STRONG AND RADIATIVE MESON DECAYS IN A GENERALIZED NAMBU–JONA-LASINIO MODEL

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We investigate strong and radiative meson decays in a generalized Nambu–Jona-Lasinio model. The one loop order calculation provides a satisfactory agreement with the data for the mesonic spectrum and for radiative decays. Higher order effects for strong decays of $\rho$ and $K^*$ are estimated to be large. We also discuss the role of the flavour mixing determinantal interaction.

1Work supported in part by Deutsche Forschungsgemeinschaft, Schweizerischer Nationalfonds, JNICT no. PMCT/C/CEN/72/90, GTAE and CERN no. PCERN-FAE-74-91.
2Heisenberg fellow
1 Introduction

The Nambu–Jona-Lasinio model [1] and generalizations thereof [2,3,4] have been used extensively to study the properties of mesons in free space and at finite temperatures and densities. It is an effective field theory of non-linearly interacting quarks which exhibits spontaneous and explicit dynamical chiral symmetry breaking. In the case of three flavours, it is mandatory to incorporate the ’t Hooft six-fermion interaction to describe the breaking of the axial $U(1)$ symmetry [3]. Mesons are bound quark-antiquark pairs in this approach and their properties can readily be calculated by solving the pertinent Bethe-Salpeter equations. However, no systematic study of three-point functions like the strong and radiative meson decays, has been performed so far. Previous attempts were limited to the leading term in the momentum expansion of the underlying quark-meson vertices [5] and thus do not account for the full dynamics of the model. Furthermore, these decays are also a good testing ground to find out the limitations of the model. This will be discussed in some detail later on.

In what follows, we will work in flavour $SU(3)$ and use the following Lagrangian

$$
\mathcal{L} = G_1[(\bar{\psi}_i \lambda_i \psi)^2 + (\bar{\psi}_i \gamma_5 \lambda_i \psi)^2] \\
+ G_2[(\bar{\psi}_a \lambda_a \gamma_\mu \psi)^2 + (\bar{\psi}_a \lambda_a \gamma_5 \gamma_\mu \psi)^2] \\
+ K[\text{det}\{\bar{\psi}(1 + \gamma_5)\psi\} + \text{det}\{\bar{\psi}(1 - \gamma_5)\psi\}] 
$$

(1)

where the flavour index $i$ runs from 0 to 8 with $\lambda_0 = \sqrt{2/3} \cdot 1$, the $\lambda_a$ are color matrices ($a = 1, \ldots, 8$). As it stands, the Lagrangian is characterized by a few parameters: The two four-fermion coupling constants $G_1$ and $G_2$, the six-fermion coupling $K$ and the cutoff $\Lambda$, which is necessary to regularize the divergences. We will use a covariant four-momentum cutoff $\Lambda = 1$ GeV. Furthermore, to account for the explicit symmetry breaking, a quark mass term has to be added. We work in the isospin limit $m_u = m_d$ and will use the current quark masses to fit the meson spectrum. Clearly, the coupling $G_1$ is related to the properties of the pseudoscalar Goldstone bosons, $G_2$ can be fixed from the $\rho$-meson mass and $K$ is necessary to give the $\eta\eta'$ mass splitting. This completely specifies the model and we are now at a point to consider its dynamical content.

2 Formalism

The basic object to consider is the triangle diagram which describes the coupling of the decaying meson ($M_1$) into the other mesons ($M_{2,3}$) or another meson ($M_2$) and a photon $\gamma$ or two photons $\gamma_1, \gamma_2$ via the quark loop. Let us first consider the strong decays. Dropping all prefactors, the transition amplitude for the process $M_1 \rightarrow M_2 M_3$ can be evaluated by working out (cf. fig.1)

$$
\Gamma(M_1 \rightarrow M_2 M_3) = Tr(\Gamma_{M_1} S_F \Gamma_{M_2} S_F \Gamma_{M_3} S_F) 
$$

(2)
where \( \Gamma_M \) gives the \( i \)th meson-quark-antiquark vertex and \( S_F \) the propagator of the constituent quarks. The latter follows from minimizing the effective potential to one loop. The Bethe-Salpeter vertex functions relevant for our considerations are of scalar, pseudoscalar and vector type

\begin{align*}
\Gamma_S &= g_S 1 \otimes I \\
\Gamma_P &= g_P (1 + h_P \not{p}) \gamma_5 \otimes I \\
\Gamma_V &= g_V \gamma_\mu \otimes I
\end{align*}

Here, \( I \) is a generic symbol for the isospin structure and we have introduced scalar, pseudoscalar and vector \( Mq\bar{q} \)-couplings. The coupling \( h_P \) stems from the pseudoscalar-axial vector meson mixing. This is discussed in more detail in refs. [2,4,6,7]. Since we work in the isospin limit, no scalar-vector mixing arises in the \( SU(2) \) subgroup. In the scalar and pseudoscalar channels, a further complication is due to the \( \lambda_0 \lambda_8 \) mixing which has already been discussed in ref.[8] in some detail. The solution of the corresponding Bethe-Salpeter equations is standard and we do not exhibit any details here.

