Implementation of linear, double-linear, and nonlinear fatigue
damage accumulation rules for fatigue life prediction of
offshore drilling top-drive tie-rods

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Abstract: The offshore oil and gas industry has been exposed to major challenges over the last
decade, particularly demanding cost cuts and more effective technical solutions. Predictive
systems and remaining life assessments for both machine and structural components are known
to be one of the core areas that has gathered much attention lately. This paper focuses on
multiaxial fatigue that is a problem in number of engineering structures and equipment. The
ability to properly assess and quantify multiaxial fatigue of offshore equipment and structures
has major benefits for owners of engineering assets both in terms of technical and safety
integrity. Traditionally, Palmgren-Miner’s damage rule is used for life estimation involving
multiaxial fatigue due to its ease of use. There are however some known shortcomings with
Palmgren-Miner’s rule namely: it does not consider the loading sequences. This for instance can
result in overestimation of fatigue life in scenarios where stress amplitudes are decreasing, and
moreover underestimation of fatigue life in scenarios where stress amplitudes increase. This
research work thus involves closely studying the application of Palmgren-Miner’s linear damage
rule, Manson’s double linear damage rule, and Subramanyan’s non-linear damage rule for the
purpose of enhancing the predictability of damage as well as accuracy of fatigue life assessments.
Each of these techniques were applied for tie-rods of an offshore drilling top-drive having a
known stress history. The loading histories were provided by a drilling company. The torque and
axial force values are transformed to stress components for the corresponding critical spots. The
“rainflow” counting technique is applied to the obtained stress histories and the mean stress
effect is considered for the damage accumulation calculations. The fatigue life prediction of the
three models are justified and discussed. This research work was aimed at contributing towards
the utilization of more robust fatigue life estimation techniques such as Manson’s double linear
damage rule and simplified non-linear damage rules which capture the true nature of the fatigue
that equipment’s are subjected to, and thus to improve the reliability of the fatigue life
predictions.

1. Introduction
Condition monitoring and equipment maintenance are essential activities which must be carried out by
managers of modern engineering assets, equipment and infrastructure. These activities are important to
ensure that the structural integrity of an asset is maintained as well as to ensure an asset performs its
function and at the same time can maintain resilience when it is exposed to sudden shocks or changes.
In a nutshell, condition monitoring of critical equipment is essential for companies to stay competitive.
and meet the strict HSE regulations set by the Petroleum Safety Authority (PSA) and other regulators for operating on the Norwegian Continental Shelf (NCS). To fully harness the advantages of condition monitoring, correct models need to be used to determine the structural integrity of equipment and infrastructure that are being utilised for operations. This is so important because using wrong models and assumptions will result in scheduled maintenance or replacement being initiated for equipment which could have been used for many more years without failure or can underestimate the fatigue damage increasing the risk when using such equipment.

Structural integrity of assets is an important issue and a key cost factor in the oil and gas industry. Several time-scheduled non-destructive testing (NDT) techniques are used to inspect integrity (fatigue, wear, corrosion) of structural members (for example every 5 years), this has some significant drawbacks such as: it is time consuming, requiring often full disassembly of equipment, while few failures are found in these structures. There usually is a great need for competence outside the crew to carry out this analysis – which drive high costs and results in significant downtime for structural engineers to access these equipment’s.

It has been observed that these periodical inspections usually add more failures to the system (due to coupling errors). Due to this reason the industry has moved into transferring inspection scheduling from ‘time-based’ inspections to ‘condition-based monitoring’ of equipment usage by applying ‘physics based’ fatigue or wear mathematical condition focused (machine learning & AI) data driven models.

The challenge with such models especially physics and statistics based) are as follows:
- These involve high level of intellectual property (IP) and this is seldomly shared.
- These are often hard-coded and difficult to evolve, when data contradicts conservative model assumptions.
- It is difficult to document such model changes and foundation for improvements.
- It is difficult to get consensus on what to improve, when and by using what quality assurance process, involving who.
- Access to results require access to special systems, without direct coaction to what these data are used for.
- Relation between data/results is manually compared and translated to value, KPI or certification process.
- Physics based models require a lot of infrastructure / hardware power to be utilized (several layers of differential equations, hard coded according to system taxonomy).
- On other hand data driven approaches require equality of configurations, data quality and datasets to achieve complete validation.

