EXPLOSIVE NUCLEOSYNTHESIS IN THE NEUTRINO-DRIVEN ASPHERICAL SUPERNova EXPLOSION OF A NON-ROTATING 15 \( M_\odot \) STAR WITH SOLAR METALLICITY

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ABSTRACT

We investigate explosive nucleosynthesis in a non-rotating 15 \( M_\odot \) star with solar metallicity that explodes by a neutrino-heating supernova (SN) mechanism aided by both standing accretion shock instability (SASI) and convection. To trigger explosions in our two-dimensional hydrodynamic simulations, we approximate the neutrino transport with a simple light-bulb scheme and systematically change the neutrino fluxes emitted from the protoneutron star. By a post-processing calculation, we evaluate abundances and masses of the SN ejecta for nuclei with a mass number \( \leq 70 \), employing a large nuclear reaction network. Aspherical abundance distributions, which are observed in nearby core-collapse SN remnants, are obtained for the non-rotating spherically symmetric progenitor, due to the growth of a low-mode SASI. The abundance pattern of the SN ejecta is similar to that of the solar system for models whose masses range between (0.4–0.5) \( M_\odot \) of the ejecta from the inner region (\( \leq 10,000 \) km) of the precursor core. For the models, the explosion energies and the \( ^{56}\text{Ni} \) masses are \( \gtrsim 10^{53} \) erg and (0.05–0.06) \( M_\odot \), respectively; their estimated baryonic masses of the neutron star are comparable to the ones observed in neutron-star binaries. These findings may have little uncertainty because most of the ejecta is composed of matter that is heated via the shock wave and has relatively definite abundances. The abundance ratios for Ne, Mg, Si, and Fe observed in the Cygnus loop are reproduced well with the SN ejecta from an inner region of the 15 \( M_\odot \) progenitor.

Key words: hydrodynamics – methods: numerical – nuclear reactions, nucleosynthesis, abundances – supernovae: general

Online-only material: color figures

1. INTRODUCTION

The explosion mechanism of core-collapse supernovae (SNe) is still not clearly understood. Multi-dimensional effects such as standing accretion shock instability (SASI) and convection are recognized as most important to unveiling the explosion mechanism, in particular for a progenitor heavier than about 11\( M_\odot \) in its main-sequence phase (Kitaura et al. 2006; Buras et al. 2006a, 2006b). Here SASI, which is becoming very popular in current SN research, is a uni- and bipolar sloshing of the stalled SN shock with pulsational strong expansion and contraction (see, e.g., Blondin et al. 2003; Scheck et al. 2004; Ohnishi et al. 2006; Foglizzo et al. 2007; Blondin & Mazzacappa 2007; Iwakami et al. 2008, 2009; Nordhaus et al. 2010 and references therein). Some of the recent two-dimensional (2D) radiation-hydrodynamic simulations show that the delayed neutrino-driven mechanism aided by SASI and convection does work to produce aspherical explosions (Marek et al. 2009; Marek & Janka 2009; Suwa et al. 2010).

Observationally, global anisotropies and mixing as well as smaller scale clumping of the SN ejecta are common features of SN remnants like those in SN1987A (Wang et al. 2002), Cas A (Hughes et al. 2000; Willingale et al. 2002), G292.0+1.8 (Park et al. 2007), and Cygnus loop (Kimura et al. 2008; Uchida et al. 2009). Asymmetries commonly observed in the nebular emission-line profiles are considered evidence that core-collapse SNe generally occur aspherically (Maeda et al. 2008; Modjaz et al. 2008; Tanaka et al. 2009b; Taubenberger et al. 2009). Evidence for asymmetry is also obtained from spectropolarimetric observations of Type Ibc SNe at an early phase (~days) (see, e.g., Tanaka et al. 2008, 2009a and references therein).

Thus far, nucleosynthesis studies of the SN ejecta have almost successfully reproduced the solar composition and abundances of radioactives observed in SN1987A (Hashimoto 1995; Woosley & Weaver 1995; Thielemann et al. 1996; Rauscher et al. 2002). However, those spherical models have some problems, such as overproduction of neutron-rich Ni isotopes and underproductions of \( ^{44}\text{Ti} \), \( ^{64}\text{Zn} \), and light p-nuclei (Rauscher et al. 2002).

Aspherical effects on the explosive nucleosynthesis have been investigated by Nagataki et al. (1997) and Nagataki (2000). Based on 2D hydrodynamic simulations in which the explosion was triggered by some form of manual energy deposition into a stellar progenitor model outside the so-called mass cut, they evaluated the composition of the ejecta with a large nuclear reaction network. They pointed out that \( ^{44}\text{Ti} \) can be produced more abundantly in the case of jet-like explosions, compared to spherical explosions. Young et al. (2006) examined the composition of the ejecta in three-dimensional (3D) smoothed particle hydrodynamic simulations to discuss a candidate for the progenitor of Cas A. They showed that the abundances of \( ^{56}\text{Ni} \) and \( ^{44}\text{Ti} \) depend on the magnitude and asymmetry of the explosion energy as well as on the amount of fallback. The effects of the fallback on the abundances have been systematically studied in one-dimensional explosion models (Young & Fryer 2007). More recently, 3D effects have been more elaborately studied (Hungerford et al. 2003, 2005), as have the impacts of different explosions by employing a number of progenitors (Joggerst et al. 2009, 2010) or by assuming a
jet-like explosion (Couch et al. 2009; Tominaga 2009), which is one possible candidate for hypernovae (e.g., Maeda & Nomoto 2003; Nagataki et al. 2006).

In addition to the above-mentioned work, nucleosynthesis in a more realistic simulation that models the multidimensional neutrino-driven SN explosion has also been extensively studied (Kifonidis et al. 2003, 2006; Gawryszczak et al. 2010). Although only a small network has ever been included in the computations, these 2D simulations employing a light-bulb scheme (Kifonidis et al. 2003) or a more accurate gray transport scheme (Scheck et al. 2006; Kifonidis et al. 2006) have made it possible to elucidate the nucleosynthesis inside the iron core after the shock revival up to explosion in a more consistent manner. Kifonidis et al. (2006) demonstrated that SASI-aided low-mode explosions can most naturally explain the masses and distribution of the synthesized elements observed in SN1987A. The recent 3D results by Hammer et al. (2010) show that the 3D effects that affect the velocity of the ejecta as well as the growth of the Rayleigh–Taylor instability are important to correctly determining the properties of the ejecta.

In the present work, we study explosive nucleosynthesis in a non-rotating 15 $M_\odot$ star with solar metallicity by performing 2D hydrodynamic simulations that model an SASI-aided delayed explosion via a light-bulb scheme. To extract detailed information for the synthesized elements, we follow the abundance evolution by employing a large nuclear reaction network. It should be emphasized that the mass cut and the aspherical distribution of the explosion energy are evaluated from the hydrodynamic simulations, as in our previous work on the nucleosynthesis in magnetohydrodynamically driven SN explosions (Nishimura et al. 2003) where the inelastic scatterings on $^4$He via neutral currents (Haxton et al. 2006) as well as in collapsars (Fujimoto et al. 2007, 2008; Ono et al. 2009).

In Section 2, we describe a numerical code for the hydrodynamic calculation, initial conditions of the progenitor star, and properties of the aspherical explosion. In Section 3, we present a large nuclear reaction network, physical properties of SN ejecta, abundances and masses of the ejecta, and the heavy-nuclei distribution of the SN ejecta. We discuss the uncertainty in the estimate of the abundances and masses and compare and evaluate the abundances with those observed in the Cygnus loop in Section 4. Finally, we summarize our results in Section 5.

2. HYDRODYNAMIC SIMULATIONS OF AN ASPHERICAL NEUTRINO-DRIVEN SUPERNOVA EXPLOSION

2.1. Hydrodynamic Code and Initial Conditions

To calculate the structure and evolution of the collapsing star, we solve the Newtonian hydrodynamic equations

$$\frac{D\rho}{Dt} + \rho \nabla \cdot v = 0,$$  \hspace{1cm} (1)

$$\rho \frac{Dv}{Dt} = -\nabla P - \rho \nabla (\Phi + \Phi_e),$$ \hspace{1cm} (2)

$$\rho \frac{d}{dt} \left( \frac{e}{\rho} \right) = -P \nabla \cdot v + Q_{\text{ti}},$$ \hspace{1cm} (3)

$$\frac{DY_e}{Dt} = Q_{\text{N}},$$ \hspace{1cm} (4)

where $\rho$, $P$, $v$, $e$, and $Y_e$ are the mass density, the pressure, the fluid velocity, the internal energy density, and the electron fraction, respectively. We denote the Lagrange derivative as $D/Dt$. The gravitational potential of fluid and the central object with a mass of $M_{\text{in}}$, $\Phi$ and $\Phi_e$, are evaluated with

$$\Delta \Phi = 4\pi G \rho$$ \hspace{1cm} (5)

and

$$\Phi_e = -\frac{GM_{\text{in}}}{r},$$ \hspace{1cm} (6)

where $G$ is the gravitational constant. We note that $M_{\text{in}}$ continuously increases due to mass accretion through the inner boundary.

