The resemblances in mathematical structures between the optical constants of artificial electromagnetic media and some physical phenomena in field theory

Jian Qi Shen\textsuperscript{1,2} *

\textsuperscript{1} Centre for Optical and Electromagnetic Research, Joint Research Centre of Photonics of The Royal Institute of Technology (Sweden) and Zhejiang University, Zhejiang University, Hangzhou Yuhuan 310027, P.R. China
\textsuperscript{2} Zhejiang Institute of Modern Physics and Department of Physics, Zhejiang University, Hangzhou 310027, P.R. China

This paper demonstrates that there is much similarity in the mathematical formalisms between the optical constants of artificial electromagnetic media (such as chiral media, left-handed media, photonic crystals and EIT media) and some physical phenomena in field theory, including general relativity, quantum mechanics, energy band theory, etc.. The significance of such comparisons lies in that: (i) the unification in mathematical descriptions shows that many physical phenomena and effects, which seem to have no connections between them, actually share almost the same mathematical structures; (ii) it can provide clue to us on suggesting more new effects which is similar in mathematical descriptions to the familiar phenomena in other areas.

I. INTRODUCTION

This paper demonstrates that there is much similarity between the mathematical structures of optical constants of artificial electromagnetic media (such as chiral media, left-handed media, photonic crystals and EIT media) and some physical phenomena in field theory, including general relativity, quantum mechanics, energy band theory, etc.. The aim of this paper is to show that the unification in mathematical descriptions can clue us to the discovery of some new physical effects, by analogy with those in other fields.

We consider four strong resemblances in the mathematical structures between the artificial electromagnetic media and the physical phenomena in field theory:

(i) the generalization of the constitutive relation of regular media to that of chiral media resembles the extension of the flat Minkowski metric to the curved-spacetime metric of Riemann spacetime. In other words, the off-diagonal terms in the mathematical formalism of the electromagnetic energy density in chiral media arises from such a generalization;

(ii) since the stationary Maxwellian equation of the time harmonic wave strongly resembles the stationary Schrödinger equation of matter waves, if the optical refractive index of media is periodic in space, then such media will give rise to the so-called photonic band gap structure. This kind of materials is called the photonic crystals;

(iii) the negative solutions of any quadratic-form relations such as $E^2 = p^2 c^2 + m_0^2 c^4$ and $n^2 = \epsilon \mu$ will also have physical meanings and therefore should not be discarded. For example, for the former relation $E^2 = p^2 c^2 + m_0^2 c^4$, the negative energy solutions indicate the existence of the antiparticle of electron, and for the latter $n^2 = \epsilon \mu$, the negative solutions corresponds to a new kind of artificial composite materials, which possess negative refractive indices. This kind of materials is referred to as the left-handed media or negative refractive index media;

(iv) the similarity between EIT and superconductivity is interesting. We show that the effective Hamiltonian (describing the effective interaction between the two ground states of the three-level system) and the BCS Hamiltonian have features that are almost the same. In addition, we take into account the destructive interference in both Cabbibo’s theory (1963) and GIM mechanism (1970) of the Weak Interaction. We state that there exists the similar destructive interferences in CPT and EIT. The mathematical formalisms of these destructive interferences in Weak Interaction and atomic ensembles are almost alike.

The significance of the above four comparisons between the current typical subjects in materials science and those in field theory lies in that: (i) the unification in mathematical descriptions shows that many physical phenomena and effects, which seem to have no connections between them, actually share almost the same mathematical structures; (ii) it can provide clue to us on suggesting more new effects which is similar in mathematical descriptions to the familiar phenomena in other areas.

