Alpha decay calculations with a new formula

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Abstract

A new semi-empirical formula for calculations of $\alpha$ decay half-lives is presented. It was derived from Royer relationship by introducing new parameters which are fixed by fit to a set of experimental data. We are using three sets: set A with 130 e-e (even-even), 119 e-o (even-odd), 109 o-e, and 96 o-o, set B with 188 e-e, 147 e-o, 131 o-e, and 114 o-o, and set C with 136 e-e, 84 e-o, 76 o-e, and 48 o-o alpha emitters. A comparison of results obtained with the new formula and the following well known relationships: semFIS (semiempirical based on fission theory); ASAF (analytical superAsymmetric fission) model, and UNIV (universal formula) is made in terms of rms standard deviation. We also introduced a weighted mean value of this quantity, allowing to compare the global properties of a given model. For the set B the order of the four models is the following: semFIS; UNIV; newF, and ASAF. Nevertheless for even-even alpha emitters UNIV gives the 2nd best result after semFIS, and for odd-even parents the 2nd is newF. Despite its simplicity in comparison with semFIS the new formula, presented in this article, behaves quite well, competing with the others well known relationships.

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I. INTRODUCTION

A.H. Becquerel discovered, by chance, $\alpha$, $\beta$, and $\gamma$ radioactivities — the first experimental information coming from an atomic nucleus. E. Rutherford realized that the alpha particle is $^4\text{He}$. H. Geiger and J.M. Nuttal \[1\] gave in 1911 a simple relationship allowing to estimate the half-lives, but only in 1928 the right explanation was given by G. Gamow and simultaneously by R.W. Gurney and E.U. Condon, as a quantum tunnelling through the potential barrier, mostly of electrostatic nature.

As far back as 1911, Geiger and Nuttal have found, for the members of a given natural radioactive family, a simple purely empirical dependence \[1\] of the $\alpha$-decay partial half-life, $T_\alpha$, on the mean $\alpha$-particle range, $R_\alpha$, in air (at $15^\circ\text{C}$ temperature and one atmosphere pressure), which may be written as:

$$\log_{10} T_\alpha (\text{s}) = -57.5 \log_{10} R_\alpha (\text{cm}) + C \tag{1}$$

where $C$ depends on the series, e.g. $C = 41$ for the $^{238}\text{U}$ series. One has approximately $R_\alpha = 0.325 E_\alpha^{3/2}$ in which the kinetic energy of $\alpha$ particles, $E_\alpha$, is expressed in MeV and the range in air, $R$, in cm. This relationship is now of historical interest; the effect of atomic number, $Z$, upon decay rate is obscured. The one-body theory of $\alpha$-decay can explain it and to a good approximation produces a formula with an explicit dependence on the $Z$ number. Nowadays, very often a diagram of $\log T_\alpha$ versus $ZQ^{-1/2}$ is called Geiger-Nuttal plot \[2\].

There are many semiempirical relationships (see for example Refs. \[3\text{–}13\]), allowing to estimate the disintegration period if the kinetic energy of the emitted particle $E_\alpha = QA_d/A$ is known. $Q$ is the released energy and $A_d, A$ are the mass numbers of the daughter and parent nuclei. Alpha-decay half-life of an even-even emitter can also be easily calculated by using the universal curves \[14\] or the analytical supersymmetric (ASAF) model \[15\]. Some of these formulae were only derived for a limited region of the parent proton and neutron numbers. Their parameters have been determined by fitting a given set of experimental data. Since then, the precision of the measurements was increased and new $\alpha$-emitters have been discovered.

The description of data in the neighborhood of the magic proton and neutron numbers, where the errors of the other relationships are large, was improved by deriving a new formula based on the fission theory of $\alpha$-decay \[16\]. A computer program \[17\] allows to change
automatically the fit parameters, every time a better set of experimental data is available. There are many alpha emitters, particularly in the intermediate mass region, for which both the Q-values and the half-lives are well known [18, 19]. Initially it was used a set of 376 data (123 even-even (e-e), 111 even-odd (e-o), 83 odd-even (o-e), and 59 odd-odd (o-o)) on the most probable (ground state to ground state or favored transitions) α-decays, with a partial decay half-life

\[ T_{\alpha} = \frac{100}{b_{\alpha}} \frac{100}{i_p} T_t \]  

where \( b_{\alpha} \) and \( i_p \), expressed in percent, represent the branching ratio of α-decay in competition with all other decay modes, and the intensity of the strongest α-transition, respectively.

