We present general solutions to the equations of motion for a superconducting relativistic chiral string that satisfy the unit magnitude constraint in terms of products of rotations. From this result we show how to construct a general family of odd harmonic superconducting chiral loops. We further generalise the product of rotations to an arbitrary number of dimensions.

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I. PRELIMINARIES

Particle physics models where symmetry breaking is involved predict, in many cases, the existence of topological defects, which are formed when the topology of the vacuum manifold of the low energy theory is non-trivial. Cosmic strings, in particular, are line-like objects that are formed when the vacuum manifold contains unshrinkable loops. For a review see [2].

In [3] it was shown that cosmic strings can be superconducting. In the case when the charge carriers on the string are not coupled to a gauge field the action for the string and the current can be taken to be

\[ S = \int d^2\xi \sqrt{-\gamma} (-\mu + \frac{1}{2} \gamma^{ab} \phi_{,a} \phi_{,b}) \]  

where \( \mu \) is the mass per unit length of the string, \( \gamma^{ab} \) is the induced metric on the string worldsheet and \( \phi \) is the field of the charge carriers living on the string. These strings were shown in [4] and [5] to have solutions in the case when \( \gamma^{ab} \phi_{,a} \phi_{,b} = 0 \) of the form

\[ x = \frac{1}{2} [a(u) + b(v)] \]  

for the string position and

\[ \phi = \frac{1}{2} f(v) \]  

for the field living on the string with the constraints

\[ a'^2 = 1 \]  

and

\[ b'^2 + f'^2 = 1 \]  

where \( u = \sigma - \tau \) and \( v = \sigma + \tau \) and \( \sigma \) and \( \tau \) are space-like and time-like parameters respectively that parametrise the string worldsheet. These strings are called chiral because the current only moves in one direction on the string.

Comparing this to the usual Nambu-Goto case one can see that \( f(v) \) acts like a fourth component of the three-vector \( b(v) \), making chiral superconducting strings behave like Nambu-Goto ones with chiral excitations in an extra fifth dimension. Indeed, this property was used in [6] in an investigation of the properties of superconducting cosmic string cusps.

The right- and left-moving excitations, \( a' \) and \( b' \), on a regular Nambu-Goto string in Minkowski space-time are arbitrary functions that satisfy the unit magnitude constraint, \( |a'| = |b'| = 1 \). Expressions for these functions are often given as Fourier sums and the unit magnitude constraint generally gives a non-linear set of equations involving the vector coefficients of the Fourier expansion. As a result, parametrising strings beyond the first few harmonics proves to be a difficult task. Fortunately, in that case, there exists a method to generate strings involving products of rotation matrices that act on a starting unit vector so that the unit magnitude constraint is satisfied trivially.

In a recent study of the properties of chiral cosmic strings [7] it was assumed that the current is constant. The work in [8] assumed that the current takes a very simple non-constant form. As was pointed out in the latter work one could expect to have loops with varying currents if the loops are formed by intersections involving different strings or if different segments of the loop or string were at some point in causally disconnected regions. For long strings this is always the case and we therefore expect varying chiral currents to be generic.

The purpose of this work is to generalise the work in [8] for generating three dimensional unit vectors to four dimensional ones that include the current as a fourth component of \( b(\sigma + \tau) \). We start by casting the method somewhat differently and generalise it to four dimensions. From this result we construct a family of chiral superconducting odd-harmonic loops. We further generalise the product of rotations to an arbitrary number of dimensions.

In Section 2 we show how to construct an arbitrary \( N \) harmonic unit vector in four and three dimensions. In Section 3 we use these results to construct arbitrary chiral
current carrying superconducting odd harmonic loops. In Section 4 we generalise the arguments in Section 2 to an arbitrary number of dimensions and we conclude in Section 5.

