Magnetic flux of progenitor stars sets gamma-ray burst luminosity and variability

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ABSTRACT
Long-duration gamma-ray bursts (GRBs) are thought to come from the core collapse of Wolf–Rayet stars. Whereas their stellar masses $M_\star$ have a rather narrow distribution, the population of GRBs is very diverse, with gamma-ray luminosities $L_{\gamma}$ spanning several orders of magnitude. This suggests the existence of a ‘hidden’ stellar variable whose burst-to-burst variation leads to a spread in $L_{\gamma}$. Whatever this hidden variable is, its variation should not noticeably affect the shape of GRB light curves, which display a constant luminosity (in a time-average sense) followed by a sharp drop at the end of the burst seen with Swift/XRT. We argue that such a hidden variable is progenitor star’s large-scale magnetic flux. Shortly after the core collapse, most of stellar magnetic flux accumulates near the black hole (BH) and remains there. The flux extracts BH rotational energy and powers jets of roughly a constant luminosity, $L_j$. However, once BH mass accretion rate $\dot{M}$ falls below $\sim L_j/c^2$, the flux becomes dynamically important and diffuses outwards, with the jet luminosity set by the rapidly declining mass accretion rate, $L_j \sim \dot{M}c^2$. This provides a potential explanation for the sharp end of GRBs and the universal shape of their light curves. During the GRB, gas infall translates spatial variation of stellar magnetic flux into temporal variation of $L_j$. We make use of the deviations from constancy in $L_j$ to perform stellar magnetic flux ‘tomography’. Using this method, we infer the presence of magnetized tori in the outer layers of progenitor stars for GRB 920513 and GRB 940210.

Key words: MHD – methods: analytical – methods: numerical – stars: magnetic field – gamma-rays: stars.

1 INTRODUCTION
Long-duration gamma-ray bursts (GRBs) are believed to be associated with the collapse of the core of massive stars. This association is very firm in GRBs accompanied by supernovae explosions (Galama et al. 1998; Stanek et al. 2003). The nature of the supernova (Type Ic) indicates Wolf–Rayet stars of spectral type WO/C as the progenitors. Such stars have mass $\sim 10 M_\odot$ and radius of $R \sim$ a few $R_\odot$. Despite their masses have a rather narrow distribution, the GRBs they produce come in a very broad range of power. Their luminosity function extends over at least 4 orders of magnitude in $L_{\gamma}$ (Wanderman & Piran 2010), and various additional categories of GRBs have emerged in recent years (e.g. Levan et al. 2014).

The observed duration of the prompt emission episode ($\sim 1–100$ s) roughly agrees with the free-fall time-scale of the progenitor star. The GRB emission is notoriously variable; composed of a large number of intense gamma-ray pulses. Despite their erratic behaviour, GRBs have some well-defined properties which tightly constrain the models for the central engine. The time-averaged properties of the prompt emission do not evolve in a systematic way with time since the trigger (Ramirez-Ruiz & Fenimore 2000). By looking at a random segment of the GRB light curve, there is no way to tell from the amplitude and duration of the pulses and the intervals between the pulses whether the segment corresponds to the first or second half of the GRB (Ramirez-Ruiz & Fenimore 2000; Quilligan et al. 2002). This is also demonstrated by the constant slope $S$ of cumulative counts during GRBs revealing that $\int_0^\infty L_{\gamma} \, dt \sim$ constant $\times t$ or $L_j \sim$ constant (McBreen et al. 2002). In contrast, the end of the GRB is typically well defined and is marked by a steep decline in flux. This end stage is seen in X-rays with XRT on Swift, and is characterized by a steep time-dependence of luminosity, with the flux dropping by several orders of magnitude. This abrupt drop is consistent with an abrupt shutoff of the central engine (see Nousek et al. 2006; Zhang, Woosley & MacFadyen 2006).

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This GRB behaviour is hard to understand in the context of core-collapse models irrespective of whether GRBs are powered by accretion on to a black hole (BH; Woosley 1993) or by the rotation of a neutron star (Usov 1992). How can the average luminosity of the jet, for a minute or so, ignore central engine evolution when the latter is expected to evolve substantially on a similar time-scale? What marks the sudden decline in the GRB emission? What sets the GRB luminosity and what causes it to differ so much from burst to burst?

In the magnetar model of GRBs the jet luminosity can be constant for ≈1 min (set by the spin-down time-scale of the magnetar). However, it is unclear what causes GRBs to abruptly end. Moreover, jet properties (baryon loading) evolve fast with time while the average observed GRB spectra do not (see Metzger et al. 2011 for a related discussion).

If the central compact object is a BH, accretion can power GRBs either through neutrino annihilation (Popham, Woosley & Fryer 1999; Ruffert & Janka 1999; Birkl et al. 2007; Chen & Beloborodov 2007; Zalamea & Beloborodov 2011) or magnetic energy extraction (Narayan, Paczynski & Piran 1992; Meszaros & Rees 1997; Levinson & Eichler 2003). After the core collapse, the accretion rate $\dot{M}$ drops fast with time unless one makes very particular choices for the progenitor star and the accretion disc viscosity (Kumar, Narayan & Johnson 2008a,b). In any jet production mechanism, which depends on the instantaneous BH mass accretion rate, $\dot{M}$, the observed constancy of the GRB luminosity requires fine tuning: in order to obtain an approximately constant jet luminosity, the efficiency of jet production must increase with time in exactly the right way so as to precisely cancel the effect of decrease in $\dot{M}$.

This monotonic decrease of $\dot{M}$ with time challenges neutrino models where the jet power scales with accretion rate as $\dot{L}_j \propto \dot{M}^{3/4}$ (or a similar steep power; see Zalamea & Beloborodov 2011). Even a weak systematic change of the average $\dot{M}$ throughout the GRB should be easily detectable, which is not the case in the majority of the bursts. The mechanism also cannot account for the energetics of the longest bursts observed (Leng & Giannios 2014). Here, we argue that it is the magnetic flux through the collapsing star that determines the luminosity of the jet, as we illustrate in Fig. 1. Indeed, if the jet is launched via magnetic fields, the accretion rate does not always determine the jet power directly (Tchekhovskoy, Narayan & McKinney 2011; see however Krolik & Piran 2011). The available magnetic flux in the progenitor star may be much more constraining (Barkov & Komissarov 2008; Komissarov & Barkov 2009). The flux accumulates on the newly formed BH on a time-scale shorter than the time it takes for the jet to emerge from, or break through, the collapsing star.

We assume that the jet is powered by the large-scale magnetic flux via the Blandford–Znajek process (BZ; Blandford & Znajek 1977). In this picture, as in most collapsar models, the jet breakout marks the start of the prompt emission, or the GRB trigger. From then on and until the end of the GRB, for reasonable collapsar parameters, the mass of the BH $M_{\text{BH}}$, the magnetic flux through the hole $\Phi_{\text{BH}}$, and BH spin $a$ evolve little over the GRB duration. So long as the mass accretion rate $\dot{M}$ is sufficiently high to sustain the magnetic flux $\Phi_{\text{BH}}$ on the BH, jet luminosity $L_j \propto a^2\Phi_{\text{BH}}^2 M_{\text{BH}}^2$ is independent of $M$ and is approximately constant, as illustrated in Fig. 1 with horizontal segments of red lines. However, since $M$ asymptotically approaches zero, eventually $M$ becomes too low to confine the magnetic flux of a constant strength on the BH. This occurs at the equipartition between accretion and jet powers, $\dot{M}c^2 \lesssim \dot{L}_j \sim 10^{49}$ erg s$^{-1}$, or, in terms of mass accretion rate, $\dot{M} \sim 10^{-4} M_\odot$ s$^{-1}$. From this point on the accretion disc cannot hold the entirety of magnetic field on the BH anymore. The remaining flux on the BH stays in equipartition with accretion power, and the jet luminosity scales linearly with $M$, which rapidly declines towards zero and causes the GRB to end abruptly.

We start with the description of progenitor star’s structure and its core collapse in Section 2, paying particular attention to how the central BH properties – mass, spin, and magnetic flux – change in time. In Section 3, we derive the time-evolution of jet power and in Section 4 perform the comparison of our model to the observed GRB light curves. In Section 5, we conclude. Throughout the paper, we use Gaussian-cgs units.

