Performance Analysis of Work-Roll Wear Models on Hot Rolling

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**Abstract.** The purpose of this work is to evaluate the performance of some wear models, either with different mathematical formulation or different definition of the unknown wear coefficients, on the prediction of the work-roll wear amplitude in Hot Strip Mills (HSM). To achieve this goal, a classical model calibration approach based on inverse optimization has been developed to calibrate these wear models. A large industrial hot rolling database composed by roll wear amplitude measurements for both later finishing mill stands (F6 and F7) from ArcelorMittal Dofasco HSM was considered and a least-square cost function was applied to minimize the differences between both numerical and experimental results during the optimization process. The averaged roll wear gap between measurements and optimized numerical predictions was then used as a quantitative indicator to compare the performance between the wear models and identify the most suitable one for roll wear prediction. In addition, an Artificial Neural Network (ANN) approach was developed based on the most suitable wear model. Thus, roll wear predictions obtained using the ANN were compared with the ones obtained using Classical calibration to evaluate the performance of both approaches.

**Introduction**

Roll wear is a very important phenomenon that occurs during hot rolling process. This phenomenon gradually changes both work-roll surface finish and profile over the rolling campaign. As a result, it affects strip crown and flatness which are the dimensional quality indicators required by hot strip mill customers. In recent years, the development of new hard grades, more sensitive to shape defect since wider and thinner, and the increasing product quality requirements from customers call for a better prediction of roll wear to improve the accuracy of strip shape control and flatness actuators on hot strip mills [1]. However, roll wear is a complex phenomenon which may involve simultaneously several mechanisms [2]. Many investigations have been conducted to understand these mechanisms, in particular roll wear abrasion [3], adhesion [4], chemical degradation [5], thermal and mechanical fatigue [6].

Nevertheless, a proper characterization and quantification of these mechanisms under industrial rolling conditions remains difficult. Therefore, the most part of the wear models available on the literature consist of contact-mechanic based equations assuming simplified relations between some working conditions. This kind of equations is often defined by considering that few material or process parameters, such as roll hardness [7], strip surface temperature [8], roll-bite contact length [9], roll diameter [10,11] or roll shape [12], are particularly important on the wear process [13].

Nonetheless, no information about which are the most influential parameters as well as which wear formulations lead to enhanced roll wear predictions can usually be found on the literature. Furthermore, unknown wear coefficients are also included on the roll wear models to enable better model tuning and then more reliable agreement between wear calculations and measurements. Such wear coefficients may theoretically depend on rolling process factors such as, strip temperature, strip grade, roll type, roll oxidation or roll-bite lubrication [9,12,14]. Though, neither analytical relation...
linking these factors nor details about the parametrization of the wear coefficients (by considering the variation of these factors during the rolling process) are generally provided.

Thus, the main goal of this work is to compare and evaluate the performance of 3 wear models and identify the most suitable parametrization approach to correlate the unknown wear coefficient of these models with the following rolling process factors: (i) roll stand, (ii) roll type/supplier and (iii) strip grade. To do this, a large rolling database and corresponding wear measurements from ArcelorMittal Dofasco HSM are used to analyze and quantify the performance of both wear equations and wear coefficient parametrization approaches on the prediction of the roll wear amplitude.

Roll Wear Modeling

Mathematical formulation. Among the several roll wear models available on the literature, 3 wear models with different mathematical formulations have been chosen. These wear models consist of:

• Wang’s model [14] which can be defined by,

\[ \delta(x) = \sum_{i=1}^{n\text{ strips}} K_i \frac{F_i}{w_i} L_i [1 + f(x)] \]  

(1)

• Nikitenko’s model [7] which can be written as,

\[ \delta(x) = \sum_{i=1}^{n\text{ strips}} K_i \frac{P_i(x) L_i}{\pi \phi_{WR} HB_{WR}} \]  

(2)

• Cao’s model [9] which can be expressed by,

\[ \delta(x) = \sum_{i=1}^{n\text{ strips}} K_i \frac{L_i l_{ci}}{\phi_{WR}} \left( \frac{F_i}{w_i l_{ci}} \right)^a [1 + f(x)] \]  

