Artificial scaling laws of the dynamical magnetic susceptibility in heavy-fermion systems

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Abstract

We report here how artificial, thus erroneous, scaling laws of the dynamical magnetic susceptibility can be obtained when data are not treated carefully. We consider the example of the heavy-fermion system Ce$_{0.925}$La$_{0.075}$Ru$_2$Si$_2$ and we explain how different kinds of artificial scaling laws in $E/T^\beta$ can be plotted in a low temperature regime where the dynamical susceptibility is nearly temperature independent.

Key words:
Heavy-Fermion System, Quantum Phase Transition, Scaling Law, Non Fermi Liquid, Magnetic Fluctuations, Inelastic Neutron Scattering

1. Introduction

In heavy-fermion systems, a magnetic instability can be reached by tuning a parameter $\delta$ such as pressure, magnetic field or doping. This instability is obtained at a critical value $\delta_c$ of the tuning parameter and it separates a paramagnetic ground state for $\delta < \delta_c$ from a magnetically ordered ground state for $\delta > \delta_c$: a quantum phase transition (defined at $T = 0$) between those two states is thus induced at $\delta_c$. While far from $\delta_c$ the paramagnetic state is well described by Landau’s Fermi liquid theory with a strongly-renormalized effective mass $m^* \gg m_0$ ($m_0$ is the free electron mass), a non-Fermi liquid behavior is generally reported at the magnetic instability. In this regime, the specific heat, the susceptibility and the resistivity follow power or logarithmic temperature laws, which are different from the laws expected for a Fermi liquid [1]. This non-Fermi liquid behavior is probably related both to the Q-dependence of the excitation spectra and to the effects of temperature [2,3,4]. However, this regime is not yet explained for heavy fermions, its understanding being now one of the major stakes in the physics of strongly correlated electron systems. Recently, several inelastic neutron scattering studies of heavy-fermion systems at their quantum phase transition were performed with the aim to obtain scaling laws of the dynamical magnetic susceptibility $\chi(E,T)$, where $E$ is the energy transfer and $T$ the temperature [5,6,7,8,9,10]. In those works, the imaginary part $\chi''(E,T)$ of the dynamical susceptibility was found to follow scaling laws of the form:

$$T^\alpha \chi''(E,T) = f(E/T)$$

down to $T = 0$, with $\alpha < 1$. These laws were interpreted as a consequence of the divergence of the magnetic excitations when $T \to 0$ and $\delta \to \delta_c$, which is responsible for the non-Fermi liquid behavior of these critical systems. For example, the study carried out by Schröder et al. on CeCu$_{5.9}$Au$_{0.1}$ led to the conclusions that a general scaling law of the form (1) is obtained down to $T = 0$ with a unique exponent $\alpha = 0.75$ for each wavevector of the reciprocal space [6]: this work motivated several new theoretical developments, such as models based on a local criticality [11,12].

In this letter, we show how it is possible to obtain artificial scaling laws of the dynamical magnetic susceptibility when the data are not carefully analyzed. We consider already published data corresponding to the antiferromagnetic fluctuations of the heavy-fermion sys-
system \(\text{Ce}_{1-x}\text{La}_x\text{Ru}_2\text{Si}_2\) at its magnetic instability \(x_c = 7.5\%\) [13,14,15]. We explain how erroneous scaling laws can be obtained in a low temperature regime where the dynamical magnetic susceptibility depends weakly on \(T\). In such a nearly \(T\)-independent regime, \(E/T\) scaling laws are irrelevant.

2. Antiferromagnetic fluctuations in \(\text{Ce}_{0.925}\text{La}_{0.075}\text{Ru}_2\text{Si}_2\)

We recall here results concerning the antiferromagnetic fluctuations of the heavy-fermion compound \(\text{Ce}_{0.925}\text{La}_{0.075}\text{Ru}_2\text{Si}_2\) [13,14,15]. These excitations are measured at a momentum transfer \(Q_1 = (0.69,1,0)\), which corresponds to a correlated signal with the wavevector \(k_1 = (0.31,0,0)\). Knowing that the scattered intensity is proportional to the scattering function \(S(Q,E,T)\), the imaginary part of the dynamical susceptibility \(\chi''(Q,E,T)\) is deduced using the fluctuation-dissipation theorem:

\[
S(Q,E,T) = \frac{1}{\pi} \frac{1}{1 - e^{-E/k_BT}} \chi''(Q,E,T). 
\] (2)

