Exact Method for Solving Single Machine Scheduling Problem Under Fuzzy Due Date to Minimize Multi-Objective Functions

Hanan A. Cheachan¹, Mustafa T. Kadhim²

¹,²Department of Mathematics, College of Sciences, University of Mustansiriyah, Baghdad, Iraq

E-mail: most_afat2000@yahoo.com

Abstract. In this paper, the branch and bound technique used to solve a single machine scheduling problem, which is the problem of scheduling n-job on a single machine of multi-objective function with triangle fuzzy due date numbers which are formulated as $1\hat{D}_j = \text{TFN} \sum_{j=1}^{n} C_j + L_{\text{max}}$. The target of this paper is to obtained optimal sequence of our problem. The computational results are calculated by using Matlab program and compare the results with the complete enumeration method.

1. Introduction

The problem of scheduling n-job on single machine problems is important for the understanding theory of scheduling and manufacturing an initiation simple for decision makers. After scheduling problem is in truth a common of abstemious machine scheduling problems. A single machine planning issues help an opportunity to rearrange the framework to the extent that could reasonably be expected will assistance a choice creator comprehends know framework segments furthermore prerequisites. Recently, the single machine of multi-objective functions (completion time, the tardiness, the earliness, and the late work) are studied by using branch and bound method to get optimal sequence, Hanan and Alaa [18]. The multi-objective functions for machine scheduling problem are studied with good lower bound and upper bound to obtain optimal sequence and proved special cases, Hanan et al. [19]. The concept of the fuzzy due date on single machine scheduling problems has already studied by some researchers in the literature pertinent works include that Ishii et al. [1], Han et al. [2], and Ishibuchi et al. [3,4]. There are studies a maximize the total degree of satisfaction with a fuzzy due date, Stanfield et al. [5] found the optimal scheduling problem among those that do not surpass the maximum acceptable possibility of lateness in the problem with respect fuzzy due date. and S.S Lam, X. Cai [6] studied scheduling n-job on single machine with fuzzy due date to the objective of minimize the weighted earliness and tardiness function and introduced measure to evaluate the deviations of job. S.S Lam, X. Cai [7] studied fuzzy lateness function to measure the lateness of job with fuzzy due date and used genetic algorithm to find optimal scheduling. Chengyao and Dingwei found the optimal scheduling of NP-hard problem with triangular fuzzy process time [13]. Sunita studied single machine problem with fuzzy due date, and found methods to found optimal sequence of minimize cost of earliness and tardiness for small system [14]. Helen and Sumathi used branch and
bound to solve single machine scheduling problem using type-2 trapezoidal fuzzy numbers to find optimal sequence for minimizing total tardiness [15].

Abdelaziz and Ismail used a new method to solve a single machine problem with fuzzy processing time and distinct due date for an objective function minimize total earliness and total tardiness [16]. Oguzhan solved minimizes the sum of the products of earliness/tardiness in single machine problem with fuzzy due date [17]. In this paper, we are involved in the direct generalization of the traditional of the total cost of completion time and maximum lateness where the due date is represented by a triangular membership function [8]. All the jobs are released at a different point in time. The problem is to find a schedule with minimum objective function. The general problem is NP-hard in the strong sense. Our main contribution is BAB algorithm based on a lower bound.

2. Preliminaries

2.1 Fuzzy set [9]

We say \( A \) is fuzzy set on X if \( A \) is function from X to I where I = [0, 1] i.e. \( A = \{(x, \tilde{A}(x)) : x \in X, 0 \leq \tilde{A}(x) \leq 1 \text{ where } \tilde{A}(x): X \to [0,1]\} \)

2.2 Fuzzy number [9]

Let \( R \) in real line. A fuzzy set \( \tilde{A} : R \to [0,1] \) which satisfies the following conditions

- \( \tilde{A} \) is piece wise continuous
- There exist an \( x \in R \) such that \( \tilde{A}(x) = 1 \)
- If \( x_1, x_2 \in R \) and \( y \in [0,1] \) then \( \tilde{A}(y x_1+(1-y) x_2) \geq \tilde{A}(x_1) \Lambda \tilde{A}(x_2) \) then \( \tilde{A} \) is convex is called a fuzzy number. For all \( x \in R \) we define the support of \( \tilde{A} \) by \( \{x \in R: \tilde{A}(x) > 0\} \) and \( \alpha \)-cut defined by \( \tilde{A}_\alpha = \{x \in R: \tilde{A}(x) \geq \alpha\} \)

