Critical scaling in the matrix model on a Bethe tree

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Abstract

The matrix model with a Bethe-tree embedding space (coinciding at large $N$ with the Kazakov-Migdal “induced QCD” model [1]) is investigated. We further elaborate the Riemann-Hilbert approach of Ref. [2] assuming certain holomorphic properties of the solution. The critical scaling (an edge singularity of the density) is found to be $\gamma_{\text{str}} = -\frac{1}{\pi} \arccos D$, for $|D| < 1$, and $\gamma_{\text{str}} = -\frac{1}{\pi} \arccos \frac{D}{2D-1}$, for $D > 1$. Explicit solutions are constructed at $D = \frac{1}{2}$ and $D = \infty$. 
1 Introduction

Let us start with the matrix model describing surfaces embedded into an arbitrary graph $G$ defined by its incidence matrix $G_{xy}$ ($G_{xy} = 1$, if there is a link between vertices $x$ and $y$, and 0, otherwise). The standard action is

$$S = -N \sum_{xy} G_{xy} \text{tr} \left[ \Phi(x) \Phi(y) \right] + N \sum_x \text{tr} V(\Phi(x))$$

(1)

where $\Phi(x)$ is an $N \times N$ hermitian matrix attached to an $x$'th vertex; the potential, $V(\varphi)$, is an arbitrary polynomial. The partition function is defined as the integral over all field configurations

$$Z = \int \prod_{x \in G} d\Phi(x) \; e^{-S}$$

(2)

At each vertex, $\Phi(x)$ can be decomposed into diagonal, $\varphi(x)$, and angular, $S(x) \in U(N)$, parts:

$$\Phi(x) = S^+(x) \varphi(x) S(x)$$

(3)

It is convenient to introduce the on-link gauge variables

$$\Omega_{xy} = S^+(x) S(y)$$

(4)

obeying the constraint that their product along any loop equals unity.

If the graph $G$ is a tree, all the gauge variables are independent and can be integrated out by the Itzykson-Zuber formula [3]:

$$I(\phi, \psi) = \int d\Omega \; e^{N \text{tr} \phi \Omega \psi \Omega^+} = N^{-N(N-1)/2} \prod_{n=1}^{N-1} n! \frac{\det_{ab} e^{N \phi_a \psi_b}}{\Delta(\phi) \Delta(\psi)}$$

(5)

In eq. (5), $\phi$ and $\psi$ without loss of generality are real and diagonal; $\Delta(\phi) = \prod_{i<j} (\phi_i - \phi_j)$ is the Van-der-Monde determinant.

As a result, one finds a model in which a role of dynamical variables is played by $N$ eigenvalues of $\Phi(x)$. In the $N \to \infty$ limit, the saddle-point approximation is exact and, in this sense, the model is soluble.

If the graph $G$ has loops, the model becomes very complicated, and so far no reliable results have been obtained in this case. Technically, mean field does not work; physically, models of this kind describe the interaction of 2d gravity with matter having the central charge $c > 1$. 

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In the simplest case of the $c = 1$ model compactified on a ring, the angular degrees of freedom describe vortices, i.e., configurations with holes in the string world sheet wrapped along the ring. By dropping the constraint that the product of the angular variables along the ring equals unity, one discards the vortices \cite{4}. Hence, the singlet (vortex free) sector of the model is physically sensible. By dropping the constraints in the matrix model embedded in the regular $D$-dimensional lattice, Kazakov and Migdal \cite{1} obtained the model (sometimes called “induced QCD” \cite{5}) generalizing the $c = 1$ vortex-free matrix model. It can be easily seen that, in the $N \to \infty$ limit, it is equivalent to the matrix model with a Bethe-tree (BT) target space \cite{3}. A Bethe tree is, by definition, an infinite tree having the same even coordination number, $2D$, at each vertex. It is the infinite simply connected covering of the regular $D$-dimensional lattice. Therefore, the BT matrix model can be regarded as the mean field approximation for lattice scalar field theory.

