Taming the TuRMoiL: The Temperature Dependence of Turbulence in Cloud–Wind Interactions

Matthew W. Abruzzo1,2, Drummond B. Fielding3, and Greg L. Bryan1,3

1 Department of Astronomy, Columbia University, 550 West 120th Street, New York, NY 10027, USA; matthew.abruzzo@gmail.com
2 Physics and Astronomy Department, University of Pittsburgh, 3941 O’Hara Street, Pittsburgh, PA 15260, USA
3 Center for Computational Astrophysics, Flatiron Institute, 162 5th Avenue, New York, NY 10010, USA

Received 2022 October 26; revised 2024 January 11; accepted 2024 January 12; published 2024 May 6

Abstract

Turbulent radiative mixing layers play an important role in many astrophysical contexts where cool \( (\lesssim 10^3 \text{ K}) \) clouds interact with hot flows (e.g., galactic winds, high-velocity clouds, infalling satellites in halos and clusters). The fate of these clouds (as well as many of their observable properties) is dictated by the competition between turbulence and radiative cooling; however, turbulence in these multiphase flows remains poorly understood. We have investigated the emergent turbulence arising in the interaction between clouds and supersonic winds in hydrodynamic ENZO-E simulations. In order to obtain robust results, we employed multiple metrics to characterize the turbulent velocity, \( v_{\text{turb}} \).

We find four primary results when cooling is sufficient for cloud survival. First, \( v_{\text{turb}} \) manifests clear temperature dependence. Initially, \( v_{\text{turb}} \) roughly matches the scaling of sound speed on temperature. In gas hotter than the temperature where cooling peaks, this dependence weakens with time until \( v_{\text{turb}} \) is constant. Second, the relative velocity between the cloud and wind initially drives rapid growth of \( v_{\text{turb}} \). As it drops (from entrainment), \( v_{\text{turb}} \) starts to decay before it stabilizes at roughly half its maximum. At late times, cooling flows appear to support turbulence. Third, the magnitude of \( v_{\text{turb}} \) scales with the ratio between the hot phase sound-crossing time and the minimum cooling time. Finally, we find tentative evidence for a length scale associated with resolving turbulence. Underresolved mixing may cause violent shattering and affect the cloud’s large-scale morphological properties.

Unified Astronomy Thesaurus concepts: Galaxy evolution (594); Hydrodynamical simulations (767); Interstellar clouds (834); Circumgalactic medium (1879); Galactic winds (572)

1. Introduction

While scales and relevant physics may vary, interactions between regions of cooler gas and coherent flows of hotter gas are prominent in many contexts. These interactions are prevalent in the circumgalactic medium (CGM), such as high-velocity clouds (e.g., Wakker & van Woerden 1997; Putman et al. 2012), ram pressure stripping of infalling satellites (e.g., Emerick et al. 2016; Simons et al. 2020) and the resulting streams (e.g., Bland-Hawthorn et al. 2007; Bustard & Gronke 2022), or cooling flows from cosmic accretion (e.g., Mandelker et al. 2020). There are also instances of these interactions within the interstellar medium (ISM), like the stellar-wind-driven bubbles within star-forming clouds (e.g., Lancaster et al. 2021). They are also relevant to the ram pressure stripping of cluster galaxies and star formation in the tails of jellyfish galaxies (e.g., Tonneess & Bryan 2021). We take a particular interest in their role within galactic winds (e.g., Fielding & Bryan 2022).

Galactic winds are ubiquitous throughout cosmic time and play a pivotal role in galaxy evolution; they regulate star formation and transport metals out of the ISM (Somerville & Davé 2015). Observations indicate that stellar-feedback-driven winds are inherently multiphase; they are composed of comoving gas phases that vary in temperatures by orders of magnitude (see Veilleux et al. 2005 and Rupke 2018 for reviews of observational evidence).

Observations favor a model in which supernovae drive hot \( \gtrsim 10^6 \text{ K} \) winds that accelerate and entrain clouds of cool \( \sim 10^4 \text{ K} \) gas from the ISM (e.g., Chevalier & Clegg 1985). This model is complicated by hydrodynamical instabilities that drive mixing of gas between the cloud and wind. Because the timescale for mixing to destroy the cloud (by homogenizing the gas phases) is shorter than the ram pressure acceleration timescale, it is remarkably difficult to accelerate clouds before they are destroyed (Zhang et al. 2017).

Various ideas have been proposed to address this difficulty. Some, like magnetic shielding (e.g., McCourt et al. 2015; Gronnow et al. 2018; Cottle et al. 2020), may extend the cold phase lifetime by reducing mixing (see also Forbes & Lin 2019, for other mechanisms). Others are alternative acceleration mechanisms like radiation pressure (e.g., Zhang et al. 2018) or cosmic rays (e.g., Wiener et al. 2019; Brüggen & Scannapieco 2020). Another idea suggests that the remnants of destroyed clouds seed the in situ formation of clouds in cooling outflows (e.g., Thompson et al. 2015; Schneider et al. 2018; Lochhaas et al. 2021).

Radiative cooling is also known to extend the cold phase lifetime (e.g., Mellema et al. 2002; Fragile et al. 2004; Melioli 2005; Cooper et al. 2009). This work focuses on the regime in which rapid cooling acts as a mechanism that facilitates cloud survival (e.g., Marinacci et al. 2010; Armillotta et al. 2016). In this regime, cooling in a thin layer of gas at the interface between the phases is able to overcome the destructive effects of mixing (Gronke & Oh 2018). As turbulent mixing feeds hot phase material into this layer, isobaric cooling removes the temperature differential in the new material (Fielding et al. 2020). This process facilitates the transfer of mass and momentum to the cold phase providing a powerful additional acceleration source and allowing cloud
growth. Hereafter, we refer to this mechanism as turbulent radiative mixing layer (TRML) entrainment.

This topic has been extensively studied using wind tunnel setups (e.g., Gronke & Oh 2020a; Li et al. 2020; Sparre et al. 2020; Kanjilal et al. 2021; Abruzzo et al. 2022; Bustard & Gronke 2022; Farber & Gronke 2022). There has also been considerable work that focuses on a single shear layer (e.g., Kwak & Shelton 2010; Ji et al. 2019; Fielding et al. 2020; Tan et al. 2021).

The literature largely agrees that the occurrence and efficacy of TRML entrainment is controlled by three principal dimensionless numbers: (i) the density contrast \( \chi = \rho_{cl}/\rho_w \) between the cloud and the wind, (ii) the Mach number of the wind \( M_w = v_w/c_{cl, hot} \), and (iii) the cooling efficiency \( \xi = \tau_{mix}/\tau_{cool} \). Here, \( \tau_{mix} \) and \( \tau_{cool} \) specify the characteristic timescales for mixing and for cooling of the mixing layer. As in Fielding et al. (2020), we primarily consider \( \xi_{sh} = t_{\text{shear}}/t_{\text{cool}, \text{min}} \), where \( t_{\text{shear}} = R_{cl}/v_w \) is the shear time and \( t_{\text{cool}, \text{min}} \) is the minimum cooling time. In practice, our choice for \( \tau_{\text{cool}} \) is similar to the popular opinion of using \( \tau_{\text{cool}, \text{mix}} \) as the cooling time of gas within the mixing layer at \( t_{\text{max}} \approx \sqrt{\chi T_c T_w} \) and \( n_{\text{mix}} \approx \sqrt{m_{\text{mix}} n_{cl} n_{w}} \). Other studies (Li et al. 2020; Sparre et al. 2020) have suggested that the relevant cooling timescale is instead set by cooling in the hot, volume-filling wind phase. However in a follow-up to this work (Abruzzo et al. 2023), we show that the results of those other studies are explained if \( \xi \) is defined as the ratio of \( t_{\text{shear}} \) to \( t_{\text{cool}, \text{mix}} \), a cooling timescale similar to \( t_{\text{cool}, \text{mix}} \) (adapted from Farber & Gronke 2022).

Despite the obvious central importance of turbulence mechanisms underlying the operation of TRMLs, we do not yet have a clear understanding of how the turbulent velocity \( v_{\text{turb}} \) changes with the three principal dimensionless numbers (\( \chi, M_w \), and \( \xi \)) are varied. This is closely related to two fundamental unanswered questions.

(i) What is the role of cooling in driving turbulence? Shear layer studies find no or a very weak cooling time dependence of \( v_{\text{turb}} \) (Fielding et al. 2020; Tan et al. 2021). In contrast, some cloud-crushing simulations find that cooling-induced pulsations may be the dominant driver of turbulence (Gronke & Oh 2020a, 2020b). Reconciling these pictures requires a careful investigation of how \( v_{\text{turb}} \) scales with \( \xi \).

(ii) What is the timescale for turbulent mixing? Shear layer studies associate \( \tau_{\text{mix}} \) with the eddy turnover time at the outer scale, or \( \tau_{\text{mix}} \approx L_{\text{outer}}/v_{\text{turb}}(T_{\text{cl}}, \xi) \), where \( v_{\text{turb}}(T_{\text{cl}}, \xi) \) is a fixed fraction of the relative velocity for \( \chi \geq 100 \) (Fielding et al. 2020; Tan et al. 2021). This scales similarly to \( t_{\text{shear}} \). Wind tunnel studies instead link \( \tau_{\text{mix}} \) with the cloud-crushing time \( t_{cc} = \sqrt{\chi R_{cl}/v_w} \). The Kelvin–Helmholtz and Rayleigh–Taylor instabilities have growth times of order \( t_{cc} \) and destroy clouds over a few \( t_{cc} \) in the absence of cooling (Klein et al. 1994). Gronke & Oh (2018) predict cloud survival when \( \xi_{GO} = t_{cc}/t_{\text{cool}, \text{mix}} \) exceeds unity. While both choices give \( \xi \) a \( M_w R_{cl} \) scaling, the latter introduces an extra dependence on \( \chi^{-1/2} \). This discrepancy could have profound impacts on cloud survival criteria and requires a careful understanding of how \( v_{\text{turb}} \) scales with \( \chi \).

To address these questions, we investigate the turbulent properties that emerge in wind tunnel simulations of cloud–wind interactions. While turbulence in TRMLs has traditionally been treated as homogeneous (e.g., Begelman & Fabian 1990; Gronke & Oh 2018; Fielding et al. 2020), we will show that it depends not just on scale but also on phase. This has important implications for mixing and hence cloud survival. Although most previous work on TRML entrainment has focused on cloud–wind density contrasts of \( \chi = 100–300 \) (see, however, Gronke & Oh 2018, 2020a; Sparre et al. 2020), galactic winds are expected to have \( \chi \gtrsim 10^3 \) (Gronke & Oh 2020b). In this work, therefore, we place particular emphasis on higher \( \chi \) results.

In Section 2, we describe the suite of simulations used in this investigation. Videos of these simulations can be found at http://matthewabruzzo.com/visualizations/. In Section 3, we describe and compare three approaches for characterizing multiphase turbulence, followed by a description of the results from applying these methods to our simulation suite in Section 4. Subsequently, we describe the implications of our results and detail our conclusions in Sections 5 and 6.

2. Simulations

We ran a suite of 3D uniform grid hydrodynamical simulations using the ENZO-e\(^5\) code, which is a rewrite of ENZO (Bryan et al. 2014) built on the adaptive mesh refinement framework CELLO ( Bordner & Norman 2012, 2018). Our simulations employed the van Leer integrator (without constrained transport; Stone & Gardiner 2009) with second-order reconstruction and the HLLC Riemann solver.

Our simulations begin with a motionless spherical cloud embedded within a hot, uniform, laminar wind in the \( \hat{x} \) direction. We imposed an inflow condition on the upstream boundary (positive \( \hat{x} \)) and outflow conditions for the other boundaries. The cloud and wind material are initialized with \( p/\rho_w = 10^{7} \text{K cm}^{-3} \). The cloud density \( \rho_{cl} \) in all of our simulations is chosen such that \( T_{cl} = 5010 \text{K} \). This roughly corresponds to the temperature where heating starts to dominate over cooling (without self-shielding). The wind density is then determined by the desired value of \( \chi \).

