Stellar matter in the Quark-Meson-Coupling Model with neutrino trapping

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The properties of hybrid stars formed by hadronic and quark matter in β-equilibrium are described by appropriate equations of state (EoS) in the framework of the quark meson coupling (QMC) model. In the present work we include the possibility of trapped neutrinos in the equation of state and obtain the properties of the related hybrid stars. We use the quark meson coupling model for the hadron matter and two possibilities for the quark matter phase, namely, the unpaired quark phase and the color-flavor locked phase. The differences are discussed and a comparison with other relativistic EoS is done.

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During the early stage of a proto-neutron star neutrinos get trapped in it when their mean-free path is smaller than the star radius. The presence of neutrinos generally gives rise to a stiffer EoS. This may have important consequences on the evolution of the star, namely, it can happen that during the cooling process the star decays into a low-mass black hole [1]. It is worth mentioning that over 20 years ago the importance of neutrino trapping was already pointed out in [2], where the authors claimed that another important effect of including trapped neutrinos is that the collapse is gentler in its presence than it would be without it.

In the present paper we are interested in building the neutrino trapped EoS for mixed matter of quark and hadron phases. We employ the QMC model (QMC) [3, 4, 5] in order to describe the hadron phase. For the quark phase we use two distinct models, the unpaired quark model (UQM), which is given by the simple MIT bag model [6] and the color-flavor locked phase (CFL) [7, 8] in which quarks are paired near the Fermi surface forming a superconducting phase [9].

In a previous work [10] we have used the same formalism in order to study the properties of hybrid stars at $T = 0$ MeV. In this work, we verify the effect of including trapped neutrinos in the same spirit as done in [11], where we have seen that the EoS changes considerably and the mixed phase appears at higher energy densities than in the EoS built without the inclusion of neutrinos in accordance with what was seen in other works, as in [1] for instance. We have also verified that trapping keeps the electron population high so that dense matter contains more protons (and depending on the parametrization used, other positively charged particles) than matter without neutrinos. This fact was also discussed in [12].

Proto-neutron stars with a certain baryonic mass at birth keep this mass during its evolution until the final neutrino free star because most of its matter is accreted in the early stages after birth. Hence, stars with trapped neutrinos and a baryonic mass larger than the corresponding ones after deleptonization collapse to a black hole. In [11] we have described the hybrid star within a non-linear Walecka model (NLWM) [13] including the baryonic octet and a phase transition to a quark phase. Within this description the maximum baryonic masses supported by neutrino trapped EoSs are larger than the corresponding ones found in neutrino free EoSs, a fact which occurs in other EoS which include the strangeness degree of freedom [11, 12]. In what follows we investigate whether this behavior is also present in the framework of the QMC model. Because of its importance in understanding the evolution process in a star, we calculate the baryonic masses for the hybrid stars studied in the present work. We also compare the properties of the stars obtained within the UQM model and the CFL model.

We perform all the calculations at $T = 0$ MeV although the neutrino trapped phase occurs for temperatures in the interior of the star which can vary between 20-40 MeV [1, 11]. However, it was shown in [11] that the effect of trapping is much stronger than the finite temperature effect. Therefore, we believe that the main conclusions drawn in the present work are still valid for a finite temperature calculation. On the other hand, the calculation with the CFL phase will give us only an upper limit, since at finite temperature there is a phase transition from the color superconducting state to a normal phase, which, according to [16], is a second order transition with a BCS critical temperature $T_c \sim 0.57\Delta$, $\Delta$ being the gap parameter.

