Measurement based Controlled Not gate for topological qubits in a Majorana fermion quantum-dot hybrid system

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We propose a scheme to implement Controlled Not-gate for topological qubits in a quantum-dot-Majorana-fermion hybrid system. Quantum information is encoded on pairs of Majorana fermions, which live on the the interface between topologically trivial and nontrivial sections of a quantum nanowire deposited on an $s$-wave superconductor. A measurement-based two-qubit Controlled-Not gate is produced with the help of parity measurements assisted by the quantum-dot and followed by prescribed single-qubit gates. The parity measurement, on the quantum-dot and a topological qubit, is achieved by the Aharonov-Casher effect.

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Recently, physical implementation of quantum computers has attracted much attention. One of the main difficulties of scalable quantum computation is decoherence of quantum information. A promising strategy against decoherence is based on the topological idea [1] where gate operations depend only on global features of the control process, and thus largely insensitive to local noises. Topological ordered states emerge as a new kind of states of quantum matter beyond the description of conventional Landau’s theory [2]. A paradigmatic system for the existence of anyons is a kind of so-called fractional quantum Hall states [2]. Alternatively, artificial spin lattice models are also promising for observing these exotic excitations, e.g., Kitaev models [3] are most famous for demonstrating anyonic statistics [4] and braiding operations for topological quantum computation.

For universal quantum computation, one needs non-Abelian anyon to serve as qubit. With the potential applications in topological quantum computation, Majorana fermions (MF) with non-Abelian statistics have attracted strong renewed interests. MF are a kind of self-conjugate quasi-particles induced from a vortex excitation in $p_x + ip_y$ superconductor [1]. However, due to the instability of the $p$-wave superconducting states, its implementation remains an experimental challenge. Recently, it is recognized that topologically protected states may be most easily engineered in 1D semiconducting nanowires deposited on an $s$-wave superconductor [7–10], which provides the first realistic experimental setting for Kitaev’s 1D topological superconducting state [11]. Note that the quench dynamics of this model across a quantum critical point is also presented [12].

Although not universal for quantum computation, parity protected quantum logical operations schemes with a pair of MF as a topological qubit are proposed [13–19], which mainly focus on braiding operations of single qubit [13–16] and quantum information transfer between a topological qubit and a quantum bus [17–19]. As the absence of topologically ordered system that is universal for quantum computation and extremely difficult to perform certain topology changing operations, present research of universal quantum computation with MF, as well as other exotic topological qubits, need to supplement braiding operations with topologically unprotected operations. These operations can be error-corrected for a high error-rate threshold of approximately $0.14$ [20]. However, such a high error threshold may still prove difficult using unprotected operations within a topological system.

One of the main difficulties in implementing topological quantum computation lies in the difficulty of braiding of MF from different topological qubits for entangling operation. Therefore, a topological quantum bus, usually in a topological and conventional qubit hybrid architecture, would be of great help, where error rates below $0.14$ have already been achieved [21]. A topological quantum bus makes the implementation much easier as the quantum bus can transfer the quantum information between different topological qubits, and thus only braiding within a topological qubit, to realize single-qubit operation, is needed for quantum computation. Here, we propose a scheme to implement Controlled Not-gate for topological qubits in a quantum-dot-MF hybrid system. The architecture consists of 1D semiconducting wires deposited on an $s$-wave superconductor, under certain conditions, the endpoints of such wires support localized zero-energy MF. Qubit is encoded on a pairs of MF, the realization of parity measurements on a QD and a topological qubit, together with control and measurement on the QD state and single-qubit gates on the target topological qubit, is able to generate a two-qubit Controlled-Not (CNOT) gate [22]. In this sense, this scheme is a measurement-based scenario. Supplemented with arbitrary single qubit rotation, which is already proposed in such MF quantum dot hybrid system [23–24], universal quantum computation can be realized.
with one dot deposit on the superconductor of the flux qubit circuit while the other is not. We assume here that there is a galvanic isolation between the superconductor and semiconductor, so that there is no charge transfer between them [18]. Remarkably, one can also realize this QD using InAs nanowires [23], which is previously used for supporting MF, and thus reduce the experimental difficulties of implementation.

A pair of MF can be combined into a complex fermion. The fermion parity operator $n_p$ has eigenvalues $-1$ and $+1$ for states $|0\rangle$ and $|1\rangle$, respectively. But, the two states differ by fermion parity, which prevents the coherent superposition of the two. Therefore, for the purpose of quantum computation, where coherent superposition is inevitable, we combine four MF to form a topological qubit. In this way, coherent superposition is permitted for the two encoded qubit subspaces with same fermion parity. Without loss of generality, we can use the subspace with fermion parity is even as the encoded qubit states, i.e., the two states of the topological qubit are encoded as $|00\rangle$ and $|11\rangle$ of the four MF.

