Torque Ripple Reduction of a Salient-Pole Permanent Magnet Synchronous Machine With an Advanced Step-Skewed Rotor Design

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ABSTRACT In this paper, a salient-pole permanent magnet synchronous machine (SPPMSM) is introduced to high-speed trains, taking advantage of its high utilization of the reluctance torque from the unbalanced reactance. Torque characteristics, such as average torque and torque ripple, are rather important and should be carefully concerned during the optimal design of the SPPMSM. Both the eccentric air-gap and the advanced step-skewed rotor (ASSR) design are developed and implemented to reduce the torque ripple for comfort and safety issues. Compared to the conventional step-skewed rotor (CSSR), the ASSR takes advantage of the high utilization of permanent magnets (PMs), which are axial continuous assembled, and the easier manufacture with a single rotor lamination model, which is asymmetrically designed with pole shoes skewed by individual skewing angles. No skewing arrangement is applied to the rotor yoke and the PM chambers. The impacts of the eccentric air-gap and the ASSR on the torque characteristics are comprehensively investigated by finite element analysis. The results show that the proposed technique can significantly reduce the torque ripple with a slight expense of the average torque. Based on the optimal eccentric design, experiments of the SPPMSM with the non-skewed rotor, CSSR, and ASSR under various conditions are carried out to validate the estimated results.

INDEX TERMS Eccentricity design, permanent magnet synchronous machine, step-skewed rotor, torque ripple.

I. INTRODUCTION Taking advantages of high efficiency, high power density, high power factor, and low noise/vibration [1], the permanent magnet synchronous machine (PMSM) has been widely used in industrial applications. For rail traction machines, especially for high-speed applications, the PMSM has received intensive attention. To achieve higher torque, a 36-slot-4-pole salient-pole PMSM, shown in Fig.1, is proposed to utilize the reluctance torque from the unbalanced parameters of the quadrature-axis (q-axis) and direct-axis (d-axis) reactance [2]. However, the torque ripple reduction should be carefully treated for comfort and safety issues.

For the PMSM, the torque ripple is composed of the cogging torque and the mutual electromagnetic torque ripple [3], [4]. The cogging torque is the product of the interacting effect between the rotor permanent magnet (PM) field and the stator slots, while the mutual electromagnetic part is the result of the mismatch between the shape of the excitation and the back electromotive force (EMF).

Over the last few decades, many effective techniques of the torque ripple reduction for PMSMs have been extensively investigated. From the stator side, design and optimization methods are carried out on different parts, e.g., slot-opening width, auxiliary tooth, dummy slot, tooth width, and skewing [5]–[9]. Similarly, it is also proved to be effective in rotor optimization, e.g., pole shaping, pole arc width, auxiliary salient poles, and rotor skewing [9]–[18].
Among the foregoing techniques, skewing, including both the stator and the magnet poles, occurs to be more popular. However, the stator skewing leads to complicated winding manufacture and is not suitable for the performed windings or the automatic slot filling, while the rotor skewing requires permanent magnets (PMs) with particular shapes, which results in difficulties of the magnetization and the fabrication. To make the rotor skewing more practical for manufacturing, the skewing may be approximated by placing the rotor laminations and the PMs axially skewed with several discrete steps. For the step-skewed rotor design, the operating mechanism, the theoretical step-skewing angels, and the critical parameters are extensively investigated [19]. Although the step-skewing technology simplifies the manufacturing process to some extent [3], it causes axial flux leakage between adjacent rotor segments resulting in lower utilization of the PMs, which signifies a higher material cost or a worse performance. Some methods have been studied to solve the problem of the axial leakage, e.g. the insertion of non-magnetic spacers between adjacent segments, and the rotor overhang [20]. The former reduces the flux leakage, while the latter compensates for the flux leakage. Both methods increase the axial length of the rotor, which leads to a bigger volume and a lower power density. Moreover, the installation of the magnetized magnets and the steel laminations in the exact skewing angles requires extra efforts, e.g. keyway and circumferential positioning [21], [22].

This paper aims to carry out a salient-pole permanent magnet synchronous machines (SPPMSM) with a low torque ripple. The parameters of the preliminary design of the prototype are listed in Table 1. Both an eccentric air-gap and an advanced step-skewed rotor (ASSR) design with axially continuous PMs are implemented to reduce the torque ripple with an easier and more precise installation. Unlike the convectional step-skewed rotor (CSSR) design, the ASSR is achieved by step-skewed pole shoes with an axial continuous rotor yoke as well as continuous PMs. Additionally, the skewing angle of each pole can be individually designed to adapt different step numbers with a single lamination model for easier manufacture and a more precise installation. A 4-step-skewed rotor for the SPPMSM is presented to illustrate the operating mechanism and the effect of the torque ripple reduction is then analyzed using finite element analysis (FEA). Finally, prototypes with the non-skewed rotor (NSR), CSSR, and ASSR are fabricated and tested to verify the simulation results.