$Q_t$ and $Q_N$ are the source terms that describe the rate of change per unit volume in Equations (3) and (4), respectively, and will be summarized in Appendices A and B. In the present study, we take into account the absorption of electron and anti-electron neutrinos as well as neutrino emission through electron and positron captures, electron–positron pair annihilation, nucleon–nucleon bremsstrahlung, and plasmon decays. We assume that the fluid is axisymmetric and that neutrinos are isotropically emitted from the neutrino spheres with given luminosities and with the Fermi–Dirac distribution of given temperatures (Ohnishi et al. 2006). Rates for the absorption of neutrinos and neutrino emission through electron and positron captures are taken from Scheck et al. (2006, Appendix D). Geometrical factor $f_r$ is set to be

$$f_r = \frac{1}{2} \left[ 1 + \sqrt{1 - (R_v/r)^2} \right],$$ \hspace{1cm} (7)

as in Scheck et al. (2006). Here, $R_v$ is the radius of the neutrino sphere and is simply estimated with the relation, $L_v = \frac{1}{4\pi} \sigma T_{\nu}^4$. $4\pi R_v^2$ for a given set of the luminosity $L_v$ and temperature $T_{\nu}$ (Ohnishi et al. 2006), where $\sigma$ is the Stefan–Boltzmann constant. We adopt rates for the emission of neutrinos ($\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$, $\nu_e$) through pair annihilation, bremsstrahlung, and plasmon decays as in Ruffert et al. (1996, Appendix B). Moreover, we include the heating term in $Q_t$ due to the absorption of neutrinos on $^4$He and the inelastic scatterings on $^4$He via neutral currents (Haxton 1988; Ohnishi et al. 2007).

The numerical code for the hydrodynamic calculations employed in this paper is based on the ZEUS-2D code (Stone & Norman 1992; Ohnishi et al. 2006). We use a realistic equation of state (EOS) based on the relativistic mean field theory (Shen et al. 1998). For a lower density regime $(\rho < 10^5$ g cm$^{-3}$), where no data is available in the EOS table with the Shen EOS, we use another EOS, which includes contributions from an ideal gas of nuclei, radiation, and electrons and positrons with arbitrary degrees of degeneracy (Blinnikov et al. 1996). We carefully connect two EOS at $\rho = 10^5$ g cm$^{-3}$ for physical quantities to vary continuously in density at a given temperature (Fujimoto et al. 2006).

First, we perform a spherical symmetric hydrodynamic simulation of the core collapse of a 15 $M_\odot$ non-rotating star with the solar metallicity (Woosley & Weaver 1995) using a hydrodynamic code (Kotake et al. 2004) for about 10 ms after core-bounce, when the bounce shock turns into a standing accretion shock and the protoneutron star (PNS) grows to $\sim 1.2 M_\odot$. Then, we map distributions of densities, temperatures, radial velocities, and electron fractions of the spherical symmetric simulation to initial distribution for 2D hydrodynamic simulations. After the remap, the central region inside 50 km in radius is excised to follow a long-term postbounce evolution (e.g., Scheck et al. 2006; Kifonidis et al. 2006). We impose velocity perturbations on the unperturbed radial velocity in a dipolar manner and follow...
the postbounce evolution. The spherical coordinates are used in our simulations, and the computational domain is extended over 50 km $\leq r \leq 50,000$ km and $0 \leq \theta \leq \pi$, or from the Fe core to inner O-rich layers, which are covered with 500($r$) $\times$ 128($\theta$) meshes. The mass is 3.17 $M_\odot$ in the computational domain. We note that convective motion occurs at the onset of the 2D simulation with the above meshes, while the motion does not appear in the case of coarser mesh points of 500($r$) $\times$ 60($\theta$). Evolution of the explosion energy and of the mass ejection rate are very similar to those for high-resolution simulations with 300($r$) $\times$ 196($\theta$) meshes (50 km $\leq r \leq 3000$ km and $0 \leq \theta \leq \pi$) for about 350 ms after the core bounce. Therefore, the resolution of the simulations with 500($r$) $\times$ 128($\theta$) meshes seems to be appropriate for the present study. However, the resolution may be too low to follow later time evolution of the explosion toward homologous expansion (Gawryszczak et al. 2010). For the high-resolution simulation, the minimum grid size in the radial- and lateral($\theta$)-directions, $\delta r$ and $\delta \theta$, is 1 km and $\pi/196$, respectively, while $\delta r = 1$ km and $\delta \theta = \pi/128$ for our fiducial set (i.e., 500($r$) $\times$ 128($\theta$) mesh points).

### 2.2. Aspherical SN Explosion

We have performed simulations for models with the electron–neutrino luminosities, $L_{\nu e} = 3.7, 3.9, 4.0, 4.2, 4.5, 4.7, and 5.0 \times 10^{52}$ erg s$^{-1}$ for 1–2 s after the core bounce, when a shock front has reached a layer with $r = 10,000$ km in almost all directions. We take the input neutrino luminosities as above because the revival of the stalled bounce shock occurs only for models with $L_{\nu e} \geq 3.9 \times 10^{52}$ erg s$^{-1}$, and also because for models with $L_{\nu e} > 5.0 \times 10^{52}$ erg s$^{-1}$, the star explodes too early for the SASI to grow, as will be discussed later. We set $L_{\nu e} = L_{\nu e}$ and $L_{\nu x} = 0.5L_{\nu e}$, where $L_{\nu e}$ and $L_{\nu x}$ are the luminosities of anti-electron neutrino and other types ($\mu$, $\tau$, anti-$\mu$, and anti-$\tau$), respectively. We consider models with neutrino temperatures, $T_{\nu e}, T_{\nu x}$, and $T_{\nu \chi}$, as 4 MeV, 5 MeV, and 10 MeV, respectively (Ohnishi et al. 2006). The adopted neutrino luminosities are comparable to those with a more accurate transport scheme, but the temperatures are slightly higher (Marek et al. 2009; Marek & Janka 2009). We present hydrodynamic and nucleosynthetic results for cases with lower neutrino temperatures in Section 4.1.

We confirm that the explosion is highly aspherical and $l = 1$ and $l = 2$ modes are dominant as shown in Kifonidis et al. (2006), Ohnishi et al. (2006), and Scheck et al. (2006), although the shape of the explosion strongly depends on numerical details, such as mesh resolution and boundary conditions (Kifonidis et al. 2006; Scheck et al. 2006). The entropy contour of the hydrodynamic simulation is shown in Figure 1 for the case with $L_{\nu e} = 4.5 \times 10^{52}$ erg s$^{-1}$. Most of the SN ejecta have entropy less than 20 $k_B$, where $k_B$ is the Boltzmann constant. Entropy reaches 70 $k_B$ for small amounts of the ejecta.

We find that for models with $L_{\nu e} \geq 3.9 \times 10^{52}$ erg s$^{-1}$, the star explodes aspherically via the neutrino heating aided by SASI. Figure 2(a) shows explosion energies as a function of $L_{\nu e}$ for all the exploded models. The energies are estimated at an epoch of 500 ms after the explosion and slightly increase after the epoch. Kinetic and thermal energies of the explosion are also shown in Figure 2(a). The thermal energies dominate the kinetic ones.

Higher $L_{\nu e}$ makes the onset of explosion earlier and the mass of the PNS ($M_{\text{PNS}}$) smaller. Note that we estimate $M_{\text{PNS}}$ at $t = t_{\text{exp}} + 500$ ms, because $M_{\text{PNS}}$ only slightly increases later than 500 ms after $t_{\text{exp}}$. Here, $t_{\text{exp}}$ indicates the timescale when the explosion sets in, which can typically be estimated when the mass ejection rate at 100 km reaches 0.1 $M_\odot$ s$^{-1}$ in our 2D simulations. Figure 2(b) shows the explosion time $t_{\text{exp}}$ and $M_{\text{PNS}}$ as a function of $L_{\nu e}$. Except for the lowest luminosity model ($L_{\nu e} \leq 3.9 \times 10^{52}$ erg s$^{-1}$), $M_{\text{PNS}}$ is in the range of 1.54–1.70 $M_\odot$. These values are much larger than the so-called mass cuts in the spherical models of 15 $M_\odot$ progenitors by Hashimoto (1995) and Rauscher et al. (2002), which are 1.30 $M_\odot$ and 1.32 $M_\odot$, respectively. The mass of the PNS, however, becomes larger (1.68 $M_\odot$) due to the fallback of ejecta (Rauscher et al. 2002).
3. NUCLEOSYNTHESIS IN SUPERNOVA EJECTA

3.1. Nuclear Reaction Network and Initial Composition

In order to calculate the chemical composition of the SN ejecta, we need the Lagrangian evolution of physical quantities, such as density, temperature, and velocity of the material. We adopt a tracer particle method (Nagataki et al. 1997; Seitenzahl et al. 2010) to calculate the Lagrangian evolution of the physical quantities from the Eulerian evolution obtained from our simulations. The Lagrangian evolution is followed during the 2D aspherical simulation as well as during the spherical collapsing phase. To obtain information on mass elements, 6000 tracer particles are placed in the regions from 300 extending to 10,000 km (the O-rich layer). We have confirmed that the estimated energies and masses of the ejecta with 6000 particles are equal to the ones with 3000 particles within ~1% accuracy, and the obtained abundance profiles are also very similar.

Initial abundances of the particles are set to be those of the star just before the core collapse (Rauscher et al. 2002), in which 1400 nuclei are taken into account. We note that a pre-SN model in Rauscher et al. (2002) has smaller helium, carbon–oxygen, and oxygen–neon core masses, compared with those in Woosley & Weaver (1995), due to coupled effects through the inclusion of mass loss and the revisions of opacity and nuclear inputs (Rauscher et al. 2002). The mass of a particle in a layer is weighted to the mass of the layer. We note that the minimum mass of the particles is ~10^{-4} M_{\odot}. We find that more than one fifth of the particles are ejected due to the aspherical explosion.

