Runaway gas accretion and gap opening versus type I migration

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Abstract

Growing planets interact with their natal protoplanetary disc, which exerts a torque onto them allowing them to migrate in the disc. Small mass planets do not affect the gas profile and migrate in the fast type-I migration. Although type-I migration can be directed outwards for planets smaller than $20 - 30 M_\oplus$ in some regions of the disc, planets above this mass should be lost into the central star long before the disc disperses. Massive planets push away material from their orbit and open a gap. They subsequently migrate in the slower, type II migration, which could save them from migrating all the way to the star. Hence, growing giant planets can be saved if and only if they can reach the gap opening mass, because this extends their migration time-scale, allowing them to eventually survive at large orbits until the disc itself disperses.

However, most of the previous studies only measured the torques on planets with fixed masses and orbits to determine the migration rate. Additionally, the transition between type-I and type-II migration itself is not well studied, especially when taking the growth mechanism of rapid gas accretion from the surrounding disc into account. Here we use isothermal 2D disc simulations with FARGO-2D1D to study the migration behaviour of gas accreting protoplanets in discs. We find that migrating giant planets always open gaps in the disc. We further show analytically and numerically that in the runaway gas accretion regime, the growth time-scale is comparable to the type-I migration time-scale, indicating that growing planets will reach gap opening masses before migrating all the way to the central star in type-I migration if the disc is not extremely viscous and/or thick. An accretion rate limited to the radial gas flow in the disc, in contrast, is not fast enough. When gas accretion by the planet is taken into account, the gap opening process is accelerated because the planet accretes material originating from its horseshoe region. This allows an accreting planet to transition to type-II migration before being lost even if gas fails to be provided for a rapid enough growth and the gap opening mass is not reached.

Keywords: Planets, migration, Planetary formation, Planet-disk interactions, accretion
1. Introduction

Planets form in proto-planetary discs. For the terrestrial planets of the solar system, only Moon to Mars sized embryos need to be formed in the proto-solar nebula; the final assembly of Venus and the Earth is thought to take place after the gas dispersal, through giant impacts among these embryos and left-over planetesimals (Kleine et al., 2009; Jacobson et al., 2014; Raymond et al., 2014). In contrast, giant planets must have acquired their final mass while gas was still present, as they are mostly composed of this gas. In the core-accretion model (Pollack et al., 1996), the gaseous envelope is slowly accreted around a solid core of ~10 Earth masses. Hence, gas giants must experience planet-disc interactions from the mass of an embryo all the way to that of Jupiter. A result of these interactions is planetary migration, which modifies the orbital radius of a planet, and generally moves it closer to the central star.

Giant planets open a gap around their orbit, separating the disc between an inner and an outer disc (Lin and Papaloizou, 1986a). In typical proto-planetary discs, this happens for planets more massive than Saturn or Jupiter (Crida et al., 2006). Once a gap is open, the planet should be locked between the inner and outer disc, and follow the viscous evolution of the proto-planetary disc, that is in general a slow accretion towards the central star (Lin and Papaloizou, 1986b; Nelson et al., 2000). This is called type II migration, and could explain the semi-major axis distribution of giant exoplanets (e.g. Lin et al., 1996), especially when photo-evaporation of the protoplanetary disc is taken into account (Alexander and Pascucci, 2012). It should be noted that the above description is ideal, and in reality a planet can decouple more or less from the disc evolution (Crida and Morbidelli, 2007; Dürmann and Kley, 2015), and some gas can pass through the gap (Lubow and D’Angelo, 2006). Despite these recent developments, the global picture that single gap opening planets migrate slowly and roughly together with the disc still holds.

In contrast, smaller planets – which do not perturb significantly the density profile of the disc – migrate with respect to the disc, in the so-called type I migration regime (Ward, 1997). In this regime, the migration rate is proportional to the planet mass. Hence, type I migration is not a big issue for the embryos of the terrestrial planets, but has always been an issue for growing giant planets: accreting a gaseous envelope should take way longer for their solid cores than migrating all the way into their host star (the typical migration time-scale for a 30 Earth mass body is only 10,000 orbits). In fact, the first planet population synthesis models (Alibert et al., 2005; Ida and Lin, 2008; Benz et al., 2008; Mordasini et al., 2009) had to decrease the efficiency of type I migration by a factor 100 at least, if they wanted planets to survive.

In the past decade, huge progress has been made on type I migration, mainly relative to the corotation torque (see Baruteau et al., 2014, for a complete review). It has been shown that this torque can be positive, and overcome the classical, negative, differential Lindblad torque when the disc is not isothermal (Paardekooper and Mellema, 2006; Kley and Crida, 2008; Baruteau and Masset, 2008; Kley et al., 2009). Typically, this is efficient for planets in the 5 – 30M⊕ range, in the inner regions of the disc, where the radial gradient of entropy is steep (ex: at opacity transitions). An analytical formula for the torque felt by a planet in type I migration has been found by Paardekooper et al. (2011) based on 2D numerical simulations, and confirmed in 3D radiative numerical simulations by Bitsch and Kley (2011) and more recently by Lega et al. (2015), who included stellar irradiation. Combined with an accurate description of the temperature and density profiles of the protoplanetary disc and its evolution, this allows to produce migration maps, where the torque felt by a planet is given as a function of its mass and position in the disc (Bitsch et al., 2013, 2014a,b; Baillié et al., 2016). In such maps, it appears possible to block a planet at a zero-torque radius, where it can grow slowly.

However, above a critical mass Mcrit, the corotation torque always saturates and vanishes. In general Mcrit ≈ 20M⊕, depending on the opacity, density and viscosity of the disc. Therefore, the too fast type I migration problem is solved only below Mcrit. As Mcrit is 5 to 10 times smaller than the gap opening mass, the question of the fast inwards migration of giant planets, as they grow from Mcrit until they open a gap, remains open. In this paper, we address this critical question (and only this question). We study in which conditions a growing giant planet can open a gap before type I migration drives it all the way into its host star.

After having presented our set-up, code and units in section 2, we first study briefly the opening of a gap by a migrating giant planet in section 3. In this section, we study how a migrating giant planet in section 3. This study is necessary because Malik et al. (2015) recently suggested that giant planets would not be able to open their gap if they are migrating too fast. More precisely, they argue in favour of Hourigan and Ward (1984) who stated that if the planet crosses its corotation region faster than a gap opens, the gap never opens and the planet therefore remains in type I migration. If this is true, there is no hope for a giant planet to ever open a gap when its grows past Mcrit. In this case, the survival of giant planets would become a puzzle because they should never leave the type I migration regime. We show in contrast that a Jupiter mass planet does open a gap in about a hundred orbits, even if it migrates fast.

