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Chapter

Matrix Cracking of Ceramic-Matrix Composites

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Abstract

In this chapter, the matrix cracking of fiber-reinforced ceramic-matrix composites (CMCs) is investigated using the energy balance approach. The relationship between the matrix cracking stress, fiber and interface oxidation, and fiber failure is established. The effects of the fiber volume, interface shear stress and interface debonding energy, fiber failure, and oxidation temperature on the time-dependent matrix cracking stress are analyzed. The experimental matrix cracking stress of different fiber-reinforced CMCs is predicted using the present models.

Keywords: ceramic-matrix composites, matrix cracking, oxidation, interface shear stress

1. Introduction

Ceramic-matrix composites (CMCs) possess high specific strength and high specific modulus especially at elevated temperature and have already been applied in hot-section components in aeroengine [1]. However, at elevated temperature, the environment affects the mechanical performance of fiber-reinforced CMCs. The matrix cracking stress decreases with operation time due to the interface oxidation [2]. Many researchers performed investigations on matrix cracking of fiber-reinforced CMCs, i.e., the energy balance approach developed by Aveston et al. [3]; Budiansky et al. [4]; Rajan and Zok [5]; and Li [6, 7], and the fracture mechanics approach proposed by Marshall et al. [8], and McCartney [9]. At elevated temperature, the oxidative environment entered inside of fiber-reinforced CMCs through the microcrackings, and the lifetime is affected by the applied load and temperature [10, 11].

In this chapter, the effects of temperature, oxidation, and applied stress on the time-dependent matrix cracking stress of fiber-reinforced CMCs are investigated. The relationship between the matrix cracking stress, fiber and interface oxidation, and fiber failure is established. The effects of the fiber volume, interface shear stress and interface debonding energy, fiber failure, and oxidation temperature on the time-dependent matrix cracking stress are analyzed. The experimental matrix cracking stress of different fiber-reinforced CMCs are predicted.

2. Theoretical analysis

At elevated temperature, oxygen reacts with the interphase along the fiber length. The time-dependent interphase oxidation length can be determined using the following equation (11) [12].
\[ \zeta = \varphi_1 \left[ 1 - \exp \left( -\frac{\varphi_2 t}{k} \right) \right] \]  

(1)

where \( \varphi_1 \) and \( \varphi_2 \) are parameters dependent on temperature and described using the Arrhenius type laws and \( b \) is a delay factor considering the deceleration of reduced oxygen activity.

The oxidation of fiber is assumed to be controlled by diffusion of oxygen gas through matrix cracks. By assuming that the fracture toughness of the fibers remains constant and that the fiber strength \( \sigma_0 \) is related to the mean oxidized layer thickness, the time dependence of the fiber strength can be determined by the following equation (12).

\[ \sigma_0(t) = \sigma_0, t \leq \frac{1}{k} \left( \frac{K_{IC}}{Y} \right)^4 \]  

(2)

\[ \sigma_0(t) = \frac{K_{IC}}{Y \sqrt{kt}}, t > \frac{1}{k} \left( \frac{K_{IC}}{Y} \right)^4 \]  

(3)

When the fiber breaks, the stress carried by the intact and broken fibers can be determined using the global load sharing (GLS) criterion by Eq. (4) [13].

\[ \sigma_{Vf} = T[1 - P(T)] + \langle T_b \rangle P(T) \]  

(4)

where \( V_f \) denotes the fiber volume fraction, \( T \) denotes the load carried by intact fibers, \( \langle T_b \rangle \) denotes the load carried by broken fibers, and \( P(T) \) denotes the fiber failure probability.

\[ P(T) = \eta P_a(T) + (1 - \eta)P_b(T) + \langle P_c(T) \rangle \]  

(5)

where \( \eta \) denotes the oxidation fiber fraction in the oxidation region and \( P_a(T) \), \( P_b(T) \), and \( \langle P_c(T) \rangle \) denote the fracture probability of oxidized fibers and unoxidized fibers in the oxidation region and interface debonding region, respectively.