Then, the susceptibility is fitted with a single quasielastic Lorentzian shape:

\[
\chi''(Q_1,E,T) = \chi'(Q_1,T) \frac{E/\Gamma(Q_1,T)}{1 + (E/\Gamma(Q_1,T))^2},
\] (3)

where \(\chi'(Q_1,T)\) and \(\Gamma(Q_1,T)\) are respectively the static susceptibility and the relaxation rate of the antiferromagnetic fluctuations. The variations with \(T\) of those two parameters are plotted in Figure 1. We obtain that the dynamical susceptibility depends weakly on \(T\) for \(T < T_1\), with \(T_1 = 2.5\) K. In this regime, the static susceptibility and the relaxation rate are given approximately by:

\[
\chi'(Q_1,T) = \frac{C_1}{T} \quad \text{and} \quad \Gamma(Q_1,T) = k_BT_1.
\] (4)

For \(T > T_1\), the susceptibility becomes controlled by \(T\) and we have:

\[
\chi'(Q_1,T) = \frac{C_1}{T} \quad \text{and} \quad \Gamma(Q_1,T) = a_1T^{0.8}.
\] (5)

This is only in this high temperature regime that antiferromagnetic fluctuations follow a scaling law: using (3) and (5), we obtain immediately:

\[
T\chi''(Q_1,E,T) = C_1f(E/(a_1T^{0.8})),
\] (6)

where \(f(x) = x/(1 + x^2)\) and \(x = E/(a_1T^{0.8})\). In Figure 2, this scaling behavior is plotted for temperatures \(T > T_1\) and all the data collapse effectively on a single curve. This law does not enter the framework of usual quantum phase transition theories, where \(\beta\) cannot be smaller than 1 [3,4,16]. The reason is that those theories do not consider the temperature dependence of the Kondo local magnetic fluctuations. In the studies reported in References [13,14], we also showed that it is necessary to consider the spectra at different wavevectors corresponding respectively to antiferromagnetic fluctuations and to local magnetic fluctuations.

3. Artificial scaling laws

Several studies of heavy-fermion systems at their quantum phase transition report different kinds of scal-
ing laws of the general form [5,6,7,8,9,10,13,14]:

\[ T^\alpha \chi''(E, T) = f(E/T^\beta). \] (7)

In our study of Ce\textsubscript{0.925}La\textsubscript{0.075}Ru\textsubscript{2}Si\textsubscript{2}, each spectrum is analyzed separately at a given momentum transfer \( Q \) and at a given temperature \( T \). Then, for the antiferromagnetic fluctuations measured at the wavevector \( Q_1 \), we deduced the exponents \( \alpha = 1 \) and \( \beta = 0.8 \) from the temperature dependence of the static susceptibility and of the relaxation rate. In the literature, the scaling laws are often obtained as follows: \( \beta \) is fixed to 1 and \( \alpha \) is chosen for the best collapse of the data on a single curve when \( T^\alpha \chi''(E, T) \) is plotted against \( E/T^\beta \). We show here how such a method can lead to erroneous results and to contradictions with the real physics of the system. In fact, for each value of \( \beta \) it is possible to obtain an optimal value of \( \alpha \) so that the data collapse in a single curve, this artificial scaling law being sometimes restricted to some ranges of temperatures and/or energies.

Let us consider the example of the antiferromagnetic fluctuations of Ce\textsubscript{0.925}La\textsubscript{0.075}Ru\textsubscript{2}Si\textsubscript{2}. Figures 3 and 4 correspond to the plot of \( T^\alpha \chi''(Q_1, E, T) \) against \( E/T^\beta \), where \( \beta \) is fixed and \( \alpha \) is chosen for the best collapse of the data on a single curve. We see in Figure 3 that when \( \beta = 1 \), the best collapse of the data is obtained for \( \alpha = 1 \) and is followed for temperatures from 80 K down to 40 mK. In Figure 4, we fix \( \beta = 1.5 \) and the best collapse of the data is found for \( \alpha = 1.5 \), corresponding to temperatures from 16 K to 40 mK. In this second graph, a collapse of the data on a single curve is not obtained for \( T \geq 25 \) K.

\[
\chi''(Q_1, E, T) = \chi'(Q_1, 0) \frac{E/\Gamma(Q_1, 0)}{1 + (E/\Gamma(Q_1, 0))^2}. \] (8)

Knowing that \( \Gamma(Q_1, 0) \approx k_B T_1 \approx 0.3 \) meV and that our data correspond to energy transfers \( 0.4 < E < 9.5 \) meV, we can make the approximation \( E \gg \Gamma(Q_1, 0) \), so that:

\[
\chi''(Q_1, E, T) = \chi'(Q_1, 0) \left( \frac{E}{\Gamma(Q_1, 0)} \right)^{-1} \sim (E)^{-1}. \] (9)

The dynamical susceptibility can then be written as:

\[
T^\alpha \chi''(Q_1, E, T) \sim \left( \frac{E}{T^\beta} \right)^{-1}, \] (10)

which means that an optimal collapse of the data measured at the temperatures \( T < T_1 \) will be obtained for \( \alpha = \beta \), when \( T^\alpha \chi''(Q_1, E, T) \) is plotted against \( E/T^\beta \). In a log-log plot of \( \chi''(Q_1, E, T) \) as a function of \( E \), the almost \( T \)-independent spectrum which is obtained for \( T < T_1 \) corresponds to a segment of a straight line of slope -1. When \( T^\alpha \chi''(Q_1, E, T) \) is plotted against \( E/T^\alpha \), it leads to a shift of this segment on the same straight line. This simple geometric construction is the
only origin of the artificial scaling.

