Development of an Attitude Determination System for Mobile Robots

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Abstract. This paper designs an attitude determination system based on a low-cost MEMS inertial measurement unit. By introducing an adaptive Kalman filter algorithm, the outputs of accelerometers, magnetometers, and gyroscopes are combined to solve the robot's attitude angle in real time. The algorithm effectively eliminates the cumulative error of the gyro output. In addition, the output of the accelerometer is used to determine the motion state of the vehicle body, and the covariance matrix is adaptively adjusted to maintain a high-accuracy attitude output in both static and dynamic environments.

1. Introduction

The mobile robot is an integrated system combining environmental awareness, decision-making, behavior control and other functions. Mobile robots are evolving toward self-organizing, self-learning, and adaptive intelligence. The level of navigation capability is an important manifestation of the level of intelligent mobile robots. With the increasing application of mobile robots, new research topics are constantly being proposed for the research and development of mobile robots, and the navigation research of mobile robots continues to deepen and develop.

This paper presents an indirect adaptive Kalman attitude update algorithm applied to the micro inertial measurement unit. Micro-inertia devices have large divergence and random errors. The attitude angles obtained by the quaternion update algorithm based on gyro integration will produce large accumulated errors in a short time. The Kalman filter in this paper adaptively corrects the posture reliance based on gyro integration by using the measurement information of accelerometers and magnetometers, which well suppresses the divergence of the quaternion algorithm. At the same time, according to the output information of the accelerometer, the filtering parameters are adjusted adaptively so that the algorithm can maintain certain accuracy when the vehicle body is in a non-uniform motion state.
2. **Gyro integral attitude measurement**

The gyroscope integral method uses only gyroscopes data to calculate the change in the attitude of the carrier over time. The attitude angle of the carrier can be represented by a quaternion. The quaternion-based digital update algorithm has a small amount of computation and reliable operation, and is widely used in engineering practice. From the knowledge of rigid body kinematics, the quaternion updated differential equation can be expressed as,

\[ \dot{q} = \frac{1}{2} q \circ \omega(1) \]

Among equation (1), \( \circ \) is quaternion multiplication, \( \omega \) represents the gyro real-time angular velocity output, expressed in quaternion form, i.e.

\[ \omega = 0 + \omega_x i + \omega_y j + \omega_z k \quad (2) \]

According to quaternion multiplication, (2) is expanded into a matrix, as,

\[
\begin{bmatrix}
\dot{q}_0 \\
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
\omega_x & -\omega_y & -\omega_z \\
\omega_y & 0 & -\omega_z \\
\omega_z & \omega_y & 0 \\
\omega_x & \omega_y & \omega_z
\end{bmatrix}\begin{bmatrix}
q_0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix} = \frac{1}{2} [\omega] \begin{bmatrix}
q_0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix} \quad (3)
\]

In the formula, \([\omega]\) is the anti-symmetric matrix of the rotational angular velocity (vector coordinate relative to the navigation coordinate). By exponentially expanding the formula b using the Bekaa method, we get,

\[ q(t) = e^{\int_{t_0}^{t} \omega(\tau) d\tau} q(t_0) \quad (4) \]

Suppose \( [\Delta \theta] = \int_{t_0}^{t} [\omega(\tau)] d\tau \), we get,

\[ e^{[\Delta \theta]} = \left[1 - \frac{(\Delta \theta)^2}{2!} + \frac{(\Delta \theta)^4}{4!} \cdots \right] I + \frac{[\Delta \theta][\Delta \theta]}{2} \left[1 - \frac{(\Delta \theta)^2}{2!} + \frac{(\Delta \theta)^4}{4!} \cdots \right] = \cos \frac{\Delta \theta}{2} I + \frac{[\Delta \theta]}{2} \sin \frac{\Delta \theta}{2} \quad (5) \]

Combining equation (4) and equation (5), the quaternion update equation can be written in a recursive form, i.e.

\[ q(n+1) = (\cos \frac{\Delta \theta}{2} I + \frac{[\Delta \theta]}{2} \sin \frac{\Delta \theta}{2})q(n) \quad (6) \]

In order to facilitate the numerical solution, we perform the Taylor expansion for the trigonometric function, and the fourth-order approximate expression of the quaternion value update is obtained, as,

\[ q(n+1) \simeq ((1 - \frac{\Delta \theta^2}{8} + \frac{\Delta \theta^4}{384}) I + (\frac{1}{2} - \frac{\Delta \theta^2}{2})[\Delta \theta])q(n) \quad (7) \]

