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Exciton-Mechanical Mode Entanglement via Dissipation-Induced Coupling

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We analyze the entanglement between two matter modes in a hybrid quantum system consists of a microcavity, a quantum well, and a mechanical oscillator. Although the exciton mode in the quantum well and the mechanical oscillator are initially uncoupled, their interaction through the microcavity field results in an indirect exciton-mechanical mode coupling. We show that this coupling is a Fano-Agarwal-type coupling induced by the decay of the exciton and the mechanical modes caused by the leakage of photons through the microcavity to the environment. Using experimental parameters and for slowly varying microcavity field, we show that the generated coupling leads to an exciton-mechanical mode entanglement. The maximum entanglement is achieved at the avoided level crossing frequency, where the hybridization of the two modes is maximum. The entanglement is also robust to the phonon thermal bath temperature.

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Hybrid quantum systems consisting of quantum mechanical oscillators have become a platform for many interesting applications of quantum mechanics. In addition to being a tool to understand the quantum to classical transition, e.g., by creating entanglement between mechanical modes, mechanical oscillators have potential applications in quantum information processing. In this regard, there has been a growing effort in exploiting the mechanical degrees of freedom to engineer devices such as a microwave-to-optical (or vice versa) frequency converter\textsuperscript{[1,2]} and quantum memory\textsuperscript{[3,4]}. Moreover, quantum mechanical oscillator has been used as an interface to transfer a quantum state from an optical cavity to a microwave cavity\textsuperscript{[5,6]}. Interest in merging optomechanical resonators with solid-state systems has been growing; examples include, ultrastrong optomechanical coupling in GaAl vibrating disk resonator\textsuperscript{[7]}, cooling of phonons in a semiconductor membrane\textsuperscript{[8,9]}, a strong optomechanical coupling in a vertical-cavity resonator\textsuperscript{[10]}, and surface-emitting laser\textsuperscript{[11]}. Coupling a mechanical oscillator to a microcavity that consists of a quantum well has been considered in the context of generating hybrid resonances\textsuperscript{[12]} among photons, excitons, and phonons, and studying the optical bistability\textsuperscript{[13,14]}. The physics of photon-exciton coupling has extensively been studied as presented in a recent review\textsuperscript{[15]}.

In this work, we analyze the entanglement between the mechanical mode and the exciton mode in a quantum well placed at the antinode of a microcavity that is formed by distributed Bragg reflector (DBR) mirrors containing a quantum well (QW) and coupled to a mechanical motion ($x$) of the mirror. The quantum well is placed at the antinode of the microcavity so that the exciton-cavity mode coupling will be maximum. The black and the orange stripes corresponding to GaAs and AlAs layers, respectively. The microcavity is driven by a pump laser of normalized amplitude $\varepsilon_p$ and has a damping rate $\kappa$. The DBRs are shifted from the equilibrium position due to the radiation pressure force.

where the damping rate of the microcavity exceeds the cavity-exciton coupling strength. A significant amount of entanglement between the exciton and the mechanical modes can be created at the exciton-mechanical mode hybrid resonance frequencies. We find that the maximum entanglement between the two modes is achieved when the exciton and the mechanical modes hybridization is maximum. Surprisingly, the entanglement persists at high temperature of the phonon thermal bath. Our entanglement analysis is based on realistic parameters from a recent experiment\textsuperscript{[12]}.

We consider a microcavity formed by a set of distributed Bragg reflector mirrors and consists of a quantum well placed at the antinode. The microcavity is coupled to the mechanical motion of the mirror via radiation pressure force and to the exciton mode in the quantum
well. An exciton in the quantum well can be considered as a quasi-particle resulting from the interaction between one hole in the valence band and one electron in the conduction band. When the exciton radius is much smaller than the average distance between neighbouring excitons (∼ n_{ex}^{-1/2} with n_{ex} being exciton concentration), we treat the exciton as a composed boson. In general, in the weak excitation regime, where the density of the excitons is sufficiently low, the interaction between the neighboring excitons due to Coulomb interaction is weak and can be neglected. However, in the moderated driving regime, the interaction between neighbouring excitons becomes strong and nonlinear [20–25], and leads to interesting properties such as squeezing and bistability [26, 28]. In this paper we will consider the exciton as a composed boson.

