Hybrid silicon lasers for optical interconnects

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New Journal of Physics 11 (2009) 125016 (17pp)
Received 10 June 2009
Published 17 December 2009
Online at http://www.njp.org/
doi:10.1088/1367-2630/11/12/125016

Abstract. We propose high-speed modulation of hybrid silicon lasers through modulation of the photon lifetime $\tau_p$. Two structures are presented to achieve $\tau_p$-modulation by modifying the distributed loss or the feedback coefficient of the laser cavity. By using small-signal modeling and the finite-difference method, the responses in the frequency and time domains are given. It is shown that it is possible to achieve a 3 dB bandwidth of over 100 GHz and a high data transmission rate of more than 50 GHz. The theoretical analysis also shows that the chirp due to the variations of the carrier densities in the gain/modulation sections is very small.

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1. Introduction

The optical transmitter is one of the most important components in any long-haul or short-distance optical communication system and for chip-to-chip interconnects. Currently commercial transmitters are mostly based on the III–V semiconductor material system. In recent years, silicon photonics has received great attention because of the possibility of low cost optoelectronic solutions due to its compatibility with the mature complementary metal-oxide-semiconductor (CMOS) process. However, the silicon indirect bandgap means that bulk silicon is not good for light generation due to the very inefficient band-to-band radiative electron–hole recombination [1]. This has been the major hindrance to the realization of an electrically pumped Si-based laser, which is one of the key elements for transmitters. In order to achieve light emission in silicon, several methods of using silicon engineered materials such as nanocrystals [2], Si/SiO2 superlattices [3], erbium-doped silicon-rich oxides [4] and Si/SiGe quantum cascade structures [5] have been developed. The drawbacks of these approaches are that the light emission efficiency is not as high as traditional III–V emitters, or an external optical pumping source is needed.

Recently, a hybrid structure combining III–V quantum-well and silicon optical waveguides has been demonstrated as a means of fabricating electrically pumped Si-based lasers [6]. The hybrid III–V-silicon structure is comprised of a III–V quantum-well region bonded to a silicon-on-insulator (SOI) wafer. The optical waveguide is defined by two trenches in the Si layer. In this way, the hybrid structure behaves like an inverse ridge waveguide. Because the III–V region is a slab, the bonding is self-aligned and no alignment step is needed for the bonding between the III–V layer and the silicon waveguide. Electrically pumped gain could be achieved from the III–V active region since the optical mode of the hybrid structure overlaps both the III–V material and the silicon waveguide. Such a hybrid platform has been demonstrated for various Si-based active components, e.g. lasers [7], photodetectors [8], modulators and amplifiers.
Figure 1. (a) Frequency responses of the DFB-SEL and (inset) the resonance frequency and the eye diagrams of a directly modulated DBR-SEL at (b) 2.5 Gb s\(^{-1}\) and (c) 4 Gb s\(^{-1}\).

For optical transmitters, on the other hand, it is also desirable to have high-speed modulation in addition to an electrically pumped laser. A simple way is to directly modulate the injected pump current of the laser [9]. Recently we measured the characteristics of directly modulated hybrid Si lasers. Figure 1(a) shows the frequency response of the laser under a small signal modulation (\(-10\) dBm). The inset in figure 1(a) shows the resonance frequency versus the square root of the injected current above threshold. There is a roughly linear dependence, as expected. The 3 dB bandwidth at 105 mA is about 2.5 GHz. Figure 1(b) shows the measured eye diagrams for the directly modulated laser with a bit rate of 2.5 Gb s\(^{-1}\). The corresponding injected dc current \(I = 105\) mA and the extinction ratio (ER) is about 8.7 dB. With a higher injection current, it is possible to have a higher bit rate, e.g. 4 Gb s\(^{-1}\) when \(I = 135\) mA (see figure 1(c)). However, the ER is relatively low (\(\sim 5.5\) dB). From these measurement results, one sees that directly modulating the injection current has a limited speed (usually <10 GHz). Furthermore, there is a severe wavelength chirp, which results in pulse spreading due to chromatic dispersion in optical fibers. Therefore, it limits the application of directly modulated lasers in high-speed long-haul telecommunication systems.

