Virtual contributions from $D^*(2007)^0$ and $D^*(2010)^\pm$ in the $B \to D \pi h$ decays

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We study the quasi-two-body decays $B \to D^* h \to D \pi h$ with $h = (\pi, K)$ in the perturbative QCD approach and focus on the virtual contributions from the off-shell $D^*(2007)^0$ and $D^*(2010)^\pm$ in the four measured decays $B^0 \to D^0 \pi^+ \pi^-$, $B^0 \to D^0 \pi^+ K^-$, $B^- \to D^+ \pi^- \pi^-$ and $B^- \to D^+ \pi^- K^-$. For the $B^0 \to D^{*+} \pi^- \to D^0 \pi^+ \pi^-$ and $B^0 \to D^{*+} K^- \to D^0 \pi^+ K^-$ decays, their branching fractions concentrate in a very small region of $m_{D^0h^+}$ near $D^{*+}$ pole mass, and the virtual contributions from $D^{*+}$, in the region $m_{D^0h^+} > 2.1$ GeV, are about 5% of the corresponding quasi-two-body results. We define two ratios $R_{D^{*+}}$ and $R_{D^{*0}}$, from which we conclude that the flavor-$SU(3)$ symmetry will be maintained for the $B \to D^* h \to D \pi h$ decays with very small breaking at any physical value of the $m_{D^0h}$. The $B^- \to D^{*0} \pi^- \to D^0 \pi^+ \pi^-$ and $B^- \to D^{*0} K^- \to D^0 \pi^+ K^-$ decays can be employed as a constraint for the $D^{*0}$ decay width, with preferred values consistent with previous theoretical predictions for this quantity.

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I. INTRODUCTION

Three-body hadronic decays $B \to D \pi h$, with the $h$ is pion or kaon, have been suggested as a way to measure the Cabibbo-Kobayashi-Maskawa (CKM) $\theta^2$ angle $\simeq 3 \pi/4$ (which have been performed in \cite{8,10}) and angle $\beta$ \cite{11,13}. These decay processes have also been proven as an appropriate field for the studies of charm meson spectroscopy, the Belle, BaBar and LHCb Collaborations have achieved brilliant progress in identifying the excited charm states and measuring their parameters \cite{14,21}. In the amplitude analyses of $B \to D \pi h$ decays, one has the contributions from the quasi-two-body decay processes $B \to D^* h \to D \pi h$, including the $S$-wave ground states of the $c \bar{q}$ ($q = u$ or $d$) quark system, the charmed vectors $D^*(2007)^0$ and $D^*(2010)^\pm$ as the intermediate states. With the strong kinematic suppression, the charged state $D^{*+}$ may decay into $D^0 \pi^+$ or $D^+ \pi^0$, the neutral state $D^{*0}$ can decay into $D^0 \pi^0$. The natural decay mode $D^{*0} \to D^+ \pi^-$ for the $D^{*0}$ is blocked because of its pole mass and the threshold of its decay daughters.

The $D^*$ is usually studied, on the theoretical side, as the stable particle in two-body hadronic $B$ meson decays in the literature. The discussions of the factorization formula for the $B$ meson decays to $D^{(*)}$ and a light pseudoscalar or vector meson could be found in Refs. \cite{25,26}. In \cite{27}, the color-favored decays $B \to D^{(*)} \pi$ were explored within the factorization hypothesis. Using the factorization approach, the two-body decays $B \to D^* h$ have been studied in \cite{28,30}. Phenomenological studies of the $B_{d,s} \to D_{d,s} V$ decays were performed in \cite{31} within the framework of QCD Factorization. The global fits under the assumption of flavor $SU(3)$ symmetry for the charmed $B$ decays have their results in \cite{32}. Within the factorization-assisted topological-amplitude approach, the two-body decays $B \to D^{(*)} M$ have been studied in \cite{33}. The discussions of the isospin relations for the $B \to D^* K(\pi)$ could be found in Ref. \cite{33}. While in the perturbative QCD (PQCD) approach \cite{33,38}, the $B \to D^{(*)}$ form factors and the $B \to D^{(*)} M$ decays were calculated in \cite{39} and \cite{40}, respectively, the color suppressed decay modes $B^0 \to D^{(*)} \eta(\gamma)$ were analyzed in \cite{41}, the two-body $B_{d(s)} \to D_{d(s)}^{(*)} P$ and $B_{d(s)} \to D_{d(s)}^{(*)} V$ decays were studied in Ref. \cite{42} and $B$ meson decay into $D^{(*)}$ and a light scalar meson were studied in Ref. \cite{43}.

In the Dalitz plot \cite{44} analyses of the decays $B^- \to D^{+} \pi^- h^-$ (the inclusion of charge-conjugate processes is always implied) performed by Belle \cite{14}, BaBar \cite{16} and LHCb Collaborations \cite{17,21}, the virtual contributions for the $D^{+} \pi^-$ pair from the intermediate state $D^{*0}$ were found to be indispensable for the total amplitudes. The virtual contributions are the contributions from the state $D^{*0}$ for the $D^{+} \pi^-$ pair in the quasi-two-body processes $B^- \to D^{*0} \pi^- \to D^{+} \pi^- \pi^-$ and $B^- \to D^{*0} K^- \to D^{+} \pi^- K^-$ with the resonance pole mass outside the kinematically

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In the light-cone coordinates, where $B^*$ denotes possible attachments of hard gluons and the rectangle represents the vector states $D^*$, the accessible region of the phase space \([17, 21]\). That is to say, although the pole mass of $D^{*0}$ is lower than the threshold of $D^+\pi^-$ pair, the natural decay tunnel $D^{*0} \to D^+\pi^-$ is blocked, but the resonance tail will contribute to the total branching fractions of the $B^- \to D^+\pi^-h^-$ processes, and the off-shell effects were found surprisingly large in \([15]\). For the decays $B^0 \to D^{*0}h^-$, the portion of $B^0 \to D^{*+}h^-$ with the natural decay $D^{*+} \to D^0\pi^+$ were always excluded from the total three-body branching fractions by a cut of the $D^0\pi^+$ invariant mass, while the necessary off-shell effects were retained in the decay amplitudes \([15, 18, 19]\).

