The fate of pion condensation in quark matter: from the chiral limit to the physical pion mass

H. Abuki,1 R. Angliani,2,3 R. Gatto,4 M. Pellicoro,2,3 and M. Ruggieri2,5

1Institut für Theoretische Physik, J.W. Goethe Universität, D-6038 Frankfurth am Main, Germany
2Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Bari, I-70126 Bari, Italy
3Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy
4Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland

We study aspects of the pion condensation in two-flavor neutral quark matter using the Nambu–Jona-Lasinio model of QCD at finite density. We investigate the role of electric charge neutrality, and explicit symmetry breaking via quark mass, both of which control the onset of the charged pion (\(\pi^c\)) condensation. We show that the equality between the electric chemical potential and the in-medium pion mass, \(\mu_e = M_\pi^-\), as a threshold, persists even for a composite pion system in the medium, provided the transition to the pion condensed phase is of the second order. Moreover, we find that the pion condensate in neutral quark matter is extremely fragile to the symmetry breaking effect via a current quark mass \(m\), and is ruled out for \(m\) larger than the order of 10 keV.

PACS numbers: 12.38.Aw, 11.10.Wx, 11.30.Rd, 12.38.Gc

I. INTRODUCTION

In the early 1970s Migdal suggested the possibility of pion condensation in a nuclear medium [1,2]. Since then, many efforts have been made in order to clarify the in-medium pion properties affected by the pion-nucleon interaction [3], because better insight of such condensation phenomena would yield important advances not only in subnuclear physics such as that of pionic atoms [4], but also in the physics of neutron stars [5], supernovas [6], and the heavy ion collisions [7]. From the results of the experiments concerning the repulsive \(\pi N\) interaction, the simplest possibility, the \(S\)-wave pion condensation is highly unlikely to be realized in nature. In contrast, the possibility of \(P\)-wave pion condensation has been widely argued [2,8,9], even though it still remains as a matter of debate if this occurs at several times the ordinary nuclear density. Most of the studies about this issue performed so far are mainly concerned with the role of the coupling between pions and baryons in the nuclear medium [4], considering the pion itself as an elementary object. However, since the pion is considered to be the Nambu-Goldstone boson associated with chiral symmetry breaking, its internal structure and mass may also be sensitive to the modification of the QCD vacuum itself in the finite density environment [10]; the finite baryon density, and even the isospin density arising from the neutrality condition, can modify the structure of the QCD ground state, and it can in turn produce significant modifications of the pion properties in the medium.

In this article we revisit the possibility of \(S\)-wave charged pion (\(\pi^c\)) condensation starting from a microscopic model which is built with quarks as the constituents of pions, and which exhibits chiral symmetry restoration at the finite quark chemical potential \(\mu\) or temperature \(T\). This strategy also enables us to present a unified view on the possible crossover from a Bose-Einstein condensate (BEC) of \(\pi^-\) condensate to a BCS-type \((\bar{u}\gamma_5 d)\) superfluid (BCS-BEC crossover) at finite isospin density [11]. This possibility was first speculated by applying the chiral Lagrangian at low \(\mu_f\) and the perturbative QCD result at high \(\mu_f\) [12]; since then, the QCD phase structure at finite isospin density has been widely investigated using the chiral effective models [13,14,15], a lattice-based model [16], the random matrix model [17], and the lattice QCD simulations [18,19,20].

Since \(\mu\) acts as the external field producing a mismatch in \(u-d\) Fermi surfaces [18,21], our problem also closely intersects with the problem of the BCS-BEC crossover with a density imbalance in the cold atomic systems [22]. Finally, it should be noted that in Refs. [23,24] a window was found for the \(S\)-wave \(\pi^c\) condensate on the finite \(\mu\) axis in the Nambu–Jona-Lasinio (NJL) model. This was called the “gapless pion condensate” since the constituent quark presents a gapless dispersion relation, due to \(u-d\) Fermi surface mismatch. Our purpose here includes clarification of how this can be understood consistently with the number of negative results on the \(S\)-wave pion condensate obtained in the past. To this end, we derive the appropriate criterion for \(\pi^c\) condensation, and based on this criterion, we show that the \(\pi^c\) condensation found in [23] is extremely fragile to explicit chiral symmetry breaking due to a finite current quark mass.

