Length Scales of Acceleration for Locally Isotropic Turbulence

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Length scales are determined that govern the behavior at small separations of the correlations of fluid-particle acceleration, viscous force, and pressure gradient. The length scales and an associated universal constant are quantified on the basis of published data. The length scale governing pressure spectra at high wave numbers is discussed. Fluid-particle acceleration correlation is governed by two length scales; one arises from the pressure gradient, the other from the viscous force.

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Accelerations in turbulent flow are violent and are important to many types of studies. Experiments in which particles are tracked in turbulence have found accelerations as large as 1500 times that of gravity and have quantified the probability density of accelerations to show that extreme events are likely because the accelerations are highly intermittent. Understanding turbulent accelerations is essential to many studies, including dispersion and settling of particles in turbulence, other interactions of bubbles and particles in turbulence, effects on flying insects, pollutant transport, and to the applicability of Taylor's hypothesis. Turbulent accelerations are the object of several studies aimed at solving the mystery of rain onset from liquid-water clouds; the accelerations affect collision-coalescence, preferential concentration, and local supersaturation. To advance understanding of turbulent accelerations, the length scales of acceleration correlations are quantified here, and those scales are discussed in connection with the scaling of the pressure spectrum by Gotoh and Fukayama and the inertial-range scale found for fourth-order velocity structure functions by Kerr et al.

Universal scaling of the small-scales of turbulence and its analogue in other areas of physics are discussed in Ref. [12]. The K41 scaling parameters are mean energy dissipation rate $\varepsilon$ and kinematic viscosity $\nu$. Traditional scaling uses K41 scaling of small-scale statistics, and seeks the resultant $R_\lambda$ dependence of the scaled statistic $\langle u_1^2 \rangle^{1/2}$; that $R_\lambda$ dependence is akin to symmetry breaking. Here, $R_\lambda = \langle u_1^2 \rangle^{1/2} \lambda_T / \nu$ is the Reynolds number, where Taylor's length scale is $\lambda_T \equiv \left[ \langle u_1^2 \rangle / \langle (\partial u_1 / \partial x_1)^2 \rangle \right]^{1/2}$ and $\langle u_1^2 \rangle$ is the variance of one component of velocity. Since $\langle u_1^2 \rangle$ is a property of the large-scale structure, so is $R_\lambda$. A radical departure from tradition is to deduce scaling relationships from relationships amongst statistics as determined from the equations of motion. That method leads, for example, to the assertion that mean-squared pressure-gradient scales with an integral of the fourth-order velocity structure function $\langle \nu \nabla^2_\nu u_r \nu \nabla^2_\nu u_i \rangle$; for such deduced scaling, there is no residual dependence on large-scale parameters $\lambda_T$. The deductive method is extended herein to the length scales of acceleration. To quantify the deduced length scales, it is necessary here as in Ref. [13] to use existing turbulence phenomenology; that causes both $\varepsilon$ and $\lambda_T$ to appear herein. However, the deductive theory does not explicitly contain K41 scales or $R_\lambda$.

The fluid-particle acceleration $a_i$ is related to the acceleration caused by the viscous force, $\nu \nabla^2_\nu u_i$, and that caused by the pressure gradient, $-\partial_x p$, by the Navier Stokes equation: $a_i \equiv D u_i / D t = -\partial_x p + \nu \partial_x^2 u_i$, where $u_i$ is the velocity, $p$ is pressure divided by fluid density; density is constant. Here, $\nabla^2_\nu \equiv \partial_x \partial_x$ is the Laplacian operator, $\partial$ denotes differentiation with respect to the subscript variable, summation is implied by repeated Roman indices. Consider two points of measurement, $x$ and $x' \equiv x + r$, let $r \equiv |r|$, and let prime denote evaluation at $x'$; e.g., $p' \equiv p(x', t)$, $u_i' \equiv u_i(x', t)$, etc. Two-point differences are denoted by $\Delta p \equiv p - p'$, $\Delta u_i \equiv u_i - u_i'$, etc.; angle brackets, i.e., $\langle \cdot \rangle$, denote an average. Let the 1-axi be in the direction of $r$ such that the 2- and 3-axes are perpendicular to $r$. Taylor's length scale $\lambda_T$ is the length scale of the parabola that oscillates the velocity correlation at $r = 0$. Specifically, for $r = (r_1, 0, 0)$, $\langle u_1' u_1 \rangle / \langle u_1^2 \rangle = 1 - (r_1 / 2 \lambda_T)^2 + \ldots$. We consider the spatial correlation tensors of fluid-particle acceleration $\langle a_i a_j \rangle$, pressure gradient $\langle \partial_x p \partial_x' p' \rangle$, and viscous force per unit mass $\langle \nu \nabla^2_\nu u_i \nu \nabla^2_\nu u_j' \rangle$, as well as the pressure structure function $D_P (r) \equiv \langle \Delta p^2 \rangle$ and spatial spectra. We determine the length scales of those statistics. The length scales considered in this paper are, like $\lambda_T$, the length scale of the parabolas that osculate those spatial correlations at $r = 0$. Unlike $\lambda_T$, all the length scales defined here do not depend on a large-scale parameter like $\langle u_1^2 \rangle$.

