Free space propagation of concentric vortices through underwater turbid environments

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Abstract
Concentric optical vortex beams of 3-petal, 5-petal, and 6-petal spatial profiles are generated at 450 nm using a single diffractive optical element. The spatial and temporal propagation characteristics of these beams are then studied in a scattering underwater environment. Experimental results demonstrate a less than 5% reduction in the spatial pattern for turbidities in excess of 10 attenuation lengths. The temporal properties of concentric vortex beams are studied by temporally encoding an on-off keyed, non-return-to-zero (OOK-NRZ) data stream at 1.5 GHz.

Keywords: orbital angular momentum, underwater communications, singular optics, optical vortices

(Some figures may appear in colour only in the online journal)

1. Introduction: underwater environments and orbital angular momentum

The propagation of optical beams through seawater has been well studied for Gaussian beams. The attenuation of a non-scattered Gaussian beam can be characterized by the attenuation coefficient, \( c \), which is the sum of the absorption coefficient, \( a \), and the scattering coefficient, \( b \). The non-scattered portion of the beam attenuates exponentially according to Beer’s Law, \( P = P_0 \exp(-cz) \), where \( P \) is the received power, \( P_0 \) is the initial power, and \( z \) is the physical range. The product, \( cz \), is the attenuation length. The absorption spectrum of seawater limits the operation of underwater links to blue/green wavelengths [1]. Scattering can broaden the beam in both the spatial and temporal domains [2–6]. Understanding the characteristics of underwater optical propagation allows for a host of underwater sensing applications such as LIDAR, laser imaging, or laser communications.

Orbital angular momentum (OAM) is a property of light which describes the helicity of the optical phase front. Optical vortex beams possess a helical phase front given by \( \exp(\text{j} \ell \theta) \) where \( \theta \) is the azimuthal angle and \( l \) is the azimuthal mode index, imparting a phase rotation of \( \ell \times 2\pi \) onto the beam where \( l \) can take on any integer value [7]. The rotation of the phase front gives vortex beams unique properties including handedness [8] and an angular momentum proportional to \( l \) [9]. These characteristics have been exploited for optical tweezers, imaging, and quantum information processing [8, 10]. Since OAM modes are orthogonal, the bandwidth of optical communication links can be increased using spatial
division multiplexing. This has been demonstrated in atmospheric free space optical links [11–13].

It is therefore natural to ask how the unique properties of optical vortex beams behave in a complicated underwater environment. Unfortunately, there has been little work in this area; however researchers have recently demonstrated the feasibility of using OAM to spatially multiplex simultaneous data streams underwater [14–17]. This is an attractive capability for underwater communication links, as any one underwater optical channel can be inherently band-limited due to particulate scattering [3–5].

These previous demonstrations have used a single vortex and symmetric spatial profile. In this paper we explore more complex spatial profiles generated by multiple concentric optical vortices, which may represent another spatial degree to be exploited underwater. The interference pattern resulting from concentric vortices results in a periodic constructive and destructive interference around the center of the beam, resulting in ‘petal’ pattern. As will be shown, these patterns contain orientation and distance information unique to the spatial profile of each concentric vortex beam combination. Additionally, the orientation of these patterns can be manipulated due to Gouy Phase, offering interesting new modalities for precision measurements [18]. Typically, the coherent superposition of concentric modes has been achieved through complex interferometric setups or spatial light modulators (SLM) [19, 20]. The use of a free-space interferometer is heavily dependent on coherence length and therefore, precision of alignment is absolutely critical and SLMs have several disadvantages compared to diffractive phase plates which are explained below. In this work, we generate concentric vortices on a single diffractive optic such that this interference can be performed on the same device, eliminating the need of the interferometric setup. Diffractive optical elements are designed and fabricated to realize 3-petal, 5-petal, and 6-petal concentric optical vortex beams at 450 nm using a wafer-based fabrication method, allowing for the generation of several optical elements simultaneously. The spatial and temporal characteristics of these beams are then studied through water of increasing turbidity.

2. Diffractive phase plates

2.1. Design

Diffractive phase plates (DPPs) have several advantages over spatial light modulators. First, the optics have 100% fill factors, which have unparalleled efficiency. The resolution of SLMs is limited to the pixel size and coding scheme, whereas the resolution of diffractive optics is limited only by the fabrication technique used. The efficiency of a DPP is dependent on the number of discrete levels. The first-order diffraction efficiency is given by the expression

\[ \eta = \sin \left( \frac{1}{n} \right)^2 \]  

(1)

where \( N \) is the number of phase levels in the device. Additionally, because these devices are comprised of Fused Silica glass, these optical elements can be utilized in high-power optical systems, whereas SLMs could sustain damage. Additionally, using a DPP in fused silica does not have any thermal or electrical requirements, since it is a purely passive optical component.

