How fast does information leak out from a black hole?

Jacob D. Bekenstein*

Department of Physics, University of California at Santa Barbara, Santa Barbara, CA 93106

and

The Racah Institute of Physics, Hebrew University of Jerusalem, Givat Ram, Jerusalem 91904, Israel**

PACS: 97.60.Lf, 95.30.Tg, 04.60.+n, 05.90.+m

ABSTRACT

Hawking’s radiance, even as computed without account of backreaction, departs from blackbody form due to the mode dependence of the barrier penetration factor. Thus the radiation is not the maximal entropy radiation for given energy. By comparing estimates of the actual entropy emission rate with the maximal entropy rate for the given power, and using standard ideas from communication theory, we set an upper bound on the permitted information outflow rate. This is several times the rates of black hole entropy decrease or radiation entropy production. Thus, if subtle quantum effects not heretofore accounted for code information in the radiance, the information that was thought to be irreparably lost down the black hole may gradually leak back out from the black hole environs over the full duration of the hole’s evaporation.

* E-mail: jacob@cosmic.physics.ucsb.edu

** Permanent address
Following his theoretical discovery of the black hole radiation that bears his name, Hawking noted [1] that such radiation seems to contradict accepted quantum physics. If a black hole forms from matter prepared in a pure state, and then radiates away its mass in ostensibly thermal radiation, one is left with a high entropy mixed state of radiation. This contradicts the quantum dogma that a pure state will always remain pure under Hamiltonian evolution. A related contradiction follows from the interpretation of black hole entropy as the measure of the information hidden in the black hole about the ways it might have been formed [2]. Since fully thermal radiation is incapable of conveying detailed information about its source, that information remains sequestered as the black hole radiates, and when it finally evaporates away, the information is lost forever. These two contradictions are facets of the black hole information loss paradox.

Three reactions to the paradox are possible (for reviews see Refs. 3 and 4). The first is to accept the loss of information and the trasmutation of pure into mixed state as an inevitable consequence of the merging of gravity with quantum physics [1]. Specific schemes for accomplishing this have been found to be incompatible with locality or conservation of energy [5]. A second point of view [1,6] holds that black hole evaporation leaves a massive remnant of Planck dimensions which retains all the information in question. This possibility is not as conservative as it sounds. According to the bound on specific entropy or information [7], or considerations from quantum gravity [8], an object of Planck mass and dimension can hold only a few bits of information, so that the posited massive remnants cannot fit the information bill of a large evaporating black hole. Variations of the remnant idea, their merits and problems have been discussed in Refs. 8,9 and 4, among many. Yet a third view [10,11] is that exploiting subtle correlations in the radiation, the information manages to leak back out from the incipient black hole in the course of the evaporation. The leak cannot be postponed to the late stages of evaporation without incurring the problems accompanying remnants [6,4]. For information leak throughout the evaporation to be a reasonable resolution of the paradox, it must be shown that an information flow of the appropriate magnitude can come out of the black hole’s near environs. A step in this direction is taken in the present paper.

Lately these three viewpoints have been widely examined by means of the 1 + 1 dimensions dilaton-gravity model of an evaporating black hole proposed by Callan, Giddings, Harvey and Strominger [12]. This model allows explicit treatment of the quantum radiance and its backreaction on the hole (for reviews see Ref. 3). Whatever the final outcome of this type of investigations, it will be nontrivial to project the conclusions from this model to the realistic case of 1 + 3 black holes. Therefore, any new model independent aproach which can address the 1 + 3 dimensional case would be of great conceptual help. We here employ a thermodynamic argument (which fact makes it virtually model independent) to show that for the 1 + 3 dimensional Schwarzschild black hole, an outflow of information of the required magnitude to resolve the information problem is permitted in principle. We do not explore here specific mechanisms for information extraction, but note that this subjects has already received attention [13].