This paper provides insight on the current approach of using standards (API 8C "Specification of Drilling and Production Hoisting Equipment" and F.E.M. "Rules for the Design of Hoisting Appliances") by a drilling company to assess fatigue damage and the structural integrity of their top-drive members with special focus on weakest link (i.e. the tie rods of the top-drive). The current approach being used by the drilling company to access fatigue damage will be compared to two other models (Manson’s Double Linear Damage model and Subramanyan’s Non-Linear Damage model). The F.E.M 1.001 standard is based on the use of Palmgren-Miner’s model and S-N curves when calculating fatigue damage on equipment.

Generally, Miner’s rule is widely accepted as the method to access the fatigue damage due to its simplicity and ease of use in design of equipment where loading history is not available. The drawback of using this method is that it does not consider the load sequence for example an equipment might be exposed to a higher loading cycle of let’s say 900 MPa followed by 600 MPa or vice versa but the Miner approach will still predict the same damage even if the smaller load cycle comes before the higher cycle. The result of this is that the life of a structure or equipment will be underestimated when the cycles are reversed low - high load sequence and overestimated in a high - low load sequence. For random loading sequences Miner’s rule is conservative in fatigue life predictions and this is one of the main reasons why no faults are discovered in the top drives after they are dismantled.
This paper will utilize the same approach adopted by the drilling company as prescribed by the F.E.M 1.001 standard when reviewing Palmgren Miner’s approach to fatigue damage. The results of this approach will be compared to those obtained by using Manson’s Double Linear Damage model and Subramanyan’s Non-linear damage model and conclusions will be drawn from these approaches to estimating fatigue damage.

2. Objectives of the study

This paper’s focus is centred on improving process of control and validation of structural integrity of the drilling company’s top-drive tie-rods. This will be achieved by applying Palmgren Miner’s rule, Manson’s double linear damage rule and Subramanyan’s non-linear damage rule to improve damage predictions and fatigue life assessments. The results from these tests using the selected fatigue damage models will serve as a basis for optimizing the current methodology of fatigue analysis employed at the drilling company. Fatigue calculation results from using the selected fatigue damage models will be compared. Von-Mises’ equivalent stress approach will be applied to transform torque and axial forces into an equivalent stress. This approach is utilised for a variety of multiaxial fatigue problems and it is suitable for materials that exhibit ductile behaviour as is the case in this research work. Von Mises’ equivalent stress approach was utilized to get the equivalent stresses for each of the stress combinations. This approach was adopted because the axial and torsional loads obtained from the loading history of the top drive were considered to be proportional and in the same phases.

According to [1], “In situations in which we can reasonably expect an overloaded part in service to fail in the same manner as the standard tensile test bar made of the same material, it is recommended that the maximum-distortion-energy theory be used to predict ductile yielding”. The material of construction of the top-drive is high strength steel which exhibits ductile behaviour, and this is additional justification why Von-Mises equivalent stress approach was used. It should be noted that Von-Mises equivalent stress approach is an approximation and there are more robust models such as those proposed by Fatemi and Papadopoulos [2-4], however these more robust approaches are more complicated for implementation in engineering applications. Details on Von-Mises’ equivalent stress equation can be seen in the equation (1) below.

\[
\sigma_{Eq.m} = \frac{1}{\sqrt{2}} \sqrt{[\sigma_{m1} - \sigma_{m2}]^2 + [\sigma_{m2} - \sigma_{m3}]^2 + [\sigma_{m3} - \sigma_{m1}]^2}
\]

where,

- \(\sigma_{Eq.m}\) is the equivalent nominal stresses amplitude
- \(\sigma_{m1}, \sigma_{m2}\) and \(\sigma_{m3}\) are the principal mean nominal stresses acting on the material

The selected models considered in this research work will be built and coded in Python [5] to attain a shift from physics dominated differential equation-based component models to hybrid models adapted from linear algebra and vector calculus. The aim of this approach is to reduce internal IT infrastructure capacity dependence (now in cloud or big data centres, depending again on good connectivity), speed up calculations and implement machine learning techniques to improve results. Such simplification will enable execution on edge devices or within control devices.

