Measurement and Calculation of Speed Profile for Rotating Machinery

Liu Qihe
Fluid Sealing Measurement and Control Engineering Research and Development Center of Jiangsu Province, Nanjing Polytechnic Institute, 210048, Nanjing, China
13851559176@163.com

Abstract. The vibration signal angular domain resampling of rotating machinery depends on the speed profile. To obtain the speed profile, n ribbons of uniform width with black and white were pasted along the circumference of the shaft. Key phasor pulses were picked up by photoelectric sensor, and the threshold of the sampling sequence was calculated. Key phasor time vectors were calculated according to the threshold and the sampling sequence. The rotating speed was calculated by the key phasor time vector and the midpoint coefficient of the Lagrange 7 point interpolation differential. The speed profiles of single key phasor pulse, 4 key phasor pulse and digital encoder with 60 pulses per revolution are measured by using LabVIEW program simultaneously. It shows that the smoothness and the calculation accuracy of the speed profile can be improved by using multi key phasor pulse per revolution.

1. Introduction
In the vibration analysis system of rotating machinery, the conventional analysis method is FFT for extracting the characteristic frequency of the vibration signals and analyzing the spectrum, however, its precondition is the speed must be stable. Usually, the rotating machinery is in a fluctuating state due to the fluctuation caused by the motor voltage, especially in start/stop stage, and the FFT would cause frequency ambiguity, so the calculation order tracking (COT) should be used[1].

The COT is an advanced technology developed in recent years, which convert the equal time interval time-domain sampling signals to angular domain sampling signals with uniform angles by signal processing algorithm. The key of calculation order analysis is angular domain resampling, and the sampling rate depends on the speed in speed profile which is a cluster binding the time vector (the time array of the key phasor signal edge) of key phasor signal and the corresponding transient speed together.

The rotating speed measurement relating vibration order analysis acquires not only speed, but also speed profile, because the order determination and later order spectrum analysis of COT need the speed vector of speed profile. Firstly, the maximum order is calculated according to the speed vector, and then the interpolation coefficient is calculated according to the maximum order. The interpolation coefficient is used to interpolate the key phasor time vector, and the interpolation results are used for angular domain resampling. Therefore, to ensure the accuracy of order analysis, precise calculation of speed profile is a prerequisite[1].

2. Necessity of setting up multi key phasor signal per revolution
The traditional rotating speed key phasor signal is a single pulse signal generated by an eddy current sensor which mounted in a key phasor slot on the rotating shaft. This method assumes that the rotating...
speed is an average value, so it is not accurate enough, especially in acceleration and deceleration stage, because in which the angular acceleration is big. The accurately way is to use digital encoder, however, which can increase the cost and installation complexity.

Using 1 key phasor pulse, there is only one physical time vector per revolution, and the rest are interpolation points, so there are \( n \) physical time vectors per revolution using \( n \) key phasor pulses. It reduces the interpolation points and makes the key phasor time vectors used for resampling more real.

A ribbons of uniform width with black and white are pasted along the circumference of the shaft, and the key phasor pulse are picked up by photoelectric sensor, which can acquire whole period vibration signal under the control of key phasor signal economically and conveniently.

For \( n \) \((n\) can be divided by 360) key phasor pulses, assuming that the key phasor time vector of the rising edge in each key phasor pulse are \( t_0, t_1, \ldots, t_n \) (\( t_0 \) in the next revolution), taking an acceleration for example, the circular motion is a uniformly acceleration, thus \( t_1 \rightarrow t_2 \rightarrow t_1 \rightarrow t_2 \rightarrow \cdots \rightarrow t_n \rightarrow t_0 \). Assuming that the angular velocity is constant in each key phasor period, the rotating speed in each section is

\[
   r_i = \frac{60}{4(t_i - t_{i-1})} \quad (i=0, 1, 2, 3, 4),
\]

thus \( r_1 < r_2 < \cdots < r_n \), and the speed is rising. However, for 1 key phasor pulse, assuming that the angular acceleration is neglected, thus the angular velocity is a constant, and \( r = 60/(t_n - t_0) \). Obviously, increasing the number of key phasor pulses per revolution can not only make the speed curve more smoother and accurate, but also reduce the interpolation points for the later resampling time vector.

