Aharonov-Bohm-Like Oscillations in Quantum Hall Corrals

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Experimental study of quantum Hall corrals reveals Aharonov-Bohm-Like (ABL) oscillations. Unlike the Aharonov-Bohm effect which has a period of one flux quantum, $\Phi_0$, the ABL oscillations possess a flux period of $\Phi_0/f$, where $f$ is the integer number of fully filled Landau levels in the constrictions. Detection of the ABL oscillations is limited to the low magnetic field side of the $\nu_c = 1, 2, 4, 6...$ integer quantum Hall plateaus. These oscillations can be understood within the Coulomb blockade model of quantum Hall interferometers as forward tunneling and backscattering, respectively, through the center island of the corral from the bulk and the edge states. The evidence for quantum interference is weak and circumstantial.

The possibility of realizing topologically protected qubits in the fractional quantum Hall (FQH) regime has generated considerable interest in the study of interferometry in quantum Hall systems\textsuperscript{1}. The proposed qubits\textsuperscript{2, 3} seek to take advantage of the exotic quasiparticle statistics thought to occur in the FQH states found at fillings $\nu = 5/2$ [3, 4, 5] and $12/5$ [5]. Theoretical studies have shown that these quasiparticles possess a non-locality, and their exchange statistics are non-Abelian\textsuperscript{6, 7}. The nonlocality can in principle keep qubits free from decoherence, and the use of exchange statistics for computation is thought to be inherently fault tolerant\textsuperscript{1}. Experimental confirmation of the non-Abelian statistics of the $\nu = 5/2$ state in principle can be performed with Aharonov-Bohm interferometry of quasiparticles\textsuperscript{2, 8, 9}. In the proposed experiments, adiabatic transport of a mobile quasiparticle is made about a localized quasiparticle to probe their mutual exchange statistics. An even-odd effect with respect to the number of localized non-Abelian quasiparticles can demonstrate the existence of the non-Abelian statistics\textsuperscript{8, 9}.

Studies of mesoscopic corrals fabricated from GaAs/AlGaAs heterostructures have reported Aharonov-Bohm-Like (ABL) interference in the integer quantum Hall regime\textsuperscript{10, 11}. These periodic magneto-oscillations have been interpreted either as a consequence of zero-dimensional states formed as a result of constructive interference of one-dimensional electron waves traveling along the edge channels\textsuperscript{10} or as Aharonov-Bohm interference of edge electrons\textsuperscript{11}. Similar data in the FQH regime has been interpreted as interference of Laughlin quasiparticles due to fractional statistics\textsuperscript{12}.

In this paper, study of a quantum Hall interferometer fabricated from an ultra-high mobility GaAs/AlGaAs quantum well is reported. The interferometer features a pair of narrow constrictions through which a circular corral is connected to the bulk two-dimensional electron system. Longitudinal and diagonal magneto-resistances in the quantum Hall regime reveal a set of prominent, periodic oscillations reminiscent of Aharonov-Bohm effect. From the flux period scaling, the effective area of the corral and the flux period of $\Phi_0/f$, where $f$ is the integer number of fully filled Landau levels through the constrictions, have been self-consistently determined. Our findings are mostly in agreement with the predictions of the Coulomb blockade model of quantum Hall interferometers proposed by Rosenow and Halperin\textsuperscript{13}.

The corral was fabricated from a high mobility, symmetrically doped GaAs/AlGaAs quantum well. The low temperature mobility of the unprocessed sample was $28.3 \times 10^6 \text{cm}^2/\text{Vs}$ with an electron density of $n = 3.3 \times 10^{11} \text{cm}^{-2}$. The layout of the quantum corral is shown in the inset of Fig. 1. The corral was initially defined using e-beam lithography, dry etched to the depth of the two-dimensional electron system, and metallized with TiAu prior to lift-off. The diameter of the corral is $1.2 \mu\text{m}$, and the width of the two constrictions is $\sim 400$ nm. The sample was mounted on a dilution refrigerator.