The considered statistics, $\langle \partial_x p \partial_x' p' \rangle, D_P, \ldots$, are related to the fourth-order velocity structure function (i.e., $D_{ijkl} (r) \equiv \langle \Delta u_i \Delta u_j \Delta u_k \Delta u_l \rangle$) in Ref. [12]; that theory is based on the Navier Stokes equation, incompressibility, and local isotropy, without further assumptions. The theory also relates $\langle \nu \nabla^2_\nu u_i \nu \nabla^2_\nu u_j' \rangle$ to the third-order velocity structure function $D_{ijk} (r) \equiv \langle \Delta u_i \Delta u_j \Delta u_k \rangle$. The theory has been used to calculate $\langle \partial_x p \partial_x' p' \rangle$ and $\langle \nu \nabla^2_\nu u_i \nu \nabla^2_\nu u_j' \rangle$ from $D_{ijkl}$ and $D_{ijk}$.
respectively, by means of hot-wire anemometry and from direct numerical simulation (DNS) (see Figs. 12, 13 of Ref. [10]), as well as to compare $D_P$ calculated from $D_{ijkl}$ with $D_P$ calculated from DNS pressure fields [17]. The theory also determines the length scales of those statistics, which is the topic here.

Taylor series expansion is used in Ref. [14] to formulate the scaling lengths of $\langle \partial_x p \partial_x', p' \rangle$. For brevity, denote the mean-squared pressure gradient by $\chi \equiv \langle \partial_x p \partial_x' \rangle$ (sum on $i$). The longitudinal component of the pressure-gradient correlation is $\langle \partial_x p \partial_x' \rangle$ (the 1-axis is in the direction of $r$). Power series expansion of the theory’s relationship between $\langle \partial_x p \partial_x' \rangle$ and components of $D_{ijkl}$ gives

$$\langle \partial_x p \partial_x' \rangle = (\chi/3) (1 - r^2/2\lambda^2 + \cdots); \quad (1)$$

$$\lambda_1 = \sqrt[3]{\chi/36 \langle \partial_x u_1 \rangle h_Q}^{1/2}; \quad (2)$$

$$h_Q = 1 + \frac{1}{3} \left(\frac{\langle \partial_x u_1 \rangle^2}{\langle \partial_x u_1 \rangle^4} - 3 \frac{\langle \partial_x u_1 \rangle^2 \langle \partial_x u_2 \rangle^2}{\langle \partial_x u_1 \rangle^4}\right)$$

$$\left(1 - \frac{7}{16} \left(1 - \frac{\omega_k \omega_k s_{ij} s_{ij}}{\langle s_{ij} s_{ij} \rangle^2} + \frac{1}{4} \frac{\omega_k \omega_k}{\langle s_{ij} s_{ij} \rangle^2}\right)\right). \quad (3)$$

