The effects of $\delta$ mesons on the baryonic direct Urca processes in neutron star matter*

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Abstract: In the framework of relativistic mean field theory, the relativistic neutrino emissivity of the nucleonic and hyperonic direct Urca processes in the degenerate baryon matter of neutron stars are studied. We investigate particularly the influence of the isovector scalar interaction which is considered by exchanging $\delta$ meson on the nucleonic and hyperonic direct Urca processes. The results indicate that $\delta$ mesons lead to obvious enhancement of the total neutrino emissivity, which must result in more rapid cooling rate of neutron star matter.

Key words: neutron star, direct Urca processes, $\delta$ meson, neutrino emissivity

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1 Introduction

Compact stellar object studies allow us to build a better and more comprehensive understanding of the properties of nuclear matter under extreme conditions. Neutron star (NS) is an ideal model for study of dense matter physics. It is the remnants of a supernova explosion with an internal temperature as high as $10^{11}$ C$10^{12}$ K, and cooling gradually to $10^6$ K by emitting neutrinos within minutes[1–5]. Then the long-term cooling process is mainly through neutrino emission in NS interior for about $10^6$ years. The primary mechanisms in the cooling stage can be divided into two categories: enhanced neutrino processes, which include the nucleonic direct Urca (NDURCA) processes and hyperonic direct Urca (YDURCA) processes. And standard neutrino processes (such as modified Urca processes and Bremsstrahlung processes). NS cooling properties tend to be dominated by whichever reaction has the highest neutrino emissivity. It is well known that NDURCA processes produce the most powerful neutrino emissivity, the second is YDURCA processes in the NS core, which are more efficiently than standard processes [6,7]. While in these work, the formulae of neutrino emissivity for NDURCA and YDURCA processes are still in non-relativistic manner. In fact, the threshold densities for NDURCA and YDURCA processes are larger than the nuclear saturation density. Above the threshold densities, the baryonic motion is relativistic in NS core. Thus the formulae in relativistic manner should be use to calculate the neutrino emissivity for NDURCA and YDURCA processes.

Meanwhile the equation of state (EOS) must be relativistic, which can be consistent with the relativistic approach. The relativistic mean field theory (RMFT) is an effective model for studying the properties of NS[9–15]. The standard RMFT includes isoscalar-scalar meson $\sigma$, isoscalar-vector meson $\omega$ and isovector-vector meson $\rho$. The isovector scalar channel usually is not contained because it is not expected to be essential to nuclei. However, the isovector scalar interaction could have an important role for asymmetric nuclear matter. It can be introduced through a coupling to $\delta$ ($a_0(980)$) meson which was studied by many authors[16–24]. These studies have shown that $\delta$ mesons had definite contributions to isospin-asymmetric nuclear matter, especially at high densities. NSs keep away from isospin-symmetric nuclear matter due to charge neutrality. Therefore, in this work we extend the analysis the contribution of the $\delta$ field in dense asymmetric matter to the effect on NS matter. When an NS includes $\delta$ mesons, the $\delta$ field not only changes the abundance of neutrons, protons and hyperons, but also changes the bulk properties of NS, which must affect the neutrino emissivity of NDURCA and YDURCA processes.

In this work, we use the RMFT including $\sigma$, $\omega$, $\rho$ and $\delta$ mesons with additional cubic and quartic nonlinearities of $\sigma$ meson to describe baryons interacting. The $\sigma$ and $\omega$ mesons supply medium-range attractive and short-range repulsive interactions between baryons, respectively. The $\delta$ and $\rho$ mesons provide the corresponding attractive and repulsive potentials in the isovector channel, respectively. We adopt the simplest NS model,
assuming NS core consists of n, p, Λ, Σ−, Σ+, Ξ−, Ξ0, e and μ. This work mainly focuses on the effects of the δ meson on the total neutrino emissivity for NDURCA and YDRCA processes in NS core.