Next, we calculate abundances and masses of the SN ejecta. Ejecta that are located on the inner region of the star (r_{ej,cc} < 10,000 km) before the core collapse have maximum temperatures high enough for elements heavier than C to burn explosively. Here, r_{ej,cc} is the radius of the ejecta at the core collapse. We therefore follow abundance evolution of the ejecta from the inner region using a nuclear reaction network, which includes 463 nuclide from neutron, proton to Kr (Fujimoto et al. 2004). We will discuss effects of neutrino interactions on heavy nuclei and uncertainty in nuclear reaction rates on nucleosynthetic results in Sections 4.2 and 4.3, respectively. The abundances of ejecta from the outer region (r_{ej,cc} > 10,000 km) are set to be those before the core collapse (Rauscher et al. 2002). We note that the mass of the outer region, or r_{ej,cc} > 10,000 km, is 10.4 M_{\odot}. Moreover, when temperatures of the ejecta are greater than 9 \times 10^{9} K, we set chemical composition of the ejecta to be that in nuclear statistical equilibrium, whose abundances are expressed with simple analytical expressions, specified by the density, temperature, and electron fraction.

Electron fractions of the ejecta are re-evaluated during SN explosion coupled with the nuclear reaction network. The change in Y_{e} is taken into account through electron and positron captures on heavy nuclei, in addition to electron and positron captures on neutrons and protons, as well as absorption of v_{e} and \bar{v}_{e} on neutrons and protons. The captures and absorptions on neutrons and protons are also taken into account in the hydrodynamic simulations. The rates for the captures and the absorptions are adopted from Fuller et al. (1980, 1982) and Scheck et al. (2006), respectively.

It should be emphasized that post-processing electron fractions are slightly different (up to 10%) from those estimated with hydrodynamic simulations, in which the evolution of electron fractions is followed. This is because abundances of neutrons and protons in the network calculations are slightly different from those estimated with EOS in the hydrodynamic simulations. We note that 463 nuclei are taken into account in the network calculations, while only neutrons, protons, 4 He, and a representative heavier nuclide are evaluated with EOS.

In the neutrino-heating-dominated region, the abundances of neutrons and protons in the network calculations are larger than those evaluated with EOS. Hence, if we perform hydrodynamic simulations, in which abundances of nucleons are reliably evaluated with the reaction network, the neutrino heating rates in the simulations could increase compared to those in the current study, since the neutrino heating through the absorption of v_{e} and \bar{v}_{e} is dominant over the other heating reactions, and the heating rates via the absorption are proportional to the abundances of the nucleons.

The explosion energies also might increase. We emphasize that abundances of SN ejecta chiefly depend on the explosion energy and the mass of SN ejecta from an inner region, not on L_{\nu_{e}}, as will be shown later.

3.2. Physical Properties of SN Ejecta

Maximum densities, \rho_{\text{max}}, and maximum temperatures, T_{\text{max}}, are good indicators for the composition of SN ejecta (Thielemann et al. 1996). Figure 3(a) shows \rho_{\text{max}} as a function of T_{\text{max}} of the ejecta for L_{\nu_{e}} = 4.5 \times 10^{52} \text{ erg s}^{-1}. Most of the ejecta with relatively low densities (< 10^{10} g cm^{-3}) have \rho_{\text{max}} and T_{\text{max}} similar to those of ejecta in the spherical model of core-collapse SNe (Thielemann et al. 1998). For some particles that have very high densities along with high temperatures (\rho_{\text{max}} \geq 10^{10} g cm^{-3} and T_{\text{max}} \geq 10^{10} K), electron captures operate to some extent, so that these particles become slightly neutron-rich, Y_{e}(10,000 km) < 0.48. Here, Y_{e}(10,000 km) represents the electron fraction for tracer particles evaluated when the particles reach r = 10,000 km. This may be a useful quantity for measuring Y_{e} of the ejecta, since Y_{e} closely freezes out at r > 10,000 km (except through \beta-decays at a later epoch). All ejecta with T_{\text{max}} > 10^{10} K fall down to the heating region \lesssim 200–300 km to be heated via neutrinos (Figure 3(b)). For these neutrino-heated ejecta, \rho_{\text{max}} ranges from 10^{8} to 2 \times 10^{10} g cm^{-3}.

For ejecta with higher \rho_{\text{max}}, electron captures on protons proceed more efficiently to make Y_{e} smaller. Time evolution of the physical quantities of such an ejecta is shown during...
the infall of the ejecta near the cooling region \( r < 100 \text{ km} \) in Figure 4(a). As the density and temperature rise to more than \( 10^9 \text{ g cm}^{-3} \) and \( 10^{10} \text{ K} \), respectively, the electron fraction decreases due to the electron captures. When the ejecta start to be released via neutrino heating at \( t = 0.42 \text{ s} \), the electron fraction increases through the absorption of \( \nu_e \) by neutrons. Finally, \( Y_e(10,000 \text{ km}) \) becomes 0.461 for the ejecta.

On the other hand, the electron captures on protons are not efficient for a proton-rich ejecta with \( Y_e(10,000 \text{ km}) = 0.559 \), as shown in Figure 4(b). This is because the densities of the inner region \( r \leq 200 \text{ km} \) are relatively low \( \leq 10^9 \text{ g cm}^{-3} \) due to the mass ejection during an earlier phase \( \geq t_{\text{exp}} \). The electron fraction therefore remains constant and rises from 0.5 to 0.559 via the \( \nu_e \) absorption in an inner region \( r \leq 200 \text{ km} \).

The proton richness in the ejecta is caused by the small energy difference between \( \nu_e \) and \( \bar{\nu}_e \). For \( T_{\nu_e} \) and \( T_{\bar{\nu}_e} \) adopted in our simulations, the relation, \( 4 (m_n - m_p) > \epsilon_{\nu_e} - \epsilon_{\bar{\nu}_e} \), holds, which leads to \( Y_e > 0.5 \) (Fröhlich et al. 2006a), where \( m_n \) and \( m_p \) are masses of neutrons and protons, and \( \epsilon_{\nu_e} \) and \( \epsilon_{\bar{\nu}_e} \) are energies of anti-electron and electron neutrinos, respectively. We note that \( \epsilon_{\nu_e} = 15.8 \text{ MeV} \) for \( T_{\nu_e} = 5 \text{ MeV} \) and \( \epsilon_{\bar{\nu}_e} = 12.6 \text{ MeV} \) for \( T_{\nu_e} = 4 \text{ MeV} \).

Figure 5 shows masses as a function of \( Y_e(10,000 \text{ km}) \) of ejecta from the inner region \( r_{\text{ej}, \text{cc}} \leq 10,000 \text{ km} \). We find that most of the ejecta (98.8%) have electron fractions of 0.49–0.5. Small fractions of the ejecta, 0.9% and 0.3% in mass, are slightly neutron-rich \( (0.46 < Y_e < 0.49) \) and proton-rich \( (0.5 < Y_e < 0.56) \), respectively. Masses of the slightly neutron- and proton-rich ejecta are larger for models with larger \( L_{\nu_e} \), while the mass fractions of these ejecta are comparable for all the models.

### 3.3. Primary and Secondary Ejecta

Electron fraction of the ejecta with the minimum radial position \( r_{\text{min}} \leq 200–300 \text{ km} \) changes due to high neutrino flux and/or efficient \( e^\pm \) capture (Figure 4(a)). Figure 6 shows \( Y_e(10,000 \text{ km}) \) as a function of \( r_{\text{min}} \) of the ejecta for a model with \( L_{\nu_e} = 4.5 \times 10^{52} \text{ erg s}^{-1} \). Hereafter, we refer to the ejecta with \( r_{\text{min}} \leq 200 \text{ km} \) as the primary ejecta, which have high maximum densities \( \geq 10^8 \text{ g cm}^{-3} \) and temperatures \( \geq 10^{10} \text{ K} \) (Figure 3). The ejecta are heated through the neutrino heating. On the other hand, the others are referred as the secondary ejecta, heated chiefly via the shock wave driven by the primary ejecta.
It is true that $Y_e$ of the primary ejecta can change if we adopt a more accurate neutrino-transfer scheme instead of the simplified light-bulb transfer scheme. Abundances of the primary ejecta are therefore highly uncertain because of the uncertainty of their $Y_e$. On the other hand, for the secondary ejecta, $Y_e$ changes chiefly through the neutrino absorptions, but the changes in $Y_e$ are found to be less than 1% for almost all the secondary ejecta. In addition, for our typical 2D models that produce energetic explosions ($L_{\nu_e} \geq 4.5 \times 10^{52} \text{ erg s}^{-1}$), the masses of the primary ejecta occupy only about 2% ($8.7 \times 10^{-3} M_\odot$) of the ejecta from the inner region ($0.41 M_\odot$ for $r_{ej,cc} \leq 10,000 \text{ km}$). Therefore, $Y_e$ of the secondary ejecta, whose mass is much larger than that of the primary ejecta, is unlikely to be largely changed even if we use a more accurate neutrino-transfer scheme. We conclude that masses of abundant nuclei, such as $^{16}\text{O}$, $^{28}\text{Si}$, and $^{56}\text{Ni}$, do not largely change in the SN ejecta. We will discuss this point in Section 4.4.

