Engineering design of solar concentrator for transporting sunlight through optical fiber

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Abstract. This article presents the development of an optical system for the concentration of natural radiation. The optical system for the developed system of natural illumination based on fiber - Cassegrain system with a two-mirror lens - is considered. A system with a square section of mirrors is justified and calculated. Next, the reflection from the surface of the protective glass made of acrylic is evaluated and the required size of the end face of the polymer fiber used to transmit light to the room is justified. In conclusion, the radiation intensity at the output end of the polymer fiber is calculated. The system parameters were determined: a large mirror diameter of 198 mm, a small mirror diameter of 34.9 mm, a large mirror focal length of 49.5 mm, a small mirror focal length of 7.189 mm, the distance between the surfaces of the mirrors is 40 mm. The evaluation of the reflection from the surface of the protective glass made of acrylic showed that for the protective window of flat geometry the reflection coefficient is 7.74% and the convex geometry is 8.83%. The calculation of the radiation intensity at the output end of the polymer fiber showed that the total output power of the system will be determined by the number of fibers in the system.

1. Introduction

Using the simplest classification of optical systems, it is possible to divide them into lens, mirror and combined mirror-lens. The choice of the optical system based on the requirements is complicated by the fact that each type of system has its own advantages and disadvantages. At a certain diameter and focal distance, the losses for reflection and absorption in the lens system will be greater than in the mirror, however, the total efficiency of the mirror system may be higher or lower than the efficiency of the lens system depending on the degree of shielding of the mirror in the center. A mirror (reflecting) optical system is usually lighter in weight and cheaper than a lens optical system of the same size, but mirror systems may not provide such a high-quality image as a lens. For example, spherical mirrors cannot be corrected for off-axis aberrations without using a refractive correction plate, while a system of lenses with spherical surfaces is easier to balance by aberrations in the calculation. Since spherical surfaces are cheaper than aspherical surfaces, this is an important advantage. On the other hand, lens systems, unlike mirror ones, have chromatic aberrations. However, the cost advantage of reflective optics makes it necessary to revise the validity of this trend again whenever the development of a new optical system begins[1-6]. The purpose of the article is to develop an optical system for the concentration of natural radiation based on optical fiber.
2. Materials and methods
The optical system for the developed system of natural lighting based on optical fiber should be easy to manufacture, should consist of available materials that will allow making this system independently and do not depend on the suppliers or manufacturers of certain optical elements. The material for lenses must have a high refractive index in order for the system to be short-focus (compact). The dependence of the angle of refraction on the wavelength creates additional difficulties in the adjustment. Technological manufacturing of lenses with the necessary surface roughness is a difficult task. Since a material with a high refractive index (n>2) is expensive, the process of manufacturing lenses is complicated, buying custom lenses is also expensive and reveals dependence on the manufacturer and supplier, the optimal solution is to manufacture a mirror optical system. Its production does not entail large economic costs, the technological capacity of a company allows making it yourself, and the adjustment is simple, since there is no dispersion.

The simplest to manufacture is the Cassegrain system. This is a variant of a two-mirror lens (Figure 1). The main mirror of a larger diameter, concave (in the original version, parabolic) casts rays on the secondary convex of a smaller diameter (usually hyperbolic). According to the classification, the Maksutov scheme refers to the so-called pre-focal extension — that is, the secondary mirror is located between the main mirror and its focus and the full focal length of the lens is larger than that of the main one. The lens with the same diameter and focal length has almost half the length of the tube and slightly less shielding than Gregory [3,7].

![Cassegrain Lens Optical Design](image1)

**Figure 1.** Cassegrain Lens Optical Design.

![Cassegrain Lens Example](image2)

**Figure 2.** Cassegrain Lens Example.

Figure 3 shows variants of the Cassegrain system. As it can be seen from the figure, in the case of Cassegrain in the form of a large square mirror and a round small one, the area of the receiving system is used more efficiently (the light catchment area is larger) with the same body size. Therefore, it is necessary to develop a system with a square section.

![Variants of the Cassegrain system](image3)

**Figure 3.** Variants of the Cassegrain system.

In order to obtain a paraboloid of revolution with a square cross section with the desired size of the side of the square, it is necessary to develop a Cassegrain system of a larger size (Figure 4). The diameter
of a large mirror can be determined from the ratio
\[ D = (a^2 + a^2)^{1/2} = \sqrt{2} \cdot a, \]
where \(a\) – required side of the square.

