THE GALAXY-WEIGHTED SMALL-SCALE VELOCITY DISPERSION OF THE LAS CAMPANAS REDSHIFT SURVEY

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ABSTRACT

The pair-weighted relative velocity dispersion of galaxies provides a measure of the thermal energy of fluctuations of the observed galaxy distribution, but the measure is difficult to interpret and is very sensitive to the existence of rare rich clusters of galaxies. Several alternative statistical procedures have recently been suggested to relieve these problems. We apply a variant of the object-weighted statistical method of Davis, Miller, & White to the Las Campanas Redshift Survey (LCRS), which is the largest and deepest existing redshift survey that is nearly fully sampled. The derived one-dimensional dispersion on scales of \( \sim 1 \) h\(^{-1}\) Mpc is quite low: \( \sigma_1 = 126 \pm 10 \) km s\(^{-1}\), with a modest decrease at larger scales. The statistic is very stable; the six independent slices of the LCRS all yield consistent results. We apply the same statistical procedure to halos in numerical simulations of an open cosmological model and flat models with and without a cosmological constant. In contrast to the LCRS, all the models show a dispersion that increases for scales \( > 1 \) h\(^{-1}\) Mpc; it is uncertain whether this is a numerical artifact or a real physical effect. The standard cluster-normalized cold dark matter model with \( \Omega_m = 1 \), as well as a tilted variant with \( n = 0.8 \), yield dispersions substantially hotter than the LCRS value, while models with low matter density (\( \Omega_m = 0.3 \)) are broadly consistent with the LCRS data. Using a filtered cosmic energy equation, we measure \( \Omega_m \approx 0.2 \), with small-scale bias factors \( b = 1.0-1.5 \) for high-density models and \( b = 0.7-1.1 \) for low-density models.

Subject headings: dark matter — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

The small-scale thermal energy of the observed galaxy distribution is important for some cosmological models. For the past decade, the pair velocity dispersion, \( \sigma_{12}(r) \) (Davis & Peebles 1983) has been the usual measure of this quantity (e.g., Bean et al. 1983; de Lapparent, Geller, & Huchra 1988; Hale-Sutton et al. 1989; Mo, Jing, & Börner 1993; Zurek et al. 1994; Fisher et al. 1994; Marzke et al. 1995; Brainerd et al. 1996; Somerville, Primack, & Nolthenius 1997; Landy, Szalay, & Broadhurst 1998; Jing, Mo, & Börner 1998). But in spite of its widespread application and the relative ease of its measurement within large redshift surveys, the \( \sigma_{12}(r) \) statistic has a number of well-known deficiencies. Chief among these is its pairwise weighting, which gives extreme influence to rare, rich clusters of galaxies containing many close pairs with high velocity dispersions.

Alternative statistics for measuring the thermal energy distribution have been suggested by Kepner, Summers, & Strauss (1997) and Davis, Miller, & White (1997, hereafter DMW). The algorithm of Kepner et al. (1997) computes the pair-weighted dispersion as a function of the local galaxy density; this statistic demonstrates the heterogeneity of the environments of the local galaxy distribution, but it must be computed in volume-limited samples. The \( \sigma_1 \) statistic described by DMW can be estimated within a flux-limited catalog and is readily interpreted in terms of a filtered version of the cosmic energy equation. The statistic is a measure of the rms one-dimensional velocity of galaxies, with large-scale bulk-flow motions filtered out. DMW applied this statistic to the UGC catalog of optical galaxies within the Optical Redshift Survey (Santiago et al. 1995), as well as to the 1.2 Jy IRAS catalog (Fisher et al. 1995). They showed that \( \Omega_m = 1 \) simulations were far too hot to match the observed dispersion. Even when compared with simulations in which the small-scale kinetic energy had been artificially lowered by a factor of 4, the observed velocity distribution was colder than the simulated distribution.

However, the UGC catalog surveys a rather limited volume of the local universe, and the IRAS catalog is quite dilute and undersamples dense cluster regions. It is therefore of considerable interest to apply the DMW statistic to a larger, more representative redshift survey, such as the Las Campanas Redshift Survey (LCRS; Shectman et al. 1996), and to compare the results with \( N \)-body simulations of cosmological models, that are favored by current data. This paper reports the application of this new statistic to the LCRS and compares the result to a few simulations of flat and open cosmological models. In a future paper (Baker, Davis, & Ferreira, in preparation), we discuss a wider variety of models, and we discuss in more detail the comparison of the LCRS with \( N \)-body simulations and the potential applications of \( \sigma_1 \) as a cosmological probe.

2. APPLICATION OF \( \sigma_1 \) TO THE LCRS

The LCRS survey consists of 26,000 galaxies selected in a hybrid \( R \) band. The survey was conducted in six thin slices, each of size \( 1.5 \times 80^\circ \) on the sky, with median redshift \( cz = 30,000 \text{ km s}^{-1} \). The redshift accuracy of the observations is typically \( \sigma_{\text{err}} = 67 \text{ km s}^{-1} \) (Shectman et al. 1996).
which is sufficient for measuring the thermal, small-scale velocity dispersion.

For measurements of $\sigma_1$, we work with the subset of 19,306 LCRS galaxies in the redshift range $10,000 < cz < 45,000 \text{ km s}^{-1}$, with absolute magnitude $-22.5 < M < -18.5$. To estimate the random background of the neighbors about each galaxy, we used a catalog of 268,000 randomly distributed points with the same selection function as the LCRS galaxies. This random catalog includes sampling variations across the fields. Since the six slices of the LCRS are spatially separated by more than the projected separation used in the $\sigma_1$ statistic, the statistical procedure is applied to each slice individually, and the results are averaged.

