Neutrinos from supernova (SN) bursts can give rise to detectable number of nuclear recoil (NR) events through the process of coherent elastic neutrino-nucleus scattering (CEνNS) in future large (multi-ton scale) liquid xenon detectors employed for dark matter search depending on the SN progenitor mass and distance to the SN event. Here we point out that in addition to the direct NR events due to CEνNS process, there is a secondary source of nuclear recoils due to elastic scattering of the neutrinos produced through inelastic neutrino-nucleus scattering of the supernova neutrinos with the target xenon nuclei. We estimate the contribution of these supernova neutrino-induced neutrons (νIn) to the total xenon NR spectrum and find that the latter can be significantly modified at large recoil energies from that expected from the CEνNS process alone, with the νIn contribution dominating the total integral recoil energy spectrum for recoil energies $\gtrsim 20$ keV. With the capability to measure the energies of individual recoil events, sufficiently large liquid xenon detectors may be able to detect these events due to νIn process triggered by neutrinos from reasonably close by SN burst events. We also note that the νIn contribution to the recoil spectrum receives dominant contribution from the charged current interaction of the SN $\nu_e$s with the target nuclei while the CEνNS contribution comes from neutral current interactions of all the six species of neutrinos with the target nuclei. This may offer the possibility of extracting useful information about the distribution of the total SN explosion energy going into different neutrino flavors.

I. INTRODUCTION

Core collapse supernova (CCSN) explosions \cite{1} give out huge flux of neutrinos (and antineutrinos) of all flavors with energies up to a few tens of MeV over a time scale of $\sim 10$ seconds \cite{2}. These neutrinos carry almost all ($\sim 99\%$) of the gravitational energy ($\sim 10^{53}$ erg) released due to collapse of the core of the massive progenitor star. A large number of experimental facilities around the world, employing a variety of neutrino detection techniques \cite{3} \cite{4}, are waiting to detect the neutrinos from the next nearby (hopefully Galactic) CCSN, after the historic first detection of neutrinos from the supernova SN1987A located in the Large Magellanic Cloud (LMC) at a distance of $\sim 50$ kpc from Earth \cite{5}. Detection of supernova neutrinos of different flavors from a single supernova by multiple detectors has the potential to yield extremely valuable information about not only the supernova process itself but also various aspects of fundamental physics of neutrinos themselves.

Neutrinos are detected through their weak charged current (CC) and neutral current (NC) interactions with electrons and nuclei. A variety of interaction channels are possible; see, e.g., Ref. \cite{1} for a review. In this paper we consider the possibility of SN neutrino detection using elastic as well as inelastic neutrino-nucleus interaction (see, e.g., Ref. \cite{2} for reviews). In particular, we focus on the processes of coherent elastic neutrino nucleus scattering (CEνNS) \cite{7} \cite{8} and inelastic neutrino-nucleus scattering, the latter involving emission of neutrons in the final state.

The CEνNS is a process that has recently received much attention \cite{9} \cite{10} \cite{11} \cite{12} in the context of large (multi-ton scale) detectors searching for the weakly interacting massive particle (WIMP) candidates of dark matter (DM) \cite{13} through WIMP-induced nuclear recoils. In this process, neutrinos of sufficiently low (few to few tens of MeV) energy undergo coherent elastic scattering on the target nucleus with a cross section that is enhanced by the square of the number of neutrons in the target nucleus. The recoiling target nucleus of mass $M$ gets a maximum kinetic energy of $2E_\nu \gamma^2/M$, where $E_\nu$ is the neutrino energy. There is a trade-off between the cross section enhancement (requiring large mass nuclei) and the maximum recoil energy ($\propto 1/M$), with the latter typically in the region of few keV for neutrino energy of $\sim 10$ MeV and target nuclei with mass number in the region of $\sim 100$ desired for reasonably large cross section (typically, $O(10^{-39}$ cm$^2$)). Such low recoil energies, while very difficult to detect in conventional neutrino detectors, are, however, within reach of the large WIMP DM detectors. Thus, sufficiently large WIMP DM detectors with suitably chosen detector materials can also be sensitive to neutrinos from individual SN events. Importantly, the CEνNS, it being a NC process, is equally sensitive to all flavors of neutrinos (and antineutrinos), which offers a probe for estimating the total explosion energy going into neutrinos \cite{9} \cite{13}, and also possibly for demarcating...
the neutrinos originating in the different temporal phases of the neutrino emission during the SN event \[11\]. Because of these reasons, detailed calculations have been done studying the CEνNS sensitivity of the next generation WIMP DM detectors, in particular, those using liquid xenon as target detector material, to possible future nearby (Galactic) SN events; see, for example, Ref. \[12\].

