Effects of neutrino oscillations and neutrino magnetic moments 
on elastic neutrino–electron scattering

W. Grimus and P. Stockinger

Institute for Theoretical Physics, University of Vienna,
Boltzmanngasse 5, A–1090 Vienna, Austria.

Abstract

We consider elastic $\bar{\nu}_e e^-$ scattering taking into account possible effects of neutrino masses and mixing and of neutrino magnetic moments and electric dipole moments. Having in mind antineutrinos produced in a nuclear reactor we compute, in particular, the weak–electromagnetic interference terms which are linear in the magnetic (electric dipole) moments and also in the neutrino masses. We show that these terms are, however, suppressed compared to the pure weak and electromagnetic cross sections. We also comment upon the possibility of using the electromagnetic cross section to investigate neutrino oscillations.

13.15.+g, 13.40.Em, 14.60.Pq
I. INTRODUCTION

Exact knowledge of the intrinsic neutrino properties is decisive for our understanding of fundamental interactions and if and how the standard model has to be extended in the lepton sector. So far, intensive experimental efforts have been made in neutrino oscillations \[1\] to pin down neutrino masses and mixing angles \[2\]. As for neutrino magnetic moments \((\mu)\) and electric dipole moments \((d)\), elastic neutrino–electron scattering is a simple and fundamental process with great sensitivity for these quantities. This process has been used to obtain the experimental limits \(\mu_{\nu_e} < 1.8 \times 10^{-10} \mu_B \) \[3\] and \(\mu_{\nu_\mu} < 7.4 \times 10^{-10} \mu_B \) \[4\] where \(\mu_B\) is the Bohr magneton. A much greater sensitivity for \(\mu_{\nu_e}\) is planned in the MUNU experiment \[5,6\] which makes use of reactor neutrinos like the experiments evaluated in Ref. \[3\]. The relevant cross section for elastic \(\bar{\nu}_e e^-\) scattering with \(\mu_{\nu_e} \neq 0\) is given by \[7\]

\[
\frac{d\sigma}{dT} = \frac{d\sigma_w}{dT} + \frac{d\sigma_m}{dT}
\]

(1.1)

where

\[
\frac{d\sigma_w}{dT} = \frac{G_F^2 m_e}{2\pi} \left[ (G_V - G_A)^2 + (G_V + G_A)^2 \left(1 - \frac{T}{E_{\nu}}\right)^2 - (G_V^2 - G_A^2) \frac{m_e T}{E_{\nu}^2} \right]
\]\n
(1.2)

and

\[
\frac{d\sigma_m}{dT} = \frac{\alpha^2 \pi}{m_e^2} (\mu'_{\nu_e})^2 \left( \frac{1}{T} - \frac{1}{E_{\nu}} \right)
\]\n
(1.3)

with \((\mu'_{\nu_e})^2 = (\mu'^2 + d'^2)_{\nu_e}\). In these formulas, neutrino masses and mixing have not been taken into account and the electron neutrino is assumed to be a Dirac particle. Furthermore, \(T = E_{\nu'} - m_e\) is the kinetic energy of the recoil electron, magnetic and electric dipole moments are given in units of the Bohr magneton, i.e. \(\mu' \equiv \mu/\mu_B\) and \(d' \equiv d/\mu_B\), \(G_F\) is the Fermi coupling constant, \(\alpha \approx 1/137\) is the fine structure constant and \(G_{V,A} \equiv g_{V,A} + 1\) with \(g_V = 2 \sin^2 \theta_W - 1/2\) and \(g_A = -1/2\). \(g_{V,A}\) correspond to the neutral current contribution to the effective interaction Lagrangian of this process.

In Eq. (1.1) there is no interference between the weak and electromagnetic interaction. This is evident since for negligible neutrino mass the electromagnetic interaction involving magnetic and electric dipole moments leads to a helicity flip of the neutrino while the weak interaction always conserves helicity. Such an interference term is, however, only absent as long as the incident neutrino flux has no transverse polarization \[8\]. Also possible scalar exchange leads to an interference with the neutrino magnetic or electric dipole moment interaction \[9\]. Moreover, a weak–electromagnetic interference term will appear if we do not neglect neutrino masses. The subject of our investigation will be the effect of neutrino masses and mixing on elastic neutrino–electron scattering with magnetic and electric dipole moments of neutrinos.

\[1\]For \(\nu_e e^-\) scattering \(G_A\) has to be replaced by \(-G_A\).
If neutrinos are massive particles, the mass matrix in the weak basis of the neutrino fields is in general non-diagonal. After diagonalization, the left-handed neutrino fields in the weak basis are linear combinations of mass eigenfields, i.e.

\[ \nu_{\alpha L}(x) = U_{\alpha j} \nu_j L(x) \]  