### II. MECHANISM OF TORQUE RIPPLE

The analytical and numerical models are widely used to calculate the torque ripple in PM machines. The analytical models are suitable for preliminary design due to its low cost of computational efforts with an acceptable accuracy [23], [24]. The numerical ones, which are normally referred as to two-dimensional (2-D) and three-dimensional (3-D) FEA methods, can provide more accurate calculation with the comprehensive performance [25], [26]. However, FEA, especially 3-D FEA, is generally computationally intensive and time-consuming for optimal designs. The analytical model can be implemented to illustrate the mechanism of the torque ripple and hence give guidance for the optimization, while the influences of the eccentric air-gap and the ASSR on the torque ripple can be investigated by comprehensive FEA.

Analytical models based on the theory of the winding distribution are widely used to study the mechanism of the torque ripple in PM machines. The mutual effect of the stator and the rotor harmonics on the torque ripple was analyzed in [27]. The influence of the port location of the flux barriers on the torque ripple was studied in [28]. The effect of the stator magnetic motive force (MMF) on the rotor was discussed in [29]. However, the air-gap was assumed to be uniformly distributed in the models used in the aforementioned researches, which did not consider the influence of the stator notches and the rotor shape on the air-gap reluctance. The analytical model for torque ripple in this paper is established based on the spatial coordinate diagram shown in Fig. 2.

| Symbol | Parameter | Value | Unit |
|--------|-----------|-------|------|
| $D_{sa}$ | Stator outer diameter | 155 | mm |
| $D_{ro}$ | Rotor outer diameter | 104.4 | mm |
| $D_{ri}$ | Rotor inner diameter | 38 | mm |
| $\delta$ | Air-gap length | 0.8 | mm |
| $l_{stk}$ | Axial length | 100 | mm |
| $w_{ps}$ | Pole shoe width | 152 | mm |
| $w_{pm}$ | Magnet width | 39 | mm |
| $h_{pm}$ | Magnet thickness | 6 | mm |
| $n_s$ | Stator slot number | 36 | - |
| $p$ | Pole pair number | 2 | - |
| $P_r$ | Rated power | 1.5 | kW |
| $I_r$ | Rated current | 2.5 | A |
| $I_m$ | Max. current | 3.5 | A |
| $n_r$ | Rated speed | 1500 | r/min |
| PM material | NdFeB 38UH | 50WW470 | |
According to the theory of the winding distribution function, the Fourier series of the stator MMF can be expressed by (1) for symmetric three-phase sinusoidal currents.

\[ f_s = \sum_{h \neq j} f_{s,h} \cos (h \theta - h_0) \]

(1)

where \( h \) is the harmonic number, \( f_{s,h} \) is the amplitude of the \( h \)th harmonic of the stator MMF, \( \theta \) is the angle of the axis relative to the A phase in the stator coordinate system, \( h_0 \) is the transient angular position of the rotor, \( \gamma_d \) is the current phase relative to the \( d \)-axis, and \( j = 1, 2, 3, \ldots \)

The rotor MMF in the interior PM machine is composed of the one from the PMs and the other from the stator MMF. Due to the salient structure, the two parts of the rotor MMF result in stepped waves of MMFs with the same distribution shape in the air-gap. Thus, the rotor MMF can be written as an expression of even symmetric Fourier series as (2).

\[ f_r = \sum_{h=2j-1} \left( f_{mr,j} + f_{sr,j} \right) \cos (h \theta - h_0) \]

(2)

where \( f_{mr,j} \) and \( f_{sr,j} \) are the amplitudes of the \( h \)th harmonic of the rotor MMF produced by the PMs and the stator MMF, respectively.

The air-gap permeance is expressed by

\[ \Lambda = \Lambda_0 + \Lambda_s + \Lambda_r = \Lambda_0 + \sum_{h=1}^{\phi_p} \Lambda_{s,h} \cos (h \theta) - \sum_{h=2j} \Lambda_{r,h} \cos (h \theta - h_0) \]

(3)

where \( \Lambda_0 \) is the mean permeance, \( \Lambda_s \) is the influence of the stator slots, and \( \Lambda_r \) is the influence of the rotor noncircular. \( \Lambda_{s,h} \) and \( \Lambda_{r,h} \) are the amplitudes of the \( h \)th harmonic of the stator slots and the rotor noncircular, respectively. \( \phi_p \) is the number of the stator slots for each pole pair.