The formula given by Fröman [3] is limited to the region of even-even nuclei with \( Z \geq 84 \). This formula describes well the experimental data of nuclei with \( N \geq 128 \) but fails in the region of lighter α-emitters, which have not been available at the moment of its derivation. A better overall result gives a simple relationship of Wapstra et al. [4] also valid for even-even nuclei with \( Z \geq 85 \). In the new variant derived by A. Brown [10] for nuclei with \( Z \geq 72 \) the agreement with experimental data is not bad for nuclei with \( Z \geq 72 \), but large errors are obtained for lighter parent nuclei.

The formula presented by Taagepera and Nurmia [5] remains one of the best. It is exceeded by a variant due to Keller and Münzel [7]. Viola and Seaborg [6] introduced a relationship which gives excellent agreement in the region of actinides but it underestimates the lifetimes of lighter nuclei, in contrast with overestimations obtained with the first of the above mentioned formulae.

In the region of superheavy nuclei the majority of researchers prefer to use Viola-Seaborg formula. Very recently for nuclei with \( Z = 84 - 110 \) and \( N = 128 - 160 \), for which both \( Q_{\alpha}^{\text{exp}} \) and \( T_{\alpha}^{\text{exp}} \) experimental values are available, new optimum parameter values [13] have been determined. The average hindrance factors for 45 o-e \((Z = 85 - 107)\), 55 e-o \((Z = 84 - 110)\), and 40 o-o \((Z = 85 - 111, N = 129 - 161)\) nuclei were determined to be \( C_{V}^{o} = 0.437 \), \( C_{V}^{e} = 0.641 \), and \( C_{V}^{m} = 1.024 \). In this way \( T_{\alpha}^{\text{exp}} \) were reproduced by the Viola-Seaborg formula within a factor of 1.4 for e-e, 2.3 for o-e, 3.7 for e-o and 4.7 for o-o nuclei, respectively. Good results were obtained with a formula due to Royer [12] having 12 parameters \( a, b, c \) for e-e, e-o, o-e and o-o nuclei. Shell effects were not taken into account; nuclei with neutron number close to the shell closures \( N = 152 \) and 162 (namely 3 nuclei with \( N = 151(Z = 96, 98, 100) \) one with \( N = 153 \) \( Z = 98 \), and one with \( N = 161 \) \( Z = 110 \) have been omitted in the fitting.
procedure. Other omission of 3 o-e nuclei with $Z = 97, N = 146, 148$ and $Z = 101, N = 154$ was motivated by a large deviation from the average behaviour. A simple version of the Viola-Seaborg formula was proposed by Parkhomenko and Sobiczewski \[13\].

Since 1979 one of us (DNP) considered $\alpha$ decay a superasymmetric fission process \[20, 21\]. Consequently a new semiempirical formula for the alpha decay half-lives \[16\] was a straightforward finding \[16, 22\]. Moreover, the analytical and numerical superasymmetric fission (ASAF \[23\] and NUSAF) models were used together with fragmentation theory developed by the Frankfurt School, and with penetrability calculations like for $\alpha$ decay, to predict cluster (or heavy particle) radioactivity \[24–29\]. The extended calculations \[30–32\] have been used to guide the experiments and as a reference for many theoretical developments. A series of books and chapters in books, e.g. \[33–37\] are also available. A computer program \[17\] gives us the possibility to improve the parameters of the ASAF model in agreement with a given set of experimental data. The UNIV (universal curve) model was updated in 2011 \[38\].

The interest for $\alpha$D is strongly simulated by the search for heavier and heavier super heavies (SHs) — nuclides with $Z > 103$, produced by fusion reactions \[39–42\] who may be identified easily if a chain of $\alpha$D leading to a known nucleus may be measured. Recently it was shown that for superheavy nuclei with atomic numbers $Z > 121$ \[29, 43\] $\alpha$D may be stronger than CD or spontaneous fission.

A very interesting result was reported by Y.Z. Wang et al. \[44\], who compared 18 such formulae in the region of superheavy nuclei. They found: “SemFIS2 formula is the best one to predict the alpha-decay half-lives ... In addition, the UNIV2 formula with fewest parameters and the VSS, SP and NRDX formulas with fewer parameters work well in prediction on the SHN alpha-decay half-lives \[6, 13, 45–47\].”

