Novel Design of a 3-RRUU 6-DOF Parallel Manipulator

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Abstract. A novel 3-RRUU 6-DOF parallel manipulator is designed based on the multi-plane design principle, that provides larger workspace, less chance of link interference and simplified kinematic analysis. The link interference detection by pentagon identification and cross-product is demonstrated. Workspace is optimized by different orientations. The Jacobian matrix by global cartesian coordinate system is developed. A transformation matrix could convert that to Jacobian matrix of a new coordinate system.

1. Introduction

A parallel manipulator has a moving platform that is actuated and connected to the base by more than one limb. This structure has higher stiffness and precision but smaller workspace [1].

Six limbed 6-DOF parallel manipulators, such as 6-UPS [2] and 6-RUS [3], are common designs. There are some topology methods [4] [5] [6] for chain design of unique applications, that include three limbed 6-DOF manipulators with serial chain for each limb [7] [8] [9] [10]; three limbed 6-DOF manipulators with parallel actuation in each limb [11] [12] [13] [14]. Compared to the six limbed counterparts, three limbed 6-DOF manipulators usually have larger workspace and less link interference [7] [8] [9] [10] [13]. Although the chance of interference is lower, some three limbed 6-DOF manipulators [12] [14] still need to detect such conditions, generally by determining the link to link distance. The RHHR chain in [13] is free from interference, while the spherical joints connected to the moving platform might limit the motion range (e.g. the rotation around z axis).

This paper discusses the multi-plane design principle. Different from [11] where the links connected to the platform have their motions restricted in the planes perpendicular to the platform plane, this paper suggests deploying these links in three intersecting planes that coincide at a line which is perpendicular to the horizontal base plane and passes platform equilateral triangle center. The links are designed similarly as to the PRU link in HALF and HANA structure [15], this avoids using spherical joints with limited motion range [16]. The five-bar robot [17] with large workspace is employed for the limb movement. The actuating five-bar links are on the horizontal base plane. This manipulator also has the lie-flat function [18] [19] that protects it from outdoor weather damages.

2. Mechanical Design

The 3-RRUU parallel manipulator has 6 DOFs. Figure 1 shows (a) the structure and (b) the kinematic chains represented by topology diagram [20]. A structure could also be expressed by a notation [14]. Therefore the structure is denoted as 3-[(2-RR)UU], where underlined joints are actuated. Each limb has two motors $A_{Ap[i]}$ and $A_{Aq[i]}$ in hybrid chain. This increases the precision and reduces the bending of link $B_{pi}$ and $B_{qi}$. The revolute joints $R_{Ap[i]}$ and $R_{Aq[i]}$ are driven. The revolute joint $R_{Ci}$
connects to the universal joint $U_{[Di]}$ which consists two revolute joints and one of these is colinear to $R_{[Ci]}$.

![Figure 1. Mechanical design.](image)

3. Inverse Kinematics

This chapter gives the calculation of link positions, and detection of link interference.

3.1. Link Positions

The global cartesian coordinate is at base center $A_o: [0 \ 0 \ 0]^T$ (figure 2. (a)).

\[
X_o = [1 \ 0 \ 0]^T, Y_o = [0 \ 1 \ 0]^T, Z_o = [0 \ 0 \ 1]^T
\]

\[
A_{pi} = r_e \cdot [\cos(\alpha_i - \beta) \ \sin(\alpha_i - \beta)]^T
\]

\[
A_{qi} = r_e \cdot [\cos(\alpha_i + \beta) \ \sin(\alpha_i + \beta)]^T
\]

The pose of the moving platform (an equilateral triangle) is defined by its orientation $\theta_e$ and position $E_o$ from global coordinate.

\[
\theta_e = [\theta_x \ \theta_y \ \theta_z]^T \text{ and } E_o = [x_e \ \ y_e \ \ z_e]^T
\]

Let $\theta_e = \sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2}$, so that $u_x = \theta_x/\theta_e$, $u_y = \theta_y/\theta_e$, $u_z = \theta_z/\theta_e$.