To model radiative cooling, we use the GRACLE\(^6\) library (Smith et al. 2017), assuming solar metallicity and no self-shielding. Specifically, we use the tabulated heating and cooling rates for optically thin gas in ionization equilibrium with the \( z = 0 \) Haardt & Madau (2012) UV background. We turn off cooling in gas with \( T > 0.6T_w \) in our simulations with \( \chi \leq 10^3 \). This helps to avoid complications from cooling in the hot wind in our \( \chi = 100 \) simulations in which the ratio of the cooling time of the hot wind to the cooling time of the mixing layer is \( t_{\text{cool}, w}/t_{\text{cool}, \text{mix}} \approx 40 \). In higher \( \chi \) simulations, cooling of the wind fluid is so slow that this ceiling has no discernible impact. For simplicity, we also turned off heating/cooling below \( T_{cl} \).

---

\(^5\) http://enzo-e.readthedocs.io

\(^6\) https://grackle.readthedocs.io/

We only explicitly checked the effects of a cooling wind in the \( \chi = 100, M_w = 1.5 \text{runs and } \chi = 10^6, \xi_{GO} = 27.8 \text{run. We expect only minimal late-time complications in our } \chi = 1000, \xi_{GO} = 83.4 \text{run since it has } t_{\text{cool}, w}/t_{cc} \approx 51 \text{(Abruzzo et al. 2022). While our } \chi = 300, \xi_{GO} = 3.2 \text{run has the same } t_{\text{cool}, w}/t_{cc}, \text{complications may be significant since that run is close to the survival threshold. Complications are likely significant in larger } \chi = 300 \text{runs.}

---

As in Abruzzo et al. (2022), we actually define \( e_{\text{mix}} = \sqrt{\xi T_c T_w} \), where \( e \) is the specific internal energy. This definition is more consistent with the arguments of Begelman & Fabian (1990), when the mean molecular weight is not constant. Since the problem is quasi-isobaric, \( t_{\text{mix}} = \sqrt{\xi T_c R_{cl}} \). Consequently, the geometric mean of \( T_{cl} \) and \( T_w (n_{cl} \text{ and } n_w) \) tends to slightly overestimate (underestimate) the value of \( \tau_{\text{mix}} (\tau_{\text{max}}) \).
Table 1
Table of Simulations

| χ  | $M_a$ (pc) | $R_c$ | $t_{cool}$/$t_{cool,max}$ | $t_{shear}$/$t_{cool,min}$ | Survival | $R_c$/$\Delta x$ | Domain $(R_c)$ | Notes |
|---|---|---|---|---|---|---|---|---|
| 100 | 1.5 | 8.647 | ... | ... | No | 16 | 120 x 10^2 | No cooling |
| 100 | 1.5 | 5.638 | 1.34 | 0.57 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 12.1 | 2.87 | 1.23 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 22.4 | 5.32 | 2.28 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 26.75 | 6.35 | 2.72 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 44.12 | 10.48 | 4.49 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 56.38 | 13.39 | 5.73 | Yes | 4, 8, 16, 32 | 120 x 10^2 |
| 100 | 1.5 | 121.0 | 28.73 | 12.30 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 262.0 | 62.21 | 26.64 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 441.2 | 104.76 | 44.86 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 563.8 | 133.87 | 57.33 | Yes | 16 | 120 x 10^2 |
| 100 | 0.75 | 28.19 | 13.39 | 5.73 | Yes | 16 | 120 x 10^2 |
| 100 | 3.0 | 117.26 | 13.39 | 5.73 | Yes | 16 | 240 x 10^2 |
| 100 | 3.0 | 1172.6 | 133.9 | 57.3 | Yes | 16 | 240 x 10^2 |
| 100 | 6.0 | 225.52 | 13.39 | 5.73 | Yes | 16 | 240 x 10^2 |
| 100 | 1.5 | 77.27 | 10.00 | 5.55 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 121.0 | 15.66 | 8.70 | Yes | 16 | 120 x 10^2 |
| 100 | 1.5 | 316.58 | 10.00 | 15.89 | Yes | 16 | 120 x 10^2 |
| 300 | 1.5 | 13.67 | 2.50 | 0.80 | No | 16 | 120 x 10^2 |
| 300 | 1.5 | 27.34 | 5.00 | 1.61 | No | 16 | 120 x 10^2 |
| 300 | 1.5 | 54.68 | 10.00 | 3.21 | Yes | 16 | 120 x 10^2 |
| 300 | 1.5 | 164.0 | 29.99 | 9.63 | Yes | 16 | 120 x 10^2 |
| 300 | 1.5 | 546.8 | 100.01 | 32.10 | Yes | 16 | 120 x 10^2 |
| 10^4 | 1.5 | 8.647 | ... | ... | No | 16 | 120 x 10^2 |
| 10^4 | 1.5 | 86.47 | 10.00 | 2.78 | No | 16 | 240 x 10^2 |
| 10^4 | 1.5 | 173.0 | 20.01 | 5.56 | Borderline | 16 | 240 x 10^2 |
| 10^4 | 1.5 | 324.0 | 37.47 | 10.42 | Borderline | 16 | 240 x 10^2 |
| 10^4 | 1.5 | 432.5 | 50.00 | 13.90 | No | 16 | 240 x 10^2 |
| 10^4 | 1.5 | 864.7 | 100.00 | 27.81 | Yes | 4, 8, 16, 32, 64 | 240 x 10^24 |
| 10^4 | 1.5 | 2594.1 | 300.01 | 83.42 | Yes | 16 | 240 x 10^2 |
| 10^4 | 1.5 | 8.647 | ... | ... | No | 16 | 360 x 10^2 |
| 10^4 | 1.5 | 1699.5 | 10.00 | 17.28 | No | 16 | 360 x 10^2 |
| 10^4 | 1.5 | 16995.0 | 100.00 | 172.82 | Borderline | 8, 16 | 360 x 10^2 |

Note. Unless otherwise noted, all runs were initialized with $T_{cl} = 5010$ K, where $t_{cool}$, $R_c$ and $t_{cool,min}$ ~ 105. All simulations were initialized with an initial thermal pressure of $p / R = 10^6$ cm$^{-2}$ K. In each run, $t_{cool}$ is minimized at $T = 1.83 \times 10^4$ K with a value of 75.5 kyr; the sound speed at this temperature is 18.6 km s$^{-1}$. The cooling length, $t_{cool} = \epsilon \cdot t_{cool,min}$, is minimized at $T = 1.70 \times 10^4$ K with a value of 1.43 pc.

Denotes whether clouds survive (i.e., if the cold phase mass ever drops to 0). "Borderline" indicates cases where the line between survival versus destruction and rapid subsequent precipitation is fuzzy.

Denotes that $R_c$ is larger than $R_c$.

Denotes that $R_c$ is larger than $R_c$.

The mass of gas denser than $R_c$ drops to 16% (6%) of its initial value and begins monotonic growth after $21.5 t_{cl}$ (24$t_{cl}$). In an alternate version of the same run, where the domain dimensions are $120 \times 10^2$, the mass instead drops to 32% (8%) of its initial value and starts growing after $21.5 t_{cl}$ (11$t_{cl}$).

$R_c$ is larger than $R_c$.

The mass of gas denser than $R_c$ drops to 16% (6%) of its initial value and begins monotonic growth after $21.5 t_{cl}$ (24$t_{cl}$). In an alternate version of the same run, where the domain dimensions are $120 \times 10^2$, the mass instead drops to 32% (8%) of its initial value and starts growing after $21.5 t_{cl}$ (11$t_{cl}$).

Denotes whether clouds survive (i.e., if the cold phase mass ever drops to 0). "Borderline" indicates cases where the line between survival versus destruction and rapid subsequent precipitation is fuzzy.

To break initial symmetries, we initialized the density of each cell within the cloud to the average of $\rho(x)$, where

$$\rho(x) = 1 + 0.099 \sum_{i=1}^{10} \cos \left( \frac{2\pi}{\lambda_i} \cdot x + \phi_i \right).$$ (1)

For each $i$, we drew a random unit vector $\hat{e}_i$ and values for $\lambda_i$ and $\phi_i$ from $[R_c/8, R_c]$ and $[0, \pi]$. Cells on the cloud edges were initialized with sub-sampling; each subcell had a width of $R_c/128$.

Our simulations have resolutions of $R_c/\Delta x = 4, 8, 16, 32, 64$. Unless stated otherwise, results are presented for $R_c/\Delta x = 16$. By default, the wind-aligned dimension and transverse dimensions for most of our simulations’ domains had sizes of $120R_c$ and $20R_c$, respectively, corresponding to a $1920 \times 320^2$ grid at our fiducial $R_c/\Delta x = 16$ resolution. The sizes were somewhat larger (360$R_c$ and 30$R_c$) for our $\chi = 10^4$ simulations in order to minimize the impact of the bow shock reflections, prevent dense material from leaking out of the transverse boundaries in cases of shattering, and give room for tail formation. While the default dimensions are adequate for determining whether our clouds survive in runs with $M_a \gg 3$ or $\chi = 10^4$, we find that boundary effects can impact later time measurements. Thus, for such cases, with radiative cooling and $R_c/\Delta x = 4, 8, 16, 16$, we present results from runs with a wind-aligned length of $240R_c$. In all cases, the cloud was initialized at the center of the domain, and we employed a frame-tracking scheme that updated the reference frame every $t_{cool}/16$ such that the mass-weighted velocity for cells with $\rho > \sqrt{\rho_{cl}/\rho_w}$ was 0.
Table 1 presents a list of our simulations.

As we will discuss in Section 3, our measurements of \( \nu_{\text{turb}} \) involve averages over velocity properties. Thus, leakage of material from the domain could plausibly bias our measurements. However, the generality of the scaling relations derived in this work, which apply to runs that do and do not leak material, suggests that overall effects on our \( \nu_{\text{turb}} \) measurements are probably minimal. We assessed this leakage by tracing material initialized within the cloud with a passive scalar. Nearly all turbulent measurements shown in this work for our \( R_{\text{cl}}/\Delta x = 16 \) runs, where the cloud avoids destruction, are from times at which our runs retain at least 95% of the passively advected scalar. This statement does not apply to our \( \chi = 300, \xi_{\text{sh}} = 3.21 \) runs (\( \chi = 10^5, \xi_{\text{sh}} = 10.47; \chi = 10^3, \xi_{\text{sh}} = 13.9 \)) run, which retains 95% of the scalar until \( 11t_{\text{cc}} \) (11.5\( t_{\text{cc}} \); 12\( t_{\text{cc}} \)) and leaks another \( \sim 6\% \) (\( \sim 34\% \); \( \sim 17\% \)) by \( 14.5t_{\text{cc}} \) (17.5\( t_{\text{cc}} \); 18.5\( t_{\text{cc}} \)). Additionally, our \( \chi = 10^3, \xi_{\text{sh}} = 27.81 \) run retains 95% for 22\( t_{\text{cc}} \) but only loses another \( \sim 2\% \) over the subsequent 18\( t_{\text{cc}} \). Finally, our \( \chi = 10^4, \xi_{\text{sh}} = 178.82 \) case retains 95% for 8\( t_{\text{cc}} \), but only retains 42% (15%) by 15\( t_{\text{cc}} \) (27.5\( t_{\text{cc}} \)). This last case is particularly noteworthy because it starts leaking the scalar at 6.5\( t_{\text{cc}} \), which coincides with a drop in the cool phase mass.

3. Characterizing Turbulence

The primary goal of this paper is to characterize the turbulent properties of the TRML that mediate mixing and cooling between the hot wind and cold cloud. Although much effort has been devoted to understanding turbulence in single-phase media, there has been considerably less work for multiphase systems (e.g., Gronke & Oh 2022; Gronke et al. 2022; Mohapatra et al. 2022). The potential dependence of turbulent properties on both scale and the gas’s local thermodynamic state complicates the interpretation of conventional methods for characterizing \( \nu_{\text{turb}} \). There are a number of possible ways to extend existing turbulence measures; however, their novel nature means that they can be difficult to interpret and their robustness is unclear. In order to get around this difficulty, in this paper, we consider three distinct methods for characterizing our multiphase turbulent simulations. These are built around three different ideas based on (i) a filter-based technique, (ii) a geometric approach, and (iii) classic structure function ideas.

