Renormalization of electrons in bilayer cuprate superconductors

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The cuprate superconductors have a layered structure, and then the physical properties are significantly enriched when two or more two copper-oxide layers are contained in a unit cell. Here the characteristic features of the renormalization of the electrons in the bilayer cuprate superconductors are investigated within the framework of the kinetic-energy driven superconducting mechanism. It is shown that the electron quasiparticle excitation spectrum is split into its bonding and antibonding components due to the presence of the bilayer coupling, with each component that is independent. In the underdoped and optimally doped regimes, two electron Fermi surface contours deriving from the bonding and antibonding copper-oxide layers are reconstructed to form the bonding and antibonding Fermi pockets around the nodal region. However, the bonding and antibonding Fermi pockets are almost degenerate, leading to that only one Fermi pocket can be observed. In particular, the special point with the strongest intensity on the bonding (antibonding) Fermi arc reaches exactly the tip of the bonding (antibonding) Fermi arc, which in this case coincide with the bonding (antibonding) hot spot. These hot spots connected by the scattering wave vectors $\mathbf{q}$ construct an \textit{octet} scattering model, and then the enhancement of the quasiparticle scattering processes with the scattering wave vectors $\mathbf{q}$ is confirmed via the result of the autocorrelation of the electron quasiparticle excitation spectral intensities. Moreover, the peak-dip-hump structure developed in each component of the electron quasiparticle excitation spectrum along the corresponding electron Fermi surface sheet is directly related with the peak structure in the corresponding quasiparticle scattering rate except for at around the hot spots, where the peak-dip-hump structure is caused mainly by the pure bilayer coupling. Although the kink in the electron quasiparticle dispersion is present all around the electron Fermi surface sheets, when the momentum moves away from the node to the antinode, the kink energy smoothly decreases, while the dispersion kink becomes more pronounced, and in particular, near the cut close to the antinode, develops into a break separating of the fastest dispersing high-energy part of the electron quasiparticle excitation spectrum from the slower dispersing low-energy part. By comparing with the corresponding results in the single-layer case, the theory also indicates that the exotic feature of the renormalization of the electrons is particularly obvious due to the presence of the bilayer coupling.

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I. INTRODUCTION

The parent compound of cuprate superconductors is actually a Mott insulator, which is known to be the result from the very strong electron-electron correlation.\textsuperscript{13} Superconductivity then is tuned through a remarkable progression of the states of matter by doping charge carriers.\textsuperscript{20} After over thirty years of the extensive research, some issues in cuprtae superconductors have been settled, such as the d-wave type electron pairing state,\textsuperscript{2} which is one of the cornerstones of our understanding of the unconventional mechanism of superconductivity in cuprate superconductors, whereas others remains controversial, such as the nature of the electron quasiparticle excitation,\textsuperscript{3,10} which plays an essential role in the occurrence of superconductivity. In cuprate superconductors, the electrons interact strongly with various bosonic excitations resulting in the modification to both their lifetime and binding-energy, a process so-called renormalization of the electrons to form the electron quasiparticle excitations,\textsuperscript{9,10} and then all the anomalous properties,\textsuperscript{5,17} arise from the renormalization of the electrons. In this case, the systematic study of the renormalization of the electrons in cuprate superconductors allows the identification of the collective bosonic excitations mediating the interaction between the electrons that is most likely also responsible for the exceptionally high superconducting (SC) transition temperature $T_c$. The single common feature in the layered crystal structure of cuprate superconductors is the presence of the two-dimensional copper-oxide layers,\textsuperscript{5,17} and then the anomalous properties and the related superconductivity arise from the strongly correlated motion of the electron quasiparticles in these copper-oxide layers. In particular, the motion of the electron quasiparticles confined to the copper-oxide layers has been confirmed experimentally by the incoherent charge transport along the out-of-copper-oxide layer direction.\textsuperscript{15,21} However, the exotic properties are significantly enriched by the coupling between the copper-oxide layers when two or more two copper-oxide layers are contained in a unit cell.\textsuperscript{22,31} For instance, $T_c$ is very sensitive to the number of the copper-oxide layers $n$ within a unit cell i.e., for all families of cuprtae superconductors, the magnitude of the optimized $T_c$ is found experimentally to increase with the increase of $n$ per unit cell for the case of $n < 3$, and reaches the maximum for the case of $n = 3$, then decreases slightly and saturates for the case of $n > 3$, which therefore shows
clearly that the number of the copper-oxide layers dependence of $T_c$ arises from the coupling between the copper-oxide layers within a unit cell. However, up to now, the cause of this $T_c$ enhancement is still not clear. In this case, it is particularly significant to clarify the characteristic features of the renormalization of the electrons in cuprate superconductors distinctive for the families in the presence of the coupling between the copper-oxide layers within a unit cell.

In this paper, we focus on the coupling effect between the copper-oxide layers on the renormalization of the electrons in cuprate superconductors. In particular, the bilayer cuprate superconductor is an ideal system to tackle the coupling effect between the copper-oxide layers on the renormalization of the electrons, where the bilayer splitting due to the present of the bilayer interaction in a unit cell has been observed from the angle-resolved photoemission spectroscopy (ARPES) experiments in a wide doping range, with the largest value of the bilayer splitting that appears at around the antinodal region, and then decreases upon approaching to the nodal region. In particular, these ARPES experimental observations also indicate that this bilayer splitting derived the electron quasiparticle excitation spectrum into the bonding and antibonding components, which leads to the formation of antibonding electron Fermi surface (EFS) sheets corresponding to different doping levels with distinct SC gaps. Moreover, it has been shown that this bilayer coupling may play a main role in the form of the well-known peak-dip-hump (PDH) structure in the electron quasiparticle excitation spectrum of the bilayer cuprate superconductors in the overdoped regime. In this case, the natural questions are: (a) whether the antibonding component of the electron quasiparticle excitation spectrum is completely independent of the bonding component or not? (b) whether the behavior of the electron quasiparticle excitations determined by the low-energy electronic structure is universal or not? (c) why superconductivity in the presence of the bilayer coupling is enhanced?

The effect of the renormalization of the electrons in cuprate superconductors detected from ARPES experiments is quantitatively characterized by the experimentally measurable quantities such as the EFS reconstruction, the complicated line-shape in the electron quasiparticle excitation spectrum, the kinks in the electron quasiparticle dispersion, and the ARPES autocorrelation. Recently, we have studied the renormalization of the electrons in the single-layer cuprate superconductors based on the framework of the kinetic-energy driven SC mechanism, and reproduced the main features observed from the corresponding ARPES experiments, including the formation of the Fermi pockets due to the EFS reconstruction, the charge-order correlation driven by the EFS instability with the characteristic charge-order wave vector corresponding to the straight hot spots on EFS, the striking PDH structure in the electron quasiparticle excitation spectrum, and the remarkable ARPES autocorrelation and its connection with quasiparticle scattering interference (QSI). However, the significant coupling effect between the copper-oxide layers on the renormalization of the electrons has not been clarified in these discussions due to the limitation in the single-layer case. In this paper, we study the characteristic features of the energy and momentum dependence of the renormalization of the electrons in the bilayer cuprate superconductors along with this line by taking into account the bilayer coupling effect. Our results show that the electron quasiparticle excitation spectrum is split into its bonding and antibonding components due to the presence of the bilayer coupling, with each component that is independent, namely, two copper-oxide layers within the unit cell are hybridized to form the bonding and antibonding copper-oxide layers, and then the bonding copper-oxide layer is independent of the antibonding copper-oxide layer, which leads to two EFS contours deriving directly from the bonding and antibonding copper-oxide layers. However, in the underdoped and optimally doped regimes, the spectral weight on the bonding and antibonding EFS contours at around the antinodal region is gapped out by the pseudogap, and then the bonding and antibonding electron quasiparticle excitations occupy the disconnected segments located at around the nodal region to form the bonding and antibonding Fermi pockets. In particular, the bonding and antibonding Fermi pockets are almost degenerate around the nodal region except for at around the tips of the disconnected segments, this special property leads to that only one Fermi pocket can be observed. Moreover, the special point with the strongest intensity on the bonding (antibonding) Fermi arc reaches exactly the tip of the bonding (antibonding) Fermi arc, which in this case coincide with the bonding (antibonding) hot spot. These hot spots connected by the scattering wave vectors $q_i$ construct an octet scattering model, and then the enhancement of the quasiparticle scattering processes with these scattering wave vectors $q_i$ and the SC correlation are confirmed via the result of the ARPES autocorrelation. This also indicates indirectly why $T_c$ is enhanced in the presence of the coupling between the copper-oxide layers within a unit cell. In a striking analogy to the single-layer cuprate superconductors, the remarkable PDH structure in the antibonding (bonding) component of the electron quasiparticle excitation spectrum at around the antinodal and nodal regions is caused by the corresponding peak structure in the antibonding (bonding) quasiparticle scattering rate. However, this PDH structure at around the hot spots is developed mainly due to the pure bilayer coupling, which is in a clear contrast to the single-layer case. More specifically, our results also show that although the kink in the electron quasiparticle dispersion is present all around the antibonding EFS sheet, when the momentum moves away from the nodal region to the antinodal region, the dispersion kink becomes more pronounced, and in particular, near the cut close to the antinode, develops into a break separating of
the fasting dispersing high-energy part of the electron quasiparticle excitation spectrum from the slower dispersing low-energy part. Furthermore, the largest value of the kink energy appears along the nodal direction, and then decreases upon approaching to the antinodal direction.

This paper is organized as follows. The basic formalism is presented in Sec. II where we generalize the electron normal and anomalous Green’s functions obtained based on the kinetic-energy driven superconductivity from the previous case in the single-layer cuprate superconductors\textsuperscript{53} to the present case in the bilayer cuprate superconductors, and then evaluate explicitly the bonding and antibonding components of the electron quasiparticle excitation spectrum. Within this basic formalism, we discuss the characteristic features of the renormalization of the electrons in the bilayer cuprate superconductors in Sec. III, where we show clearly that the multiple electronic instabilities in the bilayer cuprate superconductors are driven by the EFS instability, including charge order with the scattering that is peaked at the straight hot spots on the EFS sheet, and then these multiple electronic instabilities coexist and compete with superconductivity. Finally, the summary and discussions are presented in Sec. IV. One Appendix is also included.

II. FORMALISM

The electron quasiparticle excitation spectrum $I_{\nu}(\mathbf{k}, \omega) \propto A(\mathbf{k}, \omega)$ can be probed by the highly sophisticated ARPES experiments\textsuperscript{53,10}, where $\mathbf{k}$ is the in-plane momentum, $\omega$ is the energy of the initial state relative to the Fermi energy, while the crucial electron spectral function $A(\mathbf{k}, \omega)$ is calculated in terms of the electron Green’s function starting from the microscopic model of the system. However, for the bilayer cuprate superconductors, the electron quasiparticle excitation spectrum has been separated into its bonding and antibonding components due to the presence of the bilayer coupling within a unit cell\textsuperscript{53,54,55}.

$$I_{\nu}(\mathbf{k}, \omega) = |M_{\nu}(\mathbf{k}, \omega)|^2 n_F(\omega) A_{\nu}(\mathbf{k}, \omega),$$

where $\nu = 1, 2$ with $\nu = 1$ ($\nu = 2$) that represents the corresponding bonding (antibonding) component, the dipole matrix element $M_{\nu}(\mathbf{k}, \omega)$ is associated with the transition from the initial to final electronic state which can be affected by such things as incident photon energy and polarization as well as the Brillouin zone of the photoemitted electrons. However, it has been shown experimentally that this dipole matrix element $M_{\nu}(\mathbf{k}, \omega)$ does not vary significantly with momentum and energy over the range of the interest\textsuperscript{53,10}. In this case, as a qualitative discussion in this paper, the magnitude of this dipole matrix element $M_{\nu}(\mathbf{k}, \omega)$ can be rescaled to the unit. $n_F(\omega)$ is the fermion distribution function which indicates only occupied states are probed in the ARPES experiments, while the electron spectral function $A_{\nu}(\mathbf{k}, \omega)$ in the SC-state is related directly with the imaginary part of the electron normal Green’s function $G_{\nu}(\mathbf{k}, \omega)$ as $A_{\nu}(\mathbf{k}, \omega) = -2\text{Im} G_{\nu}(\mathbf{k}, \omega)$.

As we have mentioned above in Sec. II the most important structure element in the cuprate superconductors is the copper-oxide layers\textsuperscript{31,17}, and then it is believed that the essential physics occurs in these copper-oxide layers\textsuperscript{31}. It is widely accepted that a realistic model to provide an acceptable description of the single copper-oxide layer is the $t$-$J$ model on a square lattice\textsuperscript{53,55}. However, for the discussions of the bilayer coupling effect on the renormalization of the electrons in the bilayer cuprate superconductors, the single-layer $t$-$J$ model should be extended by including the bilayer interaction\textsuperscript{55}.

$$H = -t \sum_{l,i\bar{\sigma}} C_{l,a}^\dagger C_{l+\bar{\nu}a\sigma} + t' \sum_{l,i\sigma} C_{l,a}^\dagger C_{l+\bar{\nu}a\sigma} - t_{\perp}(l) C_{l,a}^\dagger C_{l,b\sigma} + \mu \sum_{l,a} C_{l,a} C_{l,a\sigma} + J \sum_{l,i} S_{l,a} \cdot S_{l+\bar{\nu}a} + J_{\perp} \sum_{l} S_{l,\uparrow} \cdot S_{l,\downarrow},$$

where $(a = 1, 2)$ is the copper-oxide layer index, the summation within the copper-oxide layer is over all sites $l$, and for each $l$, over its nearest-neighbor (NN) sites $\bar{\nu}$ or the next NN sites $\hat{\tau}$, $C_{l,a\sigma}/C_{l,a\sigma}$ is electron operator that create/annihilate an electron with spin $\sigma$ at lattice site $l_a$, $S_{l,a} = (S_{l,a}^x, S_{l,a}^y, S_{l,a}^z)$ is spin operator, while $\mu$ is the chemical potential. In particular, it has been found\textsuperscript{53,10} that the observed bilayer splitting can be described approximately by the momentum dependence of $t_{\perp}(k)$ as,

$$t_{\perp}(k) = \frac{t_{\perp}}{4} (\cos k_x - \cos k_y)^2,$$

which is expected for a coherent hopping between two copper-oxide layers within a unit cell. This momentum-dependent form of the interlayer hopping\textsuperscript{3} is strongly anisotropic and also follows the theoretical prediction\textsuperscript{53,55}. Moreover, the magnetic exchange $J$ between two copper-oxide layers can be obtained in terms of the magnetic exchange $J$ within the copper-oxide layer\textsuperscript{53} $J_{\perp} = (t_{\perp}/t)^2 J$. The no double electron occupancy local constraint in this bilayer $t$-$J$ model\textsuperscript{55} is ensured by $\sum_{\sigma} C_{l,a\sigma}^\dagger C_{l,a\sigma} \leq 1$, which can be treated properly in analytical calculations within the fermion-spin theory\textsuperscript{53,56,57}, where the constrained electron operators are decoupled as $C_{l,a} = h_{l,a}^\dagger S_{l,a}^-$ and $C_{l,a\uparrow} = h_{l,a}^\dagger \sigma_{l,a}$, with the spinful fermion operator $h_{l,a} = e^{-i \chi_{l,a}} h_{l,a}$ that carries the charge of the electron together with some effects of the spin configuration rearrangement due to the presence of the doped charge carrier itself, while the spin operator $S_{l,a}$ carries the spin of the electron, then the no double electron occupancy local constraint is satisfied in analytical calculations. In this formalism, $\delta = \langle h_{l,a}^\dagger h_{l,a}\sigma \rangle = \langle h_{l,a}^\dagger h_{l,a}\sigma \rangle$ gives the total charge-carrier doping concentration.

