Backreaction and Unruh effect: New insights from exact solutions of uniformly accelerated detectors

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Using nonperturbative results obtained recently for an uniformly accelerated Unruh-DeWitt detector, we discover new features in the dynamical evolution of the detector’s internal degree of freedom, and identified the Unruh effect derived originally from time-dependent perturbation theory as operative in the ultra-weak coupling and ultra-high acceleration limits. The mutual interaction between the detector and the field engenders entanglement between them, and tracing out the field leads to a mixed state of the detector even for a detector at rest in Minkowski vacuum. Our findings based on this exact solution shows clearly the differences from the ordinary result where the quantum field’s backreaction is ignored in that the detector no longer behaves like a perfect thermometer. From a calculation of the evolution of the reduced density matrix of the detector, we find that the transition probability from the initial ground state over an infinitely long duration of interaction derived from time-dependent perturbation theory is existent in the exact solution only in transient under special limiting conditions corresponding to the Markovian regime. Furthermore, the detector at late times never sees an exact Boltzmann distribution over the energy eigenstates of the free detector, thus in the non-Markovian regime covering a wider range of parameters the Unruh temperature cannot be identified inside the detector.

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I. INTRODUCTION

The Unruh effect states that an observer while undergoing uniform acceleration in the Minkowski vacuum feels as if it lives in a thermal state at the Unruh temperature. It is usually demonstrated by way of time-dependent perturbation theory (TDPT) for a “particle detector” over an infinitely long duration of interaction [1, 2, 3]. Consider the detector as a quantum mechanical object with internal degree of freedom $Q$ coupling to a quantum field $\Phi$ by the interaction Hamiltonian

$$H_I = \lambda_0 Q(\tau)\Phi(z^\mu(\tau))$$

where $\lambda_0$ is the coupling constant, $\tau$ is the proper time for the detector and $z^\mu(\tau)$ is the trajectory of the uniformly accelerated detector (UAD) with proper acceleration $a$. Suppose initially the detector-field system is in a product state, i.e., they are uncorrelated,

$$| \tau_0 \rightarrow -\infty \rangle = | E_0 \rangle \otimes | 0_M \rangle,$$

where $| E_0 \rangle$ is the ground state of the free detector and $| 0_M \rangle$ is the Minkowski vacuum of the free field. From TDPT, the transition probability $P_{0\rightarrow n}$ from the ground state to the $n$-th excited state $| E_n \rangle$ of the detector is given to first order in $\gamma \equiv \lambda_0^2/8\pi m_0$, by [2]

$$P_{0\rightarrow n} = \frac{\hbar^2}{\lambda_0^2} \sum_{n_k} \left| \langle E_n, n_k | \int_{-\infty}^{\infty} d\tau H_I(\tau)e^{i(E_n - E_0)\tau/\hbar} | E_0, 0_M \rangle \right|^2$$

$$= \frac{\lambda_0^2}{\hbar^2} |\langle E_n | Q(0) | E_0 \rangle|^2 \mathcal{R}\left(\frac{E_n - E_0}{\hbar}\right),$$

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which is nonzero. Here
\[
\mathcal{R}(\kappa) = \int_{-\infty}^{\infty} d\tau d\tau' e^{-i\kappa(\tau-\tau')} \langle 0_M| \Phi(z(\tau)) \Phi(z(\tau'))|0_M \rangle = \frac{\hbar}{2\pi} \frac{\kappa \eta}{e^{2\pi \kappa / a} - 1}.
\]

is the response function with \( \eta \equiv \int_{-\infty}^{\infty} d\tau \) being the duration of interaction [20]. In particular, for a harmonic oscillator with renormalized natural frequency \( \Omega_0 \), all the transition probabilities with \( n > 1 \) are \( O(\gamma^2) \), so the only non-vanishing \( P \) of \( O(\gamma) \) is
\[
P_{0\rightarrow 1} = \frac{\lambda_0^2}{4\pi m_0} \frac{\eta}{e^{2\pi \eta / a} - 1}.
\]

From the Planck factor in (5) one can identify the Unruh temperature \( T_U = \hbar a / 2\pi k_B \). When \( a \to 0 \), the transition probability per unit time \( P_{0\rightarrow 1} / \eta \) vanishes, which implies that there is no excitation in an inertial detector initially prepared in its ground state [2].

According to the above calculation of the transition probability one may expect that after the coupling is switched on, the reduced density matrix of the detector would evolve continuously from the ground state to higher excited states, with the detector finally ending up in equilibrium with a Boltzmann distribution \( \sim \exp[-E_n/(k_B T_U)] \) at the Unruh temperature \( T_U \). Then one can read off the Unruh temperature from the late-time distribution inside the detector.

In this note we use the exact solutions to an Unruh-Dewitt detector [1, 2] in uniform acceleration interacting with a quantum field to show that the above described conventional scenario is good only in the Markovian regime corresponding to the limits of ultra-high acceleration or ultra-weak coupling. The transition probability calculated from the infinite-time TDPT is valid only in transient under restricted conditions. The evolution of the detector under general conditions is quite different from the above picture.

The difference between this new understanding and the conventional picture originates from the fact that the full interplay between the detector and the field is included here whereas the conventional approach based on perturbation theory operative for infinite time ignores the backreactions – the use of TDPT over indefinite time amounts to invoking a Markovian approximation which imposes rather severe limitations. The conventional wisdom is built upon conditions which cannot reflect the most general features in the full dynamics of the detector-field system made possible here by the nonperturbative solutions.

This paper is organized as follows: In Sec.II we consider a moving Unruh-DeWitt detector in 4-D Minkowski space coupled to a quantum scalar field \( \Phi \) and calculate the reduced density matrix after tracing out the field. In Sec.III we calculate the transition probability of the detector from the ground state to an excited state. We elaborate on the physical meaning of the two constants \( \Lambda_0, \Lambda_1 \) in the theory. In Sec.IV we calculate the purity and the von Neumann entropy or the entropy of entanglement of the detector-field system and identify an effective temperature \( T_{\text{eff}} \) which is ostensibly different from the Unruh temperature. We identify the range of validity of the conventional results to that which corresponds to making a Markovian approximation with its strong limitations. In Sec.V we summarize our main results with remarks pertaining to the issues raised in this paper and conclude with a comment on how our nonperturbative results may bear on a new understanding of the Hawking effect of black hole radiance.

