Polarizations of $J/\psi$ and $\psi'$ in hadroproduction at Tevatron in the $k_t$ factorization approach

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Abstract

We present a calculation for the polarizations of $J/\psi$ and $\psi'$ produced in the hadron collisions at the Fermilab Tevatron. Various color octet channels including $^1S^0_0(8)$, $^3P^J(8)$, and $^3S^1_1(8)$ as well as contributions from $\chi_{cJ}$ decays are considered in the $k_t$ factorization approach. We find that in a rather wide range of the transverse momenta of $J/\psi$ and $\psi'$, the production rates could be dominated by the $^1S^0_0(8)$ channel, and the predicted polarizations from the $^1S^0_0(8)$ channel and $\chi_{cJ}$ feeddown contributions are roughly compatible with the preliminary CDF data. This might provide a possible release from the conflict between the NRQCD collinear parton model calculations and the CDF data. PACS number(s): 12.40.Nn, 13.85.Ni, 14.40.Gx
Heavy quarkonium production in high energy collisions provide important information on both perturbative and nonperturbative QCD. In recent years, heavy quarkonium production has attracted much attention from both theory and experiment. To explain the $J/\psi$ and $\psi'$ surplus problem of large transverse momentum production at the Fermilab Tevatron [1], the color-octet production mechanism was introduced for the description of heavy quarkonium production [2] based on the NRQCD factorization framework [3]. However, most recently the CDF collaboration has reported preliminary measurements on the polarizations of the promptly produced charmonium states [4], which appear not to support the color-octet predictions that the directly produced $S$-wave quarkonia have transverse polarizations at large $p_T$ [5,6]. In [7], the authors further considered the feeddown contributions from $\chi_c$ decays, and found the prompt $J/\psi$ polarization disagree with the CDF data at large $p_T$ by 3 standard deviations. In [8], we have performed a calculation of the production rates for $J/\psi$ and $\psi'$ at Tevatron in the newly advocated $k_t$ factorization approach [9], and we find that within the color-singlet model the production rates of $J/\psi$ and $\psi'$ can be enhanced by a factor of 20 compared to the leading order predictions in the naive collinear parton model, but are still below the data by at least one order of magnitude. Therefore in order to explain the $J/\psi$ and $\psi'$ productions at Tevatron we still need to call for the contributions from color-octet channels. This conclusion has been confirmed by another independent study on the $J/\psi$ production in the $k_t$ factorization approach [10]. In this letter we will report a calculation for the contributions of color-octet channels not only to the cross sections but also to the polarizations of the $J/\psi$ and $\psi'$.

The $k_t$-factorization approach differs greatly from the conventional collinear approximation because it takes the non-vanishing transverse momenta of the scattering partons into account. This approach was previously advocated for the study of heavy quark production at high energy hadron colliders [11,12]. Recently, more phenomenological studies were performed along this direction by several groups [13–15], and some successes seem to be achieved by comparing the calculations with experimental data on heavy quark production at the DESY HERA and the Fermilab Tevatron [14,16]. Especially for $b$ and $\bar{b}$ production,
the $k_t$ factorization approach may provides a successful resolution for the long standing conflict between the NLO collinear parton model prediction and the experimental data at the Tevatron [15]. From these and the most recent studies on charmonium production [8–10], we see that for heavy quark and quarkonium production in the present $p_T$ region at the Tevatron, the improved $k_t$ factorization approach may play a very important role and bring significant enhancements compared with the conventional collinear parton model. For the $b$ quark production the enhancement is about a factor of 2, while for the color-singlet $J/\psi$ production a more dramatical enhancement, which is about a factor of 20, has been found [8]. In this paper we will work along this line, to further study $J/\psi$ polarization at the Tevatron by including the color-octet channels. We find that the $k_t$ factorization approach may provide a possible way to understand both the observed production rates and polarizations of $J/\psi$ and $\psi'$.