3. Acquisition steps of speed profile
The acquisition process of speed profile is as follows:

1. Acquire the signals.
2. Separate the key phasor pulse signals and vibration signals, and acquire waveform sampling data \( key[n] \) of key phasor signal.
3. Calculate the threshold of key phasor pulse.
4. Shape pulse.
5. Detect key phasor pulse signal, and calculate the key phasor pulse signal time vector \( T[m] \) on the rising edge.
6. Calculate rotating speed according to the time vector \( T[m] \).
7. Bind the key phasor pulse signal time vector and the corresponding rotational speed to a cluster, which is the speed profile.

4. Signal acquisition

4.1 Acquisition configuration
In the NI standard, the vibration signal acquisition equipment is DSA with anti-aliasing filter, and external trigger signal is the key phasor signal. However, there is the phasor retardance between the vibration signal and key phasor signal, so it needs to be compensated. Moreover, it is very expensive. The author tries to set the key phasor signals and the vibration signals together as an analogue input task using the low-cost NIPCI-6221 card, and then separate the key phasor signals from the acquisition signals, which is used to calculate the speed profile.

4.2 Determination of sampling rate
When the photoelectric sensor acquiring the key phasor pulse signal, the shape of the pulse edge is determined by the step response characteristics of the photoelectric sensor, not the speed of the rotor. If the sampling rate is too low, it is likely that the key phasor time vector is not located on the edge, so it is necessary to increase the sampling rate.

Assuming that the step edge duration of the photoelectric sensor is 50\( \mu \)s, the sampling period must be less than 50\( \mu \)s to ensure that a sampling point falls on the edge, that is to say, the sampling rate
must be bigger than 20 kHz. To facilitate spectral analysis of vibration signals, the sampling rate \(f_s\) generally is 25.6 kHz. For the bipolar motor with \(f_s=100\) Hz, assuming that the speed of the motor is 6000 rpm, even if the doubling frequency and harmonic factors are taken into account, the Nyquist sampling theorem, \(f_{max} < 1/2 f_s\), can be fully satisfied and there is no aliasing for acquiring both the vibration signals and key phasor pulse signals at the same time\(^5\)\(^6\).

5. Calculation of Speed Profile

5.1 Calculation of threshold

The reasonable threshold is the key of the waveform shaping and the calculation of key phasor time vector. Assuming that the key phasor pulse sampling sequence is \(key[n]\), the maximum value, \(max\), and the minimum value, \(min\), can be obtained by the functions of them in the LabVIEW, respectively. The number of the statistical sub box is \(m\), and the width of which is \(dx=(max-min)/m\). The output sequence array of the box midpoint, the output array of the sub-box statistics and the computational auxiliary array are \(x[m]\), \(h[m]\) and \(hsum[m]\), respectively.

The elements in \(x[m]\) are
\[
x[i]=\min+0.5dx+i\times dx\quad (i=0, 1, 2, \ldots, m-1)
\]
The upper and lower limits of sub-box I are
\[
dx_i = (x_i - 0.5dx, x_i + 0.5dx)
\]
The function of the sub-box statistics is
\[
y_j(x) = \begin{cases} 1 & x \in dx_i \\ 0 & \text{others} \end{cases}
\]
The elements in \(h[m]\) can be obtained by
\[
h(i) = \sum_{j=0}^{n-1} y_j(key[j]) \quad (i=0, 1, 2, \ldots, m-1)
\]
\[
hsum[k] = \begin{cases} h[k] & (k = 0) \\ hsum[k-1] + h[k] & (k = 1, 2, \ldots, m-1) \end{cases}
\]
The index \(l\) and index \(h\) can be obtained by interpolating \(hsum[m]\) with integers of \(n\times0.02\) and \(n\times0.98\) as the cross-point thresholds. Using the integers of \(index_L+(index_H-index_L+1)\times0.4\) and \((index_H-index_L+1)\times0.2\) as the index and width respectively, and the subset of array \(h[m]\) can be obtained by subset function. The maximum value and the minimum value can be obtained by the functions of them respectively, and then the corresponding elements in \(x[m]\) can be obtained by using the minimum value+\(index_L+(index_H-index_L+1)\times0.4\) as index, which is the threshold \(V_e\). The flowchart of calculating \(V_e\) is shown in Figure 1.