Thus, the torque of the interior PM machine can be expressed by (4) according to the Lorenz force theorem.

\[ T_e = \frac{p}{2} r_g l_{stk} \int B_g df_r = \frac{p}{2} r_g l_{stk} \int (f_r - f_s) \Lambda \frac{df_r}{d\theta} d\theta \]

\[ = \frac{p}{2} r_g l_{stk} \left( \int f_r \Lambda \frac{df_r}{d\theta} d\theta - \int f_s \Lambda \frac{df_r}{d\theta} d\theta \right) \]

(4)

where \( p \) is the number of the pole pairs, \( r_g \) is the radius of the air-gap, \( l_{stk} \) is the effective axial length of the stator, and \( B_g \) is the flux density of the air-gap.

By substituting (1), (2), and (3) to (4), the average torque can be express by

\[ T_{avg} = \frac{p\pi}{4} r_g l_{stk} \left( f_{s,1}^2 \Lambda_{r,2} \sin (2\gamma_d) + \sin \gamma_d \left( f_{s,1} f_{r,1} (2\Lambda_0 - \Lambda_r) + \sum_{h=1}^{\phi_p} f_{s,h} f_{r,h} (h + 1)f_{s,h+1} - (h - 1)f_{s,h-1} \right) - \sum_{h=2j+1} f_{s,h} f_{r,h} (\Lambda_{r,h+1} + \Lambda_{r,h-1}) \right) \]

(5)

where

\[ f_{r,j} = f_{mr,j} + f_{sr,j} \]

When only the fundamental harmonics of the stator MMF, the rotor MMF, and the air-gap permeance are considered, (5) can be simplified as

\[ T_{avg} = \frac{p}{2} \pi r_g l_{stk} \Lambda_0 f_{s,1} f_{r,1} \sin \gamma_d \]

(6)

To gain a clear idea of the components of the torque ripple in interior PM machines, the torque ripple is divided into three parts according to different air-gap permeance components \( \Lambda_0, \Lambda_s, \) and \( \Lambda_r \). By only considering \( \Lambda_0 \), the torque ripple can be expressed by

\[ T_{rip,0} = -\frac{p}{2} \pi r_g l_{stk} \Lambda_0 \sum_{h=j} (6h \pm 1)f_{s,6h} \pm 1 f_{r,6h} \pm 1 \times \cos (6h_0 \theta t \mp \gamma_d) \]

(7)

It can be found that the torque ripple \( T_{rip,0} \) is a result of the mutual effect between \( 6h \pm 1 \) order harmonics of the stator MMF and the rotor MMF. By considering both \( \Lambda_0 \) and \( \Lambda_r \), the torque ripple is

\[ T_{rip,0+r} = \frac{p\pi}{4} r_g l_{stk} \left( \sum_{h=6j\pm 1} hf_{s,h} \cos (h \theta) \mp \gamma_d \right) \times \left( (f_{r,h+2} - f_{r,h-2}) - 2f_{r,h} \Lambda_0 \right) + \sum_{h=j} \sum_{m=2q-1} F_1(6h \pm 1) \sin (6h_0 \theta t \mp \gamma_d) - \sum_{h=j} \sum_{m=3q} F_2(6h) \sin (6h_0 \theta t) - \sum_{h=j} \sum_{m=3q} F_2(6h \pm 2) \sin (6h_0 \theta t \pm 2\gamma_d) \]

(8)

where

\[ F_1(x) = x f_{s,m} f_{s,x} \Lambda_{r,x+m} \]

\[ F_2(x) = x f_{s,m} f_{s,x} x - m \Lambda_{r,x} \]
According to (8), the torque ripple \( T_{rip,s} \) can be reduced by adjusting \( \Lambda_s \) to make \( T_{rip,0+p} < T_{rip,0} \). The influence of the air-gap permeance \( \Lambda_s \) on the torque ripple can be deduced as

\[
T_{rip,s} = \frac{p}{4} r_g \sum_{h=1}^{n} \sum_{m=-q}^{q} \left( f_r \cdot h_{sp} \pm 1 F_3 (m n_{sp} - h_{sp} \pm 1) \times \sin (h_{sp}\omega_r t \pm \gamma_d) - f_r \cdot h_{sp} \pm 1 F_3 (h_{sp} - m n_{sp} \pm 1) \sin (h_{sp}\omega_r t \mp \gamma_d) - f_r \cdot h_{sp} \pm 1 F_3 (h_{sp} + m n_{sp} \pm 1) \sin (h_{sp}\omega_r t \mp \gamma_d) \right)
\]  

(9)

where

\[
F_3(x) = x f_3 \Lambda_{s, m} n_{sp}
\]

It can also be found that the opening slots of the stator not only leads to a cogging torque but also produce extra torque ripple by the effect of the stator MMF and the rotor MMF under load conditions.