The main purpose of the present paper is to point out that, in addition to the SN neutrino induced direct recoils of the target detector nuclei due to the CEνNS process, there is a secondary source of nuclear recoils due to the neutrino-induced neutrons (νIn) produced by the neutrinos from the same SN event through inelastic neutrino-nucleus interaction in the same target detector material. In the νIn process \[16\]–\[20\], inelastic CC interaction of the electron neutrinos (νe,τ) and electron antineutrinos (νe,τ) as well as inelastic NC interaction of neutrinos and antineutrinos of all flavors with the target nucleus leave the post-interaction target nucleus in an excited state, with its subsequent deexcitation through emission of one or more neutrons. This is possible if the original neutrino energy is sufficient to excite the nucleus beyond its single or multiple neutron emission threshold. The elastic scattering of these neutrons with the detector nuclei would then give rise to additional nuclear recoils which, as we show in this paper, can effectively make the final observable recoil spectrum significantly different from that expected from the CEνNS process alone. Specifically, we show that the total differential recoil energy spectrum develops a shoulder and an extended tail at the large recoil energy end due to contribution of neutrons from the νIn process.

Motivated by the upcoming multi-ton scale liquid xenon based DM detectors \[13\], below we discuss the CEνNS and νIn processes for xenon (specifically, the isotope \(^{132}\)Xe, as an example) as the target detector material for illustration of the effect. Although, for low (few keV) recoil energy thresholds the total number of recoil events will be dominated by those due to the direct CEνNS process, the additional contribution to the differential recoil spectrum due to neutrons from the νIn process may be detectable by the next generation liquid xenon detectors depending on the detector size and distance to the SN event. The differential recoil energy spectrum may contain important information about the SN neutrinos. In particular, as discussed in Ref. \[12\], the measurement of the differential recoil spectrum may allow reconstruction of the time-integrated differential energy spectrum and average energy of the neutrinos from the SN event. Thus, the contribution of the νIn process to the differential recoil spectrum would provide an additional handle for determination of the parameters of the neutrino flux from the SN event.

The xenon recoil spectrum due to SN neutrino induced neutrons in a given liquid xenon detector will depend on the physical dimensions of the detector volume. A proper calculation of the recoil spectrum requires a simulation of in general multiple scattering of neutrons on xenon nuclei within the context of a given liquid xenon detector configuration. In this paper we calculate the neutron-induced xenon recoil spectrum for a generic liquid xenon detector with an active volume of 1 ton of liquid xenon using the GEANT4 \[21\] simulation toolkit. We find that the xenon recoil spectrum due to neutrons from the νIn process is significantly flatter than that due to direct CEνNS process, and, therefore, although at relatively low recoil energies the CEνNS contribution dominates, the νIn contribution starts dominating the total differential (integral) recoil spectrum at recoil energies \(\gtrsim 25 \, (20)\) keV.

While the νe,τ and ν̄e,τ from the SN contribute to the νIn process through both CC and NC interactions with the nucleus, the νe,τ, ν̄e,τ, νμ,τ and ν̄μ,τ can contribute only through NC interaction because of insufficient incoming neutrino energies for production of the associated charged leptons. However, the contribution from νi CC interaction with \(^{132}\)Xe, which involves conversion of protons to neutrons inside the nucleus, is strongly suppressed due to Pauli blocking of the neutron single particle states in the neutron rich final state nucleus \[6\]. Similarly, as seen in the case of \(^{208}\)Pb \[19\], the contribution from the NC interaction of neutrinos and antineutrinos of all flavors with the target nucleus—in which the incoming neutrino or antineutrino only imparts energy to the neutrons and protons inside the nucleus without inducing interconversion between the nucleons—is also expected to be subdominant to the contribution from CC interactions of νe,τ for a target nucleus like \(^{132}\)Xe with a moderately large neutron excess \((N - Z = 24)\), again partly due to Pauli blocking effects. In the present paper, therefore, as a first approximation, we only consider the neutrons resulting from deexcitation of the final state nucleus \(^{132}\)Cs produced in the CC interaction of the νe,τ with \(^{132}\)Xe, i.e., the process \(^{132}\)Xe(νe,τ)e\(^{−}\)\(^{132}\)Cs, for which the relevant Gamow-Teller (GT) and Fermi transition strength distributions (see section \[11\] below) are available in literature. The results presented in this paper, therefore, represent a lower limit to the total contribution of the νIn process to the xenon recoil spectrum.

