SU(6) ⊃ SU(3) ⊗ SU(2) and SU(8) ⊃ SU(4) ⊗ SU(2) Clebsch-Gordan coefficients

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Tables of scalar factors are presented for 63 ⊗ 63 and 120 ⊗ 63 in SU(8) ⊃ SU(4) ⊗ SU(2), and for 35 ⊗ 35 and 56 ⊗ 35 in SU(6) ⊃ SU(3) ⊗ SU(2). Related tables for SU(4) ⊃ SU(3) ⊗ U(1) and SU(3) ⊃ SU(2) ⊗ U(1) are also provided so that the Clebsch-Gordan coefficients can be completely reconstructed. These are suitable to study meson-meson and baryon-meson within a spin-flavor symmetric scheme.

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I. INTRODUCTION

After the original SU(3) flavor symmetry, SU(6) spin-flavor symmetry was soon introduced to give rise to the quark model [1–3], reaching considerable phenomenological success. (See reviews in [4–6].) It was shown early that chiral symmetry, a powerful tool to extract information from QCD [7–9], and spin-flavor symmetry were not in conflict [10]. Recently, spin-flavor has reappeared as a natural classification symmetry of baryons within the large $N_c$ approach to QCD [11], see e.g. [12, 13]. On the other hand, in the framework of unitarized chiral models [14, 17], spin-flavor symmetry has found an important role not only for three flavors but also for four [15, 24]. The reason is that spin-flavor symmetry, in its chiral version, can naturally accommodate the chiral and heavy quark symmetries of QCD. Although the underlying SU(4) flavor symmetry is severely broken in the kinematics through the quark masses, based on experience on three flavors, it is expected that the breaking should be mild at the level of interaction amplitudes.

In view of this renewed interest in spin-flavor symmetry, we have undertaken a study of the reduction of the spin-flavor group SU(8) under SU(4) ⊗ SU(2), as required in the description of meson-meson and baryon-meson interactions. Concretely we compute the scalar factors (also called singlet factors) of the reduction SU(8) ⊃ SU(4) ⊗ SU(2) for the products 63 ⊗ 63 (meson-meson) and 120 ⊗ 63 (baryon-meson). These have not been yet computed in the literature. It is true that, following the observation in [25, 26], all scalar factors and Clebsch-Gordan coefficients of special unitary groups follow from those of the symmetric group, $S_n$, however, the irreducible representations of SU(8) considered here would require large values of $n$ for which no results are readily available.

The prescription of [27] is widely used to define standard bases of irreducible representations of SU(3) [28, 29], however, for four or more flavors the natural prescription is that of [30] (see, e.g. [31]). Previous calculations of scalar factors of SU(6) ⊃ SU(3) ⊗ SU(2) have been done using [27], for instance [32] (see also, [33–36]), so we have also computed the corresponding SU(6) ⊃ SU(3) ⊗ SU(2) scalar factors for 35 ⊗ 35 and 56 ⊗ 35 with the prescrip-
We reproduce the results in [32], up to signs due to the different choice of standard bases, and also differ in the breaking of degeneracies.

We present also the related SU(4) ⊃ SU(3) ⊗ U(1) and SU(3) ⊃ SU(2) ⊗ U(1) scalar factors, always within the prescription [30]. We check the results in [31, 37] and extend them, since we need the product 20' ⊗ 15 in SU(4) which was not computed there, along with the corresponding new SU(3) ⊃ SU(2) ⊗ U(1) scalar factors required.

In section II notation is introduced, in section III we explain how to use the tables and in section IV the method of construction of the tables and the way the phases have been fixed is described. The scalar factors are displayed in the appendices (see table of contents).

II. REPRESENTATION REDUCTIONS AND CLEBSCH-GORDAN SERIES

For the groups SU(8) and SU(6) we consider the following reductions

\[
\begin{align*}
\text{SU}_{sf}(8) & \supset \text{SU}_{1}(4) \otimes \text{SU}_{J}(2), \\
\text{SU}_{sf}(6) & \supset \text{SU}_{1}(3) \otimes \text{SU}_{J}(2),
\end{align*}
\]

with

\[
\begin{align*}
\text{SU}_{1}(4) & \supset \text{SU}_{1}(3) \otimes U_{C}(1), \\
\text{SU}_{1}(3) & \supset \text{SU}_{1}(2) \otimes U_{Y}(1), \\
\text{SU}_{1}(2) & \supset U_{I_{z}}(1), \\
\text{SU}_{J}(2) & \supset U_{J_{z}}(1).
\end{align*}
\]

The labels sf, f, J, C, I, Y, I_{z} and J_{z} refer to spin-flavor, flavor, spin, charm, isospin, hypercharge, third component of isospin and third component of spin, respectively.

The CG coefficients pick up a plus or minus sign, ξ, under exchange of the states 1 and 2,

\[
\begin{pmatrix}
R_{2} & R_{1} & R_{\sigma} \\
\mu_{2} \gamma_{2} \zeta_{2} & \mu_{1} \gamma_{1} \zeta_{1} & \mu_{\gamma} \zeta
\end{pmatrix} = \xi
\begin{pmatrix}
R_{1} & R_{2} & R_{\sigma} \\
\mu_{1} \gamma_{1} \zeta_{1} & \mu_{2} \gamma_{2} \zeta_{2} & \mu_{\gamma} \zeta
\end{pmatrix}.
\]

The phase ξ depends only on R_{1}, R_{2}, R, σ, μ, and γ.

On the other hand, the uncoupled basis can be coupled under SU(4) ⊗ SU(2)

\[
\mathcal{H}_{R_{1}} \otimes \mathcal{H}_{R_{2}} = \bigoplus_{R, \sigma} \mathcal{H}_{R, \sigma}.
\]
Because the reduction chains in Eqs. (3-6) are canonical and \( SU(3) \) is symmetry (e.g. in \( SU(2) \)), it do not imply symmetry of the representation. When there is no degeneracy label, the reduction chains \( SU(4) \supset SU(3) \otimes U(1) \) and \( SU(2) \supset U(1) \) are obtained.

The relation between the SU(8)-coupled and SU(4) \( \otimes \) SU(2)-coupled basis provides the SU(8) \( \supset \) SU(4) \( \otimes \) SU(2) scalar factors (SF):

\[
| R_1, R_2; \sigma, \mu, \gamma, \zeta \rangle = \sum_{\mu_1, \mu_2, \gamma_2, \gamma} \left( \begin{array}{ccc} R_1 & R_2 & R_\sigma \\ \mu_1 \gamma_1 & \mu_2 \gamma_2 & \mu \gamma_\gamma \end{array} \right) | R_1, \mu_1, \gamma_1; R_2, \mu_2, \gamma_2; \mu, \gamma', \zeta \rangle,
\]

(13)

\[
| R_1, \mu_1, \gamma_1; R_2, \mu_2, \gamma_2; \mu, \gamma', \zeta \rangle = \sum_{R, \sigma, \gamma} \left( \begin{array}{ccc} R_1 & R_2 & R_\sigma \\ \mu_1 \gamma_1 & \mu_2 \gamma_2 & \mu \gamma_\gamma \end{array} \right) | R_1, R_2; \sigma, \mu, \gamma, \zeta \rangle.
\]

(14)

The tables are expressed in the form of equations, in terms of state vectors as in Eq. (13), i.e. a l.h.s with the SU(8)-coupled stated and a r.h.s. with the SU(4) \( \otimes \) SU(2)-coupled state. From such equations the SF can be read off. The plus or minus sign between parenthesis displayed at the left of the l.h.s. of the equation is the phase \( \xi \) defined in Eq. (10). It is put there for the sake of presentation only (it is not meant to multiply the vector).

Regarding the notation, as compared to Eq. (13), in the tables, redundant labels have been omitted. Specifically, in the l.h.s. we omit \( R_1, R_2 \) since they are explicited in the heading of the corresponding subsection, and \( \zeta \) is also omitted. The degeneracy labels \( \sigma \) and \( \gamma \), if required, take the form of a subindex. E.g., \( |63, 15_3\rangle \) \((\sigma = a)\), or \( |1232; 15_3, 3\rangle \) \((\gamma = s)\).

In the r.h.s., \( \gamma_1 \) and \( \gamma_2 \) are not needed for the SU(8) representations \( R_1 \) and \( R_2 \) considered, namely, \( 120 \) or \( 63 \). The labels \( \mu \) and \( \zeta \) are also omitted \((\mu \) is explicited in the l.h.s.). The irreps \( (R_1, \mu_1) \) and \( (R_2, \mu_2) \) are represented by symbols of the lowest lying particles with those quantum numbers. In each case, the particle with high-

### III. EXPLANATION OF THE TABLES

For SU(8) we consider the following CG series

\[
63 \otimes 63 = 1 \oplus 63_a \oplus 720 \oplus 1232 \oplus 63_a \oplus 945 \oplus 945^*,
\]

(15)

\[
120 \otimes 63 = 120 \oplus 168 \oplus 2520 \oplus 4752,
\]

and for SU(6)

\[
35 \otimes 35 = 1 \oplus 35_a \oplus 189 \oplus 405 \oplus 35_a \oplus 280 \oplus 280^*,
\]

(16)

\[
56 \otimes 35 = 56 \oplus 70 \oplus 700 \oplus 1134.
\]
TABLE I: List of Young tableaux and irrep labels.

| Group | irrep label | Young tableau |
|-------|-------------|---------------|
| SU(8) | 1           | [ ]           |
|       | 8           | [1]           |
|       | 8'          | [1']          |
|       | 63          | [2, 1', 1]    |
|       | 120         | [3]           |
|       | 168         | [2, 1]        |
|       | 720         | [2', 1', 1']  |
|       | 945         | [3, 1']       |
|       | 945'        | [3, 2', 2']   |
|       | 1232        | [4, 2']       |
|       | 2520        | [5, 1']       |
|       | 4752        | [4, 2, 1']    |
| SU(6) | 6           | [1]           |
|       | 6'          | [1']          |
|       | 35          | [2, 1']       |
|       | 56          | [3]           |
|       | 70          | [2, 1]        |
|       | 189         | [2', 1', 1']  |
|       | 280         | [3, 1']       |
|       | 280'        | [3', 2', 2']  |
|       | 405         | [4, 2']       |
|       | 700         | [5, 1']       |
|       | 1134        | [4, 2, 1']    |
| SU(4) | 4           | [1]           |
|       | 4'          | [1']          |
|       | 15          | [2, 1']       |
|       | 20          | [2, 1]        |
|       | 20'         | [3]           |
|       | 20''        | [2']          |
|       | 36'         | [3', 2']      |
|       | 45          | [3, 1]        |
|       | 45'         | [3', 2]       |
|       | 60'         | [3', 2', 1]   |
|       | 84          | [4, 2']       |
|       | 120         | [5, 1']       |
|       | 140         | [4, 2, 1]     |
| SU(3) | 3           | [1]           |
|       | 3'          | [1']          |
|       | 6           | [2]           |
|       | 6'          | [2']          |
|       | 8           | [2, 1]        |
|       | 10          | [3]           |
|       | 10'         | [3']          |
|       | 15          | [3, 1]        |
|       | 15'         | [3, 2]        |
|       | 15'         | [4]           |
|       | 24'         | [4, 1]        |
|       | 27          | [4, 2]        |
|       | 35          | [5, 1]        |

TABLE II: Reduction of SU(8) and SU(6) irreps.

| SU(8) SU(4) SU(2) | SU(6) SU(3) SU(2) |
|-------------------|-------------------|
| 63                | 35                |
| 15                | 13                |
| 15'               | 8                |
| 20'               | 20                |
| 4                | 20                |
| 15                | 20                |
| 15'               | 20                |
| 63                | 56                |
| 8                 | 10                |
| 15                | 10                |
| 15'               | 20                |
| 20                | 20                |
| 20'               | 20                |
| 6                | 120               |
| 15                | 120               |
| 15'               | 120               |
| 20                | 20                |
| 20'               | 20                |
| 63                | 1134              |
| 8                 | 12                |

est weight is used. E.g., \((120, 20_2)\) is labeled by \(\Sigma\) and \((63, 15_3)\) is labeled by \(\rho\). The list of particle symbols is displayed in Table I and there the rows are ordered so that they run from highest (top) to lowest weight (bottom).

Further, if the degeneracy label \(\gamma'\) is required, it is put as a subindex. E.g., \(|(\Sigma, \rho)_s\rangle\) for \(\gamma' = s\). As a final notational convention, in the r.h.s, when \(R_1 = R_2\) we use a label \(S\) or \(A\) to indicate symmetrized or antisymmetrized states under exchange of particle labels. E.g.,

\[
|\langle \rho, \pi \rangle_S \rangle = \frac{1}{\sqrt{2}} |\langle \rho, \pi \rangle_a \rangle + \frac{1}{\sqrt{2}} |\langle \pi, \rho \rangle_a \rangle,
\]

\[
|\langle \rho, \omega_1 \rangle_A \rangle = \frac{1}{\sqrt{2}} |\langle \rho, \omega_1 \rangle \rangle - \frac{1}{\sqrt{2}} |\langle \omega_1, \rho \rangle \rangle.
\]

Hence, Eq. (13) are written in the appendices as

\[
|\langle \xi \rangle |R_{\gamma'; \mu, \gamma} \rangle = \sum_{\mu_1, \mu_2, \gamma'} \left( \begin{array}{c} R_1 \quad R_2 \\ \mu_1 \quad \mu_2 \end{array} \right) R_{\gamma; \mu, \gamma'} |\langle \mu_1, \mu_2 \rangle \rangle.
\]

The equations corresponding to Eq. (14) are not given as they are easily reconstructed from the equations provided (of the type Eq. (13)). E.g., from the tables for \(R_1 \otimes R_2 = 63 \otimes 63\) with \(\mu = 15_5\), one finds, for
As explained, here $|\rho,\rho\rangle = |153\otimes 153;153\rangle$.

For SU(6), SU(3) $\otimes$ SU(2) everything is similar. The particle symbols in the r.h.s. correspond now to multiplets $(R_1,\mu_1)$ and $(R_2,\mu_2)$ of $(SU(6),SU(3)\otimes SU(2))$, so for instance, $\Sigma$ stands now for $(56,8_2)$ and $\rho$ stands for $(35,8_3)$.

For SU(4) $\supset$ SU(3) $\otimes$ U(1) the SU(3) $\otimes$ U(1) irrep $\mu$ in the l.h.s. is represented as $(\nu,C)$, where $\nu$ is the SU(3) irrep and $C$ is the charm quantum number. Now $\Sigma$ stands for the (SU(4);SU(3) $\otimes$ U(1)) irrep $(20;8,0)$ while $\pi$ stands for $(15;8,0)$. $(\rho)$ is not used here. It falls in the same representation $15$ of SU(4) as $\pi$ and we choose to use the pseudoscalars to label the states.)

Finally, for SU(3) $\supset$ SU(2) $\otimes$ U(1) the SU(3) $\otimes$ U(1) irrep $\mu$ in the l.h.s. is represented as $(I,Y)$, where $I$ is the isospin and $Y$ the hypercharge, and each particle symbol labels a complete isospin-hypercharge multiplet.