We describe these approaches below, and to supplement our description of the methods, we apply each to a snapshot of an \( R_{\text{cl}}/\Delta x = 64 \) of \( \chi = 1000, \xi_{\text{sh}} = 27.8, M_w = 1.5 \), which simulates a successfully entrained cloud.

3.1. Filtering

In our first approach, we attempt to explicitly remove the bulk flows by filtering out the large-scale bulk velocities. Specifically, we estimate \( \nu_{\text{turb}} \) by applying a high-pass Gaussian filter with density weighting to each component of the velocity. The size of the filter is chosen to correspond to scales on which the bulk flow is varying, that is, approximately the cloud radius. We use density weighting (which corresponds to smoothing the momentum) in order not to be dominated by the volume-filling hot gas component.

More precisely, in this approach, the \( i \)th component of the turbulent velocity is given by

\[
\nu_{\text{turb}}(x) = \nu_i(x) = \frac{\iiint f_r(x-r) \rho(r) v_r(r) d^3r}{\iiint f_r(x-r) \rho(r) d^3r},
\]

where \( f_r(x) \) is the formula for a normalized, separable, 3D Gaussian. In short, the convolution of \( f_r(x) \) with \( \nu(r) \) (i.e., the fraction term) estimates the laminar part of \( \nu_v \), and subtracting it from \( \nu_v \) gives the turbulent part.

Throughout this work, we use a Gaussian filter with a standard deviation of \( R_{\text{cl}}/4 \); this was chosen after extensive experimentation to visually pick out turbulent regions with a minimal “bleed” into the laminar regions. Our results do not depend qualitatively on the exact choice of the filtering scale as long as it is on the order of the cloud size.

The right two panels in Figure 1 illustrate the high-pass-filtered transverse velocity components for the aforementioned simulation at 4.5\( t_{\text{cc}} \), and the center panel shows the combined magnitude of these components, \( \nu_{\text{trans,hi},i} \). The left two panels show the density and specific thermal energy slices. Note that here, as elsewhere in this paper, we use \( \equiv (p/\rho)/(\gamma - 1) \) to denote the specific thermal energy of the gas. This quantity is closely related to temperature but is easier to compare among runs with different \( \chi \) values since \( \epsilon_{\text{wind}} = \chi \xi_{\text{cl}} \) (due to variations in mean molecular mass, \( T_w < \chi T_{\text{cl}} \)). The inset panels make it readily apparent that the turbulent velocity has a clear phase dependence.

In Figure 2, we show the phase dependence explicitly (at 2.5\( t_{\text{cc}} \)), plotting the 2D distribution of mass as a function of temperature and (top) high-pass-filtered velocity including all components, \( \nu_{\text{tot,hi},i} \), and (bottom) just the transverse components, \( \nu_{\text{trans,hi},i} \), for this snapshot. Because of the spatial gradients that persist in the downstream velocity component (see the Appendix), we use \( \nu_{\text{trans,hi},i} \) to estimate \( \nu_{\text{turb}} \) in the remainder of this work.

This approach is sensitive to turbulence on scales below the high-pass-filtering limit; since this is approximately the driving scale (of order the cloud radius), we expect this to be a good measure of the turbulent properties, although it may also remove some of the contribution to the turbulence on scales just below the driving scale. One possible downside of this approach is the contribution of the bulk flow in scales at and below the filtering scales; we have explored alternate weighting schemes and find only minor differences. Although we do not have detailed scale information (except the removal of large scales), this approach does permit a very fine examination of the turbulent properties with phase (i.e., specific energy) as seen in Figure 2.

Indeed, this figure clearly shows a different dependence on specific energy below and above \( \log_{10}(\epsilon/\epsilon_\text{cl}) \approx 0.7 \), which corresponds to the peak of the cooling curve we adopt. We return to this point in Section 4. Figure 1 qualitatively shows that spatial variations in turbulence are largely explained by the spatial variations in gas phase. The main exception is the hottest phase, which is “contaminated” by unmixed, laminar gas (this is reflected in Figure 2).

Because measuring phase information is not as seamless for our other approaches, we define a set of nominal coarse phase bins to be used with them. We define the bin edges in terms of \( \log(\epsilon/\epsilon_\text{cl})/\log \chi \) to ease comparisons between runs with different \( \chi \) values. The bin edges are –\( \infty \), 1/12, 3/12, 5/12,
7/12, 9/12, and 11/12, which are illustrated by the vertical dotted lines in Figure 2.

### 3.2. Geometric

Our second approach uses the geometry of isosurfaces in the flow to characterize the turbulence. To motivate this, consider a toy model in which a cloud’s geometry is a sphere or a cylinder. The cloud is oriented such that the azimuthal angle, $\phi$, measures the angle on the plane transverse to $\hat{v}_{\text{wind}}$. While cloud acceleration and accretion (e.g., by a TRML; Fielding et al. 2020) can drive steady coherent flows along the wind and radial directions, turbulence is the only source of motion along $\phi$. In other words, we can characterize $v_{\text{turb}}$ with $v_\phi$. Tan et al. (2021) drew a similar conclusion in shearing box simulations about the utility of the dispersion of the velocity component perpendicular to shear and inflow directions.

Despite their more complex morphology, we can apply the same logic to real clouds. For a given snapshot, we employ the Lewiner et al. (2003) marching cubes algorithm to construct five topologically correct meshes of triangle facets that trace specific internal energy isosurfaces using values that coincide with the centers of the closed bins mentioned in Section 3.1. We supplement these with additional isosurfaces at values near the peak of the cooling curve (we vary the precise locations based on the $\chi$ value of the simulation). Figure 3(a) shows a cutaway visualization of several of these isosurfaces at $2.5 t_{\text{tc}}$ for our $\chi = 1000$, $\xi_{\text{sh}} = 27.8$ simulation.

For each facet, we define $v_{\phi-\text{like}} \equiv \mathbf{v} \cdot (\hat{v}_{\text{wind}} \times \hat{n})$, where $\mathbf{v}$ is the linearly interpolated velocity and $\hat{n}$ is the outward normal vector. Finally, we estimate $v_{\text{turb}}$ for an isosurface with the area-weighted standard deviation of $v_{\phi-\text{like}}$ (excluding facets with $\hat{v}_{\text{wind}} \times \hat{n} = \mathbf{0}$).

Figures 3(b) and (c) show area-weighted distributions of $v_{r-\text{like}} \equiv \mathbf{v} \cdot \hat{n}$ and $v_{\phi-\text{like}}$ for the previously mentioned simulation. The distribution for $v_{r-\text{like}}$ shows a negative mean for each isosurface, which is consistent with net inflow of gas. In contrast, the distribution for $v_{\phi-\text{like}}$ is centered on 0, which is exactly what we expect.

While this approach does not provide any scale information about turbulence, it can be used to provide detailed phase information. For example, Figure 3(d) illustrates a qualitatively similar phase dependence to the filtering measurements. However, in contrast to the filtering technique, this approach requires generation of a separate surface for each phase to be probed and so is much more computationally intensive. As is discussed in the Appendix, the main advantage of this approach is that it provides the most accurate early-time measurements.
individual phase bins. We note that both points in each pair always come from the same phase bin, and we leave consideration of cross-phase terms to future work. We omit the hottest phase bin from our analysis because a large fraction is laminar (contaminating the signal), and it is computationally expensive to compute.

We also use discrete bins of $\ell$, which depend on the cell width, $\Delta x$, in our simulations. The $i$th $\ell$ bin is centered on $\ell = i\Delta x$ and has a width of $\Delta x$. However, for $i = 0$ and 1, we have adjusted the bins such that they only contain values for pairs of cells that share a face and an edge, respectively. In other words, the $i = 0$ bin ($i = 1$ bin) only contains values for cells separated by exactly $\ell = \Delta x$ ($\ell = \sqrt[2]{\Delta x}$).

Throughout this work, we largely focus on $\sqrt{\langle (\delta v)^2 \rangle (\ell)}$ because it has a similar magnitude to our other $v_{\text{turb}}$ metrics. The top panel of Figure 4 shows $\sqrt{\langle (\delta v)^2 \rangle (\ell)}$ measured for each phase bin of our $\chi = 1000$, $\xi_{\text{sh}} = 27.8$ simulation at $2.5t_{\text{cc}}$. The peak in $\langle (\delta v)^2 \rangle (\ell)$ at $\ell \sim R_{\text{cl}}$ present in all phases (in some cases it manifests as a change in slope), is expected, since the outer scale should be of order the cloud size $R_{\text{cl}}$, although the complicated cloud structure at late times is unlikely to correspond to a narrow range for the injection of turbulence. We leave investigation of the behavior above $R_{\text{cl}}$ to future work.

We generally observe a weaker dependence on $\ell$ than the $\propto \ell^{1/3}$ scaling expected for idealized Kolmogorov turbulence (for $\sqrt{\langle (\delta v)^2 \rangle (\ell)}$), although this depends somewhat on phase. We caution that the precise $\ell$ scaling at intermediate (inertial) scales may not be a robust measurement due to the bottleneck effect, which arises for an underresolved turbulent cascade (e.g., Rennehan 2021; Mohapatra et al. 2022).

In agreement with the other measures, we also see a general decrease in the turbulent velocity with temperature.

3.4. Comparison

We have shown that much more information about the phase, scale, and spatial dependence of turbulence can be gleaned from these simulations when using metrics beyond the standard root-mean-square approach. We now compare these more refined turbulent metrics to each other.

The top row of Figure 5 shows the $v_{\text{turb}}$ phase dependence, measured with each approach, in an $R_{\text{cl}}/\Delta x = 16$ version of the run of the previously mentioned simulation at $t = 1.0t_{\text{cc}}, 5.5t_{\text{cc}}, 9.5t_{\text{cc}},$ and $17.5t_{\text{cc}}$ (see Section 4.3 for a discussion of how resolution affects our measurements).

The differing approaches achieve remarkable qualitative agreement about the magnitude, phase dependence, and temporal dependence of $v_{\text{turb}}$. In particular, all measures show that $v_{\text{turb}}$ increases rapidly with temperature at early times before transitioning to a flatter slope at later times. In addition, all approaches show very similar amplitudes. However, the approaches are clearly not interchangeable. Indeed, this plot demonstrates the utility of computing all three turbulence measures, allowing us to ascertain the robust results without overinterpreting features that are not seen in at least two of the techniques.

When considering the volume-averaged properties of the entire system, our geometric approach offers the most robust measurements because it is most resilient to biases that may arise from the gradients in the laminar component of the flow at early times (see the Appendix for more details).

In the context of phase dependence, the filtering approach clearly is the most convenient metric because it naturally
4. Results

Having established the robustness and relative merits of our turbulence metrics, we now examine what they tell us about the cloud–wind interaction. We start (in Section 4.1) by describing the phase dependence of \( v_{\text{turb}} \), its scaling with dimensionless parameters, and time dependence. Then, in Section 4.2, we briefly discuss the driving scale before turning to an evaluation of numerical convergence in Section 4.3.

For the purpose of this discussion and subsequent sections, we define the cold phase as all gas with densities of at least \( \sqrt{\rho_{\text{cl}}/\rho_w} \) (i.e., the density of the mixing layer). We also define the relative velocity, \( v_{\text{rel}} \), as the difference between \( v_w \) (at the inflow boundary) and the mass-weighted velocity of the cold phase (initially, \( v_{\text{rel}} \sim v_w \) but declines as the gas is entrained).

4.1. Turbulent Properties

Throughout this group of subsections, we will compare simulations with different parameters and \( \Delta x = R_{\text{cl}}/16 \). We first consider the phase dependence of \( v_{\text{turb}} \), then show how \( v_{\text{turb}} \) scales between simulations, and finally describe the time evolution of \( v_{\text{turb}} \).

4.1.1. Phase Dependence

We begin by presenting the phase dependence in two limiting cases of the \( \chi = 1000 \) and \( M_w = 1.5 \) cloud–wind interaction. These two cases are (i) a run without cooling (\( \xi_{\text{sh}} = 0 \)) and (ii) a run where cooling is sufficient for entrainment (\( \xi_{\text{sh}} = 27.8 \)).

