Evaluation of Improved Mode Shape Curvature-Based Damage Detection using Robust Regression Method

M H C Man¹, M A Amiruddin² and N Fawazi¹
1 Mechanical Precision Engineering, Malaysia-Japan International Institute of Technology, Universiti Teknologi Malaysia, 54100 Kuala Lumpur, Malaysia.
2 Jabatan Teknologi Kejuruteraan Mekanikal, Fakulti Teknologi Kejuruteraan Mekanikal dan Pembuatan, Universiti Teknikal Malaysia, 75450, Melaka.

E-mail: mhasrizam2@graduate.utm.my

Abstract. Structural Health Monitoring (SHM) is an important tool to maintain structures integrity and safety. Gapped Smoothing Method (GSM) is a high sensitivity algorithm for damage detection using mode shape curvature (MSC) to identify small size of damage. However, it has limited capability to localize wide size of damage with decent accuracy. The objective of this study is to improve GSM algorithm using Robust Regression i.e., Iteratively Re-weighted Least Square (IRLS) methods. Subsequently, evaluate the proposed algorithm’s capability to localize damage with different size and number of measurements of an aluminum beam numerically. Damage Estimate Reliability (DER) was used to evaluate effectiveness of the improved GSM algorithm. The obtained result shows inclusion of robust regression in the original GSM algorithm increase Damage Estimate Reliability (DER) for 50mm damage size with damage level 20% by 11.6%. IRLS reduces influence of an outlier towards estimation of undamaged MSC that cause noise around damage area (in damage index plot) therefore produce more accurate estimation of damage size. Improvement of GSM allows this method to localize different level of damage in structures.

1.0 Introduction

There has been an increasing demand for continuous real-time damage detection to monitor safety of big structure like high-rise building, aircraft, and bridge to ensure their integrity and safety. Such structure has to undergo regular inspection which is usually a time-consuming and cost ineffective procedure, for example, an aviation company has spent up to 27% of aircraft life cycle cost for inspection and repair [1].

Several traditional damage detections (e.g. visual inspection and magnetic particle inspection) are limited to detect existing crack or notch on the surface only [2]. Other non-destructive detection method, such as ultrasonic and thermography is relatively high cost of operation and limited to surface types and qualities. Hence, a cost-effective structural health monitoring (SHM) method, vibration-based damage detection methods have received increasing attention for real applications [3]. It has better applicability in SHM to provide the data in real time without operator unlike traditional non-destructive inspection (NDI) that is labor intensive process and high-cost process.

Vibration-based damage detection is one of NDI method that used vibration parameters, such as frequency, curvature mode shape and damping ratio to detect and localize and damage in structure. Natural frequency information has been used for the detection of damage [4]. It has several limitations such as complexity in structural modeling and difficulty at solving non-uniqueness solutions. [5] presented an extensive review of publications dealing with the detection of structural damage through frequency changes. It was concluded that the natural frequency changes alone may not be sufficient for a unique identification of the location of structural damage because cracks associated with similar crack lengths but at two different locations may cause the same amount of frequency change.

Study by [6] shows that mode shape curvature is more sensitive than natural frequency in localizing damage however its require baseline data/mode shape curvature from undamaged structure, later the method was improved with Gapped Smoothing Method (GSM) by [7] that capable to localize damage using only damaged mode shape curvature data. GSM was further extended by u Global Fitting
Method [8] and Fourier series [9] that shows its superiority by detecting different size of notch with lower noise level around damage area in damage index plot. This method is limited to detect damage with uniform cross-section which restrict its application as NDI in real life because of non-uniform structure. In Gapped Smoothing Method (GSM), estimation of undamaged mode shape curvature using non-robust regression method (local cubic polynomial) lead to more noise in mode shape curvature data that could hide the damage location signal.

Mode shape can be measured using relatively small number of measurement points in laboratory; however, it poses difficulty for damage detection algorithm to localize damage with acceptable accuracy [10]. Although a huge number of measurement points can be monitored by means of SLV or ESPI, the measured points produces a lot of peaks that may cause a misunderstanding with real ‘damage signal’ [11]. Hence, the study of effect of limited number of measurement points to the damage detection algorithm is important for their practicability for real life application.

The objective of this paper is to compare damage detection capability between robust and non-robust curve-fitting method. Robust curve-fitting method effectiveness and limitations will be examined numerically for different damage level, number of mode shape, different length of damage and number of measurement points.

