ANOMALOUS COUPLINGS
IN THE HIGGS-STRahlUNG PROCESS

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ABSTRACT

The angular distributions in the Higgs-strahlung process $e^+e^- \rightarrow HZ \rightarrow H\bar{f}f$ are uniquely determined in the Standard Model. We study how these predictions are modified if non-standard couplings are present in the $ZZH$ vertex, as well as lepton-boson contact terms. We restrict ourselves to the set of operators which are singlets under standard $SU_3 \times SU_2 \times U_1$ transformations, CP conserving, dimension 6, helicity conserving, and custodial $SU_2$ conserving.
1. The Higgs-strahlung process \[1\]

\[ e^+ e^- \rightarrow HZ \rightarrow H \bar{f} f \]  

(1)

together with the WW fusion process, are the most important mechanisms for the production of Higgs bosons in \( e^+ e^- \) collisions \[2,3\]. Since the \( ZZH \) vertex is uniquely determined in the Standard Model (SM), the production cross section of the Higgs-strahlung process, the angular distribution of the \( HZ \) final state as well as the fermion distribution in the \( Z \) decays can be predicted if the mass of the Higgs boson is fixed \[4\]. Deviations from the pointlike coupling can occur in models with non-pointlike character of the Higgs boson itself or through interactions beyond the SM at high energy scales. We need not specify the underlying theory but instead we will adopt the usual assumption that these effects can globally be parameterized by introducing a set of dimension-6 operators

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{\alpha_i}{\Lambda^2} O_i \]  

(2)

The coefficients are in general expected to be of the order \( 1/\Lambda^2 \), where \( \Lambda \) denotes the energy scale of the new interactions. However, if the underlying theory is weakly interacting, the \( \alpha_i \) can be significantly smaller than unity, in particular for loop-induced operators. [It is assumed \textit{a priori} that the ratio of the available c.m. energy to \( \Lambda \) is small enough for the expansion in powers of \( 1/\Lambda \) to be meaningful.]

If we restrict ourselves to operators \[5\] which are singlets under \( SU_3 \times SU_2 \times U_1 \) transformations of the SM gauge group, CP conserving, and conserving the custodial \( SU_2 \) symmetry, the following bosonic operators are relevant for the Higgs-strahlung process:

\[ O_{\partial \varphi} = \frac{1}{2} |\partial_\mu (\varphi^\dagger \varphi)|^2 \]  

(3)

\[ O_{\varphi W} = \frac{1}{2} \varphi^\dagger W^2_{\mu \nu} \varphi \]  

(4)

\[ O_{\varphi B} = \frac{1}{2} \varphi^\dagger B^2_{\mu \nu} \varphi \]  

(5)

where the gauge fields \( W^3, B \) are given by the \( Z, \gamma \) fields. This set of operators is particularly interesting because it does not affect, at tree level, observables which do not involve the Higgs particle explicitly. [It is understood that the fields and parameters are (re-)normalized in the Lagrangian \( \mathcal{L} \) in such a way that the particle masses and the electromagnetic coupling retain their physical values.]
In addition, we consider the following helicity-conserving fermionic operators which induce contact terms contributing to $e^+e^- \rightarrow ZH$:

\[ O_{L1} = (\varphi^\dagger iD_\mu \varphi)(\bar{\ell}_L \gamma^\mu \ell_L) + \text{h.c.} \] \hspace{1cm} (6)

\[ O_{L3} = (\varphi^\dagger \tau^a iD_\mu \varphi)(\bar{\ell}_L \tau^a \gamma^\mu \ell_L) + \text{h.c.} \] \hspace{1cm} (7)

\[ O_R = (\varphi^\dagger iD_\mu \varphi)(\bar{e}_R \gamma^\mu e_R) + \text{h.c.} \] \hspace{1cm} (8)

[$\ell_L$ and $e_R$ denote the left-handed lepton doublet and the right-handed singlet, respectively.]

The vacuum expectation value of the Higgs field is given by $\langle \varphi \rangle = (0, v/\sqrt{2})$ with $v = 246$ GeV, and the covariant derivative acts on the Higgs doublet as $D_\mu = \partial_\mu - ig/2\tau^a W^a_\mu + i\alpha'/2gB_\mu$. Helicity-violating fermionic operators do not interfere with the SM amplitude, so that their contribution to the cross section is suppressed by another power of $\Lambda^2$. The helicity-conserving fermionic operators modify the SM $Zee$ couplings and are therefore constrained by the measurements at LEP1; however, it is possible to improve on the existing limits by measuring the Higgs-strahlung process at a high-energy $e^+e^-$ collider since the impact on this process increases with energy $[6]$.  

$$
\begin{align*}
\text{Figure 1: Anomalous } ZZH/\gamma ZH \text{ couplings and } e^+e^- ZH \text{ contact terms in the Higgs-strahlung process.}
\end{align*}
$$