### 2 COLLAPSRAR MODELS

We consider GRB jets that are powered by the accretion on to the central BH. To compute the rate at which the BHs are fed with gas, we consider several pre-collapse stellar models described in Woosley & Heger (2006). We focus in this section on the ‘16T’ model of a pre-collapse star at 1 per cent solar metallicity. The model includes a treatment of magnetic dynamo effects and mass and angular momentum loss via stellar winds. Fig. 2(a) shows the density profile in the pre-collapse star: outside the core, $R > 10^6$ cm, the density falls off roughly as a power law, $\rho \propto r^{-2.5}$, and cuts off exponentially towards the surface, at $r_c \approx 5 \times 10^{10}$ cm. Fig. 2(b) shows the radial distribution of angular momentum. It has clear...
2.1 Temporal evolution of the GRB central engine

The collapse of the core causes the star to lose pressure support, and the star evolves on the free-fall time-scale $t_{ff} \equiv r_s/v_{ff} = (r_s^2/2GM_c)^{1/2} \sim 130$ s. We assume that shortly after the core collapses, a BH forms. The density profile of the star determines the growth rate of the BH. The rotational profile of the collapsing star, shown in Fig. 2(b), controls which layers of the star collapse directly into the BH. The total angular momentum $J(t)$ and mass $M(t)$ of the layers that collapsed determine the evolution of BH spin $a = J/Mc$. When the specific angular momentum of a layer exceeds that of a test particle orbiting at the innermost stable circular orbit (ISCO), $l_{ISCO}$, the gas hits the centrifugal barrier, circularizes and forms an accretion disc. When a disc is present, the specific angular momentum of the material that accretes on the BH is set equal to $l_{ISCO}$. We now discuss these processes in more detail.

In Fig. 3(a), we show mass inflow rate $\dot{M} = 4\pi R^2 \rho(R) v_R = 4\pi R^2 \rho(R) R/\dot{t}$ as a function of time since core collapse. For our fiducial model, the formation of the disc takes place at $t_{disc} \approx 4$ s post-core collapse and is indicated in Fig. 3(a) with red circles. As seen in Fig. 3(b), at this early time the BH accretes dense, inner layers of the star, $r \sim 10^{10}$ cm, and the mass accretion rate is very high $\dot{M} \sim M_\odot s^{-1}$. The accretion power, $\dot{E} = 10^{54}$ erg s$^{-1}$, by many orders of magnitude exceeds the upper range of GRB luminosity, $L \sim 10^{50}$ erg s$^{-1}$. This implies that, at least at early times, magnetic fields are dynamically subdominant relative to the accretion flow (we discuss this in Section 2.2.2).

We assume that the formation of the centrifugal barrier and of the disc opens up a low-density polar funnel region through which the jet emerges. The jet is powered by BH rotation and large-scale magnetic fields threading the BH (see Section 3 for a discussion of jet properties). The jet takes several seconds to drill a hole through the collapsing star. The GRB trigger takes place at the moment the jet emerges, or ‘breaks out’, of the star. For our fiducial model this happens a few seconds after disc formation, at $t \approx 7$ s after core collapse, as indicated with green squares in Fig. 3. The trigger occurs early in time, $\lesssim 10$ s after the core collapse, while mass accretion rate is still very high. We adopt the following expression for jet breakout time (Bromberg et al. 2014a; Bromberg, Granot & Piran 2014b):

$$t_{breakout} - t_{disc} = \left(\frac{L_1}{2 \times 10^{50} \text{erg s}^{-1}}\right)^{-1/3} \times \left(\frac{M_c}{15 M_\odot}\right)^{1/3} \left(\frac{r_{ff}/a}{1.25 \times 10^{10} \text{[cm]}}\right)^{2/3} \text{[s]},$$

(1)

which is the difference between the time it takes the jet head to traverse the star and the light travel time across the star. Here

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1 Note that stellar rotation profile is uncertain due to the limitations of the 1D stellar models and uncertainties in the treatment of microphysics of turbulence, magnetic fields, and stellar winds. For instance, the angular momentum $\ell = (2/3)2\pi r^2$, in models 16TH and 16TI exceeds the Keplerian value, $\ell_K$, in the outer layers of the star (see Fig. 2b). If the outer layers indeed were super-Keplerian, the star would have already expelled these outer layers long before core collapse. These extremely high values appear due to the uncertainties of angular momentum transport inside the outer layers of the star and angular momentum extraction by the stellar winds and are not physical (Woosley & Heger 2006). For this reason, for our calculations, we modify the rotational profile of the models and limit the angular momentum to be at most 10% percent of the local Keplerian value.

2 We assume that the time-scale for the material to reach the BH is dominated by the free-fall time $t_{ff}$ and not by the accretion time $t_{acc}$. We have checked that for judicial viscosity parameter $a = 0.1$, and disc aspect ratio $H/R \sim 1$, the accretion time-scale is nearly always shorter than the free-fall time of the star. In this paper, for simplicity and transparency, we ignore the modifications to the time it takes the gas to reach the BH due to the formation of an accretion disc (see Appendix A).

3 Here, we equate the inflow rate of mass from the collapsing star to the accretion rate to the BH. In effect, we ignore mass-loss that can take place through, e.g. winds from an accretion disc. Wind mass-loss is considered in Section 3.3.
Figure 3. Panel (a): mass accretion rate $\dot{M}$ versus time since core collapse $t$ for our fiducial pre-collapse stellar progenitor model, 16TI (Woosley & Heger 2006). For the sake of later discussion, the time of disc formation is indicated with a red circle, jet breakout with the green square, and emergence of a dynamically important magnetic field threading the BH (or the MAD), with the blue star, at which the GRB ends (see Section 2.2). The latter two of these times depend on progenitor star’s magnetic flux strength (described by $\Phi_\ast$) and radial distribution (described by $\gamma$, see equation 8), and the mass-loss from an accretion disc (described by $\beta$ see equation 11). Panel (b): the mapping between the radial position $r$ of a layer of gas in the progenitor star and the time $t$ since core collapse at which it reaches the BH.

Figure 4. Black hole mass $M_{\text{BH}}$ (panel a) and spin $a$ (panel b) versus time $t$ since the core collapse for our fiducial model. Red circle indicates the time of accretion disc formation and green circle indicates the time of the jet breakout, which we associate with the gamma-ray trigger. Blue stars mark the time at which the magnetic flux on the BH becomes dynamically important, a MAD forms, and the GRB ends. See Fig. 3 for details.

2.2 Available magnetic flux and jet power

An important factor that controls jet power is large-scale magnetic flux threading the BH (i.e. the flux that crosses the BH event horizon). It is conceivable that this flux is dragged to the BH by the collapsing stellar gas. We first argue that the star can contain the necessary magnetic flux to power a GRB and then show that the accretion disc can hold this flux on the BH for a minute or so. We then suggest that this time-scale is what sets the GRB duration.

2.2.1 Magnetic field strength on the hole

Can the strength of the magnetic flux on the BH be limited by the amount of magnetic flux available in the progenitor star? To find this out, we will assume that a progenitor star, similar to model 16TI of Woosley & Heger (2006), has a magnetic field of $B_{\text{surf}} \sim 10^5$ G. Since GRBs are...
2.2.2 Is jet power limited by available stellar magnetic flux or mass accretion rate?

In Section 2.2.1, we found that based on the observed energetics of GRBs, the central BH is threaded by magnetic field of order $B_0 \sim 10^{15} \text{ G}$, or magnetic flux $\Phi_{\text{BH}} \sim 5 \times 10^{27} \text{ G cm}^2$. We showed that it is conceivable that a progenitor star contains this amount of magnetic flux.

What keeps this magnetic flux on the BH? If the accretion ceased, the magnetic flux would have left the BH due to the no-hair theorem. Thus, it is the presence of an accretion disc via its pressure (ram plus thermal) that keeps the magnetic flux on the BH. Immediately after the core collapse, the mass accretion rate is so high that the pressure of the disc easily overpowers the outward magnetic pressure. However, as $M$ drops, eventually the disc pressure becomes too weak to hold the entirety of magnetic flux on the BH. This happens at a critical mass accretion rate at which accretion and jet power are comparable (Tchekhovskoy, Narayan & McKinney 2011; Tchekhovskoy, McKinney & Narayan 2012; Tchekhovskoy et al. 2014; Tchekhovskoy 2015; Tchekhovskoy, McKinney & Nemmen, in preparation).

The BH magnetic flux $M_{\text{MAD}} c^2$ is limited by the period of BH luminosity (see equation 2), which is illustrated by horizontal lines in Fig. 3. Therefore, after the jet emerges through the stellar surface $\sim 10 \text{ s}$ post-core collapse, it powers a GRB of roughly a constant power of typical long-duration GRBs, around $10^{53} \text{ erg s}^{-1}$. This accretion rate on to a newly formed BH is more than sufficient to confine the magnetic flux within the BH vicinity. And, as long as $M \geq M_{\text{MAD}}$, BH magnetic flux $\Phi_{\text{BH}}$ is determined by the available stellar magnetic flux and not by mass accretion rate, even though $M$ drops fast with time (see e.g. Fig. 3).

