SPACE-TIME TORSION AND THE ROTATION OF GALAXIES

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Abstract

Torsion effects, including a spin precession in the torsion field, are considered. Some properties of neutrinos in cosmology are discussed. In the framework of Trautman’s cosmological model with torsion estimated is a specific angular momentum of initial perturbations which proved to be of the order of the observable specific rotational moment for spiral galaxies. The results obtained are compared with those from the theories of potential and vortical perturbations in which rotation of galaxies is predicted.

1 Introduction

While considering classical effects and quantum systems in strong gravitational fields, theories alternative to General Relativity can hardly be passed over in silence. On the other hand, the modern structures (stars, galaxies, clusters of galaxies) can only be understood from cosmological outlook, viz. considering the quantum early Universe.

Below we present a model with spin and torsion which can explain the origin of galactic rotation on the basis of quantum torsion effects. The early Universe is a quantum system whose hierarchical structure is related to initial perturbations. The latter, in turn, are formed from quantum fluctuations whose specific angular momenta are $LS$-coupled on a small scale and $jj$-coupled on a large one. The $LS$-coupling is classically interpreted as a precession of $L$ and $S$ about $J$. This is a mechanism of the angular momentum transfer due to the spin precession in the torsion field. Thus as a source of galactic rotation may be massive particles with spin comprising the so-called dark matter. The predicted specific angular momenta proves
to be of the order of the observable ones for spiral galaxies. The results obtained are compared with those from the theories of potential and vortical perturbations in which rotation of galaxies is predicted.

2 Torsion Effects

Immediately after General Relativity had been created, there appeared its generalizations, viz. Einstein-Cartan’s (Cartan, 1922), Kaluza-Klein’s (Kaluza, 1921) and Weyl’s (Weyl, 1918) theories. In particular, in Einstein-Cartan’s theory (ECT) (Trautman, 1979; Ivanenko et al., 1985a; Ivanenko and Sardanashvili, 1985b; Ponomarev et al., 1985; Rodichev, 1974; Sabata, 1994) considered is a nonsymmetric connection (torsion)

\[ Q_{\mu\nu}^\lambda = \frac{1}{2}(\Gamma_{\mu\nu}^\lambda - \Gamma_{\nu\mu}^\lambda). \]  

The torsion contributes to the energy-momentum tensor of the spinning matter being its source, which results in eliminating the singularities of a gravitational field, e.g. in cosmology. Really, the energy-momentum tensor of a spin liquid has the form (Ivanenko et al., 1985a)

\[ T_{\mu\nu}^{\text{eff}} = u_\mu u_\nu (p + \rho - 2s^2) - g_{\mu\nu} (p - s^2), \]

with \( s^2 = s_{\mu\nu} s^{\mu\nu} \), where the spin \( s \) leads to an effective negative pressure and eliminates the singularity.

On the other hand, writing down the Lagrangian of the spinor field in Riemann-Cartan’s space, one can obtain a nonlinear spinor equation coinciding with Ivanenko-Heisenberg’s (Ivanenko and Sardanashvili, 1985b)

\[ \gamma^\mu D^\mu \psi + \frac{3}{8\varepsilon} (\psi \gamma_\mu \gamma_5 \psi) \gamma^\mu \gamma_5 \psi = 0 \]

where \( \varepsilon \) is a constant of interaction with the torsion field.

A relation of the torsion field to a nonlinearity has first been revealed by V.I. Rodichev (1961) for the case of the absence of a gravitational field.

Finally, the torsion \( Q \) leads to a spin precession (Yefremov, 1980)

\[ \frac{ds}{dt} = c[Qs] \]
where $s$ is the spin vector ($|s| = s$), $Q$ is the "polarized" torsion vector.

\[ Q = c\kappa S, \]

where $\kappa = \frac{8\pi G}{c^4}$, $G$ is the gravitational constant, $c$ is the velocity of light, $S$ is the matter spin density creating the torsion $Q$.

It should be noted that Einstein-Cartan’s theory is now a universally recognized generalization of General Relativity to the case of taking account of the spin of matter in the early Universe (Ivanenko et al., 1985a).

### 3 Comments on Massive Neutrinos in Cosmology

The modern upper limits to the neutrino mass (Boehm and Vogel, 1987) do not contradict a closed model of the Universe wherein it is neutrinos that determine the space-time structure on a cosmological scale, because the neutrino background by 1-2 orders exceeds the average density of the matter being observed in galaxies (Dolgov et al., 1988). Since the spinor fields describing the neutrinos are a natural source of torsion, it is not unreasonable to consider cosmology in the framework of ECT. For the massless neutrino there exists a difficulty consisting in Weyl’s equation having only a trivial solution for spherically symmetric configurations both in the framework of GR (Audretsch, 1972) and ECT (Kuchowicz, 1974). Another paradox of the massless neutrino is an appearance of "ghosts", i.e. such solutions for which the energy-momentum tensor is identically equal to zero, whereas the field and the current are nonzero (Edmonds, 1976). The paradoxes of the massless neutrino suggest an idea of these difficulties being related to the assumption that the neutrino is massless. Whence it follows that the consideration of the massive neutrino is, at least, not unreasonable.