II. REDUCED DENSITY MATRIX FOR DETECTOR

Consider an Unruh-DeWitt (UD) detector moving in (3+1) dimensional Minkowski space. The total action is given by [4]
\[
S = \int d\tau \frac{m_0}{2} [\left( \partial_\tau Q \right)^2 - \Omega_0^2 Q^2] - \int d^4x \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \lambda_0 \int d\tau \int d^4x Q(\tau) \Phi(x) \delta^4(x - z(\tau)),
\]

where \( Q \) is the internal degree of freedom of the detector, assumed to be a harmonic oscillator with bare mass \( m_0 \) and bare natural frequency \( \Omega_0 \). The scalar field \( \Phi \) is assumed to be massless, and \( \lambda_0 \) is the coupling constant.

This UD detector theory behaves like the quantum Brownian motion (QBM) of a harmonic oscillator interacting with an Ohmic bath provided by the 4-D scalar quantum field [5]. The QBM model is a useful comparison because it shows clearly the dissipative and stochastic behavior of the dynamics arising from the interplay between the system and the environment, and the influence functional treatment incorporates the backreaction of the environment on the system (which could be either the quantum field or the harmonic oscillator depending on what one is after) in a self-consistent way. Since the QBM model indicates that there are nontrivial activities at zero temperature [5, 6, 7], we caution that even for the \( a = 0 \) case the detector is not just laying idle but has interesting physical features due to its interaction with the vacuum fluctuations in the quantum field.
Unruh and Zurek [7] have studied an exactly solvable QBM model where a harmonic oscillator interacts with a massless scalar field in 2-D. They derived the exact master equation for the reduced density matrix of the system (oscillator) at a temperature determined by the initial state of the field, and observed some general features different from the conventional Markovian results (valid for an ohmic bath at ultra-high temperature). One feature is the dependence of the UV cut-off in the master equation and the reduced density matrix, and thus also in the von Neumann entropy of the system. When the interaction is switched on, the factors in the master equation and the entropy have initial “jolts” over a very short time scale corresponding to the UV cut-off frequency, which cause significant change in the coherence of the quantum state of the system, while the coherence residing in each subsystem is rapidly transferred into the correlation between them. This effect is discussed in detail in [6]. Below we will see a similar behavior occurring in UD detector theory beyond the Markovian regime.

Suppose the initial state of the system at \( \tau_0 \) is given by \([2]\), a direct product of the ground state for \( Q \) and the Minkowski vacuum for \( \Phi \). In the Schrödinger representation, this initial state is a product of Gaussian functions,

\[
\psi_0(\tau_0) = N \exp \left[ -A Q^2 - \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} B^{kk'} \Phi_k \Phi_{k'} \right].
\]

(7)

Owing to the linearity of the model, this state must evolve into a general Gaussian form

\[
\psi_\tau(\tau) = N(\tau) \exp \left[ -A(\tau) Q^2 + B^{kk'}(\tau) \Phi_k \Phi_{k'} - 2C^k(\tau) \Phi_k Q \right],
\]

(8)
after the coupling is switched on. Here we have used the DeWitt notation where each upper-lower indices pairing indicates an integration (or summation). So the density matrix for this pure state in the \( (Q, \Phi_k) \) representation can be written as \( \rho[Q, \Phi_k, Q'; \Phi_k'; \tau] = \psi_0[Q, \Phi_k; \tau]\psi_0^*[Q', \Phi_k'; \tau] \), and the reduced density matrix defined by tracing out the field \( \Phi_k \) reads

\[
\rho^R(Q, Q'; \tau) = \int \mathcal{D}\Phi_k \psi_\tau[Q, \Phi_k; \tau]\psi_\tau^*[Q', \Phi_k; \tau] = \exp \left[ -G^{ij}(\tau) Q_i Q_j - F(\tau) \right],
\]

(9)

where \( i, j = 1, 2, Q_i = (Q, Q') \). The factors \( G^{ij} \) and \( F \) could be obtained by solving the master equations [8]. But here we do not need to solve them directly, because the two-point functions of the detector have been obtained in the Heisenberg picture in Ref.[4]. (These results are placed in Appendix A for convenience.) One can reconstruct the complete evolution of the reduced density matrix using those two-point functions of the detector by solving the simple algebraic relations

\[
G^{11} + G^{22} + 2G^{12} = \frac{1}{2} \langle Q^2 \rangle
\]

(10)

\[
G^{11} + G^{22} - 2G^{12} = \frac{2}{\hbar^2} \langle Q^2 \rangle \left[ \langle P^2 \rangle \langle Q^2 \rangle - \langle P, Q \rangle^2 \right]
\]

(11)

\[
G^{11} - G^{22} = -\frac{i}{\hbar} \langle P, Q \rangle \langle Q^2 \rangle
\]

(12)

where \( \langle P, Q \rangle \equiv \frac{1}{2} \langle (PQ + QP) \rangle = \langle m_\phi \rangle (d/d\tau) \langle Q^2 \rangle \) (and \( \langle [P, Q] \rangle = -i\hbar \)). Substituting (11A4) gives the values of \( G^{ij} \) at every moment, and the factor \( F \) in (9) will be determined by a normalization condition.

To compare with the transition probability (5), the reduced density matrix (9) has to be further transformed to the representation in the basis of energy-eigenstate for the “free” harmonic oscillator \( Q \):

\[
\rho^R(Q, Q') = \sum_{m,n \geq 0} \rho^R_{m,n} \phi_m(Q) \phi_n(Q')
\]

(13)

where the wave function for the \( n \)-th excited state is

\[
\phi_n(Q) = \sqrt{\frac{\alpha}{2^n n! \sqrt{\pi}}} H_n(\alpha \Omega) e^{-\alpha^2 Q^2/2}
\]

(14)

with the Hermite polynomial \( H_n(x) \) and the real constant

\[
\alpha = \sqrt{\frac{m_\phi \Omega}{\hbar}}.
\]

