). The conservation of the energy and momentum, results to the following continuity equation for the effective energy density \( \rho \),

\[
\dot{\rho} + 3H(\rho + F(\rho)) = 3HB(H, \dot{H}, ...).
\]

For the purposes of this paper, we shall consider cosmological evolutions which can be generated by a homogeneous imperfect fluid, in which case the EoS will be of the form \( p = F(\rho) \), and as we will demonstrate, the function \( F(\rho) \) will be of the form \( F(\rho) = -\rho - f(\rho) \), and only in some LQC cases, it will have a general form \( F(\rho) \). Given a cosmological evolution in terms of its Hubble rate, we can find in a straightforward way the EoS of the imperfect homogeneous or viscous fluid that can generate such a cosmological evolution. From the FRW equations (3), the effective energy density and effective pressure of the fluid can be written as functions of the cosmic time in the following way,

\[
\rho = f_\rho(t), \quad p = f_p(t).
\]

Hence, if the first equation can be solved with respect to the cosmic time \( t \), we get \( t = f_\rho^{-1}(\rho) \), so by substituting this in the second equation of Eq. (6), the effective pressure can be expressed as a function of the effective energy density as

\[
p = f_p(f_\rho^{-1}(\rho)).
\]

In the case we just described we assumed that the resulting function \( f_p \circ f_\rho^{-1} \) depends solely on the effective energy density \( \rho \), but this happens only in the imperfect fluid case. In general, the resulting function might also contain the scale factor and the Hubble rate, and this can happen for example in the cases that the scale factor or the Hubble rate are non-invertible functions of the cosmic time. In this paper however we focus on non-viscous imperfect fluid generated cosmologies, so we will not have deal with these issues.

### B. Loop Quantum Cosmology Imperfect Fluids

The Loop Quantum Cosmology imperfect fluid formalism is an extension of the classical formalism in the case that the Friedmann equations are modified by taking into account holonomy effects. In the context of LQC, the effective Hamiltonian which describes the Universe is \([16–20]\),

\[
\mathcal{H}_{LQC} = -3V \frac{\sin^2(\lambda \beta)}{\gamma^2 \lambda^2} + V \rho,
\]

with \( \gamma \) and \( \lambda \) being the Barbero-Immirzi parameter and an arbitrary parameter with dimensions of length respectively. In addition, \( V \) is the total volume of the Universe, which is \( V = a(t)^3 \), and \( \rho \) is the effective energy density of the matter fluid present, which in our case is the imperfect fluid. Using the Hamiltonian constraint \( \mathcal{H}_{LQC} = 0 \), we obtain the following equation,

\[
\frac{\sin^2(\lambda \beta)}{\gamma^2 \lambda^2} = \frac{\rho}{3},
\]

and in conjunction with the following anticommutation identity

\[
\dot{V} = \{V, \mathcal{H}_{LQC}\} = -\gamma \frac{\partial \mathcal{H}_{LQC}}{\partial \beta},
\]

we obtain the Friedmann equation in the case of LQC \([16–20]\),

\[
H^2 = \frac{\kappa^2 \rho}{3} \left(1 - \frac{\rho}{\rho_c}\right).
\]

The parameter \( \rho_c \) encompasses all the quantum effects of the theory, and the classical limit of the Friedmann equation (10) can be obtained if \( \rho_c \) tends to infinity. The effective energy density of the imperfect fluid satisfies the continuity equation,

\[
\dot{\rho}(t) = -3H \left(\rho(t) + p(t)\right),
\]

with \( p(t) \) denoting as usual the effective pressure. Differentiating the holonomy corrected Friedmann equation (10), and using the continuity equation (11), we obtain,

\[
\dot{H} = -\frac{\kappa^2}{2}(\rho + p)(1 - 2\frac{\rho}{\rho_c}).
\]
The above equation can be solved with respect to the pressure \( p(t) \), which reads,

$$ p(t) = -\rho - \frac{2\dot{H}}{\kappa^2(1 - \frac{2}{\rho_c})}, $$ (13)