2.1 Downstream stresses

**Figure 1** shows the composite under loading of a remote uniform applied stress of \( \sigma \) with a long matrix cracking. A unit cell is extracted from the composite to analyze the microstress field of the damaged fiber-reinforced CMCs, as shown in **Figure 2**. The fiber and matrix axial stress in the interface oxidation and debonding region can be determined using the following equation.

\[ \sigma_f^D(\zeta) = \begin{cases} T - \frac{2 \tau_f}{r_f} \zeta, & \zeta \in (0, \zeta) \\ T - \frac{2 \tau_f}{r_f} (\zeta - \sigma_f^D(\zeta)), & \zeta \in (\zeta, l_d) \end{cases} \]  

(6)

\[ \sigma_m^D(\zeta) = \begin{cases} 2 \frac{V_f}{V_m} \frac{\tau_f}{r_f} \zeta, & \zeta \in (0, \zeta) \\ 2 \frac{V_f}{V_m} \frac{\tau_f}{r_f} (\zeta - \sigma_m^D(\zeta)), & \zeta \in (\zeta, l_d) \end{cases} \]  

(7)

In the fiber/matrix interface bonding region, the fiber and matrix axial stresses can be determined using the following equation.
2.2 Upstream stresses

The upstream region III as shown in Figure 1 is so far away from the crack tip that the stress and strain fields are also uniform. The axial stress of the fiber and the matrix can be determined using the following equations.

\[
\sigma_{f}^{U} = \frac{E_{f}}{E_{c}} \sigma \\
\sigma_{m}^{U} = \frac{E_{m}}{E_{c}} \sigma
\]

Figure 1.
The schematic of crack-tip, interface debonding and oxidation.

Figure 2.
The schematic of shear-lag model considering the interface oxidation and debonding.
2.3 Interface debonding

The fracture mechanics approach is used to determine the fiber/matrix interface debonding length [14].

\[
\xi_d = -\frac{F}{4\pi r_f} \frac{\partial \omega_f(0)}{\partial l_d} - \frac{1}{2} \int_0^{l_d} r_i \frac{\partial \omega(z)}{\partial l_d} dz
\]

(12)

where \( F = \pi r_f^2 \sigma / V_f \) denotes the fiber load at the matrix cracking plane, \( \omega_f(0) \) denotes the fiber axial displacement on the matrix cracking plane, and \( \nu(z) \) denotes the relative displacement between the fiber and the matrix. Substituting \( \omega_f(z = 0) \) and \( \nu(z) \) into Eq. (12), it leads to the form of Eq. (13).

\[
\frac{E_c \tau_i^2}{r_f V_m E_f E_m} (l_d - \xi) + \frac{2E_c \tau_i \tau_i}{r_f V_m E_f E_m} (l_d - \xi) - \frac{\sigma \tau_i}{2V_f E_f} (l_d - \xi) - \frac{T \tau_i}{2E_f} (l_d - \xi)
\]

(13)

Solving Eq. (13), the fiber/matrix interface debonding length is determined by Eq. (14).

\[
l_d = \left( 1 - \frac{t_f}{t_i} \right) l_d + \frac{r_f V_m E_m}{4E_c \tau_i} \left( \frac{\sigma}{V_f} + T \right) - \left\{ \frac{r_f^2 V_m E_m T^2}{4E_c \tau_i^2} \left[ V_m E_m \left( \frac{\sigma}{V_f T} + 1 \right)^2 \right] \right. \\
+ \frac{V_f E_f}{E_c} \left( \frac{\sigma}{V_f T} - \frac{\sigma}{V_f} \right) \left[ \frac{r_f V_m E_m E_m}{E_c \tau_i^2} \xi_d \right] \right\}^{\frac{1}{2}}
\]

(14)

2.4 Matrix cracking stress

The energy relationship to evaluate the steady-state matrix cracking stress is determined by Eq. (15) [4].

\[
\frac{1}{2} \int_{-\infty}^{\infty} \left[ V_f \left( \sigma_f^U - \sigma_f^D \right)^2 + V_m / E_m \left( \sigma_m^U - \sigma_m^D \right)^2 \right] dz + \frac{1}{2 \pi R^2 G_m} \int_{-l_d}^{l_d} \int_0^{\infty} \frac{r_f \tau_i(z)}{r} 2\pi r dr dz
\]

(15)

where \( \xi_m \) is the matrix fracture energy and \( G_m \) is the matrix shear modulus. Substituting the fiber and matrix axial stresses in Eqs. (6)–(11) and the fiber/matrix interface debonding length of Eq. (14) into Eq. (15), the energy balance equation leads to the form of the following equation.

\[
\eta_1 \sigma^2 + \eta_2 \sigma + \eta_3 = 0
\]

(16)

where

\[
\eta_1 = \frac{l_d}{E_c}
\]