In order to extract the most information on the involved strong and weak dynamics from the experimental data of the three-body $B$ decays, different methods have been adopted, such as the isospin, U-spin and/or flavor $SU(3)$ symmetries \([46, 53]\), the QCD factorization \([54, 71]\) and the PQCD approach \([72, 77]\) in abundant works. While three-body hadronic $B$ decays are known experimentally, in most cases, to be dominated by the low energy scalar, vector and tensor resonances, which could be analysed in the quasi-two-body framework by neglecting the three-body effects and the rescattering effects \([71, 80]\). In the quasi-two-body framework, we always assume two final states, in the three-body processes, form a single resonant state which originated from a quark-antiquark pair and then the factorization procedure can be applied \([71, 81]\). In this work, we will focus on the virtual contributions originated from off-shell $D^*$ in the measured decays $B^- \to D^+\pi^-\pi^-$, $B^- \to D^+\pi^-K^-$, $B^0 \to D^0\pi^-\pi^-$ and $\bar{B}^0 \to D^0\pi^-K^-$. The $D^*$ off-shell effects in the four decay processes $B \to D\pi h$ and the natural contributions $D^{*+} \to D^0\pi^+$ in the two $\bar{B}^0$ decays in this work shall be analysed in the quasi-two-body framework which has been detailed discussed in Ref. \([80]\) in PQCD approach. The method used in \([80]\) has been adopted in Refs. \([82, 83]\) for the studies of some quasi-two-body $B$ meson decays.

This work is organized as follows. In Sec. II, we give a brief introduction for the theoretical framework. In Sec. III, we show the numerical results. Discussions and conclusions are given in Sec. IV. The factorization formulas for the relevant quasi-two-body decay amplitudes are collected in the Appendix.

II. FRAMEWORK

In the rest frame of $B$ meson, with $m_B$ being its mass, we define momentum $p_B$ and the light spectator quark momentum $k_B$ for it as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad k_B = \left(0, \frac{m_B}{\sqrt{2}}x_B, k_{BT}\right),$$  \hspace{1cm} (1)

in the light-cone coordinates, where $x_B$ is the momentum fraction. The momenta $p_3$ and $k_3$ for the bachelor final state $h$ and its spectator quark have their definitions as

$$p_3 = \frac{m_B}{\sqrt{2}}(0, 1 - \eta, 0_T), \quad k_3 = \left(0, \frac{m_B}{\sqrt{2}}(1 - \eta)x_3, k_{3T}\right).$$  \hspace{1cm} (2)

For the state $D^*$ and the $D\pi$ pair decays from it in the Feynman diagrams, the Fig. 1 for the quasi-two-body processes $B \to D^*h \to D\pi h$, we define their momentum $p = \frac{m_B}{\sqrt{2}}(1, \eta, 0)$. Its easy to see $\eta = s/m_B^2$, with the invariant mass square $s = p^2$ for the $D\pi$ pair. The light spectator quark comes from $B$ meson and goes into intermediate state in the $D^*$ hadronization as shown in Fig. 1 (a) has the momentum $k = (\frac{m_B}{\sqrt{2}}z, 0, k_T)$. Where $x_3$ and $z$ are the corresponding momentum fractions and run from 0 to 1.

The distribution amplitudes for the $B$ meson and the bachelor final state pion or kaon in this work are the same as those widely adopted in the PQCD approach in the hadronic $B$ meson decays, one can find their expressions and the relevant parameters in Ref. \([80]\). For the longitudinal polarization structure of the $P$-wave $D\pi$ system which including the $D^*$ hadronization and the $D^* \to D\pi$ processes, based on the discussions in Refs. \([39, 42, 75, 80, 87, 88]\), one could...
write
\[
\Phi_{D\pi}^{P,\text{wave}} = \frac{1}{\sqrt{2N_c}} f_1(y + \sqrt{s}) \phi_{D\pi}(z, b, s),
\]
with the distribution amplitude
\[
\phi_{D\pi}(z, b, s) = \frac{F_{D\pi}(s)}{2\sqrt{2N_c}} \text{Re} \{ \mathcal{G}(1 - \frac{m_D^2 - m_{\pi}^2}{2}) \},
\]
where the \(a_{D\pi}\) and \(\omega_{D\pi}\) are the Gegenbauer moment and the shape parameter for the \(P\)-wave \(D\pi\) system, respectively. The time-like form factor \(F_{D\pi}(s)\) has its definition in the matrix elements
\[
\langle D(p_1)\pi(p_2) | \gamma \mu (1 - \gamma_5) q | 0 \rangle = \mathcal{F}_{D\pi}(s) + \frac{m_D^2 - m_{\pi}^2}{p^2} p_\mu F_0(s),
\]
where \(p = (p_1 + p_2), p_1(p_2)\) is the momentum for \(D(\pi)\) and \(m_D(m_{\pi})\) is the mass of \(D(\pi)\) meson. The \(F_0(s)\) is the S-wave form factor for \(D\pi\) system. With Eq. (5), by inserting the intermediate state \(D^\ast\), it’s easy to have the following expression for the form factor \(F_{D\pi}(s)\)
\[
F_{D\pi}(s) = \frac{\sqrt{2} f_{D\pi} g_{D\pi} \Gamma_{\text{BW}}}{\text{BW}(s)}. \tag{6}
\]
The \(f_{D\pi}\) above is the decay constant for \(D^\ast\), one can find its different values in Refs. [50, 93]. We adopt \(f_{D^\ast} = (250 \pm 11)\) MeV [80, 90] in the numerical calculations. The energy dependent relativistic Breit-Wigner denominator \(\text{BW}(s)\) equals to \(m_D^2 - s - im_D \Gamma(s)\), with \(m_D\) the pole mass for \(D^\ast\), and the mass dependent decay width defined as
\[
\Gamma(s) = \Gamma_0 \left( \frac{q}{q_0} \right)^2 \left( \frac{m_D}{\sqrt{s}} \right)^2 \mathcal{X}(qr_{\text{BW}}),
\]
where the barrier radius \(r_{\text{BW}} = 4.0\) GeV\(^{-1}\) as it in Refs. [17, 18, 21], the Blatt-Weisskopf barrier factor [91] is
\[
\mathcal{X}(qr_{\text{BW}}) = \sqrt{1 + \frac{(q r_{\text{BW}})^2}{(qr_{\text{BW}})^2}},
\]
and \(q = \sqrt{s - (m_D + m_{\pi})^2} / s\) is the magnitude of the momentum for the daughter state \(D\) or \(\pi\) in the rest frame of the \(D^\ast\), \(q_0\) is the value of \(q\) at \(s = m_D^2\). For the virtual contributions from the state \(D^0\), the \(m_D\), in the \(q_0\) shall be replaced with the \(m_D^0\), which has its formula in Refs. [17, 21, 92].