An alternative method is to use a high-speed external modulator. Several groups have demonstrated various types of external modulation using different structures and different materials, e.g. Mach–Zehnder modulators based on LiNbO\(_3\) (with a strong electro-optic effect) [10], III–V-based electroabsorption modulators, etc. It is also possible to realize a high-speed Si-based modulator by using electric-field-induced carrier depletion in the Si waveguide with a reverse biased P–N junction. The carrier depletion introduces a fast refractive index modulation from the free carrier plasma dispersion effect. On the basis of this principle, Liu \textit{et al} have demonstrated a Si-based MZI modulator for data transmission up to 40 Gb s\(^{-1}\) [11] with a traveling-wave design for velocity-matched co-propagation of electrical and optical signals along the waveguide. However, the length of the MZI modulator is on the order of a millimeter. In general, an approach cascading a continuous-wave laser with an external modulator will make an optical transmitter relatively large and expensive. It is more attractive to achieve a compact monolithically integrated optical transmitter that consists of both laser and modulator.

In this paper, we propose high-speed direct modulation for hybrid silicon lasers by modulating the photon lifetime \(\tau_p\). The \(\tau_p\)-modulation has been used for surface
emitting distributed Bragg reflector lasers with electro-optic modulation of the Bragg mirror reflectivity [12]. By utilizing $\tau_p$-modulation, it is possible to achieve high-speed modulation as well as low chirp.

2. Design and structure

The photon lifetime $\tau_p$ in a laser is given by [13]

$$\frac{1}{\tau_p} = v_g \left( \alpha_i + \frac{1}{l} \ln \frac{1}{R} \right),$$

where $v_g$ is the group velocity, $\alpha_i$ is the distributed loss coefficient, $l$ is the length of the optical cavity and $R$ is the feedback coefficient of the optical cavity (i.e. the mirror reflectivity in an F–P cavity or coupling coefficient in a microring cavity). From equation (1) one sees that there are several ways to modify the photon lifetime $\tau_p$, i.e. by changing the $v_g$, $l$, $R$ and $\alpha_i$. Among these ways, the most simple and effective are modifying the distributed loss $\alpha_i$ and the feedback coefficient $R$. Here, we present two ways by modifying $\alpha_i$ and $R$ for the case of the hybrid III–V/silicon laser.

2.1. $\tau_p$-modulation by modifying the distributed loss coefficient $\alpha_i$.

The distributed loss includes the scattering loss due to the sidewall roughness [14], the free carrier absorption loss, etc. Since the scattering loss is almost a constant, here, we consider to modify the carrier density in the optical waveguide. Figure 2 shows the cross section of the waveguide structure for a hybrid III–V–Si laser to be $\tau_p$-modulated by modifying the distributed loss $\alpha_i$. In this structure, there are two P–N junctions: one for the III–V layer and one for the Si layer. The P–N junction in the III–V mesa is to provide an electrical pump to the gain section. The other one, located at the two sides of the SOI ridge, is to provide the modulation signal by injecting or depleting the carrier density in the SOI ridge region. The method of depleting the carrier density is suitable for high-speed modulation [3]. Such a waveguide structure is applicable to various types of laser cavity (e.g. microring cavities or F–P cavities). The hybrid combination of the gain section and the modulation section makes it very attractive to achieve a compact transmitter. More importantly, it is possible to obtain a high-speed hybrid laser modulator due to the fast response of depleting the carrier density in silicon. In addition,
by modulating the voltage used to deplete carriers from the Si and the current injected into the III–V active layer simultaneously, the change in effective index of the waveguide due to the change of the carrier density in the laser cavity is minimized. Consequently, the chirp can be small at high modulation speeds.

2.2. $\tau_p$-modulation by modifying $R$

Another way to achieve $\tau_p$-modulation is to modify the feedback coefficient, e.g. the mirror reflectivity in an F–P cavity or coupling coefficient in a microring cavity. In comparison with the reflectivity of an F–P cavity, the coupling coefficient of a microring cavity is easier to modify. Figure 3(a) shows the schematic configuration for a microring cavity with a feedback line used here. The microring cavity provides a gain and wavelength selection (which is the same as a regular microcavity). Figure 3(b) shows the cross section of the microring waveguide [15]. The feedback line has a modulation section, which modifies the phase delay or the amplitude of the feed-back lightwave by changing the carrier density. For a high speed modulation, one effective method is to deplete the carrier density in the modulation region. Figure 3(b) shows the cross section of the modulation section. By using this structure, one could easily modify the feedback coefficient $R$ and consequently realize $\tau_p$-modulation.

