Optimal Procurement Auction for Cooperative Production of Virtual Products: Vickrey-Clarke-Groves Meet Crémier-McLean

Mingshu Cong *  
The FinTech and Blockchain Laboratory  
The University of Hong Kong  
LogiOcean Technologies, Ltd.  
miranda.cong@logiocean.com

Xi Weng  
Guanghua School of Management  
Peking University  
wengxi125@gsm.pku.edu.cn

Han Yu  
School of Computer Science & Engineering  
Nanyang Technological University  
han.yu@ntu.edu.sg

Jiabao Qu  
LogiOcean Technologies, Ltd.  
qujiabao@logiocean.com

Siu Ming Yiu  
The FinTech and Blockchain Laboratory  
The University of Hong Kong  
smyiu@cs.hku.hk

Abstract

We set up a supply-side game-theoretic model for cooperative production of virtual products. In our model, a group of producers collaboratively produce a virtual product by contributing costly input resources to a production coalition. Producers are capacitated, i.e., they cannot contribute more resources than their capacity limits. Our model is an abstraction of emerging internet-based business models such as federated learning and crowd computing. To maintain an efficient and stable production coalition, the coordinator should share with producers the income brought by the virtual product. Besides the demand-side information asymmetry, another two sources of supply-side information asymmetry intertwined in this problem: 1) the capacity limit of each producer and 2) the cost incurred to each producer. In this paper, we rigorously prove that a supply-side mechanism from the VCG family, PVCG, can overcome such multiple information asymmetry and guarantee truthfulness. Furthermore, with some reasonable assumptions, PVCG simultaneously attains truthfulness, ex-post allocative efficiency, ex-post individual rationality, and ex-post weak budget balancedness on the supply side, easing the well-known tension between these four objectives in the mechanism design literature.

Introduction

On this flat earth created by the internet, hierarchies of companies are falling away. Production activities are no longer a process confined within the border of an enterprise. Independent businesses can cooperate seamlessly through distributed computing systems to produce valuable virtual products. For example, a fast-developing technology, federated learning (FL), enables businesses to train (produce) artificial intelligence models collaboratively (Yang et al.

*This work was supported by LogiOcean Technologies, Ltd.

Copyright © 2020, Association for the Advancement of Artificial Intelligence (www.aaai.org). All rights reserved.
revenue distributable to the supply side: on the supply side, the VCG mechanism enables us to overcome the free-rider problem by compensating producers for their contribution of input resources and achieve socially efficient allocation.

In this paper, our contributions include: 1) We set up a game-theoretical model for cooperative production of virtual products, which takes into consideration twofold supply-side information asymmetry in addition to the demand-side information asymmetry; 2) We propose a procurement auction that incentivizes participants to report their private information honestly (truthfulness) and maximizes social welfare ex post (allocative efficiency); 3) We apply neural network methods to optimizing for ex-post individual rationality and ex-post weak budget balancedness; 4) We prove with some reasonable assumptions, truthfulness, ex-post allocative efficiency, ex-post individual rationality and ex-post weak budget balancedness can be attained simultaneously.

Related Works

Our game-theoretic model is an abstraction of emerging internet-based cooperative production practices. Although underlying technologies of cooperative production are fast developing, existing studies on these new business models are quite limited. From a more general perspective, our model is a special type of cooperative games (Peleleg and Sudhölter 2007; Brandzei, Dimitrov, and Tijs 2008; Chalkiadakis, Elkind, and Wooldridge 2011), which have attracted great attention from the algorithmic game theory community (Sandholm and Lesser 1997; Conitzer and Sandholm 2006; Chalkiadakis et al. 2010; Elkind et al. 2009; Bachrach et al. 2009). Designing an optimal sharing rule is a key concern when studying cooperative games (Kronbak and Lindroos 2007; Niyato, Vasilakos, and Kun 2011; Weikard 2009).

We decouple the cooperative production game into a demand-side game and a supply-side game. On the demand side, we use the Crémer-McLean mechanism (Crémer and McLean 1985). Kosenok and Severinov (2008) (Albert, Conitzer, and Lopomo 2015) to extract consumer surplus. On the supply side, we proposed the PVCG mechanism, which borrows the idea of procurement auction from procurement games (Chen et al. 2005; Chandrashekar et al. 2007; Iyengar and Kumar 2008; Drechsel and Kimms 2010). Compared to existing research, our model takes into account three sources of information asymmetry, one on the demand side and two on the supply side, yet we simultaneously achieve truthfulness, allocative efficiency, individual rationality, and ex-post weak budget balancedness, easing the well-known tension between these objectives (Jackson 2014). We also follow the recent trend of neural-network-based mechanism design (Shen, Tang, and Zuo 2019).

Federated learning (Yang et al. 2019a; Yang et al. 2019b) and crowd computing (Beberg et al. 2009; Larson et al. 2009) are examples of the cooperative production game under study. Therefore, our work is also related to the literature on training machine learning models with strategic participants (Jia et al. 2019; Wang 2019; Cai, Daskalakis, and Papadimitriou 2015; Richardson, Filos-Ratsikas, and Faltings 2019; Westenbroek et al. 2019; Yu et al. 2020) and crowd computing from the angle of game theory (Christoforou et al. 2013).

Game Settings

We study a cooperative production game where a set of $n$ producers, denoted by $N = \{0, 1, \ldots, n-1\}$, cooperatively produce a valuable virtual product, of which copies are delivered to $m$ consumers, denoted by $M = \{n, n+1, \ldots, n+m-1\}$. A participant may be both a producer and a consumer, but we assume that its behaviors as a producer and as a consumer are independent. On the supply side, producers contribute input resources to the production coalition, e.g., labor, raw materials, equipments, etc. On the demand side, consumers are granted access to utilize the output virtual product. We introduce a parameter $x_i \geq 0$ to measure the input resources contributed by producer $i$. $x_i$ may be a vector when multiple input resources are involved. We use another parameter $y$ to measure the usefulness of the output product, which is determined by the contributed input resources from all producers, i.e., $y$ is a function of $x = (x_0, \ldots, x_{n-1})$.