The coupling constant \(g_{D\pi} = \Gamma_0 / \text{BW}(s)\) could be related to the decay width \(\Gamma_0\) in Eq. (4) for \(D^\ast\). For the total decay width of the \(D^\ast^+\), which is the sum of the partial widths of the decays \(D^\ast^+ \rightarrow D^0\pi^+, D^\ast^+ \rightarrow D^0\pi^0\) and \(D^\ast^+ \rightarrow D^+\gamma\), was firstly measured by CLEO Collaboration with \(\Gamma(D^\ast^+) = 96 \pm 4\) (stat.) \(\pm 22\) (syst.) keV [93]. A more precise measurement performed by BaBar Collaboration presented \(\Gamma(D^\ast^+) = 83.3 \pm 1.2\) (stat.) \(\pm 1.4\) (syst.) keV and \(\Gamma(D^\ast_\pi \pi) = 16.92 \pm 0.13 \pm 0.14\) [57, 68] with the isospin relation \(g_{D^\ast \pi} = g_{D^\ast_\pi \pi} = -\sqrt{2} g_{D^\ast D^\pi \pi}\). For the state \(D^0\), there is no accurate experimental result for its decay width. In the measurement of three-body decays including virtual \(D^0\) contributions, the width was fixed to 0.1 MeV by BaBar [10], the experimental upper limit of 2.1 MeV was adopted by LHCb [17, 21], while the decay width for \(D^0\) in the work [14] from Belle Collaboration was calculated from the width of the \(D^\ast\) assuming isospin invariance and HQET.

The Lorentz invariant amplitude \(A\) for the quasi-two-body \(B \rightarrow D^* h \rightarrow D\pi h\) decay processes in the PQCD approach, according to Fig. 1 is given by
\[
A = \phi_B \otimes H \otimes \phi_h \otimes \phi_{D\pi}, \tag{9}
\]
where the symbol \(\otimes\) means convolutions in parton momenta, the hard kernel \(H\) contains one hard gluon exchange as shown in Fig. 1 and the \(B\) meson \((h, D\pi\) pair\) distribution amplitude \(\phi_B\) \((\phi_h, \phi_{D\pi})\) absorbs the nonperturbative dynamics in decay processes. The differential branching fractions \((B)\) for the \(B \rightarrow D^* h \rightarrow D\pi h\) decays are [54, 58, 69]
\[
\frac{dB}{d\eta} = \frac{\tau_B q_h^3 q^2}{48\pi^3 m_B^2} A^2 / \eta^2, \tag{10}
\]
where \(\tau_B\) being the \(B\) meson mean lifetime. The magnitudes of \(h\) meson momentum \(q_h\), in the rest frame of the \(D^\ast\), is written as
\[
q_h = \frac{1}{2} \sqrt{[(m_B^2 - m_h^2)^2 - 2 (m_B^2 + m_h^2) s + s^2] / s}. \tag{11}
\]
The \(m_h\) is the mass of the bachelor meson pion or kaon. The decay amplitudes for \(B \rightarrow D^* h \rightarrow D\pi h\) are collected in the Appendix.
### III. RESULTS

In the numerical calculation, we adopt the decay constant \( f_B = 0.19 \text{ GeV} \) [100], the mean lifetimes \( \tau_{D^0} = (1.520 \pm 0.004) \times 10^{-12} \text{ s} \) and \( \tau_{D^0} = (1.638 \pm 0.004) \times 10^{-12} \text{ s} \) [99] for the B meson. The masses of the neutral and charged \( B, D, \pi \) and \( K \) mesons, the pole masses of the neutral and charged \( D^* \) and the Wolfenstein parameters \( \lambda \) and \( A \) are presented in Table I.

| TABLE I: Masses (in units of GeV) and Wolfenstein parameters [99]. |
|---------------------------------------------------------------|
| \( m_{D^0} = 5.280 \) | \( m_{B^0} = 5.279 \) | \( m_{D^0} = 1.865 \) | \( m_{D^0} = 1.870 \) | \( m_{e^0} = 0.135 \) |
| \( m_{e^0} = 0.140 \) | \( m_{e^0} = 0.498 \) | \( m_{K^0} = 0.494 \) | \( m_{B^0} = 2.007 \) | \( m_{D^+} = 2.010 \) |
| \( \lambda = 0.22453 \pm 0.00044 \) | \( A = 0.836 \pm 0.015 \) |

Utilizing the the differential branching fraction the Eq. (10) and the decay amplitudes collected in Appendix A, we obtain the branching fractions for the virtual contributions (\( B_\nu \)) in Table I of the concerned quasi-two-body decay processes \( B \rightarrow D^* s \rightarrow D^* h \rightarrow D s h \). The invariant mass of the \( D s \) system has been cut at 2.1 GeV for the results in Table I by following the step of Ref. [12], and the decay width \( \Gamma_{D^* h} = 2.1 \text{ MeV} \) which has been adopted by LHCb Collaboration in Refs. [17, 21] is employed for the two \( B^- \) decay modes. The largest error for the branching fractions in Table I comes from the \( B \) meson shape parameter uncertainty \( \omega_B = 0.40 \pm 0.04 \text{ GeV} \), the error induced by the decay constant \( f_{D^*} = (250 \pm 11) \text{ MeV} \) [54, 90] takes the second place, the uncertainty of the Wolfenstein parameter \( A \) in Table I contributes the fourth one, while the third error and the last one originated from the \( D^* \) Gegenbauer moment \( a_{D^*} = 0.50 \pm 0.10 \text{ and shape parameter} \omega_{D^0} = 0.10 \pm 0.02 \) [42, 101], respectively. There are other errors, which come from the uncertainties of the parameters in the distribution amplitudes for bachelor pion(kaon) [82], the Wolfenstein parameters \( \lambda \) [82], etc. are small and have been neglected. One has the integrated branching ratios for the two-body decays \( B^0 \rightarrow D^{*+} \pi^- \) and \( B^0 \rightarrow D^{*+} K^- \) as