According to (8) and (9), the torque ripple can be divided into two categories, one is caused by the interactional effect between the stator and rotor MMFs, while the other is caused by the harmonics of the stator slots. Thus, the eccentric air-gap and the step-skewing designs are implemented to reduce these two kinds of torque ripples, respectively. The eccentric air-gap design adjusts the distribution of the air-gap permeance by modifying the arc of the pole shoes, as shown in Fig. 3. The arc is designed with an eccentricity \( \Delta_s \), while the radius of the pole arc is reduced by the eccentricity correspondingly.

**FIGURE 3.** Diagram of the eccentric air-gap design.

A CSSR design is achieved by assembling several rotor segments with a specific skewing angle. For a CSSR, skewing angles are critical parameters to the torque ripple reduction. In order to eliminate the torque ripple \( T_{rip,s} \), the optimal skewing angle \( \theta_s \) for the whole rotor segments must be

\[
\theta_s = \frac{\text{HCF}(n_s, 2p) \cdot 2\pi}{2p \cdot n_s}
\]  

(10)

where the denominator is the highest common factor (HCF) between \( n_s \) and \( 2p \). Therefore, the optimal value of the mechanical skewing angle between adjacent segments is given by

\[
\theta_{ss} = \frac{\theta_s}{n_{seg}} - 1
\]  

(11)

where \( n_{seg} \) is the number of the rotor segments.

**III. OPTIMAL DESIGN FOR TORQUE RIPPLE REDUCTION**

**A. ECCENTRIC DESIGN OF POLE SHOE**

To improve the distribution of the air-gap flux density and make it close to the sinusoidal shape, the eccentric air-gap distribution was constructed by optimizing the rotor pole shoes with the scheme of tangential eccentric circles. In the specific design, the stator structure was first determined to be unchanged. 2-D nonlinear transient FEA was carried out to comprehensively reveal the electromagnetic characteristics under different eccentricities. Only one-fourth of the SPPMSM was modeled with antiperiodic boundary conditions to save computational cost. The appropriate eccentricity was selected based on the electromagnetic performance index of the harmonic distortion rate of the air-gap flux density. At the same time, the influence of the increase of air-gap length on the flux leakage coefficient should be considered, so as to avoid the excessive air-gap as far as possible.

**FIGURE 4.** Relation curve between the harmonic distortion rate of the air-gap flux density and the eccentricities.

The dichotomy method was used to optimize the eccentricity of the pole shoes, and the relation curve between the harmonic distortion rate of the air-gap flux density and the eccentricities was obtained and shown in Fig. 4. It can be inspected from Fig. 4 that the harmonic distortion rate of the air-gap flux density is quite significant with a value of 45.8% in the non-eccentric design. With an increasing eccentricity, the distortion rate increases after an initial decrease. The cases of 1.55mm, 1.9375mm and 2.325mm were selected for further analysis with the distortion rate lower than 38%.

The comparison results of the air-gap flux density harmonic contents with non-eccentricity and the three eccentricities are depicted in Fig. 5. In the non-eccentric design, the 3\(^{rd}\), 5\(^{th}\), 7\(^{th}\), 11\(^{th}\), 13\(^{th}\), 17\(^{th}\), and 19\(^{th}\) harmonics are the main components in the air-gap flux density. The 5\(^{th}\) and 7\(^{th}\) harmonics contribute to the 6\(^{th}\) order torque ripple. The 17\(^{th}\) and 19\(^{th}\) harmonics due to the cogging effect of the 36-slot-4-pole design are quite massive with values greater than 25%. It can be observed that the 5\(^{th}\), 7\(^{th}\), 11\(^{th}\) and 13\(^{th}\) harmonics are significantly decreased by the eccentric designs, while more 15\(^{th}\) and 21\(^{st}\) harmonics are
introduced. For 3rd harmonic, only the 1.55mm design has a slightly positive influence on the torque ripple reduction, while the other two eccentricity designs have a negative influence. Besides, the eccentricity designs are less effective for reducing the 17th and 19th harmonics, which are the results of the cogging effect. The design with an eccentricity value of 1.55mm is selected as the optimal design. Compared to the non-eccentric design, the total flux density harmonic is decreased from 45.8% to 37.7%, while the fundamental amplitude is increased from 0.797T to 0.802T.

