Fermion Mass Hierarchy and Supersymmetry Breaking in $6D$ $SO(10)$ GUT on Orbifold

Naoyuki HABA$^{1,2}$*, Yasuhiro SHIMIZU$^{2}$**

$^1$Faculty of Engineering, Mie University, Tsu, Mie, 514-8507, Japan
$^2$Department of Physics, Nagoya University, Nagoya, 464-8602, Japan

We suggest simple models which produce the suitable fermion mass hierarchies and flavor mixing angles based on the 6 dimensional $N = 1$ supersymmetric $SO(10)$ grand unified theory compactified on a $T^2/(Z_2 \times Z'_2)$ orbifold. We introduce extra vector-like heavy fields in the extra dimensions, and the suitable fermion mass hierarchies and flavor mixings are generated by integrating out these heavy fields. We consider gaugino mediation and gauge mediation supersymmetry breaking mechanisms and their flavor structures. The experimental constraints of small flavor changing neutral currents suggest where to locate the supersymmetry breaking brane in the gaugino mediation mechanism. On the other hand, the SUSY breaking masses are highly degenerated in the gauge mediation scenario, where the flavor changing neutral currents are naturally suppressed as in the ordinal four dimensional gauge mediation models.

§1. Introduction

Grand unified theories (GUTs) are very attractive models in which the three gauge groups are unified at a high energy scale. However, one of the most serious problems to construct a model of GUTs is how to realize the mass splitting between the triplet and the doublet Higgs particles in the Higgs sector. This problem is so-called triplet-doublet (TD) splitting problem. A new idea for solving the TD splitting problem has been suggested in higher dimensional GUTs where the extra dimensional coordinates are compactified on orbifolds. In these scenarios, Higgs and gauge fields are propagating in extra dimensions, and the orbifolding realizes the gauge group reduction and the TD splitting since the doublet (triplet) Higgs fields have (not) Kaluza-Klein zero-modes. A lot of attempts and progresses have been done in the extra dimensional GUTs on orbifolds. Especially, the reduction of $SO(10)$ gauge symmetry and the TD splitting solution are first considered in 6D models in Refs. 8), 9).

As for producing fermion mass hierarchies, several trials have been done in the extra dimensional GUTs on orbifolds. The model in Ref. 10) can induce the natural fermion mass hierarchies and flavor mixings based on a 6D $N = 1$ SUSY ((1,0)-SUSY) $SO(10)$ GUT where the 5th and 6th dimensional coordinates are compactified on a $T^2/Z_2$ orbifold. In this scenario, we introduce extra vector-like generations, $2 \times (\psi_{16_i} + \psi_{\overline{16}_i})$ and $(\psi_{16_5} + \psi_{\overline{16}_5})$, which propagate 6 and 5 dimensions, respectively. Assuming that 4th (5th) generation vector-like fields only couple to the 1st (2nd) generation chiral fields, the suitable fermion mass hierarchies and flavor

* haba@eken.phys.nagoya-u.ac.jp
** shimizu@eken.phys.nagoya-u.ac.jp

typeset using PTP\TeX.cls (Ver.0.85)
mixings are generated by integrating out these vector-like heavy fields. The mixing angles between the chiral fields and extra generations have been determined by the volume suppression factors. The extension of this model has been considered in Ref. [1] where the values of \( m_e, m_d, V_{us}, \) and \( V_{cd} \) have been improved by extending the vector-like extra generations and their configurations in the extra dimensions. However, there is a difficulty in this scenario. That is the lack of 5D fixed lines, which can not guarantee \( (\psi_{16s} + \psi_{\overline{16s}}) \) existing only in 5 dimensions, not spreading in 6 dimensions.

In this paper we will modify previous papers [2] by using the orbifold, \( T_2/(Z_2 \times Z_2') \), and also consider the SUSY breaking mechanism. We will consider the 6D \( N = 1 \) \( SO(10) \) GUT with vector-like matter contents on \( T_2/(Z_2 \times Z_2') \). As will be shown bellow, this modification makes no changes for the zero mode matter fields in the previous papers are useful. The gauge symmetry reduction and the TD splitting are also the same as those in the \( T_2/Z_2 \) orbifold. The gauge and Higgs fields live in 6 dimensions and the orbifolding and boundary conditions make the \( SO(10) \) gauge group be broken to \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \) and realize the TD splitting.

As for the SUSY breaking mechanisms, we will consider the gaugino and the gauge mediation scenarios. In the gaugino mediation scenario, the vector-like matter fields in extra dimensions can directly couple to the SUSY breaking fields, which induces non-universal contributions to SUSY breaking masses for the light matter fields. These non-universal SUSY breaking masses can give rise to too large flavor changing neutral currents (FCNCs). Thus, the location of the SUSY breaking brane should be determined in order to avoid the large FCNC phenomenological problems in the gaugino mediation scenario. On the other hand, the SUSY breaking masses for the light matter fields are highly degenerated in the gauge mediation scenario, where the FCNCs are naturally suppressed as in the ordinal 4D gauge mediation models.

