Using Monte Carlo to optimize variable cuts

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Abstract
A Monte Carlo method to optimize cuts on variables is presented and evaluated. The method gives a much higher signal to noise ratio than does a manual choice of cuts.

There are two important methods for refining a signal over background ratio: likelihood analysis and cut-based analysis. Likelihood analysis has the advantage of not discarding any potential signal. However, it is not as straightforward to evaluate its statistical significance compared to the cut-based analysis. Cut-based analysis on the other hand, cuts away parts of the signal in order to reduce the background evenmore.

Traditionally, variable-cuts have been sought with the help of good sense and some experimentation.

In this letter I address an automatical method for searching for optimal cuts. I have used only one simulated set of data, fairly large, and many different backgrounds. The simulated data are heavy leptons with masses $100 - 200$ GeV done at center-of-mass energies of $183 - 209$ GeV to correspond to the OPAL experiment at LEP.

The method is simple: Initially, determine which of the variables are most relevant and what their ranges are. If possible, find some minimum cuts that will leave the signal intact, while still reducing the background. This can significantly reduce the time spent on each iteration below.

The cut optimization then has the following general algorithm:

(1) Choose a random variable and change the cut randomly with a value between 0 and $T \times \text{max}$, where $T$ is initialized as $T_i = 100\%$ and $\text{max}$ as the maximum value of the variable.
(2) If this change leaves us with a higher $S/\sqrt{B}$-value, keep it, otherwise discard it.

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Decrease $T$ and restart from the beginning

A problem with this method is that it might get stuck in a local minimum somewhere. This can be remedied by storing the final cuts and the $S/\sqrt{B}$-value and then reinitializing the process, iterating until a satisfying $S/\sqrt{B}$-value is obtained. The method can be parametrized by $\Delta T$, the change in $T$ per iteration, $T_i$, the initial value of $T$ and $N_{it}$, the number of reinitializations.

Our test case is described in general in [1] and in particular in [2]. A short resumé follows here. The signal we are looking for is $e^+e^- \rightarrow \bar{\nu}N \rightarrow \nu lqq$ and the main variables are the lepton energy $E_l$, the missing energy, $E_\nu$, the invariant mass of $l$ and $\nu$, the invariant mass of the $N (= q, q, l)$ and the lepton type ($l = e, \mu$ or $\tau$). Both the signal events and the background events were subject to the full OPAL detector simulation [3] as well as some basic cuts to ensure a good quality [4]. The minimum cuts mentioned above were set to $E_l, E_\nu \geq 5$ GeV. The Monte Carlo generator EXOTIC [5] was used to generate the $e^+e^- \rightarrow \bar{\nu}N$ signal. The following masses were simulated $M_N = 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200$ GeV and for each mass the energies $E = 183, 189, 192, 196, 200, 202, 204, 205, 206, 207, 208$ GeV for all $M_N < E$. The total number of signal events surviving the initial cuts were about 350 for each pair of $(E, M_N)$. A variety of MC generators was used to study the multihadronic background from SM, see [2] and references therein. The relevant backgrounds are $qq\gamma$ (KK2f+PYTHIA 6.125), $llqq$, $eeqq$, $qqqq$, $ee\tau\tau$ (grrf 2.1) and $\gamma\gamma qq$ (HERWIG).

The traditional cut based analysis left us with some $\sim 5 - 15$ signal events and $\sim 5 - 10$ background events, i.e., $S/\sqrt{B} \sim 5$. On the other hand, the MC based method often managed to completely remove the background, while still preserving $\sim 50$ signal events. There are several ways to improve the value of $S/\sqrt{B}$ but they all come at the cost of longer execution time. The different improvements were:

- Use high $T_i$ value
- Use smaller $\Delta T$ for each iteration
- Increase the number of iterations, $N$
- Change more variables than one, before recomputing $S/\sqrt{B}$

For most of these improvements, the general behaviour was that

$$\frac{S}{\sqrt{B}} \sim 5.1 \times t^{0.37}$$

where $t$ is the time in seconds. The only exception was in increasing the number of variables, which was not profitable. The $S/\sqrt{B}$ is illustrated in Fig. 1. The values have been averaged over ten different optimization runs. For the dot-
Fig. 1. Comparison of changes in some parameters. The dot-dashed curve represents changing $\Delta T$, the dashed and solid curves represent changing $N_{it}$ with two different $T_i = 20, 100$ GeV and the dotted line is the approximate result of the traditional cut based analysis.

dashed curve, the step $\Delta T$ is modified from $2^4 - 2^{-10}$ and divided by two each time. The values of $T_i = 20\%$ and $N_{it} = 2$. We notice that the curve is levelling out asymptotically. The solid and the dashed lines both have $\Delta T = 10$ and the number of iterations goes from $N_{it} = 2$ to 512, multiplied by two each time. Furthermore, $T_i = 20\%$ for the dashed line and $T_i = 100\%$ for the solid one.

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References

[1] The OPAL Collaboration, G. Abbiendi, et al., Search for unstable heavy and excited leptons at LEP2, European Physical Journal C 14 (2000) 73–84.

[2] E. Elfgren, Heavy and excited leptons in the opal detector?, Master’s thesis, Université de Montréal (2002).

[3] J. Allison, et al., The detector simulation program for the OPAL experiment at LEP, Nuclear Instruments and Methods in Physics Research A 317 (1992) 47–74.

[4] G. Alexander, et al., Measurement of the $Z^0$ line shape parameters and the electroweak couplings of charged leptons, Z. Phys. C52 (1991) 175–208.

[5] R. Tafirout, G. Azuelos, EXOTIC - A heavy fermion and excited fermion Monte Carlo generator for $e^+e^-$ physics, Computer Physics Communications 126 (2000) 244–260.