*E-mail address: jqshen@coer.zju.edu.cn, jqshencn@yahoo.com.cn
II. CHIRAL MEDIA

According to the Maxwellian electrodynamics, the energy density of electromagnetic fields in electromagnetic media is written

$$W = \frac{1}{2} \left( \mathbf{E} \cdot \mathbf{B} + \mathbf{H} \cdot \mathbf{E} \right).$$

(2.1)

In the chiral media, however, the electromagnetic properties can be characterized by another constitutive relations. We can select

$$\mathbf{D} = \varepsilon \varepsilon_0 \mathbf{E} + \mathbf{i} \zeta \mathbf{B}, \quad \mathbf{H} = \mathbf{i} \zeta \mathbf{E} + \frac{\mathbf{B}}{\mu \mu_0}.$$  

(2.2)

So, the energy density of electromagnetic fields in such media is

$$W = \frac{1}{2} \left( \mathbf{E} \cdot \mathbf{B} \right) \left( \varepsilon \varepsilon_0 \begin{pmatrix} \varepsilon \varepsilon_0 & 0 \\ 0 & \frac{1}{\mu \mu_0} \end{pmatrix} \right) \left( \mathbf{E} \cdot \mathbf{B} \right).$$

(2.3)

This means that the electromagnetic-parameter matrix of the chiral media can be considered as the off-diagonal generalization of the diagonal electromagnetic-parameter matrix of the regular linear materials, that is, we have

$$\begin{pmatrix} \varepsilon \varepsilon_0 & \mathbf{i} \zeta \\ \mathbf{i} \zeta & \frac{1}{\mu \mu_0} \end{pmatrix} \leftarrow \begin{pmatrix} \varepsilon \varepsilon_0 & 0 \\ 0 & \frac{1}{\mu \mu_0} \end{pmatrix}.$$  

(2.4)

It should be noted that the above fact is of much interest, since it is similar to the curved-spacetime generalization for the metric from the flat spacetime: specifically, in the two dimensional Minkowski spacetime, the line element squared can be given as

$$ds^2 = \left( dx^0 \ dx^1 \right) \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \left( dx^0 \ dx^1 \right).$$

(2.5)

Note that here the metric matrix is diagonal. However, in the Riemann spacetime, the metric matrix has the off-diagonal elements $g_{10}$ and $g_{01}$, and in consequence, the line element squared is given

$$ds^2 = \left( dx^0 \ dx^1 \right) \begin{pmatrix} g_{00} & g_{10} \\ g_{01} & g_{11} \end{pmatrix} \left( dx^0 \ dx^1 \right).$$

(2.6)

Apparently, there exists a nontrivial extension of metric from the flat Minkowski spacetime to the Riemann spacetime, i.e.,

$$\begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} g_{00} & g_{10} \\ g_{01} & g_{11} \end{pmatrix}.$$  

(2.7)

This generalization is just the development of special relativity (1905) to the general relativity (1916). We think there is some interesting similarity between (2.4) and (2.7), at least in the research fashion of looking for potentially new (and general) physical phenomena, effects and mechanisms.

Hence, here we gave an illustrative example which demonstrates how one can suggest more new effects which is similar in mathematical descriptions to the familiar phenomena in other areas.

III. PHOTONIC CRYSTALS

It is well known that the stationary Maxwellian equation for a time harmonic wave in a medium is exactly analogous to the stationary Schrödinger equation of electrons in a potential field, namely, the following equation (with the magnetic permeability $\mu = 1$)
\[ \nabla^2 E + \omega^2 \epsilon_0 \mu_0 E = 0 \]  
(3.1)
can be rewritten as
\[ \nabla^2 E + \omega^2 \epsilon_0 \mu_0 E + \omega^2 \epsilon_0 \mu_0 \chi E = 0 \]  
(3.2)
with the electric susceptibility \( \chi = \epsilon - 1 \). It is clearly seen that the form of the equation (3.2) completely resembles the following stationary Schrödinger equation
\[ \nabla^2 \psi + \frac{2m}{\hbar^2} E \psi - \frac{2m}{\hbar^2} \chi \psi = 0. \]  
(3.3)

In the solid state physics, a theorem (Bloch’s theorem) relating to the quantum mechanics of crystals, where large numbers of atoms are held closely together in a lattice, states that the wave function for an electron in a periodic potential \( V(\mathbf{r}) \) can be rewritten as the product of the space harmonic factor \( \exp(i \mathbf{k} \cdot \mathbf{r}) \) and a periodic function. Thus, Bloch’s theorem is interpreted to mean that the wave function for an electron in a periodic potential field is a plane wave modulated by a periodic function. It follows that the energy function of electrons with a lattice-periodicity Hamiltonian is necessarily multivalued, and separates into branches or bands. This, therefore, means that in a crystal, electrons are influenced by a number of adjacent nuclei and the sharply defined levels of the atoms become bands of allowed energy. Each band, which represents a large number of allowed quantum states, is separated from neighboring band by a forbidden band of energies, i.e., between the bands are forbidden bands. The energy band theory is the fundamental approach to crystal and semiconductor physics.