The purpose of this article is twofold. First, we investigate the conditions for the onset of \(\pi^c\) condensation at finite density using the NJL model of QCD; in this model pions are not elementary fields, but they are described as the bound states of quarks. We find that even
in such a composite pion model with dynamical quarks, the threshold for \( \pi^c \) condensation for noninteracting elementary pion gas, \( \mu_e = M_{\pi^-} (\pm \mu_e = M_{\pi^+}) \) for positive (negative) \( \mu_e \) (with \( \mu_e \) being the electric chemical potential), holds as well. This enables us to elucidate the effect of the current quark mass in the neutral ground state, which will be summarized by drawing the phase diagram in the \((\mu, m_c)\) plane, where \( m_c \) denotes the vacuum pion mass. Furthermore we investigate the effect of \( \mu_e \) at the physical value of the current quark mass drawing a phase diagram in the \((\mu, \mu_e)\) plane. Based on these analyses, we conclude that \( \pi^c \) condensation is forbidden in realistic neutral quark matter.

II. THE MODEL

We study two-flavor quark matter at a finite chemical potential within the NJL model. The Lagrangian of the model is given by [25]

\[
\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} (i \gamma^\mu \partial_\mu - \mu) \psi + G \left( \bar{\psi} \gamma_5 \psi \right)^2 + \left( \bar{\psi} \gamma_5 \psi \right)^2 \right] \, ,
\]

(1)

Here \( e \) denotes the electron field, and \( \psi \) is the quark spinor with Dirac, color and flavor indices (implicitly summed). \( m = m_u = m_d \) is the bare quark mass and \( G \) is the coupling constant. \( \mu_e \) is the electric charge chemical potential needed to keep the system electrically neutral [26], while \( \mu_1 \) serves as the isospin chemical potential in the hadron sector, \( \mu_1 = -\mu_e \) since \( Q = \frac{2}{3}B + I_3 \). The quark chemical potential matrix \( \hat{\mu} \) is defined in flavor-color space as \( \hat{\mu} = \text{diag}(\mu - \frac{2}{3} \mu_e, \mu + \frac{1}{3} \mu_e) \odot 1_c \) where \( 1_c \) denotes the identity matrix in color space, \( \mu \) is the quark chemical potential related to the conserved baryon number. By performing the mean field approximation, we examine the possibility that the ground state develops condensation in the \( \sigma = G(\bar{\psi}\psi) \) and/or \( \pi = G(\bar{\psi}I_3 \tau^\mu \psi) \) channels, where \( \tau^\mu = \{ \tau_1, \tau_2, \tau_3 \} \) denotes the Pauli matrices. \(^1\) We find that \( \langle \tau_3 \rangle \) is always zero, as there is no driving force, so we omit it in the following. We use the notation, \( M = m - 2\sigma \) and \( N = 2 \sqrt{\sigma + \frac{1}{2}} \). In the numerical analyses, we fix \( \Lambda = 651 \text{ MeV} \) and \( G = 2.12/\Lambda^2 \) so that the model reproduces \( f_\pi = 92 \text{ MeV}, \langle \bar{u}u \rangle = -(250 \text{ MeV})^3 \), and \( m_\sigma = 139 \text{ MeV} \) in the vacuum with \( m = 5.5 \text{ MeV} \).

III. THE SYMMETRY STRUCTURE

The global symmetry of the model at \( m = m_e = 0 \) is the \( SU_L(2) \times SU_R(2) \) chiral symmetry. This symmetry will be broken spontaneously to \( SU(2) \) by the emergence of a nonzero condensate \( \sqrt{\sigma^2 + \pi^2} \). When the finite quark mass \( m \) is turned on, the original chiral symmetry is explicitly broken to the diagonal subgroup \( SU_{L+R}(2) \). This forces the chiral condensate to the \( \sigma \) direction (vacuum alignment), and the pions as Goldstone bosons acquire a mass, but they remain degenerate in the isospin triplet. The effect of a small \( \mu_1 \) is then to split the masses of the three pions, since the diagonal \( SU_{L+R}(2) \) is broken explicitly to \( U_\tau(1) \), which denotes the rotation about the isospin third axis. When \( \tau_1 \) and/or \( \tau_2 \) condensates (or simply \( N \)) become nonzero, this \( U_\tau(1) \) symmetry gets spontaneously broken and we expect one massless Goldstone boson to appear. It is interesting to note that when the discrete parity is considered together, there remains one discrete symmetry even in the pion condensed phase.