FIG. 1: Longitudinal, $R_L$, and diagonal, $R_D$, magnetoresistance of a quantum corral ($T < 10 \text{mK}$). The inset shows the layout of the interferometer. The red lines illustrate the edge state trajectory. Source-drain current is applied between contacts 1-2, the longitudinal magnetoresistance is measured between contacts 3-6, and the diagonal magnetoresistance is measured between contacts 1-6.
ABL oscillation in $R_D$ and 2 plateaus being the strongest. The amplitude of the $D$ Hall plateaus in $R_D$ and $R_D$ minima and $R_D$ define the quantum Hall state. As revealed by the $R_D$ detected at

levels, the filling factor across the constrictions, is smaller than $\nu_0$, the bulk filling factor [13].

Fig. 3 illustrates the longitudinal, $R_L$, and the diagonal, $R_D$, magnetoresistances of the quantum corral below 10 mK in temperature and under magnetic field from -1 to 13 tesla. The low field ($|B| < 1$ tesla) fluctuations arise as a consequence of interference of ballistic paths in confined devices whose mean free path exceeds the dimensions of the device [4]. Above $B > 1$ tesla, a number of zero resistance regions in $R_L$ and plateaus in $R_D$ define the quantum Hall state. As revealed by the $R_L$ minima and $R_D$ plateau, strong quantum Hall states are detected at $\nu_c = 1, 2, 3, 4$, and 6. From the overlap of the Hall plateaus in $R_D$ and the vanishing $R_L$, we find that $\nu_0 = \nu_c$ overlap occurs for these quantum Hall states. Such an overlap is not seen for other quantum Hall states as $\nu_c \neq \nu_0$ due to densitygradient [12, 16, 17]. Below the $\nu_c = 1, 2, 4$, and 6 plateaus, the ABL oscillations in $R_L$ and $R_D$ are observed.

Fig. 2 illustrates the ABL oscillations that are observed in $R_L$ on the low field side of the $\nu_c = 1, 2, 4$ plateaus, with the ABL oscillations below the $\nu_c = 1$ and 2 plateaus being the strongest. The amplitude of the ABL oscillation in $R_L$ can be $\geq 15\%$ of the background resistance for $\nu_c = 1$ and 2 ABL oscillations. Depending on temperature and thermal cycling, over $\geq 300$ periods can be detected for a single set of ABL oscillations. Interestingly no oscillations are detected on the low field side of the $\nu_c = 3, 5$, and other odd integer plateaus. At lower temperatures, ABL oscillations for even $\nu_c \leq 10$ have been detected [13].

The inset of Fig. 2: shows the linear dependence of the ABL oscillation period $\Delta B$ as a function of $B_{osc}$, the center of the magnetic field range at which the periodic oscillations are found. The values of $\Delta B$ and $B_{osc}$ scale in a way that yields a ratio of $1:1.2:1.4:1.6$ between the first four set of oscillations. Such a scaling of $\Delta B$ and $B_{osc}$ always yields the number of fully filled Landau levels, $f$, or equivalently the quantum number of the quantized Hall plateaus in $R_D$ with the ABL oscillations appearing on the low field side. Table 1 summarizes the properties of ABL oscillations from the data shown in Figs 1, 2. Similar scaling of $\Delta B$ and $B_{osc}$ has been reproduced under different illumination and cooldown conditions [13].

For a quantum Hall corral of radius $r$, the enclosed flux is $\Phi = B\pi r^2$. From the scaling of the magnetic field periods in Table 1, the period $\Delta B$ of the ABL oscillations is phenomenologically related to the flux period by $\Delta B\pi r^2 = \Phi_0/f$. Shown in Table 1, the radius calculated for each $\Delta B$ consistently yields $\sim 450 \text{nm}$ as the radius of the corral. An active radius of 450nm appears to be reasonable for the corral with diameter of 1.2 $\mu$m once the effect of side-wall depletion is accounted for. It is notable that $\Phi_0/f$ is the flux period even though the $f + 1^{st}$ Landau level is partially occupied within the constriction.