In view of the above discussion, we can inevitably draw our inspiration from the analogy between (3.2) and (3.3): if, for example, the electric permittivity (and hence the electric susceptibility \( \chi \)) of a certain media is of the lattice-periodicity structure, then the media will generate a photonic band gap phenomenon, which would be of interest to many researchers in materials science, electromagnetism and solid state physics. Indeed, during the last decades, a kind of material termed photonic crystals, which is patterned with a periodicity in dielectric constant and can therefore create a range of forbidden frequencies called a photonic band gap, focus considerable attention of a great number of investigators [1]. Such dielectric structure of crystals offers the possibility of molding the flow of light inside media. It has many impressive and striking applications, including the reflecting dielectric, resonant cavity, waveguide and so on.

The field of photonic crystals is a marriage of solid-state physics and electromagnetism. Crystal structures are citizens of solid-state physics, but in photonic crystals the electrons are replaced by electromagnetic waves. We can see that the concept of the photonic crystals could be inspired by that of the electronic crystals.

**IV. LEFT-HANDED MEDIA**

The antiparticle is a subatomic particle that has the same mass as another particle and equal but opposite values of some other property or properties. For example, the antiparticle of the electron is the positron, which has a positive charge equal in magnitude to the electron’s negative charge. Historically, the existence of antiparticles was predicted from the relativistic quantum mechanics by Dirac in 1928. It is well known that Einstein’s energy-momentum relation for a particle is of quadratic form, i.e., \( E^2 = p^2c^2 + m_0^2c^4 \). From the purely mathematical point of view, \( E \) has two roots, i.e., \( E_\pm = \pm \sqrt{p^2c^2 + m_0^2c^4} \). In the physics of the 19th century, such a negative root could be regarded as a one that has no physical meanings and could be discarded without any hesitation. In quantum mechanics, however, the completely different things may occur. By solving Dirac’s electron wave equation, one can find that there are solutions corresponding to the negative root \(-\sqrt{p^2c^2 + m_0^2c^4}\) other than the ones belonging to the positive root \(+\sqrt{p^2c^2 + m_0^2c^4}\). If someone discards the solutions corresponding to the negative energy root, then the completeness condition of the solutions of the wave equation no longer holds. Such a fact shows that the only retained solutions (corresponding to the positive energy) is not self-consistent in mathematics, and therefore no functions can be expanded as series in terms of the solutions corresponding to the positive energy root only. For this reason, we think the negative-energy solutions of Dirac’s equation should not be discarded. Further analysis shows that these negative-energy solutions belong to a particle that has a positive charge and an equal mass of the electron. This particle is just the positron, the antiparticle of the electron.

The above brief history of the prediction of antiparticle will unavoidably enlighten us about a lesson: for any physical phenomena, the mathematical relations of which possess quadratic forms, we should pay more attention to one of the two roots, which, at first sight, seems to have no explicit physical meanings. If we consider it further, it
may be possible for us to find its hidden physical meanings and even to discover another half world, which had never been familiar to us before. As far as the negative-energy solutions of Dirac’s equation is concerned, the anti-matter world was discovered by Dirac.

Now let us take into account of the quadratic relation between the optical refractive index \( n \) and the electric permittivity \( \epsilon \) \( \text{and} \) magnetic permeability \( \mu \), \( \text{i.e.,} \)

\[
n^2 = \epsilon \mu. \tag{4.1}
\]

The solutions has four categories, which are listed as follows

\[
\begin{align*}
n_1 &= +\sqrt{\epsilon \mu}, & n_2 &= -\sqrt{\epsilon \mu}, & n_3 &= +\sqrt{(-\epsilon)(-\mu)}, & n_4 &= -\sqrt{(-\epsilon)(-\mu)}.
\end{align*}
\tag{4.2}
\]

With the help of Maxwellian equations, one can discover that only the cases of \( n_1 = +\sqrt{\epsilon \mu} \) and \( n_4 = -\sqrt{(-\epsilon)(-\mu)} \) can allow the wave propagation in media. So, here we will not further consider the solutions \( n_2 \) and \( n_3 \). If both \( \epsilon \) and \( \mu \) are positive numbers, then it follows that there may exist a new kind of electromagnetic materials, which possess negative electric permittivity, negative magnetic permeability and hence negative optical index of refraction!