![Figure 4. Determination of the diameter of a large mirror.](image)

In Cassegrain two-mirror antennas, the aperture angles of large and small mirrors usually lie between 70°–90° and 15°–30° respectively. To ensure acceptable performance, the authors assume \(\psi=90^\circ\) and \(\varphi=20^\circ\) (Figure 5).

![Figure 5. Main design parameters of the Cassegrain system.](image)

For lighting and design considerations, the side of the square should be 140 mm. Then, the diameter of the large mirror is:
\[ D = \sqrt{2} \cdot 140 = 198 \ \text{mm}. \]

The eccentricity of a hyperbolic small mirror is:
\[ e = \frac{\sin \left( \frac{\psi + \varphi}{2} \cdot \frac{\pi}{180} \right)}{\sin \left( \frac{\psi - \varphi}{2} \cdot \frac{\pi}{180} \right)} = 1.428. \]

Then the diameter of the small mirror is:
\[ d = D \cdot \frac{e-1}{e+1} \cdot \frac{1}{\tan \left( \frac{\psi}{2} \cdot \frac{\pi}{180} \right)} = 34.913 \ \text{mm}. \]

The diameter ratio should be no more than 2:
\[ \frac{d}{D} = 0.176. \]
It is seen that the condition is satisfied. Focal length of a large mirror is:

\[ F = \frac{d}{4} \cdot \frac{e + 1}{e - 1} = 49.5 \text{mm}. \]

Focal length of a small mirror is:

\[ f = \frac{d}{4} \cdot \frac{\cos \left( \frac{\psi + \varphi}{2} \right) \cdot \frac{\pi}{180}}{\cos \left( \frac{\varphi}{2} \cdot \frac{\pi}{180} \right) \cdot \sin \left( \frac{\psi}{2} \cdot \frac{\pi}{180} \right)} = 7.189 \text{ mm}. \]

Equivalent lens focus is defined as:

\[ f_e = \frac{F \cdot f}{F - f} = 8.411 \text{mm}. \]

The profile of the large mirror is obtained as in Cartesian coordinates using the function (Figure 6):

\[ y(x) = \sqrt{4 \cdot F \cdot x}, \]

and for the polar by the function (Figure 7):

\[ \psi_1 = -\psi, -\psi + 5...\psi; \]

\[ \rho(\psi_1) = \frac{2 \cdot F}{1 + \cos \left( \frac{\psi_1 \cdot \pi}{180} \right)}. \]

Figure 6. Profile of a large mirror in Cartesian coordinates; straight solid limit the size of the profile diameter \( D \).

Figure 7. Profile of a large mirror in polar coordinates.

The hyperbolic profile of a small mirror is also obtained in Cartesian and polar coordinates. For the profile in Cartesian coordinates, the following function is used:
\[ c = \frac{f \cdot f}{2} = 25.842; \]
\[ a = \frac{c}{e} = 18.095; \]
\[ b = a; \]
\[ x_i = a, a + 1...26; \]
\[ y_i(x_i) = \sqrt{\frac{x_i^2 \cdot b^2}{a^2} - b^2}. \]

where \( a, b, c \) – hyperbole parameters. The profile of a small mirror in Cartesian coordinates is shown in Figure 8.

Figure 8. Profile of a small mirror in Cartesian coordinates.

The profile of a small mirror in polar coordinates is obtained using the function:

\[ \rho_i(\varphi_i) = \frac{2(f_c - f)}{1 - 2 \cdot \frac{f}{f_c} + \cos \left( \frac{\varphi_i \cdot \pi}{180} \right)} \cdot \frac{f}{f_c}. \]

A general view of the Cassegrain system in Cartesian coordinates is presented in Figure 9.

Figure 9. General view of the Cassegrain system.
The distance between the surfaces of the mirrors is 40 mm, so the foci of the mirrors are combined. Figure 10 shows the ray path in the developed Cassegrain system.

![Figure 10. Path of the rays in the developed Cassegrain system.](image)

Figure 11 shows the 3D model of a large Cassegrain mirror with a square section.

![Figure 11. 3D model of a large Cassegrain mirror.](image)

![Figure 12. 3D model of Cassegrain system.](image)

3. Results and Discussion

The designed Cassegrain system must be protected from dust and other factors leading to surface contamination. In practice, protective transparent materials are used for these purposes. The design of a protective window, as a rule, is a flat plate. Parans and ECHY used this method. Figure 13 shows the appearance of the natural lighting systems of these companies.

