This paper presents a novel analytical method to generate interaction diagrams useful for the design of reinforced concrete (RC) biaxial columns. Due to the introduction of new classes in concrete compressive strength \( f'_{c} \) with somewhat different parameters for the steel grades \( f_{y} \), it has become necessary to develop new interaction diagrams. The interaction diagram of any desired level of gamma \( \gamma \) can be generated to find the required axial load capacity \( P_{c} \) and moment capacity \( (M_{cx}, M_{cy}) \) of the columns having different reinforcement ratios \( \rho \). In addition, this study reports some numerical examples using the proposed interaction charts (Reciprocal as well as the equivalent uniaxial eccentricity method) to find the values of \( P_{c} \) and \( (M_{cx}, M_{cy}) \) for the columns subjected to biaxial bending. The obtained results are compared with a conventional method (i.e., the computer software SP-Column). The obtained results are promising, as the values obtained by the Interaction Charts and the conventional method appear to be in good agreement.

KEYWORDS
axial load capacity, biaxial columns, equivalent uniaxial eccentricity, interaction charts, moment capacity

1 INTRODUCTION

Columns are defined as the vertical compression members which transmit loads from the upper floors to the lower levels, and to the soil through the foundations.\(^1\) Depending upon the presence and absence of bending moments, columns may be classified as concentrically loaded, Figure 1, or eccentrically loaded columns, Figure 2. Eccentrically loaded columns are subject to moments, in addition to axial force. The moments can be converted to a load \( P \) and eccentricity \( e_{x} \) and \( e_{y} \). The moments can be uniaxial, as in the case when two adjacent panels are not similarly loaded, such as columns A and B in Figure 3. A column is considered to be as bi-axially loaded when the bending occurs about the \( x \) and \( y \) axes, such as in the case of corner column C in Figure 3.\(^2\) Al-Ansari and Afzal in a recent study presented an analytical model for generating Interaction diagram charts for uniaxial columns.\(^3\)

The strength of reinforced concrete (RC) column is normally expressed using interaction diagrams to relate the design axial load \( OP_{n} \) to the design bending moment \( OM_{n} \).\(^4\) Each control point on the column interaction curve \( OP_{n} = OM_{n} \), represents one combination of design axial load \( OP_{n} \) and design bending moment \( OM_{n} \) corresponding to a neutral-axis location (Figure 4).\(^5\)
**FIGURE 1**  Concentrically loaded columns

**FIGURE 2**  Eccentrically loaded Column

**FIGURE 3**  Uniaxially and biaxially loaded column
The biaxial interaction diagrams of RC rectangular columns have been investigated extensively by numerous researchers.\(^6\)-\(^{12}\) Al-Sherrawi et al.\(^{13}\) proposed the analytical model to construct the interaction diagram for strengthening of RC columns with steel jacket. Rafiq et al.\(^{14}\) introduced a new approach for designing RC biaxial column using genetic algorithms. Alvee Islam Navid et al.\(^{15}\) developed the computer programming for generating the interaction diagrams for columns subjected to biaxial bending. Rodriguez and Dario\(^{4}\) also provided a simplified proposed method to counter the evaluation of theoretical ultimate strength of RC short columns using different theoretical approaches which require extensive theoretical calculations to find the required strength. He further solved five numerical examples in detail to show the effectiveness of his proposed method. The other similar research on RC columns subjected to biaxial bending can be found elsewhere.\(^{16}-^{20}\)

Due to the introduction of new classes in concrete compressive strength \((f'_{c})\) with somewhat different parameters for the steel grades \((f_y)\), it has become necessary to develop new interaction diagrams which should coincide with the new advances in construction Industry. The previous research studies related to the interaction diagrams are only limited to English units \((\text{kips-in})\). Several numerical examples along with the proposed formulation of interaction charts are provided in English units only.

This study proposed simple analytical approach \((\text{Reciprocal method})\) for generating the interaction diagram charts for RC columns in SI units \((\text{kN-m})\) subjected to biaxial bending. This simplified method can also be used to draw the interaction charts for any desired level of gamma \((\gamma)\) to find the required axial load capacity \((P_c)\) and moment capacity \((M_{cx}, M_{cy})\) of the columns having different reinforcement ratios \((\rho)\). These charts will also help the designers to use them directly, for the countries where the construction is mostly done in the SI units, to compute the required design strengths. Also, it will be useful for the students and researchers to use the proposed SI unit interaction charts in their research.
related work. In addition to this reciprocal method, the equivalent uniaxial eccentricity method is also proposed in this study where the biaxial column moments are converted to equivalent uniaxial moment to ease the work for finding the required axial and moment capacities.

