A mathematical model for solar cookstoves: the relation between thermal and optical parameters

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Abstract. The article presents a mathematical model that relates the thermal properties of a concentration solar cooker with some optical properties of several of the materials used in its elaboration. Solar cookers are studied from the point of view of its functionality through their thermal performance, in terms of calculating the standard cooking power and thermal efficiency with field tests. Some of the materials characteristics that are important to make concentrating solar stoves are: the reflectance of the surface of the concentrator, the transmittance of the cover and the absorbance of the surface of the food container. Through an analysis of heat transfer, it is built a model for calculating thermal parameters depending on the optical parameters. A comparison of the model is made by varying the absorbance and reflectance of the coating that covers the food container. The results of the model agree with those found experimentally.

1. Introduction

Currently, harnessing the Solar Energy is an activity made in worldwide to a greater or lesser degree, to transform radiation this (energy sources) into heat or electricity, that say, on thermal solar energy or photovoltaic energy. Generally, thermal solar energy is being used to heat water, cook food or transform it into another type, as mechanical, electrical energy, these are not the only current applications, but they are the most common ones [1, 2, 3]. In Mexico and other countries, for exploiting thermal radiation in rural settings are solar heaters and solar cookstoves [4, 5]. Solar cookstoves are devices that utilize the sun’s radiation to cook food, exist many types and designs of solar cookstoves are on the market worldwide [5, 6, 7, 8, 9]. The scientific communities of numerous countries have been implementing solar cookstoves with different purposes and objectives in mind, particularly is bring this technology in homes [4, 5, 10].

Exist records of utensils for cooking with the sun, from times long past, it was around the mid-20th century that they began to enjoy their greatest development. So were necessary a protocols for evaluation of solar cooking and on 1997 the Test Standards Committee at the Third World Conference on Solar Cooking where provided protocols [11], generally evaluations of solar cookstoves are based on analyses of their thermal properties [11].

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The procedures dictated by these thermal evaluation protocols are exhaustive in terms of the meteorological characteristics that must be satisfied in order for the data to be considered suitable for analysis [11, 12, 13]. Evaluating solar cookstoves experimentally in the field is an arduous task, researchers have developed alternatives based on procedures or methods distinct; such is the case of mathematical models. Mathematical models can be entered into mathematical terms through functions that represent the physical phenomena that occur during the operation of the devices. Functions can contain thermal, optical parameters or devices materials properties and characteristics; too can take into account such aspects as the design. The literature include the development of mathematical models for both concentration and box-type solar cookers [14, 15, 16, 17], too various kinds of mathematical models including semi-empirical or simulation approaches, among others [18, 15, 19, 20]. For example, El- Sebaii, Dománski and Jaworski demonstrated a mathematical model together to show the efficiency and temperature distribution using a box-type solar cooker. Funk and Larson meanwhile, presented a parametric model of the operation of a solar cooker based on controlled parameters and uncontrolled variables. Thulasi Das, Karmakar and Rao found a mathematical model for a box-type solar cooker that helped establish the aspects of the parameters that significantly influence the process of heating a cooker[15, 18, 21, 22]. Now to develop the model presented below, considered a solar stove designed with Compound Parabolic Concentrators (CPC) [23], see Figure (1).

The collector is a CPC type concentrator of revolution Figure (2), it is made with reflective sheets characterized by a certain reflectance, that is, the radiation that is reflected to the absorber between the total radiation incident on the surface and the cover to reduce heat losses by convection, characterized by transmittance, that is, the percentage of solar radiation that reaches the collector [23]. The food container has a black coating on its surface, characterized by a certain absorbance Figure (3). Some of the most significant parameters for analyzing the performance of solar cookers it are by calculating the standard cooking power and thermal efficiency, which are made by applying established test protocols to be done in the field.

The thermal model provides a functional relationship between the thermal parameters, cooking power and thermal efficiency, in terms of reflectance, transmittance and absorbance. To test the model corresponding comparisons were made considering the variation of some of the optical parameters of tests done in the field for obtaining thermal parameters.
2. Materials and Model
The methodology was elaborated following the development of the model that relates the thermal data for standard cooking power of a solar cookstove obtained experimentally in the field with the absorbance of the coating applied to the container of the solar cookstove. Standard cooking power is represented graphically by a linear regression line as a function of the temperature difference of the liquid inside the cooking pot where the food is heated at each time interval, as stipulated in the ASAE S580 protocol.