The usefulness of the output product determines the value it brings to consumers. We use the parameter $v_j, j \in N$ to denote the valuation of participant $j \in M$ on the output product. $v_j$ is a function of $y$ and a type parameter $\theta_j \in \Theta_j$ (called valuation type) that reflects the heterogeneity among consumers. The type space is $\Theta_j = \{\theta_{j1}, \ldots, \theta_{jm}\}$ of cardinality $m_j < \infty$ and we assume that $0 \in \Theta_j$ for all $j$. We denote this function by $w(\cdot)$ and call the composite function of $w(\cdot)$ and $y(\cdot)$ the individual valuation function, denoted by $v(x, \theta_j) = w(y(x), \theta_j) = v_j$. For convenience, $\theta_j$ is such chosen that $v(x, \theta_j) \equiv 0$ when $\theta_j = 0$. We denote $\Theta = (\theta_{n}, \theta_{n+1}, \ldots, \theta_{n+m-1})$. Contributing resources to the cooperative production process incurs costs to participants. Producer $i$’s cost $c_i = c(x_i, \gamma_i)$ is a function (called individual cost function) of $x_i$ and another type parameter $\gamma_i$ that reflects the heterogeneity of producer $i$. We denote $\gamma = (\gamma_0, \ldots, \gamma_{n-1})$. In our game, producers are assumed to be capacitated, i.e., producer $i$ cannot contribute more resources than its capacity limit $x_i$, i.e., $x_i \leq \bar{x}_i$. Both the type parameters $\gamma_i, \theta_j$ and the capacity limit $\bar{x}_i$ are private information unknown to the coalition coordinator a priori. This coordinator makes the transfer payment $p_i$ to producer $i$ and $p_j$ to consumer $j$. Participants’ preference is represented by quasi-linear utilities $u_i = p_i - c_i$, $i \in N$ and $u_j = p_j + v_j, j \in M$. We use social surplus $S(x, \gamma) = \sum_{j=0}^{n+m-1} v(x, \theta_j) - \sum_{i=0}^{n-1} c(x_i, \gamma_i)$ to measure the social effect of the production coalition. Social surplus maximization implies Pareto efficiency (or allocative efficiency in the language of mechanism design).

Parameters in our model have concrete meanings in scenarios such as federated learning (FL) and crowd computing. In FL, the input resources are data. $x_i$ measures the size and quantity of the contributed dataset. (Measuring the quality of datasets is a separate problem. We do not go deeper here.) The capacity limit $\bar{x}_i$ is the best dataset owned by participant $i$. $c_i$ is the cost of collecting and cleaning data. $v_i$ is the value of the federated model to each participant (e.g., for the use case that banks use FL to train AI models
to predict credit risk, \( v_j \) is calculated as the reduced bad debt rate times the principle of loans. In crowd computing, \( x_i \) is the contributed computing power, \( c_i \) costs of computing power (e.g., electricity costs, hardware costs, etc.), and \( v_j \) is the value of the computation result (e.g., the value of protein folding result to drug development).

We put forward five assumptions on the individual valuation function \( v(x, \theta_j) \) and the individual cost function \( c(x_i, \gamma_i) \). Assumptions 1-2 are used for proving truthfulness and ex-post weak budget balancedness. Assumption 3 is smoothness and monotocity. The individual valuation function \( v(x, \theta_j) \) is a smooth and monotonic increasing function of \( x \) and \( \theta_j \). The individual cost function \( c(x_i, \gamma_i) \) is a smooth and monotonic increasing function of \( x_i \) and \( \gamma_i \).

Assumption 1-2 are used for proving truthfulness and ex-post weak budget balancedness. Prior(\( \theta \)) is identifiable and correlated, i.e., the identifiability condition holds for all consumers. The Procurement Auction theorem states that in a demand-side Cr"emer-McLean mechanism which is optimal on the demand side, we introduce an optimal supply-side procurement auction in this section. This proposed procurement auction, accompanied by the demand-side Cr"emer-McLean mechanism, maximizes social surplus by incentivizing producers to truthfully report their capacity limits and type parameters. This procurement auction consists of four steps, of which the soundness will be proved in the next section.

Step 1. Producers bid on capacity limits and cost types
As the first step, every producer submits a sealed bid for their respective capacity limits and cost types. The reported capacity limit \( \hat{x}_i \) is the maximum resources that producer \( i \) may accept. As an example, the following individual valuation function satisfies all these four assumptions:

\[
v(x, \theta_j) = \theta_j \sqrt{\sum_{k=0}^{n-1} z(x_k)^2},
\]

where \( z(x_i) \) is an arbitrary smooth and increasing function (e.g., the Cobb-Douglas function). Assumption 5 is introduced to guarantee full consumer surplus, so that we can derive the income of the coalition \(- \sum_{j=1}^{n-1} p_j\) from individual valuation functions, i.e., \(- \sum_{j=1}^{n-1} p_j = \sum_{j=1}^{n-1} v(x, \theta_j)\), because we have the following theorem:

**Theorem 1** (Cr"emer-McLean Theorem). There exists an interim individually rational and ex-post budget balanced Bayesian mechanism that extracts full consumer surplus if Prior(\( \theta \)) is identifiable and Cr"emer-McLean condition holds for all consumers.

Proofs of Theorem 1 can be found in (Cr"emer and McLean 1985) and (Kosenok and Severinov 2008). We can construct such a demand-side Cr"emer-McLean mechanism by following the constructive proof of Lemma A3 in (Kosenok and Severinov 2008) or by automated mechanism design techniques (Albert, Conitzer, and Lopomo 2015).

This decouples the supply-side mechanism design problem from the demand-side problem.

**The Procurement Auction**
As a counterpart of Cr"emer-McLean mechanism which is optimal on the demand side, we introduce an optimal supply-side procurement auction in this section. This proposed procurement auction, accompanied by the demand-side Cr"emer-McLean mechanism, maximizes social surplus by incentivizing producers to truthfully report their capacity limits and type parameters. This procurement auction consists of four steps, of which the soundness will be proved in the next section.

Step 2. The coordinator chooses the optimal acceptance ratios
Then, the coalition coordinator decides how many input resources to accept from each producer. It chooses \( x_i, i = 0, \ldots, n - 1 \) that maximize the social surplus, constrained by reported capacity limits and based on reported...
type parameters. Equivalently, the coordinator calculates the optimal acceptance ratio \( \eta_i \in [0, 1]^{\dim(x_i)} = x_i \odot \hat{x}_i \) such that \( x_i = \hat{x}_i \odot \eta_i \), where \( \odot \) and \( \oplus \) denote the element-wise multiplication and division respectively, and \([0, 1]\) denotes the interval between 0 and 1. The economic meaning of \( \eta_i \) is the ratio of input resources accepted by the coalition to those offered by the producer.

The optimal acceptance ratios \( (\eta^*_0, \ldots, \eta^*_{n-1}) = \eta^* \) are calculated according to the following formula:

\[
\eta^* = \arg\max_{\eta \in [0,1]^{\dim(x_i)} \times n} \{ S(\hat{x} \odot \eta, \hat{\gamma}, \hat{\theta}) \} \\
= \arg\max_{\eta \in [0,1]^{\dim(x_i)} \times n} \sum_{j=n}^{n+m-1} v(\hat{x} \odot \eta, \theta_j) - \sum_{i=0}^{n-1} c_i(\hat{x}_i \odot \eta_i, \gamma_i).
\]

Because different \( (\hat{x}, \hat{\gamma}, \hat{\theta}) \) results in different \( \eta^* \), \( \eta^* \) is written as \( \eta^*(\hat{x}, \hat{\gamma}, \hat{\theta}) \). Correspondingly, the maximum social surplus is denoted by \( S^*(\hat{x}, \hat{\gamma}, \hat{\theta}) = \sum_{j=n}^{n+m-1} v(\hat{x} \odot \eta^*(\hat{x}, \hat{\gamma}, \hat{\theta}), \theta_j) - \sum_{i=0}^{n-1} c_i(\hat{x}_i \odot \eta^*_i(\hat{x}, \hat{\gamma}, \hat{\theta}), \gamma_i) \). It is worth noting that although \( S'(\hat{x}, \hat{\gamma}, \hat{\theta}) \) and \( S(x, \gamma, \theta) \) both represent social surplus, they are different functions. The first parameter \( x \) in \( S(\cdot) \) is the accepted input resources, whereas the first parameter \( \hat{x} \) in \( S'(\cdot) \) is the reported capacity limits. \( x \) and \( \hat{x} \) are related by \( x = \hat{x} \odot \eta^* \).

