QUANTIFYING THE SIGNIFICANCE OF THE MAGNETIC FIELD FROM LARGE-SCALE CLOUD TO COLLAPSING CORE: SELF-SIMILARITY, MASS-TO-FLUX RATIO, AND STAR FORMATION EFFICIENCY

Patrick M. Koch\textsuperscript{1}, Ya-Wen Tang\textsuperscript{2,3}, and Paul T. P. Ho\textsuperscript{1,4}

\textsuperscript{1} Academia Sinica, Institute of Astronomy and Astrophysics, Taipei, Taiwan; pmkoch@asiaa.sinica.edu.tw
\textsuperscript{2} Observatoire Aquitain des Sciences de l’Univers, Université de Bordeaux, 2 rue de l’Observatoire, BP 89, F-33271 Floirac Cedex, France
\textsuperscript{3} CNRS, UMR 5804, Laboratoire d’Astrophysique de Bordeaux, 2 rue de l’Observatoire, BP 89, F-33271 Floirac Cedex, France
\textsuperscript{4} Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

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ABSTRACT

Dust polarization observational results are analyzed for the high-mass star formation region W51 from the largest parent cloud (~2 pc, James Clerk Maxwell Telescope) to the large-scale envelope (~0.5 pc, BIMA array) down to the collapsing core e2 (~60 mpc, Submillimeter Array). Magnetic field and dust emission gradient orientations reveal a correlation which becomes increasingly more tight with higher resolution. The previously developed polarization–intensity-gradient method is applied in order to quantify the magnetic field significance. This technique provides a way to estimate the local magnetic field force compared to gravity without the need of any mass or field strength measurements, solely making use of measured angles which reflect the geometrical imprint of the various forces. All three data sets clearly show regions with distinct features in the field-to-gravity force ratio. Azimuthally averaged radial profiles of this force ratio reveal a transition from a field dominance at larger distances to a gravity dominance closer to the emission peaks. Normalizing these profiles to a characteristic core scale points toward self-similarity. Furthermore, the polarization–intensity-gradient method is linked to the mass-to-flux ratio, providing a new approach to estimate the latter one without mass and field strength inputs. A transition from a magnetically supercritical to a subcritical state as a function of distance from the emission peak is found for the e2 core. Finally, based on the measured radius-dependent field-to-gravity force ratio we derive a modified star formation efficiency with a diluted gravity force. Compared to a standard (free-fall) efficiency, the observed field is capable of reducing the efficiency down to 10% or less.

Key words: ISM: clouds – ISM: individual objects (W51, W51 e2) – ISM: magnetic fields – polarization

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1. INTRODUCTION

The magnetic field is being recognized as a crucial component in the star formation process. Evidence for its presence and significance is growing with the advance of an increasing number of instruments capable of providing high-quality polarization observations. To date, various observations—based on different emission or absorption mechanisms over a range of wavelengths—cover scales from a few parsecs down to milli-parsecs in molecular clouds. Nevertheless, the precise role of the magnetic field and its interplay with, e.g., turbulence and gravity, remain a debated topic in the current literature. In order to make further progress, knowledge of both the field morphology and the field strength needs to be combined. It is unfortunate that the benefit of the additional polarization information beyond its mere detection and imaging purpose is often not very obvious. With the growing number of high-quality polarization observations from various instruments, it is thus paramount to investigate new methods and strategies to further explore and reveal the physics hidden in polarization data. In order to make further progress, efforts on several fronts are needed and eventually must be combined. On one hand, magnetohydrodynamic (MHD) simulations producing synthetic observational maps can characterize generic features and provide guidelines for data interpretation (e.g., Nakamura & Li 2011; Falceta-Gonçalves et al. 2008; Li & Nakamura 2004; Allen et al. 2003). It is further desirable to include radiative transfer modeling in this approach. On the other hand, methods and techniques inspired by observed data can lead to new unexpected insights in a phenomenological way. Such recent approaches are, e.g., the polarization dispersion function used to trace the relative turbulence level (Houde et al. 2009; Hildebrand et al. 2009) and the ambipolar diffusion scale isolated by comparing coexisting ion and molecular line spectra (Hezareh et al. 2011; Li & Houde 2008). Here, we are further exploring the recently developed polarization–intensity-gradient method (Koch et al. 2012). This technique leads to a local position-dependent measurement of the magnetic field strength, and it additionally provides an estimate of the local field-to-gravity force ratio in a model-independent way.

We are applying our method to a set of dust emission polarization data in the submillimeter (submm) regime, where the dust grains are thought to be aligned with their shorter axis parallel to the magnetic field lines due to radiative torques...
Regardless of the different physical scales in the JCMT, BIMA, and the Submillimeter Array (SMA; Y.-W. Tang et al. 2012, in preparation; Tang et al. 2009a, 2009b, 2010; Rao et al. 2009; Girart et al. 2009, 2006). This study is part of the program on SMA (Ho et al. 2004) to investigate the structure of the magnetic field from large to small scales.

The paper is organized as follows. Focusing on the magnetic field detections in the W51 star formation region, Section 2 describes the relevant polarization observations from the James Clerk Maxwell telescope (JCMT), the Berkeley–Illinois–Maryland Association array (BIMA), and SMA. The key results of the polarization–intensity-gradient method (Koch et al. 2012)—which serve as a starting point for this work here—are summarized in Section 3. Section 4 starts with pointing out the correlation between the magnetic field and intensity gradient orientations, and then presents our results on the magnetic field significance. Implications of our findings for the mass-to-flux ratio and the star formation efficiency are discussed in Section 5. A summary and conclusion are given in Section 6.

