The Origin of Corporate Control Power

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Abstract
How does the control power of corporate shareholder arise? On the fundamental principles of economics, we discover that the probability of a shareholder possessing optimal control power evolves in Fibonacci series pattern and emerges as the wave between $1/2$ and $2/3$ along with time in period of $12h$ ($h$ is the time interval between the state and state of the evolution). This novel feature suggests the efficiency of the allocation of corporate shareholders’ right and power. Data on the Chinese stock market support this prediction.

Keywords: Control Power; Control Right; Entrepreneurship; Probability Dynamics; Fibonacci Series Pattern

JEL Codes: A10, D01, G32, G34

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1. Introduction

A corporate shareholder has a pair of matching rights: residual claimancy (cash flow right) and residual right of control/control right (voting right; Alchian and Demsetz 1972; Grossman and Hart 1986; Hart and Moore 1990; Milgrom and Roberts 1992). Many previous works have contributed to the empirical correlation between this pair of rights and corporate behaviors/performance (La Porta et al. 1999; Claessens et al. 2000; Faccio and Lang 2002; Claessens et al. 2002; Lemmon and Lins 2003; Lin et al. 2013).

However, from the perspective of corporate shareholder, the exercise of voting right lies with the comparative advantage of one’s voting right over the right of others rather than the absolute amount of the voting right itself. It depends not only on the amount of one’s voting right itself but also that of others. It is endogenous from the competition among shareholders. That is, it is exactly “power” rather than “right”. We define it as “control power” corresponding to “control right”. No matter in logic or actuality, “control power” is the primary concept and “control right” is derivative (aroused) (Demsetz 1983), though the former is invisible, and the latter is concrete. It is the shareholder’s control power, not control right, that directly dominates corporate behaviors/performance. For understanding corporate behaviors/performance, control power is more simply and more substantive than control right.

Several prior works have chosen the Shapley-Shubik power index (\(SPI \in [0,1]\); Shapley and Shubik 1954) as an alternative for voting right to improve the measurement of control right (e.g., Nenova 2003; Edwards and Weichenrieder 2009; Andres et al. 2013; Aminadav and Papaioannou 2020). However, \(SPI\) is an instrument to measure power, and control right should only be
measured by voting right exactly.

In the endogeneity of corporate shareholder’s control power, it is critical to understand the dynamics. Unfortunately, a complete theoretical model that specifies how control power arises is still largely missing. Hence, we present a framework that quantifies how control power is determined and emerge explicitly.

2. Theoretic solution and testable hypothesis

On the fundamental principles of economics, we will derive the optimization condition of corporate shareholder’s control power and then develop the testable hypothesis. The deduction is subject to identifying and understanding of the entrepreneur ship of corporate shareholders (Knight 1921; Schumpeter 1912). Any one of the corporate large shareholders would be bound to compete for one’s optimal control power with one’s entrepreneurial talent being identified to the greatest extent among the other shareholders.

For a corporate shareholder, we could have

\[ P_r = V_r - C_r \]  

(1)

where \( r \) is the shareholder’s control power, \( P_r \) is the profit of \( r \), \( V_r \) is the revenue of \( r \), and \( C_r \) is the cost of \( r \). With \( r \) increasing, \( dC_r/dr \) (\( dV_r/dr \)) increases (decreases) by degrees.

Moreover, we could have

\[ C_r = C_{S_r} + C_{E_r} \]  

(2)

where \( S_r \) is the shares of the shareholder, \( E_r \) indicates the efforts the shareholder must make to communicate with other shareholders (to affect and prevail on them so they give up the challenge/struggle for control power), \( C_{S_r} \) is the opportunity cost of having shares \( S_r \), and \( C_{E_r} \) is
the negative utility by $E_r$. Given $C_r$ unchanged, $S_r$ and $E_r$ would be changed coordinate in the opposite direction. Along with the decreasing of $S_r$, $dE_r/dS_r$ increases by degrees (see Figure 1).

![Figure 1. Coordinate changing of $S_r$ and $E_r$ in the opposite direction.](image)

Corresponding to $E_r$, to $E_r'$ - the efforts the shareholder able to make to communicate with other shareholders, along with the decreasing of $S_r$, $dE_r'/dS_r$ decreases by degrees; And along with the increasing of the entrepreneurial talent of the shareholder ($e$), $dE_r'/dS_r$ increases by degrees.

To maximize $P_r$, the shareholder would take control power $r$ at (see Figure 2):

$$dV_r/dr = dC_r/dr$$  \hspace{1cm} (3)

$C_r$ is then determined.
Figure 2. The shareholder maximizing his profit at $dV_r/dr = dC_r/dr$.

Of his entrepreneurship, the shareholder would be most sensitive to the positive utility of control power, and least sensitive to the negative utility of efforts to negotiate and coordinate with other shareholders. Accordingly, we predict that $r$ would reach the top value with the highest probability (see Figure 2).

Furtherly, to be identified with his entrepreneurial talent by the other shareholders to the greatest extent, the shareholder will take $S_r^*$ (see Figure 3), where:

$$dE_r/dS_r = dE'_r/dS_r$$  \[4\]
Figure 3. The shareholder is identified with his entrepreneurial talent by the other shareholders to the greatest extent at $dE_r/dS_r = dE_r'/dS_r$.

The economic implication of this is that, to win optimal control power, the greater the shareholder’s entrepreneurial talent is, the few shares he could stake (mortgage) to other shareholders. For the perspective of other shareholders, it means that the weaker their entrepreneurial talent is, the more shares they are willing to entrust to the shareholder. It also means that the shareholder’s entrepreneurial talent is found and used to the greatest extent and meanwhile the capital of the shareholders is used to the greatest extent too.
That is, generally, to win optimal control power, the greater the comparative advantage of entrepreneurial talent over that of other shareholders (over the greatest one of that) the shareholder has ($R_e^*$), the less the comparative advantage of shares over that of the other shareholders (over the sum of their shares) the shareholder would get ($R_s^*$).\(^3\)

Namely, $R_e^*$, $R_s^*$ is reciprocal in competing for optimal control power. Thus, we could have:

$$\begin{pmatrix} R_e^* \\ R_s^* \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} R_e^* \\ R_s^* \end{pmatrix} = \varphi$$

(5)

$$R_e^* * R_s^* = \varphi / 2$$

(6)

where $\varphi$ is a power-related invariant.

We can understand this that the control power originates from the cumulative effect of $R_e^*$ on $R_s^*$.

Furtherly, for function (6), we can deduce that there exist three sets of solution:

\begin{equation}
\begin{pmatrix} e_0 \\ 0 \\ s_0 \end{pmatrix} \begin{pmatrix} R_e \\ R_s \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} e_1 \\ s_1 \end{pmatrix}
\end{equation}

So,

$$\begin{pmatrix} R_e \\ R_s \end{pmatrix} = \begin{pmatrix} 1/e_0 & 0 \\ 0 & 1/s_0 \end{pmatrix} \begin{pmatrix} e_1 \\ s_1 \end{pmatrix}$$

where $e_1, s_1$ is the entrepreneurial talent and the shares of a shareholder; $e_0, s_0$ is that of the other shareholders. We can understand that $e_1, e_0, R_e$ are intensive properties and measurement unavailable; $s_1, s_0, R_s$ are extensive properties and measurement available.
i. $R_e^* > 1, R_s^* > 1$.

ii. $R_e^* < 1, R_s^* \gg 1$.

iii. $R_e^* \gg 1, R_s^* < 1$. This set implicates that the shareholder with the highest entrepreneurial talent rather than the largest shares would be the one with maximal control power. It predicts the rise of dual-class share structure (Aggarwal et al., 2022).

These indicate the efficiency of the allocation of right and power among corporate shareholders with different patterns in different conditions.

However, there is no entrepreneurial talent measurement available, and then $R_e^*$ is uncomputable. Consequently, $R_s^*$ is uncomputable, and then $S_r^*$ is uncomputable.