For the SN neutrino flux to be used for numerical calculations in this paper, we use the results of the Basel-Darmstadt (BD) simulations \[22\] of a 18\(M_\odot\) progenitor star placed at a distance of 1 kpc from earth for illustration of our results. The instantaneous energy spectrum, i.e., the number of SN neutrinos of type \(\nu_i\) (with \(\nu_i \equiv \nu_e, \nu_\tau, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\)) emitted (at the neutrinosphere) per unit time per unit energy is usually parametrized in terms of the time-dependent neutrino luminosities \((L_{\nu_i}(t))\), average energies \((\langle E_{\nu_i}\rangle(t))\) and the spectral shape parameter \(\alpha_{\nu_i}(t)\) of the different neutrino flavors \[23\]. The temporal profiles of these parameters for different neutrino flavors extracted from the BD simulation results for the 18\(M_\odot\) progenitor SN used in this paper are given in graphical form in Refs. \[11\]–\[19\]–\[24\]. Note that the \(\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\) have essentially identical spectra and temporal profiles of their luminosities and average energies in the BD simulations.
For the CEνNS process, since it is equally sensitive to all the neutrino species, the flavor oscillation effects are not relevant, and the time-integrated (over the duration of the SN burst event) sum of fluxes of all the neutrino species reaching the earth is same as the sum of the number of neutrinos of all species emitted (at the neutrinospheres) per unit time and energy, divided by 4πd^2, d being the distance to the SN. On the other hand, the flux of ν_e, which determines the contribution to the ν_in process from the CC interaction of the ν_e's, would in general depend on various flavor oscillation processes, including the standard Mikheyev-Smirnov-Wolfenstein (MSW) matter-enhanced neutrino flavor oscillation (see, e.g., [24]) as well as the phenomenon of collective neutrino flavor oscillations due to ν-ν interaction in the deep interior of the SN progenitor star; see, e.g., Refs. [2 26 27] for reviews and references. However, as discussed in Refs. [24 28], for the time-integrated fluxes of different neutrino species, the collective oscillation effects are small, and the fluxes of various neutrino species reaching the earth are given essentially by the MSW oscillations in the supernova matter. The resulting expressions for the ν_e and ν_μ fluxes at earth, for normal ordering (NO) and inverted ordering (IO) of the neutrino mass hierarchy are given in Refs. [19 24], which we use in this paper. Note further that although the final 132Xe recoil spectrum due to the CEνNS and ν_in processes will individually be different for different distances and different progenitor masses of the SN, the relative amplitudes (i.e., the ratio) of the two contributions, the object of our main interest in this paper, will be independent of the distance to the SN and should also be roughly similar for different progenitor masses.

The rest of the paper is arranged as follows: In section [I] we set up the equations giving the differential recoil spectrum due to CEνNS interaction of the SN neutrinos with the target nucleus. Section [II] discusses the ν_in process, where we calculate the neutron spectrum resulting from the CC interaction of the SN ν_e's with 132Xe. The contribution of the neutrons from the ν_in process to the 132Xe recoil spectrum is then calculated and its implications discussed in section [IV] together with the recoil spectra due to the neutrons coming from the ν_in process discussed in the next section.

II. SUPERNOVA NEUTRINO INDUCED XENON RECOIL SPECTRUM DUE TO CEνNS

The differential cross section, \( \frac{d\sigma}{dE_R} \), for the CEνNS interaction of a neutrino of energy \( E_\nu \) with a target nucleus of mass \( M \) which is left with a recoil (kinetic) energy \( E_R \) is given by \[ \frac{d\sigma}{dE_R} = G_F^2 \frac{M}{4\pi} Q_W^2 \left( 1 - \frac{M E_R}{2E_\nu} \right) F^2(E_R), \] where \( G_F \) is the Fermi constant, \( Q_W = N - (1 - 4\sin^2 \theta_W) Z \) is the weak nuclear hypercharge for the nucleus with \( N \) neutrons and \( Z \) protons, \( \theta_W \) is the weak mixing angle (with \( \sin^2 \theta_W \approx 0.2386 \)), and \( F(E_R) \) is the nuclear form factor which we shall take to be of the Helm form \[ \frac{F(E_R)}{q r_n} \] where \( j_1 \) is the spherical Bessel function, \( q = (2M E_R)^{1/2} \) is the momentum transfer to the nucleus, \( s \approx 0.9 \text{ fm} \) is the nuclear skin thickness, \( r_n^2 = c^2 + \frac{5}{3} \pi^2 a^2 - 5s^2 \) is the square of the effective nuclear radius with \( c \approx 1.23 A^{1/3} - 0.60 \text{ fm} \), \( a \approx 0.52 \text{ fm} \), and \( A = N + Z \).

The differential recoil spectrum, \( \frac{dN_\nu}{dE_R} \), per ton (1000 kg) of target detector material (liquid 132Xe, in our case) for the time-integrated (over the duration of the SN event) neutrino flux is then given by

\[ \frac{dN_\nu}{dE_R} = N_{\text{Xe}} \sum_{\nu_i} \int_{E_\nu}^{E_{\nu_\text{min}}} dE_\nu \frac{d\sigma}{dE_R} \Phi_{\nu_i}(E_\nu), \]

where the \( \nu_i \) sum is over all six species of neutrinos, \( E_{\nu_\text{min}} = (M E_R/2)^{1/2} \) is the minimum energy of a neutrino that can produce a recoiling 132Xe nucleus of recoil energy \( E_R, \) \( M \) being the mass of a 132Xe nucleus, \( N_{\text{Xe}} = 4.56 \times 10^{27} \) is the number of 132Xe nuclei in one ton of liquid xenon, and \( \Phi_{\nu_i}(E_\nu) \) is the time-integrated flux of SN \( \nu_i \)s per unit energy and per unit area at earth.