(1.4)

where \( U \) is a unitary matrix, the neutrino fields \( \nu_j \) have masses \( m_j \) and the index \( \alpha \) indicates the flavours \( e, \mu \) and \( \tau \). Eq. (1.4) gives rise to the phenomenon of neutrino oscillations. Since we want to incorporate effects linear in the neutrino masses into elastic \( \bar{\nu}_e e^- \) scattering we are looking for weak–electromagnetic interference terms proportional to \( m_\nu \times \mu_\nu (d_\nu) \) in the cross section (\( m_\nu \) is a generic neutrino mass and \( \mu_\nu (d_\nu) \) is a generic neutrino magnetic (electric dipole) moment). It is therefore necessary to know the initial neutrino state up to terms linear in the neutrino masses. It has been stressed in the literature [10] that in order to have control over the initial state a safe way is to consider neutrino production and detection as a single process.

In the present paper we will proceed along this line. We will show that under certain conditions the production process, which is \( \beta \)-decay of fission products in a reactor in our case, factorizes from elastic neutrino–electron scattering in an unambiguous way. We will do the calculation for both Dirac and Majorana neutrinos and finally we will discuss the \( m_\nu \times \mu_\nu (d_\nu) \) interference term and also the effects of neutrino mixing on Eqs. (1.1–1.3).

II. THE AMPLITUDE

We consider the following combined production–detection process

\[ ^A_Z X \rightarrow ^A_{Z+1} X' + e^- S + \bar{\nu} \]

\[ \bar{\nu} + e^- \rightarrow \bar{\nu} + e^- \]  

(2.1)

where the neutrinos are produced by \( \beta \)-decay of fission nuclei \( ^A_Z X \) in a reactor and detected by elastic scattering off electrons. The initial particles \( ^A_Z X \) and \( e^- \) (detector electron) are localized and macroscopically separated by a distance \( L \). With \( e^- S \) we denote the electron which is generated in the \( \beta \)-decay and absorbed in the reactor. This is exactly the situation described in Ref. [11] and hence it is possible to use the methods and results obtained there. In this paper we will concentrate on the interference term between weak and electromagnetic interaction and calculate all terms in the cross section which depend linearly on the neutrino masses.

The weak and electromagnetic interaction Langrangians relevant for the detection are

\[ \mathcal{L}_w(x) = -\frac{G_F}{\sqrt{2}} \sum_{j,k} \bar{e}(x)\gamma^\mu (Z_{jk}^V - Z_{jk}^A \gamma_5) e(x) \bar{\nu}_j (x) \gamma_\mu (1 - \gamma_5) \nu_k (x) \]  

(2.2)

with

\[ Z_{jk}^{V,A} \equiv U_{ej}^* U_{ek} + \delta_{jk} g_{V,A} \]  

(2.3)
\[ L_{em}(x) = -\frac{1}{2} \sum_{j,k} \bar{v}_j(x) (\mu_{jk} + id_{jk}\gamma_5) \sigma_{\lambda\rho} \nu_k(x) F^{\lambda\rho}(x) + e \bar{e}(x) \gamma^\lambda e(x) A_\lambda(x), \]  

(2.4)

respectively, where \( A_\lambda \) is the electromagnetic vector potential, \( F^{\lambda\rho} \) the field strength tensor, 
\( e \) the charge of the positron, \( P_{R,L} = (1 \pm \gamma_5)/2 \) and the \( \mu_{jk} \) \( (d_{jk}) \) are the magnetic (electric dipole) moments and transition moments \( (j \neq k) \) of the mass eigenfields. For Dirac neutrinos, the amplitude for the process \( (2.1) \) in the limit of a macroscopic distance \( L \) with an antineutrino of mass \( m_\ell \) in the final state is given by \[ A_\ell = \sum_j \tilde{J}_\lambda \tilde{u}_{eS} \gamma^\lambda P_L (\gamma_j + m_j) e^{iq_j L} U_{ej} \]

\[ \times \left\{ \begin{array}{l} \sqrt{2} G_F \gamma^\mu P_L v(k_j') \tilde{u}_e(p') \gamma_\mu (Z_{j\ell}^V - Z_{j\ell}^A \gamma_5) \tilde{\psi}_e \\ -ie(\mu_{j\ell} + id_{j\ell}\gamma_5) q_\mu \sigma^{\mu\nu} v(k_j') \frac{q_\nu}{q^2} \tilde{u}_e(p') \gamma^\nu \tilde{\psi}_e \end{array} \right\} \]  

(2.5)

where \( \tilde{J}_\lambda \) is the Fourier transform of the hadronic current, \( \tilde{\psi}_e(p) \) is the Fourier transform of the wave function \( \psi_e \) of the initial detector electron, \( u_{eS} \) denotes the spinor of the source electron, \( p \) and \( p' \) the initial and final momenta, respectively, of the detector electron, \( k_j \) \( (k_j') \) the momenta of the intermediate (final) neutrinos and

\[ q = p - p', \quad k_j = \left( E_\nu, \vec{k}_j \right), \quad q_j = \sqrt{E_\nu^2 - m_j^2} \]  