The value of $Y_e$ of the primary ejecta depends on the epoch of the ejection. The primary ejecta that eject in an early phase (before an epoch of 200–300 ms after the explosion [$t_{ej} = 230 \text{ ms}$]) are mainly neutron-rich, while the primary ejecta are proton-rich in the later phase. The proton-rich primary ejecta corresponds to neutrino-driven winds that are possibly not neutron-rich but proton-rich (Fischer et al. 2010; Hüdepohl et al. 2010). Figure 7 shows $Y_e(10,000 \text{ km})$ as a function of $t(T_{\text{max}})$ of ejecta for a model with $L_{\nu_e} = 4.5 \times 10^{52} \text{ erg s}^{-1}$, where $t(T_{\text{max}})$ is defined as the time when the temperature of ejecta attains its maximum value, $T_{\text{max}}$. We note that the gas starts to be ejected just after $t = t(T_{\text{max}})$, as shown in Figure 4. We note that the dependence of $Y_e$ on the epoch of the ejection also appears in a spherical simulation of SN explosion of a star with an ONeMg core using an elaborated code taking into account an accurate neutrino transfer scheme (Kitaura et al. 2006; Wanajo et al. 2009).

**3.4. Masses and Abundances of Ejecta**

Masses of nuclei, such as $^{56}\text{Ni}$, $^{57}\text{Ni}$, $^{58}\text{Ni}$, and $^{44}\text{Ti}$, have been estimated in some SN remnants. For SN1987A, masses of $^{56}\text{Ni}$ and $^{44}\text{Ti}$ are deduced to be $\pm 0.07 M_\odot$ (Shigeyama et al. 1988; Woosley 1988) and $(1–2) \times 10^{-4} M_\odot$ (Nagataki 2000 and references therein), respectively. The estimated mass of $^{44}\text{Ti}$ is comparable to that in Cas A ($1.6^{+0.3}_{-0.5} \times 10^{-4} M_\odot$; Rau et al. 2006) and greater than that in the youngest Galactic SN remnant G1.9+0.3 ($1–7) \times 10^{-4} M_\odot$; Borkowski et al. 2010), which may originate from a Type Ia event. Figure 8 shows masses of $^{56}\text{Ni}$ and $^{44}\text{Ti}$ of the secondary ejecta ejected from the inner region ($r_{ej,cc} \leq 10,000 \text{ km}$), $M_{ej,cc}$, and masses of $^{56}\text{Ni}$ and $^{44}\text{Ti}$. The masses of the ejecta from the inner region of $r_{ej,cc} \leq 10,000 \text{ km}$ (solid line with filled squares) and masses of $^{56}\text{Ni}$ (dashed line with filled circles) and $^{44}\text{Ti}$ (dotted line with filled triangles) are shown with a value multiplied by a factor of 1, 10, and $10^4$, respectively. We find that masses of the ejecta and $^{56}\text{Ni}$ roughly correlate with the neutrino luminosities. The masses of $^{56}\text{Ni}$ and $^{44}\text{Ti}$ are less than and comparable to those in the spherical model (Rauscher et al. 2002), or $0.11 M_\odot$ and $1.4 \times 10^{-5} M_\odot$, respectively. Note that the explosion energy is $1.2 \times 10^{51} \text{ erg}$ in the model. We find that $^{44}\text{Ti}$ relative to $^{56}\text{Ni}$ is much smaller than those in the solar system, in SN1987A, and in Cas A, but comparable to that in the SN remnant G1.9+0.3. $^{56}\text{Ni}$ relative to $^{56}\text{Ni}$ is comparable to that in the solar system and in SN1987A, while $^{58}\text{Ni}$ relative to $^{56}\text{Ni}$ is overproduced compared with that in the solar system and in SN1987A, because it is abundantly produced in the slightly neutron-rich ejecta (Hashimoto 1995; Nagataki et al. 1997).

It should be emphasized that $^{44}\text{Ti}$ is underproduced in our simulations of the SN explosion, contrary to the overproduction in the previous 2D results (Nagataki et al. 1997), in which the explosion energy is aspherically and artificially added and the remnant mass is set to a value in order that the ejected mass is $10^5 M_\odot$. Therefore, we use a more accurate neutrino-transfer scheme instead of the simplified light-bulb transfer scheme. Abundances of the primary ejecta occupy only about 2% ($8.7 \times 10^{-3} M_\odot$) of the ejecta from the inner region ($0.41 M_\odot$ for $r_{ej,cc} \leq 10,000 \text{ km}$), $M_{ej,cc}$, and masses of $^{56}\text{Ni}$ and $^{44}\text{Ti}$. The masses of the ejecta from the inner region of $r_{ej,cc} \leq 10,000 \text{ km}$ (solid line with filled squares) and masses of $^{56}\text{Ni}$ (dashed line with filled circles) and $^{44}\text{Ti}$ (dotted line with filled triangles) are shown with a value multiplied by a factor of 1, 10, and $10^4$, respectively. We find that masses of the ejecta and $^{56}\text{Ni}$ roughly correlate with the neutrino luminosities. The masses of $^{56}\text{Ni}$ and $^{44}\text{Ti}$ are less than and comparable to those in the spherical model (Rauscher et al. 2002), or $0.11 M_\odot$ and $1.4 \times 10^{-5} M_\odot$, respectively. Note that the explosion energy is $1.2 \times 10^{51} \text{ erg}$ in the model. We find that $^{44}\text{Ti}$ relative to $^{56}\text{Ni}$ is much smaller than those in the solar system, in SN1987A, and in Cas A, but comparable to that in the SN remnant G1.9+0.3. $^{56}\text{Ni}$ relative to $^{56}\text{Ni}$ is comparable to that in the solar system and in SN1987A, while $^{58}\text{Ni}$ relative to $^{56}\text{Ni}$ is overproduced compared with that in the solar system and in SN1987A, because it is abundantly produced in the slightly neutron-rich ejecta (Hashimoto 1995; Nagataki et al. 1997).

It should be emphasized that $^{44}\text{Ti}$ is underproduced in our simulations of the SN explosion, contrary to the overproduction in the previous 2D results (Nagataki et al. 1997), in which the explosion energy is aspherically and artificially added and the remnant mass is set to a value in order that the ejected mass of $^{56}\text{Ni}$ reproduces the mass observed in SN1987A in Nagataki et al. (1997). The underproduction is possibly caused by a larger remnant mass in our models. This is because the explosion energy and the ratio of the explosion energy on the polar axis to that on the equatorial plane are comparable to those evaluated in our simulation, and masses of $^{44}\text{Ti}$ as well as $^{56}\text{Ni}$ have been shown to strongly depend on the value of the remnant mass in 2D calculations (Young et al. 2006).

In order to compare estimated abundances with those of the solar system (Anders & Grevesse 1989), we have integrated masses of nuclei over all the ejecta to evaluate the abundances.
of the SN ejecta. Figure 9(a) shows overproduction factors after decays as a function of the mass number, A, for \( L_\nu = 4.5 \times 10^{52} \text{erg s}^{-1} \), in which the explosion energy is \( 1.3 \times 10^{51} \text{erg s}^{-1} \) (Figure 2(a)) and \( M_{\text{ej,in}} = 0.41 M_\odot \) (Figure 8). We find that the abundance patterns of the SN ejecta is similar to that of the solar system. We note that \(^{18}\text{O}\), which is underproduced in the ejecta, can be abundantly synthesized in Type Ia SNe, and that \(^{15}\text{N}\) and \(^{19}\text{F}\) are comparatively produced if neutrino effects are taken into account (Woosley & Weaver 1995), as shown later in Figure 14. We point out that \(^{64}\text{Zn}\), which is underproduced in the spherical case (Rauscher et al. 2002), is abundantly produced in slightly aspherical systems. We note that \(^{17}\text{O}\), which is underproduced in the ejecta, has also appeared in the spherical models (Hashimoto 1995; Nagataki et al. 1995; Nomoto 1990). The time from the core collapse to the explosion, \( t_{\text{exp}} \), is \( \sim 0.2 \text{s} \) (Figure 2(b)), which is enough to grow a low-mode SASI. The growth is appropriate for explaining the distributions and velocities of nuclei observed in SN1987A, as shown in Kifonidis et al. (2006).

### 3.5. Aspherical Infall

The value of \( M_{\text{ej,in}} \) depends on \( E_{\text{exp}} \) as well as aspherical matter infall to the neutron star, while the masses of \(^{56}\text{Ni}\) correlate with \( E_{\text{exp}} \) (Figures 2(a) and 8). Figure 10 shows the position of SN ejecta before the core collapse for a model with \( L_\nu = 4.5 \times 10^{52} \text{erg s}^{-1} \). Filled squares, filled circles, filled triangles, and open triangles indicate ejecta that correspond to the complete Si burning \( (T_{\text{max,9}} \geq 5) \) where \( T_{\text{max,9}} = T_{\text{max}}/10^9 \text{K} \), the incomplete Si burning \( (4 \leq T_{\text{max,9}} < 5) \); the O burning \( (3.3 \leq T_{\text{max,9}} < 4) \), and the C/Ne burning \( (1 \leq T_{\text{max,9}} < 3.3) \), respectively. We find that whole of the iron core \( (\sim 2000 \text{km}) \) collapses to the PNS and that the infall of material to the PNS is aspherical; larger amounts of the gas infall from the upper hemisphere compared with from the lower half. Larger amounts of material aspherically infall to the neutron star for a model with \( L_\nu = 4.5 \times 10^{52} \text{erg s}^{-1} \) compared with the model with \( L_\nu = 5.0 \times 10^{52} \text{erg s}^{-1} \). Though \( E_{\text{exp}} \) is comparable, \( M_{\text{ej,in}} \) for \( L_\nu = 4.5 \times 10^{52} \text{erg s}^{-1} \) is smaller than that for \( L_\nu = 5.0 \times 10^{52} \text{erg s}^{-1} \).
3.6. Aspherical Distribution of Energy and Nuclei

Distributions of the energy and abundances of SN ejecta are highly aspherical, as one can expect from the entropy distribution (Figure 1). The distributions are shown for $L_{\nu} = 4.5 \times 10^{52}$ erg s$^{-1}$ at $t = 1.5$ s in Figure 11. The energy and abundances are presented in units of $10^{47}$ erg and normalized by the solar abundances, respectively, for logarithm scale. It should be emphasized that the distributions change during the later expansion phase (Kifonidis et al. 2006; Gawryszczak et al. 2010). We note that the secondary ejecta located in outer layers ($r_{\text{ej,cc}} > 10,000$ km) are not shown in the figures. The high-energy ejecta concentrates on the shock front, in particular in polar regions ($\theta < \pi/4, \theta > 3\pi/4$). Both Si and O burning proceed to produce $^{56}$Ni and $^{28}$Si abundantly in the region. We emphasize that a deformed shell structure forms with Fe, Si, and O layers from inside to outside in the secondary ejecta heated via the shock wave. Composition of the secondary ejecta depends chiefly on $T_{\text{max}}$, which is determined via the shock heating. The shell structure is therefore formed in the secondary ejecta, because $T_{\text{max}}$ gradually decreases from inner to outer layers, where the structure is highly aspherical and deformed. We note that Ca is co-produced with $^{28}$Si, as in the spherical models.

