Vector-Boson-Induced Neutrino Mass

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Abstract

One-loop radiative Majorana neutrino masses through the exchange of scalars have been considered for many years. We show for the first time how such a one-loop mass is also possible through the exchange of vector gauge bosons. It is based on a simple variation of a recently proposed $SU(2)_N$ extension of the standard model, where a vector boson is a candidate for the dark matter of the Universe.
The unique dimension-five operator for Majorana neutrino mass in the standard model, i.e \[1\]
\[
\mathcal{L}_5 = \frac{f_{ij}}{\Lambda} (\nu_i \phi_0 - l_i \phi^+)(\nu_j \phi_0 - l_j \phi^+) + H.c.,
\]
where \((\nu_i, l_i)\) is the usual lepton doublet with \(i = e, \mu, \tau\), and \((\phi^+, \phi^0)\) is the Higgs doublet, is realized at tree level in three ways \[2\], through the exchange of a fermion singlet (Type I), a scalar triplet (Type II), or a fermion triplet (Type III). It may also be realized in one loop in three ways \[2\] through the exchange of a scalar and a fermion. The exchange of two \(W\) bosons also contributes in two loops \[3\] if one of the neutrinos already has a nonzero Majorana mass. Here we show for the first time how a one-loop neutrino mass may also be generated through the exchange of vector gauge bosons. It is based on a simple variation of a recently proposed \(SU(2)_N\) extension \[4, 5\] of the standard model, where a vector boson is a candidate for the dark matter of the Universe \[4\].

The \(SU(2)_N\) gauge group contains no component of the electric charge operator. It is a subgroup in the decomposition of \(E_6\) to \(SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_N\). It was first discovered \[7\] from the consideration of superstring-inspired \(E_6\) models \[8\]. Its relevance for dark matter was first pointed out recently \[4\] in a nonsupersymmetric model with the imposition of a global \(U(1)\) symmetry \(S\), such that a generalized lepton number \(L = S + T_{3N}\) remains unbroken after \(SU(2)_N\) is completely broken spontaneously. In that case, one of the vector gauge bosons \(X_1\) of \(SU(2)_N\) becomes a good candidate for dark matter \[5\].

Under \(SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_N \times S\), where \(Q = T_{3L} + Y\) is the electric charge and \(L = S + T_{3N}\) is the generalized lepton number, the fermions of this nonsupersymmetric model are given by \[4\]
\[
\begin{align*}
\begin{pmatrix} u \\ d \end{pmatrix} &\sim (3, 2, 1/6, 1; 0), & \begin{pmatrix} u^c \\ d^c \end{pmatrix} &\sim (3^*, 1, -2/3, 1; 0), \\
(\begin{pmatrix} h^c \\ d^c \end{pmatrix} &\sim (3^*, 1, 1/3, 2; -1/2), & h &\sim (3, 1, -1/3, 1; 1),
\end{align*}
\]
\[
(2) \quad (3)
\]
There are five nonzero vacuum expectation values:

\[
\langle N \nu \rangle \sim (1, 2, -1/2, 2; 1/2), \quad \langle E^c \rangle \sim (1, 2, 1/2, 1; 0), \quad \langle e^c \rangle \sim (1, 1, 1, 1; -1), \quad \langle \nu^c, n^c \rangle \sim (1, 1, 0, 2; -1/2),
\]

where all fields are left-handed. The \(SU(2)_L\) doublet assignments are vertical with \(T_{3L} = \pm 1/2\) for the upper (lower) entries. The \(SU(2)_N\) doublet assignments are horizontal with \(T_{3N} = \pm 1/2\) for the right (left) entries. There are three copies of the above to accommodate the known three generations of quarks and leptons, together with their exotic counterparts.

It is easy to check that all gauge anomalies are canceled. The extra global \(U(1)\) symmetry \(S\) is imposed so that \((-1)^L\), where \(L = S + T_{3N}\), is conserved, even though \(SU(2)_N\) is completely broken. The imposition of \(S\) in this case amounts to a generalized lepton number. Such a procedure is very commonplace in model building. For example, it is used to avoid rapid proton decay in supersymmetric extensions of the standard model.