Second, we compare the growth and migration time-scales in section 4. We show analytically and with numerical simulations that once a 20 M⊕ core starts its runaway gas accretion, it should reach the gap opening mass before its semi-major axis is dramatically reduced. Finally, one may worry that the theoretical runaway accretion rate can not be sustained if the disk can not provide gas fast enough. However, we show in section 5 that if this occurs, then the planet has already opened a gap because all the gas in its horseshoe region has spread to the accretion streams. Consequently, once the runaway accretion of gas starts, nothing prevents the opening of a gap by the growing giant planet. After a discussion in section 6, we summarize our findings in section 7.
2. Units and simulations set-up

In this paper, all our simulations are performed with the FARGO-2D1D code \cite{Crida2007}, which is a 2D grid code in polar \((r-\theta)\) coordinates. The resolution is always \(dr/r = 0.01 = d\theta\), unless specified otherwise. Far from the planet, the standard 2D grid is replaced by a 1D grid, assuming azimuthal symmetry, allowing to model the viscous spreading of the disc. This is crucial for an accurate modelization of type II migration, but not necessary for a study of type I migration. Even if our focus here is not type II migration, but the transition into type II migration, we choose to use this code given the negligible computing cost of the 1D grid.

Our units are \(L\) as the arbitrary length unit (generally the initial orbital radius of the planet), the mass of the central star \(M_\star\) as the mass unit, and we set the gravitational constant \(G = 1\) so that an orbital period at \(L\) is given by \(P_L = 2\pi T\) with \(T = \sqrt{L^3}/GM_\star\) the time unit.

The equation of state is locally isothermal, because we focus on cases where the thermal part of the corotation torque would be saturated. The aspect ratio is always uniform (no flaring) with the usual value \(h = H/r = 0.05\), so that the sound speed is given by \(c_s = 0.05 \sqrt{(L/T)}\). Unless stated otherwise, the gas viscosity is given by \cite{Shakura1973}'s prescription, with a rather standard value of \(\alpha = 10^{-3}\). With this setting, a Jupiter mass planet (that is: a planet whose mass is \(M_J = 10^{-3} M_\star\), or mass ratio to the star is \(q = 10^{-3}\)) has a gap opening parameter as defined by \cite{Crida2006} of \(2/3 < 1\), so we expect this planet to open a gap. The threshold for gap opening according to this criterion (that is: the gas density in the middle of the gap should be 10% of the unperturbed gas density) is 0.436 \(M_J\). The initial surface density of the gas disc is always of the form \(\Sigma(r) = \Sigma_0 \times (L/r)^{3}\). With such a slope of the surface density profile and an \(\alpha\)-prescription for the viscosity in a non-flared disc, the viscous torque exerted by the disc inside any radius \(r\) on the disc outside \(r\) is independent of \(r\). Hence, the viscous torque on any elementary ring is zero, and the disc is at equilibrium, with no radial viscous drift of the gas. Of course, at the inner (resp. outer) edge of the disc, there is no support from an inner (resp outer) disc and the gas will spread. This perturbation to the density profile will propagate inside the disc profile at the viscous rate. Nonetheless, for most of the times we will consider here, the disc around the planetary orbit should not spread, and type II migration is expected to be very slow.

The semi-major axis of the planet is noted \(a\), and its orbital angular velocity is \(\Omega = \sqrt{GM_\star/ a^3}\). The gravitational potential of the planet is smoothed using the usual so-called \(\epsilon\)-smoothing, with \(\epsilon = 0.6 r_H\) where \(r_H = (q/3)^{1/3}a\) is the Hill radius of the planet. The self-gravity of the gas disc is not taken into account. Whenever the torque exerted by the disc on the planet is computed, the region within 0.6 \(r_H\) is excluded, using a smooth Fermi function, as prescribed by \cite{Crida2009}.

3. Gap opening by migrating giant planets

3.1. Fixed mass with forced migration

To start with, we have performed simulations in which a Jupiter mass planet is thrown\footnote{More precisely, the mass of the planet is smoothly increased from 0 to its final value \(M_\rho\) over the first period \(P_L\). It remains constant afterwards.} into an unperturbed disc of initial density \(\Sigma_0(r) = 10^{-9}(L/r) M_\star/L^2\). The 2D grid spans the radial range 0.1 – 2.5 and the 1D grid extends from 0.08 to 10. The planet starts at \(a_0 = 1\), and is forced to migrate at a constant rate \(\tau_m = -a/\dot{a}\) in such a way that it reaches \(a_1 = 0.25\) in 100 \(P_L\). The resulting perturbations to the density profile (that is: \(\Sigma(r)/\Sigma_0(r)\)) are shown in Fig. 2 after 25, 50, 75, and 100 orbits, as the dashed curves. The solid curves show for comparison the density profiles at the same times in the case where the planet remains at \(a = 1\). As one can see, the gap opening is not impeached by the migration, although the planet crosses the width of the gap every 25 orbits. The gap is actually deeper in the migrating planet case, because 100 orbits at \(r = L\) correspond to more orbits at \(r = 0.25L\). Simulations with shorter migration times (namely 50 and 25 \(P_L\)) show the exact same thing: the gap opens at the same pace, whether the planet migrates or not. We conclude that the gap opening process takes about a hundred orbits to complete, and the planet may migrate during this time, but the gap opening itself is not affected by the migration.

Note that at the inner edge of the 1D-grid, the open boundary condition allows the gas to flow inwards, and the density drops. This drop propagates outwards at the slow, viscous rate, but does not perturb our simulations: after 100 \(P_L\), the density is still the unperturbed one at 0.25 \(L\) if the planet does not migrate.

The migrating planet seems to act as a snowplough, pushing gas inside its orbit and leaving a slightly depleted outer disc behind. Although some gas can cross the planetary orbit (otherwise the outer disc would be empty from the planetary orbit up to \(r = 1\)), this behaviour is expected as the planet carries its depleted horseshoe region during its migration (see below) and repels the inner disc with its gravitational torque. If the planet was let free to migrate, it would be slowed down or blocked by the overdense inner disc, and transition into type II migration. Actually, after its forced migration during 100 \(P_L\), the planet is released free and we see it migrate slightly outward.

3.2. Forced growth with free migration

To go further, we have performed simulations in which the planet grows from \(M_0 = 3 \times 10^{-5} M_\star\) (or 10.0 \(M_\oplus\) if \(M_\star = M_\odot\)) following:

\[
M_\rho = M_0 \exp(t/\tau_g) .
\]  

We use the same code and parameters, except for a lower \(\alpha = 4 \times 10^{-4}\) (in order to slow down the viscous evolution of the disc), where the 1D grid extends from 0.02 to 5 and 2D grid from 0.3 to 2.3.
In the type I regime, the planet feels a torque Γ = 0.1aΣΩ - Aτ, where Γ₀ = (M_p/Mₗ)²ΣₐdΩh⁻² and A is a constant numerical coefficient given for a locally isothermal EOS by A = 2.5 + 1.7β - 0.1α⁻¹. Hence, assuming that a does not vary much, we can set Σ₀(L) such that the ratio of the growth and migration rates follows τₘ/τₔ = 100(M₀/Mₗ) during the simulation. Then, combining the expressions of Mₗ and ȧ leads to:

\[
\dot{a} = -2A \left( \frac{M_p}{M_l} \right) \Sigma \frac{a^3 \Omega}{M_a \ h^2} \frac{a}{\tau_m}. \tag{2}
\]

As well known, \( \tau_m \propto \left( \Sigma M_p / \sqrt{a} \right)^{-1} \), while here \( \tau_p \) is constant. Hence, assuming that \( a \) does not vary much, we can set Σ₀(L) such that the ratio of the growth and migration rates follows \( \tau_m/\tau_p = 100(M_0/M_p) \) during the simulation. Then, combining the expressions of \( M_p \) and \( \dot{a} \) leads to:

\[
M_p = M_0 \times \left[ 1 - 100 \ln(a/a_0) \right]. \tag{3}
\]

Because we want for this experiment the gap opening mass to be reached before the planet has migrated into its host star, this ratio of the migration and growth times seems appropriate.

