Momentum-resolved tunneling between Luttinger liquids

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We study tunneling between two nearby cleaved edge quantum wires in a perpendicular magnetic field. Due to Coulomb forces between electrons, the wires form a strongly-interacting pair of Luttinger liquids. We calculate the low-temperature differential tunneling conductance, in which singular features map out the dispersion relations of the fractionalized quasiparticles of the system. The velocities of several such spin-charge separated excitations can be explicitly observed. Moreover, the proposed measurement directly demonstrates the splintering of the tunneling electrons into a multi-particle continuum of these quasiparticles, carrying separately charge from spin. A variety of corrections to the simple Luttinger model are also discussed.

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The only universally accepted example of a non-Fermi liquid metallic state is the one dimensional (1D) Luttinger liquid (LL) \([1]\). Remarkably, the quasiparticle excitations of a LL are fractionalized, comprising a diverse set carrying spin separately from charge, and charge in fractions of the electron charge \(e\). LL behavior has been observed experimentally in carbon nanotubes \([2]\), through strongly energy dependent local tunneling, and more recently in GaAs quantum wires through power-law resonant tunneling lineshapes \([3]\). Evidence of charge fractionalization has also been seen in shot noise experiments using fractional quantum Hall edge states \([4]\), which are somewhat special chiral Luttinger liquids \([5]\). Despite these successes, no direct experimental evidence of fractionalization has ever been obtained in a non-chiral 1D system. In this letter, we show that measurements of the non-linear tunneling conductance between parallel Luttinger liquids in a transverse magnetic field provide a direct spectroscopic probe of fractionalization. The results, as described below, give very similar information to an ideal photoemission experiment. Indeed, some indications of spin-charge separation were seen in photoemission spectroscopy of the quasi-1D cuprate \(\text{SrCuO}_2\) \([6]\). Tunneling spectroscopy has, however, the advantages of being possible on a single, isolated 1D system and with potentially much higher resolution than photoemission.

Controlled tunneling experiments between two parallel wires have been recently conducted using cleaved-edge overgrowth by O.M. Auslaender et al \([7]\). The experimental geometry we consider is indicated schematically in Fig. 1. The two “wires” are in fact confined surface states, and electrical contact is made only to the upper wire via a Two-Dimensional Electron Gas (2DEG). With \(L' \gg L\), nearly the full electrochemical potential drop occurs between the shorter (left) segment of the upper wire and the lower wire. Because of the uniformity of the barrier, momentum along the wire is conserved during tunneling \([8]\).

![FIG. 1. Schematic experimental geometry (from Ref. 7)](image)

We assume that the barrier is sufficiently high as to establish a quasi-equilibrium state on either side of the barrier, treating tunneling across the barrier perturbatively. A non-interacting model Hamiltonian for the system neglecting tunneling is then

\[
H_0 = \sum_{\alpha} \int dx \ c_\alpha c_\alpha^\dagger \left[ -\frac{\partial^2}{2m} - U_\alpha - \mu_\alpha \right] c_\alpha, \tag{1}
\]

where \(\alpha = 1, 2\) labels the upper/lower wire, \(\alpha = \uparrow, \downarrow\) labels the electron spin, and \(U_\alpha\) and \(\mu_\alpha\) are the electrostatic and chemical potential of the \(a^{th}\) wire, respectively. We choose by convention to take \(\mu_\alpha = 0\) in equilibrium, so that \(k_\alpha = \sqrt{2mU_\alpha}\) is the Fermi momentum in wire \(\alpha\). Neglecting the energy dependence of the tunneling amplitude \(w\), the Zeeman shift (see below), and a small energy shift due to orbital magnetic effects within each wire, the tunneling Hamiltonian in the presence of a magnetic field \(B\hat{z}\) (in the gauge \(A_y = Bx\)) is

\[
H_{\text{tun}} = -w \sum_{\alpha} \int dx \ \left[ c_{1\alpha}^\dagger c_{2\alpha} e^{iQx} + c_{2\alpha}^\dagger c_{1\alpha} e^{-iQx} \right]. \tag{2}
\]

Here the magnetic wavenumber \(Q = 2\pi Bd/\phi_0\), \(d\) is the center-to-center distance of the wires, and \(\phi_0 = \hbar c/e\).
is the flux quantum. The one-dimensional (1D) tunneling current density, \( J = i e v \left(c_{1a}^\dagger c_{2a} e^{iQx} - c_{2a}^\dagger c_{1a} e^{-iQx}\right) \) can be calculated directly from Fermi’s golden rule. The result, whose gross features appear experimentally in Ref. [7], is shown in Fig. [3] taking \( \mu_1 = V, \mu_2 = 0 \). This diagram can be understood physically by considering all processes by which an electron can be transferred between the two wires, moving from an occupied to an unoccupied state. Geometrically, Fig. 2 is obtained by drawing the locus of values of \( V,Q \) for which one of the four Fermi points lies on the other parabola when shifted vertically and horizontally by \( V,Q \), respectively.

