Hydrodynamics in black brane with hyperscaling violation metric background

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Abstract

In this paper we consider a metric with hyperscaling violation on black brane background. In this background we calculate the ratio of shear viscosity to entropy density with hydrodynamics information. The calculation of this quantity lead us to constraint $\theta$ as $3 \leq \theta < 4$, and $\theta \leq 0$. In that case we show that the quantity of $\frac{\eta}{s}$ not dependent to hyperscaling violation parameter $\theta$. Our results about ratio of shear viscosity to entropy density in direct of QCD point of view agree with other works in literature as $\frac{1}{4\pi}$.

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1 Introduction

One of the important subject in modern theoretical physics is the understanding the dynamics of strongly coupled quantum field theories. Also there is a large class of real world physics systems which is described by the perturbation theory as QCD system. One of the interesting case is the state of matter discovered in heavy ion collisions, the Quark Gluon Plasma (QGP), that is known as a nearly ideal fluid. Then it is logical to look at this state of matter as a hydrodynamics theory. Hydrodynamics is an effective theory to describe the macroscopic dynamics at large distances.

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and time scales. The conserved quantities such as energy-momentum tensor $T^{\mu\nu}$ are considered to survive in such large distances and time scales. Unlike the familiar effective field theories the hydrodynamics is normally formulated in the language of equations of motion instead of an action principle, because there is dissipation in thermal media. The description in this section is basically following the review papers [1, 2, 3]. The hydrodynamic equation by conservation of energy-momentum tensor is given by,

$$\partial_{\mu} T^{\mu\nu} = 0.$$  

(1)

The conformal hydrodynamic in $d$ dimensional space-time at local rest frame is described by $d$ independent variable: temperature $T(x)$ and the $d$-velocity vector field $u^\mu(x)$, which is satisfied by $u_\mu u^\mu = -1$. So we express $T^{\mu\nu}$ in terms of $T(x)$ and $u^\mu(x)$. the hydrodynamic quantities in local rest frame are varying slowly over space-time, therefore their derivatives are very small. So, we can express the energy-momentum tensor in power of spatial derivative of local quantities. At the zeroth order (ideal fluid), $T^{\mu\nu}$ is given by,

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu},$$  

(2)

where $\epsilon$ is energy density, $P$ is the pressure and $g^{\mu\nu}$ is the metric tensor background space-time. At the next order in derivative expansion fluid energy-momentum tensor is given by,

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu},$$

$$\sigma^{\mu\nu} = P^{\alpha\beta} \left\{ \eta \left( \nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{d-1} g_{\alpha\beta} \nabla u \right) + \zeta g_{\alpha\beta} \nabla u \right\},$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu,$$  

(3)

where $\sigma^{\mu\nu}$ is proportional to derivative of $T(x)$ and $u^\mu(x)$, and it is the dissipative part of energy-momentum tensor, also $\sigma^{\mu\nu}$ is a symmetric tensor. The coefficients $\eta$ and $\zeta$ are called shear and bulk viscosity. If the system contains a conserved current, there is an additional hydrodynamic equation related to the current conservation $j^\mu$ with $\partial_\mu j^\mu = 0$. The constitutive equation is given as,

$$j = \rho u - D \nabla j^t,$$  

(4)

where $j^t = \rho$ and $D$ are charge density and the diffusion constant respectively. In the fluid rest frame, $j = -D \nabla \rho$, which is Fick’s law of diffusion.

The hydrodynamics behavior of a system is determined by transport coefficients, shear viscosity, bulk viscosity and etc. As we mentioned the QGP produced at Relativistic Heavy Ion Collision (RHIC) treat as nearly perfect fluid or a viscous fluid with very small shear viscosity. With a low ratio of shear viscosity to entropy density is very hard to describe with normal methods. The temperature of QGP produced at RHIC is almost 170MeV that, is so close to confinement temperature of QCD. So, in high temperature they are not in the weakly coupled area of QCD. In fact they are close to transition temperature of QGP in non-perturbative regime of QCD. So, the normal calculation of perturbative gauge theory is
not suitable for describing QGP. One important implement for understanding the dynamics of strongly coupled is AdS/CFT correspondence \cite{8, 9, 10}. In general the AdS/CFT correspondence relates between two system, it means that the interacting quantum field theory on one hand and string theory in a curved background on the other hand.

