Orbital upper critical field of type-II superconductors with pair breaking

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The orbital upper critical field \(H_{c2}\) is evaluated for isotropic materials with arbitrary transport and pair-breaking scattering rates. It is shown that unlike scattering which enhances \(H_{c2}\), the pair breaking suppresses the upper critical field and reduces the dimensionless ratio \(h^*(0) = H_{c2}(0)/T_c(dH_{c2}/dT)_T\) from the Helfand-Werthamer value of \(\approx 0.7\) to 0.5 for a strong pair-breaking. \(h^*(T)\) is evaluated for arbitrary transport and pair-breaking scattering. A phenomenological model for the pair-breaking suppression by magnetic fields is introduced. It shows qualitative features such as a positive curvature of \(H_{c2}(T)\) and the low temperature upturn usually associated with multi-band superconductivity.

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I. INTRODUCTION

In a seminal work Helfand and Werthamer calculated the \(H_{c2}(T)\) for isotropic materials with non-magnetic impurities. In particular, they showed that the ratio \(H_{c2}(0)/T_c, H'_{c2}(T_c) \approx 0.7\) for any impurity content. Since then, this result is broadly used to estimate \(H_{c2}(0)\) by measuring a readily accessible slope \(H'_{c2}\) at \(T_c\) although many new materials of interest are anisotropic with a substantial pair-breaking scattering.

The general \(H_{c2}(T)\) problem for materials with anisotropic Fermi surfaces and order parameters is quite complicated and applying the existing models to real materials requires knowledge of many material parameters. Analyzing the \(H_{c2}\) data, conclusions are often made just on the basis of analogy with other materials. An example is a commonly held belief that a positive curvature of the \(H_{c2}\) curve near \(T_c\) is an evidence for a multi-gap scenario analogous to the well studied MgB\(_2\).

In this work we have a less ambitious goal of solving the one-band isotropic problem in the presence of both transport and pair breaking scattering. This problem has been considered by Fulde and Maki in a more general context of correlated magnetic impurities. They, however, considered only the limit of short transport scattering time. On the other hand, clean materials with a strong pair breaking can in principle exist, CeCoIn\(_5\) is an example.

We take advantage of numerical methods now available and show that various combinations of scattering rates, \(1/\tau\) and \(1/\tau_m\) (\(\tau_m\) is the pair-breaking, e.g., spin-flip, scattering time) may cause variety of behaviors of \(H_{c2}(T)\) which might be useful interpreting the data on real materials at least on a qualitative level.

In the second, more speculative, part of this work we discuss an interesting possibility: The rate \(1/\tau_m\) of the spin-flip scattering of conducting carriers on local moments may depend on the applied field because the spin flip should be accompanied by a change of the spin associated with local moments, the energy of the latter is \(H\) dependent. We have included this possibility within our formalism and obtained variety of behaviors of \(H_{c2}(T)\) which open yet another channel in interpretation of the temperature dependence of the upper critical field.

II. THE PROBLEM OF \(H_{c2}\)

Consider an isotropic material with both magnetic and non-magnetic scatterers. The problem of the 2nd order phase transition at \(H_{c2}\) is addressed using the Eilenberger quasiclassical version of Gor’kov’s equations for normal and anomalous Green’s functions \(g\) and \(f\). At \(H_{c2}, g = 1\) and we are left with a linear equation for \(f\):

\[
\begin{align*}
(2\omega^+ + v \cdot \Pi)f &= 2\Delta/h + (f)/\tau^-, \\
\omega^+ &= \omega + \frac{1}{2\tau^+}, \quad \frac{1}{\tau^\pm} = \frac{1}{\tau} \pm \frac{1}{\tau_m}.
\end{align*}
\]

Here, \(v\) is the Fermi velocity, \(\Pi = \nabla + 2\pi i A/\phi_0\) with the vector potential \(A\) and the flux quantum \(\phi_0\). \(\Delta(r)\) is the gap function (the order parameter); the Matsubara frequencies are defined by \(\hbar\omega = \pi T(2n + 1)\) with an integer \(n; \langle \ldots \rangle\) stand for averages over the Fermi surface.