Although the Hawking radiance has thermal features, as certified by the exponential distribution
of the number of quanta emitted in each mode, and the lack of correlations between modes [14], it is not precisely of blackbody form. The would be blackbody spectrum is distorted by the mode dependence of the barrier penetration factor $\Gamma_{s\text{imp}}(\omega)$, where $s$ stands for the particle species, $j$ and $m$ for the angular momentum quantum numbers and $p$ for the polarization, with $\Gamma < 1$ in general [1]. For a Schwarzschild black hole of mass $M$, inverse temperature $\beta_{\text{bh}} = 8\pi GM/\hbar$ and entropy $S_{\text{bh}}$, the average energy in a mode is (henceforth we set $c = 1$)

$$\varepsilon_{s\text{imp}}(\beta_{\text{bh}}, \omega) = \frac{\hbar \omega \Gamma_{s\text{imp}}(\omega)}{e^{\beta_{\text{bh}} \hbar \omega} \pm 1}$$  \tag{1}$$

where henceforth the upper (lower) sign corresponds to fermions (bosons). It is as if blackbody radiation has been passed through a filter. But the analogy with filtered radiation stops there. In the laboratory the filter at the mouth of a blackbody cavity eventually heats up to the cavity’s temperature, and so eventually the emerging radiation becomes blackbody. For a black hole the distortion is permanent. Perhaps a better analogy is the radiation from a star which, generally, is far from blackbody because it comes from layers at different temperatures.

A consequence of the distortion is that, compared with blackbody radiation with the same power (but, of course, at inverse temperature different from $\beta_{\text{bh}}$), Hawking radiance is less entropic. Alternatively, for given inverse temperature, Hawking radiation, in contrast with blackbody radiation, has free energy, and useful work can be gotten out of it by reshuffling the energy among the modes. The entropic deficiency suggests that the radiance may be carrying information about the state of the quantum fields in the far past, $i.e.$, just the information that is supposed to be lost. This would, of course, be impossible if the radiance were exactly blackbody. In our stellar analogy, much is learned about a star’s atmosphere (composition and physical conditions) from the departure of its spectrum from blackbody, e.g., spectral lines.

Let us look at the question in the light of quantum communication theory (for reviews see Ref. 15). We shall adapt Lebedev and Levitin’s pioneering thermodynamic approach [16], and measure information in natural units (nits); 1 nit = $\log_2 e$ bits. To this end we consider the entropy of the Hawking radiance as entropy (uncertainty about the state) of the noise which is adulterating the signal conveying the information. The radiance power, $\dot{E}$, will be interpreted as the sum of noise and signal powers. With this scenario the maximum rate at which information can be recovered from the radiation by a suitable detector is $\dot{I}_{\text{max}} \equiv \dot{S}' - \dot{S}$ where $\dot{S}$ is the actual entropy outflow rate, while $\dot{S}'$ is the maximum entropy rate corresponding to the actual power $\dot{E}$ under the boundary conditions of the system. (Actually, if the noise is correlated with the signal, as may well be the case in the Hawking radiance, $\dot{I}_{\text{max}}$ will be larger [15]; in this case our arguments below are actually strengthened). Lebedev and Levitin considered a one dimensional communication channel. Most of our discussion will be devoted to the issue of how to define the three-dimensional channel issuing from a black hole.

For convenience we shall use the notation $i \equiv \{s\text{imp}\}$. The probability distribution for the black
hole to spontaneously emit \( n \) quanta in mode \( \{ i, \omega \} \) is given by \([14]\)

\[
p_{\text{sp}}(n) = (1 \pm e^{-\gamma})^{-1}e^{-\gamma n}
\]

where \( \gamma_i(\beta_{\text{bh}}, \omega) \) is defined by

\[
\frac{1}{e^{\gamma_i} \pm 1} = \frac{\Gamma_i(\omega)}{e^{\beta_{\text{bh}}h\omega} \pm 1}
\]

From this follows the entropy in the given mode:

\[
\sigma_i(\beta_{\text{bh}}, \omega) = \pm \ln(1 \pm e^{-\gamma_i}) + \frac{\gamma_i}{e^{\gamma_i} \pm 1}
\]

We may also reexpress Eq. (1) as

\[
\varepsilon_i(\beta_{\text{bh}}, \omega) = \frac{\hbar \omega}{e^{\gamma_i} \pm 1}
\]

The entropy outflux rate and the power may now be expressed as

\[
\dot{S} = \sum_i \int_0^\infty \sigma_i(\beta_{\text{bh}}, \omega) \frac{d\omega}{2\pi}
\]

\[
\dot{E} = \sum_i \int_0^\infty \varepsilon_i(\beta_{\text{bh}}, \omega) \frac{d\omega}{2\pi}
\]

where \( d\omega/2\pi \) is the rate at which modes of type \( i \) emanate from the hole.