Numerical examples for the selected RC columns are also illustrated using both methods (Reciprocal as well as Equivalent uniaxial method) to check the adequacy of this proposed analytical approach. Moreover, the results obtained from these selected reinforced columns are compared with the computer software Sp-Column.^21

2 INTERACTION CHARTS FORMULATION—ACI CODE DESIGN

The column cross-section subjected to biaxial bending is shown in Figure 5. The reciprocal method (Bresler’s formula) will be used to analyze the strength of the biaxial column and to check whether the obtained values of $\Phi P_n$ and $(\Phi M_{nx},\Phi M_{ny})$ are larger or equal to the values of $P_u$ and $(M_{ux},M_{uy})$, respectively.^22 Equation (1) shows the bresler’s formula to compute the axial capacity of the column

$$P_c = \frac{1}{\Phi_{Pc} + \frac{1}{\Phi_{Py}} - \frac{1}{\Phi_{PN_{max}}}}.$$  (1)

And,

$$\Phi P_{N_{max}} = 0.8 \phi (0.85 f'_c (A_g - A_{st}) + f_y A_{st}),$$

where $\Phi P_{N_{max}}$, maximum permissible column load; $A_{st}$, total area of steel; $A_g$, (gross area of cross section)—(sectional area of concrete member).

The moments in the $x$ and $y$ direction can be found as,

$$M_{cx} = P_c \times e_{uy},$$  (2)

$$M_{cy} = P_c \times e_{ux}.$$  (3)

The stress-strain distribution of a rectangular column section for the calculation of $P_c$ and $M_{cx}$ for the $x$-$x$ axis is given in Figure 6 whereas for the $y$-$y$ axis, the stress-strain relationship for the $P_c$ and $M_{cy}$ is provided in Figure 7.23,24

**FIGURE 5** Biaxial column cross-section
The resultant force $P_x$ and $P_y$ is equal to the summation of all internal forces.

$$P_x = C_{Con} - T_s + C_s.$$  

$$P_y = C_{Con} - T_s + C_s.$$  

Similarly, the resultant Moment $M_x$ and $M_y$ is equal to the summation of all internal moments in the $x-x$ and $y-y$ axis.

$$M_x = M_{conc} + M_T + M_{cs}.$$  

$$M_y = M_{conc} + M_T + M_{cs}.$$
Following steps revealed the calculation of the required internal forces and internal moments for the $x$-$x$ axis and the similar steps should be repeated to have the required forces and moments for the $y$-$y$ axis.

I. Plain concrete section:

The internal concrete compressive force $(C_{\text{conc}})_x$ is computed as

$$(C_{\text{conc}})_x = 0.8 \times (0.85 f'_c b a),$$

$$(C_{\text{conc}})_x = 0.68 f'_c b \beta c,$$  \hspace{1cm} (8)

where $C_{\text{conc}}$ internal concrete compression force; $f'_c$ compressive concrete strength; $b$ column width; $a$ depth of the compression stress block; $\beta$, $0.85 - 0.008 (f'_c - 30) \geq 0.65$; $c$, distance from extreme compression fiber to neutral axis.

Referring to Figures 6 and 7, the moment about the midpoint of the section $(M_{\text{conc}})_x$ can be computed as;

$$(M_{\text{conc}})_x = C_c \left(\frac{h}{2} - \frac{a}{2}\right),$$

$$(M_{\text{conc}})_x = 0.68 f'_c b a \left(\frac{h}{2} - \frac{a}{2}\right).$$  \hspace{1cm} (9)

The $\alpha_{1x}$ and $\beta_{1x}$ values for the plain concrete section calculated as;

Setting $\alpha_{1-\text{conc}} = \alpha_{1x} = \frac{C_{\text{conc}}}{f'_c bh} = 0.68 \times \frac{a}{h}$, \hspace{1cm} (10)

Setting $\beta_{1-\text{conc}} = \beta_{1x} = \frac{M_{\text{conc}}}{f'_c bh^2} = 0.68 \left(\frac{h}{2} - \frac{a}{2}\right) \times \frac{1}{h} \times \frac{a}{h}$. \hspace{1cm} (11)

II. Tension steel section:

The Internal Tensile force $T_{sx}$ is computed as;

$$T_{sx} = 0.68 A_{sf},$$  \hspace{1cm} (12)

where $A_{sx}$, area of tensile steel reinforcement; $f_s$, computed steel stress in tensile steel; the value of the internal moment $M_{Tx}$ is;

$$M_{Tx} = 0.68 A_{sf} \left(\frac{h}{2} - d'\right).$$  \hspace{1cm} (13)

