A Low-Complexity Space Time Block Codes Detection for Cell Free Massive MIMO Systems

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Abstract—The next generation of communication networks should deliver higher spectral efficiency and data rates for novel demands and utilizations. One of the techniques used in the fifth generation (5G) of communication networks is the Cell-free massive MIMO, where segmentation into cells is not established. Here, the BS antennas are distributed in the environment and the users can receive information from all the BSs, simultaneously. In this paper, we investigate a multiuser cell free massive MIMO system with linear decoders. We propose the Neumann series based inverse matrix approximation method in order to reduce the ZF and MMSE decoders’ hardware and computational complexities. In this regard, each user is equipped with two antennas, and also the space-time block codes (STBCs) are used in the uplink transmission for diversity gain. Besides, we obtain a lower bound on the throughput performance of the ZF decoder. Also, the performance of the proposed system is evaluated in terms of the spectral efficiency and the bit error rate (BER). The computer simulation results confirm the superior performance of the proposed system compared to that of single-antenna systems. Furthermore, the BER performance of the proposed method is close to that of the exact method.

Index Terms—Space Time Block Codes (STBC), Golden Code, Cell free, Massive MIMO, Neumann series.

I. INTRODUCTION

One of the significant techniques used in the 5G of the communication networks is Massive MIMO [1]. It uses massive antenna arrays. By increasing the number of base stations’ (BSs) antennas and exploiting spatial diversity and beamforming, the performance in terms of energy and spectral efficiency of the system improves with some simple signal processing techniques [2]. It is one solution to compress a network. Cellular massive MIMO system performance is restricted by the intercellular interference that arises from the cellular structure of this system. A multi-user cell free system with massive MIMO concept has recently been proposed as an alternative to cell-based systems. In this way we will not have the standard cellularization for wireless communication [3]. In this system, BSs are distributed all over the environment, and back-haul links connect them to a central controller unit (CPU). This backhaul link, in this paper, is assumed to have unlimited capacity. BSs are used to cover all users and only power control coefficients and user data is exchanged between BSs and CPU [4]–[6].

In point-to-point MIMO systems, the link capacity is proportional $\min(M, K)$, where $K$ and $M$ are the number of user terminal and BS antennas, respectively [7], [8]. As a result, single-antenna users will significantly reduce the actual capacity of the system. Therefore, equipping users with two antennas is suggested as a solution for improving the system spectral efficiency [9].

Dual-antenna users, provide two channels between user and BS and if the number of BS antennas is high, this channels will be independent. Because each user has two antennas, we can use space-time block code as a way to increase the diversity gain in uplink, which for example in [10]–[13] for MIMO system have been investigated [9].

On the other hand, in designing, the target algorithms, implementation feasibility and hardware limitations are usually carefully evaluated. Therefore, one of the important issues in design is the application of low computational complexity methods with quasi-optimal performance. It should be noted that most of the computational complexities of the linear decoding in the receiver is on the inverse Hermitian matrix. Therefore, assuming the service is provided to many users using inverse matrix computation algorithms, such as Cholesky Decomposition, the system will have high computational complexity. Therefore, it is recommended to use Neumann series for efficient and fast calculation of approximate inverse matrix [14].

The [9] shows that if the transmitted signals of each user are programmed correctly (By choosing the space-time block code) and the number of antennas at each BS is large, then the interference between users and two antennas of each user with linear decoders will be eliminated.

In this paper, we investigate the bit error rate (BER) and spectral efficiency performance of a cell free multiuser massive MIMO system in the uplink using STBCs with linear decoders. Finally, for decreasing the computational complexity of the
receiver, the approximate algorithm for matrix computation based on the Neumann series is used and the simulation results are evaluated and compared. So, the paper structure is as: in section II, the system model is introduced, and section III illustrates the design of STBCs, linear decoders and approximate calculation of the inverse decoding matrix. Section IV discusses the detailed results of the BER and achievable spectral efficiency. The numerical results and conclusion of paper are presented in V and VI Sections, respectively.

