QCD prediction for the non-$D\bar{D}$ annihilation decay of $\psi(3770)$

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To clarify the marked difference between BES and CLEO measurements on the non-$D\bar{D}$ decays of the $\psi(3770)$, a $^{1}D_{1}$-dominated charmonium, we calculate the annihilation decay of $\psi(3770)$ in NRQCD. By introducing the color-octet contributions, the results are free from infrared divergences. The color-octet matrix elements are estimated by solving the evolution equations. The S-D mixing effect is found to be very small. With $m_c = 1.5 \pm 0.1$ GeV our result is $\Gamma(\psi(3770) \to \text{light hadrons}) = 467 \pm 38 \pm 33 \text{GeV}$. For $m_c = 1.4$ GeV, together with the observed hadronic transitions and E1 transitions, the non-$D\bar{D}$ decay branching ratio of $\psi(3770)$ could reach about 5%. Our results do not favor either of the results of BES and CLEO collaborations, and further experimental tests are urged.

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Heavy quarkonia decays play an important role in understanding quantum chromodynamics (QCD). These include not only the determination of the running strong coupling constant $\alpha_s$ from S-wave decays $J/\psi \to gg$ and $\Upsilon \to gg$, but also the study of factorization from the P-wave annihilation decays, where appear infrared (IR) divergences in $^{1}P_{1}$ to $gg$ and $^{3}P_{1}$ to $gg$. A traditional way to treat the IR divergences was to use the quark binding energy or the gluon momentum as cut-off to estimate these IR divergences, but this is model dependent and breaks factorization of short and long distance processes. In [4], a new factorization scheme was proposed to absorb the IR logarithms by new non-perturbative parameters, the color octet matrix elements. Based on the non-relativistic nature of heavy quarkonia, an effective theory, Non-Relativistic QCD (NRQCD) was developed [5], in which the inclusive annihilation decays can be calculated in a systematic way by double expansions in terms of $\alpha_s$ and $v$, the relative velocity of quarks in heavy quarkonium. In [6], the authors calculated QCD radiative corrections to the light hadron (LH) decays of P-wave charmonium in NRQCD, and showed explicitly the cancelation of infrared divergences at the next leading order (NLO). In [7] a more complete and precise NLO calculation for the P-wave decay perturbative coefficients in NRQCD is given (see also [8]). At NLO in $\alpha_s$, the NRQCD predictions for the relative decay rates of $\chi_{cJ} \to LH$ are consistent with more updated data (see Chapter 4 of [9]). Moreover, the relativistic corrections of S and P-wave electromagnetic quarkonium decays have been given at order $\gamma$ [10]. As for the D-wave, in [11] calculations of $^{3}D_{1} \to ggg$ decays were given but suffered from IR divergences; while in [12] only the leading order (LO) color-octet contribution to $^{3}D_{J} \to LH$ was given. So for the D-wave a complete calculation for the IR cancelation and radiative correction in NRQCD is apparently needed.

Phenomenologically, for the $^{3}D_{1}$ ($J^{PC} = 1^{-+}$) charmonium state $\psi(3770)$, there is a long-standing puzzle in its non-$D\bar{D}$ decays that the $\psi(3770)$ might have substantial decays not into $D^0 \bar{D}^0$ and $D^+ D^-$. BES earlier reported two results based on different analysis methods: $\text{Br}(\psi(3770) \to \text{non-}D\bar{D}) = (14.5 \pm 1.7 \pm 5.8)\%$ [13], and $\text{Br}(\psi(3770) \to \text{non-}D\bar{D}) = (16.4 \pm 7.3 \pm 4.2)\%$ [14]. In contrast, CLEO [16] measured the cross section $\sigma(e^+e^- \to \psi(3770) \to \text{non-}D\bar{D}) = -0.01 \pm 0.08^{+0.04}_{-0.03} \text{nb}$. Very recently, with the first direct measurement on the non-$D\bar{D}$ decay, BES gives $\sigma(e^+e^- \to \psi(3770) \to \text{non-}D\bar{D}) = (0.95 \pm 0.35 \pm 0.29) \text{nb}$ and $\text{Br}(\psi(3770) \to \text{non-}D\bar{D}) = (13.4 \pm 5.0 \pm 3.6)\%$ [17]. Evidently, the two collaborations give very different results of the non-$D\bar{D}$ decay of $\psi(3770)$. Meanwhile, a number of experiments to search for the exclusive hadronic non-$D\bar{D}$ decays of $\psi(3770)$ have been done by BES [18] and CLEO [19], but no significant signals are found.