2 Models

The effective Lagrangian function for the RMFT can be written as follows:

\[
\mathcal{L} = \sum_B \bar{\Psi}_B[i\gamma_\mu \partial^\mu - (m_B - g_{\psi B} \tau \cdot \delta - g_{\sigma B} \sigma) - g_{\omega B} \gamma_\mu \omega^\mu] - g_{\rho B} \tau \cdot \rho^\mu |\Psi_B + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma)
\]

\[
+ \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu + \frac{1}{2} m_\rho^2 \rho^\mu \rho^\nu + \frac{1}{2} (\partial_\mu \delta \partial^\mu \delta - m_\delta^2 \delta^2)
\]

\[
+ \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} G_{\mu \nu} G^{\mu \nu} + \sum_l \bar{\Psi}_l[i\gamma_\mu \partial^\mu - m_B] |\Psi_l,
\]

(1)

where the potential function \(U(\sigma) = \frac{1}{2} a_\sigma^2 + \frac{1}{4} b_\sigma^4, F_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) and \(G_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \).

In the RMFT, the meson fields can be considered as classical fields, where the field operators are replaced by their expectation values. Meson field equations are obtained as follows:

\[
m_\sigma^2 \sigma + a \sigma^2 + b \sigma^3 = \sum_B \frac{g_{\psi B}}{\pi^2} \int_0^{p_B} \frac{m_\sigma^2 k^2}{\sqrt{k^2 + m_B^2}} dk,
\]

(2)

\[
m_\omega^2 \omega_0 = \sum_B g_{\omega B} \rho_B,
\]

(3)

\[
m_\rho^2 \rho_0 = \sum_B g_{\rho B} I_{3B} \rho_B,
\]

(4)

\[
m_\sigma^2 \sigma_0 = \sum_B \frac{g_{\psi B}}{\pi^2} \int_0^{p_B} \frac{m_\sigma^2 k^2}{\sqrt{k^2 + m_B^2}} dk,
\]

(5)

where \(p_B \) is the baryonic Fermi momentum, \(I_{3B} \) is the baryonic isospin projection, the baryonic density is expressed by:

\[
\rho_B = \frac{p_B^3}{3\pi^2}.
\]

(6)

The baryonic effective mass \(m_\rho^* \) is written:

\[
m_\rho^* = m_B - g_{\omega B} \sigma_0 - I_{3B} g_\rho B_\delta_0.
\]

(7)

Under β equilibrium conditions, the chemical potentials of baryons and leptons satisfy the following relationship:

\[
\mu_B = \mu_n - g_\beta \mu_e, \mu_\mu = \mu_e.
\]

(8)

At zero temperature approximation, they are expressed as follows:

\[
\mu_B = \sqrt{p_B^2 + m_B^*^2} + g_{\omega B} \omega_0 + g_\rho B \rho_0 I_{3B},
\]

(9)

\[\mu_i = \sqrt{p_i^2 + m_i^2}.\]  

(10)

NSs also need to meet the conditions of electrical neutrality and baryon number conservation, which are expressed as follows:

\[
\sum_B g_{\rho B} \rho_B - \rho_e - \rho_\mu = 0,
\]

(11)

\[
\sum_B \rho_B = \rho,
\]

(12)

where \(\rho \) is total baryonic density.

The possible neutrino emission processes consist of two successive reactions, beta decay and capture, expressed as follows:

\[B_1 \rightarrow B_2 + l + \bar{\nu}_l, B_2 + l \rightarrow B_1 + \nu_l, \]

(13)

where \(B_1 \) and \(B_2 \) are baryons, and \(l \) is a lepton. This paper focuses on electron processes. The relativistic neutrino emissivity can be given by Fermi golden rule. The relativistic expression of the energy loss \(Q_R \) per unit volume and time is expressed as:

\[
Q_R = \frac{457\pi}{10080} G_\rho^2 C^2 T^6 \Theta(p_e + p_{B_2} - p_{B_1})
\]

\times \{f_1 g_1 [(\varepsilon_{F_1} + \varepsilon_{F_2})^2 - (\varepsilon_{F_1} - \varepsilon_{F_2})(p_{B_1}^2 - p_{B_2}^2)]
\]

\[+ 2g_1^2 \mu_e m_{B_1} m_{B_2} + (f_1^2 + g_1^2)[\mu_e (2\varepsilon_{F_1} - \varepsilon_{F_2} - m_{B_1}^2 + m_{B_2}^2)
\]

\[+ \varepsilon_{F_1} p_e^2 - \frac{1}{2} (p_{B_1}^2 - p_{B_2}^2 + p_e^2)(\varepsilon_{F_1} + \varepsilon_{F_2})],
\]

(14)