Depending on the value of \( \tau_p \), the planets should evolve more or less slowly along the thick solid black curve, but as long as they follow Eqs (1) and (2) they must be on this curve. One expects the planets to depart from this track progressively as they open a gap, that is in the next ~ 100 orbits after they reach the gap opening mass (which is here 94.4 Mₘₖₗ according to the Crida et al. (2006) criterion). In particular, as mentioned in Section 2, the disc is stationary, and a planet in an ideal type II migration regime should have \( \dot{a} = 0 \) and follow a vertical line in the \( M - a \) diagram of Fig. 2. Some moderate migration rate beyond the gap opening mass could be observed (see discussion about type II migration in the introduction), but still much slower than the type I migration rate.

The dashed, coloured curves correspond to different \( \tau_p \) (and different \( \Sigma₀ \) to keep \( \tau_m/\tau_p = 100(M_0/M_p) \)). To illustrate the different speeds at which the curves are followed, symbols have been placed every 100 Pₗ on a few lines (see caption); in particular, 300 Mₘₖₗ are reached after 100, 200, 1000 Pₗ in the cases \( \tau_p = 30, 60, 300 \) respectively. The case \( \tau_p = 100 \) (plain red line with large bullets) illustrates very well the expected behaviour: type I migration up to ~ 100 Mₘₖₗ, and then transition to type II migration about 100 orbits later. In the case \( \tau_p = 300 \), the gap opening is faster than the growth of the planet, so that the planet starts carving a partial gap even before reaching 100 Mₘₖₗ and leaves the type I track to enter smoothly.

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Figure 1: Perturbed surface density profiles Σ(r)/Σ₀(r). Different colours correspond to different times, labelled in P_L, the orbital period at \( r = 1 \). Solid curves correspond to the case of a non migrating Jupiter, while dashed curves show the case of a migrating Jupiter with constant \( \tau_m = 0.1a/\dot{a} = 72 \) Pₗ, such that the planet migrates from \( a_0 = L \) to \( a_f = 0.25L \) in 100 Pₗ. The x-axis extends between 0.08 (inner edge of the 1D grid) and 3 (where the outer edge of the 2D grid is at 2.5), with a logarithmic scale for convenience because the gap width is proportional to the orbital radius.

Figure 2: Tracks of planets in the Mass–semi-major-axis plane, following Eq (1), with various, constant values of \( \tau_p \) and always \( \tau_m/\tau_p = 100(M_0/M_p) \) whatever \( \tau_p \). The thick, plain black line is the track corresponding to pure type I migration, Eq. (2). The thin horizontal lines mark 94.4 Mₘₖₗ, the gap-opening mass and 333 Mₘₖₗ = Mₗ. Symbols are placed every 100 orbits on the colour lines corresponding to \( \tau_p = 30, 60, 100, \) and 300 (only after 800 orbits in this latter case); owing to the logarithmic scale of the y-axis, the symbols are evenly spaced vertically.

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a type II like regime. In the cases where \( \tau_\sigma < 100 \), the planet appears to accelerate with respect to the expected type I migration rate as soon as it starts opening a gap. This is because of the positive feedback caused by the coorbital mass deficit (cmd) \( \text{(Masset and Papaloizou 2003)} \). Considering that for giant planets, the horseshoe width is the Hill radius, we can define the maximum possible cmd as:

\[
\text{cmd}_{\text{max}} = 2\pi a \Sigma_0(a) \times 2\bar{r}_\text{Hill} = 0.87 \Sigma_0(a) a \left( \frac{M_p}{M_j} \right)^{1/3}
\]

(4)

The cmd is proportional to \( \Sigma_0 \) (hence to \( 1/\tau_\sigma \)), and if it is larger than the planet mass, it leads to the runaway, type III migration \( \text{(Masset and Papaloizou 2003; Peplinski et al. 2008b)} \). Despite the positive feedback provided by the cmd in massive discs, all planets eventually open a deep gap and enter type II migration beyond Jupiter’s mass, even if they have migrated by more than the width of the gap by the time they reach this mass.

Only in the cases of very massive discs \( (\tau_\sigma \ll 50) \) the planets hit the inner edge of our 2D grid before they have a chance to transition to type II migration. It should be noted that less than a hundred orbits were spent after the planets reached \( 4.4 M_\oplus \), so they did not have time to open a clean gap. They may also have entered the type III migration regime:

in the case \( \tau_\sigma = 30 \), \( \Sigma_0(L) = 7.25 \times 10^{-4} M_\oplus/L^2 \), so that the cmd_{max} is \( 4.2 \times 10^{-4} M_\oplus = 140 M_\oplus \) for a \( 100 M_\oplus \) mass planet at \( a = L \). We believe that this is the kind of phenomenon that Malik et al. (2015) have seen, as already observed by Crida (2009). Actually, if the planet mass is below the inertial mass limit \( q_{\text{limit}} \) given by Ward and Hourigan (1989), then \( q < 0.42 h^{9/4} (\text{cmd}_{\text{max}}/M_j)^{3/4} \), which is about an order of magnitude smaller than cmd_{max} for \( h < 0.15 \) and cmd_{max}/\( M_j > 10^{-5} \). Thus, a strong positive feedback from the cmd, possibly leading to type III migration, is easier to reach than the inertial mass limit.

To summarize, because of the positive feedback on migration caused by the coorbital mass deficit, a planet opening a partial gap can migrate fast on very long distances in dense discs. However, this does not prevent the planet from opening a gap eventually. Either this planet is actually not massive enough to open a deep gap anyway, or it should open a deep gap but this takes about a hundred orbits; during this time, the planet may migrate far (and leave the simulation frame), but this is only a transient phase until the gap opens and the planet transitions to type II migration, as illustrated by the blue curve labelled 60 in Fig. 2. There is no such thing as crossing one’s horseshoe width faster than a gap opens: a migrating planet does not have time to reach the gap opening process. The question now is: can a planet reach the gap opening mass before being lost by type I migration when it’s above \( M_{\text{crit}} \) ?

In this section, we have assumed arbitrary growth rates. In the next section we relax this assumption and use gas accretion rates determined by previous high resolution 3D hydrodynamical simulations (Machida et al. 2010), and compare them with migration rates.