With orientation matrix $R$, the positions $E_i$ are calculated, where the universal joints are located with one rotational axis along symmetry lines $E_iE_o$ and the other rotational axis along $Y_{di}$. The platform has a normal vector $N_e$ that points upward.

\[
R = \begin{bmatrix}
    c \theta_e + u_x u_z^2(1 - c \theta_e) & u_x u_y(1 - c \theta_e) - u_z s \theta_e & u_x u_e(1 - c \theta_e) + u_y s \theta_e \\
    u_x u_y(1 - c \theta_e) + u_z s \theta_e & c \theta_e + u_y^2(1 - c \theta_e) & u_y u_e(1 - c \theta_e) - u_x s \theta_e \\
    u_x u_z(1 - c \theta_e) - u_y s \theta_e & u_y u_z(1 - c \theta_e) + u_x s \theta_e & c \theta_e + u_z^2(1 - c \theta_e)
\end{bmatrix}
\]

\[
E_i = R \cdot r_e \cdot [\cos(\alpha_i) \ \sin(\alpha_i) \ 0]^T + E_o
\]

\[
N_e = \frac{E_o \times E_1 \times E_2}{|E_o \times E_1 \times E_2|}
\]

\[
Y_{di} = Z_o \times E_i E_o \text{ and } X_{di} = Y_{di} \times Z_o
\]
Figure 2. Kinematics analysis.

As in figure 2. (b), the universal joints located at \( E_i \) and \( D_i \) each has a rotational axis along \( Y_{di} \). Links \( E_iD_i \) and lines \( E_iE_o \) lie in plane \( B_i \) which is perpendicular to horizontal plane \( C \) and horizontal plane \( D \). Let \( E_i \) be projected along \(-Z_o\) on plane \( D_o \), where \( E_oD_o \) is the intersecting line of plane \( B_1 \), plane \( B_2 \) and plane \( B_3 \). One obtains \( h_i \) and \( b_i \) in plane \( B_i \).

\[
h_i = E_i \cdot Z_o - d \quad \text{(8.a)}
\]

\[
b_i = \sqrt{e^2 - h_i^2} \quad \text{(8.b)}
\]

Position \( D_i \) indicates where the universal joint is located. The universal joint has one rotational axis along \( Z_o \) and the other rotational axis along \( Y_{di} \). As in figure 2. (c), let the five-bar mechanisms be in the plane \( C \) where the limbs are actuated. The position \( C_i \) indicates the location of revolute joint. The revolute joints rotate around \( Z_o \), and colinear with the universal joints at \( D_i \).
\[ D_i = E_i + b_i X_{di} - h_i Z_o \]
\[ C_i = D_i - d Z_o \]

The \( B_{pi} \) and \( B_{qi} \) represent the locations of two revolute joints, rotational axis along \( Z_o \).

\[ p_i = |A_{pi} C_i| \quad \text{and} \quad q_i = |A_{qi} C_i| \]  
(11.a)
\[ y_{pi} = \cos^{-1} \left( \frac{b_i^2 + p_i^2 - c_i^2}{2 b p_i} \right) \quad \text{and} \quad y_{qi} = \cos^{-1} \left( \frac{b_i^2 + q_i^2 - c_i^2}{2 b q_i} \right) \]  
(11.b)
\[ R_{pi} = \begin{bmatrix} \cos y_{pi} & \sin y_{pi} & 0 \\ -\sin y_{pi} & \cos y_{pi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad R_{qi} = \begin{bmatrix} \cos y_{qi} & -\sin y_{qi} & 0 \\ \sin y_{qi} & \cos y_{qi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  
(11.c)
\[ B_{pi} = R_{pi} \cdot A_{pi} C_i \cdot \frac{b}{p_i} + A_{pi} \quad \text{and} \quad B_{qi} = R_{qi} \cdot A_{qi} C_i \cdot \frac{b}{q_i} + A_{qi} \]  
(11.d)

One could calculate actuation angles \( \alpha_{pi} \) and \( \alpha_{qi} \) by \( A_{pi} B_{pi} \) or \( A_{qi} B_{qi} \).

