A multi-stage traveling-wave thermoacoustically-driven refrigeration system operating at liquefied natural gas temperature

K Luo¹, D M Sun¹, J Zhang¹, Q Shen¹ and N Zhang¹
¹Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou, China
sundaming@zju.edu.cn

Abstract. This study proposes a multi-stage travelling-wave thermoacoustically refrigeration system (TAD-RS) operating at liquefied natural gas temperature, which consists of two thermoacoustic engines (TAE) and one thermoacoustic refrigerator (TAR) in a closed-loop configuration. Three thermoacoustic units connect each other through a resonance tube of small cross-sectional area, achieving “self-matching” for efficient thermoacoustic conversion. Based on the linear thermoacoustic theory, a model of the proposed system has been built by using DeltaEC program to show the acoustic field characteristics and performance. It is shown that with pressurized 5 MPa helium as working gas, the TAEs are able to build a stable and strong acoustic field with a frequency of about 85 Hz. When hot end temperature reaches 923 K, this system can provide about 1410 W cooling power at 110 K with an overall exergy efficiency of 15.5%. This study indicates a great application prospect of TAD-RS in the field of natural gas liquefaction with a large cooling capacity and simple structure.

1. Introduction
Natural gas is recognized as one of the cleanest fossil fuel for its low harmful emissions. With abundant distribution around the world, natural gas has been widely used in power generation, vehicle fuel and domestic application, etc. Liquid natural gas (LNG) only takes up 1/600 of the gas state volume, which is much economical for transportation over large distances and storage in large quantities [1]. The thermoacoustically-driven refrigeration system (TAD-RS), which has no moving part, is a potential technology for the liquefaction process for its reliability and low cost in maintenance. Additionally, its external heating feature is especially adaptable for utilizing many heat sources, such as solar energy or fuel gas from remote area.

The TAD-RS usually contains one or multiple thermoacoustic engine (TAE) and one or more thermoacoustic refrigerator (TAR), sometimes the TAR can be replaced by pulse tube cooler (PTC). The thermal energy is converted into acoustic power in TAE stage and then transferred to the refrigerator, where the acoustic power is consumed to pump heat. The first TAD-RS was developed in 1990 by Radebaugh et al., the system integrated a standing-wave TAE with an orifice pulse tube cooler, which was capable of achieving a lowest temperature of 90 K [2]. Later in 2004, to further improve the system performance, a traveling-wave TAE was used to replace the previous standing-wave TAE, and coupled with three PTCs. The experimental results showed a cooling power of 3.8 kW at 150 K was obtained, with a liquefaction rate of 500 gpd [3]. However, this conventional “Stirling type” TAE
inevitably introduced large standing-wave resonance tubes, leading to low power density and poor refrigeration efficiency, which was hard for industrial application.

Since then, many researches has been carried out to achieve lower temperature and obtain higher efficiency in TAD-RS, mainly based on the concepts of reducing the loss along the system and better coupling between the TAE and TAR [4-8]. In 2002, Yazaki [9] put two regenerators in one loop tube, one of the stages used as TAE generates acoustic power, which is consumed by the other stage to pump heat from the cold heat exchanger, the lowest temperature of 246 K was obtained in experiments. In 2010, de Blok [10,11] improved the looped travelling-wave TAE by inserting 4-stage large TAE cores in one loop to reduce the viscous loss in the regenerators. Driven by the heat sources below 473K, the system was expected to realize an exergy efficiency of 40% [10]. Then they replace one of the four TAEs by one TAR equally spaced in a looped tube, driven by the heating temperature of 512 K, the integrated system achieved a net cooling power of 95.4 W at 227.5 K, with an exergy efficiency of 32% [11], verifying the feasibility of multi-stage TAD-RS driven by low-grade heat sources.

