Nonplannar Nonlinear Dust ion Acoustics Solitary and Shock Waves in a Dusty Multi-ion Plasma

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ABSTRACT

We have presented a stern theoretical exploration of the nonplannar and nonlinear propagation of dust-ion acoustic (DIA) waves in a dusty multi-ion plasma. We studied the propagation of nonplannar non-linear DIA waves by the reductive perturbation method which leads to the derivation of the Burgers [Korteweg de-Vries (K-dV)] equation for the shock (solitary) wave propagation. The shock (solitary) waves are found to be formed in a multi-ion dusty plasma due to the balance between nonlinearity and dissipation (dispersion), and that the dissipation (dispersion) arises due to the dust charge fluctuation (deviation of charge neutrality condition). We have analyzed the effects of nonplannar and nonlinear geometry on the DIA shock and solitary waves propagating in such a dusty multi-ion plasma. We have shown that the nonplanar (cylindrical and spherical) solitary and shock structures are significantly different from planner ones.

Indexing terms/Keywords
Nonplannar, Nonlinear, Solitary waves, Shock waves, Multi-ion, Dissipation

Academic Discipline And Sub-Disciplines
Bio-medical Engineering, Material Science, Health Physics.

SUBJECT CLASSIFICATION
Physics Classification, Electrical Engineering Classification

TYPE (METHOD/APPROACH)
Meteorology- WRF method
INTRODUCTION

Plasmas with a significant amount of negative ions, whose contribution cannot be neglected in any way are known as electronegative plasmas, [2,5,3,4]. These occur in both space (e.g. Saturn's rings, Arora, etc.) and laboratory devices (e.g. micro-electronic devices, photo-electronic industries, etc.) [1,2,5,3,4,6,7,8]. The electronegative plasmas, which are observed in such space and laboratory devices, are not pure in general. In most of the cases, they are contaminated by solid impurities (dust) which are not practically neutral, but are charged by absorbing plasma electrons and positive as well as negative ions [9,10,11,12,13,14,15,16]. Therefore, in general, electronegative plasmas are, in fact, dirty or dusty electronegative plasmas [17,18,10,19,12,13,14,15,16]. Recently, a number of investigations [9,10,18] have been made on the propagation characteristics of electrostatic waves in a dusty electronegative plasma containing Boltzmann electrons, inertial positive and negative ions [9,10,18], and stationary negatively [9,10] or positively [18] charged dust. These works are not valid for the plasma state where negative ions are in Boltzmann equilibrium. But a number of authors [20,21,22,23] have predicted that negative ions in such electronegative plasma states are in Boltzmann equilibrium. This prediction has been conclusively verified by a recent laboratory experiment of Ghim [23]. Recently, motivated by the experimental observation of Ghim, Mamun [24] have considered a dusty electronegative plasma system containing Boltzmann electrons, Boltzmann negative ions, cold mobile positive ions, and stationary negatively charged dust, and have studied the formation of solitary waves and double layers. The work of Mamun [25] is valid for constant dust charge. However, the charge of dust in such an electronegative plasma cannot be, in general, constant, but fluctuates with space and time [26,27,28,29,30,31]. The dust charge fluctuation is important only for those waves whose time period is comparable to the charging time period order of magnitude. To overcome the limitation of the constant dust charge, Mamun and Tasnim [29] have taken the dust charge fluctuation into account, and investigated the DIA shock and solitary waves in dusty electronegative plasma (DENP). The work of Mamun and Tasnim [29] is again limited to one dimensional geometry which may not be a realistic situation in laboratory devices, since the waves observed in laboratory device are certainly not bounded in one dimension. Therefore, in our present work, we generalize the work of Mamun and Tasnim [29] to non-planar cylindrical and spherical geometries, and we analyze the DIA shock and solitary waves in a dusty electronegative plasma (DENP) of cylindrical and spherical geometry. We show in such DENPs how the DIA shock and solitary waves in nonplanar (cylindrical and spherical) geometries differ from those in one-dimensional geometry.

