IMPROVING INTELLIGENCE OF EVOLUTIONARY ALGORITHMS USING EXPERIENCE SHARE AND REPLAY

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ABSTRACT

We propose PESA, a novel approach combining Particle Swarm Optimisation (PSO), Evolution Strategy (ES), and Simulated Annealing (SA) in a hybrid Algorithm, inspired from reinforcement learning. PESA hybridizes the three algorithms by storing their solutions in a shared replay memory. Next, PESA applies prioritized replay to redistribute data between the three algorithms in frequent form based on their fitness and priority values, which significantly enhances sample diversity and algorithm exploration. Additionally, greedy replay is used implicitly within SA to improve PESA exploitation close to the end of evolution. The validation against 12 high-dimensional continuous benchmark functions shows superior performance by PESA against standalone ES, PSO, and SA, under similar initial starting points, hyperparameters, and number of generations. PESA shows much better exploration behaviour, faster convergence, and ability to find the global optima compared to its standalone counterparts. Given the promising performance, PESA can offer an efficient optimisation option, especially after it goes through additional multiprocessing improvements to handle complex and expensive fitness functions.

Keywords Evolutionary Computation · Continuous Optimisation · Simulated Annealing · Experience Replay · Hybrid Algorithms

1 Introduction

Solving optimisation problems is at the heart of every scientific discipline to improve our understanding and interpretation of scientific findings. Evolution-based and swarm intelligence search and optimisation have been in remarkable growth over the years to tackle complex problems ranging from scientific research [1] to industry [2] and commerce [3]. Hybridizing evolutionary, swarm, and annealing algorithms [4] (the focus of this work) is an active area of research, since usually hybrid algorithms can offer several advantages over standalone algorithms in terms of stability, search speed, and exploration capabilities, where these advantages highlight the goals of these hybrid studies. Few examples of successful demonstration of hybrid algorithms in different domains are: (1) genetic algorithm (GA) and particle swarm optimisation (PSO) [5], (2) GA and Simulated Annealing (SA) [6], (3) GA and SA [7], (4) PSO and tabu search [8], (5) PSO and evolution strategies (ES) [9], and many more. For the authors’ interests on nuclear power plant design, evolutionary, swarm, and annealing algorithms have been proposed in several research studies in standalone and hybrid forms to optimise nuclear fuel assemblies and cores of light water reactors [10, 11, 12], to reduce fuel costs and improve nuclear reactor safety [13].

As improving capabilities of evolutionary and stochastic algorithms in solving optimisation problems is always a research target due to their widely usage, we propose a new algorithm called PESA (PSO, ES, and SA Algorithm). PESA hybridizes three known optimisation techniques by exchanging their search data on-the-fly, storing all their solutions in a buffer, and replaying them frequently based on their importance. The concept of experience replay

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was first introduced in deep reinforcement learning (RL) [14, 15] to improve agent learning by replaying relevant state/action pairs weighted by their temporal difference and reward values. Accordingly, we introduce experience replay into evolutionary algorithms to determine whether this concept would improve the performance. First, we hybridize PSO, ES, and SA to create a replay memory with diverse samples by taking the search advantages of each individual algorithm. Next, two modes of experience replay are applied: (1) prioritized replay to enhance exploration capabilities of PESA, and (2) “backdoor” greedy replay to improve algorithm exploitation, such that once the search is close to end, PESA prioritizes its main experience. To enhance search speed, PSO, ES, and SA are parallelized during search such that the three algorithms can run simultaneously to collect experiences, before updating the memory and executing the replay. We benchmark PESA against its standalone components: PSO, ES, and SA to show its promise. Variety of commonly used continuous benchmark functions are utilized in this paper, which also have a high dimensional nature to represent realistic optimisation problems. We evaluate PESA performance by its ability to find a known global optima, exploration/exploitation capabilities, and computational efficiency.