To facilitate easy comparison, the resulting 132Xe recoil spectrum due to CEνNS process for the BD 18M⊙ SN is displayed and discussed in section [IV] together with the recoil spectra due to the neutrons coming from the ν_in process discussed in the next section.

III. SUPERNOVA NEUTRINO INDUCED NEUTRONS IN XENON DETECTORS

As mentioned in section [I] in addition to direct nuclear recoils through the CEνNS process, there can be additional nuclear recoils due to elastic scattering of neutrons with the nuclei, the neutrons being produced through the ν_in process, i.e., inelastic neutrino-nucleus scattering of the same SN neutrinos on the target nuclei. In this section we first briefly review the basic physics of neutron emission from nuclei excited beyond their neutron emission thresholds due to inelastic neutrino-nucleus scattering, and then calculate the resulting neutron spectrum due to CC interaction of the SN ν_e's in 132Xe.

The inelastic CC interaction of a supernova ν_e with a 132Xe nucleus can produce a 132Cs nucleus in an excited state:

\[ \nu_e + ^{132}\text{Xe} \rightarrow e^- + ^{132}\text{Cs}^*, \]

where the superscript * stands for the excited state of the final state nucleus.

The differential cross section for this reaction in the \( q \to 0 \) limit (applicable for the energy range of SN neutrinos, \( q \) being the momentum transfer) can be written
as \[31, 33\]

\[
\frac{d\sigma_{\text{CC}}}{dE_n}(E_\nu, E_\gamma) = \frac{G_F^2 \cos^2 \theta_C}{\pi} p_e E_e F(Z + 1, E_\nu) \times \left[ S_F(E_\nu) + (g_A^{\text{eff}})^2 S_{\text{GT}^-}(E_\gamma) \right],
\]

(5)

where \(G_F\) is the Fermi constant, \(\theta_C\) is the Cabibbo angle, \(E_\nu = E_\nu - E_e\) is the excitation energy of the final nucleus with respect to the ground state of the parent \(^{132}\)Xe nucleus, \(E_o\) is the incoming neutrino energy, and \(p_e\) and \(E_e\) are the energy and momentum of the emitted electron, respectively. The quantities \(S_F(E_\nu)\) and \(S_{\text{GT}^-}(E_\gamma)\) are, respectively, the modulus squares of the averaged Fermi and Gamow-Teller (GT\textsuperscript{−}) transition matrix elements between the ground state of the initial nucleus \(^{132}\)Xe and the excited state of the final nucleus \(^{132}\)Cs, and \(g_A^{\text{eff}} \approx 1.26\) is the ratio of the effective axial vector to vector coupling constants of the bare nucleon in the \(q \to 0\) limit. The general expressions for \(S_F(E_\nu)\) and \(S_{\text{GT}^-}(E_\gamma)\) are given in Refs. \[31, 33\] (see also \[19\]).

The factor \(F(Z + 1, E_\nu)\) in equation (5) is the correction factor which accounts for the distortion of the outgoing electron wave function due to Coulomb interaction with the final nucleus \((N - 1, Z)\) (with the initial nucleus represented as \((N, Z)\)), and is given by \[34\]

\[
F(Z, E_\nu) = 2(1 + \gamma_0)(2p_e R)^2(\gamma_0 - 1) \times \left[ \frac{\Gamma(\gamma_0 + iy)}{\Gamma(2\gamma_0 + 1)} \right]^2 \exp(\pi y),
\]

(6)

where \(\gamma_0 = (1 - Z^2 \alpha^2)^{1/2}, y = \alpha Z E_e/p_e, R\) is the radius of final nucleus and \(\alpha\) the fine structure constant.

The Fermi transitions have the isospin selection rule \(\Delta T = T' - T = 0\) where \(T'\) and \(T\) are, respectively, the isospins of the final and initial nucleus. Also, the Fermi strength sum over the final nucleus states is equal to \((N - Z)\), and it goes almost completely to the Isobaric Analog state (IAS) of the final nucleus with a very small spread in the neighboring states due to isospin breaking essentially by Coulomb interaction. The IAS in \(^{132}\)Cs is at an excitation of 13.8 MeV \[25\]. On the other hand, the Gamow-Teller strength distribution is broad and its strength sum, which is slightly larger than \(3(N - Z)\), is distributed over states with \(T' = T - 1, T\) and \(T + 1\) i.e. with \(\Delta T = 1\). For the GT strength distribution \(S_{\text{GT}^-}\) for the final nucleus \(^{132}\)Cs we use the results of a theoretical calculation \[36\] done within the deformed Hartree-Fock formalism using the density dependent Skyrme interaction SkI3 \[37\].