§2. Fermion mass hierarchies and flavor mixings

We consider the 6D \( N = 1 \) SUSY \( SO(10) \) GUT, whose extra dimensional coordinates are compactified on a \( T^2/(Z_2 \times Z_2') \) orbifold. The structure of extra 2D spaces are characterized by reflection \( P \) (\( Z_2 \)), \( P' \) (\( Z_2' \)), and translations \( T_i \) (\( i = 1, 2 \)). Under the reflection \( P \) and \( P' \), \((z, \bar{z})\) is transformed into \((-z, \bar{z})\) and \((z, -\bar{z})\), respectively. Where \( z \equiv (x_5 + ix_6)/2 \) and \( \bar{z} \equiv (x_5 - ix_6)/2 \) with the physical space of \( 0 \leq x_5, x_6 < \pi R \). Under the translation \( T_1 \) and \( T_2 \), \((z, \bar{z})\) are transformed into \((z + 2\pi R_z, \bar{z})\) and \((z, \bar{z} + 2\pi R_{\bar{z}})\), respectively, where \( R_z \equiv (1 + i)R/2 \) and \( R_{\bar{z}} \equiv (1 - i)R/2 \). The physical space can be taken as \( 0 \leq z < \pi R_z \) and \( 0 \leq \bar{z} \leq \pi R_{\bar{z}} \). Thus, the \( T_2/(Z_2 \times Z_2') \) orbifold is just the same as the \( S_1/Z_2 \otimes S_1/Z_2' \) orbifold of a regular square. There are four fixed points at \((0, 0)\), \((\pi R_z, 0)\), \((0, \pi R_{\bar{z}})\) and \((\pi R_z, \pi R_{\bar{z}})\), and two fixed lines on \( z = 0 \) and \( \bar{z} = 0 \) on the orbifold. The bulk fields are decomposed by \( P, P' \), and \( T_i \). For examples, a 6D bulk scalar field \( \Phi(x^\mu, z, \bar{z}) \) is

---

*** We would like to thank T. Kugo for pointing out this problem.
decomposed into

\[ \Phi_{(\pm\pm)(\pm\pm)}(x^\mu, z, \bar{z}) \equiv \frac{1}{\pi R_c} \phi_{(\pm\pm)z(\pm\pm)}(x^\mu) \varphi_{(\pm\pm)}(z) \varphi_{(\pm\pm)}(\bar{z}), \]  

(2.1)

according to the eigenvalues of \((P, T_1)(P', T_2) (= (P, T_1)z \otimes (P', T_2)\bar{z})\). Where \(R_c \equiv |R_z| = |R_{\bar{z}}|\). Notice that only \(\Phi_{(\pm\pm)(\pm\pm)}\) can have massless zero-modes and survives in the low energy.

We consider the gauge multiplet and two \(10\) representation Higgs multiplets propagate in the 6D bulk, which are denoted as \(H_{10}\) and \(H'_{10}\), and the ordinal three-generation matter multiplets \(16_i, i = 1, 2, 3\) are localized on the 4D brane, \((0, 0)\).

We adopted the translations as \(T_{51} = \sigma_2 \otimes I_5\) and \(T_{52} = \sigma_2 \otimes \text{diag.}(1, 1, 1, -1, -1)\), which commute with the generators of the Georgi-Glashow \(SU(5) \times U(1)\) and the flipped \(SU(5)' \times U(1)'\) groups, respectively. Then, translations \(T_i\) make the \(SO(10)\) gauge group be broken to \(SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{X}\) and realize the TD splitting since the doublet (triplet) Higgs fields have (not) Kaluza-Klein zero-modes.

The zero mode of the 6D bulk matter field, \(\psi_{16(\pm\pm)(\pm\pm)}\), is classified into four types as

\[ \begin{align*}
\psi_{16(\pm\pm)(\pm\pm)} & \quad \text{(zero mode)} = Q, \\
\psi_{16(\pm\pm)(\pm\pm)} & \quad \text{(zero modes)} = \bar{U}, \bar{E}, \\
\psi_{16(\pm\pm)(\pm\pm)} & \quad \text{(zero modes)} = \bar{D}, \bar{N}, \\
\psi_{16(\pm\pm)(\pm\pm)} & \quad \text{(zero mode)} = L.
\end{align*} \]

(2.2)

Similarly, the zero mode of the 5D bulk field, which is propagating on the fixed line \(\bar{z} = 0\), is classified into

\[ \begin{align*}
\psi_{16(\pm\pm)} & \quad \text{(zero mode)} = Q, \bar{U}, \bar{E}, \\
\psi_{16(\pm\pm)} & \quad \text{(zero mode)} = L, \bar{D}, \bar{N},
\end{align*} \]

(2.3)

where the 2nd \pm sign represents the \(T_1\) parity.

1. **Model 0**

Now let us discuss how to generate fermion mass hierarchies and flavor mixings in three models. In three models, the 4D brand-localized Higgs fields, \(H_{16}\) and \(H'_{16}\), are introduced at \((0, 0)\), which are assumed to take vacuum expectation values (VEVs) of \(O(10^16)\) GeV in the directions of \(B - L\). We also impose the Peccei-Quinn symmetry and its charge on the multiplets: all matter multiplets have its charge 1, \(10\) representation Higgs multiplets have its charge \(-2\), and \(16\) and \(\bar{16}\) representation Higgs multiplets have its charge \(-1\). The superpotential of the Yukawa sector on the brane at \((0, 0)\) is given by

\[ W_Y = \left\{ \frac{y_{ij}^u}{M_*} H_{10}^{16} \bar{16}_i j + \frac{y_{ij}^d}{M_*} H'_{10}^{16} \bar{16}_i j \right\} \delta(z) \delta(\bar{z}), \]

(2.4)
in which $M_*$ is a ultraviolet cut-off scale of $O(10^{18})$ GeV. The index $i, j = 1 \sim 3$ shows the generation numbers. We consider all components of the Yukawa couplings in Eq.(2.4) are of order one.