Now we define the membership of a fuzzy number \( \tilde{A} \) in general:

\[
\tilde{A}(\chi) = \begin{cases} 
0 & \text{For } \chi < a \\
 f_{\tilde{A}}(\chi) & \text{For } a \leq \chi < c \\
1 & \text{For } c \leq \chi \leq d \\
g_{\tilde{A}}(\chi) & \text{For } d < \chi \leq b \\
0 & \text{For } b < \chi 
\end{cases}
\]

Where the function \( f_{\tilde{A}}(\chi) \) and \( g_{\tilde{A}}(\chi) \) are non-decreasing and non-increasing function

Definition 2.1: [12] A fuzzy number \( \tilde{A} = (d_j^l, d_j^c, d_j^v) \) is triangle fuzzy number with following membership function \( \tilde{A} \) defined by:

\[
\tilde{A}(\chi) = \begin{cases} 
0 & \text{if } \chi < d_j^l \\
\frac{\chi - d_j^l}{d_j^c - d_j^l} & \text{if } d_j^l \leq \chi < d_j^c \\
\frac{d_j^u - \chi}{d_j^u - d_j^c} & \text{if } d_j^c \leq \chi < d_j^u \\
0 & \text{if } d_j^u \leq \chi 
\end{cases}
\]

2.3 Expected distance [11]

The expected distance (ED) of two fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) is defined by:

\[
d(\tilde{A}, \tilde{B}) = \frac{1}{\alpha} \int_0^\alpha |a_\alpha - b_\alpha| + |\overline{a_\alpha} - \overline{b_\alpha}| d\alpha 
\]

Where \([a_\alpha, \overline{a_\alpha}] \ [b_\alpha, \overline{b_\alpha}] \) correspond to \( \alpha \)-cat of \( \tilde{A} \) and \( \tilde{B} \) respectively

3. Problem formulation
To schedule n jobs on a single machine of the minimize total completion time and maximum lateness with fuzzy due date. let N={1,2,…,n} set of n jobs requires $p_j$ units of processing time and each jobs is assigned with due date $D_j$ where $D_j$ is triangular fuzzy number (TFN). All the jobs are available to be processed by the machine and by starts processing without interrupted. let $\delta$ be a sequence of jobs in N so that is minimized of objective function .we address the case by following membership

$$D_j(\chi) = \begin{cases} 0 & \text{if } \chi < d_j^l \\ \chi - d_j^l & \text{if } d_j^l \leq \chi < d_j^c \\ d_j^c - d_j^l & \text{if } d_j^c \leq \chi < d_j^u \\ d_j^u - d_j^c & \text{if } d_j^u \leq \chi \\ 0 & \text{if } \chi \geq d_j^u \end{cases}$$

$L_j(C_j, \bar{D}_j) = C_j - \bar{d}_j$

We define the distance according to $\bar{D}_j$ is triangular fuzzy number $(d_j^l, d_j^c, d_j^u)$ , by using distance measure. Let $\tilde{A} = [a_\alpha, \bar{a}_\alpha]$ and $\tilde{B} = [b_\alpha, \bar{b}_\alpha]$ , than

$$d(\tilde{A}, \tilde{B}) = \frac{1}{2} \int_0^1 [(a_\alpha - b_\alpha)^+ + (\bar{a}_\alpha - \bar{b}_\alpha)^+] d\alpha + \frac{1}{2} \int_0^1 [(a_\alpha - b_\alpha)^- + (\bar{a}_\alpha - \bar{b}_\alpha)^-] d\alpha$$

Where

$$(\chi)^+ = \begin{cases} \chi & \text{if } \chi \geq 0 \\ 0 & \text{if } \chi < 0 \end{cases}$$

$$(\chi)^- = \begin{cases} 0 & \text{if } \chi \geq 0 \\ \chi & \text{if } \chi < 0 \end{cases}$$

By changing $\tilde{A}$ with $C_j$ where $C_j$ is a completion time and $\tilde{B}$ with $\bar{D}_j$ where $\bar{D}_j$ is fuzzy due date we can evaluated the following lateness function:

$$L(C_j, \bar{D}_j) = \frac{1}{2} \int_0^1 [(C_j - d_j^l)^+ + (C_j - \bar{d}_j^l)^+] d\alpha + \frac{1}{2} \int_0^1 [(C_j - d_j^c)^- + (C_j - \bar{d}_j^c)^-] d\alpha$$

