Entanglement fidelity ratio for elastic collisions in non-ideal two-temperature dense plasma

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Abstract
The quantum diffraction and symmetry effects on the entanglement fidelity (EF) of different elastic electron–electron, ion–ion and electron–ion interactions are investigated in non-ideal dense plasmas. The partial wave analysis, an effective screened interaction potential including quantum mechanical diffraction and symmetry effects are employed to obtain the EF in a non-ideal dense plasma. We show that collision energy and temperatures of electron and ion have destructive effects on the entanglement in the system. In fact, by decreasing the temperature of plasma particles, the quantum effects become more prominent and the entanglement is elevated. Also, increase in the density of plasma leads to the enhancement of entanglement ratio. Additionally, some important characteristic parameters of the scattering such as differential, transport and total cross sections are calculated.

Keywords: quantum entanglement, dense plasma, effective potential, collision, entanglement fidelity, two-temperature plasma

(Some figures may appear in colour only in the online journal)

1. Introduction
Investigation of entanglement variation has been developed in various areas of physics. Entanglement generation due to the expansion of universe and Lorentz invariance violation [1–6], entanglement degradation because of the observer acceleration [7–10], variation of entanglement resulted from the environmental interactions [11–13] are some examples in this research area. In fact, the entanglement between particles can be affected by every kinds of interaction with environment, dynamical background or environmental noises. Plasma is a highly interacting environment, because of the presence of charged particles at high densities, which can lead to the appearance of collective effects via long-range electromagnetic interactions. In quantum plasmas, the main task in the study of entanglement between an incident particle and another selected species of plasma is the definition of an appropriate potential which should include all important aspects of interactions. The study of elastic collision in plasmas has recently attracted an increasing interests since this process can reveal suitable parts of information about plasma parameters which may have essential role in the plasma diagnostic tools design [14–20]. One of the first simplest mechanisms describing interaction of charged particles in plasma is the Debye–Huckel screened potential which is known as the ideal plasma model where the energy of inter-particle interactions is small enough or comparable with the average kinetic energy of a particle [21, 22]. This model looks to be idealistic for dilute plasmas, however, by increasing the plasma density, in the so-called non-ideal plasma model, multi-particle correlations originated from the simultaneous interactions of multiple charged particles should be considered in the effective potential [23]. In this approach, avoiding to define the effective potential by the conventional Debye–Huckel model which is constructed through classical Boltzmann distribution for charged particles, the quantum
mechanical diffraction and symmetry effects resulted from the collective plasma interactions are considered [24]. The average separation of particles in the dense plasma is in the order of (or less than) the thermal de Broglie wavelength of particles which causes a high probability for the inter-particle collisions in a short distance. Therefore, it is better to consider wave aspect for the colliding particles taking into account some quantum-mechanical effects such as diffraction and symmetry [24–26]. Such quantum mechanical aspect of plasma has the vast applications in various fields including intense laser-solid density plasma interaction [27–31], ion accelerator [32], dense plasmas appeared in the core of planets [33], astrophysical and cosmological environments like white dwarf stars [34, 35], so-called ultra-small electronic devices [36] and metal nanostructures [37].

Recently, quantum entanglement or correlation among distinct quantum systems is a crucial concept for the feasibility of some new modern cutting-edge technologies like quantum information and quantum communications [38–40]. A new definition of entanglement measure in terms of wavepacket localization has been proposed by Fedorov et al [41] and this essential parameter has been called entanglement fidelity (EF). The EF for scattering processes has been widely noted because it represents that the quantum correlation is an important characteristic in realization of the quantum measurement and information processing [42, 43]. The general scattering processes from the entanglement point of view are theoretically studied in more detail in [44]. It is clear that because of the quantum effects, the physical properties of quantum plasma are different from those in classical one which is studied by the classical regime or Debye–Hückel model. Hence, it would be expected that the EF for the elastic \( \alpha-\beta \) particle species (ion or electron) collisions in non-ideal dense hot quantum plasmas would be quite different from those in ideal plasma due to the collective interactions. Moreover, it can be also anticipated that the study on the EF for elastic collisions in non-ideal dense plasmas gives valuable information about the physical properties and characteristics of the quantum screening and plasma parameters.

For the first time, Chang and Jung [45] theoretically studied the non-ideal collective and plasma screening effects on the EF of the low-energy electron–ion elastic collisions in the classical non-ideal plasma. Taking into account the nonthermal classical plasma with Lorentzian (kappa) distribution, the entanglement problem for the elastic electron–ion scattering are investigated by Shin and Jung [46]. In the same year [47], the screened pseudopotential model, considering the quantum and plasma screening effects, is employed to study the entanglement for the electron–ion interactions in strongly coupled semiclassical plasmas. Generalization of the entanglement problem for the hot quantum plasma in the isothermal state for electrons and ions is accomplished in [43]. In the complex classical isothermal dusty plasma, effect of the ion wake-field on the collisional EF has been studied by Jung and Hong [48]. In quantum plasmas, considering the spin of electrons through entering the quantum oscillation and recoil effects can change the effective electron–ion potential in the scattering phenomena. In such a situation, the collisional EF for the elastic electron–ion collision in an isothermal quantum plasma has been theoretically investigated in [49, 50].