The bottom row of Figure 5 shows the nonradiative run. In this case, the scaling of \( v_{\text{turb}} \) with \( \ell \) (or \( T \)) is consistent with a power law where \( M_{\text{turb}} \) is constant (i.e., \( v_{\text{turb}} \propto c_s \propto \sqrt{\ell} \)) throughout the cold phase’s lifetime. The amplitude of the turbulence decreases as \( v_{\text{rel}} \) drops, but there is no indication that the scaling with phase changes (note that at late times, the cold gas is entirely absent due to mixing with the hot phase, and so we cannot measure its turbulent properties). As expected, these trends are unaffected by our choice of turbulent metric.

The top row of Figure 5 shows the run with cooling. Compared to the constant-slope power-law phase dependence of turbulence in our nonradiative run, the case with cooling clearly has more complex behavior. We parameterize the phase...
dependence of systems with sufficient cooling for entrainment at a given time using a broken power law,

\[
\frac{v_{\text{turb}}(e)}{v_{\text{turb, break}}} \sim \begin{cases} 
\sqrt{e/e_{\text{break}}} & \text{if } e \leq e_{\text{break}} \\
(e/e_{\text{break}})^{\alpha} & \text{if } e_{\text{break}} \leq e \n\end{cases}
\]  \tag{3}

with a break at \(e_{\text{break}} = e_{\text{min, cool}}\), which coincides with the minimum of \(\tau_{\text{cool}}\). For now, we are just interested in \(\alpha\); the following subsections will discuss \(v_{\text{turb, break}}\).

Below \(e_{\text{break}}\) the scaling of \(v_{\text{turb}}\) is constant in time. Above \(e_{\text{break}}\) the slope of the power-law dependence, \(\alpha\), has clear time dependence. At very early times (\(t < 8\Delta x\)) the data to the left of the gray vertical dotted line lies outside of the inertial range.

This demonstrates an essential feature of the turbulence in systems with sufficient cooling for cloud survival and cloud entrainment: the cold phase has a larger turbulent Mach number and turbulent kinetic energy than the hot phase.

4.1.2. Scaling with Cloud Parameters (\(\chi\), \(M_w\), and \(\xi_{\text{sh}}\))

Now that we have established the behavior in these limiting cases, we discuss how the dimensionless numbers (\(\chi\), \(M_w\), and \(\xi_{\text{sh}}\)) affect the magnitude of \(v_{\text{turb}}\) in simulations with rapid enough cooling to ensure cloud survival. At a given stage of a cloud’s evolution (i.e., for a given value of \(v_{\text{rel}}/v_w\) or fixed time), we find that \(v_{\text{turb}}\) satisfies the scaling

\[
\frac{v_{\text{turb}}(e_{\text{break}})}{c_{\text{turb, break}}} \propto (\xi_{\text{sh}} M_w)^{\beta} \propto \left( \frac{R_{\text{cl}}}{c_{\text{turb, cool, min}}} \right)^{\beta},
\]  \tag{4}

where \(\xi_{\text{sh}}\) and \(M_w\) both refer to values used to initialize the problem. This is equivalent to saying that \(v_{\text{turb}}\) scales with the ratio between the hot phase sound-crossing time \(R_{\text{cl}}/c_{\text{turb, hot}}\) and \(\tau_{\text{cool, min}}\). The best-fit values for \(\beta\) are 0.25 and \(-0.5\) at early and late times, respectively. This change in \(\beta\) seems to coincide with a transition between temporal evolutionary stages, which we will discuss further in the next subsection and link to a change in the primary source of turbulent kinetic energy.

Figures 6–9 compare \(v_{\text{turb}}\) measurements, adjusted to remove differences captured by this scaling, for different sets of simulations. In other words, the agreement between the curves in a given panel in these figures indicates the accuracy of the adopted scaling. Because the principal dimensionless numbers clearly affect the slope of \(v_{\text{turb}}\) above \(e_{\text{break}}\), the reader should primarily consider agreement at \(e_{\text{break}}\) (denoted by a vertical line) and in colder gas. Note that unlike previous plots, the black dashed line shows \(v_{\text{turb}} \propto c(e)\) rather than \(v_{\text{turb}} = c(e)\).

First, we consider the scaling for runs with \(M_w = 1.5\). Figure 6 shows the scaling on \(R_{\text{cl}}\); the top (bottom) row shows runs with \(\chi = 100\) (\(\chi = 1000\)). The impressive overlap of the curves in each panel demonstrates that the adopted scaling works remarkably well—there are occasional differences at high \(e\), but the turbulence in the gas closest to the wind phase is the most challenging to accurately measure. The figure also clearly shows that the shape of the turbulence dependence with \(e\) changes over time, a point we will return to later.

Figures 7 and 8 provide evidence that \(v_{\text{turb}}\) depends not just on \(R_{\text{cl}}\) but on the ratio \(R_{\text{cl}}/c_{\text{turb, hot}}\) by comparing runs with different \(R_{\text{cl}}\) and \(c_{\text{turb, hot}}\) values. The variation in \(c_{\text{turb, hot}}\) come from adopting different values for \(T_{\text{cl}}\) and \(\chi\). Figure 8 provides further confirmation that \(c_{\text{turb, hot}}\) is the correct sound speed to include in this scaling because \(c_{\text{turb, hot}}\) has different \(e\)-dependence from the sound speed in the (cold) cloud phase.

Finally, Figure 9 demonstrates the \(v_{\text{turb}}\) scaling for runs that vary in \(M_w\). It largely confirms the lack of \(M_w\) dependence.

We provide a rough normalization for Equation (4) when \(v_{\text{rel}}/v_w \sim 0.75\). In this case, we find that \(v_{\text{turb}}(e_{\text{break}}) \sim 0.4 c_{\text{turb, break}} (R_{\text{cl}}/\tau_{\text{cool, min}} c_{\text{turb, hot}})^{1/4}\). The precise normalization will change when using other techniques to measure \(v_{\text{turb}}\).

All of these results are computed with the filtering metric for turbulence. We note that these scalings are somewhat less clear for geometric and \(\langle (\delta v)^2 \rangle / (\ell)\) measurements of the \(\chi = 100\) simulations (the scaling between \(\chi = 1000\) runs is clear for all metrics). For example, the geometric measurements show slightly different trends among the runs that initially lose mass and suggest that \(\beta\) never changes from 0.25 in the \(\chi = 1000\) runs. Although the latter quirk is difficult to explain, we are
encouraged by the fact that the geometric approach does show the change in $\beta$ for the $\chi=103$ simulations and the fact that the $\langle (\delta v)^2 \rangle (\ell)$ relation definitely supports $\beta=0.5$ at late times in our $\chi=100$ simulations. With that said, $\langle (\delta v)^2 \rangle (\ell=R_{cl})$ measurements do show more scatter than is present in the top row of Figure 6. While this could be physical, the coarser phase bins may also contribute to this scatter. We defer further investigation to future work.

4.1.3. Temporal Evolution

So far, we have focused on how the phase dependence of $v_{\text{turb}}$ changes with cloud properties. We turn our attention to how $v_{\text{turb}}$ changes with time in a single simulation. Given how the slope of the $v_{\text{turb}}$ broken power-law phase dependence is largely time-independent for the cold phase up to the break, we focus on $v_{\text{turb}}$ at $e = e_{\text{min,cool}} \sim e_{\text{break}}$. Figure 10 shows, for a broad range of simulations, the time evolution of $v_{\text{turb}}$ measured geometrically (we use this metric to isolate a narrow phase bin), average inflow velocity, and surface area on the same isosurface. The figure also shows the time evolution of $v_{\text{rel}}$. We do not show other types of $v_{\text{turb}}$ measures because they are less accurate at early times (see the Appendix) and do not distinguish between turbulence and gradients in inflowing gas as well as the geometric measurements.

We start by considering the turbulent evolution in a characteristic case with cloud entrainment: Figure 10(c) shows a $\chi=100$ run with $M_{\text{cl}}=1.5$ and $c_{\text{sh}}=5.73$. $v_{\text{turb}}$ has two primary evolutionary stages. Initially, in the “pre-entrained stage,” $v_{\text{turb}}$ rapidly grows until it reaches a peak value; $v_{\text{turb}}$ is sustained near this peak for a short time, and then it starts to drop off as the cloud becomes partially entrained. During the subsequent “partially entrained” phase, $v_{\text{turb}}$ stabilizes at a smaller value (within a factor of $\sim 2$ of the peak) that is sustained for the remainder of the run.
The primary source of turbulent energy during the pre-entrained stage appears to be the relative velocity. This would explain why $v_{\text{turb}}$ peaks within a few $t_{cc}$; we expect large $v_{\text{rel}}$ to drive the Kelvin–Helmholtz and Rayleigh–Taylor instabilities, which have growth rates proportional to $t_{cc}$ (Klein et al. 1994). This also explains similar rapid turbulent growth during the initial stage of the nonradiative and slow cooling simulations in panels (a) and (b) of Figure 10. Furthermore, it explains why the drop in $v_{\text{turb}}$—which indicates the transition between stages (and is most prominent in the radiative runs), follows the drop in $v_{\text{rel}}$—this is presumably because $v_{\text{rel}}$ no longer provides enough turbulent energy to sustain the peak $v_{\text{turb}}$.

The two stages of $v_{\text{turb}}$ evolution roughly coincide with the stages of areal growth identified in Gronke & Oh (2020a). The “pre-entrained” stage coincides with the rapid surface area growth dominated by the formation of the cloud’s tail. Likewise, the “partially entrained” stage roughly corresponds to the slower isotropic areal growth that occurs once the cloud is entrained. It is also noteworthy that the average inflow velocity plateaus before the slower isotropic areal growth, which is consistent with findings from Gronke & Oh (2020a).

At face value, it might seem surprising that there is net inflow in Figure 10(b), even though we know that this run is losing mass during the first $\sim 10 t_{cc}$ (see the mass evolution of the $\chi = 100, R_{\text{cl}} = 5.64$ pc run in Figure 6). However, this just illustrates that net inflow does not necessarily equate with mass growth; the inflowing gas will raise the temperature of the gas enclosed by an isosurface in the absence of sufficient cooling.

We now consider how the principal dimensionless numbers ($\xi_{\text{sh}}, \chi, M_{\text{sh}}$) affect the $v_{\text{turb}}$ evolution with time. In general, we find that these parameters only minimally affect the overall trend, so we focus on the relatively small differences that do emerge.

First, we examine variation in $\xi_{\text{sh}}$. Compared to panel (c) of Figure 10, panel (d) illustrates that more efficient cooling can increase the maximum $v_{\text{turb}}$ as well as the value of $v_{\text{turb}}$ at late times. This is consistent with the scalings from the last subsection. In this panel, $v_{\text{turb}}$ approaches $v_{\text{inflow}}$’s magnitude at late times. It is plausible that all entrained runs in the figure would show the same behavior if we had measurements for late enough times; it may just be most prominent in panel (d) because $v_{\text{inflow}}$ is elevated and the cloud is accelerated more.

Figure 6. The top row compares the dependence of $v_{\text{turb}}$ with gas phase at four representative times in the clouds’ evolution (full-size panels on the left) and bulk property evolution (small panels on the right) for a separate collection of simulations. The top row compares nine simulations with $\chi = 100, M_{\text{sh}} = 1.5$ and the same initial cloud temperature but varying $R_{\text{cl}}$ (and so varying $t_{cc}$). The leftmost $v_{\text{turb}}$ panel shows filtering measurements after $1.5 t_{cc}$, the middle two $v_{\text{turb}}$ panels show measurements when the relative velocities (between the cold and hot phases) are various fractions of its initial value, and the rightmost $v_{\text{turb}}$ panel shows measurements after $20 t_{cc}$. Data are only shown for a given simulation for $v_{\text{rel}}$ at four representative times in the clouds’ evolution roughly coincide with the $v_{\text{turb}}$ panel shows measurements at $v_{\text{rel}}/c_{\text{s}} \sim 0.2$. We note that $c_{\text{s,hot,cool, min}}$ is 6.56 pc (20.7 pc) for simulations in the top (bottom) row.
quickly. This feature may suggest that $v_{\text{turb}}$ is dominated by the radial flow at late times. We also find that higher $\xi_{\text{sh}}$ simulations have a somewhat smaller surface area.