### IV. METHOD OF CONSTRUCTION AND PHASE CONVENTIONS

The fundamental and antifundamental representations of SU(n) can be realized by the ladder operators:

$$E^i_j|k\rangle = \delta^i_k|j\rangle - \frac{1}{n}\delta^i_k|k\rangle,$$

$$E^\dagger_j|\bar{k}\rangle = -\delta^k_j|\bar{i}\rangle + \frac{1}{n}\delta^k_j|\bar{k}\rangle, \quad i,j,k = 1, \ldots, n, \quad (20)$$

which fulfill the su(n) algebra relations

$$[E^i_j, E^k_l] = \delta^i_k E^l_j - \delta^k_l E^i_j, \quad (E^i_j)^\dagger = E^j_i. \quad (21)$$

These operators act the on a tensor product representation in the usual way

$$E^i_j(|A\rangle \otimes |B\rangle) = (E^i_j|A\rangle \otimes |B\rangle) + |A\rangle \otimes (E^j_i|B\rangle). \quad (22)$$

For SU(8), using the tensor products $8^* \otimes 8$ and $8 \otimes 8 \otimes 8$ we construct the adjoint, $63$, and symmetric, $120$, representations, corresponding to mesons and baryons, respectively. The CG series of $63 \otimes 63$ and $120 \otimes 63$ are resolved by the standard method of extracting the state with highest weight and applying ladder operators on it to fill a full highest weight representation. The method is then repeated recursively for the orthogonal spaces. The same technique is applied for the other groups, SU(6), SU(4) and SU(3). As usual higher dimension always implies higher weight. The representations are ordered as in table [III].

In the CG series considered the degeneracy label $\sigma$ is often redundant. When needed the symmetric combination, label $s$, is taken to be of highest weight than the antisymmetric one, label $a$.

### A. Flavor groups

To fix the phases within an irrep of a flavor group, SU(4) or SU(3), we adopt the prescription of [20], namely, we demand that the matrix elements between basis states

$$\mu_1 = \mu_2 = 153,$$

$$\langle (\rho,\rho) \rangle = \sqrt{\frac{3}{4}}|720;155\rangle + \sqrt{\frac{1}{4}}|1232;155\rangle. \quad (19)$$

As explained, here $|\rho,\rho\rangle = |153 \otimes 153;153\rangle$. For SU(6) $\supset$ SU(3) $\otimes$ SU(2) everything is similar. The particle symbols in the r.h.s. correspond now to multi-
of the ladder operators of form $E^i_{j+1}$ should be nonnegative. This produces a standard basis, by definition. 38 Identifying, as usual, the states $|i\rangle$, $i = 1, 2, 3, 4$ with $u$, $d$, $s$, $c$ (in this order) this prescription differs from the SU(3) choice in 27.

To fix the relative phases between the irreps in the CG series of SU(4), we fix the sign of the state with highest weight, i.e., corresponding to the highest SU(3) irrep. We do this is the usual way, namely, we order the uncoupled states attending first to $\mu_1$, if necessary to $\mu_2$, and finally to $\gamma'$. The sign of the highest coupled state is then chosen so that it has a positive overlap with the highest uncoupled state. E.g., in $15 \otimes 15$ to give $15_s$, the coupled state with highest weight is $|15_s, 8, 0\rangle$. The highest uncoupled state is $|(\pi, \pi)_s\rangle$. Everything is similar for SU(3). The prescription adopted is just the natural extension of that in SU(2).

B. Spin-flavor groups

For the group SU(8) we are interested in its reduction under SU(4) $\otimes$ SU(2). Therefore the prescription in 33, devised for SU(n) $\supset$ SU(n − 1) $\otimes$ U(1), does not directly apply. We do apply 33 for the relative phases between states inside each SU(4) $\otimes$ SU(2) irrep, but the phase between two such irreps in the reduction of an irrep of SU(8) is to be fixed. Also, because the reduction SU(8) $\supset$ SU(4) $\otimes$ SU(2) is not a canonical one, an SU(4) $\otimes$ SU(2) irrep can appear several times (label $\gamma$ in Eq. 8) and this has also to be fixed. In addition, the phase of each SU(8) irrep in the CG series is to be settled.

There is no widely accepted way of defining standard bases of the irreps of SU(8) $\supset$ SU(4) $\otimes$ SU(2). Rather than introducing such general prescription we adopt some concrete choices in what follows. Everything we say here for SU(8) can be immediately translated to SU(6).

Let us consider the irrep 63 obtained from $8^* \otimes 8$ and the irrep 120 obtained from $8 \otimes 8 \otimes 8$. Their reduction under SU(4) $\otimes$ SU(2) can be looked up in Table I. For them we choose

$$
\langle \pi; 1, 0| E^u_{n, +}|\rho; 1, 1\rangle > 0,
$$

$$
\langle \omega_1; 0, 1| E^d_{n, +}|\rho; 1, 1\rangle > 0,
$$

$$
\langle \Sigma; 1, \frac{1}{2}| E^u_{n, -}|\Sigma^*; 1, \frac{3}{2}\rangle > 0.
$$

(23)

Here we have used the notation $|p, I_z, J_3\rangle$ where $p$ denotes a concrete state $R, \mu, \nu, I, J, Y, C$ (see Table III). Also $(u, +), (d, +), \ldots, (c, -)$ are the 8 labels $i$ in the ladder operators $E^i_j$ of SU(8). The two first relations fix the phases between $15_1$, $15_3$ and $15_3$ in 63, whereas the second relation fixes the phase between $20_2$ and $20_4$ in 120.

For all the SU(4) $\otimes$ SU(2) irreps produced through the products $63 \otimes 63$ and $120 \otimes 63$, we adopt a common procedure which is customary and extends that used to resolve the CG series in SU(n) $\supset$ SU(n − 1) $\otimes$ U(1). This is as follows. The uncoupled states (r.h.s. in Eq. 8 or in Eq. 13) are given a well defined order. Then the first coupled state is taken to have the maximum overlap with the first uncoupled state. (If this overlap is zero the next uncoupled state is taken instead, etc.) Next, the second coupled state is chosen as the state orthogonal to the first one having the maximum overlap with the second uncoupled state. And so on, recursively.

So, for instance, in $|63 \otimes 63; 15_3\rangle$, $|(\rho, \rho)_a\rangle$ is the highest uncoupled state (ρ being of higher weight than π and $\omega_1$) and so its coefficient is positive. In this example the label $\gamma$ was not needed. A more complicated case is $|120 \otimes 63; 4752, 20_4\rangle$ with $\gamma = s, a, b$. The relevant uncoupled states are, listed from highest to lowest, $|\Delta, \rho\rangle$, $|\Delta, \pi\rangle$, $|\langle \Sigma, \rho\rangle_a\rangle$, $|\langle \Sigma, \rho\rangle_b\rangle$, and $|\langle \Sigma, \omega_1\rangle\rangle$. Hence, $|20_4\rangle$ has positive overlap with $|\Delta, \rho\rangle$, $|20_4\rangle$ has no overlap with $|\Delta, \rho\rangle$ and positive overlap with $|\Delta, \pi\rangle$, and $|20_4\rangle$ has no overlap with $|\Delta, \rho\rangle$ or $|\Delta, \pi\rangle$ and positive overlap with $|\langle \Sigma, \rho\rangle\rangle$.

We point out that this prescription, although simple enough, does not automatically produce standard bases. The bases obtained for a given SU(8) irrep will depend on how that irrep is obtained. For instance, for the 63 obtained from $8^* \otimes 8$ we have fixed the relative phases between $15_1$, $15_3$ and $1_3$ as in Eq. 23. However, the product $63 \otimes 63$ produces two 63 irreps (namely, $63_s$ and $63_a$). For them the relative phases between $15_1$, $15_3$ and $1_3$ are fixed instead from the “coupling method” just described. Unfortunately, it turns out to violate the two inequalities in 24. And the same violation takes places for the new irrep 120 generated from $120 \otimes 63$ for the third inequality. Of course, we could just redefine the necessary signs within these 63, 63, and 120, but we prefer to be systematic rather than to enforce standard bases for particular irreps of SU(8).

Regarding the order of the uncoupled states, there is a subtlety. In general, the order depends first on $\mu_1$, then on $\mu_2$ and then on $\gamma'$. However, in $63 \otimes 120$, we attend first to $\mu_2$ (the baryon state) and then to $\mu_1$ (the meson state). This is necessary to have a well defined $\xi$ in Eq. 10 in all cases. For instance, in $63 \otimes 120; 4752, 20_4\rangle$ we want $|\pi, \Delta\rangle$ to be of higher weight than $|(\rho, \Sigma)_s\rangle$, so that under exchange of labels 1 and 2 (Eq. 10), $63 \otimes 120; 4752, 20_4\rangle$ maps to $120 \otimes 63; 4752, 20_4\rangle$ and $63 \otimes 120; 4752, 20_4\rangle$ maps to $120 \otimes 63; 4752, 20_4\rangle$.

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Appendix A: Tables of scalar factors of SU(8)

\[ |\rho\rangle = |63; 15_3\rangle, \quad |\pi\rangle = |63; 15_1\rangle, \quad |\omega_1\rangle = |63; 1_3\rangle, \]
\[ |\Delta\rangle = |120; 20_4\rangle, \quad |\Sigma\rangle = |120; 20_2\rangle. \]

1. SU(8): 63 \otimes 63

\[
(+) \quad |1; 1_1\rangle = \sqrt{\frac{5}{7}} |\rho, \rho\rangle - \sqrt{\frac{1}{21}} |\omega_1, \omega_1\rangle - \sqrt{\frac{5}{21}} |\pi, \pi\rangle \\
(+) \quad |720; 1_1\rangle = \sqrt{\frac{1}{28}} |\rho, \rho\rangle - \sqrt{\frac{15}{28}} |\omega_1, \omega_1\rangle + \sqrt{\frac{3}{7}} |\pi, \pi\rangle \\
(+) \quad |1232; 1_1\rangle = \sqrt{\frac{1}{4}} |\rho, \rho\rangle + \sqrt{\frac{5}{12}} |\omega_1, \omega_1\rangle + \sqrt{\frac{1}{3}} |\pi, \pi\rangle \quad (A1)
\]

\[
(+) \quad |63_a; 1_3\rangle = |\rho, \pi\rangle_S \\
(-) \quad |63_a; 1_3\rangle = \sqrt{\frac{15}{16}} |\rho, \rho\rangle - \sqrt{\frac{1}{16}} |\omega_1, \omega_1\rangle \\
(-) \quad |945; 1_3\rangle = \sqrt{\frac{1}{32}} |\rho, \rho\rangle + \sqrt{\frac{1}{2}} |\rho, \pi\rangle_A + \sqrt{\frac{15}{32}} |\omega_1, \omega_1\rangle \\
(-) \quad |945^*; 1_3\rangle = \sqrt{\frac{1}{32}} |\rho, \rho\rangle - \sqrt{\frac{1}{2}} |\rho, \pi\rangle_A + \sqrt{\frac{15}{32}} |\omega_1, \omega_1\rangle \quad (A2)
\]

\[
(+) \quad |720; 1_5\rangle = \sqrt{\frac{5}{8}} |\rho, \rho\rangle + \sqrt{\frac{3}{8}} |\omega_1, \omega_1\rangle \\
(+) \quad |1232; 1_5\rangle = \sqrt{\frac{3}{8}} |\rho, \rho\rangle - \sqrt{\frac{5}{8}} |\omega_1, \omega_1\rangle \quad (A3)
\]

\[
(+) \quad |63_a; 15_1\rangle = \sqrt{\frac{3}{5}} |(\rho, \rho)_s\rangle + \sqrt{\frac{1}{5}} |(\rho, \omega_1)_s\rangle - \sqrt{\frac{7}{5}} |(\pi, \pi)_s\rangle \\
(-) \quad |63_a; 15_1\rangle = \sqrt{\frac{3}{4}} |(\rho, \rho)_a\rangle - \sqrt{\frac{1}{4}} |(\pi, \pi)_a\rangle \\
(+) \quad |720; 15_1\rangle = \sqrt{\frac{1}{2}} |(\rho, \omega_1)_S\rangle + \sqrt{\frac{1}{2}} |(\pi, \pi)_S\rangle \\
(-) \quad |945; 15_1\rangle = \sqrt{\frac{1}{8}} |(\rho, \rho)_A\rangle + \sqrt{\frac{1}{2}} |(\rho, \omega_1)_A\rangle + \sqrt{\frac{3}{8}} |(\pi, \pi)_A\rangle \\
(-) \quad |945^*; 15_1\rangle = \sqrt{\frac{1}{8}} |(\rho, \rho)_A\rangle - \sqrt{\frac{1}{2}} |(\rho, \omega_1)_A\rangle + \sqrt{\frac{3}{8}} |(\pi, \pi)_A\rangle \\
(+) \quad |1232; 15_1\rangle = \sqrt{\frac{2}{5}} |(\rho, \rho)_s\rangle - \sqrt{\frac{3}{10}} |(\rho, \omega_1)_S\rangle + \sqrt{\frac{3}{10}} |(\pi, \pi)_S\rangle \quad (A4)
\]
\((+)\)  $|63;15\rangle = \sqrt{\frac{8}{15}}(\rho,\rho) + \sqrt{\frac{2}{5}}(\rho,\pi) + \sqrt{\frac{1}{15}}\omega_1,\pi\rangle$

\((-)\)  $|63;15\rangle = \sqrt{\frac{3}{8}}(\rho,\rho) + \sqrt{\frac{3}{8}}\omega_1,\pi\rangle$

\((+)\)  $|720;15\rangle = \sqrt{\frac{1}{6}}(\rho,\rho) - \sqrt{\frac{1}{2}}(\rho,\pi) + \sqrt{\frac{1}{3}}\omega_1,\pi\rangle$

\((+)\)  $|720;15\rangle = \sqrt{\frac{1}{2}}(\rho,\omega_1) - \sqrt{\frac{1}{2}}(\rho,\pi)\rangle$

\((-)\)  $|945;15\rangle = \sqrt{\frac{5}{16}}(\rho,\rho) - \sqrt{\frac{3}{80}}\omega_1,\pi\rangle$

\((-)\)  $|945;15\rangle = \sqrt{\frac{1}{10}}(\rho,\rho) - \sqrt{\frac{1}{10}}(\rho,\pi)\rangle$

\((-)\)  $|945;15\rangle = \sqrt{\frac{5}{16}}(\rho,\rho) - \sqrt{\frac{3}{80}}\omega_1,\pi\rangle$

\((-)\)  $|945;15\rangle = \sqrt{\frac{1}{10}}(\rho,\rho) - \sqrt{\frac{1}{10}}(\rho,\pi)\rangle$

\((-)\)  $|945;15\rangle = \sqrt{\frac{1}{10}}(\rho,\omega_1) - \sqrt{\frac{1}{10}}(\rho,\pi)\rangle$

\((+)\)  $|1232;15\rangle = \sqrt{\frac{3}{10}}(\rho,\rho) - \sqrt{\frac{3}{10}}\omega_1,\pi\rangle$

\((+)\)  $|1232;15\rangle = \sqrt{\frac{1}{2}}(\rho,\omega_1) + \sqrt{\frac{1}{2}}(\rho,\pi)\rangle$

\((-)\)  $|720;20\rangle = \sqrt{\frac{1}{4}}(\rho,\rho) - \sqrt{\frac{3}{4}}(\rho,\pi)$

\((-)\)  $|720;20\rangle = \sqrt{\frac{1}{4}}(\rho,\rho) - \sqrt{\frac{3}{4}}\omega_1,\pi\rangle$

\((+)\)  $|1232;20\rangle = \sqrt{\frac{3}{4}}(\rho,\rho) + \sqrt{\frac{1}{4}}\omega_1,\pi\rangle$

\((+)\)  $|720;20\rangle = \sqrt{\frac{1}{2}}(\rho,\rho) + \sqrt{\frac{1}{2}}\omega_1,\pi\rangle$

\((-)\)  $|720;20\rangle = \sqrt{\frac{1}{2}}(\rho,\rho) + \sqrt{\frac{1}{2}}\omega_1,\pi\rangle$

\((-)\)  $|945;20\rangle = \sqrt{\frac{1}{2}}(\rho,\rho) + \sqrt{\frac{1}{2}}(\rho,\omega_1)\rangle$

\((-)\)  $|945;20\rangle = \sqrt{\frac{1}{2}}(\rho,\rho) + \sqrt{\frac{1}{2}}\omega_1,\pi\rangle$

\((+)\)  $|720;20\rangle = \sqrt{\frac{1}{2}}(\rho,\rho) + \sqrt{\frac{1}{2}}(\rho,\omega_1)\rangle$