In Ref.\cite{5} the authors by using the AdS/CFT correspondence calculated the shear viscosity of the finite-temperature $N = 4$ supersymmetric Yang-Mills theory in the large $N$ in non-extremal black 3-brane. Also in Refs.\cite{1-7} are used of the AdS/CFT technique for calculating of hydrodynamics quantities. Computing of the second order derivative of transport coefficient is shown in Refs.\cite{11, 12, 13}. In Ref. \cite{14} the authors describe the infrared behavior of theories whose dual gravity that contain a black brane with non-zero Hawking temperature. They argue that the infrared behavior of these theories is survive by nothing other than hydrodynamics. Also, they obtained a general relation for transport coefficients (diffusion rate, shear viscosity) in terms of the components of the metric for a large class of metrics. In Ref.\cite{23} has been shown that the ratio of shear viscosity to entropy density has a lower bound as $\frac{\eta}{s} \geq \frac{1}{4\pi}$. Also this quantity obtained for theories such as $D_3$-brane, $M_2 - brane$, $M_5 - brane$, $D_p - brane$ \cite{3, 4, 14}. But such calculation did not attend for theories with hyperscaling violation metric background. The general review on holographic hydrodynamics is provided in \cite{1}. All the above information give us motivation to calculate the diffusion constant and ratio of shear viscosity to entropy density. So, in this paper, we review of the hyperscaling violation metric background, and also consider a black brane solution of this metric. Next we study the hydrodynamics of this metric background by using the diffusion constant and calculate ratio of shear viscosity to entropy density.

2 Review hyperscaling violation metric background

Here we are going to introduce the AdS metric background as,

$$ds^2 = \frac{R^2}{z^2} (-dt^2 + dx_i^2 + dz^2),$$

where $i = 1..3$, $R$ is the AdS scale and $z$ is holographic coordinate. This metric is invariant under a dilation of all coordinate $z \rightarrow \lambda z$ and $x_\mu \rightarrow \lambda x_\mu$. One generalization of AdS gravity theory and their conformal field theory dual, is to consider metrics which have reduced symmetries compared to anti de-Sitter space, and these theories display Lifshitz scaling symmetry with following metric \cite{16, 17},

$$ds^2 = -\frac{1}{r^{\tau}}dt^2 + \frac{1}{r^2} (dr^2 + dx_i^2),$$

which $\tau$ is dynamical exponent ($\tau = 1$ gives the AdS metric). This metric background is invariant under following scaling,

$$t \rightarrow \lambda^\tau t, \quad x_i \rightarrow \lambda x_i, \quad r \rightarrow \lambda r.$$
These metrics are produced in effective gravity theories with a negative cosmological constant, and with abelian gauge fields in the bulk. By containing an abelian gauge field and scalar dilation, the full class of these metrics are $$[18],$$

$$ds^2 = r^{-2(d-\theta)/d} \left( r^{-2(\tau - 1)} dt^2 + dr^2 + dx_i^2 \right),$$

(8)

where $$\theta$$ is hyperscaling violation exponent. This metric is not invariant under scale transformation $$[17],$$ but transform as,

$$ds = \lambda^\theta/d ds,$$

(9)

To consider a physically sensible dual field theories from gravity side, we should produce the null energy condition as $$T_{\mu\nu} n^\mu n^\nu \geq 0,$$ where the null vectors satisfy $$n^\mu n_\mu = 0 \ [16, 17].$$ From these condition we obtain ,

$$(d-\theta)(d(\tau - 1) - \theta) \geq 0,$$

$$(\tau - 1)(d + \tau - \theta) \geq 0.$$  

(10)

Aspect of holography for hyperscaling violation metric presented in Ref. $[16].$ Starting from metric (8) with hyperscaling violation (by fixing $$\tau = 1$$), the black brane solution become $[16, ?],$

$$ds^2 = \frac{r_F^{2\theta/d}}{r} \left\{ \left( \frac{r^2}{R^2} \right) (-f dt^2 + dx_i^2) + \left( \frac{R^2}{r^2} \right) f^{-1} dr^2 + R^2 d\Omega_5^2 \right\},$$

$$f(r) = 1 - \left( \frac{r_H}{r} \right)^{4-\theta},$$

(11)

where $$i = 1..3.$$ $$r_H$$ is the radius of horizon and $$r_F$$ is a dynamical scale which come from dynamical analysis. This is responsible for restoring the canonical dimensions in presence of hyperscaling violation. We note that by setting $$\theta = 0,$$ metric background (11) reduce to non-extremal $$D3$$-brane geometry in the near horizon.