Therefore, the shareholder could not fix $S_r^*$. What he could do is to change $S_r$ and $E_r$ coordinate in the opposite direction ceaselessly while keeping $C_r$ unchanged. Meanwhile, the shares of other holders will change along with that. Logically, therefore, the shareholder’ control power should be in the superimposed state of $r/\not r$ (in the indistinguishable state of $r/\not r$). Subject to this condition, the shareholder competes for $r$. That should be formulated as the superposition of the operations as follows:

$$
\begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix} +
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix} +
\begin{pmatrix}
0 & 0 \\
1 & 0
\end{pmatrix} = \begin{pmatrix}
1 & 1 \\
1 & 0
\end{pmatrix}
$$

(7)

It is the intrinsic dynamics of the evolution of the probability of a shareholder obtaining $r$. It is just the iterative matrix of Fibonacci series model. In Fibonacci series model, a pair of adult rabbits always reproduces a pair of infants, and the pair of infants always grows into a pair of adults. This describes the evolution of the adults and infants in an animal population. The
dynamics is not only sufficient but also necessary for the adults to have the highest ratio in the population. The evolution function is as follows:

\[ F_z = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^z \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{8} \]

where \( z = 0, 1, 2, 3, \ldots \).

Obviously, equation (8) describes the evolution in the natural scale. To understand the continuous dynamics of the evolution completely, we conduct analytical extension of equation (8) (Appendix I).

From equation (8), we could deduce that the probability of a shareholder obtaining \( r \) evolves in Fibonacci series pattern: \( 1/2 - 2/3 - 3/5 - 5/8 - 8/13 \ldots 34/55 \ldots \). And \( 8/13 \) is so closely to the limit \( \approx 0.618 \)((8/13)/0.618 \approx 0.995) \) that is statistically/realistically indistinguishable. We could understand that the series converges to \( 8/13 \).

However, by the impacts of uncertainties, the evolution process would be interrupted randomly in any one of the states to collapse back \( 1/2 \) (It could be understood as the inverse operation): \( 1/2 - 2/3 \ldots 1/2 \) (1 operation), \( 1/2 - 2/3 - 3/5 \ldots 1/2 \) (2 operations), \( 1/2 - 2/3 - 3/5 - 5/8 \ldots 1/2 \) (3 operations), \( 1/2 - 2/3 - 3/5 - 5/8 - 8/13 \ldots 1/2 \) (4 operations). So, the evolution process is the random superposition of these evolution process above in all possibilities. It is equivalent to the simple harmonic oscillation of \( 1/2 - 2/3 - 1/2 \) in 12 operations (the least common multiple of 1, 2, 3, 4) (Appendix M). It could be understood as the fluctuation of the evolution process by uncertainties.

Then, we could have
\[ t = h \times l \] 

where \( t \) is the time; \( l \) is the number of the operations in the evolution process; \( h \) is the time interval between the state and the state of the evolution, which is a random variable (\( > 0 \)).

Thus, we could derive that the probability of a shareholder getting \( r \) waves between 1/2 and 2/3 along with time in period of \( 12h \). It is the dynamic stability. This is universal.

Hence, we infer the testable hypothesis:

In the sample of companies in which the shareholder’s optimal choice is \( r \), the ratio of companies of a shareholder having obtained control power \( r \) (\( R_{Spi=r} \)) could be fitted by Fourier function on \( t \) in first order with maximum 2/3 and minimum 1/2, in period of 12\( h \).

3. Approaches

3.1 Measurement of shareholder’s control power

The \( SPI \) (\( SPI \in [0,1] \)) was defined as the ratio of counts of a shareholder being a pivotal voter (“being critical to the success of a winning coalition” / “turning a possible defeat into a success”; Shapley and Shubik 1954) to the number of all possible combinations (coalitions) of competitors for the power. Economically, this indicates the probability of a shareholder being the last decision-maker in all competitor coalitions. That is exactly what we want to describe for shareholders’ control power. We use \( SPI \) to measure it.

3.1.1 Calculation of the SPI. We use the program \textit{ssdirect}^4 developed by Dennis Leech (University of Warwick) and Robert Leech (Imperial College London) to calculate the

\footnote{http://homepages.warwick.ac.uk/~ecaee/ssdirect.html}
shareholders’ SPI.

3.1.2 Voting rule. In the Chinese stock market, the “one share one vote” rule is applicable. In shareholders’ meeting (the highest authority), by the half-win (1/2) principle, the board directors, members of the top decision-making body (top power body), are elected. Therefore, we take it as the voting rule to compute the shareholders’ SPI.

3.1.3 Which shareholders are scrambling for control power? In a company, shareholders’ meeting is the supreme authority institution. The shareholders decide fundamental issues through voting in it. As shown in Table 1, for the companies on main board in the Chinese stock market, from 2000 to 2021, the mean of the ratio of the share rate being present in shareholders’ meeting to the share rate of top10 shareholders is rather close to 1.0. Further, the ratio of companies with top1 shareholder’s SPI = 1 playing in top9, top10, top11 shareholders is rather close. So, we would consider in statistics that, in Chinese stock market, shareholders’ meeting is conducted almost within the top10 shareholders.

Table 1. Which shareholders are scrambling for control power?

| Year | $S_{meeting}/S_{top10}$ | $N_{sample}$ ($mean \pm 1s.d., mean + 1s.d.$) | $R_{SPI=1(top9)}$ | $R_{SPI=1(top10)}$ | $R_{SPI=1(top11)}$ | $N_{sample}$ |
|------|-------------------------|--------------------------------------------|-------------------|-------------------|-------------------|---------------|
| 2000 | 0.946                   | 0.890                                      | 0.782             | 0.775             | 0.767             | 1,030         |
| 2001 | 0.944                   | 0.865                                      | 0.783             | 0.774             | 0.769             | 1,139         |
| 2002 | 0.931                   | 0.846                                      | 0.771             | 0.765             | 0.763             | 1,210         |
| 2003 | 0.933                   | 0.851                                      | 0.743             | 0.739             | 0.732             | 1,266         |
| 2004 | 0.923                   | 0.836                                      | 0.734             | 0.726             | 0.724             | 1,326         |
| 2005 | 0.889                   | 0.820                                      | 0.685             | 0.676             | 0.672             | 1,142         |
| 2006 | 0.890                   | 0.775                                      | 0.697             | 0.688             | 0.681             | 1,255         |
| Year | $S_{\text{meeting}}$ | $S_{\text{top10}}$ | $R_{\text{SPI}=1(top9)}$ | $R_{\text{SPI}=1(top10)}$ | $R_{\text{SPI}=1(top11)}$ | $N_{\text{sample}}$ |
|------|-----------------|-----------------|-------------------|-------------------|-------------------|-----------------|
| 2007 | 0.886           | 0.134           | 0.805             | 0.737             | 0.727             | 1,295           |
| 2008 | 0.876           | 0.135           | 0.804             | 0.762             | 0.754             | 1,279           |
| 2009 | 0.871           | 0.136           | 0.807             | 0.773             | 0.764             | 1,280           |
| 2010 | 0.870           | 0.140           | 0.808             | 0.774             | 0.769             | 1,299           |
| 2011 | 0.869           | 0.141           | 0.826             | 0.779             | 0.770             | 1,354           |
| 2012 | 0.872           | 0.132           | 0.800             | 0.773             | 0.766             | 1,363           |
| 2013 | 0.870           | 0.136           | 0.806             | 0.765             | 0.758             | 1,363           |
| 2014 | 0.877           | 0.143           | 0.815             | 0.748             | 0.740             | 1,383           |
| 2015 | 0.853           | 0.142           | 0.801             | 0.718             | 0.708             | 1,438           |
| 2016 | 0.851           | 0.143           | 0.781             | 0.684             | 0.670             | 1,575           |
| 2017 | 0.851           | 0.148           | 0.785             | 0.650             | 0.637             | 1,745           |
| 2018 | 0.845           | 0.141           | 0.770             | 0.640             | 0.626             | 1,790           |
| 2019 | 0.850           | 0.151           | 0.806             | 0.634             | 0.627             | 1,826           |
| 2020 | 0.851           | 0.141           | 0.768             | 0.617             | 0.612             | 1,911           |
| 2021 | 0.857           | 0.140           | 0.765             | 0.628             | 0.621             | 2,032           |

