Superstring dominated early universe and epoch dependent gauge coupling.

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Abstract

We have explored the possibility that the universe at very early stage was dominated by (macroscopic) heterotic strings. We have found that the dimensionless parameter $G\mu$ for the heterotic strings varies from $10^{-2}$ to $10^{-4}$ as the universe evolve from the matter dominance to radiation dominance. This led to the interesting consequence of epoch dependent gauge coupling constant. The gauge coupling constant at early times was found to be much stronger than the present strong interaction.

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I. INTRODUCTION

Presently, superstring theory \cite{1} with space-time symmetry is being investigated by many authors. It is believed to unify the gravitational force with the other forces of nature. It can address some major unsolved problems in particle physics e.g. origin of the spectrum of quarks and leptons, the hierarchy problem, i.e. the existence of very small and very large mass scales etc. Several authors have explored the possible cosmological consequence of (macroscopic) superstrings \cite{2,3}. It is also conjectured that bulk of the dark matter may consists of the lightest supersymmetric particle \cite{6}. Indeed, if superstring is the 'theory of every thing' then whole universe should be contained as one of its solution. The most promising string theory is the heterotic string theory \cite{7}. The theory has gauge bosons and fermions interacting through a large unification gauge group ($E_8 \times E_8$). The theory even has natural handedness, which translate into chirality of electroweak interaction. However, the theory is Lorenze invariant only in ten dimensions, the extra six dimensions of which can be compactified on some Calabi-Yau manifold.

In the cosmological context the dimensionless parameter $G\mu$ ($G$ is the gravitational constant and $\mu$ is the string tension) plays an important role. For heterotic strings, there is a fundamental relation between the gravitational constant $G$, the string tension $\mu$, and the gauge coupling constant $g$ \cite{3,7}:

$$G\mu = g^2 / 32 \pi^2$$ (1)
giving $G \mu \sim 10^{-3}$. This is three order of magnitude larger than that of cosmic strings [8]. Is a universe with cosmic fundamental strings with $G \mu \sim 10^{-3}$ viable? Most stringent bounds on any cosmological model comes from the observed micro wave back ground anisotropy [9], the pulsar timings [10] and the nucleosynthesis analysis [11]. Strings with $G \mu \sim 10^{-3}$ if present today, would cause unacceptable distortions in the microwave background. However, in the heterotic string theories the fundamental strings become attached to axion domain walls at the QCD scale and thereafter rapidly disappear [2]. It can then evade the bounds imposed by the micro wave background anisotropy or the pulsar timings as they concern periods much later than QCD phase transition. The nucleosynthesis bound require that the gravity waves be less than 18% of total density at the time of nucleosynthesis. However, major fraction of gravity waves comes from periods well before the nucleosynthesis time. Roughly speaking there is equal contribution from each logarithmic intervals of time. For superstrings, though we can ignore the expansion after 100 MeV or so (QCD scale), as the strings are formed at Plank energies ($10^{19}$ GeV) there is a 16 decade of expansion to be considered. However, unlike cosmic strings, cosmic fundamental strings can decay into hundreds of mass less states including the graviton [12]. If one assume that the decay is equally probable in every channel, then gravity waves can be reduced by a factor of hundred and it is expected that nucleosynthesis bound can be met [4,5]. Considering all these aspects, in an earlier paper [6], we have argued that early universe can be heterotic string dominated. However, we now find that the analysis was incomplete. Deeper investigations revealed some interesting results, which will be reported in the present paper. The paper is organised as follows: in section 2, we shall describe briefly the evolution of the string universe, the results will be discussed in section 3. Summary will be given in section 4.

II. EVOLUTION OF STRING DOMINATED UNIVERSE

We assume that just after the Plank time, the universe was dominated by super strings. We also assume that all the compactification has been completed, by the time universe starts evolving. In string theory, the gravitational interaction is effectively described by not only the metric but the coupled system of metric and dilaton field. String theory also shows duality i.e. strings in very small space behave the same way as the strings in very large spaces. However presently we ignore the explicit dilaton field for the simple reason that we do not know its value. Also, as our universe begins after the compactification, its dynamics may be neglected. For the same reason, we ignore the duality and use Einstein’s gravity to describe the evolution of the universe [8,9].

