Ground state properties of $^4$He and $^{12}$C nuclei at equilibrium and at large static compression at zero temperature using Nijmegen and Reid Soft Core nucleon-nucleon interactions

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In this paper, we investigate the ground state properties (i.e., binding energy, nuclear radius, radial density distribution and single particle energies) for $^4$He and $^{12}$C nuclei at equilibrium and at large static compression at zero temperature by using two realistic different potentials namely, Nijmegen and Reid Soft Core (RSC) potentials. We carry out the calculations in No-Core Shell Model space consisting of six major oscillator shells within the framework of the Constrained Spherical Hartree-Fock (CSHF) approximations. We find out that, the computed equilibrium root mean square radii and the Hartree Fock energies for $^4$He and $^{12}$C with those two different potentials are very close to the experimental values of the nuclear radii and nuclear binding energies for the same nuclei.

I. INTRODUCTION

The nucleus is composed of fermions of spin 1/2 and isospin 1/2, called nucleons (protons and neutrons), interacting with each other through a complex interaction with a short-range repulsive core. For the purpose of investigating nuclear structure, it is convenient to represent the inter-nucleon interaction by a potential. Various potential models for the two nucleons interaction has been proposed and parameters in the model fitted using the result of N-N scattering experiments. The central problem of nuclear structure theory is the solution of Many Body Schrödinger Equation (MBSE). For Hamiltonians of interest in the nuclear case, an analytic solution is impossible (except in certain simple cases) and one is compelled to use some approximation, either in the numerical solution of the equation or specification of the Hamiltonian, or both. One has to solve the full (MBSE) numerically in an exact way as possible using variational techniques. No-Core Shell Model (NCSM) calculations have been made in light nuclei, and heavier nuclei close to closed shell have been treated.

The NCSM is based on a new variation of the well known shell model for nuclei. Historically, shell-model calculations have been made assuming closed, inert core of nucleons with only a few active valence nucleons. The interaction of these valence nucleons with the core and with other valence nucleons could not be described by microscopic interactions, as they have been developed for few-nucleon systems. Therefore, these attempts have not been completely successful in relating the effective shell model interaction to the basic nuclear interaction. This situation has been changed in 1990 with the development of the NCSM, which treats all nucleons in the nucleus as an active particles. The NCSM assumes that all nucleons are active, there is a systematic way to obtain the effective interactions from bare NN and 3N forces. This is the strength of the NCSM compared to traditional shell model calculation.

The ab-initio No-Core NCSM has been applied with realistic effective N-N interactions to light nuclei and it has been shown that the NCSM approach can be consistently applied to solve the three-nucleon as well as four-nucleon bound state problem. There are various models for nuclear potentials such as Bonn, CD-Bonn, Paris, Nijmegen, and Idaho. They all describe the observed deuteron and N-N scattering data very accurately. However, due to their strong repulsive core, none of them can be used directly in the nuclear structure calculations. To overcome this difficulty, the Brueckner G-matrix has been used traditionally as a starting point, but as is well known its energy dependence is an undesirable feature, in particular dealing with Hartree-Fock calculations.

Hartree-Fock is a proven tool for semi-realistic interactions even for the heaviest nuclei and is sufficiently flexible to handle many-body forces. Also, it is a starting point for practical many-body methods used extensively in heavier systems. Of course, there is a long history going back to Brueckner of merging the mean field method with non relativistic effective potentials (G-matrix) derived from N-N interactions.

We investigate the ground state properties of $^4$He and $^{12}$C nuclei at equilibrium and at large amplitude of compression with zero temperature using a realistic effective interaction based on different potentials namely, Nijmegen and Reid Soft Core (RSC) potentials. We perform the calculations in NCSM space consisting of six major oscillator shells (i.e., 21 single particles orbitals) within the framework of the constrained spherical Hartree-Fock (CSHF) approximations. In particular, we study the sensitivity of the ground state properties, such as binding energy, nuclear radius, radial density distribution and single particle energies to the degree of compression. The importance of this study is to investigate the effect of the potential used on softening the nuclear
equation of state.

This study also will shed some light on the behavior of nuclear matter under extreme conditions, which has its importance in astrophysical problems and to have better understanding of its behavior in nucleus-nucleus collisions as in heavy ion collisions in high-energy supercolliders. On the other hand, there are important physical motivations for investigating \(^{12}\)C. Actually, \(^{12}\)C nucleus plays an important role in neutrino studies as it is an ingredient of neutrino liquid-scintillator detectors [9].