Page [17,18] has calculated numerically the contributions of various particle species to \( \dot{E} \) and \( \dot{S} \), and states the results in terms of the dimensionless ratios \( \mu \equiv \dot{E}/(GM)^2 \hbar^{-1} \) and \( \nu \equiv \dot{S}/(\beta_{\text{bh}} \dot{E}) \). For each species of light neutrinos or antineutrinos he finds \( \mu \approx 4.090 \times 10^{-5} \) and \( \nu \approx 1.639 \) with modes having \( j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \) being the overwhelming contributors. For photons \( \mu \approx 3.371 \times 10^{-5} \) and \( \nu = 1.500 \) with modes having \( j = 1, 2, 3 \) making the dominant contribution. And for gravitons \( \mu \approx 3.84 \times 10^{-6} \) and \( \nu \approx 1.348 \) with modes having \( j = 2, 3 \) contributing overwhelmingly. If the black hole emits three species of neutrinos and three of antineutrinos (each with a single helicity), photons and gravitons, the overall numbers are \( \mu \approx 2.829 \times 10^{-4} \) and \( \nu = 1.619 \) (this last value involves the individual \( \nu \)'s weighted by the \( \mu \)'s). If there are only two light neutrino species the numbers are \( \mu \approx 2.011 \times 10^{-4} \) and \( \nu = 1.610 \). The contributions of bosons alone are \( \mu \approx 3.755 \times 10^{-5} \) and \( \nu = 1.484 \).

We now have to compare \( \dot{S} \) with the entropy rate \( \dot{S}' \) of the maximally entropic (blackbody) distribution whose power \( \dot{E}' \) equals \( \dot{E} \). A straightforward way to get \( \dot{S}' \) is to compute both it and \( \dot{E}' \) from the Boltzmann formulae for blackbody emission by assuming some effective radiating area for the black hole. This “photosphere” is not a well defined concept, depending as it does on frequency. Thus, in the scattering of high frequency (geodesically moving) quanta by our black hole, all quanta hitting within a crossection \( 27\pi G^2 M^2 \) will be captured [19]. This suggests a photospheric area \( A_{\text{phot}} \) four times as large. But \( A_{\text{phot}} \) must be larger than that. This is because each \( \Gamma_i(\omega) \) vanishes only as some power of \( \omega \) as \( \omega \to 0 \), so that quanta with fairly large impact parameter are sometimes absorbed and, therefore, must also be emitted sometimes. The modes they are in must thus be included in calculating the comparison blackbody radiation. We thus write \( A_{\text{phot}} = \xi 108\pi G^2 M^2 \) with \( \xi > 1 \).
At inverse temperature $\beta_{\text{eff}}$, a black body of area $A_{\text{phot}}$ emits power $\dot{E}' = N \pi^2 A_{\text{phot}} / (60 \beta_{\text{eff}}^4 \hbar^3)$, where $N$ is the effective number of particle species emitted. Photons and gravitons contribute 1 each to $N$; each species of fermions contributes $7/16$. Therefore, we must set $N = 37/8$ if there are three light neutrino species and $N = 15/4$ if there are two. Comparing these results for $\dot{E}'$ with Page’s results for $\dot{E}$ we obtain $\beta_{\text{eff}} = 1.230 \xi^{1/4} \beta_{bh}$ for three light neutrinos and $\beta_{\text{eff}} = 1.271 \xi^{1/4} \beta_{bh}$ for two. For blackbody radiation flowing in three space dimensions, $\dot{S}' = \frac{4}{3} \beta_{\text{eff}} \dot{E}'$. Thus, taking into account the $\nu$ factors of Page, $\dot{I}_{\text{max}} \equiv \dot{S}' - \dot{S} = (1.640 \xi^{1/4} - 1.619) \beta_{bh} \dot{E}$ for three neutrino species; for two the numerical factor is 1.694$^{\xi^{1/4}} - 1.610$. Since $\beta_{bh} \dot{E} = -\beta_{bh} \dot{M} = -\dot{S}_{bh}$ we see that even if $\xi = 1$, 1-5% of the sequestered information could come out in principle. This is just a lower bound because we actually expect $\xi$ to be larger, so that $\dot{I}_{\text{max}}$ may actually be a substantial fraction of $|\dot{S}_{bh}|$. The present method is, however, unable to tell us just how much.