The coupled exciton-optomechanical system is described by the Hamiltonian:

\[
H = \omega_p a^\dagger a + \omega_{ex} b^\dagger b + \omega_m c^\dagger c + i \varepsilon_{p}(a^\dagger e^{-i\omega_p t} - ae^{i\omega_p t}) - g_0 a^\dagger a (c + c^\dagger) + ig(a^\dagger b - ab^\dagger) + ab^\dagger bb. \tag{1}
\]

Here the operators \(a\), \(b\), and \(c\) are annihilation operators for a photon in the microcavity, an exciton in the quantum well, and a phonon in the mechanical oscillator, respectively. The microcavity is driven by strong drive with frequency \(\omega_p\), \(\omega_m\) and \(\omega_{ex}\) are the bare microcavity and exciton frequencies. For the mechanical oscillator, the resonance frequency is \(\omega_m\) and \(g_0\) is the single-photon optomechanical coupling; \(g\) is the linear exciton-cavity mode coupling, and \(2\alpha = 6\epsilon^2a_{ex}/\epsilon A\) [20] is the nonlinear coefficient describing the exciton-exciton scattering due to Coulomb interaction with \(\epsilon\), \(a_{ex}\), \(\epsilon\), and \(A\) being the electron charge, the exciton Bohr radius, the dielectric constant of the quantum well, and the quantization area, respectively. The strong drive of amplitude \(\varepsilon_p = \sqrt{\kappa P/\hbar \omega_p}\) with \(P\) and \(\kappa\) being the drive laser power and the microcavity damping rate, respectively, leads to a large steady-state optical filed in the microcavity which increases the occupation numbers in each mode and the optomechanical coupling. The resulting steady-state intracavity amplitude in turn shifts the equilibrium position of the mechanical oscillator through radiation pressure force.

In the Hamiltonian (1), the the first three terms in the first line represent the free energy of the system while the last term describes the coupling of laser drive with the microcavity. In the second line, the first term describes the photon-phonon coupling, the second term represents the linear exciton-phonon interaction, and the last term describes the exciton-exciton scattering due to the Coulomb interaction. In a frame rotating with the drive frequency \(\omega_p\), the interaction Hamiltonian (1) has the form

\[
V = -\Delta_a a^\dagger a - \Delta_{ex} b^\dagger b + \omega_m c^\dagger c - g_0 a^\dagger a (c + c^\dagger) + ig(a^\dagger b - ab^\dagger) + ab^\dagger bb + i\varepsilon_{p}(a^\dagger - a), \tag{2}
\]

where \(\Delta_a = \omega_p - \omega_a\), and \(\Delta_{ex} = \omega_p - \omega_{ex}\). Using the interaction Hamiltonian Eq. (2), we derive coupled equations for the macroscopic fields, \(\hat{a}\), \(\hat{b}\), and \(\hat{c}\). These equations are obtained by replacing the operators with classical amplitudes in the Heisenberg equations:

\[
\dot{\hat{a}} = -\frac{\kappa}{2} \hat{a} + i\Delta_a \hat{a} + g\hat{b} + ig_0 \hat{a}(\hat{c} + c^\dagger) + \varepsilon_p, \tag{3}
\]

\[
\dot{\hat{b}} = -\frac{\gamma}{2} \hat{b} + i\Delta_{ex} \hat{b} - g\hat{a} - 2i\alpha |\hat{b}|^2 \hat{b}, \tag{4}
\]

\[
\dot{\hat{c}} = -\frac{\gamma_m}{2} \hat{c} - i\omega_m \hat{c} + ig_0 |\hat{a}|^2, \tag{5}
\]

where \(\gamma\) is the exciton spontaneous emission rate, and \(\gamma_m\) is the damping rate of the mechanical oscillator. The steady state solution to the above equations read

\[
\hat{c}_s = \frac{i g_0 |\hat{a}_s|^2}{\gamma_m/2 + i\omega_m}, \tag{6}
\]

\[
\hat{a}_s = -\frac{\gamma/2 + i(\Delta_{ex} + 2\alpha I_b)}{g} \hat{b}_s, \tag{7}
\]

\[
\hat{b}_s = -\frac{g\varepsilon_p}{\kappa/4 + g^2 - \Delta_a \Delta_{ex} + i(\kappa\Delta_{ex}/2 + \gamma\Delta_a/2)} \hat{b}_s, \tag{8}
\]

