M5-branes with $3/8$ supersymmetry in pp-wave background

Harvendra Singh

Fachbereich Physik, Martin-Luther-Universität Halle-Wittenberg,
Friedemann-Bach-Platz 6, D-06099 Halle, Germany

Abstract: We construct M5-branes with $3/8$ supersymmetry and find that they preserve exactly half of the background pp-wave supersymmetries. We explicitly write down the standard as well as supernumerary Killing spinors and find that their respective numbers are also half of those for the pp-wave background. This is in line with the recent work of Dabholkar et.al. which shows that half-supersymmetric D-branes can be constructed in pp-wave backgrounds.

*e-mail: singh@physik.uni-halle.de
1. Introduction

Hpp-waves are maximally supersymmetric plane-fronted parallel wave configurations of type IIB string theory [1, 2] which can be obtained by taking the Penrose limit [3, 4] of the maximally supersymmetric $AdS_5 \times S^5$ spacetime. Note that unlike Minkowski spacetime, pp-waves are asymptotically non-flat geometries, however, string theory in these backgrounds is exactly solvable [5, 6] and have important consequences for dual conformal field theories [7]. Several quick advances have taken place in the following works [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. There also have been recent works exposing various possible pp-wave solutions in string theory [20, 23].

Usually embedding of extended branes in a Minkowskian background breaks half of the background supersymmetries. For D-brane in pp-wave background to have $1/2$ supersymmetry brane must be embedded in maximally supersymmetric Hpp-wave backgrounds. Recently in a paper Dabholkar et. el. [22] have proposed existence of $1/2$ supersymmetric Dp-branes (for $p = 3, 5$ and 7) in Hpp-wave backgrounds, for subsequent work see [28]. The existence of D-branes in pp-wave backgrounds definitely leads to the existence of M2 and M5-branes in eleven dimensional M-theory. Our interest in this paper is to find out supersymmetric 11-dimensional branes in pp-wave backgrounds. In particular we are looking for $3/8$ supersymmetric M5-brane embedded in a $3/4$ supersymmetric 11-dimensional pp-wave background. The reason for existence for such branes is provided by the existence of M5-branes in $AdS_3 \times S^3 \times T^5$ background. Note that $AdS_3 \times S^3 \times T^5$ preserves only $1/2$ of the supersymmetries while the Penrose limit of it leads to the pp-waves with $3/4$ supersymmetry [20]. Thus, in general, there is enhancement of the supersymmetries in the Penrose limit.
The paper is organised as follows. The section-2 is a quick review of the general properties of pp-wave solutions with some specific details which will be needed for the section-3. In the section-3 we obtain supersymmetric M5-brane in pp-wave background. These five-branes are circularly symmetric or have smearing along one of the transverse coordinates. We also construct explicitly the the Killing spinors. In particular we find that the ‘standard’ as well as the ‘supernumerary’ Killing spinors are exactly halved by an embedding of the five-brane in the pp-wave background. There is no mixing between the two kind of spinors. We also discuss the spacetime dependences of the Killing spinors.

2. Review: pp-waves and the traceless matter

As explained in [2] the Penrose limits [3, 4] of anti-de Sitter spacetimes $AdS_m \times S^n \times S^p \times T^q$ along a null geodesic with generic orbit (i.e. with a non-zero component along a sphere) leads to a pp-wave metric (for $m, n \geq 2$)

$$ds^2 = du dv + \rho^2 \sin^2\left(\frac{u}{2\rho}\right) \left(\sum_{\mu=1}^{m-1} (dy^\mu)^2 + \sum_{a=1}^{n-1} (dy^a)^2 + \sum_{k=1}^{p+q} (dz^k)^2\right)$$  \hspace{1cm} (2.1)

where $\rho$ is a parameter. The pp-wave solution (2.1) is written in Rosen coordinates and depends only on light-cone coordinate $u$. However, one could face a more generic situation of the following type

$$ds^2 = du dv + \sum_{a=1}^{m+n-2} \left(\frac{\sin(\rho_a u)}{2\rho_a}\right)^2 (dy^a)^2 + \sum_{k=1}^{p+q} (dz^k)^2.$$  \hspace{1cm} (2.2)

