Maximized lateral inhibition in paired magnetic domain wall racetracks for neuromorphic computing

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Abstract

Lateral inhibition is an important functionality in neuromorphic computing, modeled after the biological neuron behavior that a firing neuron deactivates its neighbors belonging to the same layer and prevents them from firing. In most neuromorphic hardware platforms lateral inhibition is implemented by external circuitry, thereby decreasing the energy efficiency and increasing the area overhead of such systems. Recently, the domain wall—magnetic tunnel junction (DW-MTJ) artificial neuron is demonstrated in modeling to be intrinsically inhibitory. Without peripheral circuitry, lateral inhibition in DW-MTJ neurons results from magnetostatic interaction between neighboring neuron cells. However, the lateral inhibition mechanism in DW-MTJ neurons has not been studied thoroughly, leading to weak inhibition only in very closely-spaced devices. This work approaches these problems by modeling current- and field- driven DW motion in a pair of adjacent DW-MTJ neurons. We maximize the magnitude of lateral inhibition by tuning the magnetic interaction between the neurons. The results are explained by current-driven DW velocity characteristics in response to an external magnetic field and quantified by an analytical model. Dependence of lateral inhibition strength on device parameters is also studied. Finally, lateral inhibition behavior in an array of 1000 DW-MTJ neurons is demonstrated. Our results provide a guideline for the optimization of lateral inhibition implementation in DW-MTJ neurons. With strong lateral inhibition achieved, a path towards competitive learning algorithms such as the winner-take-all are made possible on such neuromorphic devices.

Keywords: lateral inhibition, winner-take-all, domain wall motion, spin transfer torque, neuromorphic computing

(Some figures may appear in colour only in the online journal)

1. Introduction

Conventional von Neumann architecture has been the dominant large-scale computer architecture for the last five decades. Thanks to the rapid advancement of CMOS technology, shrinking transistor size and increased transistor density have been following Moore’s law, e.g. each smaller node brings about both performance improvement and cost reduction. However, the throughput of a von Neumann computer is largely limited by the von Neumann memory wall [1], i.e. the separation of memory and central processing unit (CPU), and the sequential mode of instruction execution [2]; also, the von Neumann computer is energy-hungry due to the intensive data transfers between CPU and memory units [3]. In order to mitigate speed and power bottlenecks in the von Neumann architecture, research efforts have been directed towards the
development of non-von Neumann computation paradigms with high parallelism and power-efficiency. The neuromorphic computing paradigm draws inspiration from the biological neural system, which consists of vast numbers of processing units, i.e., neurons, interconnected with synapses that carry the weights of neuron connectivity. Due to the in-memory computation nature and high parallelism, neuromorphic computing can outperform the von Neumann machine in speed and power efficiency [3, 4].

The fundamental block of the artificial neural network (ANN) is the artificial neuron. It electrically mimics the biological neuron whose behavior can be described by an integrate-and-fire (IF) process [5]: the neuron receives electrical signals from its neighboring cells (reception), builds up its membrane potential (integration) and, once the potential exceeds a threshold voltage, generates a spike or action potential that is sent down to one or more post-synaptic cells (firing). The IF process omits many intricate biological details in favor of essential features of behavior, and is thus particularly useful in studying neural network dynamics. Extensions of the IF process include leaky integrate-and-fire (LIF) [6], adaptive quadratic integrate-and-fire [7], and adaptive exponential integrate-and-fire [8]. Some of these approaches have been adopted in neuromorphic computing platforms [9].

Lateral inhibition (LI) is another important neuron feature, closely associated with biological sensory systems. Receptive fields of tactile, auditory, and visual systems have center-surround responses to local stimuli: neurons pick up both presence of stimuli at the center and the absence thereof in the surrounding region, enhancing the signal contrast [5]. This function can only be achieved if central neurons inhibit the activity of peripheral, less-active neighbors in the same layer.

