Multi-objective optimal power flow for active distribution network considering the stochastic characteristic of photovoltaic

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Abstract. To mitigate the impact on the distribution networks caused by the stochastic characteristic and high penetration of photovoltaic, a multi-objective optimal power flow model is proposed in this paper. The regulation capability of capacitor, inverter of photovoltaic and energy storage system embedded in active distribution network are considered to minimize the expected value of active power, the T loss and probability of voltage violation in this model. Firstly, a probabilistic power flow based on cumulant method is introduced to calculate the value of the objectives. Secondly, NSGA-II algorithm is adopted for optimization to obtain the Pareto optimal solutions. Finally, the best compromise solution can be achieved through fuzzy membership degree method. By the multi-objective optimization calculation of IEEE34-node distribution network, the results show that the model can effectively improve the voltage security and economy of the distribution network on different levels of photovoltaic penetration.

1. Introduction
As the problems of energy depletion and environmental pollution becoming increasingly serious, distributed generations (DGs) has got rapid development on account of less investment, flexible generating method and environmental compatibility [1-3]. However, large scale integration of DGs has an impact on security, economy and reliability of distribution networks (DNs). With the rapid development of active distribution network (ADN) technologies, regulating means and objectives for optimal optimization becomes more complex, and the traditional reactive power optimization is no longer applicable [4]. Considering the characteristic of the regulating equipments on ADN, the optimization is a nonlinear programming problem including continuous and discrete variables. Reference [5] proposed a new multi-agent immune algorithm based on the mechanism of antibody cluster and compete expansion by integrating the reactive power of different types of DGs and traditional voltage regulating equipments to minimize the power loss. Reference [6] considered the stochastic characteristic of photovoltaic (PV) station, took the minimum of power loss as the objective function and built mathematical model of reactive power optimization for DNs based on chance constraints, the simulation results verified the model could decrease the power loss on the premise that
the voltage satisfied the chance constraints. The models proposed above can improve the economy of DNs, but they only consider a single objective for optimization. A voltage control model was proposed in [7] with active/reactive power integrated optimization to reduce voltage deviation, power loss and environmental pollution by using micro-turbine and fuel cell. Reference [2] proposed a two-stage reactive power planning model whose objective was to minimize the power loss, leaping transmission of reactive power and the investment of the compensators considering the uncertainty of power output of PV and load. They try to achieve the optimization on the aspect of both economy and security, but actually when it comes to multi-objective optimization, they change it into a single objective optimization by giving weight, or use layered optimization method to solve it, which ignores the contradictory between the sub-objectives, also fails to provide variety of decision schemes. In light of the problem of multi-objective optimization, a probabilistic model with the DGs for DNs was established in [8] which is based on the two-point estimation method for probabilistic power flow, and used the particle swarm optimization (PSO) algorithm to obtain the Pareto frontier, but it sets the voltage offset as the objective function which fails to quantify the weakness of the voltage security on DNs.

When it refers to regulating means for optimization, the model mentioned above all consider the reactive power regulation capability of DGs, some take advantage of the active power regulation capability of micro-turbine and fuel cell, but the intermittence of power output of DGs is outstanding, sometimes these regulating means are not very effective, even have no effect. In recent years, energy storage system (ESS) has attracted the interests from the researchers because it can quickly regulate power output and flexibly store excess energy from the power system [9, 10]. Therefore, integrated utilization of the reactive power of DGs and the active power of ESS is benefit for the optimal operation of the DNs.

Motivated by the observations above, this paper proposes a multi-objective optimal power flow model considering the regulation capability of capacitor, inverter of photovoltaic and energy storage system, and sets the expect value of active power loss and probability of voltage violation as its objective functions. A probabilistic power flow based on cumulant method is introduced to calculate the value of the objectives and NSGA-II algorithm is adopted for optimization to obtain the Pareto frontier. Through fuzzy membership degree method, the best compromise solution can be achieved. Finally, the model is tested on IEEE34-node distribution network, the results show that compared with other different objective functions, the model can reduce loss more effectively under the same voltage security level; compared with other models considering different control variables, the model can keep the system operating in the optimal state on different levels of photovoltaic penetration.

