Quantum Corrections in Collective Field Theory

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We review and extend the computation of scattering amplitudes of tachyons in the $c = 1$ matrix model using a manifestly finite prescription for the collective field hamiltonian. We give further arguments for the exactness of the cubic hamiltonian by demonstrating the equality of the loop corrections in the collective field theory with those calculated in the fermionic picture.

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1. Introduction

In the past several years, matrix models have revolutionized the study of two-dimensional string theory and quantum gravity. (For a recent review, see [1]). By mapping the $c = 1$ string to a theory of non-relativistic free fermions, the one dimensional hermitian matrix model has allowed the calculation of the free energy and correlation functions to all orders in the loop expansion. A very enlightening description of 2D strings is the field theoretic formulation of Das and Jevicki [2]. Using the bosonic collective field representation, they derived a cubic hamiltonian, which upon normal ordering, receives an additional contribution from a linear tadpole term. This field theory reproduces the tree and loop diagrams of tachyon scattering, the tachyon field being related to the collective field via a non-local field redefinition which accounts for the external leg factors one finds in the tachyon scattering amplitudes, i.e.

$$\langle \prod_{i=1}^{N} T(q_i) \rangle = \left( \prod_{i=1}^{N} -\mu |q_i|/2 \frac{\Gamma(-|q_i|)}{\Gamma(|q_i|)} \right) A(q_1, \cdots, q_N),$$  \hspace{1cm} (1.1)

where $A$ is the Euclidean continuation of collective field $S$–matrix element.

Although much work has gone into rederiving the results of the fermionic (and Liouville) methods using collective fields, there remains some doubt as to whether this simple cubic hamiltonian applies without modification at higher loops [3]. Indeed to date, the only calculation of scattering amplitudes at loop level has been the two-point function. Additionally, the integrals in the calculations of [4,5] are unpleasant because of divergences which must be carefully regularized. In this paper, we follow the approach proposed in [6], which suggests that we take the double scaling limit of the matrix model with $\mu < 0$, or equivalently, with $\mu > 0$ but taking as the natural coordinate the conjugate momentum $p$ of the classical coordinate $\lambda$ [7]. The theory thus defined is manifestly finite. We rederive many of the results of [4,5] in this finite formalism, and we also include a new result, the calculation of the three-point function at one-loop. Our results further validate the claim that the cubic hamiltonian is not renormalized, and that the bosonization procedure is finite and exact to all orders of perturbation theory.

This paper is organized as follows: In §2 we briefly review the derivation of the collective hamiltonian. We show that a double-scaled theory with negative $\mu$ eliminates the spurious divergences. Using this manifestly finite formalism, we calculate the four-point and five-point tree amplitudes in §3. We verify the agreement with matrix model
results by Euclidean continuation and inclusion of the external leg factors (1.1). §4 is devoted to loop corrections: We compute the two- and three-point functions at one loop. Agreement with matrix model results is again confirmed. We conclude in §5 with some remarks about the future applications of the bosonic calculations. In the appendix we list for reference some useful integrals needed in the computations.

2. The Collective Field Approach

We now review the derivation of the collective field hamiltonian using the method of Gross and Klebanov [8]. Consider the second quantized hamiltonian for a system of free fermions with Planck constant $1/\beta$,

$$
\hat{h} = \int d\lambda \left\{ \frac{1}{2\beta^2} \frac{\partial \Psi^\dagger}{\partial \lambda} \frac{\partial \Psi}{\partial \lambda} + U(\lambda) \Psi^\dagger \Psi - \mu (\Psi^\dagger \Psi - N) \right\}. \quad (2.1)
$$