The primary ejecta, however, do not form such a structure. This is because physical properties of the ejecta, which are mainly heated via neutrinos to be $>10^{10}$ K (Figure 4), are independent from their positions; thus, their compositions are also independent. A fraction of the primary ejecta that are mainly composed of Fe and Ni is found to be mixed into the deformed layers, while a small amount of the secondary ejecta falls toward the neutron star near the equatorial plane. Consequently, the ejecta with abundant O and/or Si appear in the central region ($r < 2000$ km). Due to such mixings and infall, some of the Si- and O-rich ejecta can penetrate more deeply into the Fe-rich ejecta, which is in sharp contrast to spherical models.

We have averaged energy and masses of nuclei of the inner ejecta over a radial direction. We note that the energy and masses of the outer ejecta ($r_{\text{ej,cc}} > 10,000$ km) are not included in an averaging procedure. If we include ejecta from the outer layers, the asymmetry of $^{16}$O becomes small because of the existence of spherically distributed $^{16}$O in the layers. Figure 12(a) shows the averaged energy and masses of $^{16}$O, $^{28}$Si, and $^{56}$Ni. Asphericity of the energy is prominent; the ratio of the energy at $\theta = \pi/2$ to that at $\theta = 0$ is $\sim 4$, and the ratio of the maximum to the minimum energy is $\sim 8$. We note that the ratios are comparable to those in 2D models (Nagataki et al. 1997, models A2 and A3). Only a small amount of $^{56}$Ni exists near the equatorial plane ($\theta \sim (0.8–1.6)$ rad.), where the energy is lower compared with that near the polar axis. The asymmetry of $^{28}$Si mass is similar to that of the energy and smaller than that of $^{56}$Ni. Mass distribution of $^{44}$Ti is very similar to that of $^{56}$Ni, as...
in spherical models, because both nuclei are produced in the secondary ejecta through $\alpha$-rich freezeout. The peaks of $^{28}$Si mass around the polar directions are misaligned with those of $^{56}$Ni around $\theta \sim \pi/4$ and $3\pi/4$, although the misalignment could disappear due to the lateral motion of the ejecta during the later explosion phase ($\lesssim 300$ s) as pointed out in Gawryszczak et al. (2010). We note that the lateral motion does not follow during the later phase in the present study.

In addition to the mass fractions of nuclei, number ratios of nuclei relative to O ($^{12}$O) are important quantities, because the ratios are indicative of the production and destruction mechanism of nuclei. Angular distributions of the number ratios are shown for the inner ejecta ($r_{ij,cc} \lesssim 10,000$ km) averaged over a radial direction for $L_{\nu_e} = 4.5 \times 10^{52}$ erg s$^{-1}$ at $t = 1.5$ s. Solid, dashed, dotted, and dash-dotted lines indicate the ratios of $^{20}$Ne, $^{28}$Si, iron, and $^{24}$Mg, respectively. The iron consists of $^{56}$Ni, $^{56}$Fe, and $^{54}$Fe (dotted line) is similar to that in the spherical case, because Mg is produced during both the stellar evolution and the explosion through the same process, or the O/Ne burning, with which N(Mg)/N(O) becomes $\sim 0.05$ for a 15 $M_{\odot}$ progenitor. Here, N(X) is the number of nuclei, X. The ratio of Si ($^{28}$Si, dashed line) anti-correlates with that of Fe (the sum of $^{56}$Ni, $^{56}$Fe, and $^{54}$Fe; dotted line). This is because Fe is produced via the burning of Si.

### 4. DISCUSSIONS

#### 4.1. Dependences on Neutrino Temperatures

We have assumed the neutrino spheres with given luminosities and with the Fermi–Dirac distribution of neutrino temperatures, and considered models with neutrino temperatures, $T_{\nu_e}$, $T_{\nu_x}$, and $T_{\bar{\nu}_e}$ as 4 MeV, 5 MeV, and 10 MeV, respectively. The temperatures are, however, slightly lower and change with time, as shown in simulations using a more elaborate numerical code (Marek et al. 2009; Marek & Janka 2009). In order to evaluate the dependence of hydrodynamic and nucleosynthetic results on neutrino temperatures, we have performed simulations for three models with lower neutrino temperatures: $T_{\nu_e} = 3.6$ MeV, $T_{\nu_x} = 4.5$ MeV, and $T_{\bar{\nu}_e} = 9$ MeV, and $L_{\nu_e} = (4.0, 4.5, \text{and } 5.0) \times 10^{52}$ erg s$^{-1}$. We find that, for a given $L_{\nu_e}$, the explosions are much weaker for a lower $T_e$ model, in spite of only 10% changes in neutrino temperatures. Table 1 summarizes hydrodynamic and nucleosynthetic properties for the nine models that produce explosions, in which $T_e$ s are changed in the three ways mentioned above. The explosion energies are $(0.35, 0.35, \text{and } 0.45) \times 10^{51}$ erg for models with $L_{\nu_e} = (4.0, 4.5, \text{and } 5.0) \times 10^{52}$ erg s$^{-1}$, respectively. Mass of the ejecta from the inner region ($r_{ij,cc} \lesssim 10,000$ km), $M_{ej,\text{in}}$, and $^{56}$Ni mass are $0.11\sim0.21 M_{\odot}$ and $0.011\sim0.017 M_{\odot}$, respectively. Figure 13 shows overproduction factors for the low $T_e$ model with $L_{\nu_e} = 5.0 \times 10^{52}$ erg s$^{-1}$, which is the explosion energy and the ejecta mass from the inner region are comparable to the above model, or $0.45 \times 10^{51}$ erg and 0.020 $M_{\odot}$, respectively. We conclude that the abundances of the ejecta do not directly depend on $T_e$ but mainly depend on the explosion energy and the ejecta mass.

#### 4.2. Effects of the $\nu$ Interactions on Heavy Nuclei

Effects of neutrino interactions on SN ejecta have previously been investigated for spherical SN explosion (Woosley et al. 2010). We have considered the $\nu$ interactions on heavy nuclei in the framework of the present hydrodynamic and nucleosynthetic simulations. Table 1 lists properties of the explosions for the nine models that produce explosions, in which $T_e$ is changed in the three ways mentioned above. The explosion energies are $(0.35, 0.35, \text{and } 0.45) \times 10^{51}$ erg for models with $L_{\nu_e} = (4.0, 4.5, \text{and } 5.0) \times 10^{52}$ erg s$^{-1}$, respectively. Mass of the ejecta from the inner region ($r_{ij,cc} \lesssim 10,000$ km), $M_{ej,\text{in}}$, and $^{56}$Ni mass are $0.11\sim0.21 M_{\odot}$ and $0.011\sim0.017 M_{\odot}$, respectively. Figure 13 shows overproduction factors for the low $T_e$ model with $L_{\nu_e} = 5.0 \times 10^{52}$ erg s$^{-1}$, which is the explosion energy and the ejecta mass from the inner region are comparable to the above model, or $0.45 \times 10^{51}$ erg and 0.020 $M_{\odot}$, respectively. We conclude that the abundances of the ejecta do not directly depend on $T_e$ but mainly depend on the explosion energy and the ejecta mass.

### Table 1

| $T_{\nu_e}$ | $T_{\nu_x}$ | $L_{\nu_e}$ | $E_{\exp}$ | $t_{\exp}$ | $M_{pNS}$ | $M_{ej,\text{in}}$ | $M(^{56}\text{Ni})$ | $M(^{44}\text{Ti})$ |
|------------|------------|-------------|------------|------------|------------|----------------|----------------|----------------|
| 4.0        | 3.9        | 0.45        | 1.20       | 0.84       | 0.20       | 1.85e-2       | 5.29e-6       |
| 4.0        | 4.0        | 0.80        | 0.31       | 1.70       | 0.34       | 2.72e-2       | 1.31e-5       |
| 4.0        | 5.0        | 1.02        | 0.31       | 1.65       | 0.39       | 4.51e-2       | 8.04e-6       |
| 4.0        | 4.5        | 1.30        | 0.23       | 1.62       | 0.41       | 5.49e-2       | 1.52e-5       |
| 4.0        | 4.7        | 1.03        | 0.21       | 1.54       | 0.49       | 5.10e-2       | 1.14e-5       |
| 4.0        | 5.0        | 1.30        | 0.18       | 1.56       | 0.48       | 5.44e-2       | 1.05e-5       |
| 3.6        | 4.5        | 0.35        | 1.41       | 1.93       | 0.11       | 1.10e-2       | 6.95e-6       |
| 3.6        | 4.5        | 0.35        | 1.19       | 1.87       | 0.17       | 1.40e-2       | 5.53e-6       |
| 3.6        | 4.5        | 0.50        | 0.45       | 0.62       | 0.21       | 1.73e-2       | 6.38e-6       |

**Notes.** Each column shows $T_{\nu_e}$, $T_{\nu_x}$, $L_{\nu_e}$, $E_{\exp}$, $t_{\exp}$, and masses ($M_{pNS}$, $M_{ej,\text{in}}$, $M(^{56}\text{Ni})$, and $M(^{44}\text{Ti})$), in units of MeV, MeV, $10^{52}$ erg s$^{-1}$, $10^{51}$ erg, s, and $M_{\odot}$, respectively.