From here we can switch to the new set of coordinates $(dx^+, dx^-, x^a, z^k)$

$$x^- = u/2, \quad x^+ = v - \frac{c}{4} u - \frac{1}{4} \sum_a \frac{\sin(2\rho_a u)}{2\rho_a} y^a y^a, \quad x^a = \frac{y^a \sin(\rho_a u)}{2\rho_a}, \quad z^k = z^k,$$  \hspace{1cm} (2.3)

which are slightly different from those given in [2] as we have included an arbitrary constant $c$. In new coordinates we get the familiar form of the pp-wave metrics (or Cahen-Wallach spacetimes)

$$ds^2 = 2dx^+ dx^- + W(dx^-)^2 + \sum_{a=1}^{m+n-2} (dy^a)^2 + \sum_{k=1}^{p+q} (dz^k)^2,$$

$$W = c - 4 \sum_{a=1}^{m+n-2} \rho_a^2 (y^a)^2$$  \hspace{1cm} (2.4)
where \( c \) is an arbitrary positive constant which later on will be set equal to one. Note that for metric (2.4) the nonvanishing component of the Ricci tensor is \( R_{--} \) while the curvature scalar is vanishing. Therefore the Einstein’s equation cannot be satisfied without matter fields. The choice of the matter fields has to be such that the energy momentum tensor is traceless \( (T^{\mu}_{\mu} = 0) \) as curvature is vanishing. It has been shown in [20, 23] that special choices of the parameters \( \rho_a \) and the matter fields give rise to many supersymmetric pp-waves solutions in string theory. Further the choice of \( W \) in (2.4) could still be of more general form. For example \( W \) could be taken as

\[
W = c + f_-(x^-) + [H_1(y^{\alpha_1}) - m_1 \sum_{a_1=1}^{d_1} (y^{\alpha_1})^2] + [H_2(y^{\alpha_2}) - m_2 \sum_{a_2=1}^{d_2} (y^{\alpha_2})^2] + \cdots
\]

where \( H_i \)’s are the harmonic functions over the respective Euclidean planes and \( f_-(x^-) \) is an arbitrary function of \( x^- \). For such a wave solution the Ricci tensor becomes

\[
R_{--} = m_1 d_1 + m_2 d_2 + \cdots
\]

where \( d_i \) are the dimensionalities of the respective homogeneous Euclidean coordinate patches. When parameters \( m_i \) are chosen such that \( R_{--} \) vanishes we obtain pure gravitational pp-waves [29] which are Ricci flat and without matter fields. But in this paper we restrict ourselves to \( m_i \geq 0 \) and \( R_{--} \) nonvanishing.

2.1 3/4 Supersymmetric pp-wave background

We consider the supersymmetric case of 11-dimensional pp-wave solution which follows by taking the appropriate Penrose limit of half-supersymmetric \( AdS_3 \times S^3 \times T^5 \) supergravity solution. Corresponding pp-wave is given by [20]

\[
d s_{11}^2 = 2 d x^+ d x^- + W (d x^-)^2 + \sum_{a=1}^{4} (d x^a)^2 + \sum_{\alpha=5}^{8} (d y^\alpha)^2 + (d y^9)^2.
\]

\[
G_4 = 2 m \ d x^- (d x^1 d x^2 d y^9 + d x^3 d x^4 d y^9), \quad W = c - m^2 \sum_{a=1}^{4} (x^a)^2.
\]

It has been shown in [20, 23] that above wave solution preserves 24 supersymmetries. Thus, in general, pp-wave limits of the \( AdS \) spacetimes are accompanied with the enhancement of the supersymmetries.

Let us now focus on the Killing spinors for the pp-wave background (2.7). These Killing spinors have been worked out in [20, 23]. We write them here more explicitly as we shall require them in the next section. We write down the tangent space metric as \( d s^2 = 2 e^+ e^- + e^a e^a + e^\alpha e^\alpha + e^9 e^9 \), where tangent space indices are taken same as the space-time indices. The basis elements are given by \( e^+ = d x^+ + (W/2) d x^- , \ e^- = d x^- , \ e^a = d x^a , \ e^\alpha = d y^\alpha, \ e^9 = d y^9 \). It is easy to find that only non-vanishing spin
connections are $\omega^{+a} = \frac{1}{2} \partial_a W dx^-$. Then the Killing spinors are obtained by solving the following supersymmetry variations

$$\delta \Psi_M = \nabla_M \epsilon - \frac{2}{(4!)^2} \left( G_P Q_R S \Gamma^{PQRS}_M - 8 G_{MNPQ} \Gamma^{NQP} \right) \epsilon = 0 .$$