The parity operation acts as \( \pi \rightarrow P\pi = -\pi \). On the other hand, \( Z_2 \subset U_\tau(1) \) defined by \( U \in Z_2 \Leftrightarrow U = e^{i\pi_0}, \) also flips the sign of the \( \pi^c \) condensate. Hence, the combined transformation, a \( \text{rotated parity} \ (P') = P \cdot Z_2, \) remains unbroken. The new symmetry breaking pattern of the \( \pi^c \) condensate can be specified as \( U_\tau(1) \times P \pi^c_{\tau_0} \rightarrow P' \). The new parity \( P' \) is analogous to what is discussed in [27]. Finally, \( \mu \) breaks the charge conjugation, causing stress which produces a mismatch in \( u \) and \( d \) densities that are equal at small (large) \( \mu (\mu_L) \).

IV. PION CONDENSATION AT FINITE \( \mu_1 \) AND AT FINITE \( \mu \)

Before discussing \( \pi^c \) condensation in the medium, it is instructive to review the classical arguments at \( \mu = 0 \) [12]. The shift of the vacuum energy caused by the pion fields at the lowest nonvanishing order in the fields \( \pi \) is \( \delta \Omega = (m_\pi^2 + \mu_\pi^2) \pi_{\tau_0}^2/4 + (m_\pi^2 - \mu_\pi^2) \pi_{\tau_+} \pi_{\tau_-}/2 \). From this we can infer that in the chiral limit \( (m_\pi = 0) \) an infinitesimally small value of \( \mu_1 \) favors condensation, \( \langle \pi_+ \pi_- \rangle \neq 0 \). In the general case of \( m_\pi \neq 0 \) and \( \mu_\pi \neq 0 \), the condition \( |\mu_e| > m_\sigma \) needs to be fulfilled so that the \( \pi^c \) condensation turns out to be energetically favored. In this kind of approach, using chiral perturbation theory, the chiral \( SU(2) \) multiplets are elementary fields whose internal structures do not suffer from any quantum corrections. On the other hand, the same threshold at \( \mu = 0 \) is obtained also in the NJL-type model [11].

We now consider neutral quark matter at \( \mu 
eq 0 \) and \( T = 0 \). We are interested in the relation between the threshold of \( \pi^c \) condensation at finite density and the in-medium pion masses in the neutral ground state. We have shown in Ref. [26] that at the physical point \( m = 5.5 \text{ MeV} \), there is no room for \( \pi^c \) condensation in the neutral phase. Similar conclusion is also obtained in [21]. This picture changes if we lower the current quark mass. We discuss this point in detail later. For our present purpose it is enough to state that we need a current quark mass of the order of 10 keV. In Fig. 1 \( M \) and \( N \) in the neutral phase as a function of \( \mu \) for \( m = 10 \)

\(^1\) In this study we restrict ourselves to the homogeneous ground state, although some interesting possibilities of spatially modulated condensate are argued in Ref. [26].
for simplicity. For small $N$, the pole condition is $\Gamma_{\pi^+\pi^-}(Q_0; \mu, \mu_c, T) = 0$ with $\Gamma_{\pi^+\pi^-}(Q_0; \mu, \mu_c, T) = \frac{m}{2G_M} - N_c Q_0^2 J(Q_0; \mu, \mu_c, T)$ being the inverse $\pi^+$ propagator, $Q_0 = \omega - \mu_c$, $N_c$ being the number of colors, and $J$ being the polarization defined by

$$J(Q_0; \mu, \mu_c, T) = \int_{-\infty}^{\infty} dE \frac{\pi^{+M} - \pi^{-M}}{2\pi^2} \frac{1}{E^2 - Q_0^2/4} \left[ 1 - \sum_{\ell=\pm} \left( \frac{Q_0 + 2\ell E}{2Q_0} f_F(E - t\mu_e) + \frac{Q_0 - 2\ell E}{2Q_0} f_F(E - t\mu_d) \right) \right],$$

where $f_F(x)$ is the Fermi distribution. We note that the polarization function is not symmetric under $Q_0 \to -Q_0$ if there is a finite isospin density in the system, $(\bar{\psi}^T \tau_3 \psi) \neq 0$; this asymmetry is responsible for the mass splitting of $M_{\pi^+}$ and $M_{\pi^-}$. The excitation gaps for $\pi^+$ and $\pi^-$ are accordingly given by $(M_{\pi^+} + \mu_c)$ and $(M_{\pi^-} - \mu_c)$, corresponding to positive and negative solutions to the BS equation in $\omega$. From Fig. 1 we notice that the transition to the pion condensed phase is of second order and it occurs at the point where $M_{\pi^-} = \mu_c$.