![Figure 13. Natural lighting systems of the companies ECHY and Parans, respectively.](image)

After analyzing possible ways to manufacture a protective window and body, Solargy experts suggested using a hollow tube made of a translucent material (acrylic) as the body and protective window. Until this point, no such technical solutions were used in natural lighting systems. The implementation of this technical solution is economically justified, i.e. this method of solving the problem is the least expensive, since it contains the smallest number of technological operations[9-11].
It is necessary to evaluate how the reflection of the protective window surface depends on its geometry, and the difference in the use of flat and convex surfaces. To estimate the reflectivity known Fresnel formulas are used [6]. Since the size of the large mirror was 140 mm, the diameter of the hollow translucent tube was 200 mm. Figure 14 shows a cross-section of a 3D model of a natural lighting system. It can be seen from the figure that the maximum angle of direct sunlight is 55°. Based on these data, we will calculate the reflection coefficient for a normal incidence for a flat surface and a reflection coefficient in the range of angles of 0°-55°. For a protective window of a flat geometry, the reflection coefficient with regard to reflection from two boundaries is:

\[ R_p = 2 \cdot \left( \frac{n-1}{n+1} \right)^2 \cdot 100 = 7.745\% , \]

where \( n \) - acrylic refractive index 1.49.

The reflection coefficient of the first boundary of the protective window with a convex geometry is equal to:

\[ R(a) = \frac{1}{2} \cdot \frac{\sin \left( \frac{a \cdot \pi}{180} - a \cdot \sin \left( \frac{a \cdot \pi}{180} \right) \right)^2 + \left( \sin \left( \frac{a \cdot \pi}{180} + a \cdot \sin \left( \frac{a \cdot \pi}{180} \right) \right) \right)^2}{\sin \left( \frac{a \cdot \pi}{180} - a \cdot \sin \left( \frac{a \cdot \pi}{180} \right) \right)^2 + \left( \sin \left( \frac{a \cdot \pi}{180} + a \cdot \sin \left( \frac{a \cdot \pi}{180} \right) \right) \right)^2} \cdot \left( \sin \left( \frac{a \cdot \pi}{180} \right) \right) \cdot 180 \]

where \( a \) – angles of incidence of sunlight from 0 to 55°.

**Figure 14.** Cross section of the developed system.

After passing the first border of the window, the rays experience refraction and on the second border they fall at angles equal to:

\[ a \cdot \sin \left( \frac{a \cdot \pi}{180} \right) \]

\[ b(a) = \frac{180 \cdot \sin \left( \frac{a \cdot \pi}{180} \right)}{\pi} . \]
Then the reflection coefficient from the second face of the protective window of a convex geometry is equal to:

\[
R_2(a) = \frac{1}{2} \left( \sin \left( \frac{b(a) \cdot \pi}{180} \right) - a \cdot \sin \left( \sin \left( \frac{b(a) \cdot \pi}{180} \right) \cdot n \right) \right)^2 + \frac{1}{2} \left( \sin \left( \frac{b(a) \cdot \pi}{180} + a \cdot \sin \left( \sin \left( \frac{b(a) \cdot \pi}{180} \right) \cdot n \right) \right) \right)^2 \frac{1}{2} \left( \sin \left( \frac{b(a) \cdot \pi}{180} \right) - a \cdot \sin \left( \sin \left( \frac{b(a) \cdot \pi}{180} \right) \cdot n \right) \right)^2 + \frac{1}{2} \left( \sin \left( \frac{b(a) \cdot \pi}{180} + a \cdot \sin \left( \sin \left( \frac{b(a) \cdot \pi}{180} \right) \cdot n \right) \right) \right)^2 .
\]

The sum of the average values of the reflection coefficients from the two boundaries is equal to:

\[ R_v = (R_f + R_p) \cdot 100 = 8.883\% . \]

The difference of the reflection coefficients from a flat and convex surface is:

\[ \Delta = R_f - R_p = 1.138\% . \]

Thus, the difference in reflection coefficients from a flat and convex protective window is only 1%, which does not have a significant effect on the optical characteristics of the system.

We Consider Transmitting Light from Cassegrain to Fiber. After sunlight is concentrated by a Cassegrain lens, it must be transferred to optical fiber. From the calculation of the Cassegrain system, it is known that the focus is in the center of the surface of the large mirror. This is done for the convenience of fiber fasteners and simpler alignment of the optical system. It is necessary to estimate the required size of the end face of the polymer fiber used to transmit light to the room. From the drawing and calculations it is known that the light spot formed by concentrated solar radiation has a diameter of the order of 5 mm.