2 Methodology

The workflow of PESA is sketched in Figure 1, which shows how the algorithms are connected to each other through the replay memory. The choices of PSO, ES, and SA are not arbitrary. PSO [16] is known to excel in continuous optimisation, ES [17] (which originated from the genetic algorithms) performs well in both continuous/discrete spaces, while SA [18] is introduced to ensure exploration due to its stochastic nature, and more importantly to drive PESA exploitation to the best solution found, as will be described in the next section.

![Diagram of PESA algorithm workflow](image)

Figure 1: PESA algorithm workflow. Red arrows represent memory feedback, $\alpha_{\text{backdoor}}$ is the probability to use backdoor greedy replay in SA. $\eta$ and $\eta'$ are the number of PSO particles surviving from previous generation and from the memory, respectively. $\mu$ and $\mu'$ are the number of ES individuals surviving from previous generation and from the memory, respectively. $\lambda$ is the full size of ES offspring. All hyperparameters are defined in detail later in this section.

### 2.1 Evolutionary, Swarm, and Annealing Computation

Evolution strategies (ES) [17] are inspired by the theory of natural selection. In this work, the well-known ($\mu$, $\lambda$) strategy is used, which is indeed similar to the continuous GA in terms of the operators (e.g., crossover, mutation, and reproduction). However, the mutation strength of each gene/attribute in the individual ($\vec{x}$) is not constant, but learned during the evolution. Accordingly, each individual in the population carries a strategy vector of same size as ($\vec{x}$), where the strategy vector is adapted during the evolution, and controls the mutation strength of each attribute. We adopt log-normal adaptation for the strategy vector (see [17]), where the min and max strategies are bounded between $1/n$ and $n$. 

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Two modes of experience replay are relevant to this work: (1) greedy and (2) prioritized replay. Greedy replay sorts the memory from min to max (assuming a minimization problem), and replays the sample(s) with lowest fitness every time. Intuitively, replaying always the best samples could lead to improvement early on, but will restrict the diversity of the search, and likely to end with premature convergence (i.e., methods converge to a local optima). Therefore, greedy replay in PESA is used in a special form with SA described later. Prioritized replay balances between uniform and fully prioritized replay. Notice here we describe \( \alpha = 1 \) as “fully prioritized” replay rather than greedy replay (i.e., min operator). When \( \alpha = 1 \), the best samples (lowest fitness) are “more likely” to be replayed due to their larger weight, but in greedy replay these quality samples are “100%” replayed (because of applying the \( \min \) operator). Next, \( p_i \) refers to the priority of sample \( i \), which can take different forms. For our case, we choose a prioritization based on sample rank:

\[
p_i = \frac{1}{\text{rank}(i)}
\]
where \( \text{rank}(i) \) is the sample rank based on fitness value when the memory is sorted from minimum fitness to maximum. After sorting, sample 1 has \( p_1 = 1 \), sample 2 has \( p_2 = 0.5 \), and so on. According to [15], rank-based prioritization is robust since it is insensitive to outliers. Now, to balance between exploration/exploitation in prioritized replay, we start PESA evolution with small value of \( \alpha = \alpha_{\text{init}} = 0.01 \) (more exploration), and then gradually increasing until \( \alpha = \alpha_{\text{end}} = 1.0 \) (more exploitation) by the end of evolution. After a generation \( k \), priorities are calculated by Eq.(6), sampling probability is determined by Eq.(5), and replay is performed by sampling from a non-uniform distribution with probabilities \( \{P_1, P_2, ..., P_D\} \). Notice that redundant samples (if any) are removed from the memory before re-sampling to avoid biasing the replay toward certain samples.