In this fermion-spin representation, the bilayer $t$-$J$
model [2] can be rewritten as,

\[
H = t \sum_{l \sigma a} \left( h_{l+\sigma a} \right)^\dagger h_{l \sigma a} S_{l \sigma a} + \frac{1}{t} \sum_{l \sigma a} \left( h_{l+\sigma a} \right)^\dagger h_{l \sigma a} S_{l \sigma a}^\dagger \left( h_{l \sigma a} \right)^\dagger h_{l \sigma a} S_{l \sigma a}^\dagger \left( h_{l \sigma a} \right)^\dagger h_{l \sigma a} S_{l \sigma a}
\]

\[
- t' \sum_{l \sigma a} \left( h_{l+\sigma a} \right)^\dagger h_{l \sigma a} S_{l \sigma a} S_{l+\sigma a}^\dagger + \frac{1}{t} \sum_{l \sigma a} \left( h_{l+\sigma a} \right)^\dagger h_{l \sigma a} S_{l \sigma a} S_{l+\sigma a}^\dagger + \frac{1}{t} \sum_{l \sigma a} \left( h_{l \sigma a} \right)^\dagger h_{l+\sigma a} S_{l \sigma a} S_{l+\sigma a}^\dagger + \frac{1}{t} \sum_{l \sigma a} \left( h_{l \sigma a} \right)^\dagger h_{l+\sigma a} S_{l \sigma a} S_{l+\sigma a}^\dagger
\]

\[
+ \sum_{l \sigma a \neq l' \sigma a} \left( h_{l \sigma a} \right)^\dagger h_{l \sigma a} S_{l \sigma a} S_{l' \sigma a}^\dagger + \left( h_{l' \sigma a} \right)^\dagger h_{l \sigma a} S_{l' \sigma a} S_{l \sigma a}^\dagger
\]

\[
- \mu \sum_{l \sigma a} h_{l \sigma a} S_{l \sigma a} + J_{\text{eff}} \sum_{l \sigma a} S_{l \sigma a} \cdot S_{l+\sigma a}
\]

\[
+ J_{\text{eff} \perp} \sum_{l} S_{l \uparrow} \cdot S_{l \downarrow}, \quad (4)
\]

with \( J_{\text{eff}} = J(1-\delta)^2 \), \( J_{\text{eff} \perp} = J_{\perp}(1-\delta)^2 \). In the following discussions, the parameters in this bilayer \( t-J \) model are chosen as \( t/J = 2.5, t'/t = 0.28 \), and \( t_{\perp}/t = 0.3 \). However, when necessary to compare with the experimental data, we take \( J = 100 \text{ meV} \), which is the typical value of the bilayer cuprate superconductors [5][6].

For a microscopic description of the SC-state of cuprate superconductors, the kinetic-energy-driven SC mechanism has been established based on the \( t-J \) model in the fermion-spin representation [22][28], where the interaction between charge carriers directly from the kinetic energy by the exchange of spin excitations generates the formation of the \( d \)-wave charge-carrier pairs, while the \( d \)-wave electron pairs originate from the \( d \)-wave charge-carrier pairing state are due to the charge-spin recombination, and then these electron pairs condense into the \( d \)-wave SC-state. However, for the bilayer cuprate superconductors, there are two coupled copper-oxide layers within a unit cell. In this case, the charge-carrier normal and anomalous Green’s functions are the matrices [5], and can be expressed as, \( g(k, \omega) = g_L(k, \omega) + g_T(k, \omega) \sigma_x \) and \( \hat{\Gamma}(k, \omega) = \Gamma_L(k, \omega) + \Gamma_T(k, \omega) \sigma_x \), respectively, where \( \sigma_x \) is the Pauli matrix, \( g_L(k, \omega) \) [\( \Gamma_L(k, \omega) \)] and \( g_T(k, \omega) \) [\( \Gamma_T(k, \omega) \)] are the corresponding longitudinal and transverse parts of the charge-carrier normal (anomalous) Green’s function, respectively. Within the framework of the kinetic-energy-driven SC mechanism, these longitudinal and transverse components of the full charge-carrier normal and anomalous Green’s functions for the bilayer \( t-J \) model [1] have been evaluated, and are given explicitly in Ref. [5][8].

In order to the discussions of the electronic properties, we need to calculate the electron normal and anomalous Green’s functions \( \tilde{G}(k, \omega) = G_L(k, \omega) + G_T(k, \omega) \sigma_x \) and \( \tilde{\Sigma}(k, \omega) = \bar{\Theta} \bar{T}(k, \omega) \sigma_x \), which are the convolutions of the spin Green’s function and the corresponding charge-carrier normal and anomalous Green’s functions in the fermion-spin representation. However, in corresponding to the bonding and antibonding components of the electron quasiparticle excitation spectrum in Eq. [1], these electron normal and anomalous Green’s functions can be also rewritten in the bonding-antibonding representation as, \( G_{\nu}(k, \omega) = G_L(k, \omega) + (-1)^{\nu+1}G_T(k, \omega) \) and \( \tilde{\Sigma}_{\nu}(k, \omega) = \tilde{\Theta}_\nu(k, \omega) + (-1)^{\nu+1} \tilde{\Theta}_{\nu}(k, \omega) \). Following the kinetic-energy-driven SC mechanism, a full charge-spin recombination scheme has been developed recently [23], where a charge carrier and a localized spin are fully recomposed into a physical electron. According to this full charge-spin recombination scheme, \( \tilde{G}(k, \omega) \) have obtained the electron normal and anomalous Green’s functions of the single layer \( t-J \) model in the fermion-spin representation. In particular, these electron normal and anomalous Green’s functions of the single-layer \( t-J \) model have been employed to discuss the renormalization of the electrons in the single-layer cuprate superconductors [23][28] and the obtained results are well consistent with the corresponding experimental results. Following these previous discussions for the single-layer case [23][28], the electron normal and anomalous Green’s functions of the bilayer \( t-J \) model [1] in the bonding-antibonding representation can be evaluated explicitly as [see the Appendix A].

\[
G_{\nu}(k, \omega) = \frac{1}{\omega - \varepsilon_{k}^{(\nu)} - \Sigma_{\nu \text{ph}}(k, \omega) - \left[ \Sigma_{\nu \text{pp}}(k, \omega, \omega) \right]^2 / \left[ \omega + \varepsilon_{k}^{(\nu)} + \Sigma_{\nu \text{ph}}(k, -\omega) \right]}, \quad (5a)
\]

\[
\tilde{\Sigma}_{\nu}(k, \omega) = \frac{-\Sigma_{\nu \text{pp}}(k, \omega)}{\left[ \omega - \varepsilon_{k}^{(\nu)} - \Sigma_{\nu \text{ph}}(k, \omega) \right] \left[ \omega + \varepsilon_{k}^{(\nu)} + \Sigma_{\nu \text{ph}}(k, -\omega) \right] - \left[ \Sigma_{\nu \text{pp}}(k, \omega, \omega) \right]^2}, \quad (5b)
\]

where \( \varepsilon_{k}^{(\nu)} = \varepsilon_{k} + (-1)^{\nu}t_{\perp}(k) \) is the bare dispersion relation, with \( \varepsilon_{k} = -Z\gamma_{k} + Zt'\gamma_{k} + \mu \), \( \gamma_{k} = (\cos k_x + \cos k_y) / 2 \), \( \gamma_{k} = \cos k_x \cos k_y \), and the number of the NN or next NN sites \( Z \), while \( \Sigma_{\text{ph}}(k, \omega) \) and \( \Sigma_{\text{pp}}(k, \omega) \) are the electron self-energies in the particle-hole and particle-particle channels, respectively, which are the generally complex quantities describing the influence of the interaction between the electrons mediated by the exchange of spin excitations on the propagation of the electron quasiparticle, and are evaluated explicitly in terms of the full charge-spin recombination in the Appendix A.

According to the above electron normal Green’s function in Eq. (5a), the electron spectral function
\[ A_\nu(k, \omega) = -2\text{Im}G_\nu(k, \omega) \]

in the bonding-antibonding representation can be evaluated explicitly as,

\[ A_\nu(k, \omega) = \frac{2\Gamma_\nu(k, \omega)}{|\omega - E_\nu(k, \omega)|^2 + \Gamma_\nu^2(k, \omega)}, \]

with the corresponding quasiparticle scattering rate \( \Gamma_\nu(k, \omega) \) and the renormalized band structures (then the renormalized dispersion relation) \( E_\nu(k, \omega) \) that can be expressed explicitly as,

\[
\Gamma_\nu(k, \omega) = \left| \text{Im}\Sigma_{ph}^{(\nu)}(k, \omega) - \frac{[\Sigma_{pp}^{(\nu)}(k, \omega)]^2\text{Im}\Sigma_{ph}^{(\nu)}(k, -\omega)}{[\omega + \varepsilon_k^{(\nu)} + \text{Re}\Sigma_{ph}^{(\nu)}(k, -\omega)]^2 + [\text{Im}\Sigma_{ph}^{(\nu)}(k, -\omega)]^2} \right|, \tag{7a}
\]

\[
\bar{E}_\nu(k, \omega) = \varepsilon_k^{(\nu)} + \text{Re}\Sigma_{ph}^{(\nu)}(k, \omega), \tag{7b}
\]

\[
\text{Re}\Sigma_{ph}^{(\nu)}(k, \omega) = \text{Re}\Sigma_{ph}^{(\nu)}(k, \omega) + \frac{[\Sigma_{pp}^{(\nu)}(k, \omega)]^2[\omega + \varepsilon_k^{(\nu)} + \text{Re}\Sigma_{ph}^{(\nu)}(k, -\omega)]}{[\omega + \varepsilon_k^{(\nu)} + \text{Re}\Sigma_{ph}^{(\nu)}(k, -\omega)]^2 + [\text{Im}\Sigma_{ph}^{(\nu)}(k, -\omega)]^2}, \tag{7c}
\]

respectively, where \( \text{Re}\Sigma_{ph}^{(\nu)}(k, \omega) \) and \( \text{Im}\Sigma_{ph}^{(\nu)}(k, \omega) \) are the real and imaginary parts of the electron self-energy \( \Sigma_{ph}^{(\nu)}(k, \omega) \), respectively. Substituting this electron spectral functions \( A_\nu(k, \omega) \) into Eq. (1), we therefore obtain the quasiparticle excitation spectrum \( I_\nu(k, \omega) \). In this case, the energy and lifetime renormalization of the electrons in the bilayer cuprate superconductors are directly described by the renormalized dispersion relation \( E_\nu(k, \omega) \) and the quasiparticle scattering rate \( \Gamma_\nu(k, \omega) \), respectively. This is why the ARPES spectral line shape can give direct access to the lifetime of the electron quasiparticle excitation and can provide insight into the nature of the underlying interaction between the electrons.[9][10]

### III. Quantitative Characteristics of Renormalization of Electrons

In this section, we discuss the energy and momentum dependence of the renormalization of the electrons in the presence of the bilayer coupling, and show how the bilayer interaction gives some additional effects on the EFS reconstruction, the PDH structure in the electron quasiparticle excitation spectrum, the kink in the electron quasiparticle dispersion, and the ARPES autocorrelation in the bilayer cuprate superconductors.

#### A. Electron Fermi surface instability

EFS is the zero-energy contour in momentum-space that separates the filled electronic states from the empty electronic states. Superconductivity is one of several phenomena, including different ordered electronic states,[12][13][14] that arise from the interaction of electrons near EFS.[10][22][23]. The EFS topology is therefore crucial to the understanding of these phenomena and their relationships. Experimentally, the intensity of ARPES spectra at zero energy is usually used to map out the underlying EFS,[9][10], i.e., the underlying EFS is determined by looking at the electron quasiparticle excitation spectrum \( I(k, \omega = 0) \) to map out the locus of the maximum in the intensity of \( I(k, \omega = 0) \). However, as shown in Eq. (1), the electron quasiparticle excitation spectrum \( I(k, \omega) \) in the bilayer cuprate superconductors has been separated into the corresponding bonding and antibonding components due to the presence of the bilayer coupling,[32][33] which complicates the physical properties of EFS. For instance, in the self-consistent renormalized mean-field (MF) level [see, Eq. (A11)], the bonding and antibonding components of the electron quasiparticle excitation spectrum in the normal-state consist of the delta function peaks located at the precise energy and momentum given by the self-consistent renormalized MF band structure \( I_1^{(0)}(k, \omega) = 2\pi n_F(\omega)Z_F^{(1)}(\omega - \varepsilon_k^{(1)}) \) and \( I_2^{(0)}(k, \omega) = 2\pi n_F(\omega)Z_F^{(2)}(\omega - \varepsilon_k^{(2)}) \), respectively, and then the intensities of \( I_1^{(0)}(k, 0) = Z_F^{(1)} \) and \( I_2^{(0)}(k, 0) \) at zero binding-energy can be used to map out the underlying EFS, which is plotted in Fig. 1 where the doping concentration \( \delta = 0.12 \) and temperature \( T = 0.002 J \). In comparison with the corresponding self-consistent renormalized MF result of the single closed EFS contour in the single-layer case,[49][50] there are two closed EFS contours \( k_F^{(B)} \) and \( k_F^{(A)} \) in the bilayer case deriving directly from the bonding and antibonding copper-oxide layers. These bonding and antibonding EFS contours correspond to different charge-carrier doping level.[35][41] The existence of two closed bonding and antibonding EFS contours \( k_F^{(B)} \) and \( k_F^{(A)} \) are due to the presence of the bilayer coupling. In particular, the peaks with the same height distribute uniformly along with both the bonding and antibonding EFS contours, reflecting a fact that these two closed bonding and antibonding EFS contours \( k_F^{(B)} \) and \( k_F^{(A)} \) in the bilayer case are necessarily two surfaces in momentum-space on which the electron lifetime becomes infinitely long in the limit as one approaches these bonding and antibonding EFS contours. However, in corre-
Corresponding to the form of the interlayer coherent hopping in Eq. (3), the separation between the bonding and antibonding EFS contours $k^{(B)}_{F0}$ and $k^{(A)}_{F0}$ has a largest value at around the antinodes, and then it smoothly decreases with the move of the momentum away from the antinodal region, eventually disappearing at around the nodes. In the bilayer cuprate superconductors, two copper-oxide layers in the unit cell are equivalent, and they are hybridized. However, these self-consistent renormalized MF results also show that the electrons in the bonding and antibonding bands have the different EFS contours except for at around the nodes, in other words, the bonding copper-oxide layer is independent completely of the antibonding copper-oxide layer.

