Performance Analysis of IOS-Assisted NOMA System with Channel Correlation and Phase Errors

Tianxiong Wang, Mihai-Alin Badiu, Gaojie Chen, Senior Member, IEEE, and Justin P. Coon, Senior Member, IEEE,

Abstract—In this paper, we investigate the performance of an intelligent omni-surface (IOS) assisted downlink non-orthogonal multiple access (NOMA) network with phase quantization errors and channel estimation errors, where the channels related to the IOS are spatially correlated. First, upper bounds on the average achievable rates of the two users are derived. Then, channel hardening is shown to occur in the proposed system, based on which we derive approximations of the average achievable rates of the two users. The analytical results illustrate that the proposed upper bound and approximation on the average achievable rate of the strong user are asymptotically equivalent in the number of elements. Furthermore, it is proved that the average achievable rates with correlated and uncorrelated channels are asymptotically equivalent for a large number of elements. Simulation results corroborate the theoretical analysis and show that the channel hardening effect appears even for a few elements. The impact of channel correlation on the system performance in terms of average achievable rates is negligible for a large number of elements.

Index Terms—Intelligent omni-surface (IOS), NOMA, spatial correlation, average achievable rate

I. INTRODUCTION

The future sixth-generation (6G) is expected to support billions of connected devices with significant demands in spectrum efficiency, reliability, latency, and connectivity [1]. To handle these challenging requirements, a wide range of technologies, including massive multiple-input multiple-output (MIMO), network densification, and millimeter-wave (mm-wave) communications, have been investigated extensively in recent years [2]. Among various technologies, non-orthogonal multiple access (NOMA) has been regarded as a superior multiple access technology in the future wireless networks due to its promising advantages in terms of spectrum efficiency [3]–[5]. For example, multiple users could share the same resource block simultaneously in NOMA [6]. As pointed in [7], NOMA has better performance than orthogonal multiple access (OMA) when the channels gains of users are remarkably different. Conventionally, the channel gains of the users are stochastic and uncontrollable, determined by the wireless propagation environment [8], [9]. As a result, the applications of NOMA are limited in conventional wireless networks [10].

With the rapid development of metasurfaces, reconfigurable intelligent surfaces (RISs) have been recognized as a promising technology in the next-generation wireless communications networks [11]. Physically, a RIS consists of a large number of passive reflecting elements, each of which can shift the phases of the incident electromagnetic (EM) waves. Through intelligent phase shifting, a RIS could increase or decrease the composite channel gains of different users [12], [13]. Due to the advantage of RIS in reconfiguring the channels, many existing works investigated the potential of applying RIS in NOMA systems to broaden the applications of NOMA. The authors in [14] considered a RIS-aided NOMA network and analyzed the uplink outage probability of the system. Considering coherent and random phase shifting, the authors in [15] examined the impact of these two phase-shifting schemes on the outage performance of a RIS-assisted NOMA system. In [16], the capacity of a RIS-empowered NOMA network with multiple users was analyzed. The authors in [17] optimized the passive beamforming at RIS for minimizing the transmit power and compared the performance between RIS-assisted NOMA and RIS-assisted OMA.

However, the above works considered the reflecting-only RIS, which requires users located on the same side as the transmitter. As a result, the users located on the other side of the reflecting-only RIS are blocked, which limits the service coverage of RIS. The concept of intelligent omni-surface (IOS) with dual signal refraction and reflection functions was proposed to tackle this problem in some recent works. In contrast to the conventional reflecting-only RIS, signals impinging on one side of the IOS can be reflected and transmitted to users on the same and opposite sides of the surface as the transmitter, as illustrated in Fig. 1. A prototype of IOS was developed by NTT DOCOMO, Japan. By dynamically adjusting the space between the metasurface and the substrate, the incident radio waves can be entirely reflected, entirely transmitted, or simultaneously reflected and transmitted without attenuation. The authors in [18] optimized the spectral efficiency of a single-input single-output (SISO) communication link assisted by an IOS where the transmitting and reflecting signals share the same phase shift and showed the deployment of IOS can significantly enlarge the wireless coverage. As a step further, the joint active and passive beamforming design in a downlink IOS-assisted communication network with a multiple-antenna access point (AP) and multiple users was studied in [19], [20]. The authors in [21] proposed a new structure of IOS, also referred to as simultaneous transmitting and reflecting reconfigurable intelligent surface (STAR-RIS), where the phase shifts of the reflecting and transmitting signals can be configured independently. The authors in [22]
This paper presents novel results on the average achievable rates of an energy-splitting IOS-assisted NOMA network with correlated channels and imperfect phase adjustment. The main contributions of this paper are summarized as follows:

1) We analyze a downlink energy-splitting IOS-assisted NOMA communication system with spatially correlated channels and imperfect phase adjustment. The existing works have not covered several aspects of performance analysis and system design. First, most of the current works do not consider the effect of channel correlation in the performance analysis of IOS-assisted systems. However, as pointed in [27], the assumption of independent channels is valid only when the half-wavelength spaced elements are placed in a linear array. Hence, the performance of IOS-assisted networks with correlated channels requires more investigation. Second, the existing works on the performance analysis of IOS-assisted networks usually assume the channel estimation and phase adjustment are perfect. However, imprecise channel estimation and phase quantization may lead to phase errors in practice. Thus, it is essential to include phase errors in the performance analysis.

This paper presents novel results on the average achievable rates of an energy-splitting IOS-assisted NOMA network with correlated channels and imperfect phase adjustment. The main contributions of this paper are summarized as follows:

1) We analyze a downlink energy-splitting IOS-assisted NOMA communication system with spatially correlated channels and imperfect phase errors.

2) We derive upper bounds on the average achievable rates of the users with strong and weak channel conditions by using Jensen’s inequality, respectively.

3) We reveal the channel hardening effect in the IOS-assisted NOMA system with correlated channels and phase errors. Then, using the results of the channel hardening effect, two approximate expressions of the average achievable rates are obtained.

4) The upper bound derived from Jensen’s inequality and the approximation derived from the channel hardening effect are both proved to be asymptotically equivalent. Moreover, the average achievable rate with correlated channels is asymptotically equivalent to that with uncorrelated channels for a large number of elements.

This paper is organized as follows. In Section II, we introduce the system model used in this paper. Upper bounds on the average achievable rates of different users are analyzed in Section III. Section IV discusses the channel hardening effect, the asymptotic behavior of the average achievable rates, and the impact of quantization bits. Section V presents the numerical results, followed by the Conclusion in Section VI.

Notations: \( E[X] \) and \( \text{Var}[X] \) denote the expectation and variance of a random variable \( X \). \( \text{Cov}(X,Y) \) denotes the covariance of random variables \( X \) and \( Y \). Bold lower-case and bold upper-case letters represent vectors and matrices, respectively. \( \text{mod}(a,b) \) denotes the remainder of division \( a \) by \( b \). \( \lfloor a \rfloor \) represents the integer closest to \( a \) that is less than or equal to \( a \). \( \mathbb{C}^{N \times 1} \) denotes the \( N \times 1 \) complex vectors space. \( A^T \) and \( A^H \) respectively stand for the transpose and Hermitian transpose of matrix \( A \). \( \text{diag}(\mathbf{a}) \) represents a diagonal matrix with the elements of \( \mathbf{a} \) on its main diagonal. \( \xrightarrow{\Delta} \), \( \xrightarrow{P} \) and \( \xrightarrow{\text{def}} \) stand for ‘approaching to a particular value’, ‘convergence in probability’ and ‘definition’, respectively.