Under the $k_t$ factorization approach, a process can be factorized as the hard scattering cross section convoluted with the so called unintegrated gluon distribution of the proton. For example, the polarized cross section for a charmonium state $H$ at the Tevatron can be written as the following form,

$$
\frac{d\sigma(p\bar{p} \to H^{(\lambda)}X)}{d^2P_T dy} = \int dx_1 dx_2 dq_1T dq_2T \frac{f(x_1; q_1^2 T)}{q_1^2 T} \frac{f(x_2; q_2^2 T)}{q_2^2 T} \times \hat{\sigma}(g_1^* g_2^* \to H^{(\lambda)} + X),
$$

(1)

where $P_T$ and $y$ are the transverse momentum and rapidity of the charmonium state $H$, and $\lambda$ denotes its helicity. $f(x; q^2_T)$ is the unintegrated gluon distribution function, which can be related to the conventional gluon distribution by

$$
xg(x, \mu^2) = \int \mu^2 dq_T^2 f(x; q_T^2).
$$

(2)

In Eq. (1), the hard scattering cross section $\hat{\sigma}(H^{(\lambda)})$ describes the production of $H$ from two off-shell gluons fusion process, which will depend on the incident gluons’ longitudinal momentum fractions $x_1$, $x_2$ and transverse momenta $q_{1T}$, $q_{2T}$. In the collinear approximation, the cross section $\hat{\sigma}$ does not depend on $q_{1T}$ and $q_{2T}$, and then Eq. (1) will go back to the conventional collinear parton model result for the production of state $H$. However, as
mentioned above, the incident partons’ \( q \) may play an important role for heavy quarkonium production, and may not be neglected. To derive the cross section \( \hat{\sigma} \), we closely follow the BFKL approach in which gauge invariance of the interaction vertices is guaranteed order by order. For example, to get the gauge invariant amplitude for \( J/\psi \) production from \( ^3S_1^{(8)} \) channel, we must include extra term contributions (see Fig. 1(c) of [8]) which will finally result in an effective (Lipatov) vertex for three-gluon interaction.

For direct \( J/\psi \) production at the Tevatron, because the color-singlet contribution underestimates the experimental data [8] by at least an order of magnitude in the \( k_t \) factorization approach, we need to consider the color-octet contributions, for which we will have the LO (leading order) gluon fusion processes,

\[
gg \to c\bar{c}[^1S_0^{(8)}, ^3P_{J}^{(8)}, ^3S_1^{(8)}] \to J/\psi + X,
\]

where various possible color-octet channels for the \( c\bar{c} \) are considered. Note that these \( 2 \to 1 \) subprocesses do not contribute to \( J/\psi (\psi') \) production at large \( p_T \) in the collinear parton model. However, in the \( k_t \) factorization approach, these processes contribute to large \( p_T \) production, because the initial partons’ transverse momenta will lead to \( J/\psi \) having transverse momentum. In this context, the transverse momentum of \( J/\psi \) will be the sum of the transverse momenta of the two incident partons, \( P_T^{\psi} = q_{1T} + q_{2T} \). For \( \chi_{cJ} \) production, the inclusive production rates have been calculated in [11], where they find there is no need for the color-octet contributions to \( \chi_{cJ} \) production. So, in this paper we will also only consider the color-singlet contributions to \( \chi_{cJ} \) polarizations, and then calculate their feeddown contributions to \( J/\psi \) polarization. For numerical calculations, we set \( m_c = 1.5 GeV \) and choose the unintegrated gluon distribution of [14]. We set the scales \( \mu^2 \) for the strong coupling constant \( \alpha_s(\mu^2) \) in the hard scattering cross section \( \hat{\sigma} \) to be \( q_{1T}^2 \) for the interaction vertex associated with the incident gluon \( q_1 \), and \( q_{2T}^2 \) for the vertex associated with \( q_2 \) [8, 12, 13, 14].

We first give the numerical results for the \( P_T \) distributions of the inclusive cross sections for direct \( J/\psi \) and \( \psi' \) production at the Tevatron compared with the experimental data. Fig. 1 is for \( J/\psi \), and Fig. 2 for \( \psi' \). As a qualitative estimate we will use
just one individual color-octet channel to fit the cross sections observed by CDF. The solid lines correspond to the \(1S_0^{(8)}\) channel by taking the color-octet matrix elements as:

\[
\langle O_8^{(1S_0)} \rangle = 0.10 GeV^3 \quad \text{for } J/\psi \quad \text{and} \quad \langle O_8^{(1S_0)} \rangle = 0.03 GeV^3 \quad \text{for } \psi',
\]

while other matrix elements are set to be zero, \(\langle O_8^{(3S_1)} \rangle = 0\). The dashed lines correspond to the \(3P_1^{(8)}\) channel:

\[
\langle O_8^{(3P_1)} \rangle = 0.025 GeV^3 \quad \text{for } J/\psi \quad \text{and} \quad \langle O_8^{(3P_1)} \rangle / m_c^2 = 0.005 GeV^3 \quad \text{for } \psi',
\]

while \(\langle O_8^{(3S_1)} \rangle = 0\). The dotted lines correspond to the \(3S_1^{(8)}\) channel:

\[
\langle O_8^{(3S_1)} \rangle = 0.007 GeV^3 \quad \text{for } J/\psi \quad \text{and} \quad \langle O_8^{(3S_1)} \rangle = 0.0025 GeV^3 \quad \text{for } \psi',
\]

