Tricritical behaviour of Ising spin glasses with charge fluctuations

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We show that tricritical points displaying unusual behaviour exist in phase diagrams of fermionic Ising spin glasses as the chemical potential or the filling assume characteristic values. Exact results for infinite range interaction and a one loop renormalization group analysis of thermal tricritical fluctuations for finite range models are presented. Surprising similarities with zero temperature transitions and a new $T = 0$ tricritical point of metallic quantum spin glasses are derived.

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Spin glass phase transitions can have important effects on a variety of characteristics of fermionic systems at low temperatures including nonmagnetic properties, and vice versa. This has been emphasized in a series of recent theoretical articles on quantum spin glass transitions [1–5]. Considerable interest in these problems was raised by experimental results for heavy fermion systems [6,7]. Spin glasses known to exist between antiferromagnetic–fermion phase diagrams [8] may also be considered under the aspect of interplay of magnetic and electronic properties.

In this Letter we report results on existence and properties of tricritical fermionic spin glass transitions which happen to emerge as the fermion concentration, relevant for the effective spin dilution, is moved through a characteristic value. We analyzed in detail the tricritical behaviour of the Ising spin glass (denoted by ISG$_f$) on fermionic space with 4 states per site instead of the usual two of SK models for example. Results are given i) for infinite-range-, and (by use of the renormalization group) ii) for finite-range spin interaction, and iii) for a metallic model with additional electron hopping hamiltonian. The two nonmagnetic states per site provided by the fermionic space allow the system to adjust its effective random spin dilution according to quantum statistics. On decreasing the effective spin density and hence $T_c$ the system is driven through a tricritical point into a regime of discontinuous spin glass transitions. The tricritical point (TCP) turns out to be particularly interesting, since quantities such as density of states, fermion concentration (for given $\mu$ and vice versa), local susceptibility, and spin correlation function behave nonanalytically at the tricritical spin glass transition, and thereby change substantially critical properties of those quantities which typically define and signal spin glass transitions. Experimental observation is also favoured by several aspects such as increased upper critical dimension $d_c^{(u)} = 8$ (instead of a $d_c^{(u)}$ decreasing from 4 to 3 in conventional $\phi^4$–theories [9]), which allows critical fluctuations in 3D to have much stronger effects than the usual logarithmic corrections and the occurrence of phase separation in the 1st order regime, a phenomenon shared by the phase diagram of the BEG-model for $He^3 – He^4$ mixtures [10]. We also compare with other classical spin–1 models [11].

Understanding the key features of the ISG$_f$ as the simplest model which takes spin glass order and charge correlations into account provides a useful guide to phase diagrams of spin glasses allowing for thermally activated hopping or metallic conductivity. It appears to be generic for the behaviour of an even larger class of models, in a way comparable with the BEG–model [10]. The ISG$_f$ is defined by the hamiltonian $H = -\frac{1}{2} \sum J_{ij} \sigma_i \sigma_j - \mu \sum n_i$ with spins $\sigma = n_\uparrow - n_\downarrow$, the fermion number operator $n = n_\uparrow + n_\down\downarrow$, and gaussian–distributed $J_{ij}$ with variance $J$. All spin– and charge–correlations of this model are static. As an example the local susceptibility $\chi_{\mu}(t)$ has the Fourier transform $\chi(\omega) = \beta(\tilde{q} - \int q(x) \delta_{\omega 0}$ with $\tilde{q} = \langle \sigma_i(t) \sigma_i(t') \rangle$ and $q(x)$ denoting the Parisi solution of the ISG$_f$. In contrast to the standard 2–states per site Ising spin glass this 4–states per site model feels quantum statistics due to the relative occupation of magnetic and nonmagnetic states. Quantum dynamics however enters in the electron Greens function and correlations being defined with an odd number of equal–time fermion operators. The fermionic path integral representation not only produces the correct spin–static field theory for the spin glass transition, it also helped us to find a close relationship between nonanalytical behaviour of the density of states or fermion concentration and the special features of the tricritical spin glass transition. In the same way we identify the surprising similarity between this classical tricritical theory for finite $T_c$ and the recently analyzed quantum theory of the $T = 0$–quantum paramagnet to spin glass transition of a metallic model.