2.1. Method

We now briefly describe our procedure, similar to that of DMW, for determining $\sigma_1$. For each galaxy $i$ in a slice of the survey, we lay down a cylinder centered on the galaxy in redshift space. Let $r_p$ be the projected radius of the cylinder and $v_i$ its half-length along the redshift direction. For neighboring galaxies $j$ within the cylinder, we construct the distribution $P(|\Delta v|)$, which counts the number of neighbors with redshift separation in a redshift bin centered at $\Delta v = v_j - v_i$. The counts accumulated in $P(|\Delta v|)$ are weighted by the inverse selection function $w_i$ (although equal weighting yields virtually identical results). We subtract from this distribution the background distribution, $B(|\Delta v|)$, which counts the number of weight neighbors expected for an unclustered galaxy distribution. We are interested in the width of the overall distribution, $D(|\Delta v|)$, constructed by an appropriately weighted sum over the $N_g$ galaxies:

$$D(|\Delta v|) = \frac{1}{N_g} \sum_{i=1}^{N_g} w_i [P(|\Delta v|) - B(|\Delta v|)] ,$$

where the weight for galaxy $i$ is denoted by $w_i$.

In order to make the statistic object-weighted rather than pair-weighted, we wish to normalize the distributions by the number of neighbors, $N_{\text{ex}}$, in excess of the random background; i.e.,

$$w_i^{-1} = N_{\text{ex},i} = \sum_{\Delta v} [P(|\Delta v|) - B(|\Delta v|)] .$$

This, however, presents a problem for galaxies that do not have enough neighbors to ensure that the sum is positive. DMW dealt with this problem by deleting these objects from consideration, but under half of the LCRS galaxies have at least one excess neighbor for $r_p = 1 \text{ h}^{-1} \text{ Mpc}$, and these galaxies are a biased sample because they populate overdense regions. It is therefore desirable to modify the statistic to include galaxies with fewer neighbors.

We achieve a more inclusive statistic by considering separately the distributions of high- and low-density objects; that is, only galaxies with $N_{\text{ex}} \geq 1$ are included in the sum for $D_{\text{hi}}$, while only galaxies with $N_{\text{ex}} < 1$ are included in the sum for $D_{\text{lo}}$. We then weight the galaxies in the combined distribution according to

$$w_i = \begin{cases} A_{\text{hi}} N_{\text{ex},i}^{-1} & N_{\text{ex},i} \geq 1, \\ A_{\text{lo}} & N_{\text{ex},i} < 1. \end{cases}$$

To obtain an object-weighted distribution, we wish to choose the normalization constants $A_{\text{hi}}$ and $A_{\text{lo}}$ so that the high- and low-density distributions are weighted in proportion to the number of objects included in each distribution. This will be the case if we normalize the distributions so that the area under the distributions is proportional to the number of objects.

To determine the appropriate areas, it is first necessary to subtract baselines measured at large $|\Delta v|$. In the case of the LCRS data, the baselines for the combined $D(|\Delta v|)$ of all galaxies are negligible, as we expect if the selection function of the random catalogs is a good estimate of the true selection function. However, galaxies in the high-density distributions, $D_{\text{hi}}$, tend to be drawn from overdense cylinders, leading to positive tails. For the same reason, the low-density distributions have negative tails. The $N$-body simulations also show negative tails because the periodicity of the box requires the correlation function to be significantly negative at large scales (however, we have verified that our results for the dispersion do not change if the box size is increased by a factor of 2).

Our limit $v_i$ is large enough that the tails of the distributions are quite flat, and we can reliably estimate the baselines, denoted $D(|\infty|)$, by averaging over the ranges $\pm (v_i, v_i - 500 \text{ km s}^{-1})$; the derived width $\sigma_1$ is insensitive to the size of these ranges. Note that the subtraction of the constant baselines does not affect the derived widths and is done only to obtain a sensible weighting for the high- and low-density distributions. We take

$$A_{\text{hi}} = \frac{N_{\text{hi}}}{N_g} \sum_{\Delta v} \frac{1}{D_{\text{hi}}/D_{\text{lo}}} ,$$

and similarly for $A_{\text{lo}}$. Here $N_{\text{hi}}$ and $N_{\text{lo}}$ are the number of galaxies with $N_{\text{ex}} \geq 1$ and $N_{\text{ex}} < 1$, respectively; thus, $N_{\text{hi}} + N_{\text{lo}} = N_g$. The denominator represents the area under the baseline-subtracted distribution, and the numerator maintains the desired object weighting in the combined distribution. Scaling $D_{\text{hi}}$ and $D_{\text{lo}}$ by the constants $A$ also does not affect the derived widths for these distributions; rather, it merely alters the weighting of the two in the combined distribution.

This procedure, in contrast to that of DMW, allows us to include all of the available data, yielding an unbiased, object-weighted measure of the thermal energy of the galaxy distribution. It is the object-weighting that differentiates our procedure from the more traditional measure of the pair dispersion, $\sigma_1(r)$; all galaxies (not pairs) are assigned equal weight in our statistic $\sigma_1$.

We measure the width of the distribution, $D(|\Delta v|)$, using the convolution procedure outlined by DMW (their eq. [18]), in which a velocity-broadening function, $f(v)$, is convolved with the two-point correlation function, $\xi(r)$, to produce a model $M(|\Delta v|) = \int_0^{r_p} dr 2\pi r d\pi \int_{-\infty}^{\infty} dy \xi(\sqrt{r^2 + y^2}) f(|\Delta v - y|)$.

