Quantum Field Theory Solution To The Gauge Hierarchy
And Cosmological Constant Problems

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Abstract

A quantum field theory formalism is reviewed that leads to a self-
consistent, finite quantum gravity, Yang-Mills and Higgs theory, which
is unitary and gauge invariant to all orders of perturbation theory. The
gauge hierarchy problem is solved due to the exponential damping of the
Higgs self-energy loop graph for energies greater than a scale \( \Lambda_H \leq 1 \)
TeV. The cosmological constant problem is solved by introducing a fun-
damental quantum gravity scale, \( \Lambda_G \leq 10^{-4} \) eV, above which the virtual
contributions to the vacuum energy density coupled to gravity are ex-
ponentially suppressed, yielding an observationally acceptable value for
the particle physics contribution to the cosmological constant. Classical
Einstein gravity retains its causal behavior as well as the standard agree-
ment with observational data. Possible experimental tests of the onset of
quantum nonlocality at short distances are considered.

1 Introduction

We shall base our study of the hierarchy problems on a finite quantum
field theory (FQFT) which is gauge invariant, finite and unitary to all orders
of perturbation theory [1-16]. In contrast to superstring theory and membrane
theory [17, 18], our FQFT formalism does not require the existence of extra di-
mensions to guarantee its consistency, so we shall develop our field theory in a
four-dimensional spacetime. This has the advantage over string and membrane
theory that we do not have to concern ourselves with the problems of com-
 pactification. In string theory there are numerous ways to compactify to lower
dimensions and it is difficult at present to justify any unique method for achiev-
ing this compactification. Moreover, as has been learned during the recent past,
the string scale is not uniquely fixed at the Planck scale [19]. String theory is
not a quantum field theory in the usual sense. All attempts to formulate a string
field theory have so far been unsuccessful. Such theories demonstrate instabili-
ties equivalent to those met with in \( \phi^3 \) field theories, in which the Hamiltonian
is unbounded from below [20].

Our quantum field theory has strictly local tree graphs and nonlocal quan-
tum loop graphs. The FQFT gauge formalism is applied to guarantee a self-
consistent quantum gravity theory coupled to the Yang-Mills, Higgs and spinor
fields. The formalism is free of tachyons and unphysical ghosts and satisfies uni-
tarity to all orders of perturbation theory. It could incorporate supersymmetry
if required, in the form of a supergravity theory, but we shall not do so here, in order to aim for as minimal a scheme as possible. No attempt is made to unify gauge fields with gravity, or to extend the standard model, for we wish to focus on the hierarchy problems.

It is commonly believed that local quantum field theory is the only way to guarantee a consistent Poincaré invariant quantum mechanics \[21\]. If one is willing to give up the notion of a strictly local observable, then this belief can be shown to be incorrect. The issue depends on the support of the field operators and for nonlocal field theories, it can be shown that it is impossible to construct observables whose support is a compact set. Such theories emerge as “quasi-local” field theories whose local behavior only acts at distances much larger than a certain length scale \( \ell \). Nonlocal field theories were the subject of considerable study in the 60s and 70s, because it was thought that they could cure the problems of non-renormalizable field theories\[22\].

Recently, there has been renewed interest in nonlocal field theories in connection with string theory, M-theory and Little String Theory (LST) \[36, 24\]. The LSTs are generated by decoupling gravity and other bulk modes from five-branes. One takes, for example, \( N \) coincident five-branes and considers the limit \( g_s \to \infty, M_s \to \infty \), where \( g_s \) is the string coupling at spatial infinity and \( M_s = 1/\sqrt{\alpha'} \) is the string scale. It was shown by Kapustin \[24\] that LSTs do not possess local observables and that due to the exponentially increasing density of states, the Wightman functions are not polynomially bounded. However, he showed that the nonlocality can be accommodated by choosing a space of test functions different from the usual Schwartz space. We shall consider these issues in more detail in Sect. 4.

The standard gauge symmetry of local quantum field theory is generalized to a nonlocal transformation consisting of an inhomogeneous term, which preserves the local, quadratic part of the action, and a nonlocal homogeneous term, which generates a variation of the free field action that cancels the inhomogeneous variation of the nonlocal action. This generalized gauge transformation is similar to the nonlocal gauge transformation of string field theory \[20\]. The key to the success of string theory lies in the nature of this generalized gauge invariance. Its existence guarantees the raison d’être of gauge symmetry in quantum field theory, which is to decouple unphysical vector and tensor quanta while maintaining Poincaré invariance.

The fundamental gauge hierarchy problem is resolved because the finite scalar Higgs self-energy loop graphs are damped exponentially at high energies above the physical Higgs scale \( \Lambda_H \) set by the FQFT formalism and by choosing \( \Lambda_H \leq 1 \) TeV. The constant \( \Lambda_H \) enters naturally, because it sets the physical non-localizable energy scale of the Higgs particle quantum loop graphs.

The cosmological constant problem is considered to be the most severe hierarchy problem in modern physics \[25, 26, 27, 28\]. The problem arises because, in contrast to classical Newtonian gravity theory, the Einstein gravitational Lagrangian \( \mathcal{L}_{\text{grav}} \) is not invariant under the translation \( \mathcal{L}_{\text{grav}} \to \mathcal{L}_{\text{grav}} + C \), where \( C \) is a constant identified with the cosmological constant \( \lambda \). Many attempts to solve this hierarchy problem have been made \[25, 26, 27, 28\], and most recently
there has been a proposal to solve the problem by postulating a composite graviton connected to string theory \[29\]. A model based on (3+1) branes and a five-dimensional bulk has also recently been proposed\[30, 27\].

Solving the cosmological constant problem appears to demand a low-energy mechanism to cancel soft photon loop contributions. How can we obtain such a mechanism in the low-energy framework without destroying the familiar successes of the standard model?

We shall propose a quantum gravity solution to the problem based on FQFT. We can define an effective cosmological constant

$$
\lambda_{\text{eff}} = \lambda + \lambda_{\text{vac}},
$$

(1)

where \(\lambda\) is the “bare” cosmological constant in Einstein’s classical field equations, and \(\lambda_{\text{vac}}\) is the contribution that arises from the vacuum density \(\lambda_{\text{vac}} = 8\pi G \rho_{\text{vac}}\). Already at the standard model electroweak scale \(\sim 10^2\) GeV, a calculation of the vacuum density \(\rho_{\text{vac}}\) results in a discrepancy with the observational bound

$$
\rho_{\text{vac}} < 10^{-47}\ (\text{GeV})^4,
$$

(2)

of order \(10^{55}\), resulting in a a severe fine tuning problem, since the virtual quantum fluctuations giving rise to \(\lambda_{\text{vac}}\) and the “bare” cosmological constant \(\lambda\) must cancel to an unbelievable degree of accuracy. If we choose the quantum gravity scale \(\Lambda_G \leq 10^{-4}\) eV, then our quantum gravity theory leads to an exponential damping of gravitational vacuum polarisation for \(p^2 \gg \Lambda_G^2\), where \(p^2\) is the square of the Euclidean graviton momentum. This suppresses the cosmological constant \(\lambda_{\text{vac}}\) below the observational bound (2). Since the graviton tree graphs in our FQFT are identical to the standard point like, local tree graphs of perturbative gravity, we retain classical, causal GR and Newtonian gravity theory, and the measured value of the gravitational constant \(G\). Only the quantum gravity loop graphs are suppressed above energies \(\leq 10^{-4}\) eV. Thus, at very low energies or large distances, the point-like, local graviton dominates giving rise to classical Newtonian and GR dynamics.

The scales \(\Lambda_H\) and \(\Lambda_G\) are determined by the quantum non-localizable nature of the Higgs particle and the gauge particles \(W\) and \(Z\) of the standard model as compared to the graviton. The Higgs particle radiative corrections have a nonlocal scale at \(\ell_H \sim 10^{-16}\) cm, whereas the graviton radiative corrections are localizable down to a large length scale \(\ell_G \leq 1\) cm. Thus, the fundamental energy scales in the theory are determined by the underlying physical nature of the particles and fields and do not correspond to arbitrary cut-offs, which destroy the gauge invariances of the field theory. The underlying explanation of these physical scales must be sought in a more fundamental theory.

In Section 2, we describe the basic local action of the theory and in Section 3, we provide a review of FQFT as a perturbative quantum field theory. The nonlocal quantum behavior of the theory is considered in detail in Section 4, and in Section 5, we discuss a possible experimental test of the onset of nonlocality by detecting CPT asymmetries. In Sections 6 and 7, we develop the formalism for Yang-Mills gauge theory and quantum gravity. In Section 8, we turn our
attention to the resolution of the gauge hierarchy problem in the Higgs sector, while in Section 9, we analyze the results of gluon and gravitational vacuum polarization calculations. In Section 10, we use FQFT quantum gravity to resolve the cosmological constant problem and in Section 11, we end with concluding remarks.

2 The Action

We shall begin with the four-dimensional action

\[ W = W_{\text{grav}} + W_{\text{YM}} + W_{\text{H}} + W_{\text{Dirac}} + W_{M}, \]

where

\[ W_{\text{grav}} = -\frac{2}{\kappa^2} \int d^4x \sqrt{-g} (R + 2\lambda), \]

\[ W_{\text{YM}} = -\frac{1}{4} \int d^4x \sqrt{-g} \text{Tr}(F^2), \]

\[ W_{\text{H}} = -\frac{1}{2} \int d^4x \sqrt{-g}[D_\mu \phi^i D^\mu \phi^i + V(\phi^2)], \]

\[ W_{\text{Dirac}} = \frac{1}{2} \int d^4x \sqrt{-g} \bar{\psi} \gamma^\mu e_a^\mu \left[A_i^\mu \phi^i - \omega_\mu \psi - D(A_{i\mu}) \psi \right] + \text{h.c.} \]

Here, we use the notation: \( \mu, \nu = 0, 1, 2, 3 \), \( g = \det(g_{\mu\nu}) \) and the metric signature of Minkowski spacetime is \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \). The Riemann tensor is defined such that

\[ R^\lambda_{\mu\nu\rho} = \partial_\rho \Gamma_\mu^\lambda - \partial_\mu \Gamma^\lambda_{\nu\rho} + \Gamma_\mu^\alpha \Gamma_\rho^\lambda \Gamma_{\nu\alpha} - \Gamma_\rho^\alpha \Gamma_\nu^\lambda \Gamma_\mu^\alpha. \]

Moreover, h.c. denotes the Hermitian conjugate, \( \bar{\psi} = \psi^\dagger \gamma^0 \), and \( e_a^\mu \) is a vierbein, related to the metric by

\[ g_{\mu\nu} = \eta_{ab} e_a^\mu e_b^\nu, \]

where \( \eta_{ab} \) is the four-dimensional Minkowski metric tensor associated with the flat tangent space with indices a,b,c,... Moreover, \( F^2 = F_{i\mu\nu} F^{i\mu\nu} \). \( R \) denotes the scalar curvature, \( \lambda \) is the cosmological constant and

\[ F_{i\mu\nu} = \partial_\nu A_{i\mu} - \partial_\mu A_{i\nu} - e f_{ikl} A_{k\mu} A_{l\nu}, \]

where \( A_{i\mu} \) are the gauge fields of the Yang-Mills group with generators \( f_{ikl} \), \( e \) is the coupling constant and \( \kappa^2 = 32\pi G \) with \( c = 1 \). We denote by \( D_\mu \) the covariant derivative operator

\[ D_\mu \phi^i = \partial_\mu \phi^i + e f^{ikl} A_k^\mu \phi^l. \]

The Higgs potential \( V(\phi^2) \) is of the form leading to spontaneous symmetry breaking

\[ V(\phi^2) = \frac{1}{4} g(\phi^i \phi^i - K^2)^2 + V_0, \]
where $V_0$ is an adjustable constant and the coupling constant $g > 0$.

