Statistical Properties & Different Methods Of Estimation Of A New Extended Weighted Frechet Distribution.

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**Article History**: Received: 11 January 2021; Accepted: 27 February 2021; Published online: 5 April 2021

**Abstract**: In this paper, we introduce a new distribution called the extended weighted Frechet distribution, which we obtain by applying the Azzalini method and deduced some statistical properties such as mean, variance, coefficients of variation, coefficient of skewness, and coefficient of kurtosis. The parameters of the new distribution were estimated by the following estimation methods: Maximum Likelihood Method (MLE) and percentile method. We used the Monte Carlo simulation to compare the performances of the proposed estimators obtained through methods of estimation.

**Keywords**: Azzalini’s method, extended weighted Frechet distribution, Percentiles method.

1. **Introduction**

   Recently, the known distributions did not represent the data obtained from the real world, which became more complex than before, as the problems associated with that data made it more difficult in terms of representation and modeling. Therefore, the matter required all researchers in the field of statistics to develop and expand these distributions in order to infer through them the results that are unbiased and consistent. As well as to obtain application flexibility for these distributions in data representation and modeling.

   The proposed distribution called the expanded weighted Frechet distribution that we obtained from applying Azzalini’s method is more flexible. Azzalini (1985) introduced the skew-normal distribution when adding a parameter to impart more flexibility to the normal distribution. Also, M. K. Shakhtrah (2012) introduced a new class of distributions called two-parameter exponential distribution (TWE). This new class of distributions generalizes the weighted exponential distribution (WE) proposed by Gupta and Kundu (2009). It turns out that (TWE) better than (WE) application of the two groups from real data. Abbas Mahdavi & Leila Jabari (2017) proposed a new model using Azzalini’s method and demonstrated through the application that the proposed model is better and more flexible in application to real data. Abdulhakim A. Al-Batmain (2020) proposed a new model for Rayleigh distribution was called type I, the half logistic Rayleigh distribution, and the new model was applied to real data where the data was adopted (63) views from previous research by researchers (Kundu and Raqap, 2009) to clarify the importance and flexibility of the new Rayleigh distribution. In this paper, a new extended weighted Frechet distribution has been proposed and studied. Frechet distribution was introduced by Maurice Frechet (French mathematician) (1878-1973). The Frechet distribution has many applications in modeling and analyzing events such as earthquakes, floods and wind speeds, diseases, as well as in engineering fields to analyzing the statistical behavior of engineering material properties.

   The Frechet distribution is inverted Weibull distribution, also called the extreme value distribution. The probability density function (pdf) and the cumulative distribution function (CDF) for Frechet distribution are (Kamran Abbas and Tang Yincai, 2012):

   \[
   f(x) = \frac{\lambda}{\theta} \left(\frac{\theta}{x}\right)^{\lambda+1} e^{-\left(\frac{\theta}{x}\right)^\lambda} \quad x > 0 \tag{1}
   \]

   \[
   F(x) = e^{-\left(\frac{\theta}{x}\right)^\lambda} \quad x > 0 \tag{2}
   \]

   Where: $\lambda > 0$ is the shape parameter of the distribution. $\theta > 0$ is the scale parameter distribution.

2. **Azzalini’s Method**

   Mathematician Azzalini, A. (1985) introduced this method by inserting an additional parameter into the normal distribution to obtain a new distribution called the skew-normal distribution to achieve more flexibility in the normal distribution function as it is an extension of it. After that, many researchers inserted the shape parameter into
non-symmetric distributions such as the T-skew, Skew-Cauchy by (Gupta et al., 2002), and Skew-logistic distribution by (Nadarajah 2009).

The general formula for Azzaliní’s method is[9]:

$$f(x) = \frac{f(x) F(\alpha)}{P(\alpha X_1 > X_2)}$$

... (3)

where:

- $f(x)$ is the probability density function of frechet distribution.
- $F(\alpha)$ is the cumulative distribution function after adding $\alpha$ parameter.
- $X_1, X_2$: is independent and identical random variables. Then, the conditional probability density function of $X = X_1$ given $\alpha X_1 > X_2$.

Observe that:

$$P(\alpha X_1 > X_2) = \int_0^\infty \left[ \int_0^\infty f_2(X_2) dX_2 \right] f_1(X_1) dX_1$$

$$P(\alpha X_1 > X_2) = \int_0^\infty \left[ \int_0^\infty x \theta \left( \frac{\theta}{x} \right)^\lambda e^{-\left( \frac{\theta}{x} \right)^\lambda} dX_2 \right] \left( \frac{\theta}{X} \right)^\lambda e^{-\left( \frac{\theta}{X} \right)^\lambda} dX_1$$

Then:

$$P(\alpha X_1 > X_2) = \left[ -\frac{1}{\alpha^\lambda + 1} e^{-\left( \frac{\alpha^\lambda + 1}{\alpha^\lambda} \right)^\lambda} \right]_0^\infty$$

$$P(\alpha X_1 > X_2) = \frac{\alpha^\lambda}{(\alpha^\lambda + 1)^\lambda}$$

By (4) in the (3), we get the probability density function to the extended weighted frechet distribution new (pdf).

$$f(x, \lambda, \theta, \alpha) = \frac{\lambda}{\theta} \frac{\alpha^\lambda + 1}{\alpha^\lambda} \left( \frac{\theta}{x} \right)^\lambda e^{-\left( \frac{\theta}{x} \right)^\lambda} \left( \frac{\alpha^\lambda + 1}{\alpha^\lambda} \right)$$

... (5)

from eq. (5) we found the cumulative function to the new distribution by integration:

$$F(x, \lambda, \theta, \alpha) = \frac{x^\lambda}{\theta} \frac{\alpha^\lambda + 1}{\alpha^\lambda} \left( \frac{\theta}{x} \right)^\lambda e^{-\left( \frac{\theta}{x} \right)^\lambda} \left( \frac{\alpha^\lambda + 1}{\alpha^\lambda} \right)$$

... (6)

### 3. Characteristics of the new extended weighted Frechet Distribution

#### 3.1: mean

The mean can be obtained by finding The $r^{th}$ moment about the origin when the value of $r = 1$:

$$\mu_r = E(x^r) = \int_0^\infty x^r f(x) dx$$

... (7)

By (1) in the (7), we get:

$$\mu_r = E(x^r) = \int_0^\infty x^r \frac{\lambda}{\theta} \frac{\alpha^\lambda + 1}{\alpha^\lambda} \left( \frac{\theta}{x} \right)^\lambda e^{-\left( \frac{\theta}{x} \right)^\lambda} \left( \frac{\alpha^\lambda + 1}{\alpha^\lambda} \right) dx$$

The $r^{th}$ moment about the origin of extended weighted Frechet distribution is:

$$\mu_r = E(x^r) = \theta^r k^2 \Gamma \left( 1 - \frac{r}{\lambda} \right)$$

At $r=1$, we obtain the mean:

$$\mu_1 = E(x) = \theta k^2 \Gamma \left( 1 - \frac{1}{\lambda} \right)$$

... (8)

#### 3.2: Variance

The variance can be obtained by finding The $r^{th}$ moment about the mean when the value of $r = 2$:

$$E(x - \mu)^r = \int_0^\infty (x - \mu)^r f(x) dx$$

... (9)

