Concept of Productivity Levels in Modeling Sugar Beet Yields

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Abstract

Various approaches have been used to model the productive potential of sugar beets under the conditions of unstable moistening of the steppe zone of Russia. This paper considers the general theoretical approach to the functional description of most of the processes of plant growth and development in ontogenesis, as well as of any organism, which is determined by the conversion of bioclimatic resources into the biological mass of plants. Through mathematical modeling, the potential productivity of sugar beets in the absence of limiting factors, with optimal provision of plant life factors, was determined. The second level of modeling sugar beet productivity was performed for the conditions of unstable moistening in the steppe zone of Russia, where soil moisture is a factor limiting the productivity of the crop. To predict productivity in conditions of moisture deficiency, the study used plant organs and soil processes as a model, since they determine the availability of water and nutrients for the plant root system. Given the practical applications of the data for real production conditions, the data obtained in the first and second levels of crop productivity modeling were compared with the actual yield data obtained empirically. The maximum rates of dry matter accumulation for the conditions of the steppe zone of Russia in the absence of limiting factors was not limited to the supply of photosynthetically active radiation (PAR) and could produce up to 16 t/ha of dry matter root crops. With a moisture deficit during the beginning of row closing (from the 45th to the 75th day of the growing season), there was almost no increase in the dry matter of the plant mass, which reduced the potential productivity of sugar beets by 50%.

Keywords: crop modeling, sugar beet, potential crop, photosynthesis, moisture supply

1. Introduction

In agricultural science and practice, methods for predicting crop productivity based on methods of mathematical statistics are widely used [1–3]. These methods involve processing a large amount of statistical information on crop productivity in retrospect and do not take into account the correspondence of the bioclimatic potential of the region with the biological requirements of crops to growing conditions.
The conceptual prerequisites for the formation of a productivity model are to identify biological characteristics and growing conditions and should ensure the use of the model in the entire time range of the growing season of a crop.

Using the concept of productivity level as the degree of realization of genetic potential allows differentiating the modeling approaches existing in domestic and foreign studies into four levels of culture productivity \([4–6]\). At the first level, a culture, when it reaches its potential, has practically no limitations in the factors of growth and development, absorbed radiation is the only factor determining the growth rate. At the second level, plant growth, at least part of the time, is limited by a lack of moisture. Such conditions are most likely in conditions of unstable natural moisture. At the third level, the realization of production potential is constrained by a lack of nitrogen and moisture. At the fourth level, in addition to the listed limitations, a significant role is played by the lack of other elements of mineral nutrition \([7]\).

Of fundamental importance for using the concept of productivity in the formalization of biological processes are analytical models that are of practical value for the relatively simple growing conditions of the first and second levels. The formation of productivity models of the third and fourth levels, due to the growing influence of a significant number of endogenous factors, is possible only on an empirical basis.

**2. Methods and Equipment**

**2.1. Methods**

**2.1.1. Diagrammatic representation**

The general theoretical approach to the functional description of most of the processes of plant growth in ontogenesis, like any organism, is determined by the transformation of environmental nutrients into the biomass of organs. Therefore, the production process in its most general form can be represented as a closed two-component growth model proposed by J. France and J. Thornley \([8]\) (Figure 1). The system “medium - plant” has no inputs and outputs. It is assumed that growth occurs due to the conversion of the material and energy of the first component into the material of the second. On the other hand, De Wit \([9]\) proposed an approach that is more characterized by a description of the accumulation of dry matter, and to a lesser extent, by morphogenetic development.

In this case, the accumulation of dry matter should be equal to the increment:

\[
\frac{dW}{dt} = -\frac{dS}{dt}
\]  

(1)
where $W$ is dry matter; $S$ is the substrate determined by the parameters of agroclimatic conditions; $t$ is the time parameter.

Under conditions when the amount of growth energy is constant and does not depend on the amount of dry mass $W$, growth is proportional to the nutrient medium $S$, growth is irreversible and the growth rate can be represented as

$$\frac{dW}{dt} = -kS$$

(2)

where $k$ is a constant value.

After integration and elementary transformations of formula (1) we get:

$$W = W_f(1 - e^{-kt})$$

(3)

where $W_f$ is dry matter saturation level.

