RENORMALIZATION OF THE $f_0(980)$ and $a_0(980)$ SCALAR RESONANCES IN A NUCLEAR MEDIUM

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Abstract

The meson meson interaction in the scalar sector in the presence of a nuclear medium is studied with particular attention to the change of the properties of the $f_0(980)$ and $a_0(980)$ resonances. By using a chiral unitary approach which generates the $f_0$ and $a_0$ resonances and reproduces their free properties, we find that the position of the resonances in the medium is barely changed but their widths are considerably broadened. New many body corrections generating from higher orders in the chiral Lagrangian plus the contribution from N*hh excitations, not considered before in connection with the $\pi\pi$ interaction in the nuclear medium, are also investigated.

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1 Introduction

The scalar resonances produced in the scattering of pseudoscalar mesons have been a permanent source of discussion\[1, 2\]. They are advocated as ordinary $q\bar{q}$ states \[3\], $q^2\bar{q}^2$ states \[4\], $K\bar{K}$ molecules \[5, 6\], glueballs \[7\] and/or hybrids \[8\]. The properties of these resonances, like their mass, width, partial decay widths and their influence in different reactions are closely tied to their nature, although different interpretations are sometimes possible.

The properties of the resonances are modified in the presence of a nuclear medium. They get a selfenergy which changes the position of the peak, the decay width and in addition introduces new decay channels. The predictions for the modification of these properties are also closely related to the hypothesis made on the nature of the resonance. On the other hand, the knowledge of the renormalization of the resonance properties is an essential tool in order to interpret correctly experiments where pairs of pseudoscalar mesons are produced in nuclei close to the energy regions where the resonances appear. This would be the case for the production of two pions which can lead to the $\rho$ resonance in $L=1$ and isospin $I=1$, or the $\sigma$ and the $f_0(980)$ in $L=0$ and $I=0$, or the production of $\pi\eta$ which can lead to the $a_0(980)$ resonance in $L=0$ and $I=1$.

In particular, the renormalization of the $\rho$ meson properties in nuclei has witnessed a spectacular effort both theoretical and experimental after the suggestion that there should be a universal scaling of the masses \[9\]. Although not supported experimentally\[1\] and also challenged theoretically by more recent calculations, the hypothesis certainly had a stimulating effect which has lead to a series of thorough and detailed studies which have set the issue on firmer grounds \[10, 11, 12, 13, 14\]. Most works would conclude that the $\rho$ mass does not appreciably change in the medium, but the width is substantially increased. This latter change alone might be sufficient to explain the spectra of dileptons produced in heavy ion collisions \[15\], although further theoretical and experimental work is being done.

Another front where there has been much progress is the renormalization of the $\pi\pi$ scattering amplitude in a nuclear medium and its possible relationship to the enhancement of the $\pi\pi$ invariant mass distribution close to threshold, seen in the experiments of pion induced two pion production in nuclei \[13\]. It was suggested in \[17\] that the $\pi\pi$ interaction could develop a singularity just below the two pion threshold which would correspond to a kind of Cooper pair. Since this shows up in the scalar and isoscalar channel it was also interpreted as a drop of the sigma mass in the nuclear medium. More refined calculations which have payed special attention to the chiral

\[1\]The pion selfenergy is repulsive instead of attractive, as suggested for the $\rho$ in \[1\] on the basis of the value for the nucleon effective mass in the nucleus, which actually has a different meaning than the one attributed to the $\rho$ effective mass in that work.
constraints of the $\pi\pi$ amplitude appreciably weaken the renormalization of the amplitude, although there is still an appreciable enhancement of the imaginary part close to threshold $[13, 19, 20]$. Although there were initial hopes that this alone could explain the experimental features $[21]$ of the $(\pi, \pi\pi)$ reaction in nuclei, more detailed calculations have shown that some additional mechanisms may be required $[22]$.

The former examples show two fronts where the medium properties of mesons are thoroughly investigated. In the present paper we want to pay attention to the modification of the meson scalar resonances $f_0(980)$ and $a_0(980)$ inside the nuclear medium. The $f_0(980)$ resonance has the same quantum numbers as the $\sigma$, only its mass is much larger, and conversely the width is much smaller, of the order of $40$-$100$ MeV (versus $m_{\sigma} = 450 MeV$ and a width of around $450$ MeV according to the PDG book $[2]$). The $a_0(980)$ in the $I=1$ channel has about the same mass as the $f_0(980)$ and a width of the order of $50$-$100$ MeV.

