Comment on “Dynamic properties in a family of competitive growing models”

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The article [Phys. Rev. E 73, 031111 (2006)] by Horowitz and Albano reports on simulations of competitive surface-growth models RD+X that combine random deposition (RD) with another deposition X that occurs with probability p. The claim is made that at saturation the surface width w(p) obeys a power-law scaling w(p) ∝ 1/pδ, where δ is only either δ = 1/2 or δ = 1, which is illustrated by the models where X is ballistic deposition and where X is RD with surface relaxation. Another claim is that in the limit p → 0±, for any lattice size L, the time evolution of w(t) generally obeys the scaling w(p, t) ∝ (L/ps/pδ)F(p2t/L2), where F is Family-Vicsek universal scaling function. We show that these claims are incorrect.

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In Ref.[1] the following scaling ansatz is proposed:

\[ w^2(p, t) \propto \frac{L^{2\alpha}}{p^{2\beta}} F \left( \frac{p^{2\beta} t}{L^2} \right), \tag{1} \]

where w(p, t) are time evolutions of surface width in competitive growth models RD+X when a random deposition (RD) process is combined with process X, and p ∈ (0; 1] is the selection probability of process X. The function F(·) represents Family-Vicsek universal scaling. The ansatz (1) has been studied previously [2, 3, 4] by examples where X represented either Kardar-Parisi-Zhang or Edwards-Wilkinson universal process. The new claim that is being made in Ref.[1] is that a nonuniversal and model-dependent exponent δ in Eq.(1) must be only of two values, either δ = 1 or δ = 1/2, for models studied in Ref.[1]. To show that this claim is not correct we performed (1 + 1) dimensional simulations of RD+X models when X is ballistic deposition (BD) and when X is random deposition with surface relaxation (RDSR), and performed scaling in accordance to Ref.[1]. Our results are presented in figs.[1,3]

Our data have been obtained on L site lattices (L is indicated in the figures) with periodic condition, starting from initially flat substrates, and averaged over 400 to 600 independent configurations. The time t is measured in terms of the deposited monolayers. Simulations have been carried up to t = 10^7, and the surface width at saturation has been averaged over the last 5000 time steps. The data sets are for ten equally spaced selection probabilities p from p = 0.1 to p = 1, where p = 0 would be for RD process with no X present, and p = 1 is for process X in the absence of RD. The data have been scaled in L with the theoretical values of universal roughness exponent α and dynamic exponent z of the universality class of process X. The RDSR algorithm used in our simulations is given in Ref.[5] (Sec.5.1). The BD algorithm used as X=BD1 is the nearest-neighbor (NN) sticking rule found in Ref.[5] (Sec.2.2), and the BD algorithm used as X=BD2 is the next-nearest-neighbor (NNN) sticking rule found in Ref.[5] (Sec.8.1).

Saturation. Saturation data (fig.1) show that in special cases an approximate power law w(p) ∝ 1/pδ may be observed. However, this is not a principle. Even if the data can be fit to the power law in p only one of our examples shows a reasonable fit with δ ≈ 1 (seen in fig.1a). When X=BD1 the data in fig.1b show δ < 1/2. The other two examples shown in fig.1b defy a linear fit.

FIG. 1: (color on line) Interface width at saturation in the RD+X model vs the selection probability p of process X. (a) X is RDSR: the case when both RD and RDSR deposits are of unit height (diamonds, RDSR1; L = 500); and, the case when RDSR deposits are of unit height and RD deposits are of twice that height (squares, RDSR2; L = 100). (b) X is BD: the case of the NNN rule (circles, BD2); and, the case of the NN rule (triangles, BD1). In RD+BD simulations L = 500. Solid line segments connecting data points (symbols) are guides for the eye. The dashed lines give reference slopes.
would prevail in the RD limit of $p$ with the power-law prefactors evident in fig. 4 of Ref. [1].

In these cases there is no power law of the type claimed in Ref. [1]. This absence of power-law scaling in $p$ is also evident in fig. 4 of Ref. [1].

**The RD limit.** Another claim of Ref. [1] is that Eq. (1) with the power-law prefactors $p^\delta$ (where $\delta = 1$ or $1/2$) would prevail in the RD limit of $p \to 0^+$, and that such a scaling would be universal. We tested these claims in simulations of RD+BD models and found the evidence to the contrary (figs. 2–3). In order to prove the absence of power-law scaling via Eq. (1) in the RD limit we present in figs. 2–3 the original $w^2(p, t)$ data before scaling. These original data show that parameter $p$, $p \in (0; 1]$, assigns an order in the set of all curves $w^2(p, t)$ in such a way that $w^2(1, t)$ is the lowest lying curve, and at $p = 0$ the initial transients become the RD universal evolution $w^2_{RD}(0, t) \propto t$. The region between the boundaries $w^2(1, t)$ and $w^2_{RD}(0, t)$ is densely covered by the curves $w^2(p, t)$ because $p$ takes on continuous values. The pattern shown in figs. 2–3, for $p \in [0.1; 1]$ extends down to values that are infinitesimally close to $p = 0$, i.e., to the entire range of $p$. If the simulations are stopped at infinitesimally small $p'$ the width $w^2(p', t)$ is always the highest lying curve in figs. 2–3. In other words, the smaller the $p'$ the higher the saturation value of $w^2(p', t)$. But there is no bounding highest curve $w^2(p', t)$ in this set since the boundary $w^2(0, t)$ is the RD evolution. This order is reversed under the scaling of Eq. (1) when we set $\delta = 1/2$, following Ref. [1]. The outcome of this scaling is seen in figs. 2–3: the boundary $w^2(1, t)$, i.e., the lowest-lying curve in figs. 2–3, is mapped onto the highest-lying curve in the image of this scaling seen in figs. 2–3; and, a higher-lying curve $w^2(p, t)$ before scaling in figs. 2–3 is mapped onto a lower-lying curve after scaling in figs. 2–3. In this scaling, the initial transients become ever longer as $p$ becomes ever smaller and closer to $p = 0$, as seen in the insert in fig. 2a. For any range of $p$, also in the limit $p \to 0^+$, the image of this scaling demonstrates no data collapse. This image is shown in the insert in fig. 3a. Hence, for RD+BD models Eq. (1) with $\delta = 1/2$ does not produce data collapse.

In some instances of model X, however, Eq. (1) can give an approximate data collapse [2, 3] but then $\delta$ is not restricted to the two values postulated in Ref. [1]. For example, for the RD+BD1 model such scaling can be obtained with $\delta \approx 0.41$ [3] (note, $0.4 < \delta < 0.5$ is seen in fig. 1b). But for the RD+BD2 model there is no value of $\delta$ that would produce data collapse when nonuniversal prefactors in Eq. (1) are expressed as a power law $p^\delta$. We have demonstrated that such scaling does not generally exist and if occasionally it is observed it is a property of particular model.

In summary, the form of the nonuniversal prefactors as seen in universal Eq. (1) is a fit and is not a principle. The exponent $\delta$ in Eq. (1) is model dependent, and the prefactor that enters may have other forms than $p^\delta$.

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[1] C. M. Horowitz and E. V. Albano, Phys. Rev. E, 73, 031111 (2006).
[2] A. Kolakowska et al., Phys. Rev. E, 70, 051602 (2004).
[3] A. Kolakowska et al., Phys. Rev. E, 73, 011603 (2006).
[4] A. Kolakowska and M. A. Novotny, cond-mat/0511688
[5] A.-L. Barabasi and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, 1995).