Governing Equations:

We consider the nonlinear propagation of low-frequency nonplanar DIA waves in an collisionless, unmagnetized dusty electronegative plasma (DENP) composed of Boltzmann electrons, Boltzmann negative ions, inertial positive ions, and charge fluctuating stationary dust. We are interested in the propagation of a low phase speed (in comparison with the electron and negative ion thermal speeds), long wavelength [in comparison with λDM = (Te/4πne2)0.5] perturbation mode on the time scale of the ion acoustic wave time period and of 'e' being the magnitude of the electron charge] perturbation mode on the time scale of the ion acoustic (IA) waves. The time scale of these IA waves is much faster than the dust plasma period so that dust can be assumed to be stationary. The nonlinear dynamics of this low-frequency, purely electrostatic nonplanar DIA waves in such a DENP is described by

\[
\frac{\partial n_i}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 n_i u_i \right) = 0, \tag{1}
\]

\[
\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial r} (ru_i) = -\frac{\partial \phi}{\partial r} - \sigma_i \frac{\partial n_i}{\partial r}, \tag{2}
\]

\[
\left[ 1 - \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) \right] = \mu_e \exp(\phi) + \mu_n \exp(\sigma_n \phi), \quad -n_i + \mu_1 (1 + Z_d), \tag{3}
\]

\[
\frac{\partial Z_d}{\partial t} = -(I_e + I_n + I_i), \tag{4}
\]

where v=0 for a one-dimensional geometry and v=1 (2) for a nonplanar cylindrical (spherical) geometry, \(n_i\) is the positive ion number density normalized by its equilibrium value \(n_{i0}\), \(u_i\) is the positive ion fluid speed normalized by the ion-acoustic speed \(C_i = (T_i/m_i)^{0.5}\), \(\phi\) is the electrostatic wave potential normalized by \(T_e/e\), \(Z_d\) (normalized by \(Z_{d0}\)) is perturbed part of the number of electrons residing on the dust grain surface, \(Z_{d0}\) is the equilibrium part of the number of electrons residing on the dust grain surface, \(I_e, I_n, \text{ and } I_i\) (normalized by \(Te/e\)), respectively, electron, negative ion, and positive ion current, \(\gamma\) is the nonplanar space variable normalized by \(\lambda_{DM}\) and "I" is the time variable normalized by the plasma period \(\tau_i^{-1} = (m_i/4\pi n_{i0}e^2)^{0.5}\), \(\mu_e = 1/(1 + \alpha + \beta)\), \(\sigma_i = T_i/T_e\), \(\mu_n = \alpha/(1 + \alpha + \beta)\), \(\mu_d = \beta/(1 + \alpha + \beta)\), \(\alpha = n_{d0}/n_{e0}\), \(\beta = Z_{d0}n_{d0}/n_{e0}\), \(n_{d0}\), and \(n_{e0}\) are ion respectively, electron, negative ion, and dust number densities at equilibrium, \(T_i, T_n\) is the positive (negative) ion thermal energy, and \(m_i\) is the ion mass. The normalized electron, negative ion, and positive ion currents \(I_e, I_n, \text{ and } I_i\) are given by
We note that \( I_{e0} = I_{\Phi 0}, \Phi_0 = 0, Z_{d0} = 0, I_{n0} = I_{n0}(\Phi = 0, Z_{d0} = 0), \) and \( I_{i0} = I_{i0}(n_i = 1, Z_{d0} = 0) \) are, respectively, electron, negative ion, and positive ion collection currents at equilibrium.

**Shock Waves**

To study cylindrical and spherical DIA shock waves in a DENP by using (1)-(4), we employ the reductive perturbation technique [31,32]. To do so, we first introduce the stretched coordinates of Maxon and Viecelli [32]

\[
\xi = -\epsilon (r + V_p t),
\]
\[
\tau = \epsilon^2 t,
\]

where \( \epsilon \) is a smallness parameter \( 0 < \epsilon < 1 \) measuring the weakness of the dispersion, and \( V_p \) (normalized by \( C_i \) is the phase speed of the perturbation mode, and expand \( n_i, u_i, \Phi \) and \( Z_d \) in power series of \( \epsilon \) viz.

\[
n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots,
\]
\[
u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots,
\]
\[
\Phi = \epsilon \Phi^{(1)} + \epsilon^2 \Phi^{(2)} + \cdots,
\]
\[
Z_d = \epsilon Z_d^{(1)} + \epsilon^2 Z_d^{(2)} + \cdots,
\]