**Algorithm 1** Evolution Strategy \((\mu + \mu', \lambda)\) in PESA

1: 1: Given ES hyperparameters: \( \mu, \mu', \lambda, \text{MUT}, CX \)
2: for a generation \( k \) do
3: 3: Apply crossover operator to the population \((\mu + \mu')\) with probability \( CX \)
4: 4: Apply mutation operator to the population \((\mu + \mu')\) with probability \( \text{MUT} \)
5: 5: if \( \text{MUT} + CX < 1 \) then
6: 6: 6: Apply reproduction to the population \((\mu + \mu')\) with probability \( 1 - \text{MUT} - CX \)
7: 7: Generate final offspring \( \lambda \) from the population \((\mu + \mu')\)
8: 8: Evaluate fitness of the offspring
9: 9: Select \( \mu \) individuals with best fitness for next generation
10: 10: Return the selected individuals \( \mu = \{(x_1, y_1), (x_2, y_2), ..., (x_\mu, y_\mu)\} \)

As in Figure 1, prioritized replay is used with all three algorithms at the beginning of each generation. For ES, the replay memory provides \( \mu' \) individuals at the beginning of every generation, which mix with original \( \mu \) individuals (from previous generation) using crossover, mutation, and reproduction operations (see Figure 1). Therefore, the \((\mu, \lambda)\) algorithm becomes \((\mu + \mu', \lambda)\) for PESA, as \( \mu' \) individuals are mixed with \( \mu \) before forming the next \( \lambda \) offspring. The ES algorithm in PESA is given in Algorithm 1.

The prioritized replay for PSO is similar to ES, where the swarm particles at every generation constitute from \( \eta \) particles of the previous generation and \( \eta' \) particles from the memory, before going through velocity and position updates as described in section 2.1. In both cases (ES, PSO), the values of \( \mu' \) and \( \eta' \) can be tuned for better performance. The PSO algorithm in PESA is given in Algorithm 2.

**Algorithm 2** Particle Swarm Optimisation \((\eta + \eta')\) in PESA

1: 1: Given PSO hyperparameters: \( \eta, \eta', c_1, c_2 \)
2: for a generation \( k \) do
3: 3: Update velocity of swarm particles \( (\eta + \eta') \) with constriction coefficient
4: 4: Generate new swarm by updating all particle positions \( (\eta + \eta') \)
5: 5: Evaluate fitness of the swarm
6: 6: Select \( \eta \) particles with best fitness for next generation
7: 7: Return the selected particles \( \eta = \{(x_1, y_1), (x_2, y_2), ..., (x_\eta, y_\eta)\} \)

For SA, prioritized replay is used to make an initial guess for the chain before starting a new annealing generation. Given SA is running in series in this work, a single chain is used. Once the generation is done, SA updates the memory by the last pair \((\bar{x}_{\text{last}}, y_{\text{last}})\) and best pair \((\bar{x}_{\text{best}}, y_{\text{best}})\) observed by the chain in that generation. The SA algorithm in PESA is given in Algorithm 3.