**Notes:**

$S_{\text{meeting}} = \frac{\text{share rate present in the annual meeting} + \text{share rate present in the extraordinary meeting}}{\text{number of annual and extraordinary shareholders' meetings in the year}}$;

$S_{\text{top10}} =$ share rate of top10 shareholders at the end of year; $N_{[\text{mean} - 1\text{s.d}, \text{mean} + 1\text{s.d}]} =$ number of companies of $S_{\text{meeting}}/S_{\text{top10}}$ in $[\text{mean} - 1\text{s.d}, \text{mean} + 1\text{s.d}]$; $R_{\text{SPI}=1(top9)}$, $R_{\text{SPI}=1(top10)}$, $R_{\text{SPI}=1(top11)} =$ the ratio of companies with top1 shareholder’s SPI = 1 playing in top9, top10, top11(taking $S_{\text{meeting}}-S_{\text{top10}}$ (if < 0, taking as 0) as the share rate of the 11th player) shareholders respectively; $N_{\text{sample}} =$ total number of companies.

**3.2 Sampling**

When the top1 shareholder’s share rate is $\geq 50\%$, his SPI is 1, and no effort is needed to negotiate and coordinate with other shareholders. So, our sample is composed of companies in
which the top1 shareholder’s share rate is < 50%.

All data for this study are derived from the database of the China Stock Market and Accounting Research.

3.3 Grouping of the natural experiments

The Chinese stock market comprises private-owned and state-owned companies on the main board for the established companies as well as the small and medium-sized enterprises (SME) board and growth enterprises market (GEM) board for the start-ups. The state-owned shareholders’ decisions on adding/reducing shares often be interfered and restricted by the government\(^5\). However, it is difficult for the government to interfere private-owned shareholders’

\(^5\) Order of China Securities Regulatory Commission (CSRC) No. 86: Administrative Measures for the Reform of Non-tradable Shares in Listed Companies, September 4, 2005.
http://www.csrc.gov.cn/pub/newsite/flb/flfg/bmgf/ssgs/gqfz/201012/t20101231_189887.html

Order of State-owned Assets Supervision and Administration Commission of the State Council (SASAC) and China Securities Regulatory Commission (CSRC) No. 19: Interim Administrative Measures for the State-owned Shares Transferring of Listed Companies, July 1, 2007.
http://www.csrc.gov.cn/pub/newsite/flb/flfg/bmgz/ssl/201012/t20101231_189730.html

Order of State-owned Assets Supervision and Administration Commission of the State Council (SASAC), Ministry of Finance of the People's Republic of China (MOF) and China Securities Regulatory Commission (CSRC) No. 36: Administrative Measures for the Supervision over the State-owned Equity of Listed Companies, July 1, 2018.
behaviors. In SME and GME, under the condition of fewer listing requirements and less regulation/supervision than in main board, the start-ups’ shareholders would conduct accounting fraud widespread. So, from the normal viewpoint of the shareholding, the share adding/reducing behavior of the holders of the start-ups would deviate from the normal pattern predicted by our theory (Liu and Zhao 2011; Jia 2017; He 2019). Thus, we take private-owned companies on the main board as the control group and focus on them by considering them as the normal firms whose shareholders’ behavior is not distorted ($b = 0$, Appendix I); and take state-owned companies on the main board as well as state-owned and private-owned companies on the SME and GEM board as the natural experiment groups (whose shareholders’ behavior is distorted) in contrast ($b \neq 0$, Appendix I).

3.4 The optimal control power of corporate shareholder

We can understand the choice of top1 shareholder’s optimal control power by the distribution of $SPI$ value.

http://www.csirc.gov.cn/pub/newsite/zjhxwfb/xwdd/201805/t20180518_338364.html

6 Order of China Securities Regulatory Commission (CSRC) No. 32: Administrative Measures for Initial Public Offering and Listing, May 18, 2006.

http://www.csirc.gov.cn/pub/zjhpublic/zjh/200804/t20080418_14502.html

Order of China Securities Regulatory Commission (CSRC) No. 61: Interim Administrative Measures for Initial Public Offering and Listing on GEM Board, May 1, 2009.

http://www.csirc.gov.cn/pub/zjhpublic/G00306202/cyb/200911/t20091117_170416.html
We find that the distribution of top1 shareholder’s SPI is not normal but rather nearly power-law. In about 46–66% of the private-owned companies on the main board, the top1 shareholder’s SPI = 1 (Table 2). This implies that top1 shareholder’s own choice is more likely to be SPI = 1.

Table 2. Top1 shareholder’s SPI = 1.

| Year | Main board | SME and GEM boards |
|------|------------|--------------------|
|      | Private-owned | State-owned | Private-owned | State-owned |
| N_{SPI=1}/N_{sample} | N_{sample} | N_{SPI=1}/N_{sample} | N_{sample} | N_{SPI=1}/N_{sample} | N_{sample} |
| 1992 | 1.000       | 1          | 0.444       | 9          |
| 1993 | 0.500       | 28         | 0.600       | 70         |
| 1994 | 0.531       | 32         | 0.628       | 94         |
| 1995 | 0.575       | 40         | 0.615       | 117        |
| 1996 | 0.644       | 87         | 0.660       | 232        |
| 1997 | 0.561       | 114        | 0.678       | 314        |
| 1998 | 0.508       | 124        | 0.689       | 357        |
| 1999 | 0.493       | 146        | 0.668       | 391        |
| 2000 | 0.500       | 166        | 0.664       | 458        |
| 2001 | 0.489       | 184        | 0.669       | 502        |
| 2002 | 0.482       | 226        | 0.673       | 514        |
| 2003 | 0.442       | 294        | 0.675       | 508        |
| 2004 | 0.470       | 334        | 0.651       | 525        | 0.429       | 28       | 0.000       | 5       |
| 2005 | 0.463       | 337        | 0.650       | 540        | 0.459       | 37       | 0.400       | 10      |
| 2006 | 0.491       | 389        | 0.696       | 644        | 0.426       | 68       | 0.500       | 22      |
| 2007 | 0.569       | 404        | 0.725       | 661        | 0.409       | 127      | 0.590       | 39      |
| 2008 | 0.610       | 395        | 0.749       | 658        | 0.465       | 172      | 0.568       | 44      |
| 2009 | 0.664       | 402        | 0.742       | 648        | 0.483       | 238      | 0.623       | 53      |
| 2010 | 0.663       | 412        | 0.755       | 661        | 0.442       | 475      | 0.507       | 73      |
| Year | Max  | Min  | Mean | S.D  | \( N_{[\text{mean}-1\text{s.d},\text{mean}+1\text{s.d}]} \) | \( N_{\text{sample}} \) |
|------|------|------|------|------|------------------------------------------------|-----------------|
| 2011 | 0.662| 444  | 0.750| 0.439| 0.565                                              | 85              |
| 2012 | 0.639| 452  | 0.754| 0.457| 0.585                                              | 94              |
| 2013 | 0.630| 465  | 0.746| 0.506| 0.594                                              | 96              |
| 2014 | 0.601| 504  | 0.748| 0.540| 0.653                                              | 98              |
| 2015 | 0.551| 584  | 0.731| 0.502| 1,016                                              | 106             |
| 2016 | 0.515| 660  | 0.694| 0.468| 1,138                                              | 118             |
| 2017 | 0.475| 827  | 0.695| 0.446| 1,347                                              | 126             |
| 2018 | 0.479| 873  | 0.673| 0.450| 1,363                                              | 143             |
| 2019 | 0.485| 897  | 0.669| 0.464| 1,404                                              | 185             |
| 2020 | 0.454| 963  | 0.673| 0.468| 1,484                                              | 227             |
| 2021 | 0.466| 1,029| 0.682| 0.466| 1,605                                              | 269             |

Notes: \( N_{SPI=1} \)-number of companies with the top1 shareholder’s \( SPI = 1 \).