Table 1. The constants of NDURCA and YDRCA processes. \(\sin^2\theta_C = 0.231 \pm 0.003, F = 0.477 \pm 0.012, D = 0.756 \pm 0.011\)

| Process Transition | C | f1 | g1 |
|-------------------|---|----|----|
| 1                 | n → p e \bar{\nu}_e | cos\theta_C | 1 | F + D |
| 2                 | \Lambda → p e \bar{\nu}_e | sin\theta_C | -\sqrt{3/2} | -\sqrt{3/2}(F + D/3) |
| 3                 | \Sigma^+ → n e \bar{\nu}_e | sin\theta_C | -1 | -(F + D) |
| 4                 | \Sigma^- → \Lambda e \bar{\nu}_e | cos\theta_C | 0 | \sqrt{3/2}D |
| 5                 | \Sigma^- → \Sigma^0 e \bar{\nu}_e | cos\theta_C | \sqrt{2} | \sqrt{3/2}E |
| 6                 | \Xi^- → \Lambda e \bar{\nu}_e | sin\theta_C | \sqrt{3/2} | \sqrt{3/2}(F + D/3) |
| 7                 | \Xi^- → \Sigma^0 e \bar{\nu}_e | sin\theta_C | \sqrt{1/2} | (F + D)/\sqrt{2} |
| 8                 | \Xi^- → \Sigma^+ e \bar{\nu}_e | sin\theta_C | 1 | F + D |
| 9                 | \Xi^- → \Sigma^0 e \bar{\nu}_e | cos\theta_C | 1 | F - D |

Table 2. Parameter set. \(f_i = (g_i/m_i^3)(fm^2)i = \sigma, \omega, \rho, \delta, \) and \(A = a/g_0^2 (fm^{-1}), B = b/g_0^2\)

| Parameter | f_\sigma | f_\omega | f_\rho | f_\delta | A | B |
|-----------|----------|----------|--------|----------|---|---|
| without \(\delta\) | 10.33 | 5.42 | 0.95 | 0.00 | 0.033 | -0.0048 |
| with \(\delta\) | 10.33 | 5.42 | 0.95 | 2.50 | 0.033 | -0.0048 |
and YDURCA processes in NSs. Next, we will give the
effect on the inclusion of the effective mass of nucleon and hyperon. This article focuses
on the effect of the inclusion of the neutrino emissivity for NDURCA and YDURCA processes with and without $\delta$ mesons. The properties of NSs are obtained with the parameter sets presented in Table 2. The hyperon couplings are represented as the ratios to the nucleon coupling $g_{\omega N} = 0.7 g_{\omega N}$, $g_{\rho N} = 0.7 g_{\rho N}$, and $g_{\omega N} = 0.7 g_{\omega N}$.

The mass-radius relations of NSs with and without $\delta$ mesons are compared in Fig. 1. The maximum masses $M_{\text{max}}$, the corresponding radius $R$ and center density $\rho_c$ with and without $\delta$ mesons are displayed in Table 3. From Fig. 1 and Table 3, one can find that the inclusion of $\delta$ mesons leads to the larger radius for an NS with the same mass.

Fig. 1. The mass-radius relations for NSs with and without $\delta$ mesons. The dots stand for the NSs maximum masses for the two cases.

| Parameter | $M_{\text{max}}/M_\odot$ | $R$ (km) | $\rho_c$ ($\text{g/cm}^3$) |
|-----------|----------------|--------|----------------|
| without $\delta$ | 1.72 | 10.40 | 1.23 |
| with $\delta$ | 1.70 | 10.58 | 1.20 |

Table 3. The maximum masses $M_{\text{max}}$, the corresponding radius $R$ and center density $\rho_c$ with and without $\delta$ mesons.

where $G_F = 1.436 \times 10^{-49}$ erg cm$^3$ is the weak-coupling constant, $f_1$ and $g_1$ are the vector and axial-vector constants, and $C$ is the Cabibbo factor. $\mu_e$ is the chemical potential of an electron, and $p_{B_1}$, and $p_{B_2}$, $p_c$ are the Fermi-momenta of baryons $B_1$, $B_1$, and electrons, respectively. $\varepsilon_{f_1}$ and $\varepsilon_{f_2}$ are the kinetic energy of baryons $B_1$ and $B_1$. When $x \geq 0$, $\Theta(x) = 1$, and when $x < 0$, $\Theta(x) = 0$. The parameters $C$, $f_1$, and $g_1$ for NDURCA and YDURCA in NS matter are listed in Table 1 [26].

