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State estimation-based control of COVID-19 epidemic before and after vaccine development

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ABSTRACT

In this study, a nonlinear robust control policy is designed together with a state observer in order to manage the novel coronavirus disease (COVID-19) outbreak having an uncertain epidemiological model with unmeasurable variables. This nonlinear model for the COVID-19 epidemic includes eight state variables (susceptible, exposed, infected, quarantined, hospitalized, recovered, deceased, and insusceptible populations). Two plausible scenarios are put forward in this article to control this epidemic before and after its vaccine invention. In the first scenario, the social distancing and hospitalization rates are employed as two applicable control inputs to diminish the exposed and infected groups. However, in the second scenario after the vaccine development, the vaccination rate is taken into account as the third control input to reduce the susceptible populations, in addition to the two objectives of the first scenario. The proposed feedback control measures are defined in terms of the hospitalized and deceased populations due to the available statistical data, while other unmeasurable compartmental variables are estimated by an extended Kalman filter (EKF). In other words, the susceptible, exposed, infected, quarantined, recovered, and insusceptible individuals cannot be identified precisely because of the asymptomatic infection of COVID-19 in some cases, its incubation period, and the lack of an adequate community screening. Utilizing the Lyapunov theorem, the stability and bounded tracking convergence of the closed-loop epidemiological system are investigated in the presence of modeling uncertainties. Finally, a comprehensive simulation study is conducted based on Canada's reported cases for two defined timing plans (with different treatment rates). Obtained results demonstrate that the developed EKF-based control scheme can achieve desired epidemic goals (exponential decrease of infected, exposed, and susceptible people).

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1. Introduction

Lately, the study of social networks and the epidemiological analysis of diseases have attracted considerable attention, and extensive research has been carried out in these areas due to their prominent role in human life [1]. Social network studies have been conducted by researchers to analyze disease transmission, which enabled them to comprehend the behavior and geographic bridges of corresponding possible spread [2–6]. In addition, social networks provide practical methodologies to simulate diseases epidemic by developing mathematical models to predict critical features of outbreaks (including the size and variation of its response).

Mathematical models have been developed as practical tools for predicting the evolution of an epidemic by synthesizing disease information and assessing its therapy process and transmission control [7]. An epidemiological model has been proposed in [8] for HIV/AIDS considering all people in the community as susceptible cases. Since HIV may be transmitted only by body fluids’ contact, its spread can be successfully controlled with teaching programs and protective care policies, which result in wise behaviors of individuals as modeled in their work. A compartmental model of influenza having vaccination and antiviral treatment has been developed in [9] together with corresponding mathematical analyses. A generalized susceptible–exposed–infected–recovered (SEIR) model for the malaria outbreak has been presented in [10] considering time-varying human and mosquito populations. Another mathematical transmission model of the hepatitis C virus (HCV) among people who inject drugs has been investigated in [11].
The 2019 novel coronavirus, termed as COVID-19 or SARS-CoV-2 by the World Health Organization (WHO), causes severe acute respiratory difficulties, chest pain/pressure, and loss of speech/movement as its serious symptoms. SARS-CoV-2 affects the respiratory system similar to the influenza virus, with some common symptoms such as cough, fever, exhaustion, and breathlessness. The rapid pandemic of COVID-19 all over the world has become a matter of grave concern and has hugely altered the lifestyle and social behavior of humans from the beginning of 2020. Indeed, it poses considerable economic, environmental, and political challenges for the entire human population. This is mainly due to the fact that the best possible defense strategy against this pandemic is social distancing and most governments all around the world have been forced to impose lock-down policies, which definitely lead to significant political and economic disruptions. In addition, the COVID-19 pandemic exerts a profound impact on people’s primary care provisions such as prenatal care for pregnant women, immunizing children against other viruses via vaccination, and assisting individuals with physical and mental disorders. Consequently, considerable research effort has been made to investigate precise mathematical models for the outbreak of this newborn virus and rapid estimation of its future transmission and mortality rates. In this regard, Ivorra et al. [12] have developed a compartmental model for COVID-19 considering the fraction of detected cases over the real total infected cases as an essential factor. Yang et al. [13] have also tried to predict the COVID-19 epidemic having population migration information and using an artificial intelligence (AI) approach that was trained on the 2003 SARS spread data. In order to characterize the dynamics of the COVID-19 outbreak and plan for clinical progression and quarantine intervention measures, an SEIR epidemiological model (with susceptible, exposed, infectious, and recovered compartments) has been proposed in [14].

Some other research studies have been performed on the control of infectious diseases as a significant concern of different human communities [15–18]. For instance, a nonlinear robust adaptive integral-sliding-mode control (ISM) scheme has been put forward to control the malaria outbreak with an uncertain dynamic model in [19]. A nonlinear compartmental model for an infectious disease with applying optimal control method was proposed in [20]. A compartmental dynamic model of asymptomatic carrier zika virus was proposed in [21], and an optimal control method was developed considering bednets, insecticide spray and treatment as three control inputs. An effective networked epidemic control system has been designed in [22] based on a swarm-based stochastic optimization policy for a modified susceptible–exposed–infected–vigilant (SEIV) model of infectious diseases spread. However, there are always some considerable differences between the realistic behavior of disease and the response of its mathematical model that arise from dynamic uncertainties and inaccuracies, which are rooted in various factors such as seasonal effects and the population structure. Early resolution of uncertainty during an epidemic can lead to effective decision making and the rapid prioritization of actions [23]. Accordingly, the need for the development of robust and/or adaptive control methods [24–26] emerges to control dynamical systems in the presence of modeling uncertainties. For instance, an adaptive control scheme [29] has been employed for cancer chemotherapy to reduce the tumor volume. A robust super-twisting sliding-mode control for an uncertain HIV infection model was proposed in [30] to decrease the concentration level of infected cells. In order to control the insulin delivery for Type 1 diabetes, a data-driven robust model-predictive control has been suggested in [31] to automatically regulate the insulin therapy considering model uncertainties correspond to daily diet and behaviors of the patient, which significantly affect the blood glucose. In addition, without reliable incidence measures and substantial measurement feedback, it is impossible to predict the epidemic’s growth rate, which makes its transmissibility estimate highly uncertain [32]. For this reason, a state estimation-based robust control method has been implemented in [33] based on the H-infinity Kalman Filter for the acute inflammatory response without requiring the measurement of the entire state vector of its model. The extended Kalman filter (EKF) technique has been used as the state observer to approximate the likelihood of Ebola disease spread [34]. A state estimation-based optimal control strategy has been presented in [35] that only needs the numbers of exposed and infected humans as the feedback signals to control the influenza epidemic in an interactive society.

In this study, employing a new generalized epidemiological SEIR model of COVID-19 [36], a state estimation-based nonlinear robust control policy is developed for the first time to control the COVID-19 epidemic in the presence of modeling uncertainties and incomplete measurement feedback. In order to tackle the problem of identifying susceptible, exposed, infectious, and infected populations as a serious measurement limitation on these groups in the real situation, an observer-based control strategy is proposed in this work. In this framework, the numbers of hospitalized and deceased people with bounded inaccuracies in statistical data are taken into account as the only available feedback signals while other compartmental variables (states) are estimated through the EKF observer.

This article puts forward two control schemes to prevent the COVID-19 outbreak before and after the vaccine development. The first strategy aims to reduce and eradicate the number of exposed and infected humans by employing social distancing and hospitalization & treatment as two primary control measures. The second strategy is proposed to be implemented after the vaccine development having the vaccination rate as the third control input in addition to the previous ones in the first strategy. The closed-loop system stability and the bounded tracking convergence to the desired populations are guaranteed using the Lyapunov method. The robustness of the controlled system in the presence of observation errors and modeling uncertainties is proven in the stability analysis for both control scenarios.

The present paper is organized as follows. Section 2 describes the nonlinear mathematical model of the COVID-19 outbreak, state estimation algorithm, and the problem formulation. Section 3 is devoted to the design and details of the proposed control policies, and the corresponding Lyapunov stability analysis. Section 4 put forward various numerical simulations considering the current state of the COVID-19 outbreak in Canada as the initial condition, demonstrating the effectiveness of the suggested control policy. Finally, the concluding remarks are summarized in Section 5.