After a minute or so the situation qualitatively changes: $M$ drops below $M_{\text{MAD}}$, the magnetic flux on the hole becomes dynamically important, and parts of the flux, which used to thread the BH, diffuse out. The remaining BH magnetic flux obstructs gas infall and leads to a magnetically arrested disc (MAD) (Igumenshchev, Narayan & Abramowicz 2003; Narayan, Igumenshchev & Abramowicz 2003; Tchekhovskoy, Narayan & McKinney 2011; see also Bisnovatyi-Kogan & Ruzmaikin 1974, 1976). The remaining BH magnetic flux is determined by the instantaneous $M$ via equation (4).

$$\Phi_{\text{BH}, \text{MAD}} = 5 \times 10^{27} \text{ G cm}^2 \left( \frac{M}{10^{51} \text{ M}_\odot \text{ s}^{-1}} \right)^{1/2} \times \left( \frac{M_{\text{BH}}}{10 \text{ M}_\odot} \right)^{-1} \frac{h/r}{0.2} \chi,$$

where $\chi = 1.4(1 - 0.38 \Omega_{\text{BH}}) \sim 1$ is a high-order correction.

In terms of magnetic flux threading the BH, we have

$$\dot{M}_{\text{MAD}} = 10^{-4} M_\odot \text{ s}^{-1} \left( \frac{\Phi_{\text{BH}}}{5 \times 10^{27} \text{ G cm}^2} \right)^{2} \left( \frac{M_{\text{BH}}}{10 \text{ M}_\odot} \right)^{-2} \times \left( \frac{h/r}{0.2} \right)^{-1} \chi^{-2},$$

There is also the question of rare events (Guetta, Piran & Waxman 2005), such a strong surface magnetic field might not be uncommon among GRB progenitors.

Part of the core, with radius $r_{\text{coll}} \sim 3 \times 10^5 \text{ cm}$ (see Fig. 3b), directly collapses and forms a BH of mass $M_{\text{BH}} \approx 4 M_\odot$ (Fig. 4a) and gravitational radius $r_g = GM_{\text{BH}}/c^2 \approx 6 \times 10^4 \text{ cm}$. Assuming dipolar field in the star, the magnetic field strength at $r = r_{\text{coll}}$ before the collapse is $B_{\text{coll}} = (r_{\text{coll}}/r_g)^2 B_{\text{surf}} \approx 5 \times 10^{10} \text{ G}$. If the magnetic field is frozen into the stellar envelope, the direct collapse leads to the field strength at the BH horizon, $B_{\text{BH}} \approx (r_{\text{coll}}/r_g)^2 B_{\text{surf}} \approx 10^{15} \text{ G}$, or the magnetic flux through the BH of $\Phi_{\text{BH}} \approx 5 \times 10^{27} \text{ G cm}^2$. The resulting jet power,

$$L_{\text{BZ}} \approx \frac{k f}{4 \pi c} \Phi_{\text{BH}}^2 \Omega_{\text{BH}} ,$$

is $\sim 10^{56} \text{ erg s}^{-1}$, where $k \approx 0.05$ and $\Omega_{\text{BH}} = ac/(2 r_{\text{H}})$ is the angular frequency of BH horizon (Tchekhovskoy, Narayan & McKinney 2010). We will refer to the expression for BZ power, given by equation (2) with $f = 1$, as the second-order ‘BZ2’ approximation: it is accurate for $a \lesssim 0.95$. For our numerical models, which we describe in Section 3, we will make use of a more accurate sixth-order ‘BZ6’ expression for jet power that is valid for all values of spin and includes a high-order correction factor, $f = 1 + 0.350_{\Omega_{\text{H}}} - 0.580_{\Omega_{\text{H}}}$, where $\omega_{\text{BH}} = a/[1 + (1 - a^2)^{1/2}]$ is the dimensionless rotational frequency of BH event horizon.

Thus, a simple estimate (2) shows that the large-scale magnetic flux contained by a progenitor star is sufficient to account for the power of typical long-duration GRBs, around $10^{56} \text{ erg s}^{-1}$. This also implies that GRBs seem to require a strongly magnetized pre-collapse core with $B \gtrsim 10^7 \text{ G}$. Given that some white dwarfs are observed with surface magnetic fields in excess of $10^9 \text{ G}$ (Kawka et al. 2007; Külebi et al. 2009) such values may be reasonable.
and produces jets of energy efficiency, or dimensionless power, given by equation (3):

$$\eta_{\text{MAD}} = \frac{L_{j,\text{MAD}}}{M c^2} \approx a^2 h_{0.2},$$

(6)

i.e. there is a linear relation of $\dot{M}$ and jet power $L_{j,\text{MAD}}$ in the MAD regime. Since $\dot{M}$ rapidly decreases in time, after the MAD onset we expect the jet power $L_{j,\text{MAD}}$ to do so as well. Note that in our numerical models described in Section 3, we will use a more accurate expression for jet energy efficiency,

$$\eta_{\text{MAD}} = 3\omega_0^2 (1 - 0.38 \omega_0)^2 (1 + 0.35 \omega_0 - 0.58 \omega_0^2) h_{0.2},$$

(7)

which we obtain by combining the high-order accurate versions of equations (2) and (5). Equation (6) is a quadratic approximation to the dependence (7).

### 3 EVOLUTION OF JET LUMINOSITY DURING PROMPT PHASE

#### 3.1 Magnetic flux and its distribution in the progenitor

We have argued that GRB jets are powered by the rotational energy of the central BH extracted by large-scale magnetic flux (see equation 2). Within this framework, in order to get a handle on what determines the luminosity of a GRB, we need to have a better understanding of the processes that control the accumulation of large-scale vertical magnetic flux on the BH.

The growth of the magnetic flux on the BH depends on the very uncertain magnetic field configuration in the progenitor star. However, since the flux is brought in with the accreting gas, it is reasonable to parametrize it to scale with the collapsed mass of the star:

$$\Phi_{\text{BH}}(t) = \Phi_s \left(\frac{M_{\text{collapsed}}}{M_*}\right)^{\gamma},$$

(8)

where $\gamma \sim 1$ parametrizes the radial distribution of magnetic flux inside of the star and $\Phi_s$ the total magnetic flux threading the star. If we now combine equation (8) with equation (2), we obtain

$$L_{BZ} \propto \frac{a^2 \Phi_{\text{BH}}}{M_{\text{BH}}^2} \propto a^2 M_{\text{BH}}^{\gamma-1},$$

(9)

where we loosely approximated $M_{\text{BH}} \approx M_{\text{collapsed}}$. Since BH mass changes by a factor of few in the course of GRB (see Fig. 4a), equation (9) suggests that the power of the jets changes by the same (small) factor of a few, and this might account for the apparent constancy of GRB prompt emission we set out to explain. We now explore this possibility in detail.

In Figs 6(a) and (b), we plot the time-dependence of jet power, or luminosity $L_j$, and the magnetic flux $\Phi_{\text{BH}}$ threading the BH, for our fiducial scenario, $\gamma = 1$. We consider different values of progenitor magnetic flux strength, $\Phi_s = 10^{26.5}, 10^{27}, 10^{27.5}, 10^{28}$ G cm$^2$, as labelled on the figure and shown with black lines of different types. The behaviour of jet power is very similar, for the entire range of the magnetic flux strength in the progenitor star. After the trigger, which we indicate with the green squares, and until the MAD formation, which we show with blue stars, both magnetic flux and jet power change by about a factor of 2. During this time the strength of magnetic flux on the BH, shown in Fig. 6(b), is determined by the magnetic flux supply in the progenitor star and not by mass accretion rate. Note that for our fiducial model, with $\gamma = 1$, by equation (9) we would expect no dependence of jet power on $M_{\text{BH}}$ (and time) at all. As we will see later, there is actually some dependence, caused by the differences between $M_{\text{BH}}$ and $M_{\text{collapsed}}$ (see Fig. 5). However, for the entire range we considered, $0 \leq \gamma \leq 2$, the variation of $L_j$ during the GRB is rather weak. After about 100 s, a MAD forms, and the luminosity drops fast since the disc cannot hold on to the BH magnetic flux anymore and the GRB turns off. We discuss this in detail in Section 3.2.

Note that variation by a factor of $\sim 30$ of the flux $\Phi_s$ available in the progenitor star, reproduces the full range of observed GRB luminosities ($L_j \propto \Phi_s^2$), from the weakest to the most powerful ones. This range corresponds to surface (core) field strength of the progenitor ranging from $10^3$ G to $3 \times 10^4$ G ($3 \times 10^6 - 10^8$ G). Indeed, such a factor of $\sim 30$ is well within the observed range of variation in the surface field strength of massive stars. The field strength is the only property of the progenitor star that we are aware of, that can plausibly give the large variety of GRB energetics, as observed (see Section 3.5).

