Numerical Investigation for the Laminar Flow Effects over Rough Surface

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Abstract. In this paper, the heat transfer effect of rough surface in laminar flow is introduced by fully developed fields for solving the Navier–Stokes equations of incompressible flow in assuming two-dimension using the direction splitting method. Firstly, the algorithm of the incompressible Navier–Stokes equations with pressure correct is carried out. Secondly, the effects of pressure drop and heat transfer and are investigated and discussed in different rough surface elements which are configured with triangular and rectangular element. The Reynolds number, roughness element spacing, and roughness height are also considered as the factors which affect the heat transfer. The results indicate that the parallel present method reaches the stage of basically stable state rapidly and accurately. Compared with the smooth surface, the global performance of heat transfer is improved by the roughness surface since the pressure drop is lost. The effects of triangular element roughness surface on laminar flow and heat transfer are much stronger than rectangular element roughness surface when the spacing of the rough is changed. The flow over the triangular element roughness surface becomes stronger, which contributes to enhancement heat transfer meanwhile increase the pressure drop when the roughness height is higher.

Keywords: Heat Transfer, Roughness Surface, Direction Splitting

1. Introduction

Heat transfer in roughness surface is more and more important due to its many enhanced practical applications, such as industry and heat dissipation of electronic component.

Many researcher have researched enhance heat transfer by rough surface. In1980s, Tuckerman and Pease [1] worked on rough surface firstly. Sultan [2] investigates that the perforated holes can induce the heat transfer enhancement by experiment. It is shown that the average the coefficient of heat transfer increased to 33.15% by using different area ratio of the opening hole in experiment. Qu et al. [3] experimentally researched the water flow in trapezoidal channels. Kandlikar et al. [4] researched
the effect on enhanced heat transfer factors in low Re in small diameter tube by different surface roughness. Jubran et al. [5] investigate that the convective of heat transfer and pressure drop in square and rectangular surfaces. They found that rectangular modules can enhance heat transfer more than the square surfaces. Garimella [6] studied the heat transfer effect of vortex generators with an array discrete heat tubes by experiment results. The results show that the vortex generator can heavily enhance the heat transfer more than 40%.

In numerical simulation, Fu and Tong [7] found that heated blocks can enhanced strongly heat transfer with the frequency of the cylinder. Yang [8] found that the position of vortices are around the roughness surface actively and migrated downstream gradually by CFD simulation. KO [9] found that the effect of pressure drop and heat transfer in an uniformly heated rectangular surface with porous wall-mounted baffles. Compared to smooth heated channel, it shows that the devices can increase to 300%. Wang [10] studied the friction characters of different roughness elements in single phase channel by CFD simulation. Yapici [11] and Bilen et al. [12] studied the heat transfer of roughness surface which fitted with rectangular blocks by different angle. It shows that the angle and the Reynolds number and are the most efficient parameters in all factors. Croce and Agaro [13, 14] studied the effect of the pressure drop and heat transfer in roughness surface element by finite element CFD simulation. It shows that the spacing of roughness elements which is considered for enhanced the heat transfer.

As above knowledge, how to solve Navier–Stokes equations economically and accurately is popular. Guermond and Minev [15] studied a method to solve the incompressible Navier–Stokes equations by using direction-splitting-based fractional time stepping. The target of the present work is to research this method of the flow field and heat transfer effect on rough surfaces, and confirm the effects on heat transfer, pressure drop, and flow field by different geometric configurations and flow parameters using this method.

2. Heat Transfer of Rough Surface

2.1 Mathematical Model

![Figure 1. Schematic of rough surface](image)

The continuum, momentum, and energy equation for transfer in laminar flow is:

- Continuity equation: \( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \)  

- Momentum equation: \( \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \)  

(1)  

(2)
\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) 
\]
(3)

\[
\frac{\partial \Theta}{\partial t} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{1}{Pr \cdot Re} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right) 
\]
(4)

The non-dimensional variables of equation are defined as follow.