2. Epidemiological model of COVID-19, state estimation, and problem formulation

2.1. Nonlinear mathematical model of COVID-19 spread

In this section, the generalized epidemiological SEIR model of COVID-19 [36] is presented with seven states including the susceptible (S), exposed (E), infected (I), quarantined (Q), recovered (R), deceased (D), and susceptible (P) population. A conceptual flow diagram for visualization and the relations of its dynamics is shown in Fig. 1. The susceptible compartment (S), enters the exposed group (E) when the infected people (I) comes into contact with them at a rate of $\beta$. Besides, improvement of public health such as cancellation of large public gatherings, wearing face masks, and
school closures, results in a corresponding decrease in the susceptible population ($S$) with protection rate $\alpha$. The people in the exposed group ($E$) proceeds to the infective compartment at the rate $\gamma$. Likewise, the infected individuals ($I$) leave the compartment at the rate $\delta$ and go to the quarantined group ($Q$). Finally, quarantined members ($Q$) go to recovered class ($R$) with cure rate $\lambda$, whereas the rest proceeds to the deceased compartment ($D$) with mortality rate $\kappa$. Consequently, COVID-19 epidemic dynamics with seven variables are defined as follows

$$
\begin{align*}
\dot{S}(t) &= -\beta S(t)I(t) - \alpha S(t) \\
\dot{E}(t) &= \beta S(t)I(t) - \gamma E(t) \\
\dot{I}(t) &= \gamma E(t) - \delta I(t) \\
\dot{Q}(t) &= \delta I(t) - \lambda Q(t) - \kappa Q(t) \\
\dot{R}(t) &= \lambda Q(t) \\
\dot{D}(t) &= \kappa Q(t) \\
\dot{P}(t) &= \alpha S(t)
\end{align*}
$$

(1)

In this paper, the dynamics (1) is modified to include the control inputs of social distancing ($\sigma(t)$), hospitalization & treatment ($\tau(t)$) and vaccination ($\nu(t)$) rates together with a hospitalized compartment ($H$) as follows:

$$
\begin{align*}
\dot{S}(t) &= -\beta S(t)I(t)(1 - \sigma(t)) - \alpha S(t) - \nu(t)S(t) \\
\dot{E}(t) &= \beta S(t)I(t)(1 - \sigma(t)) - \gamma E(t) \\
\dot{I}(t) &= \gamma E(t) - \delta I(t) - \tau(t)I(t) \\
\dot{Q}(t) &= \delta I(t) - \lambda Q(t) - \kappa Q(t) \\
\dot{H}(t) &= \tau(t)I(t) - \epsilon H(t) - \eta H(t) \\
\dot{R}(t) &= \lambda Q(t) + \nu(t)S(t) + \epsilon H(t) \\
\dot{D}(t) &= \kappa Q(t) + \eta H(t) \\
\dot{P}(t) &= \alpha S(t)
\end{align*}
$$

(2)

Similar to the dynamics of the quarantined compartment ($Q$), the hospitalized people transfer to the recovered class ($R$) with the cure rate of $\lambda$, whereas the rest proceeds to the deceased compartment ($D$) with the mortality rate of $\kappa$. The control input $\sigma(t) \in [0, 1]$ is the rate of reduction in contact between susceptible ($S$) and infected ($I$) compartments. The control input $\tau(t)$ denotes the rate of hospitalization & treatment of the infected people. In the same way, $\nu(t)$ is the rate of vaccination of susceptible people during the treatment period.

In this article, we investigate two different scenarios for controlling the outbreak of COVID-19. The first scenario mainly concerns the lack of vaccination for COVID-19 and try to prevent its outbreak by utilizing social distancing ($\sigma(t)$) and hospitalization ($\tau(t)$) of the infected people. The second scenario is proposed to investigate the predictable effect of vaccination ($\nu(t)$) for susceptible people ($S$), after its development, in addition to the previous control inputs; this, in turn, reduces social distancing. Fig. 2 depicts the schematic diagram of the developed nonlinear adaptive controller.

2.2. State estimation using extended Kalman filter (EKF)

In this section, the EKF algorithm [37] is employed to estimate the compartmental states of the COVID-19 dynamics (2). The reason why a state estimation is required is two-fold. (a) It is impossible to accurately measure the numbers of susceptible ($S$), susceptible ($P$), exposed ($E$), and infected ($I$) people. (b) Other states’ measurements are typically noisy, so it might not be convenient to use them in the real-time control design.

In general, the nonlinear mathematical model of the COVID-19 epidemic (2) and its state measurement would be described as follows

$$
\begin{align*}
\dot{x} &= f(x, u, \theta, w(t), t) \text{ with } w(t) \sim (0, Q) \\
y &= h(x, v) \text{ with } v(t) \sim (0, R)
\end{align*}
$$

(3)

(4)

where $f(\cdot)$ denotes the outbreak dynamics and $h(\cdot)$ is the measurement equation, with $y \in \mathbb{R}^{2 \times 1}$ as the hospitalized ($H$) and deceased ($D$) populations. The variables $w \in \mathbb{R}^8$ and $v \in \mathbb{R}^2$ are the zero-mean Gaussian white process noise and the measurement noise, respectively, and $Q, R \in \mathbb{R}^{2 \times 2}$ are their covariance matrices. The initial estimation of this filter is presented as

$$
\begin{align*}
\hat{x}_0 &= \epsilon [x_0] \\
P_0 &= \epsilon \left[ (x_0 - \hat{x}_0) (x_0 - \hat{x}_0)^T \right]
\end{align*}
$$

(5)

where $\epsilon[\cdot]$ is the expected value operation and $P_0$ stands for the initial covariance matrix of the state estimation error. Besides, $x_0$ and $\hat{x}_0$ represent the initial values of the state vector and its estimation.

The time-update relations for the estimated state vector and the estimation error’s covariance matrix are given as

$$
\begin{align*}
\dot{\hat{x}} &= f(\hat{x}, u, 0, t) + K(y - h(\hat{x}, 0, t)) \\
P &= AP + PA^T + LQI - PHI (MRM^{-1}) \text{ (6)}
\end{align*}
$$

(6)

where $A$ and $L$ are the Jacobian matrices of $f$ w.r.t $x$ and $w$. Similarly, $H$ and $M$ are the partial derivatives of $h$ w.r.t $x$ and $v$. Furthermore, $K$ is the Kalman gain that is defined as

$$
K = PHI (MRM^{-1})^{-1}
$$

(7)

Although the convergence of the estimator to the true states cannot be guaranteed, to carry out a detailed stability analysis, the following common assumption is made to prove the boundedness of the estimation error using a continuous-time extended Kalman filter.
Property 1. The norm of the Kalman filter estimation error is bounded by a positive scalar $\|\Delta \hat{e}\| \leq \varepsilon$ if the initial estimation and the disturbing noise terms are bounded [38]. Therefore, all of the estimation errors, denoted by $\Delta \hat{e}_i$, $i = S, E, I, Q, R, D, P$ (e.g. $E + \Delta \hat{e}_E = \hat{E}$), are bounded.

The next section will formulate a nonlinear state-estimation-based control while utilizing the estimates of the system’s states provided by the EKF algorithm.

3. Development of state-estimation-based nonlinear control schemes

3.1. First control scenario

In this scenario, the rates of social distancing $\sigma(t)$ and hospitalization $\tau(t)$ of infected people are employed as two applicable control inputs to manage the COVID-19 outbreak.