The numerical results shown in Fig. 1 can be understood by a closer inspection to the BS equation and the gap equation for pion condensate. Although the argument below applies even to the $T \neq 0$ case, we set $T = 0$ for simplicity. For small $N$, we expand the free energy of the ground state up to fourth order in the charged pion condensate $N = 2\sqrt{2\pi^+ \pi^-}$,

$$\Omega = \Omega_0 + \alpha(\mu_c) \frac{N^2}{2} + \beta(\mu_c) \frac{N^4}{4},$$

where $\Omega_0$ is the free energy at $N = 0$. It can be shown that the second order coefficient $\alpha$ has the same structure as the $\pi^+$ propagator, with $Q_0$ replaced by $-\mu_c$, i.e., $\alpha(\mu_c) = \frac{1}{2} \Gamma_{\pi^+\pi^-}(-\mu_c, \mu_c)$. Provided that the transition to the $\pi^-$ condensed phase is of second order, the critical condition is given by the linearized gap equation, $\Omega(\mu_c) = 0$, that determines the line of the critical electric chemical potential $\mu_c^c(\mu_c)$ in the $\mu_c$ plane. From the equality of the pion propagator and $\alpha$ noted above, one immediately obtains the threshold $M_{\pi^+} = \pm \mu_c$ for positive (negative) $\mu_c$. This is because $\Gamma_{\pi^+\pi^-}(-\mu_c, \mu_c) = 0$ is also satisfied on the critical line $\mu_c^c(\mu)$, and this solution corresponds to the negative (positive) root of the BS when $\mu_c > 0$ ($\mu_c < 0$).

The above argument holds either with or without the neutrality condition. The neutrality condition determines the neutrality line $\mu_c^\text{neut}(\mu_c)$ in the $\mu_c$ plane. At $m = 10$ keV, $\mu_c^c(\mu_c)$ and $\mu_c^\text{neut}(\mu_c)$ intersect at the point $(\mu_c, \mu_c) \equiv (324, 10)$ MeV. This fact is somehow implicit in Fig. 1 where the physical quantities only along $\mu_c^\text{neut}(\mu_c)$ is sketched. We see from the figure that, as soon as $\mu$ exceeds the value of the vacuum constituent quark mass $M = 310$ MeV, $\mu_c$ becomes finite due to the appearance of quark Fermi surfaces. Accordingly, the mass degeneracy in the pion sector gets lifted.

When the critical point $\mu_c = 324$ MeV ($\equiv \mu_{c1}$, hereafter) is reached, $\mu_c$ hits $M_{\pi^-}$ from below, resulting in the gapless excitation in the $\pi^-$ mode. Thereafter, $\pi^-$ mode in the $N = 0$ phase changes into the exact Nambu-Goldstone boson associated with $\tau_3(1)$ breaking; the excitation stays gapless due to the mixing of $(\sigma, \pi^+, \pi^-)$ modes in the BS equation for $N \neq 0$ [11].

We also notice that the increase of $\mu_c$ is somehow quenched as soon as the system experiences the transition to the pion condensed phase. This is reasonable because the $\pi^-$ condensate tries to accommodate the $d$ quark Fermi surface that helps cancel the positive charge

---

2 The critical point $\mu_{c1}$ for the transition from the chiral symmetry broken phase to the pion condensed phase corresponds to the critical baryon density $n_B \sim 0.1 n_s$, with $n_s$ being the nuclear saturation density $0.16$ fm$^{-3}$. The electron fraction is $Y_e \equiv n_e/n_B \sim 0.0006$. These values are unconventionally small, which is simply attributed to the fact that we are working near the unphysical chiral limit, $m = 10$ keV.
arising from $u$ quarks; then less electrons are needed. The pion condensed phase continues up to $\mu = 330$ MeV ($\equiv \mu_{c1}$, hereafter) where the system encounters the first order phase transition into the almost chirally symmetric quark matter phase; the phase is characterized by $M \sim m$. Since we are working near the chiral limit $m = 10$ keV, the mass splitting in the sigma and neutral pion sector is tiny. In fact $M_{\pi\sigma}$ is only slightly larger than $M_{\pi}$; the difference is less than 1 MeV which is actually invisible in the figure.