The phase delay, \( \Delta \phi \), experienced by a phase front traveling a distance \( d \) through a medium with refractive index \( n \) can be calculated using

\[ \Delta \phi = \frac{2\pi}{\lambda} (n_1 - n_0), \]  

(2)

where \( \lambda \) describes the wavelength of light and \( n_0 \) is the refractive index of the external medium. The \( \text{mod}(2\pi) \) is taken of the phase profile, \( P(\theta) \), in order to compress the optical element. Therefore, the maximum phase delay experienced by the phase front will be \( 2\pi \) radians. Utilizing this fact, equation (2) can be manipulated to find the propagation depth corresponding to a phase delay of \( 2\pi \) radians:

\[ d_{2\pi} = \frac{\lambda}{n_1 - n_0}. \]  

(3)

The concentric DPPs consist of the combination of two vortex phase plates, one inside the other. The inner portion has counter-clockwise phase wrap of \( (n \times 2\pi) \) and the outer region has a clockwise phase wrap of \( (-m \times 2\pi) \), where \( m \) and \( n \) represent the respective azimuthal mode indices. The ideal representation of the phase profile of a concentric DPP is given by

\[ P(\theta) = \begin{cases} 
\exp(jn\theta), & 0 \leq r < r_1 \\
\exp(-jm\theta), & r_1 \leq r < r_2 \\
1, & r_2 \leq r 
\end{cases} \]  

(4a)

where \( \theta \) is the azimuthal angle, \( \theta = \tan^{-1}(y/x) \), and \( r \) is the distance from the center of the optical phase element, \( r = \sqrt{x^2 + y^2} \). This phase element represented in equation (4a) is illuminated with a Gaussian beam, resulting in the following amplitude and phase in rectilinear coordinates:

\[ u(r, \theta) = \begin{cases} 
\exp(-2r^2/w_0^2) \exp(jn\theta), & 0 \leq r < r_1 \\
\exp(-2r^2/w_0^2) \exp(-jm\theta), & r_1 \leq r < r_2 \\
\exp(-2r^2/w_0^2), & r_2 \leq r 
\end{cases} \]  

(4b)

proportionate percent of power is passed through the inner and outer portions of the DPP in order to form the interference patterns, illustrated in figure 1. The limits of expression (4b) are derived from the power in a normalized Gaussian Beam, defined as

\[ P(r) = 1 - \exp\left( \frac{-2r^2}{w_0^2} \right). \]  

(5)

In equation (5), for a circle with radius \( r = 0.59w_0 \), approximately 50% of the power in the incident Gaussian
Beam is contained inside the ring. When \( r = 2w_0 \), the circle contains approximately 100% of the power. These values give the ideal limits for \( r_1 \) and \( r_2 \) in expression (5) respectfully.

The resulting intensity profiles shown in figure 1 experience a slight left-handed rotation due to the natural divergence of the beam [19]. By exploiting Gouy phase, this handed nature can be used to determine distance location by observing the rotation angle. When these profiles are passed through a lens, the petals no longer display rotation. After Propagating past the focal plane, the phase shifts and the resulting output patterns appear to rotate in the opposite direction, shown in figure 2.

Additionally, it can be observed that the lobes themselves do rotate with propagation before and after the focus. For the case of superimposed optical vortices where the relative phase, \( \delta \), of the two beams remains constant, the resulting angular change in the profile position between two points along the propagation direction is defined as

\[
\Delta \phi_z = \left| \frac{n_1 - n_2}{n_0 - n_2} \right| \delta \zeta,
\]

where \( \zeta = \tan^{-1}(z/z_R) \) and \( z_R \) describing the Raleigh range [19]. By combining this property with image processing technologies, these intensity profiles could be used to probe an environment and obtain ranging information. This is a unique artifact associated with these beams; however, a fundamental question exists as to how these beams hold up in turbid environments and can they withstand temporal modulations in the range necessary for optical data communication links.