3.1.1. Control design

A state-estimation-based sliding mode control is developed to decrease the number of exposed ($\hat{E}$) and infected ($\hat{I}$) people using the social distancing ($u_1 = \sigma(t)$) and the hospitalization ($u_2 = \tau(t)$), respectively, via tracking their desired descending values. State-space representation of the COVID-19 epidemic including these two control rates is represented based on (2) as

$$
\dot{\hat{S}} = -\beta SI (1 - u_1) - \alpha S
$$

$$
\dot{\hat{E}} = \beta SI (1 - u_1) - \gamma E
$$

$$
\dot{\hat{I}} = \gamma E - \delta I - u_2 I
$$

$$
\dot{\hat{Q}} = \delta I - \lambda Q - \kappa Q
$$

$$
\dot{\hat{H}} = u_2 I - \xi H - \eta H
$$

$$
\dot{\hat{R}} = \lambda Q + \zeta H
$$

$$
\dot{\hat{D}} = \kappa Q + \eta H
$$

$$
\dot{\hat{p}} = \alpha S
$$

The dynamics of exposed ($\hat{E}$) and infected ($\hat{I}$) compartments can be rewritten in terms of the control inputs $u_1$ and $u_2$ as

$$
\dot{\hat{E}} = \frac{\hat{E}}{\beta SI} + 1 - \frac{\gamma E}{\beta SI} = u_1
$$

$$
\dot{\hat{I}} - \delta + \gamma \frac{\hat{E}}{\hat{I}} = u_2
$$

Now, the nonlinear robust control laws for the rates of social distancing and hospitalization, considering uncertainty on the actual dynamic parameters and inaccuracy of state estimation, are defined as

$$
u_1 = \frac{-\left(\hat{E} - \sigma_1 \hat{E}\right)}{\beta SI} + 1 - \frac{\hat{E}}{\beta SI} + \frac{\xi_1}{\beta SI} \tanh(k_1 \hat{E})
$$

$$
u_2 = \frac{-\left(u_2 - \sigma_2 I\right)}{I} - \delta + \frac{\hat{E}}{I} + \frac{\xi_2}{I} \tanh(k_2 \hat{I})
$$

where $\hat{p}$, $\hat{g}$ and $\hat{d}$ are the estimated parameters of the COVID-19 spread dynamics as the accurate values of these parameters are considered to be unknown for any specific community in the proposed control scheme. $\hat{S}$, $\hat{E}$ and $\hat{I}$ are the estimation of epidemic states obtained by the EKF observer. $\xi_1$ and $\xi_2$ are positive constants to compensate for the estimation errors and uncertain dynamic terms. In addition, $\hat{E}$ and $\hat{I}$ are the tracking errors of the exposed and infected populations that are formulated as

$$
\hat{E} = \hat{E} - E_d, \quad \hat{I} = \hat{I} - I_d
$$

where $E_d$ and $I_d$ are the desired values for the number of exposed and infected individuals. Regarding the above equations, the true tracking errors between the desired and actual state values are defined as

$$E - E_d = \hat{E} - \Delta \hat{e}_E, \quad I - I_d = \hat{I} - \Delta \hat{e}_I
$$

where $\Delta \hat{e}_E = \hat{E} - E$ and $\Delta \hat{e}_I = \hat{I} - I$ are the estimation errors of the state observer. It will be proven that employing the proposed nonlinear robust control laws (11) and (12), the exposed $E$ and infected $I$ populations will converge to their desired values in the presence of modeling uncertainties of the COVID-19 outbreak.

3.1.2. Closed-loop dynamics

Now, the closed-loop dynamics of the controlled process using the proposed state-estimation-based scheme is analyzed in the presence of uncertainties and bounded estimation errors. Accordingly, the control laws (11) and (12) are substituted in the exposed (9) and infected (10) dynamics, respectively. Besides, adding and subtracting the terms $\frac{\hat{E}}{\beta SI}$ and $\frac{\hat{I}}{I}$ to the dynamics of exposed and infected compartments, respectively,

$$
\frac{\hat{E}}{\beta SI} + 1 - \frac{\gamma E}{\beta SI} + \frac{\hat{E}}{\beta SI} - \frac{\hat{E}}{\beta SI} = \frac{\hat{E} - \sigma_1 \hat{E}}{\beta SI} + 1 - \frac{\gamma E}{\beta SI} + \frac{\xi_1}{\beta SI} \tanh(k_1 \hat{E})
$$

$$
\frac{\hat{I}}{I} - \delta + \gamma \frac{\hat{E}}{\hat{I}} + \frac{\hat{I}}{I} = \frac{\hat{I} - \sigma_2 \hat{I}}{I} - \delta + \gamma \frac{\hat{E}}{\hat{I}} + \frac{\xi_2}{I} \tanh(k_2 \hat{I})
$$

By rearranging the above equations, one can write

$$
\frac{\hat{E}}{\beta SI} + \frac{\hat{E}}{\beta SI} - \frac{\hat{E}}{\beta SI} = \frac{\hat{E} + \sigma_1 \hat{E}}{\beta SI} - \frac{\xi_1}{\beta SI} \tanh(k_1 \hat{E}) = \gamma E - \frac{\gamma E}{\beta SI} + \frac{\xi_1}{\beta SI} \tanh(k_1 \hat{E})
$$

$$
\frac{\hat{I}}{I} + \frac{\hat{I}}{I} - \frac{\hat{I}}{I} = \frac{\hat{I} + \sigma_2 \hat{I}}{I} - \frac{\xi_2}{I} \tanh(k_2 \hat{I}) = \delta - \gamma E - \frac{\gamma E}{I} + \frac{\xi_2}{I} \tanh(k_2 \hat{I})
$$

Therefore, the closed-loop dynamics of the controlled system is simplified and obtained as

$$
\dot{\hat{E}} = -\sigma_1 \hat{E} - \xi_1 \tanh(k_1 \hat{E}) + \frac{\hat{E}}{\beta SI} + \frac{\xi_1}{\beta SI} \tanh(k_1 \hat{E})
$$

$$
\dot{\hat{I}} = -\sigma_2 \hat{I} - \xi_2 \tanh(k_2 \hat{I}) + \frac{\hat{I}}{I} - \frac{\xi_2}{I} \tanh(k_2 \hat{I})
$$

in which the uncertainty terms $\Delta_i$, $i = 1, \ldots, 4$, are described as follows

$$
\Delta_1 = \dot{\hat{E}} - \frac{\hat{E}}{\beta SI}\dot{\hat{E}}
$$

$$
\Delta_2 = \dot{\hat{I}} - \frac{\hat{I}}{I}\dot{\hat{I}}
$$

$$
\Delta_3 = \dot{\hat{E}} - \frac{\hat{E}}{\beta SI}\dot{\hat{E}} + \frac{1}{I}\gamma E
$$

$$
\Delta_4 = \dot{\hat{I}} - \frac{\hat{I}}{I}\dot{\hat{I}} - \frac{\hat{E}}{\hat{I}}\dot{\hat{E}} + \frac{1}{I}\gamma E
$$

Next, boundedness properties of the above uncertainty terms will be studied in detail to facilitate the stability proof.

- **Properties of the uncertainty terms**

To determine the limits of the uncertainty terms (21), we first take the following property into account.
Property 2. The total population of any particular community, at any time \( t \), would remain constant based on the epidemiological dynamics (8):
\[
S(t) + E(t) + I(t) + Q(t) + H(t) + R(t) + D(t) + P(t) = N
\] (22)

Also, it is worth mentioning that all compartmental variables have positive values (populations). Thus, it can be concluded that all of these variables remain bounded during the outbreak management time due to the boundedness of the total population \( N \).

Remark 1. According to Properties 1 and 2, the estimates of all system variables are bounded.

Remark 2. According to Properties 1 and 2, and also the boundedness of the desired scenario, it is concluded that \( \dot{E} \) and \( \dot{I} \) are bounded. (See Eq. (14)).

Lemma 1. The terms \( \dot{S}/S \) and \( \dot{I}/I \) are bounded.

Proof. Based on Property 1, \(|\Delta \hat{E}_S| \leq \varepsilon_1 \) and \(|\Delta \hat{E}_I| \leq \varepsilon_2 \). Additionally, due to the positiveness of all compartmental variables, the following inequalities are obtained:
\[
\frac{|\Delta \hat{E}_S|}{S} \leq \frac{\epsilon_1}{S} \leq \epsilon'_1 \quad (23)
\]
\[
\frac{|\Delta \hat{E}_I|}{I} \leq \frac{\epsilon_2}{I} \leq \epsilon'_2 \quad (24)
\]
in which \( \epsilon'_1 \) and \( \epsilon'_2 \) are also bounded. By reformulating the above equations and adding the terms \( S \) and \( I \) to both sides of Eqs. (23) and (24) and dividing them by \( S \) and \( I \), respectively, we have
\[
(1 - \epsilon'_1) < \frac{\dot{S}}{S} < (1 + \epsilon'_1) \quad (25)
\]
\[
(1 - \epsilon'_2) < \frac{\dot{I}}{I} < (1 + \epsilon'_2) \quad (26)
\]
This substantiates the boundedness of \( \dot{S}/S \) and \( \dot{I}/I \). \( \square \)

Then, each uncertainty term in (21), \( A_i \) for \( i = 1 - 4 \), will be expanded and expressed as a linear function of tracking errors \( \dot{E} \) and \( \dot{I} \) plus another bounded term.

