Possible measurement of $P$-states probability in the ground state $^4$He nucleus

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Abstract

Using the experimental data on total $S=1$ transitions cross-sections for $^4$He($\gamma, p)^3$H and $^4$He($\gamma, n)^3$He reactions as the base, the paper discussed the possibility of measuring the probability of $^3P_0$-states in the ground state $^4$He nucleus. The analysis of the experimental data has suggested the conclusion, that within the statistical error, the ratio of the cross section of the reaction in the collinear geometry to the cross section of the electrical dipole transition with the spin $S=0$ at the angle of nucleons emission $\theta_N=90^\circ$ $\nu_p$ and $\nu_n$ in the range of photon energies $22 \leq E_\gamma \leq 100$ MeV doesn’t depend from the photon energy. This is in agreement with the assumption that the $S=1$ transitions can originate from $^3P_0$ states of the $^4$He nucleus. Average values of magnitude $\nu_p$ and $\nu_n$ in the mentioned photon energy range are calculated $\nu_p=0.01\pm0.002$ and $\nu_n=0.015\pm0.003$. The errors are statistical only.

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1 Introduction

The model-independent calculation of the ground state of the nucleus, and also of its scattering states, can be carried out on the basis of realistic inter-nucleonic forces and exact methods of solving the many-nucleon problem. The $^4$He nucleus can serve as a good test for setting this approach to work. In [1] have calculated the ground states of the lightest nuclei using the realistic $NN$ Argonne AV18 [2] and CD Bonn [3] potentials, and also, the $3N$ forces UrbanaIX [4] and Tucson-Melbourne [5, 6]. The calculations were carried out using the Faddeev-Yakubovsky (FY) technique [7, 8], which was generalized by Gloeckle and Kamada (GK) [9] to the case of taking into account two- and three-nucleon forces. The authors have estimated the error of nuclear binding energy calculations for $^4$He to be $\sim$50 keV. The calculated binding energy appeared to be $\sim$200 keV higher than the experimentally measured value. In view of this, the authors drew the conclusion that there is a possible contribution of the $4N$ forces that could have a repulsive character. Another possible explanation of this result might be the inconsistency of the data on $NN$ and $3N$ forces.

The tensor part of the $NN$ interaction and the $3NF$’s forces generate the $^4$He nuclear states with nonzero orbital momenta of nucleons. Table 1 gives the probabilities of $^1S_0$, $^3P_0$
and $^5D_0$ states of the $^4\text{He}$ nucleus calculated in [1] (notation: $^{2S+1}L_J$). The calculations gave the probability of $^5D_0$ states having the total spin $S=2$ and the total orbital momentum of nucleons $L=2$ of the $^4\text{He}$ nucleus to be $\sim 16\%$, and the probability of $^3P_0$ states having $S=1$ and $L=1$ to be $0.75\%$. It is obvious from Table 1 that the consideration of the $3\text{NF}$’s contribution increases the probability of $^3P_0$ states by a factor of $\sim 2$.

Table 1. The $^1S_0$, $^3P_0$, and $^5D_0$ states probabilities for the ground state $^4\text{He}$ nucleus (in percentage terms).

| Interaction     | $^1S_0$, % | $^3P_0$, % | $^5D_0$, % |
|-----------------|------------|------------|------------|
| AV18            | 85.87      | 0.35       | 13.78      |
| CD-Bonn         | 89.06      | 0.22       | 10.72      |
| AV18+UIX        | 83.23      | 0.75       | 16.03      |
| CD-Bonn+TM      | 89.65      | 0.45       | 9.9        |

In [10], Kievsky et al. have calculated the ground states of the lightest nuclei by the method of hyperspherical harmonic, using the $\text{NN}$ and $3\text{N}$ potentials calculated from the effective field theory. Various variants of the mentioned potentials predict the contribution from $^3P_0$ states of the $^4\text{He}$ nucleus to be between 0.1% and 0.7%. Thus, the measurement of the probability of states with nonzero orbital momenta of nucleons can provide a new information about inter-nucleonic forces.

