Spectral synthesis of circumstellar disks - application to white dwarf debris disks

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Abstract. Gas and dust disks are common objects in the universe and can be found around various objects, e.g. young stars, cataclysmic variables, active galactic nuclei, or white dwarfs. The light that we receive from disks provides us with clues about their composition, temperature, and density. In order to better understand the physical and chemical dynamics of these disks, self-consistent radiative transfer simulations are inevitable. Therefore, we have developed a 1+1D radiative transfer code as an extension to the well-established model atmosphere code PHOENIX. We will show the potential of the application of our code to model the spectra of white dwarf debris disks.

1. Introduction
The detection of infrared excesses in association with the metal rich white dwarfs GD 362 and G29-38 has introduced the picture of debris disks around these stars. The high metal content observed in these evolved stars’ atmospheres requires a steady or recent source of enriched matter, since the metals could only survive depletion by gravitational settling for a very short time. This source is believed to be provided by accretion of dusty material from a disrupted solid body, such as an asteroid. The IR excess is a strong indication for the formation of an accretion disk. To understand the origin of the dust, as well as to constrain the accretion rate and thus models for the gravitational diffusion of heavy elements in the atmosphere, comparison of observations with model spectra are necessary.

2. Standard accretion model
We adopt the standard accretion model for geometrically thin disks, i.e. height $H \ll$ radius $R$ (Shakura & Syunyaev 1973). Matter is assumed to rotate with Kepler velocity and viscous shear decelerates inner and accelerates outer parts leading to accretion of matter and outward transportation of angular momentum. Turbulent cells smaller than the disk height $H$ are the proposed origin of kinematic viscosity. The height averaged kinematic viscosity is usually described by

$$\nu = \alpha c_s H ,$$

where $0 \leq \alpha \leq 1$ is the angular momentum transfer efficiency.
3. Model calculations
We use a 1+1D disk ring structure approximation. The disk is divided into rings and each ring is assumed to be plane-parallel and to have constant physical properties from its inner to its outer radius. Radiative transfer and disk structure are calculated vertically from the disk’s midplane to the top layer. Gas and dust are in chemical equilibrium and dust is assumed not to settle in the disk.

![Disk structure as adopted for our numerical calculations. Radial disk ring extensions are chosen so that the structure varies only little over ring width. The disk is thin ($H \ll R$) so that vertical and radial structure can be separated.](image)

Basic program input parameters are the radius $R_*$ and mass $M_*$ of the central star, the distance $R$ between star and disk ring, the mass accretion rate $\dot{M}$ in the disk, and the Reynolds number $Re$ as a measure for the mean kinematic viscosity $\nu = \sqrt{GM_*/R}/Re$.

The iterative calculation of a disk ring model atmosphere starts by either constructing a grey start model for disks (after Hubeny 1990) or using a given PHOENIX model. We have further adopted the following conservation and constraint equations:

**Hydrostatic equilibrium:** This equation has to be used in a form to account for the varying surface gravity, i.e.

$$\frac{dP}{dm} = \frac{GM_*}{R^3} z .$$

**Energy conservation:** Unlike ordinary stellar atmospheres, mechanical energy is released in disk atmospheres in every layer. Therefore, the flux is not conserved and we have to consider energy equilibrium of the form

$$E_{\text{mech}} = E_{\text{rad}} + E_{\text{conv}} .$$

**Irradiation:** Also irradiation from the central star can be considered. As input source serves one or a combination of black body spectra of given effective temperatures. A PHOENIX spectrum can also be used as input. Discrete rays that receive radiation from the star are determined and assigned a fraction of the flux incident to the disk. The irradiation geometry is shown in Figure 2.

4. Results
We have calculated a simple disk model consisting of three disk rings. The rings are in hydrostatic equilibrium and the mass accretion rate is set according to Becklin et al. (2005) to $\dot{M}_* = 10^{11} \text{ g s}^{-1} \approx 2 \cdot 10^{-15} M_\odot \text{ yr}^{-1}$. This small accretion rate leads to small surface densities ($\approx$ disk mass) and the disk rings are optically thin in the continuum. The vertical temperature distribution is almost constant and only “artificially” brought to a value reasonably in agreement with the blackbody temperature fit to the infrared excess by setting an unrealistically high viscosity value.

In order to estimate the mean dust grain size, we have performed various computations varying this parameter. This allows us to get an idea of the influence on the strength of the
silicate feature at 10 $\mu$m and the 13 $\mu$m emission. We have varied the mean dust grain size between 0.01 $\mu$m and 10 $\mu$m. The resulting spectra of these simulations are shown in Figure 3. For our refined models of GD 362 we have chosen a mean dust grain size of 0.01 $\mu$m.

Since the disk slabs are optically thin, the continuum flux is in a reasonable approximation linear with the optical depth. Therefore, the fitting of the models’ continuum to the Spitzer spectrum can be adjusted by varying the mass accretion rate. We simply used a fudge factor to account for the missing flux.

In Figure 4 we show the co-added models folded to the resolution of the instrument. The innermost ring is directly irradiated and heated up by the white dwarf and is assumed to be isothermal (700 K), thus shielding the disk parts behind it. The inner ring extends from $9.3 - 10 R_\ast$. The following three rings go from $10 - 70 R_\ast$ and have almost isothermal temperature structures between 670 K and 500 K.
5. Discussion
We have shown that our disk model atmosphere code is in principal applicable to the modelling of white dwarf debris disks. Our models calculated for the parameters of GD 362 show a reasonable fit to IR observations of the objects. Masses and accretion rates of WD debris disks are however small and therefore make the model calculations challenging.

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