Later such looped multi-stage TAD-RS were proposed to analysis the properties [12-14]. However, those studies mainly focus on low heating temperature with small cooling capacity, which is hardly possible to achieve low temperature below LNG temperature. In 2015, L.M Zhang [15] proposed a symmetrical TAD-RS integrating three stage TAEs with three identical PTCs placed on the branch of the TAE loop, a cooling power of 0.88 kW at 110 K and a maximum exergy efficiency of 8% at 130 K were achieved experimentally. However the system didn’t obtain the expected efficiency because separated PTCs are hard to concentrate cooling power. In 2016, they proposed an asymmetrical TAD-RS with three TAEs and one single PTC output, according the simulation, a cooling power over 1300W and a total exergy efficiency of 15.7% were achieved at 110 K with 20.2 kW input heat at 923 K [16].

It’s worth noting that, although it’s easier to achieve good matching by placing PTC on the branches of the TAE loop, there are inevitably high exergy losses in the phase shifter of PTC (about 13% in their simulation [16]), limiting the overall performance.

In addition, previous three engines and one refrigerator configuration often has lower performance on the engine placed between the other two engines, because the middle engine tends to be in pressure trough and volumetric speed peak, meaning low acoustic impedance in the thermoacoustic core, which usually causes high acoustic losses [12]. In order to achieve high system efficiency, we remove the middle engine and choose two engines and one refrigerator configuration so both the engines can be placed on the high impedance region. This system also characterizes with PTC-like TAR structure (with relatively long and thin thermal buffer tube or pulse tube), which overcomes the high exergy loss of branch type TAD-RS by recycling the acoustic power of the TAR outlet back into the loop. In following sections, numerical investigations are conducted on the distribution of key acoustic characteristics to analysis the system performance and properties.

2. System configuration
The Figure.1 presents a schematic of the multi-stage traveling-wave TAD-RS. It includes two TAE and one TAR, connecting each other with resonance tube (RT) of relatively small cross-sectional area. The thermoacoustic core of TAE or TAR consists of a main ambient heat exchanger (MAHX), a regenerator (REG), a high-temperature exchanger (HHX) in TAE or a cold heat exchanger (CHX) in TAR, thermal buffer tube (TBT), and a secondary ambient heat exchanger (SAHX). The RT is a long hollow tube, connecting three thermoacoustic core to form a closed-loop, the x axis starts from the MAHX in TAE1 as shown in Figure.1.

When the axial temperature gradient in the regenerators of TAEs exceed a critical value, a self-excited thermoacoustic oscillation begins and acoustic power is amplified in TAEs. The good design of MAHX and HHX is important to guarantee the rapid temperature gradient across the regenerator, which is essential for thermoacoustic conversion; REG is filled up with compact porous medium, to realize efficient heat exchange between the solid and gas, the hydraulic radius of regenerator is usually comparable or smaller than the viscous penetration depth and heat penetration depth (related to thermal properties, frequency, etc.) [17]. Similarly in the regenerator of TAR, where temperature is much lower
than TAE, the hydraulic radius is even smaller; Especially, the structure of TAR in this study is different from conventional design, instead of same diameter of the thermoacoustic core, it has rather long and thin TBT or pulse tube like in PTC, which meets the different need for phase shifting in refrigerator; The RT is a key component in TAD-RS because it has the function of both transferring the acoustic power and shifting phase for thermoacoustic core.

As the transmission of acoustic power is shown in Figure 1, acoustic power is generated and amplified through two TAEs and then consumed in the TAR to obtain cooling power, the remaining acoustic work from the outlet of TAR is recycled back to TAE1. In addition, considering the DC-flow usually exists in looped configuration has a significant negative impact on the system performance, an elastic membrane is placed in the loop to eliminate the DC-flow.

In sum, for an efficient TAD-RS, firstly low viscous loss in regenerator should be achieved, in this study by enlarging the cross-sectional area of the core to decrease the velocity; then the traveling-wave phase difference between pressure and volumetric flow rate (close to 0°) should be met to realize efficient thermoacoustic conversion, because of the different acoustic impedance distribution and absence of extra phase shifter in the system, the locations of each thermoacoustic core are very important to achieve “self-matching” between TAEs and TAR.

**Figure 1.** Schematic of a multi-stage traveling-wave thermoacoustic refrigeration system. Abbreviated labels are: thermoacoustic engine (TAE), resonance tube (RT), main ambient heat exchanger (AHX), regenerator (REG); high-temperature heat exchanger (HHX); cold heat exchanger (CHX), thermal buffer tube (TBT), secondary ambient heat exchanger (SAHX).