By (1) in the (9), we get:

$$E(x - \mu)^r = \int_0^\infty (x - \mu)^r \frac{\lambda}{\theta} \frac{\alpha^\lambda + 1}{\alpha^\lambda} \left( \frac{\theta}{x} \right)^\lambda e^{-\left( \frac{\theta}{x} \right)^\lambda} \left( \frac{\alpha^\lambda + 1}{\alpha^\lambda} \right) dx$$
The $r^{th}$ moment about the mean of extended weighted Frechet distribution is:

$$E(x - \mu)^r = \sum_{i=0}^{r} \binom{r}{i} \theta^{i} \left( k^\frac{1}{\lambda} \right) (-\mu)^{r-i} \Gamma \left( 1 - \frac{i}{\lambda} \right)$$

At $r = 2$, we obtain the variance:

$$E(x - \mu)^2 = \sum_{i=0}^{2} \binom{2}{i} \theta^{i} \left( k^\frac{1}{\lambda} \right) (-\mu)^{2-i} \Gamma \left( 1 - \frac{i}{\lambda} \right) = \text{var}(x)$$

3.3: Coefficients of Variation

The mathematical formula for the coefficient of variation is:

$$C.V = \frac{\sigma}{\mu} \times 100$$

$$C.V = \sqrt{\frac{\Gamma \left( 1 - \frac{2}{\lambda} \right) - \left( \Gamma \left( 1 - \frac{1}{\lambda} \right) \right)^2}{\Gamma \left( 1 - \frac{1}{\lambda} \right)}} \times 100$$

3.4: Coefficient of Skewness

The mathematical formula for the Coefficient of Skewness is:

$$C.S = \frac{E(x - \mu)^3}{\sigma^3}$$

$$C.S = -\theta^3 k^\frac{1}{\lambda} \Gamma \left( 1 - \frac{1}{\lambda} \right)^3 + 3\theta k^\frac{1}{\lambda} \theta k^\frac{1}{\lambda} \Gamma \left( 1 - \frac{2}{\lambda} \right) - 3\theta^3 k^\frac{2}{\lambda} \Gamma \left( 1 - \frac{1}{\lambda} \right) + \theta^3 k^\frac{3}{\lambda} \Gamma \left( 1 - \frac{3}{\lambda} \right)$$

$$\theta^3 k^\frac{3}{\lambda} \left( \Gamma \left( 1 - \frac{2}{\lambda} \right) - \Gamma \left( 1 - \frac{1}{\lambda} \right)^2 \right)^{3/2}$$

3.5: Coefficient of Kurtosis

The mathematical formula for the Coefficient of Kurtosis is:

$$K.S = \frac{E(x - \mu)^4}{\sigma^4}$$

$$K.S = \frac{\left( -3 \Gamma \left( 1 - \frac{1}{\lambda} \right)^4 + 6 \Gamma \left( 1 - \frac{1}{\lambda} \right)^2 \Gamma \left( 1 - \frac{2}{\lambda} \right) - 4 \Gamma \left( 1 - \frac{1}{\lambda} \right) \Gamma \left( 1 - \frac{3}{\lambda} \right) + \Gamma \left( 1 - \frac{4}{\lambda} \right) \right)}{\left( \Gamma \left( 1 - \frac{2}{\lambda} \right)^2 - 2\Gamma \left( 1 - \frac{2}{\lambda} \right) \Gamma \left( 1 - \frac{1}{\lambda} \right)^2 + \Gamma \left( 1 - \frac{1}{\lambda} \right)^4 \right)}$$

4. Estimation

4.1: Maximum Likelihood Method (MLE)

If $x_1, x_2, ..., x_n$ are random variables distributed in the extended weighted Frechet distribution, then:

$$f(x, \lambda, \theta, \alpha) = \frac{\lambda}{\theta} \left( 1 + \alpha^{-\lambda} \right) \left( \frac{\theta}{\lambda x} \right)^{\lambda+1} e^{-(\theta/x)^{\lambda}} (1 + \alpha^{-\lambda})$$

$$L(x_1, x_2, ..., x_n, \lambda, \theta, \alpha) = \prod_{i=1}^{n} f(x_i, \lambda, \theta, \alpha)$$

$$L(x, \lambda, \theta, \alpha) = \frac{\lambda^n}{\theta} \left( 1 + \alpha^{-\lambda} \right) \prod_{i=1}^{n} \left( \frac{\theta}{\lambda x_i} \right)^{\lambda+1} e^{-(\alpha/x_i)^{\lambda}}$$

$$\ln L = n \ln \lambda - n \ln \theta + n \ln(1 + \alpha^{-\lambda}) + (\lambda + 1) \sum \ln \left( \frac{\theta}{\lambda x_i} \right) - (1 + \alpha^{-\lambda}) \sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda} \ln \left( \frac{\theta}{\lambda x_i} \right) + \sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda} \alpha^{-\lambda} \ln \alpha$$

$$\frac{\partial \ln L}{\partial \lambda} = n \lambda - n \alpha^{-\lambda} \ln(\alpha) + \sum \ln \left( \frac{\theta}{\lambda x_i} \right) - \sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda} \ln \left( \frac{\theta}{\lambda x_i} \right) - \alpha^{-\lambda} \sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda} \ln \left( \frac{\theta}{\lambda x_i} \right) + \sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda} \alpha^{-\lambda} \ln \alpha$$

To estimate the parameter ($\theta$):

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} + \frac{\alpha^{-\lambda} \sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda-1} (\frac{1}{\lambda x_i}) - \lambda \alpha^{-\lambda} \sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda-1} (\frac{1}{\lambda x_i})}{\sum \left( \frac{\theta}{\lambda x_i} \right)^{\lambda} (\frac{\theta}{\lambda x_i})}$$

To estimate the parameter ($\alpha$):
\[ \frac{\partial \ln L}{\partial \alpha} = -\frac{n(-1)a}{(1+a^{-1})} + \lambda a^{-1} \sum \left( \frac{\theta}{x_i} \right)^{\lambda} \] ... (12)

The MLE \( \hat{\lambda}, \hat{\theta} \) and \( \hat{\alpha} \) can be obtained by solving the likelihood eqs.

\[ \frac{\partial \ln L}{\partial \lambda} \bigg|_{\lambda=\hat{\lambda}} = 0 \quad \text{and} \quad \frac{\partial \ln L}{\partial \theta} \bigg|_{\theta=\hat{\theta}} = 0 \quad \text{and} \quad \frac{\partial \ln L}{\partial \alpha} \bigg|_{\alpha=\hat{\alpha}} = 0 \]

Clearly, it is difficult to solve the equations (10), (11), and (12) therefore applying Newton-Raphson’s method.