How a superstring universe will evolve? Here we take a cue from the cosmic string study [13]. A cosmic string network quickly evolve into a scaling solution. Since action for both the cosmic string and the cosmic superstring are purely geometrical Nambu action, amenable to classical description, it is possible that cosmic fundamental string network also quickly evolve into scaling era, provided other things are similar [8]. Unlike in cosmic string, there is no infinitely long superstring in heterotic string theory. All the strings consists of loops only. However, if the loops are very large $>>$ horizon, they behave essentially like a infinitely long string. Thus we start our universe initially consists of long string. They can chop off loops, which in turn decay by emitting radiation. And we look for scaling solution.
We assume the standard hot cosmological model with spatially flat Robertson-Walker metric with scale factor $a(t)$. Expansion is governed by the Einstein equation,

$$\dot{a}/a = 8\pi G \rho /3$$  \hspace{1cm} (2)$$

here $\rho = \rho_m + \rho_{rad}$. $\rho_m$ and $\rho_{rad}$ are the matter and radiation energy density respectively. We further divide $\rho_m$ into two parts, the long string energy density $(\rho_L)$ and the loop energy density $(\rho_l)$. We also define a radiation fraction $r$ such that,

$$r = \rho_{rad}/(\rho_L + \rho_l + \rho_{rad})$$  \hspace{1cm} (3)$$

In the scaling era Einstein eq. can be solved to yield \[4,5\],

$$a(t) \propto t^{2/(r+3)}$$  \hspace{1cm} (4)$$

The Hubble parameter ($H$), the Hubble radius ($R_H$) and the horizon ($d_H$) can be obtained as,

$$H = \dot{a}/a = 2/((r+3)t)$$  \hspace{1cm} (5)$$

$$R_H = (r + 3)t/2$$  \hspace{1cm} (6)$$

$$d_H = a(t) \int dt' a(t')^{-1} = (r + 3)t/(r + 1)$$  \hspace{1cm} (7)$$

The main idea of scaling solutions \[10\] is that, there is a single length scale in the problem, which we take to be the Hubble radius ($R_H$). All the other scales are determined in terms of $R_H$. We define a length scale $\xi$ on the string; $\rho_L = \mu / \xi^2$. As long as the reconnection is frequent between the strings, which should be the case for $\xi << R_H$, it will keep the network random so that $\xi$ will be related by a constant factor of order unity to the typical radius of curvature on the string. The existence of scaling solution can be argued thus: when $\xi << R_H$, the long strings rapidly chop off loops and the long string density falls, and $\xi$ grows faster than $R_H$. If $\xi >> R_H$, then chopping off becomes infrequent, the string density rises and $\xi$ falls relative to $R_H$. It must be mentioned that the above discussion assume the intercommuting probability to be unity.

In the one scale hypothesis we can obtain analytical expressions for the long string, loops and radiation energy densities \[4,5\]. For completeness purpose we briefly discuss the procedure.

The time evolution of long string energy density is described by the equation \[4,5,14\],

$$\frac{d\rho_L}{dt} = -3H\rho_L + H/R_H\xi\rho_L - C\rho_L/\xi$$  \hspace{1cm} (8)$$

the 1st and 3rd term on the r.h.s. represent the loss in long string energy density due to the Hubble expansion and chopping off of loops. The 2nd term represent the gain in energy density due to stretching of strings as a result of expansion. In the scaling era, eq (8) can be integrated to obtain the long string energy density as \[4,5,14\],

$$\rho_L = \gamma_s^2 \mu / R_H^2$$  \hspace{1cm} (9)$$

with the scaling density,
\[ \gamma_s = \frac{1}{2C} \left[ r + \sqrt{r^2 + 4C} \right] \] (10)