The spinor field is minimally coupled to the gauge potential $A_{\mu}$, and $\mathcal{D}$ is a matrix representation of the gauge group $SO(3,1)$. The spin connection $\omega_\mu$ is

$$\omega_\mu = \frac{1}{2} \omega_{\mu ab} \Sigma^a,$$

(13)

where $\Sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$ is the spinor matrix associated with the Lorentz algebra $SO(3,1)$. The components $\omega_{\mu ab}$ satisfy

$$\partial_\mu e^a_\sigma + \Gamma^\mu_{\nu\sigma} e^\nu_a - \omega_\mu^a e^\rho_a = 0,$$

(14)

where $\Gamma^\mu_{\nu\sigma}$ is the Christoffel symbol. The field equations for the gravity-Yang-Mills-Higgs-Dirac sector are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = - \frac{1}{4} \kappa^2 T_{\mu\nu},$$

(15)

$$g^\rho\nu \nabla_\rho F^{i}_{\mu\nu} = g^\rho\mu \left( \partial_\rho F^i_{\mu\nu} - \Gamma^i_{\rho\nu} F^i_{\sigma\nu} - \Gamma^i_{\rho\mu} F^i_{\sigma\nu} \right),$$

(16)

$$\frac{1}{\sqrt{-g}} D_\mu [\sqrt{-g} g^{\mu\nu} D_\nu \phi^i] = \left( \frac{\partial V}{\partial \phi^2} \right) \phi^i,$$

(17)

$$\gamma^a e^\mu_a [\partial_\mu - \omega_\mu - \mathcal{D}(A_\mu)] \psi = 0.$$  

(18)

The energy-momentum tensor is

$$T_{\mu\nu} = T_{\mu\nu}^{\text{YMH}} + T_{\mu\nu}^{\text{Dirac}} + T_{\mu\nu}^{\text{M}},$$

(19)

where

$$T_{\mu\nu}^{\text{YMH}} = \text{Tr}(F_{\mu\sigma} F^{\sigma}_{\nu}) + D_\mu \phi^j D_\nu \phi^j,$$

$$- \frac{1}{2} g_{\mu\nu} \left[ \frac{1}{2} \text{Tr}(F^2) + D_\sigma \phi^j D^\sigma \phi^j + V(\phi^2) \right],$$

(20)

$$T_{\mu\nu}^{\text{Dirac}} = - \bar{\psi} \gamma_\mu [\partial_\nu - \omega_\nu - \mathcal{D}(A_\nu)] \psi,$$

(21)

and $T_{\mu\nu}^{\text{M}}$ is the energy-momentum tensor of non-field matter.

3 Finite Quantum Field Theory Formalism

An important development in nonlocal FQFT was the discovery that gauge invariance and unitarity can be restored by adding series of higher interactions. The resulting theory possesses a nonlinear, field representation dependent gauge invariance which agrees with the original local symmetry on shell but is larger off shell. Quantization is performed in the functional formalism using an analytic
and convergent measure factor which retains invariance under the new symmetry. An explicit calculation was made of the measure factor in QED [2], and it was obtained to lowest order in Yang-Mills theory [3]. Kleppe and Woodward [9] obtained an ansatz based on the derived dimensionally regulated result when \( \Lambda \to \infty \), which was conjectured to lead to a general functional measure factor in FQFT gauge theories.

In contrast to string theory, we can achieve a genuine quantum field theory, which allows vertex operators to be taken off the mass shell. The finiteness draws from the fact that factors of \( \exp[\mathcal{K}(p^2)/2\Lambda^2] \) are attached to propagators which suppress any ultraviolet divergences in Euclidean momentum space, where \( \Lambda \) is an energy scale factor. An important feature of FQFT is that only the quantum loop graphs have nonlocal properties; the classical tree graph theory retains full causal and local behavior.

A convenient formalism which makes the FQFT construction transparent is based on shadow fields [5, 9]. We shall consider the 4-dimensional spacetime to be approximately flat Minkowski spacetime. Let us denote by \( f_i \) a generic local field and write the standard local action as

\[
W[f] = W_F[f] + W_I[f],
\]

where \( W_F \) and \( W_I \) denote the free part and the interaction part of the action, respectively, and

\[
W_F[f] = \frac{1}{2} \int d^4x f_i K_{ij} f_j.
\]

In a gauge theory \( W \) would be the Becchi, Rouet, Stora, Tyutin (BRST) gauge-fixed action including ghost fields in the invariant action required to fix the gauge [31]. The kinetic operator \( \mathcal{K} \) is fixed by defining a Lorentz-invariant distribution operator

\[
\mathcal{E} \equiv \exp\left(\frac{\mathcal{K}}{2\Lambda^2}\right)
\]

and the shadow operator:

\[
\mathcal{O}^{-1} = \frac{\mathcal{K}}{\mathcal{E}^2 - 1}.
\]

Every local field \( f_i \) has an auxiliary counterpart field \( h_i \), and they are used to form a new action

\[
W[f, h] \equiv W_F[\hat{f}] - P[h] + W_I[f + h],
\]

where

\[
\hat{f} = \mathcal{E}^{-1} f, \quad P[h] = \frac{1}{2} \int d^4x h_i h_j \mathcal{O}^{-1}_{ij} h_j.
\]

By iterating the equation

\[
h_i = \mathcal{O}_{ij} \frac{\delta W_I[f + h]}{\delta h_j}
\]
the shadow fields can be determined as functions, and the regulated action is derived from
\[ \hat{W}[f] = W[f, h(f)]. \]  
\hspace{1cm} (28)

We recover the original local action when we take the limit \( \Lambda \to \infty \) and \( \hat{f} \to f, h(f) \to 0 \).

The expression \((27)\) can be developed into a series expansion for \( h_i[f] \). The regularized action is found by substituting into it the classical solution \( h_i[f] \). Expanding \( \hat{W} \) in powers of \( f \) gives the kinetic term \( W_F[\hat{f}] \), together with an infinite series of interaction terms the first of which is just \( W_I[f] \). Since \( O \) is an entire function of \( K \) the higher interactions are also entire functions of \( K \). This is important for preserving unitarity.

Quantization is performed using the definition
\[ \langle 0 | T^* (O[f]) | 0 \rangle_E = \int [Df] \mu[f] (gauge fixing) O[\hat{f}] \exp(i\hat{W}[f]). \]  
\hspace{1cm} (29)

On the left-hand side we have the regulated vacuum expectation value of the \( T^* \)-ordered product of an arbitrary operator \( O[f] \) formed from the local fields \( f_i \). The subscript \( E \) signifies that a regulating Lorentz distribution has been used. Moreover, \( \mu[f] \) is a measure factor and there is a gauge fixing factor, both of which are needed to maintain perturbative unitarity in gauge theories.

The new Feynman rules for FQFT are obtained as follows: The vertices remain unchanged within the regularized action, but every leg of a diagram is connected either to a regularized propagator,
\[ \frac{i \mathcal{E}^2}{\mathcal{K} + i \epsilon} = -i \int_1^\infty d\tau \exp \left( -\frac{\mathcal{K}}{\Lambda^2} \right), \]  
\hspace{1cm} (30)
or to a shadow propagator,
\[ -i \mathcal{O} = \frac{i(1 - \mathcal{E}^2)}{\mathcal{K}} = -i \int_0^1 d\tau \exp \left( -\frac{\mathcal{K}}{\Lambda^2} \right). \]  
\hspace{1cm} (31)

We shall also attach a factor \( \mathcal{E}(p^2) \) to every external leg connected to a loop, which is unity on shell. The formalism is set up in Minkowski spacetime and loop integrals are formally defined in Euclidean space by performing a Wick rotation. This facilitates the analytic continuation; the whole formalism could from the outset be developed in Euclidean space.

In FQFT renormalization is carried out as in any other field theory. The bare parameters are calculated from the renormalized ones and \( \Lambda \), such that the limit \( \Lambda \to \infty \) is finite for all noncoincident Green’s functions, and the bare parameters are those of the local theory. The regularizing interactions are determined by the local operators.

The regulating Lorentz distribution function \( \mathcal{E} \) must be chosen to perform an explicit calculation in perturbation theory. We do not know the unique choice of \( \mathcal{E} \). However, once a choice for the function is made, then the theory and the
perturbative calculations are uniquely fixed. A standard choice in early FQFT papers is \cite{1, 2}:
\[ E_m = \exp\left(\frac{\partial^2 - m^2}{2\Lambda^2}\right). \] (32)

An explicit construction for QED was given using the Cutkosky rules as applied to FQFT whose propagators have poles only where $K = 0$ and whose vertices are entire functions of $K$. The regulated action $\hat{W}[f]$ satisfies these requirements which guarantees unitarity on the physical space of states. The local action is gauge fixed and then a regularization is performed on the BRST theory.

The infinitesimal transformation
\[ \delta f_i = T_i(f) \] (33)
generates a symmetry of $W[f]$, and the infinitesimal transformation
\[ \hat{\delta} f_i = \mathcal{E}^2_{ij} T_j(f + h[f]) \] (34)
generates a symmetry of the regulated action $\hat{W}[f]$. To see this consider the transformations
\[ \delta f_i = \mathcal{E}^2_{ij} T_j[f + h], \quad \delta h_i = (1 - \mathcal{E}^2_{ij}) T_j[f + h]. \] (35)

Adding these two transformations gives
\[ \delta (f + h)_i = T_i[f + h]. \] (36)

Then, (35) is a symmetry of the action $W[f, h]$. We have
\[ \delta W[f, h] = \int d^4x \left\{(f_i + h_i)\mathcal{K}_{ij} T_j[f + h]\right\} \]
\[ + \frac{\delta W[f + h]}{\delta f_i} T_i[f + h] = \delta W[f + h]. \] (37)

It follows that $\delta W[f, h] = 0$ is a consequence of the assumed invariance $\delta W[f + h] = 0$. Now $\hat{W}[f]$ is invariant under (34), for we have
\[ \hat{\delta} h_i[f] = (1 - \mathcal{E}^2_{ij}) T_j[f + h[f]] - L_{ij}[f + h[f]] \frac{\delta T_i}{\delta f_k}[f + h[f]] \mathcal{E}^2_{kl} \frac{\delta \hat{W}[f]}{\delta f_l}, \]
where
\[ L^{-1}_{ij} = O^{-1}_{ij} - \frac{\delta^2 W_j[f]}{\delta f_i \delta f_j}. \]

It follows that FQFT regularization preserves all continuous symmetries including supersymmetry. The quantum theory will preserve symmetries provided a suitable measure factor can be found such that
\[ \hat{\delta}([D f] \mu[f]) = 0. \] (38)
Moreover, the interaction vertices of the measure factor must be entire functions of the operator $\mathcal{K}$ and they must not destroy the FQFT finiteness.

In FQFT tree order, Green’s functions remain local except for external lines which are unity on shell. It follows immediately that since on shell tree amplitudes are unchanged by the regularization, $\hat{W}$ preserves all symmetries of $W$ on shell. Also all loops contain at least one regularizing propagator and therefore are ultraviolet finite. Shadow fields are eliminated at the classical level, for functionally integrating over them would produce divergences from shadow loops. Since shadow field propagators do not contain any poles there is no need to quantize the shadow fields.

In FQFT, the on shell tree amplitudes agree with the local, unregulated action, while the loop amplitudes disagree. This seems to contradict the Feynman tree theorem [32], which states that loop amplitudes of local field theory can be expressed as sums of integrals of tree diagrams. If two local theories agree at the tree level, then the loop amplitudes agree as well. However, the tree theorem does not apply to nonlocal field theories. The tree theorem is proved by using the propagator relation

$$D_F = D_R + D^+$$  \hspace{1cm} (39)$$

to expand the Feynman propagator $D_F$ into a series in the on shell propagator $D^+$. This decomposes all terms with even one $D^+$ into trees. The term with no $D^+$s is a loop formed with the retarded propagator and vanishes for local interactions. But for nonlocal interactions, this term generally survives and new physical effects occur in loop amplitudes, which cannot be predicted from the local on shell tree graphs.

4 Quantum Nonlocal Behavior in FQFT

It appears on general grounds that interacting strings are nonlocal [33-35, 36]. Nonlocality in open string theory can arise from the non-commutativity of space-time coordinates

$$[x^\mu, x^\nu] = i\theta\epsilon^{\mu\nu}. \hspace{1cm} (40)$$

This nonlocality in string theory is closely associated with the string uncertainty principle

$$\Delta x \Delta t \geq \alpha'.$$  \hspace{1cm} (41)$$

Nonlocality has also been associated with the formation of black hole horizons and the lack of commutativity of spatial coordinates and time [24]. The horizon responds to incoming matter before it comes in.

Kapustin [24] has recently shown that LSTs are quasi-local field theories whose infrared limit can approach local field theories in the large $\hat{W}$. The exponential growth of Wightman functions (Green’s functions) in momentum space is a characteristic feature of nonlocal field theories. The corresponding test functions in x-space are real analytic and cannot possess compact support.
The Wightman functions \([40]\) or vacuum expectations values of products of field operators \(\phi(p)\):

\[
W_n(q_1, \ldots, q_{n-1}) = \langle 0 | \phi(q_1) \phi(q_2) \ldots \phi(q_{n-1}) | 0 \rangle,
\]

grow exponentially with momenta for nonlocal field theories. By the positivity of energy, \(W_n\) vanishes when any of its arguments are outside the forward light cone. Inside the forward light cone \(W_n\) is bounded by

\[
\exp[\ell((|q_1| + \ldots |q_{n-1}|))],
\]

where \(|q| = \sqrt{q^2}\) and \(\ell\) is a length scale. In the case of LST models, the length scale is given by \(\ell \sim \sqrt{N/M_s}\) where \(N\) is the number of coincident five-branes.