Notation: Vectors and matrices are written in Bold lowercase letters and boldface with capital letters, respectively. The Hermitian by the superscripts H. K x K identity matrix is represented as I_K. The notation ||·|| and E(·) are used to represent Euclidean norm and the random variable expectation, respectively. The σ ~ CN(0, C) is complex Gaussian vector.

II. SYSTEM MODEL

In this paper we study a cell free multiuser massive MIMO system, in which the CPU communicates with all the BSs through a backhaul link, and only user data and power control coefficients is exchanged between them. It is assumed that the backhaul link between the BSs and the CPU is an error-free link with unlimited capacity. The channel coefficients from the m-th antenna in l-th BS to the j-th antenna in k-th user is g_{mljk} = β_{mljk}h_{mljk}, (1 ≤ k ≤ K, j = 1, 2, 1 ≤ m ≤ M). In this system h_{mljk} denotes the small scale fading that is assumed i.i.d. and is specified as h_{mljk} ~ CN(0, I_m) and β_{mljk} models path loss and shadowing effects.

Because the large-scale fading coefficients changes very slowly and the distance between the antennas of each user is small, so we can assume that: β_{mljk} = β_j and for normalization of average power in BS, consider β_1 ≥ β_2 ≥ ... β_k.

Assuming each user selects STBC to transmit signals, the l-th BS received signal is as follows:

\[ Y_l = \sum_{k=1}^{K} (\sqrt{\rho} H_l B_{lk} X_{lk}) + W_l, \]  

where H_{lk} = (h_{mljk})_{M \times 2} and B_{lk} = β_j I_2. ρ shows the received signal to noise ratio (SNR) and 1/\sqrt{ρ} is the normalization factor of the transmitted signal energy at each time slot. \[ X_l \]_2 × T , W \]_M × T and \[ Y_l \]_M × T are the k-th user transmitted-code, noise and received signal matrices, respectively, where M represents the number of each BS antennas and T is the number of time slots. Noise matrix entries are i.i.d., which have a zero-mean and unit variance complex Gaussian distribution.

III. GOLDEN CODE, LINEAR DECODERS AND NEUMANN SERIES

We first discuss the code design criteria and linear decoders. In next subsection, we introduce the method of computational complexity reduction using the Neumann series.

A. GOLDEN CODE AND LINEAR DECODERS

The Golden code as a STBC was firstly investigated for a simple 2 x 2 MIMO system in [10]. This code is made by a special set of algebra called cyclic division algebra. The transmitted signal matrix using the Golden code in the T = 2 time slot is as follows:

\[ X_k = \begin{bmatrix} a_k (x_{k1} + b_k x_{k2}) \\ c_k (x_{k3} + d_k x_{k4}) \\ c_k (x_{k1} + d_k x_{k2}) \end{bmatrix}, \]  

(2)

Where x_{kl} coefficients are the distinctive symbols that k-th user transmits and a_k, b_k, c_k, d_k, γ are constants.

One of the important properties of large number of algebraic structures of space-time codes is the exchangeability of structure (correlations) that is expressed as, if the system model matrices are \[ X \in C^{M \times T}, H \in C^{N \times M}, Y \in C^{N \times T}, W \in C^{N \times T}. \] They can be shown as: \[ X \in C^{L \times 1}, H \in C^{NT \times L}, \bar{Y} \in C^{NT \times 1}, \bar{W} \in C^{NT \times 1}. \] Where H contains both the channel coefficients and the effect of transmitted code. Distinctive transmitted symbols in each code block in T time slots are represented as L. The exchangeability of structure is feasible for Golden code [15].