Now let us turn to the history of research of this new electromagnetic media: more recently, a kind of artificial composite metamaterials (the so-called left-handed media) having a frequency band where the effective permittivity and the effective permeability are simultaneously negative attracts considerable attention of many authors both experimentally and theoretically [2–12]. In 1967\(^1\), Veselago first considered this peculiar medium and showed from Maxwellian equations that such media possessing negative simultaneously negative \( \epsilon \) and \( \mu \) exhibit a negative index of refraction [13]. It follows from the Maxwell’s curl equations that the phase velocity of light wave propagating inside this medium is pointed opposite to the direction of energy flow, that is, the Poynting vector and wave vector of electromagnetic wave would be antiparallel, \( \text{i.e.,} \) the vector \( \mathbf{k} \), the electric field \( \mathbf{E} \) and the magnetic field \( \mathbf{H} \) form a left-handed system; thus Veselago referred to such materials as “left-handed” media, and correspondingly, the ordinary medium in which \( \mathbf{k} \), \( \mathbf{E} \) and \( \mathbf{H} \) form a right-handed system may be termed the “right-handed” one. Other authors call this class of materials “negative-index media (NIM)” [14], “double negative media (DNM)” [5] and Veselago’s media. It is readily verified that in such media having both \( \epsilon \) and \( \mu \) negative, there exist a number of peculiar electromagnetic and optical properties, for instance, many dramatically different propagation characteristics stem from the sign change of the optical refractive index and phase velocity, including reversal of both the Doppler shift and Cherenkov radiation, anomalous refraction, modified spontaneous emission rates, unconventional photon tunnelling effect, amplification of evanescent wave and even reversals of radiation pressure to radiation tension [3]. In experiments, this artificial negative electric permittivity media may be obtained by using the \textit{array of long metallic wires} (ALMWs) [15], which simulates the plasma behavior at microwave frequencies, and the artificial negative magnetic permeability media may be built up by using small resonant metallic particles, \( \text{e.g.,} \) the \textit{split ring resonators} (SRRs), with very high magnetic polarizability [16–18]. A combination of the two structures yields a left-handed medium. Recently, Shelby \textit{et al.} reported their first experimental realization of this artificial composite medium, the permittivity and permeability of which have negative real parts [4]. One of the potential applications of negative refractive index materials is to fabricate the so-called “superlenses” (perfect lenses): specifically, a slab of such materials may have the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner [19,20].

In the paper [21], we have demonstrated that the electromagnetic wave propagation in the negative refractive index media behaves like that of antiphotons, which implies that in certain artificial composite metamaterials \( \text{such as the left-handed media} \) the complex vector field theory will be a very convenient theoretical tool for taking into consideration the wave propagation behavior \( \text{e.g.,} \) scattering, transmission and refraction.

The investigation of the antiparticle solutions of Dirac equation means the discovery of another half matter world. Similarly, the consideration of the negative optical constants \( (\epsilon, \mu, n) \) means the discovery of another half material world.

\(^1\)Note that, in the literature, some authors mentioned the wrong year when Veselago suggested the \textit{left-handed media}. They claimed that Veselago proposed or introduced the concept of \textit{left-handed media} in 1968 or 1964. On the contrary, the true history is as follows: Veselago’s excellent paper was first published in Russian in July, 1967 [Usp. Fiz. Nauk 92, 517-526 (1967)]. This original paper was translated into English by W.H. Furry and published again in 1968 in the journal of Sov. Phys. Usp. [13]. Unfortunately, Furry stated erroneously in his English translation that the original version of Veselago’ work was first published in 1964.
V. EIT MEDIA

Controlling the phase coherence in ensembles of multilevel atoms has led to the observation of many striking phenomena in the propagation of near-resonant light. These phenomena include the coherent population trapping (CPT), lasing without inversion, electromagnetically induced transparency (EIT), and anomalously slow and anomalously fast pulse velocities [22]. Here we will briefly discuss the coherent population trapping (CPT) and electromagnetically induced transparency (EIT).

