Dynamics of impurity, local and non-local information for two non-identical qubits

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From the separability point of view the problem of two atoms interact with a single cavity mode is investigated. The density matrix is calculated and used to discuss the entanglement and to examine the dynamics of the local and non-local information. Our examination concentrated on the variation in the mean photon number and the ratio of the coupling parameters. Furthermore, we have also assumed that the atomic system is initially in the ground states as well as in the intermediate states. It has been shown that the local information is transferred to non-local information when the impurity of one qubit or both is maximum.

Keywords: Qubit, Entanglement, Impurity, Local and non-Local Information.

1 Introduction

As well known quantum entanglement is a non-local correlation in quantum system which plays a fundamental role in almost all efficient protocols of quantum computation and quantum information processing. This is due to the fact that the quantum states of two or more coupled objects have to be described with reference to each other, even though the individual objects may be spatially well separated. It is widely recognized that the control of quantum entanglement leads to new classes of measurement, communication and computational systems, and in some cases dramatically to perform the non-quantum analogies.

The research on quantifying entangled states has been considered by several authors [1, 2, 3]. To quantify entanglement we have to know whether the states are pure or mixed states. Therefore, if the entangled state is in a pure state, then it is sufficient to use von Neumann entropy [4], which has a unique measurement. Here we may refer to the quantum computation of trapped ions introduced by Cirac and Zoller [5]. It can be regarded as a potentially powerful technique for storage and
manipulation of quantum information. In this case the information is usually stored in the spin states of an array of trapped ions and manipulated using laser pulses. Reasonably long coherence times can be achieved, compared to achievable switching rates [6], where individual qubits are addressed through spatial separation of the ions.

Experimental implementations of this scheme have succeeded in performing simple two-qubit logic gates [7, 8] and preparing entangled states [9]. Recently, there has been an ongoing effort to characterize qualitatively and quantitatively the entanglement properties of condensed matter systems and apply them in quantum communication and information. For example, an important emerging field is the quantum entanglement in solid state system such as spin chains which are the natural candidates for the realization of the entanglement compared with the other physics system. Also the problem of atom-field interaction or trapped ions interacting with laser light were suggested as one promising candidate to build a small scale quantum computer. In fact these type of problems have attracted considerable attention due to its potential application in high-resolution spectroscopy, as well as the high-precision atomic fountain clock and the high-precision spin polarization measurements.

The main purpose of the present work is to discuss the degree of entanglement for the above quantum system. Therefore we have to calculate the dynamical operators from which we are able to reach our goal. This can be achieved either by find the solution of the equations of motion in the Heisenberg picture or to employ the wave function in the Schrödinger picture to derive the unitary operator. In this context the later method will be adopted and this can be seen in the following section. Section III is devoted to discuss Peres’s and degree of entanglement. This is followed by section VI where the impurity as well as the dynamics of information is considered. Finally, in section V our conclusion is given.

2 The Model

During the last decade many theoretical and experimental efforts have been done in order to study processes involving atoms inside a cavity, stimulated by the experimental realization of a multi-photon micromaser [34]. In the rotating wave approximation, the interaction of the cavity mode with the injected atoms is described by
the Hamiltonian

$$\hat{H} = \hbar \omega \hat{a}^\dagger \hat{a} + \hbar \sum_{i=1}^{2} \left( \frac{\omega^{(i)}}{2} \hat{\sigma}_{z}^{(i)} + \lambda_i ( \hat{\sigma}_{-}^{(i)} \hat{a}^\dagger + \hat{\sigma}_{+}^{(i)} \hat{a} ) \right).$$  (1)

