QUANTUM INTERFERENCE AND MONTE-CARLO SIMULATIONS OF MULTIPARTICLE PRODUCTION

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Abstract: We show that the effects of quantum interference can be implemented in Monte-Carlo generators by modelling the generalized Wigner functions. A specific prescription for an appropriate modification of the weights of events produced by standard generators is proposed.

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1. A quantitative analysis of multiparticle data requires using generators of computer "events". The trouble is that the models employed in this context are essentially classical and that their predictions are inadequate and sometimes turn out to be misleading in the study of those aspects of multiparticle production where quantum interference is important [1]. This is a serious problem because, as evidenced by the studies of the so-called intermittency phenomenon [2], the momentum-space short-range correlations are to a large extent dominated by the Hanbury-Brown-Twiss (HBT) effect [3], i.e. by quantum interference.

We would like to emphasize that this does not mean at all that these correlations are "trivial" as is sometimes claimed. On the contrary, since quantum interference is sensitive to the space-time development of the collision process it yields precious information thereon and its effects should be examined with most attention.

The aim of this communication is to outline a way of implementing the effects of quantum statistics in Monte-Carlo simulations of multiparticle phenomena. One could wonder whether this is possible at all, since the Monte-Carlo method deals with probabilities while quantum interference is a consequence of adding amplitudes whose phases are essential. We will show, however, that one can devise a systematic and practical approach to the problem if Wigner functions and not scattering amplitudes are used to describe the multiparticle system.

2. We shall now write the n-particle spectrum in terms of the Wigner function, first for distinguishable and then for identical secondaries.

Let \( \psi(q; \alpha) \) denote a wave-function describing a stationary state produced at high-energy. Here \( q \) refers to the momenta \( q_1, q_2, ..., q_n \) of the \( n \) produced spinless particles and \( \alpha \) denotes all other parameters, assumed irrelevant for our problem. For the moment, the secondaries are supposed to be distinguishable.

The n-particle spectrum is, of course

\[
\Omega_0(q) = \sum_\alpha |\psi(q; \alpha)|^2, \quad q = (q_1, ..., q_n)
\]  

The weights generated by a Monte-Carlo algorithm are directly proportional to \( \Omega_0(q) \).

Going over to the coordinate representation we write
\[ \Omega_0(q) = \int dx dx' e^{i q \cdot (x - x')} \rho(x; x'), \quad x = (x_1, \ldots, x_n) \] (2)

where

\[ \rho(x; x') = \sum_{\alpha} \hat{\psi}(x; \alpha) \hat{\psi}^*(x'; \alpha) \] (3)

is a density matrix, \( \hat{\psi} \) being the Fourier transform of \( \psi \). Eq. (2) can further be rewritten as

\[ \Omega_0(q) = \int dx^+ W(q; x^+) \] (4)

where [notation: \( x^+ = \frac{1}{2}(x + x') \), \( x^- = x - x' \)]

\[ W(q; x^+) = \int dx^- e^{i q \cdot x^-} \rho(x; x') \] (5)

is the generalized Wigner function [4], the quantum analog of the classical Boltzmann phase-space density. It is real and it gives the observable spectrum when integrated, as seen in (4).

Let us now assume that the secondaries are all identical (the generalisation of the discussion to the case where there are several species of identical secondaries is straightforward). Let \( P \) denote some arbitrary permutations of the integers 1, ..., \( n \) and \( q_P \) the corresponding permutation of the momenta \( q_1, \ldots, q_n \). Once the wave function \( \psi(q; \alpha) \) is symmetrized with respect to the momenta of produced bosons, eq. (2) becomes

\[ \Omega(q) = \frac{1}{n!} \sum_{P, P'} \int dx dx' e^{i (x \cdot q_P - x' \cdot q_{P'})} \rho(x; x') \] (6)

which is rewritten as

\[ \Omega(q) = \frac{1}{n!} \sum_{P, P'} \int dx^+ e^{i x^+ \cdot (q_P - q_{P'})} W \left( \frac{q_P + q_{P'}}{2}; x^+ \right) \] (7)