2. OBSERVATIONS AND SOURCE DESCRIPTION

The W51A cloud is one of the most active and luminous high-mass star formation sites in the Galaxy. Located at a distance of ~7 kpc (Genzel et al. 1981), 1″ is equivalent to ~0.03 pc. Chrysostomou et al. (2002) measured the polarization at 850 μm with SCUBA on JCMT across the molecular cloud at a scale of 5 pc with a binned resolution θ ≈ 9′.3. Their observation encompasses two cores with polarized emission detected both in the cores and the region in between them. At this scale, the polarization appears to be organized but with a morphology that changes from the dense cores to the surrounding more diffuse areas. Lai et al. (2001) reported a higher angular resolution (3″) polarization map at 1.3 mm obtained with BIMA, which resolved out large-scale structures. In contrast to the larger scale JCMT map, the polarization appears to be uniform across the envelope at a scale of 0.5 pc, resolving the eastern core in the JCMT map into the sources W51 e2 and e8. W51 e2 is one of the strongest mm/submm continuum sources in the W51A region. In the highest angular resolution map obtained with the SMA at 870 μm with θ ≈ 0′.7, the polarization patterns appear to be pinched in e2 and also possibly in e8 (Tang et al. 2009b). In particular, the structure detected in the collapsing core e2 reveals hourglass-like features. These data currently provide the highest angular resolution information on the morphology of the magnetic field in the plane of sky obtained with the emitted polarized light in the W51A star formation site. A statistical analysis based on a polarization structure function (of second order) shows a turbulent to mean magnetic field ratio which decreases from the larger (BIMA) to the smaller (SMA) scales from ~1.2 to ~0.7 (Koch et al. 2010), possibly demonstrating that the role of magnetic field and turbulence evolves with scale. Collapse signatures in W51 have been reported for various molecules in Rudolph et al. (1990), Ho & Young (1996), Zhang & Ho (1997), Young et al. (1998), Zhang et al. (1998), and Solins et al. (2004).

Figure 1 reproduces the dust continuum Stokes I maps from the JCMT, BIMA, and SMA observations, with their original (phase) centers at right ascension (J2000) α = 19h23m39″.0, declination (J2000) δ = 14°31′08″.0: α = 19h23m44″.2, δ = 14°30′33″.4; and α = 19h23m43″.95, δ = 14°30′34″.00. Overlaid in red are the magnetic field segments, rotated by 90° with respect to the originally detected polarization orientations. Typical measurement uncertainties of individual position angles (P.A.s) are in the range of 5°–10° and 3°–9° for SMA and for BIMA, respectively. The median uncertainty in the JCMT data is about 5°, with a few outliers between 30° and 40°. Additionally shown are the intensity gradient orientations in blue, which are relevant for the further analysis in the following sections.

We remark that W51A was also observed with the 350 μm polarimeter Hertz at the Caltech Submillimeter Observatory with a resolution of ~20″ (Dotson et al. 2010). For our purposes here, we have found these data to be in agreement with and equivalent to the JCMT observation. They are, thus, not further discussed here.

3. A MODEL-INDEPENDENT APPROACH: THE POLARIZATION–INTENSITY-GRADIENT METHOD

Various forces are interacting in molecular clouds. Maps of observed dust emission are reflecting the overall result of gravity, pressure, and magnetic field forces, and possible additional constituents. Dust emission and magnetic field morphologies are left with a geometrical imprint by the combined effect of all these forces. Interestingly, polarization orientations are often observed to be close to tangential to dust emission intensity contours. Therefore, magnetic field and intensity gradient directions show a correlation, where the difference δ in their orientations can be linked to the magnetic field strength (Figure 3 in Koch et al. 2012). In this method, various components in the ideal MHD force equation are identified in dust polarization and Stokes I maps. In particular, a change in emission intensity (gradient) is assumed to be the result of the transport of matter driven by the combination of all the forces in the MHD equation. Adopting this, it then follows that the gradient in the dust emission Stokes I intensity defines the resulting direction of motion in the MHD force equation. A force triangle, where the vector sum of all forces is set equal to the intensity gradient, can then be constructed (Figure 3 in Koch et al. 2012). As a result, the total magnetic field strength as a function of position in a map can be calculated as

$$B = \sqrt{\frac{\sin \psi}{\sin \alpha} \left(\nabla P + \rho \nabla \phi\right) 4\pi R},$$  

where the angle $\psi$ is the difference in orientations between the gravitational pull and the intensity gradient, and the angle $\alpha$ is the difference between the polarization and the intensity gradient orientations. $\rho \nabla \phi$ and $\nabla P$ are the gravitational pull and the pressure gradient, respectively. $R$ is the magnetic field radius. Generally, all variables are functions of positions in a map. In

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5 The Submillimeter Array is a joint project between the Smithsonian Astrophysical Observatory and the Academia Sinica Institute of Astronomy and Astrophysics, and is funded by the Smithsonian Institution and the Academia Sinica.

6 The source names W51A (G49.5-0.4) or W51 are used synonymously in the following sections.

7 Regardless of the different physical scales in the JCMT, BIMA, and the SMA observations, areas in any map identified by clear emission peaks are denoted as cores, here and in the following sections.

8 The reduced JCMT data are available at http://cdsarc.u-strasbg.fr/viz-bin/qcat?J/A+A/A/. 
the case of W51 e2, when neglecting the pressure gradient, the field strengths vary between \( \sim 2 \) mG and \( \sim 20 \) mG with a radial profile \( B(r) \sim r^{-1/2} \) (Koch et al. 2012). The field strength averaged over the e2 core is \( \sim 7.7 \) mG.

In order to determine the field strength \( B \) in Equation (1), the mass (gravitational potential \( \phi \)) and the pressure gradient, if significant, need to be known. Contrary to this absolute field strength measure, the relative importance of the field compared to the other forces is directly imprinted in the field and intensity morphologies. With the magnetic field tension force \( F_B = B^2/4\pi R \) and the gravitational and pressure forces \( |F_G + F_P| = |\rho \nabla \phi + \nabla \rho| \), Equation (1) is rewritten as

\[
\left( \frac{F_B}{|F_G + F_P|} \right)_{\text{local}} = \left( \frac{\sin \psi}{\sin \alpha} \right)_{\text{local}} \equiv \Sigma_B, \tag{2}
\]
where we have introduced $\Sigma_B$ to define the field significance. The polarization–intensity-gradient method thus provides a way to estimate the local magnetic field significance relative to other forces in a model-independent way. It is based on a generally valid (ideal) MHD equation, but it is independent of any molecular cloud/core models. Furthermore, this technique to extract the field significance is free of any necessity of mass and field strength measurements. The ratio in Equation (2) is solely based on measured angles which reflect the geometrical imprint of the various forces. Consequently, this also means that molecular clouds with accordingly scaled masses and field strengths can show identical morphologies. This self-similar (or scale-free) property is lifted when calculating the field strength $B$ for a particular cloud mass in Equation (1).