However, the bonding and antibonding components of the electron quasiparticle excitation spectrum $I_1(k,0)$ and $I_2(k,0)$ in Eq. (1) are modified by the inclusion of the energy and momentum dependence of the bonding and antibonding electron self-energies $\Sigma^{(1)}_{ph}(k,\omega)$ and $\Sigma^{(2)}_{ph}(k,\omega)$, respectively, which captures the strong interaction between the electrons by the exchange of spin excitations in the bilayer cuprate superconductors. This strong interaction between the electrons therefore produces a strong redistribution of the spectral weight in the electron quasiparticle excitation spectrum [35,52], leading to the bonding and antibonding EFS contours reconstruction. To see these bonding and antibonding EFS contours reconstruction more clearly, we plot the EFS mapping from both $I_1(k,0)$ and $I_2(k,0)$ in Eq. (1) at zero binding-energy for $\delta = 0.12$ with $T = 0.002J$ in Fig. 1. With respect to the self-consistent renormalized MF result in Fig. 1 the closed bonding and antibonding EFS contours $k^{(B)}_{F0}$ and $k^{(A)}_{F0}$ have been reconstructed, where the single bonding EFS contour $k^{(B)}_{F0}$ is divided into two bonding EFS sheets $k^{(B)}_{F}$ and $k^{(B)}_{BS}$ due to the presence of the bonding electron self-energy, while the single antibonding EFS contour $k^{(A)}_{F0}$ is separated into two antibonding EFS sheets $k^{(A)}_{F}$ and $k^{(A)}_{BS}$ by the antibonding electron self-energy, which leads to form four EFS sheets with the peaks on four EFS sheets that gain a finite width in a way dependent on the momentum of the electron quasiparticle excitations. However, the lifetime renormalization determined by the quasiparticle scattering rate is angular dependent, i.e., the spectral intensity of the electron quasiparticle excitations on these four EFS sheets at around the antinodal region are suppressed heavily, leading to that the spectral intensity on these four EFS sheets at around the antinodal region is too weak to be observed experimentally [39,41]. In this case, these four EFS sheets are truncated to form the disconnected segments around the nodal region instead of the closed EFS contours. More specifically, the antibonding EFS sheet $k^{(A)}_{F}$ intersects the antibonding EFS sheet $k^{(A)}_{BS}$ at the tips of the disconnected segments on $k^{(A)}_{F}$ and $k^{(A)}_{BS}$ to form an antibonding Fermi pocket, while the bonding EFS sheet $k^{(B)}_{F}$ intersects the bonding EFS sheet $k^{(B)}_{BS}$ at the tips of the disconnected segments on $k^{(B)}_{F}$ and $k^{(B)}_{BS}$ to form a bonding Fermi pocket, where the disconnected segment around the nodal region at the antibonding EFS sheet $k^{(A)}_{F}$ (the bonding EFS sheet $k^{(B)}_{F}$) is so-called the antibonding (bonding) Fermi arc, and is actually the front side of the antibonding (bonding) Fermi pocket [42,50], while the other at the antibonding
EFS sheet $k_{BS}^{(A)}$ (the bonding EFS sheet $k_{BS}^{(B)}$) around the nodal region is defined as the back side of the antibonding (bonding) Fermi pocket. The result in Fig. 2 also shows that in the underdoped and optimally doped regimes, the formation of the antibonding Fermi pocket around the nodal region due to the antibonding EFS contour reconstruction is completely independent of the formation of the bonding Fermi pocket due to the bonding EFS contour reconstruction. However, the bilayer splitting between the bonding and antibonding bands in Eq. (3) is rather weak around the nodal region, and disappears exactly at around the node, this leads to that the bonding and antibonding Fermi pockets are almost degenerate around the nodal region except for at around the tips of the Fermi arcs. In this case, the boundary between the bonding and antibonding Fermi pockets around the nodal region is too dim to be observed. This is also why only one Fermi pocket can be observed experimentally in the underdoped and optimally doped bilayer cuprate superconductors[23,24]. Moreover, we have also discuss this bonding and antibonding EFS contours reconstruction in the normal-state, and the results show that this unusual EFS contours reconstruction appeared in the SC-state also persists into the normal-state. On the other hand, in the heavily overdoped regime, the renormalization of the electrons within each copper-oxide layer is weakened, and then as in the self-consistent renormalized MF case, the bonding (antibonding) Fermi arc covers the full length of the bonding (antibonding) EFS contours, leading to that two EFS contours induced mainly by the bilayer coupling are observed experimentally.

B. Octet scattering model and related coexistence and competition between multiple electronic orders and superconductivity

The result in Fig. 2 on the other hand also shows that the special point with the strongest intensity on the bonding Fermi arc (the antibonding Fermi arc) does not reach the node, but instead reaches exactly the tip of the bonding Fermi arc (the antibonding Fermi arc), which in this case coincide with the hot spot on the bonding (antibonding) EFS sheet. To see the positions of these hot spots on the tips of the antibonding and bonding Fermi arcs more clearly, we plot map of $I_1(k,0)$ and $I_2(k,0)$ in the $[k_x,k_y]$ plane at zero binding-energy for $\delta = 0.12$ with $T = 0.002J$ in Fig. 3 where although the corresponding spectral densities on the antibonding and bonding EFS sheets are almost the same, the highest peak height at the bonding (the antibonding) EFS sheet, marked by the black (red) circle, reaches exactly the tip of the bonding (the antibonding) Fermi arc, which conforms well to the ARPES experiments[39,46,47,80,81], where the observed result in the bilayer cuprate superconductors indicates that the sharp quasiparticle peaks with largest spectral weight appear always at off-node place. More specifically, these hot spots connected by the scattering wave vectors $q_i$ shown in Fig. 2 construct an octet scattering model[50,57,22], reflecting a fact that the hot spots are the special points on the EFS sheets that are displaced from each other by the scattering wave vectors $q_i$. In this case, a bewildering variety of competing phases described by the quasiparticle scattering processes with the scattering wave vectors $q_i$ therefore are driven by the EFS instability. This is also why a remarkable phenomenon of cuprate superconductor is the coexistence and competition between the multiple nearly-degenerate electronic orders and instabilities and superconductivity[23,32]. Furthermore, in comparison with the corresponding case in the single-layer cuprate superconductors[50,52], the quasiparticle scattering processes with the scattering wave vectors $q_i$ and the SC correlation are enhanced in the bilayer cuprate superconductors. This follows a fact that for a given quasiparticle scattering process, there are four corresponding scattering wave vectors: the scattering wave vector $q_i^{(AA)}$ connected between the antibonding hot spots, the scattering wave vector $q_i^{(AB)}$ connected between the antibonding and bonding hot spots, the scattering wave vector $q_i^{(BA)}$ connected between the bonding and antibonding hot spots, and the scattering wave vector $q_i^{(BB)}$ connected between the bonding hot spots. The quasiparticle scattering processes with the corresponding four scattering wave vectors $q_i^{(AA)}$, $q_i^{(AB)}$, $q_i^{(BA)}$, and $q_i^{(BB)}$ contribute to the same electronic instability, and then this given electronic instability is enhanced. In particular, the quasiparticle scattering process with the scattering wave vector $q_i$ connected the hot spots on the straight Fermi arcs shown in Fig. 2 describes the
pairing of electrons and holes at \( \mathbf{k} \uparrow \) and \( \mathbf{k} + \mathbf{Q}^{(\alpha \alpha')} \uparrow \), with \( Q_{\text{HS}}^{(\alpha \alpha')} = q_i^{(\alpha \alpha')} \), \( Q_{\text{HS}}^{(\alpha \beta \alpha')}, \) \( Q_{\text{HS}}^{(\beta \alpha \alpha')}, \) and \( Q_{\text{HS}}^{(\beta \beta \alpha')} \), which is responsible for the charge-order formation. On the other hand, the electron pairing at \( \mathbf{k} + \mathbf{Q}^{(\alpha \alpha')} \downarrow \) and \( -\mathbf{k} - \mathbf{Q}^{(\alpha \alpha')} \downarrow \) states related with the wave vector \( q_i \) shown in Fig. 2 is responsible for the SC correlation. These two special processes therefore indicate a coexistence and competition between charge order and superconductivity just as they have been observed from a wide variety of measurement techniques. Moreover, the average value of these four quasiparticle scattering wave vectors \( \bar{Q}_{\text{HS}} = (Q_{\text{HS}}^{(\alpha \alpha')} + Q_{\text{HS}}^{(\beta \beta \alpha')} + Q_{\text{HS}}^{(\beta \alpha \alpha')} + Q_{\text{HS}}^{(\beta \beta \alpha')})/4 \) at the doping concentration \( \delta \approx 0.12 \) is found to be \( \bar{Q}_{\text{HS}} \approx 0.31 \) (here we use the reciprocal units), which is well consistent with experimental average value \( \bar{Q}_{\text{CO}} \approx 0.256 \) obtained in the underdoped bilayer cuprate superconductors \( \text{Bi}_2\text{Sr}_2\text{La}_n\text{CuO}_{6+\delta} \). Superconductivity in cuprate superconductors occurs on adding charge carriers to the copper-oxide layers. As a natural consequence of this adding charge-carrier progression, the charge-order wave vector \( \bar{Q}_{\text{CO}} \) is also doping dependent, where we find that as the doping concentration is increased, \( Q_{\text{HS}} \) decreases almost linearly, also in qualitative agreement with the experimental data of the doping dependence of the charge-order wave vector \( Q_{\text{CO}} \) observed from the bilayer cuprate superconductors \( \text{Bi}_2\text{Sr}_2\text{La}_n\text{CuO}_{6+\delta} \). The enhancement of the electronic orders and SC correlation in the bilayer cuprate superconductors will be discussed further in Sec. III.F.3.

The essential ingredients to develop the bonding and antibonding EFS reconstructions in the bilayer cuprate superconductors are the following two: the pseudogap effect as in the single-layer cases and the bilayer coupling. The electron self-energy in the particle-hole channel \( \Sigma^{(\nu)}(\mathbf{k}, \omega) \) in Eq. (5) in the bonding-antibonding representation can be also rewritten as

\[
\Sigma^{(\nu)}(\mathbf{k}, \omega) \approx \frac{\Delta^{(\nu)}(k)^2}{\omega + \epsilon_{0k}}, \tag{8}
\]

where the corresponding dispersion relation \( \epsilon_{0k}^{(\nu)} \) and the momentum dependence of the pseudogap \( \Delta^{(\nu)}(k) \) are obtained directly from the electron self-energy \( \Sigma^{(\nu)}(\mathbf{k}, \omega) \) in Eq. (5) and its antisymmetric part \( \Sigma^{(\nu)}(\mathbf{k}, \omega) \) as \( \epsilon_{0k}^{(\nu)} = -\Sigma^{(\nu)}(\mathbf{k}, 0) / \Sigma^{(\nu)}(\mathbf{k}, 0) \) and \( \Delta^{(\nu)}(k) = \Sigma^{(\nu)}(\mathbf{k}, 0) / \sqrt{-\Sigma^{(\nu)}(\mathbf{k}, 0)} \), respectively. The onset of this momentum-dependent pseudogap \( \Delta^{(\nu)}(k) \) is therefore identified as the reduction of the closed EFS contour to the Fermi pocket. This in turn shows that the emergence of the pseudogap \( \Delta^{(\nu)}(k) \) is the microscopic mechanism underlying multiple electronic orders driven by the EFS instability in cuprate superconductors. The expression in Eq. (8) also shows that the imaginary part of \( \Sigma^{(\nu)}(\mathbf{k}, \omega) \) is related directly with the pseudogap as

\[
\text{Im} \Sigma^{(\nu)}(\mathbf{k}, \omega) \approx 2\pi|\Delta^{(\nu)}(k)|^2 \delta(\omega + \epsilon_{0k}^{(\nu)}), \tag{9}
\]

which therefore shows that the momentum dependence of the quasiparticle scattering rate \( \Gamma_{\nu}(\mathbf{k}, \omega) \) in Eq. (5) is dominated by the momentum dependence of the pseudogap \( \Delta^{(\nu)}(k) \).