In this work we intend to study how may be improved a formula developed by G. Royer \[48\] by adding few parameters fitted to experimental data. We shall use three data sets, say: A (130 e-e, 119 e-o, 109 o-e, and 96 o-o), set B (188 e-e, 147 e-o, 131 o-e, and 114 o-o), and set C with 136 e-e, 84 e-o, 76 o-e, and 48 o-o alpha emitters.set C with 136 e-e, 84 e-o, 76 o-e, and 48 o-o alpha emitters. The set A was developed by one of us (DA), the set B belongs to DNP’s group, and the set C was taken from G. Royer \[48\]; few Q-values have been updated using the AME12 evaluation of experimental atomic masses \[18\]. Comparison with ASAF, UNIV, and semFIS will be made using both A, B and C data sets.
II. NEW FORMULA

FIG. 1. The differences of $\log_{10} T_{\text{theor}} - \log_{10} T_{\text{exp}}$ in four groups of alpha emitters versus the neutron number of the daughter $N_d$. New formula. Vertical dashed lines are marking magic numbers of neutrons, either spherical or deformed.

The Royer formula [48] is defined as

$$T_{1/2} = a + b \frac{A^{1/6}}{\sqrt{Z}} + \frac{cZ}{\sqrt{Q_\alpha}}$$

(3)

with initial parameters $a = -27.657; -28.408; -27.408$, and $-24.763$, $b = -0.966; -0.920; -1.038$, and $-0.907$, and $c = 1.522; 1.519; 1.581$, and $1.410$ for e-e, e-o, o-e, and o-o, respectively. The rms standard deviation for 130 e-e, 119 e-o, 109 o-e, and 96 o-o was $\sigma = 0.560, 1.050, 0.871,$ and 0.926, respectively.

The new relationship is obtained by introducing $I = (N - Z)/A$ and the new parameters
\[ T_{1/2} = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + dI + eI^2 \]  

where initially for the set A the parameters \( a, b, c, d, e \) are given in table I.

Before optimization, with our set of 580 \( \alpha \) emitters, and the initial values of the parameters \( a = -27.989, b = -0.940, c = 1.532, d = -5.747, e = 11.336 \) for even-even nuclei we got the following values of rms standard deviations, \( \sigma = 0.5547 \). After optimization for e-e emitters, with \( a = -27.837, b = -0.94199975, c = 1.5343, d = -5.7004, e = 8.785 \) the agreement was improved: \( \sigma = 0.540 \). The order of optimization of the 5 parameters was: \( a; e; d; c, \) and \( b \).

We may compare the results obtained by using the set A, B and C in tables I and II. We can see a slight improvement by using the set B.

**TABLE I. Optimization of coefficients using the set A (454 data). New formula.**

| Group | n   | \( \sigma \) | a      | b      | c     | d     | e     |
|-------|-----|---------------|--------|--------|-------|-------|-------|
| e-e   | 130 | 0.557         | -27.884| -0.952 | 1.533 | -4.101| 6.285 |
| e-o   | 119 | 0.961         | -26.160| -1.140 | 1.559 | 17.756| 37.055|
| o-e   | 109 | 0.816         | -27.800| -0.897 | 1.535 | 15.319| 30.443|
| o-o   | 96  | 0.915         | -24.292| -0.911 | 1.409 | -3.418| 7.640 |

**TABLE II. Optimization of coefficients using the set B.**

| Group | n   | \( \sigma \) | a      | b      | c     | d     | e     |
|-------|-----|---------------|--------|--------|-------|-------|-------|
| e-e   | 188 | 0.540         | -27.837| -0.94199975| 1.5343| -5.7004| 8.785 |
| e-o   | 147 | 0.678         | -28.2245| -0.8629 | 1.53774| -21.145| 53.890|
| o-e   | 131 | 0.522         | -26.8005| -1.10783| 1.5585| 14.8525| -30.523|
| o-o   | 114 | 0.840         | -23.6354| -0.891  | 1.404 | -12.4255| 36.9005|
### III. ASAF

The half-life of a parent nucleus AZ against the split into a cluster $A_eZ_e$ and a daughter $A_dZ_d$

$$T = \left[ \frac{(h \ln 2)}{(2E_v)} \right] e^{\exp(K_{ov} + K_s)}$$

is calculated by using the WKB quasiclassical approximation, according to which the action integral is expressed as

$$K = \frac{2}{\hbar} \int_{R_a}^{R_b} \sqrt{2B(R)E(R)}dR$$

with $B = \mu$ — the reduced mass, $K = K_{ov} + K_s$, and $E(R)$ replaced by $[E(R) - E_{corr}] - Q$. $E_{corr}$ is a correction energy similar to the Strutinsky [49] shell correction, also taking into account the fact that Myers-Swiatecki’s liquid drop model (LDM) [50] overestimates fission barrier heights, and the effective inertia in the overlapping region is different from the reduced mass. The turning points of the WKB integral are:

$$R_a = R_i + (R_t - R_i)[(E_v + E^*)/E_0^{01/2}$$

$$R_b = R_t E_c \{1/2 + [1/4 + (Q + E_v + E^*)E_l/E_c^{21/2}] / (Q + E_v + E^*) \}$$

where $E^*$ is the excitation energy concentrated in the separation degree of freedom, $R_i = R_0 - R_e$ is the initial separation distance, $R_t = R_e + R_d$ is the touching point separation distance, $R_j = r_0 A_j^{1/3}$ ($j = 0, e, d$; $r_0 = 1.2249$ fm) are the radii of parent, emitted and daughter nuclei, and $E_0^0 = E_i - Q$ is the barrier height before correction. The interaction energy at the top of the barrier, in the presence of a non negligible angular momentum, $\hbar l$, is given by:

$$E_i = E_c + E_l = e^2 Z_e Z_d / R_t + \hbar^2 l(l + 1) / (2\mu R_t^2)$$

### TABLE III. Optimization of coefficients using the set C.

| Group | n | $\sigma$ | a     | b     | c     | d     | e     |
|-------|---|---------|-------|-------|-------|-------|-------|
| e-e   | 136 | 0.298  | -26.661 | -1.151 | 1.591 | 14.680 | -50.779 |
| e-o   | 84  | 0.914  | -32.567 | -0.851 | 1.691 | -32.307 | 111.787 |
| o-e   | 76  | 0.859  | -30.303 | -1.006 | 1.749 | -41.203 | 124.968 |
| o-o   | 48  | 0.810  | -25.542 | -1.139 | 1.619 | -21.016 | 109.613 |
The two terms of the action integral $K$, corresponding to the overlapping ($K_{ov}$) and separated ($K_s$) fragments, are calculated by analytical formulas (approximated for $K_{ov}$ and exact for $K_s$ in case of separated spherical shapes within the LDM):

$$K_{ov} = 0.2196(E_0^0 A_c A_d / A)^{1/2} (R_t - R_i) \left[ \sqrt{1 - b^2} - b^2 \ln \frac{1 + \sqrt{1 - b^2}}{b} \right]$$  \hspace{1cm} (10)$$

$$K_s = 0.4392 [(Q + E_v + E^*) A_c A_d / A]^{1/2} R_b J_{rc} ; \quad b^2 = (E_v + E^*) / E_0^0$$  \hspace{1cm} (11)$$

$$J_{rc} = (c) \arccos \sqrt{(1-c+r)/(2-c)} - [(1-r)(1-c+r)]^{1/2} + \sqrt{1-c} \ln \frac{2 \sqrt{(1-c)(1-r)(1-c+r)} + 2 - 2c + cr}{r(2-c)}$$  \hspace{1cm} (12)$$

where $r = R_t / R_b$ and $c = r E_c / (Q + E_v + E^*)$. In the absence of the centrifugal contribution ($l = 0$), one has $c = 1$.

The choice $E_v = E_{corr}$ allows to get a smaller number of parameters. It is evident that, owing to the exponential dependence, any small variation of $E_{corr}$ induces a large change of $T$, and thus plays a more important role compared to the preexponential factor variation due to $E_v$. Shell and pairing effects are included in $E_{corr} = a_i(A_c)Q$ ($i = 1, 2, 3, 4$ for even-even, odd-even, even-odd, and odd-odd parent nuclei). For a given cluster radioactivity there are four values of the coefficients $a_i$, the largest for even-even parent and the smallest for the odd-odd one (see figure 1 of [31]). The shell effects for every cluster radioactivity is implicitly contained in the correction energy due to its proportionality with the $Q$ value, which is maximum when the daughter nucleus has a magic number of neutrons and protons.

With only few exceptions, in the region of nuclei far from stability, measured $\alpha$-decay partial half-lives are not available. In principle we can use the ASAF model to estimate these quantities. Nevertheless, slightly better results can be obtained by using semFIS [51]. The potential barrier shape similar to that we considered within the ASAF model was calculated by using the macroscopic-microscopic method [52], as a cut through the PES at a given mass asymmetry, usually the $^{208}$Pb valley or not far from it.