2 The analisys of the experimental data about cross-section $^4\text{He}(\gamma, p)^3\text{H}$ and $^4\text{He}(\gamma, n)^3\text{He}$ reactions in the collinear geometry

Here, we discuss the possibility of measuring the probability of $P(^3P_0)$ states of the ground state $^4\text{He}$ nucleus through the studies of two-body ($\gamma, p$) and ($\gamma, n$) reactions of $^4\text{He}$. In these reactions, transitions matrix elements of two types, with spins $S=0$ and $S=1$ of the final state of the particle system, may take place. It is known [11], that at the electromagnetic interaction the spin-flip of hadronic particle system is significantly suppressed. The $S=1$ transitions may originate from $^3P_0$ nuclear states with no spin-flip. Maybe such transitions can occur also from $^1S_0$ or $^5D_0$ states of the $^4\text{He}$ nucleus as a result of different channels of the reaction coupled, for example, with the existence of states with nonzero orbital momenta of nucleons of the residual nucleus and from the secondary effects. It can be supposed that the cross section of the ($\gamma,N$) reaction doesn’t depend on the total spin of the ground state of $^4\text{He}$ nucleus. Then the ratio

$$\alpha = \frac{\sigma(^3M_{1,2})}{\sigma_{\text{tot}}(\gamma, N)}$$

of the total cross sections of the transitions with the spin $S=1$ to the total cross section $\sigma_{\text{tot}}(\gamma, N)$ of the reaction, after the subtraction of the contribution of other possible mechanisms of formation of transitions with spin $S=1$, can be sensible to contribution of $P$-wave component in the wave function of $^4\text{He}$ nucleus. The indexes $(1,2)$ are total momentums $1^-,1^+$ and $2^+$ of final-state of particle system at the transitions $S=1$.

In the $E1$, $E2$ and $M1$ approximation, the laws of conservation of the total momentum and parity for two-body ($\gamma, p$) and ($\gamma, n$) reactions of $^4\text{He}$ nuclear disintegration permit the
occurrence of two multipole transitions $E_1^1P_1$ and $E_2^1D_2$ with the spin $S=0$ and four transitions $E_1^3P_1$, $M_1^3D_1$, $M_1^3S_1$ and $E_2^3D_2$ with the spin $S=1$ of final-state particles. According to the present experimental data the sum of total cross sections of transitions with the spin $S=1$ is $\sim 10^{-2}$ of the total cross section of the reaction. The nucleon emission distributions in the polar angle for each of the mentioned transitions are presented in Table 2.

**Table 2: Angular distributions for $E_1$, $E_2$ and $M_1$ multipoles.**

| Spin of the final-states | Multipole transition | Angular distribution |
|--------------------------|----------------------|---------------------|
| $S=0$                    | $|E_1^1P_1|^2$       | $\sin^2 \theta$    |
|                          | $|E_2^1D_2|^2$       | $\sin^2 \theta \cos^2 \theta$ |
| $S=1$                    | $|E_1^3P_1|^2$       | $1+\cos^2 \theta$  |
|                          | $|M_1^3S_1|^2$       | const               |
|                          | $|M_1^3D_1|^2$       | $5-3\cos^2 \theta$ |
|                          | $|E_2^3D_2|^2$       | $1-3\cos^2 \theta + 4\cos^4 \theta$ |

It can be seen from Table 2 that the reaction cross-section in the collinear geometry can be due only to $S=1$ transitions, at that $d\sigma(0^0)=d\sigma(180^0)$. For the purpose of determining the reaction cross-section in the collinear geometry, an analysis was made of the information available in the literature about differential cross sections of the $^4\text{He}(\gamma, p)^3\text{H}$ and $^4\text{He}(\gamma, n)^3\text{He}$ reactions in the photons energy range up to the meson-producing threshold.

In [12, 13], the reaction products were registered at nucleon-exit polar angles $0^0 \leq \theta_N \leq 180^0$, using chambers placed in the magnetic field. However, the number of events registered in those experiments was insufficient for measuring the reaction cross-section in the collinear geometry (the cross-section estimation is shown by full circles in Fig.1).

Jones *et al.* [14] have measured the differential cross section for the $^4\text{He}(\gamma, p)^3\text{H}$ reaction at tagged photon energies between 63 and 71 MeV (triangles in Figs. 1 and 2). The reaction products were registered by means of a wide-acceptance detector LASA. The measurements were performed in the interval of polar proton-exit angles $22.5^0 \leq \theta_p \leq 145.5^0$. The lack of data for large and small angles of proton escape has led to significant errors in the measurement of the reaction cross-section in the collinear geometry.

