Development characteristics of inner rail corrugation in small radius curve of Metro

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Abstract—This paper mainly studies the influence of different curve positions of metro's small radius curve and the tidality of metro passenger flow on the development of inner rail corrugation. Based on the material friction and wear theory, the rail wear is simulated and analyzed to analyze the time-frequency characteristics of rail corrugated wear. And the development characteristics of rail corrugation are revealed by the analysis of the natural frequency response of the rail and the vibration mode analysis. The results show that the rail corrugation disease first appears on the circular curve, and the occurrence of corrugation is related to the lateral first-order and second-order bending vibrations of the rail. In addition, the tidal nature of subway passenger flow leads to the extension of the rail surface corrugation at its initial position, producing a series of sub-waves with different wave depths.

1. INTRODUCTION
Rail corrugation is a kind of rail head surface defect, which is a kind of irregular phenomenon that appears on the surface of the rail. Its shape shows a wave-like shape and has a strong periodicity\cite{1}. Rail corrugation on metro lines can cause severe vibration of vehicles, bogies and wheelsets, and accelerate the damage of locomotive, vehicle and track components. In addition, rail corrugation leads to severe vibration of the train, which reduces the comfort of passengers and causes annoying howling\cite{2}. Therefore, it is of great significance to study the development characteristics of rail corrugation on the small radius curve for improving the safety and comfort of metro operation.

Researchers in the field of rail transit in various countries have carried out a lot of research on the generation and development mechanism of rail corrugation on metro lines. Grassie et al. \cite{3,4} summarized and improved the classification method, characteristics, causes and corresponding treatment measures of rail corrugation. Correa and Oyarzabal \cite{5} analyzed the growth trend of corrugation for four rail structures of European high-speed railways based on a linear corrugation prediction model. The results show that the vibration characteristics of the wheelset are crucial to the development of corrugation. Suda\cite{6} shows that the growth of rail corrugation depends on the natural frequency of the system. Reference \cite{7} carried out the reproducibility test of rail corrugation based on the experimental platform, and the results show that the generation of rail corrugation is the result of the wheel set self-excited vibration amplitude being greater than the longitudinal axis length of the contact ellipse. Refs \cite{8}\cite{9}\cite{10} based on the theory of friction self-excited vibration, put forward the theory that...
friction self-excited vibration occurs when the wheelset runs on the rail, which leads to the formation of rail corrugation. [11] believes that a certain vibration mode of the track structure leads to the formation of corrugation at the corresponding wavelength on the rail surface. Jin et al. [12] reproduced the rail short wave wear through the test, and found that the formation of the corrugation is related to the resonance characteristics of the track structure. Wang Anbin et al. [13] put forward the theory of ‘wheel rail flexibility difference’, and considered that the wheel rail flexibility difference, rail discontinuous support and rail support stiffness change are the main reasons for rail wave wear.

2. THEORETICAL MODEL OF RAIL WEAR

The track structure can be regarded as a periodic discrete support structure. Considering the vibration frequency response characteristics of the periodic structure, when a vehicle passes on the track, it inevitably causes periodic changes in the contact force between the wheels and rails. This periodic wheel-rail contact force and wheel-rail contact area cause initial periodic wear on the rail surface. In addition, the small radius curve track structure as a special line form, different curve positions and changes in subway passenger flow affect the attenuation of the rail vibration, which in turn affects the development of rail surface wear. To explore the development characteristics of rail grinding on small radius curves, this section studies the calculation model of rail wear on small radius curves based on vehicle-track coupling dynamics and material friction and wear theory.

According to the Archard material wear model, the rail wear mass \( \Delta m_i \) caused by the wheel \( i \) passing through the track position \( x \) can be expressed as [14]:

\[
\Delta m_i(x) = k W_i(x) \tag{1}
\]

where \( k \) is the material wear coefficient, \( W_i(x) \) is the friction work done by wheel \( i \) at position \( x \) of the track.

Assuming that the friction power \( P_i(x) \) between the wheel set \( i \) and the rail remains constant in a short time step \( \Delta t \), then

\[
W_i(x) = P_i(x) \cdot \Delta t \tag{2}
\]

Considering the effect of spin on wear, then

\[
P_i(x) = T_x \cdot C_x + T_y \cdot C_y + M_z \cdot \xi_z \tag{3}
\]

where \( T_x, T_y \) are the longitudinal and transverse creep forces respectively, \( M_z \) is the moments around the \( z \)-axis between wheels and rails; \( C_x, C_y, \xi_z \) are the longitudinal, transverse and spin creepage, respectively.

