Nonequilibrium transport through a quantum dot weakly coupled to Luttinger liquids

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Abstract

We study the nonequilibrium transport through a quantum dot weakly coupled to Luttinger liquids (LL). A general current expression is derived by using nonequilibrium Green function method. Then a special case of the dot with only a single energy level is discussed. As a function of the dot’s energy level, we find that the current as well as differential conductance is strongly renormalized by the interaction in the LL leads. In comparison with the system with Fermi liquid (FL) leads, the current is suppressed, consistent with the suppression of the electron tunneling density of states of the LL; and the outset of the resonant tunneling is shifted to higher bias voltages. Besides, the linear conductance obtained by Furusaki using master equation can be reproduced from our result.

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Tunneling phenomena in mesoscopic systems have been intensively studied for years both experimentally and theoretically. Since early 1990s, the tremendous progress in nanofabrication and the discovery of novel one-dimensional (1D) materials, especially the carbon nanotubes, have led to a revived interest in studying the tunneling in systems including 1D conductors.

It is well-known that at low temperatures the two-dimensional (2D) and three-dimensional (3D) interacting electron systems can be described by Landau’s Fermi liquid (FL) theory. However, the interacting electron systems in one dimension (1D) is quite different, it behaves as Luttinger liquid (LL), for that theory predicts a series of unique characters, e.g., the absence of Landau quasiparticles, spin-charge separation, suppression of the electron tunneling density of states (DOS), anomalous power laws for transport coefficients with interaction-dependent exponents, etc. Naturally, one may ask whether the (resonant) tunneling between LLs is different from that of FLs.

Recently, Furusaki reported a theoretical study of the resonant tunneling through a quantum dot (QD) weakly coupled to two LL leads (hereafter as LL-QD-LL). Assuming that the tunneling is an incoherent sequential tunneling, the author derived the linear conductance using the master equation approach. Similar studies have been done by other authors earlier. In this paper, we present an investigation for a LL-QD-LL system. Differently, we consider the tunneling as a coherent process and study the nonequilibrium current through the system with a finite bias voltage applied on the two LL leads. By using nonequilibrium Green function technique, a general current formula is derived. We find that both the current and differential conductance are strongly renormalized by the interaction in the LL leads. In current-voltage characteristics, the current is first suppressed, consistent with the suppression of the electron tunneling density of states; and the outset of the resonant tunneling is found to be shifted to higher bias voltages, comparing to the system FL-QD-FL. Finally, the linear conductance obtained by Furusaki using master equation can be reproduced from our result.

The Hamiltonian of the system can be split into three parts: \( H = H_{\text{leads}} + H_d + H_T \), where \( H_{\text{leads}} = H_L + H_R \) denotes the left and right LL leads in its standard form (see [1]), \( H_d = \sum_n \varepsilon_n d_n^+ d_n \) is the Hamiltonian of the QD, with \( \{d_n^+, d_n\} \) the creation/annihilation operators of the n-th energy level in the dot (for simplicity, the intradot Coulomb interaction has been neglected). \( H_T \) is the tunneling Hamiltonian and can be written as
where $\Sigma_{n,l}$ is the tunneling constant. The spin indices have been suppressed since they are not important in the following discussion.

Defining the mixed casual Green function $G_{n,\lambda}(t_1, t_2) = -i \langle T d_n(t_1) \psi^+_\lambda(t_2) \rangle_H$ and Green functions of all the other types in the standard form, \[13\] the current from left LL lead into the QD under a bias voltage $-V$ applied on the right lead can be evaluated as follows (in units of $\hbar = 1$):

$$J_L = e \left\langle \frac{dN_L}{dt} \right\rangle = -2e \sum_n \text{Re} \left[ t^*_n \Sigma_n G_{n,L}^<(t, t) \right],$$

where $G_{n,L}(\tau, \tau') = \sum_m \int_d d\tau G_{n,m}(\tau, \tau_1) t_{m,L} g_L(\tau_1, \tau')$ is the Dyson equation for contour Green function $G_{n,L}(\tau, \tau'), G_{n,m}(\tau, \tau') = -i \langle T_c d_n(\tau) d^+_m(\tau') \rangle_H$ is the full Green function of the QD, $g_L(\tau, \tau') = -i \langle T_c \psi_L(\tau) \psi^+_L(\tau') \rangle_H$ is the local Green function at the end point of the left LL lead without the coupling to the QD, and $T_c$ is the contour-ordering operator.

