Testing Supersymmetry in the Associated Production of CP-odd and Charged Higgs Bosons

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In the Minimal Supersymmetric Standard Model (MSSM), the masses of the charged Higgs boson \( H^\pm \) and the CP-odd scalar \( A \) are related by \( M_{H^\pm}^2 = M_A^2 + m_W^2 \). Furthermore, because the coupling of \( W^- A - H^+ \) is fixed by gauge interaction, the tree level production rate of \( gg \rightarrow W^\pm A \) depends only on one supersymmetry parameter – the mass \( M_A \) of \( A \). We show that to a good approximation this conclusion also holds at the one-loop level. Consequently, this process can be used to distinguish MSSM from its alternatives (such as a general two-Higgs-doublet model) by verifying the above mass relation, and to test the prediction of the MSSM on the product of the decay branching ratios of \( A \) and \( H^\pm \) in terms of only one single parameter – \( M_A \).

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One of the commonly discussed new physics models is the Minimal Supersymmetric Standard Model (MSSM). To describe an experimental data in the framework of the MSSM usually requires introducing more than one supersymmetry (SUSY) parameters. Hence, the usual practice is to compare data to a pre-selected class of MSSM in which certain well-defined relations among the SUSY parameters are assumed in order to reduce the number of independent variables needed for discussion.

An interesting question to ask is “Can one find a process to test SUSY models at colliders without making many assumptions on the choice of SUSY parameters?” To answer that, let us consider the Higgs sector of the model. In the MSSM, because of the supersymmetry, two Higgs doublets have to be introduced in its Higgs sector. Although the MSSM Higgs sector resembles the one in a type-II two-Higgs-doublet model (THDM) [1], it has a very specific feature – all the Higgs self couplings \( \lambda_i \) are fixed by the electroweak gauge couplings \( g \) and \( g' \), as required by supersymmetry. Hence, at the tree level, only two additional free parameters appear in the Higgs sector of the MSSM. We may take \( M_A \) (the mass of the CP-odd Higgs boson \( A \)) and \( \tan \beta \) (the ratio of the two vacuum expectation values) as these two free parameters.

One of the striking features resulted from the requirement of supersymmetry is that the mass of the lightest CP-even Higgs boson (\( h \)) has to be less than the mass \( m_Z \) of the weak gauge boson \( Z \) at the Born level, although a large radiative correction due to a heavy top quark can push this bound up to about 130 GeV in the MSSM [3]. This result is interesting when compared to the theoretical bounds on the mass of the SM Higgs boson. Requiring the SM be a well defined theory up to the Planck scale (about 10^{19} GeV), the Higgs boson mass has to be approximately between 130 GeV and 180 GeV [3,4]. Therefore, a light Higgs boson with its mass less than about 130 GeV can be a signal of the supersymmetric models, especially the MSSM. However, it is also known that such a light Higgs boson can exist in various non-SUSY models, such as a general THDM or the Zee model, even when the cutoff scale of the model is close to the Planck scale [5]. Hence, the existence of a light Higgs boson by itself cannot rule out models other than the MSSM (or its extensions).

Another striking feature resulted from the requirement of supersymmetry is that the masses of the charged Higgs boson \( H^\pm \) and the CP-odd scalar \( A \) are strongly correlated. At the Born level, they are related by the mass of the \( W^\pm \) boson \( (m_{W^\pm}) \) as

\[
M_{H^\pm}^2 = M_A^2 + m_W^2.
\]

For comparison, the corresponding mass relation in a general THDM is \( M_{H^\pm}^2 = M_A^2 + \frac{1}{2}(\lambda_5 - \lambda_4)v^2 \), where \( v \) is the weak scale (246 GeV) and \( \lambda_{4,5} \) are two free parameters of the model [5]. Therefore, the mass relation (5) can be a strong criterion to discriminate the MSSM from its alternatives, e.g., a general THDM.

To test the mass relation (5), we propose to study the associated production of \( A \) and \( H^\pm \) at high energy hadron colliders, e.g., \( pp \rightarrow AH^\pm \) at the Fermilab Tevatron (a 2 TeV proton-antiproton collider) and \( pp \rightarrow AH^\pm \) at the CERN LHC (a 14 TeV proton-proton collider). As to be explained below, this process has the following unique features: (i) its Born level rate generally depends on the masses of \( A \) and \( H^\pm \). Because of the mass relation (5), the MSSM prediction of the Born level rate only depends on one (in contrast to two or more) SUSY parameter – \( M_A \); (ii) the kinematic acceptance (therefore, the detection efficiency) of the signal events do not depend on the choice of other SUSY parameters because both \( A \) and \( H^\pm \) are spin-0 (pseudo-)scalar particles so that the kinematic distributions of their decay particles can be accurately modeled; (iii) it can constrain MSSM parameters by examining the product of the Higgs boson decay branching ratios (in contrast to the product of decay branching ratios and production rate); (iv) both \( M_A \) and \( M_{H^\pm} \) can be reconstructed from its final state to test the mass relation (5); (v) finally, the electroweak radiative corrections to its production rate and to the mass relation (5) are generally smaller than the expected ex-
experimental errors, such as the di-jet invariant mass resolution.