Similarly, the effect of the eccentric design can be found in the comparison of the back EMF waveforms under no-load operation, shown in Fig. 6. The eccentricity design makes a contribution to the reduction of the harmonic distortion in back EMF, which leads to a lower torque ripple. The total harmonic distortion rate is significantly decreased from 31.0% to 8.6%.

The corresponding waveforms of the cogging torque waveforms, together with their main harmonics under no-load operation are obtained and illustrated in Fig. 7. It can be observed from Fig. 7(a) that the cogging torque is suppressed by the eccentric design with peak-to-peak (P-P) value decreased by 57.4% (from 1904.03mNm to 810.40mNm). For the SPPMSM with 36 slots and 4 poles, the first- and second-order torque harmonics are the 18th and 36th in one electrical cycle [15]. The mean-square amplitudes (MSAs) of 1st, 2nd, 3rd, 4th, 5th, and 6th harmonics are reduced by 44.4%, 64.8%, 87.6%, 81.3%, 49.6%, and 68.6%, respectively. Fig. 8 presents the effect of the eccentric design on the torque under rated current excitation with the maximum torque per ampere (MTPA) control. It can be found that the torque waveforms of the NSR have rather peculiar and asymmetric profiles due to the eccentric air-gap, and hence result in rather high peak values in certain rotor positions. It is obvious that the eccentric design has slight effect on the torque profile and the average torque, which is 10.62Nm and 10.52Nm for the non-eccentric and the eccentric designs, respectively. Meanwhile, the P-P torque ripple rate is reduced by a range 90.0% to 69.0%. It can be observed from Fig. 8(b) that the MSAs of 1st, 2nd, 3rd, 4th, and 5th harmonics are reduced by 20.2%, 32.5%, 30.1%, 54.8%, and 71.4%, respectively. Although the eccentric design contributes to the torque ripple reduction to some extent, the torque ripples are quite significant, which implies that the machine would suffer from fatal mechanical vibrations. Therefore, further optimization designs of the torque ripple reduction, such as skewing rotor, are essential for the SPPMSM.

B. ADVANCED STEP-SKEWED ROTOR DESIGN

For a CSSR design, shown in Fig. 9, the step-skewing is achieved by rotating the rotor lamination with four skewing
angles designed as $\theta_s/2$, $-\theta_s/6$, $+\theta_s/6$, and $+\theta_s/2$ ("−" for rotating counterclockwise and "+" for clockwise). In this way, the PMs of each pole are also separated into four step-skewed segments. Extra efforts are needed to install laminations and pre-magnetized PMs with a high precision of skewing angles. Moreover, the existence of the axial leakage flux lowers the utilization of PMs. To make the rotor fabrication easier, an ASSR design is proposed to keep the PMs axially continuous. Moreover, the rotor can be assembled by a single lamination model using particular arrangements. The mechanism is illustrated by an example of a 36-slot-4-pole SPPMSM. It can be deduced from (10) that the skewing angle $\theta_s$ for the SPPMSM should be $\pi/18$.

![Diagram of the ASSR design. (a) Diagram of rotor lamination, in which the red dot line is the sketch of the non-skewing rotor. (b) Arrangement of 4 rotor segments with axially continuous PMs.](image)

Based on this proposed method, 2-step-skewed and 8-step-skewed rotors with a single lamination model can be easily achieved by the same skewing angle ($\theta_s/2$) on each pole shoe and four different skewing angles ($\theta_s/14$, $3\theta_s/14$, $5\theta_s/14$, and $\theta_s/2$) on corresponding four pole shoes, respectively. For a SPPMSM with $p$ pairs of poles, it can be deduced that the single lamination model arrangement requires the step numbers $n_{seg}$ to be

$$n_{seg} = k \cdot F(2p)$$

where $k = 1$ or 2, and $F(2p)$ is the factor of $2p$ other than 1.

More examples of the ASSR design with a single lamination model are listed in Table 2. Normally, the effect of the torque ripple reduction, as well as the difficulty of the manufacture, increases with a greater $n_{seg}$. The comprehensive factors, such as the stator slot number, pole arc width, and mechanical strength, should be carefully concerned during the optimization of $n_{seg}$. In this paper, the 4-step arrangement was chosen as a tradeoff between the better electromagnetic performance and the simpler manufacture.