4.2: Maximum product of spacing estimator method

\[ F(x) = e^{-\left( \frac{x}{\theta} \right)^{\lambda} \left( \frac{a}{a+1} \right) \left( \frac{x}{x_1} \right)^{\lambda}} \]

\[ G = \left( \prod_{i=1}^{n+1} D_i \right) \]

\[ D_i = F(x_i) \]

\[ D_i = F(x_i) - F(x_{i-1}) = F(x(2m)) \quad i = 2, ..., m \]

\[ \ln G = \frac{1}{n+1} \left[ -\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_1} \right)^{\lambda} + \sum_{i=2}^{n} \left( e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_i} \right)^{\lambda}} - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_{i-1}} \right)^{\lambda}} \right) + \ln \left( 1 - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_n} \right)^{\lambda}} \right) \right] \]

\[ \frac{\partial \ln(G)}{\partial \alpha} = \left( \frac{1}{n+1} \right) \left[ -\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_1} \right)^{\lambda} + \sum_{i=2}^{n} \left( e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_i} \right)^{\lambda}} - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_{i-1}} \right)^{\lambda}} \right) + \ln \left( 1 - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_n} \right)^{\lambda}} \right) \right] \]

\[ \frac{\partial \ln(G)}{\partial \theta} = \left( \frac{1}{n+1} \right) \left[ -\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_1} \right)^{\lambda} + \sum_{i=2}^{n} \left( e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_i} \right)^{\lambda}} - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_{i-1}} \right)^{\lambda}} \right) + \ln \left( 1 - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_n} \right)^{\lambda}} \right) \right] \]

\[ \frac{\partial \ln(G)}{\partial x} = \left( \frac{1}{n+1} \right) \left[ -\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_1} \right)^{\lambda} + \sum_{i=2}^{n} \left( e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_i} \right)^{\lambda}} - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_{i-1}} \right)^{\lambda}} \right) + \ln \left( 1 - e^{-\left( \frac{a+1}{a} \right) \left( \frac{\theta}{x_n} \right)^{\lambda}} \right) \right] \]

The MLE \( \hat{\lambda}, \hat{\theta} \) and \( \hat{\alpha} \) can be obtained by solving the likelihood eqs.

\[ \frac{\partial \ln L}{\partial \lambda} \bigg|_{\lambda=\hat{\lambda}} = 0 \quad \text{and} \quad \frac{\partial \ln L}{\partial \theta} \bigg|_{\theta=\hat{\theta}} = 0 \quad \text{and} \quad \frac{\partial \ln L}{\partial \alpha} \bigg|_{\alpha=\hat{\alpha}} = 0 \]

Clearly, it is difficult to solve the equations (13), (14), and (15) therefore applying Newton-Raphson’s method.

4.3: Method of Cramer-Von Mises Minimum

The CVME, as a type of minimum distance estimator, has less bias than the other minimum distance estimators.
\[
C(\lambda, \alpha, 0) = \frac{1}{12n} + \sum_{i=1}^{n} \left[ F(x_i, \lambda, \alpha, 0) - \frac{2i - 1}{2n} \right]^2
\]
\[
C(\lambda, \alpha, 0) = \frac{1}{12n} + \sum_{i=1}^{n} \left[ -\frac{\theta}{\lambda} \left( \frac{\alpha^{i+1}}{\alpha^{\frac{1}{\lambda}}} \right) - \frac{2i - 1}{2n} \right]^2
\]
\[
\frac{\partial C(\lambda, \alpha, 0)}{\partial \lambda} = 2 \sum_{i=1}^{n} e^{-\left( \frac{\alpha^{i+1}}{\alpha^{\frac{1}{\lambda}}} \right)}
\]
\[
- \frac{2i - 1}{2n} \left[ e^{-\left( \frac{\alpha^{i+1}}{\alpha^{\frac{1}{\lambda}}} \right)} \left( \frac{\theta}{\lambda} \left( \frac{\alpha^{\frac{1}{\lambda}}}{\alpha^{i+1}} \right) \ln \alpha - \frac{\alpha^{i+1} + 1}{\alpha^{i+1} \alpha^{\frac{1}{\lambda}}} \right) \right]
\]
\[
+ \left( \frac{\alpha^{i+1} + 1}{\alpha^{i+1}} \right) \left( \frac{\theta}{\lambda} \ln \left( \frac{\theta}{\lambda} \right) \right)
\]
And
\[
\frac{\partial C(\lambda, \alpha, 0)}{\partial \alpha} = 2 \sum_{i=1}^{n} e^{-\left( \frac{\alpha^{i+1}}{\alpha^{\frac{1}{\lambda}}} \right)} \left[ e^{-\left( \frac{\alpha^{i+1}}{\alpha^{\frac{1}{\lambda}}} \right)} \left( \frac{\theta}{\lambda} \left( \frac{\alpha^{\frac{1}{\lambda}}}{\alpha^{i+1}} \right) \right) \right]
\]
And
\[
\frac{\partial C(\lambda, \alpha, 0)}{\partial \alpha} = -2 \sum_{i=1}^{n} e^{-\left( \frac{\alpha^{i+1}}{\alpha^{\frac{1}{\lambda}}} \right)} \left( \frac{\alpha^{i+1} + 1}{\alpha^{i+1} \alpha^{\frac{1}{\lambda}}} \right) \left( \frac{\theta}{\lambda} \left( \frac{\alpha^{i+1}}{\alpha^{\frac{1}{\lambda}}} \right) \right)
\]
The MLE \( \hat{\lambda} \), \( \hat{\theta} \), and \( \hat{\alpha} \) can be obtained by solving the likelihood eqs.
\[
\frac{\partial C(\lambda, \alpha, 0)}{\partial \lambda} |_{\hat{\lambda} = \lambda} = 0, \quad \frac{\partial C(\lambda, \alpha, 0)}{\partial \theta} |_{\hat{\theta} = \theta} = 0 \quad \text{and} \quad \frac{\partial C(\lambda, \alpha, 0)}{\partial \alpha} |_{\hat{\alpha} = \alpha} = 0
\]
Clearly, it is difficult to solve the equations (19), (20), therefore applying Newton-Raphson’s method.

5. Simulation Study

We ran a simulation to compare the behavior of the estimates with respect to mean squares of error (MSEs) using the rank method, as this method is based on selecting the lowest order for the sum of the total and partial ranks of all estimation methods for a set of imposed parameter values \( (\hat{\lambda}, \hat{\theta}, \hat{\alpha}) \). The results were as shown in Table (1).

| Estimated parameters | \( \lambda \) | \( \theta \) | \( \alpha \) | \( \lambda \) | \( \theta \) | \( \alpha \) | \( \lambda \) | \( \theta \) | \( \alpha \) |
|---------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( N \) | \( ti \) | Real | Mle | MSE | MPS | MSE | CV | MD | MSE |
| 13.298 | 4 | 1.7 | 2.7 | 7.2 | 9 | 2.2 | 3.9 | 3.5 | 2 |
| 12.885 | 3 | 2.0 | 0.1985 | 0.2578 | 0.0302 | 7 | 2.266 | 0.000812 | 0.19 | 86 |
| 12.526 | 0 | 1.884 | 0.2565 | 0.0228 | 7 | 2.266 | 0.000612 | 0.18 | 85 |
| 12.400 | 7 | 1.844 | 0.2406 | 0.0074 | 3 | 0.208 | 0.000512 | 0.18 | 45 |
| 12.379 | 6 | 1.833 | 0.2404 | 0.0073 | 4 | 0.202 | 0.000412 | 0.18 | 34 |
| 11.140 | 1 | 1.787 | 0.2291 | 0.0032 | 4 | 0.199 | 0.000312 | 0.17 | 80 |

