LOTTTTTUCE: Layer Oriented Tip-Tilt Turbulence Tomography using Covariance and Elevation

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Abstract. LOTTTTTUCE is based upon the fact that turbulence at the pupil produces correlated tip-tilt motion over the entire field (averaging the tip-tilt across the widest field possible gives the strength of the turbulence at the telescope), while the on-axis (any axis) image motion measures the integrated tip-tilt over the line of sight (single stars provide the variance of the tip-tilt, which allows to infer the integrated seeing). Between these two extremes, the amount of correlation across a given field size is the integral of the turbulence from the ground to the altitude where the tip-tilt decorrelates over the meta-pupil. Differentiating the altitude-integrated tip-tilt with respect to altitude generates an estimate of tip-tilt (hence turbulence, assuming Kolmogorov properties) at each altitude. Alternately, the 3D Fourier transform of a data cube containing the time evolution of the tip (or tilt) across the field allows to determine the amount of energy for “field” frequencies (in other words, the integrated seeing across each same size patches) and the temporal spectrum of each of these features. Differentiating the spectrum with respect to spatial frequency would provide the amount of energy, as well as speed and direction, of each layer. The LOTTTTTUCE method is a novel method of measuring the vertical turbulence profile that uses wide field tip-tilt information such as that provided by Pan-STARRS. However, the method also has limitations due to tip-tilt decorrelation as a function of meta-pupil overlap, finite outer scale, and non-Kolmogorov power spectrum.

1. By way of an introduction
Although this method should work (see below), simulations have not yet yielded any satisfactory results. Nonetheless, these simulations, as well as handling new forms of data (tip-tilt data cubes across a wide field are not commonly available and manipulating such data reveals correlations and relationships which are not immediately apparent) have revealed much more information than the originally simple algorithm described in the abstract. It would have been easy to withdraw this paper from publication, but null results are an important aspect of the scientific process.

The term “null result” comes from the Latin nullus resultarum which does not imply no result, but rather that the results do not support the underlying hypotheses, that the experimental outcome does not show an otherwise expected effect.
The reason this method should work in principle is that it is an extension of a demonstrated method originally developed by Flicker & Rigaut [1]: They used the adaptive optics corrected isoplanatic angle to determine the altitude integrated $r_0$, as well as its first moment, i.e. its mean height. The LOTTTTUCE method proposes to extend higher orders of isoplanatism (though using tip-tilt, i.e. isokinetism only), to obtain the higher moments, and hence a vertical profile, of the turbulence.

The general context in which this method is relevant is the importance of turbulence profiling. Any sensor measuring tip-tilt across wide fields (such as GLAO or MOAO but may be even more appropriately, Pan-Starrs [2]) will generate data that contains information about the turbulence profile. For instance in GLAO, the ground layer is given by the commands to the deformable mirror, while the residual isokinetism from the wavefront sensor measurements could be used to determine the turbulence. This will be useful for PSF reconstruction. Being able to separate turbulent layers and speeds would also have applications in predictive algorithms for MOAO.

2. Simulations

The ideal data to test out the LOTTTTUCE method would be a pair of data cubes containing $[\text{tip}(x,y,t)]$ and $[\text{tilt}(x,y,t)]$. A GLAO code developed to simulate very wide field AO systems (e.g. Imaka [3]) named instant_GLAO was modified to produce a grid of instantaneous PSF stars seen through a layered atmosphere. Originally, instant_GLAO was developed to simulate complex effects and implementation issues of very wide field AO, such as a tiled or
misconjugated DM, field aberrations seen by different wavefront sensors, etc. All these features were turned off, as well as the adaptive optics correction (i.e. a non AO, sky only simulation). Instead, each instantaneous PSF was recorded and the tip and tilt were determined by measuring the shift in centroid.

Using a 1.8m telescope with a field of view of 1°, a grid of 32x32 PSF were generated for 1024 time steps at 200Hz. In the simplest case, the model atmosphere typically consisted of two layers, one at a few hundreds of meters altitude and the other a few kilometers high moving perpendicular to each other at a few ms⁻¹. These PSFs are shown in Figure 2. Because only two layers are present, two different scales of correlation should be present in the centroid data. Centroid cubes are shown on Figure 3. Based on such data, different methods were developed to try to retrieve the vertical turbulence profile, which are described below (section 3).

3. Methods to retrieve the turbulence profile

3.1. Multi-scale tip-tilt averaging

The simplest approach to retrieve the turbulence profile consists in averaging the instantaneous tip-tilt into different size boxes (if N is the size of a box, there will be N² boxes of 32/Nx32/N pixels each) and measuring the time variance of the signals. The largest box (the variance of the average) is a measure of the variance of the tip-tilt at the ground and the average of the (32)² smallest boxes of single star/pixel measurements (the average of the variances) provides the altitude integrated seeing. We can write an equation of the form:

$$\sigma^2_{\text{tip}}(N) \approx 1.03 \int_0^h \left( \frac{D}{r_0(h)} \right)^{\frac{5}{3}} \cdot dh$$

with the size of the box-averaged field N proportional to the altitude h of the layer. Plots of the field-size dependent isokinetism (Figure 4) indeed show a monotonic increase of the variance with N, but it is surprisingly smooth and differentiating with respect to N does not retrieve the layers introduced in the simulations.