### 4 Cosmological Model with Torsion

The cosmology with torsion has been investigated by A. Trautman (1973) who arrived at the conclusion that the torsion eliminates the singularity and stops the collapse (in the case of a closed model) at the minimum radius...
\( R \sim 1 \text{ cm} \), with the matter density \( \rho \sim 10^{55} \text{ gcm}^{-3} \). These values are obtained assuming that the source of torsion is \( 10^{80} \) nucleons with polarized spins. In the framework of ECT the following formulae were used for Friedmann's universe:

\[
R_{\text{min}} = \left( \frac{3G\hbar^2 N}{8mc^4} \right)^{\frac{1}{4}},
\]

\[
\rho_{\text{max}} = \frac{4m^2c^4}{3\pi^2G\hbar^2},
\]

where \( N \) is the number of nucleons, \( m \) is their rest mass. Formulae (6)-(7) also remain valid for a chaotic spin distribution with \( <S> = 0 \) and \( <S^2> \neq 0 \) (Ponomarev et al., 1985).

If we assume the neutrinos with the rest energy \( m_\nu c^2 \sim 35 \text{ eV} \) (Lyubimov, 1980) to be a source of torsion, then we shall obtain the values of parameters as follows:

\( R_{\text{min}} \sim 2 \cdot 10^5 \text{ cm}, \rho_{\text{max}} \sim 4 \cdot 10^{39} \text{ gcm}^{-3} \). It is easy to see that the separation of neutrinos \( l_{\text{min}} = (\rho_{\text{max}}/m_\nu)^{-1/3} \sim 10^{-24} \text{ cm} \) which much less than the Compton wavelength \( \lambda = \frac{\hbar}{m_\nu c} \sim 10^{-6} \text{ cm} \). This means that the problem should be considered at least in terms of quantum theory.

5 Estimation of the Specific Angular Momentum

From quantum mechanics it is known that for light atoms there occurs the Russell-Saunders or \( LS \)-coupling (Landau and Lifshitz, 1963) when

\[
L = \sum_i l_i, \quad S = \sum_i s_i, \quad J = L + S.
\]

This is called a vector model of the atom to which in terms of classical mechanics corresponds a precession of the vectors \( L \) and \( S \) about the total angular momentum vector \( J \) (Blokhintsev, 1976).

On the other hand, from ECT it is known that the spin of a test body precesses in the torsion field (see Sec. 1) with the frequency

\[
\Omega_{pr} = cQ.
\]

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From (5) for $S = \frac{\hbar}{2} n$ we obtain $Q = \frac{\hbar}{2} c \kappa n$, where $n$ is the density of the number of particles with the spin $\frac{\hbar}{2}$. For the precession frequency we obtain

$$\Omega_{pr} = \frac{4 \pi \hbar G n}{c^2}.$$  \hspace{1cm} (9)

Notice that the quantity of the spin of a test body does not enter in formula (9).

From classical mechanics it is known (Landau and Lifshitz, 1978) that the precession of a symmetric top occurs about the direction of its total angular momentum with the frequency

$$\Omega_{pr} = \frac{J}{I}.$$  \hspace{1cm} (10)

where $I$ is the moment of inertia. Using the relation

$$I \simeq MR^2,$$  \hspace{1cm} (11)

we obtain for the specific angular momentum of the top

$$\frac{J}{M} \simeq \Omega_{pr} r^2.$$  \hspace{1cm} (12)

where $M$ is the top mass, $R$ is its effective radius.

Hence, if we assume that the initial perturbations corresponding to protogalaxies with the spin moment $S$ and the orbital moment $L$ could be added (in terms of quantum mechanics), i.e. could precess (in terms of classical mechanics) about the total angular momentum $J$ as well as the uncompensated spin $s$ of a test body precesses about the ”polarized” torsion $Q$ being created by the particles having a spin, then to estimate their specific angular momentum, we shall be able to use formula (12), where the torsion $Q$ plays the role of the total angular momentum $J$, i.e. in ECT by analogy with the $LS$-coupling we have $Q = \frac{L+S}{c^2}$. This means that the spins of small perturbations are added into the total torsion vectors of protogalaxies. The specific angular momentum in formula (12) is related to the total torsion vector that in a selfconsistent system comprises uncompensated spin angular momenta of the initial perturbations being their source at the same time. Thus from (8) and (10) we have

$$J = cIQ$$  \hspace{1cm} (13)
where

\[ I = \rho \int_0^a r^2 dV, \quad (14) \]

\[ V = 2\pi^2 r^3 \] for a closed model.