Before any other model was published, there were estimations of the half-lives for more than 150 decay modes, including all cases experimentally confirmed until now. A comprehensive table was produced by performing calculations within that model. Subsequently, the numerical predictions of the ASAF model have been improved by taking better account of the pairing effect in the correction energy, deduced from systematics in four groups of
parent nuclei (even - even, odd - even, even - odd and odd - odd). In a new table, published in 1986, cold fission as cluster emission has been included. The systematics was extended in the region of heavier emitted clusters (mass numbers \(A_e > 24\)), and of parent nuclei far from stability and superheavies. Since 1984, the ASAF model results have been used to guide the experiments and to stimulate other theoretical works.

**TABLE IV. Parameters of ASAF model; data — the set A.**

| Group | n  | \(C_{i1}\) | \(C_{i2}\) | \(C_{i3}\) | \(C_{i4}\) | \(D_i\) | \(y50_i\) |
|-------|----|------------|------------|------------|------------|---------|----------|
| e-e   | 130| 0.985286   | 0.017996   | 0.027056   | 0.030373   | 0.001215| -0.018261|
| e-o   | 119| 1.011020   | -0.027134  | 0.074588   | 0.051785   | 0.141828| -0.139752|
| o-e   | 109| 0.990173   | 0.063476   | 0.112658   | 0.041385   | 0.087355| -0.168133|
| o-o   | 96 | 0.989577   | 0.025327   | 0.226867   | 0.043133   | 0.989809| -0.296017|

**TABLE V. Parameters of ASAF model for the two other data sets: B, and C.**

| Group | n  | \(C_{i1}\) | \(C_{i2}\) | \(C_{i3}\) | \(C_{i4}\) | \(D_i\) | \(y50_i\) |
|-------|----|------------|------------|------------|------------|---------|----------|
| e-e   | 188 or 136| 5.62810   | -2.81718   | 1.53065   | -3.97164   | 5.666667| 0.03680  |
| e-o   | 147 or 84 | 4.28000   | -2.16122   | 1.55363   | -4.07848   | 3.933333| 0.03200  |
| o-e   | 131 or 76 | 4.85400   | -2.63110   | 1.56753   | -2.92537   | 4.800000| 0.03440  |
| o-o   | 114 or 48 | 3.70000   | -1.66474   | 1.46448   | -4.60082   | 2.933333| 0.03000  |

**TABLE VI. Standard deviations for ASAF model with the set A, B, and C.**

| Group | \(\sigma^A_{ASAF}\) | \(\sigma^B_{ASAF}\) | \(\sigma^C_{ASAF}\) |
|-------|---------------------|---------------------|---------------------|
| e-e   | 0.731               | 0.415               | 0.438               |
| e-o   | 1.069               | 0.713               | 1.426               |
| o-e   | 1.044               | 0.637               | 1.336               |
| o-o   | 1.041               | 0.876               | 1.069               |
IV. UNIV (UNIVERSAL FORMULA)

In cluster radioactivity and $\alpha$-decay the (measurable) decay constant $\lambda = \ln 2/T$, can be expressed as a product of three (model dependent) quantities

$$\lambda = \nu S P_s$$

where $\nu$ is the frequency of assaults on the barrier per second, $S$ is the preformation probability of the cluster at the nuclear surface, and $P_s$ is the quantum penetrability of the external potential barrier. The frequency $\nu$ remains practically constant, the preformation differs from one decay mode to another but it is not changed very much for a given radioactivity, while the general trend of penetrability follows closely that of the half-life. The external part of the barrier (for separated fragments), essentially of Coulomb nature, is much wider than the internal one (still overlapping fragments).

According to Ref. [53] the preformation probability can be calculated within a fission model as a penetrability of the internal part of the barrier, which corresponds to still overlapping fragments. One may assume as a first approximation, that preformation probability only depends on the mass number of the emitted cluster, $S = S(A_e)$. The next assumption is that $\nu(A_e, Z_e, A_d, Z_d) = \text{constant}$. In this way one arrives at a single straight line universal curve on a double logarithmic scale

$$\log T = -\log P_s - 22.169 + 0.598(A_e - 1)$$

where

$$-\log P_s = c_{AZ} \left[ \arccos \sqrt{r} - \sqrt{r(1-r)} \right]$$

with $c_{AZ} = 0.22873(\mu_A Z_d Z_e R_b) ^{1/2}$, $r = R_t/R_b$, $R_t = 1.2249(A_d^{1/3} + A_e^{1/3})$, $R_b = 1.43998 Z_d Z_e/Q$, and $\mu_A = A_d A_e / A$.