Solutions \(f\) and \(\Delta\) of Eq. (1) should satisfy the self-consistency equation:

\[
\frac{\Delta}{2\pi T \ln T_c} T = \sum_{\omega > 0} \left( \frac{\Delta}{\hbar\omega} - \langle f \rangle \right),
\]

where \(T_c\) is the critical temperature in the absence of pair-breaking scattering. In zero field, Eq. (1) yields

\[
\langle f \rangle = \frac{\Delta}{\hbar\omega_m}, \quad \omega_m = \omega + \frac{1}{\tau_m}.
\]

Substituting this in Eq. (3) one obtains an equation for the actual \(T_c\) which together with Eq. (3) allows one to exclude \(T_{c0}\):

\[
\frac{\Delta}{2\pi T \ln T_c} T_c = \sum_{\omega > 0} \left( \frac{\Delta}{\hbar\omega} - \langle f \rangle \right), \quad \omega' = \omega + \frac{t}{\tau_m}
\]

where \(T_c\) is the actual (suppressed by magnetic impurities) critical temperature and \(t = T/T_c\).
The general scheme for finding $H_c2(T; \tau, \tau_m)$ is as follows: The solution of Eq. (1) is written in the form:

$$f = \frac{2}{\hbar} \int_0^\infty d\eta e^{-\eta(2\omega^+ + v \mathbf{n})} \left( \Delta + \frac{\hbar f(\eta)}{2\tau} \right). \quad (6)$$

Taking average over the Fermi surface of both sides we have:

$$F = \frac{2}{\hbar} \int_0^\infty d\eta e^{-2\omega^+} \left( e^{-\eta v \mathbf{n}} \right) \left( \Delta + \frac{\hbar F}{2\tau} \right), \quad (7)$$

where $F = \langle f \rangle$. As argued in Refs. 1 and 10 both $\Delta$ and $F$ satisfy at $H_{c2}(T)$ a linear equation $-\xi^2 \Pi^2 \Delta = \Delta$ which gives $H_{c2} = \phi_0/2\pi\xi^2$. This allows one to manipulate the exponential operator to the form

$$e^{-\eta v \mathbf{n}} \Delta = \Delta \exp \left( -\frac{\eta^2 v^2}{4 \xi^2} \right), \quad (8)$$

and the same for $F$: $v_\perp$ is the Fermi velocity projection onto the plane perpendicular to $\mathbf{H}$. The Fermi sphere average of this expression is readily found:

$$\langle e^{-\alpha^2 \sin^2 \theta} \rangle = \frac{\sqrt{\pi}}{2\alpha} e^{-\alpha^2} \text{Erfi}(\alpha), \quad \alpha = \frac{\eta v}{2\xi}, \quad (9)$$

where $\theta$ is the polar angle on the sphere, Erfi$(\alpha) = \text{erfi}(i\alpha)/i = (2/\sqrt{\pi}) \int_0^\alpha dt e^{t^2}$. Substituting this in (7) we find $F(r) \propto \Delta(r)$:

$$F = \frac{2\tau - \Delta}{\hbar} J, \quad \frac{J(\xi, T, \tau^+)}{\hbar^2} = \frac{\sqrt{\pi}}{2} \int_0^\infty d\eta e^{-2\omega^+} \frac{e^{-\alpha^2}}{\alpha} \text{Erfi}(\alpha). \quad (10)$$

Hence, we have the self-consistency relation:

$$\frac{1}{2\tau} \ln \frac{T_c}{T} = \sum_{\omega > 0} \left( \frac{1}{\hbar\omega} - \frac{2\tau - J}{\hbar(\tau - J)} \right), \quad (11)$$

which is an equation for $\xi(T; \tau, \tau_m)$. It is readily seen that this equation reduces to the standard form for nonmagnetic scattering if one sets $\tau_m \to \infty$.

The integral $J$ is convergent; this is seen from the power series$^{11}$

$$\frac{\text{Erfi}(\alpha)}{\alpha} e^{-\alpha^2} = \frac{2}{\sqrt{\pi}} \sum_{n=0}^\infty \frac{(-2)^n \alpha^{2n}}{(2n + 1)!}, \quad (13)$$

which gives a constant for $\alpha \to 0$. We can use this expansion to recast $J$ in a different form. To this end, substitute it in Eq. (11) and integrate:

$$J = \frac{1}{2\omega^+} \sum_{n=0}^\infty \frac{(-1)^n n!}{2n + 1} \left( \frac{\omega}{2\xi \omega^+} \right)^{2n}. \quad (14)$$

The sum here belong to Borel summable types.$^{12}$ It has been studied by HW and can be written as an integral

$$J = \frac{2\gamma}{\nu} \int_0^\infty du e^{-u^2} \tan^{-1} \left( \frac{\nu}{2\xi \omega^+} u \right). \quad (15)$$

