Interference between the Atmospheric and Solar Oscillation Amplitudes

Patrick Huber, Hisakazu Minakata and Rebekah Peset

Center for Neutrino Physics, Department of Physics, Virginia Tech, Blacksburg, Virginia 24061, USA

(Dated: December 6, 2019)

We propose to detect the interference effect between the atmospheric-scale and solar-scale waves of neutrino oscillation, one of the key consequences of the three-generation structure of leptons. In vacuum, we show that there is a natural and general way of decomposing the oscillation amplitude into these two oscillation modes. The nature of the interference is clearest in the $\bar{\nu}_e$ disappearance channel since it is free from the CP-phase $\delta$. We find that the upcoming JUNO experiment offers an ideal setting to observe this interference with more than 4 $\sigma$ significance, even under conservative assumptions about the systematic uncertainties.

INTRODUCTION

It is a remarkable feature of nature that the fundamental fermions, quarks, and leptons come into our world in the form of three generations; with various important consequences. Among them, the most dramatic one would be to provide a mechanism for CP violation [1]. The Standard Model of particle physics with the three family of quarks, not two, allows existence of a phase, the Kobayashi-Maskawa (KM) phase [2], by which CP symmetry is broken and indeed CP violation has been observed [3-5]. If a similar phase exists in the lepton sector of neutrino oscillation, the three generation structure allows for the existence of two independent mass squared differences: $\Delta m^2_{21}$ and $\Delta m^2_{31}$ of the larger $\Delta m^2_{31}$ drive the atmospheric neutrino experiments [6-10] and the solar $\Delta m^2_{21}$-driven reactor neutrino oscillation [11], as well as the $\Delta m^2_{21}$-matter potential induced flavor conversion [14-15] inside the sun [16].

Building on this success, in this Letter, we wish to add a new item to the list of nontrivial consequences of the three generation structure: Quantum interference between the atmospheric-scale and the solar-scale waves of neutrino oscillation. So far, the existence of the small $\Delta m^2_{21}$ effects in atmospheric and long-baseline (LBL) accelerator neutrino experiments and, similarly, the effects of the larger $\Delta m^2_{31}$, as well as the $\theta_{13}$ mixing effect, in the solar neutrino observation have been recognized as small sub-leading effects. The simultaneous full existence of both the $\Delta m^2_{31}$ and $\Delta m^2_{21}$ waves and their mutual interference, if observed, would establish another consequence of the three generation structure of neutrinos embedded into $\nu$SM.

THE ATMOSPHERIC AND SOLAR AMPLITUDES

Our first task is to define what the atmospheric and solar amplitudes are in neutrino oscillation. In this Letter, we restrict our discussion to vacuum, as a similar generic definition is not available — in fact, very likely not existing — in matter [17]. The flavor basis $S$ matrix elements $S_{\alpha\beta}$ ($\alpha, \beta = e, \mu, \tau$), which describe the neutrino flavor transformation $\nu_\beta \rightarrow \nu_\alpha$, can be written under the ultra-relativistic approximation of neutrinos as

$$S_{\alpha\beta} = U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{31}}{2E}} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{21}}{2E}}$$

where $E$ is the energy and $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ ($i, j = 1, 2, 3$) denote the mass squared differences of neutrinos. $U_{\alpha i}$ is the element of the lepton flavor mixing matrix which relates the flavor and the mass eigenstates of neutrinos as $\nu_\alpha = U_{\alpha i} \nu_i$. In Eq. (1), we factor out $e^{-im^2_{ij}/2E}$ for simplicity of the expression, which of course does not alter the physical observables. The oscillation probability of the process $\nu_\beta \rightarrow \nu_\alpha$ is given by $P(\nu_\beta \rightarrow \nu_\alpha : x) = |S_{\alpha\beta}|^2$. Hereafter, again for simplicity of the expressions, we define

$$\Delta_{ji} \equiv \frac{\Delta m^2_{ij}}{2E}$$

We take a heuristic way to find the appropriate definitions of the atmospheric and solar amplitudes. Let us first discuss the appearance channel, $\alpha \neq \beta$. The $S$ matrix elements in Eq. (2) can be rewritten as

$$S_{\alpha\beta} = U_{\alpha 3} U^*_{\beta 3} (e^{-i \Delta_{31} x} - 1) + U_{\alpha 2} U^*_{\beta 2} (e^{-i \Delta_{21} x} - 1)$$

due to unitarity, $U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} + U_{\alpha 3} U^*_{\beta 3} = 0$. Then, we claim that

$$S_{\alpha\beta}^{\Delta m^2_{31}} = U_{\alpha 3} U^*_{\beta 3} (e^{-i \Delta_{31} x} - 1)$$