We now turn to the problem of reading out the two qubit states, which is one of the preliminary requirements for quantum computation purpose. As the states of the two pairs of MF in a topological qubit are always the same, detecting one of them can then fulfill the purpose of distinguish them. As noted above, the two states $|0\rangle$ and $|1\rangle$ are different in fermion parity, so they can be distinguished by $n_p$. Without loss of generality, we choose to detect the pair of $\gamma_1$ and $\gamma_2$ while move $\gamma_3$ and $\gamma_4$ out of the flux qubit circuit, also our measurement circuit, as shown in Fig. 1. Note that the topological property of the wire will not be interrupted by the junctions if the Josephson junctions’ thickness is much smaller than the superconducting coherence length $\xi$. To measure the parity of $n_p$, we use the suppression of macroscopic quantum tunneling by the Aharonov-Casher effect [24]: a Josephson vortex encircling a superconducting island picks up a phase $\phi = nq/e$ determined by the total charge $q$ coupled capacitively to the superconductor, which includes both the charge on the superconducting island and on a nearby gate electrode.

Following Ref. [14, 24], we consider a superconducting flux qubit with three Josephson junctions, as shown in Fig. 1 where junctions 1 and 3 have the same Josephson coupling energy $E_J$ while that of junction 2 is $\alpha E_J$. The charging energy of the islands is much larger than $E_J$ so that the considered qubit works in the flux regime. The gauge-invariant phase drops of the three junctions $\phi_1$ and $\phi_3$ are related to the total magnetic flux $\Phi_z$ through the flux qubit loop by the constraint $\phi_1 + \phi_2 + \phi_3 = 2\pi \Phi_z/\Phi_0$ with $\Phi_0 = h/2e$ being the flux quanta. On the condition that the size of the qubit is sufficiently small, the flux generated by the circular supercurrent along the loop can be neglected. Then, the enclosed flux of the qubit contour comes solely from the external magnetic filed. Then, the
superconducting energy of the flux qubit reads

\[ U = -E_J \left[ \cos \phi_1 + \cos \phi_3 + \alpha \cos \left( 2\pi \frac{\Phi_x}{\Phi_0} - \phi_1 - \phi_3 \right) \right], \]

which exists two lowest degenerate states, the supercurrent of which flows clockwise and counterclockwise, respectively. These states are usually defined as the superconducting flux qubit states. The qubit states correspond to the potential energy minima in Eq. (2), and thus the tunneling between the two states requires quantum phase slips.

When \( \alpha > 1 \), the two energy minima are connected by two different tunneling paths, differ by \( 2\pi \) in \( \phi_1 \) and \( -2\pi \) in \( \phi_3 \). The interference between the two tunneling paths constitutes the circulation of a Josephson vortex around both superconducting islands \( L \) and \( R \). According to Aharonov-Casher effect, the acquired phase is \( \psi = \pi q/e \) with \( q = \sum_{i=L,R} (en_i + q_i) \) being the total charge on the two islands \( en_i \) and gate capacitors \( q_i \) to the qubit loop. As we chose junctions 1 and 3 have the same Josephson coupling energy, the two tunneling paths have the same amplitude. The interference between the two tunneling paths produces an oscillating tunnel splitting of the two levels of the flux qubit

\[ \Delta = \Delta_0 \cos \left( \frac{\psi}{2} \right), \]

where \( \Delta_0 \) is the tunnel splitting associated with one path. Therefore, if \( q \) is an odd (even) multiple of the electron charge \( e \), the two tunneling paths interfere destructively (constructive), and thus the tunnel splitting is minimum (maximal).

As we only need to distinguish maximal from minimal tunnel splitting, the flux qubit does not need to have a high quality factor. In addition, \( \Delta_0 \approx 100\mu eV \approx 1K \) for parameters in typical experiments of flux qubits [24], which should be readily observable by microwave absorption. To make sure the total charge is solely comes from the two superconducting islands, one would first calibrate the charge on the gate capacitor to zero, i.e., \( q_i = 0 \), by maximizing the tunnel splitting in the absence of vortices in the island. The read-out is nondestructive, which is necessary for our proposed way of implementation the two-qubit CNOT gate. Meanwhile, it is insensitive to sub-gap excitations in the superconductor as they do not change the fermion parity.

