Counterterms for the Dirichlet Prescription of the AdS/CFT Correspondence

W. Mück* and K. S. Viswanathan†
Department of Physics, Simon Fraser University, Burnaby, B.C., V5A 1S6 Canada

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Abstract

We illustrate the Dirichlet prescription of the AdS/CFT correspondence using the example of a massive scalar field and argue that it is the only entirely consistent regularization procedure known so far. Using the Dirichlet prescription, we then calculate the divergent terms for gravity in the cases $d = 2, 4, 6$, which give rise to the Weyl anomaly in the boundary conformal field theory.

*E-mail: wmueck@sfu.ca
†E-mail: kviswana@sfu.ca
1 Introduction

It has been stated in most papers on this subject that the correspondence between a field
theory on anti-de Sitter space (AdS) and a conformal field theory (CFT) on its horizon is
formally described by the formula \[1, 2\]
\[
\int \mathcal{D}\phi \, e^{-I_{\text{AdS}}[\phi]} = \left\langle \exp \int d^d x \, \phi_0(x) \mathcal{O}(x) \right\rangle,
\] (1)
where the functional integral on the left hand side is over all fields \(\phi\), which asymptotically
approach \(\phi_0\) on the AdS horizon. On the right hand side, \(\phi_0\) couples as a current to some
boundary conformal field \(\mathcal{O}\). In the classical approximation the left hand side is identical
to \(\exp(-I[\phi_0])\), where \(I[\phi_0]\) is the on-shell action evaluated as a functional of the boundary
value. Thus, the formula (1) enables one to calculate correlation functions of the field
\(\mathcal{O}\) in the boundary conformal field theory. This rather formal identific
ation of partition functions
needs refinement due to the fact that \(I[\phi_0]\) is divergent as a result of the divergence of the
AdS metric on the horizon. Let us choose the conventional representation of anti-de Sitter
space, namely the upper half space \(x^0 > 0, x \in \mathbb{R}^d\) with the metric
\[
ds^2 = \frac{1}{(x^0)^2} \left[ (dx^0)^2 + (d\mathbf{x})^2 \right].\] (2)

The horizon is given by \(x^0 = 0\), but in order to regularize the action one considers the space
restricted to \(x^0 > \epsilon\). The regularized on-shell action will be a function of \(\epsilon\). Moreover, the
terms which diverge in the limit \(\epsilon \to 0\) can be isolated and cancelled with counterterms.
The remaining finite result is identified with the right hand side of eqn. (1). There is a
subtlety concerning the proper choice of boundary values, but consistency forces us to use
the boundary values at \(x_0 = \epsilon\) (We call this the Dirichlet prescription). This subtlety and
its resolution is illustrated for the example of the massive scalar field in section 2.

The Dirichlet prescription of the AdS/CFT correspondence has been used to successfully
calculate the two-point functions of scalar fields \(1, 3, 4\), spinors \(3\), vector fields \(2\), Rarita
Schwinger fields \(2\) and gravitons \(7\). It must be noted that the subtlety mentioned above
affects neither the finite terms in the two-point functions for massless scalar and vector fields,
gravitons, spinors and Rarita Schwinger fields \(2, 3, 4, 10, 11\), nor higher point correlators
(cf. \[2\] and references therein).

More recently, attention has been brought to the divergent contributions, which have to
be cancelled by counterterms \(3, 4, 5, 6, 7, 8, 9, 20, 21\). Of particular importance are
terms, which are logarithmically divergent, since those counterterms are not invariant under
conformal or Weyl scaling transformations. Hence, the presence of a logarithmic divergence
leads to a conformal or Weyl anomaly in the finite part of the action. The Weyl anomaly
has recently been calculated for the cases \(d = 2, 4, 6\ \[3, 4\]. However, the authors of these
papers used a regularization, which does not consistently address the subtlety mentioned
above. Therefore, we present in section 3 the calculation of the divergent terms for free
gravity using the Dirichlet prescription. Our results for the terms relevant to the Weyl
anomaly in \(d = 2, 4, 6\) agree with those of \(3, 4\), but we regard this as a coincidence
particular to gravitons. Finally, we urge the reader to consult the appendix for our notation
and for a review of the time slicing formalism, which is used in section 3.
2 The Regularization Procedure