The Higgs sector consists of one bidoublet, two doublets, and one \(SU(2)_R\) triplet:

\[
\left(\begin{array}{c}
\phi_1^0 \\
\phi_1^+ \\
\phi_3^0 \\
\phi_3^-
\end{array}\right) \sim (1, 2, -1/2, 2; 1/2), \quad \left(\begin{array}{c}
\phi_2^0 \\
\phi_2^+ \\
\phi_2^0
\end{array}\right) \sim (1, 2, 1/2, 1; 0),
\]

\[
(\chi_1^0, \chi_2^0) \sim (1, 1, 0, 2; -1/2), \quad \left(\begin{array}{c}
\Delta_1^0/\sqrt{2} \\
\Delta_0^0 \\
-\Delta_2^0/\sqrt{2}
\end{array}\right) \sim (1, 1, 0, 3; 1).
\]

In the following, we differ from the previous proposal by imposing an extra \(Z_2\) symmetry under which \(u^c\) and \(\phi_2\) are odd, but all other particles are even. The allowed Yukawa couplings are thus

\[
(d\phi_1^0 - u\phi_1^-)d^c - (d\phi_3^0 - u\phi_3^-)h^c, \quad (u\phi_2^0 - d\phi_2^+)^c u^c, \quad (h^\dagger\chi_2^0 - d^\dagger\chi_1^0)h,
\]

\[
(EE^c - NN^c)\chi_2^0 - (eE^c - \nu N^c)\chi_1^0, \quad (E^c\phi_1^- - N^c\phi_1^0)n^c - (E^c\phi_3^- - N^c\phi_3^0)\nu^c,
\]

\[
(N\phi_3^- - \nu\phi_1^- - E\phi_3^0 + e\phi_1^0)e^c, \quad n^c\nu^c\Delta_1^0 + (n^c\nu^c + \nu^c n^c)\Delta_2^0/\sqrt{2} - \nu^c\nu^c\Delta_3^0.
\]

There are five nonzero vacuum expectation values: \(\langle \phi_1^0 \rangle = v_1, \langle \phi_2^0 \rangle = v_2, \langle \Delta_1^0 \rangle = u_1,\) and \(\langle \chi_2^0 \rangle = u_2,\) corresponding to scalar fields with \(L = 0,\) as well as \(\langle \Delta_3^0 \rangle = u_3,\) which breaks \(L\) to \((-1)^L.\) Thus \(m_d, m_e\) come from \(v_1,\) and \(m_u\) comes from \(v_2,\) whereas \(m_h, m_E (= -m_{NN^c})\)
come from $u_2$, the $N^e n^e$ mass terms from $v_1$, and $n^e, \nu^c$ Majorana masses from $u_1$ and $u_3$. The scalar fields $\phi_3^0^-$ and $\Delta_2^0$ have $L = 1$, whereas $\chi_1^0$ has $L = -1$ and $\Delta_3^0$ has $L = 2$. The imposed $Z_2$ symmetry is broken spontaneously by $v_2$ and softly by the trilinear scalar term $\phi_2^0(\phi_1^0 \chi_2^0 - \phi_3^0 \chi_1^0) - \phi_2^+(\phi_1^0 \chi_2^0 - \phi_3^0 \chi_1^0)$.

There are five neutral fermions per family. The $3 \times 3$ mass matrix spanning the $L = 0$ fermions ($N, N^e, n^e$) is given by

$$\mathcal{M}_N = \begin{pmatrix} 0 & -m_E & 0 \\ -m_E & 0 & m_1 \\ 0 & m_1 & m_{n^e} \end{pmatrix}, \quad (11)$$

where $m_E$ comes from $u_2$, $m_1$ from $v_1$, and $m_{n^e}$ from $u_1$. The $2 \times 2$ mass matrix for the $L = \pm 1$ fermions ($\nu, \nu^c$) at tree level is given by

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 \\ 0 & m_{\nu^c} \end{pmatrix}, \quad (12)$$

However, a radiative Dirac mass linking $\nu$ and $\nu^c$ will appear in one loop. Together with the large $\nu^c$ mass, a small Majorana mass for $\nu$ will thus be generated through the usual seesaw mechanism.

