CONSTRAINING THE COLD DARK MATTER SPECTRUM NORMALIZATION IN FLAT DARK ENERGY COSMOLOGIES

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ABSTRACT

We study the relation between the rms mass fluctuations on 8 $h^{-1}$ Mpc scales and $\Omega_m$ using the recent clustering results of XMM-Newton soft (0.5–2 keV) X-ray sources, which have a median redshift of $z \sim 1.2$. The relation can be represented in the form $\sigma_8 = 0.34(\pm0.01)\Omega_m^{0.7}$, where $\gamma \equiv \gamma(\Omega_m, w)$, and it is valid for all $w < -\frac{1}{3}$ models. By combining the X-ray clustering and SN Ia data we find that the model that best reproduces the observational data is that with $\Omega_m \approx 0.26$, $w = -0.90$, and $\sigma_8 \approx 0.73$, which is in excellent agreement with the recent 3 yr Wilkinson Microwave Anisotropy Probe results.

Subject headings: cosmological parameters — large-scale structure of universe

1. INTRODUCTION

The combination of the recently acquired, high-quality observational data on galaxy clustering, the Type Ia supernova (SN Ia) Hubble relation, and the cosmic microwave background (CMB) fluctuations strongly support a universe with flat geometry and a currently accelerated expansion due to the combination of a low matter density and a dark energy component (for a review see, e.g., Riess et al. 1998; Perlmutter et al. 1999; Percival et al. 2002; Efstathiou et al. 2002; Spergel et al. 2003, 2006; Tonry et al. 2003; Schuecker et al. 2003; Riess et al. 2004; Tegmark et al. 2004; Seljak et al. 2004; Allen et al. 2004; Basilakos & Plionis 2005; Blake et al. 2006; Wilson et al. 2006; see also Lahav & Lidelle 2006).

From the theoretical point of view various candidates of the exotic “dark energy” have been proposed, most of them described by an equation of state $p_w = wp_0$ with $w < -\frac{1}{3}$ (see Peebles & Ratra 2003 and references therein). Note that a redshift dependence of $w$ is also possible, but present measurements are not precise enough to allow meaningful constraints (e.g., Dicus & Repko 2004; Wang & Mukherjee 2006). From the observational point of view and for a flat geometry, a variety of studies indicate that $w < -0.8$ (e.g., Tonry et al. 2003; Riess et al. 2004; Sanchez et al. 2006; Spergel et al. 2006; Wang & Mukherjee 2006 and references therein).

Another important cosmological parameter is the normalization of the cold dark matter (CDM) power spectrum in the form of the rms density fluctuations in spheres of radius $8 \ h^{-1}$ Mpc, the so-called $\sigma_8$. A tight relation between $\sigma_8$ and the $\Omega_m$ has been derived mainly using the cluster abundance with $\sigma_8 \approx 0.52\Omega_m^{-0.52}$ for a $\Lambda$ cosmology (Eke et al. 1996). Also, generalizing to take into account dark energy models (with $w \geq -1$), Wang & Steinhardt (1998) found $\sigma_8 \approx 0.58\Omega_m^{0.45}$

In this Letter we use the clustering of high-$z$ X-ray active galactic nuclei (AGNs) to estimate a new normalization of the CDM spectrum, valid for spatially flat cosmological models and also for $w \leq -1$ (the so-called phantom models). Finally, combining our results with SN Ia data (Tonry et al. 2003), we put strong constraints on the value of the equation of state parameter.

In a previous paper (Basilakos et al. 2005) we derived the angular correlation function of the soft (0.5–2 keV) XMM-Newton X-ray sources using a shallow (2–10 ks) wide-field survey (survey area $\sim 2.3$ deg$^2$). A full description of the data reduction, source detection, and flux estimate are presented in Georgakis et al. (2004). Here we describe only the basic points. The survey contains 432 point sources within an effective area of $\sim 2.1$ deg$^2$ (for $f_x \geq 2.7 \times 10^{-14}$ ergs cm$^{-2}$ s$^{-1}$), while for $f_x \geq 8.8 \times 10^{-15}$ ergs cm$^{-2}$ s$^{-1}$ the effective area of the survey is $\sim 1.8$ deg$^2$. The details of the correlation function estimate, the various biases that should be taken into account (the amplification bias and integral constraint), the survey luminosity, and selection functions, as well as issues related to possible stellar contamination, are presented in Basilakos et al. (2005). The redshift selection function of our X-ray sources was derived using the soft-band luminosity function of Miyaji et al. (2000) and assuming the realistic luminosity dependent density evolution of X-ray AGNs, and it predicts a characteristic depth of $\Omega_m \approx 1.2$ for our sample (for details see Basilakos et al. 2005).