(3)

where \( \delta \) is the cumulated roll wear amount of \( n \) strips, \( F \) is the rolling force, \( w \) is the strip width, \( L \) is the rolled length, \( P \) is the specific force applied to the roll by strip at the distance \( x \) from the roll center, \( \phi_{WR} \) is the work-roll diameter, \( HB_{WR} \) is the Brinell hardness of the work-roll surface, \( l_c \) is the arc contact length in the roll-bite, \( f(x) \) is a function that describes the non-uniform wear repartition over the roll width, \( K \) is the wear tuning coefficient and \( a \) stands for an exponent tuning parameter. Note that, the roll wear models described by contact-mechanic based equations are usually derived from Archard principles [15] that stipulates that wear (volume loss) is proportional to the normal (rolling) force by the sliding length \( L \). So, the selected wear equations basically differ (i) on the use of distinct influential material and process parameters or (ii) on the definition of the wear profile over the roll width. In this work, only the wear amplitude at the roll center (\( x=0 \)) is evaluated and so, Eqs. (1) and (3) are simplified by neglecting the term corresponding to function \( f(x) \). This function only deals with the roll wear distribution from the center to the roll borders and assumes value 0 at the roll center.

Wear coefficients parametrization. The wear tuning coefficient \( K \) of the chosen wear models can be defined by relating multiple rolling factors. The roll type and supplier play a crucial role on the wear generated during hot rolling. On the one hand, High Speed Steel (HSS) and High Chromium (HiCr) roll types are commonly used for early stands (F1-F4), while Indefinite Chill (ICDP) roll type is used for later stands (F5-F7). On the other hand, work-roll microstructure varies from supplier to supplier even for each roll type. As a result, the work-rolls used for early and later finishing stands and from different suppliers present distinct thermo-mechanical behavior and wear resistance.
According to [10], strip grade seems to have some influence on the roll wear as well. Indeed, a large mix of strip grades with different hardness can be rolled over a rolling campaign. So, a different work-roll wear contribution may eventually be expected in function of the strip grade.

Moreover, roll stand and its upper/bottom part should also be distinguished. It should enable to indirectly account for both strip temperature and abrasive oxide scale effects which vary for top/bottom strip surfaces and from stand to stand.

Hence, the wear tuning coefficient $K$ is parametrized here based on strip grade, roll type/supplier and roll stand. However, 3 different parametrization approaches have been established to link these rolling factors. The first parametrization approach (named as Option 1) is defined as,

$$K = k_{Fi} \times k_{Rj} \times k_{Sh}$$

with,

$$k_F \in \{k_{F1}, k_{F2}, ..., k_{Fn}\}$$

$$k_R \in \{k_{R1}, k_{R2}, ..., k_{Rn}\}$$

$$k_S \in \{k_{S1}, k_{S2}, ..., k_{Sn}\}$$

where $k_{Fi}$, $k_{Rj}$ and $k_{Sh}$ are constant values respectively for the stand $i$, roll type/supplier $j$ and strip grade $h$.

The second parametrization approach (called as Option 2) is expressed by,

$$K = k_{Fi} \times k_{RjSh}(k_R, k_S)$$

with,

$$k_{RS} = \begin{bmatrix}
    k_{S1R1} & k_{S1R2} & ... & k_{S1Rp} \\
    k_{S2R1} & k_{S2R2} & ... & k_{S2Rp} \\
    \vdots & \vdots & \ddots & \vdots \\
    k_{SnR1} & k_{SnR2} & ... & k_{SnRp}
\end{bmatrix}$$

where the constant values of tensor $k_{RS}$ are a function of both roll type/supplier and strip grade factors.

The last parametrization approach defined (named as Option 3) consists of the simplification of the first approach disregarding the strip grade factor. Thus, Option 3 is given by,

$$K = k_{Fi} \times k_{Rj}$$

This later wear parametrization option was defined with the purpose of verifying the effect of not considering the strip grade information on the roll wear prediction.