II. System Model

In this paper, we consider an IOS-assisted communication network, where a single-antenna transmitter (TX) communicates with two single-antenna users. Two users are located on different sides of the IOS. As shown in Fig. 1, the reflecting user \( R \) is located on the same side of the IOS as TX, while the transmitting user \( T \) is on the other side. It is assumed that obstacles block the direct links from the TX to the users. This set-up reflects scenarios where IOSs are deployed on the windows or building facades, serving users located inside and outside of the buildings simultaneously.
A. IOS Implementations

We consider an IOS with \( N = N_h N_e \) elements placed in a planar rectangular array. As illustrated in Fig. 2, the IOS is on the \( yOz \) plane with \( N_h \) elements per row and \( N_e \) elements per column. Without loss of generality, we assume each element is of size \( S = l \times w \), where \( l \) and \( w \) are the horizontal and vertical lengths of each element. The IOS is equipped on a building of height \( l_0 \), and the coordinate of the \( n \)th element can be written as

\[
a_n = [0, y(n)l, z(n)w + l_0],
\]

where

\[
y(n) = \text{mod}(n - 1, N_h),
\]

\[
z(n) = \left[ \frac{n - 1}{N_h} \right].
\]

\( y(n) \) and \( z(n) \) are the indexes of the \( n \)th element on the \( y \)-axis and \( z \)-axis.

Each IOS element is able to adjust phase shifts for the user \( T \) and user \( R \) independently, as introduced in [22]–[24]. Denoting the phase shifts imposed on the incident signals by the \( n \)th element in the transmitting and reflecting modes as \( \theta_n \) and \( \psi_n \), the transmitting and reflecting coefficients matrices can be written as

\[
\Theta = \alpha \text{diag} (e^{j\theta_1}, ..., e^{j\theta_N}),
\]

\[
\Psi = \beta \text{diag} (e^{j\psi_1}, ..., e^{j\psi_N}),
\]

where \( \alpha \) and \( \beta \) are the transmitting and reflecting amplitude coefficients of the IOS. The IOS is assumed to be passive with no energy dissipation. Hence, the amplitude coefficients of each element satisfy [26]

\[
\alpha^2 + \beta^2 = 1.
\]

B. Channel and Signal Models

1) Small Scale Fading: Let \( \mathbf{h} = (h_1, ..., h_N)^T \in \mathbb{C}^{N \times 1} \), \( \mathbf{g} = (g_1, ..., g_N)^T \in \mathbb{C}^{N \times 1} \) and \( \mathbf{r} = (r_1, ..., r_N)^T \in \mathbb{C}^{N \times 1} \) denote the normalized small scale fading of the TX-IOS, IOS-user \( T \) and IOS-user \( R \) links, respectively. IOS operates in the far-field of TX and users, thus the EM waves impinging on the IOS are plane waves with the wave vector [28]

\[
\mathbf{v}(\varrho, \varphi) = \frac{2\pi}{\lambda} [\cos(\varrho) \cos(\varphi), \sin(\varrho) \cos(\varphi), \sin(\varphi)],
\]

where \( \varrho \) and \( \varphi \) are the azimuth and elevation angles; \( \lambda \) is the wavelength.

Due to the multi-path propagation, the small scale fading \( \mathbf{h} \) is a superposition of the independent array response of each multi-path component \( \mathbf{c}_k \) as [29]

\[
\mathbf{h} = \sum_{k=1}^{K} \mathbf{c}_k = \sum_{k=1}^{K} \frac{l_k}{\sqrt{K}} \mathbf{c}_k,
\]

where \( l_k / \sqrt{K} \) is a complex random variable that stands for the attenuation and phase-rotation of the \( k \)th multi-path component and \( \mathbf{c}_k = [\exp(j \mathbf{va}_1^T), \exp(j \mathbf{va}_2^T), ..., \exp(j \mathbf{va}_K^T)]^T \).

In non-LoS scenarios, \( l_k, k \in \{1, 2, ..., K\} \), are independent and identical (i.i.d.) random variables with zero mean and unit variance. In a rich scattering environment, there are infinite number of multi-path components (\( K \rightarrow \infty \)). Hence, with the help of central limit theorem [27], we have

\[
\mathbf{h} \sim \mathcal{CN}(0_N, \mathbf{R}),
\]

where \( \mathbf{R} \) is the covariance matrix of \( \mathbf{h} \), defined as

\[
\mathbf{R} = \mathbb{E} [\mathbf{h}\mathbf{h}^H] = \frac{1}{K} \mathbb{E} \sum_{k=1}^{K} \mathbf{c}_k\mathbf{c}_k^H.
\]

Using the result in [27, Prop. 1], each element in \( \mathbf{R} \) can be written as

\[
[R]_{m,n} = \frac{\sin(\frac{2\pi}{\lambda} |\mathbf{a}_m - \mathbf{a}_n|)}{\frac{2\pi}{\lambda} |\mathbf{a}_m - \mathbf{a}_n|}.
\]

In similar ways, we can have

\[
g \sim \mathcal{CN}(0_N, \mathbf{R}),
\]

\[
r \sim \mathcal{CN}(0_N, \mathbf{R}).
\]

2) Large Scale Fading: The pathloss of the TX-IOS-user \( T \) and TX-IOS-user \( R \) links can be written as [30]

\[
\eta_T = \frac{A_t}{d_t^\chi}, \quad \eta_R = \frac{A_r}{d_r^\chi},
\]

where \( A_t \) and \( A_r \) stand for the intercepts at a reference distance of 1m of the TX-IOS-user \( T \)-user \( R \) links, respectively; \( \chi \) is the pathloss exponent; \( d_t \) and \( d_r \) denote the distances of the TX-IOS, IOS-user \( T \) and IOS-user \( R \) links, respectively.

3) Signal Models: In the NOMA transmission scheme, TX broadcasts the superimposed signals of the two users\(^1\) as

\[
s = \sqrt{P} (q_t s_t + q_r s_r),
\]

where \( s_t \) and \( s_r \) denote the signals of user \( T \) and user \( R \) with unit power, i.e., \( \mathbb{E} [s_t^2] = \mathbb{E} [s_r^2] = 1 \). \( P \) is the total transmit power of TX. \( q_t \) and \( q_r \) are the transmit power allocation coefficients for user \( T \) and user \( R \) at TX, satisfying \( q_t^2 + q_r^2 = 1 \). Hence, the received signals at user \( T \) and user \( R \) can be written as

\[
y_T = \sqrt{\eta_T} g_T^T \mathbf{h} \sqrt{P} (q_t s_t + q_r s_r) + n_t,
\]

\[
y_R = \sqrt{\eta_R} g_R^T \mathbf{h} \sqrt{P} (q_t s_t + q_r s_r) + n_r,
\]

where \( n_t \) and \( n_r \) are the additive white Gaussian noise at user \( T \) and user \( R \) with zero mean and variance \( \sigma_0^2 \).

In the NOMA transmission scheme, one of the two users performs successive interference cancellation (SIC) by decoding the other user’s signal, subtracting it from the received signal, and decoding its own signal. The optimal decoding

\(^1\)Note that more than two users can be selected to perform NOMA, but the performance might be degraded since the co-channel interference of the NOMA scheme can be severe and the SIC can be very complex. To optimize the performance of NOMA, hybrid NOMA can be implemented, by combining NOMA with conventional OMA, such as TDMA, FDMA, CDMA.
order corresponds to the ascending order of the channel gains of the two users. However, the order of the channel gains is dictated by the small scale fading which fluctuates quickly and renders the analysis complicated. It is worth highlighting that we do not aim to investigate the optimal decoding order in this work. Alternatively, the decoding order is assumed to be fixed as \((R, T)\). Specifically, user \(R\) decodes its signal directly by treating the signal of user \(T\) as interference, and user \(T\) performs SIC. A typical application scenario of IOS is to deploy IOS on the facades of buildings to serve indoor \((T)\) and outdoor \((R)\) users simultaneously. Since the distance of the IOS-user \(T\) channel is smaller than that of the IOS-user \(R\) channel, the large scale fading of the indoor user \((T)\) is less severe than that of the outdoor user \((R)\), which supports the practicability of the assumption. Furthermore, this assumption is also adopted in many other similar works, such as [6], [10], [31], [32]. Based on the principles of NOMA transmission scheme, user \(R\) is allocated with more transmit power to ensure that user \(T\) could perform SIC successfully, i.e., \(q_t < q_r\). Hence, signal to interference plus noise ratio (SINR) of user \(T\) decoding the signal of user \(R\) in the SIC process can be written as

\[
\gamma_t = \frac{\gamma_t}{\gamma_t} = \frac{P_0 q_t^2} {\gamma_t^2} \frac{|g_T^T \Theta_h|^2}{|g_T^T \Theta_h|^2 + \sigma_0^2}. \tag{14}
\]

After subtracting the signal of user \(R\) from the received signal, user \(T\) is able to decode its own signal with signal to noise ratio (SNR)

\[
\gamma_t = \gamma_0 q_t^2 \left| g_T^T \Theta_h \right|^2, \tag{15}
\]

where \(\gamma_0 = P/\sigma_0^2\) is the transmit SNR at TX.