Further, we find normal distribution (\( N(0.466,0.165^2) \)) in the sample of private-owned firms on the main board with the top1 shareholder’s control power \( SPI < 1 \) (Table 3). This normal distribution is robust and independent of the ratio of companies with the top1 shareholder’s control power \( SPI = 1 \). It suggests that this result has not occurred because of the top1 shareholder’s own choice but rather because of a defeat in the competition for \( SPI = 1 \), which they can’t but accept.

Table 3. Description of the sample of private-owned firms on the main board with top1 shareholder having \( SPI < 1 \).
| Year | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|      | 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800| 0.800|
|      | 0.197| 0.218| 0.225| 0.224| 0.221| 0.144| 0.144| 0.147| 0.149| 0.148| 0.143| 0.143| 0.154| 0.159| 0.160| 0.169| 0.164| 0.197| 0.137| 0.135| 0.135| 0.135| 0.160| 0.142|
|      | 0.434| 0.450| 0.471| 0.462| 0.474| 0.474| 0.480| 0.470| 0.474| 0.462| 0.468| 0.500| 0.470| 0.470| 0.507| 0.484| 0.483| 0.478| 0.468| 0.474| 0.464| 0.474| 0.459| 0.459|
|      | 0.188| 0.162| 0.157| 0.157| 0.163| 0.161| 0.162| 0.157| 0.158| 0.151| 0.153| 0.165| 0.161| 0.157| 0.173| 0.170| 0.166| 0.161| 0.177| 0.173| 0.166| 0.176| 0.156| 0.159|
|      | 0.706| 0.710| 0.720| 0.672| 0.676| 0.687| 0.734| 0.675| 0.707| 0.712| 0.713| 0.591| 0.718| 0.701| 0.637| 0.662| 0.647| 0.669| 0.657| 0.687| 0.675| 0.661| 0.669| 0.711|
|      | 17   | 31   | 50   | 61   | 74   | 83   | 94   | 117  | 164  | 177  | 181  | 198  | 174  | 154  | 135  | 139  | 150  | 163  | 172  | 201  | 262  | 320  | 434  | 455  | 462  | 526  | 550  |
| Mean | 0.797 | 0.163 | 0.466 | 0.165 | 0.681 |
|------|-------|-------|-------|-------|-------|
| (S.D) | (0.017) | (0.032) | (0.023) | (0.009) | (0.031) |

Notes: $N_{[\text{mean}-1s.d,\text{mean}+1s.d]}$ = number of companies with top1 shareholder’s SPI in $[\text{mean} - 1s.d, \text{mean} + 1s.d]$. 

All of these indicate that, for almost every company in the Chinese stock market, the choice of the top1 shareholder’s optimal control power is $SPI = 1$. This result supports our theoretical prediction above. Moreover, this gives us the opportunity to test our hypotheses using the ratio of companies with top1 shareholder that have obtained control power with $SPI = 1$, $R_{SPI=1}$.

3.5 The time interval between the state and the state of the evolution, $h$

$h$ is a random variable. When we use the data of end of the year, it would only be a positive integer as 1, 2, … $n$ years. Further, due to three-year election cycle of directors in listed company⁷, $h$ would be more likely to be 1 or 2 years and more likely to have the probability of 1/2. So, we could consider $h = 1.5$ years statistically.

4. Results

Using data of the Chinese stock market for 1996–2021 (sample size ≥ 50), we obtained a good fit with $R_{SPI=1}$ on $t$ with Fourier function in first order (Figure 4a). The result could not refute our hypothesis statistically.

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⁷ The Company Law of the People’s Republic of China (Revision 2018). The Sixth Meeting of the Standing Committee of the 13th National People’s Congress, October 26, 2018. http://www.npc.gov.cn/zgwh/npc/nw/2018-11/05/content_2065671.htm.
Figure 4. Fitting of $R_{SPI=1}$, $M_{top1}$, and $M_{top2-10}$ on $t$ with Fourier function in first order for private-owned firms on the main board in 1996–2021
As shown in Figure 1, $S_r$ and $E_r$ are changed coordinate in the opposite direction. That would make the changing of the top2-10 shareholders’ shares ($S_{2-10}$).

Subject to

$$S_r = A \cos \left( \frac{2\pi}{T} t + \pi \right)$$  \hspace{1cm} (10)

$$E_r = B \cos \left( \frac{2\pi}{T} t \right)$$  \hspace{1cm} (11)

where $t$ is time.

Then

$$S_{2-10} \propto A \cos \left( \frac{2\pi}{T} t + \pi \right) \cdot B \cos \left( \frac{2\pi}{T} t \right)$$

$$= \frac{A+B}{2} \left[ \cos \left( \frac{4\pi}{T} t + \pi \right) + \cos \pi \right]$$

$$= \frac{A+B}{2} \left[ \cos \left( \frac{2\pi}{T/2} t + \pi \right) - 1 \right]$$  \hspace{1cm} (12)

Therefore, for the sample, we can furtherly predict:

i. The period of mean of the top2-10 shareholders’ share rate ($M_{top2-10}$) on $t$ is $1/2$ of that of the top1 shareholder’s share rate ($M_{top1}$).

And the comparative advantage of the top1 shareholder’s voting right over the top2-10 shareholders determines that his control power ($SPI$) is equal to 1 or not. Therefore,

ii. The period of mean of the top2-10 shareholders’ share rate ($M_{top2-10}$) on $t$ is equal to that of $R_{SPI=1}$ but with phase position difference $\pi$.

The results meet our predictions (Figure 4b, 4c and Appendix Table A.1).

Statistically, as the main founder of the firm, the top1 shareholder’s entrepreneurial talent is
significantly greater than that of the others. So, in the subsample of the companies with the top1 holder’s $SPI = 1$, we could predict that:

a. The ratio of the top1 holder’s shares to the top2-10 holder’s shares ($\frac{\text{top1 holder’s shares}}{\text{top2–10 holder’s shares}}$) would present power-law distribution.

And in the subsample of the companies with the top1 holder’s $SPI < 1$, because of the striving and chasing for shares among the shareholders, we could predict that:

b. $\frac{\text{top1 holder’s shares}}{\text{top2–10 holder’s shares}}$ would be uniformly distributed.

The results meet our predictions (Appendix Table A.2 and Table A.3).

Only in our theoretic framework, we could deduce that the top1 shareholder’s shares and the top2-10 shareholders’ shares would appear as these patterns. This proves that our theory is not only sufficient but also necessary for our hypothesis.

Further, since $t = 1.5l$, we could have

$$R_{SPI=1}(t) = R_{SPI=1}(1.5l) \quad (13)$$

Then,

$$0.553 + 0.060 \cos\left(\frac{2\pi}{17.357} t\right) - 0.083 \sin\left(\frac{2\pi}{17.357} t\right) = 0.553 + 0.060 \cos\left(\frac{2\pi}{11.571} l\right) -$$

$$0.083 \sin\left(\frac{2\pi}{11.571} l\right) \quad (14)$$

$$\left(\frac{2\pi}{17.357}\right)^2 \left[-0.060 \cos\left(\frac{2\pi}{17.357} t\right) + 0.083 \sin\left(\frac{2\pi}{17.357} t\right)\right] = \frac{1}{(1.5)^2} \left(\frac{2\pi}{11.571}\right)^2$$
\[ \ast \left[ -0.060 \cos\left( \frac{2\pi}{11.571} l \right) + 0.083 \sin\left( \frac{2\pi}{11.571} l \right) \right] \]  

(15)

Meanwhile, we could have

\[
\frac{\partial^2 R_{SPI=1}}{\partial t^2} = \left( \frac{2\pi}{17.357} \right)^2 \left[ -0.060 \cos\left( \frac{2\pi}{17.357} t \right) + 0.083 \sin\left( \frac{2\pi}{17.357} t \right) \right] 
\]

(16)

\[
\frac{\partial^2 R_{SPI=1}}{\partial l^2} = \left( \frac{2\pi}{11.571} \right)^2 \left[ -0.060 \cos\left( \frac{2\pi}{11.571} l \right) + 0.083 \sin\left( \frac{2\pi}{11.571} l \right) \right] 
\]

(17)

Namely, we have

\[
\frac{\partial^2 R_{SPI=1}}{\partial t^2} - \frac{1}{h^2} \cdot \frac{\partial^2 R_{SPI=1}}{\partial l^2} = 0
\]

(18)

This is the wave equation of \( R_{SPI=1} \).