**Step 3. Producers contribute accepted input resources to the production coalition**

In this step, producers are required to contribute \( \hat{x} \odot \eta^* \) units of input resources to the production coalition. Since in the first step, producer \( i \) has promised to offer at most \( \hat{x}_i \) units of input resources, if it cannot contribute \( \hat{x}_i \odot \eta^*_i \leq \hat{x}_i \), we impose a high punishment on it. With the contributed input resources, producers collaboratively produce the output virtual product, bringing value \( v(\hat{x} \odot \eta^*, \theta_j) \) to consumer \( j \in M \).

**Step 4. The coordinator makes transfer payments to participants according to the PVC G sharing rule**

In this final step, the coordinator pays producers according to the PVC G sharing rule. The PVC G payment

\[
p_i(\cdot) = \tau_i(\cdot) + h_i^*(\cdot)
\]

is composed of two parts, the VCG payment \( \tau_i \) and the optimal adjustment payment \( h_i^* \). The VCG payment is designed to induce truthfulness, i.e., the reported capacity limits \( \hat{x} \) and reported cost type \( \hat{\gamma} \) are equal to the true capacity limits \( x \) and true cost type \( \gamma \). The adjustment payment is optimized so that ex-post individual rationality and ex-post weak budget balancedness can also be attained.

With \( \eta^* \) calculated in Step 2, the VCG payment \( \tau_i \) to producer \( i \) is:

\[
\tau_i = S^*(\hat{x}, \hat{\gamma}, \hat{\theta}_i) - S^*_{-i}(\hat{x}_{-i}, \hat{\gamma}_{-i}, \hat{\theta})
+ c(\hat{x}_i \odot \eta_i^*(\hat{x}, \hat{\gamma}, \hat{\theta}), \gamma_i)
+ n \prod_{j=0}^{n-1} | v(\hat{x} \odot \eta^*(\hat{x}, \hat{\gamma}, \hat{\theta}), \theta_j) - v(\hat{x}_{-i} \odot \eta^*_{-i}(\hat{x}_{-i}, \hat{\gamma}_{-i}, \hat{\theta})) |
\]

where \( (\hat{x}_{-i}, \hat{\gamma}_{-i}) \) denotes the reported capacity limits and the reported cost types excluding producer \( i \). \( \eta^*_{-i} \) and \( S^*_{-i}(\hat{x}_{-i}, \hat{\gamma}_{-i}, \hat{\theta}) \) are the corresponding optimal acceptance ratios and maximum social surplus. Note that \( \eta^*_{-i} \) is different from \( \eta^*_i \): the former maximizes \( S(\hat{x}_{-i} \odot \eta_{-i}, \hat{\gamma}_{-i}, \theta) \), whereas the latter is the component of \( \eta^* \) that maximizes \( S(\hat{x} \odot \eta, \hat{\gamma}, \hat{\theta}) \). \( \tau = (\tau_0, \ldots, \tau_{n-1}) \) is a function of \( (\hat{x}, \hat{\gamma}, \hat{\theta}) \), written as \( \tau(\hat{x}, \hat{\gamma}, \hat{\theta}) \).

The adjustment payment \( h_i(\hat{x}_{-i}, \hat{\gamma}_{-i}, \hat{\theta}) \) is a function of \( (\hat{x}_{-i}, \hat{\gamma}_{-i}, \hat{\theta}) \). The optimal adjustment payments \( (h^*_0(\cdot), \ldots, h^*_{n-1}(\cdot)) = h^*(\cdot) \) are determined by solving the following functional equation (a type of equation in which the unknowns are functions instead of variables; refer to [Rassias 2012] for more details):

\[
\sum_{i=0}^{n-1} \text{ReLU}( - (S^*(x, \gamma, \theta) - S_{-i}^*(x_{-i}, \gamma_{-i}, \theta)))
- h_i(x_{-i}, \gamma_{-i}, \theta)
+ \text{ReLU}( \sum_{i=0}^{n-1} | S^*(x, \gamma, \theta) - S_{-i}^*(x_{-i}, \gamma_{-i}, \theta) | )
\]

where \( \text{supp}(\text{Prior}(x, \gamma, \theta)) \) is the support of the prior distribution \( \text{Prior}(x, \gamma, \theta) \) of the true parameters \( (x, \gamma, \theta) \) estimated by the coalition coordinator. Support is a terminology from measure theory, defined by \( \text{supp}(\text{Prior}(x, \gamma, \theta)) = \{(x, \gamma, \theta) | \text{Prior}(x, \gamma, \theta) > 0 \} \). In general, there is no closed-form solution to Eq. 8, so we employ neural network techniques to learn the solution. We will go into more details later.

### Theoretical Analyses

Theoretical analyses in this section are organized through the following strand:

1. First (in Proposition 1 and Proposition 2), we prove that for arbitrary \( h_i(\hat{x}_{-i}, \hat{\gamma}_{-i}, \hat{\theta}) \), \( i = 0, \ldots, n-1 \), the PVC G payments guarantee supply-side truthfulness (dominant strategy incentive compatibility) and maximize social surplus (allocative efficiency).

2. Second (in Proposition 3 and Proposition 4), given that Crémé-McLean Theorem (Theorem 1) holds on the demand side, we derive two inequality constraints on the
3. Lastly (in Theorem 2 and Corollary 4), we prove the existence of at least one solution $h^*(\cdot)$ to Eq. [9] The PVCG payment corresponding to this solution attains truthfulness, ex-post allocative efficiency, ex-post individual rationality, and ex-post weak budget balancedness simultaneously on the supply side.

First, we prove that for arbitrary $h_i(\bar{x}_{-i}, \gamma_{-i}, \theta)$, the payment $p_i(\cdot) = \tau_i(\cdot) + h_i(\cdot)$ encourages all producers to report their capacity limits and cost types truthfully.

**Proposition 1** (Dominant strategy incentive compatibility).

For every producer $i$, truthfully reporting its capacity limit $\bar{x}_i$ and cost type $\gamma_i$ is its dominant strategy, i.e.,

$$p_i((\bar{x}_i, \bar{x}_{-i}), (\gamma_i, \gamma_{-i}), \theta)$$

$$- c(\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta), (\gamma_i, \gamma_{-i})), \gamma_i) \geq p_i(\hat{x}, \gamma_i, \theta) - c(\hat{x}_i \odot \eta_i^*(\hat{x}_i, \gamma_i, \theta), \gamma_i)$$

$$, \forall \bar{x_i} \in N, \bar{x}, \gamma_i, \gamma_i, \theta.$$ \hspace{1cm} (9)

**Proof.** We differentiate the cases where $\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta) > \bar{x}_i$ with those where $\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta) \leq \bar{x}_i$.