With the help of the above expression of the electron self-energy \( \Sigma^{(\nu)}(\mathbf{k}, \omega) \) in Eq. (5), the electron normal and anomalous Green’s functions in Eq. (5) in the SC-state with the coexisting pseudogap state can be reexpressed explicitly as

\[
G_{\nu}(\mathbf{k}, \omega) = \left( \frac{U_{1k}^{(\nu)} + V_{1k}^{(\nu)}(\omega - E_{1k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k))}{\omega - E_{1k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k)} \right) + \left( \frac{U_{2k}^{(\nu)} + V_{2k}^{(\nu)}(\omega - E_{2k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k))}{\omega - E_{2k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k)} \right), \tag{10a}
\]

\[
\Gamma_{\nu}^{(\nu)}(\mathbf{k}, \omega) = -\frac{a_{1k}^{(\nu)} \Delta_{\nu}^{(\nu)}(k)}{2E_{1k}^{(\nu)}(\omega - E_{1k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k))} \left( \frac{1}{\omega - E_{1k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k)} - \frac{1}{\omega + E_{1k}^{(\nu)}} \right) + \frac{a_{2k}^{(\nu)} \Delta_{\nu}^{(\nu)}(k)}{2E_{2k}^{(\nu)}(\omega - E_{2k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k))} \left( \frac{1}{\omega - E_{2k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k)} - \frac{1}{\omega + E_{2k}^{(\nu)}} \right), \tag{10b}
\]

where \( a_{1k}^{(\nu)} = \frac{(E_{1k}^{(\nu)} - E_{0k}^{(\nu)})/(E_{1k}^{(\nu)} + E_{2k}^{(\nu)})}{(E_{1k}^{(\nu)} - E_{0k}^{(\nu)})/(E_{1k}^{(\nu)} - E_{2k}^{(\nu)})} \), the electron pair gap \( \Delta_{\nu}^{(\nu)}(k) = \Sigma^{(\nu)}(\mathbf{k}, \omega = 0) = \Delta_{L}^{(d)}(k) + (-1)^{1+\nu} \Delta_{T} \), with \( \gamma_{L}^{(d)} = [\cos k_x - \cos k_y]/2 \). However, as we have mentioned above, the electron quasiparticle excitation spectrum has been split into its bonding and antibonding components by the bilayer coupling, with each component that is independent. In this case, the bonding electron quasiparticle excitations in the SC-state with the coexisting pseudogap state become superpositions of four bonding-eigenstates with the corresponding four bonding-energy eigenvalues \( E_{1k}^{(1)}, -E_{1k}^{(1)}, E_{2k}^{(1)} \), and \( -E_{2k}^{(1)} \), while the antibonding electron quasiparticle excitations in the SC-state with the coexisting pseudogap state become superpositions of four antibonding-eigenstates with the corresponding four antibonding-energy eigenvalues \( E_{1k}^{(2)}, -E_{1k}^{(2)}, E_{2k}^{(2)} \), and \( -E_{2k}^{(2)} \), where the energy eigenvalues \( E_{1k}^{(\nu)} = \sqrt{|K_{1k}^{(\nu)} + K_{2k}^{(\nu)}|/2}, E_{2k}^{(\nu)} = \sqrt{|K_{1k}^{(\nu)} - K_{2k}^{(\nu)}|/2}, \) and the kernel functions,

\[
K_{1k}^{(\nu)} = \epsilon_{k}^{(\nu)} + \epsilon_{0k}^{(\nu)} + 2\Delta_{\nu}^{(\nu)}(k), \quad K_{2k}^{(\nu)} = \epsilon_{k}^{(\nu)} + \epsilon_{0k}^{(\nu)} + 4\Delta_{\nu}^{(\nu)}(k) + \Delta_{\nu}^{(\nu)}(k), \quad \text{and} \quad b_{1k}^{(\nu)} = \epsilon_{k}^{(\nu)} - \epsilon_{0k}^{(\nu)} + 2\Delta_{\nu}^{(\nu)}(k), \quad b_{2k}^{(\nu)} = \epsilon_{k}^{(\nu)} - \epsilon_{0k}^{(\nu)} + \Delta_{\nu}^{(\nu)}(k), \]

while the coherence factors in the SC-state with the coexisting pseudogap state can be obtained.
as,

\[ U^{(v)/2}_{1k} = \frac{1}{2} \left[ a_{v k}^{(v)} \left( 1 + \frac{\varepsilon_{v k}^{(v)}}{E_{1k}^{(v)}} \right) - a_{-v k}^{(v)} \left( 1 + \frac{\varepsilon_{-v k}^{(v)}}{E_{1k}^{(v)}} \right) \right], \quad (11a) \]

\[ \Gamma^{(v)/2}_{1k} = \frac{1}{2} \left[ a_{v k}^{(v)} \left( 1 - \frac{\varepsilon_{v k}^{(v)}}{E_{1k}^{(v)}} \right) - a_{-v k}^{(v)} \left( 1 - \frac{\varepsilon_{-v k}^{(v)}}{E_{1k}^{(v)}} \right) \right], \quad (11b) \]

\[ U^{(v)/2}_{2k} = \frac{1}{2} \left[ \varepsilon_{2k}^{(v)} \left( 1 + \frac{\varepsilon_{2k}^{(v)}}{E_{2k}^{(v)}} \right) - a_{-v k}^{(v)} \left( 1 + \frac{\varepsilon_{-v k}^{(v)}}{E_{2k}^{(v)}} \right) \right], \quad (11c) \]

\[ \Gamma^{(v)/2}_{2k} = \frac{1}{2} \left[ \varepsilon_{2k}^{(v)} \left( 1 - \frac{\varepsilon_{2k}^{(v)}}{E_{2k}^{(v)}} \right) - a_{-v k}^{(v)} \left( 1 - \frac{\varepsilon_{-v k}^{(v)}}{E_{2k}^{(v)}} \right) \right], \quad (11d) \]

with \( a_{v k}^{(v)} = [\Delta_{PG}^{(v)}(k)]^2 / (E_{1k}^{(v)} - E_{2k}^{(v)})^2 \). These coherence factors are constrained by the sum rule for any wave vector \( k \), i.e., \( U^{(v)2}_{1k} + V^{(v)2}_{1k} + U^{(v)2}_{2k} + V^{(v)2}_{2k} = 1 \).

From the above expression of the electron normal and anomalous Green’s function in Eq. [10], it is easy to find that on the first bonding (antibonding) EFS sheet \( k_{F}^{(B)} \) \( (k_{F}^{(A)}) \) as shown in Fig. [2] the bonding (antibonding) energy eigenvalue \( E_{1k}^{(B)} \) \( (E_{1k}^{(A)}) \) along \( k_{F}^{(B)} \) \( (k_{F}^{(A)}) \) vanishes, while on the second bonding (antibonding) EFS sheet \( k_{BS}^{(B)} \) \( (k_{BS}^{(A)}) \) as shown in Fig. [2] the bonding (antibonding) energy eigenvalue \( E_{2k}^{(B)} \) \( (E_{2k}^{(A)}) \) along \( k_{BS}^{(B)} \) \( (k_{BS}^{(A)}) \) is equal to zero. These results therefore reflect a fact that in analogy to the single-layer case[34][35] the bonding (antibonding) energy band splitting is induced by the bonding (antibonding) pseudogap, which leads to form two bonding (antibonding) EFS sheets \( k_{F}^{(B)} \) and \( k_{BS}^{(B)} \) \( (k_{F}^{(A)} \) and \( k_{BS}^{(A)} \)) in momentum space as shown in Fig. [2]. Moreover, both the bonding and antibonding electron self-energies \( \Sigma_{ph}^{(1)}(k, \omega) \) and \( \Sigma_{ph}^{(2)}(k, \omega) \) originated in the electron’s coupling to spin excitations are strongly momentum dependent, indicating that both the bonding and antibonding pseudogaps \( \Delta_{PG}^{(1)}(k) \) and \( \Delta_{PG}^{(2)}(k) \) are also strongly dependent on momentum.

FIG. 4: (Color online) The map of (a) the bonding and (b) the antibonding quasiparticle scattering rates at \( \delta = 0.12 \) with \( T = 0.002J \) for \( t/J = 2.5 \), \( t'/t = 0.28 \), and \( t_{\perp}/t = 0.3 \).

The quasiparticle scattering rate \( \Gamma_{1}(k, \omega) \) in Eq. [7a] is dominated by the imaginary part of the electron self-energy \( \text{Im} \Sigma_{ph}^{(v)}(k, \omega) \) [then the pseudogap \( \Delta_{PG}^{(v)}(k) \)]. In order to show whether the spectral weight at around the antinodal region is gapped out by the pseudogap and the hot spot appears always at off-node place, we plot the map of the intensity of (a) the bonding quasiparticle scattering rate \( \Gamma_{1}(k,0) \) and (b) the antibonding quasiparticle scattering rate \( \Gamma_{2}(k,0) \) at \( \delta = 0.12 \) with \( T = 0.002J \) in Fig. [4] where the quasiparticle scattering occurs mainly along the EFS sheets. However, both \( \Gamma_{1}(k,0) \) and \( \Gamma_{2}(k,0) \), with the scattering much stronger at around the antinodal region than the nodal region, are strongly dependent on the Fermi angle. To see these anisotropic quasiparticle scattering rates along the EFS sheets more clearly, we plot the angular dependence of (a) \( \Gamma_{1}(k_{F}^{(B)} , 0) \) along the bonding EFS sheet \( k_{F}^{(B)} \) and (b) \( \Gamma_{2}(k_{F}^{(A)} , 0) \) along the antibonding EFS sheet \( k_{F}^{(A)} \) from the corresponding antinode to the node at \( \delta = 0.12 \) with \( T = 0.002J \) in Fig. [5] in comparison with the experimental result[35] of the quasiparticle scattering rate along EFS sheet observed from the underdoped bilayer cuprate superconductors Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (insets). It is thus shown that both \( \Gamma_{1}(k_{F}^{(B)} , 0) \) and \( \Gamma_{2}(k_{F}^{(A)} , 0) \) have a strong angular dependence, with \( \Gamma_{1}(k_{F}^{(B)} , 0) \) and \( \Gamma_{2}(k_{F}^{(A)} , 0) \) that exhibit the strongest scattering at the corresponding bonding and antibonding antinodes \( k_{AN}^{(B)} \) and \( k_{AN}^{(A)} \) respectively. More interestingly, \( \Gamma_{1}(k_{F}^{(B)} , 0) \) and \( \Gamma_{2}(k_{F}^{(A)} , 0) \) exhibit the weakest scattering at the corresponding bonding and antibonding hot spots \( k_{HS}^{(B)} \) and \( k_{HS}^{(A)} \), respectively. It should be emphasized that at around the bonding (antibonding) hot spot, the corresponding dispersion relation in the electron self-energy in Eq. [8] \[ \varepsilon_{1}^{(1)}(k_{HS}^{(B)}) \approx -\varepsilon_{1}^{(1)}(k_{HS}^{(A)}), \varepsilon_{2}^{(2)}(k_{HS}^{(B)}) \approx -\varepsilon_{2}^{(2)}(k_{HS}^{(A)}) \], and then the magnitude of the bonding (antibonding) pseudogap \( \Delta_{PG}^{(1)}(k_{F}^{(B)}) \) \( \Delta_{PG}^{(2)}(k_{F}^{(A)}) \) is
anomalously small. As we have shown in the single-layer case, this pseudogap \( \Delta^{(\nu)}(k) \) can be also identified as the momentum dependence of the charge-order gap. In this case, this result of the anomalously small bonding (antibonding) charge-order gap at around the bonding (antibonding) hot spots is well consistent with the experimental results of the momentum dependence of the charge-order gap observed at the tips of the Fermi arcs \( \delta \) however, they directly contradict the standard charge-order picture where an energy gap is expected at precisely that point. Furthermore, we have also calculated the angular dependence of \( \Gamma_1(k_{BS}, 0) \) along the bonding EFS sheet \( k_{BS}^{(B)} \) and \( \Gamma_2(k_{BS}, 0) \) along the antibonding EFS sheet \( k_{BS}^{(A)} \) from the corresponding antinode to the node, and found that the angular dependence of \( \Gamma_1(k_{BS}, 0) \) and \( \Gamma_2(k_{BS}, 0) \) resemble those of \( \Gamma_1(k_{F}, 0) \) and \( \Gamma_2(k_{F}, 0) \), respectively. These angular dependence of \( \Gamma_1(k_{BS}, 0) \) and \( \Gamma_2(k_{BS}, 0) \) therefore suppress heavily the spectral weight of the electron quasiparticle excitation spectrum on the bonding (antibonding) EFS sheets at around the antinodes \( \nu_1 \), however, they reduce modestly the spectral weight on the bonding (antibonding) EFS sheets at around the nodes \( \nu_2 \), and have the weakest scattering at around the hot spots, which lead to that the most part of the quasiparticles reside at around the bonding (antibonding) hot spots, and then these hot spots connected by the scattering wave vectors \( q_i \) construct an octet scattering model. Furthermore, the tips of the disconnected segments on the bonding EFS sheets \( k_{F}^{(B)} \) and \( k_{BS}^{(B)} \) (the antibonding EFS sheets \( k_{F}^{(A)} \) and \( k_{BS}^{(A)} \)) converges on the bonding (antibonding) hot spots to form the bonding (antibonding) Fermi pockets around the nodal region. Concomitantly, this instability of the EFS sheets thus drives the formation of the multiple electronic orders with the wave vectors \( q_i \), connecting the hot spots on the EFS sheets. As in the single-layer case \( \delta \), these multiple electronic orders also coexist and compete with superconductivity in the bilayer cuprate superconductors. On the other hand, the positions of the bonding and antibonding hot spots in the bilayer cuprate superconductors are doping dependent. For a better understanding of the evolution of the positions of the bonding and antibonding hot spots with doping, we have made a series of calculations for the bonding and antibonding components of the electron quasiparticle excitation spectrum \( I_1(k, \omega) \) and \( I_2(k, \omega) \) with the different doping concentrations, and the results show that when doping is increased, the positions of the bonding and antibonding hot spots move towards to the corresponding bonding and antibonding antinodes, respectively, which thus induces the decrease of the charge-order wave vector connecting the parallel hot spots on the EFS sheets with the increase of doping \( \delta \). In the normal-state, where the electron pair gap \( \Delta^{(\nu)}(k, \omega) = 0 \), the quasiparticle scattering rate \( \Gamma_\nu(k, \omega) \) in Eq. (7a) is reduced as the normal-state quasiparticle scattering rate \( \Gamma_\nu(k, \omega) = |\text{Im} \Sigma^{(\nu)}_{\text{ph}}(k, \omega)| \). However, the main behavior of the angular dependence of \( \Gamma_\nu(k, 0) \) in the SC-state is dominated by the imaginary part of the electron self-energy \( \text{Im} \Sigma^{(\nu)}_{\text{ph}}(k, \omega) \) [then the pseudogap \( \Delta^{(\nu)}_{\text{ph}}(k) \)]. In this case, the main feature of the above octet scattering model and the related multiple electronic orders driven by EFS instability can persist into the normal-state.

C. Line-shape in electron quasiparticle excitation spectrum

We now turn to discuss the complicated line-shape in the electron quasiparticle excitation spectrum of the bilayer cuprate superconductors. One of the most characteristic features of the electron quasiparticle excitation spectrum has been the so-called PDH structure observed firstly on the bilayer cuprate superconductors Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\) at around the antinodal region. This PDH structure consists of a sharp electron quasiparticle excitation peak at the lowest binding-energy, a broad hump at the higher binding-energy, and a spectral dip between them. Later, this outstanding PDH structure was also found in YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) and in other families of cuprate superconductors. This well-known PDH structure now has been identified along the entire EFS, and is a hallmark of the spectral line-shape of the ARPES spectrum in cuprate superconductors. Considering the dip as a consequence of very strong scattering of the electron quasiparticles by bosonic mode immediately nomates the appropriate bosonic excitations for the role of the electron pairing glue. Several scenarios have been proposed: the strong electron-phonon coupling, the bilayer splitting effect, and the pseudogap effect. However, the origin of this PDH structure detected by the ARPES experiments is still under debate.