In neuromorphic computing, LI is crucial to the winner-take-all (WTA) algorithm [10, 11]: in a neuron layer, mutual inhibition of the neurons should be strong such that only the most active neuron can produce a spiking output. The system’s ability to pick a winner is necessary to competitive learning [12–14], pattern recognition [15, 16], and general-purpose self-organizing networks [17]. It has also been shown to greatly improve the computing power of a neural network: for example, in one CMOS implementation of vector matrix multiplication, it was shown that including WTA gave a one-layer neural network the computing power equivalent to a two-layer neural network [18]. LI is also a well-known mechanism of \( k \)-WTA, a more general form of WTA which selects \( k \) most active neurons as winners. CMOS implementations of LI typically require additional circuit components such as differential amplifiers [19], a global reference voltage [20], or feedback loops [10]; in a hybrid memristor-MOS crossbar array [21], the inhibitory relation between neurons is realized by recurrently connecting neurons with memristor synapses. While LI or WTA functionalities have been successfully realized in these hardware platforms, the following drawbacks exist: (1) peripheral circuitry reduces power efficiency; (2) circuit design and layout are of great complexity; and (3) occupied chip area significantly increases with larger neuron numbers and connectivity. The overhead and energy cost is non-negligible in larger systems: for example, in one CMOS-based WTA implementation, five additional transistors are required per output neuron of a layer [22]. Therefore, an energy-efficient, simple, and scalable LI implementation is highly desirable.

Recently, a LIF neuron called domain wall—magnetic tunnel junction (DW-MTJ) neuron was demonstrated in simulation to be intrinsically inhibitory via magnetic interactions [23]. The neuron prototype is based on the three-terminal magnetic DW logic device [24] shown in the figure 1(a) side-view cartoon. It consists of a perpendicularly magnetized wire containing a single DW and an MTJ sitting on top of the wire. When current of density \( J_r \) is applied to the wire, the DW propagates along the \(+z\) direction through spin transfer torque (STT) or spin orbit torque (SOT). The MTJ defines the firing point of the neuron: when the DW moves past the junction, the wire magnetization under the MTJ is aligned with the top pinned ferromagnet layer, switching the MTJ resistance state low and generating a spiking current \( I_{OUT} \) at the MTJ output terminal, which can be grounded at the subsequent device. Since DW velocity \( v_{DW} \) increases with current density \( J_z \), the neuron with higher current density has a higher chance to fire, and is therefore more active. Such DW-MTJ neurons have been shown in simulation to have energy-efficient behavior at both the device [25] and circuit [26] levels.

The inhibitory relation between a pair of DW-MTJ neurons is illustrated by figure 1(b). The two neurons are referred to as the neuron of interest Neuron I and its neighbor Neuron N, each with a single DW named DW\(_I\) and DW\(_N\) respectively. The DWs are driven by electrical current with density \( J_{D,1} < J_{D,2} \), so that the DW velocity \( v_{DW,I} < v_{DW,N} \) and the active Neuron N...
will be the first to fire. DW$_I$ falls behind DW$_N$ and is subjected to a stray field $\vec{B}_{\text{stray}}$ from Neuron N in the $-\hat{z}$ direction; on the contrary, DW$_N$ experiences a stray field of Neuron I of same magnitude in the $+\hat{z}$ direction. Thus, the magnetic force experienced by a DW is determined by its relative position with its neighbor. We will show that if the magnitude of the stray field in $-\hat{z}$ is carefully chosen, it can serve as an inhibitory force to prevent the firing of the inactive neuron (Neuron I); on the contrary, the stray field in $+\hat{z}$ has much less impact on DW motion.

This work focuses on investigating the LI mechanism, maximizing LI in a pair of DW-MTJ neurons, and understanding the design parameters to tune the LI based on material and device parameters. Current- and field-driven DW motion is first simulated for the two-neuron system to find the optimal stray field magnitude for LI. The results are then explained by modeling the velocity characteristic of current-driven domain wall motion in a single neuron in response to an external magnetic field. We further quantify our simulation results with calculations based on the Landau–Lifshitz-Gilbert (LLG) equation, which reveal the impact of device geometry and material parameters on the magnitude of the largest achievable LI. Finally, we will propose a DW-MTJ neuron array architecture which exhibits LI-induced neuron competition.