![Figure 15. Course of the rays in the transmission of luminous flux to focon.](image)

In order to transmit the maximum part of the light flux from the Cassegrain lens to the optical fiber, it is necessary to take a polymer fiber with a focon. The diameter of the end face of the focon should not be less than the light spot. For example, the optimal size for the focon of optical polymer fibers is 7-10 mm. According to the information provided by the “Technology Center of Polymeric Optical Fiber”, there is the possibility of making focons of this diameter.

The fiber aperture, according to this center, is \( NA = 0.45 \). This means that the maximum possible angle of incidence on the fiber end, the field of which gives rise to the effect of total internal reflection, is equal to arcsine (NA) = 26.7°. The Cassegrain lens is designed in such a way that the maximum angle of incidence at the fiber end is 18°, which allows the entire flux of radiation to experience the effect of total internal reflection and spread through the fiber with minimal losses.
Next, we calculate the radiation intensity at the output end of the polymer fiber. When calculating fiber output power, the concept of decibel (dB) is used. Initially, dB was used to estimate the power ratio, and in the canonical, customary sense, the value expressed in dB assumes the logarithm of the ratio of two powers and is calculated by the formula:

$$ dB = 10 \cdot \log\left( \frac{P_1}{P_0} \right), $$

where $P_1$ – the measured radiation power; $P_0$ – reference power taken for zero.

Accordingly, the transition from dB to the power ratio is carried out according to the formula:

$$ \frac{P_1}{P_0} = 10^{(0.1 \cdot dB)}, $$

and the power $P_1$ can be found with a known reference power $P_0$ by the expression[12]:

$$ P_1 = P_0 \cdot 10^{(0.1 \cdot dB)}, $$

When using data “Tver” of dependence of attenuation in dB from wavelength in the polymeric optical fiber produced by them it is possible to define intensity of output radiation. For this purpose it is necessary, using a formula for determination of output power, to make calculation for each wavelength:

$$ Q = P_{in} \cdot 10^{\left[0.1 \cdot \left(\frac{dB}{1000}\right) \cdot L\right]}, $$

where $P_{in}$ – input power concentrated at the end of the fiber; $dB$ – the attenuation value of the radiation in the fiber for each wavelength (Table 1); $L$ – the fiber length in meters.

The obtained $Q$ for each wavelength must be multiplied by the corresponding fractions of the energy of each wavelength in the emission spectrum, shown in Table 1.

| No. | Wavelength, $\lambda$, nm | Attenuation of each wavelength, $T_{dB}$, dB/km | $Q_i$ | The proportion of energy of each wavelength in the visible spectrum, $\chi_i$ |
|-----|--------------------------|-----------------------------------------------|------|----------------------------------|
| 1   | 400                      | 890                                           | 0.99795 | 0.025                           |
| 2   | 410                      | 730                                           | 0.99832 | 0.026                           |
| 3   | 420                      | 620                                           | 0.99857 | 0.027                           |
| 4   | 430                      | 550                                           | 0.99873 | 0.027                           |
| 5   | 440                      | 500                                           | 0.99885 | 0.028                           |
| 6   | 450                      | 465                                           | 0.99893 | 0.028                           |
| 7   | 460                      | 435                                           | 0.99900 | 0.028                           |
| 8   | 470                      | 413                                           | 0.99905 | 0.029                           |
| 9   | 480                      | 415                                           | 0.99904 | 0.029                           |
| 10  | 490                      | 418                                           | 0.99904 | 0.029                           |
| 11  | 500                      | 420                                           | 0.99903 | 0.029                           |
| 12  | 510                      | 415                                           | 0.99904 | 0.029                           |
| 13  | 520                      | 410                                           | 0.99906 | 0.029                           |
| 14  | 530                      | 400                                           | 0.99908 | 0.029                           |
| 15  | 540                      | 380                                           | 0.99913 | 0.029                           |
| 16  | …                        | …                                             | …     | …                               |

After multiplying the values, it is necessary to add:

$$ Z_m = Q_m \cdot \chi_m; $$

$$ P_{out} = \sum Z. $$

The value of $P_{out}$ will be the output power of the end of the polymer fiber.

