Multi-Party Dynamic State Estimation that Preserves Data and Model Privacy

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Abstract—In this paper we focus on the dynamic state estimation which harnesses a vast amount of sensing data harvested by multiple parties and recognize that in many applications, to improve collaborations between parties, the estimation procedure must be designed with the awareness of protecting participants’ data and model privacy, where the latter refers to the privacy of key parameters of observation models. We develop a state estimation paradigm for the scenario where multiple parties with data and model privacy concerns are involved. Multiple parties monitor a physical dynamic process by deploying their own sensor networks and update the state estimate according to the average state estimate of all the parties calculated by a cloud server and security module. The paradigm taps additively homomorphic encryption which enables the cloud server and security module to jointly fuse parties’ data while preserving the data privacy. Meanwhile, all the parties collaboratively develop a stable (or optimal) fusion rule without divulging sensitive model information. For the proposed filtering paradigm, we analyze the stabilization and the optimality. First, to stabilize the multi-party state estimator while preserving observation model privacy, two stabilization design methods are proposed. For special scenarios, the parties directly design their estimator gains by the matrix norm relaxation. For general scenarios, after transforming the original design problem into a convex semi-definite programming problem, the parties collaboratively derive suitable estimator gains based on the alternating direction method of multipliers (ADMM). Second, an optimal collaborative gain design method with model privacy guarantees is provided, which results in the asymptotic minimum mean square error (MMSE) state estimation. Finally, numerical examples are presented to illustrate our design and theoretical findings.

Index Terms—Privacy, dynamic state estimation, multiple parties, additively homomorphic encryption.

I. INTRODUCTION

A. Background

Cyber-Physical systems (CPSs) have nowadays been seen in numerous applications, including smart power grid, autonomous vehicles, intelligent transportation, healthcare, and environment monitoring, etc. [1]. Privacy issues arise when information that is generated by some party needs to be shared with and stored by other parties. For the purpose to have better understanding of dynamic system states and further manipulate its input to make the system output behave in some desired manner, sensors are deployed at the sites of interest to sequentially collect measurements and send them to remote devices. In some cases, measurements are privacy-sensitive because they directly contain sensitive information. For example, measurements could be medical records of personal physiological data, such as the skin temperature and blood pressure of patients in a smart hospital. In other cases, measurements are private in that one can infer sensitive information from them. A particular example is that the habits, financial status, and more of a household could be learned from the daily power data recorded by a meter [2]. If some cybercriminals obtain and then exploit these data in an unlawful manner, it would probably cause serious consequences [3].

The privacy of data has been extensively investigated in various research fields, such as data science [4], [5], machine learning [6]–[8], control and systems [9]–[11], etc., where the encryption acts as a mainstreaming technique to maintain the confidentiality of data. Other than the matters of direct data leakage, in the process of utilizing data to extract meaningful insights or make decisions, there also exists the risk of hidden information disclosure, yet having not drawn proper attention it deserves. For example, Tramèr et al. [12] demonstrated the possibility of simple and successful model extraction attacks against popular machine learning models. The hidden privacy disclosure is a common challenge when we control physical systems, perform parallel computation, or apply artificial intelligence algorithms on shared computing platforms, and cannot be fully overcome only by the traditional encryption schemes. For instance, the average consensus is a fundamental algorithm for distributed computing and control, where nodes in a network constantly communicate to learn the average of their initial values in an iterative manner. There could be an undesirable disclosure of a node’s initial state to other nodes. Mo and Murray [13] discussed the initial-value disclosure under the maximum likelihood estimation theory where random noises are used to blur the consensus process. Liu et al. [14], [15] proposed privacy-preserving gossip algorithms with consistent summation of network node values. They showed that even if eavesdroppers possess the full network structure and flow knowledge, they are unable to reconstruct the network initial node values.

B. Our Work and Contributions

We focus on dynamic state estimation which harnesses a vast amount of sensing data harvested by various parties and takes place on a third-party computational platform, termed the secure multi-party dynamic state estimation problem. The computing framework is economically motivated by the scale advantage, theoretically rooted in the distributed dynamic state...
estimation and data fusion, and technically rooted from cloud computing and Internet-of-Things, with the purpose to protect the privacy of sensitive information. Specifically, multiple parties monitor a physical process by deploying their own sensor networks, respectively. These parties may be companies in competition with each other, or institutions whose data are protected by law. The key point is that they may achieve better estimation performance if they can obtain additional information from others. On one hand, they understand that sharing information is good for all, but on the other hand, they are unwilling, or unable to expose their sensitive information to other parties or a third party without trustworthiness.

The following example of dynamic state estimation in power grid unveils that the concept of privacy preserving should not only be confined to data, but also be extended to key model parameters. In power systems, the technological advances enable sensors to promptly track the dynamics of the system states. The state-transition model is assumed to be linear [16]:

\[ x(k+1) = Ax(k) + w(k), \quad (1) \]

where \( x(k) \) is the vector of the bus voltage magnitude and angle at time step \( k \), \( w(k) \) captures the state noise, and \( A \) characterizes the discrete-time dynamic system model. As a consequence of the power industry restructuring, some of the grid have been administered by independent system operators [17]. Operators in different areas observe the system of the grid have been administered by independent system operators [17]. Operators in different areas observe the system state by deploying their own sensing devices at buses, e.g., phasor measurement units and meters. The observation model for operator \( i \) is assumed to be linear:

\[ y_i(k) = C_i x(k) + v_i(k), \quad (2) \]

where \( C_i \) is the linearized observation matrix and \( v_i(k) \) is the measurement noise. These operators may obtain a more accurate real-time estimate of the overall grid if they could share data. However, operators in different areas are reluctant to share network data and measurements due to competition. The internal measurements and internal line parameters of each area are hidden from all other areas [18]. They worry about that the conventional collaborative dynamic state estimation may lead to the leakage of their trade secrets and potential competitors would learn their technical competence.

The state estimation problem where a group of sensors monitor a dynamic process has been extensively investigated in centralized [19]–[21] and distributed [22]–[26] settings. When there are privacy concerns in the context of state estimation, Gonzalez-Serrano et al. [27] developed an extended Kalman filter based on encrypted measurements, which was combined with additively homomorphic encryption. They considered a scenario with a data owner who provides privacy-protected measurements, and an algorithm owner who processes the encrypted data to estimate the process state. Based on a similar setup, Zamani et al. [28] proposed a secure Luenberger observer combined with the Paillier encryption such that the estimation could be performed on the encrypted data directly. Song et al. [29] utilized compressive privacy schemes to prevent the fusion center from inferring private states accurately while still allowing it to estimate public states with good accuracy. In summary, the existing literature mainly focused on the data privacy. However, our vision in multi-party state estimation, for example, the aforementioned dynamic state estimation in power grid, is that the sensitive information to be protected consists of not only local data to be shared but also the parameters of observation models, which has been seldom discussed in the literature, to the best of our knowledge.