Further, we could have the probability density function of the distribution of top1 shareholder’s control power:

\[
p(SPI) = \begin{cases} 
0.553 + 0.060 \cos\left( \frac{2\pi}{17.357} t \right) - 0.083 \sin\left( \frac{2\pi}{17.357} t \right), & SPI = 1 \\
\left[ 1 - \left( 0.553 + 0.060 \cos\left( \frac{2\pi}{17.357} t \right) - 0.083 \sin\left( \frac{2\pi}{17.357} t \right) \right) \right] \ast \frac{1}{0.165\sqrt{2\pi}} e^{-\frac{(SPI-0.466)^2}{2\cdot0.165^2}}, & SPI \in (0,1)
\end{cases}
\]

(19)

This is the probability evolution of top1 shareholder’s control power.

In contrast, the fittings in the state-owned firms on the main board as well as the state-owned and private-owned firms on the SME and GEM board deviate from the patterns in the private-owned firms in the main board. The results are consistent with our predictions (Appendix I, Appendix Figure A.1, Table A.4, Figure A.2, A.3, Table A.5, A.6).
$R_{SPI=1}$ is not correlative to the stock market index and the economic growth of China. Moreover, $M_{top1}$ and $M_{top2−10}$ have negative correlation with the stock market index (Appendix Figure A.4 and Table A.7). This explains the independent and universal feature of $R_{SPI=1}$ and the dependence of $M_{top1}$ and $M_{top2−10}$ on the opportunity cost of shareholding.

5. Conclusions

Our framework for corporate shareholder’s control power differs from the empirical models that fit curves to phenomenological data. In contrast, we theoretically and quantitatively deduce that the probability of a shareholder obtaining optimal control power evolves in Fibonacci series pattern. It is a story of right transfer and power origin among corporate shareholders. It captures the salient, dynamic, and universal features of normal firm. This specific pattern, as the benchmark, demonstrates the elemental dynamics of the shareholder’s control right and power. It indicates that the shareholder’s control power comes from the cumulative effect of the comparative advantage of entrepreneurial talent over other shareholders on the comparative advantage of shares over other shareholders. It could be understood that the nature of firm is the finding and using of entrepreneurial talent. It implies the efficiency of the allocation of right and power among corporate shareholders. This deduction can be applied to understand the efficiency equivalence of different patterns of allocation of right and power, such as “one share one vote”, “dual-class share structure”, “limited partnership”, and different capital structure, and so on, under zero transaction cost condition.

Furtherly, our framework could also be extended to understand the power allocation in classical enterprise (the origin of shareholder’s residual claimancy and residual right of control).
We could unify it as follows:

**Table 4. The unification of our framework to classical enterprise**

| who holds power? | non-comparative advantage factor |
|------------------|---------------------------------|
|                  | I holding shares | II entrepreneurial talent | III labor |
| comparative advantage factor | holding shares | yes | yes or no |
| entrepreneurial talent | yes | | yes or no |
| labor | no | no | |

Here, in a company, there are two partners each having three factors: capital (shares), entrepreneurial talent and labor which have different property characteristics (Table 5).

**Table 5. The factors with different property characteristics**

| measurement available | holding shares | entrepreneurial talent | labor | unknown factor? |
|-----------------------|----------------|-----------------------|-------|-----------------|
| mortgageable          | yes            | no                    | no    | yes             |

Table 4 shows the control power allocation in different status of the comparative advantages and the property characteristics of the factors. The column I and II describe the power allocation in classical enterprise: capital employs labor (the shareholder has the right of residual claimancy and residual control-the control power over labor). The column III describes the control power allocation among shareholders studied in our paper.

Furthermore, we may generalize this theory to analyze the behaviors of politic parties.

We quantitatively confirm this in line with the data for the Chinese stock market.
Nevertheless, how well it captures the fundamental feature of corporate control power in other markets such as in the US or Europe remains to be tested.

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Appendix

Appendix M: The analytic extension of $F_z = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^z \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ \ (z = 0, 1, 2, 3, ...)

We could have the Taylor expansion of $F_z = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^z \begin{pmatrix} 1 \\ 1 \end{pmatrix}$:

$$F_z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \left[ \ln \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right]^k \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

We extend $z = a + ib$ defined in the complex domain. That describes the process of the operation in higher dimension which corresponds with the practical actions of the shareholders continuously. $a, b$ are defined in real domain. $a = a_\rho + a_\tau$, $a_\rho$ defined in integer domain, $a_\tau \in [0, 1)$ defined in real domain. It means that $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^a$ could be expressed as $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{a_\rho}$ by taking $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{a_\tau}$ as base space. Namely,

$$F_z = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^z \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{a+ib} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{ib} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^a \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{ib} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{a_\tau} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{a_\rho} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Here, $a > 0$ describes the iteration of the operation and $a < 0$, that of the inverse operation. And $b$ is the function of corporate shareholders’ behaviors. $+i, -i$ describe the distortion of corporate shareholders’ behaviors.
We let \( a_\tau = \cos \frac{\pi}{2} \alpha \ (\alpha \in (0,1]) \). Namely,

\[
\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^{a_\tau} = \sum_{k=0}^{\infty} \frac{a_\tau^k}{k!} \left[ \ln \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right]^k
\]

\[
= \sum_{k=0}^{\infty} \left( \frac{\cos \frac{\pi}{2} \alpha}{k!} \right)^k \left[ \ln \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right]^k
\]

Here, \( \alpha \) corresponds (describes) all the possibilities (the uncertainties) of the probability evolution process.

Furtherly,

\[
\ln \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \varphi - \left( \frac{\sqrt{5}^{-1} - 1}{2} \right) \pi i & 2\varphi - \sqrt{5}^{-1} \pi i \\ 2\varphi - \sqrt{5}^{-1} \pi i & -\varphi + \left( \frac{\sqrt{5}^{-1} + 1}{2} \right) \pi i \end{pmatrix}
\]

where \( \varphi = \ln \left[ \frac{(\sqrt{5}^{-1})(\sqrt{5}^{-1})(\sqrt{5}+1)}{2\sqrt{5} \cdot 2\sqrt{5}} \right] \)

the trace of \( \ln \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \):

\[
\text{tr} \left( \ln \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right) = i\pi
\]

\[
= \ln(-1)
\]

\[
= \ln(i \cdot i), \text{ or } \ln((-i) \cdot (-i))
\]

and the eigenvector of \( \ln \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \):
\[ V_1 = \left( \begin{array}{c} -\ln \left( \frac{2^{20\sqrt{5}}(\sqrt{5} - 3)^{10}}{2^{10}(\sqrt{5} + 1)^{20\sqrt{5}}} \right) + \pi(5 + \sqrt{5})i \\ \sigma \end{array} \right) \]

\[ V_2 = \left( \begin{array}{c} \ln \left( \frac{(\sqrt{5} - 1)^{20\sqrt{5}}(\sqrt{5} + 3)^{10}}{2^{20\sqrt{5}2^{10}}(\sqrt{5} + 3)^{20\sqrt{5}}} \right) + \pi(5 - \sqrt{5})i \\ \sigma \end{array} \right) \]

where \( \sigma = 2 \left( \ln \left( \frac{(\sqrt{5} - 1)^{10}(\sqrt{5} - 1)^{10}(\sqrt{5} + 1)^{10}(\sqrt{5} + 1)^{10}}{2^{20\sqrt{5}}} \right) - \pi \sqrt{5}i \right) \)

the eigenvalue of \( \ln \left( \begin{array}{cc} 1 & 1 \\ 1 & 0 \end{array} \right) \): 

\[ D_1 = \sqrt{5} \ln \left( \frac{\sqrt{5} + 3}{2} \right) \]

\[ D_2 = -\sqrt{5} \ln \left( \frac{\sqrt{5} + 3}{2} \right) + \pi i \]