On the other hand, user \(R\) decodes its own signal directly with SNR

\[
\gamma_r = \frac{\gamma_r}{\gamma_r} = \frac{P_0 q_r^2} {\gamma_r^2} \frac{|r^T \Psi_h|^2}{|r^T \Psi_h|^2 + \sigma_0^2}. \tag{16}
\]

4) Phase Adjustments: It can be proved that \(\gamma_{t-r}, \gamma_t\) and \(\gamma_r\) in (14), (15) and (16) are maximized when the phase shifts \(\hat{\theta}_n\) and \(\hat{\psi}_n\) are controlled to co-phase the transmitting and reflecting links, respectively [24]. Thus, the optimal phase shifts can be written as

\[
\hat{\theta}_n = -\arg (h_n) - \arg (g_n), \quad n \in \{1, \ldots, N\}, \tag{17a}
\]
\[
\hat{\psi}_n = -\arg (r_n) - \arg (g_n), \quad n \in \{1, \ldots, N\}. \tag{17b}
\]

However, the phase adjustment may not be perfect in practice due to imperfect channel knowledge and phase quantization, which leads to the phase error \(\{\phi_n^u\}_{n=1}^N, \ u \in \{t, r\}\) at each IOS element [8]. These two types of errors are introduced as follows:

- Channel estimation errors: Channel estimation may not be precise in practice. And the phase error \(\{\phi_n^u\}_{n=1}^N, \ u \in \{t, r\}\) is modeled by i.i.d. Von Mises random variables with zero mean and concentration parameter \(\kappa_u\) [33], of which the probability density function (PDF) can be written as

\[
f_{\phi_u^u}(\theta) = \frac{e^{\kappa_u \cos(\theta)}}{2\pi I_0(\kappa_u)}, \quad -\pi < \theta < \pi, \tag{18}
\]

where \(I_0(\cdot)\) is the modified Bessel function of the first kind of order zero. PDF of \(\phi_u^u\) is symmetric around zero and gets more concentrated with the increase of \(\kappa_u\), which also means the channel estimation is more accurate.

- Phase quantization errors: Assuming that each element of the IOS is a \(b\)-bit phase shifter, the phase error can be modeled by i.i.d. random variables uniformly distributed over \(\{-\pi, \pi\}\), whose distribution is symmetric around zero [34].

Due to phase errors, the realistic phase shifts at each element can be written as

\[
\hat{\theta}_n = \theta_n + \phi_n^t, \quad \hat{\psi}_n = \psi_n + \phi_n^r. \tag{19}
\]

5) Average Achievable Rate: After phase adjustment, \(\gamma_{t-r}, \gamma_t\) and \(\gamma_r\) can be rewritten as

\[
\gamma_t = \gamma_0 q_t^2 \gamma_t \alpha^2 H_t, \tag{20a}
\]
\[
\gamma_{t-r} = \frac{\gamma_{t-r}}{\gamma_{t-r}} = \frac{P_0 q_t^2 \gamma_r \alpha^2 H_r}{P_0 q_t^2 \gamma_r \alpha^2 H_t + \sigma_0^2}, \tag{20b}
\]
\[
\gamma_r = \frac{\gamma_r}{\gamma_r} = \frac{P_0 q_r^2 \gamma_r \beta^2 H_r}{P_0 q_r^2 \gamma_r \beta^2 H_t + \sigma_0^2}, \tag{20c}
\]

where

\[
H_t = \left| \sum_{n=1}^N |g_n| |h_n| e^{j\phi_n^t} \right|^2, \quad H_r = \left| \sum_{n=1}^N |r_n| |h_n| e^{j\phi_n^r} \right|^2. \tag{21}
\]

The average achievable rates of user \(T\) and user \(R\) can be written as [6]

\[
R_t = \mathbb{E}[\log_2(1 + \gamma_t)], \tag{22}
\]
\[
R_r = \mathbb{E}[\min \{\log_2(1 + \gamma_{t-r}), \log_2(1 + \gamma_r)\}]. \tag{23}
\]

III. AVERAGE ACHIEVABLE RATE ANALYSIS

In this section, we analyze the average achievable rates of user \(T\) and user \(R\), respectively.

A. Average Achievable Rate of User \(T\)

It is mathematically intractable to derive the exact closeform expression of \(R_t\). Alternatively, we use Jensen’s inequality to obtain an upper bound \(R_t^*\) as

\[
R_t^* = \log_2 \left( 1 + \mathbb{E}[\gamma_t] \right)
\]
\[
= \log_2 \left( 1 + \gamma_0 q_t^2 \gamma_t \alpha^2 \mathbb{E}[H_t] \right), \tag{24}
\]

where \(\mathbb{E}[H_t]\) can be further decomposed as (25) shown at the top of the next page. Since the channels \(h\) and \(g\) are correlated, we need to figure out \(\mathbb{E}[|h_n| |h_t|]\) and \(\mathbb{E}[|g_n| |g_t|]\). The result is summarized in the following proposition.
Proposition 1. For \( n \neq i \), \( \mathbb{E}[|w_n| | w_i|] \), \( w \in \{h, g, r\} \) can be expressed in terms of \( \mathbb{E}[w_n w_i^*] \) as

\[
\mathbb{E}[|w_n| | w_i|] = \left( \frac{\mathbb{E}[|w_n w_i^*|^2]}{2} - \frac{1}{2} \right) K \left( \mathbb{E}[|w_n w_i^*|^2] \right) + E \left( |\mathbb{E}[|w_n w_i^*|^2]| \right).
\]

(26)

For \( n = i \),

\[
\mathbb{E}[|w_n| | w_i|] = 1.
\]

(27)

\( K(\cdot) \) and \( E(\cdot) \) are the complete elliptic integral of the first kind and the second kind, respectively.

Proof: See Appendix A.

Corollary 1. Under the proposed system settings, \( \frac{\pi}{4} \leq \mathbb{E}[|w_n| | w_i|] \leq 1 \), for all \( n, i \).

Proof: See Appendix B.

Since the covariance matrices of \( h \) and \( g \) are equal, it can be learned that the matrices \( \mathbb{E}[|h| | h|^T] \) and \( \mathbb{E}[|g| | g|^T] \) are equal, and we denote each by \( R \). Applying Proposition 1, an upper bound on the average achievable rate of user \( T \) can be obtained, which is summarized in the following proposition.

Proposition 2. An upper bound on the average achievable rate of user \( T \) can be written as

\[
R_T^* = \log_2 \left( 1 + \gamma_0 q_t^2 \eta_t \alpha^2 \left( N(1 - \epsilon_t^2) + \epsilon_t^2 \text{tr} (\bar{R} \bar{R}) \right) \right).
\]

(28)

where

\[
\epsilon_t = \mathbb{E}[\cos(\phi_t^i)], \quad n \in \{1, 2, ..., N\}.
\]

(29)

For phase estimation errors,

\[
\epsilon_t = \frac{I_1(\kappa_t)}{I_0(\kappa_t)}.
\]

(30)

For phase quantization errors,

\[
\epsilon_t = \frac{2^b \sin \left( \frac{\pi}{2^b} \right)}{\pi}.
\]

(31)

Proof: See Appendix C.