4. Runaway gas accretion versus type I migration

In the standard core accretion model \( \text{(Pollack et al. 1996)} \), once a solid core of \( \sim 10 M_\oplus \) is formed (phase 1), it accretes slowly a gaseous envelope (phase 2). When the mass of the envelope exceeds that of the core, the envelope becomes unstable and collapses onto the core: more gas comes in and the planet undergoes runaway gas accretion (phase 3). Phase 2 is limited by the cooling rate to evacuate the energy of the accretion of solids, and could last millions of years. In this case, \( \tau_\sigma \gg \tau_\text{m} \) and the planet migrates almost at constant mass. A planet in this regime would just go wherever type I migration drives it, eventually being trapped at a zero-torque radius. In this section we assume that the core has outgrown phase 2 and starts its runaway gas accretion in phase 3. At this point the planet becomes massive enough so that its corotation torque saturates. During this growth and migration phase, the planet generally loses a major part of its semi-major axis in planet population synthesis studies, making this part of its evolution crucial for the final orbital position of the planet at disc dispersal (Bitsch et al. 2015).

4.1. Analytical study

Machida et al. (2010) found an accretion rate in the runaway regime:

\[
M_{p,10} = \Sigma H^2 \Omega \times \min\{0.14; 0.83 (r_H/H)^{5/2}\},
\]

(5)

where \( r_H \) is the Hill radius of the planet, \( H \) is the disc scale height, and the index \( M_{10} \) denotes Machida et al. (2010)’s rate.
Let us compute the ratio of the typical times for the growth of \( M_p \) and the migration: \( G \equiv \frac{\delta t}{\delta a}/M_p,\min/M_c \). Using Eq. (2), and Eq. (5) one finds:

\[
G = -\frac{A M_p}{h^2 M_c \min(0.14; 0.83(\pi H/H) )^{9/2}}.
\]

This ratio is independent of \( \Sigma, a, \alpha, \Omega \). \( G \) is only a function of \( h \) (that we have assumed uniform here) and \( M_p \). For reference, with \( h = 0.05 \) and \( A = 2.7, \tau_f = \tau_{\text{m}} \) for \( M_p = 0.4 M_J \). Planets smaller than that grow faster in runaway regime than they should migrate in type I migration, hence they should reach the gap opening mass (0.436 \( M_J \) with \( h = 0.05 \) and \( \alpha = 10^{-3} \)) before being lost.

Assuming a planet is in type I migration, following Eq. (2) and assuming it grows in the runaway mode following Eq. (5), one can compute analytically the reduction of its semi-major axis while it grows from a mass \( M_i \) to a mass \( M_f \):

\[
\ln \left( \frac{a_f}{a_i} \right) = \int_{M_i}^{M_f} \frac{dM}{M} \ln a = \int_{M_i}^{M_f} \frac{dM}{M} G \frac{dM_p}{M_c}.
\]

Note \( M_i = 3(0.14)^{2/3} h^3 M_c \) the mass at which the transition occurs in Eq. (5) = that is: for \( M_p > M_i \), the min term in Eq. (5) is 0.14 and for \( M_p < M_i \), the second argument of the min should be considered. For \( h < 0.176, M_i < 0.5 M_J \), so in reasonable discs, the transition will occur before half a Jupiter mass is reached, and we will consider hereafter that \( M_i < M_f \). For \( h < 0.0403 \), \( M_i < 20 M_g \), so in thin discs it is possible that \( M_i < M_f \). In this case, the above integration is straightforward and leads to:

\[
\ln \left( \frac{a_f}{a_i} \right) = -\frac{A}{0.28 h^4} \left\{ \frac{M_f}{M_i} - \left( \frac{M_j}{M_i} \right)^2 \right\}^{3/2}.
\]

If \( M_f > M_i > M_j \), the integral should be split and in the end:

\[
\ln \left( \frac{a_f}{a_i} \right) = -\frac{9}{2} \left( \frac{0.14}{0.83} \right) A h^2 \left\{ 2 \left( \frac{M_i}{M_f} \right)^2 - 1 \right\} + 4 \left( 1 - \frac{M_j}{M_f} \right) \left\{ 2 \left( \frac{M_i}{M_f} \right)^2 + 3 \right\}.
\]

Taking \( A = 2.5, M_i = 20 M_g \) and \( M_f = 0.5 M_J \) or \( M_j = M_f \), the ratio \( a_f/a_i \) provided by Eq. (8) for \( h > 0.0403 \) and Eq. (7) for \( h < 0.0403 \) is plotted as a function of \( h \) in Fig. 3. The case \( M_i = 0 \) actually corresponds to Eq. (9) and is shown as the thin, long-dashed curve; it shows that the influence of \( M_i \) is actually small, compared to the influence of \( M_f \). However, the real key parameter appears to be \( h \); in thin discs \( h < 0.035 \), migration is faster than growth and the planets are lost before they reach half a Jupiter mass. In thicker discs \( h > 0.045 \), \( a_f/a_i > 0.6 \) and standard type I migration should not make the planets migrate by more than 40% of their initial semi-major axis by the time they reach half a Jupiter mass.

As we are interested in reaching the gap opening mass, it is tempting to replace \( M_f \) by the gap opening mass \( M_{\text{gap}} \), which is a function of \( h \), as given by Eq. (10) of [Baruteau et al. 2014]:

\[
\frac{M_{\text{gap}}}{M_e} = \frac{100}{\mathcal{R}} (X + 1)^{1/3} \left( X - 1 \right)^{-1/3},
\]

with \( \mathcal{R} = \alpha^2 \Omega/\nu = 1/(a h^2) \) and \( X = \sqrt{1 + 3 h^3} \). Substituting \( M_f \) in Eq. (9) by \( M_{\text{gap}} \) given by Eq. (10), we can compute an upper boundary of \( a_f/a_i \) as a function of \( h \) and \( \alpha \), independent of \( M_i \). Fig. 3 shows this result as a colour map; it shows that for low enough \( h \) and \( \alpha \), a planet reaches the gap opening mass before its semi-major axis would be halved by pure type I migration. In particular, the green double-dashed curve corresponds to \( \alpha = 0.4 \) contour line. For any fixed \( h \), the growth rate is determined; then, as \( \alpha \) increases, \( M_{\text{gap}} \) increases so that migration has more time to reduce the semi-major axis. The black and white curves overlaid on the colour map show contours of \( M_{\text{gap}} \) as given by Eq. (10). For \( \alpha = 10^{-2} \), whatever \( h \), type I migration can divide the semi major axis by more than an order of magnitude by the time \( M_{\text{gap}} \) is reached in the runaway growth regime. Conversely, for a fixed \( \alpha \), the increase of \( h \) leads to a larger gap opening mass but also a more favourable growth to migration times ratio; as a consequence, even in thin discs \( h > 0.1 \) where \( M_{\text{gap}} > M_f \), planets in runaway gas accretion can theoretically reach the gap opening mass before being lost into their host star, even if their migration is more important in thicker discs. In standard discs \( h \sim 0.05, \alpha \sim \text{a few } \times 10^{-3} \), \( M_{\text{gap}} \) is between Saturn and Jupiter’s masses, and is reached in the runaway growth while the orbital radius is roughly halved by type I migration.