A crucial feature of the LQC formalism is that in the limit \( \rho_c \to \infty \), the Friedmann equations (10) and (12) yield the classical Friedmann equations. Indeed, in the limit \( \rho_c \to \infty \), the equations (10) and (13) become,

$$ \dot{H} = -\frac{\kappa^2}{2}(\rho + p), \quad H^2 = \frac{\kappa^2 \rho}{3}, $$ (14)

which are identical to the classical equations (3). Having equations (10) and (13) at hand, and also given the Hubble rate of a specific cosmological evolution, we can easily find which imperfect fluid can generate such an evolution. An important feature is that in the limit \( \rho_c \to \infty \) the resulting EoS of the LQC imperfect fluid must be identical to the corresponding classical EoS which generates the same cosmological evolution. As we will demonstrate in the following sections, this indeed happens in all the examples we shall present. In order to find the LQC EoS of the imperfect fluid, it is necessary to find the explicit functional dependence of the energy density as a function of time. By solving the Friedmann equation (10) with respect to \( \rho \) we obtain,

$$ \rho = \frac{\kappa^2 \rho_c \pm \sqrt{-12\dot{H}^2 \kappa^2 \rho_c + \kappa^4 \rho_c^2}}{2\kappa^2}, $$ (15)

so there are two solutions, so this point is crucial in order to choose the correct one. In order to do so, we can use the feature that the loop quantum corrected quantities should be identical to the classical ones in the limit \( \rho_c \to \infty \).

The classical expression of the EoS \( w_{\text{eff}} = p/\rho \) as a function of the cosmic time is,

$$ w_{\text{eff}} = -1 - \frac{2\dot{H}}{3H^2}, $$ (16)

so the loop quantum corrected expression should be identical to this. The loop quantum corrected EoS as a function of time is,

$$ w_{\text{eff}} = -1 - \frac{\dot{H}}{3H^2} \pm \frac{\rho_c \dot{H}}{3H^2 \sqrt{\rho_c (\rho_c - 12\dot{H}^2)}}, $$ (17)

and therefore, in order for the expressions (14) and (16) to be identical in the limit \( \rho_c \to \infty \), the energy density as a function of the cosmic time should be,

$$ \rho = \frac{\kappa^2 \rho_c - \sqrt{-12\dot{H}^2 \kappa^2 \rho_c + \kappa^4 \rho_c^2}}{2\kappa^2}. $$ (18)

Noticing the functional form of the effective pressure \( p \), we can see that the only term not being expressed as a function of the energy density \( \rho \), is the derivative of the Hubble rate \( \dot{H} \). So by finding this derivative as a function of the cosmic time \( t \), we can have the resulting EoS as a function of \( \rho \), and this can easily be done by inverting Eq. (18), and this finding \( t = t(\rho) \). It is conceivable however that the case for which the function \( \rho(t) \) is invertible corresponds to the imperfect fluid scenario, and if it is not invertible the fluid has bulk viscosity since the function \( \dot{H} \) cannot be expressed as an explicit function of the energy density \( \rho \). Practically, the equations (18) and (13) will enable us to find the EoS of the imperfect fluid that generates a given cosmological evolution. In the next sections we shall investigate which imperfect fluids can generate various inflationary and bouncing cosmologies.

Before we close this section we need to discuss an important issue, with regard to the EoS in LQC. Particularly, an important property which is needed in the analysis of LQC imperfect fluids, is that the EoS does not change when one goes to LQC. This is an issue that requires special attention, as it was discussed in [65]. As it was discussed in [65], the assumption we made about the EoS, is true only when inverse volume modifications are ignored. If only holonomy modifications are present, only then there is no change in the EoS.

### III. INFLATIONARY AND BOUNCE COSMOLOGY MODELS FROM CLASSICAL AND LQC IMPERFECT FLUIDS

#### A. Inflationary Models

In this section we shall employ the methods we described in the previous two sections in order to realize some well-known inflationary cosmologies. The first is the nearly \( R^2 \) inflationary evolution which is a quasi-de Sitter cosmology
in which case the Hubble rate and the scale factor read,
\[ H(t) \simeq H_i - \frac{M^2}{6} (t - t_i), \quad a(t) = e^{H_i (t - t_i)^2}. \] (19)

where \( t_i \) is the time instance that inflation starts and in addition \( H_i \) is the Hubble rate at \( t = t_i \). It can be shown that in the context of \( F(R) \) gravity, the cosmological evolution can be realized by the \( F(R) \) gravity of the form
\[ F(R) = R + \frac{1}{6M^2} R^2, \]
and due to the fact that during the inflationary phase, the terms \( \ddot{H} \) and \( \dot{H} \) are neglected. Another inflationary solution we shall present and study is the so-called intermediate inflation scenario \[46–49\], in which case the scale factor is,
\[ a(t) = e^{At}, \] (21)

with \( A \) being a constant parameter and \( 0 < f < 1 \). Let us see how these two inflationary cosmologies can be realized by classical and loop quantum cosmology imperfect fluids. We start off with the classical picture, in which case we shall calculate the slow-roll indices and the resulting observational inflation indices.

