Hadron structure effect in finite nuclei

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The quark-meson coupling model, based on a mean-field description of non-overlapping nucleon bags bound by the self-consistent exchange of $\sigma$, $\omega$ and $\rho$ mesons, is reviewed. In particular, I present the changes in the hadron masses of the (non-strange) vector mesons and the nucleon in a nucleus. I also give a new, simple scaling relation among the changes of the hadron masses.

§1. Introduction

As the nuclear environment changes, hadron properties are nowadays expected to be modified $\Box$. In particular, the variation of the light vector-meson mass is receiving a lot of attention, both theoretically and experimentally. Recent experiments from the HELIOS-3, CERES and NA50 collaborations at SPS/CERN energies have shown that there exists a large excess of the lepton pairs in central heavy-ion collisions $\Box$. On the other hand, even in low-energy nuclear physics, some attempts to measure the variation of hadron masses in nuclei are now underway $\Box$.

Recently we have developed the quark-meson coupling (QMC) model $\Box$ to treat the variation of hadron properties in finite nuclei $\Box$. The QMC model may be viewed as an extension of QHD $\Box$ in which the nucleons still interact through the exchange of scalar and vector mesons. However, the mesons couple not to point-like nucleons but to confined quarks (in the bag). In studies of infinite nuclear matter $\Box$ it was found that the quark degrees of freedom in the nucleon give an acceptable value for the incompressibility once the coupling constants are chosen to reproduce the correct saturation energy and density for symmetric nuclear matter. This is a significant improvement on QHD at the same level of sophistication.

In this report I briefly review the QMC model and show some numerical results for static properties and variation of the hadron masses in some nuclei $\Box$. I also give a new, simple scaling relation among the changes of the hadron masses in a nucleus.

§2. The quark-meson coupling model

Let us suppose that a free nucleon (at the origin) consists of three light (u and d) quarks under a (Lorentz scalar) confinement potential, $V_c$. Then, the Dirac equation for the quark field, $\psi_q$, is given by

$$\left[ i \gamma \cdot \partial - m_q - V_c(r) \right] \psi_q(r) = 0,$$

where $m_q$ is the bare quark mass.
Next we consider how Eq.(1) is modified when the nucleon is bound in static, uniformly distributed (iso-symmetric) nuclear matter. In the QMC model it is assumed that each quark feels scalar, $V^s_q$, and vector, $V^v_q$, potentials, which are generated by the surrounding nucleons, as well as the confinement potential. Since the typical distance between two nucleons around normal nuclear density is surely larger than the typical size of the nucleon (its radius $R_N \sim 0.8$ fm), the interaction (except for the short-range part) between the nucleons should be colour singlet, e.g., a meson-exchange potential. Therefore, this assumption seems appropriate when the baryon density, $\rho_B$, is not high. If we use the mean-field approximation for the meson fields, Eq.(1) may be rewritten as

$$[i\gamma \cdot \partial - (m_q - V^s_q) - V_c - \gamma_0 V^v_q]\psi_q = 0.$$  (2)

The potentials generated by the medium are constants because the matter distributes uniformly. As the nucleon is static, the time-derivative operator can be replaced by the quark energy, $-i\epsilon_q$. By analogy with the procedure applied to the nucleon in QHD, if we introduce the effective quark mass by $m^\star_q = m_q - V^s_q$, the Dirac equation, Eq.(2), can be rewritten in the same form as that in free space, with the mass $m^\star_q$ and the energy $\epsilon_q - V^s_q$, instead of $m_q$ and $\epsilon_q$. In other words, the vector interaction has no effect on the nucleon structure except for an overall phase in the quark wave function, which gives a shift in the nucleon energy. This fact does not depend on how to choose the confinement potential, $V_c$. Then, the nucleon energy (at rest), $E_N$, in the medium is given by $E_N = M_N^\star(V^s_q) + 3V^v_q$, where the effective nucleon mass, $M_N^\star$, depends on only the scalar potential $V^s_q$.

Now we extend this idea to finite nuclei. The solution of the general problem of a composite, quantum particle moving in background scalar and vector fields that vary with position is extremely difficult. One has, however, a chance to solve the particular problem of interest to us, namely light quarks confined in a nucleon which is itself bound in a finite nucleus, only because the nucleon motion is relatively slow and the quarks highly relativistic [4]. Thus the Born-Oppenheimer approximation is naturally suited to the problem.