Working in the grand canonical ensemble our detailed analysis of the ISG$_f$ shows that upon increasing the chemical potential from $\mu = 0$ at half-filling the line of continuous spin–glass transitions given by $T_c = J\tilde{q}(T_c)$ is only realized up to a tricritical point located at $T_{c:3} = J/3$. Beyond this point, ie for $T_c < \frac{1}{3}$ one enters the domain of discontinuous spin–glass tran-
sitions. Fig. 1 displays this most interesting part of the phase diagram which surrounds the TCP. All relevant zero-field properties can be concluded from our result for the saddle-point free energy and expanded around the TCP, which reads (the reader may also extract the tricritical behaviour from the simpler approximate form using $q(x) = q_{EA}$ constant)

$$f - f_{TCP} = \mu - \mu_{c3} + J\left\{ \frac{3}{2} r_g g - r_\tau \tau^2 \right\} \delta \bar{q} + \frac{3}{2} \left( \delta \bar{q} \right)^2 - \int_0^1 dx q^2(x) - \frac{3}{2} \int_0^1 dx [x q^3(x) + 3q(x)] \int_0^x dy q^2(y)$$

$$- 3\delta \bar{q} \int_0^1 dx q^2(x) + \frac{1}{4} \left( \delta \bar{q} \right)^3 - \frac{y_1}{4} \int_0^1 dx q^4(x)$$

(1)

where $\delta \bar{q} \equiv \bar{q} - \bar{q}_{TCP}, g J = \mu - \mu_{c3} + (\zeta^{-1} J - \mu_{c3}) \tau$ as nonordering field, and $\tau = \frac{T - T_{CP}}{T_{CP}}$. The constants are given by $r_g = \frac{2\nu}{c}, r_\tau = 2\nu(1 - \frac{3}{4}\zeta^{-2})$ with $\zeta = \tanh(\mu_{c3}/T_{CP}) \approx 0.9938$, and $\mu_{c3} = \frac{1}{4} \ln(2e\exp(\frac{3}{2})) \approx 0.9611$ as the characteristic chemical potential locating the TCP. The average filling factor corresponding to $\mu_{c3}$ is evaluated as $< \nu_{c3} > \approx 1.6625$. By symmetry w.r.t. $\mu = 0$ one also finds a TCP for less than half-filling at $< \nu_{c3} > \approx 0.3375$. Thus the low- and the high-filling domains host discontinuous spin glass transitions. We shall argue below that this is a rather general phenomenon.

FIG. 1. Vicinity of tricritical point (TCP) for positive chemical potential. Continuous spin glass transitions occur on curve (a) above the TCP (thick unbroken line). Below the TCP first order thermodynamic transitions take place on curve (c). Curve (d), starting at the TCP, and curve (b) below the TCP limit the existence regime of ordered and disordered phases respectively.

The particularity of the TCP stems from the fact that the replica–diagonal fields $Q^{ab}$ become critical in addition to the usual replica–overlap fields $Q^{ab}$ with $a \neq b$. On approaching the TCP with $T \rightarrow T_{CP}$ and $\mu$ fixed for example, the saddle–point solution $\bar{q} = q_{SP}^{ab}$, as obtained from Eq. (1), develops nonanalytical behaviour given by

$$\delta \bar{q} \equiv \bar{q}(T) - \bar{q}(T_{CP}) = A_\pm (\pm \tau)^{3/2} + O(\tau)$$

(2)

with $A_+ = \sqrt{3} A_- \approx 0.1999$. This behaviour is at the origin of the crossover from the critical exponents $-\alpha = \beta_q = \gamma = 1$, valid for $T_c > T_{CP}$, to the tricritical ones $\alpha_3 = \beta_3 = \gamma_3 = \frac{1}{2}$ obtained here for $T \rightarrow T_{CP}$. The free energy given above can be cast into the scaling form $f = \tau^{2-\alpha} G(\frac{\bar{q}}{\tau})$ for the disordered phase with $\alpha = -1$ and crossover exponent $\phi = 2$. In the tricritical regime the singular part behaves like $f_{sing}^{TCP} \sim \frac{g^2}{\tau}$ whence $\alpha_3 = \frac{1}{2}$ from above. In contrast to crystal–field split spin glasses [10] the quartic coefficient $q_{y1}$ of our free energy, Eq. (3), is nonzero and one obtains the Parisi solution $q(x) = \frac{2}{3g} x$ for $0 \leq x \leq x_1$ and $q(x) = q(1)$ for $x_1 \leq x \leq 1$. The plateau height is found to satisfy $q(1) = \delta \bar{q} + O(\delta \bar{q}^2)$. Consequently, plateau and break–point scale like $\sqrt{|\tau|} + O(\tau)$ at the TCP, while linear $\tau$-dependence is reserved to $T_c > T_{CP}$. Adapting the notation of [11] we express our result for the irreversible response $q(1) - \int_0^1 dx q(x) \sim |\tau|^{\frac{3}{2}}$ in terms of the exponent $\beta_3 = 1$ for $T \rightarrow T_{CP}$ and $\beta_3 = 2$ for $T \rightarrow T_c > T_{CP}$.