Note, if neutrinos are Majorana particle [11], there is an option of having CP violation with only two generations of leptons.
is the atmospheric amplitude, and
\[ S^\text{sol}_{\alpha\beta} \equiv U^*_{\alpha1} U_{\beta2} \left( e^{-i\Delta_{21} x} - 1 \right) \] (6)
is the solar amplitude. The atmospheric amplitude, by definition, describes neutrino oscillation due to non-vanishing \( \Delta m^2_{21} \), and the solar amplitude the one caused by \( \Delta m^2_{31} \). Therefore, the obtained expressions for them are entirely natural ones.

Due to a difference in unitarity in disappearance channels, \( U_{\alpha1} U^*_{\alpha1} + U_{\alpha2} U^*_{\alpha2} + U_{\alpha3} U^*_{\alpha3} = 1 \), the \( S \) matrix has a slightly different expression when it is written in terms of the atmospheric and the solar amplitudes,
\[ S_{\alpha\alpha} = 1 + |U_{\alpha3}|^2 \left( e^{-i\Delta_{31} x} - 1 \right) + |U_{\alpha2}|^2 \left( e^{-i\Delta_{21} x} - 1 \right) = 1 + S^\text{atm}_{\alpha\alpha} + S^\text{sol}_{\alpha\alpha} \] (7)
where \( S^\text{atm} \) and \( S^\text{sol} \) are defined by extending the definition in (5) and (6), by setting \( \beta = \alpha \). They, of course, satisfy the conditions \( S^\text{atm}_{\alpha\alpha} \to 0 \) when \( \Delta_{31} \to 0 \), and \( S^\text{sol}_{\alpha\alpha} \to 0 \) when \( \Delta_{21} \to 0 \), respectively.

Now, we try to elevate the heuristic definitions into the general definition of \( S^\text{atm}_{\alpha\beta} \) and \( S^\text{sol}_{\alpha\beta} \). For a given \( S \) matrix element \( S_{\alpha\beta} \)

1. The atmospheric and the solar amplitudes are defined, respectively, as
\[ S^\text{atm}_{\alpha\beta} = \lim_{\Delta m^2_{31} \to 0} S_{\alpha\beta}, \quad S^\text{sol}_{\alpha\beta} = \lim_{\Delta m^2_{21} \to 0} S_{\alpha\beta}. \] (8)

2. We demand the completeness condition
\[ S_{\alpha\beta} = \delta_{\alpha\beta} + S^\text{atm}_{\alpha\beta} + S^\text{sol}_{\alpha\beta}, \]
where \( \delta_{\alpha\beta} \) denotes the Kronecker delta function. Consistency requires the so obtained amplitudes to satisfy
\[ \lim_{\Delta m^2_{31} \to 0} S^\text{atm}_{\alpha\beta} = \lim_{\Delta m^2_{21} \to 0} S^\text{sol}_{\alpha\beta} = 0. \]
The second condition, the completeness condition, demands that decomposition of the oscillation amplitude into the atmospheric and the solar amplitudes is complete. We only have the three neutrino states and therefore the two independent \( \Delta m^2 \), the atmospheric \( \Delta m^2_{31} \) and the solar \( \Delta m^2_{21} \). Then, there should be the two independent amplitudes, not more, not less.

\( \nu_\mu \to \nu_e \) AND \( \nu_e \to \nu_e \) CHANNELS

To obtain a sense on what the atmospheric and the solar amplitudes are, we write down their explicit forms in the \( \nu_\mu \to \nu_e \) and \( \nu_e \to \nu_e \) channels by using the flavor mixing matrix using the Particle Data Group (PDG) convention [5]. We leave the discussions of the other channels to ref. [17].

