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SIMULATION OF PIPELINE RANDOM RESPONSE TO STRAY CURRENTS
EFFECTS PRODUCED BY D.C. TRACTION SYSTEM

Abstract
The paper presents a method of the simulation of the pipeline potential shift produced by D.C. traction stray currents which are stochastic in character. The calculation model presented is based on the deterministic model used in the earth-return circuit theory combined with the non-deterministic approach based on the Monte Carlo procedure. The model of the equivalent rail with current energization and the concept of superposition allow one to consider more complicated D.C. railway systems using a segmental approximation of the complex railway route and taking into account a number of substations and loads at any location. A locomotive position and a load current are assumed to be independent random variables in the non-deterministic approach. Using simulation program developed random characteristics of a pipeline response e.g. maximum, minimum, median and mean values can be obtained. Hence the pipeline regions more exposed to corrosion risk can be determined.

Keywords: D.C. traction, complex geometry, stochastic stray currents, earth return circuit, pipeline potential shift, simulation, Monte Carlo method.

Streszczenie
W artykule przedstawiono metodę symulacji potencjału rurociągu generowanego przez prądy błądzące o losowym kierunku przepływu i wartości. Przedstawiony model zrealizowano w oparciu o metodę deterministyczną w połączeniu z procedurą Monte Carlo. Model załatając szyn wykorzystuje zasilanie prądowne oraz zasadę superpozycji. Pozwala to rozważyć złożone układy z zastosowaniem segmentowej aproksymacji trasy kolejowej oraz uwzględnieniu wielu podstacji i pojazdów w dowolnej lokalizacji. Zakłada się, że pozycja lokomotyw i prąd obciążenia są niezależnymi zmiennymi losowymi w podejściu niedeterministycznym. Wykorzystując zaproponowaną metodę przedstawiono charakterystyki odpowiedzi rurociągu, tj. minimalne, maksymalne, medianę, a wartości średnie potencjału. Na tej podstawie można wyznaczyć rejon ryjów rurociągów zagrożony korozją elektrochemiczną.

Słowa kluczowe: Trakcja prądu stałego, złożona geometria, stochastyczne prądy błądzące, obwód ziemnopowrotny, potencjał rurociągu, symulacja, metoda Monte Carlo.
1. Introduction

The electromagnetic compatibility of components of electric traction system is a criterion participating more and more in the decision of network planning and operation. D.C. electrified traction systems are a potential source of stray currents. The important problem, technically, is to evaluate the harmful effects (electrolytic corrosion) that an electrified railway has on nearby earth-return circuits (e.g. pipelines).

The stray currents from the D.C. rail-return circuit may flow into the earth and into the underground structure, returning to the rails or negative feeder taps in the vicinity of the substation or power plant. The general nature of the stray current problem is illustrated schematically in Fig. 1 [10, 16].

![Fig. 1. Generation of stray currents: a) a railway system, b) railway system equivalents](image)

When a metallic structure is electrically influenced by stray currents, the potential of the structure shifts in the positive or negative direction, where the current leaves or enters the metal surface, Fig. 1a. The key problem in the evaluation of the new foreign structure response to the stray currents interference consist in the determination of the potential shift of the structure with respect to the adjacent (local) earth.

To predict the potential shift due to the stray current influence, calculation methods/tools can be used, especially at design stage of new traction lines or pipelines. The existing simulation models presented in the literature are mainly based on the deterministic approach, e.g. an analytical method of calculation basing on the complete field method of solution of the transmission-line problem. The analysis is applicable to any D.C. railway system in which tracks can be represented by a single earth return circuit (equivalent rail) with current (shunt) energization [10, 11, 16].

