The external circuit effect on the steady states of the Pierce diode

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Abstract. It was found out that the inclusion of a reactive load in the external circuit of a plasma diode with an electron beam can greatly change the modes of its operation. We demonstrated that the presence of an inductance in the external circuit of the Pierce diode leads to appearance of a new dispersion branch. The growth rates and frequencies for this branch are calculated. The frequencies corresponding to unstable states of this branch exceed the plasma frequency. The threshold of instability is shown to shift to smaller inter-electrode distance values with decreasing in the inductance magnitude and can be lower than the threshold of the Pierce diode with a short circuit.

Introduction

The alternator based on thermionic energy converter (TIC) was proposed at Ioffe Institute. The effect of an electron instability development which leads to the electron current cut-off was used. In order to realize such a generator the electrodes are bridged over an inductance, which accumulates periodically an energy and then sends it into a load as voltage pulses of about tenfold contact potential (see, e. g., Ref. [1]). This generator can only operate in the collisionless regime when the electron flow with small velocity spread moves over the diode plasma. Therefore, it is very important to study effects of the inductance load on the TIC operation.

In the case of the over-neutralized regime when there are more ions than electrons on the emitter, the steady states of the TIC are characterized by a monotonically increasing potential distribution. It exhibits a quasineutral plasma region with nearly constant potential, embedded between two space-charge sheaths adjacent to the electrodes. The electrons are greatly accelerated by a potential jump near the emitter, and enter into the plasma keeping a small velocity spread. In this case steady states of a TIC can be successfully modelled by means of the Pierce-like diode in which injected electrons have the beam-like velocity distribution [2].

The current paper studies stability features of the Pierce diode steady states with an external circuit inductance. Earlier, similar problem was numerically studied in Ref. [3]. It was shown that the presence of an external circuit inductance leads to an instability even at the inter-electrode distance \(d < \pi \lambda_D\). The objective of our study was to clarify details of this phenomenon.

Statement of the problem

We consider the planar diode in which the non-relativistic and mono-energetic electron beam enters from the emitter \((z = 0)\) with the density \(n_b\) and the injection velocity \(v_b\) perpendicularly to the emitter surface and moves without collisions in the self-consistent electric field \(E\). The
field \( E \) can be calculated from the scalar potential \( \varphi \) which depends on the coordinate \( z \) only. We also assume that the diode region is uniformly occupied by infinitely massive immobile ions of constant density \( n_i = n_b \) (Fig.1). The electric field with constant potential everywhere in the
diode plasma region corresponds to the steady state solution for the diode. In order to study the effect of the external circuit on stability features of the diode it is convenient to consider the empedance of the diode \( Z(\omega, d) = -\tilde{\Phi}(\omega, d)/(\tilde{j}S) \). Here \( \omega \) is the external frequency, \( \tilde{\Phi}(\omega, d) \) is the perturbation amplitude of the collector potential, \( \tilde{j} \) is the total current density perturbation, and \( S \) is the electrode surface area.

**Results**

We suppose that all perturbations are proportional to \( \exp(-i\omega t) \), so one can obtain for the diode impedance [4]

\[
Z(\delta, \Omega) = \frac{R_0 i}{\Omega(\Omega^2 - 1)^2} \left\{ \left[ (1 + \Omega^2) \sin \delta + 2i\Omega \cos \delta \right] e^{i\Omega\delta} - \Omega^2 (1 - \Omega^2) \delta - 2i\Omega \right\}.
\] (1)

Here \( R_0 = v_b/(\epsilon_0 \omega_b^2 S) \) has the same units as resistance, \( \Omega = \omega/\omega_b \), \( \omega_b = v_b/\lambda_D \) is the characteristic frequency, and \( \delta = d/\lambda_D \). The frequency \( \Omega \) in Eq. (1) is complex: \( \Omega = \Gamma + i\Omega \). To express all relevant variables in terms of dimensionless quantities, we introduce energy and length units which are the kinetic energy of electrons at the emitter \( W_b \) and the beam Debye length \( \lambda_D \), respectively

\[
\lambda_D = \left[ \frac{2\epsilon_0 W_b}{e^2 n_b} \right]^{1/2} \approx 0.3238 \times 10^{-2} \frac{V_b^{3/4}}{J_b^{1/2}} \text{[cm]}, \quad W_b = mv_b^2/2.
\] (2)

Here, the beam current density \( j_b = en_b v_b \) and the accelerating voltage \( V_b = W_b/e = mv_b^2/(2e) \) creating a beam are taken in Amperes per square cm and Volts, respectively; \( e \) and \( m \) are the electron charge and mass; the free-space permittivity \( \epsilon_0 = 8.854 \times 10^{-12} \text{C}^2/\text{Nm}^2 \). The characteristic frequency \( \omega_b = v_b/\lambda_D \approx 0.3754 \times 10^8 J_b^{1/4}/V_b^{1/4} \text{[c]} \).