Figs 7(a) and (b) explores the effect of different radial distributions of $\Phi_s$ in the progenitor star. For this, we plot the time-dependence of $L_j$ and $\Phi_{\text{BH}}$ for three different values of the $\gamma$ parameter (defined in equation 8): $\gamma = 0, 1, 2$. The variation in $\gamma$ affects neither the time of GRB turnoff nor the characteristic power of the GRB, but it does change the early-time trend of jet power dependence on time: for $\gamma = 0$, shown with the thin solid line, jet power decreases with time, before levelling off at a constant value. Thus, even though the magnetic flux on the BH is constant in this case (equation 8), jet power changes: this is because in addition to...
Panels (a) and (b) show, respectively, jet power and magnetic flux $L = (\text{see equation } 4)$, i.e. the considered. This is $\dot{\alpha}$ and $\beta \propto \gamma = \frac{r}{\Phi_1} M$, stronger $\gamma \propto \dot{\beta}$ is the $0, 0.5, and 1$. Fig. $L_s \propto \dot{\alpha} 8 r 1$ and 2, both $t \beta > (10)$ $(11) = \frac{r}{\Phi_1} \propto \dot{\beta}$ is a characteristic radius at which the disc wind, at least $2012 \gamma 0$ (thin), $1$ (medium-thickness), $2$ (thick solid line). Connected $\dot{\alpha} 0.5$ (McKinney et al. $\dot{\alpha} 8 \beta$ means no mass-loss, and all else being equal, we would expect the GRB to be more powerful and last for a shorter time. However, this effect is diluted by the fact that more powerful jets breakout faster and thus we start seeing GRB earlier (see equation 1). We will also see later (Section 3.4) that changes in stellar angular momentum profile strongly affect the jet breakout time and the GRB duration.

### 3.3 Mass-loss from the disc

So far we have assumed that all the mass from the collapsing star makes it to the BH. In reality, stellar material settles into a disc with an outer size $r_D$, which is set by the value of the specific angular momentum $\ell$ of the stellar material, $r_D \approx \ell^2 / GM$, where $M$ is the enclosed mass. For outer layers of the star, $r_D$ can be much larger than $r_{\text{ISCO}}$, and substantial mass-loss can take place between the two radii (Blandford & Begelman 1999; McKinney, Tchekhovskoy & Blandford 2012; Narayan et al. 2012; Sadowski et al. 2013, 2014).

We model the effect of mass-loss in a standard way, assuming that winds take away a fraction of locally accreting mass, i.e. we parameterize $M$ at the BH as (e.g. Blandford & Begelman 1999)

$$\dot{M} = \left( \frac{r_0}{r_D} \right)^\beta \dot{M}(r_D),$$

(11)

where $r_0 = 10 r_\odot$ is a characteristic radius at which the disc wind, which carries away mass from the disc, starts. The value of $\beta$ controls the strength of mass-loss: $\beta = 0$ means no mass-loss, $\beta = 1$ means very strong mass-loss. GR numerical simulations suggest $\beta \approx 0.5$ (McKinney et al. 2012; Narayan et al. 2012).

We show the effects of disc mass-loss in Figs 8 and 9 for three different values of mass-loss parameter: $\beta = 0, 0.5, and 1$. Fig. 8(a) shows that the presence of mass-loss suppresses BH mass accretion rate: the thick solid line, which corresponds to the case $\beta = 0$ and does not include any mass-loss (see equation 11), lies above the other two lines for $\beta = 0.5$ and 1, both of which include mass-loss. The suppression of $M$ for $\beta > 0$ leads to the suppression of BH mass growth: as is clear from Figs 8(a) and (b), the higher the value of $\beta$, the stronger the mass-loss, the smaller the BH mass. However, the plausible range of $\beta$ we considered results in only a modest change in $M_{\text{BH}}$, by at most a factor of 2. Fig. 8(c) shows that mass-loss does not strongly affect BH spin, either: its value levels off at $a \approx 1$ soon after jet breakout for the entire range of $\beta$ considered. This is because by the time mass-loss becomes substantial (i.e. $r_0 > r_D$), the BH was already spun up to a near-maximum spin, $a \gtrsim 0.9$.

Fig. 9(a) shows that jet power, $L_j \propto M_{\text{BH}}^2$, is slightly increased for $\beta > 0$, due to the suppression of BH mass growth. Fig. 9(b) to turn off at least as steeply as the bursts are observed to decline, which is consistent with our findings.

What sets the duration of the GRB in our model? There are at least two parameters: the strength of the magnetic flux threading the star, $\Phi_1$, and the density distribution in the outer layers of the star, which sets the late-time dependence of $M$. Since the strength of $\Phi$, can be inferred from GRB jet luminosity via equation (2), the end of the GRB signals the time at which $M = M_{\text{MAD}}$ (see equation 4), i.e. the accretion flow on to the BH enters the MAD regime. Since MADs around rapidly spinning BHs give $L_j \approx MC^2$ (see equation 3), we can estimate mass accretion rate at the end of the GRB as

$$\dot{M}_{\text{turnoff}} = \frac{L_j}{c^2} = 0.6 \times 10^{-4} M_\odot \text{ s}^{-1} \frac{L_j}{10^{39} \text{ erg s}^{-1}}.$$  

(10)

Since, according to equation (4), we have $M_{\text{MAD}} \propto \Phi_{\text{BH}}^{1/2}$, stronger BH magnetic flux translates into a higher mass accretion rate at GRB shutoff. Therefore, for larger $\Phi_{\text{BH}}$ and all else being equal, we would expect the GRB to be more powerful and last for a shorter time. However, this effect is diluted by the fact that more powerful jets breakout faster and thus we start seeing GRB earlier (see equation 1). We will also see later (Section 3.4) that changes in stellar angular momentum profile strongly affect the jet breakout time and the GRB duration.

### 3.2 Steep decline phase

Steep decline stage is routinely seen by Swift/XRT as a rapid drop-off of GRB luminosity at the end of the GRB, as $L_j \propto r^{-3-5}$. In our model, the steep decline stage starts at the formation of the MAD. In this steep decline stage, we are likely seeing the shutoff of the central engine: the decrease in mass accretion rate below $M_{\text{MAD}}$ (see equation 4) leads to the MAD state, in which the jet power tracks the rapidly declining $M$. Thus, we can conclude that the observed steep decline is the natural result of the central BH accreting the outer layers of the star. In fact, we find that the jet power drops extremely steeply, $L_j \propto M \propto r^{-20}$ or so. The resulting gamma-ray luminosity, $L_{\gamma}$, will experience a shallower decline, depending on the size of the emitting region (Kumar & Panaite 2000). Thus, in order to explain the observed decline, $L_{\gamma} \propto r^{-3-5}$, the jet has

![Figure 7. Panels (a) and (b) show, respectively, jet power and magnetic flux versus time, for different distributions of magnetic flux in the progenitor star: $y = 0$ (thin), 1 (medium-thickness), 2 (thick solid line). Connected red circles show the times of disc formation, green squares the times of jet breakout (=GRB trigger), and blue stars the times of MAD formation (=end of GRB). Variations in spatial distribution of magnetic flux lead to at most variations by factors of a few in GRB luminosity for the different models we considered.](https://academic.oup.com/mnras/article-abstract/447/1/327/987092/fig7)
Figure 8. Mass accretion rate $\dot{M}$, BH mass $M_{\text{BH}}$, and BH spin, $a$ as a function of time for the case when we explore the effect of mass-loss in a wind from an accretion disc. The mass-loss is modelled through suppression of $\dot{M}$ versus radius, $\dot{M} = (10 r_g/r_D)^\beta \dot{M}_D$, where $r_D$ is the outer radius of the accretion disc. We explore three different $\beta$ values: 0, 0.5, and 1.

shows that mass-loss does not change the amount of magnetic flux brought to the BH, i.e. the three solid lines, each for a different value of the mass-loss parameter $\beta$, are on top of each other. This is to be expected, since in our model the time-dependence of $\Phi_{\text{BH}}$ is determined by the initial distribution of magnetic flux in the progenitor star and therefore depends neither on mass-loss nor the choice of $\beta$ (see equation 11).

Whereas mass-loss only weakly affects the GRB luminosity, it does affect the duration of the prompt emission: the larger the mass-loss, the earlier the MAD onset, the shorter the GRB. Figs 8 and 9 indicate the MAD onset time with blue stars: for a stronger mass-loss (larger values of $\beta$), the MAD forms at an earlier time. This is to be expected, since mass-loss diminishes mass accretion rate and therefore BH magnetic flux, which remains unaffected by mass-loss, becomes dynamically important at an earlier time.

Thus, all our previous conclusions hold in the presence of mass-loss. Summing up, jet power throughout the GRB duration is rather constant. The presence of mass-loss causes the remnant BH to be smaller, and therefore, for the same available magnetic flux the jet power to be slightly larger. Since mass-loss reduces $\dot{M}$ at the BH, the disc turns MAD earlier and therefore the GRB duration is shortened. The effects of the mass-loss on GRB light curves are thus moderate at most.