2. Methods

We model a pair of side-by-side magnetic wires with perpendicular magnetic anisotropy (PMA), each containing a single DW driven by electrical current via STT. We assume the MTJ output has negligible contribution to stray field and is omitted in the simulation. Both wires have dimensions 5 $\mu$m $\times$ 50 nm $\times$ 1.3 nm in length ($x$), width ($y$) and thickness ($z$), and are spaced s nm apart in $\hat{y}$. The wire width is chosen to be large enough to investigate this effect for feasibility-fabricated prototypes; the results can be scaled to smaller widths and spacings. All simulations are carried out in Muma3 [27]. Simulation cell size is 2 nm $\times$ 5 nm $\times$ 1.3 nm and material parameters are those of CoFeB [28]: saturation magnetization $M_s = 1273$ emu cm$^{-3}$, anisotropy constant $K_U = 1 \times 10^7$ erg cm$^{-3}$, exchange stiffness $A_{ex} = 1.3 \times 10^{-6}$ erg cm$^{-1}$, Gilbert damping constant $\alpha = 0.02$, STT non-adiabatic parameter $\beta = 0.04$, and spin polarization $P = 0.72$. As above, whether a neuron can be inhibited depends on the magnitude of its activity relative to its neighbor’s activity. In terms of the DW-MTJ neuron whose activity is encoded in DW velocity $v_{DW}$, LI can be quantified based on the reduction of $v_{DW}$ when a neuron is inhibited:

$$LI = \frac{v_{DW}(\text{non-inhibition}) - v_{DW}(\text{inhibition})}{v_{DW}(\text{non-inhibition})} \times 100\%. \tag{1}$$

LI can be regarded as an intrinsic parameter of the inhibition neuron device. Denoting the DWs in the two wires as the DW of interest DW$_I$ and its neighbor DW$_N$, the two conditions of DW$_I$ motion are: (a) inhibition condition $J_A < J_N$ and (b) non-inhibition condition $J_A > J_N$. At simulation time $t = 0$ ns, a Neél-type DW is initialized at $x = 0$ nm for each wire to satisfy the fair start condition; electrical currents are then applied to both wires driving DW$_I$ and DW$_N$ along $+\hat{x}$. For inhibition condition $J_A = 2.2 \times 10^{12}$ A m$^{-2}$ and $J_N = 4 \times 10^{12}$ A m$^{-2}$; for non-inhibition condition $J_A = 2.2 \times 10^{12}$ A m$^{-2}$ and $J_N = 0$ A m$^{-2}$. DW positions and velocities are extracted from the time evolution of the wire magnetization and LI is then calculated according to equation (1).

3. Results and discussion

We first investigate the dependence of LI on the magnitude of magnetostatic interaction. For this purpose, we increase neuron spacing $s$ from 10 nm to 150 nm by steps of 10 nm and simulate the corresponding DW$_I$ velocity $v_{DWI}$. In figure 2(a), DW$_I$ position ($x$) as a function of time ($t$) for $s = 30$ nm, 60 nm, 90 nm, and 120 nm under inhibition and non-inhibition conditions are compared. Two distinct $x$-$t$ characteristics of the inhibited DW$_I$ motion (red solid curves) are observed as $s$ increases: in the strong magnetic interaction regime $s = 30$, 60 nm, $x$-$t$ of DW$_I$ motion is non-linear due to Walker breakdown (WB) [29] i.e. DW magnetization orientation (DW azimuthal angle $\varphi$) precesses in the $x$-$y$ plane as DW$_I$ translates along $\hat{x}$. This type of DW motion occurs when the external magnetic field is larger than the Walker field. In the weak magnetic interaction regime $s = 90$, 120 nm, $x$-$t$ of DW$_I$ motion is linear except in the first $\approx 10$ ns of simulation. During this short settling period, the DW angle changes from its initial configuration ($\varphi = 0^\circ$, Neél wall) to an orientation determined by the strength of magnetic stray field, and remains unchanged onwards. This corresponds to the below-Walker breakdown (sub-WB) or steady DW motion, for which the DW experiences a magnetic field smaller than the Walker field. On the other hand, for each simulated spacing, $x$-$t$ of non-inhibited DW$_I$ motion (blue dotted curves) is non-linear, indicating WB condition for all these cases. DW$_I$ velocities $v_{DWI}$ for the inhibition and non-inhibition cases and the derived LI according to equation (1) as a function of $s$ are shown in figure 2(b). $v_{DWI}$ is taken as an average value in case of the non-linear motion, and as the settled constant velocity in case of the linear motion. For the chosen material and geometry parameters, at $s = 90$ nm we see that $v_{DWI}$ is drastically reduced from 79 ms$^{-1}$ under non-inhibition condition to 20 ms$^{-1}$ under inhibition condition, and LI is at a maximum of 75%. Based on neuron geometry, material and spacing $s = 90$ nm, we estimate the stray field acting on DW$_I$ in inhibition case to be $H_s = -9$ Oe [30]. Compared to the degree of LI shown in [23], here LI is largely enhanced by means of optimizing wire interaction strength.