We illustrate the regularization procedure with the example of the free massive scalar field, whose action is given by

\[ I = \frac{1}{2} \int d^{d+1}x \sqrt{g} \left( D_\mu \phi D^\mu \phi + m^2 \phi^2 \right), \]  

(3)

and whose equation of motion with the metric (2) is

\[ [x_0^2 \partial_\mu \partial_\mu - x_0 (d-1) \partial_0 - m^2] \phi = 0. \]  

(4)

The solution of eqn. (4), which does not diverge for \( x_0 \to \infty \) is given in terms of the mode

\[ x_0^\frac{d}{2} e^{-i k \cdot x} K_\alpha(k x_0), \quad \text{where} \quad \alpha = \sqrt{\frac{d^2}{4} + m^2} \]

and \( K_\alpha \) is a modified Bessel function. Let us isolate the leading behaviour for small \( x_0 \) by defining

\[ \hat{\phi}(x) = x_0^{\frac{d}{2} - \alpha} \hat{\phi}(x). \]  

(5)

Then, \( \hat{\phi} \) has a finite limit as \( x_0 \) goes to zero. However, one must take care to express the regularized on-shell action in terms of the boundary value at \( x_0 = \epsilon \). This is easiest done by using

\[ \hat{\phi}(x) = \left( \frac{x_0}{\epsilon} \right)^{\alpha} \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot x} \frac{K_\alpha(k x_0)}{K_\alpha(k \epsilon)} \phi_\epsilon(k), \]  

(6)

which satisfies \( \hat{\phi}(x, \epsilon) = \phi_\epsilon(x) \). Consider the regularized on-shell action, which is

\[ I(\epsilon) = -\frac{1}{2} \int d^d x \epsilon^{-2\alpha} \left[ \left( \frac{d}{2} - \alpha \right) \phi_\epsilon^2 + \epsilon \phi_\epsilon \partial_0 \hat{\phi}_\epsilon \right] \]  

(7)

The first term on the right hand side is divergent and must be cancelled with a counterterm. The second term might contain other divergent terms, but also gives rise to the finite term

\[ I_{\text{fin}} = -\alpha c_\alpha \int d^d x d^d y \frac{\phi_\epsilon(x) \phi_\epsilon(y)}{|x - y|^{d+2\alpha}}. \]  

(8)

where \( c_\alpha = \Gamma(d/2 + \alpha)/[\pi^\frac{d}{2} \Gamma(\alpha)] \).

On the other hand, there appears to be a slightly different, and in our view not entirely consistent, prescription. Essentially, it expresses \( \hat{\phi} \) in terms of the boundary value \( \phi_0 \) at \( x_0 = 0 \), which can be done by writing

\[ \hat{\phi}(x) = 2^{1-\alpha} \frac{\Gamma(\alpha)}{\Gamma(\alpha)} \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot x} K_\alpha(k x_0) \phi_0(k). \]  

(9)
For small $x_0$ this can be expanded as
\[
\hat{\phi}(x) = \phi_0(x) + x_0^{2\alpha} c_\alpha \int d^d y \frac{\phi_0(y)}{|x-y|^{d+2\alpha}} + \mathcal{O}\left(x_0^{2n}, x_0^{2(\alpha+n)}\right). \tag{10}
\]
Substituting eqn. (10) into eqn. (7) one obtains
\[
I = -\frac{1}{2} \int d^d x \, \epsilon^{2\alpha} \left(\frac{d}{2} - \alpha\right) \phi_0^2 - \frac{d}{2} c_\alpha \int d^d x d^d y \frac{\phi_0(x)\phi_0(y)}{|x-y|^{d+2\alpha}} + \mathcal{O}\left(\epsilon^{2(\alpha-n)}, \epsilon^{2n}\right). \tag{11}
\]
Obviously, the finite term in eqn. (11) does not agree with eqn. (8), except for $d = 2\alpha$, i.e. for $m = 0$. The reason for the discrepancy is that the first term on the right hand side of eqn. (7), which is purely divergent in the Dirichlet prescription, contributes to the finite term, if eqn. (9) is used. Ignoring this spurious contribution (by including it into the counterterm), the finite terms coincide. Thus, one must accept that counterterms are to be expressed in terms of $\phi_\epsilon$, not $\phi_0$, which is the essence of the Dirichlet prescription.