2. Probabilistic model of load and PV

2.1. Probabilistic model of load

Error caused by the forecasting and measurement of load is unavoidable, so its active power \( P_L \) and reactive power \( Q_L \) should be described as random variables. Many researches demonstrate that these two random variables can be described by a normal distributed function.

2.2. Probabilistic model of PV

Some researches have shown that the solar irradiance \( r \) in a certain period of time can be described by a Beta distribution function as following:

\[
\begin{align*}
    f_R\left(\frac{r}{r_{\text{max}}}\right) &= \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{r}{r_{\text{max}}}\right)^{\alpha - 1} \left(1 - \frac{r}{r_{\text{max}}}\right)^{\beta - 1} \\
    (1)
\end{align*}
\]

In (1), \( f_R(\cdot) \) is the PDF of the random variable \( R \), \( r \) and \( r_{\text{max}} \) represent the measured value and the maximum of the solar irradiance, \( \Gamma(\cdot) \) is the Gamma function, \( \alpha \) and \( \beta \) are two shape parameters of Beta distribution.
The power output of PV is mainly determined by $r$, and their mathematical relation can be expressed by the equation as following:

$$P_{PV} = A \eta r$$  \hspace{1cm} (2)

where $A$ is the area of PV cells, $\eta$ is the conversion efficiency. According to (1) and (2), the PDF of the active power of PV can be expressed by the equation as following:

$$f_{p_{PV}}(p_{PV}) = \frac{1}{A^{\alpha}} \cdot \frac{1}{\Gamma(\alpha + \beta)} \left( \frac{P_{PV}}{P_{PV_{max}}} \right)^{\alpha-1} \left( 1 - \frac{P_{PV}}{P_{PV_{max}}} \right)^{\beta-1}$$  \hspace{1cm} (3)

Define $\lambda_{PV}$ as the power factor of PV, so the PDF of the reactive power of PV can be expressed by

$$f_{q_{PV}}(q_{PV}) = \frac{\lambda_{PV}}{P_{PV_{max}}^{\alpha/2}} \left( \frac{q_{PV}}{q_{PV_{max}}} \right)^{\alpha-1} \left( 1 - \frac{q_{PV}}{q_{PV_{max}}} \right)^{\beta-1}$$  \hspace{1cm} (4)

3. Multi-objective optimal power flow model

Multi-objective optimal power flow model mainly consists of objectives, equality constraints and inequality constraints. It uses the probabilistic method to quantify the economy and voltage security of the DNs and considers the regulation capability of capacitor, inverter of PV and ESS for optimization, which can ensure the accuracy and objectiveness of the optimization results.

3.1. Objectives

Expect value of the active power loss and the probability of voltage violation are set as the two objectives, mathematically

$$F_1 = \min \bar{P}_{loss} = \min \left( \sum_{i=1}^{N_g} \bar{P}_{G_i} - \sum_{i=1}^{N_L} \bar{P}_{L_i} \right)$$

$$F_2 = \min \left[ \max \left( P_{V_{1,\text{violation}}}, P_{V_{2,\text{violation}}}, \ldots, P_{V_{n,\text{violation}}} \right) \right]$$

In (5), $\bar{P}_{G_i}$ is expect value of the active power of generators at bus $i$, $\bar{P}_{L_i}$ is expect value of the active power of load at bus $i$, $P_{V_{n,\text{violation}}}$ is the probability of voltage violation at bus $i$.