\[
\Delta_{ex}(I_b) = \Delta_a - \frac{2g_0^2\gamma_m}{\omega_m^2 + (\gamma/m)^2/2} g^2 \Delta_{ex} I_b^2, \tag{9}
\]

\[
\Delta_{ex}(I_b) = \Delta_{ex} + 2\alpha I_b, \tag{10}
\]

where \(I_b = |\hat{b}_s|^2\) is the steady state exciton number in the quantum well. Note that the Eq. (3) yields nonlinear equation for \(I_b\) in the form

\[
\frac{I_b}{g^2} \left[ (\frac{\gamma\Delta_{ex}}{4} + g^2 - \Delta_a \Delta_{ex})^2 + \left(\frac{\kappa}{2} \Delta_{ex} + \frac{\gamma}{2} \Delta_a\right)^2 \right] = |\varepsilon_p|^2. \tag{11}
\]

The nonlinear equation for \(I_b\) is a signature that the exciton number can exhibit bistability [13, 14] behaviour for a certain parameter regime. In the following, we discuss exciton-mechanical mode entanglement in the regime where the system is stable.

The nonlinear quantum Langevin equations can be linearized by writing the operators as the sum of the steady state classical mean value plus a fluctuating quantum part: \(a = \bar{a}_s + \delta a, b = \bar{b}_s + \delta b, c = \bar{c}_s + \delta c\). The linearized Langevin equations of the fluctuation operators then read

\[
\dot{\delta a} = -\frac{\kappa}{2} \delta a + i\Delta_a \delta a + g\delta b + G(\delta c + \delta c^\dagger) + \sqrt{\kappa} \delta a_in, \tag{12}
\]

\[
\dot{\delta b} = -\frac{\gamma}{2} \delta b + i\Delta_{ex} \delta b - g\delta a - 2i\alpha \delta b^\dagger \delta b + \sqrt{\Gamma} \delta b_in, \tag{13}
\]

\[
\dot{\delta c} = -\frac{\gamma_m}{2} \delta c + i\omega_m \delta c + G(\delta a^\dagger - \delta a) + \sqrt{\gamma} \delta c_in, \tag{14}
\]

where \(G = g_0\sqrt{n_s}\) is the many-photon optomechanical coupling with \(n_s = |\bar{a}_s|^2\) being the steady state mean photon number in the microcavity. For simplicity, we have chosen the phase of the coherent drive such that \(\bar{a}_s = -i|\bar{a}_s|\). Here \(a_{in}, b_{in}, c_{in}\) are the Langevin noise operators for the microcavity, exciton, and the mechanical modes, respectively. All noise operators have
mode evolutions describe the dynamics of the exciton and the mechanical modes in the adiabatic regime, where the microcavity damping rate is larger than the exciton-mechanical mode coupling. In contrast to the exciton and the mechanical modes, it is more convenient to use the cavity damping of photons through the microcavity to the environment, also known as the Purcell effect [18,19]. Note that the relaxation rate of the exciton is increased by γ_b as result of interaction with the cavity mode; γ_c = 4g^2/k[1 + (2Δ_a/κ)^2] is the effective damping rate of the mechanical mode. In contrast to the exciton mode evolution, the cavity-induced relation does not affect the decay term in the Langevin equation for δc, it does however appear in the noise terms as manifested in Eq. (13). Note also that the cavity-exciton coupling shifts the exciton frequency by δω_{ex} = γ_b Δ_a/κ. Similarly, the cavity-mechanical mode coupling gives rise to a shift δω_m = 2γ_c Δ_a/κ in the mechanical mode frequency; λ_{bc(c)} = (1 + 2i Δ_a/κ)[1 + (2Δ_a/κ)^2]^{-1/2} is the contribution of the cavity-induced dissipation to the noise operator of the exciton (mechanical) mode and finally

$$ G_{bc} = \sqrt{\gamma_b \gamma_c} \left(1 + 2i \Delta_a/\kappa\right) $$

is the effective exciton-mechanical mode cross coupling. Notice that the cross coupling depends on the effective decay rates γ_b and γ_c induced by the photon leakage through the microcavity, which is similar to the Fano-Agarwal effect [16,17]. Dissipation-induced coupling has extensively been explored in quantum optics in creating coherence in three-level atomic systems [31,32]. Here we exploit the dissipation-induced coupling to entangle two matter modes: the exciton and the mechanical modes.