Our target of this section is to derive the time-dependent density matrix which enables us to discuss some of the statistical properties of the present model. This can be reached from the solution of the Schrödinger picture. For this reason let us assume that the initial state of the two atoms are prepared in a superposition state which can be written as

$$|\psi_{12}(0)\rangle = a_1 a_2 |e, e\rangle + a_1 b_2 |e, g\rangle + b_1 a_2 |g, e\rangle + b_1 b_2 |g, g\rangle,$$  (2)

where $|\psi_{12}(0)\rangle = |\psi_{1}(0)\rangle |\psi_{2}(0)\rangle$ and we have defined

$$|\psi_{i}(0)\rangle = a_i |e\rangle + b_i |g\rangle, \quad i = 1, 2.$$  (3)

Also we have considered the field to be initially in the coherent state i.e

$$|\psi_{f}(0)\rangle = \sum_{n=0}^{\infty} q_n |n\rangle, \quad q_n = \frac{\alpha^n}{\sqrt{n!}} \exp(-\frac{1}{2}|\alpha|^2).$$  (4)

Now we can write the time evolution of the wave function in the form

$$|\psi(t)\rangle = U(t) |\psi_{12}(0)\rangle \otimes |\psi_{f}(0)\rangle$$  (5)

where $U(t) = \exp(-i\hat{H}t/\hbar)$ is the time-dependent unitary operator. Since the invariant sub-space of the global system can be considered as a set of complete basis of the atom-field, therefore one can expand the wave function for the present system to take the form

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} q_n [A_n(t)|e, e, n\rangle + B_n(t)|e, g, n+1\rangle + C_n(t)|g, e, n+1\rangle + D_n(t)|g, g, n+2\rangle$$  (6)

where the coefficients $A_n(t), B_n(t), C_n(t)$ and $D_n(t)$ are given by

$$\begin{pmatrix} A_n(t) \\ B_n(t) \\ C_n(t) \\ D_n(t) \end{pmatrix} = \begin{pmatrix} U_{11}(t) & U_{12}(t) & U_{13}(t) & U_{14}(t) \\ U_{21}(t) & U_{22}(t) & U_{23}(t) & U_{24}(t) \\ U_{31}(t) & U_{32}(t) & U_{33}(t) & U_{34}(t) \\ U_{41}(t) & U_{42}(t) & U_{43}(t) & U_{44}(t) \end{pmatrix} \begin{pmatrix} a_1 a_2 \\ a_1 b_2 \\ b_1 a_2 \\ b_1 b_2 \end{pmatrix}$$  (7)
and the entities \( U_{ij}(t) \) are calculated to take the form

\[
U_{11}(t) = -\frac{1}{(\mu_n - \nu_n)} \left[ \beta_n^2 (1 + r^2) \cos(\sqrt{\mu_n} t) - \cos(\sqrt{\nu_n} t) + [\mu_n \cos(\sqrt{\mu_n} t) + \nu_n \cos(\sqrt{\nu_n} t)] \right],
\]

\[
U_{12}(t) = -\frac{ir}{(\mu_n - \nu_n)} \left[ (\Delta_n \beta_n + \gamma_n \mu_n) \frac{\sin(\sqrt{\mu_n} t)}{\sqrt{\mu_n}} + (\Delta_n \beta_n - \gamma_n \nu_n) \frac{\sin(\sqrt{\nu_n} t)}{\sqrt{\nu_n}} \right],
\]

\[
U_{13}(t) = \frac{i}{(\mu_n - \nu_n)} \left[ (\Delta_n \beta_n - \gamma_n \mu_n) \frac{\sin(\sqrt{\mu_n} t)}{\sqrt{\mu_n}} - (\Delta_n \beta_n - \gamma_n \nu_n) \frac{\sin(\sqrt{\nu_n} t)}{\sqrt{\nu_n}} \right],
\]

\[
U_{14}(t) = -\frac{2r\Delta_n}{(\mu_n - \nu_n)(1 - r^2)} \left[ \cos(\sqrt{\mu_n} t) - \cos(\sqrt{\nu_n} t) \right],
\]

\[
U_{22}(t) = -\frac{1}{(\mu_n - \nu_n)} \left[ (r^2 \beta_n^2 + \gamma_n^2 - \mu_n) \cos(\sqrt{\mu_n} t) - (r^2 \beta_n^2 + \gamma_n^2 - \nu_n) \cos(\sqrt{\nu_n} t) \right],
\]