\footnote{Note that in the limit \( h \to 0 \), the right member of Eq. (10) converges towards \( -3 A \ln h \), but does not diverge.}
This study, being purely analytic, relies on 2 simplified hypothesis: (i) the planets migrate in pure type I migration, as given by Eq. (2); (ii) they grow in the runaway regime according to Eq. (5). It allows to explore the parameter space in the $h - \alpha$ plane, and to understand the role of these two parameters – which are the only free ones in the final equation, with $A$, which can be seen as the ratio of the migration and growth rates normalized to Eq. (2) and Eq. (5): $\alpha = (\Gamma / \Gamma_0) / (M_s / M_{\text{r, M10}})$. Namely, $\alpha$ is only responsible for gap opening, while $h$ determines the growth and migration rates as well as gap opening. However, we have seen in section 3.2 that in dense discs, when planets start opening a partial gap, they leave the pure type I regime, and they can migrate faster. Consequently, in next subsection, we explore with numerical simulations the role of the surface density of the disc, which had disappeared from the equations here, and explore the effect of a decreased accretion rate.

4.2. Numerical study

To study numerically the competition between migration, runaway gas accretion and gap opening, we perform simulations in which the increase of the planet mass follows Eq. (5). As for the value of $\Sigma$ in Eq. (5), we take the initial density of the disc at the planet location, $\Sigma_0(\alpha)$ because in Machida et al. (2010)’s shearing box, the density profile can not be perturbed by the planet (no gap opening). Furthermore, we are mostly interested in the phase before the gap opening, where the density close to the planet is actually little perturbed : we come back to this choice later. Note that here, we prescribe this growth rate to the planet mass, without removing the gas from the disc. We set the initial mass of the planet to $6 \times 10^{-5} M_\oplus = 20 M_\oplus$, which is roughly the mass at which planets in the outer disc start their runaway gas accretion phase in Bitsch et al. (2015) (see also Lambrechts et al. 2014). In these simulations, the 1D grid extends from $0$ to $50$ and the 2D grid from $1.0$ to $25.126$, and the planet starts at $10$. As usual, the disc parameters are $h = 0.05$ and $\alpha = 10^{-3}$, resulting in $M_{\text{gap}} = 0.436 M_\oplus$, and $a_f/a_\iota \gtrsim 0.72$ according to Fig. 4 independent of $\Sigma_0$.

The results of these simulations are shown in Fig. 5 in the $M_\oplus - a$ plane as the thick curves, with dots every $1000 P_L$. Each curve corresponds to a different value of $\Sigma_0$, given by the key. All our planets reach the gap opening mass before being lost into their central star.

Concerning migration, all planets in our simulations eventually open a gap and almost stop migrating, as expected for ideal type II migration in our disc. In the case $\Sigma_0 = 3 \times 10^{-5}$, the final semi major axis is $\sim 8$, in agreement with the analytic estimate above. We also recover the same trend as previously: at a given mass, planets in denser discs migrate slightly faster because of the positive feedback exerted by the coorbital mass deficit of the horseshoe region, hence they end further inwards. Note that with $\Sigma_0 = 10^{-4}$, $\text{cmd}_{\max} = 4 M_\oplus$ at $\alpha = 10$ for a $0.1 M_\oplus$ planet. This was not taken into account in the analytical calculations of previous subsection, nor was considered the time needed to open the gap and transition from the type I to the type II regime.

4.3. Influence of the accretion rate

Gas accretion can influence slightly the migration rate of giant planets (e.g. Nelson et al. 2000, Peplinski et al. 2008). In the frame of our study, the main effect of the accretion rate is on the growth of the planet. Admittedly, Machida et al. (2010)’s
accretion rate is probably an upper estimate: their simulations considered a shearing box with constant density boundaries, and did not take the gap opening into account (which only plays a role after a gap is opened, though). Therefore, taking always $\Sigma_0 = 10^{-4}$, we have divided the accretion rate given by Eq. (5) by 2, 5 and 10, in order to look for a threshold below which accretion would win over accretion. This would be equivalent to multiply $A$ by the corresponding number in Eq. (9), hence more migration is expected. We have also made a simulation in which the value of $\Sigma$ in Eq. (5) is the actual azimuthally averaged value of $\Sigma$ at $a$, smaller than $\Sigma_0(a)$ when a gap opens. Finally, we have also prescribed an accretion rate equal to the radial gas flow through the disc, or the accretion rate by the star:

$$M_p = M_{\text{disc}} = 3\alpha \pi \Sigma_0.$$  

Indeed, Lubow and D’Angelo (2006) show that the planet can accrete a maximum of 80% of what the disc’s accretion rate is. For this reason, many authors (especially in the population synthesis field, e.g. Dittkrist et al. 2014, Bitsch et al. 2015) consider that the accretion of a giant planet is limited by the accretion rate of the disc. We come back to this issue in the next section.

The results are shown in Fig. 6. First, using the perturbed value of the density $\Sigma(a)$ in Machida et al. (2010)’s formula instead of the unperturbed value $\Sigma_0(a)$, does not change much until the planet reaches $\sim 0.2 M_J$ (compare the green curve with + and the blue curve with o). At larger masses, the planet starts carving a gap, so accretion is slower with $\Sigma(a)$ than with $\Sigma_0(a)$, and the blue curve does not rise as fast as the green one. Consequently, $0.4 M_J$ are reached after slightly more migration, but then the transition to type II migration occurs: the circular symbols are very close to each other, indicating a slow migration. Therefore, using $\Sigma$ or $\Sigma_0$ in Eq. (5) is a key issue for the final mass of giant planets, but in both cases the accretion rate is fast enough to open a gap before being lost by type I migration. Actually, this choice only matters in a phase where a significant gap is already opening, so it does not affect our answer to the question we study here.

Second, as expected, the smaller the accretion rate, the longer the planet stays in type I migration and the further it migrates. In particular, if the accretion rate can not exceed a tenth of Machida et al. (2010)’s prescription, or is limited to the gas flow through the disc, the planet loses 80% of its semi-major axis before reaching the gap opening mass. In order for planets to have a chance of opening a gap before migrating significantly, the runaway accretion rate should be of the same order of magnitude as the one found by Machida et al. (2010). This is consistent with the results of Bitsch et al. (2015) who used an accretion rate limited to 80% of Eq. (11) and found that giant planets observed today at $\sim 5$ AU should be born at a few tens of AU and migrate a lot.

In fact, using $M_p = M_{\text{disc}}$ given by Eq. (11) instead of $M_{p,M10}$ given by Eq. (5), one finds $G = -\frac{A}{3\alpha \pi h^2 M_*}$, hence

$$\ln (\frac{a_i}{a}) = -\frac{A}{6\alpha h^2} \left( \frac{M_1}{M_*} \right)^2 - \left( \frac{M_0}{M_*} \right)^2.$$ 

and the initial to final semi-major axis ratio to reach the gap opening follows:

$$\ln (\frac{a_i}{a}) > -\frac{A}{6\alpha h^2} \left( \frac{M_{\text{gap}}}{M_*} \right)^2.$$  

This is shown in Fig. 7 and illustrates the fact that the accretion rate given by Eq. (11) is too low in most discs for the gap opening mass to be reached before type I migration drives the growing planet close to its host star. Keep in mind that the outcome of Fig. 7 is strongly influenced by the migration rate given by $A$, which we determined here for pure isothermal discs with a density slope of $\alpha_\Sigma = 1$. In reality, the inward migration can be significantly reduced by the entropy driven corotation torque (Baruteau and Masset, 2008), or a shallower profile of the density profile. This may allow giant planet formation far away from the central star also in discs with higher viscosity and $\alpha$, like in Bitsch et al. (2015). Still, this viscous accretion rate induces much more migration than the unlimited runaway growth Eq. (5). Hence, in the next section, we address the question of the sustainability of a high accretion rate until a gap opens.