5.2 Calculation of key phasor pulse time vector of speed profile

Both the calculation speed and the resampling of signals of the motor require precise key phasor pulse time vector \(^2\). The flowchart of the algorithm for obtaining the precise key phasor pulse time vector is shown in Figure 2.

The obtained key phasor time vector is the time value, \(t_i\), of the first sampling point larger than the threshold on the edge of each pulse, which is an integer multiple of the sampling period. Because the key phasor pulse edge is steep, so \(V_e-V_i\) is larger. To obtain an accurately key phasor time vector, the result of linear interpolation between each key phasor time vector \(t_i\) and \(t_{i+1}\) of its direct precursor point is used as the key phasor time vector.

The algorithm for obtaining the key phasor pulse time vector \(t_k\) when through the threshold using the linear interpolation method is
\[ t_k = \left( t_i + \frac{V_k - V_i}{V_{i+1} - V_i} \right) \times T_s \]

where \( T_s \) is the sampling period, \( t_i \) is the sampling time, \( V_k \) is the threshold, and \( V_i < V_k < V_{i+1} \).

After scanning all the data of the sampling sequence array of the key phasor pulse, the corresponding vector \( Y[m] \) in whole sampling period to the rising edge of each key phasor signal can be obtained. After interpolation, the relatively accurate key phasor time vector \( T[m] \) can be obtained, which can be can be used to calculate the speed profile and in the angular domain resampling.

```
Key[n]; y=0, i=0, j=0, k=0, m=1000, x[m], h[m], hsum[m], max, min

dx=(max-min)/m
x[i]=min+(0.5+i)dx; i++;

if i<m
    T
    i=0;
else
    F

if i<m
    T
    j<n
else
    F

key[j]>(x[i]-0.5dx) && key[j]<(x[i]+0.5dx)

if T
    y=y+1;
else
    y=y+0;

j++;

if k<m
    h[k]=y;
else
    F

k++;

Output threshold V_k
```

Figure 1 Flowchart of calculating \( V_k \)
5.3 Calculation of Speed in Speed Profile

The speed profile is a combination of the key phasor pulse time vector and the corresponding speed at the corresponding time. All of the maximum order calculation, later cascade analysis, trajectory analysis and bode diagram analysis require the speed profile, it is necessary to calculate the speed corresponding to the key phasor pulse time vector accurately[3].

According to the acquisition process of key phasor pulse, the arrival time of speed pulse is related to the angle of cumulative rotation. The angular interval between the arrival times of two adjacent pulses is the same, which is $2\pi/n$, where $n$ is the number of pulses per rotation.

According to the circular kinematics, the angular velocity is:

$$\omega(t_k) = \frac{d\theta}{dt} \bigg|_{t=t_k} = \frac{1}{\frac{d\theta}{d\theta_k}} \bigg|_{\theta=\theta_k} = \frac{1}{r_k}$$

The relationship between the angular velocity and rotational speed is:

$$r(t_k) = \frac{60}{2\pi} (t_k)$$

The angular interval $d\theta=2\pi/n$ corresponding to the key phasor time vector $[t_0, t_1, t_2, \ldots, t_{m-1}]$ is known, and $r'(\theta)$ can be calculated by Lagrange numerical differentiation method. According to the value of the time vector and the angular interval $d\theta$, the speed of the pulse at the corresponding time can be calculated. To improve the accuracy of calculation, the Lagrange 7-point interpolation is used to calculate the first order differential at the key phasor time vector$^[4]$

According to the value of $[t_0, t_1, t_2, \ldots, t_{m-1}]$ given by function $t=f(\theta)$ on the node, the interpolation polynomial $r=L_m(\theta)$ is established, and the value $L_m'(\theta)$ is taken as the approximate value of $f'(\theta)$.