\[
U_{23}(t) = \frac{r\delta_n}{(\mu_n - \nu_n)(1 + r^2)} \left[ \cos(\sqrt{\mu_n} t) - \cos(\nu_n t) \right],
\]

\[
U_{24}(t) = \frac{i}{(\mu_n - \nu_n)} \left[ (\Delta_n \gamma_n - \beta_n \mu_n) \frac{\sin(\sqrt{\mu_n} t)}{\sqrt{\mu_n}} - (\Delta_n \gamma_n - \beta_n \nu_n) \frac{\sin(\sqrt{\nu_n} t)}{\sqrt{\nu_n}} \right],
\]

\[
U_{33}(t) = -\frac{1}{(\mu_n - \nu_n)} \left[ (r^2 \gamma_n^2 + \beta_n^2 - \mu_n) \cos(\sqrt{\mu_n} t) - (r^2 \gamma_n^2 + \beta_n^2 - \nu_n) \cos(\sqrt{\nu_n} t) \right],
\]

\[
U_{34}(t) = -\frac{ir}{(\mu_n - \nu_n)} \left[ (\Delta_n \gamma_n + \beta_n \mu_n) \frac{\sin(\sqrt{\mu_n} t)}{\sqrt{\mu_n}} + (\Delta_n \gamma_n - \beta_n \nu_n) \frac{\sin(\sqrt{\nu_n} t)}{\sqrt{\nu_n}} \right],
\]

\[
U_{44}(t) = -\frac{1}{(\mu_n - \nu_n)} \left[ \gamma_n^2 (1 + r^2) \cos(\sqrt{\mu_n} t) - \cos(\sqrt{\nu_n} t) + [\mu_n \cos(\sqrt{\mu_n} t) + \nu_n \cos(\sqrt{\nu_n} t)] \right],
\]

(8)

The other components of the unitary operator can be deduced from the relation \( U_{ij}(t) = U_{ji}(t) \), for \( i \neq j = 1, 2, 3, 4 \). In the above equation we have used the abbreviations

\[
\mu_n = \frac{1}{2} \left( \delta_n + \sqrt{\delta_n^2 - 4\Delta_n^2} \right), \quad \nu_n = \frac{1}{2} \left( \delta_n - \sqrt{\delta_n^2 - 4\Delta_n^2} \right),
\]

\[
\delta_n = \sqrt{2n + 3(1 + r^2)}, \quad \Delta_n = \sqrt{(n + 1)(n + 2)(1 - r^2)},
\]

and defined \( \gamma_n = \sqrt{n + 1}, \beta_n = \sqrt{n + 2} \) while we set \( r = \lambda_2/\lambda_1 \).

Having obtained the explicate time-dependent unitary operator we are therefore in position to derive the density operator for the field or for the atomic system. In fact this can be achieved if we trace out either the field or the atom. Since we are interested in this communication to discuss the purity as well as the local and non-local information for the atomic system. Therefore we have to trace over the field to obtain the density matrix for the atom where \( \rho_{12} = tr_f \{ \rho_a \} \), and \( \rho_a(t) = |\psi(t)\rangle \langle \psi(t)| \).

In what follows we turn our attention to employ the results obtained to discuss some statistical properties for the present system. This will be seen in the
Figure 1: The Positive partial transpose criterion, PPT for atomic system is prepared initially in the ground state $|gg\rangle$ for a coupling operator, $r = 0.1$ (dash line) and $r = 0.8$ (solid curve). (a) For $\bar{n} = 5$ (b) For $\bar{n} = 10$.