3 Divergent Terms for Gravity

3.1 General Formalism

The gravity action is given by
\[
I = -\int d^{d+1} x \sqrt{\tilde{g}} \left[ \tilde{R} + \frac{d(d-1)}{l^2} \right] + 2 \int d^d x \sqrt{g} \left[ H + \frac{d-1}{l} \right], \tag{12}
\]
where the cosmological constant has been set equal to $2\Lambda = -d(d-1)/l^2$. The last term in the boundary integral can be considered as the first counterterm. As for our calculation of the finite part of the action we use the time slicing formalism, which is summarized in the appendix. Let us choose $\rho = X^0$ as time coordinate and use the gauge
\[
n = \frac{l}{2\rho}, \quad n^i = 0. \tag{13}
\]
After isolating the leading behaviour of $g_{ij}$ for small $\rho$ (which can be found from the equation of motion) by defining
\[
g_{ij} = \frac{1}{\rho} \hat{g}_{ij}, \tag{14}
\]
the equation of motion (A.16) becomes
\[
\hat{l}^2 \hat{R}_{ij} + (d-2) \hat{g}_{ij} - 2\rho \hat{g}_{ij}'' + 2\rho \hat{g}^{kl} \hat{g}_{ik} \hat{g}_{lj} - \hat{g}^{kl} \hat{g}_{kl} \left(\rho \hat{g}_{ij}'' - \hat{g}_{ij}\right) = 0. \tag{15}
\]
Here, $\hat{R}_{ij} = R_{ij}$ is the Ricci tensor of the time slice hypersurface. Raising an index with the metric $\hat{g}^{ij}$ we realize that it is handy to define the quantity
\[
h_{ij}^{ik} = \hat{g}^{ik} \hat{g}_{kj}, \tag{16}
\]
In fact, eqn. (15) becomes
\[ l^2 \hat{R}_j^i + (d - 2)h_j^i + h\delta_j^i - \rho \left( 2h_j^{i'} + hh_j^i \right) = 0. \] (17)

Similarly, rewriting the constraints (A.13) and (A.14) using eqns. (13), (14) and (16) one obtains
\[ l^2 \hat{R} + 2(d - 1)h + \rho \left( h^j_i h_i^j - h^2 \right) = 0 \] (18)

and
\[ D_i h - D_j h_i^j = 0, \] (19)

respectively.

In the AdS/CFT correspondence we have to calculate the on-shell value of the action (12) as a functional of prescribed boundary values \( \hat{g}_{ij} \), where the boundary is given by \( \rho = \epsilon \).

First, the on-shell action is easily found to be
\[ I(\epsilon) = \frac{d}{l} \int \rho d^d x \sqrt{\hat{g}} \rho^{-1 - \frac{d}{2}} + \frac{2}{l} \int d^d x \sqrt{\hat{g}} \epsilon^{-\frac{d}{2}}(\epsilon h - 1). \] (20)

In order to find the singular terms in the limit \( \epsilon \to 0 \), we differentiate eqn. (20) with respect to \( \epsilon \), leading to
\[ \frac{\partial I}{\partial \epsilon} = \int d^d x \sqrt{\hat{g}} \epsilon^{-\frac{d}{2}} \left[ l \hat{R} + \frac{d - 1}{l} h \right]. \] (21)

We have made use of the trace of the equation of motion (17) in order to simplify this expression. One can find the singular terms by calculating \( h \) from eqns. (17), (18) and (19) as a power series in \( \epsilon \), keeping only terms of order smaller than \( \epsilon^{\frac{d}{2}} \). Thus, for odd \( d \), eqn. (21) contains only singular terms proportional to powers \( \epsilon^{-n + \frac{d}{2}} \). On the other hand, for even \( d \), eqn. (21) contains a term proportional to \( 1/\epsilon \), which yields a corresponding term proportional to \( \ln \epsilon \) in \( I \). This logarithmic divergence is the source of the Weyl anomaly in the regularized finite action.