(2.6)

where \( \vec{l} \) is the unit vector pointing from the neutrino source to the detection point. Apart from the factor \( \exp(iq_j L) \), the amplitude \( (2.3) \) in the approximation of macroscopic separation of the neutrino source and the detector is just the sum over the products of production and detection amplitude of antineutrinos with mass \( m_j \). The definitions

\[ M_{j\ell} = -i(\mu_{j\ell} + id_{j\ell}\gamma_5) q_\mu L_{em}^{\mu\nu} \sigma^{\mu\nu}, \]  

(2.7)

\[ W_{j\ell} = \gamma^\mu P_L L_{\mu j\ell} \]  

(2.8)

with

\[ L_{em}^{\mu} = \frac{e}{q^2} \tilde{u}_e(p') \gamma_\nu \tilde{\psi}_e \]  

(2.9)

and

\[ L_{\mu j\ell} = \sqrt{2} G_F \bar{u}_e(p') \gamma_\mu (Z_{j\ell}^V - Z_{j\ell}^A \gamma_5) \tilde{\psi}_e \]  

(2.10)

\[ 2p = (\sqrt{m_\ell^2 + p'^2}, \vec{p}) \]
will serve in the following to simplify our notations as far as possible and to easily keep track of the interactions involved in the various terms of the cross section. With the help of these definitions the amplitude (2.5) is rewritten as

\[
A_\ell = \sum_j \tilde{J}_\lambda \bar{u}_e s \gamma^\lambda P_L(-k_j + m_j)e^{i q_j \cdot L} U_{ej}[W_{j\ell} + M_{j\ell}] v(k_{\ell}').
\]  

(2.11)

In the next section we will show that under certain conditions also the cross section can be factorized into a product of production and detection cross section.

III. THE CROSS SECTION

In order to simplify the calculations it is necessary to make an approximation for the state of the detector electron. We will assume henceforth that the momentum spread of this electron is negligible and thus use for its description simply a spinor of an electron at rest. Summing over the spins of the leptons and over the absolute squares of the amplitudes \(A_\ell\) we obtain

\[
\sum_\ell \sum_{\text{spins}} A_\ell A_\ell^* = \text{Tr}\left\{\hat{J}_S \bar{J}_P L(-k_j + m_j)(W_{j\ell} + M_{j\ell})(k'_{\ell} - m_{\ell})(\hat{W}_{\ell n} + \hat{M}_{\ell n})(-k_n + m_n)\right\} \\
\times e^{i(q_j - q_n) \cdot L} U_{ej} U_{en}^*
\]  

(3.1)

where

\[
\hat{W}_{\ell n} \equiv \gamma^\mu P_L L_{\mu\nu}^*, \quad \hat{M}_{\ell n} \equiv i(\mu_n^* + i d_{n\ell}^*) q_{\mu\nu} L_{\mu\nu}^{\text{em}} \sigma^{\mu\nu}
\]  

(3.2)

and

\[
\hat{J} \equiv \tilde{J}_\lambda \gamma^\lambda, \quad \hat{J}^* \equiv \tilde{J}_\lambda^* \gamma^\lambda.
\]  

(3.3)

The momentum of the electron from the source is denoted by \(p_S\).

Now we will approximate the right-hand side of Eq. (3.1) by confining ourselves to terms which are at most linear in the masses of the neutrinos. Fixing the spatial neutrino momenta, the neutrino energies depend quadratically on the neutrino masses for \(m_\nu^2 \ll k_\nu^2\) where \(k_\nu\) denotes a generic neutrino momentum. Therefore, linear dependence on the neutrino masses derives from the explicit mass factors \(m_{j,\ell,n}\) in Eq. (3.1) and the linear approximation amounts to neglecting all neutrino masses in the neutrino 4-momenta:

\[
k_j = k \ \forall j, \quad k'_{\ell} = k' \ \forall \ell
\]  

(3.4)

with

\[
k^2 = k'^2 = 0.
\]  

(3.5)

Note, however, that we keep the squares of neutrino masses in the phase factors \(e^{i(q_j - q_n) \cdot L}\) to allow for neutrino oscillations. Eq. (3.1) becomes in this approximation
\[
\sum_{\ell} \sum_{\text{spins}} A_{\ell} A_{\ell}^* = e^{i(q_j - q_n)L} U_{e\ell} U_{e\nu}^* \text{Tr} \left\{ \hat{J} \psi_S \hat{J} P_L \left[ k'(W_{j\ell} + M_{j\ell}) k' \right] \right. \\
\left. - k(W_{j\ell} + M_{j\ell}) m_\ell (\tilde{W}_{\ell n} + \tilde{M}_{\ell n}) k - m_j (W_{j\ell} + M_{j\ell}) k'(\tilde{W}_{\ell n} + \tilde{M}_{\ell n}) \right\}.
\]

