Supersymmetric Grand Unification and Lepton Universality in
$K \to \ell \nu$ Decays

John Ellis $^1$, Smaragda Lola $^2$ and Martti Raidal $^3$

$^1$ Theory Division, Physics Department, CERN 1211, Geneva 23, Switzerland
$^2$ Department of Physics, University of Patras, GR-26500 Patras, Greece
$^3$ National Institute of Chemical Physics and Biophysics
Ravala 10, Tallinn 10143, Estonia

Abstract

Motivated by the prospects for an improved test of lepton universality in $K \to \ell \nu$ decays by the NA62 experiment at CERN, we study predictions for the possible lepton non-universality in $K \to \ell \nu$ decays in supersymmetric models. Violations of $\mu - e$ universality in this process may originate from mixing effects in the right-handed slepton sector, providing a unique window into this aspect of supersymmetric flavour physics in the large-$\tan \beta$ region. Minimal unification scenarios with universal soft supersymmetry-breaking terms at the GUT scale would predict negligible violation of lepton universality. However, lepton non-universality may be observable in non-minimal grand unified models with higher-dimensional terms contributing to fermion masses, in which case renormalization effects above the GUT scale may enhance the mixing among the right-handed sleptons. This could lead to observable lepton non-universality in $K \to \ell \nu$ decays in specific regions of the parameter space with high $\tan \beta$, large $A$ terms and small charged Higgs boson mass. Observable non-universality in $K \to \ell \nu$ decays would be correlated with a large value of $BR(\tau \to e\gamma)$. The experimental upper limit on the electric dipole moment of the electron could be reconciled with leptogenesis, if the latter occurs at a relatively low scale, which would also alleviate the cosmological gravitino problem. Even if lepton non-universality is not seen in the near future, one may nevertheless obtain significant constraints on the model parameters and unknown aspects of right-handed fermion and sfermion mixing.
1 Introduction

A large number of experiments have measured neutrino oscillations [1, 2, 3, 4, 5], thereby providing important information on the neutrino mass differences and mixing angles [6]. Within the framework of supersymmetry, massive neutrinos lead to charged-lepton-flavour violation (LFV) via radiative corrections to sfermion masses [7, 8], that may be observable in forthcoming experiments. The predictions of models of massive neutrinos for processes such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\tau \rightarrow \mu\gamma$, $\mu \rightarrow e$ conversion on heavy nuclei and sparticle decays at the LHC [9] have been studied extensively [8]. These predictions are frequently very close to the current experimental limits [8, 10, 11, 12], and may be further refined by requiring successful leptogenesis [13, 14] and sneutrino inflation [15].

In addition to charged-lepton decays, rare decays of mesons are also of potential interest. In a previous work, we studied in detail rare kaon decays [16] to $\mu e$ pairs with and without accompanying pions, finding that radiative corrections related to neutrino mixing may induce significant rates, even when starting from universal initial conditions for the soft terms at a high-energy input scale. In these examples, as well as in most low-energy LFV processes, the relevant mixing arose dominantly from left-handed slepton mixing induced via the renormalization-group equations (RGEs).

It has recently been pointed out that mixing effects in the right-handed sfermion sector can be probed very sensitively by checks on $\mu - e$ universality in the decays

$$ K \rightarrow \ell\nu , \ell \equiv e, \mu $$

which can be generated by flavour non-universality in an effective $\tilde{\nu}_\ell R H^+$ coupling [17, 18, 19]. In general, the uses of meson decays as probes of physics beyond the Standard Model (SM) are complicated by hadronic uncertainties. However, working with the ratios of the electronic and muonic decay modes, in this case $R_K \equiv \Gamma(K \rightarrow e\nu)/\Gamma(K \rightarrow \mu\nu)$, the hadronic uncertainties cancel to a large extent, allowing a precise confrontation between theory and experiment. The current bound on $R_K$ is given by [20]:

$$ R^{exp}_K = (2.457 \pm 0.032) \cdot 10^{-5}, $$

which is to be compared with the SM prediction $R^{SM}_K = (2.472\pm0.001) \cdot 10^{-5}$ [21]. The NA62 experiment at CERN now plans a significant improvement in the experimental accuracy, expecting to reduce the uncertainty in $R_K$ to $\pm 0.003$.

Any violation of $\mu - e$ universality in $K \rightarrow \ell\nu$ decays would constitute unambiguous evidence for new physics. In particular, within a supersymmetric framework, it would provide crucial information on right-handed slepton mixing, thereby complementing in an important way the other LFV processes studied previously [17]. As we discuss later, in schemes with universal soft scalar masses at the GUT scale, the experimental bounds on other LFV processes imply that RGE effects below the GUT scale would be insufficient to generate non-negligible $\mu - e$ non-universality in $K \rightarrow \ell\nu$ decays. However, right-handed slepton mixing and $\mu - e$ non-universality might arise through RGE effects above the GUT scale [22] in models where universality of the soft supersymmetry-breaking contributions to the right-handed slepton masses is assumed at some higher input ‘gravity’ scale $M_{grav}$. This might
then lead to observable non-universality in $K \to \ell \nu$ if $\tan \beta$ is large and other conditions on the supersymmetric model parameters are also met. However, the simultaneous presence of both left- and right-slepton flavour mixings, together with very large values of $\tan \beta$, would in general imply too large rates for the LFV decays end electric dipole moments (EDMs) of charged leptons. Therefore, in order for non-universality to be observable in $K \to \ell \nu$, consistency with the present bounds on LFV imposes non-trivial conditions on the flavour physics as well as the supersymmetry-breaking pattern.

In this paper we study the patterns of the soft supersymmetry breaking terms required for obtaining observable renormalization induced $\mu - e$ non-universality effects in the NA62 experiment at CERN. As an initial condition we assume the SUSY breaking parameters to be flavour universal at $M_{\text{grav}} > M_{\text{GUT}}$ and consider the RGE running of the soft supersymmetry-breaking mass parameters both above and below the GUT scale. We assume the seesaw mechanism \cite{23} with three singlet neutrinos, and we use the observed neutrino masses and mixing angles as inputs. We apply a parameterization via a Hermitian matrix $H$ \cite{24} employing the orthogonal parameterization \cite{11} to calculate the corresponding singlet neutrino Yukawa couplings $Y_\nu$ and masses $M_N$. This parameterization greatly facilitates keeping the RGE-induced left-handed slepton flavour structure under control.

Within the minimal supersymmetric SU(5) GUT, the flavour mixing of the right-handed sfermions is RGE-induced above the GUT scale by the Cabibbo-Kobayashi-Maskawa matrix. However, it is well known that this minimal SU(5) GUT relates $m_e$ and $m_\mu$ incorrectly to $m_d$ and $m_s$. This defect can be cured by adding supplementary terms in the $d$-quark and charged-lepton mass matrices and in the coloured triplet Higgs Yukawa couplings originating from higher-order, non-renormalizable terms in the effective superpotential below $M_{\text{grav}}$ \cite{25}. The corrected Yukawa couplings leave their imprint on the flavour structure of the right-slepton supersymmetry breaking parameters via renormalization above the GUT scale.