Even though this model is nonsupersymmetric, $R$ parity as defined in the same way as in supersymmetry, i.e. $R \equiv (-)^{3B+L+2j}$, still holds, so that the usual quarks and leptons (including $\nu^c$) have even $R$, whereas $h, h^e, (N, E), (E^c, N^e)$, and $n^c$ have odd $R$. As for the scalars, $(\phi_1^0, \phi_1^\pm), (\phi_2^0, \phi_2^\pm), \chi_2^0, \Delta_1^0$, and $\Delta_3^0$ have even $R$, whereas $(\phi_3^0, \phi_3^\pm), \chi_1^0$, and $\Delta_2^0$ have odd $R$. After spontaneous symmetry breaking, the gauge boson mass eigenstates are the $W^\pm$ and $X_{1,2}$, and 3 neutral $R$-even vector bosons

$$A = c_W B + s_W W_3,$$

$$Z = s_W c_\alpha B - c_W c_\alpha W_3 + s_\alpha X_3,$$

$$Z' = -s_W s_\alpha B + c_W s_\alpha W_3 + c_\alpha X_3, \quad (13)$$
where $B$ is the $U(1)_Y$ gauge vector field, $c_W = \cos \theta_W$, $c_\alpha = \cos \alpha$, etc.; with masses
\[
\begin{align*}
m^2_W &= \frac{1}{2} g^2 (v_1^2 + v_2^2), \\
m^2_{X_{1,2}} &= \frac{1}{2} g_N^2 [u_2^2 + v_1^2 + 2(u_1 \mp u_3)^2], \\
m^2_A &= 0, \\
m_Z &= a_+ - \sqrt{a_+^2 + b^2}, \\
m_{Z'} &= a_+ + \sqrt{a_+^2 + b^2},
\end{align*}
\]
(14)

where
\[
\begin{align*}
a_+ &= \frac{1}{4} g_N^2 \left( 4u_1^2 + u_2^2 + 4u_3^2 + v_1^2 \right) \mp \frac{1}{4} \left( g_1^2 + g_2^2 \right) (v_1^2 + v_2^2), \\
b &= \frac{1}{2} g_N \sqrt{g_1^2 + g_2^2} v_1^2, \\
\tan 2\alpha &= \frac{b}{a_-}.
\end{align*}
\]
(15)

In the limit $u_i \gg v_j$ for all $i, j$, $\alpha \simeq 0$ and $Z' \simeq X_3$.

Whereas the usual gauge bosons have even $R$, two of the $SU(2)_N$ gauge bosons $X_{1,2}$ have odd $R$ and $X_3(\simeq Z')$ has even $R$. Assuming that $X_1$ is the lightest particle of odd $R$, it becomes a good candidate for dark matter [5]. There is also $Z - Z'$ mixing in this model, given by $-\sqrt{g_1^2 + g_2^2} v_1^2 / (u_2^2 + 4u_1^2 + 4u_3^2)$. This is constrained by precision electroweak data to be less than a few times $10^{-4}$ [9]. If $m_{Z'} \sim 1$ TeV, then $v_1$ should be less than about 10 GeV. Now $m_b$ comes from $v_1$, so this model implies that $\tan \beta = v_2 / v_1$ is large and the Yukawa coupling of $bb^c \phi_1^0$ is enhanced.