Table (1): Simulation results for \( (\lambda = 2, \theta = 4, \alpha = 4) \)
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#### Estimated parameters

| N  | ti  | Real  | Mle Mse MPS Mse CV MD MSE |
|----|-----|-------|---------------------------|
| 75 | 13.298 1 0.1988 | 0.2779 0.0375 0.471 0.0780 0.20 0.0001 (1) |
| 4 25 | 0.1988 | 0.2779 0.0359 0.470 0.0746 0.20 0.0001 (1) |
| 12.885 1 0.1988 | 0.2776 0.0352 0.461 0.0746 0.20 0.0001 (1) |
| 12.542 1 0.1986 | 0.2769 0.0294 0.459 0.0712 0.20 0.0001 (1) |
| 12.526 1 0.1986 | 0.2769 0.0294 0.459 0.0712 0.20 0.0001 (1) |
| 12.513 1 0.1985 | 0.2591 0.0109 0.452 0.0688 0.20 0.0001 (1) |
| 12.400 1 0.1985 | 0.2589 0.0108 0.451 0.0682 0.20 0.0001 (1) |
| 12.379 1 0.1984 | 0.2495 0.0066 0.450 0.0640 0.20 0.0001 (1) |
| 11.969 2 0.1974 | 0.2459 0.0054 0.448 0.0638 0.20 0.0001 (1) |
| 11.140 8 0.1955 | 0.2407 0.0040 0.446 0.0635 0.20 0.0001 (1) |
| 10.927 7 0.1925 | 0.2392 0.0037 0.434 0.0625 0.19 0.0001 (1) |

#### Estimated parameters

| N  | ti  | Real  | Mle Mse MPS Mse CV MD MSE |
|----|-----|-------|---------------------------|
| 10 0 | 13.298 4 0.1988 | 0.2914 0.0569 0.357 0.0255 0.21 0.0004 (1) |
| 12.885 3 0.1988 | 0.2910 0.0430 0.357 0.0253 0.21 0.0004 (1) |
| 12.542 1 0.1988 | 0.2908 0.0411 0.356 0.0251 0.21 0.0004 (1) |
| 12.526 0 0.1988 | 0.2880 0.0402 0.356 0.0251 0.21 0.0004 (1) |

- $\hat{\lambda}$, $\hat{\theta}$, $\hat{\alpha}$ represent the estimated parameters.
- Mle, MSE, MPS, CV, MD, MSE are the maximum likelihood estimation, mean square error, mean product of skewness, coefficient of variation, median deviation, mean square error, respectively.
- The numbers in parentheses indicate the number of decimal places.

#### Summary

| $\sum Rank$ | $20^{(2)}$ | $30^{(3)}$ | $10^{(1)}$ |
|-------------|-------------|-------------|-------------|
| Estimated parameters | $\hat{\lambda}$ | $\hat{\theta}$ | $\hat{\alpha}$ | $\hat{\lambda}$ | $\hat{\theta}$ | $\hat{\alpha}$ | $\hat{\lambda}$ | $\hat{\theta}$ | $\hat{\alpha}$ |
| 1.8 6 | 2.6 2 | 7.2 6 | 3.5 3 | 3.7 6 | 3.9 9 | 2.1 2 | 3.8 5 | 3.85 |

- The numbers in parentheses indicate the number of decimal places.
- The values above the diagonal represent the sum of the rank of the parameters.
- The values below the diagonal represent the estimated parameters.

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### Estimated parameters

| N  | Ti  | Real | \( \lambda \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \lambda \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \lambda \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \lambda \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \lambda \) | \( \hat{\theta} \) | \( \hat{\alpha} \) |
|----|-----|------|--------------|----------------|----------------|------------|----------------|----------------|------------|----------------|----------------|------------|----------------|----------------|------------|----------------|----------------|
| 35 |     |      | 2.1          | 2.6            | 6.2            | 3.6        | 5.3            | 1.5            | 3.1        | 4.1            | 3.8            | 3.1        | 4.1            | 3.8            | 3.1        | 4.1            | 3.8            |
|    |     |      | 0.250        | 0.0865(3)      | 0.286          | 0.0296(2)   | 0.28        | 0.0291(2)      | 0.28          | 0.0261(2)   | 0.274          | 0.0246(2)      | 0.27        | 0.0201(1)      | 0.26          | 0.0191(1)    | 0.26          | 0.0170(1)    |
| 75 |     |      | 2.3          | 2.6            | 6.2            | 3.5        | 4.1            | 3.7            | 3.2        | 4.1            | 3.7            | 3.2        | 4.1            | 3.7            | 3.2        | 4.1            | 3.7            |

Table (2): Simulation results for \( (\lambda = 3.5, \theta = 4, \alpha = 3.5) \)
### Table (3): Simulation results for \((\lambda = 3.5, \theta = 5, \alpha = 2.5)\)

|                | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) |
|----------------|--------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| \(N = 10\)    | \(\checkmark\)    | \(\checkmark\)    | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  |
| \(n = 10\)    | \(\checkmark\)    | \(\checkmark\)    | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  |
| \(\checkmark\) | \(\checkmark\)    | \(\checkmark\)    | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  |
| \(\checkmark\) | \(\checkmark\)    | \(\checkmark\)    | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  |
| \(\checkmark\) | \(\checkmark\)    | \(\checkmark\)    | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  | \(\checkmark\)  |

Statistical Properties & Different Methods Of Estimation Of A New Extended Weighted Frechet Distribution.
| Rank | Mle | MSE | MPS | MSE | CV MD | MSE |
|------|-----|-----|-----|-----|-------|-----|
| 1    | 0.2642 | 0.2755 | 0.0473 | 0.264 | 0.00000 | 0.3307 | 0.0064(2) |
| 2    | 0.2629 | 0.2313 | 0.0230 | 0.263 | 0.00000 | 0.3272 | 0.0064(2) |
| 3    | 0.2627 | 0.2251 | 0.0230 | 0.262 | 0.00000 | 0.3271 | 0.0062(2) |
| 4    | 0.2589 | 0.2094 | 0.0229 | 0.258 | 0.00000 | 0.3189 | 0.0061(2) |
| 5    | 0.2559 | 0.1965 | 0.0229 | 0.255 | 0.00000 | 0.3161 | 0.0053(2) |
| 6    | 0.2544 | 0.1833 | 0.0229 | 0.254 | 0.00000 | 0.3146 | 0.0046(2) |
| 7    | 0.2507 | 0.1691 | 0.0229 | 0.250 | 0.00000 | 0.3027 | 0.0042(2) |
| 8    | 0.2505 | 0.1667 | 0.0228 | 0.250 | 0.00000 | 0.2951 | 0.0040(2) |
| 9    | 0.2494 | 0.1578 | 0.0225 | 0.249 | 0.00000 | 0.2946 | 0.0027(2) |
| 10   | 0.2485 | 0.1355 | 0.0224 | 0.249 | 0.00000 | 0.2929 | 0.0027(2) |