Now, substituting (5)-(15) into (1)-(4), we develop equations in various powers of \( \epsilon \). We have for the lowest order of \( \epsilon \):
and \( C_1 = \mu_e + \mu_d \alpha_1 \). Equation (19) represents the linear dispersion relation for the DIA waves modified by the presence of the charge fluctuating stationary dust. To the next higher order of \( \varepsilon \), one obtains another set of coupled equations for \( u_i^{(2)} \), \( n_i^{(2)} \), \( Z_d^{(2)} \) and \( \Phi^{(2)} \), which along with (16)-(19), can be reduced to a nonlinear dynamical equation

\[
\frac{\partial \Phi}{\partial \tau} + \frac{\nu}{2\tau} \Phi + A \frac{\partial \Phi}{\partial \xi} = C \frac{\partial^2 \Phi}{\partial \xi^2},
\]

where \( \Phi = \Phi^{(1)} \), \( A \) is the nonlinear coefficient, and \( C \) is the dissipation coefficient. The nonlinear and dissipation for our present purpose read

\[
A = \frac{S_p^2}{2V_p} [A_1 + A_2 + A_3 + A_4 + A_5],
\]

\[
C = \frac{\mu_d V_p \beta_1 S_1 S_2^2}{2V_p I_d S_3},
\]

where \( A_1 = P/S_2^2, \) \( A_2 = (2V_p^0/\beta_1 \gamma_i \mu_d S_1)/(I_d S_2 S_3), \) \( A_3 = -C_2/S_3, \) \( A_4 = -S_4(\sigma_n - \gamma_n S_1)/S_2, \)

\[
A_5 = -\mu_d \alpha_1 (1 - \gamma_e S_1)/S_3 S_5, \]

\[
C_2 = \mu_e + \mu_n \sigma_n^2, \]

\[
P = 3V_p^2 - \sigma_i, \]

\[
S_1 = \alpha_1 + \beta_1 [\sigma_n - \frac{1}{S_2}], \]

\[
S_2 = V_p^2 - \sigma_i, \]

\[
S_3 = 1 + \mu_d \beta_1, \]

\[
S_4 = \beta_1 \mu_d - (\alpha_1 \mu_d)/(1 - \sigma_n), \]

and \( S_5 = 1 - \sigma_n \). Equation (20)
Figure 1: Time evolution of cylindrical $\nu=1$ DIA shock (positive) waves for $n_{i0}=10^9$ cm$^{-3}$, $n_e=10^7$ cm$^{-3}$, $n_{e0}=10^6$ cm$^{-3}$, $r_d=5$ micro meter, $Z_d=0.5$ times $10^4$, and $U_0=0.1$.

Figure 2: Time evolution of spherical $\mu=2$ DIA shock (positive) waves for parameters given in figure-1.
Figure 3  Time evolution of cylindrical (µ=1) DIA shock (negative) waves for $n_{io} = 2 \times 10^9$ cm$^{-3}$, $n_{ne} = 10^6$ cm$^{-3}$, $r_d = 5$ micro meter, $Z_d = 0.5 \times 10^4$, and $U_0 = 0.1$

Figure 4 Time evolution of spherical (µ=2) DIA shock (negative) waves for parameters given in figure-3