For the Almeida–Thouless line at tricriticality we find $H_{JT} = \frac{80}{67}(\frac{1}{2}(1 - \frac{2}{3}\zeta^{-1}J \tanh(\mu_{c3}/T_{CP})))/3^{1/2}3^2/2 + O(\tau^2)$. If $T_{CP} \rightarrow T_{CP}^{ISG}$ we obtain the critical exponent $\theta_3 = \frac{2}{3}$ near $T_{CP}$, while $\theta = \frac{1}{2}$ for all $T > T_{CP}$. These values do not satisfy the scaling relation $\theta_3 = \frac{2}{\beta_{\Delta 3}}$ with $\beta_{\Delta 3} = 1 + (\gamma_3 - \alpha_3)/2$. Along the lines described by D. Fisher and H. Stompinsky [11], this problem of mean–field exponents will be resolved below by the renormalization group analysis of the coupling $y_1$ of the finite–range and finite–dimensional ISG.

The fermionic nature of the ISG$_f$ and of related models on Fock space calls for a representation in terms of electron Green’s functions. We shall see that even if charge–fields do not occur or do not become critical together with the spin–fields, nonanalytic behaviour at the spin–glass transition can become not only observable but also intimately related to quantities defined in the charge–sector. We mention our result for the density of states (DOS), which is given in the disordered phase by

$$< \rho_{\sigma}(\epsilon) > = \frac{\text{ch}(\beta \mu) + \text{ch}(\beta(\epsilon + \mu))}{\sqrt{2\pi q J(\text{ch}(\beta \mu) + \text{exp}(\frac{\beta(\epsilon + \mu)}{2}))}} e^{-\frac{(\epsilon + \mu)^2}{2}}.$$  

(3)

For $T < T_c$ we note that $< \rho > = 0$ on both sides of the TCP. Transitions at $T_c > T_{CP}$ obey $\delta q \sim (1/\tau)$ instead. The average filling factor $< \nu > = \text{tr} < n_{\sigma} >$ obeys the relation $< \nu > = 1 + \tanh(\beta \mu)(1 - \bar{q})$ (we find that this relation is invariant under replica symmetry breaking). For fixed $\mu$ this implies that $< \nu >$ shows nonanalytical $\sqrt{|\tau|}$–behaviour near the TCP, and the same holds for the electronic density of states. Collecting the results by
we conclude that all these quantities diverge with the MF–exponent $\kappa_3$ of the specific heat both from above and below $T_{c3}$. The local susceptibility $\chi = \frac{1}{\eta} (1 - \frac{d\tilde{q}}{dT})$ has a divergent slope for $T \downarrow T_{c3}$ but a finite one for $T \uparrow T_{c3}$ due to our result $q(\tau) = \delta\tilde{q} + O(\delta\tilde{q}^2)$. Inferring Eq.(L) one finds $d\chi/dT \sim \tau^{-\frac{1}{2}} \theta(\tau) + O(\tau)$. It is thus the nonanalytic change of the fermion concentration, which yields in combination with the ubiquitous replica–overlap critical fluctuations the special tricritical behaviour as the TCP is approached. What happens if one wishes to consider $\mu$ as a function of given average filling $< \nu >$? Again the exact relation given above, which holds true if $\mu(< \nu >)$ is constrained to be nonrandom, shows that now the chemical potential will acquire $\sqrt{\tau}$–corrections. The other conclusions remain unchanged.

The fermionic picture also allows to study correlations of the statistical fluctuations of the density of states. The DOS–cumulant $< \delta \rho_\alpha^x(\epsilon) \delta \rho_\beta^y(\epsilon') >$ with $\delta \rho \equiv \rho - < \rho >$ is sensitive to the spin glass transition. We obtain $< \delta \rho^x(0) \delta \rho^y(0) >= \frac{A(0,0,\nu)}{q^2} + O(q^4)$ with $A(0,0;0) \leq |\mu| \leq |\mu_{c3}|$) decreasing monotonously from the maximum value 0.6592 at half filling to 0.3613 at the tricritical point. Hence this cumulant, being defined in the charge sector, decays to zero at the spin–glass transition quadratically in $\tau$ for $T_{c1} > T_{c3}$ and linearly at the TCP. Beyond this point it is discontinuous at the transition. The DOS–cumulant also feels replica–symmetry breaking and becomes proportional to $q^2(\tau)$. In a nonvanishing magnetic field the DOS–cumulant becomes nonzero also above $T = T_{c1}(H = 0)$.