2.2. Fabrication

Three different diffractive phase plate designs are used to convert a Gaussian beam into the desired interference patterns. The devices used in this work have an inner vortex profile with a diameter of 1.25 mm and an outer vortex profile with a diameter of 5 mm. The ideal height profile of a concentric DPP with refractive index \( n \) is derived from equation (4) and is expressed as

\[
h(\theta) = \begin{cases} 
\frac{n_1 \lambda \theta}{|n_1 - n_2|2\pi}, & 0 < r \leq 0.625 \text{ mm} \\
\frac{m \lambda \theta}{|n_1 - n_2|2\pi}, & 0.625 \text{ mm} < r < 2.5 \text{ mm}
\end{cases}
\]

where \( r \) gives the radius in millimeters, \( n_1 \) is the refractive index of the substrate, and \( n_0 \) gives the refractive index of the surrounding medium. For this work, \( n_0 = 1 \) for air. The phase profile of the diffractive optics was designed by taking \( h(\theta) \text{ mod}(2\pi) \).

This profile is then divided into 16 discrete levels, indicating four iterative etches in the fabrication process. This results in a calculated diffraction efficiency of 98.7% using equation (1). Similarly, the diffraction efficiency is 95.0% for an 8-level DPP and is 99.7% for a 32-level device. The 16-level devices, illustrated in figure 3, are chosen over the other options to balance the ease of fabrication and diffraction efficiency.

A photolithographic method is used pattern the wafer, which is then etched in an inductively coupled plasma tool to
transfer this profile into a fused silica. For each depth in the etching process the wafer is cleaned, coated in photoresist, exposed using an i-line projection lithography tool, developed, and plasma etched. An etch process consisting of three steps resulting in an 8-level device is illustrated in figure 4.

For operation at 450 nm, the refractive index of fused silica is 1.466. Using equation (3), a phase delay of $2\pi$ radians occurs after propagation through approximately 960 nm of fused silica glass. This depth is divided into four etches: 60 nm, 120 nm, 240 nm, and 480 nm. The total etch depth is therefore approximately 900 nm. This is to ensure a proper phase wrapped profile. Precise etch depths and alignment of each layer is crucial, especially when designed for operation at shorter wavelengths. In this case, any small deviation may be on the order of magnitude as the wavelength and will result in a loss of efficiency. The etch depth profile of the fabricated devices is given in figure 5.

2.3. Simulation

A numerical implementation of the Rayleigh-Sommerfeld diffraction integral [21] is used to simulate the development of these beams through a non-scattering environment, and is given by

$$E_P(x, y) = \int \int u(\xi, \eta) \times \frac{1}{j\lambda z} \left[1 + \left(\frac{x - \xi}{z}\right)^2 + \left(\frac{y - \eta}{z}\right)^2\right] d\xi d\eta,$$

(8)

where $E_P$ is the electric field at the observation point $P(x, y)$, $u(\xi, \eta)$ is the electric field within the aperture defined by equation (3) and $z$ is the distance from the input plane where the phase optic is illuminated to the plane where the point $P(x, y)$ is located.

For this propagation, $r = 0.625$ mm and $\omega_0 = 1.15$ mm. These are the design specifications used in the experiment and are not optimized as in the ideal case. Instead, approximately 45% of the Gaussian power passes through the inner vortex while 55% passes through the outer. This was due to optical system limitations. When the ratios of $r$ to $\omega_0$ are altered, the resulting interference patterns change...
slightly as shown in figures 6(a)–(c). By significantly changing this ratio, the intensity profiles are minimally impacted. Additionally, the resulting power passing through the inner vortex as a function of incident Gaussian beam waist is plotted in figure 6(d). The concentric vortices examined in this paper and the interference of OAM modes are quite similar in nature and have yet to be investigated for their propagation properties through turbid media or underwater propagation.

Additionally, these beams must travel some distance before the interference patterns form properly as can be seen in figure 1. Intensity profile images were captured in air at different distances from the phase plate and compared to simulation results for the same corresponding distances, illustrated in figure 7, in order to demonstrate the development of the spatial profile.

3. Propagation through turbid media

The spatial and temporal propagation characteristics of concentric vortex beams underwater are studied next. To
investigate the spatial properties of the experimental setup in figure 8 is used. The light source was a mounted ThorLabs LDM9LP laser diode operating at 450 nm, corresponding to the approximate absorption minimum in clear ocean water. The output power was controlled electrically with a DC Bias current. The output from the fiber pigtail is connected to a collimator and then expanded using a Keplerian telescope system to achieve a collimated Gaussian with a 2.3 mm diameter. The Gaussian beam is then passed through the diffractive phase plate and propagates 1.11 meters in air, a distance selected such that the interference between inner and outer generated vortex beams have propagated far enough to give a distinct petal pattern. The beam then passes through a PVC pipe 3 m long and 5 inches in diameter.