Experiments

An industrial rolling database from ArcelorMittal Dofasco HSM for the stands F6 and F7 top and bottom rolls was used for this study. This database is composed by 395 rolling campaigns which correspond to a total of 51710 rolled strips. Roll wear profiles were measured on roll grinders at the end of each rolling campaign and after work-rolls fully cooled down, to avoid any effect of the roll thermal expansion. Thereby, a total of 1580 roll wear profile measurements have been acquired (395 campaigns x 4 roll stands). From these measured wear profiles, only the wear amplitude at the roll center is used to be compared with the numerical calculations of each wear model. Moreover, 9 different strip grades have been rolled and 8 different roll type/suppliers have been used over this rolling campaign database. In this way, the wear parametrization options 1, 2 and 3 lead respectively to 21, 76 and 12 wear coefficients to calibrate.

Note that roll wear model analysis found in the literature were usually carried out only considering a few rolling campaigns [8,10–12]. Thus, this database composed by a larger number of rolling campaigns with distinct strip rolled schedules enables a more reliable evaluation of roll wear model predictions.
Model Calibration Framework

A classical calibration approach based on inverse parameters optimization is applied to tune the wear models. However, not all the rolling database is used on the model’s calibration. In fact, the rolling database and corresponding wear measurements from Dofasco HSM are randomly split into two sets. The first dataset, which corresponds to 2/3 of the rolling campaigns, is named as training set and is applied for the calibration of the wear models. While the second dataset, which consists in the remaining 1/3 rolling campaigns, is used for the validation of the models through a performance analysis of both wear equations and wear coefficient parametrization approaches on the prediction of the roll wear amplitude. This performance analysis is made by quantifying the averaged wear gap between measurements and predictions. Note that the validation dataset allows for a more robust comparison and evaluation between the different wear models since it was not used to fit the wear coefficients and so, gives an unbiased effectiveness estimation of the calibrated wear models.

Inverse parameters optimization. The wear models are calibrated by an inverse optimization process which searches for the set of input wear coefficients leading to the smallest gap between wear amplitude measurements and calculations. This inverse optimization process combines an optimization algorithm, responsible for updating the unknown wear coefficients, with a cost function, responsible for calculating the difference between experimental and numerical wear amplitudes. During the optimization, the wear coefficients set is iteratively updated with the aim of minimizing the cost function value. Figure 1 summarizes the inverse optimization process. Both optimization process framework and wear models have been developed in MATLAB.

The cost function ($F_{\text{cost}}$) used for the calibration of the wear models is defined by,

$$F_{\text{cost}} = \sum_{j=1}^{n_{\text{stands}}} \frac{1}{n_{\text{camp}}} \sum_{i=1}^{n_{\text{camp}}} \left( \delta_{i,j}^{\text{Meas}} - \delta_{i,j}^{\text{Calc}} \right)^2$$

where $n_{\text{stands}}$ is the number of roll stands, $n_{\text{camp}}$ is the number of rolling campaigns and $\delta_{i,j}^{\text{Meas}}$ and $\delta_{i,j}^{\text{Calc}}$ are respectively the measured and calculated roll wear amplitude values for the rolling campaign $i$ belonging to the roll stand $j$. This cost function is minimized by using the Levenberg-Marquardt (L-M) gradient-based algorithm. This type of optimization algorithm is characterized (i) by using the information of the derivative of the objective function to successively update the solution and (ii) by a quick convergence in the vicinity of the solution [16].

![Fig. 1. Inverse optimization framework applied for wear model calibration.](image-url)

Optimization conditions. The L-M gradient-based algorithm may lead to an optimization solution dependent on the initial starting point. Due to this reason, 3 optimizations starting with different initial sets of wear coefficients have been carried out for each wear formulation and wear parametrization option to investigate the robustness of the optimized solutions. The same 3 initial sets of wear
coefficients have been used for all the model calibrations to assure identical starting optimization conditions. Constrained optimization is also considered by defining lower and upper bound limits for each wear tuning coefficient.

During the optimization process, the derivatives of the objective function are calculated numerically through a forward finite difference scheme with a perturbation value of $5 \times 10^{-3}$. In addition, the maximum number of evaluations allowed corresponds to 10 times the number of wear coefficients of the wear model being subjected to calibration.