Introducing chiral fermions $\Psi_L$ and $\Psi_R$, (2.1) can be shown to be composed of chiral parts, $\mathcal{H} = 2\beta \hat{h} = \mathcal{H}_L + \mathcal{H}_R$, i.e. the left and right movers do not mix. The mixing of chiralities occurs only through the boundary conditions. Upon bosonizing the fermion fields, we find, as $\beta \to \infty$,

$$
\mathcal{H} := \frac{1}{2} \int_0^\infty d\tau : \left[ P^2 + (X')^2 - \frac{\sqrt{\pi}}{\beta v^2} \left( PX'P + \frac{1}{3} (X')^3 \right) - \frac{1}{2\beta \sqrt{\pi}} X' \left( \frac{v''}{3v^3} - \frac{(v')^2}{2v^4} \right) \right] : \quad (2.2)
$$

where $v(\lambda)$ is the velocity of the classical trajectory of a particle at the Fermi level in the potential $U(\lambda)$,

$$
v(\lambda) = \frac{d\lambda}{d\tau} = \sqrt{2(\mu_F - U(\lambda))}. \quad (2.3)
$$

In the double scaling limit, all that survive are this cubic interaction and linear tadpole, both of order $g_{st} = 1/(\beta \mu)$, where $\mu$ is defined as $\mu_c - \mu_F$, $\mu_c$ being the height of the potential barrier $U$. The equivalence of (2.2) with the Das-Jevicki hamiltonian was demonstrated in [8].

In the usual double-scaled theory, $v(\tau) = \sqrt{2\mu} \sinh \tau$ near the quadratic maximum, and (2.2) diverges at the turning point $\tau = 0$. One can either carefully regularize the theory near the turning point [4,5,9] or approach the double scaling limit with $\mu < 0$ [3,4]. In this paper we shall utilize the latter method, which renders the entire perturbative expansion manifestly finite. In [3,4], it is shown that positive and negative $\mu$ are related by a simple interchange of the classical coordinate $\lambda$ with its conjugate momentum $p$. 

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This interpretation allows us to apply this method of extracting finite hamiltonians to any potential, in particular, to the deformed matrix model of \cite{11}. For negative $\mu$, there is no turning point, and $v(\tau) = \sqrt{2|\mu|} \cosh \tau$ near the quadratic maximum. Equation (2.2) becomes

$$H = \frac{1}{2} \int_{-\infty}^{\infty} d\tau \left[ P^2 + (X')^2 - \frac{\sqrt{\pi}}{2\beta|\mu| \cosh^2 \tau} \left( PX'P + \frac{1}{3}(X')^3 - \frac{1 - \frac{3}{2} \tanh^2 \tau}{12\beta|\mu| \sqrt{\pi} \cosh^2 \tau} X' \right) \right],$$

and the divergence at $\tau = 0$ has disappeared.

Since the above hamiltonian does not mix the chiralities, we will consider only the scattering of right movers described by

$$H_R := \frac{1}{4} \int_{-\infty}^{\infty} d\tau \left[ (P - X')^2 + \frac{\sqrt{\pi}}{6\beta|\mu| \cosh^2 \tau} (P - X')^3 + \frac{1 - \frac{3}{2} \tanh^2 \tau}{12\beta|\mu| \sqrt{\pi} \cosh^2 \tau} (P - X') \right].$$

We follow the methods of \cite{4,5} in calculating scattering amplitudes, using the hamiltonian formalism to evaluate Feynman diagrams. We will use the canonical oscillator basis

$$X(t, \tau) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{4\pi|k|}} \left( a(k)e^{i(k\tau - |k|t)} + a^+(k)e^{-i(k\tau - |k|t)} \right),$$

$$P(t, \tau) = -i \int_{-\infty}^{\infty} \frac{dk}{\sqrt{4\pi|k|}} \left( a(k)e^{i(k\tau - |k|t)} - a^+(k)e^{-i(k\tau - |k|t)} \right),$$

with $[a(k), a^+(k')] = \delta(k - k')$. Inserting this into (2.3), $H_R$ assumes the form $H_R = H_2 + H_3 + H_1$, with

$$H_2 = \int_{0}^{\infty} dk a^+(k) a(k),$$

$$H_3 = \frac{i}{24\pi|\mu|} \int_{0}^{\infty} dk_1dk_2dk_3 \sqrt{k_1k_2k_3} \left[ f(k_1 + k_2 + k_3) a(k_1) a(k_2) a(k_3) - 3f(k_1 + k_2 - k_3) : a(k_1) a(k_2) a^+(k_3) : \right] + \text{h.c.},$$