2.0 Methodology

2.1 Gapped Smoothing Method (GSM)

The work in this study is based on improvement of Gapped Smoothing Method (GSM) by [7, 8]. GSM estimated mode shape curvature, \( \phi''^d \) using the central finite difference equation from damaged structure:

\[
\phi''^d = \frac{u_{i-1} - 2u_i + u_{i+1}}{\Delta x^2} \tag{1}
\]

Where, \( u_i \) is the transverse displacement data at point nodes \( i \) and \( \Delta x \) is the displacement between two consecutive nodes. In GSM, damage location was detected from damage index (DI) plot that calculated from difference between damaged and undamaged mode shape curvature.

\[
\delta[m] = |\phi''^d - \phi''^u| \tag{2}
\]

Where, undamaged mode shape curvature was estimated using cubic polynomial regression using this equation.

\[
\phi''^u = a_0 + a_1x + a_2x^2 + a_3x^3 \tag{3}
\]

Although this method is sensitive to detect small size of crack [7, 8] shows GSM method create smeared noise near damage location for wide size of damage because estimation of undamaged mode shape curvature using cubic polynomial that has localized effect which induced noise in detection signal. This causes this method unsuitable for detecting different size of damage with good accuracy.

2.2 Fourier series Approximation

Commonly, MSC damage detection require baseline data from undamaged structure which may rarely be available in real life. Fourier series is a means to estimate undamaged structure MSC through approximation on a mode shape curvature data of the damaged structure. Fourier series is a transformation to describe a periodic signal by splitting it in a harmonic series comprising of sine and cosine terms that has following form [12]:

\[
\phi''^u = y = \frac{a_0}{2} + a_1 \cos(n\sigma x_j) + b_1 \sin(n\sigma x_j) \tag{4}
\]

Where,

\[
\sigma = 2\pi f^m \Delta t, \Delta t = 1 \tag{5}
\]

\( f^m \) is the fundamental frequency obtained from Spectrum Analysis (FFT) in LABVIEW using the MSC data for every mode shape of damaged structure. \( n \) is the number of harmonics or order of a series, it has range of order 1-8 [12]. Order of 1 used in this study.
Where $x_j$ is the nodal point and in this study total of 149, 74, 49 measurement points are used as nodal points. The obtained coefficient values $a_0$, $a_1$, $b_1$ and $\sigma$ are used to reconstruct the approximation of mode shape curvature data for undamaged structure.

### 2.3 Robust Regression: Iteratively Re-weighted Least Square

Although Fourier series could be used to estimate undamaged mode shape curvature (MSC) data. The estimated undamaged MSC data is sensitive to noise (outlier) in damaged MSC data which cause false detection in interpreting the damage index plot. Robust Regression procedures has been proposed in this study to dampen the influence of outlier in the Fourier series approximation. Iterative Re-weighted Least Square (IRLS) is one of Robust Regression method and solved using $L_1$ estimator [13].

$$L_1 = \sum |y_i - (\beta_0 + \beta_1 x_i)|$$  (9)

IRLS uses the weighted least squares procedures. The weight is based on residual instead of weight based on error variances (i.e. WLS) [13]. IRLS steps as follows:

1. Choose weight function
2. Obtain starting weight
3. Use starting weight in weighted least square
4. Use residual in step 3 to obtain revised weight
5. Continue iteration until converge

$$\phi'' u = \hat{y} = \beta_0 + \beta_1 \cos \frac{2\pi}{n} (x_j - 1) + \beta_2 \sin \frac{2\pi}{n} (x_j - 1)$$  (11)

Equation (11) can be solving using the following terms:

$$\beta = (x'w'wx)^{-1}x'w'wy$$  (12)

$$x = \begin{bmatrix} 1 & \cos \sigma_1 & \sin \sigma_1 \\ 1 & \cos \sigma_2 & \sin \sigma_2 \\ \vdots & \vdots & \vdots \\ 1 & \cos \sigma_n & \sin \sigma_n \end{bmatrix}$$  (13)

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$  (14)

$$w = \begin{bmatrix} w_1 \\ 0 \quad \cdots \quad 0 \\ \vdots \quad \ddots \quad \vdots \\ 0 \quad 0 \quad \cdots \quad w_n \end{bmatrix}$$  (15)

W is weight Function, two widely used weight functions are the Huber and bisquare [13]. This study using Huber because it less sensitive to starting value compared than Bisquare and the Huber weight function is defined as following [13]:

$$w = \begin{cases} 1 & |u_i| \leq 1.345 \\ \frac{|u_i|}{1.345} & |u_i| > 1.345 \end{cases}$$  (16)

Where $u_i$ is scaled residual

$$u_i = \frac{e_i}{MAD}$$  (17)
Where MAD is median absolute deviation of the residuals from their median, \( \text{MAD} = \frac{1}{0.675} \text{median}(|e_i - \text{median}(e_i)|) \). Calculation of Structural Irregularity Index (SII) using equation (18) from [8]:

\[
\text{Structural Irregularity Index, } \delta_i^A = \frac{N}{M} \sum_{m=1}^{M} \left( \frac{\delta_i^m}{\sum_{i=1}^{N} \delta_i^m} \right)
\]  

(18)

Where \( \delta_i^A \) is the averaged SII at a grid point of \( i \); \( N \) is the number of all grid points and \( M \) is the number of all modes.