The CPT phenomenon can be observed in a simple three-level atomic system. The Hamiltonian of such a three-level quantum system in the interaction picture is given as

$$H_1 = g_{23}E_a (|2\rangle\langle3| + |3\rangle\langle2|) + g_{13}E_b (|1\rangle\langle3| + |3\rangle\langle1|),$$

(5.1)

where $|1\rangle$ and $|2\rangle$ denote the ground states of this three-level system, and $|3\rangle$ the excited state. Here the $E_a$ field is tuned to resonance with the transition between atomic levels $|2\rangle$ and $|3\rangle$, while the $E_b$ field excites the transition between levels $|1\rangle$ and $|3\rangle$. In order to realize the atomic coherent population trapping, the excited state $|3\rangle$ should be empty, and the atomic initial state should be

$$|\Psi(t = 0)\rangle = \frac{1}{\sqrt{(g_{23}E_a)^2 + (g_{13}E_b)^2}} (g_{23}E_a |1\rangle - g_{13}E_b |2\rangle).$$

(5.2)

It follows from (5.2) that in this state, the three-level quantum system will not evolve. The physical reason for this is as follows: since the probability amplitude of level $|1\rangle$ is $g_{23}E_a$, and the interaction strength between levels $|1\rangle$ and $|3\rangle$ is $g_{13}E_b$, the driving contribution of level $|1\rangle$ to $|3\rangle$, which is the product of probability amplitude and interaction strength, is given

$$A_{1\rightarrow3} = g_{23}g_{13}E_a E_b.$$  

(5.3)

In the meanwhile, the probability amplitude of level $|2\rangle$ is $-g_{13}E_b$, and the coupling strength between levels $|2\rangle$ and $|3\rangle$ is $g_{23}E_a$. So, the driving contribution of level $|2\rangle$ to $|3\rangle$ is

$$A_{2\rightarrow3} = -g_{23}g_{13}E_a E_b.$$  

(5.4)

Thus, the total driving contributions of the two ground states $|1\rangle$ and $|2\rangle$ to the excited state $|3\rangle$ vanishes, i.e.,

$$A_{\text{tot}} = A_{1\rightarrow3} + A_{2\rightarrow3} = 0.$$  

(5.5)

It is apparently seen that the mechanism of atomic CPT is just the destructive interference. Such a state (5.2), which can be said to be decoupled to the electromagnetic fields, is called the dark state (non-coupling state or trapped state). Once the three-level atom is in the dark state, the transitions from the ground states to the excited will not occur, and consequently this atomic ensemble will not absorb the near-resonant laser fields propagating inside it.

EIT is such a quantum optical phenomenon that if we propagate one laser beam through a medium and it will get absorbed; but if we propagate two laser beams instead through the same medium and neither will be absorbed. Thus the opaque medium is turned into a transparent one. The physical essence of EIT is just the CPT. According to the theoretical analysis of multilevel atomic phase coherence, the requirement of the occurrence of EIT is that the strength of coupling light is much stronger than that of probe light [23,24]. This requirement can be interpreted as follows: it follows that if the laser field $E_a$ is much stronger than $E_b$, then the dark state

$$|\Psi(t = 0)\rangle \rightarrow |1\rangle,$$  

(5.6)

which means that under this condition ($E_a \gg E_b$), one of the ground levels (i.e., $|1\rangle$) can just be thought of as the dark state. Under this condition, the EIT atomic vapor allows the probe light to propagate without dissipation through the medium. From the fully quantum point of view, the interaction Hamiltonian of the three-level atom interacting with the two quantized light fields is given

$$V = g'_{23} (a^\dagger |2\rangle\langle3| + a|3\rangle\langle2|) + g'_{13} (b^\dagger |1\rangle\langle3| + b|3\rangle\langle1|),$$  