The backdoor greedy replay for SA in Figure 1 and Algorithm 3 has two main benefits: (1) ensuring PESA exploitation at the end of evolution and (2) providing more guidance to SA that relies extensively on random-walk. Unlike prioritized replays that occur explicitly at the beginning of every generation of all algorithms including SA, backdoor replay obtained this name since it occurs implicitly during SA generation, by giving the SA chain a choice between its regular random-walk or the best quality sample in the memory with probability \( \text{rand} \sim U[0, 1] < \alpha_{\text{backdoor}} \). Since SA tends to accept low quality solutions early on, but tightens the acceptance criteria at the end by rejecting low quality solutions, this means SA will implicitly drive PESA to always converge to the best solution in the memory once the evolution is close to end. Second, as SA lacks the learning capabilities of PSO and ES (velocity update, crossover, etc.), the backdoor replay will frequently correct the SA chain to focus the search in relevant space regions as found by her sisters (PSO, ES) so far during the evolution. We typically recommend small value for \( \alpha_{\text{backdoor}} \) (i.e., \(< 0.15\)), since obviously large values lead to repetitive greedy replays, which in turn cause a bias in the chain, eventually leading to premature convergence in SA.
Algorithm 3 Simulated Annealing in PESA

```plaintext
1: • Given SA hyperparameters: $T_{\text{max}}, T_{\text{min}}, \alpha_{\text{backdoor}}$
2: • Draw initial state from the memory, $\theta' = (\tilde{x}_0, y_0)$
3: • Set $\tilde{x}_{\text{prev}} \leftarrow \tilde{x}_0$, $E_{\text{prev}} \leftarrow y_0$
4: for a generation $k$ do
5:   if $\text{rand} \sim U[0,1] < \alpha_{\text{backdoor}}$ then
6:     • Draw the best sample from the memory $(\tilde{x}', y')$
7:     • Set $\tilde{x} \leftarrow \tilde{x}'$, $E \leftarrow y'$
8:   else
9:     • Perform a random walk as next chain state $(\tilde{x})$
10:    • Evaluate fitness $E$ for the new state
11:   if $\Delta E = E - E_{\text{prev}}$ then
12:     if $\Delta E < 0 \text{ OR } \exp(-\Delta E / T) > \text{rand} \sim U[0,1]$ then
13:         • Accept the candidate
14:     else
15:         • Set $E_{\text{prev}} \leftarrow E$, $\tilde{x}_{\text{prev}} \leftarrow \tilde{x}$
16:     • Reject the candidate and restore previous state
17:     • Set $E_{\text{prev}} \leftarrow E_{\text{prev}}$, $x_{\text{prev}} \leftarrow x_{\text{prev}}$
18:     • Anneal $T$ between $T_{\text{max}}$ and $T_{\text{min}}$
19: • Return the last chain state $(\tilde{x}^{\text{last}}, y^{\text{last}})$ and best state $(\tilde{x}^{\text{best}}, y^{\text{best}})$
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2.3 PESA Algorithm

By combining all parts presented in sections 2.1-2.2, PESA algorithm can be constructed as given in Algorithm 4. The flow of PESA can be summarised in three main phases:

1. Warmup (lines 1-4 in Algorithm 4): Hyperparameters of all individual algorithms (ES, PSO, SA) are specified. The memory is initialized with a few warmup samples ($N_{\text{warmup}}$) and maximum capacity ($D_{\text{max}}$).

2. Evolution (lines 8-17 in Algorithm 4): The three algorithms, ES, PSO, and SA are executed in parallel according to Algorithm 1, Algorithm 2, and Algorithm 3, respectively. Each individual algorithm runs its iterations in serial.

3. Memory management (lines 6-7, 18-20 in Algorithm 4): This phase involves updating the memory with new samples, calculating and updating sample priority, annealing the prioritization coefficient ($\alpha$), and cleaning the memory from duplicates.

Algorithm 4 PESA Algorithm with Prioritized Replay