In spite of its relevance as a source of information on the nature of these resonances and the interest that it should have in the analysis of the two meson production in nuclei, the surprising fact is that there are neither theoretical nor experimental studies on this issue, which contrast with the large efforts devoted to the study of the $\rho$ meson properties in an energy region very close to the one where these resonances appear. The reasons for this might be simply technical. On the one hand the confusion about the nature of these states was a deterrent. On the other hand, the study of the properties of these resonances in nuclei, which couple both of them largely to the $K\bar{K}$ system, had their own problems since the question of the $K^-$ selfenergy in a nuclear medium was itself unclear $[23, 24, 25, 26]$. Fortunately things have changed in both fronts recently, to the point that one can count on reasonable models with which to tackle the problem.

One of the areas which has witnessed an important progress in recent years is the meson meson interaction by means of Chiral Perturbation Theory ($\chi PT$), which is supposed to be the effective theory of QCD at low energies $[27]$. The theory has proved rich in applications to strong, weak and electromagnetic processes in which pairs of mesons appear at small energies $[27, 28, 29, 30]$. Yet, implicit to the perturbative nature of the theory is the fact that it does not generate poles in the scattering amplitudes and hence is unsuited to study the energy regions where meson resonances appear. In this respect, there have been recent advances which have shown the usefulness of $\chi PT$ as a means to constrain non perturbative unitary methods. For instance, by means of the inverse amplitude method (IAM) $[31]$ and the chiral Lagrangians, one can obtain the $\sigma$ and $\rho$ mesons in the $\pi\pi$ scattering and the $K^*$ resonance in $K\pi$ scattering. The generation of the $f_0(980)$ and $a_0(980)$ resonances, however, required the extension of the IAM to coupled channels, which was done in $[32, 33, 34]$ where these two mesons were also generated. In
addition, all meson meson properties up to about 1.2 GeV were reproduced using simply the standard $O(p^2)$ and $O(p^4)$ chiral Lagrangians as input.

Advances have also been made using the hypothesis of resonance saturation of Ref. [35], which states that the information of the $O(p^4)$ chiral Lagrangian is tied to the exchange of resonances which survive in the large $N_c$ limit. By allowing such genuine (preexisting to the unitarization, or multiple scattering of the mesons) resonances and using the $O(p^2)$ chiral Lagrangian in addition, together with a proper unitarization scheme based on the N/D method, it was shown in [36] that a good description of all the meson meson information up about 1.5 GeV could be accomplished. In this respect it is interesting to observe that the use of the lowest order $\chi PT$ Lagrangian, properly unitarized by means of the Bethe Salpeter equation, together with an appropriate cut off to regularize the loops, is able to reproduce all the information of the meson meson scattering in the scalar sector up to about 1.2 GeV and gives the same results as the more general methods reported above. This is possible due to the large weight of the lowest order Lagrangian in the scalar sector, in contrast to what happens in the vector sector where higher orders play an essential role.

The meson baryon interaction, from the point of view of $\chi PT$, has also witnessed much progress [28, 29, 30, 37]. On the other hand the meson baryon interaction has been the ground for application of the chiral unitary techniques, using the Lippmann Schwinger summation [38, 39] or the Bethe Salpeter integral equation [40] and more recently the IAM in [41] or the N/D method [12].

The work of [38, 39] initiated a fruitful line which has allowed to link the $K^-N$ interaction with its coupled channels at low energies, plus the properties of the $\Lambda(1405)$ resonance, with the information of the chiral Lagrangians. The work of [40] added more channels in the coupled channels set and proved that it was possible to get a good reproduction of the low energy data in terms of the lowest order chiral Lagrangian and a suitable cut off for the loops.

This progress in the elementary reaction also stimulated work directed to obtain the selfenergy of the kaons in nuclei. It was soon realized that the consideration of Pauli blocking in the intermediate nucleon states lead to a shift towards higher energies of the $\Lambda(1405)$ resonance and, as a consequence, to an appreciable attraction on the kaon [24, 25]. However, the large kaon attraction which was found suggested that the problem had to be solved selfconsistently. This was done in [26], where it was found that the selfconsistent treatment rendered the resonance to the same free position, although it appeared with a substantially larger width. This problem has been further pursued in [43] where all elements of [26] were considered but the renormalization of the pions in the intermediate states was also taken into account. With all these ingredients a $K^- \text{nucleus}$ optical potential was found which has proved consistent with the information of kaonic atoms [44].
Thus, the situation at present is that one has the information and the means to generate the scalar mesons from the chiral Lagrangians and also the interaction of the kaons with the nucleus, the two ingredients that blocked progress in the topic of the scalar meson renormalization in nuclei, and this is the problem that we shall tackle in this paper.