(3.6)

Now we would like to separate neutrino production from neutrino scattering by representing Eq. (3.6) as a product of two factors describing production and scattering, respectively, conforming with the usual situation. Actually, this can easily be done for the two terms in the square brackets in Eq. (3.6) which have \( k \) on both sides. Since now neutrinos are “massless” we can write

\[
v(k, s_+) \bar{v}(k, s_+) = P_L k
\]

(3.7)

where \( v(k, s_+) \) represents an antineutrino with positive helicity. Using Eq. (3.7) we can split the first two terms of Eq. (3.6) into a product of production and detection trace and obtain for these terms

\[
e^{i(q_j - q_n)L} U_{e\ell} U_{e\nu}^* \text{Tr} \left\{ \hat{J} \psi_S \hat{J} P_L \right\} \\
\times \left\{ \text{Tr}[W_{j\ell} k' \tilde{W}_{\ell n} P_L k] + \text{Tr}[M_{j\ell} k' \tilde{M}_{\ell n} P_L k] + \left. - m_\ell \text{Tr}[(W_{j\ell} \tilde{M}_{\ell n} + M_{j\ell} \tilde{W}_{\ell n}) P_L k] \right\}.
\]

(3.8)

After omitting the production process, the first two terms of Eq. (3.8) represent the detection of the antineutrino via the pure weak and the pure electromagnetic interaction, respectively. If neutrino mixing is neglected these terms give rise to the well-known cross section (1.1). The third term in (3.8) gives the interference terms between weak and electromagnetic interaction which are proportional to the masses of the final neutrinos.

It remains to consider the last two terms in Eq. (3.6) which depend on the masses of the intermediate neutrinos. It turns out that with the method employed before factorization cannot be achieved in this case. Nevertheless, factorization is obtained after integration over the momentum \( k' \) of the outgoing neutrino. Two points seem to be operative for this goal:

- rotational invariance around the axis neutrino source – neutrino detection point defined by \( \vec{k} \),
- detector electron in its rest frame.

To perform an averaging according to the first of these points we define an orthonormal basis with the three vectors \( \vec{n}, \vec{a}, \vec{b} \) where

\[
\vec{n} \equiv \frac{\vec{k}}{E_{\nu}}.
\]

(3.9)

Then we can write

\[
\frac{\vec{k}'}{E'_{\nu}} = \cos \gamma \vec{n} + \sin \gamma \vec{m}(\phi)
\]

(3.10)
with
\[ \vec{m}(\phi) \equiv \cos \phi \vec{a} + \sin \phi \vec{b}. \]  

(3.11)

With this notation we obtain in the rest frame of the detector electron the relation
\[
\frac{1}{2\pi} \int_0^{2\pi} k' d\phi = \frac{E'_\nu}{E_\nu} \cos \gamma k' + \frac{E'_\nu}{m_e} (1 - \cos \gamma) \vec{p}'
\]

(3.12)

where
\[ 1 - \cos \gamma = \frac{m_e T}{E_\nu E'_\nu}. \]

(3.13)

Note that also \( \vec{p}' \) contains the angle \( \phi \) because \( \vec{p}' = \vec{k}' - \vec{k} \). Using Eq. (3.12), after some algebra also in the last two terms of Eq. (3.6) neutrino production can be factorized from subsequent scattering. Thus the production process can be totally separated off and also the weak–electromagnetic interference terms can be written as a cross section for scattering alone.

Finally, the elastic differential antineutrino–electron cross section in the laboratory frame of the electron is given by
\[
\frac{d\sigma}{dT} = \frac{d\sigma_w}{dT} + \frac{d\sigma_m}{dT} + \frac{d\sigma_{wm}}{dT}
\]

(3.14)

with
\[
\frac{d\sigma_w}{dT} = \frac{d\sigma_w(\vec{\nu}_e e^-)}{dT} + \left| \sum_j e^{iq_j L} |U_{e j}|^2 \right|^2 \left( \frac{d\sigma_w(\vec{\nu}_e e^-)}{dT} - \frac{d\sigma_w(\vec{\nu}_e e^-)}{dT} \right)
\]

\[
= \frac{m_e G_F^2}{2\pi} \left\{ (g_V - g_A)^2 + (g_V + g_A)^2 (1 - \frac{T}{E_\nu})^2 - m_e \frac{T}{E_\nu}^2 (g_V^2 - g_A^2) \right.
\]

\[ + \left. \sum_j e^{iq_j L} |U_{e j}|^2 \left[ 4 \left( 1 - \frac{T}{E_\nu} \right)^2 (1 + g_V + g_A) - 2m_e \frac{T}{E_\nu} (g_V - g_A) \right] \right\}, \]