Accordingly, we could understand that \( \left( \begin{array}{cc} 1 & 1 \\ 1 & 0 \end{array} \right)^{a_{\tau}} \) is the fractal structure with quintic symmetry, which dimension is \( a_{\tau} = \cos \frac{\pi}{2} \alpha \).
Appendix Figures:

Figure A.1: Fitting of $R_{SPI=1}$, $M_{top1}$, and $M_{top2-10}$ on $t$ with a Fourier function in the first order for state-owned firms on the main board in 1996–2021.
Figure A.2: Fitting of $R_{SPI=1}$, $M_{top1}$, and $M_{top2-10}$ on $t$ with a Fourier function in the first order for private-owned firms on the SME and GEM boards in 2004–2021.
Figure A.3: Fitting of $R_{SPI=1}$, $M_{top1}$, and $M_{top2-10}$ on $t$ with a Fourier function in the first order for state-owned firms on the SME and GEM boards in 2004–2021.
Figure A.4: Economic climate of China in 1996–2021. GDP = gross domestic product.
Appendix Tables:

Table A.1: Descriptions of the share rate of the top10 shareholders for private-owned firms on the main board.

| Year | Top1 Shareholder Mean | Top1 Shareholder S.D. | Top2-10 Shareholders Mean | Top2-10 Shareholders S.D. | N_{sample} |
|------|-----------------------|-----------------------|---------------------------|---------------------------|------------|
| 1992 | 0.382                 | .                     | 0.268                     | .                         | 1          |
| 1993 | 0.294                 | 0.141                 | 0.263                     | 0.148                     | 28         |
| 1994 | 0.300                 | 0.118                 | 0.262                     | 0.141                     | 32         |
| 1995 | 0.302                 | 0.110                 | 0.273                     | 0.130                     | 40         |
| 1996 | 0.316                 | 0.114                 | 0.250                     | 0.130                     | 87         |
| 1997 | 0.306                 | 0.110                 | 0.257                     | 0.126                     | 114        |
| 1998 | 0.298                 | 0.105                 | 0.275                     | 0.122                     | 124        |
| 1999 | 0.303                 | 0.105                 | 0.275                     | 0.122                     | 146        |
| 2000 | 0.297                 | 0.105                 | 0.276                     | 0.118                     | 166        |
| 2001 | 0.292                 | 0.105                 | 0.271                     | 0.114                     | 184        |
| 2002 | 0.287                 | 0.100                 | 0.280                     | 0.118                     | 226        |
| 2003 | 0.282                 | 0.089                 | 0.284                     | 0.118                     | 294        |
| 2004 | 0.284                 | 0.089                 | 0.286                     | 0.118                     | 334        |
| 2005 | 0.282                 | 0.084                 | 0.278                     | 0.118                     | 337        |
| 2006 | 0.273                 | 0.089                 | 0.246                     | 0.118                     | 389        |
| 2007 | 0.268                 | 0.100                 | 0.221                     | 0.118                     | 404        |
| 2008 | 0.265                 | 0.105                 | 0.207                     | 0.118                     | 395        |
| 2009 | 0.259                 | 0.105                 | 0.184                     | 0.110                     | 402        |
| 2010 | 0.254                 | 0.100                 | 0.186                     | 0.114                     | 412        |
| 2011 | 0.258                 | 0.105                 | 0.192                     | 0.122                     | 444        |
| 2012 | 0.258                 | 0.105                 | 0.196                     | 0.126                     | 452        |
| 2013 | 0.259                 | 0.110                 | 0.204                     | 0.126                     | 465        |
| Year | Value 1 | Value 2 | Value 3 | Value 4 | Total |
|------|---------|---------|---------|---------|-------|
| 2014 | 0.264   | 0.110   | 0.214   | 0.126   | 504   |
| 2015 | 0.271   | 0.110   | 0.244   | 0.130   | 584   |
| 2016 | 0.276   | 0.114   | 0.270   | 0.130   | 660   |
| 2017 | 0.288   | 0.110   | 0.299   | 0.134   | 827   |
| 2018 | 0.288   | 0.110   | 0.297   | 0.130   | 873   |
| 2019 | 0.285   | 0.109   | 0.291   | 0.127   | 897   |
| 2020 | 0.280   | 0.106   | 0.297   | 0.128   | 963   |
| 2021 | 0.278   | 0.106   | 0.293   | 0.127   | 1029  |
Table A.2: The distribution of top1 holder's shares in the subsample $SPI = 1$ for private-owned firms on the main board.

| Year | top1 holder's shares | top2−10 holder's shares | $N_{sample}$ |
|------|----------------------|-------------------------|-------------|
|      | [1.0,1.1) | [1.1,1.2) | [1.2,1.3) | [1.3,1.4] | [1.4,1.5) | [1.5,1.6] | [1.6,1.7) | [1.7,1.8] | [1.8,1.9) | [1.9,2.0) | ...... | [10.5,10.6) | ...... |
| 1996 | 0.143 | 0.018 | 0.054 | 0.054 | 0.000 | 0.054 | 0.054 | 0.054 | 0.018 | 0.089 | ...... | 0.000 | ...... | 56 |
| 1997 | 0.094 | 0.047 | 0.031 | 0.078 | 0.063 | 0.063 | 0.047 | 0.016 | 0.031 | 0.031 | ...... | 0.016 | ...... | 64 |
| 1998 | 0.175 | 0.048 | 0.032 | 0.079 | 0.016 | 0.032 | 0.079 | 0.032 | 0.032 | 0.048 | ...... | 0.000 | ...... | 63 |
| 1999 | 0.111 | 0.097 | 0.028 | 0.028 | 0.083 | 0.056 | 0.083 | 0.014 | 0.042 | 0.028 | ...... | 0.000 | ...... | 72 |
| 2000 | 0.133 | 0.120 | 0.060 | 0.036 | 0.084 | 0.024 | 0.048 | 0.036 | 0.036 | 0.012 | ...... | 0.000 | ...... | 83 |
| 2001 | 0.156 | 0.067 | 0.111 | 0.022 | 0.111 | 0.044 | 0.011 | 0.033 | 0.078 | 0.000 | ...... | 0.000 | ...... | 90 |
| 2002 | 0.193 | 0.055 | 0.073 | 0.073 | 0.055 | 0.064 | 0.028 | 0.055 | 0.037 | 0.018 | ...... | 0.000 | ...... | 109 |
| 2003 | 0.146 | 0.108 | 0.069 | 0.069 | 0.069 | 0.023 | 0.046 | 0.046 | 0.023 | 0.008 | ...... | 0.000 | ...... | 130 |
| 2004 | 0.140 | 0.121 | 0.070 | 0.083 | 0.057 | 0.032 | 0.045 | 0.032 | 0.070 | 0.019 | ...... | 0.000 | ...... | 157 |
| 2005 | 0.154 | 0.083 | 0.077 | 0.058 | 0.090 | 0.038 | 0.045 | 0.032 | 0.064 | 0.032 | ...... | 0.000 | ...... | 156 |
| 2006 | 0.110 | 0.079 | 0.047 | 0.068 | 0.068 | 0.052 | 0.068 | 0.031 | 0.047 | 0.005 | ...... | 0.000 | ...... | 191 |
| 2007 | 0.091 | 0.083 | 0.061 | 0.065 | 0.052 | 0.043 | 0.039 | 0.030 | 0.030 | 0.009 | ...... | 0.004 | ...... | 230 |
| 2008 | 0.087 | 0.050 | 0.054 | 0.071 | 0.062 | 0.037 | 0.021 | 0.025 | 0.041 | 0.033 | ...... | 0.004 | ...... | 241 |
| 2009 | 0.082 | 0.060 | 0.060 | 0.064 | 0.026 | 0.034 | 0.030 | 0.026 | 0.037 | 0.045 | ...... | 0.004 | ...... | 267 |
| 2010 | 0.055 | 0.081 | 0.048 | 0.066 | 0.055 | 0.029 | 0.037 | 0.037 | 0.018 | 0.026 | ...... | 0.000 | ...... | 273 |
| 2011 | 0.082 | 0.082 | 0.061 | 0.044 | 0.024 | 0.044 | 0.031 | 0.027 | 0.034 | 0.027 | ...... | 0.000 | ...... | 294 |
| 2012 | 0.066 | 0.062 | 0.073 | 0.059 | 0.024 | 0.042 | 0.021 | 0.038 | 0.031 | 0.024 | ...... | 0.000 | ...... | 289 |
| 2013 | 0.061 | 0.075 | 0.082 | 0.044 | 0.041 | 0.058 | 0.024 | 0.038 | 0.020 | 0.027 | ...... | 0.000 | ...... | 293 |
| 2014 | 0.076 | 0.059 | 0.076 | 0.059 | 0.046 | 0.046 | 0.030 | 0.030 | 0.026 | 0.023 | ...... | 0.000 | ...... | 303 |
| 2015 | 0.099 | 0.081 | 0.081 | 0.068 | 0.047 | 0.050 | 0.043 | 0.037 | 0.056 | 0.025 | ...... | 0.000 | ...... | 322 |
| 2016 | 0.121 | 0.094 | 0.074 | 0.062 | 0.062 | 0.053 | 0.038 | 0.056 | 0.026 | 0.041 | ...... | 0.000 | ...... | 340 |
| 2017 | 0.107 | 0.094 | 0.104 | 0.064 | 0.056 | 0.069 | 0.074 | 0.064 | 0.033 | 0.041 | ...... | 0.003 | ...... | 393 |
| 2018 | 0.103 | 0.100 | 0.079 | 0.089 | 0.067 | 0.055 | 0.065 | 0.045 | 0.050 | 0.050 | ...... | 0.000 | ...... | 418 |
| 2019 | 0.113 | 0.094 | 0.090 | 0.071 | 0.055 | 0.062 | 0.057 | 0.044 | 0.034 | 0.032 | ...... | 0.000 | ...... | 435 |
| 2020 | 0.110 | 0.101 | 0.098 | 0.073 | 0.080 | 0.069 | 0.041 | 0.037 | 0.041 | 0.027 | ...... | 0.000 | ...... | 437 |
| 2021 | 0.106 | 0.090 | 0.088 | 0.088 | 0.077 | 0.061 | 0.040 | 0.035 | 0.040 | 0.029 | ...... | 0.000 | ...... | 479 |
Table A.3: The distribution of \( \text{top1 holder's shares} \) in the subsample \( SPI < 1 \) for private-owned firms on the main board.