```plaintext
1: • Set hyperparameters of ES, SA, PSO
2: • Set replay parameters: $\alpha_{\text{backdoor}}, \alpha_{\text{init}}, \alpha_{\text{end}}$
3: • Construct the replay memory with size $D_{\text{max}}$ and initialize with warmup samples $N_{\text{warmup}}$
4: • Set $\alpha \leftarrow \alpha_{\text{init}}$
5: for GEN $i = 1$ to $N_{\text{gen}}$ do
6:   • Calculate $p_i = 1/\text{rank}(i)$ and priorities $P(i) = p_i^\alpha / \sum_D p_i^\alpha$
7:   • With probabilities $P(i)$, draw samples $\mu', \eta'$, and $\theta'$
8: for Parallel Process 1: ES do
9:   • Given $\mu' = \{(\tilde{x}_1, y_1), ..., (\tilde{x}_\mu, y_\mu)\}$, run ES Algorithm 1
10: • Obtain ES population $\mu = \{(\tilde{x}_1, y_1), (\tilde{x}_2, y_2), ..., (\tilde{x}_\mu, y_\mu)\}$
11: for Parallel Process 2: PSO do
12: • Given $\eta' = \{(\tilde{x}_1, y_1), \tilde{x}_\eta, y_\eta)\}$, run PSO Algorithm 2
13: • Obtain PSO population $\eta = \{(\tilde{x}_1, y_1), (\tilde{x}_2, y_2), ..., (\tilde{x}_\eta, y_\eta)\}$
14: for Parallel Process 3: SA do
15: • Given $\theta' = (\tilde{x}', y')$, run SA Algorithm 3
16: • Obtain SA population $\theta = (\tilde{x}^{\text{last}}, y^{\text{last}})$
17: • Obtain SA best population $\theta^{\text{best}} = (\tilde{x}^{\text{best}}, y^{\text{best}})$
18: • Update the memory with samples $\mu, \eta, \theta$, and $\theta^{\text{best}}$
19: • Remove duplicated samples from the memory
20: • Anneal $\alpha$ between $\alpha_{\text{init}}, \alpha_{\text{end}}$
```

As can be noticed from Algorithm 4, algorithm-based parallelism can be observed in PESA (see steps 8, 11, and 14 in Algorithm 4), which involves running the three algorithms simultaneously. This feature exists in the algorithm, but not
activated in this work, since the functions investigated are too fast-to-evaluate. Internal parallelization of each algorithm is left for future, since again the benchmark functions tested in this work are cheap-to-evaluate, so multiprocessors of each algorithm is also not advantageous.

With an effort to minimize the number of hyperparameters in PESA, we assume similar size of generation across algorithms. For example, lets assume a generation of 60 individuals assigned for each algorithm. In this case, the ES population has size $\lambda = 60$, PSO swarm has 60 particles, SA chain has size of $C_{\text{size}} = 60$ steps. Although previous assumptions do not necessarily guarantee the most optimal performance, the authors believe that such symmetry in PESA has two main advantages. First, the burden of hyperparameter optimisation is significantly reduced. Second, algorithm dynamics is improved as internal algorithms will finish their generation roughly same time, which allows them to stay up to date with the memory. This is clearly under the assumption that fitness evaluation cost $(y)$ is roughly the same regardless of the input value $(x)$.

## 3 Numerical Tests

The mathematical forms of selected benchmark functions are given in Table 1, which are all well-known optimisation benchmarks [20]. All functions have $n$-dimensions nature, where here we select $n = 50$ to represent a more high-dimensional problem. All functions have a known global minima at $f(\vec{x}) = 0$, except for the Ridge and Exponential functions, which have their global minima at -5 and -1, respectively.

| Name     | Formula                                                                 | $\vec{x}$ Range   | Global Optima     |
|----------|--------------------------------------------------------------------------|-------------------|-------------------|
| (1) Cigar | $f(\vec{x}) = x_0^2 + 10n \sum_{i=1}^{n} x_i^2$                       | $[-10, 10]^n$     | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (2) Sphere | $f(\vec{x}) = \sum_{i=1}^{n} x_i^2$                                           | $[-100, 100]^n$    | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (3) Ridge | $f(\vec{x}) = x_0 + (\sum_{i=1}^{n} x_i^2)^{0.5}$                      | $[-5, 5]^n$       | $\vec{x}^* = [-5, 0, ..., 0]$, $f(\vec{x}^*) = -5$ |