1. When $\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta) > \bar{x}_i$,

   it is impossible for producer $i$ to contribute the accepted amount of input resources $\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta)$ because this exceeds its capacity limit. In this case, producer $i$ suffers a high punishment, i.e., $p_i(\hat{x}_i, \gamma_i, \theta)$ is a large negative number. Hence, the right side of Eq. [9] becomes extremely negative and Eq. [9] holds. This shows a rational producer will not choose $\hat{x}_i$ such that $\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta) > \bar{x}_i$.

2. When $\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta) \leq \bar{x}_i$,

   we aim to prove that by truthfully reporting $\bar{x}_i$, $\gamma_i$, and $\theta_i$, the utility of producer $i$ is at least the same as before.

By definition,

$$S^*(\hat{x}, \gamma, \theta) = \max_{\eta \in [0,1]^{\dim(x)} \times \eta} \{n+m-1 \cdot v(\hat{x} \odot \eta, \theta) - \sum_{j=1} v(\hat{x}_j \odot \eta_j, \gamma_j) \} \forall \hat{x}, \gamma, \theta.$$  \hspace{1cm} (10)

In particular, for $\eta = ((\bar{x}_i \odot \eta_i^*(\bar{x}_i, \gamma_i, \theta)) \odot \bar{x}_i, \eta_{-i}^*(\bar{x}_i, \gamma_i, \theta)) \in [0, 1]^{\dim(x)} \times \eta$, Eq. [10] holds. Therefore,

$$S^*((\bar{x}_i, \bar{x}_{-i}), (\gamma_i, \gamma_{-i}), \theta)$$

$$\geq \sum_{j=n}^{n+m-1} v((\bar{x}_i, \bar{x}_{-i}) \odot \eta_j, \theta_j) - c(\bar{x}_i \odot \eta_i, \gamma_i)$$

$$- \sum_{k=0, \neq i}^{n} c(\bar{x}_k \odot \eta_k, \gamma_k), \forall \eta \in [0, 1]^{\dim(x)} \times \eta$$ \hspace{1cm} (11)

Adding $h_i(\bar{x}_{-i}, \gamma_{-i}, \theta_{-i}) - S^*_i((\bar{x}_{-i}, \gamma_{-i}, \theta_{-i}))$ to both sides of Eq. (11) and substituting $p_{i}^*(\cdot) + v(\cdot) - c(\cdot) = S^*(\cdot) - S^*_i(\gamma) + h_i(\cdot)$, we get Eq. (9).

From Proposition 1, we know reported parameters equal true parameters, i.e., $\hat{\bar{x}}, \gamma, \theta = (\bar{x}, \gamma, \theta)$. Therefore, we can use these two sets of parameters interchangeably in the remaining parts of this paper.

**Proposition 2** (Ex-post social surplus maximization / allocative efficiency): PVCG maximizes social surplus ex post.

**Proof.** Suppose $x^{**} = \arg \max x \in x \in S(x, \gamma, \theta)$ and $S^{**} = S(x^{**}, \gamma, \theta)$. We aim to prove that PVCG results in ex-post social surplus no less than $S^{**}$.

By definition of $\eta^*(\hat{x}, \gamma, \theta)$,

$$S(\hat{x} \odot \eta^*(\hat{x}, \gamma, \theta), \gamma, \theta) \geq S(\hat{x} \odot \eta, \gamma, \theta), \forall \eta.$$ \hspace{1cm} (12)

Incentive compatibility guarantees $\hat{x} = \bar{x}$, $\hat{\gamma} = \gamma$, and $\theta = \theta$. Therefore,

$$S(\hat{x} \odot \eta^*(\hat{x}, \gamma, \theta), \gamma, \theta) \geq S(\hat{x} \odot \eta, \gamma, \theta), \forall \eta.$$ \hspace{1cm} (13)

Particularly, Eq. [13] holds for $\eta = x^{**} \in [0, 1]^{\dim(x)} \times x$, i.e.,

$$S(\hat{x} \odot \eta^*(\hat{x}, \gamma, \theta), \gamma, \theta) \geq S(\hat{x} \odot x^{**} \odot \bar{x}, \gamma, \theta) = S(x^{**}, \gamma, \theta) = S^{**}.$$ \hspace{1cm} (14)

The left side of Eq. [14] is the ex-post social surplus achieved by PVCG, while the right side is the maximum social surplus across all possible $x \leq \bar{x}$, given $(\bar{x}, \gamma, \theta)$. Hence, the right side is also no less than the left side. Therefore, the left side equals the right side.

Furthermore, given that incentive compatibility has been proved in Proposition 1 and Theorem 1, the following two propositions provide sufficient and necessary conditions for ex-post individual rationality and ex-post weak budget balancedness.

**Proposition 3** (Condition for ex-post individual rationality): PVCG is ex-post individual rational (IR) for all producers if.f. the true capacity limits $\bar{x}$, the true type parameters $\gamma$, $\theta$, and the adjustment payments $h_i(\cdot)$ satisfy

$$h_i(\bar{x}_{-i}, \gamma_{-i}, \theta)$$

$$\geq -[S^*(\hat{x}, \gamma, \theta) - S^*_i(\bar{x}_{-i}, \gamma_{-i}, \theta)], \forall i \in N.$$ \hspace{1cm} (15)
Proof. According to truthfulness proved in Proposition 1 and Theorem 1, we use \( \hat{x}, \gamma, \theta \) to substitute \( \bar{x}, \gamma, \theta \) in Eq. 15 and Eq. 17. Then, the ex-post utility of producer \( i \) becomes
\[
u_i(\hat{x}, \gamma, \theta) = p_i(\hat{x}, \gamma, \theta) - c(\hat{x}_i \circ \eta^*_i(\hat{x}, \gamma, \theta), \gamma_i) \quad (16)
\]
\[
= S^*(\hat{x}, \gamma, \theta) - S_{\bar{x}_i}^*(\bar{x}_i, \gamma_i, \theta) + h_i(\bar{x}_i, \gamma_i, \theta).
\]

Ex-post IR requires \( u_i(\hat{x}, \gamma, \theta) \geq 0, \forall i \), which is equivalent to the inequality in Eq. 15.