Our approach in Ref.[4] was to start with a classical nucleon and to allow its internal structure to adjust to minimise the energy of three quarks in the ground-state of a system under constant scalar and vector fields, with values equal to those at the centre of the nucleon. In our case, the MIT bag model was used to describe the nucleon structure (see also Ref. [8]). Of course, the major problem with the MIT bag (as with many other relativistic models of nucleon structure) is that it is difficult to boost. We therefore solve the bag equations in the instantaneous rest frame (IRF) of the nucleon – using a standard Lorentz transformation to find the energy and momentum of the classical nucleon bag in the nuclear rest frame. Having solved the problem using the meson fields at the centre of the nucleon, one can use perturbation theory to correct for the variation of the scalar and vector fields across the nucleon bag. In first order perturbation theory only the spatial components of the vector potential give a non-vanishing contribution. This extra term is a correction to the spin-orbit force [4].

As shown in Refs.[4, 5], the basic result in the QMC model is that, in the scalar and vector fields, the nucleon behaves essentially as a point-like particle with an effective mass $M_N^\star$, which depends on the position through only the scalar field, moving in the vector potential.

Let us consider that the scalar and vector potentials in Eq.(2) are mediated by the $\sigma$ and $\omega$ mesons, and introduce their mean-field values, which now depend on position $\vec{r}$, by $V^s_q(\vec{r}) = g^s_q \sigma(\vec{r})$ and $V^v_q(\vec{r}) = g^v_q \omega(\vec{r})$, respectively, where $g^s_q$ ($g^v_q$) is the coupling constant of the quark-$\sigma$ ($\omega$) meson. Furthermore, we shall add the isovector, vector meson, $\rho$, and
the Coulomb field, \( A(\vec{r}) \), to describe finite nuclei realistically \[1, 3\]. Then, the effective lagrangian density for finite nuclei, involving the quark degrees of freedom in the nucleon and the (structureless) meson fields in MFA, would be given by

\[
\mathcal{L}_{\text{QMC-I}} = \bar{\psi} \left[ i \gamma \cdot \partial - M_{N}^{*} - g_{\sigma} \omega \gamma_{0} - \frac{g_{b}}{2} \tau_{3}^{N} b \gamma_{0} - \frac{e}{2} (1 + \tau_{3}^{N}) A \gamma_{0} \right] \psi \\
- \frac{1}{2} \left( (\nabla \sigma)^{2} + m_{\sigma}^{2} \sigma^{2} \right) + \frac{1}{2} \left( (\nabla \omega)^{2} + m_{\omega}^{2} \omega^{2} \right) + \frac{1}{2} \left( (\nabla b)^{2} + m_{b}^{2} b^{2} \right) + \frac{1}{2} (\nabla A)^{2},
\]

where \( \psi \) and \( b \) are respectively the nucleon and the \( \rho \) fields. \( m_{\sigma}, m_{\omega} \) and \( m_{b} \) are respectively the (constant) masses of the \( \sigma, \omega \) and \( \rho \) mesons. \( g_{\sigma} \) and \( g_{b} \) are respectively the \( \omega \)-\( N \) and \( \rho \)-\( N \) coupling constants, which are given by \( g_{\omega} = 3g_{b}^{0} \) and \( g_{b} = g_{b}^{0} \) (where \( g_{b}^{0} \) is the quark-\( \rho \) coupling constant). We call this model the QMC-I model. If we define the field-dependent \( \sigma \)-\( N \) coupling constant, \( g_{\sigma}(\sigma) \), by

\[
M_{N}^{*}(\sigma(\vec{r})) = M_{N} - g_{\sigma}(\sigma(\vec{r})) \sigma(\vec{r}),
\]

where \( M_{N} \) is the free nucleon mass, it is easy to compare with QHD. The difference between QMC-I and QHD lies only in the coupling constant \( g_{\sigma} \), which depends on the scalar field in QMC-I while it is constant in QHD. However, this difference leads to a lot of favorable results, notably the nuclear incompressibility \[3\].

