F0 fluxbranes, F-walls and new brane worlds

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Abstract

We complete the list of fluxbrane solutions in classical supergravities by introducing
Melvin type space-times supported by antisymmetric forms of rank $D-1$ and their
pseudoscalar duals. In IIB theory these solutions belong to the same family as the
seven-brane and D-instanton. In current notation, a fluxbrane supported by the $D-1$
form is an F0 brane, its euclidean continuation is a cylindrical background which
“interacts” with a pointlike instanton. The general F0 brane can have a transverse space
$S_k \times R^{(D-k-2)}$ with $1 \leq k \leq D-2$. For $k=1$ we find the complete solution containing
four parameters, three of them associated with pointlike singularities on the Melvin-
type background. The S-dual to the F0 brane in ten dimensions is the F8 fluxbrane of
codimension one, or F-wall, similar fluxbranes exist in any dimensions if an appropriate
form field is present. F-walls contain naked singularities unless one introduces source
branes. In such a way one obtains new brane-world type solutions with two bulk scalar
fields. Their relation to the supersymmetric brane-worlds is discussed.

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1 Introduction

Multidimensional generalizations of the Melvin magnetic universe [1], currently known as fluxbranes, were suggested by Gibbons and Witshire in 1986 [2] and generalized for the presence of dilaton by Gibbons and Maeda [3]. Their relevance in quantum theory was emphasized in [4, 5] where quantum creation of branes in the fluxbrane background was considered. More recently fluxbrane solutions were found to be important in the superstring theory [4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Their applications include construction of exact string backgrounds [1, 7], demonstration of duality between type 0A and IIA string theories [10], brane nucleation [5, 27], tachyon condensation [23, 26, 27] and others. Some of these solutions (but not all) are obtainable by compactification of vacuum spaces on twisted circles [5], other were found as exact solutions of supergravity equations. A rather complete list of (non-intersecting) fluxbrane solutions in classical ten and eleven-dimensional supergravities was given in [20], this includes F6 and F3 branes in $D = 11$ and solutions corresponding to rank $1 − 7$ antisymmetric potentials in IIA and IIB theories. The transverse space of $F_p$ fluxbranes may have spherical or cylindrical topology $S_k × R^{8−p−k}$. Analytic solutions of the Melvin type exist only for $k = 1$, in other cases regular solutions were found numerically.

Here we add to the list of one-component fluxbranes new solutions corresponding to so-called exotic branes. In type IIB theory these are associated with the Ramond-Ramond scalar and its dual eight-form potential, which give rise to the D-instanton and the sevenbrane [28, 29, 30]. Another exotic brane is the eight-brane, usually interpreted as belonging to massive IIA supergravity [31]. An alternative interpretation uses a formulation of type II supergravities including a nine-form potential [32], note also the more recent “brane and bulk” formulation of ten-dimensional supergravities including all Ramond-Ramond potentials of rank $p = 0, ..., 9$ [33].

Magnetic fluxbranes in IIB theory carrying the flux of an eight-form, according to current notation [12], are F0 branes, with a pointlike support, so they present a new type of Lorentzian solutions of physical interest. It may seem counterintuitive to think about such configurations as fluxbranes, but in fact they preserve the main fluxbrane property — the finiteness of the form flux, and the space is still of the Melvin type in the cases where analytic solutions exist. Another natural question is that of the object with which the F0 branes are entitled to interact. Similarly with F1 fluxbranes which interact with a point charge, by dimensional counting F0 fluxbranes should interact with an instanton, in other words, putting the instanton on the (euclidean continuation of the) F0 fluxbrane will modify
the D-instanton action. This is, however, not in the stream of the present investigation, so we do not discuss it further. For recent generalizations of instanton solutions to arbitrary dimensions (with the same action as ours) and to (A)dS spaces see [34].

We show that dilatonic F0 branes exist in any dimension \( D \) with transverse spaces \( S_k \times R^{(D-k-2)} \) for \( 1 \leq k \leq D - 2 \), and we find analytic solutions for \( k = 1 \) and \( k = D - 2 \). In the first case our solution is generic and contains (after gauge fixing) four arbitrary parameters. For certain values of these parameters one has a regular space of the Melvin type, while generically pointlike naked singularities are present. In particular, in \( D = 3 \) this is a static Melvin-type magnetic universe supported by the Maxwell field. In four dimensions such a solution is supported by the Kalb-Ramond field which can be equally associated with the NS potential.

Electric fluxbranes associated with an eight-form in ten dimensions are F8 branes, alternatively they can be viewed as magnetic solutions supported by a RR scalar. These have the well-known structure of domain walls. F-walls turn out to be singular on some nine-dimensional hyperplane, so it is suggestive to introduce the source eight-brane(s). We show that a single brane of negative tension matches indeed to the F-wall producing an otherwise non-singular space-time. The situation is similar in any dimension in the presence of a scalar potential. It is worth noting that the source brane does not carry the charge of the fluxbrane form field, which in this case is not of the correct rank. To compare the situation with the recently studied charged supersymmetric branes [35, 36, 37, 38], we also construct a solution for a source eight-brane carrying the RR charge of a nine-form potential. This is similar to the charged Randall-Sundrum flat brane-world without bulk cosmological constant, its five-dimensional counterpart is reminiscent of the solutions discussed in [39]. Our solution containing a single source brane is regular outside the brane in the case of a negative brane tension. More complicated regular solutions should exist, including several source branes of both positive and negative tensions.

## 2 F0 branes

In this paper we will essentially use the formulation of Ref. [20]. Consider the coupled system of \( D \)-dimensional gravity, an antisymmetric form of rank \( D - 1 \) and the dilaton:

\[
S = \int \left( R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{(D-1)!} e^{a \phi} F_{[D-1]}^2 \right) \sqrt{-g} \, d^D x, \tag{2.1}
\]

where \( F_{[D-1]} = dA_{[D-2]} \) and \( a \) is an arbitrary dilaton coupling. The F0-brane spacetime we are looking for can be parametrized by three functions \( A(r), B(r), C(r) \) of the radial
variable as follows
\[ ds^2 = -e^{2A} dt^2 + e^{2B} dr^2 + e^{2C}(r^2 d\Omega_k^2 + \delta^{mn} dy_m dy_n), \]  
(2.2)

with \( \Omega_k, (1 \leq k \leq D - 2), \) being the spherical volume element, and \( m, n = 1, \ldots, D - 2 - k \). The F0-brane is centered at the origin, the \((D - 1)\)-dimensional transverse space is the product of the \(k + 1\)-dimensional ball and the flat euclidean space of dimension \(D - 2 - k\).