Either in the MSSM or the THDM, the coupling of $W^+A$ is induced from the gauge invariant kinetic term of the Higgs sector:

$$L_{\text{int}} = \frac{g}{2} W^+_{\mu} (A\partial^\mu H^- - H^- \partial^\mu A) + h.c., \quad (2)$$

so that the coupling strength of $W^+H^-A$ is completely determined by the weak gauge coupling $g$. (In contrast, the coupling constants relevant to the interactions of $W$-boson (or $Z$-boson) and neutral Higgs bosons depend on $\beta$ and $\alpha$, where $\alpha$ characterizes the mixing between the two CP-even Higgs bosons $h$ and $H$.) Thus, the Born level production rate of a $AH^\pm$ pair at hadron colliders only depends on $M_A$ and $M_{H^\pm}$. Since in the MSSM these masses are strongly correlated, cf. Eq. (1), the production rate of $p\bar{p}, pp \rightarrow AH^\pm$ only depends on one SUSY parameter, which can be taken as $M_A$.

At the Tevatron and the LHC, the dominant constituent process for the production of a $AH^\pm$ pair is $q\bar{q} \rightarrow W^\pm + A H^\pm$. For a given $M_A$, the cross section $\sigma(p\bar{p}, pp \rightarrow AH^\pm)$ is completely determined. Its squared amplitude, after averaging over the spins and colors is

$$\overline{|M|^2} = \frac{4}{3} m_W^3 G_F^2 \frac{s}{(s^2 - m_W^2)^2 + m_W^2 P^2 \sin^2 \theta}, \quad (3)$$

where $P = \sqrt{E_A^2 - M_A^2}$ with $E_A = (s + M_A^2 - M_{H^\pm}^2)/(2\sqrt{s})$ and $\theta$ is the polar angle of $A$ in the center-of-mass (c.m.) frame of $A$ and $H^\pm$. We note that for the $c\bar{b} \rightarrow AH^\pm$ subprocess, in addition to the CKM (Cabibbo-Kobayashi-Maskawa) suppressed s-channel W-boson diagram, there is a t-channel diagram, that depends on $\tan \beta$. However, the $c\bar{b} \rightarrow AH^\pm$ contribution to the inclusive $AH^\pm$ rate is small. For example, its contribution to the total rate is less than $0.01\%$ and $0.1\%$ at the Tevatron and the LHC, respectively, for $\tan \beta \approx 10$ and $M_A = 90$ GeV. For a smaller $\tan \beta$, its contribution becomes negligible. Hence, we shall ignore its contribution in the following discussion. In Fig. 1, we show the inclusive production rate of $AH^\pm$ as a function of $M_A$. Here, the CTEQ5M1 parton distribution functions (PDF) are used and both the renormalization and the factorization scales are chosen to be the invariant mass ($\sqrt{s}$) of the $AH^\pm$ pair. A next-to-leading order (NLO) QCD correction is also included, which typically increases the LO rate by about $20\%$ (when the same set of PDF is used).

It is trivial to model the kinematic acceptance (therefore, the detection efficiency) of the signal event. This is because both $A$ and $H^\pm$ are spin-0 bosons. Therefore, if the signal is not found, knowing the luminosity of the collider, the detection efficiency, and the theoretical production rate, one can conclude from the data whether the constraint on the production and branching ratio of $A$ and $H^\pm$ is a function of $M_A$. For example, if the decay mode of $A \rightarrow b\bar{b}$ and $H^\pm \rightarrow \tau^\pm \nu_\tau$ is studied and no excess is found for a given mass bin of $M_A$ (hence, $M_{H^\pm}$) when comparing with the experimental data, then one can constrain the MSSM by demanding the product of the branching ratios, $\text{Br}(A \rightarrow b\bar{b}) \times \text{Br}(H^\pm \rightarrow \tau^\pm \nu_\tau)$, to be bounded from above as a function of $M_A$. Needless to say that applying the same strategy, one can constrain the product $\text{Br}(A \rightarrow X) \times \text{Br}(H^\pm \rightarrow Y)$ for any decay mode $X$ and $Y$ predicted by the MSSM as a function of only one SUSY parameter $- M_A$.

In case that a signal is found, the analysis is slightly more complicated. In the MSSM, the mass of the heavier CP-even Higgs boson ($H$) is not very different (less than about 10 GeV) from $M_A$ for $M_A \gtrsim 120$ GeV and $\tan \beta > 10$. In this case, $q\bar{q} \rightarrow AH^\pm$ can produce the similar final states as $q\bar{q} \rightarrow AH^\pm$. Generally, the coupling of $W^\pm HH^\pm$ depends on $g$ and $\sin(\alpha - \beta)$. However, for $M_A \approx 190$ GeV and $\tan \beta > 10$, $\sin^2(\alpha - \beta) \approx 1$ and the production rate of $HH^\pm$ is almost the same as $AH^\pm$ in the MSSM. When both of them decay into the same decay channel, it will be very difficult to separate the production of $AH^\pm$ from $HH^\pm$ unless a fine mass resolution can be achieved experimentally. Nevertheless, studying different decay channels can help to separate these two production modes. For instance, a heavy $H$ can decay into a $ZZ$ pair at the Born level, but $A$ cannot.

In conclusion, if no signal is found experimentally, a conservative bound on the product of the decay branching ratios of $A$ and $H^\pm$ can be derived for a CP-conserving model. This is because in a CP-conserving model, the $AH^\pm$ and $HH^\pm$ production modes do not interfere even if the masses of $A$ and $H$ are about the same. (We note that $A$ is a CP-odd scalar, while $H$ is CP-even.)