II. SOLUTION TO THE UNIT MAGNITUDE CONSTRAINT IN TERMS OF PRODUCTS OF ROTATIONS

We can think of the Euclidean 4-vector
\[ \vec{b}' = \begin{pmatrix} b'_w \\ b'_z \\ b'_x \\ b'_y \end{pmatrix} \]  
(6)
as having unit magnitude according to (5) with \( b_w = f \). Consider the vector \( \vec{b}'_N \), that can be constructed from a finite sum of Fourier components,
\[ \vec{b}'_N(v) = \mathbf{Z} + \sum_{n=1}^{N} \{ A_n \cos n v + B_n \sin n v \}. \]  
(7)
The Fourier coefficients satisfy the set of \( 4N + 1 \) nonlinear relations derived in (6)
\[ \sum_{n=m-N}^{N} (\alpha_n \cdot \alpha_{m-n} - \beta_n \cdot \beta_{m-n}) = 4\delta_{m0} \]  
(8)
with \( m = 0, 1, ..., 2N \),
\[ \sum_{n=m-N}^{N} (\alpha_n \cdot \beta_{m-n} - \beta_n \cdot \alpha_{m-n}) = 0 \]  
(9)
with \( m = 1, ..., 2N \),
\[ \alpha_n = \alpha_{-n} = A_n, \beta_n = -\beta_{-n} = B_n, \quad n \neq 0, \]  
(10)
and
\[ \alpha_0 = 2\mathbf{Z}, \beta_0 = 0. \]  
(11)
These equations can be obtained from the constraint equation (6). The total number of degrees of freedom in the coefficients in (6) is \( 8N + 4 \) so the remaining number of degrees of freedom after satisfying the constraint is \( 4N + 3 \). Below we show how to construct (6) from a product of rotation matrices by generalising a modified version of the three dimensional method presented in (6) to four dimensions.

The constraint equations (8) and (9) for \( m = 2N \) and \( m = 2N - 1 \) are
\[ A_N \cdot A_{N-1} - B_N \cdot B_{N-1} = 0, \]
\[ A_N \cdot B_{N-1} + B_N \cdot A_{N-1} = 0 \]  
(13)
respectively. It follows from (12) that the highest harmonic is a circle that lives on some arbitrary plane. Clearly, we can introduce coordinates such that \( A_N \) and \( B_N \) lie on the \( w \) and \( x \) axes
\[ A_N = a\hat{w}, \quad B_N = a\hat{x} \]  
(14)
making the highest harmonic a circle of radius \( a \) on the \( w-x \) plane. This puts the vector \( \vec{b}'_N \) into the so-called standard form (6). Let \( R_{wx}(\theta) \) be a matrix that rotates the \( w-x \) plane by an angle \( \theta \). Acting on \( \vec{b}'_N \) with, \( R_{wx}(-v) \) in these coordinates lowers the highest harmonic term of \( \vec{b}'_N \),
\[ R_{wx}(-v) \begin{pmatrix} a \cos Nv \\ a \sin Nv \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a \cos (N-1)v \\ a \sin (N-1)v \\ 0 \\ 0 \end{pmatrix}. \]  
(15)
This is not sufficient to verify that the overall harmonic content has been lowered because the \( N - 1 \) terms of (6) could still give us an \( N \) harmonic through trigonometric identities. We now show, however, that this is not the case.

The constraint equations for \( m = 2N - 1 \) (16) give us using (14) the conditions on the coefficients,
\[ (A_{N-1})_w = (B_{N-1})_x, \quad (A_{N-1})_x = -(B_{N-1})_w. \]  
(16)
It is not too difficult to see that using the conditions (16) when acting with \( R_{wx}(-v) \) on the \( N - 1 \) terms of (6) does not lead to the creation of \( N \) harmonic terms,
\[ R_{wx}(-v) \begin{pmatrix} A_{N-1} \cos (N-1)v + B_{N-1} \sin (N-1)v \\ A_{N-1} \sin (N-1)v + B_{N-1} \cos (N-1)v \end{pmatrix} \]  
(17)
\[ = \begin{pmatrix} A_{N-1} \cos (N-2)v - A_{N-1} \sin (N-2)v \\ A_{N-1} \sin (N-2)v + A_{N-1} \cos (N-2)v \end{pmatrix} \]  
\[ = \begin{pmatrix} A_{N-1} \cos (N-1)v + (B_{N-1})_y \sin (N-1)v \\ A_{N-1} \sin (N-1)v + (B_{N-1})_y \cos (N-1)v \end{pmatrix}. \]  
(18)
Clearly, the \( N - 2 \) terms will not yield an \( N \) harmonic term when acted on by \( R_{wx}(-v) \).