### 3. Numerical simulation and discussion

#### 3.1. Simulation setting

The simulation is calculated by using the DeltaEC program [18], one dimensional simulation model based on linear thermoacoustics [17]. To validate the model in this study, we first calculated a model compared to de Blok’s refrigerator experiment while the parameters are kept the same with those in the references [10,11], Table 1 compares the results from the calculation and experiment for a curtain operating condition, the working gas is helium of 2.7 MPa, the temperature of HHX and AHX in engine stage are fixed at 169 °C and 12 °C, whereas the temperature of CHX and AHX in refrigerator stage are -33.7 °C and 19.2 °C. The calculated engine efficiency and refrigerator COP are 0.12 and 1.12, and the overall system efficiency is 0.138, which shows a good agreement with experiment. Then the model
can be used to calculate the present configuration by keeping the same calculation targets and varying the thermoacoustic stages and temperature in the heat exchangers.

In the present model, the heating temperature in HHX is set to be 923 K, the temperatures in MAHX and SAHX both are 293 K, and the temperature in CHX is fixed at 110 K. The system charges with 5 MPa of pressurized helium gas, with a frequency around 85 Hz. Resonance tubes are assumed to be adiabatic. In order to simplify the calculation process, we choose the same dimensions for TAE cores and the same diameter for resonance tubes. Based on the program, the multi-stage traveling-wave TAD-RS proposed is designed and optimized with above working conditions, the final optimized dimensions are listed in table 2.

Table 1. A comparison between the performance of the calculation and the experiment in Refs.[10,11]

| Stage | Description          | Unit | Experiment | Simulation |
|-------|----------------------|------|------------|------------|
| Engine| Temperature of HHX   | °C   | 169        | 169        |
|       | Temperature of HHX   | °C   | 12         | 12         |
|       | Thermal efficiency   | -    | 0.10       | 0.12       |
|       | Relative Carnot efficiency | - | 0.29       | 0.35       |
| Refrigerator| Temperature of HHX | °C   | -33.7      | -33.7      |
|       | Temperature of HHX   | °C   | 19.2       | 19.2       |
|       | COP                  | -    | 1.42       | 1.15       |
|       | Relative Carnot efficiency | - | 0.32       | 0.26       |
| Whole system | Overall system efficiency | - | 0.142      | 0.138      |

Table 2. Detailed geometry of the thermoacoustically-driven refrigeration system (TAD-RS)*

| Stage | Component | Diameter | Length | Porosity | Other dimensions                   |
|-------|-----------|----------|--------|----------|-----------------------------------|
| TAE   | AHX       | 60       | 22%    | Shell-tube type, hydraulic diameter $d_h=0.8$ |
|       | REG       | 60       | 80%    | Stainless mesh, wire diameter $d_w=0.05$       |
|       | HHX       | 76       | 80     | 36%      | Plated-fin type, channel width $y_0=0.8$ |
|       | TBT       | 125      |        |          |                                    |
|       | SAHX      | 40       | 14%    | Shell-tube type, hydraulic diameter $d_h=1$    |
| TAR   | AHX       | 60       | 22%    | Shell-tube type, hydraulic diameter $d_h=0.8$ |
|       | REG       | 130      | 50     | 80%      | Stainless mesh, wire diameter $d_w=0.03$ |
|       | CHX       | 40       | 17%    | Plated-fin type, channel width $y_0=0.3$      |
|       | TBT       | 200      |        |          |                                    |
|       | SAHX      | 40       | 14%    | Shell-tube type, hydraulic diameter $d_h=1$    |
| Others| RT1       | 3800     |        |          |                                    |
|       | RT2       | 24       |        | 1000     |                                    |
|       | RT3       | 3500     |        |          |                                    |

*a The listed dimensions are all in millimetres (mm).

The acoustic power flow, $W_a$, can be calculated as

$$W_a = \text{Re} [ p_1 \cdot U_1 ]$$  \hspace{1cm} (1)

Where the $p$ is the pressure, $U$ is the volumetric flow rate, and subscript 1 represents the first order, Re[ ] means the real part of a complex number.