Substituting equations (2)(3) into equation (1) and setting a sufficiently short calculation step, the wear depth \( d_i(x) \) of the rail surface at the corresponding position at that time can be calculated, ie

\[
d_i(x) = \frac{\Delta m_i(x)}{A \rho} = \frac{k(T_x \cdot C_x + T_y \cdot C_y + M_z \cdot \xi_z) \cdot \Delta t}{A \rho} \tag{4}
\]

where \( A \) is the contact area between the wheel \( i \) and the rail at the \( x \) position; \( \rho \) is the density of the rail material.

Therefore, the depth \( D(x) \) of wear on the rail surface caused by a single carriage passing through the rail position \( x \) can be expressed as:

\[
D(x) = \sum_{i=1}^{4} d_i(x) \tag{5}
\]

The rail wear depth calculated by formula (5) is superimposed with the original track irregularity, and added to the rail surface as a new rail irregularity to calculate the rail wear again, then the next rail wear under the influence of the previous wear can be obtained. By repeating the calculation process, the cumulative wear on the rail surface caused by the second train passing by can be obtained.
The above calculation process can be realized in UM simulation environment based on the vehicle track coupling dynamic simulation model. The model is detailed below.

3. SIMULATION MODEL

3.1. Wheel-rail finite element model
In order to calculate the natural vibration frequency response characteristics of small radius curved track, the finite element model of wheel rail system is established in this section. Combined with frequency response analysis and modal analysis, the influence of rail natural vibration on rail corrugation development is studied.

Under the premise of ensuring the analysis accuracy and avoiding excessive calculation, the wheelset and rail are modeled according to the actual size of 1:1 through the three-dimensional software SolidWorks, and then imported into the ABAQUS software for wheel-rail assembly. This model uses C3D8I hexahedral elements for meshing, with a total of 578,752 nodes and 498,648 elements. See Table 1 for material modeling parameters. In addition, a grounding spring is added directly between the fastener and the rail contact surface, and the actual supporting structure under the rail is simulated by changing the stiffness and damping of the spring. The wheel rail contact model with initial support stiffness and damping is shown in Fig. 1.

![Figure 1. Finite element model of wheel-rail contact](image)

3.2. Vehicle-track coupling dynamics model
In this section, the vehicle-track coupling dynamic model is built to study the development of the rail surface corrugation after $n$ trains passing by.

The vehicle model takes Guangzhou Metro A-type car structure as the modeling prototype. In order to accurately describe the characteristics of curved track, flexible track structure is used in the model. The Timoshenko beam with shear deformation is adopted as the rail model, which can reflect the influence of high frequency vibration and is suitable for the study of rail corrugation. The fastener part is simulated by bushing force element. The three-dimensional solid element model of the structure under the track (track slab and foundation) is established by finite element software, and then imported into UM pretreatment as a subsystem of dynamic model. The connection between Track Bed Slab and rail is simulated by force element, and the stiffness and damping in transverse / longitudinal / vertical directions are considered. The Kik-Piotrowski contact model is used for wheel-rail contact normal simulation calculation [15], which has fast calculation speed and high reliability. For the calculation of wheel-rail tangential contact, the calculation of wheel-rail tangential stress and creep force is derived based on FASTSIM algorithm [16].

4. DEVELOPMENT CHARACTERISTICS OF RAIL CORRUGATION

4.1. Rail corrugation at different positions on the curve
Considering the requirements of metro design code, the transition curve section with corresponding length should be set for curve track with different radius. In this section, considering the line characteristics of straight line section, transition curve section and circular curve part on small radius curve, three measuring points (straight and gentle point (T), slow circle point (s) and circle midpoint (c)) are introduced to analyze the development characteristics of rail corrugation. The setting of line conditions is shown in Table 1.
| TABLE I. TRACK PARAMETERS |
|---------------------------|
| Radius/m | Superelevation/m | Transition curve /m | Circular curve /m | Speed/(km/h) |
| 250      | 0.12             | 45                   | 200               | 50           |
|          |                  |                      |                   |              |
| Vertical stiffness of structure under rail (MN/m) | Lateral stiffness of structure under rail (MN/m) | Vertical damping of structure under rail / (kNs/m) | Lateral damping of structure under rail / (kNs/m) |
| 75       | 13               | 20.8                 | 8.7               | -            |

Fig. 2 and Fig. 3 are the vertical and transverse frequency response characteristic curves of rail at three different positions on the above small radius curve calculated based on the finite element model, representing the vibration response of rail under unit excitation.

![Figure 2. Vertical frequency response of rail](image1)

![Figure 3. Lateral frequency response of rail](image2)

It can be seen that the frequency response amplitude of the rail of the transition curve on the curve is significantly higher than that of the circular curve, which means that under the same excitation, the rail of the transition curve is more prone to different modes of vibration. The transverse vibration frequency of rail is lower than the vertical vibration frequency of rail, and the transverse vibration of rail is mostly concentrated in 60Hz ~ 110Hz. The vertical vibration response frequency of rail is slightly higher, with 160Hz ~ 300Hz as the main frequency band.