Our aim here is to investigate the relation between the normalization of the CDM power spectrum ($\sigma_8$) and $\Omega_m$ in flat cosmologies with $w \leq -\frac{1}{3}$. A thorough study of the theoretical clustering predictions from different flat cosmological models to the actual observed angular clustering of distant X-ray AGNs was presented in Basilakos & Plionis (2005). For the purpose of this study we use Limber’s formula, which relates the angular, $w(\theta)$, and the spatial, $\xi(r)$, correlation functions. In the case of a spatially flat universe, Limber’s equation can be written as

$$w(\theta) = \frac{2H_0}{c} \int_0^\infty \left( \frac{1}{N} \frac{dN}{dz} \right) E(z) \xi(r) dz,$$

with $E(z) = [\Omega_m(1 + z)^3 + (1 - \Omega_m)(1 + z)^{3(1+w)}]^{1/2}$. Also, $r$ is the physical separation between two sources, having an angular separation, $\theta$, given by $r = (1 + z)^{-1} (u^2 + \theta^2)^{1/2}$ (small angle approximation). The number of objects within a shell $(z, z + dz)$ and in a given survey of solid angle $\Omega$, is

$$\frac{dN}{dz} = \Omega_x r^2(z) n_{\phi}(x) \left( \frac{\Omega_x}{H_0} \right) E^{-1}(z).$$
where \( n_s \) is the comoving number density at zero redshift and \( x(z) \) is the coordinate distance

\[
x(z) = \frac{c}{H_0} \int_0^z \frac{dy}{E(y)}. \tag{3}
\]

Finally, the selection function \( \phi(x) \) (the probability that a source at a distance \( x \) is detected in the survey) is estimated by integrating the appropriate Miyaji et al. (2000) luminosity function.

2.1. The Evolution of Clustering

It is well known (Kaiser 1984; Benson et al. 2000) that according to linear biasing the correlation function of the mass tracer (\( \xi_{\text{m}} \)) and dark matter (\( \xi_{\text{DM}} \)) are related by

\[
\xi_{\text{m}}(r, z) = b^2(z)\xi_{\text{DM}}(r, z), \tag{4}
\]

where \( b(z) \) is the bias evolution function. Here we use the bias evolution model of Basilakos & Plionis (2001, 2003), where we also compared in detail our model with that of Mo & White (1996) and Matarrese et al. (1997). As an example, our model predicts an AGN bias that is greater by \( \sim 30\% \) and lower by \( \sim 35\% \) than that of Matarrese et al. (1997) at \( z = 0 \) and 3, respectively (see Basilakos et al. 2005). We remind the reader that our bias model is based on linear perturbation theory and the Friedmann-Lemaître solutions of the cosmological field equations. For the case of a spatially flat cosmological model, our general bias evolution can be written as

\[
b(z) = A E(z) + C E(z) \int_z^\infty \frac{(1 + y)^3}{E^3(y)} dy + 1. \tag{5}
\]

Note that our model gives a family of bias curves, due to the fact that it has two unknowns (the integration constants \( A, C \)). The value of \( C \) is approximately found to be \( \approx 0.004 \), as was determined and tested in Basilakos & Plionis (2003). Note that

\[
E(0) = 1 \quad \text{and} \quad A = b_0 - 1 - C \int_0^\infty (1 + y)^3 E^3(y) dy,
\]

where \( b_0 \) is the bias at the present time. We have tested the robustness of our results by increasing \( C \) by a factor of 10 and 100 to find differences of only \( \sim 5\% \) in the fitted values of \( \Omega_m \) and \( b_0 \). This behavior can be explained from the fact that the dominant term in the right-hand side of equation (5) is the first term \([\propto (1 + z)^{1/2}] \), while the second term has a slower dependence on redshift \([\propto (1 + z)] \).