(17)
$\eta_2 = -\frac{2V_fT_d}{E_c}$ (18)

$\eta_3 = \frac{V_f l_d T^2}{E_f} - \frac{V_f}{E_f} \left( \frac{2\tau_i}{r_f} \right) \zeta (2l_d - \zeta) T - \frac{V_f}{E_f} \left( \frac{2\tau_i}{r_f} \right) (l_d - \zeta)^2 T$

$+ \frac{4}{3} \left( \frac{V_f E_c}{V_m E_t E_m} \right) \left( \frac{r_f}{r_t} \right)^2 \left( l_d - \zeta \right)^3 + \left( \frac{r_f}{r_t} \right)^2 \zeta^3$

$\times \left[ l_d - \left( 1 - \frac{r_f}{r_t} \right) \zeta - \frac{4V_f l_d}{V_m E_t E_m} \right] \frac{\tau_i}{r_f} \zeta (l_d - \zeta)$ (19)

3. Result and discussion

The effects of fiber volume, fiber/matrix interface debonding energy, interface shear stress, fiber strength, and oxidation temperature on the matrix cracking stress, interface oxidation, and interface debonding are analyzed.

3.1 Effect of the fiber volume on matrix cracking stress

The matrix cracking stress, fiber/matrix interface debonding length, and fiber/matrix interface oxidation length versus the oxidation time curves corresponding to different fiber volume of $V_f = 30\%$ and $35\%$ are shown in Figure 3. When the fiber volume increases, the matrix cracking stress and the fiber/matrix interface oxidation length increase, and the fiber/matrix interface debonding length decreases.

When the fiber volume is $V_f = 30\%$, the matrix cracking stress decreases from $\sigma_{mc} = 86$ to $44$ MPa after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$; the fiber/matrix interface debonding length first decreases from $l_d/r_f = 6.9$ to $6.7$ after $t = 2$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$ and then increases to $l_d/r_f = 9.0$ after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$, and the fiber/matrix interface oxidation length increases from $\zeta/l_d = 0$ to $0.94$ after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$.

When the fiber volume is $V_f = 35\%$, the matrix cracking stress decreases from $\sigma_{mc} = 97$ to $50$ MPa after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$; the fiber/matrix interface debonding length first decreases from $l_d/r_f = 5.8$ to $5.7$ after $t = 1.7$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$ and then increases to $l_d/r_f = 8.4$ after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$, and the fiber/matrix interface oxidation length increases from $\zeta/l_d = 0$ to $1.0$ after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$.

3.2 Effect of the fiber/matrix interface debonding energy on matrix cracking stress

The matrix cracking stress, fiber/matrix interface debonding length, and fiber/matrix interface oxidation length versus the oxidation time curves corresponding to different interface debonding energies of $\xi_d/\xi_m = 0.1$ and $0.2$ are shown in Figure 4. When the fiber/matrix interface debonding energy increases, the matrix cracking stress and the interface oxidation length increase, and the interface debonding length decreases.

When the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.1$, the matrix cracking stress decreases from $\sigma_{mc} = 86$ MPa to $32$ MPa after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ C$; the fiber/matrix interface debonding length
Figure 3.
(a) The matrix cracking stress versus the oxidation time, (b) the fiber/matrix interface debonding length versus the oxidation time, and (c) the fiber/matrix interface oxidation length versus the oxidation time corresponding to different fiber volume of $V_f = 30$ and 35%. 

Hysteresis of Composites
Figure 4.
(a) The matrix cracking stress versus the oxidation time, (b) the fiber/matrix interface debonding length versus the oxidation time, and (c) the fiber/matrix interface oxidation length versus the oxidation time corresponding to different interface debonding energy of $\xi_d/\xi_m = 0.1$ and 0.2.
first decreases from \( l_d/r_f = 6.9 \) to 6.7 after \( t = 1.7 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \) and then increases to \( l_d/r_f = 9.6 \) after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \), and the fiber/matrix interface oxidation length increases from \( \zeta/l_d = 0 \) to 0.88 after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \). When the fiber/matrix interface debonding energy is \( \zeta_d/\zeta_{em} = 0.2 \), the matrix cracking stress decreases from \( \sigma_{mc} = 134 \) to 87 MPa after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \); the fiber/matrix interface debonding length first decreases from \( l_d/r_f = 5.5 \) to 5.4 after \( t = 1.1 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \) and then increases to \( l_d/r_f = 8.8 \) after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \), and the fiber/matrix interface oxidation length increases from \( \zeta/l_d = 0 \) to 0.96 after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \).