If the real part of the impedance turns negative for any parameter values of the system the diode may operate as the generator. In this case parameters of the external circuit should be naturally chosen in such a way that its impedance \( Z_{\text{ext}}(\omega) \) is matched with the diode one:

\[
ReZ(\omega, d) + ReZ_{\text{ext}}(\omega) \leq 0, \quad ImZ(\omega, d) + ImZ_{\text{ext}}(\omega) = 0.
\] (3)
Note, that the frequency $\omega$ in Eq. (3) is real. Thus, using Eq. (1) we obtain following formulas for the Pierce diode impedance:

$$ReZ = \frac{4R_0}{\Omega(1-\Omega^2)^2} \left( \Omega \cdot \sin \frac{\delta}{2} \cdot \cos \frac{\Omega \delta}{2} - \sin \frac{\Omega \delta}{2} \cdot \cos \frac{\delta}{2} \right) \left( \sin \frac{\delta}{2} \cdot \cos \frac{\Omega \delta}{2} - \Omega \cdot \sin \frac{\Omega \delta}{2} \cdot \cos \frac{\delta}{2} \right),$$

$$ImZ = \frac{R_0}{\Omega(1-\Omega^2)^2} \left( (1+\Omega^2) \cdot \sin \delta \cdot \cos \Omega \delta - 2\Omega \cdot \sin \Omega \delta \cdot \cos \delta - \Omega^2 (1-\Omega^2) \right)$$

Figure 2 shows generation regions of the Pierce diode. One can see that diode can produce oscillations with frequency both lower and higher than the plasma one ($\Omega = 1$). The generation regions of the diode being closely related to the diode oscillatory solutions of the dispersion equation for the Pierce diode with the short circuit (see Fig. 3).

Now study in more detail the effect of an external circuit on a stability features of the diode. At $\Gamma = 0$ the diode impedance (1) can vanish if $ReZ$ and $ImZ$ become zero simultaneously. However, one can see from Eq. (4) that for the parameter values corresponding to $ReZ = 0$ the image part of $Z$ equals to

$$ImZ(\Omega, \delta) = \frac{R_0}{\Omega(\Omega^2 - 1)} \left( \delta \Omega^2 \pm \sin \delta \right).$$

It is seen from Eq. (5) that $ImZ$ can vanish only for $\Omega = (|sin\delta|/\delta)^{1/2} < 1$. Really, in the case of a short circuit ($R = 0$, $L = 0$ and $C = \infty$) the growth rate of the oscillatory branch can vanish if only $\omega < \omega_b$. On the other hand, if the external circuit includes reactive elements the oscillatory instability can develop at $\Omega > 1$, too. For example, for a purely inductive load ($ImZ_{ext} = -i\omega L$) the $\Gamma(\delta)$-dependence can vanish at $\Omega > 1$ when parameters $\Omega$ and $\delta$ obey by equations

$$\tan \frac{\Omega \delta}{2} = \Omega^2 \tan \frac{\delta}{2}, \quad \frac{\Omega^2 \delta \mp \sin \delta}{\Omega^2 (\Omega^2 - 1)} = L.$$

Here $L = L/L_0$ is the dimensionless inductance with $L_0 = 5.3277 \cdot 10^{-8} V_0^{5/4}/(S J_0^{3/2})[H]$. For example, at $V_0 = 100V$, $J_0 = 1A/cm^2$, and S=10 cm$^2$ a magnitude of $L_0$ is about $2\mu$H.

Let’s calculate dispersion curves of the diode. Figure 3 illustrates the dispersion branches $\Gamma(\delta)$ and $\Omega(\delta)$ for the Pierce diode with the short circuit (see, e.g., [4]). We see that in this
case an instability can develop only when $\delta > \pi$. On the other hand, if an inductance is placed in the external circuit of the diode then the new oscillatory branch appears in addition to the old ones. The frequency for the new branch being higher than the plasma one, i.e. $\omega_b$ (Fig. 4). Left boundary of the instability region for this branch ($\Gamma \geq 0$) shifts to a smaller $\delta$-values with decreasing in inductance. This fact is the same as in the Bursian diode in the presence of an external inductance [5]. At a certain magnitude of $\bar{L}$ this boundary becomes less then the instability threshold for the diode with the short circuit, i.e. $\pi$.

![Figure 4. New branch brought by inductance $\bar{L}$. $\bar{L} = 0.05$ (curve 1), 0.1 (2), 0.5 (3), 1.0 (4).](image)

**Conclusion**

In order to create an alternative current directly in the TIC, its electrodes are bridged over an inductance with a magnitude of about several units of $\mu$H. Feasibility of such a generator is based on the current cut-off effect being resulted in the electron instability development in collisionless plasma diodes. Presence of a reactive external circuit is able to effect on the instability development and performance of the alternator. The study carried out shows that the external inductance gives rise to a new unstable eigenmode, its frequency being higher than the plasma one. As we can conclude from our results, by varying the external inductance we can control the value of current at which cut-off occurs. New threshold of the instability may be both higher and lower that the Pierce one. In order to calculate the optimum magnitude of the inductance one has to calculate the non-linear time-dependent process in the diode plasma taking into account a time variation of the collector potential caused by the presence of the inductance.

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**References**

[1] Babanin V I et al 1997 32rd IECEC AIChe Proc. 427
[2] Ender A Ya et al 2006 Phys. Plasmas 13 113506
[3] Kuhn S and Hørhager M 1986 J. Appl. Phys. 60 1952
[4] Ender A Ya and Kuznetsov V I 2015 Plasma Phys. Rep. 41 905
[5] Kuznetsov V I and Gerasimenko A B 2018 J. Phys. Conf. Series 1038 012127