1. Classical Imperfect Fluid Description

We start our analysis with the intermediate inflation cosmology \[21\], and we shall assume that the EoS of the imperfect fluid that realizes this cosmology is of the form,
\[ p = -\rho - f(\rho), \] (22)
so our task is to find the analytic form of the function \( f(\rho) \). The classical Friedmann equations \[43\] for the intermediate inflation yield,
\[ \rho = \frac{3A^2 f^2 t^{-2+2f}}{\kappa^2}, \] (23)

so by solving with respect to \( t \) and substituting in the expression for the pressure, the pressure as a function of \( \rho \) is,
\[ p = -\rho - B \rho^{\frac{-2+2f}{2+2f}}, \] (24)

and therefore, the function \( f(\rho) \) is equal to \( f(\rho) = B \rho^{\frac{-2+2f}{2+2f}} \), where we set \( B \) equal to,
\[ B = 2 \frac{3^{\frac{-2+2f}{2+2f}}} {2^{\frac{-2+2f}{2+2f}}} A \frac{1}{\kappa^2} (-1 + f) \frac{1}{\kappa^2} \frac{1}{\kappa^2}. \] (25)

By repeating the same procedure for the quasi-de Sitter cosmology of Eq. \[19\], the resulting EoS of the imperfect fluid that realizes this cosmology has a very simple form, which is,
\[ p = -\rho + \frac{M^2}{3\kappa^2}, \] (26)

so in this case the function \( f(\rho) \) is constant, that is, \( f(\rho) = -\frac{M^2}{3\kappa^2} \). For the classical imperfect fluid cosmology we can calculate the inflationary observational indices, and particularly the spectral index of primordial curvature perturbations \( n_s \), the scalar-to-tensor ratio \( r \) and the running of the spectral index \( \alpha_s \). We shall use the formalism
of Ref. 3, and as it was shown, the indices in terms of the function $f(\rho(N))$ and $\rho$, are equal to,

$$
\begin{align*}
  n_s - 1 &= -9\rho(N) f(\rho(N)) \left( \frac{\dot{f}(\rho(N)) - 2}{2\rho(N) - f(\rho(N))} \right)^2 + \frac{6\rho(N)}{2\rho(N) - f(\rho(N))} \left\{ \frac{f(\rho(N))}{\rho(N)} \right\} \\
  &+ \frac{1}{2} \left( f'(\rho(N)) \right)^2 + f'(\rho(N)) - \frac{5 f(\rho(N)) f'(\rho(N))}{\rho(N)} + \left( \frac{f(\rho)}{\rho(N)} \right)^2 + \frac{1}{3} f'(\rho(N)) \\
  &\times \left[ \left( f'(\rho(N)) \right)^2 + f'(\rho(N)) f''(\rho(N)) - 2 \frac{f(\rho(N)) f'(\rho(N))}{\rho(N)} + \left( \frac{f(\rho(N))}{\rho(N)} \right)^2 \right],
\end{align*}
$$

where $N$ is the e-folding number and the exact form of the functions $J_1$ and $J_2$ can be found in the Appendix. The latest Planck data 59 suggest that the values of the observational indices are constrained as follows,

$$
n_s = 0.9644 \pm 0.0049, \quad r < 0.10, \quad \alpha_s = -0.0057 \pm 0.0071,
$$

so we shall compare the observational data to values of the observational indices for the two imperfect fluids cases we found earlier. The calculation can be simplified in the case that the function $f(\rho(N))$ satisfies,

$$
\frac{f(\rho)}{\rho} \ll 1,
$$

a constraint that can be satisfied by both cases we presented earlier. In this case, the observational inflationary indices are simplified as follows 3,

$$
n_s \simeq 1 - 6 \frac{f(\rho)}{\rho(N)}, \quad r \simeq 24 \frac{f(\rho)}{\rho(N)}, \quad \alpha_s = -9 \left( \frac{f(\rho)}{\rho(N)} \right)^2.
$$