To test the MSSM relation (1) via the $b\bar{b}\nu\nu$ mode, if the signal is sufficiently large as compared to the backgrounds, a resonance bump should be observed (at a value $M_{b\bar{b}}$) in the distribution of the $b\bar{b}$ invariant mass. Then, by searching for the corresponding Jacobian peak (at a value $M_{\tau\nu}$) in the distribution of the transverse mass of the $\tau\nu$ pair, one can test the MSSM by examining whether $\langle M_{\tau\nu} \rangle$ is consistent with $\sqrt{(M_{b\bar{b}})^2 + m_W^2}$ within the accuracy of the mass resolution of the detector. By testing this mass relation via the process $p\bar{p}, pp \rightarrow AH^\pm$, one can discriminate the MSSM from its alternative (e.g., a general THDM).

In order to prove that the proposed process can be used to test the MSSM and is sensitive to only one SUSY parameter $- M_A$, we assume that the $A$ and $H^\pm$ couplings to the $W$-boson production at hadron colliders, except at a different invariant mass.
parameter – $M_A$, we have to show that the SUSY electroweak correction, which occurs at the one-loop level, to the production rate is small. (The final state SUSY QCD correction does not contribute until the two-loop level.) Specifically, we need to consider two essential points: (a) what is the typical size of the radiative correction to the coupling of $W^\mp A H^\mp$? (b) does the mass relation (9) hold beyond the Born level? In the following, we shall have a more detailed discussion on these important questions. A brief summary is that as long as the typical SUSY mass scale is at the order of a few TeV or below, the one-loop correction to the $W^\pm AH^\mp$ vertex can modify the production rate of the $AH^\pm$ pair at most by a few percent. Therefore, the electroweak correction is smaller than (i) the expected statistical and systematic errors of the experimental data, or (ii) the uncertainty in the theory prediction of the production rate originated from the parton distribution functions (which is estimated to be about 6% at the Tevatron and 5% at the LHC for $M_A = 120$ GeV, when applying the prescription presented in Ref. [5]), or (iii) the higher order ($\alpha_s^2$ or above) QCD correction (which is estimated to be about 10% at the Tevatron and less than a percent at the LHC for $M_A = 120$ GeV, when varying the factorization scale around $\sqrt{s}$, the c.m. energy of the subprocess $qq' \rightarrow AH^\pm$, by a factor of 2). Furthermore, as long as the typical SUSY mass scales are at the order of a couple of TeV or below, the radiative correction to the mass relation (9) is generally smaller than the typical mass resolution of the experimental measurement (which is about 10 GeV for a 100 GeV Higgs boson decaying into jets).

The dominant one-loop electroweak corrections to the $qq' \rightarrow AH^\pm$ process come from the loops of top ($t$) and bottom ($b$) quarks as well as their supersymmetric partners, i.e. stops ($\tilde{t}_{1,2}$) and sbottoms ($\tilde{b}_{1,2}$), in the MSSM. This is due to their potentially large couplings to Higgs bosons.

In the following, we discuss the quark- and squark-loop radiative corrections to the effective coupling of $W^\pm AH^\mp$ and to the mass relation (9). In our calculation, we adopt the on-shell renormalization scheme developed by Dabelstein in Ref. [1] (see Appendix A).

The part of one-loop effective coupling of $W^\pm AH^\mp$, that is relevant to the production process $qq' \rightarrow H^+A$, can be written as

$$M^{\mu}_{WHA}(q^2) = -\frac{g}{2}(p_A - p_H)^\mu \left[1 + F^{(1)}(q^2)\right],$$

(4)

where $q^\mu$, $p_A^\mu$ and $p_H^\mu$ are the incoming momenta of $W^\pm$, $A$ and $H^\mp$, respectively, and $g$ is the effective weak gauge coupling evaluated at $q^2$. Hence, the radiative correction to the cross section of the sub-process $qq' \rightarrow AH^\pm$ at the one-loop order is

$$K^{(1)}(q^2) = 2Re F^{(1)}(q^2).$$

(5)

The detailed calculation for $F^{(1)}(q^2)$ is summarized in Appendix B. As shown in Eqs. (21), (49) and (50), the quark-loop contribution is proportional to the squared Yukawa coupling constants $y_{c(t)}^2(= 2m_t^2\cot^2 \beta/\bar{v}^2)$ and $y_b^2(= 2m_b^2 \tan^2 \beta/\bar{v}^2)$. In the large $m_t$ or large $m_b/\tan \beta$ limit, it can be written as

$$F^{(1)}_{\text{quark}} \sim \frac{N_c}{16\pi^2} \left[-\frac{1}{4} y_t^2 + \frac{1}{2} \left(\frac{3}{2} - \ln\frac{m_t^2}{m_b^2}\right) y_b^2\right],$$

(6)

where $N_c(=3)$ is the number of colors. Since $y_t^2$ and $y_b^2$ are at most $\mathcal{O}(1)$ for $\tan \beta \sim 1$ and $m_t/m_b$, respectively, $F^{(1)}_{\text{quark}}$ is at most a few percent for $1 \lesssim \tan \beta \lesssim m_t/m_b$.