The atmospheric and solar amplitudes as defined in Eqs. (5) and (6), respectively, can be written in the \( \nu_\mu \to \nu_e \) channel as
\[ S^\text{atm}_{\mu e} = 2s_{23} c_{13} s_{13} e^{-i\frac{\Delta_{21} x}{2}} \sin \frac{\Delta_{31} x}{2}, \]
\[ S^\text{sol}_{\mu e} = 2s_{12} c_{13} e^{-i\frac{\Delta_{21} x}{2}} \left( c_{12} c_{23} - s_{12} s_{23} s_{13} e^{-i\delta} \right) \sin \frac{\Delta_{21} x}{2}. \] (9)
The oscillation probability consists of two terms, each amplitude squared and summed and the interference term:
\[ P(\nu_\mu \to \nu_e) = |S^\text{atm}_{\mu e} + S^\text{sol}_{\mu e}|^2 = P_{\mu e}^\text{non-int-fer} + P_{\mu e}^\text{int-fer}, \] (10)
where
\[ P_{\mu e}^\text{non-int-fer} = |S^\text{atm}_{\mu e}|^2 + |S^\text{sol}_{\mu e}|^2 = s_{23}^2 \sin^2 \theta_{13} \sin^2 \frac{\Delta_{31} x}{2} \right] \right) \sin \frac{\Delta_{21} x}{2}, \]
\[ P_{\mu e}^\text{int-fer} = 2Re \left( (S^\text{atm}_{\mu e}^*) S^\text{sol}_{\mu e} \right) \]
\[ = 8 \left[ J_c \left( \frac{\delta \Delta_{21} x}{2} \right) - s_{23}^2 c_{13}^2 s_{13}^2 \cos \left( \frac{\Delta_{21} x}{2} \right) \right] \sin \frac{\Delta_{21} x}{2} \sin \frac{\Delta_{31} x}{2}. \] (11)
We note that the interference term, the second equation of (11), displays the key feature of the problem. That is, it consists of two terms: One that depends on \( \delta \) and another that does not. Therefore, observing effect of \( \delta \) is due to the quantum interference between the atmospheric and the solar amplitudes, but only a part of the total effect. A claim of observation of the quantum interference between the atmospheric and the solar amplitudes requires the observation of both terms in (11) with the correct magnitudes; i.e., a measurement of \( \delta \) is not the same as a measurement of the interference effect.

Now, we discuss the \( \nu_e \to \nu_e \) channel, which due to CPT-invariance is identical to the \( \nu_e \to \nu_e \) channel. The atmospheric and solar amplitudes are written as
\[ S^\text{atm}_{ee} = 2s_{13}^2 e^{-i\frac{\Delta_{31} x}{2}} \sin \frac{\Delta_{31} x}{2}, \]
\[ S^\text{sol}_{ee} = 2s_{12}^2 c_{13}^2 e^{-i\frac{\Delta_{21} x}{2}} \sin \frac{\Delta_{21} x}{2}. \] (12)
Due to un-oscillated “1” in Eq. (7), the \( \nu_e \) survival probability \( P(\nu_e \to \nu_e) \) takes a slightly complicated form, but can be written in the similar form as in the appearance channel,
\[ P(\nu_e \to \nu_e) = P_{ee}^\text{non-int-fer} + P_{ee}^\text{int-fer}, \] (13)
where
\[ P_{ee}^\text{non-int-fer} = 1 + |S^\text{atm}_{ee}|^2 + |S^\text{sol}_{ee}|^2 + 2Re \left( S^\text{atm}_{ee} + S^\text{sol}_{ee} \right) \]
\[ = 1 - \sin^2 \theta_{13} \sin^2 \frac{\Delta_{31} x}{2} - 4s_{12}^2 c_{13}^2 \left( 1 - s_{12}^2 c_{13}^2 \right) \sin^2 \frac{\Delta_{21} x}{2}, \]
\[ P_{ee}^\text{int-fer} = 2Re \left( (S^\text{atm}_{ee}^*) S^\text{sol}_{ee} \right) \]
\[ = 2 \sin^2 \theta_{13} \sin \frac{\Delta_{31} x}{2} \cos \frac{\Delta_{32} x}{2} \sin \frac{\Delta_{21} x}{2}. \] (14)
HOW TO OBSERVE THE QUANTUM INTERFERENCE EFFECT

We briefly discuss how to pin down the quantum interference effect between the atmospheric and solar amplitudes. Once we obtain the expression of the oscillation probability as

\[ P(\nu_\beta \to \nu_\alpha) = P^\text{non-int-fer}_{\beta\alpha} + P^\text{int-fer}_{\beta\alpha}, \]  

one can define a "test oscillation probability" by introducing the \( q \) parameter as

\[ P(\nu_\beta \to \nu_\alpha) = P^\text{non-int-fer}_{\beta\alpha} + q P^\text{int-fer}_{\beta\alpha}. \]  

By fitting the data with the test oscillation probability \[15\], we would obtain 1-dimensional \( \chi^2 \) (1 DOF) for the \( q \) parameter. We note that, in the case of appearance experiments, we marginalize over \( \delta \) as well as the other mixing parameters in the experimentally allowed ranges.