Considering the k-th user matrix as \[ H_{lk} = [h_{lk1}, h_{lk2}] \] and with the (2) insertion into (1) and the Golden code exchangeability of structure feature, we will have the following relation for the received signal:

\[ Vec(Y_l) = \sum_{k=1}^{K} \sqrt{\frac{2}{\rho}} \beta_k \bar{H}_{lk} x_k + Vec(W_l), \]  

(3)

Where the k-th user channel matrix is as follows:

\[ \bar{H}_{lk} = \begin{bmatrix} a_k h_{lk1} & a_k b_k h_{lk1} \\ c_k h_{lk2} & c_k d_k h_{lk2} \end{bmatrix}, \]  

(4)

and \[ x_k = (x_{k1}, x_{k2}, x_{k3}, x_{k4}) \] is the k-th user vector.

The used Golden code coefficients, according to [9], are as follow:

\[ a_k = (1 + j(1 - b_k))/\sqrt{5} \]
\[ b_k = (1 + \sqrt{5})/2 \]
\[ c_k = (1 + j(1 - d_k))/\sqrt{5} \]
\[ d_k = (1 - \sqrt{5})/2 \]
\[ γ_k = j. \]  

(5)
According to large-M analysis \((M \rightarrow \infty)\), it is easily proven that:

\[
\begin{align*}
\hat{H}_M^H \hat{H}_k & \overset{a.s.}{\rightarrow} \text{diag}(p, s, p, s), \\
\hat{H}_{4K}^H \hat{H}_k & \overset{a.s.}{\rightarrow} 0,
\end{align*}
\]

where \(p = |a_k|^2 + |c_k|^2\), \(s = |a_kb_k|^2 + |c_kb_k|^2\) and "a.s." represents "almost surely" [9].

We define the matrices of the system coefficients as follows:

\[
G_t = [\beta_{11} \hat{H}_{11}, \ldots, \beta_{4K} \hat{H}_{4K}],
\]

\[
\tilde{X}_t = (x^t_{11}, x^t_{12}, \ldots, x^t_{4K}),
\]

where \(G\) and \(\tilde{X}\) are the \(2M \times 4K\) matrix and \(4K\) vector, respectively.

Then we can write (3) as follows:

\[
Vec(Y_t) = \left(\frac{\sqrt{p}}{2} G_t \tilde{X}_t\right) + Vec(W_t).
\]

According to the decoder matrices, the decoding by the \(A_t\) matrix is as follows:

\[
A_t Vec(Y_t) = \left(\frac{\sqrt{p}}{2} A_t G_t \tilde{X}_t\right) + A_t Vec(W_t),
\]

Where \(A_t\) is

\[
A_t = \begin{cases} 
(G_t^H G_t)^{-1} G_t^H & \text{ZF}, \\
(G_t^H G_t + 2I_{4K} / \rho)^{-1} G_t^H & \text{MMSE}.
\end{cases}
\]

B. NEUMANN SERIES

Define the \(Z_t\) matrix as follow:

\[
Z_t = \begin{cases} 
\hat{G}_t^H \hat{G}_t & \text{ZF}, \\
\hat{G}_t^H \hat{G}_t + 2I_{4K} / \rho & \text{MMSE}.
\end{cases}
\]

Much of the computational complexity of linear decoding is on the inverse Hermitian matrix \(Z_t \in \mathbb{C}^{2M \times 2M}\) calculation. A noteworthy point is that the computation of \(Z_t^{-1}\) by usual methods such as Cholesky decomposition needs \(O(M^3)\) computation, therefore implementation of such algorithms to serve many users may have limitations. In terms of hardware constraints, an algorithm based on the Neumann series was first proposed in [16] to approximate the \(Z_t\) matrix. According to [17] if \(Z_t = D + E\), where diagonal entries of \(Z_t\) are located in diagonal matrix \(D\) and remainder of the \(Z_t\) matrix entries are in \(E\) matrix, then the Neumann series for inverse calculation will be as follows:

\[
\tilde{Z}_{tR}^{-1} = \sum_{r=0}^{R-1} (-D^{-1}E)^r D^{-1},
\]