We also calculate the squark-loop contribution. As compared to the quark effects, the squark effects are rather complex due to the additional free (SUSY) parameters. The mass eigenstates $\tilde{f}_{1,2}$ ($\tilde{f} = \tilde{t}$ or $\tilde{b}$) of the squarks are obtained from the weak eigenstates $\tilde{f}_{L,R}$ by diagonalizing the mass matrices defined through (10)

$$\mathcal{L}_{\text{mass}} = -\left(\tilde{f}_{L}, \tilde{f}_{R}\right) \left(\begin{array}{cc} M^2_L & m_f X_f \\ m_f X_f & M^2_R \end{array}\right) \left(\begin{array}{c} \tilde{f}_L \\ \tilde{f}_R \end{array}\right),$$

(7)

where, $M^2_L = M^2_Q + m_f^2 + (m_Z^2 \cos 2\beta)(T_{fL} - Q_f s_W^2)$ and $M^2_R = M^2_{U,D} + m_f^2 + (m_Z^2 \cos 2\beta)Q_f s_W^2$. In this expression, $M^2_Q$, $M^2_{U,D}$ (for $\tilde{f} = \tilde{t}$) and $M^2_{U,D}$ (for $\tilde{f} = \tilde{b}$) are the soft-breaking masses for $\tilde{f}_L$, $\tilde{f}_R$ and $\tilde{b}_L$, respectively; $s_W = \sin \theta_W$ with $\theta_W$ being the weak mixing angle; $T_{fL}$ and $Q_f$ are the isospin and the electric charge of the quark $f_L$. Moreover, $X_f = A_f - \mu \cot \beta$ and $X_b = A_b - \mu \tan \beta$, where $A_f$ ($A_b$) is the trilinear $A$-term for $f$ ($b$), and $\mu$ is the SUSY invariant higgsino mass [10]. For completeness, we have listed all the relevant squark and Higgs bosons couplings in Appendix C, so that the

2 The other form factor, $(p_A + p_H)^\mu$, does not contribute to this process for massless quarks.
squark-loop contributions to $F^{(1)}(s)$, cf. [13], can be directly read out from the Eqs. (9), (10) and (11).

To examine the effect of one-loop electroweak corrections, we shall discuss two limiting cases below. Firstly, we consider the cases with $\mu = A_t = A_b = 0$, i.e., the cases without stop mixing ($|X_t| = 0$) and sbottom mixing ($|X_b| = 0$). Under this scenario, the masses of squarks are proportional to $M^2$, and all the relevant couplings between squarks and Higgs bosons are independent of the soft-breaking masses $M_Q$, $M_U$, and $M_D$ (see Appendix C). Thus, the squark-loop effect is decoupled and its contribution is very small for a large value of $M$, where $M \gg M_Q \simeq M_U \simeq M_D$. (Throughout this paper we denote $M$ as the typical scale of the soft-breaking masses.) For a smaller $M$, $F^{(1)}_{\text{squark}}$ becomes larger. However, $M$ cannot be too small because a small $M$ implies light squarks whose masses are already bounded from below by the direct search results [12]. Furthermore, as to be shown later, the case with a small $M$ is also strongly constrained by the $\rho$ parameter measurement.

Secondly, we consider the case with a large stop mixing, assuming $m_t |X_t| \sim M^2 \gg m_\tilde{t}_2^2$. Such a large stop mixing leads to a large mass splitting between $t_1$ and $t_2$ so that $m_{\tilde{t}_1} \simeq \mathcal{O}(m_Z)$ and $m_{\tilde{t}_2} \simeq \sqrt{2}M$, while $m_{\tilde{b}_{1,2}} \simeq M$. The leading squark contribution to $F^{(1)}(q^2)$ can be expressed as

$$F^{(1)}_{\text{squark}} \simeq -\frac{N_c}{16\pi^2} \left[ \left( \frac{3}{4} - \ln 2 \right) \left( \frac{Y_t}{M} \right)^2 + \left( \frac{13}{6} - 3\ln 2 \right) \left( \frac{Y_b}{M} \right)^2 \right].$$

(8)

with $Y_t = \frac{\mu}{m_t}(A_t \cot \beta + \mu)$ and $Y_b = \frac{\mu}{m_b}(A_b \tan \beta + \mu)$. Since in this case $|A_t| \simeq |M^2/m_t \pm |\mu|\cot \beta|$, we have $|Y_t| \sim \mathcal{O}(M^2/v)$ for $|\mu| \lesssim M$ and $1 \lesssim \tan \beta$. When $|A_b| \simeq |A_t|$ and $\tan \beta \sim m_t/m_b$, we find $|Y_b| \approx \mathcal{O}(M^2/v)$. Thus, with a large stop mixing ($m_t |X_t| \sim M^2$), $F^{(1)}_{\text{squark}}$ is proportional to the soft-breaking mass scale $M$, and does not decouple in the large $M$ limit. However, the $\rho$-parameter (or the $T$-parameter) can also strongly constrain such kind of model. With a large stop mixing ($M^2 \simeq m_t |X_t|$), the squark contribution (cf. Appendix D) to the $\rho$-parameter is

$$\Delta \rho_{\text{squark}} \simeq (2.2 \times 10^{-5}) \frac{M^2}{v^2}. \quad (9)$$

Since any new physics contribution to the $\rho$-parameter has to be bounded by data as [14]

$$-1.7 < \Delta \rho_{\text{new}} \times 10^3 < 2.7, \quad \text{at 2}\sigma \text{ level}, \quad (10)$$

the scale $M$ cannot be too large in this case. Consequently, the above $F^{(1)}_{\text{squark}}$ is constrained to be smaller than a few percent as long as $\mu^2$ is not much larger than $M^2$.