The $\alpha_{2x}$ and $\beta_{2x}$ values for the tension steel section are calculated as;

Setting $\alpha_{2x} = \frac{T_s}{f'_c bh} = 0.68 A_{sf} = \rho_2 \frac{0.68 f_s}{f'_c}$,

$$\alpha_{2x} = \rho_2 \frac{0.68 f_y}{f'_c},$$  \hspace{1cm} (14)

Setting $\beta_{2x} = \frac{M_T}{f'_c bh^2} = 0.68 \frac{A_{sf} \left(\frac{h}{2} - d'\right)}{f'_c bh^2}$.

$$\beta_{2x} = 0.68 \frac{A_{sf} \left(\frac{h}{2} - d'\right)}{f'_c bh^2}. $$  \hspace{1cm} (15)
Substituting the value of \( \alpha_{2x} \) in Equation (15),

\[
\beta_{2x} = \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_{2x},
\]

where \( f_y \), yield stress of reinforcing steel; \( d' \), distance from extreme compression fiber to centroid of reinforcing steel.

### III. Compression steel section:

The Internal compressive force \( C_{sx} \) is computed as

\[
C_{sx} = 0.8 A'_{sx} f'_s,
\]

where \( A'_{sx} \), area of compression steel reinforcement; \( f'_s \), computed compressive stress in compression steel; the value of the internal moment \( M_{Tx} \) is;

\[
M_{Tx} = 0.8 A'_{sx} f'_s \left( \frac{h}{2} - d' \right).
\]

The \( \alpha_{3x} \) and \( \beta_{3x} \) values for the compression steel section are calculated as;

Setting \( \alpha_{3x} = \frac{C_s}{f'_s bh} = \frac{0.8 A'_{sx} f'_s}{f'_s bh} = 0.8 \frac{A'_{sx}}{f'_s} \times \frac{f'_s}{f'_c} = \rho_3 \frac{0.8 f'_s}{f'_c}, \)

\[
\alpha_{3x} = \rho_3 \frac{0.8 f'_s}{f'_c},
\]

Setting \( \beta_{3x} = \frac{M_{Cx}}{f'_c bh^2} = 0.8 \frac{A'_{sx} f'_s \left( \frac{h}{2} - d' \right)}{f'_c bh^2}. \)

Substituting the value of \( \alpha_{3} \) in Equation (20)

\[
\beta_{3x} = \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_{3x}.
\]

The similar steps I to III should be repeated to find the values of \( \alpha \) and \( \beta \) in y-direction.

### 2.1 Construction of interaction chart

The column axial load capacity \( P_x \) is summation of all internal forces in the x-direction

\[
P_x = \Theta P_{nx},
\]

where \( P_{nx} = C_{Con} - T + C_S \).

Therefore, \( \alpha = \alpha_{1x} - \alpha_{2x} + \alpha_{3x} \)

\[
a = 0.68 \times \frac{a}{h} - \rho_2 \frac{0.68 f_y}{f'_c} + \rho_3 \frac{0.8 f_y}{f'_c},
\]

\[
P_x = \Theta a \ b \ h.
\]
For the moment capacity, the column moment capacity $M_c$ is summation of all internal moments in x-direction $M_x$.

$$M_x = \Omega M_{nx},$$

where $M_{nx} = M_{Con} + M_T + M_{C_s}$.

Therefore, $\beta = \beta_{1x} + \beta_{2x} + \beta_{3x}$

$$\beta = 0.68 \left( \frac{h}{2} - \frac{a}{2} \right) \times \frac{1}{h} \times \frac{a}{h} + \left( \frac{1}{2} - \frac{d'}{h} \right) a_2 + \left( \frac{1}{2} - \frac{d'}{h} \right) a_3,$$

(24)

$$M_x = \Omega \beta b h^2.$$  

(25)

Computing the values of $\alpha$ and $\beta$ from the above Equations (23) and (25) for the x-x axis.

$$\alpha = \frac{P_x}{\Omega bh} = \frac{P_{N_x}}{A_g}, \quad \beta = \frac{M_x}{\Omega bh^2} = \frac{M_{N_x}}{A_g h}.$$  