II. METHODOLOGY AND PARAMETERS

Our nuclear system consists of \(A\) Nucleons (\(N\) neutrons and \(Z\) protons) of spin \(s = 1/2\), isospin \(\tau = 1/2\) and mass \(m\) each. The Hamiltonian of the system consists of the single particle energy and a two-body interaction:

\[
H = \sum_{i=1}^{A} t_i + \sum_{i<j}^{A} V_{ij}
\]  

(1)

Where \(t_i\) denotes the single particle kinetic energy operator, and \(V_{ij}\) is the two-body interaction term which consists of two body interaction and the Coulomb potential \((V = V_{NN} + V_{C})\). The labels enclosed within the brackets refer to particle coordinates, where the restriction \(i < j\) in the second sum in equation (1) take care of the fact that the interaction has to be summed counting each pair only once. In this work, we use a no core-effective Hamiltonian; that is all nucleons are active.

In principle, if one solves the many-body problem in the full Hilbert space, then one gets the exact solution, however, this is not possible for nuclei with \(A > 4\). Therefore, we truncate the Hilbert space to finite model space. The price one has to pay is to define an effective Hamiltonian \((H_{eff})\) based on the study presented in [16] [17].

The detailed calculations of the effective Hamiltonian, \(H_{eff}\), model space, calculation procedures, and strategy have been extracted from references [16] [17]. Based on these studies, the two-body matrix elements are scaled to an optimal value of \(\omega\), the adjusting parameters \((\lambda_1, \lambda_2, \text{and } \hbar \omega)\) for \(^4\)He and \(^{12}\)C nuclei in a given model space at equilibrium, are presented in Table I. We notice that the value of \(\lambda_1\) is less than unity, this is because the fact that kinetic energy operator \((T_{rel})\) is a positive definite operator and if it is normalized by itself into a finite model space this will reduce its magnitude. Moreover, we notice that the value of \(\lambda_2\) is greater than unity in order to compensate for the lack of sufficient binding when we truncate the full Hilbert space to a finite model space. In our calculations, we use a large model space consisting six major shells; i.e. 21 nucleon orbitals each orbital has definite quantum numbers, \(n, s, J\).

III. RESULTS AND DISCUSSION

A. Results of \(^4\)He

The parameters \(\lambda_1, \lambda_2, \text{and } \hbar \omega\) which have been used for \(^4\)He are presented in Table I. With these parameters we find that the equilibrium root mean square radius \((r_{rms})\) and \(E_{HF}\) using RSC (Nijmegen) potentials are, \(r_{rms} = 1.46\text{fm} \ (r_{rms} = 1.46\text{fm})\), and \(E_{HF} = -28.296\text{MeV} \ (E_{HF} = -28.296\text{MeV})\), respectively. We remind the reader here that the experimental nuclear radius for \(^4\)He, is \(r_{rms} = 1.46\text{fm}\) and the measured binding energy is \(E_{bind} = -28.296\text{MeV} [18, 19]\). We found that the occupied orbitals are \(0s_{1/2}\) in agreement with the standard shell model.

In Figure 1, the \(E_{HF}\) energies using RSC and Nijmegen potentials are presented as a function of \(r_{rms}\). In Figure 2, the single particle energies \(SPEs\) are displayed as a function of \(r_{rms}\). Moreover, Figure 3 represents the radial density distribution for neutrons \((\rho_n)\), protons \((\rho_p)\), and \(\rho_{total} = \rho_n + \rho_p\), at equilibrium (i.e. at \(r_{rms} = 1.46\text{fm}\)) using Nijmegen potential, while Figure 4 displays the total radial density distribution \(\rho_{total}\) at equilibrium \((r_{rms} = 1.46\text{fm})\) and at large static compression \((r_{rms} = 1.33\text{fm})\) using Nijmegen potential. In addition, Figure 5 displays the total radial density distribution at equilibrium \((r_{rms} = 1.46\text{fm})\) and at large static compression \((r_{rms} = 1.24\text{fm})\) using RSC potential. In Figure 6, we compare \(\rho_{total}\) for the two potentials (Nijmegen and RSC) at equilibrium \((r_{rms} = 1.46\text{fm})\). Finally, In Figure 7 we compare \(\rho_{total}\) for the two potentials (Nijmegen and RSC) at large static compression \((r_{rms} = 1.33\text{fm})\).