3 Peres’s criterion and degree of entanglement

In this section, we discuss the entanglement between the two atoms. As we know the atoms in the atomic system are called separable (uncorrelated) if all the spectrum of the partial transpose of the atomic system is non-negative. Therefore, it will be more convenient to use the Peres-Horodecki’s criterion to study the spectrum behavior of the partial transpose of the state $\rho_{12}(t)$ \[11\]. In the meantime, to quantify the amount of entanglement in the correlated states, one may use the measurement introduced by K.Zyczkowski \[12\]. Here, the degree of entanglement (DOE) will be measured according to

$$DOE := \sum_{i=1}^{N} |\eta_i| - 1,$$

where $\eta_i, (i = 1, 2, 3, ...N)$ denotes the eigenvalues of the partial transposed matrix $\rho^T$. Note that for any separable matrix all eigenvalues are positive and its trace is unity and hence $DOE$ is equal to zero. On the other hand, the maximally entangled states is one for a system belonging to a $2 \times 2$ where the spectrum of eigenvalues $\eta_i$ consists of $\{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$, which in fact is consistent with the present system. Therefore, the above equation is adequate for our case to measure the degree of entanglement, however, we have to use a computational program due to the complicated expression for the density matrix. To do so we have plotted some figures to
see the effect of the ratio of the coupling parameter \( r = \lambda_2 / \lambda_1 \) and the mean photon number \( \bar{n} \) on the separability as well as on the degree of entanglement.

The behavior of the positive partial transpose, PPT is plotted in Fig(1) against the scaled time \( \lambda_1 t \). In Fig. (1a) and for a fixed value of the mean photon number \( \bar{n} = 5 \), we considered the cases in which \( r = 0.1 \) (dash line) and 0.8 (solid line). In these two cases we have assumed that the atomic system is initially in the ground state. For \( r = 0.1 \) the entanglement occurs after onset of the interaction, however, for short periods of time. This is followed by a period of disentanglement between the atoms. The second period of entanglement has been observed, however, for longer time compared with the first period. Moreover, the entanglement in this case is apparent which means that as the period of time increases the entanglement gets pronounced. This conclusion is limited for a certain period of time where the atoms get separable again. Furthermore, the irregular fluctuations which can be seen in the function behavior indicates that the energy response to the correlation between the atoms gets weak. This leads to a long instability and the separability behavior can be reported. As time goes on the entanglement between the atoms is seen for a long time. The situation is different when we increase the value of the coupling parameter \( r = 0.8 \). This is realized from the solid curve where a small amount of entanglement can be seen just for a short period of time. This is followed by a long period of separability between the atoms. The entanglement starts again to be seen at later time with amount greater than that of the case in which \( r = 0.1 \), but for short periods of time. This means that the amount of entanglement is effected by the value of the coupling parameter. Similar behavior can be seen for the case in which the mean photon number \( \bar{n} = 10 \), however, slight different observation can be observed. For example, for \( r = 0.1 \) we observe an increase in the amount of entanglement occurred in the second period of time compared with the case of \( \bar{n} = 5 \). While for \( r = 0.8 \) the period of entanglement is shifted with decreasing in its amount, see Fig. (1b). This behavior is due to the usual effect of the Rabi oscillations where the photon number plays a crucial role in its value.

Now let us discuss the entanglement when the atomic system is initially prepared in the partially entangled state such that

\[
|\psi_{12}(0)\rangle = \cos \theta |e, e\rangle + \sin \theta |g, g\rangle, \quad (11)
\]

where \( \theta = \pi/3 \). In this case and for \( r = 0.1, 0.8 \) and \( \bar{n} = 5 \). In this case there are different features have been seen. For example, strong entanglement is realized for
both cases with maximum value at $\sim -0.42$ occurred after onset of the interaction. This is followed by decreasing in the degree of entanglement for both cases too, which means that the atoms start to be separable. For the case $r = 0.1$ the atoms get separable faster than the case of $r = 0.8$, however the function shows irregular fluctuations in the period of separability.

On the other hand, for the case in which $r = 0.8$ more stability can be realized for the disentanglement period. Also the second period of entanglement is occurred for $r = 0.1$ faster than the case when $r = 0.8$. However, with maximum value of the entanglement is less than that observed of the first period of correlation. Furthermore, the general behavior shows irregular fluctuations which referees to the energy exchange between the atoms and the field, see Fig.(2a). Similar behavior can be reported for the case in which $\bar{n} = 10$, but with more fluctuations for the case in which $r = 0.1$, see Fig.(2b).