(i) Term \( A_1 \): According to Lemma 1 and also the boundedness of the system parameters, the term \( \beta \hat{S}/SI \) is also bounded. Thus, \( A_1 \) can be rewritten as
\[
A_1 = \dot{E} - \frac{\beta \hat{S}}{SI} E = \dot{E} - \mu_1 \dot{E} \quad (27)
\]
\[
= \hat{E} - \mu_1 \dot{E} - \mu_1 \beta \hat{S}U_1 - \mu_1 \beta SI + \mu_1 \gamma E + \mu_1 SI u_1 + \mu_1 \beta SI \hat{E}
\]
Now, substituting the control law (11) and Eq. (14) into Eq. (27) leads to
\[
A_1 = \mu_1 \gamma E - \dot{\hat{E}} \quad (28)
\]
\[
= \mu_1 \gamma (E_d + \hat{E} + \Delta \hat{E}) - \dot{\hat{E}} (E_d + \hat{E})
\]
As a result, the term \( A_1 \) has an alternative form of \( A_1 = A_1 \hat{E} + W_1 \), where
\[
A_1 = \mu_1 \gamma - \dot{\hat{E}}
\]
\[
W_1 = (\mu_1 \gamma - \dot{\hat{E}}) E_d - \mu_1 \gamma \Delta \hat{E}
\]
Regarding Properties 1 and 2, all the terms in \( A_1 \) and \( W_1 \) are bounded and consequently \( A_1 \) is bounded in terms of the linear function of \( \|E\| \) and the bounded term \( W_1 \):
\[
\|A_1\| \leq \bar{A}_1 \|\hat{E}\| + W_1 \quad (30)
\]
where \( \|A_1\| \leq \bar{A}_1 \) and \( \|W_1\| \leq \bar{W}_1 \).

(ii) Term \( A_2 \): Based on Eq. (14), \( A_2 \) in (21) can be reformulated as
\[
A_2 = \gamma \dot{\hat{E}} - \frac{\beta SI}{\beta SI} \gamma E
\]
\[
= \gamma \dot{\hat{E}} - \mu \gamma E = \gamma \dot{\hat{E}} - \mu \gamma (E_d + \hat{E} + \Delta \hat{E}) \quad (31)
\]
\[
= -\mu \gamma \dot{E} + \gamma \hat{E} - \mu \gamma (E_d - \Delta \hat{E})
\]
Thereby, the term \( A_2 \) can be expressed as \( A_2 = A_2 \hat{E} + W_2 \) in which
\[
A_2 = -\mu \gamma
\]
\[
W_2 = \gamma \hat{E} - \mu \gamma (E_d - \Delta \hat{E}) \quad (32)
\]
As stated in Properties 1 and 2, all the terms in \( A_2 \) and \( W_2 \) are bounded. As a result, \( A_2 \) is bounded as the linear function of \( \|\hat{E}\| \) together with the bounded term \( W_2 \):
\[
\|A_2\| \leq \bar{A}_2 \|\hat{E}\| + W_2 \quad (33)
\]
where \( \|A_2\| \leq \bar{A}_2 \) and \( \|W_2\| \leq \bar{W}_2 \).

(iii) Term \( A_3 \): Similar to \( A_1 \), employing Lemma 1 and considering the boundedness of the term \( \dot{I}/I \), \( A_3 \) in (21) can be written as
\[
A_3 = \frac{1}{\beta SI \big(\delta - \hat{\delta}\big)} + \frac{\mu_2 \gamma E}{\beta SI \big(\delta - \hat{\delta}\big)} \quad (34)
\]
\[
= \frac{1}{\beta SI \big(\delta - \hat{\delta}\big)} + \frac{\mu_2 \gamma E}{\beta SI \big(\delta - \hat{\delta}\big)} \quad (35)
\]
Now, having the control law (12) and the true tracking errors (14), \( A_3 \) is reformulated as
\[
A_3 = \mu_2 \beta SI \big(\delta - \hat{\delta}\big) + \mu_2 \beta SI \big(\delta - \hat{\delta}\big) \quad (36)
\]
Taking the boundedness of parameters estimations into account and having Properties 1 and 2, it is concluded that all the terms in \( A_3 \) and \( W_3 \) are bounded. Thus, \( A_3 \) is bounded in terms of a linear function of \( \|I\| \) plus the bounded term \( W_3 \):
\[
\|A_3\| \leq \bar{A}_3 \|I\| + W_3 \quad (37)
\]
where \( \|A_3\| \leq \bar{A}_3 \) and \( \|W_3\| \leq \bar{W}_3 \).

(iv) Term \( A_4 \): Due to Eq. (14), \( A_4 \) in (21) is presented as
\[
A_4 = \big(\delta - \hat{\delta}\big) + \frac{1}{\beta SI \big(\delta - \hat{\delta}\big)} + \frac{\mu_2 \gamma E}{\beta SI \big(\delta - \hat{\delta}\big)} \quad (38)
\]
\[
= \big(\delta - \hat{\delta}\big) (I + \hat{l}_4) - \gamma \hat{E} + \mu_2 \gamma E
\]
\[
= \big(\delta - \hat{\delta}\big) I + \hat{l}_4 - \gamma \hat{E} + \mu_2 \gamma E
\]
Similar to the previous uncertainty term, \( A_4 \) can be rewritten as
\[
A_4 = \bar{A}_4 I + W_4 \quad (39)
\]
Since the parameters estimation is bounded and according to Properties 1 and 2, one can deduce that all terms in \( A_4 \) and \( W_4 \)
are bounded. As a result, \( A_4 \) is bounded as a linear function of \( \| I \| \) plus the bounded term \( W_4 \):

\[
\| \Delta_4 \| \leq A_4 \| I \| + \tilde{W}_4
\]

in which \( A_4 \leq \tilde{A}_4 \) and \( \| W_4 \| \leq \tilde{W}_4 \).

### 3.1.3. Stability analysis

The Lyapunov theorem is employed to prove the stability of the controlled system and tracking convergence to the desired numbers of exposed (\( E \)) and infected (\( I \)) people. To this aim, the following Lyapunov function candidate is utilized:

\[
V(t) = \frac{1}{2} \left[ \tilde{E}^2 + \tilde{I}^2 \right] \geq 0
\]

**Lemma 2.** Having a as a positive constant, the following inequality is always valid for any \( e \in \mathbb{R} \) [39]:

\[
0 \leq |e| - e \tanh(e/a) \leq ca
\]

where \( c = 0.2785 \).

**Theorem 1.** Consider the closed-loop dynamics (19) and (20), and the bounds of the uncertainty terms \( \Delta_i \) for \( i = 1 \) to \( 4 \), the boundedness of the tracking errors for the exposed (\( E \)) and infected (\( I \)) populations is guaranteed.

**Proof.** The time derivative of the Lyapunov function (41) is

\[
\dot{V}(t) = \tilde{E} \ddot{E} + \tilde{I} \ddot{I}
\]

Substituting the closed-loop dynamics (19) and (20) into (43), and considering Lemma 2, one can write:

\[
\dot{V}(t) \leq - (\sigma_1 - \tilde{A}) \| \tilde{E} \|^2 + (\tilde{W} - \zeta_1) \| \tilde{E} \| - (\sigma_2 - \tilde{B}) \| \tilde{I} \|^2 + (\tilde{W} - \zeta_2) \| \tilde{I} \| + ca (\zeta_1 + \zeta_2)
\]

where \( \tilde{A} = \tilde{A}_1 + \tilde{A}_2, \tilde{B} = \tilde{A}_2 + \tilde{A}_4, \tilde{W} = W_1 + W_2, \) and \( \tilde{W} = W_3 + W_4 \). Hence, the time derivative of the Lyapunov function is negative definite \( V(t) < 0 \) outside the following region:

\[
\| \tilde{E} \| \leq \frac{-D_2 + \sqrt{D_2^2 + 4D_3D_4}}{2D_1} = \mathcal{M}_1 \| \tilde{I} \| \leq \frac{-D_3 + \sqrt{D_3^2 + 4D_4D_5}}{2D_4} = \mathcal{M}_2
\]

where

\[
\begin{align*}
D_1 &= (\sigma_1 - \tilde{A}) \\
D_2 &= -(\tilde{W} - \zeta_1) \\
D_3 &= -(\sigma_2 - \tilde{B}) \\
D_4 &= (\sigma_2 - \tilde{B}) \\
D_5 &= -(\tilde{W} - \zeta_2) \\
D_6 &= -(\sigma_1 - \tilde{A}) (\| \tilde{E} \|^2 + (\tilde{W} - \zeta_1) \| \tilde{E} \| + ca (\zeta_1 + \zeta_2))
\end{align*}
\]