We now have a string in the form of (6) with \( N \to N - 1 \). Its highest harmonic is therefore also a circle of some other radius living on some other arbitrary plane. Armed with this knowledge we can see that the unit magnitude constraints are such that in general we can write
\[ \vec{b}'_N = R_{P_N}(v)\vec{b}'_{N-1} \]  
(18)
where \( R_{P_N}(v) \) is a rotation by an angle \( v \) on the plane \( P_N \) where the highest harmonic lives. By induction it must be that
\[ \vec{b}'_N = \prod_{i=N}^1 R_{P_i}(v)\vec{b}'_0 \]  
(19)
where the \( R_P(v) \) are rotations by an angle \( v \) on arbitrary planes \( P_i \) and \( B_0' \) is an arbitrary constant unit vector in 4 dimensions.

To specify a plane in 4 dimensions one needs to specify a direction on the plane (3 angles), a linearly independent direction (2 angles), but now the plane is overspecified by internal rotations \((-1\) angle), giving a total of 4 degrees of freedom for the matrices \( R_P(v) \). For an \( N \) harmonic vector, one therefore has 4\( N \) parameters in the rotators and 3 parameters in the constant unit vector \( B_0' \) giving a total of \( 4\, N + 3 \) parameters which checks perfectly with \( 8\, N + 4 \) vector coefficients in \( B \) minus the \( 4\, N + 1 \) constraints \( B \) and \( B' \).

The form of the \( R_P(v) \) matrices is quite simple. Generally, if we want to transform, say, a rotation by an angle \( v \) on the \( w\times \) plane to one on an arbitrary plane we will need to perform a transformation of the type

\[
R_P(v) = E_i R_{wx}(v) E_i^T.
\]  

(20)

For the purpose of finding the form of the rotators \( E_i \) it is easiest to envision the inverse process to the one we are seeking, namely the rotation of an arbitrary oriented plane to lie on the \( w\times \) plane. Let’s consider first the projection of the four dimensional arbitrary plane onto the \( (x, y, z) \) subspace. We can perform a rotation by some angle \( \alpha \) about the \( z \) axis \( (R_{xy}(\alpha)) \) until the vector perpendicular to the projected plane lies on the \( y\times z \) plane and perform a further rotation by an angle \( \beta \) about the \( x \) axis \( (R_{yz}(\beta)) \) until that vector lies on the \( z \) axis. At this stage the projected plane lies wholly on the \( x\times y \) plane. The ranges of both \( \alpha \) and \( \beta \) from 0 to \( \pi \) are sufficient to perform these transformations. After performing these two rotations, our original four dimensional plane lies entirely in the \( (w, x, y) \) subspace and we can repeat an analogous process to the one above to rotate it into the \( w\times x \) plane. A rotation by an angle \( \gamma \) about the \( y \) axis \( (R_{yx}(\gamma)) \) puts the vector perpendicular to the plane on the \( x\times y \) plane and a rotation by an angle \( \delta \) about \( w \) \( (R_{xy}(\delta)) \) makes that vector parallel with the \( y \) axis. For the first of these rotations a range of \( \gamma \) from 0 to \( \pi \) is sufficient, for the second rotation, however, matters are slightly different. If the plane we were trying to rotate was featureless it would be enough for the range of \( \delta \) to be from 0 to \( \pi \). In fact this is not the case. The plane contains a circle in \( w \) which can be oriented clockwise or anti-clockwise on the \( w\times x \) plane and therefore the final rotation on the \( x\times y \) plane in general requires an angle that ranges from 0 to \( 2\pi \).