$$H_1 = -\frac{i}{48\pi|\mu|} \int_{0}^{\infty} dk \sqrt{k} g(k) \left(a(k) - a^+(k)\right),$$

where

$$f(k) = \int_{-\infty}^{\infty} d\tau \frac{1}{\cosh^2 \tau} e^{ik\tau} = \frac{\pi k}{\sinh(\pi k/2)},$$

$$g(k) = \int_{-\infty}^{\infty} d\tau \frac{1 - \frac{3}{2} \tanh^2 \tau}{\cosh^2 \tau} e^{ik\tau} = \frac{\pi (k^3 + 2k)}{4\sinh(\pi k/2)}.$$
For our purposes of perturbatively calculating scattering amplitudes, we may either use old fashioned time-ordered diagrams, or use the Feynman rules defined by the cubic interaction $\mathcal{H}_3$ and the linear tadpole $\mathcal{H}_1$ (relevant at the loop level) with the propagators \[4\]

\[
\langle T(a(k_1,t)a^\dagger(k_2,0)) \rangle = \delta(k_1 - k_2) \int \frac{dE}{2\pi} \frac{i}{E - k_1 + i\epsilon} e^{-iEt},
\]

\[
\langle T(a^\dagger(k_1,t)a(k_2,0)) \rangle = \delta(k_1 - k_2) \int \frac{dE}{2\pi} \frac{-i}{E + k_1 - i\epsilon} e^{-iEt}.
\] (2.9)

3. Review of Tree Calculations

As a warm up, let us evaluate the tree level $S$–matrix,

\[ S = 1 - 2\pi i \delta(E_i - E_f)T. \] (3.1)

Various authors have used the collective field in deriving the exact $S$–matrix \[4, 8–13\]. In this section, we will only calculate the four-point and five-point functions. Each right-moving massless particle has energy equal to momentum, $E = k > 0$, and in what follows, we shall use them interchangeably.

Let us consider scattering of two particles of momenta $k_1$ and $k_2$ into particles of momenta $k_3$ and $k_4$. From second order perturbation theory, we easily find for the $s$-channel

\[
T^{(s)} = \frac{g_{st}^2}{16\pi^2} k_1 k_2 k_3 k_4 \int_{-\infty}^{\infty} dk \frac{f^2(k_1 + k_2 - k)}{k_1 + k_2 - k + i\epsilon \text{sgn}(k)}. \] (3.2)

To evaluate the integral in (3.2), we use

\[ \frac{1}{x \pm i\epsilon} = \mathcal{P} \frac{1}{x} \mp i\pi \delta(x) \] (3.3)

and the integral listed in the appendix. We find

\[
T^{(s)} = -\frac{g_{st}^2}{16\pi^2} k_1 k_2 k_3 k_4 \left( \frac{8\pi}{3} + 4\pi i |k_1 + k_2| \right). \] (3.4)

Likewise, one may evaluate the contribution from the $t$- and $u$-channels, which have similar forms. Summing over all three channels, we obtain for the total amplitude

\[
S(k_1, k_2; k_3, k_4) = -\frac{g_{st}^2}{2} \delta(k_1 + k_2 - k_3 - k_4) \prod_{j=1}^{4} k_j (|k_1 + k_2| + |k_1 - k_3| + |k_1 - k_4| - 2i). \] (3.5)
Under Euclidean continuation $k_i \rightarrow i|q_i|$, where $q_i > 0$ for an incoming particle and $q_i < 0$ for an outgoing particle. Upon inclusion of the external leg factors, the Euclidean result agrees with the fermion calculations [14,15].