As for the bulk matter fields, we introduced 5D bulk fields on $\tilde{z} = 0$, $(\psi_{16}^{++}, \psi_{16}^{c+}) + (\psi_{\overline{16}}^{++}, \psi_{\overline{16}}^{c+})$, which contains $10 + \overline{10}$ of $SU(5)$ as the zero modes, which are regarded as the 5th generation fields denoted by $\psi_{16^5}$ and $\overline{\psi}_{\overline{16}^5}$. The 5D bulk fields have the non-chiral structures since the 5D $N = 1$ SUSY corresponds to the 4D $N = 2$ SUSY. We assume that the 5th generation fields interact only with the 2nd generation matter fields. In 6D bulk, we introduce 6D bulk vector-like fields, $(\psi_{16}^{(++)}, \psi_{16}^{c(-)}), (\psi_{\overline{16}}^{(++)}, \psi_{\overline{16}}^{c(-)})$, which are regarded as the 4th generation fields denoted by $\psi_{16^4}$ and $\overline{\psi}_{\overline{16}^4}$, respectively. We assume that the 4th generation fields interact with only the 1st generation matter fields.

Then, in addition to the superpotential in Eq.(2.4), the following interactions between the chiral and extra generation fields on the 4D brane, $(0, 0)$,

$$W_6 = H_{16}H_{\overline{16}} \left\{ \frac{y_{14}}{M_2^2} \psi_{16^4} \psi_{\overline{16}^4} + \frac{y_{14}}{M_2^2} \psi_{16^4} \psi_{\overline{16}^4} + \frac{y_{14}}{M_2^2} \psi_{16^4} \psi_{\overline{16}^4} + \frac{y_{14}}{M_2^2} \psi_{16^4} \psi_{\overline{16}^4} \right\} \delta(z)\delta(\tilde{z}). \tag{2.5}$$

Where we assume that the vector-like masses which mix the 4th and the 5th generations are forbidden by the fundamental theory. Below the compactification scale, the interactions in Eq.(2.5) induce the mass terms for the Kaluza-Klein zero-modes of vector-like matter fields as,

$$W_4 \simeq \frac{v_N^2}{M_*} \left\{ \epsilon_1^4 \left( Q_4^{(0)} \overline{Q}_4^{(0)} + U_4^{(0)} \overline{U}_4^{(0)} + E_4^{(0)} \overline{E}_4^{(0)} \right) + \epsilon_2^4 \left( Q_1 \overline{Q}_4^{(0)} + U_1 \overline{U}_4^{(0)} + E_1 \overline{E}_4^{(0)} \right) + \epsilon_3^2 \left( Q_5^{(0)} \overline{Q}_5^{(0)} + U_5^{(0)} \overline{U}_5^{(0)} + E_5^{(0)} \overline{E}_5^{(0)} \right) \right\} \tag{2.6}$$

where $\langle H_{16} \rangle = \langle H_{\overline{16}} \rangle \equiv v_N$. $\epsilon_i$s are the volume suppression factors which are given by

$$\epsilon_1 = \epsilon_2 \equiv 1/\sqrt{\pi R_c M_*}. \tag{2.7}$$

These volume suppression factors play crucial roles for generating the fermion mass matrices in the low energy range. Now we set $1/R_c = O(10^{16})$ GeV, which means $\epsilon_1 \simeq \lambda^2 \sim 0.04$, where $\lambda$ is the Cabibbo angle, $\lambda \sim 0.2$.

After integrating out the heavy fields, the model gives the following mass matrices in the up quark sector, the down quark sector, and the charged lepton sector.

---

$^{\dagger}$ The zero modes of the 4th and 5th generation fields have vector-like masses of $\epsilon_1 v_3/M_0$ and $\epsilon_2^2 v_3/M_0$, respectively. Since these zero modes form $SU(5)$ multiples, we can expect that the gauge coupling unification is not spoiled.
\[ m_u^l \simeq \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v, \quad m_d^l \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix} \bar{\tau}, \quad m_e^l \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \end{pmatrix} \bar{\tau}, \]

respectively. \( \bar{\tau} \) and \( v \) are the vacuum expectation values of the weak Higgs doublets.

We write the mass matrices in the basis that the left-handed fermions are to the left and the right-handed fermions are to the right. We notice that all elements in the mass matrices have \( O(1) \) coefficients. The fermion mass hierarchies are given by

\[ m_t : m_c : m_u \simeq 1 : \lambda^4 : \lambda^8, \quad m_b : m_s : m_d \simeq m_\tau : m_\mu : m_e \simeq 1 : \lambda^2 : \lambda^4, \]

with the large \( \tan \beta \). The mass matrix of three light neutrinos \( m_{\nu}^{(l)} \) through the see-saw mechanism is given by

\[ m_{\nu}^{(l)} \simeq \frac{m_D^T m_D}{M_R} \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{v^2}{M_R}. \]

\( M_R \) is about \( 10^{14} \) GeV induced from the interaction

\[ W_{MN} = \frac{y_{ij}^N}{M_s} H_{16} \bar{H}_{16} \epsilon_i^j \epsilon _i^j \delta(z) \delta(\bar{z}) \]

at \((0,0)\). We can obtain the suitable mass scale \( (O(10^{-1}) \text{ eV}) \) for the atmospheric neutrino oscillation experiments, by taking account of the \( SO(10) \) relation, \( y_u \simeq y_\nu \).

As for the flavor mixings, the CKM and the MNS matrices are given by

\[ V_{CKM} \simeq \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad V_{MNS} \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \]

which realize the suitable flavor mixings roughly in order of magnitudes. They give us a natural explanation why the flavor mixing in the quark sector is small while the flavor mixing in the lepton sector is large. However, they suggest too small Cabibbo angle and too large \( V_{e3} \). For the suitable values of them, we need suitable choice of \( O(1) \) coefficients in mass matrices as in Ref. [28]. Or, if \( O(1) \) coefficients are not determined by a specific reason (symmetry) in the fundamental theory, it is meaningful to see the most probable hierarchies and mixing angles by considering random \( O(1) \) coefficients.\(^{29}\) Anyway, if the fermion mass hierarchies and flavor mixing angles should determined from the fundamental theory in order (power of \( \lambda \)) not by tunings of \( O(1) \) coefficients, we should modify this scenario. We show two examples of the modifications below.