To quantify the amount of entanglement contained in the atomic system we have plotted figures (3). The system is considered to be initially in the ground state using the same value of the parameters as before. For example, in Fig.(3a) we have considered $r = 0.1$ (dash line) and $r = 0.8$ (solid line) and the mean photon number $\bar{n} = 5$. In this figure the system shows weak entanglement for short period of time after onset of the interaction. This is followed by short period of time where the atoms are separable and consequently the amount of entanglement is zero. This means that the energy of the system is totally transferred to the field. However, strong correlation between the atoms can be seen in the second period of

Figure 2: The same as Fig.(1), but the system is The PPT criterion for atomic system is prepared initially in a partial entangled state.
Figure 3: The degree of entanglement, DOE for atomic system is prepared initially in the ground state for $R = 0.1$ (solid curve line) and $R = 0.8$ (dot curve) (a) For $\bar{n} = 5$ and (b) for $\bar{n} = 10$ (c) The same as Fig.(a) but for small values of the scaled time.
entanglement compared with the first period. In the meantime, the maximum value of entanglement in this case approaches 0.17. Also, we observe a slight changes of fluctuations in this period which reflects the exchange of the energy between the two atoms. In fact this is the longest period of entanglement during the considered time and also it is the most pronounced one. The third period of correlation is short with a small amount of entanglement and occurred after long period of time. This is followed by the fourth period of correlation (short but with maximum value \( \sim 0.17 \)) and then the system starts showing a successive periods of entanglement with decreasing in their amounts, see Fig.3a. Increasing the value of the coupling parameter ratio \( r = 0.8 \), different observation can be reported where a small amount of entanglement can be seen after a short period of time.

On contrary to the previous case the strong entanglement occurred after considerable period of time with maximum value approaches 0.36. Also a regular fluctuations with different maximum values can be seen, which means that the exchange of the energy between the atoms and the field is nearly stable. This behavior should be expected since the values of the coupling parameters get close to each other. We have also examined the case in which \( \bar{n} = 10 \), for \( r = 0.1 \) and \( r = 0.8 \). Increasing the value of the mean photon number would effect on the Rabi frequency and hence on the system behavior. This can be realized from the figure (3b) where the amount of entanglement for \( r = 0.1 \) increased compared with the case \( \bar{n} = 5 \). In the meantime, for the case \( r = 0.8 \), we can see decreasing in the amount of entanglement which occurred after considerable period of time.