In any case, the angle factor $\sin \psi/\sin \alpha = \Sigma_B$ provides a quantitative criterion as to whether the magnetic field can prevent an area in a molecular cloud from gravitational collapse ($\Sigma_B > 1$) or not ($\Sigma_B < 1$). As further demonstrated in Koch et al. (2012), $\Sigma_B$ is only minimally or even not at all affected by projection effects.

4. RESULTS

4.1. Magnetic-field–Intensity-gradient Correlation

The dust continuum Stokes $I$ maps of JCMT, BIMA, and the SMA are shown in the left column panels in Figure 1. In this sequence of subsequently higher resolution maps, the JCMT main core (40° in size) is resolved into two cores (~8°) in the BIMA observation, where the northern core is further resolved into W51 e2 (~2°) in the SMA map. Overlaid on the dust continuum maps are the magnetic field segments (red) and the intensity gradient segments (blue). For completeness, the SMA result for W51 e2 is reproduced from Koch et al. (2012). A tight correlation between magnetic field P.A. and the corresponding intensity gradient P.A. (angle $\delta < \pi/2$ in Figure 3 in Koch et al. 2012) is obvious for many of the pairs. The correlation coefficients—in the definition of Pearson’s linear correlation coefficient—are 0.71, 0.72, and 0.95 for the JCMT, BIMA, and the SMA data, respectively. Thus, there is seemingly a trend for an increasing alignment between magnetic field and intensity gradient orientations with smaller scales, with the tightest correlation found for the collapsing core. This correlation is being investigated in a separate work on a larger data sample (P. M. Koch et al. 2012, in preparation). It suffices to mention here that the above correlation can possibly serve as an indicator for the role of the magnetic field through the evolutionary stages of a molecular cloud.

4.2. Relative Magnetic Field Significance from Large to Small Scales

Maps of the magnetic field to gravitational force ratio, $\Sigma_B = \sin \psi/\sin \alpha$, are displayed in the right column panels in Figure 1. The pressure gradient force, $\nabla P$, typically being small compared to gravity, is omitted here. For the JCMT and BIMA data, two gravity centers at the two emission peaks in each map are assumed in order to calculate $\psi$. The angle $\psi$ is then simply measured in between the intensity gradient direction (left column panels in Figure 1) and the gravity center direction. For W51 e2 (SMA data), a single gravity center at the emission peak in the SMA map is adopted. The angle $\alpha = \pi/2 - \delta$, where $\delta$ is the difference in between the intensity gradient and the magnetic field orientations, is deduced from the left column panels in Figure 1. All three maps—originating from three different instruments, one single dish and two interferometers—show clear differences in the ratios between core regions and areas in between the cores or further away. With the largest mapping area, the JCMT observation reveals very distinct features. With the exception of only two segments in the eastern core, both core regions show ratios below 0.5. In between the two cores the ratios reveal peaks in between 1.5 and 5, with large stretches in the northeast–southwest direction larger than 1. The BIMA observation is rather restricted to the cores, with an area in the northern core without polarization detection. Nevertheless, both cores clearly show ratios below 1, except one segment in the southern core. The four segments in the east–west direction (around $y$-offset $\sim -4$) reveal ratios larger or around one. Though limited to a few segments only, this still points toward a trend of increased ratios outside the immediate core areas. The detection by the SMA mostly resolves polarization features in the e2 core. The ratio averaged over the core is $\sim 0.2$. The northwest extension shows two segments with a ratio larger than 1 and an average of about 0.8. If assuming an additional new core being formed here, the ratios get reduced to about 0.5. In any case, the main collapsing core very clearly shows ratios below 1, whereas the more distant northwest extension shows larger values. We remark that the total outflow mass in e2 (1.4 $M_\odot$; Shi et al. 2010) is negligibly small compared to the total core mass of about 220 $M_\odot$ (Tang et al. 2009b). Additionally, with a dust-to-gas ratio of about 1–100, the outflow is unlikely to be detected in the dust continuum with the SMA sensitivity. Consequently, we do not expect the magnetic field and the dust continuum Stokes $I$ morphologies to be significantly affected by outflows. Our analysis, therefore, still quantifies the field to gravitational force ratio. The SMA e8 core is not further analyzed here because only a few polarization segments are detected (Tang et al. 2009b).

In summary, the cores in the JCMT, BIMA, and SMA data have average ratios of 0.33, 0.45 (east, west), 0.38, 0.49 (north, south), and $\sim 0.2$ (main core), respectively. The other areas reveal significantly and systematically larger ratios ($\gtrsim 1$). Despite probing three very different physical scales, the three data sets equally reveal a minor role of the magnetic field, $F_B < F_G$, in the center regions, and an increasingly more significant field, $F_B \gtrsim F_G$, at larger distances.