Focus will be placed on building these models in a way that it is user-friendly and easily adaptable to other equipment apart from the tie-rods on the top drives. This will be necessary to ensure that calculations are quick and allow for the use of machine learning techniques to further optimize the results from this model.
Figure 1. Research methodology.

Figure 2. Illustration showing the suspension system of the top drive with the tie-rods which are analysed in this research work.
3. Review of selected fatigue damage models

3.1 Palmgren-Miner’s linear damage model

Miner’s rule was popularized in 1945 by Miner [6] in his work on fatigue involving tension-tension axial fatigue data for aircraft skin material, this was a development of an earlier cumulative damage model proposed by Palmgren in 1924 [7]. Palmgren Miner’s model operates on two major assumptions: the load spectra is assumed to be fully reversed sinusoidal cycle and it is assumed that the total work absorbed by the system will result in failure occurring. This model postulates that “where there are \( k \) different stress magnitudes in a spectrum, \( S_i \) \( (1 \leq i \leq k) \), each contributing \( n_i(S_i) \) cycles, then if \( N_i(S_i) \) is the number of cycles to failure of a constant stress reversal \( S_i \) (determined by uniaxial fatigue tests), failure occurs when damage(D) i.e. the ratio of the applied cycles to the number of cycles to failure is equal to 1” [6]. This can be further explained with the equations below.

\[
D = \sum_{i=1}^{k} \frac{n_i}{N_i} \\
D = \left(1 - \sum_{i=1}^{k} \frac{n_i}{N_i}\right)
\]

Where:
- \( n_i \) is the number of cycles accumulated at stress \( S_i \).
- \( N_i \) is the number of cycles a material can take until failure at the given stress
- \( D \) is the fraction of life consumed by exposure to the cycles at the different stress levels.

Palmgren-Miner’s linear damage model is the most popular model used in various industries for analysing fatigue life and expressing the damage that a material is being subjected to, this is due to the simplicity in utilizing this model. Also, most of the stress-life curves available in standards such as DNV-RP-C203 were developed from experimental data based on Miners rule.

To overcome Miner’s rule shortcomings various probabilistic methods have been used to counteract the load sequence effects thus resulting in acceptable predictions for fatigue life under random loading [8-11]

3.2 Manson’s double linear damage model

This model was developed by Manson in 1967, where he considered fatigue to be occurring in two major stages namely: crack initiation and crack propagation. The main assumption in this initial postulate by Manson, was that the crack initiation period \( N_p \) could be used to express the total life of a material by using (4) and (5). Various revisions have been made to this model until 1981 where Manson abandoned the use of the terminologies of crack initiation and crack propagation, rather he chose to call this Phase I and Phase II, he also presented equations to implement the double linear damage rule. This was done to achieve simplicity in the application of this principle in comparison to the damage curve approach that he had suggested in his previous publications [12, 13]. Manson’s double linear damage rule can be regarded as Miner’s rule applied to two phases of fatigue damage. According to Manson, [14] when block loading exceeds two levels the following equations can be used to compute damage in the two phases.

Total fatigue life is expressed as:

\[
N_f = N_I + N_{II}
\]

The relationship between Phase I damage and total fatigue life is then expressed as

\[
N_I = N_f \exp (ZN_I\Phi)
\]

\( Z \) and \( \Phi \) are constants and can be determined from the knee points of the curve for Phase I damage. The knee points are the same for all materials and can be determined from the maximum and minimum lives present in the loading cycle.
\[ N_{I, N_1, f} = N_{1,f} \left( \frac{n_1}{N_1} \right)_{\text{knee}} = 0.35N_{1,f} \left( \frac{n_1}{N_2} \right)_{\text{knee}} \]  
\[ N_{I, N_2, f} = N_{2,f} \left( 1 - \frac{n_2}{N_2} \right)_{\text{knee}} = N_{2,f} \left( 1 - 0.65 \left( \frac{n_1}{N_2} \right)_{\text{knee}} \right) \]  

By substituting equations (6) and (7) into equations (4) and (5) a solution can be obtained for the constants \( Z \) and \( \Phi \) as shown by the equations below.