2 The meson meson interaction in the nuclear medium

Here, we briefly review the formalism for the meson meson interaction which was developed in [20]. We start by taking the states $\pi\pi$ and $K\bar{K}$, which we label 1 and 2. In the $I=0$ channel they are given by

$$ |K\bar{K} > = -\frac{1}{\sqrt{2}} |K^+(-\bar{q})K^-(-\bar{q}) + K^0(-\bar{q})\bar{K}^0(-\bar{q}) > $$

$$ |\pi\pi > = -\frac{1}{\sqrt{6}} |\pi^+(-\bar{q})\pi^-(-\bar{q}) + \pi^0(-\bar{q})\bar{\pi}^0(-\bar{q}) >,$$

and the $K\bar{K}$, $\pi\eta$ which we label 1 and 2, in the isospin $I=1$ channel,

$$ |K\bar{K} > = -\frac{1}{\sqrt{2}} |K^+(-\bar{q})K^-(-\bar{q}) - K^0(-\bar{q})\bar{K}^0(-\bar{q}) > $$

$$ |\pi\eta > = |\pi^0(-\bar{q})\eta(-\bar{q}) >,$$

where $\bar{q}$ is the momentum of the particles in the CM of the pair. We follow the convention $|\pi^+ > = -|1,1>$ and $|K^- > = -|\frac{1}{2}, -\frac{1}{2}>$ isospin states. In the $I=0$ case we neglect the $\eta\eta$ channel. Its contribution has been assessed in [36, 45] and it has only relevance at energies beyond 1.2 GeV, hence, as done in [6], we shall omit this channel here too.

The Bethe Salpeter (BS) equation is given by

$$ T = VGT. $$

Eq. 3 is meant as a coupled channel equation with two channels in each of the isospin states. It is an integral equation, meaning that the term $VGT$ involves one loop integral where both $V$ and $T$ would appear off shell. However, it was shown in [8, 10] that, for the free case, these amplitudes could be factorized on shell out of the integral and the off shell part was absorbed by a renormalization of the coupling constants. Thus, the equation becomes purely algebraic and is quite easy to solve. In Eq. 3, the formal product of $VGT$ inside the loop integral becomes then the product of $V, G$ and $T$, with $V$ and $T$ the on shell amplitudes, and the function $G$ is given by the diagonal matrix

$$ G_{ii} = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_{1i}^2 + i\epsilon} \frac{1}{(P-q)^2 - m_{2i}^2 + i\epsilon} $$

5
where $P$ is the total fourmomentum of the meson-meson system.

The BS equation sums the series of diagrams depicted in Fig. 1. In the nuclear medium, one has to add the diagrams depicted in Fig. 2, which stem from the interaction of the pions with the medium, through $ph$ and $\Delta h$ excitation. We neglect here the small s-wave pion-nucleon interaction. The interesting finding of [17] and [20] was that the contact terms involving $\Delta h$ excitation of diagrams (b)(c)(d) canceled exactly the off shell contribution from the meson meson vertices in the term of Fig. 2(a). Hence technically one only had to evaluate the diagrams of the free type (Fig. 1) and those of Fig. 2(a) (plus higher order iterations) in order to evaluate the pion pion scattering amplitude in the nuclear medium.

In [20], only the pions were renormalized inside the medium since the work was only concerned with the meson-meson interaction at low energies where the kaons do not play much of a role. However, in order to address now the question of the $f_0(980)$ and $a_0(980)$ resonances we shall have to take into account the kaon selfenergy in the medium. Since we are close to the two kaon threshold, the s-wave part of the kaon selfenergy is the relevant ingredient. For completeness, we shall also consider the p-wave selfenergy due to $\Sigma h$ or $\Lambda h$ excitation, as in [43], although it does not play an important role in the present problem.

The loop involving pions is done as in [20]. This provides the $\pi\pi$ matrix element of the $G$ function in the medium. As for the $KK$ intermediate states we have an asymmetric situation. In particular, the selfenergy of the $K$ requires special care. We will use the results of the detailed model of Ref. [13]. On the other hand, the $K$ selfenergy can be accounted for in a
much simpler way since there are no resonances with strangeness $S = 1$ and the $KN$ interaction is quite smooth. Altogether, $t\rho$ gives a very reasonable approximation to the $K$ selfenergy. By taking results from [23] or [40] we can write

$$\Pi = \frac{1}{2}(t_{KP} + t_{Kn})\rho \approx 0.13m^2_K\rho/\rho_0$$  \hspace{1cm} (5)$$

where $t$ is the elastic $K$-nucleon amplitude, $\rho$ is the nuclear density and $\rho_0$ is the normal nuclear density.