The distinct features of the field-to-gravity force ratio in the right panels in Figure 1 are further analyzed in the following. Azimuthally averaged radial profiles, binned at half of the beam resolution, are displayed in Figure 2. The profiles are centered at the dust continuum peaks. From the JCMT data, the main (eastern) peak covering the BIMA observation and the SMA e2 core is selected. On the next smaller scale, the BIMA southern core is chosen because its force ratio in the center has a smaller uncertainty than the northern core. As already manifest in the right panels in Figure 1, a clear difference between center and outer regions is seen. Both the JCMT and SMA profiles show ratios across the center ($\sim 0.3$ for JCMT and $\sim 0.2$ for SMA) with relatively little variations. This is followed by a rather abrupt increase over a short distance (one bin) by a factor of $\sim 2$ (JCMT) and $\sim 4$ (SMA). A plateau, with ratios $\sim 0.8$ for both JCMT and SMA, then extends over distances comparable to or larger than the center regions. The JCMT data, with the largest mapped area, then show another abrupt change to a ratio beyond 1. No polarized emission is detected for W51 e2 at larger distances. The case of BIMA is less clear, but still shows values in the center of $\sim 0.1$ and $\sim 0.6$, followed by a plateau around 0.6 and an increase to larger than 1 at the largest distances. For
From the JCMT and BIMA observations, the results for the main peak and the southern peak, respectively, are shown. Binned values are connected with lines for visual guidance only. Errors for the force ratios are calculated by propagating the measurement uncertainties in the intensity emission profile as a function of distance from the emission peak for the JCMT western and the BIMA northern core. Nevertheless, values are clearly below one in the core region with a trend to grow at larger distances up to about 0.9. Similarly to the eastern core, the western core in the JCMT data shows ratios around 0.2–0.6 up to about 0.9. Further away from the peak, ratios grow up to about 5 with some oscillatory behavior remaining larger than 1. Thus, despite being less pronounced, the two additional cores here also reveal a transition in their force ratios between central and more distant ratios. In particular, in the overlapping region between the BIMA northern core and the SMA e2 core, the ratios show similar trends regardless of the different field morphologies (hourglass-like for SMA, more uniform for BIMA).

Errors in the field-to-gravity force ratio are calculated by propagating the measurement uncertainties $\Delta\psi$ and $\Delta\alpha$ through Equation (2). The measured magnetic field P.A. uncertainty, in the range of a few degrees to $\sim10^\circ$, is assumed for $\Delta\alpha$. A typical uncertainty of $3^\circ$–$5^\circ$ in $\Delta\psi$ results after interpolation.
when calculating the intensity gradients. This leads to errors for individual ratios in Figure 2 of $\sim \pm 4\%$ to $\pm 19\%$, $\sim \pm 4\%$ to $\pm 40\%$, and $\sim \pm 2\%$ to $\pm 20\%$ (with a single outlier at $\pm 60\%$) for the SMA, BIMA, and the JCMT data, respectively. After averaging in each bin, the errors are typically reduced due to the sample variance factor, resulting in average errors of $\sim \pm 10\%$ or less. Average errors of $\sim \pm 20\%$ remain for two bins in the JCMT data. A 10\% measurement uncertainty is conservatively estimated for the emission intensity when calculating errors for its radial profiles. Binned errors are then typically at the percent level (Figure 2). Except for the single central value for the force ratio in the BIMA northern core ($\sim \pm 50\%$ error), similar errors are present in Figure 3.

Finally, it is important to keep in mind that the correlation presented in Section 4.1 (based on one angle between two orientations) can be affected by projection effects. Generally, all values are integrated along the line of sight. The field significance $\Sigma_B$ presented here is much less or not at all affected by projection, because it is based on the ratio of two angles (Koch et al. 2012). It nevertheless still deals with quantities averaged along the line of sight.

5. DISCUSSION

5.1. Self-similar Profiles

Given the common features in the radial profiles of the magnetic to gravitational force ratio (Figures 2 and 3), we address here the question of self-similarity. We focus on the JCMT main core where the BIMA and SMA data provide higher resolution follow-up observations. The BIMA southern core is adopted due to its better statistics. All three data sets in Figure 2 show a rather constant ratio across their cores (plateau-like or a single binned radius), followed by a plateau with a larger ratio before eventually the ratio grows to values larger than or around one. With the cores in the maps of Figure 1 being clearly identified, we choose to normalize the distances from the peaks to units of core sizes, i.e., all profiles are aligned to one normalized core size. The normalization radii, chosen to be those bins where the emission intensity profiles in Figure 2 flatten out, are $21'85, 3'42$, and $1'06$ for the JCMT, the BIMA, and the SMA data, respectively. At these radii, the emissions are $\sim 15\%–20\%$ of the peak emissions. Figure 4 shows the result.

Even with some uncertainty left in the normalization, it is obvious that the aligned profiles show a close resemblance. This self-similarity suggests that the interplay between magnetic and gravitational force is independent of the three different scales probed here. Thus, the analysis here quantifies the relative magnetic field significance from being generally dominant or comparable to gravity at distances twice the core size to minor or negligible inside the core. We stress that this result seems to hold generally for all of the different field morphologies analyzed here, including even the more irregular cases shown in Figure 3.

5.2. Mass-to-flux Ratio Derived with Polarization–Intensity-gradient Method

In this section, we investigate the connection between the force ratio in Equation (2) and the mass-to-flux ratio for a molecular cloud. The mass scale associated with the amount of magnetic flux $\Phi$ threaded by a self-gravitating cloud was introduced in Mestel & Spitzer (1956). The magnetic critical mass $M_c = \Phi/2\pi G^{1/2}$, with $\Phi = \pi R^2 B$ where $R$ is the cloud radius, defines the maximum mass that can be supported by the magnetic field against gravitational collapse if no other forces are present (e.g., Shu et al. 1999). This leads to the notions of magnetically supercritical clouds when $M > M_c$.
Figure 5. Illustration of the various mass-to-flux ratios as derived in Section 5.2. Every small circle of radius $R_ℓ$ represents a local mass-to-flux ratio ($m_ℓ/φ_ℓ$, Equation (4)). Its size is determined by the beam resolution. The differential mass-to-flux ratio $(ΔM/Δφ(r)$, Equation (5)) at radius $r$ is calculated by azimuthally averaging all the local ratios of the hatched red circles at radius $r$. The four solid red circles symbolize differential ratios at four different radii. The integrated mass-to-flux ratio ($M/φ(r)$, Equation (7)) within radius $r$ is shown with the blue hatched circle. Concentric blue circles with growing radii symbolize integrated ratios for increasingly larger volumes of the cloud. Azimuthally averaged ratios for each radius are displayed in the profiles in Figure 6. A single global ratio ($M/φ$, Equation (8)) is calculated from the black circle encompassing the entire cloud.

