Quantum key distribution without alternative measurements and rotations

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Abstract

A quantum key distribution protocol based on entanglement swapping is proposed. Through choosing particles by twos from the sequence and performing Bell measurements, two communicators can detect eavesdropping and obtain the secure key. Because the two particles measured together are selected out randomly, we need neither alternative measurements nor rotations of the Bell states to obtain security.

Key words: quantum key distribution, quantum cryptography, entanglement swapping
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1 Introduction

As a kind of important resource, entanglement \cite{1} is widely used in the research of quantum information, including quantum communication, quantum

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cryptography and quantum computation. Entanglement swapping [2], abbreviated by ES, is a nice property of entanglement. That is, by appropriate Bell measurements, entanglement can be swapped between different particles. For example, consider two pairs of particles in the state of $|\Phi^+\rangle$, equivalently, $|\Phi^+\rangle_{12} = |\Phi^+\rangle_{34} = 1/\sqrt{2}(|00\rangle + |11\rangle)$, where the subscripts denote different particles. If we make a Bell measurement on 1 and 3, they will be entangled to one of the Bell states. Simultaneously, 2 and 4 will be also projected onto a corresponding Bell state. We can find the possible results through the following process:

$$|\Phi^+\rangle_{12} \otimes |\Phi^+\rangle_{34} = \frac{1}{2}(|00\rangle + |11\rangle)_{12} \otimes (|00\rangle + |11\rangle)_{34}$$
$$= \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle)_{1324}$$
$$= \frac{1}{2}(|\Phi^+\Phi^+\rangle + |\Phi^-\Phi^-\rangle + |\Psi^+\Psi^+\rangle + |\Psi^-\Psi^-\rangle)_{1324} (1)$$

It can be seen that there are four possible results: $|\Phi^+\rangle_{13}|\Phi^+\rangle_{24}$, $|\Phi^-\rangle_{13}|\Phi^-\rangle_{24}$, $|\Psi^+\rangle_{13}|\Psi^+\rangle_{24}$ and $|\Psi^-\rangle_{13}|\Psi^-\rangle_{24}$. Furthermore, these results appear with equal probability, that is, 1/4. For further discussion about ES, please see Refs.[3,4,5,6].

Quantum cryptography is the combination of quantum mechanics and cryptography. It employs fundamental theories in quantum mechanics to obtain unconditional security. Quantum key distribution (QKD) is an important research direction in quantum cryptography. Bennett and Brassard came up with the first QKD protocol (BB84 protocol) in 1984 [7]. Afterwards, many protocols were presented [8,9,10,11,12,13,14,15,16,17,18,19,20,21,22]. Recently, several QKD schemes based on ES were proposed [23,24,25,26,27,28,29,30]. In Refs.[23,24,25] the author introduced a protocol without alternative measurements. It was simplified [26] and generalized [27] before long, and its security was proved in Ref.[28]. Besides, by ES, doubly entangled photon pairs [29] and previously shared Bell states [30] can be used to distribute secure key.

In this Letter we propose a QKD protocol based on ES, which needs neither alternative measurements [29] nor rotations of the Bell states [25,26,27]. The security against the attack discussed in Ref.[24] is assured by a special technique, that is, random grouping (RG). See Sec.2 for the details of this protocol. The security against general individual attack is analyzed in Sec.3 and a conclusion is given in Sec.4.

2 The QKD protocol

The particular process of this scheme is as follows:
1. Prepare the particles. Alice generates a sequence of EPR pairs in the state \( |\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \). For each pair, Alice stores one particle and sends the other to Bob.

2. Detect eavesdropping.

(1) Having received all the particles from Alice, Bob randomly selects a set of particles out and makes Bell measurements on them by twos.

(2) Bob tells Alice the sequence numbers and measurement results of the pairs he measured.

(3) According to the sequence numbers, Alice performs Bell measurements on the corresponding pairs, and compares her results with Bob’s. For example, consider one of the pairs Bob measured, in which the sequence numbers of the two particles are \( m \) and \( n \), respectively. Then Alice measures her \( m \)-th and \( n \)-th particles in Bell basis, and compares the two outcomes. As discussed in Sec.1, if these particles were not eavesdropped, Alice and Bob should obtain the same results. With this knowledge, Alice can determine, through the error rate, whether there is any eavesdropping. If there are no eavesdroppers in the channel, Alice and Bob proceed with the next step.