3.3 Effect of the fiber/matrix interface shear stress on the matrix cracking stress

The matrix cracking stress, fiber/matrix interface debonding length, and the fiber/matrix interface oxidation length versus the oxidation time curves corresponding to different fiber/matrix interface shear stress of \( \tau_i = 20 \) and 10 MPa are shown in Figure 5. When the fiber/matrix interface shear stress in the slip region increases, the matrix cracking stress and the fiber/matrix interface oxidation length increase, and the fiber/matrix interface debonding length decreases.

When the fiber/matrix interface shear stress is \( \tau_i = 10 \) MPa, the matrix cracking stress decreases from \( \sigma_{mc} = 67 \) to 43 MPa after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \); the fiber/matrix interface debonding length first decreases from \( l_d/r_f = 8.0 \) to 7.7 after \( t = 2.6 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \) and then increases to \( l_d/r_f = 9.1 \) after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \), and the fiber/matrix interface oxidation length increases from \( \zeta/l_d = 0 \) to 0.93 after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \).

When the fiber/matrix interface shear stress is \( \tau_i = 20 \) MPa, the matrix cracking stress decreases from \( \sigma_{mc} = 102 \) to 45 MPa after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \); the fiber/matrix interface debonding length first decreases from \( l_d/r_f = 6.2 \) to 6.0 after \( t = 1.7 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \) and then increases to \( l_d/r_f = 8.9 \) after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \), and the fiber/matrix interface oxidation length increases from \( \zeta/l_d = 0 \) to 0.95 after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \).

The matrix cracking stress, fiber/matrix interface debonding length, and the fiber/matrix interface oxidation length versus the oxidation time curves corresponding to different fiber/matrix interface shear stress of \( \tau_f = 1 \) and 5 MPa are shown in Figure 6. When the fiber/matrix interface shear stress in the slip region increases, the matrix cracking stress and the fiber/matrix interface oxidation length increase, and the fiber/matrix interface debonding length decreases.

When the fiber/matrix interface shear stress is \( \tau_f = 1 \) MPa, the matrix cracking stress decreases from \( \sigma_{mc} = 86 \) to 28 MPa after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \); the fiber/matrix interface debonding length first decreases from \( l_d/r_f = 6.9 \) to 6.7 after \( t = 1.6 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \) and then increases to \( l_d/r_f = 9.8 \) after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \), and the fiber/matrix interface oxidation length increases from \( \zeta/l_d = 0 \) to 0.86 after \( t = 10 \) h oxidation at elevated temperature of \( T_{em} = 800^\circ C \).
Figure 5.
(a) The matrix cracking stress versus the oxidation time, (b) the fiber/matrix interface debonding length versus the oxidation time, and (c) the fiber/matrix interface oxidation length versus the oxidation time corresponding to different interface shear stress of $\tau_i = 10$ and 20 MPa.
Figure 6.
(a) The matrix cracking stress versus the oxidation time, (b) the fiber/matrix interface debonding length versus oxidation time, and (c) the fiber/matrix interface oxidation length versus the oxidation time corresponding to different interface shear stress of $\tau_f = 1$ and 5 MPa.
When the fiber/matrix interface shear stress is $\tau_f = 5$ MPa, the matrix cracking stress decreases from $\sigma_{mc} = 86$ to 44 MPa after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C; the fiber/matrix interface debonding length first decreases from $l_d/r_f = 6.9$ to 6.7 after $t = 2$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C and then increases to $l_d/r_f = 9$ after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C, and the fiber/matrix interface oxidation length increases from $\zeta/l_d = 0$ to 0.94 after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C.

3.4 Effect of the fiber strength on matrix cracking stress

The matrix cracking stress, fiber/matrix interface debonding length, and fiber/matrix interface oxidation length versus the oxidation time curves corresponding to different fiber strength of $\sigma_0 = 1$ and 2 GPa are shown in Figure 7. When the fiber strength increases, the matrix cracking stress and the fiber/matrix interface debonding length increase, and the fiber/matrix interface oxidation length decreases.