\((+)\)  $|720;20\rangle = \sqrt{\frac{1}{2}}(\rho,\rho)$

\(\)
\(-\) \(|945; 45_1\rangle = \sqrt{\frac{3}{4}}|\rho, \rho\rangle - \sqrt{\frac{1}{4}}|\pi, \pi\rangle
\)

\(-\) \(|945^*; 45_1\rangle = \sqrt{\frac{1}{4}}|\rho, \rho\rangle + \sqrt{\frac{3}{4}}|\pi, \pi\rangle
\)

\(+\) \(|720; 45_3\rangle = \sqrt{\frac{1}{2}}|\rho, \rho\rangle + \sqrt{\frac{1}{2}}|\rho, \pi\rangle_A
\)

\(-\) \(|945; 45_3\rangle = |\rho, \pi\rangle_S
\)

\(+\) \(|1232; 45_3\rangle = \sqrt{\frac{1}{2}}|\rho, \rho\rangle - \sqrt{\frac{1}{2}}|\rho, \pi\rangle_A
\)

\(-\) \(|945^*; 45_5\rangle = |\rho, \rho\rangle
\)

\(-\) \(|945^*; 45_1\rangle = \sqrt{\frac{3}{4}}|\rho, \rho\rangle + \sqrt{\frac{1}{4}}|\pi, \pi\rangle
\)

\(-\) \(|945^*; 45_1\rangle = \sqrt{\frac{1}{4}}|\rho, \rho\rangle - \sqrt{\frac{3}{4}}|\pi, \pi\rangle
\)

\(+\) \(|720; 45_5\rangle = \sqrt{\frac{1}{2}}|\rho, \rho\rangle - \sqrt{\frac{1}{2}}|\rho, \pi\rangle_A
\)

\(-\) \(|945^*; 45_5\rangle = |\rho, \pi\rangle_S
\)

\(+\) \(|1232; 45_5\rangle = \sqrt{\frac{1}{2}}|\rho, \rho\rangle + \sqrt{\frac{1}{2}}|\rho, \pi\rangle_A
\)

\(-\) \(|945^*; 45_5\rangle = |\rho, \rho\rangle
\)

\(+\) \(|1232; 84_1\rangle = \sqrt{\frac{3}{4}}|\rho, \rho\rangle + \sqrt{\frac{1}{4}}|\pi, \pi\rangle
\)

\(+\) \(|1232; 84_1\rangle = \sqrt{\frac{1}{4}}|\rho, \rho\rangle - \sqrt{\frac{3}{4}}|\pi, \pi\rangle
\)

\(-\) \(|945; 84_3\rangle = \sqrt{\frac{1}{2}}|\rho, \rho\rangle + \sqrt{\frac{1}{2}}|\rho, \pi\rangle_A
\)

\(-\) \(|945^*; 84_3\rangle = \sqrt{\frac{1}{2}}|\rho, \rho\rangle - \sqrt{\frac{1}{2}}|\rho, \pi\rangle_A
\)

\(+\) \(|1232; 84_3\rangle = |\rho, \pi\rangle_S
\)

\(+\) \(|1232; 84_5\rangle = |\rho, \rho\rangle
\)

\(+\) \(|168; 4^*_2\rangle = \sqrt{\frac{3}{4}}|\Sigma, \rho\rangle - \sqrt{\frac{1}{4}}|\Sigma, \pi\rangle
\)

\(+\) \(|4752; 4^*_2\rangle = \sqrt{\frac{1}{4}}|\Sigma, \rho\rangle + \sqrt{\frac{3}{4}}|\Sigma, \pi\rangle
\)

2. SU(8): 120 ⊗ 63

\(+\) \(|168; 4^*_2\rangle = \sqrt{\frac{3}{4}}|\Sigma, \rho\rangle - \sqrt{\frac{1}{4}}|\Sigma, \pi\rangle
\)

\(+\) \(|4752; 4^*_2\rangle = \sqrt{\frac{1}{4}}|\Sigma, \rho\rangle + \sqrt{\frac{3}{4}}|\Sigma, \pi\rangle
\)
\[-\) |4752; 4^4\rangle = |\Sigma, \rho\rangle \] (A20)

\[\begin{align*}
(+) \ |120; 20_2\rangle &= \sqrt{\frac{32}{77}}|\Delta, \rho\rangle + \sqrt{\frac{256}{1001}}|(\Sigma, \rho)_s\rangle - \sqrt{\frac{21}{143}}|(\Sigma, \rho)_a\rangle + \sqrt{\frac{1}{11}}|\Sigma, \omega_1\rangle \\
&+ \sqrt{\frac{13}{77}}|(\Sigma, \pi)_a\rangle

(+) \ |168; 20_2\rangle &= \sqrt{\frac{2}{5}}|\Delta, \rho\rangle - \frac{4}{65}|(\Sigma, \rho)_s\rangle + \frac{15}{52}|(\Sigma, \rho)_a\rangle - \frac{1}{20}|\Sigma, \omega_1\rangle \\
&+ \sqrt{\frac{12}{65}}|(\Sigma, \pi)_s\rangle - \frac{1}{65}|(\Sigma, \pi)_a\rangle

(+) \ |2520; 20_2\rangle &= \sqrt{\frac{2}{33}}|\Delta, \rho\rangle - \frac{625}{1716}|(\Sigma, \rho)_s\rangle + \frac{1}{143}|(\Sigma, \rho)_a\rangle + \frac{3}{11}|\Sigma, \omega_1\rangle \\
&- \frac{11}{52}|(\Sigma, \pi)_s\rangle + \frac{12}{143}|(\Sigma, \pi)_a\rangle

(+ |4752; 20_{m, 2}\rangle &= \sqrt{\frac{13}{105}}|\Delta, \rho\rangle - \frac{121}{35490}|(\Sigma, \rho)_s\rangle - \frac{35}{338}|(\Sigma, \rho)_a\rangle - \frac{27}{910}|\Sigma, \omega_1\rangle \\
&- \sqrt{\frac{343}{1690}}|(\Sigma, \pi)_s\rangle - \frac{3174}{5915}|(\Sigma, \pi)_a\rangle

(- |4752; 20_{m, 2}\rangle &= \sqrt{\frac{213}{676}}|(\Sigma, \rho)_s\rangle + \frac{4900}{11999}|(\Sigma, \rho)_a\rangle + \frac{108}{923}|\Sigma, \omega_1\rangle - \sqrt{\frac{5929}{47996}}|(\Sigma, \pi)_s\rangle \\
&- \sqrt{\frac{432}{11999}}|(\Sigma, \pi)_a\rangle

(+ \ |4752; 20_{b, 2}\rangle &= \sqrt{\frac{13}{284}}|(\Sigma, \rho)_a\rangle - \frac{147}{284}|\Sigma, \omega_1\rangle - \sqrt{\frac{256}{923}}|(\Sigma, \pi)_s\rangle + \sqrt{\frac{147}{923}}|(\Sigma, \pi)_a\rangle \] (A21)

\[-\) |168; 20_4\rangle &= \sqrt{\frac{1}{2}}|\Delta, \rho\rangle - \frac{3}{10}|\Delta, \pi\rangle + \frac{8}{65}|(\Sigma, \rho)_s\rangle + \frac{27}{520}|(\Sigma, \rho)_a\rangle \\
&- \frac{1}{40}|\Sigma, \omega_1\rangle

(- \ |2520; 20_4\rangle &= \sqrt{\frac{5}{24}}|\Delta, \rho\rangle + \sqrt{\frac{7}{8}}|\Delta, \pi\rangle + \frac{2}{39}|(\Sigma, \rho)_s\rangle - \frac{25}{104}|(\Sigma, \rho)_a\rangle \\
&+ \sqrt{\frac{3}{8}}|\Sigma, \omega_1\rangle

(- \ |4752; 20_{m, 4}\rangle &= \sqrt{\frac{7}{24}}|\Delta, \rho\rangle + \sqrt{\frac{7}{40}}|\Delta, \pi\rangle - \sqrt{\frac{578}{1365}}|(\Sigma, \rho)_s\rangle + \sqrt{\frac{7}{520}}|(\Sigma, \rho)_a\rangle \\
&- \frac{27}{280}|\Sigma, \omega_1\rangle

(+ \ |4752; 20_{m, 4}\rangle &= \sqrt{\frac{2}{5}}|\Delta, \pi\rangle + \sqrt{\frac{24}{65}}|(\Sigma, \rho)_s\rangle + \sqrt{\frac{81}{520}}|(\Sigma, \rho)_a\rangle - \frac{3}{40}|\Sigma, \omega_1\rangle

(+ \ |4752; 20_{b, 4}\rangle &= \sqrt{\frac{3}{91}}|(\Sigma, \rho)_s\rangle - \sqrt{\frac{7}{13}}|(\Sigma, \rho)_a\rangle - \sqrt{\frac{3}{7}}|\Sigma, \omega_1\rangle \] (A22)

\[(+) \ |4752; 20_6\rangle = |\Delta, \rho\rangle \] (A23)
(-) $|168; 20_2\rangle = \sqrt{\frac{7}{10}}|\Delta, \rho\rangle - \sqrt{\frac{1}{10}}|\Delta, \omega_1\rangle + \sqrt{\frac{1}{20}}|\Sigma, \rho\rangle + \sqrt{\frac{3}{20}}|\Sigma, \pi\rangle$

(-) $|2520; 20_2\rangle = \sqrt{\frac{7}{48}}|\Delta, \rho\rangle + \sqrt{\frac{3}{16}}|\Delta, \omega_1\rangle + \sqrt{\frac{1}{6}}|\Sigma, \rho\rangle - \sqrt{\frac{1}{2}}|\Sigma, \pi\rangle$

(-) $|4752; 20_{s,2}\rangle = \sqrt{\frac{37}{240}}|\Delta, \rho\rangle + \sqrt{\frac{189}{2960}}|\Delta, \omega_1\rangle - \sqrt{\frac{847}{1110}}|\Sigma, \rho\rangle - \sqrt{\frac{7}{370}}|\Sigma, \pi\rangle$

(+) $|4752; 20_{a,2}\rangle = \sqrt{\frac{24}{37}}|\Delta, \omega_1\rangle + \sqrt{\frac{3}{148}}|\Sigma, \rho\rangle + \sqrt{\frac{49}{148}}|\Sigma, \pi\rangle$ (A24)

(+) $|120; 20_4\rangle = \sqrt{\frac{5}{11}}|\Delta, \rho\rangle - \sqrt{\frac{5}{77}}|\Delta, \omega_1\rangle - \sqrt{\frac{3}{11}}|\Delta, \pi\rangle + \sqrt{\frac{16}{77}}|\Sigma, \rho\rangle$

(+) $|2520; 20_4\rangle = \sqrt{\frac{1}{204}}|\Delta, \rho\rangle + \sqrt{\frac{21}{88}}|\Delta, \omega_1\rangle + \sqrt{\frac{5}{22}}|\Delta, \pi\rangle + \sqrt{\frac{35}{66}}|\Sigma, \rho\rangle$

(+) $|4752; 20_{s,4}\rangle = \sqrt{\frac{13}{24}}|\Delta, \rho\rangle + \sqrt{\frac{27}{728}}|\Delta, \omega_1\rangle + \sqrt{\frac{5}{26}}|\Delta, \pi\rangle - \frac{125}{546}|\Sigma, \rho\rangle$

(-) $|4752; 20_{a,4}\rangle = \sqrt{\frac{60}{91}}|\Delta, \omega_1\rangle - \sqrt{\frac{4}{13}}|\Delta, \pi\rangle - \sqrt{\frac{3}{91}}|\Sigma, \rho\rangle$ (A25)

(-) $|2520; 20_6\rangle = \sqrt{\frac{9}{16}}|\Delta, \rho\rangle - \sqrt{\frac{7}{16}}|\Delta, \omega_1\rangle$

(-) $|4752; 20_{a,6}\rangle = \sqrt{\frac{7}{16}}|\Delta, \rho\rangle + \sqrt{\frac{9}{16}}|\Delta, \omega_1\rangle$ (A26)

(-) $|4752; 36_{s,2}\rangle = |\Sigma, \rho\rangle$

(+) $|4752; 36_{a,2}\rangle = |\Sigma, \pi\rangle$ (A27)

(+) $|4752; 36_4\rangle = |\Sigma, \rho\rangle$ (A28)

(+) $|2520; 60_2\rangle = \sqrt{\frac{3}{4}}|\Sigma, \rho\rangle + \sqrt{\frac{1}{4}}|\Sigma, \pi\rangle$

(+) $|4752; 60_2\rangle = \sqrt{\frac{1}{4}}|\Sigma, \rho\rangle - \sqrt{\frac{3}{4}}|\Sigma, \pi\rangle$ (A29)

(-) $|4752; 60_4\rangle = |\Sigma, \rho\rangle$ (A30)

(+) $|4752; 120_2\rangle = |\Delta, \rho\rangle$ (A31)

(-) $|2520; 120_4\rangle = \sqrt{\frac{3}{8}}|\Delta, \rho\rangle - \sqrt{\frac{3}{8}}|\Delta, \pi\rangle$

(-) $|4752; 120_4\rangle = \sqrt{\frac{5}{8}}|\Delta, \rho\rangle + \sqrt{\frac{3}{8}}|\Delta, \pi\rangle$ (A32)

(+) $|2520; 120_6\rangle = |\Delta, \rho\rangle$ (A33)

(-) $|2520; 140_2\rangle = \sqrt{\frac{1}{3}}|\Delta, \rho\rangle - \sqrt{\frac{1}{6}}|\Sigma, \rho\rangle + \sqrt{\frac{1}{2}}|\Sigma, \pi\rangle$

(-) $|4752; 140_{s,2}\rangle = \sqrt{\frac{2}{3}}|\Delta, \rho\rangle + \sqrt{\frac{1}{12}}|\Sigma, \rho\rangle - \sqrt{\frac{1}{4}}|\Sigma, \pi\rangle$

(-) $|4752; 140_{a,2}\rangle = \sqrt{\frac{3}{4}}|\Sigma, \rho\rangle + \sqrt{\frac{1}{4}}|\Sigma, \pi\rangle$ (A34)
\[ (+) \quad |2520; 140_4\rangle = \sqrt{\frac{5}{24}}|\Delta, \rho\rangle + \sqrt{\frac{1}{8}}|\Delta, \pi\rangle - \sqrt{\frac{2}{3}}|\Sigma, \rho\rangle \]

\[ (+) \quad |4752; 140_{s,4}\rangle = \sqrt{\frac{19}{24}}|\Delta, \rho\rangle - \sqrt{\frac{5}{152}}|\Delta, \pi\rangle + \sqrt{\frac{10}{57}}|\Sigma, \rho\rangle \]

\[ (-) \quad |4752; 140_{a,4}\rangle = \sqrt{\frac{16}{19}}|\Delta, \pi\rangle + \sqrt{\frac{3}{19}}|\Sigma, \rho\rangle \quad (A35) \]

\[ (-) \quad |4752; 140_6\rangle = |\Delta, \rho\rangle \quad (A36) \]

### Appendix B: Tables of scalar factors of SU(6)

\[ |\rho\rangle = |35; 8_3\rangle, \quad |\pi\rangle = |35; 8_1\rangle, \quad |\omega_1\rangle = |35; 1_3\rangle, \]
\[ |\Delta\rangle = |56; 10_4\rangle, \quad |\Sigma\rangle = |56; 8_2\rangle. \]