The magnetic ansatz for the F0-brane reads
\[ F_{[D-1]} = f(r) dr \wedge \epsilon[k] \wedge dy_1 \wedge \cdots \wedge dy_{D-2-k}, \]  
(2.3)

where \( \epsilon[k] \) is the volume element of the unit \(k\)-sphere. The \(D - 1\)-form field equation
\[ \partial_\mu \left( \sqrt{-g} e^{a \phi} F^{\mu \nu_1 \cdots \nu_9} \right) = 0, \]  
(2.4)

is then easily solved with the above ansatz to give
\[ f = 2\tilde{b} e^{H + 2(B-A) - a \phi}, \]  
(2.5)

where \( \tilde{b} \) is an integration constant, and
\[ H = A - B + (D - 2)C + k \ln r. \]  
(2.6)

The non-zero components of the Ricci tensor for the metric \(2.2\) are given in \(20\), the resulting Einstein equations read
\[ A'' + H' A' - 2 \tilde{b}^2 e^{2(B-A) - a \phi} = 0, \]  
(2.7)
\[ C'' + H' C' + r^{-2} \left( rH' - 1 - (k - 1)e^{2B - 2C} \right) = 0, \]  
(2.8)
\[ (A'' + A'^2 - A'B') + (D - 2)(C'' + C'^2 - B'C') \]  
\[ + k r^{-1} (2C' - B') + \frac{1}{2} \phi'^2 = 0. \]  
(2.9)

In addition, for all \( k < D - 2 \) the vanishing of the \(R_{yy}\) component of the Ricci tensor leads to the equation
\[ C'' + H' C' = 0. \]  
(2.10)

The dilaton equation is similar to \(2.7\):
\[ \phi'' + H' \phi' = 2a \tilde{b}^2 e^{2(B-A) - a \phi}. \]  
(2.11)
2.1 $k = 1$

We start with the simplest case $k = 1$ (assuming $D > 3$), in which the system of equations can be integrated in a closed form. In this case, Eq. (2.8) combined with (2.10) gives

$$H = \ln(r/l), \quad l = \text{const.} \quad (2.12)$$

Substituting this into Eq. (2.10) we find

$$C = \gamma \ln(r/r_0), \quad r_0 = \text{const}, \quad (2.13)$$

and consequently

$$B = A + \gamma(D - 2) \ln(r/r_0) + \ln l. \quad (2.14)$$

Integrating a linear combination of Eqs. (2.7) and (2.11) we find the following relation between $\phi$ and $A$:

$$\phi = aA + \beta \ln(r/r_1), \quad \beta, r_1 = \text{const}. \quad (2.15)$$

Using this together with the two previous relations in the Einstein equation (2.9), we arrive at the following non-linear equation for the dilaton function

$$\ddot{\phi} + \frac{a}{2} \dot{\phi}^2 - 2\alpha \dot{\phi} + 2\alpha \beta - a \frac{D - 3}{D - 2}(\alpha^2 - 1) = 0, \quad (2.16)$$

where the dot denotes the derivative with respect to the logarithmic variable $\tau = \ln r$, and $\alpha = 1 + \gamma(D - 2)$. This is a first order equation for $\dot{\phi}$, which is solved by

$$\phi = \phi_0 + \frac{2}{a} \left[ \alpha(\tau + \tau_0) + \ln(2 \cosh \kappa(\tau + \tau_0)) \right], \quad (2.17)$$

with

$$\kappa^2 = \alpha^2 - a\beta \alpha + \frac{a^2 D - 3}{2(D - 2)}(\alpha^2 - 1). \quad (2.18)$$

Here $\phi_0$ and $\tau_0$ are two new integration constants, and it is convenient to rename

$$\tau_0 = \ln b. \quad (2.19)$$

So far we have not used the Einstein equation (2.7). Substituting (2.13), (2.17), (2.13) and (2.14) into (2.7), one finds

$$\bar{b} = \frac{2kb}{|a|} (br_0)^{\alpha - 1} e^{a\phi_0/2}. \quad (2.20)$$
As a result, we obtain the following solution

\[ e^{2A} = e^{2\phi_0/a} (br)^{4(\alpha-\kappa)/a^2} \left( \frac{r_1}{r} \right)^{2\beta/a} \left( 1 + (br)^{2\kappa} \right)^{4/a^2}, \]  
\[ (2.21) \]

\[ e^{2B} = l^2 \left( \frac{r}{r_0} \right)^{2(\alpha-1)} e^{2A}, \]  
\[ (2.22) \]

\[ e^{2C} = \left( \frac{r}{r_0} \right)^{2(\alpha-1)/(D-2)} e^{2A}, \]  
\[ (2.23) \]

\[ e^{a\phi/2} = e^{a\phi_0/2} (br)^{\alpha-\kappa} \left( 1 + (br)^{2\kappa} \right), \]  
\[ (2.24) \]

\[ f = \frac{4\kappa}{|a|} e^{-a\phi_0/2} (br_0)^{1-\alpha} (br)^{2\kappa-1} \left( 1 + (br)^{2\kappa} \right)^{-2}, \]  
\[ (2.25) \]

depending on arbitrary constants \( r_0, r_1, l, b, \alpha, \kappa, \phi_0 \) (\( \beta \) being related to \( \kappa \) by (2.18)). Without loss of generality, one can fix the scale of time and of the radial coordinate \( r \), and choose a normalization point for the dilaton so that

\[ r_0 = r_1 = b^{-1}, \quad \phi_0 = 0, \]  
\[ (2.26) \]

so finally we have a four-parametric family of solutions. Evidently, \( b \) is the fluxbrane field strength parameter, the physical sense of other parameters is yet to be clarified.

Remarkably, the magnetic field \( F \) depends only on the parameters \( b \) and \( \kappa \). It derives from the potential

\[ A_{[D-2]} = \frac{2}{|a|b} \frac{(br)^{2\kappa}}{1 + (br)^{2\kappa}} d\varphi \wedge dy_1 \wedge \cdots \wedge dy_{D-3}. \]  
\[ (2.27) \]

The corresponding magnetic flux per unit (D-3)-dimensional volume is finite,

\[ \Phi = \oint_{r \to \infty} A_{[D-2]} = \frac{4\pi}{|a|b} dy_1 \wedge \cdots \wedge dy_{D-3}. \]  
\[ (2.28) \]

Solutions regular in the origin are obtained for \( \gamma = 0 \ (\alpha = 1) \), \( \beta = 0 \ (\kappa = 1) \) and \( l = 1 \).

