EXTRACTING PERIODIC TRANSIT SIGNALS FROM NOISY LIGHT CURVES USING FOURIER SERIES

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ABSTRACT

We present a simple and powerful method for extracting transit signals associated with a known transiting planet from noisy light curves. Assuming the orbital period of the planet is known and the signal is periodic, we illustrate that systematic noise can be removed in Fourier space at all frequencies by only using data within a fixed time frame with a width equal to an integer number of orbital periods. This results in a reconstruction of the full transit signal, which on average is unbiased despite no prior knowledge of either the noise or the transit signal itself being used in the analysis. The method therefore has clear advantages over standard phase folding, which normally requires external input such as nearby stars or noise models for removing systematic components. In addition, we can extract the full orbital transit signal \((360^\circ)\) simultaneously, and Kepler-like data can be analyzed in just a few seconds. We illustrate the performance of our method by applying it to a dataset composed of light curves from Kepler with a fake injected signal emulating a planet with rings. For extracting periodic transit signals, our presented method is in general the optimal and least biased estimator and could therefore lead the way toward the first detections of, e.g., planet rings and exo-trojan asteroids.

Key words: methods: data analysis – Planetary systems

1. INTRODUCTION

Our understanding of where exoplanets form, their orbital distribution, and even properties such as mass and radius has greatly improved over recent years (Lissauer et al. 2014; Laughlin & Lissauer 2015). Several methods have contributed to this progress, from mass sensitive measurements including pulsar timings (Wolszczan & Frail 1992), radial velocimetry (Mayor & Queloz 1995; Mayor et al. 2014) and microlensing (Bond et al. 2004), to transit photometry (Charbonneau et al. 2000; Henry et al. 2000), which in turn depends on the planet radius. Recent developments have primarily been driven by photometric surveys operating from both the ground (OGLE (Udalski et al. 2002), HAT (Bakos et al. 2002) and WASP (Pollacco et al. 2006)), and space (CoRoT (Auvergne et al. 2009), Kepler (Borucki et al. 2010)). The emerging picture is that exoplanets are not rare, but exist with a wide variety of masses, orbital sizes, and eccentricities, and often in multiplanet configurations which can be very different from our own solar system (Laughlin & Lissauer 2015). Despite the success in detecting planets, there are still major gaps in our understanding of how they form and their subsequent evolution. The problem covers all aspects of astrophysics from the formation and evolution of the protoplanetary disk (see, e.g., Turner et al. 2014), to late time gravitational dynamics including both strong field planet–planet scattering and secular evolution (Beaugé et al. 2012). Due to this complexity, observations play a crucial role in driving the field.

In this paper we focus on photometric measurements and how to improve the scientific output from transit light curves. One of the present challenges is to reduce the noise to a level where “higher-order photometric effects” (Perryman 2011), with relative flux variations of order \(10^{-4}\), can be resolved. This level of precision could lead to the first detection of exomoons (Kipping et al. 2012; Simon et al. 2012; Heller 2014; Hippke 2015), planet rings (Barnes & Fortney 2004), and exo-trojan asteroids (Laughlin & Chambers 2002), as well as teach us about the rotation of the transiting planets (Zhu et al. 2014) and even their atmospheres (Hui & Seager 2002; Sidis & Sari 2010), to name a few examples. However, such variations are very hard to resolve with present observations and noise removal techniques.

Motivated by these challenges, here we present a fast and effective method for extracting small flux variations associated with a known planet transit, from light curves with random and systematic noise. The noise represents all components which are not associated with the transit signal itself but such as instrumental noise and real stellar flux variations. By assuming the transit signal is periodic, we illustrate how one can remove systematic noise in a model independent way, which makes it possible to resolve finer flux variations than standard phase folding techniques.\(^5\) This could directly improve the ongoing search for planetary rings and trojan-asteroids, as well as lead to better measurements of any phenomena associated with the planet transit itself.