Though our discussion in this paper covers both the appearance and the disappearance experiments in vacuum, the analysis of the appearance channel in accelerator LBL experiments requires treatment of the matter effect \[17\], which is beyond the scope of this Letter.

The experimental setting of JUNO \[18\] is uniquely suited for our purpose of observing the interference effect between the atmospheric and solar oscillations. In JUNO the solar and the atmospheric oscillation effects coexist with their full magnitudes at the same detector. Both oscillations are fully developed and have left the linear regime of \( \sin \frac{\Delta m^2_{\alpha\beta}}{2E} \). Even though the atmospheric oscillation may be small wiggles over the long-wavelength solar oscillation, the very good energy resolution of the JUNO detector aims at its precision measurement. Therefore, JUNO is the ideal experiment for the purpose of detecting the atmospheric - solar interference effect. It very likely is the best choice among all possible experiments, ongoing or planned, in vacuum and in matter.

Here, we describe in detail the procedure of our statistical analysis. Using GLoBES \[20\, 21\], we set up an experiment with two detectors: a JUNO-like far detector with a fiducial mass of 20 kt and an energy resolution of 3%/\( \sqrt{E} \) at a distance of 53 km from a nuclear reactor source with a total power of 36 GWth and a TAO-like near detector with a fiducial mass of 1 ton and an energy resolution of 1.7%/\( \sqrt{E} \) at a distance of 30 m from a 4.6 GWth nuclear reactor core. For each detector, we use a model for non-linear effects in the reconstruction of the positron energy like that described in Ref. \[22\] up to cubic terms. To account for the uncertainties in the reactor antineutrino flux prediction, we conservatively introduce a nuisance parameter to each of our 100 energy bins with the spectrum computed before applying the energy resolution function. This is equivalent to the assumption of no prior knowledge of fluxes, as in Ref. \[22\]. For the purposes of producing simulated data, we assume the normal hierarchy to be the true hierarchy and the relevant oscillation parameters to be \( \Delta m^2_{31} = 7.54\times10^{-5} \text{ eV}^2 \), \( \Delta m^2_{32} = 2.43 \times 10^{-3} \text{ eV}^2 \), \( \theta_{12} = 33.6 \text{ deg} \), and \( \theta_{13} = 8.9 \text{ deg} \). For the analysis of the resulting data, we fit the data obtained from the oscillation probability in Eq. \[14\] with that obtained using the oscillation probability modified with the parameter \( q \), as in Eq. \[15\], by minimizing the following \( \chi^2 \) function for various values of \( q \) while allowing all nuisance and standard oscillation parameters to vary:

\[ \chi^2 = \sum_{i,j} \left( \frac{\phi_{i,j} - \phi_{i,j}^\text{fit}}{\phi_{i,j}^\text{true}} \right)^2 + \text{pull terms}, \]  

where \( \phi_{i,j}^\text{true} \) and \( \phi_{i,j}^\text{fit} \) are the simulated rate and modified rate, respectively, in the \( i \)-th energy bin for the detector specified by \( I = \text{Near, Far} \). The "pull terms," defined in Eq. \[16\], provide a penalty for \( \theta_{13} \) with an uncertainty of \( \sigma_{\theta_{13}} = 10\% \) and the nuisance parameters \( n_k \) for which uncertainties are \( \sigma_k \):

\[ \text{pull terms} = \frac{(\theta_{13,\text{true}} - \theta_{13,\text{fit}})^2}{\sigma^2_{\theta_{13}}} + \sum_k \frac{n_k^2}{\sigma^2_{n_k}}. \]  

The nuisance parameters encode the energy calibration and flux uncertainties as described in detail in Ref. \[22\].
The resulting $\chi^2$ curve is shown as the thick black line in figure [1]. The same analysis procedure is repeated except assuming that the energy calibration error for each detector is linear (blue solid line), and then without a near detector assuming perfect knowledge of detector and source systematics (gray dashed line). For the first case mentioned, the value of $\chi^2$ at $q = 0$ is 16.7, so the interference effect would be able to be seen in JUNO with a significance of more than 4$\sigma$. Note that there is a potential model-dependence, in that we assume that atmospheric oscillation experiments observe $\Delta m^2_{31}$. If, instead, we assume that they measure $\Delta m^2_{32}$, the value of $\chi^2$ at $q = 0$ is still 16.7.