**2. Model I**

In the first modification, which we call Model I, we introduce the additional vector-like 5D bulk matter fields, which are \( \psi_{16^+} + \psi_{16^-} \equiv \psi_{16^+} + \psi_{16^-} \). They are called
as the 4th generation fields and assumed to interact with only the 1st generation. Their PQ charge is 1 as the other matter fields. In this case the following terms are added to Eq. (2.5)

$$W_6 = H_{16} H_{16} + \frac{y_{14}}{M_*^2} \psi_{16} \psi_{16}^c + \frac{y_{24}}{M_*^{5/2}} 16_1 \psi_{16} \psi_{16}^c \delta(z) \delta(\bar{z}).$$ (2.14)

When $1/R_c = O(10^{16})$ GeV, which means $\epsilon_1 \sim \epsilon_2 \sim \lambda^2$, the fermion mass matrices in the low energy are given by

$$m^l_u \simeq \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \\ \lambda^4 & 1 & 1 \end{pmatrix} v, \quad m^l_d \simeq \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \lambda v,$$

$$m^l_e \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} v, \quad m^{l(l)}_\nu \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \frac{v^2}{M_R},$$ (2.15)

after integrating out the heavy vector-like fields. They induce the more realistic fermion mass hierarchies as

$$m_t : m_c : m_u \simeq 1 : \lambda^4 : \lambda^8,$$

$$m_b : m_s : m_d \simeq m_\tau : m_\mu : m_e \simeq 1 : \lambda^2 : \lambda^6,$$ (2.16)

with large $\tan \beta$. As for the neutrino sector, the rank of $2 \times 2$ sub-matrix in the 2nd and the 3rd generations in $m^{l(l)}_\nu$ should be reduced, and the light eigenvalue of this sub-matrix should be of $O(\lambda^2)$ for the LMA solar neutrino solution. This case induce the hierarchical type of neutrino mass, $m_1, m_2 \ll m_3$. Then, the CKM and the MNS matrices become

$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad V_{MNS} \simeq \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \lambda^2 \\ 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix},$$ (2.17)

where the MNS matrix has the large 1-2 and 2-3 mixings because of the assumption of the rank reduction. This case induce small value of $U_{e3}$. The CKM matrix has the same structure as in the Model 0. Needless to say, the suitable $V_{us}$ can be easily obtained by choosing the $O(1)$ coefficients.

(3). Model II

Here let us show the second modification, which we call Model II. We introduce the following bulk matter fields with PQ charge 1 in addition to the Model 0: $\psi_{16^- -} + \psi_{16^- -}^c \equiv \psi_{16_i}^{\mu} + \psi_{16_i}^{\mu}^c$ and $\psi_{16^- -} + \psi_{16^- -}^c \equiv \psi_{16_i}^{\mu} + \psi_{16_i}^{\mu}^c$ (we call them the 4th generation fields) which propagate in the 6D bulk and interact with only the 1st generation matter multiplet, $\psi_{16^- -} + \psi_{16^- -}^c \equiv \psi_{16}^{\mu} + \psi_{16}^{\mu}^c$ (we call them the 5th generation fields) which propagate in the 5D bulk ($\bar{z} = 0$) and interact with only the 2nd generation matter multiplet, $\psi_{16^- -} + \psi_{16^- -}^c \equiv \psi_{16}^{\mu} + \psi_{16}^{\mu}^c$ (we call them the 6th
Fermion Mass Hierarchy and Supersymmetry Breaking in 6D SO(10) GUT on Orbifold

... which propagate in the 5D bulk ($\bar{z} = 0$) and interact with only the 3rd generation matter multiplet. In this case, the following terms are added to Eq.(2.5),

$$W_6 = H_{16} H_{16} \left( \frac{y_{14}'}{M_q^2} \psi_{16} \psi_{16}^c + \frac{y_{14}''}{M_q^2} \psi_{16} \psi_{16}^c + \frac{y_{14}'}{M_q^2} \bar{16}_1 \psi_{16}^c + \frac{y_{14}''}{M_q^2} \bar{16}_1 \psi_{16}^c \right) \delta (z) \delta (\bar{z}).$$ (2.18)

When $1/R_c = O(10^{16})$ GeV, which means $\epsilon_1 \sim \epsilon_2 \sim \lambda^2$, the fermion mass matrices below the compactification scale become

$$m_u^l \simeq \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v, \quad m_d^l \simeq \lambda^2 \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix},$$

$$m_e^l \simeq \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \bar{v}, \quad m_{\nu}^l \simeq \lambda^4 \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \frac{v^2}{M_R}. \quad (2.19)$$

The forms of these mass matrices are the same as those of the first case of Model I except for the overall factors. Thus the suitable fermion mass hierarchies of the quark and the charged lepton sectors are the same as Eq.(2.16). The flavor mixing matrices, $V_{CKM}$ and $V_{MNS}$, are also the same as those of the first case of Model I. The different between this model and the first case of Model I exists just in the value of $\tan \beta$. This model shows the small $\tan \beta$ of $\tan \beta \sim m_t/m_b \sim 1$. The discussion of neutrino mass hierarchy and the flavor mixings are also the same as the first case of Model I.

§3. SUSY breaking and flavor mixings

Now let us discuss how to induce soft SUSY breaking terms in our model. We consider the gaugino and the gauge mediation scenarios in Model 0–II. The soft SUSY breaking terms for the matter fields depend on the two SUSY breaking scenarios and bulk matter configurations in the extra dimension.