(5.7)

where $a^\dagger$ ($a$) and $b^\dagger$ ($b$) denote the creation (annihilation) operators of $E_a$ and $E_b$ fields, respectively. The effective Hamiltonian of the interaction between the two ground states is
According to the GIM mechanism, in addition to the neutral current process of $\Delta S = 1$, there should exist the neutral current process of $\Delta S = 0$. How can we explain this conflict between theory and experiments? In 1970, Glashow et al. proposed a mechanism (which is now known as the GIM mechanism) in Weak Interaction. Cabbibo’s theory shows that in the weak interaction, the $d$ and $s$ quarks actually behave like the mixed ones $d'$ and $s'$, which are the linear combinations

$$d' = d \cos \theta_c + s \sin \theta_c, \quad s' = -d \sin \theta_c + s \cos \theta_c,$$

where $\theta_c$ denotes the Cabbibo angle. This means that the existing form of $d$ and $s$ quarks in the weak interaction is in fact the $d'$ and $s'$ rather than the $d$ and $s$ itself. So, the interaction eigenstates of the above two quarks in the weak interaction is just the $d'$ and $s'$ rather than the $d$ and $s$. Thus the interaction weak current between $d'$ and $u$ is given by

$$\bar{u}\gamma_{\mu}(1 + \gamma_5)d' = \bar{u}\gamma_{\mu}(1 + \gamma_5)(d \cos \theta_c + s \sin \theta_c).$$

The coupling constant of the $d\bar{u}$ coupling, in which the strangeness number does not change ($\Delta S = 0$), is $G_F$, and the coupling constant of the $s\bar{d}$ coupling, where the strangeness number changes ($\Delta S = 1$), is $G'_F$. Here $G_F = G_{\mu} \cos \theta_c$, $G'_F = G_{\mu} \sin \theta_c$, where $G_{\mu}$ is the coupling constant of the purely leptonic process (universal coupling constant in the weak interaction). Thus the Fermi transition amplitude of $d' \rightarrow u$ decay is proportional to

$$\cos \theta_c G_F + \sin \theta_c G'_F = G_{\mu},$$

while the transition amplitude of $s' \rightarrow u$ decay is vanishing, i.e.,

$$-\sin \theta_c G_F + \cos \theta_c G'_F = -G_{\mu}(\sin \theta_c \cos \theta_c - \cos \theta_c \sin \theta_c) = 0.$$

In other words, the $s'$ quark can be said to be steady for the case of decay into $u$. This, therefore, means that in the weak interaction, it is more reasonable and convenient for $(d', s')$ to represent the interaction eigenstate than for $(d, s)$. In 1970’s, in the experiments physicists found that all the weak-interaction neutral current processes satisfy the selection rule of $\Delta S = 0$, while the neutral current process of $\Delta S = 1$ could rarely be observed. In accordance with the uds three-quark theory, however, there should exist the neutral current process of $\Delta S = 1$. For example, the weak-interaction matrix element of neutral currents $u\bar{u} \rightarrow z^0$ and $d'\bar{d'} \rightarrow z^0$ takes the form

$$u\bar{u} + (d\bar{d} \cos^2 \theta_c + s\bar{s} \sin^2 \theta_c) + (s\bar{d} + \bar{s}d) \sin \theta_c \cos \theta_c,$$

where $u\bar{u} + (d\bar{d} \cos^2 \theta_c + s\bar{s} \sin^2 \theta_c)$ is the contribution of the $\Delta S = 0$ process, and $(s\bar{d} + \bar{s}d) \sin \theta_c \cos \theta_c$ results from the $\Delta S = 1$ process. Note that in theory there truly exists the neutral current of $\Delta S = 1$. How can we explain this conflict between theory and experiments? In 1970, Glashow et al. proposed a mechanism (which is now known as GIM mechanism) and supposed that there exists a new quark ($c$) that had not been discovered in Nature. In the GIM mechanism, $(u, d')$ and $(c, s')$ form the weak-interaction doublets, respectively, i.e.,