The new meson-meson amplitude in the medium is now given by means of the modified BS equation

$$\tilde{T} = V\tilde{G}\tilde{T},$$  \hspace{1cm} (6)$$

where the $K\bar{K}$ matrix element of $\tilde{G}$ is given by

$$\tilde{G}_{K\bar{K}} = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m^2_{\bar{K}} - \Pi_K(q^0, q, \rho)} \frac{1}{(P - q)^2 - m^2_K - \Pi_K(P^0 - q^0, q, \rho)}$$ \hspace{1cm} (7)$$

The $\bar{K}$ propagator can be written in the Lehmann representation,

$$\frac{1}{(P - q)^2 - m^2_{\bar{K}} - \Pi_K(P^0 - q^0, q, \rho)} = \int_0^\infty d\omega \frac{S_{\bar{K}}(\omega, q, \rho)}{(\omega^2 + i\epsilon)(P^0 - q^0)^2 - \omega^2 + i\epsilon}$$ \hspace{1cm} (8)$$

where $S_{\bar{K}}(\omega, q, \rho)$ is the spectral function of the $\bar{K}$ in the medium, as described in [13]. The $q^0$ and angular integrations can be done analytically and then we find

$$\tilde{G}_{K\bar{K}} = \frac{1}{2\pi^2} \int_0^\infty d\omega \int_0^{q_{max}} dq q^2 S_{\bar{K}}(\omega, q, \rho) \frac{\omega + \tilde{\omega}(q)}{\tilde{\omega}(q)(\omega - (\omega + \tilde{\omega}(q))^2) + i\epsilon}$$ \hspace{1cm} (9)$$

where $\tilde{\omega}(q)$ is given by

$$\tilde{\omega}(q) = \sqrt{q^2 + m^2_{\bar{K}} + \Pi_K}.$$ \hspace{1cm} (10)$$

For the $\pi\eta$ intermediate channel we proceed in the same way. The $\pi$ propagator is also written in terms of the Lehmann representation, as done in [20] and the $\eta$ propagator explicitly in terms of the eta selfenergy, as we have done above for the $K$. We also take the $t\rho$ approximation for the $\eta$ selfenergy. However, the $t$ matrix is not so well known in this case. We take the following amplitudes from the fit of [48]

$$t^{-1} = \frac{M}{4\pi\sqrt{s}} \left( \frac{1}{a} + r_0q_0^2 + s_0q_0^4 - iq_0 \right),$$ \hspace{1cm} (11)$$

where $M$ is the nucleon mass, $\sqrt{s}$ is the center of mass energy of the $\eta$-nucleon system, and $q_0$ is the momentum of the $\eta$ meson in the same system, assuming
$\eta$ and nucleon to be on shell, $a = 0.75 + 0.27i \text{ fm}$, $r_0 = -1.50 - 0.24i \text{ fm}$ and $s_0 = -0.10 - 0.01i \text{ fm}^3$. We use these results from $\eta N$ threshold up to a value of $\sqrt{s}$ 200 MeV above it. Outside the region of validity of this fit, we have assumed a constant selfenergy equal to the one of the closest extreme of the parametrized region. The uncertainties associated to this assumption are small since taking just a zero $\eta$ selfenergy outside that range of energies changes the results by less than 5%.

3 Other medium corrections

3.1 Tadpole terms

In this section we consider some additional many body corrections. We begin by the contribution of the diagram depicted in Fig. 3 which has been

![Tadpole $\sigma$ selfenergy diagram.](image)

Figure 3: Tadpole $\sigma$ selfenergy diagram.

considered in [49] using the linear sigma model, with sigmas and pions as elementary fields. The Lagrangian gives rise to a vertex with three sigmas as shown in the figure which produces a many body correction, assuming a certain coupling of the sigma to the nucleons, which in [49] is borrowed from the Bonn phenomenological boson exchange models of the $NN$ interaction [50].