Following Ref. [18], as shown in Fig 1, \( q \) can also have a quantum component: charge comes from the QD. Logical basis of QD can be defined as semiconductor charge qubit \( |0\rangle = |0\rangle \otimes |1\rangle \) and \( |1\rangle = |1\rangle \otimes |0\rangle \) with the electron occupies the lower and upper dot, respectively. Therefore, the qubit basis states correspond to the electron parity on the upper dot enclosed by the Josephson vortex circulation. To read-out the MF state, one can simply set the QD to \( |0\rangle \) so that its charge do not interrupt the measurement. For our purpose, we are also interested in the joint parity of QD and a pair of MF. Indeed, the flux qubit splitting energy \( \Delta \) is the same for combined topological-QD qubit states with equal joint parity [18]. Thus, measurement of the flux qubit splitting energy is equivalent to a joint parity measurement to the states of QD and a pair of MF. In Ref. [18], quantum state transfer between topological and QD qubits is achieved by standard quantum teleportation: the proposed parity measurements supplied with Hadamard gates are equivalent to Bell state measurements. Here, our scheme for CNOT gate is a measurement based one.

We next proceed to implement a CNOT gate between two topological qubits with the help of QD as an auxiliary. Here we propose a measurement-based CNOT gate operation. The relevant operations are single-qubit rotations, single-QD rotations/measurements, and effective joint parity measurements for QD and a topological qubit. The circuit for the CNOT gate is depicted in Fig. 2. The auxiliary QD is initially prepared in the state \( |0\rangle_A \). After a Hadamard gate on the QD, the first joint parity measurement \( P_1 \) in Fig. 2 is implemented on the QD and a pair of MF in the "C" qubit. After Hadamard rotation of the QD and the target qubit, the second parity measurement \( P_2 \) is implemented on the QD and a pair of MF in the "T" qubit. Then we rotate back the QD and the target qubit state by Hadamard gate. The last step is the measurement of the QD in the \( \{ |0\rangle, |1\rangle \} \) basis. The two parity measurement results, together with the measurement result of the QD determine which single-qubit gates to be operated on the control and target qubits to generate a CNOT gate. The relationship between the measurement results and the gates to be operated is summarized in the table. After completing the required gates on the corresponding qubits, it is straightforward to check that the process is a CNOT gate operation between the two qubits.

![FIG. 2: Measurement-based CNOT gate for two topological qubits. Capital letters "A" represents ancilla of QD, "H" is the Hadamard gate, "C" and "T" represent the control and target qubit, respectively. The measurement "MF" results of "A" together with the outcomes of the two joint parity measurements "P" determine which operation one has to apply on the "C" and "T" qubit in order to complete the CNOT gate. The arrowed line in the bottom represents the sequence of the process.](image-url)
TABLE I: Correspondence between the measurement results and the gates operated on the control and target qubits. "0" and "1" represent odd and even parity, respectively.

| "P_1" "P_2" result of "M" gate on "C" gate on "T" | 1 1 | 0 0 |
|------------------------------------------------|-----|-----|
|                                                | 1 1 | I   |
|                                                | 1 0 | I   |
|                                                | 0 1 | I   |
|                                                | 0 0 | I   |

Here, we want to emphasize that our implementation is different from that of Ref. [14], which is alone the line presented in Ref. [25]. In the proposal [14, 25], they use the parity measurement of two MF qubit, to entangle them, and braiding operations of MF from qubit and ancilla to implement a CNOT gate. While we use joint MF-QD parity measurements to construct the measurement-based CNOT gate, where only braiding operations of MF within topological qubits are needed. In the proposal [14, 22], only one parity measurement of two MF qubit is needed, thus it is more efficient in terms of complexity. But, it requires long range braiding operations between qubit and the ancilla [22], which is experimentally challenging. As for our introducing of a QD qubit to serve as the ancilla, it breaks the topological protect of the proposal in [25] in some sense. But, our joint MF-QD parity measurement is based on the Aharonov-Casher effect, which is also topological in nature and insensitive to local noises.

Before concluding, we want to emphasize that our scheme have the following distinct merits. (1) As in Ref. [18], one can host both MF and QD using a single InAs nanowires [23], which may thus reduce the technical challenges of implementation. (2) Here, QD serves as ancillary qubit. Therefore, only braiding between MF within a topological qubit is needed, since braiding of two MF from different topological qubits will be experimentally challenging. (3) Measurement of topological qubit uses the Aharonov-Casher effect [24], which is nondestructive. (4) Parity measurement on QD and a certain topological qubit can be achieved by moving the topological qubit into the measurement circuit by local tunable gate on quantum wire [14].

In summary, a measurement-based two-qubit Controlled-Not gate is produced with the help of joint parity measurements and followed by prescribed single-qubit gates.

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