The reaction \(^{132}\)Xe(\(\nu_e, e^-\))\(^{132}\)I initiated by the supernova \(\nu_e\)s also takes place, but the GT strength, \(S_{\text{GT}^-}\), for the reaction is only \(\approx 0.51\) \[38\], which is only a few percent of the total GT\textsuperscript{−} strength for the reaction \(^{132}\)Xe(\(\nu_e, e^-\)) discussed above. In this paper we neglect the \(\nu_e\) CC contribution compared to the contribution from \(\nu_e\) capture reaction.

The excited final nucleus \(^{132}\)Cs can decay by emitting one or more neutrons:

\[
^{132}\text{Cs} \rightarrow ^{131}\text{Cs} + n \quad \text{or} \quad ^{132}\text{Cs} \rightarrow ^{130}\text{Cs} + 2n,
\]

(7)

and so on. This emission of neutrons after the excitation of the nucleus by the weak process of neutrino capture can be considered as a two-step process involving two independent physical processes \[17, 18\]: the production of a final state excited nucleus due to absorption of the incoming neutrino by the target nucleus in the first step, and subsequent de-excitation of the final state nucleus through neutron emission in the second step.

The differential cross section for the first step, i.e., production of the excited \(^{132}\)Cs nucleus due to \(\nu_e\) capture on a target \(^{132}\)Xe nucleus is given by equation (5). With this, the energy spectrum of the emitted neutrons is calculated in the following way: The differential cross section for emission of neutrons per unit neutron (kinetic) energy \(E_n\) by a single final state nucleus due to inelastic CC interaction of an incoming \(\nu_e\) of energy \(E_\nu\) with the target nucleus can be written as

\[
\frac{d\sigma_{\text{CC}}}{dE_n}(E_\nu, E_n) = \int \frac{d\sigma_{\text{CC}}}{dE_e}(E_\nu, E_e) \frac{dN_n}{dE_n}(E_\nu, E_e) dE_e,
\]

(8)

where \(\frac{dN_n}{dE_n}(E_\nu, E_n)\) is the energy spectrum of the neutrons produced by the excited nucleus of excitation energy \(E_n\).

The total energy spectrum of the neutrons produced by the incident flux of SN neutrinos is then given by

\[
\frac{dN_{\text{n, total}}}{dE_n} = N_T \int dE_\nu \Phi_{\nu_e}(E_\nu) \frac{d\sigma_{\text{CC}}}{dE_n}(E_\nu, E_n),
\]

(9)

where \(\Phi_{\nu_e}(E_\nu)\) is the time-integrated flux spectrum (number per unit area per unit energy) of the SN \(\nu_e\)s falling on the detector, and \(N_T\) is the total number of target detector nuclei.

We calculate the neutron energy spectrum, \(\frac{dN_{\text{n, total}}}{dE_n}(E_\nu, E_n)\), for \(^{132}\)Cs\textsuperscript{−} using the fusion-evaporation Monte Carlo code PACE4 \[35\] originally developed by Gavron \[39\], and include contributions from both 1- and 2-neutron emissions. Contribution from 3-neutron emission is negligibly small compared to those of 1- and 2-neutron emissions for the relevant range of supernova neutrino energies. The resulting neutron energy spectra for the BD 18M\textsubscript{Sun} SN \(\nu_e\) flux for the two cases of NO and IO neutrino mass hierarchies are shown in Fig. \[1\].

The total number of neutrons produced is \(\sim 11.1\) and 8.3 for the NO and IO cases, respectively.

IV. XENON RECOIL SPECTRUM DUE TO SUPERNOVA NEUTRINO INDUCED NEUTRONS

The neutrons produced by inelastic CC interaction of the supernova \(\nu_e\)s with target \(^{132}\)Xe nuclei will scatter off the xenon nuclei themselves giving rise to recoiling...
\( ^{132}\text{Xe} (\nu_\nu, e^-) ^{132}\text{Cs}^+ \)

\( ^{132}\text{Cs}^- \rightarrow ^{131}\text{Cs} + n \)

\( ^{132}\text{Cs}^- \rightarrow ^{130}\text{Cs} + 2n \)

\( 18M_\odot \text{SN} \)

distance = 1 kpc

FIG. 1: Energy spectrum of neutrons emitted by excited \( ^{132}\text{Cs} \) nuclei produced due to inelastic CC interaction of supernova \( \nu_\nu \) with \( ^{132}\text{Xe} \) nuclei, for the two cases of Normal Ordering (NO) and Inverted Ordering (IO) of neutrino mass hierarchy, for SN neutrino flux given by the Basel-Darmstadt simulations [22] of a 18\( M_\odot \) progenitor supernova at a distance of 1 kpc.