Throughout our analysis, we require the magnitude and pattern of supersymmetry breaking parameters to be consistent with supersymmetric Dark Matter, the baryon asymmetry of the Universe, and with all present bounds on flavour-violating decays and EDMs. This imposes nontrivial requirements on the pattern of SUSY breaking parameters. As an example, one pattern of supersymmetry breaking which simultaneously gives the desired Higgs and sparticle mass spectrum, the correct amount of DM, and large RGE induced non-universality effects, is a tuned version of the so called Higgs boson exempt no-scale supersymmetry breaking \cite{26}. This scenario allows us to generate small charged Higgs boson masses, while keeping all other soft mass terms heavy so that all other relevant observables like $(g-2)_{\mu}$, $B \to \mu\mu$, $b \to s\gamma$ etc. are consistent with the measurements. One consequence of this sample SUSY breaking point is that supersymmetric particles would be difficult to discover at the LHC, whereas the charged Higgs boson should be relatively easily accessible at the LHC \cite{27}.

Within this framework, we find examples with values of the renormalization induced non-universality parameter $\Delta R_K$ as high as $\sim \mathcal{O}(10^{-2})$ to $(10^{-3})$, well within the reach of the NA62 experiment. However, in order to achieve this, a very constrained flavour structure for Yukawa matrices is required in order to keep LFV decays under control while generating
large non-universality effects in $K \to \ell \nu$. As a result, we find a strong correlation between the decay $\tau \to e\gamma$ and the size of $R_K$. Observation of one of them would, knowing the SUSY parameters and the mass of the charged Higgs boson, predict the other. In order to have observable $R_K$, the charged Higgs boson must be light while the other supersymmetric particle masses must be heavy in order to suppress $\tau \to e\gamma$ below the experimental bounds.

In this scenario large EDMs of charged leptons are induced if there are phases in the complex neutrino Yukawa couplings $Y_\nu$ as would be needed to generate the baryon asymmetry of the Universe via leptogenesis [28]. At large $\tan \beta$, and due to the simultaneous presence of both $\delta \tilde{m}_{LL}$ and $\delta \tilde{m}_{RR}$ flavour mixings, neutrino Yukawa-induced contributions to the $i$-th lepton EDM is strongly enhanced due to the dominant term $(\delta \tilde{m}_{LL}^2 m_\mu \tan \beta \delta \tilde{m}_{RR}^2)_{ii}$. While in the minimal SUSY seesaw models the induced charged lepton EDMs are, in the most optimistic case, a few orders of magnitude below the present experimental bound [29, 30, 31], in our scenario the EDMs can be significantly larger. Suppressing the electron EDM below the experimental bound $d_e < 1.6 \cdot 10^{-27}$ e cm [32] by assuming small phases in the neutrino Yukawa couplings is possible but, in the absence of a concrete theory of phases, may be unnatural, and would also suppress the CP asymmetry for leptogenesis. We find that the natural way to suppress the electron EDM in this scenario is related to the flavour structure of the heavy neutrino Yukawa couplings and, consequently, the flavour structure of the induced soft SUSY breaking terms. Using the $H$ parameterization of neutrino Yukawa couplings [24], assuming $H_{11} \ll H_{22,33}$ would imply $M_{N_1} \ll M_{N_{2,3}}$ and, therefore, small Yukawa couplings of the lightest heavy neutrino $N_1$. Hence, for a given $M_{N_1}$ we find an upper limit on the electron EDM. We argue that the gravitino problem [33] in supersymmetric theories, which sets upper limits on the reheating temperature of the Universe and therefore requires a relatively light $N_1$ for successful leptogenesis, also provides a solution to the EDM problem. For $M_{N_1} \sim 10^8$ (10$^6$) GeV the electron EDM is bounded as $d_e \lesssim 10^{-28}$ (10$^{-30}$) e cm which is within the reach of the proposed electron EDM experiments [34].

The outline of our paper is the following. In Section 2 we review the details of non-universality effects in the decays $K \to \ell \nu$. In Section 3 we discuss the RGE effects in supersymmetric models, including running both below and above the GUT scale. Numerical examples are given in Section 4, and we conclude in Section 5.

## 2 Sfermion Mixing, $\mu - e$ Universality in $K \to \ell \nu$ Decays and other Observables

Probing charged-lepton universality in $K \to \ell \nu$ decays [21] is interesting in view of the very promising experimental prospects and since, in a supersymmetric framework, this decay probes mass mixing between the right-handed charged sleptons, $\tilde{m}_{RR}$. In contrast to mixing between the left-handed charged sleptons, $\tilde{m}_{LL}$, rare decays and other processes have given us relatively little information yet about $\tilde{m}_{RR}$. It is clear [17] that there would be significant sfermion mixing in the presence of general non-universal soft masses at the GUT scale. Here, however, we will focus on lepton-flavour violation (LFV) induced by the
renormalization-group equations for soft terms, in particular through the effects of right-handed neutrinos below the GUT scale and through RG evolution above the GUT scale in models of grand unification.

The decay $K \to \ell \nu$ has been discussed in detail in [17, 18, 19], where its magnitude was shown to be dominated by [17]

$$R_{K}^{LFV} \simeq R_{K}^{SM} \left[ 1 - \frac{m_{K}^{2}}{M_{H}^{2}} m_{e} \Delta_{RL}^{11} \tan^{2} \beta \right]^{2} + \left( \frac{m_{K}}{M_{H}} \right) \left( \frac{m_{e}^{2}}{m_{\mu}^{2}} \right) |\Delta_{R}^{31}|^{2} \tan^{6} \beta \right],$$

where

$$\Delta_{RL}^{\ell} \simeq - \frac{\alpha}{4\pi} \mu M_{1}^{2} m_{\ell}^{2} \delta_{RR} \delta_{LL}^{\ell} I''(M_{1}^{2}, m_{L}^{2}, m_{R}^{2}),$$

and

$$\Delta_{R}^{3\ell} \simeq \frac{\alpha}{4\pi} \mu M_{1}^{2} \delta_{RR}^{\ell} \left[ I'(M_{4}^{2}, \mu^{2}, m_{R}^{2}) - (\mu \leftrightarrow m_{L}) \right].$$

In these expressions $I$ is the standard three-point one-loop integral

$$I(x, y, z) \equiv \frac{xy \log \frac{x}{y} + yz \log \frac{y}{z} + zx \log \frac{z}{x}}{(x - y)(z - y)},$$

and $I'(x, y, z) \equiv \frac{dI(x, y, z)}{dydz}$. As usual we denote

$$\delta_{XX}^{ij}(\tilde{m}_{e}^{2})_{XX}^{ij}/m_{X}^{2} \quad (X = L, R),$$

and for the rest of the paper we will drop the flavour indices in $\delta_{XX}^{ij}$. The first term in (2) features a double insertion of LFV mixing, and interferes with the SM contribution, whereas the second term clearly has no such interference. Note that we neglect a term proportional to $\Delta_{R}^{32}$, which is suppressed by a factor $m_{e}^{2}/m_{\mu}^{2}$ with respect to the term proportional to $\Delta_{R}^{31}$. Similarly, we neglect the contributions from left-slepton mixing $\Delta_{\ell}$, as those are numerically subleading [19]. In our numerical calculations we use the full expressions from [18] rather than just the dominant terms (2, 3, 4). However, the latter expressions are sufficient for discussion of the new physics non-universality effects in kaon decays.

