Neutrino-induced coherent pion production off nuclei

T. Leitner, U. Mosel and S. Winkelmann

Institut für Theoretische Physik, Universität Giessen, Germany

Abstract. All available theoretical estimates of neutrino-induced coherent pion production rely on the 'local approximation' for the Delta propagator. The validity of this approximation is scrutinized. It is found that the local approximation overestimates the neutrino-induced coherent pion production on nuclei significantly, by up to 100%.

Keywords: neutrino-nucleus interactions, coherent scattering, pion production
PACS: 25.30.Pt, 24.10.-i

INTRODUCTION

By scattering electroweak probes with nuclei, pions can be produced either coherently, leaving the nucleus intact, or incoherently. The former one has attracted considerable attention in the last years, both theoretically [1, 2, 3, 4, 5] and experimentally [6, 7, 8]. While there is compelling evidence for NC coherent pion production, no evidence for CC coherent pion production could be found. However, all these experimental analyses suffer from the fact that the coherent fraction is not accessible directly but has to be extracted from data assuming specific models for incoherent pion production. Furthermore, the theoretical models for coherent scattering used in the experimental analyses overpredict the measured rates.

The above mentioned theoretical models can be classified into two classes: first, the PCAC models which relate the coherent pion production to a forward scattering amplitude via PCAC assuming that specific nuclear effects play no role, besides providing nuclear size information. Second, models based on nuclear structure which start from a theoretical description of the nuclear structure and sum the pion production amplitude coherently over all target nucleon states. Both classes rely on the so-called local approximation which allows one to factorize out the nuclear form factor. In the following we investigate the impact of the local approximation for neutrino-induced processes. For further details, we refer the reader to [9] and references therein.

FULL CALCULATION VS. LOCAL APPROXIMATION

Our model assumes that pions are dominantly created via the \( \Delta (1232) \) resonance. Then, the hadronic current for a nucleon is given by

\[
J_\mu^{\text{nucleon}} (p \bar{q}) = i f \frac{C^\Delta F (p_\Delta^2) \bar{u}(p) k_\pi G_\alpha \Gamma_{\alpha \beta}(p_\Delta) \Gamma^\beta \mu \bar{u}(p) \bar{q} u(p)}{m_\pi};
\]  

(1)
with the pion momentum $k\pi$, the nucleon’s final and initial momenta, $p^0$ and $p$, and the transferred four-momentum $q = p^0 - p$. Thus, the $\Delta$ momentum reads $p_\Delta = p + q$. $G_{\alpha\beta}$ represents the full Rarita-Schwinger propagator

$$G_{\alpha\beta} = \frac{1}{p_\Delta^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta} P_{\alpha\beta};$$

where $P_{\alpha\beta}$ is the usual Rarita-Schwinger projection operator. The vertex function $\Gamma^{\beta\mu}$ denotes the standard electroweak vertex structure with vector and axial contributions including the resonance excitation form factors. $f = m_\pi$ is the $N\Delta\pi$ coupling constant, $F(p^2_\Delta)$ a form factor for the $\Delta$ and $C^A$ contains isospin factors (cf. [9] for details).

The single particle current (1) has to be summed over all occupied single-particle states in the target nucleus (full calculation), so that

$$J_{\text{nucleus}}^\mu(q) = \sum_i \frac{Z}{2} \hbar^2 \mathcal{P}_{1i}(p, q)$$

where the bound-state-spinors $\psi_i(p)$ are obtained in a Walecka-type mean field model [10] and replace the free-particle-spinors $u(p)$ in (1) (same for $\bar{u}(p^0)$). Note that the momentum integration extends also over the $\Delta$ propagator since $p_\Delta = p + q$.

The 'local approximation' now consists of fixing the momentum of the initial nucleon state in the product $G_{\alpha\beta}(p_\Delta) \Gamma^{\beta\mu}(p, q)$ to some value — here we use

$$p^0 = (q - k\pi = 2) \quad p^0_\Delta = (q + k\pi = 2):$$

With that, the momentum of the $\Delta$ resonance is determined, and the $\Delta$ propagator can be moved out of the integral and even out of the sum. This approximation basically consists of suppressing the propagation of the $\Delta$ resonance and corresponds to the assumption of a very heavy $\Delta$ resonance. Consequently, the $W; Z + N; \pi + N$ vertex becomes local. In an $r$-space representation, the current in the local approximation reads

$$J_{\text{nucleus}}^\mu(q) = \frac{i}{m_\pi} C^A \frac{k_\pi^\alpha}{p^2_\Delta - M_\Delta^2 + iM_\Delta \Gamma_\Delta} \sum_i \int d^3r e^{i(q \cdot k_\pi)} \tau_{\text{tr}} \rho(r) P_{\alpha\beta}(p^0_\Delta) \Gamma^{\beta\mu}(p, q) \psi_i(p)$$

Here the trace is taken over the Dirac indices and $\rho(r)$ is the diagonal element of the one-body density matrix. This is the final result in the local approximation. Equation (5) shows that the nuclear form factor has been factorized out; all the other (non-local) densities present in the full expression (3) no longer appear.