Here we consider the nucleon mass in matter further. The nucleon mass is a function of the scalar field. Because the scalar field is small at low density it can be expanded in terms of \( \sigma \) as

\[
M_{N}^{*} = M_{N} + \left( \frac{\partial M_{N}^{*}}{\partial \sigma} \right)_{\sigma=0} \sigma + \frac{1}{2} \left( \frac{\partial^{2} M_{N}^{*}}{\partial \sigma^{2}} \right)_{\sigma=0} \sigma^{2} + \cdots.
\]

Since the interaction Hamiltonian between the nucleon and the \( \sigma \) field at the quark level is given by \( H_{\text{int}} = -3g_{b}^{0} \int d\vec{r} \bar{\psi}(\vec{r}) \sigma \psi(\vec{r}) \), the derivative of \( M_{N}^{*} \) with respect to \( \sigma \) is given by

\[
-3g_{b}^{0} \int d\vec{r} \bar{\psi}(\vec{r}) \sigma \psi(\vec{r}) = \frac{1}{3} g_{N}(0) S_{N}(\sigma),
\]

where we define the quark-scalar density in the nucleon, \( S_{N}(\sigma) \), which is itself a function of the scalar field. Because of a negative value of the derivative the nucleon mass decreases in matter at low density.

Furthermore, we define the scalar-density ratio, \( S_{N}(\sigma)/S_{N}(0) \), to be \( C_{N}(\sigma) \) and the \( \sigma \)-\( N \) coupling constant at \( \sigma = 0 \) to be \( g_{\sigma} \) (i.e., \( C_{N}(\sigma) = S_{N}(\sigma)/S_{N}(0) \) and \( g_{\sigma} = g_{\sigma}(\sigma = 0) = 3g_{b}^{0} S_{N}(0) \)). Using these quantities, the nucleon mass is rewritten as

\[
M_{N}^{*} = M_{N} - g_{\sigma} \sigma - \frac{1}{2} g_{\sigma} C_{N}(0) \sigma^{2} + \cdots.
\]

In general, \( C_{N} \) is a decreasing function because the quark in matter is more relativistic than in free space. Thus, \( C_{N}(0) \) takes a negative value. If the nucleon were structureless \( C_{N} \) would not depend on the scalar field, that is, \( C_{N} \) would be constant (\( C_{N} = 1 \)). Therefore, only the first two terms in the right hand side of Eq.\[3\] remain, which is exactly the same as the equation for the effective nucleon mass in QHD (see also Ref.\[4, 10\]). By taking the heavy-quark-mass limit in QMC we can reproduce the QHD results \[3, 4\].

We have considered the effect of nucleon structure. It is however true that the mesons are also built of quarks and anti-quarks, and that they may change their properties in matter. To incorporate the effect of meson structure in the QMC model, we suppose that the vector mesons are again described by a relativistic quark model with common scalar and vector mean-fields \[3\], like the nucleon (see Eq.\[3\]). Then, again the effective vector-meson mass in matter, \( m_{v}^{*} \) (\( v = \omega \) or \( \rho \)), depends on only the scalar mean-field.
However, for the scalar (σ) meson it may not be easy to describe it by a simple quark model (like a bag) because it couples strongly to the pseudoscalar (2π) channel, which requires a direct treatment of chiral symmetry in medium [1]. Since, according to the Nambu–Jona-Lasinio model or the Walecka model, one might expect the σ-meson mass in medium, \( m_\sigma^* \), to be less than the free one, we shall here parametrize it using a quadratic function of the scalar field (by hand) [5]:

\[
\left( \frac{m_\sigma^*}{m_\sigma} \right) = 1 - a_\sigma(g_\sigma \sigma) + b_\sigma(g_\sigma \sigma)^2,
\]

with \( g_\sigma \sigma \) in MeV, and we introduce two parameters, \( a_\sigma \) (in MeV\(^{-1}\)) and \( b_\sigma \) (in MeV\(^{-2}\)). We will determine these parameters later.

Using these effective meson masses, we can find a new lagrangian density for finite nuclei, which involves the structure effects of both the nucleon and mesons in MFA:

\[
\mathcal{L}_{\text{QMC-II}} = \bar{\psi} \left[ i \gamma \cdot \partial - M_N^* - g_\omega \omega \gamma_0 - \frac{g_\rho}{2} \gamma_3 \gamma_0 - \frac{c}{2} (1 + \gamma_3 \gamma_0) A \gamma_0 \right] \psi
- \frac{1}{2} \left[ (\nabla \sigma)^2 + m_\sigma^* \sigma^2 \right] + \frac{1}{2} \left[ (\nabla \omega)^2 + m_\omega^* \omega^2 \right] + \frac{1}{2} \left[ (\nabla b)^2 + m_\rho^* \rho^2 \right] + \frac{1}{2} (\nabla A)^2.
\]

We call this model QMC-II. Note that all the masses depend on the scalar mean-field.

### §3. Numerical results

In this section, we will show our numerical results using the QMC-II model. (For QMC-I, see Ref. [1].)