These results suggest that for a given neuron geometry, maximum LI is achieved when the magnetic interaction strength coincides with the WB field.\textsuperscript{4} This is confirmed by...
approximating the influence of the neighboring neuron as a uniform vertical magnetic field \( H \) and simulating the response of current-driven DW velocity of a single neuron to such field. Figure 3 shows \( v_{DW} \) as a function of \( H \) ranging from \(-100 \) Oe to \(+100 \) Oe. For each data point, current density \( J_e \) is held at \( 2.2 \times 10^{12} \) A m\(^{-2}\) in simulation. A sub-WB steady motion regime characterized by a high DW mobility \( dv_{DW}/dH \) is observed. The regime is bounded by two Walker limits \( H_{WL} = -9 \) Oe \( \pm 1 \) Oe and \( H_{WU} = -1 \) Oe \( \pm 1 \) Oe, respectively, corresponding to lower-bound and upper-bound of \( v_{DW} \). It is worth noting that even in the absence of an external magnetic field \( (H = 0 \) Oe) DW motion is already in WB regime; when \( H < 0 \) is applied, DW motion can be pushed back to the sub-WB regime. Due to the high DW mobility in this regime, \( v_{DW} \) can either be significantly increased (neuron excitation, \( H_e = H_{WU} \)) or decreased (neuron inhibition, \( H_e = H_{WL} \)). Thus, the maximum LI is achieved when the magnetic interaction strength is equal to \( H_{WL} \), in good agreement with the optimal stray field of \(-9 \) Oe determined from the two-wire simulation.

Having demonstrated the optimized magnetic interaction strength for LI given a set of device parameters, we next focus on maximizing LI in terms of geometric and material parameters. Time evolution of ferromagnet magnetization \( \vec{M} \) is governed by the LLG equation. For the magnetic wire with a one-dimensional DW propagating in \( +\hat{x} \), the LLG equation takes the form:

\[
\frac{\partial \vec{M}}{\partial t} = \gamma \vec{H}_{eff} \times \vec{M} + \alpha \frac{\vec{M} \times \partial \vec{M}}{\partial t} - u \frac{\partial \vec{M}}{\partial \vec{x}} + \beta u \frac{\vec{M} \times \partial \vec{M}}{\partial \vec{x}},
\]

(2)

Here \( \gamma \) is gyromagnetic ratio, \( H_{eff} \) is the total effective magnetic field including external field \( \vec{H}_{ext} \) and demagnetization field \( \vec{H}_d \), and \( u = (g\mu_B P/2eM_e)J_e \) is proportional to current density and has the dimensions of velocity, where \( g \) is the Landé g-factor, \( \mu_B \) is the Bohr magneton, and \( e \) is the electron charge.

We use the macroscopic approach described in [31] which treats DW propagation as the result of different torques acting on DW; this yields the relation between the DW angle \( \phi \), vertical external field \( H_z \), and \( u \):

\[
\sin 2\phi = \frac{H_z + (\beta - \alpha) \frac{u}{\gamma s}}{2\pi \alpha M_s K_{\perp}}
\]

(3)

where \( \delta \) is the DW width and hard axis anisotropy \( K_{\perp} = N_x - N_y \) is the difference of DW demagnetization factors in \( \hat{x} \) and \( \hat{y} \) directions, and is proportional to the demagnetization energy difference between Neél- and Bloch-type DWs [32].