The two-point correlation function of the LCRS is well approximated by $\xi(r) = (r/r_0)^\gamma$, where $r_0 = 5 \text{ h}^{-1} \text{ Mpc}$ and $\gamma = 1.8$ (Jing et al. 1998), while for the $N$-body simulations we use the cylindrically averaged mass correlation function, $\xi_c(|\Delta v|)$, measured directly from the particle distribution. We find that an exponential broadening function (see Diaferio & Geller 1996; Sheth 1996; Juszkiewicz, Fisher, & Szapudi 1998),

$$f(v) = \frac{1}{\sigma_1} \exp\left(-\frac{|v|}{\sigma_1}\right) ,$$
provides a much better fit to the LCRS data and all N-body models than does a Gaussian. Here we have defined the width \( \sigma_1 \) such that it is a measure of the rms velocity of individual galaxies in one dimension (with bulk motions on scales \( \gtrsim 1 \, h^{-1} \) Mpc filtered out). The (object-weighted) rms difference in velocity between any two galaxies is then \( \sigma_1 \sqrt{2} \) (DMW call this quantity, which is equal to the rms dispersion of the distribution, \( f_1 \) the “intrinsic” dispersion, \( \sigma_1 \); we will work exclusively with \( \sigma_1 \) to avoid confusion). The three-dimensional dispersions are larger by an additional factor of \( \sqrt{3} \).

We perform a nonlinear \( \chi^2 \)-minimization fit to determine the width, \( \sigma_1 \), and amplitude of the model, \( M(\Delta v) \). Before fitting, we convolve the model with a Gaussian of rms \( \sigma_{\text{err}} \sqrt{2} = 95 \, \text{km s}^{-1} \) to account for the LCRS redshift measurement uncertainties; the factor of \( \sqrt{2} \) converts from the measurement uncertainty for individual redshifts to the uncertainty for redshift differences, which are accumulated in \( D(\Delta v) \). We also include baseline terms in the model that are constant and linear in \( \Delta v \), for a total of four fit parameters. The linear term is necessary for the LCRS because for \( v_1 \) constant and linear in \( v_2 \); we expect if the exponential broadening function of \( v_2 \) degree to which sample variance affects the result. We also include baseline terms in the model that widths of the individual slices serves as a check on the widths. The second to last line of Table 1 gives the mean and standard deviation of the mean for separate fits to the six slices, while the last line gives the result of a single fit to the combined distribution of all galaxies. Note that the dispersion measured for objects with excess neighbors (\( N_{\text{ex}} \geq 1 \)) is clearly higher than that measured for objects with fewer neighbors. This behavior is expected because objects with more neighbors are found in regions of higher density, which tend to be hotter.

The fit to the LCRS \( D(\Delta v) \), shown in Figure 2, is quite good, with \( \chi^2 = 117/96 = 1.22 \); the probability of \( \chi^2 \) exceeding this value is \( 1 - P(\chi^2 | v) = 7\% \). The best-fitting Gaussian \( f(o) \) is much worse, with \( \chi^2 = 1.84 \) and \( 1 - P(\chi^2 | v) = 10^{-6} \).

Based on the mean of the six slices, we adopt \( \sigma_1 = 126 \pm 10 \, \text{km s}^{-1} \). This value has been computed for \( r_p = 1 \, h^{-1} \) Mpc and \( v_l = 2500 \, \text{km s}^{-1} \). The results are quite insensitive to cylinder length, ranging only from \( 117 \pm 14 \, \text{km s}^{-1} \) at \( v_l = 1500 \, \text{km s}^{-1} \) to \( 132 \pm 13 \, \text{km s}^{-1} \) at \( v_l = 3500 \, \text{km s}^{-1} \). Our chosen value of \( v_l = 2500 \, \text{km s}^{-1} \) is large enough to

![Fig. 1. —Galaxy-weighted velocity distribution, \( D(\Delta v) \), for the six LCRS slices.](image)

### Table 1

The \( \sigma_1 \) statistic for the six LCRS slices

| SLICE | DECL. (deg) | \( \text{All} \) (km s\(^{-1}\)) | \( N_{\text{ex}} \geq 1 \) (km s\(^{-1}\)) | \( N_{\text{ex}} < 1 \) (km s\(^{-1}\)) | \( N_{\text{ex}}/N_{\text{ex}} \) | \( N_{\text{ex}} \) |
|-------|------------|-------------------|----------------|----------------|----------------|----------------|
| 1     | \( -3 \)   | 96                | 184            | 53             | 0.44           | 3540           |
| 2     | \( -6 \)   | 103               | 255            | 73             | 0.34           | 2067           |
| 3     | \( -12 \)  | 163               | 273            | 129            | 0.44           | 3754           |
| 4     | \( -39 \)  | 117               | 181            | 94             | 0.41           | 3265           |
| 5     | \( -42 \)  | 136               | 178            | 112            | 0.44           | 3503           |
| 6     | \( -45 \)  | 142               | 171            | 131            | 0.43           | 3177           |
| Mean\( ^a \) | \( 126 \pm 10 \) | 207 \pm 18 | 99 \pm 13 | 0.42 | 3218 |
| 1-6° | \( 136 \) | 208               | 101            | 0.42           | 19306          |

\( ^a \) Here \( N_{\text{ex}}/N_{\text{ex}} \) is the fraction of galaxies with \( N_{\text{ex}} \geq 1 \), and \( N_{\text{ex}} \) is the total number of galaxies.

\( ^b \) Mean and standard deviation of the mean of the independent slices.

\( ^c \) Result of a fit to the entire data set.
allow a clean measure of the tails of the distribution without significant nonlinearities in the baseline gradient due to variations in the selection function.