\[
L_{ref} = H, X = \frac{x}{H}, Y = \frac{y}{H}, t_{ref} = \frac{H}{U_0}, Pr = \frac{v}{a}
\]

\[
U_r = U_0/U, U = \frac{u}{U_r}, V = \frac{v}{U_r}, P = \frac{p}{\rho U_r^2}
\]

\[
T_r = \min(T_{in}, T_{out}), \Theta = \frac{T-T_0}{\Delta T}, Re = \frac{U_0 H}{v}
\]
(5)

where, \( L_{ref} \) is the characteristic length, \( t_{ref} \) is the characteristic time, \( u \) and \( v \) are the components of the velocity vector along the \( x \), \( y \) directions respectively, \( p \) is the fluid pressure, \( \rho \) is the fluid viscosity, \( a \) is the thermal diffusivity, \( T \) is the fluid temperature, \( U_0 \) is the mean velocity of inlet flow, \( T_{in} \) is the temperature of inlet flow, \( T_{out} \) is the temperature of outlet flow.

The fluid of the inlet condition is fully developed with a parabolic profile \( U_0(Y) = 6Y (1-Y) \) and the boundary conditions are:

\[
X = 0, 0 \leq Y \leq A, U = 0, V = 0, \frac{\partial \Theta}{\partial X} = 0 \\
X = 0, A \leq Y \leq H, U = 1, V = 0, \Theta = 0 
\]
(6)

Where \( U=1 \) means the maximum velocity in the channel model. Meanwhile, the outlet channel can be defined as fully developed flow, and the boundary condition of outlet can be defined as:

\[
X = L, 0 \leq Y \leq A, U = 0, V = 0, \frac{\partial \Theta}{\partial X} = 0 \\
X = L, A \leq Y \leq H, \frac{\partial U}{\partial X} = 0, \frac{\partial V}{\partial Y} = 0, \frac{\partial \Theta}{\partial X} = 0 
\]
(7)

The boundary conditions of wall and slipping condition can be defined as:

\[
Y = 0, U = 0, V = 0, \Theta = 1
\]
(8)

\[
Y = H, \frac{\partial U}{\partial Y} = 0, V = 0, \frac{\partial \Theta}{\partial Y} = 0 
\]
(9)

Where \( \Theta = 1 \) means that \( \Theta \) is the maximum temperature near the wall in channel. \( T_e \) is the minimum temperature between inlet and wall. All the dimensionless parameters of heat transfer and flow field are characterized based on the height of channel, and velocity of inlet.

The dimensionless parameters of Reynolds number should characterize as the channel height in the flow field and transfer. The transfer of internal laminar flow characteristics are characterized as the Nusselt number. The local Nu number of the roughness surface where the wall located can be characterized by local temperature gradient as:

\[
Nu_e = \frac{1}{\Theta} \left. \frac{\partial \Theta}{\partial n} \right|_{surface}
\]
(10)
Mean face Nusselt number is:

\[ \overline{Nu} = \frac{1}{S_f} \int_0^1 Nu \, dx \]  

Where \( S_f \) is the face area, the Poiseuille number of laminar internal flow is characterized as:

\[ Po = f \overline{Re} = \frac{1}{S_f} \int_0^1 f(x) \overline{Re}(x) \, dx \]  

Where \( \overline{Re} \) is the Reynold number, \( f \) is the friction factor of the wall, and \( S_f \) is the face area.

2.2 Numerical Method

The equations are solved by the finite difference. The second-order accuracy time discretization scheme is:

\[ \frac{\partial f^{n+1}}{\partial t} = \frac{3 f^{n+1} - 4 f^n + f^{n-1}}{2\Delta t} + O(\Delta t^2) \]  

\[ f^{n+1} = 2 f^n - f^{n-1} + O(\Delta t^2) \]  

So all the equations can be defined that:

\[ \frac{3 f^{n+1} - 4 f^n + f^{n-1}}{2\Delta t} + 2(V \cdot \nabla f)^n - (V \cdot \nabla f)^{n-1} = \nabla^2 f^{n+1} \]  

Which can be written as Helmholtz:

\[ (C_f \nabla^2 - \lambda) f^{n+1} = S_f + O(\Delta t^2) \]  

where \( \lambda = \frac{3}{2\Delta t} S_f = \frac{-4 f^n + f^{n-1}}{2\Delta t} + 2(V \cdot \nabla f)^n - (V \cdot \nabla f)^{n-1} \)

(1) \( f \) is \( U \), \( C_f = \overline{Re}^{-1} \), \( S_f = S_U \)