The vehicle-track coupling dynamic model is used to obtain the wheel-rail contact data after the vehicle passes, and the inner rail wear after \( n \) train passes is calculated according to the theoretical model of the small radius curve rail wear calculation, as shown in Fig. 4. It can be clearly seen from the figure that the linear rail wear is very small after 10,000 operating vehicles pass, and the wear depth is less than 0.01mm, while the wear of the circular curve part is 1mm ~ 1.6mm, and the maximum wear depth is 1.8mm. The wear of the circular curve is much higher than the wear of the rails at other positions of the curve.
The $1/3$ frequency doubling spectrum analysis of rail wear on the circular curve in the figure is carried out in order to explore the frequency characteristics of rail periodic wear on the circular curve, as shown in Fig. 5.

Ignoring the low frequency part of wear caused by the low frequency characteristics of track structure in the figure, it can be seen that the wear of inner rail in circular curve presents two characteristic frequencies, namely $53\text{Hz}$ and $100\text{Hz}$. The low-frequency wear of $53\text{Hz}$ is the main vibration frequency of corrugated wear, and the secondary frequency of $100\text{Hz}$ is the secondary vibration frequency. Compared with the rail vibration frequency response curves shown in Fig. 2 and Fig. 3, it can be seen that the characteristic frequency of rail surface corrugation is close to the transverse natural vibration frequency of $60\text{ Hz}$ and $100\text{ Hz}$ of curved rail, and the corresponding rail vibration mode is shown in Fig. 6. Vibration at $60\text{Hz}$ and $110\text{Hz}$ respectively correspond to the first-order and second-order transverse bending vibration of small radius curve rail, that is to say, with the increase of passing trains, the corrugation formed on the rail surface is related to the first and second-order transverse bending vibration of the rail.

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![Figure 4: Wear of inner rail in small radius curve](image.png)

Figure 4. Wear of inner rail in small radius curve

![Figure 5: 1/3 octave frequency spectrum of internal rail corrugation in circular curve](image.png)

Figure 5. $1/3$ octave frequency spectrum of internal rail corrugation in circular curve

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![Figure 6: Vibration mode of circular curve rail at 60Hz](image.png)

(a) Vibration mode of circular curve rail at $60\text{Hz}$

![Figure 6: Vibration mode of circular curve rail at 100Hz](image.png)

(b) Vibration mode of circular curve rail at $100\text{Hz}$

Figure 6. Vibration modal analysis of circular curve rail
4.2. Rail corrugation under different loads.

In this section, the axle load in the simulation model is adjusted based on three different passenger flow conditions: no-load, full load and overload. Different axle loads are used to represent different passenger flow states of metro vehicles, and time-frequency analysis is conducted on the wear of inner rail of Metro curve under different axle loads.

Fig. 7 shows the rail wear on the inner curve of the circular curve. The wear curve of the rail surface in Fig. 7 (a) shows a clear periodicity, and the long-wave wear caused by the low-frequency vibration characteristics of the rail structure is particularly prominent. At the same time, it is accompanied by some short-wave wear, and the periodicity of these short-wave wear is also more obvious, as shown in Fig. 7 (b).

The short wave periodic wear on the rail surface does not disappear with the change of vehicle axle load, but extends. For example, after the no-load metro train passes, the corrugation on the rail surface appears at 194.1m, while when the overloaded metro train passes, the corrugation on the rail surface appears at 193.97m. This shows that the extension of the corrugation on the rail surface is related to the tidal variation of subway passenger flow.

The inner rail wear under the above different load conditions is subjected to 1/3 frequency doubling to obtain the spectrum curve shown in Fig. 8. It can be seen that the spectrum characteristics of the rail wear under different load conditions are basically the same. Ignoring the low-frequency characteristics caused by the rail structure, the rail wear is dominated by the vibration frequencies of 60 Hz and 100 Hz. This is consistent with the spectral characteristics of the circular rail wear shown in Fig. 6, that is, as the passing train increases, the corrugation formed on the rail surface is related to the lateral first-order and second-order bending vibrations of the rail.
Figure 8. 1/3 octave frequency spectrum of rail wear under different axle load

To sum up, the change of subway passenger flow leads to the change of initial corrugation position and initial wear depth, which means that the rail corrugation extends on the rail surface with the tidal change of subway passenger flow, forming a series of wave abrasion diseases with different wave depths.

5. CONCLUSION
In this paper, the simulation of rail wear in small radius curve is carried out on the basis of the rail wear theory. The results show that the rail corrugation first appears on the circular curve. The first-order and second-order bending vibration of curved track structure at 60Hz and 100Hz eventually leads to rail corrugated wear. While the tidal characteristics of the subway passenger flow affects the extension of rail corrugation, and the main vibration frequency of that does not change with the change of the train load, which reflects the fixed frequency of rail corrugated wear.

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