The following alternative approach is able to set an upper bound on $\dot{I}_{\text{max}} / |\dot{S}_{bh}|$. We shall compare Page’s $\dot{S}$ with the entropy flow $\dot{S}'$ in the same angular momentum modes of a blackbody distribution with inverse temperature $\beta_{\text{eff}}$ determined by the equality of the powers (in the relevant set of modes only). This is different from the above calculation which compared with blackbody modes having sharply defined directions (coming from the black hole). The new comparison should overestimate $\dot{S}'$ and consequently $\dot{I}_{\text{max}}$ because blackbody radiation populating a finite number of angular momentum modes assigns substantial weight to modes with $\omega \to 0$; for nonzero orbital angular momentum these correspond to arbitrarily large impact parameter, and are thus not related to the black hole. These spurious modes broaden the phase space and so artificially increase the entropy rate $\dot{S}'$. Later on we shall show how to repair part of this problem.

To calculate the blackbody quantities we replace for each mode $\gamma_i \to \beta_{\text{eff}} \hbar \omega$ in Eq. (4) and Eq. (5). The integral of the logarithmic term in $\sigma_i$ of Eq. (6) can be combined with the other term by integration by parts. Using

$$\int_0^\infty \frac{x \, dx}{e^x + 1} = \frac{\pi^2 (3 + 1)}{24}$$

one can cast the results in the form

$$\frac{\dot{S}'}{2\beta_{\text{eff}}} = \dot{E}' = \frac{\pi}{12 \hbar \beta_{\text{eff}}} \sum_i g_i$$

where $g_i = 1$ or $\frac{1}{2}$ for a boson or fermion mode, respectively. Summing over the particle species and modes which Page considered, we obtain $\sum_i g_i = 90$ for three light neutrinos and $\sum_i g_i = 78$ for two. Equating $\dot{E}'$ with Page’s $\dot{E}$ gives $\beta_{\text{eff}} = 11.48 \beta_{bh}$ and $\dot{S}' = 22.97 \beta_{bh} \dot{E}$ if there are three light neutrinos while $\beta_{\text{eff}} = 12.68 \beta_{bh}$ and $\dot{S}' = 25.36 \beta_{bh} \dot{E}$ if there are two. Using the cited values of $\nu$ we conclude that

$$\dot{I}_{\text{max}} = \dot{S}' - \dot{S} = 21.35 |\dot{S}_{bh}|$$

5
for three light neutrinos. If there are only two, the numerical factor is 23.75.

Although the above figure for $\dot{I}_{\text{max}}$ is an overestimate, it is so large as to suggest that an information leak of sufficient magnitude to resolve the information problem is allowed. For example, if $\dot{I}$, the actual information outflow rate, amounts to $1.619|\dot{S}_{\text{bh}}|$ throughout the course of evaporation of a massive black hole down to $M \approx 1 \times 10^{14}$ g (when the emission of massive particles becomes important and most of the initial black hole entropy has disappeared [17]), the outgoing information equals the total Hawking radiance entropy. Hence, given an appropriate mechanism, the radiation can end up in a pure state.

But how exaggerated is the above bound on $\dot{I}_{\text{max}}$? We shall not attempt to exclude the low frequency modes by hand from our calculation; such a task would be fraught with ambiguities. Rather we ask, if it were possible to modify the curvature barrier surrounding the black hole, and consequently to modify the $\Gamma_i(\omega)$, what would be the most entropic spectrum that could come out of the black hole? As we shall see presently, the answer is not blackbody: the $\Gamma_i(\omega)$ cannot all be unity. However, the new spectrum is more relevant for comparison than the pure blackbody one because, for given angular momentum, it does suppress low frequency modes.

