Local properties of local multiplicity distributions

S.V. Chekanov

High Energy Physics Institute Nijmegen (HEFIN),
University of Nijmegen/NIKHEF, NL-6525 ED,
Nijmegen, The Netherlands

Abstract

Some aspects of applications of bunching parameters are dis-
cussed. It is investigated to what extent Monte-Carlo models, which
have been tuned to reproduce global event-shape variables and
single-particle inclusive distributions, agree with each other.

Presented at the International School-Seminar “The actual problems of
particle physics” (NCPHEP-Dubna) Gomel, Belarus,
August 8-17, 1997

1 Introduction

One of the simplest observable, which contains information about the dy-
namics of multiparticle production beyond single-particle densities, is the
multiplicity distribution. While the study of the multiplicity distribution
$P_N$ in full phase space deals with limited dynamical information influenced
by charge- and energy-momentum conservation, the investigation of the
evolution of the probabilities $P_n(\delta)$ of detecting $n$ particles in ever smaller
sizes $\delta$ of phase-space windows (bins) can provide detailed information on
QCD multihadron production without these trivial constraints. A devia-
tion of this distribution from that expected for purely independent particle
production can be attributed to dynamical local multiplicity fluctuations.

The important quest behind such a study is the understanding of
the origin of short-range correlations between final-state particles, leading

*On leave from Institute of Physics, AS of Belarus, Skaryna av.70, Minsk 220072,
Belarus.
to the appearance of dynamical multiparticle spikes in individual events. As a consequence of these correlations, the normalized factorial moments (NFMs)

\[ F_q(\delta) \equiv \frac{\langle n^{[q]} \rangle}{\langle n \rangle^q}, \]

\[ \langle n^{[q]} \rangle = \sum_{n=q}^{\infty} n^{[q]} P_n(\delta), \quad n^{[q]} = n(n-1)\ldots(n-q+1), \]

of the local multiplicity distribution \( P_n(\delta) \) exhibit a power-like increase with decreasing \( \delta \), namely \( F_q(\delta) \propto \delta^{-\phi_q} \). The constants \( \phi_q \) are called intermittency indices. This phenomenon reflects the peculiarity of \( P_n(\delta) \) to become broader with decreasing \( \delta \). Since NFMs satisfy the scaling property \( F_q(\lambda\delta) = \lambda^{-\phi_q} F_q(\delta) \), this is widely regarded as evidence that the correlations exhibit a self-similar underlying dynamics.

Experimentally, local fluctuations in \( e^+e^- \)-processes have already been studied by the TASSO, HRS, CELLO, OPAL, ALEPH, DELPHI and L3 Collaborations [2]. The data do exhibit approximate power-like rise of the NFMs with a saturation at small \( \delta \). The conclusion has been reached that such a phenomenon is a consequence of the multi-jet structure of events, i.e., groups of particles with similar angles resulting in spikes of particles as seen in selected phase-space projections. The hard gluon radiation significantly affects the NFMs, so that they have stronger increase in 3-jet events than in 2-jet events. It has been found that for the statistics used at that time current Monte-Carlo models can, in general, describe the data, even without additional tuning.

Recently, it has been realized that the factorial-moment method poorly reflects the information content of local fluctuations, since the NFM of order \( q \) contains a trivial contamination from lower-order correlation functions (see reviews [3]). As a result, rather different event samples can exhibit a very similar behavior of the NFMs. The fact that subtle details in the behavior of \( P_n(\delta) \) are missing, together with the small statistics used, may be the reason why different Monte-Carlo models can reasonably describe the local fluctuations measured in \( e^+e^- \) annihilation so far.

Another shortcoming of the factorial-moment measurement is that in moving to ever smaller phase-space bins, the statistical bias due to a finite event sample \( (N_{\text{ev}} \neq \infty) \) becomes significant, especially for high-order moments \( q \). This is because in actual measurements the NFMs at small bin size are determined by the first few terms in (2). In most cases this
leads to a significant underestimate of the measured NFMs with respect to their true values.

Cumulants are a more sensitive statistical tool (see [3] and references therein). However, their measurement is rather difficult and was rarely attempted. Besides, the cumulants are expected to be influenced by the statistical bias to even larger degree, since they are constructed from the factorial moments of different orders $q$.