A modest decrease in $\sigma_1$ is evident as $r_p$ increases above $r_p = 1 \, h^{-1} \, \text{Mpc}$ (see Table 2). Although the $D(\Delta v)$ distributions are very insensitive to $r_p$, the averaged correlation function, $\xi_v(\Delta v)$, becomes broader as $r_p$ increases. As a result, smaller values of $r_p$ provide a cleaner measure of the true (real-space) velocity broadening on small scales, but decreasing $r_p$ below $1 \, h^{-1} \, \text{Mpc}$ reduces the signal-to-noise ratio, since most galaxies have too few neighbors. The background subtraction also becomes cleaner as $r_p$ is reduced.

Note that for the larger value of $r_p = 2 \, h^{-1} \, \text{Mpc}$ used by DMW, our result is $\sigma_1 = 114 \, \pm \, 10$ km s$^{-1}$. If, as in the DMW analysis, we do not account for broadening due to redshift measurement errors, the result increases to $\sigma_1 = 136 \, \pm \, 9$ km s$^{-1}$. Since the two surveys have comparable redshift uncertainties, our LCRS result is perfectly consistent with the value $\sigma_1 = 130 \, \pm \, 15$ km s$^{-1}$ that DMW derived for the much smaller UGC catalog.

3. COMPARISON TO N-BODY MODELS

We have completed a suite of N-body simulations designed to predict the small-scale velocity dispersion in a variety of cosmological models. Here we discuss the results of a few of these models: the "standard" cold dark matter (SCDM) model and a tilted variant (TCDM), a model with a cosmological constant $\Lambda$ (ACDM), and an open model (OCDM). The cosmological parameters for these models are listed in Table 3. All models are approximately normalized to the present-day abundance of clusters; the ACDM and TCDM models also satisfy the COBE normalization. The SCDM model is known to fail a number of cosmological tests and is included for historical reasons, and only ACDM is fully consistent with current limits from high-redshift supernovae (Perlmutter et al. 1999). We note that on the scales relevant for our simulations, the TCDM power spectrum is indistinguishable from a $\tau$CDM spectrum with shape parameter $\Gamma = 0.2$. A broader range of models and a more detailed discussion of the simulations can be found in Baker et al. (in preparation).

Initial power spectra were obtained using the CMBFAST code (Seljak & Zaldarriaga 1996). The simulations were evolved on a $128^3$ mesh using a P$^4$M code (Brieu, Summers, & Ostriker 1995), in which short-range forces are computed using a special-purpose GRAPE-3AF board (Okumura et al. 1993). We chose a box of size $L = 50 \, h^{-1} \, \text{Mpc}$ to match the length of the LCRS cylinders; with $N_p = 64^3$ particles, this gives a mass resolution of $1.3 \times 10^{11} \Omega_m h^{-1} \, \text{M}_{\odot}$, where $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$. A Plummer force softening of $\epsilon = 50 \, h^{-1}$ kpc was used. The simulations were started at redshifts $z_i = 15$ (for $\Omega_m = 1$) or $z_i = 19$ (for $\Omega_m = 0.3$) and evolved to $z = 0$ in 1500 time steps, using $p = a$ as the integration variable.

The simulations are converted to "redshift" space by adding the velocities, $v_i$, along one of the three coordinates $i$ to the positions $x_i$: $x_i \rightarrow x_i + v_i/H$, where $H$ is the Hubble constant. Periodic boundary conditions are applied at the box edges. We then apply exactly the same statistical pro-

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TABLE 2

| $r_p$ (h$^{-1}$ Mpc) | $\sigma_1$ (km s$^{-1}$) | $N_h/N_s$ |
|----------------------|-------------------------|-----------|
| 0.5                  | 136 $\pm$ 10            | 0.23      |
| 1                    | 126 $\pm$ 10            | 0.42      |
| 1.5                  | 107 $\pm$ 8             | 0.55      |
| 2                    | 96 $\pm$ 12             | 0.63      |
| 2.5                  | 99 $\pm$ 13             | 0.68      |

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TABLE 3

| Model      | $\Omega_m$ | $\Omega_\Lambda$ | $n$ | $h$ | $\sigma_8$ |
|------------|------------|-------------------|-----|-----|------------|
| SCDM       | 1          | 0                 | 1   | 0.5 | 0.7        |
| TCDM       | 1          | 0                 | 0.8 | 0.7 | 0.7        |
| ACDM       | 0.3        | 0.7               | 1   | 1   | 1          |
| OCDM       | 0.3        | 0                 | 1   | 0.7 | 1          |

Note.—Table shows matter density, $\Omega_m$; cosmological constant, $\Omega_\Lambda$; tilt, $n$, where $P(k) \propto k^n$; Hubble constant, $H_0 = h/100$ km s$^{-1}$ Mpc$^{-1}$; and rms mass fluctuation $\sigma_8$ in spheres of radius 8 h$^{-1}$ Mpc.

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2 CMBFAST (U. Seljak & M. Zaldarriaga 1998) is available at: http://arcturus.mit.edu/~matias/CMBFAST/cmbfast.html.
procedure for determining $\sigma_1$ as for the LCRS, except that the selection function is now unity.

3.1. Tests of $\sigma_1$ Measurements

We have used our $N$-body simulations to perform a number of checks on the robustness of our method for determining the small-scale velocity dispersion. One test is to ask how well our model is able to account for the redshift measurement uncertainties in the LCRS. To simulate these uncertainties, we added Gaussian random velocities of rms $\sigma_{str}$ along the "redshift" coordinate in the simulations. We then make two determinations of $\sigma_1$, which should ideally be equal. In one determination, the random velocities have been added and we perform an extra Gaussian convolution in the model to account for them. In the other, no random velocities are added and no Gaussian convolution is necessary. We find that the two widths agree quite well, to within $10\%$ for $\sigma_1$ over the range of interest for $\sigma_1 (100-300 \, \text{km} \, \text{s}^{-1})$. The agreement improves as $\sigma_1$ increases and the uncertainties contribute relatively less to the width of the observed velocity distribution.