Sometimes this universal curve is misinterpreted as being a Geiger-Nuttal plot. Nowadays by Geiger-Nuttal diagram one understands a plot of $\log T$ versus $ZQ^{-1/2}$, or versus $Q^{-1/2}$. In this kind of systematics the experimental points are scattered. Nevertheless, for a given atomic number, $Z$, or for the members of a natural radioactive series, it is still possible to get a single straight line.

The strong shell effect at the magic neutron number $N = 126$, which was ignored when the approximation $S = S(A_e)$ was made to give a pronounced underestimation of the half-lives in the neighborhood of $N = 126.$
For $\alpha$-decay of even-even nuclei, $A_e = 4$, one has

$$\log T = -\log P_s + c_{ee}$$

(16)

where $c_{ee} = \log S_\alpha - \log \nu + \log(\ln 2) = -20.375$. We can find new values for $c_{ee}$ and we also can extend the relationship to even-odd, odd-even, and odd-odd nuclei, by fitting a given set of experimentally determined alpha decay data.

V. SEMFIS (SEMIEMPIRICAL RELATIONSHIP BASED ON FISSION THEORY OF $\alpha$-DECAY)

Mainly the $Z$ dependence was stressed by all formulae, in spite of strong influence of the neutron shell effects. The neighborhood of the magic numbers of nucleons is badly described by all these relationships.

TABLE VII. Standard deviations of calculated half-lives ($\log_{10} T_\alpha(s)$) with UNIV after optimization compared to experimental data in four groups of parent nuclei: even-even; even-odd; odd-even, and odd-odd. The set A, B, and C.

| Group | n  | $\sigma^A_{UNIV}$ | n  | $\sigma^B_{UNIV}$ | n  | $\sigma^C_{UNIV}$ |
|-------|----|-------------------|----|-------------------|----|-------------------|
| e-e   | 130| 0.560             | 188| 0.223             | 136| 0.287             |
| e-o   | 119| 1.050             | 147| 0.533             | 84 | 1.384             |
| o-e   | 109| 0.871             | 131| 0.442             | 76 | 1.269             |
| o-o   | 96 | 0.926             | 114| 0.609             | 48 | 1.494             |

The SemFIS formula based on the fission theory of $\alpha$-decay gives

$$\log T = 0.43429K_s\chi - 20.446$$

(17)

where

$$K_s = 2.52956Z_{da}[A_{da}/(AQ_\alpha)]^{1/2}[\arccos \sqrt{x} - \sqrt{x(1 - x)}];$$

$$x = 0.423Q_\alpha(1.5874 + A_{da}^{1/3})/Z_{da}$$

(18)

and the numerical coefficient $\chi$, close to unity, is a second-order polynomial.
TABLE VIII. B_k parameters of semFIS formula obtained by fitting the data evaluated by Rytz.

| Group | σ   | B_1            | B_2            | B_3            | B_4            | B_5            | B_6            |
|-------|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| e-e   | 0.223| 0.993119       | -0.004670      | 0.017010       | 0.045030       | 0.018102       | -0.025097      |
| o-e   | 0.533| 1.000560       | 0.010783       | 0.050671       | 0.013919       | 0.043657       | -0.079999      |
| e-o   | 0.442| 1.017560       | -0.113054      | 0.019057       | 0.147320       | 0.230300       | -0.101528      |
| o-o   | 0.609| 1.004470       | -0.160560      | 0.264857       | 0.212332       | 0.292664       | -0.401158      |

TABLE IX. B_k parameters of semFIS formula obtained by fitting the set C.

| Group | σ   | B_1            | B_2            | B_3            | B_4            | B_5            | B_6            |
|-------|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| e-e   | 0.287| 0.993119       | -0.004670      | 0.017010       | 0.045030       | 0.018102       | -0.025097      |
| o-e   | 1.384| 1.017560       | -0.113054      | 0.019057       | 0.147320       | 0.230300       | -0.101528      |
| e-o   | 1.269| 1.000560       | 0.010783       | 0.050671       | 0.013919       | 0.043657       | -0.079999      |
| o-o   | 1.494| 1.004470       | -0.160560      | 0.264857       | 0.212332       | 0.292664       | -0.401158      |

\[ \chi = B_1 + B_2 y + B_3 z + B_4 y^2 + B_5 yz + B_6 z^2 \]  \quad (19)

in the reduced variables y and z, expressing the distance from the closest magic-plus-one neutron and proton numbers \( N_i \) and \( Z_i \):

\[ y \equiv (N - N_i)/(N_{i+1} - N_i) ; \ N_i < N \leq N_{i+1} \]  \quad (20)