The new strategy we propose is based on removing systematic noise in Fourier space. We show this becomes especially effective if one only uses data within a fixed time frame starting from the middle of the first planet transit and ending in the last. In this way any signal associated with the transiting planet will be constrained to discrete and separate frequencies in Fourier space, which makes the noise separation possible. Our method does not rely on any prior knowledge of either the noise or signal, which also makes it fast enough that several quarters of Kepler data can be analyzed in seconds. If the noise is not directly correlated with the periodic signal, our method will, on average, return an unbiased reconstruction of the full orbital transit signal. Fourier transforms have previously been used for, e.g., finding short period planets (Sanchis-Ojeda et al. 2014), as well as removing noise in exoplanet emission spectra (Swain et al. 2010; Zellem

\(^5\) Phase folding techniques in this paper loosely refer to methods which use out-of-transit fits and stacking in real space to reduce systematic and random noise, respectively.

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et al. 2014), but has to our knowledge not been applied for extracting the transit shape itself in the way we propose.

This paper is organized as follows. In Section 2 we describe how the observed flux, possible signal, and noise are related. The notation and assumptions from this section will be used in the remaining parts. In Section 3 we describe our new Fourier method step by step and how it can be applied to reconstruct any periodic transit shape. The performance of the method is shown in Section 4, where we apply it to reconstruct a fake light signal into real Kepler light curves. Conclusions are represented as a Fourier series

2. TRANSIT SIGNALS AND LIGHT CURVE OBSERVATIONS

We consider a planetary system consisting of a star with a single planet. If observed from the right position, the planet will move in front of the star during its orbit, which gives rise to periodic dips in the observed light curve. We denote this as the main transit. Objects associated with the planet such as rings, moons, and trojan-asteroids can likewise block parts of the stellar light, and can therefore be indirectly seen as small flux variations on top of the main transit signal. A few examples are shown in Figure 1. In this section we describe the assumptions we make about how the observed flux, transit signal, and noise are related to each other.

2.1. Flux from a Star with an Orbiting Planet

For this paper we consider observations of a star with one transiting planet. The observed flux \( F_{\text{obs}} \) can in the majority of cases be modeled as

\[
F_{\text{obs}} = F_\star + \Delta F + F_\star I^S,
\]

where \( F_\star \) is the mean flux from the star, \( \Delta F \) is considered noise, and \( I^S \) is the fractional flux modulation caused by orbiting objects, including the planet. The modulation term \( I^S \) is typically negative, but reflections from the planet surface can give rise to positive modulations. The noise term \( \Delta F \) is assumed to be small compared to \( F_\star \), and represents all systematic and random flux variations which are not associated with the true transit signal. By defining the observable quantity \( I(t) = F_{\text{obs}}/F_\star - 1 \) and the relative noise contribution \( I(t)^N = \Delta F/F_\star \), we can rewrite Equation (1) as

\[
I(t) = I(t)^S + I(t)^N.
\]

In the remaining part of the paper we generally refer to \( I(t) \) as the data, \( I(t)^S \) as the signal, and \( I(t)^N \) as the noise. In the following sections we show that if the signal \( I(t)^S \) is periodic, then one can make a precise reconstruction of its transit shape by working in Fourier space.

3. RECONSTRUCTION OF PERIODIC TRANSIT SIGNALS

We now present our new method for extracting periodic transit signals. The method relies on the basic assumption that the transit signal must be strictly periodic, with the same period as the orbiting planet. Besides that, there are no restrictions on the shape of the signal, i.e., the method can be used to reconstruct transits with any shape. The assumed periodicity makes it possible to work in Fourier space, which enables us to remove systematic noise components. This is in contrast to standard phase folding techniques where the real space stacking is only optimal for reducing random noise. In the sections below we describe our proposed Fourier method step by step.

3.1. Representing Data as a Fourier Series

The general strategy is to extract the periodic transit signal in Fourier space. We therefore first consider the data \( I(t) \) represented as a Fourier series

\[
I(t) = \sum_{n=1}^{N/2} a_n \cos \left( \frac{2\pi n(t - \tau_f)}{T_f - \tau_f} \right) + \sum_{n=1}^{N/2} b_n \sin \left( \frac{2\pi n(t - \tau_f)}{T_f - \tau_f} \right),
\]

where \( t \) is time, \( N \) is the number of data points, and \( \tau_f, \tau_f \) are denoting the starting time and ending time of the dataset used in the analysis, respectively. We refer to this fixed time frame as the “data window” and denote its width, \( |\tau_f - \tau_b| \), by \( T_w \). The importance of the width and location of this window is discussed in Section 3.2. From our signal and noise model shown in Equation (2) and by the linearity of the Fourier transformation, the Fourier coefficients can be divided up into signal and noise as

\[
a_n = a_n^S + a_n^N, \quad b_n = b_n^S + b_n^N.
\]

The idea is now to separate the signal and the noise in the Fourier domain, as shown in Equation (4). In the section below we show that a specific choice for \( \tau_f \) and \( \tau_b \) will make this approach possible.