If we could manipulate the $\Gamma_i(\omega)$, the largest entropy flow $\dot{S}'$ would be obtained with the $\Gamma_i(\omega)$ as large as possible. This is seen by differentiating $\sigma_i$ [Eq. (4)] with respect to $\gamma_i$, and transforming the derivative to one with respect to the corresponding $\Gamma_i(\omega)$ with help of Eq. (3); the result is positive definite. We are thus interested in the hypothetical situation when all the $\Gamma_i(\omega)$ are as large as physically possible. For fermion modes no reason is known to prevent $\Gamma_i(\omega)$ from approaching unity. However, for boson modes the value of $\Gamma_i(\omega)$ is subject to a bound.

This bound stems from the formula

$$\Gamma_i(\omega) = (1 - e^{-\beta_{\text{bh}}\hbar\omega}) \Gamma_{i0}(\omega)$$  \hspace{1cm} (12)

where $1 - \Gamma_{i0}$ is the probability that a single incident quantum is scattered back from the black hole [20,21]. Formula (12) follows by combinatorics from the interpretation in terms of a combination of scattering, and spontaneous and stimulated emission of the conditional probability $p(m|n)$ that the Schwarzschild black hole returns outward $m$ quanta in a mode which had $n$ incident ones. The $p(m|n)$ has been obtained independently by information theoretic [20] and field theoretic [22] methods. It turns out to be impossible to understand its form as due to a combination of Hawking emission and scattering [20]; the inclusion of stimulated emission supplies the missing element. The stimulated emission depresses the value of $\Gamma_i(\omega)$ under the naive absorption probability $\Gamma_{i0}(\omega)$. In fact, because $\Gamma_{i0} \leq 1$, $\Gamma_i(\omega) \leq 1 - e^{-\beta_{\text{bh}}\hbar\omega}$. (The case $\Gamma_i = \Gamma_{i0} = 1$ is not actually excluded by the considerations of Ref. 20, but a $\Gamma_i$ close to unity is not allowed).

In view of the above, let us compare the actual $\dot{S}$ with the $\dot{S}'$ of a spectrum of the form of Eq. (1) with inverse temperature $\beta_{\text{eff}}$ and having $\Gamma_i(\omega) = 1$ (perfect blackbody) for all fermion modes, but $\Gamma_i(\omega) = 1 - e^{-\beta_{\text{eff}}\hbar\omega}$ for all boson modes. This is the closest a black hole emission spectrum could come to blackbody, and thus gives the largest $\dot{S}'$ for given power. Note that the new comparison
spectrum is poor in low frequency bosons as compared with the blackbody spectrum. Thus we have
gone part of the way towards repairing the problem noted earlier. Since the fermions are blackbody
as in our previous calculation, we shall just concentrate on the boson contributions to \( \hat{S} \) and \( \hat{S}' \). In
what follows the subscript “b” stands for bosons.

We shall first compute the boson contribution \( \hat{E}'_b \). From the chosen \( \Gamma_i(\omega) \) it follows that
\[ \varepsilon_i(\beta_{\text{eff}}, \omega) = \hbar \omega e^{-\beta_{\text{eff}} \hbar \omega}. \]
Thus Eq. (7) gives \( \hat{E}'_b = (2\pi \hbar \beta_{\text{eff}}^2)^{-1} \sum_b g_i \). The sum over the boson
modes calculated by Page is 54. Equating the result to his \( \hat{E}_b \) determines that \( \beta_{\text{eff}} = 19.04 \beta_{\text{bh}} \). We
now compare \( \varepsilon_i \) with Eq. (5) to determine that \( e^{\gamma_i} = 1 + e^{\beta_{\text{eff}} \hbar \omega} \). It then follows from Eq. (4) that
\[ \sigma_i(\beta_{\text{eff}}, \omega) = \ln(1 + e^{-\beta_{\text{eff}} \hbar \omega}) + e^{-\beta_{\text{eff}} \hbar \omega} \ln(1 + e^{\beta_{\text{eff}} \hbar \omega}) \]
for boson modes. After integration by parts, Eq. (6) gives
\[ \hat{S}'_b = \left( \pi/24 + \ln 2/\pi \right) \sum_b g_i \beta_{\text{eff}} \]
(13)
Comparing with Page’s result for \( \hat{S}_b \) gives
\[ (\hat{I}_{\text{max}})_b = \hat{S}'_b - \hat{S}_b = 5.382 |\hat{S}_{bh}| \]
where we have used the value of the total \( \hat{S}_{bh} \) including the contribution of three light neutrino
species.