To examine the effect from the stop and sbottom loops to the production rate of $AH^\pm$, we consider 4 sets of SUSY parameters, as listed in Table 1, which give the largest allowed deviation in the $\rho$-parameter. Set 1 and Set 2 represent the cases without either a stop mixing ($|X_t| = 0$) or a sbottom mixing ($|X_b| = 0$), and Set 3 and Set 4 are the cases with a large stop mixing ($m_t |X_t| \sim M^2$) and $m_{\tilde{b}} \simeq 100$ GeV. The $K^{(1)}(s)$ factor, as defined in Eq. (6), is shown in Fig. 2 as a function of the invariant mass ($\sqrt{s}$) of the constituent process for $M_A = 90$ GeV. It is clear that the squark-loop contribution to $K^{(1)}(s)$ dominates the squark-loop contribution. For the above sets of SUSY parameters (Set 1-4), the squark-loop contribution is smaller than the quark-loop contribution by about a factor of 100. Generally, the squark contributions are at most a few percent, unless $|\mu|$ is taken to be very large as compared to the scale $M$.

We have checked that this conclusion does not change when our assumption of $M_Q \simeq M_U \simeq M_D$ is relaxed to some extent. Including both the quark- and squark-loop contributions to $K^{(1)}(s)$, we found that the correction to the hadronic cross section of $H^+A$ production in the invariant mass region just above the $H^+A$ threshold, where the constituent cross section is the largest, is at a percent level. In summary, we illustrated that to be consistent with the low-energy data and the direct search results for stops and sbottoms, the one-loop electroweak correction to the production rate of $pp$, $pp \rightarrow AH^\pm$ is small (at most a few percent).

Next, we discuss the one-loop corrections to the mass relation [1]. Let us parameterize the deviation from the tree-level relation by $\delta$, so that at the one-loop order

$$M_{H^\pm} = \sqrt{M_A^2 + m_W^2 (1 + \delta)}. \quad (11)$$

We note that in our renormalization scheme (see Appendix A), $M_A$ and $m_W$ are the input parameters, but $M_{H^\pm}$ is not. The one-loop corrected mass of the charged Higgs boson $M_{H^\pm}$ can be obtained by solving

$$0 = \text{Det} \begin{vmatrix} \Gamma_{ij}^{(2)}(p^2) & \Gamma_{ij}^{(2)}(H^-)(p^2) \\ \Gamma_{ij}^{(2)}(H^+)(p^2) & \Gamma_{ij}^{(2)}(H^+)(p^2) \end{vmatrix}, \quad (12)$$

where $\Gamma_{ij}^{(2)}(p^2)$ represent the renormalized two-point functions in the basis of the renormalized Goldstone boson ($G^\pm$) and charged Higgs boson ($H^\pm$) fields. Here,

| \text{Set} | \text{Set 1} | \text{Set 2} | \text{Set 3} | \text{Set 4} |
|----------|----------|----------|----------|----------|
| $M_Q = M_D = M_D$ (GeV) | 106 | 84 | 408 | 409 |
| $\tan \beta$ | 2 | 40 | 2 | 40 |
| $A_t = A_b$ (GeV) | 0 | 0 | +1261 | +1119 |
| $\mu$ | 0 | 0 | +300 | +300 |

$\Delta \rho_{\text{squark}} \times 10^3 = 2.72, 2.70, 2.71, 2.70$

TABLE I. The SUSY (input and output) parameters used in Fig. 2.
The notation “Det” denotes taking the determinant of the $2 \times 2$ matrix. One of the solution of the above equation is $p^2 = 0$, which corresponds to the charged Goldstone mode, and another is $M_{H^\pm}^2$. At the one-loop level, the pole mass of the charged-Higgs boson can be calculated from

$$M_{H^\pm}^2 = M_A^2 + m_W^2 + \Pi_{AA}(M_A^2) - \Pi_{H^+H^-}(M_A^2 + m_W^2) + \Pi_{WW}(m_W^2),$$

where $\Pi_{\phi\phi}(q^2) (\phi = A, H^\pm, \text{and } W)$ are the self-energies. For completeness, we list the quark and squark contributions to the self-energies of $A$ and $H^\pm$ in Appendix E.

When $A_{t,b}$ and $\mu$ are zero (i.e., no-mixing case), the leading contribution (which is proportional to the forth power of heavy quark mass) to $\delta$ is found to be

$$\delta \sim \frac{N_c}{8\pi^2v^2} \frac{m_t^2m_b^2}{M_A^2 + m_W^2} \frac{1}{\sin^2\beta \cos^2\beta} \left(1 + \ln \frac{M^2}{m_t^2}\right).$$

This correction is substantial for $\tan\beta \simeq m_t/m_b$ and $M^2 \gg m^2$. Applying Eq. (13) with the complete expression of $\Pi_{\phi\phi}(q^2)$, we found $\delta$ to be less than 4.9% for $2 < \tan\beta < 40, M < 2000 \text{ GeV}$ and $M_A > 90 \text{ GeV}$. Our result agrees well with Ref. [13], in which the approximate formula were presented for $M^2 \gg m_t^2$.

For the cases with a nonzero $A_{t,b}$ and $\mu$, $\delta$ receives extra contributions, which are proportional to $A_{t,b}^2/M^4, A_{t,b}\mu^2/M^4$ and $\mu^4/M^4$ [14], [15] originated from the squark couplings [cf. Eqs. (29), (30), (33) and (34)] and squark masses. For the Set 3 and Set 4 parameters listed in Table 1, $\delta$ is less than 5.3% and 3.6% for $M_A > 90 \text{ GeV}$, respectively. In summary, as long as $|A_{t,b}|$ and $|\mu|$ are not too large as compared to $M$, in a wide range of the parameter space that is allowed by the available experimental and theoretical constraints, $\delta$ does not exceed 7-10%.