On the other hand, the $\chi PT$ Lagrangian involves only pseudoscalar meson fields. The sigma can be generated dynamically through the rescattering of the pions. The closest analog to the diagram of Fig. 3 is given by the series of Fig. 4 where the sigma is generated to the left and right of the nucleon-hole loop by means of the iteration of the chiral Lagrangian. In order to evaluate

![Some tadpole diagrams contributing to the meson-meson scattering amplitude.](image)

Figure 4: Some tadpole diagrams contributing to the meson-meson scattering amplitude.
the contribution of these diagrams we use the chiral Lagrangians involving the octet of baryons and the octet of pseudoscalar mesons[28, 29, 30]. The terms needed come from the covariant derivative terms of the Lagrangian. After some trivial algebra, we find that for the scalar channel these terms are proportional to $\bar{p}\gamma^\mu p - \bar{n}\gamma^\mu n$ and therefore would vanish in symmetric nuclear matter after summation over protons and neutrons.

Considering higher order corrections, there is another possible way to generate an analogous structure as it is shown in Fig. 5.

\begin{center}
\begin{tikzpicture}
\draw[thick,dashed] (0,0) -- (1,1);
\draw[thick,dashed] (1,0) -- (0,1);
\draw[thick,dashed] (-1,0) -- (0,1);
\draw[thick,dashed] (0,-1) -- (1,0);
\end{tikzpicture}
\end{center}

Figure 5: Higher order tadpole diagram.

Here one also needs the coupling of the sigma, generated through pion-pion interaction, with the nucleons. The scalar isoscalar exchange in the $NN$ interaction has been addressed in [51, 52] only at the perturbative level. In [53] it has been revisited taking into account all the meson meson rescattering which generates the sigma. In this latter work one finds an attraction (not generated in the perturbative approach) at intermediate distances, but weaker than in the Bonn model because an appreciable repulsion sets up already at distances like 0.7 fm, which grows fast at small distances. However, the relevant magnitude would be the strength of the potential in momentum space at zero momentum, which is what is met in Fig. 5 and there the strength of the potential of [53] is about one order of magnitude smaller than for the Bonn potential. Hence, from this source we also get a negligible contribution to the modification of the $\pi\pi$ scattering in the nuclear medium.

### 3.2 Roper-hole excitation

Finally, we will consider the excitation of resonances from the occupied nucleon states by the pair of mesons. This requires, in our case, a resonance which decays into a nucleon and two pions (two mesons in general) in s-wave and with I=0. There is such a candidate at the energies where we are concerned which is the $N^*(1440)$ Roper resonance. The branching ratio for this decay is small but it plays a very important role in the $\pi N \rightarrow \pi\pi N$ reaction [54, 55, 56, 57, 58]. It has also been shown that this mechanism is the dominant in the $NN \rightarrow NN\pi\pi$ reaction close to threshold [59, 60]. In the present problem it could contribute via the mechanism depicted in Fig. 6.
Figure 6: First order resonance-hole contribution to the meson meson scattering.

The evaluation of this mechanism is straightforward in the $SU(2)$ sector. In the first place we take the Lagrangian suggested in [57] which reads

\[ \mathcal{L}_{N^*N\pi\pi} = c_1^* \bar{\psi} N^* \chi + \psi N - \frac{c_2^*}{M^*} \left( \partial_\mu \partial_\nu \bar{\psi} N^* \right) u^\mu u^\nu \psi N + h.c. \] (12)

where

\[ \chi_+ = m_\pi^2 \left( 2 - \frac{\vec{\phi}^2}{f^2} + \ldots \right); \quad u_\mu = -\frac{1}{f} \vec{\tau} \partial_\mu \vec{\phi} + \ldots \] (13)

and $\vec{\phi}$ is the pion field. The value of the $c_1^*$ and $c_2^*$ constants is not fully determined by the partial decay width of the $N^*$, which only tells us that these values satisfy the equation of an ellipse. One needs to find extra constraints and in [59] it was found that the best set that lead to agreement with the $\pi^- p \rightarrow \pi^+ \pi^- p$ reaction was $c_1^* = -7.27 GeV^{-1}$ and $c_2^*=0$. This was corroborated in [22] by looking at all the different isospin channels. With the $c_1^*$ term alone we obtain the Lagrangian, after we expand up to two pion fields

\[ \mathcal{L}_{N^*N\pi\pi} = -\frac{1}{f^2} c_1^* m_\pi^2 (\pi^0 \pi^0 + 2\pi^+ \pi^-) (\bar{p}^* p + \bar{n}^* n) + h.c. \] (14)

There is no experimental information on the $N^* NK \bar{K}$ coupling. For simplicity we will assume the simple generalization of the previous Lagrangian

\[ \mathcal{L} = \frac{1}{2} c_1^* Tr(BB)Tr(\chi_+) \] (15)