An intriguing feature of quantum Hall corrals is that it exhibits an extended sequence of ABL oscillations in the second Landau level. Fig. 3a shows $R_D$ with ABL oscillations beginning immediately on the low field side of $\nu_c = 2$ plateau and lasting until nearly the $\nu_c = 3$ plateau. Fig. 3b shows $\Delta R_D$, which is $R_D$ with a smooth background subtracted. $\Delta R_D$ shows apparent amplitude modulation with the strongest oscillations found just below the $\nu_c = 2$ plateau. Fig. 3c shows the magnetic field period $\Delta B$ determined from $\Delta R_D$ data in Fig. 3b. Each period was determined by Fourier analysis of a different window of

| $\Delta B (mT)$ | 6.45 $\pm 0.18$ | 3.30 $\pm 0.24$ | 1.61 | 1.09 |
|----------------|-----------------|-----------------|-----|-----|
| $B_{osc} (T)$  | 11.71           | 5.89            | 2.94| 1.97|
| $\Delta B_{osc} (T)$ | 2.00 | 0.60 | 0.16 | 0.06 |
| $\Delta B_1 / \Delta B_2$ | 1 | 0.51 | 0.25 | 0.17 |
| $B_{osc} / B_{osc}^1$ | 1 | 0.50 | 0.25 | 0.17 |
| $1/f$ | 1 | 1/2 | 1/4 | 1/6 |
| $r (nm)$ | 452 | 447 | 452 | 449 |
magnetic field. A reduction of 10% in $\Delta B$ between $\nu_c = 2$ and 3 is found with some scatter. Changing periods may mean that different areas are being probed. Indeed, we see there is a sudden change where $\nu_c \approx 5/2$ where one might expect a rearrangement of densities in the corral due to the condensation of the 5/2 state.

Fig. 4 illustrates the striking asymmetry between the low and the high field sides of $\nu_c = 2$ plateau. The conductance $G = 1/R_D$ shows a particularly strong set of ABL oscillations on the low magnetic field side of the $\nu_c = 2$ plateau. The upper inset of Fig. 4 shows an expanded view of $G$ on the immediate, low field side of the $\nu_c = 2$ plateau. A series of equally spaced, Coulomb blockade-like conductance peaks is observed as the magnetic field is reduced from the $\nu_c = 2$ plateau. The lower inset of Fig. 4 shows an expanded view of conductance on the high magnetic field side of the $\nu_c = 2$ plateau. Unlike the low field side, there are no ABL oscillations as the conductance exhibits irregular, jagged dips. Fourier analysis shows no dominant oscillation frequency.

Our experiment establishes the following notable features of the ABL oscillations: (i) ABL oscillations are observed only on the low magnetic field side of the $\nu_c = 1, 2, 4, 6,$ and other even integer Hall plateaus. Except below the $\nu_c = 1$ plateau, ABL oscillations are noticeably absent below odd integer Hall plateaus. (ii) No ABL oscillations are observed on the immediate, high field side of Hall plateaus. (iii) The ABL oscillations possess a flux period of $\Phi_0/f$ with $f$ is the number of fully filled Landau levels. The $f+1^{st}$ Landau level is partially occupied within the constrictions. (iv) A typical set of oscillations terminates within 300 periods or less. (v) Below the $\nu_c = 2$ plateau, the ABL oscillations persists over nearly the entire second Landau level. (vi) There is a noticeable amplitude modulation and associated variation of the oscillation period for an extended set of oscillations. (vii) The ABL oscillations can be detected even when the constriction is in a compressible state, i.e., there is no quantized Hall state. This is most clear below the $\nu_c = 1$ and 2 plateaus.

The features noted above establish that the ABL oscillations are not the Aharonov-Bohm effect, which possesses a flux period of $\Phi_0$ and two copropagating interference trajectories. Similarly, the noninteracting model of Ref. 14 predicts flux period of $\Phi_0$, and is not consistent with our results. In contrast, our findings are mostly in agreement with the Coulomb blockade model of quantum Hall interferometers proposed by Rosenow and Halperin[18]. In the simplest version of this model, the constriction consists of an incompressible fluid at integer filling fraction $f$ and the corral contains a compressible island at somewhat higher density. Transport through the system is effected by (A) forward tunneling from the bulk through the island, (B) backscattering from the edge state through the island, and (C) backscattering between edge states via tunneling at the constrictions.