When the fiber strength is $\sigma_0 = 1$ GPa, the matrix cracking stress decreases from $\sigma_{mc} = 38$ to 18 MPa after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C; the fiber/matrix interface debonding length first decreases from $l_d/r_f = 14.9$ to 14.7 after $t = 2$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C and then increases to $l_d/r_f = 16.3$ after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C, and the fiber/matrix interface oxidation length increases from $\zeta/l_d = 0$ to 0.52 after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C.

When the fiber strength is $\sigma_0 = 2$ GPa, the matrix cracking stress decreases from $\sigma_{mc} = 43$ to 21 MPa after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C; the fiber/matrix interface debonding length first decreases from $l_d/r_f = 15.3$ to 15.2 after $t = 2.3$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C and then increases to $l_d/r_f = 16.6$ after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C, and the fiber/matrix interface oxidation length increases from $\zeta/l_d = 0$ to 0.51 after $t = 10$ h oxidation at elevated temperature of $T_{em} = 800^\circ$C.

3.5 Effect of the temperature on matrix cracking stress

The matrix cracking stress, fiber/matrix interface debonding length, and fiber/matrix interface oxidation length versus the oxidation time curves corresponding to different oxidation temperature of $T_{em} = 600$ and 800°C are shown in Figure 8. When the oxidation temperature increases, the matrix cracking stress decreases, and the fiber/matrix interface oxidation length and the interface debonding length increase.

When the oxidation temperature is $T_{em} = 600^\circ$C, the matrix cracking stress decreases from $\sigma_{mc} = 86.3$ to 73 MPa after $t = 10$ h oxidation, the fiber/matrix interface debonding length decreases from $l_d/r_f = 6.9$ to $6.7$ after $t = 10$ h oxidation, and the fiber/matrix interface oxidation length increases from $\zeta/l_d = 0$ to 0.19 after $t = 10$ h oxidation.

When the oxidation temperature is $T_{em} = 800^\circ$C, the matrix cracking stress decreases from $\sigma_{mc} = 86$ to 44 MPa after $t = 10$ h oxidation; the fiber/matrix interface debonding length first decreases from $l_d/r_f = 6.9$ to 6.7 after $t = 2$ h oxidation and then increases to $l_d/r_f = 9$ after $t = 10$ h oxidation, and the fiber/matrix interface oxidation length increases from $\zeta/l_d = 0$ to 0.94 after $t = 10$ h oxidation.
Figure 7.
(a) The matrix cracking stress versus the oxidation time, (b) the fiber/matrix interface debonding length versus the oxidation time, and (c) the fiber/matrix interface oxidation length versus the oxidation time corresponding to different fiber strength of $\sigma_0 = 1$ and 2 GPa.
Figure 8.
(a) The matrix cracking stress versus the oxidation time, (b) the fiber/matrix interface debonding length versus the oxidation time, and (c) the fiber/matrix interface oxidation length versus the oxidation time corresponding to different oxidation temperature of $T_{\text{em}} = 600$ and $800^\circ$C.
4. Experimental comparison

Barsoum et al. [15] investigated the matrix crack initiation in fiber-reinforced CMCs. The experimental and predicted matrix cracking stress versus the fiber volume corresponding to different fiber/matrix interface debonding energy of SiC/borosilicate, SiC/LAS, and C/borosilicate composites is shown in Figures 9–11.

For the SiC/borosilicate composite, the predicted matrix cracking stress with the fiber/matrix interface debonding energy range of $\xi_d/\xi_m = 0.05, 0.1, \text{ and } 0.2$ agrees with the experimental data corresponding to the fiber volume changing from 10 to 50%, as shown in Figure 9. When the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.05$, the matrix cracking stress increases from $\sigma_{mc} = 49.4 \text{ MPa}$ at the fiber volume of $V_f = 10\%$ to $\sigma_{mc} = 334 \text{ MPa}$ at the fiber volume of $V_f = 50\%$; when the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.1$, the matrix cracking stress increases from $\sigma_{mc} = 52.2 \text{ MPa}$ at the fiber volume of $V_f = 10\%$ to $\sigma_{mc} = 371 \text{ MPa}$ at the fiber volume $V_f = 50\%$; and when the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.2$, the matrix cracking stress increases from $\sigma_{mc} = 57.1 \text{ MPa}$ at the fiber volume of $V_f = 10\%$ to $\sigma_{mc} = 428 \text{ MPa}$ at the fiber volume of $V_f = 50\%$.