Next we investigate the five-point function. Consider the scattering of four particles of momenta $k_1, k_2, k_4$, and $k_5$ into one with momentum $k_3$. The basic building block is

$$
\frac{ig^3_{st}}{64\pi^3} \int_{-\infty}^{\infty} dp_1 dp_2 \frac{p_1 p_2 f(k_1 + k_2 - p_1) f(k_3 - p_1 + p_2) f(k_4 + k_5 + p_2)}{(k_1 + k_2 - p_1 + i\epsilon \text{sgn}(p_1))(k_4 + k_5 + p_2 - i\epsilon \text{sgn}(p_2))}
$$

where $p_1$ and $p_2$ are the internal momenta. Again we utilize the integrals collected in the appendix and find that (3.6) yields

$$
\frac{ig^3_{st}}{8\pi} \left( \frac{4}{3} (k_1 + k_2)(k_4 + k_5) - \frac{2i}{3} (k_1 + k_2 + k_4 + k_5) - \frac{8}{15} \right)
$$

As in the four-point case, we must sum over the inequivalent permutations of the momenta. It is shown in [5] that when we restrict the kinematic region, we obtain exact agreement with the calculations in Liouville theory.

4. One Loop Corrections

In order to determine the exactness of the bosonization procedure in §2, we study the quantum corrections to the scattering amplitudes with the hope that (2.3) is sufficient to reproduce all the results of the fermionic theory. Let us first look at the two-point function at one-loop given by the diagrams in fig. 1. The contribution to $T$ from the one-loop graph in fig. 1a is

$$
T_a = \frac{g^2_{st} k^2}{32\pi^2} \int_0^{\infty} dk_1 dk_2 k_1 k_2 \left( \frac{f^2(k_1 + k_2 - k)}{k - k_1 - k_2 + i\epsilon} - \frac{f^2(k_1 + k_2 + k)}{k + k_1 + k_2 - i\epsilon} \right),
$$

Fig. 1: The contributions to the two-point function of order $g^2_{st}$.
where the two terms come from the different time orderings of the two vertices. One may of course also derive (4.1) using the Feynman rules (2.9). In that case, one needs to perform an integral over the energy, which is conserved at the vertices, unlike the momentum. After changing variables to $s = k_1 + k_2$ and $k_2$, and integrating over $k_2$, this becomes

$$T_a = \frac{g_{st}^2 k^2}{192\pi^2} \int_{-\infty}^\infty ds \ s^3 \ \frac{f^2(k - s)}{k - s + i\epsilon \text{sgn}(s)} = -\frac{g_{st}^2 k^2}{48\pi} (ik^3 + 2k^2 + \frac{8}{15})$$  \hspace{1cm} (4.2)$$

where the integral is evaluated using (3.3) and integrals shown in the appendix.

The contribution to $T$ from the tadpole graph fig. 1b is

$$T_b = \frac{g_{st}^2 k^2}{192\pi^2} \int_{-\infty}^\infty dp \ p \ f(p) \ g(p) \frac{-p + i\epsilon \text{sgn}(p)}{-p + i\epsilon \text{sgn}(p)} = -\frac{g_{st}^2 k^2}{48\pi} \left( \frac{7}{15} \right)$$  \hspace{1cm} (4.3)$$

This added to (4.2) gives

$$T = -\frac{g_{st}^2 k^2}{48\pi} (ik^3 + 2k^2 + 1).$$  \hspace{1cm} (4.4)$$

By continuing to Euclidean space and including the external leg factors, we again obtain complete agreement with the fermionic result [14,15].

![Fig. 2: The contributions to the three-point function of order $g_{st}^2$.](image)

We now turn to the more difficult calculation of the $2 \rightarrow 1$ amplitude at one loop, where $k_1$ and $k_2$ are the incoming momenta. This is another non-trivial check on the exactness of the Hamiltonian (2.2). There are three types of diagrams, shown in fig. 2. For fig. 2a, we have six integrals, corresponding to the six possible time orderings for the three
vertices. Thus