The loop production in the scaling era is described by the equation \[4,5,14\],

\[ \frac{d\rho(l)}{dt} = -3H\rho(l) + \frac{\mu}{\xi^4} f_{\text{off}}(l/\xi) - \frac{\mu}{\xi^4} f_{\text{rec}}(l/\xi) \] (11)

where \( \rho(l)dl \) is the energy density partitioned in loops of length \( l \) and \( l+dl \). The 1st and 3rd terms on r.h.s. represents the decrease in energy density due to Hubble expansion and reconnection of loops with long strings. The 2nd term gives the gain in energy density due to chopping off of long strings. The chopping efficiency \( C \), satisfies the energy sum rule,

\[ C = \int dx f(x) = \int dx (f_{\text{off}}(x) - f_{\text{rec}}(x)) \] (12)

From statistical mechanical considerations, Albrecht and Turok [14], obtained the chopping off function as,

\[ f_{\text{off}}(x) = Ax^{-1/2} \exp(-Bx) \] (13)

with \( A \) and \( B \) smoothly interpolating between radiation and matter era.

The reconnection function can be written as [14],

\[ f_{\text{rec}}(l/\xi) = k\xi^2 l\rho(l)/\mu \] (14)

by noting that the probability per unit time, for any loop of length \( l \) to hit a long string is \( \approx kl/\xi^2 \) (with long string density \( \approx \xi^{-2} \)). Here \( k \) is a factor of order unity, determining the geometrical cross section for a loop to hit the long string.

The slow decay of loops into gravitational radiation can also be taken into account. In analogy with cosmic string, the energy loss per unit time due to gravitational radiation can be assumed to be \( \Gamma G\mu^2 \) [8], with \( \Gamma \) a characteristic constant for the loops. Eq.11 can be integrated to obtain the energy density of the loops as [5],

\[ \rho_l = A(r) \frac{2r}{r+3} \gamma_s^3 B \frac{2\pi}{\sqrt{\gamma_s}} \int_{x_1}^{x_2} dx x^{-3(r+1)/(r+3)} I_0 \left( \frac{R_H x}{B\gamma_s} \right) \mu/R_H^2 \] (15)

where,

\[ I_0 \left( \frac{R_H x}{B\gamma_s} \right) = \int_0^{\infty} (z + \frac{x}{B}) \exp\left[-(B + \frac{2k\gamma_s}{r+3})z\right] dz \] (16)

and

\[ x_1 = 2B\gamma_s/(r+3) \] (17)

\[ x_2 = x_1 \left( 1 + \frac{r+3}{(r+1)\Gamma_{\text{eff}}} \right) \] (18)

\[ \Gamma_{\text{eff}} = \Gamma G\mu \] (19)

Energy density of radiation emitted from the loops can be calculated easily [4,5,14] and we write down the result only. At a time \( t >> t_{\text{Plank}} \).
\[
\rho_{rad} = A(r) \frac{2}{(1-r)(r+3)} \gamma_s^4 B^{\frac{3\mp 8}{3}} \Gamma_{eff} X_0 Y_0 \mu / R_H^2
\]  

(20)

where,

\[
X_0 = \int x^{\frac{4-x}{1+6}} \exp(-x) I_0[R_H x / B \gamma_s]
\]  

(21)

\[
Y_0 = \int g(x) x^{\frac{1-x}{1+6}} dx
\]  

(22)

\[
g(x) = x^7 e^{-x} / 6!
\]  

(23)

We now have all the relations needed to evaluate the energy density of the universe. The undetermined parameters of the model are, the chopping efficiency \( C, \Gamma_{eff} (= \Gamma G \mu) \) and \( k \), the geometric factor of loop reconnection function. They will be determined in the next section.

### III. RESULTS AND DISCUSSIONS

The chopping efficiency (C) of cosmic fundamental strings are unknown. Even for cosmic strings, the chopping efficiency is not determined well. Thus while the simulation study of Albrecht and Turok [14] gave \( C=0.74 \), much larger value, \( C=0.16 \) was obtained in the simulation studies of Bennett and Bouchet [13]. In our previous analysis [8], both the values were used. However, it is possible to obtain the (maximum) chopping efficiency for cosmic fundamental strings in the one scale hypothesis [5]. The essential 'one scale' assumption is that the time scale for loops production is just the scale length \( \xi \) of the infinite strings [8] (i.e. \( C \) is a constant). This has been verified to a good accuracy in the simulation studies also [14]. Then using the sum rule eq.(12) and demanding that \( C \) remain constant at all \( r \), we can find \( k \). This induces \( r \)-dependence on \( k \). We further demand that \( k \), being a geometric parameter, should be positive. The maximum value of \( C \) for which \( k \) remain positive at all \( r \) is found to be \( C=0.153 \). This is the maximum chopping efficiency for cosmic fundamental strings. Chopping efficiency greater than this lead to unacceptable, negative \( k \). We note that the maximum chopping efficiency thus obtained is very close to the value obtained by Bennet and Bouchet [13], in their simulation studies.