(2.8)

For above background these reduce to the following set of equations

$$\partial_+ \epsilon = 0,$$

$$[\partial_- + \frac{1}{4} \partial_a W \gamma_+ \gamma^a + \frac{m}{6} \gamma_9 \Theta (\gamma_+ \gamma_- + 1)] \epsilon = 0$$

$$[\partial_a - \frac{m}{12} \gamma_9 (3 \Theta \gamma_a - \gamma_a \Theta) \gamma_+] \epsilon = 0$$

$$[\partial_\alpha - \frac{m}{6} \Theta \gamma_\alpha \gamma_+] \epsilon = 0$$

$$[\partial_\beta - \frac{m}{3} \Theta \gamma_9 \gamma_+] \epsilon = 0$$

(2.9)

where $\Theta = (\gamma_1 \gamma_2 + \gamma_3 \gamma_4)$ and all small $\gamma$ matrices are undressed. Now there are two kind of solutions of the above equations. One corresponds to taking $\gamma_+ \psi = 0$, these are called ‘standard’ Killing spinors. This condition keeps 16 spinors out of the set of total 32. For these spinors except $\partial_- \epsilon + \cdots = 0$ all other equation can be trivially satisfied. These sixteen standard killing spinors are [20, 23]

$$\epsilon = e^{-\frac{m}{2} \gamma_9 \Theta x^- \psi} , \quad \gamma_+ \psi = 0 .$$

(2.10)

All these spinors depend on $x^-$ except those which are annihilated by $\Theta$. Rest of the Killing spinors, for which $\gamma_+ \chi \neq 0$, are usually called as ‘supernumerary’ Killing spinors. These can be constructed out of the sixteen spinors $\chi$ with a condition $\Theta \chi = 0$. For these spinors all but the equations $\partial_\alpha \epsilon + \cdots = 0$ and $\partial_\beta \epsilon + \cdots = 0$ are to be solved. These solutions can be written in the simplified form as

$$\epsilon = \left( 1 + \frac{m}{4} \gamma_9 \Theta \gamma^a \gamma_+ x^a \right) \chi, \quad \theta \chi = \chi ,$$

(2.11)

with $\theta = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_+. \gamma_+$. Due to the condition $\theta \chi = \chi$ half of the $\chi$’s are vanishing and we are left with only 8 supernumerary Killing spinors. Thus total number of standard and the supernumerary Killing spinors is 24. All these Killing spinors are independent of the transverse $y^a$ and $y^9$ coordinates. Since $\psi = \psi_+ + \psi_-$, with $\theta \psi_+ = \pm \psi_+$, then $\Theta$ will automatically annihilate half of the 16 standard Killing spinors. These 8 standard Killing spinors will then be independent of the $x^-$. Thus 8 standard Killing spinors as well as all the supernumerary Killing spinors are independent of $x^-$. In conclusion, supersymmetry of the pp-wave background is more than that of the corresponding $AdS_3 \times S^3 \times T^5$ background. Thus there is enhancement of supersymmetries of $AdS$ spacetime in the Penrose limit. In the next section we shall embed M5-branes in this pp-wave background and we will find that the number of supersymmetries is halved.

---

1We are in the frame where $(\gamma_+)^2 = (\gamma_-)^2 = 0$ and $[\gamma_+, \gamma_-] = 2$ and the projector is $\gamma_- \gamma_+$. Note that $\gamma_+$ is not a projector.
2.2 String coupling:

Before closing this review let us also describe the importance of the constant $c$ in the function $W$ which we have incorporated in our backgrounds by simply using modified coordinate change rules in \( (2.3) \). Consider the pp-wave solution \( (2.7) \) which can be dimensionally reduced along any of the isometry directions. In particular the reduction along $x^-$ would give following deformed D0-branes with 16 supercharges

$$
\begin{align*}
\begin{split}
ds_{10}^2 &= -W^{-\frac{1}{2}}dx^+dx^- + W^{\frac{1}{2}} \left[ \sum_{a=1}^{4} (dx^a)^2 + \sum_{\alpha=1}^{4} (dy^\alpha)^2 + (dy^9)^2 \right] \\
H^{NS}_3 &= 2m(dx^1dx^2dy^9 + dx^3dx^4dy^9), \\
e^{2\phi} &= W^{3/2}, & A_1 &= W^{-1}dx^+.
\end{split}
\end{align*}
$$