(15)
Here $\Omega_r$ is the renormalized natural frequency [4]. Following the method given by Ruiz [9], the matrix elements $\rho_{m,n}^R$ could be extracted by comparing the coefficients of $s_2^n s_2^m$ on both sides of the following equation,

$$\sum_{m,n} \sqrt{\frac{2m+n}{m!n!}} \rho_{m,n}^R s_2^m s_2^n = \frac{\alpha}{\sqrt{2G}} \exp \left( \frac{1}{G} [\tilde{g}^{11} s_2^1 + \tilde{g}^{22} s_2^2 + 2\tilde{g}^{12} s_1 s_2] \right),$$  \hspace{1cm} (16)

where

$$\tilde{g}^{11} = (\tilde{g}^{22})^* = \frac{\alpha^4}{4} - \frac{i\alpha^2 (P, Q)}{4h^2 (Q^2)} + \frac{i\alpha^2 (P, Q)}{2h (Q^2)},$$  \hspace{1cm} (17)

$$\tilde{g}^{12} = -\frac{\alpha^2}{2} \left[ \frac{1}{4 (Q^2)} - \frac{(P, Q)}{h^2 (Q^2)} \right],$$  \hspace{1cm} (18)

$$\tilde{G} = \frac{\alpha^4}{4} + \frac{\alpha^2}{2} \left[ \frac{1}{4 (Q^2)} + \frac{(P, Q)}{h^2 (Q^2)} - \frac{(P, Q)}{h^2 (Q^2)} + \frac{(P, Q)}{4h^2 (Q^2)} \right].$$  \hspace{1cm} (19)

### III. TRANSITION PROBABILITY

The transition probability from the initial ground state to the first excited state is the $m = n = 1$ component of the reduced density matrix in energy eigenstate representation [32]:

$$\rho_{1,1}^R = \frac{\hbar \left[ \langle P^2 \rangle (Q^2) - \langle P, Q \rangle^2 - (h^2/4) \right]}{\left\{ [\hbar^{-1} \alpha^{-2} (P^2) + (h/2)] [\hbar \alpha^2 (Q^2) + (h/2)] - \langle P, Q \rangle^2 \right\}^{3/2}}.$$ \hspace{1cm} (20)

Expanding the two-point functions of the detector [A.1 - A.4] in terms of $\gamma$, $\langle Q^2 \rangle$ and $\langle P^2 \rangle$ look like

$$\langle Q^2 \rangle \approx \frac{\hbar}{2m_0\Omega_r} + \gamma \langle Q^2 \rangle \langle P^2 \rangle \langle P, Q \rangle + O(\gamma^2),$$  \hspace{1cm} (21)

$$\langle P^2 \rangle \approx \frac{\hbar}{2m_0\Omega_r} + \gamma \langle P^2 \rangle \langle P, Q \rangle + O(\gamma^2),$$

and $\langle P, Q \rangle \sim O(\gamma)$, which yield

$$\rho_{1,1}^R \approx \frac{\gamma}{2\hbar m_0\Omega_r} \left[ \langle P^2 \rangle \langle P, Q \rangle \right] + O(\gamma^2).$$ \hspace{1cm} (22)

When $\eta \equiv \tau - \tau_0 \gg a^{-1}$ the approximate value up to the first order of $\gamma \equiv \lambda_0^2/8\pi m_0$ becomes

$$\rho_{1,1}^R \mid_{\gamma \eta \to 0} \approx \frac{\lambda_0^2}{4\pi m_0} \left[ \frac{\eta}{e^{2\pi \Omega_r/a} - 1} + \frac{\Lambda_1 + \Lambda_0 - 2\ln(a/\Omega_r)}{2\pi \Omega_r} \right]$$ \hspace{1cm} (23)

from [A.1 - A.4]. Here $\Lambda_0$ and $\Lambda_1$ are large constants in [A.3] and [A.4]: $\Lambda_1$ denotes the time resolution/frequency cut-off of this detector theory, while $\Lambda_0$ denotes the time scale of switching on the interaction.

### A. Range of validity in conventional results

We see that the first term of (23) gives the conventional transition probability [5] from TDPT over infinite time. Only when $\Omega, \eta \gg \Lambda_1, \Lambda_0$, or $a$ is extremely large, can the second term in (23) be neglected and [5] recovered. Hence the conventional transition probability [5] is valid only in the limits of (a) ultra-high acceleration ($a \gg \Omega_r$ and $\Lambda_1 \ll a \eta \ll a^{-1}$) or (b) ultra-weak coupling ($a^{-1}, \Omega_r^{-1} \Lambda_1 \ll \eta \ll \gamma^{-1}$).

Note that, in obtaining (23), we have assumed $a^{-1} \ll \eta \ll a^{-1}$, when the system is still in transient. If $a < \eta$, the conventional transition probability [5] has no chance to dominate at all. Mathematically, the first term in (23) is contributed by the poles at $\pm i\eta$ or $\pm i\gamma$ in the $\kappa$-integrations of two-point functions (see Eqs. (60) and (67) in [4]). But when $a < \gamma$ the poles at $\pm ia$ would be closer to the real $\kappa$ axis than the poles at $\pm \Omega \pm i\gamma$, while the poles on
the imaginary axis $\kappa = \pm ina$, $n \in N$ become very dense for small $a$, so their contributions dominate the result and the first term of (23) becomes unimportant.

In particular, the $a = 0$ case is beyond the reach of TDPT over infinite time shown in Section I, and the conventional wisdom from perturbation theory that no transition occurs in an inertial detector is untenable. In contrast, our result indicates that the evolution of $\rho_{1,1}^R$ with $a = 0$ behaves qualitatively similar to those cases with nonzero acceleration $[4]$. This agrees with the expectation from the observation that the UD detector theory is a special case of quantum Brownian motion $[3]$, where there is nontrivial interplay between the oscillator and the quantum field at zero temperature.

B. Essential roles of $\Lambda_0$ and $\Lambda_1$

We see the presence of two constants $\Lambda_0, \Lambda_1$ and may wonder whether they have some real physical meaning or are just part of a calculational tool or artifact. We will address these concerns here. To begin with, the presence of large constants corresponding to frequency cut-offs in the coincidence limit of two-point functions of the detector is a common feature for detector-field theories and quantum Brownian motion. For example, the Raine-Sciama-Grove(RSG) model in (1+1)D $[13]$, in which the detector acts like a harmonic oscillator in a sub-Ohmic bath, also has two-point functions of the detector with dependence of large constants due to the infrared cut-off.