Remark 2. When the phase errors are uniformly distributed over \([-\pi, \pi]\), i.e., \( \epsilon_t = 0 \), the upper bound on the average achievable rate of user \( R \) is

\[
R_T^* = \log_2 \left( 1 + \gamma_0 q_t^2 \eta_t \alpha^2 N \right).
\]

(33)

It can be seen that \( R_T^* \) scales with \( \log_2(N) \) when the phase errors are uniformly distributed over \([-\pi, \pi]\), which coincides with the proposition in [35].

Corollary 2. Under the proposed system settings, \( \mathbb{E}[H_1] \) satisfies

\[
N + \frac{\pi^2 N(N - 1)}{16} \epsilon_t^2 \leq \mathbb{E}[H_1] < N + N(N - 1) \epsilon_t^2.
\]

(34)

Proof: The conclusion is obtained by substituting the result in Corollary 1 into (25).

Remark 3. It can be learned from Corollary 2 that \( R_T^* \) scales with \( \log_2(N^2) \) when \( \epsilon_t \neq 0 \). Compared with Remark 2, it can be seen that even imperfect phase adjustment is able to provide performance gain, which scales with \( \log_2(N) \). This is a known result for the system with uncorrelated channels [8], and it is shown here that it still works for correlated channels. Simulation results will illustrate this point in Section V.

B. Average Achievable Rate of User \( R \)

It appears that obtaining the close-form expression of \( R_r \) in (23) is intricate. Alternatively, we first analyze \( R_r \) for large transmit SNR \( \gamma_0 \) and then seek for an upper bound based on Jensen’s inequality.

1) Analysis of Large Transmit SNR: \( \gamma_{t \rightarrow r} \), defined in (20b), can be rewritten as

\[
\gamma_{t \rightarrow r} = \frac{q_r^2}{q_t^2 + (\gamma_0 \eta_t \alpha^2 H_1)^{-1}}.
\]

(35)

As \( \gamma_0 \rightarrow \infty \), we can have

\[
\gamma_{t \rightarrow r} \rightarrow \frac{q_r^2}{q_t^2}.
\]

(36)

The same conclusion can be derived for \( \gamma_r \): \( \gamma_r \rightarrow q_r^2/q_t^2 \) as \( \gamma_0 \rightarrow \infty \). Hence,

\[
R_r \rightarrow \log_2 \left( 1 + \frac{q_r^2}{q_t^2} \right), \quad \gamma_0 \rightarrow \infty.
\]

(37)
2) Analysis of Finite Transmit SNR: The min function is concave [36]. Therefore, Jensen’s inequality can be applied on $R_r$ to obtain an upper bound on $R_r$ as

$$R_r \leq \min \{ \log_2 (1 + \mathbb{E}[\gamma_{t-r}]), \log_2 (1 + \mathbb{E}[\gamma_r]) \}. \tag{38}$$

Referring to (20b) and (20c), (38) is hard to deal with due to the common terms in the numerators and denominators of $\gamma_{t-r}$ and $\gamma_r$. Specifically, the common term in the numerator and denominator of $\gamma_{t-r}$ is $H_t$, and the common term of $\gamma_r$ is $H_r$. Fortunately, it can be proved that $\gamma_{t-r}$ and $\gamma_r$ are concave to their respective common terms. Thus, Jensen’s inequality can be applied again, which gives

$$R_r \leq \min \left\{ \log_2 \left(1 + \frac{q_r^2}{q_t^2 + f_{t-r}} \right), \log_2 \left(1 + \frac{q_r^2}{q_t^2 + f_{t-r}} \right) \right\}. \tag{39}$$

Following the similar procedures in the Propositions 1 and 2, an upper bound $R^*_r$ can be written as

$$R^*_r = \min \left\{ \log_2 \left(1 + \frac{q_r^2}{q_t^2 + f_{t-r}} \right), \log_2 \left(1 + \frac{q_r^2}{q_t^2 + f_{t-r}} \right) \right\} = \begin{cases} \log_2 \left(1 + \frac{q_r^2}{q_t^2 + f_{t-r}} \right), & f_t < f_r, \\ \log_2 \left(1 + \frac{q_r^2}{q_t^2 + f_{t-r}} \right), & f_t \geq f_r, \end{cases} \tag{40}$$

where

$$\epsilon_r = \mathbb{E}[\cos(\phi_n^r)], \ n \in \{1, 2, ..., N\}, \tag{41a}$$

$$f_t = \gamma_0 \eta t \alpha^2 \left( N(1 - \epsilon_t) + \epsilon_t^2 \text{tr} \left( \mathbf{R} \mathbf{R}^H \right) \right), \tag{41b}$$

$$f_r = \gamma_0 \eta r \beta^2 \left( N(1 - \epsilon_r) + \epsilon_r^2 \text{tr} \left( \mathbf{R} \mathbf{R}^H \right) \right). \tag{41c}$$

IV. LARGE ARRAY ANALYSIS

In this section, we show that the channel hardening effect appears in the proposed system and revisit the average achievable rates for a large number of IOS elements.

A. Channel Hardening Analysis

As defined in [27], the channel hardening effect in a RIS-assisted network refers that the received SNR variations average out and the received SNR is approximately equal to $N^2$ times a constant as $N$ goes large. The channel hardening effect in the proposed system model with correlated channels and imperfect phase adjustment is presented in the following proposition.

**Proposition 3.** The composite channel gains associated with user $T$ and user $R$, namely $H_t$ and $H_r$, satisfy

$$\frac{1}{N^2} H_t \xrightarrow{P} \frac{\pi^2}{16} \epsilon_t^2, \ N \rightarrow \infty, \tag{42a}$$

$$\frac{1}{N^2} H_r \xrightarrow{P} \frac{\pi^2}{16} \epsilon_r^2, \ N \rightarrow \infty, \tag{42b}$$

with the convergence in probability. Hence, for non-uniform phase errors over $[-\pi, \pi]$, $H_t$ and $H_r$ can be approximated by a constant as

$$H_t \approx \frac{\pi^2}{16} N^2 \epsilon_t^2, \ H_r \approx \frac{\pi^2}{16} N^2 \epsilon_r^2, \tag{43}$$

when the number of IOS elements is large.

**Proof:** See Appendix D.

**Remark 4.** Under the correlation profile proposed in (9), the channel correlation does not play a role in the conclusion of Proposition 3. And, in the case of uncorrelated channels, the same approximations on $H_t$ and $H_r$ can derived by using the conventional law of large numbers.

B. Average Achievable Rates for Large Number of Elements

We revisit the average achievable rates when the number of elements is large. By using (43), the average achievable rate of user $T$ and user $R$ with non-uniform phase errors over $[-\pi, \pi]$ can be approximated as

$$R_t \approx R_t^* = \log_2 \left(1 + \frac{\pi^2 N^2 \gamma_0 \epsilon_t^2 \eta \alpha^2}{16} \right), \tag{44}$$

$$R_r \approx R_r^* = \begin{cases} \log_2 \left(1 + \frac{\pi^2 N^2 \gamma_0 \epsilon_r^2 \eta \beta^2}{16} \right), & \epsilon_r^2 \eta \alpha^2 < \epsilon_r^2 \eta \beta^2, \\ \log_2 \left(1 + \frac{\pi^2 N^2 \gamma_0 \epsilon_r^2 \eta \beta^2}{16} \right), & \epsilon_r^2 \eta \alpha^2 \geq \epsilon_r^2 \eta \beta^2. \end{cases} \tag{45}$$

**Remark 5.** The impact of phase quantization bits on the average achievable rate can be analyzed by using the result in (44). The average achievable rate improvement of user $T$ brought by adding one bit for phase quantization can be expressed as

$$R_b = \log_2 \left(\frac{16 + \pi^2 N^2 \gamma_0 \epsilon_b \eta \alpha^2}{16 + \pi^2 N^2 \gamma_0 \epsilon_b \eta \alpha^2} \right), \tag{46}$$

where

$$\epsilon_b = \frac{2^b \sin \left( \frac{\pi}{2^{b+1}} \right)}{\pi}. \tag{47}$$

For large number of elements, we can have

$$R_b \sim f(b), \ N \rightarrow \infty, \tag{48}$$

where

$$f(b) = \log_2 \left(\frac{4 \sin^2 \left( \frac{\pi}{2^{b+1}} \right)}{\sin^2 \left( \frac{\pi}{2^{b}} \right)} \right). \tag{49}$$

$f(b)$ is always positive. However,

$$f'(b) = -\frac{\pi}{2^{b+1}} \tan \left( \frac{\pi}{2^{b+1}} \right) < 0. \tag{50}$$

Therefore, adding the number of quantization bits can always improve the average achievable rate of user $T$, but the rate of improvement decreases.
C. Asymptotic Analysis

Relations between $R_t^*$ and $R_t^+$ as well as between $R_r^*$ and $R_r^+$ are analyzed and summarized in the following proposition.