| Year | \( [0.9,1.0] \) | \( [0.8,0.9] \) | \( [0.7,0.8] \) | \( [0.6,0.7] \) | \( [0.5,0.6] \) | \( [0.4,0.5] \) | \( [0.3,0.4] \) | \( [0.2,0.3] \) | \( [0.1,0.2] \) | \( [0.0,0.1] \) | \( N_{\text{sample}} \) |
|------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1996 | 0.129          | 0.065          | 0.129          | 0.226          | 0.129          | 0.065          | 0.129          | 0.129          | 0.000          | 0.000          | 31             |
| 1997 | 0.140          | 0.140          | 0.160          | 0.240          | 0.060          | 0.140          | 0.060          | 0.060          | 0.000          | 0.000          | 50             |
| 1998 | 0.148          | 0.180          | 0.180          | 0.148          | 0.082          | 0.164          | 0.016          | 0.082          | 0.000          | 0.000          | 61             |
| 1999 | 0.162          | 0.189          | 0.135          | 0.189          | 0.095          | 0.135          | 0.068          | 0.027          | 0.000          | 0.000          | 74             |
| 2000 | 0.145          | 0.193          | 0.157          | 0.205          | 0.108          | 0.048          | 0.120          | 0.012          | 0.012          | 0.000          | 83             |
| 2001 | 0.160          | 0.170          | 0.160          | 0.181          | 0.138          | 0.117          | 0.032          | 0.032          | 0.011          | 0.000          | 94             |
| 2002 | 0.145          | 0.154          | 0.145          | 0.205          | 0.120          | 0.120          | 0.060          | 0.043          | 0.090          | 0.000          | 117            |
| 2003 | 0.165          | 0.152          | 0.152          | 0.238          | 0.079          | 0.116          | 0.061          | 0.024          | 0.012          | 0.000          | 164            |
| 2004 | 0.153          | 0.153          | 0.158          | 0.181          | 0.141          | 0.096          | 0.090          | 0.023          | 0.006          | 0.000          | 177            |
| 2005 | 0.177          | 0.133          | 0.199          | 0.177          | 0.133          | 0.105          | 0.061          | 0.011          | 0.006          | 0.000          | 181            |
| 2006 | 0.172          | 0.217          | 0.152          | 0.182          | 0.101          | 0.086          | 0.066          | 0.020          | 0.005          | 0.000          | 198            |
| 2007 | 0.132          | 0.155          | 0.218          | 0.126          | 0.161          | 0.086          | 0.057          | 0.034          | 0.029          | 0.000          | 174            |
| 2008 | 0.136          | 0.130          | 0.234          | 0.117          | 0.162          | 0.117          | 0.052          | 0.026          | 0.026          | 0.000          | 154            |
| 2009 | 0.207          | 0.126          | 0.163          | 0.126          | 0.163          | 0.104          | 0.052          | 0.037          | 0.022          | 0.000          | 135            |
| 2010 | 0.101          | 0.173          | 0.173          | 0.180          | 0.137          | 0.065          | 0.101          | 0.065          | 0.007          | 0.000          | 139            |
| 2011 | 0.100          | 0.180          | 0.173          | 0.113          | 0.187          | 0.107          | 0.067          | 0.067          | 0.007          | 0.000          | 150            |
| 2012 | 0.104          | 0.147          | 0.160          | 0.153          | 0.166          | 0.129          | 0.074          | 0.067          | 0.000          | 0.000          | 163            |
| 2013 | 0.087          | 0.157          | 0.163          | 0.099          | 0.215          | 0.105          | 0.087          | 0.081          | 0.006          | 0.000          | 172            |
| 2014 | 0.144          | 0.119          | 0.129          | 0.154          | 0.144          | 0.139          | 0.104          | 0.050          | 0.015          | 0.000          | 201            |
| 2015 | 0.130          | 0.137          | 0.134          | 0.153          | 0.141          | 0.153          | 0.099          | 0.038          | 0.015          | 0.000          | 262            |
| 2016 | 0.119          | 0.119          | 0.131          | 0.153          | 0.134          | 0.147          | 0.131          | 0.050          | 0.016          | 0.000          | 320            |
| 2017 | 0.154          | 0.101          | 0.131          | 0.157          | 0.143          | 0.134          | 0.101          | 0.067          | 0.012          | 0.000          | 434            |
| 2018 | 0.127          | 0.125          | 0.160          | 0.141          | 0.143          | 0.119          | 0.119          | 0.053          | 0.013          | 0.000          | 455            |
| 2019 | 0.106          | 0.134          | 0.171          | 0.128          | 0.134          | 0.143          | 0.117          | 0.058          | 0.009          | 0.000          | 462            |
| 2020 | 0.080          | 0.143          | 0.158          | 0.143          | 0.150          | 0.158          | 0.116          | 0.048          | 0.006          | 0.000          | 526            |
| 2021 | 0.096          | 0.140          | 0.140          | 0.155          | 0.142          | 0.162          | 0.100          | 0.056          | 0.009          | 0.000          | 550            |
Table A.4: Descriptions of the share rate of the top10 shareholders for state-owned firms on the main board.