3.4 Effect of stellar rotation

An important, but poorly understood, property of GRB progenitor stars is their rotation. There is observational indication of anticorrelation between stellar metallicity and the occurrence of GRBs (see, e.g. Modjaz et al. 2008). This can be understood if rapid rotation of progenitor is required for its core collapse to result in a GRB: higher metallicity leads to stronger stellar winds, which extract stellar angular momentum and slow down stellar rotation. In fact, there are many competing models, and the underlying physical processes are not agreed upon.

Motivated by this, we explore the effect of stellar rotation on the emergent GRB luminosity within our model. We started by varying the stellar rotation rate, $\omega$, by a constant factor relative to $\omega_r(r)$, the rotation rate in our fiducial model. Fig. 10(a) shows the analogue of Fig. 6(a) computed for $\omega = 0.23 \omega_r$ (see also Fig. 11). The decrease in stellar rotation causes the jet to break out at a much later time. This is because the lower angular momentum of the stellar envelope means that most of the star collapses into a BH directly, bypassing the formation of an accretion
Jet power and magnetic flux versus time, for slower stellar rotation than in our fiducial model ($\omega = 0.23 \omega_*$) and different values of magnetic flux in the progenitor star, $\Phi_*$. Comparison to Fig. 6 shows that slower rotation does not noticeably affect the power of the jets. However, it does lead to a later time of debris circularization and disc formation (shown with red circles) and jet breakout (shown with green squares that are barely visible under the red circles). Since the time of MAD formation (shown with blue stars) stays the same, slower rotation of a progenitor leads to shorter GRBs.

The above example makes it clear that the standard GRB light curve profile is extremely resilient, even when the stellar rotation is extremely slow: the jet power follows a plateau, which abruptly ends in a steep decline. It could be a concern that the presence of large-scale magnetic field in a progenitor star might lead to the slowdown of the stellar core by magnetic torques and even potentially result in solid-body-like rotation of the star as a whole. Can GRBs be produced in such an unfavourable scenario?

To explore this, we consider solid-body rotation in the progenitor star, with a constant angular velocity, $\omega = \text{constant}$. We choose the rotation rate such that the surface layers of the star rotate at 10 per cent of Keplerian value, which corresponds to surface rotational velocity $v_\ast \approx 200 \text{ km s}^{-1}$. As Fig. 12(a) illustrates, the mass accretion rate is unaffected by the rotation profile (c.f. Fig. 3a). Since for solid-body rotation most of the angular momentum is carried by the outer layers of the star, the collapse of the core leads to essentially a non-spinning BH, $a(t = 1 \text{ s}) \approx 0.003 \ll 1$ (see Fig. 12b), which is too low a spin to power a jet of substantial power. Thus, naively, it would seem that this scenario is hopeless for producing a GRB!

However, this is not so: just as we saw above in the case of slow rotation, the GRB does not start until the infalling gas hits the centrifugal barrier and forms an accretion disc and a jet. In this model, this happens around $t \approx 57 \text{ s}$ (marked with red circle in Fig. 12a), and the jet breaks out of the star and the GRB starts at $t \approx 76 \text{ s}$ (indicated by the green square). By this late time, the central BH receives most of the angular momentum carried by the outer layers of progenitor star, and BH spin saturates at a respectable value, $a \approx 0.4$. Jet power, shown in Fig. 13(a), and BH magnetic flux, shown in Fig. 13(b), saturate at near-constant values standard for GRBs: $L_j \lesssim 10^{50} \text{ erg s}^{-1}$ and $\Phi \sim 10^{27} \text{ G cm}^2$. The end of the GRB is marked by the onset of MAD and occurs at approximately the same time as in our fiducial model, $t_{\text{MAD}} \approx 100 \text{ s}$. Thus, the duration of the GRB is shortened down to $\approx 40 \text{ s}$ from $\approx 100 \text{ s}$ in our fiducial model.
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GRB properties versus time for a model with a solid-body rotation profile in the progenitor star, with surface rotation speed of 200 km s\(^{-1}\), which is a characteristic rotational speed for O-stars. Panel (a): time-dependence of \(M\) for a progenitor star with a solid-body rotation profile. \(M(t)\) is the same as in our fiducial model, but disc formation and jet breakout happen at a later time due to the lower angular momentum of the progenitor star (compared to Fig. 3). Panel (b): BH spin, \(a\), versus time. Because for solid-body rotation \(\ell \propto \tau^2\), most of the angular momentum resides in the outer layers of the star and reaches the BH late in the course of the core collapse; in fact, direct collapse into BH of the core leads to an essentially non-rotating BH: \(a \approx 3 \times 10^{-2}\) for \(t \approx 1\) s. However, by the time the accreting material hits the centrifugal barrier and circularizes, \(t \approx 60\) s, BH spin reaches a respectably high value, \(a \approx 0.4\).

Thus, the details of radial distribution of angular momentum inside the star are not important for the observational appearance of the GRB: for instance, GRB light curves are essentially identical for the slow stellar rotation, with \(\omega = 0.23\omega_\star\) (see Fig. 11) and the case of solid-body rotation, with \(\omega = 200\) km s\(^{-1}\) (see Fig. 12). The only major underlying difference is that the latter results in an approximately twice as low BH spin and a somewhat shorter GRB. For instance, for the weakest stellar magnetic flux value we considered, \(\Phi_\star = 10^{26.5}\) G cm\(^2\), the jet does not break out of the star at all (see Figs 12a and b) and thus no normal GRB is produced. Therefore, for a normal GRB to occur, it is important to have both sufficient stellar magnetic flux and stellar angular momentum.

The slower the rotation, the shorter the GRB duration. As the angular momentum of the progenitor is reduced, the duration of a core-collapse GRB can become shorter than \(\approx 2\) s, which is the standard divide between the populations of short- and long-duration GRBs (Kouveliotou et al. 1993). This can make long-duration GRBs appear as short GRBs (see also Bromberg et al. 2012, 2013). For very slow rotation, the jets either barely have enough time to break out of the star or fail to break out of the star and can result in a low-luminosity GRB or an X-ray flash. If the analogy with low-luminosity GRBs holds, such a failed or near-failed GRB can be

accompanied by a supernova explosion (Stanek et al. 2003; Modjaz et al. 2006).

So far we have assumed that for the jet to form, the accreting material needs to hit a centrifugal barrier. What happens if the gas never hits the centrifugal barrier and the disc does not form? In such a limit of very slow rotation, it might seem that the jet never forms and there would be no GRB at all, without the disc, there is no low-density funnel through which the jet can escape. However, this is not the case. In fact, the low-density funnel is only a necessity for jet formation when the magnetic field is not dynamically important. However, once a MAD forms, a jet will be produced even by a non-rotating accretion flow in the absence of a low-density funnel (Komissarov & Barkov 2009; Tchekhovskoy et al. 2014). We estimate that the energetics of such events is of order \(L_{\text{MAD}} / 20\), or about 5% of the energetics of normal GRBs. (Here, the factor of 1/20 comes from integrating the luminosity time-dependence \(L_j \propto t^{-20}\) around the onset of the MAD regime.)

Figure 12. GRB properties versus time for a model with a solid-body rotation profile in the progenitor star, with surface rotation speed of 200 km s\(^{-1}\), which is a characteristic rotational speed for O-stars. Panel (a): time-dependence of \(M\) for a progenitor star with a solid-body rotation profile. \(M(t)\) is the same as in our fiducial model, but disc formation and jet breakout happen at a later time due to the lower angular momentum of the progenitor star (compared to Fig. 3). Panel (b): BH spin, \(a\), versus time. Because for solid-body rotation \(\ell \propto \tau^2\), most of the angular momentum resides in the outer layers of the star and reaches the BH late in the course of the core collapse; in fact, direct collapse into BH of the core leads to an essentially non-rotating BH: \(a \approx 3 \times 10^{-2}\) for \(t \approx 1\) s. However, by the time the accreting material hits the centrifugal barrier and circularizes, \(t \approx 60\) s, BH spin reaches a respectably high value, \(a \approx 0.4\).