Since most of the analytical calculations and formulae used in the present work have already been given in [10] they will be omitted here. As mentioned above, we have used the QMC model with the inclusion of hyperons for the hadron phase. In this model, the nucleon in nuclear medium is assumed to be a static spherical MIT bag in which quarks interact with the scalar and vector fields, $\sigma$, $\omega$ and $\rho$ and these fields are treated as classical fields in the mean
The quark field, $\psi_q(x)$, inside the bag then satisfies the equation of motion:

$$\left[ i \partial_x - (m_0^q - g_0^q\sigma) - g_0^q \omega \gamma^0 + \frac{1}{2} g_0^q \tau_3 \rho_{03} \right] \psi_q(x) = 0 \quad q = u, d, s,$$

where $m_0^q$ is the current quark mass, and $g_0^q$, $g_0^q_\sigma$, and $g_0^q_{\tau_3}$ denote the quark-meson coupling constants. After enforcing the boundary condition at the bag surface, the transcendental equation for the ground state solution of the quark (in $s$-state) is $\beta_q(x_q) = \beta_q h_q(x_q)$ which determines the bag eigenfrequency $\omega_q$, where

$$\beta_q = \sqrt{(\Omega_q - R_B m_0^q)/(\Omega_q + R_B m_0^q)},$$

with $\Omega_q = (x_q^2 + R^2 m_0^q)^{1/2}$; $m_q^0 = m_0^q - g_0^q \sigma_0$, is the effective quark mass. The energy of the nucleon bag is $M_B^2 = 3 \frac{\Omega_q^2}{\beta_q^2} - \frac{\Omega_q}{\beta_q} + \frac{4}{3} \pi R_B^3 B_B$, where $B_B$ is the bag constant and $Z_B$ parameterizes the sum of the center-of-mass motion and the gluonic corrections. The bag radius, $R_B$, is then obtained through the stability condition for the bag. An interesting fact related to the QMC model is that the bag volume changes in the medium through the mean value of the $\sigma$-field. This also implies that the bag eigenvalues are also modified. The onset of hyperons depends on the conditions of chemical equilibrium and charge neutrality discussed below and also on the meson-hyperon coupling constants for which we have chosen the hyperon coupling constants constrained by the binding of the $\Lambda$ hyperon in nuclear matter, hyper-nuclear levels and neutron star masses ($x_q = 0.7$ and $x_\omega = x_\rho = 0.783$) and assumed that the couplings to the $\Sigma$ and $\Xi$ are equal to those of the $\Lambda$ hyperon [12]. The leptons either in the hadron or in the quark phase are considered as free Fermi gases and they do not interact with the hadrons, the mesons or with the quarks.

The condition of chemical equilibrium is imposed through the two independent chemical potentials for neutrons $\mu_n$ and electrons $\mu_e$ and it implies that the chemical potential of baryon $B_i$ is $\mu_{B_i} = Q_{\mu_B}^i \mu_n - Q_{\mu_e}^i \mu_e$, where $Q_{\mu_B}^i$ and $Q_{\mu_e}^i$ are, respectively, the electric and baryonic charge of baryon or quark $i$. Charge neutrality implies $\sum_i \mu_{B_i}^e + \sum_i q_i \mu_i = 0$, where $q_i$ stands for the electric charges of leptons. If neutrino trapping is imposed to the system, the beta equilibrium condition is altered to $\mu_{B_i} = Q_{\mu_B}^i \mu_n - Q_{\mu_e}^i (\mu_e - \mu_{\nu_e})$. In this work we have not included trapped muon neutrinos. Because of the imposition of trapping the total leptonic number is conserved, i.e., $Y_L = Y_e + Y_{\nu_e} = 0.4$.

For the quark phase we consider two models. First of all, we take the quark matter EoS as in [8] in which $u$ and $d$ s quark degrees of freedom are included in addition to electrons. Up and down quark masses are set to zero and the strange quark mass is taken to be either 150 or 200 MeV so that we are able check the effect of the $s$-quark mass. In chemical equilibrium $\mu_d = \mu_s = \mu_u + \mu_e$. In terms of neutron and electron charge chemical potentials $\mu_n$ and $\mu_e$, one has $\mu_u = \frac{2}{3} \mu_n - \frac{2}{3} \mu_e$, $\mu_d = \frac{1}{3} \mu_n + \frac{1}{3} \mu_e$, and $\mu_s = \frac{1}{3} \mu_n + \frac{1}{3} \mu_e$. In the energy density for the quark matter EoS a term $+B$ and in the pressure a term $-B$ are inserted. This term is responsible for the simulation of confinement. For the Bag model, we have taken $B^{1/4} = 150$ and 200 MeV.