2 The matrix model on a Bethe lattice

In this paper we consider the matrix model with a BT target space. In the large $N$ limit, the ground state should be homogeneous. The free energy per lattice site is given by the matrix integral

\[ F = \log \int d^{N^2} X \ e^{-N \text{tr} V(X)} \left[ I(X) \right]^{2D} \]  

where $I(X)$ obeys the equation

\[ I(X) = \int d^{N^2} Y \ e^{N \text{tr} (XY - V(Y))} \left[ I(Y) \right]^{2D-1} \]  

In terms of eigenvalues of $X$ we find

\[ F = \log \int \prod_{n=1}^N dx_n \Delta^2(x) \ e^{-N \sum_i V(x_i)} \left[ I(X) \right]^{2D} \]  

The saddle-point equation for this integral reads

\[ \frac{2}{N} \sum_{j \neq i} \frac{1}{x_i - x_j} - V'(x_i) + 2D w_i = 0 \]
where
\[ w_k = \frac{1}{N} \frac{\partial}{\partial x_k} \log I(X). \] (10)

Let us assume that, in the \( N \to \infty \) limit, the eigenvalues distribution

\[ \rho(x) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \delta(x - x_n) \] (11)

has a finite connected support. Then we can introduce the function

\[ f(x) = \int dy \frac{\rho(y)}{x - y} \] (12)

where the integral goes over the support of \( \rho(y) \) (\( \text{Supp } \rho(y) \)). \( f(x) \) is holomorphic at the infinity \( f(x) = \frac{1}{x} + 0(\frac{1}{x^2}) \) and has one cut on the real axis where

\[ \text{Im } f(x) = \pi \rho(x) \] (13)

In terms of \( f(x) \), Eq. (9) takes the form

\[ 2 \text{Re } f(x) - V'(x) + 2Dw(x) = 0 \] (14)

which is valid, strictly speaking, only on \( \text{Supp } \rho(x) \).

\[ w(x) = \frac{\partial}{\partial x} \frac{\delta}{\delta \rho(x)} \lim_{N \to \infty} \frac{1}{N^2} \log I(X) \] (15)

is the large \( N \) limit of the function (10).

To close the system of equations, we should calculate \( w(x) \) as a functional of \( f(x) \), i.e., to solve Eq. (7). However, we can avoid doing these complicated calculations. As was shown in Ref. [2] (see also [6]), in the \( N \to \infty \) limit, the following equation holds

\[ 1 = (z - w(x))W(z, x) - \int dy \rho(y) \frac{W(z, x) - W(z, y)}{x - y} \] (16)

where \( W(z, x) \) is the \( N \to \infty \) limit of the mean value of diagonal elements of the resolvent matrix

\[ W_k(z) = \frac{1}{I(X)} \left[ z - \frac{1}{N} \frac{\partial}{\partial X} \right]_{kk} I(X) \] (17)
The large $z$ expansion of this function has the form

$$W(z, x) = \frac{1}{z} + \frac{w(x)}{z^2} + \ldots$$  \hfill (18)

As long as the ground state is considered, the following homogeneity condition holds

$$\int dx \, \rho(x) W(z, x) = f(z)$$  \hfill (19)

Eq. (16) is valid only on $\text{Supp} \, \rho(x)$. However, all functions can be analytically continued into the whole complex plane. Following Ref. [2], let us introduce the function

$$F(z, x) = 1 - \int dy \frac{\rho(y) W(z, y)}{x - y} \equiv 1 - \lim_{N \to \infty} e^{-F} \int d^{N^2} X \, d^{N^2} Y$$

$$\frac{1}{N} \text{tr} \left[ \frac{1}{z - X} \frac{1}{x - Y} \right] e^{N \text{tr}(XY - V(X) - V(Y))} \left[ I(X) I(Y) \right]^{2D-1}$$  \hfill (20)

where the integral goes over $\text{Supp} \, \rho(x)$. By construction, $F(z, x)$ is symmetric $F(z, x) = F(x, z)$.

At $z$ sufficiently large, $F(z, x)$ as a function of $x$ is holomorphic everywhere except the support, where its imaginary part equals

$$\text{Im} \, F(z, x) = -\pi \rho(x) W(z, x)$$  \hfill (21)

The real part can be determined from Eq. (16)

$$\text{Re} \, F(z, x) = (z - w(x) - \text{Re} \, f(x)) W(z, x)$$  \hfill (22)