Figures 3 and 4 correspond effectively to two cases of artificial scaling laws of the form (10). Moreover, those erroneous laws are not only obtained for the low temperature data, but also for parts of the high temperature data. Indeed, Figure 3 is characterized for $T > T_1$ by a collapse of the data in a single curve, since the values $\alpha = 1$ and $\beta = 1$ are very close to the correct values $\alpha = 1$ and $\beta = 0.8$. In Figure 4, the exponents $\alpha = 1.5$ and $\beta = 1.5$ are quite different from the correct values $\alpha = 1$ and $\beta = 0.8$. As a consequence, the artificial scaling law is obtained only up to 16 K and for $T \geq 25$ K, the data do not collapse any more in a single curve. We recall that the only $E/T$ scaling behavior of the antiferromagnetic fluctuations of Ce$_{0.925}$La$_{0.075}$Ru$_2$Si$_2$ corresponds to the temperature regime $T > T_1$, with the exponents $\alpha = 1$ and $\beta = 0.8$, as explained in Section 2.

Other kinds of artificial scaling laws can also be obtained for non Lorentzian spectra: at sufficiently small temperatures, when the dynamical magnetic susceptibility is nearly $T$-independent such as, instead of (9):

$$\chi''(Q, E, T) \sim (E)^{-\gamma},$$

we obtain immediately:

$$T^\alpha \chi''(Q, E, T) \sim \left(\frac{E}{T^\alpha \gamma}\right)^{-\gamma}.$$  \hspace{1cm} \text{(12)}

An artificial scaling law can thus be plotted with $\beta = \alpha / \gamma$. When $\beta$ is fixed, a collapse of the low temperature data in a single curve is consequently obtained for an exponent $\alpha = \beta \gamma$. This signifies that, when $\beta$ is fixed to 1, an artificial scaling law of the form (1) is obtained with the exponent $\alpha = \gamma$.

Finally, depending on the temperature and energy ranges which are considered, but also on the shape of the spectra, several kinds of erroneous scaling laws can be plotted. Equations (10) and (12) are just two examples of such artificial scaling laws. We illustrate the former case using the graphs of the Figures 3 and 4, where $\alpha = 1$ and $\alpha = 3/2$, respectively. Those two graphs could be taken as proofs of diverging fluctuations for $T \to 0$. Moreover, the plot of Figure 4 could be abusively interpreted as a verification of quantum phase transition theories for an itinerant system [3,4,16]. Actually, for the spatial dimension $d = 3$ and the critical exponent $z = 2$ corresponding to Ce$_{1-x}$La$_x$Ru$_2$Si$_2$, such models predict i) that the anomalous exponents of the scaling law (7) are $\alpha = 3/2$ and $\beta = 3/2$, ii) that the fluctuations should diverge for $T \to 0$ and iii) that this law would not been verified for temperatures bigger than the Kondo temperature, which is here equal to 18 K [13,14]. However, we know that this is not verified in Ce$_{0.925}$La$_{0.075}$Ru$_2$Si$_2$ since the fluctuations saturate below $T_1 = 2.5$ K and thus no scaling can be obtained in the low temperature regime of the magnetic fluctuations. This illustrates perfectly the dangers of a unique graphical determination of scaling laws of the form $T^\alpha \chi''(E, T) = g(E/T^\beta)$.

4. Conclusion

In this letter, we used the specific example of the antiferromagnetic fluctuations of Ce$_{0.925}$La$_{0.075}$Ru$_2$Si$_2$ to show how erroneous $E/T^\beta$ scaling laws of the dynamical magnetic susceptibility can be plotted. Such artificial scaling laws are obtained when the exponents $\alpha$ and $\beta$ are only determined graphically, as done in Section 3, and come just from a simple geometric construction. This can lead to misunderstand the physics of the system; the erroneous scaling laws shown here are plotted without any divergence of the fluctuations, while such $E/T^\beta$ scaling laws obtained down to $T = 0$ should be associated to diverging fluctuations. We stress thus that a scaling behavior can only be established after a precise study of the temperature dependence of the dynamical magnetic susceptibility and of the parameters which characterize it, as shown in Section 2. A great care has also to be given to the $Q$-dependence of those spectra and to the corresponding scaling laws [13,14].

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