2 Local Properties

An important step towards an improvement of experimental measurements of the local multiplicity distribution was made in [4,5], where it was shown that any complex distribution can be represented as

$$P_n(\delta) = P_0(\delta) \frac{\lambda^n}{n!} L_n, \quad L_n = \prod_{i=2}^{n} \eta_i^{n-i+1}(\delta),$$

where $\lambda = P_1(\delta)/P_0(\delta)$. The factor $L_n$ measures a deviation of the distribution from a Poisson with $L_n = 1$. Non-poissonian fluctuations exhibit themselves as a deviation of $L_n$ from unity. The $L_n$ is constructed from the bunching parameters (BPs)

$$\eta_q(\delta) = \frac{q}{q-1} \frac{P_q(\delta) P_{q-2}(\delta)}{P_{q-1}(\delta)}, \quad q > 1.$$  (3)

The values of the BPs and NFMs for most popular distributions are shown in Table 1. The most interesting observation is that while the NFM is an “integral” characteristic of the $P_n(\delta)$ and the BP is a “differential”, both tools have values larger than unity if the distribution is broader than a Poisson. Generally, however, one should not expect that all BPs are larger than unity for a broad distribution; BPs probe the distribution locally, i.e. they are simply determined by the second-order derivative from the logarithm of $P_n(\delta)$ with respect to the $n$. Note, that in the case of local distributions, the width of distributions is mainly determined by $\eta_2(\delta)$. This observation is based on the simple fact that $P_n(\delta)$ ceases to be bell-shaped at sufficiently small $\delta$. 

3
BPs are more sensitive to the variation in the shape of $P_n(\delta)$ with decreasing $\delta$ than are the NFMs \[[6]\]. In the case of intermittent fluctuations, one should expect $\eta_2(\delta) \propto \delta^{-d_2}$. For multifractal local fluctuations, the $\eta_q(\delta)$ are $\delta$-dependent functions for all $q \geq 3$, while for monofractal behavior $\eta_q(\delta) = const$ for $q \geq 3$ \[[4]\].

From an experimental point of view, the BPs have the following important advantages \[[5]\]:

1) They are less severely affected by the bias from finite statistics than the NFMs, since the $q$th-order BP resolves only the behavior of the multiplicity distribution near multiplicity $n = q - 1$;

2) For the calculation of the BP of order $q$, one needs to know only the $q$-particle resolution of the detector, not any higher-order resolution.

The problem we are dealing with in this paper is to investigate whether different Monte-Carlo (MC) models, which were tuned to reproduce the global-shape variables and single-particle inclusive densities, can lead to the same structure of the local multiplicity fluctuations which are determined by many-particle inclusive densities. We study JETSET 7.4 PS \[[7]\], ARIADNE 4.08 \[[8]\] and HERWIG 5.9 \[[9]\] models. The models have been tuned as described above by the L3 Collaboration \[[10]\].

| Distribution       | $P_n$                                      | NFMs  | BPs                      |
|--------------------|--------------------------------------------|-------|--------------------------|
| Pos. Binomial      | $C_n^N p^n (1 - p)^{N-n}$                  | $\prod_{i=1}^q (1 - \frac{i}{N}) < 1$ | $\frac{q-1-N}{q-2-N} < 1$ |
| Poisson            | $p^n \exp(-p)/n!$                         | 1     | 1                        |
| Neg. Binomial      | $\frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} p^n (1 + p)^{-(k+n)}$ | $\prod_{i=1}^q (1 + \frac{i}{k}) > 1$ | $\frac{q-1+k}{q-2+k} > 1$ |
| Geometric          | $p^n (p + 1)^{-n-1}$                      | $\prod_{i=1}^q (1 + i) > 1$            | $\frac{q}{q-1} > 1$           |

Table 1. NFMs and BPs for positive-binomial, Poisson, negative-binomial and geometric distributions.
3 Monte-Carlo Analysis

1) Horizontal BPs:

In order to reduce the statistical error on the observed local quantities when analyzing experimental data, we use the bin-averaged BPs [4, 5]:

\[ n_q(M) = \frac{q}{q-1} \frac{\bar{N}_q(M)\bar{N}_{q-2}(M)}{N_{q-1}^2(M)}, \]  
\( q \geq 2 \)  \hspace{1cm} (4)

\[ \bar{N}_q(M) = \frac{1}{M} \sum_{m=1}^{M} N_q(m, \delta), \]  \hspace{1cm} (5)

where \( N_q(m, \delta) \) is the number of events having \( q \) particles in bin \( m \) and \( M = \Delta/\delta \) is the total number of bins (\( \Delta \) represents the size of full phase-space volume). To be able to study non-flat distributions, like for rapidity, we have to carry out a transformation from the original phase-space variable to one in which the underlying density is approximately uniform, as suggested by Bialas, Gadzinski and Ochs [11].