\( ^{132}\text{Xe} \) nuclei. From Fig. 1 we see that the neutrons produced by the supernova neutrinos typically have energies of a few MeV with a spectrum peaking at \( \sim 2 \) MeV. At these neutron energies, elastic scattering dominates [30]. The mean-free-path (m.f.p) of neutrons of energies in the range of few 100 keV to few MeV in liquid xenon is \( O(10) \) cm. So, depending on the dimensions of the active volume of liquid xenon and the point of production of the neutron within the detector, a neutron of a given initial energy may undergo multiple scattering within the detector giving rise to multiple xenon recoils with different recoil energies. To take this into account we perform a simulation of neutron scattering on xenon nuclei using the GEANT4 simulation toolkit [21] as described below.

A. GEANT4 simulation of neutron scattering on xenon nuclei

Guided by the typical designs of the liquid xenon detectors being used in dark matter search experiments [13], we consider 1 ton of liquid \( ^{132}\text{Xe} \) contained in a cylindrical volume of aspect ratio (diameter-to-height) of 1:1. With a density of \( \sim 2.953 \text{ g/cm}^3 \) of liquid xenon, both the diameter and height of the cylinder are taken to be \( \sim 75.4 \) cm. As far as the neutron induced xenon recoils are concerned, we find no significant differences in the results for the recoil spectra whether we consider natural xenon or one of its isotope (here \( ^{132}\text{Xe} \)). Neutrons of any given initial energy are generated homogeneously within the liquid xenon volume with randomly chosen initial positions and with isotropic initial velocity directions.

Although at the neutron energies of our interest the elastic scattering dominates [30], inelastic processes can also occur whereby, for example, the incident neutron is absorbed by a \( ^{132}\text{Xe} \) nucleus and re-emitted together with a gamma ray, resulting in a small recoil of the nucleus. The emitted neutron can then again scatter elastically or inelastically on another xenon nucleus depending on its energy, and so on. Each neutron is tracked until it hits and exits the outer boundary of the simulation volume.

To understand the nature of the final recoil spectrum, we first study the recoil spectra generated by initially monoenergetic neutrons. Figure 2 shows the xenon recoil spectra for four different initial energies of neutrons, namely, 0.01, 0.1, 1 and 5 MeV. It is seen that while at very low energies the recoil spectrum is almost flat, the spectrum becomes increasingly skewed towards low recoil energies as the initial neutron energy increases. This directly reflects the behavior of the angular distribution of the recoiling nucleus in the elastic scattering of neutrons on \( ^{132}\text{Xe} \) nuclei, with relatively more forward (small angle) scattering events taking place than backward events as the neutron energy increases [40].

Next, we show in Figure 3 the probability distribution of the number of \( ^{132}\text{Xe} \) recoils produced by neutrons of different initial energies. It can be seen that for all initial neutron energies, 1–2 scattering events dominate, with the probability of larger number of scattering falling off faster with number of scattering events for increasing initial energy of the neutron. Note, however, that the multiple scattering events (greater than 1–2 scatterings), although less probable than 1–2 scattering events, can still contribute significantly to the total recoil spectrum because each such event produces a larger number of recoils per initial neutron than the 1–2 scattering events.

The final differential recoil spectrum generated by the
SN $\nu_e$ induced neutrons whose initial energies are randomly sampled from the spectrum shown in Fig. 1, is shown in Fig. 4 together with the direct recoil spectrum of $^{132}$Xe due to the CE$\nu$NS process given by eq. 3. For simplicity, for the neutron-induced recoil spectrum, we only show the results for the case of normal ordering (NO) of neutrino mass hierarchy; similar calculations can be done for the IO mass hierarchy case.

For comparison, Fig. 4 also shows the “average” recoil spectra calculated by assuming each neutron to undergo, on the average, one (curve marked “$\nu_e$:1-scatt.”) and three (curve marked “$\nu_e$:3-scatt.”) elastic scatterings on $^{132}$Xe nuclei. The angular distribution of the recoiling $^{132}$Xe nucleus is assumed to be uniform, and the $^{132}$Xe nucleus and the scattered neutron are assigned with their average energies after each scattering event. With these simplifying assumptions, the average recoil spectrum is calculated as follows:

From simple kinematics of non-relativistic elastic scattering, the recoil energy, $E_R$, received by a nucleus of mass $m_A$ due to elastic scattering of a neutron of kinetic energy $E_n$, can be written as

$$E_R = \xi E_n, \quad \text{with} \quad \xi = \frac{4m_n m_A}{(m_n + m_A)^2} \frac{1 - \cos \theta_{\text{CM}}}{2},$$

where $m_n$ is the neutron mass and $\theta_{\text{CM}}$ is the scattering angle in the center-of-momentum (CM) frame. If we assume isotropic scattering in the CM frame, then the average recoil energy $\langle E_R \rangle$ is simply given by

$$\langle E_R \rangle = \langle \xi \rangle E_n, \quad \text{with} \quad \langle \xi \rangle = \frac{2m_n m_A}{(m_n + m_A)^2} = 0.015$$

for $^{132}$Xe. Dropping the angular brackets “$\langle \cdot \rangle$” for average quantities, and considering only the average values of the quantity $\xi$ and the energies of the recoiling nucleus and the scattered neutron, we see that if each neutron of initial energy $E_n$ undergoes on the average only one scattering, then after this one scattering we would have one $^{132}$Xe recoil of energy $E_{R,1} = \xi E_n$ for every such neutron, and the original neutron would be left with the energy $E_{n,1} = (1 - \xi) E_n$. Similarly, if each neutron of initial energy $E_n$ undergoes on the average two scatterings, then after these two scatterings we would have on the average two $^{132}$Xe recoils for every original neutron of energy $E_n$: one recoil of energy $E_{R,1} = \xi E_n$ and the other with energy $E_{R,2} = \xi (1 - \xi) E_n$, with the original neutron being left with the energy $E_{n,2} = (1 - \xi)^2 E_n$, and so on. So, if each neutron of energy $E_n$ undergoes on the average $n_{\text{sc}}$ scatterings, then the final “average” $^{132}$Xe recoil spectrum, $\frac{dN_{\text{Xe}}}{dE_{\text{R}}} \langle E_R \rangle$, can be written as

$$\left( \frac{dN_{\text{Xe}}}{dE_{\text{R}}} \right) \langle E_R \rangle = \sum_{k=1}^{n_{\text{sc}}} \frac{dN_n}{dE_n} \left( E_n = \frac{E_R}{\xi (1 - \xi)^{(k-1)}} \right) \times \frac{1}{\xi (1 - \xi)^{(k-1)}},$$

where the last factor on the right hand side comes from the derivative $\frac{dE_n}{dE_R}$. 

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**FIG. 3:** Distribution of the number of $^{132}$Xe recoils produced (i.e., the number of scattering events suffered) by a neutron of different initial energy.

**FIG. 4:** $^{132}$Xe differential recoil spectrum due to supernova neutrino induced CE$\nu$NS and the spectrum due to neutrons produced by inelastic CC interaction of supernova $\nu_e$s with $^{132}$Xe nuclei (solid curve marked “$\nu_e$:GEANT4”). For comparison, the neutron-induced recoil spectra calculated by assuming one ($\nu_e$:1-scatt.) and three ($\nu_e$:3-scatt.) average number of elastic scattering for every neutron are also shown. In calculating these “average” recoil spectra, the angular distributions of the recoiling $^{132}$Xe nucleus and the scattered neutron are assumed to be uniform, with the $^{132}$Xe nucleus and the scattered neutron assigned their average energies after each scattering event (see text). The $\nu_e$ induced neutron energy spectrum used in this figure corresponds to the case of normal ordering (NO) of neutrino mass hierarchy shown in Fig. 1. All the curves correspond to SN neutrino flux given by the Basel-Darmstadt simulations [22] of a $18 M_\odot$ supernova progenitor at a distance of 1 kpc.
In reality, of course, the scattering of the neutrons is a random process—some neutrons may directly escape the detector without interacting, some may undergo single scattering, some others double scattering, and so on, depending on the detector geometry and the production point and propagation direction of the neutron within the detector. The resulting $^{132}$Xe recoil spectrum, calculated with the help of the GEANT4 simulation toolkit that allows one to automatically include the effects of multiple scattering and angular distributions of the scattered neutron and recoiling nucleus in each scattering event, is represented by the solid curve marked “$\nu_e In$:GEANT4” in Fig. 4. However, it is interesting to note that the average recoil spectrum with $n_{sc} = 1$ roughly reproduces the GEANT4 simulation results at large recoil energies beyond $\sim 30$ keV, although it grossly underestimates the actual recoil spectrum at lower recoil energies. This is in broad agreement with the fact that most neutrons on the average suffer only 1–2 scatterings (see Fig. 3). The failure of the average spectrum to reproduce the simulation results at lower recoil energies can be understood from the fact that, for relatively higher energy neutrons (which are responsible for producing the higher energy recoils), the elastic scattering is dominated by forward scattering events, i.e., by small angle and hence low energy recoils (see Fig. 2), which the average spectrum by construction misses because of the underlying simplifying assumption of uniform angular distribution of the scattering process. In addition the effect of multiple scattering is missing in the average spectrum. Nevertheless, as seen from Fig. 4, the simple treatment embodied in the average spectrum does allow one to make a rough analytical estimate of the neutron-induced recoil spectrum, albeit only at large recoil energies where, interestingly, the neutron-induced recoil spectrum in fact dominates over the CE$\nu$NS recoil spectrum.