In the present work, we theoretically study the quantum and screening effects on the EF for the elastic \( \alpha-\beta \) particles collision in non-ideal dense plasma. For short-range interactions of particles, the effect of surrounding plasma medium on the potential of particles collisions is taken into account through some quantum considerations which lead to some corrections on the classical Debye screening terms. In the effective interaction model, these quantum corrections are related to the quantum diffraction and symmetry effects. To save the generality of problem, temperature of each sort of plasma particles is considered different that can be happened in the non-equilibrium plasma where the different species cannot reach to a thermodynamic equilibrium with each other. Also, partial wave method is employed to investigate the EF for the elastic particle collisions as a function of the thermal de Broglie wavelength and projectile energy. The EF rate is studied for different kinds of particles interaction including electron–electron, electron–ion and ion–ion collisions. The paper is organized as follows: we summarize the evaluation of EF in a plasma in section 2. Effective potential for a dense plasma is introduced in section 3. Two different regimes are recognizable with different effective potentials. We focus on deriving analytical relationships and evaluate the EF in section 4 and in more detail in sections 4.1 and 4.2. In section 5, we have obtained some characteristic parameters of scattering. The conditions of appearance of non-ideal (coupled) and quantum plasmas are discussed in section 6. Finally, we conclude the paper in section 7.

### 2. Entanglement fidelity

In this section, we briefly introduce an interesting phenomenon in plasma which is called the entanglement. In order to give a quantitative measurement for entanglement, we define the EF. First, we consider the collision of a particle by a scattering potential. The stationary-state Schrödinger equation for the potential in quantum collision processes can be written as

\[
(\nabla^2 + k^2)\psi_\ell(r) = \frac{2\mu}{\hbar^2} V(r)\psi_\ell(r),
\]

where \( \psi_\ell(r) \) is the solution of the scattered wave equation, \( k = \sqrt{2\mu E/\hbar^2} \) is the wave number, \( \mu \) is the reduced mass of the collision system, \( V(r) \) is scattering potential, \( E = \mu v^2/2 \) is the kinetic energy of the projectile, \( v \) is the collision velocity, and \( \hbar \) is the rationalized Plank constant. Here the final state wave function \( \psi_\ell(r) \) would be represented by the partial wave expansion [51, 52] in the following form

\[
\psi_\ell(r) = \sum_{l = -\infty}^{\infty} i^l (2l + 1) D_l(k) P_l(\cos \theta) R_l(r),
\]

where, \( D_l(k) \) is the expansion coefficient, \( i \) is the pure imaginary number, \( R_l(r) \) is the solution of the radial wave equation, \( P_l(\cos \theta) \) is the Legendre polynomial of order \( l \), and \( l \) is the angular momentum quantum number. For a spherically
symmetric potential $V(r)$, it has been shown that the radial wave equation and the expansion coefficient $D_l(k)$ are given by [42, 45]

$$\left[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \right) - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2} V(r) + k^2 \right] R_l(r) = 0,$$

(3)

$$D_l(k) = (2\pi)^{3/2} \left[ 1 + \frac{2\mu k}{\hbar^2} \int_0^\infty dr \int_0^\infty d\theta j_l(k \theta) V(r) \right]^{-1}$$

(4)

respectively, where $j_l(\theta)$ is the spherical Bessel function of order $l$, for solving equation (3) we use the well-known method of Green function. For this purpose, we should solve the following equation

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \right) R_l(r) = \frac{\delta(r - r')}{r^2},$$

(5)

in which the radial part of wave function can be obtained as

$$R_l(r) = j_l(kr) + \frac{2\mu k}{\hbar^2} \int_0^\infty dr' r'^2 j_l(kr') V(r') R_l(r').$$

(6)

The Green function of equation (5) is

$$g(r, r') = k j_l(kr) j_l(kr').$$

(7)

Asymptotic form of the radial wave function can be achieved by the phase-shift $\delta_l$ such as $R_l(r) \propto (kr)^{-(l+1)} \sin(kr - \pi l/2 + \delta_l)$. In fact, equation (7) is an integral equation and we use the s-wave approximation and the asymptotic form in the following. The entanglement generation by the scattering processes has been investigated by Mishima et al [42]. It has been shown that the collisional EF for the scattering process can be represented by

$$f_k \propto \left[ \int_0^\infty dr \int_0^\infty d\theta j_0(k \theta) \right]^2,$$

(8)

Now, the collisional EF in the low energies for elastic collisions between $\alpha$ and $\beta$ two different or the same species particles, in a plasma with an appropriate effective potential can be evaluated as follows

$$f_{k\alpha} \propto \left[ \int_0^\infty dr \int_0^\infty d\theta j_0(k \theta) \right]^2,$$

(9)

where $\varphi_{\alpha\beta}(r)$ describes the effective interaction potential between the projectile $\alpha$ and the screened $\beta$ particles.