After using Langreth theorem of analytic continuation, and the Fourier transformation, the current can then be expressed as:

$$J_L = -2e \sum_{n,m} \text{Re} \int \frac{d\omega}{2\pi} \left[ t^*_n \left( G_{n,m}^r(\omega) g_L^<(\omega) + G_{n,m}^< \right) t_{m,L} \right].$$

The Dyson equation for $G_{n,m}(\omega)$ is $G_{n,m}(\omega) = \delta_{n,m} g_n(\omega) + \sum_l g_n(\omega) \Sigma_{n,l}(\omega) G_{l,m}(\omega)$, where $\Sigma_{n,l}(\omega) = \sum_{\lambda=L/R} t^*_{n,\lambda} t_{l,\lambda} \Sigma_{n,\lambda} g_\lambda(\omega)$ is the irreducible self-energy and $g_n(\omega)$ is the free Green function of the dot without the coupling between the QD and leads. Using Langreth theorem and noticing $\Sigma^{r,a} = \pm \frac{1}{2}(\Sigma^> - \Sigma^<)$, the current expression, Eq.(3), can be reduced to

$$J_L = e \sum_{n,m} \int \frac{d\omega}{2\pi} t^*_n \left\{ \left[ (g^r)^{-1} - \Sigma^r \right]^{-1} g_L^> \Sigma^< - g_L^< \Sigma^> \right\} n,m \right] \left[ (g^a)^{-1} - \Sigma^a \right]^{-1} t_{m,L}. \tag{4}$$

Neglecting the off-diagonal tunneling matrix elements and their energy-dependence, which is equivalent to take $t^*_n t_{m,L} = \delta_{n,m} |t_{n,L}|^2$, \[14\] then Eq.(3) reduces to

$$J_L = e \sum_n |t_{n,L}|^2 \int \frac{d\omega}{2\pi} \frac{g_L^> g_L^< - g_R^> g_R^<}{(g_n^r)^{-1} - \Sigma_{n,n}^r} \left( (g_n^a)^{-1} - \Sigma_{n,n}^a \right), \tag{5}$$

which describes the nonequilibrium current in LL-QD-LL, and is the the central result of this work. Similarly, one can easily obtain the current from right LL lead to the QD, $J_R$, 

\[H_T = \sum_{n,\lambda=L/R} (t_{n,\lambda} d^+_n \psi_\lambda + h.c.), \tag{1}\]
just by changing the subscript in Eq.(5) \( L \rightarrow R \), and easily to see \( J_L + J_R = 0 \), the current conservation, as expected.

As an important example, let us consider the quantum dot with only a single energy level, i.e., \( H_d = \varepsilon d^+d \). After taking a contour integral, Eq.(5) reduces to

\[
J = J_L = -J_R = e|t_Lt_R|^2 \left[ \frac{g_L^< (\varepsilon) g_R^> (\varepsilon) - g_L^> (\varepsilon) g_R^< (\varepsilon)}{\sum_{\lambda=L/R} |t_{\lambda}|^2 |g_{\lambda}^< (\varepsilon) - g_{\lambda}^> (\varepsilon)|} \right] + O(|t_Lt_R|^2). \tag{6}
\]

In which \( g_L^{<,>}(\varepsilon) \) and \( g_R^{<,>}(\varepsilon) \) are the lesser (greater) Green functions of the left and right leads without the couplings to the dot, and can be directly quoted from: \[9\]

\[
g_L^{<,>}(\varepsilon) = \pm i\frac{T}{|t_L|^2} e^{\mp \frac{\varepsilon}{2T}} \gamma_L(\varepsilon),
\]

\[
g_R^{<,>}(\varepsilon) = \pm i\frac{T}{|t_R|^2} e^{\mp \frac{\varepsilon - eV}{2T}} \gamma_R(\varepsilon - eV),
\]

where \( T \) is the temperature, and \( V \) the bias voltage. \( \gamma_{L/R}(\varepsilon) \) is defined as \( \gamma_{L/R}(\varepsilon) = \frac{\Gamma_{L/R}}{2\pi T} \frac{\pi T}{\Gamma(1/\gamma_{L/R})} \), here \( \Gamma(x) \) is the Gamma function, \( g_{L/R} \) are interaction parameters characterizing the left/right LL liquids, which should not be confused with the Green functions with the same symbols, \( \Gamma_{L/R} \) describes the effective level broadening of the dot, proportional to \( |t_{L/R}|^2 \), and \( \Lambda \) is the high-energy cutoff or a band width. \[9\]