The bottom row of Figure 10 shows data for a set of runs with $\chi = 10^3$ and varying entries of $\xi_{\text{sh}}$. In simulations in which the cloud survives, the transition between evolutionary stages of $v_{\text{turb}}$ happens at larger $v_{\text{rel}}/v_w$ when $\chi$ is larger. This transition appears to roughly coincide with the time at which the value of $\beta$, from Equation (4), increases from 0.25. Differences in $v_{\text{turb}}$’s magnitude are qualitatively consistent with the scaling given in that equation.

Finally, the middle row of Figure 10 compares runs with varying $M_w$. Increasing $M_w$ appears to increase $v_{\text{turb}}$’s initial growth rate, $v_{\text{turb}}$’s magnitude, and the duration over which $v_{\text{turb}}$’s maximum magnitude is sustained. There is also some indication that higher $M_w$ simulations may also have larger inflow rates and larger surface areas, even at late times.

Independent of $\xi_{\text{sh}}$, $\chi$, and $M_w$, Figure 10 illustrates that the acceleration timescale is tightly correlated with the stages of areal growth (the surface area and $v_{\text{rel}}$ curves feature abrupt slope changes at similar times). In contrast, the transition between $v_{\text{turb}}$ stages appears less tightly coupled with the acceleration timescale as the principal dimensionless numbers are changed. We attribute this mostly to the fact that the cold acceleration timescale is tightly correlated with the stages of entrained clouds in our various simulations have different wind-aligned lengths, we know that changes in these numbers alter the cloud’s differential acceleration. Consider the temporal evolution of the volume-averaged $v_{\text{turb}}$ measurements for a narrow phase bin of a very coherently accelerated cloud and a less coherently accelerated cloud. One would naturally expect that $v_{\text{turb}}$ measurements might spend more time near its maximum value in one of these cases. It is not much of a stretch to assume that $v_{\text{rel}}$ might be fairly different when $v_{\text{turb}}$ starts to decrease (i.e., begins transitioning between stages). Thus, we would find different coupling between $v_{\text{rel}}$’s evolution and $v_{\text{turb}}$’s evolution in these cases.

4.2. Evolution of the Driving Scale

We now briefly revisit the velocity structure function in order to investigate how the turbulent driving scale varies with time. The bottom panel of Figure 4 shows $\sqrt{\langle (\delta v)^2 \rangle (\ell)}$ for the $R_{\ell}/\Delta x = 64$ run of our $\chi = 1000$, $M_w = 1.5$, $\xi_{\text{sh}} = 27.8$ simulation when the cloud is mostly entrained in the wind ($v_{\text{rel}}/v_w \sim 0.27$). Comparisons with the top panel ($v_{\text{rel}}/v_w \sim 0.94$) reveal that the outer scale of turbulent driving, which coincides with the peak $\langle (\delta v)^2 \rangle (\ell)$, does not change substantially from early to late times. Although we do not show it, we confirmed similar behavior in the $R_{\ell}/\Delta x = 32$ runs of our $\chi = 100$, $M_w = 1.5$, $\xi_{\text{sh}} = 5.73$ simulations for similar values of $v_{\text{rel}}$ and at times when the cloud is more entrained.

We note that it is unclear why the $9/12 \leq e/e_{cl} < 11/12$ phase bin’s $\langle (\delta v)^2 \rangle (\ell \sim 0.3 R_{\ell})$ measurement, from the lower panel, is smaller than comparable measurements for other phase bins. This feature also appears in the $R_{\ell}/\Delta x = 32$ version of this simulation. In contrast, this feature is absent from the aforementioned $\chi = 100$ run; in that case, $\langle (\delta v)^2 \rangle (\ell)$ is always larger for a given $\ell$ in hotter gas.

4.3. Convergence

In this section, we discuss how numerical resolution impacts our various measurements. We primarily compare the measurements among different resolution runs of our $\chi = 1000$, $\xi_{\text{sh}} = 27.8$ simulation, varying $R_{\ell}/\Delta x$ from 4 to 64.

Figure 7. Like the top row of Figure 6 except that the pictured simulations primarily vary the cloud temperature. Each simulation has $\chi = 100$ and $M_w = 1.5$. We expect at higher resolution that the power-law slope below $e_{\text{break}}$ in the purple curve will be closer to 0.5 (i.e., the slope of the dashed black line).
Figure 8. Like the top row of Figure 6 except that the pictured simulations primarily vary in \( \chi \). We have made two compromises in presenting these data. First, we fix \( \beta \) to 0.25 for all panels. This is done as a simplification because \( \beta \) changes on a timescale related to \( \chi \). Second, the rightmost \( v_{\text{turb}} \) panel compares simulations at a fixed value of \( v_{\text{rel}}/v_\text{esc} \) rather than at a fixed time. The last panel typically compares the simulations at a point in evolution when \( v_{\text{turb}} \) stabilizes (see Section 4.1.3). However, that time seems to come much later in our \( \chi = 10^3 \) simulation, after the simulation terminates. While we include the \( \chi = 10^3 \) run for completeness, strong resolution dependence (see Table 1) and the atypical shape of the cool phase mass evolution may indicate that it is not well converged. As noted in Section 2, some material that started in the cloud leaks out of the domain at 6.5\( t_{\text{cc}} \), which coincides with the large drop-off in cool phase mass.

4.3.1. Turbulence Metrics

The large panels in the top row of Figure 12 compare the phase dependence of \( v_{\text{turb}} \) using filtering measurements of \( v_{\text{turb}} \) at various points in the cloud’s lifetime. The figure shows that resolution appears to slightly affect the magnitude and the slope of the \( v_{\text{turb}} \) phase dependence above \( e_{\text{break}} \). Importantly, the figure also suggests that the occurrence of a negative slope of the phase dependence is likely a resolution effect. The full phase dependence of \( v_{\text{turb}} \) is well converged for \( R_{\text{cl}}/\Delta x \gtrsim 32 \). These same conclusions apply to our other turbulence metrics (shown in the other rows).

Figure 13 shows how resolution affects the temporal evolution of various quantities. The top panels show convergence in the total cold phase mass\(^{10} \) and \( v_{\text{rel}} \). The only noteworthy feature is that rapid growth begins slightly sooner at higher resolutions. However, the surface area measurements are not converged at all; they increase more rapidly for higher-resolution runs. These results are consistent with the findings of Gronke & Oh (2020a) for a \( \chi = 10 \) simulation.

There are some differences in the \( v_{\text{turb}} \) evolution. While low-resolution runs have a strong, sharp peak followed by a flat region, higher-resolution runs have a moderate peak with a gradual descent. With that said, there seems to be convergence for \( R_{\text{cl}}/\Delta x \gtrsim 16 \), and all of the runs qualitatively agree with our picture that there are two stages of \( v_{\text{turb}} \) evolution. The average inflow velocity measurements are similar overall but do show some significant differences—it is somewhat unclear what the relevant trends are. We defer further investigation of inflow velocity convergence to future work.

4.3.2. Phase Structure

Resolution strongly affects the 2D thermodynamic \( e-p \) phase-space distribution. Previous work (e.g., Fielding et al. 2020; Abruzzo et al. 2022) established that gas in \( \chi \lesssim 100 \) simulations is roughly distributed along the isobar that is bounded by the properties of the cloud and wind. However, a pressure decrement emerges in the phase distribution at points along this isobar where cooling is not resolved.

Each point in the internal energy–pressure (\( e-p \)) phase space has an associated cooling length scale \( c_f \ell_{\text{cool}} \). Figure 14 shows that the size of the pressure decrement scales inversely with how well \( c_f \ell_{\text{cool}} \) is resolved. The figure also suggests that resolving the minimum cooling length scale (i.e., \( \Delta x \lesssim \ell_{\text{cool}} \sim \min(c_f \ell_{\text{cool}}) \)), which is equivalent to the “shattering” length scale (McCourt et al. 2018), is adequate to largely remove the pressure decrement for \( \chi \sim 100 \), which is consistent with results from prior works (e.g., Abruzzo et al. 2022).

Underresolved cooling is not the sole reason for the gas distribution’s deviations from the pressure isobar. Ji et al. (2019) previously argued that it is actually the sum of the turbulent pressure, \( \rho_{\text{turb}}^2 \), and thermal pressure that should match the external pressure. Figure 14 illustrates the median turbulent pressure as a function of \( e \) with dashed lines. In both \( \chi \) cases, the turbulent pressure shows clear convergence in our higher-resolution runs. The turbulent pressure’s lack of dependence on \( e \) below \( e_{\text{break}} \) and inverse correlation with \( e \) above \( e_{\text{break}} \) (at the pictured time) are consistent with the scaling described in Equation (3). The factor of \( \sim 3 \) difference in the maximum values (i.e., at \( e \sim e_{\text{break}} \)) of the turbulent pressures between the two \( \chi \) cases helps explain why the \( \chi = 1000 \) case has larger deviations in the thermal pressure from the external pressure. This difference is consistent with the scaling from Equation (4). For context, we expect \( v_{\text{turb}} \) at \( e_{\text{break}} \) to be a factor of \( \sim 4.85^{2/\beta} \) larger in this \( \chi = 1000 \) case, although the value of \( \beta \) is ambiguous; Figure 6 suggests that these particular \( \chi = 1000 \) and 100 runs should have \( \beta = 0.5 \) and 0.25 at 11.5\( t_{\text{cc}} \).

At the finite resolutions of our simulations, there is a decrement in the total pressure in our simulations. While the turbulent pressure shows convergence at our highest resolutions, the size of the thermal pressure decrement has not yet converged. Thus, at infinite resolution, it is plausible that the total pressure of the gas is constant. In short, the minimum \( c_f \ell_{\text{cool}} \) along the segment of the pressure isobar, connecting the cloud and the wind phase properties, specifies the grid-scale requirement for fully resolving phase structure. Remarkably, the degree to which we resolve \( \ell_{\text{cool}} \) appears to have minimal

\(^{10} \) As an aside, we do see some indications that resolution may strongly affect a cloud’s fate in other simulations close to the survival threshold. However, we defer further investigation to future work.
Figure 9. Like the top row of Figure 6 except that the pictured simulations primarily vary \( M_\infty \). The solid (dotted) lines show data from simulations with \( R_\text{d}/M_\infty = 37.6 \text{ pc} \) (376 pc) and \( \zeta_\text{bound} = 5.73 \) (57.3). We note that the \( c_{\text{bc}, \text{hot, cool, min}} \) is 6.56 pc for all simulations in this plot. As we will show in panels (e)–(h) of Figure 10, \( v_{\text{turb}} \) evolves more slowly in higher \( M_\infty \) runs. Consequently, the “late times” panel shows data from \( M_\infty = 0.75, 1.5 \) runs at \( 20\zeta_\text{bc} \) and data from \( M_\infty = 3, 6 \) runs at \( 30\zeta_\text{bc} \) (we did not run the \( M_\infty = 6 \) simulation to late enough times or with a long enough domain for an optimal late-time comparison).

4.3.3. Turbulent Structure and Cloud Morphology

Our results hint that underresolving turbulence might influence various properties of these interactions. To illustrate this, we turn to Figure 15, which shows measurements taken from different resolution runs of our \( \chi = 10^3 \), \( \zeta_\text{bc} = 27.8 \) simulation at 7.5\( \zeta_\text{bc} \). Each panel in the top row displays lines of data that come from each resolution. Subsequent rows just show measurements taken from a single resolution.

Figure 15(a) shows the first-order velocity structure function, \( \langle |\delta v| \rangle (\ell) \), measured for gas in the \( 1/12 \leq \log_2 e/\ell_\text{d} < 3/12 \) (8.1 \( \times 10^3 \) K \( \leq T \leq 2.0 \times 10^4 \) K) phase bin at various resolutions. Values are divided by the bin’s maximum sound speed, and the gray region denotes the width of the bin. \( \langle |\delta v| \rangle (\ell) \) specifies the average magnitude of the velocity differences\footnote{Unlike for our \( \langle (\delta v)^2 \rangle (\ell) \) measurements, these calculations use the 3D velocity vectors.} for pairs of points separated by a length scale \( \ell \).