Another integral representation is given in Ref. 13:

$$J = \sqrt{\frac{\nu \xi}{2}} \int_0^\infty \frac{dt}{1 + t^2} \text{erfc} \left( \frac{2\xi \omega^+}{\nu} t \right). \quad (16)$$

We now introduce dimensionless variables

$$t = \frac{T}{T_c}, \quad \nu = \frac{\hbar^2 v^2}{4\pi^2 T_c^2 \xi^2}, \quad H_{c2} = \frac{\hbar^2 v^2}{2\pi T_c^2 \phi_0}, \quad (17)$$

and the scattering parameters

$$\rho_m = \frac{\hbar}{2\pi T_c \tau_m}, \quad \rho = \frac{\hbar}{2\pi T_c \tau}, \quad \rho^+ = \rho \pm \rho_m. \quad (18)$$

Note that $\rho, \rho_m$ involve the actual $T_c$, they differ from used often scattering parameters defined via $T_{c0}$.

The self-consistency Eq. (12) in dimensionless form reads:

$$-\ln t = \sum_{n=0}^\infty \left( \frac{1}{n + 1/2 + \rho_m} - \frac{2tI}{1 - \rho - I} \right), \quad (19)$$

$$I = \sqrt{\frac{\pi}{\gamma}} \int_0^{\infty} dz \text{erfc} z \left( \frac{1}{z^2 + 1} \right), \quad (20)$$

This can be solved numerically for $h(t)$ for any combination of scattering parameters $\rho$ and $\rho_m$.

**A. \ T \to T_c**

As $T \to T_c$, $h \to 0$ and the parameter

$$s = \sqrt{h / \gamma}, \quad (21)$$

can be considered as small. The integral $I$ can then be evaluated:

$$I = \sqrt{\frac{\pi}{\gamma}} \int_0^{\infty} \frac{dz \text{erfc} z}{z^2 + 1} \approx \sqrt{\frac{\pi}{\gamma}} \int_0^{\infty} \frac{dz (1 - z^2 s^2) \text{erfc} z}{1 - z^2 s^2} \left[ \frac{1}{\gamma} - \frac{s^2}{3} \right]; \quad (22)$$

erfc$(z)$ effectively truncates the integration domain to approximately $z \ll 2$, so that the expansion of $(1 + z^2 s^2)^{-1}$ in powers of $z^2 s^2$ is justified. We then obtain keeping only the terms $\sim s^2$ in Eq. (19):

$$-\ln t = \psi \left( \frac{\rho_m + 1/2}{t} \right) - \psi \left( \rho_m + 1/2 \right) + \frac{\hbar}{3 \rho^2} \psi \left( \frac{\rho_m + 1/2}{2t} \right) - \psi \left( \frac{\rho^+ + 1/2}{2t} \right) + \frac{\rho^+}{2t} \psi \left( \frac{\rho_m + 1/2}{2} \right). \quad (23)$$

Expanding this in powers of $1 - t \ll 1$, we obtain the slope at $t = 1$:

$$-\frac{dh}{dt} \bigg|_{t=1} = 3 \rho^2 \left[ 1 - \rho_m \psi \left( \frac{\rho_m + 1/2}{2} \right) - \psi \left( \frac{\rho^+ + 1/2}{2} \right) + \frac{\rho^+}{2} \psi \left( \frac{\rho_m + 1/2}{2} \right) \right]. \quad (24)$$
iron-based superconductors. It is suggested as evidence of a pair breaking present in many parameters. Hence, one can say that in a broad domain of scattering $\rho < \rho_m < 10$. Fig 1 shows the slopes according to Eq. (24). One observes that the pair-breaking scattering depresses the slopes $h'$ at $T_c$, just the opposite to what the transport scattering does. We see that (i) for weak transport scattering, $\rho_m$ the slopes are nearly independent of the magnetic scattering $\rho_m$, and (ii) for strong pair-breaking scattering (roughly $\rho_m > 4$) the slopes remain low even if the transport scattering intensifies.