(A color version of this figure is available in the online journal.)

and magnetically subcritical clouds when $M < M_0$. Contrary to the mass-to-flux ratio—which is typically applied to the entire (global) molecular cloud—the force ratio in Equation (2) is a local criterion comparing the field significance with gravity at a specific location in a cloud. Neglecting pressure gradients and explicitly writing out the local field force and gravity force terms, it can be expressed as a ratio of local flux over local mass:

$$\sin \frac{\psi_ℓ}{\sin α_ℓ} = \frac{Φ_ℓ^2}{m_ℓ^2} \cdot \frac{m_ℓ}{M_ℓ} \cdot \frac{1}{R_{B,ℓ}} \cdot \frac{R_{G,ℓ}^2}{R_ℓ^2} \cdot \frac{1}{4π^3 G}$$  \hspace{1cm} (3)

where the lower index $ℓ$ refers to a local quantity. $M_ℓ$ is the gravitating mass leading to the local gravitational force $F_{G,ℓ} = G(m_ℓ M_ℓ/R_{G,ℓ}^2)$ acting upon a local mass element $m_ℓ$ at a distance $R_{G,ℓ}$. $Φ_ℓ$ is the flux associated with the local mass within a flux tube of radius $R_ℓ$ (Figure 5). $R_{B,ℓ}$ is the local field radius. $G$ and $η$ are the gravitational constant and a numerical unit conversion factor, respectively. Equation (3) is generally valid. However, in order to make further use of it, the local mass element $m_ℓ$ and $M_ℓ$ need to be specified. We therefore make the basic assumption of spherical symmetry. The enclosed mass within a distance $r$ from the center is then $M_ℓ(r) = \int_0^r 4πr'^2 ρ(r') dr' \equiv N_ℓ m_ℓ(r)$, where $N_ℓ$ is the number of local flux tubes with an average local mass $m_ℓ(r)$ where both depend on the integration radius $r$. A local mass-to-flux ratio then results from Equation (3):

$$\frac{m_ℓ}{φ_ℓ} = \left( \frac{\sin \psi_ℓ}{\sin α_ℓ} \right)^{-1/2} \cdot \left( \frac{m_ℓ(r)}{m_ℓ(r)} \right)^{1/2} \cdot R_{B,ℓ}^{-1/2} \cdot R_ℓ^{-1} \cdot \left( \frac{4π^3}{G} \right)^{-1/2} \cdot \left( \frac{η}{G} \right)^{1/2} \cdot \left( \frac{m_ℓ(r)}{m_ℓ(r)} \right)^{1/2} \cdot \left( \frac{R_ℓ^2}{R_{B,ℓ}^2} \right),$$  \hspace{1cm} (4)

where we have used $M_ℓ(r) = N_ℓ m_ℓ(r)$ with $N_ℓ = R_{G,ℓ}^2 / R_ℓ^2$.

We note that the above expression is still fairly general and valid for any density profile $ρ(r)$. In particular, since the ratio depends on the ratio of the mass profile over a mean mass, only the functional form of the density profile is relevant, but not its absolute value. Introducing a mean local mass, $m_ℓ(r)$, allows us to write the local mass-to-flux ratio with the explicit spherical radius dependence only in the second term on the right-hand side of Equation (4). Besides the spherical symmetry assumption for the mass distribution, Equation (4) is basically valid at any position in a molecular cloud. The average inverse of the force ratio decreases with larger radius (Figure 4), and $(m_ℓ(r)/m_ℓ(r))^{1/2} \leq 1$ is monotonically decreasing for centrally peaked density profiles, with unity in the center. The local mass-to-flux ratio in Equation (4) is thus decreasing with larger radius from the center. Azimuthally averaging Equation (4) and normalizing it to the critical mass-to-flux ratio, we get the local normalized mass-to-flux ratio—for individual subvolumes of fixed size $R_ℓ$ in a cloud—as a function of radius:

$$\left( \frac{ΔM}{ΔΦ} \right)_{norm} = \left( \frac{\sin \psi_ℓ}{\sin α_ℓ} \right)^{-1/2} \cdot \left( \frac{R_ℓ}{R_{B,ℓ}} \right)^{-1/2} \cdot \left( \frac{m_ℓ(r)}{m_ℓ(r)} \right)^{1/2} \cdot \left( \frac{R_ℓ^2}{R_{B,ℓ}^2} \right)^{-1/2} \cdot \left( \frac{4π^3}{G} \right)^{-1/2} \cdot \left( \frac{η}{G} \right)^{1/2} \cdot \left( \frac{m_ℓ(r)}{m_ℓ(r)} \right)^{1/2} \cdot \left( \frac{R_ℓ^2}{R_{B,ℓ}^2} \right), \hspace{1cm} (5)$$

where $⟨\cdots⟩_r$ denotes azimuthal averaging at radius $r$. When normalizing to a local flux tube, we can simply set $R_0 \equiv R_ℓ$.

We note that it is also possible to proceed directly with Equation (3). Assuming that both $m_ℓ$ and $M_ℓ$ are proportional to the integrated dust emission with an identical conversion factor, the ratio $m_ℓ/M_ℓ$ can directly be calculated from a dust continuum emission map. In this way, it is valid for any arbitrary cloud shape. This is identical to the approach outlined in Koch et al. (2012), where a local gravity direction is derived for any cloud shape. Equation (3), in its most general form, then leads to a map of local mass-to-flux ratios. However, we are here aiming at revealing changes in the mass-to-flux ratios with radius. Therefore, the spherical symmetry assumption and azimuthal averaging are appropriate (Equation (5)). In practice, we will calculate $m_ℓ/M_ℓ$ from the dust emission maps as mentioned above.
Furthermore, as found in Koch et al. (2012), \( R_{B,\ell} \) is roughly constant over an observed map, and similar to \( R_s \), which we identify with the beam resolution of an observation. With this simplification, \( R_{B,\ell}^{1/2} R_s^{-1} / R_s^{-3/2} \approx 1 \). Assuming the cloud gravitating mass to be proportional to the dust emission, \( m_s / M_s \) is derived from the observed emission intensity profiles in Figure 2. As already indicated with the notation, Equation (5) quantifies the differential mass-to-flux ratio as a function of radius.