In our paper, we develop a paradigm of privacy-aware multi-party dynamic state estimation. The notion of privacy contains data and model privacy. In particular, each party’s sensitive information, including their locally generated estimates and key parameters of the observation models (i.e., \( C_i \) and \( R_i \)), is kept secret. To the best of our knowledge, it is the first time that model privacy is taken into account in the context of multi-party dynamic state estimation. In the paradigm we demonstrate how a fusion protocol that taps additively homomorphic encryption enables the parties to jointly estimate the dynamic process while preserving their own data privacy. In general, the data fusion is accomplished by a cloud server and security module. The data producers (i.e., the multiple parties) upload encrypted messages to the cloud server, and the cloud server performs arithmetical operations in the ciphertext space and pass the calculated result to a security module. Finally, the security module is responsible for decrypting and sending back the result to all the parties. We also demonstrate how the parties together develop a stable (or asymptotic MMSE) fusion rule without revealing observation model parameters to others. This procedure is essentially a stabilization problem for a linear filtering system which needs to be solved in a collaborative manner with privacy awareness.

The contribution of this paper is multi-fold.

1) We highlight the importance of model privacy, i.e., protecting key parameters of the observation models in some multi-party state estimation applications, as the observation models may also be privacy-sensitive. The concept of model privacy, which has begun to be recognized in machine learning communities, has been introduced to dynamic state estimation for the first time in this paper. The notation of privacy considered in this work is two-leveled, and it indicates the need of careful state estimation algorithm design for preserving both levels.

2) We investigate the estimator stabilization of the proposed paradigm. The fundamental stabilization problem of a linear filtering system is resolved in a collaborative manner with model-privacy awareness. Specifically, methods based on the matrix norm relaxation and ADMM are designed for multiple parties to collaboratively decide their estimator gains without disclosing model parameters.

3) We also propose an optimal gain design method with model-privacy awareness to achieve the asymptotic MMSE estimation. Essentially, it aims at obtaining the optimal steady-state Kalman gain without disclosing model parameters. To achieve this, we formulate a semi-definite programming problem and solve it by ADMM.

The remainder of this paper is organized as follows. In the rest part of this section, we present some preliminaries regarding encryption schemes in the cryptography field. Section II provides the system model, proposes the secure multi-
party filtering protocol, and lists the problems of interests. Section III and Section IV focus on the main results including the convergence and the MMSE optimality analysis. Section V provides simulations and interpretations. Section VI draws conclusions.

Notations: $\mathbb{R}^n$ is the $n$-dimensional Euclidean space. $\mathbb{R}^{n \times m}$ denotes the set of $n \times m$ real matrices. $\mathbb{S}^+_n$ (or $\mathbb{S}^{++}_n$) is the set of $n \times n$ positive semi-definite (definite) matrices. When $X \in \mathbb{S}^+_n$ (or $\mathbb{S}^{++}_n$), we simply write $X \succeq 0$ ($X \succ 0$). The identity matrix with size $n$ is represented by $I_n$. The superscript $\top$, $\text{Tr}\{\cdot\}$, $\|\cdot\|_p$, and $\|\cdot\|_2$ stand for the transpose, trace, Frobenius norm, and spectral norm of a matrix, respectively. $\mathbb{E}[\cdot]$ denotes the mathematical expectation of a random variable. To save the writing space, some entries of the symmetric matrix are denoted by $\ast$ which can be recovered by the matrix symmetry.

C. Preliminaries: Homomorphic Encryption

In this subsection, we introduce the homomorphic encryption, which is the key supporting technique of this paper to protect the sensitive information of multiple parties.

The encryption mechanism is crucial to preserve the message privacy. Typically, in a secure message transmission, the encryption key is public which is usually broadcast to every party, and it is different from the decryption key which is only kept by the security module, secretly. Each party can use the public key, i.e., $f_e(\cdot)$, to encrypt a message, but only the security module with knowledge of the secret key, i.e., $f_d(\cdot)$, can decode the message:

$$f_d(f_e(\pi_1)) = \pi_1 \quad (3)$$

for some message $\pi_1$. It is not necessary for a conventional encryption mechanism to allow operations on encrypted messages without decryption. As a consequence, the parties’ privacy will be sacrificed when they need to collaborate with other parties or utilize cloud services to process sensitive messages. To tackle such challenges, Rivest et al. [30] used the term “homomorphism” for the first time to describe special encryption functions, which permit encrypted messages to be operated on without preliminary decryption. Inspired by [30], a burgeoning literature [31–39] is devoted to investigating the homomorphic scheme in the cryptographic field.

The homomorphic encryption is an encryption scheme which allows a third party (e.g., a cloud server) to apply certain operations on encrypted messages without decryption first. The homomorphic property means that given encryptions $f_e(\pi_1)$, $f_e(\pi_2)$ of messages $\pi_1$, $\pi_2$, and an operator “$\cdot$”, the result of the operation “$\cdot$” based on encrypted messages, when decrypted, matches the result of the operation “$\circ$” directly performed on the original plaintext, i.e.,

$$f_d(f_e(\pi_1) \cdot f_e(\pi_2)) = \pi_1 \circ \pi_2. \quad (4)$$

Based on the number of allowed operations on the encrypted messages, the homomorphic encryption can be categorized into two main types: partially homomorphic encryption (PHE) and fully homomorphic encryption (FHE). Several widely used PHE schemes are RSA [31], GM [32], ElGamal [33], Benaloh [34], Paillier [35], etc. For example, RSA is multiplicatively homomorphic, i.e., $f_d^{RSA}(f_e^{RSA}(\pi_1) \times f_e^{RSA}(\pi_2)) = \pi_1 \times \pi_2$, and Paillier is additively homomorphic, i.e., $f_d^{Paillier}(f_e^{Paillier}(\pi_1) + f_e^{Paillier}(\pi_2)) = \pi_1 + \pi_2$. FHE was first proposed by Gentry [36] which allows arbitrary operations on encrypted messages based on ideal lattices. Follow-up works [37–39] proposed new schemes to tackle its bottlenecks such as massive computational cost and complicated concepts.