In common units, the slope

$$\frac{dH_{c2}}{dT} \bigg|_{T_c} = \frac{2\pi \varphi_0}{h^2 v^2} T_c \frac{dh}{dT} \bigg|_{t=1}.$$ (25)

Hence, one can say that in a broad domain of scattering parameters

$$\frac{dH_{c2}}{dT} \bigg|_{T_c} \propto T_c$$ (26)

provided roughly $\rho_m > 4$. This feature, in fact, has been suggested as evidence of a pair breaking present in many iron-based superconductors.$^{14-16}$

B. Strong pair breaking, $T_c \to 0$

When $\tau_m$ is close to the critical value where $T_c \to 0$, $H_{c2}$ can be calculated analytically in the whole temperature range $0 < T < T_c$. Formally, the simplification comes about because in this domain all $\rho$’s are large. Then, $s = \sqrt{h'/\gamma}$ is small due to large $\gamma$. Eq. (22) and (23) are still valid and one can do sums in Eq. (19) keeping only terms $O(s^2)$. We can utilize the asymptotic expansion $\psi(x + 1/2) = \ln x + 1/24x^2 + O(1/x^4)$ to obtain:

$$h = \frac{1}{8} \left( \frac{\rho^-}{\rho_m} \right)^2 \left( \frac{\rho^-}{2\rho_m} + \ln \frac{2\rho_m}{\rho^+} \right)^{-1} (1 - t^2).$$ (27)

It is worth noting that here the ratio

$$h^*(0) = \frac{H_{c2}(0)}{H_c^2(0)} |_{T_c} = \frac{h(0)}{h'(1)} = \frac{1}{2}.$$ (28)

The value $h(0; \rho, \rho_m)$ as given in Eq. (27) in fact depends only on the ratio $\rho/\rho_m$ and varies from the minimum of $1/4\ln(4/e) = 0.647$ corresponding to $\rho/\rho_m \ll 1$, through the unity at $\rho/\rho_m = 1$, to $\rho/4\rho_m$ for $\rho/\rho_m \gg 1$. For the gapless regime with $\rho \gg \rho_m$, Eq. (27) reduces to the result of Abrikosov and Gor’kov.$^{17}$

C. Numerical results

Equations (19) and (20) can be solved numerically for any $\rho$ and $\rho_m$. Numerical results were obtained using Matlab and Mathematica. Attention has to be paid to the number of summation terms in Eq. (19). At low temperatures as many as 5000 terms were needed.

Representative examples of such calculations are given in Fig. 2 and 3. Parameters for these graphs are chosen not because they are realistic, but rather to demonstrate evolution of $h(t)$ with changing scattering parameters $\rho$ and $\rho_m$. We also show the HW ratios $h^*(t) = h(t)/h'(1)$ for both clean and dirty transport limits. One clearly sees that this ratio, which is close to 0.7 for purely transport scattering, drops to $\approx 0.5$ for a strong pair-breaking. It is worth noting that actual $H_{c2}(T)$ given in Eq. (17) is $\propto T_c^2$, the latter being suppressed by pair-breaking scattering. Hence, the plots of $h(t)/h'(1)$ are valuable in particular.

Having solved for $h(t; \rho, \rho_m)$ one can collect the zero-$T$ values $h(0; \rho, \rho_m)$. This calculation should be done with care because the sums over $\omega$ in Eq. (19) are logarithmically divergent and should be truncated at $n$ corresponding to the Debye frequency $\omega_D$: $n_D = h\omega_D(2\pi T)$, i.e., it diverges at $t = 0$. The calculation then can be done for a small but finite $t$ as shown in Fig. 4.

One can now construct the HW ratio $h^*(0) = h(0)/h'(1)$ for any $\rho$ and $\rho_m$ with the result shown in Fig. 5. At $\rho_m = 0$ we have the standard HW behavior of $h^*(0)$ which is close to 0.73 for the clean limit and reduces to 0.69 at the dirty side. With the pair-breaking increasing, $h^*(0)$ approaches 0.5 for large $\rho_m$.

III. MODEL OF FIELD DEPENDENT SPIN-FLIP SCATTERING

The rate $1/\tau_m$ of the spin-flip scattering of conducting carriers on local moments may depend on the field because the spin flip should be accompanied by a change
of the spin associated with local moments, the energy of the latter is $H$ dependent. If $\delta \mu$ is the local moment change, the probability of the pair-breaking scattering should contain a factor $\exp(-\delta \mu H/T)$. This factor should enter the magnetic scattering parameter: $\rho_m = \rho_{m0} \exp(-\delta \mu H/T)$. Hence, the pair-breaking scattering becomes weaker with increasing $H$.