Accordingly, \( \tilde{E} \) and \( \tilde{I} \) converge to the following compact set within a finite time \( T \) and remain there \( \forall t \geq T \):

\[
\mathcal{S} = \left\{ (\tilde{E}, \tilde{I}) : \| \tilde{E} \| \leq \mathcal{M}_1 \land \| \tilde{I} \| \leq \mathcal{M}_2 \right\}
\]

### 3.2. Second control scenario

The second scenario is formulated to be employed since the vaccine development in which the vaccination rate \( v(t) \) is taken into account as the third control input to immunize and diminish the susceptible individuals in addition to the two other control objectives defined for the first scenario.
where $\varsigma_3$ is a positive constant to compensate for the estimation errors and uncertain dynamic terms. By reformulating the above equation, one can write:

$$
\dot{\hat{S}} = \ddot{\hat{S}} - \frac{\dddot{S}}{S} - \zeta_1 \frac{\dddot{S}}{S} - (\dot{S} + \sigma_3 \ddot{S}) \tanh(k_j \hat{S})
$$

(54)

Therefore, the closed-loop dynamics of the susceptible population is obtained as

$$
\dot{\hat{S}} = -\sigma_3 \ddot{S} - \zeta_3 \dddot{S} \tanh(k_j \hat{S}) + \dddot{S} - \frac{\dddot{S}}{S}
$$

(55)

where

$$
\Delta_5 = \ddot{\hat{S}} - \dddot{S} \frac{S}{\dot{S}} = -\mu_3 \ddot{S}
$$

(56)

Then, the properties of these two terms of uncertainty need to be studied in detail.

- **Properties of the uncertainty terms**

  (i) Term $\Delta_5$: According to Lemma 1, we can extend the term $\Delta_5$ in (56) as

$$
\Delta_5 = \ddot{\hat{S}} - \dddot{\hat{S}} \frac{\dot{S}}{S} = \ddot{\hat{S}} - \mu_3 \ddot{S}
$$

(57)

Now, by substituting the control law (50) and the true tracking error (52) into (57), it can be written that

$$
\Delta_5 = \mu_3 \ddot{S} \left( \hat{\beta} I (1 - u_i) - \hat{\beta} I (1 - u_i) + \alpha - \hat{\alpha} \right)
$$

$$
= \mu_3 \ddot{\hat{S}} \left( \ddot{\hat{S}} + S_d + \Delta \hat{S} \right) \left( \hat{\beta} I (1 - u_i) - \hat{\beta} I (1 - u_i) + \alpha - \hat{\alpha} \right)
$$

$$
= \mu_3 \ddot{\hat{S}} \left( \hat{\beta} I (1 - u_i) - \hat{\beta} I (1 - u_i) + \alpha - \hat{\alpha} \right)
$$

(58)

Therefore, $\Delta_5 = \mu_3 \ddot{\hat{S}} + W_5$ in which

$$
\Delta_5 = \mu_3 \left( \hat{\beta} I (1 - u_i) - \hat{\beta} I (1 - u_i) + \alpha - \hat{\alpha} \right)
$$

$$
W_5 = \mu_3 \left( S_d + \Delta \hat{S} \right) \left( \hat{\beta} I (1 - u_i) - \hat{\beta} I (1 - u_i) + \alpha - \hat{\alpha} \right)
$$

(59)

According to Properties 1 and 2, and also Remark 2, all of the terms in Eq. (11) are bounded. Hence, one can conclude that $u_i$ is bounded. As a result, all terms in $\Delta_5$ and $W_5$ are bounded, and consequently $\Delta_5$ is bounded as a linear function of $\|\hat{S}\|$ and $W_5$:

$$
\|\Delta_5\| \leq \bar{A}_5 \|\hat{S}\| + W_5
$$

(60)

where $\|\Delta_5\| \leq \bar{A}_5$ and $\|W_5\| \leq \bar{W}_5$.

(ii) Term $\Delta_6$: Using Eq. (51), the term $\Delta_6$ in (56) can be represented as

$$
\Delta_6 = \ddot{\hat{S}} \left( \hat{\beta} I - \beta I \right) (1 - u_i) + \ddot{\hat{S}} (\alpha - \hat{\alpha})
$$

$$
= \ddot{\hat{S}} \left( \ddot{\hat{S}} + S_d \right) \left( \hat{\beta} I - \beta I \right) (1 - u_i) + (\alpha - \hat{\alpha})
$$

$$
= \ddot{\hat{S}} \left( \hat{\beta} I - \beta I \right) (1 - u_i) + (\alpha - \hat{\alpha})
$$

$$
+ S_d \left( \hat{\beta} I - \beta I \right) (1 - u_i) + (\alpha - \hat{\alpha})
$$

(61)

This can be summarized as $\Delta_6 = A_6 \ddot{\hat{S}} + W_6$ where

$$
A_6 = \left( \hat{\beta} I - \beta I \right) (1 - u_i) + (\alpha - \hat{\alpha})
$$

$$
W_6 = S_d \left( \hat{\beta} I - \beta I \right) (1 - u_i) + (\alpha - \hat{\alpha})
$$

(62)

As described for Eq. (59), all terms in $A_6$ and $W_6$ are also bounded, which result in the boundedness of $\Delta_6$:

$$
\|\Delta_6\| \leq \bar{A}_6 \|\hat{S}\| + \bar{W}_6
$$

(63)

in which $\|\Delta_6\| \leq \bar{A}_6$ and $\|W_6\| \leq \bar{W}_6$.

### 3.2.3 Stability analysis

Similar to the first control scenario, the Lyapunov theorem is utilized to ensure the stability of the closed-loop system and the tracking convergence to the desired populations of exposed (E), infected (I) and susceptible (S) groups. Therefore, the following Lyapunov function candidate is taken into account

$$
V(t) = \frac{1}{2} \left( E^2 + I^2 + S^2 \right) \geq 0
$$

(64)

**Theorem 2.** Consider the closed-loop dynamics (19), (20), and (55), and the determined bounds of uncertainty terms $\Delta_i$ for $i = 1$ to 6, the boundedness of the tracking errors for the exposed (E), infected (I) and susceptible (S) compartments is ensured. When

**Proof.** The time derivative of the Lyapunov function (64) is

$$
\dot{V}(t) = \ddot{E} \ddot{E} + \ddot{I} \ddot{I} + \dddot{S} \ddot{S}
$$

(65)

Substituting the closed-loop dynamics (19), (20), and (55) into (65) and considering Lemma 2, one can write:

$$
\dot{V}(t) \leq - (\sigma_1 - \bar{A}) \|\ddot{E}\|^2 + (\bar{W} - \bar{C}) \|\ddot{E}\|
$$

$$
- (\sigma_2 - \bar{B}) \|\dddot{I}\|^2 + (\bar{W} - \bar{C}) \|\dddot{I}\|
$$

$$
- (\sigma_3 - \bar{C}) \|\ddot{S}\|^2 + (\bar{W} - \bar{C}) \|\ddot{S}\| + (\sigma_1 + \sigma_2 + \sigma_3)
$$

(66)

where $\bar{A} = \bar{A}_1 + \bar{A}_2$, $\bar{B} = \bar{A}_3 + \bar{A}_4$, $\bar{C} = \bar{A}_5 + \bar{A}_6$, $\bar{W} = \bar{W}_1 + \bar{W}_2$, $\bar{W} = \bar{W}_3 + \bar{W}_4$ and $\bar{W} = \bar{W}_5 + \bar{W}_6$. Then, similar to the previous section, the time derivative of the Lyapunov function is obtained negative definite ($\dot{V}(t) < 0$) outside the following region:

$$
\|\ddot{E}\| \leq -\frac{\bar{D}_2 + \sqrt{\bar{D}_2^2 + 4\bar{D}_1 \bar{D}_3}}{2\bar{D}_1} = H_1
$$

$$
\|\ddot{I}\| \leq -\frac{\bar{D}_2 + \sqrt{\bar{D}_2^2 + 4\bar{D}_1 \bar{D}_3}}{2\bar{D}_1} = H_2
$$

$$
\|\ddot{S}\| \leq -\frac{\bar{D}_2 + \sqrt{\bar{D}_2^2 + 4\bar{D}_1 \bar{D}_3}}{2\bar{D}_1} = H_3
$$