**Proposition 4.** For large number of IOS elements with non-uniform phase errors over $[-\pi, \pi)$, the following asymptotic equivalence relations hold:

\[ R_t^* \sim R_t^+, \quad N \to \infty, \quad (51a) \]

\[ R_r^* \sim R_r^+, \quad N \to \infty. \quad (51b) \]

**Proof:** See Appendix E.

Next, we consider the relation between $R_t$ and $R_t^*$.

**Proposition 5.** For large number of IOS elements with non-uniform phase errors over $[-\pi, \pi)$, $R_t$ and $R_t^*$ satisfy the asymptotic equivalence, i.e.,

\[ R_t \sim R_t^*, \quad N \to \infty. \quad (52) \]

Further, due to the transitive property of asymptotic analysis and Proposition 4, $R_t$ and $R_t^+$ are also asymptotically equivalent, i.e.,

\[ R_t \sim R_t^+, \quad N \to \infty. \quad (53) \]

**Proof:** See Appendix F.

**Remark 6.** Eq.(52) in Proposition 5 does not hold when the phase errors are uniformly distributed over $[-\pi, \pi)$. In this case, $R_t^*$ only acts as an upper bound to $R_t$, The reason is clarified in Appendix F.

**Remark 7.** Due to changing the order of computing expectation and minimum, the relation between $R_r$ and $R_r^*$ cannot be analyzed similarly. However, the high SNR regime is of more interests from the perspective of engineering. In the high SNR regime, $R_r$ can be well approximated by $\log_2 \left( 1 + q_r^2/q_t^2 \right)$.

With the conclusions in Propositions 4 and 5, the relation between the average achievable rates of user $T$ with correlated channels, $R_t$, and uncorrelated channels, $R_{t,u}$, is presented in the following proposition.

**Proposition 6.** For large number of IOS elements with non-uniform phase errors over $[-\pi, \pi)$, $R_t$ is asymptotically equivalent to $R_{t,u}$, i.e.,

\[ R_t \sim R_{t,u}, \quad N \to \infty. \quad (54) \]

**Proof:** Denote the approximate expression of $R_{t,u}$ derived from the channel hardening effect as $R_{t,u}^h$. Referring to Remark 4, we have $R_t^* = R_{t,u}^h$. Then, following the similar procedures in Propositions 4 and 5, we can obtain $R_t \sim R_{t,u}$ as $N \to \infty$. \(\square\)

V. Numerical Results

In this section, we present numerical results to evaluate the accuracy of the proposed analysis by Monte Carlo (MC) simulations and show the impacts of phase errors and channel correlations on the system performance.

A. Simulation Setup

Unless otherwise stated, the simulation parameters are set as follows. The distances of the TX-IOS, IOS-user $T$ and IOS-user $R$ links are set to be $d_T = 10$ m, $d_R = 5$ m and $d_r = 10$ m. The pathloss exponent $\chi$ is 2.4 and the carrier frequency is 3 GHz with the wavelength $\lambda = 0.1$ m. Each IOS element is of size $l = \frac{\lambda}{2}$ and $w = \frac{\lambda}{2}$. The energy splitting coefficients of user $T$ and user $R$ are given as $\alpha = 0.8$ and $\beta = 0.6$, respectively. The intercepts at unit distance of the transmitting and reflecting channels are $\Lambda_t = \Lambda_r = -30$ dB. The transmit power allocation coefficients are $q_t = 0.6$ and $q_r = 0.8$. The noise power is $\sigma_n^2 = -50$ dBm.

B. Average Achievable Rates versus Number of IOS Elements

We investigate the average achievable rates versus the number of IOS elements with different phase quantization bits. IOS elements are assumed to be placed in a uniform rectangular array with four elements per column, i.e., $N_v = 4$. The number of elements per row varies in $N_h \in [1, 25]$. The transmit power is set to be 20 dBm. As benchmarks, the systems with perfect phase adjustment, i.e., $\phi_n^0 = 0$, and with uniform phase error over $[-\pi, \pi)$, are both shown. In the legend, ‘Jensen’ and ‘Hardening’ represent the upper bound and approximation derived in Sec. III and Sec. IV; ‘MC Simulations’ denotes the MC simulation results.

As shown in Fig. 3, the proposed upper bound and approximation on $R_t$ are very accurate even for a small number of elements. Increasing the number of elements can increase the average achievable rate of user $T$, but the slope of the curve becomes flat, which can be concluded from the log relation between $R_t$ and $N^2$ in Remark 3. As expected in Remark 5, adding the number of quantization bits is able to improve system performance. However, the performance gain diminishes with the increase of quantization levels. For example, when the number of elements is 60, the average achievable rates of user $T$ are 5 bit/s/Hz and 9.7 bit/s/Hz for uniform phase errors and 1-bit phase shifting, respectively. However, compared with 1-bit phase shifting, the performance gain brought by 2-bit phase shifting is much smaller with 10.7 bit/s/Hz, and it is very close to the performance of perfect phase shifting with 11 bit/s/Hz. Besides, as expected in Remark 6, $R_t^*$ can only act as an upper bound when the phase errors are uniformly distributed over $[-\pi, \pi)$.

Fig. 4 illustrates the average achievable rate of user $R$ against the number of elements. The proposed upper bound is relatively loose for a small number of elements and becomes tight as the number of elements grows large. ‘Upper bound’ in the legend stands for $\log_2 (1 + q_r^2/q_t^2)$. As can be seen from Fig. 4, the average achievable rate of user $R$ converges to $\log_2 (1 + q_r^2/q_t^2)$ with increasing numbers of elements.
C. Average Achievable Rates versus Transmit SNR

The relation between average achievable rates and transmit SNR is explored in Fig. 5, where a rectangular IOS with $N_v = 2$ elements per column and $N_h = 8$ elements per row is used. The transmit SNR lies in the interval $\gamma_0 \in [20, 100]$ dB. Channel estimation errors are considered by setting the concentration parameter of the Von Mises distribution to $\kappa_u \in \{1, 2\}$, $u \in \{t, r\}$. It can be observed from Fig. 5 that the average achievable rate of user $T$ increases with the transmit SNR, as expected. The proposed upper bound and approximation are very similar. In addition, in Fig. 6, the average achievable rate of user $R$ grows and converges to the upper bound $\log_2(1 + q_r^2/q_t^2)$ with increasing SNR. This observation corroborates our discussion in Sec. III.

D. Correlated Channels versus Uncorrelated Channels

The impact of channel correlations on the system performance is investigated in Fig. 7, where a 1-bit phase adjustment is performed at the IOS. We consider a rectangular IOS, where each IOS element is of size $5 \text{ cm} \times 5 \text{ cm}$ and is placed in a $5 \times N_h$ rectangular array, with $N_h$ varying in the interval $[1, 20]$. We assume the wavelength $\lambda = 20 \text{ cm}$. According to (9), the channels are correlated in this case. The average achievable rate with uncorrelated channels is provided as a benchmark. As shown in Fig. 7, the average achievable rates of uncorrelated and correlated channels are close to each other, even for a small number of elements. For example, when $N = 15$, the performance gap is $0.09 \text{ bit/s/Hz}$. Moreover, the performance gap of the average achievable rates between uncorrelated and correlated channels decreases with an increase in the number of elements, as expected from Proposition 6. For example, for $N = 90$, the performance gap decreases to $0.02 \text{ bit/s/Hz}$.