5. Gap opening induced by accreting giant planets

Kley (1999) provided a recipe for implementing accretion of gas by a planet in 2D simulations, which has been widely used in the literature. At every time-step, gas is removed from the cells inside 0.45 Hill radius of the planet and added to the planet mass, at such a rate that 2/3 of the gas is accreted by the planet within a given time that can be set by a parameter (generally an orbital period). The same applies for cells in an annulus between 0.45 and 0.75 Hill radius of the planet, with
an accretion time two times longer. The linear momentum of
the gas removed from the cells should also be transferred
onto the planet, within the limits of what the disc can provide.

The gas accreted from the planet originates
material from the inner
Szulágyi et al., 2014). These streamlines are fed from material
circulating and the horseshoe streamlines (see e.g. Fig. 10 of
Machida et al. 2010)’s Eq. (5) in the case where Kley (1999)’s
recipe would promote a too fast accretion. The corresponding
curves are the thin curves with + of figure 5 as a function of time.

Figure 5 shows the accretion rates of these planets as a func-
tion of time. The initial accretion rate, marked with a dot, is
proportional to the initial surface density, as these planets all
have the same initial mass. The accretion rate then increases as
inwards towards regions where \( \Sigma_0(r) \) is larger. At some point
though, the planets have depleted their horseshoe region, and
the gas supply limits the accretion rate, which then keeps de-
creasing as the gap deepens. At this turnover point, the planet
has reached a mass of respectively 0.99, 0.96, 0.64, 0.36 \( M_J \)
in the cases where \( \Sigma_0(L) = 2, 1, 0.3, 0.1 \times 10^{-4} M_L/L^2 \).

In the lightest disc simulation above, it appears that the planet empties its horseshoe region before it reaches the gap opening mass. This is illustrated even more clearly by another set of simulations, where \( \Sigma_0(L) = 10^{-4} \) and the planet starts at \( a = L \), so that \( \text{cmd}_{\text{max}} = 0.087 M_J \) only, for a Jupiter mass
planet. Here, the resolution is \( dr = 0.005 \) uniform, \( d\theta = 0.01 \),
the 2D grid extends from 0.4 to 2.2 and the 1D grid from 0.02
to 50; again, \( h = 0.05 \) and \( \alpha = 10^{-6} \). In figure 9 the short
dashed curve corresponds to an imposed mass growth given by
Eq. 5, and no gas removed from the disc. There is one dot
every 1000 \( P_L \) along the curve. Migration slows down as the
gas accreted from the planet grows, because it opens a gap thanks to its gravitation (and here, the cmd is negligible). The red
plain curve, however, corresponds to a case where the accretion is performed using Kley (1999)’s recipe, but limited to Machida et al. (2010)’s

Figure 8: Same as Fig. 4 where the planet grows at the rate given by Eq. 11 instead of the runaway regime of Eq. 5. The planets still migrate in type I migration following Eq. 2 with \( A = 2.5 \). Colour map : \( a_f/a_i \) given by Eq. 12.

Lines : contours of the gap opening mass \( M_{\text{gap}} \) as given by Eq. 19.

Figure 7: Same as Fig. 4, where the planet grows at the rate given by Eq. 11 instead of the runaway regime of Eq. 5. The planets still migrate in type I migration following Eq. 2 with \( A = 2.5 \). Colour map : \( a_f/a_i \) given by Eq. 12.

Lines : contours of the gap opening mass \( M_{\text{gap}} \) as given by Eq. 19.
rate; here, gas is removed from the disc and added to the planet’s mass. It is not until 900 $P_L$ that the accretion rate drops due to lack of gas in the Hill sphere of the planet (the first dot is almost at the same height for all the curves), but already before this point, the planet migrates slower than in the previous case.

From this point on, the planet migrates much slower than in the previous case, as indicated by how close to each other the dots are. This planet is not in type I migration, but in type II. Indeed, type II migration in this simulation is slow but inwards, as our inner disc is accreted by the central star. However, the gas accretion rate of the planet is so low that it barely reaches a Jupiter mass before migrating down to $r = 0.5 \, L$.

Figure 10 shows the mass evolution of the planet in the cases shown in Fig. 9. In the blue case, accretion is almost linear in time. In the red case, it looks more logarithmic. In fact, the accretion rate after 900 $P_L$ can be very well fitted by the following expression:

$$\frac{d(M_p/M_j)}{d(t/P_L)} = 6\pi \times 10^{-8} \left(\frac{t/P_L}{1000}\right)^{-0.05} + 1.6 \times 10^{-8} \exp\left(-\frac{t/P_L - 900}{140}\right),$$

where $t$ is the time since the beginning of the simulation. The green curve in figures 9 and 10 is the track of a planet whose mass evolution is given by Eq. (5) before 900 $P_L$, and by Eq. (13) after, but no gas is removed from the disc. Its mass evolution follows closely the red one, but its migration is much faster. By the time it reaches $M_p = 0.6 \, M_j$, it has migrated about twice as much as in the red case (0.314 versus 0.177 inward), and it reaches $a = 0.5$ with as mass of 0.78 $M_j$ versus 0.93 $M_j$ in the case where gas is removed from the disc (red curve). This illustrates the importance of taking material off the gas disc when a planet accretes. By emptying its horseshoe region as it grows, a planet can save a large fraction of the type I migration it should suffer otherwise.

6. Discussion

The accretion rate of giant planets is still very uncertain. Machida et al. (2010)’s rate is probably an overestimate, essentially because the gap opening and decrease of gas supply is not taken into account (which would not be a concern for the beginning of the runaway growth phase anyway), and also because of the simplified equations of state used (although this parameter seems to have little influence on their results, but see also Klahr and Kley, 2006; Uribe et al., 2013). Shall this rate turn out to be overestimated by an order of magnitude, the migration time-scale would be too short compared to the accretion time-scale. However, other works tend to confirm their result and the order of magnitude of the accretion rate in the runaway regime (e.g. Ayllon and Bate, 2009; Gressel et al., 2013; Szulágyi et al., 2014).

On the other hand, Kley (1999)’s recipe in 2D simulations probably underestimates the gas supply, because gas actually constantly flows in the corotation region through a 3D meridional circulation (Morbidelli et al., 2014), which is not modelled by 2D simulations. The before mentioned simulations by Lubow and D’Angelo (2006) calculating the mass flow through the gap of an accreting planet were also performed in 2D. Taking the meridional circulation into account would only result in larger accretion rates onto planets which have opened a gap. Thus, this would not affect our conclusion that they can grow to $M_j$ at their own pace while being in type II migration.