1. **SU(6): 35 \otimes 35**

\[ (+) \quad |1; 1_1\rangle = \sqrt{\frac{24}{35}}|\rho, \rho\rangle - \sqrt{\frac{3}{35}}|\omega_1, \omega_1\rangle - \sqrt{\frac{8}{35}}|\pi, \pi\rangle \]

\[ (+) \quad |189; 1_1\rangle = \sqrt{\frac{1}{60}}|\rho, \rho\rangle - \sqrt{\frac{8}{15}}|\omega_1, \omega_1\rangle + \sqrt{\frac{9}{20}}|\pi, \pi\rangle \]

\[ (+) \quad |405; 1_1\rangle = \sqrt{\frac{25}{84}}|\rho, \rho\rangle + \sqrt{\frac{8}{21}}|\omega_1, \omega_1\rangle + \sqrt{\frac{9}{28}}|\pi, \pi\rangle \quad (B1) \]

\[ (+) \quad |35_s; 1_3\rangle = |\rho, \pi\rangle \]

\[ (-) \quad |35_a; 1_3\rangle = \sqrt{\frac{8}{9}}|\rho, \rho\rangle - \frac{1}{9}|\omega_1, \omega_1\rangle \]

\[ (-) \quad |280; 1_3\rangle = \sqrt{\frac{1}{18}}|\rho, \rho\rangle + \sqrt{\frac{1}{2}}|\rho, \pi\rangle + \sqrt{\frac{4}{9}}|\omega_1, \omega_1\rangle \]

\[ (-) \quad |280^*; 1_3\rangle = \sqrt{\frac{1}{18}}|\rho, \rho\rangle - \sqrt{\frac{1}{2}}|\rho, \pi\rangle + \sqrt{\frac{4}{9}}|\omega_1, \omega_1\rangle \quad (B2) \]

\[ (+) \quad |189; 1_5\rangle = \sqrt{\frac{2}{3}}|\rho, \rho\rangle + \sqrt{\frac{1}{3}}|\omega_1, \omega_1\rangle \]

\[ (+) \quad |405; 1_5\rangle = \sqrt{\frac{1}{3}}|\rho, \rho\rangle - \sqrt{\frac{2}{3}}|\omega_1, \omega_1\rangle \quad (B3) \]
\begin{align}
(+) \quad |35_s; 8_1\rangle &= \sqrt{\frac{15}{32}}(\rho, \rho)_s + \sqrt{\frac{3}{8}}(\rho, \omega_1)_s - \sqrt{\frac{5}{32}}(\pi, \pi)_s \\
(-) \quad |35_a; 8_1\rangle &= \sqrt{\frac{2}{4}}(\rho, \rho)_a - \sqrt{\frac{1}{4}}(\pi, \pi)_a \\
(+) \quad |189; 8_1\rangle &= \sqrt{\frac{1}{48}}(\rho, \rho)_s - \sqrt{\frac{5}{12}}(\rho, \omega_1)_s - \sqrt{\frac{9}{16}}(\pi, \pi)_s \\
(-) \quad |280; 8_1\rangle &= \sqrt{\frac{1}{8}}(\rho, \rho)_a + \sqrt{\frac{1}{2}}(\rho, \omega_1)_A + \sqrt{\frac{3}{8}}(\pi, \pi)_a \\
(-) \quad |280^*; 8_1\rangle &= \sqrt{\frac{1}{8}}(\rho, \rho)_a - \sqrt{\frac{1}{2}}(\rho, \omega_1)_A + \sqrt{\frac{3}{8}}(\pi, \pi)_a \\
(+) \quad |405; 8_1\rangle &= \sqrt{\frac{49}{96}}(\rho, \rho)_s - \sqrt{\frac{5}{24}}(\rho, \omega_1)_s + \sqrt{\frac{9}{32}}(\pi, \pi)_s \tag{B4}
\end{align}

\begin{align}
(+) \quad |35_s; 8_3\rangle &= \sqrt{\frac{9}{16}}(\rho, \rho)_s + \sqrt{\frac{5}{16}}(\rho, \pi)_s + \sqrt{\frac{1}{8}}(\omega_1, \pi)_s \\
(-) \quad |35_a; 8_3\rangle &= \sqrt{\frac{5}{18}}(\rho, \rho)_a + \sqrt{\frac{2}{9}}(\rho, \omega_1)_s + \sqrt{\frac{1}{7}}(\rho, \pi)_a \\
(+) \quad |189; 8_s, 3\rangle &= \sqrt{\frac{1}{8}}(\rho, \rho)_a - \sqrt{\frac{5}{8}}(\rho, \pi)_s + \sqrt{\frac{1}{4}}(\omega_1, \pi)_s \\
(+) \quad |189; 8_a, 3\rangle &= \sqrt{\frac{1}{2}}(\rho, \omega_1)_A - \sqrt{\frac{1}{2}}(\rho, \pi)_A \\
(-) \quad |280; 8_s, 3\rangle &= \sqrt{\frac{13}{36}}(\rho, \rho)_s - \sqrt{\frac{5}{117}}(\rho, \omega_1)_s + \sqrt{\frac{4}{13}}(\rho, \pi)_s - \sqrt{\frac{5}{52}}(\pi, \pi)_A \\
&\quad - \sqrt{\frac{5}{26}}(\omega_1, \pi)_A \\
(-) \quad |280; 8_a, 3\rangle &= \sqrt{\frac{9}{26}}(\rho, \omega_1)_s - \sqrt{\frac{5}{20}}(\rho, \pi)_A - \sqrt{\frac{2}{13}}(\rho, \pi)_s - \sqrt{\frac{4}{13}}(\omega_1, \pi)_A \\
(-) \quad |280^*; 8_s, 3\rangle &= \sqrt{\frac{13}{36}}(\rho, \rho)_s - \sqrt{\frac{5}{117}}(\rho, \omega_1)_s - \sqrt{\frac{4}{13}}(\rho, \pi)_s - \sqrt{\frac{5}{52}}(\pi, \pi)_A \\
&\quad + \sqrt{\frac{5}{26}}(\omega_1, \pi)_A \\
(-) \quad |280^*; 8_a, 3\rangle &= \sqrt{\frac{9}{26}}(\rho, \omega_1)_s + \sqrt{\frac{5}{20}}(\rho, \pi)_A - \sqrt{\frac{2}{13}}(\rho, \pi)_s + \sqrt{\frac{4}{13}}(\omega_1, \pi)_A \\
(+) \quad |405; 8_s, 3\rangle &= \sqrt{\frac{5}{16}}(\rho, \rho)_a - \sqrt{\frac{1}{16}}(\rho, \pi)_s - \sqrt{\frac{5}{8}}(\omega_1, \pi)_s \\
(+) \quad |405; 8_a, 3\rangle &= \sqrt{\frac{1}{2}}(\rho, \omega_1)_A + \sqrt{\frac{1}{2}}(\rho, \pi)_A \tag{B5}
\end{align}

\begin{align}
(+) \quad |189; 8_5\rangle &= \sqrt{\frac{5}{6}}(\rho, \rho)_s - \sqrt{\frac{1}{6}}(\rho, \omega_1)_s \\
(-) \quad |280; 8_5\rangle &= \sqrt{\frac{1}{2}}(\rho, \rho)_a - \sqrt{\frac{1}{2}}(\rho, \omega_1)_A \\
(-) \quad |280^*; 8_5\rangle &= \sqrt{\frac{1}{2}}(\rho, \rho)_a + \sqrt{\frac{1}{2}}(\rho, \omega_1)_A \\
(+) \quad |405; 8_5\rangle &= \sqrt{\frac{5}{6}}(\rho, \rho)_s + \sqrt{\frac{5}{6}}(\rho, \omega_1)_s \tag{B6}
\end{align}
\begin{align*}
\langle - | 280; 10_1 \rangle &= \sqrt{\frac{1}{4}} | \rho, \rho \rangle - \sqrt{\frac{3}{4}} | \pi, \pi \rangle \\
\langle - | 280^*; 10_1 \rangle &= \sqrt{\frac{3}{4}} | \rho, \rho \rangle + \sqrt{\frac{1}{4}} | \pi, \pi \rangle \\
\langle + | 189; 10_3 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle + \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle - | 280; 10_3 \rangle &= | \rho, \pi \rangle_S \\
\langle + | 405; 10_3 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle - \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle - | 280^*; 10_5 \rangle &= | \rho, \rho \rangle \\
\langle - | 280; 10^*_1 \rangle &= \sqrt{\frac{3}{4}} | \rho, \rho \rangle + \sqrt{\frac{1}{4}} | \pi, \pi \rangle \\
\langle - | 280^*; 10^*_1 \rangle &= \sqrt{\frac{1}{4}} | \rho, \rho \rangle - \sqrt{\frac{3}{4}} | \pi, \pi \rangle \\
\langle + | 189; 10^*_3 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle - \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle - | 280^*; 10^*_3 \rangle &= | \rho, \pi \rangle_S \\
\langle + | 405; 10^*_3 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle + \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle - | 280^*; 10^*_5 \rangle &= | \rho, \rho \rangle \\
\langle - | 280; 27_3 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle - \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle - | 280^*; 27_3 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle + \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle + | 405; 27_3 \rangle &= | \rho, \pi \rangle_S \\
\langle + | 405; 27_5 \rangle &= | \rho, \rho \rangle \\
\langle - | 280; 27_5 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle - \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle - | 280^*; 27_5 \rangle &= \sqrt{\frac{1}{2}} | \rho, \rho \rangle + \sqrt{\frac{1}{2}} | \rho, \pi \rangle_A \\
\langle + | 405; 27_5 \rangle &= | \rho, \rho \rangle \\
\langle - | 280; 70_1 \rangle &= \sqrt{\frac{3}{4}} | \Sigma, \rho \rangle - \sqrt{\frac{1}{4}} | \Sigma, \pi \rangle \\
\langle - | 1134; 12 \rangle &= \sqrt{\frac{1}{4}} | \Sigma, \rho \rangle + \sqrt{\frac{3}{4}} | \Sigma, \pi \rangle
\end{align*}

2. SU(6): 56 \otimes 35

\begin{align*}
\langle - | 70; 1_2 \rangle &= \sqrt{\frac{3}{4}} | \Sigma, \rho \rangle - \sqrt{\frac{1}{4}} | \Sigma, \pi \rangle \\
\langle - | 1134; 1_2 \rangle &= \sqrt{\frac{1}{4}} | \Sigma, \rho \rangle + \sqrt{\frac{3}{4}} | \Sigma, \pi \rangle
\end{align*}
\[
(+)
\left| 1134: 1_4 \right> = \left| \Sigma, \rho \right>
\]  
(B17)

\[
(+)
\left| 56: 8_2 \right> = \sqrt{\frac{4}{9}} \left| \Delta, \rho \right> + \sqrt{\frac{2}{9}} \left( \Sigma, \rho \right)_s - \sqrt{\frac{8}{144}} \left( \Sigma, \rho \right)_a + \sqrt{\frac{1}{48}} \left( \Sigma, \omega_1 \right)
+ \sqrt{\frac{2}{15}} \left( \Sigma, \pi \right)_a
\]

\[
(+)
\left| 70: 8_2 \right> = \sqrt{\frac{5}{12}} \left| \Delta, \rho \right> - \sqrt{\frac{5}{60}} \left( \Sigma, \rho \right)_s + \sqrt{\frac{25}{96}} \left( \Sigma, \rho \right)_a - \sqrt{\frac{1}{12}} \left( \Sigma, \omega_1 \right)
+ \sqrt{\frac{5}{32}} \left( \Sigma, \pi \right)_a - \sqrt{\frac{1}{32}} \left( \Sigma, \pi \right)_a
\]

\[
(+)
\left| 700: 8_2 \right> = \sqrt{\frac{1}{18}} \left| \Delta, \rho \right> - \sqrt{\frac{49}{144}} \left( \Sigma, \rho \right)_s + \sqrt{\frac{5}{144}} \left( \Sigma, \rho \right)_a + \sqrt{\frac{5}{18}} \left( \Sigma, \omega_1 \right)
- \sqrt{\frac{3}{16}} \left( \Sigma, \pi \right)_s + \sqrt{\frac{5}{48}} \left( \Sigma, \pi \right)_a
\]

\[
(+)
\left| 1134: 8_{b,2} \right> = \sqrt{\frac{1}{12}} \left| \Delta, \rho \right> - \sqrt{\frac{1}{96}} \left( \Sigma, \rho \right)_s - \sqrt{\frac{49}{480}} \left( \Sigma, \rho \right)_a - \sqrt{\frac{1}{60}} \left( \Sigma, \omega_1 \right)
- \sqrt{\frac{9}{32}} \left( \Sigma, \pi \right)_s - \sqrt{\frac{81}{160}} \left( \Sigma, \pi \right)_a
\]

\[
(-)
\left| 1134: 8_{a,2} \right> = \sqrt{\frac{3}{8}} \left( \Sigma, \rho \right)_s + \sqrt{\frac{49}{120}} \left( \Sigma, \rho \right)_a + \sqrt{\frac{1}{15}} \left( \Sigma, \omega_1 \right) - \sqrt{\frac{1}{8}} \left( \Sigma, \pi \right)_s
- \sqrt{\frac{1}{40}} \left( \Sigma, \pi \right)_a
\]

\[
(+)
\left| 1134: 8_{b,2} \right> = \sqrt{\frac{1}{60}} \left( \Sigma, \rho \right)_a - \sqrt{\frac{8}{15}} \left( \Sigma, \omega_1 \right) - \sqrt{\frac{3}{4}} \left( \Sigma, \pi \right)_s + \sqrt{\frac{1}{5}} \left( \Sigma, \pi \right)_a
\]  
(B18)

\[
(-)
\left| 70: 8_4 \right> = \sqrt{\frac{25}{48}} \left| \Delta, \rho \right> - \sqrt{\frac{5}{16}} \left( \Delta, \pi \right) + \sqrt{\frac{5}{48}} \left( \Sigma, \rho \right)_s + \sqrt{\frac{1}{48}} \left( \Sigma, \rho \right)_a
- \sqrt{\frac{1}{24}} \left( \Sigma, \omega_1 \right)
\]

\[
(-)
\left| 700: 8_4 \right> = \sqrt{\frac{5}{24}} \left| \Delta, \rho \right> + \sqrt{\frac{1}{8}} \left( \Sigma, \rho \right)_s + \sqrt{\frac{1}{24}} \left( \Sigma, \rho \right)_a - \sqrt{\frac{5}{24}} \left( \Sigma, \rho \right)_a
+ \sqrt{\frac{5}{12}} \left( \Sigma, \omega_1 \right)
\]

\[
(-)
\left| 1134: 8_{a,4} \right> = \sqrt{\frac{13}{48}} \left| \Delta, \rho \right> + \sqrt{\frac{45}{208}} \left( \Delta, \pi \right) - \sqrt{\frac{245}{624}} \left( \Sigma, \rho \right)_s + \sqrt{\frac{25}{624}} \left( \Sigma, \rho \right)_a
- \sqrt{\frac{25}{312}} \left( \Sigma, \omega_1 \right)
\]

\[
(+)
\left| 1134: 8_{a,4} \right> = \sqrt{\frac{9}{26}} \left| \Delta, \pi \right> + \sqrt{\frac{6}{13}} \left( \Sigma, \rho \right)_s + \sqrt{\frac{5}{78}} \left( \Sigma, \rho \right)_a - \sqrt{\frac{5}{39}} \left( \Sigma, \omega_1 \right)
\]

\[
(-)
\left| 1134: 8_{b,4} \right> = \sqrt{\frac{2}{3}} \left( \Sigma, \rho \right)_a + \sqrt{\frac{4}{3}} \left( \Sigma, \omega_1 \right)
\]  
(B19)