In Figure 1, we notice that, using RSC potential that reducing the volume of the nucleus by about 24% (at equilibrium \(r_{rms} = 1.46\text{fm}\), and at large static compression, \(r_{rms} = 1.33\text{fm}\)), compared to its volume at equilibrium reduces the binding energy by about 8% (at equilibrium, \(E_{HF} = -28.296\text{MeV}\), and at large static compression \(E_{HF} = -25.956\text{MeV}\)), but when the nuclear volume is reduced by about 417%, the change in the nuclear binding energy by about 3%. However, when we use Nijmegen potential that will reduce the volume by about 24% compared to its volume at equilibrium reduces the binding energy by about 25%, and when we reduce the nuclear volume by about 17% the change in the nu-

| Nucleus Potential | \(\hbar \omega\) | \(\lambda_1\) | \(\lambda_2\) |
|------------------|---------------|--------------|--------------|
| \(^4\)He          |               |              |              |
| Nijmegen         | 15.700        | 0.990        | 1.041        |
| RSC              | 17.872        | 0.980        | 1.186        |
| \(^{12}\)C        |               |              |              |
| Nijmegen         | 8.454         | 0.976        | 1.200        |
| RSC              | 10.104        | 0.973        | 1.420        |

TABLE I. Values of adjusting Parameters of the effective Hamiltonian \((H_{eff})\) for \(^4\)He and \(^{12}\)C by using Nijmegen and RSC penitentials in six shells to get an agreement between HF results and experimental data [13] [19].
FIG. 1. \( E_{HF} \) using RSC and Nijmegen potentials in six-oscillator Shells as a function of \( r_{rms} \) for \(^4\text{He}\)

![Graph showing \( E_{HF} \) vs. \( r_{rms} \) for \(^4\text{He}\) using RSC and Nijmegen potentials.]

FIG. 2. Single particle energy (S.P.E) for \(^4\text{He}\) in six-oscillator shells as a function of \( r_{rms} \) using Nijmegen and (RSC) potentials.

![Graph showing S.P.E vs. \( r_{rms} \) for \(^4\text{He}\) using Nijmegen and RSC potentials.]

FIG. 3. Radial density distribution for \(^4\text{He}\) as a function of nucleus radius \( r(\text{fm}) \) at equilibrium \((r = 1.46\text{fm})\). Using Nijmegen potential.

![Graph showing radial density distribution for \(^4\text{He}\) at equilibrium.]

FIG. 4. Total radial density distribution \( \rho_{\text{total}} \) for \(^4\text{He}\) at equilibrium \((r_{rms} = 1.46\text{fm})\) and large static compression \((r_{rms} = 1.33\text{fm})\) as a function of nucleus radius \( r(\text{fm}) \). Using Nijmegen potential.

![Graph showing total radial density distribution for \(^4\text{He}\) at equilibrium and large compression.]

FIG. 5. Total radial density distribution \( \rho_{\text{total}} \) for \(^4\text{He}\) at equilibrium \((r_{rms} = 1.46\text{fm})\) and large static compression \((r_{rms} = 1.24\text{fm})\) as a function of nucleus radius \( r(\text{fm}) \) using RSC potential.

![Graph showing total radial density distribution for \(^4\text{He}\) at equilibrium and large compression using RSC potential.]