3. Obtain the key. Bob makes Bell measurements on his left particles by twos. It should be emphasized that each pair he measures is selected out randomly. Bob records the sequence numbers of all these pairs and sends the record to Alice. Alice then measures her corresponding particles in Bell basis. As discussed in the above paragraphs, their measurement results would be identical. Subsequently, Alice and Bob can obtain the raw key from these results. For example, \( |\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle \) and \( |\Psi^-\rangle \) are encoded into 00, 01, 10 and 11, respectively. After error correction and privacy amplification [32], the raw key will be processed into ideal secret key.

Thus the whole QKD protocol is finished. By this process, Alice and Bob can obtain secure key. In this scheme, we use \( |\Phi^+\rangle \) as the initial state. In practice, any other Bell state is competent and the communicators can even utilize various states for different pairs. It should be emphasized that, however, the various initial states cannot improve the efficiency of QKD (the alleged “high efficiency” in Ref.[30] is a mistake [31]). In fact, our protocol works in a deterministic manner and then has full efficiency in the sense that one qubit-transmission brings one key bit. That is, except for the detection particles, the users can obtain 1 bit (raw) key per qubit-transmission in our protocol, which is higher than the BB84 protocol (0.5 bit).

To compare the efficiency of our protocol with that of others deeply, we can employ Cabello’s definition of QKD efficiency [19]. Let us give a simple example to implement the above protocol and then calculate its efficiency. Suppose
Alice and Bob deal with four EPR pairs (denoted as pairs 1,2,3,4, respectively) in one step. More specifically, Alice sends four particles (each from one of the four EPR pairs) to Bob and announces a classical (random) bit (0 or 1) after Bob received this group of particles. If the classical bit is 0, they perform ES on the pairs 1,3 and 2,4 to obtain the key. Otherwise they perform ES on the pairs 1,4 and 2,3. In this example, Alice and Bob get four key bits by transmitting four qubit and one cbit (classical bit). Obviously, the efficiency equals to 0.8, which is relatively higher (For instance, the efficiency of the famous protocols in Ref.[9], [7], [11], [8], [13], [23] is < 0.25, 0.25, ≤ 0.33, 0.5, 0.5, 0.67, respectively. See Table I in Ref.[19] for details).

3 Security

The above scheme can be regarded as secure because the key distributed can not be eavesdropped imperceptively. There are two general eavesdropping strategies for Eve. One is called “intercept and resend”, that is, Eve intercepts the legal particles and replaces them by her counterfeit ones. For example, Eve generates the same EPR pairs and sends one particle from each pair to Bob, thus she can judge Bob’s measurement results as Alice does in step 3. But in this case there are no correlations between Alice’s particles and the counterfeit ones. Alice and Bob will get random measurement results when they detect eavesdropping in step 2. Suppose both Alice and Bob use $s$ pairs to detect eavesdropping, the probability with which they obtain the same results is only $(1/4)^s$. That is, Eve will be detected with high probability when $s$ is big enough. The second strategy for Eve is to entangle an ancilla with the two-particle state that Alice and Bob are using. At some later time she can measure the ancilla to gain information about the measurement results of Bob. This kind of attack seems to be stronger than the first strategy. However, it is invalid to our protocol as we prove below.

Because each particle transmitted in the channel is in a maximally mixed state, there are no differences among all these particles for Eve. Furthermore, Eve does not know which two particles Bob will put together to make a Bell measurement. As a result, what she can do is to make the same operation on each particle. Let $|\varphi\rangle_{ABE}$ denote the state of the composite system including one certain EPR pair and the corresponding ancilla, where the subscripts $A$, $B$ and $E$ express the particles belonging to Alice, Bob and Eve, respectively. Note that each ancilla’s dimension is not limited here, and Eve is permitted to build all devices allowed by the laws of quantum mechanics. What we want to show is that $|\varphi\rangle_{ABE}$ must be a product of a two-particle state and the ancilla if the eavesdropping introduces no errors into the QKD procedure, which implies that Eve will gain no information about the key by observing the ancilla. Conversely, if gaining information about the key, Eve will invariably
introduce errors.