The method, similarly to the “field approach” – e.g. the Boundary Element Method [1, 2, 13] is an alternative to the approximate method in which the equivalent rail with current energization is modeled as a large multinode electrical equivalent circuit with lumped parameters. This circuit is a chain of basic circuits, which are equivalents of homogenous sections of the rail [3, 4, 7, 9, 14, 15]. It should be pointed out, that the simulations presented by deterministic approach refer to the chosen point of time, i.e. at the time \( t = \text{const.} \). In reality flowing stray currents are stochastic in character, meaning that the current as well as the flow direction change at random. Different from the existing models, which are based on a deterministic approach, the paper [7] presents a non-deterministic approach to study of effects generated on buried pipelines located in stray currents area. The method bases on random and statistical aspects of stray current, which are captured by Monte Carlo approach.
The objective of the paper is to present problems of the modeling of stochastic stray currents effects generated by D.C. electrified railways forming geometrically complex routes. The model of the equivalent rail with current energization and the concept of superposition allow one to consider more complicated D.C. railway systems using a segmental approximation of the complex railway route and taking into account a number of substations and loads at any location. The special concern will be however given to the simulation of a pipeline response i.e. the pipeline potential shift produced by stochastic stray currents. The calculation model is based on the deterministic model combined with the non-deterministic approach based on the Monte Carlo procedure, in which a locomotive position and a load current are independent random variables.

The analysis described in the paper may be useful in understanding effects on metal installation buried in the stochastic stray current area. The non-deterministic simulation model presented can be especially useful in the design stage of new earth return circuit (pipelines) buried in the stray current area, when frequent alterations are made as the design progresses. The efficiency of the simulation program developed is demonstrated by illustrative calculations.

2. Current and potential excited in a rail by current energization

Solution for current and potential can be obtained using a rail modeled as a circuit with distributed parameters [10, 11, 16]. The system shown in Fig. 1b may be applied directly by superposition in building up electrified railway system. In this system tracks are represented by a single conductor – equivalent to a rail continuously in contact with the earth through the track ballast. The conductor is energized with the currents $I_0$ and $(-I_0)$ by a feeder station and a load at points $x = x_0$ and $x = x_L$, respectively.

The starting point for the analytical solution for current and potential along an equivalent rail located along the $x$ – axis of the Cartesian coordinate system is, according to the multi-conductor line theory, the system of linear differential equations:

$$\begin{align*}
-\frac{dV_r(x)}{dx} &= ZI_r(x) - E_s(x) \\
-\frac{dI_r(x)}{dx} &= YV_r(x) - J_s(x)
\end{align*}$$

where:

- $V_r$ – denotes the rail potential,
- $I_r$ – the rail current,
- $Z$ – the longitudinal impedance (resistance) per unit length (p.u.l.),
- $Y$ – the p.u.l. shunt admittance (conductance),
- $E_s, J_s$ – the p.u.l. external sources (longitudinal and shunt, respectively) driving the homogeneous line. The details of the circuit with earth return parameters can be found in the literature, e.g. [5, 6, 10–12, 16].
Consider the case of a finite rail extending from \( x = x_1 \) to \( x = x_2 \). The rail is energized with the current \( I_0 \) at \( x = x_0 \) and is open circuited on both ends. The solution of the eqn (1) for the current along the rail, taking into account the boundary conditions:

\[
I_r(x_1) = I_r(x_2) = 0
\]

is given in the form:

\[
I_r(x) = -\text{sign}(x-x_0) \frac{I_0}{2} e^{-\Gamma|x-x_0|} + A_1 e^{-\Gamma x} + B_1 e^{\Gamma x}
\]

where:

- \( A_1, B_1 \) – constants which are to determine from the boundary conditions,
- \( \Gamma \) – the propagation constant and

\[
\text{sign}(x-x_0) = \begin{cases} 
-1 & \text{when } x-x_0 < 0 \\
1 & \text{when } x-x_0 > 0 
\end{cases}
\]

Finally the constants \( A_1 \) and \( B_1 \) become:

\[
A_1 = -\frac{I_0}{2} \frac{ch(x_2-x_0)}{sh\Gamma L} e^{\Gamma x_1}, \quad B_1 = \frac{I_0}{2} \frac{ch(x_0-x_1)}{sh\Gamma L} e^{-\Gamma x_2}
\]

where \( L = x_2 - x_1 \) denotes the rail length.

Potential along the equivalent rail can be calculate from the relationship:

\[
V_r(x) = -\frac{1}{Y} \frac{dI_r(x)}{dx}
\]

For the case of current energization of the rail by a vehicle at the point \( x = x_L \) (Fig. 1b), currents and potentials are calculated from the equations (3, 5 and 6) with \( I_0 = -I_0 \) and \( x_0 = x_L \), respectively.