\[
\Phi = \frac{1}{\ln \left( \frac{N_{1,f}}{N_{2,f}} \right)} \ln \left[ \ln \left( \frac{0.35 \left( \frac{N_{1,f}}{N_{2,f}} \right)_{\text{knee}}^{0.25}}{1 - 0.65 \left( \frac{n_1}{N_2} \right)_{\text{knee}}^{0.25}} \right) \right] \]  
\[
Z = \frac{\ln \left( 0.35 \left( \frac{N_{1,f}}{N_{2,f}} \right)_{\text{knee}}^{0.25} \right)}{N_{1,f}} \]  

Therefore, \( N_{II} \) can be expressed as:

\[
N_{II} = N_f - N_I = N_f \left( 1 - \exp \left( ZN_f^\Phi \right) \right) \]  

Where:

- \( N_f \) is the total number of cycles to failure
- \( N_I \) is the number of cycles to failure in phase one
- \( N_{II} \) is the number of cycles to failure in phase two

Manson’s double linear damage model has been shown to conform with experimental results as demonstrated by the research work carried out by Manson et al for NASA [14] and other publications from notable researchers within the fatigue subject area [15, 16]. By utilizing two linear damage phases for fatigue, the ease in the use of Miner’s rule can be carried over into this model, like it was stated earlier the double linear damage rule is similar to applying Miner’s rule in two phases of damage. It eliminates the deficiencies that are present in Miner’s rule by having the co-ordinates for the knee-point of the S-N-curve and due to its linear nature, application of this model for designing components or analysing fatigue is easily done.

The drawbacks when utilizing this model is that the knee-point of the S-N curve has to be determined to properly implement this model, also Manson’s model does not take into account the retardation mechanisms for crack growth [17, 18] and mixed mode cracks [19-21].

### 3.3 Subramanyan’s non-linear damage model

This non-linear model was developed by S. Subramanyan in 1976, in this model the concept of iso-damage lines that converge at the knee-point of an S-N curve is utilized when analysing fatigue damage. This postulate of iso-damage lines is where Subramanyan’s model deviates from Miner’s rule, because in Miner’s rule it is assumed that a constant damage line lies on an S-N curve and this constant damage line is parallel to the S-N curve for all the stress and number of cycles to failure combinations [22]. This cumulative damage model operates under the assumption of 100% damage existing on the S-N curve of a material. When stress and equivalent number of cycles to failure below the endurance limit of a material are read off an S-N curve their combinations will result in no damage (0% damage). The interval between the S-N curve (100 % damage) and stress and cycle combinations below the endurance limit (0% damage) will have a set of straight iso-damage lines which will converge at the knee-point of the S-N curve. Revisions were made to this model in 1978 to account for the reduction in the endurance limit of a material at various stress levels [23].
According to Subramanyan’s model, the condition for fatigue failure is:

\[ C_i + \{ C_{i-1} + [C_{i-2} + \cdots + \left( C_2 + C_1 \alpha_1 \right)^{\alpha_2} \cdots ]^{\alpha_{i-2}} \}^{\alpha_{i-1}} = 1 \quad (11) \]

The cycle ratio of this stress loading combinations is the ratio between the loading cycles and the number of cycles to failure which is obtained from the S-N curve.

\[ C_i = \frac{n_i}{N_i} \quad (12) \]

\[ \alpha_i = \frac{\log N_e - \log N_{i+1}}{\log N_e - \log N_i} \quad (13) \]

Where:
- \( n_i \) is the number of counted load cycles at the given stress level \( n \)
- \( N_i \) is the number of cycles to failure at a given stress level \( n \) on the S-N curve
- \( C_i \) is the cycle ratio at a given stress level \( i \)
- \( N_e \) is the number of cycles at the knee point of the S-N curve

Subramanyan’s model has shown slightly non-conservativeness when compared to actual experimental results for SAE 4130 [16] which makes its usage acceptable. However, application of this model to analyse fatigue must be carried out with caution. According to Fatemi and Yang [24] Subramanyan’s model is not applicable in cases where the stress amplitudes are near the endurance limit of the material, because S-N curves exhibit non-linearity close to their knee-point and because there is singularity at the knee point since all iso-damage lines will converge at this point.