FIG. 2. Tunneling current in the \( V-Q \) plane for the non-interacting model. Regions of non-zero current are shaded, and \( dI/dV \) has delta-function singularities along the boundaries between shaded and unshaded regions.

**Zero bias features occur** when \( (A) Q = \pm (k_F + 1, k_F) \), \( (B) Q = \pm (k_F - 1, k_F) \). It is in the low-energy region of the \( V-Q \) plane near these four features that universal features arise in the interacting system, to which we now turn. Focusing on the the low-bias regime, we first decompose the electron fields into right and left movers, \( c_{Rao}(x) = c_{Rao}(x) e^{ik_F x} + c_{Lao}(x) e^{-ik_F x} \), which are described by the Luttinger Hamiltonian (taking \( \mu_a = 0 \) for simplicity)

\[
H_0 = -i \sum_a \int dx \; v_{Ra} \left[ c_{Rao}^\dagger \partial_x c_{Rao} - c_{Lao}^\dagger \partial_x c_{Lao} \right],
\]

where \( v_{Ra} = k_{Fa}/m \). In general, these right- and left-moving Fermions undergo a diverse set of scattering processes mediated by the Coulomb interaction (screened by the 2DEG). A systematic study of these terms reveals an important simplification due to the experimental fact that the width \( W \) of a typical cleaved edge quantum wire is large compared to the Fermi wavelength \( \lambda_F \) \((k_FW \gg 1)\). We also expect the 2D screening length \( \lambda_s \gg W \). These properties imply a strong suppression of two-electron backscattering processes. We therefore adopt a forward scattering model retaining only the strongest (unsuppressed) interactions,

\[
H_{\text{int}} = \frac{1}{2} \sum_{ab} \int dx \; n_a(x) V_{ab} n_b(x),
\]

with \( H = H_0 + H_{\text{int}} \), and \( n_a = \sum_c c_{Rao}^\dagger c_{Rao} + c_{Lao}^\dagger c_{Lao} \). Eq. 3 neglects momentum dependence of the forward-scattering interactions, and is hence valid for \( eV \ll \hbar v_F/\lambda_F \). The interactions \( V_{ab} \) can be roughly estimated as \( V_{11} \approx V_{22} \approx (2e^2/\hbar \lambda_F W/\hbar) \), \( V_{12} \approx (2e^2/\hbar \lambda_F \ln(\lambda_F/d)/\hbar) \), where \( \hbar \) is the electron charge, \( \epsilon \) the wires dielectric constant. Experimentally, \( d - W \ll W \), so the inter-wire interaction \( V_{12} \) is not negligible. Thus the tunneling conductance does not in fact probe the spectral properties of two decoupled 1DEGs. Nevertheless, all properties of the interacting Hamiltonian \( H_0 + H_{\text{int}} \) can be calculated exactly by bosonization (LL theory).

From now on we will focus on the current at zero temperature around the point A of Fig. 3. An analogous description can be obtained around each of the low bias points of fig. 3 [13]. To lowest order in perturbation in \( w \), we can write the current density \( J = 2e|w|^2 (J_+ - J_-) \), where \( J_+ - J_- \) are positive functions which satisfy (time-reversal symmetry) \( J_+(Q,V) = J_-(-Q,-V) \), and \( J_+(J_-) \) is nonzero only for \( V > 0 \) \((V < 0)\). Around point A, \( q = Q - (k_F + k_F) \) is small, and

\[
J_+ = \Re \int_{-\infty}^{+\infty} dx \int_{0}^{\infty} dt \; e^{i(V+\delta t)t+q x} C_{E}^{R \rightarrow L}(x,\tau - t + i\epsilon)
\]

where \( C_{E}^{R \rightarrow L}(x,\tau) \) is the Euclidean correlation function

\[
C_{E}^{R \rightarrow L}(x,\tau) = \langle c_{R1}^\dagger c_{L2}(x,\tau) c_{L1}^\dagger c_{R2}(0,0) \rangle.
\]