As line (b) of Fig.1 merges with line (a) at the TCP, the replica–diagonal fields $Q^{ab}$ become critical together with the off–diagonal fields, a phenomenon very unusual with the off–diagonal fields, a phenomenon very unusual for classical thermal spin glass transitions and so far only known to occur in special limits [13]. In addition $Q^{aa}$ appears linearly in the Lagrangian and hence plays a special role. These crucial features are surprisingly shared by the thermal tricritical theory and the $T \rightarrow 0$ theory for metallic spin glass–[13] and for the transverse field Ising spin glass transitions [4]. These crosslinks with $T \rightarrow 0$–quantum phase transitions with irrelevant quantum dynamics (for dynamic exponents $z > 2$) are best appreciated in field theory. We derived the Lagrangian for the tricritical and finite range ISG[1] and a Lagrangian of the same structure is obtained for generalized models (eg with a transport mechanism) at finite temperature by integrating out the dynamical degrees of freedom

$$L = \frac{1}{\tau} \int d^d x \sum_{(\kappa_2)^2} Q^{aa} + \frac{1}{2} \sum Q^{aa} (-\nabla^2 + u)Q^{aa}$$

- $\frac{1}{\tau} \sum' Q^{aa} Q^{bb} - \frac{\kappa_1}{3} \sum (Q^{aa})^3$

- $- \frac{\kappa_3}{3} Tr' Q^3 - \kappa_2 \sum' Q^{aa} Q^{ab} Q^{ba} + \frac{4}{3} \sum (Q^{ab})^4$, (5)

where $4(\tau(\nu))^{(0)} = (\tau(\nu))^{(0)} = (\tau(\nu))^{(2)}$ and $\eta(0) = 0$ denote the bare coefficients at tricriticality. One fourth order term relevant for replica symmetry breaking is kept. Replicas under $\sum'$ or $Tr'$ are distinct. The $Q^{aa} Q^{bb}$–coupling is renormalization group generated as in the metallic quantum spin glass and leads to a common upper critical dimension $d^{(u)}_{sr} = 8$. A shift of the fields removing the redundant $(Q^{ab})^2$–mass term was performed at each step of our one–loop renormalization group analysis. Since $Q^{aa}$ and $Q^{bb}$–fields may possess different anomalous dimensions, they are kept separately in the Lagrangian. Apart from the striking similarities between the tricritical ISG[1] Lagrangian and the metallic quantum case there are also some important differences:

i) Time–integrals are absent, each $Q$–field may be viewed as $Q_{\omega = 0, \omega = 0}(x)$.

ii)Unlike the metallic model the linear term is time–independent.

iii) Three relevant cubic couplings appear instead of one in the metallic case.

iv) The dangerously irrelevant (DI) $u \int d\tau (Q^{aa})^2$ quantum mechanical interaction of the metallic case [3] turns into a relevant mass term for the ISG[1].

While in case of the metallic $T = 0$–transition the $u$–interaction was seen to render the nonlinear susceptibility $\chi_{nl}$ less divergent than the spin glass susceptibility $\chi_{SG}$. The $ISG$ shows $\chi_{nl} \sim \chi_{SG}$ due to property iv). This was confirmed by our MF calculation of the small field behaviour of the magnetization $m$ which yields to $O(H^3)$ $m = \frac{H}{\tau} - (\frac{27}{2} (1 - \frac{\eta}{4}) th(\frac{3\tau}{4})) \frac{3H^2}{\tau^3}$ for $T > T_{c3}(H = 0), \mu = \mu_{c3}$ near the tricritical point. DUE TO $\chi_{SG} \sim \tau$ near $T_c > T_{c3}$ and $\chi_{SG} \sim \sqrt{\tau}$ the $Q$–propagator $1/(k^2 + m^2)$ yields the MF correlation length exponent $\nu_3 = \frac{1}{2}$ at the TCP and $\nu = \frac{1}{2}$ in the continuous regime above. As for the metallic $T = 0$–transition the MF–exponents violate hyperscaling even in the upper critical dimension $d_{sr}^{(u)} = 8$ (TCP of ISG[1]). The $\frac{1}{\tau} Q^{aa} Q^{bb}$–term is at the origin of this violation for both the tricritical point of the $ISG$ and for the $T = 0$–metallic spin–glass transition. In the metallic case ($z = 4$) the replacement of dimension $d$ by the ‘quantum’–mechanical dimension $d_{qm} = d + 2z - \theta_t$ yields modified hyperscaling relations which are satisfied by MF–exponents in $d = d_{sr}^{(u)} = 8$. The same result is obtained for the tricritical $ISG$ by replacing $d \rightarrow d - \theta_t$. In both cases $\theta_t$ denotes the dimension of the DIC t.