C. SIMULATION AND EXPERIMENTAL RESULTS

The optimal eccentric design is employed in the NSR, CSSR, and ASSR designs, which share the same pole arc length. 3-D FEA of the NSR, CSSR, and ASSR was carried out to study the extra axial flux leakage by the step-skewing designs. Since it was difficult to obtain the accurate amount of the axial leakage between the adjacent segments, a specific leakage coefficient $\eta_l$ was introduced to evaluate the leakage according to the valid flux across the rotor laminations. The coefficient $\eta_l$ was calculated by (13).

$$\eta_l = 1 - \frac{\Phi_{skew}}{\Phi_n}$$

where $\Phi_{skew}$ is the valid flux of the step-skewed rotor and $\Phi_n$ is the one of the non-skewing rotor.

Fig. 11 presents the results of the back EMF waveforms of the NSR, CSSR, and ASSR designs, which are 1.86% and 0.33% respectively, which indicates that the ASSR design has higher utilization of the PMs than the CSSR design. It is imaginable that the ASSR design can do better with larger skewing angles or a smaller value of $n_{seg}$. Fig. 12 shows the results of the back EMF waveforms of the NSR, CSSR, and ASSR designs. The cogging effect on the back EMF is significantly suppressed by both the CSSR and the ASSR designs, while the amplitude of the ASSR design is slightly higher than the CSSR one. The harmonic distortion rate is decreased by the same range from 8.6% to 3.5% for the CSSR and ASSR designs, which indicates a good effect on the torque ripple reductions.

The prototype SPPMSM with three different rotors, including the NSR, CSSR, and ASSR, shown in Fig. 13, was fabricated for the experimental validation of the rotor step-skewing technique for the torque ripple reduction. The torque performance of the SPPMSM under different current
TABLE 2. Examples of the advanced step-skewed rotor design with different pole pair and segment numbers.

| $2p$ | $n_{seg}$ | Design 1 | Design 2 | Design 3 | Design 4 |
|------|-----------|----------|----------|----------|----------|
| 4    | 2         | 2.A      | 2.A+7/6  | 2.A+7/6  | 2.A      |
| 4    | 4         | 3.A      | 3.A+7/6  | 3.A+7/6  | 3.A      |
| 4    | 8         | 4.A      | 4.A+7/6  | 4.A+7/6  | 4.A      |
| 6    | 2         | 5.A      | 5.A+7/6  | 5.A+7/6  | 5.A      |
| 6    | 3         | 6.A      | 6.A+7/6  | 6.A+7/6  | 6.A      |
| 6    | 4         | 7.A      | 7.A+7/6  | 7.A+7/6  | 7.A      |
| 6    | 6         | 8.A      | 8.A+7/6  | 8.A+7/6  | 8.A      |
| 6    | 12        | 11.A     | 11.A+7/6 | 11.A+7/6 | 11.A     |

FIGURE 11. Comparison results of the axial flux leakage of the prototype SPPMSM with the NSR, CSSR, and ASSR.

FIGURE 12. Effect of the CSSR and ASSR designs on the back EMF waveform under no-load operation.

The test-rig was mainly constructed of a twin drag system, including the SPPMSM and an induction machine (IM). The SPMMSM was driven by an inverter under two distinct controls, while the IM was dragged by the SPPMSM with a direct current feedback system. The commonly used $i_d = 0$ control is not suitable for the proposed SPPMSM due to the unavailability of the reluctance torque. For high-speed train applications, the MTPA control is used in the situation of the start-up and acceleration to make full use of the current excitation for the maximum torque output. The FW control contributes to reduce the inverter capacity and widen the speed range by enlarging the demagnetization current.
The cogging torque waveforms of the prototype SPPMSM with the NSR, CSSR, and ASSR from non-linear FEA and experimental tests are obtained and shown in Fig. 15. It can be observed from Fig. 15(a)-Fig. 15(c) that the cogging torque profiles from FEA and test results are in great agreement. The cogging torque waveforms from the FEA results of the NSR and CSSR designs are normal and symmetric. The FEA results in Fig. 15(c) demonstrate that the positive and negative parts of the cogging torque waveform from the ASSR design are different, but the mean value is still zero. The ASSR design breaks the magnetic symmetries of the axial interactions resulting in an asymmetric cogging torque waveform. Additionally, the cogging torque profile of the ASSR shows glaring discrepancies with the others, which indicates more higher order harmonics of the cogging torque than the others. Compared to the NSR, the P-P values of the cogging torque are decreased by 63.1% and 60.0% by the CSSR and ASSR, respectively. It can be found in Fig. 15(d) that 1\textsuperscript{st} and 2\textsuperscript{nd} harmonics are remarkably reduced by the CSSR and ASSR designs. Compared with the CSSR, the ASSR takes disadvantage on 3\textsuperscript{rd}, 4\textsuperscript{th}, and 5\textsuperscript{th} harmonics. The experimental results validate that the cogging torque can be effectively reduced by both the CSSR and the ASSR designs.

