The equilibrium state of the dense electron-nuclear plasma in the self-gravitational field. The stellar mass distribution and stellar magnetic fields.

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Abstract
The equilibrium of dense plasma in a self-gravitation is considered. The obtained results radically distinguish from the point of view which is commonly accepted in the astrophysical society. It is important that all these results were obtained without any disputable speculative assumptions. They were obtained on the standard physical base by standard formal methods. The novelty of the obtained results is based on a rejection of the oversimplified ideal gas approximation which is usually accepted for a star interior description and on a taking into consideration the electron-nuclear plasma features. It was shown that there is the minimum for plasma energy at a density and a temperature which determines the equilibrium state of plasma in the self-gravitation at zero gradient of the general parameters of plasma. This effect plays an important role for astrophysics. It enables to explain the mechanism of the star magnetic field generation and to make a prediction for the spectrum of star masses with a quite satisfactory agreement for the observation data.

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1 The current hypothesis
Now it is conventionally accepted to think that the density and the temperature of a star interior substance are growing depthward in a star and can amount to tremendous values at its central core. It seems that this growing is a necessary condition of an equilibrium of a star substance in the self-gravity.

The substance exists as hot dense electron-nuclear plasma at high pressures and temperatures of a star interior. At this condition in the zero approximation, plasma can be considered as a Boltzmann ideal gas with energy

$$E = \frac{3}{2}kTN$$  (1)
where $N$ is the particle number.

Since the direct inter-nuclear interaction in plasma is small, it can be neglected and one can write the equilibrium equation in the form [1]:

$$\mu_e + m'\psi = \text{const}$$

(2)

where $\mu_e$ is the electron chemical potential, $m' = \gamma/n_e$ is the mass of the substance related to one electron, $n_e$ is the electron gas density, $\gamma$ is the mass density of plasma, $\psi$ is the Newton gravitational potential. Because $\Delta\psi = -4\pi G\gamma$ in a spherically symmetric case, Eq.(2) is reduced to

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\mu_e}{dr} \right) = -4G\pi\gamma m'.$$

(3)

The electron gas chemical potential

$$\mu_e = kT\ln \left[ \frac{n_e^{2} \sqrt{2\pi\hbar^{2}/mkT}}{3/2} \right]$$

(4)

is the function of the particle density and the temperature only [1]. At the consideration of the hot non-degenerate plasma in an ideal gas approximation, one can deduce an unambiguous conclusion from the equilibrium equation (Eq.(3)): the balance of plasma particles in the self-gravitation demands temperature and density increasing depthward in a star.

The interparticle interaction is completely neglected in the ideal gas approximation by definition. It is an oversimplified assumption for electron-nuclear plasma, although at the first sight it is acceptable because the interparticle interactions are small in comparison with the ideal gas energy (Eq.(1)). But the allowance of these interactions has a principal importance because they form a stable equilibrium state of hot plasma. It falls out of the consideration when the ideal gas approximation is used.

## 2 The density and temperature of a hot dense plasmas in the equilibrium state

### 2.1 The steady-state density of a hot dense plasma

There are two main characteristic features which we must take into account at hot dense plasma consideration. The first of them is related to the quantum properties of the electron gas. The second feature is concerned with the presence of positively charged nuclei inside the electron gas of plasma.

#### 2.1.1 The correction for Fermi-statistics

The estimation of the electron gas energy in the Boltzmann case ($kT \gg E_F$) can be obtained by a series expansion of the full energy of the non-relativistic Fermi-particle system [1]:

""
\[ E = \frac{2^{1/2}V m_e^{3/2}}{\pi^2 h^3} \int_0^\infty \frac{\varepsilon^{3/2} d\varepsilon}{e^{(\varepsilon-\mu_e)/kT} + 1}, \]  
\[ (5) \]

where \( \varepsilon \) is the energy of a particle. In the Boltzmann case, \( \mu_e < 0 \) and \( |\mu_e/kT| \gg 1 \) and the integrand at \( e^{\mu_e/kT} \ll 1 \) can be expanded into a series according to their powers \( e^{\mu_e/kT - \varepsilon/kT} \).

As a result, taking into account its quantum properties, the hot electron gas full energy obtains the form \( \text{[1]} \)

\[ E = \frac{3}{2} kT \left[ 1 + \frac{\pi^{3/2}}{4} \left( \frac{a_0 e^2}{kT} \right)^{3/2} n_e \right], \]  
\[ (6) \]

where \( a_0 = \frac{k^2}{m_e^2} \) is Bohr radius.