As $F(z, x)$ is real on the real axis out of the cut, we find from the Riemann-Schwartz principle that

$$F(z, \overline{x}) = \overline{F(z, x)}$$  \hfill (23)

and, hence, the ratio of limiting values of $F(z, x)$ above and below the cuts is independent of $W(z, x)$

$$\frac{F(z, x + i0)}{F(z, x - i0)} = \frac{z - w(x) - \text{Re} \, f(x) + i \text{Im} \, f(x)}{z - w(x) - \text{Re} \, f(x) - i \text{Im} \, f(x)}$$  \hfill (24)
At the infinity we have by construction

\[ F(z, x) = 1 - \frac{f(z)}{x} + 0\left(\frac{1}{x^2}\right) \] (25)

The function having all needed properties is given by the formula

\[ F(z, x) = \exp \oint_C \frac{dy}{2\pi i} \frac{1}{y - x} \log[z - w(y) - f(y)] \] (26)

where the contour \( C \) encircles the cut of \( f(y) \) in the positive direction leaving aside all other singularities (whatever they are), that is, \( F(z, x) \) is defined perturbatively by its expansion in inverse powers of \( z \) and \( x \).

For Eq. (26) to make sense, the function \( u(x) = w(x) + f(x) \) has to be holomorphic. However, \( w(x) \) depends through the saddle-point equation (14) on \( \text{Re} f(x) \). Hence, we should be able to introduce two holomorphic functions \( f_1(x) \) and \( f_2(x) \) such that, on \( \text{Supp} \rho(x) \), \( f_1(x) = \text{Re} f(x) \) and \( f_2(x) = \text{Im} f(x) \). Off the support, both \( f_1(x) \) and \( f_2(x) \) are holomorphic (i.e., obey the Cauchy-Riemann equations separately); on it \( \text{Im} f_1(x) = \text{Im} f_2(x) = 0 \). Everywhere in the complex plane \( f(x) = f_1(x) + if_2(x) \), and we define

\[ u(y) = \frac{1}{2D} V'(y) + \frac{D - 1}{D} f_1(y) + if_2(y) \]

(27)

Let \( \text{Supp} \rho(x) = (a, b) \), then we can rewrite Eq. (26) as

\[ F(z, x) = \exp - \int_a^b dy \frac{1}{2\pi i} \frac{1}{y - x} \log \left[ \frac{z - u(y)}{z - \bar{u}(y)} \right] = \]

\[ \exp \int_a^b \frac{du}{2\pi i} \frac{1}{u - z} \log \left[ \frac{x - y(u)}{z - \bar{y}(u)} \right] \] (28)

where \( y(x) \) is the inverse function: \( u(y(x)) = x \). For \( F(z, x) \) to be symmetric, \( F(z, x) = F(x, z) \), the following remarkable equation has to hold

\[ y(x) = \bar{u}(x) \]

(29)

or, equivalently,

\[ u(\bar{u}(x)) = x \]

(30)

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This equation has previously been obtained by Luochen (unpublished) and rederived by Matytsin within a quite different approach [8].

In general, the inverse function \( y(x) \) is not unique. However, we are interesting in critical behavior consisting in the divergence of a perturbative expansion in a non-gaussian part of the potential \( V(x) \). In the gaussian case, \( V_0(x) = \frac{m^2}{2}x^2 \), the solution was found by D.Gross [7]:

\[
f(x) = \frac{a}{2}x + i\sqrt{a - \frac{a^2x^2}{4}}
\]

with 2 possible values of \( a \)

\[
a_\pm = \frac{1}{2D - 1} \left[ m^2(D - 1) \pm D\sqrt{m^4 - 4(2D - 1)} \right]
\]

The condition that a solution has to describe perturbations around this semi-circular saddle-point fixes a branch of \( y(x) \) unambiguously.

From the expansion (25), we find Master Field Equation in the form

\[
f(z) = \oint C \frac{dy}{2\pi i} \log[z - w(y) - f(y)]
\]

which is satisfied automatically, if Eq. (30) holds.