2) Generalized integral BPs:

To study the distribution for spikes, we will consider the generalized integral BPs [5] using the squared pairwise four-momentum difference \( Q_{12}^2 = -(p_1 - p_2)^2 \). In this variable, the definition of the BPs is given by

\[ \chi_q(Q^2) = \frac{q}{q-1} \frac{\Pi_q(Q^2)\Pi_{q-2}(Q^2)}{\Pi_{q-1}^2(Q^2)}, \]  \hspace{1cm} (6)

where \( \Pi_q(Q^2) \) represents the number of events having \( q \) spikes of size \( Q^2 \) in the phase-space of variable \( Q_{12}^2 \), irrespective of how many particles are inside each spike. To define the spike size, we shall use the so-called Grassberger-Hentschel-Procaccia counting topology for which a many-particle hyper-tube is assigned a size \( Q^2 \) corresponding to the maximum of all pairwise distances (see [5] for details). For purely independent particle production, with the multiplicity distribution characterized by a Poissonian law, the BPs (6) are equal to unity for all \( q \).

3.1 In rapidity variable

In order to study fluctuations inside jets, in most investigations the fluctuations have been measured in the rapidity \( y \) defined with respect to the
thrust or sphericity axis \cite{2}. The Monte Carlo analysis for this variable is performed in the full rapidity range $|Y| \leq 5$. Fig. 1 shows the results for the BPs \cite{4} for rapidity after the Bialas-Gazdzicki-Ochs transformation. The second-order BP for JETSET model decreases with increasing $M$ up to $M \approx 20$, which is found to correspond to the value of $M$ at which the maximum of the multiplicity distribution $P_n(\delta)$ first occurs at $n = 0$. At large $M$, all BPs show a power-law increase with increasing $M$, $\eta_0 \sim M^{\alpha_q}$. This indicates that the fluctuations in $y$ defined with respect to the thrust axis are multifractal scale invariant.

![Figure 1](image)

Figure 1. BPs as a function of the number $M$ of bins in rapidity defined with respect to the thrust axis. The shaded areas represent the statistical and systematical errors on the JETSET predictions.

Note that the conclusion that fluctuations have a multifractal structure is possible without the necessity of calculating the intermittency indices $\phi_q$. In contrast, to reveal multifractality with the help of the NFMs, one first needs to carry out fits of the NFMs by a power law.
HERWIG predictions (dashed lines) significantly overestimate the second-order BP obtained from LUND MCs. Since, for small phase-space cells, the second-order BP is determined by the dispersion of the distribution [4, 5], this means that the HERWIG produces too broad local multiplicity distributions. Such a result confirms that obtained by the ALEPH Collaboration [12].

To study the disagreement between Monte-Carlo models in more detail, one can split \( \eta_2 \) into two BPs:

\[
\eta_2 = \eta_2^{(\pm\pm)} + \eta_2^{(+-)}.
\] (7)

Here \( \eta_2^{(\pm\pm)} \) is defined by (4) with \( N_2(m, \delta) = N_2^{(\pm\pm)}(m, \delta y) \), \( N_2^{(\pm\pm)}(m, \delta y) \) being the number of events having like-charged two-particle combinations inside bin \( m \) of size \( \delta y \). Analogously, \( \eta_2^{(+-)} \) is constructed from the number of events \( N_2^{(+-)}(m, \delta y) \) having unlike-charged two-particle combinations. Note that due to a combinatorial reason, \( \eta_2^{(\pm\pm)} < \eta_2^{(+-)} \).

![Figure 2. The second-order BP as a function of the number M of bins in rapidity defined with respect to the thrust axis for like-charged and unlike-charged particle combinations.](image)

Fig. 2 shows that \( \eta_2^{(\pm\pm)} \) and \( \eta_2^{(+-)} \) indeed behave completely differently. While \( \eta_2^{(\pm\pm)} \) shows the expected rise, \( \eta_2^{(+-)} \) shows a strong decrease at low
and the onset of an increase at large $M$. The structure of $\eta_2$ observed in Fig. 2 is a combination of these two effects.