These theoretical prerequisites for constructing functional dependencies were used in this work to describe the processes of sugar beet growth under the influence of the studied factors at the first and second levels of productivity modeling, identified to the agroclimatic conditions of the steppe zone of Russia.

The zonal cultivation conditions of any crop are characterized by geographical latitude and longitude. They determine the magnitude and daily duration of the PAR, as well as the amount of precipitation. The first of these values (latitude and longitude) are fixed and do not depend on the season, the second—soil moisture and precipitation—are either time series (for retrospective analysis) or predicted probability distribution functions.

The identification of a crop when modeling the level of productivity is determined by its type ($C_1 - C_4$), as well as by a parameter characterizing the efficiency of interception of the light flux by the vegetation cover in the field. The architectonics of sugar beet culture is characterized by a spherical angular distribution of leaves, in which there is a uniform distribution of the density of the leaf surface along the sine of the angle of incidence. Therefore, the proportion of the leaf surface that receives direct radiation between the two sines of the angle of incidence is equal to their difference. As a result, the interaction between the canopy formed by leaves and the light flux is one of the most effective in comparison with other cultures, because the spatial distribution of sugar beet leaves (architectonics of the culture) provides minimal shading of the lower
leaves by the upper ones, while the attenuation of the light flux practically does not occur. In accordance with the accepted classification, sugar beets are classified as type 
\( C_3 \) [10, 11].

In accordance with the accepted conceptual prerequisites, biological features and conditions for the growth of sugar beets under conditions of unstable moistening of the steppe zone of Russia, analytical models of the first two levels of the production process have been developed.

3. Results

At the first level of modeling sugar beet productivity, it is assumed that growth occurs in the absence of limiting factors while optimally supplying plants with nutrients and soil moisture, without weeds, pests and diseases. In this case, the PAR becomes the only factor determining the productivity of the culture, which determines the dynamics of the potential productivity of the culture.

The energy of solar radiation in the radiation range of 400-700 nm absorbed by the leaves of plants, as a result of photosynthesis, turns into a mass of aboveground and underground organs. Crop productivity is determined by the dependence of the dry matter mass of leaves and roots on the area of the photosynthetic surface. In turn, the efficiency of light use by sugar beet crops is determined by the synthesis of carbohydrates used for respiration, maintenance and growth of structural biomass.

To describe the photosynthesis of leaves, the J. Goodriaan equation [12] and specific values of the parameters of the external conditions for growing the steppe zone of Russia were used:

\[
F_n = (F_m + R_d)(1 - \exp(-\frac{H\epsilon}{F_m + R_d})) - R_d
\]

where \( F_n \) is true assimilation of \( \text{CO}_2 \) by leaves, kg/ha/h; \( F_m \) is maximum true assimilation rate for leaves at high radiation intensity, kg/ha/h (for plant \( C_3 \) it equals \( 11 \times 10^{-9} \) kg/J\(^{-1}\)); \( R_d \) is dark respiration, kg/ha/h; \( H \) is absorbed radiation flux in the range of 400-700 nm (ФАР), J.m\(^{-2}\) of leaf surface; \( \epsilon \) is initial efficiency of using radiation for fixing \( \text{CO}_2 \) in leaves, kg/J.

The specific value of the PAR is determined by the geographical latitude and time of day. In accordance with the conditions of the steppe zone of Russia (47-48° north latitude), the dependence of the true \( \text{CO}_2 \) assimilation for a single leaf of sugar beet on the PAR of different periods of the day was determined by formula (4) (Table 1).
TABLE 1: Calculation of true CO\textsubscript{2} assimilation for sugar beet leaves as a function of absorbed radiation flux in the range of 400-700 nm of PAR.

| Absorbed radiation flux [J.m\textsuperscript{-2} of leaf surface (\(H\))] | Intermediate parameters *) | True assimilation of CO\textsubscript{2} \((F_{n})\) [kg/ha/h] |
|---|---|---|
| 0 | 0 | -0.14 |
| 0.5 | 0.58 | 0.56 | 0.68 |
| 1 | 1.16 | 0.31 | 1.07 |
| 1.5 | 1.74 | 0.18 | 1.28 |
| 2 | 2.31 | 0.1 | 1.4 |
| 2.5 | 2.89 | 0.06 | 1.47 |
| 3 | 3.47 | 0.03 | 1.51 |
| 3.5 | 4.05 | 0.02 | 1.53 |

*) Note: \(F_{m} = 11 \times 10^{-9}\) kg/J; \(R_{d} = 11 \times 10^{-8}\) kg/J; \(\varepsilon = 1.4 \times 10^{-8}\) kg/J

With PAR over 2 J, saturation of true assimilation sets in, and a further increase in active radiation has practically no effect on the increase in sugar beet productivity in the steppe zone of Russia (Figure 2).