We can as well obtain the equation

\[
f(z) = -\oint C \frac{dy}{2\pi i} \log \left[ z - w(y) - \bar{f}(y) \right]
\]

then, obviously,

\[
\oint C \frac{dy}{2\pi i} \log \left[ \left( x - \frac{1}{2D}V'(y) - \frac{D - 1}{D}f_1(y) \right)^2 + \left( f_2(y) \right)^2 \right] = 0
\]

and we conclude that

\[
f_2(x) = \sqrt{\Phi(x)}
\]

where \( \Phi(x) \) is holomorphic inside the contour \( C \). Critical behavior takes place when (i) a branching point or (ii) a zero of \( \Phi(x) \) approach the support of the eigenvalues density (its edges correspond to zeros of \( \Phi(x) \)), in other words, when two singularities pinch the contour. In the first (i) case, the order of the branching point increases, which is characteristic of \( c < 1 \) models.
The second (ii) possibility corresponds to the shrinking of a handle of the Riemann surface of \( f(x) \), as in the \( c = 1 \) matrix model.

3 Critical scaling

In matrix models, critical behavior is determined by a scaling near an edge of the eigenvalues density exactly at a critical point, when

\[
\rho(x) \propto (-x)^{1+\gamma} \quad x < 0
\]

where \( \gamma \equiv -\gamma_{str} > 0 \) is a critical exponent (for simplicity, we have assumed that the branching point of \( f(x) \) is at \( x = 0 \)). In the small vicinity, \( |x| \ll 1 \), we can expand all functions in powers of \( x \). For example, (if \( 0 < \gamma < 1 \))

\[
u(x) \approx ax + b \left( \frac{D-1}{D} \cos \pi \gamma + i \sin \pi \gamma \right) e^{-i\pi \gamma} x^{1+\gamma} + \ldots
\]

and

\[
\bar{u}(x) \approx ax + b \left( \frac{D-1}{D} \cos \pi \gamma - i \sin \pi \gamma \right) e^{-i\pi \gamma} x^{1+\gamma} + \ldots
\]

Expanding Eq. (30), we find

\[
x = \bar{u}(u(x)) \approx a^2 x + ab e^{-i\pi \gamma} e^{x^{1+\gamma}} \left[ \frac{D-1}{D} \cos \pi \gamma + i \sin \pi \gamma + \left( \frac{D-1}{D} \cos \pi \gamma - i \sin \pi \gamma \right) a^\gamma \right]
\]

Hence, \( a^2 = 1 \). If we take \( a = 1 \), \( a^\gamma = 1 \), the only solution is \( \gamma = \frac{1}{2} \), which is the well-known one-matrix-model exponent. Indeed, this solution should always be possible. For example, at \( D = \frac{1}{2} \), it corresponds to the disordered phase of the Ising model.

The second possibility is \( a = -1 \), which can be interpreted as a quadratic top of the effective potential, \( V_{eff} = \frac{1}{2D} V'(x) + \frac{D-1}{D} f_{1}^{\text{reg}}(x) \), where \( f_{1}^{\text{reg}}(x) \) is a regular at \( x = 0 \) part of \( f_1(x) \). Therefore, one should expect critical behavior in this case. In principle, we can take 2 branches of the function \( a^\gamma \). In the first case, \( a^\gamma = e^{i\pi \gamma} \), we find the equation

\[
\cos \pi \gamma = D
\]

which makes sense for \( |D| < 1 \) and fits 3 exactly soluble cases: \( D = 0, \frac{1}{2} \) and 1.
The second possibility is $a^\gamma = e^{-i\pi\gamma}$. In this case, a simple algebra gives the equation

$$\cos \pi \gamma = \frac{D}{2D - 1}$$

which formally makes sense for $D > 1$ and $D < \frac{1}{3}$.

4 Explicit solution at $D = \frac{1}{2}$ and $\infty$

In the previous section we have determined possible types of singularities of $\rho(x)$. However, it would be instructive to have a non-trivial global solution to the problem.

Let us suppose that there exist holomorphic coordinates $\lambda$ in which the function $f(x)$ becomes an integral function:

$$f(x(\lambda)) \equiv \varphi(\lambda) = \sum_{k \geq 1} \varphi_k \lambda^k$$

and the infinity of the $x$-plane is mapped to the center of the $\lambda$-plane (i.e., $x \sim \frac{1}{\lambda}$, when $\lambda \to 0$). It means that, geometrically, the Riemann surface of $f(x)$ is a sphere with one puncture.