In the EoS taking into consideration a CFL quark paired phase, the quark matter is treated as a Fermi sea of free quarks with an additional contribution to the pressure arising from the formation of the CFL condensates. The density of the three types of quarks are identical and the electron density is zero, as shown in [7]. The expressions for the energy density and pressure depend on a gap parameter $\Delta$ which is taken to be 100 MeV [9].

Once the hadron and quark phases are well established, we have to construct the mixed phase, imposing charge neutrality globally, $\chi \rho_{QP}^{UP} + (1 - \chi) \rho_{QP}^{UP} + \rho_e^{QP} = 0$, where $\rho_e^{QP}$ is the charge density of the phase $i$, $\chi$ is the volume fraction occupied by the quark phase and $\rho_q^{QP}$ is the electric charge density of leptons. We consider a uniform background of leptons in the mixed phase since Coulomb interaction has not been taken into account. According to the Gibbs conditions for phase coexistence, the baryon chemical potentials, temperatures and pressures have to be identical in both phases, i.e., $\mu_{HP,n} = \mu_{QP,n} = \mu_n$, $\mu_{HP,e} = \mu_{QP,e} = \mu_e$, $T_{HP} = T_{QP}$, $P_{HP}(\mu_n, \mu_e, T) = P_{QP}(\mu_n, \mu_e, T)$, reflecting the needs of chemical, thermal and mechanical equilibrium, respectively.

In fig. 4 the EoSs obtained with both quark models are displayed for different $B$ values and two strange quark masses with neutrino trapping ($Y_{\nu_e} = 0.4$). For the sake of comparison we have also plotted one EoS with no neutrinos ($Y_{\nu_e} = 0$), and the EoS obtained with a NLWM formalism for the hadronic phase [11] and an UQM with $m_s = 150$ MeV and $B^{1/4} = 190$ MeV, with and without neutrino trapping. As already discussed in [11, 12] the EoSs are harder if neutrino trapping is imposed, independently of the model used. A larger $s$-quark mass and a larger $B$ parameter make the quark EoSs harder in the mixed phase, a fact that manifests itself on the maximum mass stellar configuration. The main differences between the QMC formalism and the NLWM are: a) the NLWM EoS is harder at low densities and softer at intermediate densities due to the presence of hyperons; b) the transition to a pure quark phase occurs at lower densities in the NLWM. This behavior has consequences on the properties of the corresponding families of stars.

In order to better understand the importance of the neutrinos when neutrino trapping is imposed, in fig. 2 the fraction of neutrinos is shown. The behavior encountered for the neutrino fraction if the UQM is used resembles the one shown in [11]: the population of neutrinos decreases in the hadron phase and, contrary to [11], only increases in the mixed and quark phases. The highest yields are of the order of 0.16. In this model, for smaller values of the
TABLE I: Star properties

| Model          | $B^{1/4}$ (MeV) | $m_s$ (MeV) | $\frac{M_{\text{max}}}{M_\odot}$ | $\frac{M_\odot}{M_\odot}$ | $R$ (km) | $\varepsilon_0$ (MeV) |
|----------------|-----------------|-------------|-------------------------------|-----------------------------|---------|---------------------|
| QMC+UQM        | 190             | 150         | 1.94                          | 2.15                        | 12.09   | 5.47                |
| QMC+UQM        | 190             | 200         | 1.98                          | 2.20                        | 12.02   | 5.59                |
| QMC+UQM        | 200             | 150         | 1.99                          | 2.22                        | 11.97   | 5.63                |
| QMC+UQM        | 200             | 200         | 2.02                          | 2.26                        | 11.89   | 5.77                |
| QMC+UQM ($Y_{\nu_e} = 0$) | 190             | 150         | 1.73                          | 1.93                        | 12.44   | 4.89                |
| QMC+CFL        | 190             | 150         | 1.64                          | 1.83                        | 12.84   | 4.5                 |
| QMC+CFL        | 190             | 200         | 2.00                          | 2.22                        | 12.59   | 5.06                |
| QMC+CFL        | 200             | 150         | 1.80                          | 1.99                        | 10.86   | 7.32                |
| QMC+CFL        | 200             | 200         | 1.84                          | 2.03                        | 11.10   | 6.97                |
| QMC+CFL ($Y_{\nu_e} = 0$) | 190             | 150         | 1.83                          | 2.02                        | 11.36   | 6.57                |
| QMC+CFL ($Y_{\nu_e} = 0$) | 200             | 200         | 1.87                          | 2.07                        | 11.57   | 6.20                |

