1. Introduction

Weyl semimetals (WSMs) have recently attracted intensive attention due to their exotic band structure [1]. The anomalous band structure of WSMs manifests itself in their topological properties such as protected Weyl nodes and Fermi arc surface states [2, 3]. The conduction and valance bands in WSMs touch each other at Weyl nodes characterized by linear dispersion around the Fermi level [4]. The Weyl nodes appear in pairs with different chiralities separated in momentum or energy space in WSMs with broken time reversal symmetry (TRS) [5] or spatial inversion symmetry (SIS) [6], respectively. The WSM phase has been observed in topologically non-trivial materials including non-magnetic samples such as TaAs [7], NbAs [8], NbP [9] and magnetic compounds Y$_3$Ir$_2$O$_7$ [3] and Eu$_2$Ir$_2$O$_7$ [10]. Moreover, the so-called type-II WSM phase has been observed recently in materials such as WTe$_2$ [11] and MoTe$_2$ [12] having open Fermi surfaces. Some exotic effects such as chiral anomaly [13], the anomalous Hall effect [14, 15] and negative magnetoresistance [16] arise from the non-trivial topological band structure of WSMs. In particular, an unusual optical response emerges in WSMs due to the coupling of the electrical and magnetical properties originating from the chiral anomaly [17–22].

WSMs are promising materials for photonics and plasmonics applications due to the broad tunability of their chemical potential. Surface plasmon polaritons (SPPs) at the surface of WSMs have been studied theoretically [23–28] and have been observed at visible wavelengths in WTe$_2$ [29]. SPPs are collective electromagnetic and electronic charge excitations confined to the interface of a conductor with a dielectric [30]. SPPs are employed in optoelectronic devices.
such as surface plasmon resonance sensors [31] and scanning near-field optical microscopy [32]. It has been shown that an unconventional plasmon mode exists in WSMs due to the chiral anomaly which can be used as a signature of the WSM phase [23]. The interface of two adjacent WSMs with different magnetization orientations hosts low-loss localized guided SPPs [24]. It has been demonstrated that the SPP dispersion depends on the Weyl node separation in energy or momentum space [25]. In a WSM with broken TRS the Weyl node separation vector acts as an effective external magnetic field. Furthermore, it has been predicted that giant non-reciprocal waveguide electromagnetic modes exist in WSM thin films in the Voigt configuration [26]. The thickness of the WSM thin film and the dielectric contrast of the outer insulators can be used to fine-tune the SPP dispersion and its non-reciprocal property [27]. Recently, we have studied the SPP dispersion and its properties in a WSM waveguide comprised of two TRS WSMs in different configurations [28]. In particular, it has been shown that tremendous unidirectional SPP modes are hosted by this structure and can be tuned by the chemical potential and topological parameters of the WSMs. It is worth noting that these intriguing features of SPP modes in WSMs emerge without needing the application of high external magnetic fields. On the other hand, SPP modes on the surface of topological insulators (TIs) [33, 34], exhibiting a bulk gap and gapless surface states protected by TRS, as the first member of the family of topological materials have been investigated theoretically [35–39] and experimentally [40, 41]. Due to identical linear Dirac electronic dispersion of the surface states in TI and graphene, these materials exhibit very similar dispersion for SPP modes. But, the spin-momentum locking in the TIs gives rise to the charge and spin density waves leading to spin-coupled surface plasmons or spin plasmons [35, 36]. Moreover, the exciting magneto-optical Kerr effect in TIs observed by employing a ferromagnetic coupling or an external magnetic field results in the generation of a novel transverse SPP mode in addition to the usual longitudinal one [37–39].

In this paper we study the interplay of the SPP modes existing at the interfaces of two WSMs with broken TRS and SIS connected via a dielectric layer in the slot waveguide geometry. We consider both Voigt and Faraday configurations for SPP propagation at the interface of the WSM with broken TRS. We demonstrate that the interplay of the SPP modes localized at independent interfaces of two WSMs with different symmetries leads to unidirectional SPP modes in the waveguide geometry for frequencies above the bulk plasmon frequency. However, we find a non-reciprocal bidirectional dispersion for SPP modes with frequencies below the bulk plasmon frequency. In particular, our analysis reveals that these chiral SPP modes can be tuned by the topological parameters and chemical potentials of the two WSMs. In addition, we observe a gapfull SPP dispersion in the Faraday configuration. We show that the gap of the dispersion can be controlled by the physical parameters of the WSMs. These exotic features originating from intrinsic topological properties of WSMs may be employed practically in optical devices.