In a next step, we derive an integrated mass-to-flux ratio, i.e., we are asking whether the entire cloud inside a certain radius \( r \) is magnetically supercritical. We then have to evaluate

\[
\frac{M}{\Phi}(r) = \sum_{\ell=1}^{N_{\ell}} \frac{m_{\ell}}{\phi_{\ell}} = M_s(r) \cdot \sum_{\ell=1}^{N_{\ell}} \phi_{\ell}^{-1}.
\]

\( \phi_{\ell} \) can be expressed with Equation (4). The summation is over all local mass and flux elements within the radius \( r \), where the summation limit \( N_{\ell} \) depends on \( r \). After some algebra and after normalizing with the critical mass-to-flux ratio, we find

\[
\left( \frac{M}{\Phi}(r) \right)_{\text{norm}} = \left( \sin \psi_{\ell} / \sin \alpha_{\ell} \right)^{-1/2} \cdot R_{B,\ell}^{1/2} \cdot \frac{R_s^{-3/2}}{r^{3/2}} \cdot \pi^{-1/2}.
\]

Eq. (7) states that the integrated mass-to-flux ratio is calculated by adding bin-averaged force ratios which are weighted with a mass function and \( N_{ri} \), where the latter one results from the integrated volume growing with \( r \). The normalization is with respect to \( r \), i.e., the increasing volume (with growing radius \( r \)) is normalized with its corresponding critical mass-to-flux ratio. \( R_s \) is constant and again set by the beam resolution. \( R_{B,\ell} \) is also roughly constant. When omitting the summation, setting \( N_{ri} \equiv 1 \) and \( r \equiv R_s \) and replacing \( m(r) \) with \( m(r_i) \), Equation (7) reduces to the differential mass-to-flux ratio in Equation (5).

Finally, a global mass-to-flux ratio for the entire cloud as a single entity can be derived from Equation (4) in the limiting case where all (small) local quantities grow to cloud-size: \( m_{\ell} \rightarrow M_s \) and \( \langle R_{B,\ell} \rangle \approx \langle R_s \rangle \approx \langle R_c \rangle \equiv R \). The force ratio needs to be averaged over the entire cloud: \( \langle \sin \psi_{\ell} / \sin \alpha_{\ell} \rangle \rightarrow \langle \sin \psi / \sin \alpha \rangle \). This then leads to the global mass-to-flux ratio, normalized to the critical mass-to-flux ratio with \( R_0 \):

\[
\left( \frac{M}{\Phi} \right)_{\text{norm}} = \left( \sin \psi / \sin \alpha \right)^{-1/2} \cdot \left( \frac{R}{R_0} \right)^{-3/2} \cdot \pi^{-1/2}.
\]

Eq. (8) states that the integrated mass-to-flux ratio is driven by the increasing volume. It therefore grows with radius. As expected, this ratio is generally larger than the differential ratio of a subvolume because mass scales with volume whereas flux scales with area. The slight mismatch in the innermost bin results from different weightings for the two ratios. Finally, the global mass-to-flux ratio is close to the integrated ratio at large radii. The difference here results from the incomplete sampling of \( N_{ri} \) in azimuth for those bins which contain some depolarization zones. It is important to remark that both the integrated and global ratios present average values for the entire cloud. Therefore, they are likely biased toward some subvolumes. Whereas
where the gravitational force ratio (Figure 2) with the additional mass ratio, \((m_i(r)/\bar{m}_i(r))^{1/2} \leq 1\). The mass ratio is derived from the integrated emission profiles in Figure 2. A clear transition between magnetically supercritical in the center to subcritical at larger distances is revealed. Open circles correspond to the individual ratios before averaging and before multiplying with the mass ratio. Thus, they are local upper limits (Equation (5)). The dashed line is the integrated mass-to-flux ratio (Equation (7)) normalized to the critical mass-to-flux ratio with its radius corresponding to each binning radius. The single global mass-to-flux ratio (Equation (8)), averaging all force ratios and normalizing to the core radius, is displayed with the filled circle. Indicated with black dashed lines are the critical mass-to-flux ratio and the core size (normalized to one). When propagating uncertainties through \(\Delta\Sigma_B = \pm 10\%\) as the force ratios drop in the center area, errors grow larger by a factor of about two to three, i.e., reaching \(\pm 20\% - \pm 30\%\). For clarity, error bars are not displayed here.

(A color version of this figure is available in the online journal.)

**Figure 6.** Various mass-to-flux ratios normalized to the corresponding critical mass-to-flux ratios (Section 5.2) for W51 e2. The ratios are shown as a function of distance from the peak normalized to the core size. The solid line displays the normalized azimuthally averaged differential mass-to-flux ratios (Equation (5), converted to the magnetic field to gravitational force ratios (Figure 2) with the additional mass ratio, \(m_i(r)/\bar{m}_i(r))^{1/2} \leq 1\). The mass ratio is derived from the integrated emission profiles in Figure 2. A clear transition between magnetically supercritical in the center to subcritical at larger distances is revealed. Open circles correspond to the individual ratios before averaging and before multiplying with the mass ratio. Thus, they are local upper limits (Equation (5)). The dashed line is the integrated mass-to-flux ratio (Equation (7)) normalized to the critical mass-to-flux ratio with its radius corresponding to each binning radius. The single global mass-to-flux ratio (Equation (8)), averaging all force ratios and normalizing to the core radius, is displayed with the filled circle. Indicated with black dashed lines are the critical mass-to-flux ratio and the core size (normalized to one). When propagating uncertainties through \(\Delta\Sigma_B = \pm 10\%\) as the force ratios drop in the center area, errors grow larger by a factor of about two to three, i.e., reaching \(\pm 20\% - \pm 30\%\). For clarity, error bars are not displayed here.