(1). Model 0

As for the gaugino mediation scenario, SUSY is broken at a spatially different place from our living brane, $(0, 0)$, in extra dimensions. In our 6D theory, there are three fixed points where we can put a SUSY breaking field, $S = \theta^2 F$. Since the gauge multiplets live in the 6D bulk, the gauginos receive the SUSY breaking masses through a direct interaction with $S$. When $S$ is located at the fixed point $(\pi R_z, 0)$, it is possible to write an interaction as,

$$\mathcal{L} = \frac{1}{M_3^2} \int dz d\bar{z} \int d^2 \theta S W_{ij} \delta (z - \pi R_z) \delta (\bar{z}). \quad (3.1)$$
This interaction gives rise to the gaugino masses as $M_{\tilde{g}i} = \epsilon_1^2 F/M_* \equiv \epsilon_1^2 \tilde{m}$. The cases of $S$ at other fixed points can be considered in the same way. Since the chiral matter fields $\mathbf{16}_i$ are localized on the 4D brane $(0,0)$, they do not couple directly to $S$. Thus, $\mathbf{16}_i$ receive SUSY breaking masses only through the renormalization effect of the gaugino masses. On the other hand, the 6D bulk matter fields can have the direct coupling to $S$ as

$$\mathcal{L} = \frac{1}{M_*^4} \int dz d\bar{z} d^4 \theta S^\dagger \delta \left( \psi^\dagger_{\mathbf{16}_5} \psi_{\mathbf{16}_5} + \psi^\dagger_{\mathbf{16}_5} \psi_{\mathbf{16}_5} \right) \delta(z - \pi R_{\tilde{z}}) \delta(\bar{z}).$$

This induces the SUSY breaking mass to the zero-modes of the 6D matter fields as

$$\mathcal{L}_{\text{soft}} = -\epsilon_1^4 m_{\tilde{m}}^2 \left( \bar{Q}_4^{(0)} \tilde{Q}_4^{(0)} + \bar{U}_4^{(0)} \tilde{U}_4^{(0)} + \bar{E}_4^{(0)} \tilde{E}_4^{(0)} + \bar{Q}_4 \tilde{Q}_4 + \bar{U}_4 \tilde{U}_4 + \bar{E}_4 \tilde{E}_4 \right),$$

at the compactification scale. The 5D bulk matter fields can also have the direct coupling with $S$ as

$$\mathcal{L} = \frac{1}{M_*^4} \int dz d\bar{z} d^4 \theta S^\dagger \left( \psi^\dagger_{\mathbf{16}_5} \psi_{\mathbf{16}_5} + \psi^\dagger_{\mathbf{16}_5} \psi_{\mathbf{16}_5} \right) \delta(z - \pi R_{\tilde{z}}) \delta(\bar{z}),$$

which induces the SUSY breaking for the zero modes as

$$\mathcal{L}_{\text{soft}} = -\epsilon_2^2 m_{\tilde{m}}^2 \left( \bar{Q}_5^{(0)} \tilde{Q}_5^{(0)} + \bar{U}_5^{(0)} \tilde{U}_5^{(0)} + \bar{E}_5^{(0)} \tilde{E}_5^{(0)} + \bar{Q}_5 \tilde{Q}_5 + \bar{U}_5 \tilde{U}_5 + \bar{E}_5 \tilde{E}_5 \right).$$

After integrating out the vector-like heavy fields, the soft SUSY breaking mass terms of the light matter fields become

$$m_{\tilde{10}}^2 = \left( \begin{array}{ccc} \epsilon_1^4 & 0 & 0 \\ 0 & \epsilon_2^2 & 0 \\ 0 & 0 & \tilde{m}^2 \end{array} \right), \quad m_{\frac{5}{2}}^2 = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right),$$

where $\tilde{10} = (\tilde{Q}, \tilde{U}, \tilde{E})$ and $\frac{5}{2} = (\tilde{D}, \tilde{L})$. They are soft masses around the compactification scale. Due to quantum corrections from the compactification scale to the electroweak scale, additional SUSY breaking masses of order the gaugino masses are added into the diagonal elements of Eq. (3.4). These soft SUSY breaking masses, however, are not phenomenologically acceptable. That is because, when we take the gaugino masses, $M_{\tilde{g}i} = \epsilon_1^2 \tilde{m}$, as $O(10^2)$ GeV, Eq. (3.4) suggests $(m_{\tilde{10}})_{22} = O(10^2)$ TeV. Such a heavy soft mass induces the color instability, that is negative mass squared of the stop through the 2-loop renormalization effects. As for other choices of the locations of $S$, at $(0, \pi R_{\tilde{z}})$ or $(\pi R_{\tilde{z}}, \pi R_{\tilde{z}})$, through the interaction in Eq. (3.2), the SUSY breaking masses for the light matter fields become

$$m_{\tilde{10}}^2 = \left( \begin{array}{ccc} \epsilon_1^4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right), \quad m_{\frac{5}{2}}^2 = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right).$$

The radiative induced masses of order the gaugino masses are added in the diagonal elements in Eq. (3.7). Thus, the mass difference between the first and the second
Fermion Mass Hierarchy and Supersymmetry Breaking in 6D SO(10) GUT on Orbifold

generation left-handed down-type squarks is of order the gaugino mass. Notice that the experimental constraint from the $K^0\overline{K}^0$ mixing is given by

$$\sin^2 2\theta_{12} \left( \frac{\Delta m^2_{12}}{m_d^2} \right)^2 \left( \frac{10 \text{TeV}}{m_d} \right)^2 \lesssim 1,$$