For infinite nuclear matter we take the Fermi momenta for protons and neutrons to be \( k_{F_i} \) \((i = p \text{ or } n)\). This is defined by \( \rho_i = k_{F_i}^3 / (3 \pi^2) \), where \( \rho_i \) is the density of protons or neutrons, and the total baryon density, \( \rho_B \), is then given by \( \rho_p + \rho_n \). (Note that the mean-field values for the mesons are constant.)

From the lagrangian density Eq. (8), we can easily find the total energy per nucleon, \( E_{\text{tot}} / A \), and the values of the \( \omega \) and \( \rho \) fields (which are respectively given by baryon number conservation and the difference in proton and neutron densities, \( \rho_3 = \rho_p - \rho_n \)). On the other hand, the scalar mean-field is given by a self-consistency condition (SCC): \( \left( \frac{dE_{\text{tot}}}{d\sigma} \right) = 0 \).

Now we need a model for the structure of the hadrons involved. We use the MIT bag model in static, spherical cavity approximation [12]. In the model, the bag constant \( B \) and the parameter \( z_N \) in the familiar form of the MIT bag model lagrangian are fixed to reproduce the free nucleon mass (\( M_N = 939 \text{ MeV} \)) under the condition that the hadron mass be stationary under variation of the free bag radius (\( R_N \) in the case of the nucleon). Furthermore, to fit the free vector-meson masses, \( m_\omega = 783 \text{ MeV} \) and \( m_\rho = 770 \text{ MeV} \), we introduce new \( z \)-parameters for them, \( z_\omega \) and \( z_\rho \). In the following we choose \( R_N = 0.8 \text{ fm} \) and the free quark mass \( m_q = 5 \text{ MeV} \). We then find that \( B^{1/4} = 170.0 \text{ MeV} \), \( z_N = 3.295 \), \( z_\omega = 1.907 \) and \( z_\rho = 1.857 \). (Variations of the quark mass and \( R_N \) only lead to numerically small changes in the calculated results [1].)

Next we must choose the two parameters in the parametrization for the \( \sigma \)-meson mass in matter (see Eq. (5)). Here we consider three parameter sets: (A) \( a_\sigma = 3.0 \times 10^{-4} \text{ (MeV}^{-1}) \) and \( b_\sigma = 100 \times 10^{-8} \text{ (MeV}^{-2}) \), (B) \( a_\sigma = 5.0 \times 10^{-4} \text{ (MeV}^{-1}) \) and \( b_\sigma = 50 \times 10^{-8} \text{ (MeV}^{-2}) \),

\[ a_\sigma \text{ and } b_\sigma \text{ in MeV, and we introduce two parameters, } a_\sigma \text{ (in MeV}^{-1}) \text{ and } b_\sigma \text{ (in MeV}^{-2}) \text{. We will determine these parameters later.} \]

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- \frac{1}{2} \left[ (\nabla \sigma)^2 + m_\sigma^* \sigma^2 \right] + \frac{1}{2} \left[ (\nabla \omega)^2 + m_\omega^* \omega^2 \right] + \frac{1}{2} \left[ (\nabla b)^2 + m_\rho^* \rho^2 \right] + \frac{1}{2} (\nabla A)^2.
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\[ a_\sigma \text{ and } b_\sigma \text{ in MeV, and we introduce two parameters, } a_\sigma \text{ (in MeV}^{-1}) \text{ and } b_\sigma \text{ (in MeV}^{-2}) \text{. We will determine these parameters later.} \]
(C) \( a_\sigma = 7.5 \times 10^{-4} \) (MeV\(^{-1}\)) and \( b_\sigma = 100 \times 10^{-8} \) (MeV\(^{-2}\)). The parameter sets A, B and C give about 2%, 7% and 10% decreases of the \( \sigma \) mass at saturation density, respectively \[5\].

Now we are in a position to determine the coupling constants. \( g_\sigma \) and \( g_\omega \) are fixed to fit the binding energy \((-15.7\) MeV\) at the saturation density \((\rho_0 = 0.15\) fm\(^{-3}\)) for symmetric nuclear matter. Furthermore, the \( g_\rho \) is used to reproduce the bulk symmetry energy, 35 MeV. We take \( m_\sigma = 550\) MeV. The coupling constants and some calculated properties for matter in QMC-II are listed in Table 1. The last three columns show the relative changes (from their values at zero density) of the nucleon-bag radius \((\delta R_N^*/R_N)\), the lowest eigenvalue \((\delta x_N^*/x_N)\) and the root-mean-square radius (rms radius) of the nucleon calculated using the quark wave function \((\delta r_q^*/r_q)\) at saturation density. We note that the nuclear incompressibility is higher