Proposition 4 (Condition for ex-post weak budget balancedness). PVCG is ex-post weakly budget balanced (WBB) on the supply side i.f.f. the true capacity limits \( \hat{x} \), the true type parameters \( \gamma, \theta \), and the adjustment payments \( h(\cdot) \) satisfy
\[
\sum_{i=0}^{n-1} h_i(\bar{x}_i, \gamma_i, \theta) \leq S^*(\hat{x}, \gamma, \theta)
\]
\[
- \sum_{i=0}^{n-1} [S^*(\hat{x}, \gamma, \theta) - S_{\bar{x}_i}^*(\bar{x}_i, \gamma_i, \theta)].
\]

\[
(17)
\]

Proof. According to truthfulness proved in Proposition 1 and Theorem 1, we use \( \hat{x}, \gamma, \theta \) to substitute \( \bar{x}, \gamma, \theta \) in Eq. 15 and Eq. 17. Then, the ex-post total payment to all producers is
\[
\sum_{i=0}^{n-1} p_i(\hat{x}, \gamma, \theta) = \sum_{i=0}^{n-1} [\tau_i(\hat{x}, \gamma, \theta) + h_i(\bar{x}_i, \gamma_i, \theta)]
\]
\[
= \sum_{i=0}^{n-1} [S^*(\hat{x}, \gamma, \theta) - S_{\bar{x}_i}^*(\bar{x}_i, \gamma_i, \theta)]
\]
\[
+ c(\hat{x}_i \circ \eta^*_i(\hat{x}, \gamma, \theta), \gamma_i) + h_i(\bar{x}_i, \gamma_i, \theta)].
\]

As explained before, Theorem 1 transforms the WBB condition to \( \sum_{i=0}^{n-1} p_i(\hat{x}, \gamma, \theta) \leq \sum_{j=n}^{n+m-1} v(\hat{x} \circ \eta(\hat{x}, \gamma, \theta), \theta_j) \). This inequality, together with the definition \( S^*(\hat{x}, \gamma, \theta) = \sum_{j=n}^{n+m-1} v(\hat{x} \circ \eta(\hat{x}, \gamma, \theta), \theta_j) - \sum_{i=0}^{n-1} c(\hat{x}_i \circ \eta^*_i(\hat{x}, \gamma, \theta), \gamma_i) \), transforms Eq. 18 to Eq. 17.

Then, we prove Eq. 8 is a sufficient and necessary condition for Proposition 3 and Proposition 4.

Corollary 1. Under PVCG, a sufficient and necessary condition for ex-post IR and ex-post WBB to coexist on the supply side is
\[
LOSS = Loss1 + Loss2 = 0,
\]
where
\[
Loss1 = \sum_{i=0}^{n-1} \text{ReLU}[-(S^*(\hat{x}, \gamma, \theta) - S_{\bar{x}_i}^*(\bar{x}_i, \gamma_i, \theta))]
\]
\[
- h_i(\bar{x}_i, \gamma_i, \theta)] \quad \text{and}
\]
\[
Loss2 = \sum_{i=0}^{n-1} \text{ReLU}[S^*(\hat{x}, \gamma, \theta) - S_{\bar{x}_i}^*(\bar{x}_i, \gamma_i, \theta)]
\]
\[
+ h_i(\bar{x}_i, \gamma_i, \theta)] - S^*(\hat{x}, \gamma, \theta)].
\]

Proof. Loss1 \( \geq 0, \) Loss2 \( \geq 0, \forall \hat{x}, \gamma, \theta. \) Loss1 = 0 i.f.f. Eq. 15 holds; Loss2 = 0 i.f.f. Eq. 17 holds.

From Corollary 1, we know that if the adjustment payments \( h(\hat{x}, \gamma, \theta) \) are such chosen that Eq. 19 holds for true capacity limits \( \hat{x} \) and true type parameters \( \gamma, \theta \), then PVCG attains IR and WBB simultaneously. However, these true parameters are unknown to the coalition coordinator a priori, we need to find the optimal adjustment payment function \( h^*() \) such that for all possible \( \hat{x} \) and \( \gamma \) drawn from their respective prior distributions, Eq. 19 holds. This is equivalent to solve the functional equation in Eq. 8. When such a functional solution exists, it minimizes the expected value of \( LOSS = Loss1 + Loss2. \)

Corollary 2. The solution \( h^*(\cdot) \) to Eq. 8 if existing, is also a solution to the following minimization problem:
\[
h^*(\cdot) = \arg \min_{h(\cdot)} \mathbb{E}_{(\hat{x}, \gamma, \theta)} \{LOSS\},
\]
where the expectation is over the prior distribution of \( (\hat{x}, \gamma, \theta) \).

Proof. For the solution \( h^*(\cdot) \) to Eq. 8,
\[
LOSS \equiv 0, \forall (\hat{x}, \gamma, \theta) \in \text{supp(Prior}(\hat{x}, \gamma, \theta)).\]

Therefore, \( \mathbb{E}_{(\hat{x}, \gamma, \theta)} \{LOSS\} = 0. \)

For all other function \( h(\cdot) \), since \( LOSS \geq 0, \forall (\hat{x}, \gamma, \theta) \), we have \( \mathbb{E}_{(\hat{x}, \gamma, \theta)} \{LOSS\} \geq 0. \)

From Corollary 3, we can find the functional solution \( h^*(\cdot) \) to Eq. 8 by minimizing the expected \( LOSS. \) The remaining problem is to prove the existence of such a solution.

Theorem 2. The following inequality is a sufficient and necessary condition for the existence of \( h^*(\cdot) \) such that the PVCG payments \( p(\cdot) = \tau(\cdot) + h^*(\cdot) \) satisfy WBB and IR for all \( (\hat{x}, \gamma, \theta) \in \text{supp(Prior}(\hat{x}, \gamma, \theta)). \)
\[
\sum_{i=0}^{n-1} [S^*(\hat{x}, \gamma, \theta) - S^*(\min \hat{x}_i, x_{-i}, \text{max} \gamma_i, \gamma_{-i}, \theta)]
\]
\[
\leq S^*(\hat{x}, \gamma, \theta), \forall (\hat{x}, \gamma, \theta) \in \text{supp(Prior}(\hat{x}, \gamma, \theta). \]

where \( \min \hat{x}_i \) and \( \text{max} \gamma_i \) are the extreme values of \( \hat{x} \) and \( \gamma \) on their support set \( \text{supp(Prior}(\hat{x}, \gamma, \theta). \)

In order to prove Theorem 2, we introduce the following lemma first.

Lemma 1. The maximum social surplus monotonically increases with \( \hat{x} \), and monotonically decreases with \( \gamma \), for every producer \( i \).

Proof. Suppose \( \hat{x}' \geq \hat{x} \), then, by definition,
\[
S^*(\hat{x}', \gamma, \theta) \geq S(\hat{x}' \circ \eta, \gamma, \theta), \forall \eta \in [0, 1]^{\text{dim}(\hat{x}) \times n}. \]
Particularly, Eq. 13 holds for \( \eta = \eta^*(\hat{x}, \gamma, \theta) \circ \hat{x} \circ \hat{x}' \in [0, 1]^{\text{dim}(\hat{x}) \times n} \), i.e.,
\[
S^*(\hat{x}', \gamma, \theta) \geq S(\hat{x}' \circ \eta^*(\hat{x}, \gamma, \theta) \circ \hat{x} \circ \hat{x}', \gamma, \theta)
\]
\[
= S^*(\hat{x}, \gamma, \theta). \]

Therefore, \( S^*(\hat{x}, \gamma, \theta) \) increases with \( \hat{x}_i \).

That \( S^*(\hat{x}, \gamma, \theta) \) decreases with \( \gamma_i \) is due to the envelope theorem, which results in
\[
\frac{\partial S^*}{\partial \gamma_i}|_{\eta=\eta^*} = \frac{\partial S}{\partial \gamma_i}|_{\eta=\eta^*} = -\frac{\partial \hat{x}_i}{\partial \gamma_i}|_{\eta=\eta^*} \leq 0. \]
Now we can prove Theorem 2 as follows.