We quantify the underlying matter distribution clustering by presenting the spatial correlation function of the mass \( \xi_{\text{M}(r, z)} \) as the Fourier transform of the spatial power spectrum \( P(k) \):

\[
\xi_{\text{M}(r, z)} = \left( \frac{1 + z}{2\pi} \right)^{3/2} \int_0^\infty k^2 P(k) \frac{\sin (kr)}{kr} dk, \tag{6}
\]

where \( k \) is the comoving wavenumber. Note that the parameter \( \epsilon \) parameterizes the type of clustering evolution (e.g., de Zotti et al. 1990). In this work we utilize a clustering behavior that is constant in comoving coordinates (\( \epsilon = -1.2 \)).

As for the power spectrum, we consider that of CDM models, where

\[
P(k) = P_c (k) \propto T^2(k) \text{ with scale-invariant } (n = 1) \text{ primeval inflationary fluctuations (we verified that a small change of } n, \text{ e.g., } n = 0.95 \text{ according to the 3 yr Wilkinson Microwave Anisotropy Probe [WMAP] does not produce appreciable differences in our results). In particular, we use the transfer function parameterization as in Bardeen et al. (1986), with the corrections given approximately by Sugiyama (1995) while the normalization of the power spectrum is given by

\[
P_0 = 2\pi^2 \sigma_8^2 \int_0^\infty T(k) k^{n+1} W^2(kR) dk, \tag{7}
\]

where \( \sigma_8 \) is the rms mass fluctuation on \( R = 8 h^{-1} \text{ Mpc scales and } W(kR) \) is the window function given by

\[
W(kR) = \frac{3(\sin kR - kR \cos kR)}{(kR)^3}. \tag{8}
\]

Note that we also use the nonlinear corrections introduced by Peacock & Dodds (1994).

3. COSMOLOGICAL CONSTRAINTS

3.1. X-Ray AGN Clustering Likelihood

Following the same notations as in Basilakos & Plionis (2005) in order to constrain the cosmological parameters, we use a standard \( \chi^2 \) likelihood procedure to compare the measured XMM-Newton soft source angular correlation function (Basilakos et al. 2005) with the prediction of different spatially flat cosmological models. The likelihood estimator\(^4\) is defined as

\[
L_AGN(e) \propto \exp \left[-\frac{\chi^2_{\text{AGN}}(e)}{2}\right]
\]

with

\[
\chi^2_{\text{AGN}}(e) = \sum_{i=1}^8 \frac{\left[ w_{\text{obs}}(\theta, j) - w_{\text{fit}}(\theta, j) \right]^2}{\sigma_i^2}, \tag{9}
\]

where \( e \) is a vector containing the cosmological parameters that we want to fit and \( \sigma_i \) is the uncertainty of the observed angular correlation function. We make clear that we work within the framework of a flat cosmology with primordial adiabatic fluctuations and baryonic density of \( \Omega_m h^2 = 0.022(\pm 0.002) \) (e.g., Kirkman et al. 2003; Spengler et al. 2006). Also, using the results of the Hubble Space Telescope key project (Freedman et al. 2001), we fix the Hubble constant to its nominal value, i.e., \( h = 0.72 \), derived also by our previous AGN clustering analysis (Basilakos & Plionis 2005). In that work the \( 1 \sigma \) uncertainty of the marginalized value of \( h \) was found to be only \( \sim 0.03 \). Note that since we fix in the following analysis the values of both \( h \) and \( \Omega_m \), we do not take into account their quite small uncertainties.