Where $[d_j^l, d_j^c, d_j^u]$ according to $\alpha$-cut

To derive the fuzzy lateness cost function, we have four cases:

- **Case (1):**
  - if $C_j < d_j^l$
    - For $C_j - (d_j^l + (d_j^c - d_j^l)\alpha$)
      - If $\alpha = 0$ then $C_j - d_j^l < 0$
      - If $\alpha = 1$ then $C_j - d_j^c < 0$
    - For $C_j - (d_j^l + (d_j^c - d_j^u)\alpha$)
      - If $\alpha = 0$ then $C_j - d_j^l < 0$
      - If $\alpha = 1$ then $C_j - d_j^c < 0$
  - Then by equation (1) we get
    $$L(C_j, \bar{D}_j) = \frac{1}{2} \int_0^1 [(C_j - d_j^l)^- + (C_j - \bar{d}_j^l)^-] d\alpha$$
    $$= \frac{1}{2} \int_0^1 [(C_j - (d_j^l + (d_j^c - d_j^l)\alpha) + C_j - (d_j^l + (d_j^c - d_j^u)\alpha)] d\alpha$$
\begin{align*}
&= \frac{1}{2} \left[ C_j \alpha - d_j' \alpha - \frac{1}{2} d_j^c \alpha^2 + \frac{1}{2} d_j' \alpha^2 + C_j \alpha - d_j^u \alpha - \frac{1}{2} d_j^c \alpha^2 + \frac{1}{2} d_j' \alpha^2 \right]_0^n \\
&= \frac{1}{2} \left[ 2C_j - \frac{1}{2} d_j' - d_j^c - \frac{1}{2} d_j^u \right] \\
&= C_j - \frac{1}{4} \left[ d_j' + 2d_j^c + d_j^u \right]
\end{align*}

- Case 2:
  If $d_j' \leq C_j < d_j^c$ then:
  
  For $C_j - (d_j' + (d_j^c - d_j')\alpha$

  If $\alpha = 0$ then $C_j - d_j' \geq 0$
  
  If $\alpha = 1$ then $C_j - d_j^c < 0$

  For $C_j - (d_j^u + (d_j^c - d_j')\alpha$

  If $\alpha = 0$ then $C_j - d_j^u < 0$
  
  If $\alpha = 1$ then $C_j - d_j^c < 0$

  Then $T = (d_j^c - d_j')\alpha - d_j' \geq 0$

  $C_j - d_j' \geq (d_j^c - d_j')\alpha$ Than $\alpha \leq \frac{c_j-d_j'}{d_j^c-d_j'}$

  Than $[0, \frac{c_j-d_j'}{d_j^c-d_j'}] \geq 0, \left[ \frac{c_j-d_j'}{d_j^c-d_j'}, 1 \right]$

  by using equation (1)

\begin{align*}
\tilde{L}(C_j, \partial_j) &= \frac{1}{2} \int_a \left[ (C_j - d_j') + (C_j - d_j^c) \right] d\alpha + \frac{1}{2} \int_a \left[ (C_j - d_j') - (C_j - d_j^c) \right] d\alpha \\
&= \frac{1}{2} \int_a \left[ (C_j - (d_j' + (d_j^c - d_j')\alpha)) d\alpha + \frac{1}{2} \int_a \left[ (C_j - (d_j' + (d_j^c - d_j')\alpha)) d\alpha \\
&+ \frac{1}{2} \int_a \left[ (C_j - (d_j^u + (d_j^c - d_j')\alpha)) d\alpha \\
&= \frac{1}{2} \left[ C_j \alpha - d_j' \alpha - \frac{1}{2} d_j^c \alpha^2 + \frac{1}{2} d_j' \alpha^2 + C_j \alpha - d_j^u \alpha - \frac{1}{2} d_j^c \alpha^2 + \frac{1}{2} d_j' \alpha^2 \right]_0^n \\
&= \frac{1}{2} \left[ 2C_j - \frac{1}{2} d_j' - d_j^c + \frac{1}{2} d_j^u \right] \\
&= C_j - \frac{1}{4} \left[ d_j' + 2d_j^c + d_j^u \right]
\end{align*}

- Case (3)
  If $d_j^c \leq C_j < d_j^u$ then:

  For $C_j - (d_j' + (d_j^c - d_j')\alpha$

  If $\alpha = 0$ then $C_j - d_j' > 0$
If $\alpha = 1$ then $C_j - d_j^c > 0$
For $C_j - (d_j^c + (d_j^c - d_j^u))\alpha$
If $\alpha = 0$ then $C_j - d_j^u < 0$
If $\alpha = 1$ then $C_j - d_j^c > 0$

Then $C_j - (d_j^u + (d_j^c - d_j^u))\alpha \geq 0$

$\Rightarrow (d_j^u - d_j^c)\alpha \geq d_j^u - C_j$

$\Rightarrow \alpha \geq \frac{d_j^u - C_j}{d_j^u - d_j^c}$

Then $\left[0, \frac{d_j^u - C_j}{d_j^u - d_j^c}\right] \leq 0$, $\left[\frac{d_j^u - C_j}{d_j^u - d_j^c}, 1\right] \geq 0$

Then by using equation (1)

$\mathcal{L}(C_j, \bar{B}_j) = \frac{1}{2} \int_0^1 \left\{(C_j - \frac{d_j^c}{\alpha})^+ + \left(C_j - \frac{d_j^u}{\alpha}\right)^+\right\} d\alpha + \frac{1}{2} \int_0^1 \left\{(C_j - \frac{d_j^c}{\alpha})^- + \left(C_j - \frac{d_j^u}{\alpha}\right)^-\right\} d\alpha$

$= \frac{1}{2} \int_0^1 \left[C_j - (d_j^u + (d_j^c - d_j^u))\alpha\right] d\alpha + \frac{1}{2} \int_0^1 \left[C_j - (d_j^u + (d_j^c - d_j^u))\alpha\right] d\alpha$

$= \frac{1}{2} \left[C_j \alpha - d_j^l \alpha - \frac{1}{2} (d_j^c - d_j^l)\alpha^2\right]_b \frac{d_j^u - C_j}{d_j^u - d_j^c} + \frac{1}{2} \left[C_j \alpha - d_j^u \alpha - \frac{1}{2} (d_j^c - d_j^u)\alpha^2\right]_0 \frac{d_j^u - C_j}{d_j^u - d_j^c}$

Then we get

$C_j - \frac{1}{4} \left[d_j^l + 2d_j^c + d_j^u\right]$

• Case (4)
  : if $d_j^u < C_j$

For $C_j - (d_j^l + (d_j^c - d_j^l))\alpha$

If $\alpha = 0$ then $C_j - d_j^l > 0$
If $\alpha = 1$ then $C_j - d_j^c > 0$

For $C_j - (d_j^u + (d_j^c - d_j^u))\alpha$

If $\alpha = 0$ then $C_j - d_j^u > 0$
If $\alpha = 1$ then $C_j - d_j^c > 0$

Then by equation (1) we get

$\mathcal{L}(C_j, \bar{B}_j) = \frac{1}{2} \int_0^1 \left\{(C_j - \frac{d_j^c}{\alpha})^+ + \left(C_j - \frac{d_j^u}{\alpha}\right)^+\right\} d\alpha$

$= \frac{1}{2} \int_0^1 \left\{(C_j - (d_j^l + (d_j^c - d_j^l))\alpha) + C_j - (d_j^u + (d_j^c - d_j^u))\alpha\right\} d\alpha$

$= \frac{1}{2} \left[C_j \alpha - d_j^l \alpha - \frac{1}{2} d_j^c \alpha^2 + \frac{1}{2} d_j^l \alpha^2 + C_j \alpha - d_j^u \alpha - \frac{1}{2} d_j^c \alpha^2 + \frac{1}{2} d_j^u \alpha^2\right]_b$
\[
\begin{align*}
&= \frac{1}{2} \left[ 2C_j - \frac{1}{2}d_j^l - d_j^c - \frac{1}{2}d_j^u \right] \\
&= C_j - \frac{1}{4} \left[ d_j^l + 2d_j^c + d_j^u \right]
\end{align*}
\]

Than from case (1, 2, 3, 4) we get
\[
L(C_j, \bar{D}_j) = C_j - \frac{1}{4} \left[ d_j^l + 2d_j^c + d_j^u \right]
\]

Where \( C_j \) is completion time of jobs \( j \) under a sequence \( \delta \)

Using the traditional notion, we denote by
\[
\Delta \bar{D}_j = TFN(\sum_{j=1}^{n} C_j + L_{max})
\]

the problem formulated by
\[
\text{Min } F = \text{Min} \sum_{j=1}^{n} C_j + L_{max}
\]

subject to:
\[
\begin{align*}
& C_j \geq P_j; \quad j=1,2,...,n \\
& C_j = C_{j-1} + P_j; \quad j=2,3,...,n \\
& L_j = C_j - d_j
\end{align*}
\]