### 3.2. Link Interference

There are some literatures [12] [14] [21] [22] [23] discussing the link interference detection, where the main algorithm is to keep safe distance between links. One needs to check that three five-bar mechanisms have no link interference. The interference detection in this analysis has three sections.

Section 1 is to limit \( \alpha_{pi} \) and \( \alpha_{qi} \) ranges so that the moving sides of the link will not collide with the base arc of a neighboring link. Such as in figure 3. (a), the side of \( A_{qi} B_{qi} \) interferes with the arc of \( A_{pi} \). Let \( a_{in} = 2 r_a \sin \beta \) and \( a_{ex} = 2 r_a \sin (\frac{\pi}{3} - \beta) \) that are the lengths of hexagon sides. One needs to verify that \( a_{in} > 2 r_m \) and \( a_{ex} > 2 r_m \) so that it has enough distance for two motors with radius \( r_m \).  

\[ a_{min} = \sqrt{b^2 + 4 r_b^2} \quad \text{and} \quad a_{max} = b + 2 r_b \]  
(12.a)
\[ y_{in} = g(a_{in}) \quad \text{and} \quad y_{ex} = g(a_{ex}) \]  
(12.b)

| Conditions | \( g(x) \) |
|------------|-------------|
| \( x \in (0, a_{min}] \) | \( \sin^{-1} \left( \frac{2 r_b}{a_x} \right) \) |
| \( x \in [a_{min}, a_{max}] \) | \( \cos^{-1} \left( \frac{b^2 + a_x^2 - 4 r_b^2}{2 \cdot a_x \cdot b} \right) \) |
| \( x \in [a_{max}, \infty) \) | 0 |

Table 1 gives \( y_{in} \) and \( y_{ex} \) as offsets on \( \alpha_{pi} \) and \( \alpha_{qi} \), to be interference free for section 1.

\[ \alpha_{pi} \text{ and } \alpha_{qi} \in \left[ y_{in}, \frac{240}{180} \pi - y_{ex} \right] \]  
(12.c)

Section 2 is to wipe out the conditions where \( \alpha_{pi} \) and \( \alpha_{qi} \) are within ranges but there exists intersection of links in a loop (see loop 1 and loop 2 in figure 2. (d)). An algorithm of pentagon identification could easily find intersection. For concave or convex pentagons, the sum \( \theta_i \) should be \( 3\pi \) to be interference free for section 2.

\[ \theta_i = \alpha_{pi} + \beta_{qi} + \alpha_{pi} + \beta_{qi} + y_i \]  
(13)
Section 3 is to avoid the moving arc of a link interfering with the moving side of a link in another loop. Such as in figure 3. (a), the arc of $\mathbf{B}_{p1}$ interferes with the side of $\mathbf{A}_{q3}\mathbf{B}_{q3}$. For when $\alpha_{pi} + \alpha_{qk} > 5\pi/3$ ($i$ and $k$ index two loops in clockwise order), the cross-product method could conveniently calculate the distance between the two. The link $\mathbf{B}_{pi}\mathbf{C}_i$ or $\mathbf{B}_{qk}\mathbf{C}_i$ are in different levels, thus are exempted form examination. The criteria in table 2 need to be met to be interference free for section 3.

Table 2. Cross product criteria.

| Conditions | Criteria |
|------------|----------|
| $\alpha_{pi} > \alpha_{qk}$ | $Z_o \cdot (\mathbf{B}_{pi}\mathbf{A}_{qk} \times \mathbf{B}_{pi}\mathbf{B}_{qk}) - 2r_bb > 0$ |
| $\alpha_{pi} < \alpha_{qk}$ | $Z_o \cdot (\mathbf{B}_{qk}\mathbf{B}_{pi} \times \mathbf{B}_{qk}\mathbf{A}_{pi}) - 2r_bb > 0$ |

4. Workspace Optimization

The workspace is evaluated by setting three orientations $\theta_{ej}$ ($j = 1, 2, 3$) of the platform, and then searching for all eligible positions of the $\mathbf{E}_o$ within the searching ranges. The number $n_{ej}$ of the eligible positions indicate the volume of the workspace by orientation $\theta_{ej}$.