Having these at hand we can directly calculate the observational indices, which for the imperfect fluid EoS of Eq. 23 are equal to,

$$
n_s \simeq \frac{4 + f(-4 + N)}{fN}, \quad r \simeq \frac{16(-1 + f)}{fN}, \quad \alpha_s = -4 \frac{(1 - f)^2}{f^2 N^2}
$$

where we used the fact that the function $\rho(N)$ in the case of the intermediate inflation is equal to,

$$
\rho(N) = 3 \frac{A^2 N^{2+2f}}{f^2 N^{2+2f}}.
$$

By using the values $(N, f) = (50, 0.9)$ (recall that $0 < f < 1$), the observational indices become,

$$
n_s \simeq 1.00889, \quad r \simeq -0.0355556, \quad \alpha_s \simeq -0.0000197531,
$$

and as it can be seen the values do not satisfy any of the Planck constraints 30. Equivalently, for the quasi-de Sitter case, the observational indices are,

$$
n_s \simeq 1 + \frac{2M^2}{3H_s^2 - M^2 N + H_i M^2 t_i}, \quad r \simeq \frac{8M^2}{3H_s^2 - M^2 N + H_i M^2 t_i}, \quad \alpha_s \simeq \frac{M^4}{(3H_s^2 - M^2 N + H_i M^2 t_i)^2},
$$

so by using the values $(N, H_i, t_i, M) = (60, 10^{10}, 10^{-60}, 10^{10.5})$, the observational indices become,

$$
n_s \simeq 0.966499, \quad r \simeq 0.134003, \quad \alpha_s \simeq -0.000280577,
$$

and as it can be seen the values do not satisfy any of the Planck constraints 30.
and it can be seen that only the spectral index of primordial curvature perturbations agrees with the Planck data.

Another interesting inflationary scenario was studied in Ref. [62], and it involves some canonical scalar field models with a constant rate of roll, to which we refer to as “constant-roll inflationary models”. These models are phenomenologically interesting since these are similar to some well known inflationary scenarios in the literature. Particularly, one of these models is similar to the solution found in [60], while the other one is similar to hilltop inflation [62]. In addition, these models have appealing observational features, since the observational indices in some cases are compatible with observational data. The main assumption of these scalar models is that there is a constant rate of roll in which case the following relation holds true $\dot{\phi}/H\dot{\phi} = -3 - \alpha$, where the parameter $\alpha$ measures the deviation from the flat potential. We shall not describe in detail all the features of the models, since the theoretical framework is not related directly to our work, so we just use the resulting cosmological evolutions, which we shall realize by using classical and quantum imperfect fluids. Details on the scalar models we present here can be found in [62]. The first inflationary model found in [62], has the following scale factor,

$$a(t) = \sinh^{1/(3+\alpha)}((3 + \alpha)Mt),$$

where the parameter $\alpha > -3$ and also $M$ is an integration constant which determines the amplitude of the power spectrum of curvature perturbations. The second inflationary solution has the following scale factor [62],

$$a(t) = \cosh^{-1/(3+\alpha)}((3 + \alpha)Mt),$$

where in this case $\alpha < -3$. Using the imperfect fluid reconstruction method we described earlier, the resulting EoS which realizes the cosmology [38] is equal to,

$$p = -\rho - \frac{6M^2}{\kappa^2} - \frac{2M^2\alpha}{\kappa^2} + 2\rho + \frac{2\alpha\rho}{3}.$$  

We can calculate the observational indices for this case, and their analytic form is,

$$n_s \approx 13 + 4\alpha - \frac{12M^3(3 + \alpha)^2}{\kappa^2 \text{arcsin} (e^{(3+\alpha)M})},$$

$$r \approx \frac{16(3 + \alpha)(3M^3(3 + \alpha) - \kappa^2 \text{arcsin} (e^{(3+\alpha)M}))}{\kappa^2 \text{arcsin} (e^{(3+\alpha)M})},$$

$$\alpha_s \approx -\frac{4(3 + \alpha)^2(-3M^3(3 + \alpha) + \kappa^2 \text{arcsin} (e^{(3+\alpha)M}))^2}{\kappa^4 \text{arcsin} (e^{(3+\alpha)M})^2},$$

so by using $\alpha = -2.99$, $M \sim \kappa$ and $N = 50$, the values of the observational indices are,

$$n_s \approx 0.966913, \quad r \approx 0.132348, \quad \alpha_s \approx -0.000273689.$$  

At is can be seen, only the spectral index of primordial curvature perturbations is compatible with the Planck data, and this picture occurs by choosing other values of the parameters. Notice that as $\alpha$ approaches zero, the spectrum loses its red tilt and approaches the scale invariant value $n_s = 1$. With regards to the second model [39], the EoS which realizes this cosmological evolution is,

$$p = -\rho + \frac{6M^2}{\kappa^2} + \frac{2M^2\alpha}{\kappa^2} - \left(2 + \frac{2\alpha}{3}\right)\rho.$$  