It should be pointed out, that for the case of other kind of the boundary conditions, e.g. defined by impedances of finite value at rail both ends, the constants can be evaluated in similar way.

To demonstrate range of changes of rail potential and current values, sample results of a deterministic simulation are presented, Fig. 2. For a straight finite 10 km length rail two cases were considered. In both cases vehicle current was \( I = 1 \) A and the station was located at point \( x_0 = 0 \) km. In case 1 the vehicle was located at \( x_L = 1 \) km, whereas in case 2 the position of the vehicle was \( x_L = 9 \) km.

![Fig. 2. Potential and current along the rail as a function of the vehicle position](image)
Despite to the simple layout, the presented values have changed in wide range. It follows that the use of random algorithms for calculating stray currents effects on nearby earth return circuits and the estimation of the electrochemical corrosion risk due to the D.C. stray currents is justified.

3. Scalar potential in the earth due to current in the equivalent rail

The knowledge of the earth potential of the electric flow field in the vicinity of the tracks is required for the evaluation of stray currents effects on nearby structures. The potential (primary potential) can be obtained by the technique used in the earth return circuit theory, when the conductor with earth return carries a longitudinal current [8, 10, 11, 16]. The basic circuit for the calculation of the earth potential is shown in Fig. 3.

![Fig. 3. Equivalent rail with longitudinal current flow on the earth surface](image)

The equivalent rail is placed on the earth surface and is carrying the longitudinal current $I_r(x)$ which flows in the positive direction of the $x$ axis lying along the rail. The rail can be regarded as a set of current elements of length $d\tau$. From each element an elementary leakage current $(-dI_r(\tau)/d\tau)$ flows into the earth with the conductivity $\gamma$, producing the elementary scalar potential. In the observation point $P(x, y, z)$ the scalar potential can be determined from the expression:

$$dV^0_e(P) = -\frac{1}{2\pi\gamma r} \frac{dI_r(\tau)}{d\tau}$$

where $r$ is the distance from the current element (source point) to the observation point.

If a finite rail extending from $x = x_1$ to $x = x_2$, is energized with the current $I_0$ at $x = x_0$ and open circuited on both ends, the current along the rail is described by eqn. (3). Thus the scalar potential can be determined from the following expression:

$$V^0_e(x, y, z) = \frac{I_0\Gamma}{4\pi\gamma} \left[ e^{-\gamma x_0} \int_{x_1}^{x_0} \frac{e^{\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau - e^{\Gamma x_0} \int_{x_0}^{x_1} \frac{e^{-\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau - \frac{ch\Gamma(x_2-x_0)}{sh\Gamma L} e^{\Gamma x_1} \int_{x_1}^{x_2} \frac{e^{-\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau + \frac{ch\Gamma(x_0-x_1)}{sh\Gamma L} e^{-\Gamma x_2} \int_{x_1}^{x_2} \frac{e^{\Gamma\tau}}{\sqrt{(x-\tau)^2 + y^2 + z^2}} d\tau \right]$$

(8)
4. Calculation of earth potential generated by D.C. traction of complex geometry

The models of the equivalent rail with current energization shown in Fig. 1b and the concept of superposition allow one to consider more complicated D.C. railway systems using a segmental approximation of the complex railway route and taking into account greater number of substations and loads at any location. The earth potential in the observation point has to be evaluated for each rail segment with leakage current applying each time a new coordinate system and transforming appropriately boundary conditions and coordinates of energization points.

Consider the arbitrary configuration of the D.C. railway system, as shown in Fig. 4a. For calculation purposes, the current path is divided into straight-line segments. For simplicity consider only the $k$-th segment of the current path. It is convenient to define two different Cartesian reference systems: the first one $x, y, z$ is a reference system (external reference system), the second one $x', y', z'$ is referred to the $k$-th segment, Fig. 4 b. It should be noted, that the reference coordinate system can be arbitrary located in the space, it is however reasonable to locate the $xy$ plane on the earth surface.

