Judging the Judges:
A General Framework for Evaluating the Performance of International Sports Judges

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Abstract—The monitoring of judges and referees in sports has become an important topic due to the increasing media exposure of international sporting events and the large monetary sums involved. In this article, we present a method to assess the accuracy of sports judges and estimate their bias. Our method is broadly applicable to all sports where panels of judges evaluate athletic performances on a finite scale.

We analyze judging scores from eight different sports with comparable judging systems: diving, dressage, figure skating, freestyle skiing (aerials), freestyle snowboard (halfpipe, slopestyle), gymnastics, ski jumping and synchronized swimming. With the notable exception of dressage, we identify, for each aforementioned sport, a general and accurate pattern of the intrinsic judging error as a function of the performance level of the athlete. This intrinsic judging inaccuracy is heteroscedastic and can be approximated by a quadratic curve, indicating increased consensus among judges towards the best athletes.

Using this observation, the framework developed to assess the performance of international gymnastics judges is applicable to all these sports: we can evaluate the performance of judges compared to their peers and distinguish cheating from unintentional misjudging. Our analysis also leads to valuable insights about the judging practices of the sports under consideration. In particular, it reveals a systemic judging problem in dressage, where judges disagree on what constitutes a good performance.

Keywords: Sports judges, quantifying accuracy, judging panels, heteroscedasticity, general framework

I. INTRODUCTION

In October 1964, the communication satellite 'Syncom 3' enabled the first cross-Pacific telecasting of the Olympic Games in Tokyo. Since then, the ever-expanding commercialization and instantaneous media exposure of sporting events have dictated the public perception of sports. Athletes can become world-wide heroes or laughing stocks within seconds. In many sports, judging decisions can make the difference between victory and defeat, bringing fame and fortune for the winners and lifetime disappointment for the losers. This, as well as judging and refereeing scandals such as the 2002 Winter Olympics figure skating scandal and the 2007 NBA betting scandal, has led to increased scrutiny of judges and referees, as athletes, coaches, fans, officials and sponsors all wish for accurate and fair judges. Price and Wolfers [1] quote former NBA Commissioner David Stern claiming that NBA referees "are the most ranked, rated, reviewed, statistically analyzed and mentored group of employees of any company in any place in the world". Assessing the skill level of sports judges objectively is a difficult endeavor and there is little literature on the topic.

This is the third in a series of three articles on sports judging. In the first article [2], we model the intrinsic judging error of international gymnastics judges as a function of the performance level of the gymnasts using heteroscedastic random variables. We then develop a marking score to quantify the accuracy of international gymnastics judges. In the second article [3], we leverage the heteroscedasticity of the judging error of gymnastics judges to improve the assessment of national bias in gymnastics. In this article, we extend prior work in gymnastics and present a general framework to evaluate the performance of international sports judges, applicable to all sports where panels of judges evaluate athletes on a finite scale. We leverage data from diving, dressage, figure skating, freestyle skiing (aerials), freestyle snowboard (halfpipe, slopestyle), gymnastics, ski jumping and synchronized swimming international competitions. Our main observation is that for all these sports except dressage, the standard deviation of the judging error is heteroscedastic, and we can model it accurately using a concave quadratic equation: judges are more precise when evaluating outstanding or atrocious performances than when evaluating mediocre ones.

We provide evidence that the implemented scoring systems lead to objective, precise and reliable judging. The exception is dressage, where judges increasingly disagree with each other as the performance quality improves, which implies a lack of objectivity compared to the other sports we analyze.

II. RELATED WORK ON JUDGING SKILL AND HETEROSCEDASTICITY

The vast majority of research assessing judging skill in sports focuses on consensus and consistency within groups. In 1979, Shrout [4] introduced the idea of intra-class correlation. This technique was used to evaluate judging in figure skating [5], artistic gymnastics [6,8] and rhythmic gymnastics [9]. The dependence between the variability of the judging error and performance quality had never been properly studied until our work in gymnastics [2], although it was observed in prior work. Atiković et al. [8] notice a variation of the marks deviation by apparatus in artistic gymnastics. Leandro et al. [9] observe that the deviation of scores is smaller for the best athletes in rhythmic gymnastics, and Looney [5] notices the
same thing in figure skating. Besides sports, heteroscedasticity of judging variability as a function of the score is apparent in the data of wine tasting scores [10].

Even though heteroscedasticity is often under-reported in scientific research [11], it is a well-known property and appears in countless fields such as chemistry [12], economics [13] and finance [14]. In most cases, heteroscedasticity arises as a scale effect, i.e., the variance is linked to the size of the observable. This is not the case in our work, where the variability of the judging error depends on the quality of the performance – a completely different source of variability than the scale effect.

