On the magnitude of the energy flow inherent in zero-point radiation

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I. INTRODUCTION

Sparnaay’s 1958 experiments exposed the existence of zero-point radiation, which Nernst had considered as a possibility in 1916. However, this was not Sparnaay’s intention, as he was only trying to check Casimir’s hypothesis about the mutual attraction of two uncharged conductor plates placed very close together in a vacuum. This attraction must disappear when the temperature there was still some attraction not accounted for by Casimir’s hypothesis, being independent of temperature and obeying a very simple law, it is directly proportional to the surface area of the plates, and inversely proportional to the fourth power of the distance “d” between them. Sparnaay observed this force when the plates were located in a near-perfect vacuum at near-zero absolute temperature. For a distance of \( 5 \times 10^{-5} \) cm, he was able to measure a force of 0.196 g cm s\(^{-2}\), and deduced the formula

\[ f = \frac{k_s}{d^4}; \quad \text{where} \quad k_s = 1.3 \times 10^{-18} \text{ erg cm} \]

In a near-perfect vacuum and at near absolute zero temperature, which means the absence of any “photon gas”, the phenomenon observed by Sparnaay could only be produced by a radiation inherent in space. This could only be the case if its spectrum is relativistically invariant, which could only happen if its spectral density function is inversely proportional to the cubes of the wavelengths. In other words, if the number of photons of wavelength \( \lambda \), which strike a given area within a given time is inversely proportional to \( \lambda^3 \).

A function of spectral density inversely proportional to the cubes of the wavelengths, implies a distribution of energies which is inversely proportional to the fourth power of the wavelengths, because the photons energies are inversely proportional to their wavelengths. In 1969, Timothy H. Boyer showed that the spectral density function of zero-point radiation is

\[ f_\varphi(\lambda) = \frac{1}{2\pi^2} \frac{1}{\lambda^4}; \quad (1) \]

where \( \lambda_n \) is the number giving the measurement of wavelength \( \lambda \). This function produces the next for the corresponding energies

\[ E_\varphi(\lambda) = \frac{1}{2\pi^2} \frac{hc}{\lambda} \frac{1}{(\lambda_n)^3}. \quad (2) \]

For \( \lambda \to 0 \), \( E_\varphi(\lambda) \to \infty \). There must therefore be a threshold for \( \lambda \), which will hereafter be designated by the symbol \( q_\lambda \).

In the Sparnaay effect, the presence of a force which is inversely proportional to the fourth power of the distance “d” between the plates, while the photons distribution of energies is inversely proportional to the fourth power of the wavelengths, leads us to infer that the cause of the apparent attraction must lie in some factor which varies inversely with wavelength and which produces a different behaviour in photons of wavelength equal to or greater than “d”, since:

\[ \sum_{n=1}^{\infty} k n^{-5} \to \frac{k}{4d^4} \]

The factor in question turns out to be the proportion of photons which are reflected, not absorbed; that is the coefficient of reflection \( \rho \), while the different behaviour is caused by the obstacle which each plate offers to the reflection by the inner plate of the other one, of photons of wavelengths equal to or greater than “d”. As we know, the energy which in transferred by a reflected photon is double that transferred by one which is absorbed.

II. INTERPRETATION OF THE EXPERIMENTAL RESULTS

We now come to the title of this paper, an attempt to find the magnitude of the energy flow which is in-
herent in zero-point radiation; two radiations with the same spectrum, i.e., with the same energy distribution by wavelength, can have very different energy flows.

The results of Sparnaay’s experiments provide a link with reality which is enough to show the intensity of the energy flow belonging to zero-point radiation. So, remembering the requirement that in zero-point radiation there must exist a threshold, \( q_e \), for the wavelength of the photons, we will now test the hypothesis that the results agree with an energy flow which would produce the incidence in a hypothetical area \((q_\lambda)^2\) of:

One photon of wavelength \( q_\lambda \), per \( xq_\tau \) \( (q_\tau = q_\lambda/c) \)

One photon of wavelength \( 2q_\lambda \), per \( x^2q_\tau \ldots \)

One photon of wavelength \( nq_\lambda \), per \( xn^2q_\lambda \)

This radiation implies an energy flow of:

\[
W_\varphi = \frac{1}{x} \frac{hc}{q_\lambda q_\tau} \left( 1 + \frac{1}{2^3} + \cdots + \frac{1}{n^4} \right) = \frac{1}{x} \frac{h^2}{9nq_\lambda} \text{ per } (q_\lambda)^2. \tag{3}
\]