Supported by the finding that the one-loop electroweak corrections to the $W^\pm A H^\mp$ coupling and to the mass relation $M_{H^\pm}^2 = M_A^2 + m_W^2$ are generally smaller than the other theoretical errors (such as the parton distribution function uncertainties) and the expected experimental errors (such as the mass resolution of Higgs boson decaying into jets), we anticipate that our conclusions based upon a Born level analysis should also hold well at the loop level. Namely, studying the process $p\bar{p}, pp \to W^{\pm*} \to AH^\pm$ allows us to distinguish the MSSM from its alternatives by verifying the mass relation (10) and checking its production rate. If a signal is not found, studying this process can provide an upper bound on the product of the decay branching ratios of $A$ and $H^\pm$ as a function of the only one SUSY parameter $- M_A$.

To detect the signal event, it is necessary to suppress its potentially large backgrounds. For example, for the $t\bar{t}b\bar{b}$ backgrounds can be largely reduced by having a good $b$-tagging and tau selection (by using the nature of $\tau$ polarization, which differs between a parent $H^\pm$ and $W^\pm$ (14)). We expected that its observability is relatively easy at the LHC and is a challenging task at the Tevatron for the signal event rate is small. Clearly, a detailed Monte Carlo analysis is needed to calculate the significance of the signal event at a collider. This will be deferred to a future study.

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APPENDICES

A. Renormalization

In this paper, we adopt the on-shell renormalization scheme developed by Dabelstein [19] to calculate the one-loop electroweak corrections. The standard model parameters are fixed by defining $\alpha_{em}, m_W$ and $m_Z$, and the additional SUSY parameters in the Higgs sector[20] are

$$\delta \sim \frac{N_c}{8\pi^2v^2} \frac{m_t^2m_b^2}{M_A^2 + m_W^2} \frac{1}{\sin^2\beta \cos^2\beta} \left(1 + \ln \frac{M^2}{m_t^2}\right).$$

There are 7 parameters in the Higgs sector of the MSSM. They are $g', g, v_1, v_2, m_1, m_2,$ and $m_3$. Beyond the Born level, the wavefunction renormalization factors $Z_{H_1}$ and $Z_{H_2}$ also need to be introduced to renormalize the theory, where $H_1$ and $H_2$ denote the two Higgs doublets in the model.
fixed by the following renormalization conditions: (1) the
tadpole contributions ($T_{H^+} = 0$, $T_{H^0} = 0$), (2) the
on-shell condition for the mass of $A$, (3) the on-shell con-
dition for the wavefunction of $A$, (4) a renormalization
condition on $\tan \beta$ (which requires $\delta v_1/v_1 = \delta v_2/(v_2)$), and
(5) a vanishing $A - Z$ mixing for an on-shell $A$.

**B. Calculation of $F^{(1)}(q^2)$**

The one-loop correction to the renormalized form fac-
tor of the $W^\pm H^\mp A$ vertex, apart from the effective weak
gauge coupling $g$, can be written as

$$F^{(1)}(q^2) = Z_A^{1/2} Z_{H^+}^{1/2} \{1 + \delta F_{WHA} + F^{1PI}_{WHA}(M_A^2, M_{H^+}^2, q^2)\},$$

(15)

where $\tilde{Z}_{AA}$ and $\tilde{Z}_{H^+ H^-}$ are the finite wavefunction fac-
tors for the renormalizations of the external Higgs bosons
$A$ and $H^\pm$. In our scheme,

$$\tilde{Z}_{AA} = 1,$$

$$\tilde{Z}_{H^+ H^-} = 1 - \Pi_{H^+ H^-}(M_A^2 + m_W^2) + \Pi'_{AA}(M_A^2),$$

(16)

where $\Pi'_{AA}(M_A^2)$ denotes taking the derivative of the two
point function $\Pi_{AA}(k^2)$ of the CP-odd scalar $A$ with respect
to $k^2$ at $k^2 = M_A^2$, etc. The terms inside the curly bracket of Eq. (15) arise from the renormalized
vertex function of $WHA$. $F^{1PI}_{WHA}(p_A^2, p_H^2, q^2)$ represents
the one-loop contribution of the one-particle-irreducible
(1PI) diagrams with $p_A^2, p_H^2, q^2$ as the four-momentum
squares of the incoming $A$, $H^\mp$ and $W^\pm$ particles, respec-
tively. $\delta F_{WHA}$ is the counterterm contribution resulting from
the field renormalization of $H^+$ and $A$:

$$H^+ A \to H^+ A \left(1 + \frac{1}{2} \delta Z_{H^+} + \frac{1}{2} \delta Z_A\right).$$

(18)

In terms of the independent counterterms fixed by the
renormalization scheme, the wavefunction counter-
terms $\delta Z_{H^+}$ and $\delta Z_A$ can be written as $(\sin^2 \beta) \delta Z_{H^+}$(cos $^2 \beta) \delta Z_A$, which is found to be equal to $-\frac{1}{2} \Pi'_{AA}(M_A^2)$.