The effective $ZZH$ and the induced $\gamma ZH$ interactions (Fig.1, left diagram) may be written

\[ \mathcal{L}_{ZZH} = g_Z M_Z \left( \frac{1 + a_0}{2} Z_\mu Z^\mu H + \frac{a_1}{4} Z_{\mu\nu} Z^{\mu\nu} H \right) \] \hspace{1cm} (9)

\[ \mathcal{L}_{\gamma ZH} = g_Z M_Z b_1 Z_{\mu\nu} A^{\mu\nu} H \] \hspace{1cm} (10)

where $g_Z = M_Z \sqrt{4\sqrt{2}G_F}$. Additional operators $Z_\mu Z^{\mu\nu} \partial_\nu H$ and $Z_{\mu\nu} A^{\mu\nu} \partial_\nu H$ are redundant in this basis: They may be eliminated in favor of the other operators and the contact terms by applying the equations of motion. The remaining coefficients are given by

\[ a_0 = -\frac{1}{2} \alpha \varphi^2 v^2 / \Lambda^2 \] \hspace{1cm} (11)
\[
a_1 = 4g_Z^{-2} \left( c_W^2 \alpha_W + s_W^2 \alpha_B \right) / \Lambda^2 \tag{12}
\]
\[
b_1 = 4g_Z^{-2} c_W s_W (\alpha_W - \alpha_B) / \Lambda^2 \tag{13}
\]

where \( s_W \) and \( c_W \) are the sine and cosine of the weak mixing angle, respectively.

In the same way the \( e\bar{e}HZ \) contact interactions (Fig. 4, right diagram) can be defined for left/right-handed electrons and right/left-handed positrons

\[
\mathcal{L}_{eeZH} = g_Z M_Z \left[ c_L \bar{e}_L \bar{Z} e_L H + c_R \bar{e}_R \bar{Z} e_R H \right] \tag{14}
\]

with

\[
c_L = -2g_Z^{-1} (\alpha_{L1} + \alpha_{L3}) / \Lambda^2 \tag{15}
\]
\[
c_R = -2g_Z^{-1} \alpha_R / \Lambda^2 \tag{16}
\]

Some consequences of these operators for Higgs production in \( e^+e^- \) collisions have been investigated in the past. Most recently, the effect of novel \( ZZH \) vertex operators and \( \ell \bar{\ell}ZH \) contact terms on the total cross sections for Higgs production has been studied in Ref. [6]. The impact of vertex operators on angular distributions has been analyzed in Refs. [7] and [8]. We expand on these analyses by studying the angular distributions for the more general case where both novel vertex operators and contact interactions are present. The analysis of angular distributions in the Higgs-strahlung process (11) allows us to discriminate between various novel interactions. In fact, the entire set of parameters \( a_0, a_1, b_1 \) and \( c_L, c_R \) can be determined by measuring the polar and azimuthal angular distributions as a function of the beam energy if the electron/positron beams are unpolarized. As expected, the energy dependence of the polar angular distribution is sufficient to provide a complete set of measurements if longitudinally polarized electron beams are available\(^1\).

2. Total cross section and polar angular distribution. Denoting the polar angle between the \( Z \) boson and the \( e^+e^- \) beam axis by \( \theta \), the differential cross section for the process \( e^+e^- \rightarrow ZH \) may be written as

\[
\frac{d\sigma^{L,R}}{d \cos \theta} = \frac{G_F^2 M_Z^4}{96 \pi s} (v_e \pm a_e)^2 \lambda^{1/2} \frac{3 \lambda \sin^2 \theta \left( 1 + \alpha_{L,R}^{L,R} \right) + 6 \left( 1 + \beta_{L,R}^{L,R} \right) M_Z^2 / s}{(1 - M_Z^2 / s)^2} \tag{17}
\]

\(^1\)Since we can restrict ourselves to helicity-conserving couplings, as argued before, additional positron polarization need not be required.
and the integrated cross section
\[
\sigma = \frac{G_F^2 M_Z^4}{96 \pi s} (v_e \pm a_e)^2 \lambda^{1/2} \left[ 1 + \alpha^{L,R} \right] + 12 \left( 1 + \beta^{L,R} \right) \frac{M_Z^2 / s}{(1 - M_Z^2 / s)^2}
\] (18)

The integrated cross section is modified by the new interaction terms as follows. The coefficient \(a_0\) just renormalizes the SM cross section. By contrast, the contact interactions grow with \(s\). [The ratio \(s/\Lambda^2\) is assumed to be small enough for the restriction to dimension-6 operators to be meaningful.]

The operators \(O_{\varphi W}, \ O_{\varphi B}\) affect the coefficient in the cross section which is independent of \(\theta\). They damp the fall-off of this term, changing the \(1/s^2\) to a \(1/s\) behavior; however, these contributions remain subleading since they are associated
with transversely polarized Z bosons which are suppressed at high energies compared with the longitudinal components. To illustrate the size of the modifications $\alpha(s)^{L,R}$ and $\beta(s)^{L,R}$, we have depicted these functions in Fig.3(a) for the special choice $\alpha_i = 1$.