Solving the coupling equations self-consistently at a fixed baryon density $\rho$, one can obtain a list of physical quantities, such as the particle fraction, effective mass, nucleon Fermi momentum. Then the total relativistic neutrino emissivity for NDURCA and YDURCA processes can be obtained.

3 Discussion

A primary contribution for NSs including $\delta$ mesons is changing the EOS, which must lead to the change of the bulk properties of NSs [17]. We substitute EOS into the Tolman-Oppenheimer-Volkoff (TOV) equations [27, 28], the mass-radius relations for NSs can also be obtained. The most important physical quantities for the neutrino emissivity of NDURCA and YDURCA processes are the particle fraction, Fermi momentum and effective mass of nucleon and hyperon. This article focuses on the effect of the inclusion of $\delta$ mesons on NDURCA and YDURCA processes in NSs. Next, we will give the numeric results of the relativistic neutrino emissivity for NDURCA and YDURCA processes with and without $\delta$ mesons.

Fig. 2. The particle fractions of baryons and leptons as a function of the baryon density $\rho$ with and without $\delta$ mesons. The two vertical lines represent the maximum center densities of the maximum masses NS with and without $\delta$ mesons.

The particle fraction $Y_i$ as a function of the baryonic density $\rho$, with or without $\delta$ mesons is shown in Fig. 2. As seen in Fig. 2, whether or not the $\delta$ meson is included, the $\Sigma$, $\Lambda$ and $\Xi$ appear one by one. This is because that a hyperonic type is populated only if its chemical potential exceeds its lowest energy state in NS matter, e.g. $\mu_\Sigma - q_\delta \mu_e \geq m_\Sigma^\text{B} + g_{\omega B} \omega_0 + g_{\rho B} \rho_0 \delta_0$. So when the thresholds are reached, additional hyperonic species are populated. As shown in Fig.2, the inclusion of $\delta$ mesons changes the baryonic threshold conditions, the threshold densities of $\rho$, $\Lambda$, $\Sigma^0$, $\Sigma^-$, and $\Xi^0$ with $\delta$ mesons are shifted to lower densities. And the fractions $Y_\rho$, $Y_\Lambda$, $Y_{\Sigma^0}$, $Y_{\Sigma^-}$ and $Y_{\Xi^0}$ with $\delta$ mesons are larger than the corresponding values without $\delta$ mesons. Then according to Eq. (6), when the $\delta$ field is included, $\rho_{\rho}$, $\rho_\Lambda$, $\rho_{\Sigma^0}$, $\rho_{\Sigma^-}$ and $\rho_{\Xi^0}$ are larger than the corresponding values without the $\delta$ field. From Fig. 2, we can also see that the inclusion of $\delta$ mesons makes the fractions $Y_\rho$, $Y_{\Sigma^0}$, $Y_{\Sigma^-}$ and $Y_{\Xi^0}$ decrease due to the charge neutrality and
$\beta$-equilibrium conditions. The changes of baryonic fractions must change the neutrino emissivity of NDURCA and YDURCA processes in NS matter. The relativistic neutrino emissivity of NDURCA and YDURCA processes as a function of the baryonic density $\rho$ with and without $\delta$ mesons is plotted in Fig. 3. One can find that the threshold densities of reactions 1-5 changing. Furthermore, the inclusion of $\delta$ mesons makes the fractions 6-9 with $\delta$ mesons would never have occurred within stable NSs, because they occur at higher densities which is larger than the center densities of the maximum masses NS. As shown in Fig. 4, whether or not the $\delta$ meson is included in NS matter, reactions 2 and 3 occur as long as $\Lambda$ and $\Sigma^-$ hyperons appear. Namely, the triangle condition $p_{B_\delta} + p_E > p_{B_\delta}$ in Eq. (14) is satisfied automatically for reactions 2 and 3 if $\Lambda$, $\Sigma^-$ hyperons appear in NS core. The occurrence of reactions 2 and 3 does not need $Y_{\Lambda}$, $Y_{\Sigma^-}$ reach a certain quantity, they are much more likely to happen than the other direct Urca reactions. While the reactions 4 and 5 (or 4-7) with (or without) $\delta$ mesons occur later. The reactions 6 and 7 only occur in NSs without inclusion of $\delta$ mesons. As shown in Fig. 4, when $\delta$ mesons are included, the relativistic neutrino emissivity $Q_R$ is obviously larger than the corresponding values without $\delta$ mesons. It may be concluded that the inclusion of $\delta$ mesons would accelerate the nonsuperfluid NS cooling in most mass ranges of happening NDURCA and YDURCA processes.