\[
B(B^0 \rightarrow D^{*+} \pi^-) = (2.54 \pm 0.23 \text{ GeV}) \times 10^{-3},
\]

and

\[
B(B^0 \rightarrow D^{*+} K^-) = (2.05 \pm 0.15 \text{ GeV}) \times 10^{-4},
\]

from the corresponding quasi-two-body decays by integrating the whole physical region of the \( D^0 \pi^- \) invariant mass and considering the data \( B(D^{*+} \rightarrow D^0 \pi^+) = 67.7\% \) [82]. The two results above predicted by PQCD agree well with the branching fractions \( (2.74 \pm 0.13) \times 10^{-3} \) and \( (2.12 \pm 0.15) \times 10^{-4} \) for the two-body decays \( B^0 \rightarrow D^{*+} \pi^- \) and \( B^0 \rightarrow D^{*+} K^- \) in the Review of Particle Physics 2016, respectively.

| TABLE II: The PQCD predictions of the virtual contributions from \( D^* \) state in the \( D s \) invariant mass region \( \sqrt{s} > 2.1 \text{ GeV} \) for the \( B \rightarrow D^* s \rightarrow D s h \) decays. |
|-------------------------------------------------------------|
| \( B_\nu \) | Unit | \( B_\nu \) |
|-------------------------------------------------------------|
| \( B^0 \rightarrow D^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^- \) | (10^{-4}) | 0.81(0.94)(10^{-4})(4) \text{ GeV} \] |
| \( B^0 \rightarrow D^{*+} K^- \rightarrow D^0 \pi^+ K^- \) | (10^{-4}) | 0.72(0.57)(10^{-4})(4) \text{ GeV} |
| \( B^- \rightarrow D^{*0} \pi^- \rightarrow D^0 \pi^- \pi^- \) | (10^{-4}) | 1.91(0.55)(10^{-4})(4) \text{ GeV} |
| \( B^- \rightarrow D^{*0} K^- \rightarrow D^0 \pi^- K^- \) | (10^{-4}) | 1.48(0.65)(10^{-4})(4) \text{ GeV} |

For the denominator \( D_{BW}(s) \) in the Eq. (8), we have \( |m_{D^*} - s| \gg |m_{D^*} \Gamma(s)| \) when \( \sqrt{s} > 2.1 \text{ GeV} \) even if the \( D^{*0} \) decay width is 2.1 MeV. As a result, the variation of the \( r_{BW} \) from 4.0 GeV to 1.6 GeV is 14% [14, 14] in Eq. (7) makes the virtual contributions for the \( B \rightarrow D^* s \rightarrow D s h \) decays in Table I essentially unchanged. The same situation will happen again because of the same reason when one replaces Blatt-Weisskopf barrier factor, the Eq. (8), with the exponential form factor (EFF) \( F(z) = \exp(- (z - z')) \) for the denominator \( D_{BW}(s) \), where \( z \) and \( z' \) have their expressions in Ref. [16]. The EFF \( R(m^2(D^*0)) = e^{-(\beta_1 + \beta_2)m^2(D^*)} \) with the free parameters \( \beta_1 \) and \( \beta_2 \), has been used in the experimental Dalitz plot analyses [16] to describe the contributions from the off-shell \( D^*(2010)^- \) and the general \( D^0 \pi^- \) P-wave. We don’t tend to employ an EFF to replace the time-like form factor of Eq. (6) because the EFF will bring us an unknown parameter and reduce the ability of theoretical prediction. As a test of the effect of \( m_{D^*}^{\text{eff}} \) instead of \( m_{D^*} \) for the virtual contributions in the decays involving \( D^{*0} \) in this work, we employ the value \( m_{D^*} + \Delta m \) to replace the \( m_{D^*}^{\text{eff}} \) for \( q_0 \) in the Eq. (8). When \( \Delta m \) is \( \pm 0.5 \text{ GeV} \) or even \( \pm 1.0 \text{ GeV} \), the results for \( B^- \rightarrow D^{*0} \pi^- \rightarrow D^+ \pi^- \pi^- \) and \( B^- \rightarrow D^{*0} K^- \rightarrow D^+ \pi^- K^- \) are almost the same as they in Table I the variations are found less than 0.1% for the corresponding values.
The differential branching fractions for the process $\bar{B}^0 \rightarrow D^{\ast+}\pi^- \rightarrow D^0\pi^+\pi^-$ (left) and $\bar{B}^0 \rightarrow D^{\ast+}K^- \rightarrow D^0\pi^+\pi^-$ (right). The two small diagrams are for the corresponding virtual contributions in the $m_{D\pi}$ region (2.1 ~ 3.5) GeV.

The distributions of the branching ratios for the quasi-two-body decays $\bar{B}^0 \rightarrow D^{\ast+}\pi^- \rightarrow D^0\pi^+\pi^-$ and $\bar{B}^0 \rightarrow D^{\ast+}K^- \rightarrow D^0\pi^+\pi^-$ in the $D\pi$ pair invariant mass $m_{D\pi}$ (equals to $\sqrt{s}$) are shown in Fig. 2. These diagrams reveal that the main portion of the values of Eq. (14) and Eq. (15) concentrate in a very small region of the $D^0\pi^\pm$ invariant mass. Take $\bar{B}^0 \rightarrow D^{\ast+}K^- \rightarrow D^0\pi^+\pi^-$ as an example, we have {92.7%, 92.9%, 93.6%} of its quasi-two-body branching ratio in the $m_{D\pi^\pm}$ region ($m_{D\pi^+} - \delta_m, m_{D\pi^+} + \delta_m$) when $\delta_m = \{2.5, 3.5, 5.0\}$ MeV. The distinct feature of the diagrams with two very sharp peaks located at the mass of $D^{\ast+}$ in Fig. 2 is dominated by the tiny decay width $\Gamma(D^{\ast+}) = 83.3$ keV [17, 18] for $D^{\ast+}$ and the $D^0\pi^\mp$ threshold which is so close to $m_{D^{\ast+}}$. These two points result in dramatic difference of the curves when comparing with the differential branching fractions predicted in Ref. [82] for the $B \rightarrow D_s^0(2400)h \rightarrow Ds\pi$ decays with the broad resonant state. The differential branching fractions for the $B^- \rightarrow D^{*0}\pi^- \rightarrow D^+\pi^-\pi^-$ and $B^- \rightarrow D^{*0}K^- \rightarrow D^+\pi^-\pi^-$ are shown in Fig. 3. The $d\mathcal{B}/dE_{D\pi}$ values at the point $m_{D\pi} = 3.5$ GeV are about 5% of the values at $m_{D\pi} = 2.1$ GeV for both the decays $B^- \rightarrow D^{*0}\pi^- \rightarrow D^+\pi^-\pi^-$ and $B^- \rightarrow D^{*0}K^- \rightarrow D^+\pi^-\pi^-K^-$. 