Although the renormalization of the electrons in cuprate superconductors is characterized by a strong momentum-dependent anisotropy between the electron quasiparticle excitations along the EFS, the information revealed by ARPES experiments has shown that at around the antinodal, the hot spot, and the nodal regions of EFS contain the essentials of the whole low-energy electron quasiparticle excitations of cuprate superconductor. In this case, we have made a series of calculations for the bonding (antibonding) component of the electron quasiparticle excitation spectrum \( I_1(k, \omega) \) along with the bonding (antibonding) EFS sheet \( k_{F}^{(B)} \) (\( k_{F}^{(A)} \)) from the bonding (antibonding) antinode to the node. In Fig. 6, we firstly plot the results of the antibonding component of the electron quasiparticle excitation spectrum \( I_2(k, \omega) \) as a function of energy at (a) the antibonding antinode, (b) the antibonding hot spot, and (c) the node in \( \delta = 0.12 \) with \( T = 0.002J \). At around the antibonding antinode (see Fig. 5a), a very
sharp antibonding peak emerges at the lowest binding-energy corresponding to the SC peak, followed by an antibonding dip and then an antibonding hump in the higher binding-energy, leading to form a PDH structure in the antibonding component of the electron quasiparticle excitation spectrum. However, the positions of the antibonding peak, the antibonding dip, and the antibonding hump are momentum dependent. In particular, the position of the antibonding hump moves appreciably towards the antibonding peak as the momentum moves from the antibonding antinode to the antibonding hot spot, and eventually this antibonding hump is incorporated with the antibonding peak at around the antibonding hot spot in the antibonding component of the electron quasiparticle excitation spectrum (see Fig. 6), which leads to an absence of the PDH structure at around the antibonding hot spots. More importantly, this PDH structure in the antibonding components of the electron quasiparticle excitation spectrum is gradually developed again as the momentum moves from the antibonding hot spot to the node (see Fig. 6). This evolution of the PDH structure with momentum in the antibonding component of the electron quasiparticle excitation spectrum in the bilayer cuprate superconductors is in a striking analogy to that in the quasiparticle excitation spectrum of the single-layer case.

As a complement of the analysis of the spectral line-shape in the electron quasiparticle excitation spectrum of the bilayer cuprate superconductors, the results of the bonding component of the electron quasiparticle excitation spectrum \( I_1(k, \omega) \) as a function of energy at (a) the bonding antinode, (b) the bonding hot spot, and (c) the node in \( \delta = 0.12 \) with \( T = 0.002J \) for \( t/J = 2.5, t'/t = 0.28, \) and \( t_{\perp}/t = 0.3, \) where the arrows indicate the positions of the dip, while AN, HS, and N in the insets denote the bonding antinode, bonding hot spot, and node, respectively.

It is remarkable that the evolution of the PDH structure with momentum in the bonding component of the electron quasiparticle excitation spectrum in the bilayer cuprate superconductors is almost the same as that in the antibonding component of the electron quasiparticle excitation spectrum, including the location of the dip energy, and then both the results in the bonding and antibonding components of the electron quasiparticle excitation spectrum along the entire bonding and antibonding EFS sheets except for at around the bonding and antibonding hot spots are well consistent with the experimental observations on the bilayer cuprate superconductors. These results in Fig. 6 and Fig. 7 confirm again that the antibonding component of the electron quasiparticle excitation spectrum in the bilayer cuprate superconductors is completely independent of the bonding component.

Although the PDH structure in the bonding (antibonding) component of the electron quasiparticle excitation spectrum vanishes at around the bonding (antibonding) hot spots, however, the contribution from both the bonding and antibonding components of the electron quasipar-
the PDH structure at around the antibonding hot spots. To see this special feature more clearly, we plot the results of the bonding and antibonding components of the quasiparticle excitation spectrum \( f_1(k, \omega) \) (blue line) and \( f_2(k, \omega) \) (red line) as a function of energy at the antibonding hot spot in \( \delta = 0.12 \) with \( T = 0.002J \) in Fig. 8. It is obvious that the PDH structure at around the antibonding hot spot is mainly caused by the pure bilayer coupling\(^{12-14} \), with the very sharp low-energy peak that is closely related to the antibonding component of the quasiparticle excitation spectrum, while the hump is directly formed by the bonding component. In particular, this PDH structure at the antibonding hot spot originates from the single-layer cuprate superconductors,\(^{52,53} \) where the PDH structure is absent from the hot spots at any doping levels. Our results in Fig. 6 and Fig. 8 thus show that the PDH structure in the bilayer cuprate superconductors is significantly modulated by the hybridization of two copper-oxide layers within the unit cell, resulting the PDH structure that can be present all around EFS.

The above obtained results of the PDH structure in the electron quasiparticle excitation spectrum at around the antinodal, the hot spot, and the nodal regions are well consistent with the experimental observations on the bilayer cuprate superconductors.\(^{25,102,107,109} \) In particular, some distinctive features in Fig. 6, Fig. 7 and Fig. 8 can be also summarized as: (i) the electron quasiparticle excitation spectrum in the bilayer cuprate superconductors is split into its bonding and antibonding components, with each component itself that contributes to the low-energy spectral line-shape; (ii) although the PDH structure in the bonding (antibonding) component of the electron quasiparticle excitation spectrum is absent from the bonding (antibonding) hot spots, the total contribution from both the bonding and antibonding components of the electron quasiparticle excitation spectrum generates the PDH structure at around the antibonding hot spots, with the sharp low-energy peak that is closely related to the antibonding component of the quasiparticle excitation spectrum, while the hump is directly formed by the bonding component.

The bonding (antibonding) component of the electron quasiparticle excitation spectrum in Eq. 1 then the electron spectral function in Eq. 6 has a well-defined resonance character, where \( I_\nu(k, \omega) \) exhibits the peaks when the incoming photon energy \( \omega \) is equal to the renormalized quasiparticle excitation energy, i.e.,

\[
\omega - \tilde{E}_\nu(k, \omega) = 0,
\]

and then the weights of these peaks are dominated by the inverse of the quasiparticle scattering rate \( \Gamma_\nu(k, \omega) \). In other words, the spectral line-shape in the electron quasiparticle excitation spectrum is determined by both the renormalized band structures \( \tilde{E}_\nu(k, \omega) \) and the quasiparticle scattering rate \( \Gamma_\nu(k, \omega) \).

In the electron quasiparticle excitation spectrum, a quasiparticle with a long lifetime is observed as a sharp peak in intensity, and a quasiparticle with a short lifetime is observed as a broad hump, while the dip in the electron quasiparticle excitation spectrum except for at around the bonding (antibonding) hot spots implies that the quasiparticle scattering rate \( \Gamma_\nu(k, \omega) \) has a peak structure with the sharp peak located at around the dip energy. To see this point more clearly, we first plot the antibonding quasiparticle scattering rate \( \Gamma_2(k, \omega) \) as a function of energy at (a) the antibonding antinode, (b) the antibonding hot spot, and (c) the node for \( \delta = 0.12 \) with \( T = 0.002J \) in Fig. 9. At around the antibonding antinode, the antibonding quasiparticle scattering rate \( \Gamma_2(k, \omega) \) reaches a sharp peak at the binding-energy of \( \omega_{AB} \sim 0.136 \), and then the weight of this sharp peak decreases rapidly away from this binding-energy \( \omega_{AB} \), in good agreement with the corresponding experimental results\(^{122} \) of the energy dependence of the quasiparticle scattering rate at around the antinodal region observed on the bilayer cuprate superconductors \( Bi_2Sr_2CaCu_2O_{8+\delta} \). In particular, the position of this sharp peak in \( \Gamma_2(k, \omega) \) is just corresponding to the position of the antibonding dip in the antibonding component of the electron quasiparticle excitation spectrum as shown in Fig. 6. In other words, the peak structure in \( \Gamma_2(k, \omega) \) induces an intensity depletion in the antibonding dip region. However, the peak in \( \Gamma_2(k, \omega) \) at around the antinode become progressively broader as the antibonding hot spot is approached, and then the peak structures in \( \Gamma_2(k, \omega) \) vanish eventually at the antibonding hot spots as shown in Fig. 9b, which therefore leads to that the PDH structure in the antibonding component of the quasiparticle excitation spectrum is absent from the antibonding hot spot region as shown in Fig. 9. In particular, the peak in \( \Gamma_2(k, \omega) \) is gradually de-
veloped again as the momentum moves away from the antibonding hot spot to the node (see Fig. 9), which induces the emergence of the PDH structure at around the nodal region (see Fig. 6).

In Fig. 10 we plot the bonding quasiparticle scattering rate \( \Gamma_1(\mathbf{k}, \omega) \) as a function of energy at (a) the bonding antinode, (b) the bonding hot spot, and (c) the node for \( \delta = 0.12 \) with \( T = 0.002J \). Obviously, the evolution of the bonding quasiparticle scattering rate \( \Gamma_1(\mathbf{k}, \omega) \) with momentum in Fig. 10 is the same as that of the antibonding quasiparticle scattering rate \( \Gamma_2(\mathbf{k}, \omega) \) in Fig. 9. This special momentum dependence of the peak structure in the bonding quasiparticle scattering rate therefore induces the emergence of the striking PDH structure in the bonding component of the quasiparticle excitation spectrum shown in Fig. 7.

### D. Kink in dispersion

The first indication of the renormalization of the electrons in cuprate superconductors was observed in the ARPES measurements on the bilayer cuprate superconductor \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta \), where the presence of the kink in the electron quasiparticle dispersion is identified along the nodal direction \( \mathbf{k} \parallel \langle 110 \rangle \). Later, this kink is found to be present all around EFS, and appears in all families of cuprate superconductors, with the energy scale (in the energy range 50 ~ 80 meV) at which the kink occurs that is similar for these cuprate superconductors with one or more copper-oxide layers per unit cell. Although it is believed that the kink in the electron quasiparticle dispersion is due to the coupling of the electrons to particular bosonic excitations, the nature of these bosonic excitations remains controversial, where two main proposals are disputing the explanations of the origin of the kink. In one of the proposals, the kink is associated to the phonon dispersion \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta \), while in the other, the kink is related to the spin excitation. In this subsection, we show within the framework of the kinetic-energy driven superconductivity that the electron quasiparticle dispersion is affected by the spin excitation, and then the kink in the electron quasiparticle dispersion associated with the renormalization of the electrons is electronic in nature, while the bilayer coupling leads to the enhancement of the kink effect.

To show how the kink dispersion evolves as the momentum moves along EFS, we plot the intensity maps of the antibonding component of the electron quasiparticle excitation spectrum as a function of binding-energy along (a) the nodal cut, (b) the hot spot cut, and (c) the antinodal cut at \( \delta = 0.12 \) with \( T = 0.002J \) in the upper panel of Fig. 11. In the lower panel, we plot the corresponding antibonding dispersions along (d) the nodal cut.
cut, (e) the hot spot cut, and (f) the antinodal cut extracted from the positions of the lowest-energy electron quasiparticle excitation peaks in (a), (b) and (c), respectively. The arrow indicates the kink position.

FIG. 11: (Color online) Upper panel: the intensity maps of the antibonding component of the electron quasiparticle excitation spectrum as a function of binding-energy along (a) the nodal cut, (b) the hot spot cut, and (c) the antinodal cut at δ = 0.12 with T = 0.002J for t/ t* = 2.5, t'/t = 0.28, and t_⊥/ t = 0.3. Lower panel: the antibonding electron quasiparticle dispersions along (d) the nodal cut, (e) the hot spot cut, and (f) the antinodal cut extracted from the positions of the lowest-energy electron quasiparticle excitation peaks in (a), (b) and (c), respectively. The arrow indicates the kink position.

at around the energy \( \omega_{kink} \sim 0.7J = 70 \text{ meV} \) in the nodal direction and \( \omega_{kink} \sim 0.5J = 50 \text{ meV} \) near the antinode, also in good agreement with the experimental observations\(^{110,112}\). These results also indicate that the characteristic kink energy decreases smoothly as the momentum moves from the nodal region to the antinodal region.

In the framework of the kinetic-energy driven superconductivity, the renormalization of the electrons originates from the coupling between the electrons and spin excitations. However, the quantity that describes this coupling is the real and imaginary parts of the electron self-energy \( \Sigma^{(\nu)}_{ph}(k, \omega) \). In this case, above obtained kink in the electron quasiparticle dispersion relation in Eq. (7b) that does not originate from the bare dispersion relation \( \varepsilon_{n}(k) \) must be due to the slope changes in the real part of the modified electron self-energy \( \text{Re}\Sigma^{(\nu)}_{ph}(k, \omega) \) in Eq. (7c) and the drop in the quasiparticle scattering rate \( \Gamma_{\nu}(k, \omega) \) in Eq. (7a). To reveal the deep relation between the kink and electron self-energy more clearly, we plot the real part of the antibonding modified electron self-energy \( \text{Re}\Sigma^{(\nu)}_{ph}(k, \omega) \) as a function of binding-energy along (a) the nodal dispersion, (b) the hot spot dispersion, and (c) the antinodal dispersion as shown in Fig. 11d, Fig. 11e, Fig. 11f, respectively, at \( \delta = 0.12 \) with \( T = 0.002J \) in the upper panel of Fig. 12, where the red arrow indicates inflection point (then the point of the slope changes). In the lower panel, we plot the corresponding antibonding quasiparticle scattering rate \( \Gamma_{2}(k, \omega) \) as a function of binding-energy along (d) the nodal dispersion, (e) the hot spot dispersion, and (f) the antinodal dispersion, where
the blue arrow denotes the peak position (then the point of the drop in the quasiparticle scattering rate). The results in Fig. 12 therefore show that a slope change in the real part of the modified electron self-energy \( \text{Re} \Sigma_{\text{ph}}^{(2)}(k, \omega) \) is present all around the antibonding EFS sheet, while the corresponding quasiparticle scattering rate \( \Gamma_{2}(k, \omega) \) exhibits a peak structure except for \( \Gamma_{2}(k, \omega) \) along the nodal direction, where \( \Gamma_{2}(k, \omega) \) exhibits a gentle reduction with the decrease of binding-energy. In this case, we therefore find that the appearance of the kink in the electron quasiparticle dispersion is always related to the slope change in \( \text{Re} \Sigma_{\text{ph}}^{(2)}(k, \omega) \), i.e., the position of the kink in the electron quasiparticle dispersion shown in Fig. 11 is the exact same as that for the corresponding inflection point in \( \text{Re} \Sigma_{\text{ph}}^{(2)}(k, \omega) \) shown in Fig. 12. This inflection point is the point of the slope change in \( \text{Re} \Sigma_{\text{ph}}^{(2)}(k, \omega) \) as shown in Fig. 12 and therefore leads to the emergence of the kink in the electron quasiparticle dispersion. This is why the kink in the electron quasiparticle dispersion marks the crossover between two different slopes. On the other hand, the position of the kink in the electron quasiparticle dispersion shown in Fig. 11 is also the same as that for the corresponding peak in the quasiparticle scattering rate \( \Gamma_{2}(k, \omega) \) shown in Fig. 12 except for along the nodal direction, i.e., there is an exact one to one correspondence between the kink position shown in Fig. 11 and the peak position in \( \Gamma_{2}(k, \omega) \) shown in Fig. 12 except for \( \Gamma_{2}(k, \omega) \) along the nodal cut. This peak in the quasiparticle scattering rate suppresses heavily the spectral weight at around the kink, and then the weak spectral intensity appears always at around the kink along the EFS sheet cut except for along the nodal direction. This is why the experimentally observed kink is always related to the drop in the quasiparticle scattering rate except for along the nodal cut. More specifically, the scattering along the antinodal dispersion for the binding-energy less than the kink energy is stronger than that of the corresponding scattering along the nodal dispersion, while the scattering along the nodal dispersion for the binding-energy above the kink energy is stronger than that of the corresponding scattering along the antinodal dispersion, which leads to that the spectral weights along the nodal dispersion above the kink energy and along the antinodal dispersion less than the kink energy are suppressed heavily.