**Results and Discussion**

In this section, both calibration results and performance analysis are presented and discussed for each wear model. The influence of the initial guess on finding the optimum set of wear coefficients is discussed as well.

Robustness of the model calibrations. As previously mentioned, 3 optimizations with different initial sets of coefficients have been carried out for each wear model. Such optimization results are given by Table 1 for the wear model defined by the Wang’s wear equation linked to the wear coefficients option 1 (21 coefficients). This table lists the initial and final cost function values as well as the averaged wear gap results obtained for both training and validation sets. In the case of Set 1, the initial wear coefficients were all set equal to 1 to promote a starting optimization from isotropic wear conditions between the several wear factors. In the case of Set 2, a same random initial value was defined for all the wear coefficients belonging to a specific wear factor group but these initial values are different among the factor groups (stand, roll type/supplier and strip grade). In the case of set 3, empirical initial wear coefficients were defined for both stand and roll type/supplier factors while a same initial wear coefficient value was set for all the strip grades.

Table 1. Optimizations carried out for the wear model defined by Wang’s equation and wear coefficients option 1.

| SET 1 | SET 2 | SET 3 |
|-------|-------|-------|
| Units | Initial | Optimal | Initial | Optimal | Initial | Optimal |
| $F_{\text{cost}}$ | 0.1519 | 0.0167 | 0.1840 | 0.0192 | 0.5231 | 0.0163 |
| $F_{\text{cost}}$ reduction | - | -89.0% | - | -89.6% | - | -96.9% |
| Avg. Wear Gap (Train. set) | - | 46.3 | - | 49.4 | - | 45.5 |
| Avg. Wear Gap (Val. set) | - | 45.8 | - | 48.1 | - | 45.0 |

Despite the different initial parameter sets and initial cost function values, it can be seen from Table 1 that the optimization algorithm converged each time towards almost the same final cost solution. Also, comparing both initial and final cost function values given at the bottom of Table 1, it is observed that a cost function reduction of about 89-97% was obtained for these 3 optimizations. It was then found that the optimization process is robust enough to provide reliable wear model calibrations.

By using both initial and optimal wear coefficients from Set 3, roll wear amplitudes were calculated and are depicted in Figure 2. This figure shows that the optimized wear coefficients lead to an enhanced reproduction of the experimental roll wear amplitude when compared with the numerical wear calculations obtained by using the initial wear coefficients. These wear calculations reveal the significant improvement made by the optimization process from the initial guess to the optimum result.
Performance of the roll wear formulations. The calibration of the roll wear formulations (cf. Eqs. (1)-(3)) was carried out by considering the wear parametrization option 1 (cf. Eq.(4)) which corresponds to 21 wear coefficients to tune. Among the 3 optimizations made for each wear formulation by using the initial coefficients sets listed in Table 1, the one starting with the initial set 1 led to the overall best model calibrations. So, the calibrated model results obtained from initial set 1 were used to evaluate the performance of the different roll wear formulations. Table 2 lists (i) the final cost function values as well as (ii) the averaged gap between predicted and measured roll wear amplitudes obtained by both training and validation datasets for each model formulation. From this table, it is shown that no very significant differences exist between the 3 wear model predictions since the final cost function values as well as the averaged wear gaps obtained are quite similar. Indeed, the averaged wear gap between the measured wear amplitudes and the predictions of the 3 wear formulations are in-between ≈46-51 µm for the validation dataset.

However, Wang’s model calculations tend to promote better roll wear amplitude predictions (≈46 µm) while, in opposition, Cao’s model calculations led to the worst ones (≈51 µm). From a performance analysis point of view, it appears then that the Wang’s model, which has a simpler formulation than the 2 other models since it needs less process parameters, is able to give same or even better quality of roll wear predictions. So, in the case of the 3 roll wear models investigated, it seems that a more complex roll wear formulation including several influential material or process parameters does not necessarily lead to an enhanced roll wear calculation.