Figure 13. GRB properties versus time for a model with a solid-body rotation profile in the progenitor star, with surface rotation speed of 200 km s\(^{-1}\), which is a characteristic rotational speed for O-stars. Panel (a): GRB luminosity versus time for different values of stellar magnetic flux, as labelled (see also Fig. 6). The relatively modest solid-body rotation leads to a delayed formation of disc and jet: around 60 s post-collapse. Despite this delay, the light curve follows the standard GRB template, with a constant-luminosity plateau lasting about 40 s (shorter for weaker bursts) and followed by an abrupt shutoff. For the weakest magnetic flux, \(\Phi_\star = 10^{26.5}\) G cm\(^2\), the jet is so weak that it does not make it out of the star before the accretion turns off. Panel (b): BH magnetic flux \(\Phi_{\text{BH}}\) as function of time, for different values of total stellar magnetic flux, \(\Phi_\star\). The magnetic flux time-dependence is identical to that in our fiducial model shown in Fig. 6b. Thus, the primary effect of the solid-body rotation is the delayed time of disc and jet formation and shortening of the GRB duration.
Progenitor magnetic flux and GRB light curves

Figure 14. BH mass and spin versus time since the trigger for different stellar progenitor models: 16TH (thick solid red lines), 16TI (medium-thickness solid black lines), and 16TJ (thin cyan lines). In all cases, we use our fiducial model parameters: $\gamma = 1$, $\beta = 0$, $\Phi_* = 10^{27.5}$. Panel (a) shows BH mass change is between 10 per cent for model 16TH and 50 per cent for models 16TI and 16TJ. BH spin remains approximately constant in time (to better than 20 per cent) and levels off to a high value: $a \approx 0.9$ for model 16TH and $a \approx 1$ for models 16TI and 16TJ.

Thus, on the energetics grounds it is conceivable that such events could indeed be the counterparts of low-luminosity GRBs.

Summarizing, GRBs appear to be successfully produced over a wide range of progenitor rotation rates and profiles. The lower the angular momentum of the progenitor star, the shorter is the GRB that results. However, regardless of the progenitor rotation rate and profile, GRBs robustly show an early-time plateau followed by the sharp decline. Thus, the observational appearance of GRBs appears to be insensitive to the details of angular momentum distribution inside the progenitor star. How do other parameters of the progenitor affect the GRB?

3.5 Effect of stellar progenitor model

We will now investigate the effect of the pre-collapse stellar model on GRB light curves. For simplicity, in all cases we choose our fiducial parameters: $\gamma = 1$, $\beta = 0$ (no mass-loss), and $\Phi_* = 10^{27.5}$. Fig. 14 shows the time-dependence of BH mass (panel a) and spin (panel b) for three different progenitor models: 16TH, 16TI, and 16TJ. The temporal profiles of $M_{\text{BH}}$ and $a$ are qualitatively similar in all three models: both mass and spin level off early in the course of the GRB and do not change thereafter. In all models BH spin ends up at a rather high value, $a \gtrsim 0.9$.

Fig. 15(a) shows that $M$ drops by several orders of magnitude during the GRB. However, $M$ does not directly control jet power, $L_j$ (see Section 2.2.2). For this reason, $L_j$ remains mostly constant throughout the GRB, as is clear from Fig. 15(b). All three progenitor models lead to virtually indistinguishable shapes of prompt emission light curves: all of them show about 50 per cent variation at the very beginning of the GRB, after which the emission levels off to a constant. In all cases, at the end of the GRB, the jet power abruptly drops. This happens when $M$ drops below $M_{\text{MAD}}$ (see equation 4), after which jet power begins to track the rapidly decreasing mass accretion rate. The durations of the GRBs for the three different stellar models we have considered are within a factor of 2 of each other and in all cases are set by the free-fall time-scale of the outer layers of the progenitor star.

Figure 15. Time variation of various quantities for different stellar progenitor models (see the legend and caption to Fig. 14 for details). In all cases, we use our fiducial model parameters: $\gamma = 1$, $\beta = 0$, $\Phi_* = 10^{27.5}$. Panel (a): shows mass accretion rate $\dot{M}$ on a logarithmic scale: it changes by many orders of magnitude during the GRB and differs from one model to another by 1–2 orders of magnitude. Panel (b): solid lines show the power of the jets $L_j$ on a linear scale. In contrast to $\dot{M}$, the power remains relatively constant and changes between the models by at most a factor of 2. This is because during the GRB, jet power is decoupled from the widely varying mass accretion rate. To help us visually quantify the constancy of $L_j$, the lighter-coloured dashed lines and the right y-axis show cumulative light curves of jet power, $E_j(<t) = \int_0^t L_j dt'$, for the three models. That $E_j(<t)$ shows little curvature throughout the prompt emission is a reflection of the near-constancy of its slope, or jet power.
In summary, all basic properties like $M_{\text{BH}}$, $a$, and $M$ do not differ substantially between different stellar models. For a fixed magnetic flux through the progenitor, $\Phi_*$, the jet power $L_j$ differs by factors of just a few. The magnetic flux therefore appears to be the only quantity that can feasibly account for the huge range of observed GRB luminosity. Note, however, that the progenitor properties such as rotation can have an effect on the GRB duration (Section 3.4) and may have an indirect effect on the field strength (e.g. the faster rotators can have stronger magnetic fields). Since the origin of stellar magnetism is not well understood, we do not explore such cross-correlations in this work.

4 COMPARISON WITH GRB LIGHT CURVES

The GRB emission is made up of a superposition of many pulses. The pulse properties, such as peak flux, duration, inter-pulse intervals, do not change in any systematic way during the burst. As a result, cumulative photon counts during GRBs increase, on average, linearly with time: $\int_{0}^{t} L_{\gamma} \, dt \sim \text{constant} \times t$ or $L_{\gamma} \sim \text{constant}$ (McBreen et al. 2002). Here, we show that our model reproduces this observation.

In Fig. 15(b), we plot along the right y-axis the cumulative light curves of our simulated GRBs with dashed lines. The cumulative light curves are mostly straight lines throughout the prompt emission, indicating the near-constancy of jet power. We will now apply this type of visual analysis to several representative BATSE light curves.

McBreen et al. (2002) demonstrated that the cumulative count rate increases linearly with time in a sample of ~500 bright BATSE bursts. Here, we repeat their analysis for representative GRBs, 940817, 940210, 920513, and compare the results to the predictions of our model.

We chose GRBs 940817 and 940210 because they are rather clean examples of the linear increase of the radiated energy versus time followed by a sharp turn-off of the GRB emission. In Figs 16 and 18, we overplot the cumulative count rate from these bursts with those for our model (we rescaled the time axis to fit the observed duration). We adopt our fiducial model parameters: the magnetic flux distribution index of $\gamma = 1$, the total magnetic flux of $\Phi_* = 10^{27.5}$ G cm$^{-2}$, no mass-loss from the disc ($\beta = 0$), along with the 16TI progenitor star model. The results for other combinations of model parameters, $\gamma$, $\beta$, and $\Phi_*$, and progenitor models are similar (not shown).

As is illustrated by Figs 16 and 18, our models naturally account for the near constancy of the burst luminosity over its duration as well the steep decline of the jet power at $t \approx t_{\text{wag}} \sim 60$ s and 30 s, respectively. For comparison, in Figs 17 and 19 we show the implied gas and magnetic pressures as functions of distance in the pre-collapse star for the same two models. The gas pressure dominates magnetic pressure in the star by a typical factor of ~10$^9$.

4.1 Magnetic tomography of progenitor stars

From Fig. 20, it is clear that gross properties of the cumulative count rate from GRB 920513 can also be reproduced by this simplest version of the model. However, the cumulative light curve has either a pronounced bump at $t - t_{\text{wag}} \sim 40$ s or a deficit at $t - t_{\text{wag}} \sim 60$ s.

Such a deviation from the simple prescription where magnetic flux scales with mass in the progenitor star can be used as a probe of the stellar field structure. To account for the relative weakness of the burst at the interval $50 \lesssim t \lesssim 80$ s, we introduce an additional torus-shaped component of poloidal magnetic field centred at $t - t_{\text{wag}}$.

$\Phi_{BH}^{GRB920513}(t) = \Phi_* \left( \frac{M_{\text{collapsed}}}{M_*} \right)$

\begin{equation}
\times \left\{ 1 - f_0 \exp \left[ - \frac{(t - t_{\text{wag}} - t_0)}{\Delta t_0} \right] \right\}.
\end{equation}

where the term in the curly braces represents BH magnetic flux suppression factor. For illustration, we performed a by-eye fit and obtained the following parameters $f_0 = 0.55$, $t_0 = 63$ s, and $\Delta t_0 = 23$ s. Here, $f_0$ is a dimensionless parameter equal to the dimensionless magnetic flux carried by the knot, i.e. the ratio of the magnetic flux in the knot to the net flux in the star. Thus, $f_0 = 1$ means that, as
the knot is consumed by the BH, the jet power vanishes completely, a value of \( f_0 \lesssim 1 \) leads to a substantial suppression of BH magnetic flux and GRB luminosity, whereas \( f_0 \ll 1 \) leads to a small dip in the flux and luminosity.