The resulting dependence reflects the true assimilation of CO\textsubscript{2} for any particular point in time and is the basis for determining the productivity of the culture for a day and for the entire growing season.

The dependence of the incoming PAR \(S\) (about half of all the incoming radiation measured by Kipp radiometer on a clear day and 60% on a cloudy day) on the height of the sun \(\beta\) in the absence of cloudiness can be represented as the following expression:

\[
S = 640 \sin \beta \exp(-K_{atm}/\sin \beta)
\]  

(5)
where $K_{atm}$ is a coefficient depending on the transparency of the atmosphere ($K_{atm} = 0.1$ for high transparency; $K_{atm} = 0.18$ for high humidity and dust).

The height of the sun $\sin \beta$ depends on the geographical latitude, serial number of the day, and time of day as follows:

$$\sin \beta = \sin \lambda \sin \delta + \cos \lambda \cos \delta \cos(2\pi(t_h + 12)/24)$$  \hspace{1cm} (6)

where $\lambda$ is latitude, degrees; $\delta$ is declination of the sun, degrees; $t_h$ is local time of day, h.

Longitude of the day has a significant effect on plant growth. The need to describe it as a function of the calendar period $h_n$ follows from the need to model photosynthesis over the entire period of culture growth:

$$h_n = \frac{2}{360} \arccos(-\tan \lambda \cdot \tan \delta)$$  \hspace{1cm} (7)

The declination of the sun during the year for days varies as follows:

$$\delta = -23.4 \cos(2\pi(t_d + 10)/365)$$  \hspace{1cm} (8)

where $\delta$ is the serial number of the day starting January 1 ($\delta$ is expressed in degrees).

The average daily photosynthetic radiation activity (PAR) for a given geographical latitude also depends on the serial number of the day:

$$\bar{J}_N = (\bar{J}_{113} + \bar{J}_{296}) + (\bar{J}_{113} - \bar{J}_{296}) \sin(\frac{N - 21}{365} \cdot 360)$$  \hspace{1cm} (9)

where $N=1$ corresponds to March 1; $N=113$ is June 21; $N=296$ is December 21; $J_N$ is the long-term average value of the daily PAR, falling on a horizontal surface per day with serial number $N$.

In accordance with the obtained true assimilation of CO$_2$, the dependence of the PAR on the periods of the year was determined for geographical conditions of 46° and 48° N (steppe zone of Russia) (Figure 3). The growth rate of sugar beet productivity increases from May 10 to June 21 (from seedlings to closing of the rows), after which they begin to decline at a higher rate than in the previous period.

Biological yield is defined as the difference between the intensity of photosynthesis and total respiration \[13–15\]. Plants when breathing lose some of the products of photosynthesis and the total mass. Respiratory rate $R$ is defined as the sum of the shares of total photosynthesis ($P$) and current biomass ($W$):

$$R = aW + b$$  \hspace{1cm} (10)

where $a$ and $b$ are constants.
A constant fraction of photosynthesis products $\phi$ is spent on leaf growth, the rest goes to the root, ensuring its growth, maintenance of vital functions, sugar accumulation, etc.

According to the concept of balance formulated by J. France and J. Thornley, a plant receives resource of a nutrient medium $\Delta s$ over time $\Delta t$ and completely expends it in the same time for growth $\Delta s_g$ and maintenance $\Delta s_m$:

$$\Delta s = \Delta s_g + \Delta s_m$$  \hspace{1cm} (11)

Two components are distinguished in the respiration of organisms: respiration of growth and respiration of maintenance. The biomass increment $\Delta w$ consumes a part of the component $\Delta s_g$ and the result of the transformation of the nutrient medium into the plant substance $Y$ is determined by the formula:

$$Y = \frac{\Delta w}{\Delta S} = \frac{\Delta w}{\Delta w + \Delta S_g + \Delta S_m}$$  \hspace{1cm} (12)

The results obtained represent an analytical model of the first level of productivity. Using it, a quantitative assessment of the full productivity of sugar beets in the steppe zone of Russia is given (Figure 4).