Without loss of generality, suppose the Schmidt decomposition [33] of $|\varphi\rangle_{ABE}$ is in the form

\[
|\varphi\rangle_{ABE} = a_1|\psi_1\rangle_{AB}|\phi_1\rangle_E + a_2|\psi_2\rangle_{AB}|\phi_2\rangle_E + a_3|\psi_3\rangle_{AB}|\phi_3\rangle_E + a_4|\psi_4\rangle_{AB}|\phi_4\rangle_E
\]  

(2)

where $|\psi_i\rangle$ and $|\phi_j\rangle$ are two sets of orthonormal states, $a_k$ are non-negative real numbers ($i, j, k = 1, 2, 3, 4$).

Because $|\psi_i\rangle$ are two-particle (four-dimensional) states, they can be written as the linear combinations of $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$. Let

\[
|\psi_1\rangle = b_{11}|00\rangle + b_{12}|01\rangle + b_{13}|10\rangle + b_{14}|11\rangle
\]
\[
|\psi_2\rangle = b_{21}|00\rangle + b_{22}|01\rangle + b_{23}|10\rangle + b_{24}|11\rangle
\]
\[
|\psi_3\rangle = b_{31}|00\rangle + b_{32}|01\rangle + b_{33}|10\rangle + b_{34}|11\rangle
\]
\[
|\psi_4\rangle = b_{41}|00\rangle + b_{42}|01\rangle + b_{43}|10\rangle + b_{44}|11\rangle
\]  

(3)

where $b_{pq}$ ($p, q = 1, 2, 3, 4$) are complex numbers. Then $|\varphi\rangle_{ABE}$ can be written, thanks to Eqs. (2) and (3), as

\[
|\varphi\rangle_{ABE} = |00\rangle_{AB} \otimes (a_1b_{11}|\phi_1\rangle + a_2b_{21}|\phi_2\rangle + a_3b_{31}|\phi_3\rangle + a_4b_{41}|\phi_4\rangle)_{E}
\]
\[
+ |01\rangle_{AB} \otimes (a_1b_{12}|\phi_1\rangle + a_2b_{22}|\phi_2\rangle + a_3b_{32}|\phi_3\rangle + a_4b_{42}|\phi_4\rangle)_{E}
\]
\[
+ |10\rangle_{AB} \otimes (a_1b_{13}|\phi_1\rangle + a_2b_{23}|\phi_2\rangle + a_3b_{33}|\phi_3\rangle + a_4b_{43}|\phi_4\rangle)_{E}
\]
\[
+ |11\rangle_{AB} \otimes (a_1b_{14}|\phi_1\rangle + a_2b_{24}|\phi_2\rangle + a_3b_{34}|\phi_3\rangle + a_4b_{44}|\phi_4\rangle)_{E}
\]  

(4)

For convenience, we define four vectors (not quantum states) as follows:

\[
v_l = (a_1b_{1l}, a_2b_{2l}, a_3b_{3l}, a_4b_{4l}) \quad l = 1, 2, 3, 4
\]  

(5)

Consider any two sets of particles on which Alice and Bob will do ES, the state of the system is $|\varphi\rangle_{ABE} \otimes |\varphi\rangle_{ABE}$. According to the properties of ES, we can calculate the probability with which each possible measurement-result-pair is obtained after Alice and Bob measured their particles in Bell basis. For example, observe the event that Alice gets $|\Phi^+\rangle$ and Bob gets $|\Psi^+\rangle$, which corresponds to the following item in the expansion:

\[
\frac{1}{2} |\Phi^+\rangle_A |\Psi^+\rangle_B \otimes \sum_{r,s=1}^{4} (a_rb_{r1}a_s b_{s2} + a_rb_{r2} a_s b_{s1} + a_rb_{r3} a_s b_{s4} + a_rb_{r4} a_s b_{s3}) |\phi_r \phi_s\rangle_E
\]  

(6)
Therefore, this event occurs with the probability

\[ P(\Phi_A^+ \Psi_B^+) = \frac{1}{4} \sum_{r,s=1}^{4} |a_r b_s + a_s b_r + a_r b_s + a_s b_r|^2 \]  \hspace{1cm} (7)

However, this event should not occur. In fact, if Eve wants to escape from the detection of Alice and Bob, any results-pair other than \( \Phi^+ \Phi^+ \), \( \Phi^- \Phi^- \), \( \Psi^+ \Psi^+ \) and \( \Psi^- \Psi^- \) should not appear. Let \( P(\Phi_A^+ \Psi_B^+) = 0 \), we then have, from Eqs.(7) and (5),

\[ v_1^T v_2 + v_2^T v_1 + v_3^T v_4 + v_4^T v_3 = 0 \]  \hspace{1cm} (8)

in which \( v_i^T \) is the transpose of \( v_i \).