Although the above figure may overestimate \( (\hat{I}_{\text{max}})_b \), we must still add a contribution from
fermions to get the total \( \hat{I}_{\text{max}} \). Thus our earlier impression from Eq. (11) that the departure of
Hawking radiance from blackbody is enough to permit a large information outflux stands. Gradual
escape of the sequestered information (equal to \( S_{bh} \)) and reconstitution of a pure radiation state
by the time the hole has evaporated away seem feasible, provided some quantum mechanism codes
the information in the radiation. It would be surprising if nature has not taken advantage of this
opportunity to obviate the information problem. There remains the task of identifying the mechanism
of information leak. The prominent part played by the curvature barrier in deforming the blackbody
spectrum makes processes associated with it, such as stimulated emission, likely culprits.

I thank Gary Horowits, Don Page and Andy Strominger for informative conversations, and Jim
Hartle for hospitality in Santa Barbara.

References

[1] S. W. Hawking, Phys. Rev. D 14, 2460 (1976) and Commun. Math. Phys. 87, 395 (1982).
[2] J. D. Bekenstein, Phys. Rev. D 7, 2333 (1973).
[3] J. Harvey and A. Strominger, “Quantum aspects of black holes”, Chicago preprint EFI-92-
41, hep-th/9209055; S. Giddings, “Toy models for black hole evaporation”, UCSB-TH-92-36,
hep-th/9209113.
[4] J. Preskill, “Do black holes destroy information?” CALT-68-1819, hep-th/9209058.
[5] T. Banks, M. E. Peskin and L. Susskind, Nucl. Phys. B244, 125 (1984).
[6] Y. Aharonov, A. Casher and S. Nussinov, Phys. Lett. 191B, 51 (1987).
[7] J. D. Bekenstein, Phys. Rev. D 23, 287 (1981); J. D. Bekenstein and M. Schiffer, Phys. Rev. D 39, 1109 (1989).
[8] S. Giddings, Phys. Rev. D 46, 1347 (1992).
[9] L. Susskind and L. Thorlacius, Nucl. Phys. B382, 123 (1992); T. Banks, A. Dabholkar, M.R. Douglas, and M. O’Loughlin, Phys. Rev. D 45, 3607 (1992); T. Banks, A. Strominger and M. O’Loughlin, “Black hole remnants and the information puzzle”, RU-92-40 and hep-th/9211030.
[10] D. Page, Phys. Rev. Letters 44, 301 (1980).
[11] G.’t Hooft, Nucl. Phys. B256, 727 (1985) and B335, 138 (1990).
[12] C.G. Callan, S.B. Giddings, J.A. Harvey, and A. Strominger, Phys. Rev. D 45, R1005 (1992).
[13] J. Preskill, P. Schwarz, A. Shapere, S. Trivedi and F. Wilczek, Mod. Phys. Letters A6, 2353 (1991).
[14] L. Parker, Phys. Rev. D 12, 1519 (1975); J. D. Bekenstein, Phys. Rev. D 12, 3077 (1975) R. M. Wald, Commun. Math. Phys. 45, 9 (1975); S. W. Hawking, Phys. Rev. D 13, 191 (1976).
[15] Y. Yamamoto and H. A. Haus, Revs. Mod. Phys. 58, 1001 (1986); J. D. Bekenstein and M. Schiffer, Int. Journ. Mod. Phys. C 1, 355 (1990).
[16] D. S. Lebedev and L. B. Levitin, Dokl. Akad. Nauk SSSR 149, 1299 (1963) [Sov. Phys. Dokl. 8, 377 (1963)].
[17] D. N. Page, Phys. Rev. D 13, 198 (1976).
[18] D. N. Page, Phys. Rev. D 14, 3260 (1976).
[19] C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation (Freeman, San Francisco, 1973).
[20] J. D. Bekenstein and A. Meisels, Phys. Rev. D 15, 2775 (1977).
[21] J. D. Bekenstein, in To Fulfill a Vision, ed. Y. Ne’eman (Addison-Wesley, Reading, Mass., 1981).
[22] P. Panangaden and R. M. Wald, Phys. Rev. D 16, 929 (1977).