Keeping in mind these considerations we can quite generally write

\[
E_i = R_{xy}(\alpha_i) R_{yz}(\beta_i) R_{yx}(\gamma_i) R_{xy}(\delta_i)
\]  

(21)

where \( \alpha_i, \beta_i, \gamma_i \) range from 0 to \( \pi \) and \( \delta_i \) ranges from 0 to \( 2\pi \). Then

\[
\tilde{B}'_N(v) = \prod_{i=N}^1 E_i R_{wx}(v) E_i^T \tilde{B}_0'.
\]  

(22)

In order to construct the entire chiral string, we also need to find the form of \( a'(u) \) in \( B \). The constraints \( B \) can be satisfied using a product of rotations that can be found from analogous arguments to the ones in the preceding section. For \( M \) harmonics this yields

\[
a'_M(u) = \prod_{i=M}^1 D_i R_{xy}(u) D_i^T a_0
\]  

(23)

with the rotator

\[
D_i = R_{xy}(\phi_i) R_{yz}(\theta_i)
\]  

(24)

where the angles \( \theta_i \) range from 0 to \( \pi \), the angles \( \phi_i \) from 0 to \( 2\pi \) and \( R \) are the three dimensional rotation matrices.

III. AN APPLICATION: A FAMILY OF ODD HARMONIC SUPERCONDUCTING CHIRAL LOOPS

A. Overall Orientation Freedom

Both expressions for the oppositely moving excitations on the string \( (22) \) and \( (23) \) include overall orientation freedom. In some applications, for instance self-intersection or gravitational radiation analyses, only the shape of a loop, and not its orientation, is important. In this case the inclusion of overall orientation freedom of the right-moving and left-moving excitations separately is unnecessary: All that matters is the relative orientation between \( a' \) and the spatial part of \( B' \).

Overall orientation freedom of the loop is set by an Euler matrix \( Q \) that acts only on \( a' \) and the spatial components of \( B' \) and contains three angles. We can use this freedom to standardise the vectors in some way. We will choose the plane of the last rotation in \( a' \) and \( B' \) to contain some coordinate axis (this will enable us in the following sub-section to use the arbitrariness in the origin of \( u \) and \( v \) to eliminate more parameters).

The four-dimensional circle that constitutes the highest harmonic of \( B' \) projected onto three-dimensional space typically looks like an ellipse. We can perform ordinary three-dimensional rotations on it to put it in some convenient form. In particular, we can rotate the ellipse on the \( x\times y \) plane until its major axis, say, lies on the \( y\times z \) plane and further rotate on the \( y\times z \) plane until it lies entirely on the \( z \) axis. We still have one more rotation left in \( Q \) which we choose to be a rotation on the \( x\times y \) plane. This leaves the major axis of the ellipse on the \( z \)-axis but orients the circle of the highest harmonic in \( a' \) such that it contains the \( x \)-axis.

We now construct a four-dimensional planar rotator that contains the \( z \)-axis but is otherwise arbitrary. Starting with \( R_{wx}(v) \), we see that we can perform an arbitrary planar rotation on it in the \( (w, x, y) \) subspace that preserves the \( z \)-axis. This requires only two rotations.
Therefore, to rotate about a plane that contains the z-axis but is otherwise arbitrary we can use
\[ R_{xy}(\gamma)R_{wx}(\delta)R_{wz}(v)R_{wx}(\delta)R_{xy}(-\gamma) \] (25)
where \( \delta \) ranges from 0 to \( \pi \) and \( \gamma \) from 0 to \( 2\pi \). This procedure eliminates two of the parameters in the last planar rotation of \( \tilde{b}' \).

In three dimensions to rotate by an angle \( u \) on a plane that contains the \( x \)-axis, but is otherwise arbitrary, we may use
\[ R_{yz}(\theta)R_{xy}(u)R_{yz}(-\theta) \] (26)
where \( \theta \) ranges from 0 to \( 2\pi \). This procedure eliminates one of the parameters in the last planar rotation of \( a' \).