The baryonic matrix $B$ and the mesonic matrix $\chi_+$ can be found in Ref. [29]. Expanding this Lagrangian up to terms with two meson fields we obtain and keeping only those terms relevant to our calculation we get

\[ \mathcal{L} = -\frac{c_1^*}{f^2} \left\{ m_\pi^2 (\pi^0 \pi^0 + 2\pi^+ \pi^-) + m_K^2 (K^0 \bar{K}^0 + K^+ \bar{K}^-) \right\} (\bar{p}^* p + \bar{n}^* n) + h.c. \] (16)
We can see that the strength of the coupling to $K\bar{K}$ is $\frac{m_{\pi}^2}{m_{\pi}^2}$ times that of the charged pions. Other simple generalizations of the $SU(2)$ Lagrangian, i.e. $Tr(BX+B)$, also provide that scaling, although with some different numeric factors and charge combinations. With the choice of Eq. [16], the contribution of the mechanism of Fig. 6 to the meson meson scalar isoscalar interaction is then given by

$$V'_{ij(N^*N)} = (c_1^*)^2 \frac{4m_i^2 m_j^2}{f^4} U(p^0, \vec{p} = 0, \rho)$$  \hspace{1cm} (17)$$

where $i, j$ stand for the physical meson states, $m_i, m_j$ are the masses of the initial and final mesons and $U(p^0, \vec{p} = 0, \rho)$ is the complex Lindhard function for the excitation of resonances, taken from [61], which in the limit of $\vec{p}=0$, which one has for a meson pair in their center of mass frame, has the simple expression

$$U(p^0, \vec{p} = 0, \rho) = \frac{\rho}{p^0 - m_{N^*} + m_N + \frac{iN^*_N}{2}}.$$  \hspace{1cm} (18)$$

This leads to an additional contribution to the $\pi\pi$ "potential" used in the Bethe Salpeter equation which is given by the matrix elements for $I=0$ (for $I=1$ one gets zero contribution)

$$V'_{11} = 2(c_1^*)^2 \frac{4m_{\pi}^4}{f^4} U(p^0, \vec{p} = 0, \rho)$$  \hspace{1cm} (19)$$

$$V'_{12} = \sqrt{3}(c_1^*)^2 \frac{4m_{\pi}^2 m_{\pi}^4}{f^4} U(p^0, \vec{p} = 0, \rho)$$  \hspace{1cm} (20)$$

$$V'_{22} = \frac{3}{2}(c_1^*)^2 \frac{4m_{\pi}^4}{f^4} U(p^0, \vec{p} = 0, \rho)$$  \hspace{1cm} (21)$$

Now one must be cautious to use the empirical value of the $c_1^*$ parameter when the potential of the former equations is put into the kernel of the BS equation. Indeed, at the level of terms linear in the density one will generate the diagrams of the figure

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7.png}
\caption{Higher order resonance-hole contribution to the meson meson scattering.}
\end{figure}

This means that we are generating the iteration of the free mesons to the right and the left of the $N^*h$ excitation. As a consequence the empirical
coupling should be related to a bare one by

\[ c_1^* = c_{1B}^* (1 + G_{\pi\pi} T_{\pi\pi,\pi\pi}^{I=0} + \sqrt{\frac{4 m_k^2}{3 m_{\pi}^2}} G_{K\bar{K}} T_{K\bar{K},\pi\pi}^{I=0}) \]  

(22)

Thus the calculations must be done using the bare coupling, since the free pion interaction renormalizes it to the effective values demanded by the \( N^* \) decay into two pions in s-wave.

4 Results and discussion

In the first place let us discuss the results in the I=1 channel. In figs. 8,9 we show the real and imaginary parts of the \( K\bar{K} \rightarrow K\bar{K} \) and \( \pi\eta \rightarrow \pi\eta \) amplitudes for different values of the nuclear density at energies around the \( a_0(980) \) meson. An inspection to the imaginary part of Fig. 8 seems to indicate that the peak of its magnitude, corresponding to the apparent mass of the \( a_0(980) \) resonance, moves to low energies as the density increases, producing a shift of about \(-50\) MeV at \( \rho = \rho_0 \). This shift is also visible in the real part by looking at the point where the real part changes sign. The apparent width measured from the imaginary part of the amplitude becomes bigger as the density increases and for \( \rho = \rho_0 \) becomes as large as 200 MeV from an apparent free width of around 90 MeV. This behaviour is, however, not reproduced in the \( \pi\eta \rightarrow \pi\eta \) amplitude.