A second test of the method is to compare velocity widths measured in real space with those measured in cylinders in redshift space. For this test, we replace the velocities of the simulation particles with velocities drawn from a random exponential distribution of a given rms $\sigma$. It is straightforward to show that the velocity distribution appropriate for the difference distribution $D(\Delta v)$ is then

$$f(v) = \frac{1}{2\sigma^2} \left( |v| + \frac{\sigma}{\sqrt{2}} \right) \exp \left( -\sqrt{2} \frac{|v|}{\sigma} \right). \quad (7)$$

Using this form in the redshift-space model (eq. [5]), we find that our procedure recovers the true velocity dispersion with an accuracy better than $10\%$ for $\sigma_1$ in the range $100-300 \, \text{km} \, \text{s}^{-1}$.

We can also test the extent to which our measurement of $\sigma_1$ in the long redshift-space cylinders is contaminated by motions on scales larger than $1 \, h^{-1} \, \text{Mpc}$. First, we construct distributions analogous to $D(\Delta v)$, but measured in real space, with neighbors drawn from spheres of radius $1 \, h^{-1} \, \text{Mpc}$ in the simulations. These are compared to distributions with neighbors drawn from the long cylinders, also measured in real space. The widths of these distributions agree to within $1\%$, and we conclude that the contamination from large scales is negligible.

Finally, it is important to consider the extent to which $\sigma_1$ is affected by observational selection effects in the LCRS. One such effect is caused by fiber collisions; galaxies within $55''$ of a fiber location could not be observed. This effect tends to exclude galaxies in dense regions and would be expected to lower the measured velocity dispersion. Jing et al. (1998) found that neglecting fiber collisions caused the pair dispersion to be underestimated by about $3\%$. This is already comfortably smaller than our uncertainty in $\sigma_1$, but in the case of our object-weighted statistic, where galaxies are weighted equally regardless of density, we expect the fiber collisions to be considerably less important. In contrast, $\sigma_{12}$ weights pairs equally, and a missing galaxy in a high-density region represents the loss of many more pairs than a missing galaxy in a low-density region would.

3.2. Selection of Galaxies from the Mass Distribution

We can easily compute $\sigma_1$ for particles in the simulations, but the observed small-scale dispersion of galaxies, which correspond in some way to halos in the simulations, will in general differ from that of the mass. The internal velocity dispersions of galaxies are not included in the observed statistic; moreover, the galaxy population may be a biased tracer. In order to test whether our simulations can reproduce the LCRS result for $\sigma_1$, it is therefore important to identify "galaxies" within the $N$-body simulations. Unfortunately, the process of galaxy formation includes baryonic physics on a wide range of scales not probed by our dark-matter-only simulations. For the present work, we define galaxies using a simple phenomenological model, which we expect to yield results similar to those of larger gas-dynamical simulations.

We first apply the standard friends-of-friends (FOF; Davis et al. 1985) algorithm to the simulations, with a linking length of $0.2 \, \text{mesh cells}$ and a minimum group size of $N \geq 10$, corresponding to halos with mass $M \geq 10^{12} \, \text{M}_\odot \, h^{-1}$. We have also considered the HOP method (Eisenstein & Hut 1998) for defining halos, but we obtain similar results for reasonable parameter choices and do not discuss them here.

Our limited resolution and the nature of the FOF algorithm lead to a serious and well-known overmerging problem, in which a large cluster containing many galaxies will be identified as a single halo. This drastically lowers the small-scale velocity dispersion, because the motions of galaxies within clusters are neglected. To remedy this situation, we split halos with more than $N_1$ particles by randomly selecting particles from within the halos and identifying these particles as galaxies. Halos identified in this way will again include the internal motions of galaxies, but since the splitting is only applied to large, hot halos ($N_1 \gg 10$), we expect these internal motions to have a negligible effect on our result. Small halos with fewer than $N_1$ particles are taken to be individual galaxies.

For comparison with the LCRS, we choose a set of halos that resemble the LCRS galaxies as closely as possible. Some $N$-body models yield a correlation function, $\xi(r)$, that is too steep, and it is therefore advantageous to select halos that are antibiased on small scales (Jing et al. 1998). We accomplish this through our halo-splitting procedure by drawing random particles with a probability $p$, which has a power-law dependence on the number of particles, $N$, in the parent halo: $p(N) = N^{-a-1} \, N^{-2}$, with $a > 0$. The number of galaxies per unit halo mass then falls as $N^{-a}$ for large halos. We choose parameters $N_1$ and $a$ that simultaneously mimic the power-law shape of the LCRS correlation function and produce approximately the correct number density of galaxies, $n \approx 0.02 \, \text{h}^3 \, \text{Mpc}^{-3}$, implying 2500 galaxies per simulation volume. Increasing $a$ tends to flatten the correlation function on small scales and yields fewer halos; increasing $N_1$ at fixed $a$ tends to lower the correlation amplitude and also yields fewer halos. This behavior is illustrated in Figure 3 for the $\Lambda$CDM model.

Figure 4 shows the correlation functions for our selected halos in each of the models. In the low-density models, we are able to select halos that match the LCRS $\xi(r)$ quite well. The normalization of the high-density models is such that $\xi(r)$ always falls below the LCRS power law on large scales. The TCDM halos match well at $r \leq 2 \, h^{-1} \, \text{Mpc}$. In the SCDM model, we are unable to reproduce exactly the shape of $\xi(r)$ without falling too far below the LCRS amplitude and producing too few halos. However, the differences in $J_2$ (see § 3.4) computed from these correlation functions show
that this mismatch should affect our estimate of $\Omega_m$ by at most 30%.