The corresponding vector that we have to fit is \( e = (\Omega_m, w, \sigma_8, b_0) \). In this Letter we sample the various parameters as follows: the matter density \( \Omega_m \in [0.01, 1] \) in steps of 0.01, the equation of state parameter \( w \in [-2, -0.35] \) in steps of 0.05, the rms matter fluctuations \( \sigma_8 \in [0.4, 1.4] \) in steps of 0.02, and the X-ray source bias at the present time \( b_0 \in [0.5, 3] \) in steps of 0.05. Note that in order to investigate possible equations of state, we have allowed the parameter \( w \) to take values below \( -1 \). Such models correspond to the so-called phantom cosmologies (e.g., Caldwell 2002; Corasaniti et al. 2004).

The resulting best-fit parameters are presented in Table 1. It is important to note that our estimate for the \( \sigma_8 \) parameter is in very good agreement with that derived (\( \sigma_8 \approx 0.74 \)) by the recent 3 yr WMAP results (Spengler et al. 2006). In Figure 1 we present the 1, 2, and 3 \( \sigma \) confidence levels in the \( (\Omega_m, \sigma_8, (\sigma_8, b_0)) \) and \( (\Omega_m, \sigma_8, b_0) \) planes.

\(^4\) Likelihoods are normalized to their maximum values.
(\(\Omega_m, b_0\)) planes by marginalizing the first one over \(b_0\) and \(w\), the second one over \(\Omega_m\) and \(w\), and the last one over \(\sigma_8\) and \(w\); the dot in Figure 1 corresponds to the best-fitted values.\(^5\)

Therefore, using the clustering properties of our XMM-Newton sources and allowing for the first time values \(w < -1\) (phantom models) we derive the \((\Omega_m, \sigma_8)\) relation, which can be fit (within the 1 \(\sigma\) uncertainty) by

\[
\sigma_8 = 0.34(\pm 0.01) \Omega_m^{-\gamma(\Omega_m, w)}, \quad (10)
\]

with

\[
\gamma(\Omega_m, w) = 0.22(\pm 0.04) - 0.40(\pm 0.05)w - 0.052(\pm 0.04)\Omega_m.
\]

In Figure 2 we present the results of the likelihood analysis for different values of \(w\) (points with error bars) and the previous fit as a continuous line.

Note that equation (10) produces \(\sigma_8\) values that are significantly smaller than the usual cluster normalization (Wang & Steinhardt 1998) but are in good agreement with the 3 yr WMAP results; for example, for \(w = -1\) and \(\Omega_m = 0.28\) we get \(\sigma_8 \approx 0.73 \pm 0.03\). It should be mentioned that in our previous work (Basilakos & Plionis 2005) we had imposed a high \(\sigma_8\) normalization, based on the cluster abundance, while here we leave the \(\sigma_8\) parameter free.

The lower right panel of Figure 1 shows the 1, 2, and 3 \(\sigma\) confidence levels (solid lines) in the \((\Omega_m, w)\)-plane by marginalizing over the \(\sigma_8\) and the bias factor at the present time. It is evident that \(w\) is degenerate with respect to \(\Omega_m\) and that all the values in the interval \(-2 \leq w \leq -0.35\) are acceptable within the 1 \(\sigma\) uncertainty. Therefore, in order to put further constraints on \(w\), we also use a sample of 172 supernovae (see Tonry et al. 2003).

\[\text{TABLE 1} \]

| Data                  | \(\Omega_m\) | \(w\)       | \(\sigma_8\) | \(b_0\) | \(\chi^2/\text{dof}\) |
|-----------------------|--------------|-------------|--------------|---------|----------------------|
| XMM-Newton            | 0.28 \(\pm\) 0.03 | \(w = -1\) | 0.75 \(\pm\) 0.03 | 2.0\(^{+0.29}_{-0.25}\) | 0.90 |
| XMM-Newton/SN Ia      | 0.26 \(\pm\) 0.04 | \(-0.90^{+0.04}_{-0.05}\) | 0.73 | 2.0 | 0.87 |

Notes.—The first column indicates the data used (the last row corresponds to the joint analysis). Errors of the fitted parameters represent 1 \(\sigma\) uncertainties. Therefore, in order to put further constraints on \(w\), we also use a sample of 172 supernovae (see Tonry et al. 2003).