At least two kinds of non-$D\bar{D}$ decays of $\psi(3770)$ have been observed. The hadronic transitions $\psi(3770) \to \pi^+ \pi^- J/\psi$ was first observed by BES with a branching ratio of $(0.34 \pm 0.14 \pm 0.09)\%$ [21], and was later confirmed by CLEO with a somewhat smaller branching ratio $\text{Br}(\psi(3770) \to \pi^+ \pi^- J/\psi) = (0.189 \pm 0.020 \pm 0.020)\%$ [22], and the $\pi^0 \pi^0 J/\psi$ and $J/\psi J$ modes were also seen with each having a branching ratio of about one half of that of $\pi^+ \pi^- J/\psi$ [22]. These results are within the range of theoretical predictions based on the QCD multipole expansion for hadronic transitions [24]. With the total width of $23.0 \pm 2.7$ Mev for $\psi(3770)$ [23], the width of all hadronic transitions is about $100 - 150$ KeV. Another kind of non-$D\bar{D}$ decays of $\psi(3770)$ are the E1 transitions $\psi(3770) \to \gamma + \chi_{cJ} (J=0,1,2)$, and their widths are measured by CLEO to be $172 \pm 30, 70 \pm 17, < 21$ KeV for $J=0,1,2$ respectively [24], which are in good agreement with predicted values 199, 72, 3.0 KeV in a QCD-inspired potential model calculation with relativistic corrections [25] (see also [26, 27]). The width of all E1 transitions $\psi(3770) \to \gamma + \chi_{cJ} (J=0,1,2)$ is about $250 \pm 50$ KeV. The above mentioned hadronic and E1 transitions only contribute $350 - 400$ KeV and 1.5-1.8% to the non-$D\bar{D}$ decay width and branching ratio of $\psi(3770)$.

To clarify the puzzle of $\psi(3770)$ non-$D\bar{D}$ decay, in this letter we will give a complete infrared safe NLO QCD corrections to the annihilation decay rate of the $\psi(3770)$...
in the framework of NRQCD. Since $v^2 \sim \alpha_s(m_c) \approx 0.3$ in charmonium, the relativistic corrections are also important and should be considered in the future work.

The $\psi(3770)$ can be viewed as a $1^3D_1$ dominated state with a small admixture of $2^3S_1$, and expressed as (see e.g. [25, 26])

$$|\psi(3770)\rangle = \cos \theta |1^3D_1\rangle + \sin \theta |2^3S_1\rangle,$$
$$|\psi(3686)\rangle = -\sin \theta |1^3D_1\rangle + \cos \theta |2^3S_1\rangle,$$

(1)

where $\theta$ is the S-D mixing angle and it is about $(12 \pm 2)^\circ$ by fitting the leptonic decay widths of $\psi(3770)$ and $\psi(3686)$. Then the LH decay width of $\psi(3770)$ is

$$\Gamma(\psi(3770) \to LH) = \cos^2 \theta \Gamma(1^3D_1 \to LH) + \sin^2 \theta \Gamma(2^3S_1 \to LH) + IF,$$

(2)

where $IF$ stands for the S-D interference term. The calculation of S-wave decay at order $\alpha_s^3$ and leading order in $v^2$ is trivial, and it gives

$$\Gamma(2^3S_1 \to LH) = \frac{|R_{2S}(0)|^2}{4\pi} \frac{4\alpha_s^3(\pi^2 - 9)}{81m_c^2},$$