(3.8)

where $m_d^2 = (m_{d_1}^2 m_{d_2}^2)^{1/2}$. In Model 0, the flavor mixing between the first and the second generation left-handed down-type squarks is $\sin \theta_{12} \sim \lambda^2$ from Eq.(2.8), and $(\Delta m^2_{12}/m_d^2) \sim 1$. Therefore the large FCNC can be avoided when the gaugino mass are $\geq O(1) \text{TeV}$ with $S$ at $(0, \pi R)$ or $(\pi R, \pi R)$.\n
Next we consider the gauge mediation scenario.\n
We assume that the messenger sector is localized on the 4D brane $(\pi R_z, 0)$ and introduce $N$ pairs of vector-like messenger fields, $5^i_M$ and $\bar{5}^i_M$, which are 5 and $\bar{5}$ representations of the SU(5) group, respectively. A $U(1)$ in the bulk can transmit the SUSY breaking effects.\n
We consider the following superpotential for the messenger sector.

$$W = \sum_\alpha \lambda_\alpha S 5^i_M \bar{5}_\alpha^i \delta(z - \pi R_z) \delta(\bar{z}),$$

(3.9)

where $S = M + \theta^2 F_S$ is a spurion superfield, which represents the SUSY breaking. We assign the vanishing PQ charge for $5^i_M$, $\bar{5}^i_M$, and $S$. The gaugino and sfermion masses are induced to the MSSM sector through the SM gauge interactions as

$$M_{\tilde{g}_a} \simeq N \frac{\alpha_a}{4\pi} \frac{F_S}{M},$$

(3.10)

$$(m_{\tilde{f}}^2)_{ij} \simeq 2N \sum_\alpha C_\alpha(\tilde{f}) \left( \frac{\alpha_a}{4\pi} \right)^2 \left| \frac{F_S}{M} \right|^2 \delta_{ij},$$

where $a(= 1 - 3)$ represents the gauge groups and $C_\alpha(\tilde{f})$ is the quadratic Casimir for the sfermions. Since the sfermion masses are determined by the gauge quantum number, the SUSY breaking masses are the same for the vector-like heavy fields. As a result, the SUSY breaking masses for the light matter fields are universal around the messenger scale. It is because the vector-like matter fields in the bulk do not have the coupling of $W = \frac{1}{M^2} S \psi_{16} \psi_{\bar{16}} \delta(z - R_z) \delta(\bar{z})$ in the superpotential due to the PQ symmetry in which all matter fields have charge 1. The non-universal contribution to the sfermion masses is induced from the interaction,

$$\int \frac{dz}{M^2} \int d^4z S^* \psi_{16}^* \psi_{\bar{16}} \delta(z - R_z) \delta(\bar{z}) \sim \epsilon_1 (\frac{F_S}{M})^2 \tilde{Q}_{1}^{(0)} \bar{Q}_{1}^{(0)}.$$\n
However, such a non-universal effect is $\epsilon_1 (\frac{F_S}{M})^2 \simeq O(10^{-14}) \text{GeV}^2$ with $\sqrt{F_S} = O(10^7) \text{GeV}$, which is negligible compared to the contribution in Eq.(3.10). Thus, the flavor mixing for sfermions are naturally suppressed as in the 4D gauge mediation scenario.\n
\[\text{†† Here we do not specify the dynamical SUSY breaking sector since the mass spectra of the MSSM fields do not depend the detail of them.}\]
(2). Model I
In Model I, we consider the gaugino mediation scenario with $S$ being localized on the 4D brane $(\pi R_z, 0)$ at first. The 6D matter fields obtain the SUSY breaking masses as in Eq.(3.3). For 5D matter fields, $\psi_{16_5}$ and $\psi_{\overline{16}_5}$, the SUSY breaking masses are induced as in Eq.(3.5). In addition, the interaction between $\psi_{16_4}^\mu$, $\psi_{\overline{16}_4}^\mu$, and $S$, 

$$L = \frac{1}{M_s^4} \int dz d\bar{z} d^4 \theta S^\dagger S \left(\psi_{16_4}^\mu \psi_{16_4}^\mu + \psi_{\overline{16}_4}^\mu \psi_{\overline{16}_4}^\mu\right) \delta(z - \pi R_z) \delta(\bar{z}),$$

induces the SUSY breaking terms 

$$L_{\text{soft}} = -\epsilon_2^2 \frac{m^2}{m^2} \left(\tilde{L}_5(0) + \tilde{D}_5(0) + \tilde{N}_5(0)\right) \left(\tilde{L}_5(0) + \tilde{D}_5(0) + \tilde{N}_5(0)\right) \delta(z - \pi R_z) \delta(\bar{z}).$$

After integrating out the vector-like heavy fields, the soft SUSY breaking masses for the light matter fields are given by 

$$m_{10}^2 = \left(\begin{array}{ccc} e_1^4 & 0 & 0 \\ 0 & e_2^4 & 0 \\ 0 & 0 & 0 \end{array}\right) m^2, \quad m_{\overline{10}}^2 = \left(\begin{array}{ccc} e_1^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) m^2. \quad (3.12)$$

They suggest that the above mass spectra suffer from the large FCNC problem as in Model 0. For the cases where $S$ is localized on the 4D brane $(0, \pi R_z)$ or $(\pi R_z, \pi R_z)$, the FCNC problem can be solved with gaugino mass $\geq O(1)$ TeV, since $\epsilon_2^2$ above vanish in Eq.(3.13). These are the same situations as in Model 0. While the gauge mediation mechanism works well as in Model 0.