We note that in $\delta F_{WHA}$ the contributions from the counter-
terms of the weak gauge coupling and the wavefunc-
tion renormalization of the $W$-boson are not included,
because they should be combined with the $W$-boson self
energy contribution to derive the running weak gauge
coupling $\tilde{g}(q^2)$. In our numerical calculation, we use

$$\tilde{g}^2 = 4\sqrt{2} m_W^2 G_F.$$

In summary, the one-loop electroweak correction to
$F^{(1)}(q^2)$ is found to be

$$F^{(1)}(q^2) = F_{WHA}^{1PI}(M_A^2, M_{H^\pm}^2, q^2)$$

$$-\frac{1}{2} \Pi'_{H^+ H^-}(M_A^2 + m_W^2) - \frac{1}{2} \Pi'_{AA}(M_A^2),$$

(19)

In the above equation, the top- and bottom-loop contribu-
tion to $F_{WHA}^{1PI}$ is given by

$$F_{WHA}^{1PI(\text{quark})}(q^2, p_A^2, p_H^2)$$

$$= \sum_{f f f'} F_{WHA}^{1PI}(q^2, p_A^2, p_H^2),$$

(20)

with

$$F_{WHA}^{1PI}(p_A^2, p_H^2, q^2) = \frac{N_c}{16\pi^2} g_f^2 \left\{ p_A^2 C_{31}^{ff'} - p_H^2 C_{32}^{ff'} \right\}$$

$$+ (2p_A \cdot p_H - p_A^2) C_{33}^{ff'} - (2p_A \cdot p_H - p_H^2) C_{34}^{ff'}$$

$$+ (D + 2)(C_{35}^{ff'} - C_{36}^{ff'}) + p_A^2 C_{21}^{ff'} - (2p_A \cdot p_H)$$

$$+ p_H^2 C_{22}^{ff'} - 2p_A^2 C_{23}^{ff'} - (D - 2) C_{24}^{ff'} - m_f^2 C_{11}^{ff'}$$

$$- (q^2 + m_f^2) C_{12}^{ff'} - (q^2 + m_{f'}^2) C_{13}^{ff'} \frac{1}{16\pi^2} g_{ff'}^2 m_f m_{f'} C_{01}^{ff'}.$$
The coupling constants among the weak-eigenstate squarks and the Higgs bosons are defined through the Lagrangian

$$\mathcal{L} = \cdots + \lambda[f_i^*, f_j, \phi, \ldots] f_i \tilde{f}_j \phi, \ldots + \cdots.$$  \hfill (26)

Hence, the coupling constants for the mass-eigenstate squarks are a linear combination of the couplings for the weak-eigenstate squarks, and

$$\lambda[f_i^*, f_j, \phi, \ldots] = \lambda[f_i^* O_i^T \tilde{f}_j O_j^L \phi, \ldots] = O_i^T \tilde{f}_j \phi, \ldots] (\lambda[f_i^*, \tilde{f}_j \phi, \ldots].$$  \hfill (27)

The relevant couplings $\lambda[f_i^*, f_j, \phi, \ldots]$, denoted as $\lambda[f_i^*, f_j, \phi, \ldots]$, are listed below.

$$\lambda_{i\tilde{i},LH} = -\frac{\sqrt{v}}{v} (m_{\tilde{t}} \sin 2\beta - m_{\tilde{b}} \tan \beta - m_{\tilde{q}} \cot \beta),$$  \hfill (28)

$$\lambda_{i\tilde{i},Rh} = -\frac{\sqrt{v}}{v} (A_t \cot \beta + \mu),$$  \hfill (29)

$$\lambda_{i\tilde{i},LH} = -\frac{2\sqrt{v}}{v} (A_b \tan \beta + \mu),$$  \hfill (30)

$$\lambda_{i\tilde{i},Rh} = \lambda_{i\tilde{i},LH} = \lambda_{i\tilde{i},LH} = \lambda_{i\tilde{i},Rh} = \frac{2\sqrt{v}}{v} (v \sin 2\beta),$$  \hfill (31)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = \lambda_{f_i^*, f_j, f_{\tilde{A}}} A = 0, (\tilde{f} = \tilde{t}, \tilde{b}),$$  \hfill (32)

$$\lambda_{f_i^*, f_j, f_{\tilde{B}}} A = \frac{m_{\tilde{t}}}{v} (A_b \tan \beta + \mu),$$  \hfill (33)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = \frac{m_{\tilde{b}}}{v} (A_t \cot \beta + \mu),$$  \hfill (34)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{t}}}{v} \tan 2\beta + \frac{g_2^2}{4} (T_b - Q_b s_W^2) \cos 2\beta,$$  \hfill (35)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{b}}}{v} \tan 2\beta + \frac{g_2^2}{4} Q_b s_W^2 \cos 2\beta,$$  \hfill (36)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{t}}}{v} \cos 2\beta + \frac{g_2^2}{4} (T_t - Q_t s_W^2) \cos 2\beta,$$  \hfill (37)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{b}}}{v} \cos 2\beta + \frac{g_2^2}{4} Q_t s_W^2 \cos 2\beta,$$  \hfill (38)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{t}}}{v} \tan 2\beta + \frac{g_2^2}{2} (T_b - Q_b s_W^2) \cos 2\beta,$$  \hfill (39)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{b}}}{v} \tan 2\beta + \frac{g_2^2}{2} Q_b s_W^2 \cos 2\beta,$$  \hfill (40)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{t}}}{v} \tan 2\beta + \frac{g_2^2}{2} (T_t - Q_t s_W^2) \cos 2\beta,$$  \hfill (41)

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = -\frac{m_{\tilde{b}}}{v} \tan 2\beta + \frac{g_2^2}{2} Q_t s_W^2 \cos 2\beta,$$  \hfill (42)

where $T_t$, $T_b$, $Q_t$ and $Q_b$ are $\frac{1}{2}$, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{2}{3}$, respectively, and

$$\lambda_{f_i^*, f_j, f_{\tilde{A}}} A = \lambda_{f_i^*, f_j, f_{\tilde{A}}} A = \lambda_{f_i^*, f_j, f_{\tilde{A}}} A = \lambda_{f_i^*, f_j, f_{\tilde{A}}} A.$$  \hfill (43)

for $I, J = L, R$ and $\tilde{f} = \tilde{t}, \tilde{b}$.