Similarly, let the probabilities of \( \Phi_A^- \Psi_B^- \), \( \Phi_A^- \Psi_B^- \) and \( \Phi_A^- \Psi_B^- \) equal to 0, we get

\[ v_1^T v_2 - v_2^T v_1 + v_3^T v_4 - v_4^T v_3 = 0 \]  \hspace{1cm} (9)

\[ v_1^T v_2 + v_2^T v_1 - v_3^T v_4 - v_4^T v_3 = 0 \]  \hspace{1cm} (10)

\[ v_1^T v_2 - v_2^T v_1 - v_3^T v_4 + v_4^T v_3 = 0 \]  \hspace{1cm} (11)

From Eqs.(8)-(11), we can obtain

\[ v_1^T v_2 = v_2^T v_1 = v_3^T v_4 = v_4^T v_3 = 0 \]  \hspace{1cm} (12)

That is,

\[ \begin{cases} v_1 = 0 \quad \text{or} \quad v_2 = 0 \\ v_3 = 0 \quad \text{or} \quad v_4 = 0 \end{cases} \]  \hspace{1cm} (13)

For the same reason, we can obtain the following results:

(1) Let the probabilities of \( \Psi_A^+ \Phi_B^+ \), \( \Psi_A^+ \Phi_B^- \), \( \Psi_A^- \Phi_B^+ \) and \( \Psi_A^- \Phi_B^- \) equal to 0, we can get

\[ \begin{cases} v_1 = 0 \quad \text{or} \quad v_3 = 0 \\ v_2 = 0 \quad \text{or} \quad v_4 = 0 \end{cases} \]  \hspace{1cm} (14)

(2) Let the probabilities of \( \Phi_A^+ \Phi_B^- \) and \( \Phi_A^- \Phi_B^+ \) equal to 0, we then have

\[ v_1^T v_1 - v_2^T v_2 + v_3^T v_3 - v_4^T v_4 = 0 \]  \hspace{1cm} (15)
\[ v_1^T v_1 + v_2^T v_2 - v_3^T v_3 - v_4^T v_4 = 0 \]  \hspace{1cm} (16)

And then

\[ \begin{cases} v_1 = \pm v_4 \\ v_2 = \pm v_3 \end{cases} \] \hspace{1cm} (17)

(3) Let the probabilities of \( \Psi_A^+ \Psi_B^− \) and \( \Psi_A^− \Psi_B^+ \) equal to 0, we can get the same conclusion as Eq.(17).

Finally, we can obtain three results from Eqs.(13), (14) and (17):

1. \( v_1 = v_2 = v_3 = v_4 = 0 \);
2. \( v_1 = v_4 = 0 \) and \( v_2 = \pm v_3 \);
3. \( v_2 = v_3 = 0 \) and \( v_1 = \pm v_4 \)

That is, each of these results makes Eve succeed in escaping from the detection of Alice and Bob. Now we observe what the state \( |\varphi\rangle_{ABE} \) is by putting these results into Eq.(4). If the first result holds, we have \( |\varphi\rangle_{ABE} = 0 \), which is meaningless for our analysis. Consider the condition where the second result holds, \( |\varphi\rangle_{ABE} \) can be written as:

\[ |\varphi\rangle_{ABE} = (|01\rangle \pm |10\rangle)_{AB} \otimes (a_1 b_{12} |\phi_1\rangle + a_2 b_{22} |\phi_2\rangle + a_3 b_{32} |\phi_3\rangle + a_4 b_{42} |\phi_4\rangle)_E \] \hspace{1cm} (18)

It can be seen that \( |\varphi\rangle_{ABE} \) is a product of a two-particle state and the ancilla. That is, there is no entanglement between Eve’s ancilla and the legal particles, and Eve can obtain no information about the key. Similarly, we can draw the same conclusion when the third result holds.