In [15], (cross in Fig.1), a monoenergetic photon beam in the energy range from 21.8 to 29.8 MeV and nearly a $4\pi$ time projection chamber were used to measure the total and differential cross-sections for photodisintegration reactions of $^4\text{He}$ nucleus. The authors found that the M1 strength was about $2\pm 1\%$ of the E1 strength.

The differential cross sections for two-body ($\gamma, p$) and ($\gamma, n$) reactions have been measured by Arkatov *et al.* [16], [17] in the bremsstrahlung photon energy range from the reaction threshold up to $E_\gamma=150$ MeV. The reaction products were registered with the help of a diffusion chamber, placed in the magnetic field in the interval of polar nucleon-exit angles $0^0 \leq \theta_N \leq 180^0$. Later on, Nagorny *et al.* [18] reprocessed this experiment, using a new program for geometric remodeling of events, a more powerful (for that time) computer, and also, an upgraded particle track measuring system. The number of the processed events was increased by a factor of 3, and amounted to $\sim 3 \cdot 10^4$ for each of the ($\gamma, p$) and ($\gamma, n$) reaction channels. The differential cross-sections were measured with a 1 MeV step up to $E_\gamma=45$ MeV, and with a greater step at higher energies, as well as with a 10° c.m.s. step in the polar nucleon-exit angle. The authors have published their data on the differential cross-sections at photon energies of 22.5, 27.5, 33.5, 40.5, 45, and 49 MeV. The comprehensive data of Arkatov *et al.* on the differential
The differential cross-section for these reactions in the c.m.s. can be presented as:

$$\frac{d\sigma}{d\Omega} = A[\sin^2 \theta(1 + \beta \cos \theta + \gamma \cos^2 \theta) + \varepsilon \cos \theta + \nu],$$

where $\nu=[d\sigma(0^\circ)+d\sigma(180^\circ)]/2d\sigma(90^\circ)$, and $\varepsilon=[d\sigma(0^\circ)-d\sigma(180^\circ)]/2d\sigma(90^\circ)$, here $d\sigma(90^\circ)$ is a cross section $E1^1P_1$ transition at the nucleon emission angle $\theta_N = 90^\circ$.

It can be supposed, that transition $M1^3S_1$ is the main one only at the reaction threshold [20]. In the majority of works, it was supposed that the $E2^3D_2$ amplitude is the smallest one. This suggestion is confirmed by experimental hints [21]. Assuming that basic transitions, which give a contribution to ratio $\nu$, are electric dipole transitions with spin $S=1$ and $S=0$ and executing integration of proper angular distributions over the solid angle, we obtain:

$$\alpha = \frac{\sigma(M1^3D_1)}{\sigma(E1^1P_1)} = \frac{3d\sigma(0^\circ)}{d\sigma(90^\circ)} = 3\nu$$

The ratio $\nu$ was calculated as a result of least-squares fitting (LSM) of expression (2) to the experimental data on differential cross-sections [19] (with a double step in the photon energy). The results of calculations are presented in Figs. 1 and 2 as open circles. It can be seen from the figures that within the statistical errors the ratio of the cross-section in collinear geometry of the $S=1$ transitions to the cross-sections at polar nucleon-exit angle $\theta_N=90^\circ$ reaction in the photon energy region $22 \leq E_\gamma \leq 100$ MeV is independent of the photon energy. This is in agreement with the assumption that these transitions might originate from $P$-states of the $^4$He nucleus. The average values of the ratio $\nu$ in the mentioned photon energy range are calculated to be $\nu_p=0.019\pm0.002$ and $\nu_n=0.028\pm0.003$. The average values of the coefficients are $\varepsilon_p=0.0\pm0.002$ and $\varepsilon_n=-0.001\pm0.003$.

The calculated $\nu_p$ and $\nu_n$ values may be displaced as a result of experimental data histogramming, and also, due to polar nucleon-exit angle measurement errors, which were $\delta\theta_n=0.5^\circ \div 1^\circ$, as reported in ref. [22]. In this connection, by the use of simulation we have determined the corrections for the histogramming step as $10^\circ$ and for $\delta\theta_N=1^\circ$ [23]. Taking into account these corrections, we have $\nu_p=0.01\pm0.002$ and $\nu_n=0.015\pm0.003$.

