Spinons and holons on the lattice
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We point out that the dynamical fermion mass generation in the 3D compact U(1) lattice gauge theory with charged fermion and scalar fields ($\chi U\varphi$ model) may be of relevance for the spinon-holon theory with local gauge symmetry in the condensed matter physics. However, many properties of the $\chi U\varphi$ model are uncertain, so we make some conjectures to motivate their future investigation. Most probably, for strong gauge coupling the model is by universality equivalent to the familiar 3D four-fermion coupling models with $N_f = 2$. Available numerical results indicate that at the intermediate and weak gauge coupling two universality classes with new interesting physics may arise. One of them is associated with a tricritical point which probably exists in the phase diagram of the $\chi U\varphi$ model. The other one is determined by the dynamical fermion mass generation in the compact QED$_3$, which is insufficiently understood but of much interest by itself.

1. Controversial background

One of the attempts to understand the high temperature superconductors in the condensed matter physics is based on the strongly coupled 3D U(1) gauge theory. The gauge symmetry arises essentially through the Hubbard-Stratonovich transformation of the nearest-neighbour four-fermion coupling (see ref. [1] for an explanation), and the strongly coupled U(1) gauge field is thus different from the electromagnetic field. Including the idea of spin-charge separation, this U(1) gauge field couples to fermion and boson fields (spinons and holons, respectively) and is naturally compact. The kinetic term is absent in the original formulation, i.e. $\beta = 1/g^2 = 0$, but may arise during a renormalization or if some degrees of freedom are integrated out.

Whether this framework, originated by P. W. Anderson [2], is really suitable for achieving the original goal is a highly controversial issue [3]. But at least it is still being advocated by some experts in most respected journals. For a recent example with a valuable exposition of the approach and a list of earlier references see [4,5]. A related idea is that of a conjectured new infrared fixed point in QED$_3$ [7].

The present author cannot take a position in the controversy, but he wants to point out how natural its persistence is: the involved mechanisms include highly complex interplay of Higgs mechanism, dynamical mass generation (DMG), confinement and screening. Analytic arguments may miss important points. This is at least our experience from the numerical simulation of a lattice model, the $\chi U\varphi$ model [8]. This model resembles the 3D spinon-holon system coupled by the U(1) gauge field, though simplified by omitting chemical potential used in some cases [8].

2. $\chi U\varphi$ model

The $\chi U\varphi$ model (see ref. [8] for a more detailed description) is defined on a 3D cubic lattice. The action reads:

$$S_{\chi U\varphi} = S_\chi + S_U + S_\phi, \quad (1)$$

with

$$S_\chi = \frac{1}{2} \sum_x \chi_x \sum_{\mu=1}^3 \eta_{\mu x} (U_{x,\mu} \chi_{x+\mu} - U_{x-\mu} U^\dagger_{x,\mu} \chi_{x-\mu}) + \hat{m}_0 \sum_x \chi_x \chi_x, \quad (2)$$

$$S_U = \beta \sum_{x,\mu < \nu} (1 - \text{Re} U_{x,\mu\nu}) , \quad (3)$$

$$S_\phi = -\kappa \sum_x \sum_{\mu=1}^3 (\phi_x U_{x,\mu} \phi_{x+\mu} + \text{h.c.}) . \quad (4)$$
Here $\chi$ is the Kogut-Susskind fermion field. Because of doubling our model describes two four-component fermions ($N_f = 2$). We stress that the charges of the matter fields exclude a direct Yukawa coupling between them. We are mainly interested in the limit of vanishing bare fermion mass $\hat{m}_0$ (in lattice units), but allowing nonvanishing $\hat{m}_0$ is important for better understanding of the model, as well as for technical reasons. $U_{x,\mu}$ represent the compact U(1) link variables, and $U_{x,\mu\nu}$ is their plaquette product. The scalar field $\phi$ has, for simplicity, frozen length $|\phi| = 1$. Its hopping parameter $\kappa$ vanishes, if the square of the bare mass of the scalar field is $+\infty$, and is infinite, if the bare mass squared is $-\infty$. Thus large values of $\kappa$ correspond to the Higgs region, whereas small ones correspond to the confinement region of the phase diagram.