with \( W(q; x^+) \) defined in (4). Thus we observe that the same Wigner function determines the spectrum before and after the symmetrization. This has a simple physical reason: All information, compatible with the rules of quantum mechanics, about what happens in the full phase-space of coordinates and momenta (in contrast to the momentum-space alone) is encoded in the
Wigner function. This information includes the phases of different waves and therefore all that is needed to predict the interference patterns. A further advantage of the Wigner function is that it appeals directly to one’s intuition. A few words of caution are necessary at this point, however.

Contrary to the Boltzmann phase-space density, the Wigner function is locally not positive definite. It can actually oscillate quite violently. The oscillations integrate to zero in (5) but can conspire with oscillating terms in the integrand of (7) to contribute significantly to the result. This is how quantum mechanics shows up in the problem. Thus, it is clear that regarding the Wigner function as a phase-space density is possible only when the function is smoothed by averaging the oscillations out. Physically it means appropriate smearing of coordinates and momenta. The price to pay is that, in general, the resulting probabilistic model can only be trusted when the momentum differences appearing in (7) are not too large.

3. The standard Monte-Carlo algorithms rest on models of momentum-space densities. The goal to achieve is to correct the weights of Monte-Carlo events, once they have been generated according to the distribution $\Omega_0(q)$. Our proposal consists in going from $\Omega_0(q)$ to $\Omega(q)$ by modelling the Wigner function.

Writing

$$W(q; x^+) = \Omega_0(q) w(q; x^+),$$

we see that, if the Wigner function is regarded as a phase-space density, $w(q; x^+)$ has the meaning of a conditional probability: given that the particles with momenta $q_1, \ldots q_n$ are present in the final state, $w$ is the probability that they are produced at the points $x_1^+, \ldots, x_n^+$. The problem is to construct a viable model for $w(q; x^+)$. In the absence of additional information it seems reasonable to start with the working assumption that the likelihood to radiate a particle from a given space point is statistically independent of what happens to other particles. This means that $w$ factorizes:

$$w(q; x^+) = \prod_j w(q_j, x_j^+)$$

where

$$\int d^3 x \ w(q, x) = 1$$
Substituting (8)-(9) into (7) one finds

$$
\Omega(q) = \frac{1}{n!} \sum_{p, p'} \Omega_0\left(\frac{q_p + q_{p'}}{2}\right) \prod_j \hat{w}[q_j, (q_p - q_{p'})_j]
$$

(11)

where

$$
\hat{w}(q, \Delta) = \int d^3 x \, e^{ix \cdot \Delta} w(q, x)
$$

(12)

Eq. (11) and the reality of $w(q, x)$ imply $\hat{w}(q, 0) = 1$ and $\hat{w}(q, \Delta) = \hat{w}^*(q, -\Delta)$, respectively. This guarantees that $\Omega(q)$ calculated from (11) is real.

Eq. (11) can be used as it stands when one has an explicit formula for $\Omega_0(q)$. In practice, however, $\Omega_0(q)$ is constructed iteratively by a Monte-Carlo algorithm and at a given stage of the simulation it is computed for one configuration of momenta. To deal with this complication we observe that one does not make a big error by replacing in (11) $\Omega_0(q_p + q_{p'})$ by $\Omega_0(q_p)$. Indeed, those terms in eq. (11) where this approximation is poor are suppressed by the rapidly decreasing factors $\hat{w}$ and thus need not be calculated with a great precision. The equation (11) now becomes

$$
\Omega(q) = \frac{1}{n!} \sum_{\mathcal{P}} \{\Omega_0(q_{\mathcal{P}}) \sum_{\mathcal{P}'} \prod_j \hat{w}[q_j, (q_p - q_{p'})_j]\}
$$

(13)

Thus, once a configuration $q_{\mathcal{P}}$ of momenta has been generated by the original algorithm, the weight of the event in question has to be multiplied by the correction factor $\text{Re}\{\sum_{\mathcal{P}'} \prod_j \hat{w}[q_j, (q_p - q_{p'})_j]\}$ in order to take care of the HBT interference. This is the result sought. In (13) the sum over $\mathcal{P}$ just expresses formally the fact that even in a classical model the labelling of identical particles is merely a matter of convention.