LL theory then gives

\[
C_{E}^{R \rightarrow L}(x,\tau) = \sum_{a=1,2} \left( v_{sa} \tau - \epsilon_a i x \right)^{-1/2} \left( v_{ca} \tau + \epsilon_a i x \right)^{-\eta-\theta_a}
\]

where \( \epsilon_a = (-1)^{a+1} \), and \( a_0 \) (of order \( W \)) is a small distance cut-off for the LL description. Fractionalization is evident formally in Eq. 4 through the multiple branch points characterized by distinct charge and spin velocities, \( v_{c1/2} \) and \( v_{s1/2} \). For two independent wires \((V_{12} = 0)\), it is well-known that \( \eta = 1/4 \) and \( \theta_1, \theta_2 > 1/4 \). In the present case, the strong interactions between the wires make \( \eta \) interaction-dependent, and \( \theta_1, \theta_2 \) can take values smaller than \( 1/4 \). Indeed in the case of two coupled identical wires \((V_{11} = V_{22})\), these exponents take the values \( \theta_1 = 1/(4/2(V_{11} + V_{12})/(\pi v_F)) \), \( \theta_2 = \sqrt{1 + 2(V_{11} - V_{12})/(\pi v_F)} \), \( \eta = 0 \). General but non-illuminating formula are postponed to [13].

Letting \( x \to tu \) in Eq. 5 and integrating over \( t \) gives

\[
J_+(Q,V) \propto \Gamma(1 - 2\theta_1 - 2\theta_2) \times \Re \int_{-\infty}^{+\infty} du \; h(u),
\]

where the complex function \( h(u) \) is defined by

\[
h(u) = [\delta - i(V + qu)]^{2\theta_1 + 2\theta_2 - 1} C_{E}^{R \rightarrow L}(u,\epsilon).
\]
The contour of integration in (1) can then be deformed according to Fig. 3. As $h(u)$ vanishes faster than $|u|^{-2}$ at infinity, we are reduced to the integral of $h(u)$ on the contour 3 (see Fig. 3). More explicitly in the situation where $v_{s1} < v_{c1} < v_{s2} < v_{c2}$, and for e.g. $q > 0$, the remaining integral can be written as the sum of real integrals:

$$J_+(Q, V) \propto 2 \sin(2\pi(\theta_1 + \theta_2))\Gamma(1 - 2\theta_1 - 2\theta_2)$$

$$\times \sum_{\alpha=1}^{3} \theta(V - q v_{c\alpha}) \sin \phi_{\alpha} \int_{-v_{c\alpha}}^{-v_{s\alpha}} |h(u)| du,$$

with $v_1 = v_{c1}, v_2 = v_{s2}, v_3 = v_{c2}, v_4 = +\infty$ and $\phi_1 = \pi(\theta_1 - \eta), \phi_2 = \pi(\theta_1 - \eta + \frac{3}{2}), \phi_3 = \pi(\theta_1 + \theta_2 + 1/2)$. Similar expressions for $J_+$ and $J_-$ can be derived for any other order of the velocities, and for the other $V \simeq 0$ points of Fig. 2. A typical result is presented in Fig. 4.

The density plot of the differential conductance (per unit length) $G = dJ/dV$ in Fig. 4 directly exhibits evidence of electron fractionalization. First, non-analytic features appear along rays (3 per quadrant) whose slope gives the two charge and spin velocities. Second, $dJ/dV$ is non-zero for any bias above the threshold $|V| > v^*|q|$, with $v^* = \min(v_{s2}, v_{c1}, v_{c2})\Theta(V) + \min(v_{s1}, v_{c1}, v_{c2})\Theta(-V)$. Both these properties can be understood from kinematics. Each tunneling event corresponds to a transfer of charge $\pm e$ and spin $\pm 1/2$ from one wire to the other, accompanied by the addition of momentum $q$ and energy $eV$. This creates a combination of (6 total) fractional “chargons” and “spinons”. The final state with the least energy for a given momentum $q$ gives all the momentum to the slowest appropriate “particle”, which determines the threshold. Moreover, since the final states are six-particle excitations, kinematics allows any energy above $v^*|q|$, explaining the non-zero weight in $dJ/dV$ as a multi-particle continuum. Detailed expressions for the singularities along the various rays in Fig. 4 can be extracted from Eq. 7.