For an estimate we take $\delta \mu \sim \mu_B$, $\mu_B$ is the Bohr magneton. Then, writing the Boltzmann factor in our dimensionless units as $\exp(-\delta \mu H/T) = \exp(-a h/t)$ one estimates $a \sim 0.03 T_c (K)$. Setting in our equations for $h(t)$ the parameter

$$\rho_m = \rho_{m0} e^{-a h/t}$$

we can study qualitatively how the field suppression of spin-flip scattering affects $h(t)$. We note that our results for the slopes of $H_{c2}$ at $T_c$ are not affected by this change since there $h \to 0$. On the other hand, as $t \to 0$, the new $\rho_m$ vanishes, i.e., the spin-flip scattering is completely “frozen out”. In the following we will call the constant $a$ the “pair-breaking freezing parameter”.

A few examples are given below to illustrate field effects upon the pair-breaking and their influence on the behavior of $h(t)$. The first interesting feature of the $h(t)$ curve is shown in Fig.6: the positive curvature of $h(t)$ at high and intermediate temperatures. Traditionally, this feature is associated with the multi-band superconductivity, as is the case of MgB$_2$. We now see that the positive curvature of $h(t)$ can be present in a one-band isotropic material due to the pair-breaking scattering and its suppression by the field.

Figure 7 shows a set of three curves corresponding to the same magnetic scattering $\rho_{m0} = 1$, the same pair-breaking freezing parameter $a = 0.1$, but different transport scattering $\rho = 0.2, 5$. A feature of these curves

![Figure 2](image1.png)

![Figure 3](image2.png)
worth noting is nearly linear temperature dependence in a broad temperature domain. This feature is seen in many iron-based materials;\textsuperscript{18,19} our work therefore suggests that the near-linear behavior of $H_{c2}(T)$ might be related to pair-breaking.

IV. D-WAVE

We show here that the problem of $H_{c2}$ in a d-wave material with a spherical Fermi surface in the presence of impurities is simpler than for the s-wave symmetry, because in all relations for $H_{c2}$ transport and pair-breaking scattering rates enter only via $\rho^{+} = \rho + \rho_m$.

Within a popular approximation, the effective coupling responsible for superconductivity is assumed factorizable: $V(k_F, k_F') = V_0 \Omega(k_F) \Omega(k_F')$.\textsuperscript{20} One looks for the order parameter in the form $\Delta(r, T; k_F) = \Psi(r, T) \Omega(k_F)$. The self-consistency equation takes the form:

$$\frac{\Psi}{2\pi T} \ln \frac{T_{c0}}{T} = \sum_{\omega > 0} \left( \frac{\Psi}{h\omega} - \langle \Omega f \rangle \right).$$  \hspace{1cm} (30)

$\Omega(k_F)$ describes the variation of $\Delta$ along the Fermi surface and is normalized: $\langle \Omega^2 \rangle = 1$. For the d-wave, $\Omega = \sqrt{2} \cos 2\varphi$ and $\langle \Delta \rangle = 0$.

The Elenberger Eq. (1) holds for any symmetry of the
order parameter $\Delta$. Taking the average of Eq. (1) in zero field over the Fermi surface we obtain $\langle f \rangle = 0$ and $f = \Delta/\hbar \omega^+$. Substituting this in Eq. (30) we obtain for the actual critical temperature\textsuperscript{14,21}

$$
\ln \frac{T_{c0}}{T_c} = \psi \left( \frac{\rho^+ + 1}{2} \right) - \psi \left( \frac{1}{2} \right).
$$

(31)

Combining this with Eq. (30) one can exclude $T_{c0}$.

The same derivation as above results in the dimensionless upper critical field $\bar{h}$

$$
- \ln t = \sum_{n=0}^{\infty} \left( \frac{1}{n + 1/2 + \rho^+ / 2} - 2t I \right),
$$

(32)

This can be solved numerically for $h(t)$ for any $\rho$ and $\rho_m$ which in fact enter only via $\rho^+ = \rho + \rho_m$.

One can obtain slopes $h'(1)$ at the critical temperature in the same manner as for s-wave treatment above:

$$
\left. \frac{dh}{dt} \right|_{t=1} = 24 \left[ 1 - \frac{\rho^+}{2} \psi' \left( \frac{\rho^+ + 1}{2} \right) \right] / \psi'' \left( \frac{\rho^+ + 1}{2} \right).
$$

(33)

In the clean limit, this yields $h' = -12/7\zeta(3)$ in agreement with the general clean limit formulas for the d-wave.\textsuperscript{5} For a strong $T_c$ suppression when $\rho^+ \to \infty$, we get $h' = -2$, so that the actual slope at $T_c$ vanishes as $dH_{c2}/dT \propto T_c/h' \to 0$.