\(\langle O_8^{(3S_1)} \rangle / m_c^2 = 0\). For \(J/\psi\) production, from Fig. 3 we can see that the \(1S_0^{(8)}\) and \(3P_1^{(8)}\) channels can both rather well describe the Tevatron data, especially for the shapes of the differential cross sections. Whereas the \(3S_1^{(8)}\) channel gives an inadequate shape for the differential cross sections compared with the experimental data. For \(\psi'\) production, at first sight all the three curves seem to roughly agree with data. However, we can still see that the \(3S_1^{(8)}\) channel overestimates the production rate of \(\psi'\) at large \(p_T\). For both \(J/\psi\) and \(\psi'\) the theoretical predictions for \(3S_1^{(8)}\) channel drop more slowly as \(p_T\) increases than the experimental data. This is mainly because for \(3S_1^{(8)}\) channel there are additional terms (see Fig. 1b and c of [8]), which give main contributions at large \(p_T\), but these terms do not contribute to the \(1S_0^{(8)}\) and \(3P_1^{(8)}\) processes. The disagreements between theoretical predictions for the \(3S_1^{(8)}\) contributions and experimental data show that in the present \(p_T\) region \((p_T < 20 GeV)\) at the Tevatron, the \(3S_1^{(8)}\) channel may not be the dominant one to the direct \(J/\psi\) and \(\psi'\) production in the \(k_t\) factorization approach. This result is very different from the collinear parton model calculations, where the \(3S_1^{(8)}\) channel overwhelmingly dominates the \(S\)-wave quarkonium production at moderate and large transverse momentum at the Tevatron [2][8].

This difference has significant consequence on the polarization predictions of \(J/\psi\) and \(\psi'\). We recall that in the collinear parton model the NRQCD formalism predicts \(S\)-wave quarkonia being transversely polarized dominantly due to the \(3S_1^{(8)}\) gluon fragmentation contributions [3][8]. However, in the \(k_t\) factorization approach, the \(3S_1^{(8)}\) channel may not dominate the \(S\)-wave quarkonium production, so the previous conclusion of transverse po-
larization at large $P_T$ may be changed. In the following we will study the polarizations of $J/\psi$ and $\psi'$ in the $k_t$ factorization approach. In Fig. 3, we present our result for the $\psi'$ polarization at Tevatron. Because it receives no feeddown contributions from higher lying charmonium states, the polarization of $\psi'$ is only due to the direct production from the color-octet processes. In this figure, we give each individual contributions from these octet processes in the $k_t$ factorization approach. The solid line is for the $^1S_0^{(8)}$ contribution, the dashed line for the $^3P_J^{(8)}$, and the dotted-dashed line for the $^3S_1^{(8)}$. From this figure we can see that the $^3S_1^{(8)}$ channel contributes to the transversely polarized $\psi'$ as it contributes in the collinear parton model approach. Whereas the $^1S_0^{(8)}$ channel contributes to the unpolarized, and $^3P_J^{(8)}$ contributes to the longitudinally polarized. If $^1S_0^{(8)}$ could dominate $\psi'$ production at the Tevatron (see Fig. 2), the $\psi'$ will be unpolarized. This is consistent with the experimental data, which show that in the available $p_T$ region the $\psi'$ seems to be unpolarized (though with large errors). So, the experimental data on $\psi'$ polarization support the $^1S_0^{(8)}$ dominance on $\psi'$ production.