**Figure 3.** (a) The structure of the microcavity for a $\tau_p$-modulated laser realized by modifying the feedback coefficient $R$; (b) the cross section of the modulation section and (c) the cross section of the gain section.
3. Small signal modeling

In order to give an analysis for the proposed $\tau_p$-modulated lasers, in the following section we use the well-known rate equations, which are given by [16]

\[
\frac{dN}{dt} = -\frac{g_0(N - N_t)S}{1 + \varepsilon S} + \frac{I}{qV} - \frac{N}{\tau_n}, \tag{2a}
\]

\[
\frac{dS}{dt} = \frac{\Gamma g_0(N - N_t)S}{1 + \varepsilon S} - \frac{S}{\tau_p} + \frac{\beta \Gamma N}{\tau_n}, \tag{2b}
\]

where $N$ and $S$ are the electron and photon densities in the active layer, $I$ is the drive current, $q$ is the electron charge, $V$ is the active layer volume, $g_0$ is the gain coefficient, $N_t$ is the electron concentration at transparency, $\tau_n$ and $\tau_p$ are electron and photon lifetimes, $\Gamma$ is the mode confinement factor, $\varepsilon$ is gain suppression factor and $\beta$ is the fraction of spontaneous emission coupled into the lasing mode.

3.1. The steady state

For the steady state, one has

\[
0 = \frac{dN_0}{dt} = -\frac{g_0(N_0 - N_t)S_0}{1 + \varepsilon S_0} + \frac{I_0}{qV} - \frac{N_0}{\tau_n}, \tag{3a}
\]

\[
0 = \frac{dS_0}{dt} = \frac{\Gamma g_0(N_0 - N_t)S_0}{1 + \varepsilon S_0} - \frac{S_0}{\tau_p} + \frac{\beta \Gamma N_0}{\tau_n}. \tag{3b}
\]

From equation (3b),

\[
N_0 - N_t = \frac{S_0/\tau_p - \beta \Gamma N_t/\tau_n}{\Gamma g_0 S_0[1/(1 + \varepsilon S_0) + \beta/(g_0 S_0 \tau_n)]}. \tag{4}
\]

The formula above is the same as equations (2) in [17]. From equations (3a) and (3b),

\[
0 = (1 - \beta) \Gamma \frac{g_0 S_0}{1 + \varepsilon S_0}(N_0 - N_t) - \frac{S_0}{\tau_p} + \beta \Gamma \frac{I_0}{qV},
\]

By inserting equation (4) into the above equation, the solution is as follows:

\[
S_0 = (-b_0 + \sqrt{b_0^2 - 4a_0 c_0})/(2a_0), \tag{5}
\]

where

\[
a_0 = (\varepsilon/\tau_n + g_0)/\tau_p,
\]

\[
b_0 = -\Gamma g_0 \frac{I_0}{qV} + \frac{1}{\tau_n} \frac{1}{\tau_p} + \frac{1}{\tau_n} g_0 \Gamma N_t - \left( g_0 N_t + \varepsilon \frac{I_0}{qV} \right) \frac{1}{\tau_n} \beta \Gamma,
\]

\[
c_0 = -\beta \Gamma (I_0/qV)/\tau_n.
\]
3.2. The $\tau_p$-modulation case

Here, we consider the small-signal model of a $\tau_p$-modulated laser. We assume that there is a $\tau_p$ modulation of

$$\frac{1}{\tau_p} = \frac{1}{\tau_{p0}} + \frac{1}{\Delta \tau_p} e^{j\omega t}$$