The analytic expressions of the resulting observational indices for this case are,

$$n_s \approx -11 - 4\alpha + \frac{12M^3(3 + \alpha)^2}{\kappa^2 \text{arccosh} (e^{(3+\alpha)M})},$$

$$r \approx -\frac{16(3 + \alpha)(3M^3(3 + \alpha) - \kappa^2 \text{arccosh} (e^{-N(3+\alpha)})]}{\kappa^2 \text{arccosh} (e^{-N(3+\alpha)})},$$

$$\alpha_s \approx -\frac{4(3 + \alpha)^2(-3M^3(3 + \alpha) + \kappa^2 \text{arccosh} (e^{-N(3+\alpha)})^2}{\kappa^4 \text{arccosh} (e^{-N(3+\alpha)})^2},$$

so by using $\alpha = -3.00000001$, $M/\kappa \approx 10^{-2}$ and $N = 60$, the values of the observational indices are,

$$n_s \approx 1, \quad r \approx -1.60014 \times 10^{-14}, \quad \alpha_s \approx -4.00071 \times 10^{-24},$$

hence the resulting spectrum is exactly scale invariant.

In conclusion, in this section we have shown that many interesting inflationary scenarios coming from various theoretical contexts, can be realized by classical cosmological imperfect fluids, however even though these scenarios were originally viable in the context of their theoretical frameworks, their viability is not guaranteed in the imperfect fluid description. Hence, this shows that even though there can be many theoretical realizations of various cosmologies, the observational validity of the cosmological evolution cannot be guaranteed for all the theoretical modified gravity descriptions. In Table V we gathered all the results of this section, and we now proceed to the LQC imperfect fluid description of these inflationary cosmologies.
so by inverting this, the function \( t = t(p) \) is equal to,

\[
\rho(t) = \frac{\rho_c}{2} - \frac{-12A^2f^2t^{-2+2f}k^2\rho_c + \kappa^4\rho_c^2}{2\kappa^2},
\]

so by inverting this, the function \( t = t(p) \) is equal to,

\[
t(p) = 12^{\frac{1}{2-2f}} \left( \frac{-4k^4\rho^2 + 4k^4\rho_c^2}{A^2f^2k^2\rho_c} \right)^{-\frac{1}{2-2f}}
\]

and therefore by substituting this to the effective pressure of Eq. (13), the resulting imperfect fluid EoS is equal to,

\[
p = -\rho - \frac{2.3^{-2+2f}A(-1+f)\rho_c\left(\frac{k^2\rho(\rho+c)}{A^2f^2\rho_c}\right)^{-\frac{1}{2-2f}}}{\kappa^2(-2p+c)}.
\]

An important feature that will validate our results is that the EoS of Eq. (18) should be identical to the classical EoS given in Eq. (21), in the limit \( \rho_c \to \infty \), and as it can be seen, the two expressions (18) and (21) are identical in the limit \( \rho_c \to \infty \). The rest of the LQC imperfect fluid equations of state that realize the inflationary cosmologies of Eqs. (19), (28) and (39), can be found in Table II By looking the resulting expressions for the equations of state appearing in Table II these are identical to the corresponding ones in Table I when the limit \( \rho_c \to \infty \) is taken.

### 2. LQC Imperfect Fluid Description

In the case of LQC imperfect fluids, the procedure to obtain the EoS of the imperfect fluids that realize the inflationary cosmology we studied in the previous section is straightforward. So we study one of the cases in detail and the rest of the results can be found in Table III Consider for example the intermediate inflation case (21), and by applying the LQC reconstruction method we presented in a previous section, we easily find that the function \( \rho(t) \) given in Eq. (18) is equal to,

\[
\rho(t) = \frac{\rho_c}{2} - \frac{-12A^2f^2t^{-2+2f}k^2\rho_c + \kappa^4\rho_c^2}{2\kappa^2},
\]

so by inverting this, the function \( t = t(p) \) is equal to,

\[
t(p) = 12^{\frac{1}{2-2f}} \left( \frac{-4k^4\rho^2 + 4k^4\rho_c^2}{A^2f^2k^2\rho_c} \right)^{-\frac{1}{2-2f}}
\]

and therefore by substituting this to the effective pressure of Eq. (13), the resulting imperfect fluid EoS is equal to,

\[
p = -\rho - \frac{2.3^{-2+2f}A(-1+f)\rho_c\left(\frac{k^2\rho(\rho+c)}{A^2f^2\rho_c}\right)^{-\frac{1}{2-2f}}}{\kappa^2(-2p+c)}.
\]

An important feature that will validate our results is that the EoS of Eq. (18) should be identical to the classical EoS given in Eq. (21), in the limit \( \rho_c \to \infty \), and as it can be seen, the two expressions (18) and (21) are identical in the limit \( \rho_c \to \infty \). The rest of the LQC imperfect fluid equations of state that realize the inflationary cosmologies of Eqs. (19), (28) and (39), can be found in Table II By looking the resulting expressions for the equations of state appearing in Table II these are identical to the corresponding ones in Table I when the limit \( \rho_c \to \infty \) is taken.

