Tensor force induced isospin-dependence of short-range nucleon-nucleon correlation and high-density behavior of nuclear symmetry energy

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Quantitative information on the tensor force induced isospin-dependence of short-range nucleon-nucleon correlation (SRC) extracted from recent J-Lab experiments was used in constraining the high-momentum tails of single nucleon momentum distributions in both symmetric nuclear matter (SNM) and pure neutron matter (PNM). Its effects on the Equations of State of SNM and PNM as well as the nuclear symmetry energy are investigated. It is found that the tensor force induced isospin-dependence of SRC softens significantly the nuclear symmetry energy especially at supra-saturation densities.

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I. INTRODUCTION

Recently, much efforts have been devoted to study the isospin-dependence of short-range nucleon-nucleon correlations (SRC) in nuclei both experimentally [1–4] and theoretically [5–10]. For reviews, see, e.g., Refs. [11, 12]. It is particularly exciting to note that experiments at the Jefferson Lab (J-Lab) have shown that about 20% of nucleons in $^{12}\text{C}$ are correlated. Most interestingly, the strength of np SRC is about 20 times that of pp (nn) SRC [3]. Theoretical studies have revealed that the dominance of np over pp (nn) SRC is a direct consequence of the tensor force acting in the np deuteron-like state $^2\text{H}$ [3]. Moreover, it was found that [6, 10] the isospin-dependence of SRC is robust and does not depend on the exact parameterization of the nucleon-nucleon force, the type of nucleus, or the exact ground-state wave function used to describe the nucleons [3]. Because the local density of SRC pairs in nuclei is estimated to reach that expected in the core of neutron stars, it has been repeatedly speculated that the observed isospin-dependence of the SRC may have significant effects on the Equation of State (EOS) of cold dense neutron-rich nucleonic matter and thus properties of neutron stars [2, 7]. Nevertheless, a quantitative evaluation of the effects has not been carried out yet.

Nuclear symmetry energy $E_{\text{sym}}(\rho)$, which encodes the energy related to neutron-proton asymmetry in the nuclear matter EOS, is a vital ingredient in the theoretical description of neutron stars and of the structure of neutron-rich nuclei and reactions involving them. Since the density-dependence of $E_{\text{sym}}(\rho)$ is still the most uncertain part of the EOS of neutron-rich nucleonic matter especially at supra-saturation densities, to better determine the $E_{\text{sym}}(\rho)$ has become a major goal of both nuclear physics and astrophysics [13, 24]. The $E_{\text{sym}}(\rho)$ can be approximated as the difference between the energy per nucleon in pure neutron matter (PNM) and symmetric nuclear matter (SNM), i.e., $E_{\text{sym}}(\rho) = E_{\text{PNM}}(\rho) - E_{\text{SNM}}(\rho)$. The isospin-dependence of SRC affects differently the EOSs of PNM and SNM at high densities, it is thus expected to play an important role in determining the high density behavior of $E_{\text{sym}}(\rho)$ [13, 24, 25]. However, the strength of the isospin-dependence of SRC may allow us to better understand effects of the tensor force on the EOS and symmetry energy of dense neutron-rich nucleonic matter. In this work, within a phenomenological model for nuclear matter we show that the tensor force induced isospin-dependence of SRC softens significantly the EOS of SNM especially at suprasaturation densities. However, it has almost no effect on the EOS of PNM. Consequently, the tensor force acting in neutron-proton isosinglet pairs significantly lowers the symmetry energy of dense neutron-rich nucleonic matter.