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \cos \theta_c + s \sin \theta_c \end{pmatrix}, \quad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d \sin \theta_c + s \cos \theta_c \end{pmatrix}.$$
\[ u\bar{u} + c\bar{c} + [(d\bar{d} + s\bar{s}) \cos^2 \theta_c + (s\bar{s} + d\bar{d}) \sin^2 \theta_c] + (s\bar{d} + \bar{s}\bar{d} - s\bar{d} - \bar{s}\bar{d}) \sin \theta_c \cos \theta_c, \tag{5.16} \]

where \( u\bar{u} + c\bar{c} + [(d\bar{d} + s\bar{s}) \cos^2 \theta_c + (s\bar{s} + d\bar{d}) \sin^2 \theta_c] \) is the contribution of the neutral current of \( \Delta S = 0 \), and the contribution of \( \Delta S = 1 \) is

\[ (s\bar{d} + \bar{s}\bar{d} - s\bar{d} - \bar{s}\bar{d}) \sin \theta_c \cos \theta_c = 0, \tag{5.17} \]

which demonstrates that the neutral current of \( \Delta S = 1 \) is automatically eliminated and will therefore not exist in the weak interaction. In a word, the introduction of \( c \) quark leads to the destructive interference of the neutral current of \( \Delta S = 1 \) among the weak-interaction doublets \( (u, d') \) and \( (c, s') \). Thus, the GIM mechanism resolves successfully the above conflict between the uds three-quark theory and the experiments.

We think the destructive interference is one of the common features in EIT (CPT) and Cabbibo’s theory and GIM mechanism.

**Acknowledgements** This work was supported by the National Natural Science Foundation of China under Project No. 90101024 and 60378037.

[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987); E. Yablonovitch and T.J. Gmitter, Phys. Rev. Lett. **63**, 1950 (1989); P. Villeneuve and M. Piche, Phys. Rev. B **46**, 4969 (1992).
[2] D.R. Smith, W.J. Padilla, D.C. Vier et al., Phys. Rev. Lett. **84**, 4184 (2000).
[3] V.V. Klimov, Opt. Comm. **211**, 183 (2002).
[4] R.A. Shelby, D.R. Smith, and S. Schultz, Science **292**, 77 (2001).
[5] R.W. Ziolkowski, Phys. Rev. E **64**, 056625 (2001).
[6] J.A. Kong, B.L. Wu, and Y. Zhang, Appl. Phys. Lett. **80**, 2084 (2002).
[7] N. Garcia and M. Nieto-Vesperinas, Opt. Lett. **27**, 885 (2002).
[8] J.Q. Shen, Phys. Scr. **68**, 87 (2003).
[9] L.F. Shen and S.L. He, Phys. Lett. A **309**, 298 (2003).
[10] C.R. Simovski and S.L. He, Phys. Lett. A **311**, 254 (2003).
[11] J Lu and S.L. He, Microwave Opt. Technol. Lett. **37**, 292 (2003).
[12] C.R. Simovski, P.A. Belov, and S.L. He, IEEE Trans. Antennas Propagat. (special issue on metamaterials) 1 (2003).
[13] V.G. Veselago, Sov. Phys. Usp. **10**, 3966 (1968).
[14] J. Gerardin and A. Lakhtakia, Phys. Lett. A **301**, 377 (2002).
[15] J.B. Pendry, A.J. Holden, D.J. Robbins, and W.J. Stewart, J. Phys. Condens. Matter **10**, 4785 (1998).
[16] J.B. Pendry, A.J. Holden, W.J. Stewart, and I. Youngs, Phys. Rev. Lett. **76**, 4773 (1996).
[17] J.B. Pendry, A.J. Holden, D.J. Robbins, and W. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[18] S.I. Maslovski, S.A. Tretyakov, and P.A. Belov, Inc. Microwave Opt. Tech. Lett. **35**, 47 (2001).
[19] J.B. Pendry, Phys. Rev. Lett. **85**, 4785 (2000).
[20] G.W. t’ Hooft, Phys. Rev. Lett. **87**, 249701 (2001).
[21] J.Q. Shen, arXiv: cond-mat/0308349 (2003).
[22] T. Purdy and M. Ligare, arXiv: quant-ph/0204173 (2002).
[23] S.E. Harris, Phys. Today **50**(7), 36 (1997).
[24] M. Lukin, S. Yellin, A. Zibrov, and M. Scully, Laser Physics **9**, 759 (1999).
[25] J.Q. Shen, H.Y. Zhu, and H.L. Zhu, Laser & Infrared (in Chinese) **32**, 315 (2002).
[26] N. Cabbibo, Phys. Lett. **10**, 531 (1963).