We now turn to a discussion of the numerous effects left out of the above treatment. The two most significant corrections can be treated exactly. First, the spectral density is rounded on a scale set by temperature (the rounding can be calculated exactly to describe detailed line-shapes). Experimentally, for $e^2/v_F$ of $O(1)$, LL behavior is expected to be manifested for $eV < \epsilon_F \sim 10^{-20} mV$ in cleaved edge samples. Thus for experiments with $T$ in the Kelvin range, the thermal rounding is minimal except at very low bias.

As $k_B T \gg 1$, the dominant impurity process is elastic forward scattering. This is easily included by convolving $G(V, q) (in q)$ calculated above with a Lorentzian of half-width $\Delta q \equiv 1/\ell_{el}$, and has several consequences. The singularities at $V > 0$ are rounded over an energy width $1/\ell_{el} \sim v_{s1} / \ell_{el}$. Non-vanishing weight also appears outside the kinematically allowed region. At low bias and temperature, this weight is itself singular and displays a pronounced low bias conductance dip. In particular, for $eV, k_B T, v^* q \ll 1/\ell_{el}$, $G(V, q, \ell_{el}) \sim |w|^2 |\ell_{el}|(1 + 2q^2/\ell_{el}^2)|T^2 F_\beta(eV/k_B T)$, with $0 < \beta = 2\theta_1 + 2\theta_2 - 1 < 1$, where $F_\beta(X)$ is the well-known scaling function for point-contact tunneling, satisfying $F_\beta(0) = 1$ and $F_\beta(X) \sim X^\beta$ for $X \gg 1$. 

Even at $T = 0$, effects not included in the LL model can broaden the spectral function. First, consider the ef-
fects of non-forward-scattering interactions between electrons at the edge. In the generic situation, \( k_{e1} \neq k_{e2} \), and charge/spin-density-wave coupling (e.g. \( \mathcal{H}_{CDW} = V_{CDW}\psi_{L1}^\dagger \psi_{L2}^\dagger \psi_{R1} \psi_{R2}^\dagger \)) between the two wires is forbidden. The dominant residual term is then the “exchange” interaction (we reserve the usual term “backscattering” for impurity effects discussed below) within a single wire, e.g. \( \mathcal{H}_{ex,i} = -\lambda_i \psi_L^i \sigma^i \psi_{L,R}^i \), where \( \lambda_i \) characterizes the dimensionless backscattering strength in wire \( i \). In the experimentally relevant situation, \( \lambda_i \sim (k_p W)^{-1} \ll 1 \), so this is a weak interaction. Formally, such exchange interactions are known to be marginally irrelevant in the renormalization group sense, and a simple perturbative (bosonized “phonon” self-energy) estimate suggests a resulting “lifetime” scaling approximately linearly with energy \( \omega, (1/\tau_{ex,i})^2 \sim \lambda^2 \omega^2 / (c_1 + c_2 \lambda^2 \ln^2 (\epsilon F/\omega)) \), where \( c_1/2 \) are order one constants. The magnitude of \( 1/\tau_{ex,i} \), however, is small, due both to the smallness of \( \lambda_i \) and the additional logarithmic suppression at low energies. Other “internal” corrections to the forward-scattering model, such as band curvature, are strongly irrelevant, and a similar self-energy estimate shows that they contribute only negligibly \( 1/\tau_{curves} \sim O(\omega^2/\epsilon_F) \) terms to the lifetime.

Electrons in the wires also interact with those in the bulk 2DEG. Expressing the Coulomb interaction in the basis of wire and bulk states, one finds several distinct scattering channels. Most interesting are charge \( \pm 2e \) decay processes, in which an electron/hole in one of the wires decays into an electron/hole in the 2DEG and an electron/hole pair. Because the 2DEG is a Fermi-liquid, however, strong phase-space restrictions reduce the rate of such decays at low energies: crudely, \( 1/\tau_e \sim c_e \omega^2 / \epsilon_F \), where the \( O(1) \) constant \( c_e \) depends upon the density of the 2DEG, etc. Interestingly, if \( k_{F,bulk} < 3k_{F,i} \), such processes are kinematically forbidden (\( c_e = 0 \)). Similar, albeit slightly less restrictive kinematic conditions reduce the phase space for charge \( \pm 2e \) decay, in which Cooper pairs (hole pairs) are scattered between the wires and 2DEG. All such processes in which charge is transferred are further reduced by LL orthogonality catastrophe effects. Finally, there are forward-scattering processes in which no charge but only energy and small momenta are transferred between the wires and 2DEG. Because the Coulomb potential suppresses long-wavelength charge fluctuations in the 2DEG, we expect this also to be a weak effect, and will explore it in greater detail in a future publication.