The dependence of the deviation from universality in the $K \to \ell \nu$ decay rates on $\Delta_{RL}^{11}$ and $\Delta_{RL}^{31}$ is not complicated; it is clear from the formulae (2, 3, 4) that larger rates are expected for large $\tan \beta$, a light ‘heavy’ Higgs mass $M_{H}$, large $\mu$ (note, in particular, that the dominant $\Delta_{RL}^{\ell}$ contribution is proportional to $\mu$), and small slepton masses (in order to avoid suppressions in the three-point loop functions; this can be true for the right-handed staus, in particular). Specifically, for $M_{H} = 180$ GeV and $\tan \beta = 50$, one obtains

$$\delta R_{K}^{LFV} \simeq 10^{7}[(\Delta_{R}^{31})^{2} + (\Delta_{RL}^{11})^{2} - 0.0006 \Delta_{RL}^{11}].$$

In general:

(i) For the range of parameters where $\delta_{LL,RR}$ have small and comparable magnitudes, the interference term proportional to $\Delta_{RL}^{11}$ would be expected to dominate over $(\Delta_{R}^{31})^{2}$ and $(\Delta_{RL}^{11})^{2}$. 

5
(ii) In the case that $\delta_{LL} \ll \delta_{RR}$, $\delta R^L_{K}^{LFV}$ scales as the $(\Delta^{31}_R)^2$ and thus $(\delta_{RR})^2$ terms.

(iii) For larger $\delta_{RR,LL}$, both quadratic and linear terms may be important in $R_K$.

Barring a cancellation, an experimental measurement with an error $\Delta R_K \sim 0.003$ would provide sensitivity to $\Delta^{11}_{RR} \sim 5 \times 10^{-7}$ with a significantly smaller $(\Delta^{31}_R)^2$. On the other hand, for a very small $\delta_{LL}$ and thus $\Delta^{11}_{RL}$, the sensitivity to $\Delta R_K \sim 0.003$ is compatible with $\Delta^{31}_R \sim 1.7 \times 10^{-5}$ (which for the above quoted optimal set of supersymmetric parameters would correspond to $\delta_{RR} \sim 0.12$).

Because of the prefactors in $\Delta^{31}_R$, unless $x = y = z$ to a great accuracy (which is not expected, in view of RGE effects), one would typically expect $\Delta^{31}_R \leq 10^{-3}$ even in models with enhanced non-universalities, such as in [35]. Values of this order of magnitude are potentially interesting for experiment. However, if non-diagonal scalar terms are induced only by RGE effects, one expects rather smaller values of the $\delta^{ij}_{XX}$ (and hence $\Delta^{11}_{RR}$ and $\Delta^{31}_R$) than in models where universality is explicitly violated. In typical scenarios, one expects the RGE-generated right-handed mixings to be small, whereas the $\delta_{LL}$ are found to be generically larger. Nevertheless, it is clear that if there is a signal in $K \to \ell \nu$ decays in the near future, this would imply a non-negligible right-handed slepton mixing, and would inevitably lead to very constrained scenarios, particularly for models with universal initial conditions for the soft terms: models with large $\tan \beta$, light right-handed staus and large $A$-terms would be favored.

Moreover, since observable non-universality effects in $K$ decays would require non-negligible $\tau - e$ mixing in the RR slepton sector, the LFV decays $\tau \to e\gamma$ must inevitably be large, and correlated with the non-universality. This among others would imply strong constraints on the latter from the bound $BR(\tau \to e\gamma) < 1.1 \times 10^{-7}$ [36].

Before passing to the details of the calculation of the $\delta^{ij}_{XX}$ in GUT scenarios, we give a feeling for the magnitudes of $\delta_{RR}$ and $\delta_{LL}$ required to see a signal in $K \to \ell \nu$ decays, for realistic points in the supersymmetric parameter space. This is done in Fig. 1, which shows contour plots of the calculated deviation from universality in $R_K$, as functions of the $\delta^{ij}_{LL,RR}$. Contour plot (a), on the left side, indicates that if $\delta_{LL}$ and $\delta_{RR}$ were to be comparable, and the NA62 experiment reaches the expected sensitivity of 0.003, it would be possible to observe non-universality for slepton mixing parameters $\delta = \mathcal{O}(0.04 - 0.05)$, for a feasible set of parameters with a light Higgs boson and a light right-handed third-generation slepton mass. However, from the RGE running one would naively expect that the left-slepton mixing would be larger, and simultaneous mixing in the $LL$ and $RR$ channels would tend to generate unacceptably large flavour violation in channels that are strongly constrained (particularly $\mu \to e\gamma$, which must be kept under control in any LFV SUSY model). This would imply that for non-negligible non-universality in kaon decays $\delta_{LL}$ would have to be small. This would then correspond to solutions with a dominant right-handed slepton mixing, as in the contour plot (b) on the right.

We examine in subsequent sections the magnitudes of right-handed slepton mixing that arise in various theoretical scenarios. We first discuss briefly the non-universal corrections to the soft sfermion masses that are induced in the presence of non-zero $A$-terms, by RGE effects between $M_{GUT}$ and low energies in sample seesaw neutrino-mass models for the complete $3 \times 3$ mixing. In this case the RR mixing is too small for any observable effect.
Figure 1: Contour plots of the Lepton-Flavour-Violating (LFV) correction to the Standard Model Value of $R_K$ (denoted by $\delta R_K$) as a function of the soft term mixing parameters, for $\tan \beta = 50$. We assume for illustration $M_H = 180$ GeV, $M_1 = 190$ GeV, $\mu = 650$ GeV, $m_L = 300$ GeV and $m_R = 200$ GeV. In (a) the left and right slepton mixing are comparable, while in (b) the right-handed slepton mixing dominates.

Subsequently, we consider the possible effects of RGE running above the GUT scale, where the overall right-handed mixing may be amplified in some GUT scenarios.

3 RGE effects below and above the GUT Scale

In supersymmetric seesaw models of neutrino masses, the RGEs between the GUT scale and the heavy singlet neutrino mass scale generate non-universalities in the soft supersymmetry-breaking scalar masses. These may have implications for rare kaon decays that involve two charged leptons, as well as charged leptons and pions, as have been studied in [16]. In that work only the dominant left-handed slepton mixing was considered, neglecting the subdominant mixing in the right-handed slepton sector.