| (4) Ackley | $f(\vec{x}) = 20 - 20 \exp \left(-0.2 \sqrt{\frac{1}{\sum_{i=1}^{n} x_i^2}}\right) - \exp \left(\frac{1}{\sum_{i=1}^{n} \cos(2\pi x_i)\right) + e$ | $[-32, 32]^n$     | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (5) Bohachevsky | $f(\vec{x}) = \sum_{i=1}^{n} \left(x_i^2 + 2x_i^4 - 0.3 \cos(3\pi x_i) - 0.4 \cos(4\pi x_i) + 0.7\right)$ | $[-100, 100]^n$   | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (6) Griewank | $f(\vec{x}) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \left(\frac{x_i}{\sqrt{i}}\right) + 1$ | $[-600, 600]^n$   | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (7) Brown | $f(\vec{x}) = \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} (x_i^2)^{1/\alpha}$ | $[-1, 4]^n$        | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (8) Exponential | $f(\vec{x}) = -\exp(-0.5 \sum_{i=1}^{n} x_i^2)$               | $[-1, 1]^n$       | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = -1$ |
| (9) Zakharov | $f(\vec{x}) = \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} (0.5x_i)^2 \sum_{i=1}^{n} (x_i)^4$ | $[-5, 10]^n$      | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (10) Salomon | $f(\vec{x}) = 1 - \cos(2\pi \sqrt{\sum_{i=1}^{n} x_i^2}) + \frac{1}{4} \sum_{i=1}^{n} x_i^4$ | $[-100, 100]^n$   | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0$ |
| (11) Quartic | $f(\vec{x}) = \sum_{i=1}^{n} x_i^4 + \text{random}(0, 1)$ | $[-1.28, 1.28]^n$ | $\vec{x}^* = \vec{0}$, $f(\vec{x}^*) = 0 + \text{noise}$ |
| (12) Levy | $f(\vec{x}) = \sin^2(\pi w_1) + \sum_{i=1}^{n-1} \left[(w_i - 1)^2 + 10 \sin^2(\pi w_1, 1) + \right.$ | $[-10, 10]^n$     | $\vec{x}^* = \vec{1}$, $f(\vec{x}^*) = 0$ |
|           | $\left. + (w_n - 1)^2 + 1 + \sin^2(2\pi w_n)\right)$, $w_1 = 1 + (x, -1)/4$ |                  |                  |

The hyperparameters selected to perform the tests are as follows: For ES ($CX = 0.6$, $MUT = 0.15$, $\lambda = 60$, $\mu = 30$, $\mu' = 30$), for PSO ($c_1 = 2.05$, $c_2 = 2.05$, $\eta = 30$, $\eta' = 30$), for SA ($T_{\text{max}} = 10000$, $T_{\text{min}} = 1$, $\chi = 0.1$, $C_{\text{size}} = 60$), while for replay parameters ($\alpha_{\text{init}} = 0.01$, $\alpha_{\text{end}} = 1.0$, $\alpha_{\text{backdoor}} = 0.1$). The tests are executed with $N_{\text{gen}} = 100$ generations and $N_{\text{warmup}} = 500$ samples. The previous hyperparameters yielded satisfactory performance for all test functions. It is very important highlighting that the hyperparameters, initial starting points, and number of generations are preserved between PESA and the standalone algorithms to isolate their effects. This means that any difference in performance comes purely from the “experience share and replay”, the contribution of this work.

The convergence of fitness results is plotted in Figure 1, which compares PESA against the standalone algorithms given the prescribed hyperparameters. For brevity, the plotted results include the minimum fitness found in each generation, since it is our end goal, so the first generation point is not necessarily the initial guess in all algorithms. In addition, log-scale is used for some functions (Cigar, Sphere, Brown, etc.) to reflect a better scale, where we set $10^{-2}$ as a lower bound to represent the zero global minima. Obviously, the results clearly show PESA outperforming the standalone ES, PSO, and SA by a wide margin in terms of number of generations to reach global minima. First, in all benchmarks, PESA successfully found and converged to the global optima, while the standalone algorithms failed to do so in most cases. In addition, PESA tends to converge much faster to the feasible region, thanks to the collaborative environment that PESA creates. We can notice that PSO is the most competitive algorithm to PESA. This is expected, since PSO was developed for continuous optimization, which is the feature of all the considered benchmarks. However, PSO alone seems to converge slowly. Due to the high dimensionality, standalone SA struggles to resolve the search space or to converge to a relevant solution in all cases. Standalone ES performance in Figure 1 is bounded between PSO and SA in most cases, except for the Levy function at which ES outperforms PSO.
Figure 2: Comparison of fitness convergence of PESA against ES, PSO, and SA in standalone form for minimization of 12 continuous benchmarks, dimensionality $n = 50$ (See Table 1)

To demonstrate PESA exploration capabilities, Figure 3 plots the Ackley fitness mean and $1 - \sigma$ standard deviation for PESA against standalone ES, PSO, and SA. The statistics are calculated based on "all" individuals within each generation (i.e., low and high quality solutions). Obviously, PESA features a much bigger error bar than the standalone algorithms, which reflects sample diversity within each PESA generation. The diversity of samples helps PESA to have better exploration of the search space, and overall better performance. On the other hand, the very small error bar of ES in Figure 3 implies more exploitative behavior than PSO, which fairly explores the space.

