Let $(X, d)$ be a bounded metric space. The object of the investigation is the metric invariant

$$\delta(X) = \inf_{p \in X} \sup_{q \in X} d(p, q)$$

arising in approximation theory. In the case when $X$ is a regular closed curve $\Gamma$ in the plane $\mathbb{R}^2$, it is conjectured that $L(\Gamma) \geq \pi \delta(\Gamma)$, where $L(\Gamma)$ is the length of $\Gamma$ and $d$ is the standard restricted Euclidean metric. The author proves this conjecture in the case when $\Gamma$ is a convex curve of class $C^2$ and all curvature centers of $\Gamma$ lie in the interior of $\Gamma$. In this case, it is shown that the equality $L(\Gamma) = \pi \delta(\Gamma)$ holds if and only if $\Gamma$ is of constant breadth. Approximation of $\Gamma$ by polygons is also studied and the related estimates of $\delta(\Gamma)$ are obtained, including numerical experiments.

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MSC:

51K05 General theory of distance geometry
52A10 Convex sets in 2 dimensions (including convex curves)
52A40 Inequalities and extremum problems involving convexity in convex geometry
53A04 Curves in Euclidean and related spaces
57Q55 Approximations in PL-topology
52A27 Approximation by convex sets
51E21 Blocking sets, ovals, $k$-arcs

Keywords:
metric invariant; relative Chebyshev radius; isoperimetric inequality

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