\( \Delta \theta, [\Delta \theta] \) are the angle increment modulus and the angle increment anti-symmetric matrix, respectively, where

\[ \Delta \theta^2 = \Delta \theta_{bx}^2 + \Delta \theta_{by}^2 + \Delta \theta_{bz}^2 \quad (8) \]

\[ [\Delta \theta] = \int_{t_0}^{t} [\omega(\tau)] d\tau \approx \begin{bmatrix} 0 & -\Delta \theta_{bx} & -\Delta \theta_{by} & -\Delta \theta_{bz} \\ \Delta \theta_{bx} & 0 & -\Delta \theta_{bz} & -\Delta \theta_{by} \\ \Delta \theta_{by} & -\Delta \theta_{bx} & 0 & \Delta \theta_{bz} \\ \Delta \theta_{bz} & \Delta \theta_{by} & -\Delta \theta_{bx} & 0 \end{bmatrix} \quad (9) \]
In the above equation, $\Delta \theta_x, \Delta \theta_y, \Delta \theta_z$ are the angular increments of $n, n+1$ moments along the three axes of the carrier. The triaxial angular velocity of the gyro output at $n, n+1$ moments is known as $\omega_{bi(i=x,y,z)}(n,n+1)$. The angular increment can be approximated as,

$$
\Delta \theta_i = \int_n^{n+1} \omega_{bi} dt \approx \frac{\omega_{bi}(n) + \omega_{bi}(n+1)}{2} \ , i = x, y, z \quad (10)
$$

This paper uses the Mti MEMS IMU produced by Xsens. It integrates three-axis magnetometers, three-axis micro-machined accelerometers, and three-axis silicon micro-machined vibratory gyroscopes. Each sensor can output calibrated three-dimensional line acceleration, angular velocity, and geomagnetic intensity data in real-time after internal DSP fusion processing. Mti outputs triaxial angular rate information in real time at a frequency of 50 Hz. The angular increment for each axis can be calculated by (10). As long as we know the quaternion value at the previous moment, we can calculate the quaternion value at the current moment based on (7). There is a one-to-one correspondence between quaternion and attitude angle. Knowing the initial time pose information, the quaternion-based three-axis gyro update algorithm flow is as follows,

![Diagram of quaternion-based three-axis gyro update algorithm.](image)

Figure 1. Quaternion-based three-axis gyro update algorithm.

In the stationary state, we use the angular rate to solve the attitude angle in real time. The output result is shown in figure 2.

![Graph of 3D attitude angle solution outputs.](image)

Figure 2. 3D Attitude angle solution outputs.

In the stationary state, the drift of the attitude angle is approximately linear with time. The zero drifts for heading angle, roll angle, and attitude angle are approximately 0.872°/min, 0.982°/min, and 0.545°/min. Due to the large measurement noise of the MEMS gyroscope device, the attitude angle error obtained by simply using angular rate integration for posture update is divergent with time. Other sensor data must be introduced to correct the error.

### 3. Acceler-magnetometer attitude estimation

Mti also offers real-time magnetometers and acceleration outputs. Knowing the output of the triaxial acceleration and magnetic field strength, the attitude of the carrier (pitch, roll, heading, respectively) can be estimated, as,

$$
\theta = a \sin \left( \frac{a_x}{g} \right), \quad \gamma = a \sin \left( \frac{a_y}{g \cos(\theta)} \right)
$$

(11)
\[ \psi_m = \tan^{-1}(M_y^b, M_x^b) = \tan^{-1}(M_y^b \cos \gamma + M_x^b \sin \gamma, M_y^b \cos \theta + M_x^b \sin \gamma - M_z^b \cos \gamma \sin \theta) \] 

The direction of the geomagnetic field points to magnetic north, which also differs by a magnetic declination from true north. The true northbound direction should be the magnetic heading plus the magnetic declination correction. Using accelerometer and magnetometer output, the calculated attitude angle is shown in figure 3.

\[ \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \dot{\omega}_3 & -\dot{\omega}_2 \\ -\dot{\omega}_3 & 0 & \dot{\omega}_1 \\ \dot{\omega}_2 & -\dot{\omega}_1 & 0 \end{bmatrix} \begin{bmatrix} \delta \phi \\ \delta \theta \\ \delta \psi \end{bmatrix} + \begin{bmatrix} \delta \omega_1 \\ \delta \omega_2 \\ \delta \omega_3 \end{bmatrix} \] (12)

4. Kalman filter fusion attitude estimation algorithm

Based on the above experimental results, there is an accumulated error in gyro integration-based posture update, which cannot be used alone for a long period of time. Meanwhile, there is random noise in the attitude angle information calculated by the addition and magnetometer, and it can only be applied when the carrier is outside the static plane or the acceleration is small. In this paper, a Kalman filtering algorithm based on attitude angle error is proposed. The angular velocity, accelerometer, and magnetometer outputs are combined to estimate the dynamic 3D pose of the carrier in real time. The algorithm can suppress the cumulative error caused by elimination of gyro zero drift, and overcome the simple estimation of the distortion of the attitude estimation under dynamic conditions using the accelerometer/magnetometer.