Jaffe \([46]\) defined a test function space \(\tilde{S}_g\) in momentum space, which is convenient to use when discussing nonlocal field theories, in which all functions are infinitely differentiable and for which all the norms are finite. Given a positive function \(g(t)\) which is entire, Jaffe showed that if \(g(t)\) satisfies

\[
\int_0^\infty \frac{dt}{1 + t^2} \ln \frac{g(t^2)}{1 + t^2} < \infty,
\]

then the Fourier transform of \(\tilde{S}_g\) has functions with compact support, strictly local quantum fields can be defined and a local quantum field theory can be formulated. On the other hand, if (44) is not satisfied, then there are no test functions with compact support and we have a nonlocal quantum field theory.

Our choice of the entire function \(E(p^2)\) in the factor, Eq. (24), will not lead to a test function space that satisfies the condition (44). We can choose a function \(E(p^2)\) which will provide a test function space that leads to a quasi-local quantum field theory, as defined by Kapustin, and in the earlier work by Iofa and Fainberg. In the present work, we have chosen \(K(p^2) = -(p^2 + m^2)\), because it leads to a simplification of calculations in perturbation theory. But this is purely a technical issue, and we can certainly adopt entire functions \(K(p^2)\) which lead to quasi-local field operators, which only violate locality at short distances.

The commutator for a scalar field operator \(\phi(x)\):

\[
[\phi(x), \phi(y)] = W_2(x - y) - W_2(y - x)
\]

in our theory will not vanish outside the light cone for space-like separations \((x - y)^2 > 0\). Indeed, it will satisfy

\[
[\phi(x), \phi(y)] \sim \delta((x - y)^2 - \ell^2) \text{sign}(x_0 - y_0).
\]

In FQFT, it can be argued that the extended objects that replace point particles (the latter are obtained in the limit \(\Lambda \to \infty\)) cannot be probed because of a Heisenberg uncertainty type of argument. The FQFT nonlocality only occurs at the quantum loop level, so there is no noncausal classical behavior. In FQFT the strength of a signal propagated over an invariant interval \(\ell^2\) outside the light cone would be suppressed by a factor \(\exp(-\ell^2\Lambda^2)\).
Nonlocal field theories can possess non-perturbative instabilities. These instabilities arise because of extra canonical degrees of freedom associated with higher time derivatives. If a Lagrangian contains up to $N$ time derivatives, then the associated Hamiltonian is linear in $N - 1$ of the corresponding canonical variables and extra canonical degrees of freedom will be generated by the higher time derivatives. A nonlocal theory can be viewed as the limit $N \to \infty$ of an $N$th derivative Lagrangian. Unless the dependence on the extra solutions is arbitrarily choppy in the limit, then the higher derivative limit will produce instabilities [47]. The condition for the smoothness of the extra solutions is that no invertible field redefinition exists which maps the nonlocal field equations into the local ones. String theory does satisfy this smoothness condition as can be seen by inspection of the S-matrix tree graphs. In FQFT the tree amplitudes agree with those of the local theory, so the smoothness condition is not obeyed.

It was proved by Kleppe and Woodard [3] that the solutions of the nonlocal field equations in FQFT are in one-to-one correspondence with those of the original local theory. The relation for a generic field $v_i$ is

$$v_i^{\text{nonlocal}} = V_{ij}v_j^{\text{local}}. \quad (47)$$

Also the actions satisfy

$$W[v] = \hat{W}[V^2v]. \quad (48)$$

Thus, there are no extra classical solutions. The solutions of the regularized nonlocal Euler-Lagrange equations are in one-to-one correspondence with those of the local action. It follows that the regularized nonlocal FQFT is free of higher derivative solutions, so FQFT can be a stable theory.

Since only the quantum loop graphs in the nonlocal FQFT differ from the local field theory, then FQFT can be viewed as a non-canonical quantization of fields which obey the local equations of motion. Provided the functional quantization in FQFT is successful, then the theory does maintain perturbative unitarity.

### 5 Experimental Tests of Nonlocality

In order to solve the Higgs and cosmological constant radiative stability, hierarchy problems, we have relaxed the assumption of microcausal locality in our FQFT. A scale of nonlocality $\Lambda$ is set for the graviton ($\Lambda_G \leq 10^{-3}$ eV), the Higgs particle ($\Lambda_H \leq 1$ TeV) and the standard model gauge particles ($\Lambda_{GP} \gg 1$ TeV). We do not understand the fundamental physics which is the source of these non-locality scales but, as we shall see, given these scales we can potentially solve the radiative stability problems in a fully gauge invariant, finite and unitary fashion, including the gravitational stability of the cosmological constant.

Supersymmetry and technicolor models have been proposed to solve the Higgs gauge hierarchy problem. The mass scales for supersymmetry are set "by hand", so to speak, according to when we expect supersymmetry breaking to set in, allowing super-partners to be detected, and when technicolor fermions form
condensates, allowing us to detect technicolor particles. No known fundamental physics tells us what these mass scales are. We can only guess their magnitude above certain obvious intermediate energy bounds. Experiments already tend to disfavour technicolor models, and if the large hadron colliders do not detect super-partners below 2-3 TeV, then this would kill the possibility of using supersymmetric models to explain the radiative stability of the Higgs particle.

Can we experimentally detect the onset of nonlocality? We could do this by checking dispersion relations for scattering amplitudes at high energies. We expect that the non-vanishing of commutators of field operators outside the light cone will decrease exponentially with the spacelike distance, so violations of nonlocality will be small, and changes of analyticity of the scattering amplitudes from the standard microcausal analyticity properties will correspondingly be small. Another possible signature of nonlocality is a violation of CPT invariance. This is a fundamental theorem of local quantum field theory [48, 40]. There have been suggestions that CPT invariance could be broken in quantum gravity [49]. Moreover, there have been several studies of meson decays with the prospects of detecting CPT invariance breaking at K-meson and B-meson factories [50]. Let us investigate how CPT invariance could be violated by nonlocality. Consider a complex, nonlocal Heisenberg-picture scalar field operator $\Phi(x)$. The K"allen-Lehmann representation is given by the vacuum expectation value \[ \langle 0 | \Phi(x) \Phi^\dagger(y) | 0 \rangle = \int_0^\infty d\mu^2 \rho(\mu^2) \tilde{\Delta}_+(x - y; \mu^2), \] (49) where \[ \tilde{\Delta}_+(x - y; \mu^2) = \frac{1}{(2\pi)^3} \int_0^\infty d^4p \exp[ip \cdot (x - y)] \Pi(x - y) \delta(p^0) \delta(p^2 + \mu^2), \] (50) and $\Pi(x - y)$ is an entire analytic function with $\Pi(x) > 0$ for real $x$. The spectral function $\rho$ is defined by \[ \sum_n \delta^4(p - p_n) |\langle 0 | \Phi(0) | n \rangle|^2 = \frac{1}{(2\pi)^3} \theta(p^0) \rho(-p^2) \] (51) with $\rho(-p^2) = 0$ for $p^2 < 0$. We also have \[ \langle 0 | \Phi^\dagger(y) \Phi(x) | 0 \rangle = \int_0^\infty d\mu^2 \bar{\rho}(\mu^2) \tilde{\Delta}_+(y - x; \mu^2), \] (52) where \[ \sum_n \delta^4(p - p_n) |\langle n | \Phi^\dagger(0) | 0 \rangle|^2 = \frac{1}{(2\pi)^3} \theta(p^0) \bar{\rho}(-p^2). \] (53) Let us define \[ \alpha(\mu^2) = \rho(\mu^2) - \bar{\rho}(\mu^2). \] (54) The vacuum expectation value of the commutator is \[ \langle 0 | [\Phi(x), \Phi^\dagger(y)] | 0 \rangle \]
\[
\int_0^\infty d\mu^2 \{ \rho(\mu^2)[\hat{\Delta}_+(x-y;\mu^2) - \hat{\Delta}_+(y-x;\mu^2)] + \alpha(\mu^2)\hat{\Delta}_+(y-x;\mu^2) \}. \tag{55}
\]

For spacelike separations \((x-y)^2 > 0\), the function \(\hat{\Delta}_+(x-y;\mu^2) = \hat{\Delta}_+(y-x;\mu^2)\) and it does not vanish. For (55) to vanish for spacelike separations, we must have \(\alpha(\mu^2) = 0\). This is a nonperturbative proof of the CPT theorem, for states with \(p^2 = -\mu^2\) have the quantum numbers of the particle associated with \(\Phi\), and there must be corresponding states with \(p^2 = -\mu^2\) that have the quantum numbers of the anti-particle described by the operator \(\Phi^\dagger\) \[21\]. For strictly local field operators \(\Phi\), the commutator

\[
[\Phi(x), \Phi(y)] = 0 \tag{56}
\]

for spacelike separations \((x-y)^2 > 0\). However, we assumed that the \(\Phi(x)\) were nonlocal field operators, so there will be a violation of the CPT theorem when \(\alpha(\mu^2) \neq 0\), and we have for spacelike separation

\[
\langle 0 \vert [\Phi(x), \Phi^\dagger(y)] \vert 0 \rangle = \int_0^\infty d\mu^2 \alpha(\mu^2)\Delta_F(y-x;\mu^2). \tag{57}
\]

The vacuum expectation value of the time-ordered product is

\[
\langle 0 \vert T \{ \Phi(x)\Phi^\dagger(y) \} \vert 0 \rangle = i \int_0^\infty d\mu^2 \rho(\mu^2)\Delta_F(x-y;\mu^2) + i \int_0^\infty d\mu^2 \alpha(\mu^2)\theta(y^0 - x^0)\Delta_F(y-x;\mu^2), \tag{58}
\]

where \(\Delta_F\) is the Feynman propagator

\[
-i\Delta_F(x-y;\mu^2) = \theta(x^0 - y^0)\Delta_+(x-y;\mu^2) - \theta(y^0 - x^0)\Delta_+(y-x;\mu^2). \tag{59}
\]

For a nonlocal interaction

\[
V_{\text{NL}} = \int d^3x \mathcal{H}_{\text{NL}}(\bar{x},0), \tag{60}
\]

the commutator \([\text{CPT}, V_{\text{NL}}]\) will not in general vanish. The masses and decay rates of particles and anti-particles will not be equal for CPT invariance violating processes. For the discrete symmetries of nature, violations have been observed for C, P and the combined CP symmetries. Two types of CP symmetry violation have been observed for K-mesons. An active pursuit to detect CPT asymmetries in meson decays is presently underway.

## 6 Finite Quantum Yang-Mills Theory

Let us now review the finite quantization of the Yang-Mills sector in four-dimensional Minkowski flat space. The gauge field strength \(F_{\mu
u}\) is invariant under the familiar transformations:

\[
\delta A_{\mu} = -\partial_\mu \theta_i + e f_{ikl} A_{k\mu} \theta_l. \tag{61}
\]
To regularize the Yang-Mills sector, we identify the kinetic operator

\[ K_{ik}^{\mu\nu} = \delta_{ik} (\partial^2 \eta^{\mu\nu} - \partial^{\mu} \partial^{\nu}). \]

The regularized action is given by

\[ \hat{W}_{YM}[A] = \frac{1}{2} \int d^4 x \left\{ \hat{A}_{i\mu} K_{ik}^{\mu\nu} \hat{A}_{k\nu}^{\prime} - B_{ik}[A] (O_{ik}^{\mu\nu})^{-1} B_{k\nu}[A] \right\} + W_{YM}[A + B[A]], \]  

(62)

where \( B_{ik} \) is the Yang-Mills shadow field, which satisfies the expansion

\[ B_{ik}[A] = O_{ik}^{\mu\nu} \frac{\delta W_{YM}[A + B]}{\delta B_k^{\nu}} \]

\[ = O_{ik}^{\mu\nu} e f_{klm} [A_{i\nu} \partial_\sigma A_m^{\sigma} + A_{i\sigma} \partial_\nu A_m^{\sigma} - 2 A_{i\nu} \partial_\sigma A_m^{\sigma}] + O(e^2 A^3). \]  

(63)

The regularized gauge symmetry transformation is

\[ \delta A_i^{\mu} = (E_{ik}^{2\mu\nu}) \left\{ - \partial_\nu \theta_k + e f_{klm} (A_{i\nu} + B_{l\nu}[A]) \theta_m \right\}. \]

The extended gauge transformation is neither linear nor local.