As the separation \( \ell \) decreases, so does the velocity difference. On scales comparable to the cloud radius, \( \ell \sim R_\text{cl} \), the slope and normalization of \( \langle |\delta v| \rangle (\ell) \) are remarkably well converged.\footnote{The slope and normalization are less well converged at earlier times \( (\ell \lesssim 4.5\zeta_\text{bc}) \).} As the separation approaches the grid scale, the velocity differences are damped by numerical dissipation. Where this numerical dissipation kicks in relative to the sound speed appears to have a major impact on the morphology of the system. In reality, the true physical viscosity of these systems is uncertain but is likely to be much less than the effective numerical viscosity even in our highest-resolution simulation.

On large scales, the velocity differences are greater than the sound speed, but at small enough separations, the velocity differences become subsonic. We define the turbulent sonic length, \( \ell_{\text{turb, sonic}} \), as the scale at which \( \langle |\delta v| \rangle (\ell) \) passes through the point \( \langle |\delta v| \rangle (\ell_{\text{turb, sonic}}) = c_\text{s} \). Extrapolating the slope from large separations, we can estimate \( \ell_{\text{turb, sonic}} \) in the limit of infinite resolution (and very small viscosity), which, in this case, falls around 0.07\( R_\text{cl} \). This \( \ell_{\text{turb, sonic}} \) is not resolved by the simulations with \( R_\text{d}/\Delta x = 4 \) or 8, which is resolved by the \( R_\text{d}/\Delta x = 16 \) simulation, and is fairly well resolved by the \( R_\text{d}/\Delta x = 32 \) and 64 simulations. When \( \Delta x > \ell_{\text{turb, sonic}} \), the average velocity difference in a given phase bin can be supersonic at the viscous scale (i.e., between adjacent cells).

Panels (d), (f), (h), (j), and (l) show the distribution of velocity difference magnitudes in the previously mentioned phase bin, measured at \( \ell = \Delta x \); the average values of these distributions give the leftmost points of the curves in panel (a). These panels illustrate that as \( \ell_{\text{turb, sonic}}/\Delta x \) decreases, fewer pairs of cells have supersonic velocity differences. They also show that some grid-scale supersonic velocity differences persist when \( \ell_{\text{turb, sonic}} \) is barely resolved.

We now investigate the question: what are the consequences of underresolving \( \ell_{\text{turb, sonic}} \)? Panels (e), (g), (i), (k), and (m) of Figure 15 show the projected density of these simulations. The dramatic differences in these maps suggest that the degree to which \( \ell_{\text{turb, sonic}} \) is resolved may be linked to morphological differences between simulations. We find that the cold phase in higher-resolution simulations is composed of more large-scale structures and has a narrower transverse extent, whereas in lower-resolution simulations, the cold phase is clumpier and more dispersed. The cold phase in the low-resolution simulations has effectively shatted, while in the higher-resolution simulations that have \( \ell_{\text{turb, sonic}}/\Delta x > 1 \), the cold phase remains more intact (McCourt et al. 2018). These effects support a picture in which underresolving turbulence intensifies shattering by enabling the presence of supersonic velocity differences on the grid scale. This also naturally explains the wider dispersal of cold gas in low-resolution simulations, since the most intense shattering causes an explosive breakup of clouds (Gronke & Oh 2020b). Physically, supersonic grid-scale velocity differences will lead to large pressure imbalances that will in turn promote the dispersal as opposed to coagulation of cold cloudlets (Gronke & Oh 2022).

Using Equation (4), which captures how \( v_{\text{turb}} \) scales with \( v_{\text{rel}}/v_\infty \) and \( \ell_{\text{turb, min}} \), we can write a rough scaling relation for \( \ell_{\text{turb, sonic}} \). Assuming that \( \langle |\delta v| \rangle (\ell) \propto \ell^4 \), we find for cold gas
with \( c_{\text{cl}} \leq c \leq c_{\text{break}} \) that

\[
\frac{\ell_{\text{turb, sonic}}}{R_{\text{cl}}} \propto \left( \frac{\ell_{\text{cool, min}}}{\ell_{\text{shear}}} \right)^{3/\zeta} \mathcal{M}_w^{-1/\zeta}(\zeta). \tag{5}
\]

For sake of convenience, we take \( \zeta = 1/3 \) (the scaling for Kolmogorov turbulence), which is close to what is found in the simulations (see Figure 15(a)). At early times in the “pre-entrained” stage (e.g., when \( v_{\text{cl}}/v_w \sim 0.75 \)), \( \beta = 1/4 \), which yields a precise prediction for the turbulent sonic length:

\[
\left( \frac{\ell_{\text{turb, sonic}}}{R_{\text{cl}}} \right)_{\text{pre-entrained}} \approx 1.2 \left( \frac{c_{\text{sh, hot}} \ell_{\text{cool, min}}}{R_{\text{cl}}} \right)^{3/4}. \tag{6}
\]

The normalization is measured empirically in our \( \chi = 1000 \), \( \xi_{\text{sh}} = 27.8 \) simulation. Note that we focus on the \( \langle |\delta v| \rangle / \ell \rangle \) measurements from the same phase bin that includes \( c_{\text{break}} \), since, as we have shown above, this is the region of phase space where these scalings are robust, but the general trends will be the same for other bins below \( c_{\text{break}} \). Due to the fact that we have only measured the length where \( \langle |\delta v| \rangle / \ell \rangle \) is equal to the maximum sound speed of the phase bin, this relation should be considered an upper limit on \( \ell_{\text{turb, sonic}} \).

It is more intuitive to compare this relation against other known length scales, like the minimum radius for cloud survival, \( R_{\text{cl,crit}} \), or the minimum cooling length. For fixed cloud properties, we find that \( \ell_{\text{turb, sonic}} / R_{\text{cl,crit}} \propto \ell_{\text{cool, min}} P^{3/4} c_{\text{cl}}^{1.5} \chi^{-1.2} \mathcal{M}_w^{-1.7} \). This demonstrates that the turbulent sonic length tends to be more difficult to resolve in runs with larger \( \chi \) and higher \( \mathcal{M}_w \). If we assume that \( \ell_{\text{cool}} = \min(c_{\text{break}} \ell_{\text{cool}}, \text{cool, min}) \) and \( c_{\text{break}} \sim c_{\text{cool, min}} \), we find that \( \ell_{\text{turb, sonic}} / \ell_{\text{cool}} \sim c_{\text{cl}}^{1.3} \mathcal{M}_w^{0.4} \sqrt{\chi} \).

Given our table of simulations, it should be clear that \( \ell_{\text{turb, sonic}} \) exceeds \( \ell_{\text{cool}} \) in all of our entrained runs.

We find that this relation reproduces the value of \( \ell_{\text{turb, sonic}} \) measured from the \( R_{\text{cl}} / \Delta x \sim 32 \) run of our \( \chi = 1000 \), \( \xi_{\text{sh}} = 5.73 \) simulation to within \( \sim 50\% \). The lower-resolution runs of that simulation all resolve \( \ell_{\text{turb, sonic}} \), and we are encouraged that none of them show signs of shattering (the transverse extent is fairly consistent among runs). In the \( R_{\text{cl}} / \Delta x \sim 16 \) run of our \( \chi = 10^4 \), \( \xi_{\text{sh}} = 172.8 \) simulation, we find that \( \ell_{\text{turb, sonic}} \) is smaller than the grid scale when \( v_{\text{cl}}/v_w \sim 0.75 \), which is consistent with the relation’s prediction. We note that both resolution runs of this simulation clearly shatter. We performed a few spot-checks with a handful

\[
13 \text{ This assumes that } R_{\text{cl, crit}} \text{ has the scaling from the Li et al. (2020)/Sparre et al. (2020) survival criteria, since this does an accurate job of predicting cloud survival (see Section 5.5). Survival criteria have the generic form } \\
\tau_{\text{cool}} / \tau_{\text{shear}} = q \text{ and thus } R_{\text{cl, crit}} \propto \tau_{\text{cool}} / \tau_{\text{shear}} / q. \text{ In this case, } \tau_{\text{cool}} \sim \max(\ell_{\text{cool, min}}, \ell_{\text{cl, crit}}) \text{ and } q \propto R_{\text{cl}}^{0.5} \ell_{\text{cool, min}}^{0.5} c_{\text{cl}}^{0.5} / \tau_{\text{shear}}^{0.25}. \text{ For } T_{\text{w}} \geq 10^6 \text{ K, } \ell_{\text{cool, min}} \text{ roughly scales as } e^{-p/2}, \text{ and the mean molecular weight, } \mu, \text{ is constant. Putting this together yields } \\
R_{\text{cl, crit}} \propto \chi_2^2 c_{\text{cl}}^{2.7} \mathcal{M}_w^{0.9} \mu^{-1.3} \ell_{\text{cool}} \text{ when } T_{\text{w}} \geq 10^6 \text{ K.}
\]
5. Discussion

5.1. Phase Dependence of Turbulence

We have demonstrated for the first time that the turbulent velocity, $v_{\text{turb}}$, in a mixing layer follows a broken power-law dependence on temperature or internal energy. A major implication of this finding is that the turbulent kinetic energy density is not constant across gas phase. Consider the ratio of the turbulent kinetic energy densities in the hot and cold phases, or $\epsilon = \rho_w v_{\text{turb}}(T_w)^2/(\rho_{\text{mix}} v_{\text{turb}}(T_d)^2)$. Per Equation (3), this evaluates to $\epsilon = (v_{\text{break}}/v_w)^{1-2\epsilon}$. We remind the reader that $\alpha$, the power-law slope above $v_{\text{break}}$, starts out near 1/2 at the earliest times and decreases to ~0 at a rate that depends on the principal dimensionless numbers. Thus, during the bulk of the cloud–wind interaction, the cold phase has a larger turbulent kinetic energy density (i.e., $\epsilon < 1$). This contradicts (explicit and implicit) assumptions that $\epsilon = 1$ in multiple works on TRMLs.

For example, we consider the arguments that lead to the expression for the temperature of the mixing layer, $T_{\text{mix}} \sim \sqrt{T_d T_w}$ (Begelman & Fabian 1990; Gronke & Oh 2018). This relation derives from the average of the cold and hot phase temperatures, weighted by the mass flux from each phase into the mixing layer. The derivation assumes that each phase’s mass flux scales with the respective $v_{\text{turb}}$ values. Because the derivation involves arguments equivalent to assuming $\epsilon = 1$, it overestimates the hot phase’s $v_{\text{turb}}$ and consequently the mass flux when compared against the values for the cold phase. Thus, $\sqrt{T_d T_w}$ overestimates $T_{\text{mix}}$ and the size of the discrepancy is inversely correlated with $\epsilon$. Because the value of $T_{\text{cool}}$ is commonly monotonic between $T_{\text{cool, min}}$ and $T_{\text{cool, max}}$, the overestimate of $T_{\text{cool, mix}}$ by an amount also negatively correlated with $\epsilon$.

In another case, Fielding et al. (2020) explicitly assumes that $\epsilon = 1$. The only practical implication is that their quoted measurement of $f_{\text{turb}} = v_{\text{turb}}/v_{\text{rel}}$ is too large by a factor of $\sqrt{\epsilon}$. Thus, $f_{\text{turb}}$ might have a weak dependence on the shape of the cooling curve. In their analysis of clouds in a turbulent medium, Gronke et al. (2022) also assumes $\epsilon = 1$, but this may be valid since they consider externally driven turbulence.

5.2. Observable Predictions

It may be possible to observe $v_{\text{turb}}$’s broken power-law phase dependence in real-world systems. For example, previous studies have already placed constraints on temperature and nonthermal motion in the CGM of other galaxies by measuring the widths of absorption lines for elements with different atomic masses (e.g., Rudie et al. 2019; Qu et al. 2022). Similar measurements may also be possible for high-velocity clouds, for which there is an abundance of absorption (e.g., Fox et al. 2004) and emission line data (e.g., Tufte et al. 1998; Hill et al. 2009). One could also imagine using 21 cm emission or Mg II absorption to extend such an analysis to probe the turbulent properties down to lower temperatures, where gas is atomic (e.g., Marchal et al. 2021).