Thus, $\lambda_1$ is the scaling length for the longitudinal component $\langle \partial_x p \partial_x' \rangle$, analogous with Taylor’s scale. In Eq. (3), $h_Q$ is given in terms of 3 of the 4 fourth-order invariants determined by Siggia [16], where $s_{ij}$ is the rate of strain tensor and $\omega_k$ is the vorticity. The invariant missing from Eq. (3) is the one that Siggia [17] expected to vary with $R_\lambda$ differently than the other 3, so there is no conflict with the expectation expressed in Ref. [14] that $h_Q$ becomes a constant at large Reynolds number. Also, note that $h_Q = \left(\langle V^2 \rangle \right)/1575 \left(\langle s_{ij} s_{ij} \rangle \right)^{2}$. The corresponding length scale of the transverse component of the pressure gradient correlation is $3^{1/2} \lambda_1$ (Ref. [14] Eq. (61)), and that of $\langle \partial_x \partial_x' \rangle$ is $(9/5)^{1/2} \lambda_1$. The analogous length scale of $D_P (r)$ is $\sqrt{6} \Lambda_1$ (Ref. [14] Eq.(38), et seq.). The above shows that to cause pressure-gradient correlations to coincide near the origin of a graph, one divides them by $\chi$ and uses $r/\lambda_1$ on the abscissa; for $D_P (r)$, $D_P (r) / \chi \lambda_1^2$ should be on the ordinate with $r/\lambda_1$ on the abscissa. The pressure spectrum is the sine transform of the pressure-gradient correlation [Ref. [14] Eqs. (15), (22a)]. Because $\lambda_1$ scales the pressure-gradient correlation at small $r$, it follows that to cause pressure spectra to coincide at viscous-range wave numbers, one should divide the spectra by $\chi \lambda_1^2$ and use $k \lambda_1$ on the abscissa ($k$ is wave number). A Reynolds number limitation for this to be so is discussed near the end.

To calculate $\Lambda_1$ using Eq. (4) we must evaluate $h_Q$ using Eqs. (3) or (4). Siggia’s DNS data for $R_\lambda \sim 60$ to 90 gives a single value $h_Q \approx 0.28$. The DNS data of Siggia’s [16], [23] give a single value $h_Q \approx 0.15$, and that of Ref. [16], as well as to compare from direct numerical simulation (DNS) (see Figs. 12, 13 of Ref. [10]), as well as to compare $D_P$ calculated from $D_{ijkl}$ with $D_P$ calculated from DNS pressure fields [17]. The theory also determines the length scales of those statistics, which is the topic here.

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is necessary to use the coefficient 3.5 instead of 3.1 in \( \chi \approx 3.1H_\kappa \varepsilon^{3/2}v^{-1/2}F^{0.79} \). That is, to be consistent with the data of both Refs. [24] and [25], we use, for \( R_\lambda \gtrsim 400 \),

\[ \chi \approx 3.5H_\kappa \varepsilon^{3/2}v^{-1/2}F^{0.79} \approx 4AH_\kappa \varepsilon^{3/2}v^{-1/2}R_\lambda^{0.25} \quad (6) \]

Evidence that \( H_\kappa \) is a constant for \( R_\lambda \gtrsim 80 \) and that its value is about 0.65 is given in Ref. [13]. For \( H_\kappa = 0.65 \), Eq. (6) agrees quantitatively with the DNS data in Table 1 of Ref. [8] for \( R_\lambda \geq 387 \); also, Eq. (3) is a good approximation of the DNS data for \( R_\lambda \) as small as 200. Substitute Eq. (3) in Eq. (5) to obtain, for \( R_\lambda > 400 \),

\[ \lambda_1 \approx 4.7\eta (H_\kappa / F)^{0.21}h_Q^{1/2} \approx 6.8\eta R_\lambda^{-0.033} \quad (7) \]

For the right-most expression in Eq. (7), \( H_\kappa = 0.65 \), \( h_Q = 0.3 \), and \( F \approx 1.18R_\lambda^{0.31} \) were used. For the case of low Reynolds numbers, Ref. [15] obtains

\[ \chi \approx 0.11\varepsilon^{3/2}v^{-1/2}R_\lambda \quad \text{for} \quad R_\lambda \lesssim 20 \quad (8) \]

which agrees with the DNS data of Ref. [16]. For low Reynolds numbers, use Eq. (5) in Eq. (4) to obtain

\[ \lambda_1 \approx 0.83\eta (R_\lambda / Fh_Q)^{1/2} \quad \text{for} \quad R_\lambda < 20 \quad (9) \]

None of Eqs. (4) to (8) is K41 scaling because of their \( R_\lambda \) dependence. Figure 2 shows \( \lambda_1 \) increasing relative to \( \eta \) as \( R_\lambda \) increases from 9 to attain a maximum near \( R_\lambda \approx 150 \), beyond which \( \lambda_1 \) gradually decreases relative to \( \eta \) in agreement with Eq. (7).