In the lowest order approximation, Eq.(6) reduces to

\[
J = eT \frac{\gamma_L(\varepsilon) \gamma_R(\varepsilon - eV)}{\gamma_L(\varepsilon) \cosh\left(\frac{\varepsilon}{2T}\right) + \gamma_R(\varepsilon - eV) \cosh\left(\frac{\varepsilon - eV}{2T}\right)} \sinh\left(\frac{eV}{2T}\right), \tag{8}
\]

Eq.(8) describes the nonequilibrium current of LL-QD-LL under a finite bias voltage. In the following, we present the numerical studies based on Eq.(8) for the symmetric case, i.e., \( g_L = g_R = g \), \( t_L = t_R \) and \( \Gamma_L = \Gamma_R \).

Fig.1 shows the current vs bias voltage at a fixed value \( \frac{\varepsilon}{2T} = 5 \) for different interaction parameters of the LL leads, \( g = 0.9, 0.7, \frac{1}{2}, \frac{1}{3}, \frac{1}{10} \). The system with FL leads is also shown for comparison (see the curve with \( g = 1 \) in Fig.1). We find the following features: (1) For FL-QD-FL, a steep increase occurs around \( \frac{eV}{2T} = 5 \), which is consistent with the suppression of the electron tunneling of states of the LL. The suppression becomes stronger with the decrease of \( g \) (or the increase of the interactions in the leads). (2) The outset of the resonant tunneling varies with the change of \( g \), from \( eV = \varepsilon \) for \( g = 1 \) (i.e., the FL leads) to \( eV = 2\varepsilon \) for \( g \sim 0 \) (i.e., the strongest interacting LL leads). A current plateau appears again for \( g \rightarrow 0 \) around \( eV = 2\varepsilon \). (3) Somehow, the curves with \( g \lesssim \frac{1}{2} \) intersect approximately at a
point \( (eV/2T = 10, J/J_{\text{max}} = 1/2) \), which may indicate the peculiar behavior for strong interacting case.

The second feature above mentioned can also be seen from Fig.2, in which the differential conductance vs \( \varepsilon/2T \) for different interaction parameters of LL, \( g = 1, 1/2, 1/10 \), is presented. The location of the peak of conductance is shifted from \( \varepsilon = eV \) to \( \varepsilon = eV/2 \) with the increase of \( g \).

The nonequilibrium current as a function of \( \varepsilon/2T \) at a fixed bias is shown in Fig.3. Comparing to the current of FL-QD-FL, these curves are strongly renormalized by the interactions of the LL leads: instead a mesa for FL-QD-FL, a peak occurs at \( \varepsilon = eV \); and the half-width of the peak becomes narrower with the increase of the interactions of electrons in the LL leads. The dependence of the peak current on \( T \) and \( \varepsilon \) can be obtained by estimating the current \( J \) at \( \varepsilon = eV/2 \). Because the asymptotic limit of the Gamma function is \( \Gamma(x) \sim x^{1/2}e^{-x}\sqrt{2\pi} \) (for \( |x| \to \infty \), and \( |\text{arg} \ x| \leq \pi - \delta, \delta > 0 \)), one finds that at low temperatures \( T \ll \xi/2 \), the current behaves as \( J \sim (eV/2)^{1/g-1} \), whereas at high temperatures \( T \gg \xi/2 \), \( J \sim (eV/2A)^{1/g-2} \). Similar scaling forms have been derived by Chamon and Wen by considering the incoherent sequential tunneling. \[ \text{[12]} \]

