Hybrid automatic repeat request-based intelligent reflecting surface-assisted communication system

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The intelligent reflecting surface (IRS) is an emerging technique to extend wireless coverage. In this letter, the performance of the hybrid automatic repeat request (hybrid ARQ) for an IRS-assisted system is analysed. More specifically, the outage performance of the IRS-aided system using hybrid ARQ protocol with chase combining is studied. The asymptotic analysis also shows that the outage performance is better and improves linearly by increasing the number of reflectors of the IRS-aided system. The results also verify the potential of combining the ARQ scheme in the link layer of the IRS-aided system and demonstrate that a very small change of path loss condition can impact the performance largely.

Introduction: Intelligent reflecting surface (IRS) is a revolutionary enabling technique to significantly improve the system performance of the wireless system. By functioning as a reconfigurable lens or mirror for the electromagnetic waves and intelligently reconfiguring the radio wave propagation environment, the IRS can improve system coverage especially when line-of-sight (LoS) is not guaranteed [1]. To this end, a number of researches have been devoted to investigating the performance of IRS under different setups. The ergodic capacity and bit error rate (BER) performance of an IRS-assisted dual-hop unmanned aerial vehicle (UAV) communication system are investigated in [2]. The analysis on coverage, probability of signal-to-noise ratio (SNR) gain, and delay outage rate of IRS-aided communication system is conducted in [3]. The secrecy performance of IRS-assisted millimetre wave system is studied in [4].

It is well known that system performance can be further improved with the incorporation of the automatic repeat request (ARQ) scheme in the link layer of the system [5]. The ARQ mechanism is a well-established retransmission technique that has been applied in virtually all modern communication systems. The ARQ mechanism can be interpreted as channels with sequential feedback, where the performance can be enhanced by resending data that has been impaired by unfavourable channel conditions with the use of both error correction and error detection codes. Depending on whether the retransmission includes new redundancy bits from the channel encoder or the retransmitted packet is identical to the original transmission, the hybrid ARQ can be further categorised into the following two types: Hybrid ARQ with incremental redundancy (IR) and hybrid ARQ with chase combining (CC) [5]. Despite the omnipresence of hybrid ARQ and the great potential of IRS, a thorough literature search indicates that the performance of IRS-aided communication with hybrid ARQ has not been investigated yet to the authors’ best knowledge. To fill the gap, we study the outage performance of IRS-aided system with hybrid ARQ with CC in this letter.

System and channel models: The investigated system model for the IRS-aided communication system is shown in Figure 1, where a source node $S$ tries to communicate with a destination node $D$. There exists no LoS path (i.e. no LoS scenario) between nodes $S$ and $D$ due to blockage. Instead, an IRS $R$ composed of $N$ reflecting elements is placed between $S$ and $D$ to facilitate the transmission between them. For simplicity, both $S$ and $D$ are assumed to be equipped with only one antenna. The full channel state information (CSI) of the links $S-R$ and $R-D$ is available to the IRS $R$ such that maximised SNR at $D$ can be achieved. As in [3], it is also assumed that the signals reflected only once by the IRS are much more significant than the signals reflected multiple times, which are thus ignored. We also consider the case as in [6–9], where the reflectors of the IRS are separated large enough such that the elements of the vectors $\mathbf{h}_{SR}$ and $\mathbf{h}_{RD}$ are independent. Further, Rayleigh fading is considered for the links between the IRS and $S$ and between the IRS and $D$. The choice of Rayleigh fading is justified by the fact that in practice LoS between IRS and communicating entities cannot always be guaranteed, and scattering from the surrounding environments generally should not be overlooked [10]. Therefore, the IRS-based transmission leads to a double Rayleigh fading distribution from $S$ to $D$. It should be noted that when different fading distributions are followed, the central limit theorem (CLT) can still be invoked as in this letter when $N$ is sufficiently large, and therefore the main findings obtained in this letter will still be valid.

