Use of the quantum cluster algorithm and scaling dynamics in magnetic resonance imaging for prostate cancer staging

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Abstract. Malignant tumors in the prostate generally have amorphous geometries and roughness is evident on their surface. These characteristics, together with the escalation dynamics of this type of lesion, provide information on their degree of malignancy or stage, therefore, their understanding leads to establishing an adequate diagnosis and a good radiotherapeutic implementation. The geometry present in prostate cancers leads to an analysis based on scales and a fractal study. In the present work, an in vivo diagnosis of prostate cancer will be made with magnetic resonance images, three-dimensional, using the cluster quantum algorithm and the calculation of the critical exponents of local roughness and the fractal dimension, which will allow staging said lesions. and whose results are consistent with those reported in the literature.

1. Introduction
A powerful tool for the study and diagnosis of cancer pathologies is based on the interpretation of medical images, these store very valuable information and for their extraction their quality must be improved, which is a fundamental task of image processing and with this more it is necessary to identify and characterize the tissues where these types of lesions are found, leading to a safer and more precise diagnosis [1,2]. For its diagnosis, invasive techniques are commonly used, such as digital rectal examination, transrectal ultrasound, among others, in addition, non-invasive techniques with medical imaging such as computerized axial tomography (CAT) or magnetic resonance imaging (MRI) can be used, with clear benefits for patients [3].

Cancer cells, unlike normal cells, their mitotic process takes place almost without limit, which implies a loss of their ability to die, increasing the probability of generating tumors [4,5]. The disorderly growth of these cell colonies forms an irregular contour, which can only be quantified with the fractal geometry whose dimension describes the smallest of the sizes of the irregularity present in these lesions. In this sense, the degree of roughness of the lesion is added through the critical exponent of local roughness that accounts for the dynamics of its scaling [6,7].

Prostate cancer is one of the leading causes of death worldwide, being very common in white Caucasian men or in black men and over 60 years of age [4,5]. In this work, a staging, in vivo and in 3D, with magnetic resonance images, of prostate cancer is carried out, using scaling analysis, fractal geometry and the quantum cluster method. The results obtained are in accordance with those reported in the literature, which generates reliability in the improvement of diagnoses and treatments.
2. Theory

In Parzen estimator and quantum clustering algorithm, if we take a potential \( V(\vec{X}) \) and the function \( \varphi(\vec{X}) \) that satisfy the time-independent Schrödinger equation [8], that is Equation (1).

\[
-\frac{1}{2\sigma^2} \nabla^2 \varphi + V(\vec{X}) \varphi = E \varphi = 0, \tag{1}
\]

where \( \varphi(\vec{X}) \) is a positive function which correspond the Parzen estimator [9].

On the other hand, in fractal geometry and scaling dynamics tumor-host, the interface is the part with the highest cellular activity and therefore the width of this interface must be studied according to its roughness surface, which is characterized by Equation (2) [10,11].

\[
W(s) \sim S^{\alpha_{loc}}, \tag{2}
\]

where \( W \) is the width of the interface, \( s \) is the area of the tumor-host interface, and \( \alpha_{loc} \) is the critical local roughness exponent [12]; temporal evolution was not considered because the images obtained only account for a single instant of time, not an interval.

3. Materials and methods

To classify the images, the radiotherapy protocol for prostate was considered, that is, contiguous, non-overlapping sections of contrasted images obtained in the axial anatomical plane that contained the entire tumor volume were taken. In this work, 100 to 170 slice images were selected per patient, whose minimum pixel sizes were between \( 256 \times 256 \) and a resolution plane of less than 1 mm and slice thickness ranging between 1 mm and 2 mm. And the associated voxel corresponds to a value close to 1 mm\(^3\). For the study of adenocarcinomas whose sizes were within the diameters of 1 cm and 3.5 cm according to the pathological results of said lesions. The image was taken in such a way that it encompasses the entire lesion as shown in Figure 1.

![Figure 1. Selection of volume of interest in an adenocarcinoma.](image)

The use of contrast means to obtain the images is due to the fact of highlighting the greater cellular activity in the tumor and therefore the definition of tumor volume. In this work, only well-defined contrast images were considered, without considering their histological origin; all images with poor contrast were not taken into account. The segmentation process of the images containing the tumor volume was made section by section, using the cluster quantum clustering algorithm. In this grouping process, the classes are not known and the exploration is carried out from the same data, dividing them into similar groups for classification [13]. Each group is labeled by a vector with n-dimensional characteristics, which is the obtaining of the potential value from the Schrödinger equation according to the Parzen estimator, which corresponds to the definition of the centroid \( \mu_j \). The flowchart in Figure 2 presents the quantum clustering algorithm for image processing.
Initially, the effect generated by the size of the data must be considered to determine the critical local roughness exponent $\alpha_{loc}$. A high uncertainty occurs when evaluating Equation (2) for a small tumor-host interface; therefore, to delimit the interface, different diameters of the spheres were used, obtaining a different number of interface points for each of them. In each sphere, the number of points contained in the maximum width of the interface was calculated, that is, in $W(4\pi)$, a value that is constant because the width of the interface only depends on the discretization imposed by the sphere of the image matrix, as shown in Figure 3.