To study the entanglement between the exciton and the mechanical modes, it is more convenient to use the quadrature operators defined by, δx_a = δb^\dagger - δb)/√2, δy_a = i(δb^\dagger - δb)/√2, δx_m = (δc^\dagger + δc)/√2, and δy_m = i(δc^\dagger - δc)/√2 and similar definitions for fluctuation operators x_j, in, y_j, in (j = a, b). The equations for these quadrature operators in matrix form read

$$ \dot{u} = Ru + \eta, $$

where $u = (δx_a, δy_a, δx_m, δy_m)^T$ is vector of quadrature operators and $\eta = (F^a_{x,in}, F^a_{y,in}, F^{bc}_{x,in}, F^{bc}_{y,in})$ with $F^a_{x,in} = -Re(λ_b)x_{a,in} + Im(λ_c)x_{a,in} + \sqrt{γ_b}\sqrt{γ_c}x_{m,in}, F^a_{y,in} = -Re(λ_b)y_{a,in} - Im(λ_c)x_{a,in} + \sqrt{γ_b}\sqrt{γ_c}y_{m,in}, F^{bc}_{x,in} = \sqrt{γ_b}\sqrt{γ_c}x_{m,in}$, and $F^{bc}_{y,in} = -2Re(λ_c)y_{a,in} - 2Im(λ_c)x_{a,in} + \sqrt{γ_b}\sqrt{γ_c}y_{m,in}$. The diffusion matrix $R$ is given by

$$ R = \begin{pmatrix} -\Gamma_b^+ T & -\Delta_+^* & Re(G_{bc}) & 0 \\ \Delta_+ & -\Gamma_b T & -Im(G_{bc}) & 0 \\ -Im(G_{bc}) & -Re(G_{bc}) & -\omega_m & -2\omega_m \\ -\omega_m & \omega_m & 2\omega_m & -2\omega_m \end{pmatrix}, $$

where $\Gamma_b^\pm = Γ_b ± 4aIm(\tilde{b}_m^2)$ and $\tilde{Δ}_+ = Δ_+ - γ_b Δ_a/κ ± 2aRe(\tilde{b}_m^2)$.

We focus on the steady-state entanglement between the exciton and the mechanical modes. For this, one needs to find a stable solution for Eq. (10), so that it reaches a unique steady state independent of the initial conditions. Since we have assumed $a_{in}, b_{in},$ and...
$c_{in}$ to be zero-mean Gaussian noises and the corresponding equations for fluctuations $\delta x_{j,in}$ and $\delta y_{j,in}$ are linearized, the quantum steady state for fluctuations is simply a zero-mean Gaussian state, which is fully characterized by a correlation matrix $V_{ij} = \langle [u_i(\infty)u_j(\infty) + u_j(\infty)u_i(\infty)] \rangle / 2$. The solution to Eq. (10) is stable and reaches the steady state when all of the eigenvalues of $R$ have negative real parts. For all results presented in this work, the stability has been checked using the non-linear equation mentioned earlier. When the system is stable the correlation matrix satisfies Lyapunov equation $RV + VR^T = -D$, where

$$D = \begin{pmatrix}
\frac{\varGamma}{\sqrt{\gamma_c}} & 0 & 0 & 0 \\
0 & \frac{\varGamma}{\sqrt{\gamma_c}} & 0 & \sqrt{\gamma_c^2 + \Delta_c^2} (2n_{th} + 1) \\
0 & 0 & \frac{\varGamma}{\sqrt{\gamma_c}} & 0 \\
\sqrt{\gamma_c^2 + \Delta_c^2} (2n_{th} + 1) & 0 & 0 & \sqrt{\gamma_c^2 + \Delta_c^2} (2n_{th} + 1)
\end{pmatrix}$$

and the elements of the drift matrix $D$ are obtained using the correlations of the noise operators $\sigma_{ij}$ defined earlier. Note that the cavity-induced dissipation terms contribute to the drift matrix. Notably, the off-diagonal element $\sqrt{\gamma_c^2 + \Delta_c^2} = \text{Re}(G_{bc})$ contributes to the correlation between the exciton and the mechanical modes.