It is important to underline that the correction for Fermi-statistics is positive, because it takes into account that an electron cannot occupy energetic positions filled by other electrons and the resulting pressure is more than the pressure of an ideal gas at exactly the same density and temperature.

2.1.2 The correction for a correlation of charged particles in plasma

At high temperature, the plasma particles tend to uniform space distribution. At this limit, the energy of ion-electron interaction tends to zero. Some correlation in space distribution of particles arises as the positively charged particle groups around itself preferably particles with negative charges and vice versa. It is accepted to estimate the energy of this correlation by the method developed by Debye-Hückel for strong electrolytes \( \text{[1]} \). The energy of a charged particle inside plasma is equal to \( e\varphi \), where \( e \) is the charge of a particle, and \( \varphi \) is the electric potential induced by other particles on the particle under consideration.

This potential inside plasma is determined by the Debye law \( \text{[1]} \):

\[ \varphi(r) = \frac{e}{r} e^{-r/r_D}, \]  
\[ (7) \]

where the Debye radius is

\[ r_D = \sqrt{\frac{kT}{4\pi e^2 n_e}}, \]  
\[ (8) \]

For small values of ratio \( \frac{r}{r_D} \), the potential can be expanded into a series

\[ \varphi(r) = \frac{e}{r} - \frac{e}{r_D} + ... \]  
\[ (9) \]

The following terms are converted into zero at \( r \to 0 \). The first term of this series is the potential of the considered particle. The second term is a potential induced by other particles of plasma on the charge under consideration. And so the correlation energy of plasma is \( \text{[1]} \)
\[ \delta E_{\text{corr}} = -N_e \frac{\pi^{1/2} e^3 kT}{n_e} \left[ Z^2 \frac{n_n}{kT} + \left( \frac{\partial n_e}{\partial \mu_e} \right)_{N,T} \right]^{3/2}, \] (10)

where \( Z \) is the nuclear charge and \( n_n = \frac{n}{2} \) is the density of nuclei.

Since the chemical potential of the Boltzmann ideal gas at high temperature

\[ \frac{d\mu_e}{dn_e} = kT \] (11)

and

\[ \delta E_{\text{corr}} = -N_e \left( \frac{\pi n_e}{kT} \right)^{1/2} (Z + 1)^{3/2} e^3 \] (12)

Since the space correlation an attraction between unlike charges is prevalent over a repulsion of like charges, the plasma pressure is below the pressure of the ideal gas at the same parameters. By this reason this correction has negative sign.

### 2.1.3 The density of hot dense plasma at the equilibrium state

Finally, the full energy of plasma in consideration of both main corrections on inter-particle interaction given by

\[ E = \frac{3}{2} kTN_e \left[ 1 + \frac{\pi^{3/2} n_e}{4} \left( \frac{a_0 e^2}{kT} \right)^{3/2} n_e - \frac{2\pi^{1/2}}{3} \left( \frac{Z + 1}{kT} \right)^{3/2} e^3 n_e^{1/2} \right] \] (13)

At a constant full number of particles in the system and at a constant temperature, the equilibrium state exists at the minimum of energy

\[ \left( \frac{\partial E}{\partial n_e} \right)_{N,T} = 0, \] (14)

what allows one to obtain the steady-state value of density of hot non-relativistic plasma

\[ n_\star = \frac{16(Z + 1)^3}{9\pi^2 a_0^3} \approx 2 \cdot 10^{24} (Z + 1)^3 \text{ cm}^{-3}, \] (15)

The Fermi-energy of electron gas of equilibrium plasma at this density is

\[ \varepsilon_F(n_\star) = \left( \frac{16}{3} \right)^{2/3} \frac{me^4}{2\hbar^2} (Z + 1)^2 \approx 1.5 \frac{e^2}{a_0} (Z + 1)^2 \] (16)
2.2 Equilibrium temperature of a hot non-relativistic star

As the steady-state value of the density of hot non-relativistic plasma is known, we can obtain a steady-state value of the temperature of hot non-relativistic plasma. According to the virial theorem \[1, 3\], the potential energy \( U \) of particles with Coulomb interaction is equal to their double kinetic energy \( T \) with opposite sign

\[
U = -2T \tag{17}
\]

and their full energy is equal to kinetic energy with opposite sign. Neglecting small corrections at a high temperature, one can write the full energy of hot dense plasma as

\[
E_{\text{plasma}} = U + \frac{3}{2} kT N_e = -\frac{3}{2} kT N_e. \tag{18}
\]