The remainder of the paper is organized as follows. In section 2 we introduce our theoretical model and give the necessary equations for deriving SPP dispersion. Section 3 is devoted to presenting the results and related discussions. Finally, we end with a conclusion in section 4.

2. Theoretical model and equations

The unique optical response of a WSM originating from its intrinsic topological nature is characterized by the term $\theta(r, t) = 2(b, r - bt)$ which is called the axion angle [19]. Here, $b$ expresses the vector connecting two Weyl points in the momentum space of a WSM with broken TRS, whereas $b_0$ is the separation of them in energy for a WSM with broken SIS. The topological properties of WSMs result in a modified displacement electric field given by [25]

$$D = \left( \varepsilon_\infty + \frac{4\pi \sigma}{\omega} \right) E + \frac{i\varepsilon^2}{\pi \hbar \omega} (\nabla \theta) \times E + \frac{i\varepsilon^2}{\pi \hbar \omega} \phi B, \quad (1)$$

where $E$, $B$, $\varepsilon_\infty$ and $\sigma$ are the electric field, the magnetic field, the static dielectric constant and the conductivity of the WSM, respectively. The second and third terms of equation (1) originate from the topological properties of the WSM corresponding to the anomalous Hall effect and the chiral magnetic effect, respectively [1], whereas the first term of this equation is the usual displacement field for a normal metal.

For a WSM with broken TRS ($b \neq 0$), the chiral anomaly causes an anisotropic optical response characterized by a dielectric tensor $\varepsilon(\omega)$, with diagonal and off-diagonal terms which we will denote by $\varepsilon(\omega)$ and $\varepsilon_b(\omega)$, respectively. Also, a WSM with broken SIS ($b_0 \neq 0$) represents an anisotropic optical response with the same diagonal terms but having different off-diagonal terms represented by $\varepsilon_{b_0}(\omega)$.

The SPP localized at the interface of a WSM with a dielectric is described by an electric field of the form

$$E = (E_x, E_y, E_z) e^{i\mathbf{r} \cdot \mathbf{q} - \kappa z} e^{-i\omega t}, \quad (2)$$

which decays exponentially away from the interface in the $z$ direction and propagates in the interface along the direction of $\mathbf{q} = (q_x, q_y, 0)$. This electric field should be satisfied by the wave equation

$$\nabla \times (\nabla \times E) = -\frac{1}{c^2} \frac{\partial^2 D}{\partial t^2}. \quad (3)$$

This equation can be expressed as a matrix equation $\mathbf{ME} = 0$, with $\mathbf{M}$ given by

$$\mathbf{M} = \begin{pmatrix}
q_x^2 - \kappa^2 & -q_x q_y & \mp i q_x \kappa \\
-q_x q_y & q_y^2 - \kappa^2 & \mp i q_y \kappa \\
\mp i q_x \kappa & \mp i q_y \kappa & q_x^2 + q_y^2
\end{pmatrix} - \frac{\omega^2}{c^2} \varepsilon(\omega), \quad (4)$$

with a positive sign for $\varepsilon < 0$ and a negative one for $\varepsilon > 0$. The decay constant ($\kappa$) is determined by the condition $\text{det}(\mathbf{M}) = 0$. The decay constant on the WSM side depends on the relative direction of the vectors $\mathbf{q}$ and $\mathbf{b}$.

The system under consideration is a slot waveguide depicted in figure 1. The slot waveguide has been constructed...
of two semi-infinite WSMs, media I and III in the figure, connected by a dielectric layer (medium II) with thickness $a$ and a dielectric constant $\varepsilon_d$.

We consider that TRS is broken in the upper WSM (medium I) while in the lower one (medium III) SIS is broken. The relative direction of the vectors $\mathbf{b}$ and $\mathbf{q}$ results in different configurations for SPP propagation. We study two different Voigt and Faraday configurations at the upper interface. In the Voigt configuration $\mathbf{b}$ is parallel to the surface but perpendicular to $\mathbf{q}$. By considering $\mathbf{q} = (0, q, 0)$ and $\mathbf{b} = (b, 0, 0)$, the matrix $\mathbf{M}$ is expressed as

\[
\hat{\mathbf{M}}_V = \begin{pmatrix}
    q^2 - \kappa_1^2 & 0 & 0 \\
    0 & -\kappa_1^2 & -iq\varepsilon_1 \\
    0 & -iq\varepsilon_1 & q^2
\end{pmatrix} - k_0^2\varepsilon_V(\omega),
\]

with

\[
\varepsilon_V(\omega) = \begin{pmatrix}
    \varepsilon & 0 & 0 \\
    0 & \varepsilon & i\varepsilon_b \\
    0 & -i\varepsilon_b & \varepsilon
\end{pmatrix}.
\]