The comparison of the predicted virtual contributions with the experimental measurements are presented in the Table III. The theoretical errors are added in quadrature. For the $\bar{B}^0 \rightarrow D^{*+}\pi^- \rightarrow D^0\pi^+\pi^-$ decay, the branching fraction of the two-body subprocess $\bar{B}^0 \rightarrow D^{\ast+}\pi^-$ was found to be $(2.22 \pm 0.04 \pm 0.19) \times 10^{-3}$, which is about
87% of the Eq. (12) and 81% of the corresponding data in [99] for the central value, in the $m_{D\pi^+}$ region within 3 MeV of the nominal $D^+ - D^0$ mass difference in Ref. [15] by Belle Collaboration, and the relevant quasi-two-body virtual contribution is $(0.88 \pm 0.13) \times 10^{-4}$. In the Ref. [16] for the same decay process, with $m_{D\pi^+} > 2.1$ GeV, the $D^0\pi^+P$-wave contribution is $(9.21 \pm 0.56 \pm 0.24 \pm 1.73\%$ (isobar model) and $(9.22 \pm 0.58 \pm 0.67 \pm 0.75\% (K$-matrix model) of the total branching fraction $(8.46 \pm 0.14 \pm 0.29 \pm 0.40) \times 10^{-4}$, that is about $0.78 \times 10^{-4}$ as shown in Table III which is close to the PQCD prediction. For the decay $B^0 \rightarrow D^{*+}K^- \rightarrow D^0\pi^+K^-$, in Ref. [18], within 2.5 MeV of the $D^* - D^0$ mass difference to remove background containing $D^{*+} \rightarrow D^0\pi^+$, the virtual contribution was treated as part of the background with $D\piP$-wave nonresonant contributions as a result of $(0.81 \pm 0.15 \pm 0.27 \pm 0.09) \times 10^{-5}$ which is slightly larger than the corresponding result in Table III. For the $B^- \rightarrow D^{*0}\pi^- \rightarrow D^+\pi^-\pi^-$ decay, the Belle Collaboration provided $(2.23 \pm 0.32) \times 10^{-4}$ for the branching fraction of the virtual contribution in [14] with the parameterization $F(q) = \exp(-r_{BW}(q - q_0))$ for $D^{*0} \rightarrow D^+$ form factor. The same form factor was adopted for the same decay process in [16] by BaBar Collaboration, while $(10.1 \pm 1.4)\%$ of the total branching fraction $(1.08 \pm 0.03) \times 10^{-3}$, about $1.09 \times 10^{-4}$, was obtained for the same virtual contribution, which is about half of the corresponding result in Table III. In Ref. [21], LHCb presented the experimental result $(1.09 \pm 0.07 \pm 0.24 \pm 0.07) \times 10^{-4}$ for the same virtual contribution. As for the $B^- \rightarrow D^{*0}K^- \rightarrow D^+\pi^-K^-$ decay, LHCb presented the experimental result $(5.6 \pm 1.7 \pm 1.0 \pm 1.1 \pm 0.4) \times 10^{-6}$ in Ref. [17] for the virtual contribution which is only about $1/3$ of the corresponding PQCD prediction.

For the quasi-two-body processes $B^0 \rightarrow D^{*+}K^- \rightarrow D^0\pi^+K^-$ and $B^0 \rightarrow D^{*+}\pi^- \rightarrow D^0\pi^+\pi^-$, we have an identical step $D^{*+} \rightarrow D^0\pi^+$, the difference of these two decay modes originated from the bachelor particles pion and kaon. Assuming factorization and flavor-$SU(3)$ symmetry, one has the ratio $R_{D^{*+}}$ for the branching fractions of these two processes as

$$R_{D^{*+}} = \frac{B(B^0 \rightarrow D^{*+}K^- \rightarrow D^0\pi^+K^-)}{B(B^0 \rightarrow D^{*+}\pi^- \rightarrow D^0\pi^+\pi^-)} \approx \left| \frac{V_{us}}{V_{ud}} \right|^2 \frac{f_{K^+}}{f_{\pi^+}}$$

(14)

With the result

$$\left| \frac{V_{us}}{V_{ud}} \right| \frac{f_{K^+}}{f_{\pi^+}} = 0.276$$

(15)

in Review of Particle Physics [99], one has $R_{D^{*+}} \approx 0.076$. The PQCD predicted branching ratios provide

$$R_{D^{*+}} = 0.081^{+0.000\,(\omega_B)^{+0.001\,(a_{D\pi})}}{0.000\,(\omega_B)^{-0.000\,(a_{D\pi})}}.$$

(16)

It’s clear that the breaking effects of the flavor-$SU(3)$ symmetry is quite small for $R_{D^{*+}}$. The small errors induced by the uncertainties of $\omega_B$ and $a_{D\pi}$ for $R_{D^{*+}}$ are caused by the cancellation, which means the increase or decrease for the values of these parameters will result in nearly identical change of the weight at the same direction for the branching ratios of these two decays. And the errors of $R_{D^{*+}}$ come from the $f_{D^+}$, Wolfenstein parameter $A$ and $\omega_{D\pi}$ are zeros for the same reason. The result of Eq. (16) is consistent with the data $(7.6 \pm 0.34 \pm 0.29)\%$ presented by BaBar [102] and $(0.074 \pm 0.015 \pm 0.006)$ announced by Belle [103]. The energy dependent $R_{D^{*+}}$ is shown as the left diagram in Fig. A similar ratio $R_{D^{*0}}$, which has the definition as

$$R_{D^{*0}} = \frac{B(B^- \rightarrow D^{*0}K^- \rightarrow D^+\pi^-K^-)}{B(B^- \rightarrow D^{*0}\pi^- \rightarrow D^+\pi^-\pi^-)} \approx \left| \frac{V_{us}}{V_{ud}} \right|^2 \frac{f_{K^+}}{f_{\pi^+}},$$