Hence

\[ I = \frac{3}{5}a^2 M \quad (15) \]

where \( M = 2\pi^2 \rho a^3 \).

For the specific angular momentum we obtain the formula

\[ \frac{J}{M} = \frac{12\pi h G}{5c^2} na^2 = \frac{3}{5} \Omega \rho a^2 \quad (16) \]

similar to (12).

The neutrino background, on the one hand, is a source of the gravitational field, and on the other hand, is a quantum system, at least for the early Universe. Hence a correct description of its behaviour is possible only in the framework of quantum theory. To estimate the specific angular momentum, we shall consider the spin precession of initial perturbations to occur in the neighbourhood of the minimum radius of the Universe. In formula (16) we assume that \( n = n_{max}, \ a = R_{min} \). From (6), (7) we have

\[ n_{max} = \frac{4mc^4}{3\pi^2 Gh}, \quad R_{min} = \left( \frac{3Gh^2 N}{8mc^4} \right)^{\frac{1}{2}} \]

where \( n_{max} = \frac{\rho_{max}}{m} \).

Hence

\[ \frac{J}{M} = \frac{16mc^2}{5\pi h} \left( \frac{3Gh^2 N}{8mc^4} \right)^{\frac{3}{4}}. \quad (17) \]

Using the formulae (Zel’dovich and Novikov, 1975)

\[ N = nV, \quad V = 2\pi^2 a_0^3, \quad a_0 = \frac{c}{H_0 \sqrt{\Omega - 1}}, \quad \Omega = \frac{8\pi Gmn}{3H_0^2}, \]

where \( a_0 \) is the scale factor, \( H_0 \) is the Hubble constant and \( \Omega \) is the average density in units of the critical one (the index "0" corresponds to the present epoch), we can obtain

\[ N = \frac{3\pi c^3}{4H_0 Gm (\Omega - 1)^3} \quad (18) \]
and finally express $\frac{J}{M}$ in terms of $\Omega$ as follows:

$$\frac{J}{M} = \frac{6}{5} \sqrt[3]{\frac{12}{\pi}} \left( \frac{c\Omega}{H_0\lambda} \right)^{\frac{2}{3}} \frac{S}{M}$$

(19)

where the Compton wavelength of spin particles $\lambda = \frac{\hbar}{mc}$, the specific spin moment $\frac{S}{M} = \frac{\hbar}{2m}$.

From this formula it follows that $J \gg S$ for $\Omega - 1 \ll 1$ since $\frac{c}{H_0\lambda} \gg 1$. Note that $\frac{J}{S}$ does not depend on $G$ which enters only via $\Omega$ being of the order of unity. Hence, if $J = L + S$, then $J \approx L$. This means an angular momentum transfer due to the spin precession in the torsion field. For $\Omega - 1 \ll 1$ the required $\frac{J}{M}$ is always achievable by tuning the mass of the particle being a source of the spin. For example, for $\Omega - 1 = 10^{-2}$, $m_{\nu}c^2 = 13$ eV, $n_{\nu} = 450$ cm$^{-3}$ (a massive neutrino), $H_0 = 75$ km $\cdot$ s$^{-1}$Mps$^{-1}$ we obtain $\frac{J}{M} = 2 \cdot 10^{29}$ cm$^2$s$^{-1}$ which is close to the corresponding value for spiral galaxies (for our Galaxy $K = 5 \cdot 10^{29}$ cm$^2$s$^{-1}$ (Ozernoy, 1978) since the specific angular momenta of protogalaxies are conserved and equal to those of galaxies observable at present. Angular momenta of protogalaxies are $jj$-coupled into the total momentum of a closed Universe equal to zero.

The observed anisotropy of the microwave background radiation sets an upper bound on the density and velocity perturbations of clusters but not galaxies. For the clusters of galaxies the averaged angular momenta are close to zero and do not contribute to the observed $\Delta T/T \sim 10^{-5}$, i.e. only due to density perturbations.

6 Conclusion

We have shown that the problem of the origin of galactic rotation is solvable in the framework of a cosmology taking account of spin and torsion. It has partly been solved in the theories of potential and vortical perturbations (Ozernoy, 1978; Gurevich and Chernin, 1978; Vorontsov-Velyaminov, 1972). In the latter case considered are chaotic supersonic turbulent motions of the matter density and velocity perturbations having spin and orbital rotations about each other. We see that this picture qualitatively resembles that considered above in the framework of ECT. In this connexion we notice paper (Soares, 1981) where generation of a macroscopic asymmetry of neutrinos...
due to the macroscopic vortical field of matter is considered, i.e. this is a process in some sense inverse to ours.

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