**Proof of Theorem 2** We prove sufficiency first. If Eq. 23 holds, we aim to prove

\[ h_i((\bar{x}, \gamma, \theta)) \equiv -[S^*((\min \bar{x}, \bar{x}_{-i}), (\max \gamma_{i}, \gamma_{-i}), \theta) - S_{\gamma_{i}}^*((\bar{x}_{-i}, \gamma_{-i}), \theta)], i = 0, \ldots, n - 1 \text{ satisfy Eq. 15 and 17 for all } (\bar{x}, \gamma, \theta) \in \text{supp}(\text{Prior}(\bar{x}, \gamma, \theta)). \]

In this case, all \((\bar{x}, \gamma, \theta)\) satisfy Eq. 15 because of the monotonic property proved in Lemma 1, i.e.,

\[ S^*((\min \bar{x}, \bar{x}_{-i}), (\max \gamma_{i}, \gamma_{-i}), \theta) \leq S^*(\bar{x}, \gamma, \theta), \forall \bar{x}, \gamma, \theta \in \text{supp}(\text{Prior}(\bar{x}, \gamma, \theta)). \] (27)

Meanwhile, Eq. 17 becomes

\[ \sum_{i=0}^{n-1} \{ -[S^*((\min \bar{x}, \bar{x}_{-i}), (\max \gamma_{i}, \gamma_{-i}), \theta) - S_{\gamma_{i}}^*((\bar{x}_{-i}, \gamma_{-i}), \theta)] \leq S^*(\bar{x}, \gamma, \theta) \leq \sum_{i=0}^{n-1} [S^*(\bar{x}, \gamma, \theta) - S_{\gamma_{i}}^*((\bar{x}_{-i}, \gamma_{-i}), \theta)], \] (28)

which can be deduced from Eq. 23.

To prove necessity, we set \( \bar{x} = (\min \bar{x}, \bar{x}_{-i}) \) and \( \gamma = (\max \gamma_{i}, \gamma_{-i}) \) in Eq. 15 and get a necessary condition:

\[ h_i((\bar{x}, \gamma, \theta)) \geq -[S^*((\min \bar{x}, \bar{x}_{-i}), (\max \gamma_{i}, \gamma_{-i}), \theta) - S_{\gamma_{i}}^*((\bar{x}_{-i}, \gamma_{-i}), \theta)], \forall \bar{x}, \gamma, \theta \in \text{supp}(\text{Prior}(\bar{x}, \gamma, \theta)). \] (29)

Eq. 29 together with Eq. 17 results in Eq. 23.

There are some useful corollaries of Theorem 2.

**Corollary 3.** The functional solution \( h^*(\cdot) \) to Eq. 8 exists if the prior estimation on \( \bar{x}, \gamma \) is accurate enough.

**Proof.** When the prior estimation on \( \bar{x}, \gamma \) is accurate enough, \( \bar{x}_{i} \) approaches to \( \min \bar{x} \) and \( \gamma_{i} \) approaches to \( \max \gamma_{i} \); hence, the left side of Eq. 23 approaches 0, while the right side approaches a positive number.

**Corollary 4.** If the individual valuation function is super additive and with decreasing cross marginal returns, there exists at least one solution \( h^*(\cdot) \) to Eq. 8 such that dominant incentive compatibility, ex-post allocative efficiency, ex-post individual rationality, and ex-post weak budget balance ness are simultaneously attained on the supply side.

**Proof.** According to Lemma 1,

\[ S^*((\min \bar{x}, \bar{x}_{-i}), (\max \gamma_{i}, \gamma_{-i}), \theta) \geq S^*((0, \bar{x}_{-i}), (\max \gamma_{i}, \gamma_{-i}), \theta) = S^*((0, \bar{x}_{-i}), \gamma, \theta). \] (30)

Hence, a sufficient condition for Eq. 23 to hold in this case is

\[ \sum_{i=0}^{n-1} [S^*(\bar{x}, \gamma, \theta) - S^*((0, \bar{x}_{-i}), \gamma, \theta)] \leq S^*(\bar{x}, \gamma, \theta), \forall \bar{x}, \gamma, \theta \in \text{supp}(\text{Prior}(\bar{x}, \gamma, \theta)). \] (31)

For arbitrary \((\bar{x}, \gamma, \theta) \in \text{supp}(\text{Prior}(\bar{x}, \gamma, \theta))\), suppose \( \eta^* \) maximizes \( S(\bar{x} \odot \eta^*, \gamma, \theta) \). The corresponding optimal social surplus are denoted by \( S^*(\bar{x} \odot \eta^*, \gamma, \theta) \), i.e.,

\[ S^*(\bar{x}, \gamma, \theta) = S((\bar{x} \odot \eta^*, \gamma, \theta) = \sum_{j=n}^{n+m-1} v(\bar{x} \odot \eta^*, \theta_j) - \sum_{i=0}^{n-1} c(\bar{x}_i \odot \eta^*_i, \gamma_i). \] (32)

We use \( \eta^*_i \) and \( \eta^*_i \) to denote the components of \( \eta^* \).

According to the definition of \( S^*((0, \bar{x}_{-i}), \gamma, \theta) \), we know

\[ S^*((0, \bar{x}_{-i}), \gamma, \theta) \geq S((0, \bar{x}_{-i}) \odot \eta, \gamma, \theta), \forall \eta, \] (33)

which is particularly true for \( \eta = \eta^* \).

According to Assumption 8 and 2,

\[ v(\bar{x} \odot \eta^*, \theta_j) = v((0, \bar{x}_{-i}) \odot \eta^*, \theta_j) = v((\bar{x}_i \odot \eta^*_i, \bar{x}_{-i} \odot \eta^*_{-i}, \theta_j) - v((0, \bar{x}_{-i} \odot \eta^*_{-i}, \theta_j) \leq v((\bar{x}_i \odot \eta^*_i, 0, \ldots, 0), \theta_j) - v((0, 0, \ldots, 0), \theta_j) = v(\bar{x}_i \odot \eta^*_i, \theta_j), \forall \eta \in N, j \in M. \] (34)