It should be noted that the IRS can function in the same manner as a relay. However, an IRS and a relay have quite different structures and working principles, which leads to different analysis and performance for them [2]. For an IRS, it can be better perceived as part of the so-called smart environment, which simply reflects passively the signals with the configuration on the phase to redirect the signals to the desired receiver [6]. Instead, a relay needs to process the received signals before further forwarding the signals to the desired receiver. A detailed comparison between the IRS-aided and relay-aided communication is done in [11].

For communication with the aid of IRS consisting of $N$ reflecting elements, the received signal at node $D$ can be expressed as [3]

$$y = \sqrt{P_S} \cdot \mathbf{h}_{SR}^H \cdot \mathbf{h}_{RD} \cdot s + w_D,$$

(1)

where $P_S$ is the transmit power of $S$, $s$ is the transmitted signal with unit energy, $w_D$ is the zero-mean additive white Gaussian noise with variance $N_0$, $\omega_D = \text{diag}(\omega_1, \ldots, \omega_N) = \text{diag}(\phi_1 e^{j2\pi \phi_1}, \ldots, \phi_N e^{j2\pi \phi_N})$ is the diagonal matrix consisting of the reflection coefficient produced by each reflection element of the IRS, $\mathbf{h}_{SR}$ is the vector with the channel gain from $S$ to each element of IRS and the vector $\mathbf{h}_{RD}$ includes the channel gain from each element of IRS to $D$, respectively.

The channel gains for the link between the $i$th element of IRS and $D$ can be formulated as [2]

$$h_{SR,i} = \frac{a_i \phi_i^n}{d_i^n}, \quad h_{RD,i} = \frac{\beta_i e^{j\theta_i}}{d_i^n},$$

(2)

where $a_i$ and $\beta_i$ are the amplitudes of the corresponding channel gain and follow independent and identical distributed (i.i.d.) Rayleigh distribution, $\theta_i$ and $\phi_i$ are the phases of the corresponding channel gain, $n$ is the path loss exponent, $d_i$ is distances between $S$ and $R$ and $d_j$ is the distance between $R$ and $D$. It is assumed that $S$ and $D$ are in the far field of the IRS, and the IRS is located far enough from $S$ and $D$ such that the distance between $S$ (or $D$) and all elements of IRS can be regarded as the same.
Performance analysis: Referring to Equation (1), the received instantaneous SNR $\gamma$ at the node $D$ can be written as

$$
\gamma = \left| \sqrt{P_S} \cdot h_{k\omega}^\dagger \cdot h_{k\omega} \right|^2_{N_0^2} = P_S \cdot \sum_{i=1}^N \alpha_i \beta_i \cdot \tilde{\omega}(\phi_i) e^{i(\phi_i - \delta_i - \theta)}^2_{N_0^2}.
$$

(3)

With the ideal phase shifting of the IRS (i.e. $\tilde{\omega}(\phi_i) = 1$ and $\phi_i = \theta_i + \phi_i$), the maximum SNR which $D$ can be achieved is given by

$$
\gamma = \frac{P_S \cdot \sum_{i=1}^N \alpha_i \beta_i^2}{N_0 d_i^2} = \gamma \cdot \left( \sum_{i=1}^N \alpha_i \beta_i^2 \right)^2.
$$

(4)

where $\gamma = \frac{P}{N_0 d_i^2}.$

With the random variables (RVs) $\alpha_i$ and $\beta_i$ being i.i.d. Rayleigh distributed with parameter $\gamma$, the CLT can be applied for a very large number of reflection elements in the IRS (i.e. $N \gg 1$). In this case, it holds that the RV $\sum_{i=1}^N \alpha_i \beta_i$ becomes a Gaussian RV with mean $\frac{\gamma}{N}$ and variance $N \cdot (1 - \frac{1}{e})$. Then, it is straightforward to show that the RV $\gamma$ in Equation (4) becomes a non-central-$\chi^2$ distributed RV with the probability density function (PDF) and cumulative distribution function (CDF) given by [3]