![](image)

| N  | Ti | Real | Rank | Mle | MSE | MPS | MSE | CV MD | MSE |
|----|----|------|------|-----|-----|-----|-----|-------|-----|
| 75 | 12.525 | 0.2643 | 30(3) | 0.2746 | 0.0496(3) | 0.185 | 0.0062(2) | 0.3279 | 0.0056(2) |
|    | 11.679 | 0.2643 | 10(1) | 0.2559 | 0.0302(3) | 0.185 | 0.0062(2) | 0.3252 | 0.0056(2) |
|    | 11.427 | 0.2643 | 20(2) | 0.2491 | 0.0239(3) | 0.185 | 0.0062(2) | 0.3246 | 0.0056(2) |
|    | 9.5865 | 0.2643 | 30(3) | 0.2290 | 0.0226(3) | 0.185 | 0.0062(2) | 0.3243 | 0.0056(2) |
|    | 9.4554 | 0.2641 | 10(1) | 0.2277 | 0.0225(3) | 0.185 | 0.0062(2) | 0.3229 | 0.0056(2) |
|    | 9.3709 | 0.2634 | 20(2) | 0.2273 | 0.0223(3) | 0.184 | 0.0062(2) | 0.3221 | 0.0054(2) |
|    | 8.9241 | 0.2632 | 30(3) | 0.2254 | 0.0223(3) | 0.184 | 0.0062(2) | 0.3218 | 0.0053(2) |
|    | 8.7807 | 0.2630 | 10(1) | 0.2205 | 0.0219(3) | 0.184 | 0.0062(2) | 0.3218 | 0.0053(2) |
|    | 8.6519 | 0.2620 | 20(2) | 0.2150 | 0.0218(3) | 0.184 | 0.0061(2) | 0.3205 | 0.0053(2) |
|    | 8.6278 | 0.2606 | 30(3) | 0.2019 | 0.0218(3) | 0.183 | 0.0060(2) | 0.3180 | 0.0052(2) |

Mahdi Wahhab Neamah¹, Nahla Hadi Abdul-Sahib²
**Estimated parameters**

| n  | ti  | Real | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \lambda \) | \( \theta \) | \( \alpha \) |
|----|-----|-----|------------------|------------------|-----------------|--------|--------|--------|
| 3  | 11.648 | 0.2643 | 2.6 | 3.0 | 6.1 | 3.3 | 5.1 | 2.5 |
| 7  | 11.335 | 0.2642 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 0  | 10.157 | 0.2641 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 1  | 10.034 | 0.2640 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 1  | 10.034 | 0.2640 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 1  | 9.9475 | 0.2640 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 1  | 9.4164 | 0.2637 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 0  | 9.5078 | 0.2631 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 1  | 9.1838 | 0.2618 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 1  | 9.1231 | 0.2618 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |
| 1  | 8.5764 | 0.2618 | 2.6 | 3.0 | 8.8 | 3.3 | 5.1 | 2.5 |

\[
\sum \text{Rank} = 30^{(1)} \quad 10^{(1)} \quad 20^{(2)}
\]

### Table (4): Simulation results for \( (\lambda = 4, \theta = 6.5, \alpha = 3) \)

| Estimated parameters | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \lambda \) | \( \theta \) | \( \alpha \) |
|----------------------|------------------|------------------|------------------|--------|--------|--------|
| N  | Ti  | Real | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \lambda \) | \( \theta \) | \( \alpha \) |
| 9  | 15.842 | 0.2323 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |
| 6  | 15.702 | 0.2316 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |
| 2  | 13.034 | 0.2302 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |
| 4  | 12.581 | 0.2264 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |
| 0  | 11.699 | 0.2234 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |
| 7  | 9.7573 | 0.2220 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |
| 3  | 9.6687 | 0.2220 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |
| 5  | 9.3736 | 0.2185 | 2.6 | 4.1 | 6.3 | 7.04 | 7.2 | 3.0 |

\[
\sum \text{Rank} = 30^{(1)} \quad 10^{(1)} \quad 20^{(2)}
\]
| N  | Ti | Real | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) |
|----|----|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 19.332 | 7 | 0.232 | 0.196 | 0.0218 | 0.232 | 0.0035 | 0.27 | 0.0021 |
| 16.528 | 3 | 0.232 | 0.183 | 0.0199 | 0.232 | 0.0034 | 0.27 | 0.0021 |
| 15.890 | 7 | 0.232 | 0.178 | 0.0198 | 0.231 | 0.0034 | 0.26 | 0.0021 |
| 12.418 | 3 | 0.232 | 0.164 | 0.0197 | 0.231 | 0.0033 | 0.26 | 0.0021 |
| 12.217 | 1 | 0.232 | 0.164 | 0.0197 | 0.230 | 0.0033 | 0.26 | 0.0021 |
| 12.090 | 0 | 0.230 | 0.163 | 0.0194 | 0.229 | 0.0033 | 0.26 | 0.0021 |
| 11.439 | 4 | 0.230 | 0.162 | 0.0194 | 0.229 | 0.0033 | 0.26 | 0.0021 |
| 11.237 | 6 | 0.230 | 0.159 | 0.0193 | 0.228 | 0.0032 | 0.26 | 0.0020 |
| 11.059 | 4 | 0.229 | 0.155 | 0.0191 | 0.227 | 0.0031 | 0.26 | 0.0020 |
| 11.026 | 5 | 0.229 | 0.146 | 0.0190 | 0.225 | 0.0028 | 0.26 | 0.0020 |

| n  | ti | Real | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) |
|----|----|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 16.444 | 7 | 0.232 | 0.200 | 0.0182 | 0.232 | 0.000000 | 0.23 | 0.000004 |
| 15.672 | 9 | 0.232 | 0.197 | 0.0182 | 0.232 | 0.000000 | 0.23 | 0.000003 |
| 13.344 | 3 | 0.232 | 0.187 | 0.0182 | 0.232 | 0.000000 | 0.23 | 0.000003 |
| 13.137 | 1 | 0.231 | 0.173 | 0.0181 | 0.231 | 0.000000 | 0.23 | 0.000003 |
| 12.993 | 6 | 0.231 | 0.171 | 0.0181 | 0.231 | 0.000000 | 0.23 | 0.000003 |
| 12.272 | 8 | 0.231 | 0.166 | 0.0181 | 0.231 | 0.000000 | 0.23 | 0.000003 |
Statistical Properties & Different Methods Of Estimation Of A New Extended Weighted Frechet Distribution.

| n     | Ti    | Real | 2.3 | 3 | 7 | 31 | 3 | 11 | 65 | 21 |
|-------|-------|------|-----|---|---|----|---|----|----|----|
| 2     | 12.145| 0.230| 0.156| 6 | 0.0181| 31 | 0.230| 0.000000| 11 | 23 |
| 3     | 0.000031 | 23 |
| 5     | 0.0181| 31 | 0.000031 | 23 |
| 1     | 0.230| 7 | 0.153| 9 | 0.0180| 31 | 0.230| 0.000000| 11 | 23 |
| 1     | 0.000031 | 23 |
| 7     | 0.230| 6 | 0.149| 6 | 0.0180| 31 | 0.230| 0.000000| 11 | 23 |
| 1     | 0.000031 | 23 |
| 2     | 10.956| 0.230| 0.147| 5 | 0.0180| 31 | 0.230| 0.000000| 11 | 23 |
| 1     | 0.000031 | 23 |