3.2. Constraints

3.2.1. Equality Constraints.

The equation constraints of the active power and reactive power flow equations are given by

$$P_{IS} - V_i \sum_{j \in \Gamma} V_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) = 0$$

$$Q_{IS} - V_i \sum_{j \in \Gamma} V_j \left( G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right) = 0$$  \hspace{1cm} (6)

3.2.2. Inequality Constraints.

The variables in this model consists of control variables and state variables. The control variables include the reactive power outputs of capacitors, the active power output of the ESS and the power factor of PV. The inequality constraints of control variables are given as following:
\[
\begin{aligned}
Q_{C(i)\text{min}} & \leq Q_{C(i)} \leq Q_{C(i)\text{max}} \\
P_{\text{ESS}(j)\text{min}} & \leq P_{\text{ESS}(j)} \leq P_{\text{ESS}(j)\text{max}} \\
\lambda_{\text{PV}(k)\text{min}} & \leq \lambda_{\text{PV}(k)} \leq \lambda_{\text{PV}(k)\text{max}}
\end{aligned}
\] (7)

In (7), \(Q_{C(i)}\) is the reactive power outputs of capacitors at bus \(i\), \(P_{\text{ESS}(j)}\) is the active power output of the ESS at bus \(j\), \(\lambda_{\text{PV}(k)}\) is the power factor of PV at bus \(k\), \(Q_{C(i)\text{min}}, Q_{C(i)\text{max}}, P_{\text{ESS}(j)\text{min}}, P_{\text{ESS}(j)\text{max}}, \lambda_{\text{PV}(k)\text{min}}\) and \(\lambda_{\text{PV}(k)\text{max}}\) represent the lower bound and the upper bound of \(Q_{C(i)}, P_{\text{ESS}(j)}, \lambda_{\text{PV}(k)}\).

The state variables include the node voltage, the branch transmission power and the leaping transmission of reactive power. The inequality constraints of state variables are given as following:

\[
\begin{aligned}
V_{i\text{min}} & \leq V_i \leq V_{i\text{max}} \\
|I_j| & \leq |I_{j\text{max}}| \\
Q_S & \geq 0
\end{aligned}
\] (8)

where \(V_i\) is the voltage amplitude at node \(i\), \(I_j\) is the current of branch \(j\), \(Q_S\) is the leaping transmission of reactive power. \(V_{i\text{min}}, V_{i\text{max}}\) represent the minimum and the maximum of \(V_i\), \(I_{j\text{max}}\) represent the maximum of \(I_j\).

4. Solution

4.1. NSGA- II

NSGA-II, also known as non-dominated sorting genetic algorithm [11], has high efficiency, rapid speed of convergence and strong capability of global search. The specific steps are as follows:

4.1.1. Coding. The decimal system is used to code for each gene on the chromosome and each gene represents each control variable. When coding, make sure that the value of each control variable should be in the range set before.

4.1.2. Initialization of Population. Generate the value of each control variable randomly and calculate the value of objective functions. Then the first solution can be obtained, and so on.

4.1.3. Non-domination Rank and Crowding-distance. The non-domination rank of each solution can be calculated through the fast non-dominated sorting approach. If the value of objective functions is closer to the Pareto frontier, the non-domination rank of the individual will be higher. The crowding-distance of an individual is the average distance of two individuals on either side of this individual along each of the objectives.

4.1.4. Selection, Crossover and Mutation. The aim of selection is to select the individual with higher non-domination rank in different ranks, and the individual with bigger crowding-distance in the same non-domination rank. In order to prevent the prematurity and improve the diversity of the new population, also to avoid trapping into the local optimization, crossover and mutation will be operated to form children population after selection.

4.1.5. Elite Strategy. The elite strategy is to avoid superior individuals losing when optimizing. Firstly, combine the parent population \(P_t\) of size \(N\) and its children population \(Q_t\) of the same size \(N\) to form a new population \(R_t\). Then calculate its non-domination rank and crowding-distance and select the top \(N\) individuals as the new parent population \(P_{t+1}\). Finally, obtain the new children population \(C_{t+1}\) by the selection, crossover and mutation refer before.
4.1.6. **Pareto Frontier and Best Compromise Solution.** Suppose control vector \( X \) and objective function \( F_i(X) \) \((i=1, 2, \ldots, n)\), if a vector \( X' \) exists and it satisfies the following relation:

\[
F_i(X') \leq F_i(X)
\]

then we can draw a conclusion that \( X' \) is the Pareto optimal solution. The Pareto frontier is a set of optimal solutions like \( X' \) and the best compromise solution can be obtained through fuzzy membership degree method, see reference [12].