3 Hydrodynamics and ratio of shear viscosity to entropy density

As we know the hydrodynamics is an effective theory for the describing the macroscopic dynamics at large distances and it is a proper method for explain of matter discovered in heavy ion collision ($QGP$). In the hydrodynamics we have some important quantities such as shear viscosity, which plays important role in physics of early universe. One of most important problems of $$QGP$$ is shear viscosity, such quantity play important role in strongly coupled thermal gauge theories which is achieved by the AdS/CFT correspondence. There are several approach to obtain the ratio of shear viscosity to entropy density. The most known approach for the such quantity is Kubo formula . But in this paper, in order to obtain the ratio of shear viscosity to entropy density we use diffusion constant for the corresponding background. Now we are going to calculate the ratio of shear viscosity to entropy density
for metric background (11) as Ref. [14]. Here we note that, for general metric background we have following expression,

$$ds^2 = g_{00}(r) dt^2 + g_{rr}(r) dr^2 + g_{xx}(r) dx_i^2,$$

so, the diffusion constant can be obtained by following relation [14],

$$D = \sqrt{-g(r) g_{xx}^2(r_H) \int_{r_H}^\infty dr \frac{-g_{00}(r) g_{rr}(r) g_{xx}^2(eff)(r_H)}{-g(r)}}. \quad (12)$$

where $g_{eff}$ is an effective gauge coupling which is coming from the action of the gauge field dual to the conserved current and, can be a function of the radial coordinate $r$. In generally we can say that for the obtaining the $D$, we have to give perturbation to the corresponding background metric. If we give perturbation to diagonal component $g_{eff} = \text{const}$ and for metric background (11) one can obtain $D$ as,

$$D = r_H^{-1/3} R^2 \int_{r_H}^\infty drr^{\theta/3-3} = \frac{1}{2 - \theta/3} R^2 \quad \text{and,} \quad \theta < 6. \quad (13)$$

As we know in equation (10) we got $\tau = 1$ correspond to metric background (11). In that case we have two conditions in $d = 3$,

$$\theta \leq 0 \quad \text{and,} \quad \theta \geq 3. \quad (14)$$

These two conditions and the finite form of integral (13) and equation (14) lead us to pick up $\theta$ as following,

$$3 \leq \theta < 6 \quad \text{and,} \quad \theta \leq 0. \quad (15)$$

Here also note that in case of $\theta = 0$ on can achieve the $D$ as a D3-brane system [14]. In second step we are going to calculate the shear mode of the stress-energy tensor. In that case we have effectively $g_{eff} = g_{xx}$ [3, 14]. So, we have

$$D = r_H^{-3/2} R^2 \int_{r_H}^\infty drr^{\theta-5} = \frac{1}{4 - \theta} R^2 \quad \text{and,} \quad \theta < 4, \quad (16)$$

where by considering of the conditions of (14), we have, $3 \leq \theta < 4$, and $\theta \leq 0$. Now we are going to investigate the ratio of shear viscosity to entropy density, we use $\eta/s = T D$ ($T$ is the Hawking temperature) [3, 14].

The Hawking temperature of metric background (11) can be obtained by surface gravity as,

$$\kappa^2 = -\frac{1}{2} (\nabla^\mu \chi^\nu)(\nabla_\mu \chi_\nu), \quad \beta = \frac{1}{T} = \frac{2\pi}{\kappa}, \quad (17)$$

where $\kappa$ is the surface gravity and $\chi$ is the killing vector field. Since the metric background (11) is independent of time, so $\partial / \partial t$ is a killing vector for this metric. Thus we can obtain the surface gravity and temperature as,

$$\kappa = \frac{1}{2} \sqrt{-g^{tt} g^{rr}(g_{tt,r})^2} = \frac{4 - \theta}{2} \frac{r_H}{R^2}, \quad T = \frac{4 - \theta}{4\pi} \frac{r_H}{R^2} \quad (18)$$
where $\theta = 0$ give the result of D3-brane geometry. So by using the equations (13), (16) and (18) one can obtain the diffusion constant as $D = \frac{4 - \theta}{2 - \theta / 3 \pi T}$ and $D = \frac{1}{4 \pi T}$. We note here in case of shear mode the diffusion constant not dependent to hyperscaling violation. Also from relations of (16) and (18) we obtain the ratio of shear viscosity to entropy density,

$$\frac{\eta}{s} = \frac{1}{4 \pi},$$

(19)

In generally, the ratio of shear viscosity to entropy density not dependence to hyperscaling violation in case of this paper. Also, we can say that the $\theta$ will be constraint with $3 \leq \theta < 4$, and $\theta \leq 0$.

4 Conclusion

In this work, we considered hyperscaling violation metric background. In this background we calculated the ratio of shear viscosity to entropy density with hydrodynamics information. The calculation of this quantity lead us to constraint $\theta$ as $3 \leq \theta < 4$, and $\theta \leq 0$. In that case we have shown that such quantity with the first order calculation of hydrodynamics not dependent to hyperscaling violation parameter $\theta$. Our results about ratio of shear viscosity to entropy density completely agree with other works as [3, 14]. The advantage of this approach for the calculation of $D$ and $\frac{\eta}{s}$ is much simpler than of AdS/CFT and Kubo’s approaches.

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