Considering the efficiency of workspace (largest workspace made by minimum limb sizes), cost function $f_j$ is taken. Pareto method [24] is used to search for the best result of each cost function with non-sacrifice to another cost function.

$$f_j = -\frac{n_{ej}}{(\Sigma_{i=1}^n x_i)^3}$$

The basic modeling parameters are given in table 3. Table 4 gives the ranges of the optimization variables $x_1$-$x_6$, considering that each revolute or universal joint is at least 0.05 m from the center of a body it connects to. Table 5 gives the three orientations, and the $\mathbf{E}_o$ search ranges by each orientation.

Table 6 shows the results of the optimization and the cost function value, where $F = f_1 + f_2 + f_3$. The four results are selected from the final optimization solutions, with result 1 for the best of $f_1$; result 2 for the best of $f_2$; result 3 for the best of $f_3$; and result 4 for the best of $F$ overall. The result 4 is chosen for the full workspace plotting in $\theta_{e1}$, $\theta_{e2}$ and $\theta_{e3}$ in figure 4.
Table 3. Modeling parameters.

| Parameters | Values | Units |
|------------|--------|-------|
| $d$        | 0      | m     |
| $r_b$      | 0.015  | m     |
| $r_c$      | 0.010  | m     |
| $r_m$      | 0.025  | m     |
| $\alpha_i$ | $(4i - 1) \cdot \pi / 6$ | rad |

Table 4. Optimization variables.

| Variables | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\beta$ |
|-----------|-------|-------|-------|-------|-------|-------|---------|
| Ranges    | [0.1, 0.7] | [0.05, 0.7] | [0.1, 0.7] | [0.05, 0.7] | [0.1, 0.7] | [0.1, 0.7] | $[\pi / 18, 5\pi / 18]$ |
| Units     | m     | m     | m     | m     | m     | m     | rad     |

Table 5. Motion and search ranges.

| Orientations and positions | Values | Units |
|----------------------------|--------|-------|
| $\theta_{e1}$             | [0 0 0] | rad   |
| $\theta_{e2}$             | $[\pi / 6 0 0]$ | rad   |
| $\theta_{e3}$             | [0 0 $\pi / 6$] | rad   |
| $x_e$                     | -0.1: 0.05: 0.1 | m     |
| $y_e$                     | -0.1: 0.05: 0.1 | m     |
| $z_e$                     | 0.1: 0.05: 0.3  | m     |

Table 6. Optimization results.

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $f_1$ | $f_2$ | $f_3$ | $F$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
| Result 1 | 0.361 | 0.051 | 0.102 | 0.207 | 0.218 | 0.296 | -126.574 | -89.204 | -309.804 |
| Result 2 | 0.345 | 0.051 | 0.102 | 0.208 | 0.219 | 0.303 | -119.825 | -119.825 | -99.644 | -339.294 |
| Result 3 | 0.386 | 0.051 | 0.079 | 0.245 | 0.232 | 0.344 | -120.724 | -85.939 | -111.52 | -318.179 |
| Result 4 | 0.348 | 0.051 | 0.102 | 0.205 | 0.219 | 0.300 | -121.277 | -117.488 | -101.07 | -339.829 |

Figure 4. Full workspace in $\theta_{e1}$, $\theta_{e2}$, $\theta_{e3}$ of result 4.

5. Jacobian-based Stiffness
In figure 5. (a), from the global cartesian coordinate at $A_o$, the moving platform center $E_o$ has a linear velocity $v_{eo}$ and an angular velocity $\omega_{eo}$, that contribute to the linear velocity at $E_1$. The linear velocity at $D_1$ could be divided to $v_{xdi}$ and $v_{ydi}$, which are along $X_{di}$ and $Y_{di}$ respectively.

A method of dot multiplication [25] [26] can be used since $v_{xdi}$ is not perpendicular to $E_1D_i$.  