In this paper, encryption schemes which are additively homomorphic are adopted to calculate the summation of messages from multiple parties without the leakage of parties’ sensitive information. This privacy-preserving property is denoted by

$$f_d(f_e(\pi_1) + f_e(\pi_2)) = \pi_1 + \pi_2, \quad (5)$$

where “$+$” is the corresponding operator on the ciphertext for certain additively homomorphic encryption. Throughout this paper, we neglect the slight performance degradation induced by the quantization required before the encryption. Wa aim to get some inspiring insights theoretically.

II. Secure Multi-Party State Estimation

Consider a discrete-time linear time-invariant (LTI) process in (1), where $x(k) \in \mathbb{R}^n$ is the state of the physical process and $w(k) \in \mathbb{R}^m$ is i.i.d. zero-mean Gaussian process noise with covariance $Q \succ 0$. There are $N$ parties deploying their own sensor networks to monitor the process. Each party $i$, $i \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$, has its own observation equation (2),

$$\begin{align*}
C_i \in \mathbb{R}^{m_i \times n} \quad \text{is the observation matrix of party} \ i,
\end{align*}$$

where the concrete form is decided by the configuration of the sensors that belong to party $i$. $y_i(k) \in \mathbb{R}^{m_i}$ is the associated measurements, and $v_i(k) \in \mathbb{R}^{m_i}$ denotes i.i.d. zero-mean Gaussian measurement noise with covariance $R_i \succ 0$, which is uncorrelated with $w(k)$ and $v_j(k)$ if $j \neq i$. The initial
Algorithm 1 Secure Multi-Party Filtering Protocol

1: Public input: \(N\);
2: Private input: \(y_i(k), C_i\), and \(K_i\) for each party \(i\);
3: Initialization: \(\hat{x}_i(0)\) for all \(i \in \mathcal{N}\);
4: Security module: broadcast \(f_e(\cdot)\) to all the \(N\) parties;
5: for \(k = 1, 2, \ldots\) do
6:   for each party \(i \in \mathcal{N}\) do in parallel
7:       Update local state estimate \(\hat{x}_i^-(k)\):
8:       \[\hat{x}_i^-(k) = A\hat{x}_i(k-1) + K_i(y_i(k) - C_iA\hat{x}_i(k-1))\]
9:   end for
10: Cloud server: perform “\(\oplus\)” on received messages;
11: Cloud server: broadcast \(f_e(\hat{x}_i^-(k))\) to cloud server;
12: Security module: encrypt the received message by \(f_e(\cdot)\) and divide the encrypted result by \(N\);
13: Security module: broadcast \(\bar{x}(k)\) to all the \(N\) parties where
14: \[\bar{x}(k) = \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i^-(k)\]
15: for each party \(i \in \mathcal{N}\) do in parallel
16:   Synchronize state estimate \(\hat{x}_i(k)\) with \(\bar{x}(k)\):
17: \[\hat{x}_i(k) = \bar{x}(k)\]
18: end for

state \(x(0)\) is zero-mean Gaussian with covariance \(\Pi_0 \succeq 0\), and is uncorrelated with \(w(k)\) and \(v_i(k)\) for all \(k \geq 0\) and \(i \in \mathcal{N}\).

A. Secure Dynamic State Estimation Paradigm

We propose a secure multi-party state estimation paradigm. As depicted in Fig. 1 the whole filtering procedure involves a physical process, multiple parties together with their own sensors, a cloud server, and a security module. At the beginning, the public key \(f_e(\cdot)\) is broadcast from the security module to all the parties. First, at every time epoch \(k\), after updating the local state estimate \(\hat{x}_i^-(k)\) based on the measurement \(y_i(k)\) collected from party \(i\)’s own sensor network as Eq. (6) shows, party \(i\) encrypts its local state estimate \(\hat{x}_i^-(k)\) by \(f_e(\cdot)\) and uploads the encrypted message \(f_e(\hat{x}_i^-(k))\) to the cloud server. Second, according to the chosen encryption scheme, the cloud server performs the corresponding operator “\(\oplus\)” on all the encrypted messages received and transmits the generated result \(f_e(\hat{x}_i^-(k)) \oplus \cdots \oplus f_e(\hat{x}_N^-(k))\) to the security module. Since the cloud server does not hold the secret key, no more information about the parties’ original messages can be inferred from the encrypted messages, which demonstrates the privacy-preserving property of the homomorphic encryption. Then the security module decrypts the received message using the secret key \(f_d(\cdot)\), divides the decrypted result by the parties’ total number \(N\), and broadcasts it to all the parties, which is exactly the average of all the local state estimates, i.e., \(\bar{x}(k)\), as shown by Eq. (7). Finally, each party \(i\) synchronizes its state estimate \(\hat{x}_i(k)\) with the average state estimate \(\bar{x}(k)\) as Eq. (8) shows.

The secure multi-party state estimation paradigm is summarized in Algorithm 1. The matrix \(K_i \in \mathbb{R}^{n \times m_i}\) is the linear estimator gain of party \(i\) to be decided. The initial state estimate \(\hat{x}_i(0)\) is i.i.d. zero-mean Gaussian with covariance \(\Pi_0 \succeq 0\). During the whole filtering procedure, only the average state estimate is accessible to each party \(i\), and in general, no more knowledge about the other parties’ sensitive information including their observation parameters (i.e., \(C_i\) and \(R_i\)) and their local state estimates \(\hat{x}_j(k)\) where \(j \in \mathcal{N}\) and \(j \neq i\) can be learned, which is one of the main differences from the existing literature on the state estimation with multiple parties [19]–[26]. For example, when the information filter is adopted for multi-party state estimation, the unexpected disclosure of encrypted messages may reveal the estimation quality of the party, i.e., a larger \(C_iR_i^{-1}C_i\) implies a better estimation performance. As a contrast, for our proposed protocol, even when the cloud server and security module conspire to get access to the data of some parties, the model privacy is still preserved.

B. Problems of Interests

In the multi-party state estimation paradigm, the local state estimates from all the parties need to be averaged and the additively homomorphic encryption is introduced to preserve each party’s data privacy. To run the filtering protocol, each party’s local estimator gain \(K_i\) needs to be carefully designed.

Based on standard results in control theory, it is necessary to have the knowledge of \(C_i\)’s, and \(R_i\)’s in some particular settings, to design \(K_i\)’s. If we design \(K_i\)’s in a centralized manner, each party should share its own \(C_i\) (and \(R_i\)) and inevitably, it leads to model privacy loss. Therefore, the first and most crucial problem that we need to address is how to design \(K_i\)’s to stabilize the linear filtering system without disclosing each party’s model privacy. Moreover, along this line of research, we are interested in finding a design method to achieve the MMSE multi-party dynamic state estimation. We hope that our findings on performance analysis can serve as motivations for multiple parties to work together to bring the proposed paradigm into life.