Let us explore the physical meaning of $\Lambda_0$ and $\Lambda_1$. Since $\Lambda_0$ corresponds to the time scale of switching on the interaction, it could be finite in real processes, and for every finite value of $\Lambda_0$, the $\Lambda_0$ terms in all two-point functions vanish at late times (see $[13]$ and $[14]$). Hence $\Lambda_0$ will not be present in the late-time results. On the other hand, $\Lambda_1$ is a constant of time, appearing from the very beginning and never decays. One way to see that it is a quantity of real physical meaning is that if $\Lambda_1$ was subtracted naively, the uncertainty principle will be violated, namely, $\Delta P \Delta Q = \sqrt{\langle P^2 \rangle \langle Q^2 \rangle} < \hbar/2$ at late times for $a$ is small enough, as shown in FIG.1.

Actually (22) is formally identical to the first-order transition probability from TDPT for a UAD with finite duration of interaction $(\tau_0, \tau)$ $[10]$, so is the result (23). To verify this, first note that the $O(\gamma)$ term in (22) is the $O(\Lambda_0^2)$ term of

$$\frac{1}{\hbar \Omega_r} \langle E_0, 0_M | \hat{H}_Q(\eta) | E_0, 0_M \rangle.$$  \hspace{1cm} (24)

Here $\hat{H}_Q(\eta)$ is defined by

$$\hat{H}_Q(\eta) \equiv e^{-\frac{s}{\hbar} \hat{H}_Q} e^{\frac{s}{\hbar} \hat{H}_Q} \hat{H} e^{-\frac{s}{\hbar} \hat{H}_Q} e^{\frac{s}{\hbar} \hat{H}_Q},$$  \hspace{1cm} (25)

where $\hat{H}(\tau)$ is the total Hamiltonian for the combined system, $\hat{H}_Q$ is the Hamiltonian for the "free" detector so that $| E_0 \rangle$ is an eigenstate of $\hat{H}_Q$. Then in the interaction picture it is straightforward to show that

$$\rho_{1,1}^R \approx \frac{\lambda_0^2}{\hbar \Omega_r} \int_{\tau_0}^{\tau} \int_{\tau_0}^{\tau} d\tau_1 d\tau_2 (E_1 - E_0) \langle E_1 | \hat{Q}(0) | E_0 \rangle^2 e^{-\frac{\hbar}{2} (E_1 - E_0)(\tau_1 - \tau_2)} \langle 0_M | \Phi(z(\tau_1)) \Phi(z(\tau_2)) | 0_M \rangle,$$  \hspace{1cm} (26)

from which one recovers the finite-time transition probability from TDPT (cf. Eq.3) since $E_1 - E_0 = \hbar \Omega_r$. Mathematically the large constants $\Lambda_0$ and $\Lambda_1$ are formally the same as the divergences found in Ref.10. In Ref.11 it has been shown that these divergences can be tamed if one switches on and off the interaction smoothly, so can $\Lambda_0$. Nevertheless, the physical meaning of $\Lambda_1$ here is totally different from those divergences of the finite-time UAD. Here we are looking at the real-time causal evolution problem ("in-in" formulation) rather than a scattering transition amplitude ("in-out" formulation) problem. Also in our set-up we never turn off the coupling and $\Lambda_1$ is present at every moment.

As was already found in Ref.12 the constant $\Lambda_1$ changes the scenario of the evolution of the system in an essential way. $\Lambda_1$ is present whenever the coupling is on, so right after the initial moment the values of $\rho_{m,n}^R$, $m, n > 0$ jump from zero to large numbers depending on $\Lambda_1$, in a time scale indistinguishable from zero at the level of precision of this theory. This means that the initial distribution of $\rho_{m,n}^R$ peaked at the element $\rho_{0,0}^R$ would upon the switch-on of the coupling collapse rapidly (rather than smoothly diffuse) into a distribution widely spread (if $\Lambda_1$ is large) over the whole density matrix. After that the density matrix begins to redistribute itself and finally settles into a steady state.

IV. PURITY, ENTROPY AND EFFECTIVE TEMPERATURE IN DETECTOR

After substituting the late-time two-point functions $[17]$ and $[18]$ with $\Lambda_1$ kept in $\langle P^2 \rangle$ into $[10]$, we find that, in steady state, the diagonal terms of $\rho_{m,n}^R$ in the free eigenstate representation do not assume the form of a Boltzmann
distribution, while the off-diagonal terms of $\rho_{m,n}^R$ do not vanish at late times. Therefore the free eigenstates are no longer orthogonal when interactions are introduced.

To define an effective temperature and the entropy of entanglement, one should choose another set of orthogonal states which have a similar form as the energy eigenstate of the free detector but with $\alpha$ in (15) replaced by

$$\alpha = \left[ \frac{\langle P^2 \rangle}{\hbar^2 \langle Q^2 \rangle} \right]^{1/4}.$$ (27)

Since our initial state is Gaussian, one can further apply a canonical transformation introduced by Unruh and Zurek [7] to diagonalize the reduced density matrix into

$$\tilde{\rho}_{m,n}^R = \hbar \delta_{mn} \frac{(U - \hbar/2)^n}{(U + \hbar/2)^{n+1}},$$ (28)

where

$$U \equiv \sqrt{\langle P^2 \rangle \langle Q^2 \rangle} - \langle P \cdot Q \rangle^2.$$ (29)

is the uncertainty function and $U \geq \hbar/2$ is the Robertson-Schrödinger uncertainty relation. From (28) one immediately obtains the purity $\mathcal{P}$ for the reduced density matrix [9],

$$\mathcal{P} = Tr[\tilde{\rho}_{m,n}^R \tilde{\rho}_{m,n}^R] = \frac{\hbar/2}{U}.$$ (30)

In this case $\mathcal{P}^{-1}$ is the ratio of the uncertainty function $U$ to its minimal value $\hbar/2$, hence one always has $\mathcal{P} \in (0, 1)$ because of the uncertainty principle. Also the von Neumann entropy or the entropy of entanglement reads

$$S \equiv -Tr(\tilde{\rho}^R \ln \tilde{\rho}^R) = \left( \frac{U}{\hbar} + \frac{1}{2} \right) \ln \left( \frac{U}{\hbar} + \frac{1}{2} \right) - \left( \frac{U}{\hbar} - \frac{1}{2} \right) \ln \left( \frac{U}{\hbar} - \frac{1}{2} \right)$$ (31)

which is a measure of the entanglement between the detector and the quantum field.