The first difference one may observe is the presence of a larger background, even in free space, which makes it not resemble a Breit Wigner resonance so much. For this channel, the \( a_0 \) resonance does not move much with the density and the width becomes very large already at small densities, to the point that at densities of the order of one half \( \rho_0 \) the resonant shape is practically lost. Given the fact that the \( a_0(980) \) meson is usually observed in mass distributions of \( \pi\eta \) in the final state of some reaction, the relevant magnitude entering the cross section of these reactions close to the resonance is the modulus squared of the \( \pi\eta \rightarrow \pi\eta \) amplitude [62] plotted in Fig. 10. What we observe there is, indeed, that the resonance melts very fast as the density increases and at densities of the order of \( \rho_0/2 \) there is practically no resonant trace left. The fast disappearance of this relatively narrow resonance in nuclei is probably one of the most striking predictions for this channel.

The I=0 channel has a richer structure as a function of the density. In Fig. 11 we show the amplitude \( \pi\pi \rightarrow \pi\pi \) in a range of energies from 200 MeV to 1100 MeV. The figure shows results at low energies already discussed in [20]. As one can see in the figure, there is an accumulation of strength in the imaginary part below threshold which was first pointed out in [11] and has also been predicted in other approaches [19]. The relationship of this increased strength to the enhanced two pion distribution in \((\pi,2\pi)\) reactions
Figure 8: Real and imaginary part of the kaon-antikaon scattering amplitude in the I=1 channel as a function of the center of mass energy for different nuclear densities. Solid line, free amplitude; long dashed line, $\rho = \rho_0/8$; short dashed line, $\rho = \rho_0/2$; dotted line, $\rho = \rho_0$; dashed dotted line, $\rho = 1.5\rho_0$.

in nuclei [16] at small invariant masses has been discussed in [21, 22], but according to the detailed calculation of [22] it is not enough to reproduce the experimental data.

The intermediate region of energies is quite interesting and no much attention has been given to it so far. There one can see a drastic decrease of the strength of the imaginary part as the density increases. This reduction could lead to appreciable changes in the two pion production reactions in nuclei, like the $(\gamma, 2\pi)$ reaction for which experiments are already becoming available [63]. The region of the $f_0(980)$ resonance is also interesting. By looking both at the dip of the imaginary part of the amplitude, as well as to the position of the zero of the real part, we can see that the position of the resonance does not change when the density increases. We observe,
however, a gradual melting of the dip of the imaginary part which comes as an interference between the background of the $\sigma$ meson contribution and the contribution of the $f_0(980)$ resonance.

It is interesting to discuss what happens when we introduce the $N^*h$ excitation, which is a novel ingredient with respect to the approach of [20]. In Fig. 12 we show the results in which the SU(3) version of the coupling of the $N^*$ to $N\pi\pi$ and $NK\bar{K}$ is used. We can see that the inclusion of this new ingredient barely modifies the results of the amplitude in all the range of energies shown. These changes amount to about a 10 per cent increase of the strength of the imaginary part of the amplitude in the region around 300 to 400 MeV. We do not show it here, but point out that using the SU(2) version of the $N^*$ coupling to $N\pi\pi$, in which the $N^*$ only couples to $N\pi\pi$, practically does not change the results with respect to those in which the $N^*$ is allowed to couple to pions and kaons.

The role of the $N^*h$ excitation is more apparent in the $K\bar{K} \rightarrow \pi\pi$ and

Figure 9: Same as Fig. 8 for $\pi\eta$ scattering.
$K\bar{K} \rightarrow K\bar{K}$ amplitudes. In Fig. [13] we show the $K\bar{K} \rightarrow \pi\pi$ amplitude for different densities. This amplitude is better suited than the $\pi\pi \rightarrow \pi\pi$ in order to show the $f_0(980)$ resonance because it does not have a large background. Notice that in this inelastic channel the $f_0(980)$ shows up as a Breit Wigner contribution rotated 90 degrees. Thus, the roles of the real and imaginary parts of the amplitude are interchanged [64]. We can see that as the density increases the position of the resonance barely moves. The width, however, grows with the density from a free value of around 30 MeV to about 100 MeV at $\rho = \rho_0$. If we include the $N^*h$ contribution in the SU(2) formulation there are no appreciable changes with respect to those shown in the figure. The results are however quite different if we include the $N^*h$ contribution in the SU(3) formulation. These results are shown in Fig. [14]. We can see there that around 300 to 400 MeV a resonant like structure develops with the imaginary part showing a negative peak and the real part changing fast.
Figure 11: Real and imaginary part of the pion-pion scattering amplitude in the I=0 channel as a function of the center of mass energy for different nuclear densities. Meaning of the lines as in Fig. 8