Table 1: Coupling constants and calculated properties for symmetric nuclear matter at normal nuclear density \((m_q = 5\) MeV, \(R_N = 0.8\) fm and \(m_\sigma = 550\) MeV). The effective nucleon mass, \(M_N^*\), and the nuclear incompressibility, \(K\), are quoted in MeV. The bottom row is for QHD.

| type | \(g_\omega^2/4\pi\) | \(g_\sigma^2/4\pi\) | \(g_\rho^2/4\pi\) | \(M_N^*\) | \(K\) | \(\delta R_N^*/R_N\) | \(\delta x_N^*/x_N\) | \(\delta r_q^*/r_q\) |
|------|----------------|----------------|----------------|--------|-----|----------------|----------------|----------------|
| A    | 3.84           | 2.70           | 5.54           | 801    | 325 | -0.01         | -0.11          | 0.02           |
| B    | 3.94           | 3.17           | 5.27           | 781    | 382 | -0.01         | -0.13          | 0.02           |
| C    | 3.84           | 3.31           | 5.18           | 775    | 433 | -0.02         | -0.14          | 0.02           |
| QHD  | 7.29           | 10.8           | 2.93           | 522    | 540 | —             | —              | —              |

than that in QMC-I \((K \sim 200 – 300\) MeV) \[4\] \[5\]. However, it is still much lower than in QHD. As in QMC-I, the bag radius of the nucleon shrinks a little, while its rms radius swells a little. On the other hand, because of the \( \sigma \) field, the eigenvalue is reduced more than 10\% (at \( \rho_0 \)) from that in free space.

In the present model it is possible to calculate masses of other hadrons like hyperons in medium \[5\] (see also Ref. \[13\].) In that case, we assume that for the hyperons themselves we again use the MIT bag model, and that the strange quark in the hyperon does not directly couple to the scalar field in MFA, as one would expect if the \( \sigma \) meson represented a two-pion-exchange potential. It is also assumed that the addition of a single hyperon to nuclear matter of density \( \rho_B \) does not alter the values of the scalar and vector mean-fields, namely, we take the local-density approximation to the hyperons.

In general, as in the nucleon case (see Eq.(5)), we can expand the effective hadron mass in nuclear matter at low \( \rho_B \) as

\[
M_j^* = M_j + \left( \frac{\partial M_j}{\partial \sigma} \right)_{\sigma=0} \sigma + \frac{1}{2} \left( \frac{\partial^2 M_j^*}{\partial \sigma^2} \right)_{\sigma=0} \sigma^2 + \cdots,
\]

\[
\simeq M_j - \frac{n_0}{3} g_\sigma \sigma - \frac{n_0}{6} g_\sigma C_j'(0) \sigma^2,
\]

(9)

where \( j \) stands for \( N, \omega, \rho, \Lambda, \Sigma, \Xi, \) etc., \( n_0 \) is the number of non-strange quarks in the hadron \( j \) and \( C_j(\sigma) = S_j(\sigma)/S_j(0) \) is the scalar-density ratio defined by the quark-scalar density, \( S_j \), in \( j \). Since the strength of the scalar mean-field, \( g_\sigma \sigma \), at low \( \rho_B \) can be approximated by a linear function of the density \[5\] as \( g_\sigma \sigma \approx 200\) (MeV) \((\rho_B/\rho_0)\), we find the hadron mass at low \( \rho_B \):

\[
\left( \frac{M_j^*}{M_j} \right) \simeq 1 - c_j \left( \frac{\rho_B}{\rho_0} \right),
\]

(10)
where \( c_{N,v,\Lambda,\Sigma,\Xi} = 0.21, 0.17, 0.12, 0.11, 0.05 \), respectively.