#### References

Fujimoto et al. (2010). We note that the lateral motion does not follow during the later explosion phase ($\lesssim 300$ s) as pointed out in Gawryszczak et al. (2010). We note that the lateral motion does not follow during the later phase in the present study.
although the production of these elements is efficient in the
outer layers through neutrino interactions (Woosley et al. 1990; Yoshida et al. 2008). Moreover, neutrino absorptions on D may be important for nucleosynthesis as well as the dynamics of the SN explosion (Nakamura et al. 2009).

4.3. Dependences of Abundances on Nuclear Reaction Rates
We have adopted reaction rates mainly taken from the REACLIB database in our nuclear reaction network presented in Section 3.1. The database was recently updated and is continuously maintained by the Joint Institute for Nuclear Astrophysics (JINA) REACLIB project (Cyburt et al. 2010). We have calculated abundances of the ejecta for $L_{\nu_e} = 4.5 \times 10^{52}$ erg s$^{-1}$, adopting reaction rates taken from the JINA REACLIB V1.0 database. Figure 15 shows ratios of the abundances of the ejecta, adopting reaction rates in the JINA REACLIB V1.0 database to those in REACLIB database. We find that the differences are small, up to 30\%, between abundances with the JINA REACLIB V1.0 and REACLIB databases. If we use newly evaluated $\alpha$-capture rates on $^{40}$Ca and $^{44}$Ti, yields of $^{44}$Ti are likely to be lower (Hoffman et al. 2010).

4.4. Uncertainty of Abundances of the Ejecta
As shown in Section 3.3, the ejecta consist of the primary ejecta, whose $Y_e$ and thus abundances are highly uncertain, and the secondary ejecta, which have a relatively definite composition and are much heavier than the primary ejecta. In order to clarify the uncertainty of the estimate of abundances of the ejecta, we have evaluated abundances of the ejecta integrated over ejecta without the primary ejecta and have compared the abundances with those summed over all the ejecta (Figure 9(a)). Figure 16 shows ratios of the abundances of all the ejecta to those without the primary ejecta for $L_{\nu_e} = 4.5 \times 10^{52}$ erg s$^{-1}$. We find that abundances of most of the nuclei with $A \leq 70$ do not greatly change within a factor of two. The overproduction factors are therefore very similar to those in Figure 9(a). The ratios are greater than 1.5 for $^{43}$Ca, $^{45}$Sc, $^{47}$Ti, $^{50}$Ti, $^{54}$Cr, $^{60}$Ni, $^{64}$Zn, $^{66}$Zn, and $^{70}$Ge, in particular, 9.0, 2.8, and 3.1 for $^{64}$Zn, $^{66}$Zn, and $^{70}$Ge, respectively. These nuclei are therefore chiefly synthesized in the primary ejecta, not in the secondary ejecta. Moreover, $^{50}$Ti, $^{54}$Cr, $^{60}$Ni, $^{64}$Zn, $^{66}$Zn, and $^{70}$Ge are synthesized in neutron-rich primary ejecta, while $^{43}$Ca, $^{45}$Sc, $^{47}$Ti, and $^{60}$Ni...
are abundantly produced in proton-rich primary ejecta through \( p\nu \)-processes (Fröhlich et al. 2006a, 2006b; Pruet et al. 2006; Wanajo 2006).

In short, abundances of \(^{64}\text{Zn}\), \(^{66}\text{Zn}\), and \(^{70}\text{Ge}\) are highly uncertain, but those of the other nuclei are relatively definite. If a fraction of the primary ejecta could become much larger, abundances might be highly uncertain, in particular for the nuclei that are produced in the primary ejecta. The fraction is, however, unlikely to be much larger, because the ejection of the secondary ejecta is driven by the shock wave caused by the primary ejecta. It should be noted that the mass fractions of the primary ejecta to the secondary ejecta are comparable for all the models, although the explosion energies and ejected masses from the inner region are diverse among the models.

Moreover, matter near the PNS is blown off via neutrino-driven winds during later evolution of the star (>2 s). The ejecta could be proton-rich rather than neutron-rich (\( Y_e \sim 0.5–0.6 \)) and have high entropy (>50\( k_B \); Fischer et al. 2010; Hüdepohl et al. 2010). The \( p\nu \)-process could operate in the ejecta to synthesize light \( p \)-nuclei (Fröhlich et al. 2006a, 2006b; Pruet et al. 2006; Wanajo 2006). However, the process may not make large contributions to the ejected masses of abundant nuclei, since the mass ejection rate through the winds is small at the later epoch (Hüdepohl et al. 2010).

4.5. Comparison with Observations of Abundances of SN Remnants

Recently, Kimura et al. (2009) and Uchida et al. (2009) analyzed the metal distribution of the Cygnus loop using the data obtained by the Suzaku and XMM-Newton observations. The progenitor of the Cygnus loop is a core-collapse SN explosion whose progenitor mass ranges from \( \sim 12–15 M_\odot \). The ejecta distributions are asymmetric to the geometric center; the ejecta of O and Ne are distributed more in the northwest rim, while the ejecta of Si and Fe are distributed more in the southwest of the Cygnus loop. Since the material in this middle-aged SN remnant has not yet been completely mixed, the observed asymmetry is considered to still remain a trace of inhomogeneity produced at the moment of explosion. This evidence may allow us to speculate that the asymmetry of the heavy element observed in the Cygnus loop may come from globally asymmetric explosions explored in this work. We try to seek relevance in the following.

Figure 17 shows number ratios of Ne, Mg, Si, and Fe relative to O observed in the Cygnus loop and those of ejecta for \( L_{\nu_e} = 4.5 \times 10^{52} \) erg s\(^{-1}\) at \( t = 1.5 \) s. The solid line indicates ratios observed in the Cygnus loop, while dotted, dashed, and dash-dotted lines indicate ratios averaged over ejecta from an inner equatorial region \( (r_{\text{ej,cc}} \leq 10,000 \) km and \( \pi/3 < \theta < 2\pi/3 \)), ejecta from an inner polar region \( (r_{\text{ej,cc}} \leq 10,000 \) km and \( \theta \geq 2\pi/3 \)) and all the ejecta, respectively. (A color version of this figure is available in the online journal.)

\[ \frac{N(\text{Ne})}{N(O)} \text{ and } \frac{N(\text{Mg})}{N(O)} \text{ are independent from an averaging region and are comparable to all our aspherical models. These ratios are therefore concluded to be determined during the hydrostatic evolution of the progenitor and are thus a good indicator for the progenitor mass. On the other hand, ratios of Si and Fe relative to O are much higher in the inner region, in particular in the inner lower region, compared with the outer region (>10,000 km). } \]

\[ \frac{N(\text{Si})}{N(O)} \text{ and } \frac{N(\text{Fe})}{N(O)} \text{ averaged over all the ejecta are much smaller than those observed in the Cygnus loop. Therefore, incomplete radial mixing of Fe and Si with O is required to explain the high Fe and Si ratios in the Cygnus loop. In fact, the averaged abundances of the Cygnus loop have a correlation with the Si- and Fe-rich regions (Kimura et al. 2009; Uchida et al. 2009). Our results suggest that the abundance ratios for Ne and Mg as well as for Si and Fe observed in the Cygnus loop could be well reproduced with the SN ejecta from the inner region of a 15 \( M_\odot \) progenitor.}

4.6. Three-dimensional Effects

In order to draw a robust conclusion to the findings obtained in the current 2D simulations, it is indispensable to move on to 3D simulations (e.g., Blondin & Mezzacappa 2007; Iwakami et al. 2008, 2009; Nordhaus et al. 2010; Wongwathanarat et al. 2010).
In 2D, the growth of SASI and the large-scale convection tend to develop along the coordinate symmetry axis preferentially, thus suppressing the anisotropies in explosions as well as in the resulting explosive nucleosynthesis. An encouraging piece of news to us is that the 3D simulations cited above are, at least, in favor of the SASI-aided low-mode explosions, in which the resulting explosive nucleosynthesis.

By incorporating the present scheme into the 3D simulations of Iwakami et al. (2008, 2009), we plan to clarify these issues as a sequel to this study.

5. SUMMARY

We have investigated the explosive nucleosynthesis in the delayed neutrino-driven, aspherical SN explosion aided by SASI, based on 2D, axisymmetric hydrodynamic simulations of the explosion of a non-rotating $15 M_\odot$ star. We employed a hydrodynamic code with a simplified light-bulb neutrino transport scheme. We have approximately taken into account neutrino heating and cooling as well as the evolution of electron fraction due to weak interactions, both in the hydrodynamic simulations and nucleosynthetic calculations. Neutrinos are assumed to be isotropically emitted from the neutrino spheres with given luminosities and with the Fermi–Dirac distribution of given temperatures. We have performed simulations with the temperatures and luminosities of $\nu_e$, $\bar{\nu}_e$, and $\nu_x$ constant in time. We have followed abundance evolution of SN ejecta using the nuclear reaction network coupled with an evolution equation of the electron fraction of the ejecta.