### 4. Solve the problem by using BAB algorithm

Consider branch and bound algorithm as the general solution of single machine problem. The problem is seemed as a tree include nodes, and each node has a possibility to emerge into a partial sequence. To select the monster partial sequence, we must calculate the lower bound of all partial sequence node with the lowest lower bound is chosen. This the procedure continued till the least node. After obtaining an order where the job on a machine is scheduling, if the nodes having the upper lower bounds then no more branching is possible the aim of this problem is to find the minimum objective function.

#### 4.1 decomposition of the problem

To solve a problem we can make decomposition of the problem by braiking it to up smaller ones, let the problem define by \( \Delta \bar{D}_j \) where \( i=1,2 \)

\[
Z_1 = \text{min} \sum_{j=1}^{n} C_j
\]

subject to:
\[
\begin{align*}
& C_j \geq P_j; \quad j=1,2,...,n \\
& C_j = C_{j-1} + P_j; \quad j=2,3,...,n
\end{align*}
\]

And
\[
Z_2 = L_{max}(C_j, \bar{d}_j)
\]

subject to:
\[
\begin{align*}
& C_j \geq P_j; \quad j=1,2,...,n \\
& C_j = C_{j-1} + P_j; \quad j=2,3,...,n \\
& L_j = C_j - d_j
\end{align*}
\]

#### 4.2 Lower bound

The lower bound calculating by sequence of jobs in SPT order to get minimum value total completion time to sub problem \( Z_1 = \text{min} \sum_{j=1}^{n} C_j \) and \( Z_2 = L_{max}(C_j, \bar{d}_j) \) calculated by sequence of jobs in EDD order to get minimum maximum lateness with fuzzy due date . Hence the lower bound calculated by:
\[
LB = \sum_{j=1}^{n} C_j (SPT) + L_{max}(EDD)
\]

#### 4.3 Upper bound
To find the upper bound can be compete the first and second upper bound by SPT and EDD order sequence respectively, then

\[ UB (1) = \sum_{j=1}^{n} C_j(SPT) + L_{\max}(SPT) \]

\[ UB (2) = \sum_{j=1}^{n} C_j(EDD) + L_{\max}(EDD) \]

UB (3) this upper bound was found by applying average high ranking methodology (AHR) Heuristic algorithm by following steps;

Step (1): We find (AHR) of fuzzy due date time (a,b,c) of all the jobs

Step (2): The second step arrange the jobs increasing order of average high ranking, if we have two jobs has same order consider the lowest due date

Step (3): We used step 2 after obtained the sequence to find the value of objective function

The upper bound competed by

UB = Min (UB1, UB2,UB3)

5. Computational results

In this section, the computational results are given in below table tested by coding then Matlab R2017a and run on core i7 at CPU @2.40GHs 4GB computer. In the table given the results of optimal values by complete enumeration method and branch and bound method for n=3,4,5,6,7,8,9,10 as the following table

| n  | EX  | CEM  | TIME  | B&B   | TIME  | NOD  |
|----|-----|------|-------|-------|-------|------|
| 4  | 1   | 123.25 | 0.018 | 123.25 | 0.122 | 27   |
|    | 2   | 164  | 0.024 | 164   | 0.160 | 27   |
|    | 3   | 140  | 0.025 | 140   | 0.127 | 10   |
|    | 4   | 77   | 0.027 | 77    | 0.159 | 27   |
|    | 5   | 53.75 | 0.025 | 53.75 | 0.158 | 27   |
| 5  | 1   | 184.75| 0.026 | 184.75| 0.172 | 49   |
|    | 2   | 184.5 | 0.032 | 184.5 | 0.206 | 51   |
|    | 3   | 155  | 0.026 | 155   | 0.137 | 15   |
|    | 4   | 216.25| 0.029 | 216.25| 0.123 | 15   |
|    | 5   | 82.75| 0.026 | 82.75 | 0.156 | 27   |
| 6  | 1   | 180.5 | 0.051 | 180.5 | 0.138 | 21   |
|    | 2   | 270.75| 0.051 | 270.75| 0.157 | 36   |
|    | 3   | 80.5  | 0.046 | 80.5  | 0.276 | 113  |
|    | 4   | 196  | 0.034 | 196   | 0.199 | 67   |
|    | 5   | 244.5 | 0.052 | 244.5 | 0.146 | 21   |
| 7  | 1   | 332.25| 0.143 | 332.25| 0.459 | 139  |
|    | 2   | 353.25| 0.105 | 353.25| 0.158 | 28   |
|    | 3   | 253.5 | 0.107 | 253.5 | 0.370 | 104  |
|    | 4   | 259.25| 0.104 | 259.25| 0.378 | 118  |
|    | 5   | 206.75| 0.152 | 206.75| 0.432 | 118  |
| 8  | 1   | 195.75| 0.820 | 195.75| 5.879 | 438  |
|    | 2   | 270  | 0.753 | 270   | 3.983 | 334  |
|    | 3   | 277.25| 0.792 | 277.25| 1.951 | 168  |
|    | 4   | 312.75| 0.795 | 312.75| 0.521 | 36   |
6. Conclusion
We solved the problem Q with fuzzy due date by using branch and bound method for several different example for each n–job and comparison between (branch and bound method) and (complete enumeration method) to show the optimal value are equal in n ≤ 10 in this paper.