In addition, the roll wear amplitudes predicted by the 3 wear formulations for the rolling validation dataset are illustrated by Figure 3. This figure shows that all the formulations tend to overestimate the roll wear measurements for wear amplitude values below 300 µm, while underestimated predictions tend to be observed above this wear amplitude value. It may be because the roll wear amount calculated for a given rolled strip is independent of the strip position in the rolling campaign schedule. In other words, the amount of wear generated by a same rolled strip (and same rolling conditions) at the beginning and at the end of the rolling campaign should be different due to the weakness of the thermo-mechanical roll properties. This effect is not taken into account by the wear model approaches presented here. Thus, for larger wear amplitude values usually from longer rolling campaigns, the roll wear measurements tend to be underpredicted.
Performance of the wear coefficient parametrizations. The different wear coefficient parametrization options (cf. Eqs. (4)-(8)) were investigated by using the Wang’s mathematical formulation (cf. Eq. (1)) since this one led to better wear predictions. So, 3 Wang’s model calibrations were made for each parametrization option by using the initial coefficients sets listed in Table 1. Note that these initial sets were adapted for each parametrization option in order to start the calibrations from the same initial cost function values. Best calibration results were obtained from initial set 1 and an overview of these results is given by Table 3. It can be seen from this table that the parametrization option 2 which is defined by 76 unknown wear coefficients enabled best calibration results (training dataset) since it reached smaller final cost function value and smaller averaged gap between wear measurements and predictions (~43 µm). But, concerning the validation dataset, Option 1 which is defined by 21 coefficients led to same averaged wear prediction (~46 µm) than Option 2. Such wear predictions for the validation dataset are depicted by Figure 4. This figure allows for a qualitative analysis of the predicted results and it shows that similar (i) wear amplitude values and (ii) wear dispersion behaviour were obtained from the 3 coefficient parametrization options. Therefore, in terms of model performance, the parametrization option 1 was identified as the most suitable one due to the better compromise between number of unknown coefficients and wear predictions.

Table 3. Calibration and validation results obtained by the 3 wear coefficient parametrization options.

| Option 1 (21 coef.) | Option 2 (76 coef.) | Option 3 (12 coef.) | Units |
|---------------------|---------------------|---------------------|-------|
| Optimal $F_{cost}$ | 0.0167              | 0.0143              | 0.0181 | µm$^2$ |
| Avg. Wear Gap (Train. set) | 46.3                | 43.1                | 48.4   | µm    |
| Avg. Wear Gap (Val. set) | 45.8                | 45.2                | 49.4   | µm    |

Fig. 3. Roll wear amplitudes at stands a) F6 and b) F7 calculated by the different roll wear formulations for the rolling validation dataset.

Fig. 4. Roll wear amplitudes at stands a) F6 and b) F7 calculated by the different wear coefficient parametrization options for the validation dataset.
Furthermore, validation results listed in Table 3 also show that the averaged wear calculations using the parametrization option 3 were just \( \approx 4 \mu m \) worse than the ones obtained by option 1. This result reveals that the effect of not considering the strip grade factor on the wear coefficients parametrization led to a deterioration of the wear amplitude predictions around 8% for this validation dataset. Note that the parametrization option 3 is a simplified version of the option 1 by disregarding the strip factor and then is just defined by 12 wear coefficients. So, this smaller parametrization option may be more interesting to predict roll wear in hot strip mills only producing similar kind of strip family grades.

**Artificial Neural Network vs. Classical Calibration**

An Artificial Neural Network (ANN) approach was also developed and applied with the purpose of comparing the roll wear predictions calculated from this approach with the ones obtained by the classical calibration approach used above. The ANN, which was coded in Python, computed the corresponding wear for each coil by accounting for roll type/supplier and strip grade factors as well as rolling process parameters, such as hardness, forces, temperatures, speeds, thickness reduction, etc.

First ANN trials to predict wear were not very conclusive, mostly due to the lack of learning data. To improve performances, the Wang’s wear model (cf. Eq. (1)) was exploited and the ANN was then used to estimate the wear parameter \( K \) for each coil. This approach provided much better results, which can be explained as the ANN did not have to learn, from a reduced database, the impact of \( \frac{F_i}{w_i}L_i \) on individual wear. This final ANN structure is represented by Figure 5.