\[
(+)
\left| 1134: 8_b \right> = \left| \Delta, \rho \right>
\]  
(B20)
\[\left| 70; 10_2 \right\rangle = \sqrt{\frac{2}{3}} |\Delta, \rho\rangle - \sqrt{\frac{1}{6}} |\Delta, \omega_1\rangle + \sqrt{\frac{1}{24}} |\Sigma, \rho\rangle + \sqrt{\frac{1}{8}} |\Sigma, \pi\rangle \]
\[\left| 700; 10_2 \right\rangle = \sqrt{\frac{1}{6}} |\Delta, \rho\rangle + \sqrt{\frac{1}{6}} |\Delta, \omega_1\rangle + \sqrt{\frac{1}{24}} |\Sigma, \rho\rangle - \sqrt{\frac{1}{2}} |\Sigma, \pi\rangle \]
\[\left| 1134; 10_{a,2} \right\rangle = \sqrt{\frac{1}{6}} |\Delta, \rho\rangle + \sqrt{\frac{1}{6}} |\Delta, \omega_1\rangle - \sqrt{\frac{2}{3}} |\Sigma, \rho\rangle \]
\[\left| 1134; 10_{a,2} \right\rangle = \sqrt{\frac{1}{2}} |\Delta, \omega_1\rangle + \sqrt{\frac{7}{8}} |\Sigma, \rho\rangle + \sqrt{\frac{3}{8}} |\Sigma, \pi\rangle \]  \( \text{(B21)} \)

\[\left| 56; 10_4 \right\rangle = \sqrt{\frac{4}{9}} |\Delta, \rho\rangle - \sqrt{\frac{1}{9}} |\Delta, \omega_1\rangle - \sqrt{\frac{4}{45}} |\Delta, \pi\rangle + \sqrt{\frac{8}{45}} |\Sigma, \rho\rangle \]
\[\left| 700; 10_4 \right\rangle = \sqrt{\frac{1}{72}} |\Delta, \rho\rangle + \sqrt{\frac{2}{9}} |\Delta, \omega_1\rangle + \sqrt{\frac{5}{24}} |\Delta, \pi\rangle + \sqrt{\frac{5}{9}} |\Sigma, \rho\rangle \]
\[\left| 1134; 10_{a,4} \right\rangle = \sqrt{\frac{13}{24}} |\Delta, \rho\rangle + \sqrt{\frac{2}{39}} |\Delta, \omega_1\rangle + \sqrt{\frac{81}{520}} |\Delta, \pi\rangle - \sqrt{\frac{49}{195}} |\Sigma, \rho\rangle \]
\[\left| 1134; 10_{a,4} \right\rangle = \sqrt{\frac{8}{13}} |\Delta, \omega_1\rangle - \sqrt{\frac{24}{65}} |\Delta, \pi\rangle - \sqrt{\frac{1}{65}} |\Sigma, \rho\rangle \]  \( \text{(B22)} \)

\[\left| 700; 10_6 \right\rangle = \sqrt{\frac{1}{2}} |\Delta, \rho\rangle - \sqrt{\frac{1}{2}} |\Delta, \omega_1\rangle \]
\[\left| 1134; 10_6 \right\rangle = \sqrt{\frac{1}{2}} |\Delta, \rho\rangle + \sqrt{\frac{1}{2}} |\Delta, \omega_1\rangle \]  \( \text{(B23)} \)

\[\left| 700; 10^5_2 \right\rangle = \sqrt{\frac{3}{4}} |\Sigma, \rho\rangle + \sqrt{\frac{1}{4}} |\Sigma, \pi\rangle \]
\[\left| 1134; 10^5_2 \right\rangle = \sqrt{\frac{7}{4}} |\Sigma, \rho\rangle - \sqrt{\frac{3}{4}} |\Sigma, \pi\rangle \]  \( \text{(B24)} \)

\[\left| 1134; 10^4_4 \right\rangle = |\Sigma, \rho\rangle \]  \( \text{(B25)} \)

\[\left| 700; 27_2 \right\rangle = \sqrt{\frac{1}{3}} |\Delta, \rho\rangle - \sqrt{\frac{1}{6}} |\Sigma, \rho\rangle + \sqrt{\frac{1}{2}} |\Sigma, \pi\rangle \]
\[\left| 1134; 27_{s,2} \right\rangle = \sqrt{\frac{2}{3}} |\Delta, \rho\rangle + \sqrt{\frac{1}{12}} |\Sigma, \rho\rangle - \sqrt{\frac{1}{4}} |\Sigma, \pi\rangle \]
\[\left| 1134; 27_{a,2} \right\rangle = \sqrt{\frac{3}{4}} |\Sigma, \rho\rangle + \sqrt{\frac{1}{4}} |\Sigma, \pi\rangle \]  \( \text{(B26)} \)

\[\left| 700; 27_4 \right\rangle = \sqrt{\frac{5}{24}} |\Delta, \rho\rangle + \sqrt{\frac{1}{8}} |\Delta, \pi\rangle - \sqrt{\frac{2}{3}} |\Sigma, \rho\rangle \]
\[\left| 1134; 27_{s,4} \right\rangle = \sqrt{\frac{19}{24}} |\Delta, \rho\rangle - \sqrt{\frac{5}{152}} |\Delta, \pi\rangle + \sqrt{\frac{10}{57}} |\Sigma, \rho\rangle \]
\[\left| 1134; 27_{a,4} \right\rangle = \sqrt{\frac{16}{19}} |\Delta, \pi\rangle + \sqrt{\frac{3}{19}} |\Sigma, \rho\rangle \]  \( \text{(B27)} \)

\[\left| 1134; 27_6 \right\rangle = |\Delta, \rho\rangle \]  \( \text{(B28)} \)

\[\left| 1134; 35_2 \right\rangle = |\Delta, \rho\rangle \]  \( \text{(B29)} \)
\[( - ) \quad |700; 35_4\rangle = \sqrt{\frac{3}{8}}|\Delta, \rho\rangle - \sqrt{\frac{5}{8}}|\Delta, \pi\rangle \]
\[( - ) \quad |1134; 35_4\rangle = \sqrt{\frac{5}{8}}|\Delta, \rho\rangle + \sqrt{\frac{3}{8}}|\Delta, \pi\rangle \quad \text{ (B30)} \]
\[( + ) \quad |700; 35_b\rangle = |\Delta, \rho\rangle \quad \text{ (B31)} \]

Appendix C: Tables of scalar factors of SU(4)

\[|\pi\rangle = |15; 8, 0\rangle, \quad |D\rangle = |15; 3^*, 1\rangle, \quad |\bar{D}\rangle = |15; 3, -1\rangle, \quad |\eta_c\rangle = |15; 1, 0\rangle, \]
\[|\Sigma\rangle = |20; 8, 0\rangle, \quad |\Sigma_c\rangle = |20; 6, 1\rangle, \quad |\Xi_c\rangle = |20; 3^*, 1\rangle, \quad |\Xi_{cc}\rangle = |20; 3, 2\rangle, \quad |\Delta\rangle = |20^c; 10, 0\rangle, \quad |\Delta_c\rangle = |20^c; 6, 1\rangle, \quad |\Xi^c_{cc}\rangle = |20^c; 3, 2\rangle, \quad |\Omega_{ccc}\rangle = |20^c; 1, 3\rangle. \]

1. SU(4): 15 \otimes 15

\[\begin{align*}
(+ ) & \quad |1; 1, 0\rangle = \sqrt{\frac{8}{15}}|\pi, \pi\rangle - \sqrt{\frac{2}{3}}|D, \bar{D}\rangle_A - \sqrt{\frac{1}{15}}|\eta_c, \eta_c\rangle \\
(+ ) & \quad |15^a; 1, 0\rangle = \sqrt{\frac{4}{9}}|\pi, \pi\rangle + \sqrt{\frac{1}{3}}|D, \bar{D}\rangle_A + \sqrt{\frac{2}{9}}|\eta_c, \eta_c\rangle \\
(- ) & \quad |15^a; 1, 0\rangle = |D, \bar{D}\rangle_A \\
(+ ) & \quad |84; 1, 0\rangle = -\sqrt{\frac{1}{45}}|\pi, \pi\rangle - \sqrt{\frac{4}{15}}|D, \bar{D}\rangle_A + \sqrt{\frac{32}{45}}|\eta_c, \eta_c\rangle \quad \text{ (C1)} \\
(+ ) & \quad |15^a; 3, -1\rangle = \sqrt{\frac{8}{9}}|\pi, \bar{D}\rangle_A + \sqrt{\frac{1}{9}}|\bar{D}, \eta_c\rangle_A \\
(- ) & \quad |15^a; 3, -1\rangle = \sqrt{\frac{2}{3}}|\pi, \bar{D}\rangle_A + \sqrt{\frac{1}{3}}|\bar{D}, \eta_c\rangle_A \\
(- ) & \quad |45^*; 3, -1\rangle = -\sqrt{\frac{1}{3}}|\pi, \bar{D}\rangle_A + \sqrt{\frac{2}{3}}|\bar{D}, \eta_c\rangle_A \\
(+ ) & \quad |84; 3, -1\rangle = -\sqrt{\frac{1}{9}}|\pi, \bar{D}\rangle_A + \sqrt{\frac{8}{9}}|\bar{D}, \eta_c\rangle_A \quad \text{ (C2)} \\
(- ) & \quad |45; 3, 2\rangle = |D, D\rangle \quad \text{ (C3)} \\
(- ) & \quad |45^*; 3^*, -2\rangle = |\bar{D}, \bar{D}\rangle \\ 
(+ ) & \quad |15^a; 3^*, 1\rangle = -\sqrt{\frac{8}{9}}|\pi, D\rangle_A + \sqrt{\frac{1}{9}}|D, \eta_c\rangle_A \\
(- ) & \quad |15^a; 3^*, 1\rangle = \sqrt{\frac{2}{3}}|\pi, D\rangle_A - \sqrt{\frac{1}{3}}|D, \eta_c\rangle_A \\
(- ) & \quad |45; 3^*, 1\rangle = \sqrt{\frac{1}{3}}|\pi, D\rangle_A + \sqrt{\frac{2}{3}}|D, \eta_c\rangle_A \\
(+ ) & \quad |84; 3^*, 1\rangle = \sqrt{\frac{1}{9}}|\pi, D\rangle_A + \sqrt{\frac{8}{9}}|D, \eta_c\rangle_A \quad \text{ (C5)}
\end{align*} \]
\[
\begin{align*}
(+) & \quad |84; 6, -2\rangle = |\bar{D}, \bar{D}\rangle \\
(+) & \quad |20'; 6, 1\rangle = |\pi, D\rangle_A \\
(-) & \quad |45; 6, 1\rangle = |\pi, \bar{D}\rangle \\
(+) & \quad |20'; 6^*, -1\rangle = |\pi, \bar{D}\rangle_A \\
(-) & \quad |45^*; 6, -1\rangle = |\pi, \bar{D}\rangle \\
(+) & \quad |84; 6^*, 2\rangle = |D, D\rangle \\
(+) & \quad |15_a; 8, 0\rangle = \sqrt{\frac{5}{7}}|\pi, \pi\rangle_s - \sqrt{\frac{1}{7}}|\pi, \eta_c\rangle_s + \sqrt{\frac{1}{3}}|D, \bar{D}\rangle_S \\
(-) & \quad |15_a; 8, 0\rangle = \sqrt{\frac{3}{4}}|\pi, \pi\rangle_a - \sqrt{\frac{1}{4}}|D, \bar{D}\rangle_A \\
(+) & \quad |20'; 8, 0\rangle = \sqrt{\frac{5}{12}}|\pi, \pi\rangle_s + \sqrt{\frac{1}{3}}|\pi, \eta_c\rangle_s - \sqrt{\frac{1}{4}}|D, \bar{D}\rangle_S \\
(-) & \quad |45; 8, 0\rangle = \sqrt{\frac{1}{8}}|\pi, \pi\rangle_a + \sqrt{\frac{1}{2}}|\pi, \eta_c\rangle_A + \sqrt{\frac{3}{8}}|D, \bar{D}\rangle_A \\
(-) & \quad |45^*; 8, 0\rangle = -\sqrt{\frac{1}{8}}|\pi, \pi\rangle_a + \sqrt{\frac{1}{2}}|\pi, \eta_c\rangle_A - \sqrt{\frac{3}{8}}|D, \bar{D}\rangle_A \\
(+) & \quad |84; 8, 0\rangle = -\sqrt{\frac{1}{36}}|\pi, \pi\rangle_s + \sqrt{\frac{5}{9}}|\pi, \eta_c\rangle_s + \sqrt{\frac{5}{12}}|D, \bar{D}\rangle_S \\
(-) & \quad |45; 10, 0\rangle = |\pi, \pi\rangle \\
(-) & \quad |45^*; 10^*, 0\rangle = |\pi, \pi\rangle \\
(-) & \quad |45; 15, -1\rangle = |\pi, \bar{D}\rangle_A \\
(+) & \quad |84; 15, -1\rangle = |\pi, \bar{D}\rangle \\
(-) & \quad |45^*; 15^*, 1\rangle = |\pi, D\rangle_A \\
(+) & \quad |84; 15^*, 1\rangle = |\pi, D\rangle \\
(+) & \quad |84; 27, 0\rangle = |\pi, \pi\rangle \\

\end{align*}
\]

2. SU(4): 20 ⊗ 15

\[
\begin{align*}
(-) & \quad |4^*; 1, 0\rangle = -\sqrt{\frac{1}{5}}|\Sigma, \pi\rangle + \sqrt{\frac{1}{5}}|\Xi_c, \bar{D}\rangle \\
(+) & \quad |36^*; 1, 0\rangle = -\sqrt{\frac{1}{5}}|\Sigma, \pi\rangle - \sqrt{\frac{1}{5}}|\Xi_c, \bar{D}\rangle \\
(-) & \quad |20'; 1, 3\rangle = -|\Xi_{cc}, D\rangle \\
(+) & \quad |36^*; 3, -1\rangle = -|\Sigma, \bar{D}\rangle \\
\end{align*}
\]
\((+)\) \[ |20_a; 3, 2\rangle = -\sqrt{\frac{121}{234}} (\Sigma_c, D) + \sqrt{\frac{1}{39}} (\Xi_c, D) + \sqrt{\frac{289}{702}} |\Xi_{cc}, \pi\rangle + \sqrt{\frac{16}{351}} |\Xi_{cc}, \eta_c\rangle \]

\((-)\) \[ |20_a; 3, 2\rangle = \sqrt{\frac{8}{39}} (\Sigma_c, D) + \sqrt{\frac{4}{13}} (\Xi_c, D) + \sqrt{\frac{32}{117}} |\Xi_{cc}, \pi\rangle - \sqrt{\frac{25}{117}} |\Xi_{cc}, \eta_c\rangle \]

\((-)\) \[ |20'; 3, 2\rangle = -\sqrt{\frac{2}{9}} (\Sigma_c, D) + \sqrt{\frac{1}{3}} (\Xi_c, D) - \sqrt{\frac{8}{27}} |\Xi_{cc}, \pi\rangle - \sqrt{\frac{4}{27}} |\Xi_{cc}, \eta_c\rangle \]

\((+)\) \[ |140; 3, 2\rangle = \sqrt{\frac{1}{18}} (\Sigma_c, D) + \sqrt{\frac{1}{3}} (\Xi_c, D) - \sqrt{\frac{1}{54}} |\Xi_{cc}, \pi\rangle + \sqrt{\frac{16}{27}} |\Xi_{cc}, \eta_c\rangle \] (C19)

\((-)\) \[ |4^*; 3, 1\rangle = -\sqrt{\frac{4}{15}} (\Sigma, D) - \sqrt{\frac{2}{5}} (\Sigma_c, \pi) - \sqrt{\frac{2}{45}} (\Xi_c, \pi) - \sqrt{\frac{4}{45}} |\Xi_{cc}, \eta_c\rangle + \sqrt{\frac{1}{5}} |\Xi_{cc}, \bar{D}\rangle \]

\((+)\) \[ |20_a; 3^*, 1\rangle = -\sqrt{\frac{9}{52}} (\Sigma, D) - \sqrt{\frac{13}{24}} (\Sigma_c, \pi) + \sqrt{\frac{49}{312}} (\Xi_c, \pi) + \sqrt{\frac{4}{39}} |\Xi_{cc}, \eta_c\rangle - \sqrt{\frac{1}{39}} |\Xi_{cc}, \bar{D}\rangle \]