(where $\frac{1}{\Delta \tau_p} \ll \frac{1}{\tau_{p0}}$ due to the small variation of the distributed loss coefficient $\alpha_i$ or the feedback coefficient $R$ of the optical cavity). Then the photon density $S$ and the carrier density $N$ are given as

$$N = N_0 + \Delta N e^{j\omega t}$$
$$S = S_0 + \Delta S e^{j\omega t},$$

where $\Delta N \ll N_0$ and $\Delta S \ll S_0$. Substituting these expressions into the rate equations and yields the following results for the carrier density $\Delta N$ and the photon density $\Delta S$:

$$\Delta N = -\frac{C}{j\omega + B} \Delta S,$$

$$\Delta S = \frac{S_0(j\omega + B)/\Delta \tau_p}{-\omega^2 + j\omega(A + B) + AB + CD},$$

where

$$A = 1/\tau_{p0} - \Gamma g_0(N_0 - N_t)/(1 + \varepsilon S_0)^2,$$
$$B = g_0 S_0/(1 + \varepsilon S_0) + 1/\tau_n,$$
$$C = g_0(N_0 - N_t)/(1 + \varepsilon S_0)^2,$$
$$D = \Gamma g_0 S_0/(1 + \varepsilon S_0) + \beta \Gamma/\tau_n.$$

According to the expression (6b) for the photon density $\Delta S$, from $\frac{d|\Delta S|^2}{d\omega} = 0$, one obtains the frequency $\omega_p$, where the peak of the frequency response is located:

$$\omega_p = \frac{1}{2a_1} (-b_1 + \sqrt{b_1^2 - 4c_1}),$$

where

$$a_1 = 1,$$
$$b_1 = 2B^2,$$
$$c_1 = B^4 - 2B^2CD - 2ABCD - C^2D^2.$$

In comparison with the results for the mode of injection–current modulation [16], the difference is a term of $j\omega$ in the numerator. Consequently, the response due to the $\tau_p$-modulation decays at high frequencies only as $1/\omega$, which is similar to the results of modulating the distributed Bragg mirror reflectivity for surface-emitting laser diodes shown in [12]. In contrast, for the conventional current modulation, the intensity response decays as $1/\omega^2$. This is why the present $\tau_p$-modulation has much larger 3 dB bandwidth than the conventional current modulation.

In the following section, we give some calculation results from the small-signal model developed above. We choose the parameters shown in table 1 for the laser modulator as an example. The threshold current $I_{th} = 0.0268$ A from the formula $I_{th} = (q V/\tau_n) [N_t + 1/(\tau_{p0} \Gamma g_0)]$. 

New Journal of Physics 11 (2009) 125016 (http://www.njp.org/)}
Table 1. The parameters used for the calculations.

| Symbol | Value                                      |
|--------|--------------------------------------------|
| $V$    | $200 \times 1.0 \times 0.15 \mu m^3$      |
| $\tau_{p0}$ | 1.7887 ps                                  |
| $\tau_n$ | 1 ns                                       |
| $N_t$  | $1 \times 10^{18} \text{m}^{-3}$          |
| $g_0$  | $1.0 \times 10^{-12} \text{s}^{-1} \text{m}^3$ |
| $\Gamma$ | 0.1                                         |
| $\beta$ | $10^{-4}$                                   |
| $\epsilon$ | $8 \times 10^{-23} \text{m}^3$            |
| $\alpha_i$ | 25 cm$^{-1}$                               |
| $\lambda$ | 1.55 $\mu m$                               |

Figure 4. Frequency responses for the $\tau_p$-modulated laser with different $\epsilon$ values when $\beta = 10^{-4}$ and $I = 5I_{th}$.

As an example, first we choose an injected current $I = 5I_{th}$. Figure 4 shows the calculated frequency responses for different values of $\epsilon$. From this figure, it is clear that the frequency response has a peak enhancement (due to the electron–photon resonance [12]) at a specific frequency $f_p = \omega_p/(2\pi)$ as expected, where $\omega_p$ is predicted by equation (7). For a small $\epsilon$ (e.g. $1 \times 10^{-23} \text{m}^3$), the peak enhancement is very significant (>10 dB). Such a significant peak enhancement may lead to degradation of the eye-diagram performance due to the overshoot effect [18]. From figure 4, one also sees that the peak $S'(f_p)$ decreases when the $\epsilon$ value increases and thus the eye-diagram will be improved by increasing $\epsilon$. This depression of peak due to the increase of $\epsilon$ is similar to what is observed in conventional directly current-modulated lasers. However, for the case of I-modulation, a larger $\epsilon$ value will introduce a significant reduction of 3 dB bandwidth. In contrast, in the $\tau_p$-modulation case, the response at high frequency does not change much when the $\epsilon$ value increases. Thus, the $\epsilon$ value has little influence on the 3dB bandwidth in a $\tau_p$ modulated laser.