In order to quantify the bipartite entanglement, we employ the logarithmic negativity $E_N$, a measure of bipartite entanglement $[33, 35]$. For continuous variables, $E_N$ is defined as

$$E_N = \max[0, -\ln 2\chi],$$

where $\chi = 2^{-1/2} \left[ \sigma - \sqrt{\sigma^2 - 4\det V} \right]^{1/2}$ is the lowest simplistic eigenvalue of the partial transpose of the $4 \times 4$ correlation matrix $V$ with $\sigma = \det V_A + \det V_B - 2 \det V_{AB}$ $[37]$. Here $V_A$ and $V_B$ represent the exciton and the mechanical modes, respectively, while $V_{AB}$ describes the correlation between the two modes. These matrices are elements of the $2 \times 2$ block form of the correlation matrix $V = \begin{pmatrix} V_A & V_{AB} \\ V_{AB}^T & V_B \end{pmatrix}$. The exciton and the mechanical modes are entangled when the logarithmic negativity $E_N$ is positive.

We numerically studied the exciton-mechanical mode entanglement by exploiting the indirect coupling mediated by the cavity field. Using realistic parameters from a recent microcavity experiment $[12]$, we plot in Fig. 2 the logarithmic negativity $E_N$ as a function of the normalized detuning $\Delta_\omega/\omega_m = 1.05$ (red solid curve), 1.10 (blue dashed curve), 1.15 (green dotdashed curve), and 1.20 (magenta dotted curve). Here we used the thermal photon number $n_{th} = 100$. Notice that the maximum entanglement for $\Delta_\omega/\omega_m = 1.20$ in (a) appears at the power ($P \approx 17.8 \mu W$), where the maximum hybridization between the two modes occurs. All the other parameters are as in Fig. 2.

In order to study the dependence of the generated entanglement on the applied input laser power, we plot in Fig. 3 the logarithmic negativity versus power for different values of the cavity-laser detuning. As can be seen from this figure, to obtain a maximum entanglement for a given cavity-laser detuning one has to apply a certain laser power strength. Naively, one would expect that an increase in the coupling strength (due to an increase in power) to increase the entanglement. We however find that there exists an optimum amount of power that is needed to obtain the maximum entanglement for the realistic set of parameters $[12]$. These peaks of the entanglement at different values of the laser power strength and detuning can be explained in terms of the exciton-mechanical mode hybrid resonances. The peaks appear at laser powers where the maximum repulsion between the eigenstates of the two modes occur [see, e.g., Fig. 3 (b)], indicating that the maximum entanglement is achieved at the maximum of hybridization.

The optimized entanglement over the input power as a function of detuning and for different values of the ther-
entanglement occurs when the thermal phonon numbers are varied. This is because the effective coupling [see Eq. (15)] between the exciton and the mechanical mode depends on the cavity-induced damping rates. These damping rates rely on the number of phonons, thus changing the resonance frequency at which maximum hybridization occurs.

We note that the exciton-mechanical mode entanglement can be detected by measuring the optomechanical entanglement [38–40] and the photon-exciton entanglement. From application view point, the generated entangled state has potential in one-way continuous-variable (CV) quantum computation [41]. By forming a cluster of entangling gates, it is possible to implement CV quantum computation using our system. The exciton-mechanical mode entanglement might have advantages over that obtained between optical modes [41] due to the robustness of the entanglement as well as the availability of semiconductor and micro-electromechanical (MEMS) technologies. The exciton-mechanical mode entanglement also means entanglement with mechanical oscillator or MEMS, a significant progress towards entanglement of macroscopic objects. Achieving entanglement in excitons against its large decoherence is an important step forward as it opens up new possibilities of merging quantum information with existing matured and ubiquitous technologies of semiconducting devices.

In conclusion, we have analyzed the entanglement between two matter modes (exciton and mechanical modes) in a hybrid quantum system consists of a microcavity, a quantum well, and a quantum mechanical oscillator. We have shown that although the exciton and the mechanical modes are initially uncoupled, their interaction with the common microcavity field results in dissipation-induced indirect coupling. This indirect coupling is responsible for the entanglement between the exciton and the mechanical modes. Maximum entanglement is achieved in the adiabatic regime where the microcavity damping rate is larger than the coupling strengths and when the two modes form a complete hybridization. Recent successful experiments [9, 10, 12] in coupling mechanical systems with microcavity pave the way for the realization of the proposed entanglement generation between exciton and the mechanical modes via dissipation-induced coupling.

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