The actual dependence of productivity turned out to be slightly lower than theoretical, which is explained by the difference in actual periods of real agroclimatic conditions from the requirements of the first level of productivity (absence of restrictions on the hydration and mineral nutrition of plants). However, the difference between the final results—theoretical and actual productivity at the time of harvesting—does not exceed 10%.

To predict productivity in conditions of moisture deficiency, it is necessary to model the work of plant organs and soil processes, since they determine the availability
of water and nutrients for the root system of the plant. Based on the physical and physiological aspects of transpiration and photosynthesis, de Wit showed that the relationship between the accumulation of dry matter and water consumption depends on the level of radiation throughout the growing season:

\[ P/W = W/E_0 \]  \hspace{1cm} (13)

where \( P \) dry matter accumulation, kg; \( W \) water loss by plants, kg/day; \( E_0 \) is average daily evaporation of water from an open water surface, kg/ha/day; \( M \) is the coefficient of water consumption of the plant, kg/ha/day.

Under the influence of moisture deficiency, the growth rate of the plant’s aboveground organs is lower than optimal; in turn, the growth of the root system is enhanced. The parameters of the root vital activity model are determined by the spatial (vertical) distribution of soil moisture and its availability; vegetation transpiration rate is determined by evaporation and leaf surface index.

![Graph showing accumulation of dry matter of roots](image)

**Figure 4:** Accumulation of dry matter of roots in the conditions of the first level of productivity.

The methodology for determining the parameters during the formation of the model of the second level of productivity in the de Wit empirical equation (13) took into account the following factors:

- firstly, the value of the plant’s water consumption coefficient \( M \) (kg/ha/day) depends on the type of sugar beet and the conditions of its cultivation, in particular, on mineral nutrition. As shown by experiments, the value of \( M \) varies in the range of 85-60 kg/ha/day;

- secondly, the value of \( E_0 \) is the average daily evaporation of water from an open water surface (kg/ha/day), its average speed from an open surface depends on a deficit of air humidity (about 40,000 kg/ha/day) and is established from hydrometeorological reference books for the zone cultivation. Then the accumulation of 1
kg of dry matter will be associated with water loss due to transpiration (40000/85 = 470 kg). Multiplying this value by the growth rate of dry matter, as defined in the previous section, have determined the need for culture in water.

Knowing the moisture reserves in the soil at the date of emergence, the predicted or actual value of precipitation, we compared the required and available amount of water and analytically assessed the moisture supply deficit. The degree of growth retardation is determined by the ratio of the required and the available amount of moisture.

Guided by this technique and the results of measurements of soil moisture, precipitation, and dynamics of dry matter accumulation, model parameters (formula 13) for the second level of sugar beet productivity were identified in a specific situation. Figure 5 presents a quantitative assessment of the yield of sugar beet roots according to the growing season of the crop for moderate water deficiency conditions characterizing the second level of productivity.

![Figure 5: Accumulation of dry matter of sugar beet roots in conditions of moisture deficit (steppe zone of Russia).](image)

4. Discussion

Analyzing the results obtained, it is necessary to note the difference in the rate of accumulation of the second-level crop from the first: the dry matter grows at a variable rate. The increase in dry matter here is limited by moisture deficit, and during the periods of the 45-75th day of vegetation, when precipitation did not fall and soil moisture decreased to minimum values, there was almost no increase in dry matter, which formed shallow sections on the experimental and theoretical graphs. After precipitation during the 80-90th day of vegetation, the moisture deficit ceased to limit the growth of sugar
beets, the growth rate of dry matter resumed at a rate determined by the conditions of the model of the first productivity level. The discrepancy between the model obtained and the actual results of productivity is quite explainable.

5. Conclusion

Based on the analytical model of sugar beet productivity, it was found that the maximum rates of dry matter accumulation for the conditions of the steppe zone of Russia, in the absence of limiting factors, are not limited to the intake of PAR and allow obtaining up to 16 t/ha of dry matter of root crops. With a moisture deficit during the beginning of row closure (from the 45th to the 75th day of the growing season), there is almost no increase in the dry matter of the plant mass, which reduces the potential productivity of sugar beets by 50%.

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Conflict of Interest

The authors have no conflict of interest to declare.

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