\[
T_a = k_1 k_2 k_3 \left\{ \frac{-i g_{st}^3}{64 \pi^3} \right\} \left\{ \begin{array}{l}
\int_0^\infty dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} \frac{\mathcal{F}}{(k_1 - q_{12} - q_{13} + i \epsilon)(k_3 - q_{13} - q_{23} + i \epsilon)} \\
- \int_{-\infty}^0 dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} \frac{\mathcal{F}}{(k_2 + q_{12} - q_{23} + i \epsilon)(k_3 - q_{13} - q_{23} + i \epsilon)} \\
+ \int_0^\infty dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} \frac{\mathcal{F}}{(k_1 - q_{12} - q_{13} + i \epsilon)(k_2 + q_{12} - q_{23} - i \epsilon)} \\
- \int_{-\infty}^0 dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} \frac{\mathcal{F}}{(k_1 - q_{12} - q_{13} - i \epsilon)(k_2 + q_{12} - q_{23} + i \epsilon)} \\
+ \int_0^\infty dq_{12} \int_{-\infty}^0 dq_{13} \int_{-\infty}^\infty dq_{23} \frac{\mathcal{F}}{(k_1 - q_{12} - q_{13} - i \epsilon)(k_3 - q_{13} - q_{23} - i \epsilon)} \\
- \int_{-\infty}^0 dq_{12} \int_{-\infty}^0 dq_{13} \int_{-\infty}^\infty dq_{23} \frac{\mathcal{F}}{(k_1 - q_{12} - q_{13} - i \epsilon)(k_3 - q_{13} - q_{23} - i \epsilon)} \\
\end{array} \right\}
\] (4.5)

where $\mathcal{F} = q_{12} q_{13} q_{23} f(k_1 - q_{12} - q_{13}) f(k_2 + q_{12} - q_{13}) f(k_3 - q_{13} - q_{23})$. The difficulty in evaluating (4.5) lies in the fact that some of the limits of integration are not infinite, and under a change of variables such as $x = k_1 - q_{12} - q_{13}$, they acquire a finite $k$ dependence. Under such circumstances, it would be difficult to compute the integrals in (4.5). In order to circumvent this problem, we use the following identity

\[
\frac{1}{(k_1 - q_{12} - q_{13} \pm i \epsilon)(k_2 + q_{12} - q_{23} \mp i \epsilon)} = \frac{1}{(k_3 - q_{13} - q_{23})} \left( \frac{1}{k_1 - q_{12} - q_{13} \pm i \epsilon} + \frac{1}{k_2 + q_{12} - q_{23} \mp i \epsilon} \right). \tag{4.6}
\]

This allows the third and fourth integrals in (4.5) to be split into four, and upon combining some regions of integration, equation (4.5) is transformed to

\[
T_a = k_1 k_2 k_3 \left\{ \frac{-i g_{st}^3}{64 \pi^3} \right\} \left\{ \begin{array}{l}
\left( \int_0^\infty dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} - \int_{-\infty}^0 dq_{12} \int_0^\infty dq_{13} \int_{-\infty}^\infty dq_{23} \right) \frac{\mathcal{F} f(k_3 - q_{13} - q_{23})}{k_1 - q_{12} - q_{13} + i \epsilon \text{sgn}(q_{12})} \\
\left( \int_0^\infty dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} - \int_{-\infty}^0 dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} \right) \frac{\mathcal{F} f(k_3 - q_{13} - q_{23})}{k_2 + q_{12} - q_{23} - i \epsilon \text{sgn}(q_{12})} \\
- i\pi \left( \int_0^\infty dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} + \int_{-\infty}^0 dq_{12} \int_0^\infty dq_{13} \int_{-\infty}^\infty dq_{23} \right) \frac{\mathcal{F} \delta(k_3 - q_{13} - q_{23})}{k_1 - q_{12} - q_{13} + i \epsilon \text{sgn}(q_{12})} \\
+ i\pi \left( \int_0^\infty dq_{12} \int_0^\infty dq_{13} \int_0^\infty dq_{23} + \int_{-\infty}^0 dq_{12} \int_0^\infty dq_{13} \int_{-\infty}^\infty dq_{23} \right) \frac{\mathcal{F} \delta(k_3 - q_{13} - q_{23})}{k_2 + q_{12} - q_{23} - i \epsilon \text{sgn}(q_{12})} \right\}. \tag{4.7}
\]
We now follow the same procedure as in the one-loop two-point function: Make the change of variables to $s = q_{12} + q_{13}$, $t = q_{23} - q_{12}$, and $q_{12}$, and integrate over $q_{12}$. Equation (4.7) reduces to