3.2. The AGN+SN Ia Likelihoods

Here we combine the X-ray AGN clustering properties with the SN Ia data by performing a joined likelihood analysis and marginalizing the X-ray clustering results over \(\sigma_8\) and \(b_0\) (see Table 1), and thus the vector \(c\) now becomes \(c \equiv (\Omega_m, w)\). The SN Ia likelihood function can be written as

\[
\mathcal{L}^{\text{SN Ia}}(c) \propto \exp[-\chi^2_{\text{SN Ia}}(c)/2]
\]

with

\[
\chi^2_{\text{SN Ia}}(c) = \sum_{i=1}^{172} \left[ \log D_L^{\text{obs}}(z_i, c) - \log D_L^{\text{calc}}(z_i) \right]^2 / \sigma_i.
\]

Fig. 1.—Likelihood contours in the following planes: \((\Omega_m, \sigma_8)\) (top right), \((\sigma_8, b_0)\) (top left), \((\Omega_m, b_0)\) (bottom left), and \((\Omega_m, \sigma_8)\) (bottom right). The contours are plotted where \(-2 \ln \mathcal{L}/\mathcal{L}_{\text{max}}\) is equal to 2.30, 6.16, and 11.83, corresponding to 1, 2, and 3 \(\sigma\) confidence levels. Finally, the thick contours correspond to the SN Ia likelihoods.

Fig. 2.—The \((\Omega_m, \sigma_8)\)-plane using different values for the equation of state parameter (points). The errors correspond to 1 \(\sigma\) (2.30) confidence levels. The continuous line corresponds to the fit given by eq. (10).
where $D_L(z)$ is the dimensionless luminosity distance, $D_A(z) = H_0 (1 + z) x(z)$, and $z_i$ is the observed redshift. The thick lines in Figure 1 represent the 1, 2, and 3 $\sigma$ confidence levels in the $(\Omega_m, \omega)$-plane. We find that the best-fit solution is $\Omega_m = 0.30 \pm 0.04$ for $\omega < -1$, in complete agreement with previous SN Ia studies (Tonry et al. 2003; Riess et al. 2004).

We now join the likelihoods

$$L^{\text{joint}}(\Omega_m, \omega) = L^{\text{AGN}} L^{\text{SN Ia}},$$

which peak at $\Omega_m = 0.26 \pm 0.04$ with $\omega = -0.90^{+0.10}_{-0.05}$. Using equation (10) we find that the normalization of the power spectrum that corresponds to these cosmological parameters is $\sigma_8 = 0.73$. It should be pointed out that our results are in excellent agreement with those derived by Spergel et al. (2006) using the recent WMAP (3 yr) data: $\Omega_m \simeq 0.24$, $\omega \simeq -0.97$, and $\sigma_8 \simeq 0.74$.

Many other recent analyses using different combinations of data seem to agree with the former cosmological model. For example, Sanchez et al. (2006) used the WMAP (1 yr) CMB anisotropies in combination with the Two Degree Field Galaxy Redshift Survey power spectrum and found $\Omega_m \simeq 0.24$ and $\omega \simeq -0.85$, while Wang & Mukherjee (2006) using the 3 yr WMAP data together with SN Ia and galaxy clustering results found $\omega \simeq -0.9$ (see also Wilson et al. 2006).

4. CONCLUSIONS

We have combined the clustering properties of distant X-ray AGNs, identified as soft (0.5–2 keV) point sources in a shallow ~2.3 deg$^2$ XMM-Newton survey, with the SN Ia data. From the X-ray AGN clustering likelihood analysis alone we have estimated the normalization of the CDM power spectrum and found that the rms density fluctuation in spheres of radius 8 $h^{-1}$ Mpc is fitted by

$$\sigma_8 \simeq 0.34(\pm 0.01)\Omega_m^{-0.22+0.40n_{+0.052}Q_n},$$

which is also valid for phantom models ($\omega < -1$). Furthermore, a joined likelihood analysis between the X-ray and SN Ia data provides a best model fit with $\Omega_m \simeq 0.26$ and $\omega \simeq -0.90$, which corresponds to $\sigma_8 \simeq 0.73$, in agreement with the recent 3 yr WMAP results (Spergel et al. 2006).

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