\((-)\) \[ |20_a; 3^*, 1\rangle = \sqrt{\frac{16}{39}} (\Sigma, D) + \sqrt{\frac{32}{117}} (\Xi_c, \pi) - \sqrt{\frac{4}{45}} |\Xi_{cc}, \eta_c\rangle - \sqrt{\frac{1}{13}} |\Xi_{cc}, \bar{D}\rangle \]

\((+)\) \[ |36^*; 3, 1\rangle = \sqrt{\frac{1}{40}} (\Sigma, D) - \sqrt{\frac{3}{80}} (\Sigma_c, \pi) - \sqrt{\frac{121}{240}} (\Xi_c, \pi) + \sqrt{\frac{2}{15}} |\Xi_{cc}, \eta_c\rangle - \sqrt{\frac{3}{10}} |\Xi_{cc}, \bar{D}\rangle \]

\((+)\) \[ |140; 3^*, 1\rangle = \sqrt{\frac{1}{8}} (\Sigma, D) + \sqrt{\frac{1}{48}} (\Sigma_c, \pi) + \sqrt{\frac{2}{45}} (\Xi_c, \pi) + \sqrt{\frac{1}{15}} |\Xi_{cc}, \eta_c\rangle + \sqrt{\frac{1}{6}} |\Xi_{cc}, \bar{D}\rangle \] (C20)

\((+)\) \[ |20_a; 6, 1\rangle = \sqrt{\frac{289}{936}} (\Sigma, D) - \sqrt{\frac{125}{5016}} (\Sigma_c, \pi) - \sqrt{\frac{49}{351}} (\Xi_c, \eta_c) - \sqrt{\frac{13}{48}} |\Xi_c, \eta_c\rangle + \sqrt{\frac{121}{468}} |\Xi_{cc}, \bar{D}\rangle \]

\((-)\) \[ |20_a; 6, 1\rangle = \sqrt{\frac{8}{39}} (\Sigma, D) + \sqrt{\frac{80}{117}} (\Sigma_c, \pi) - \sqrt{\frac{4}{45}} |\Xi_{cc}, \eta_c\rangle - \sqrt{\frac{1}{5}} |\Xi_{cc}, \bar{D}\rangle \]

\((-)\) \[ |20'; 6, 1\rangle = -\sqrt{\frac{2}{9}} (\Sigma, D) - \sqrt{\frac{5}{27}} (\Sigma_c, \pi) + \sqrt{\frac{1}{3}} (\Xi_c, \eta_c) + \sqrt{\frac{1}{45}} |\Xi_{cc}, \eta_c\rangle - \sqrt{\frac{1}{9}} |\Xi_{cc}, \bar{D}\rangle \]

\((-)\) \[ |60^*; 6, 1\rangle = \sqrt{\frac{1}{8}} (\Sigma, D) - \sqrt{\frac{5}{48}} (\Sigma_c, \pi) + \sqrt{\frac{1}{4}} (\Xi_c, \eta_c) - \sqrt{\frac{3}{16}} |\Xi_c, \eta_c\rangle - \sqrt{\frac{1}{4}} |\Xi_{cc}, \bar{D}\rangle \]

\((+)\) \[ |140; 6, 1\rangle = \sqrt{\frac{5}{36}} (\Sigma, D) + \sqrt{\frac{1}{210}} (\Sigma_c, \pi) + \sqrt{\frac{10}{27}} (\Xi_c, \eta_c) + \sqrt{\frac{5}{24}} |\Xi_{cc}, \eta_c\rangle + \sqrt{\frac{5}{18}} |\Xi_{cc}, \bar{D}\rangle \] (C21)

\((-)\) \[ |60^*; 6^*, -1\rangle = (\Sigma, \bar{D}) \] (C22)

\((+)\) \[ |36^*; 6^*, 2\rangle = \sqrt{\frac{7}{4}} (\Xi_c, D) - \sqrt{\frac{3}{4}} |\Xi_{cc}, \pi\rangle \]

\((+)\) \[ |140; 6^*, 2\rangle = \sqrt{\frac{3}{4}} (\Xi_c, D) + \sqrt{\frac{1}{4}} |\Xi_{cc}, \pi\rangle \] (C23)

\((+)\) \[ |20_a; 8, 0\rangle = \sqrt{\frac{65}{96}} (\Sigma, \pi)_s + \sqrt{\frac{25}{1248}} (\Sigma, \pi)_a + \sqrt{\frac{1}{156}} (\Sigma, \eta_c) - \sqrt{\frac{289}{1248}} |\Xi_c, \bar{D}\rangle + \sqrt{\frac{27}{416}} |\Xi_c, \bar{D}\rangle \]

\((-)\) \[ |20_a; 8, 0\rangle = \sqrt{\frac{8}{13}} (\Sigma, \pi)_a + \sqrt{\frac{1}{13}} (\Sigma, \eta_c) - \sqrt{\frac{2}{13}} (\Xi_c, \bar{D}) - \sqrt{\frac{1}{13}} |\Xi_c, \bar{D}\rangle \]

\((+)\) \[ |36^*; 8, 0\rangle = -\sqrt{\frac{5}{32}} (\Sigma, \pi)_s - \sqrt{\frac{9}{32}} (\Sigma, \pi)_a + \sqrt{\frac{1}{4}} (\Sigma, \eta_c) - \sqrt{\frac{9}{32}} |\Xi_c, \bar{D}\rangle - \sqrt{\frac{1}{32}} |\Xi_c, \bar{D}\rangle \]

\((-)\) \[ |60^*; 8, 0\rangle = \sqrt{\frac{5}{32}} (\Sigma, \pi)_a - \sqrt{\frac{1}{32}} (\Sigma, \pi)_s + \sqrt{\frac{1}{4}} (\Sigma, \eta_c) + \sqrt{\frac{9}{32}} |\Xi_c, \bar{D}\rangle - \sqrt{\frac{9}{32}} |\Xi_c, \bar{D}\rangle \]

\((+)\) \[ |140; 8, 0\rangle = -\sqrt{\frac{1}{96}} (\Sigma, \pi)_s + \sqrt{\frac{5}{96}} (\Sigma, \pi)_a + \sqrt{\frac{5}{12}} |\Xi_c, \eta_c\rangle + \sqrt{\frac{5}{96}} |\Xi_c, \bar{D}\rangle + \sqrt{\frac{15}{32}} |\Xi_c, \bar{D}\rangle \] (C24)
\begin{align}
(+) \ |140; 8, 3\rangle &= |\Xi_{cc}, D\rangle & (C25) \\
(-) \ |20'; 10, 0\rangle &= \sqrt{\frac{7}{3}}|\Sigma, \pi\rangle - \sqrt{\frac{2}{3}}|\Sigma_{\bar{c}}, \bar{D}\rangle \\
(+) \ |140; 10, 0\rangle &= \sqrt{\frac{1}{3}}|\Sigma, \pi\rangle + \sqrt{\frac{2}{3}}|\Sigma_{\bar{c}}, \bar{D}\rangle & (C26) \\
(-) \ |60^*; 10^*, 0\rangle &= |\Sigma, \pi\rangle & (C27) \\
(+) \ |140; 15, -1\rangle &= |\Sigma, \bar{D}\rangle & (C28) \\
(-) \ |60^*; 15, 2\rangle &= \sqrt{\frac{7}{2}}|\Sigma_{\bar{c}}, D\rangle - \sqrt{\frac{1}{2}}|\Xi_{cc}, \pi\rangle \\
(+) \ |140; 15, 2\rangle &= \sqrt{\frac{7}{2}}|\Sigma_{\bar{c}}, D\rangle + \sqrt{\frac{1}{2}}|\Xi_{cc}, \pi\rangle & (C29) \\
(+) \ |36^*; 15^*, 1\rangle &= \sqrt{\frac{3}{8}}|\Sigma, D\rangle - \sqrt{\frac{9}{16}}|\Sigma_{\bar{c}}, \pi\rangle - \sqrt{\frac{1}{16}}|\Xi_{cc}, \pi\rangle \\
(-) \ |60^*; 15^*, 1\rangle &= \sqrt{\frac{1}{4}}|\Sigma, D\rangle + \sqrt{\frac{3}{8}}|\Sigma_{\bar{c}}, \pi\rangle - \sqrt{\frac{3}{8}}|\Xi_{cc}, \pi\rangle \\
(+) \ |140; 15^*, 1\rangle &= \sqrt{\frac{3}{8}}|\Sigma, D\rangle + \sqrt{\frac{1}{16}}|\Sigma_{\bar{c}}, \pi\rangle + \sqrt{\frac{9}{16}}|\Xi_{cc}, \pi\rangle & (C30) \\
(+) \ |140; 24^*, 1\rangle &= |\Sigma_{cc}, \pi\rangle & (C31) \\
(+) \ |140; 27, 0\rangle &= |\Sigma, \pi\rangle & (C32) \\
\end{align}

3. SU(4): $20' \otimes 15$

\begin{align}
(-) \ |20'; 1, 3\rangle &= \sqrt{\frac{4}{7}}|\Xi_{cc}^*, D\rangle - \sqrt{\frac{3}{7}}|\Omega_{cc}, \eta_{\bar{c}}\rangle \\
(+) \ |120; 1, 3\rangle &= \sqrt{\frac{3}{7}}|\Xi_{cc}^*, D\rangle + \sqrt{\frac{4}{7}}|\Omega_{cc}, \eta_{\bar{c}}\rangle & (C33) \\
(+) \ |20; 3, 2\rangle &= \sqrt{\frac{2}{9}}|\Sigma_{\bar{c}}, D\rangle - \sqrt{\frac{8}{27}}|\Xi_{cc}, \pi\rangle - \sqrt{\frac{4}{27}}|\Xi_{cc}, \eta_{\bar{c}}\rangle + \sqrt{\frac{1}{3}}|\Omega_{cc}, \bar{D}\rangle \\
(-) \ |20'; 3, 2\rangle &= \sqrt{\frac{32}{63}}|\Sigma_{\bar{c}}^*, D\rangle + \sqrt{\frac{32}{189}}|\Xi_{cc}^*, \pi\rangle - \sqrt{\frac{25}{189}}|\Xi_{cc}^*, \eta_{\bar{c}}\rangle - \sqrt{\frac{4}{21}}|\Omega_{cc}, \bar{D}\rangle \\
(+) \ |120; 3, 2\rangle &= \sqrt{\frac{3}{14}}|\Sigma_{\bar{c}}^*, D\rangle + \sqrt{\frac{1}{14}}|\Xi_{cc}^*, \pi\rangle + \sqrt{\frac{4}{7}}|\Xi_{cc}, \eta_{\bar{c}}\rangle + \sqrt{\frac{1}{7}}|\Omega_{cc}, \bar{D}\rangle \\
(-) \ |140; 3, 2\rangle &= \sqrt{\frac{1}{18}}|\Sigma_{\bar{c}}, D\rangle - \sqrt{\frac{25}{54}}|\Xi_{cc}, \pi\rangle + \sqrt{\frac{4}{27}}|\Xi_{cc}, \eta_{\bar{c}}\rangle - \sqrt{\frac{1}{3}}|\Omega_{cc}, \bar{D}\rangle & (C34) \\
(+) \ |20; 3^*, 1\rangle &= \sqrt{\frac{7}{3}}|\Sigma_{\bar{c}}, \pi\rangle - \sqrt{\frac{1}{3}}|\Xi_{cc}, \bar{D}\rangle \\
(-) \ |140; 3^*, 1\rangle &= \sqrt{\frac{7}{3}}|\Sigma_{\bar{c}}, \pi\rangle + \sqrt{\frac{2}{3}}|\Xi_{cc}, \bar{D}\rangle & (C35)
\end{align}
\[<20; 3^*, 4> = \Omega_{ccc}, D\]  \hfill (C36)

\[<20; 6, 1> = \sqrt{\frac{5}{9}}(\Delta, D) - \sqrt{\frac{5}{27}}(\Sigma_c^*, \pi) - \sqrt{\frac{4}{27}}(\Sigma_c^*, \eta_c) + \sqrt{\frac{1}{9}} \Omega_{ccc}, \bar{D}\]  \hfill (C37)

\[<20; 6^*, 1> = \sqrt{\frac{20}{63}}(\Delta, D) + \sqrt{\frac{80}{189}}(\Sigma_c^*, \pi) - \sqrt{\frac{1}{189}}(\Sigma_c^*, \eta_c) - \sqrt{\frac{16}{63}} \Omega_{ccc}, \bar{D}\]  \hfill (C38)

\[<20; 8, 0> = \sqrt{\frac{5}{6}}(\Delta, \pi) - \sqrt{\frac{7}{6}}(\Sigma_c^*, \bar{D})\]  \hfill (C39)

\[<20; 8, 0> = \sqrt{\frac{5}{6}}(\Delta, \pi) + \sqrt{\frac{7}{6}}(\Sigma_c^*, \bar{D})\]  \hfill (C40)

\[<20; 8, 0> = \sqrt{\frac{16}{21}}(\Delta, \pi) + \sqrt{\frac{1}{21}}(\Delta, \eta_c) - \sqrt{\frac{4}{21}}(\Sigma_c^*, \bar{D})\]  \hfill (C41)

\[<20; 10, 0> = \sqrt{\frac{16}{21}}(\Delta, \pi) + \sqrt{\frac{1}{21}}(\Delta, \eta_c) + \sqrt{\frac{9}{21}}(\Sigma_c^*, \bar{D})\]  \hfill (C42)

\[<20; 10, 0> = \sqrt{\frac{16}{21}}(\Delta, \pi) - \sqrt{\frac{1}{21}}(\Delta, \eta_c) - \sqrt{\frac{9}{21}}(\Sigma_c^*, \bar{D})\]  \hfill (C43)

\[<20; 10, 0> = -\sqrt{\frac{1}{6}}(\Delta, \pi) + \sqrt{\frac{2}{5}}(\Delta, \eta_c) - \sqrt{\frac{7}{6}}(\Sigma_c^*, \bar{D})\]  \hfill (C44)

\[<140; 3^*, 3> = \sqrt{\frac{1}{4}}(\Sigma_c^*, \bar{D}) + \sqrt{\frac{3}{4}}(\Sigma_c^*, \pi)\]  \hfill (C45)

\[<140; 5^*, 1> = |\Sigma_c^*, \pi\rangle\]  \hfill (C46)

\[<140; 27, 0> = |\Delta, \pi\rangle\]  \hfill (C47)

\[<140; 27, 0> = |\Delta, \pi\rangle\]  \hfill (C48)
Appendix D: Tables of scalar factors of SU(3)

1. SU(3): $3 \otimes 3$

\begin{align*}
(+) \quad |6; 0, -\frac{4}{3} \rangle &= |\bar{D}_s, \bar{D}\rangle \\
(--) \quad |3^*; 0, \frac{4}{3} \rangle &= |\bar{D}, \bar{D}\rangle \\
(--) \quad |3^*; \frac{1}{2}, -\frac{1}{3} \rangle &= |\bar{D}, \bar{D}_s\rangle_A \\
(+) \quad |6; \frac{1}{2}, -\frac{1}{3} \rangle &= |\bar{D}, \bar{D}_s\rangle_S \\
(+) \quad |6; 1, \frac{2}{3} \rangle &= |\bar{D}, \bar{D}\rangle
\end{align*}

2. SU(3): $3 \otimes 3^*$

\begin{align*}
(--) \quad |1; 0, 0 \rangle &= \sqrt{\frac{2}{3}}|\bar{D}, D\rangle + \sqrt{\frac{1}{3}}|\bar{D}_s, D_s\rangle \\
(+) \quad |8; 0, 0 \rangle &= -\sqrt{\frac{1}{3}}|\bar{D}, D\rangle + \sqrt{\frac{2}{3}}|\bar{D}_s, D_s\rangle \\
(+) \quad |8; \frac{1}{2}, -1 \rangle &= |\bar{D}_s, D\rangle \\
(+) \quad |8; \frac{1}{2}, 1 \rangle &= |\bar{D}, D_s\rangle \\
(+) \quad |8; 1, 0 \rangle &= |\bar{D}, D\rangle
\end{align*}