As the plasma temperature is high enough, the pressure of the black radiation cannot be neglected. The full energy of a star depending on the plasma energy and the black radiation energy is

\[
E_{\text{total}} = -\frac{3}{2} kT N_e + \pi^2 \left( \frac{kT}{\hbar c} \right)^3 V kT \tag{19}
\]

The equilibrium temperature of a body consisting of hot non-relativistic plasma is determined by the energy minimum condition

\[
\left( \frac{\partial E_{\text{total}}}{\partial T} \right)_{N,V} = 0. \tag{20}
\]

It gives the following value of the equilibrium temperature

\[
T_* = \left( \frac{10}{\pi^4} \right)^{1/3} \left( Z + 1 \right) \frac{\hbar c}{k_0 q} \approx 2 \cdot 10^7 (Z + 1) \text{ K}. \tag{21}
\]

All substances usually have a positive thermal capacity. Therefore the minimal energy for such substances exists at \( T = 0 \). The existence, in our case, of the energy minimum at the finite temperature \( T_* \neq 0 \) is not confusing. Each small part of a star has a positive thermal capacity, but a gravitational interaction of these parts between themselves results in a situation where the thermal capacity of a star as the whole becomes negative at some temperature and a star energy decreases with an increased temperature. As a result there are two branches of the temperature dependence of a star energy - with the negative capacity at low temperatures and with a positive capacity at high temperatures. Between them, at some finite temperature \( T_* \) there is the minimum of the energy.

The steady-state values of the density and the temperature for hot non-relativistic plasma have been considered above. One can see, that the criterion of hot plasma

\[
kT \gg \varepsilon_F \tag{22}
\]
is satisfied according of Eq.16 if the nuclear charge is not too large

\[
\frac{kT_\star}{\varepsilon_F(n_\star)} = \left(\frac{45}{16\pi^4}\right)^{1/3} \frac{\hbar c}{(Z + 1)e^2} \approx \frac{0.3}{Z + 1} \alpha^{-1} \approx \frac{40}{Z + 1},
\]

where \(\alpha = e^2/\hbar c = 1/137\) is the fine structure constant.

3 The equilibrium of a dense electron-nuclear plasma

According to the definition (Eq.11) plasma chemical potential in equilibrium state at constant temperature and density must also be constant:

\[
\mu(n_\star, T_\star) = \text{const}
\]

or

\[
\nabla \mu = 0
\]

How can one obtain an agreement between this condition and the equation of equilibrium in gravitation field (Eq.2)?

The condition of the equilibrium of plasma in gravity field can be obtained if the role of non-gravitational fields, for example, the electric field, is taken into account. More precisely, the equilibrium equation (Eq.2) must include all fields, which can have an impact on particles, for example, electric field for system of charged particles:

\[
\mu + \sum_i q_i \phi_i = \text{const}
\]

(26)

(where \(q_i\) and \(\phi_i\) are charge and potential of any nature (gravitational, electric)).

In a spherical symmetric case at \(\mu = \text{const}\) it reduces to

\[
G\gamma m' = \rho q
\]

(27)

where \(\rho = q \cdot n_e\) is the electric charge density and

\[
q = G^{1/2}m'
\]

is the charge induced in plasma cell by gravity (related to one electron). One should not think that the gravity field really induces in plasma some additional charge. We can rather speak about electric polarization of plasma that can be described as some redistribution of internal charges in the plasma body.

Essentially it stays electrically neutral as a whole, because the positive charge with volume density

\[
\rho = G^{1/2}\gamma
\]

(29)
is concentrated inside the charged plasma core and the corresponding negative electric charge exists on its surface. Since

\[ 4\pi \rho = \text{div} \mathbf{E} \]  

(30)

and

\[ -4\pi G\gamma = \text{div} \mathbf{g} \]  

(31)

the equilibrium equation can be rewritten as

\[ \gamma \mathbf{g} + \rho \mathbf{E} = 0 \]  

(32)

where

\[ \mathbf{E} = \frac{\mathbf{g}}{G^{1/2}}. \]  

(33)

It must be noted, the using of Thomas-Fermi approximation gives possibility to consider the balance in plasma cells in more detail [4].

### 3.0.1 The equilibrium density of another kind of dense plasmas

The above consideration of equilibrium of the hot non-relativistic plasma in a self-gravity is characterized by its obviousness, but the similar equilibrium is not the characteristic property for this kind of plasma only.

The direct consideration of the plasma equilibrium in Fermi-Thomas approximation shows that the application of a gravity field to plasma induces its electric polarization ([4], [12]), so the equilibrium equation in the form Eq. (32) is applicable to all kind of dense plasmas - relativistic or non-relativistic and simultaneously degenerate or non-degenerate ones.