III. JUDGING SYSTEMS AND DATASET

We analyze eight sports with comparable judging systems: diving (including high-diving) [15, 16], dressage (Grand Prix, Grand Prix Special & Grand Prix Freestyle) [17], figure skating [18], freestyle skiing (aerials) [19], freestyle snowboard (halfpipe, slopestyle) [20], gymnastics (acrobatic, aerobic, artistic, rhythmic, trampoline) [21, 26], ski jumping [27] and synchronized swimming [28]. For all these sports, it is impossible to evaluate performances in an automated fashion, and a panel of judges evaluate the athletes. Each judge in the panel reports an individual mark within a closed finite range following predefined judging guidelines. Although the implementation details such as the precise judging guidelines, the marking range and the step size vary, all the judging systems incorporate

1) An execution evaluation of performance components;
2) Penalty deductions for mistakes.

The judging guidelines of each sport are the embodiment of the performance quality, mapped to a closed finite nominal mark. In particular, they embed the concept of perfection: a component or routine whose mark is the maximum possible value is considered perfect.

Although the judging guidelines try to be as deterministic and accurate as possible, every reported mark inevitably remains a subjective approximation of the actual performance level. Moreover, live judging is a noisy process, and judges within a panel rarely agree with each other. This introduces variability in the marking process. Every judging system accounts for this deviation with an aggregation of the judging panel marks to ensure an accurate and fair evaluation of the athletes. This aggregation, which varies for each sport, is also used to discard outlier marks, and is the most effective way to decrease the influence of erratic, biased or cheating judges.

Table I shows the typical judging panel and size of our dataset per sport.

The Fédération Internationale de Gymnastique (FIG) and Longine[f] provided the data for all the gymnastics disciplines. It includes 21 continental and international competitions from the 2013–2016 Olympic cycle, ending with the 2016 Rio Olympic Games. We gathered data for all the other sports from publicly available sources at official federation or result websites[f]. The data only comprises professional and international competitions. When available, we include all results of official World cup events, international championships and Olympic Games from January 2013 to August 2017 in the analysis. None of the sports has a gender-specific scoring system, thus we include men and women competitions in the dataset.

Table II excludes some of the gathered data to ensure comparability among the sports as follows. First, the analysis focuses on the execution and artistry components of performances, thus we exclude difficulty scores of technical elements from the sample. Acrobatic and aerobic gymnastics have separate marks for the execution and artistry components; we split the marks in our analysis, but do not distinguish between them because judges have the same judging behavior in both instances [2]. In dressage, we only consider events at the Grand Prix level, which includes 'Grand Prix', 'Grand Prix Special' and 'Freestyle' competitions. Judges in figure skating and synchronized swimming evaluate the execution of multiple components of a performance separately, which we group together in our analysis. Scores in aerials competitions consist of a 'Air', 'Form', and 'Landing' mark, although this granularity is not available for all competitions. We add the three marks together when analyzing total scores, and study the components separately when they are available.

IV. METHODS

We perform the same analysis for every sport and discipline. Table III summarizes our notation. Let $s_{p, j}$ be the

| Sport                        | Size of judging panel | Number of performances | Number of marks |
|------------------------------|-----------------------|------------------------|-----------------|
| Aerials                      | 3 or 5                | 7'079                  | 53'543          |
| Diving                       | 7                     | 19'111                 | 133'777         |
| Dressage                     |                        |                        |                 |
| - GP & GP Special            | 5 or 7                | 5'500                  | 28'382          |
| - GP Freestyle               | 5 or 7                | 2'172                  | 11'162          |
| Figure skating               | 9                     | 5'076                  | 228'420         |
| Freestyle snowboard          |                        |                        |                 |
| - Halfpipe                   | 3, 4, 5 or 6          | 4'828                  | 22'389          |
| - Slopestyle                 | 3, 4, 5 or 6          | 2'524                  | 12'374          |
| Gymnastics                   |                        |                        |                 |
| - Acrobatics                 | 6 or 8                | 756                    | 4'870           |
| - Aerobics                   | 6 or 8                | 934                    | 6'072           |
| - Artistic                   | 4, 6 or 7             | 11'940                 | 78'696          |
| - Rhythmic                   | 4, 5 or 7             | 2'841                  | 19'052          |
| - Trampoline                 | 4 or 5                | 1'986                  | 9'654           |
| Ski jumping                  | 5                     | 12'381                 | 61'905          |
| Synchronized swimming        | 5, 6 or 7             | 3'882                  | 42'576          |

1[www.fig-gymnastics.com]
2[www.longines.com]
3[www.data.fis-ski.com (aerials, halfpipe, ski jumping, slopestyle); www.omegatiming.com (diving); data.fei.org (dressage); www.isu.org (figure skating); www.swimming.org (synchronized swimming); all retrieved September 1, 2017.]
mark from judge \( j \) for performance \( p \). For each performance, we need a control score \( c_p \), providing an objective measure of performance quality. We use the median panel score \( c_p \triangleq \text{median} s_{p,j} \) in our analysis. The median is more robust than the mean or the trimmed mean against misjudgments and biased judges, and in the aggregate provides a good approximation of performance quality. In some sports such as gymnastics \(^2\), more accurate control scores are derived using video analysis post competition, however these are not accessible for our analysis.