We summarize our results as follows:

1. The stalled shock revives due to the neutrino heating aided by SASI for cases with $L_{\nu_e} \geq 3.9 \times 10^{52}$ erg s$^{-1}$, and the aspherical shock passes through the outer layers of the star ($\geq 10,000$ km). Evaluated explosion energies roughly correlate with neutrino luminosities. For models with larger luminosities, the explosion occurs earlier and the mass of a neutron star becomes lighter.

2. The whole of the iron core of the progenitor collapses to the PNS. The infall of material to the star is aspherical. Larger amounts of the gas infall from directions with lower explosion energies.

3. Abundances of the neutrino-heated ejecta are highly uncertain, particularly for neutron-rich ones, due to uncertainty in the estimate of $Y_e$. On the other hand, the shock-heated ejecta has definite abundances, which depend mainly on the maximum temperature. The uncertainty in the estimate of the masses and abundances of abundant nuclei in the SN ejecta is small because of the small fraction of the neutrino-heated ejecta.

4. The abundance pattern of the SN ejecta is similar to that of the solar system for cases with the mass of the ejecta from the inner region ($\leq 10,000$ km), $M_{\odot, in} = (0.4–0.5) M_\odot$, which corresponds to models with a high explosion energy of $\geq 10^{51}$ erg. Masses of a neutron star remnant, estimated to be $(1.54–1.62) M_\odot$ for these models, are comparable to the baryonic mass of the neutron star observed in neutron-star binaries. $E_{\nu \nu}/M_\odot$ evaluated for the models ($(0.92–1.2) \times 10^{50}$ erg $M_\odot^{-1}$) are comparable to the estimate in SN1987A.

5. Underproduction of $^{44}$Ti and overproduction of $^{62}$Ni, which appear in spherical models, are also shown in our 2D calculations. The overproduction of $^{62}$Ni possibly inherits the uncertainty not only in the progenitor model but also of the change in $Y_e$ during the SN explosion. On the other hand, $^{64}$Zn, which is underproduced in a spherical model, is found to be abundantly produced in our 2D model, although the abundance and mass are uncertain.

6. Distributions of nuclei and energy are highly aspherical in the SN ejecta, although the progenitor is non-rotating and has spherical symmetric configuration. The shock-heated ejecta forms an aspherical and deformed shell-like structure composed of Fe, Si, and O from inside to outside. The neutrino-heated ejecta do not have any definite structure and fractions of the ejecta are mixed into the shell-like structure of the shock-heated ejecta.

7. The asymmetry of the $^{28}$Si mass distribution is similar to that of the explosion energy and smaller than that of $^{56}$Ni. $^{40}$Ca and $^{44}$Ti are accompanied by $^{28}$Si and $^{56}$Ni, respectively.

APPENDICES

In Appendices A and B, we summarize our treatment of the rate of change in $Y_e$ and energy per unit volume, $Q_e$, and $Q_{E\nu}$, respectively. We take into account the absorption of electron and anti-electron neutrinos as well as neutrino emission through electron and positron captures, electron–positron pair annihilation, nucleon–nucleon bremsstrahlung, and plasmon decays. Moreover, we include the heating term in $Q_e$ due to the absorption of neutrinos on $^4$He and the inelastic scatterings on $^4$He via neutral currents (Haxton 1988; Ohnishi et al. 2007).

We chiefly follow the treatments in Appendix D of Scheck et al. (2006) and in Appendix B of Ruffert et al. (1996 and references therein).

APPENDIX A

THE RATE OF CHANGE IN SPECIFIC ENERGY, $Q_e$

A.1. Neutrino Absorption Processes

The heating rate per unit volume through the absorption of $\nu_e$ on neutrons is described as

$$Q_{\nu e} = \sigma c^2 \frac{L_{\nu e}}{4\pi r^2 c^2 f_{\nu e}} \left( \frac{\epsilon_{\nu e}^3}{\langle \epsilon_{\nu e} \rangle} \right) + \frac{\Delta^2 (\langle \epsilon_{\nu e} \rangle)}{\langle \epsilon_{\nu e} \rangle} \Theta(\langle \epsilon_{\nu e} \rangle), \quad (A1)$$

where $\Delta = (m_n - m_p)c^2$, $n_n$ is the number density of neutrons and $\sigma = 4G_F^2 m_n^2 \beta^2 / \pi c^2 = 1/3 (3\alpha_0 c^2 + 1) \sigma_0 (m_n c^2)^2$, with the Fermi coupling constant $G_F$, the reduced Planck constant $\hbar$, the electron mass $m_e$, $\alpha_0 = 1.254$, and $\sigma_0 = 1.76 \times 10^{-44}$ cm$^2$. 

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We have followed abundance evolution of SN ejecta using the nuclear reaction network coupled with an evolution equation of the electron fraction of the ejecta.

We summarize our results as follows:

1. The stalled shock revives due to the neutrino heating aided by SASI for cases with $L_{\nu_e} \geq 3.9 \times 10^{52}$ erg s$^{-1}$, and the aspherical shock passes through the outer layers of the star ($\geq 10,000$ km). Evaluated explosion energies roughly correlate with neutrino luminosities. For models with larger luminosities, the explosion occurs earlier and the mass of a neutron star becomes lighter.

2. The whole of the iron core of the progenitor collapses to the PNS. The infall of material to the star is aspherical. Larger amounts of the gas infall from directions with lower explosion energies.

3. Abundances of the neutrino-heated ejecta are highly uncertain, particularly for neutron-rich ones, due to uncertainty in the estimate of $Y_e$. On the other hand, the shock-heated ejecta has definite abundances, which depend mainly on the maximum temperature. The uncertainty in the estimate of the masses and abundances of abundant nuclei in the SN ejecta is small because of the small fraction of the neutrino-heated ejecta.

4. The abundance pattern of the SN ejecta is similar to that of the solar system for cases with the mass of the ejecta from the inner region ($\leq 10,000$ km), $M_{\odot, in} = (0.4–0.5) M_\odot$, which corresponds to models with a high explosion energy of $\geq 10^{51}$ erg. Masses of a neutron star remnant, estimated to be $(1.54–1.62) M_\odot$ for these models, are comparable to the baryonic mass of the neutron star observed in neutron-star binaries. $E_{\nu \nu}/M_\odot$ evaluated for the models ($(0.92–1.2) \times 10^{50}$ erg $M_\odot^{-1}$) are comparable to the estimate in SN1987A.

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APPENDICES

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We chiefly follow the treatments in Appendix D of Scheck et al. (2006) and in Appendix B of Ruffert et al. (1996 and references therein).
Here, the $n$th energy moment of $v_\gamma$, $\langle \epsilon^n_\gamma \rangle$, is given by

$$\langle \epsilon^n_\gamma \rangle = (k_B T_\gamma)^n \frac{F_{2n}(\eta_e)}{F_2(\eta_e)}, \quad (A2)$$

and a factor for the Pauli blocking, $\Theta(\langle \epsilon^n_\gamma \rangle)$, is approximated as

$$\Theta(\langle \epsilon^n_\gamma \rangle) = 1 - f_{FD} \left( \frac{\langle \epsilon^n_\gamma \rangle + \Delta}{k_B T_\gamma \eta_e} \right), \quad (A3)$$

where $\eta_e$ is the chemical potential of $v_e$ in units of $k_B T_\gamma$ and is set to be 0, $\eta_e$ is the chemical potential of electrons in units of $k_B T_\gamma$, $k_B$ is the Boltzmann constant, and $F_n(\eta)$ is defined as

$$F_n(\eta) \equiv \int_0^\infty dx x^n f_{FD}(x, \eta), \quad (A4)$$

using the Fermi–Dirac distribution function,

$$f_{FD}(x, \eta) = \frac{1}{1 + \exp(x - \eta)}. \quad (A5)$$

The energy emission rates of $v_e$ and $v_\gamma$ per unit volume via electron–positron pair annihilations are given by

$$Q_{v_e} = \frac{Q_{v_e}^p}{(C_V - C_A) + (C_V + C_A - 2) \left( \frac{8\pi}{\hbar c^2} \right)^2} \times \frac{\sigma_{e^+ e^-}(k_B T_\gamma)^9}{(m_{e^+}c^2)^2} \times [F_3(\eta_e)F_3(-\eta_e) + F_3(\eta_e)F_3(-\eta_e)]P_{pair,v_e}^2, \quad (A13)$$

where $C_A = \frac{1}{4}$, $C_V = \frac{1}{2} + 2\sin^2 \theta_W$, and $\sin^2 \theta_W = 0.23$. Here, $P_{pair,v_e}$ and $P_{pair,v_\gamma}$ are factors for the phase space blocking of $v_e$ and $v_\gamma$ for the pair processes, respectively, and the factor for $v_\gamma$ is approximately expressed as

$$P_{pair,v_\gamma} \simeq \left\{ 1 + \exp \left[ \left( \frac{1}{2} F_3(\eta_e) + \frac{2}{3} F_3(-\eta_e) \right)\eta_e \right] \right\}^{-1}. \quad (A14)$$

The energy emission rate per unit volume via the pair annihilations is given by

$$Q_{v_e} = \frac{(C_V - C_A)^2 + (C_V + C_A - 2)^2 \left( \frac{8\pi}{\hbar c^2} \right)^2}{18} \times \frac{\sigma_{e^+ e^-}(k_B T_\gamma)^9}{(m_{e^+}c^2)^2} \times [F_3(\eta_e)F_3(-\eta_e) + F_3(\eta_e)F_3(-\eta_e)]P_{pair,v_e}^2. \quad (A15)$$

### A.4. Nucleon–Nucleon Bremsstrahlung Processes

The energy emission rate per unit volume through nucleon–nucleon bremsstrahlung of a single neutrino pair is well approximated with

$$Q_{v_\gamma} = 1.04 \times 10^{30} \left( \frac{X_n^2 + X_p^2}{2} + \frac{28}{3} X_n X_p \right) \left( \frac{10^{14} \rho \text{ g cm}^{-3}}{1 \text{ MeV}} \right)^{2.5} \text{ erg cm}^{-3} \text{ s}^{-1}, \quad (A16)$$

where $\xi$ is set to be 0.5 (Burrows et al. 2000), and $X_n$ and $X_p$ are the mass fraction of neutrons and protons, respectively.