Let us briefly elaborate on the connection between the various transition amplitudes and meson-meson coupling constants. Consider first the decay of a scalar (\( S \)) into two pseudoscalars (\( P \)). The transition amplitude is a purely scalar function, called \( T_S \), and we have

\begin{align*}
\Gamma(S \to PP) &= \frac{|\vec{p}_c||T_S|^2}{8\pi E_S^2} \\
G_{SPP}^2 &= \frac{|T_S|^2}{4\pi}
\end{align*}

with \( |\vec{p}_c| \) the momentum of an outgoing particle in the rest frame of the decaying particle, \( |\vec{p}_c| = ((s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)/4s)^{1/2} \) and \( s = m^2(M_1) \). For the decay of a vector (\( V \)) into two pseudoscalars, one has

\begin{align*}
T_\mu(V \to PP) &= (p_1 - p_2)_\mu G_{VPP} \\
\Gamma(V \to PP) &= \frac{|\vec{p}_c|^3 G_{VPP}^2}{6\pi m_V^2}
\end{align*}

Finally, for the reaction \( V \to \tilde{V}P \) we find

\begin{align*}
T_{\mu\nu}(V \to \tilde{V}P) &= \epsilon_{\alpha\beta\mu\nu} p_{\tilde{V}}^\alpha q_\beta F \\
\Gamma(V \to \tilde{V}P) &= \frac{|\vec{p}_c|^3}{3} \frac{F^2}{4\pi}
\end{align*}

In the case of the radiative decays, one can use the same formalism since the photon behaves much like a vector particle. Since our Lagrangian contains no tensor interaction term, the pertinent photon-quark-antiquark vertex takes the minimal form

\begin{equation}
\Gamma_\gamma = \frac{e}{2}(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8)
\end{equation}

For the decay of a pseudoscalar into two photons, we have a structure similar to the one for \( V \to \tilde{V}P \), the only difference being that in the formula for the width \( \Gamma(P \to \gamma\gamma) \) one has a factor \( 1/2 \) instead of \( 1/3 \) which is the reduction in polarization degrees of freedom for a massless particle.
3 Estimates of higher order effects

The description of mesons as $q\bar{q}$ pairs has been proven to be quite successful in the calculation of the meson mass spectrum. However it may not account for other properties of some mesonic resonances, such as their decays. We anticipate that this is indeed the case for the strong decays $\rho \to \pi\pi$ and $K^* \to K\pi$, which come out very small in the one loop calculation. The insufficiency of a $q\bar{q}$ description of the mesons was already emphasized by Krewald et al [9] in the context of the pion electromagnetic form factor.

Due to the failure of the one loop approximation for the calculation of those decays, it is necessary to estimate the magnitude of higher order effects, such as the two loop corrections. The full calculation is, for the moment, out of the scope of the present work. A simple estimate of such effects for the mesonic decays can be obtained by calculating a dressed meson propagator, as shown in fig.2. The dashed loop refers to $\pi\pi$ or $K\pi$ states, in the case of the $\rho$ and $K^*$ propagators, respectively. By bare propagator we denote the meson described as a $q\bar{q}$ state.

The full propagator reads

$$G_{\alpha\beta} = G_{\alpha\beta}^0 + G_{\alpha\lambda}^0 \Sigma_{\lambda\mu} G_{\mu\beta}$$  \hspace{1cm} (8)

where the bare propagator is

$$G_{\alpha\beta}^0 = \frac{g_{\alpha\beta} - \hat{q}_\alpha \hat{q}_\beta}{q^2 - m_V^2} = \frac{T_{\alpha\beta}}{q^2 - m_V^2}$$  \hspace{1cm} (9)

and $\Sigma_{\lambda\mu}$ is the meson loop given by

$$\Sigma_{\lambda\mu} = 8 G_{VPP}^2 \int_{\Lambda_1} \frac{d^4k}{(2\pi)^4} ((2k - q)_\lambda) ((2k - q)_\mu) S(k, m_{P1}) S(k - q, m_{P2})$$  \hspace{1cm} (10)

with $S(k, m_{P1})$ the propagators of the mesons obtained in the one loop order, $m_{P1}$ and $m_{P2}$ being their masses. The remaining factors in the integrand correspond to the coupling of the vector mesons to the pseudoscalars in the first order calculation (5). The covariant cutoff $\Lambda_1$ needs not to be the same as $\Lambda$. One obtains:

$$G_{\alpha\beta} = \frac{T_{\alpha\beta}}{q^2 - m_V^2 - C}$$  \hspace{1cm} (11)

with $C \equiv C \left( G_{VPP}^2, q^2, m_{P1}, m_{P2}, \Lambda_1 \right)$. The quantity $C$ has a cut for $q^2 > (m_{P1} + m_{P2})^2$ and one rewrites $G_{\alpha\beta}$ as

$$G_{\alpha\beta} = T_{\alpha\beta} \frac{Z}{q^2 - \tilde{m}_V^2 - i \text{Im} C (\tilde{m}_V^2) Z}$$  \hspace{1cm} (12)

provided that the renormalization factor $Z$ is roughly constant around the physical mass $\tilde{m}_V$ and where the renormalization factor is:

$$Z = \left( 1 - \frac{d}{dq^2} \text{Re} C \right)^{-1} \bigg|_{q^2 = \tilde{m}_V^2}$$  \hspace{1cm} (13)

The decay width of the vector meson is finally
\[ \Gamma (V \rightarrow PP) = \frac{\text{Im} C(m_V^2)}{m_V} Z. \] (14)

Using this scheme the amplitudes for radiative decays of \( \rho \) and \( K^* \) have then to be also multiplied by \( \sqrt{Z} \) (wave function renormalization).

4 Results and discussion

At the one loop level we use the meson spectrum and decay constants to fix the parameters of the model. For \( \Lambda = 1 \text{ GeV}, G_1 \Lambda^2 = 3.95, G_2 \Lambda^2 = 5.43, K \Lambda^5 = 42, m_u = m_d = 4 \text{ MeV} \) and \( m_s = 115 \text{ MeV} \), we find the following meson masses (the experimental values are given in parentheses for comparison): \( M_\pi = 136.5(139.6), M_K = 497.5(497.7), M_\eta = 549(548.8), M_{\eta'} = 936(957.5), M_\rho = 775(768.3), M_\omega = 764(781.95), M_{K^*} = 898(891.6), M_\Phi = 990(1019.41) \) and \( M_{a_0} = 970(983.3) \) (all in MeV). For the pion and kaon decay constants, we have \( F_\pi = 93.9 \text{ MeV} \) and \( F_K = 96.6 \text{ MeV} \), i.e. the ratio \( F_K/F_\pi \) is too small, which is a common feature in this kind of models. We find an overall satisfactory description of the meson spectrum together with reasonable values for the vacuum expectation values of the scalar quark densities \( \bar{u}u \) and \( \bar{s}s \), \( - < \bar{u}u >^{1/3} = 272 \text{ MeV} \) (225 \pm 35) and \( < \bar{s}s >/ < \bar{u}u > = 0.74(0.8 \pm 0.2) \).

In table 1, we show the results for the strong decays in comparison to the empirical values. Obviously, for states in the quark-antiquark continuum the results are not reliable as indicated by the decay \( \Phi \rightarrow \pi \rho \). Also, for our set of parameters the decay \( \Phi \rightarrow \bar{K}K \) is kinematically forbidden. The large width of the \( \Phi \rightarrow \pi \rho \) decay is due to the too strong flavour mixing induced by the six-fermion interaction proportional to \( K \). This could presumably be cured by including more terms in the Lagrangian like e.g. in ref.[6].

As for the strong decays of \( \rho \) and \( K^* \) the one loop order calculation is clearly insufficient to account for the respective empirical widths. Using the simple approximation scheme, described in the previous section, to include the second order effects and approximating the meson propagators in the loop by propagators of structureless particles, the results improve by about a factor of 2. We think, therefore, that it is mandatory to consider a more complex multiquark structure for the \( \rho \) and \( K^* \) mesons. The strong \( \rho \) decay width including second order effects is still quite small as compared to the experimental one, but this number should be understood only as a guide for the order of magnitude of higher loop corrections. We notice that the parameters could have been adjusted in order to have the correct decay width for the \( \rho \) and the KSFR relation fulfilled, but at the cost of having bad values for the radiative decays.