There are sixteen supersymmetries because 16 Killing spinors which are independent of the $x^-$ coordinate in \( (2.7) \) survive after compactification. There is a constant flux of NS-NS 3-form in these D0-brane solutions that leads to the deformation. Note that $x^+$ coordinate of pp-wave after compactification plays the role of the time coordinate. From \( (2.12) \) it is clear that for string coupling and the geometry to be well defined $W$ must be non-negative. The constant $c$ is related to the background value of the string coupling $g_s$ at the origin $x^a = 0$. Although the pp-wave solution \( (2.7) \) holds good without the constant $c$ (as it can be absorbed by the shifts $dx^+ \rightarrow dx^+ - c/2dx^-$), but it becomes an important parameter after compactification along $x^-$ when we try to make contact with D0-branes in \( (2.12) \). A reduction of \( (2.7) \) along any of the transverse coordinates $y^\alpha$, $y^9$ would give rise to pp-wave solutions of type IIA string theory.

A similar conclusion follows if we consider maximally supersymmetric Hpp-wave background in type IIB theory \[1\] which upon T-duality along $x^-$ (although this has no Killing isometries along $x^-$) describes deformed type IIA fundamental strings in presence of constant $F_4$-flux

$$
\begin{align*}
\begin{split}
\begin{align*}
ds_{10}^2 &= W^{-1}(-dx^+dx^+ + dx^-dx^-) + \sum_{a=1}^{8} (dx^a)^2. \\
B_{+-} &= W^{-1}, & F_4 &= 2m(dx^1dx^2dx^3dx^4 + dx^5dx^6dx^7dx^8), \\
e^{2\phi} &= W^{-1}, & W &= c - m^2 \sum_{a=1}^{8} x^2_a.
\end{align*}
\end{split}
\end{align*}
$$

Here again $W$ gets related to the string coupling in type IIA string theory.

3. M5-branes in pp-wave background

Our objective in this paper is to construct solitonic M5-brane solutions in M-theory in supersymmetric pp-wave backgrounds. There can be many ways to construct such
solutions, we follow here the most obvious and simple procedure which involves first writing down the intersecting M2/M5/M5 brane configuration [30]

\[ ds_{11}^2 = f^{-\frac{2}{3}} H_1^{\frac{1}{3}} H_2^{\frac{2}{3}} (-dt^2 + dz^2) + f^{-\frac{2}{3}} H_1^{\frac{2}{3}} H_2^{\frac{2}{3}} (dy^9)^2 + \]

\[ f^{\frac{1}{3}} H_1^{\frac{2}{3}} H_2^{\frac{2}{3}} \sum_{a=1}^{4} (dx^a)^2 + f^{\frac{1}{3}} H_1^{-\frac{4}{3}} H_2^2 \sum_{a=5}^{8} (dy^a)^2 , \]

\[ G_4 = [d f^{-1} dt dz + *d H_1 + *d H_2] dy^9 \]  

(3.1)

where Hodge * operations are defined over 4-dimensional flat \( x^a \) and \( y^\alpha \) coordinate patches respectively.\(^2\) The harmonic functions satisfy the equation

\[ (H_2 \nabla_x + H_1 \nabla_y) f = 0, \quad H_1 = 1 + \frac{Q_1}{x^2}, \quad H_2 = 1 + \frac{Q_2}{y^2} \]  

(3.2)

where \( \nabla \)'s are Laplacians defined over two four-plane. For this \( f = H_1 H_2 \) and \( f = H_1 \) are the two most obvious solutions of (3.2). For the latter case (3.1) becomes

\[ ds_{11}^2 = H_2^{-\frac{1}{3}} \left( H_1^{-1} (-dt^2 + dz^2) + H_1 \sum_{a=1}^{4} (dx^a)^2 \right) + H_2^{\frac{2}{3}} \left( \sum_{a=5}^{8} (dy^a)^2 + (dy^9)^2 \right) , \]

\[ G_4 = [d H_1^{-1} dt dz + *d H_1 + *d H_2] dy^9 \]  

(3.3)