Table (5): Simulation results for \( \lambda = 3, \theta = 6.5, \alpha = 4 \)

| Estimated parameters | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) |
|----------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| n       | Ti     | Real | \( \lambda \) | \( \theta \) | \( \alpha \) | \( \lambda \) | \( \theta \) | \( \alpha \) | \( \lambda \) | \( \theta \) | \( \alpha \) |
| 20.566  | 5.0    | 0.1776 | 0.203 | 5/3 | 0.0251 | 3/3 | 0.110 | 0 | 0.0051 | 1/2 | 0.21 | 23 |
| 20.348  | 0.1773 | 0.174 | 9/3 | 0.0078 | 3/3 | 0.109 | 9 | 0.0050 | 2/3 | 0.21 | 22 |
| 16.159  | 4.0    | 0.174 | 9/3 | 0.0078 | 3/3 | 0.109 | 6 | 0.0050 | 2/3 | 0.20 | 95 |
| 15.449  | 5.0    | 0.1716 | 0.158 | 6/3 | 0.0078 | 3/3 | 0.109 | 6 | 0.0048 | 2/3 | 0.20 | 79 |
| 14.073  | 4.0    | 0.1699 | 0.149 | 1/3 | 0.0078 | 3/3 | 0.108 | 5 | 0.0046 | 2/3 | 0.20 | 63 |
| 11.111  | 6.0    | 0.1671 | 0.139 | 3/3 | 0.0078 | 3/3 | 0.107 | 1 | 0.0045 | 2/3 | 0.20 | 03 |
| 10.979  | 4.0    | 0.1664 | 0.128 | 5/3 | 0.0078 | 3/3 | 0.106 | 8 | 0.0043 | 2/3 | 0.19 | 40 |
| 10.541  | 3.0    | 0.1657 | 0.126 | 7/3 | 0.0078 | 3/3 | 0.106 | 0 | 0.0043 | 2/3 | 0.19 | 20 |
| 9.5079  | 4.0    | 0.1648 | 0.119 | 8/3 | 0.0078 | 3/3 | 0.103 | 8 | 0.0042 | 2/3 | 0.18 | 68 |
| 8.5734  | 4.0    | 0.1626 | 0.102 | 6/3 | 0.0076 | 3/3 | 0.103 | 1 | 0.0041 | 2/3 | 0.18 | 44 |

| Estimated parameters | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) |
|----------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| n       | Ti     | Real | \( \lambda \) | \( \theta \) | \( \alpha \) | \( \lambda \) | \( \theta \) | \( \alpha \) | \( \lambda \) | \( \theta \) | \( \alpha \) |
| 25.788  | 4.0    | 0.1774 | 0.204 | 6/3 | 0.0269 | 3/3 | 0.170 | 5 | 0.0001 | 1/2 | 0.21 | 09 |
| 21.625  | 7.0    | 0.1772 | 0.192 | 5/3 | 0.0168 | 3/3 | 0.170 | 3 | 0.0005 | 1/1 | 0.21 | 02 |
| 20.640  | 9.0    | 0.1770 | 0.187 | 9/3 | 0.0135 | 3/3 | 0.170 | 1 | 0.0005 | 1/1 | 0.20 | 99 |

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Table (6): Simulation results for \((\lambda = 2, \theta = 4, \alpha = 2)\)

| N  | Ti | Real | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\hat{\lambda}\) | \(\hat{\theta}\) | \(\hat{\alpha}\) |
|----|----|------|---------------------|-----------------|-----------------|---------------------|-----------------|-----------------|---------------------|-----------------|-----------------|---------------------|-----------------|-----------------|
| 35 | 8.1785 | 0.1833 | 0.409 | 0.1486 | 0.179 | 0.00005 | 0.33 | 0.0346 | 1.92 |

| \(\sum_{i} \text{Rank}\) | \(30^{(3)}\) | \(10^{(1)}\) | \(20^{(2)}\) |
|--------------------------|---------|---------|---------|
| Estimated parameters     | \(\lambda\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\lambda\) | \(\hat{\theta}\) | \(\hat{\alpha}\) | \(\lambda\) | \(\hat{\theta}\) | \(\hat{\alpha}\) |
| Real | 2.3 | 4.0 | 6.5 | 7 | 2.9 | 6.5 | 7 | 3.2 | 6.3 | 2 | 3.73 |
| | Mle | MSE | MPS | MSE | CV | MD | MSE |
| 21.497 | 4 | 0.1774 | 0.207 | 0.0233 | 0.127 | 0.0026 | 0.18 | 0.00004 | 1 |
| | 20.302 | 2 | 0.1777 | 0.204 | 0.0204 | 0.127 | 0.0026 | 0.18 | 0.00004 | 1 |
| | 16.647 | 6 | 0.1774 | 0.195 | 0.0135 | 0.127 | 0.0026 | 0.18 | 0.00004 | 1 |
| | 16.321 | 3 | 0.1776 | 0.182 | 0.0066 | 0.127 | 0.0026 | 0.18 | 0.00004 | 1 |
| | 16.909 | 9 | 0.1772 | 0.179 | 0.0066 | 0.127 | 0.0025 | 0.18 | 0.00004 | 1 |
| | 14.965 | 7 | 0.1770 | 0.175 | 0.0066 | 0.126 | 0.0025 | 0.18 | 0.00004 | 1 |
| | 14.767 | 8 | 0.1766 | 0.164 | 0.0066 | 0.126 | 0.0025 | 0.18 | 0.00004 | 1 |
| | 14.250 | 4 | 0.1760 | 0.161 | 0.0066 | 0.126 | 0.0025 | 0.18 | 0.00004 | 1 |
| | 14.113 | 2 | 0.1759 | 0.155 | 0.0066 | 0.126 | 0.0025 | 0.18 | 0.00004 | 1 |
| | 12.927 | 4 | 0.1756 | 0.154 | 0.0066 | 0.126 | 0.0025 | 0.18 | 0.00004 | 1 |
| \(\sum_{i} \text{Rank}\) | \(30^{(3)}\) | \(20^{(2)}\) | \(10^{(1)}\) |

Mahdi Wahhab Neamah \(^1\), Nahla Hadi Abdul-Sahib \(^2\)
### Statistical Properties & Different Methods Of Estimation Of A New Extended Weighted Frechet Distribution.

|               | 3       | 4   | 8   | 71     |
|---------------|---------|-----|-----|--------|
|               | 0.355   | 0.0831 | 0.179 | 0.00005 |
|               | 0.345   | 0.0732 | 0.179 | 0.00005 |
|               | 0.318   | 0.0500 | 0.179 | 0.00005 |
|               | 0.294   | 0.0338 | 0.179 | 0.00005 |
|               | 0.270   | 0.0204 | 0.179 | 0.00005 |
|               | 0.243   | 0.0099 | 0.178 | 0.00003 |
|               | 0.239   | 0.0085 | 0.178 | 0.00003 |
|               | 0.222   | 0.0076 | 0.178 | 0.00003 |
|               | 0.181   | 0.0074 | 0.177 | 0.00003 |