The torque characteristic of the prototype SPPMSM with the NSR, CSSR, and ASSR are measured and compared with the corresponding FEA results in Fig. 16. Fig. 16(a) and Fig. 16(b) present the torque waveforms under 2.5A current excitation with the MTPA and FW controls, respectively. Similarly, Fig. 16(c) and Fig. 16(d) illustrate the ones under 3.5A current excitation with the MTPA and FW conditions, respectively. It can be observed in Fig. 16(a)-Fig. 16(d) that the torque waveforms with both the MTPA and the FW controls have peculiar and asymmetric profiles. The torque profiles from the FEA and experimental results of different MTPA conditions, especially the ones of the CSSR and ASSR, have better agreements than the corresponding ones of different FW conditions. However, there are still glaring discrepancies between the FEA and experimental results, especially the ones of the NSR. The discrepancies are mainly the results of the manufacture defects, installation uncertainty of the test-rig, and the ideal machine model without any mechanical tolerance in 3-D FEA. The amplitudes and tendencies of the torque waveforms under the MTPA conditions are nearly amplified in the same proportion as the current. Moreover, the distortion rates of the torque waveforms under the MTPA conditions are smaller than the corresponding ones under the FW conditions, which indicates that the FW control may introduce more high order harmonics of the torque ripple into the SPPMSM.

The average torque and the P-P torque ripple rate values are calculated from the results in Fig. 16(a)-Fig. 16(d) and depicted in Fig. 16(e) and Fig. 16(f). It can be inspected from Fig. 16(e) that the average torque values from the test and FEA results under four operations are in great agreement with trivial errors in the range from 0.54% to 3.90%. By comparing the experimental results of the CSSR and ASSR to the corresponding ones of the NSR, the average torque values under the MTPA conditions are reduced by a range from 0.16% to 1.93%, while the ones under the FW conditions are reduced by a range from 5.40% to 13.25%. Moreover, the average torques of the CSSR under the MTPA conditions are 1.40% and 1.93% lower than the corresponding ones of the NSR under 2.5A and 3.5A excitations respectively, while the ones of the ASSR are 0.16% and 1.34% lower. It can be observed from the comparison results between the P-P torque ripple rates with the MTPA and FW controls that the FW control introduce more torque ripples into the prototype SPPMSM, especially for the ASSR with asymmetric pole shapes. In addition, the average torque values of the ASSR are higher than the ones of the CSSR, which implies that the continuous PMs are effective in reducing axial flux leakage. It can be found in Fig. 16(f) that the highest torque ripple rate appears to exceed 50%, which hints that the prototype with the NSR would potentially suffer from severe mechanical vibrations. By comparing the experimental results of the CSSR and ASSR to the corresponding ones of the NSR, it can be easily found in Fig. 16(f) that the torque ripples in the prototype SPPMSM with the CSSR are significantly reduced by a range from 90.61% to 95.24%, while the ones with the ASSR are reduced by a range from 85.55% to 92.77%. Both the maximum torque ripple reduction by the CSSR and ASSR occurs in the MTPA condition at rated current. The effects of the torque ripple reduction by the step-skewing technologies are lower with a higher current excitation. For the prototype SPPMSM under

![FIGURE 15. Cogging torque waveforms and their spectra of the prototype SPPMSM with the NSR, CSSR, and ASSR from FEA and experimental tests.](image-url)
rated current excitation with the MTPA control, the torque ripples are remarkably reduced by 95.24% and 92.77% at the corresponding expenses of 1.40% and 0.16% drop on the average torque by the CSSR and ASSR designs, respectively.

Furthermore, the spectra of the experimental results from Fig. 16(a)-Fig. 16(d) are derived and compared in Fig. 17 on a logarithmic scale. The MSA values in Fig. 17(a)-17(d) are listed in Table 3, in which the reduction of the torque harmonics by both the CSSR and the ASSR designs in

**FIGURE 16.** Torque waveforms together with their torque characteristics of the prototype SPPMSM with the NSR, CSSR and ASSR in different conditions.

**FIGURE 17.** Spectra of the torque waveforms of the NSR, CSSR, and ASSR from experimental results in different conditions on a logarithmic scale.