2In practice, the channels associated with the IOS are uncorrelated when the elements are placed in a linear array, and the spacing of elements equals $\lambda/2$ times integers. Besides, the covariance matrix $R$ of uncorrelated channels in (9) is the identity matrix.
VI. Conclusions

This paper proposed an IOS-assisted downlink NOMA network with spatially correlated channels and phase errors and analyzed the average achievable rates of two users. Specifically, we investigated an upper bound on the average achievable rates using Jensen’s inequality. The channel hardening effect in the proposed system was exploited, from which an approximation on the average achievable rates was derived. The proposed upper bound and approximation were proved to be asymptotically equivalent. It was shown that the average achievable rates with correlated and uncorrelated channels converge to the same value for large numbers of elements. Simulation results were presented to validate the analysis and showed that the impact of channel correlations on the average achievable rates is small for a large number of elements.

Appendix A

In the proposed system model, $h_n$ can be written as

$$h_n = R_n \exp(j \vartheta_n) = X_n + j Y_n, \quad n \in \{1, 2, ..., N\}, \quad (A.1)$$

where $R_n$, $\vartheta_n$, $X_n$ and $Y_n$ denote the amplitude, phase angle, real part and imaginary part of $h_n$, respectively; $X_n$ and $Y_n$ are uncorrelated normal random variables, each with zero mean and variance as

$$\tau \triangleq \text{Var}[X_n] = \text{Var}[Y_n] = \frac{1}{2}. \quad (A.2)$$

For the channels associated with two different IOS elements, i.e., $h_n$ and $h_i$, it can be obtained that

$$\begin{align*}
E[X_n X_i] &= E[\{h_n\}\{h_i\} \cos(\vartheta_n) \cos(\vartheta_i)], \quad (A.3a) \\
E[Y_n Y_i] &= E[\{h_n\}\{h_i\} \sin(\vartheta_n) \sin(\vartheta_i)] \\
&= E[\{h_n\}\{h_i\} \cos(\vartheta_n + \pi/2) \cos(\vartheta_i + \pi/2)]. \quad (A.3b)
\end{align*}$$

Since $\vartheta_n$, $n \in \{1, 2, ..., N\}$, are uniformly distributed over $[-\pi, \pi]$ and the above functions are periodic, we can have

$$\tau_1 \triangleq E[X_n X_i] = E[Y_n Y_i]. \quad (A.4)$$

Similarly,

$$\begin{align*}
E[X_n Y_i] &= E[\{h_n\}\{h_i\} \cos(\vartheta_n) \sin(\vartheta_i)], \\
E[Y_n X_i] &= E[\{h_n\}\{h_i\} \sin(\vartheta_n) \cos(\vartheta_i)] \\
&= -E[\{h_n\}\{h_i\} \cos(\vartheta_n + \pi/2) \sin(\vartheta_i + \pi/2)]. \quad (A.5a)
\end{align*}$$

Hence,

$$\tau_2 \triangleq E[X_n Y_i] = -E[Y_n X_i]. \quad (A.6)$$

For $n \neq i$, the joint distribution of $(X_n, Y_n, X_i, Y_i)$ can be written as

$$f(x_n, y_n, x_i, y_i) = \frac{1}{(2\pi)^2 \sqrt{\det C}} \exp \left(-\frac{1}{2} \mathbf{v} C^{-1} \mathbf{v}^T\right), \quad (A.7)$$

where $\mathbf{v} = [x_n, y_n, x_i, y_i]$; $C$ is the covariance matrix of $\{X_n, Y_n, X_i, Y_i\}$, given by

$$C = \begin{pmatrix} \tau & 0 & \tau_1 & \tau_2 \\ 0 & \tau & -\tau_2 & \tau_1 \\ \tau_1 & -\tau_2 & \tau & 0 \\ \tau_2 & \tau_1 & 0 & \tau \end{pmatrix}. \quad (A.8)$$

The determinant and inverse of $C$ can be calculated as

$$\det\{C\} = (\tau^2 - \tau_1^2 - \tau_2^2)^2, \quad (A.9)$$

$$C^{-1} = \frac{1}{\tau^2 - \tau_1^2 - \tau_2^2} \begin{pmatrix} \tau & 0 & \tau_1 & \tau_2 \\ 0 & \tau & -\tau_2 & \tau_1 \\ \tau_1 & -\tau_2 & \tau & 0 \\ \tau_2 & \tau_1 & 0 & \tau \end{pmatrix}. \quad (A.10)$$

Taking (A.9) and (A.10) into (A.7), the joint distribution of $(X_n, Y_n, X_i, Y_i)$ can be obtained. Then, using the theorem of the transformation of random variables:

$$f(r_n, r_i, \vartheta_n, \vartheta_i) = \left|\det(J(r_n, r_i, \vartheta_n, \vartheta_i))\right| f(x_n, y_n, x_i, y_i), \quad (A.11)$$

where

$$J(r_n, r_i, \vartheta_n, \vartheta_i) = \frac{\partial(x_n, y_n, x_i, y_i)}{\partial(r_n, r_i, \vartheta_n, \vartheta_i)} \quad (A.12)$$

is the Jacobian matrix for the transformation

$${x_n} = r_n \cos(\vartheta_n), {y_n} = r_n \sin(\vartheta_n), \quad x_i = r_i \cos(\vartheta_i), y_i = r_i \sin(\vartheta_i), \quad (A.13)$$

we can derive the joint distribution of $(R_n, R_i, \vartheta_n, \vartheta_i)$ as

$$f(r_n, r_i, \vartheta_n, \vartheta_i) = \frac{r_n r_i}{(2\pi)^2 (\tau^2 - q^2)} \exp \left[-\frac{\tau (r_n^2 + r_i^2) - 2r_n r_i \cos(\vartheta_i - \vartheta_n - \xi)}{2 (\tau^2 - q^2)}\right], \quad (A.14)$$

where

$$\xi = \arctan\left(\frac{\tau_2}{\tau_1}\right), \quad (A.15a)$$

$$q^2 = \tau_1^2 + \tau_2^2 = \frac{\{E[|h_n h_i^*|^2]\}^2}{4}. \quad (A.15b)$$
Using [37, eq.6.31.1, eq.7.621.4] and after some mathematical manipulation, \( \mathbb{E}[R_n R_i] \) can be expressed as

\[
\mathbb{E}[R_n R_i] = \int_0^\infty \int_0^\infty \int_0^{2\pi} \int_0^{2\pi} r_n r_i f(r_n, r_i, \vartheta_n, \vartheta_i) d\vartheta_n d\vartheta_i dr_n dr_i \]

\[
= \left( \frac{q^2}{\tau} - \tau \right) K \left( \frac{q^2}{\tau^2} \right) + 2\tau E \left( \frac{q^2}{\tau^2} \right).
\]

(A.16)

For \( n = i \), it is easy to show that

\[
\mathbb{E}[R_n R_i] = 2\tau = 1.
\]

(A.17)

The same conclusions can be derived for the channels \( g \) and \( r \) following the framework. Thus, the statement in Proposition 1 is proved.