For the UQM, these critical masses are always higher than for the CFL model, because it also corresponds to harder EoS. The radii and central energy density depend on the model and on the strange quark mass. The radii values are larger for the UQM model and the central energy density are larger for the CFL model, again due to the fact that the UQM EoSs are harder. Comparing the results of table I with the ones presented in [11], where neutrino trapping was not considered, one can see that the inclusion of trapping makes the gravitational masses reasonably higher. The same conclusion was drawn in [11] where the calculations were performed with different relativistic models. In the Table I we have also included properties of maximum mass stars obtained within the NLWM for the hadronic phase [11]. Two conclusions are in order: the maximum baryonic masses obtained within the NLWM are larger and the difference of maximum masses for trapped and untrapped matter is smaller for the QMC ($\sim 0.2M_\odot$) than for the NLWM ($\sim 0.4M_\odot$). This means that the number of stars that would decay into a black hole is much smaller in the QMC model and is probably due to the fact that no hyperons are formed in the interior of stars obtained with QMC for $m_s = 150$ MeV and $B^{1/4} = 190$ MeV contrary to the NLWM case. If the quark phase is a CFL state the baryonic mass difference between the neutrino rich stars and neutrino poor is greater than in the UQM, ($\sim 0.35M_\odot$). This is understood because the greater flux of neutrinos carries out more energy. However for a finite temperature calculation we expect a smaller effect.

A similar analysis was done in [14], where the authors used a derivative coupling model with hyperons for the
hadron phase and the Bag model for the quark matter. They did not obtain any mixed phase for bag values larger than $B^{1/4} = 190$ MeV in contrast with the present work. Moreover, the maximum masses shown in [14] ($\sim 1.6 M_\odot$) and the differences between maximum masses in neutrino rich and neutrino poor stellar matter are lower than in our calculations.

In fig. 3 we display the baryonic masses versus the gravitational masses for both models. It is seen that neutrino trapped EoSs give rise to greater gravitational masses for the same baryonic mass. The mass difference reflects the binding energy released during the deleptonization and cooling stage [1]. The maximum baryonic mass of the neutrino rich EoS are larger than the neutrino poor. This will lead to a blackhole formation during the leptonization period for stars with baryonic masses greater than the maximum baryonic mass of the neutrino poor EoS. This behavior has been encountered in other EoS which include strange matter, namely hyperon and/or quark matter [11, 12].

In summary, we have investigated the effects of neutrino trapping in the properties of neutron stars within the QMC framework, including the possibility of hyperon formation and a transition to an unpaired quark-phase or a CFL phase. We have concluded that within the QMC model with hyperons for the hadronic matter, either the hyperons only occur at very high densities or in very small amounts at lower densities (e.g., UQM). Another important point is that the maximum mass of a neutrino rich neutron star decreases after neutrino diffusion leading to the formation of a low mass black-hole. This mass reduction is smaller for a quark phase described within an unpaired quark phase than a CFL phase. If the quark phase is in a CFL state a large fraction of neutrinos is expected in the mixed and quark phases which will carry away more energy as they diffuse out. We also point out that the amount of neutrinos present in the CFL phase is almost the double in comparison with the amount found in the UQM phase. At finite temperature the effect will not be so strong and a self-consistent finite temperature calculation should be performed. We have also seen that the mass reduction of the maximum mass stars during to neutrino diffusion is smaller within a QMC formalism than in a NLWM formalism.

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FIG. 1: Equation of state obtained with the QMC model plus (a) UQM (b) CFL.

FIG. 2: Neutrino fraction for the EoS obtained with the QMC model plus (a) UQM (b) CFL.

FIG. 3: Baryonic mass versus gravitational mass obtained from the EoS with QMC plus (a) UQM (b) CFL.