From another point of view, we can derive an effective relation between the errors introduced in the key and the information gained by Eve as in Ref.[34]. Consider any two EPR pairs on which Alice and Bob will perform ES, for example, \( \Phi_{12}^+ \) and \( \Phi_{34}^+ \), where particles 1, 3 and 2, 4 belong to Alice and Bob respectively. As we know, when Alice and Bob make Bell measurements on these particles, the marginal statistics of the measurement results are independent of the measurement order. Suppose Alice makes her measurement before Bob, the state of 2, 4 will thus be projected onto one of the Bell states \( |\xi\rangle \). Because of Eve’s intervention, these two particles will be entangled into Eve’s ancilla and it follows that the state \( |\xi\rangle \) becomes a mixed state \( \rho \). The information Bob can gain from \( \rho \) is bounded by the Holevo quantity \( \chi(\rho) \) [33]. Let \( I_{Eve} \) denote the information Eve can obtain, then \( I_{Eve} \leq \chi(\rho) \). (Obviously, Eve can
not gain more information about Bob’s measurement result than Bob.) From

$$\chi(\rho) = S(\rho) - \sum_i p_i S(\rho_i)$$  \hspace{1cm} (19)$$

we know $S(\rho)$ is the upper bound of $\chi(\rho)$. “High fidelity implies low entropy” [34]. Suppose

$$F(|\xi\rangle, \rho)^2 = \langle \xi | \rho | \xi \rangle = 1 - \gamma$$  \hspace{1cm} (20)$$

where $F(|\xi\rangle, \rho)$ is the fidelity [35] of the states $|\xi\rangle$ and $\rho$, $0 \leq \gamma \leq 1$. Therefore, the entropy of $\rho$ is bounded above by the entropy of a diagonal density matrix $\rho_{max}$ with diagonal entries $1 - \gamma, \gamma/3, \gamma/3, \gamma/3$. The entropy of $\rho_{max}$ is

$$S(\rho_{max}) = -(1 - \gamma) \log_2 (1 - \gamma) - \gamma \log_2 \frac{\gamma}{3}$$  \hspace{1cm} (21)$$

Then we have

$$I_{Eve} \leq -(1 - \gamma) \log_2 (1 - \gamma) - \gamma \log_2 \frac{\gamma}{3}$$  \hspace{1cm} (22)$$

Let us discuss the connection between the fidelity $F(|\xi\rangle, \rho)$ and the detection probability $d$. When Alice and Bob detect eavesdropping, only $|\xi\rangle$ is the correct result, whereas any other Bell state will be regarded as an error. Since $F(|\xi\rangle, \rho)^2 = 1 - \gamma$, the detection probability $d = \gamma$. From Eq.(22), we get

$$I_{Eve} \leq -(1 - d) \log_2 (1 - d) - d \log_2 \frac{d}{3}$$  \hspace{1cm} (23)$$

It can be seen from this relation that when $d = 0$, i.e., Eve introduces no error to the key, she will obtain no information, which is in agreement with the above result. When $\gamma > 0$, i.e., Eve can gain some of Bob’s information, but she has to face a nonzero risk $d = \gamma$ of being detected. When $\gamma = 3/4$, we have $S(\rho_{max}) = 2$, which implies that Eve has the chance to eavesdrop on all of Bob’s information. In this case, however, the detection probability is no less than 3/4 per ES for eavesdropping detection. For example, when Eve intercepts all the particles and resends new particles from her own EPR pairs, she will get all of the information about Bob’s key while introduce 3/4 error rate per ES.

To sum up, our protocol can resist the eavesdropping with ancilla.
4 Conclusion

We have presented a full-efficiency QKD protocol based on ES. The security against the attack discussed in Ref.[24] is assured by the technique of RG instead of requiring alternative measurements [29] or rotations of the Bell states [25,26,27]. Furthermore, this technique brings us another advantage. That is, it is unnecessary to randomize the initial Bell states as in Refs.[23,25], which leads to less Bell measurements in our protocol. For instance, to distribute two key bits, Alice and Bob make two Bell measurements in our protocol, while in Refs.[23,25] they must make three.

On the other hand, we have to confess that our protocol has a disadvantage, i.e., it uses a sequence of entangled states instead of a single quantum system [25,26,27] to generate the key. Nevertheless, it is not a fatal problem. Many QKD protocols work in this model, for example, the famous E91 protocol [8]. Furthermore, each pair of particles is still in one of the Bell states and can be reused in other applications after QKD.

In practical implementations, our scheme needs complete Bell states analysis. Though Bell measurement has not been generally accomplished [36], it was experimentally realized based on some certain techniques [37,38,39]. Furthermore, the realizations of entanglement swapping has been proposed [6,40]. Therefore, our scheme is within the reach of current technology.

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