3. SU(3): $3^* \otimes 3^*$

\begin{align*}
(--) \quad |3^*; 0, -\frac{2}{3} \rangle &= -|D, D\rangle \\
(+) \quad |6^*; 0, \frac{4}{3} \rangle &= |D_s, D_s\rangle \\
(--) \quad |3^*; \frac{1}{2}, \frac{1}{3} \rangle &= |D, D_s\rangle_A \\
(+) \quad |6^*; \frac{1}{2}, \frac{1}{3} \rangle &= |D, D_s\rangle_S \\
(+) \quad |6^*; 1, -\frac{2}{3} \rangle &= |D, D\rangle
\end{align*}
4. SU(3): $6 \otimes 3$

\[
(+) \quad |10; 0, -2\rangle = |\Omega_c, \bar{D}_s\rangle \quad (D13)
\]

\[
(-) \quad |8; 0, 0\rangle = |\Xi'_c, \bar{D}\rangle \quad (D14)
\]

\[
(-) \quad |8; \frac{1}{2}, -1\rangle = \sqrt{\frac{1}{3}}|\Xi'_c, \bar{D}_s\rangle - \sqrt{\frac{2}{3}}|\Omega_c, \bar{D}\rangle
\]
\[
(+) \quad |10; \frac{1}{2}, -1\rangle = \sqrt{\frac{2}{3}}|\Xi'_c, \bar{D}_s\rangle + \sqrt{\frac{1}{3}}|\Omega_c, \bar{D}\rangle \quad (D15)
\]

\[
(-) \quad |8; \frac{1}{2}, 1\rangle = |\Sigma_c, \bar{D}\rangle \quad (D16)
\]

\[
(-) \quad |8; 1, 0\rangle = \sqrt{\frac{2}{3}}|\Sigma_c, \bar{D}_s\rangle - \sqrt{\frac{1}{3}}|\Xi'_c, \bar{D}\rangle
\]
\[
(+) \quad |10; 1, 0\rangle = \sqrt{\frac{1}{3}}|\Sigma_c, \bar{D}_s\rangle + \sqrt{\frac{2}{3}}|\Xi'_c, \bar{D}\rangle \quad (D17)
\]

\[
(+) \quad |10; \frac{3}{2}, 1\rangle = |\Sigma_c, \bar{D}\rangle \quad (D18)
\]

5. SU(3): $6 \otimes 3^*$

\[
(-) \quad |3; 0, -\frac{2}{3}\rangle = \sqrt{\frac{1}{2}}|\Xi'_c, D\rangle + \sqrt{\frac{1}{2}}|\Omega_c, D_s\rangle
\]
\[
(+) \quad |15; 0, -\frac{2}{3}\rangle = -\sqrt{\frac{1}{2}}|\Xi'_c, D\rangle + \sqrt{\frac{1}{2}}|\Omega_c, D_s\rangle \quad (D19)
\]

\[
(+) \quad |15; \frac{1}{2}, -\frac{5}{3}\rangle = |\Omega_c, D\rangle \quad (D20)
\]

\[
(-) \quad |3; \frac{1}{2}, \frac{1}{3}\rangle = \sqrt{\frac{3}{4}}|\Sigma_c, D\rangle + \sqrt{\frac{1}{4}}|\Xi'_c, D_s\rangle
\]
\[
(+) \quad |15; \frac{1}{2}, \frac{1}{3}\rangle = -\sqrt{\frac{1}{4}}|\Sigma_c, D\rangle + \sqrt{\frac{3}{4}}|\Xi'_c, D_s\rangle \quad (D21)
\]

\[
(+) \quad |15; 1, -\frac{2}{3}\rangle = |\Xi'_c, D\rangle \quad (D22)
\]

\[
(+) \quad |15; 1, \frac{1}{3}\rangle = |\Sigma_c, D_s\rangle \quad (D23)
\]

\[
(+) \quad |15; \frac{2}{2}, \frac{1}{3}\rangle = |\Sigma_c, D\rangle \quad (D24)
\]
6. $\text{SU}(3)$: $6 \otimes 8$

\[\begin{align*}
(-) \hspace{1em} |6; 0, -\frac{1}{3}\rangle &= \sqrt{\frac{3}{5}}|\Xi'_c, \bar{K}\rangle + \sqrt{\frac{2}{5}}|\Omega_c, \eta\rangle \\
(+) \hspace{1em} |24^*; 0, -\frac{4}{3}\rangle &= -\sqrt{\frac{2}{5}}|\Xi'_c, \bar{K}\rangle + \sqrt{\frac{3}{5}}|\Omega_c, \eta\rangle \\
(+) \hspace{1em} |3^*; 0, \frac{2}{3}\rangle &= \sqrt{\frac{3}{4}}|\Sigma_c, \pi\rangle + \sqrt{\frac{1}{4}}|\Xi'_c, K\rangle \\
(-) \hspace{1em} |15^*; 0, \frac{2}{3}\rangle &= -\sqrt{\frac{1}{4}}|\Sigma_c, \pi\rangle + \sqrt{\frac{3}{4}}|\Xi'_c, K\rangle \\
(+) \hspace{1em} |24^*; \frac{1}{2}, -\frac{7}{3}\rangle &= |\Omega_c, \bar{K}\rangle \\
(-) \hspace{1em} |3^*; \frac{1}{2}, -\frac{1}{3}\rangle &= \sqrt{\frac{2}{3}}|\Sigma_c, \bar{K}\rangle - \sqrt{\frac{3}{10}}|\Xi'_c, \bar{K}\rangle + \sqrt{\frac{3}{10}}|\Xi'_c, \eta\rangle - \sqrt{\frac{1}{4}}|\Omega_c, K\rangle \\
(-) \hspace{1em} |6; \frac{1}{3}, -\frac{4}{3}\rangle &= \sqrt{\frac{9}{20}}|\Sigma_c, \bar{K}\rangle + \sqrt{\frac{9}{40}}|\Xi'_c, \pi\rangle + \sqrt{\frac{1}{40}}|\Xi'_c, \eta\rangle + \sqrt{\frac{3}{10}}|\Omega_c, K\rangle \\
(-) \hspace{1em} |15^*; \frac{1}{2}, -\frac{4}{3}\rangle &= -\sqrt{\frac{1}{24}}|\Sigma_c, \bar{K}\rangle + \sqrt{\frac{25}{48}}|\Xi'_c, \pi\rangle + \sqrt{\frac{3}{16}}|\Xi'_c, \eta\rangle - \sqrt{\frac{1}{4}}|\Omega_c, K\rangle \\
(+) \hspace{1em} |24^*; \frac{1}{2}, -\frac{1}{3}\rangle &= -\sqrt{\frac{2}{15}}|\Sigma_c, \bar{K}\rangle - \sqrt{\frac{1}{15}}|\Xi'_c, \pi\rangle + \sqrt{\frac{3}{5}}|\Xi'_c, \eta\rangle + \sqrt{\frac{1}{5}}|\Omega_c, K\rangle \\
(-) \hspace{1em} |15^*; \frac{1}{3}, \frac{5}{3}\rangle &= |\Sigma_c, K\rangle \\
(-) \hspace{1em} |15^*; 1, -\frac{4}{3}\rangle &= \sqrt{\frac{1}{3}}|\Xi'_c, \bar{K}\rangle - \sqrt{\frac{2}{3}}|\Omega_c, \pi\rangle \\
(+) \hspace{1em} |24^*; 1, -\frac{4}{3}\rangle &= \sqrt{\frac{2}{3}}|\Xi'_c, \pi\rangle + \sqrt{\frac{1}{3}}|\Omega_c, \pi\rangle \\
(-) \hspace{1em} |6; 1, \frac{2}{3}\rangle &= \sqrt{\frac{3}{5}}|\Sigma_c, \pi\rangle - \sqrt{\frac{1}{10}}|\Sigma_c, \eta\rangle + \sqrt{\frac{3}{10}}|\Xi'_c, K\rangle \\
(-) \hspace{1em} |15^*; 1, \frac{2}{3}\rangle &= \sqrt{\frac{1}{3}}|\Sigma_c, \pi\rangle + \sqrt{\frac{1}{5}}|\Xi'_c, \eta\rangle - \sqrt{\frac{1}{6}}|\Xi'_c, K\rangle \\
(+) \hspace{1em} |24^*; 1, \frac{2}{3}\rangle &= -\sqrt{\frac{1}{15}}|\Sigma_c, \pi\rangle + \sqrt{\frac{2}{5}}|\Sigma_c, \eta\rangle + \sqrt{\frac{8}{15}}|\Xi'_c, K\rangle \\
(-) \hspace{1em} |15^*; \frac{3}{2}, -\frac{1}{3}\rangle &= \sqrt{\frac{2}{3}}|\Sigma_c, \bar{K}\rangle - \sqrt{\frac{1}{3}}|\Xi'_c, \pi\rangle \\
(+ ) \hspace{1em} |24^*; \frac{3}{2}, -\frac{1}{3}\rangle &= \sqrt{\frac{1}{3}}|\Sigma_c, \bar{K}\rangle + \sqrt{\frac{2}{3}}|\Xi'_c, \pi\rangle \\
(+ ) \hspace{1em} |24^*; \frac{3}{2}, \frac{5}{3}\rangle &= |\Sigma_c, K\rangle \\
(+ ) \hspace{1em} |24^*; 2, \frac{2}{3}\rangle &= |\Sigma_c, \pi\rangle
\end{align*}\]
7. SU(3): $8 \otimes 3$

$\begin{align*}
\text{(−)} \quad |3; 0, -\frac{2}{3}\rangle &= \sqrt{\frac{3}{4}} |K, \bar{D}\rangle - \sqrt{\frac{1}{4}} |\eta, \bar{D}\rangle \\
\text{(+) } |15; 0, -\frac{2}{3}\rangle &= \sqrt{\frac{1}{4}} |K, \bar{D}\rangle + \sqrt{\frac{3}{4}} |\eta, \bar{D}\rangle \\
\text{(−)} \quad |6^*; 0, \frac{4}{3}\rangle &= |K, \bar{D}\rangle \\
\text{(+) } |15; \frac{1}{2}, -\frac{5}{3}\rangle &= |K, \bar{D}_s\rangle \\
\text{(−)} \quad |3; \frac{1}{2}, \frac{1}{3}\rangle &= \sqrt{\frac{9}{16}} |\pi, \bar{D}\rangle - \sqrt{\frac{3}{8}} |K, \bar{D}_s\rangle + \sqrt{\frac{1}{16}} |\eta, \bar{D}\rangle \\
\text{(−)} \quad |6^*; \frac{1}{2}, \frac{1}{3}\rangle &= \sqrt{\frac{3}{8}} |\pi, D\rangle + \sqrt{\frac{1}{4}} |K, \bar{D}_s\rangle - \sqrt{\frac{3}{8}} |\eta, \bar{D}\rangle \\
\text{(+) } |15; \frac{1}{2}, \frac{1}{3}\rangle &= \sqrt{\frac{1}{16}} |\pi, \bar{D}\rangle + \sqrt{\frac{3}{8}} |K, \bar{D}_s\rangle + \sqrt{\frac{9}{16}} |\eta, \bar{D}\rangle \\
\text{(−)} \quad |6^*; 1, -\frac{2}{3}\rangle &= \sqrt{\frac{1}{2}} |\pi, \bar{D}_s\rangle - \sqrt{\frac{1}{2}} |K, \bar{D}\rangle \\
\text{(+) } |15; 1, -\frac{2}{3}\rangle &= \sqrt{\frac{1}{2}} |\pi, \bar{D}_s\rangle + \sqrt{\frac{1}{2}} |K, \bar{D}\rangle \\
\text{(+) } |15; 1, \frac{4}{3}\rangle &= |K, \bar{D}\rangle \\
\text{(+) } |15; \frac{3}{2}, \frac{1}{3}\rangle &= |\pi, \bar{D}\rangle \\
\text{(+) } |15; \frac{3}{2}, \frac{1}{3}\rangle &= |\pi, \bar{D}\rangle
\end{align*}$

8. SU(3): $8 \otimes 3^*$

$\begin{align*}
\text{(−)} \quad |6^*; 0, -\frac{4}{3}\rangle &= -|K, D\rangle \\
\text{(−)} \quad |3^*; 0, \frac{2}{3}\rangle &= \sqrt{\frac{2}{4}} |K, D\rangle + \sqrt{\frac{1}{4}} |\eta, D_s\rangle \\
\text{(+) } |15^*; 0, \frac{2}{3}\rangle &= -\sqrt{\frac{2}{4}} |K, D\rangle + \sqrt{\frac{3}{4}} |\eta, D_s\rangle \\
\text{(−)} \quad |3^*; \frac{1}{2}, -\frac{4}{3}\rangle &= \sqrt{\frac{9}{16}} |\pi, D\rangle + \sqrt{\frac{3}{8}} |K, D_s\rangle - \sqrt{\frac{1}{16}} |\eta, D\rangle \\
\text{(−)} \quad |6; \frac{1}{2}, -\frac{2}{3}\rangle &= -\sqrt{\frac{3}{8}} |\pi, D\rangle + \sqrt{\frac{1}{4}} |K, D_s\rangle - \sqrt{\frac{3}{8}} |\eta, D\rangle \\
\text{(+) } |15^*; \frac{1}{2}, -\frac{1}{3}\rangle &= -\sqrt{\frac{1}{16}} |\pi, D\rangle + \sqrt{\frac{3}{8}} |K, D_s\rangle + \sqrt{\frac{9}{16}} |\eta, D\rangle \\
\text{(+) } |15^*; \frac{1}{2}, \frac{5}{3}\rangle &= |K, D_s\rangle
\end{align*}$
\[ (+) \quad |15^*; 1, -\frac{1}{3}\rangle = |\bar{K}, D\rangle \quad \text{(D46)} \]

\[ (-) \quad |6; 1, \frac{2}{3}\rangle = \sqrt{\frac{1}{2}}|\pi, D_s\rangle - \sqrt{\frac{1}{2}}|K, D\rangle \]

\[ (+) \quad |15^*; 1, \frac{2}{3}\rangle = \sqrt{\frac{1}{2}}|\pi, D_s\rangle + \sqrt{\frac{1}{2}}|K, D\rangle \quad \text{(D47)} \]

\[ (+) \quad |15^*; \frac{2}{3}, -\frac{1}{3}\rangle = |\pi, D\rangle \quad \text{(D48)} \]