$$T_a = k_1k_2k_3 \left( \frac{-ig_{st}^3}{676\pi^3} \right) \left\{ \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dt s^3(s + 2t) f(k_1 - s) f(k_2 - s) f(k_3 - s - t) \frac{(k_3 - s - t + i\epsilon \sgn(s))(k_3 - s - t + i\epsilon \sgn(s + t))}{(k_3 - s - t + i\epsilon \sgn(s))(k_3 - s - t + i\epsilon \sgn(s + t))} \right\} + \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dt t^3(t + 2s) f(k_1 - s) f(k_2 - s) f(k_3 - s - t) \frac{(k_3 - s - t + i\epsilon \sgn(t))(k_3 - s - t + i\epsilon \sgn(t))}{(k_3 - s - t + i\epsilon \sgn(t))(k_3 - s - t + i\epsilon \sgn(t))} \right\}.$$  

It is now straightforward to evaluate these integrals, using (3.3) and the integrals tabulated in the appendix. We need only calculate the first integral in (4.8) since the second is the same with $k_1$ and $k_2$ interchanged. The result is

$$T_a = \frac{-ig_{st}^3}{48\pi}k_1k_2k_3 \left( \frac{-1}{3} \left( k_1^4 + 2k_1^3k_2 + 2k_1k_2^3 + k_2^4 + \frac{2i}{3} \left( k_1^3 + k_2^3 + k_3^3 \right) + \frac{8}{5} \left( k_1^2 + k_1k_2 + k_2^2 \right) + \frac{16}{35} \right) \right).$$

Next, we look at fig. 2b, where we have a loop on the external leg $k_1$. After making the substitution $s = q_1 + q_2$ and integrating over $q_2$, we have

$$T_{b_1} = k_1k_2k_3 \left( \frac{-ig_{st}^3}{676\pi^3} \right) \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} p s^3 f(k_1 - p) f(k_1 - s) f(p - s) \frac{(k_1 - s + i\epsilon \sgn(s))(k_1 - p + i\epsilon \sgn(p))}{(k_1 - s + i\epsilon \sgn(s))(k_1 - p + i\epsilon \sgn(p))}.$$  

Using the integrals in the appendix, this gives

$$T_{b_1} = \frac{-ig_{st}^3}{48\pi}k_1k_2k_3 \left( \frac{-k_1^4}{3} + \frac{4ik_1^3}{3} + \frac{6k_1^2}{5} + \frac{4ik_1}{15} + \frac{32}{105} \right).$$

We must, of course, also include the contributions from the diagrams where the loop is attached to $k_2$ and $k_3$, $T_{b_2}$ and $T_{b_3}$ respectively. The total contribution of all three diagrams is

$$T_b = T_{b_1} + T_{b_2} + T_{b_3} = \frac{-ig_{st}^3}{48\pi}k_1k_2k_3 \left( \frac{-k_1^4}{3} + \frac{4ik_1^3}{3} + \frac{6k_1^2}{5} + \frac{4ik_1}{15} + \frac{32}{105} \right) + \frac{6}{5} \left( k_1^2 + k_2^2 + k_3^2 + \frac{4i}{15} \right) \left( k_1 + k_2 + k_3 \right) + \frac{32}{35} \right).$$

Finally, there are the three diagrams where a tadpole is attached on an external leg, as shown for the case of the tadpole on $k_1$ in fig. 2c. For the tadpole on $k_1$, we have

$$T_{c_1} = k_1k_2k_3 \left( \frac{-ig_{st}^3}{768\pi^3} \right) \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dq \left( \frac{pqg(p)f(k_1 - q)f(k_1 + p - q)}{(-p + i\epsilon \sgn(p))(k_1 - q + i\epsilon \sgn(q))} \right) = \frac{-ig_{st}^3}{48\pi}k_1k_2k_3 \left( \frac{7ik_1}{30} + \frac{22}{105} \right).$$