We functionally quantize the Yang-Mills sector using

\[ \langle 0 \mid T^\star (O[A]) \mid 0 \rangle_E = \int [DA] [\mu[A] (gauge fixing) O[\hat{A}] \exp(i\hat{W}_{YM}[A])]. \]  

(64)

To fix the gauge we use Becchi-Rouet-Stora-Tyutin (BRST) invariance. The ghost structure of the BRST action comes from exponentiating the Faddeev-Popov determinant. Since the FQFT algebra fails to close off-shell, we need to introduce higher ghost terms into both the action and the BRST transformation. In Feynman gauge, the local BRST Lagrangian is

\[ L_{YM BRST} = - \frac{1}{2} \partial_\mu A_{i\nu} \partial^{\mu} A_i^{\nu} - \partial^{\mu} \bar{\eta}_i \partial_\mu \eta_i + e f_{ikl} \partial^{\mu} \bar{\eta}_i A_{k\mu} \eta_l \]

\[ + e f_{ikl} \partial_\mu A_{i\nu} A_{k\mu} A_l^{\nu} - \frac{1}{4} e^2 f_{ikl} f_{lmn} A_{i\mu} A_{k\nu} A_{m\mu} A_l^{\nu}. \]  

(65)

It is invariant under the global symmetry transformation:

\[ \delta A_{i\mu} = (\partial_\mu \eta_i - e f_{ikl} A_{k\mu} \eta_l) \delta \zeta, \]

\[ \delta \eta_i = - \frac{1}{2} e f_{ikl} \eta_k \eta_l \delta \zeta, \]

\[ \delta \bar{\eta}_l = - \partial_\mu A_l^{\mu} \delta \zeta, \]

where \( \zeta \) is a constant anticommuting c-number.
The gluon and ghost kinetic operators are
\[ K_{ik}^{\mu\nu} = \delta_{ik} \eta^{\mu\nu} \partial_2, \]
\[ K_{ik} = \delta_{ik} \partial_2, \] (66)

The gluon propagator and the shadow gluon propagator are given by
\[ D_{ik}^{\mu\nu}(p^2) = \frac{-i\delta_{ik}\eta^{\mu\nu}}{p^2 - i\epsilon} \exp \left( -\frac{p^2}{\Lambda_{YM}^2} \right), \] (67)
\[ D_{ik}^{\text{shad}}(p^2) = \frac{-i\delta_{ik}\eta^{\mu\nu}}{p^2 - i\epsilon} \left[ 1 - \exp \left( -\frac{p^2}{\Lambda_{YM}^2} \right) \right], \] (68)

where \( \Lambda_{YM} \) denotes the FQFT Yang-Mills energy scale.

The regularized BRST action is
\[ \hat{W}_{YM} [A, \bar{\eta}, \eta] = \int d^4x \left\{ -\frac{1}{2} \partial_\mu \hat{A}_i^{\mu} \partial_\nu \hat{A}_i^{\mu} - \frac{1}{2} B_{i\mu} \hat{O}^{-1} B_i^{\mu} \right. \]
\[ - \bar{\partial}_\mu \hat{\eta}_i \partial_\mu \hat{\eta}_i - \bar{\chi}_i \hat{O}^{-1} \chi_i \left\} \right. + W_{YM}^I [A + B, \bar{\eta} + \bar{\chi}, \eta + \chi], \] (69)

where \( \chi \) is the ghost shadow field.

The regularizing, nonlocal BRST symmetry transformation is
\[ \hat{\delta} A_{i\mu} = \bar{E}^2 \left\{ (\partial_\mu \eta_i + \partial_\mu \chi_i) - e f_{ikl} (A_{k\mu} + B_{k\mu})(\eta_l + \chi_l) \right\} \delta \zeta, \]
\[ \hat{\delta} \eta_i = -\frac{1}{2} e f_{ikl} \bar{E}^2 (\eta_k + \chi_k)(\eta_l + \chi_l) \delta \zeta, \]
\[ \hat{\delta} \bar{\eta}_i = -\bar{E}^2 (\partial_\mu A_i^{\mu} + \partial_\mu B_i^{\mu}) \delta \zeta. \] (70)

The existence of a suitable invariant measure factor implies that the necessary Slavnov-Taylor identities also exist.

Kleppe and Woodard \[5\] have obtained the invariant measure factor for the regularized Yang-Mills sector to first order in the coupling constant \( e \):
\[ \ln(\mu[A, \bar{\eta}, \eta]) = -\frac{1}{2} e^2 f_{ilm} f_{kklm} \int d^4 x A_{i\mu} \mathcal{M} A_i^{\mu} + O(e^3), \] (72)

where
\[ \mathcal{M} = \frac{1}{16\pi^2} \int_0^1 \frac{d\tau}{(\tau + 1)^2} \exp \left( \frac{\tau - \delta^2}{\tau + 1} \right) \left\{ \frac{2 + 6\tau}{\tau + 1} - 3 \right\}. \] (73)
7 Finite Perturbative Quantum Gravity

As is well known, the problem with perturbative quantum gravity based on a point-like graviton and a local field theory formalism is that the theory is not renormalizable \[41, 42\]. Due to the Gauss-Bonnet theorem, it can be shown that the one-loop graviton calculation is renormalizable but two-loop is not \[43\]. Moreover, gravity-matter interactions are not renormalizable at any loop order.

We shall now formulate the gravitational sector in more detail as a FQFT. This problem has been considered previously in the context of four-dimensional GR \[1, 2, 16\]. We shall expand the gravity sector about flat Minkowski spacetime. In fact, FQFT can be formulated as a perturbative theory by expanding around any fixed, classical metric background \[41\].

\[
g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu},
\]

where \(\bar{g}_{\mu\nu}\) is any smooth background metric field, e.g. a de Sitter spacetime metric. For the sake of simplicity, we shall only consider expansions about flat spacetime. Since the gravitational field is weak up to the Planck energy scale, this expansion is considered justified; even at the standard model energy scale \(E_{\text{SM}} \sim 10^{2}\text{ GeV}\), we have \(\kappa^2 E_{\text{SM}}^2 \sim 10^{-33}\). Also, at these energy scales the curvature of spacetime is very small. However, if we wish to include the cosmological constant \(\lambda\), then we cannot strictly speaking expand about flat spacetime, because such an expansion of the Einstein field equations will lead to the result that \(\lambda = 0\). This is to be expected, because the cosmological constant produces a curved spacetime even when the energy-momentum tensor \(T_{\mu\nu} = 0\). Therefore, we should in this case use the expansion (74). But for energy scales encountered in particle physics, the curvature is very small, so we can approximate the perturbation calculation by using the flat spacetime expansion and trust that the results are valid in general for curved spacetime backgrounds including the cosmological constant.

As in ref. \[10\], we will regularize the GR equations using the covariant shadow field formalism. Let us define \(g^{\mu\nu} = \sqrt{-g}g^{\mu\nu}\). It can be shown that \(\sqrt{-g} = \sqrt{-\bar{g}}\), where \(g = \det(g^{\mu\nu})\) and \(\partial_{\rho}g = g_{\alpha\beta}\partial_{\rho}g^{\alpha\beta}\). We can then write the local gravitational action \(W_{\text{grav}}\) in the form \[44\]:

\[
W_{\text{grav}} = \int d^4x \mathcal{L}_{\text{grav}} = \frac{1}{2\kappa^2} \int d^4x \left[(g^{\rho\sigma}g_{\lambda\mu}g_{\kappa\nu})
- \frac{1}{2}g^{\rho\sigma}g_{\lambda\mu}g_{\kappa\nu} - 2\delta_{\lambda\mu}\delta_{\kappa\nu}g^{\rho\sigma}\partial_{\rho}g^{\lambda\nu}
- \frac{1}{\alpha\kappa^2}\partial_{\rho}g^{\mu\nu}\partial_{\lambda}g^{\eta\nu\lambda} + \bar{C}^{\mu\nu\lambda\rho}X_{\mu\nu\lambda}C^\rho,\right]
\]

where we have added a gauge fixing term with the parameter \(\alpha\), \(C^{\mu}\) is the Fadeev-Popov ghost field and \(X_{\mu\nu\lambda}\) is a differential operator.

We expand the local interpolating graviton field \(g^{\mu\nu}\) as

\[
g^{\mu\nu} = \eta^{\mu\nu} + \kappa \gamma^{\mu\nu} + O(\kappa^2).
\]
Then,
\[ g_{\mu\nu} = \eta_{\mu\nu} - \kappa \gamma_{\mu\nu} + \kappa^2 \gamma_{\mu}^{\alpha} \gamma_{\alpha\nu} + O(\kappa^3). \] (77)

The gravitational Lagrangian density is expanded as
\[ \mathcal{L}_{grav} = \mathcal{L}^{(0)} + \kappa \mathcal{L}^{(1)} + \kappa^2 \mathcal{L}^{(2)} + \ldots. \] (78)

We obtain
\[ \mathcal{L}^{(0)} = \frac{1}{2} \partial_{\sigma} \gamma_{\lambda\rho} \partial^{\sigma} \gamma^{\lambda\rho} - \partial_\lambda \gamma^{\rho\kappa} \partial_\kappa \gamma_{\rho}^{\lambda} - \frac{1}{4} \partial_\rho \partial^{\rho} \gamma \]
\[ - \frac{1}{\alpha} \partial_\rho \gamma^{\rho} \partial_\gamma \kappa^{\lambda} + \hat{C}^{\lambda} \partial_\sigma \partial^{\sigma} C_\lambda, \] (79)
\[ \mathcal{L}^{(1)} = \frac{1}{4} (4 \gamma_{\lambda\mu} \partial^{\rho} \gamma^{\kappa\sigma} \partial_\rho \gamma_{\lambda}^{\kappa} + 2 \gamma_{\mu\sigma} \partial^{\rho} \gamma^{\kappa\mu} \partial_\rho \gamma_{\sigma}^{\kappa} + 4 \gamma_{\mu\sigma} \partial_\lambda \gamma^{\kappa\mu} \partial_\kappa \gamma_{\sigma}^{\lambda}) \]
\[ + \hat{C}^{\nu} \gamma_{\kappa\rho} \partial^{\rho} \partial^{\nu} C_\nu + \hat{C}^{\nu} \partial^{\kappa} \gamma_{\kappa\rho} \partial^{\rho} C_\nu - \hat{C}^{\nu} \partial^{\lambda} \partial^{\mu} \gamma_{\mu\nu} C_\lambda - \hat{C}^{\nu} \partial^{\mu} \gamma_{\mu\nu} \partial^{\lambda} C_\lambda, \] (80)
\[ \mathcal{L}^{(2)} = \frac{1}{4} (4 \gamma_{\kappa\lambda} \gamma^{\alpha\nu} \partial^{\rho} \gamma^{\lambda\kappa} \partial_\rho \gamma_{\nu}^{\alpha} + (2 \gamma_{\lambda\mu} \gamma_{\kappa\nu} - \gamma_{\mu\kappa} \gamma_{\sigma\nu}) \partial^{\rho} \gamma^{\kappa\sigma} \partial_\rho \gamma_{\lambda}^{\mu} \]
\[ - 2 \gamma_{\lambda\kappa} \gamma^{\alpha\nu} \partial^{\rho} \gamma^{\lambda\kappa} \partial_\rho \gamma_{\nu}^{\alpha} - 2 \gamma_{\sigma\kappa} \gamma^{\alpha\nu} \partial^{\rho} \gamma^{\lambda\kappa} \partial_\rho \gamma_{\nu}^{\alpha} \gamma_{\sigma}^{\lambda} \]
\[ + \gamma^{\alpha\nu} \gamma_{\rho}^{\mu} \partial^{\rho} \partial_\sigma \gamma_{\nu\lambda} \gamma_{\rho}^{\mu} \partial_\sigma \gamma_{\rho}^{\lambda} - 2 \gamma_{\mu\nu} \gamma^{\alpha\nu} \partial^{\rho} \gamma^{\kappa\mu} \partial_\rho \gamma_{\nu}^{\kappa}). \] (81)

where \( \gamma = \gamma^{\alpha}_{\alpha}. \)

In the limit \( \alpha \to \infty, \) the Lagrangian density \( \mathcal{L}_{grav} \) is invariant under the gauge transformation
\[ \delta \gamma_{\mu\nu} = X_{\mu\nu\lambda} \xi^{\lambda}, \] (82)
where \( \xi^{\lambda} \) is an infinitesimal vector quantity and
\[ X_{\mu\nu\lambda} = \kappa \left( - \partial_\lambda \gamma_{\mu\nu} + 2 \eta_{\lambda(\mu \kappa) \nu} \partial^{\kappa} \right) + (\eta_{\mu\nu} \partial_\alpha) - \eta_{\mu\nu} \partial_\lambda. \] (83)

However, for the quantized theory it is more useful to require the BRST symmetry. We choose \( \xi^{\lambda} = C^{\lambda} \sigma, \) where \( \sigma \) is a global anticommuting scalar. Then, the BRST transformation is
\[ \delta \gamma_{\mu\nu} = X_{\mu\nu\lambda} C^{\lambda} \sigma, \quad \delta \hat{C}^{\nu} = - \partial_\mu \gamma^{\mu\nu} \left( \frac{2\sigma}{\alpha} \right), \quad \delta C_\nu = \kappa C^{\mu} \partial_\mu C_\nu \sigma. \] (84)

We now substitute the operators
\[ \gamma_{\mu\nu} \to \hat{\gamma}_{\mu\nu}, \quad C_\lambda \to \hat{C}_\lambda, \quad \hat{C}_\nu \to \hat{\hat{C}}_\nu, \] (85)
where
\[ \hat{\gamma}_{\mu\nu} = \mathcal{E}^{-1} \gamma_{\mu\nu}, \quad \hat{C}_\lambda = \mathcal{E}^{-1} C_\lambda, \quad \hat{\hat{C}}_\lambda = \mathcal{E}^{-1} C_\lambda. \] (86)

As in the case of the Yang-Mills sector, the on shell propagators are unaltered from their local antecedents, while virtual particles are nonlocal. This destroys the gauge invariance of e.g. graviton-graviton scattering and requires an iteratively defined series of “stripping” vertices to ensure the decoupling of
all unphysical modes. Moreover, the local gauge transformations have to be
extended to nonlinear, nonlocal gauge transformations to guarantee the over-all
invariance of the regularized amplitudes. Cornish has derived the primary grav-

ton vertices and the BRST symmetry relations for the regularized $\hat{W}_{\text{grav}}$,
where
\[ \hat{\gamma} = E^{-1}\gamma, \quad P_{\text{grav}} (s) = \int d^4x G (s_i O_{ij}^{-1} s_j), \]

\( s \) denotes the graviton shadow field, and \( G \) denotes the detailed expansion of the contributions formed from the shadow field.