Similarly, the values of $\alpha$ and $\beta$ for the y-y axis are to be formulated as

$$\alpha = \frac{P_y}{\Omega bh} = \frac{P_{N_y}}{A_g}, \quad \beta = \frac{M_y}{\Omega bh^2} = \frac{M_{N_y}}{A_g h}.$$  

The value of Gamma ($\gamma_x$) and ($\gamma_y$) for the column interaction chart is computed as

$$\gamma_x = \frac{h_x - 2d'}{h_x}, \quad \gamma_y = \frac{h_y - 2d'}{h_y}.$$  

| Sr. no | $\gamma$ | $\frac{d'}{h}$ |
|--------|----------|----------------|
| 1      | 0.6      | 0.2            |
| 2      | 0.7      | 0.15           |
| 3      | 0.8      | 0.1            |
| 4      | 0.9      | 0.05           |

**TABLE 1** Values of $\gamma$ VS $\frac{d'}{h}$

**FIGURE 8** Column interaction diagram ($\beta - \alpha$) for $\gamma = 0.6$
Table 1 describes the values of $\frac{d}{h}$ obtained against different values of $\gamma$ which will be used in Equation (24).

Thus, by assigning different values to $\frac{x}{h}$ and substituting in Equations (22) and (24), the $(\alpha - \beta)$ curve can be constructed (Figure 8). The interaction chart (Figure 8) is for column section having $\gamma = 0.6$, $f_y = 415$ MPa and $f'_c = 30$ MPa.

The column interaction diagram for the remaining values of $\gamma = 0.7, 0.8,$ and $0.9$ with $f'_c = 30$ Mpa and $f_y = 415$ MPa are displayed in Figures 9-11, respectively. These generated interaction charts for RC columns in SI units ($kN-m$) can be used to compute the required axial and moment capacities of biaxial columns. The formulas revealed in this study can be utilized to draw the Interaction diagram of any desired level of gamma ($\gamma$), having different values of concrete compressive strength ($f'_c$) and steel yield strength ($f_y$).

**FIGURE 9** Column interaction diagram $(\beta - \alpha)$ for $\gamma = 0.7$  

**FIGURE 10** Column interaction diagram $(\beta - \alpha)$ for $\gamma = 0.8$
3 | NUMERICAL EXAMPLES—(RECIPROCAL METHOD)

The following steps need to be followed to compute the values of $P_x$ and $P_y$ to be used in the Bresler's formula for an economical design.

**Step-1**: Find the value of $\beta_u$ from the moments and the given cross-section. $\beta_u = \frac{M_x}{A_g h}$

**Step-2**: Find the value of $\alpha_u$ from the axial load and the given cross-section. $\alpha_u = \frac{P_u}{A_g}$

**Step-3**: Extend a line through point $(\beta_u, \alpha_u)$ from the origin (0,0) to the desired $\rho$ line.

**Step-4**: Determine the new points $(\beta, \alpha)$ on the desired $\rho$ line.

**Step-5**: Compute $P_x = \rho \alpha b h$ and $M_x = \rho \beta b h^2$

**Step-6**: Repeat the steps 1-5 to compute $P_y$ and $M_y$.

**Step-7**: Utilize the values of the $P_x$ and $P_y$ in the Bresler's Equations (1-3) to find the column capacity $P_c$ and the required moments $M_{cx}$ and $M_{cy}$.

An example is illustrated to compare the results of reciprocal method (Bresler method) obtained using the $(\beta - \alpha)$ column interaction charts with the finite element software (SP-Column).

**Example:**

A rectangular column section C-1 of 350 mm $\times$ 700 mm with $\phi = 0.65$ is loaded externally with an axial load of $P_u = 2500$ kN and with an external moments of $M_{cx} = 250$ kN-m and $M_{cy} = 150$ kN-m. The concrete compressive strength and steel yield strength are $f'_c = 30$ MPa and $f_y = 400$ MPa, respectively. Determine the column strength $P_c$ and $M_{cx}$ and $M_{cy}$ having the reinforcement ratio of $(\rho = 0.01)$.

**Solution:**

The values of the $P_c$ and $(M_{cx}, M_{cy})$ are determined by following the steps 1 to 7. The $\alpha$ and $\beta$ values for $(\rho = 0.01)$ are reflected in column interaction diagram Figure 12.

The results obtained are compared with the Finite Element software (SP column) and are listed in Tables 3 and 4.