A few remarks are in order with regards to the nature of pion condensed phase obtained here: (i) Having a look at the fermionic spectrum, we can see that it is of gapless type \[23\]. (ii) An investigation of the spectral functions in the bosonic sector leads us to conclude that it is the BEC of the real $u$ quark matter phase; the phase is characterized by $\mu_{c1}$.

Then, \( (\omega = 0) \) in the $\pi^-$ propagator is realized as an isolated bound state pole if $2M > \mu_c$, while in the opposite case it is realized as a soft mode peak whose width goes to zero only when its momentum $Q$ goes to zero. Actually the former is realized here, so the system is the BEC; the isospin chemical potential $|\mu_{c1}| = \mu_c$ is too small to realize the BCS-like superfluid state $\text{23, 28}$. Let us now turn to point (i). The dispersion relations for quasiquarks in the pion condensed phase are given by

\[
E_{\bar{u}}(p) = \sqrt{(\sqrt{M^2 + p^2} + \mu_c/2)^2 + N^2} - (\mu - \mu_c/6),
\]

\[
E_{\bar{d}}(p) = \sqrt{(\sqrt{M^2 + p^2} - \mu_c/2)^2 + N^2} - (\mu - \mu_c/6),
\]

where the subscript $\bar{u}$ ($\bar{d}$) represents the quasiquark which has a large $u$-quark ($d$-quark) content at large $p$.

Since we have already noted that $M > \mu_c/2$ in the BEC, each quasiquark energy has a minimum at $p = 0$. Then we see that both quasiquarks have a blocking region in $p$-space, defined by the condition $E_{\bar{u}/\bar{d}}(p) < 0$; the constituent $u$ and $d$ quarks are, accordingly, accumulated in those regions. From (i) and (ii), we may conclude that the system is the BEC of the real $\bar{u}\bar{d}$ bound state ($\pi^-$) formed in the charge neutral background characterized by the Fermi seas of $u$, $d$ quarks, and electrons.

V. THE ROLE OF THE CURRENT QUARK MASS

In the past years, a large number of the calculations dealing with the issue of $\pi^-$ condensation have been performed in the chiral limit. This treatment is simple from the point of view of calculations but somehow unphysical. In this section we investigate the role of the current quark mass in $\pi^-$ condensation. In this analysis we fix the cutoff $\Lambda$ and the coupling $G$ to the values specified above and we treat $m$ as a free parameter. As a consequence, the pion mass at $\mu = T = 0$, $m_{\pi}$ in the following, is a free parameter as well. In Fig. 2 we report the phase diagram in $(\mu, m_{\pi})$ plane in the neutral case. The solid line represents the border between the two regions where chiral symmetry is broken and restored. The bold dot is the critical endpoint of the first order transition. The shaded region indicates the region where $\pi^-$ condensation occurs. In the chiral limit ($m_{\pi} = 0$) our results are in good agreement with those obtained in Ref. \[23\]. As we discussed in the previous section, there exist two critical values of the quark chemical potential, $\mu_{c1}$ and $\mu_{c2}$, corresponding to the onset and vanishing of $\pi^-$ condensation, respectively. Increasing the current quark mass results in the shrinking of the shaded region till the point $\mu_{c1} \equiv \mu_{c2}$ for $m_{\pi} \sim 9$ MeV, corresponding to a current quark mass of $m \sim 10$ keV. Hence we have shown that the gapless $\pi^-$ condensation is extremely fragile with respect to the symmetry breaking effect of the current quark mass; this can be understood by observing the fact that the increase of quark mass leads to a drastic magnification of the vacuum pion mass at small $m$ because of $m_{\pi} \propto \sqrt{m}$, while the change in quark mass in keV scale brings about no significant modification in $\mu_c$.