There should exist an involution transformation of the $\lambda$-plane, $\pi(\lambda)$, preserving a curve of real $x$’s,

$$\pi(\pi(\lambda)) = \lambda \quad x(\pi(\lambda)) = x(\lambda),$$

such that, on the curve (and only on it), the action of $\pi$ coincides with the complex conjugation:

$$\overline{\varphi(\lambda)} = \varphi(\pi(\lambda))$$

Then, as $f(\bar{x}) = \bar{f}(x)$, we can identify

$$\varphi_1(\lambda) \equiv f_1(x(\lambda)) = \frac{1}{2} [\varphi(\lambda) + \varphi(\pi(\lambda))]$$

$$\varphi_2(\lambda) \equiv f_2(x(\lambda)) = \frac{1}{2i} [\varphi(\lambda) - \varphi(\pi(\lambda))]$$

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The functions $u(x)$ and $\bar{u}(x)$ define an invertible map of the $\lambda$-plane onto the $\mu$-plane and back, correspondingly, determined by the equations

$$
x(\mu) = \frac{1}{2D} V'(x(\lambda)) + \frac{2D-1}{2D} \varphi(\lambda) - \frac{1}{2D} \varphi(\pi(\lambda))
$$

$$
x(\lambda) = \frac{1}{2D} V'(x(\mu)) - \frac{1}{2D} \varphi(\mu) + \frac{2D-1}{2D} \varphi(\pi(\mu))
$$

The simplest realization of this scheme,

$$
x(\lambda) = \frac{1}{\lambda} + a\lambda \quad \varphi(\lambda) = \lambda,
$$

(47)

(48)

corresponds to the semi-circle solution (31). Here,

$$
\pi(\lambda) = \frac{1}{a\lambda}
$$

(49)

For the gaussian potential, $V'(x) = m^2 x$, equations (47) take the form

$$
\frac{1}{\mu} + a\mu = \frac{1}{2Da}(m^2a - 1)\frac{1}{\lambda} + \frac{1}{2D}(m^2a + 2D - 1)\lambda
$$

$$
\frac{1}{\lambda} + a\lambda = \frac{1}{2Da}(m^2a + 2D - 1)\frac{1}{\mu} + \frac{1}{2D}(m^2a - 1)\mu
$$

(50)

from which we find

$$
\mu = \frac{2Da}{m^2a - 1}\lambda
$$

(51)

together with the self-consistency equation for $a$:

$$
(m^2a + 2D - 1)(m^2a - 1) = (2Da)^2
$$

(52)

which gives the solution (32).

Now, let us consider the non-gaussian potential $\frac{1}{2D} V'(x) = m^2 x + gx^2$ and try the ansatz

$$
x(\lambda) = \frac{1}{\lambda} + c + a\lambda + b\lambda^2
$$

(53)

From the power counting, we find the form of $\varphi(\lambda)$:

$$
\varphi(\lambda) = \lambda + \varphi_2\lambda^2 + \varphi_3\lambda^3 + \varphi_4\lambda^4
$$

(54)
In this case, $\pi(\lambda)$ obeys the quadratic equation

$$b\lambda\pi^2(\lambda) + (a\lambda + b\lambda^2)\pi(\lambda) - 1 = 0 \quad (55)$$

which has two solutions

$$\pi_{\pm}(\lambda) = \frac{1}{2} \left[ -\frac{a}{b} - \lambda \pm \sqrt{\frac{4}{b\lambda} + \left(\frac{a}{b} + \lambda\right)^2} \right] \quad (56)$$

We have to use $\pi_+(\lambda)$ for the conjugation, because it is an involution: $\pi_+(\pi_+(\lambda)) = \lambda$. Its asymptotic behavior is

$$\pi_+(\lambda) = \begin{cases} \frac{1}{\sqrt{b\lambda}}, & \text{when } \lambda \to 0 \\ \frac{1}{b\lambda^2}, & \text{when } \lambda \to \infty \end{cases} \quad (57)$$

It is natural to suppose that zero and the infinity are fixed points of the function $\mu(\lambda)$ ($\mu(0) = 0$ and $\mu(\infty) = \infty$) as it was in the gaussian case. We see then that a number of terms on the right hand side of Eq. (47) have to cancel identically, which gives us the asymptotics of $\mu(\lambda)$:

$$\mu(\lambda) = \begin{cases} \sqrt{g\lambda}, & \text{when } \lambda \to 0 \\ \sqrt{bg\lambda^2}, & \text{when } \lambda \to \infty \end{cases} \quad (58)$$

together with the answer for $\varphi(\lambda)$:

$$\varphi(\lambda) = \lambda + [m^2b + g(a^2 + 2bc)]\lambda^2 + 2abg\lambda^3 + b^2g\lambda^4 \quad (59)$$

If $D \neq \frac{1}{2}$, we find contradictions and the equations have no solution. One could have expected it from the very beginning, because the ansatz (38) is well known in the orthogonal polynomials approach to the two-matrix model and gives $\gamma_{str} = -\frac{1}{3}$ [3].