Our present understanding of the phase diagram in the limit $\hat{m}_0 = 0$ is shown in fig. 1. The DMG occurs in the Nambu phase, whereas in the Higgs phase the fermions remain massless. It is not clear where the boundary between these phases lies for $\beta > 1$. The data is consistent with two possibilities indicated by dashed lines. Correspondingly, properties of the region denoted by X are unclear. At least one of the dashed lines must be a phase transition, one can be a mere crossover.

3. Limit cases

For $\beta = 0$, the gauge and scalar fields can be integrated out exactly, and one ends up with a lattice version of a three-dimensional four-fermion model, the Gross-Neveu or Thirring model (see ref. [8] for a discussion of these alternatives). For our purposes, the important properties of this model are the second order phase transition at $m_0 = 0, \kappa \simeq 1$, below which the DMG takes place, and nonperturbative renormalisability in its vicinity, allowing a continuum limit. At small nonzero values of $\beta$ these properties persist.

For $\kappa = 0$, the scalar field is absent and the model is equivalent to the compact QED$_3$ with $N_f = 2$ fermions. For $\hat{m}_0 \to \infty$, it reduces to the pure compact QED$_3$. This is a confining theory, presumably with some gauge-ball spectrum.

When matter fields are dynamical, various gauge singlets, i.e. unconfined states are possible, in particular the fermion $F = \phi \chi$, which in the Nambu phase acquires mass through DMG. $F$ would presumably be the electron in the spinon-holon context.

In the weak gauge coupling limit, $\beta = \infty$, the fermions are free with mass $\hat{m}_0$, and $S_{\phi}$ reduces to the XY$_3$ model. It has a phase transition at $\kappa \simeq 0.27$.

At $\hat{m}_0 = \infty$, the model reduces to the three-dimensional compact U(1) Higgs model. For its recent investigation and references to earlier numerical studies see [10]. Data suggests that the phase transition of the XY$_3$ model continues as the Higgs phase transition of uncertain order to finite values of $\beta$. (A different, I think improbable scenario without the Higgs phase transition has been proposed recently in ref. [6]). At some small $\beta$, there is a critical end point.

The (presumed) knowledge about these limit cases is usually used in analytic arguments about what happens inside. However, some new phenomena can be conjectured.
4. A tricritical point?

The data\cite{8} suggests that in the interval $0 \leq \beta \simeq 0.8$ the phase transition with DMG stays in the same universality class, that of the four-fermion model. Our first conjecture is that the universality class nevertheless changes at higher $\beta$, presumably before $\beta \simeq 1.3$, because a tricritical point may be encountered.

The argument is based on an analogy with a similar model in 4D, the $\chi U \phi_4$ model\cite{11}. There the line of transitions with DMG meets the lines of endpoints of the Higgs phase transitions which exist at finite $\hat{m}_0$. Such a common point of several critical lines (tricritical point) in the middle of the phase diagram has not been predicted by any analytic approach, but has been found in a large numerical simulation\cite{12}. Tricritical points are known to be described by universality classes different from those of any of the entering critical lines.

We expect also in the $\chi U \phi_3$ model critical endpoints of the Higgs phase transitions for finite $\hat{m}_0$. It would be a challenge to find evidence for them and to check whether they meet the DMG transition line. If so, a new universality class of DMG would be established in 3D. Its properties might be qualitatively similar to those found in 4D\cite{12}. In particular, the massive fermion would be accompanied by a massive scalar gauge ball. Could this be of some interest for the condensed matter physics?

5. Still another universality class of DMG, or a new fixed point in QED$_3$?

The nature of the region X is of much interest. Because $\kappa$ is small, one can neglect the scalar field nearly in the whole region X. Then the question is what are the properties of compact QED$_3$ at large $\beta$. One would expect that these are well known. This is not the case, however, because the perturbation expansion fails to grasp important properties of that theory even for $\beta \to \infty$.

On the basis of analytic arguments it is expected that for small $N_F < N_F^c$ the DMG holds in the whole range of $\beta$ including $\beta \to \infty$, whereas for $N_F > N_F^c$ it ends at some finite $\beta$. This are properties similar to QCD$_4$. For noncompact QED$_3$ one expects $N_F^c \simeq 3 - 4$ (see references in\cite{8}).

In our study of the compact model with $N_F = 2$, we have found for $\beta \simeq 1.3$ an indication of a phase transition (vertical dashed line in fig.\cite{1}), which would mean a substantially smaller value of $N_F^c$ than expected in the noncompact case. Naturally, because of the small sizes of our lattices, we cannot exclude that the condensate rapidly but analytically decreases around $\beta \simeq 1.3$ to a small but nonvanishing value. It is very difficult to distinguish numerically such a crossover from a genuine phase transition. Therefore the DMG phase transition could also take place on the horizontal line in fig.\cite{1}.