When the inner half of the magnetic knot accretes \( (t - t_{\text{trig}} \sim t_0 - \Delta t_0 = 40 \, \text{s}) \), the total BH magnetic flux gets reduced by a factor of \((1 - f_0)^{-1} \approx 2\) and the jet power by a factor of \((1 - f_0)^{2} \approx 5\). Subsequent accretion of the outer half of the knot \( (t - t_{\text{trig}} \sim t_0 + \Delta t_0 = 86 \, \text{s}) \) replenishes the BH magnetic flux and leads to the recovery of jet power. Note that the magnetic knot fully resides under the stellar surface and so does not affect the surface magnetic field strength.

GRB 940210 light curve shows two prominent deficits, or quiescent intervals, as seen in Fig. 18: one at \( t - t_{\text{trig}} \sim 5-10 \, \text{s} \) and another one at \( t - t_{\text{trig}} \sim 15-20 \, \text{s} \). Similar to GRB 920513, we can model these quiescent intervals by introducing magnetic knots into an otherwise large-scale magnetic flux distribution in the star. As each magnetic knot is accreted by the BH, the knot’s magnetic flux cancels out a substantial fraction of BH magnetic flux and leads to a depression in GRB luminosity. We model the magnetic flux in the knots similar to equation (12):

\[
\Phi_{\text{BH}}^{\text{GRB940210}}(t) = \Phi_\ast \left( \frac{M_{\text{collaps}}}{M_\ast} \right) \times \prod_{i=1,2} \left\{ 1 - f_i \exp \left[ -\frac{(t - t_{\text{trig}} - t_i)^2}{\Delta t_i} \right] \right\} .
\]

Here, we choose \( f_1 = 0.85, t_1 = 6.1 \, \text{s}, \Delta t_1 = 2.9 \, \text{s} \), and \( f_2 = 0.55, t_2 = 18.6 \, \text{s}, \Delta t_2 = 3.9 \, \text{s} \) (we give the precise values for reproducibility; however, qualitative behaviour of the light curve is insensitive to the details of the fit). Thick green line in Fig. 22 shows the predicted
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6.2 s. The radial profile of gas and magnetic pressures corresponding to
3.1 s, \( \gtrsim 447 \), (net stellar flux), and smaller (but still large) scales,
\( \sigma \) and \( 2010 \), i.e. the flux in the knot is
24
\( f > f_1 \) gives a rather good fit
20
\( \sigma \) and
22
\( \gtrsim 1 \) gives a rather good fit
18
\( f_1 \). For this reason, it is surprising that for
19
\( f > f_1 \), i.e. the flux in the knot is lower than the net flux through the star. How does a knot know about the net magnetic flux through the star?

We tried and were unable to obtain a visually good fit to the
20
\( f > 1 \). The reason for this is clear. For a knot with \( f > 1 \), the magnetic flux on the BH would vanish twice: once at the beginning and once at the end of the consumption of the knot by the BH. This would lead to the cumulative GRB light curve that flattens at the beginning and at the end of the quiescent interval and steepens in between. No such behaviour is obviously present in Fig. 20: in fact, the cumulative light curve appears to rise continuously, suggesting that the knot that led to the quiescent interval in GRB 920513 light curve contained a relatively small magnetic flux compared to that on the BH. The situation is different with GRB 940210, whose light curve is shown in Fig. 22. The light curve appears to show precisely the type of behaviour indicative of a large flux in the knot, at least for the second quiescent interval of the two. Indeed, we find that \( f_1 \) is a good fit to the cumulative light curve, as seen in Fig. 24, with the rest of the parameters similar to those used for the fit shown in Fig. 22: \( f_1 = 0.85, t_1 = 6.2 \) s, \( \Delta t_1 = 3.1 \) s, \( t_2 = 18.7 \) s, and \( \Delta t_2 = 3.1 \) s. Fig. 25 shows that the magnetic pressure in the second knot (indicated with the rightmost vertical grey stripe) is higher than in Fig. 23, reflecting the larger magnetic flux in the knot.

In the above, we assumed that the polarity of magnetic flux in the knots is negative, i.e. once a knot is accreted, it leads to a depression in BH magnetic flux and a quiescent interval in the GRB light curve. However, it is possible that in some cases the polarity of magnetic flux in the knots is positive. This then would lead to ‘hyper-active’ intervals and substantial differences in GRB luminosity from one active interval to another. Statistical analysis of GRB light curves can potentially be used to determine the rates of occurrence of the two polarities of the magnetic knots, thereby providing us with valuable and otherwise not directly accessible information on the magnetic field structure inside of GRB progenitor stars.

So far we have considered magnetic flux inhomogeneities over two different scales inside the progenitor star: large-scales, comparable to \( r_s \) (net stellar flux), and smaller (but still large) scales, around \( 0.1 r_s \) (magnetic knots). It is conceivable that spatial distribution of stellar magnetic flux follows a continuous distribution.

Figure 22. Similar to Fig. 18 but with the addition of two magnetic knots, which are centred at \( t - t_{\text{trig}} \sim 5 \) and \( \sim 20 \) s and indicated by vertical grey stripes and containing magnetic fluxes that make \( \sim 85 \) and \( \sim 55 \) per cent that through the star. Stellar magnetic flux distribution versus radius is given by equation (13). As these knots are consumed by the BH, BH magnetic flux, and GRB power are depressed.

Figure 23. The radial profile of gas and magnetic pressures corresponding to Fig. 22. Each of the two magnetic knots is seen as a pair of bumps on top of the underlying, monotonically decreasing magnetic pressure profile (which is shown in Fig. 19). Vertical grey stripes indicate the 1\( \sigma \) extent of the knots that matches the extent of the bumps in magnetic pressure.
This spatial distribution of magnetic flux in a progenitor star leads to temporal distribution of quiescent intervals. Thus, temporal analysis of GRB light curves is warranted to constrain this distribution and thereby to improve our understanding of the geometry of magnetic fields in the progenitors of GRBs.

GRB light curves show variability over a wide range of time-scales, from milliseconds to tens and hundreds of seconds. Until now, we have focused on how the properties of a GRB evolve on a time-scale similar to its overall duration ($t_{\text{GRB}} \sim 30 \text{ s}$). We showed that the rotation of the star and the free-fall time of its outer layers set the GRB duration. We suggested that the spatial distribution of large-scale inhomogeneities of the magnetic field in the outer layers of the star leads to quiescent intervals in GRB light curves lasting $\gtrsim$ few seconds. For completeness, here we speculate on what drives faster GRB variability, e.g. GRB pulses on $t_{\text{p}} \lesssim 1 \text{ s}$ time-scales. Such GRB variability may originate from the jet interactions with the stellar material. If the collapsing star is not axisymmetric, it can imprint perturbations, e.g. bends, on to the jet. Alternatively, if the jet is strongly magnetized, current-driven instabilities may lead to non-axisymmetric structures as the jet propagates through the star (Levinson & Begelman 2013; Bromberg et al. 2014a). Beaming effects can result in large temporal changes of the observed radiation even for modest changes of the Lorentz factor or the opening angle of the jet. Since the characteristic time-scale for sound or Alfvén waves to cross the jet is $\sim 1 \text{ s}$, short time-scale variability may, in part, be due to the jet propagation through the collapsar.

Another process that may modulate the jet power on a short time-scale is the excursions of the accumulated magnetic flux from the BH into the inner accretion disc and vice versa. The accretion disc may switch between a neutrino-cooled disc and a thick, advective disc multiple times during the GRB. Such transitions can have a profound effect on the jet power and GRB luminosity. Neutrino cooling results in a rather thin disc in the inner $R_{\text{in}} \sim 30 r_{\text{g}}$ (Chen & Beloborodov 2007). When the inner disc is thin, most of BH magnetic flux may diffuse outwards throughout $R_{\text{in}}$, leaving only a small fraction of the magnetic flux on the BH and powering the jet. On the other hand, a thick disc can push most of the available flux into the BH, which can result in flares. Incidentally, the accretion time-scale for the neutrino-cooled disc is a fraction of a second and may be or relevance for the typical duration of such gamma-ray pulses.

5 CONCLUSIONS

GRBs are characterized by the sudden onset of emission that lasts for $T_{\text{GRB}} \sim 1 \text{ min}$ and is followed by a sharp turn off. The time-averaged properties of the prompt emission such as luminosity do not show any clear systematic trends during the GRB. Within the collapsar framework, the trigger of the burst can be naturally associated with the moment at which the jet breaks through the surface of the collapsing star. The trigger takes place at time $t_{\text{trigger}} \sim 10 \text{ s}$ after core collapse (MacFadyen & Woosley 1999; Bromberg et al. 2014a,b). Since $T_{\text{GRB}} \gtrsim t_{\text{trigger}}$, one expects that the accretion rate $\dot{M}$ at the BH evolves (drops) appreciably over $T_{\text{GRB}}$. As a result, in any model for which the jet power directly depends on the accretion rate $\dot{M}$, one would expect the burst to become fainter throughout its duration until it is not detectable anymore. Such a behaviour is not observed.