4.2. **PPF based on cumulant method**

In this paper, the probabilistic power flow (PPF) based on cumulant method is applied to calculate the expect value of active power loss and probability of voltage violation. The linear power flow model is adopted for power flow calculation, and Gram-Charlier expansion is used to gain the PDF and CDF of the state variables, such as node voltage, active line flow and so on. The specific mathematical procedure of the PPF based on cumulant method can see the reference [13, 14].

5. **Case Study**

This paper selects IEEE34-node distribution network as the simulation example as shown in Figure 1. In this case, the base capacity is 1MVA and the base voltage is 24.9kV (1.03p.u.). Line parameters, expected value of load refer to reference [16] and the standard deviation is the 10% of the expected value. Two groups of switchable shunt capacitors are connected to node 4, 10, 13, 23 and 28, the capacity of each group is 5kvar. A photovoltaic-energy storage system are connected to node 34, of which the maximum charge and discharge power of energy storage system is 200kW, and the power factor of PV inverter ranges from lagging 0.85 to leading 0.85. Some parameters for NSGA-II are set as following: size of population is 100, maximum of evolution generation is 50, rate of crossover is 0.9, and rate of mutation is 0.1. The voltage security range is set as [0.95, 1.05] for the probability of voltage violation calculation.

![Figure 1. IEEE34-node distribution network](image)

This paper uses HOMER software to obtain the data of solar irradiance in Guangzhou, China (23°6' N, 113°2' E). Assuming that the solar irradiance follows Beta distribution [15], then the shape parameter \( \alpha \) is 1.54, \( \beta \) is 0.96, the maximum of solar irradiance \( r_{max} \) is 1.03kW/m². The PV penetration is defined as the ratio of expected value of active power output of PV and expected value of active power of load on the distribution network, shown as following:

\[
\rho_{PV} = \frac{P_{PV}}{P_L} = \frac{A \eta r_{max} P_{LL}}{P_L}
\]

(10)

And three levels of PV penetration are set as following: low penetration (10%), medium penetration (40%) and high penetration (80%), where the corresponding PV parameters are shown as following:
Table 1. PV parameters correspond to three levels of PV penetration

| PV parameters                  | Low penetration (10%) | Medium penetration (40%) | High penetration (80%) |
|--------------------------------|-----------------------|--------------------------|------------------------|
| Area of PV cells (m²)          | 500                   | 2000                     | 4000                   |
| Conversion efficiency          | 13%                   | 13%                      | 13%                    |
| Expected value of active       | 41.26                 | 165.05                   | 330.09                 |
| power output of PV (kW)        |                       |                          |                        |

In order to verify the validity of the proposed model, this paper carries out the following two-part simulation. The first part is to verify the validity of the objective functions, and four kinds of optimization model—Model A, Model B, Model C and Model D are set in this paper, of which the corresponding objectives and solving algorithms are shown in Table 1. \( F_{10} \), \( F_1 \) represent expected value of active power loss before and after optimization respectively; \( F_{20} \), \( F_2 \) represent probability of voltage violation before and after optimization respectively; \( F_{30} \), \( F_3 \) represent voltage satisfaction before and after optimization; \( a \), \( b \) represent the weight of the two objectives respectively, of which the value is 0.4 and 0.6 respectively. Model D is the proposed model in this paper. In addition, the function of voltage satisfaction is shown in equation (11):

\[
V_{\text{satisfy}} = \sum_{i=1}^{N} \left( V_i - \frac{V_{\text{max}} + V_{\text{min}}}{2} \right)^2
\]

where \( N \) is the total number of system nodes; \( V_{\text{min}}, V_{\text{max}} \) represent the minimum and the maximum of voltage in node \( i \) respectively, which are 0.95 and 1.05 respectively.