Further, in steady state, $\langle P, Q \rangle \to 0$, so (28) becomes

$$\tilde{\rho}_{m,n}^R \xrightarrow{\gamma \Omega_{\nu} \gg 1} \hbar \delta_{mn} \frac{\left[ \sqrt{\langle P^2 \rangle \langle Q^2 \rangle} - \hbar/2 \right]^n}{\left[ \sqrt{\langle P^2 \rangle \langle Q^2 \rangle} + \hbar/2 \right]^{n+1}} \equiv \delta_{mn} \rho_{0,0}^R \times e^{-nM_{\nu}/k_B T_{\text{eff}}}. $$ (32)

Now the diagonal terms assume a Boltzmann distribution with the effective temperature

$$T_{\text{eff}} = \frac{k_B}{\hbar \Omega_{\nu}} \ln \left( \frac{\sqrt{\langle P^2 \rangle \langle Q^2 \rangle} + \hbar/2}{\sqrt{\langle P^2 \rangle \langle Q^2 \rangle} - \hbar/2} \right)^{-1}.$$ (33)

This is not surprising: Even for such a simple system containing only two coupled harmonic oscillators, the ground state of the total system also looks like a thermal state with some effective temperature in view of the reduced density matrix for one of the oscillator [12].
FIG. 2: The evolution of the entropy of entanglement $S(\eta)$ in (31) for a detector initially in the ground state. At late times, the entropy converges to a large number if $\Lambda_1$ is large. Here the parameters are taken to be $a = 2, \gamma = 0.1, \Omega = 2.3, m_0 = 1, \Lambda_1 = \Lambda_0 = 10000$, and $\hbar = c = 1$.

A. Realistic cases: $\Omega \Lambda_1 \gg a, \gamma$

Since $\Lambda_1$ corresponds to the cut-off frequency of the theory, in real processes it is most likely that $\Omega \Lambda_1 \gg a, \gamma$. The evolution of the entropy of entanglement in this case is shown in FIG. 2. At the initial moment the entropy of entanglement has a sudden jump from zero to a large number $\sim O(\ln \Lambda_1)$ while the entropy converges to a number of the same order at late times. Indeed, for $\Lambda_1 \gg 1$, the late-time entropy of entanglement has approximately the value

$$S \approx \frac{1}{2} \ln \Lambda_1 + 1 + \frac{1}{2} \ln \left\{ \frac{\gamma}{\pi^2} + \frac{i}{\Omega} \right\} \left( 1 + \frac{\gamma - i\Omega}{a} \right) - \frac{i}{\Omega} \left( 1 + \frac{\gamma + i\Omega}{a} \right),$$

(34)

which is dominated by the $\ln \Lambda_1$ term (the $\Lambda_0$ term dies out at late times as long as $\Lambda_0$ is finite). This indicates that the constant $\Lambda_1$ is due to the entanglement between the detector and the infinitely many degrees of freedom of the field.

For $\Lambda_1 \gg 1$, the effective temperature is approximately

$$T_{\text{eff}} \approx \frac{\hbar \Omega_r}{\pi k_B} \sqrt{\gamma \Lambda_1} \left[ \frac{a}{\Omega_r^2} + \frac{i}{\Omega} \psi \left( 1 + \frac{\gamma - i\Omega}{a} \right) - \frac{i}{\Omega} \psi \left( 1 + \frac{\gamma + i\Omega}{a} \right) \right]^{1/2} + O(\Lambda_1^{-1/2}).$$

(35)

which, as we can see, is totally different from the Unruh temperature: It is determined not only by the proper acceleration $a$, but also by the properties of the detector ($\Omega_r$), the interaction ($\gamma$), and the frequency cutoff of this theory ($\Lambda_1$). Moreover $T_{\text{eff}}$ is very large ($O(\sqrt{\Lambda_1})$) for large $\Lambda_1$ and non-vanishing even when $a \to 0$.

B. Inertial detector: $a = 0$

Substituting (A11) and (A12) into (30), (31) and (33), one can write down in closed form the purity, entropy of entanglement and the effective temperature of an inertial detector ($a = 0$). These quantities are still nontrivial. In particular, in the ultra-weak coupling limit $\gamma \Lambda_1 \ll 1$, one has

$$S \approx \frac{\gamma (\Lambda_1 - 1)}{\pi \Omega_r} \left( 1 - \ln \frac{\gamma (\Lambda_1 - 1)}{\pi \Omega_r} \right) + O(\gamma^2),$$

(36)

$$T_{\text{eff}} \approx \frac{\hbar \Omega_r}{-k_B \ln \left[ \frac{\gamma (\Lambda_1 - 1)}{\pi \Omega_r} + O(\gamma^2) \right]}$$

(37)

[Note that $\lim_{x \to a^+} \ln(-1 + x) = -i\pi$.] Both quantities go to zero as $\gamma \to 0$, when the description of the combined system returns to the nearly free theory.

C. Conventional result: the Markovian regime

As we mentioned in Section III.A, the transition probability calculated from TDPT over infinite time is valid only in the ultra-high acceleration (temperature) limit or the ultra-weak coupling limit, both corresponding to the Markovian
regime for this model (with the quantum field as an Ohmic bath). In these limits the entropy of entanglement and effective temperature do have the approximate values expected in the conventional picture:

(a) Ultra-high acceleration (temperature) limit, \( a \gg \gamma_{1,}, \Omega \). In this limit, the late-time entropy of entanglement reads

\[
S \approx \ln \frac{a}{2\pi \Omega} + 1 + O(a^{-1}),
\]

and the effective temperature

\[
T_{\text{eff}} \approx T_{U} + \frac{\hbar \gamma}{k_{B}a} \left( \Lambda + \gamma_{c} - \ln \frac{a}{\Omega} \right) + O(a^{-1}),
\]

is very close to the Unruh temperature \( T_{U} = \hbar a/2\pi k_{B} \).