$$
\begin{align*}
\gamma(x) &= \frac{1}{2\sigma^2} \cdot \left( \frac{x}{\gamma} \right)^{\gamma-1} \cdot \exp\left( -\frac{x + \lambda \gamma}{2\sigma^2} \right) \cdot I_\gamma \left( \frac{\sqrt{x \cdot \gamma}}{\sqrt{\sigma \gamma}} \right), \\
\gamma_{\text{CDF}}(x) &= 1 - Q_{\chi^2} \left( \frac{\sqrt{N \gamma}}{\sqrt{\gamma \sigma^2}} \right),
\end{align*}
$$

(5)

where $\lambda = (\frac{\gamma}{\sigma^2})^2$ and $\sigma^2 = N \cdot (1 - \frac{1}{e})$. $I_\gamma (\cdot)$ is the modified Bessel function of the first class with order $\gamma$, and $Q_{\chi^2} (\cdot)$ is the generalised Marcum Q-function, which can be efficiently evaluated with Matlab and Mathematica.

For hybrid ARQ with CC, the packet received in the 4th transmission round is combined with the previous received packets and decoding is performed on the combined packet. We assume that information-theoretic capacity-achieving channel coding is used. With the optimal combining of the received signals, the mutual information is obtained by combining received SNR over the previous and current transmission rounds [14]. The accumulated SNR $\gamma_{\text{K}}$ at the node $D$ after K ARQ rounds can be written as

$$
\gamma_{\text{K}} = \sum_{k=1}^K \gamma_k = \sum_{k=1}^K \gamma_k \cdot \left( \sum_{i=1}^N \alpha_i \beta_i \right)^2.
$$

(7)

1It should be noted that ideal phase shifting require perfect knowledge of CSI at the IRS, which might be challenging. The results obtained under the assumption of ideal phase shifting clearly represent the upper bound of the performance, which provides insights on the potential of IRS-aided performance. An incomplete summary of channel estimation approaches for IRS is given in [6, sec. VI.B]. The phase error can potentially come from imperfect phase estimation as well as the use of discrete phase shifters. It is worth mentioning that limited quantisation bits for the IRS can already provide satisfactory performance improvement as demonstrated in [12, fig. 2]. It is also proven in [13] that the required number of phase shifters to meet some fixed performance target decreases as the number of IRS reflectors grows (e.g. five quantisation bits already result in very insignificant performance degradation for an IRS of $N = 300$ [13, fig. 5]). This implies that even with imperfect phase shifting, the IRS can potentially provide significant performance improvements.

Fig. 2 $P_{\text{out}}$ under varying number of automatic repeat request round where $\gamma_1$ is the SNRs for the link from $S$ to $D$ via IRS at the 4th round. The PDF and CDF of the RV $\gamma_{\text{K}}$ is given in Equations (5) and (6), respectively.

The total accumulated mutual information, $I_{\text{K}}$, can be expressed as

$$
I_{\text{K}} = \log_2 (1 + \gamma_{\text{K}}) = \log_2 \left( 1 + \sum_{k=1}^K \gamma_k \right).
$$

(8)

An outage after K ARQ rounds implies that the accumulated total mutual information $I_{\text{K}}$ is still less than the transmission rate $R$. Mathematically, the outage probability $P_{\text{out}}^{(K)}$ after K ARQ rounds can be written as [15]

$$
P_{\text{out}}^{(K)} = \Pr \left( I_{\text{K}} < R \right) = \Pr \left( \log_2 \left( 1 + \sum_{k=1}^K \gamma_k \right) < R \right) = \Pr \left( \gamma_{\text{K}} < \theta \right) = F_{\gamma_{\text{K}}} (\theta).
$$

(9)

where $\theta = 2^R - 1$.