The resulting metric

\[ ds^2 = (1 + b^2 r^2)^{4/a^2} (-dt^2 + dr^2) + r^2 d\varphi^2 + \delta^{mn} dy_m dy_n \]  
\[ (2.29) \]

is nothing else than the smeared \( (1 + 2) \)-dimensional dilatonic Melvin-type solution. The corresponding dilaton and the form field are

\[ e^{a\phi/2} = 1 + (br)^2, \]  
\[ (2.30) \]

\[ F_{D-1} = \frac{4bkr}{|a|} \left( 1 + (br)^2 \right)^{-2} dr \wedge \epsilon_{[k]} \wedge dy_1 \wedge \cdots \wedge dy_{D-2-k}. \]  
\[ (2.31) \]
2.2 $2 \leq k \leq D - 3$

For $k \neq 1$ there is no hope to find the general solution analytically (except in the ultimate spherically symmetric case $k = D - 2$, see below), so one is led to numerical calculations. Still, a particular solution of the type discussed in [20] can be found by making the ansatz $C \equiv 0$, $\phi = aA$ which is consistent with the set of equations. Then one can derive a decoupled second order differential equation for $A$:

$$
\left(\frac{2k}{r} A'' + \frac{k^2 A'}{2} + 2k A'^2 - a^2 r^2 A'^2 \right) (k - 1)e^{(a^2 + 2)A}
+ 2b^2 r \left(2r^2 A'' - 2r A' (2k - 1) - 2k(k - 1) + a^2 r^2 A' \right) = 0 \quad , \quad (2.32)
$$

while $B$ is given in terms of $A$ as follows

$$
e^{2B} = \frac{2k A' + k(k - 1) - a^2 r^2 A'^2 / 2}{k(k - 1) + 2b^2 r^2 e^{-(a^2 + 2)A}}.
\quad (2.33)
$$

This system admits an analytic solution

$$
2A = \gamma \ln \left(\frac{2b^2 r^2}{\gamma (\gamma + k - 1)}\right),\quad (2.34)
$$

$$
2B = \ln \left(1 + \frac{\gamma}{k - 1}\right), \quad \gamma = \frac{2}{a^2 + 2}.\quad (2.35)
$$

This solution is singular at the origin, but it serves as an attractor for regular ones that can be found numerically. The latter are quite similar to more general solutions found in [20] so we do not give them here.

2.3 $k = D - 2$

Another case in which some progress can be made is that of spherical symmetry, $k = D - 2$. Putting $C = E - \ln r$, so that

$$
H = A - B + kE, \quad (2.36)
$$

we find that Eqs. (2.8) and (2.9) take the simpler form

$$
E'' + H' E' = (k - 1)e^{2(B - E)}, \quad (2.37)
$$

$$
A'' + A'^2 - A'B' + k(E'' + E'^2 - E'B') + (1/2)\phi'^2 = 0. \quad (2.38)
$$

We see that Eq. (2.37) is of the same form as Eqs. (2.7) and (2.11) provided

$$
E = A + \frac{a}{2}\phi + \epsilon \quad \quad (2.39)
$$
with constant $\epsilon$. Furthermore, in the case $k = D - 2$ one can always use radial coordinate transformations to choose a gauge such that

$$H = \ln(r/l).$$  \hspace{1cm} (2.40)

Note that this condition does not fix the gauge completely, the condition (2.40) being, up to an additive constant, invariant under reparametrizations $r \to \tilde{r} = r^p$, which lead to $\tilde{H} = H + (p - 1) \ln r + \text{const} = \ln r + \text{const}$. As in the case $k = 1$, the comparison of Eqs. (2.7) and (2.11) now leads to

$$\phi = aA + \beta \ln(r/r_1),$$  \hspace{1cm} (2.41)

where $\beta$ is some constant. On account of (2.39) it follows that

$$E = \gamma^{-1} A + \frac{a\beta}{2} \ln(r/r_1) + \epsilon,$$  \hspace{1cm} (2.42)

with $\gamma^{-1} = 1 + a^2/2$. Eqs. (2.7) and (2.37) are then equivalent provided

$$e^{2\epsilon} = \frac{(k - 1)\gamma}{2b^2}.$$  \hspace{1cm} (2.43)

Inserting these relations in Eq. (2.38), we arrive at the first order differential equation with respect to the variable $\tau = \ln(br)$:

$$\ddot{A} + (1 - k)a\beta \dot{A} + \frac{\beta^2}{2} \left(1 - \frac{k^2}{k + \gamma}\right) + \frac{1 - k - \gamma}{\gamma} \dot{A}^2 = 0.$$  \hspace{1cm} (2.44)

For $\beta \neq 0$ (the solution in the case $\beta = 0$ is a gauge transform of the power-law solution (2.34), (2.35)), this is solved by

$$\dot{A} = \alpha + \kappa \delta \cot \kappa (\tau + \tau_0),$$  \hspace{1cm} (2.45)

with

$$\alpha = (k - 1)\delta \frac{a\beta}{2}, \quad \delta = -\frac{\gamma}{k + \gamma - 1}, \quad \kappa^2 = \frac{\beta^2}{2} \left(k - 2 + \frac{1}{k + \gamma}\right).$$  \hspace{1cm} (2.46)

At this point, we may further fix the gauge by choosing $\beta = -2/a\delta$ so that $\alpha = 1 - k$. Integrating (2.43) (with convenient choices for the integration constants) we obtain the solution as

$$e^{2A} = (br)^{-2(k-1)} \sin \kappa \tau \left(\frac{1}{k + \gamma - 1}\right)^{2\gamma},$$  \hspace{1cm} (2.47)

$$e^{2B} = \sin \kappa \tau \left(\frac{2(k + \gamma)}{k + \gamma - 1}\right)^{2\gamma},$$  \hspace{1cm} (2.48)

$$e^{2C} = b^2 e^{2A} \sin \kappa \tau \left(\frac{1}{k + \gamma - 1}\right)^{2\gamma},$$  \hspace{1cm} (2.49)

$$e^{a\phi} = (br)^{2k} \sin \kappa \tau \left(\frac{2(k + \gamma)}{k + \gamma - 1}\right)^{2\gamma}.$$  \hspace{1cm} (2.50)
(τ = ln(br)). Inserting these in (2.7), we check that this equation is satisfied provided

$$
\bar{b}^2 = \frac{\gamma \kappa^2}{2(k + \gamma - 1)}.
$$

(2.51)