The exergy efficiency is used to assess the effectiveness of energy resource utilization in a thermal system. The total exergy efficiency of two TAEs, $\eta_t$, the exergy efficiency of the TAR, $\eta_r$, and the total exergy efficiency of the system, $\eta_s$, are defined as:
\[ \eta_c = \frac{\Delta W_a}{Q_h} \left( \frac{T_h-T_a}{T_h} \right) \]  

\[ \eta_r = \frac{Q_e}{\Delta W_a} \left( \frac{T_c}{T_a-T_c} \right) \]  

\[ \eta_s = \eta_c \eta_r = \frac{Q_e}{Q_h} \left( \frac{T_h-T_a}{T_h-T_a-T_c} \right) \]

Where \( Q_h \) represents the total heat input of two HHXs, \( Q_e \) is the heat output or cooling power obtained in CHX, and \( \Delta W_a \) is the acoustic power difference between the inlet and outlet of the TAR core.

In order to compare the performance between TAE1 and TAE2, the exergy efficiency in regenerator of TAE1 and TAE2, \( \varepsilon_{e1} \) and \( \varepsilon_{e2} \) respectively, are defined as:

\[ \varepsilon_{e1} = \frac{\Delta W_{e1}}{Q_{h1}} \left( \frac{T_h-T_a}{T_h} \right) \]  

\[ \varepsilon_{e2} = \frac{\Delta W_{e2}}{Q_{h2}} \left( \frac{T_h-T_a}{T_h} \right) \]

Where \( Q_{h1} \) and \( Q_{h2} \) represent the heat input of the HHX of TAE1 and TAE2 respectively, \( \Delta W_{e1} \) and \( \Delta W_{e2} \) are the acoustic power difference between the inlet and outlet of the REG of TAE1 and TAE2, respectively.

### 3.2. Acoustic field characteristics

The acoustic field characteristics of the design of table 1, including the distributions of the pressure amplitude, the volumetric velocity amplitude, the phase angle, the acoustic power, and the normalized acoustic impedance, in the system, are shown in Figure 2.

Figure 2 (a) shows the pressure amplitude, which contains two peaks and two troughs within one wavelength, indicating the sound wave in the system is not pure travelling wave but with some standing-wave component, the standing wave ratio is 3.2 (defined as the ratio of the maximum to the minimum of pressure amplitude [19]), future work needs to be done to decrease the strong acoustic reflection. Three thermoacoustic cores all are located in high-pressure amplitude region, sharp drops appear in all three regenerators (marked in grey strip) due to the viscous loss, and the drop is more notable in the regenerator of TAE2 because of the higher volumetric speed in TAE2 core.

Figure 2 (b) shows the distribution of volumetric speed in the system. Owing to the thermoacoustic conversion, the volumetric velocity both rises significantly in two regenerators of TAE, implying the thermal energy is converted into acoustic power. While the volumetric velocity drops dramatically in the regenerator of TAR, where the acoustic power is consumed to obtain cooling power. As discussed above, for high volumetric velocity in a looped system, a key to reduce the viscous loss is to decrease the velocity amplitude, while a sufficient volumetric velocity is required for efficient thermoacoustic conversion, therefore enlarging cross-sectional area of thermoacoustic core is very necessary for achieving high efficiency.

Figure 2 (c) shows the distribution of the phase of pressure \( \phi(p_1) \), the phase of volumetric velocity \( \phi(U_1) \), and their difference \( \phi(p_1-U_1) \). The phase angle ranges from -180°~180° thus sudden changes of 360° for \( \phi(p_1) \) and \( \phi(U_1) \) occur along the axis. The range of \( \phi(p_1-U_1) \) overall is -60°~60°, implying the system is a traveling-wave domain system. The range of phase difference in the regenerator of TAE1 is -48.1°~16.0°, -31.0°~10.0°, in the regenerator of TAE2, and -12.5°~32.4° in the regenerator of TAR. Although some phase differences are not exact 0°, compared with previous studies [12~14,16], the acoustic field is close enough to the traveling-wave for efficient thermoacoustic conversion.