There seems to be a faint trend in the results, apparent when changing the number of layers or their height but this is not sufficient to reliably retrieve the input turbulence height or strength. This may be due to the fact that the tip-tilt angular correlations are in fact much larger than the meta-pupil at most altitudes. Also, the tip-tilt angular correlation function is most likely very smooth, translating into a very broad altitude response. The tip-tilt angular correlations are also very sensitive to the outer scale of the turbulence, L₀. It is expected that the outer scale close to the ground varies with altitude and there is no reason to assume that the free atmosphere outer scale is correlated to the ground. For instance, we can assume that the dome seeing outer scale has to be smaller than the diameter of the dome itself. All of this could readily be included in the simulations, but the disappointing results shown in Figure 4 are unfortunately just for the simplest of cases…
Figure 4: Multi-scale box-averaging field tip-tilt for model atmospheres with two and three layers. Top left: 40% of turbulence at 0m and 60% at 2000m; top right, 70% at 150m and 30% at 2000m. Bottom left: 40% at 0m, 30% at 1200m and 30% at 2000m; bottom right, 20% at 0m, 40% at 800m and 40% at 2000m. In each case, the cumulative seeing obtained from $\sigma_{\text{tip-tilt}}(N)$ is shown (left, tip=green, tilt=red, focus=blue, r.m.s. tip-tilt=black) as well as its derivative with respect to altitude, which should be the vertical turbulence profile.

3.2. Covariance method

The multi-scale averaging method only uses the temporal statistical information, but we have do access to the full time series of tip tilt measurements, so it may be possible to extract more detailed information by studying the tip-tilt spatio-temporal covariances. Because these are field covariances (and not pupil covariances as one is more accustomed to in SLODAR work), it is worth pointing out that the effect of altitude on layers is to broaden the covariance peak for low-altitude turbulence and to narrow the covariance peak as the altitude increases (turbulence at the ground produces a constant plane). Furthermore, at t=0, all the covariance peaks associated with each layer will be superposed; it is only by looking at temporal covariance that the wind speeds and directions can be extracted, although this is non-trivial as it ideally requires fitting multiple Gaussian component. Because turbulence at the pupil introduces a constant plane, it may not be possible to extract the direction of the ground layer, though its speed can possibly be recovered from the decay time of the constant.

Figure 5: time evolution of the tip-tilt covariance function for three layers at 0m, 200m and 700m. Ground layer turbulence produces a constant plane, which decreases in time. Higher layers produce sharper peaks, which move across the covariance as a function of their speed, height and direction.

Although this method appears to contain more information, it is not trivial to retrieve the turbulence profile, as quantification would require fitting Gaussian components to the data. More information also seems to be available for example from one time step to the next (e.g. being able to track a correlation peak through the time dimension) as shown in the 3D projection of the temporal data.
This is known as the autocovariance and is work in progress, as at least conceptually, it may open the path for a truly layer-oriented version of the turbulence profiling (see Section 4).

3.3. 3D FTT
The last method that was developed used a 3-dimensional Fourier transform of the tip and tilt cubes. The result is the temporal and spatial power spectrum of the tip and tilt. Each spatial frequency corresponds to a given spatial scale of field variations (for example the average tip-tilt is the frequency zero, corresponding to the DC bias; the highest spatial frequency corresponds to very small scale variations of the signal, in other words, small isokinetism introduced by high altitude layers). The temporal dimension of the spectrum should then provide information related to the speed of the layers/spatial frequencies.

One difficulty of this method is that the phase screens don’t evolve, but simply translate. A temporally translating phase screen will produce a plane of constant phase in the plane orthogonal to the 3D direction of motion. Examples of cuts through the amplitude and phase of the 3D FFT of the tip and tilt cubes are shown in Figure 7.
Another apparently trivial difficulty is being able to visualize the phase in 3 dimensions. Iso-phase surfaces might be useful, but because the phase wraps modulo $2\pi$ at high frequency, it hides the coherent phase information at lower frequencies. It can again be seen that the data is very rich, but in this case it is the difficulty in extracting the relevant information (profile, wind speed), which makes this method impractical.

Taking the inverse Fourier transform of the modulus squared of the previous step produces the spatiotemporal autocorrelation of the tip and tilts across the field. In theory, individual layers at different heights and speeds produce autocorrelation peaks that narrow with layer altitude (similar to the covariance). Different attempts at visualizing dual and triple layers are shown in Figure 8.

![Figure 8: FFT spatiotemporal autocorrelation of two and three layer atmosphere. Left to right: 60% at 0m and 40% at 2000m; 60% at 1000m and 40% at 2000m; 40% at 15000m and 60% at 2000m; 40% at 0m, 30% at 12000m and 30% at 2000m; 20% at 0m, 40% at 800m and 40% at 2000m. As the altitude of the layer increases, the correlation peaks narrows (it is a constant plane for ground turbulence). The direction and angle of the correlation peaks give the direction and angular velocity components respectively of the velocity vector of each layer.](image)

### 4. Why it doesn’t work and future work

Although the tip-tilt correlations across the field do contain information regarding the vertical turbulence profile, the response function is most likely too smooth to be able to reconstruct useful profiles. The information is encoded in the data (and correlations can be seen “by eye”), but a different method is needed for this concept to work quantitatively on realistic turbulence profiles. For example, instead of tip and tilt, it might be possible to use higher orders of aberrations (because they smaller correlation lengths). This requires looking at a different observable than the centroid (for example, focus is encoded in the second moment of the PSF, shown in the dashed blue line of Figure 4); or by using low order wavefront sensors on field stars - but this is no different than SLODAR!

The conference was very stimulating at generating new ideas and discussions: a suggestion which has not yet been tried out was made by Miska LeLouarn to build the equivalent of an interaction matrix between field covariances and single layer altitudes and then invert it, to retrieve the altitudes from the measured field covariances. Another very interesting discussion with Aglae Kellerer indicated that the layer oriented MACO approach to solar observations[4] might be applicable if the notion of extended object is applied to star fields. How to translate measurements on partially illuminated high-altitude meta-pupils into useful tip-tilt correlations remains to be determined and will be the focus of this ongoing work.

### References

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