Let us now turn to the radiative decays. In table 2, our results are summarized. We find an overall satisfactory description of the data, the main exceptions being the widths for \( \Phi \rightarrow \eta \gamma \) and for the \( K^* \rightarrow K \gamma \). In the first case this is, again, an artifact of the interaction Lagrangian used and also, since the \( \Phi \) lies in the unphysical quark-antiquark continuum, should not be considered significantly troublesome. The fact that the ratios \( \Gamma_{\gamma PP}/\Gamma_{\gamma PP} \) and \( \Gamma_{\gamma PP}/\Gamma_{\gamma PP} \) are larger for the \( K^* \) decays than for the \( \rho \) decays (both strong and radiative) is consistent with the small ratio \( F_K/F_\pi \). We have also investigated the case \( K = 0 \) (no six-fermion term). Reducing the strange quark mass to 80 MeV [2], which is necessary to find a decent fit to the spectrum, one is not able to get a satisfactory description of strong and radiative decay widths. After finishing this work, we became
aware of a preprint by Takizawa and Krewald [10], who deal with the radiative decays $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ in a similar model. Their Lagrangian contains the four–fermion interaction proportional to $G_1$ and the six–fermion determinantal interaction. While their results are similar to ours, we disagree with their conclusions at various places. First, in the case of $\pi^0 \to \gamma\gamma$ they remove the cut–off to find agreement with the current algebra prediction. This is, however, not a consistent procedure since once the cut–off is introduced, the effective theory is defined and should not be altered in the process of calculating various quantities. Second, for calculating the width of $\eta \to \gamma\gamma$, they use the empirical $\eta$ mass, which is 17 per cent larger than the value they find within the model. This, of course, alters substantially the result for this particular width. Comparing their results with ours, we also find a satisfactory description for these two particular radiative decays. It should be obvious, however, that the model is somewhat too crude too draw as far reaching conclusions as done in ref.[10]. As long as one is not able to properly account for the $SU(3)$ breaking effects in the pseudoscalar decay constants, it is doubtful that one can make firm quantitative statements about such breaking effects in other processes.

In summary, we have used the generalized three-flavour NJL model to calculate strong and radiative meson decays (three-point functions) taking the full solution to the Bethe-Salpeter equations in the one-loop approximation. A reasonable agreement to the experimental data is obtained, with the exception of the $\rho$ and $K^*$ decays. This seems to indicate that a more complex multiquark structure should be accounted for these mesons. This conjecture is supported by a simple estimate of the two loop order corrections. We also demonstrated the importance of the flavour-mixing determinantal six–fermion interaction. Further studies including also the effects of isospin–breaking seem necessary to understand some fine details of the mesonic interactions at low energies and to gain insight into effects of $SU(3)$ breaking on various observables.

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### Tables

|          | $\rho \rightarrow \pi\pi$ | $K^{*+} \rightarrow \pi^0K^0$ | $K^{*+} \rightarrow \pi^0K^+$ | $a_0 \rightarrow \pi\eta$ | $\phi \rightarrow \pi\rho$ |
|----------|-----------------------------|---------------------------------|--------------------------------|---------------------------|-----------------------------|
| NJL(I)   | 52.0                        | 18.0                            | 9.0                            | 74.5                      | 1.5                         |
| NJL(II)  | 94.0                        | 38.2                            | 19.1                           | -                         | -                           |
| Exp.     | 151.5 ± 1.2                 | 38.6 ± 0.6                      | 16.7 ± 0.3                     | 57 ± 11                   | 0.6 ± 0.3                   |

Table 1: Strong meson widths for various decays in units of MeV: (I) calculated in one loop order; (II) with estimates of two loop order included.

|          | $\pi^0 \rightarrow \gamma\gamma$ | $\eta \rightarrow \gamma\gamma$ | $\rho^+ \rightarrow \pi^+\gamma$ | $\rho \rightarrow \eta\gamma$ |
|----------|-----------------------------------|----------------------------------|----------------------------------|-------------------------------|
| NJL      | $7.9 \cdot 10^{-3}$               | 0.77                             | 60.1                             | 60.4                          |
| Exp.     | $(7.7 \pm 0.6) \cdot 10^{-3}$     | 0.46 ± 0.04                      | 67.1 ± 7.6                       | 57.6 ± 10.7                   |
|          | $\omega \rightarrow \pi^0\gamma$ | $\omega \rightarrow \eta\gamma$ | $K^{*+} \rightarrow K^+\gamma$ | $\Phi \rightarrow \eta\gamma$ |
| NJL      | 762                               | 6.3                              | 92.0                             | 259.0                        |
| Exp.     | 716.6 ± 43.0                      | 4.0 ± 1.9                        | 50.3 ± 4.6                       | 56.7 ± 2.8                   |

Table 2: Anomalous and radiative meson decay widths in units of keV, calculated in the one loop order. The second order corrected decays are $\Gamma_{\rho\pi\gamma} = 63$ keV and $\Gamma_{K^{*}\pi\gamma} = 98.9$ keV ($\Lambda_1 = 1.3$ GeV).

### Figure Captions

Fig. 1: Quark triangle diagram to calculate the strong meson decays. In the case of radiative decays, one has to substitute the third meson $M_3$ by a photon, and for the anomalous decays $M_2$ and $M_3$ by two photons.

Fig. 2: The dressed vector meson propagator (thick line) includes a 2-pseudoscalar excitation (dashed line), which is a 2nd order effect. The one loop order ($q\bar{q}$ state) is denoted by the thin line, the bare meson propagator.