It should be pointed out that our calculation is valid only when the higher order contributions \( \gtrsim O(|t Lt_{R}|^2) \) can be neglected. The pole of the integrand in Eq.(4) satisfies the equation \( (g^r(\omega))^{-1} - \Sigma^r(\omega) = 0 \). Neglecting the shift of the energy level of the QD, the lowest order solution is \( \omega = \varepsilon + Im \Sigma^r(\varepsilon) \). Thus, the higher order correction can be neglected only if \( \left| \frac{Im \Sigma^r(\varepsilon + Im \Sigma^r(\varepsilon)) - Im \Sigma^r(\varepsilon)}{Im \Sigma^r(\varepsilon)} \right| \ll 1 \), or equivalently, \( \left| \frac{\gamma(\varepsilon + Im \Sigma^r(\varepsilon)) - \gamma(\varepsilon)}{\gamma(\varepsilon)} \right| \ll 1 \). Using the asymptotic limit of the Gamma function, one finds that at low temperatures \( T \ll \frac{\xi}{2}, \frac{eV-\epsilon}{2A} \), the condition reduces to \( \frac{\Gamma}{T} \left( \frac{\xi}{2A} \right)^{1/g-1} \ln \left( \frac{\xi}{2A} \right) \ll 1 \) and \( \frac{\Gamma}{T} \left( \frac{eV-\epsilon}{2A} \right)^{1/g-1} \ln \left( \frac{eV-\epsilon}{2T} \right) \ll 1 \); whereas at high temperatures \( T \gg \frac{\xi}{2}, \frac{eV-\epsilon}{2A} \), \( \frac{\Gamma}{T} \left( \frac{\xi}{2} \right)^{1/g-2} \ll 1 \). Therefore, our evaluations validate only when \( T \neq 0 \) and \( \varepsilon, eV, T \ll \Lambda. \) \[ \text{[16]} \]

Finally, as a quick check, we can obtain the linear conductance \( \sigma \) from Eq.(7) by simply taking \( V \to 0^+ \), which gives:

\[
\sigma = e^2 \frac{1}{2 \cosh \left( \frac{\varepsilon}{2T} \right)} \frac{\gamma_L(\varepsilon) \gamma_R(\varepsilon)}{\gamma_L(\varepsilon) + \gamma_R(\varepsilon)}.
\]

This is exactly the linear conductance obtained by Furusaki, \[ \text{[8]} \] and was confirmed by experiment recently. \[ \text{[17]} \]

In conclusion, we derive a general formula for nonequilibrium dc current through a quantum dot weakly coupled to two Luttinger liquids. We consider the tunneling as a coherent process and use nonequilibrium Green function technique, different from Chamon and Wen
and Furusaki. In symmetric case and for the dot with a single energy level, we find that both the current and the differential conductance are strongly renormalized by electron-electron interactions in the LL leads. We hope that further experiments may provide evidence for these theoretical predictions.

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FIGURE CAPTIONS

Fig. 1 Current, scaled by its maximum at $\frac{eV}{2T} = 5$, vs bias voltage for $g = 0.9, 0.75, \frac{1}{2}, \frac{1}{3}, \frac{1}{10}$. $\varepsilon$ is the energy level of the dot, $g$ is the interaction parameters of the leads. Curves for $g = \frac{1}{2}, \frac{1}{3}, \frac{1}{10}$ intersect approximately at a point $\left(\frac{eV}{2T} = 10, \frac{J}{J_{\text{max}}} = \frac{1}{2}\right)$. The case with FL leads ($g = 1$) is also shown for comparison.

Fig. 2 Differential conductance scaled by its maximum at $\frac{eV}{2T} = 10$ as a function of $\frac{eV}{2T}$ for $g = 1, \frac{1}{2}, \frac{1}{10}$. The location of the maximum is shifted away from $\varepsilon = eV$ for $g = 1$ to $\varepsilon = \frac{eV}{2}$ for $g \sim 0$.

Fig. 3 Current vs $\frac{eV}{2T}$ for $g = 1, \frac{1}{2}, \frac{1}{10}$. The current is scaled by its maximum at bias voltage $\frac{eV}{2T} = 10$. For $g < 1$ (Luttinger liquid), a peak occurs at $\varepsilon = \frac{eV}{2}$, and the half width of the peak becomes narrower with the decrease of $g$. The curve with $g = 1$ (Fermi liquid) is also shown for comparison.
Fig. 1

![Graph showing the relationship between $J/J_{\text{max}}$ and $eV/2T$ for different values of $g$. The legend includes lines for $g=1$, $g=0.9$, $g=0.75$, $g=1/2$, $g=1/3$, and $g=1/10$. The graph illustrates the decrease in $J/J_{\text{max}}$ as $eV/2T$ increases for each value of $g$.](image_url)
Fig. 2

The graph shows the normalized conductance $G/G_{\text{max}}$ as a function of $\varepsilon/2T$ for different values of $g$:

- $g=1$
- $g=1/2$
- $g=1/10$
Fig. 3