Then, we can derive

\[ S^*(\bar{x}, \gamma, \theta) = S((\bar{x} \odot \eta^*, \gamma, \theta) = \sum_{j=n}^{n+m-1} v(\bar{x} \odot \eta^*, \theta_j) - \sum_{i=0}^{n-1} c(\bar{x}_i \odot \eta^*_i, \gamma_i) \geq \sum_{j=n}^{n+m-1} \sum_{i=0}^{n-1} v(\bar{x}_i \odot \eta^*_i, \theta_j) - \sum_{i=0}^{n-1} c(\bar{x}_i \odot \eta^*_i, \gamma_i) \geq \sum_{j=n}^{n+m-1} \sum_{k=0}^{n-1} \sum_{i=0}^{n-1} [v(\bar{x} \odot \eta^*, \theta_j) - v((0, \bar{x}_{-i}) \odot \eta^*, \theta_j)] - c(\bar{x}_i \odot \eta^*_i, \gamma_i) \]

\[ = \sum_{j=n}^{n+m-1} \sum_{k=0}^{n-1} \sum_{i=0}^{n-1} [v((0, \bar{x}_{-i}) \odot \eta^*, \theta_j) - c(0 \odot \eta^*_i, \gamma_i) - \sum_{k=0}^{n-1} c(\bar{x}_k \odot \eta^*_k, \gamma_k)] \]

\[ = \sum_{j=0}^{n-1} \sum_{i=0}^{n-1} S((\bar{x}, \gamma, \theta) - S^*((0, \bar{x}_{-i}), \gamma, \theta)] \geq \sum_{i=0}^{n-1} [S^*(\bar{x}, \gamma, \theta) - S^*((0, \bar{x}_{-i}), \gamma, \theta)], \] (35)

where the first inequality is due to super additivity, the second inequality is a result from Eq. 34, and the third inequality is from Eq. 32 and 33. Eq. 35 is true for arbitrary \((\bar{x}, \gamma, \theta) \in \text{supp}(\text{Prior}(\bar{x}, \gamma, \theta))\), so we get Eq. 31 which is a sufficient condition for the existence of \( h^*(\cdot) \) as a solution to 8.
Lastly, it is worth pointing out even for those cases where there is no solution to Eq. 22(i.e., Assumption 3-5 do not hold), an optimal $h^*$ can still be found by minimizing the expected LOSS in Eq. 22. Such an optimal $h^*$ guarantees the expected deviation from the IR and WBB constraints are minimized.

**Learning the Optimal Adjustment Payments**

Corollary 2 informs us that we can learn the optimal adjustment payments $h^*(\cdot)$ by minimizing the expected LOSS in Eq. 22. Also, we know neural networks can approximate arbitrary continuous functions to arbitrary precisions (Fujinahashi 1989). Therefore, we construct $n$ neural networks $\text{NET}_i^h, i \in N$ to approximate $h_i(\cdot), i \in N$. Output nodes of these $n$ networks, denoted by $\text{NET}_i^h, o, i \in N$, are combined into a single composite neural network in Figure 1 with the following loss function:

$$
\text{LOSS} = \frac{1}{T} \sum_{t=0}^{T-1} \left( \sum_{i=0}^{n-1} \text{ReLU}[\left( (S^* - S^*_{i}) - \text{NET}_i^h, o \right] + \text{ReLU}\left( \left( (S^* - S^*_{i}) + \text{NET}_i^h, o \right) - S^* \right) \right),
$$

where each sample $(\hat{x}^t, \gamma^t, \theta^t)$ is drawn from their prior distribution $\text{Prior}(\hat{x}, \gamma, \theta)$ and $T$ is the sample size. For the $t$th sample, $\tau^t = \tau(\hat{x}^t, \gamma^t, \theta^t)$, $S^t = S^*(\hat{x}^t, \gamma^t, \theta^t)$, and $S^*_{-i} = S^*(\hat{x}^t_{-i}, \gamma^t_{-i}, \theta^t_{-i})$. Since we only need synthetic data to train this network, we can generate as many data as needed. As a result, LOSS can be minimized to its theoretical minimum almost perfectly in experiments.

To illustrate the effectiveness of this neural network method and to show the correctness of Corollary 2 we learned the adjustment payments for a hypothetical scenario. We set the individual valuation functions and individual cost functions as follows:

$$
v(x) = \theta_i \sqrt{n \sum_{k=0}^{n-1} x_k} + c_i(x_i, \gamma_i) = \gamma_i x_i, i \in N.
$$

Note that the individual valuation function in Eq. 37 is a special case of that in Eq. 3 where $z(x_i)$ is set to be $z(x_i) = \sqrt{x_i}$. We report the experiment results for $n = 10, m = 2$, Prior($\hat{x}_i$) = Uniform$[0, 5]$, $i \in N$, Prior($\gamma_i$) = Uniform$[0, 1]$, $i \in N$, and Prior($\theta_j$) = $[0, 1], j \in M$. We let $\text{NET}_i^h, o, i \in N$ each have three 10-dimensional hidden layers.

The loss curve is shown in the left figure of Fig. 2. The training loss fast converges to 0 after 40 iterations, as expected. After we obtain the trained networks $\text{NET}_i^h, o, i \in N$, we can use trained networks to calculate PVCG payments $p(\hat{x}, \gamma, \theta)$ for any reported $(\hat{x}, \gamma, \theta)$. For illustration, we draw $p_0$, the payment to participant 0, with respect to $\hat{x}_0$ and $\gamma_0$ in the right figure in Fig. 2 fixing parameters of other participants at $\gamma_i \equiv 2.5, \gamma_i \equiv 0.5, i \in N, \neq 0, \theta_j \equiv 0.5, j \in M$. We can see that $p_0$ increases with $\gamma_0$. This aligns well with our expectation that the higher the capacity limit a producer reports, the more input resources are contributed by this producer; hence, it receives higher payments. Also, $p_0$ remains constant with $\gamma_0$ when $\gamma_0$ is below a threshold and sharply drops to around 0 when $\gamma_0$ passes the threshold. This implies that the payment to a producer should only be affected by its contribution to the cooperative production process rather than its cost, but if a producer’s cost is too high, the optimal social choice is to exclude this producer from the production coalition and thus pay it nothing.

**Conclusions and Future Work**

In this paper, we presented the Procurement-VCG (PVCG) mechanism for incentivizing producers to collaboratively produce some valuable virtual products. PVCG incentivizes capacitated producers to truthfully report their capacity limits and their cost types, based on which we optimize for social surplus and guarantee ex-post allocative efficiency. With some reasonable assumptions, we prove that PVCG is ex-post individually rational and ex-post weakly budget balanced. To our knowledge, PVCG is among the very few mechanisms in the mechanism design literature that attain these four objectives simultaneously.

Since this paper focuses on theoretical analyses, we reported limited experimental results. When the number of producers increases, we require more advanced algorithms to learn the PVCG payments efficiently. Besides, for non-analytic individual valuation functions and individual cost functions, calculating the optimal social surplus is compu-
tationally expensive as well. These computational issues are beyond the scope of this paper and left for future research. Also, in order to apply PVCG to scenarios such as federated learning, some special-purpose engineering designs (e.g., a sandbox) are required to measure the required parameters in our model, e.g., the input datasets and the output model. These implementation details will be discussed in our subsequent research. Moreover, in our following work, we will report simulation results that compare PVCG with other sharing rules for cooperative games, e.g., Shapley (Jia et al. 2019), Labour Union (Gollapudi et al. 2017), etc.

References

Albert, Conitzer, and Lopomo 2015 Albert, M.; Conitzer, V.; and Lopomo, G. 2015. Assessing the robustness of cremer-mclean with automated mechanism design. In Twenty-Ninth AAAI Conference on Artificial Intelligence.

Bachrach et al. 2009 Bachrach, Y.; Meir, R.; Zuckerman, M.; Rothe, J.; and Rosenschein, J. S. 2009. The cost of stability in weighted voting games. In Proceedings of The 8th International Conference on Autonomous Agents and Multiagent Systems-Volume 2, 1289–1290. International Foundation for Autonomous Agents and Multiagent Systems.