The formula of the Lagrange 7-point interpolation is:

$$L_7(\theta) = \sum_{i=0}^{6} f(\theta_i) \prod_{j=0, j\neq i}^{6} \left( \frac{\theta-\theta_j}{\theta_i-\theta_j} \right)$$

The basis function is
\[ I_i(\theta) = \frac{1}{6} \sum_{j=0, j \neq i}^{6} \left( \frac{\theta - \theta_j}{\theta_i - \theta_j} \right) \quad (i=0,1,\ldots,6) \]

Assuming that
\[ \theta = \theta_0 + ih \quad (i=0, 1, \ldots, 6; \ h = 2\pi/n) \]

Take \( I_3(\theta) \) for example, and

\[ I_3(\theta) = \frac{h(i-1)(i-2)(i-4)(i-5)(i-6)}{36} \]

Thus, \( I_0(\theta), I_1(\theta), I_2(\theta), I_3(\theta), I_4(\theta), I_5(\theta) \) are the same.

Derivative of Eq.(3) to \( \theta \), there are

\[ i_{\theta} = \frac{1}{h} \]

\[ L_6(\theta) = \frac{\theta}{6h} \frac{d^2}{d\theta^2} \]

Similarly, \( l_0'(\theta), l_1'(\theta), l_2'(\theta), l_3'(\theta), l_4'(\theta), l_5'(\theta), l_6'(\theta) \) can be calculated respectively. Thus,

\[ L_6(\theta) = \sum_{i=0}^{6} l_i(\theta)f(\theta) \]  

Substituting \( i=0, 1, \ldots, 6, \) and \( t_0=f(\theta_0), \ldots, t_6=f(\theta_6) \) into Eq.(4), the derivative formula of Lagrange 7-point interpolation is as follows

\[
\begin{bmatrix}
I_0'(\theta_0) \\
I_1'(\theta_1) \\
I_2'(\theta_2) \\
I_3'(\theta_3) \\
I_4'(\theta_4) \\
I_5'(\theta_5) \\
I_6'(\theta_6)
\end{bmatrix} = \frac{1}{180h} \begin{bmatrix}
-441 & 1080 & -1350 & -1200 & -675 & 216 & -30 \\
-30 & -231 & 450 & 300 & 150 & -45 & 6 \\
6 & -72 & -105 & -240 & -90 & 24 & -3 \\
-3 & 27 & -135 & 0 & 135 & -27 & 3 \\
3 & -24 & 90 & 240 & 105 & 72 & -6 \\
-6 & 45 & -150 & -300 & -450 & 231 & 30 \\
30 & -216 & 675 & 1200 & 1350 & -1080 & 441
\end{bmatrix} \begin{bmatrix}
t_0 \\
t_1 \\
t_2 \\
t_3 \\
t_4 \\
t_5 \\
t_6
\end{bmatrix}
\]

\[ t'(\theta) \approx L_6'(\theta) \]

When \( 3 \leq i \leq N-3 \), the numerical differential formula of the time at the center point is:

\[ t'(\theta) \approx \frac{1}{180h} \begin{bmatrix}
-3 & 27 & -135 & 0 & 135 & -27 & 3 \\
3 & -216 & 675 & 1200 & 1350 & -1080 & 441
\end{bmatrix} \begin{bmatrix}
t_{i-2} \\
t_{i-1} \\
t_i \\
t_{i+1} \\
t_{i+2} \\
t_{i+3}
\end{bmatrix}
\]

To make both the starting point and the last point of the time vectors be a center of point 7, three points must be interpolated in front of \( t_0 \) and behind of \( t_{m-1} \), respectively, and the linear fitting method is adopted for interpolations.

6. Measurement of speed

6.1 Measurement method

The AKC-215 torque/speed sensor with a digital encode mounted on the rotating shaft manufactured by China academy of aerospace aerodynamics is used to measure the rotational speed, which transmits 60 pulses per revolution. The frequency converter is used to adjust the speed, and controlled by the external voltage input mode. The experimental rig is shown in Figure 3.
According to the design principle of NI acquisition, the different analog input channels in NI PCI-6221 are used for inputting the vibration signals in X and Y directions, 1 pulse photoelectric key phasor signal per revolution, 4 pulse photoelectric key phasor signal per revolution and the rotational speed signal in AKC-215, and then an analog input acquisition task was set, and the mode of continuous and differential is used to acquire the waveform.