3.3. Results for $\sigma_1$

The results for $\sigma_1$ for our four cosmological models are listed in Table 4. We see that the mass in the two $\Omega_m = 1$

models is far too hot on $1\,h^{-1}\text{Mpc}$ scales, with $\sigma_1$ well over twice the LCRS value. The spectral tilt of the TCDM model has very little effect on the small-scale velocities, since the result is nearly identical to the SCDM result. The mass in the low-$\Omega_m$ models, on the other hand, is also hotter than the LCRS, but only by a factor of about 1.5.

The halos in the simulations are somewhat cooler than the mass, with small-scale dispersions lower by factors in the range 0.7–0.9. The CDM halos come closest to the LCRS value; at 143 km s$^{-1}$, they are only marginally (1.7 $\sigma$) hotter than the LCRS. The open model produces velocity dispersions that are slightly higher than the CDM model, while the halos in the $\Omega_m = 1$ models are again much hotter than the LCRS data.

| Model   | $N_s$ | $\alpha$ | $N_{\text{halos}}$ | Mass (km s$^{-1}$) | Halos (km s$^{-1}$) |
|---------|-------|----------|---------------------|--------------------|---------------------|
| SCDM    | 80    | 0.00     | 2624                | 310                | 269                 |
| TCDM    | 40    | 0.25     | 2351                | 320                | 293                 |
| CDM     | 80    | 0.25     | 1901                | 188                | 143                 |
| OCDM    | 80    | 0.20     | 2128                | 197                | 158                 |

Note.—Results are listed for all particles and for the $N_{\text{halos}}$ halos identified using our splitting procedure with parameters $N_s$ and $\alpha$. 

![Figure 3](image.png)

Fig. 3.—Two-point correlation functions multiplied by $(r/r_0)_{\xi(r)}^r$ for $\Lambda$CDM simulation halos for a range of $N_s$ and $\alpha$. Here $r_0 = 5.1\,h^{-1}\text{Mpc}$, and the LCRS $\xi(r)$ appears as a solid horizontal line. In both plots, the dotted curve at bottom shows $N_s = \infty$ (no halo splitting), and the solid curve shows the mass correlation function. In the left plot, $N_s$ is held fixed at 80, while $\alpha$ takes on the values 0, 0.25, and 0.5 (top to bottom). In the right plot, $\alpha$ is held fixed at 0.25, while $N_s$ takes on the values 10, 20, 40, and 80 (top to bottom).

![Figure 4](image.png)

Fig. 4.—Two-point correlation functions for the mass (dotted lines), halos (dashed lines), and LCRS (solid lines), plotted as in Fig. 3. Halos were selected using the parameters listed in Table 4.
Figure 5 shows that the exponential $f(v)$ provides an excellent fit to the velocity distributions measured in the simulations in redshift space. We show distributions for the $N$-body mass particles and for the halos. The halo distributions are noisier because there are many fewer halos than mass particles in the simulation volumes. The distributions for the SCDM and OCDM models are nearly indistinguishable from the TCDM and $\Lambda$CDM distributions, respectively, and are not shown.

We have also computed $\sigma_1$ for galaxies drawn using more sophisticated semianalytical techniques from a large Virgo simulation (Benson et al. 2000) of the $\Lambda$CDM model. This simulation has a mass resolution better than ours by about a factor of 2, and the box length is nearly 3 times as large. The result is 126 km s$^{-1}$, only slightly lower than our value of 143 km s$^{-1}$. This suggests that our procedure for defining galaxies is reasonable. The Virgo result exactly matches the LCRS dispersion, which suggests that the small-scale velocity dispersion predicted by the $\Omega_m = 0.3$ flat model is in fact perfectly consistent with the observational data. Further details of this comparison will be presented in a future work (Baker et al., in preparation).

As noted in § 2.2, the LCRS velocity width decreases somewhat as the limiting radius $r_{p,\text{max}}$ is increased. In Figure 6, we show this scale dependence measured in independent cylindrical shells of width 1 $h^{-1}$ Mpc, where the limits on the radial integration in the model (eq. [5]) have
been adjusted appropriately. Although the measured LCRS $D(\Delta v)$ shows little scale dependence, the integrated correlation function broadens with scale, leading to a smaller measured velocity width.

None of the $N$-body models, however, are able to reproduce the scale dependence observed in the LCRS. The halos drawn from the Virgo simulation, which show very little scale dependence, come closest, while the other models tend to show an increase in velocity dispersion with scale. Only the $\Lambda$CDM model is shown in Figure 6, but we find similar discrepancies for the other models as well. Although the $\Omega_m = 0.3 \Lambda$CDM model produces a reasonable match to the velocity dispersion on very small scales, none of the models seem able to reproduce the observed coldness of the velocities on intermediate scales of $\sim 1-3$ $h^{-1}$ Mpc. At present, it is unclear whether this discrepancy is due to problems with the galaxy-selection procedure, the resolution of the simulations, or a more fundamental flaw in the cosmological models.