9. SU(3): 8 \otimes 8

\[ (-) \quad |10; 0, -2\rangle = -|\bar{K}, K\rangle \quad \text{(D49)} \]

\[ (+) \quad |1; 0, 0\rangle = \sqrt{\frac{3}{8}}|\pi, \pi\rangle - \sqrt{\frac{1}{2}}|K, \bar{K}\rangle_A - \sqrt{\frac{1}{8}}|\eta, \eta\rangle \]

\[ (+) \quad |8_s; 0, 0\rangle = -\sqrt{\frac{3}{5}}|\pi, \pi\rangle - \sqrt{\frac{1}{5}}|K, \bar{K}\rangle_A - \sqrt{\frac{1}{5}}|\eta, \eta\rangle \]

\[ (-) \quad |8_s; 0, 0\rangle = |K, \bar{K}\rangle_S \quad \text{(D50)} \]

\[ (+) \quad |27; 0, 0\rangle = -\sqrt{\frac{1}{40}}|\pi, \pi\rangle - \sqrt{\frac{3}{10}}|K, \bar{K}\rangle_A + \sqrt{\frac{27}{40}}|\eta, \eta\rangle \]

\[ (-) \quad |10^*; 0, 2\rangle = |K, K\rangle \quad \text{(D51)} \]

\[ (+) \quad |8_s; \frac{1}{2}, -1\rangle = -\sqrt{\frac{9}{10}}|\pi, \bar{K}\rangle_A - \sqrt{\frac{1}{10}}|K, \eta\rangle_S \]

\[ (-) \quad |8_s; \frac{1}{2}, -1\rangle = \sqrt{\frac{1}{2}}|\pi, \bar{K}\rangle_S + \sqrt{\frac{1}{2}}|\bar{K}, \eta\rangle_A \quad \text{(D52)} \]

\[ (-) \quad |10; \frac{1}{2}, -1\rangle = -\sqrt{\frac{1}{2}}|\pi, \bar{K}\rangle_S + \sqrt{\frac{1}{2}}|\bar{K}, \eta\rangle_A \]

\[ (+) \quad |27; \frac{1}{2}, -1\rangle = -\sqrt{\frac{1}{10}}|\pi, \bar{K}\rangle_A + \sqrt{\frac{9}{10}}|\bar{K}, \eta\rangle_S \]

\[ (+) \quad |8_s; \frac{1}{2}, 1\rangle = \sqrt{\frac{9}{10}}|\pi, K\rangle_A - \sqrt{\frac{1}{10}}|K, \eta\rangle_S \]

\[ (-) \quad |8_s; \frac{1}{2}, 1\rangle = \sqrt{\frac{1}{2}}|\pi, K\rangle_S - \sqrt{\frac{1}{2}}|K, \eta\rangle_A \]

\[ (-) \quad |10^*; \frac{1}{2}, 1\rangle = \sqrt{\frac{1}{2}}|\pi, K\rangle_S + \sqrt{\frac{1}{2}}|K, \eta\rangle_A \]

\[ (+) \quad |27; \frac{1}{2}, 1\rangle = \sqrt{\frac{1}{10}}|\pi, K\rangle_A + \sqrt{\frac{9}{10}}|K, \eta\rangle_S \quad \text{(D53)} \]

\[ (+) \quad |27; 1, -2\rangle = |\bar{K}, K\rangle \quad \text{(D54)} \]
\[ (+) \ |8_s; 1, 0\rangle = \sqrt{\frac{2}{5}} |\pi, \eta\rangle_S - \sqrt{\frac{3}{5}} |K, \bar{K}\rangle_S \]
\[ (-) \ |8_u; 1, 0\rangle = \sqrt{\frac{2}{3}} |\pi, \pi\rangle - \sqrt{\frac{1}{3}} |K, \bar{K}\rangle_A \]
\[ (-) \ |10; 1, 0\rangle = -\sqrt{\frac{1}{6}} |\pi, \pi\rangle + \sqrt{\frac{1}{2}} |\pi, \eta\rangle_A - \sqrt{\frac{1}{3}} |K, \bar{K}\rangle_A \]
\[ (-) \ |10^*; 1, 0\rangle = \sqrt{\frac{1}{6}} |\pi, \pi\rangle + \sqrt{\frac{1}{2}} |\pi, \eta\rangle_A + \sqrt{\frac{1}{3}} |K, \bar{K}\rangle_A \]
\[ (+) \ |27; 1, 0\rangle = \sqrt{\frac{3}{5}} |\pi, \eta\rangle_S + \sqrt{\frac{2}{5}} |K, \bar{K}\rangle_S \]  
\[ (+) \ |27; 1, 2\rangle = |K, K\rangle \]
\[ (-) \ |10^*; \frac{3}{2}, -1\rangle = |\pi, \bar{K}\rangle_A \]
\[ (+) \ |27; \frac{3}{2}, -1\rangle = |\pi, K\rangle_S \]
\[ (-) \ |10; \frac{3}{2}, 1\rangle = |\pi, K\rangle_A \]
\[ (+) \ |27; \frac{3}{2}, 1\rangle = |\pi, K\rangle_S \]
\[ (+) \ |27; 2, 0\rangle = |\pi, \pi\rangle \]  

10. SU(3): $10 \otimes 3$

\[ (+) \ |15'; 0, -\frac{8}{3}\rangle = |\Omega, \bar{D}_s\rangle \]  
\[ (-) \ |15; 0, -\frac{2}{3}\rangle = |\Xi^*, \bar{D}\rangle \]
\[ (-) \ |15; \frac{1}{2}, -\frac{5}{3}\rangle = \sqrt{\frac{1}{4}} |\Xi^*, \bar{D}_s\rangle - \sqrt{\frac{3}{4}} |\Omega, \bar{D}\rangle \]
\[ (+) \ |15'; \frac{1}{2}, -\frac{5}{3}\rangle = \sqrt{\frac{3}{4}} |\Xi^*, \bar{D}_s\rangle + \sqrt{\frac{1}{4}} |\Omega, \bar{D}\rangle \]
\[ (-) \ |15; \frac{1}{2}, \frac{1}{3}\rangle = |\Sigma^*, \bar{D}\rangle \]
\[ (-) \ |15; 1, -\frac{2}{3}\rangle = \sqrt{\frac{1}{2}} |\Sigma^*, \bar{D}_s\rangle - \sqrt{\frac{1}{2}} |\Xi^*, \bar{D}\rangle \]
\[ (+) \ |15'; 1, -\frac{2}{3}\rangle = \sqrt{\frac{1}{2}} |\Sigma^*, \bar{D}_s\rangle + \sqrt{\frac{1}{2}} |\Xi^*, \bar{D}\rangle \]
\[ (-) \ |15; 1, \frac{1}{3}\rangle = |\Delta, \bar{D}\rangle \]
\[ (-) \ |15; \frac{3}{2}, -\frac{1}{3}\rangle = \sqrt{\frac{3}{4}} |\Delta, \bar{D}_s\rangle - \sqrt{\frac{1}{4}} |\Sigma^*, \bar{D}\rangle \]
\[ (+) \ |15'; \frac{3}{2}, -\frac{1}{3}\rangle = \sqrt{\frac{1}{4}} |\Delta, \bar{D}_s\rangle + \sqrt{\frac{3}{4}} |\Sigma^*, \bar{D}\rangle \]
\[ (+) \ |15; 2, \frac{1}{3}\rangle = |\Delta, \bar{D}\rangle \]
11. SU(3): $10 \otimes 3^*$

\begin{align*}
\begin{array}{ll}
\text{(-)} & |6; 0, -\frac{1}{3} \rangle = \sqrt{\frac{2}{5}}|\Xi^*, D\rangle + \sqrt{\frac{3}{5}}|\Omega, D_s\rangle \\
\text{(+) } & |24^*; 0, -\frac{4}{3} \rangle = -\sqrt{\frac{3}{5}}|\Xi^*, D\rangle + \sqrt{\frac{2}{5}}|\Omega, D_s\rangle \\
\text{(D68)} & \\
\text{(+) } & |24^*; \frac{1}{2}, -\frac{7}{3} \rangle = |\Xi^*, D\rangle \\
\text{(D69)} & \\
\text{(-)} & |6; \frac{1}{2}, -\frac{1}{3} \rangle = \sqrt{\frac{2}{5}}|\Sigma^*, D\rangle + \sqrt{\frac{3}{5}}|\Xi^*, D_s\rangle \\
\text{(+) } & |24^*; \frac{1}{2}, -\frac{4}{3} \rangle = -\sqrt{\frac{3}{5}}|\Sigma^*, D\rangle + \sqrt{\frac{2}{5}}|\Xi^*, D_s\rangle \\
\text{(D70)} & \\
\text{(+) } & |24^*; 1, -\frac{4}{3} \rangle = |\Xi^*, D\rangle \\
\text{(D71)} & \\
\text{(-)} & |6; 1, \frac{2}{3} \rangle = \sqrt{\frac{4}{5}}|\Delta, D\rangle + \sqrt{\frac{4}{5}}|\Xi^*, D_s\rangle \\
\text{(+) } & |24^*; 1, \frac{5}{3} \rangle = -\sqrt{\frac{4}{5}}|\Delta, D\rangle + \sqrt{\frac{4}{5}}|\Xi^*, D_s\rangle \\
\text{(D72)} & \\
\text{(+) } & |24^*; \frac{3}{2}, -\frac{1}{3} \rangle = |\Sigma^*, D\rangle \\
\text{(D73)} & \\
\text{(+) } & |24^*; \frac{3}{2}, \frac{5}{3} \rangle = |\Delta, D_s\rangle \\
\text{(D74)} & \\
\text{(+) } & |24^*; 2, \frac{2}{3} \rangle = |\Delta, D\rangle \\
\text{(D75)} & \\
\end{array}
\end{align*}

12. SU(3): $10 \otimes 8$

\begin{align*}
\begin{array}{ll}
\text{(-)} & |10; 0, -2 \rangle = \sqrt{\frac{1}{2}}|\Xi^*, \bar{K}\rangle + \sqrt{\frac{1}{2}}|\Omega, \eta\rangle \\
\text{(+) } & |35; 0, -2 \rangle = -\sqrt{\frac{1}{2}}|\Xi^*, \bar{K}\rangle + \sqrt{\frac{1}{2}}|\Omega, \eta\rangle \\
\text{(D76)} & \\
\text{(+) } & |8; 0, 0 \rangle = \sqrt{\frac{3}{5}}|\Sigma^*, \pi\rangle + \sqrt{\frac{2}{5}}|\Xi^*, K\rangle \\
\text{(-)} & |27; 0, 0 \rangle = -\sqrt{\frac{3}{5}}|\Sigma^*, \pi\rangle + \sqrt{\frac{2}{5}}|\Xi^*, K\rangle \\
\text{(D77)} & \\
\text{(+) } & |35; \frac{1}{2}, -3 \rangle = |\Omega, \bar{K}\rangle \\
\text{(D78)} & \\
\end{array}
\end{align*}
\begin{align*}
(+) \quad |8; \frac{1}{2}, -1\rangle &= \sqrt{\frac{1}{3}} |\Delta, \pi\rangle + \sqrt{\frac{1}{5}} |\Xi^*, \pi\rangle + \sqrt{\frac{1}{15}} |\Sigma^*, K\rangle \\
(-) \quad |10; \frac{1}{2}, -1\rangle &= \sqrt{\frac{1}{2}} |\Delta, \pi\rangle + \sqrt{\frac{1}{5}} |\Xi^*, \pi\rangle + \sqrt{\frac{1}{5}} |\Xi^*, \eta\rangle - \sqrt{\frac{2}{5}} |\Omega, K\rangle \\
(-) \quad |27; \frac{1}{2}, -1\rangle &= -\sqrt{\frac{1}{20}} |\Sigma^*, K\rangle + \sqrt{\frac{49}{80}} |\Xi^*, \pi\rangle + \sqrt{\frac{9}{80}} |\Xi^*, \eta\rangle - \sqrt{\frac{9}{40}} |\Omega, K\rangle \\
(+) \quad |35; \frac{1}{2}, -1\rangle &= -\sqrt{\frac{1}{4}} |\Sigma^*, K\rangle - \sqrt{\frac{1}{10}} |\Xi^*, \pi\rangle + \sqrt{\frac{9}{16}} |\Xi^*, \eta\rangle + \sqrt{\frac{1}{8}} |\Omega, K\rangle \\
(+) \quad |8; \frac{3}{2}, 1\rangle &= \sqrt{\frac{1}{5}} |\Delta, \pi\rangle + \sqrt{\frac{4}{5}} |\Sigma^*, K\rangle \\
(-) \quad |27; \frac{1}{2}, 1\rangle &= -\sqrt{\frac{1}{5}} |\Delta, \pi\rangle + \sqrt{4 \frac{1}{5}} |\Sigma^*, K\rangle \\
(-) \quad |27; 1, -2\rangle &= \sqrt{\frac{3}{4}} \Xi^*, \bar{K}) - \sqrt{\frac{3}{4}} |\Omega, \pi\rangle \\
(+) \quad |35; 1, -2\rangle &= \sqrt{\frac{3}{4}} \Xi^*, \bar{K}) + \sqrt{\frac{1}{4}} |\Omega, \pi\rangle \\
(+) \quad |8; 1, 0\rangle &= \sqrt{\frac{8}{15}} |\Delta, \bar{K}\rangle - \sqrt{\frac{2}{15}} |\Sigma^*, \pi\rangle + \sqrt{\frac{1}{5}} |\Xi^*, \eta\rangle - \sqrt{\frac{2}{15}} |\Xi^*, K\rangle \\
(-) \quad |10; 1, 0\rangle &= \sqrt{\frac{1}{3}} |\Delta, \bar{K}\rangle + \sqrt{\frac{1}{3}} |\Sigma^*, \pi\rangle + \sqrt{\frac{1}{3}} \Xi^*, K\rangle \\
(-) \quad |27; 1, 0\rangle &= -\sqrt{\frac{1}{20}} |\Delta, \bar{K}\rangle + \sqrt{\frac{9}{20}} |\Sigma^*, \pi\rangle + \sqrt{\frac{3}{10}} |\Sigma^*, \eta\rangle - \sqrt{\frac{1}{5}} |\Xi^*, K\rangle \\
(+) \quad |35; 1, 0\rangle &= -\sqrt{\frac{1}{12}} |\Delta, \bar{K}\rangle - \sqrt{\frac{1}{12}} |\Sigma^*, \pi\rangle + \sqrt{\frac{1}{2}} |\Sigma^*, \eta\rangle + \sqrt{\frac{1}{3}} |\Xi^*, K\rangle \\
(-) \quad |27; 2, 1\rangle &= |\Delta, K\rangle \\
(-) \quad |27; \frac{3}{2}, -1\rangle &= \sqrt{\frac{1}{2}} |\Sigma^*, \bar{K}\rangle - \sqrt{\frac{1}{2}} |\Xi^*, \pi\rangle \\
(+) \quad |35; \frac{3}{2}, -1\rangle &= \sqrt{\frac{1}{2}} |\Sigma^*, \bar{K}\rangle + \sqrt{\frac{1}{2}} |\Xi^*, \pi\rangle \\
(-) \quad |10; \frac{1}{2}, 1\rangle &= \sqrt{\frac{5}{8}} |\Delta, \pi\rangle - \sqrt{\frac{5}{8}} |\Delta, \eta\rangle + \sqrt{\frac{1}{4}} |\Sigma^*, K\rangle \\
(-) \quad |27; \frac{1}{2}, 1\rangle &= \sqrt{\frac{5}{16}} |\Delta, \pi\rangle + \sqrt{\frac{9}{16}} |\Delta, \eta\rangle - \sqrt{\frac{1}{8}} |\Sigma^*, K\rangle \\
(+ ) \quad |35; \frac{1}{2}, 1\rangle &= -\sqrt{\frac{1}{16}} |\Delta, \pi\rangle + \sqrt{\frac{5}{16}} |\Delta, \eta\rangle + \sqrt{\frac{5}{8}} |\Sigma^*, K\rangle \\
(-) \quad |27; 2, 0\rangle &= \sqrt{\frac{3}{4}} |\Delta, \bar{K}\rangle - \sqrt{\frac{3}{4}} |\Xi^*, \pi\rangle \\
(+ ) \quad |35; 2, 0\rangle &= \sqrt{\frac{1}{4}} |\Delta, \bar{K}\rangle + \sqrt{\frac{5}{4}} |\Xi^*, \pi\rangle \\
\end{align*}
\begin{align}
(+) \quad |35; 2, 2\rangle &= |\Delta, K\rangle \\
(+) \quad |35; \frac{5}{2}, 1\rangle &= |\Delta, \pi\rangle
\end{align}

(\text{D87}) \quad (\text{D88})

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[37] We only differ in the value of $\xi$ (see Eq. \textbf{10} below) for the two states $|i\rangle$ in $|\epsilon\rangle = |20 \oplus 15$ for SU(4).

[38] In this sense, the basis $|i\rangle$ in Eq. \textbf{10} is standard, whereas $|\epsilon\rangle$ is not.