II. NUCLEON MOMENTUM DISTRIBUTION INCLUDING EFFECTS OF SHORT-RANGE NUCLEON-NUCLEON CORRELATIONS

To treat properly SRC effects has been a challenging task for nuclear many-body theories. For the present study, the most relevant question is how the SRC affects the one-body nucleon momentum distribution $n(k)$. In fact, this question has been studied both experimentally and theoretically for a long time. Fortunately, many interesting results have already been well established and can be used reliably in our present work. For a comprehensive review, we refer the reader to the book by Antonov et al. [32]. The $n(k)$ contains information not only about the nuclear mean-field but also the SRC. For a Fermi gas of uncorrelated nucleons at zero temperature, the $n(k)$ is simply a step function of one for $k \leq k_F$, and zero for $k > k_F$ where $k_F$ is the nucleon Fermi momen-

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The SRC, however, will deplete states below the Fermi surface and make the states above it partially occupied, leading to a high-momentum tail in $n(k)$. This feature has been well established in various microscopic and phenomenological models. Thus, in a nucleus, the momentum of a low-momentum nucleon with $k \leq k_F$ is balanced by the rest of the nucleus; however, a high-momentum nucleon with $k \geq k_F$ is almost always balanced by only one other nucleon and the two nucleons form a correlated pair. It is also well known that the SRC can be caused by either the repulsive core of the central force or the tensor part of the nucleon-nucleon interaction. The experimental finding that the $np$ SRC dominates over the $nn$ ($pp$) one indicates that the tensor force instead of the repulsive core is mainly responsible for the high-momentum tail of $n(k)$.

![Graph showing n(k) distribution for SNM and PNM](image)

**FIG. 1:** (Color online) Single nucleon momentum distribution in symmetric nuclear matter and pure neutron matter.

To investigate how the strength of tensor force affects the high-momentum tail of $n(k)$, some phenomenological methods have been shown to be particularly useful. Instead of seeking for exact many-body wave functions containing the SRC as in microscopic many-body theories, the phenomenological methods include the SRC in a physically very transparent way and allow one to easily examine effects of the tensor force on the momentum distribution $n(k)$. For instance, Dellagiacoma et al. have derived formulas for $n(k)$ explicitly including the tensor force induced SRC for finite nuclei. It was shown that the probability of finding nucleons with high momentum increases with the strength of tensor force (or equivalently the percentage of D-wave mixture). With the same D-wave percentage, it was also found that the high-momentum tails of $n(k)$ for different nuclei are very close to each other. This finding is consistent with the conclusion of microscopic calculations that the behavior of $n(k)$ at high momenta ($k > 2 \text{ fm}^{-1}$) is almost independent of the mass number. Moreover, the high momentum behavior of $n(k)$ for symmetric nuclear matter was shown to be very similar to those of finite nuclei. Results obtained using the y-scaling analysis of inclusive electron scattering data for $^2\text{H}, ^3\text{He}, ^{12}\text{C}, ^{56}\text{Fe}$ also support this conclusion.

Here we parameterize the $n(k)$ of nucleons in both SNM and PNM as

$$n(k) = \begin{cases} a & (k \leq k_F) \\ e^{b k} & (k > k_F), \end{cases}$$

under the normalization condition that

$$\frac{3}{k_F^3} \int_{0}^{\infty} n(k)k^2 dk = 1.$$ 

The $a$ and $b$ are parameters determined using the experimental findings from the J-Lab experiments. For SNM, we take 80% (20%) nucleons as uncorrelated (correlated) as indicated by the J-Lab experiments and model calculations as we discussed earlier. Thus, for SNM we have

$$\frac{3}{k_F^3} \int_{0}^{k_F} n(k)k^2 dk = 0.8,$$

$$\frac{3}{k_F^3} \int_{k_F}^{\infty} n(k)k^2 dk = 0.2.$$ 

The required parameter $a = 0.8$ and $b$ is obtained numerically from the integration $\frac{3}{k_F^3} \int_{k_F}^{\infty} e^{b k} k^2 dk = 0.2$. The value of $b$ depends on the Fermi momentum and thus the density. At nuclear matter saturation density, $b = -1.899$. Shown in the left panel of Fig.1 is the $n(k)$ of symmetric nuclear matter at saturation density. The blue dashed line denotes the $n(k)$ in an ideal Fermi gas. The red solid line is our parameterization. The black dotted line is the parameterization given by Ciofi degli Atti et al., which is a fit to the result of variational many-body calculations. It is clearly seen that our parameterization is very close to the microscopic single nucleon momentum distribution. For PNM, the SRC is induced only by the repulsive core. Utilizing the experimental finding that only 2% of nucleons in $^{12}\text{C}$ can form the $nn$- or $pp$- type SRC pairs, we have for PNM