VI. THE PHASE DIAGRAM IN THE PHYSICAL CASE

As a final investigation, in Fig. 3 we draw the phase diagram of neutral matter in $(\mu, m_{\pi})$ plane in the physical limit where the current quark mass is tuned to $m = 5.5$ MeV. At each value of $(\mu, m_{\pi})$ we compute the chiral and pion condensates by minimization of the thermodynamical potential. The solid line represents the first or-
order transition from the $\pi^c$ condensed phase to the chiral symmetry broken phase without the $\pi^c$ condensate. The bold dot is the critical endpoint for the first order transition, after which the second order transition sets in. The dashed line indicates the first order transition between the two regions where chiral symmetry is broken and restored, respectively. The dot-dashed line is the neutrality line manifestly shows the impossibility of finding a $\pi^c$ condensate in a physical situation.

VII. CONCLUSIONS

In this article we have studied the phenomenology of $\pi^c$ condensation in two-flavor neutral quark matter using a Nambu–Jona-Lasinio model of QCD. In particular, we have clarified the role of the quark mass $m$ and the electric chemical potential $\mu_e$. Our central results is that the threshold to the $\pi^c$ condensed phase, $\mu_e = M_{\pi^c}$, persists even for a composite pionic gas in the presence of dynamical quarks provided the transition to the $\pi^c$ condensed phase is of second order, which is indeed true up to $\mu \approx 300$ MeV. Furthermore, we have found that the possibility of $\pi^c$ condensation is ruled out in neutral quark matter with a current quark mass larger than the order of 10 keV. We have given a natural explanation for this fact by using the threshold criterion noted above. As a further result we have computed the phase diagram in the $(\mu, \mu_e)$ plane at the physical quark mass. We find that the transition to the $\pi^c$ condensed phase is of second order up to $\mu \approx 300$ MeV, after which it turns into a first order one. By superimposing the neutrality line onto the phase diagram, we have shown that neutral quark matter never meets the regions of the nonvanishing $\pi^c$ condensate. Thus, we conclude that in neutral quark matter at the physical quark mass, the NJL model does not allow charged pion condensation in the ground state.

In the present analyses, we have focused on the modification of pion propagation due to the character change of the QCD ground state with respect to the finite baryon (and isospin) density environment. It should be noted, however, that in our restricted treatment, the contribution to finite baryon density comes only from constituent quarks and not from nucleons. The existence of nucleons, the bound states of constituent quarks, should make S-wave $\pi^c$ condensation even less favorable. This is because the Tomozawa-Weinberg interaction between the nucleon and the pion is isospin odd, giving rise to repulsive pion self-energy at the physical situation where the neutron density is larger than the proton density, i.e., $\langle n^1n \rangle > \langle p^1p \rangle$. It would be interesting, though challenging, to investigate pion condensation by taking into account nucleon degrees of freedom within the NJL model. The prescription made in [31] may be useful.

Acknowledgements.— We thank D. Blaschke, T. Brauner, P. Colangelo, K. Fukushima, D. Rischke, A. Sedrakian, A. Ohnishi, and T. Tatsumi for a careful reading of the manuscript. The work of H. A. was supported by the Alexander von Humboldt Foundation. Part of numerical calculations was performed using the facilities of the Frankfurt Center for Scientific Computing.