Damage Estimate Reliability (DER) is introduced to quantify the proposed damage identification method [9]. The whole interval along the axis of the beam (x axis) is split into 2 parts; the 1st part (a) is the one which does not contain damage, namely 0 mm < x < 450 mm, 500 mm < x < 750 mm and 800 mm < x < 1250 mm (part a) and the 2nd part (b), containing damage, is 450 mm < x < 500 mm and 750 mm < x < 800 mm.

In each of these parts standardized damage indices (SIIs) from equation (18) of a respective approximation function are summed and divided by the number of data points in this interval, giving average amplitude of SDI (\( \overline{\text{SDI}}_i \)). DER is equal to average SDI in damage (part b) divided by average SDI in all parts combined. It is expressed in percentage in Equation (19).

\[
\text{DER}_i = \frac{\overline{\text{SDI}}_i(b)}{\overline{\text{SDI}}_i(a) + \overline{\text{SDI}}_i(b)} \times 100\%
\]  

(19)

2.4 Finite Element Analysis (FEA)

An aluminum beam with dimensions of 1250mm x 50mm x 5.25 mm is modeled in ABAQUS. The beam has double notches at location 450-500mm and 750-800mm as shown in Figure 1. The beam has young modulus, Poisson’s ratio, and density of 69.5GPa, 0.31 and 2708kg/m³, respectively.

Figure 1 Geometry and dimensions of tested aluminum beam

The beam was modeled using 1D beam element, B32 that has 3 degrees of freedom, which are translations along the X and Y axes and rotation along the Z axis at each node. Frequency analysis was performed to determine mode shape up to 30, this range included the first until eleven bending modes. The beam is constructed by means of 148 equal length elements (i = 149 nodes). Four cases are considered in this study:

- Case 1-Damage Level: Damage Level was defined by ratio between notch depths with beam thickness. It was defined by changing the notch depth by 1mm, 2mm and 3mm which correspond to damage level of 20%, 40% and 60%, respectively.
- Case 2-Number of Mode Shape: Initially all 11 modal frequencies and its corresponding mode shapes are used to calculate DER in Case 1, then in Case 2 the number of modal frequencies and its corresponding mode shape used to calculate the DER value was reduced to 9, 7, 5, 3 and 1 for damage level of 20%.


3.0 Result and Discussions

Robust curve-fitting method effectiveness and limitations has been examined numerically for different damage level, number of mode shape, damage size and number of measurement points. The calculated Damage Estimate Reliability (DER) are used to compare effectiveness of the proposed MSC damage detection using Robust Regression compared to GSM and Fourier series method.

3.1 Case 1: Damage Level

Table 1 and Figure 2 shows DER value for damage level of 20%, 40% and 60% using different MSC damage detection method. It was noted from Table 1 that the GSM has the highest DER value for all damage level. GSM has highest DER because for every damage level it has highest average structural irregularity index (SII) plot as shown in Figure 2. For damage level of 60%, DER calculated with 11 number of mode shape for IRLS has less noise in the damage area (red dot line) compared to GSM as shown in Figure 2 c). For lowest damage level (20%), it was noted IRLS has lower DER value compared to GSM because of increased noise in undamaged area nevertheless the GSM shows false peak at the boundary of the beam same as reported by [8] which may give false detection during interpretation.

Table 1 DER for the measurement point of 149, 11 number of mode shape and 50mm damage size for different damage level: 20%, 40% and 60%.

| Damage Level | GSM   | Fourier | IRLS  |
|--------------|-------|---------|-------|
| 20%          | 84.92 | 71.24   | 74.05 |
| 40%          | 88.46 | 80.09   | 84.23 |
| 60%          | 91.62 | 86.45   | 90.08 |

GSM

Fourier Series

Robust Regression (IRLS)
3.2 Case 2: Mode Shape Number

Calculation of DER using high number of mode shape created high noise in undamaged area for Fourier and IRLS as shown in Figure 2 a), therefore effect of number of mode shape to the DER was investigated. Table 2 shows DER value for measurement point of 149 and 20% damage level using different number of mode shape. It was noted that DER calculated with fundamental mode shape using Robust Regression produce the highest magnitude of DER by 10.4% and 4.9% compared to GSM and Fourier method, respectively. Figure 3 f) shows that for fundamental mode shape, very minimal noise in undamaged area compared to GSM. It shows that the improved GSM algorithm with fundamental mode shape became less sensitive to outlier and the deviation between the estimated of undamaged MSC and the damaged MSC is smaller compared to Fourier as shown in Figure 4.