The function $\hat{w}(q, \Delta)$ is unknown. It can either be taken from a model [5] or, perhaps more reliably, be determined by fitting 2-body HBT correlations. The weight of an event is then completely determined and the procedure can be used to study the implications of quantum statistics for other aspects of the production process.

4. Adopting the probabilistic interpretation of the Wigner function, the proposed approach allows an intuitive interpretation of the results in terms of the space-time structure of the region of particle emission. As already mentioned this is meaningful when momentum differences are not too large.
It should be clear that a simple physical meaning can be ascribed to \( w(q, x) \) rather than to \( \hat{w}(q, \Delta) \). Let us briefly outline how one might proceed in modelling \( w(q, x) \). We imagine that a particle with momentum \( q \) is emitted from a diffuse source centered at \( x = x_0(q) \). We are thus led to write

\[
w(q, x) = P[q, x - x_0(q)]
\]  

(14)

The simplest choice for \( P(q, x) \) would be to take a Gaussian

\[
P_G(x) = \frac{1}{\pi^{\frac{3}{2}} \sigma_x \sigma_y \sigma_z} e^{-\left(\frac{x_x^2}{\sigma_x^2} + \frac{x_y^2}{\sigma_y^2} + \frac{x_z^2}{\sigma_z^2}\right)}
\]

(15)

with \( \sigma = \sigma(q) \), to take into account the possible dependence of the shape of the source on \( q \). More generally one can set

\[
P(q, x) = \int d^3 \sigma H(q, \sigma) P_G(x)
\]

(16)

This gives

\[
\hat{w}(q, \Delta) = e^{i \Delta \cdot x_0(q)} \int d^3 \sigma H(q, \sigma) e^{-\frac{1}{4} \left(\sigma_x^2 \Delta_x^2 + \sigma_y^2 \Delta_y^2 + \sigma_z^2 \Delta_z^2\right)},
\]

(17)

a formula that seems general enough to accommodate all physically reasonable choices for \( \hat{w} \).

Notice that, for obvious physical reasons, the distribution (14) can only depend on differences \( x_0(q) - x_0(q') \). This is indeed the case as can be seen by observing that \( \sum_j (q_P - q_{P'})_j = 0 \) for any two permutations \( P \) and \( P' \).

A further simplification is obtained by assuming that the place where a particle is produced depends at most on some global characteristic of the collision, like the total energy, to give an example. This is presumably a good assumption as long as the source can be regarded as static. Technically, it amounts to neglect the first argument of \( \hat{w}(q, \Delta) \).

5. A few comments are in order:

(i) The proposed approach, even used in its simplest version, enables one to incorporate into a Monte-Carlo simulation the collective nature of the HBT effect.

(ii) Our ansatz (9) is to be considered as a working assumption to be upgraded when more information is available (e.g. if two identical particles result from the decay of the same resonance).
(iii) The ansatz can be checked against the data on higher order correlations. This may lead to a discovery of hitherto entirely unknown correlation structures in the space-time development of the collision and is, therefore, potentially of great interest.

(iv) The motivation of our probabilistic approach to the Wigner function is mostly phenomenological. It would be, of course, highly desirable to invest more effort in developing theoretically based models of the Wigner function.

To summarize, we have shown that the effects of quantum interference in multiparticle production can be naturally expressed in terms of generalized Wigner functions. Such formulation allows one to incorporate these effects into Monte-Carlo generators and to give them an intuitive interpretation in terms of the space-time development of the interaction. It also explicitly demonstrates that in order to obtain theoretically founded predictions for the HBT correlations one should compute the Wigner functions from the underlying theory.

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