The first two processes (A and B) occur via tunneling through the center island and are enhanced when the Coulomb blockade condition is satisfied, i.e., when the energy of $N$ and $N+1$ electrons on the island is equal. Since the constrictions are quantized at filling fraction
\( f \), addition of a flux quantum \( \Phi_0 \) transports \( f \) electrons into the central compressible island region, satisfying the Coulomb blockade condition \( f \) times. Thus these processes show oscillating conductivity with a magnetic flux period of \( \Phi_0/f \).

On the lower magnetic field side of the plateau, the Rosenow-Halperin model predicts that either forward tunneling from the bulk (A) or backscattering from the edge (B) should be predominant. The positive conductance peaks shown in the low field side of \( \nu_c = 2 \) plateau shown in Fig. 4 identify each peak as a forward tunneling event that increases the overall conductance. Below the \( \nu_c = 1, 2, 4, \) and 6 plateaus, conductance uniformly increases, suggesting predominance of forward scattering in ABL oscillation immediately below these Hall plateaus. This is quite natural since on the low field side of the plateau the island and the bulk are particularly close together \[18\].

On the high field side of Hall plateaus, the model favors backscattering either via tunneling through the island (B) or tunneling at the constrictions (C). Since backscattering reduces the overall conductance, the negative conductance features found above the \( \nu_c = 2 \) plateau shown in Fig. 4 can be interpreted as a sign of backscattering. However, the irregularity of the signal prevents us from making a clean identification of which process is involved. In particular, we cannot identify any particular region as being due to interactions associated with process (C). It is, of course, possible that the complicated and irregular nature of the signal is a result of several competing processes.

A salient feature of the ABL oscillations is that it is detected below the \( \nu_c = 1, 2, 4, 6 \) and other even integer Hall plateaus, but not below other odd plateaus. Clearly, spin plays a nontrivial role in whether or not ABL oscillations can be observed. This is likely because the smaller exchange gap for odd integers (compared with the cyclotron gap of even integers) makes the physical distance between the 2n-1 and 2n+1 edges smaller than that between the 2n and 2n+1 edges. Thus, in the forward scattering process (A) the distance required for a particle to tunnel from the bulk to the island could be smaller on the low field side of an even plateau than on the low field side of an odd plateau. We note, however, that for process (B) such an even-odd effect would not seem likely as long as the tunneling from the edge to the bulk preserves spin.

The apparent modulation of the intensity of the ABL oscillations could be from any one of a number of sources. Such modulation could caused by slow changes in the overlap of wavefunctions involved in the relevant tunneling process, by slow rearrangement of densities, or by beating between competing tunneling processes.

In contrast to the model described by Rosenow-Halperin \[18\], we detect ABL oscillations when the filling factors inside the constrictions are in a compressible regime. Indeed, for the ABL below \( \nu = 2 \) roughly the same period oscillations are seen almost all the way to \( \nu = 3 \). While this is slightly outside of the orthodox model, it allows us to conclude that throughout this region: (a) The tunneling is always single electron tunneling. (b) The oscillations are from a Coulomb blockade origin. (c) Addition of a flux quantum \( \Phi_0 \) continues to add an integer number of \( f \) electrons to the island, even though the constriction may have a partially filled \( f+1 \)st Landau level. This result is quite interesting, since in a sample of this high mobility, one might expect FQH states in the constrictions. Indeed, (as shown in Fig. 4 there appears to be an effect in the flux period when the constrictions have the density of the FQH state (although evidence for this is somewhat weak). If the constriction is really quantized at filling \( \nu_c \), one might expect that addition of a flux quantum adds charge \( \nu_c e \). However, this does not appear to be the case.

In summary, we have studied the ABL oscillations in quantum Hall corrals. The ABL oscillations are most prominent in the low field side of the \( \nu = 1, 2, 4, 6 \ldots \) quantum Hall plateaus. They can be detected over extended ranges of magnetic fields, including over the compressible filling factors. Their flux periods is \( \Phi_0/f \), where \( f \) is the integer number of fully filled Landau levels through the constriction. These features establish that ABL oscillations do not arise from Aharonov-Bohm effect. Instead, these oscillations can be identified as electron tunneling peaks due to Coulomb blockade through the island in the corral. It will be necessary to distinguish Coulomb blockade effects from interference effects in the future.

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