clear binding energy by about 4%. This means that the nuclear equation of state becomes stiffer as we compress the nucleus. We also note that, at large compression the nuclear binding energy using Nijmegen potential is larger than the binding energy using RSC potential. In Figure 2, we notice that, the ordering of the orbits is in exact agreement with the orbits ordering of the standard shell model. The gap is very clear between the shells, and also the splitting of the levels in each shell is an indicator that the L-S coupling is strong enough in RSC and Nijmegen potentials. This is also clear from shifting down the \( 0f_{7/2} \) orbit from the p-f shell to the sd shell. The L-S coupling becomes stronger as we increase the static load on the nucleus. Finally, we notice that, the levels curve up as we compress the nucleus more and more and the levels curve up more rapidly using Nijmegen potential. This means
that the kinetic energy becomes more influential than the attractive means field of the nucleon. Figure 3, shows that the neutron and proton densities are almost the same except in the interior region, where the neutrons are denser than protons. Actually, this difference in densities is attributed to the Coulomb repulsion between the protons. Figure 4, represents the total nuclear radial density distribution $\rho_{\text{total}}$ at equilibrium (i.e. $r_{\text{rms}} = 1.46\text{fm}$) and at ($r_{\text{rms}} = 1.33\text{fm}$) (a volume reduction by about 75%) using Nijmegen potential. The nuclear density becomes denser in the interior and less dense in the exterior, (i.e. close to the surface of the nucleus). This means that, as we increase the load more and more, the surface of the nucleus becomes more and more responsive. Figure 5, displays the total nuclear radial density distribution $\rho_{\text{total}}$ at equilibrium (i.e. $r_{\text{rms}} = 1.46\text{fm}$) and at ($r_{\text{rms}} = 1.24\text{fm}$) (a volume reduction by about 61%) using RSC potential. In addition, Figure 6 displays the total radial density distribution total at equilibrium ($r_{\text{rms}} = 1.46\text{fm}$), at two different potentials (Nijmegen and RSC). We notice that, in the interior region the $\rho_{\text{total}}$ using RSC potential is larger than $\rho_{\text{total}}$ using Nijmegen potential, but this difference is very small. In the exterior region $\rho_{\text{total}}$ for both potential are nearly the same. Figure 7, displays the total radial density distribution $\rho_{\text{total}}$ at large static compression ($r_{\text{rms}} = 1.33\text{fm}$) for the two different potentials (Nijmegen and RSC). Finally, we see, first, it is clear that from Figures 6 and 7 as the nucleus is compressed; the nuclear density become denser in the interior and less dense in the exterior for both potentials (Nijmegen and RSC), and second, the nuclear density becomes denser in the interior with Nijmegen potential than with RSC potential, and less dense in the exterior with Nijmegen potential than with RSC potential. This means that as we increase the load more and more, the surface of the nucleus becomes more and more responsive, and it is possible to compress the nucleus to a smaller radius using RSC potential than Nijmegen potential.

**B. Results of $^{12}$C**

The calculations proceed in the same manner as the calculations for $^{4}$He. The values of the parameters $\lambda_1$, $\lambda_2$, and $\hbar \omega$ to obtain the agreement between the $E_{\text{HF}}$ and $r_{\text{rms}}$, and the experimental binding energy and the nuclear radii [20] are listed in Table I. In the input data file, we change the mass number from 4 to 12, number of protons and neutrons are 6, and the occupied orbits are $0s_{1/2}$ and $0p_{3/2}$ . With the adjusting parameters (i.e. $\lambda_1$, $\lambda_2$, and $\hbar \omega$), we find an equilibrium ($r_{\text{rms}}$) and $E_{\text{HF}}$ using RSC (Nijmegen) potentials, $r_{\text{rms}} = 2.3508\text{fm}$ ($r_{\text{rms}} = 2.3498\text{fm}$), and $E_{\text{HF}} = -92.174\text{MeV}$ ($E_{\text{HF}} = 92.167$), respectively. The experimental nuclear radius $r_{\text{exp}} = 2.35\text{fm}$ and the binding energy is $E_{\text{bind}} = -92.162\text{MeV}$ [20]. We find the occupied orbital are $0s_{1/2}$ and $0p_{3/2}$, in agreement with the standard shell model.

The $E_{\text{HF}}$ energies as a function of $r_{\text{rms}}$ using RSC and Nijmegen potential are displayed in Figure 8. The single particle energies (SPE) as a function of the $r_{\text{rms}}$ are displayed in Figure 9. Moreover, Figure 10 displays the radial density distribution for neutrons $\rho_n$, protons $\rho_p$, and $\rho_{\text{total}} = \rho_n + \rho_p$, at equilibrium (i.e. at $r_{\text{rms}} = 2.35\text{fm}$) using Nijmegen potential. Figure 11 represents the radial density distribution at equilibrium($r_{\text{rms}} = 2.35\text{fm}$) and at large static compression ($r_{\text{rms}} = 2.26\text{fm}$) using Nijmegen potential. In Figure 12, we compare $\rho_{\text{total}}$ at two different values of $r_{\text{rms}}$, at $r_{\text{rms}} = 2.35\text{fm}$ (i.e. equilibrium) and at $r_{\text{rms}} = 2.06\text{fm}$ using RSC potential. In Figure 13, we compare $\rho_{\text{total}}$ at two different potentials (Nijmegen and RSC) at equilibrium ($r_{\text{rms}} = 2.35\text{fm}$).
Furthermore, in Figure 14, we compare total at two different potentials (Nijmegen and RSC) at large static compression \((r_{rms} = 2.26 \text{ fm})\).