In the Faraday configuration $\mathbf{b} = (b, 0, 0)$ is parallel to the interface and $\mathbf{q} = (0, q, 0)$. In this case the matrix $\mathbf{M}$ is given by

\[
\hat{\mathbf{M}}_F = \begin{pmatrix}
    q^2 - \kappa_1^2 & 0 & 0 \\
    0 & -\kappa_1^2 & -iq\kappa_1 \\
    0 & -iq\kappa_1 & q^2
\end{pmatrix} - k_0^2\varepsilon_F(\omega),
\]

with

\[
\varepsilon_F(\omega) = \begin{pmatrix}
    \varepsilon & 0 & i\varepsilon_b \\
    0 & \varepsilon & 0 \\
    -i\varepsilon_b & 0 & \varepsilon
\end{pmatrix}.
\]

Since the lower WSM (medium III) is assumed to have broken SIS ($\mathbf{b} = 0, b_0 \neq 0$), the off-diagonal terms of the dielectric tensor result from the third term of equation (1). In this case the matrix $\mathbf{M}$ can be written in the following form:

\[
\hat{\mathbf{M}}_{bo} = \begin{pmatrix}
    q^2 - \kappa_1^2 & 0 & 0 \\
    0 & -\kappa_1^2 & +iq\kappa_1 \\
    0 & +iq\kappa_1 & q^2
\end{pmatrix} - k_0^2\varepsilon_{bo}(\omega, q),
\]

with

\[
\varepsilon_{bo}(\omega, q) = \begin{pmatrix}
    \varepsilon & -\frac{\kappa_1}{\varepsilon} \varepsilon_{bo} & +\frac{iq}{\varepsilon} \varepsilon_{bo} \\
    +\frac{\kappa_1}{\varepsilon} \varepsilon_{bo} & \varepsilon & 0 \\
    -\frac{iq}{\varepsilon} \varepsilon_{bo} & 0 & \varepsilon
\end{pmatrix}.
\]

In spite of the two former cases, in this case the dielectric tensor has a direct dependence on the propagation wave vector $\mathbf{q}$. The diagonal terms of the dielectric tensors in equations (6), (8) and (10) are equal to $\varepsilon(\omega) = \varepsilon_{\infty}(1 - \frac{\Omega_\mathbf{p}^2}{\omega^2})$ and the off-diagonal terms are determined by $\varepsilon_{bo}(\omega) = \varepsilon_{\infty} \frac{\Omega_\mathbf{b} \Omega_\mathbf{p}}{\omega^2}$ and $\varepsilon_{bo}(\omega) = \varepsilon_{\infty} \frac{\Omega_\mathbf{b} \Omega_{\mathbf{p}}}{\omega^2}$. Here $\Omega_\mathbf{p}^2 = \frac{4\alpha}{\varepsilon_{\infty}} (\frac{\omega_q^2}{\varepsilon})^2$ refers to the bulk plasmon frequency with $\alpha = \varepsilon_{\infty} \frac{\varepsilon_{bo}}{\varepsilon_{\infty}}$, $\Omega_{\mathbf{b}} = \frac{2\varepsilon_{bo} \Omega_\mathbf{p}}{\varepsilon_{\infty}}$, and $k_0 = \frac{\omega}{c}$ is the wave vector in the vacuum. It should be noted that we have ignored the effect of the carrier scattering in the dielectric tensor of the WSMs due to the very low carrier scattering rates measured in this class of materials [9, 10].