### B. Bouncing Models from Classical and LQC Imperfect Fluids

The big bounce cosmology offers an appealing and promising alternative to the inflationary paradigm, since in the big bounce case the initial singularity is avoided. This is because the Universe never reaches the size zero, so effectively all crushing types of singularities are avoided. In this section we shall investigate how various bouncing models can be realized from classical and LQC imperfect fluids. In the case of classical imperfect fluids, we shall also check the viability of the resulting cosmology by calculating the observational indices. The models we shall investigate are: the well known matter bounce scenario [39–45], the superbounce scenario [54, 55], and finally the singular bounce [56–58]. The matter bounce scenario and the singular bounce scenario have quite appealing observational properties in the
context of modified gravity and LQC, and one of the aims of this section is to directly check whether the observational viability of these models still occurs even in the context of imperfect fluid cosmology.

We start off the analysis with the matter bounce scenario, in which case the scale factor is,
\[
a(t) = \left(\frac{3}{4} \kappa^2 \mu^2 + 1\right)^{\frac{1}{3}},
\]
where \(\mu\) is a mass scale. Employing the imperfect fluid reconstruction techniques we described in the previous sections, the resulting classical imperfect fluid EoS that realizes the matter bounce cosmology is,
\[
p = \frac{1}{2} \left(\mu + \frac{\sqrt{\kappa^2 \mu^2 (\mu - 4 \rho)}}{\kappa^2 \mu} - 4 \rho\right).
\]
Accordingly, a direct calculation of the observational indices yields,
\[
n_s \approx 1 + \frac{9}{2} N^2 \kappa^2 \mu, \quad r \approx -18 N^2 \kappa^2 \mu, \quad \alpha_s \approx -\frac{16 - 4 \kappa - 3 N^2 \kappa^3 \mu + 9 N^4 \kappa^4 \mu^2}{64 N^2 \kappa^3 \mu^2}.
\]
By looking the functional form of the observational indices, we can easily conclude that the resulting cosmology is not viable, since there is no way that the power spectrum can be scale invariant or even nearly scale invariant.

The second bouncing cosmology we shall investigate is the superbounce cosmology \([54, 55]\), in which case the scale factor is,
\[
a(t) = (t - t_s)^{2/c^2},
\]
where \(c\) is a parameter which satisfies \(c > \sqrt{6}\) and \(t_s\) the bouncing time instance. Using the imperfect fluid reconstruction method we described earlier, the resulting EoS which realizes the cosmology \((52)\) is very simple and has the following functional form,
\[
p = -\rho + \frac{c^2 \rho}{3}.
\]
We can calculate the observational indices for this case, and their analytic form is,
\[
n_s \approx 1 + 2c^2, \quad r \approx -8c^2, \quad \alpha_s \approx -c^4.
\]
Hence, in this case too, the observational features of the superbounce cosmology in the context of imperfect fluids are not appealing at all, since no scale invariant power spectrum occurs.

The most interesting case for bouncing cosmologies in the context of imperfect fluids is the singular bounce evolution, in which case the scale factor is,
\[
a(t) = e^{(t-t_s)\alpha},
\]
where \(\alpha > 1\) and \(t = t_s\) is the bouncing point which coincides with the singular point. The singular bounce is called singular because at the bouncing point a Type IV singularity occurs \([61]\). Before proceeding in the imperfect fluid realization of this cosmology, it is worth noticing that the singular bounce and the intermediate inflation scenario look quite similar, at least their function forms. However, the difference is that the singular bounce starts with a contraction, and the parameter \(\alpha\) has to be \(\alpha = 2n/(2m + 1)\), in order to avoid complex values in the scale factor and Hubble rate. In the intermediate inflation case, the evolution starts at \(t = 0\) and the initial singularity is a Type II