Note that $\eta_2$ is strongly effected by the Bose-Einstein (BE) interference incorporated into the JETSET generator\(^1\). This is not unexpected since $\eta_2 \sim P_2/P_1^2$, which is very similar to the correlation functions used for Bose-Einstein studies.

Let us remind that in order to model the BE interference in JETSET, the momenta of identical final-state particles are shifted to reproduce the expected two-particle correlation function. The main disadvantage of such \textit{ad hoc} method is that it spoils overall energy-momentum conservation and it is necessary to modify also momenta of non-identical particles to compensate for this. This effect in JETSET model can be seen in Fig. 2.

The strong anti-bunching tendency seen for unlike-charged particles at $M < 30$ can be attributed to resonance decays and to chain-like particle production along the thrust axis, as expected from the QCD-string model \cite{13}. The latter effect leads to local charge conservation with an alternating charge structure. Evidence for this effect was recently observed by DELPHI \cite{14}. As a result, there is a smaller rapidity separation between unlike-charged particles than between like-charged and $\eta_2^{(+\pm)}$ is much larger than $\eta_2^{(\pm\pm)}$ at small $M$. Having correlation lengths $\delta y \sim 0.5 - 1.0$ in rapidity, the resonance and the charge-ordering effects, however, become smaller with increasing $M$.

Note that to distinguish the NFMs calculated for different charge combinations in a bin-splitting technique is difficult due to insufficient sensitivity of this tool and a purely combinatorial reason.

### 3.2 In the four-momentum difference

The study of BPs described above can help us to understand a tendency of the particles to be grouped into spikes inside small phase-space intervals. Another question is how the multiplicity of these spikes fluctuates from event to event when the spike size goes to zero. To study this, we will use the BPs defined in (9).

Fig. 3 shows the behavior of $\chi_q$ as a function of $-\ln Q^2$. The full lines represent the behavior of the BPs in the Poissonian case. In contrast, all BPs obtained from the Monte Carlo models rise with increasing $-\ln Q^2$.

\(^1\)Here and below, we show JETSET predictions with the BE interference disabled after the retuning of this model to describe global-shape variables.
Figure 3. Generalized integral BPs as a function of the squared four-momentum difference $Q^2$ between two charged particles.

Figure 4. Generalized second-order BP as a function of the squared four-momentum difference $Q^2$ between two charged particles.
(decreasing $Q^2$). This corresponds to a strong bunching effect of all orders, as expected for multifractal fluctuations. The anti-bunching effect ($\chi_q < 1$) for small $-\ln Q^2$ is caused by the energy-momentum conservation constraint $[5]$.

To learn more about the mechanism of multiparticle fluctuations in $Q^2_{12}$ variable, we present in Fig. 4 the behavior of the second-order BP as a function of $-\ln Q^2$ for multiparticle hyper-tubes (spikes) made of like-charged and those of unlike-charged particles, separately. A significant difference is observed for like-charged combinations between HERWIG and LUND MCs.

4 Discussion

Local multiplicity fluctuations in Monte Carlo models have been studied by means of bunching parameters. Since all high-order BPs show a power-like rise with decreasing the size of phase-space interval, none of the conventional multiplicity distributions given in Table 1 can describe the local fluctuations observed in the MC models.

For $e^+e^-$ interactions, one can be confident that, at least on the parton level of this reaction, perturbative QCD can give a hint for the understanding of the problem. Analytical calculations based on the DLLA of perturbative QCD show that the multiplicity distribution of partons in ever smaller opening angles is inherently multifractal $[15]$. Qualitatively, this is consistent with our results on the BPs for rapidity. Quantitatively, however, the QCD predictions disagree with the $e^+e^-$ data and MC models $[16]$.

In this paper we show that the power-law behavior of BPs is mainly due to like-charged particles. JETSET gives the same power-law trend even without the BE effect. This means that the intermittency observed for like-charged particles appears to be largely a consequence of QCD parton showers and hadronization.

The predictions of the ARIADNE 4.08 model are comparable with those of the JETSET 7.4 PS model. This is essentially due to the same implementation of hadronization, which is based for both models on string fragmentation.