It is essential that one can find the constancy of density and chemical potential of another kind of plasmas in an equilibrium condition.

For a cold non-relativistic plasma the kinetic energy of electron is

\[ E_k = \frac{3}{5} E_F = \frac{3}{10} (3\pi^2)^{2/3} a_0 e^2 n_e^{2/3}. \]  

(34)

Its potential energy is

\[ E_p \approx -e^2 n_e^{1/3}. \]  

(35)

According to the virial theorem \( E_k \approx -E_p \). Thus the equilibrium electron density is

\[ n_e \approx a_0^{-3}. \]  

(36)

and it does not depend on temperature.

The relativistic plasma exists at a huge pressure which induces the neutronization of substance. For this process the density

\[ n_e = \frac{\Delta^3}{3\pi^2 (e\hbar)^3} \]  

(37)
is characteristic \( \Delta \). (Where \( \Delta \) is the difference of nuclear bonding energy of neighbouring interacting nuclei.) Since the difference

\[
\Delta \approx m_e c^2
\]  

(38)

the equilibrium of relativistic plasma density (at condition of a homogeneous mixing of reacting substance)

\[
n_e \approx \frac{1}{3\pi^2}(\alpha a_0)^{-3} \approx 10^{30}, \text{cm}^{-3}
\]  

(39)

where \( \alpha = e^2/\hbar c \) is the fine structure constant, \( a_0 = \hbar^2/m_e e^2 \) is the Bohr radius.

A similar consideration can be extended on the neutron matter if it is considered as electron-proton plasma in the neutron environment.

## 4 The giro-magnetic ratio of stars

Gravitation produces a redistribution of free charges in plasma inside a star. Essentially, a star as a whole conserves its electric neutrality. However, as the star rotates about its axis, positive volume charges are moving on smaller radial distances than the surface negative charge. It induces a magnetic field which can be measured.

The magnetic moment of the surface spherical layer, which carries the charge \( Q \), is

\[
\mu = \frac{Q \Omega R^2}{3c},
\]  

(40)

where \( \Omega \) is rotational velocity, \( Q = \frac{4\pi}{3}\rho R^3 \).

The magnetic moment induced by a volume charge is small because it is concentrated in the small central core for the most part.

On the other hand, the angular momentum of a star is approximately

\[
L \approx \frac{2}{5}M\Omega R^2
\]  

(41)

and the giro-magnetic ratio of a star is expressed through the world constants only:

\[
\vartheta \approx \frac{\mu}{L} \approx \frac{\sqrt{G}}{c}
\]  

(42)

It can be verified by the measurement data.

The values of giro-magnetic ratio for all celestial bodies (for which they are known today) are shown in Fig. 1.

The data for planets are taken from [6], the data for stars are taken from [7], and those for pulsars - from [8]. Therefore, for all celestial bodies - for planets and their satellites, for Ap-stars and several pulsars, which angular momenta themselves change within the limits of more than 20 orders - calculated values of

| Celestial Body | Giro-Magnetic Ratio | Reference |
|---------------|---------------------|-----------|
| Planet        | \( \vartheta \)     | [6]       |
| Star          | \( \vartheta \)     | [7]       |
| Pulsar        | \( \vartheta \)     | [8]       |
Figure 1: The observed values of the magnetic moments of celestial bodies vs. their angular momenta. On the ordinate, the logarithm of the magnetic moment over $Gs \cdot cm^3$ is plotted; on the abscissa the logarithm of the angular momentum over $erg \cdot s$ is shown. The solid line illustrates Eq. (42). The dash-dotted line is the fitting of the observed values.
the gyromagnetic ratio Eq. (42) with a logarithmic accuracy quite satisfactorily agrees with measurements.

5 The stellar mass distribution

5.1 The mass of star consisting of hot non-relativistic dense plasma

Inside the stellar core consisting of hot dense plasma the gravitational force is counterbalanced by the electric force (Eq. (42)) and there is no gradient of pressure. The absence of a pressure gradient inside the hot star core does not mean the absence of pressure. It is not difficult to show that if the gravity force inside a star is compensated by the electric force, the negative energy of gravitational field is canceled by the energy of the electric field. The non-compensated part of the total energy is the energy of gravitational field outside a star. This field has the energy

\[
E_G = -\frac{GM^2}{2R^*}
\]

where \(M^*\) and \(R^*\) are the mass and the radius of the hot plasma core. This external gravitational field endeavours to compress a star.