3.2. Choosing the Optimal Data Window

The Fourier coefficients obtained from using the full data set \( I(t) \) will in general have broad features, and the underlying
signal will overlap with the noise spectrum, even if the signal has strictly periodic components. This is mainly due to "window effects" where the position and width of the data window interfere with the Fourier modes of the data. Separating the coefficients as illustrated in Equation (4), is therefore not possible in general.

However, what is usually not appreciated is that the data window actually can be freely chosen to highlight features of interest in the data. In our case, we can use our knowledge of where the main transits occur in the light curve to construct a window which will not affect the representation of the signal in Fourier space. Following this idea, one can show that the minimum overlap between noise and signal generally can be achieved by choosing a data window such that

\[ \tau_i = \text{time of first transit} \]
\[ \tau_f = \text{time of last transit}, \]  

(5)

where the transit time here refers to the time at the center of the main planet transit. There are two reasons for why this window can significantly improve the reconstruction of the signal. First, the frequencies related to the signal will now be confined to single frequency bins with no broadening because \( T_w \) is now a multiple of the period of the planet. Second, since the time window is now fixed to start in the middle of a transit, the obtained signal coefficients, \( a_n^S \) and \( b_n^S \), not only describe the odd and even contributions to the data, but also the symmetry of the signal itself. For example, if we search for a signal which is symmetric around the main transit (as the one shown in Figure 1), then we know right away that all the \( b_n^S \) must equal zero. This gives us a perfect separation of noise and signal in the \( b_n \) coefficient space, which greatly improves the extraction of the signal. The top plot in Figure 2 illustrates the proposed window applied to data from Kepler, where the top center plot shows how this results in Fourier coefficients with distinct and isolated signal peaks.

### 3.3. Reconstruction of a Symmetric Transit Signal

To make the presentation of the method as clear as possible and to shorten the algebra, we now assume that the transit shape of the signal is symmetric with respect to the main transit. This is a special case, but in fact represents a large class of real astrophysical transit phenomena including the main planet transit, secondary eclipse, planetary rings, and trojan-asteroids. Figure 1 shows an example. In Section 3.4 we describe how to extend the following procedure to reconstruct a signal with arbitrary transit shape.

#### 3.3.1. Identifying Noise and Signal in Fourier Space

By assuming the transit shape is symmetric, the signal periodic, and using the window proposed in Equation (5), we then know that only cosine terms will be needed for the reconstruction. Inserting this case into Equation (3) and using the linearity relation between signal and noise shown in Equation (4), we can now express the data described by the

\[ \tilde{f}(t) = a_0^S + \sum_{n \text{ odd}} a_n^S \cos\left(\frac{2\pi n}{T_w} t + \phi_n^S\right) + \sum_{n \text{ even}} b_n^S \cos\left(\frac{2\pi n}{T_w} t + \phi_n^S\right). \]  

The frequency bins with clear narrow components separated into a noise component \( a_n^N \) (red) and a signal component \( a_n^S \) (blue). The noise contribution in the signal bins is here estimated by using linear interpolation as described in Section 3.3.2. This way of removing systematic noise makes our method on average an unbiased estimator of the true transit signal, in contrast to normal phase folding. Bottom: the full reconstructed signal shown up to the Nyquist frequency. The blue symbols denote \( a_n^S \) (even part) coefficients, where the green symbols denote \( b_n^S \) (odd part) coefficients.
cosine terms as
\[
I_c(t) = \sum_{n=1}^{N/2} a_n^N \cos \left( \frac{2\pi nt}{T_w} \right) + \sum_{m=1}^{N/2} a_{mN_0}^S \cos \left( \frac{2\pi (mN_0)t}{T_w} \right),
\]
where \(N_0\) denotes the number of full planet orbits within the data window. From this equation it is now clear that the noise can contribute at all \(n = \{1, 2, 3\ldots\}\), but the signal only contributes at \(n = mN_0\) where \(m = \{1, 2, 3\ldots\}\). In other words, the signal is now constrained to discrete and separate frequency bins in Fourier space. The fraction these signal bins cover of the total available frequency space is \(\sim 1/(2N_0)\), which is simply the ratio of the number of signal bins, \(N/(2N_0)\), to the total number of bins, \(N\). This means that if \(N_0 = 10\) then the signal only appears in 5% of the total available space, i.e., 95% of the modes in Fourier space can be labeled as pure noise. An example of this signal and noise separation is shown in the middle panels of Figure 2. In the next section we illustrate how the noise contribution can be estimated in the signal bins, which leads to the final extraction of the signal.