As mentioned earlier, the geometric cross section \( k \) now depends on the radiation fraction (\( r \)). In fig.1, we have shown the variation of \( k \) with the radiation fraction for \( C=0.153 \). \( k \) increases with the radiation fraction. The variation of \( k \) with \( r \) can be understood easily. As \( r \) increases, string density drops and geometric cross section \( k \) has to increase to keep chopping efficiency fixed.

The other parameter of the model (\( \Gamma_{eff} = \Gamma G \mu \)) is obtained from the self-consistency condition that [8,5],

\[
r(\rho_L + \rho_t + \rho_{rad}) - \rho_{rad} = 0
\]  

(24)

The condition demand that the motion of the strings and the radiation in the background space-time determines the space-time itself [4]. This aspect reflects the fundamental nature of string theory that the space-time is unified with matter.
In fig. 2, we have shown the radiation fraction \( r \) as a function of \( \Gamma_{\text{eff}} \). As \( \Gamma_{\text{eff}} \) increases, \( r \) increases, till a maximum value \( r_{\text{max}} \approx 0.8 \) is reached. With further increase of \( \Gamma_{\text{eff}} \), \( r \) decreases. We thus find that for two values of \( \Gamma_{\text{eff}} \), the universe can stay at a particular radiation fraction. One can then distinguish two regions, in region I (below \( \Gamma_{\text{eff}} = 10^{-3} \)), radiation fraction increases with \( \Gamma_{\text{eff}} \) and in region II (above \( \Gamma_{\text{eff}} = 10^{-3} \)), it decreases. If we can form the universe at \( r_{\text{max}} \approx 0.8 \), then it can go either way. Physics will be different in different region. Later we shall argue that the region I is unphysical. The finding that \( r_{\text{max}} \approx 0.8 \), is in agreement with ref. [3]. Radiation can not increase indefinitely in presence of strings.

In fig. 3, we have shown the variation of long string, loops and radiation energy density, with the radiation fraction \( (r) \). The two regions are shown separately. In both the region, energy densities of different components of the universe behave in similar fashion. They increase with \( r \). As expected the energy density of radiations picks up very fast. However, in region I, energy densities attained are much higher than in region II. Now, there is another self-consistency condition arising from the use of the flat universe model. For the flat universe, the density should be equal to the critical density,

\[
\rho = \rho_{\text{crit}} = \frac{3}{8\pi G \mu} \mu/R_H^2
\]  

(25)

This relation can be used to obtain \( \Gamma \) and \( G\mu \) separately. In fig. 4, variation of \( G\mu \) and \( \Gamma \) with the radiation fraction are shown. In both the region, dimensionless constant \( G\mu \) decreases as the radiation fraction increases. As \( G\mu \) is related to the coupling constant (see eq. [4], this indicate that in a string dominated universe, the coupling constant is epoch dependent. Before we elaborate on this result, let us note the behavior of the decay constant \( \Gamma \). The behavior of \( \Gamma \) in region I and II are completely different. While in region I, \( \Gamma \) increases exponentially by six orders of magnitude as the radiation fraction increases from 0 to 0.8, it remains approximately constant in region II. In fig. 5, the same result is shown in a different fashion. Here, we have shown the decay constant \( \Gamma \) as a function of the dimensionless constant \( G\mu \). In region I, \( \Gamma \) varies from \( 10^{-5} \) to 10 as \( G\mu \) increases from \( 10^{-4} \) to \( 10^{-2} \). In region II, for the same range of \( G\mu \), \( \Gamma \) remains approximately constant. Now \( \Gamma \) is characteristic of the loops and we expect it to be constant and independent of \( \mu \), as obtained in region II. We thus argue that the region I is unphysical and only region II is physical and need to be considered. That in region II, \( \Gamma \) remain constant gave credence to our model. Value of \( \Gamma \approx 30 \), obtained in the present calculation is also interesting. This value is similar to the value obtained in the cosmic string case for gravity wave emission [8]. As cosmic fundamental strings can decay into hundreds of massless states, two order of magnitude larger \( \Gamma \) is expected. Small value of \( \Gamma \) indicate that the zero mode emission for cosmic fundamental strings is not efficient. This may have bearing on lightest supersymmetric particle as a candidate for dark matter.