To proceed, we assume universal initial conditions for the soft terms at the GUT scale, and consider the RGEs including neutrino Yukawa couplings. We also assume a single common mass scale $M_N$ for the heavy singlet neutrinos (which may easily be modified, see [20]). Then, in the leading-logarithmic approximation the RGE-induced soft supersymmetry-breaking terms are given by

\[
(\delta \tilde{m}^2_M)_{ij} \approx -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu Y_\nu + Y_e Y_e)_{ij} \log \frac{M_{\text{GUT}}}{M_N},
\]

\[
(\delta \tilde{m}^2_E)_{ij} \approx -\frac{1}{4\pi^2}(3m_0^2 + A_0^2)(Y_e Y_e)_{ij} \log \frac{M_{\text{GUT}}}{M_N},
\]
The trilinear soft supersymmetry-breaking terms $A_{e, 
u}$ are assumed to be related by universal factors $A_0$ to the corresponding Yukawa couplings $Y_{e, 
u}$. From this equation, it becomes already clear that large values of $A_0$ could lead to enhanced RGE corrections to soft masses, a feature that we will use in our considerations.

The above equations hold for fully universal initial conditions. However, well-motivated models with deviations from Higgs-scalar fermion universality, as in [26], induce additional corrections linked to $S = (M_{Hu}^2 - M_{Hd}^2) + T r_F (m_Q^2 - 2m_U^2 + m_E^2 + m_D^2 - m_L^2)$, where the trace runs over flavours.

It is possible, even likely, that the GUT scale lies significantly below the scale $M_{grav}$ at which gravitational effects can no longer be neglected. In specific models, $M_{grav}$ might be identified with either the Planck mass $M_P = 1.2 \times 10^{19}$ GeV or some lower string unification scale $M_{string} \sim 10^{18}$ GeV. In general, the renormalization of couplings at scales between $M_{grav}$ and $M_{GUT}$ may induce significant flavour-violating effects, particularly in the $\delta_{RR}$, which can be calculated in any specific supersymmetric GUT.

The simplest example is provided by the minimal supersymmetric SU(5) GUT, whose superpotential contains terms of the form $e^c u^c H$, where the $H$ is a colour-triplet Higgs field that is expected to have a mass $\sim M_{GUT}$. This gives rise to one-loop diagrams that renormalize the right-handed slepton masses between $M_{GUT}$ and $M_{grav}$. In the leading-logarithmic approximation, these take the form [12]:

$$\langle \delta \tilde{m}_{E}^{2} \rangle_{ij} \approx - \frac{3}{8\pi^2} \lambda_{u_3}^2 V_{U}^{3i} V_{U}^{3j} (3m_0^2 + A_0^2) \log \frac{M_{grav}}{M_{GUT}},$$

for $i \neq j$, where $V_U$ denotes the mixing matrix in the corresponding couplings in the basis where the $u$-quark and charged-lepton masses are diagonal. This is to be compared with the corresponding corrections to left-handed slepton masses, which are proportional to $V_D$, the Dirac neutrino mixing matrix in the basis where the $d$-quark and charged-lepton masses are diagonal, and are given by

$$\langle \delta \tilde{m}_{L}^{2} \rangle_{ij} \approx - \frac{1}{8\pi^2} \left( \lambda_{v_3}^2 V_{D}^{*3i} V_{D}^{3j} \log \frac{M_{grav}}{M_{\nu_3}} + \lambda_{v_2}^2 V_{D}^{*2i} V_{D}^{2j} \log \frac{M_{grav}}{M_{\nu_2}} \right) (3m_0^2 + A_0^2).$$

Finally the leading-logarithmic renormalization of the $A_e$ terms is given by

$$\delta A_{e}^{ij} \approx - \frac{3}{8\pi^2} A_0 \left( \lambda_{e_1} V_{D}^{*3i} V_{D}^{3j} \lambda_{\nu_3}^2 \log \frac{M_{grav}}{M_{\nu_3}} + \lambda_{e_1} V_{D}^{*2i} V_{D}^{2j} \lambda_{\nu_2}^2 \log \frac{M_{grav}}{M_{\nu_2}} \right) + 3 \lambda_{e} V_{U}^{3i} V_{U}^{3j} \lambda_{u_3}^2 \log \frac{M_{grav}}{M_{GUT}}.$$

One must appeal to a specific GUT model for the structures of the mixing matrices $V_{U,D}$. In the case of minimal supersymmetric SU(5), as already remarked, the $d$-quark mass matrix is the transpose of the charged-lepton mass matrix, and $V_D$ is simply the unit matrix. On the other hand, $V_U$ is non-trivial, and related to the familiar CKM matrix. We recall
that in minimal SU(5) matter fields are arranged in \( \bar{5} \) ((\( L, d^c \))_i) and 10 supermultiplets ((\( Q, u^c, e^c \))_i), the \( d \)-quark and charged-lepton masses arise from \( 10 - \bar{5} - \bar{H} \) couplings \( \lambda_5 \), and the \( u \)-quark masses arise from \( 10 - 10 - H \) couplings \( \lambda_{10} \). The theory may be written in a basis where the \( \lambda_5 \) are diagonal, and hence also the \( d \)-quark and charged-lepton masses. In this basis, the \( d \)-quark triplets in the 10 supermultiplets are rotated relative to the \( u \)-quark triplets and the \( u^c \) anti-triplets by the familiar CKM matrix \( V_{CKM} \), and the \( u \)-quark triplets and the \( u^c \) antitriplets are related by a diagonal phase matrix \( U \) with unit determinant \( [37] \). It is clear from the forms of the equations \([9, 11]\) that the phase matrix \( U \) is irrelevant for our considerations in this paper, though it might have played a role in generating the baryon asymmetry of the universe \([38]\).

In this simplest \( SU(5) \) one has \( m_b = m_\tau \) (a successful relation), \( m_s = m_\mu \) and \( m_d = m_e \) (unsuccessful relations) \([39]\). The latter predictions can be modified by taking into account possible non-renormalizable fourth-order terms in the effective superpotential, of the form \( \bar{H} - 10 - 24 - \bar{5} \) \([40]\), which make different contributions to the \( d \)-quark and charged-lepton mass matrices:

\[
\lambda(10 - \bar{5} - H) + \lambda'(H - 10 - 24 - \bar{5}) \rightarrow \lambda \bar{v}(dd^c + e^c e) + \lambda \bar{v} V(2dd^c - 3e^c e),
\]

implying that, in the basis where \( m_d \) is diagonal,

\[
m_e = m_d^D - 5\lambda' \bar{v} V,
\]

where the matrix of couplings \( \lambda' \) is non-diagonal, in general. Then, the diagonalization of \( m_e^D = V_{eR} m_e V_{eL}^+ \) gives

\[
m_e^D = V_{eR}(m_d^D - 5\lambda' \bar{v} V) V_{eL}^+.
\]

Hence, in this modification of the simplest \( SU(5) \) model, the diagonalization of charged lepton mass matrix is not any more given by \( V_{CKM} \) and the model can realistically reproduce the observed phenomenology. In a similar manner \([25]\), the colour-triplet-induced \( e^c u^c \) mixing receives potentially large corrections for the first two generations. Parametrizing the non-renormalizable correction to this mixing by \( V_{uR} \), the RGE induced right-slepton mixing is not given by \( V_{CKM} \) as in the minimal model, but by the product

\[
V_R = V_{CKM} V_{uR}^+.
\]

As \( V_{uR} \) is not constrained at present, we assume that all possible values of mixing angles parameterizing \( V_R \) are allowed.

