Oversampling Effect Analysis of Correlation Metric in Selected Mapping Scheme with Nonlinearity

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Abstract—In this letter, oversampling effect is analyzed when correlation (CORR) metric is used in the selected mapping (SLM) scheme with the presence of nonlinearity. In general, 4 times oversampling is enough for estimating continuous signal. But, we can use 2 times oversampling with similar bit error rate (BER) performance. Therefore, we can reduce the computational complexity half roughly. Simulation results show that BER performance when 2 times oversampling is used is almost the same as 4 times oversampling is used. On the other hand, BER performance when 1 times oversampling (Nyquist rate) is used is degraded than 2 times oversampling or 4 times oversampling are used. By deriving the Pearson correlation coefficient, simulation results can be confirmed.

Index Terms—Bit error rate (BER), orthogonal frequency division multiplexing (OFDM), oversampling, selected mapping (SLM), solid state power amplifier (SSPA).

I. INTRODUCTION

F OR many years, orthogonal frequency division multiplexing (OFDM) has received enormous attention. Due to the fact that OFDM has high peak-to-average power ratio (PAPR), many papers tried to mitigate the PAPR problem. Partial transmit sequence (PTS) and selected mapping (SLM) can be a good answer to solve the PAPR problem [1]. PTS and SLM generates alternative OFDM signal sequences. Among them, the one with the minimum PAPR is selected for transmission. Since these schemes have high computational complexity, low complexity schemes are proposed. Besides, many other techniques such as clipping, tone reservation, and error rate (BER) performance when the presence of nonlinearity are also proposed.

Then, an oversampled input symbol sequence is analyzed when the presence of nonlinear HPA in SLM scheme. Simulation result is given in Section IV and Section V concludes this letter.

II. OVERVIEW OF SLM WITH CORR METRIC

Binary data sequences are modulated by M-ary quadrature amplitude modulation (QAM) constellation. Then, an oversampled input symbol sequence $X = [X_0, X_1, \cdots, X_{N-1}, 0, \cdots, 0]$ is inverse Fourier transformed (IFFT), where $N$ is the number of subcarriers. The $n$th time domain sample of OFDM signal sequence can be expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j \frac{2\pi kn}{L \cdot N}}$$

where LN is the IFFT size, L is the oversampling factor, and $n = 0, 1, \cdots, LN - 1$.

Fig. 1 shows a block diagram of SLM scheme with CORR metric. Conventional SLM scheme generates $U$ alternative OFDM signal sequences by componentwise multiplication of input symbol sequence and $U$ different phase sequences. Let $P^{(u)} = \{P_0^{(u)}, P_1^{(u)}, \cdots, P_{L-1}^{(u)}\}$ be the $u$th phase sequence and the elements are $P_k^{(u)} = e^{j \phi_k^{(u)}}$ where $\phi_k^{(u)} \in [0, 2\pi)$, $k = 0, 1, \cdots, N - 1$, and $u = 0, 1, \cdots, U - 1$. For implementation, we use $P_k^{(u)} = \pm 1$.

Each alternative sequence is $LN$-point IFFT$x^{(u)} = \{x_0^{(u)}, x_1^{(u)}, \cdots, x_{L-1}^{(u)}\}$ and passes through the solid state power amplifier (SSPA) model. Here, the polynomial model is used for SSPA model. The HPA polynomial model is expressed as a third order nonlinearity. Thus, $n$th element of the output of the HPA can be represented by

$$y_n \approx \alpha_1 x_n + \alpha_3 |x_n|^2$$

where $\alpha_1 = 1$ and $\alpha_3 = -0.1769$ are the polynomial coefficients [5]. Then, CORR calculate $U$ single point correlation between input $x^{(u)}$ and output $y^{(u)}$ of the SSPA model. The CORR computation is given as

$$P_{xy}^{(u)} = \sum_{n=0}^{LN-1} x_n^{(u)} y_n^{(u)} = \alpha_1 \sum_{n=0}^{LN-1} |x_n^{(u)}|^2 + \alpha_3 \sum_{n=0}^{LN-1} |x_n^{(u)}|^4$$

where $*$ indicates complex conjugation. Among $U$ $P_{xy}^{(u)}$, the signal with maximum CORR is selected for transmission. PTS scheme with CORR is treated in [5], but in this letter, we treat

![Fig. 1. A block diagram of SLM scheme with CORR metric.](image-url)
SLM scheme with CORR since SLM scheme is a general scheme.

In [8], analytic BER of SLM scheme is described when nonlinear HPA is presented.

\[ P_b = \frac{4}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3}{M - 1} \text{SDNR}} \right) \] (4)

According to [4], for better BER performance, SDNR should be large. To do so, \( E[x_n^{(u)}y_n^{(u)}] \) should be large and it is identical to CORR metric. Thus, CORR metric is the optimal in terms of BER performance. Thus, we focus on CORR metric among many metrics targeting the BER performance and describe the oversampling effect in SLM scheme with nonlinearity.