2.1. Description of the model
To conduct the experimental protocol, the cooking pot of the solar cookstove (Figure 3) where the food is placed was filled with water and placed in the concentration region of the CPC. Collector CPC has a plastic cover, this serves to preserve the temperature inside the solar stove, because it protects it from the cold currents of the wind. The model is supported on an energy balance between energy from the Sun that enters the CPC, with the energy used to heat the water. The model is based on an energy balance between energy since the Sun that enters the CPC with the energy used to heat the water. The energy that makes the water heat comes from different directions; directly from the sun's rays, from the reflectors that reflect a part of the sun's rays towards the pot, due to the design of the CPC.

![Figure 3. Solar cookstove setup.](image)

The equation for the energy balance of the water inside the solar cookstove’s cooking pot can be expressed, in general, with the following expression [24]:

$$m_w C_w \frac{dT_w}{dt} = F\left[I - AU_L(T_w - T_a)\right]$$  \hspace{1cm} (1)

Where:

- $m_w$ is the mass of the water in the cooking pot (kg)
- $C_w$ is the specific heat of water
- $T_w$ is the water temperature (°C)
- $T_a$ is the ambient temperature (°C)
- $F$ is the heat exchange factor between the pot and the water
- $A$ is the surface of the cooking pot (m$^2$)
- $U_L$ is the total coefficient of heat loss ($\frac{W}{m^2K}$)
- $I$ is the total radiation that impacts the pot ($\frac{W}{m^2}$)

Total radiation ($I$) is the sum of direct plus concentrated radiation, as described in the following expression:

$$I = \tau a I_b + \tau a \left(\frac{A_x}{A}\right) \rho^n I_b$$  \hspace{1cm} (2)
Where:

\( \alpha \) is the absorbance of the coating on the surface of the absorbing pot
\( I_b \) is the direct incident radiation
\( T \) is the transmittance of the plastic cover
\( A_c \) is the area of the aperture of the solar collector (CPC)
\( \rho \) is the reflectance of the mirrors on the collector
\( n \) is the average number of reflections on the CPC

\[
P_c = FA \left( \tau \alpha I_b + \tau a \left( \frac{A_c}{A} \right) \rho^n I_b - U_l \Delta T \right)
\]  
(3)

The left side of the equation (1) represents the cooking power \( (P_c) \) which can be rewritten by multiplying it by the normalization factor, \( \left( \frac{l_o}{I_p} \right) \) where \( l_o \) is the radiation used to standardize the cooking power heat, according to the ASAE protocol, has the value of 700 W/m². So:

\[
P_s = P_c \left( \frac{l_o}{I_p} \right) = \left( m_{cw} \frac{dT}{dt} \right) \left( \frac{l_o}{I_p} \right)
\]  
(4)

Too, the standard cooking power can be re-written in terms of the temperature difference \( \Delta T = T_w - T_a \).

So, substituting equation (3) into equation (4) we have to:

\[
P_s(\Delta T) = \left( \frac{l_o}{I_p} \right) FA \left( \tau \alpha I_b + \tau a \left( \frac{A_c}{A} \right) \rho^n I_b - U_l \Delta T \right)
\]  
(5)

According to the ASAE 580 protocol, standard cooking power should be presented in a graph form that displays the experimental data for this parameter as a function of the temperature difference that exists between the water and the ambient temperatures \( \Delta T \) point-by-point. A straight line is then adjusted by means of the linear regression:

\[
P_s(\Delta T) = b - m\Delta T
\]  
(6)

This is to say that standard cooking power is represented by a straight line, where is the slope of the line and \( b \) its intercept. This suggests, directly, that the left side of the equation (5) can be considered, as a first approximation, as being linear in terms of \( \Delta T \). Recent studies on this stove showed the cooking powers by varying the coating on the container, the soot-based coatings and black paint.