The virial theorem connects the gravitational energy of a star and pressure inside it [1]

\[
E_G = -3\int PdV,
\]

or in our case (P=const)

\[
E_G = -3PV^*,
\]

where \(V^* = \frac{4\pi R^3}{3}\) is the volume of the star core. Taking into account of the black radiation pressure we obtain

\[
\frac{GM^2}{6RV} = kT^*n^* + \frac{\pi^2}{45}\frac{(kT^*)^4}{(hc)^3}
\]

Finally, for the mass of the star core consisting from hot dense plasma we have

\[
M^* = \frac{5^{1/2}3^{3/2}}{2\pi^{3/2}}\left(\frac{hc}{Gm_p}\right)^{3/2}\left(\frac{Z}{A}\right)^2 m_p \approx 5.42 M_{Ch}\left(\frac{Z}{A}\right)^2,
\]

where \(M_{Ch} = \left(\frac{hc}{Gm_p}\right)^{3/2} m_p = 3.42 \cdot 10^{33} g\) is the Chandrasechar mass. It is important to underline that the obtained stellar mass estimation (Eq. (47)) is depending only on one variable parameter \(A/Z\).

One can note that the equilibrium radius of the star core
\[ R_* = \left( \frac{3/2}{2} \right)^{1/6} \left( \frac{10}{\pi} \right)^{1/6} \left( \frac{hc}{Gm_p^2} \right)^{1/2} \frac{a_0}{(Z+1)A/Z}. \] (48)

also depends on variables \( A \) and \( Z \) only.

### 5.2 The mass of a star consisting of cold relativistic plasma

With then increase of the density, the plasma can turn into a relativistic state. It occurs when Fermi momentum of electrons satisfies

\[ p_F = (3\pi^2)^{1/3} n_{e}^{1/3} \hbar > m_e c \] (49)

This value of momentum corresponds to the steady-state density of a substance under neutronization (Eq. (39)) at \( n_* \approx 10^{39} \text{ cm}^{-3} \). At temperature

\[ T \ll \frac{mc^2}{k} \approx 10^{10} \text{ K}. \] (50)

it can be considered as cold.

As the pressure of the relativistic electron gas is

\[ P_R = \frac{(3\pi^2)^{1/3}}{4} n^{4/3} \hbar c, \] (51)

according to Eq. (45), the pressure balance obtains the form:

\[ \frac{GM^2}{6R_*V_*} = \frac{(3\pi^2)^{1/3}}{4} n^{4/3} \hbar c, \] (52)

Therefore, the relativistic degenerate star in equilibrium state must have the steady value of mass

\[ M_* = 1.5^{5/2} \pi^{1/2} \left( \frac{hc}{Gm_p^2} \right)^{3/2} \frac{m_p}{(A/Z)^2} \approx \frac{4.88M_{\odot}c}{(A/Z)^2} \] (53)

at the radius corresponding to Eq. (39)

\[ R_* \approx \left( \frac{hc}{Gm_p^2} \right)^{1/2} \frac{a_0}{(A/Z)} \approx \frac{10^{-2}R_{\odot}}{A/Z}. \] (54)

The objects which have such masses and density are best suited to the dwarfs.

### 5.3 The comparison of calculated star masses with observations

The comparison of calculated results with the data of measurements is shown in Fig. 2. There is a large quantity of star mass measurements but only those, which was obtained from the measurement of binary star parameters, have a
Figure 2: The mass distribution of binary stars [9]. On abscissa, the logarithm of the star mass over the Sun mass is shown. Solid lines mark masses which agree with selected values of $A/Z$ from Eq.(47) for stars. The dotted lines mark masses which agree with selected values of $A/Z$ from Eq.(53) for dwarfs.

sufficient accuracy only. The mass distribution of visual and eclipsing binary stars [9] is shown in Fig.2. On abscissa, the logarithm of the star mass over the Sun mass is plotted. Solid lines mark masses which agree with selected values of $A/Z$ for stars from Eq.(47). The dotted lines mark $A/Z$ for dwarfs from Eq.(53).

According to the existing knowledge, the hydrogen inside dwarfs is fully burnt out. In full agreement with it in Fig.2 there are hydrogen stars and there are not dwarfs with $A/Z = 1$, whereas there are both - helium-deuterium stars and dwarfs - with $A/Z = 2$. Attention should be attracted to the fact that there is the peak in the star distribution consisting of heavy nuclei with $A/Z = 2.8$. The nuclei with $A/Z > 2.8$ are absent in the terrestrial condition. This ratio of $A/Z$ is the limit value for the stable nuclei at a relatively small pressure. One can expect that at high pressure which can induce the neutronization process, the neutron-excess nuclei with large $A/Z$ ratio obtain stability. Attention is attracted to the peak for dwarfs with $A/Z = 3$ where the Sun is placed. It seems that stars composed of nuclei with $A/Z$ ratio up to 7 are presented in this spectrum however this question is complicated and demands a special and more attentive consideration.
5.4 Masses of star cores, composed by another kinds of plasma.