$$\frac{3}{2k_F^3} \int_{0}^{2k_F} n(k)k^2 dk = 0.98,$$

$$\frac{3}{2k_F^3} \int_{2k_F}^{\infty} n(k)k^2 dk = 0.02.$$ 

The resulting parameter $a = 0.98$. Again, the parameter $b$ is density-dependent and has to be found numerically. At nuclear matter saturation density, $b = -2.435$. The corresponding $n(k)$ for PNM is shown in the right panel of Fig.1. As mentioned above, the big difference in $n(k)$ for SNM and PNM is caused by the tensor-force induced SRC in SNM.

### III. EFFECTS OF SHORT-RANGE NUCLEON-NUCLEON CORRELATIONS ON THE EOS AND SYMMETRY ENERGY OF DENSE NEUTRON-RICH NUCLEONIC MATTER

We now examine effects of the isospin-dependence of SRC on the EOS and symmetry energy of dense neutron-rich nuclear matter.
rich nucleonic matter by using the single nucleon momentum distributions in SNM and PNM constrained by the J-Lab experiments within a phenomenological approach. With momentum-dependent nuclear interactions, not only the kinetic but also the potential energy are affected by the high-momentum tail due to the SRC. Let us first examine separately effects of the SRC on the kinetic and potential energy per nucleon in both SNM and PNM. While the average kinetic energy per nucleon in a Fermi gas of independent nucleons is simply $E_{\text{kin}} = \frac{3}{2} \frac{\hbar^2 k^2}{2m}$, with correlated nucleons it is given by

$$E_{\text{kin}} = \alpha \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk,$$

(5)

where $\alpha = \frac{3}{k_F^2}$ and $\frac{3}{k_F^2}$ for SNM and PNM, respectively. We compare in Fig. 2 the average kinetic energies for both SNM and PNM calculated with and without the SRC. As one expects, the SRC increases the kinetic energy of the Fermi gas of uncorrelated nucleons. However, for PNM, the increase in kinetic energy per nucleon is very small. The different changes in $E_{\text{kin}}$ due to the SRC induced only by the repulsive core is significantly for SNM but only little for PNM because of the strong isospin-dependence of the SRC. More quantitatively, for SNM at saturation density, the $E_{\text{kin}}$ with the SRC ($E_{\text{kin}}(k_F^0) \approx 40$ MeV) is about twice of that ($E_{\text{kin}}(k_F^0) \approx 22$ MeV) for the Fermi gas of uncorrelated nucleons. However, for PNM, the increase in $E_{\text{kin}}$ due to the SRC induced only by the repulsive core is very small. The different changes in $E_{\text{kin}}$ for SNM and PNM due to the isospin-dependence of the SRC has a dramatic effect on the kinetic part of the nuclear symmetry energy. In the Fermi gas without the SRC, the kinetic energy contribution to the symmetry energy is $E_{\text{sym}} = E_{\text{kin}}(\text{PNM}) - E_{\text{kin}}(\text{SNM}) = (2 - 1) \frac{3}{2} \frac{\hbar^2 k^2}{2m}$, which is always positive with increasing density. However, because the SRC induced increase in $E_{\text{kin}}$ is much larger for SNM than PNM, the kinetic contribution to the symmetry energy now becomes negative. Thus, the kinetic part of $E_{\text{sym}}$ and its density-dependence are strongly affected by the tensor-force induced SRC.