Associating the gravitational energy release during the accretion on to the BH with the power of the GRB jets is hard on energetic grounds. Wolf–Rayet stars have their masses varying over a narrow range. The BH grows by several solar masses over the first minute past core collapse with the accretion power released $\gtrsim 10^{52} \text{ erg}$. GRB jets, on the other hand, come in a broad range of energies $\sim 10^{50}-52 \text{ erg}$. Therefore, another parameter, and not the accreted mass, sets the GRB energetics and dispersion in their properties. We argue that this parameter is stellar magnetic flux.

To show this, we explored a number of models for collapsing Wolf–Rayet stars and demonstrated that, by the time the jet breaks through the collapsing star and the GRB becomes visible, the mass and spin of the BH are close to their asymptotic values, i.e. $M_{\text{BH}} \sim 10 M_{\odot}$, $a \sim 1$, respectively. For reasonable assumptions about magnetic flux distribution in the star, the same holds true for the total magnetic flux $\Phi_{\text{BH}}$ at the BH. Since the jet power in the BZ model depends only on these three weakly changing parameters ($M_{\text{BH}}, a, \Phi_{\text{BH}}$), this explains the rough constancy of GRB luminosity during the burst.

But what causes the sharp turn off of the GRBs? Since mass accretion rate decreases asymptotically to zero, eventually, the accretion rate drops to such a low level that the disc cannot confine
the BH magnetic flux anymore. Then, the flux diffuses out into the disc, and the disc enters the MAD state (see Section 3.2). In MADs, jet power is proportional to mass accretion rate, and the jet power drops fast with time as \( L_j \propto M \propto r^{-2.02} \). Thus, it is the transition to the MAD regime causes the abrupt turnoff of the GRBs seen with Swift (see Lazzati, Perna & Begelman 2008 for an alternative mechanism).

What leads to the large burst-to-burst variation of GRB luminosity? In our model, the jet power is set by the magnetic flux available in the progenitor star. Typical BH magnetic flux of \( \Phi_{\text{BH}} \sim 10^{17.5} \text{ G cm}^{-2} \) is required to account for the observed luminosity of bright bursts, which implies a progenitor with surface magnetic field strength of \( B \sim 10^4 \text{ G} \). Given the large observed variation in surface stellar magnetic field strength, it is conceivable that \( \Phi_{\text{BH}} \) varies by a factor of \( \gtrsim 30 \) between different progenitors, and since jet power scales as \( \Phi_{\text{BH}}^2 \), such a variation translates into 3 orders of magnitude variation in jet power, which is more than sufficient to account to the full observed range of GRB luminosities.

In addition to power, the GRBs show a large variety in their durations, from a few to a hundred seconds. What can cause such diversity? In our model, the GRB starts at the jet break out time and ends abruptly when the disc enters the MAD state. The jet breakout time is controlled by the longest of disc formation and jet propagation times. In most cases, it takes \( \sim 10^3 \) s for the disc to form, which sets the trigger time, after which the GRB is detectable. It takes longer for the disc to form in progenitor stars with slow rotation. In such cases, the formation of the disc (and jet) can be substantially delayed, by tens of seconds, thereby shortening the GRB duration. Other than that, the slow rotation does not leave noticeable imprint on the GRB light curves, thus core-collapse GRBs with durations much shorter than the free-fall time of the outer layers of the progenitor star (which is around 100 s), might be suggestive of low angular momentum of their progenitors (however, see Lazzati et al. 2013).

What can cause slow stellar rotation? One possibility is that magnetic torques couple the rotation of different layers inside the star, slow down the rotation of the core and potentially lead to near solid-body rotation of the progenitor, with the outer layers of the star carrying most of the stellar angular momentum. We showed that even such an extreme scenario can be capable of leading a long-duration GRB, suggesting that GRBs can be more robust than previously thought. Thus, in order to get a successful long-duration GRB, the progenitor star needs to contain sufficient angular momentum for the accretion flow to encounter a centrifugal barrier and spin-up the central BH to a moderate spin. If high metallicity is conducive to the production of strong stellar winds, which remove stellar angular momentum, low stellar metallicity might be conducive to the production of long-duration GRBs. In the limit of extremely low stellar angular momentum, infalling stellar material does not encounter a centrifugal barrier, and an accretion disc does not form. In this scenario, the jet forms just before the onset of MAD, and we suggest that such extreme cases can be counterparts of failed or low-luminosity GRBs. These considerations may indicate that the flux through the progenitor is at least as decisive of a factor for a successful GRB as the rotation of the collapsing star.

The MAD formation time (=the time at which GRB ends) is very close to the free-fall time of the outer layers of the star. Thus, another factor that controls the GRB duration is the radius of the progenitor star, with larger stars leading to longer GRBs. In this work, we neglected the effects of accretion disc formation on the GRB duration, which we estimate are insignificant, unlessstellar rotation makes up a substantial fraction of Keplerian velocity. If the stellar rotation is near-Keplerian, an accretion disc of a large size and long accretion time can form, thereby extending the central engine activity time beyond the free-fall time of the outer layers of the star.

We used our model to probe the structure of the magnetic flux in the star. By fitting the light curve of GRB 940917, we infer that the total magnetic flux through the BH varies little throughout the burst. The same conclusion holds for the majority of bright GRBs that exhibit approximately linear cumulative count curves (McBreen et al. 2002). We argue that quiescent intervals seen in many GRB light curves arise due to inhomogeneities of the magnetic flux distribution inside the star. In at least one case, GRB 920513, there is a clear deviation from linear cumulative curve during the second half of that GRB. We infer the presence of a magnetized torus, or a ‘knot’, in the outer half of the star. In another case, GRB 940210, which shows two such deviations, we infer the presence of two such knots. While the magnetic field structure in the stellar interiors is an open question, knot-like magnetic flux configurations are expected from MHD stability arguments (Braithwaite & Spruit 2004; Braithwaite 2006). Statistical analysis of a larger number of GRB light curves might be a powerful probe of stellar magnetic structure.

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APPENDIX A: TIME-SCALES INVOLVED IN CORE COLLAPSE AND ACCRETION

As we discussed previously (see footnote 1), rotation in progenitor models is uncertain, and in fact it can exceed the Keplerian value (see Fig. 2b). Thus, in our work, we limit the rotation to be at most 10 per cent of Keplerian value. The resulting angular momentum profile is shown in panel (a) with the solid line; the Keplerian angular momentum is shown with the dashed line. We assume that gas falls freely from its initial position $r$, until it hits the centrifugal barrier at the circularisation radius, $R_{\text{circ}}$; the ratio of the two is shown in panel (b). Inside of that radius, gas travels viscously in the form of an accretion disc, with the accretion time $t_{\text{acc}} \approx r/v_r$, that is much shorter than the free-fall time, $t_{\text{ff}}$, as seen in panel (c). In the plot, we set $h/r = 1$ and $\alpha = 0.1$. Since gas transit time though the accretion disc is negligible compared to the free-fall time, we for simplicity ignore it.

![Figure A1](https://academic.oup.com/mnras/article-abstract/447/1/327/987092/fig)
We assume that gas falls freely from its initial position $r$, until it hits the centrifugal barrier at the circularization radius, $r_D$,

$$\frac{r_D}{r} = \left( \frac{\ell}{\ell_K} \right)^2 .$$  \hspace{1cm} (A1)

The ratio (A1) is shown in Fig. A1(b). Inside of $r_D$, gas travels viscously in the form of an accretion disc, with the accretion time

$$t_{\text{acc}} \approx \frac{r_D}{v_y} \approx \alpha^{-1} \left( \frac{R_{\text{circ}}}{r_g} \right)^{1/2} \left( \frac{\ell}{r} \right)^{-2} \frac{r_g}{c} .$$  \hspace{1cm} (A2)

Fig. A1(c) shows the ratio if accretion time to free-fall time, $t_{\text{ff}} = (2r_g/r)^{1/2}$:

$$\frac{t_{\text{acc}}}{t_{\text{ff}}} = \left( \frac{r_D}{r} \right)^{3/2} \left( \frac{h}{r} \right)^{-2} = \frac{2^{3/2}}{\alpha} \left( \frac{\ell}{\ell_K} \right)^{3/2} \left( \frac{h}{r} \right)^{-2} .$$  \hspace{1cm} (A3)

In Fig. A1, for illustration we choose $h/r = 1$ and $\alpha = 0.1$. Since gas transit time through the accretion disc is negligible compared to the free-fall time, we for simplicity ignore the accretion time. That said, there could be scenarios in which $t_{\text{acc}} \gg t_{\text{ff}}$ (e.g. if stellar rotation is close to Keplerian). In such extreme cases, one does need to account for the accretion time.