(A color version of this figure is available in the online journal.)

these ratios provide a description as to whether a cloud as an entity is in a subcritical or a supercritical state, they cannot properly assess the state of individual subvolumes. This question can only be further addressed with the differential mass-to-flux ratio. The finding here clearly demonstrates the differences and the need for a measurement of the local field significance.

5.3. Dynamical Implication and Star Formation Efficiency

In this section, we aim at investigating further implications of the local force ratio \(\sin \psi/\sin \alpha\) (Figure 4) and the differential mass-to-flux ratio (Figure 6). We limit the discussion here to the collapsing core W51 e2. Global and/or average cloud properties are typically used to characterize its state, e.g., in order to determine whether a system is gravitationally bound or whether it can collapse or not. With the local criteria—the magnetic field to gravity force ratio in Equation (2) and the differential mass-to-flux ratio in Equation (4)—we have tools to go one step further. We cannot only address the question whether a cloud collapses or not, but we can even ask: Where does a cloud collapse? How does it collapse? Does all of the gas take part in the collapse? And finally, depending on the answers to these questions, what are the consequences for the inferred star formation efficiency?

The change with radius in the magnetic field to gravity force ratio (Figure 4) clearly demonstrates that gravity is not everywhere equally efficient to initiate and to keep driving a collapse. The result in Figure 4 suggests and quantifies an effective gravitational force which is reduced by the presence of the magnetic field. Indeed, on the theory side, a concept of diluted gravity (by the magnetic field) was put forward by Shu & Li (1997). Thus, we can define an effectively acting gravitational force \(F_e\) as

\[
F_e(r) = \left(1 - \frac{\sin \psi}{\sin \alpha}\right) \cdot F_{\text{max}}(r) \equiv \langle \epsilon_G \rangle \cdot F_{\text{max}}(r),\tag{9}
\]

where \(\langle \cdot \cdot \cdot \rangle_{\alpha}\) denotes azimuthal averaging at radius \(r\) and \(F_{\text{max}}(r)\) is the maximum non-diluted gravitational force resulting from an enclosed spherically symmetric mass distribution within \(r\). \(F_e\) is only defined where \(\sin \psi(r)/\sin \alpha(r) \leq 1\), which possibly holds only for a limited area. In Equation (9), we have introduced the gravity efficiency factor \(\epsilon_G \in [0, 1]\).

Besides the diluted gravity which slows down the accretion and collapse process, the fraction of volume of a cloud (or core) that actually can collapse is relevant, i.e., where is the cloud/core magnetically supercritical? The differential mass-to-flux ratio (Figure 6) provides a local criterion that answers this question. The increasing ratio toward the center indicates that only the central part of the core has accumulated enough mass to overcome the magnetic flux. By reading off the radius where

\[10\, \text{Unlike the gravity efficiency factor } \epsilon_G, \text{ the field significance } \Sigma_B \text{ in Equation (2) is not constrained to an upper limit of one.}\]
the core state changes from magnetically sub- to supercritical, the potentially collapsing volume and its associated mass can be estimated. This defines a volume efficiency, $\epsilon_V \in [0, 1]$, compared to the maximum efficiency $\epsilon_V \equiv 1$ where the entire core can collapse.

Based on the two findings described above, we proceed to estimate a star formation efficiency with the assumptions: (1) the available effective gravitational force is diluted by the presence of the magnetic field and (2) only the regions where the differential mass-to-flux ratio is larger than one will eventually collapse and form stars. We choose to reference our estimate to the well-known pressureless free-fall collapse of a spherical gas sphere. The dynamics in this case are governed by the momentum equation $(DV/Dt) = -G(M_r/r^2)$, where the convective derivative is $(DV/Dt) = (\partial v/\partial t) + v(\partial v/\partial r)$ for the gas infall velocity $v$. The enclosed mass within radius $r$ is $M_r = \int_0^r 4\pi r^2 \rho(r') dr'$ with the gas density $\rho$. With the diluted gravity, the momentum equation reads

$$\frac{DV}{Dt} = -\langle \epsilon_G \rangle_r \cdot \frac{G M_r}{r^2}.$$  \hspace{1cm} (10)

Technically, the gravity efficiency factor $\langle \epsilon_G \rangle_r$ could be absorbed into an effective density $\rho_e(r)$. Therefore, the general solution (for any $M_r$) of Equation (10) for a fluid element starting at a distance $r_0$ and reaching the center at $r = 0$ at the time $t$ is still valid. The time $t_s$ in the presence of the magnetic field is then

$$t_s = \frac{\pi}{2} \sqrt{\frac{r_0^3}{2GM_r \langle \epsilon_G \rangle_r}} \approx \langle \epsilon_G \rangle_r^{-1/2} \cdot t.$$  \hspace{1cm} (11)

Strictly speaking, $\langle \epsilon_G \rangle_r$ in the above equation is still a function of radius. In writing the right-hand side we have assumed it to be averaged within an appropriate radius. The average accretion rate $\langle M_r \rangle$, assuming the entire mass $M$ within a cloud/core to be accreted and to collapse, is $\langle M_r \rangle = M/t$. Adding the volume efficiency factor $\epsilon_V$ together with Equation (11), the effective accretion rate $\langle M_\epsilon \rangle$, corrected for our observed magnetic field features, becomes

$$\langle M_\epsilon \rangle = \frac{M_\epsilon}{t} \approx \frac{\epsilon V \cdot M}{\langle \epsilon_G \rangle_r^{-1/2} \cdot t}.$$ \hspace{1cm} (12)

The resulting star formation efficiency relative to the standard free fall one is then expressed as

$$\frac{\langle M_\epsilon \rangle}{\langle M \rangle} \approx \epsilon_V \cdot \langle \epsilon_G \rangle_r^{1/2}.$$ \hspace{1cm} (13)