(3.15)

\[
\frac{d\sigma_m}{dT} = \frac{\alpha^2 \pi}{m_e^2} \sum_\ell \left| \sum_j e^{iq_j L} U_{e j}(\mu'_{j\ell} + i d'_{j\ell}) \right|^2 \left( \frac{1}{T} - \frac{1}{E_\nu} \right)
\]

(3.16)

and
\[
\frac{d\sigma_{wm}}{dT} = \frac{\alpha G_F}{\sqrt{2} E_\nu m_e} \Re \left\{ \sum_{j,n,\ell} e^{i(q_j - q_n) L} U_{e j} U_{e n}^* \right.\]

\[ \times \left( m_\ell (\mu'_{j\ell} + i d'_{j\ell}) \left[ \left( \frac{m_e}{E_\nu} - \frac{T}{E_\nu} \right) Z_{n\ell}^V + (2 - \frac{T}{E_\nu}) Z_{n\ell}^A \right] \right.\]

\[ + m_j (\mu'_{j\ell} - i d'_{j\ell}) \left[ \left( \frac{m_e T}{2 E_\nu^2} - 1 \right) Z_{n\ell}^V + \left( \frac{m_e T}{2 E_\nu^2} + 1 \right) Z_{n\ell}^A \right] \left. \right\} \right\}. \]

(3.17)
In Eq. (3.15), \( \frac{d\sigma_w}{dT}(\bar{\nu}_e e^-) \) is identical with the expression given in Eq. (1.2), the elastic \( \bar{\nu}_e e^- \) cross section without effects of neutrino oscillations.

If neutrino mixing is neglected, i.e. \( U \) is the unit matrix, Eqs. (3.15) and (3.16) reproduce the cross sections (1.2) and (1.3). In contrast to the pure weak and electromagnetic contributions in Eq. (3.14), the interference terms (3.17) depend explicitly on the neutrino masses. This linear dependence on \( m_\nu \) is obviously needed to provide the necessary helicity flip to make interference between the weak amplitude (no helicity flip) and the amplitude for magnetic and electric dipole moments (helicity flip) possible. The terms in the second line of Eq. (3.17) are proportional to the masses of the final state neutrinos whereas the terms in the third line contain the masses of the incident neutrinos. In the latter terms the details of the neutrino production mechanism enter. Thus in the interference cross section the information that the neutrinos are produced by a static \( \beta \)-decay source is incorporated.

The validity of Eq. (3.17) for other production mechanisms is not guaranteed.

**IV. MAJORANA NEUTRINOS**

If neutrinos are Majorana particles it is usual to define the magnetic (electric dipole) moment part of the electromagnetic interaction Lagrangian (2.4) with an additional factor 1/2. The neutrino mass eigenfields now fulfill the Majorana condition \( (\nu_i) = \nu_i \). Moreover, the matrices \((\mu_{jk})\) and \((d_{jk})\) are antisymmetric which means that the neutrinos possess only transition moments.

Since in the Majorana case also \( \nu_j \nu_j^T \) can be Wick-contracted the above mentioned factor 1/2 is cancelled and an additional weak term \( A^M_\ell \) occurs in the amplitude (2.3):

\[
A^M_\ell = \sum_j \tilde{J}_\lambda \bar{\nu}_e \gamma^\lambda P_L(-\gamma_j + m_j) e^{i q_j L} U_{e j} \times (-1) \sqrt{2} G_F \gamma^\mu P_R(v(k'_j) \bar{\nu}_e(p') \gamma_\mu (Z^V_{ij} - Z^A_{ij} \gamma_5)) \tilde{\psi}_e.
\]

This additional amplitude is proportional to \( m_j \) and generates in our approximation only one additional term in the \( \bar{\nu}_e e^- \) cross section which is a weak–electromagnetic interference term. It is given by

\[
\frac{d\sigma^M_{wm}}{dT} = \frac{\alpha G_F}{\sqrt{2} E_{\nu} m_e} \text{Re} \left\{ \sum_{j,n,\ell} e^{i(q_j - q_n) L} U^*_{en} U_{ej} m_n (\mu'_{j\ell} + i d'_{j\ell}) \times \left[ \left( \frac{m_e T}{2E_{\nu}^2} + \frac{T}{E_{\nu}} - \frac{m_e}{E_{\nu}} - 1 \right) Z^V_{ln} + (1 - \frac{T}{E_{\nu}} - \frac{m_e T}{2E_{\nu}^2}) Z^A_{ln} \right] \right\}
\]

and has to be added to Eq. (3.17) in the Majorana case.

**V. DISCUSSION**

In this paper we only consider limits on magnetic moments of neutrinos derived in terrestrial experiments. It should be kept in mind, however, that cosmological and astrophysical
considerations result usually in more stringent limits (for reviews see e.g. Refs. [12]) although with assumptions beyond those made in terrestrial scattering experiments.

The pure weak cross section Eq. (3.13) with neutrino oscillations incorporated is also relevant in the KAMIOKANDE experiment for the detection of solar [13] and atmospheric [14] neutrino fluxes.