where \(R\) is the order of the Neumann series and \(\tilde{Z}_{tR}^{-1}\) is the \(R\)-term approximation of \(Z_t^{-1}\). If the maximum modulus of eigenvalues of matrix \((I - D^{-1}Z_t)\) is less than 1, then (12) converges, and also the approximation will be closer to \(R \rightarrow \infty\) [17]. Moreover, convergence will occur faster if the eigenvalues are lower, That would be true as long as the ratio \(\gamma = \frac{M}{R}\) is high [18]. Neumann series is a iterative method with low computational complexity. So unlike the usual methods it is hardware friendly [17]. For example, the approximation for \(R = 2\) will be as follows:

\[
\tilde{Z}_{t2}^{-1} = D - \frac{(D^{-1}E)D^{-1}}{p_0} - \frac{(D^{-1}E)D^{-1}}{p_1}
\]

The number of calculations is related to the \(p_0\) part is \(2M\) divisions and for part \(p_1\) is \(12M^2 - 6M\) Multiplications, while calculating \(Z_t\) with exact methods requires \(O(M^3)\) computation.

IV. SPECTRAL EFFICIENCY AND BER OF THE SYSTEM

Now, the system’s spectral efficiency and BER is investigated, when dual-antenna users exploiting Golden code exchange information with the BSs and SBSs, after decoding, sends the information to the CPU for final processing.

A. BER PERFORMANCE

Estimated signal \(x_{lki}\) will be as follows:

\[
\hat{x}_{lki} = \arg\min_x \left\| (A_t vec(Y_{k(i)})) - \frac{\sqrt{P}}{2} x_l(\hat{a}_{l(i),k(i)}) \right\|
\]

where \(\hat{a}_{l,j} = (A_t G_t)_{i,j}\) and \(x_l\) represents the \(l\)-th entry of vector \(x\).

Each BS quantizes the user data and forwards quantized signals to the CPU to extract the final data of each user.

B. SPECTRAL EFFICIENCY

Using equations (8) and (14) we have the following equation for the arbitrary user.

\[
\begin{align*}
\mathbf{s}_{k(i),l} &= (A vec(Y_{k(i)})) - \frac{\sqrt{P}}{2} (\hat{a}_{k(i),k(i)} x_{k(i)}) + (\hat{b}_{k(i)}), \\
&= \mathbf{DesiredSignal} + \mathbf{Interference},
\end{align*}
\]

where

\[
\hat{u}_{k(i),l} = \sum_{q=1}^{4K} \sqrt{\frac{P}{2}} (\hat{a}_{k(i),q}) l(x_q) + ((A vec(W))_{k(i)}),
\]

where \(k(i)\) is the \(i\)-th symbol of \(k\)-th user and it is expressed as \(k(i) = 4(k-1) + i\). As discussed in II, the received data at each BS are independently decoded and then these signals are transmitted to the CPU via a back-haul link. Therefore, the signal required to extract \(k\)-th user data is equal to \(r_{k(i)}\), that is expressed as follows:

\[
\begin{align*}
\mathbf{r}_{k(i)} &= \sum_{l=1}^{L} \mathbf{s}_{k(i),l},
\end{align*}
\]

To calculate the whole system’s spectral efficiency according to the above relation, we need to calculate the variance of the noise and interference. Due to the independence of the transmitted signals with each other, \(E(|x_i|^2) = 1\) and their
independence from \( \mathbf{W} \), according to (15) and (17), we can write as follow:

\[
E(|\tilde{\mathbf{u}}_{k(i)}|^2) = \frac{\rho}{2} \sum_{l=1}^{L} \sum_{q=1}^{4K} \left| (\mathbf{a}_{k(i),q} \tilde{\mathbf{g}}_l) \right|^2 + \left| (\mathbf{a}_{k(i)}) \right|^2,
\]

where \( \mathbf{a}_i \) and \( \tilde{\mathbf{g}}_j \) are the \( i \)-th row and \( j \)-th column of \( \mathbf{A}_l \) and \( \mathbf{G}_l \) matrices, respectively.