| SUM Rank      | 25(2.5) | 10(1) | 25(2.5) |
|---------------|---------|-------|---------|
| Estimated parameters | λ | θ | α | λ | θ | α | λ | θ | α |
| N | Ti | Real | Mle | MSE | MPS | MSE | CV | MD | MSE |
|---|----|------|-----|-----|-----|-----|-----|-----|-----|
| 8.4581 | 0.1831 | 0.385 | 0.1328 | 0.148 | 0.0023 | 0.33 | 0.0332 | 0.2 |
| 8.2550 | 0.1830 | 0.369 | 0.1107 | 0.148 | 0.0021 | 0.33 | 0.0331 | 0.2 |
| 8.1843 | 0.1827 | 0.361 | 0.1011 | 0.148 | 0.0020 | 0.33 | 0.0331 | 0.2 |
| 7.4805 | 0.1824 | 0.334 | 0.0718 | 0.148 | 0.0018 | 0.33 | 0.0330 | 0.2 |
| 7.4152 | 0.1822 | 0.332 | 0.0699 | 0.148 | 0.0017 | 0.33 | 0.0328 | 0.2 |
| 7.3718 | 0.1820 | 0.332 | 0.0693 | 0.148 | 0.0017 | 0.33 | 0.0328 | 0.2 |
| 7.1257 | 0.1818 | 0.329 | 0.0666 | 0.147 | 0.0017 | 0.33 | 0.0327 | 0.2 |
| 7.0404 | 0.1807 | 0.321 | 0.0596 | 0.147 | 0.0017 | 0.33 | 0.0317 | 0.2 |
| 6.9611 | 0.1803 | 0.313 | 0.0522 | 0.147 | 0.0017 | 0.33 | 0.0311 | 0.2 |
| 6.9460 | 0.1798 | 0.291 | 0.0359 | 0.147 | 0.0017 | 0.33 | 0.0305 | 0.2 |

### Additional Table

|               | 2      | 3.8 | 1.8 | 26 | 3.3 | 1.93 |
|---------------|--------|-----|-----|----|-----|------|
|               | 0.147  | 0.147 | 0.147 | 0.147 | 0.147 | 0.147 |
|               | 0.179  | 0.179 | 0.179 | 0.179 | 0.179 | 0.179 |
|               | 0.177  | 0.177 | 0.177 | 0.177 | 0.177 | 0.177 |
|               | 0.178  | 0.178 | 0.178 | 0.178 | 0.178 | 0.178 |
|               | 0.179  | 0.179 | 0.179 | 0.179 | 0.179 | 0.179 |

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| n   | Ti | Real | Mle   | MSE  | MPS | MSE  | CV MD | MSE  |
|-----|----|------|-------|------|-----|------|-------|------|
| 10  |    |      | 8.2464| 0.1833|     |      |       |      |
|     |    |      | 0.389 | 0.1286 | 0.149 | 0.0022 | 0.25 | 0.0066 |
|     |    |      | 0.387 | 0.1235 | 0.149 | 0.0020 | 0.25 | 0.0066 |
|     |    |      | 0.376 | 0.1067 | 0.149 | 0.0019 | 0.25 | 0.0065 |
|     |    |      | 0.353 | 0.0779 | 0.149 | 0.0017 | 0.25 | 0.0064 |
|     |    |      | 0.349 | 0.0734 | 0.149 | 0.0016 | 0.25 | 0.0063 |
|     |    |      | 0.339 | 0.0639 | 0.149 | 0.0016 | 0.25 | 0.0063 |
|     |    |      | 0.316 | 0.0444 | 0.148 | 0.0016 | 0.25 | 0.0063 |
|     |    |      | 0.309 | 0.0396 | 0.148 | 0.0016 | 0.25 | 0.0062 |
|     |    |      | 0.297 | 0.0315 | 0.148 | 0.0016 | 0.25 | 0.0062 |
|     |    |      | 0.293 | 0.0292 | 0.148 | 0.0016 | 0.25 | 0.0060 |

\[ \sum \text{Rank} = 30^{(1)}, 10^{(1)}, 20^{(2)} \]

Table (7): Simulation results for \( \lambda = 4.5, \theta = 5, \alpha = 3 \)
### Statistical Properties & Different Methods Of Estimation Of A New Extended Weighted Frechet Distribution.

| \( \sum \text{Rank} \) | 30\(^{[3]} \) | 10\(^{[1]} \) | 20\(^{[2]} \) |
|------------------------|-----------|-----------|-----------|
| \( \text{Estimated parameters} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) |
| \text{Mle} | MSE | MPS | MSE | CV | MD | MSE |
| --- | --- | --- | --- | --- | --- | --- |
| 75 | | | | | | |
| \( N \) | \( Ti \) | Real | 2.8 | 3.1 | 5.5 | 4 | 3.8 | 5.2 | 3.0 | 6 | 4.7 | 4.9 | 2.89 |
| 14.782 | 9 | 0.3383 | 0.254 | 9 | 0.0477\(^{[3]} \) | 0.194 | 2 | 0.0208\(^{[2]} \) | 0.38 | 98 | 0.0038\(^{[1]} \) |
| 12.098 | 6 | 0.3383 | 0.237 | 3 | 0.0475\(^{[3]} \) | 0.194 | 2 | 0.0208\(^{[2]} \) | 0.38 | 89 | 0.0038\(^{[1]} \) |
| 11.581 | 3 | 0.3383 | 0.231 | 1 | 0.0474\(^{[3]} \) | 0.194 | 1 | 0.0208\(^{[2]} \) | 0.38 | 79 | 0.0038\(^{[1]} \) |
| 9.0316 | 4 | 0.3382 | 0.213 | 4 | 0.0470\(^{[3]} \) | 0.194 | 1 | 0.0208\(^{[2]} \) | 0.38 | 72 | 0.0038\(^{[1]} \) |
| 8.8925 | 3 | 0.3381 | 0.212 | 3 | 0.0469\(^{[3]} \) | 0.194 | 0 | 0.0207\(^{[2]} \) | 0.38 | 70 | 0.0037\(^{[1]} \) |
| 8.8044 | 3 | 0.3368 | 0.211 | 9 | 0.0468\(^{[3]} \) | 0.193 | 6 | 0.0205\(^{[2]} \) | 0.38 | 58 | 0.0037\(^{[1]} \) |
| 8.3578 | 3 | 0.3366 | 0.210 | 3 | 0.0467\(^{[3]} \) | 0.193 | 4 | 0.0205\(^{[2]} \) | 0.38 | 46 | 0.0036\(^{[1]} \) |
| 8.2203 | 3 | 0.3363 | 0.206 | 0 | 0.0454\(^{[3]} \) | 0.193 | 4 | 0.0205\(^{[2]} \) | 0.38 | 36 | 0.0036\(^{[1]} \) |
| 8.0988 | 3 | 0.3348 | 0.201 | 3 | 0.0452\(^{[3]} \) | 0.193 | 3 | 0.0202\(^{[2]} \) | 0.38 | 04 | 0.0035\(^{[1]} \) |
| 8.0763 | 3 | 0.3330 | 0.190 | 1 | 0.0452\(^{[3]} \) | 0.192 | 8 | 0.0198\(^{[2]} \) | 0.37 | 98 | 0.0035\(^{[1]} \) |