\[ z \equiv (Z - Z_i)/(Z_{i+1} - Z_i) ; \ Z_i < Z \leq Z_{i+1} \]  \quad (21)

with \( N_i = \ldots, 51, 83, 127, 185, 229, \ldots \) , \( Z_i = \ldots, 29, 51, 83, 115, \ldots \) , and \( Z_{da} = Z - 2 \) , \( A_{da} = A - 4 \) . The coefficients \( B_i \) obtained by using a high-quality selected set of alpha-decay data [54] are given in the Table VIII. Better agreement with experimental results are obtained in the region of superheavy nuclei by introducing other values of the magic numbers plus one unit for protons (suggesting that the next magic number of protons could be 126 instead of 114): \( Z_i = \ldots, 83, 127, 165, \ldots \). With the SemFIS formula, Rurarz [11] have made predictions for nuclei far from stability with \( 62 < Z < 76 \). In the variant of Ref. [9] the shell effect on the formation factor was approximated by an empirical relationship.

Practically for even-even nuclei, the increased errors in the neighborhood of \( N = 126 \), present in all other cases, are smoothed out by SemFIS formula using the second order...
polynomial approximation for $\chi$. They are still present for the strongest $\alpha$-decays of some even-odd and odd-odd parent nuclides. In fact for non-even number of nucleons the structure effects became very important, and they should be carefully taken into account for every nucleus, not only globally. An overall estimation of the accuracy, gives the standard rms deviation of log $T$ values:

$$\sigma = \left\{ \sum_{i=1}^{n} [\log(T_i/T_{exp})]^2/(n-1) \right\}^{1/2}$$ (22)

The parameters $\{B_k\}$ of the SemFIS formula could be automatically improved, for a given set of experimental data, by using the computer program described in the Ref. [17]. The partial $\alpha$-decay half-lives plotted in this figure are lying in the range of $10^{-7}$ to $10^{25}$ seconds. One can see the effect of the spherical and deformed neutron magic numbers of the daughter nuclei $N_d = 126, 152, 162$ particularly clear for even-even and even-odd nuclides. For the large set of alpha emitters the following values of the rms errors have been obtained: log $T$: 0.19 for SemFIS formula; 0.33 for the universal curve; 0.39 for ASAF model, and 0.43 for numerical superasymmetric (NuSAF) model [15].

There are many parameters of the SemFIS formula introduced in order to reproduce the experimental behaviour around the magic numbers of protons and neutrons, which could be a drawback in the region of light and intermediate alpha emitters. In the region of superheavies these characteristics may be conveniently used to get informations concerning the next magic numbers of protons and neutrons which are not well known until now. When accurate experimental values of $Q$ and $T$ are available in the region centered on $Z = 114 – 126, N = 172 – 184$, the SemFIS formula may be used to estimate whether the right value of the spherical magic number is $Z = 114, Z = 120, Z = 126$, and $N = 172$, or $N = 184$, due to the high sensitivity of $\chi$ to the values of $Z_i$ and $N_i$ (see eqs. [17, 21]).

In figure 2 we plotted the individual errors: differences of $\log_{10} T_{theor} - \log_{10} T_{exp}$ in four groups of alpha emitters. Vertical dashed lines are marking magic numbers of neutrons, either spherical (50, 82, 126) or deformed (152, 162, 172). One can see that the best result is always obtained for even-even alpha emitters.

Superheavy (SH) nuclei, with atomic numbers $Z = 104 – 118$, are decaying mainly by $\alpha$ decay and spontaneous fission. They have been produced in cold fusion or hot fusion ($^{48}$Ca projectile) reactions [39, 42]. In a systematic study of $\alpha$-decay energies and half-lives of superheavy nuclei it was shown [44] that our semFIS (semiempirical formula based on
VI. COMPARISON OF RESULTS OBTAINED WITH THE NEW FORMULA, SEMFIS, UNIV, AND ASAF.

We present in figure 3 the results obtained using the sets A, B, and C, respectively. A global indicator for a given model could be the weighted mean value

\[ \sigma_{newF}^A = \frac{130\sigma_{e-e} + 119\sigma_{e-o} + 109\sigma_{o-e} + 96\sigma_{o-o}}{454} = 0.8008 \]  

(23)
FIG. 3. Standard rms deviations of the four models: new=newF; ASAF; UNIV, and semFIS in four groups of e-e, e-o, o-e, o-o emitters, as well as the global parameter, uding the three sets of experimental data: A; B, and C.