From equation (3), \( \phi \) can have a time-independent solution...
\( \varphi = \varphi_0 \) only when the condition \(|\sin 2\varphi| \leq 1\) is satisfied; otherwise \( \varphi \) must be a time-varying quantity \( \varphi = \varphi(t) \). We therefore obtain the two Walker limits \( H_{\text{WU}} \) and \( H_{\text{WL}} \):

\[
H_{\text{WU}} = 2\pi \alpha M_s K_{\perp} - (\beta - \alpha) \frac{u}{\gamma \delta},
\]

and the conditions for steady and WB motions:

- steady, \( H_{\text{WL}} \leq H_z \leq H_{\text{WU}} \)
- WB, \( H_z < H_{\text{WL}} \) or \( H_z > H_{\text{WU}} \).

Accordingly, instantaneous \( v_{\text{DW}} \) is a function of \( \varphi_0 \) or \( \varphi(t) \):

- steady, \( v_{\text{DW}} = 2\pi \gamma \delta M_s K_{\perp} \sin 2\varphi_0 + u \)
- WB, \( v_{\text{DW}}(t) = \frac{1}{1 + \alpha} \times [2\pi \gamma \delta M_s K_{\perp} \sin 2\varphi(t) + (1 + \alpha \beta)u + \alpha \gamma \delta H_z] \).

Given that \( \alpha, \beta \ll 1 \) and that the stray field magnitude is far smaller than the wire saturation field \( H_z \ll 2\pi M_s \), equation (7) takes the approximate form:

\[
v_{\text{DW}}(t) \approx 2\pi \gamma \delta M_s K_{\perp} \sin 2\varphi(t) + u.
\]

Comparing equations (6) and (8) validates that given a weak stray field, instantaneous \( v_{\text{DW}} \) has the same dependence on \( \varphi \) for steady and WB motions. This is confirmed by extracting the \((\varphi, v_{\text{DW}})\) relation from two-wire simulation results (figure 4). For \( s = 60 \text{ nm} \), \( DW \) has WB motion and \( \varphi \) changes in the range of \((0, 2\pi)\); as a result, \( v_{\text{DW}} \) changes with \( \varphi \) and reaches the minimum \( v_{\text{min}} = 20 \text{ m}\text{s}^{-1} \) at \( \varphi_{\text{WL}} = -\pi/4 \) and the maximum \( v_{\text{max}} = 125 \text{ m}\text{s}^{-1} \) at \( \varphi_{\text{WU}} = +\pi/4 \). For the larger spacings \( s = 90 \text{ nm}, 110 \text{ nm}, 130 \text{ nm}, \) and \( 150 \text{ nm}, \) the stray field from the neighboring wire brings \( DW \) to the steady motion regime, and \( \varphi \) eventually settles to a fixed value. In such cases the \((\varphi, v_{\text{DW}})\) relations are represented by single dots located on the \( s = 60 \text{ nm} \) curve. Notably, \( v_{\text{min}} \) at \( \varphi_{\text{WL}} = -\pi/4 \) is achieved for \( s = 90 \text{ nm} \). This confirms the drastic lowering of the \( v_{\text{DW}} \) at the optimized spacing earlier visible in figure 2(b), and therefore the large and maximized LI, arises from the neighboring wire’s stray magnetic field setting \( \varphi \) to the minimum velocity angle.

Equation (6) can therefore be used to select the material and geometry parameters to maximize LI. Besides tuning the DW angle \( \varphi \), the minimum velocity is equal to \( -2\pi \gamma \delta M_s K_{\perp} + u \). Figure 5 summarizes the influence of saturation magnetization \( M_s \), wire width \( w \), and anisotropy constant \( K_{\perp} \) on the largest achievable LI. For each set of parameters, a two-wire simulation is carried out and LI has been maximized in terms of wire spacing \( s \).

Figure 5(a) shows that the LI is maximized for smaller wire widths \( w \). We attribute the LI dependence on \( w \) to the change of \( K_{\perp} \). As is mentioned, \( K_{\perp} \) is proportional to the demagnetization energy difference of Bloch and Neél walls. Bloch wall energy increases as \( w \) becomes smaller because of the larger surface poles induced on the sidewalls, thereby increasing \( K_{\perp} \). The impact of \( M_s \) on LI is also shown in figure 5(a).