### 3.3.2. Reconstruct Signal

From using the fixed data window proposed in Section 3.2, the signal has now been constrained to only appear in \(\sim 1/(2N_0)\) of the available Fourier space. However, noise will in general still be present in the bins where the signal is now localized. The question is if we can remove this noise contribution to improve the reconstruction of the true signal. We start by using Equation (4) to express the signal coefficients as
\[
a_{mN_0}^S = a_{mN_0} - a_n^N,
\]
where \(a_{mN_0}\) is obtained directly from data. If one now can estimate \(a_n^N\), then the underlying signal \(a_{mN_0}^S\) can be extracted. The random part of the noise can of course not be removed in individual bins, but we here show that it is possible to correct for systematic components since these on average appear as smooth features in Fourier space. The idea is to use the spectral shape of the noise outside the signal bins \((n = mN_0)\) to predict the value inside the bins \((n = mN_0)\). In other words, we suggest that the systematic contribution to the noise spectrum can on average be estimated by interpolating across the signal bins in Fourier space, using the regions outside the bins as a baseline. The regions over which we have to interpolate are always just single frequency bins with unit width, no matter how broad the signal is in real space. The optimal interpolation scheme depends on the properties of the noise; however, we find that a simple linear interpolation is by far the most robust when using real data. Using a linear interpolation scheme, the estimate for the true signal coefficients is now given by
\[
a_{mN_0}^S \approx a_{mN_0} - \left( a_{mN_0-1} + a_{mN_0+1} \right)/2,
\]
where the last term is just the average of the neighbor bins around \(a_{mN_0}\). The \(a_n\) coefficients can directly be found from
data by

$$a_x = \sum_{j=1}^{N} I(t_j) \cos \left( \frac{2\pi x t_j}{T_w} \right), \quad 0 \leq t_j \leq T_w, \quad (9)$$

where the summation must be performed over the fixed time window as indicated. If the data is nonuniformly sampled in time, one must use a slightly more general approach for estimating the coefficients. This is further discussed in the Appendix.

It is often useful to also consider the real space representation of the reconstructed signal. This representation is simply given by

$$I(t)^{S} \approx \sum_{m=1}^{N/[2N_0]} a_{mN_0}^{S} \cos \left( \frac{2\pi m t}{T_0} \right), \quad (10)$$

where $T_0$ is the orbital time of the planet, and the signal coefficients $a_{mN_0}^{S}$ are given by Equation (8). The real space representation has the advantage that the different signal components can separate in time along the planetary orbit (see, e.g., Figure 3). However, the drawback is that single outliers among the Fourier coefficients generate global periodic variations, which makes it very hard to distinguish noise in Fourier space from a possible interesting signal identified in real space. This is different from the Fourier space representation where all the signal components share the same frequency bins. A model comparison must therefore be performed by fitting in Fourier space instead of real space. A blind search for unexpected signals is on the other hand probably best to perform in real space, where the different parts of the signal often separate and therefore become easier to visually identify.

Our estimate of the signal shown in Equation (8) is of course not always perfect, but it is expected to be good on average. This means that if one has several datasets from the same system, the stacked set of reconstructed signals in Fourier space will on average be an unbiased estimate of the true signal. Standard phase folding is not unbiased in the same way, since the stacking in real space is only optimal for removing random uncorrelated noise. A noise contribution with, e.g., a sinusoidal shape, has highly correlated values in real space and is therefore not averaging to zero if divided up in orbital segments and stacked, unless the divisions are matched in phase and size to the noise shape. However, in Fourier space such a contribution is basically only occupying a single bin and can therefore in general be fully removed without touching the signal bins. Masking out the signal regions in real space and using the data outside to estimate systematic noise is sometimes possible in phase folding, but highly limited to cases where the location of the signal is already known and constrained to a small region out of the full orbit. These restrictions do not apply to our method, which makes it possible to remove noise at all frequencies and reconstruct the full orbital signal. Our new method is illustrated step by step in Figure 2 with 