Equation [20] is the Burgers equation modified by an extra term $(\nu/2\tau)\Phi$ arising due to the effect of the non-planer cylindrical $\nu=1$ or spherical $\nu=2$ geometry. An exact analytic solution of (20) is not possible. Therefore, we have numerically solved (20), and have studied the effects of cylindrical ($\nu=1$) and spherical ($\nu=2$) geometry on time dependent DIA shock waves in dusty electronegative plasmas. The results are displayed in figures (1-4), where $x=\xi$, and $Y=\Phi$. It is obvious that for a large value of $\tau$, the term $(\nu/2\tau)\Phi$ is negligible. Therefore, we start with a large (negative) value of $\tau$ (viz. $\tau = -30/\tau^*$), and at this negative large value of $\tau$ we choose the stationary shock solution of (20) [without the term $(\nu/2\tau)\Phi$ as our initial pulse:
where $\Phi_0 = U_0/\Delta$ and $\Delta = 2C/U_0$ are, respectively, the height and thickness (normalized by $\lambda_0$) of the DIA shock waves moving with the speed $U_0$. It is obvious that the formation of these DIA shock waves is due to the dust charge fluctuation. The basic features (shock height and shock thickness) of 1D planar DIA shock structures have been studied elsewhere [31]. Our main interest here is to examine the effects of nonplanar (cylindrical or spherical) geometry on these DIA shock structures. We have chosen the parameters corresponding to the recent laboratory electronegative plasma experiments [11,12,10,35] viz. $T_e = 0.69$ eV, $T_i = 0.345$ eV, $T_{e0} = 0.0069$ eV, $n_{e0} = (10^3-10^5)$ cm$^{-3}$, $U_0 = 0.1$, $\mu_i = 39 m_p$, where $m_p$ is the proton mass. The results are displayed in Figures (1-4). It is obvious from Figures (1-4) that in a nonplanar (cylindrical and spherical) DENP system the DIA shock structures exist with both positive ($\Phi > 0$) and negative ($\Phi < 0$) potential. Thus, the formation of shock structures can be categorized into two parametric regimes (one corresponds to positive shock profiles and the other corresponds to negative shock profiles). The polarity of the shock structures independent of the geometry which have been chosen for our present problem (1D, cylindrical, or spherical) but the effects of nonplanar geometry on the shock height have been explicitly observed. We first consider an example of the parametric regime $[n_{e0} = 10^3$ cm$^{-3}$, $n_{e0} = (2*10^4-6*10^5)$ cm$^{-3}$, $n_{e0} = (1*10^5-5*10^5)$ cm$^{-3}$, $r_e = (10-50)$ $\mu$m, and $Z_d = 1^10^{-3}-1^10^{-4}$ for the formation of DIA shock structures with positive potential. Figures (1) and (2) show how the effects of the cylindrical $\nu = 1$ and spherical $\nu = 2$ geometry modify the time dependent DIA shock (positive) structures in DENPs. The numerical solutions of (20) reveal that for a large value of $\tau$ (e.g. $\tau = 30/\tau_1$), the cylindrical and spherical shock structures are similar to 1D planner ones. This is because for a large value of $\tau$ the term $\nu^2\Phi/2\tau$, which is due to the effect of the cylindrical ($\nu = 1$) or spherical ($\nu = 2$) geometry, becomes negligible. However, as the value of $\tau$ decreases, the term $\nu^2\Phi/2\tau$ becomes important, and both the cylindrical and spherical DIA shock (positive) waves significantly differ from 1D planner ones. It is found that as the value of $\tau$ decreases, the amplitude of these localized pulses increases. It is also found that the amplitude of the cylindrical shock (positive) structures is larger than that of the 1D planner ones, but smaller than that of the spherical ones. We now consider the example of the parametric regime $[n_{e0} = 10^3$ cm$^{-3}$, $n_{e0} = (5*10^4-7*10^4)$ cm$^{-3}$, $n_{e0} = (5*10^4-10^5)$ cm$^{-3}$, $r_e = (0.1-2)$ $\mu$m, and $Z_d = (0.5*10^{-3}-0.9*10^{-4})$ for observing the existence of shock structures with negative potential. The time variation of negative shock structure with height in cylindrical (spherical) geometry is shown in Figure 3 (4). It is found that as the value of $\Delta\tau$s decreases, the amplitude of these localized pulses increases. It is also found that the amplitude of the cylindrical shock (negative) structures is larger than that of the 1D planner ones, but smaller than that of the spherical ones.

### Solitary Waves

To derive a dynamical equation for the nonplanar (cylindrical and spherical) DIA solitary waves from our basic equations (1)-(4), we again employ the reductive perturbation technique [31] with another stretched coordinates of Maxon and Viarelli [32]

$$\xi = -e^{1/2}(r + V_p t),$$
$$\tau = e^{3/2}t.$$  \hspace{1cm} (24)

$$\nu \frac{\partial \Phi}{\partial \tau} + \frac{\nu}{2r} \Phi + A \Phi \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = 0,$$  \hspace{1cm} (26)

where $\Phi = \Phi^1$ and the nonlinear coefficient $A$ is exactly the same as appeared in (21), and the dispersion coefficient $B$ is given by

$$B = \frac{(V_p^2 - \sigma_i^2)}{2V_p(1 + \mu_d\beta_1)}.$$  \hspace{1cm} (27)