It may also be possible to perform a similar exercise for gas in multiphase galactic outflows (e.g., Strickland & Heckman 2009; Reichardt Chu et al. 2022). Additionally, one can perform more straightforward comparisons against observational measurements of turbulent measurements in $\sim 10^8$ K gas. However, given the simplifying assumptions in this work (described further in Section 5.8) and the fact that the drivers of turbulence may vary between different systems, such comparisons must be interpreted with great caution. Nevertheless, we find it encouraging that there is evidence that the Perseus molecular cloud has transonic...
turbulent Mach number (Burkhart et al. 2015), just like we see in a fair number of our simulations. We also find it encouraging that studies of CGM clouds (e.g. Rudie et al. 2019; Qu et al. 2022) recover nonthermal broadening measurements within a factor of a few tens of km s\(^{-1}\), which nicely matches the turbulent velocities in our simulations. We leave further comparisons to future work.

5.3. What Drives Mixing?

We now return to one of the motivating questions, the origin of turbulence in the flow. From the results in this paper, the short answer appears to be that both shear and cooling drive the turbulence responsible for mixing. As we conclude in Section 4.1.1, shear is the primary driver of turbulence at early times. After the cloud becomes partially entrained, \(v_{\text{turb}}\) falls off before stabilizing at a lower value. The long-term support of a nonzero \(v_{\text{turb}}\) value, as \(v_{\text{rel}}\) goes to 0, suggests that some form of “cooling-induced mixing” mechanism takes over. To put this another way, the primary source of turbulent kinetic energy changes with time. At early times, turbulent kinetic energy primarily comes from the large relative shear velocities between fluid elements. At late times, it instead comes from the radial kinetic energy of inflowing material.

Possible origins for the late-time turbulence include rapid cooling-driven pulsations in the cloud (Gronke & Oh 2020a)\(^{14}\) or simply the net radial inflow driven by the initial shear-driven turbulence. This latter explanation is supported by the correlation of \(v_{\text{turb}}\)’s late-time magnitude with \(v_{\text{inflow}}\), which itself correlates with a run’s cooling efficiency. We plan to provide a detailed analysis of the temporal evolution of \(v_{\text{turb}}\) and its dependence on \(v_{\text{rel}}\) in a follow-up work.

A few other features are consistent with this conclusion. First, we find the rapid growth of surface area when shear primarily drives mixing and subsequent stabilization at a roughly constant value when mixing is primarily driven by pulsations or radial inflow to be consistent. Second, the minimal variance in the driving scale as the cloud is elongated is also consistent. At early times, the driving scale is linked with the length of the wind-aligned axis of the cloud, of order \(R_d\). Because the cloud’s transverse extent does not change much with time, the typical radial separation between opposite inflow “fronts” of the clouds should still be of order \(R_d\) at late times. Finally, the saturation of the inflow velocity after cooling-driven mixing has fully developed fits into this picture since the shear-driven contribution will have become subdominant.

Gronke & Oh (2020a) noted that the anticorrelation between the cold cloud mass growth rate and \(v_{\text{rel}}\) might suggest that shear-driven turbulence from the Kelvin–Helmholtz instability might not fuel mass growth and instead might be a competing destructive process. However, our most efficiently cooling \(M_w = 1.5\) runs with \(\chi = 100, 300,\) and 1000 have significant \(v_{\text{rel}}\) when they start monotonically growing. In other words, mass growth at early times in these runs should primarily arise from shear-driven turbulence. With that said, mass growth is

\(^{14}\)We did not save snapshots frequently enough to test our simulations for their presence.
5.4. What Is the Mixing Timescale?

The canonical estimates for the characteristic mixing timescale are \( t_{\text{cc}} \) and \( t_{\text{shear}} \). We find that the turbulent velocity scales with \( R_{\text{cl}} \sqrt{\langle v_{\text{rel}}^2 \rangle} \), where \( \beta \) is 0.25 at early times and 0.5 at late times. Notably, it has no dependence on \( M_w \) for most of the cloud’s evolution. Therefore, the characteristic mixing time has no \( v_{\text{rel}} \) dependence.

With that said, the initial value of \( M_w \) does affect the temporal evolution of \( v_{\text{turb}} \). Figure 10 also provides some indications that the magnitude of \( v_{\text{turb}} \) may have some dependence on \( M_w \) at very early times. Comparing panels (g) to (h) (as well as (f) to (g)) reveals that the peak value of \( v_{\text{turb}} \) when \( v_{\text{rel}} > 0.8 \), is larger in the higher \( M_w \) run by more than the factor of \( \sqrt{2} \) expected by Equation (4) from differences in \( R_{\text{cl}} \).

5.5. Survival Criterion

There has been great interest in the literature about the minimum radius for cloud survival (e.g., Gronke & Oh 2018; Li et al. 2020; Sparre et al. 2020; Kanjilal et al. 2021; Abruzzo et al. 2022; Farber & Gronke 2022). We provide more firm conclusions about this topic in Abruzzo et al. (2023). However, we do note that the our results are most consistent with the
predictions of Li et al. (2020) with the corrections described by Sparre et al. (2020) for supersonic winds.

5.6. Convergence

What does it mean to resolve the cloud–wind interaction? The obvious ideal is to achieve pointwise convergence, but this is generally prohibitively computationally expensive except in rare cases (e.g., Lecoanet et al. 2016). Short of this ultimate goal, there are lesser gradations of convergence that depend on the question at hand. The easiest quantity to achieve convergence in is the net mass growth of the cold phase. We show in Figure 15(c) that the mass growth is fairly well converged for resolutions of $R_{c1}/\Delta x \gtrsim 8$. This likely corresponds to some minimum threshold to resolve any turbulent mixing and is consistent with previous findings (e.g., Gronke & Oh 2020a). The hardest quantity to achieve convergence in is the 2D $p-e$ phase distribution, which requires resolving the minimum cooling length ($\ell_{\text{cool}}$; also known as the shattering length). Therefore, if one is interested in simply capturing the
total amount of mass in the cold phase, then the resolution requirements are much less onerous than if one is interested in capturing the detailed phase structure (or cloud morphology). The details of the phase structure can be extremely important for comparisons to observations since the pressure decrement that develops in underresolved simulations occurs in precisely the region traced by commonly observed ions, such as Mg II (e.g., Burchett et al. 2021; Nelson et al. 2021).

Here we propose an intermediate convergence criterion for the large-scale morphology of cold structures that requires resolving the turbulent sonic length $\ell_{\text{turb, sonic}}$ by several cells. This is in general less stringent than the requirement to resolve the minimum cooling length. At face value, the difficulty of resolving $\ell_{\text{turb, sonic}}$ in galaxy-scale simulations suggests that the detailed morphological properties of cool ($\sim 10^4$ K) gas involved in TRML entrainment within galactic outflows and the CGM are unlikely to be correct. However, the implications of accurately capturing the morphology may be more complex in more realistic systems because of the way cloud shape and size couple to other physical processes absent in our simulations. For example, in systems in which the hot phase is itself turbulent, such as in galactic wind simulations (e.g., Schneider et al. 2020), underresolved $\ell_{\text{turb, sonic}}$ may lead to artificially shattered clouds, which will in turn be more likely to be destroyed than if they were able to remain coherent. Therefore, having $\Delta x < \ell_{\text{turb, sonic}}$ may prove to be essential for determining the overall phase structure and evolution of turbulent multiphase flows that are ubiquitous in and around galaxies.

This discussion about large-scale morphological convergence of cool gas in larger-scale models deserves elaboration on two finer points. First, it assumes applicability of our results about the emergent turbulent properties in the cloud–wind interactions; we discuss how the equilibrium $T_{\text{cl}}$ and shape of $t_{\text{cool}}(T)$ affect this in the next subsection (Section 5.7). Second, we are extrapolating from simulations of isolated clouds, whereas larger-scale models often include multiple clouds in an outflow (e.g., Cooper et al. 2008; Kim & Ostriker 2018; Schneider et al. 2020). This is not an issue when the intercloud spacing is large enough for clouds to be treated individually, albeit with a hot phase that is already turbulent from upstream interactions. However, more work is required to make predictions when the intercloud separation is small (such work might use a multicloud setup akin to Altazas et al. 2012; Banda-Barragán et al. 2020).

5.7. Comparison to Prior Work

At early times, when the Kelvin–Helmholtz instability is the primary driver of mixing, one might expect similarities between our runs and the TRML simulations of Fielding et al. (2020) and Tan et al. (2021). Unfortunately, it is difficult to draw direct comparisons since those works highlight properties after reaching a quasi-steady state. In contrast, our runs never reach such a state since $v_{\text{rel}}$ evolves with time. More meaningful comparisons could be made if the cloud was in a potential that was tuned to maintain $v_{\text{rel}}$ at late times. Additionally, Tan et al. (2021) point out that we would likely expect different $v_{\text{turb}}$ scaling to be dependent on geometry. Nevertheless, we find the presence of inflowing gas at early times to be encouraging (especially when juxtaposed with our adiabatic runs that do not have net inflow). The fact that $v_{\text{turb}}$ and the inflow velocity show signs of scaling with cooling efficiency is also encouraging.

Likewise, we expect similarities with Gronke & Oh (2020a) at late times when turbulence is driven by cooling-induced mixing. Although we broadly see similar qualitative evolution in the surface area, detailed comparisons of other properties are challenging. While both works measured $v_{\text{inflow}}$, we expect differences in our methodologies will complicate comparisons of these quantities at late times. Gronke & Oh (2020a) used $v_{\text{inflow}} \sim \dot{m}_{\text{cold}}/(\rho u)$, while we directly measure the velocity component normal to the $e_{\text{max}}$ isosurface (the scaling does not change much if we use the $e_{\text{break}}$ isosurface). In other words, their measurements are weighted by mass flux, and ours are weighted by surface area. We expect that this difference in methodology explains why our results indicate that inflow starts much earlier in our runs; early-time inflow that does not correspond to mass growth will not be picked up by their measurements. Because our work focused on measuring $v_{\text{turb}}$ rather than $v_{\text{inflow}}$, we defer detailed scaling of $v_{\text{inflow}}$ to follow-up work.

Gronke & Oh (2020a) found the cold phase mass evolution’s convergence in an $M_w = 6$ simulation run at $R_{\text{cl}}/\Delta x = 8$ and 32 to be quite poor. In contrast, we found the cold phase mass evolution in our $R_{\text{cl}}/\Delta x = 8$ and 16 runs of our $M_w = 6$ simulation to be fairly well converged. While it is possible that we could see differences at higher resolution, it is plausible that this difference could arise from differences in the cloud temperature. The clouds in Gronke & Oh (2020a) had a temperature of $T_{\text{cl}} = 4 \times 10^4$ K. This translates to values of $t_{\text{cool, min}}$ and $e_{\text{cl}}$ that are factors of 5$15$ and $1.1$ larger. Consequently, we expect $\ell_{\text{turb, sonic}}/R_{\text{cl,crit}}$ to be 7.3 times smaller in their simulations, which means they could be underresolving $R_{\text{cl}}$ according to our new resolution criterion.

More generally, one might ask “How does the choice of $T_{\text{cl}}$ affect our results?” given that the equilibrium $T_{\text{cl}}$ varies$15$ greatly among cloud-crushing and galactic outflow studies. For context, this work focuses on runs with $T_{\text{cl}} < 5 \times 10^4$ K, while other works commonly include simulations with $T_{\text{cl}} < 10^5$ K (e.g., Li et al. 2020; Schneider et al. 2020; Kanjilal et al. 2021; Abruzzo et al. 2022) or $T_{\text{cl}} < 4 \times 10^4$ K (e.g., Gronke & Oh 2018, 2020a; Abruzzo et al. 2022). We expect that the applicability of our results is more strongly tied to the shape of $t_{\text{cool}}(T)$ over $T_{\text{cl}} \lesssim T < T_{\text{break}}$ than the precise value of $T_{\text{cl}}$. Figure 7 suggests that our results are minimally affected when $t_{\text{cool}}(T_{\text{cl}})$ exceeds the minimum value of $t_{\text{cool}}$ computed over the temperature range. However, the applicability is less clear when $t_{\text{cool}}(T)$ is minimized at $T_{\text{cl}}$ (i.e., if $T_{\text{cl}} \gtrsim 2 \times 10^4$ K for $p/k_B = 10^5$ K cm$^{-3}$, $Z_{\odot}, z = 0$) or at a value of $T$ exceeding $\sqrt{T_{\text{cl}} T_w}$. Finally, we note that some works also consider conditions with $T_{\text{cl}} < 500$ K (e.g., Banda-Barragán et al. 2021; Farber & Gronke 2022). Further investigation is required to understand the applicability of our results in this context, but our above discussion about $t_{\text{cool}}(T_{\text{cl}})$’s shape is relevant.