The systematic error of the data might be caused by the inaccuracy in the measurement of the resolution on the polar angle of the nucleon emission. In particular, the difference in coefficients $\nu_p$ and $\nu_n$ might be conditioned by the fact that resolution of the neutron $\delta\theta_n$ emission angle was worse than of the angle of the proton $\delta\theta_p$ emission. Besides, a small number of events in some histogramming steps, especially at high photon energies, can lead to a systematic error specified by the use of the LSM method. The conclusion in [24] about large errors in the cross-section measurements in the collinear geometry had been based on early works of Arkatov et al. [16], [17].

The cross-sections of spin $S=1$ transitions can be measured by means of polarization observables. For example, the transitions $E1^1P_1$ and $E2^1D_2$ with the spin $S=0$ exhibit the asymmetry of the cross-section with linearly polarized photons $\Sigma(\theta)=1$ at all polar nucleon-exit angles, except $\theta_N=0^\circ$ and $180^\circ$. The difference of the asymmetry $\Sigma$ from unity can be due to spin $S=1$ transitions. With an aim of separating the contributions from $E1^1P_1$, $M1^3D_1$ and
$M1^3S_1$ transitions, Lyakhno et al. [21] performed a combined analysis of both the experimental data on the cross-section asymmetry $\Sigma(\theta)$ and the data on differential cross-sections for the reactions under discussion [19] at photon energies $E_{\gamma}^{peak} = 40, 56$ and 78 MeV. Because of small cross-sections of spin $S=1$ transitions, the errors of asymmetry $\Sigma(\theta)$ measurements have led to considerable errors in the cross-section measurements of these transitions.

The authors of [20, 25, 26] investigated the reactions of radiative capture of polarized protons by tritium nuclei. In [25], Wagenaar et al. have investigated the capture reaction at proton energies $0.8 \leq E_p \leq 9$ MeV. They came to the conclusion that the main transition with spin $S=1$ is $M1^3S_1$. In [20], this reaction was investigated by Pitts at the proton energy $E_p=2$ MeV. It was concluded that the main transition with $S=1$ is $E1^3P_1$. These contradictory statements were caused by considerable statistical and systematic errors of the experimental data. Within the experimental errors, the data obtained in the studies of $(\gamma, N)$ and $(\bar{p}, \gamma)$ reactions are in satisfactory agreement between themselves [21].
Figure 2: Coefficients $\varepsilon_p$ and $\varepsilon_n$. Triangle-data from [14]; Open circles-data from [19]. The errors are statistical only.

3 Conclusions

In [20] it was found that the transitions with spin $S=1$ can be conditioned by the contribution of meson exchange currents (MEC). It should be noticed, that the MEC contribution depends on the photon energy [27]. Despite the considerable MEC contribution into the total cross section of the reaction, contribution of the spin-flip of the hadronic particle system can be insignificant. The weak dependence of the ratio of $S=1$ to $S=0$ transitions cross-sections from the photon energy in the energy region from the reaction threshold up to $E_\gamma \sim 100$ MeV (this corresponding to the nucleon momentum $P_N \sim 350$ MeV/c) may point to an insignificant contribution to the total cross-section of spin $S=1$ transitions by the final-state particle interactions, and also, by other photon energy-dependent reaction mechanisms. The present experimental data coincide with the supposition that contribution $^3P_0$ components of the ground state of $^4$He nucleus to the formation of the transitions with the spin $S=1$ at the two-body $(\gamma,N)$ reaction can be considerable and these data can be used for measurement of contribution $P$-wave component in the wave function $^4$He nucleus.

A number of investigations, e.g. [28]-[32], were made into the reaction $^2$H($\vec{d},\gamma)^4$He with an aim of measuring the probability of $^5D_0$ states of the $^4$He nucleus. This reaction permits the occurrence of three types of transitions with $S=0$, 1 and 2. In this connection, the analysis
of the experimental data on this reaction may be more complicated than that of the two-body \((\gamma, N)\) reaction. It might be reasonable to perform a combined analysis of these reactions. The detailed theoretical calculations of these reactions are required.

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