The difference \( e_{p,j} \triangleq s_{p,j} - c_p \) is the judging error of judge \( j \) for performance \( p \). For a given discipline \( d \), we group the judging errors by control score \( c \) and calculate the standard deviation \( \sigma_d(c) \), quantifying how judges agree or disagree with each other for a given performance quality \( c \). We then approximate the standard deviation as a function of performance quality with a polynomial of second degree \( \hat{\sigma}_d(c) \) using a weighted least-squares quadratic regression.

### V. Results and Discussion

#### A. The general pattern of heteroscedasticity

Figures 1-15 show the standard deviation of the judging marks \( \sigma_d(c) \) and the weighted least-squares quadratic regression curve \( \hat{\sigma}_d(c) \) as a function of performance \( c \) for diving, figure skating, halfpipe, ski jumping, slopestyle, trampoline, acrobatic gymnastics, aerobic gymnastics, artistic gymnastics, rhythmic gymnastics, synchronized swimming, aerials (total and component scores) and dressage (regular and freestyle to music events), respectively\(^3\). Each figure includes the scaleless weighted coefficient of determination \( r^2 \) quantifying the goodness of fit of the regression. The weighted \( r^2 \) are high, ranging from 0.24 (Dressage GP Freestyle) to 0.98 (artistic gymnastics). Each figure also shows the weighted root-mean-square deviation (RMSD) quantifying the average discrepancy between the approximated deviation and the measured values. The RMSD depends of the scale of the marking range and cannot be compared between different sports.

With the notable exception of dressage, all sports exhibit the same heteroscedastic pattern: panel judges disagree the most when evaluating mediocre performances, and their judging error decreases as the performance quality improves towards perfection. The behavior for the worst performances depends on the sport. On the one hand, sports such as diving (Figure 1), trampoline (Figure 6) and snowboard (Figures 3 and 5) have many aborted or missed routines (splashing the water, stepping outside the trampoline boundaries after a jump, falling during the run). These routines result in very low marks, and the concave parabola is clearly visible for these sports, indicating smaller judging variability for performances close to zero. Smaller variability for atrocious and outstanding performances is not surprising: they either contain less components to evaluate or less errors to deduct. In both cases, this decreases the number of potential judging errors, as opposed to performances in the middle of the scoring range. On the other hand, gymnastics performances (Figures 7-10) and synchronized swimming routines (Figure 11) barely receive a score in the lower half of the possible interval. Without these bad performances close to the minimum possible score, the quadratic fit \( \hat{\sigma}_d(c) \) does not decrease towards the left border of the scoring range and can even be slightly convex.

Aerials is of particular interest because it exhibits both possible behaviors for the worst performances, which is not obvious from Figure 12. More precisely, the total score in aerials is the combined sum of three independent components: ‘Air’, ‘Form’ & ‘Landing’. Even though athletes do often fall when landing, this only influences their ‘Landing’ score and not the other two components. Figure 13 shows the aerials.

\(^{\text{Note that for some sports we aggregate close quality levels (control scores) to improve the visibility of the figures. We do the analysis without the aggregation.}}\)
Figure 3. Standard deviation of judging marks versus performance quality in halfpipe.

Figure 4. Standard deviation of judging marks versus performance quality in ski jumping.

Figure 5. Standard deviation of judging marks versus performance quality in slopestyle.

Figure 6. Standard deviation of judging marks versus performance quality in trampoline.

Figure 7. Standard deviation of judging marks versus performance quality in acrobatic gymnastics.

Figure 8. Standard deviation of judging marks versus performance quality in aerobic gymnastics.
judging errors split per component. The variability of the 'Landing scores', which are evenly distributed among the possible scoring range, closely follows the concave parabola, whereas the 'Air' and 'Form' components have right skewed distributions because low marks are rarely given. For these two components the quadratic regression is closer to what we observe in gymnastics or figure skating. Aerials shows at the component level what we observe at the sport level: the shape of the parabola depends on the presence or absence of performances whose quality is close to zero.

### B. The special case of dressage

Surprisingly, the observed general pattern of heteroscedasticity does not apply to dressage. Figure 14 shows the results for standard dressage GP and GP Special competitions, whereas Figure 15 shows the results for 'GP Freestyle to music' events.