The power of natural light on the surface of the Earth is defined as:
where \( \eta \) – the fraction of natural light in solar radiation, as defined above; \( W \) – the value of solar radiation in kWh/m\(^2\) \cdot year; \( H \) – the number of hours the sun shines in the sky.

With solar radiation of 1000 kWh/m\(^2\) year and the sun shining about 1000 hours, the value of \( P \) is equal to:

\[
P = \frac{\eta \cdot W \cdot 1000}{H} = 468.742 \text{ W/m}^2.
\]

Then the input power concentrated at the end of the polymer fiber is:

\[
P_m = (a^2 \cdot P \cdot 10^{-4}) \cdot \tau \cdot \rho_1 \cdot \rho_2 \cdot \xi,
\]

where \( a \) – the side of the large Cassegrain mirror in mm; \( P \) – the power of natural light on the surface of the earth; \( \tau \) – the transmittance of a hollow translucent tube; \( \rho_1 \) – the reflection coefficient of a large Cassegrain system; \( \rho_2 \) – the reflection coefficient of a small mirror; \( \xi \) – the transmission coefficient of radiation from the lens to the optical fiber.

It follows that the output power at the fiber end will be determined by the expression:

\[
Q = P_m \cdot 10^{\left[0.1 \left(-\frac{d B \cdot L}{1000}\right)\right]},
\]

\[
Z_m = Q_m \cdot x_m;
\]

\[
P_{out} = \sum Z.
\]

where \( P_m \) – the input power concentrated at the fiber end, \( dB \) – the attenuation value of the radiation in the fiber for each wavelength (Table 1); \( L \) – the fiber length in meters; \( Z \) – the fraction of the power of each wavelength introduced into the total radiation power \( P_{out} \).

This calculation allows estimating the output radiation power at the end of a single fiber with a known input power. The total output power of the system will be determined by the number of fibers in the system.

4. Conclusion

The study developed a system of natural lighting based on fiber-Cassegrain system with a two-mirror lens. A system with a square section of mirrors is justified and calculated. The system parameters were determined: a large mirror diameter of 198 mm, a small mirror diameter of 34.9 mm, a large mirror focal length of 49.5 mm, a small mirror focal length of 7.189 mm, the distance between the surfaces of the mirrors is 40 mm. The evaluation of the reflection from the surface of the protective glass made of acrylic showed that for the protective window of flat geometry the reflection coefficient is 7.74% and the convex geometry is 8.83%. The calculation of the radiation intensity at the output end of the polymer fiber showed that the total output power of the system will be determined by the number of fibers in the system.

References

[1] Optics. Thermal imaging systems. [Electron recourse]. Available at: http://leg.co.ua/arhiv/raznoe-arhiv/sistemy-teplovideniya-21.html

[2] Yezhova V V, Andreev L N 2011 Applied theory of aberrations St. Petersburg ITMO 2 52

[3] Astronomical portal. [Electron recourse]. Available at: http://www.galactic.name/articles/reflecting_telescope.php

[4] Zhao Y, Ma X, Xue B, Li F, He Y, Lv J, Yan X, Yu J, Xiang M 2018 Temperature analysis of Cassegrain optical antenna for space laser communication Optical Eng. 57(7)

[5] Zhou M, Yang H, Jiang P, Qin Y, Caiyang W, Mao S, Cao B 2019 Structure design of a conic lens pair for improving the transmission efficiency of a Cassegrain antenna Applied Optics 58(13) 3410–3417
[6] Polyanin A A 1996 Quick reference for engineers and students M.:. *International Education Program* 1 432

[7] Gao G-P, Yang C, Hu B, Wang S-F, Zhang R-F 2018 Design of a High-Gain and Low-Profile Quasi-Cassegrain Antenna Based on Metasurfaces *IEEE Antennas and Wireless Propagation Letters* 17(8) 1435–1439

[8] ECHY company website. [Electron recourse]. Available at:: http://www.echy.fr/

[9] Twidell J 1990 *Renewable Energy Sources* (Moscow: Energoatomizdat) p 392

[10] Duffy J A 1977 *Thermal processes using solar energy* (Moscow: Mir) p 420

[11] Klammt S, Neyer A, Mueller HFO 2012 Microoptics for efficient redirection of sunlight *Applied Optics* 51 2051–2056

[12] Internet security portal. [Electron recourse]. Available at: http://secandsafe.ru/stati/zaschita_informacii/chto_takoe_decibel_db