This leaves \( a' \) and \( \tilde{b}' \) in the form
\[ \tilde{b}'_N(v) = \tilde{R}^b_{P_N}(v) \prod_{i=N-1}^1 E_iR_{wx}(v)E_i^T \tilde{b}'_0. \] (27)
with
\[ \tilde{R}^b_{P_N}(v) = R_{wx}(\gamma)R_{xy}(\delta)R_{wz}(v)R_{xy}(-\delta)R_{wx}(-\gamma) \] (28)
and
\[ a'_M(u) = \tilde{R}^a_{P_M}(u) \prod_{i=M-1}^1 D_iR_{xy}(u)D_i^T a'_0 \] (29)
with
\[ \tilde{R}^a_{P_M}(u) = R_{yz}(\theta)R_{xy}(u)R_{yz}(-\theta). \] (30)

B. The Origin of \( u \) and \( v \)

The conditions on the coefficients of the highest harmonics \([12]\) only specify the planar rotation up to a phase so that generally we can take \( \tilde{R}^b_{P_N}(v + \beta) \) and \( \tilde{R}^a_{P_M}(u + \alpha) \) in \([27]\) and \([28]\).

We consider the action of this extra planar rotation matrix on \( \tilde{b}'_N(v) \). It can be verified to be
\[ \tilde{b}'_N(v) = \tilde{R}^b_{P_N}(v) \prod_{i=N-1}^1 P'_i(v) \tilde{R}^b_{P_N}(\beta) \tilde{b}'_0. \] (31)
where
\[ P'_i(v) = \tilde{R}^b_{P_N}(\beta)P_i(v)\tilde{R}^b_{P_N}(-\beta) \] (32)
with analogous expressions for \( a'_M(u) \). It is important to note that the effect of replacing \( \tilde{R}_P(v) \) with \( P'_i(v) \) is to make the same transformation on each of the planes of rotation, in other words, to rotate on some other set of planes. Since we can express any rotation on a plane using \([21]\), however, the effect of the matrices \( \tilde{R}^b_{P_N}(\beta) \) and \( \tilde{R}^a_{P_M}(\alpha) \) on the rotators can be ignored.

Since the planar rotations \( \tilde{R}^b_{P_N} \) and \( \tilde{R}^a_{P_M} \) both contain one of the coordinate axes we can choose \( \beta \) and \( \alpha \) so that the \( z \) component of \( \tilde{b}'_0 \) and the \( x \) component \( a'_0 \) vanish. This leaves them in the form
\[ \tilde{R}^b_{P_N}(\beta)\tilde{b}'_0 = \begin{pmatrix} \cos \theta_b \\ \cos \phi_b \sin \theta_b \\ \sin \phi_b \sin \theta_b \end{pmatrix} = R_{xy}(\phi_b)R_{wx}(\theta_b)\tilde{w} \] (33)
and
\[ \tilde{R}^a_{P_M}(\alpha)a'_0 = \begin{pmatrix} 0 \\ \cos \theta_a \\ \sin \theta_a \end{pmatrix} = R_{yz}(\theta_a)\tilde{y} \] (34)
where \( \phi_b \) and \( \theta_a \) range from 0 to \( 2\pi \) and \( \theta_b \) ranges from 0 to \( \pi \). This means we can write \( a' \) and \( \tilde{b}' \) as
\[ a'_M(u) = \tilde{R}^a_{P_M}(u) \prod_{i=M-1}^1 R_P(u)R_{yz}(\theta_a)\tilde{y}. \] (35)
and
\[ \tilde{b}'_N(v) = \tilde{R}^b_{P_N}(v) \prod_{i=N-1}^1 R_P(v)R_{xy}(\phi_b)R_{wx}(\theta_b)\tilde{w}. \] (36)

C. The Center of Mass Constraint

In order to construct string loops, apart from solving the unit magnitude constraint, we need to satisfy the center of mass constraint, namely that the loop should be closed and that we want to work in the rest frame of the loop. These constraints imply that the center of mass term will be generated. Since our starting center of mass terms to zero, and to apply further rotation matrices in such a way as to ensure that the center of mass term is zero. Here we will proceed along similar lines.

Generally, if the starting unit vector lies somewhere on the plane of the first rotation, \( i = 1 \) in \([27]\) and \([28]\), no center of mass term will be generated. Since our starting unit vectors are given by \([33]\) and \([34]\), we need to choose the first planes of rotation appropriately.