4.1. State update equation

The state update equation based on attitude angle error can be expressed as,

\[ \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \dot{\omega}_3 & -\dot{\omega}_2 \\ -\dot{\omega}_3 & 0 & \dot{\omega}_1 \\ \dot{\omega}_2 & -\dot{\omega}_1 & 0 \end{bmatrix} \begin{bmatrix} \delta \phi \\ \delta \theta \\ \delta \psi \end{bmatrix} + \begin{bmatrix} \delta \omega_1 \\ \delta \omega_2 \\ \delta \omega_3 \end{bmatrix} \] (12)

4.2. Measurement equation

We define the true value of the attitude as \( A \) and the estimated attitude from angular rate integration as \( A + \delta A \). \( A + W \) is the attitude estimation obtained by the accelerometer& magnetometer. \( W \) stands for the white noise. The measurement equation based on gyro attitude estimation error can be obtained from substitution of the two.

\[ \dot{Z}(\delta A) = (A + \delta A) - (A + W) = \delta A + (-W) = \delta A + W \] (13)

4.3. Filter matrix and parameter determination

Suppose the attitude error as state variable. The discredited state and measurement equation can be expressed as,
\[ x_k = (I - \Omega_0 \Delta T)x_{k-1} + w_{k-1} \]  
(14)

\[ z_k = Ix_k + v_k \]

Where, the state transition matrix and measurement matrix are

\[
A = \begin{bmatrix}
1 & -\hat{\Omega}_0 \Delta T & \hat{\Omega}_0 \Delta T \\
\hat{\Omega}_0 \Delta T & 1 & -\hat{\Omega}_1 \Delta T \\
-\hat{\Omega}_1 \Delta T & \hat{\Omega}_1 \Delta T & 1
\end{bmatrix},
H = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Where, \( \Delta T \) is the sampling period. Assuming the components between process noise and measurement noise are uncorrelated, the process noise covariance matrix and the measurement noise covariance matrix can be represented by diagonal matrices, i.e.

\[
Q = \begin{bmatrix}
w_1^2 & 0 & 0 \\
0 & w_2^2 & 0 \\
0 & 0 & w_3^2
\end{bmatrix},
R = \begin{bmatrix}
v_1^2 & 0 & 0 \\
0 & v_2^2 & 0 \\
0 & 0 & v_3^2
\end{bmatrix}
\]

Where, \( w_1, w_2, w_3 \) denote the error of the angular rate measurement; \( v_1, v_2, v_3 \) denote the attitude error measured by accelerometer&magnetometer. Through static measurement calculations, the angular rate measurement error is 0.1°/s. The pitch and roll angle errors are 1°/s, and the heading angle error is 3°/s. Converting the unit to rad/s, we get the specific matrix, i.e.

\[
Q = \begin{bmatrix}
(10^{-4})^2 & 0 & 0 \\
0 & (10^{-4})^2 & 0 \\
0 & 0 & (10^{-4})^2
\end{bmatrix},
R = \begin{bmatrix}
(0.017)^2 & 0 & 0 \\
0 & (0.017)^2 & 0 \\
0 & 0 & (0.05)^2
\end{bmatrix}
\]

(17)

5. Adaptive fusion algorithm and test results

When the carrier is stationary or moving at a constant speed, the attitude angle error can be predicted according to the state transition matrix, measurement matrix, and error covariance given in the previous section. In the non-uniform state, due to the existence of interference acceleration items, the attitude angle information of the carrier cannot be accurately measured by accelerometer&magnetometer. When applying the filter process, the motion state of the carrier should be judged first, if the output of the accelerometer satisfies,

\[
|\sqrt{a_x^2 + a_y^2 + a_z^2} - g| < \varepsilon
\]

(18)

It means that the carrier is in an approximately stationary and uniform state. At this time, the attitude error is predicted and updated according to the Kalman filtering process. If the formula is not satisfied, the carrier is in a non-uniform state and the measured covariance matrix will increase significantly. Only the gyro angular rate output is used to predict the attitude error, and no measurement update is performed. The fusion algorithm is verified by the actual MTI output data. Collect 1400 seconds of MTI output data. The MTI moves with the carrier. The comparison of attitude angle results obtained by quaternion update, addition magnetometer and fusion algorithm is shown in figure 4.
6. Conclusions

This paper designs an attitude determination system based on a low-cost MEMS inertial measurement unit. Due to accumulation of errors, the attitude obtained by gyro calculation becomes significantly distorted with time. And there exists large white noise in attitude calculated by accelerometer & magnetometer. The Kalman fusion filter algorithm combines the outputs of the above two methods, which can effectively eliminate the cumulative error and random noise, resulting in a more accurate attitude angle solution.

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