We next draw comparisons with works that studied multiphase gas in turbulent box simulations. For example, Gronke et al. (2022) initialized a pressure-confined cloud ($T_{\text{cl}} = 4 \times 10^4$ K) in a hot ambient background and studied how the system evolved while driving turbulence in the hot phase. Mohapatra et al. (2022) studied the turbulent properties of multiphase gas (comparable to ICM conditions)

---

$^{15}$ The value is commonly controlled by setting a temperature floor or turning off cooling below a certain temperature.
that emerged from driven turbulence and radiative cooling in a box of initially hot ($T = 4 \times 10^6$ K) gas. These studies respectively observed that the amplitude of the first- and second-order velocity structure functions ($\langle \delta v \rangle (\ell)$ and ($\langle (\delta v)^2 \rangle (\ell)$) have lower amplitudes in the cold phase gas than in the other phases, which is in good qualitative agreement with our results. We note that the sub-Kolmogorov scaling of our $\langle (\delta v)^2 \rangle (\ell)$ measurements is more consistent with the hydrodynamic volume-weighted heating run from Mohapatra et al. (2022) than the mass-weighted run. However, as mentioned in Section 3.3, the driving scale is not sufficiently resolved to remove the bottleneck effect’s influence on the slope of $\langle (\delta v)^2 \rangle (\ell)$. To be more concrete, we note that Mohapatra et al. (2022) illustrated that the driving scale must be resolved by more than 192 cells in a nonradiative turbulence simulation to remove the bottleneck effect’s influence on the slope. For that reason, we refrain from making detailed comparisons.

5.8. Caveats

This work made a number of simplifying assumptions and omitted a variety of potentially relevant physical effects that could potentially modify our results. Future work should consider the following.

Other sources of turbulence. We only analyzed the turbulence that emerged from two phases that initially had coherent velocities without turbulence. In reality, external processes, like supernovae, can drive turbulence in the wind; this likely alters the interaction’s evolution and makes survival more difficult (e.g., Schneider et al. 2020). Additionally, differences in the initial cloud structure due to turbulent driving before encountering a wind can affect the rates at which mixing destroys clouds (e.g., Schneider & Robertson 2017; Banda-Barragán et al. 2019).

Thermal conduction. The omission of thermal conduction from our simulations will certainly affect the morphology of the cold phase (e.g., Brüggen & Scannapieco 2016; Li et al. 2020). However, we take solace in the fact that mass transfer through the TRML will be minimally affected in simulations where cooling is fast relative to the mixing time (Tan et al. 2021).

Magnetic fields. It is well known that magnetic fields can extend the lifetime of clouds (e.g., Dursi & Pfrommer 2008; McCourt et al. 2015). Banda-Barragán et al. (2018) showed that magnetic fields have a stabilizing effect on initially turbulent clouds embedded in a laminar wind. While realistic magnetic field strengths do not seem to strongly affect the criteria for survival through rapid cooling, they do have a number of other effects that will almost certainly affect the system’s turbulent properties (Gronke & Oh 2020a). Among others, such effects include nonthermal support, which could alter cooling properties; suppression of the Kelvin–Helmholtz instability; and alteration of cloud morphology, leading to higher surface areas (Gronke & Oh 2020a).

Cosmic rays. Cosmic rays were also omitted from our simulations. They are a known source of nonthermal pressure support, which may alter cooling properties (Butsky et al. 2020). They can also provide another mechanism for accelerating clouds (Wiener et al. 2019; Huang et al. 2022).

Gravity. Our simulations neglected the effects of gravity because we generally expect our $\chi \lesssim 10^3$ simulations to be Jeans stable. However, one could imagine that external gravitational fields could sustain an elevated shear velocity (Tan et al. 2023) and consequently influence the system’s turbulent properties.

More realistic cooling. All of our simulations assume simplified equilibrium cooling and neglect self-shielding. However, given that all our simulations where the cloud survives have $N_{H_1} > 10^{17.2} \text{cm}^{-2}$, self-shielding may be relevant. Including more realistic cooling could modify our results (Farber & Gronke 2022), but we leave that for future work.

Viscosity. Our simulations do not have explicit viscosity (Li et al. 2020; Jennings & Li 2021). This may affect turbulent properties near the scale of turbulent dissipation.

6. Conclusion

We have investigated the multiphase turbulent properties that emerge from interactions between cool clouds and hot supersonic flows (or winds). The relative efficiency of turbulent mixing and radiative cooling in mixing layers governs the outcome of such interactions. To address difficulties associated with characterizing multiphase turbulence, our analysis employed three distinct methods to measure $v_{\text{turb}}$. We found the following primary results for simulations in which cooling is sufficient for the cloud to survive the interaction and become entrained.

1. Radiative cooling dramatically changes the $v_{\text{turb}}$ temperature scaling. In nonradiative simulations, $v_{\text{turb}}$ has a scaling consistent with the sound speed’s temperature scaling: $v_{\text{turb}} \propto c_s \propto \sqrt{T}$. In runs with sufficient cooling for entrainment; this scaling only applies for gas colder than $T_{\text{break}}$, the temperature where $T_{\text{break}}$ is minimized. Above $T_{\text{break}}$, the power-law slope starts near 0.5 and flattens to $\sim 0$. Consequently, cold gas generally has a larger turbulent Mach number and turbulent kinetic energy than hot gas.

2. $v_{\text{turb}}$ has two stages of temporal evolution. The shear velocity initially drives rapid growth of $v_{\text{turb}}$ at early times in the “pre-entrained” phase. As the cloud becomes partially entrained, $v_{\text{turb}}$ drops off before stabilizing at a lower value, one that is of comparable magnitude to the average inflow velocity.

3. When comparing different simulations at given points in its evolution, $v_{\text{turb}}(T_{\text{break}})/c_{s,\text{break}}$ scales with $(\chi_{\text{cl}} M_{\text{cl}})^3$ or $(\chi_{\text{cl}} c_{s,\text{break}})/(\rho_{\text{cool, min}})^3$. At early times, $\beta \approx 1/4$, while at late times, $\beta \approx 1/2$.

4. The driving scale is of order the cloud radius throughout the cloud’s entire evolution.

5. The grid scale should exceed the minimum cooling length, $l_{\text{cool}} \sim \min(c_{s,\text{cool}}) \rho$, to resolve 2D phase structure. The 1D temperature phase structure is remarkably well converged at lower resolutions.

6. Our simulations suggest the existence of a minimum length scale for resolving turbulence, $l_{\text{turb,sonic}}$, for clouds with an equilibrium temperature of $5 \times 10^5 \lesssim (T_{\text{cl}}/K) \lesssim 2 \times 10^5$. Underresolving this scale seems to artificially amplify the violence of shattering. When this scale is resolved, the entrained cool phase is composed of larger clouds.

\footnote{For the reader’s convenience, we describe phase dependence in terms of temperature even though the majority of this work primarily considers specific internal energy.}
Acknowledgments

We thank M. Gronke for useful discussions and for sharing some sample code to compute the velocity structure function. We are grateful to James Bordner, Mike Norman, and the other ENZO-E developers. G.L.B. acknowledges support from the NSF (AST-2108470, AST-2307419, XSEDE), a NASA TCAN award, and the Simons Foundation. D.B.F. is supported by the Simons Foundation through the Flatiron Institute.

Software: numpy (Harris et al. 2020), matplotlib (Hunter 2007), yt (Turk et al. 2011), scipy (Virtanen et al. 2020), pandas (McKinney 2010), scikitimage (van der Walt et al. 2014), fitMPI (http://fftmpi.sandia.gov), Launcher Utility (Wilson & Fonner 2014), GRACKLE (Smith et al. 2017), ENZO-E (https://github.com/enzoproject/enzo-e).

Appendix

Robustness of Metrics at Early Times

Our approaches for characterizing $v_{turb}$ all build on the idea that a velocity field can be decomposed into a laminar part and a turbulent part. Consider an ideal turbulent flow in which the laminar part of the velocity field is uniform. In this scenario, the magnitude of the laminar part sets the average of the velocity field, and the turbulent part sets the dispersion in the velocity values. For this reason, our methods for measuring a spatially averaged $v_{turb}$ (in a given gas phase) all measure this dispersion in one way or another.

Unfortunately, the flows considered in this work are more complex: the laminar portion of the flow has spatial gradients. Figure 16(a) illustrates these gradients for several velocity components measured on the $log_e e_{1/3} = 1/6$ isosurface of our $\chi = 1000$, $\xi_{sh} = 27.8$ simulation at $0.5t_{cc}$. In more detail, the panel shows the conditional distributions$^{17}$ of multiple velocity components as a function of $cos(\theta_{\text{spherical}})$, where $\theta_{\text{spherical}}$ is the polar angle measured from the center of the inflow boundary.

Unless they are removed, such gradients can dominate or inflate the dispersion of the global velocity distribution, which can bias our measurements of $v_{turb}$. Figure 16(b) suggests that this is less of an issue after early times (once $v_{turb}$ has had time to grow) because the dispersion from turbulence is larger relative to the laminar variations. However, it is clear that these gradients still remain problematic in the wind-aligned velocity component. Figure 11(b) shows that large variations in the wind-aligned velocity persist to later times, even as the cloud is accelerated.

We expect our $v_{turb}$ measurements from our geometric approach to be unaffected by this issue because it estimates $v_{turb}$ from the dispersion in $v_{\phi}$-like, which maintains a mean of 0 at all times. However, the laminar variations will bias the measurements using our other approaches at early times. While one might expect our filtering measurements to be resilient to this effect, because it uses a local estimate of the laminar flow, at least some bias will remain given that these early-time gradients are most naturally described in spherical components. Throughout this work, we elect to just focus on turbulence in velocity components orthogonal to the wind direction in our filtering and $(\langle \delta v^2 \rangle (\ell))$ measurements in order to avoid biases from the wind-aligned velocity component.

17 These distributions were approximated with kernel density estimation.

Figure 16. The probability density functions of several velocity components (in the cloud’s rest frame), as measured on the $log_e e_{1/3} = 1/6$ isosurface for our $\chi = 1000$, $\xi_{sh} = 27.8$, $R_v/\Delta x = 64$ simulation at multiple times. The dotted lines show the mean values of $v_{r}$-like and $v_{\phi}$-like as functions of $cos(\theta_{\text{spherical}})$. The vertical extent of a contour arises from turbulence (and is somewhat inflated by asymmetries in the flow). At early times, estimating $v_{turb}$ from the variance in any velocity component other than $v_{\phi}$-like without explicitly accounting for these laminar variations will yield overestimates.

As an aside, the resilience of our geometric approach to these biases is related to the definition of the velocity components. Consider $\hat{u}_{r}$-like, which we define the unit vector parallel to the specific internal energy gradient (i.e., $\hat{u}_{r}$-like $= \nabla e/|\nabla e|$). Because this vector is always normal to the specific internal energy isosurfaces, we can define $v_{r}$-like and $v_{\phi}$-like at arbitrary locations using $v_{r}$-like $= -v \cdot \hat{u}_{r}$-like and $v_{\phi}$-like $= v \cdot (\hat{\omega}_{\text{wind}} \times \hat{u}_{r}$-like). Future work may wish to perform filtering or compute the structure function in terms of these components.

ORCID iDs

Matthew W. Abruzzo @ https://orcid.org/0000-0002-7918-3086
Drummond B. Fielding @ https://orcid.org/0000-0003-3806-8548
Greg L. Bryan @ https://orcid.org/0000-0003-2630-9228

References

Abruzzo, M. W., Bryan, G. L., & Fielding, D. B. 2022, ApJ, 925, 199
Abruzzo, M. W., Fielding, D. B., & Bryan, G. L. 2023, arXiv:2307.03228
Altizas, R., Pittard, J. M., Hartquist, T. W., Falle, S. A. E. G., & Langton, R. 2012, MNRAS, 425, 2212
Armillotta, L., Fraternali, F., & Marinacci, F. 2016, MNRAS, 462, 4157

21