3.4. Filtered Cosmic Energy Equation

The $\sigma_1$ statistic is ideally suited to the application of the cosmic energy (Layzer-Irvine) equation filtered on small scales. As shown by DMW, we expect $\sigma_1^2 \propto \Omega_m J_{2,m}$ in the absence of velocity bias, where

$$J_2 = \int_{r_{\min}}^{r_{\max}} dr \xi(r) .$$

(8)

The subscript $m$ means that $J_2$ is computed from $\xi_m(r)$, the correlation function for the underlying mass. We can write this in terms of the measured $\xi(r)$ of an observed sample $j$ by defining an effective bias $b_j^2 = J_{2,j} / J_{2,m}$. If we then compare $\sigma_{1,j}$ measured for sample $j$ with $\sigma_{1,N}$ measured for the underlying mass in an $N$-body simulation with mass density parameter $\Omega_N$, we can measure the parameter

$$\frac{\Omega_m}{b_j^2} = \left( \frac{\sigma_{1,j}}{\sigma_{1,N}} \right)^2 \left( \frac{J_{2,N}}{J_{2,j}} \right) \Omega_N .$$

(9)

If in addition we can choose a sample of $N$-body halos that matches the correlation function of the sample $j$, then we have a direct measure of $\Omega_m$:

$$\Omega_m = \left( \frac{\sigma_{1,j}}{\sigma_{1,N}} \right)^2 \Omega_N ,$$

(10)

where $\sigma_{1,N}$ is now measured for the $N$-body halos rather than the underlying mass.

The results of combining the LCRS dispersion, $\sigma_1 = 126 \pm 10$ km s$^{-1}$, with our four cosmological $N$-body models are listed in Table 5. Based on the halos in each of the four simulations, we derive consistent values of $\Omega_m \approx 0.2$. Note that the errors listed for $\Omega_m$ are 1 $\sigma$ uncertainties derived solely from the LCRS $\sigma_1$ result; they do not include any systematic errors in the model results. The fact that we derive similar values of $\Omega_m$ from each of the different models is an important consistency check, and gives us confidence that our method is indeed a sensitive probe of the matter density.

Table 5 also lists the values of $\Omega_m/b^2$ derived by comparing the LCRS dispersion with the dispersion of the $N$-body mass. The integral $J_2$ converges rather slowly, and its value is quite sensitive to the integration limits $r_{\min}$ and $r_{\max}$. A reasonable lower limit is $r_{\min} = 0.1$ $h^{-1}$ Mpc, which eliminates from the analysis the internal velocity dispersion of typical galaxies and includes only the dispersion of galaxies moving relative to each other. We might also take $r_{\max}$ to be slightly larger than $1$ $h^{-1}$ Mpc, since the length of the redshift-space cylinders means that there will be some contribution to $\sigma_1$ from larger scales (although we have measured this effect in the simulations and have found that it is very small). The ranges shown for $\Omega_m/b^2$ were obtained by allowing $r_{\min}$ and $r_{\max}$ to vary over the ranges $0.05-0.2$ and $1-5$ $h^{-1}$ Mpc, respectively. Our results for the high-density models are consistent with the value $\Omega_m/b^2 = 0.14 \pm 0.05$ found by DMW, who only considered an $\Omega_m = 1$ model.

The parameter $\Omega_m/b^2$ is approximately equal to $\beta^2$, where $\beta \approx \Omega_m^{0.6}/b$ is the parameter measured by large-scale flow analyses. We find $\beta \approx 0.3-0.4$ for the two high-density models, and $\beta \approx 0.45-0.55$ for the two low-density models. These ranges are generally consistent with some large-scale flow determinations (e.g., Willick & Strauss 1998; Baker et al. 1998; Davis, Nusser, & Willick 1996), but not with the POTENT analyses, which prefer $\beta \approx 1$ (e.g., Sigad et al. 1998). Of course, the bias may in general depend on scale, in which case our small-scale result need not match the $\beta$ values measured using flows on much larger scales.

Finally, we can combine the values of $\Omega_m$ and $\Omega_m/b^2$ to obtain an estimate of the bias of the galaxy distribution on small scales. Our high-density models require biases $b = 1.0-1.5$, while the low-density models are slightly anti-biased, $b = 0.7-1.1$. These ranges are consistent with the biases measured directly from the correlation functions of the simulations.

3.5. Effects of Streaming Velocities

Although our goal is to measure the particle distribution function from redshift-space information alone, we must do this by considering the relative motions of pairs of galaxies, for which we expect mean streaming as well as thermal motions. As defined in equation (6), our model does not account for a nonzero first moment of the velocity distribution of pairs of galaxies. However, the first moment will, in general, be nonnegligible, because of the mean tendency of galaxies to approach each other, and it will contaminate a measurement of the second moment. On small scales in virialized clusters, for example, the infall velocity approximately cancels the Hubble expansion, so its presence can affect our measurements on $1 h^{-1}$ Mpc scales by an amount of order 100 km s$^{-1}$. Jing & Borner (1998) have shown that the effect of the streaming motions on the estimate of the pairwise velocity dispersion can be dramatic, increasing $\sigma_{12}$ from $\sim 400$ to $580$ km s$^{-1}$ at $1 h^{-1}$ Mpc separation.

The effects of the streaming motions can be incorporated into our analysis by writing the distribution function in equation (5) as

$$f(\nu) = \frac{1}{\sigma_1} \exp \left( -\frac{|\nu - \bar{\nu}_1|}{\sigma_1} \right) ,$$

(11)

where $\bar{\nu}_1$ is the mean object-weighted streaming velocity, which is a function of separation, and $\sigma_1'$ is the second moment of the streaming-corrected velocity distribution. The form of $\bar{\nu}_1$ is unknown but can be measured directly from $N$-body simulations.

Our estimate of $\sigma_1$ with $\bar{\nu}_1 = 0$ will be smaller than $\sigma_1'$ because streaming motions tend to cause objects to pile up
at small velocity separations in redshift space. However, $\sigma_1$ has the advantage that it is a model-independent statistic, relying only on the assumption of an exponential velocity distribution. The comparison of the data with $N$-body models is also consistent; to the extent that the models describe the real universe, the same streaming motions will be present in both the data and the models, and will affect the estimates of $\sigma_1$ similarly. Incorporating a nonzero $\overline{v}_1$ introduces model dependencies into the measurement, and there is no guarantee that the infall measured in the $N$-body simulations matches that of the real universe.