Let us take a quick look on star cores composed by another kinds of plasma. As before we will proceed from the virial theorem ratio (45).

5.4.1 Celestial bodies composed by cold non-relativistic plasma

The cold plasma exists at temperatures $T << T_F$ and has a pressure

$$P = \frac{(3\pi^2)^{2/3} \hbar^2}{5m} \left( \frac{\gamma}{m_p A/Z} \right)^{5/3}.$$  

(56)

It gives a possibility to represent the equilibrium equation (45) as

$$\frac{GM^2}{2R} = 3V \frac{(3\pi^2)^{2/3} \hbar^2}{5m} \left( \frac{\gamma}{m_p A/Z} \right)^{5/3}.$$  

(57)

and to obtain the expression for a mass of the body

$$M = M_{Ch} \left( \frac{\hbar}{mc} \right)^{3/2} \left( \frac{\gamma}{m_p} \right)^{1/2} \frac{6^{3/2}9\pi}{4(A/Z)^{5/2}}.$$  

(58)

As the degenerate non-relativistic plasma has the density $\gamma \approx 1 \text{ g/cm}^3$, we obtain

$$M \approx 1.26 \cdot 10^{-3} \frac{M_{Ch}}{(A/Z)^{5/2}} = \frac{4.3 \cdot 10^{30}}{(A/Z)^{5/2}} \text{ g}.$$  

(59)

At this density, the degeneration temperature $T_F \approx 10^5 K$. Thus the temperature of the object under consideration should not exceed several thousands degrees. Among celestial bodies, only planets possess these properties.

The comparison of the obtained estimation (59) and the measured data for Solar system planets is shown in Fig. 3. It is seen that the obtained estimation is in agreement with the value of masses of large planets at $A/Z \approx 2$. It is important to note that according to Eq. (59), planets with masses more than $10^{31} \text{ g}$ must not exist. In reality only the Jupiter has the mass of this level.

5.4.2 Do stars consisting of hot ultra-relativistic plasma represent quasars?

A star may be considered as a hot one if the temperature of radiation inside it is much higher than the degeneration temperature of its electron gas:

$$\frac{T_R}{T_F} \gg 1.$$  

(60)
Figure 3: The dependence of the core mass of planets over ratio $A/Z$ (Eq. (59)) at $\gamma = 1 \, g/cm^3$. On the ordinate, the logarithm of mass (over 1 $g$) is plotted.

Figure 4: The mass distribution of galaxies [10]. In abscissa, the logarithm of the galaxy mass over the Sun mass is shown.
For non-relativistic star, this ratio is approximately equal to $1/3\alpha$ (Eq. 23). At this condition, the pressure of degenerate electron gas can be neglected and the equilibrium equation takes the form

$$\frac{GM^2}{6RV} = \frac{\pi^2}{45} \left(\frac{kT}{\hbar c}\right)^4 \approx \left(\frac{T_R}{T_F}\right)^3 kT_R n$$  \hspace{1cm} (61)

It gives a possibility to estimate the mass of a hot ultra-relativistic star

$$M \approx \left(\frac{T_R}{T_F}\right)^6 M_{Ch}$$  \hspace{1cm} (62)

According to the existing knowledge, among compact celestial objects only quasars have masses of this level.

Apparently it is an agreed-upon opinion that quasars represent a relatively short stage of the evolution of galaxies. If to adhere to this hypothesis and because of the lack of information about quasar mass distribution, we can use the distribution of masses of galaxies to check our estimation (Fig 4).

It can be seen that obtained quasar equilibrium conditions (Eq. (61), Eq. (62)) is in agreement with the observation data at

$$10 < \frac{T_R}{T_F} < 100$$  \hspace{1cm} (63)

and, as for non-relativistic hot stars, some maximum exists at the equilibrium condition of (Eq. 23):

$$\frac{T_R}{T_F} \approx \frac{1}{3\alpha}$$  \hspace{1cm} (64)

Thus, the conclusion that quasars consist of a hot relativistic plasma is not contradict with the observations.