Fig. 3. The hyperonic effective masses as a function of the baryonic density $\rho$ with and without $\delta$ mesons.

Fig. 4. The relativistic neutrino emissivity $Q_R$ as a function of the baryonic density $\rho$ with and without $\delta$ mesons in NS matter. The vertical two lines represent the center densities of the maximum masses NS with and without $\delta$ mesons.

4 Conclusions

We have shown the influence of $\delta$ mesons on the relativistic neutrino emissivity for NDURCA and YDURCA processes by adopting the RMFT in NS matter. Results show that the inclusion of $\delta$ mesons makes the baryonic threshold densities change in NS matter, which lead to the threshold densities of reactions 1-5 changing. Furthermore, the inclusion of $\delta$ mesons makes the fractions 6-9 with $\delta$ mesons would never have occurred within stable NS matter. The fractions of $\rho$, $\Lambda$, $\Sigma^0$, $\Sigma^-$ increase, which leads to an obvious enhancement of the relativistic neutrino emissivity in NS matter. Thus the $\delta$ mesons would speed up the nonsuperfluid NS cooling rate in most mass ranges of happening NDURCA and YDURCA processes.

References

1. Boguta J 1981 Phys. Lett. B 106 255
2. Glendenning N K 1985 Astrophys. J. 293 470
3. Kaminker A D, Haensel P, Yakovlev D G 2001 Astron. Astrophys. 373 L17
4. Leinson L B 2002 Nucl. Phys. A 707 543
5. Yakovlev D G, Pethick C J 2004 Ann. Rev. Astron. Astrophys. 42 169
6. Prakash M, Prakash M, Lattimer J M and Pethick C J 1992 Astrophys. J. 390 L77
7. Haensel P and Gnedin O Y 1994 Astron. Astrophys. 290 458
8. Yakovlev D G, Kaminker A D, Gnedin O Y and Haensel P 2001 Phys. Rep. 354 1
9. Schaffner J and Mishustin I N 1996 Phys. Rev. C 53 1416
10. Bednarek I and Manka R 2005 J. Phys. G: Nucl. Part. Phys. 31 1009
11 Mi A J, Zuo W, Burgio G et al 2007 Chin. Phys. B 16 1934
12 Ding W B, Liu G Z, Zhu M F et al 2008 Chin. Phys. Lett. 25 458
13 Ding W B, Liu G Z, Zhu M F et al 2010 Commun. Theor. Phys. 54 500
14 Xu Y, Liu G Z, Wu Y R et al 2011 Chin. Phys. Lett. 28 079701
15 Yu Z and Ding W B 2011 Commun. Theor. Phys. 55 643
16 Xu Y, Liu G Z, Wang H Y et al 2012 Chin. Phys. Lett. 29
17 Kubis S and Kutschera M 1997 Phys. Lett. B 399 191
18 Kubis S, Kutschera M and Stachniewicz S 1998 Acta Phys. Pol. B 29 809
19 Liu B, Greco V and Baran V 2002 Phys. Rev. C 65 045201
20 Menezes D P and Providência C 2004 Phys. Rev. C 70 058801
21 Liu B, Guo H, di Toro M and Greco V 2005 Eur. Phys. J. A 25 293
22 Yu Z, Liu G Z, Zhu M F et al 2009 Chin. Phys. Lett. 26 022601
23 Xu Y, Liu G Z, Wu Y R et al 2012 Plasma Sci. Technol. 14 375
24 Xu Y, Liu G Z, Liu C Z et al 2013 Chin. Phys. Lett. 30 062101
25 Leinson L B and Perez A 2001 Phys. Lett. B 518 15
26 Prakash M, Prakash M, Lattimer J M, et al. The Astrophysical Journal, 1992, 390 L77
27 R. C. Tolman 1939 Phys. Rev. 55 364
28 J. R. Oppenheimer and G. M. Volkoff 1939 Phys. Rev. 55 374