These non-renormalizable corrections also change the forms of the fermion mass matrices, and hence the predictions of this type of flavour texture model within minimal SU(5). For example, the predictions on new physics effects in \( B_s - \bar{B}_s \) mixing \([41]\) will be modified and the direct relation between the latter and lepton flavour violating observables is lost. Thus, these corrections would also affect the renormalization between the GUT and heavy-neutrino mass scales. These effects would also be important for the \( \delta_{LL}^{ij} \), but we do not consider them here.

Another GUT scenario is flipped \( SU(5) \), in which the fields \( Q_i, d^c_i \) and \( u^c_i \) of each family belong to a 10 representation of \( SU(5) \), the \( u^c_i \) and \( L_i \) belong to \( \bar{5} \) representations, and the \( e^c_i \) fields belong to singlet representations of the group.
In this case, one would expect that large right-handed slepton mixing could be accommodated more easily. These particle assignments imply a symmetric down-quark mass matrix, and a charged-lepton mixing matrix that is not directly correlated with that of the quarks, and the corresponding mixing angle and phase analysis has been carried out in [42]. However, the correlation between left-handed charged leptons and right-handed $u$ quarks, as well as the direct link between the neutrino and $d$-quark mass matrices, makes it hard to find a phenomenological model with a $U(1)$ flavour group that also accommodates the solar and atmospheric neutrino data, without fine tuning of the flavour charges [43, 44]. In string-inspired versions of flipped SU(5), natural solutions to the complete fermion data have been found [45], but the large number of zero entries in the mass matrices imposed by string selection rules leave room only for minimal flavour mixing, and we do not study them further here.

4 SUSY (Flavour) Parameter Space and Numerical Examples

In order to induce observable non-universality effects in $K \rightarrow \ell\nu$ decays due to RGE effects below and above the GUT scale, while respecting all available experimental constraints on flavour conserving and violating processes and cosmology, both the mass pattern of supersymmetry breaking terms as well as their RGE induced flavour structure must be non-trivially constrained. In this Section we first provide an example of a supersymmetry breaking scenario which satisfies all such constraints. Subsequently, we discuss the necessary flavour pattern of the soft supersymmetry breaking parameters and we find how such flavour structures can be RGE induced above and below the GUT scale. We find that the necessary flavour structure for our scenario is very tightly constrained.

The study of models with a mass spectrum that could potentially lead to large $K \rightarrow \ell\nu$ decays has been motivated for independent reasons. For instance, the WMAP benchmark scenarios with universal supersymmetry-breaking soft terms studied in [46] include some in the $\chi - \tilde{\tau}_1$ coannihilation region, which have light right-handed staus. However, these scenarios generally predict high masses for the heavier Higgs bosons, leading to a suppression of non-universality in $K \rightarrow \ell\nu$ decays. This then suggests moving to the study of models that deviate from the minimal schemes, e.g., by breaking the universality of the soft supersymmetry-breaking masses in the Higgs sector [47]. Indeed, soft universality in the Higgs sector is not as well motivated as for the sfermion masses. Moreover, large values for the $A$-terms would also allow smaller heavy Higgs masses [48].

When looking for input SUSY parameters at some high scale that are consistent with supersymmetric Dark Matter and with all experimental constraints, we consider the following region of the free parameters:

$$m_0 \ll M_2 < |M_{H_u}| \approx |M_{H_d}|, \quad A_0; \quad \tan \beta > 50, \quad sign(\mu).$$  \hspace{1cm} (15)

This scenario resembles the so-called Higgs boson exempt no-scale supersymmetry breaking scenario [26]. In this scheme, all the RGE-corrected SUSY breaking masses at low scale
are large, thus explaining why no SUSY particles have been observed so far. The Higgs mass parameters $|M_{H_d}|^2 \approx |M_{H_u}|^2 < 0$ are negative, triggering the electroweak symmetry breaking. However, the light charged Higgs mass (as well as the correct scale of the electroweak symmetry breaking) are obtained due to the large cancellations between the RGE-corrected SUSY parameters, and are thus tuned. Since $m_0$ is smaller than all other parameters of the model, RGE-induced LFV is generated by the large parameters $M_{H_u}$, $A_0$ via

$$
(\delta \tilde{m}_L^2)_{ij} \approx -\frac{1}{8\pi^2}(M_{H_u}^2 + A_0^2)(Y_\nu^\dagger \log \frac{M_{\text{grav}}}{M_N} Y_\nu)_{ij},
$$

(16)

$$
(\delta \tilde{m}_E^2)_{ij} \approx -\frac{3}{8\pi^2} \lambda_{u3}^2 V_R^{3i} V_R^{*3j} (M_{H_u}^2 + A_0^2) \log \frac{M_{\text{grav}}}{M_{\text{GUT}}},
$$

(17)

where $V_R$ is the mixing matrix $V_R$ corrected by the higher-dimensional operators and we have assumed that the top quark Yukawa coupling $\lambda_{u3}$ does not receive large corrections. Thus, large off-diagonal elements in both the left- and right-slepton mass matrices are to be expected at low energies.

As a representative example, we take the parameter set appearing in Table 1 which results in a heavy sparticle spectrum, but a light charged Higgs boson mass. The Higgs mass is fine-tuned, so small changes in the parameters would alter the charged Higgs boson mass significantly. Moreover, since small changes in $M_{\tilde{t}_L}, |M_{H_d}|, |M_{H_u}|, A_0$ do not drastically alter the rest of the model parameters, in this scenario, the charged Higgs boson mass can essentially be considered as a free parameter. For example, increasing (decreasing) the Higgs mass parameter $|M_{H_u}|$ by 5 GeV compared to the value in Table 1 would result to a charged Higgs boson of 235 GeV (159 GeV). In the numerical study that follows, we perturb the parameters (15) around the values of Table 1 so that a light charged Higgs boson is obtained.