(b) Ultra-weak coupling limit, \( \gamma_{A} \ll a, \Omega \). In this limit, the late-time entropy of entanglement and effective temperature read

\[
S \approx \frac{\pi \Omega}{a} \coth \frac{\pi \Omega}{a} - \ln \left[ 2 \sinh \frac{\pi \Omega}{a} \right] + \frac{2a}{\Lambda} \left( \Lambda - \ln \frac{a}{\Omega} - \text{Re} \left[ \psi \left( \frac{i\Omega}{a} \right) + \frac{i\Omega}{a} \psi^{(1)} \left( \frac{i\Omega}{a} \right) \right] \right) + O(\gamma^{2}),
\]

\[
T_{\text{eff}} \approx T_{U} + \frac{\hbar \gamma a^{2}}{k_{B}a^{2} \pi^{2} \Omega^{2}} \sinh^{2} \frac{\pi \Omega}{a} \left( \Lambda - \ln \frac{a}{\Omega} - \text{Re} \left[ \psi \left( \frac{i\Omega}{a} + \frac{i\Omega}{a} \psi^{(1)} \left( \frac{i\Omega}{a} \right) \right) \right] \right) + O(\gamma^{2}).
\]

Again the effective temperature of the detector is very close to the Unruh temperature.

It is well-known that the quantum mechanical time-dependent perturbation theory invokes the equivalent of a Markovian approximation (e.g., as in the derivation of the Pauli master equation \([13]\)). Indeed, both the above limits correspond to the Markovian regime of the quantum Brownian motion, its dynamics being described by the master equation of Caldeira and Leggett \([14]\) (in contradistinction to the non-Markovian master equation of Hu, Paz, and Zhang \([6]\)). There is admittedly a fluctuation-dissipation relation between the fluctuations in the field and the dynamics of the detector and thus backreaction is present, but with ultra-weak coupling it is restricted to the linear response regime (which has limited domain of applicability compared to the full nonequilibrium dynamics of the combined system). Here, the thermal bath is only slightly affected by the back reaction from the detector to the field, namely, the detector acts essentially as a test particle in the field. It is only under these special assumptions that the detector can come to equilibrium with an approximate thermal field at the Unruh temperature. Beyond this regime, the dynamics of the detector-field system has a totally different character.

V. REMARKS

We conclude with a few remarks:

a. Working range of Unruh-DeWitt detectors. Conventional results from time-dependent perturbation theory over an infinitely long duration of interaction are trustworthy only in the ultra-high acceleration (or temperature) limit and the ultra-weak coupling limit, under the Markovian regime. In the non-Markovian regimes, the large constants \( \Lambda_{1} \) and \( \Lambda_{0} \) alter the scenario fundamentally. With the presence of the large constant \( \Lambda_{1} \), which reflects the entanglement between the detector and the infinitely many degrees of freedom of the field, the evolution of the reduced density matrix of the detector collapses (sudden rapid change), as shown here, rather than diffuses (smooth gradual change), as conjured in the conventional picture. Furthermore, the detector at late times never sees an exact Boltzmann distribution over the energy eigenstates of the free detector, let alone the Unruh temperature. The strong interplay between the detector and the field makes the late-time quantum states for the detector-field system highly entangled and never factorizable (an analogy in quantum optics is the photon-atom bound state \([13]\).)

In equilibrium conditions, one can diagonalize the reduced density matrix and define an effective temperature \([35]\), which is usually large (\( \sim O(\sqrt{\Lambda_{1}}) \)) and does not vanish even for an inertial detector in Minkowski vacuum as long as the coupling is on. Only in the Markovian regime indicated above could this effective temperature get very close to the Unruh temperature.

b. Correspondence with QBM. For a uniformly accelerated UD detector in (3+1)D with proper acceleration \( a \) the Unruh effect \([1, 2, 3]\) attests that it should behave the same way as an inertial UD detector in contact with a thermal bath at Unruh temperature \( T_{U} \), or more precisely, as an inertial harmonic oscillator in contact with an Ohmic
bath at $T_B$ [8]. This is clear from an examination of the integral in the derivation of the two-point functions of the detector, for example, Eq.(60) in Ref.[4]:

$$\langle Q(\tau)Q(\tau') \rangle_v \sim \frac{\lambda^2 \hbar}{(2\pi)^2 m_0^2} \int \frac{dk}{1 - e^{-2\pi k/a}} \langle \cdots \rangle,$$  \hspace{1cm} (42)

Therefore for an inertial UD detector in contact with an Ohmic bath at temperature $T$, one can simply substitute $2\pi k_B T/\hbar$ for $a$ in the above results to get its purity, entropy and effective temperature.

However, examining this from the vantage point of the exact solutions we obtained, we see the above statements are accurate only at the initial moment. After the coupling is switched on, the quantum state of the field will have been changed by the detector, so the field is no longer in the Minkowski vacuum and it does not make exact sense to say that the detector is immersed in a thermal state (or any state defined in the test-field description, i.e., where the field is assumed not to be modified by the presence of the detector).

A theorem by Bisognano and Wichmann (BW) [16] states that the Minkowski vacuum, which is uniquely characterized by its invariance under all Poincaré translations, is a Kubo-Martin-Schwinger (KMS) state with respect to all observables confined to a Rindler wedge. One may wonder why it does not apply here. The reason is that the BW theorem refers to the vacuum state of a quantum field alone, not the combined detector-field system. Even when the combined system is in a steady state, the quantum state of the interacting field is not invariant under spatial translations in Minkowski space, hence is not covered under the assumption of the BW theorem pertaining to Poincaré invariance.