Neglecting the contributions of $v_{1,2}$ compared to $u_{1,2,3}$, the would-be Goldstone bosons for the longitudinal components of $X_{1,2}$ are given by ($c_1 = \cos \gamma_1$, etc.)
\[
\begin{align*}
G_1 &= c_1 \chi_{11} - s_1 \Delta_{21} = \frac{u_2 \chi_{11} - \sqrt{2}(u_1 - u_3) \Delta_{21}}{\sqrt{u_2^2 + 2(u_1 - u_3)^2}}, \\
G_2 &= c_2 \chi_{1R} + s_2 \Delta_{2R} = \frac{u_2 \chi_{1R} + \sqrt{2}(u_1 + u_3) \Delta_{2R}}{\sqrt{u_2^2 + 2(u_1 + u_3)^2}},
\end{align*}
\]
(16)
and the orthogonal linear combinations are physical scalars $H_{1,2}$. The one-loop radiative neutrino mass is then obtained from the exchange of $X_{1,2}, G_{1,2}$, and $H_{1,2}$ with the $(N, N^c, n^c)$ fermions, as shown in Figs. 1 and 2. Note that both diagrams vanish if $m_1 = 0$ in Eq. (11). As it is, they are finite and calculable.

Figure 1: One-loop generation of neutrino mass from the vector bosons $X_{1,2}$.

Figure 2: One-loop generation of neutrino mass from the Goldstone bosons $G_{1,2}$ and Higgs bosons $H_{1,2}$.

The calculation of the $\nu\nu^c$ radiative mass is straightforward. Using the Feynman gauge, we find that it is given by

$$m_1 m_N m_{n^c} \left\{ g_N^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_N^2)(k^2 - m_{n^c}^2)} \left( \frac{1}{k^2 - m_{X_1}^2} + \frac{1}{k^2 - m_{X_2}^2} \right) + \frac{1}{\sqrt{2u_1 u_2}} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_N^2)(k^2 - m_{n^c}^2)} \left( \frac{c_1 s_1}{k^2 - m_{X_1}^2} - \frac{c_1 s_1}{k^2 - m_{H_1}^2} + \frac{c_2 s_2}{k^2 - m_{X_2}^2} - \frac{c_2 s_2}{k^2 - m_{H_2}^2} \right) \right\} .$$

(18)
Note that the $H_{1,2}$ contributions are crucial in the above to make the second integral finite. Using

\[ \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - a)} \frac{1}{(k^2 - b)} \frac{1}{(k^2 - c)} = f(a, b, c) \]

\[ = \frac{1}{16\pi^2 i} \left[ \frac{a \ln a}{(a - b)(a - c)} + \frac{b \ln b}{(b - a)(b - c)} + \frac{c \ln c}{(c - a)(c - b)} \right], \tag{19} \]

\[ \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - a)(k^2 - b)} \left( \frac{1}{k^2 - c} - \frac{1}{k^2 - d} \right) = c \ f(a, b, c) - d \ f(a, b, d), \tag{20} \]

we obtain

\[ m_1 m_N m_{\nu} \left\{ g_N^2 \left[ f(m_N^2, m_{\nu}^2, m_{X_1}^2) + f(m_N^2, m_{\nu}^2, m_{X_2}^2) \right] \right. \]

\[ + \left. \frac{c_1 s_1}{\sqrt{2}u_1 u_2} \left[ m_{X_1}^2 f(m_N^2, m_{\nu}^2, m_{X_1}^2) - m_{H_1}^2 f(m_N^2, m_{\nu}^2, m_{H_1}^2) \right] \right. \]

\[ + \left. \frac{c_2 s_2}{\sqrt{2}u_1 u_2} \left[ m_{X_2}^2 f(m_N^2, m_{\nu}^2, m_{X_2}^2) - m_{H_2}^2 f(m_N^2, m_{\nu}^2, m_{H_2}^2) \right] \right\}. \tag{21} \]

Assuming that all heavy particles are roughly equal in mass and $g_N \simeq g_2$, then

\[ m_{\nu} \simeq \frac{g_N^2 m_1}{8\pi^2} \simeq \frac{\alpha m_1}{2\pi \sin^2 \theta_W}. \tag{22} \]

Let $m_1 \simeq 0.1$ GeV and $m_{\nu} \simeq 1$ TeV, then the seesaw neutrino mass is about 0.3 eV.