In Fig.(3c), we plot the degree entanglement for small scale of the scaled time, where we consider the system is initially prepared in the ground state. Since we start from a product state, the degree of entanglement is zero at \( \lambda_1 t = 0 \). As the time increases the degree of entanglement increases. For large values of the coupling constant \( r = 0.8 \), the degree of entanglement is increased faster and reaches its maximum value(0.009), round the scaled time \( \lambda_1 t \approx 0.3 \). Then the sudden death phenomenon of entanglement is seen for \( \lambda_1 t > 0.3 \) \([14]\), where the degree of entanglement is decreased quickly until completely vanishes. On the other hand for small values of \( r = 0.1 \), the degree of entanglement is slightly increased reaching to its maximum value (\( \approx 0.001 \)), which is very small comparing by the previous case. Then the degree of entanglement decays smoothly till completely vanishes. This type of decay is seen for behavior of the two qubit pair in Bloch channels \([15]\).
Now let us turn our attention to consider the system to be initially in the intermediate state, equation (15). For this reason we have plotted figures (4a) and (4b) for the mean photon number $\bar{n} = 5$ and $\bar{n} = 10$, respectively. In Fig.(4a) one can see that the entanglement reaches its maximum after onset of the interaction for $r = 0.1$ (dash line) and $r = 0.8$ (solid line) and then decreases its value to reach the minimum (zero value) where the correlation between the atoms disappeared. However, for the case $r = 0.8$ we can see that after certain period of time a weak entanglement starts to appear followed with period of increasing in its value beside irregular fluctuations. This means that the atoms start to gain and exchange the energy until the entanglement reaches its maximum at $\sim 0.57$. As soon as the entanglement reaches its maximum, the atoms start again to loss part of their energy and the amount of entanglement begins to decrease until to reach the minimum at zero. After this period of entanglement between the atoms, the system shows fluctuations for the rest of the considered time which reflects the exchange of the energy between the atoms and the field. Similar behavior can be reported for $r = 0.1$, where the entanglement in this case occurred at later time to that of $r = 0.8$, see Fig.(4a).
Increasing the value of the mean photon number $\bar{n} = 10$, one can easily realize that the maximum value for $r = 0.1$ and $r = 0.8$ occurs after onset of the interaction as in the case $\bar{n} = 5$ with the same value, see Fig.(4b). We can also report that the long period of entanglement between the atoms occurs first in the case $r = 0.1$. Furthermore, the general behavior for both cases $r = 0.1$ and $r = 0.8$ are nearly the same but with fluctuations less than the case in which $\bar{n} = 5$. It should be noted that in this case there is some delay in the second period of the entanglement between the atoms compared with the previous case and the period of entanglement for $r = 0.8$ is greater than the period for $r = 0.1$. Comparing these results with that obtained for the identical atoms [13], we can see that the generated state contained in the identical atoms is much larger than that obtained in the non-identical atoms. This is due to the equal possibilities of interaction of the two atoms together.

In Fig.(4c), we plot the degree of entanglement for small scale of time, where we assume the same case which shown in Fig.(4a). From this figure we see that at the zero value of the scaled time, $\lambda_1 t$, the degree of entanglement is $\simeq 0.78$, which represents the degree of entanglement of the initial state. As the interaction goes on, the degree of entanglement is slightly increased and then decays as the time increases. The sudden death of entanglement [14] is seen clearly for small values of the coupling parameter($r = 0.1$). For large values of the coupling parameter ($r = 0.8$), the degree of entanglement decrease smoothly. So one can say that the survival time of entanglement increases as one increases the the coupling constant.

4 Impurity and the dynamics of the information

Our target in this section is to quantify the amount of the local and non-local information for the present system. To reach this goal, let us first define the local and non-local information which can be defined from the following situation: Suppose we have a source supplies two users Alice and Bob with qubit (atoms) to code their information. In this case we can say that these information are a local information. On the other hand, if we assume that the atoms are forced to pass through a cavity, then the two atoms will entangled with each other and then interact with the cavity field. As a resultant of this interaction the local information will be transferred between the two atoms and whence the atomic system will behaves as an entangled state or as a separable state. In this case the information is called non-local information. Since the main purpose of the users is to distill as much
Figure 5: (a) The impurity $\xi$, for the density operators $\rho_1$ (dot curve), $\rho_2$ (dash-dot curve) and $\rho_{12}$ (solid curve) with $R = 0.1$ and $\bar{n} = 10$ (b) The non-local information, $I_{\text{non-Local}}$ (for short $I_n$) for $\rho_{12}$ (solid curve), the dot and dash dot curve are the local information, $I_{\text{Local}}$ (for short $I_l$) for $\rho_1$ and $\rho_2$ respectively, where $R = 0.8$ and $\bar{n} = 10$.

as possible from the information. Therefore to reach this target one may seek to quantify the amount of the local information. For this reason let us use the measure introduced in reference [16] to quantify such information which is defined by

$$I_{\text{Local}} = (2F_0(\rho) - 1)^2$$  \hspace{1cm} (12)

where

$$F_0(\rho) = \max_{A \in SU(2)} \langle \phi | A \rho A^\dagger | \phi \rangle,$$  \hspace{1cm} (13)

and $|\phi\rangle$ is the initial state of the atomic system, while $A$ is the unitary operator used by Alice and Bob to maximize their local information. The total local information for the two qubit is given by

$$I^l_{\text{Local}} = \sum_{j=1}^{2} I^j_{\text{Local}},$$  \hspace{1cm} (14)

and the non-local information is defined such as

$$I_{\text{non-Local}} = 2 - \sum_{j=1}^{2} I^j_{\text{Local}}$$  \hspace{1cm} (15)