The positive result of the LSND experiment [15] for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions together with the negative results of all other experiments on this channel, in particular, the experiments of Ref. [16], and the negative result of the Bugey $\bar{\nu}_e$ disappearance experiment [17] restricts the neutrino mass-squared difference $\Delta m^2$ responsible for this transition to $0.3 \lesssim \Delta m^2 \lesssim 2.2$ eV$^2$ (see also Ref. [18]). Since the neutrino oscillation length is given by $l_{osc} \approx 2.5 \text{m} \times (E_\nu/1 \text{MeV})(1 \text{eV}^2/\Delta m^2)$ all reactor experiments are at a distance $L \gtrsim l_{osc}$ from the reactor core where terms in the $\bar{\nu}_e$ flux are more or less averaged out. Thus the $\bar{\nu}_e$ flux is reduced by a factor $1 - \frac{1}{2} \sin^2 2\theta$ with $10^{-3} \lesssim \sin^2 2\theta \lesssim 4 \times 10^{-2}$ determined by the LSND experiment and the above range of $\Delta m^2$. The transition probability is given by $\sin^2 2\theta \sin^2(\pi L/l_{osc})$ in a two-flavour oscillation formalism. This flux reduction seems to be too small to be detected by presently planned $\bar{\nu}_e e^-$ scattering experiments [19].

Admitting also sterile neutrinos then Eq. (3.13) gets modified to

$$\frac{d\sigma_w}{dT} = (1 - P_{\bar{\nu}_e \rightarrow \bar{\nu}_s}(L)) \frac{d\sigma_w(\bar{\nu}_e e^-)}{dT} + P_{\bar{\nu}_e \rightarrow \bar{\nu}_s}(L) \left( \frac{d\sigma_w(\bar{\nu}_e e^-)}{dT} - \frac{d\sigma_w(\bar{\nu}_\mu e^-)}{dT} \right)$$  \hspace{1cm} (5.1)

with survival ($\alpha = \beta$) or transition ($\alpha \neq \beta$) probabilities $P_{\nu_\alpha \rightarrow \nu_\beta}$. Here $\alpha$ and $\beta$ denote not only the flavours $e, \mu$ and $\tau$ but also sterile degrees of freedom ($s$). Apart from the MUNU experiment, all presently operating experiments with reactor neutrinos have $\bar{\nu}_e + p \rightarrow e^+ + n$ as detection reaction (see, e.g. Ref. [19]) and thus measure the survival probability $P_{\nu_e \rightarrow \nu_e}$. Combining the results of these experiments with future results from elastic $\bar{\nu}_e e^-$ scattering and neglecting possible neutrino magnetic (electric dipole) moments would therefore allow to obtain information on $P_{\nu_e \rightarrow \nu_e}$, the transition probability for $\bar{\nu}_e$ into sterile neutrinos. This situation is similar to the SNO experiment [20] for the solar $\nu_e$ flux. If the dynamical zero in $d\sigma_w(\bar{\nu}_e e^-)/dT$ [21] can be exploited then even elastic $\bar{\nu}_e e^-$ scattering alone would be sufficient.

In order to discuss the cross section $\sigma_m$ (3.16) it is useful to define the oscillation amplitude

$$\bar{A}_{\alpha\beta}(L) \equiv \sum_j U_{\beta j}^* U_{\alpha j} e^{iq_jL}$$  \hspace{1cm} (5.2)

such that the transition or survival probabilities in neutrino oscillations are given by

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = |\bar{A}_{\alpha\beta}(L)|^2.$$ \hspace{1cm} (5.3)

With Eq. (5.2) we can rewrite $\sigma_m$ by using

$$\sum_\ell |U_{e\ell} e^{iq_\ell L}(\mu'_{j\ell} + id'_{j\ell})|^2 = \sum_\beta |\bar{A}_{e\alpha} \bar{\mu}_{\alpha\beta}|^2 \text{ with } \bar{\mu}_{\alpha\beta} \equiv U_{\alpha j}(\mu'_{j\ell} + id'_{j\ell}) U^*_{\beta\ell}.$$ \hspace{1cm} (5.4)

It is obvious that $\sigma_m$ is sensitive to neutrino oscillations as long as $\bar{\mu}_{\alpha\beta}$, the neutrino “moment” matrix in flavour space, or, equivalently, $\mu'_{j\ell} + id'_{j\ell}$ is not proportional to the unit matrix. In this case, due to conservation of probability, we obtain
\[ \sum_{\beta} |A_{e\alpha} \bar{\mu}_{\alpha\beta}|^2 \propto \sum_{\beta} |A_{e\beta}|^2 = \sum_{\beta} P_{\bar{\nu}_e \rightarrow \nu_\beta}(L) = 1 \] (5.5)

and the dependence of \( \sigma_m \) on \( L \) disappears.