Since all the particles inside the loop in Figs. 1 and 2 have odd $R$, the radiative Dirac neutrino mass is “scotogenic”, i.e. caused by darkness. The first such example \cite{10} was that of a radiative Majorana neutrino mass using a second scalar doublet $(\eta^+, \eta^0)$ which is odd under an extra $Z_2$ symmetry together with three neutral fermion singlets $N_{1,2,3}$, now often referred to as radiative seesaw. Both $\sqrt{2} \text{Re}(\eta^0)$ and $N_1$ have been studied as possible dark-matter candidates. In the $SU(2)_N$ model \cite{11 15}, $X_1$ was studied as the first example of a vector-boson dark-matter candidate not from exotic physics. As mentioned above, this scenario can also be realized here with radiative neutrino mass generation. However, depending on the region of parameter space, $H_1$ is also a possible dark-matter candidate, which we consider below.
The complete Higgs potential of this model is given by

\[ V = \mu_1^2 Tr(\phi_{13}^\dagger \phi_{13}) + \mu_2^2 \phi_1^\dagger \phi_1 + \mu_3^2 \chi \chi^\dagger + \mu_4^2 Tr(\Delta^\dagger \Delta) + (\mu_5^2 \det \Delta + H.c.) \]

\[ + (\mu_2 \phi_1^\dagger \phi_1 + \mu_2 \chi \Delta \chi^\dagger + \mu_3 \bar{\chi} \Delta \chi^\dagger + H.c.) + \frac{1}{2} \lambda_1 [Tr(\phi_{13}^\dagger \phi_{13})]^2 + \frac{1}{2} \lambda_2 (\phi_1^\dagger \phi_1)^2 \]

\[ + \frac{1}{2} \lambda_3 Tr(\phi_{13}^\dagger \phi_{13} \phi_{13}^\dagger \phi_{13}) + \frac{1}{2} \lambda_4 (\chi \chi^\dagger)^2 + \frac{1}{2} \lambda_5 [Tr(\Delta^\dagger \Delta)]^2 + \frac{1}{4} \lambda_6 Tr(\Delta^\dagger \Delta - \Delta \Delta^\dagger)^2 \]

\[ + f_1 \chi \phi_{13}^\dagger \phi_{13} \chi^\dagger + f_2 \bar{\chi} \phi_{13}^\dagger \phi_{13} \chi^\dagger + f_3 \phi_1^\dagger \phi_1 \phi_{13}^\dagger \phi_{13} + f_4 \phi_1^\dagger \phi_1 \bar{\phi}_{13}^\dagger \phi_{13} + f_5 (\phi_1^\dagger \phi_1)(\chi \chi^\dagger) \]

\[ + f_6 (\chi \chi^\dagger) Tr(\Delta^\dagger \Delta) + f_7 \chi (\Delta^\dagger \Delta - \Delta \Delta^\dagger) \chi^\dagger + f_8 (\phi_1^\dagger \phi_1) Tr(\Delta^\dagger \Delta) \]

\[ + f_9 Tr(\phi_{13}^\dagger \phi_{13}) Tr(\Delta^\dagger \Delta) + f_{10} Tr(\phi_{13}^\dagger \phi_{13} - \Delta \Delta^\dagger) \phi_{13}^\dagger, \]

where

\[ \tilde{\phi}_2 = \left( \begin{array}{c} \phi_2^0 \\ -\phi_2^- \end{array} \right), \quad \tilde{\phi}_{13} = \left( \begin{array}{cc} \phi_1^+ & -\phi_1^- \\ \phi_3^0 & -\phi_3^- \end{array} \right), \quad \tilde{\chi} = (\chi_2^0, -\chi_1^0), \]