Some more examples for the biaxial columns with different column sizes are also solved with the $(\beta - \alpha)$ chart using the reciprocal method (Bresler's formula) and the results obtained are later compared with the Computer Software. These columns are having different reinforcement ratios $(\rho)$ with different values of gamma ($\gamma_x$ and $\gamma_y$). The load input data for these columns are given in Table 2.
The results obtained for the interaction charts using the reciprocal method (Bresler’s formula) for the x-x axis and y-y axis are shown in Table 3.

The axial load values \( P_x \) and \( P_y \) as well as the moment values \( M_x \) and \( M_y \) obtained from Table 3 are used in the Bresler’s formula Equations (1-3) to compute the required values of \( P_c \) and \( (M_{cx}, M_{cy}) \). These rectangular column sections (C1 to C6) are also analyzed with the computer software (SP-Column) having the same input load values mentioned in Table 2. The required column capacities obtained from the reciprocal method using the
**TABLE 4** Bresler biaxial column design result (reciprocal method)

| Col no. | Axial load $P_u$ (kN) | Moment in x-direction $M_{ux}$ (kN-m) | Moment in y-direction $M_{uy}$ (kN-m) | Interaction charts | Computer software (SP column) |
|---------|------------------------|--------------------------------------|--------------------------------------|-------------------|--------------------------------|
| C1      | 2500                   | 250                                  | 120                                  | 2714              | 271                            | 130                             | 130                             | 2805                            | 280.5                           | 125                             |
| C2      | 1500                   | 200                                  | 90                                   | 1527              | 203                            | 91.62                           | 1751                            | 206                             | 103                             |
| C3      | 1700                   | 200                                  | 100                                  | 2035              | 239                            | 120                             | 1751                            | 206                             | 103                             |
| C4      | 500                    | 50                                   | 30                                   | 736.4             | 73.6                           | 44.2                            | 873                             | 87.4                            | 52.4                             |
| C5      | 400                    | 60                                   | 40                                   | 447               | 67                             | 44.7                            | 517                             | 77.6                            | 51.8                             |
| C6      | 1500                   | 300                                  | 300                                  | 1804              | 360                            | 360                             | 2240                            | 448                             | 448                             |

Proposed Interaction charts are compared with the values obtained from SP-Column and the results are depicted in Table 4.

### 4. EQUIVALENT UNIAXIAL ECCENTRICITY METHOD

These biaxial columns can also be analyzed using the equivalent uniaxial eccentricity method. In this method, the biaxial eccentricities $e_x$ and $e_y$ are replaced by an equivalent uniaxial eccentricity, $e_{ox}$ or $e_{oy}$ and the column is designed for the uniaxial bending.\(^{25}\)

If $e_x/b \geq e_y/h$, $e_{ox} = e_x + \alpha \frac{e_y}{h_b}$

where $b$, width of the column section and $h$, height of the column cross-section.

Also,

for $\frac{P_u}{f_y' A_g} \leq 0.4$, $\alpha = \left(0.5 + \frac{P_u}{f_y' A_g}\right) \frac{f_y + 300}{700} \geq 0.6$,

and for $\frac{P_u}{f_y' A_g} > 0.4$, $\alpha = \left(1.3 - \frac{P_u}{f_y' A_g}\right) \frac{f_y + 300}{700} \geq 0.5$.

For the cases, where the condition $e_x/b \geq e_y/h$ is not satisfied, either the axes may be interchanged ($e_x$ becomes $e_y$ and vice versa) or the equation may be written for $e_y$.

The equivalent uniaxial moments can be computed using the Equation (26).

$$M_{uox} = P_u \times e_{oy} \quad \text{or} \quad M_{uoy} = P_u \times e_{ox}.$$ \hspace{1cm} (26)

This method has certain restrictions which are as follows:

1. This method is only applicable for columns symmetrical about both the axes and the ratio of their sides ($b/h$) should be between 0.5 and 2.0.
2. The resulting reinforcement is to be placed in all four faces of the column.

### 4.1 Numerical examples

The selected six columns (C1-C6) in this study are also analyzed using this equivalent uniaxial eccentricity method and the results obtained using this method are compared with the Bresler method as well as with the finite element software (SP-Column).
TABLE 5  Equivalent eccentricity method design results

| Column | Check conditions | \( P_u \) (kN) | \( M_{uax} \) (kN-m) | Interaction chart (eccentricity method) |
|--------|-----------------|----------------|----------------------|---------------------------------------|
|        |                 | \( \gamma \)   | \( \alpha \)         | \( \beta \) \( P_c \) (kN) | \( M_{cox} \) (kN-m) |
| C1     | Satisfied       | 2500           | 300                  | 0.8  16.866  2.722  2878  348     |
| C2     | Satisfied       | 1500           | 268                  | 0.6  11.272  4.613  1528  275     |
| C3     | Satisfied       | 1700           | 261.5                | 0.7  16.373  5.0528 1995  308     |
| C4     | Satisfied       | 500            | 61                   | 0.8  19.213  5.89  999  122.5     |
| C5     | Satisfied       | 400            | 85                   | 0.7  7.800  5.585  457  98        |
| C6     | Not satisfied   | —              | —                    | —                  —                  —                  —                  —   |