Equation (26) is a modified Korteweg-de Vries (mKdV) equation describing the nonlinear propagation of the DIA solitary waves in the dusty electronegative plasma under consideration. The extra term $\nu^2\Phi/2\tau$ in (26) is due to the effect of the non-planar cylindrical ($\nu = 1$) or spherical ($\nu = 2$) geometry. We have numerically solved (26), and have studied the effects of cylindrical ($\nu = 1$) and spherical ($\nu = 2$) geometries on time dependent DIA solitary waves. The results are displayed in figures (5-8), where $x = \xi$, and $Y = \Phi$. We should note that for a large value of $\tau$, the term $\nu^2\Phi/2\tau$ is negligible. So, in our numerical analysis, we start with a large (negative) value of $\tau$ (viz. $\tau = 30^{-1}$), and at this large (negative) value of $\tau$, we choose the stationary solitary wave solution of (26) [without the term $\nu^2\Phi/2\tau$] as our initial pulse:
\[ \Phi(\nu = 0) = \phi_0^{(1)} \text{sech}^2 \left[ \frac{\xi - U_0 \tau}{\Delta} \right] , \]  

(28)

where \( \Phi_0 = 3U_0/A \) and \( \Delta = (4B/U_0)^{0.5} \) are, respectively, the amplitude and the width of the stationary solitary waves for \( \nu = 0 \). The general expressions for the coefficients \( A \) [by using (21)] are used to have some numerical appreciations of our results, viz. the solitary wave height in nonplanar geometry is numerically analyzed. The parameters corresponding to the recent laboratory electronegative plasma experiment \([11,12,10,32]\), which are used to analyze the shock structures, have chosen.

Figure 5: Time evolution of cylindrical (\( \nu = 1 \)) DIA solitary (positive) waves for \( n_0 = 2 \times 10^9 \text{ cm}^{-3} \), \( N_{n0} = 10^8 \text{ cm}^{-3} \), \( n_{e0} = 10^6 \text{ cm}^{-3} \), \( r_d = 30 \mu \text{m} \), \( Z_d = 5 \times 10^3 \), and \( U_0 = 0.1 \).

Figure 6: Time evolution of spherical (\( \mu = 2 \)) DIA solitary (positive) waves for parameters given in figure-5
Figure 7 Time evolution of cylindrical (ν=1) DIA solitary (positive) waves for $n_0=1.5\times10^{10}\text{cm}^{-3}$, $n_{\text{eq}}=1.5\times10^6\text{cm}^{-3}$, $r_d=2\mu\text{m}$, $Z_d=5\times10^3$, and $U_0=0.1$

Figure 8 Time evolution of spherical (ν=2) DIA solitary (negative) waves for parameters given in figure-7
DISCUSSION

We have considered a consistent and realistic DENP (containing Boltzmann electrons, Boltzmann negative ions, inertial positive ions, and charge fluctuating stationary dust), and investigated the nonlinear propagation of the low phase speed DIA waves by the reductive perturbation method. We have studied the time dependent DIA shock and solitary structures in DENPs of non-planner geometries. It is found that the basic features of cylindrical and spherical shock and solitary structures are different from those of 1D planar structures. The cylindrical DIA shock and solitary waves travel faster than the planer ones, but slower than the spherical ones. The amplitude of the cylindrical DIA shock and solitary waves in DENPs is larger than that of the planer ones, but smaller than that of the spherical ones. It has been observed that depending on the value of dust charge density, the dusty electronegative plasma system (under consideration) supports either positive or negative shock/solitary potential structures. It is shown that the presence of negatively charged dust causes to change the polarity of the nonlinear coefficient "A", and thus causes to form the negative shock (solitary) potential structures. It is a matter of great interest that the existence of positive or negative shock (solitary) waves are independent of the geometrical system whether the planar or nonplanar geometry has been considered. In our numerical analysis, we have used the ranges corresponding to the experimental conditions of Ghim and Hershkowitz [22]. We finally propose to perform a new laboratory experiment by the experimental set up of nGhim and Hershkowitz [cite{22}], which will be able to detect the nonplanar DIA shock or solitary structures, and to identify their basic features predicted in this theoretical investigation. It may also be added here that our investigation is valid for small amplitude DIA shock and solitary waves and for unmagnetized and uniform dusty electronegative plasma system. However, arbitrary amplitude DIA shock or solitary waves in uniform/nonuniform dusty plasma with or without the effects of obliqueness and external magnetic field are also problems of recent interest for many space and laboratory dusty plasma situations, but beyond the scope of our present investigation.

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