The corresponding magnetic potential is

$$
A_{[k]} = -\frac{2b}{\kappa b} e^{\kappa \epsilon} \cot \kappa \epsilon.
$$

(2.52)

The solution (2.47)-(2.50) is defined only in disjoint sectors \( n\pi < \kappa \tau < (n + 1)\pi \). In such a sector, the spacetime metric behaves near the formal singularity \( \kappa \tau = n\pi \) as

$$
ds^2 \sim -R^{2\gamma} dt^2 + R^2 + b^2 e^{2\kappa \epsilon} R^2 d\Omega^2_k, \quad (R \to \infty)
$$

(2.53)

(we have put \( \kappa \tau = n\pi + R^{-(k+\gamma-1)} \), and neglected irrelevant multiplicative constants). We see that the behavior near the singularity is governed by the attractor (2.34), (2.35). A first integral for geodesic motion in such a metric is

$$
\dot{R}^2 + \frac{L^2}{R^2} = E^2 R^{-2\gamma} - \sigma
$$

(2.54)

(\( \sigma = +1, 0, \text{ or } -1 \) for timelike, null, or spacelike geodesics). Remembering that \( \gamma = (1 + a^2/2)^{-1} > 0 \), we see that timelike geodesics and nonradial \((L \neq 0)\) null geodesics are reflected by a potential barrier before reaching infinity, while spacelike and radial null geodesics reach infinity for an infinite value of the affine parameter. The behavior being similar at the other end \( \kappa \tau \to (n + 1)\pi \), it follows that the spacetime (2.47)-(2.49) is geodesically complete.

3 Particular cases

3.1 1+2 dilaton Melvin

In 1+2 dimensions the action (2.1) corresponds to the Einstein-Maxwell dilaton theory. In this case, \( k = 1 = D - 2 \), so the analytic solution with \( k = 1 \) is the general one. One can also fix \( \alpha = 1 \) \((C = 0)\) as a gauge condition. Then the solution can be rewritten as follows

$$
ds^2 = \left(\frac{r}{r_1}\right)^{2(\kappa^2 + 1)/a^2} \left[ \left(\frac{r}{r_b}\right)^{\kappa} + \left(\frac{r_b}{r}\right)^{\kappa}\right]^{4/a^2} (-dt^2 + t^2 dr^2) + r^2 d\varphi^2,
$$

(3.1)

where \( r_b = 1/b \) and we removed the constant dilaton factor by time rescaling. This spacetime is supported by the following Maxwell two-form

$$
F = \frac{4\kappa}{|a|} e^{-a\phi_0/2} r_b \left[ \left(\frac{r}{r_b}\right)^{\kappa} + \left(\frac{r_b}{r}\right)^{\kappa}\right]^{-2} dr \wedge d\varphi,
$$

(3.2)
and dilaton

\[ e^{a\phi/2} = e^{a\phi_0/2} \frac{r}{r_b} \left[ \left( \frac{r}{r_b} \right)^\kappa + \left( \frac{r_b}{r} \right)^\kappa \right]. \tag{3.3} \]

We note some overlap in this point with the recent paper [40] which appeared after our calculations had been completed.

For \( \kappa = 0 \) we recover the singular solution for a massless gravitating scalar field in 2+1 dimensions [41], [42]

\[ ds^2 = \frac{r^2}{a^2} (-dt^2 + dr^2) + r^2 d\varphi^2, \quad \phi = \frac{2}{a} \ln r, \quad F = 0. \tag{3.4} \]

In this case \( a \) is an arbitrary integration constant. Up to a coordinate transformation \( r \to r^p \), the behavior (3.4) dominates the asymptotic behavior of (3.1) \((p = 1 + \kappa)\)

\[ ds^2 \sim r^{2(1+\kappa)/2}a^2 (-dt^2 + dr^2) + r^2 d\varphi^2, \quad e^{a\phi/2} \sim r^{1+\kappa}, \quad F \sim r r^{-2(1+\kappa)} dr \wedge d\varphi, \tag{3.5} \]

as well as the small \( r \) behavior \((p = 1 - \kappa)\)

\[ ds^2 \sim r^{2(1-\kappa)/2}a^2 (-dt^2 + dr^2) + r^2 d\varphi^2, \quad e^{a\phi/2} \sim r^{1-\kappa}, \quad F \sim r r^{-2(1-\kappa)} dr \wedge d\varphi. \tag{3.6} \]

So the general solution (3.1) may be viewed as a soliton interpolating between two “vacua” where the magnetic field is negligible.

For \( \kappa = 1 \) we get the regular dilaton-Melvin solution. Coming back to the initial parametrization, and taking \( \phi_0 = 0 \), we obtain

\[ ds^2 = (1 + b^2 r^2)^{4/2} (-dt^2 + l^2 dr^2) + r^2 d\varphi^2, \tag{3.7} \]

\[ F = \frac{4br}{|a| (1 + b^2 r^2)^2} dr \wedge d\varphi, \tag{3.8} \]

\[ e^{a\phi/2} = (1 + b^2 r^2). \tag{3.9} \]

The flux of the magnetic field

\[ \Phi = \oint \oint_{r \to \infty} A_\varphi d\varphi, \tag{3.10} \]

where

\[ A_\varphi = \frac{2br}{1 + b^2 r^2} \tag{3.11} \]

is the one-form potential generating the two-form field \( F = dA \), is finite

\[ \Phi = \frac{4\pi}{ab}. \tag{3.12} \]
For \( a = 2 \) this solution can be obtained by a twisted compactification of flat four-dimensional space, similarly to the case of usual dilatonic Melvin solutions in higher dimensions. Starting with
\[
ds^2_4 = -dt^2 + dr^2 + r^2d\varphi^2 + dy^2
\]  
(3.13)
and performing dimensional reduction with respect to the Killing vector
\[
K = \partial_y + b\partial_\varphi,
\]  
(3.14)
one finds
\[
ds^2_4 = e^{-\phi}ds^2_3 + e^{\phi}(dy + A_\varphi d\varphi)^2
\]  
(3.15)
where
\[
A_\varphi = \frac{br^2}{1 + b^2r^2}, \quad e^{\phi} = 1 + b^2r^2.
\]  
(3.16)
Similarly, the general solution (3.1) with \( a = 2 \) is a twisted compactification of the cylindrically symmetric Levi-Civita metric
\[
ds^2_4 = r^{(\kappa^2-1)/2}(-dt^2 + dr^2) + (1 + \kappa) d\varphi^2 + r^{1-\kappa} dy^2.
\]  
(3.17)