The Signal to Interference plus Noise Ratio (SINR) is calculated as follows:

\[
SINR_{k(i)} = \frac{\frac{1}{\rho} \sum_{l=1}^{L} \sum_{q=1}^{4K} \left| (\mathbf{a}_{k(i),q} \tilde{\mathbf{g}}_l) \right|^2}{\sum_{l=1}^{L} \sum_{q=1}^{4K} \left| (\mathbf{a}_{k(i),q}) \right|^2}.
\]

The spectral efficiency lower bound in uplink for an arbitrary user is as follows:

\[
SE_k \geq \frac{1}{2} \sum_{i=1}^{L} \log_2(1 + SINR_{k(i)}),
\]

where factor \( \frac{1}{2} \) comes from the fact that we send four symbols in two time slots.

V. SIMULATION RESULTS

Here, we analyze and compare our theoretical results with the computer simulation. In this simulations, users with golden code exchange information with BSs. We consider the number of BSs to be four and the users can connect to all BSs.

A. BER SIMULATION

In this simulation, the BER of the proposed system with the usual (exact) structure and the approximate calculation of the inverse matrix using Neumann series with \( R = 2 \) is investigated. Assume that \( M = 256 \) and \( K = 10 \). Each user has two antennas. The used modulation for each user is BPSK. The ZF and MMSE are the decoders used at each BS. Results are compared with the state which users have only single antenna. The used modulation for any user in this mode is 4QAM for providing fairness in bandwidth efficiency compared to the dual-antenna mode. The change of BER versus SINR are described in Fig. 2 and Fig. 3. As can be seen from the figures, the system BER in dual-antenna mode have better performance than a single-antenna mode. This indicates that diversity gain in uplink is earned for dual-antenna users, and it is seen in the figures that the approximation method using the Neumann series with \( R = 2 \) based on (13), performs close to the exact structure.

B. SPECTRAL EFFICIENCY SIMULATION

Now, the system’s spectral efficiency is considered with the linear decoders. Assume that \( K = 10 \) and \( \rho = 10 \). Large scale fading coefficients \( \beta_{ik} \) are chosen uniformly and randomly in interval \([0, 1]\). BS antennas varies from 50 to 500. The simulation is based on (19) and (20) formulas. The change of spectral efficiency versus BS antennas are described in Fig. 4. We have also performed a single-antenna users mode based on (9) and (27) in paper [19] to compare performance. As can be seen from the Fig. 4, dual-antenna users have better performance in term of spectral efficiency than single-antenna users, and the approximation method using the Neumann series with \( R = 2 \) based on (13), performs close to the exact method, that is a completely hardware-friendly approach to alleviate the decoder’s computational complexity.

VI. CONCLUSION

Cell-free multiuser massive MIMO system performance with dual-antenna users using STBC in terms of BER and spectral efficiency with ZF and MMSE decoders were investigated. The lower bound of throughput for the considered system was derived. The simulation results show that the dual-antenna mode
performs better than the single-antenna mode in terms of the BER and the spectral efficiency, at the same system. Also, it is shown that the performance of a given system with the proposed method is close to the exact matrix inverse, with less computational complexity using Neumann series with $R = 2$.

REFERENCES

[1] E. Björnson et al., “Massive mimo networks: spectral, energy, and hardware efficiency,” *Foundations and Trend in Signal Process.*, vol. 11, no. 3-4, pp. 154–655, 2017.

[2] P. Pasang, M. Atshahar, and M. M. Feghhi, “Blind downlink channel estimation of multiuser multi-cell massive mimo system in presence of the pilot contamination,” *AEU - Int. J. of Electronics and Commun.*, vol. 117, p. 153099, Apr. 2020.

[3] G. Interdonato et al., “Ubiquitous cell free massive mimo communications,” *EURASIP J. on Wireless Commun. and Net.*, vol. 2019, no. 1, p. 197, 2019.