Figure 5 shows the frequency responses for the $\tau_p$-modulated laser with different injection current values, $I = [2, 4, 6, 8, 10] \times I_{th}$. Here, we choose $\beta = 10^{-4}$ and $\epsilon = 8 \times 10^{-23}$ as an example. When the injection current increases from $2I_{th}$ to $10I_{th}$, the peak of the frequency...
response decreases and the 3 dB bandwidth increases. This is similar to the conventional current-modulated laser.

In order to investigate the influence of the injection current on the bandwidth, in figures 6(a) and (b), we plotted the curves of $f_{0 \text{dB}} \sim (I-I_{\text{th}})^{1/2}$, and $f_{3 \text{ dB}} \sim (I-I_{\text{th}})^{1/2}$, respectively, where $f_{0 \text{dB}}$ and $f_{3 \text{ dB}}$ are the 0 dB and 3 dB bandwidths. Here $f_{x \text{dB}}$ is defined as the frequency where the response is smaller by $x$ dB than the zero-frequency response. We also consider the cases where $\varepsilon = [1, 2, 4, 8, 16] \times 10^{-23}$. From these figures, one sees both the 0 dB and 3 dB bandwidths increase very sharply when $(I-I_{\text{th}})^{1/2}$ increases from 0 to 0.4. This indicates that it is possible to have a large bandwidth with a low injection current. In this range, the bandwidths are almost the same for different $\varepsilon$ values. When the current increases further, the 0 dB bandwidth decreases. A smaller $\varepsilon$ value introduces a larger 0 dB bandwidth. The 3 dB bandwidth (see figure 6(b)) could be as high as 150 GHz when $(I-I_{\text{th}})^{1/2}$ is increased to 0.5 $A^{1/2}$. When the injected current is increased further, the 3 dB bandwidth changes slightly and the $\varepsilon$ value does not influence the 3 dB bandwidth significantly.

4. Time-domain response of $\tau_p$-modulation

Here, we use the finite-difference method to give a numerical calculation for the response of a $\tau_p$-modulated laser in the time domain. The partial differentials in equations (2a) and (2b) are given by the following different forms:

\[
\frac{dN}{dr} \bigg|_{r=(m+1/2)\Delta t} = \frac{N_{m+1} - N_m}{\Delta t} = \frac{\Delta N_m}{\Delta t},
\]

\[
\frac{dS}{dr} \bigg|_{r=(m+1/2)\Delta t} = \frac{S_{m+1} - S_m}{\Delta t} = \frac{\Delta S_m}{\Delta t},
\]

\[
N_{r=(m+1/2)\Delta t} = \frac{N_{m+1} + N_m}{2} = N_m + \frac{\Delta N_m}{2},
\]

\[
S_{r=(m+1/2)\Delta t} = \frac{S_{m+1} + S_m}{2} = S_m + \frac{\Delta S_m}{2}.
\]
Figure 6. The bandwidth for the $\tau_p$-modulated laser as the injection current varies when $\beta = 10^{-4}$: (a) the 0 dB bandwidth and (b) the 3 dB bandwidth.

The photon density and carrier density at the $(m + 1)$th step (i.e. $t = (m + 1)\Delta t$) are given by:

$$
\begin{align*}
N_{m+1} &= N_m + \Delta N_m, \\
S_{m+1} &= S_m + \Delta S_m,
\end{align*}
$$

where

$$
\Delta N_m = a \Delta S_m + b, \quad (8a)
$$

and

$$
\Delta S_m = \frac{-B_1 - \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \quad (8b)
$$