The terminating points of the $k$-th segment have in the reference (unprimed) system the coordinates $(x_k, y_k, z_k)$ and $(x_{k+1}, y_{k+1}, z_{k+1})$ respectively. The segment lies in the $xy$ plane, $\phi_k$ is the angle between the segment and the $x$ – axis (angle measured anticlockwise), $l_k$ is its length and taking into account that the circuit segment is parallel to the $xy$ plane ($z_k = z_{k+1}$)

$$l_k = \sqrt{(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2}$$ (9)

Assuming that the substation or the vehicle is located inside the $k$-th segment as shown in Fig.1b at points $x = x_0$ or $x = x_L$ in the reference system, the coordinates of the current
energization with currents $I_0$ or $(-I_0)$ should be transformed into the current – primed coordinate system giving:

$$x'_0 = (x_0 - x_k) \cos \phi_k \quad x'_L = (x_L - x_k) \cos \phi_k \tag{10}$$

Similarly the coordinates of the end points of the equivalent rail, given in the reference coordinate system, $x_1$ and $x_2$ after the transformation into the current coordinate system are

$$x'_1 = -\sum_{i=1}^{k-1} l_i \quad x'_2 = \sum_{i=k}^n l_i$$

where $n$ denotes the number of segments the rail is divided into.

The current along the rail can be now determined in the primed coordinate system from the expression (3), whereas the constants $A_1$ and $B_1$ are defined by eqn. (5) with $L = \sum_{i=1}^n l_i$.

Taking into account the relation (8) the scalar potential in the earth due to current flowing in the $k$-th segment energized with the current $I_0$ at the point $x'_0$ (substation) can be determined from the following expression:

$$V^0_{ek}(P) = \frac{I_0 \Gamma}{4\pi l} \left[ -e^{-\Gamma x_0} \int_0^{x_2} \frac{e^{\Gamma \tau}}{\sqrt{(x' - \tau)^2 + y^2 + z^2}} \, d\tau - e^{-\Gamma x_0} \int_{x_0}^i \frac{e^{\Gamma \tau}}{\sqrt{(x' - \tau)^2 + y^2 + z^2}} \, d\tau + \right.$$  
$$\left. - \frac{ch \Gamma (x'_2 - x'_0)}{sh \Gamma L} e^{\Gamma x_i} \int_0^{i_k} \frac{e^{-\Gamma \tau}}{\sqrt{(x' - \tau)^2 + y^2 + z^2}} \, d\tau - \frac{ch \Gamma (x'_0 - x'_i)}{sh \Gamma L} e^{-\Gamma x_i} \int_0^{i_k} \frac{e^{\Gamma \tau}}{\sqrt{(x' - \tau)^2 + y^2 + z^2}} \, d\tau \right]$$

On the other hand the earth scalar potential due to current flowing in the $k$-th segment

with the current $(-I_0)$ at the point $x'_L$ (vehicle) can be calculated from the eqn. (12) with $I_0 = -I_0$ and $x'_0 = x_l$, respectively.

It should be noted that in order to calculate the earth potential in the observation point $P(x, y, z)$, the coordinate transformation (transposition and rotation) should be taken into account, i.e.

$$x = x' \cos \phi - y' \sin \phi + x_k, \quad y = x' \sin \phi + y' \cos \phi + y_k \tag{13}$$

where the origin $O'$ of $x'y'$ coordinate system has coordinates $(x_k, y_k)$ relative to the reference $xy$ coordinate system and the $x'$ axis makes an angle $\phi$ with the positive x axis.

Finally, when the number of current energizations (substations and loads) of the equivalent rail is $N$ and the rail is divided into $n$ segments the earth potential can be calculated from the relation:

$$V^0_e(P) = \sum_{k=1}^n \sum_{m=1}^N V^0_{ek,m}(P) \tag{14}$$
5. Current and potential along a pipeline buried in the d.C. Stray currents area

Current and potential along the pipeline located in an electric flow field \( E(x) = -dV_e/dx \) due to stray currents can be calculated using a pipeline modeled as a circuit with lumped parameters [4, 7–9, 15].

Assuming a segment of the length \( l \) of the pipeline to be homogeneous (e.g., \( Z_p, Y_p = \text{const.} \)), it is possible to model the segment by a \( \pi \) – two port, as shown in Fig. 5 [4, 9] with the series impedance and the shunt admittance:

\[
Z_p = Z_{op} \sinh(\Gamma_p l),\ Y_p = \frac{2 \tanh \left( \frac{\Gamma_p l}{2} \right)}{Z_{op}}
\]

where:
- \( Z_{op} \) – characteristic impedance,
- \( \Gamma_p \) – propagation constant of the pipeline.