The regularized Lagrangian density up to order \( \kappa^2 \) is invariant under the extended BRST transformations [2]:

\[
\begin{align*}
\hat{\delta}_0 &\gamma_{\mu\nu} = X^{(0)}_{\mu\nu\lambda}(C^\lambda \sigma) = (\partial_\nu C_\mu + \partial_\mu C_\nu - \eta_{\mu\nu} \partial_\lambda C^\lambda)\sigma, \\
\hat{\delta}_1 &\gamma_{\mu\nu} = \kappa \mathcal{E}^2 X^{(1)}_{\mu\nu\lambda} C^\lambda \sigma = \kappa \mathcal{E}^2 (2\gamma_{\rho\mu}(\partial^\rho C_\nu) - \partial_\lambda \gamma_{\mu\nu} C^\lambda - \gamma_{\mu\nu} \partial_\lambda C^\lambda), \\
\hat{\delta}_0 &\overline{C}^\nu = 2\partial_\mu \gamma^{\mu\nu} \sigma, \\
\hat{\delta}_1 &C_\nu = \kappa \mathcal{E}^2 C^\mu \partial_\mu C_\nu \sigma.
\end{align*}
\]

The order \( \kappa^2 \) transformations are

\[
\begin{align*}
\hat{\delta}_2 &\gamma_{\mu\nu} = \kappa^2 \mathcal{E}^2 [2\partial^\rho C_\mu D^{\text{shad}}_{\rho\nu\rho\lambda}(B^{\nu\lambda} + H^{\nu\lambda}) \\
&- C^{\rho} D^{\text{shad}}_{\mu\nu\rho\lambda}(\partial_\rho B^{\kappa\lambda} + \partial_\rho H^{\kappa\lambda}) - \partial_\rho C^{\rho} D^{\text{shad}}_{\mu\nu\rho\lambda}(B^{\nu\lambda} + H^{\nu\lambda}) \\
&+ 2\gamma_{\rho\mu}(D^{\text{shad}}_{\nu\rho\kappa} G^{\rho} H^{\kappa} - \partial_\rho \gamma_{\mu\nu} D^{\text{shad}}_{\rho\nu\rho\kappa} H^{\kappa} - \gamma_{\mu\nu} D^{\text{shad}}_{\rho\nu\rho\kappa} \partial_\rho H^{\kappa})\sigma, \\
\hat{\delta}_2 &C_\nu = -\kappa^2 \mathcal{E}^2 (\partial_\mu C_\nu D^{\text{shad}}_{\rho\nu\rho\kappa} G^{\rho} H^{\kappa} + C_\mu D^{\text{shad}}_{\nu\rho\rho\kappa} G^{\rho} H^{\kappa})\sigma.
\end{align*}
\]

Here, we have

\[
\begin{align*}
H^{\alpha\beta} &= - (\partial_\alpha \bar{C}_\rho \partial_\beta C^\rho + \partial_\rho \bar{C}^{(\alpha} \partial_\beta C^{\rho)} + \partial_\rho \partial_{(\alpha} \bar{C}^{\beta)} C^{\rho)}, \\
H^\rho &= \gamma_{\lambda\kappa} \partial_\lambda \partial^\rho C^\kappa + \partial_\nu \gamma_{\lambda\kappa} \partial^\lambda \partial^\rho C_\nu - \partial_\nu \partial_\lambda \gamma_{\rho\kappa} C^\lambda - \partial_\nu \gamma_{\rho\kappa} \partial_\lambda C^\lambda, \\
\bar{H}^\rho &= \partial_\lambda \bar{C}_{\rho} \partial_\gamma \gamma_{\lambda\kappa} + \partial_\lambda \partial^\gamma \partial^\rho \bar{C}_{\gamma \kappa} + \partial_\nu \partial^\gamma \partial^\rho \gamma_{\nu \kappa}.
\end{align*}
\]

Because we have extended the gauge symmetry to nonlinear, nonlocal transformations, we must also supplement the quantization procedure with an invariant measure

\[
\mathcal{M} = \Delta (g, \bar{C}, C) D[g_{\mu\nu}] D[\bar{C}_\lambda] D[C_\sigma]
\]

such that \( \delta \mathcal{M} = 0 \).

As we have demonstrated, the quantum gravity perturbation theory is invariant under the FQFT generalized, nonlinear field representation dependent transformations. It is unitary and finite to all orders in a way similar to the non-Abelian gauge theories formulated using FQFT. At the tree graph level all unphysical polarization states are decoupled and nonlocal effects will only occur in graviton and graviton-matter loop graphs. Because the gravitational tree graphs are purely local there is a well-defined classical GR limit. The finite quantum gravity theory is well-defined in four real spacetime dimensions.

We quantize by means of the path integral operation

\[
\langle 0 | T^s(O[g]) | 0 \rangle = \int [Dg] [g] [(\text{gauge fixing}) O[\hat{g}] \exp(iW_{\text{grav}}[g])]
\]
The quantization is carried out in the functional formalism by finding a measure factor $\mu[g]$ to make $[Dg]$ invariant under the classical symmetry. To ensure a correct gauge fixing scheme, we write $W_{\text{grav}}[g]$ in the BRST invariant form with ghost fields; the ghost structure arises from exponentiating the Faddeev-Popov determinant $[45]$. The algebra of extended gauge symmetries is not expected to close off-shell, so one needs to introduce higher ghost terms (beyond the normal ones) into both the action and the BRST transformation. The BRST action will be regularized directly to ensure that all the corrections to the measure factor are included.

8 A Resolution of The Higgs Hierarchy Problem

It is time to discuss the Higgs sector hierarchy problem $[53]$. The gauge hierarchy problem is related to the spin 0+ scalar field nature of the Higgs particle in the standard model with quadratic mass divergence and no protective extra symmetry at $m = 0$. In standard point particle, local field theory the fermion masses are logarithmically divergent and there exists a chiral symmetry restoration at $m = 0$. Writing $m_H^2 = m_{0H}^2 + \delta m_H^2$, where $m_{0H}$ is the bare Higgs mass and $\delta m_H$ is the Higgs self-energy renormalization constant, we get for the one loop Feynman graph in $D = 4$ spacetime:

$$\delta m_H^2 \sim \frac{g^2}{32\pi^2} M_c^2,$$

where $M_c$ is a cutoff parameter. If we want to understand the nature of the Higgs mass we must require that

$$\delta m_H^2 \leq O(m_H^2),$$

i.e. the quadratic divergence should be cut off at the mass scale of the order of the physical Higgs mass. Since $m_H \simeq \sqrt{2}gv$, where $v = \langle \phi \rangle_0$ is the vacuum expectation value of the scalar field $\phi$ and $v = 246$ GeV from the electroweak theory, then in order to keep perturbation theory valid, we must demand that $10$ GeV $\leq m_H \leq 350$ GeV and we need

$$M_c = M_{\text{Higgs}} \leq 1 \text{ TeV},$$

where the lower bound on $m_H$ comes from the avoidance of washing out the spontaneous symmetry breaking of the vacuum.

Nothing in the standard model can tell us why (109) should be true, so we must go beyond the local standard model to solve the problem. $M_c$ is an arbitrary parameter in point particle field theory with no physical interpretation. Since all particles interact through gravity, then ultimately we should expect to include gravity in the standard model, so we expect that $M_{\text{Planck}} \sim 10^{19}$ GeV should be the natural cutoff. Then we have using (109) and $g \sim 1$:

$$\frac{\delta m_H^2 (M_{\text{Higgs}})}{\delta m_H^2 (M_{\text{Planck}})} \approx \frac{M_H^2}{M_{\text{Planck}}^2} \approx 10^{-34},$$
which represents an intolerable fine-tuning of parameters. This ‘naturalness’ or hierarchy problem is one of the most serious defects of the standard model.

There have been two strategies proposed as ways out of the hierarchy problem. The Higgs is taken to be composite at a scale $M_c \simeq 1$ TeV, thereby providing a natural cutoff in the quadratically divergent Higgs loops. One such scenario is the ‘technicolor’ model, but it cannot be reconciled with the accurate standard model data, nor with the smallness of fermion masses and the flavor-changing neutral current interactions. The other strategy is to postulate supersymmetry, so that the opposite signs of the boson and fermion lines cancel by means of the non-renormalization theorem. However, supersymmetry is badly broken at lower energies, so we require that

$$\delta m_H^2 \sim \frac{9}{32\pi^2} |M^2_{c\text{bosons}} - M^2_{c\text{fermions}}| \leq 1 \text{ TeV}^2,$$

or, in effect

$$|m_b - m_f| \leq 1 \text{ TeV}.$$  

This physical requirement leads to the prediction that the supersymmetric partners of known particles should have a threshold $\leq 1$ TeV.

A third possible strategy is to introduce the FQFT formalism, and realize a field theory mechanism which will introduce a natural physical scale in the theory $\Lambda_H \leq 1$ TeV, which will protect the Higgs mass from becoming large and unstable.

Let us consider the regularized scalar field FQFT Lagrangian in Minkowski spacetime

$$\hat{\mathcal{L}}_S = \frac{1}{2} \hat{\phi} (\partial^2 - m^2) \hat{\phi} - \frac{1}{2} \rho \hat{\phi} \hat{\phi} + \frac{1}{2} Z^{-1} \delta m^2 (\phi + \rho)^2 - \frac{1}{24} g_0 (\phi + \rho)^4, \quad (110)$$

where $\phi = Z^{1/2} \phi_R$ is the bare field, $\phi_R$ is the renormalized field, $\hat{\phi} = \mathcal{E}^{-1} \phi$, $\rho$ is the shadow field, $m_0$ is the bare mass, $Z$ is the field strength renormalization constant, $\delta m^2$ is the mass renormalization constant and $m$ is the physical mass.

The regularizing operator is given by

$$\mathcal{E}_m = \exp \left( \frac{\partial^2 - m^2}{2\Lambda_H^2} \right), \quad (111)$$

while the shadow kinetic operator is

$$\mathcal{O}^{-1} = \frac{\partial^2 - m^2}{\mathcal{E}_m^2 - 1}. \quad (112)$$

Here, $\Lambda_H$ is the Higgs scalar field energy scale in FQFT, which determines the scale of nonlocalizability of the Higgs particle.

The full propagator is

$$-i \Delta_R(p^2) = \frac{-i \mathcal{E}_m^2}{p^2 + m^2 - i\epsilon} = -i \int_1^\infty \frac{d\tau}{\Lambda_H^2} \exp \left[ -\tau \left( \frac{p^2 + m^2}{\Lambda_H^2} \right) \right], \quad (113)$$
whereas the shadow propagator is

\[
i\Delta_{\text{shadow}} = \frac{\mathcal{E}_m^2 - 1}{p^2 + m^2} = -i \int_0^1 \frac{d\tau}{\Lambda_H^2} \exp\left[-\tau \left(\frac{p^2 + m^2}{\Lambda_H^2}\right)\right]. \tag{114}\]

Let us define the self-energy \(\Sigma(p)\) as a Taylor series expansion around the mass shell \(p^2 = -m^2\):

\[
\Sigma(p^2) = \Sigma(-m^2) + (p^2 + m^2) \frac{\partial \Sigma}{\partial p^2}(-m^2) + \tilde{\Sigma}(p^2), \tag{115}\]

where \(\tilde{\Sigma}(p^2)\) is the usual finite part in the point particle limit \(\Lambda_H \to \infty\). We have

\[
\tilde{\Sigma}(-m^2) = 0, \tag{116}\]

and

\[
\frac{\partial \tilde{\Sigma}(p^2)}{\partial p^2}(p^2 = -m^2) = 0. \tag{117}\]

The full propagator is related to the self-energy \(\Sigma(p^2)\) by

\[
- i\Delta_R(p^2) = -i\mathcal{E}_m^2 \frac{1 + \mathcal{O}(\Sigma(p^2))}{p^2 + m^2 + \Sigma(p^2)} = \frac{-iZ}{p^2 + m^2 + \Sigma_R(p^2)}. \tag{118}\]

Here \(\Sigma_R(p^2)\) is the renormalized self-energy which can be written as

\[
\Sigma_R(p^2) = (p^2 + m^2) \left[\frac{Z}{\mathcal{E}_m^2(1 + \mathcal{O})} - 1\right] + \frac{Z\Sigma}{\mathcal{E}_m^2(1 + \mathcal{O})}. \tag{119}\]

The 1PI two-point function is given by

\[
- i\Gamma_R^{(2)}(p^2) = i[\Delta_R(p^2)]^{-1} = \frac{i[p^2 + m^2 + \Sigma_R(p^2)]}{\mathcal{E}_m^2[1 + \mathcal{O}(\Sigma(p^2))]} \tag{120}\]

Since \(\mathcal{E}_m \to 1\) and \(\mathcal{O} \to 0\) as \(\Lambda_H \to \infty\), then in this limit

\[
- i\Gamma_R^{(2)}(p^2) = i[p^2 + m^2 + \Sigma(p^2)], \tag{121}\]

which is the standard point particle result.