An analog output task is used. The bipolar variable frequency motor is used, which controlled by the frequency converter. An output voltage in an analog output channel is as a regulating voltage of frequency converter. The input voltages of $v_l=1.18$ V and $v_h=9.87$ V corresponding to 500rpm and 3000rpm are pre-measured.

The analog voltage output task is used to input 1.18 as a DAQmx Write.vi date, so that the analog output voltage of NI PCI-6221 is 1.18 V, which can be as the external voltage of frequency converter to control the initial the rotating speed of the motor. When the speed of shaft is stable, 9.87 V is used as an input date of DAQmx Write.vi for the analog output task. The analog input and output tasks are started and executed in parallel, simultaneously. During the acquisition process, the speed measured by AKC-215 is monitored, and the measurement is stopped when it is bigger than 2900 rpm, which makes the acceleration process become natural. In the measurement process, the shift register is used to save the vibration signal, two key phasor pulse signals and speed signal waveforms from AKC-215, and the calculated speed profile data are recorded in the file with TDMS format in LabVIEW.

6.2 Analysis of results
The results of speed profile show that the pair of $\langle$speed, time vector$\rangle$ is 715 for 1 pulse per revolution, and the pair of $\langle$speed, time vector$\rangle$ is 2858 for 4 pulses per revolution. Theoretically, the number of pulse of 4 pulses per revolution should be 4 times+1 than that of 1 pulse, which is caused by the phasor difference at the beginning and the end of the period.

Considering that there are too many data in the speed profile in the whole speed-up stage, to display the pair $\langle$speed, time vector$\rangle$ of speed profile clearly, the data subsets of 8 cycles of 1 key phasor pulse per revolution, 4 key phasor pulse per revolution and AKC-215 of speed profile are displayed and output simultaneously.

The speed profiles of 1 key phasor pulse per revolution, 4 key phasor pulse per revolution and AKC-215 are shown in figure 4. It can be seen that the smoothness and linearity of the speed profile are improved with the increase of the number of pulses per revolution.
7. Conclusions
The N ribbons of uniform width with black and white were pasted along the circumference of the axis, and the photoelectric sensor was used to pick up the key phasor pulses. Compared with the digital encode, it is not only economical and feasible, simple installation, but also can obtain the accurate results. Compared with using eddy current sensor to obtain the key phasor pulse mounted in a slot, the rotation stability of the axis is not destroyed, and n key phasor pulse per rotation can be generated. There is no accumulative error for measuring speed profile by using n key phasor pulses per revolution, which improves the smoothness of speed profile, increases the physical key phasor time vector, and reduces the number of interpolation for later angular domain resampling.

Acknowledgments
This work is supported by the 5th “333” High-Level Talents Cultivation Project of Jiangsu Province (BRA2017482).

References
[1] Guo, Y., Qin, S.R., Bao, P., Ji, Y.B. 2003 Order tracking of rotating machinery based on instantaneous frequency estimation, J. Chinese Journal of Mechanical Engineering, 39(3), pp 32-35. (In Chinese)
[2] Li, Z.X., GUO, Y., Liu, W.B. 2007 Study on Obtaining Thresholds Precise Time Mark of Tacho Pulses, J. Journal of Kunming University of Science and Technology (Science and Technology), 32(6), pp12-16 (In Chinese)
[3] Huang S.P., Qin, S.R., Guo, Y. 2009 Calculating of rotating speed of rotating machinery and method of key phasor pulse processing, J.China Measurement & Test, 35(3), pp101-103 (In Chinese)
[4] Wang, Y. 2009 Extrapolation method of five-point numerical formulas for two-order derivative, J. Journal of Tianjin University of Technology, 25(4), pp37-39 (In Chinese)
[5] Cong, H., Wu, G.P., Rao, G.Q., Feng, F.Z. 2012 A method to avoid order aliasing in COT based on filtering, J. Journal of Vibration and Shock, 31(12), pp42-44 (In Chinese)
[6] Yang, T.Q., Zheng, H.Q., Gong, L.H., Tang L.W. 2011 Sampling rate setting criteria for computed order analysis, J. Chinese Journal of Construction Machinery, 9(1), pp14-18(In Chinese)