We notice in Figure 8, that, reducing the volume of the nucleus to about 12% compared to its volume at equilibrium reduces the binding energy by about 2% using RSC potential. However, when reduce the nuclear volume by about 6% the change in the nuclear binding energy by about 1%. But, by using Nijmegen potential for the same reduction in nuclear volume compared to the volume at equilibrium reduces the binding energy by 11% and 1%. Thus, that means that the nuclear equation of state becomes stiffer as we compress the nucleus, using RSC potential softening the equation of state more than using Nijmegen potential.

In Figure 9, we notice that the ordering of the orbits in agreement with the orbital ordering of the standard shell model. In addition, we notice that the splitting of the levels in each shell is an indicator that the L-S coupling is strong enough in both potentials. This is also clear from shifting down the \(0f_{7/2}\) orbit from the p-f shell to the s-d shell. This L-S coupling becomes weaker as we increase the static load on the nucleus. On the other hand, the orbits curve up as we increase the load on the nucleus. The SPE levels curve up more rapidly when using Nijmegen potential. This was discussed in the results of \(^4\text{He}\), and that is because the kinetic energy of the nucleon which is positive quantity becomes more influential than the attractive mean field of the nucleon. In addition, we realize that the SPE’s when using Nijmegen potential are less bound than the (SPE) with using RSC potential, especially when we compress the nucleus more and more.

Figure 10 shows the radial density distribution for neutron \(\rho_n\), protons \(\rho_p\), and their sum \(\rho_{\text{total}}\) as a function of the radial distance from the center of the nucleus at equilibrium (i.e. \(r_{rms} = 2.35 \text{ fm}\)). We notice that the neutron’s and proton’s densities are almost the same except in the interior region, where the neutrons are denser than protons. This difference in densities is attributed to Coulomb repulsion between the protons. Also, Figure 11 displays the total radial density distribution \(\rho_{\text{total}}\) nucleus at equilibrium (i.e. \(r_{rms} = 2.35 \text{ fm}\)) and at \(r_{rms} = 2.26 \text{ fm}\), the volume reduction is about 11%) using Nijmegen potential. Obviously, that as the nucleus is compressed; the nuclear density becomes denser in the interior and less dense in the exterior (i.e. closer to the surface of the nucleus). This means that, as we increase the load more and more, the surface of the nucleus becomes more responsive.

Figure 12 displays the nuclear total radial density distribution \(\rho_{\text{total}}\) at equilibrium (i.e. \(r_{rms} = 2.35 \text{ fm}\)) and at \(r_{rms} = 2.06 \text{ fm}\), the volume reduction is about 67%) using RSC potential. Interestingly, we notice the same features as in Figure 11, that is the interior of the nucleus becomes more dense than the exterior as we compress the nucleus. There is one difference though: that the increase in nuclear density is more pronounced using RSC than when using Nijmegen potential.

Figure 14 displays the total radial density distribution \(\rho_{\text{total}}\) at large static compression \((r_{rms} = 2.26 \text{ fm})\) with the two potentials (Nijmegen and RSC). We draw the following conclusions. First, it is clear that as the nucleus compressed; the nuclear density become denser in the interior and less dense in the exterior for both potentials. Second, the increase in nuclear density with compression in the interior is more pronounced using RSC potential than with Nijmegen potential and less dense in the exterior using RSC potential than with Nijmegen potential. Finally, Figure 13 displays the total radial density distribution \(\rho_{\text{total}}\) at equilibrium \((r_{rms} = 2.35 \text{ fm})\) with the two potentials (Nijmegen and RSC). We notice from this figure that the interior region the nuclear density is larger using RSC than when using Nijmegen potential. The situation is reversed in the exterior.
FIG. 10. Radial density distribution for $^{12}$C as a function of nuclear radius $r$(fm) at equilibrium ($r_{rms} = 2.35 fm$) using Nijmegen potential.

FIG. 11. Total radial density distribution $\rho_{total}$ for $^{12}$C at equilibrium ($r_{rms} = 2.35 fm$) and large static compression ($r_{rms} = 2.26 fm$) as a function of nuclear radius $r$(fm). using Nijmegen potential.