The mass renormalization is determined by the propagator pole at \(p^2 = -m^2\) and we have

\[
\Sigma_R(-m^2) = 0. \tag{122}\]

Also, we have the condition

\[
\frac{\partial \Sigma_R(p^2)}{\partial p^2}(p^2 = -m^2) = 0. \tag{123}\]

The renormalized coupling constant is defined by the four-point function \(\Gamma_R^{(4)}(p_1, p_2, p_3, p_4)\) at the point \(p_i = 0\):

\[
\Gamma_R^{(4)}(0, 0, 0, 0) = g. \tag{124}\]
The bare coupling constant $g_0$ is determined by

$$Z^2 g_0 = g + \delta g(g, m^2, \Lambda_H^2).$$

(125)

Moreover,

$$Z = 1 + \delta Z(g, m^2, \Lambda_H^2),$$

$$Zm_0^2 = Zm^2 - \delta m^2(g, m^2, \Lambda_H^2).$$

A calculation of the scalar field mass renormalization in D-dimensional space gives [9]:

$$\delta m^2 = \frac{g}{2^{D+1} 1 \pi D/2} m^{D-2} \Gamma \left(1 - \frac{D}{2}, \frac{m^2}{\Lambda_H^2}\right) + O(g^2),$$

(126)

where $\Gamma(n, z)$ is the incomplete gamma function:

$$\Gamma(n, z) = \int_z^\infty dt t^n \exp(-t) = (n-1) \Gamma(n-1, z) + z^{n-1} \exp(-z).$$

(127)

We have

$$\Gamma(-1, z) = - \ln(z) - \gamma + \frac{z^2}{2 \cdot 2!} + \frac{z^3}{3 \cdot 3!} - \ldots,$$

(129)

where $\gamma$ is Euler’s constant. For large positive values of $z$, we have the asymptotic expansion

$$E_i(z) \sim \exp(-z) \left[ \frac{1}{z} - \frac{1}{z^2} + \frac{2!}{z^3} - \ldots \right].$$

(130)

Thus, for small $m/\Lambda_H$ we obtain in $D = 4$ spacetime:

$$\delta m^2 = \frac{g}{32 \pi^2} \left[ \Lambda_H^2 - m^2 \ln \left( \frac{\Lambda_H^2}{m^2} \right) - m^2 (1 - \gamma) + O\left( \frac{m^2}{\Lambda_H^2} \right) \right] + O(g^2),$$

(131)

which is the standard quadratically divergent self-energy, obtained from a cutoff procedure or a dimensional regularization scheme.

We have for $z \to \infty$:

$$\Gamma(a, z) \sim z^{a-1} \exp(-z) \left[ 1 + \frac{a-1}{z} + O\left( \frac{1}{z^2} \right) \right],$$

(132)

so that for $m \gg \Lambda_H$, we get in four-dimensional spacetime

$$\delta m^2 \sim \frac{g}{32 \pi^2} \left( \frac{\Lambda_H^2}{m^2} \right) \exp\left( - \frac{m^2}{\Lambda_H^2} \right).$$

(133)

Thus, the Higgs self-energy one loop graph falls off exponentially fast for $m \gg \Lambda_H$. We have succeeded in stabilizing the radiative corrections to the Higgs sector, solving the Higgs hierarchy problem for $\Lambda_H \leq 1$ TeV.
9 Gluon and Gravitational Vacuum Polarization

A calculation of the one-loop gluon vacuum polarization in FQFT gives the tensor in D-dimensions

$$\Pi_{ik}^{\mu\nu}(p) = \frac{g^2}{2D_p D/2} f_{ilm} f_{klm} (p^2 \eta^{\mu\nu} - p^\mu p^\nu) \Pi(p^2),$$  \hspace{1cm} (134)

where $p$ is the gluon momentum and

$$\Pi(p^2) = 2 \exp \left( -\frac{p^2}{\Lambda_{YM}^2} \right) \int_0^{1/2} dy \Gamma(2 - D/2, yp^2/\Lambda_{YM}^2) \left[ y(1 - y) + 1 \right]^{D/2 - 2} \times [2(D - 2)y(1 - y) - \frac{1}{2}(D - 6)].$$  \hspace{1cm} (135)

We observe that $\Pi_{ik,\mu}^{\mu}(0) = 0$ a result that is required by gauge invariance and the fact that the gluon has zero mass.

The dimensionally regulated gluon vacuum polarization result is obtained by the replacement

$$\Gamma(2 - D/2, yp^2/\Lambda_{YM}^2) \rightarrow \Gamma(2 - D/2)$$

and choosing $p^2 \ll \Lambda_{YM}^2$. In four-dimensions we get

$$\Pi(p^2) = 2 \exp(-p^2/\Lambda_{YM}^2) \int_0^{1/2} dy E_i(yp^2/\Lambda_{YM}^2)[4y(1 - y) + 1],$$  \hspace{1cm} (137)

where we have used the relation

$$\Gamma(0, z) \equiv E_i(z) = \int_z^\infty dt \exp(-t)t^{-1}.$$  \hspace{1cm} (138)

By using the behavior for large Euclidean momentum $p^2 \gg \Lambda_{YM}^2$:

$$E_i(yp^2/\Lambda_{YM}^2) \sim \frac{\Lambda_{YM}^2}{p^2} \exp(-yp^2/\Lambda_{YM}^2),$$  \hspace{1cm} (139)

we find from (137) that

$$\Pi(p^2) \sim \frac{\Lambda_{YM}^2}{p^2} \exp \left( -\frac{p^2}{\Lambda_{YM}^2} \right) \left[ \exp(-p^2/2\Lambda_{YM}^2) + \frac{4\Lambda_{YM}^2}{p^2} \right] \Lambda_{YM}^2 - \frac{16\Lambda_{YM}^4}{p^6}. $$  \hspace{1cm} (140)

Thus, the gluon vacuum polarization is exponentially damped for $p^2 \gg \Lambda_{YM}^2$.

The lowest order contributions to the graviton self-energy in FQFT will include the standard graviton loops, the shadow field graviton loops, the ghost field loop contributions with their shadow field counterparts, and the measure.
loop contributions. In the regularized perturbative gravity theory the first order vacuum polarization tensor $\Pi^{\mu\nu\rho\sigma}$ must satisfy the Slavnov-Ward identities [54]:
\[ p_\mu p_\rho D^{\mu\nu\alpha\beta}(p) \Pi_{\alpha\beta\gamma\delta}(p) D^{\gamma\delta\rho\sigma}(p) = 0. \] (141)

By symmetry and Lorentz invariance, the vacuum polarization tensor must have the form
\[
\Pi_{\alpha\beta\gamma\delta}(p) = \Pi_1(p^2) \eta_{\alpha\beta} \eta_{\gamma\delta} + \Pi_2(p^2) \eta_{\alpha\gamma} \eta_{\beta\delta} + \Pi_3(p^2) \eta_{\alpha\delta} \eta_{\beta\gamma} + \Pi_4(p^2) \eta_{\alpha\gamma} \eta_{\beta\delta} + \Pi_5(p^2) \eta_{\alpha\delta} \eta_{\beta\gamma}.
\] (142)

The Slavnov-Ward identities impose the restrictions
\[
\Pi_2 + \Pi_4 = 0, \quad 4(\Pi_1 + \Pi_2 - \Pi_3) + \Pi_5 = 0.
\] (143)

The basic lowest order graviton self-energy diagram is determined by [55, 56, 57, 58, 59]:
\[
\Pi^1_{\mu\nu\rho\sigma}(p) = \frac{1}{2} \kappa^2 \exp\left(-\frac{p^2}{\Lambda_G^2}\right) \int d^4q \mathcal{U}_{\mu\nu\alpha\beta\gamma\delta}(p, -q, q - p) D^{\alpha\beta\gamma\delta}(q)
\times D^{\gamma\delta\rho\sigma}(p - q) \mathcal{U}_{\lambda\tau\xi\rho\sigma}(q, p - q, -p),
\] (144)
where $\mathcal{U}$ is the three-graviton vertex function
\[
\mathcal{U}_{\mu\nu\rho\delta\sigma}(q_1, q_2, q_3) = \frac{1}{2} [g_2(\rho q_3)\eta_{\rho(\delta\eta)\sigma} - \frac{2}{D-2} \eta_{\mu\nu} \eta_{\delta\tau}]
q_1(\rho q_3)\eta_{\rho(\delta\eta)\nu} - \frac{2}{D-2} \eta_{\mu\nu} \eta_{\delta\tau} + [],
\] (145)
and the ellipsis denote similar contributions.

To this diagram, we must add the ghost particle diagram contribution $\Pi^2$, the shadow diagram contribution $\Pi^3$ and the measure diagram contribution $\Pi^4$. The dominant finite contribution to the graviton self-energy will be of the form
\[
\Pi_{\mu\nu\rho\sigma}(p) \sim \kappa^2 \Lambda_G^4 \exp\left(-\frac{p^2}{\Lambda_G^2}\right) Q_{\mu\nu\rho\sigma}(p^2)
\sim \frac{\Lambda_G^4}{M_{PL}^2} \exp\left(-\frac{p^2}{\Lambda_G^2}\right) Q_{\mu\nu\rho\sigma}(p^2),
\] (146)
where $M_{PL}$ is the reduced Planck mass and $Q(p^2)$ is a finite remaining part.

For renormalizable field theories such as quantum electrodynamics and Yang-Mills theory, we will find that in FQFT the loop contributions are controlled by the incomplete $\Gamma$-function. If we adopt an “effective” quantum gravity theory expansion in the energy [57], then we would expect to obtain
\[
\Pi_{\mu\nu\rho\sigma}(p) \sim \kappa^2 \exp\left(-\frac{p^2}{\Lambda_G^2}\right) \mathcal{F}((2 - D/2, p^2/\Lambda_G^2) Q_{\mu\nu\rho\sigma}(p^2),
\] (147)
where $\mathcal{F}$ denotes the functional dependence on the incomplete $\Gamma$-function. By making the replacement

$$
\mathcal{F}(\Gamma(2 - D/2), p^2/\Lambda_G^2) \rightarrow \mathcal{F}(\Gamma(2 - D/2)),
$$

we would then obtain the second order graviton loop calculations using dimensional regularization [55, 56, 57, 58, 59, 41]. The dominant behavior will now be $\ln(\Lambda_G^2/q^2)$ and not $\Lambda_G^4$. However, in a nonrenormalizable theory such as quantum gravity, the dimensional regularization technique may not provide a correct result for the dominant behavior of the loop integral and we expect the result to be of order $\Lambda_G^4$. Indeed, it is well known that dimensional regularization for massless particles removes all contributions from tadpole graphs and $\delta^4(0)$ contact terms. On the other hand, FQFT takes into account all leading order contributions and provides a complete account of all counterterms. Because all the scattering amplitudes are finite, then renormalizability is no longer an issue.

The function

$$Q_{\mu\sigma\nu\rho}(p^2) \sim p^4$$

as $p^2 \rightarrow 0$. Therefore, $\Pi_{\mu\nu\rho\sigma}(p^2)$ vanishes at $p^2 = 0$ as it should from gauge invariance and for massless gravitons.

In Euclidean momentum space, which we can reach by a Wick rotation, we see that for $p^2 \gg \Lambda_G^2$ the graviton self-energy (146) is exponentially damped and the quantum gravity loop corrections are negligible for energies greater than $\Lambda_G$.

It is often argued in the literature on quantum gravity that the gravitational quantum corrections scale as $\alpha_G = GE^2$, so that for sufficiently large values of the energy $E$, namely, of order the Planck energy, the gravitational quantum fluctuations become large. We see that in FQFT this will not be the case, because the finite quantum loop corrections become negligible in the high energy limit provided the perturbative approximation is valid. Of course, the contributions of the tree graph exchanges of virtual gravitons can be large in the high energy limit, corresponding to strong classical gravitational fields. It follows that for high enough energies, a classical curved spacetime would be a good approximation, at least until the perturbation calculations break down.