The following steps need to be followed to find the required values of \( P_c \) and \( M_c \) using this method.

a. Satisfy the conditions for this method.
b. Calculate \( e_x \) and \( e_y \) to find the equivalent uniaxial eccentricity \( e_{ox} \) or \( e_{oy} \)
c. Find the equivalent uniaxial moments (\( M_{uox} \) or \( M_{uoy} \)).
d. Follow the steps (1-5), mentioned in the Reciprocal method, to design it as a uniaxial column.
e. Compute \( P_c = \theta abh \) and \( M_{cox/y} = \theta bh^2 \)

The biaxial moments are converted to the uniaxial moment either \( M_{uox} \) or \( M_{uoy} \) and columns are analyzed as uniaxial columns. The results obtained using this method are displayed in Table 5.

5  | RESULTS AND DISCUSSION

The results obtained from the column Interaction charts using both methods (Reciprocal and Equivalent eccentricity method) showed a safe and conservative column design. The axial load capacity obtained from both methods is compared with the SP-Column software and the results are displayed in the bar chart (Figure 13).

For column C-6, the results are only displayed for the Reciprocal method and the computer software, as this column does not satisfy the conditions for the equivalent eccentricity method. The axial load capacity for the above six columns showed promising results with a difference range of 5% to 13%, respectively.

The moment capacity results for the Bresler’s method and the computer software are displayed in the bar chart (Figure 14). The moments value obtained in both directions (x-x and y-y axis) using the Reciprocal method are quite close with the computer software results.

For the equivalent eccentricity method, the moments (\( M_{cox} \)) are compared with the \( M_{cx} \) of the reciprocal method and with the finite element software (Figure 15). In this method, the original moments (\( M_{ux} \) and \( M_{uy} \)) are converted to uniaxial moments (\( M_{uox} \) along the x-x axis. The \( M_{uox} \) values in equivalent eccentricity method are slightly higher.

**FIGURE 13**  Axial load capacity comparison (C1-C6)
than the $M_{ux}$ (reciprocal method). This shows that the moment capacity results obtained from the equivalent uniaxial eccentricity method are relatively higher than the Bresler’s formula and computer software.

6 | CONCLUSION

In this study, the analytical method is derived for generating the interaction diagrams for design of RC biaxial columns. The interaction diagram of any desired level of gamma ($\gamma$), concrete compressive strength ($f'_C$) and steel yield strength ($f_y$) can be generated to find the required axial load capacity ($P_c$) and moment capacity ($M_{cx}, M_{cy}$). Bresler’s method and the equivalent uniaxial eccentricity method are used to analyze the biaxial column using these interaction charts.

Six (RC) columns are analyzed in this study, which are subjected to biaxial bending. These columns are analyzed having different reinforcement ratios ($\rho$) and with different load capacity conditions. Using the ($\beta - \alpha$) interaction charts, the results obtained with the reciprocal method (Bresler’s formula) showed a good agreement with the computer software results. The average variation of analytically computed values to the finite element software was not more than 15%, showing relatively satisfactory results.

Equivalent eccentricity method is good, conservative and reliable method to design the columns subjected to biaxial bending, because it designs the column for bigger moment. Using this method, column can be designed as a uniaxial column, but this method is limited to certain conditions. The axial load capacity results obtained using this method also showed promising results as the $P_c$ values obtained are quite close with the ones obtained from Bresler’s method and with the finite element software. For the moment capacity analysis, the moments are slightly bigger than the Bresler’s method and the computer software.

In short, the developed interaction charts ($\beta - \alpha$) can be used to determine the required $P_c, M_{cx}$, and $M_{cy}$ for the preliminary design of reinforced biaxial concrete columns.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this article.
AUTHOR CONTRIBUTIONS
Mohammed Salem Al-Ansari lead formal analysis and supervision and equally contributed to conceptualization, data curation, investigation, methodology, validation, and Writing—review and editing. Muhammad Shekaib Afzal lead software and equally contributed to data curation, validation, writing original draft, review, and editing of this article.

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