We studied tricritical fluctuation in a 1-loop renormalization group analysis for the Lagrangian [13]. Anomalous dimensions $\eta$ and $\bar{\eta}$ are introduced to account for $Q^{(u)\pm b}$– and for $Q^{aa}$–fluctuations, respectively. These exponents and the one for the DIC t are given by $\eta = \frac{2\eta_0}{(1 + \eta_0)}, \bar{\eta} = \frac{2\eta_0}{(1 + \eta_0)}, \theta_t = 2$. We obtain the following RG flow equations
we find the tricritical point of the metallic spin glass gaussian random hopping at discontinuous low– and high–filling–regimes. At $T = 0$ metallic Ising spin glass show tricritical behaviour on the $\mu$–field models [4]. The runaway flow is expected within the first order regime, $u(0) < 0$, but a strong coupling tricritical fixed point limiting the known second order regime is still to be found.

The RG for the DIC $y_4$ showed that its long–distance behaviour is dominated by a $\kappa^4$–contribution (like in [10] but) for $d^c(u) = 8 < d < 10$. This leads to the modified MF exponent $\theta_3 = \frac{8}{d-4}$, which satisfies the scaling relation $\theta_3 = 2/\beta_\Delta$, in $d^c(u) = 8$ and reduces to the MF–result in 10 dimensions. The dimensional shift by 2 in comparison with Ref. [11] is due to coupling $u$.

For given nonrandom chemical potential the frustrated spin interaction generates weakly nongaussian statistical fluctuations of the fermion filling and vice versa, half filling exempted. Imposing instead a Gaussian $\delta\mu$–distribution one finds the present problem mapped onto the $Q$–static approximation [12] of a metallic Ising spin glass in the limit of electron hopping range zero. This limit turns random hopping matrix elements into random site–local energies, which are equivalent to the fluctuating chemical potential and render the metallic Ising spin glass classical and static. The main new effect of $\mu$–randomness is the generation of a classical $T = 0$–transition at $< (\delta\mu)^2 >, \varepsilon = (\frac{\langle 1 \rangle^2}{8\sqrt{2\pi}})$ for $< \mu > = 0$.

We find that the random- $\mu$ ISG$_f$–model and the metallic Ising spin glass show tricritical behaviour on the $T = 0$–axis as well as in their thermal transitions with discontinuous low– and high–filling–regimes. At $T = 0$ we find the tricritical point of the metallic spin glass with gaussian random hopping at

$$E_F = \sqrt{1 - \frac{5}{8}E_0}, \quad J_c = \frac{3\pi E_0}{32}[1 - \frac{E_F^2}{E_0^2}]^2, \quad (7)$$

where $2E_0$ denotes the width of the semielliptic electronic band. Quantum dynamical corrections can be approximated by a generalized Miller–Huse method [4]. Details of the present work will be given elsewhere.

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[1] S. Sachdev and J. Ye, Phys. Rev. Lett. 70, 3339 (1993).
[2] J. Ye, S. Sachdev, and N. Read, Phys. Rev. Lett. 70, 4011 (1993).
[3] A. Sengupta and A. Georges, Phys. Rev. B 52 10295 (1995).
[4] D. S. Fisher and H. Sompolinsky, Phys. Rev. Lett. 54 1063 (1985).
[5] D. Huse and J. Miller, Phys. Rev. Lett. 54 10286 (1985).
[6] M. B. Maple et al., J. Low Temp. Phys. 95, 225 (1994).
[7] H. von Loehneysen et al., Phys. Rev. Lett. 72, 3262 (1994).
[8] F. C. Chou, N. R. Belk, M. A. Kastner, R. J. Birgeneau, and A. Aharony, Phys. Rev. Lett. 75, 2204 (1995).
[9] M. Blume, V. J. Emery, and R. B. Griffiths, Phys. Rev. A 4, 1071 (1971).
[10] P. J. Mottishaw, D. Sherrington, J. Phys. C 18, 5201 (1985).
[11] D. S. Fisher and H. Sompolinsky, Phys.Rev.Lett.54, 1063 (1985).
[12] J. E. Green, A. J. Bray, M. A. Moore, J. Phys. A 15, 2307 (1982).
[13] R. Oppermann and M. Binderberger, Ann. Phys. 3, 494 (1994).
[14] D. Huse and J. Miller, Phys. Rev. Lett. 70, 3147 (1993).
[15] I. D. Lawrie and S. Sarbach, in Phase Transitions and Critical Phenomena edited by C. Domb and J. L. Lebowitz (Academic Press, London, 1984), p.1