The ANN uses (i) 2 hidden layers of neurons defined by a total of 20 neurons, using selu and sigmoid activation functions, and (ii) recurrent architecture to account for the cumulative wear calculated from previous strips as input data. Recurrence supplies information about the current wear of the roll to the ANN, providing the ability to modulate the individual wear based on the state of the roll. This ANN structure was determined by trials, where the small size of the dataset led to a small network size too.

![Fig. 5. Artificial Neural Network applied to predict work-roll wear amplitude.](image)

The ANN was trained for each roll stand by using the training dataset and was then tested on the validation dataset. Figure 6 shows the wear predictions obtained by the Artificial Neural Network for the validation dataset. In addition, roll wear predictions from the classical calibration (CC) using the Wang’s model and the coefficients parametrization option 1 were also included in this figure. From a qualitative point of view, it appears that similar wear prediction trends were obtained by both ANN and classical calibration approaches. Despite ANN approach is based on a recurrent architecture which accounts for the cumulated wear along the rolling campaign, no clear enhanced reproduction of roll wear for larger wear amplitude values (>300 µm) was observed comparatively to the classical calibration.
Fig. 6. Roll wear amplitudes at stands a) F6 and b) F7 calculated by both Artificial Neural Network (ANN) and Classical Calibration (CC) approaches for the rolling validation dataset.

Table 4 lists the averaged wear gap between wear amplitude measurements and predictions for both training and validation datasets. It can be seen from this table that the ANN led to a much better performance (+25%) than the classical calibration on the roll wear prediction of the training dataset. However, quite similar roll wear predictions were obtained by both approaches for the validation dataset since ANN was just 7.6% better than the classical calibration approach. While the classical calibration approach led to the same performance on both training and validation sets, the ANN approach revealed some overfitting for the training dataset. Nevertheless, this work is a first shot at ANN wear models and additional data could greatly improve their performances, by reducing the overfitting for instance.

Table 4. Averaged wear gap results obtained by both Artificial Neural Network and Classical Calibration approaches.

|                      | Avg. Wear Gap | Units |
|----------------------|---------------|-------|
|                      | Training Set  | Validation Set |
| Classical Calibration (CC) | 46.3          | 45.8   | µm    |
| Artificial Neural Network (ANN) | 34.7          | 42.3   | µm    |
| ∆ prediction         | 25.1          | 7.6    | %     |

The results from the validation dataset show that both approaches had almost equivalent wear prediction performance for this rolling database.

On one side, the classical calibration approach has the advantage of the wear coefficients are known once calibration is made and so, a relationship between these coefficients can be established. It is particularly interesting when plant purchases work-rolls from new suppliers, since wear coefficients for these rolls could be easily assigned after a few rolling campaigns.

On the other side, ANN have important potential for bigger training database and additional data which, assorted with bigger ANN structure, could lead to much better roll wear predictions. The ANN described here shall not be taken as an overall statement on their performances for such wear applications. Further experiments on databases of bigger size shall be pursued.

Thus, the selection of the most suitable approach is then dependent on the user needs and size of the rolling database available for calibration.

Conclusion

In this work, the performance of 3 roll wear models as well as 3 different options to parametrize the unknown wear coefficients of these models were compared by using an industrial rolling database for finishing stands F6 and F7. The roll wear models consisted of contact-based mechanic equations while the wear coefficient parametrization options consisted of distinct parameter sets relating (i) roll stand, (ii) roll type/supplier and (iii) strip grade factors. The performance analysis revealed that:

- averaged wear amplitude gap in-between 46-51 µm were achieved by the 3 wear models
• all the wear models led to underestimated predictions for roll wear amplitudes above 300 μm
• Wang’s wear model with simplest formulation (since defined by less process parameters) was able to give similar or even better results than the 2 other wear models evaluated here
• wear coefficients parametrization distinguishing strip grades could improve roll wear predictions in about 8%

In addition, roll wear predictions obtained from ANN approach were also compared with the ones obtained from a classical calibration approach to identify the most performant one. The developed ANN led to an improvement of the roll wear predictions of about 7.6% for the rolling database used in this work. Tough, enhanced performance of ANN on the roll wear prediction could be expected if larger rolling databases are used.

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