3. Azimuthal distributions. The azimuthal angle $\phi_*$ of the fermion $f$ is defined as the angle between the $[e^-,Z]$ production plane and the $[Z,f]$ decay plane (Fig.2). It corresponds to the azimuthal angle of $f$ in the $Z$ rest frame with respect to the $[e^-,Z]$ plane. On general grounds, the $\phi_*$ distribution must be a linear function of $\cos \phi_*$, $\cos 2\phi_*$, and $\sin \phi_*$, $\sin 2\phi_*$, measuring the helicity components of the decaying spin-1 $Z$ state. The coefficients of the sine terms vanish for CP invariant theories. The $\cos \phi_*$ and $\cos 2\phi_*$ terms correspond to P-odd and P-even combinations of the fermion currents. The general azimuthal distributions are quite involved [4,7,8]. We therefore restrict ourselves to the simplified case in which all polar angles are integrated out, i.e., the polar angle $\theta$ of the $Z$ boson in the laboratory frame and the polar angle $\theta_*$ of $f$ in the $Z$ rest frame. In this way we find for the azimuthal $\phi_*$ distribution:

$$\frac{d\sigma^{L,R}}{d\phi_*} \sim 1 \mp \frac{9\pi^2}{32} \frac{2v_f a_f}{v_f^2 + a_f^2} \frac{\gamma}{\gamma^2 + 2} \left(1 + f_1^{L,R}\right) \cos \phi_* + \frac{1}{2(\gamma^2 + 2)} \left(1 + f_2^{L,R}\right) \cos 2\phi_* \quad (21)$$

with

$$f_1(s)^{L,R} = M_Z \sqrt{s} \frac{(\gamma^2 - 1)(\gamma^2 - 2)}{\gamma(\gamma^2 + 2)} \left[a_1 + \frac{4s_{WCW}}{v_e \pm a_e} \left(1 - \frac{M_Z^2}{s}\right) b_1\right] \quad (22)$$

$$f_2(s)^{L,R} = 2M_Z \sqrt{s} \frac{\gamma(\gamma^2 - 1)}{\gamma^2 + 2} \left[a_1 + \frac{4s_{WCW}}{v_e \pm a_e} \left(1 - \frac{M_Z^2}{s}\right) b_1\right] \quad (23)$$

The cross section flattens with increasing c.m. energy in the Standard Model, i.e. the coefficients of $\cos \phi_*$ and $\cos 2\phi_*$ decrease asymptotically proportional to $1/\sqrt{s}$ and $1/s$, respectively. The anomalous contributions modify this behavior: The $\cos \phi_*$ term receives contributions which increase proportional to $\sqrt{s}$ with respect to the total cross section, while the $\cos 2\phi_*$ term receive contributions from the transversal couplings that approaches a constant value asymptotically. The size of the new terms in $f_{1,2}^{L,R}$ is shown in Fig.3(b) as a function of the energy. [The special choice $\alpha_i = 1$ we have adopted for illustration, implies $f_{1,2}^L = f_{1,2}^R$.]

4. It is instructive to study the high-energy behavior of the coefficients in the limit $M_Z^2 \ll s \ll \Lambda^2$. In this case we obtain the simplified relations:

$$\alpha(s)^{L,R} \simeq \mp s \cdot 8s_{WCW} c_{L,R} + O(v_e) \quad (24)$$
\[ \beta(s)^{L,R} \simeq \alpha(s)^{L,R} + s (a_1 \mp 4 s_W c_W b_1) + \mathcal{O}(v_e) \] (25)

and

\[
\begin{align*}
  f_1(s)^{L,R} & \simeq \frac{s}{2} (a_1 \mp 4 s_W c_W b_1) + \mathcal{O}(v_e) \quad (26) \\
  f_2(s)^{L,R} & \simeq s (a_1 \mp 4 s_W c_W b_1) + \mathcal{O}(v_e) \quad (27)
\end{align*}
\]

Terms which are proportional to \( v_e = -1 + 4 s_W^2 \) are suppressed by an order of magnitude. If longitudinally polarized electrons are available, the asymptotic value of the coefficients \( a_1, b_1, c_L \) and \( c_R \) can be determined by measuring the polar angular distribution without varying the beam energy. The analysis of the azimuthal \( \phi_\ast \) distribution provides two additional independent measurements of the coefficients \( a_1 \) and \( b_1 \). On the other hand, the set of measurements remains incomplete for fixed energy if only unpolarized electron/positron beams are used at high energies; in this case the coefficients cannot be disentangled completely without varying the beam energy within the preasymptotic region.

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Figure 3: Coefficients of the angular distributions as a function of the beam energy. Parameters are described in the text; in particular, $\alpha_i = 1$ has been chosen in the effective Lagrangian Eq. (3). [The $L, R$ coefficients of the azimuthal distribution coincide for the special choice $\alpha_i = 1$.]