At this point, Eq. (47) take the form

$$\frac{1}{\mu} + c + a\mu + b\mu^2 = b^2g\lambda^4 + 2abg\lambda^3 + (m^2b + g(a^2 + 2bc))\lambda^2 + ((m^2 + 2cg)a + bg)\lambda + m^2c + (2a + c^2)g + \{b^2g\lambda^3 + abg\lambda^2 + (m^2 + 2cg)b\lambda + 2bg + (m^2 + 2cg)a - 1\} \pi_+(\lambda) \quad (60)$$

and

$$\frac{1}{\lambda} + c + a\lambda + b\lambda^2 = \frac{g}{\mu^2} + (m^2 + 2gc)\frac{1}{\mu} + (2a + c^2)g + m^2c + \sqrt{\frac{b}{g}} \mu \quad (61)$$
Let us assume that the function \( \mu(\lambda) \) obeys a quadratic equation, then there is no ambiguity left and the ansatz having correct asymptotic behavior is of the form

\[
\mu(\lambda) = -\sqrt{bg\lambda\pi_-(\lambda)} = \sqrt{bg\lambda\pi_+(\lambda)} + \sqrt{\frac{g}{b}}(a\lambda + b\lambda^2) \tag{62}
\]

It can be easily checked that this function does satisfy Eqs. (60,61) provided the parameters obey the following three constraints

\[
\begin{align*}
  a &= \sqrt{\frac{g}{b}}(m^2 + 2gc) \\
  gc^2 + (m^2 - 1)c + 2ag &= 0 \\
  2bg - \sqrt{\frac{g}{b}} + (m^2 + 2gc)a - 1 &= 0
\end{align*} \tag{63}
\]

which completes the solution in the \( D = \frac{1}{2} \) case.

If \( D = \infty \), Eq. (47) takes a similar form and the same ansatz (53) should work. In this case, it is convenient to introduce the function \( \lambda(\mu) \) which coincides with Eq. (62):

\[
\lambda(\mu) = -\sqrt{bg\mu\pi_-(\mu)} \tag{64}
\]

and, in complete analogy with the previous case, we find

\[
\varphi(\lambda) = \lambda - [m^2b + g(a^2 + 2bc)]\lambda^2 - 2abg\lambda^3 - b^2g\lambda^4 \tag{65}
\]

where parameters obey the equations

\[
\begin{align*}
  a &= \sqrt{\frac{g}{b}}(m^2 + 2gc) \\
  gc^2 + (m^2 - 1)c + 2ag &= 0 \\
  2bg - \sqrt{\frac{g}{b}} + (m^2 + 2gc)a + 1 &= 0
\end{align*} \tag{66}
\]

The only difference between Eq. (63) and Eq. (66) is the sign before 1 in the last equation.

At the critical point we have

\[
x(\lambda) = \frac{(1 + s\lambda)^3}{\lambda} \tag{67}
\]

and a simple algebra gives

\[
s^2 = \begin{cases} 
  \frac{1}{10}, & \text{when } D = \frac{1}{2} \\
  -\frac{1}{10}, & \text{when } D = \infty
\end{cases} \tag{68}
\]
The critical coupling, $g_c$, is to be determined from the equation

$$\sqrt{sg_c} = 1$$

Hence, at $D = \infty$, $g_c$ is complex.

5 Discussion

The main result of this paper, Eq. (41), looks very natural. After the identification $D = \frac{n}{2}$, it coincides with the Kostov’s solution of the $O(n)$ matrix model [10]. It is a standard matrix-model singularity. The second branch (42) corresponds to a perturbation expansion around a local maximum of the free energy [7] (only this saddle-point of the gaussian model is non-singular at $D = \infty$). Moreover, we have found that the critical coupling in this case is complex (at least, at $D = \infty$). This means, presumably, that, at $D > 1$, our model is in the usual sense non-critical as all solved so far matrix models.

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