### 3.2 \( d = 2 \)

There is not really much to do for \( d = 2 \). In fact, the divergent term in eqn. (21) is obtained from the leading order solution for \( h \). Using the constraint (18) one finds
\[ h = -\frac{l^2}{2} \hat{R} + \mathcal{O}(\epsilon). \] (22)

Hence, the divergent term in the action is
\[ I_{\text{div}} = \ln \frac{l}{2} \int d^d x \sqrt{\hat{g}} \hat{R}. \] (23)
3.3 \( d = 4 \)

Starting from the constraint (18) one finds

\[ h = \frac{-1}{6} \left( l^2 \hat{R} + \epsilon \left( h^i_j h^j_i - h^2 \right) \right). \]

(24)

Here, the leading order behaviour of the term in parentheses is sufficient. The equation of motion (17) gives

\[ h^i_j = \frac{l^2}{2} \left( \hat{R}^i_j - \frac{1}{6} \delta^i_j \hat{R} \right) + \mathcal{O}(\epsilon), \]

which in turn yields

\[ h^i_j h^j_i - h^2 = \frac{l^4}{4} \left( \hat{R}^i_j \hat{R}^j_i - \frac{1}{3} \hat{R}^2 \right) + \mathcal{O}(\epsilon). \]

Hence, one finds

\[ I_{\text{div}} = - \int d^d x \sqrt{\hat{g}} \left[ \frac{l^3}{2 \epsilon} \hat{R} + \ln \epsilon \left( \frac{l^3}{8} \left( \hat{R}^i_j \hat{R}^j_i - \frac{1}{3} \hat{R}^2 \right) \right) \right]. \]

(25)

3.4 \( d = 6 \)

The constraint (18) yields

\[ h = \frac{-1}{10} \left( l^2 \hat{R} + \epsilon \left( h^i_j h^j_i - h^2 \right) \right). \]

(26)

We have to calculate the term in parentheses up to order \( \epsilon \). Starting from the equation of motion (17) we obtain

\[ h^i_j = \frac{-l^2}{4} \left( \hat{R}^i_j - \delta^i_j \hat{R} \right) + \epsilon \left[ \frac{2}{4} h^i_j + \frac{l^4}{40} \left( \hat{R}^i_j - \delta^i_j \hat{R} \right) + \delta^i_j \frac{1}{10} \left( h^k_i h^k_j - h^2 \right) \right] + \mathcal{O}(\epsilon^2), \]

which in turn yields

\[ h^i_j h^j_i - h^2 = \frac{l^4}{16} \left( \hat{R}^i_j \hat{R}^j_i - \frac{3}{10} \hat{R}^2 \right) - \frac{\epsilon}{8} \left( 2l^2 \hat{R}^i_j h^i_j' - \frac{l^2}{5} \hat{R} h' + \frac{15 l^6}{400} \hat{R} \hat{R}^i_j \hat{R}^j_i - \frac{29 l^6}{4000} \hat{R}^3 \right) + \mathcal{O}(\epsilon^2). \]

(27)

The quantities \( h' \) and \( h^i_j' \) can be found by differentiating the equation of motion (17) with respect to \( \rho \), leading to

\[ h' = -\frac{1}{8} \left( l^2 \hat{R}' - h^2 \right) + \mathcal{O}(\epsilon), \]

(28)

\[ h^i_j' = -\frac{1}{2} \left( l^2 \hat{R}^i_j' - \frac{l^2}{8} \hat{R}' \delta^i_j + \frac{1}{8} \delta^i_j h^2 - h h^i_j \right) + \mathcal{O}(\epsilon). \]

(29)
The missing quantity $\hat{R}_j'$ is given by

$$\hat{R}_j' = \frac{1}{2} \left( \hat{R}_k^j h^k_j - \hat{R}_j^k h^j_k \right) - \hat{R}_j^{ik} h^k_i + \frac{1}{2} D^i D_j h^i - \frac{1}{2} D^k D_k h^j_j,$$

(30)

where we have used the constraint (19). Taking the trace of eqn. (30) yields

$$\hat{R}' = -\hat{R}_j h^j_i.$$

(31)