In contrast to recent models of branes and strings in which the higher-dimensional compactification scale is lowered to the TeV range [19], we retain the classical GR gravitation picture and its Newtonian limit. It is perhaps a radical notion to entertain that quantum gravity becomes weaker as the energy scale increases towards the Planck scale $\sim 10^{19}$ Gev, but there is, of course, no known experimental reason why this should not be the case in nature. However, we do not expect that our weak gravity field expansion is valid at the Planck scale when $GE^2 \sim 1$, although the exponential damping of the quantum gravity loop graphs could still persist at the Planck scale. This question remains unresolved until a nonperturbative solution to quantum gravity is found.

It is worth noting that in the framework of an effective gravitational field theory [57], the leading lowest order loop divergence can be “renormalized” by being absorbed into two parameters $c_1$ and $c_2$. For a non-flat spacetime
background metric $\bar{g}_{\mu \nu}$, the divergent term at one loop due to graviton and ghost loops is given by (11):

$$L_{\text{div}}^{\text{1loop}} = \frac{1}{8\pi^2\epsilon} \left[ \frac{1}{120} \bar{R}^2 + \frac{7}{20} \bar{R}_{\mu \nu} \bar{R}^{\mu \nu} \right],$$  \hspace{1cm} (150)

where $\epsilon = 4 - D$ and the effective field theory renormalization parameters are

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}, \quad c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}.$$  \hspace{1cm} (151)

10 A Quantum Gravity Resolution of the Cosmological Constant Problem

Zeldovich [60] showed that the zero-point vacuum fluctuations must have a Lorentz invariant form

$$T_{\text{vac} \mu \nu} = \lambda_{\text{vac}} g_{\mu \nu},$$  \hspace{1cm} (152)

consistent with the equation of state $\rho_{\text{vac}} = -p_{\text{vac}}$. Thus, the vacuum within the framework of particle quantum physics has properties identical to the cosmological constant. In quantum theory, the second quantization of a classical field of mass $m$, treated as an ensemble of oscillators each with a frequency $\omega(k)$, leads to a zero-point energy $E_0 = \sum_k \frac{1}{2} \hbar \omega(k)$. The experimental confirmation of a zero-point vacuum fluctuation was demonstrated by the Casimir effect [61].

A simple evaluation of the vacuum density obtained from a summation of the zero-point energy modes gives

$$\rho_{\text{vac}} = \frac{1}{(2\pi)^2} \int_0^{M_c} d k k^2 (k^2 + m^2)^{1/2} \sim \frac{M_c^4}{16\pi^2},$$  \hspace{1cm} (153)

where $M_c$ is the cutoff. Already at the level of the standard model, we get $\rho_{\text{vac}} \sim (10^2 \text{GeV})^4$ which is 55 orders of magnitude larger than the bound (2). To agree with the experimental bound (3), we would have to invoke a very finely tuned cancellation of $\lambda_{\text{vac}}$ with the “bare” cosmological constant $\lambda$, which is generally conceded to be theoretically unacceptable.

We can understand this result by using the language of Feynman graphs. To avoid undue technical issues in FQFT, we shall consider initially the basic lowest order vacuum fluctuation diagram computed from the matrix element in flat Minkowski spacetime

$$M_{(2)}^{(0)} \sim g^2 \int d^4 p d^4 p' d^4 k \delta(k + p - p') \delta(k + p - p')$$

$$\times \frac{1}{k^2 + m^2} \text{Tr} \left( \frac{i \gamma^\sigma p_\sigma - m_f \gamma^\mu \gamma^\nu p'_\nu - m_f \gamma_\mu}{p^2 + m_f^2} \frac{i \gamma^\sigma p'_\sigma - m_f \gamma^\mu}{p'^2 + m_f^2} \gamma_\mu \right)$$

$$\times \exp \left[ - \left( \frac{p^2 + m_f^2}{\Lambda_{\text{SM}}^2} \right) - \left( \frac{p'^2 + m_f^2}{\Lambda_{\text{SM}}^2} \right) - \left( \frac{k^2}{\Lambda_{\text{SM}}^2} \right) \right],$$  \hspace{1cm} (154)

27
where $g$ is a coupling constant associated with the standard model. We have considered a closed loop made of a standard model fermion of mass $m_f$, an antifermion of the same mass and an internal standard model boson propagator of mass $m$; the scale $\Lambda_{\text{SM}} \sim 10^2 - 10^3$ GeV. This leads to the result

$$M_{(2)}^{(0)} \sim 16\pi^4 g^2 \delta^4(a) \int_0^\infty dp \int_0^\infty dp' \int_0^p d\tau \left( \frac{-P^2 + p^2 + p'^2 + 4m_f^2}{(P + a)(P - a)} \right) \frac{1}{(p^2 + m_f^2)(p'^2 + m_f^2)} \exp \left[ -\left( \frac{p^2 + p'^2 + 2m_f^2}{\Lambda^2_{\text{SM}}} - \frac{P^2}{\Lambda^2_{\text{SM}}} \right) \right],$$

(155)

where $P = p - p'$ and $a$ is an infinitesimal constant which formally regularizes the infinite volume factor $\delta^4(0)$. We see that $\rho_{\text{vac}} \sim M_{(2)}^{(0)}$ is finite and $M_{(2)}^{(0)} \sim \Lambda^4_{\text{SM}}$. To maintain gauge invariance and unitarity in FQFT, we must add to this result the contributions from the ghost diagram, the shadow diagram and the measure diagram.

In flat Minkowski spacetime, the sum of all disconnected vacuum diagrams $C = \sum_n M_n^{(0)}$ is a constant factor in the scattering S-matrix $S' = SC$. Since the S-matrix is unitary $|S'|^2 = 1$, then we must conclude that $|C|^2 = 1$, and all the disconnected vacuum graphs can be ignored. However, due to the equivalence principle gravity couples to all forms of energy, including the vacuum energy density $\rho_{\text{vac}}$, so we can no longer ignore these virtual quantum fluctuations in the presence of a non-zero gravitational field.

Let us now consider the dominant contributions to the vacuum density arising from the graviton loop corrections. As explained above, we shall perform the calculations by expanding about flat spacetime and trust that the results still hold for an expansion about a curved metric background field, which is strictly required for a non-zero cosmological constant. Since the scales involved in the final answer, including the predicted smallness of the cosmological constant, correspond to a very small curvature of spacetime, we expect that our approximation is justified.

We shall adopt a simple model consisting of a massive vector meson $V_\mu$, which has the standard model energy scale $\sim 10^2 - 10^3$ GeV. We have for the vector field Lagrangian density

$$L_V = -\frac{1}{4}(-g)^{-1/2}g^{\mu\nu}g_{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} + m_V^2 V_\mu V^\mu,$$

(156)

where

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu.$$  

(157)

We include in the Lagrangian density an additional piece $-\frac{1}{2}(\partial_\mu V_\mu)^2$, and the vector field propagator has the form

$$D_{\mu\nu}^V = \frac{\eta_{\mu\nu}}{p^2 + m_V^2 - i\epsilon} \exp \left[ -(p^2 + m_V^2)/\Lambda^2_{\text{SM}} \right]$$

$$= \frac{\eta_{\mu\nu}}{\Lambda^2_{\text{SM}}} \int_0^\infty \frac{d\tau}{\Lambda^2_{\text{SM}}} \exp \left[ -\tau(p^2 + m_V^2)/\Lambda^2_{\text{SM}} \right],$$

(158)
while the shadow propagator is
\[ D_{\mu\nu}^{\text{shad \ V}} = \frac{\eta_{\mu\nu}}{p^2 + m_V^2} \left[ 1 - \exp \left( -\frac{(p^2 + m_V^2)}{\Lambda_{\text{SM}}^2} \right) \right] \]
\[ = \eta_{\mu\nu} \int_0^1 \frac{d\tau}{\Lambda_{\text{SM}}^2} \exp \left( -\tau(p^2 + m_V^2)/\Lambda_{\text{SM}}^2 \right). \tag{159} \]

The graviton-V-V vertex in momentum space is given by
\[ V_{\alpha\beta\lambda\sigma}(p, q_1, q_2) = \eta_{\lambda\sigma} q_1(\alpha q_2 \beta) - \eta_{\sigma}(\beta q_1 q_2 - q_1 q_2 \lambda), \tag{160} \]
where \( q_1, q_2 \) denote the momenta of the two Vs connected to the graviton with momentum \( p \). We use the notation \( A_{(\alpha B_{\beta})} = \frac{1}{2}(A_{\alpha} B_{\beta} + A_{\beta} B_{\alpha}) \).

The lowest order correction to the graviton vacuum loop will have the form
\[ \Pi_{\mu\nu\rho\sigma}^{V}(p) = -\kappa^2 \exp \left( -\frac{p^2}{\Lambda_G^2} \right) \int d^4q V_{\mu\nu\lambda\alpha}(p, -q, -p) D_{\nu\lambda}^{V,W}(q) \]
\[ \times V_{\rho\sigma\delta}(q, -p, -q) D_{\rho\delta}^{V,W}(q). \tag{161} \]

We obtain
\[ \Pi_{\mu\nu\rho\sigma}^{V}(p) = -\kappa^2 \exp \left( -\frac{p^2}{\Lambda_G^2} \right) \int d^4q \frac{1}{(q^2 + m_V^2)((q - p)^2 + m_V^2)} K_{\mu\nu\rho\sigma}(p, q) \]
\[ \times \exp \left( -(q^2 + m_V^2)/\Lambda_{\text{SM}}^2 \right) \exp \left( -[(q - p)^2 + m_V^2]/\Lambda_{\text{SM}}^2 \right), \tag{162} \]
where in D-dimensions
\[ K_{\mu\nu\rho\sigma}(p, q) = p_\alpha p_\beta p_\rho p_\sigma + q_\alpha p_\beta p_\rho p_\sigma - q_\alpha q_\beta p_\rho q_\sigma + (1 - D) q_\alpha q_\beta q_\rho q_\sigma \]
\[ - (1 + D) p_\alpha q_\beta q_\rho q_\sigma + (D - 1) p_\alpha q_\beta p_\rho q_\sigma + D q_\alpha q_\beta q_\rho q_\sigma. \tag{163} \]

As usual, we must add to (161) the contributions from the fictitious ghost particle diagrams, the shadow field diagrams and the invariant measure diagram.

We observe that from power counting of the momenta in the integral (162), we obtain
\[ \Pi_{\mu\nu\rho\sigma}^{V}(p) \sim \kappa^2 \Lambda_{\text{SM}}^4 \exp \left( -\frac{p^2}{\Lambda_G^2} \right) N_{\mu\nu\rho\sigma}(p^2) \]
\[ \sim \frac{\Lambda_{\text{SM}}^4}{M_{\text{PL}}^2} \exp \left( -\frac{p^2}{\Lambda_G^2} \right) N_{\mu\nu\rho\sigma}(p^2), \tag{164} \]
where \( N(p^2) \) is a finite remaining part of \( \Pi_{\mu\nu\rho\sigma}^{V}(p) \). We have as \( p^2 \to 0 \):
\[ N_{\mu\nu\rho\sigma}(p^2) \sim p^4. \tag{165} \]
Thus, $\Pi^{\nu}_{\mu\rho\sigma}(p)$ vanishes at $p^2 = 0$ as it should because of gauge invariance and the massless graviton.

For four-dimensional Euclidean momenta $p^2 \gg \Lambda^2_G$, $\Pi^{\nu}_{\mu\rho\sigma}(p)$ is exponentially damped. At some value of the external graviton momentum $p$, when $\Lambda^2_{SM}$ could begin to become significant, the exponential damping suppresses this contribution. If we choose $\Lambda_G \leq 10^{-4}$ eV, then due to the damping of the gravitational vacuum polarization loop graph in the Euclidean limit $p^2 \gg \Lambda^2_G$, the cosmological constant contribution is suppressed sufficiently to satisfy the bound (3), and it is protected from large unstable radiative corrections. Thus, FQFT provides a solution to the cosmological constant problem at the energy level of the standard model and possible higher energy extensions of the standard model. The universal fixed FQFT gravitational scale $\Lambda_G$ corresponds to the fundamental length $\ell_G \leq 1$ cm at which virtual gravitational radiative corrections are cut off.

We observe that the required suppression of the vacuum diagram loop contribution to the cosmological constant, associated with the vacuum energy momentum tensor at lowest order, demands a low fundamental energy scale $\Lambda_G \leq 10^{-4}$ eV, which controls the quantum gravity loop contributions. This is essentially because the external graviton momenta are close to the mass shell, requiring a low energy scale $\Lambda_G$. This seems at first sight a radical suggestion that quantum gravity corrections are weak at energies higher than $\leq 10^{-4}$ eV, but this is clearly not in contradiction with any known gravitational experiment. Indeed, as has been stressed in recent work on large higher dimensions, there is no experimental knowledge of gravitational forces below 1 mm. In fact, we have no experimental knowledge at present about the strength of graviton radiative corrections. The standard model experimental agreement is achieved for standard model particle states close to the mass shell. However, we expect that the dominant contributions to the vacuum density arise from standard model states far off the mass shell. In our perturbative quantum gravity theory, the tree graphs involving gravitons are identical to the tree graphs in local point graviton perturbation theory, retaining classical, causal GR and Newtonian gravity. In particular, we do not decrease the strength of the classical, large distance gravity force.