4. NUMERICAL EXAMPLE: A PLANET WITH RINGS

We here give an example of how well a transit signal can be reconstructed using our Fourier method. For this illustration we inject a fake signal into 12 quarters of data from the Kepler object 010264660. This object already has a planet, as seen in Figure 2, which we therefore remove before injecting our fake signal. Besides the clean transit signal from the planet itself, we also include a wider and weaker transit component centered around the planet to emulate a ring system. This gives rise to wide wings at ingress and egress, which are one of the main indications for rings (Barnes & Fortney 2004; Tüsken & Valio 2011). For illustrative purposes, we chose the injected signal to have a symmetric shape with respect to the main transit. The interesting question is if our method is precise enough to tell if a ring is present or not.

Results are summarized in Figure 3. There are several ways to combine the 12 datasets, here we use the approach of applying our method to each of the datasets individually. As seen in the panel showing the Fourier coefficients, each realization has some scatter, but the average of the 12 reconstructed sets is consistent with the true signal shape. As a result, the contributions from rings are easily seen both in Fourier space (top center panel) and in real space (bottom panel). How to compare the extracted signal to models including proper error propagation, will be discussed in an upcoming paper.

5. CONCLUSIONS

We present a simple yet powerful approach for extracting the shape of periodic transit signals associated with a known planet from an observed light curve. The method is based on the idea of separating signal and noise in Fourier space, which especially enables us to remove systematic noise components. We show that this becomes possible if only data within a fixed time window with a width equal to an integer number of orbital periods is used in the analysis. In this case, the signal separates out and is described by a significantly smaller number of frequencies than the full dataset. In addition, the signal frequencies are easily identified in Fourier space, and $1 - (1/N_0)$ of the total space can be labeled as pure noise as a result. For extracting the true transit signal, we suggest that the noise contribution across the frequency bins of the signal can be estimated by using the regions outside the bins as a baseline. We illustrate this by successfully extracting a fake transit signal injected into real Kepler light curves.

Our proposed method for removing systematic noise does not rely on either input noise models or nearby field stars for calibration. This makes our method not only model independent, but also fast enough that several quarters of Kepler data can be analyzed in seconds. Furthermore, we can extract the full orbital transit signal without applying any masking or out-of-transit fitting. Our method has some limitations which depend on the data quality and the noise properties, and one could in general also benefit from combining it with other techniques such as the complementary trend filtering algorithm (Kovács et al. 2005). This will be explored in an upcoming
paper where we apply the method to real data, to search for rings, trojans and other higher-order photometric effects associated with the main orbiting planet.

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APPENDIX

Observationally obtained light curves are often sampled nonuniformly in time and include gaps with no data. The Fourier coefficients can therefore not be obtained using standard FFT methods or by the simple relation shown in Equation (9). Instead, one has to perform a joint least squares fit of sine and cosine functions to the data. Due to the linearity of the Fourier expansion, this fit is simple and can with great advantage be formulated using matrices. Following this approach we can now express the data as a matrix multiplication

\[ I_i \approx C_i a_n + S_i b_n = T_i R_k, \]

where \( C_i = \cos(2\pi n_i/T_i) \), \( S_i = \sin(2\pi n_i/T_i) \), \( T_i \) is their joined matrix, and \( R_k = [a_n, b_n] \) is the vector of coefficients we want to fit for. By minimizing the difference squared between our expansion and the data, one can now express the solution to the vector \( R_k \) as

\[ R_k \approx (T^T T)^{-1} T^T I, \]

where the superscript T here denotes the transpose. If the data is uniformly sampled then the matrix \( T^T T \) equals the identity matrix, and the solution reduces to the simple Fourier relation shown in Equation (9). Only in this case can a FFT solver be used. Data from the Kepler telescope is very close to being uniformly sampled; however, there are many gaps with missing data. One must therefore use this general fitting approach to keep the frequencies under exact control. All examples in this paper are based on this general fitting.

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