### A.5. Plasmon Decay Processes

The emission rates of $v_e$ and $v_\gamma$ per unit volume via plasmon decay are given by

$$R_{v_e}^{pl} = R_{v_\gamma}^{pl} = \frac{\alpha^3}{3 \alpha_s (\hbar c)^5} \frac{\sigma_{e^+ e^-}(k_B T_\gamma)^9 \gamma^6 e^{-\gamma}}{\ln(1 + \gamma) P_{plas,v_e}^2 P_{plas,v_\gamma}^2}, \quad (A17)$$

where $\alpha_s = 1/137.036$ is the fine-structure constant, $\gamma = \gamma_0 \sqrt{\eta_e^2 + \pi^2/3}$ with $\gamma_0 = 2\sqrt{\alpha_s / 3\pi} = 5.565 \times 10^{-2}$, and $P_{plas,v_e}$ and $P_{plas,v_\gamma}$ are factors for the phase space blocking of $v_e$ and $v_\gamma$ for plasmon decay processes, respectively. The factor for $v_\gamma$ is approximately expressed as

$$P_{plas,v_\gamma} \simeq \left\{ 1 + \exp \left[ (1 + \gamma - \eta_e) \right] \right\}^{-1}. \quad (A18)$$

The emission rate of $v_\gamma$ per unit volume via plasmon decay is given by

$$R_{v_e}^{pl} = (C_V - 1)^2 \frac{4\pi^3}{3 \alpha_s (\hbar c)^5} \frac{\sigma_{e^+ e^-}(k_B T_\gamma)^9 \gamma^6 e^{-\gamma}(1 + \gamma) (P_{plas,v_e})^2}{\ln(1 + \gamma) P_{plas,v_e}^2}. \quad (A19)$$
The energy emission rates of $\nu_i$ per unit volume via plasmon decay are given by

$$Q_{\nu_i}^{pl} = R_{\nu_i}^{pl} \cdot k_B T \left( 1 + \frac{\nu^2}{2(1 + \nu)} \right). \quad (A20)$$

A.6. Neutrino Absorption on Helium and Inelastic Neutrino–Helium Scatterings

In addition to the heating processes through the neutrino absorption on nucleons, the heating processes due to neutrino–helium interactions are taken into account, as in Ohnishi et al. (2007). Through the absorption of $\nu_e$ and $\bar{\nu}_e$ and the neutrino–helium inelastic scatterings on nuclei via neutral currents, $\nu + (A, Z) \rightarrow \nu + (A, Z)^*$, the heating rate per unit volume, $Q^\alpha$, is evaluated as

$$Q^\alpha = \frac{\rho X_A}{m_B} \frac{31.6 \text{ MeV}}{(\tau/10^7 \text{ cm})^2} \times \left[ \frac{L_{\nu_e}}{10^{52} \text{ erg s}^{-1}} \left( \frac{5 \text{ MeV}}{T_{\nu_e}} \right) A^{-1} \left( \frac{\sigma^\alpha \langle \epsilon E_{\nu} \rangle_{\text{ex}}}{T_{\nu_e}} + \frac{\sigma^\beta \langle \epsilon E_{\bar{\nu}} \rangle_{\text{ex}}}{T_{\nu_e}} \right) \right] \left( \frac{10^{-40} \text{ cm}^2 \text{ MeV}}{4 \pi r^2 c f_{\nu}} \right) \left( \frac{\langle \epsilon }{e} \rangle_{\nu_e} \right)^{\epsilon^2} + 2 \Delta \langle \epsilon }{e} \rangle_{\nu_e} + \Delta^2 \langle \epsilon }{e} \rangle_{\nu_e}^2 \right), \quad \Theta(\langle \epsilon }{e} \rangle_{\nu_e}) \right) \right), \quad (B2)$$

and

$$R_{\nu_e}^{\alpha} = \frac{\sigma c L_{\nu_e} n_B}{4 \pi r^2 c f_{\nu}} \left( \frac{\langle \epsilon }{e} \rangle_{\nu_e}^2 + 2 \Delta \langle \epsilon }{e} \rangle_{\nu_e} + \Delta^2 \langle \epsilon }{e} \rangle_{\nu_e}^2 \right), \quad (B3)$$

respectively.

The emission rate of $\nu_e$ per baryon through the electron capture on protons is given by

$$R_{\nu}^{\nu_e} = \frac{1}{2} \sigma c n_p n_e \left[ \frac{\langle \epsilon }{e} \rangle_{\nu_e}^2 + 2 \Delta \langle \epsilon }{e} \rangle_{\nu_e} + \Delta^2 \langle \epsilon }{e} \rangle_{\nu_e}^2 \right], \quad (B4)$$

and that of $\bar{\nu}_e$ via the positron capture on neutrons by

$$R_{\nu}^{\bar{\nu}_e} = \frac{1}{2} \sigma c n_p n_e \left[ \frac{\langle \epsilon }{e} \rangle_{\nu_e}^2 + 2 \Delta \langle \epsilon }{e} \rangle_{\nu_e} + \Delta^2 \langle \epsilon }{e} \rangle_{\nu_e}^2 \right]. \quad (B5)$$

REFERENCES

Anders, E., & Grevesse, N. 1989, Geochim. Cosmochim. Acta, 53, 197
Blinnikov, S. I., Dunina-Barkovskaya, N. V., & Nadyozhin, D. K. 1996, ApJS, 106, 171
Blondin, J. M., & Mezzacappa, A. 2007, Nature, 445, 58
Blondin, J. M., Mezzacappa, A., & DeMarino, C. 2003, ApJ, 584, 971
Borkowski, K. J., Reynolds, S. P., Green, D. A., Hwang, U., Petre, R., Krishnamurthy, K., & Willett, R. 2010, ApJ, 724, L161
Busas, R., Janka, H.-T., Rampp, M., & Kifonidis, K. 2006a, A&A, 457, 281
Busas, R., Rampp, M., Janka, H.-T., & Kifonidis, K. 2006b, A&A, 447, 1049
Burrows, A., Young, T., Pinto, P., Eastman, R., & Thompson, T. A. 2000, ApJ, 539, 865
Couch, S. M., Wheeler, J. C., & Milosavljević, M. 2009, ApJ, 696, 953
Cyburt, R. H., et al. 2010, ApJS, 189, 240
Fischer, T., Whitehouse, S. C., Mezzacappa, A., Thielemann, F.-K., & Liebendörfer, M. 2010, A&A, 517, A80
Foglizzo, T., Galletti, P., Schek, L., & Janka, H.-T. 2007, ApJ, 654, 1006
Fröhlich, C., et al. 2006a, ApJ, 637, 415
Fröhlich, C., Martinez-Pinedo, G., Liebendörfer, M., Thielemann, F.-K., Bravo, E., Hix, W. R., Langanke, K., & Zinner, N. T. 2006b, Phys. Rev. Lett., 96, 142502
Fujimoto, S., Hashimoto, M., Arak, K., & Matsuba, R. 2004, ApJ, 614, 817
Fujimoto, S., Hashimoto, M., Kotake, K., & Yamada, S. 2007, ApJ, 656, 382
Fujimoto, S., Kotake, K., Yamada, S., Hashimoto, M., & Sato, K. 2006, ApJ, 664, 1040
Fujimoto, S., Nishimura, N., & Hashimoto, M. 2008, ApJ, 680, 1350
Fuller, G. M., Fowler, W. A., & Newman, M. J. 1980, ApJS, 42, 447
Fuller, G. M., Fowler, W. A., & Newman, M. J. 1982, ApJS, 48, 279
Gawryszczak, A., Guzman, J., Plewa, T., & Kifonidis, K. 2010, A&A, 521, 38
Goriely, S., Arnold, M., Borzov, I., & Rayet, M. 2001, A&A, 375, L35
Hammer, N. J., Janka, H.-T., & Müller, E. 2010, ApJ, 714, 1371
Hashimoto, M. 1995, Prog. Theor. Phys., 94, 663

APPENDIX B

THE RATE OF CHANGE IN ELECTRON FRACTION, $Q_N$

The rate of change in $Y_e$ per unit volume, $Q_N$, is evaluated as

$$Q_N = \left( R_{\nu_e}^{a} - R_{\nu_e}^{b} - R_{\nu_e}^{c} + R_{\nu_e}^{\text{MB}} \right) \frac{\rho}{\rho}, \quad (B1)$$

where $R_{\nu_e}^{a}$ and $R_{\nu_e}^{b}$ are the absorption rates of $\nu_e$ and $\bar{\nu}_e$, respectively, and $R_{\nu_e}^{c}$ and $R_{\nu_e}^{\text{MB}}$ are the emission rates of $\nu_e$ and $\bar{\nu}_e$ through the capture of electrons on protons and that of positron on neutrons, respectively. We ignore the variations of the electron fraction by the neutrino absorption on $^4$He, since they are minor and give no qualitative difference to the dynamics (Ohnishi et al. 2007).