D. Squark contributions to the $\rho$ parameter

The squark one-loop contribution to the $\rho$ parameter is given by

$$\Delta \rho = \rho - 1 = -4\sqrt{2} G_F \Re[\Delta \Pi^{(1)}_T (0) - \Delta \Pi^{(3)}_T (0)],$$  \hfill (45)

with

$$\Delta \Pi^{(1)}_T (q^2) = \frac{N_c}{16\pi^2} \sum_{f = t, b, i, j = 1}^2 T_f \lambda_i^2 |O_i^{T L}|^2 |O_j^{L L}|^2 \times B(q^2; m_{f_i}^2, m_{f_j}^2),$$  \hfill (46)

$$\Delta \Pi^{(3)}_T (q^2) = \frac{N_c}{32\pi^2} \sum_{i, j = 1}^2 (U_{i i})^2 |D_f^{L L}|^2 B(q^2; m_{i_i}^2, m_{i_j}^2),$$  \hfill (47)

where $O_i^{T L} = U_{i i}$ and $O_i^{L L} = D_{f i}$; $B(q^2; m_{i_i}^2, m_{i_j}^2) = A(m_{i_i}^2) + A(m_{i_j}^2) - 4B_{22}(q^2; m_{i_i}^2, m_{i_j}^2)$. By using the expression

$$B(0; m_{t_1}^2, m_{T_2}^2) = -\frac{1}{2} (m_{t_1}^2 + m_{T_2}^2) + \frac{m_{t_1}^2 m_{T_2}^2}{m_{t_1}^2 + m_{T_2}^2} \ln \frac{m_{t_1}^2}{m_{T_2}^2},$$  \hfill (48)

Eq. (46) is deduced under the assumption that $M^2 = M_Q^2 \simeq M_U^2 \simeq M_D^2 \gg m_{i_i}^2$ and the stop mixing is large $(m_{i_1} X_{i_1} \simeq M^2$ and $m_b X_{b_1} \simeq 0)$, so that $m_{i_1} \sim O(m_Z)$, which yields $m_{i_1} \sim \sqrt{2} M$, and $m_{i_2} \sim m_{i_2} \sim M$.

E. Self energies

The (top and bottom) quark-loop contributions to the self-energies $\Pi_{AA}(q^2)$ and $\Pi_{HiH'} (q^2)$ are expressed in terms of the Passarino-Veltman functions \cite{5} by

$$\Pi_{iA}^{(iA)} (q^2) = -\frac{N_c}{16\pi^2} \sum_{f = t, b} 2y_f^2 \{ B_{12}(q^2, m_f, m_f) + B_{22}(q^2, m_{f_1}, m_{f_2}) + m_{f_1}^2 B_0(q^2, m_{f_1}, m_{f_2}) \},$$  \hfill (49)

$$\Pi_{iA}^{(iA)} (q^2) = -\frac{N_c}{16\pi^2} 2 (y_t^2 + y_b^2) \{ B_{12}(q^2, m_t, m_t) + B_{22}(q^2, m_{t_1}, m_{t_2}) + m_{t_1}^2 B_0(q^2, m_{t_1}, m_{t_2}) \},$$  \hfill (50)

The stop- and sbottom-loop contributions are given by

$$\Pi_{iA}^{(iA)} (q^2) = -\frac{N_c}{16\pi^2} \sum_{f = t, b} \sum_{i, j = 1}^2 \lambda[f_i^*, f_j, A] \lambda[f_j^*, f_i, A] B_0(q^2, m_{f_i}^2, m_{f_j}^2),$$  \hfill (51)

$$\Pi_{iA}^{(iA)} (q^2) = -\frac{N_c}{16\pi^2} \sum_{f = t, b} \sum_{i, j = 1}^2 \lambda[f_i, f_j^*, A] A(m_{f_i}),$$  \hfill (52)

$$\Pi_{iA}^{(iA)} (q^2) = -\frac{N_c}{16\pi^2} \sum_{i, j = 1}^2 \lambda[f_i, f_j, A] A(m_{f_i}),$$  \hfill (53)
\begin{align}
\times \lambda[\tilde{t}^*_i, \tilde{b}_j, H^+] \lambda[\tilde{b}^*_j, \tilde{t}_i, H^-] B_0(q^2, m_{\tilde{t}_i}, m_{\tilde{b}_j}) \quad (53)
\end{align}

\begin{align}
- \frac{N_c}{16\pi^2} \sum_{\tilde{f} = \tilde{t}, \tilde{b}} \sum_{i=1}^2 \lambda[\tilde{f}^*_i, \tilde{f}_i, H^+, H^-] A(m_{\tilde{f}_i}) \quad (54)
\end{align}

The self-energy \(\Pi_{WW}(q^2)\) of the \(W\) boson was already presented in the literature. For example, the quark-loop contribution can be found in Ref. [16], and the squark-loop contribution in Ref. [15].

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