| \( \sum \text{Rank} \) | 30\(^{[3]} \) | 20\(^{[2]} \) | 10\(^{[1]} \) |
|------------------------|-----------|-----------|-----------|
| \( \text{Estimated parameters} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) | \( \hat{\lambda} \) | \( \hat{\theta} \) | \( \hat{\alpha} \) |
| \text{Mle} | MSE | MPS | MSE | CV | MD | MSE |
| --- | --- | --- | --- | --- | --- | --- |
| 100 | | | | | | |
| \( n \) | \( Ti \) | Real | 2.9 | 3.1 | 5.2 | 9 | 4.2 | 5.1 | 3.2 | 8 | 4.7 | 4.9 | 2.86 |
| 12.029 | 6 | 0.3383 | 0.269 | 9 | 0.0433\(^{[3]} \) | 0.320 | 3 | 0.0064\(^{[2]} \) | 0.34 | 17 | 0.00001\(^{[1]} \) |
| 11.409 | 4 | 0.3382 | 0.264 | 5 | 0.0433\(^{[3]} \) | 0.320 | 3 | 0.0064\(^{[2]} \) | 0.34 | 17 | 0.00001\(^{[1]} \) |
| 9.6816 | 3 | 0.3381 | 0.250 | 9 | 0.0433\(^{[3]} \) | 0.320 | 2 | 0.0063\(^{[2]} \) | 0.34 | 17 | 0.00001\(^{[1]} \) |
| 9.5347 | 6 | 0.3378 | 0.231 | 6 | 0.0432\(^{[3]} \) | 0.320 | 2 | 0.0062\(^{[2]} \) | 0.34 | 11 | 0.00001\(^{[1]} \) |
| 9.4336 | 8 | 0.3378 | 0.228 | 8 | 0.0432\(^{[3]} \) | 0.320 | 2 | 0.0060\(^{[2]} \) | 0.34 | 11 | 0.00001\(^{[1]} \) |
| 8.9303 | 5 | 0.3372 | 0.222 | 5 | 0.0432\(^{[3]} \) | 0.320 | 1 | 0.0060\(^{[2]} \) | 0.34 | 05 | 0.00001\(^{[1]} \) |
| 8.8427 | 5 | 0.3363 | 0.208 | 9 | 0.0430\(^{[3]} \) | 0.319 | 3 | 0.0059\(^{[2]} \) | 0.33 | 95 | 0.00001\(^{[1]} \) |
| 8.6137 | 2 | 0.3354 | 0.205 | 2 | 0.0429\(^{[3]} \) | 0.319 | 2 | 0.0051\(^{[2]} \) | 0.33 | 91 | 0.00001\(^{[1]} \) |
| 8.5531 | 6 | 0.3354 | 0.198 | 6 | 0.0427\(^{[3]} \) | 0.319 | 0 | 0.0050\(^{[2]} \) | 0.33 | 91 | 0.00001\(^{[1]} \) |
| Estimated parameters | \( \lambda \) | \( \hat{\lambda} \) | \( \alpha \) | \( \hat{\alpha} \) | \( \lambda \) | \( \hat{\lambda} \) | \( \alpha \) | \( \hat{\alpha} \) |
|----------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| N  | Ti | Real | Mle | MSE | MPS | MSE | CVM | MSE |
| 15.534 | 0.194 | 0.0522(3) | 0.3519 | 0.0249(2) | 0.248 | 1 | 0.0039(1) |
| 15.427 | 0.193 | 0.180(2) | 0.3460 | 0.0232(3) | 0.245 | 1 | 0.0039(1) |
| 12.969 | 0.190 | 0.0143(2) | 0.3372 | 0.0224(3) | 0.244 | 3 | 0.0039(1) |
| 12.476 | 0.187 | 0.0070(2) | 0.3233 | 0.0181(3) | 0.242 | 7 | 0.0037(1) |
| 11.460 | 0.185 | 0.0069(2) | 0.3164 | 0.0177(3) | 0.240 | 8 | 0.0035(1) |
| 9.0421 | 0.182 | 0.0069(2) | 0.3096 | 0.0168(3) | 0.229 | 4 | 0.0028(1) |
| 8.9282 | 0.181 | 0.0069(2) | 0.3018 | 0.0135(3) | 0.228 | 2 | 0.0026(1) |
| 8.5483 | 0.181 | 0.0069(2) | 0.2883 | 0.0112(3) | 0.217 | 4 | 0.0021(1) |
| 7.6380 | 0.180 | 0.0069(2) | 0.2821 | 0.0102(3) | 0.211 | 6 | 0.0019(1) |
| 6.8036 | 0.177 | 0.0068(2) | 0.2686 | 0.0087(3) | 0.210 | 7 | 0.0016(1) |

Table (8): Simulation results for \( \lambda = 2.5, \theta = 5, \alpha = 3.5 \)

| Estimated parameters | \( \lambda \) | \( \hat{\lambda} \) | \( \alpha \) | \( \hat{\alpha} \) | \( \lambda \) | \( \hat{\lambda} \) | \( \alpha \) | \( \hat{\alpha} \) |
|----------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| n  | Ti | Real | Mle | MSE | MPS | MSE | CVM | MSE |
| 17.567 | 0.193 | 0.0530(3) | 0.1894 | 0.00002(1) | 0.245 | 7 | 0.0034(2) |
| 16.027 | 0.193 | 0.0371(3) | 0.1890 | 0.00002(1) | 0.245 | 5 | 0.0034(2) |
| 15.570 | 0.193 | 0.0314(3) | 0.1890 | 0.00002(1) | 0.245 | 2 | 0.0034(2) |
| 12.293 | 0.193 | 0.0169(3) | 0.1888 | 0.00002(1) | 0.244 | 5 | 0.0034(2) |
| 12.065 | 0.193 | 0.0160(3) | 0.1887 | 0.00002(1) | 0.242 | 7 | 0.0034(2) |
| 11.918 | 0.193 | 0.0158(3) | 0.1884 | 0.00002(1) | 0.241 | 5 | 0.0034(2) |
| 11.149 | 0.191 | 0.0146(3) | 0.1864 | 0.00002(1) | 0.241 | 5 | 0.0034(2) |
Table (9): Partial and overall ranks of all estimation methods for various combinations of \((\hat{\lambda}, \hat{\theta}, \hat{\alpha})\)

| Paramerters case          | N   | Method | MLE  | PSM  | CVM |
|---------------------------|-----|--------|------|------|-----|
| \((\hat{\lambda} = 2, \hat{\theta} = 4,\hat{\alpha} = 4)\) |     |        |      |      |     |
|                           | 35  | 3      | 2    | 1    |     |
|                           | 75  | 2      | 3    | 1    |     |
|                           | 100 | 3      | 2    | 1    |     |
| \(\lambda = 3.5, \theta = 4,\alpha = 3.5\)          |     |        |      |      |     |
|                           | 35  | 3      | 2    | 1    |     |
|                           | 75  | 3      | 1    | 2    |     |
|                           | 100 | 3      | 1    | 2    |     |
| \(\lambda = 3.5,\theta = 5,\alpha = 2.5\)          |     |        |      |      |     |
|                           | 35  | 3      | 1    | 2    |     |
|                           | 75  | 3      | 2    | 1    |     |
|                           | 100 | 3      | 1    | 2    |     |
From table (10), we can observe that:
- The maximum product spacing of the estimator method is the best method over the rest of the estimation methods.
- The sample size (n=100) is the best over the rest of the sample sizes.
- And the values imposed for the parameters when (\( \lambda = 4 \), \( \theta = 6.5 \), \( \alpha = 3 \)).

**Conclusion**

In this paper, we proposed a new distribution derived from the Frechet distribution called the new extended weighted Frechet distribution. The properties of the new distribution were derived through mathematical and statistical operations. We also estimated the parameters of the new distribution by estimation methods Max. Production spacing of estimator method, and estimation of failure function by simulation.

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