\[\begin{array}{|c|c|}
\hline
\text{Bouncing Model} & \text{Classical Imperfect Fluid Equation of State} \\
\hline
\hline
a(t) = \left(\frac{3}{4} \kappa^2 \mu^2 + 1\right)^{\frac{1}{3}} & p = \frac{1}{2} \left(\mu + \frac{\sqrt{\kappa^2 \mu^2 (\mu - 4 \rho)}}{\kappa^2 \mu} - 4 \rho\right) \\
\hline
a(t) = (t - t_s)^{2/c^2} & p = -\rho + \frac{c^2 \rho}{3} \\
\hline
a(t) = e^{(t-t_s)\alpha} & n_s \approx 1 + \frac{9}{2} N^2 \kappa^2 \mu, \quad r \approx -18 N^2 \kappa^2 \mu, \quad \alpha_s \approx -\frac{16 - 4 \kappa - 3 N^2 \kappa^3 \mu + 9 N^4 \kappa^4 \mu^2}{64 N^2 \kappa^3 \mu^2}.
\end{array}\]
TABLE IV: The LQC Imperfect Fluid Equations of State for Various Bouncing Cosmologies

| Bouncing Model | LQC Imperfect Fluid Equation of State |
|----------------|---------------------------------------|
| $a(t) = \left(\frac{3\kappa^2 \mu t^2 + 1}{4}\right)^{\frac{1}{2}}$ | $p = \frac{\kappa^2 \mu (\mu - 4\rho_p) \rho_p + \sqrt{\kappa^4 \mu^3 (\mu - 4\rho_p)^2}}{2\\kappa^2 \mu (-2\rho_p + \rho_c)}$ |
| $a(t) = (t - t_s)^{2/c^2}$ | $p = -\rho - \frac{\kappa^2 \mu (\mu - 4\rho_p) \rho_p}{3\sqrt{\kappa^4 \mu^3 (\mu - 4\rho_p)^2}}$ |
| $a(t) = e^{\beta (t - t_0)^{\gamma}}$ | $p = -\rho - 2 \left(\frac{1}{2(-1 + \alpha)} \sqrt{(4\rho_c + \rho_{\text{H}} + \rho_{\text{b}}) (1 + \alpha) \rho_c^2 (1 + \alpha) \rho_{\text{H}}^2 (1 + \alpha) \rho_{\text{b}}^2 (1 + \alpha)} \right)$ |

singularity, since $0 < f < 1$. This issue deserves a more detailed analysis, so we defer this to a future work. Using the imperfect fluid reconstruction method, the resulting EOS which realizes the cosmology is the following,

$$p = -\rho - 2 \left(\frac{1}{2(-1 + \alpha)} \sqrt{(4\rho_c + \rho_{\text{H}} + \rho_{\text{b}}) (1 + \alpha) \rho_c^2 (1 + \alpha) \rho_{\text{H}}^2 (1 + \alpha) \rho_{\text{b}}^2 (1 + \alpha)} \right).$$

(56)

The analytic forms of the observational indices are the following,

$$n_s \approx 1 - \frac{4(-1 + \alpha)}{N \alpha}, \quad r \approx \frac{16(-1 + \alpha)}{N \alpha}, \quad \alpha_s \approx -\frac{4(-1 + \alpha)^2}{\alpha^2 N^2},$$

(57)

so by using $\alpha = 2$ and $N = 60$, the resulting values of the observational indices are,

$$n_s \approx 0.966667, \quad r \approx 0.133333, \quad \alpha_s \approx -0.000277778.$$  

(58)

So the resulting power spectrum is a red tilted scale invariant spectrum, however the scalar-to-tensor ratio is excluded from the Planck data. Nevertheless this scenario is the only bounce cosmology which yields a nearly scale invariant power spectrum. In Table IV we gathered the resulting classical imperfect fluid equations of state.

In a similar way, by using the LQC cosmology theoretical framework, the resulting equations of state can be easily found for all the bouncing cosmologies we studied earlier, and we presented the results in Table IV since the procedure is the same as in the inflationary case. As it can be seen in Table IV the LQC imperfect fluid equations of state become identical to the corresponding classical equations of state, in the limit $\rho_c \to \infty$.

IV. CONCLUSIONS

Motivated by the fact that the Universe has an equation of state which is around the phantom divide line value $w = -1$, in this paper we investigated how some well defined inflationary and bouncing cosmologies can be realized by imperfect fluids. Particularly, with regards to the inflationary scenarios, we examined the intermediate inflation scenario, the Starobinsky inflation scenario and two constant-roll inflationary scenarios. With regards to the bouncing cosmologies, we examined the matter bounce scenario, the superbounce and the singular bounce scenario. We found the imperfect fluid description for each of the aforementioned cosmologies, and non of these is described by a viscous fluid. We used two theoretical frameworks for the imperfect fluids, the classical cosmology framework and the LQC framework. In the case of the classical cosmology, and with regards to the inflationary scenarios, the Starobinsky inflationary scenario and one of the constant-roll scenarios resulted to a spectral index of primordial curvature perturbations compatible with the observational data, however the rest of the indices were not compatible with the observations. With respect to the bouncing cosmologies, only the singular bounce yielded a spectral index compatible with the latest Planck data, but in this case too, the rest of the indices were not compatible with the data.