4. Results and discussion

This section contains the results obtained from using Miner’s linear damage rule, Manson’s double linear damage rule and Subramanyan’s non-linear damage rule. The comparison of results obtained from the various fatigue damage models will focus on two (2) key areas, damage results, and number of loading blocks until failure results. Here, emphasis will be placed on how the fatigue damage models compare to each other on the Main well and Auxiliary well. The key difference between these two is that the top-drive for the Main well was used for more operations than the top-drive on the Auxiliary well.

4.1 Comparison of Fatigue Damage Results

In Figure 3(a) the fatigue damage results from top drive on the Main well of Rig 1 shows slight increases in damage from Manson’s DLDR and Subramanyan’s NLDR. Damage increases by 50 percent from 0.065 to 0.13 in Manson’s DLDR model, while in Subramanyan’s model there is an 8 percent increase in damage from 0.096 to 0.104. In Miner’s linear damage rule there is a 60 percent increase in damage from 3.81E-6 to 9.08E-6.

From these damage results it can be observed that Subramanyan’s non-linear model and Manson’s DLDR are quite close in their estimates for accumulated damage. Subramanyan’s model gives the most damage accumulation of 0.2 which is slightly higher than damage accumulation in Manson’s DLDR model (0.195). Meanwhile, the accumulated damage from Miner’s rule (1.289E-5) is quite low in comparison to Manson and Subramanyan’s models. This has significant implications on the operational life of this top-drive, because according to the damage accumulation obtained from Miner’s rule this equipment can go on in operation without experiencing fatigue failure for a much longer time in comparison to the damage accumulation results from Manson’s double linear damage rule and Subramanyan’s non-linear damage which predict a shorter operational life for the top-drive. These deviations in the damage accumulation results from Subramanyan’s model, Manson’s and Miner’s model are in agreement with variable amplitude tests that have been carried out by [8-11] that show
Miner’s rule overestimates fatigue life when there is a decrease in applied stresses in a high-low loading sequence.

**Figure 3.** Estimates of damage versus the number of remaining cycles until failure from the selected fatigue damage models for Rig 1 Main Well (a) and Auxiliary Well (b) in 2014.

From the fatigue damage results shown in figure 3(b) for the top drive on the Auxiliary well on Rig 1, it can be observed that the accumulated damage results from Manson’s DLDR and Subramanyan’s model are more conservative than the accumulated damage predicted by Miner’s rule. Here, the accumulated damage from Subramanyan’s model is 0.208 while that from Manson’s DLDR is 0.47. Meanwhile, the accumulated damage results obtained from using Miner’s rule is 9.71E-6 which is significantly lower than the accumulated damage results obtained from Manson’s DLDR or Subramanyan’s model, this continues in the trend of overestimation of the remaining fatigue life by Miner’s rule that has been seen in the previous results.

In summary, the damage results obtained from the rigs for the year 2014 shows close correlations between the damage results obtained using Manson’s double linear damage rule and Subramanyan’s non-linear damage rule with Subramanyan’s model being slightly less conservative than Manson’s
model. In the results obtained, Miner’s rule consistently overestimated the fatigue life for the analysed top-drive members in comparison to the fatigue life estimates gotten from Manson’s and Subramanyan’s models.

Given that these three models consider different mechanisms and assumptions when measuring damage this brings into question how much information can be derived from their damage estimates. It is because of this that the number of loading blocks until failure was computed using each model. It is believed that this will be more informative when comparing the results from these three fatigue damage models. This comparison using the number of loading blocks until failure will be shown the next section of this discussion.

4.2 Comparison of the number of loading blocks until failure results for Miner’s linear damage rule and Manson’s double linear damage rule models

In this comparison the analysed top-drive members are subjected to a stress loading history which results in damage when these applied stresses are above the endurance limit of the top-drive. This loading history is assumed to represent one (1) loading block of applied stresses on the top drive, from this analogy the number of loading blocks until failure is computed as the number of applied stress loading blocks that will result in a damage value of unity (1).

For Miner’s rule and Manson’s DLDR, the number of loading blocks until failure was computed as the inverse of the total damage accumulation, where in Manson’s model this was computed for the two damage phases considered in this model. While in Subramanyan’s model the stress block iterations were done until damage was equal to unity (1), the stress block where damage was equal to (1) was taken as the predicted number of loading blocks until failure.