(3)

where $R_{2S}(0)$ is the $2^3S_1$ wave function at the origin. The S-D interference term $IF$ in Eq.(2) is infrared finite at leading order in $v^2$ and $\alpha_s$, and can be obtained by combining the $1^3D_1 \to 3g$ with $2^3S_1 \to 3g$ amplitudes

$$IF = 2\sin \theta \cos \theta \left[ \frac{5(-240 + 71\pi^2)\alpha_s^3}{324m_c^4} \frac{R_{2S}(0)}{\sqrt{4\pi}} \sqrt{\frac{1}{8\pi}} R_D''(0) \right],$$

(4)

where $R_D''(0)$ is the second derivative of the $1^3D_1$ wave function at the origin.

We now proceed with the calculation of the main part, the D-wave quarkonium LH decay. In NRQCD, the inclusive annihilation decay of $1^3D_1$ at leading order in $v^2$ is factorized as

$$\Gamma(1^3D_1 \to LH) = 2\text{Im} f(3D_1^{[1]}) H_{D1} + \sum_{J=0}^{2} 2\text{Im} f(3P_J^{[8]}) H_{P_{8J}} + 2\text{Im} f(3S_1^{[8]}) H_{SS} + 2\text{Im} f(3S_1^{[1]}) H_{S1},$$

(5)

where $\text{Im} f(n)$ is the imaginary part of the $Q\bar{Q} \to Q\bar{Q}$ scattering amplitude, and can be calculated perturbatively. And the corresponding non-perturbative matrix elements are

$$H_{D1} = \langle \psi(3^1D_1) | H | \psi(3^1D_1) \rangle_{m_c},$$
$$H_{P_{8J}} = \langle \psi(3^1P_J) | H | \psi(3^1P_J) \rangle_{m_c},$$
$$H_{SS} = \langle \psi(3^1S_1) | H | \psi(3^1S_1) \rangle_{m_c},$$
$$H_{S1} = \langle \psi(3^1S_1) | H | \psi(3^1S_1) \rangle_{m_c},$$

(6)

where $H$ is $\psi(1^3D_1)$. Those four-fermion operators of S-wave and P-wave are defined in [3], and here we only give the definition of the D-wave four-fermion operator (the normalization of the color singlet four-fermion operators agree with those in [3]):

$$O_{1}^{(3D_1)} = \frac{3}{10N_c} \psi^i T^i \chi \chi^j T^j \psi,$$

(7)

where $T^i = \alpha^i S^{ij}$ and $S^{kl} = (m_c^2 \bar{D}^{ij} D^j - \frac{1}{2} \bar{D}^2 \delta^{ij})$.

We calculate the short distance coefficients at order $\alpha_s^3$, and details of our calculation will be given elsewhere. The S-wave and P-wave short-distance coefficients have been calculated in [3], and our calculated results agree with theirs. The D-wave short distance coefficients presented here are new, and they are

$$2\text{Im} f(3S_1^{[8]}) = \frac{4\alpha_s^3(\pi^2 - 9)}{81},$$

(8a)

$$2\text{Im} f(3S_1^{[1]}) = \frac{4\alpha_s^3(\pi^2 - 9)}{81},$$

(8b)

$$2\text{Im} f(3P_0^{[8]}) = \frac{5\alpha_s^3}{1296} (648\pi + \alpha_s(9096 - 464N_f + 63\pi^2 + 2520\ln 2 + 2592\beta_0 \ln \frac{\mu}{2m_c^2} + 96N_f \ln \frac{2m_c^2}{\mu}),$$

(8c)

$$2\text{Im} f(3P_1^{[8]}) = \frac{5\alpha_s^3}{648} (10176 - 64N_f - 414\pi^2 + 48N_f \ln \frac{2m_c^2}{\mu}),$$

(8d)

$$2\text{Im} f(3P_2^{[8]}) = \frac{\alpha_s^3}{648} (432\pi + \alpha_s(12561 - 464N_f - 774\pi^2 + 1008\ln 2 + 1728\beta_0 \ln \frac{\mu}{2m_c^2} + 240N_f \ln \frac{2m_c^2}{\mu}),$$