Above solution has a near horizon limit \( x \to 0 \) in which the solution becomes M5-brane with anti-de Sitter world-volume

\[ ds_{11}^2 = H_2^{-\frac{1}{3}} \left[ AdS_3(Q_1) \times S^3(Q_1) \right] + H_2^{\frac{2}{3}} \left( \sum_{a=5}^{8} (dy^a)^2 + (dy^9)^2 \right) , \]

\[ G_4 = [2Q_1 \Omega(AdS_3) - 2Q_1 \Omega(S^3) + *d H_2] dy^9 \]  

(3.4)

where \( \Omega(M) \) represents the volume form of unit \( M \) space. Note that \( AdS_3(Q_1) \) and \( S^3(Q_1) \) have equal size and is given by \( Q_1 \). This solution preserves eight supersymmetries. We shall write down the corresponding Killing spinors in the next subsection. Thus there is no enhancement (doubling) of the supercharges in the near horizon limit \( x \to 0 \). This solution represents a solitonic M5-brane which has a world volume wrapped on \( AdS_3 \times S^3 \) and is asymptotically \( (y \to \infty) \) the \( AdS_3 \times S^3 \times T^5 \) space-time which has 16 supersymmetries. There is an over all isometry direction \( y^9 \) and therefore these M5-branes are different from the usual ones. These are like (smeared) circularly symmetric M5-branes. Also this construction is some what unique and we have checked that with \( AdS_3 \times S^3 \) world volume there are no solutions which depend upon all the five transverse coordinates.

Having obtained such a configuration of M5-branes, we would like to find out what will happen to these solutions in the Penrose limit. The Penrose limit of the

\(^2\)A similar construction was done in [27] for NS5 branes in pp-wave background.
asymptotic $AdS_3 \times S^3 \times T^5$ geometry is given in (2.7). For (3.4) we take the Penrose (scaling) limit in which scaling parameter $\lambda \rightarrow 0$ and is accompanied with the scalings $Q_2 \rightarrow \lambda^2 Q_2$, $y^\alpha \rightarrow \lambda y^\alpha$, $y^9 \rightarrow \lambda y^9$. In order to obtain nontrivial scaling limit we have to first express the $AdS_3 \times S^3$ part of the spacetime in suitable light-cone coordinates $U, V, X^a$ as done in [2], follow it with the scalings $U \rightarrow u$, $V \rightarrow \lambda^2 v$, $X^a \rightarrow \lambda x^a$ and then take the limit $\lambda \rightarrow 0$. Using coordinate change rules (2.3) we get

$$d\mathbf{s}_{11}^2 = \lambda^2 \left[ H_2^{-\frac{1}{2}} F_{AdS} F_S \epsilon_0 \right.\]

$$F_{AdS} = [e^\frac{\gamma_7}{2} P_+ + (e^{-\frac{\gamma_7}{2}} + e^\frac{\gamma_7}{2} (t\gamma_t + z\gamma_z)\gamma_r) P_- ]$$

$$F_S = e^{-\frac{\gamma_7}{2} \gamma_9 \gamma_7 \gamma_8 \gamma_8} e^{-\frac{\gamma_7}{2} \gamma_8 \gamma_8 \gamma_9} e^{-\frac{\gamma_7}{2} \gamma_8 \gamma_8 \gamma_9} e^{-\frac{\gamma_7}{2} \gamma_8 \gamma_8 \gamma_9}$$

(3.6)

where all small $\gamma$-matrices are undressed, $P_\pm = \frac{1}{2}(1 \pm \gamma_t \gamma_z \gamma_y \gamma_y)$, with $\gamma_t$, $\gamma_z$, $\gamma_r$, $\gamma_\theta$, $\gamma_\psi$, $\gamma_\psi$ being along $AdS_3 \times S^3$ in the same order (we have set $Q_1 = 1$). The constant spinor $\epsilon_0$ satisfies the constraints

$$\gamma_y \gamma_y \gamma_y \gamma_y \gamma_y \gamma_y \gamma_y \gamma_y \epsilon_0 = -\epsilon_0, \quad \gamma_t \gamma_z \gamma_r \gamma_\theta \gamma_\phi \gamma_\psi \epsilon_0 = -\epsilon_0.$$  

(3.7)

These twin conditions break the supersymmetries to one-quarter. The two sets of operators in (3.4) commute with each other. These operators also commute with $F_{AdS}$ and $F_S$ as well. When the charge of M5-branes vanishes (i.e. $Q_2 = 0$) the first condition drops out and the supersymmetry is increased to sixteen. Thus embedding of the branes in $AdS_3 \times S^3 \times T^5$ explicitly breaks half of the supersymmetries. This

\footnote{Anti-de Sitter metric can be written as $e^{2\tau}(-dt^2 + dz^2) + dr^2$ and $S^3$ line element is taken to be $d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2.$}
should also be the case when the five-branes are considered in pp-wave background. In particular it is interesting to know what happens to the standard and the supernumery Killing spinors when five-branes are embedded in pp-wave background.