For direct current the electrical parameters of a pipeline segment can be defined:

\[
Z_p = R_p Y_p^{-1} = G_i^{-1} + G_e^{-1}
\]

where:
- \( R_p \) – longitudinal pipeline resistance,
- \( G_i \) – pipeline insulation conductance,
- \( G_e \) – pipeline shunt conductance related to the soil conductivity.

The whole rail length can be subdivided into elementary cells which may have different lengths or different specific parameters.

If the pipeline is subjected to the electric field with the potential \( V_e^0 \), the passive model (Fig. 5 a) has to be completed by the voltage sources acting in shunt branches of the \( \pi \) – two port, Fig. 5 b.

![Fig. 5. a) \( \pi \) – two port model of an elementary homogeneous segment of a pipeline, b) chain of \( \pi \) – two ports modeling a pipeline with voltage sources representing the pipeline energization](image)

After being divided into sections the pipeline can be composed of such basic two-ports which define the nodes and branches of the network model, which is well suited for computer - aided circuit analysis using simulation programs. The number of subdivisions of the pipeline can theoretically be as large as required, according to the wanted degree of discrimination in the potential and current computation.
The key problem in the evaluation of a foreign structure response to the stray currents interference consist in the determination the potential shift of the structure with respect to the adjacent (local) earth, which is described by equation:

\[ V_b(x) = \frac{1}{G'_i} \frac{dI(x)}{dx} \]  

(17)

where:

- \( G'_i \) – unit-length pipeline insulation conductivity.

To calculate this quantity the current flowing between pipeline and earth is required, Fig. 5b. To determine the current nodal analysis is used.

6. Incorporating Monte carlo procedure

An electrified D.C. railway system (producer of stray currents) and a nearby underground pipeline (victim of stray current interference) create a conductively coupled system of earth return circuits. Almost all parameters of the system present random characteristics. The outflow of stray currents into the ground depends on the properties of electric traction return circuits: the actual load of traction circuits i.e., the load of each electric locomotive, their number and position on the route, type and quality of rails and subgrade, and also the structure and conductivity of the surrounding environment, etc. Similarly, such parameters as conductance of pipeline insulation, soil structure and conductivity (seasonal changed), groundings along the pipeline route, insulating flanges, etc. influence electrical parameters (mutual conductance, series and shunt resistances, propagation coefficients) of the coupled earth return circuits. It is assumed in the paper, as in [7], that two stochastic quantities: locomotive position and a load current are most useful, as independent random variables characterized by suitable probability distribution, for the estimation of stochastic stray currents effects on affected pipelines.

The method proposed is intended as a tool for estimation of location of anodic/cathodic zones along a pipeline buried in stochastic stray current area. The calculation model is based on the deterministic block of models described in sections 2-5 combined with the non-deterministic approach based on the Monte Carlo procedure, in which the independent random variables are treated as input parameters for calculation of random characteristics of pipeline responses (output parameters of deterministic block) e.g. potential shift along an affected pipeline. The values of the pipeline responses compose statistical distributions and each of them can be suitably processed, thus obtaining significant parameters like maximum, minimum, median and mean values. Hence the pipeline regions more exposed to corrosion risk can be estimated. The application of the method presented shall be illustrated by an example in the sequel. Calculation algorithm developed is shown in Fig. 6.

The first step is insertion the parameters of pipeline, electric traction and simulation geometry. On this basis are next calculated rail and pipeline mathematical models. These models are used to calculate output quantities in a static state for one drawn position of the vehicle and load current. For this purpose is used a random number generator, which is available in the development...
environment MATLAB. Numerous tests have shown that the distribution of random numbers is evenly in the search range. Repeating many times calculations for static system, taking into account different current values and the position of the vehicle results in obtaining the entire spectrum of the results of a given size including information on the incidence of specific values (earth potential, pipeline potential to the adjacent earth, leakage current).