To investigate the dynamic of the impurity for the individual states of the qubits as well as the total state, one can use the definition

$$\xi_k = 1 - tr\{\rho_k^2\}, \hspace{1cm} k = 1, 2, 12$$  \hspace{1cm} (16)
This quantity $\xi_k$, measures the distance between the given state and its purity, for more details see [17, 18]. Using similar parameters as in the previous section we have plotted the dynamics of the impurity, the local and non-local information for the individual qubits and the total state. For example, in figures (5a, b) we have considered the case in which the atomic system is prepared in the ground state with fixed value of the photon number $\bar{n} = 10$, while the ratio of the coupling parameter $r = 0.1$. From these figures one can see that after onset of the interaction the impurity for one individual qubit $\rho_2$ increases while it decreases for the total state $\rho_{12}$ so that both of them approaches each other without intersection. In the meantime the impurity of the qubit $\rho_1$ starts to increase, however, after short period of time to intersect with the individual qubit $\rho_2$ as well as with the total state $\rho_{12}$. This is followed by certain periods of time where similar behavior can be seen between the total states $\rho_{12}$ and individual qubit $\rho_1$, where the amount of increment in $\rho_{12}$ is nearly equal to the amount of decrement in $\rho_1$. This behavior disappeared at later time where one can see irregular fluctuations with interference between the impurity of individual qubits $\rho_1, \rho_2$ and the total state $\rho_{12}$, see Fig.(5a). This means that the information between each qubits as well as between the total states are lost.

On the other hand we have plotted Fig.(5b) to display the dynamics of the local and non-local information keeping all parameters unchanged. In this case it is also seen that at $t = 0$, the local information for both qubit is maximum, while for the total state is zero. However, the individual qubits $\rho_1$ and $\rho_2$ decrease their values after onset of the interaction which leads to increase the value of the total state $\rho_{12}$. Furthermore, the individual qubits show simultaneously fluctuations with decreasing in their value as the time increased. However, after considerable period of time one can see disturbance in the local information for the individual qubit $\rho_1$ corresponding to irregular rapid fluctuations in the non-local information. This means that the non-local information would increase its minimum as soon as local information start to die out, see Fig.(5b).

Different behavior can be observed for impurity as well as local and non-local information when we increase the ratio of the coupling parameter $r = 0.8$. For example, after short period of time one can see from figure (5c) that a considerable reduction in the total state $\rho_{12}$ to reach value below 0.4, this is corresponding to an increase in the impurity for each qubits $\rho_1$ and $\rho_2$ individually. This emphasis on the fact that the impurity of the total state is sensitive to the variation of the dynamics of the impurity of each qubit. Also it is noted that the impurity behavior of both
qubits is almost the same up to the time $t \sim 12$. This in fact is due to the closed value of the coupling parameters to each other. At later time we have realized that there is an interference between the impurity for the qubits and the total state. This would leads to an increase in the noise of the system and consequently reduction in the amount of the information received. This phenomenon is also observed for $r = 0.1$, but with more reduction in the value of the impurity of each individual qubits.

In figure (5d) we have plotted the local and non-local information for $r = 0.8$ and $\bar{n} = 10$, where we can see drastic reduction for the non-local information value corresponding to increasing in the local information value for each qubits. This is followed by a long period of time where the local information keeps its value at minimum, whereas the non-local information at maximum. This means that there is no transmission for any information during this period. At later time one of the qubits starts to show increasing in its value followed by the second qubit. In this case it is easy to observe that the value of the non-local information is decreased. Thus we may conclude that increasing the value of the coupling parameter ratio leads to reduction and delay in transmitting the information.