Setting the determinant of $\mathbf{M}$ matrices in equations (5), (7) and (9) to zero, the decay constant $\kappa$ for the three aforementioned different configurations is obtained. For the Voigt configuration the decay constant should be a positive and real number and is obtained as

\[
\kappa_{V+}^2 = q^2 - k_0^2 \varepsilon, \quad \kappa_{V-}^2 = q^2 - k_0^2 \varepsilon, \quad (11)
\]

where $\varepsilon_v = (\varepsilon^2 - \varepsilon_0^2)/\varepsilon$ is the Voigt dielectric function. For the Faraday configuration it reads

\[
\kappa_{F+}^2 = q^2 - k_0^2 \varepsilon + \frac{q^2}{\varepsilon} \left(k_0^4 \varepsilon^4 + \frac{q^2}{\varepsilon} \varepsilon_0^2 \varepsilon^3 \right)^{1/2},
\]

and for the case of $b_0 \neq 0$ is given by

\[
\kappa_{bo}^2 = q^2 - k_0^2 \varepsilon - \frac{1}{4} k_0^2 b^2 \varepsilon_0^2 + \frac{1}{2} k_0^2 b^2 \varepsilon_0^2 \varepsilon^4 + 4 \kappa_0^2 b^2 \varepsilon_0^2 \varepsilon^2 \right)^{1/2}.
\]

In the dielectric layer, which is an isotropic medium, the dielectric tensor is diagonal with elements equal to constant quantity $\varepsilon_d$. In this medium the decay constant is specified by $\kappa_2 = \sqrt{q^2 - k_0^2 \varepsilon_d}$.

### 2.1 SPP modes in the Voigt configuration

First, we consider the slot waveguide with Voigt configuration in the upper interface. The electric field amplitude in the three media can be written as a combination of the terms in the form of equation (2) with the appropriate decay constants in the related media [28]. Applying the continuity condition for
tangential components of the electric field, denoted by vector $\hat{E}_t$, we arrive at the following set of linear equations at the upper interface ($z = a/2$)

$$
\begin{pmatrix}
1 & 0 & -e^{\kappa_1z/2} & 0 & -e^{\kappa_2z/2} & 0 & 0 & 0 \\
0 & 1 & 0 & -e^{\kappa_1z/2} & 0 & -e^{\kappa_2z/2} & 0 & 0 \\
0 & -\frac{\eta_A}{\kappa_2} & 0 & -e^{\kappa_1z/2} & 0 & -e^{\kappa_2z/2} & 0 & 0 \\
-\kappa_1 & 0 & -\kappa_2 e^{\kappa_1z/2} & 0 & \kappa_2 e^{-\kappa_1z/2} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\hat{E}_t
\end{pmatrix} = 0
$$

(14)

and for the lower interface ($z = -a/2$) it reads

$$
\begin{pmatrix}
0 & 0 & -e^{-\kappa_1z/2} & 0 & -e^{-\kappa_2z/2} & 0 & 1 & 1 \\
0 & 0 & 0 & -e^{-\kappa_1z/2} & 0 & -e^{-\kappa_2z/2} & 0 & 0 \\
0 & 0 & -\kappa_2 e^{-\kappa_1z/2} & 0 & \kappa_2 e^{-\kappa_1z/2} & 0 & \kappa_{b_1} & \kappa_{b_2}
\end{pmatrix}
\begin{pmatrix}
\hat{E}_t
\end{pmatrix} = 0
$$

(15)

where $\varepsilon_{11} = -\kappa_{-} - (iq\beta_1 + \kappa_{-})$, $A_{12} = (iq\beta_{12} - \kappa_{b_2} \pm \chi_{1,2})$ with $\beta_1 = [(q^2 - k_0^2)/(q^2 - k_0^2)]$, $\beta_{12} = [(-(iq\kappa_{b_2})/(q^2 - k_0^2)]$, $\chi_{1,2} = [((iq\kappa_{b_2})/(q^2 - k_0^2)]$. Applying these simplifications in equations (14) and (15) we have

$$
\begin{align*}
(16)
\end{align*}
$$

This relation reveals the chiral nature of the SPP propagation in the considered structure. Dependence of the dispersion relation to the sign of propagation wave vector ($q$) results in different SPP dispersion for the forward and backward directions. The asymptotic frequencies for SPP dispersion, frequencies of SPP modes at the long wavelength limit $|q| \to \infty$, are obtained by taking the limit of $a/|q| \to 1$ in equation (16). This leads to equations $\varepsilon - \varepsilon_b + \varepsilon_d = 0$ and $\varepsilon = 0$, giving the asymptotic frequencies as

$$
\omega_{\text{ad}}^2 = \frac{\sqrt{\omega_a^2 - \varepsilon_b^2}}{\varepsilon_d + \varepsilon_c},
$$

$$
\omega_{\text{ad}}^2 = \frac{\sqrt{\omega_a^2 - \varepsilon_b^2}}{\varepsilon_d + \varepsilon_c},
$$