Fig. 6. Flowchart with algorithm of the proposed stochastic method

7. Examples of calculation

The following case has been proposed to present the algorithm developed. A rail section is modeled by equivalent finite 10 km length earth return circuit with parameters $Z' = 0.02 \, \Omega/\text{km}$ and $Y' = 0.76 \, \text{S/km}$. A substation is located at point $x_0 = 0$ km. Two vehicles are running along the rail section. A pipeline with the diameter 355.6 mm has a length of 1.0 km, and its electrical parameters are: $Z_p' = 0.02 \, \Omega/\text{km}$, $Y_p' = 0.011 \, \text{S/km}$. The pipeline is buried in the soil with conductivity 0.01 S/m at 1.0 m depth, and its center is located 10 meters away of the rail middle section central point. The pipeline is open circuited at its both ends. The angle between pipeline and the rail is $\alpha = 45^\circ$, Fig. 7. Calculations of the pipeline potential to the adjacent earth have been curried out for $n = 1000$ samples with randomly chosen load $I_0 \in (0, \ldots, 1000 \, \text{A})$ and the position of the vehicles $x_L \in (0, \ldots, 10 \, \text{km})$, assuming that the calculation points are located at 0, 250, 500, 750 and 1000 m from the left end of the pipeline. The histograms of the pipeline potentials are shown in Fig.8, and minimum, maximum, average and median values of pipeline potential to the adjacent earth are summarized in Table 1.
Table 1. Summary of minimum, maximum, average and median values of pipeline potential to the adjacent earth, calculated at points located along the pipeline

| Location of the calculation point $x_p$ [m] | Min value [V] | Max value [V] | Average [V] | Median [V] |
|------------------------------------------|---------------|---------------|-------------|------------|
| 0                                       | 0,000         | 1,295         | 0,365       | 0,329      |
| 250                                      | 0,000         | 0,643         | 0,187       | 0,170      |
| 500                                      | –1,880        | 0,013         | –0,383      | –0,307     |
| 750                                      | –0,336        | 0,088         | –0,068      | –0,052     |
| 1000                                     | –0,566        | 0,682         | 0,021       | 0,031      |

The calculations have been performed to estimate the anodic and cathodic zones along the pipeline. On the basis of the histograms and the results shown in the Table 1, it can be determined that the positive pipeline potential to the adjacent earth values occur at end points of the pipeline. Along sections of the pipeline lying between points $x_p \in (0–350$ m) and $x_p \in (900–1000$ m) the anodic zones can be expected. The highest and positive values of the potential average and median values are obtained at point located nearest to the substation ($x_p = 0$ m). As one would expect the negative potential occurs at the middle point of the pipeline, and the cathodic zone spreads between points $x_p \in (350$ m–900 m).

For the above calculation example, it was proposed to change the parameters of the insulation of the central pipeline segment (from 400 to 600 m). The initial conductivity value of the insulation was reduced a ten times. The rest of the pipeline insulation value remains unchanged. Calculations of the pipeline potential to the adjacent earth have been curried out for $n = 1000$ samples with randomly chosen load $I_0 \in (0, \ldots, 1000$ A) and the position of the vehicles $x_L \in (0, \ldots, 10$ km). The average value of the potential to the adjacent earth decreased both at the beginning and at the end of the pipeline. In the end section of the pipeline the mean values changed the sign to negative, which resulted in the conversion from anode to cathode zone. The result was presented in Fig. 9.
Fig. 8. Histograms and average value of the pipeline potential to the adjacent earth at points located along the pipeline

Fig. 9. The average value of the pipeline potential to the adjacent earth as a function of the insulation conductivity
8. Final remarks

The method proposed is intended as a tool for estimation of location of anodic/cathodic zones along a pipeline buried in stochastic stray current area. The calculation model is based on the deterministic block of models combined with the non-deterministic approach based on the Monte Carlo procedure, in which the independent random variables are treated as input parameters for calculation of random characteristics of pipeline responses (output parameters of deterministic block) e.g. potential shift along an affected pipeline. The values of the pipeline responses compose statistical distributions and each of them can be suitably processed, thus obtaining significant parameters like maximum, minimum, median and mean values. Hence the pipeline regions more exposed to corrosion risk can be estimated.