The average potential energy per nucleon $E_{\text{pot}}$ can be calculated using potential energy density functionals. For extracting information about the EOS of dense nuclear matter from heavy-ion reactions, several momentum-dependent single-particle potentials, such as the GBD (Gale-Bertsch-Das Gupta) [38], BGBD (Bombaci-Gale-Beretsch-Das Gupta) [39] and MDI (Momentum-Dependent Interaction) [40] phenomenological potentials, have been widely used. In calculating the corresponding EOSs with these potentials only the step function for the momentum distribution of a Fermi gas was used. Thus, effects of the SRC were not considered. Here we use the MDI interaction and the nucleon momentum distributions including the SRC effects. The MDI potential energy per nucleon can be written as

$$E_{\text{pot}} = \frac{A}{2} \frac{\rho}{\rho_0} + \frac{B}{\sigma + 1} \frac{\rho^\sigma}{\rho_0^\sigma} + \frac{C}{\rho \rho_0} \int_0^\infty \int_0^\infty \frac{n(k_1)n(k_2)}{1 + (k_1 - k_2)^2/\Lambda^2} dk_1 dk_2,$$

(6)

where $\sigma = 4/3$ and the parameter $\Lambda = 1.0k_F^0$. [40]. The $n(k_1)$ and $n(k_2)$ are the one-body momentum distribution of nucleon-1 and nucleon-2. For SNM, as usual, the parameters $A = -114.079$ MeV, $B = 107.154$ MeV and $C = -0.0274$ MeV are obtained from fitting its three empirical saturation properties, namely the vanishing pressure, the average binding energy of $E = -16$ MeV, and the incompressibility of $K_0 = 220$ MeV at the saturation density $\rho_0 = 0.16 fm^{-3}$. For PNM, we obtain the three parameters $A = -120.570$ MeV, $B = 102.080$ MeV and $C = -7.61 \times 10^{-4}$ MeV by satisfying the well-established theoretical constraints on the low density PNM EOS given in Refs. [41, 42], namely, $E = 1.7$ MeV at $k_F = 0.4$ fm$^{-1}$ and $E = 4.2$ MeV at $k_F = 0.8$ fm$^{-1}$, as well as the symmetry energy $E_{\text{sym}} = 31$ MeV at $\rho_0$. Shown in Fig. 3 is a comparison of $E_{\text{pot}}$ with and without the SRC for both SNM and PNM. Similar to the case of $E_{\text{kin}}$, the
SRC increases the magnitude of $E_{pot}$ significantly for SNM but has negligible effect for PNM. Consequently, the tensor force induced SRC also softens the potential part of the symmetry energy $E_{sym}^{pot} = E_{PNM}^{pot} - E_{SNM}^{pot}$. Shown in Fig. 4 are the EOSs of SNM and PNM with and without the SRC. The corresponding symmetry energies are presented in Fig. 5. It is interesting to see that the tensor force induced SRC has a significant impact on the symmetry energy especially at supra-saturation densities. The symmetry energy obtained in calculations with the SRC is much softer especially for $k_F > 2.0$ fm$^{-1}$. Microscopic many-body theories have shown clearly that the isosinglet nucleon-nucleon interaction dominates the symmetry energy $[43, 44]$. It was also known that the tensor force acting in the isosinglet channel affects significantly the high-density behavior of nuclear symmetry energy $[27, 31]$. The finding here that the SRC softens the symmetry energy is thus not surprising qualitatively. In fact, the short-range repulsive tensor force may even lead to negative symmetry energies at supra-saturation densities as first predicted by Pandharipande and Gade in 1972 within a variational many-body approach $[27]$. However, we never had before the kind of quantitative information on the isospin-dependence of the SRC due to the tensor force and the repulsive core from experiments. The results presented here thus represent a significant progress in our understanding about the tensor force effects on the EOS and symmetry energy of dense neutron-rich nucleonic matter.

**IV. SUMMARY**

In summary, the quantitative information on the tensor force induced isospin-dependence of SRC from the recent J-Lab experiments was used in constraining the high-momentum tails of single nucleon momentum distributions in SNM and PNM. The latter are then used in evaluating the EOSs of SNM and PNM as well as the symmetry energy of neutron-rich nucleonic matter. It is found that the isospin-dependence of SRC stiffens significantly the EOS of SNM but has little effect on that of PNM. Consequently, the nuclear symmetry energy especially at supra-saturation densities is significantly softened. This finding confirms the critical role of the tensor force acting in neutron-proton isosinglet pairs in determining the high-density behavior of nuclear symmetry energy predicted by various microscopic and phenomenological nuclear many-body theories.

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