For the SiC/LAS composite, the matrix cracking stress with the interface debonding energy range of $\xi_d/\xi_m = 0.01, 0.02, \text{ and } 0.03$ agrees with the experimental data corresponding to the fiber volume changing from 30 to 50%, as shown in Figure 10. When the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.01$, the matrix cracking stress increases from $\sigma_{mc} = 228 \text{ MPa}$ at the fiber volume of $V_f = 30\%$ to $\sigma_{mc} = 423 \text{ MPa}$ at the fiber volume of $V_f = 50\%$; when the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.02$, the matrix cracking stress increases from $\sigma_{mc} = 257 \text{ MPa}$ at the fiber volume of $V_f = 30\%$ to $\sigma_{mc} = 489 \text{ MPa}$ at the fiber volume of $V_f = 50\%$; and when the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.03$, the matrix cracking stress increases from $\sigma_{mc} = 281 \text{ MPa}$ at the fiber volume of $V_f = 30\%$ to $\sigma_{mc} = 542 \text{ MPa}$ at the fiber volume of $V_f = 50\%$.

For the C/borosilicate composite, the predicted matrix cracking stress with the fiber/matrix interface debonding energy range of $\xi_d/\xi_m = 0.01, 0.02, \text{ and } 0.03$
agrees with the experimental data corresponding to the fiber volume changing from 30 to 55%, as shown in Figure 11. When the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.01$, the matrix cracking stress increases from $\sigma_{mc} = 136$ MPa at the fiber volume of $V_f = 30\%$ to $\sigma_{mc} = 438$ MPa at the fiber volume of $V_f = 55\%$; when the fiber/matrix interface debonding energy is $\xi_d/\xi_m = 0.02$, the matrix cracking stress increases from $\sigma_{mc} = 156$ MPa at the fiber volume of $V_f = 30\%$ to $\sigma_{mc} = 471$ MPa at the fiber volume of $V_f = 55\%$; and when the fiber/matrix interface
debonding energy is $\xi_d/\xi_m = 0.03$, the matrix cracking stress increases from $\sigma_{mc} = 173$ MPa at the fiber volume of $V_f = 30\%$ to $\sigma_{mc} = 504$ MPa at the fiber volume of $V_f = 55\%$.

Yang [16] investigated the mechanical behavior of C/SiC composite after unstressed oxidation at elevated temperature of 700°C in air atmosphere. The composite was divided into two types based on the interface bonding, i.e., strong interface bonding and weak interface bonding. For the C/SiC with the strong interface bonding, the matrix cracking stresses of C/SiC corresponding to the proportional limit stresses in the tensile curves are 37, 30, and 20 MPa corresponding to the cases of without oxidation, 4 h oxidation, and 6 h unstressed oxidation, respectively. For the C/SiC with the weak interface bonding, the matrix cracking stresses...
of C/SiC corresponding to the proportional limit stresses in the tensile curves are 27, 20, and 13 MPa corresponding to the cases of without oxidation, 2 h oxidation, and 6 h unstressed oxidation, respectively. The experimental and predicted matrix cracking stresses of C/SiC composite with strong and weak interface bonding after unstressed oxidation at elevated temperature of 700°C in air are shown in Figure 12a and b, respectively. The matrix cracking stress decreases 18.9% after oxidation for 4 h, 46% after oxidation for 6 h for C/SiC with strong interface bonding, 25.9% after oxidation for 1 h, and 51.8% after oxidation for 6 h for C/SiC with weak interface bonding. The theoretical predicted results agreed with experimental data. The strong interface bonding can be used for oxidation resistant of C/SiC composite at elevated temperature.

5. Conclusion

In this chapter, the time-dependent matrix cracking of fiber-reinforced CMCs was investigated using the energy balance approach. The relationship between the matrix cracking stress, fiber and interface oxidation, and fiber failure was established. The effects of the fiber volume, interface shear stress and interface debonding energy, fiber failure, and oxidation temperature on the time-dependent matrix cracking stress were analyzed. The experimental matrix cracking stress of different fiber-reinforced CMCs was predicted.

1. When the fiber volume, fiber/matrix interface debonding energy and interface shear stress increase, the matrix cracking stress and the fiber/matrix interface oxidation length increase, and the fiber/matrix interface debonding length decreases.

2. When the fiber strength increases, the matrix cracking stress and the fiber/matrix interface debonding length increase, and the fiber/matrix interface oxidation length decreases.

3. When the oxidation temperature increases, the matrix cracking stress decreases, and the fiber/matrix interface oxidation length and the interface debonding length increase.

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