Earlier we established that if we want a rotation by an angle \( u \) on a plane that contains the \( x \)-axis we can use \([26]\). If we want an arbitrary rotation that contains the \( y \)-axis instead we may use
If we now decide we want a rotation by a unit vector living in the subspace, but is otherwise arbitrary, because this is the situation of (38). If we start with a rotator that contains a vector lying somewhere in the \( (w, x, y) \) subspace, it is not hard to see that the first rotator in (35) must be

\[
R_{P_1}(v) = R_{xy}(\theta_b)R_{wx}(\phi_b)R_{yz}(\gamma_1)R_{xy}(\delta_1)R_{wx}(v)R_{xy}(-\delta_1)R_{yz}(-\gamma_1),
\]

(39)

where \( \delta_1 \) ranges from 0 to \( \pi \) and \( \gamma_1 \) ranges from 0 to \( 2\pi \), it is not hard to see that the first rotator in (35) must be

\[
R_{P_1}(u) = R_{y,z}(\theta_a)R_{x,z}(\phi_1)R_{x,y}(u)R_{x,z}(-\phi_1)R_{y,z}(-\theta_a).
\]

(38)

where \( \phi_1 \) ranges from 0 to \( 2\pi \).

To find the first rotator in (33) we proceed analogously.

If we consider the projection of the plane onto the subspace given by the last three coordinates \( (d - 2, d - 1, d) \) one can see that a rotation by \( \alpha_d \) about the \( d \) axis \( (R_{d-2,d-1}(\alpha_d)) \) until the vector perpendicular to the projected plane lies in the \( d - d - 1 \) plane followed by a rotation by an angle \( \beta_d \) about the \( d - 2 \) axis \( (R_{d-1,d}(\beta_d)) \) is sufficient to rotate the projected plane out of the \( d \) axis. To perform these transformations it is sufficient for the angles to range from 0 to \( \pi \). We can repeat this procedure by moving up \( d - 2 \) times in the coordinates until the plane lies entirely in the 1-2 plane as desired, ensuring that the range of the angle in the very last rotation \( R_{2,3}(\beta_3) \) is from 0 to \( 2\pi \) to account for the fact that we are dealing with an oriented plane. We can then write the rotator as

\[
E = R_{d-2,d-1}(\alpha_d)R_{d-1,d}(\beta_d)...R_{1,2}(\alpha_3)R_{2,3}(\beta_3).
\]

(43)

The number of parameters introduced by such a product is \( 2(d - 2) \) per harmonic plus \( (d - 1) \) parameters to specify the initial unit vector giving a total of \( 2N(d - 2) + d - 1 \) which checks with \( d(2N + 1) \) degrees of freedom in the vector coefficients of the Fourier series minus \( 4N + 1 \) constraints.

In the case of Nambu-Goto strings in an arbitrary number of dimensions \( d \) one would span both right and left moving excitations according to

\[
a'_{N}(u) = \prod_{i=1}^{N} E_i R_{1,2}(u) E_i^T a_0
\]

(44)

and

\[
b'_{N}(v) = \prod_{i=1}^{N} E_i R_{1,2}(v) E_i^T b_0
\]

(45)

with the \( E_i \) given by the a choice of (3) appropriate to the desired number of dimensions.

V. CONCLUSIONS

We have generalised the solution to the unit magnitude constraint presented in (3) from three to four dimensions, casting it somewhat differently, in an effort to arrive at a general parametrisation of chiral superconducting strings with a finite number of harmonics. We have further shown how to construct loop solutions that satisfy the center of mass constraint and exclude overall orientation freedom. This result is useful because in studies of the properties of chiral loops, such as self-intersection and gravitational radiation properties, overall orientation of the loop is unimportant.

Studies of chiral cosmic string loops with constant currents \( \delta \) and simple varying currents \( \beta \) have been performed. Generally, however, we expect the current to be arbitrarily varying when loops are formed by intersections involving different strings or if different segments
of the loop or string were at some point in causally disconnected regions. This is a fairly generic situation and a study of the properties of more general chiral loops should account for these variations.

Along the way, we have found that our modification of the method lends itself readily to a generalisation to arbitrary dimensions. We use such a generalisation to present solutions that could be useful in the investigation of classical relativistic strings in higher dimensions as well as strings in 3+1 Minkowski space with currents and charges induced by Kaluza-Klein compactification \[11\] when the back-reaction from the gauge fields can be considered negligible.

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