How does the Universe expand?

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Quantization of gravity suggests that a finite region of space has a finite number of degrees of freedom or ‘bits’. What happens to these bits when spacetime expands, as in cosmological evolution? Using gravity/field theory duality we argue that bits ‘fuse together’ when space expands.

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A quantum cosmologist, arguing very naïvely, finds the following puzzle. He takes a spacelike region which has a proper volume \( V \) at time \( t \). Quantum gravity tells him that there is a cut-off at planck length \( l_p \), so he has \( N = V/l_p^3 \) ‘cells’ or ‘bits’. He thus expects the quantum theory to be described by a Hilbert space with dimension \( \sim e^N \). But the volume \( V \) expands, and for \( t' > t \) seems to have \( N' = V'/l_p^3 > N \) bits. Since in quantum mechanics the dimension of the Hilbert space must stay fixed, he concludes that no standard quantum theory can handle cosmological expansion.

The simplest retort to this argument is – there is no cutoff at planck length; fourier modes of quantum fields have wavelengths that go all the way down to \( \lambda = 0 \). But wait: this assumption leads to trouble with black holes. In Hawking’s computation of radiation [1] it is assumed that vacuum modes with \( \lambda << l_p \) exist; such modes dilate as they evolve near the horizon and eventually become the radiation quanta. This computation, if correct, gives information loss. Faced with this failure of quantum mechanics we exclaim: But it is wrong to use transplanckian modes so naïvely! In a ‘correct’ derivation of radiation the modes with \( \lambda << l_p \) are to be replaced by nonlocal data; nonlocality occurs across such large distances that information in the singularity (\( r = 0 \)) is encoded in the radiation that is apparently leaving from \( r = R_{\text{Schwarzschild}} \).

Today most physicists would probably agree that in some sense there are a finite number of degrees of freedom in a finite region; in fact holography suggests that \( N \) is even smaller, given by the surface area of the region in planck units [2]. We are really up against the most basic question: What are the ‘bits’ making up spacetime, and how do these bits behave when spacetime deforms, as for example in cosmological expansion? Without a quantum description of spacetime we cannot understand why \( \Lambda \) is finite (and small), or what determines the wavefunction of the Universe. Inflation expands a planck volume by an enormous factor and derives \( \delta \rho/\rho \) by freezing vacuum fluctuations; this makes it imperative to have some insight into how degrees of freedom reshuffle when spacelike slices ‘stretch’.

To address this issue we look at a different system where we also see a ‘stretching’ of space but where we also have an exact quantum description of spacetime.

Maldecena’s duality [3] says that string theory (describing quantum gravity) on certain spacetimes has an exact dual description as a field theory. One case where this duality has been established in detail is the ‘D1-D5 system’ [4] [5] [6]. The field theory is described by an ‘effective string’ which has winding number \( N \) around a circle of length \( L \). This string can be wound as \( N \) separately closed loops (fig.1a), or joined up into a single ‘multi-wound’
string (fig.1b), or more generally have $m$ ‘component strings’, each of which winds $n_i$ times before closing:

$$\sum_{i=1}^{m} n_i = N.$$  \hspace{1cm} (1)

All these states have the same mass and charge, but from fig.1 we see that their dual gravity descriptions have ‘throats’ that ‘stretch’ to different depths [6]. We will shortly argue that each of the $m$ component strings is a ‘bit’, but we already see the moral: *When spacetime stretches, bits fuse together.*

To see why component strings are ‘bits’ consider the dynamics in the field theory and gravity descriptions [6]. A graviton incident on the effective string gets absorbed, its energy converted to a left moving vibration (L) and a right moving vibration (R). In fig.1a the L and R excitations travel around the loop, collide and exit the string after time $\Delta t_{string} = L/2$, while in fig.1(b) the string loop is $N$ times longer and $\Delta t_{string} = NL/2$. 
In the *gravity* description the incident quantum just falls down the throat and returns back up, in each case after a time

\[ \Delta t_{\text{gravity}} = \Delta t_{\text{string}} = \frac{L}{2} \langle n_i \rangle = \frac{L}{2} \frac{N}{\langle m \rangle} \]  

(2)

If we have *two* pairs of vibrations \((L, R)\) and \((L', R')\) in the field theory then they interact only if both pairs are carried by the *same* component string. This makes the effective interaction strength \(G_{\text{eff}}^{\text{string}} \sim 1/m\).

In the dual geometries there is a compact direction of length \(L'\) (not drawn). \(L'\) is smaller for longer throats, and we again find

\[ G_{\text{eff}}^{\text{gravity}} = \frac{G}{L'} \sim \frac{1}{m} \]  

(3)

\((G\) is Newton’s constant.)

When the number of quanta in the throat exceeds \(\sim m\) we find that the gravitational backreaction becomes order unity and a horizon forms. In the dual field theory the presence of more than \(m\) excitations means that two or more excitations would be forced to live on the same component string – the ‘bits’ are all ‘used up’.

To summarize, longer throats are described by fewer component strings (bits), and each particle placed in the throat corresponds to exciting one component string.

Let us now apply these lessons to cosmological expansion. Imagine that the region marked in fig.2a inflates as shown in fig.2b. A spherical wave sent in towards \(r = 0\) in the spacetime of fig.2a returns back in some time \(\Delta t_1\). For the spacetime of fig.2b it would return after \(\Delta t_2 \gg \Delta t_1\).
In this case we do not know the map to a field theory dual, but based on intuition from the exact duality studied above we expect the following:

- **Fusion of bits:** The marked region in fig.1a is described by $N_i$ bits; these fuse together to give $N_f$ bits for the inflated region in fig.2b with

  \[ N_f < N_i \]  

\[ (4) \]

- **Evolution equation:** The D1-D5 states pictured in fig.1 are stable, but if we break supersymmetry either by adding energy or by modifying the action then we get an interaction Hamiltonian $H_{\text{int}}$ which cuts and joins loops \[ (5) \]

Such would be the fundamental equation describing the evolution of spacetime at the quantum level. (The particles on the spacetime are excitations of the bits, and these excitations get rearranged when there is a merger of the bits they live on.) Einstein’s equations give only an effective low energy description which treats spacetime as an infinitely stretchable smooth manifold.

- **Increase of effective coupling:** When the space expands and bits fuse we expect that the effective coupling $G_{\text{eff}}$ will increase. The simplest way to realize this would be to have a compact direction (as in the D1-D5 system) whose length shrinks as the visible directions stretch.

Physicists have long sought to describe spacetime in terms of discrete bits \[ [7] \]. A crucial result of our analysis is that bits are ‘dynamic’ objects that must join and split as spacetime deforms. Near the cosmological singularity ($t = 0$) space is minimally stretched, so $m$ will be large and the excitations representing different particles in spacetime will be typically carried by different bits. (This would validate the proposal \[ [8] \] that there be a decoupling of dynamics between nearby points when $t \to 0$.) At the other extreme, expansion will stop when all bits have fused together ($m = 1$). This is a very nonperturbative quantum gravity effect, and cannot be seen by solving the classical Einstein equations or by performing a semiclassical quantization of gravity.
References

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