We now turn to discuss the constraints on the flavour structure of the SUSY mass matrices. In general, as Section 3 indicates, non-vanishing neutrino masses and large mixing in the neutrino sector imply large LFV effects in the LL slepton sector in SUSY seesaw models. Moreover, the simultaneous presence of LL and RR slepton mixing in the large-tan $\beta$ regime implies enormous enhancement of LFV decay rates. As the RR mixing is necessary for observable non-universality in the kaon decays under discussion, we have to forbid any significant mixing in the LL slepton sector. This is greatly facilitated by using the parameterization of neutrino seesaw parameters, in terms of effective light neutrino observables and an auxiliary Hermitian matrix $H$ [22] that can be related directly to low-energy observables, including the processes that violate lepton number. Indeed, the Hermitian matrix $H$ in the leading-logarithmic approximation can be regarded as the FLV mixing in the LL slepton sector, and is given by

$$
H_{ij} = \sum (Y_\nu^*)_{ki} (Y_\nu)_{kj} \log \frac{M_{\text{GUT}}}{M_{N_k}}.
$$

(18)

Observable neutrino masses and mixing can be obtained for

$$
H = \begin{pmatrix}
H_{11} & 0 & 0 \\
0 & H_{22} & 0 \\
0 & 0 & H_{33}
\end{pmatrix},
$$

(19)
Table 1: Sample supersymmetric particle spectrum that may lead to enhanced non-universality in $K \to \ell\nu$ decays. The mass parameters are in GeV units.

which minimizes at leading-logarithmic level all flavour mixings in the left slepton sector. Eq. (19) also implies that the CP violation in the neutrino sector is entirely linked to leptonic CP violation in the light neutrino sector, i.e., to the Dirac phase $\delta$ and to the two Majorana phases $\beta_{1,2}$ of the light neutrino mass matrix. These phases give rise to CP violation consistent with leptogenesis [14], as well as to electric dipole moments of charged leptons [29, 30]. Consequently, in this scenario, high-energy CP violation in $N_i$ decays can, in principle, be tested through low-energy measurements.

As already discussed, in contrast to the left-slepton sector, large flavour mixings in the right-slepton sector must exist in order to generate observable non-universality in the $K \to \ell\nu$ decays. Specifically, as discussed earlier, the mixing must be large in the $\tau - e$ sector; such a mixing could be induced due to the RGE running above the GUT scale. However if, in addition, considerable mixing exists in the $\mu - e$ or $\mu - \tau$ sectors, the stringent experimental bounds from $\mu \to e\gamma$ decays would rule out the scenario. Thus the phenomenological requirements are such that only $(1 \leftrightarrow 3)$ LFV mixing is allowed in the RR sector. The above considerations indicate that the SU(5) GUT model must be non-minimal and fine-tuned in the flavour sector above the GUT scale. In practice this implies that the non-renormalizable corrections [25] to the coloured triplet Higgs Yukawa couplings must be such that the corrected mixing matrix (14) is, in the standard parameterization, described with the mixing angles $\theta_{12}^R = \theta_{23}^R = 0$, $\theta_{13}^R \neq 0$. We do not speculate on the origin of such a flavour pattern, we just comment that such a model is consistent with phenomenology and allowed by model building [25].

| Input Parameters | Value |
|------------------|-------|
| $m_{1/2}$        | 1000  |
| $m_0$            | 200   |
| $\tan \beta$    | 50    |
| $|M_{Hu}|$        | 2550  |
| $|M_{Hd}|$        | 2500  |
| $A_0$            | 3000  |

| (s)-particle masses | Value |
|---------------------|-------|
| $M_1$               | 432   |
| $m_{x_1}$           | 425   |
| $\mu$               | 2394  |
| $m_{h0}$            | 115   |
| $M_{H^+}$           | 201   |
| $m_{e_L}; m_{\mu_L}$| 681   |
| $m_{e_R}; m_{\mu_R}$| 432   |
| $m_{\tau_1}$        | 505   |
| $m_{\tau_2}$        | 852   |
In conclusion, the flavour constraints on the non-universality parameter $\Delta_{R}^{31}$, the decay $BR(\tau \rightarrow e\gamma)$ and the EDM of the electron can depend only on Eq. (19) which controls the heavy neutrino parameters and on the $(1 \leftrightarrow 3)$ mixing in the RR slepton sector. Thus the flavour structure is essentially fixed, implying particularly strong correlations between the relevant observables.

Although the flavour construction presented above eliminates the most constraining RGE-induced LFV decay $\mu \rightarrow e\gamma$ at the leading-logarithmic level, dangerous $(1 \leftrightarrow 2)$ mixing appears beyond the leading-logarithmic approximation. Our first concern is to check that our numerical calculations are consistent with all the present bounds on LFV decays. In the left panel of Fig. 2 we present a scatter plot of the branching ratios for $BR(\mu \rightarrow e\gamma)$ and $BR(\tau \rightarrow e\gamma)$ which are obtained for the SUSY point of Table 1 by randomly generating all the free neutrino seesaw parameters and the right-mixing angle $\theta_{13}^{R}$. Since we work in the large-$\tan \beta$ regime, the value of $BR(\mu \rightarrow e\gamma)$ generated beyond the leading-logarithmic level can be as large as the present experimental bound. The decay rate of $\tau \rightarrow e\gamma$ is directly controlled by $\theta_{13}^{R}$, which constrains its value. There is no correlation between $BR(\mu \rightarrow e\gamma)$ and the other observables of the model and, thus, both $BR(\mu \rightarrow e\gamma)$ and $BR(\tau \rightarrow e\gamma)$ can be suppressed below the present bound by our flavour construction.

However, the right panel of Fig. 2 in which we plot the non-universality parameter $R_{K}^{LFV}$ as a function of $BR(\tau \rightarrow e\gamma)$ for three different charged Higgs boson masses, indicates a strong correlation between these observables. We conclude that non-universality of this magnitude is indeed observable in the NA62 experiment. Detection of non-universality in $K \rightarrow \ell\nu$ decays would allow estimating the LFV rates in the tau sector, provided that the charged Higgs boson mass is determined at the LHC.

We now recall a couple of well-known generic problems in SUSY: the supersymmetric CP problem and the cosmological gravitino problem. In our scenario the parameterization (19) solves both of them provided $H_{11} \ll H_{22,33}$. Indeed, the EDM of electron, $d_e$, is proportional to $H_{11}$, and its smallness suppresses this EDM independently of the phases arising in other sectors of the theory. At the same time, in this parameterization, Eq. (19) also determines the heavy neutrino mass spectrum. If $H_{11} \ll 1$, the seesaw mechanism implies that the $N_1$ mass has to be small, allowing a low reheating temperature of the Universe and solving the gravitino problem. Thus, if our construction is correct, there should be a correlation between the maximal $d_e$ and the lightest neutrino mass. In such a case, the standard Fukugita-Yanagida leptogenesis mechanism [28] cannot provide the observed baryon asymmetry of the universe due to too small $M_{N_1}$ [49]. In our SUSY scenario, therefore, resonant leptogenesis or “soft leptogenesis” [50] turn out to be the favoured leptogenesis mechanisms.