Liquid antacid was used to increase turbidity in the water channel. The attenuation coefficient of each solution was determined by measuring the laser power before and after the water channel, and attenuation according to Beer’s Law was verified. After transmission through the channel, the attenuated beam is imaged onto a WATEC 902-H2 visible CCD camera. The collected images are processed in MATLAB® in order to calculate the correlation coefficient between the clear-water case and turbid cases.

The calculation of the coefficient was made using the 2-dimensional form of the Pearson Product moment, given by

$$r = \frac{\sum_{m} \sum_{n} (A_{mn} - \bar{A})(B_{mn} - \bar{B})}{\sqrt{\left(\sum_{m} \sum_{n} (A_{mn} - \bar{A})^2\right) \left(\sum_{m} \sum_{n} (B_{mn} - \bar{B})^2\right)}}. \tag{9}$$

In this equation, A and B are two M by N images, where $A_{mn}$ corresponds to the pixel value in image A at location [m, n], $B_{mn}$ gives the pixel value in image B at location [m, n], and the means of image A and image B are expressed as

$$\bar{A} = \frac{1}{MN} \sum_{m} \sum_{n} A_{mn}, \tag{10a}$$

$$\bar{B} = \frac{1}{MN} \sum_{m} \sum_{n} B_{mn}. \tag{10b}$$

This is undertaken for the whole image where $M = 512$ pixels and $N = 480$ pixels and the pixel pitch is 9 µm.

Individual shifts in the image were corrected for prior to computing the correlation coefficients. It is important to adjust for any slight misalignments of the CCD array from successive measurements. Rotation of the images was not an issue, since these beams do not show any rotation with increasing turbidity.

The intensity profiles of the propagated concentric vortices after propagation through clear and highly turbid water are shown in figure 9. These images are juxtaposed with simulated beam development for in-air propagation generated using equation (4b). The clear water images approximately match that of these simulations. The irregular intensity distribution between the petals can be attributed to a slightly oblong incident Gaussian beam. After propagation through the turbid environment the low frequency information in the intensity profile sees very little distortion. The scattering is obvious from the high frequency information as a result of the scattering medium. In order to quantify this scattering contribution, a correlation coefficient is computed for the beams without scattering agents added to the water and at various points with increasing scattering as a result of increasing the liquid antacid concentration.

The correlation coefficients calculated for each case are compared in figure 10. As the attenuation coefficient of the water channel is increased, the correlation coefficient for each beam decreases. As a result, the scattered light reduces the contrast of the image. These correlation coefficients are comparable to that of a Gaussian beam despite having a more complex intensity profile. Each calculated coefficient is well over 0.9, where 1 corresponds to perfect correlation and 0 to no correlation. This indicates excellent preservation of the spatial profiles despite propagation through extremely turbid conditions.

To study the temporal characteristics of concentric vortex beams in turbid water, the original setup was modified to the design indicated in figure 11 in order to implement an underwater communications link. An AC signal consisting of a 32-bit, pseudo-random, M-series, on-off keying, non-return-to-zero (OOK-NRZ) bit sequence operating at 1.5 GHz was generated using a Tektronix AWG615, Arbitrary Waveform Generator. This signal was then amplified using a 10 dB Picosecond Pulse Labs 5828-MP amplifier, and combined with a DC bias current through a bias tee located in the ThorLabs LDM9LP pigtailed laser diode mount. The AWG output signal was optimized for the diode such that after the amplifier the AC current was 50 mA peak-to-peak with a DC bias of 60 mA.