### 3.2 F0 branes in IIB theory

In the case of IIB theory, \( D = 10, a = -2 \) and F0 fluxbranes correspond to the eight-form potential \( A_8 \) dual to the RR scalar. There are eight different solutions generated by the form field
\[
F_{[9]} = f(r) dr \wedge \epsilon_{[k]} \wedge dy_1 \wedge \cdots \wedge dy_{8-k}.
\]  
(3.18)
with \( k = 1, \ldots, 8 \). We find the following metric for \( k = 1 \):
\[
ds^2 = (br)^{\alpha+\beta-\kappa} \left(1 + (br)^{2\kappa}\right) \left(-dt^2 + l^2(br)^{2(\alpha-1)} dr^2\right) + (br)^{(\alpha-1)/4} \left(r^2d\varphi^2 + \delta^{mn} dy_m dy_n\right),
\]  
(3.19)
with \( \kappa^2 = \alpha^2 + 2\beta\alpha + 7(\alpha^2 - 1)/4 \), while the dilaton and the form field are
\[
e^{-\phi} = (br)^{\alpha-\kappa} \left(1 + (br)^{2\kappa}\right),
\]  
(3.20)
\[
f = \frac{2\kappa (br)^{2(\kappa-1)}}{\left(1 + (br)^{2\kappa}\right)^2}.
\]  
(3.21)
The \( \kappa = \alpha = l = 1 \) solution regular at the origin reads
\[
e^{2A} = 1 + b^2r^2,
\]  
(3.22)
\[
e^{-\phi} = 1 + b^2r^2,
\]  
(3.23)
\[
f = 2br \left(1 + b^2r^2\right)^{-2}.
\]  
(3.24)
The corresponding space-time actually is nothing else than the 1+2 dilaton Melvin metric smeared in seven extra dimensions

\[ ds^2 = (1 + b^2 r^2) (-dt^2 + dr^2) + r^2 d\varphi^2 + \delta^{mn} dy_m dy_n . \tag{3.25} \]

Numerical solutions for \( k \neq 1 \) approach the following asymptotics:

\[ A = \frac{1}{6} \ln \frac{18b^2 r^2}{3k - 2}, \quad B = \frac{1}{2} \ln \left( 1 + \frac{1}{3(k - 1)} \right) . \tag{3.26} \]

### 3.3 \( D = 4 \) dilaton-axion F0 branes

In four spacetime dimensions, the relevant theory is the dilaton-axion system coupled to gravity which for \( a = 1 \) is associated with the toroidally compactified heterotic string. Here the metric for the \( k = 1 \) solution has cylindrical symmetry and is just a smeared 1 + 2 solution, while for \( k = 2 \) one has the spherically symmetric spacetime

\[ ds^2 = -e^{2A} dt^2 + e^{2B} dr^2 + r^2 d\Omega^2 , \tag{3.27} \]

where the metric functions \( A, B \) can be found numerically as described above. Special solutions are the geodesically complete solutions

\[ ds^2 = -(br)^{-2} (\sin \kappa \tau)^{-\frac{2\gamma}{1 + \gamma}} dt^2 + (\sin \kappa \tau)^{-\frac{2(2 + \gamma)}{1 + \gamma}} dr^2 + e^{2c} (br)^2 (\sin \kappa \tau)^{-\frac{2}{1 + \gamma}} d\Omega^2 , \tag{3.28} \]

with \( \tau = \ln r, \quad \gamma = (1 + a^2/2)^{-1}, \quad \kappa = (1 + \gamma)/a \gamma \sqrt{1 + \gamma/2}, \quad e^{2c} = a^2 \gamma^2 (1 + \gamma/2)/(1 + \gamma), \) and the singular attractor

\[ ds^2 = -r^{2\gamma} dt^2 + (1 + \gamma) dr^2 + r^2 d\Omega^2 . \tag{3.29} \]

For \( a = 0 \ (\gamma = 1) \) this last metric has previously been considered by Hayward \[44\], in the context of gravitational collapse of a scalar field (actually the axion field dual to the three-form); it is continuously self-similar, and the singularity \( r = 0 \) is null.

### 4 F-walls

The S-dual solutions to the F0 branes are F-walls of codimension one. The most natural way to construct them is to pass to the dual action

\[ S_1 = \int \left( R - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} e^{\phi} F^2_{[1]} \right) \sqrt{-g} d^D x , \tag{4.1} \]
where $F_1 = dA_0$, with $A_0$ being the RR scalar in IIB theory (with $a = 2$, for generality we continue to assume the dimension $D$ and dilaton coupling $a$ to be arbitrary). Assuming $SO(D-2,1)$ Lorentz symmetry on the world-volume, we choose the metric in the gauge

$$ds^2 = e^{2A} \eta_{ab} dx^a dx^b + dy^2,$$  \hspace{1cm} (4.2)

where $A$ is the function of the transverse coordinate $y$ and $a, b = 0, \cdots, D - 2$. The form field is simply

$$F_1 = f(y) dy,$$  \hspace{1cm} (4.3)

and the solution for $f$ is readily obtained

$$f = 2b e^{-(D-1)A-a\phi}.$$  \hspace{1cm} (4.4)

The Ricci tensor has the following non-zero components:

$$R_{ab} = -\eta_{ab} \left[A'' + (D-1)A'^2\right] e^{2A},$$  \hspace{1cm} (4.5)

$$R_{yy} = -(D-1)(A'' + A'^2),$$  \hspace{1cm} (4.6)

and the corresponding Einstein equations read

$$R_{ab} = 0,$$  \hspace{1cm} (4.7)

$$R_{yy} = \frac{1}{2} \left( \phi'^2 + e^{a\phi} f^2 \right).$$ \hspace{1cm} (4.8)

Therefore, one has a decoupled equation for the metric function

$$A'' + (D-1)A'^2 = 0,$$  \hspace{1cm} (4.9)

and a constraint

$$2(D-1)(D-2)A'^2 = \phi'^2 + e^{a\phi} f^2.$$  \hspace{1cm} (4.10)

The dilaton equation reads

$$\phi'' + (D-1)A'\phi' = \frac{a}{2} e^{a\phi} f^2.$$  \hspace{1cm} (4.11)

It is convenient to substitute

$$A = \frac{1}{D-1} \ln Z.$$  \hspace{1cm} (4.12)