However, even if the moment matrix \( \bar{\mu}_{\alpha\beta} \) is not proportional to the unit matrix it is probably quite difficult to exploit this property for experiments on neutrino oscillations because by order of magnitude we would expect \( |\bar{\mu}_{\alpha\beta}| \sim |\bar{\mu}_{\beta\alpha}| \lesssim 10^{-10} - 10^{-9} \) for \( \alpha = e, \mu \) and \( \beta = e, \mu, \tau \). For the \( \tau \) neutrino the limit is \( \mu_{\nu_e} < 5.4 \times 10^{-7} \mu_B \) [22] which translates in our framework into \( |\bar{\mu}_{\tau\beta}| \sim |\bar{\mu}_{\beta\tau}| \lesssim 10^{-6} \). Consequently, only \( \bar{\mu}_{\tau\tau} \) could be as large as the limit from Ref. [22]. In addition, there are no terrestrial limits on the magnetic moments and electric dipole moments for neutrino degrees of freedom which are sterile. If there are neutrino oscillations into the \( \tau \) neutrino and/or sterile neutrinos these transitions could be enhanced by corresponding large magnetic moments in the electromagnetic cross section Eq. (3.16). One should, however, take into account that then the same electromagnetic cross section is also present in the KAMIOKANDE experiment measuring the solar [23] and atmospheric neutrino fluxes in which neutrinos with large magnetic moments of order \( 10^{-6} \) can therefore not be present in large quantities. It has been noted [21] that the dynamical zero in the \( \bar{\nu}_e e^- \) scattering can be used to discover new physics and, in particular, to study neutrino magnetic moments.

The order of magnitude of the interference term \( \sigma_{wm} \), Eqs. (3.17) and (4.2), is estimated by considering the factors appearing in it which leads to

\[ \frac{\alpha G_F m_\nu \mu'_\nu}{\sqrt{2} m_e} (\hbar c)^2 \approx 4.59 \times 10^{-51} \text{ cm}^2 \times (m_\nu/1 \text{ eV}) \left( \mu'_\nu/10^{-10} \right) \] (5.6)

where \( \mu'_\nu \) denotes a generic neutrino magnetic moment or electric dipole moment in units of \( \mu_B \). Note that in Eq. (5.6) we have put \( m_e \) in the denominator rather than \( E_\nu \) as expected from a superficial look at Eq. (3.17) or (4.2). The reason is that a factor \( 1/E_\nu \) in Eq. (5.6) would suggest that \( \sigma_{wm} \) rises for \( E_\nu \rightarrow 0 \). This is not the case because the upper boundary of \( T \) goes to zero like \( E_\nu^2 \) in this limit:

\[ 0 \leq T \leq T_{\text{max}} = \frac{2E_\nu^2}{2E_\nu + m_e}. \] (5.7)

Eq. (5.6) correctly indicates the behaviour of the integrated interference cross section \( \sigma_{wm} \) for \( E_\nu \rightarrow 0 \) and \( E_\nu \rightarrow \infty \).

Eq. (5.6) has to be compared with analogous quantities

\[ \frac{G_F^2 m_\nu E_\nu}{2\pi} (\hbar c)^2 \approx 4.31 \times 10^{-45} \text{ cm}^2 \times (E_\nu/1 \text{ MeV}) \] (5.8)

and

\[ \frac{\alpha^2 \pi \mu'^2_\nu}{m_e^2} (\hbar c)^2 \approx 2.49 \times 10^{-45} \text{ cm}^2 \times (\mu'_\nu/10^{-10})^2 \] (5.9)

determining the orders of magnitude of \( \sigma_w \) and \( \sigma_m \), respectively. Although in \( \sigma_{wm} \) the neutrino moments appear only linearly, this term is about six orders of magnitude smaller.
than $\sigma_w$ and $\sigma_m$ for $m_\nu \sim 1$ eV. The latter two cross sections are of the same order of magnitude for $E_\nu \sim 1$ MeV and $\mu'_\nu \sim 10^{-10}$ which is the reason that in elastic neutrino–electron scattering it is possible that stringent bounds on neutrino moments can be obtained experimentally. The interference term $\sigma_{wm}$ can only be of similar order of magnitude for neutrino oscillations into degrees of freedom with very large magnetic moments as discussed before and neutrino masses e.g. in the keV range which are not favoured by present hints for neutrino oscillations.