(3). Model II
As for the gaugino mediation scenario with $S$ being located at $(\pi R_z, 0)$ in Model II, the 6D matter fields, $\psi_{16_4}$, $\psi_{\overline{16}_4}$, $\psi_{16_4}$, and $\psi_{\overline{16}_4}$, obtain the SUSY breaking masses as in Eq.(3.3). For the additional 6D fields, $\psi_{16_4}$, $\psi_{\overline{16}_4}$, $\psi_{16_4}$, and $\psi_{\overline{16}_4}$, they have a direct coupling to $S$, 

$$L = \frac{1}{M_s^4} \int dz d\bar{z} d^4 \theta S^\dagger S \left(\psi_{16_4}^\mu \psi_{16_4}^\mu + \psi_{\overline{16}_4}^\mu \psi_{\overline{16}_4}^\mu + \psi_{16_4}^\mu \psi_{16_4}^\mu + \psi_{\overline{16}_4}^\mu \psi_{\overline{16}_4}^\mu\right) \delta(z - \pi R_z) \delta(\bar{z}),$$

which induces SUSY breaking masses as 

$$L_{\text{soft}} = -\epsilon_1^2 \frac{1}{m^2} \left(\tilde{m}_4^{(0)} + \tilde{D}_4^{(0)} + \tilde{N}_4^{(0)}\right) \left(\tilde{m}_4^{(0)} + \tilde{D}_4^{(0)} + \tilde{N}_4^{(0)}\right) \delta(z - \pi R_z) \delta(\bar{z}).$$

On the other hand, the 5D matter fields have the interaction, Eq.(3.5). 

$$L = \frac{1}{M_s^4} \int dz d\bar{z} d^4 \theta S^\dagger S \left(\psi_{16_5}^\mu \psi_{16_5}^\mu + \psi_{\overline{16}_5}^\mu \psi_{\overline{16}_5}^\mu + \psi_{16_5}^\mu \psi_{16_5}^\mu + \psi_{\overline{16}_5}^\mu \psi_{\overline{16}_5}^\mu\right) \delta(z - \pi R_z) \delta(\bar{z}).$$
which induces the SUSY breaking masses,

\[ \mathcal{L}_{\text{soft}} = - e_2^2 \overline{m}^2 \left( \bar{L}_5^{(0)\dagger} \bar{L}_5^{(0)} + \bar{D}_5^{(0)\dagger} \bar{D}_5^{(0)} + \bar{N}_5^{(0)\dagger} \bar{N}_5^{(0)} + \bar{L}_5^{(0)\dagger} \bar{L}_5^{(0)} + \bar{D}_5^{(0)\dagger} \bar{D}_5^{(0)} + \bar{N}_5^{(0)\dagger} \bar{N}_5^{(0)} + \bar{L}_6^{(0)\dagger} \bar{L}_6^{(0)} + \bar{D}_6^{(0)\dagger} \bar{D}_6^{(0)} + \bar{N}_6^{(0)\dagger} \bar{N}_6^{(0)} \right) \]  

After integrating out the vector-like heavy fields, the SUSY breaking masses for the light matter fields are given by

\[ m_{10}^2 = \begin{pmatrix} e_1^4 & 0 & 0 \\ 0 & e_2^2 & 0 \\ 0 & 0 & e_2^2 \end{pmatrix} \overline{m}^2, \quad m_{5}^2 = \begin{pmatrix} e_1^4 & 0 & 0 \\ 0 & e_2^2 & 0 \\ 0 & 0 & e_2^2 \end{pmatrix} \overline{m}^2. \]  

These mass spectra also suffer from the phenomenological problem as in Models 0 and I. As for the cases where \( S \) is localized on the 4D brane \((0, \pi R_z)\) or \((\pi R_z, \pi R_z)\), the FCNC problem can be avoidable when gaugino mass \( \geq O(1) \) TeV as in the Models 0 and I, since \( e_2^2s \) vanish in Eq.(3.18). These are the same situations as in Models 0 and I. On the other hand, the gauge mediation in Model II works well as in Models 0 and I.

\section{Summary and Discussion}

In this paper, we have shown three models based on the 6D \( N = 1 \) SUSY \( SO(10) \) GUT where the 5th and 6th dimensional coordinates are compactified on a \( T^2/(Z_2 \times Z'_2) \) orbifold. The gauge and Higgs fields live in 6 dimensions while ordinal chiral matter fields are localized in 4 dimensions. We have shown briefly three models which can produce the suitable fermion mass hierarchies and flavor mixings. In these models, the three-generation chiral matter fields are localized at the 4D wall, and the suitable fermion mass hierarchies and flavor mixings are generated by integrating out vector-like heavy generations.

As for the SUSY breaking mechanisms, we have considered the gaugino and the gauge mediation scenarios. In the gaugino mediation scenario, the vector-like matter fields in extra dimensions can directly couple to the SUSY breaking fields, which induces non-universal contributions to SUSY breaking masses for the light matter fields. These non-universal SUSY breaking masses can give rise to too large flavor changing neutral currents (FCNCs). Thus, the location of the SUSY breaking brane should be determined in order to avoid the large FCNC phenomenological problems in the gaugino mediation scenario. The condition of the gaugino mass \( \geq O(1) \) TeV is also needed. On the other hand, the SUSY breaking masses for the light matter fields are highly degenerated in the gauge mediation scenario, where the FCNCs are naturally suppressed as in the ordinal 4D gauge mediation models.

Finally we comment on other SUSY breaking scenarios. The gravity mediation scenario gives rise the non-universal corrections to the soft SUSY masses in general. It is because the SUSY breaking effects are mediated by “Yukawa” interactions not by gauge interactions. The “Yukawa” interactions among the bulk fields always receive...
the volume suppressions, which violate the degeneracy of the soft SUSY breaking masses. The Scherk-Schwarz SUSY breaking \[ \text{might also give the non-negligible effects of breaking degeneracy, since the first and the second generation fields are mainly composed by the bulk fields in our three models.} \]

Acknowledgment

We would like to thank T. Kugo and Y. Nomura for helpful discussions. This work is supported in part by the Grant-in-Aid for Science Research, Ministry of Education, Culture, Sports, Science and Technology, of Japan (No. 14039207, No. 14046208, No. 14740164).