**B. Discussions**

From Fig. 4 we see that below $\sim 25$ keV recoil energy, the CE$\nu$NS process dominates the recoil spectrum by up to more than two orders of magnitude over the $\nu In$ contribution. However, the CE$\nu$NS recoil spectrum falls rapidly with increasing recoil energy, and above $\sim 25$ keV the $\nu In$ contribution starts to dominate. The integral numbers of recoil events as a function of the detector threshold energy for both CE$\nu$NS and $\nu In$ processes are shown in Fig. 5 where it is seen that for thresholds beyond $\sim 20$ keV the total number of recoil events can significantly differ from that due only to CE$\nu$NS.

The dominance of the $\nu In$ contribution to the total number of recoil events at large recoil energies can be seen more clearly from Fig. 6 where we plot the ratio $r$ of the $\nu In$ contribution to the total number of recoil events. Note that the ratio $r$ is independent of the distance to the SN event, and to a large extent should also be independent of the SN progenitor mass provided that supernovae of different masses emit neutrinos of different flavors roughly in the same ratios.

From Fig. 5 we see that for a $18M_\odot$ SN at a distance of 1 kpc, a 1-ton detector will yield, on the average $\sim 10$ (20), 7 (6) and 6 (2) recoil events above 15, 20 and 25 keV, respectively, from $\nu In$ (CE$\nu NS$). The number of
neutrons produced within the detector volume through the $\nu$In process will scale as $M_{\text{Det}}/d^2$, where $M_{\text{Det}}$ is the active liquid xenon mass in the detector. However, the xenon recoil spectrum and the total number of xenon recoils produced by the neutrons will not simply scale with $M_{\text{Det}}$, since the number of scattering a neutron may undergo would depend on the dimensions of the detector volume — larger dimensions would generally provide for more number of scatterings. Nevertheless, a rough (probably conservative) estimate of the number of events expected in a larger detector may be obtained by nominally scaling the number of events for a 1 ton detector to a larger detector with the same aspect ratio of its physical dimensions. With this, for a future large liquid xenon detector of the class of the proposed DARWIN collaboration detector, for example, with an active target mass of ~40 ton, we may expect ~16 (32), 11 (10) and 10 (3) recoil events above 15, 20 and 25 keV, respectively, from $\nu$In (CE$\nu$NS) for the SN at a distance of e.g., 5 kpc, while the numbers would be reduced to ~4 (8), 3 (2) and 2 (1) for the same SN at a distance of e.g., 10 kpc. With the capability to measure the energies of the individual recoil events, and with the general expectation that detection efficiency is generally an increasing function of the recoil energy of the events, these future detectors may offer the possibility of measuring the dominant contributions of the $\nu$In process at relatively large recoil energies.

Nuclear recoil (NR) events from background sources including those from detector components, surface contaminants, cosmic ray muon-induced neutrons, atmospheric neutrinos, and so on, are expected to be small over the duration of a typical SN burst event. For example, the LZ experiment estimates a total of ~1.03 NR events in the 6–30 keV NR energy range from all background sources for a fiducial mass of 5.6 ton of liquid xenon over 1000 live days of exposure. This works out to ~2.1 $\times$ 10$^{-9}$ NR events/ton/sec. For a 40 ton active mass class experiment like DARWIN, this nominally gives a number of ~8.5 $\times$ 10$^{-7}$ NR events from background sources over the duration of ~10 sec of a typical SN burst, well below the expected numbers of SN neutrino induced NR events due to both CE$\nu$NS and $\nu$In processes estimated above.

V. SUMMARY AND CONCLUSIONS

Future large 10-ton class liquid xenon detectors for dark matter search will also be sensitive to supernova neutrino induced nuclear recoil events through the CE$\nu$NS process. In this paper we have pointed out that in addition to the direct nuclear recoil events due to CE$\nu$NS, there is a secondary source of nuclear recoil events due to neutrons produced through inelastic neutrino-nucleus interaction of the supernova neutrinos with the detector nuclei. In such inelastic interactions, the neutrino scatters off the target nucleus leaving the final state nucleus in an excited state which subsequently decays by emission of one or more neutrons if the initial neutrino energy is sufficient to excite the nucleus above its neutron emission threshold. We have estimated the contribution of this neutrino-induced neutron ($\nu$In) process to the total nuclear recoil energy spectrum due to supernova neutrinos in liquid xenon detectors using the GEANT4 simulation toolkit and find that this can significantly modify the overall recoil spectrum at large recoil energies (> 25 keV) from that expected from the CE$\nu$NS process alone. Since the $\nu$In contribution to the recoil spectrum receives dominant contribution from the SN $\nu_e$s while the CE$\nu$NS contribution comes from all the six species of neutrinos, the measurement of the $\nu$In contribution by future large liquid xenon detectors may offer the possibility of extracting useful information about the total SN explosion energy going into different neutrino flavors.

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