Interestingly enough, integrating the first term of Eq. (13) from $t = 900 \, P_L$ to infinity (the second term being negligible beyond $t = 1000 \, P_L$) converges to a total of $M_{as} = 3.8 \, M_j$, and only 1.12 $M_j$ in one million orbits. But one should keep in mind that Kley (1999)’s recipe is pessimistic in terms of accretion rate, and that the numerical coefficient in Eq. (13) should be proportional to $\Sigma_0$ so that the final mass could be anything. In addition, in an other simulation where the same accretion recipe was applied, we found a power index of $-0.95$ for the accretion rate as a function of time, which diverges (still
slowly). The idea that giant planets grow roughly logarithmically in time is attractive to explain why most extrasolar giant planets have not reached several Jupiter masses, but gas accretion is a complex process, and the question of the final mass of giant planets is not the object of this paper.

We additionally performed simulations where we have taken the angular momentum exchange between the accreted material and the planet into account. It turned out that the angular momentum transferred from the accreted material onto the planet is of the order of one percent and thus negligible. This value was also found in the 2D studies of Dürrmann and Kley (2015).

7. Conclusion

In this paper, we have performed 2D, locally isothermal simulations of embryos of giant planets migrating in their protoplanetary disc, while accreting gas in the runaway phase, starting at \(0.06 M_\odot = 20 M_J\). Our study can be summarized as the following points:

1. We stress that even a fully formed Jupiter mass planet, once released in a disc, opens a gap in \(\sim 100\) orbits. This remains true even if the planet migrates by more than its horseshoe width during this process.

2. We remind that the transition between the fast type I and the slower type II migration regimes is not smooth in massive discs: due to the positive feedback from the coorbital mass deficit (Masset and Papaloizou, 2003), the planet migrates faster than the type I rate as it starts opening a gap, before it sharply transitions to the type II regime. In very massive discs, the planet could have time to migrate very far or enter the type III migration regime before opening its gap.

3. We find that the runaway growth rate given by Machida et al. (2010) is comparable to the type I migration rate in discs with typical aspect ratios \((h \gtrsim 0.04)\), so that once in runaway growth, a gaseous planet can reach half the mass of Jupiter without being lost into its host star by type I migration. More specifically, for \(a \lesssim 0.005 - h/30\), the gap opening mass \(M_{\text{gap}}\) can be reached in runaway growth while losing less than 60% of the semi-major axis in pure type I migration (see Fig. 9).

4. A planet in the runaway growth regime can open a gap before it reaches the critical mass to do so gravitationally. Indeed, as the planet accretes gas flowing from the separatrix around its horseshoe region, this region spreads and empties.

5. Independent of whether a planet has emptied its corotation region by accreting the gas it encompassed or by repelling the gas gravitationally, it is then in type II migration, not type I any more.

6. All planets start their runaway growth embedded in the disc, hence at this time, Machida et al. (2010)’s rate applies. Only once the planet’s corotation region empties does \(M_p\) decrease. The total mass available in the corotation region is roughly \(\Sigma(a) a^2\) for a Jupiter mass planet, which can be of the order of Jupiter’s mass far enough from the star or in massive enough discs. Hence, limiting the planetary accretion rate to the accretion rate through the disc towards the central star is not appropriate until all this available mass has been taken.

To synthesize these points, either a giant planet is in type I migration but then it is fully embedded in the disc and accretes fast enough to open a gap before migrating too far, or its accretion is limited by the gas supply from the disc because it has opened a deep gap and is in the type II migration regime. Thus, a planet in the runaway growth is saved from type I migration. Its subsequent type II migration and its final mass and position remain open questions that we do not address in this paper.

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Alexander, R. D., Pascucci, I., May 2012. Deserts and pile-ups in the distribution of exoplanets due to photoevaporative disc clearing. MNRAS 422, 82–86.

Alibert, Y., Mordasini, C., Benz, W., Winisdoerffer, C., Apr. 2005. Models of giant planet formation with migration and disc evolution. A&A 434, 343–353.

Ayliffe, B. A., Bate, M. R., Feb. 2009. Gas accretion on to planetary cores: three-dimensional self-gravitating radiation hydrodynamical calculations. MNRAS 393, 49–64.

Baillie, K., Charnoz, S., Pantin, E., May 2016. Trapping planets in an evolving protoplanetary disk: preferred time, locations, and planet mass. A&A 590, A60.

Baruteau, C., Crida, A., Paardekooper, S.-J., Masset, F., Guilet, J., Bitsch, B., Nelson, R., Kley, W., Papaloizou, J., 2014. Planet-Disk Interactions and Early Evolution of Planetary Systems. Protostars and Planets VI, 667–689.

Baruteau, C., Masset, F., Jan. 2008. On the Corotation Torque in a Radiatively Inefficient Disk. ApJ 672, 1054–1067.

Benz, W., Mordasini, C., Alibert, Y., Naef, D., Aug. 2008. Giant planet population synthesis: comparing theory with observations. Physica Scripta Volume T 130 (1), 014022.

Bitsch, B., Crida, A., Morbidelli, A., Kley, W., Dobbs-Dixon, I., Jan. 2013. Stellar irradiated discs and implications on migration of embedded planets. I. Equilibrium discs. A&A 549, A124.
Bitsch, B., Kley, W., Dec. 2011. Range of outward migration and influence of the disc’s mass on the migration of giant planet cores. A&A 536, A77.

Bitsch, B., Lambrechts, M., Johansen, A., Oct. 2015. The growth of planets by pebble accretion in evolving protoplanetary discs. A&A 582, A112.

Bitsch, B., Morbidelli, A., Lega, E., Crìda, A., Apr. 2014a. Stellar irradiated discs and implications on migration of embedded planets. II. Accreting-discs. A&A 564, A135.

Bitsch, B., Morbidelli, A., Lega, E., Kretke, K., Crìda, A., Oct. 2014b. Stellar irradiated discs and implications on migration of embedded planets. III. Viscosity transitions. A&A 570, A75.

Crida, A., Jun. 2009. Minimum Mass Solar Nebulae and Planetary Migration. ApJ 698, 606–614.

Crida, A., Baruteau, C., Kley, W., Masset, F., Aug. 2009. The dynamical role of the circumplanetary disc in planetary migration. A&A 502, 679–693.

Crida, A., Morbidelli, A., May 2007. Cavity opening by a giant planet in a protoplanetary disc and effects on planetary migration. MNRAS 377, 1324–1336.

Crida, A., Morbidelli, A., Masset, F., Apr. 2006. On the width and shape of gaps in protoplanetary disks. Icarus 181, 587–604.

Crida, A., Morbidelli, A., Masset, F., Jan. 2007. Simulating planet migration in globally evolving disks. A&A 461, 1173–1183.

Dittkrist, K.-M., Mordasini, C., Klahr, H., Alibert, Y., Henning, T., Jul. 2014. Impacts of planet migration models on planetary populations. Effects of saturation, cooling and stellar irradiation. A&A 567, A121.