Integration of Eq. (4.9), which is equivalent to $Z'' = 0$, gives

$$Z = a(y + y_0),$$  \hspace{1cm} (4.13)
where $\alpha$ and $y_0$ are two integration constants. Introducing a new independent variable $\tau$ such that
\[
\frac{d\tau}{dy} = \frac{1}{Z},
\]
the dilaton equation becomes
\[
\ddot{\phi} - 2ab^2e^{-a\phi} = 0,
\]
while the constraint (4.10) reads
\[
\frac{\dot{\phi}^2}{2} + 2b^2e^{-a\phi} = \alpha^2D - 2 \frac{D-2}{D-1}.
\]
The solution to Eqs. (4.15) and (4.16) is
\[
\phi = \frac{2a}{\nu} \ln \left( \frac{ab}{\nu a \cosh [\nu a(\tau + \tau_1)]} \right), \quad \nu = a \sqrt{\frac{D-2}{2(D-1)}},
\]
where $\tau_1$ is an integration constant. Finally, from (4.14) the variable $\tau$ is related to $y$ by
\[
\tau + \tau_1 = \alpha^{-1} \ln[c(y + y_0)],
\]
with a new parameter $c$, so that the solution takes the form
\[
e^{2A} = (\alpha(y + y_0))^{2/(D-1)},
\]
\[
e^{a\phi/2} = \frac{ab}{2\nu a} \left[ (c(y + y_0))^\nu + (c(y + y_0))^{-\nu} \right],
\]
\[
f = \frac{8\nu^2 \alpha}{a^2 b(y + y_0)} \left[ (c(y + y_0))^\nu + (c(y + y_0))^{-\nu} \right]^{-2}.
\]
It has a naked singularity at $y = -y_0$.

The F8 fluxbrane of IIB theory corresponds to $D = 10, a = 2$, so $\nu = 4/3$. Letting for simplicity $\alpha = 1, y_0 = 0$, we obtain:
\[
ds^2 = y^{2/9} \eta_{ab} dx^a dx^b + dy^2,
\]
\[
e^{\phi} = \frac{3b}{4} (cy)^{4/3} + (cy)^{-4/3}),
\]
\[
f = \frac{32}{9by} (cy)^{4/3} + (cy)^{-4/3})^{-2}.
\]
After the coordinate transformation $z = 3/4y^{4/3}$ and a suitable rescaling of $x^a$, one can cast this solution into the ‘Melvin-like’ form:
\[
ds^2 = z^{1/6} \eta_{ab} dx^a dx^b + z^{-1/2} dz^2,
\]
\[
e^{\phi} = \frac{3b}{4cz} (1 + c^2 z^2),
\]
\[
f = \frac{8c^2 z^{5/4}}{3b} (1 + c^2 z^2)^{-2}.
\]
5 Adding source brane

The nature of the singularity in the F-wall solutions suggests that one can try to improve these solutions by introducing a source brane. In fact, a non-singular metric can be obtained by orbifolding the above solution at some point away from the singularity. In physical terms, we add to the lagrangian \((4.1)\) a source brane term and (for more generality and to make contact with the Randall-Sundrum setup) a (negative) cosmological constant

\[
S = S_1 - \int \left(2\Lambda + 2\lambda e^{\beta\phi} \delta(y)\right) \sqrt{-g} \, d^Dx. \tag{5.1}
\]

Then, Eqs. \((4.9)\) and \((4.11)\) are replaced by

\[
A'' + (D-1)A^{'}^2 = -\frac{\lambda_0}{D-2} \delta(y) - \frac{2\Lambda}{D-2}, \tag{5.2}
\]

\[
\phi'' + (D-1)A'\phi' = \frac{a}{2} e^{\alpha\phi} f^2 + 2\lambda_0 \beta \delta(y), \tag{5.3}
\]

where

\[
\lambda_0 = \lambda e^{\beta\phi(0)}, \tag{5.4}
\]

while the constraint equation now reads

\[
2(D-1)(D-2)A^{'}^2 + 4\Lambda = \phi'^2 + e^{\alpha\phi} f^2. \tag{5.5}
\]

Note that the form field does not interact with the brane, so the solution \((4.4)\) remains true.

Using again the parametrization \((4.12)\), we find for \(Z\) the equation

\[
Z'' - \kappa^2 Z = -\lambda_0 \frac{D-1}{D-2} Z \delta(y), \tag{5.6}
\]

where

\[
\kappa^2 = -\frac{2(D-1)\Lambda}{D-2}, \tag{5.7}
\]

and \(\Lambda\) is assumed to be negative. For \(\beta = 0\) and in the absence of the form field \((b = 0)\) it is consistent to set the dilaton to zero. In this case a particular solution will be the flat RS brane-world in the AdS bulk, it is given by

\[
Z = \alpha e^{-\kappa|y|}, \quad \alpha > 0, \quad \kappa > 0, \tag{5.8}
\]

the matching condition at \(y = 0\) leading to

\[
2\kappa = \lambda_0 \frac{D-1}{D-2}. \tag{5.9}
\]

The tension \(\lambda\) of the brane thus has to be positive.
Adding $F_{[1]}$ and the dilaton (i.e. two bulk scalar fields) changes the situation as follows. Introducing the variable $\tau$ as in (4.14), one obtain the dilaton equation
\[ \ddot{\phi} - 2ab^2 e^{-a\phi} = 2\lambda_0 \beta(y). \tag{5.10} \]
The first integral (which does not contain discontinuities) replacing (4.16) is
\[ \frac{\dot{\phi}^2}{2} + 2b^2 e^{-a\phi} = 2\mu^2, \tag{5.11} \]
where from the constraint equation (5.5)
\[ \mu^2 = \frac{D - 2}{2(D - 1)} \left( Z'^2 - \kappa^2 Z^2 \right), \tag{5.12} \]
$\kappa$ being still given by (5.7). In view of this relation, the exponential solution of the $Z$-equation is no longer valid, instead we should take the following solution to Eq. (5.6):
\[ Z = \alpha \sinh \kappa(|y| + y_0), \tag{5.13} \]
so that
\[ \mu^2 = \frac{D - 2}{2(D - 1)} \kappa^2 \alpha^2. \tag{5.14} \]
Now the matching condition on the brane gives
\[ -2\kappa \coth \kappa y_0 = \lambda_0 \frac{D - 1}{D - 2}, \tag{5.15} \]
leading to a negative brane tension. The solution to the dilaton equation now reads
\[ \phi = \frac{2}{a} \ln \left[ \frac{b}{\mu} \cosh a\mu(|\tau| + \tau_0) \right], \tag{5.16} \]
and the matching condition reads
\[ \frac{2\mu \tanh(a\mu\tau_0)}{\alpha \sinh \kappa y_0} = \lambda_0 \beta. \tag{5.17} \]
The dependence $\tau(y)$ normalized so that $\tau(0) = 0$ reads
\[ \tau = \frac{\epsilon(y)}{\kappa \alpha} \ln \left[ \frac{\tanh (\kappa(|y| + y_0)/2)}{\tanh(\kappa y_0/2)} \right]. \tag{5.18} \]
This is valid for any non-zero bulk cosmological constant. The limit $\Lambda = 0$ corresponds to taking $\kappa \to 0$ with $\kappa \alpha$ fixed. Then
\[ \tau = \frac{\epsilon(y)}{\kappa \alpha} \ln \left( \frac{|y| + y_0}{y_0} \right), \tag{5.19} \]
with
\[ y_0 = \frac{2}{|\lambda_0| \frac{D - 2}{D - 1}}, \tag{5.20} \]
and
\[ Z = \kappa \alpha (|y| + y_0). \tag{5.21} \]
Again, the matching condition gives a negative brane tension.
6 Charged RS walls