Here, for each \( M_s \) examined, we keep PMA quality factor \( Q = K_{\text{U}}/2\pi M_s^2 = 1 \) by choosing \( K_{\text{U}} \) such that both \( \delta \) and \( K_{\perp} \) are mainly determined by wire aspect ratio \( w/t \). For all \( w \) the LI is

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**Figure 4.** Dependence of instantaneous DW velocity \( v_{\text{DW}} \) on DW angle \( \varphi \), extracted from the two-wire simulations. \( v_{\text{DW}} \) is plotted as spokes out from \( v_{\text{DW}} = 0 \text{ m}\text{s}^{-1} \) at center, vs. \( \varphi \). Both precessional (\( s = 60 \text{ nm} \)) and steady (\( s = 90 \text{ nm}, 110 \text{ nm}, 130 \text{ nm}, \) and \( 150 \text{ nm} \)) motions are shown.
The largest for highest $M_s = 1273$ emu cm$^{-3}$. According to equation (6), LI should be proportional to $M_s$; however, no substantial difference of LI is observed between $M_s = 1193$ erg cm$^{-3}$ and $M_s = 1114$ emu cm$^{-3}$: this is because although we keep $Q = 1$ to suppress the change of $\delta$, its slight increase for $M_s = 1114$ emu cm$^{-3}$ compared to $M_s = 1193$ emu cm$^{-3}$ is still sufficient to compensate for the reduction in $M_s$.

In figure 5(b), the saturation field is held at $M_s = 1273$ emu cm$^{-3}$ and LI is compared to the anisotropy constant $K_U$ for three wire widths: $w = 30$ nm, 40 nm and 50 nm. For each $w$, since hard axis anisotropy $K_{H}$ is independent of $K_{U}$, the decrease of LI with higher $K_{U}$ is mainly due to the shrinking of DW width $\delta$. Thus, by choosing small $w$ and keeping $Q$ close to 1, DW motion can be almost entirely halted by an inhibition of more than 90%, as is shown for $w = 30$ nm, $M_s = 1273$ emu cm$^{-3}$ and $Q = 1$.

We here compare the magnitude of LI achieved in this work with that in [23]. In the previous work, an external magnetic field of $-200$ Oe is applied to the neuron to implement its leaking functionality. In such strong field the DW-MTJ neuron can only operate in the WB regime, wherein DW mobility is much lower compared to that of the linear regime. Therefore, adjacent neurons must be spaced extremely close to yield substantial LI. In order to implement the leaking feature in DW-MTJ neuron while maintaining a large LI, field-free implementation of leaking employing shape or anisotropy gradients in the neuron design [33, 34] can be adopted. It is worth noting that in these leaking implementations the DW can be already largely inhibited in the wire portion close to the starting point, where wire width $w$ and anisotropy constant $K_{U}$ are both small. Therefore, LI will not be degraded by the gradient-induced larger $w$ and $K_{U}$ close to the firing point.

To show the potential of DW-MTJs to implement WTA functionality, we next demonstrate LI behavior in a DW-MTJ array consisted of e.g. $N = 1000$ side-by-side DW-MTJs arranged as shown in figure 6. We assume that DW motion in each neuron is only influenced by its two immediate neighbors, as will be validated by the array design parameters. The proposed layout adopts alternating neuron spacings $s_1$ and $s_2$ ($s_2 > s_1$) instead of uniform neuron spacings: such design is used to break the spatial symmetry of two immediate neighbors and ensure that they never create a zero net field on the central neuron.

Recalling from the previous simulation results that a neuron can be inhibited only if it experiences a total stray field in $-\mathbf{Z}$, two cases of LI and the corresponding stray fields can be identified for a neuron in the array (figure 6): (a) strong-field LI, where a device’s DW is less active than both its neighbors, $H_{ZS} = -(|H_{Znear}| + |H_{Zfar}|)$ (e.g. black triangle neuron in figure 6); (b) weak-field LI, where a device’s DW is less active than its near neighbor but more active than its far neighbor, $H_{ZW} = -(|H_{Znear}| - |H_{Zfar}|)$ (e.g. grey triangle neuron in figure 6). $s_1$ and $s_2$ should be optimized to yield maximum LI, but since $H_{ZS} \neq H_{ZW}$, only one case of LI can be maximized. To reduce area overhead of the neuron array, we choose to maximize weak-field LI, i.e. choosing $s_1$ and $s_2$ such that $H_{ZW}$ is tuned to the optimal inhibition field $H_{WL}$. Strong-field LI is therefore moderate since in this case total stray field is far above the Walker field.