From Sparnaay’s experiments and our earlier argument, this energy flow should cause the difference between the flows transmitted to the outer faces, and those transmitted to their inner faces to be:

\[
\Delta W_\lambda = \rho \left( \frac{hc}{x} \frac{1}{q_\lambda q_\tau} \right) \left[ \frac{1}{n^5} + \frac{1}{(n+1)^5} + \cdots + \frac{1}{n^5} \right]_{m \to \infty}
\]

per \( q_\lambda \) of surface \( \lambda \)

where \( n \) is the distance between the plates expressed in \( q_\lambda \). Hence

\[
\Delta W_\lambda = \frac{h^2}{9nq_\lambda} \frac{1}{c} \text{ per } (q_\lambda)^2. \tag{4}
\]

To simplify the next stages of reasoning we will use the system of measurement \((e, m_e, c)\), in which the basic magnitudes are the quantum of electric charge \( e \), the electron mass \( m_e \), and the speed of light \( c \). In this system the units of length and time are respectively:

\( l_e = e^2/m_ec^2 \),
\( t_e = c^2/m_ec^3 \); producing \( h = \frac{2\pi m_e l_e^2}{\alpha} \).

Clearly, if \( \lambda = k_\lambda q_\lambda \), we have \( \lambda = c/t_e = k_\lambda q_\lambda/c = k_\lambda q_\tau \). On the other hand if \( n \) is the number representing the measurement of the distance between the plates in \( q_\lambda \), the number which represents it in \( l_e \) is \( D_{l_e} \). \( D_{l_e}k_\lambda \) being equal to \( n \). It must be remembered that \( k_\lambda \) is certainly a very large number; there have been detected cosmic rays photons with wavelengths up to \( 10^{20} \) eV, which would imply \( k_\lambda \) values of the order of \( 2.273 \times 10^{11} \). The true value of \( k_\lambda \) could be much greater than this.

If in (5) we substitute \( q_\lambda \) by \( l_e/k_\lambda \), \( q_\tau \) by \( t_c/k_\lambda \); \( n \) by \( D_{l_e}k_\lambda \), and \( h \) by \( \frac{2\pi m_e l_e^2}{\alpha} t_c \), and remember that the flow per \( (l_e)^2 \) is \( (k_\lambda)^2 \) times greater than the flow per \( (q_\lambda)^2 \), we can write

\[
\Delta W_{(e,m_e,c)} = \frac{\rho}{2} \frac{m_e l_e^2}{x} \frac{1}{\alpha} \frac{1}{(l_e/k_\lambda)(t_c/k_\lambda)} \frac{1}{4D_{l_e}^4(k_\lambda)^2(l_e)^2}. \tag{6}
\]

That is

\[
\Delta W_{(e,m_e,c)} = \frac{\rho}{2} \frac{m_e l_e^2}{x} \frac{1}{\alpha} \frac{1}{D_{l_e}^4(k_\lambda)^2} \text{ per } (l_e)^2. \tag{6}
\]

In (6) we find that the factor \( k_\lambda \) has disappeared. If this were not so, we would arrive at absurd results, since the value of \( \rho \) would be provided by:

\[
\rho = \frac{\Delta W_{(e,m_e,c)}}{2\alpha x D_{l_e}^4(k_\lambda)^2}. \tag{7}
\]

For \( a > 0 \) \( \rho \) would be huge and for \( a < 0 \) \( \rho \) would be tiny; it must be remembered that \( k_\lambda \) must be greater than \( 2.273 \times 10^{11} \). The values of \( \rho \) must always be less that 1, and for a radiation where photons predominate with wavelengths slightly greater than \( 5 \times 10^{-5} \) cm, these values must be greater than 0.4.

Since we must consider the energy flows for both the plates, we find that they produce an apparent force of attraction which can be expressed by:

\[
F = \frac{2\Delta W_{(e,m_e,c)}}{c} = \frac{\alpha x}{D_{l_e}^4} \frac{m_e l_e}{c} \text{ per } (l_e)^2. \tag{7}
\]

In order to express (7) in terms of \( g \) s per \( cm^2 \), we have to include the transformation coefficients \( 1 \) cm = \( k_{l_e} l_e \); \( 1 \) \( s=\tau \), \( 1 \) g = \( k_{m_e} m_e \), remembering to multiply by \((k_\lambda)^2\) in order to move to forces per \( cm^2 \), and that \( D_{l_e} = k_{l_e} d \) cm, from which we obtain the following force per \( cm^2 \):

\[
F_{(e,g,s)} = \frac{\alpha x}{D_{l_e}^4} \frac{(k_\lambda)^2}{k_{m_e} d^2} \frac{1}{c} \text{ g cm}. \tag{8}
\]