III. OVERSAMPLING EFFECT

Based on [7], oversampled signal can be achieved by linear combination of Nyquist rate samples. The impulse response of interpolator for the \( L \) times oversampling is defined as

\[ h_L[n] = \frac{\sin(\pi n/L)}{\pi n/L} \] (5)

where we use the limited filter length \( (L) \) in practice. Then, the output of the interpolator can be expressed as

\[ \tilde{x}_L[n_L] = \sum_{k=-(n_L-L)/L}^{(n_L+L)/L} x_1[k]h_L[n_L - Lk] \] (6)

where \( x_1[n_L] \) is \( n_L \)th element of \( L \) times oversampled signal \( x \). In this letter, we use \( I = 2 \) for analysis.

Let us consider 16 times oversampled signal because we can regard it as a continuous signal. Then, the interpolation matrix \( M = [M_1, M_2, \cdots, M_{16}] \) can be written as [7] at the top of the page 3. For example, \( \tilde{x}_{16}[16m+1] = h_{16}[17]x_{16}[16(m-1)] + h_{16}[1]x_{16}[16m] + h_{16}[16-15]x_{16}[16(m+1)] + h_{16}[16-31]x_{16}[16(m+2)] \), where \( 0 \leq m \leq N - 1 \).

We can only express 16 samples among \( 16N \) samples without loss of generality. Then, we can rewrite (3) as

\[ \tilde{R}^{(u)}_{xy} = \sum_{s=0}^{15} \tilde{x}[16m + s]\tilde{y}[16m + s]^* \] (8)

There are 16 terms and we assign alphabets, \((a)\) to \((p)\), to each terms, i.e., \((a) = \tilde{x}[16m + 1]^*(u)\tilde{y}[16m + 1]^*(s)\) and \((p) = \tilde{x}[16m + 15]^*(u)\tilde{y}[16m + 15]^*(s)\). For better understanding, let us consider only \((i)\) term and write it concretely as \((3)\) by \((4)\) and \((7)\). With same way, we can compute all the other terms.

Now, let us consider \( L \) times oversampling for computing CORR metric. We only need \((a)\) term for Nyquist rate sampling. \((a) + (i), (a) + (e) + (i) + (m)\), and \((a) + (b) + \cdots + (p)\) terms are needed to compute \((3)\) for 2 times, 4 times, and 16 times oversampling, respectively.

Table II shows the coefficients of the equations to compute CORR metric when \( L \) times oversampling is used. Indices are arranged in numerical order \((1-16)\). For example, in \((2)\), the coefficients of index 1, index 3, and index 6 are 0.04, -0.13, and 0.39, respectively. To compute the correlation between sequences derived from \((8)\) with different \( L = \{1, 2, 4, 16\} \), we only consider the coefficients of each equation.

Now, we use the Pearson correlation coefficient \((r)\). \( r \) is a measure of linear correlation between two sequences \((\text{variables})\) and is giving a value \([-1, 1]\), where 0 is no correlation and 1\((1-1)\) is total positive\((0-)\) negative correlation.

Pearson correlation coefficient between two sequences \((x, y)\) is defined as

\[ r = \frac{\sum_i(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i(x_i - \bar{x})^2}\sum_i(y_i - \bar{y})^2} \] (10)

where \( \bar{x} \) denotes the sample mean and \( x_i \) denotes the \( i \)th element of the sequence \( x \).

Table II shows the Pearson correlation coefficients between two sequences obtained from Table I when different \( L \) is used. We can regard when \( L = 16 \) is used as a continuous signal which can estimate the nonlinear effect of HPA well. When comparing \( L = 1 \) and \( L = 16 \), \( r \) is 0.6023, which has very low correlation. On the other hand, the value of \( r \) when comparing \( L = 2 \) & \( L = 16 \) and \( L = 4 \) & \( L = 16 \) are 0.9133 and 0.9839, respectively, which have very high correlation. Therefore, we can expect that 2 times and 4 times oversampling can be used instead of using 16 times oversampling.