Due to the limitation of computation resources and time, it is not practical to perform full micromagnetic simulation on a 1000 neuron array. We instead adopt a simulation methodology that combines micromagnetic simulation and numerical calculation: first, three-neuron simulations are carried out in Mumax3 to extract the strong-field and weak-field LI; then, the LI parameters are used to compute $\nu_{PW}$ changes due to LI in a neuron array. This allows us to efficiently model LI in a large neuron array and loop over many possible input current configurations.

In the Mumax3 simulation, neuron geometry and material parameters are the same as those described in section 2; the two spacings are chosen as $s_1 = 60$ nm and $s_2 = 110$ nm to yield desired maximum weak-field LI. Such $s_1$ and $s_2$ also validate the assumption of nearest-neighbor interaction, because the additional stray field from a second-nearest neighbor spaced at distance $s_1 + s_2 + w = 220$ nm is too weak to
Figure 6. Schematic of an array of \(N = 1000\) DW-MTJ neurons, with modulated neuron spacings \(s_1\) and \(s_2\). Neurons with strong-field inhibition (black triangle), weak-field inhibition (grey triangle) and no inhibition (white triangle) depending on their DW position relative to their nearest neighbors are marked.

exert any influence on DW motion. Similar to the method described in the two-neuron simulations, different combinations of current densities are applied to the three wires to create strong-field, weak-field and non-inhibition conditions. Strong- and weak-field LIs are determined to be \(LI_S = 13\%\) and \(LI_W = 75\%\), respectively. In the array simulation, a series of normally-distributed input current densities \(\{J_i\}_i\) \((i = 1, 2, \ldots, N)\) are first generated and randomly assigned to each neuron. The intrinsic DW velocities, that is, \(v_{DW}\) in case of no magnetic interaction between neurons, follow the same distribution since \(v_{DW}\) scales linearly with \(J_e\). Then, the inhibition condition of each neuron is evaluated based on their relative DW velocities compared to their immediate neighbors. Accordingly, \(v_{DW}\) is reduced by proportion \(LI_S, LI_W\) or kept unchanged in case of strong-field, weak-field, or non-inhibition conditions.

The array simulation results are shown in figure 7. Since we are most concerned with the firing statistics of the neuron array instead of the spatial distribution of \(v_{DW}\), intrinsic and inhibited \(v_{DW}\)_s are both ranked in ascending order. It can be seen that \(v_{DW}\) reduction resulting from LI is more significant for smaller intrinsic \(v_{DW}\)_s, while the largest \(v_{DW}\)_s remain unchanged. The interpretation of this result is two-fold: first, \(v_{DW}\) contrast in the array is enhanced due to inhibition, because the less active neurons have a larger probability of being inhibited; second, from the perspective of circuit applications, if we define a fixed firing time window \(\Delta T\) during which spike signals of more than one neuron are accepted, less neurons are able to fire within \(\Delta T\) due to the LI-facilitated competition. This feature makes our proposed DW-MTJ array a promising candidate for implementing a \(k\)-WTA layer, where \(k\) most active neurons are allowed to generate output signals. It is also worth pointing out that implementing hard WTA with the proposed DW-MTJ array is still challenging, since the neuron interaction range is not sufficient to achieve global inhibition.

4. Conclusions

An energy-efficient implementation of strong lateral inhibition in artificial neural networks is crucial to building competitive learning networks with emerging devices. This work proposes a method to maximize lateral inhibition in the domain wall—magnetic tunnel junction (DW-MTJ) neuron. By optimizing
spacing between a pair of DW-MTJs, DW velocity is reduced by as large as 90% under inhibition conditions (i.e. 90% lateral inhibition). Since this large inhibition does not require strong magnetostatic interaction strength in our implementation, adjacent DW-MTJs can be spaced further apart, enabling the fabrication of such devices with standard nanopatterning techniques. We also propose a DW-MTJ array architecture, whose inhibition behavior is suitable for implementing $k$-winner-take-all. This work establishes a materially-feasible basis for intrinsic lateral inhibition in DW-MTJs, motivating experimental efforts to achieve such lateral inhibition in the presence of defects and device imperfections. The results can lead to future implementations of powerful neuro-inspired networks employing winner-take-all layers.

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