In conclusion, we have calculated the weak–electromagnetic interference terms for elastic $\bar{\nu}_e e^-$ scattering of neutrinos with masses, mixing and magnetic and/or electric dipole moments. In lowest order these terms are linear in the neutrino masses. To study such interference terms it is necessary to know the initial neutrino state in $\bar{\nu}_e e^-$ scattering, which is a superposition of neutrino mass eigenstates and corresponding helicity states, to the same precision, i.e. up to terms linear in the neutrino masses. To this end we have investigated neutrino production and scattering as a single combined process. Though naturally in this way production and scattering processes are entangled it is easy to see that for pure weak and electromagnetic antineutrino–electron scattering, the production and scattering processes factorize as expected at order $m_\nu^0$. This is not the case anymore at the next non-trivial order $m_\nu^2$. (There are no contributions linear in $m_\nu$.) As for weak–electromagnetic interference which occurs at order $m_\nu$ we could show that for the usual experimental set-up of experiments with reactor neutrinos (neutrino source and electron target at rest, rotational invariance around the axis source – detector) production factorizes from scattering as well. In this way we obtained a weak–electromagnetic interference cross section which adds to the pure weak and electromagnetic $\bar{\nu}_e e^-$ scattering cross sections. However, it turns out that for neutrino masses of order 1 eV and magnetic moments of order $10^{-10} \mu_B$ this interference cross section is approximately six orders of magnitude suppressed compared to the pure weak and electromagnetic cross sections for neutrino energies $\sim 1$ MeV. Thus under the usual assumptions one can neglect this term in elastic neutrino–electron scattering. We have also commented on the possibility of using the electromagnetic neutrino cross section as a means for the investigation of neutrino oscillations. This could be promising for a large magnetic moment (or electric dipole moment) of the $\tau$ neutrino.

ACKNOWLEDGMENTS

We thank S. M. Bilenky for drawing our attention to the problem studied in this paper.
REFERENCES

[1] B. Pontecorvo, Sov. Phys. JETP 26, 984 (1968); S.M. Bilenky and B. Pontecorvo, Phys. Rep. 41, 225 (1978); S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. 59, 671 (1987).
[2] F. Boehm, Nucl. Phys. B (Proc. Suppl.) 48, 148 (1996); F. Vannucci, ibid., 154 (1996).
[3] A. V. Derbin, Yad. Fiz. 57, 236 (1994) [Phys. of Atomic Nuclei, 57, 222 (1994)].
[4] D. A. Krakauer et al., Phys. Lett. B 252, 177 (1990); R. C. Allen et al., Phys. Rev. D 47, 11 (1993).
[5] C. Broggini (MUNU Coll.), Nucl. Phys. B (Proc. Suppl.) 35, 441 (1994).
[6] F. Ould–Saada, in Proceedings of the Summer School on Physics with Neutrinos, Zuoz, Switzerland, 1996, edited by M. P. Locher (Paul Scherrer Institut, Villigen, 1996).
[7] D. Yu. Bardin, S. M. Bilenky and B. Pontecorvo, Phys. Lett. B 32, 68 (1970); A. V. Kyuldjiev, Nucl. Phys. B 243, 387 (1984); P. Vogel and J. Engel, Phys. Rev. D 39, 3378 (1989).
[8] R. Barbieri and G. Fiorentini, Nucl. Phys. B 304, 909 (1988).
[9] P. Stockinger and W. Grimus, Phys. Lett. B 327, 327 (1994).
[10] J. Rich, D. Lloyd Owen and M. Spiro, Phys. Rep. 151, 267 (1987); B. Kayser, Nucl. Phys. B (Proc. Suppl.) 19, 177 (1991); C. Giunti, C. W. Kim and U. W. Lee, Phys. Rev. D 45, 2414 (1992); C. Giunti, C. W. Kim, J. A. Lee and U. W. Lee, Phys. Rev. D 48, 4310 (1993); J. Rich, Phys. Rev. D 48, 4318 (1993).
[11] W. Grimus and P. Stockinger, Phys. Rev. D 54, 3414 (1996).
[12] R. N. Mohapatra and P. B. Pal, Massive Neutrinos in Physics and Astrophysics (World Scientific, Singapore, 1991).
[13] K. S. Hirata et al., Phys. Rev. D 44, 2241 (1991).
[14] Y. Fukuda et al., Phys. Lett. B 335, 237 (1994).
[15] C. Athanassopoulos et al., Phys. Rev. Lett. 77, 3082 (1996).
[16] J. Kleinfeller, Nucl. Phys. B (Proc. Suppl.) 48, 207 (1996); L. Borodovsky et al., Phys. Rev. Lett. 68, 274 (1992).
[17] B. Achkar et al., Nucl. Phys. B 434, 503 (1995).
[18] S. M. Bilenky, C. Giunti and W. Grimus, preprint UWTh-1996-42 [hep-ph/9607372], to appear in Z. Phys. C, and preprint UWTh-1997-11 [hep-ph/9705300].
[19] K. Zuber, hep-ph/9406364.
[20] Sudbury Neutrino Observatory Coll., Phys. Lett. B 194, 321 (1987).
[21] J. Segura et al., Phys. Rev. D 49, 1633 (1994).
[22] A. M. Cooper–Sarkar et al., Phys. Lett. B 280, 153 (1992).
[23] A. M. Mourão, J. Pulido and J. P. Ralston, Phys. Lett. B 285, 364 (1992).