Reference
[1] H Ishii, M Tada, T Masuda 1992 Two scheduling problems with fuzzy due dates Fuzzy Sets and Systems 46 pp 339–347
[2] S Han, H Ishii and S Fujii 1994 One machine scheduling system with fuzzy due dates European journal Operation Research 79 pp 1–12
[3] H Ishibuchi, N Yamamoto, T Murata and H Tanaka 1994 Genetic algorithms and neighborhood search algorithms for fuzzy flow shop scheduling problems Fuzzy Sets and Systems 94 pp 81–100.
[4] H Ishibuchi, N Yamamoto, S Misaki and H Tanaka 1994 Local search algorithms for flow shop scheduling with fuzzy due dates Internat Journal Product. Econ 33 pp 53–66
[5] P M Stanfield, R E King and J A Joines 1996 Scheduling arrivals to a production system in a fuzzy environment European journal Operation Research 93 pp 75–87
[6] S S Lam and X Cai 2002 single machine scheduling with nonlinear lateness cost function and fuzzy due date nonlinear analysis: real word applications 3 pp 307-316
[7] S S Lam and X Cai 1994 Minimizing Earliness and Tardiness of Job Completions about a fuzzy Due Date Research Grants Council of Hong Kong under grants no. CUHK 278/943 and CUHK 356/963
[8] W Pedrycz 1994 Why triangular membership functions Fuzzy Sets and Systems 64 pp 21-30
[9] D Dubois and H Prade 1980 Fuzzy Sets and Systems Academic Press 144 in mathematics in science and engineering
[10] Yu-Jie Wang 2015 Ranking triangle and trapezoidal fuzzy number based on the relative preference relation applied mathematical modeling 39 (2) pp 586-599
[11] S Heilpern 1992 The expected value of a fuzzy number Fuzzy Sets and Systems 47 pp 81–86
[12] W Pedrycz 1994 Why triangular membership functions Fuzzy Sets and Systems 64 pp 21-30
[13] C Wanga, D Wanga, W H Ip and D W Yuenc 2002 The single machine ready time scheduling problem with fuzzy processing times Fuzzy Sets and Systems 127 pp 117–129
[14] Sunita Gupta 2011 single machine scheduling with distinct due dates under fuzzy environment international journal of enterprise computing and business systems (online) 1 (2) pp 2230-8849
[15] R Helen and R Sumathi 2014 branch and bound technique for single machine scheduling problem using type-2 trapezoidal fuzzy numbers international journal of scientific and research publications 4 (12)
A Hamad and I Abaker 2017 new method for solving single machine scheduling problem with fuzzy processing time and distinct due date international journal of science, engineering and technology research (ijsetr) 6 (11)

O A ARIK1 2019 Dissatisfaction levels of earliness and tardiness durations by relaxing common due date on single machine scheduling problems Journal of Multidisciplinary Modeling and Optimization 2 (1) pp 1-15

H A Chachan and A S Hameed 2019 Exact Methods for Solving Multi-Objective Problem on Single Machine Scheduling Iraqi Journal of Science 60 (8) pp 1802-1813

H A Cheachan, F H Ali and M H Ibrahim 2020 Branch and Bound and Heuristic Methods for Solving Multi-Objective Function for Machine Scheduling Problems 6th international engineering conference Sustainable Technology and Development Erbil Iraq