Both alternatives are interesting. Analogy to noncompact QED$_3$ with fermions suggests that, for $N_F = 2$, DMG might persist until $\beta = \infty$. Then, provided the transition on the horizontal dashed line in fig.\cite{1} is continuous, a continuum theory with DMG would be obtained also here. It would contain again the unconfined massive fermion $F$, since its mass appears to scale (fig.\cite{8}). Thus it would represent still another universality class with DMG.

The data do really suggest a continuous phase
transition on the horizontal line. However, there is something strange with it: as seen in fig. 2b, the fermion condensate appears to increase with $\kappa$ across the transition, though, usually, decreasing mass of the scalar field suppresses this condensate. Such a behaviour requires a clarification.

The analogy to noncompact QED$_3$ (whose properties are far from certain anyhow) might be misleading, and DMG might end at the vertical dashed line. In this case, there would be no unconfined fermion of finite mass in the corresponding continuum theory. However, it would mean a new insight into the properties of compact QED$_3$, presumably implying the existence of a new fixed point in this theory, as conjectured e.g. in refs. [7].

It is known [3, 4] that pure compact QED$_3$ has no phase transition and, as $\beta \to \infty$, it is confining via a linear potential. String tension and a scalar gauge ball mass scale in this limit, but the scales separate [4]. Such a rare scale separation might occur also at the new fixed point of the full QED$_3$.

These alternatives may or may not be of relevance for the condensed matter physics. But they certainly belong to interesting open questions in 3D gauge theories.

6. Conclusion

It is remarkable that after 20 years of numerical studies of lattice gauge theories so many open questions about the flatland QED with matter fields remain unanswered. I think the main reason lies in the subtlety of the problems: to distinguish between crossovers and genuine phase transitions, between weak first order and second order transitions, to demonstrate scaling behaviour whose form is not predicted by some reliable analytic means, etc. Attempts to understand 4D abelian lattice gauge theories have got stuck because of similar subtleties [3]. In some sense the study of the QED$_3$ with matter fields is more difficult than the QCD calculations.

The other reason may be a low priority assignment. If so, I hope to have contributed to its reconsideration.

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REFERENCES

1. E. Fradkin, *Field Theories of Condensed Matter Systems*, Ch. 6.3, 6.4 (Addison-Wesley 1991).
2. P. W. Anderson, *Science* **235** (1987) 1196; G. Baskaran, Z. Zou and P. W. Anderson, *Solid State Commun.* **63** (1987) 973; G. Baskaran and P. W. Anderson, *Phys. Rev. B* **37** (1988) 580.
3. Your local condensed matter experts.
4. D. H. Kim and P. A. Lee and X.-G. Wen, *Phys. Rev. Lett.* **79** (1997) 2109.
5. D.H. Kim and P. A. Lee, *Ann. Phys.* **272** (1999) 130;
6. N. Nagaosa and P. A. Lee, *Phys. Rev. B* **61** (2000).
7. I. J. R. Aitchison and N. E. Mavromatos, *Phys. Rev. B* **53** (1996) 9321; N. E. Mavromatos and J. Papavassiliou, *Phys. Rev. D* **60** (1999) 125008.
8. I. M. Barbour, E. Focht, W. Franzki, J. Jersák, and N. Psycharis, *Phys. Rev. D* **58** (1998) 074507.
9. I. M. Barbour, W. Franzki and N. Psycharis, *Nucl. Phys. B* (Proc. Suppl.) **63** (1998) 712.
10. K. Kajantie, M. Karjalainen, M. Laine, and J. Peisa, *Nucl. Phys. B* **520** (1998) 345.
11. C. Frick and J. Jersák, *Phys. Rev. D* **52** (1995) 340.
12. W. Franzki and J. Jersák, *Phys. Rev. D* **58** (1998) 034508; *Phys. Rev. D* **58** (1998) 034509.
13. A. M. Polyakov, *Phys. Lett. B* **59** (1975) 82.
14. M. Göpfert and G. Mack, *Comm. Math. Phys.* **82** (1982) 545.
15. J. Jersák, Study of compact abelian lattice gauge theories, [hep-lat/0010014].