The performance of the filtering protocol depends on the state estimation error and its corresponding error covariance. The state estimation error covariance for party \(i\) is defined as

\[P_i(k) \triangleq \mathbb{E}[(x(k) - \hat{x}_i(k)) (x(k) - \hat{x}_i(k))^\top].\] (9)

Intuitively, a smaller error covariance implies a higher state estimation quality on average. After Eq. (6), all of the parties’ state estimates are synchronized with \(\bar{x}(k)\) and become the same, and the average state estimate \(\bar{x}(k)\) is updated as:

\[\bar{x}(k) = \frac{1}{N} \sum_{i=1}^{N} \left[(A - K_iC_iA)\bar{x}(k-1) + K_iC_iAx(k-1) + K_iC_iw(k-1) + K_iv_i(k)\right].\] (10)
The state estimation error $\tilde{e}(k) \triangleq x(k) - \hat{x}(k)$ is denoted by

$$
\tilde{e}(k) = \frac{1}{N} \sum_{i=1}^{N} \left[ (A - K_i C_i) \tilde{e}(k - 1) + (I_n - K_i C_i) w(k - 1) - K_i v_i (k) \right],
$$

where $\mathbb{E}[\tilde{e}(k)] = 0$. Its corresponding error covariance $\overline{P}(k)$, which is defined as $\overline{P}(k) = \mathbb{E}[\tilde{e}(k)\tilde{e}(k)^\top]$, and also equals to $P^i(k)$ for all $i \in \mathcal{M}$, has the following dynamic:

$$
\overline{P}(k) = \frac{1}{N^2} \sum_{i=1}^{N} (I_n - K_i C_i) (A \overline{P}(k - 1) A^\top + Q) \sum_{i=1}^{N} (I_n - K_i C_i)^\top + \frac{1}{N^2} \left( \sum_{i=1}^{N} K_i R_i K_i^\top \right) \sum_{i=1}^{N} M_i \left( A \overline{P}(k - 1) A^\top + Q \right) \sum_{i=1}^{N} M_i^\top + \frac{1}{N^2} \left( \sum_{i=1}^{N} S_i \right)^\top.
$$

(12)

For notational simplicity, we denote $M_i \triangleq I_n - K_i C_i$ and $S_i \triangleq K_i R_i K_i^\top \geq 0$ for all $i \in \mathcal{M}$.

In the following two sections, the error covariance $\overline{P}(k)$ is adopted as a metric to evaluate the stability and optimality of our proposed secure multi-party state estimator.

### III. STABILIZATION OF MULTI-PARTY DYNAMIC STATE ESTIMATOR WITH PRIVACY AWARENESS

In this section, we analyze the stability of the error covariance $\overline{P}(k)$. First, an assumption is provided to guarantee the existence of parties’ estimator gains which stabilize the estimator. Second, two methods are provided on how to choose these linear estimator gains collaboratively without leakage of model privacy.

According to Eq. (12), the convergence of the state estimation error covariance $\overline{P}(k)$ is determined by the spectral radius of $\frac{1}{N} \sum_{i=1}^{N} M_i A$. Based on standard results on the discrete Lyapunov equation, it can be concluded that if and only if

$$
\rho \left( \frac{1}{N} \sum_{i=1}^{N} M_i A \right) < 1,
$$

(13)

$\overline{P}(k)$ converges to a unique positive semi-definite value. To ensure the convergence of $\overline{P}(k)$, each party should carefully design its local estimator’s linear gain $K_i$ such that the matrix $\frac{1}{N} \sum_{i=1}^{N} M_i A$ is stable. The following assumption provides a sufficient and necessary condition for the existence of such gain $K_i$’s. For notational convenience, we define

$$
C \triangleq \begin{bmatrix} C_1^\top & C_2^\top & \cdots & C_N^\top \end{bmatrix}^\top.
$$

(14)

**Assumption 1**: The pair $(A, C)$ is detectable.

**Remark 1**: The pair $(A, C)$ being detectable is equivalent to $(A, CA)$ being detectable. To see this, first, since $(A, CA)$ is detectable, one can always find a suitable $K$ such that $A - KCA$ is stable. Note that $(I_n - KCA)A$ and $A(I_n - KC)$ have the same eigenvalues, and therefore $A - AKC$ is also stable. Obviously, the pair $(A, C)$ is detectable. Conversely, from [40], if $(A, C)$ is detectable, then for any $\bar{\mathbf{R}} > 0$ and $\bar{\mathbf{Q}} \geq 0$ satisfying that $(A, \sqrt{\bar{\mathbf{Q}}})$ is stabilizable, the discrete-time algebraic Riccati equation (DARE) $X = AXA^\top + \bar{\mathbf{Q}} - AXC^\top (CXC^\top + \bar{\mathbf{R}})^{-1} CXA^\top$ has a unique solution $\hat{X} \succeq 0$. Let $K \triangleq \hat{X}C^\top (C\hat{X}C^\top + \bar{\mathbf{R}})^{-1}$, and one can conclude that $A - KCA$ is stable according to the standard Kalman filter results. By this construction method, we find a suitable $K$ to make $A - KCA$ stable and therefore, $(A, CA)$ is detectable.

Assumption [1] is arguably the weakest assumption in the context of dynamic state estimation. Otherwise, all the parties cannot even achieve a stable estimator under a centralized setup. Remark 1 suggests that Assumption [1] is equivalent to the detectability of $(A, CA)$. When $(A, C)$ is detectable, we can always find a matrix $K$, where $K = \frac{1}{N} [K_1 \ K_2 \ \cdots \ K_N]$ with each $K_i$ being of proper dimension in accordance with $C_i$, that makes $A - KCA$ stable. These $K_i$’s guarantee the feasibility of (13).

In the following, we will develop two collaboratively design methods to stabilize the multi-party state estimator without disclosing sensitive model information (i.e., the observation matrix $C_i$) for the sake of protecting model privacy.