[1] A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2210 (1971) [Sov. Phys. JETP 36, 1052 (1973)]; A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rept. 192, 179 (1990).
[2] R.F. Sawyer, Phys. Rev. Lett. 29, 382 (1972); D.J. Scalapino, Phys. Rev. Lett. 29, 386 (1972). G. Baym and D.K. Campbell in: Mesons and Nuclei, v. 3, eds. M. Rho and D. Wilkinson (North Holland, 1979).
[3] T. Kumihiro, T. Muto, R. Tamagaki, T. Tatsumi and T. Takatsuka, Prog. Theor. Phys. Suppl. 112, 1 (1993).
[4] H. Toki, S. Hirenzaki, T. Yamazaki and R. S. Hayano, Nucl. Phys. A 501, 653 (1989); P. Kienle and T. Yamazaki, Prog. Part. Nucl. Phys. 52, 85 (2004).
[5] J. B. Hartle, R. F. Sawyer and D. J. Scalapino, Astrophys. J. 199, 471 (1975); A. B. Migdal, G. A. Sorokin, O. A. Markin and I. N. Mishustin, Phys. Lett. B 65, 423 (1976); O. Maxwell, G. E. Brown, D. K. Campbell, R. F. Dashen and J. T. Manassah, Astrophys. J. 216, 77 (1977).
[6] C. Ishizuka, A. Ohnishi, K. Tsubakihara, K. Sumiyoshi and S. Yamada, J. Phys. G 35 (2008) 085201.
[7] J. Zimanyi, G. I. Fai and B. Jakobsson, Phys. Rev. Lett. 43, 1705 (1979); S. Krewald and J. W. Negele, Phys. Rev. C 21, 2385 (1980).
[8] T.E.O. Ericson and W. Weise, Pions and Nuclei (Oxford University Press, Oxford, 1988).
[9] T. Wakasa et al., Phys. Rev. C 55, 2909 (1997); T. Suzuki and H. Sakai, Phys. Lett. B 455, 25 (1999).
[10] T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994); S. P. Klevansky, Rev. Mod. Phys. 64 (1992) 649.
[11] L. y. He, M. Jin and P. f. Zhuang, Phys. Rev. D 71, 116001 (2005).
[12] D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 86, 592 (2001).
[13] D. Toublan and J. B. Kogut, Phys. Lett. B 564, 212 (2003); A. Barducci, R. Casalbuoni, G. Pettini and L. Ravagli, Phys. Rev. D 69, 096004 (2004); L. He and P. Zhuang, Phys. Lett. B 615, 93 (2005); H. J. Warringa, D. Boer and J. O. Andersen, Phys. Rev. D 72, 014015 (2005).
[14] H. Mao, N. Petropoulos and W. Q. Zhao, J. Phys. G 32, 2187 (2006); J. O. Andersen, Phys. Rev. D 75, 065011 (2007).
[15] J. O. Andersen and T. Brauner, Phys. Rev. D 78, 014030 (2008).
[16] Y. Nishida, Phys. Rev. D 69, 094501 (2004).
[17] B. Klein, D. Toublan and J. J. M. Verbaarschot, Phys. Rev. D 72, 015007 (2005).
[18] J. B. Kogut and D. K. Sinclair, Phys. Rev. D 66, 034505 (2002); Phys. Rev. D 70, 094501 (2004).
[19] P. de Forcrand, M. A. Stephanov and U. Wenger, PoS LAT2007, 237 (2007).
[20] W. Detmold, K. Orginos, M. J. Savage and A. Walker-Loud, Phys. Rev. D 78, 054514 (2008).
[21] L. He, M. Jin and P. Zhuang, Phys. Rev. D 74, 036005 (2006).
[22] S. Giorgini, L. P. Pitaevskii and S. Stringari, Rev. Mod. Phys. 80, 1215 (2008).
[23] D. Ebert and K. G. Klimekno, Eur. Phys. J. C 46, 771 (2006).
[24] J. O. Andersen and L. Kjellingstad, arXiv:hep-ph/0701033.
[25] H. Abuki, M. Ciminale, R. Gatto, N. Depolito, G. Nardulli and M. Ruggieri, Phys. Rev. D 78, 014002 (2008); H. Abuki, M. Ciminale, R. Gatto, G. Nardulli and M. Ruggieri, Phys. Rev. D 77, 074018 (2008); H. Abuki, R. Anglani, R. Gatto, G. Nardulli and M. Ruggieri, Phys. Rev. D 78, 034034 (2008).
[26] E. Nakano and T. Tatsumi, Phys. Rev. D 71, 114006 (2005).
[27] A. Kryjevski, D. B. Kaplan and T. Schafer, Phys. Rev. D 71, 034004 (2005).
[28] Y. Nishida and H. Abuki, Phys. Rev. D 72, 096004 (2005); H. Abuki, Nucl. Phys. A 791, 117 (2007).
[29] G. f. Sun, L. He and P. Zhuang, Phys. Rev. D 75, 096004 (2007).
[30] For review, see for example, T. Muto, Prog. Theor. Phys. Suppl. 153, 174 (2004).
[31] W. Bentz, T. Horikawa, N. Ishii and A. W. Thomas, Nucl. Phys. A 720, 95 (2003).