We then study the $J/\psi$ polarization. The polarization data of $J/\psi$ are for the prompt production, i.e., including the direct production and the feeddown contributions from $\chi_{cJ}$ and $\psi'$ decays. The $\psi'$ feeddown contributions to $J/\psi$ polarization can be obtained from the above $\psi'$ polarization calculations multiplied by the decay branching ratio of $\psi'$ to $J/\psi$. (Note that the observed transitions of $\psi'$ to $J/\psi$ involve no spin flips, so this part of contribution has the same behavior as $\psi'$ polarization.) To obtain the feeddown contributions from the $\chi_{cJ}$ decays, we must convolute the $\chi_{cJ}$ polarized cross sections with their decay branching ratios of $\chi_{cJ} \to J/\psi\gamma$. We find that in the $k_t$ factorization approach $\chi_{cJ}$ feeddown contributes to $J/\psi$ being a little transversely polarized. The polarization parameter $\alpha$ for $J/\psi$ from $\chi_{cJ}$ decays approaches to 0.5 at large $p_T$. This is different from the collinear parton model calculations which predict that $\alpha$ approaches to about 0.24 at large $p_T$ [19, 24]. (Note that the polarizations of $\chi_{cJ}$ themselves are very interesting for testing the $k_t$ factorization approach, and has been presented in [20]). All of these contributions including the direct contributions and the feeddown contributions to the $J/\psi$ polarization are plotted in Fig. 4.
As in Fig. 3 for $\psi'$, we also plot in this figure the individual contributions from the three color-octet processes for the direct polarizations, and then combine them with the $\chi_{cJ}$ and $\psi'$ feeddown contributions for the prompt polarizations compared with the experimental data from the CDF collaboration. The thinner lines are for the direct polarizations, and the thicker lines for the prompt polarizations. The solid lines are for $1S_0^{(8)}$ contributions, the dashed lines for $3P_J^{(8)}$, the dotted-dashed lines for $3S_1^{(8)}$, and the dotted line for the $\chi_{cJ}$ feeddown contribution. We see that in general the polarizations increase as $p_T$ increases. For example, for the $3P_J^{(8)}$ direct production the value of polarization parameter $\alpha(\psi)$ (being longitudinal) varies from $-0.63$ at $p_T = 5\, GeV$ to $-0.38$ at $p_T = 20\, GeV$. For the $3S_1^{(8)}$ direct production $\alpha(\psi)$ (being transverse) varies from 0.72 to 0.96 in the same $p_T$ range. The $\chi_c$ feed down contribution will reduce the extremities of $\alpha(\psi)$ in direct productions. From this figure we find again that the $1S_0^{(8)}$ dominance together with the $\chi_c$ feed down contribution will lead to the predicted prompt $J/\psi$ polarization in rough agreement with the Tevatron data except the one at about $p_T = 16.7\, GeV$ which, however, has very large errors. We note that this seemingly nontrivial behavior of $J/\psi$ polarization at $p_T = 16.7\, GeV$ (being longitudinal) can not be explained consistently with the moderate $p_T$ data (being transverse) in our approach. If the CDF data at $p_T = 16.7\, GeV$ is further confirmed by CDF or D0 experiments in the future with higher statistics, the above mechanism may fail.

Concluding our analysis, we find that the $1S_0^{(8)}$ channel dominance is the only reasonable realization of the polarization data of $J/\psi$ and $\psi'$ in the $k_T$ factorization approach. To see this point more clearly, we can perform a $\chi^2$ analysis for theoretical comparison with the experimental data. For $\psi'$ polarization data, $1S_0^{(8)}$ channel gives the smallest value of $\chi^2$ ($\chi^2/dof = 0.61$) as expected. Moreover, inclusion of other channels does not improve the fit. For example, including the $3S_1^{(8)}$ contribution would make the fit even worse, and when including $3P_J^{(8)}$ contribution the fit is in the same manner as $1S_0^{(8)}$ alone but with a little higher $\chi^2/dof$ (= 0.83). For $J/\psi$ polarization, the result for the fit is similar. I.e., $1S_0^{(8)}$ alone gives the smallest $\chi^2$ value ($\chi^2/dof = 1.85$), and inclusion of $3P_J^{(8)}$ gives a even worse fit, while inclusion of $3S_1^{(8)}$ gives the same quality fit as $1S_0^{(8)}$ alone with a little higher $\chi^2/dof$.
So, with the available statistics at present, the polarization data of $J/\psi$ and $\psi'$ may indicate a strong signal of $1S_0^{(8)}$ dominance for $J/\psi$ and $\psi'$ production.