For the application of the cosmic energy equation, it is in fact more appropriate to use $\sigma_1$ rather than $\sigma_1'$, because contributions from both random thermal motions and mean streaming motions are already included. On the other hand, $\sigma_1'$ is a better measure of the truly thermal energy of the galaxy distribution. We can estimate it by using equation (11) with an appropriate model for $\overline{v}_1$. For the mean pairwise velocity, the simple form

$$\overline{v}_{12}(r) = \frac{F H_0 r}{1 + (r/r_0)^2}$$  \hspace{1cm} (12)

(Davis & Peebles 1983) is often used, where $F$ is a numerical factor, typically $F = 1 - 1.5$. Another expression has been proposed more recently by Juszkiewicz, Springel, & Durrer (1999):

$$\overline{v}_{12}(r) = -\frac{F H_0 r}{1 + (r/r_0)^2} \left[ 1 + \alpha \frac{\xi_1}{\xi_0} \right],$$  \hspace{1cm} (13)

where $f \approx \Omega_{\text{m},0.6}$ and $\alpha \approx 1.2 - 0.65 \gamma_0$, with $\gamma_0 \equiv -d \ln \xi/d \ln r_{\xi}=1$, and

$$[1 + \xi_1(r)]\frac{\xi_1}{\xi_0} = \frac{3}{F^2} \int_0^r dx \chi^2 \xi(x).$$  \hspace{1cm} (14)

These two forms for $\overline{v}_{12}(r)$ are nearly equal at small scales, $r \leq 10 \ h^{-1} \ Mpc$, if we set $F = 1.8 \Omega_{\text{m},0.6}$; note that $F = 1$ corresponds to streaming motions that just cancel the Hubble expansion on small scales.

Table 6 shows that the streaming correction has a substantial effect on the derived LCRS velocity width, with $\sigma_1'$ rising to $201 \pm 13 \ km \ s^{-1}$ for $F = 1$ and $261 \pm 15 \ km \ s^{-1}$ for $F = 1.8$. The $\chi^2$ statistic worsens somewhat for $F > 1$. The $N$-body models show similar behavior. We caution, however, that the streaming-corrected dispersions are model-dependent and are not an appropriate measure of the single-particle dispersion for use with the cosmic energy equation, which is defined in the comoving frame of the universe. This is in contrast to analyses of the pair dispersion, where it is appropriate to use the cosmic virial theorem, defined in the mean streaming frame.

### 4. CONCLUSIONS

Although the potential of small-scale cosmological velocities as a cosmological probe has long been recognized, the application of pair-weighted statistics is problematic. We apply an extended version of the more stable galaxy-weighted statistic of DMW to the Las Campanas Redshift Survey. We derive a one-dimensional rms velocity for individual galaxies relative to their neighbors of $\overline{v}_1 = 160 \pm 10 \ km \ s^{-1}$ on scales of $1 \ h^{-1} \ Mpc$.

Using this new statistic, we find that the observed velocities remain quite cold relative to the predictions of high-$\Omega_{\text{m}}$ $N$-body simulations. Tilting the power spectrum to reduce the initial power on small scales does little to resolve this discrepancy. We have also examined flat and open models with $\Omega_{\text{m}} \approx 0.3$; these models produce significantly lower dispersions than the high-density models. Combining the LCRS data with the predictions based on halos in the simulations, we measure consistent values of $\Omega_{\text{m}} \sim 0.2$ for all models, and we can rule out $\Omega_{\text{m}} = 1$ with a high degree of confidence. Our result suggests that the extremely cold dispersion measured in the vicinity of the Local Group (Schlegel, Davis, & Summers 1994; Governato et al. 1997) might be a local anomaly, since currently popular low-density models can reproduce the observed mean dispersion on $1 \ h^{-1} \ Mpc$ scales. On the other hand, at slightly larger separations, we find evidence that all of the models may again be too hot relative to the observations.

In future, it will be extremely useful to apply our statistic to upcoming redshift surveys, such as the Sloan Digital Sky and Two-Degree Field (2dF) surveys, which will contain enough galaxies to compute $\sigma_1$ precisely for different subsamples of the galaxy population. The Deep Extragalactic Probe (DEEP; Davis & Faber 1998) and other surveys at high redshift will also provide a measure of the evolution of $\sigma_1$, which can be used to place additional constraints on cosmological parameters and the bias of the galaxy distribution.

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### Table 5

| Model      | $\Omega_m$ | $\Omega_{\text{m}}/b^2$ | $b$ |
|------------|------------|--------------------------|-----|
| SCDM       | 0.22 ± 0.03| 0.10-0.14                | 1.2-1.5 |
| TCDM       | 0.18 ± 0.03| 0.12-0.15                | 1.0-1.3 |
| $\Lambda$CDM | 0.23 ± 0.04| 0.20-0.27                | 0.8-1.1 |
| OCDM       | 0.19 ± 0.03| 0.20-0.29                | 0.7-1.0 |

### Table 6

| $F$   | $\sigma_{1,\text{LCRS}}$ | $\chi^2$ | $\sigma_{1,\text{ACDM}}$ |
|-------|--------------------------|----------|--------------------------|
| 0.0    | 126 ± 10                 | 1.22     | 143                      |
| 0.5    | 162 ± 12                 | 1.20     | 195                      |
| 1.0    | 201 ± 13                 | 1.25     | 245                      |
| 1.5    | 239 ± 14                 | 1.35     | 292                      |

Note: Results are listed for the LCRS and for halos drawn from our $\Lambda$CDM simulation.
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