Figure 2 (d) represents the distribution of acoustic power in the system. In the regenerator of each TAE, the acoustic power rises dramatically due to the thermoacoustic conversion. As shown, about 3603 W and 5515 W of acoustic power is gained in REG1, and REG2, respectively. While in the regenerator
of TAR, about 5727 W acoustic power is consumed to obtain cooling power. It’s worth noting that RTs contributes large acoustic loss in the system, about 986,661,662 W acoustic power are consumed in RT1, RT2 and RT3, respectively.

Figure 2 (e) shows the normalized impedance $Z_n$ ($Z_n=|p/v|/\rho_{mc}$, where $v$ is the velocity of the gas, $\rho_{mc}$ is the mean density and $c$ is the sound speed). According to the definition of $Z_n$, the abrupt rise of the normalized impedance is due to enlarging the cross-sectional area of thermoacoustic core, which is beneficial for reducing the viscous loss thus improving the performance. As shown, high normalized impedance $Z_n$ appears in thermoacoustic core, with average values of 13.4 in TAE1, 8.3 in TAE2 and 21.2 in TAR, close to the value recommend by Gardner and Swift [20] for efficient thermoacoustic conversion.

![Figure 2](image)

**Figure 2.** Distribution of acoustic characteristics along the x axis of the apparatus. (a) Pressure amplitude, (b) volumetric velocity amplitude, (c) phase angle and phase difference, (d) acoustic power, (e) normalized acoustic impedance, (Regenerator regions are marked in grey strip).

3.3. Performance analysis

Figure. 3 gives the cooling power and exergy efficiencies under different cooling temperature. As shown, the cooling power drops almost linearly with the cooling temperature decreasing, generally the exergy efficiency of TAE1 and TAE2 are relatively stable around 60% from cooling temperature of 100 K to 120 K, implying the good matching between the loads (TAR) with TAE. The exergy efficiency of
TAR drops significantly as the cooling temperature decreases. When cooling temperature reaches 110 K, the system achieves a cooling power of 1411 W, with an overall exergy efficiency of 15.5%. The result is close to the performance of the single PTC output TAD-RS proposed by J.Y XU [16], although it has a higher total TAEs exergy efficiency of 49.6% at 110 K, mainly due to the optimization of different diameter of TAEs and RTs, and the PTC in their system has a lower exergy efficiency of 31.6% at 110 K, mainly caused by the large exergy loss in the phase shifter (about 13%). Considering that without the asymmetrical structure optimization for the TAE cores in our study, the total TAEs exergy efficiency \( \eta_e \) is reasonably lower for 39.5% at 110 K in our study, and because the absence of phase shifter in system, the acoustic power from the TAR outlet is recycled into the loop instead of being dissipated in phase shifter on the branch, the TAR in our study has a higher exergy efficiency \( \eta_r \) of 39.2% at 110 K, therefore this agreement validates our model. Again, considering those stages locate differently in acoustic field, there are still some possible room for performance improvement by setting different TAE structure.

![Figure 3. Cooling power and exergy efficiency with different cooling temperature](image)

4. Conclusion
This paper introduces a multi-stage traveling-wave TAD-RS with over 1 kW cooling capacity at liquefied natural gas temperature. This system is characterized by closed-loop configuration with multiple TAE core and one PTC-like TAR, which implies the feasibility for liquefied process of natural gas with a large cooling capacity and compact structure.

In this paper, numerical simulations based on DeltaEC program are conducted to design and investigate the system. With 5 MPa of pressurized helium gas, a cooling power of 1411 W and an overall exergy efficiency of 15.5% is achieved at 110 K, with an input heat of 22.2 kW at 923 K. The distribution of key acoustic characteristics, such as pressure amplitude, volumetric velocity amplitude and phase difference, etc., are represented to analysis the system performance. Large viscous losses could occur in TAE core since now two TAEs have the same dimensions while locate differently in acoustic field, future work can be done to uplift the TAEs’ performance by setting different dimensions of TAEs and RTs.

With the guidance of this theoretical analysis, experimental work will soon be carried out to verify the accuracy of the model predictions and the practicability of multi-stage thermoacoustically-driven refrigeration system for liquefied natural gas.

5. References
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