Actually the Planck factor in (42) is a consequence of the BW theorem. Nevertheless, it is derived from only the free-field-solution part of the complete interacting field (see Eqs. (28), (56) and (58) in Ref. [4]). Here the factor is not distorted by the interaction simply because the field is linear and the coupling is bilinear. For nonlinear fields or couplings it would have a non-Planckian spectrum and the departure from the conventional picture would be more pronounced.

c. Backreaction and memory effects. It is common knowledge in nonequilibrium statistical mechanics [17] that for two interacting subsystems the two ordinary differential equations governing each subsystem can be written as an integro-differential equation governing one such subsystem, thus rendering its dynamics non-Markovian, with the memory of the other subsystem’s dynamics registered in the nonlocal kernels (which are responsible for the appearance of dissipation and noise should the other subsystem possesses a much greater number of degrees of freedom and are coarse-grained in some way). Thus inclusion of back-action self-consistently in general engenders non-Markovian dynamics. For our problem the two subsystems are the detector and the quantum field. Combining Eqs. (28), (30) and (10) in Ref.[4], we can write down the equation of motion for the detector evolution functions,

$$(\partial^2_t + \Omega_0^2)q^{(+)}(\tau; k) = \frac{\lambda_0}{m_0} \left[ f_0^{(+)}(z(\tau); k) + \lambda_0 \int_{\tau_0}^\infty d\tau' G_{ret}(z(\tau); z(\tau')) q^{(+)}(\tau'; k) \right],$$

which is an integro-differential equation. The backreaction to the detector is registered through the retarded Green’s function $G_{ret}$ of the field. Various approximations are usually invoked to solve this equation, amongst which the most common is the Markovian or memoryless approximation. This is what enters in the conventional derivation of the Unruh effect, but as we see above, is a very special and nongeneric subcase.

d. Hiding $\Lambda_1$? One may wonder why the large constant $\Lambda_1$ cannot be absorbed by any parameter of this theory so one can renormalize $T_{eff}$. To begin with, as we mentioned in Sec.111B, the UD detector theory is not a fundamental theory to meet the renormalizability requirement, and the presence of cut-offs as physical parameters is an expected feature which characterizes the range of validity of this semiclassical theory, just like the Compton wavelength of the electron acting as a cut-off in quantum optics. Second, the large constant $\Lambda_1$ is not present in the renormalized stress-energy tensor of the field induced by the detector [4], thus $T_{eff}$ may not be a directly measurable quantity. The interference between the vacuum fluctuations and their back reaction cancels this cut-off dependence so that $\Lambda_1$ is not observable outside of the detector.

In some cases, though, $\Lambda_1$ term can be subtracted from the physical quantities of the detector after these quantities are worked out. For example, one can subtract $\Lambda_1$ from the detector energy defined in Eq.(82) of Ref.[4] because during a physical process only the difference of energy matters.

e. Hawking effect. It is tempting to see if analogous descriptions can be made and implications drawn for the Hawking effect. For a UD detector at rest in a static gravitational field, the response of the detector is similar. The simplest example is the UD detector fixed at radius $r$ far from a Schwarzschild black hole, while the scalar field is in Hartle-Hawking (HH) vacuum (which is the counterpart of Minkowski vacuum for the uniformly accelerated
detector in Minkowski space \(^{[3]}\)). The response function per unit time for a massless scalar field in the Schwarzschild background is given by Candelas (Section V in Ref.\(^{[19]}\)). According to \(^{[19]}\), when \(r \to \infty\), the response function for HH vacuum is exactly the same as the one in the uniform acceleration case (Eq.(58) in \(^{[4]}\)) with the Unruh temperature \(a/2\pi k_B\) replaced by the Hawking temperature \(T_{H} \equiv 1/8\pi k_B M\) for the Schwarzschild black hole with mass \(M\). Therefore the results in this paper can be directly applied to the case of UD detector fixed at \(r \to \infty\) outside a Schwarzschild black hole by neglecting its back reaction to spacetime. The effective temperature \(T_{\text{eff}}\) read off from such a detector in steady state is given by \(^{[13]}\) with \(a\) replaced by \(1/4M\) rather than the Hawking temperature \(T_{H}\).

For the case with the detector sitting very close to the event horizon, due to the effective potential barrier in the radial equation for the field, one has to consider the back-scattering of the retarded field induced by the detector in addition to the vacuum fluctuations described by the response function. We expect that the effective temperature read off from the detector in steady state would be even more complicated but definitely different from \(T_{H}\). We hope to report on this investigation in a later paper.

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**APPENDIX A: TWO-POINT FUNCTIONS OF INERTIAL DETECTORS IN MINKOWSKI VACUUM**

Recall that for a uniformly accelerated UD detector with proper acceleration \(a\) \(^{[4]}\), once we choose the factorized initial state \(\ket{2}\), the two-point functions split into two part, \((\ldots) = (\ldots)_a + (\ldots)_v\). The coincidence limits of the two-point functions of the detector with respect to its ground state (with \(q_0\) in Ref.\(^{[4]}\) being zero) read

\[
\langle Q(\eta)^2 \rangle_v = \lim_{\nu \to \frac{1}{2}} \frac{\nu}{\nu} \langle \{Q(\eta), Q(\eta')\} \rangle_v
= \frac{2\hbar \gamma}{\pi m_0 \Omega^2} \theta(\eta) \Re \left\{ \left( \Lambda_0 - \ln \frac{\hbar}{\Omega} \right) e^{-2\gamma \eta \sin^2 \Omega \eta} \right. \\
+ \frac{\hbar}{2} \left[ \frac{F_{\gamma+i\Omega} e^{-\gamma i\eta} \left( \frac{i}{\gamma} - \ln \frac{\hbar}{\Omega} \right)}{\gamma + i\Omega + a} - \frac{F_{\gamma-i\Omega} e^{-\gamma i\eta} \left( \frac{i}{\gamma} + 1 - e^{-2i\Omega \eta} \right)}{\gamma + i\Omega - a} \right] \\
\left. \left( 1 + \frac{i}{\gamma} \right) e^{i\eta \gamma} - e^{-i\eta \gamma} \right\} \\
+ \frac{1}{4} \left[ \frac{\hbar}{\gamma} e^{-2\gamma \eta} \left( \frac{\Omega}{\gamma} + 1 - e^{-2i\Omega \eta} \right) \right] \left( \psi_{\gamma+i\Omega} + \psi_{\gamma-i\Omega} \right) \\
- \left( - \frac{\hbar}{\gamma} e^{-2\gamma \eta} \left( \frac{\Omega}{\gamma} + 1 - e^{-2i\Omega \eta} \right) \right) i\pi \coth \frac{\pi}{\hbar \gamma} (\Omega - i\eta), \right. (A3)
\]