(17)

2.2. SPP modes in the Faraday configuration

Employing the same procedure explained for the Voigt configuration, we can obtain the following set of linear equations for tangential electric field amplitudes at the upper interface ($z = +a/2$),

$$
\begin{pmatrix}
-\alpha \eta_A & -\alpha \eta_A & -e^{-\kappa_2z/2} & 0 & -e^{-\kappa_1z/2} & 0 & 0 & 0 \\
-\kappa_{f+} & -\kappa_{f-} & -e^{\kappa_2z/2} & 0 & -e^{\kappa_1z/2} & 0 & 0 & 0 \\
0 & 0 & -e^{\kappa_2z/2} & 0 & -e^{\kappa_1z/2} & 0 & 0 & 0 \\
0 & 0 & -e^{\kappa_1z/2} & 0 & -e^{\kappa_2z/2} & 0 & \kappa_{b_1} & \kappa_{b_2}
\end{pmatrix}
\begin{pmatrix}
\hat{E}_t
\end{pmatrix} = 0
$$

(18)

and at the lower interface ($z = -a/2$) we have

$$
\begin{pmatrix}
0 & 0 & -e^{-\kappa_1z/2} & 0 & -e^{-\kappa_2z/2} & 0 & 1 & 1 \\
0 & 0 & -\beta e^{-\kappa_1z/2} & 0 & -\beta e^{-\kappa_2z/2} & 0 & \kappa_{b_1} & \kappa_{b_2}
\end{pmatrix}
\begin{pmatrix}
\hat{E}_t
\end{pmatrix} = 0
$$

(19)
3. Results and discussion

Numerical solution of the dispersion relations give dispersion curves for SPP modes in the slot waveguide with Voigt and Faraday configurations. In numerical calculation we adopt the measured parameters for Eu$_2$B$_6$O$_7$ as typical parameters for a WSM: $\varepsilon_{\infty} = 13$, $E_f = 0.15$ eV, $v_f = 10^8$ m s$^{-1}$, $\Omega_p = 60.92$ THz, $\varepsilon_0 = 1.0$ and for $\omega_0 = 0.5 \Omega_p$, $\omega_b = 0.1$, 0.25, 0.5, 1.0 $\Omega_p$. The bulk plasmon dispersions related to the upper WSM in Voigt configuration are indicated by the thin black dashed lines and that for the WSM with broken SIS ($b_0 \neq 0$) is shown by the thin red dashed line.

where $\beta = k_2^2 \varepsilon_d$, $\alpha = k_2^2 \varepsilon_b$, $A_{k_b} = \frac{q_0^2 - \kappa_b^2 - \kappa_2^2}{q_0^2 + \kappa_b^2}$, $\eta = \frac{q_0 \kappa_b^2}{k_2 \varepsilon_b \varepsilon_d}$.

$B_{1,2} = iq\theta_{1,2} - \kappa_b \chi_{1,2}$ with $\theta_{1,2}$ and $\chi_{1,2}$ as defined earlier. Setting the determinant of the coefficient matrix of equations (18) and (19) to zero once again gives rise to a lengthy equation for the dispersion relation of the Faraday configuration. In this case, the dispersion relation does not depend on the sign of $q$ and thus the resultant SPP modes should be reciprocal. In the non-retarded limit, $q \gg k_b$, we have $A_{f_1} = 0$, $B_{f_2} = 0$, $\kappa_{b_{f_1}} = \kappa_{f_2} = \kappa_2 = q$ and the dispersion relation reduces to

$$
\varepsilon [\varepsilon + \varepsilon_d \tanh (a |q|)] = 0.
$$

Again, by taking the long wavelength limit $a |q| \gg 1$, we get the equations $\varepsilon = 0$ and $\varepsilon + \varepsilon_d = 0$, which lead to the asymptotic frequencies for Faraday configuration as follows:

$$
\omega_{b_{ff}} = \Omega_p \varepsilon_{\infty} + \varepsilon_d
$$

$$
\omega_{a_{ff}} = \Omega_p.
$$

In the next section we present our numerical results for the SPP dispersion for both Voigt and Faraday configurations. We will show that the topological properties provide a feasible way to stably tune SPP propagation without needing the application of an external magnetic field.