Although the kink is present all around EFS, there are some subtle differences for the physical origin of the kinks along the nodal, the hot spot, and the antinodal directions. At the nodal direction, the bilayer coupling is absent, and then the less visible kink in the antibonding electron quasiparticle dispersion is caused mainly by the renormalization of the electrons originated from the coupling between the electrons and spin excitations within the copper-oxide layer. However, the actual minimum of the quasiparticle scattering rate locates exactly at the hot spot regime as shown in Fig. 5b, where the quasiparticle scattering is very weak, and then the kink along the hot spot cut is induced mainly by the bilayer coupling. On the other hand, near the antinode, both the quasiparticle scattering rate and bilayer coupling exhibit the largest value, this large band splitting together with the particularly strong quasiparticle scattering lead to a break separating of the fastest dispersing high-energy part of the electron quasiparticle excitation spectrum located at...
the antibonding EFS sheet $k_{ph}^{(A)}$ from the slower dispersing low-energy part located at the antibonding EFS sheet $k_p^{(A)}$, and then these two branches approach one another at the kink energy. This separation of the band dispersion becomes most prominent at the antinode and results in the most strong strength of the kink. In other words, although the kink in the electron quasiparticle dispersion is present all around the antibonding EFS sheet, the kink along the hot spot cut is different from the kink along the nodal or antinodal cut, while the dispersion kink near the antinode becomes more pronounced due to the presence of the strong bilayer coupling. The above analysis therefore also explains why the kink effect near the antinode is more pronounced than that along the nodal cut.

As a complement of the above analysis of the kink in the electron quasiparticle dispersion in the SC-state, we have also discussed the dispersion kink in the normal-state, where the electron pair gap $\Delta(E_F)$ is zero, and then the real part of the modified electron self-energy $\text{Re}\Sigma_{ph}^{(r)}(k, \omega)$ and the quasiparticle scattering rate $\Gamma_{\nu\nu}(k, \omega)$ in Eq. (7a) are reduced as $\text{Re}\Sigma_{ph}^{(r)}(k, \omega) = \text{Re}\Sigma_{ph}^{(r)}(k, \omega)$ and $\Gamma_{\nu\nu}(k, \omega) = \text{Im}\Sigma_{ph}^{(r)}(k, \omega)$, respectively. In this case, we have calculated the antibonding component of the electron quasiparticle excitation spectrum in the normal-state along the nodal, the hot spot, and the antinodal cuts, and the related real and imaginary parts of the antibonding electron self-energy $\text{Re}\Sigma_{ph}^{(2)}(k, \omega)$ and $\text{Im}\Sigma_{ph}^{(2)}(k, \omega)$ along the nodal, the hot spot, and the antinodal dispersions with the same set of parameters as in Fig. 11 and Fig. 12 except for in the normal-state, and the obtained intensity maps of the antibonding component of the electron quasiparticle excitation spectrum and the related results of $\text{Re}\Sigma_{ph}^{(2)}(k, \omega)$ and $\text{Im}\Sigma_{ph}^{(2)}(k, \omega)$ are almost the same as that of the corresponding maps in the SC-state in Fig. 11 and results of $\text{Re}\Sigma_{ph}^{(2)}(k, \omega)$ and $\Gamma_{\nu\nu}(k, \omega)$ in Fig. 12 respectively. These results therefore confirm that the kink emerged in the SC-state can persist into the normal-state, and is caused by the same electron self-energy (then the pseudogap) effect generated by the coupling between the electrons and spin excitations, also in good agreement with the experimental observation.410–121 Our these results also indicate that the kink in the electron quasiparticle dispersion is totally unrelated to superconductivity.410–121

E. ARPES autocorrelation

The ARPES autocorrelation $\bar{C}(q, \omega) = (1/N) \sum_k I(k + q, \omega)/I(k, \omega)$ is defined as the autocorrelation of the electron quasiparticle excitation spectral intensities at two different momenta, separated by a momentum transfer $q$, at a fixed energy $\omega$, where the summation of momentum $k$ is restricted within the first Brillouin zone.66 This ARPES autocorrelation is effectively the momentum-resolved joint density of states in the electronic state, and can give us important new insights into the renormalization of the electrons. On the other hand, the scanning tunneling spectroscopy (STS) provides the information on the local density of states (LDOS) of the electronic state as a function of energy, and although the measured data obtained in real-space, these data can be inverted in terms of the Fourier transform (FT) to provide the momentum-space picture of the renormalization of the electrons. This technique is complementary to the photoemission in that it reveal local variations of the renormalized electrons in cuprate superconductors.101–117 In particular, the ARPES experimental results46–111 have shown that the ARPES autocorrelation exhibits discrete spots in momentum-space, which are directly related with the wave vectors $q_i$ connecting the hot spots on EFS shown in Fig. 2 and are well consistent with the QSI peaks observed from the FT-STS experiment.127–131 In this subsection, we discuss the characteristic feature of the ARPES autocorrelation in the bilayer cuprate superconductors and its connection with the QSI measured from the FT-STS experiments.

In the bilayer cuprate superconductors, the experimentally measurable ARPES autocorrelation $\bar{C}(q, \omega)$ can be described in terms of the antibonding and bonding components of the electron quasiparticle excitation spectrum as,

$$\bar{C}(q, \omega) = \sum_{\nu\nu'} \bar{C}^{(\nu\nu')}(q, \omega),$$  \hspace{1cm} (12a)

$$\bar{C}^{(\nu\nu')}(q, \omega) = \frac{1}{N} \sum_k I_\nu(k + q, \omega) I_{\nu'}(k, \omega),$$  \hspace{1cm} (12b)

where the components $\bar{C}^{(11)}(q, \omega)$, $\bar{C}^{(12)}(q, \omega)$, $\bar{C}^{(21)}(q, \omega)$, and $\bar{C}^{(22)}(q, \omega)$ are the autocorrelation functions of the bonding-bonding spectral intensities, the bonding-antibonding spectral intensities, the antibonding-bonding spectral intensities, and the antibonding-antibonding spectral intensities, respectively. For a better understanding of the characteristic feature of the ARPES autocorrelation in the bilayer cuprate superconductors, we first plot the maps of these four components of the ARPES autocorrelation (a) $\bar{C}^{(11)}(q, \omega)$, (b) $\bar{C}^{(12)}(q, \omega)$, (c) $\bar{C}^{(21)}(q, \omega)$, and (d) $\bar{C}^{(22)}(q, \omega)$ in the $[q_x, q_y]$ plane for the binding-energy $\omega = 0.12J$ at $\delta = 0.15$ with $T = 0.002J$ in Fig. 13. Obviously, the sharp peaks or so-called discrete spots emerge in all components $\bar{C}^{(11)}(q, \omega)$, $\bar{C}^{(12)}(q, \omega)$, $\bar{C}^{(21)}(q, \omega)$, and $\bar{C}^{(22)}(q, \omega)$ at almost the same positions, where the joint density of states is highest. Moreover, these discrete spots in $\bar{C}^{(11)}(q, \omega)$, $\bar{C}^{(12)}(q, \omega)$, $\bar{C}^{(21)}(q, \omega)$, and $\bar{C}^{(22)}(q, \omega)$ can be described by the octet scattering model shown in Fig. 2 and therefore are directly correlated with the wave vectors $q_i$ connecting the bonding-bonding hot spots, $q_i$ connecting the bonding-antibonding hot spots, $q_i$ connecting the antibonding-bonding hot spots, and $q_i$ connecting the bonding-bonding hot spots.
FIG. 13: (Color online) The autocorrelations of (a) the bonding-bonding spectral intensities, (b) the bonding-antibonding spectral intensities, (c) the antibonding-bonding spectral intensities, and (d) the antibonding-antibonding spectral intensities in the $[q_x, q_y]$ plane for $\omega = 0.12J$ at $\delta = 0.15$ with $T = 0.002J$ for $t/J = 2.5$, $t'/t = 0.28$, and $T/t = 0.3$.

FIG. 14: (Color online) (a) The maps of the ARPES autocorrelation in a $[q_x, q_y]$ plane for $\omega = 0.12J$ at $\delta = 0.15$ with $T = 0.002J$ for $t/J = 2.5$, $t'/t = 0.28$, and $T/t = 0.3$. (b) The corresponding experimental results of the optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ for $\omega = 12$ meV taken from Ref. [16].

FIG. 15: (Color online) (a) The map of the intensity of the ARPES autocorrelation in momentum-space for $\omega = 0.18J$ at $\delta = 0.15$ with $T = 0.002J$ for $t/J = 2.5$, $t'/t = 0.28$, and $T/t = 0.3$, and $J = 100$ meV. (b) The corresponding experimental result of the optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ for $\omega = 18$ meV taken from Ref. [16].

The antibonding-antibonding hot spots on the EFS sheets, respectively. In particular, the autocorrelation pattern in each component $C^{(\nu\nu')}(\mathbf{q}, \omega)$ is in a striking analogy to that in the single-layer case.\cite{17,12}

With the above results in Fig. 13, we now turn to discuss the experimentally measurable ARPES autocorrelation $C(\mathbf{q}, \omega)$ in the bilayer cuprate superconductors. In Fig. 14, we plot the map of the ARPES autocorrelation $C(\mathbf{q}, \omega)$ in a $[q_x, q_y]$ plane for the binding-energy $\omega = 12$ meV at $\delta = 0.15$ with $T = 0.002J$. For a comparison, the corresponding experimental result\cite{15} of the ARPES autocorrelation observed from the optimally doped bilayer cuprate superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ for the

bind-energy $\omega = 12$ meV is also shown in Fig. 14). It is very interesting to note that the obtained autocorrelation pattern in Fig. 14a is well consistent with the corresponding pattern in Fig. 14, obtained from the ARPES experimental observation on the bilayer superconductor.\cite{16}

Moreover, in comparison with the autocorrelation pattern for each component $C^{(\nu\nu')}(\mathbf{q}, \omega)$ in Fig. 13, it is thus shown clearly that the weight of the ARPES autocorrelation peaks (the discrete spots) are enhanced, reflecting a fact that the quasiparticle scattering processes with the scattering wave vectors $\mathbf{q}_i$ shown in Fig. 2 are enhanced, including the enhancement of the SC correlation as we have mentioned in Sec. III B. It therefore would be reasonable to expect its connection with the higher $\Gamma_c$ in the bilayer cuprate superconductors. In other words, this may be why $\Gamma_c$ in the bilayer cuprate superconductors is higher than that in the single-layer case.

To see these sharp peaks with the enhanced weights at the discrete spots more clearly, we plot the intensity map of $C(\mathbf{q}, \omega)$ in the $[q_x, q_y]$ plane for the binding-energy $\omega = 18$ meV at $\delta = 0.15$ with $T = 0.002J$ in Fig. 15, in comparison with the corresponding experimental result\cite{15} of the ARPES autocorrelation of the optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ for the binding-energy $\omega = 18$ meV in Fig. 15, where the sharp autocorrelation peaks with the enhanced weights are located exactly at the corresponding discrete spots of $C(\mathbf{q}, \omega)$, which further confirms that the sharp peaks in $C(\mathbf{q}, \omega)$ are closely associated with the corresponding wave vectors $\mathbf{q}_i$ connecting the antibonding and bonding hot spots on the EFS sheets shown in Fig. 2. To analyze the evolution of the dispersion of the ARPES autocorrelation peaks, we have also made a series of calculations for the momentum and energy dependence of $C(\mathbf{q}, \omega)$ with different $\mathbf{q}_i$, and the results show that the sharp autocorrelation peaks in $C(\mathbf{q}, \omega)$ disperse smoothly with energy, and these dispersion autocorrelation peaks follow the same evolution of the corresponding hot spots on the EFS sheets with energy, also in good agreement with the ARPES experimental observations.\cite{16}

Now we turn to discuss the direct connection between the discrete spots in the ARPES autocorrelation
The quasiparticles scattering from impurities in cuprate superconductors interfere with one another, producing a standing wave pattern in the inhomogeneous part $\delta\rho^{(v\nu')}(\mathbf{q},\omega)$ for FT LDOS.\cite{124,131} The dispersion of the peaks in $\delta\rho^{(v\nu')}(\mathbf{q},\omega)$ as a function of bias voltage $\omega$ is analyzed in terms of the electron quasiparticle excitation spectrum and yields the information about EFS. In the single-layer cuprate superconductors, the inhomogeneous part $\delta\rho^{(v\nu')}(\mathbf{q},\omega)$ for FT LDOS in the presence of a single point-like impurity scattering potential has been evaluated based on the kinetic-energy-driven SC mechanism.\cite{124} and then the main features of QSI are qualitatively reproduced. In particular, it has been shown that in the case of the presence of the strong scattering potential, the momentum-space structure of the QSI patterns is well consistent with the momentum-space structure of the ARPES autocorrelation patterns. The essential physics of the intimate connection between the ARPES autocorrelation and QSI in the bilayer cuprate superconductors is the same as in the single-layer case\cite{124} except for the enhancement of the weights of the ARPES autocorrelation peaks in the presence of the bilayer coupling. This follows a fact that although the electron quasiparticle excitation spectrum is split into its bonding and antibonding components by the bilayer coupling in Eq. (3), the bonding component of the electron quasiparticle excitation spectrum is independent of the antibonding one. In this case, the essential properties of the inhomogeneous part $\delta\rho^{(v\nu')}(\mathbf{q},\omega)$ for FT LDOS obtained in terms of the antibonding and bonding components of the electron normal and anomalous Green’s functions (then the antibonding and bonding components of the electron spectral function) in the bilayer cuprate superconductors in the presence of a single point-like impurity scattering potential is almost the same as in the single-layer case in the presence of a single point-like impurity scattering potential,\cite{124} and then the experimentally measurable inhomogeneous part $\delta\rho^{(v\nu')}(\mathbf{q},\omega)$ for FT LDOS is also almost the same as in the single-layer case, although the weights of the QSI peaks are enhanced as in the case of the enhanced weights of the ARPES autocorrelation peaks. This is why as in the single-layer case\cite{124}, the momentum-space structure of the QSI patterns in the bilayer cuprate superconductors is also well consistent with the momentum-space structure of the ARPES autocorrelation patterns in the case of the presence of the strong scattering potential, and then the intimate connection between the ARPES autocorrelation and QSI is a universal feature of cuprate superconductors.