and

$$
A_1 = \left[ \frac{1}{\Delta t} + \frac{1}{2\tau_n} \right] \frac{\varepsilon a}{2} + \frac{g_0a}{4},
$$
By using the formulae above, we give a simulated eye diagram for the proposed \( \tau_p \)-modulated laser with the parameters shown in table 1. For this calculation, the photon lifetime has a Gaussian-type pulse modulation with a bit rate of 50 Gb s\(^{-1}\). We choose \( \tau_{p, on} = 1.073 \) ps (\( R = 0.156 \)) and \( \tau_{p, off} = 1.788 \) ps (\( R = 0.4 \)) for the on and off states, respectively. The injection current is kept constant (\( I_0 = 0.15 \) A). Figure 7 shows the simulated eye diagram at a bit rate of 50 Gb s\(^{-1}\). From this figure, one sees that the eye is open at 50 Gb s\(^{-1}\). It is possible to have higher bit rate according to the 3 dB bandwidth shown in figure 6(a). In order to give a comparison, we also show the eye diagram for the conventional I-modulated laser with a bit rate of 5 Gb s\(^{-1}\). The currents for the on and off states are \( I_{on} = 0.15 \) A and \( I_{off} = 0.05 \) A, respectively. The simulation result shows that it is possible to achieve a data rate of several Gbps, which is similar to what we observed from the measurement results (see figure 1(b)). However, the eye diagram becomes close for a relatively high bit rate (e.g. 10 Gb s\(^{-1}\)).

### 5. Chirp analysis of the \( \tau_p \)-modulated laser

In the \( \tau_p \)-modulated laser, there are two sources of chirp. One is the variation of the carrier density in the gain section and the other is the phase shift in the modulation section.
5.1. Chirp due to the variation of the carrier density in the gain section

The complex refractive index of the active layer is given by \( n = n' + jn'' \). The relationship between \( n' \) and \( n'' \) is given by

\[
\frac{\partial n'}{\partial N} = \zeta \frac{\partial n''}{\partial N},
\]

where \( N \) is the carrier density and \( \zeta \) is the linewidth enhancement factor. The gain \( g \) is related with \( n'' \) by

\[
g = n'' \frac{4\pi}{\lambda},
\]

and thus

\[
\Delta g = \Delta n'' \frac{4\pi}{\lambda} = \Delta n' \frac{4\pi}{\zeta \lambda}
\]

or,

\[
\Delta n' = \frac{\zeta \lambda}{4\pi} \Delta g.
\]

The laser frequency shift \( \Delta v \) associated with \( \Delta n' \) is given by \( \frac{\Delta f}{f} = -\frac{\Delta n'}{n''} \). Thus, one has

\[
\Delta f = -\frac{\zeta}{4\pi} v_g \Delta g.
\]

From [19], one has an approximation of \( g(N) \approx a(N-N_i) \), where \( a = g_0/[v_g(1+\varepsilon S)] \) [16, 19], where \( v_g \) is the group velocity. Thus

\[
\Delta g \approx a \Delta N.
\]

Inserting equation (12) into equation (11), one has

\[
\Delta f = -\frac{\zeta}{4\pi} \frac{g_0}{1+\varepsilon S} \Delta N.
\]

Here \( \Delta N \) is the change of the carrier density, which can be obtained from equation (6a).

Figures 8(a)–(c), respectively, show the calculated frequency shift \( \Delta f \), the power variation \( \Delta p \) and the chirp-power ratio (CPR = \( \Delta f / \Delta p \)) as the modulation frequency increases (solid curves). The parameters used for the calculation is given in table 1. And the linewidth enhancement factor \( \zeta \) is assumed to be 4.5, which is from [19]. From these figures, one sees the chirp due to the variation of the carrier density in the gain section decreases as the modulation frequency increases. In order to give a comparison, the case for current modulation is also calculated (dashed curves in figures 8(a)–(c)). It is shown that the chirp for the I-modulation case increases linearly as the modulation frequency increases.

5.2. The chirp due to the variation of the carrier density in the modulation section

5.2.1. The case of \( \tau_p \)-modulation by modifying \( \alpha \). In this case, the absorption loss \( \alpha \) is modified by depleting carriers in the SOI ridge region. When the carrier concentration changes, both the real and imaginary parts of the refractive index of the Si change. For Si, the changes of refractive
Figure 8. The frequency variation (a), the optical power variation (b) and the CPR (c) as the frequency of \(\tau_p\)-modulation increases.