![Figure 4](image-url)
\[ E_i D_i \cdot (v_{eo} + \omega_{eo} \times E_a E_i) = E_i D_i \cdot v_{xdt} \]  

(15.a)

So that

\[ v_{xdt} = \frac{E_i D_i (v_{eo} + \omega_{eo} \times E_a E_i)}{E_i D_i X_{dt}} \cdot X_{dt} \]  

(15.b)

The angular velocity \( \omega_{eo} \) could be divided to \( \omega_{e1}, \omega_{y1} \text{ and } \omega_{z1} \), that are along \( E_a E_i, Y_{dt} \text{ and } Z_o \) respectively. Components \( \omega_{e1} \text{ and } \omega_{y1} \) have no contribution to \( v_{ydi} \), except \( \omega_{z1} \).

\[ v_{eo} \cdot Y_{dt} \cdot Y_{dt} + \omega_{z1} \times E_o D_i = v_{ydi} \], where \( \omega_{z1} = \left( \omega_{eo} \cdot Z_o - \frac{E_o E_i Z_o}{E_o E_i X_{dt}} \cdot \omega_{eo} \cdot X_{dt} \right) \cdot Z_o \]  

(16.a)

So that

\[ v_{ydi} = v_{eo} \cdot Y_{dt} \cdot Y_{dt} + \left( \omega_{eo} \cdot Z_o - \frac{E_o E_i Z_o}{E_o E_i X_{dt}} \cdot \omega_{eo} \cdot X_{dt} \right) \cdot Z_o \times E_o D_i \]  

(16.b)

The angular velocity \( \omega_{pi} \cdot Z_o \text{ and } \omega_{qi} \cdot Z_o \) are the actuation angular velocities at \( A_{pi} \text{ and } A_{qi} \).

\[ B_{pi} C_{i} \cdot (v_{xdi} + v_{ydi}) = B_{pi} C_{i} \cdot \left( Z_o \times A_{pi} B_{pi} \right) \cdot \omega_{pi} \]  

(17)

So that,

\[ \omega_{pi} = J_{pvi}^T \cdot v_{eo} + J_{pwi}^T \cdot \omega_{eo} \]  

(18.a)

where,

\[ J_{pvi} = \frac{B_{pi} C_i X_{di}}{B_{pi} C_i (Z_o \times A_{pi} B_{pi})} \cdot \frac{E_i D_i}{E_i D_i X_{di}} + \frac{B_{pi} C_i Y_{di} Y_{dt}}{B_{pi} C_i (Z_o \times A_{pi} B_{pi})} \]  

(18.b)

\[ J_{pwi} = \frac{B_{pi} C_i X_{di}}{B_{pi} C_i (Z_o \times A_{pi} B_{pi})} \cdot \frac{E_i D_i}{E_i D_i X_{di}} + \frac{B_{pi} C_i (Z_o \times E_o D_i) Z_o}{B_{pi} C_i (Z_o \times Z_o B_{pi})} - \frac{B_{pi} C_i (Z_o \times E_o D_i) X_{di}}{B_{pi} C_i (Z_o \times A_{pi} B_{pi})} \]  

(18.c)

**Figure 5.** Jacobian matrix analysis.

One could have the similar for \( \omega_{qi}, J_{qv1} \text{ and } J_{qw1} \). The Jacobian matrix is obtained as below.

\[ J = \begin{bmatrix} J_{pvi} & J_{pv2} & J_{pv3} & J_{qv1} & J_{qv2} & J_{qv3} \\ J_{pwi} & J_{pw2} & J_{pw3} & J_{qw1} & J_{qw2} & J_{qw3} \end{bmatrix}^T \]  

(19.a)

So that,

\[ \begin{bmatrix} \omega_p \\ \omega_q \end{bmatrix} = J \cdot [v_{eo} \omega_{eo}] \]  

(19.b)
The stiffness of manipulator at \( E_o \) by the global cartesian coordinate at \( A_o \) is given as \( K_a \) which is related to the stiffness of actuator \( K_q = k_q \cdot I_6 \) through Jacobian matrix \( J \) [27].