Table 2 DER for measurement point of 149, 20% damage level 50mm damage size with different number of mode shape.

| Mode Shape | 11 | 9  | 7  | 5  | 3  | 1  |
|------------|----|----|----|----|----|----|
| Ratcliffe  | 84.92 | 85.21 | 85.43 | 85.70 | 85.60 | 85.68 |
| Fourier    | 71.24 | 73.03 | 74.93 | 79.20 | 84.27 | 90.95 |
| IRLS       | 74.05 | 75.99 | 78.55 | 82.87 | 88.08 | 95.61 |

Figure 2 Structural Irregularity Index (SII) Plot for measurement point of 149, 11 number of mode shape, 50mm damage size for different damage level: a) 20%, b) 40% and c) 60%.
Figure 3 Structural Irregularity Index (SII) Plot for measurement point of 149, 20% Damage Level and 50mm damage size for number of mode shape: a) 11, b) 9, c) 7, d) 5, e) 3 and f) 1.
4.0 Conclusions
This paper proposed improve the existing Gapped Smoothing Method by [7] to estimate undamaged curvature mode shape data using damaged curvature mode shape data using Robust Regression. Numerical analysis has been conducted using aluminum beam with 20-60% damage, 50mm damage size and 149 measurement points to demonstrate the feasibility of proposed method.

For lowest damage level (20%) with 11 mode shape, IRLS has lower DER compared to GSM because of increased noise in undamaged area and IRLS shows less noise around damage area for all damage level due to better estimation of undamaged mode shape curvature using Robust Regression. It was noted that when fundamental mode shape used to calculate DER for the damage level 20%, Robust Regression method shows highest DER by 10.4% and very minimal noise in undamaged area compared to GSM because estimation of undamaged MSC using Robust Regression is less sensitive towards outlier at low number of mode shape.

However, this study is only limited to numerical analysis. Hence, experimental works has to be done to validate the proposed MSC damage detection using Robust Regression is recommended for future study.

Acknowledgements
The authors would like to express their appreciation to the Ministry of Higher Education of Malaysia (MOHE), Universiti Teknikal Malaysia Melaka (UTeM), and Universiti Teknologi Malaysia (UTM) for providing the facilities and funding to support the experimental and FEA simulation tasks. This work is supported by the Malaysian Government under the Fundamental Research Grant Scheme (FRGS/1/2018/TK03/UTM/02/12).

References
[1] Hall S, Conquest T. The Total Data Integrity Initiative—Structural Health Monitoring. The Next Generation. 2nd Proceedings of the USAF ASIP Conference 1999.
[2] Chen Y, Olutunde O. Damage Detection using Modal Frequency curve and squared residual wavelet coefficients-based damage indicator. mechanical systems and signal processing 2017;83:21.
[3] Zou Y, Tong I, Steven G. Vibration-based model-dependent damage (delamination) identification and health monitoring for composite structures—a review. Journal of Sound and Vibration 2000;230:22.
[4] Crema L, Castellani A, Coppotelli G. Damage localization in composite material structures by using eigenvalue measurements. American Society of Mechanical Engineers (ASME) energy sources technology conference and exhibition, Houston, TX (United States) 1995:5.
[5] Salawu O. Detection of structural damage through changes in frequency: A review. Engineering Structures 1997;19:6.
[6] Pandey A, Biswas M, Samman M. Damage detection from changes in curvature mode shapes. Journal of Sound and Vibration 1991;14:12.
[7] Ratcliffe C. Damage Detection Using a Modified Laplacian Operator on Mode Shape Data. Journal of Sound and Vibration 1997;204:13.
[8] Yoon M, Heider D, Gillespie J, Ratcliffe C, Crane R. Local Damage Detection with the Global Fitting Method Using Mode Shape Data in Notched Beams. Journal of Nondestructive Evaluation 2009;28:12.
[9] Rucevskis S, Janeliukstis R, Akishin P, Chate A. Mode shape-based damage detection in plate structure without baseline data. Structural Control & Health Monitoring 2016;23:1180.
[10] Fan W, Qiao P. Vibration-based Damage Identification Methods: A Review and Comparative Study. Structural Health Monitoring 2011;10:29.
[11] Qiao P, Lestari W, Shah M, Wang J. Dynamics-based damage detection of composite laminated beams using contact and noncontact measurement systems. Journal of Composite Materials, 2007;41:36.
[12] Esfandiari R. Numerical Methods for Engineers and Scientists Using MATLAB (First Edition). CRC Press 2003.
[13] Kutner M, Nachtsheim C, Nachtsheim C, Neter J. Applied Linear Regression Models (4th Edition). McGraw-Hill Education 2004.