Beberg et al. 2009 Beberg, A. L.; Ensign, D. L.; Jayachandran, G.; Khaliq, S.; and Pande, V. S. 2009. Folding @ home: Lessons from eight years of volunteer distributed computing. In IEEE International Symposium on Parallel & Distributed Processing.

Branzei, Dimitrov, and Tijs 2008 Branzei, R.; Dimitrov, D.; and Tijs, S. 2008. Models in cooperative game theory, volume 556. Springer Science & Business Media.

Cai, Daskalakis, and Papadimitriou 2015 Cai, Y.; Daskalakis, C.; and Papadimitriou, C. 2015. Optimum statistical estimation with strategic data sources. In Conference on Learning Theory, 280–296.

Chalkiadakis et al. 2010 Chalkiadakis, G.; Elkind, E.; Markakis, E.; Polukarov, M.; and Jennings, N. R. 2010. Cooperative games with overlapping coalitions. Journal of Artificial Intelligence Research 39:179–216.

Chalkiadakis, Elkind, and Wooldridge 2011 Chalkiadakis, G.; Elkind, E.; and Wooldridge, M. 2011. Computational aspects of cooperative game theory. Synthesis Lectures on Artificial Intelligence and Machine Learning 5(6):1–168.

Chandrashekar et al. 2007 Chandrashekar, T. S.; Narahari, Y.; Rosa, C. H.; Kulkarni, D. M.; Tew, J. D.; and Dayama, P. 2007. Auction-based mechanisms for electronic procurement. IEEE Transactions on Automation Science and Engineering 4(3):297–321.

Chen et al. 2005 Chen, R. R.; Roundy, R. O.; Zhang, R. Q.; and Janakiraman, G. 2005. Efficient auction mechanisms for supply chain procurement. Management Science 51(3):467–482.

Christoforou et al. 2013 Christoforou, E.; Anta, A. F.; Georgiou, C.; Mosteiro, M. A.; and Sánchez, A. 2013. Crowd computing as a cooperation problem: an evolutionary approach. Journal of Statistical Physics 151(3-4):654–672.

Conitzer and Sandholm 2006 Conitzer, V., and Sandholm, T. 2006. Complexity of constructing solutions in the core based on synergies among coalitions. Artificial Intelligence 170(6-7):607–619.

Crémer and McLean 1985 Crémer, J., and McLean, R. P. 1985. Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent. Econometrica 53:345–361.

Drechsl and Kimms 2010 Drechsl, J., and Kimms, A. 2010. Computing core allocations in cooperative games with an application to cooperative procurement. International Journal of Production Economics 128(1):310–321.

Elkind et al. 2009 Elkind, E.; Goldberg, L. A.; Goldberg, P. W.; and Wooldridge, M. 2009. On the computational complexity of weighted voting games. Annals of Mathematics and Artificial Intelligence 56(2):109–131.

Funahashi 1989 Funahashi, K-I. 1989. On the approximate realization of continuous mappings by neural networks. Neural networks 2(3):183–192.

Gollapudi et al. 2017 Gollapudi, S.; Kollias, K.; Panigrahi, D.; and Platsika, V. 2017. Profit sharing and efficiency in utility games. In Proceedings of the 25th Annual European Symposium on Algorithms (ESA’17).

Iyengar and Kumar 2008 Iyengar, G., and Kumar, A. 2008. Optimal procurement mechanisms for divisible goods with capacitated suppliers. Review of Economic Design 12(2):129.

Jackson 2014 Jackson, M. O. 2014. Mechanism theory. Available at SSRN 2542983.

Jia et al. 2019 Jia, R.; Dao, D.; Wang, B.; Hubis, F. A.; Hynes, N.; Gurel, N. M.; Li, B.; Zhang, C.; Song, D.; and Spanos, C. 2019. Towards efficient data valuation based on the shapley value. CoRR, arXiv:1902.10275.

Kosenok and Severinov 2008 Kosenok, G., and Severinov, S. 2008. Individually rational, budget-balanced mechanisms and allocation of surplus. Journal of Economic Theory 140(1):126–161.

Kronbak and Lindroos 2007 Kronbak, L. G., and Lindroos, M. 2007. Sharing rules and stability in coalition games with externalities. Marine Resource Economics 22(2):137–154.

Larson et al. 2009 Larson, S. M.; Snow, C. D.; Shirts, M.; and Pande, V. S. 2009. Folding @ home and genome@ home: Using distributed computing to tackle previously intractable problems in computational biology. arXiv preprint arXiv:0901.0866.

Niyato, Vasilakos, and Kun 2011 Niyato, D.; Vasilakos, A. V.; and Kun, Z. 2011. Resource and revenue sharing with coalition formation of cloud providers: Game theoretic approach. In 2011 11th IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing, 215–224. IEEE.

Peleg and Sudhölter 2007 Peleg, B., and Sudhölter, P. 2007. Introduction to the theory of cooperative games, volume 34. Springer Science & Business Media.

Rassias 2012 Rassias, T. 2012. Functional equations and inequalities, volume 518. Springer Science & Business Media.
[Richardson, Filos-Ratsikas, and Faltings 2019] Richardson, A.; Filos-Ratsikas, A.; and Faltings, B. 2019. Rewarding high-quality data via influence functions. CoRR, arXiv:1908.11598.

[Sandholm and Lesser 1997] Sandholm, T. W., and Lesser, V. R. 1997. Coalitions among computationally bounded agents. Artificial intelligence 94(1):99–138.

[Shen, Tang, and Zuo 2019] Shen, W.; Tang, P.; and Zuo, S. 2019. Automated mechanism design via neural networks. In Proceedings of the 18th International Conference on Autonomous Agents and Multiagent Systems, 215–223. International Foundation for Autonomous Agents and Multiagent Systems.

[Wang 2019] Wang, G. 2019. Interpret federated learning with shapley values. the 1st International Workshop on Federated Machine Learning for User Privacy and Data Confidentiality.

[Weikard 2009] Weikard, H.-P. 2009. Cartel stability under an optimal sharing rule. The manchester school 77(5):575–593.

[Westenbroek et al. 2019] Westenbroek, T.; Dong, R.; Ratliff, L. J.; and Sastry, S. S. 2019. Competitive statistical estimation with strategic data sources. CoRR, arXiv:1904.12768.

[Yang et al. 2019a] Yang, Q.; Liu, Y.; Chen, T.; and Tong, Y. 2019a. Federated machine learning: Concept and applications. ACM Transactions on Intelligent Systems and Technology (TIST) 10(2):12.

[Yang et al. 2019b] Yang, Q.; Liu, Y.; Cheng, Y.; Kang, Y.; Chen, T.; and Yu, H. 2019b. Federated Learning. Morgan & Claypool Publishers.

[Yu et al. 2020] Yu, H.; Liu, Z.; Liu, Y.; Chen, T.; Cong, M.; Weng, X.; Niyato, D.; and Qiang, Y. 2020. A fairness-aware incentive scheme for federated learning. In Proceedings of the 3rd AAAI/ACM Conference on Artificial Intelligence, Ethics, and Society (AIES-20), 393–399.