In Figure 3, a fluctuation is observed for the interface whose size is less than 1,000 points and stabilization are achieved for sizes greater than these. Consequently, this work will only take into account all those images that exceed 1,000 points in the maximum width of the tumor-host interface.

Given that the symmetry of the tumor-host interface is not necessarily spherical, this induces certain artifacts in the images, which leads to significant variations in the maximum width of the interface in $W(4\pi)$ and therefore in the calculation of $\alpha_{loc}$. To quantify this effect, $W(4\pi)$ was calculated for ellipsoid-type geometries, which were simulated for the tumor-host interface. For this fact, each interface is taken as a mass distribution system, to which an inertial tensor is associated that allows the calculation of its own values. In this sense, the anisotropy present in the tumor-host was determined according to Equation (3) [12].

$$\sigma = \left( \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} \right)^{\frac{1}{2}},$$  \hspace{1cm} (3)

where $\sigma_1, \sigma_2, \sigma_3$ are the eigenvalues of the associated inertia tensor with the tumor.

As shown in Figure 4, anisotropy values lower than 0.7 show stable behavior. If limited to tumor-host interfaces with spherical symmetry, the standard deviation of $\alpha_{loc}$ should be small compared to the elliptical geometry, which occurred in the interfaces analyzed in this work.

To calculate $\alpha_{loc}$, the choice of the image data must take into account the following aspects: (a) start with a sampling sphere whose scale exceeds the minimum limit defined by the image matrix; (b) the estimation is made when the number of points at the tumor-host interface exceed 1,000; (c) calculations restricted to the tumor-host interface will be below an anisotropy $\sigma = 0.7$. Finally, a linear regression is performed on Equation (3) that allows determining the local roughness exponent $\alpha_{loc}$ whose valid results present a square correlation coefficient, $r^2$ with $r^2 > 0.99$.

The fractal dimension ($d_f$) was determined for each of the cases on average as shown in Figure 5.

![Flow diagram for the quantum clustering algorithm.](image_url)
4. Results and discussion
The scaling analysis was done on images obtained in 12 patients with prostate tumor lesions, which were histologically classified into two (2) groups, that is, adenocarcinoma IIC (6 cases) and adenocarcinoma IIB (6 cases). In all the cases studied, fractal behavior was present at the tumor-host interface with values of its fractal dimension between 1.93 and 2.01.

Regarding the calculation of the critical exponent $\alpha_{loc}$, it was done based on Figure 6 and Figure 7 where the behavior of a power law is seen with its respective variations depending on the histological group, which is summarized in Table 1.

The potential values obtained from the solution of the Schrödinger equation correspond to the values of the centroids that were taken for the segmentation of the image and therefore in the determination of the calculations of the fractal dimension and critical roughness exponent shown in Table 1.
Table 1. Geometric characterization of the tumor-host interface.

| Type                 | Cases | $d_f$         | $\alpha_{loc}$ |
|----------------------|-------|---------------|----------------|
| Adenocarcinoma IIC   | 6     | $2.01 \pm 0.01$ | $0.96 \pm 0.01$ |
| Adenocarcinoma IIB   | 6     | $1.93 \pm 0.02$ | $0.88 \pm 0.02$ |

The possible correlation between the local roughness exponent that could classify the studied tumors according to their stage is shown in Figure 8. The fractal dimension present at the tumor-host interface is between the ranges $1.93 \pm 0.02$ and $2.01 \pm 0.01$ for the cases studied. Table 1 shows that only stage IIC adenocarcinomas fulfill the characteristic of a ballistic growth model, because the sum of the local roughness exponent and fractal dimension generate the Euclidean dimension where the lesion is embedded [7,14]. All the $\alpha_{loc}$ values shown in Table 1 for the different lesions studied are consistent with those obtained by [15]. The difference in values for the local roughness exponent $\alpha_{loc}$ is due to the location of the tissue where the tumor is located [12]. The analysis of Table 1 leads to a possible estimation of the degree of malignancy of the lesion from the critical exponents of local roughness, $\alpha_{loc}$.

Figure 8. Shape of represent the stage of prostate tumors according to the local roughness exponent $\alpha_{loc}$.

5. Conclusion

In this work, scaling analysis and the cluster quantum algorithm were used for the in vivo processing of prostate tumors using magnetic resonance images, where the different types of IIB and IIC adenocarcinomas were analyzed according to the morphology of the interface. of the lesions or tumor-host, thereby determining the geometric properties present such as the fractal dimension and the critical exponent of local roughness. The results show good agreement with some reported in the literature of in vitro and in vivo works. The relationship between the roughness exponent and the fractal dimension was also analyzed, whose arithmetic sum for the case of IIC adenocarcinomas corresponds to the type of ballistic growth.

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