\[
\langle \dot{Q}(\eta)^2 \rangle_v = \frac{2\hbar \gamma}{\pi m_0 \Omega^2} \theta(\eta) \Re \left\{ \left( \Lambda_0 - \ln \frac{\hbar}{\Omega} \right) \Omega^2 + \left( \Lambda_0 - \ln \frac{\hbar}{\Omega} \right) e^{-2\gamma \eta} \Omega \cos \Omega \eta \gamma \sin \Omega \eta \right. \\
+ \frac{\hbar}{2} \left[ \frac{F_{\gamma+i\Omega} e^{-\gamma i\eta} \left( \frac{i}{\gamma} - \ln \frac{\hbar}{\Omega} \right)}{\gamma + i\Omega + a} - \frac{F_{\gamma-i\Omega} e^{-\gamma i\eta} \left( \frac{i}{\gamma} + 1 - e^{-2i\Omega \eta} \right)}{\gamma + i\Omega - a} \right] \\
\left. \left( 1 + \frac{i}{\gamma} \right) e^{i\eta \gamma} - e^{-i\eta \gamma} \right\} \\
+ \frac{1}{4} \left[ \frac{\hbar}{\gamma} e^{-2\gamma \eta} \left( \frac{\Omega}{\gamma} + 1 - e^{-2i\Omega \eta} \right) \right] \left( \psi_{\gamma+i\Omega} + \psi_{\gamma-i\Omega} \right) \\
- \left( - \frac{\hbar}{\gamma} e^{-2\gamma \eta} \left( \frac{\Omega}{\gamma} + 1 - e^{-2i\Omega \eta} \right) \right) i\pi \coth \frac{\pi}{\hbar \gamma} (\Omega - i\eta), \right. (A4)
\]

Here again \(\eta \equiv \tau - \tau_0\) is the duration of interaction, \(\gamma \equiv \lambda_0^2/8\pi m_0\), \(\Omega \equiv \sqrt{\Omega_0^2 - \gamma^2}\), \(F_s(y)\) is defined by the hypergeometric function as

\[
F_s(y) \equiv 2F1 \left( 1 + \frac{y}{a}, 1, 2 \right), \quad (A5)
\]
and

\[ \psi_s \equiv \psi \left( 1 + \frac{s}{a} \right) \quad \text{(A6)} \]

is the digamma function. The large constant \( \Lambda_0 \equiv -\ln \Omega |\gamma_0 - \tau_0'\| - \gamma_e \) with the Euler’s constant \( \gamma_e \) corresponds to the time scale of switching-on the interaction, so \( \Lambda_0 \) could be finite in real processes, and for every finite value of \( \Lambda_0 \), the terms containing \( \Lambda_0 \) in (A3) and (A4) vanish as \( \gamma \eta \to \infty \). The other large constant \( \Lambda_1 \equiv -\ln \Omega |\tau - \tau'| - \gamma_e \) corresponds to the time-resolution or the cut-off frequency of this theory. Note that here we use slightly different definitions of \( \Lambda_1 \) and \( \Lambda_0 \) from those of Ref.[4] to make the arguments in logarithm functions dimensionless. Note also that the above results are valid when \( \Omega |\tau - \tau'| \ll 1 \) and \( \Omega |\gamma_0 - \tau_0'\| \ll 1 \). Beyond this regime, the form of the above “coincidence” limit of two-points functions should be modified.

At late times (\( \gamma \eta \gg 1 \)), the two-point functions of the detector with respect to the initial ground state \( \langle \ldots \rangle_a \) die away, and the two-point functions of the detector saturate to

\[ \langle Q^2 \rangle \to \frac{\hbar}{2\pi m_0 \Omega} \text{Re} \left[ \frac{ia}{\gamma + i\Omega} - 2i \psi_{\gamma + i\Omega} \right], \quad \text{(A7)} \]

\[ \langle P^2 \rangle = m_0^2 \langle \dot{Q}^2 \rangle \to \frac{\hbar m_0}{2\pi \Omega} \text{Re} \left\{ \left( \Omega - i\eta \right)^2 \left[ \frac{ia}{\gamma + i\Omega} - 2i \psi_{\gamma + i\Omega} \right] \right\} + \frac{2}{\pi} \hbar m_0 \gamma \left( \Lambda_1 - \ln \frac{a}{\Omega} \right), \quad \text{(A8)} \]

which are identical to the results of the quantum Brownian motion of a harmonic oscillator in contact with an Ohmic bath at the Unruh temperature initially [8]. To satisfy the uncertainty principle for all \( a \) (see FIG.1), here we keep the constant \( \Lambda_1 \), which was subtracted in Ref.[4] because of the observation that \( \Lambda_1 \) will not be seen outside of the detector.

When \( a \to 0 \), the regular terms in the two-point functions diverge as \( \ln a \) and cancel the ones following \( \Lambda_0 \) and \( \Lambda_1 \), so the two-point functions remain well-behaved. In this limit the two-point function (A3) continuously approaches

\[ \langle Q(\eta)^2 \rangle_v |_{a=0} = \frac{\hbar \gamma}{\pi m_0 \Omega} \theta(\eta) \text{Re} \left\{ \left( \Lambda_1 \Omega + \Lambda_0 e^{-2\gamma \eta} \sin^2 \Omega \eta \right) \right\}, \quad \text{(A9)} \]

where \( \Gamma \) is the incomplete gamma function, while (A4) goes to

\[ \langle Q(\eta)^2 \rangle_v |_{a=0} = \frac{\hbar \gamma}{\pi m_0 \Omega} \theta(\eta) \text{Re} \left\{ \left[ \Lambda_1 \Omega^2 + 2 \Lambda_0 e^{-2\gamma \eta} \cos \Omega \eta \right] \right\}, \quad \text{(A10)} \]

At late times, one has

\[ \langle Q^2 \rangle |_{a=0} \to \frac{\hbar i}{2\pi m_0 \Omega} \ln \frac{\gamma - i\Omega}{\gamma + i\Omega}, \quad \text{(A11)} \]

\[ \langle P^2 \rangle |_{a=0} \to \frac{\hbar m_0}{\pi} \left\{ \frac{i}{2\Omega} (\Omega^2 - \gamma^2) \ln \frac{\gamma - i\Omega}{\gamma + i\Omega} + \gamma \left( 2 \Lambda_1 - \ln \left( 1 + \frac{\gamma^2}{\Omega^2} \right) \right) \right\}. \quad \text{(A12)} \]

Substituting this into (30), (41) and (33), one obtains the purity, entropy of entanglement and effective temperature for the inertial detector in the Minkowski vacuum.

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