**APPENDIX B**

Define

\[
f(x) = \left( \frac{x}{2} - \frac{1}{2} \right) K(x) + E(x).
\]

(B.1)

It can be proved that \( f(x) \) is an increasing function for \( x \geq 0 \). Since

\[
0 \leq |\mathbb{E}[w_n w_i^*]|^2 \leq 1,
\]

taking (B.2) into (26), we can have

\[
\frac{\pi}{4} \leq \mathbb{E}[w_n w_i^*] \leq 1.
\]

(B.3)

**APPENDIX C**

Since

\[
\mathbb{E} \left[ \sum_{n=1}^{N} |g_n|^2 |h_n|^2 \right] = N,
\]

we can have

\[
\mathbb{E} \left[ \sum_{n=1}^{N} \sum_{i=n+1}^{N} |g_n| |h_n| |g_i| |h_i| \right] = \frac{\text{tr} (\hat{R} \hat{R})}{2} - N.
\]

(C.2)

Therefore,

\[
\mathbb{E} \left[ \sum_{n=1}^{N-1} \sum_{i=n+1}^{N} |g_n| |h_n| \cos(\phi_n^i) |g_i| |h_i| \cos(\phi_i^i) \right] = \frac{\gamma^2}{2} (\text{tr} (\hat{R} \hat{R}) - N).
\]

(C.3)

Moreover,

\[
\mathbb{E} \left[ \sum_{n=1}^{N-1} \sum_{i=n+1}^{N} |g_n| |h_n| \sin(\phi_n^i) |g_i| |h_i| \sin(\phi_i^i) \right] = 0,
\]

(C.4)

due to the symmetrical distribution around zero of \( \phi_n, n \in \{1, ..., N\} \). Substituting (C.1), (C.3) and (C.4) into (25), we can obtain (28). Thus, the proof is complete.

**APPENDIX D**

To begin with, a Lemma regarding the sum of correlated random variables is presented as follows.

**Lemma 1** ([38]). Consider a sequence of random variables \( \{W_k\} \). If \( \{W_k\} \) satisfies

- \( \mathbb{E}[W_k] = a \),
- \( \text{Var}[W_k] \) is bounded,
- \( \text{Cov}[W_i, W_j] \to 0 \) as \( |i - j| \to \infty \),

then,

\[
\frac{1}{K} \sum_{k=1}^{K} W_k \to a, \ K \to \infty,
\]

(D.1)

with the convergence in probability.

**Proof:** Even though this is a known result, we include a proof because the obtained scaling of the variance with \( K \) is used in this paper in the proof of Proposition 4. Since the variance is bounded, the covariance of \( W_i \) and \( W_j \) is bounded as

\[
\text{Cov} \{W_i, W_j\} \leq c_0,
\]

(D.2)

where \( c_0 \) is a constant. Denoting \( V_K = \sum_{k=1}^{K} W_k \), the variance of \( V_K \) can be written as

\[
\text{var}[V_K] = \sum_{k=1}^{K} \text{Cov} \{W_k, W_k\} + 2 \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \text{Cov} \{W_i, W_j\}
\]

\[
\leq K c_0 + 2 \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \text{Cov} \{W_i, W_j\}.
\]

(D.3)

Since \( \text{Cov} \{W_i, W_j\} \to 0 \) when \( |i - j| \to \infty \), we can find \( K_0 \) such that for \( |i - j| > K_0 \), \( \text{Cov} \{W_i, W_j\} < \varepsilon \), for any \( \varepsilon > 0 \). Hence,

\[
\left| \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \text{Cov} \{W_i, W_j\} \right| \leq \sum_{j=i+1}^{K-1} \text{Cov} \{W_i, W_j\} \leq K K_0 c_0 + K^2 \varepsilon.
\]

(D.4)

Therefore, the variance of \( V_K/K \) satisfies

\[
\frac{\text{Var}[V_K]}{K} \leq \frac{c_0}{K} + \frac{2K_0 c_0}{K} + 2\varepsilon.
\]

(D.5)

It can be seen that \( \text{Var}[V_K/K] \to 0 \), as \( K \to \infty \). Thus, the proof of Lemma 1 is complete.

The composite channel associated with user \( T \) is analyzed first. Let \( Y_n = |g_n| |h_n|^2 e^{j\phi_n^e} \). We can have

\[
\mathbb{E}[Y_n] = \frac{\pi}{4} a_t,
\]

(D.6)

and the variance of \( Y_n \) is bounded. Covariance of \( \{Y_n, Y_i\} \) can be written as

\[
\text{Cov} \{Y_n, Y_i\} = \mathbb{E}[|g_n| |h_n|^2 e^{j\phi_n^e} |g_i| |h_i|^2 e^{-j\phi_i^e}] - \mathbb{E}[|g_n| |h_n|^2 e^{j\phi_n^e}] \mathbb{E}[|g_i| |h_i|^2 e^{-j\phi_i^e}].
\]

(D.7)
From (9) and (26), it can be learned that
\[ \mathbb{E}[|w_n| |w_i|] \rightarrow \mathbb{E}[|w_n|] \mathbb{E}[|w_i|] = \frac{\pi}{4}, \quad |n-i| \rightarrow \infty, \quad (D.8) \]
where \( w \in \{ h, g, r \} \). Given that the phase errors are i.i.d. with symmetric distribution around zero, it can be proved that
\[ \text{Cov}\{Y_n, Y_i\} = \mathbb{E}[|g_n||g_i|] \mathbb{E}[|h_n||h_i|] \epsilon_t^2 - \mathbb{E}[|g_n||g_i|] \mathbb{E}[|h_n|] \mathbb{E}[|h_i|] \epsilon_t^2 \rightarrow 0, \quad |n-i| \rightarrow \infty. \quad (D.9) \]
Hence, using Lemma 1, we obtain
\[ \frac{1}{N} \sum_{n=1}^{N} |g_n| |h_n| e^{j\phi_n} \rightarrow \frac{\pi}{4} \epsilon_t, \quad N \rightarrow \infty. \quad (D.10) \]

With the continuous mapping theorem, (42a) in Proposition 3 can be obtained. Following the same framework, (42b) can be derived.

**APPENDIX E**

Let
\[
A_N = \sum_{n=1}^{N} |g_n| |h_n| \cos(\phi_n^s), \quad (E.1a)
\]
\[
B_N = \sum_{n=1}^{N} |g_n| |h_n| \sin(\phi_n^s), \quad (E.1b)
\]
The expectations of \( A_N \) and \( B_N \) are \( \mathbb{E}[A_N] = \frac{\pi}{4} N \epsilon_t \) and \( \mathbb{E}[B_N] = 0. \mathbb{E}[A_N^2 + B_N^2] \) can be written as
\[
\mathbb{E}[A_N^2 + B_N^2] = \mathbb{E}^2[A_N] + \text{var}[A_N] + \mathbb{E}^2[B_N] + \text{var}[B_N]. \quad (E.2)
\]
As shown in Lemma 1, the variances of \( A_N \) and \( B_N \) scale with \( N \). Thus, when the phase errors are non-uniform over \([-\pi, \pi]\), \( \mathbb{E}[A_N^2 + B_N^2] \) satisfies the asymptotic equivalence as
\[
\mathbb{E}[A_N^2 + B_N^2] \sim \frac{\pi^2}{16} N^2 \epsilon_t^2, \quad N \rightarrow \infty. \quad (E.3)
\]
Thus, with the properties of asymptotic analysis, we can obtain (51a) in Proposition 4. Following the similar procedures, (51b) can be derived as well.

**APPENDIX F**

As shown in Proposition 3, the random variable \( G_N = \frac{1}{N^2} H_t \) satisfies
\[
G_N \stackrel{P}{\rightarrow} \frac{\pi^2}{16} \epsilon_t^2, \quad N \rightarrow \infty, \quad (F.1)
\]
with the convergence in probability. As shown in Proposition 4 and its proof, the expectation and variance of \( G_N \) satisfy
\[
\mathbb{E}[G_N] \rightarrow \frac{\pi^2}{16} \epsilon_t^2, \quad \text{Var}[G_N] \rightarrow 0, \quad N \rightarrow \infty. \quad (F.2)
\]
Thus, when the phase errors are non-uniform over \([-\pi, \pi]\), the expectation and variance of \( H_t \) satisfy
\[
\text{Var}\left[\frac{H_t}{E[H_t]}\right] \rightarrow 0, \quad N \rightarrow \infty. \quad (F.3)
\]
Invoking [39, Theorem 4], the asymptotic equivalence of \( R_t \) and \( R_t^* \) can be obtained. With the transitive property of asymptotic analysis, the asymptotic equivalence of \( R_t \) and \( R_t^* \) also holds.