(67)
### Table 1
Parameters of the epidemiological COVID-19 model.

| Parameter | Description | Value |
|-----------|-------------|-------|
| $\alpha$ | Protection rate | 0.172 |
| $\beta$ | Transmission rate | 1 |
| $\gamma^{-1}$ | Latent time | 2 |
| $\delta^{-1}$ | Quarantine time | 0.1 |
| $\kappa$ | Cure rate of quarantined people | 0.025 |
| $\zeta$ | Mortality rate of quarantined people | 0.004 |
| $\eta$ | Cure rate of hospitalized people | 0.92 |
| $s_0$ | Initial values of susceptible case | 37.5 Million |
| $i_0$ | Initial values of infected case | 35000 |
| $q_0$ | Initial values of quarantined case | 34500 |
| $h_0$ | Initial values of hospitalized case | 40000 |
| $r_0$ | Initial values of recovered case | 47000 |
| $p_0$ | Initial values of deceased case | 7000 |
| $\tilde{p}_0$ | Initial values of insensitive case | 37500 |

where $\tilde{D}_1 = (\sigma_1 - \tilde{A})$

$$D_2 = -(W' - \zeta_1)$$

$$D_3 = -(\sigma_2 - $B$) \|\tilde{E}\|^2 + (W'' - \zeta_2) \|\tilde{I}\| - (\sigma_1 - \tilde{C}) \|\tilde{S}\|^2$$

$$D_4 = (\sigma_2 - $B$)$$

$$D_5 = -(W'' - \zeta_2)$$

$$D_6 = -(\sigma_1 - \tilde{A}) \|\tilde{E}\|^2 + (W'' - \zeta_1) \|\tilde{I}\| - (\sigma_1 - \tilde{C}) \|\tilde{S}\|^2$$

$$D_7 = (\sigma_3 - \tilde{C})$$

$$D_8 = -(W'' - \zeta_1)$$

$$D_9 = -(\sigma_1 - \tilde{A}) \|\tilde{E}\|^2 + (W'' - \zeta_1) \|\tilde{I}\| - (\sigma_2 - $B$) \|\tilde{I}\|^2$$

$$D_9 = -(\sigma_1 - \tilde{A}) \|\tilde{E}\|^2 + (W'' - \zeta_2) \|\tilde{I}\| + ca (\zeta_1 + \zeta_2 + \zeta_3)$$

Therefore, $\tilde{E}$, $\tilde{I}$, and $\tilde{S}$ converge to the following compact set within a finite time $T$ and remain there $\forall t \geq T$:

$$\mathcal{S} = \left\{ \tilde{E}, \tilde{I}, \tilde{S} : \|\tilde{E}\| \leq \mathcal{K}_1 \land \|\tilde{I}\| \leq \mathcal{K}_2 \land \|\tilde{S}\| \leq \mathcal{K}_3 \right\}$$

## 4. Simulation studies

In this section, simulation results are presented to verify the effectiveness of the proposed robust control approach for the COVID-19 epidemic in Canada while considering the first and second control scenarios for two defined timing plans (120 and 240 days). The initial values of the state variables are taken from the daily statistics reports of WHO as of the time of writing this article. The initial values along with the model parameters have been introduced by Peng et al. [38] and [40] as listed in Table 1. The following exponential functions are considered for the desired reduction scenarios of susceptible ($S$), exposed ($E$) and infected ($I$) people

$$S_d = (S_0 - S_f) \exp(-a_s t) + S_f$$

$$E_d = (E_0 - E_f) \exp(-a_e t) + E_f$$

$$I_d = (I_0 - I_f) \exp(-a_t t) + I_f$$

where $S_f$, $E_f$, and $I_f$ are the desired final populations of susceptible ($S$), exposed ($E$), and infected ($I$) compartments, respectively, which have been considered zero; $S_0$, $E_0$, $I_0$ are the initial population of these compartments; $a_s = 0.04$, $a_e = 0.08$, and $a_t = 0.04$ are the desired reduction rates for the first timing plan (120 days); and $a_s = 0.025$, $a_e = 0.045$, and $a_t = 0.025$ are the ones for the second timing plan (240 days). Besides, as mentioned previously, due to the fundamental limitations in measuring the numbers of susceptible ($S$), susceptible ($P$), exposed ($E$), and infected ($I$) people, it would be difficult to estimate the exact initial population of these compartments. Thus, the initial estimated values of susceptible ($S$), exposed ($E$), and infected ($I$) and in turn, quarantined ($Q$), recovered ($R$), and susceptible ($P$) groups are assumed to be $\tilde{S}_0 = 3.75 Million$, $\tilde{E}_0 = 30 000$, $\tilde{I}_0 = 35 000$, $\tilde{Q}_0 = 40 000$, $\tilde{R}_0 = 51 500$, and $\tilde{P}_0 = 375 000$, respectively.

Referring to the previous argument, in the real situation only the number of hospitalized individuals ($H$), who receive health care in clinical settings, and deceased ($D$) people are measurable. Hence, we disturb the hospitalized ($H$) and deceased ($D$) populations with a white Gaussian measurement noise as the feedback signal and try to estimate the other states using the EKF. Therefore, the observer matrix, the measurement noise covariance, the preprocess noise covariance, and the initial covariance of the estimation are considered as $h = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $R = 0.012$, $Q = I_6$, and $P_0 = 10I_6$, respectively.

The model parameters are perturbed by 70%. In the first control scenario, we decide to only employ the social distancing ($u_1$) and hospitalization ($u_2$) to decrease the populations of exposed ($E$) and infected ($I$) groups along with their desired reduction trajectory (see (71) and (72)). Figs. 3(a) and 3(b) illustrate the convergence of exposed and infected people during the two timing plans (120 and 240 days), respectively while utilizing the laws
Fig. 5. Control inputs applied to the COVID-19 epidemic during the first control scenario including (a) social distancing rate $u_1$ and (b) hospitalization rate $u_2$.

Fig. 6. Uncontrolled states of the COVID-19 epidemic during the first control scenario: (a) susceptible $S$, (b) quarantined $Q$, (c) hospitalized $H$, (d) recovered $R$, (e) deceased $D$, and (f) insusceptible $P$ populations.

It can be seen that the estimated states converge to their actual values demonstrating the EKF’s performance.

Fig. 4 depicts the tracking errors of exposed and infected groups with respect to their desired values ($E_d$ and $I_d$) for the two timing plans (120 and 240 days). The control inputs are shown in Figs. 5(a) and 5(b). To have a proper convergence of the tracking errors, the controller gains are tuned to be $\sigma_1 = \sigma_2 = 1$, $\zeta_1 = \zeta_2 = 2$, and $k_1 = k_2 = k_3 = 4$. As seen in Fig. 5, social distancing ($u_1$) and hospitalization rate ($u_2$) take higher values in the first timing plan (120 days) in comparison with the second ones (240 days). It is evident that social distancing never converges to zero. It implies that before the vaccine development, the social distancing policy should remain and it is definitely the most effective way of slowing down the spread of the virus.

Fig. 6 depicts the response of other states of the COVID-19 epidemiological model for the first control scenario. As predicted, by implementing only the social distancing policies, the number of susceptible people has a slow decay rate. In fact, the exposed people enter the susceptible compartment. However, due to the low protection rate ($\alpha$), the reduction rate of the susceptible people ($S$) is slow. Furthermore, utilizing the hospitalization rate $u_2$, the infected population ($I$) converges to zero (Fig. 3(b)). Thereby, based on the dynamics (8), the quarantined ($Q$) and hospitalized ($H$) compartments approaches zero, and consequently, the increase rate of the deceased compartment ($D$) also converges to zero, as seen in Figs. 6(b) and 6(c). Furthermore, the recovered people ($R$) increase bypassing the time.
As described above, the second control scenario studies the probable effect of the vaccination of susceptible people ($S$) after its development, in addition to the previous control inputs: social distancing ($u_1$) and hospitalization ($u_2$). Fig. 7 shows the convergence of susceptible ($S$), exposed ($E$), and infected ($I$) people, under the proposed control laws (11), (12) and (50) in the second control scenario for the two timing plans (120 and 240 days). In addition, the tracking errors of susceptible individuals with respect to its desired value ($S_d$) is shown in Fig. 8.