3.1. The slot waveguide with Voigt configuration

First we consider the case of Voigt configuration at the upper interface. In figure 2 we have exhibited the SPP dispersion in Voigt configuration for both $q > 0$ and $q < 0$ for different values of $\omega_b = 0.1$, 0.25, 0.5, 1.0 $\Omega_p$ and $\omega_b = 0.5 \Omega_p$. As expected from the dispersion relation (equation (16)), the SPP dispersion is non-reciprocal in this case and depends on the propagation direction. There are higher and lower SPP dispersion bands for both $q > 0$ and $q < 0$. In addition, there exists a reciprocal SPP dispersion band lying on the light line corresponding to the lower WSM with broken SIS. The lower SPP dispersion curves start from the zero frequency and continuously tend to their asymptotic frequency given by $\omega_{b_{ff}}$ in equation (17). The higher SPP dispersion curves terminate when they intersect with the bulk plasmon modes which have been indicated by the black thin dashed lines and red thin dashed line for the upper and lower WSMs, respectively. These dispersion curves approach to their asymptotic frequencies given by $\omega_{a_{ff}}$ in equation (17) for $q > 0$ and $\omega_{a_{ff}}$ for $q < 0$. The asymptotic frequencies depend on the physical parameters of WSMs such as the Weyl node separation vector ($b$) and chemical potential ($\mu$). Thus, it is obvious that we can tune the SPP modes by changing the WSM parameters. The dispersion curves lying on the light line start from the zero frequency and end when they coincide with bulk plasmon modes belonging to the lower WSM. It is remarkable that the SPP modes with frequencies below the $\Omega_p$ are non-reciprocal but propagate bidirectionally while the SPP modes with frequencies above the bulk plasmon frequency are chiral and propagate unidirectionally. This fascinating result is in contrast with the SPP modes hosted by a single interface of a WSM and a dielectric in Voigt configuration [25]. The latter structure supports SPP modes for $q < 0$ with frequencies above the bulk plasmon frequency, while these modes disappear in the waveguide structure due to their propagation in the bulk of the lower WSM with broken SIS. This is a striking result which shows the profound tunability of SPP modes in the waveguide structure by involving a second WSM with broken SIS. By increasing $\omega_b$ the dispersion curves for $q > 0$ shift toward higher frequencies. This shift is more significant for curves above the bulk plasmon frequency. For $q < 0$ the dispersion curves show an inverse behavior and their frequencies decrease by increasing $\omega_b$.

The normalized intensity of the $y$ component of the electric field for lower and higher bands of SPP dispersion have been depicted respectively in figures 3(a) and (b) as a function of the $z$ position. The profile of the electric field for $\omega_b = 0.1 \Omega_p$ (at frequencies of 46.42 THz for $q > 0$ and 44.69 THz for $q < 0$) and $\omega_b = 1.0 \Omega_p$ (at frequencies of 49.70 THz for $q > 0$ and 33.62 THz for $q < 0$) are indicated by the solid and dashed lines, respectively. The electric field profile in figures 3(a) and (b) reveals that for the SPP modes related to a positive wave number (the black lines), the lower (higher) bands are mostly localized at the lower (higher) interface while the SPP modes with negative wave number (the red lines) have the inverse behavior. A remarkable feature is that the SPP modes...
become highly confined to the corresponding interfaces with increasing $\omega_b$.

Decay constants for SPP modes of the bands with frequencies below the bulk plasmon frequency are pure, real and positive in the upper WSM with Voigt configuration ($\kappa_{v \pm}$), which ensures decaying of the electric field away from the interface. Meanwhile those for the lower WSM with broken SIS ($\kappa_{b0 \pm}$) are complex conjugates of each other, which causes oscillatory decay the electric field into the lower WSM. Therefore, these SSP modes are categorized as generalized surface waves [28]. On the other hand, decay constants for SPP modes having frequencies above the bulk plasmon frequency (that only exist for $q > 0$) are real and positive, and these SPP modes are known as normal surface waves. To illustrate the dependence of the decaying constants on the wave vector, we have plotted the real and imaginary parts of the reduced decay constants $\beta_{v -}$ as a function of $q$ for lower and higher bands of SPP dispersion for $q > 0$ in figures 4(a) and (b), respectively. For the lower band, figure 4(a), $\beta_{v -}$ is real, while $\beta_{b0 -}$ is complex with an imaginary part.