Dürmann, C., Kley, W., Feb. 2015. Migration of massive planets in accreting disks. A&A 574, A52.

Gressel, O., Nelson, R. P., Turner, N. J., Ziegler, U., Dec. 2013. Global Hydromagnetic Simulations of a Planet Embedded in a Dead Zone: Gap Opening, Gas Accretion, and Formation of a Protoplanetary Jet. ApJ 779, 59.

Hourigan, K., Ward, W. R., Oct. 1984. Radial migration of preplanetary material - Implications for the accretion time scale problem. Icarus60, 29–39.

Ida, S., Lin, D. N. C., Jan. 2008. Toward a Deterministic Model of Planetary Formation. IV. Effects of Type I Migration. ApJ 673, 487–501.

Jacobson, S. A., Morbidelli, A., Raymond, S. N., O’Brien, D. P., Walsh, K. J., Rubie, D. C., Apr. 2014. Highly siderophile elements in Earth’s mantle as a clock for the Moon-forming impact. Nature 508, 84–87.

Klahr, H., Kley, W., Jan. 2006. 3D-radiation hydro simulations of disk-planet interactions. I. Numerical algorithm and test cases. A&A 445, 747–758.

Kleine, T., Touboul, M., Bourdon, B., Nimmo, F., Mezger, K., Palme, H., Jacobsen, S. B., Yin, Q.-Z., Halliday, A. N., Sep. 2009. Hf-W chronology of the accretion and early evolution of asteroids and terrestrial planets. Geochimica Cosmochimica Acta 73, 5150–5188.

Kley, W., Mar. 1999. Mass flow and accretion through gaps in accretion discs. MNRAS 303, 696–710.

Kley, W., Bitsch, B., Klahr, H., Nov. 2009. Planet migration in three-dimensional radiative discs. A&A 506, 971–987.

Kley, W., Crida, A., Aug. 2008. Migration of protoplanets in radiative discs. A&A 487, L9–L12.

Lambrechts, M., Johansen, A., Morbidelli, A., Dec. 2014. Separating gas-giant and ice-giant planets by halting pebble accretion. A&A 572, A35.

Lega, E., Morbidelli, A., Bitsch, B., Crida, A., Szulágyi, J., Sep. 2015. Outwards migration for planets in stellar irradiated 3D discs. MNRAS 452, 1717–1726.

Lin, D. N. C., Bodenheimer, P., Richardson, D. C., Apr. 1996. Orbital migration of the planetary companion of 51 Pegasus to its present location. Nature 380, 606–607.

Lin, D. N. C., Papaloizou, J., Aug. 1986a. On the tidal interaction between protoplanets and the primordial solar nebula. I. Self-consistent nonlinear interaction. ApJ 307, 395–409.

Lin, D. N. C., Papaloizou, J., Oct. 1986b. On the tidal interaction between protoplanets and the protoplanetary disk. III. Orbital migration of protoplanets. ApJ 309, 846–857.

Lubow, S. H., D’Angelo, G., Apr. 2006. Gas Flow across Gaps in Protoplanetary Disks. ApJ 641, 526–533.

Machida, M. N., Kokubo, E., Inutsuka, S.-I., Matsumoto, T., Jun. 2010. Gas accretion onto a protoplanet and formation of a gas giant planet. MNRAS 405, 1227–1243.

Malik, M., Meru, F., Mayer, L., Meyer, M., Mar. 2015. On the Gap-opening Criterion of Migrating Planets in Protoplanetary Disks. ApJ 802, 56.

Masset, F. S., 2008. Planet Disk Interactions. In: Goupil, M.-J., Zahn, J.-P. (Eds.), EAS Publications Series. Vol. 29 of EAS Publications Series. pp. 165–244.

Masset, F. S., Papaloizou, J. C. B., May 2003. Runaway Migration and the Formation of Hot Jupiters. ApJ 588, 494–508.

Morbidelli, A., Szulágyi, J., Crida, A., Lega, E., Bitsch, B., Tanigawa, T., Kanagawa, K., Apr. 2014. Meridional circulation of gas into gaps opened by giant planets in three-dimensional low-viscosity disks. Icarus232, 266–270.
Mordasini, C., Alibert, Y., Benz, W., Jul. 2009. Extrasolar planet population synthesis. I. Method, formation tracks, and mass-distance distribution. A&A 501, 1139–1160.

Nelson, R. P., Papaloizou, J. C. B., Masset, F., Kley, W., Oct. 2000. The migration and growth of protoplanets in protostellar discs. MNRAS 318, 18–36.

Paardekooper, S., Baruteau, C., Crida, A., Kley, W., Jan. 2010. A torque formula for non-isothermal type I planetary migration - I. Unsaturated horseshoe drag. MNRAS 401, 1950–1964.

Paardekooper, S., Baruteau, C., Kley, W., Jan. 2011. A torque formula for non-isothermal Type I planetary migration - II. Effects of diffusion. MNRAS 410, 293–303.

Paardekooper, S.-J., Nov. 2014. Dynamical corotation torques on low-mass planets. MNRAS 444, 2031–2042.

Paardekooper, S.-J., Mellema, G., Nov. 2006. Halting type I planet migration in non-isothermal disks. A&A 459, L17–L20.

Pepliński, A., Artymowicz, P., Mellema, G., May 2008a. Numerical simulations of type III planetary migration - I. Disc model and convergence tests. MNRAS 386, 164–178.

Pepliński, A., Artymowicz, P., Mellema, G., May 2008b. Numerical simulations of type III planetary migration - II. Inward migration of massive planets. MNRAS 386, 179–198.

Pierens, A., Dec. 2015. Fast migration of low-mass planets in radiative discs. MNRAS 454, 2003–2014.

Pollack, J. B., Hubickyj, O., Bodenheimer, P., Lissauer, J. J., Podolak, M., Greenzweig, Y., Nov. 1996. Formation of the Giant Planets by Concurrent Accretion of Solids and Gas. Icarus 124, 62–85.

Raymond, S. N., Kokubo, E., Morbidelli, A., Morishima, R., Walsh, K. J., 2014. Terrestrial Planet Formation at Home and Abroad. Protostars and Planets VI, 595–618.

Shakura, N. I., Sunyaev, R. A., 1973. Black holes in binary systems. Observational appearance. A&A 24, 337–355.

Szulágyi, J., Morbidelli, A., Crida, A., Masset, F., Feb. 2014. Accretion of Jupiter-mass Planets in the Limit of Vanishing Viscosity. ApJ 782, 65.

Uribe, A. L., Klahr, H., Henning, T., Jun. 2013. Accretion of Gas onto Gap-opening Planets and Circumplanetary Flow Structure in Magnetized Turbulent Disks. ApJ 769, 97.

Ward, W. R., Apr. 1997. Protoplanet Migration by Nebula Tides. Icarus 126, 261–281.

Ward, W. R., Hourigan, K., Dec. 1989. Orbital migration of protoplanets - The inertial limit. ApJ 347, 490–495.