[4] E. Nayezi, A. Ashikhmin, T. Marzetta, and H. Yang, “Cell free massive mimo systems,” in *49th Asilomar Conf. on Signals, Systems and Computers*, pp. 695–699, 2015.

[5] H. Q. Ngo et al., “Cell free massive mimo versus small cells,” *IEEE Trans. on Wireless Commun.*, vol. 16, no. 3, pp. 1834–1850, 2017.

[6] H. Ngo, A. Ashikhmin, H. Yang, E. Larsson, and T. Marzetta, “Cell free massive mimo: Uniformly great service for everyone,” in *16th int. workshop on signal process. advances in wireless commun. (IEEE SPAWC)*, pp. 201–205, 2015.

[7] E. Telatar, “Capacity of multi antenna gaussian channels,” *European trans.on telecom.*, vol. 10, no. 6, pp. 585–595, 1999.

[8] G. J. Foschini, “Layered space time architecture for wireless communication in a fading environment when using multi-element antennas,” *Bell labs tech. j.*, vol. 1, no. 2, pp. 41–59, 1996.

[9] H. Wang, X. Yue, D. Qiao, and W. Zhang, “A massive mimo system with space time block codes,” in *Int. Conf. on Commun. in China (ICCC)*, pp. 1–5, 2016.

[10] L. Luzzi, G. Rekaya, J. Belfiore, and E. Viterbo, “The golden code: A 2 × 2 full rate space time code with non vanishing determinants,” *IEEE Trans. on Inf. theory*, 2008.

[11] L. Shi, W. Zhang, and X.-G. Xia, “Space time block code designs for two user mimo x channels,” *IEEE Trans. on Commun.*, vol. 61, no. 9, pp. 3806–3815, 2013.

[12] W. Wang and W. Zhang, “Signal shaping and precoding for mimo systems using lattice codes,” *IEEE Trans. on Wireless Commun.*, vol. 15, no. 7, pp. 4625–4634, 2016.

[13] M. R. ghavidel Aghdam, B. M. Tazekand, R. Abdolee, and M. M. Feghhi, “Space-time block coding in millimeter wave large-scale mimo-noma transmission scheme,” *International Journal of Communication Systems*, Accepted for publication, DOI:10.1002/dac.4392, 2020.

[14] F. Rosario, F. A. Monteiro, and A. Rodrigues, “Fast matrix inversion updates for massive mimo detection and precoding,” *IEEE Sig. Proc. Let.*, vol. 23, no. 1, pp. 75–79, 2015.

[15] Y. Wu and R. Calderbank, “Code diversity in multiple antenna wireless communication,” *IEEE J. of Sel. Topics in Sig. Process.*, vol. 3, no. 6, pp. 928–938, 2009.

[16] M. Wu, B. Yin, A. Vosoughi, C. Studer, J. R. Cavallaro, and C. Dick, “Approximate matrix inversion for high throughput data detection in the large scale mimo uplink,” in *IEEE Int. Symp. on Circuits and Sys. (ISCAS)*, pp. 2155–2158, 2013.

[17] M. Wu, B. Yin, G. Wang, C. Dick, J. R. Cavallaro, and C. Studer, “Large scale mimo detection for 5gpp lte: algorithms and fpga implementations,” *IEEE J. of Sel. Topics in Sig. Process.*, vol. 8, no. 5, pp. 916–929, 2014.

[18] D. Zhu, B. Li, and P. Liang, “On the matrix inversion approximation based on neumann series in massive mimo systems,” in *IEEE int. conf. on commun. (IEEE ICC)*, pp. 1763–1769, 2015.

[19] T. H. Nguyen, T. K. Nguyen, H. D. Han, et al., “Optimal power control and load balancing for uplink cell free multi user massive mimo,” *IEEE access*, vol. 6, pp. 14462–14473, 2018.