(17)

for the quasi-two-body decays $B^- \rightarrow D^{*0}K^- \rightarrow D^+\pi^-K^-$ and $B^- \rightarrow D^{*0}\pi^- \rightarrow D^+\pi^-\pi^-$ is shown as the right diagram of Fig. in the $m_{D\pi}$ region $(2.03 \sim 3.50)$ GeV. An interesting conclusion could be made from the $R_{D^-}$ lines in Fig. is that the flavor-$SU(3)$ symmetry will be maintained at any physical point of the invariant mass $m_{D\pi}$ for the concerned quasi-two-body decay $B \rightarrow D^+h \rightarrow D\pi h$ decays. The ratio between the branching fractions of the two-body decays $B^- \rightarrow D^{*0}K^-$ and $B^0 \rightarrow D^{*0}\pi^-$ were measured to be $(7.930 \pm 0.110(stat) \pm 0.560(syst)) \times 10^{-4}$ by LHCb [104] and $0.0813 \pm 0.0040(stat) \pm 0.0031(syst)$ at BaBar [103], which are close to the result

$$R_{D^{*0}} = 0.077^{+0.000\,(\omega_B)^{+0.000\,(a_{D\pi})}}{0.000\,(\omega_B)^{-0.000\,(a_{D\pi})}}.$$

(18)

in the region $(2.1 \sim 3.5)$ GeV deduced from the results in Table III.
quasi-two-body branching ratios. The $\Gamma_D$ and $\Gamma_D^*$ subprocess.

Although there is no direct measurement for the $\Gamma_D$, we have theoretical results $58$ keV $[106]$, $59.6 \pm 1.2$ keV $[107]$, $55.9 \pm 1.6$ keV $[108]$ and $53 \pm 5 \pm 7$ keV $[109]$. If we replace the decay width $2.1$ MeV with $\Gamma_D = 53$ keV $[109]$ and considering the factor $\mathcal{B}(D^0 \rightarrow D^0\pi^0) = 64.7\%$ $[99]$, we have

$$\mathcal{B}(B^- \rightarrow D^{*0}\pi^-) = (5.03^{+2.28}_{-1.44}(\sqrt{s_B})^{-0.43}(f_{D^*}^{+0.31})^{+0.31}_{-0.23}(a_{D\pi}) \pm 0.18(A)^{+0.18}_{-0.28}(\omega_D)) \times 10^{-3},$$

$$\mathcal{B}(B^- \rightarrow D^0K^-) = (3.99^{+1.75}_{-1.21}(\sqrt{s_B})^{+0.36}_{-0.34}(f_{D^*}^{+0.17})^{+0.17}_{-0.24}(a_{D\pi}) \pm 0.14(A)^{+0.09}_{-0.07}(\omega_D)) \times 10^{-4},$$

as the two-body branching ratios, which are consistent with the data $\mathcal{B}(B^- \rightarrow D^{*0}\pi^-) = (4.90 \pm 0.17) \times 10^{-3}$, $\mathcal{B}(B^- \rightarrow D^0K^-) = 3.97^{+0.31}_{-0.28} \times 10^{-4}$ $[99]$, $[104]$, $[110]$. The virtual contributions with $\Gamma_D = 53$ keV $[109]$ are

$$\mathcal{B}_v(B^- \rightarrow D^{*0}\pi^- \rightarrow D^0\pi^0\pi^-) = (0.99^{+0.45}_{-0.30}(\omega_B) \pm 0.09(f_{D^*}^{+0.06})^{+0.06}_{-0.05}(a_{D\pi}) \pm 0.04(A)^{+0.02}_{-0.01}(\omega_D)) \times 10^{-4},$$

$$\mathcal{B}_v(B^- \rightarrow D^0K^- \rightarrow D^0\pi^0K^-) = (0.77^{+0.34}_{-0.24}(\omega_B) \pm 0.07(f_{D^0}^{+0.04})^{+0.04}_{-0.03}(a_{D\pi}) \pm 0.03(A)^{+0.02}_{-0.01}(\omega_D)) \times 10^{-5},$$

in the region $m_{D^{*0}\pi^-} > 2.1$ GeV. The percentage of the virtual contributions are all about $3\%$ of the corresponding quasi-two-body branching ratios. The $\Gamma_{D^*}$ dependent branching ratios of $B^- \rightarrow D^{*0}\pi^-$ and $B^- \rightarrow D^0K^-$ with the
subprocess $D^{*0} \to D^0\pi^0$ are shown in Fig. 5: the dash-dot curves are the PQCD predictions, the blue lines and the gray bands are the data with their errors from [99]. The branching ratios in Fig. 5 can be exploited to constrain the $D^{*0}$ decay width which could be read as $\Gamma_{D^{*0}} \approx 53$ keV from these two diagrams. The detailed discussion including the impacts of different parameter uncertainties about $D^{*0}$ decay width in the three-body hadronic $B$ meson decays shall be left for the future study. It must be pointed out that, the changes are tiny for the two virtual contributions involving $D^{*0}$ in the Table II when we adopt 53 keV for $\Gamma_{D^{*0}}$, the reason is that the $|m_{D}, \Gamma(s)|$ shall be less than $1/10^4$ of $|m_{D}, s|$ even if $\Gamma_{D^{*0}}$ equals to 2.1 MeV when $m_{D}^{2} - s$ is larger than 2.1 GeV. The small branching fractions for the two-body decays $B^{-} \to D^{*0}\pi^{-}$ and $B^{-} \to D^{*0}K^{-}$ with the subprocess $D^{*0} \to D^0\pi^0$ in this work with $\Gamma_{D^{*0}} = 2.1$ MeV is caused by the insufficient contributions in the $m_{D^{*0}} > 2.1$ GeV.