In order to solve the severe cosmological constant hierarchy problem, we have been led to the surprising conclusion that, in contrast to the conventional folklore, quantum gravity corrections to the classical GR theory are negligible at energies above $\leq 10^{-4}$ eV, a result that will continue to persist if our perturbative calculations can be extrapolated to near the Planck energy scale $\sim 10^{19}$ GeV. Since the cosmological constant problem already results in a severe crisis at the energies of the standard model, our quantum gravity resolution based on perturbation theory can resolve the crisis at the standard model energy scale and well beyond this energy scale.
11 Conclusions

The ultraviolet finiteness of perturbative quantum field theory in four-dimensions is achieved by applying the FQFT formalism. The nonlocal quantum loop interactions reflect the quantum, non-point-like nature of the field theory, although we do not specify the nature of the extended object that describes a particle. Thus, as with string theories, the point-like nature of particles is “fuzzy” in FQFT for energies greater than the scale \( \Lambda \). One of the features of superstrings is that they provide a mathematically consistent theory of quantum gravity, which is ultraviolet finite and unitary. FQFT focuses on the basic mechanism behind string theory’s finite ultraviolet behavior by invoking a suppression of bad vertex behavior at high energies, without compromising perturbative unitarity and gauge invariance. FQFT provides a mathematically consistent theory of quantum gravity at the perturbative level. If we choose \( \Lambda_G \leq 10^{-4} \) eV, then quantum radiative corrections to the classical tree graph gravity theory are perturbatively negligible to all energies greater than \( \Lambda_G \), provided that the perturbative regime is valid.

The important gauge hierarchy problem, associated with the Higgs sector, is solved by the exponential damping of the Higgs self-energy in the Euclidean \( p^2 \) domain for \( p^2 \gg \Lambda_H^2 \), and for a \( \Lambda_H \) scale in the electroweak range \( \sim 10^2 - 10^3 \) GeV. A damping of the vacuum polarization loop contributions to the vacuum energy density-gravity coupling at lowest order can resolve the cosmological constant hierarchy problem, if the gravity loop scale \( \Lambda_G \leq 10^{-4} \) eV, by suppressing virtual gravitational radiative corrections above the energy scale \( \Lambda_G \).

We must still set the physical scale \( \Lambda_{YM} \), which controls the size of radiative loop corrections in the Yang-Mills sector of FQFT. We expect this scale to be much larger than the electroweak scale \( \sim 10^2 - 10^3 \) Gev, and it could be as large as grand unification theory (GUT) scales \( \sim 10^{16} \) Gev, allowing for possible GUT unification schemes.

Recently, new supernovae data have strongly indicated a cosmic acceleration of the present universe \cite{62}. This has brought the status of the cosmological constant back into prominence, since one possible explanation for this acceleration of the expansion of the universe is that the cosmological constant is non-zero but very small. We can, of course, accommodate a small non-zero cosmological constant by choosing carefully the gravity scale \( \Lambda_G \). Indeed, this new observational data can be viewed as a means of determining the size of \( \Lambda_G \).

Our quantum field theory formalism has helped to resolve two critical hierarchy problems in modern physics, given two parameters \( \Lambda_H \sim 10^2 - 10^3 \) GeV and \( \Lambda_G \leq 10^{-4} \) GeV. These parameters will hopefully be explained by a more fundamental non-perturbative theory.

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References

[1] J. W. Moffat, Phys. Rev. D41, 1177 (1990).
[2] D. Evens, J. W. Moffat, G. Kleppe and R. P. Woodard, Phys. Rev. D43, 49 (1991).
[3] J. W. Moffat and S. M. Robbins, Mod. Phys. Lett. A6, 1581 (1991).
[4] G. Kleppe and R. P. Woodard, Phys. Lett. B253, 331 (1991).
[5] G. Kleppe and R. P. Woodard, Nucl. Phys. B388, 81 (1992).
[6] N. J. Cornish, Mod. Phys. Lett. 7, 631 (1992).
[7] N. J. Cornish, Mod. Phys. Lett. 7, 1895 (1992).
[8] B. Hand, Phys. Lett. B275, 419 (1992).
[9] G. Kleppe and R. P. Woodard, Ann. of Phys. 221, 106 (1993).
[10] M. A. Clayton, L. Demopolous and J. W. Moffat, Int. J. Mod. Phys. A9, 4549 (1994).
[11] J. Paris, Nucl. Phys. B450, 357 (1995).
[12] J. Paris and W. Troost, Nucl. Phys. B482, 373 (1996).
[13] G. Saini and S. D. Joglekar, Z. Phys. C76, 343 (1997).
[14] S. D. Joglekar, hep-th/0003104, hepth/0003077.
[15] A. Basu and S. D. Joglekar, hep-th/0004128.
[16] J. W. Moffat, hep-th/9808091. Talk given at the XI International Conference on Problems in Quantum Field Theory, Dubna, Russia, July, 1998. Proceedings published by World Scientific, Singapore, 1999; J. W. Moffat, Talk given at the IV Workshop on Quantum Chromodynamics, June, 1998, eds. H. M. Fried and B. Müller. Proceedings published by World Scientific, Singapore, 1999.
[17] M. Green and J. H. Schwarz, Phys. Lett. B149, 117 (1984); D. Gross, J. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54, 502 (1985); P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258, 46 (1985); M. Green, J. H. Schwarz and E. Witten, Superstring Theory,
[18] J. Polchinski, “TASI Lectures on D-branes,” \texttt{hep-th/9611050} (1996); P. Townsend, “Four Lectures on M-theory,” \texttt{hep-th/9612121} (1996).

[19] E. Witten, Nucl. Phys. B\textbf{471}, 135 (1996); J. D. Lykken, Phys. Rev. D\textbf{54}, 3693 (1996); I. Antoniadis, Phys. Lett. B\textbf{246}, 377 (1990); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B\textbf{429}, 263 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D\textbf{59}, 105002 (1999); K. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. B\textbf{537}, 47 (1999); L. Randall and R. Sundrum, Phys. Rev. Lett. \textbf{83}, 3370 (1999), \texttt{hep-th/9905222}.

[20] E. Witten, Nucl. Phys. B\textbf{268}, 253 (1986); B\textbf{276}, 291 (1986); M. Saadi and B. Zwiebach, Ann. Phys. (N.Y.) \textbf{192}, 213 (1989); T. Kugo, H Kunitomo, and K. Suehiro, Phys. Lett. \textbf{226}, 48 (1989); Nucl. Phys. B\textbf{337}, 434 (1990).

[21] S. Weinberg, The Quantum Theory of Fields, Vol. 1: Foundations, Cambridge University Press, 1995.

[22] For a review, see: V. Ya. Fainberg and M. A. Soloviev, Ann. Phys. \textbf{113}, 421 (1978); M. Z. Iofa and V. Ya. Fainberg, Zh. Eksp. Teor. Fiz. \textbf{56}, 1644 (1969) [Sov. Phys. JETP \textbf{29}, 880 (1969); M. A. Soloviev, Theor. Math. Phys. \textbf{7}, 183 (1971); \textit{ibid.} \textbf{20}, 299 (1974); F. Constantinescu and J. G. Taylor, J. Math. Phys. \textbf{15}, 824 (1974); G. V. Efimov, Comm. Math. Phys. \textbf{5}, 42 (1967).

[23] N. Seiberg, Phys. Lett. B\textbf{408}, 98 (1997), \texttt{hep-th/9705221}; M. Berkooz, M. Rozali, and N. Seiberg, Phys. Lett. B\textbf{408}, 105 (1997), \texttt{hep-th/9704089}.

[24] A. Kapustin, \texttt{hep-th/9912044}.

[25] S. Weinberg, Rev. Mod. Phys. \textbf{61}, 1 (1989).

[26] V. Sahni and A. Starobinski, \texttt{astro-ph/9904396}.

[27] E. Witten, \texttt{hep-ph/0002297}.

[28] S. M. Carroll, \texttt{hep-th/0004075}.

[29] R. Sundrum, JHEP 9907, 001 (1999), \texttt{hep-ph/9708329}.

[30] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundrum, \texttt{hep-th/0001197}.

[31] C. Becchi, A. Rouet, and R. Stora, Comm. Math. Phys. \textbf{42}, 127 (1975); I. V. Tyutin, Lebedev Institute preprint N39 (1975).
[32] R. P. Feynman, Acta Phys. Pol. 24, 697 (1963); Magic Without Magic, edited by J. Klauder (Freeman, New York, 1972), p. 355; Feynman Lectures on Gravitation, edited by B. Hatfield, (Addison-Wesley publishing Co. 1995.)

[33] L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D48, 3743 (1993), hep-th/9306069.

[34] D. A. Lowe, L. Susskind, and J. Uglum, Phys. Lett. B327, 226 (1994), hep-th/9402136.

[35] D. A. Lowe, J. Polchinski, L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D52, 6997 (1995), hep-th/9506138.

[36] N. Seiberg, L. Susskind, and N. Toumbas, hep-th/0005015.

[37] N. Seiberg, L. Susskind, and N. Toumbas, hep-th/0005040.

[38] N. Seiberg and E. Witten, JHEP 9909, 032 (1999), hep-th/9906142 v3.

[39] G. ’t Hooft, Int. J. Mod. Phys. A11, 4623 (1996).

[40] R. F. Streater and A. S. Wightman, PCT, Spin and Statistics, and All That, Redwood City, USA, published by Addison-Wesley, 1989.

[41] G. ’t Hooft and M. Veltman, Ann. Inst. Henri Poincaré, 30, 69 (1974).

[42] S. Deser and P. van Nieuwenhuizen, Phys. Rev. Phys. Rev. 10, 401 (1974); Phys. Rev. 10, 411 (1974).

[43] M. Goroff and A. Sagnotti, Nucl. Phys. B266, 709 (1986).

[44] J. N. Goldberg, Phys. Rev. 111, 315 (1958).

[45] E. S. Fradkin and I. V. Tyutin, Phys. Rev. D2, 2841 (1970).

[46] A. M. Jaffe, Phys. Rev. 158, 1454 (1967).

[47] D. A. Eliezer and R. P. Woodard, Nucl. Phys. B325, 389 (1989).

[48] G. Lüders, Ann. Phys. (N.Y.) 2, 1 (1957); W. Pauli, Niels Bohr and the Development of Physics, eds. W. Pauli, L. Rosenfeld, and V. Weisskopf (McGraw Hill, New York (1955)); R. Jost, Helv. Phys. Acta 31, 263 (1958).

[49] J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, Talk presented by J. Ellis, at the Workshop on K Physics, Orsay, France, May-June, 1996, hep-ph/9607434.

[50] J. S. Bell and J. Steinberger, Proceedings of the Oxford International Conference on Elementary Particles, 1965, p.195; K. C. Chou, W. F. Palmer, E. A. Paschos, and Y. L. Wu, hep-ph/9911466; A. I. Sanda, hep-ph/9902355; P. Huet and M. E. Peskin, Nucl. Phys. B434, 3 (1994).
[51] G. Källen, Helv. Phys. Acta 25, 417 (1952); Quantum Electrodynamics (Springer-Verlag, Berlin, 1972); H. Lehmann, Nuovo Cimento 11, 342 (1954).

[52] T. de Donder, La Grafique Einsteinienne (Gauthier-Villars, Paris, 1921); V. A. Fock, Theory of Space, Time and Gravitation (Pergamon, New York, 1959).

[53] L. Susskind, Phys. Rep. 104, 181 (1984); E. Gildener, Phys. Rev. 14, 1667 (1976); E. Gildner and S. Weinberg, Phys. Rev. 15, 3333 (1976).

[54] D. M. Capper and M. R. Medrano, Phys. Rev. 9, 1641 (1974).

[55] D. M. Capper, G. Leibbrandt, and M. R. Medrano, Phys. Rev. 8, 4320 (1973).

[56] M. R. Brown, Nucl. Phys. B56, 194 (1973).

[57] J. F. Donoghue, Phys. Rev. D50, 3874 (1994).

[58] D. M. Capper, M. J. Duff, and L. Halpern, Phys. Rev. 10, 461 (1974).

[59] M. J. Duff, Phys. Rev. 9, 1837 (1974).

[60] Ya. B. Zeldovich, Pis’ma Zh. Eksp. Teor. Fiz. 6, 883 [JETP Lett. 6, 316 (1967)].

[61] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51, 635 (1948).

[62] S. Perlmutter et al., Nature 391, 51 (1998); Ap. J. 517, 565 (1999); A. Riess, et al., Astron. Journ. 117 207 (1998); B. Schmidt et al., Ap. J. 507, 46 (1998); P. Garnavich et al., Ap. J. 509, 74 (1998).