We now study quantitatively the above qualitative statements. We first recall that in the minimal SUSY seesaw model (without right-slepton mixings induced above the GUT scale) one finds strong correlations between the generated baryon asymmetry, the RGE-induced electron electric dipole moment $d_e$ and $BR(\tau \rightarrow e\gamma)$ [14]. The maximally allowed values of $d_e$ are a few orders of magnitude below the present experimental bound. This correlation occurs because in the minimal SUSY seesaw model all these observables are generated by the dominant $(1 \leftrightarrow 3)$ mixing. In our scenario the $(1 \leftrightarrow 3)$ mixing occurs in the right-
Figure 2: Correlations of $BR(\mu \to e\gamma)$ (left panel) and $R^{LFV}_K$ (right panel) with $BR(\tau \to e\gamma)$ for the SUSY points $\tan \beta = 50$, $M^2_2 = 1000$ GeV, $m_0 = 200$ GeV, $A_0 = 3000$ GeV, $|M_{H_d}| = 2500$ GeV, and for three values of $|M_{H_u}| = 2545$ GeV (upper band, black dots), $|M_{H_u}| = 2550$ GeV (middle band, red dots), $|M_{H_u}| = 2555$ GeV (upper band, green dots), The remaining parameters are randomly generated.

Figure 3: Dependence of the electron electric dipole moment $d_e$ on $BR(\tau \to e\gamma)$ (left panel) and on the lightest neutrino mass $M_{N_1}$ (right panel). The parameters and the colour code are as in Fig. 2.

slepton sector, and such a correlation is expected to be absent. In the left panel of Fig. 3
we present a scatter plot of the values of $d_e$ and $BR(\tau \rightarrow e\gamma)$ for the same parameters as in Fig. 2. In our scenario, the predicted values of $d_e$ can easily exceed the present bound $d_e < 1.6 \cdot 10^{-27}$ e cm and the CP-violating and LFV observables are not correlated. However, as explained above, there is a correlation between $d_e$ and $M_{N_1}$, as can be seen in the right panel of Fig. 3. Indeed, for a fixed $M_{N_1}$ there is an upper bound on the electron EDM. Therefore our scenario relates the solution of the SUSY CP problem to the gravitino problem - if $M_{N_1}$ is small enough to be generated thermally at reheating, the electron EDM is suppressed. Thus, avoidance of the gravitino problem in SUSY models could also explain why $d_e$ has not been observed so far.

5 Conclusions

In view of the expected improvements in measurements of $K \rightarrow \ell\nu$ decays by the NA62 experiment, we have studied the expected violation of lepton universality in these decays, in supersymmetric models. Unlike flavour-violating decays, which mainly probe the left-handed sector of the theory, a violation of universality in $K \rightarrow \ell\nu$ originates directly from mixing effects in the right-handed slepton sector. In this respect, it would provide a unique probe into this aspect of supersymmetric flavour physics, particularly for large $\tan \beta$.

Unless universality in the scalar soft terms is violated, $K \rightarrow \ell\nu$ decays can give observable rates only in non-minimal grand unified models; this would occur through a combination of RGE effects above the GUT scale and higher-dimensional terms that enhance the mixing among the right-handed sleptons. Even in this case, we are limited to very specific regions of the parameter space, with large $A$ terms and small Higgs masses. Moreover, the very strong bounds from several flavour-violating processes would require a significant suppression of left-handed slepton mixing, would further limit the already constrained the supersymmetric parameter space, and would imply fine-tuned solutions.

We find that, in the scenario under consideration, the flavour structure of the soft supersymmetry breaking terms induced by RGE effects both below and above the GUT scale is essentially fixed. This implies strong correlations between different lepton-flavour-violating processes. In particular, should the NA62 experiment at CERN discover the non-universality effects, observable rates for $\tau \rightarrow e\gamma$ can be predicted. At the same time, the electron EDM naturally exceeds the present experimental bound unless the lightest heavy neutrino mass is sufficiently small, as seen in Fig. 3. In this scenario the solution to the supersymmetric gravitino problem is, due to the constrained flavour structure of the neutrino Yukawa couplings, related to the LFV observables and EDMs. In particular, the expected future experimental sensitivity to the electron EDM will put an upper limit to the lightest heavy neutrino mass and to solve (or rule out this solution) to the gravitino problem.

In view of the above, one may hope for either of the following:
(i) to see a deviation from lepton universality in the near future, which would imply that we must focus on a very constrained set of solutions in the SUSY parameter space;
(ii) to obtain further constraints on the model parameters and unknown aspects of right-
handed fermion and sfermion mixing.

Acknowledgements We thank P. Paradisi for useful communication and discussions. S. Lola and M. Raidal would like to thank the CERN Theory Division, where a significant amount of this research has been performed. The research of S. Lola is funded in part by the FP6 Marie Curie Excellence Grant MEXT-CT-2004-014297 and that of M. Raidal by the ESF grant No. 6140. The work of J. Ellis and S. Lola was supported in part by the European Union through the Marie Curie Research and Training Network UniverseNet (MRTN-CT-2006-035863).

References

[1] Y. Fukuda et al., Super-Kamiokande Collaboration, Phys. Rev. Lett. 81 (1998) 1562, Phys. Rev. Lett. 82 (1999) 1810 and Phys. Rev. Lett. 82 (1999) 2430.

[2] Q. R. Ahmad et al., SNO Collaboration, Phys. Rev. Lett. 87 (2001) 071301 and Phys. Rev. Lett. 89 (2002) 011301.

[3] K. Eguchi et al., KamLAND Collaboration, Phys. Rev. Lett. 90 (2003) 021802; T. Araki et al., KamLAND Collaboration, Phys. Rev. Lett. 94 (2004) 081801.

[4] M.H. Ahn et al., K2K Collaboration, Phys. Rev. Lett. 90 (2003) 041801.

[5] D.G. Michael et al., MINOS collaboration, Phys. Rev. Lett. 97 (2006) 191801.

[6] For an extensive list of references on the neutrino oscillation, reactor and accelerator data, and for related global fits, see: M. Maltoni, T. Schwetz, M.A.Tortola, J.W.F. Valle, New J. Phys. 6 (2004) 122; M.C. Gonzalez-Garcia, Phys. Scripta T121 (2005) 72.

[7] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961; L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267 (1986) 415.

[8] For a complete review and references see, M. Raidal et al., arXiv:hep-ph/0801.1826.

[9] N. Arkani-Hamed, et al., Phys. Rev. Lett. 77 (1996) 1937 and Nucl. Phys. B 505 (1997) 3; J. Hisano et al., Phys. Rev. D 60 (1999) 055008. I. Hinchliffe and F. E. Paige, Phys. Rev. D 63 (2001) 115006; D. F. Carvalho et al., Phys. Lett. B 618 (2005) 162.

[10] D. Tommasini et al., Nucl. Phys. B444 (1995) 451; J. Hisano et al., Phys. Lett. B 357 (1995) 579 and Phys. Rev. D 53 (1996) 2442; J. R. Ellis et al., Eur.Phys.J.C14 (2000) 319; J. L. Feng, Y. Nir, and Y. Shadmi, Phys.Rev.D61 (2000) 113005; S. Baek et al., Phys. Rev. D64 (2001) 095001; S. Lavignac, I. Masina and C. A. Savoy, Phys. Lett. B 520 (2001) 269.