FIG. 16: (Color online) (a) The intensity map of the ARPES autocorrelation in the $[q_1,q_7]$ plane for $\omega = 0.16J$ at $\delta = 0.15$ with $T = 0.002J$ for $t'/t = 2.5$, $t'/t = 0.28$, $t_1/t = 0.3$, and $J = 100$ meV. (b) The experimental result of the quasiparticle scattering interference pattern $Z(q,\omega = 16$ meV) for the optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ taken from Ref. 130.

IV. SUMMARY AND DISCUSSIONS

Within the framework of the kinetic-energy driven superconductivity, we have studied the renormalization of the electrons in the bilayer cuprate superconductors by taking into account the effect due to the presence of the bilayer coupling, where the obtained characteristic features can be summarized clearly as: (a) the electron quasiparticle excitation spectrum in the bilayer cuprate superconductors is split into its bonding and antibonding components by the bilayer coupling, with each component that is independent, namely, although two copper-oxide layers within the unit cell are hybridized to form the bonding and antibonding copper-oxide layers, the bonding component of the electron quasiparticle excitation spectrum in the bonding copper-oxide layer is independent of the antibonding component of the electron quasiparticle excitation spectrum in the antibonding copper-oxide layer; (b) in the underdoped and optimally doped regimes, the spectral weight of the bonding (antibonding) EFS sheets at around the antinodal region is gaped out.
by the corresponding bonding (antibonding) pseudogap, and then the bonding (antibonding) electron quasiparticle excitations occupy the disconnected segments (Fermi arcs) located around the nodal region to form the bonding (antibonding) Fermi pocket. However, the bonding and antibonding Fermi pockets are almost degenerate, leading to that only one Fermi pocket can be observed; (c) the special point with the strongest intensity on the bonding (antibonding) Fermi arc which in this case coincide with the bonding (antibonding) hot spot. In particular, these bonding and antibonding hot spots connected by the scattering wave vectors $\mathbf{q}_i$ construct an octet scattering model, and then the enhancement of these quasiparticle scattering processes with the scattering wave vectors $\mathbf{q}_i$ and the SC correlation are confirmed via the result of the ARPES autocorrelation; (d) in a striking analogy to the single-layer cuprate superconductors, the PDH structure developed in each component of the electron quasiparticle excitation spectrum along the corresponding EFS sheet is directly related with the peak structure in the corresponding quasiparticle scattering rate except for at around the hot spots, where the PDH structure is caused mainly by the pure bilayer coupling, and is in a clear contrast to the single-layer case; (e) although the kink in the electron quasiparticle dispersion is present all around the antibonding EFS sheet, when the momentum moves from the node to the antinode, the kink energy smoothly decreases, while the dispersion kink becomes more pronounced, and in particular, near the cut close to the antinode, develops into a break separating of the fast dispersing high-energy part of the electron quasiparticle excitation spectrum located at $\mathbf{k}_{BS}^{(A)}$ from the slower dispersing low-energy part located at $\mathbf{k}_{p}^{(A)}$. By comparing with the corresponding results in the single-layer case, our present results also indicate that the exotic feature of the renormalized electrons in the bilayer cuprate superconductors is particularly obvious due to the presence of the bilayer coupling, reflecting a fact that the bilayer interaction has significant contributions to the renormalization of the electrons in the bilayer cuprate superconductors.

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Appendix A: Bonding and antibonding electron self-energies in the bilayer cuprate superconductors

In this Appendix, we derive explicitly the bonding and antibonding electron self-energies $\Sigma_{\text{ph}}^{(1)}(\mathbf{k}, \omega)$ and $\Sigma_{\text{ph}}^{(2)}(\mathbf{k}, \omega)$ in the particle-hole channel and the bonding and antibonding electron self-energies $\Sigma_{\text{pp}}^{(1)}(\mathbf{k}, \omega)$ and $\Sigma_{\text{pp}}^{(2)}(\mathbf{k}, \omega)$ in the particle-particle channel in Eq. (5) of the main text. Following the kinetic-energy driven superconductivity\cite{58,66–68} it has been shown\cite{58} that for the bilayer $t$-$J$ model (1), the interaction between the charge carriers directly from the kinetic energy by the exchange of spin excitations induces the charge-carrier pairing state, where the self-consistent equations that satisfied by the charge-carrier normal and anomalous Green’s functions have been obtained as,

$$
\tilde{g}(\mathbf{k}, \omega) = \tilde{g}^{(0)}(\mathbf{k}, \omega) + \tilde{g}^{(0)}(\mathbf{k}, \omega)[\tilde{\Sigma}_{\text{ph}}^{(h)}(\mathbf{k}, \omega)\tilde{g}(\mathbf{k}, \omega) - \tilde{\Sigma}_{\text{pp}}^{(h)}(\mathbf{k}, \omega)\tilde{\Gamma}^\dagger(\mathbf{k}, \omega)],
$$

(A1a)

$$
\tilde{\Gamma}^\dagger(\mathbf{k}, \omega) = \tilde{g}^{(0)}(\mathbf{k}, \omega)\tilde{\Sigma}_{\text{ph}}^{(h)}(\mathbf{k}, -\omega)\tilde{\Gamma}^\dagger(\mathbf{k}, \omega) + \tilde{\Sigma}_{\text{pp}}^{(h)}(\mathbf{k}, \omega)\tilde{g}(\mathbf{k}, \omega),
$$

(A1b)

where $\tilde{g}^{(0)}(\mathbf{k}, \omega) = g^{(0)}_L(\mathbf{k}, \omega) + \sigma_x g^{(0)}_T(\mathbf{k}, \omega)$ is the MF charge-carrier normal Green’s function, with the corresponding longitudinal and transverse parts $g^{(0)}_L(\mathbf{k}, \omega)$ and $g^{(0)}_T(\mathbf{k}, \omega)$, respectively, that have been given explicitly in Ref. 58, while $\tilde{\Sigma}_{\text{ph}}^{(h)}(\mathbf{k}, \omega) = \Sigma_{\text{ph}}^{(h)}(\mathbf{k}, \omega) + \Sigma_{\text{ph}}^{(h)}(\mathbf{k}, \omega)\sigma_x$ and $\tilde{\Sigma}_{\text{pp}}^{(h)}(\mathbf{k}, \omega) = \Sigma_{\text{pp}}^{(h)}(\mathbf{k}, \omega) + \Sigma_{\text{pp}}^{(h)}(\mathbf{k}, \omega)\sigma_x$ are the charge-carrier self-energies in the particle-hole and particle-particle channels, with $\Sigma_{\text{ph}}^{(h)}(\mathbf{k}, \omega)$ and $\Sigma_{\text{ph}}^{(h)}(\mathbf{k}, \omega)$ that are the corresponding longitudinal and transverse parts of the charge-carrier self-energy in the particle-hole channel, and $\Sigma_{\text{pp}}^{(h)}(\mathbf{k}, \omega)$ and $\Sigma_{\text{pp}}^{(h)}(\mathbf{k}, \omega)$ that are the corresponding longitudinal and transverse parts of the charge-carrier self-energy in the particle-particle channel. In particular, these longitudinal and transverse parts of the charge-carrier self-energies have been obtained explicitly from the spin bubble as\cite{58,66–68}. 

$$
\Sigma_{\text{ph}}^{(h)}(\mathbf{k}, \omega_n) = \frac{1}{N^2} \sum_{\mathbf{p}, \mathbf{q}} R^{(1)}_{\mathbf{p}+\mathbf{q}+\mathbf{k}} \frac{1}{i \beta} \sum_{i \omega_m} g_L(\mathbf{p} + \mathbf{k}, i \omega_m + i \omega_n)\Pi_{LL}(\mathbf{p}, \mathbf{q}, i \omega_m)
$$

$$
+ R^{(2)}_{\mathbf{p}+\mathbf{q}+\mathbf{k}} \frac{1}{i \beta} \sum_{i \omega_m} g_T(\mathbf{p} + \mathbf{k}, i \omega_m + i \omega_n)\Pi_{TL}(\mathbf{p}, \mathbf{q}, i \omega_m),
$$

(A2a)
\[ \Sigma^{(h)}_{\text{ph}T}(k, i\omega_n) = \frac{1}{N^2} \sum_{p, q} \left[ R^{(1)}_{p+q+k} \frac{1}{\beta} \sum_{ip_m} g_T(p + k, ip_m + i\omega_n) \Pi_{TT}(p, q, ip_m) \right] \]
\[ + R^{(2)}_{p+q+k} \frac{1}{\beta} \sum_{ip_m} g_L(p + k, ip_m + i\omega_n) \Pi_{LT}(p, q, ip_m) \]
\[\text{(A2b)}\]
\[ \Sigma^{(h)}_{\text{pp}L}(k, i\omega_n) = \frac{1}{N^2} \sum_{p, q} \left[ R^{(1)}_{p+q+k} \frac{1}{\beta} \sum_{ip_m} \Gamma^\dagger_L(p + k, ip_m + i\omega_n) \Pi_{LL}(p, q, ip_m) \right] \]
\[ + R^{(2)}_{p+q+k} \frac{1}{\beta} \sum_{ip_m} \Gamma^\dagger_L(p + k, ip_m + i\omega_n) \Pi_{LT}(p, q, ip_m) \]
\[\text{(A2c)}\]
\[ \Sigma^{(h)}_{\text{pp}T}(k, i\omega_n) = \frac{1}{N^2} \sum_{p, q} \left[ R^{(1)}_{p+q+k} \frac{1}{\beta} \sum_{ip_m} \Gamma^\dagger_T(p + k, ip_m + i\omega_n) \Pi_{TT}(p, q, ip_m) \right] \]
\[ + R^{(2)}_{p+q+k} \frac{1}{\beta} \sum_{ip_m} \Gamma^\dagger_T(p + k, ip_m + i\omega_n) \Pi_{LT}(p, q, ip_m) \]
\[\text{(A2d)}\]

where \( \omega_n \) and \( p_m \) are the fermionic and bosonic Matsubara frequencies, respectively, \( R^{(1)}_k = [Z(t'k - t'q_k)]^2 + t'^2 \) \( R^{(2)}_k = 2Z(t'k - t'q_k)t_\perp(k) \), and the spin bubbles \( \tilde{\Pi}_{\alpha',\alpha}(p, q, ip_m) \) \( \alpha = L, T, \alpha' = L, T \) have been evaluated explicitly as \[ 58 \] \[ \tilde{\Pi}_{\alpha}(p, q, ip_m) = \frac{1}{\beta} \sum_{ip_m} D^{(0)}_{\alpha}(q, ip_m) \]
\[ \times D^{(0)}_{\alpha}(q + p, ip_m + ip_m), \quad (A3) \]

with the bosonic Matsubara frequency \( g_m \) and the corresponding longitudinal and transverse parts of the spin Green’s function \( D^{(0)}_L(q, \omega) \) and \( D^{(0)}_T(q, \omega) \) in the self-consistent renormalized MF level, respectively, that have been evaluated as,
\[ D^{(0)}_L(k, \omega) = \frac{1}{2} \sum_{\nu=1,2} \frac{B^{(\nu)}_k}{\omega^2 - \omega^{(\nu)^2}}, \quad (A4a) \]
\[ D^{(0)}_T(k, \omega) = \frac{1}{2} \sum_{\nu=1,2} \frac{B^{(\nu)}_k}{\omega^2 - \omega^{(\nu)^2}}, \quad (A4b) \]

where the spin excitation spectrum \( \omega^{(\nu)}_k \) and function \( B^{(\nu)}_k \) have been given explicitly in Ref. \[ 58 \].

Based on the kinetic-energy-driven SC mechanism, we \[ 63 \] have developed a full charge-spin recombination scheme, where a charge carrier and a localized spin are fully recombined into a constrained electron. In particular, within this full charge-spin recombination scheme, we have realized that the coupling form between the electrons and spin excitations is the same as that between the charge carriers and spin excitations, which therefore implies that the form of the self-consistent equations satisfied by the electron normal and anomalous Green’s functions is the same as the form in Eq. \[ (A1) \] satisfied by the charge-carrier normal and anomalous Green’s functions. In other words, we can perform a full charge-spin recombination in which the charge-carrier normal and anomalous Green’s functions \( \tilde{g}(k, \omega) \) and \( \tilde{\Gamma}(k, \omega) \) in Eq. \[ (A1) \] are replaced by the electron normal and anomalous Green’s functions \( G(k, \omega) \) and \( \tilde{\Pi}(k, \omega) \), respectively, and then we obtained explicitly the self-consistent equations satisfied by the electron normal and anomalous Green’s functions for the bilayer \( t-J \) model \[ 2 \] as,
\[ \tilde{G}(k, \omega) = G^{(0)}(k, \omega) + \tilde{G}^{(0)}(k, \omega)\tilde{\Pi}_{\text{ph}}(k, \omega)G(k, \omega) \]
\[ - \tilde{\Sigma}_{\text{pp}}(k, \omega)\tilde{\Pi}_{\text{pp}}(k, \omega), \quad (A5a) \]
\[ \tilde{\Pi}(k, \omega) = G^{(0)}(k, -\omega)\tilde{\Sigma}_{\text{ph}}(k, -\omega)\tilde{\Pi}_{\text{ph}}(k, \omega) \]
\[ + \tilde{\Sigma}_{\text{pp}}(k, \omega)\tilde{G}(k, \omega), \quad (A5b) \]

where \( G^{(0)}(k, \omega) = G^{(0)}_L(k, \omega) + \sigma_x G^{(0)}_T(k, \omega) \) is the MF electron normal Green’s function, with the corresponding longitudinal and transverse parts \( G^{(0)}_L(k, \omega) \) and \( G^{(0)}_T(k, \omega) \), respectively, that can be obtained directly from the bilayer \( t-J \) model \[ 2 \] as,
\[ G^{(0)}_L(k, \omega) = \frac{1}{2} \sum_{\nu=1,2} \frac{1}{\omega - \varepsilon^{(\nu)}_k}, \quad (A6a) \]
\[ G^{(0)}_T(k, \omega) = \frac{1}{2} \sum_{\nu=1,2} \frac{-1}{\omega - \varepsilon^{(\nu)}_k}, \quad (A6b) \]