Since \( k_{l_e} = 3.548692 \times 10^{12} \); \( k_{l_e} = 1.063871 \times 10^{23} \); \( k_{m_e} = 1.097668 \times 10^{27} \); \( d = 5 \times 10^{-5} \); by inserting these values in (8), we reach

\[
F_{(e,g,s)} = \frac{\rho}{x} \frac{15.893023}{s^2} \text{ g cm}. \tag{9}
\]

Zero-point radiation comes from all directions within the half of space which is faced by the outer side of each plate, and its normal component is given, for each direction, by \( F \sin \alpha \), where \( \alpha \) is the angle between the plane of the plate and the direction in question. If we imagine a sphere of radius \( F \), it is not hard to see that the number with measures the sum of the normal components is the number which measures the volume of the hemisphere of radius \( F \), where \( F \) is the number measuring \( F \) in the system of units employed.
However $\Delta W_\lambda$, as shown in (5), is not the flow which arrives from each and every direction in the said half-space, but the sum of all the flows which arrive from all those directions. In order to make the necessary correction we have only to divide the number $\frac{2\pi}{3} (F_e)^3$ by $2\pi (F_e)^2$, which represents the area of the hemisphere of radius $F_e$, this amounts to dividing the total sum between the total of the directions to be considered. The resulting value is

$$F_0 = \frac{1}{3} F_{(c,g,s)} = \frac{\rho}{x} 5.297674 \frac{\text{g cm}}{s^3} \quad (10)$$

However, this relation, which makes it easier to visualize the problem, is not the correct one, because the radiation which strikes at an angle $\varphi$ from the vertical has a trajectory between the plates which is given by $d/\cos \varphi$ (Fig. 1).

Therefore the value of $\Delta W_\lambda$ to be considered is:

$$\Delta W_{\lambda\varphi} = \frac{hc \rho}{q \lambda q_t} \frac{1}{x} \left[ \frac{1}{n^6/\cos^5 \varphi} + \frac{1}{(n/\cos \varphi + 1)^5} + \cdots \right]$$

+ \frac{1}{n/\cos \varphi + m)^5}, \quad (11)$$

replacing (4), so that we obtain:

$$\Delta W_{\lambda\varphi} = \frac{\rho}{x} \frac{hc}{q \lambda q_t} \frac{1}{4n^4/\cos^4 \varphi},$$

from which we reach:

$$F_{\varphi (c,m,e,c)} = \frac{\rho \pi \cos^4 \varphi m_s l_c}{\alpha x} \frac{D^4}{l_c^2} \text{ per } (l_c)^2, \quad (12)$$

replacing (7).

This force is equal to $F_{(c,m,e,c)} \cos^4 \varphi$ and its component normal to the plane of the plates is $F_{(c,m,e,c)} \cos^5 \varphi$. On Fig. 2 it can be seen that to each point $P$ on the circumference of radius $F$ there corresponds a point $P_{\varphi}$ whose coordinates are those of $P$ multiplied by $\cos^4 \varphi$. The points $P_{\varphi}$ form a line within the circle, and the sum of the component normal to the plates is now given by the volume produced by the area enclosed within that line and by the axis of coordinates when it turns around the axis $OY$, instead of the volume produced by the quadrant of circle turning around that axis. The volume in question is given by the product of the said area and the circle described by its center of gravity $\mathcal{F}$ (Fig. 2).

The parametric coordinates of the curve thus described are

$$x = F_e \cos^4 \varphi \sin \varphi \quad y = F_e \cos^5 \varphi$$

To the ordinate $F_e \cos^5 \varphi$ there corresponds the differential element of abscissa $F_e (\cos^5 \varphi - 4 \cos^3 \varphi \sin^2 \varphi)$. Therefore

$$S = (F_e)^2 \int_0^{\pi/2} (\cos^{10} \varphi - 4 \cos^8 \varphi \sin^2 \varphi) d\varphi;$$

whence:

$$S = 0.214757 (F_e)^2$$

The circle described by the center of gravity as it turns round axis $OY$ is one whose radius is the abcissa of that center of gravity, which is given by:

$$x_G S = \int_0^{\pi/2} x dy = \frac{5}{2} (F_e)^3 \int_0^{\pi/2} (\cos^{12} \varphi \sin^3 \varphi) d\varphi;$$