Thus, substituting everything back into eqn. (27) we find

$$h^j_i h^i_j - h^2 = \frac{l^4}{16} \left( \hat{R}_j^i \hat{R}_i^j - \frac{3}{10} \hat{R}^2 \right) - \frac{\epsilon^6}{32} \left( \frac{1}{20} \hat{R} D_k D^k \hat{R} + \frac{1}{5} \hat{R}_j^i D^j D_i \hat{R} - \frac{1}{2} \hat{R}_j^i D_k D^k \hat{R}_i^j \right. - \hat{R}_j^{ik} \hat{R}_i^l \hat{R}_k^l + \frac{1}{2} \hat{R}_j^i \hat{R}_k^l \hat{R}_i^l \hat{R}_k^l - \frac{3}{50} \hat{R}^3 \right) + O(\epsilon^2).$$

(32)

Finally, substituting eqns. (26) and (32) into eqn. (21) we obtain the result

$$I_{div} = \int d^d x \sqrt{\hat{g}} \left[ -\frac{l^4}{4\epsilon^2} \hat{R} + \frac{l^3}{32\epsilon} \left( \hat{R}_j^i \hat{R}_i^j - \frac{3}{10} \hat{R}^2 \right) + \ln \frac{\epsilon}{l^4} \left( \frac{1}{20} \hat{R} D_k D^k \hat{R} + \frac{1}{5} \hat{R}_j^i D^j D_i \hat{R} - \frac{1}{2} \hat{R}_j^i D_k D^k \hat{R}_i^j \right. - \hat{R}_j^{ik} \hat{R}_i^l \hat{R}_k^l + \frac{1}{2} \hat{R}_j^i \hat{R}_k^l \hat{R}_i^l \hat{R}_k^l - \frac{3}{50} \hat{R}^3 \right) \right].$$

(33)

4 Conclusions

In this paper, we first explained the regularization procedure for the AdS/CFT correspondence. This was done using the example of a massive scalar field. The regularization procedure involves considering a family of surfaces as space time boundary, which tend towards the AdS horizon for some limit $\epsilon \to 0$. When using the cut-off, one must express all counterterms in terms of the boundary values of the AdS fields at the cut-off boundary, not the asymptotic horizon value. Our example demonstrates the importance of this step and thus shows that the “Dirichlet prescription” is the only entirely consistent one known so far.

Then, we calculated the divergent terms for AdS gravity for $d = 2, 4, 6$ using the Dirichlet prescription. We found agreement with earlier results, whose derivation did not properly address the boundary value subtlety [13], or used different techniques [13, 17, 20]. The fact that the subtlety does not affect the result should be regarded as a coincidence, as in the other cases mentioned in the introduction. In fact, we calculated some divergent terms for the scalar field and found that they generically disagree for the correct and asymptotic boundary values – even in the massless case, where the finite terms coincide.

As in our calculation of the finite term [4], which yields the two-point function of CFT energy momentum tensors, the time slicing formalism proves a valuable tool for the gravity part of the AdS/CFT correspondence. Moreover, we found that the calculation was greatly simplified by considering the derivative of the action, eqn. (21).

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A Time Slicing Formalism

Let us begin with a review of basic geometric relations for immersed hypersurfaces \[23\]. Let a hypersurface be defined by the functions \( X^\mu(x^i), (\mu = 0, \ldots, d, i = 1, \ldots d) \) and let \( g_{\mu\nu} \) and \( g_{ij} \) be the metric tensors of the imbedding manifold and the hypersurface, respectively. The tangents \( \partial_i X^\mu \) and the normal \( N^\mu \) of the hypersurface satisfy the following orthogonality relations:

\[
\begin{align*}
\tilde{g}_{\mu\nu} \partial_i X^\mu \partial_j X^\nu &= g_{ij}, \\
\partial_i X^\mu N_\mu &= 0, \\
N_\mu N^\mu &= 1.
\end{align*}
\]