We now consider the spatial correlation of the viscous force because it is part of the fluid-particle correlation as well as intrinsic interest [18]. The spatial correlation of \( \nu \nabla^2 u \) is [18] [18]:

\[ V_{ij} (r) \equiv \langle \nu^2 \nabla^2 u_i \nabla^2 u_j \rangle = -\frac{\nu^2}{2} \nabla^2 \nabla^2 D_{ij} (r) \quad (10) \]

\[ = -(\nu/4) \nabla^2 \partial_r D_{ijk} (r), \quad (11) \]

where \( \nabla^2 \) is the Laplacian operator in \( r \)-space. Then, for the longitudinal components of \( V_{ij} (r) \), Eq. (11) yields [14]

\[ V_{11} (r) = (\nu/2r^3) [D_{111} (r) + 2D_{122} (r) - 5\partial_r D_{122} (r) - r^2 \partial_r^2 D_{122} (r)]. \quad (12a) \]

A similar formula applies to the transverse component \( V_{22} (r) \). [15]. Power series expansion gives \( D_{111} (r) = \left( \partial_{x_1} u_1 \right)^3 r^3 \left[ 1 - (r^2 / (2\lambda_{D_{111}}^2)) \right] + \cdots \), where

\[ \lambda_{D_{111}} = \left[ -4 \left( \partial_{x_1} u_1 \right)^3 \right] \left[ 3 \left( \partial_{x_1} u_1 \right)^2 \partial_{x_1} u_1 + 2 \left( \partial_{x_1} u_1 \right)^2 \partial_{x_1} u_1 \right] \right]^{1/2} \quad (13) \]

Then, Eq. (12a) yields

\[ V_{11} (r) = \frac{35}{6} \nu \left( \partial_{x_1} u_1 \right)^3 \left[ 1 - (r^2 / 2\lambda_{V_{111}}^3) \right] + \cdots, \]

where \( \lambda_{V_{111}} = \lambda_{D_{111}} / \sqrt{18/5} \) is the length scale of the oscillating parabola of \( V_{11} (r) \). The corresponding expansions of the transverse component of \( V_{11} (r) \) and \( V_{11} (r) \) give length scales \( \lambda_{V_{111}} / \sqrt{7} \) and \( \lambda_{V_{111}} / \sqrt{5/3} \), respectively. Kolmogorov’s equation can relate the velocity-derivative moments in Eq. (13) to other high-order derivative moments; this gives the same result as would beginning with the relationship of \( V_{11} (r) \) to \( D_{11} (r) \) in Eq. (10). The difficulty of observing the initial fall-off of \( V_{11} (r) \), and hence its length scale \( \lambda_{V_{111}} \) is clearly illustrated in Fig. 3 of Ref. [13] and Fig. 13 of Ref. [18] and their discussions; one requires a spatial resolution substantially finer than \( \eta \). Such fine resolution data is not yet available. The high-order derivative moments in the denominator of Eq. (13) likewise require fine spatial resolution; their Reynolds-number dependence has not been investigated, but it would be surprising if they obey K41 scaling.

Despite these difficulties, we can obtain the \( r \)-values at which \( V_{11} (r) / V_{11} (0) = 0.5 \), and similarly for the transverse component, from the graphs of those correlation coefficients in Fig. 13 of Ref. [15]; the \( r \)-values are 3.5\( \eta \) and 2.5\( \eta \) for longitudinal and transverse components, respectively. Limited to \( R_\lambda = 230 \) of the DNS run in Fig. 13 of Ref. [18], 3.5\( \eta \) and 2.5\( \eta \) are the first estimates of these length scales. It is not suggested that these length scales maintain fixed ratio to \( \eta \) as \( R_\lambda \) varies.