(8e)

$$2\text{Im} f(3D_1^{[1]}) = \frac{321\pi^2 - 8032 - 29184 \ln \frac{2\Lambda}{2m_c} \alpha_s^3}{5832},$$

(8f)

where $\beta_0 = \frac{11N_c - 2N_f}{6}$, $N_c = 3$, $N_f$ is the number of flavors of light quarks. $\mu$ and $\mu_\Lambda$ are renormalization
and factorization scales respectively. We consider ten processes to get the short distance coefficients in Eq.[8], including $gg$, $gqg$, $q\bar{q}$, and $q\bar{q}g$ final states. The contributions of $q\bar{q}$ and $q\bar{q}g$ processes are labeled by the powers of $N_f$.

After calculating the short distance coefficients, we come to determine the long-distance matrix elements. In the P-wave charmonium decay, at leading order in $v^2$ there are two four-fermion operators $H1$ and $H8[4]$, while in the case of D-wave, there are four independent matrix elements under the heavy-quark spin-symmetry. They are $H_{D1}$, $H_{P8}$, $H_{S8}$, $H_{S1}$, where

$$H_{PS} = \frac{5}{3m_c^2} \left[ \frac{5}{3m_c^2} \right] \left( \frac{\alpha_s(\mu_A)}{\alpha_s(\mu)} \right) H_{D1},$$

$$H_{S8} = \frac{C_F B_F}{3 \lambda_0^2} \left( \frac{5}{3m_c^2} \right)^2 \ln^2 \left( \frac{\alpha_s(\mu_A)}{\alpha_s(\mu)} \right) H_{D1},$$

$$H_{S1} = \frac{C_F B_F}{4 N_c} \left( \frac{5}{3m_c^2} \right)^2 \ln^2 \left( \frac{\alpha_s(\mu_A)}{\alpha_s(\mu)} \right) H_{D1},$$

where $C_F = \frac{4}{3}$, $B_F = \frac{1}{2}$. We choose the region of validity of the evolution equation: the lower limit $\mu_A = m_c v$ and the upper limit $\mu_A$ of order $m_c$.

With both the obtained short distance coefficients and long distance matrix elements, we predict the LH decay width of $\chi_{cJ}$. The renormalization proceeds by using the $\overline{MS}$ scheme for the coupling constant $\alpha_s$ and the on shell scheme for the charm quark mass. For convenience, we take the factorization scale $\mu_A$ to be the same as the renormalization scale $\mu$ of order $m_c$. We choose the pole mass $m_c = 1.5\text{GeV}$, $v^2 = 0.3$, $\mu_A = m_c v$, $\mu = 0.3 m_c$, $\alpha_s(2 m_c) = 0.249$, $N_f = 3$, $\Lambda_{QCD} = 390\text{MeV}$, $H_{D1} = \frac{1/2}{8\pi m_c^2} = 0.786 \times 10^{-3}\text{GeV}$. At $O(\alpha_s^3)$, the LH decay involves three subprocesses $(3P_0)s \to gg, (3P_2)s \to gg, (3S_1)s \to q\bar{q}$, and the decay width is estimated to be

$$\Gamma(3D_1 \to LH) = 205\text{MeV}.$$  \hspace{1cm} (10)

At $O(\alpha_s^3)$, there will be seven more subprocesses $(3S_1)1s \to gg, (3P_1)s \to gg, (3P_2)s \to q\bar{q}g, (3D_1)1 \to ggg$ involved, and the result turns to be

$$\Gamma(3D_1 \to LH) = 436\text{MeV}.$$  \hspace{1cm} (11)

Our result shows that in NRQCD factorization the NLO QCD correction is even larger than the LO result. The numerical values for all subprocesses are listed in Table 1. If we choose $\mu_A = m_c$, $\alpha_s(m_c) = 0.369$, the values of LO and NLO (the sum of LO contribution plus NLO correction) become 0.28 and 0.68 MeV respectively. The renormalization scale $\mu$ dependence of the decay rate is shown in Fig.(1). We see that the $\mu$-dependence at $O(\alpha_s^3)$ is rather mild when $\mu > 0.9 m_c$. For simplicity we take $\mu = 2m_c$, where the logarithm term $\ln \frac{m_c}{2m_c}$ = 0.