Index and of absorption coefficient due to the variation of the carrier density are given by [20]

\[
\Delta n = -8.8 \times 10^{-22} \Delta N_e - 8.5 \times 10^{-18} (\Delta N_h)^{0.8},
\]

\[
\Delta \alpha = 8.5 \times 10^{-18} \Delta N_e + 6.0 \times 10^{-18} \Delta N_h,
\]

where \(\Delta N_e\) and \(\Delta N_h\) are the carrier density of electrons and holes, respectively. This introduces a change of the effective index of the hybrid waveguide in the cavity. The wavelength shift and the total loss are estimated by

\[
\Delta \lambda = (\Delta n/n) \Gamma_{Si}(l_{mod}/L_{cav}) \lambda,
\]

\[
\alpha = \alpha_0 + \Gamma_{Si} \Delta \alpha,
\]

where \(\Gamma_{Si}\) is the power confinement ratio in silicon layer of the hybrid waveguide. In our calculation, we assume that \(\Gamma_{Si} = 0.3\) and \(l_{mod} = l_{cav} = 200\ \mu m\). Figures 9(a) and (b) show the calculated photon lifetime \(\tau_p\) and the wavelength variation \(\Delta \lambda\) as the carrier density varies from \(10 \times 10^{17}\ cm^{-3}\) to 0. From this result, one sees the photon lifetime \(\tau_p\) is only modulated slightly from 1.788 to 1.686 ps even when there is a high modulation of the carrier density (varying from 0 to \(10 \times 10^{17}\ cm^{-3}\)). Since a larger modulation of photon lifetime \(\tau_p\) is desirable, one could choose a longer cavity or larger variation of carrier density. However, usually the modulation range of carrier density in Si layer is lower than \(1 \times 10^{17}\ cm^{-3}\). On the other hand, the wavelength variation due to the carrier-density modulation of \(10 \times 10^{17}\ cm^{-3}\) is up to 0.31 nm.

In order to minimize the chirp, a potential solution is to modulate the carrier concentrations in the III–V region and the Si region by the same drive signal simultaneously. When the signal depleted the carriers from the Si waveguide, it would increase the injection current into the III–V gain section at the same time. In this case, the refractive index in the Si region would
The carrier density (cm$^{-3}$)
$\lambda \Delta (\text{nm})$
$\tau$ $p$ $s^p( )$

Figure 9. For the case of $\tau_p$-modulation by modifying $\alpha_i$: (a) the variation of the photon lifetime $\tau_p$ and (b) the wavelength variation $\Delta \lambda$.

increase while that in the III–V region would decrease. Therefore, the effective index would change slightly, and consequently the chirp could be minimized. Furthermore, the modulation of the photon lifetime would be enhanced.

5.2.2. For the case of $\tau_p$-modulation by modifying $R$. In our design, we use a MZI-coupled microring laser (as shown in figure 3(a)). In this structure, the feedback coefficient $R$ of the microring laser is modified by changing the amplitude or phase delay of the arm of the MZI coupler. The transmission coefficient of an MZI is given by

$$
t = \exp \left( j \phi_1 \right) \left[ (1 - \kappa^2) - \kappa^2 \gamma \exp \left( j \Delta \phi_{21} \right) \right],
$$

where $\kappa$ is the coupling coefficient for coupler used in the MZI, $\gamma = \exp (-\alpha \ell_{\text{mod}})$ (here $\alpha$ is the absorption coefficient due to carrier injection and $\ell_{\text{mod}}$ is the modulation length), $\phi_1$ is the phase delay of arm #1 and $\Delta \phi_{21}$ is the phase difference between the two arms. One has

$$
\phi_1 = n_{\text{eff}} L_1 2\pi / \lambda,
$$

$$
\Delta \phi_{21} = \Delta \phi_{21(0)} + \Delta n_{\text{eff}} \ell_{\text{mod}} 2\pi / \lambda,
$$

where $\Delta \phi_{21(0)}$ is the phase difference when there is no modulation, $\Delta \phi_{21(0)} = n_{\text{eff}} L_{21} 2\pi / \lambda$. From equation (1),

$$
t = |t| \exp \left[ j (\phi_1 + \delta \phi_{\text{MZI}}) \right] = |t| \exp \left( j \phi_{\text{MZI}} \right),
$$

where

$$
\phi_{\text{MZI}} = \phi_1 + \delta \phi_{\text{MZI}},
$$

$$
\delta \phi_{\text{MZI}} = \tan^{-1} \left[ \frac{-\kappa^2 \gamma \sin(\Delta \phi_{21})}{(1 - \kappa^2) - \kappa^2 \gamma \cos(\Delta \phi_{21})} \right].
$$