The fluid-particle acceleration correlation is [14]

\[ A_{ij} (r) \equiv \langle a_i a_j' \rangle = \langle \partial_x p \partial_x' \partial_x' \rangle + V_{ij} (r). \quad (14) \]

As a result of the differing length scales of \( \langle \partial_x p \partial_x' \rangle \) and \( V_{ij} (r) \), the theory requires that the initial fall-off of the longitudinal component \( A_{11} (r) \) be described by two length scales; the same is true for the transverse component and for \( A_{ii} (r) \). For example, consider the trace \( A_{ii} (r) \) and \( R_\lambda = 230 \). From the above discussion we have \( \lambda_{V_{111}} = 3.5\eta \) such that \( \lambda_{V_{111}} / \sqrt{5/3} = 2.7\eta \). From Fig. 2 at \( R_\lambda = 230 \) we have that the length scale of \( \langle \partial_x p \partial_x' \rangle \) is \( \sqrt{5/3} \lambda_{V_{111}} = 7.6\eta \). Also, \( V_{ii} (0) / \chi = 0.015 \) at \( R_\lambda = 230 \) [13][18]. Thus, \( A_{ii} (r) \) has an initial rapid but small-amplitude decay (1.5% of the total) from \( V_{ii} (r) \), having scale 2.7\( \eta \), followed by the larger amplitude and more gradual decay of \( \langle \partial_x p \partial_x' \rangle \) with scale...
7.6η. It is not implied that the two length scales maintain a fixed ratio as \( R_\lambda \) varies; the example above is for \( R_\lambda = 230 \).

We noted above that to cause pressure spectra to coincide at viscous-range wave numbers, one should divide the spectra by \( \chi \frac{\lambda^2}{\eta} \) and use \( k \lambda_1 \) on the abscissa. However, the Reynolds number must be large enough that, at high wave numbers, the pressure spectrum has negligible contributions from the sine transform of the pressure-gradient correlation at \( r \) on the order of the integral scale. How large is large enough? From the pressure spectra from isotropic DNS shown in Fig. 5 of Gotoh and Fukayama [8], where \( k \eta \) is the abscissa, combined with the weak variation of \( \lambda_1/\eta \) shown in Fig. 2 for \( 100 < R_\lambda < 400 \), it seems that \( R_\lambda \gg 200 \) is large enough. Of course, no such limitation applies to scaling of \( \langle \partial_x p \partial_x \xi \partial_x \eta \rangle \) and \( D_p(r) \) with the parameters \( \chi \) and \( \lambda_1 \).

Kerr et al. [9] identify, by empirical means, a length scale within the inertial range of fourth-order velocity structure functions \( \langle D_{ijkl}(r) \rangle \) such that scaling exponents should be determined only from \( r \) greater than that scale. In the inertial range, the divergence of \( D_{ijkl}(r) \) equals the pressure-gradient velocity-velocity structure function which is, in turn, related to \( A_{ij}(r) \) (Eqs. 9, 10, A4, A5 of Ref. [25]). Taylor series expansion of those relationships shows that \( \lambda_1 \) is a scale of, at least, linear combinations of the nonzero components of \( D_{ijkl}(r) \). From Fig. 2 of Ref. [9], their empirical scale is about 5 times \( \lambda_1 \); on the other hand, \( \lambda_1 \) is only the initial roll-off of \( \langle \partial_x p \partial_x \xi \partial_x \eta \rangle \). Further work is needed to establish a causal relationship between the scales, if one exists.

The theory [14] predicts the scaling with \( \chi \) that was found empirically in Refs. [13] and [8]. Even for large Reynolds numbers, dividing \( \langle \partial_x p \partial_x \xi \partial_x \eta \rangle \) by \( \chi \lambda_1^2 \) and similarly for pressure spectra is not K41 scaling. The approximation Eq. (6) contains, in addition to K41 scaling parameters, the factor \( F \) and thereby dependence on \( R_\lambda \). The deviation of both \( \chi \) and \( F \) from K41 scaling is large for large variations of Reynolds number. Whether or not \( \lambda_1/\eta \) continues the weak downward trend shown in Fig. 2 or becomes constant as \( R_\lambda \to \infty \) is also a topic for further investigation, as is the relationship of \( \eta \) to the viscous-force scale \( \lambda_{V_1} \).

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