IV. CONCLUSION

In this work, we studied the quasi-two-body decays $B \to D^{*}h \to D\pi h$ and focused on the virtual contributions originated from off-shell $D^{*}(2007)^0$ and $D^{*}(2010)^{\pm}$ in the decays of $B^{-} \to D^{*+}\pi^{-}\pi^{-}$, $B^{-} \to D^{*+}\pi^{-}K^{-}$, $\bar{B}^{0} \to D^{0}\pi^{+}\pi^{-}$ and $\bar{B}^{0} \to D^{0}\pi^{+}K^{-}$ which have been measured by Belle, BaBar and LHCb Collaborations. For the $\bar{B}^{0} \to D^{*+}\pi^{-} \to D^{0}\pi^{+}\pi^{-}$ and $\bar{B}^{0} \to D^{*+}K^{-} \to D^{0}\pi^{+}K^{-}$ decays, we found that the main portions of their quasi-two-body branching fractions concentrate in a very small region of the $D^{0}\pi^{+}$ invariant mass, the percentage is larger than 90% for the branching ratios in the realm of 2.5 MeV around the pole mass of $D^{*+}$. And the virtual contributions from $D^{*+}$, in the region of $m_{D^{0}\pi^{+}} > 2.1$ GeV, are about 5% of the integrated values for the corresponding quasi-two-body results by considering $B(D^{*+} \to D^{0}\pi^{+}) = 67.7\%$. The virtual contributions in this work for $B^{-} \to D^{0}\pi^{-} \to D^{*+}\pi^{-}\pi^{-}$ and $B^{-} \to D^{0}\pi^{-} \to D^{*+}\pi^{-}K^{-}$ decay modes were found to be 3.9% and 3.7% of the two-body data for $B^{-} \to D^{0}\pi^{-}$ and $B^{-} \to D^{0}K^{-}$ in Review of Particle Physics, respectively.

From the ratios $R_{D^{*+}}$ and $R_{D^{*0}}$ defined between the quasi-two-body decays including $D^{*+}$ and $D^{*0}$ as the intermediate states, respectively, we concluded that the flavor-$SU(3)$ symmetry will be maintained with very small breaking at any physical value of the invariant mass $m_{D\pi}$ for the concerned $B \to D^{*}h \to D\pi h$ decays. We found that the decays $B^{-} \to D^{*0}\pi^{-} \to D^{0}\pi^{0}\pi^{-}$ and $B^{-} \to D^{*0}K^{-} \to D^{0}\pi^{0}K^{-}$ have strong dependence on the $D^{*0}$ decay width for their branching fractions which could be employed as a constraint for $\Gamma_{D^{*0}}$. 2.1 MeV for $D^{*0}$ decay width will make the branching ratios of the quasi-two-body decays $B^{-} \to D^{*0}\pi^{-} \to D^{0}\pi^{0}\pi^{-}$ and $B^{-} \to D^{*0}K^{-} \to D^{0}\pi^{0}K^{-}$ be highly underestimated because of the insufficient contributions in the $m_{D^{0}\pi^{0}}$ region near the $D^{*0}$ pole mass. With $\Gamma_{D^{*0}} = 53$ keV, we predicted

$$B_v(B^{-} \to D^{*0}\pi^{-} \to D^{0}\pi^{0}\pi^{-}) = (0.99_{-0.13}^{+0.05}(\omega_B) \pm 0.09(f_{D^*})_{-0.08}^{+0.06}(a_{D\pi}) \pm 0.04(A)_{-0.01}^{+0.02}(\omega_{D\pi})) \times 10^{-4}, \quad (23)$$

$$B_v(B^{-} \to D^{*0}K^{-} \to D^{0}\pi^{0}K^{-}) = (0.77_{-0.21}^{+0.34}(\omega_B) \pm 0.07(f_{D^*})_{-0.05}^{+0.04}(a_{D\pi}) \pm 0.03(A)_{-0.01}^{+0.01}(\omega_{D\pi})) \times 10^{-5}, \quad (24)$$

as the virtual contributions, which are about 3% of the corresponding quasi-two-body results.

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Appendix A: DECAY AMPLITUDES

The concerned quasi-two-body decay amplitudes are given, in the PQCD approach, by

$$A(B^0 \rightarrow \pi^- [D^*(2010)^+] \rightarrow D^0 \pi^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[ \left( \frac{C_1}{3} + c_2 \right) F_{TD^*} + c_1 M_{TD^*} + \left( c_1 + \frac{C_2}{3} \right) F_{\pi^*} + c_2 M_{\pi^*} \right], \quad (A1)$$

$$A(\bar{B}^0 \rightarrow K^- [D^*(2010)^+] \rightarrow D^0 \pi^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* \left[ \left( \frac{C_1}{3} + c_2 \right) F_{TD^*} + c_1 M_{TD^*} \right], \quad (A2)$$

$$A(B^- \rightarrow \pi^- [D^*(2007)^0] \rightarrow D^+ \pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[ \left( \frac{C_1}{3} + c_2 \right) F_{TD^*} + c_1 M_{TD^*} + \left( c_1 + \frac{C_2}{3} \right) F_{\pi^*} + c_2 M_{\pi^*} \right], \quad (A3)$$

$$A(B^- \rightarrow K^- [D^*(2007)^0] \rightarrow D^+ \pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* \left[ \left( \frac{C_1}{3} + c_2 \right) F_{TD^*} + c_1 M_{TD^*} + \left( c_1 + \frac{C_2}{3} \right) F_{\pi^*} + c_2 M_{\pi^*} \right], \quad (A4)$$

$$A(\bar{B}^- \rightarrow \pi^- [D^*(2007)^0] \rightarrow D^0 \pi^0) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[ \left( \frac{C_1}{3} + c_2 \right) F_{TD^*} + c_1 M_{TD^*} + \left( c_1 + \frac{C_2}{3} \right) F_{\pi^*} + c_2 M_{\pi^*} \right], \quad (A5)$$

$$A(\bar{B}^- \rightarrow K^- [D^*(2007)^0] \rightarrow D^0 \pi^0) = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* \left[ \left( \frac{C_1}{3} + c_2 \right) F_{TD^*} + c_1 M_{TD^*} + \left( c_1 + \frac{C_2}{3} \right) F_{\pi^*} + c_2 M_{\pi^*} \right], \quad (A6)$$

in which $G_F$ is the Fermi coupling constant, $V$’s are the CKM matrix elements, the Wilson coefficients $c_1$ and $c_2$ will appear in convolutions in momentum fractions and impact parameters $b$.