\[
\sigma_{new}^{B} = \frac{188\sigma_{e-e} + 147\sigma_{e-o} + 131\sigma_{o-e} + 114\sigma_{o-o}}{580} = 0.6299
\]

(24)

\[
\sigma_{new}^{C} = \frac{136\sigma_{e-e} + 84\sigma_{e-o} + 76\sigma_{o-e} + 48\sigma_{o-o}}{344} = 0.6438
\]

(25)

Similarly for the other models

\[
\sigma_{ASAF}^{A} = \frac{130\sigma_{e-e} + 119\sigma_{e-o} + 109\sigma_{o-e} + 96\sigma_{o-o}}{457} = 0.9591
\]

(26)

\[
\sigma_{ASAF}^{B} = \frac{188\sigma_{e-e} + 147\sigma_{e-o} + 109\sigma_{o-e} + 114\sigma_{o-o}}{580} = 0.6313
\]

(27)
\[
\sigma_{ASAF}^C = \frac{136 \sigma_{e-e} + 84 \sigma_{e-o} + 76 \sigma_{o-e} + 48 \sigma_{o-o}}{344} = 0.9657
\]
\[
\sigma_{UNIV}^A = \frac{130 \sigma_{e-e} + 119 \sigma_{e-o} + 109 \sigma_{o-e} + 96 \sigma_{o-o}}{457} = 0.8937
\]
\[
\sigma_{UNIV}^B = \frac{188 \sigma_{e-e} + 147 \sigma_{e-o} + 131 \sigma_{o-e} + 114 \sigma_{o-o}}{580} = 0.5634
\]
\[
\sigma_{UNIV}^C = \frac{136 \sigma_{e-e} + 84 \sigma_{e-o} + 76 \sigma_{o-e} + 48 \sigma_{o-o}}{344} = 0.8958
\]
\[
\sigma_{semFIS}^A = \frac{130 \sigma_{e-e} + 119 \sigma_{e-o} + 109 \sigma_{o-e} + 96 \sigma_{o-o}}{457} = 0.9135
\]
\[
\sigma_{semFIS}^B = \frac{188 \sigma_{e-e} + 147 \sigma_{e-o} + 131 \sigma_{o-e} + 114 \sigma_{o-o}}{580} = 0.4269
\]
\[
\sigma_{semFIS}^C = \frac{136 \sigma_{e-e} + 84 \sigma_{e-o} + 76 \sigma_{o-e} + 48 \sigma_{o-o}}{344} = 0.9402
\]

From these results we may say that globally semFIS, with \( \sigma_{semFIS}^B = 0.4269 \), is the best one, followed in order by UNIV, \( \sigma_{UNIV}^B = 0.5634 \), ASAF, \( \sigma_{newF}^B = 0.6299 \), and \( \sigma_{ASAF}^B = 0.6313 \). Nevertheless, it is interesting to observe that for even-even alpha emitters newF gives the 3rd best result after semFIS (\( \sigma_{newF_{e-e}} = 0.298 \) compared to \( \sigma_{semFIS_{e-e}} = 0.223 \), and \( \sigma_{semFIS_{e-e}} = 0.287 \)), and for odd-even parents the 2nd is newF with \( \sigma_{newF_{o-e}} = 0.522 \) compared to \( \sigma_{semFIS_{o-e}} = 0.442 \). newF is also better than ASAF for o-o nuclides, when \( \sigma_{newF_{o-o}} = 0.840 \) and \( \sigma_{ASAF_{o-o}} = 0.876 \).

VII. CONCLUSIONS

The accuracy of the new formula was increased after optimization of the five parameters in the order: \( a; e; d; c, \) and \( b \). The SemFIS formula taking into account the magic numbers of nucleons, the analytical super-asymmetric fission model and the universal curves may be used to estimate the alpha emitter half-lives in the region of superheavy nuclei. The dependence on the proton and neutron magic numbers of the semiempirical formula may be
exploited to obtain informations about the values of the magic numbers which are not well known until now.

We introduced a weighted mean value of the rms standard deviation, allowing to compare the global properties of a given model. In this respect for the set B the order of the four models is the following: semFIS; UNIV; newF, and ASAF. Nevertheless for even-even alpha emitters UNIV gives the 2nd best result after semFIS, and for odd-even parents the 2nd is newF.

The quality of experimental data was also tested, as one can see by comparing the three sets (A, B, C). The set B with large number of emitters (580) gives the best global result. It is followed by the set A (454) three times and the set C (344).

Despite its simplicity in comparison with semFIS the new formula, presented in this article, behaves quite well, competing with the others well known relationships discussed in the Ref. [44].

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