**TABLE 3.** Experimental results of the harmonics in the prototype SPPMSM with the NSR, CSSR, and ASSR in different conditions.

| Harmonic order | MTPA @2.5A | FW @2.5A | MTPA @3.5A | FW @3.5A |
|----------------|------------|------------|------------|------------|
|                | NSR        | CSSR       | ASSR       | NSR        | CSSR       | ASSR       | NSR        | CSSR       | ASSR       | NSR        | CSSR       | ASSR       |
| MSA (N·m)      | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  | MSA (N·m)  |
| Red. (%)       | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   | Red. (%)   |
| 1st            | 2.06       | 0.07       | 96.7       | 0.04       | 98.2       | 0.50       | 18.3       | 0.05       | 90.2       | 0.25       | 0.07       | 97.0       | 0.04       | 98.4       | 0.66       | 0.04       | 93.6       | 0.05       | 92.2       |
| 2nd            | 0.78       | 0.01       | 98.2       | 0.02       | 96.9       | 0.32       | 0.18       | 43.6       | 0.01       | 95.9       | 0.57       | 0.01       | 97.5       | 0.03       | 94.2       | 0.33       | 0.005      | 98.5       | 0.006      | 98.2       |
| 3rd            | 0.37       | 0.04       | 90.4       | 0.02       | 93.8       | 0.12       | 0.07       | 43.0       | 0.02       | 87.0       | 0.37       | 0.04       | 88.6       | 0.01       | 96.6       | 0.14       | 0.02       | 87.2       | 0.02       | 82.7       |
| 4th            | 0.17       | 0.009      | 94.7       | 0.01       | 92.8       | 0.03       | 0.02       | 33.3       | 0.02       | 37.8       | 0.12       | 0.004      | 96.7       | 0.01       | 90.9       | 0.05       | 0.002      | 96.3       | 0.02       | 61.3       |
| 5th            | 0.04       | 0.002      | 95.1       | 0.007      | 83.7       | 0.009      | 0.01       | -9.7       | 0.008      | 12.1       | 0.07       | 0.005      | 92.8       | 0.01       | 86.0       | 0.007      | 0.002      | 69.6       | 0.01      | -103.3     |
| 6th            | 0.12       | 0.003      | 97.2       | 0.007      | 93.9       | 0.006      | 0.01       | -91.6      | 0.009      | -44.2      | 0.05       | 0.004      | 92.5       | 0.008      | 85.7       | 0.02       | 0.001      | 93.3       | 0.005      | 76.3       |

*Abbreviation of the word “Reduction”.

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different conditions are derived and listed as percentages. It can be inspected from Fig. 17(a), Fig. 17(c) and Table 3 that the harmonics in the prototype SPPMSM with the MTPA control are significantly alleviated by more than 83.7% through the rotor step-skewing technologies. Although the CSSR and ASSR can effectively reduce the total harmonic contents in the SPPMSM with the FW controls, some high-order harmonics are even increased by the rotor step-skewing technologies.

IV. CONCLUSION

An optimization process for reducing the torque ripple in a SPPMSM for high-speed train applications is presented. 2-D and 3-D FEA are adopted to evaluate the effects of the eccentric air-gap, CSSR, and ASSR designs on the cogging torque and torque ripple reductions. Some conclusions can be drawn as follows.

The eccentric air-gap in the SPPMSM has a great influence on the 5th, 7th, 11th, and 13th order harmonics of the air-gap flux density. With an increasing eccentricity, the effectiveness of the eccentric design decreases after an initial increase. Taking the total distortion rate and lower harmonics of the air-gap flux density into consideration, 1.55mm is suggested for the proposed SPPMSM. Both the peak value and the harmonic contents of the cogging torque waveforms are significantly reduced by the eccentric design. Meanwhile, the torque ripple rate is reduced from 90.0% to 69.0% at the expense of 0.9% drop at the average torque by the eccentric design. Also, the torque waveforms from the eccentric prototype have peculiar and asymmetric profiles.

Combining the eccentric technology, the ASSR with step-skewed pole shoes and axial continuous PMs is employed to reduce the torque ripple. The mechanism of the ASSR with a single lamination model is illustrated by examples of different pole pairs and step numbers. The step number can be determined by (12), but also constrained by the skewing angles, the pole arc width, and the manufacturing complexity. The torque performances of the SPPMSM with the NSR, CSSR, and ASSR are evaluated by FEA and experimental tests. Both the CSSR and ASSR are proved to be effective in reducing the torque ripples. The torque ripple can be remarkably alleviated by a range from 94.82% to 95.24% and 90.77% to 92.77% at the expenses of 1.40% to 1.93% and 0.16% to 1.34% drop on the average torque by the CSSR and ASSR designs with the MTPA control. Compared with the CSSR, the proposed ASSR design is proved to take advantage of a higher utilization of the PMs with smaller axial flux leakage. In addition, easier and more precise installation can be achieved by a single rotor lamination design of the ASSR.

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