and the $\mu_3^2$, $\mu_{23}$ terms break $L$ softly to $(-1)^L$. Assuming $u_1 = u_3$, and using the details provided in Ref. [4], we find that $H_1 = \sqrt{2} \Delta_{2I}$ to a very good approximation, i.e. neglecting the terms of order $v_{1,2}/u_{1,2}$. In that case,

\[ m_{H_1}^2 = 4\lambda_6 u_1^2 + 2\mu_3^2 - (\mu_{12} + \mu_{23}) \frac{u_2^2}{2u_1}, \]

\[ m_{H_2}^2 = -(\mu_{12} + \mu_{23}) \frac{8u_1^2 + u_2^2}{2u_1}. \]

Assuming that $H_1$ is the lightest of all particles of odd $R$, we now estimate its annihilation cross section in the early Universe and its spin-independent cross section with nuclei at underground experiments.

The relic density of $H_1$ is related to the $H_1 H_1$ annihilation cross section times the relative velocity of the two particles as they become non-relativistic. This comes mainly from the contact interactions

\[ \frac{1}{2} f_8 H_1^2 (\phi_2^\dagger \phi_2 + \phi_2^0 \phi_2^0) + \frac{1}{2} f_9 H_1^2 (\phi_1^\dagger \phi_1^+ + \phi_1^0 \phi_1^0), \]

and the $\chi_{2R}$, $\Delta_{1R}$ and $\Delta_{3R}$ $s$–channel exchanges; the $t$– and $u$–channel exchanges being suppressed by factors $\sim v_i/u_j$. Taking $f_{7,10} = 0$, $\mu_{12} = \mu_{23}(=\mu)$ to simplify the expressions,
the $H_1 H_1 \rightarrow \phi_1 \phi_1^\dagger$ amplitude is then given by

$$-i f_9 - 4 i f_2 f_6 u_2^2 (s + 8 \lambda_6 u_1^2 + 2 \xi) + 2 f_9 u_2^2 [(s - 2 \lambda_4 u_1^2) (\lambda_5 + 2 \lambda_6) + 2 f_6 (f_6 u_2^2 + \xi)] + 4 \xi f_2 u_1^2 (\lambda_5 + 2 \lambda_6) \over s^2 - 2 s (2 \lambda_5 u_1^2 + \lambda_4 u_2^2 - \xi) - 8 (f_6 - \lambda_4 \lambda_5) u_1^2 u_2^2 - 4 \xi (4 f_6 u_1^2 + \lambda_4 u_2^2 + 2 u_1 \mu), $$

(28)

where $\xi = \mu u_2^2 / u_1$, with the first term coming from the contact interaction. The corresponding amplitude for $H_1 H_1 \rightarrow \phi_2 \phi_2^\dagger$ is obtained by replacing $f_2 \rightarrow f_5$, $f_9 \rightarrow f_8$.

In order to get an estimate of the relic-density constraints for this model, we consider two simple cases. First suppose $|f_9|, |\lambda_5|, |\lambda_6| \ll 1$: in this case

$$\sigma_{\phi_1} = |f_9|^2 / 16 \pi \beta s, \quad \sigma_{\phi_2} = |f_6|^2 / 16 \pi \beta s, $$

(29)

where $s \beta = \sqrt{s - 4 m_{H_1}^2}$. We use the standard expressions [11],

$$\langle \sigma v \rangle = x \over 8 \pi m_{H_1}^2 K_2(x) \int_{4 m_{H_1}^2}^{\infty} ds \sqrt{s} (s - 4 m_{H_1}^2) K_1 \left( s / T \right) \left( \sigma_{\phi_1} + \sigma_{\phi_2} \right), $$

$$\Omega_{DM} h^2 = 1.06 \times 10^9 \text{GeV}^{-1} \frac{x f}{\sqrt{g_* M_{Pl} \langle \sigma v \rangle}}, \quad x f = \log \left( 0.038 \frac{\langle \sigma v \rangle M_{Pl} m_{H_1}}{\sqrt{g_* x f}} \right), $$