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Again we must include the diagrams with the tadpole attached to $k_2$ and $k_3$, which gives a total contribution of

$$T_c = \frac{-ig_{st}^3}{48\pi}k_1k_2k_3\left(\frac{7i}{30}(k_1 + k_2 + k_3) + \frac{22}{35}\right). \quad (4.14)$$

By including the factor of $-2\pi i\delta(k_1 + k_2 - k_3)$ and adding equations (4.9), (4.12), and (4.14), we get the total three-point function at one loop:

$$S(k_1, k_2; k_3) = -\frac{g_{st}^3}{24}\delta(k_1 + k_2 - k_3)k_1k_2k_3\left((1 + ik_3)(2 + ik_3)(1 + k_1^2 + k_2^2 - ik_3)\right). \quad (4.15)$$

Upon the Euclidean continuation $k_j \to i|q_j|$, and inclusion of the external leg factors, this agrees with the 3-tachyon correlator of the non-relativistic fermion calculation [14,15]:

$$\langle T(q_1)T(q_2)T(q_3)\rangle = -\frac{1}{24(\beta\mu)^3}\delta(\sum_{j=1}^{3} q_j)\prod_{j=1}^{3} \left(\frac{\Gamma(1 - |q_j|)\mu|q_j|/2}{\Gamma(|q_j|)}\right)(|q_3| - 1)(|q_3| - 2)(q_1^2 + q_2^2 - |q_3| - 1). \quad (4.16)$$

The agreement of the bosonic calculations of collective field theory with the fermionic results gives us confidence that the bosonization procedure and the simple cubic Hamiltonian are indeed exact, to all orders in perturbation theory.

5. Conclusion

In this paper we have reviewed computations of scattering amplitudes using collective field theory in a manifestly finite formalism. We have also presented some new calculations. Furthermore, these results were shown to coincide with their fermionic counterparts, providing evidence that the bosonization procedure is finite and exact.

One drawback of the collective field theory calculations is that the evaluation of higher point or higher loop amplitudes become increasingly laborious. As a purely computational tool, the bosonized theory does not compare favorably with its fermionic parent. The methods of [15] are much more powerful in their applications. However, the bosonic theory deserved study in its own right as a simple string field theory, where different backgrounds may be studied. Recently, Jevicki and Yoneya [14] proposed a deformed matrix model and conjectured that it describes the 2D black hole solution of critical string theory. It is an interesting step towards understanding the $c = 1$ theory in other backgrounds. The results
of this paper may be applied to that model, and it would be interesting to show exactly how the one-loop three-point function vanishes, as found in [16].

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Appendix A. Some Useful Integrals
In this appendix we list some of the integrals necessary to evaluate the various diagrams in this paper. The first integral we encounter occurs in the calculation of the four-point function and many subsequent integrals:

\[
\int_{-\infty}^{\infty} dx f^2(x) = \frac{8\pi}{3}.
\]  

(A.1)

For the five-point function integrals, we have

\[
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \frac{f(x) f(y) f(x-y)}{xy} = \frac{8}{3} \pi^2,
\]

\[
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy f(x) f(y) f(x-y) = \frac{64}{15} \pi^2,
\]

(A.2)

where in evaluating the integrals, we need to use the integral definition of \( f(x) \), equation (2.8).

In addition to (A.1), the following is needed for the loop diagram (4.2):

\[
\int_{-\infty}^{\infty} dx x^2 f^2(x) = \frac{32\pi}{15}.
\]

(A.3)

For the tadpole integral in (4.3), one finds

\[
\int_{-\infty}^{\infty} dx f(x) g(x) = \frac{28\pi}{15}.
\]

(A.4)
Below we list the integrals required for the evaluation of diagram fig. 2a and fig. 2b:

\[
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \frac{x f(x) f(y) f(x-y)}{y} = \frac{32}{15} \pi^2, \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy x^2 f(x) f(y) f(x-y) = \frac{512}{105} \pi^2, \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy x^3 f(x) f(y) f(x-y) = \frac{128}{21} \pi^2.
\]  

(A.5)

Finally, we have the tadpole integral (4.14), for which we must have

\[
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy f(x) g(y) f(x+y) = \frac{352}{105} \pi^2.
\]  

(A.6)
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