#### A. Stabilization Design Method I: Matrix Norm Relaxation

The first design method (Algorithm 2) utilizes the matrix norm relaxation and additively homomorphic encryption. For any induced matrix norm $\| \cdot \|$, there holds:

$$
\rho \left( \frac{1}{N} \sum_{i=1}^{N} M_i A \right) \leq \left\| \frac{1}{N} \sum_{i=1}^{N} M_i A \right\| \leq \frac{1}{N} \sum_{i=1}^{N} \| M_i A \|. \quad (17)
$$

**Algorithm 2 Stabilization Design Method I**

1. Public input: $N$;
2. Private input: $C_i$ for each party $i$;
3. Security module: broadcast $f_\epsilon(\cdot)$ to all the $N$ parties;
4. For each party $i \in \mathcal{M}$ do in parallel
5. Calculate estimator gain $K_i$:
   
   $$
   K_i = \arg \min_{X \in \mathbb{R}^{n	imes n}} \| (I_n - X C_i) A \|
   $$

(15)

6. Encrypt $\| (I_n - K_i C_i)^{-1} \|$ by $f_\epsilon(\cdot)$;
7. Upload $f_\epsilon(\| (I_n - K_i C_i)^{-1} \|)$ to cloud server;
8. end for
9. Cloud server: perform “$\oplus$” on received messages;
10. Cloud server: transmit $f_\epsilon(\| (I_n - K_i C_i)^{-1} \|)$ to security module;
11. Security module: decrypt the received message by $f_d(\cdot)$ and divide the decrypted result by $N$;
12. Security module: tell whether

$$
\frac{1}{N} \sum_{i=1}^{N} \| (I_n - K_i C_i) A \| < 1
$$

(16)

and broadcast the result to all the $N$ parties.
Note that the local estimator gain $K_i$ could be determined such that $\|M_i A\| < 1$ for each party $i$, then (13) holds trivially and $\mathcal{P}(k)$ converges. However, the condition $\|M_i A\| < 1$, $\forall i \in \mathcal{N}$ restricts the choice of the linear gain $K_i$, and sometimes such a $K_i$ may even not exist. We propose a simple design method in Algorithm 2. This algorithm is primitive and optional. Similarly to Algorithm 1 in Section II-A, after each party decides its estimator gain $K_i$, the average $\frac{1}{N} \sum_{i=1}^{N} \|M_i A\|$ can be calculated with the help of the cloud server and the security module, which does not leak any private $\|M_i A\|$. If the average is less than 1, then the candidate estimator gains are acceptable and a bounded estimation error covariance is guaranteed. This method is applicable when 

$$\min_{\{K_i\}} \frac{1}{N} \sum_{i=1}^{N} \| (I_n - K_i C_i) A \| < 1.$$ 

### B. Stabilization Design Method II: ADMM

The second design method (Algorithm 3), which solves a formulated convex semi-definite programming problem by ADMM, is applicable to general cases under Assumption 1. The design purpose is to find the estimator gain $K_i$’s such that $A - KCA$ is stable, where 

$$K \triangleq \frac{1}{N} [K_1 \quad K_2 \quad \ldots \quad K_N]$$

and $C$ is as defined in Eq. (12). For notational convenience, $m \triangleq \sum_{i=1}^{N} m_i$. Basically, how to find a $K$ to stabilize $A - KCA$ has already been well investigated in the control field, for example, the pole placement problem [41]. However, parties’ concerns about their sensitive information, e.g., $C_i$, prohibit a centralized design. To disclose as little sensitive information as possible, we formulate an equivalent convex problem and solve it by ADMM.

First, we transform the collaborative gain design problem into an optimization problem. The stabilization of $A - KCA$ is equivalent to the following condition according to the standard results on the Lyapunov equation

$$\exists \Delta \in \mathbb{S}^{n_+}, \text{ and } K \in \mathbb{R}^{n \times m},$$

s.t. $\Delta - (A - KCA) \Delta (A - KCA) ^\top > 0$, 

which is further equivalent to

$$\exists \Delta \in \mathbb{S}^{n_+}, \text{ and } K \in \mathbb{R}^{n \times m},$$

s.t. $\begin{bmatrix} \Delta^{-1} & A - KCA \\ (A - KCA)^\top & -I \end{bmatrix} > 0$, 

due to the Schur complement condition for positive definiteness. By multiplying $\begin{bmatrix} \Delta^{-1} & I_n \end{bmatrix}$ on both sides of (20), we have the following equivalent condition:

$$\exists \Delta \in \mathbb{S}^{n_+}, \text{ and } K \in \mathbb{R}^{n \times m},$$

s.t. $\begin{bmatrix} \Delta^{-1} & A - KCA \\ (A - KCA)^\top & -I \end{bmatrix} > 0$.

For notational brevity, we denote $H \triangleq \Delta^{-1}$ and $Z_i \triangleq \Delta^{-1} K_i$. By algebraic manipulation and change of variables, the problem to find suitable $H$ and $Z_i$’s is equivalent to an optimization problem as follows.

**Problem 1:**

$$\min_{\{Z_i\}, \{U_i\}, U_0, H} \mathcal{I}_\Omega (\{U_i\}, U_0, H),$$

s.t. $Z_i C_i A = U_i$, $\forall i \in \mathcal{N}$.

The ADMM algorithm is remarkably successful in solving convex programs which minimize a sum of $N$ convex objectives whose variables are linked by some constraints. Although the convergence of ADMM for $N = 2$ is a standard result, the result for scenario where $N \geq 3$ is not that obvious. In particular, it was shown by Chen et al. [42] that ADMM for $N \geq 3$ fails to converge in general. To convert the multi-block problem into an equivalent two-block problem, we introduce an indicator function

$$\mathcal{I}_\Omega (\{U_i\}, U_0, H) = \left\{ \begin{array}{ll} 0, & \text{if } (\{U_i\}, U_0, H) \in \Omega, \\ +\infty, & \text{otherwise} \end{array} \right.$$ 

where the convex set $\Omega$ is given by

$$\Omega \triangleq \left\{ (\{U_i\}, U_0, H) : U_0 \succ 0, \quad H \succ 0, \quad N \left[ \begin{array}{cc} H & HA \\ (HA)^\top & H \end{array} \right] - \sum_{i=1}^{N} \left[ \begin{array}{cc} 0 & U_i \\ U_i^\top & 0 \end{array} \right] = U_0 \right\}$$

for $U_i \in \mathbb{R}^{n \times n}$, $i \in \mathcal{N}$. By variable splitting, an equivalent two-block problem is formulated as follows.

**Problem 2:**

$$\min_{\{Z_i\}, \{U_i\}, U_0, H} \mathcal{I}_\Omega (\{U_i\}, U_0, H),$$

s.t. $Z_i C_i A = U_i$, $\forall i \in \mathcal{N}$.