References

1) see for examples,
   Y. Kawamura, Prog. Theor. Phys. 103 (2000), 613; ibid 105 (2001), 691; ibid 105 (2001), 999;
   G. Altarelli and F. Feruglio, Phys. Lett. B 511 (2001), 257;
   A. B. Kobakhidze, Phys. Lett. B 514 (2001), 131;
   L. J. Hall and Y. Nomura, Phys. Rev. D 64 (2001), 055003;
   Y. Nomura, D. Smith and N. Weiner, Nucl. Phys. B 613 (2001), 147;
   A. Hebecker and J. March-Russell, Nucl. Phys. B 613 (2001), 3;
   R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D66 (2002), 045025;
   R. Barbieri, L. J. Hall and Y. Nomura, Nucl. Phys. B 624 (2002), 63;
   L. J. Hall, H. Murayama and Y. Nomura, hep-th/0107245;
   N. Haba, T. Kondo, Y. Shimizu, T. Suzuki and K. Ukai, Prog. Theor. Phys. 106 (2001), 1247;
   Y. Nomura, Phys. Rev. D65 (2002), 085036.
2) L. J. Hall and Y. Nomura, Phys. Rev. D65 (2002), 125012.
3) A. Hebecker and J. March-Russell, Nucl. Phys. B 625 (2002) 128.
4) N. Haba, Y. Shimizu, T. Suzuki and K. Ukai, Prog. Theor. Phys. 107 (2002) 151
5) R. Dermisek and A. Mall, Phys. Rev. D 65 (2002), 055002.
6) T. Li, Nucl. Phys. B619 (2001), 75.
7) T. Li, Phys. Lett. B 520 (2001), 377;
   L. J. Hall, Y. Nomura and D. Smith, Nucl. Phys. B639 (2002), 307;
   T. Watari and T. Yanagida, Phys. Lett. B 519 (2001), 164.
8) T. Asaka, W. Buchmüller and L. Covi, Phys. Lett. B523 (2001), 199.
9) L. J. Hall, Y. Nomura, T. Okui and D. Smith, Phys. Rev. D 65 (2002), 035008.
10) N. Haba, T. Kondo and Y. Shimizu, Phys. Lett. B531 (2002) 245.
11) N. Haba, T. Kondo and Y. Shimizu, Phys. Lett. B535 (2002) 271.
12) L. J. Hall, J. March-Russell, T. Okui and D. Smith, hep-ph/0108161.
13) N. Arkani-Hamed, T. Gregoire and J. Wacker, JHEP 0203 (2002), 055.
14) T. Kawamoto and Y. Kawamura, hep-ph/0106163.
15) A. Hebecker and J. March-Russell, Phys. Lett. B 541 (2002) 338.
16) L. Hall and Y. Nomura, hep-ph/0205067.
17) S. M. Barr and I. Dorsner, Phys. Rev. D 66 (2002) 065013.
18) L. Hall and Y. Nomura, hep-ph/0207079.
19) A. Hebecker, J. March-Russell and T. Yanagida, arXiv:hep-ph/0208249.
20) R. Barbieri, L. Hall, G. Marandella, Y. Nomura, T. Okui, and S. Oliver, M. Papucci, hep-ph/0208153.
21) C. H. Albright and S. M. Barr, arXiv:hep-ph/0209173.
22) Q. Shafi and Z. Tavartkiladze, arXiv:hep-ph/0210181.
23) H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, (1974), 438.
24) S. M. Barr, Phys. Lett. B 112 (1982), 219; Phys. Rev. D 40 (1989), 2457;
   J. P. Derendinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 139 (1984), 170;
Fermion Mass Hierarchy and Supersymmetry Breaking in 6D SO(10) GUT on Orbifolds

I. Antoniadis, J. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B 194 (1987), 231; J. L. Lopez and D. V. Nanopoulos, hep-ph/9511264.

25) T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979) p.95; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979) p.315.

26) N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973), 652.

27) Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962), 870.

28) K. S. Babu and S. M. Barr, Phys. Lett. B 381 (1996), 202; S. M. Barr, Phys. Rev. D 55 (1997), 1659; M. J. Strassler, Phys. Lett. B 376 (1996), 119;
N. Haba, Phys. Rev. D 59 (1999), 035011; K. Yoshioka, Mod. Phys. Lett. A 15 (2000), 29; J. Hisano, K. Kurosawa, and Y. Nomura, Nucl. Phys. B 584 (2000), 3.

29) N. Haba and H. Murayama, Phys. Rev. D 63 (2001), 053010.

30) J. Sato and T. Yanagida, Phys. Lett. B 493 (2000), 356; J. Sato and K. Tobe, Phys. Rev. D 63 (2001), 116010.

31) F. Vissani, JHEP 9811 (1998), 025.

32) D. E. Kaplan, G. D. Kribs and M. Schmaltz, Phys. Rev. D 62 (2000) 035010.

33) M. Dine and A. E. Nelson, Phys. Rev. D 48 (1993) 1277; M. Dine, A. E. Nelson and Y. Shirman Phys. Rev. D 51 (1995) 1362; M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D 53 (1996) 2658.

34) N. Arkani-Hamed and H. Murayama, Phys. Rev. D 56 (1997) 6733; K. Agashe and M. Graesser, Phys. Rev. D 59 (1999) 015007.

35) Y. Nomura and T. Yanagida, Phys. Lett. B 487 (2000) 140.

36) J. Scherk and J. H. Schwarz, Phys. Lett. B 82 (1979), 60; Nucl. phys. B 153 (1979), 61;

37) P. Fayet, Phys. Lett. B159 (1985), 121; P. Fayet, Nucl. Phys. B263 (1986), 87.