We shall in the sequel use a tilde to label quantities relating to the \( d + 1 \) dimensional space time manifold and leave those relating to the hypersurface undecorated. Moreover, we use the symbol \( D \) to denote a covariant derivative with respect to whatever indices may follow. Then, there are the equations of Gauss and Weingarten, which define the second fundamental form \( H_{ij} \) of the hypersurface,

\[
\begin{align*}
D_i \partial_j X^\mu &\equiv \partial_i \partial_j X^\mu + \tilde{\Gamma}^\mu_{\lambda\nu} \partial_i X^\lambda \partial_j X^\nu - \Gamma^k_{ij} \partial_k X^\mu = H_{ij} N^\mu, \\
D_i N^\mu &\equiv \partial_i N^\mu + \tilde{\Gamma}^\mu_{\lambda\nu} \partial_i X^\lambda N^\nu = -H^j_i \partial_j X^\mu.
\end{align*}
\]

The second fundamental form describes the extrinsic curvature of the hypersurface and is related to the intrinsic curvature by another equation of Gauss,

\[
\tilde{R}_{\mu\nu\lambda\rho} \partial_i X^\mu \partial_j X^\nu \partial_k X^\lambda \partial_l X^\rho = R_{ijkl} + H_{il} H_{jk} - H_{ik} H_{jl}.
\]

Furthermore, it satisfies the equation of Codazzi,

\[
\tilde{R}_{\mu\nu\lambda\rho} \partial_i X^\mu \partial_j X^\nu N^\lambda \partial_k X^\rho = D_i H_{jk} - D_j H_{ik}.
\]

In the time slicing formalism \[24, 25\] we consider the bundle of immersed hypersurfaces defined by \( X^0 = \text{const.} \), whose tangent vectors are given by \( \partial_i X^0 = 0 \) and \( \partial_i X^\mu = \delta^\mu_i \) \( (\mu = 1, \ldots d) \). One conveniently splits up the metric as (shown here for Euclidean signature)

\[
\tilde{g}_{\mu\nu} = \begin{pmatrix} n_i n^i + n^2 & n_j \\ n_i & g_{ij} \end{pmatrix},
\]

whose inverse is given by

\[
\tilde{g}^{\mu\nu} = \frac{1}{n^2} \begin{pmatrix} 1 & -n^j \\ -n^i & n^2 g^{ij} + n^i n^j \end{pmatrix}
\]

and whose determinant is \( \tilde{g} = n^2 g \). The quantities \( n \) and \( n^i \) are called the lapse function and shift vector, respectively. The normal vector \( N^\mu \) satisfying eqns. (A.2) and (A.3) is given by

\[
N_\mu = (-n, 0), \quad N^\mu = \frac{1}{n}(-1, n^i),
\]
where the sign has been chosen such that the normal points outwards on the boundary ($n > 0$ without loss of generality). Then, one obtains the second fundamental form from the equation of Gauss (A.4) as

$$H_{ij} = \frac{1}{2n}(g'_{ij} - D_in_j - D_jn_i), \quad (A.11)$$

where the prime denotes a derivative with respect to the time coordinate ($X^0$).

The advantage of the time slicing formalism is that one removes the diffeomorphism invariance in Einstein’s equation by specifying the lapse function $n$ and shift vector $n^i$ and thus obtains an equation of motion as well as constraints for the physical degrees of freedom $g_{ij}$. Consider Einstein’s equation without matter fields,

$$\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R} = -\tilde{g}_{\mu\nu} \Lambda. \quad (A.12)$$

Multiplying it with $N^\mu N^\nu$ and using the equation of Gauss (A.6) as well as the relation (A.3) one obtains the first constraint,

$$R + H_{ij}H^{ij} - H^2 = 2\Lambda, \quad (A.13)$$

where $H = H^i_i$. Similarly, multiplying with $N^\mu \partial_i X^\nu$, using the equation of Codazzi (A.7) and the relation (A.2) yields the second constraint,

$$D_i H - D_j H^j_i = 0. \quad (A.14)$$

Finally, rewriting eqn. (A.12) in the form

$$\tilde{R}_{\mu\nu} = \frac{2}{d-1} \tilde{g}_{\mu\nu} \Lambda \quad (A.15)$$

and projecting out its tangential components we obtain the equation of motion

$$\tilde{R}_{ij} = \frac{2}{d-1} g_{ij} \Lambda. \quad (A.16)$$

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