Here $\phi_{\text{MZI}}$ is the phase response of an MZI. $|t|^2$ is similar to the reflectivity of the mirror in an F–P cavity and thus the effective absorption loss is given by $\alpha = \frac{t}{L_{\text{cav}}} \ln \frac{1}{|t|^2}$. The total loss is then given by $\alpha = \alpha_0 + \alpha_i$. When the MZI coupler is modulated, the photon lifetime for the MZI-coupled MRR cavity will be changed. For an MZI-coupled MRR, the phase shift in

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the modulated MZI coupler will cause a wavelength shift. For a resonant wavelength $\lambda_m$ of MZI-coupled MRR,

$$\phi_{\text{MZI}} + n_{\text{eff}} L'_{\text{cav}} 2\pi / \lambda_m = m 2\pi,$$

where $L'_{\text{cav}} = L_{\text{cav}} - L_1$ (in which $L_{\text{cav}}$ is the cavity length). Inserting equation $(15a)$ into the equation above,

$$\delta \phi_{\text{MZI}(\lambda_m)} + n_{\text{eff}} L_{\text{cav}} 2\pi / \lambda_m = m 2\pi.$$

For the case without modulation,

$$\delta \phi_{\text{MZI}(\lambda_m)} + n_{\text{eff}} L_{\text{cav}} 2\pi / \lambda_m = m 2\pi.$$  \hspace{1cm} (17a)

For the case with modulation,

$$\delta \phi'_{\text{MZI}(\lambda_m)} + n_{\text{eff}} L_{\text{cav}} 2\pi / \lambda'_m = m 2\pi.$$  \hspace{1cm} (17b)

From equation $(17a)$ and $(17b)$,

$$\Delta \lambda = \lambda_m - \lambda'_m \approx \frac{\delta \phi_{\text{MZI}(\lambda_m)} - \delta \phi'_{\text{MZI}(\lambda_m)}}{2\pi n_{\text{eff}} L_{\text{cav}}} \lambda_m^2.$$

Here, we consider the method of using electric-field-induced carrier depletion in a Si waveguide with a reverse biased P–N junction. Assuming that the modulation length $L_{\text{mod}} = 400 \mu$m and that the maximum carrier density $N = 10^{17} \text{cm}^{-3}$. For the MZI coupler, we assume that $\Delta \phi_{21(0)} = 1.309$ and $\kappa = 0.635$, which are chosen to have desirable photon lifetimes ($\tau_{p,\text{on}} = 1.073 \text{ps}$ ($R = 0.156$) and $\tau_{p,\text{off}} = 1.788 \text{ps}$ ($R = 0.4$)) when modulated. The calculated photon lifetime $\tau_p$ and the wavelength variation $\Delta \lambda$ are shown in figures 10(a) and (b) as the carrier density varies. From this figure, it is clear that when the carrier density decreases from $0.7 \times 10^{17} \text{cm}^{-3}$ to 0, the photon lifetime increases from 1.788 to 1.073 ps and the wavelength variation is smaller than 0.024 nm.

**Figure 10.** For the case of $\tau_p$-modulation by modifying $R$: (a) the variation of photon lifetime $\tau_p$ and (b) the wavelength variation $\Delta \lambda$. 

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6. Conclusion

We have proposed two methods of realizing a high speed $\tau_p$-modulated hybrid silicon laser by modulating the distribution loss $\alpha_i$ or the feedback coefficient $R$. By using the small signal modeling and FDM-based numerical simulation, the modulation responses in the frequency and time domains have been calculated. It has been shown that it is possible to achieve a 3 dB bandwidth of over 100 GHz by using $\tau_p$-modulation. The simulation also showed an open eye diagram even when the data rate reaches 50 GHz. Additionally, the chirp due to variations of the carrier densities in the gain section and the modulation section has also been analyzed. It has been shown that the chirp is very low even at very high frequencies.

Acknowledgments

This work was sponsored by the Defense Advanced Research Projects Agency (DARPA) under contract number HR0011-08-1-0006. The authors acknowledge useful conversations with Mike Haney, Larry Coldren, Mark Rodwell and Di Liang.

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