Using the results for each case, were calculated the on the slopes of the linear approximation that is used to determine standard cooking power and intercepts to have functions like equation (6) [23]. Rewriting equation (2), the total radiation that impacts the cooking pot is re-written as follows:

\[
I(\alpha) = \tau \alpha I_b \left[ 1 + \left( \frac{A_c}{A} \right) \rho^n \right]
\]  
(7)

Now, re-writing equation (6) with equation (7), the standard cooking power is expressed as:

\[
P_s(\Delta T) = \left( \frac{l_o}{I_p} \right) FA \left\{ \tau I_b \left[ 1 + \left( \frac{A_c}{A} \right) \rho^n \right] - U_l \Delta T \right\}
\]  
(8)

This operation reveals a linear relation between the cooking power and the absorbance of the surface of the cooking pot that contains, in this case, water. In addition, the total coefficient of heat loss, \( U_l \), and the heat exchange factor between the liquid and the cooking pot, \( F \), can be determined in terms of the slope and intercept of the straight line from the linear regression. Thus, the coefficients of equations (6) and (8) are equal. So the slope equation \( (m) \) and the point of intersection \( (b) \) remain as:
These operations represent a two-equation system with two unknowns; namely, the total coefficient of heat loss \((UL)\), and the heat exchange factor \((F)\). To calculate the thermal efficiency in terms of optical parameters, it proceeds by applying the first and second law of thermodynamics:

\[
\eta = \frac{Q_u}{E_t}
\]  

(11)

Where:

\(Q_u\) is energy that was needed to heat the water (90ºC)
\(E_t\) is energy entering through the top surface of the collector

Rewriting \(Q_u\) by cooking power and \(E_t\) by energy entering through the top surface of the collector, into the equation (11) we have to:

\[
\eta = \frac{P_c}{I_bA}
\]  

(12)

Now, substituting \(P_c\) into equation (12), the above equation is as:

\[
\eta = F\tau \alpha \left[ 1 + \left( \frac{\Delta T}{A} \right) \rho^n \right] \frac{UL}{I_b}(T_w - T_a)
\]  

(13)

Thus, equation (3) and (13) respectively, are written in terms of optical parameters: \(P_c = P_c(\alpha, \rho, \tau)\); \(\eta = \eta(\alpha, \rho, \tau)\).

This means that knowing the values of the optical parameters, absorbance of the surface of the absorber, the reflectance of the reflective surface of the collector and transmittance of the cover, the cooking power and thermal efficiency values of the solar cooker can be roughly calculated, without requiring laborious standard field-testing protocols.

Equations (3) and (13) are the instantaneous cooking power and thermal efficiency. In the case of standard cooking power, it is evaluated at a temperature difference between water and the environment of 50 °C. For thermal efficiency average applying theorem of the mean value for integrals:

\[
\bar{\eta} = \frac{\int_{20^\circ C}^{70^\circ C} \eta d(\Delta T)}{50^\circ C}
\]  

(14)

The limits of integration correspond to the difference in water temperatures, at the beginning (20ºC equal at temperature environment) and at the end (90ºC) of the experimentation. So, integrating and evaluating equation (14), we have:

\[
\bar{\eta} = F\tau \alpha A \left[ 1 + \left( \frac{\Delta T}{A} \right) \rho^n \right] - \frac{70\, FUL}{2I_b}
\]  

(15)

The expression (15) is obtained for the average thermal efficiency of solar stove, considering the temperature difference between the temperature water and the environment is 70 ° C.

3. Results Analysis

With mathematical modeling, it is observed how the equations of the cooking power and the thermal performance can be had in terms of the optical parameters.

Thus, giving the corresponding values of the optical properties it possible to determine, the total coefficients
of heat loss and the heat exchange factors \((U_L,F)\), solving the system of equations (9) - (10) and substituting the optical properties \((\alpha, \rho, \tau)\). To test the model has been used different absorbance values, keeping fixed the values of the reflectance of the reflectors of the collector mirrors and transmittance of the cover. To this end they have been considered the optical parameters of coatings elaborated with three types of soot (from resin, sugarcane and traditional forest biomass soot) and temperature resistant black paint.

Table (1), shows the experimental data obtained in laboratory.

**Table 1.** Absorbance of different types of solar absorption coatings in the solar spectrum, obtained experimentally in the laboratory [23].

| Type of coating            | \(\alpha\) (Absorbance) |
|----------------------------|--------------------------|
| Resin                      | 0.977                    |
| Sugarcane                  | 0.974                    |
| Forest biomass soot        | 0.972                    |
| High-temperature black paint| 0.954                    |

In addition to the optical properties, for obtained \((U_L,F)\) it is necessary to know the surface area of the cooking container and the number of reflections: the area of the aperture of the solar collector (CPC) and the number of reflections on the CPC: The area of the cooking pot is approximately 0.2 m², considering the dimensions of the pressure cooking pot: diameter \(d = 20\) cm, height \(h = 20\) cm; the average number of reflections on the CPC is \(A_c = 0.5\) m²; and according to Duffie and Beckman (2013), the average number of reflections on the CPC, the value of \(n = 0.7\).