RESULTS

In the following, we compare the full calculation, based on Eq. (3) with a propagating $\Delta$, with the results of the local approximation [Eq. (5)] for the target nucleus $^{12}$C. To
FIGURE 1. CC induced pion angular distribution for a neutrino energy of 500 MeV (1000 MeV) and target $^{12}$C. The dashed curve gives the result of the calculation using the local approximation [cf. Eq. (5)]; the solid curve gives the result of a fully dynamic calculation [cf. Eq. (3)]. All curves are without pion final state interactions.

isolate the effects of the local approximation, both calculations are done in the plane wave approximation in which the produced pion is taken to be a free particle. We also do not include in-medium changes of the $\Delta$ spectral function in the propagator [Eq. (2)]. Both calculations use the same nuclear structure model, i.e., the density and momentum distributions are calculated consistently in the same relativistic mean field model.

Figure 1 shows a comparison of the full calculation with the results obtained by using the local approximation for the angular distribution of the produced pions at $E_\nu = 500$ and 1000 MeV. The difference between the full and the approximate calculation is larger at lower energies. At $E_\nu = 500$ MeV, the difference is dramatic over a wide angular range and amounts to a factor of $1.7$ at zero degrees. At $E_\nu = 1000$ MeV, we find that the local approximation gives a cross section for very forward angles that is about 20% larger than that obtained in the full calculation.

The pion momentum distribution for induced by CC muon neutrinos of $E_\nu = 500$ MeV is shown in the top panel of Fig. 2. It is seen that the local approximation overestimates the full result by a factor of about 2.5 at the peak. We find qualitatively similar results for NC induced coherent pion production (bottom panel). The slight shift downward relative to the fully dynamical result is a consequence of the local approximation which assumes a very heavy $\Delta$ thus minimizing any recoil effects. We finally note that our curves agree quantitatively with the recent results of the Ghent group [11]. While all these results were obtained in calculations without pion final state interactions (fsi) the recent calculations by Nakamura et al.[12] show that the local approximation fails as badly when the pion fsi are taken into account.
CONCLUSIONS

The local approximation, used from the start in all presently available microscopic calculations, overestimates the coherent neutrino-induced pion production significantly and involves errors which can reach up to 100% in the neutrino energy regime relevant to present experiments (MiniBooNE, T2K).

This work has been supported by the Deutsche Forschungsgemeinschaft (DFG).

REFERENCES

1. S. K. Singh, M. Sajjad Athar and S. Ahmad, Phys. Rev. Lett. 96, 241801 (2006).
2. L. Alvarez-Ruso, L. S. Geng, S. Hirenzaki and M. J. Vicente Vacas, Phys. Rev. C 75, 055501 (2007).
3. J. E. Amaro, E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D 79, 013002 (2009).
4. C. Berger and L. M. Sehgal, Phys. Rev. D 79, 013002 (2009), arXiv:0812.2653.
5. E. A. Paschos and D. Schalla, arXiv:0903.0451.
6. K2K, M. Hasegawa et al., Phys. Rev. Lett. 95, 252301 (2005).
7. SciBooNE, K. Hiraide et al., Phys. Rev. D 78, 112004 (2008).
8. MiniBooNE, A. A. Aguilar-Arevalo et al., Phys. Lett. B 664, 41 (2008).
9. T. Leitner, U. Mosel and S. Winkelmann, Phys. Rev. C 79, 057601 (2009).
10. W. Peters, H. Lenske and U. Mosel, Nucl. Phys. A 640, 89 (1998).
11. C. Praet, Modeling quasi-free neutrino-nucleus reactions for accelerator-based experiments, PhD thesis, Universiteit Gent, 2009, available online at http://inwpent5.ugent.be/papers/phdchristophe.pdf.
12. S. Nakamura, talk at NUINT 09, Sitges, Spain, May 2009, talk available at http://nuint09.ifae.es/Agenda.html