The ansatz (4.2) of the previous section is actually the same as for the D8 brane in the massive IIA theory [32, 31], but the form field is different, actually the D8 brane can be obtained by adding a non-dynamical \( A_9 \) potential in ten dimensions, or, more generally, \( A_{[D-1]} \) in \( D \) dimensions. Recently this problem attracted attention in connection with supersymmetric generalizations of the Randall-Sundrum setup [35, 36, 33, 39]. To clarify the relationship of our F-wall solutions with BPS brane worlds we construct here the one-brane charged RS type solution, without a bulk cosmological constant, in any number of dimensions \( D \).

Our action is

\[
S_D = \int \left( R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2D!} e^{a\phi} F_{[D]}^2 \right) \sqrt{-g} d^D x \\
- 2\lambda \int e^{\phi} \sqrt{-g_{D-1}} d^{D-1} x - q \int A_{[D-1]},
\]

while the ansatz for the space-time metric is again (4.2), so the difference with the previous section is that the form field now interacts with the brane (the last term in (6.1)). Clearly, the \( D \)-form in \( D \) dimensions

\[
F_{[D]} = f(y) dy \wedge dt \wedge dx_1 \wedge \cdots \wedge dx_{D-2}
\]

is trivial and the corresponding term in the lagrangian acts merely as a dilaton potential. However, we have to be careful about the jump condition on the source brane. From the form equation

\[
\partial_y f(y) e^{-{(D-1)A + a\phi}} = -q\delta(y),
\]

it follows that

\[
f = 2be^{(D-1)A - a\phi},
\]

with the discontinuous parameter

\[
b = b_+ \theta(y) + b_- \theta(-y), \quad b_\pm = \text{const},
\]

and the jump condition

\[
b_+ - b_- = -\frac{q}{2}.
\]

Substituting this into the other equations one arrives at the following system:

\[
(D - 2)[A'' + (D - 1)A'^2] = -2b^2 e^{-a\phi} - \lambda_0 \delta(y),
\]

\[
2(D - 1)(D - 2)A'^2 = \phi'^2 - 4b^2 e^{-a\phi},
\]

\[
\phi'' + (D - 1)A' \phi' = -2ab^2 e^{-a\phi} + 2\beta \lambda_0 \delta(y).
\]
First we assume the relation
\[ \beta = -\frac{a}{2}, \]  
(6.10)
which results from imposing the scale invariance of the action (6.1). Again it is convenient to introduce the function \( Z(y) \) via Eq. (4.12). Then, as can be easily seen, the dilaton equation and Eq. (6.7) imply that \( \phi \) and \( A \) are proportional modulo the solution of the homogeneous equation:

\[ \phi = a(D - 2)A + d(\tau + \tau_0), \quad (d, \tau_0 = \text{const}), \]  
(6.11)
where \( \tau \) satisfies Eq. (4.14). The remaining equations are easily solved in the special case \( d = 0 \). Combining (6.11) with Eq. (6.7) one obtains the following equation for \( Z \):

\[ Z'' + 2b^2 \frac{D - 1}{D - 2} Z^{2\nu - 1} = -\lambda_0 \frac{D - 1}{D - 2} Z \delta(y), \]  
(6.12)
with
\[ \nu = -1 + \frac{a^2(D - 2)}{2(D - 1)}. \]  
(6.13)
The constraint equation (6.8) then implies

\[ Z'^2 - \kappa^2 Z^{-2\nu} = 0, \]  
(6.14)
where
\[ \kappa^2 = \frac{4b^2(D - 1)^2}{(D - 2)(a^2(D - 2) - 2D + 2)}. \]  
(6.15)
For this to be consistent, one has to impose the condition

\[ a^2 > \frac{2(D - 1)}{D - 2} \]  
(6.16)
\((\nu > 0)\), which we assume to hold. The solution to Eqs. (6.12) and (6.14) is readily obtained,

\[ Z = [\kappa(1 + \nu)(|y| + y_0)]^{1/(1+\nu)}. \]  
(6.17)
Here the parameter \( \kappa \) is continuous on the brane provided
\[ b_+ = -b_- = -\frac{q}{4}, \]  
(6.18)
and we have to accommodate for the source term in Eq. (6.12). This is achieved imposing the following condition on the parameter \( y_0 \):

\[ 4 = -\lambda_0 a^2 y_0. \]  
(6.19)
In the case of a single source brane one has to demand positivity $y_0 > 0$ to ensure regularity of the metric, so in this case the appropriate brane tension is negative. However, in a many-brane setup such as in \([39]\) one can achieve a non-singular metric including positive tension branes as well.