Fig. 9 illustrates rates of social distancing ($u_1$) for the exposed group ($E$), hospitalization ($u_2$) for the infected people ($I$), and vaccination ($u_3$) for the susceptible compartments ($S$) which are obtained from the control laws (11), (12), and (50). The controller parameters are adjusted as $\sigma_1 = \sigma_2 = \sigma_3 = 1$, $\xi_1 = \xi_2 = \xi_3 = 2$, and $k_1 = k_2 = k_3 = 4$. It can be easily noticed from Fig. 9 that the vaccination of susceptible people ($S$) prepares the way of reducing social distancing rate in comparison with the previous scenario (shown in Fig. 5).

The time integral of the terms $\sigma(t)$$\beta SL$, $\tau(t)L$, and $v(t)L$ in Eq. (48) during the second control scenario are shown in Fig. 10. These integral values indicate the number of people who should keep a safe space between themselves (for physical distancing), the treated population, and the vaccinated individuals in the control process. As seen in Fig. 10, the total numbers of people who need to do social distancing, be treated and get vaccinated are 438,830, 126,280, and 37,088,000, respectively, at the end of this control scenario for 240 days.

In addition, the response of other states of the COVID-19 epidemiological model in the second control scenario for the second timing plans (120 and 240 days) is shown in Fig. 11. Moreover, the estimator innovations in the first control scenario are presented in Fig. 12. For the sake of brevity, the variation of these innovations in the second control scenario are not illustrated, while they have similar behavior to the ones obtained for the first scenario.

To demonstrate the response of the COVID-19 epidemic in the absence of control measures, the populations of exposed, infected, quarantined, and deceased compartments are obtained and illustrated for this case in Fig. 13. Due to the herd immunity achieved by natural infection, the number of exposed people increases until 23rd day and then decreases to around zero by 51st day when the infected population experiences its maximum with 35.7 million people (95% of the whole population), as seen in Figs. 13(a) and 13(b). After this time, the population of quarantined people has a smooth overshoot (Fig. 13(c)). More importantly, the numbers of deaths and recovered cases rise continuously. As shown in Figs. 13(d) and 13(e), after 500 days, the deceased individuals increase to 2.5 million and the recovered ones grow to 15.8 million. However, employing the proposed control policy for 240 days, we have shown in Figs. 11(d) and 11(c) that the deceased population can be managed to be less than 18.5 thousand (0.7% of 2.5 million in the open-loop condition), and the recovered people are 37.3 million (235% of 15.8 million in the open-loop condition). This implies the significant benefits of using this robust observer-based control strategy for the COVID-19 pandemic achieving a huge decline in the death rate and considerably faster immunity for the community (convergence of the recovered population to its maximum possible value).

5. Conclusion

In this paper, a state estimation-based nonlinear robust control method was designed to appropriately adjust the use of
social distancing, hospitalization, and vaccination in order to prevent the COVID-19 epidemic, having an uncertain epidemiological model with unmeasurable variables. To this end, two plausible scenarios were proposed to fight this epidemic before and after its vaccine invention. In the first scenario, the use of social distancing and hospitalization have only been taken into account to decrease the exposed and infected groups. While in the second scenario, the vaccine development is taken into account and the third control input is defined as the vaccination rate to diminish the susceptible population, in addition to the two objectives of the first scenario.

The control laws were designed based on the nonlinear model of COVID-19 with eight state variables. Fundamental limitations such as its incubation period and the lack of an adequate community screening, cause significant problems to precisely identify the populations of susceptible, insusceptible, exposed, and infected compartments. Thereby, the hospitalized and deceased populations were utilized as the only feedback signals, and the system states were estimated using an EKF-based observer. A Lyapunov analysis was presented to ensure the stability of the closed-loop system and the convergence of tracking errors in the presence of the modeling uncertainties.
Fig. 11. Uncontrolled states of the COVID-19 epidemic during the second control scenario including (a) quarantined $Q$, (b) hospitalized $H$, (c) recovered $R$, (d) deceased $D$, and (e) susceptible $P$ populations.

Fig. 12. Innovation values of the EKF estimator for hospitalized and deceased populations.

The proposed state estimation-based nonlinear robust controller achieved multiple objectives such as state estimation, tracking control and robustness against uncertainties. The simulation studies demonstrated that although the initial estimations of state variables were different from their real values, the proposed controller succeeded to prevent the COVID-19 epidemic in two suggested plausible scenarios. Besides, the vaccination of susceptible people in the second scenario resulted in reducing the required social distancing rate compared to the first scenario considerably. While the susceptible, exposed, and infected populations were controlled in this paper, a minimization of the required vaccination, social distancing, and hospitalization rates has not been investigated yet. In future work, an optimization problem will be synthesized to investigate optimal control inputs while estimating the COVID-19 epidemic states using a hybrid state estimation algorithm, in the presence of discrete measurements. Although the observability of the employed nonlinear COVID-19 epidemic model was investigated via simulation studies, other analytical assessments can be conducted for this purpose in future studies.

CRediT authorship contribution statement

Arman Rajaei: Conception and design of study, Acquisition of Data, Analysis and/or interpretation of data, Writing - original draft. Mahsa Raeiszadeh: Acquisition of Data, Analysis and/or interpretation of data, Writing - original draft. Vahid Azimi: Analysis and/or interpretation of data, Writing - review & editing. Mojtaba Sharifi: Conception and design of study, Analysis and/or interpretation of data, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Fig. 13. Open-loop response of the COVID-19 epidemic in the absence of control measures: (a) Exposed $E$, (b) infected $I$, (c) quarantined $Q$, (d) deceased $D$, and (e) recovered $R$ compartments.

References

[1] M.R. Sanatkar, W.N. White, B. Natarajan, C.M. Scoglio, K.A. Garrett, Epidemic threshold of an SIS model in dynamic switching networks, IEEE Trans. Syst. Man Cybern. Syst. A 46 (3) (2016) 345–355, http://dx.doi.org/10.1109/TSMC.2015.2488061.

[2] K.M.A. Kabir, J. Tanimoto, Analysis of epidemic outbreaks in two-layer networks with different structures for information spreading and disease diffusion, Commun. Nonlinear Sci. Numer. Simul. 72 (2019) 565–574, http://dx.doi.org/10.1016/j.cnsns.2019.01.020.

[3] T. Khan, G. Zaman, M. Ikhlaq Chohan, The transmission dynamic and optimal control of acute and chronic hepatitis B, J. Biol. Dyn. 11 (1) (2017) 172–189, http://dx.doi.org/10.1080/17513758.2016.1256441.

[4] E.H. Russell, E.C. Dangerfield, C.A. Gilligan, N.J. Cunniffe, Applying optimal control theory to complex epidemiological models to inform real-world disease management, Philos. Trans. R. Soc. B 374 (1776) (2019) http://dx.doi.org/10.1098/rstb.2018.0284.

[5] S. Saha, G.P. Samanta, Modelling and optimal control of HIV/AIDS prevention through PrEP and limited treatment, Physica A 516 (2019) 280–307, http://dx.doi.org/10.1016/j.physa.2018.10.033.

[6] F. Brauer, Some simple epidemic models, Math. Biosci. Eng. 3 (1) (2006) 1–15, http://dx.doi.org/10.3934/mbe.2006.3.1.

[7] M. Sharifi, H. Moradi, Nonlinear robust adaptive sliding mode control of influenza epidemic in the presence of uncertainty, J. Process Control 56 (2017) 48–57, http://dx.doi.org/10.1016/j.jprocont.2017.05.010.

[8] P. Di Giamberardino, L. Compagnucci, C. De Giorgi, D. Iacoviello, Modeling the effects of prevention and early diagnosis on HIV/AIDS infection diffusion, IEEE Trans. Syst. Man Cybern. Syst. A 49 (10) (2019) 2119–2130, http://dx.doi.org/10.1109/TSMC.2017.2749138.

[9] J. Arino, F. Brauer, P. van den Driessche, J. Watmough, J. Wu, A model for influenza with vaccination and antiviral treatment, J. Theoret. Biol. 253 (1) (2008) 118–130, http://dx.doi.org/10.1016/j.jtbi.2008.02.026.

[10] G. Ngwa, A mathematical model for endemic malaria with variable human and mosquito populations, Math. Comput. Model. 32 (2000) (2009) 747–763.