The epoch dependent gauge coupling obtained in the present calculation can have several interesting consequences. Let us first see whether the result is physical or not? We note that we do not have explicit dilaton field in the theory. In string theory, the coupling constant is determined by the expectation value of the dilaton field. Apparently, it is surprising that though the theory do not have explicit dilaton field, it gives epoch dependent gauge coupling. However, the theory do contain expectation value of the dilaton field through the
string tension $\mu$. The epoch dependent coupling constant also have a natural explanation. As the universe expands, the strings has to be stretched to keep themselves cosmologically relevant, consequently $\mu$ decreases. In quantum field theory also, the coupling constants are effectively constant only at certain energy. They are energy dependent. In an expanding universe, temperature of the universe can be considered as the relevant energy scale for the coupling constant. Then as the universe expands, radiation fraction increases, temperature of the universe decreases, resulting into the epoch dependent gauge coupling constant.

We note that, epoch dependent coupling was obtained in our earlier analysis [5] and also in the analysis of Minakata and Mashino [4]. It was not elaborated upon. Tseytlin and Vafa [17] also obtained a similar result in a recent calculation. They have considered string cosmology taking into account of dilaton field and string duality. One of the result of their paper is that the dilaton field is not a constant field but evolve. They have not elaborated on this point, but the result indicate an epoch dependent coupling constant.

There is a close similarity of our result with the Large number theory (LNT) of Dirac. In LNT also, Dirac found that a combination of $G$ and fundamental particle masses changes from epoch to epoch, requiring either $G$ or fundamental particle masses to be epoch dependent. Dirac choose to keep the particle masses fixed and allow the gravitational constant to change from epoch to epoch. Interestingly, in the present model also, in a different context, we find that $G\mu$ changes from epoch to epoch. Since we are compelled to keep $G$ fixed (Einstein theory), we have to change $\mu$, leading to epoch dependent gauge coupling constant.

In fig.6, we have shown the variation of the string coupling constant as a function of the radiation fraction for both the region I and II. The coupling constant was calculated as, [4],

$$\alpha = g^2/4\pi = 8\pi G\mu$$

(26)

Behavior of $\alpha$ in I and II are similar, only magnitude differ. In the physical region II, as the universe evolve from matter to radiation dominance, the coupling constant changes from a high value of $\sim 0.46$ to a low value of $\sim 0.005$. For comparison purpose, we have shown the three coupling constants ($\alpha_1, \alpha_2$ and $\alpha_3$) of the standard model [18]. Also shown is the currently accepted standard model unified coupling constant ($\alpha_{GUT} = \approx 1/24$ at the GUT scale. Interactions at early times were very much strong, much stronger than the present day strong interactions. As the universe expands, the interaction become weaker. The standard model unified interaction is reached at $r \approx 0.6$. It becomes further weakened as universe expands and become weaker than the (present) weak interaction at $r = r_{max} \approx 0.8$.