In both figures, judging errors are the lowest for average performances and the parabola is convex. For standard events in Figure 14 we first observe that the judges increasingly

\*Some competitions in our dataset are not split per component, thus we excluded them from Figure 13.*
disagree as the performance quality decreases. This is similar to what we observe in gymnastics and synchronized swimming and due to the fact that there are no easy to judge performances close to the lower boundary of the marking range (in dressage the lowest possible score is 0 and there is no control score below 45 in our dataset). However, judges also increasingly disagree as the performance quality approaches perfection, which is extraordinary. The judging behavior in ‘GP Freestyle to music’ events in Figure [15] is similar but less pronounced. Furthermore, there are a few exceptional performances for which the judging errors decreases again, although this might be due to a mathematical truisum: when the median mark is almost perfect, at least half the panel marks must be between the median and maximal scores, hence also perfect or close to perfect.

We did additional analyses to understand this extraordinary behavior, and found that it appears at all levels of competition in our dataset, and for every judging position around the arena. The Fédération Équestre Internationale (FEI) dressage states that “Dressage is the ultimate expression of horse training and elegance. Often compared to ballet, the intense connection between both human and equine athletes is a thing of beauty to behold.” Elegance and beauty are highly subjective, and the subjectivity of dressage judges is not new (consult, for instance, [29]). The simplest and most uncomfortable explanation is that judges fundamentally disagree on what constitutes an above average dressage performance. This might be due to imprecise or overly subjective judging guidelines, or to the unwillingness of judges to apply said guidelines objectively. No matter the reason, our analysis reveals a clear and systemic judging problem in dressage, and we recommend that the FEI thoroughly reviews its judging practices and how it monitors its judges. We presented and discussed our results with FEI officials, who independently observed the same judging behavior and are considering major changes leading to more precise codes of points.

VI. FROM HETEROSEDASTICITY MODEL TO JUDGING THE JUDGES

The knowledge of the heteroscedasticity model of the judging error \( \hat{\sigma}_d(c) \) makes it possible for federations to evaluate the accuracy of their judges for all the sports we analyze in this article, exactly as was done in gymnastics [2]. We note that in practice it is often better to use weighted least-squares exponential regressions instead quadratic ones since they are more accurate for the best performances [2].

The marking score of judge \( j \) for performance \( p \) is given by

\[
m_{p,j} \triangleq \frac{\hat{\sigma}_d(c_p)}{\sigma(c_p)}.
\]

This scales the error of the judge for a specific performance as a fraction of the estimated standard deviation of the judging error for a specific discipline \( d \) and performance quality \( c_p \), and allows to compare judging errors for different quality levels and disciplines in an unbiased fashion. The overall marking score \( M_j \) of judge \( j \) is the mean squared error of all his/her judging errors in the dataset, i.e.,

\[
M_j \triangleq \sqrt{E[m_{p,j}^2]}.
\]

We can calculate the marking score of a judge for a specific competition, or longitudinally for multiple competitions over many years. The higher the marking score \( M_j \), the more a judge misjudges performances compared to his/her peers. A judge always marking one standard deviation \( \hat{\sigma}_d(c_p) \) away from the median has a marking score of 1, and a perfect judge has a marking score of 0.

We can use the marking score to detect outlier misjudgments, for instance judging errors greater than \( 2 \cdot \hat{\sigma}_d(c_p) \cdot M_j \). This flags \( \approx 5\% \) of the evaluations and adjusts the threshold based on the intrinsic variability of each judge: an accurate judge has a lower outlier detection threshold than an erratic judge. This is important to differentiate erratic but honest judges from accurate but sometimes highly biased judges. However, we must note that when using the median as the control score, a bad marking score for a single performance is not necessarily an judging error but can also mean that the judge is out of consensus. A more accurate control score is necessary to remove the ambiguity. Finally, we can also integrate the approximated standard deviation \( \hat{\sigma}_d(c_p) \) and the

\[\text{From http://www.fei.org/disciplines/dressage.}\]
marking score $M_j$ in bias analyses, as we did in our study of national bias in gymnastics [3].

VII. Conclusion

In this article, we study sports for which performances are evaluated by panels of judges within a finite interval, and model the judging error using heteroscedastic random variables. With the exception of dressage, consensus among judges increases with the quality level of the performance, and we can approximate the standard deviation of the judging error accurately using a quadratic equation. Our analysis of dressage judges further shows that they increasingly disagree as the performance level increases, indicating a significant amount of subjectivity in the judging process compared to other sports with similar judging systems. Estimating and modeling the intrinsic heteroscedasticity could also be used for other judging processes within a finite scale such as the evaluation of movies or wines, although in these instances there is no clear notion of control score indicating the true performance level.

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