We again write down the tangent space metric as $ds^2 = 2e^+e^- + e^a e^a + e^9 e^9$, where tangent space indices are taken same as the space-time indices. The basis elements are now given by $e^+ = H^{-1/2}_2(dx^+ + W dx^-)$, $e^- = H^{-1/2}_2 dx^-$, $e^a = H^{-1/2}_2 dx^a$, $e^a = H^7_2 dy^a$, $e^9 = H^3_2 dy^9$. Correspondingly the spin connections are

$$\omega^+ = \frac{1}{2} \partial_\alpha W dx^- , \quad \omega^a = -\frac{1}{6} \partial_\alpha H_2 H^{-2}_2 (dx^+ + \frac{W}{2} dx^-) , \quad \omega^{-a} = -\frac{1}{6} \partial_\alpha H_2 H^{-2}_2 \partial^a dx^- ,$$

$$\omega^{\alpha a} = -\frac{1}{6} \partial_\alpha H_2 H^{-2}_2 \partial^a dx^a , \quad \omega^{\alpha \beta} = \frac{1}{3} \partial_\alpha H_2 H^{-1}_2 dy^\alpha , \quad \omega^{9_\alpha} = \frac{1}{3} \partial_\alpha H_2 H^{-1}_2 dy^9 .$$

With these spin connections we solve for the Killing equations in (2.8). We find for (3.5) the standard Killing spinors ($\gamma^+ \psi = 0$) are given by

$$\epsilon = H^{-1/2}_2(y) e^{-\frac{1}{2} \gamma^9 \Theta} x^- \psi , \quad \bar{\Gamma} \psi = -\psi , \quad (3.8)$$

which are 8 in number while the supernumerary ones ($\gamma^+ \psi \neq 0$) are given by

$$\epsilon = H^{-1/2}_2(y) (1 + \frac{1}{4} \gamma^9 \Theta \gamma_a \gamma^a) \chi , \quad \Theta \chi = 0 , \quad \bar{\Gamma} \chi = -\chi , \quad (3.9)$$

with $\bar{\Gamma} = \gamma^y \gamma^6 \gamma^7 \gamma^8 \gamma^9 \epsilon$. Note that the $\Theta$ and $\bar{\Gamma}$ commute with each other which is crucial. Thus the number of supernumery killing spinors is only four and the total number of the Killing spinors for M5-brane embedded in pp-wave background becomes twelve. All of these spinors are independent of the coordinate $y^9$ only. All supernumery Killing spinors and half of the standard ones are also independent of the $x^-$ coordinate. That is total of 8 Killing spinors are independent of $x^-$. These will survive if we compactify the $x^-$ direction on a circle.

In conclusion we have shown that both standard as well as the supernumery Killing spinors exist for the five-brane background (3.5) but their numbers are reduced by half due to the additional condition $\gamma^y \gamma^6 \gamma^7 \gamma^8 \gamma^9 \epsilon = -\epsilon$ in the transverse space. This condition was absent for pp-wave background in (2.7).

The compactification of (3.5) along the $y^9$ coordinate will give NS5-branes in pp-wave background of type IIA, which has been considered by Kumar et.al. [27]. All the Killing spinors in eqs. (3.8) and (3.9) will survive in this compactification. Thus we have provided the M-theory relationship for NS5-branes in pp-wave backgrounds having 24 supersymmetries. The existence of half supersymmetric (with 16 susy) D3, D5 and D7-branes was recently shown by Dabholkar et.al. [22] where the branes are embedded in maximally supersymmetric Hpp-wave backgrounds [1]. However, our M5-branes only preserve 12 supersymmetries which is half of the amount preserved by the asymptotic pp-wave background. It would be interesting to see if half supersymmetric M5-branes can be embedded into maximally supersymmetric Mpp-wave backgrounds.
Acknowledgments

I would like to thank A. Micu and S. Theisen for useful discussions. This work is supported by AvH (the Alexander von Humboldt foundation).