### 5.4.3 The mass of a star consisting of neutron matter

Dwarfs and quasars may be considered as stars where the process of neutronisation is just beginning. Finally, at nuclear density, plasma turns into neutron matter.

It is agreed that a pulsar is a star consisting of neutron matter with some impurity of other particles - electrons and protons. Evidently, at nuclear density neutrons and protons are indistinguishable inside pulsars as well as inside a huge nucleus. The neutron matter can be considered as a kind of plasma where electrons and protons exist in a neutron environment. Under this condition a very small impurity of electrons and protons (about a level of $10^{-18}$) is enough to induce a sufficient electric polarization and to balance the action of gravity.

It is known from nuclear physics, that a nuclear matter is incompressible one. By this reason one can expect that its mass density inside neutron star, similarly as for atomic nuclei, approximately equals $3 \cdot 10^{14} g/cm^3$ at partial density $1.8 \cdot 10^{38} cm^{-3}$. At this density it may be considered as cold at $T \ll$
$10^{12} K$. It is important to note that the neutron gas is not ultra-relativistic at the nuclear density. It is a relativistic gas only when the Fermi momentum of neutrons is

$$\frac{p_F}{m_p c} \approx 0.36.$$  \hspace{1cm} (65)

The pressure of this neutron gas is described by the complicated equation in general case [1]:

$$P = \frac{m^4 c^5}{32 \pi^2 \hbar^3} \left[ \frac{1}{3} \text{sh} \xi - \frac{8}{3} \text{sh} \frac{\xi}{2} + \xi \right],$$  \hspace{1cm} (66)

where $\xi = 4A r \text{sh} \frac{p_F}{m_p c}$.

The equilibrium equation

$$\frac{GM_{\text{pulsar}}^2}{6RV} = \frac{m_p^4 c^5}{32 \pi^2 \hbar^3} \left( \frac{1}{3} \text{sh} \xi - \frac{8}{3} \text{sh} \frac{\xi}{2} + \xi \right)$$  \hspace{1cm} (67)

gives the equation for the neutron star mass

$$M_{\text{pulsar}} = \frac{3^4}{2^7} M_{Ch} F,$$  \hspace{1cm} (68)

where

$$F = \left[ \frac{\frac{1}{3} \text{sh} \xi - \frac{8}{3} \text{sh} \frac{\xi}{2} + \xi}{\left( \text{sh} \frac{\xi}{4} \right)^4} \right]^{3/2}.$$  \hspace{1cm} (69)

At $\frac{p_F}{m_p c} \approx 0.36$ we have $F \approx 0.64$ and

$$M_{\text{pulsar}} = 1.32 M_\odot,$$  \hspace{1cm} (70)

where $M_\odot$ is the Sun mass. This estimation is in a good agreement with the pulsar mass measuring data [11], which is shown in Fig. (5). In the upper scale the density of neutron matter according to Eqs. (68)-(69) is plotted. One can see that the result of the pulsar masses measurement is in full agreement with the incompressible nuclear matter hypothesis.
Figure 5: The mass distribution of pulsars [11]. In lower abscissa, the logarithm of the pulsar mass over the Sun mass is shown. In upper abscissa, the density of the substance in $g \cdot cm^{-3}$ according to Eqs. (68)–(69) is plotted.
6 The classification of stellar objects

The commonly accepted classification of stars is based on their external indicators: surface temperature, luminosities, characteristic properties of radiation. That is why the classification of stars according to equation of states of their substance may represent a more physical, stringent and consistent method. First of all, this makes evident in the determination of the number of classes into which all celestial objects of the Universe may be divided.

The matter may exist in seven states.

Atomic substances may have
1. A condensed state (solid or liquid) at low temperatures.
2. A gas state at a high temperature.

The electron-nuclear plasma may have four states. They are
3. The non-relativistic and simultaneously degenerate (cold) plasma.
4. The non-relativistic and non-degenerate (hot) plasma.
5. The relativistic and degenerate (cold) plasma.
6. The relativistic hot plasma, consisting of the relativistic electron gas and the radiation which temperature is larger than the degeneration temperature of the electron gas.

And furthermore
7. The neutron matter with the nuclear density can have degenerate state.

Nowadays, assumptions of the existence of the substance in a state other than indicated above seem to be unjustified. Consequently, the classification of all celestial bodies in accordance with possible states of substance should be performed dividing these objects into seven classes.