[11] J. A. Casas and A. Ibarra, Nucl. Phys. B 618 (2001) 171.
[12] J. Hisano and D. Nomura, Phys. Rev. D 59 (1999) 116005.

[13] For a recent review and a list of references, see S. Davidson, E. Nardi and Y. Nir, Phys. Rept. 466 (2008) 105. See also:
   R. Barbieri, et al., Nucl. Phys. B 575 (2000) 61; J. Ellis, J. Hisano, S. Lola and M. Raidal, Nucl. Phys. B621 (2002) 208; J. Ellis and M. Raidal, Nucl. Phys. B643 (2002) 229; T. Endoh et al., Phys. Rev. Lett. 89 (2002) 231601; W. Buchmüller, P. Di Bari and M. Plumacher, Nucl. Phys. B 643 (2002) 367 [Erratum-ibid. B 793 (2008) 362, Nucl. Phys. B665 (2003) 445 and Annals Phys. 315 (2005) 305; G.C. Branco, et al., Nucl. Phys. B640 (2002) 202 and Phys. Rev. D67 (2003) 073025; S. Pascoli, S.T. Petcov and W. Rodejohann, Phys. Rev. D68 (2003) 093007; L. Velasco-Sevilla, JHEP 0310 (2003) 035; G. F. Giudice et al., Nucl. Phys. B 685 (2004) 89; S. T. Petcov, W. Rodejohann, T. Shindou and Y. Takanishi, Nucl. Phys. B 739 (2006) 208.

[14] J. R. Ellis and M. Raidal, Nucl. Phys. B 643 (2002) 229; F. R. Joaquim, I. Masina and A. Riotto, Int. J. Mod. Phys. A 22 (2007) 6253; S. Davidson, J. Garayoa, F. Palorini and N. Rius, JHEP 0809 (2008) 053.

[15] H. Murayama, et al., Phys. Rev. Lett. 70 (1993) 1912 and Phys. Rev. D 50 (1994) 2356; J. R. Ellis, M. Raidal and T. Yanagida, Phys. Lett. B 581 (2004) 9; P. H. Chankowski, et al., Nucl. Phys. B 690 (2004) 279; S. Antusch, M. Bastero-Gil, S. F. King and Q. Shafi, Phys.Rev.D71 (2005) 083519.

[16] A. Belyaev, et al., Eur. Phys. J. C 22 (2002) 715 and \texttt{arXiv:hep-ph/0107046}.

[17] A. Masiero, P. Paradisi and R. Petronzio, Phys. Rev. D 74 (2006) 011701; A. Masiero, P. Paradisi and R. Petronzio, arXiv:hep-ph/0807.4721.

[18] A. Brignole and A. Rossi, Nucl. Phys. B 701 (2004) 3.

[19] P. Paradisi, JHEP 0602 (2006) 050 and JHEP 0608 (2006) 047.

[20] M. Antonelli et al., FlaviaNet Working Group on Kaon Decays, arXiv:hep-ph/0801.1817; Flavianet kaon WG, \texttt{http://www.lnf.infn.it/wg/vus/}.

[21] W.J. Marciano and A. Sirlin, Phys. Rev. Lett. 71 (1993) 3629; M.Finkemeier, Phys. Lett. B 387 (1996) 391.

[22] R. Barbieri and L. J. Hall, Phys. Lett. B 338 (1994) 212; R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. B 445 (1995) 219.

[23] P. Minkowski, Phys. Lett. B 67 (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (eds. A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[24] S. Davidson and A. Ibarra, JHEP 0109 (2001) 013; J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Rev. D 66 (2002) 115013.

[25] J. Hisano, D. Nomura, Y. Okada, Y. Shimizu and M. Tanaka, Phys. Rev. D 58 (1998) 116010.

[26] J. L. Evans, D. E. Morrissey and J. D. Wells, Phys. Rev. D 75 (2007) 055017; E. J. Chun, J. L. Evans, D. E. Morrissey and J. D. Wells, arXiv:hep-ph/0804.3050.

[27] G. L. Bayatian et al. [CMS Collaboration], J. Phys. G 34 (2007) 995.

[28] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.

[29] J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B 528 (2002) 86.

[30] I. Masina, Nucl. Phys. B 671 (2003) 432.

[31] Y. Farzan and M. E. Peskin, Phys. Rev. D 70 (2004) 095001.

[32] B. C. Regan, E. D. Commins, C. J. Schmidt and D. DeMille, Phys. Rev. Lett. 88 (2002) 071805.

[33] J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 145 (1984) 181; M. Y. Khlopov and A. D. Linde, Phys. Lett. B 138 (1984) 265.

[34] D. DeMille et al., Phys. Rev. A 61 (2000) 052507.

[35] A. Brignole and A. Rossi, Phys. Lett. B 566 (2003) 217.

[36] B. Aubert et al., Phys. Rev. Lett. 96 (2006) 041801.

[37] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Phys. Lett. B 88 (1979) 320.

[38] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Phys. Lett. B 80 (1979) 360.

[39] M. S. Chanowitz, J. R. Ellis and M. K. Gaillard, Nucl. Phys. B 128 (1977) 506; A. J. Buras, J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B 135 (1978) 66.

[40] J. R. Ellis and M. K. Gaillard, Phys. Lett. B 88 (1979) 315.

[41] See, for example, J. Hisano and Y. Shimizu, Phys. Lett. B 565 (2003) 183; M. Ciuchini, A. Masiero, P. Paradisi, L. Silvestrini, S. K. Vempati and O. Vives, Nucl. Phys. B 783 (2007) 112; T. Goto, Y. Okada, T. Shindou and M. Tanaka, Phys. Rev. D 77 (2008) 095010; P. Ko, J. h. Park and M. Yamaguchi, arXiv:0809.2784 [hep-ph].

[42] J. R. Ellis, J. L. Lopez, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B 308 (1993) 70.

[43] S. Lola and G.G. Ross, Nucl. Phys. B553 (1999) 81.
[44] J. R. Ellis, M. E. Gomez and S. Lola, JHEP 0707 (2007) 052.

[45] J. R. Ellis, G. K. Leontaris, S. Lola and D. V. Nanopoulos, Eur. Phys. J. C 9 (1999) 389.

[46] M. Battaglia et al., Eur. Phys. J. C 33 (2004) 273.

[47] See, for example, A. De Roeck, J. R. Ellis, F. Gianotti, F. Moortgat, K. A. Olive and L. Pape, Eur. Phys. J. C 49 (2007) 1041 [arXiv:hep-ph/0508198].

[48] J. R. Ellis et al., Phys. Lett. B 653 (2007) 292 and JHEP 0710 (2007) 092.

[49] S. Davidson and A. Ibarra, Phys. Lett. B 535 (2002) 25.

[50] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, Phys. Rev. Lett. 91 (2003) 251801; G. D’Ambrosio, G. F. Giudice and M. Raidal, Phys. Lett. B 575 (2003) 75.