Arising from the fact that a non-central-$\chi^2$ RV results from the sum of squares of several i.i.d. Gaussian RVs with non-zero mean, then the sum of several i.i.d. non-central-$\chi^2$ RVs also has a distribution of the same type with the parameters being the sums of the corresponding parameters of the summands. Therefore, the outage probability after K ARQ rounds can be expressed, after some mathematical manipulations, as

$$
P_{\text{out}}^{(K)} = 1 - Q_{\chi^2} \left( \frac{\sqrt{NK \pi}}{\sqrt{16 \pi^2}} \cdot \left( \frac{16\theta}{N \cdot (16 \cdot \pi^2) \cdot \gamma} \right)^2 \right).
$$

(10)

Asymptotic analysis: Next, we consider the system performance when the number of reflectors $N$ is large.

The following asymptotic expression of generalised Marcum-Q function holds as $q \to 0$:

$$
Q_{\chi^2} (p, q) \equiv 1 - \frac{q^{2m}}{2^m \cdot \Gamma(m) \cdot m} \cdot \exp \left( -\frac{p^2}{2} \right) + o(q^{2m})
$$

(11)

When $N$ is sufficiently large, by utilising the above asymptotic expression for Equation (10), we have

$$
P_{\text{out}}^{(K)} \approx 1 - \exp \left( -\frac{C_1 N}{N^2} \cdot \frac{C_2}{\gamma} \right).
$$

(12)

where $C_1 = \frac{K \pi}{\Gamma(1-\frac{1}{2})}$ and $C_2 = (\frac{16\pi}{\Gamma(1-\frac{1}{2})})^2 \cdot (2^{\frac{1}{2}} \cdot \Gamma(\frac{1}{2}) \cdot \cdot K)^{-1}$.

The result in Equation (12) shows that when the reflector number is sufficiently large, the following holds: $P_{\text{out}} \approx a \cdot N \cdot b \cdot \log (N)$, where $P_{\text{out}}$ is in log scale, $a$ and $b$ are some constants related to $K$.

Numerical analysis and discussions: Figure 2 illustrates the outage performance of the IRS-aided system under different numbers of ARQ round. It can be seen that compared to the case where ARQ is not
of 10 gain becomes smaller. For instance, referring to the outage probability performance. However, with the increasing of the ARQ round, the SNR utilised, the hybrid ARQ scheme can greatly improve the system per-

Fig. 3 $P_{out}$ versus varying number of reflecting elements $N$

Fig. 4 $P_{out}$ versus varying path loss exponent $n$

ences. In this study, we investigate the performance of hybrid ARQ with CC over the IRS-aided communication system. The results represent the best performance that can be achieved with the ARQ based IRS-aided system under Rayleigh fading. The results clearly verify the potential of combining the ARQ scheme in the link layer of the IRS-aided system as well as demonstrates the sensitivity of the propagation environment on the system performance. The potential of the IR to support the low-SNR mode is evidenced by the observation that the required transmit SNR, to obtain a certain performance target, can be lessened significantly as the numbers of reflectors or ARQ rounds increase.

Future work can include the analysis of such system under more general fading channels (e.g. Fisher–Snedecor $F$ [16], fluctuating two-ray fading [17], $\alpha$-$\eta$-$\kappa$-$\mu$ fading [18] etc) and under the assumption of imperfect phase shifting [12] at the IRS as well as under the assumption of correlated fading among the reflectors at the IRS.

**Conclusion:** In this study, we investigate the performance of hybrid ARQ with CC over the IRS-aided communication system. The results represent the best performance that can be achieved with the ARQ based IRS-aided system under Rayleigh fading. The results clearly verify the potential of combining the ARQ scheme in the link layer of the IRS-aided system as well as demonstrates the sensitivity of the propagation environment on the system performance. The potential of the IR to support the low-SNR mode is evidenced by the observation that the required transmit SNR, to obtain a certain performance target, can be lessened significantly as the numbers of reflectors or ARQ rounds increase.

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