Next, we calculate the field at $T = 0$. To this end, we transform Eq. (32):

$$
- \ln t = \sum_{n=0}^{\infty} \left( \frac{1}{n + 1/2 + \rho^+ / 2} - \frac{1}{n + 1/2} \right)

+ \sum_{n=0}^{\infty} \frac{1}{n + 1/2} - 2t \sum_{n=0}^{\infty} I.
$$

(34)

The first sum here is expressed in terms of di-gamma functions. The divergent sum $\sum (n + 1/2)^{-1}$ is truncated at $n_{\text{max}} = \hbar \omega_D / 2\pi T$ to give $\ln(2eC\hbar \omega_D / \pi T)$ where $\omega_D$ is the Debye frequency and $C = 0.577$ is the Euler constant. The last sum in Eq. (34) is replaced with an integral according to $2\pi T \sum \to \int_0^{\hbar \omega_D} d\omega$. Since $\gamma = \hbar \omega / \pi T + \rho^+$, the integration over $\hbar \omega$ can be replaced with integration over $\gamma$. Collecting all terms we obtain an equation for $h(0)$ as a function of $\rho^+$:

$$
\psi \left( \frac{\rho^+ + 1}{2} \right) + \ln 2 = \frac{\sqrt{\pi}}{2} \int_0^{\infty} dz \operatorname{erfc}(z) \ln \left( z^2 h + \rho^+ \right).
$$

(35)

Fig. 8 shows that, in fact, $h(0) \approx 1$ for all $\rho^+$ within 5% accuracy. Physical significance of the shallow minimum in $h(0; \rho^+)$ is not clear.

Fig. 9 shows the HW ratio $h'(0) = h(0)/h'(1)$ as a function of $\rho^+$ for a d-wave superconductor. We note that the HW ratio in the clean limit at $t = 0$ is the same for d- and s-waves (for a Fermi sphere) and for a strong pair breaking it approaches 0.5, as is the case for s-wave.

V. DISCUSSION

To summarize, we have solved the problem of the orbital upper critical field $H_{c2}(T)$ for the isotropic case and any combination of transport and pair-breaking scattering rates, $\rho$ and $\rho_m$. The simplicity of the model notwithstanding, $H_{c2}(T, \rho, \rho_m)$ show a number of interesting features.

The pair-breaking scattering depresses the slopes of the dimensionless upper critical field $h'$ at $t = 1$, just the opposite to what the transport scattering does. The suppression is pronounced even more in common units since $dH_{c2}/dT \propto h'(1)T_c$ and $T_c$ is suppressed too. For purely transport scattering, $\rho_m = 0$, the slopes increase.
FIG. 9. (Color online) The HW ratio \( h^*(0) = h(0)/h'(1) \) as a function of \( \rho^* \) for a d-wave superconductor.

with increasing \( \rho \) as they should. For weak transport scattering, the slopes \( h'(1) \) are nearly independent of \( \rho_m \) and for a strong pair breaking (roughly, \( \rho_m > 4 \)) they remain low even if the transport scattering intensifies.

For a strong pair breaking, \( h = h(0)(1 - t^2) \) with \( h(0) \) given in Eq. (27) which depends only on the ratio \( \rho/\rho_m \). Then, if in a material the temperature dependence of \( H_c2 \) is close to \( (1 - t^2) \), one can determine \( \rho/\rho_m = \tau_m/\tau \), the ratio of scattering rates, from the experimental \( h(0) \). In this case \( \rho_m \gg 1 \) and the transport scattering has practically no effect upon the HW scaled field \( h^*(t) = h(t)/h'(1) \)

The problem of \( H_c2(T) \) for the d-wave order parameter in the presence of impurities turns out to be simpler than for s-wave. The reason is that the scattering rates \( \rho \) and \( \rho_m \) enter the theory only as a sum, see Eq. (31) for the \( T_c \) suppression and Eqs. (32) and (20) containing only \( \rho^+ \).

Intriguing in particular is the similarity of the curves for \( H_c2(T; \rho, \rho_m, a) \) with account for possible “freezing out” of the spin-flip scattering by the field, with two-band scenarios without pair-breaking scattering as discussed, e.g., in Ref. 22. We are far from claiming that our model can be literally applied to real materials, it is too simple and the field freezing of the pair-breaking is introduced in a profoundly qualitative manner. Still, in our view possibility of the field suppression of the spin-flip scattering should not be discarded. In fact, this possibility, if confirmed, makes interpretation of \( H_c2 \) curves even less definite as far as extracting material characteristics from the shape of these curves.

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