FIG. 12. Total radial density distribution $\rho_{total}$ for $^{12}$C at equilibrium ($r_{rms} = 2.35 fm$) at large static compression ($r_{rms} = 2.06 fm$) as a function of nuclear radius $r$(fm) using RSC potential.

FIG. 13. Total radial density distribution $\rho_{total}$ for $^{12}$C at equilibrium ($r_{rms} = 2.35 fm$) as a function of nuclear radius $r$(fm) using Nijmegen and RSC potentials.

IV. CONCLUSION

The ground state properties of $^{4}$He and $^{12}$C nuclei have been investigated at equilibrium and at large amplitude of compression using a realistic effective interaction based on two different potentials namely, Nijmegen and Reid Soft Core (RSC). The calculations were performed in no-Core model space consisting of six major oscillator shells (i.e. 21 single particles orbitals) within the framework of the constrained spherical Hartree-Fock (CSHF) approximations. Specifically, the sensitivity of the ground state properties, such as binding energy, nuclear radius, radial density distribution and the single particle energy, to the degree of compression were investigated.

We find that, the equilibrium root mean square radius $r_{rms}$ for $^{4}$He equals to 1.46 fm, and the corresponding Hartee-Fock Energy $E_{HF}$ equals to $-28.296 MeV$ using both potentials (Nijmegen and RSC) in a good agreement with the experimental results of nuclear radius of $r_{rms} = 1.46 fm$, and experimental binding energy of $-28.296 MeV$ [18, 19]. For the case of $^{12}$C, we find that the equilibrium $rms$ radius equals to 2.351 fm and 2.35 fm, where the corresponding $E_{HF}$ are $-92.174 MeV$ and $-92.167 MeV$ using RSC and Nijmegen potentials, respectively. However, the experimental nuclear radius for $^{12}$C equals to 2.35 fm and value of the binding energy equals to $-92.162 MeV$ [20].

For $^{4}$He, with maximum compression used the minimum $r_{rms}$ radii obtained are 1.244 fm, and 1.327 fm and the corresponding EHF are $-11.260 MeV$ and $-20.804 MeV$ using RSC and Nijmegen potentials respectively. On the
FIG. 14. Total radial density distribution $\rho_{\text{total}}$ for $^{12}\text{C}$ at large static compression ($r_{\text{rms}} = 2.26 \text{fm}$) using Nijmegen and RSC potentials.

On the other hand, for $^{12}\text{C}$, the minimum $r_{\text{rms}}$ radii obtained are 2.063 fm, and 2.255 fm, and the corresponding $E_{\text{HF}}$ are $-49.579 \text{MeV}$ and $-82.444 \text{MeV}$ for using RSC and Nijmegen potentials respectively. For both nuclei, it is possible to compress the nucleus to a smaller size using RSC than using Nijmegen potential. At equilibrium, the neutrons and protons densities are almost the same except in the interior region; where neutrons are more dense than protons. This difference in densities is attributed to Coulomb repulsion between protons. For $^4\text{He}$, and at equilibrium, the radial density distribution are the same except in the interior region using RSC and Nijmegen potential; where it is larger with RSC than with Nijmegen potential. At large compression the situation is reversed especially in the interior region; the radial density distribution becomes larger than the radial density distribution when using RSC potential. For the case of $^{12}\text{C}$, and at equilibrium, with the two potentials (Nijmegen and RSC) the nuclear density is larger using RSC than when using Nijmegen potential. The situation is reversed in the exterior. Moreover, for both nuclei (i.e. $^4\text{He}$ and $^{12}\text{C}$), the ordering of the orbit is in exact agreement with the orbit ordering of the standard shell model. The gap is very clear between the shells. The splitting of the levels in each shell is an indicator that L-S coupling is strong enough in RSC and Nijmegen potentials. This is also clear from shifting down the $0f_{7/2}$ orbit from the p-f shell to the s-d shell. This L-S coupling becomes stronger as we increase the static load on the nucleus. In addition, these levels curve up as we compress the nucleus more and more and these levels curve up more rapidly when using Nijmegen potential. This indicates that the kinetic energy of the nucleus which is a positive quantity becomes more and more influential than the attractive means field of the nucleon. Finally, we notice that the SPEs becomes larger when using Nijmegen potential than the (SPE) with using RSC potential, especially at large compressions.

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