around a zero value. This is a reflection of the $N^*h$ excitation which in this case is magnified because of the large coupling of the $N^*h$ excitation to $K\bar{K}$, as we saw in section 3. As we discussed there, we found a large coupling, of the order of $m_K^2/m_\pi^2$ that of the pion, based on a generalization to SU(3) of the pion coupling. We also saw that there were ambiguities, but any simple generalization led to a coupling of this order of magnitude. In spite of this huge coupling, we saw no visible effects in the $\pi\pi \to \pi\pi$ amplitude. Here it shows clearly in a large medium change of the $\pi\pi \to K\bar{K}$ amplitude. The effects in the $K\bar{K} \to K\bar{K}$ amplitude are even more pronounced.

In Fig. 12 we show the $K\bar{K} \to K\bar{K}$ in the energy region around the $f_0$ resonance. The density effects around the pole can be appreciated better and one can see that even at $\rho = \rho_0$ the shape of the resonance is not lost, but the width increases to about 100 MeV at $\rho_0$. 

16
As we have said, there are some elements of uncertainty tied to the extrapolation of the $N^*$ coupling to kaons, and it would be worth trying to find some observable consequences of the assumptions made. Since the larger effects are seen in a region where the kaons are far off shell, one can only hope to observe indirect effects on reactions were the kaons appear as intermediate states. The large values obtained in the amplitudes will certainly be softened by the small weight of a $K\bar{K}$ propagator where the two kaons are quite off shell, so one should not expect drastic changes. Yet, even moderate changes might be relevant in some processes like the $(\pi, 2\pi)$ reaction in nuclei, where it was shown in [22] that there were large cancellations between terms to give a final result smaller than the contribution of individual terms, such that any small changes in one of them might alter the final balance. The finding of indirect evidence of this directly unobservable $N^*$ coupling to $N$ and kaons would be an important test of particle symmetries.

In any case the interesting medium effects found here, independent of the
still unknown couplings, would certainly call for devoted experiments from which we could learn more about the nature of the scalar resonances and the way the meson meson interaction is changed in a medium. Reactions like $\gamma p \rightarrow \pi\pi p$ have already been suggested as a means to observe the scalar resonances [65]. Their extension using nuclear targets is certainly feasible and, together with other experiments, should be encouraged.

5 Conclusions

In section 3 we addressed the question of new contributions to the $\pi\pi$ scattering in a nuclear medium beyond those already considered in other approaches. One of the terms considered in which a nucleon loop is attached to the four meson vertex was found to be zero for symmetric nuclear matter. Other
Figure 14: Same as Fig. 13, including $N^*h$ excitations.

possible mechanisms which would simulate a three sigma vertex coupling were also estimated to be much weaker than previously suggested. These results would further strengthen those obtained in \cite{20, 22}, which would in turn mean that the experimental problem of the enhanced invariant $\pi\pi$ mass close to threshold would not be solved yet.

The main topic of the present paper has been the discussion of the renormalization of the properties of the scalar meson resonances, concretely the $f_0(980)$ and the $a_0(980)$ resonances, in the nuclear medium. The renormalization required the use of the kaon selfenergy for which we have used a recent one deduced from chiral Lagrangians and which is consistent with the information of kaonic atoms. We have systematically tried to use the chiral unitary formalism in the different aspects of the problem, be the generation of the resonances through the meson meson interaction given by the chiral meson Lagrangians, or the meson baryon interaction, which for the most delicate case, the one of the $K^-$, is also obtained by means of a nonperturbative
Figure 15: Real and imaginary part of the kaon-kaon scattering amplitude in the $I=0$ channel as a function of the center of mass energy for different nuclear densities. Meaning of the lines as in Fig. 8

... chiral approach. The results obtained are interesting, we do not observe an appreciable change of the position of either resonance. However, the widths are substantially changed. In the case of the $f_0(980)$ resonance the width passes from 30 MeV in the free case to about 100 MeV at normal nuclear matter. In the case of the $a_0(980)$ resonance the width grows so fast with density that even at $\rho_0/2$ there is practically no trace of the resonance. The next step should be the search for these effects in nuclear experiments which can help shed new light on the nature of these resonances and the behaviour of kaons in nuclear matter.
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