while \( \tilde{\Sigma}_{\text{ph}}(k, \omega) = \Sigma_{\text{ph}L}(k, \omega) + \sigma_x \Sigma_{\text{ph}T}(k, \omega) \) and \( \tilde{\Sigma}_{\text{pp}}(k, \omega) = \Sigma_{\text{pp}L}(k, \omega) + \sigma_x \Sigma_{\text{pp}T}(k, \omega) \) are electron self-energies in the particle-hole and particle-particle channels, respectively, with the corresponding longitudinal and transverse parts \( \Sigma_{\text{ph}L}(k, \omega) \) and \( \Sigma_{\text{ph}T}(k, \omega) \) \( \Sigma_{\text{pp}L}(k, \omega) \) and \( \Sigma_{\text{pp}T}(k, \omega) \), respectively, in the particle-hole (particle-particle) channel are obtained directly from the corresponding channels of the charge-carrier self-energies in Eq. \[ (A2) \] by the replacement of the charge-carrier normal and anomalous Green’s functions \( \tilde{g}(k, \omega) \) and \( \tilde{\Gamma}(k, \omega) \) with the corresponding electron normal and anomalous Green’s functions \( G(k, \omega) \) and \( \tilde{\Pi}(k, \omega) \) as,
\[ \Sigma_{\text{phL}}(k, i\omega_n) = \frac{1}{N^2} \sum_{p,q} \left[ R_{\text{pp}}^{(1)}(p+q+k) \beta \sum_{ip_m} G_L(p+k, ip_m+i\omega_n) \Pi_{\text{LL}}(p,q,ip_m) \right] + R_{\text{pp}}^{(2)}(p+q+k) \beta \sum_{ip_m} G_T(p+k, ip_m+i\omega_n) \Pi_{\text{TT}}(p,q,ip_m), \]

\[ \Sigma_{\text{phT}}(k, i\omega_n) = \frac{1}{N^2} \sum_{p,q} \left[ R_{\text{pp}}^{(1)}(p+q+k) \beta \sum_{ip_m} \Sigma_1^T(p+k, ip_m+i\omega_n) \Pi_{\text{LL}}(p,q,ip_m) \right] + R_{\text{pp}}^{(2)}(p+q+k) \beta \sum_{ip_m} \Sigma_1^T(p+k, ip_m+i\omega_n) \Pi_{\text{TT}}(p,q,ip_m), \]

\[ \Sigma_{\text{ppL}}(k, i\omega_n) = \frac{1}{N^2} \sum_{p,q} \left[ R_{\text{pp}}^{(1)}(p+q+k) \beta \sum_{ip_m} \Sigma_1^L(p+k, ip_m+i\omega_n) \Pi_{\text{LL}}(p,q,ip_m) \right] + R_{\text{pp}}^{(2)}(p+q+k) \beta \sum_{ip_m} \Sigma_1^L(p+k, ip_m+i\omega_n) \Pi_{\text{TT}}(p,q,ip_m). \]

However, as we have mentioned in Eq. (1), in the case of the coherent coupling of the copper-oxide layers within the unit cell, the more appropriate classification in the bilayer coupling case is in terms of the physical quantities within the basis of the bonding and antibonding components. With the help of the above longitudinal and transverse parts of the electron self-energies in the particle-hole and particle-particle channels, the corresponding bonding and antibonding parts of the electron self-energies in the particle-hole and particle-particle channels can be expressed explicitly in the bonding-antibonding representation as,

\[ \Sigma^{(\nu)}_{\text{ph}}(k, \omega) = \Sigma_{\text{phL}}(k, \omega) + (-1)^{\nu+1} \Sigma_{\text{phT}}(k, \omega), \]

\[ \Sigma^{(\nu)}_{\text{pp}}(k, \omega) = \Sigma_{\text{ppL}}(k, \omega) + (-1)^{\nu+1} \Sigma_{\text{ppT}}(k, \omega), \]

respectively, and then the corresponding bonding and antibonding parts of the electron normal and anomalous Green’s functions now can be obtained as quoted in Eq. (5).

Now our goal is to evaluate these bonding and antibonding parts of the electron self-energies in the particle-hole and particle-particle channels. According to our previous discussions of the renormalization of the electrons in the single-layer cuprate superconductors, the bonding and antibonding electron self-energies \( \Sigma^{(1)}_{\text{pp}}(k, \omega) \) and \( \Sigma^{(2)}_{\text{pp}}(k, \omega) \) in the particle-particle channel represent the bonding and antibonding electron pair gaps, respectively, while the bonding and antibonding electron self-energies \( \Sigma^{(1)}_{\text{ph}}(k, \omega) \) and \( \Sigma^{(2)}_{\text{ph}}(k, \omega) \) in the particle-hole channel represent the bonding and antibonding electron quasiparticle coherence, respectively. In particular, the electron self-energy \( \Sigma^{(\nu)}_{\text{pp}}(k, \omega) \) is an even function of energy \( \omega \), while the electron self-energy \( \Sigma^{(\nu)}_{\text{ph}}(k, \omega) \) is not. For convenience in the following calculations, the electron self-energy \( \Sigma^{(\nu)}_{\text{ph}}(k, \omega) \) can be separated into two parts as \( \Sigma^{(\nu)}_{\text{ph}}(k, \omega) = \Sigma^{(\nu)}_{\text{phL}}(k, \omega) + \omega \Sigma^{(\nu)}_{\text{phO}}(k, \omega) \) with the corresponding symmetric part \( \Sigma^{(\nu)}_{\text{phS}}(k, \omega) \) and antisymmetric part \( \omega \Sigma^{(\nu)}_{\text{phO}}(k, \omega) \), respectively, and then both \( \Sigma^{(\nu)}_{\text{phL}}(k, \omega) \) and \( \Sigma^{(\nu)}_{\text{phO}}(k, \omega) \) are an even function of energy \( \omega \). Following the common practice, the electron quasiparticle coherent weight in the bilayer cuprate superconductors therefore can be obtained as \( Z^{(\nu)}_{\text{phL}}(k, \omega) = Z^{(\nu)}_{\text{phS}}(k, \omega) + (-1)^{\nu+1} Z^{(\nu)}_{\text{phO}}(k, \omega) \), with \( Z^{(\nu)}_{\text{phL}}(k, \omega) = 1 - \Sigma_{\text{phL}}(k, \omega) \) and \( Z^{(\nu)}_{\text{phO}}(k, \omega) = \Sigma_{\text{phO}}(k, \omega) \). However, as we have mentioned in Eq. (1), in the case of the coherent coupling of the copper-oxide layers within the unit cell, the more appropriate classification in the bilayer coupling case is in terms of the physical quantities within the basis of the bonding and antibonding components. With the help of the above longitudinal and transverse parts of the electron self-energies in the particle-hole and particle-particle channels, the corresponding bonding and antibonding parts of the electron self-energies in the particle-hole and particle-particle channels can be expressed explicitly in the bonding-antibonding representation as,

\[ \Sigma^{(\nu)}_{\text{phL}}(k, \omega) = \Sigma_{\text{phL}}(k, \omega) + (-1)^{\nu+1} \Sigma_{\text{phT}}(k, \omega), \]

\[ \Sigma^{(\nu)}_{\text{ppL}}(k, \omega) = \Sigma_{\text{ppL}}(k, \omega) + (-1)^{\nu+1} \Sigma_{\text{ppT}}(k, \omega), \]

[ ]
and antibonding parts of the self-consistent renormalized MF electron normal and anomalous Green’s functions now can be obtained directly from Eq. (5) as,

\[ G_{\nu}^{(\text{RMF})}(k, \omega) = Z_{F}^{(\nu)} \left( \frac{I_{k}^{(\nu)} + V_{k}^{(\nu)}}{\omega - E_{k}^{(\nu)}} \right) \]  \hspace{1cm} (A9a)

\[ \Im_{\nu}^{(\text{RMF})}(k, \omega) = -Z_{F}^{(\nu)} \frac{\Delta_{Z}^{(\nu)}(k)}{2E_{k}^{(\nu)}} \times \left( \frac{1}{\omega - E_{k}^{(\nu)}} - \frac{1}{\omega + E_{k}^{(\nu)}} \right) \]  \hspace{1cm} (A9b)

where the coherence factors \( U_{k}^{(\nu)} = (1 + \frac{\varepsilon_{k}^{(\nu)}}{E_{k}^{(\nu)}})/2 \) and \( V_{k}^{(\nu)} = (1 - \frac{\varepsilon_{k}^{(\nu)}}{E_{k}^{(\nu)}})/2 \), \( \varepsilon_{k}^{(\nu)} = Z_{F}^{(\nu)} \), \( E_{k}^{(\nu)} = \sqrt{(\varepsilon_{k}^{(\nu)})^2 + (\Delta_{Z}^{(\nu)}(k))^2} \), and \( \Delta_{Z}^{(\nu)}(k) = Z_{F}^{(\nu)} \Delta(k) \). In particular, in the normal-state, the electron pair gap \( \Delta^{(\nu)}(k) = 0 \), and then the bonding and antibonding parts of the self-consistent renormalized MF electron normal Green’s functions in Eq. (A9) are reduced as,

\[ G_{\nu}^{(\text{RMF})}(k, \omega) = Z_{F}^{(\nu)} \frac{1}{\omega - \varepsilon_{k}^{(\nu)}} \]  \hspace{1cm} (A10)

with the related bonding and antibonding components of the self-consistent renormalized MF electron quasiparticle excitation spectrum,

\[ I_{1}^{(0)}(k, \omega) = 2\pi n_{F}(\omega) Z_{F}^{(1)}(\omega - \varepsilon_{k}^{(1)}) \]  \hspace{1cm} (A11a)

\[ I_{2}^{(0)}(k, \omega) = 2\pi n_{F}(\omega) Z_{F}^{(2)}(\omega - \varepsilon_{k}^{(2)}) \]  \hspace{1cm} (A11b)

respectively.

Substituting these bonding and antibonding parts of the self-consistent renormalized MF electron normal and anomalous Green’s functions in Eq. (A9) into Eqs. (A7) and (A8), we therefore obtain explicitly the bonding and antibonding parts of the electron self-energies in the particle-hole and particle-particle channels as,

\[ \Sigma_{ph}^{(\nu)}(k, \omega) = \frac{1}{N^2} \sum_{\mu\nu\mu'\nu'_{123}} \left( -\mu^{\nu} G_{\nu'_{123}}(k_{ph}^{\nu}) \right) \left[ \frac{F_{\mu\nu\mu'\nu'_{123}}(k_{ph}^{\nu})}{\omega - E_{k_{ph}^{\nu}}^{(\nu)} + \omega_{\mu\nu'_{123}}^{(\nu)}} + \frac{F_{\mu\nu\mu'\nu'_{123}}(k_{ph}^{\nu})}{\omega - E_{k_{ph}^{\nu}}^{(\nu)} + \omega_{\mu'\nu'_{123}}^{(\nu)}} \right] \]  \hspace{1cm} (A12a)

\[ \Sigma_{pp}^{(\nu)}(k, \omega) = -\frac{1}{N^2} \sum_{\mu\nu\mu'\nu'_{123}} \left( -\mu^{\nu} G_{\nu'_{123}}(k_{pp}^{\nu}) \right) \left[ \frac{\Delta_{Z}^{(\nu)}(k_{pp}^{\nu})}{2E_{k_{pp}^{\nu}}} \left[ \frac{F_{\mu\nu\mu'\nu'_{123}}(k_{pp}^{\nu})}{\omega - E_{k_{pp}^{\nu}}^{(\nu)} + \omega_{\mu\nu'_{123}}^{(\nu)}} + \frac{F_{\mu\nu\mu'\nu'_{123}}(k_{pp}^{\nu})}{\omega - E_{k_{pp}^{\nu}}^{(\nu)} + \omega_{\mu'\nu'_{123}}^{(\nu)}} \right] \right] \]  \hspace{1cm} (A12b)

with \( n_{1B}^{(\nu)}(p, q) = n_{B}(\omega_{p+q}^{(\nu)}) - n_{B}(-\mu_{p+q}^{(\nu)}) \) and \( n_{2B}^{(\nu)}(p, q) = [1 + n_{B}(\omega_{p}^{(\nu)})]n_{B}(-\mu_{p+q}^{(\nu)}) \), and the boson and fermion distribution functions \( n_{B}(\omega) \) and \( n_{F}(\omega) \), respectively. In this case, the longitudinal and transverse parts of the electron pair gap parameter, the electron quasiparticle coherent weight, and the chemical potential satisfy following self-consistent equations,

\[ \Delta_{L} = \frac{4}{N^3} \sum_{\mu\nu\mu'\nu'_{123}} \left( -\mu^{\nu} G_{\nu'_{123}}(k_{pp}^{\nu}) \right) \left[ \frac{F_{\mu\nu\mu'\nu'_{123}}(k_{pp}^{\nu})}{\omega_{\mu\nu'_{123}}^{(\nu)}} \right] \left[ \frac{F_{\mu\nu\mu'\nu'_{123}}(k_{pp}^{\nu})}{\omega_{\mu'\nu'_{123}}^{(\nu)}} + \frac{F_{\mu\nu\mu'\nu'_{123}}(k_{pp}^{\nu})}{\omega_{\mu\nu'_{123}}^{(\nu)}} \right] \]  \hspace{1cm} (A13a)
$$\bar{\Delta}_T = -\frac{1}{N^3} \sum_{\nu \nu' \nu'' k p q} (-1)^{\mu+\nu} \Omega_{\nu \nu' \nu'' k p q}^1 \frac{\Delta_{\nu}^{(1)}(k+p)}{2 E_{k+p}^{(1)}} \left( \frac{F_{\nu' \nu'' \nu}(k,p,q)}{E_{k+p}^{(1)} + \omega_{\mu pq}} + \frac{F_{\nu' \nu'' \nu}(k,p,q)}{E_{k+p}^{(1)} - \omega_{\mu pq}} \right),$$

(A13b)

$$\frac{1}{Z_F^{(\nu)}} = 1 + \frac{1}{N^2} \sum_{\nu \nu' \nu'' k p q} (-1)^{\mu+\nu} \Omega_{\nu \nu' \nu'' k p q}^1 \left( \frac{F_{\nu' \nu'' \nu}(k_0,p,q)}{(E_{k_0+p}^{(1)} + \omega_{\mu pq})^2} + \frac{F_{\nu' \nu'' \nu}(k_0,p,q)}{(E_{k_0+p}^{(1)} - \omega_{\mu pq})^2} \right),$$

(A13c)

$$1 - \delta = \frac{1}{2N} \sum_{k,\nu} Z_F^{(\nu)} \left( 1 - \frac{z(k)}{E_F^{(\nu)}} \tanh \left[ \frac{1}{2} \beta E_F^{(\nu)} \right] \right),$$

(A13d)

with $k_0 = [\pi, 0]$. These equations have been solved self-consistently, and then all order parameters and chemical potential are determined by the self-consistent calculation.
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