The variables in Problem 2 can be grouped into two blocks: $(\{U_i\}, U_0, H)$ and $(\{Z_i\})$, so that ADMM can directly apply. The augmented Lagrangian where $\Lambda_i \in \mathbb{R}^{n \times n}$ is the corresponding Lagrange multiplier for Problem 2 is given by:

$$\mathcal{L}_\gamma (\{Z_i\}, \{U_i\}, U_0, H, \{\Lambda_i\}) = \mathcal{I}_\Omega (\{U_i\}, U_0, H)$$

$$+ \sum_{i=1}^{N} \mathbb{R} \{ \Lambda_i^\top (Z_i C_i A - U_i) \} + \frac{\gamma}{2} \sum_{i=1}^{N} \|Z_i C_i A - U_i\|_F^2,$$

which incorporates a quadratic penalty of the constraint scaled by a parameter $\gamma > 0$ into the Lagrangian. Since $Z_i$’s are fully decoupled, the resulting subproblem is decomposed into $N$ separated subproblems, which can be solved in parallel. The details are summarized in Algorithm 3.

Recall that we aim to collaboratively design the linear estimator gains without exposing sensitive model information such that the state estimation error covariance $\mathcal{P}(k)$ converges. By adopting Algorithm 3 all the parties compute their gains together with the cloud server. First, the cloud server updates the variables $(\{U_i\}, U_0^\ast, H^\ast)$ and feedbacks $(U_i^\ast, \Lambda_i^\ast)$ to each party $i$, respectively. And then each party updates its own $Z_i^\ast$ in parallel. Note that at each iteration $k$, party $i$ can upload the value of $Z_i^\ast C_i A$ to the cloud server. There is no need for party $i$ to expose its own observation matrix $C_i$ to others. In
other words, the sensitive information of all parties is protected by the designed method. After the convergence of Algorithm 3 each party obtains its estimator gain by solving the convex subproblem:

\[
\left\{ \left(U_i^I, U_0^I, H^I \right) \right\} = \arg \min_{\left\{ \left(U_i^I, U_0^I, H^I \right) \right\}} - \sum_{i=1}^{N} \text{Tr} \left\{ \left( \Lambda_i^{t-1} \right)^\top \left( Z_i^{t-1} C_i A - U_i \right) \right\} + \frac{\gamma}{2} \sum_{i=1}^{N} \left\| Z_i^{t-1} C_i A - U_i \right\|_F^2 , \text{s.t.} \quad \left\{ \left(U_i^I, U_0^I, H^I \right) \right\} \in \Omega ;
\]

where

\[
\left[ \begin{array}{c}
Z_i^t \arg \min_{Z_i^t} - \text{Tr} \left\{ \left( \Lambda_i^{t-1} \right)^\top \left( Z_i C_i A - U_i^I \right) \right\} + \frac{\gamma}{2} \left\| Z_i C_i A - U_i^I \right\|_F^2 ;
\end{array} \right.
\]

privacy. Recall that by adopting the proposed secure multi-party dynamic state estimation paradigm, the average state estimate \( \bar{x}(k) \) (see Eq. (10)) evolves as:

\[
\bar{x}(k) = A\bar{x}(k-1) + K \left[ y(k) - CA\bar{x}(k-1) \right] ,
\]

where \( y(k) \triangleq \left[ y_1^T(k) \ y_2^T(k) \ \cdots \ y_N^T(k) \right]^\top \), C and K are as defined in Eq. (14) and (18), respectively. If the pair \((A, C)\) is detectable and \( Q > 0 \), then the optimal steady-state Kalman gain \( K^* \) asymptotically achieves MMSE estimation and is given by \( K^* \triangleq P_{\text{pri}} C^\top (C P_{\text{pri}} C^\top + R)^{-1} \), where \( P_{\text{pri}} \geq 0 \) is the solution to the DARE \( \dot{\gamma}(X) = X \) for \( \dot{\gamma}(X) \triangleq AXA^\top + Q - AXC^\top (C X C^\top + R)^{-1} CXA^\top \). Here, \( R \triangleq \text{diag} \{ R_1, R_2, \ldots, R_N \} \). In this section, we target at obtaining the optimal steady-state Kalman gain \( K^* \) while protecting the observation parameters \( C_i \) and \( R_i \) of party \( i \). To establish this, we need the following Lemma 1 as preliminaries. The remaining results in this section are developed under Assumption 2.

**Assumption 2**: The pair \((A, C)\) is detectable and \( A \) is nonsingular.

**Remark 3**: Note that the process dynamic matrix \( A \) is always nonsingular if the LTI process (11) is a discretization from a continuous-time state space model, i.e., \( \dot{x}_c(t) = A_c x_c(t) + w_c(t) \). One can check that the dynamic matrix of the corresponding discretized model is \( A = e^{A T_s} \) where \( T_s \) is the sampling time (11), and \( A \) must be nonsingular.

For notational convenience, the operator \( \phi(\cdot, \cdot) : \mathbb{R}^{n \times m} \times \mathbb{S}_+^n \rightarrow \mathbb{S}_+^m \) is defined as:

\[
\phi(Y, X) = (A - A Y C) X (A - A Y C)^\top + Q + A Y R + A^\top Y A^\top .
\]

**Lemma 1**: Under Assumption 2 the following statements are equivalent.

IV. **ASYMPTOTIC MMSE SECURE MULTI-PARTY STATE ESTIMATION**

When statistics of the noise process are known a priori, the MMSE estimator, which minimizes a quadratic error cost function in a Bayesian setting, is recognized as a popular choice. In this section, we will discuss MMSE secure multi-party dynamic state estimation design that preserves model

**Algorithm 3 Stabilization Design Method II**

1: **Public input**: \( N, \gamma \);  
2: **Private input**: \( C_i \) for each party \( i \);  
3: **Initialization**: \( Z_i^0 \) and \( \Lambda_i^0 \) for all \( i \in \mathcal{N} \);  
4: for iteration \( i = 1, 2, \ldots \) do  
5: Cloud server: update \( \left\{ \left(U_i^I, U_0^I, H^I \right) \right\} \) by solving the convex subproblem:

\[
\left\{ \left(U_i^I, U_0^I, H^I \right) \right\} = \arg \min_{\left\{ \left(U_i^I, U_0^I, H^I \right) \right\}} - \sum_{i=1}^{N} \text{Tr} \left\{ \left( \Lambda_i^{t-1} \right)^\top \left( Z_i^{t-1} C_i A - U_i \right) \right\} + \frac{\gamma}{2} \sum_{i=1}^{N} \left\| Z_i^{t-1} C_i A - U_i \right\|_F^2 , \text{s.t.} \quad \left\{ \left(U_i^I, U_0^I, H^I \right) \right\} \in \Omega ;
\]

6: Cloud server: feedback \( \left(U_i^I, \Lambda_i^{t-1} \right) \) to each party \( i \);  
7: for each party \( i \in \mathcal{N} \) do in parallel  
8: Update \( Z_i \) by solving the convex subproblem:

\[
Z_i^t \arg \min_{Z_i^t} - \text{Tr} \left\{ \left( \Lambda_i^{t-1} \right)^\top \left( Z_i C_i A - U_i^I \right) \right\} + \frac{\gamma}{2} \left\| Z_i C_i A - U_i^I \right\|_F^2 ;
\]