Figure 3. The normalized electric field intensity of SPP modes of (a) lower and (b) higher bands as a function of $z$ coordinate for the waveguide width $a = 0.1$ µm and different $\omega_b = 0.1, 1.0 \Omega_p$ that are shown by solid and dashed lines, respectively. The other parameters are the same as in figure 2. The black lines denote the electric field profile for a positive wave number ($q > 0$) and the red lines are for a negative wave number ($q < 0$).

Figure 4. Real and imaginary parts of the reduced decay constants of SPP modes for (a) lower and (b) higher bands for $q > 0$ as a function of $q$ shown respectively by solid and dashed lines with $a = 0.1$ µm, $\omega_b = 1 \Omega_p$ and $\omega_{b0} = 0.5 \Omega_p$. 
In figures 7(a) and (b) the normalized localization length \( \lambda_l/a \) for different \( \omega_b = 0.1 \), 0.25, 0.5, 1 \( \Omega_p \) except small variations in the short wave vectors close to the upper interface for \( \epsilon_d = 1.0, 2.0, 3.0 \) with \( \omega_B = \omega_{b0} = 0.25 \Omega_p \) and \( a = 0.1 \mu m \). By increasing \( \epsilon_d \) a substantial decrease is seen in the lower bands and the bands lying on the light line while the higher bands remain almost intact.

3.2. The slot waveguide with Faraday configuration

Now we turn to studying the SPP dispersion of a slot waveguide with Faraday configuration at the upper interface. Since the resultant dispersion curves are reciprocal we show dispersion curves only for \( q > 0 \). Figure 9 shows the SPP dispersion approaching zero in the limit of large wave vectors. However, both of these decaying constants are real and positive for SPP modes of the upper band (figure 4(b)) and tend to the same constant value (\( \beta = 1 \)) at large wave vectors.

Moreover, the localization length (\( \lambda_l = 1/Re(\kappa) \)) normalized to the waveguide width (\( a \)) has been plotted against the wave vector in figure 5 for higher and lower SPP bands for different \( \omega_B \). By decreasing \( \omega_B \), the localization length in the upper WSM (\( \kappa_m \)) for lower (higher) SPP bands decreases (increases), while the localization length in the lower WSM for lower (higher) SPP bands remains intact (decreases) at small wave vectors. All of \( \lambda_l \) for both the higher and lower bands approach \( \lambda_l = 2a \) at large wave vectors.

To study the dependence of the SPP dispersion on the parameters of the lower WSM we have plotted them as a function of the wave vector for different \( \omega_{b0} = 0.1, 0.25, 0.5, 0.75 \Omega_p \) and \( \omega_B = 0.5 \Omega_p \) for \( a = 0.1 \mu m \) in figure 6. As we can see, there is no significant variation in SPP dispersion by changing \( \omega_{b0} \), except small variations in the short wave vectors close to the bulk plasmon dispersion curves. These variations take place due to the change in the bulk plasmon dispersion by changing \( \omega_B \). In figures 7(a) and (b) the normalized localization length (\( \lambda_l/a \)) is plotted for the lower and higher SPP bands with different \( \omega_{b0} \). It is apparent from these figures that, in spite of the negligible variation in the SPP mode frequency with changing \( \omega_{b0} \), there is a significant change in the reduced decay constant of the lower WSM. It decreases for all wave vectors by decreasing \( \omega_{b0} \).

Dependence of the SPP dispersion on the dielectric constant of the dielectric layer has been shown in figure 8, where we have plotted the SPP dispersion for \( \epsilon_d = 1.0, 2.0, 3.0 \) with \( \omega_B = \omega_{b0} = 0.25 \Omega_p \) and \( a = 0.1 \mu m \). By increasing \( \epsilon_d \) a substantial decrease is seen in the lower bands and the bands lying on the light line while the higher bands remain almost intact.
for waveguide widths $a = 0.1, 1.0 \mu m$ and the other parameters the same as in figure 2. In this case there are two bands both below the bulk plasmon frequency. The lower band starts from the origin and then approaches the asymptotic frequency $\omega_f$ as given by equation (21), but the higher band is comprised of two branches with a gap between them. The lower branch rises just to the right of the light line and terminates when it intersects the bulk plasmon dispersion, while the higher branch is a nearly flat band which starts at the bulk plasmon frequency and finally tends to its asymptotic frequency $\omega_{b0} = \Omega_p$. In figures 9(a)–(d) we compare dispersion curves for different waveguide widths and different values of $\omega_b$. As we can see from the figures, decreasing the waveguide width leads to a shift of the lower band to the lower frequencies and an increment of the gap of the higher band. Increasing $\omega_b$ does not alter the lower band but it leads to enlarging the gap of the higher band.