IV. SIMULATION RESULTS

In this section, we derive the BER performance of SLM schemes with CORR metric when different \( L \) times oversampling are used. Also, nonlinear HPA is considered. There are four BER curves in Fig. 2. Original indicates OFDM system without SLM scheme.
(9)

\[ I = \tilde{x}[16m + 8]^* \tilde{y}[16m + 8]^* = (-0.21x[16m - 16] + 0.63x[16m] + 0.63x[16m + 16] - 0.21x[16 + 32]) \\
- (-0.21y[16m - 16] + 0.63y[16m] + 0.63y[16m + 16] - 0.21y[16 + 32])^* \\
- 0.4x[16m - 16]y^*[16m - 16] - 0.33x[16m - 16]y^*[16m] - 0.33x[16m - 16]y^*[16m + 16] + 0.4x[16m - 16]y^*[16m + 32] \\
- 0.13x[16m]y^*[16m - 16] + 0.39x[16m]y^*[16m] + 0.39x[16m]y^*[16m + 16] - 0.13x[16m + 16]y^*[16m + 32] \\
- 0.13x[16m + 32]y^*[16m - 16] - 0.33x[16m + 32]y^*[16m] - 0.33x[16m + 32]y^*[16m + 16] + 0.4x[16m + 32]y^*[16m + 32]. \]

\[ (h_{16}^0 \mid h_{16}^1 \mid h_{16}^2 \mid h_{16}^3 \mid h_{16}^4 \mid h_{16}^5 \mid h_{16}^6 \mid h_{16}^7 \mid h_{16}^8 \mid h_{16}^9 \mid h_{16}^{10} \mid h_{16}^{11} \mid h_{16}^{12} \mid h_{16}^{13} \mid h_{16}^{14} \mid h_{16}^{15} \mid h_{16}^{16} \mid h_{16}^{17}) \]

V. Conclusion

References

[1] S. H. Müller, R. W. Bäuml, R. F. H. Fischer, and J. B. Höber, “OFDM with reduced peak-to-average power ratio by multiple signal representation,” Ann. Telecommun., vol. 52, no. 1–2, pp. 58–67, Feb. 1997.

[2] M. R. D. Rodrigues and I. J. Wassell, “IMD reduction with SLM and PTS to improve the error-probability performance of nonlinearly distorted OFDM signals,” IEEE Trans. Veh. Technol., vol. 55, pp. 537–548, Mar. 2006.

[3] E. Al-Dalakta, A. Al-Dweik, A. Hazmi, C. Tsimenidis, and B. Sharif, “Efficient BER reduction technique for nonlinear OFDM transmission using distortion prediction,” IEEE Trans. Veh. Technol., vol. 61, pp. 2330–2336, Jun. 2012.

[4] D. Park and H. Song, “A new PAPR reduction technique of OFDM system with nonlinear high power amplifier,” IEEE Trans. Consum. Electron., vol. 53, pp. 327–332, May 2007.

[5] E. Al-Dalakta, A. Al-Dweik, A. Hazmi, C. Tsimenidis, and B. Sharif, “PAPR reduction scheme using maximum cross correlation,” IEEE Commun. Lett., vol. 16, no. 12, pp. 2032–2035, Dec. 2012.

[6] K. Bae, C. Shin, and E. J. Powers, “Performance analysis of OFDM systems with selected mapping in the presence of nonlinearity,” IEEE Trans. Wireless Commun., vol. 12, no. 5, pp. 2314–2322, MAY 2013.

[7] C.-L. Wang, S.-J. Ku, and C.-J. Yang, “A low-complexity PAPR estimation scheme for OFDM signals and its application to SLM-based PAPR reduction,” IEEE J. Sel. Topics Signal Process., vol. 4, no. 3, pp. 637–645, Jun. 2010.

[8] C. Tellambura, “Computation of the continuous-time PAR of an OFDM signal with BPSK subcarriers,” IEEE Commun. Lett., vol. 5, pp. 185–187, May 2001.

[9] M. Sharif, M. Gharavi-Alkhansari, and B. H. Khalaj, “On the peak-to-average power of OFDM signals based on oversampling,” IEEE Trans. Commun., vol. 51, no. 1, pp. 72–78, Jan. 2003.

\[ \text{TABLE I} \]

COEFFICIENTS SET OF THE EQUATIONS TO COMPUTE CORR METRIC WHEN L TIMES OVERSAMPLING IS USED

| Index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| L = 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| L = 2 | 0.04 | −0.13 | −0.13 | 0.04 | −0.13 | 1.4 | 0.4 | −0.13 | −0.13 | 0.4 | 0.4 | −0.13 | −0.13 | 0.4 | −0.13 | −0.13 |
| L = 4 | 0.09 | −0.33 | −0.30 | 0.09 | −0.33 | 2.3 | 0.94 | −0.3 | −0.3 | 0.94 | 1.3 | −0.33 | 0.09 | −0.3 | −0.33 | 0.09 |
| L = 16 | 0.37 | −1.42 | −1.25 | 0.36 | −1.42 | 7.72 | 3.94 | −1.25 | −1.25 | 3.94 | 6.72 | −1.42 | 0.36 | −1.25 | −1.42 | 0.37 |