To explain how PESA improved the performance of the three individual algorithms, Figure 4 shows how ES, PSO, and SA perform during PESA search (i.e., NOT as standalone algorithms). First, we notice that PSO is leading PESA search early on as can be seen from the lower PSO fitness. Afterward, experience replay takes over by first guiding SA and preventing it from divergence as seen in Figure 2, and second by allowing ES and SA to lead PESA search as can be observed in some generations between 5-10. Additionally, thanks to the backdoor greedy replay, which allows SA to stay close to ES and PSO over the whole search period, investigating relevant search regions. At the end of the evolution, the three algorithms converge to each other due to the experience share of high quality solutions across the three algorithms. Unlike Figure 3, which shows PESA exploration ability, Figure 4 shows how PESA uses SA to continuously exploit toward best solutions over the whole evolution period.
Comparison of computational time between the four algorithms for three selected functions is given in Table 2. The memory management phase in the forms of sorting the memory, updating and re-sampling from the memory, calculating the priorities, annealing $\alpha$, and cleaning the memory from duplicates result in a longer run time for PESA compared to the standalone algorithms as expected. It is important mentioning that the numbers in Table 2 involve running PESA and other algorithms in serial (single core), meaning that PSO, ES, and SA parts in PESA are not running in parallel, since we found that parallelization not necessary for these benchmarks. Therefore, from Table 2, out of about $\sim 12s$ of PESA computing time, about 65% ($\sim 7$-8s) of the time is taken by the algorithms, while the rest are taken by the memory. The user can reduce the memory effect by controlling the full capacity of the memory ($D_{max}$), as interpreting large memories later in the evolution is consuming more time. The tests below in Table 2 involve registering every possible unique solution without limits (to obtain best performance), which would justify the significant portion consumed by the memory.

Table 2: Computing time in seconds required by different algorithms to run 100 generations for three selected functions

| Method   | Cigar | Sphere | Ackley |
|----------|-------|--------|--------|
| PESA (serial) | 11.7s | 11.0s  | 12.8s  |
| ES (serial)  | 1.2s  | 1.3s   | 1.2s   |
| PSO (serial) | 5.6s  | 5.2s   | 6.2s   |
| SA (serial)  | 0.5s  | 0.5s   | 0.9s   |
4 Conclusions

The concepts of experience share and replay are demonstrated through the proposed PESA algorithm to improve the search performance of evolutionary/stochastic algorithms. Experience share is performed through connecting particle swarm optimisation (PSO), evolution strategies (ES), and simulated annealing (SA) with a replay memory, storing all their observed solutions. Experience replay is conducted by re-sampling with priority coefficient from the memory to guide the learning of all algorithms. In addition, greedy replay is used in backdoor form with SA to improve PESA exploitation behavior. The validation against 12 high-dimensional continuous benchmark functions shows superior performance by PESA against standalone ES, PSO, and SA, under similar initial starting points, hyperparameters, and number of generations. PESA shows much better exploration behaviour, faster convergence, and ability to find the global optima compared to its standalone counterparts. Given the promising performance, the authors are now focusing on fully-parallelizing PESA such that ES, PSO, and SA can evaluate each generation in shorter time. This is especially important when the fitness evaluation is complex (e.g. requires computer simulation). Additionally, PESA will go through additional benchmarking against other hybrid evolutionary methods in the literature, e.g. ES/SA or RL/PSO. Lastly, combinatorial PESA version will be developed and benchmarked in solving engineering combinatorial problems with heavy constraints.

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Data Availability

The PESA GitHub repository will be released to the public once the peer-review process is done, which will include the source implementation and wide range of unit tests from benchmarks to engineering applications.

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