**SUMMARY**

In this letter we have shown that, in vacuum, a natural and general factorization of oscillation amplitudes into a solar and atmospheric part is possible for appearance and disappearance channels. This factorization is exact and not relying on the actual values of observed oscillation parameters. With this factorization, it becomes possible to define the effect of interference between the two partial amplitudes. For appearance channels, the interference term contains the CP-phase $\delta$ but also terms independent of it. In the $\nu_e$ disappearance channel, the oscillation amplitude does not depend on $\delta$ and hence the interference effect we saw has nothing to do with the CP-phase. The nature of the interference phenomena indicated by these features is a dynamical, quantum mechanical interference inside the three family of neutrinos, not particularly related to the CP-violating phase. We show, by detailed numerical calculation, that JUNO can observe this interference effect with more than 4 $\sigma$ significance.

One of the authors (H.M.) thanks Takaaki Kajita and Hiroshi Nunokawa for intriguing conversations while the concept of amplitude interference was still in its infancy. The work of P.H and R.P. is supported by the US Department of Energy Office of Science under award number DE-SC0020262.

---

1. J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Phys. Rev. Lett. 13, 138 (1964). doi:10.1103/PhysRevLett.13.138
2. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973). doi:10.1143/PTP.49.652
3. B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 87, 091801 (2001). doi:10.1103/PhysRevLett.87.091801 [hep-ex/0107013].
4. K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 87, 091802 (2001). doi:10.1103/PhysRevLett.87.091802 [hep-ex/0107061].
5. M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no. 3, 030001 (2018). doi:10.1103/PhysRevD.98.030001
6. K. Abe et al. [T2K Collaboration], arXiv:1910.03887 [hep-ex].
7. M. A. Acero et al. [NOvA Collaboration], Phys. Rev. Lett. 123, no. 15, 151803 (2019). doi:10.1103/PhysRevLett.123.151803 [arXiv:1906.04907 [hep-ex]].
8. M. Jiang et al. [Super-Kamiokande Collaboration], PTEP 2019, no. 5, 053F01 (2019) doi:10.1093/ptep/ptz015 [arXiv:1901.03230 [hep-ex]].
9. K. Abe et al. [Hyper-Kamiokande Proto-Collaboration], PTEP 2015, 053C02 (2015) doi:10.1093/ptep/ptv061 [arXiv:1502.05199 [hep-ex]].
10. R. Acciarri et al. [DUNE Collaboration], arXiv:1512.06148 [physics.ins-det].
11. E. Majorana, Nuovo Cim. 14, 171 (1937). doi:10.1007/BF02961314
12. Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81, 1562 (1998). doi:10.1103/PhysRevLett.81.1562 [hep-ex/9807003].
13. K. Eguchi et al. [KamiLAND Collaboration], Phys. Rev. Lett. 90, 021802 (2003). doi:10.1103/PhysRevLett.90.021802 [hep-ex/0212021].
14. L. Wolfenstein, Phys. Rev. D 17, 2369 (1978). doi:10.1103/PhysRevD.17.2369
15. S. P. Mikheyev and A. Y. Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985) [Yad. Fiz. 42, 1441 (1985)].
16. Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 89, 011301 (2002). doi:10.1103/PhysRevLett.89.011301 [nucl-ex/0204008].
17. P. Huber, H. Minakata, and R. Pestes, in preparation.
18. F. An et al. [JUNO Collaboration], J. Phys. G 43 (2016) no.3, 030401 doi:10.1088/0954-3899/43/3/030401 [arXiv:1507.05613 [physics.ins-det]].
19. L. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, JHEP 1901, 106 (2019) doi:10.1007/JHEP01(2019)106 [arXiv:1811.05487 [hep-ph]].
20. P. Huber, M. Lindner and W. Winter, Comput. Phys. Commun. 167, 195 (2005). doi:10.1016/j.cpc.2005.01.003 [arXiv:0407333 [hep-ph]].
21. P. Huber, J. Kopp, M. Lindner, M. Rolinec and W. Winter, Comput. Phys. Commun. 177, 432-438 (2007). doi:10.1016/j.cpc.2007.05.004 [arXiv:0701187 [hep-ph]].
22. D. V. Forero, R. Hawkins and P. Huber, arXiv:1710.07378 [hep-ph].
23. J. P. A. M. de Andr´e for JUNO, Talk given at the 27th International Workshop on Weak Interactions and Neutrinos on June 4, 2019.