With the pole mass $m_c = 1.5\text{GeV}$, $\alpha_s(2 m_c) = 0.249$, $R_{1D}^n(0)^2 = 0.015\text{GeV}^7$, and $R_{2D}^n(0)^2 = 0.529\text{GeV}^7$, we find that the three terms on the right hand side of Eq.(2) contribute 417, 5.3, 44 KeV respectively to the LH decay of $\psi(3770)$, and result in

$$\Gamma(\psi(3770) \to LH) = 467\text{KeV}.$$  \hspace{1cm} (12)

Our result shows that the D-wave LH decay is dominant, and the S-D mixing only has a very small effect on the $\psi(3770)$ LH decay. One important uncertainty of our prediction is associated with the long-distance matrix elements, especially the color-octet matrix elements. Using
the same evolution equation method in $\chi_{cJ}$ decays, we find the ratio of color octet to color singlet $P$ wave decay matrix elements agree with the lattice calculations to within about 20% and with the phenomenological values to within about 30%. This might indicate, though not compellingly, that the uncertainty related to the matrix elements calculated using the evolution equation in the D-wave decays are also about (20-30)% or (with more confidence) less than 50%. Other uncertainties such as the relativistic corrections and higher order QCD radiative corrections are beyond the scope of the present study. On the other hand, however, we find the decay rate to be sensitive to the value of the charm quark mass. If we choose the pole mass $m_c = 1.5 \pm 0.1$ GeV, $\alpha_s(\mu) = \alpha_s(2m_c)$, and fix other parameters as before, then our prediction becomes

$$\Gamma(\psi(3770) \to LH) = 467^{+187}_{-338}\text{KeV}(\pm 50\%)$$

(13)

$$\text{Br}(\psi(3770) \to LH) = (2.0^{+0.80}_{-1.56})(\pm 50\%)$$

(14)

For a small mass $m_c = 1.4$ GeV, the LH decay width and branching ratio of $\psi(3770)$ can reach 805 KeV($\pm 50\%)$ and 3.5%($\pm 50\%)$ respectively, and this could be viewed as the maximum value for the LH decay of $\psi(3770)$ in our estimation based on the calculation at leading order in $v^2$ and next-to leading order in $\alpha_s$ in NRQCD.

Together with the partial decay width of 350–400 KeV observed for hadronic transitions and E1 transitions of the $\psi(3770)$, the predicted annihilation (LH) decay width in Eq.(13) will make the total non-$D\bar{D}$ decay width of $\psi(3770)$ to be about 820–870 KeV for $m_c = 1.5$ GeV, and 1.15–1.20 MeV for $m_c = 1.4$ GeV. The latter may be viewed as the maximum value obtained in our approach for the total non-$D\bar{D}$ decay width, corresponding to a branching ratio of about 5% of the $\psi(3770)$ decay.

In summary, we have given a rigorous theoretical prediction for the LH decay of $\psi(3770)$, based on NRQCD factorization at NLO in $\alpha_s$ and LO in $v^2$. By introducing the color-octet contributions, the results are free from infrared divergences. We find that for the $\psi(3770)$ the D-wave contribution is dominant, and the effect of S-D mixing is very small. Numerically, our results do not favor either of the two experimental results measured by BES and CLEO collaborations. We hope our theoretical result can serve as a clue to clarify the long-standing puzzle of the $\psi(3770)$ non-$D\bar{D}$ decay. We urge doing more precise measurements on both inclusive and exclusive non-$D\bar{D}$ decays of $\psi(3770)$ in the future. If their total branching ratio can be as large as 10%, it will be a real challenge to our current understanding of QCD, and new decay mechanisms have to be considered.

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