9: Upload \( Z_i^t C_i A \) to cloud server;  
10: end for  
11: Cloud server: update \( \Lambda_i^t = \Lambda_i^{t-1} - \gamma \left( Z_i C_i A - U_i^I \right) \) for all \( i \in \mathcal{N} \);  
12: end for  
13: Cloud server: broadcast \( H^I \) to all the \( N \) parties;  
14: **Output**: Linear estimator gain \( K_i^t = (H^I)^{-1} Z_i^t \) for all \( i \in \mathcal{N} \).
Algorithm 4 Asymptotic MMSE Gain Design Method with Privacy Guarantees

1: Public input: $N$, $\gamma$, $m_i$ for all $i \in \mathcal{N}$;
2: Private input: $C_i$ and $R_i$ for each party $i$;
3: Initialization: $Z_i$ and $\bar{A}_i$ for all $i \in \mathcal{N}$;
4: for iteration $i = 1, 2, \ldots$ do
5:    Cloud server: update $(\{U_i\}, \{V_i\}, W, H)$ by solving the convex subproblem:
6:    
7:    
8:    
9:    
10: end for
11: Cloud server: feedback $(U_i, V_i, \bar{A}_i)$ to each party $i$;
12: for each party $i \in \mathcal{N}$ do in parallel
13:    Update $Z_i$ by solving the convex subproblem:
14: end for
15: Cloud server: broadcast $H_i$ to all the $N$ parties;
16: Output: Linear estimator gain $K_i = A^{-1}(H_i)^{-1}Z_i$ for all $i \in \mathcal{N}$.

1) $\exists \bar{K} \in \mathbb{R}^{n \times m}$, $\bar{X} \in S^m_+$ such that $\bar{X} \succeq \phi(\bar{K}, \bar{X})$;
2) $\exists \bar{Y} \in \mathbb{R}^{n \times m}$, $\bar{A} \in S^m_+$ such that

Proof: Similar to the proof of Theorem 5 in [43].

Problem 3:

$$
\min_{\{Z_i\}, H} - \text{Tr}(H),
$$

s.t. $H > 0,$

\begin{equation}
\sum_{i=1}^{N} \begin{bmatrix} A^T H_i - C_i^T Z_i \bar{H} & * & * \\
\bar{H} & 0 & 0 \\
I_m & 0 & 0 \\
0 & I_n & 0 \end{bmatrix} \succeq 0.
\end{equation}

Theorem 1: The optimal solution to Problem 3, i.e., $\{Z_i\}$ and $H^*$, gives the optimal steady-state Kalman gain $K^* = \frac{1}{\bar{N}} A^{-1}(H^*)^{-1} \left[ Z_1^* \quad Z_2^* \quad \cdots \quad Z_N^* \right]$ and the fixed point of the DARE, i.e., $P_{pri} = (H^*)^{-1}$.

Proof: We set the estimator gain $K_i = A^{-1}H_i^{-1}Z_i$, $K = \frac{1}{\bar{N}} \left[ K_1 \quad K_2 \quad \cdots \quad K_N \right]$, and let $P = H^{-1}$. The constraints in Problem 3 are equivalent to $P \succ 0$ and $P \succeq \phi(K, P)$ according to Lemma 7. Clearly, the optimal steady-state Kalman gain $K^*$ and the positive definite solution $P_{pri}$ to the DARE satisfy $P_{pri} = \phi(K^*, P_{pri})$, and thus belong to the feasible set of the optimization problem.

We now prove Theorem 1 by contradiction in two steps. First, we prove that the optimal solution reaches the equality $P = \phi(K, P)$. Suppose that $\bar{K}$ and $\bar{P}$ solve Problem 3 but $\bar{P} \neq \phi(\bar{K}, \bar{P})$. Then there exists $\bar{P} \succeq \phi(\bar{K}, \bar{P}) \succeq \bar{g}(\bar{P}) > 0$ but $\text{Tr}\{P\} > \text{Tr}\{\phi(\bar{K}, \bar{P})\}$. The inequality $\phi(\bar{K}, \bar{P}) \succeq \bar{g}(\bar{P})$.
is due to $\tilde{g}(X) = \min_Y \phi(Y, X) \preceq \phi(Y, X), \forall Y \in \mathbb{R}^{n \times m}$ (see Lemma 1 in [43]). We denote $\tilde{P} \equiv \tilde{g}(\tilde{P})$. By setting $K = \tilde{P}C(I + R\tilde{P})^{-1}$, we have $\tilde{P} \succ 0$ and $\tilde{P} \succeq \phi(\hat{K}, \hat{P})$ which satisfy the constraints. However, the inequalities $\tilde{P} \succeq \hat{P} \succ 0$ and $\text{Tr}(\tilde{P}) > \text{Tr}(\hat{P})$ imply $-\text{Tr}(\tilde{P}^{-1}) > -\text{Tr}(\hat{P}^{-1})$, which contradicts the hypothesis of optimality of $\tilde{P}$. Therefore, the optimal solution to Problem 3 must satisfy $\hat{P} = \phi(\hat{K}, \hat{P})$. Second, we show that the optimal solution $K$ and $P$ must also satisfy $K = PC(I + R\tilde{P})^{-1}$, i.e., $P = \phi(K, P) = \tilde{g}(P)$. Suppose that $\hat{K}$ and $\hat{P}$ solve Problem 3 but $\tilde{g}(P)$. Then there exists $\tilde{P} = \phi(\hat{K}, \hat{P}) > \hat{P} > 0$ but $\text{Tr}(\tilde{P}) > \text{Tr}(\hat{P})$. We denote $\tilde{P} \equiv \tilde{g}(P)$ and similar to the first step, $-\text{Tr}(\tilde{P}^{-1}) > -\text{Tr}(\hat{P}^{-1})$ can be shown, which again contradicts the hypothesis of optimality of $\tilde{P}$. Therefore, the optimal solution $K$ and $P$ to Problem 3 must also satisfy $K = PC(I + R\tilde{P})^{-1}$, i.e., $P = \tilde{g}(P)$. In other words, the optimal Kalman gain $K^*$ and the fixed point of the DARE $P_{\text{opt}}$ can be represented by the optimal solution to Problem 3 and this concludes the theorem.