The $1S_0^{(8)}$ dominance has a straightforward consequence that the other two matrix elements $\langle O_8^{(3S_1)} \rangle$ and $\langle O_8^{(3P_0)} \rangle$ must be much smaller than $\langle O_8^{(1S_0)} \rangle$. This is quite different from the naive expectation based on the NRQCD velocity scaling rules, which predict no big difference between these three matrix elements ($\langle O_8^{(1S_0)} \rangle$ scales as $v^6$, while $\langle O_8^{(3S_1)} \rangle$ and $\langle O_8^{(3P_0)} \rangle$ both scale as $v^7$). But, the values of $\langle O_8^{(\psi')} (1S_0) \rangle$ we used in Figs. 1-2 are smaller than the relevant color-singlet matrix elements by about an order of magnitude (e.g., the observed $J/\psi$ leptonic decay width leading to $\langle O_1^{(3S_1)} \rangle = 1.06 GeV^3$). This is roughly consistent with the NRQCD velocity scaling rule estimate. As for the other two matrix elements $\langle O_8^{(3S_1)} \rangle$ and $\langle O_8^{(3P_0)} \rangle$, there might be some dynamical reasons (associated with the nonperturbative evolution processes) to suppress their values and violate the velocity scaling rules [21], or there might be some new counting rules for charmonium system to suppress their values [22].

Some theoretical uncertainties must be addressed before concluding. First there is an uncertainty coming from the scale choice used for the numerical calculations. We have set the scales to be $q_{1T}^2$ or $q_{2T}^2$ which are also adopted by some previous studies [13-15]. If we set these scales both equal to $p_T^2(\psi) + m_c^2$, the cross sections for the three color-octet channels will be lowered down with the normalization factor changing from 1 to about 0.6 both for $J/\psi$ and $\psi'$. But this change has little effects on the shapes of the curves in Figs. 1-2. So, our conclusion for the $1S_0^{(8)}$ dominance will not change with the scale changes. Another uncertainty may come from the parameterizations of the unintegrated gluon distributions, for which, however, it is difficult to give an estimate because we have little knowledge about it at present. On the other hand, we note that the parameterization provided by [17] are obtained from an excellent fit to $F_2(x, Q^2)$ in a very large window of $x$ and $Q^2$ in the $k_t$ factorization approach, which may be viewed as a reliable determination of these quantities.

In conclusion, in this paper we have calculated the $J/\psi$ and $\psi'$ production rates and polarizations in hadroproduction at the Tevatron in the $k_t$ factorization approach. Various
color-octet contributions including $^1S_0^{(8)}$, $^3P_J^{(8)}$, and $^3S_1^{(8)}$ as well as contributions from $\chi_{cJ}$ decays are considered in this approach. We find that in a rather wide range of the transverse momenta of $J/\psi$ and $\psi'$, the production rates could be dominated by the $^1S_0^{(8)}$ channel, and the predicted polarizations are in rough agreement with the preliminary Tevatron data. However, to draw the final conclusion on heavy quarkonium production mechanism in high energy hadron collisions, we still need more accurate experimental data on large $p_T$ production cross sections and polarizations. We also need more solid theoretical calculations such as NLO corrections to the color-octet processes discussed above, and a more reliable estimate for the so-called unintegrated gluon distribution function. It is worthwhile to make further investigations along this direction.

While the calculation in this paper was finished we noticed a paper by Ph. Häegler et al. [10], who studied the $J/\psi$ production rate in the $k_t$ factorization approach with color octet channels. Their result is similar to ours for the calculation of cross sections.

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Figure Captions

FIG. 1. The $p_T$ distribution of direct $J/\psi$ production cross section at the Tevatron in the $k_t$ factorization approach. The solid line is for $^1S_0^{(8)}$ contribution, the dashed line for $^3P_1^{(8)}$, and the dotted-dashed line for $^3S_1^{(8)}$.

FIG. 2. The same as Fig. 1 but for $\psi'$.

FIG. 3. Prompt $\psi'$ polarization at the Tevatron.

FIG. 4. Prompt $J/\psi$ polarization at the Tevatron. The thinner lines are for the direct polarizations, and the thicker lines for the prompt polarizations.
Fig. 1

Direct $J/\psi$ production at Tevatron

$B^* d\sigma/dP_T$ (nb/GeV)

$P_T$ (GeV)

Fig. 1
Direct $\psi'$ production at Tevatron

Fig. 2
Fig. 3
$\alpha(\psi) = 3S_1^{(8)} + \chi_c$

$1S_0^{(8)} + \chi_c$

$3P_J^{(8)} + \chi_c$

Fig. 4