In figures 10(a) and (b) the normalized $y$ component of the electric field intensity is shown for two different values of $\omega_b = 1.0, 0.25 \Omega_p$ at asymptotic frequencies for both higher and lower bands, with waveguide widths $a = 0.1 \mu m$ and $a = 1.0 \mu m$ denoted respectively by the solid and the dashed-dotted lines. As we can see, in both figures the electric field profile is nearly symmetric with respect to the $z$ coordinate for a small waveguide width ($a = 0.1 \mu m$, denoted by solid lines), while it is asymmetric for a large waveguide width ($a = 1.0 \mu m$, denoted by dashed-dotted lines). The symmetric profile for the electric field arises from the effective mixing of the SPP modes at the two upper and lower interfaces at small waveguide widths. However, in the limit of a wide waveguide this hybridization is not considerable and SPP modes belonging to the lower (higher) band are highly localized at the upper (lower) interface. The electric field intensity is suppressed to zero at the middle of the dielectric layer for $\omega_b = 1.0 \Omega_p$ (figure 10(a)) and for $\omega_b = 0.25 \Omega_p$ (figure 10(b)) except the SPP modes of the higher band for small waveguide widths.

The decay constants in both upper and lower WSM media have complex values for SPP modes of both higher and lower bands. The real parts of them tend to $q$, but their imaginary parts approach zero for large wave vectors. So,
it is clear that the real parts of decay constants are more dominant than their imaginary parts at large wave vectors, while both of them play significant roles at small wave vectors. For example, figure 11(a) shows the real and imaginary parts of the reduced decay constants $\beta_f = \kappa_f / q$ and $\beta_{b0} = \kappa_{b0} / q$ of the lower band for $a = 0.1 \, \mu m$. Moreover, the localization length in the upper and lower WSMs and for both SPP modes of the higher and lower bands decay with respect to $q$ and approach $2a$ at the limit of the large wave vectors. We have shown $\lambda_{ll}/a$ in figure 11(b) as a function of $qc/\Omega_p$ for both higher and lower bands for $a = 0.1 \, \mu m$, $\omega_{b0} = 0.1 \, \Omega_p$ and $\omega_{b0} = 0.5 \, \Omega_p$.
Figure 11. (a) Real and imaginary parts of the reduced decay constants versus wave vector for lower band with $\omega_b = 0.5 \Omega_p$. $\omega_b = 0.1 \Omega_p$ and $a = 0.1 \mu m$. (b) The normalized localization length as a function of the wave vector for both lower and higher bands with $\omega_{b_0} = 0.5 \Omega_p$, $\omega_b = 0.1 \Omega_p$ and $a = 0.1, 1.0 \mu m$.

4. Conclusion

In summary, we have investigated the dispersion, the electric field profile and the localization length of SPP modes hosted by a WSM waveguide constructed of two WSMs with distinct symmetries. We found that incorporating a WSM with broken SIS drastically modifies the SPP modes corresponding to a single interface of a WSM with broken TRS in both Voigt and Faraday configurations. In the Voigt configuration inclusion of the second WSM leads to a giant unidirectional SPP mode in frequencies above the bulk plasmon frequency. The SPP modes supported by the waveguide structure in the Faraday configuration are comprised of two bands including a gapfull and one gapless band. Our investigations revealed that the SPP mode dispersion and localization can be effectively tuned by the waveguide characteristic parameters such as the topological properties of the two WSMs. The anomalous Hall effect and chiral magnetic effect which depend on the separation of the Weyl nodes in momentum space and energy determine the strength of the coupling of the electrical and magnetic properties of WSMs. The inhomogeneous optical response of WSMs originating from the transverse or Hall conductivity of these materials is estimated to be several orders of magnitude larger than that of typical magnetic dielectrics [26, 27]. Therefore, stable and efficient control of SPP propagation at the interface of WSMs can be achieved by their intrinsic topological properties. Practically, chiral quantum optics, which deals with propagation-direction-dependent light-matter interactions, has attracted great attention recently [42]. The chiral SPP modes reported here may be employed in developing unidirectional optical circuits [43] and directed excitations in ring lasers [44].

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