Hence the present article leads to the conclusion that not all theoretical frameworks lead to successful descriptions of the Universe. The main feature that renders a theoretical description viable is the compatibility with the observational data, and as we showed the imperfect fluid classical cosmology description provides partial compatibility with data, and therefore it is somehow incomplete. Note that the Starobinsky model in the context of $F(R)$ modified gravity is a viable model, and therefore the imperfect fluid case does not make it viable. However, we need to note that we have not extended the calculation of the slow-roll indices for an imperfect fluid in the context of LQC. Possibly in this case the holonomy corrected theory has something new to offer to the imperfect fluid theoretical description. This task is a non-trivial exercise, which we intend to do in a future work.

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Appendix: The Functions $J_1$ and $J_2$

In this Appendix we provide the detailed form of the functions $J_1$ and $J_2$, appearing in Eq. (29). Their detailed form is [15],

$$J_1 = \frac{f(\rho(N))}{\rho(N)} + \frac{1}{2} \left( \frac{f'(\rho(N))^2 + f^2(\rho(N))/2}{f(N)} \right) + \frac{5}{2} \frac{f(\rho(N)) f' \left( \frac{\rho(N)}{\rho(N)} \right)}{f(N)} + \left( \frac{f' \left( \frac{\rho(N)}{\rho(N)} \right)}{f(N)} \right)^2 + \frac{1}{3} \frac{\rho'(N)}{f(N)}$$

$$\times \left[ \frac{f'(\rho(N))^2 + f(\rho(N)) f''(\rho(N)) - 2 f(\rho(N)) f'(\rho(N))}{f(N)} + \left( \frac{f(\rho(N))}{f(N)} \right)^2 \right],$$

$$J_2 = \frac{45}{2} \frac{f(\rho(N))}{\rho(N)} \left( f'(\rho(N)) - \frac{1}{2} \frac{f(\rho(N))}{\rho(N)} \right) + 18 \left( \frac{f(\rho(N))}{\rho(N)} \right)^{-1} \left\{ \left( f'(\rho(N)) - \frac{1}{2} \frac{f(\rho(N))}{\rho(N)} \right)^2 + \left( \frac{f(\rho(N))}{\rho(N)} \right)^{-2} \rho'(N) \right\}$$

$$+ 3 \left\{ 4 f'(\rho(N)) - \frac{7 f(\rho(N))}{\rho(N)} + 2 \right\} \left\{ - \frac{3}{2} f'(\rho(N)) - \frac{1}{2} \frac{f(\rho(N))}{\rho(N)} \right\} + \left( \frac{f(\rho(N))}{\rho(N)} \right)^{-2} \left[ \frac{3 f'(\rho(N))^2 + 2 f(\rho(N)) f''(\rho(N))}{f(N)} \right]$$

$$- \frac{11}{2} \frac{f(\rho(N)) f'(\rho(N))}{\rho(N)} + \left( \frac{f(\rho(N))}{\rho(N)} \right)^{-2} \left[ \frac{3 f'(\rho(N))^2 + 2 f(\rho(N)) f''(\rho(N))}{f(N)} \right]$$

$$+ \left( \frac{f(\rho(N))}{\rho(N)} \right)^{-2} \left[ \frac{3 f'(\rho(N))^2 + 2 f(\rho(N)) f''(\rho(N))}{f(N)} \right] \frac{f'(\rho(N))^2 + f(\rho(N)) f''(\rho(N)) - 2 f(\rho(N)) f'(\rho(N)) + \left( \frac{f(\rho(N))}{\rho(N)} \right)^2}{f(N)}$$

$$+ \left( \frac{f(\rho(N))}{\rho(N)} \right)^{-2} \left[ \frac{3 f'(\rho(N))^2 + 2 f(\rho(N)) f''(\rho(N))}{f(N)} \right] \rho(N) - 3 \left( f'(\rho(N))^2 \right)$$

$$- 3 f(\rho(N)) f''(\rho(N)) + 6 f(\rho(N)) f'(\rho(N)) - 3 \left( \frac{f(\rho(N))}{\rho(N)} \right)^2 \right].$$

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