The amplitudes from Fig.1 are written as

$$F_{TD^*} = 8\pi C_F m_B^2 f_{(K)}(\eta - 1) \int dx_B dz \int b_d b_d b_d b_d b_d b_d b_d b_d b_d \phi_{(K)}(z, b, s)$$

$$\times \left\{ \left[ \sqrt[N]{\eta(1 - 2z) + z + 1} \right] E^{(1)}_a(t^{(1)}_d) h_{(z, b, b, b)} + (\eta + r_c) E^{(2)}_a(t^{(2)}_d) h_{(z, b, b, b)} \right\}, \quad (A7)$$

$$M_{TD^*} = 32\pi C_F m_B^4 / \sqrt{2N_c}(\eta - 1) \int dx_B dz dx_3 \int b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d \phi^{A}(z, b, s)$$

$$\times \left\{ \left[ \left[ \left[ \eta(x - z - 1) + z\sqrt{\eta} - x_b - x_3 + 1 \right] E_0(t^{(1)}_b) h^{(1)}_b(x, b) \right] \right] + [x_3(\eta - 1) + (\sqrt{\eta} - 1) + x_B] E_b(t^{(2)}_b) h^{(2)}_b(x, b) \right\}, \quad (A8)$$

$$F_{\pi^*}(K) = 8\pi C_F m_B^2 f_{(K)}(\eta - 1) \int dx_B dz dx_3 \int b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d \phi_{(K)}(z, b, s)$$

$$\times \left\{ \left[ \phi^{A}(\eta - 1)[x_3(\eta - 1) - 1] + r_0[\phi^{P}(\eta - 1)2x_3 - 1] + \phi^{T}(x_3(\eta - 1) + 1)] E^{(1)}_c(t^{(1)}_d) \right] h_{(z, b, b)}$$

$$\times [x_3(\eta - 1) - 1] + (\sqrt{\eta} - 1) + x_B] E_b(t^{(2)}_b) h^{(2)}_b(x, b) \right\}, \quad (A9)$$

$$M_{\pi^*}(K) = 32\pi C_F m_B^4 / \sqrt{2N_c}(\eta - 1) \int dx_B dz dx_3 \int b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d \phi_{(K)}(z, b, s)$$

$$\times \left\{ \left[ \phi^{A}(\eta - 1)[x_3(\eta - 1) - 1] + r_0[\phi^{P}(\eta - 1)2x_3 - 1] + \phi^{T}(x_3(\eta - 1) + 1)] E^{(1)}_c(t^{(1)}_d) \right] h_{(z, b, b)}$$

$$\times [x_3(\eta - 1) - 1] + (\sqrt{\eta} - 1) + x_B] E_b(t^{(2)}_b) h^{(2)}_b(x, b) \right\}, \quad (A10)$$

$$F_{\pi^*} = 8\pi C_F m_B^2 f_{B} \int dz dx_3 \int b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d \phi_{(K)}(z, b, s)$$

$$\times \left\{ \left[ \left[ ((1 - \eta))(1 - 1)(x_b + z - 1) + r_0[\phi^{P}(\eta - 1)2x_3 - 1] + \phi^{T}(x_3(\eta - 1) + 1)] E^{(1)}_c(t^{(1)}_d) \right] h_{(z, b, b)}$$

$$\times [x_3(\eta - 1) - 1] + (\sqrt{\eta} - 1) + x_B] E_b(t^{(2)}_b) h^{(2)}_b(x, b) \right\}, \quad (A11)$$

$$M_{\pi^*} = 32\pi C_F m_B^4 / \sqrt{2N_c}(\eta - 1) \int dx_B dz dx_3 \int b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d b_d \phi_{(K)}(z, b, s)$$

$$\times \left\{ \left[ \left[ ((1 - \eta))(1 - 1)(x_b + z - 1) - \eta \phi^{A} + r_0[\phi^{P}(\eta - 1)2x_3 - 1] + \phi^{T}(x_3(\eta - 1) + 1)] E^{(1)}_c(t^{(1)}_d) \right] h_{(z, b, b)}$$

$$\times [x_3(\eta - 1) - 1] + (\sqrt{\eta} - 1) + x_B] E_b(t^{(2)}_b) h^{(2)}_b(x, b) \right\}, \quad (A12)$$
The evolution factors in the above factorization formulas are given by
\[ E_a^{(1)}(t) = \alpha_s(t)\exp[-S_B(t) - S_C(t)]S_t(z), \quad E_a^{(2)}(t) = \alpha_s(t)\exp[-S_B(t) - S_C(t)]S_t(x_B), \quad (A13) \]
\[ E_b(t) = \alpha_s(t)\exp[-S_B(t) - S_C(t) - S_P(t)]|_{b=b_B}, \quad (A14) \]
\[ E_3^{(1)}(t) = \alpha_s(t)\exp[-S_B(t) - S_P(t)]S_t(x_3), \quad E_3^{(2)}(t) = \alpha_s(t)\exp[-S_B(t) - S_P(t)]S_t(x_B), \quad (A15) \]
\[ E_4(t) = \alpha_s(t)\exp[-S_B(t) - S_C(t) - S_P(t)]|_{b_3=b_B}, \quad (A16) \]
\[ E_5^{(1)}(t) = \alpha_s(t)\exp[-S_C(t) - S_P(t)]S_t(z), \quad E_5^{(2)}(t) = \alpha_s(t)\exp[-S_C(t) - S_P(t)]S_t(z), \quad (A17) \]
\[ E_6(t) = \alpha_s(t)\exp[-S_B(t) - S_C(t) - S_P(t)]|_{b_3=b_3}. \quad (A18) \]
in which \( S_{(B,C,P)}(t) \) are the Appendix of [42], the hard functions \( h, h_{(b,d,f)}^{(1,2)} \) and the hard scales \( t_{(c,b,i,d,a,f)}^{(1,2)} \) have their explicit expressions in the Ref. [42]. Because of the different definitions of the momenta for the initial and final states, the concerned expressions in [42] could be employed in this work only after the replacements \( \{x_1 \to x_B, b_1 \to b_B, x_2 \to z, b_2 \to h, x^2 \to \eta \} \). The parameter \( c \) in the Eq. (A1) of [42] is adopt to be 0.35 in this work according to the Refs. [111, 112].

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