After propagation through the link, the optical signal was focused onto a Menlo Systems APD210 Si Avalanche Photo-Detector (APD) using a 300 mm focal length lens. The high gain and sensitivity that APDs provide make them a popular choice in underwater optical sensors where the exponential attenuation experienced due to Beer’s Law can rapidly attenuate the received signal. Unfortunately, the price paid for high gain and sensitivity is a low maximum output current. Therefore, it is challenging to use standard bit error rate testers (BERTs) which require larger signal levels for accurate measurement. An alternate method for inferring the BER is used by capturing the received signal on a Tektronix TDS8200 digital sampling oscilloscope to create an eye diagram. A collection of 1000 waveforms was collected as an eye diagram, shown in figure 12, for each concentric vortex. The mean and standard deviation of the upper and lower rails were measured on these eye diagram using a histogram. The window used was measured from 40% to 60% of the bit period. These values were used in estimating the bit error rate (BER) of the signal using the formula for equal probability
signals for ‘1’ and ‘0’

\[
BER = \frac{1}{4} \text{erfc}\left(\frac{\mu_1 - \mu_0}{\sigma_1 \sqrt{2}}\right) + \frac{1}{4} \text{erfc}\left(\frac{\mu_0 - \mu_1}{\sigma_1 \sqrt{2}}\right) (11)
\]

where \(\mu_i\) and \(\sigma_i\) are the mean and standard deviation, respectively of the signals \(i = 0\) or \(1\), and

\[
\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) \, dt (12)
\]

and \(V_\text{th}\) is the threshold voltage, given by

\[
V_\text{th} = \frac{\mu_0 \sigma_1 + \mu_1 \sigma_0}{\sigma_1 + \sigma_0} \quad (13)
\]

The BER was measured for clean and turbid water with an attenuation coefficient of approximately \(cz \approx 0.27\) and \(cz \approx 1.48\), respectively. The field of view was limited by the 300 mm focal length lens and the 0.5 mm detector diameter. The full-angle field of view was calculated to be \(0.02^\circ\) using \(\text{AFOV} = 2 \times \tan^{-1}(h/2f)\) where \(h\) is half of the detector diameter and \(f\) is the focal length of the lens.

The BER of each eye diagram was estimated using equation (9) and is reported in table 1. The BERs shown are above the lower limit of \(10^{-9}\) for an OOK link, and below the limit of \(10^{-4}\) for a system utilizing forward error correction techniques. It is likely that the higher BERs for the larger petal numbers can be attributed to the increased divergence with petal number, resulting in a higher percentage of the signal collected by the APD. Due to the limited output current of the APD, experiments were limited to only 1.5 attenuation lengths. Therefore, it is posited from [5] that the reduction in BER is mostly due to a low signal level (i.e.—attenuation) rather than temporal dispersion arising from multiply scattered components of the 1.5 GHz modulated light. Further study is ongoing.
4. Conclusion

The creation of interfering optical vortices using apodized phase plates is a simple method for generating the coherent superposition of optical vortex beams. Segmenting the optical element eliminates the need for interferometric methods of generating interfering OAM beams. This is also important if the optical source(s) do not have a long coherence length, as is typical in GaN laser diodes in the 450–470 nm spectral range. Diffractive elements are also very efficient in the generation of such beams and can withstand high power densities if necessary. In this paper, 3-petal, 5-petal, and 6-petal concentric vortices were designed and created using diffractive phase plates; however, more complex beams are possible if required.

These beams were propagated through a water pipe to study their spatial and temporal propagation characteristics in turbid water. The spatial profiles were maintained in a multiple scattering environment, as evidenced by only a 3% drop in the spatial correlation coefficient over 13 attenuation lengths. The temporal effect of scattering on concentric beams was investigated via implementation of an underwater optical communications link at 1.5 GHz. The bit error rates realized were on the order of $10^{-4}$ for on-off keying, non-return-to-zero (OOK-NRZ) pulses from a directly modulated GaN laser diode at 450 nm over 1.5 attenuation lengths. This BER can also be further enhanced using forward error correction techniques for underwater communications [22].

Future study at longer attenuation lengths and/or wider receiver FOV will help further quantify the impact of multiple scattering, both spatially and temporally. The influence of optical turbulence is also of interest. Through characterization of the propagation of concentric vortex beams, we may better exploit the unique spatial profiles for precise underwater measurement or high bandwidth optical wireless communications underwater.

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Figure 12. Eye diagrams from the 3-Petal (a), 5-Petal (b), and 6-Petal (c) beams.

| Beam       | $cz = 0.27$ | $cz = 1.48$ |
|------------|-------------|-------------|
| 3-Petal    | 5.47E-04    | 5.34E-04    |
| 5-Petal    | 5.39E-04    | 5.71E-04    |
| 6-Petal    | 6.32E-04    | 6.74E-04    |
| Gaussian   | 5.96E-04    | 5.96E-04    |
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