We note that the above phenomenological modeling is still valid in the presence of additional force terms in the momentum equation with any density profiles. The pressureless free-fall collapse simply allows us to analytically express the results. Figure 7 shows the reduced star formation efficiency taking into account the effects of the magnetic field. The red dots indicate efficiencies estimated from the gravity dilution and the reduced collapse volume based on the Figures 4 and 6. Since $1 - (\sin(\psi(r)/\sin(a(r))) \sim 0$ at large distances, an arbitrarily low star formation efficiency would result from adopting these values for $\epsilon_G$. Instead, we adopt values averaged over the entire core (force ratio $\sim 0.33$), the central core only ($\sim 0.2$), and the outer plateau ($\sim 0.85$). We consider these values adequate to estimate and sample the gravity dilution for W51 e2. Similarly, for the volume efficiency factor we read a radius of about 1'' within which the core is magnetically supercritical compared to an overall size of about 2''. For comparison, a larger volume of about 1.5'' is considered as well. The resulting efficiencies are reduced at least to a conservative value of $\sim 0.35$ or less when still assuming the larger volume to collapse. The efficiencies drop to $\sim 0.1$ with the smaller volume, with a likely limit of

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1 For a uniform density and $\langle \epsilon_G \rangle_r \equiv 1$, Equation (11) describes a synchronized collapse with the well-known free-fall time $t_{ff} = \sqrt{3GM_0/2\pi}$. More realistic density profiles with a central concentration, e.g., $\rho(r) = \rho_0 (r/r_0)^{-\alpha}$ lead to a collapse time $t = \sqrt{GM_0 \rho_0 / 4\pi G^2 \rho_0^2}$, which increases with larger distance $r_0$ from the center and then reproduces an inside-out collapse.
about $\sim 0.05$ in combination with the largest observed gravity dilution. These values are close to the observationally inferred efficiencies of a few percent (e.g., Krumholz & Tan 2007). For comparison, recent numerical simulations by Nakamura & Li (2011) find a star formation efficiency per global free-fall time of $\sim 20\%$ to $\sim 30\%$ depending on the magnetic field strength. When they further take into account the feedback from outflows, their efficiencies can be reduced by another order of magnitude.

We finally remark that the discussion here is limited to the core $e_2$. A more complete picture, estimating the star formation efficiency starting from the largest scales of the cloud envelop, will need to dynamically link the observed self-similar profiles from Figure 4. Caution is needed here because linking structures of different size scales is non-trivial due to additional structures possibly generated by, e.g., fragmentation and turbulence. Nevertheless, this first estimate further solidifies the significance and impact of the magnetic field. Moreover, it demonstrates that the magnetic field is capable of reducing the star formation efficiency by at least one order of magnitude or even more.

6. SUMMARY AND CONCLUSION

We have applied the polarization–intensity-gradient method (Koch et al. 2012) to a set of single dish and interferometer dust continuum data of the W51 star formation region. These observations cover scales from the initial parent cloud ($\sim 2$ pc, JCMT) to the large-scale envelope ($\sim 0.5$ pc, BIMA) and to the collapsing core ($\sim 60$ mpc, SMA). Besides leading to a magnetic field strength as a function of position in a map, our new method also provides a way to estimate the local magnetic field to gravity force ratio. The technique is model independent, without any input from mass or field strength, solely making use of measured angles in dust polarization and Stokes $I$ map. Here, we have focused on this force ratio and on some of its implications. Our main results are summarized in the following.

1. Magnetic-field–intensity-gradient correlation. The correlation in the magnetic field and dust intensity gradient orientations—which served as a starting point for the new method developed in Koch et al. (2012)—is also found at larger scales in the BIMA and JCMT data. The correlation coefficients seem to be the larger the higher the resolution is, i.e., the tightest correlation is found for the collapsing core $e_2$ (Figure 1, left panels).

2. Magnetic field significance. Maps of the magnetic field to gravity force ratio show distinct features for all three data sets (Figure 1, right panels). At larger distances from the emission peaks, the field force is comparable or larger than the gravity force (ratio $\gtrsim 1$). Closer to the emission peaks, gravity becomes more dominant (ratio $< 1$). This suggests and quantifies that gravity is not everywhere acting equally efficiently, but is being more significantly opposed by the magnetic field tension force at larger distances. Based on this finding, we introduce a gravity efficiency factor. This also seems to be in agreement with the concept of gravity dilution (Figure 4). Moreover, this is in support of inside-out collapse scenarios.

3. Self-similarity. Azimuthally averaged radial profiles of the force ratios show similar features for the JCMT, BIMA, and the SMA data (Figure 2). When normalized to core sizes, these profiles closely align (Figure 4). This points toward self-similar properties in the magnetic field and gravity interplay from the large parent cloud down to the collapsing core.

4. Mass-to-flux ratio. The force ratio can be converted into a mass-to-flux ratio. In its most general form, the mass-to-flux ratio can then be expressed as the inverse square root of the force ratio multiplied by the square root of a ratio of two mass elements. With the force ratio being a local criterion, this naturally leads to a local (differential) mass-to-flux ratio for a subvolume in a molecular cloud as a function of position. An integrated and global mass-to-flux ratio can be derived by appropriately adding and averaging local quantities. Similar to the force ratio, all the various mass-to-flux ratios do not have to rely on absolute mass and field strength inputs, but they are largely model independent. This finding then only provides a criterion to decide whether a molecular cloud as an entity is magnetically supercritical or not, but it also extends this criterion to any subvolume (Figures 5 and 6). In the case of W51 $e_2$, a transition occurs from subcritical at larger distances to supercritical in the central area. This might explain why fragmentation is found to happen preferentially in the center.

5. Star formation efficiency. A reduced star formation efficiency (compared to the non-magnetized free-fall case) is derived based on two observed magnetic field properties: (1) gravity dilution (Figure 4) defines an effectively available gravity force to initiate and drive a collapse and, therefore, it increases the collapse time; and (2) the differential mass-to-flux ratio (as a function of radius) provides a hint that the magnetic field reduces the volume of supercritical and, therefore, only a limited volume can collapse and form stars (Figure 6). With these two findings a modified accretion rate can be calculated. Comparing this to a standard free-fall rate shows that the magnetic field is capable of reducing a standard star formation efficiency to $10\%$ or less.

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