Furthermore, using the MIT bag model we find that the scalar-density ratio \( C_j \) can be well approximated by a linear function of the scalar field \( \sigma \):

\[
C_j(\sigma) = 1 - a_j \times (g_\sigma \sigma),
\]

where \( a_j \) is the slope parameter for the hadron \( j \), and that the dependence of \( a_j \) on the hadrons is quite weak (it ranges around \( 8.6 \sim 9.5 \times 10^{-4} \text{ (MeV}^{-1}) \)). Therefore, if we ignore its weak dependence on the hadrons, the effective hadron mass can be rewritten in a quite simple form (from Eq.(9)):

\[
M_j^* \simeq M_j - \frac{n_0}{3} (g_\sigma \sigma) \left[ 1 - \frac{a}{2} (g_\sigma \sigma) \right],
\]

where \( a \simeq 9.0 \times 10^{-4} \text{ (MeV}^{-1}) \). This mass formula can reproduce the hadron masses in matter quite well over a wide range of \( \rho_B \) (up to \( \sim 3 \rho_0 \)). Therefore, once one knows the behaviour of \( g_\sigma \sigma \) at finite density, one can easily calculate the effective hadron mass using Eq.(11). A simple parametrization of \( g_\sigma \sigma \) is given in Ref. [14].

Since the scalar field is common to all hadrons, Eq.(11) leads to a new, simple scaling relationship among the hadron masses [5]:

\[
\left( \frac{\delta m_j^*}{\delta M_N^*} \right) \simeq \left( \frac{\delta M_j^*}{\delta M_N^*} \right) \simeq \left( \frac{\delta M_j^*}{\delta M_N^*} \right) \simeq \frac{2}{3} \text{ and } \left( \frac{\delta M_j^*}{\delta M_N^*} \right) \simeq \frac{1}{3},
\]

where \( \delta M_j^* \equiv M_j - M_j^* \). The factors, \( \frac{2}{3} \) and \( \frac{1}{3} \), in Eq.(12) come from the ratio of the number of non-strange quarks in \( j \) to that in the nucleon. It would be very interesting to see whether this scaling relationship is correct in forthcoming experiments.

![Figure 1: Changes of the nucleon, \( \sigma \) and \( \omega \) meson masses in \( {}^{40}\text{Ca} \). The nuclear baryon density is also illustrated (solid curve). The right (left) scale is for the effective mass (the baryon density). The parameter set B is used.](image)

For finite nuclei, we will show our results of some finite, closed shell nuclei. We have solved self-consistently a set of coupled non-linear differential equations [15] given by the lagrangian density Eq.(8). In Fig. [1] we present our self-consistent, full results of the changes of the nucleon, \( \sigma \) and \( \omega \) meson masses in \( {}^{40}\text{Ca} \). Table [3] gives a summary of the calculated binding
energy per nucleon \( (E/A) \), rms charge radii and the difference between nuclear rms radii for neutrons and protons \((r_n - r_p)\) for several closed-shell nuclei \([4, 5]\). While there are still some discrepancies between the results and data, the present model provides reasonable results. In particular, as in QMC-I, it reproduces the rms charge radii for medium and heavy nuclei quite well.

§4. Summary

I have reviewed the quark-meson coupling (QMC) model to include quark degrees of freedom in the hadrons involved, and have shown the density dependence of hadron masses in \(^{40}\text{Ca}\) as an example. As several authors have suggested \([1, 2]\), the hadron mass is reduced because of the scalar mean-field in medium. In the present model the hadron mass can be related to the number of non-strange quarks and the strength of the scalar mean-field (see Eq.\((11)\)). We have found a new, simple formula to describe the hadron masses in the medium, and this led to a new scaling relationship among them (see Eq.\((12)\)). We should note that the origins of the meson-mass reduction in QMC and QHD are completely different \([5, 10]\).

It would be very interesting to compare our results with forthcoming experiments on the vector-meson mass \([3]\).

At low energy an effective field theory to describe nuclei will generally contain an infinite number of interaction terms, which incorporate the compositeness of the low-energy degrees of freedom, namely the hadrons \([13]\), and it is then expected to involve numerous couplings which may be nonrenormalizable. Thus, one needs an organizing principle to find out a sensible field theory. Manohar and Georgi \([17]\) have proposed a systematic way to manage such complicated, effective field theories called “naive dimensional analysis” (NDA). NDA gives rules for assigning a coefficient of the appropriate size to any interaction term in an effective lagrangian. After extracting the dimensional factors and some appropriate counting factors using NDA, the remaining dimensionless coefficients are all assumed to be of order unity. This is the so-called naturalness assumption.

Our lagrangian density, Eq.\((8)\), provides a lot of effective coupling terms among the meson fields because the mesons have structure. In particular, the lagrangian automatically offers self-coupling terms (or non-linear terms) with respect to the \( \sigma \) field. Therefore, it is very interesting whether the QMC model gives natural coupling constants. In Ref. \([15]\), we exam-
ined the QMC model using NDA, and concluded that the QMC model is quite natural as an effective field theory for nuclei.

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