(2) \( f \) is \( V \), \( C_f = \overline{Re}^{-1} \), \( S_f = S_V \)

(3) \( f \) is \( \Theta \), \( C_f = \overline{Re} \cdot \overline{Pr}^{-1} \), \( S_f = S_\Theta \)

(16) can be transformed to

\[ (I - \frac{C_f}{\lambda} \nabla^2) f^{n+1} = -\frac{S_f}{\lambda} + O(\Delta t^3) \]  

so, we define \( f^{n+1} = f^n + O(\Delta t) \) and (17) can transform to

\[ (I - \frac{C_f}{\lambda} \frac{\partial^2}{\partial x^2})(I - \frac{C_f}{\lambda} \frac{\partial^2}{\partial y^2}) \delta f = \frac{S_f}{\lambda} + O(\Delta t^3) \]
where \( \tilde{S}_f = -\frac{S_f}{\lambda} - (I - \frac{C_f}{\lambda} \nabla^2) f^n \), \( \delta f = f^{n+1} - f^n \), \( f=U,V,\Theta \).

According to Guermond and Minev, the pressure correction can be defined that:
\[
\Delta \phi^{n+1} = -\frac{1}{\Delta t} \nabla U^{n+1},
\]
where \( \phi \) is \( P^{n+1} - P^n \).

3. Results and Discussion

The grids uniform is close to the surface highly refined for capture the high-gradient velocity, temperature, and pressure near the boundary, as shown in Fig. 2. In order to make sure the grid independence of the simulation results, four tests grids are carried out which is different grids number. The grids of about 400*300 mesh cells are considered to be reliable enough to assure mesh independence.

![Figure 2. The grid distribution in the channel](image)

Table 1 shows that the economy of parallel version between SIMPLE and present method in different grids when Re is same. It can clearly indicate that parallel present method is entered the stage of basically stable state rapidly. In grids of A, B and C, two methods are calculated, the operation speed of present method faster than SIMPLE because the parallel version can use CPU more reasonably. In grid of D, only new method can be run by given computer. So it can conclude that the present method is used in parallel economically when the space is regular.

**Table 1.** Comparison of accuracy of different schemes (Re=1000)

| grids   | method | Time step | Time in steady |
|---------|--------|-----------|----------------|
| A:200*125 | SIMPLE  | 5e10-5    | 5.6sec         |
|         | Present | 5e10-5    | 8.9sec         |
| B:400*300 | SIMPLE  | 2.5e10-5  | 12.6sec        |
|         | Present | 2.5e10-5  | 18.6sec        |
| C:800*600 | SIMPLE  | 1.25e10-5 | 32.9 sec       |
|         | Present | 1.25e10-5 | 66.9sec        |
| D:1600*1200 | SIMPLE | 0.75e10-5 | N/A            |
|         | Present | 0.75e10-5 | 236sec         |

The pressure drop including smooth, rectangular, and triangular roughness elements along these local sections are given in Fig. 3. The pressure drop effect of triangular roughness elements along the channel is higher than the rectangular surface. This is the reason that the boundary layer that near the wall can regenerate and enhance the heat transfer, meanwhile, the separation and recirculation of flow is the main reason which can increase the pressure drop. The boundary layer of roughness elements can regenerate and accompany with the flow separation generally.
Figure 3. Local pressure drops of the local sections (Re = 1500, s=0.032, h=0.04).

Fig.4 shows that the change of average Nusselt number and Poiseuille number. It can conclude that the vortex effects of triangular roughness surface elements at lower Re number are much smaller than that of larger Re number.

4. Conclusions
In this study, a new method with direction splitting based fractional time stepping which solves Navier–Stokes equations in different surfaces configured with smooth, rectangular, and triangular roughness elements is carried out. The conclusions are summarized as follows:

(1) The new method can be used in solving Navier–Stokes equations on rough surface in laminar flow more economically.

(2) The heat transfer of roughness surface is improved compared with the smooth surface when the pressure head is loss. Compare with the rectangular surface element, the pressure drop and local Nu number of triangular surface element is higher.

(3) When the Re number increases, the average Po number and Nu number of rough surface are larger. The average Po number and Nu number of triangular roughness elements increase linearly with Re number.

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