[11] A.B. Pitcher, A. Borquez, B. Skaathun, N.K. Martin, Mathematical modeling of hepatitis c virus (HCV) prevention among people who inject drugs: A review of the literature and insights for elimination strategies, J. Theoret. Biol. 481 (2019) 194–201, http://dx.doi.org/10.1016/j.jtbi.2018.11.013.

[12] B. Iveroa, M.R. Ferrández, M. Vela-Pérez, A.M. Ramos, Mathematical modeling of the spread of the coronavirus disease 2019 (COVID-19) taking into account the undetected infections. The case of China, Commun. Nonlinear Sci. Numer. Simul. 2019 (PG-105303-105303) (2020) 105303, http://dx.doi.org/10.1016/j.cnsns.2020.105303.

[13] Z. Yang, et al., Modified SEIR and AI prediction of the epidemics trend of COVID-19 in China under public health interventions, J. Thorac. Dis. 12 (3) (2020) 165–174, http://dx.doi.org/10.21037/jtd.2020.02.64.

[14] C. Hou, et al., The effectiveness of quarantine of Wuhan city against the Corona Virus Disease 2019 (COVID-19): A well-mixed SEIR model analysis, J. Med. Virol. (March) (2020) 1–8, http://dx.doi.org/10.1002/jmv.25827.

[15] O. Aghajanzadeh, M. Sharifi, A. Falsafi, Robust control strategy for HBV treatment: Considering parametric and nonparametric uncertainties, Control Appl. Biomed. Eng. Syst. (2020) 127–147, http://dx.doi.org/10.1016/b978-0-12-817461-6.00005-6.

[16] M. Sharifi, H. Moradi, Nonlinear composite adaptive control of cancer chemotherapy with online identification of uncertain parameters, Biomed. Signal Process. Control. 49 (2019) 360–374, http://dx.doi.org/10.1016/j.bspc.2018.07.005.

[17] E. Jung, S. Iwami, Y. Takeuchi, T.C. Jo, Optimal control strategy for prevention of avian influenza pandemic, J. Theoret. Biol. 260 (2) (2009) 220–229, http://dx.doi.org/10.1016/j.jtbi.2009.05.031.

[18] K. Li, H. Zhang, G. Zhu, M. Small, X. Fu, Suboptimal control and targeted constant control for semi-random epidemic networks, IEEE Trans. Syst. Man Cybern. Syst. A (2019) 1–9, http://dx.doi.org/10.1109/tsmca.2019.2916859.

[19] A. Rajaei, A. Vahidi-Moghaddam, A. Chizhahm, M. Sharifi, Control of malaria outbreak using a non-linear robust strategy with adaptive gains, IET Control Theory Appl. 13 (14) (2019) 2308–2317, http://dx.doi.org/10.1049/iet-cta.2019.5202.
[20] A. Kumar, P.K. Srivastava, Y. Dong, Y. Takeuchi, Optimal control of infectious disease: Information-induced vaccination and limited treatment, Physica A 542 (2020) http://dx.doi.org/10.1016/j.physa.2019.123196.

[21] M.A. Khan, S.W. Shah, S. Ullah, J.F. Gómez-Aguilar, A dynamical model of asymptomatic carrier zika virus with optimal control strategies, Nonlinear Anal. RWA 50 (2019) 144–170, http://dx.doi.org/10.1016/j.nonwa.2019.04.006.

[22] T.F. Zhao, W.-N. Chen, A.W.-C. Liew, T. Gu, X.-K. Wu, J. Zhang, A binary particle swarm optimizer with priority planning and hierarchical learning for networked epidemic control, IEEE Trans. Syst. Man Cybern. Syst. A (2019) 1–15, http://dx.doi.org/10.1109/tsmc.2019.2945055.

[23] S.L. Li, et al., Essential information: uncertainty and optimal control of Ebola outbreaks, Proc. Natl. Acad. Sci. USA 114 (22) (2017) 5659–5664, http://dx.doi.org/10.1073/pnas.1617482114--/DCSupplemental.

[24] A. Vahidi-Moghaddam, A. Rajaei, M. Ayati, Disturbance-observer-based fuzzy terminal sliding mode control for MIMO uncertain nonlinear systems, Appl. Math. Model. 70 (2019) 109–127, http://dx.doi.org/10.1016/j.apm.2019.01.010.

[25] A. Yousefpour, H. Jahanshahi, Fast disturbance-observer-based robust integral terminal sliding mode control of a hyperchaotic memristor oscillator, Eur. Phys. J. Spec. Top. 228 (10) (2019) 2247–2268, http://dx.doi.org/10.1140/epjst/e2019-900041-4.

[26] A. Yousefpour, A. Vahidi-Moghaddam, A. Rajaei, M. Ayati, Stabilization of nonlinear vibrations of carbon nanotubes using observer-based terminal sliding mode control, Trans. Inst. Meas. Control 42 (5) (2020) 1047–1058, http://dx.doi.org/10.1177/0142331219881547.

[27] Z. Peng, D. Wang, X. Hu, Robust adaptive formation control of underactuated autonomous surface vehicles with uncertain dynamics, IET Control Theory Appl. 5 (12) (2011) 1378–1387, http://dx.doi.org/10.1049/iet-cta.2010.0429.

[28] A. Rajaei, A. Vahidi-Moghaddam, M. Ayati, M. Baghani, Integral sliding mode control for nonlinear damped model of arch microbeams, Microsyst. Technol. 25 (1) (2019) 57–68, http://dx.doi.org/10.1007/s00542-018-3931-1.

[29] H. Moradi, M. Sharifi, G. Vossoughi, Adaptive robust control of cancer chemotherapy in the presence of parametric uncertainties: A comparison between three hypotheses, Comput. Biol. Med. 56 (2015) 145–157, http://dx.doi.org/10.1016/j.compbiomed.2014.11.002.

[30] M.K. Bera, P. Kumar, R.K. Biswas, Robust control of HIV infection by antiretroviral therapy: A super-twisting sliding mode control approach, IET Syst. Biol. 13 (3) (2019) 120–128, http://dx.doi.org/10.1049/iet-syb.2018.0563.

[31] N. Paolletti, K.S. Liu, S.A. Smolka, S. Lin, Data-Driven Robust Control for Type 1 Diabetes under Meal and Exercise Uncertainties, in: Lect. Notes Comput. Sci. (Including Subser. Lect. Notes Artif. Intell. Lect. Notes Bioinformatics), vol. 10545 LNBL, 2017, pp. 214–232, http://dx.doi.org/10.1007/978-3-319-67471-1–3.

[32] M. Lipstich, S. Riley, S. Cauchemez, A.C. Ghani, N.M. Ferguson, Managing and reducing uncertainty in an emerging influenza pandemic, N. Engl. J. Med. 361 (2) (2009) 112–115, http://dx.doi.org/10.1056/NEJMp0904380.

[33] G. Rigatos, K. Busawon, M. Abbaszadeh, Nonlinear optimal control of the acute inflammatory response, Biomed. Signal Process. Control 55 (2020) http://dx.doi.org/10.1016/j.bspc.2019.101631.

[34] D. Ndaguzu, I.S. Mbalawata, H. Haario, J.M. Tchuenche, Analysis of bias in an Ebola epidemic model by extended Kalman filter approach, Math. Comput. Simulation 142 (2017) 113–129, http://dx.doi.org/10.1016/j.matcom.2017.05.005.

[35] V. Azimi, M. Sharifi, S. Fakoorian, T.T. Nguyen, State estimation-based robust optimal control of influenza epidemics in an interactive human society, 2020.

[36] L. Peng, W. Yang, D. Zhang, C. Zhuge, L. Hong, Epidemic analysis of COVID-19 in China by dynamical modeling, 2020.

[37] D. Simon, Optimal state estimation: Kalman, H∞, and nonlinear approaches, 2006.

[38] K. Reif, S. Gunther, E. Yaz, R. Unbehauen, Stochastic stability of the continuous-time extended Kalman filter, IEEE Proc., Control Theory Appl. 147 (1) (2000) 45–52.

[39] M.M. Polycarpou, P.A. Ioannou, A robust adaptive nonlinear control design, in: 1993 American Control Conference, IEEE, 1993, pp. 1365–1369.

[40] H. Ritchie, E. Ortiz-Ospina, D. Belpietro, E. Mathieu, J. Hasell, B. Macdonald, C. Giattino, M. Roser, Mortality risk of COVID-19, 2020, https://ourworldindata.org/mortality-risk-covid.