Using the slope and intercept of the linear regressions obtained from the tests designed to calculate standard cooking power in the field, we find the values for \(m\) and \(b\), then determined the heat exchange factor, \(F\), and the total coefficient of heat exchange \((U_t)\). Obtaining the following:

\[
F \approx 0.35 \text{ and } U_L \approx 9.8 \frac{W}{m^2K}
\]

By substituting in equations (3), (13), \(U_L, F\) and evaluating at \(\Delta T = 50^\circ\)C, we obtained the standard cooking power and thermal efficiency.

The following table shows that absorbance considering the standard cooking power and the thermal efficiency obtained experimentally and the standard cooking power predicted by the model.

**Table 2.** Experimental data against data obtained with the mathematical model. Using a solar stove with a catchment area of 0.5 m².

| Type of soot             | Absorbance in laboratory | Experimental standard cooking power (W) | Model Standard cooking power (W) | Experimental thermal efficiency (%) | Model thermal efficiency (%) |
|--------------------------|--------------------------|----------------------------------------|----------------------------------|------------------------------------|-----------------------------|
| Resin                    | 0.977                    | 71                                     | 67.8                             | 25                                 | 25.2                        |
| Sugarcane                | 0.974                    | 70                                     | 71.1                             | 24.8                               | 24.7                        |
| Forest biomass soot      | 0.972                    | 75                                     | 73                               | 27                                 | 24.7                        |
| High-temperature black paint | 0.954                | 57.5                                   | 58.1                             | 21                                 | 22.8                        |

The experimental calculation for standard cooking power and thermal efficiency they had a propagation error of \(\pm 4\) W and \(\pm 2\%)\%, respectively. It is clear that the values predicted by the model are within the margin of error of the calculations. Later, another case with a larger solar stove was considered, that is, the collector area is larger considering the same optical variables. Values for reflectance \(\rho = 0.92\) and \(A = 0.62\) m². In that case with the model was obtained:

\[
F \approx 0.35 \text{ and } U_L \approx 5.5 \frac{W}{m^2K}
\]
Table 3. Experimental data against data obtained with the mathematical model.

| Type of soot          | Absorbance in laboratory | Experimental standard cooking power (w) | Model standard cooking power (w) | Experimental thermal efficiency (%) | Model thermal efficiency (%) |
|----------------------|-------------------------|----------------------------------------|----------------------------------|-----------------------------------|-----------------------------|
| Resin                | 0.977                   | 132.5                                  | 126.63                           | 35                                | 33                          |
| High-temperature black paint | 0.954                  | 116                                    | 120                              | 30                                | 30.5                        |

In this case too, the data obtained from the model were satisfactory.

4. Discussion
Mathematical model relates thermal parameters of a solar stove to a CPC collector, with some of the most important optical parameters of the materials used for its elaboration; the transmittance of the collector cover, the reflectance of the reflective surface of the collector and the absorbance of the food container coating. The model is simple and good results are obtained, especially in the absorbance values close, so can be very useful in the study of coatings. In the same way it has given good results when the reflectance of the concentrating surface has been varied. The model is also a good tool to estimate the parameters such as the heat removal factor and the coefficient of total energy losses, in agreement with those found in the literature for similar devices.

5. Conclusions
The linear model produced a result that is a good approximation of standard cooking power in relation to the solar absorbance of the surface of the cooking pot of the solar cookstove and the average radiation during the period of the experiment. Based on the optical parameters obtained experimentally in the laboratory for this case, the absorbance obtained for the surface of the absorbing container of the solar cookstove presents a good approximation to standard cooking power. This thermal parameter was used to characterize and compare different models of solar cookstoves. It was obtained by applying protocols in the field that require conducting tests with repetitions that require exposing the cookstove to solar radiation for 5-10 consecutive days, under very specific meteorological conditions.

Obviously, carrying out filed tests in this manner entails large investments in time, resources and labour. In stark contrast, the comparisons developed in the present study required only ascertaining the absorbance of the surface of the cooking pot in order to obtain a good approximation of standard cooking power, at least for the range of high-absorbance coatings tested.

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