When the phase errors are uniform over \([-\pi, \pi]\), the expectation of \( G_N \) goes to zero as \( N \) goes large. The asymptotic equivalence of \( R_t \) and \( R_t^* \) does not hold since the relation in (F.3) is not satisfied.

**REFERENCES**

[1] S. Dang, O. Amin et al., “What should 6G be?” Nat. Electron., vol. 3, no. 1, pp. 20–29, 2020.
[2] K. David and H. Berndt, “6G vision and requirements: Is there any need for beyond 5G?” IEEE Veh. Technol. Mag., vol. 13, no. 3, pp. 72–80, 2018.
[3] J. Tang, J. Luo et al., “Energy efficiency optimization for noma with swipt,” IEEE J. Sel. Areas Commun., vol. 13, no. 3, pp. 452–466, 2019.
[4] M. Alkhawatrath, Y. Gong et al., “Buffer-aided relay selection for cooperative noma in the internet of things,” IEEE Internet Things J., vol. 6, no. 3, pp. 5722–5731, 2019.
[5] L. Dai, B. Wang et al., “Non-orthogonal multiple access for 5G: solutions, challenges, opportunities, and future research trends,” IEEE Commun. Mag., vol. 53, no. 9, pp. 74–81, 2015.
[6] J. Zhu, Y. Huang et al., “Power efficient IRS-assisted NOMA,” IEEE Trans. Commun., vol. 69, no. 2, pp. 900–913, 2020.
[7] W. Wang, X. Liu et al., “Beamforming and jamming optimization for IRS-aided secure NOMA networks,” IEEE Trans. Wireless Commun., Early Access, 2021.
[8] M. Badiu and J. P. Coon, “Communication through a large reflecting surface with phase errors,” IEEE Wireless Commun. Lett., vol. 9, no. 2, pp. 184–188, 2020.
[9] C. Huang, G. Chen et al., “Buffer-aided relay selection for cooperative hybrid noma/oma networks with asynchronous deep reinforcement learning,” IEEE J. Sel. Areas Commun., Early Access, 2021.
[10] Z. Chen, Z. Ding et al., “On the application of quasi-degradation to MISO-NOMA downlink,” IEEE Trans. Signal Process., vol. 64, no. 23, pp. 6174–6189, 2016.
[11] C. Liaskos, S. Nie et al., “A new wireless communication paradigm through software-controlled metasurfaces,” IEEE Commun. Mag., vol. 56, no. 9, pp. 162–169, 2018.
[12] C. Huang, G. Chen, and K.-K. Wong, “Multi-agent reinforcement learning-based buffer-aided relay selection in irs-assisted cooperative networks,” IEEE Trans. Inf. Forensics Security, vol. 16, pp. 4101–4112, 2021.
[13] Q. Wu and R. Zhang, “Towards smart and reconﬁgurable environment: Intelligent reﬂecting surface aided wireless network,” IEEE Commun. Mag., vol. 58, no. 1, pp. 106–112, 2020.
[14] B. Tahir, S. Schwarz, and M. Rupp, “Analysis of uplink IRS-assisted NOMA under nakagami-m fading via moments matching,” IEEE Wireless Commun. Lett., vol. 10, no. 3, pp. 624–628, 2020.
[15] Z. Ding, R. Schober, and H. V. Poor, “On the impact of phase shifting designs on IRS-NOMA,” IEEE Wireless Commun. Lett., vol. 9, no. 10, pp. 1596–1600, 2020.
[16] X. Mu, Y. Liu et al., “Capacity and optimal resource allocation for IRS-assisted multi-user communication systems,” IEEE Trans. Commun., Early Access, 2021.
[17] B. Zheng, Q. Wu, and R. Zhang, “Intelligent reﬂecting surface-assisted multiple access with user pairing: NOMA or OMA?” IEEE Commun. Lett., vol. 24, no. 4, pp. 753–757, 2020.
[18] S. Zhang, H. Zhang et al., “Beyond intelligent reﬂecting surfaces: Reﬂective-transmissive metasurface aided communications for full-dimensional coverage extension,” IEEE Trans. Veh. Technol., vol. 69, no. 11, pp. 13 905–13 909, 2020.
[19] S. Zeng, H. Zhang et al., “Reconﬁgurable intelligent surfaces in 6G: Reﬂective, transmissive, or both?” IEEE Commun. Lett., vol. 25, no. 6, pp. 2063–2067, 2021.
[20] S. Zhang, H. Zhang et al., “Intelligent omni-surfaces: Ubiquitous wireless transmission by reflective-refractive metasurfaces,” IEEE Trans. Wireless Commun., Early Access, 2021.
[21] Y. Liu, X. Mu et al., “STAR: Simultaneous transmission and reflection for 360° coverage by intelligent surfaces,” 2021. [Online]. Available: https://arxiv.org/abs/2103.09104.

[22] J. Xu, Y. Liu et al., “STAR-RISs: Simultaneous transmitting and reflecting reconfigurable intelligent surfaces,” IEEE Commun. Lett., vol. 25, no. 9, pp. 3134–3138, 2021.

[23] X. Mu, Y. Liu et al., “Simultaneously transmitting and reflecting (STAR) RIS aided wireless communications,” IEEE Trans. Wireless Commun., Early Access, 2021.

[24] C. Wu, Y. Liu et al., “Coverage characterization of STAR-RIS networks: NOMA and OMA,” IEEE Commun. Lett., Early Access, 2021.

[25] M. Aldababsa, A. Khaleel, and E. Basar, “Simultaneous transmitting and reflecting intelligent surfaces-empowered NOMA networks,” 2021. [Online]. Available: https://arxiv.org/abs/2110.05311.

[26] C. Zhang, W. Yi et al., “STAR-IOS aided NOMA networks: Channel model approximation and performance analysis,” 2021. [Online]. Available: https://arxiv.org/abs/2107.01543.

[27] E. Björnson and L. Sanguinetti, “Rayleigh fading modeling and channel hardening for reconfigurable intelligent surfaces,” IEEE Wireless Commun. Lett., vol. 10, no. 4, pp. 830–834, 2020.

[28] E. Björnson, J. Hoydis, and L. Sanguinetti, “Massive MIMO networks: Spectral, energy, and hardware efficiency,” Foundations and Trends in Signal Processing, vol. 11, no. 3-4, pp. 154–655, 2017.

[29] D. Tse and P. Viswanath, Fundamentals of wireless communication. Cambridge university press, 2005.

[30] O. Özdogan, E. Björnson, and E. G. Larsson, “Intelligent reflecting surfaces: Physics, propagation, and pathloss modeling,” IEEE Wireless Commun. Lett., vol. 9, no. 5, pp. 581–585, 2019.

[31] Z. Ding and H. V. Poor, “A simple design of IRS-NOMA transmission,” IEEE Commun. Lett., vol. 24, no. 5, pp. 1119–1123, 2020.

[32] F. Fang, Y. Xu et al., “Energy-efficient design of IRS-NOMA networks,” IEEE Trans. Veh. Technol., vol. 69, no. 11, pp. 14088–14092, 2020.

[33] T. Wang, M.-A. Badiei et al., “Outage probability analysis of RIS-assisted wireless networks with Von Mises phase errors,” IEEE Wireless Commun. Lett., Early Access, 2021.

[34] T. Wang, G. Chen et al., “Study of intelligent reflective surface assisted communications with one-bit phase adjustments,” in Proc. IEEE Global Commun. Conf. (GLOBECOM), Virtual, Dec. 2020, pp. 1–6.

[35] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Trans. Wireless Commun., vol. 18, no. 11, pp. 5394–5409, 2019.

[36] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge university press, 2004.

[37] I. S. Gradshteyn and I. M. Ryzhik, Table of integrals, series, and products. Academic press, 2014.

[38] T. Cacoullos, Exercises in probability. Springer Science & Business Media, 2012.

[39] S. Sanayei, A. Nosratinia et al., “Opportunistic beamforming with limited feedback,” IEEE Trans. Wireless Commun., vol. 6, no. 8, pp. 2765–2771, 2007.