The formulas derived and program developed allow to manage cases with any complex geometry of the system D.C. traction route – underground pipeline. The necessary data for calculations are: the number of substations and vehicles, the magnitude of current energizations, coordinates of energization points, electrical parameters of the equivalent rail, the number of segments the equivalent rail is divided into and the coordinates $(x_i, y_i, z_i)$ and $(x_{i+1}, y_{i+1}, z_{i+1})$ of terminating points of each segment, coordinates of the observation point along the pipeline and its electrical parameters, earth conductivity. It should be noted that all coordinates refer to the reference system, which can be arbitrary located in the space.

The analysis described in the paper may be useful in understanding effects on metal installation buried in the stochastic stray current area. The simulation models presented can be especially useful in the design stage of new earth return circuit buried in the D.C. stray current area, when frequent alterations are made as the design progresses.

References

[1] Bortels L., Dorochenko A., Van den Bossche B., Weyns G., Deconinck J., Three-Dimensional Boundary Element Method and Finite Element Method Simulations Applied to Stray Current Interference Problems, A Unique Coupling Mechanism That Takes the Best of Both Methods, Corrosion, Vol. 63, No. 6, June 2007, 561–576.

[2] Brichau F., Deconinck J., A Numerical Model for Cathodic Protection of Buried Pipes. Corrosion, Vol. 50, No. 1, January 1994, 39–49.

[3] Charalambous C.A., Cotton I., Aylott P., A Simulation Tool to Predict the Impact of Soil Topologies on Coupling Between a Light Rail System and Buried Third-Party Infrastructure, IEEE Trans. Veh. Technol., Vol. 57, No. 3, 2008, 1404–1416.

[4] Czarnywojtek P., Machczyński W., Computer simulation of responses of earth-return circuits to the a.c. and DC external excitation, European Trans. on Electrical Power, ETEP Vol. 13, No. 3, May/June 2003, 173–184.

[5] Hill R.J., Brillante S., Leonard P.J., Railway track transmission line parameters from finite element field modeling: Shunt admittance, Proc. IEE Elect. Power Applicat., Vol. 146, No. 6, 1999, 647–660.
[6] Hill R.J., Brillante S., Leonard P.J., *Railway track transmission line parameters from finite element field modeling: Series impedance*, Proc. IEE Elect. Power Applicat., Vol. 147, No. 3, 2000, 227–238.

[7] Lucca G., *Estimating stray currents interference from DC traction lines on buried pipelines by means a Monte Carlo algorithm*, Electrical Engineering, DOI 10.1007/s00202-015-0333-6, published online: 05 April 2015.

[8] Machczyński W., Budnik K., Szymenderski J., *Assessment of DC traction stray currents effects on nearby pipelines*, The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 35, iss 4, 2016, 1468–1477.

[9] Machczyński W., Czarnywojtek P., *Computer simulation of a protection of underground conductors against stray currents*, 16th International Corrosion Congress, September 19 – 24, 2005, Beijing, China, paper 21-03, 1–8.

[10] Machczyński W., *Simulation model for drainage protection of earth–return circuits laid in stray currents area*, Electrical Engineering, Vol. 84, No 3, July 2002, 165–172.

[11] Machczyński, W., *Currents and potentials in earth return circuits exposed to alternating current electric railways*, Proc. IEE, Part B, Vol. 129, 5, 1982, 279–288.

[12] Mariscotti A., Pozzobon P., *Determination of the electrical parameters of railway traction lines: Calculation, measurements and reference data*, IEEE Trans. on Power Delivery, Vol. 19, No. 4, 2004, 1538–1546.

[13] Metwally I.A., Al-Mandhari H.M., Nadir Z., Gastli A., *Boundary element simulation of DC stray currents in oil industry due to cathodic protection interference*, European Trans. on Electrical Power, Vol. 17, Sept./Oct. 2007, 486–499.

[14] Ogunsola A., Mariscotti A., *Electromagnetic Compatibility in Railways, Analysis and management*, Springer – Verlag, Berlin Heidelberg 2013.

[15] Ogunsola A., Mariscotti A., Sandrolini L., *Estimation of stray current from a dc-electrified railway and impressed potential on a buried pipe*, IEEE Trans. on Power Delivery, Vol. 27, No. 4, 2012, 2238–2246.

[16] Sunde E.D., *Earth conduction effects in transmission system*, New York, Dover 1968.