A noticeable disagreement, however, is found between LUND and HERWIG models. The conversion of the partons into hadrons in the first models is based on the Lund String Model $[13]$. However, the hadronization
in HERWIG is modelled with a cluster mechanism [9]. This can be a rather natural candidate to explain the observed difference between local fluctuations in these models. A particular concern is the large difference between MC’s for $\eta_2$. The behavior of $\eta_2$ for not very small intervals is sensitive to low-multiplicity events, for which hadronization details could play a significant role.

Acknowledgments

I wish to express my gratitude to W.Kittel and W.J.Metzger for many useful discussions.

References

[1] A.Białas and R.Peschanski, Nucl. Phys. B273 (1986) 703; Nucl. Phys. B308 (1988) 857.

[2] TASSO Collab., W.Braunschweig et al., Phys. Lett. B231 (1989) 548; K.Sugano (HRS Collab.), Proc. Santa Fe Workshop “Intermittency in High-Energy Collisions”, Eds. F.Cooper et al. (World Scientific, Singapore, 1991) p.1; CELLO Collab., H.J.Behrend et al., Phys. Lett. B262 (1991) 351; ALEPH Collab., D.Decamp et al., Z. Phys. C53 (1992) 21; DELPHI Collab., P.Abreu et al., Phys. Lett. B247 (1990) 137; A.De Angelis, Mod. Phys. Lett. A5 (1990) 2395; DELPHI Collab., P.Abreu et al., Nucl. Phys. B386 (1992) 471; L3 Collab., B. Adeva et al., Z. Phys. C55 (1992) 39.

[3] P.Bożek, M.Ploszajczak and R.Botet, Phys. Rep. 252 (1995) 101; E.A.De Wolf, I.M.Dremin and W.Kittel, Phys. Rep. 270 (1996) 1.

[4] S.V.Chekanov and V.I.Kuvshinov, Acta Phys. Pol. B25 (1994) 1189; S.V.Chekanov, W.Kittel and V.I.Kuvshinov, Acta Phys. Pol. B27 (1996) 1739; S.V.Chekanov and V.I.Kuvshinov, J. Phys. G22 (1996) 601.

[5] S.V.Chekanov, W.Kittel and V.I.Kuvshinov, Z. Phys. C73 (1997) 517.
[6] S.V.Chekanov, V.I.Kuvshinov, J.Phys. G23 (1997) 951.

[7] T.Sjöstrand, Comp. Phys. Comm. 82 (1994) 74.

[8] L.Lönnblad, Comp. Phys. Comm. 71 (1992) 15.

[9] G.Marchesini and B.R.Webber, Nucl. Phys. B310 (1988) 461 ; G.Marchesini et al., Comp. Phys. Comm. 67 (1992) 465.

[10] J.Casaus, L3 Note 1946 (1996); Sunanda Banerjee and Swagato Banerjee, L3 Note 1978 (1996); I.G.Knowles and T.Sjöstrand (conveners), “QCD Event Generators”, Physics at LEP2, CERN-96-01, Vol. 2 (1996) p.103.

[11] A.Bialas and M.Gazdzicki, Phys. Lett. B252 (1990) 483 ; W.Ochs, Z. Phys. C50 (1991) 339.

[12] ALEPH Collab., D.Buskulic et al., Z. Phys. C69 (1995) 15.

[13] B.Andersson, G.Gustafson, G.Ingelman, T.Sjöstrand, Phys. Rep. 97 (1983) 31.

[14] DELPHI Collab., P.Abreu et al., Phys. Lett. B407 (1997) 174.

[15] W.Ochs and J.Wosiek, Phys. Lett. B289 (1992) 159 ; B304 (1993) 144 ; Z. Phys. C68 (1995) 269 ; Ph.Brax, J.-L.Meunier and R.Peschanski, Z. Phys. C62 (1994) 649 ; Yu.Dokshitzer and I.M.Dremin, Nucl. Phys. B402 (1993) 139.

[16] S.V.Chekanov (for the L3 Collaboration), Presented at the 32th Rencontres de Moriond, “QCD and High Energy Hadronic Interactions” Les Arcs, France (1997) hep-ex/9707012. S.V.Chekanov, “Local Multiplicity Fluctuations and Intermittent Structure Inside Jets’, Thesis, (1997), Nijmegen, The Netherlands.