(30)

where $x f = m_{H_1} / T_f$, and $T_f$ is the freeze-out temperature, $g_*$ the effective number of relativistic degrees of freedom at $T = T_f$. Taking $g_* = 110.75$, corresponding to a two-Higgs-doublet extension of the Standard Model, and using the experimental constraint $\Omega_{DM} h^2 = 0.11 \pm 0.018$, we find to a good approximation

$$\frac{m_{H_1}}{f} = 4.3^{+0.4}_{-0.3} \text{TeV}, \quad \bar{f} = \sqrt{|f_8|^2 + |f_9|^2 / 2}. $$

(31)

As a second example we take the case $f_6 \ll 1$ and $\mu_{3,12,23} \ll u_{1,2}$; then the total cross sections equal

$$\sigma_{\phi_1} = |f_9|^2 / 16 \pi \beta s \left( s + (r + 4) m_{H_1}^2 \right)^2, \quad \sigma_{\phi_2} = |f_6|^2 / 16 \pi \beta s \left( s + (r + 4) m_{H_1}^2 \right)^2, \quad r = \lambda_5 / \lambda_6,$$

(32)

where we take $r > 1$ to insure that $H_1$ is the lightest dark-matter candidate, and also assume $r < 4$ in order to avoid complications associated with resonant contributions to $\langle \sigma v \rangle$. Using
Eq. (30) we find

| $r$  | $m_{H_1}/\bar{f}$ (TeV) | (33) |
|------|------------------------|------|
| 2    | $18.1^{+1.8}_{-1.3}$   |      |
| 2.5  | $23.9^{+2.4}_{-1.8}$   |      |
| 3    | $34.1^{+3.2}_{-2.5}$   |      |
| 3.5  | $58.2^{+5.4}_{-4.3}$   |      |

The factor of $\sim 10$ increase compared to Eq. (31) is produced by a correspondingly larger cross section from Eq. (29) to Eq. (32): i.e. by a factor of $[(r + 8)/(r - 4)]^2$ in the limit $s = 4m_{H_1}^2$.

In underground dark-matter direct-search experiments, the spin-independent elastic cross section for $H_1$ scattering off a nucleus of $Z$ protons and $A - Z$ neutrons normalized to one nucleon is given by

$$
\sigma_0 = \frac{1}{\pi} \left( \frac{m_N}{m_{H_1} + Am_N} \right)^2 \left| \frac{Z f_p + (A - Z)f_n}{A} \right|^2,
$$

(34)

where $m_N$ is the mass of a nucleon, and $f_{p,n}$ come from Higgs exchange [12]:

$$
\frac{f_p}{m_p} = \left(-0.075 - \frac{0.925(3.51)(2)}{27}\right)\frac{\sqrt{2}}{m_\phi} \left( \frac{f_8 v_1^2 + f_9 v_1^2}{v_1^2 + v_2^2} \right),
$$

(35)

$$
\frac{f_n}{m_n} = \left(-0.078 - \frac{0.922(3.51)(2)}{27}\right)\frac{\sqrt{2}}{m_\phi} \left( \frac{f_8 v_2^2 + f_9 v_1^2}{v_1^2 + v_2^2} \right).
$$

(36)

Assuming an effective $m_\phi = 125$ GeV and using $Z = 54$ and $A - Z = 77$ for $^{131}$Xe, we find for $f_8 \simeq f_9$ with Eq. (29) that $\sigma_0 < 3.2 \times 10^{-10}$ pb, which is far below the current upper bound of the 2011 XENON100 experiment [13].

In conclusion, we have shown how the $SU(2)_N$ model of vector-boson dark matter allows also the one-loop radiative generation of a Dirac neutrino mass through vector and scalar exchange. This is the first example of such a one-loop mechanism involving vector bosons. We have also studied the scalar $H_1$ as a possible dark-matter candidate.
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