\[
K_a = J^T \cdot K_q \cdot J
\]  

(19.c)

\[
\theta_e = \begin{bmatrix} 
15 \cdot \frac{\pi}{180} & 20 \cdot \frac{\pi}{180} & 10 \cdot \frac{\pi}{180} 
\end{bmatrix}^T.
\]

Figure 6. Stiffness mapping \( \theta_e \) = \( \begin{bmatrix} 
15 \cdot \frac{\pi}{180} & 20 \cdot \frac{\pi}{180} & 10 \cdot \frac{\pi}{180} 
\end{bmatrix}^T \).

A transformation matrix is needed [19] to convert to another coordinate system. In figure 5. (b), the coordinate system is established at \( E_o \) with unit vectors \( X_{eo}, Y_{eo} \) and \( N_e \). Vector \( X_o \) from global coordinate \( A_o \) equals the \( X_{eo} \) from the platform coordinate at \( E_o \). One could transform \( J \) to \( J_e \).

\[
(X_o)_{Ao} = (X_{eo})_{Eo}
\]

(20.a)

\[
(X_{eo})_{Ao} = R \cdot (X_o)_{Ao} = R \cdot (X_{eo})_{Eo}
\]

(20.b)

\[
J_e = J \begin{bmatrix} 
R & 0_{3,3} \\
0_{3,3} & R 
\end{bmatrix}
\]

(20.c)
Similarly, in figure 5. (b) a coordinate of $\Phi$ has a vector $X_\varphi$ that rotates from $X_{eo}$ by $\varphi_y$ and $\varphi_z$.

\[ (X_o)_{Ao} = (X_\varphi)_{\varphi} \]  \hspace{1cm} (21.a)

\[ (X_\varphi)_{Ao} = R \cdot (X_\varphi)_{Eo} = R \cdot R_z \cdot R_y \cdot (X_\varphi)_{\varphi} \]  \hspace{1cm} (21.b)

\[
R_z = \begin{bmatrix}
\cos \varphi_z & -\sin \varphi_z & 0 \\
\sin \varphi_z & \cos \varphi_z & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

and

\[
R_y = \begin{bmatrix}
\cos \varphi_y & 0 & \sin \varphi_y \\
0 & 1 & 0 \\
-\sin \varphi_y & 0 & \cos \varphi_y
\end{bmatrix}
\]  \hspace{1cm} (21.c)

A vector in coordinate $A_o$ could be converted to that in coordinate $\Phi$. One could transform $J$ to $J_\varphi$.

\[ J_\varphi = J \cdot \begin{bmatrix}
R \cdot R_z \cdot R_y & 0_{3,3} \\
0_{3,3} & R \cdot R_z \cdot R_y
\end{bmatrix} \]  \hspace{1cm} (21.d)

\[ K_\varphi = J_\varphi^T \cdot K_q \cdot J_\varphi \]  \hspace{1cm} (21.e)

As one could calculate the stiffness in or around $X_\varphi$. The polar stiffness of the end effector center $E_o$ at any pose could be plotted about rotation angle $\varphi_y$ and $\varphi_z$. The unit stiffness in and around vectors $X_o, Y_o, Z_o$ from global coordinate at $A_o$ are mapped over an area of $E_o = [x_e \ y_e \ 0.2]^T$ in figure 6. (a)–(f). The unit stiffness in and around $X_\varphi$ from given coordinate of $\Phi$ is mapped about angle $[\varphi_y \ \varphi_z]$ when $E_o = [0.1 \ 0.1 \ 0.2]^T$ in figure 6. (g) and (h).

6. Conclusion
A 3-[(2-RR)UU] 6-DOF parallel manipulator is designed with multi-plane principle. The pentagon identification and cross-product method are developed for link interference detection. The workspace boundaries of different orientation angles are plotted. The unit stiffness in and around the global coordinate vectors are mapped. With a transformation matrix this stiffness could be converted to indicate stiffness in or around any vector that is a rotation angle from the global coordinate system.

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