According to Theorem 1 solving Problem 3 leads to the optimal steady-state Kalman gain which is of our interests. However, with privacy concerns, we do not directly solve it. Similar to the stabilization design method II in Section III-B we introduce an indicator function

$$I_{\tilde{P}}(\{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H}) = \begin{cases} 0, & \text{if } (\{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H}) \in \tilde{\Omega}, \\ +\infty, & \text{otherwise}, \end{cases}$$

(25)

where the convex set $\tilde{\Omega}$ is given by

$$\tilde{\Omega} \equiv \left\{ (\{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H}) : \mathbf{W} \succeq 0, \mathbf{H} \succ 0, \mathbf{2} \right\},$$

for $\Omega_i \in \mathbb{R}^{n \times n}, \nabla_i \in \mathbb{R}^{n \times m}, i \in \mathcal{N}$. By variable splitting, a two-block problem which is equivalent to Problem 3 is formulated as follows.

**Problem 4:**

$$\min_{\{Z_i\}, \{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H}} - \text{Tr}(\mathbf{H}) + I_{\tilde{P}}(\{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H}),$$

s.t. $Z_i \left[ C_i \right] \sqrt{\mathbf{R}_i} \mathbf{B}_i = [\Omega_i, \nabla_i], \forall i \in \mathcal{N}.$

Similar to the procedure in Section III-B the variables in Problem 4 can be grouped into two blocks: $(\{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H})$ and $(\{Z_i\}, \mathbf{R}_i)$, so that ADMM can directly apply. The augmented Lagrangian for Problem 4 where $X_i \in \mathbb{R}^{n \times (n + m^2)}$ is the corresponding Lagrange multiplier is given by:

$$\mathcal{L}_i \left(\{Z_i\}, \{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H}, \{\Lambda_i\}\right) = -\text{Tr}(\mathbf{H}) + I_{\tilde{P}}(\{\Omega_i\}, \{\nabla_i\}, \mathbf{W}, \mathbf{H})$$

$$- \sum_{i=1}^{N} \text{Tr}\left\{ \Lambda_i^T \left( Z_i \left[ C_i \right] \sqrt{\mathbf{R}_i} \mathbf{B}_i - [\Omega_i, \nabla_i] \right) \right\}$$

$$+ \frac{\gamma}{2} \sum_{i=1}^{N} \left\| Z_i \left[ C_i \right] \sqrt{\mathbf{R}_i} \mathbf{B}_i - [\Omega_i, \nabla_i] \right\|_F^2.$$  

Since $Z_i$’s are fully decoupled, the resulting subproblems can be solved in parallel. Algorithm 4 is proposed to solve Problem 4. After the convergence of the algorithm, each party obtains its estimator gain by $K_i = A^{-1}H^{-1}Z_i$, and $K = \frac{1}{N} \left[ K_1 \ K_2 \ \cdots \ K_N \right]$ is exactly the optimal steady-state Kalman gain $K^*$ based on Theorem 1 which means that $\hat{x}(k)$ is the asymptotic MMSE estimate. At each iteration $t$, each party uploads its $Z_i^t C_i$ and $Z_i^t \sqrt{R}_i B_i$ to the cloud server and there is no need for party $i$ to publish its own observation parameters $C_i$ and $R_i$ directly.

**Remark 4:** In all of the four algorithms proposed in our paper, when $N \geq 3$, each party can hardly learn or infer any data or model information during the procedure, since each party receives the processed information from the cloud server and security module. The message broadcast to each party is a fusion of other parties’ sensitive information. Other parties’ privacy is preserved by the fusion processes. For example, in Algorithms 1 and 2 the average is made public to all the $N$ parties. In Algorithms 3 and 4 the message transmitted to party $i$ is dependent on other parties’ parameters. To summarize, party $i$ can only infer the overall performance of the others. The sensitive information of each party is well preserved.

V. Numerical Examples

In this section, we illustrate the effective performance of our proposed secure multi-party state estimation with numerical examples. We consider a system with parameters:

$$A = \begin{bmatrix} 4.58 & 1.72 & -0.54 & -3.51 & -0.14 \\ 2.77 & 2.07 & -0.34 & -2.68 & -0.01 \\ 2.07 & 0.92 & 0.57 & -2.15 & 0.19 \\ 5.36 & 2.46 & -0.76 & -4.20 & -0.22 \\ 4.03 & 1.69 & -0.29 & -3.73 & 0.58 \end{bmatrix}$$

$$Q = 0.1 I_5.$$

To illustrate the effectiveness of the stabilization design method II based on ADMM in Section III-B we give an example where $N = 4$. Four parties deploy their own sensor networks to monitor this process. The corresponding observation matrices are generated as follows:

$$C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad R_1 = 0.10,$$

$$C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad R_2 = 0.08,$$

$$C_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad R_3 = 0.09 I_2,$$

$$C_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad R_4 = 0.06.$$

In this scenario, even if $K_i = \arg \min_{X_i} \| (I_n - X C_i) A \|_2$, Inequality (16) does not hold. We adopt Algorithm 3 to
The asymptotic MMSE design method achieves a good estimation performance, i.e., $\text{Tr}(\mathcal{P}(300)) = 0.27$.

VI. CONCLUSION AND FUTURE WORK

In this paper, we proposed a secure multi-party dynamic state estimation paradigm. During the whole procedure, the sensitive information of all the parties are well preserved. The local state estimates are protected by the additively homomorphic encryption. Different methods were provided to collaboratively design the linear estimator gains with the purpose of guaranteeing the filtering stability without leakage of observation parameters. Furthermore, an optimal gain design method was provided to reach the asymptotic MMSE estimation. Examples and simulations verified the theoretic results.

For future work, one possible direction is to provide privacy analysis on our proposed state estimation paradigm. For example, how to evaluate privacy breached by $Z_iC_iA$, $Z_iC_i$, and $Z_i\sqrt{R_i}B_i$ in Algorithms 3 and 4 would be worth exploring when the cloud server is assumed to be an adversary, or an honest but curious participant. One feasible method is to define a metric to characterize how accurate the estimates of $C_i$ and $R_i$ can be when the products are revealed. For security concerns, another possible direction is to investigate performance with imperfect communication channels and malicious adversaries. When delay and packet dropouts happen during the information transmission procedure, the estimation performance will be degraded inevitably. The alternative strategy of each party for uncertain channels is worth exploring. When there are malicious attackers or black sheep who deliberately tamper with the transmitted data to interfere with or even destroy the multi-party collaboration, it is necessary to design detection mechanisms to defend the collaboration and guarantee the performance. Furthermore, a more self-interested metric can be adopted as an extension. One may consider how to wisely fuse the local state estimate and the average state estimate for those parties who only care about their own estimation qualities in the privacy-preserving context.

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