Now let us turn our attention to discuss the effect of the mean photon number when the system is initially in the ground state. To do so we have plotted figures (6) taken into consideration $r = 0.1$ and examined two different cases $\bar{n} = 5$ and $\bar{n} = 7$. For these two cases a similar behavior to that displayed in Fig.(5a) is seen in figures (6a,c). In the meantime, it is noted that decreasing the number of photons leads to more irregular fluctuations within certain period of time, more precisely between $t = 3$ and $t = 20$. Moreover, less interference between the impurity of the individual qubits and the total state can be reported in these cases, see Figs.(6a,c). This means that to reduce the noise in the present quantum system and to improve the quality of the transmission we have to decrease the mean photon number. Also for the same values of the mean photon numbers $\bar{n} = 5$ and $\bar{n} = 7$, we have considered the local and non-local information. Simple comparison between the case in which $\bar{n} = 10$ and the cases where $\bar{n} = 5$ and $\bar{n} = 7$, we realize that the general behavior is the same. However, the main difference between them is occurred in the non-local information, see Figs.(5b) and (6b,d). For example, at $t > 15$, one can realize that during the period of the irregular fluctuations some revivals can be seen as we decrease the mean photon number. Furthermore, at $t \approx 20$ the non-local information decreases its value as we decrease the mean photon number.
Figure 6: (a) The impurity $\xi$ for the density operators $\rho_1$ (dot curve), $\rho_2$ (dash-dot curve) and $\rho_{12}$ (solid curve) with $R = 0.1$ and $\bar{n} = 10$. (b) The non-local information $I_n$ for $\rho_{12}$ (solid curve), the dot and dash dot curve are the local information $I_l$ for $\rho_1$ and $\rho_2$ respectively, where $R = 0.8$ and $\bar{n} = 10$.

Finally, let us concentrate on discussing the variation when the system is initially in the intermediate state. For this reason we have plotted figure (7a) to display the behavior of the impurity for $\bar{n} = 7$ and $r = 0.1$. In this case we note that after the onset of the interaction the total state $\rho_{12}$ decreases while the impurity of the individual qubits $\rho_1$ and $\rho_2$ increases. Also it is easy to observe that the increment in $\rho_2$ is faster than that the increment in $\rho_1$. However, as the time goes on the impurity of $\rho_2$ backs to decrease its value while the impurity of $\rho_1$ continue its increasing. This means that the individual qubits $\rho_2$ acquired part of the energy less but faster than the energy acquired by the individual qubits $\rho_1$. At later time we can see irregular fluctuations in both $\rho_1$ and $\rho_2$ as well as in $\rho_{12}$ which indicates the appearance of some noise in the transmission.

To analyze the local and non-local information we have plotted Fig.(7b). Here and after onset of the interaction we see a rapid fluctuations in a short period of time.
where the interference between local and non-local in formation is pronounced. This is followed by a period of time where we see decreasing in the local information corresponding to increasing in the non-local information. At $t > 15$ the local information increases and the non-local decreases referencing to better period of transmission.

5 Conclusion

In this contribution, we investigate the time evolution of atomic system interacting with a single cavity mode from the separability point of view. The effect of the coupling constant, between the cavity mode and the two non-identical atoms, and the mean photon number are investigated on the separable and entangled behavior of the atomic system. We find as one increases the values of the coupling constant, the possibility of generating entangled state decreases and the amount of entanglement decreases. Also, as the mean photon number increases, the entangled intervals shifted also the degree of entanglement decreases.

The dynamics of the impurity and the information is investigated for different values of the coupling and initial state of the two atoms. It is found that as the impurity increases for one qubit, decreases for the other one. Also, the transfer of the local information into non-local depends on the impurity of the individual qubits. If the impurity of one qubit is maximum, then the total information is converted to be non-local. This phenomena is too important in quantum communication as we shall see in our next work.
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