![Graph showing number of loading blocks until failure](image)

**Figure 4.** Estimates of number of loading blocks until failure from the selected fatigue damage models for Rig 1 Main Well (a) and Auxiliary Well (b).
Table 1. Estimates of number of loading blocks until failure from the selected fatigue damage models for Rig 1 Main Well (a) and Auxiliary Well (b).

(a)

| Rig   | Loading Blocks to Failure | 2014     | 2015     | 2016     |
|-------|---------------------------|----------|----------|----------|
| Rig 1 | Miners Rule               | 77563.37101 | No Damage | 37562.5037 |
|       | Manson's DLD R            | 77563.34530 | No Damage | 37558.85643 |
|       | Subramanyan's NLD R       | 155080   | No Damage | 86848    |
|       | Miners Rule               | No Damage | No Damage | No Damage |
| Rig 2 | Manson's DLD R            | No Damage | No Damage | No Damage |
|       | Subramanyan's NLD R       | No Damage | No Damage | No Damage |
|       | Miners Rule               | No Damage | No Damage | No Damage |
| Rig 3 | Manson's DLD R            | No Damage | No Damage | No Damage |
|       | Subramanyan's NLD R       | No Damage | No Damage | No Damage |

(b)

The results shown in figure 4 (a) covers the predicted number of loading blocks until failure for the Main Well on the analysed rigs. It can be observed that Miner’s rule and Manson’s DLD R show close similar predictions for the number of loading blocks until failure. This can also be seen in table 1(a), there is very little between Miner’s rule and Manson’s DLD R. Meanwhile, Subramanyan’s model shows big deviations in its predictions of number of loading blocks until failure in comparison to Miner’s rule and Manson’s DLD R. These results show a trend of Miner’s rule and Manson’s double linear damage rule having more conservative estimates for the number of loading blocks of the applied stresses that the top drive can take before it experiences fatigue failure, Subramanyan’s non-linear damage rule shows less conservative estimates. The implication of this is that the top drive will have to be recertified or replaced much earlier if the estimates from Miner’s rule and Manson’s DLD R are used in comparison to Subramanyan’s model.

The results for the Auxiliary well on the analysed rigs can be seen in figure 4 (b) and table 1(b) below. The same trend observed in the Main Well continued here except in 2016 where the predictions for the number of loading blocks until failure estimates from Miner’s rule and Manson’s double linear damage rule and Subramanyan’s non-linear damage rule are fairly equivalent. This can be due to the very low stress loading for the top drive on the Auxiliary well and its infrequency of use, from the rainflow count there was just one counted cycle (1) of a damaging stress that was applied to this top drive.

In summary, the results from the non-conservative estimates for the three (3) Rigs show that Subramanyan’s non-linear damage rule is less conservative in its assessment of the number of loading blocks until failure in comparison to Miner’s rule and Manson’s double linear damage rule which are more conservative in their estimates. This observation is in agreement with experimental results from tests carried out Lee et al on SAE 4130 [16] where it was shown that Subramanyan’s model was less conservative in its predictions of number of loading blocks until failure.
5. Conclusions
The damage accumulation results obtained from the selected models show that Miner’s rule was consistently less conservative in its prediction for the accumulated damage in comparison to Manson’s double linear damage rule and Subramanyan’s non-linear damage rule. The selected double-linear and non-linear damage accumulation rules showed close correlations in their predictions of the amount of damage that had been accumulated by the tie-rods on the drilling top-drives which was in most cases more than the damage accumulation that was predicted by Palmgren Miner’s rule. It can be concluded from these results that Palmgren Miner’s rule underestimates the amount of accumulated damage which will result in an overestimation of the number of cycles remaining before the component experiences a failure. This finding is consistent with the research carried out by [15, 24-27] where it was proven that Palmgren Miner’s rule predicts longer life for components when the stress amplitude is decreasing. Manson’s and Subramanyan's damage accumulation rules should be utilised to provide more reliable damage accumulation estimates.

It can be concluded that the results from this project will serve as an enabler for the utilization of more comprehensive fatigue life estimation techniques such as Manson’s double linear damage rule and simplified non-linear damage rules which capture the true nature of the fatigue that equipment’s are subjected to, and thus to improve the reliability of the fatigue life predictions.

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