1. If the mass of a celestial body is relatively small, the pressure at its central region is also small and plasma is absent there. In this case, the whole body consists of an atomic substance. Small celestial bodies like asteroids and satellites of the planets are related to this class. At higher mass, a non-relativistic degenerate ion-electron plasma core exists in the central region of a body. This core is covered by a mantle of atomic substance in condensed form. This picture relates to relatively small planets (like Earth) and some of their satellites.

2. If the temperature is sufficient for evaporation but lower than the ionization temperature, a celestial body is consist of a gas. Because this body can be formed as a result of a the cooling of a hot star, it may have a mass which is typical for a star with the gas density and a relatively small temperature. The very large dimensions and the small density give a possibility to classify giants into this class.

3. Large planets can be considered as objects consisting of non-relativistic and degenerate plasma. Atomic mantles play an unimportant role in their forming. The equilibrium equation leads to the limitation of their masses [35].

4. Stars consisting of hot non-degenerate non-relativistic plasma must have the internal temperature \( \approx \frac{\hbar c}{a_0 k} \approx 10^7 K \) and the masses \( \approx 10^{33} g \), depending on their nuclear composition.

5. Dwarfs, consisting of relativistic degenerate plasma and having high density cores with substance under neutralization, must have small radii. Their
states (at temperatures $T < mc^2/k \approx 10^{10} K$) are not depending on their temperatures and can be considered as steady.

6. Quasars are consist of the relativistic plasma and the high density radiation. They can exist in steady state at masses $\approx 10^6 \div 10^{12} M_{Ch}$ and temperatures $T > mc^2/k \approx 10^{10} K$.

7. Pulsars consisting of neutron matter with nuclear density $3 \cdot 10^{14} g/cm^3$ at $T < \frac{mc^2}{k} \approx 10^{10} K$ can be considered as bodies in steady state too.

One can expect that some transitional and intermediate states can exist. They fall out of this static classification, because it classifies objects in steady state only. But a static character of the classification does not prevent to review some dynamics of stellar matter.

It seems that there is no thermodynamical prohibition to suppose the existence a stellar object consisting of neutron matter or relativistic plasma with radiation at a temperature $T >> 10^{12} K$ at some starting phase. The equilibrium mass of this strange object must be on the level $10^{53} g$, i.e. all mass of the reviewing Universe can be concentrated in this compact body in the starting phase. The following cooling must disturb its equilibrium and it can decay in approximately $10^{10}$ quasars which steady-state temperatures are approximately two orders below. A cooling of quasars leads to the loss of their equilibrium and to their decay on galaxies of hot stars with masses about $M_{Ch}$ and temperatures about $10^7 K$. The following cooling of stars may lead to the birth of pulsars or dwarfs, or to the scattering of the hot star substance in small cool celestial bodies - planets, asteroids or in gas clouds, which are stable on this stage of cooling and expansion of Universe.

7 Conclusion.

The results obtained above sharply distinguish from the point of view which is commonly accepted in the astrophysical society. It is important to note that no disputable speculative assumptions were made above. All these results were obtained on the standard physical base by standard formal methods. The novelty of the developed approach to fundamental astrophysical problems is based on a rejection of the usually accepted stating that an increase of the temperature and the density depthward of a celestial body is a requirement of the equilibrium of a substance in the self-gravity field. This requirement is really applicable to the equilibrium of atomic substances and does not applicable to plasma. It is shown above that there is the minimum for electron-nuclear plasma energy at a density and a temperature which determines the equilibrium state of plasma in the self-gravitation at the zero gradient of the general parameters of plasma.

This effect provides the simple mechanism of the generation of the magnetic field by celestial bodies. It can be noted that all previous models tried to solve the other basic problem: they tried to calculate the magnetic field of a celestial body. Now space flights and the development of astronomy discovered a
remarkable and previously unknown fact: the magnetic moments of all celestial bodies are proportional to their angular momenta and the proportionality coefficient is determined by the ratio of world constants only \((\text{5, 6})\). Nowadays the explanation of this phenomenon is really the basic problem of planetary and stars magnetism. The theory developed in this paper gives a simple and standard solution to this problem.

Our approach gives a possibility to predict important properties of stars in their steady state. Starting from the equilibrium conditions it allows to calculate masses of different types of stars. Thus the masses of stars composed by non-relativistic non-degenerate plasma and dwarfs composed by relativistic degenerate plasma can be expressed by the ratio of world constants and one variable parameter \((A/Z)\) only, and this statement is in a rather good agreement with the observation data. Just as the predicted value of mass of pulsars is in full agreement with observations at the assumption of incompressibility of nuclear matter.

Some considered questions, especially the electric polarization of plasma in a self-gravitational field, are analyzed more systematically in \([4, 12]\).

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