Let us now consider some implications of an epoch dependent gauge coupling constant. If the gauge coupling is epoch dependent, several interesting things can happen. For example, spectrum of states of strings is given by [19],

$$w(m) = w_0(m\sqrt{\alpha})^{-a} e^{b m \sqrt{\alpha}}$$

(27)

where $\alpha t$ is the Regge slope, related to the string tension $\mu$ or the dimensionless parameter $G\mu$ as,

$$\alpha t = \frac{1}{2\pi \mu} = \frac{1}{2\pi G\mu m^2_{pl}}$$

(28)
As $G\mu$ changes by two order of magnitude as the universe evolve from matter to radiation dominance, the density of states will change. However, large Plank mass will ensure that the density of states of low mass particles remain unchanged, only that of heavy particles will differ from epoch to epoch. Particles with mass $m \sim m_{pl}$ will be more copiously produced in early universe. As the heavy particles will ultimately decay into lighter ones, we can say that the density of states of all the particles will be affected, if the gauge coupling is epoch dependent. Though at present, it is not possible to calculate the fundamental particle masses in the string theory, possibility can not be ruled out that fundamental particle masses are also epoch dependent. This also raises the possibility of verifying the hypothesis of string dominated early universe. As the string universe has to decay away before QCD transition we can not hope to obtain experimental signature of string domination of the universe. And as yet there seems to be no other independent way of verifying the hypothesis. However, as the nucleosynthesis analysis concerns periods much earlier than the time of nucleosynthesis, if the fundamental particle masses and mass spectra were different at early times, its results may change. It is then possible that early string dominated universe will left its imprint on the nucleosynthesis analysis.

Though as we have argued above, it is possible that the epoch dependent gauge coupling constant is physical and the universe was string dominated, we must mention the other possibility. The present model relies on the assumption of scaling solution. However, even in cosmic strings we find that there is controversy about the scaling law. While the simulations studies of Albrecht and Turok [14] agree with the one scale hypothesis, that of Bennet and Bouchet [15] disagree. It is then possible that the assumption of scaling solution for fundamental string network is wrong. The epoch dependent gauge coupling is the manifestation of the wrong assumption and the early universe was not superstring dominated.

IV. SUMMARY

To summarise, we have studied the evolution of a string dominated early universe. The evolution was followed in a flat space-time within the one scale hypothesis. The long string can chop off loops, which in turn can decay by emitting radiation. They can also get re-connected with the long string. The reconnection was treated in a manner consistent with the one scale hypothesis. In the one scale model, chopping efficiency of long strings is a constant. This condition was utilised to obtain the maximum chopping efficiency, which is very close to the value obtained by Bennet and Bouchet [15] in a simulation study of cosmic strings. For this chopping efficiency, imposing self-consistency condition that the motion of strings determine the space-time, and that the density be equal to the critical density, we have obtained the dimensionless constant $G\mu$ for the strings and its decay characteristics. We confirmed that radiation density is limited in presence of strings, a result obtained by Turok [8]. We also found that, as the string universe evolve from matter to radiation dominance, dimension less parameter $G\mu$ varies from $10^{-2}$ to $10^{-4}$. This leads to the interesting consequence of epoch dependent gauge coupling constant. This is also in agreement with the finding of Tseytlin and Vafa [17] that dilaton field evolve. We also find close similarity of the present result with Dirac’s Large number theory. Epoch dependent gauge coupling can results into epoch dependent fundamental particle masses. Density of states will also be epoch dependent. It also raises the possibility that effect of string dominated universe will
be felt on the nucleosynthesis analysis.
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FIGURES

FIG. 1. Variation of the geometric factor k for loop reconnection with long strings, with radiation fraction (r) for chopping efficiency C=0.153.

FIG. 2. Radiation fraction (r) as a function of $\Gamma_{\text{eff}}$ as obtained from the self-consistent condition.

FIG. 3. The long string ($\rho_L$), loop ($\rho_l$) and the radiation ($\rho_{\text{rad}}$) energy density, in units of $\mu/R_H^2$, as a function of radiation fraction (r).

FIG. 4. The variation of $G\mu$ and $\Gamma$ with radiation fraction obtained from the self-consistent condition $\rho = \rho_{\text{crit}}$.

FIG. 5. The relation between $\Gamma$ and $G\mu$ obtained from the self-consistent condition $\rho = \rho_{\text{crit}}$.

FIG. 6. The string coupling constant $\alpha$ as a function of the radiation fraction (r). The straight lines are drawn to show the strength of the three standard model coupling constants at $M_z$ and their unified value at the GUT scale.