A more general solution can be derived by relaxing the condition (6.10). Putting

$$B \equiv A', \quad \psi \equiv \frac{\kappa}{D-1} e^{-a\phi/2},$$

(6.20)

and combining Eqs. (6.7) and (6.8) we obtain, outside the brane,

$$\phi' = a(D - 2)\psi \frac{dB}{d\psi}.$$  

(6.21)

Inserting this in Eq. (6.8) yields the Lagrange-type differential equation

$$(1 + \nu)\psi^2 \left( \frac{dB}{d\psi} \right)^2 - B^2 - \nu \psi^2 = 0.$$  

(6.22)

To solve this, put

$$B = t\psi, \quad \frac{dB}{d\psi} = \pm f(t) \equiv \pm \frac{q}{p} \sqrt{t^2 + p^2},$$

(6.23)

with

$$p^2 = \nu, \quad q^2 = \frac{\nu}{1 + \nu}$$

(6.24)

($p > 0, q > 0$). The case $d = 0$ corresponds to $t \equiv \pm 1, f \equiv 1$. In the present case,

$$\frac{d\psi}{\psi} = \frac{dt}{\pm f(t) - t},$$

(6.25)

which is integrated, for $q \neq p$, by

$$\ln \left( \frac{\psi_\pm}{\psi_0} \right) = \frac{p}{q^2 - p^2} (p \ln |q \mp p \sin \theta| \pm q \ln(1 + \sin \theta) - (p \pm q) \ln(\cos \theta)), $$

(6.26)

where we have put $t = p \tan \theta (-\pi/2 < \theta < \pi/2)$. Also, it follows from the definitions (6.20) and from Eq. (6.21) that

$$dA = \frac{2}{(D - 2)a^2} \left( \frac{d\psi}{\psi} \mp \frac{p}{q \cos \theta} \right) d\theta, \quad dy = -\frac{2pd\theta}{(D - 2)a^2 q(q \mp p \sin \theta)\psi}.$$  

(6.27)

The resulting spacetime metric after suitable rescaling of $x^a$ is

$$ds_{\pm}^2 = \left( \frac{1 + \sin \theta}{\cos \theta} \right)^{\frac{\gamma}{\gamma - 1}} \eta_{ab} dx^a dx^b + $$

$$+ \frac{1}{((D - 1)p\psi_0)^2} \left( \frac{1 + \sin \theta}{\cos \theta} \right)^{\frac{\gamma}{\gamma - 1}} d\theta^2.$$  

(6.28)
with $\gamma = p/q = \sqrt{1+\nu}$, while the dilaton function and form potential read

$$e^{-a\phi/2} = \frac{D-1}{\kappa} \psi_0 \left[ q(1+\sin\theta)^{\pm 1}(1 \mp \gamma \sin\theta)\right]^{\mp \frac{\gamma^2}{2\gamma^2-1}}, \quad (6.29)$$

$$A_{[D-1]} = \frac{\psi_0}{b(D-2)} \left( \frac{\gamma}{\sqrt{\gamma^2-1}} \right)^{-\frac{\gamma^2}{2\gamma^2-1}-1} (1 \mp \gamma \sin\theta)^{-1}. \quad (6.30)$$

Note that $ds^2_\pm(-\theta) = ds^2_\mp(\theta)$ (the same being true for the dilaton $\phi$).

For $\gamma < 1$ ($\nu < 0$), this metric is singular for $\theta = \pi/2$ and for $\theta = -\pi/2$. Putting $\theta = \epsilon(\pi/2 - u)$, we find that for $u \to 0$, $y_\pm \sim u^{1/\gamma}$ and the metric behaves as

$$ds^2 \sim |y|^{2/(D-1)} \eta_{ab} dx^a dx^b + dy^2 \quad (y \to 0). \quad (6.31)$$

Because there are two singularities, it is not possible to obtain a regular solution with a single source brane. On the other hand, for $\gamma > 1$ ($\nu > 0$), i.e. when the condition (6.16) holds, the singularity at $\theta = \pm \pi/2$ is sent off to infinity, while a new singularity appears at $\theta = \pm \theta_0$ with $\sin \theta_0 = 1/\gamma$. So the regular RS type solution is given by

$$(ds^2, \phi, f) = (ds^2_+, \phi_+, f_+)(\theta) \quad (\theta_1 < \theta < \pi/2) \quad (6.32)$$

$$= (ds^2_-, \phi_-, f_-)(\theta) \quad (-\pi/2 < \theta < -\theta_1) \quad (6.33)$$

with $\theta_0 < \theta_1 < \pi/2$. The continuity of the metric and of the dilaton on the brane $y = 0$ ($\theta = \pm \theta_1$) is ensured by the above-mentioned symmetry together with the relation (6.18) which fixes the strength of the form field, while the full equation (6.7) gives for the jump of the connexion

$$2(D-2)A'_+(\theta_1) = 2(D-2)p \tan \theta_1 \psi_+(\theta_1) = -\lambda_0, \quad (6.34)$$

so that the brane tension is again negative. Finally, the comparison of Eqs. (6.7) and (6.9) yields

$$\beta = -\frac{a f(\theta_1)}{2 \ t(\theta_1)} = -\frac{a \sin \theta_0}{2 \sin \theta_1} > -\frac{a}{2} \quad (6.35)$$

(it follows that for $\beta = -a/2$ the $d = 0$ solution is the unique regular solution).

### 7 Conclusion

In this paper we have presented two classes of solutions to gravity coupled to a dilaton and to an antisymmetric form in arbitrary dimensions. These two classes lie at the limits of the sequence of fluxbranes of different dimensionalities.
The first class is that of zero flux branes supported by a form of rank $D - 2$. Presumably (the Euclidean continuations of) such branes are entitled to interact with instantons. We have shown that such solutions may exist with the transverse spaces $S_k \times R^{(D-k-2)}$, with $1 \leq k \leq D - 2$, and in the case $k = 1$ we obtained a complete analytic solution containing four physical parameters. One of these parameters is connected with the form field strength, while the other parameters are associated with pointlike singularities on the regular Melvin-type background. Solutions of an entirely new type are obtained in the case of a spherically symmetric transverse space $k = D - 2$. The full metric then consists of an infinite number of disjoint sectors, each of which is shown to be geodesically complete. For other values of $k$ regular solutions exist which can be found numerically.

At the other extreme of the supergravity fluxbrane sequence lies the F-wall of codimension one. Actually, this is a domain wall supported by the scalar field. Such a solution is easy to find in a closed form which reveals its singularity on a hyperplane of dimension $(D - 1)$. We explored whether one can regularize the solution by orbifolding the space at some point away from the singularity. This leads to some new brane world setup with two scalar fields. In the case where only one brane is present, the space is non-singular if the brane tension is negative (in a multibranesetup positive tension branes will be present too). The general solution was found with a bulk cosmological constant added. It is worth noting that in this setup the brane does not carry a Ramond-Ramond charge.

We have also constructed new brane-world solutions with charged branes, assuming an arbitrary coupling of the dilaton to the brane. The usual supersymmetric brane-world corresponds to a particular value of this constant. In the general case the solution turns out to be strikingly different. We found conditions for its regularity, and, as a by product, have shown uniqueness of the supersymmetric brane-world.

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