Table 2. The objective functions and solving algorithm for four kinds of optimization models

| Model | objectives | solving algorithms |
|-------|------------|--------------------|
| A     | \[ a \frac{F_1}{F_{10}} + b \frac{F_3}{F_{30}} \] | Cataclysmic GA |
| B     | \[ a \frac{F_1}{F_{10}} + b \frac{F_2}{F_{20}} \] | Cataclysmic GA |
| C     | \[ \min \{ F_1, F_3 \} \] | NSGA-II |
| D     | \[ \min \{ F_1, F_2 \} \] | NSGA-II |

The second part is to verify the validity and adaptability of control method in the proposed model. Three control methods—Method A, Method B and Method C, are set in this part. The control variables of each method are

- Method A: \( Q_C \)
- Method B: \( Q_C \) and \( Q_{PV} \)
- Method C: \( Q_C, Q_{PV} \) and \( P_{ESS} \)

And Method C is the control method of the proposed model.

5.1. Validity of objective functions

On the high level of PV penetration, the Pareto frontier of four models optimizing by the NSGA-II algorithm are drawn in Figure 2.
According to Figure 2, we can draw conclusions as following:
(1) The optimal solution for Model A does not fall in the Pareto frontier, the same with Model B. It suggests that on the one hand the models can’t always lead to the non-inferior solutions for multi-objective optimization because of the weight given without the basis, on the other hand they can’t provide variety of solutions.
(2) As a whole, the Pareto frontier obtained from Model C is higher than that of Model D in the figure, showing that under the same probability of voltage violation, Model D gets the lower expected value of active power loss. This is mainly because the voltage satisfaction can’t quantify the actual risk of voltage violation, tending to conservative optimization results. The improvement of optimization results in Model D is shown in Table 3.

### Table 3. Improvement of optimization results in Model D

| Expected value of active power loss (kW) | Probability of voltage violation |
|----------------------------------------|----------------------------------|
|                                       | 1%      | 2%      | 3%      | 4%      | 5%      |
| Model C                                | 2.70    | 2.24    | 1.97    | 1.66    | 1.56    |
| Model D                                | 2.29    | 1.84    | 1.64    | 1.41    | 1.24    |
| Improvement                            | 15.2%   | 17.9%   | 16.8%   | 15.1%   | 20.5%   |

5.2. **Validity of objective functions**

The optimization results of best compromise solutions are compared in Figure 3 and Figure 4. According to Figure 3 and Figure 4, we can draw some conclusions as following:
(1) On the aspect of economy, the optimization effect of Method C is more effective than that of the other two methods on the low and medium levels of penetration. Compared with the other two methods, the rates of loss reduction reach 82.76% and 50.74% respectively. However, the loss of Method C increases slightly compared with that before optimization on the high level of penetration. This is mainly because the active power output of PV is high and the voltage generally exceeds the limits. In order to reduce the risk of voltage violation, ESS will absorb the excess active power to reduce the probability of voltage violation, which will increase the loss. In addition, on different levels of PV penetration, the variances of the optimization results reach up to 52.95 and 43.16 respectively in Method A and B, while it is only 0.61 in Method C, suggesting that Method C has better adaptability.
(2) On the aspect of voltage security, the probability of voltage violation is nearly zero on the low and medium levels of penetration, but on the high level of penetration, the probability of system voltage violation increases sharply. In this case, the optimization effect of Method C is more effective—the probability of voltage violation drops from 8.43% to 2.22% and the rate of improvement reaches 73.67%. In addition, Method C has better adaptability compared with the other two methods on different levels of PV penetration.
6. Conclusions

In this paper, a multi-objective optimal power flow model for ADN is proposed to minimize active power loss and probability of voltage violation. NSGA-II algorithm and PPF based on cumulant method are applied to find the Pareto optimal solutions. The results show that the model has evident advantage in reducing loss and ensuring voltage security, also it has better adaptability on different levels of PV penetration. The uncertainty of other types of DG and the impact of correlation between input variables will be considered in the future.

7. References

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Acknowledgment
This work was supported in part by 2015 Science and Technology Project of China Southern Power Grid (CSGTRC-KI53010-002-XT).