References

[1] M. Blau, J. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, A new maximally supersymmetric background of IIB superstring theory, JHEP 0201 (2002) 047, hep-th/0110242.

[2] M. Blau, J. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, Penrose limits and maximal supersymmetry, hep-th/0201081.

[3] R. Penrose, Any spacetime has a plane wave as a limit, in Differential Geometry and relativity, pp.271-75, Reidel, Dordrecht, 1976.

[4] R. Güven, Plane wave limits and T-duality, Phys. Lett. B 482 (2000) 255, hep-th/0005061.

[5] R.R. Matsaev, Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background, Nucl. Phys. B 625 (2002) 70, hep-th/0112044.

[6] R.R. Matsaev and A.A. Tseytlin, Exactly solvable model of superstring in plane wave Ramond-Ramond background, hep-th/0202109.

[7] For a review see, O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Large N field theories, string theory and gravity, Phys. Rep. 323 (2000) 183, hep-th/9905111.

[8] D. Berenstein, J. Maldacena and H. Nastase, Strings in flat space and pp wave from $\mathcal{N} = 4$ super Yang Mills, hep-th/0202021.

[9] M. Blau, J. Figueroa-O’Farrill and G. Papadopoulos, Penrose limits, supergravity and brane dynamics, hep-th/0202111.

[10] N. Itzhaki, I.R. Klebanov and S. Mukhi, PP wave limit and enhanced supersymmetry in gauge theories, JHEP 0203 (2002) 048, hep-th/0202153.

[11] M. Alishahiha and M.M. Sheikh-Jabbari, Strings in PP-waves and Worldsheet Deconstruction, hep-th/0204174.

[12] J. Gomis and H. Ooguri, Penrose limit of N = 1 gauge theories, hep-th/0202157.

[13] L.A. Zayas and J. Sonnenschein, On Penrose limit and gauge theories, hep-th/0202186.

[14] M. Billo’ and I. Pesando, Boundary states for GS superstrings in an Hpp wave background, hep-th/0203028.
[15] N. Kim, A. Pankiewicz, S.-J. Rey and S. Theisen, Superstring on pp-wave orbifold from large-N quiver gauge theory, hep-th/0203080.

[16] T. Takayanagi and S. Terashima, Strings on orbifolded pp-waves, hep-th/0203093.

[17] U. Gursoy, C. Nunez and M. Schvellinger, RG flows from spin(7), CY 4-fold and HK manifolds to AdS, Penrose limits and pp waves, hep-th/0203124.

[18] S.R. Das, C. Gomez, S.-J. Rey, Penrose limit, spontaneous symmetry breaking and holography in pp-wave background, hep-th/0203164.

[19] C.S. Chu and P.M. Ho, Noncommutative D-brane and open string in pp-wave background with B-field, hep-th/0203186.

[20] M. Cvetic, H. Lü and C.N. Pope, M-theory PP-Waves, Penrose Limits and Supernumerary Supersymmetries, hep-th/0203229.

[21] D. Berenstein, E. Gava, J. Maldacena, K.S. Narain and H. Nastase, Open strings on plane waves and their Yang Mills duals, hep-th/0203249.

[22] A. Dabholkar and S. Parvizi, Dp Branes in PP-wave Background, hep-th/0203231.

[23] J. Gauntlett and C.M. Hull, pp-waves in 11-dimensions with extra supersymmetry, hep-th/0203255.

[24] P. Lee and J. Park, Open strings in pp-wave background from defect conformal field theory, hep-th/0203257.

[25] H. Lu and J.F. Vazquez-Poritz, Penrose Limits of non-standard brane intersections, hep-th/0204001.

[26] M. Hatsuda, K. Kamimura and M. Sakaguchi, Super-PP-wave Algebra from Super-AdS × S Algebras in Eleven-dimensions, hep-th/0204002.

[27] A. Kumar, R.R. Nayak and Sanjay, D-Brane Solutions in pp-wave Background, hep-th/0204025.

[28] K. Skenderis and M. Taylor, Branes in AdS and pp-wave spacetimes, hep-th/0204054.

[29] G. T. Horowitz and A. R. Steif, Spacetime singularities in string theory, Phys. Rev. Lett. 64 (1990) 266.

[30] H.J. Boonstra, B. Peeters and K. Skenderis, Brane intersections, anti-de Sitter spacetimes and dual conformal field theories, Nucl. Phys. B 533 (1998) 127, hep-th/9803231.