| Year | Top1 Shareholder | Top2-10 Shareholders | \(N_{sample}\) |
|------|------------------|-----------------------|----------------|
|      | Mean             | S.D.                  | Mean           | S.D.          |
| 1992 | 0.311            | 0.084                 | 0.332          | 0.187         | 9              |
| 1993 | 0.288            | 0.148                 | 0.222          | 0.141         | 70             |
| 1994 | 0.301            | 0.138                 | 0.210          | 0.130         | 94             |
| 1995 | 0.318            | 0.114                 | 0.240          | 0.134         | 117            |
| 1996 | 0.328            | 0.105                 | 0.237          | 0.130         | 232            |
| 1997 | 0.329            | 0.105                 | 0.224          | 0.126         | 314            |
| 1998 | 0.335            | 0.100                 | 0.223          | 0.122         | 357            |
| 1999 | 0.332            | 0.100                 | 0.234          | 0.126         | 391            |
| 2000 | 0.327            | 0.100                 | 0.232          | 0.130         | 458            |
| 2001 | 0.327            | 0.100                 | 0.227          | 0.130         | 502            |
| 2002 | 0.329            | 0.100                 | 0.232          | 0.130         | 514            |
| 2003 | 0.333            | 0.100                 | 0.234          | 0.130         | 508            |
| 2004 | 0.334            | 0.095                 | 0.243          | 0.130         | 525            |
| 2005 | 0.328            | 0.095                 | 0.241          | 0.130         | 540            |
| 2006 | 0.318            | 0.105                 | 0.205          | 0.126         | 644            |
| 2007 | 0.316            | 0.110                 | 0.193          | 0.126         | 661            |
| 2008 | 0.314            | 0.110                 | 0.185          | 0.126         | 658            |
| 2009 | 0.313            | 0.105                 | 0.186          | 0.126         | 648            |
| 2010 | 0.316            | 0.110                 | 0.184          | 0.130         | 661            |
| 2011 | 0.315            | 0.105                 | 0.184          | 0.130         | 651            |
| 2012 | 0.317            | 0.105                 | 0.185          | 0.134         | 638            |
| 2013 | 0.317            | 0.105                 | 0.191          | 0.134         | 625            |
| 2014 | 0.317            | 0.100                 | 0.191          | 0.130         | 636            |
| Year | Value_1 | Value_2 | Value_3 | Value_4 | Value_5 |
|------|---------|---------|---------|---------|---------|
| 2015 | 0.319   | 0.100   | 0.206   | 0.130   | 651     |
| 2016 | 0.317   | 0.100   | 0.220   | 0.130   | 667     |
| 2017 | 0.318   | 0.100   | 0.225   | 0.130   | 681     |
| 2018 | 0.321   | 0.100   | 0.230   | 0.134   | 675     |
| 2019 | 0.320   | 0.099   | 0.235   | 0.136   | 683     |
| 2020 | 0.319   | 0.101   | 0.234   | 0.132   | 703     |
| 2021 | 0.321   | 0.102   | 0.229   | 0.129   | 730     |
Table A.5: Descriptions of the share rate of the top10 shareholders for private-owned firms on the SME and GEM boards.

| Year | Top1 Shareholder |          |          |          |          |          |          |          |
|------|-----------------|----------|----------|----------|----------|----------|----------|----------|
|      | Mean | S.D. | Mean | S.D. | Na sample |          |          |          |
| 2004 | 0.318 | 0.109 | 0.379 | 0.109 | 28        |          |          |          |
| 2005 | 0.294 | 0.093 | 0.309 | 0.104 | 37        |          |          |          |
| 2006 | 0.307 | 0.093 | 0.350 | 0.109 | 68        |          |          |          |
| 2007 | 0.316 | 0.095 | 0.359 | 0.106 | 127       |          |          |          |
| 2008 | 0.326 | 0.102 | 0.347 | 0.106 | 172       |          |          |          |
| 2009 | 0.320 | 0.102 | 0.335 | 0.105 | 238       |          |          |          |
| 2010 | 0.314 | 0.103 | 0.354 | 0.110 | 475       |          |          |          |
| 2011 | 0.313 | 0.102 | 0.350 | 0.107 | 670       |          |          |          |
| 2012 | 0.311 | 0.101 | 0.333 | 0.112 | 775       |          |          |          |
| 2013 | 0.305 | 0.102 | 0.304 | 0.112 | 785       |          |          |          |
| 2014 | 0.301 | 0.104 | 0.289 | 0.118 | 881       |          |          |          |
| 2015 | 0.292 | 0.104 | 0.293 | 0.119 | 1,016     |          |          |          |
| 2016 | 0.283 | 0.104 | 0.296 | 0.116 | 1,138     |          |          |          |
| 2017 | 0.283 | 0.103 | 0.309 | 0.119 | 1,347     |          |          |          |
| 2018 | 0.278 | 0.102 | 0.302 | 0.115 | 1,363     |          |          |          |
| 2019 | 0.271 | 0.103 | 0.291 | 0.115 | 1,404     |          |          |          |
| 2020 | 0.266 | 0.104 | 0.278 | 0.115 | 1,484     |          |          |          |
| 2021 | 0.262 | 0.105 | 0.272 | 0.116 | 1,605     |          |          |          |
Table A.6: Descriptions of the share rate of the top10 shareholders for state-owned firms on the SME and GEM boards.

| Year | Top1 Shareholder | Top2-10 Shareholders | $N_{sample}$ |
|------|------------------|----------------------|--------------|
|      | Mean             | S.D.                 | Mean         | S.D. |  |
| 2004 | 0.290            | 0.048                | 0.451        | 0.051 | 5  |
| 2005 | 0.302            | 0.101                | 0.283        | 0.155 | 10 |
| 2006 | 0.343            | 0.107                | 0.299        | 0.119 | 22 |
| 2007 | 0.341            | 0.109                | 0.295        | 0.123 | 39 |
| 2008 | 0.335            | 0.104                | 0.287        | 0.124 | 44 |
| 2009 | 0.343            | 0.100                | 0.274        | 0.124 | 53 |
| 2010 | 0.325            | 0.101                | 0.309        | 0.144 | 73 |
| 2011 | 0.323            | 0.096                | 0.290        | 0.129 | 85 |
| 2012 | 0.326            | 0.099                | 0.275        | 0.127 | 94 |
| 2013 | 0.320            | 0.096                | 0.267        | 0.128 | 96 |
| 2014 | 0.322            | 0.099                | 0.246        | 0.121 | 98 |
| 2015 | 0.310            | 0.099                | 0.245        | 0.115 | 106 |
| 2016 | 0.306            | 0.102                | 0.255        | 0.113 | 118 |
| 2017 | 0.303            | 0.097                | 0.268        | 0.117 | 126 |
| 2018 | 0.296            | 0.101                | 0.268        | 0.114 | 143 |
| 2019 | 0.282            | 0.097                | 0.272        | 0.111 | 185 |
| 2020 | 0.278            | 0.095                | 0.263        | 0.104 | 227 |
| 2021 | 0.278            | 0.094                | 0.254        | 0.112 | 269 |
Table A.7: Correlation of $R_{SPI=1}$, $M_{top1}$, and $M_{top2−10}$ with the economic climate of China in 1996–2021 (Pearson correlation coefficient).

|                                | $R_{SPI=1}$ | $M_{top1}$ | $M_{top2−10}$ |
|--------------------------------|-------------|------------|---------------|
| Shanghai securities composite index highest in the year | 0.214       | -0.545     | -0.303        |
| (0.294)                        | (0.004)     | (0.132)    |               |
| GDP per capita (adjusted by consumer price index benchmarked against 1978; CNY) | -0.035      | -0.435     | 0.060         |
| (0.867)                        | (0.026)     | (0.771)    |               |
| Growth rate of GDP per capita (adjusted by consumer price index benchmarked against 1978; year to year) | 0.092       | -0.256     | -0.262        |
| (0.653)                        | (0.207)     | (0.196)    |               |

Notes: Figures in the cells, from top to bottom, are the correlation coefficient and p-value.