whence:

$$x_G S = (F_e)^3 \frac{1}{39} \quad \text{and} \quad x_G = 0.119395 F_e$$

The volume to be considered is therefore

$$V = (F_e)^3 \times 0.214757 \times 2\pi \times 0.119395 = 0.1611 (F_e)^3$$

While the force to be considered is

$$F = \frac{0.1611 (F_e)^3}{2\pi (F_e)^2} = 0.0256402 F_e$$

With the value of $F_e$ given by (9)

$$F = \frac{0.407513 \text{ g cm}}{s^2}. $$
Using this with the results of Sparnaay’s measurements, we obtain
\[ \frac{\rho}{x} = 0.407513 = 0.196; \quad \text{whence }\rho = 0.481x \]

Since \( x \) must be a natural number, and \( \rho \) must fall between 0 and 1 the only possible results are
\[ x = 1 \quad \rho = 0.481 \]
\[ x = 2 \quad \rho = 0.962 \]

The second of these pairs is unlikely for wavelengths close to \( 5 \times 10^{-5} \) cm. We can therefore deduce that the energy flow of zero-point radiation is the results of the incidence, in an area \((q^2)\), of:

One photon of wavelength \( q\lambda \) per \( q^2 \)
One photon of wavelength \( 2q\lambda \), per \( 2^3q^2 \)...
One photon of wavelength \( nq\lambda \), per \( n^3q^2 \)

### III. CONCLUSIONS

Returning to (3); \( W_\lambda = \frac{\pi^4}{90x} \frac{hc}{q^2 q^2} \) per \((q\lambda)^2\), and expressing it through the \((e, m_e, c)\) system and as an energy flow by \((l_e)^2\), we obtain
\[ W_{(e,m_e,c)} = \frac{\pi^4}{90x} \frac{2\pi m_e l_e^2}{\alpha} \frac{(k\lambda)^2}{(l_e/k\lambda)(l_e/k\lambda)} \]

Whence, where \( x = 1 \),
\[ W_{(e,m_e,c)} = \frac{\pi^4}{45\alpha} (k\lambda)^4 \frac{m_e c^2}{l_e} \]

and a force
\[ F_{(e,m_e,c)} = \frac{\pi^5}{45\alpha} (k\lambda)^4 \frac{m_e c}{l_e^2} \]

over an area \((l_e)^2\).

This force, expressed in the \((c,g,s)\) system and over 1 cm\(^2\) is produced by
\[
F_{(c,g,s)} = \frac{\pi^5}{45\alpha} (k\lambda)^4 \frac{(g/m_e)(cm/k_\lambda)}{(s/k_\lambda)^2} \]
\[
= 3.409994 \times 10^{34} (k\lambda)^4 \frac{g}{cm^2} \quad (13) \]

As before, the normal force per cm\(^2\) of surface is given by the third part of this value (see (10))
\[
\frac{F_{(c,g,s)}}{3} = 1.136647 \times 10^{34} (k\lambda)^4 \frac{g}{cm^2} \]

per cm\(^2\) of surface.

Given the importance of \( k\lambda \), this force is huge and, at first sight, impossible to imagine. However, the rigorous equality prevailing between the energy flows arriving from all directions of space implies that the results are systematically nil. The presence of this force can only be detected when its equality of arrival is disturbed, as a result of phenomena such as the Sparnaay effect, or through the interaction of the photons with free elemental particles. Obviously Sparnaay’s experiments implied only photons whose wavelengths were greater than \( 5 \times 10^{-5} \) cm, and zero-point radiation may include only photons with wavelengths greater than \( xq\lambda \), being \( x \) an integer perhaps very great. Nonetheless the fact that this force may be immensely great is a surprising datum in the reality of the Universe. A surprising datum from which surprising consequences may derive.

[1] M. J. Sparnaay, Physica 24, 51 (1958).
[2] H. B. J. Casimir, J. Chim. Phys. 46, 407 (1949).
[3] T. H. Boyer, Phys. Rev. 182, 1374 (1969); Sci. Am. N. 10, 42 (1985).
[4] E. Rouché and Ch. Comberousse, Traité de Geometrie Vol. 2 (Gauthier-Villars, Paris, 1949) p. 228.
[5] P. W. Milonni, The Quantum Vacuum (Acad. Press, New York, 1994) pp. 275-280.
[6] R. Alvaragonzález, Rev. Esp. Fis. 14 (4), 32 (2000).