### 3. Non ideal dense plasma and effective potential

The effective interparticle interaction potential in plasma can be derived in two different methods. In the first method, one obtains the effective potential using the solution of generalized Poisson–Boltzmann equation [54]. The second one, is related to the dielectric response function [55]. Recently, using the second method, the effective potentials of interactions of a non-ideal, non-isothermal plasma has been investigated by Ramazanov et al [24]. The effective potential of the $\alpha–\beta$ particle species (ion or electron) interaction in non-ideal dense hot plasma with effective screening taking into account quantum-mechanical diffraction and symmetry effects with a strongly coupled ion and semiclassical electron subsystems is given by [24]

$$\varphi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{r} \left[ \frac{1}{\gamma^2 (1 - (2k_D/\lambda_{ee} \gamma)^2)} \left( \frac{1}{1 - B^2 \lambda_{ab}^2} \right) e^{-B r} - \left( \frac{1}{1 - A^2 \lambda_{ab}^2} \right) e^{-A r} \right] - \frac{Z_{ee} Z_{\alpha\beta} e^2}{r} \frac{1 - \delta_{ab}^\alpha}{1 + C_{ab}^\alpha} e^{-\gamma r} \delta_{\alpha\beta},$$

(10)

where $Z_\alpha(Z_\beta)$, $e$ and $\lambda_{ab}^\alpha = \hbar \sqrt{4\pi \mu_{ab} k_j T_{j\beta}}$ are atomic numbers of particle species $\alpha$ ( $\beta$), the electron charge and thermal de-Broglie wavelength of pairs of particles $\alpha$ and $\beta$, respectively, $\mu_{ab} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass, $k_D$ is Boltzmann constant, and $T_{j\beta} = \sqrt{\epsilon_{j\beta} T_j}$ is defined by the temperatures of species $\alpha$ ($T_\alpha$) and $\beta$ ($T_\beta$). Also, $k_D = \sqrt{k_i^2 + k_j^2}$ is the screening parameter considering the contributions of electrons and ions, where $k_i = \sqrt{4\pi \mu_{ee} e^2/k_B T_e}$, $k_j = \sqrt{4\pi \mu_{ee} e^2/k_B T_i}$, and $\gamma^2 = k_i^2 + 1/\lambda_{ee}^2$. Also, $A$, $B$ and $C_{ab}^\alpha$ have been defined as

$$A^2 = \frac{\gamma^2}{2} \left( 1 + \frac{1 - (2k_D/\lambda_{ee} \gamma)^2}{\sqrt{1 - (2k_D/\lambda_{ee} \gamma)^2}} \right),$$

(11)

$$B^2 = \frac{\gamma^2}{2} \left( 1 + \frac{1 - (2k_D/\lambda_{ee} \gamma)^2}{\sqrt{1 - (2k_D/\lambda_{ee} \gamma)^2}} \right),$$

(12)

$$C_{ab}^\alpha = \frac{k_D^2 \lambda_{ab}^\alpha}{\lambda_{ee}^2 (1 - \lambda_{ab}^\alpha)^2}.$$
\[
\varphi_{\alpha \beta}(r) = \frac{Z_{\alpha} Z_{\beta} e^2}{r} \frac{d_{\alpha \beta}}{\gamma \sqrt{(2k_D/\lambda_{ee} \gamma)^2 - 1}} \times \sin(\sqrt{k_D/\lambda_{ee}} \sin(\omega/2) r + \theta_{\alpha \beta}) \times \exp[-r\sqrt{k_D/\lambda_{ee}} \cos(\omega/2)] 
- \frac{Z_{\alpha} Z_{\beta} e^2}{r} \frac{d_{\alpha \beta}}{1 + C_{\alpha \beta}} e^{-r/\lambda_{\alpha \beta}},
\]

(14)

where parameters \(d_{\alpha \beta}, \theta_{\alpha \beta}\) and \(\omega\) are

\[
\begin{align*}
\theta_{\alpha \beta} &= \arctan\left(\frac{b_{\alpha \beta}}{a_{\alpha \beta}}\right), \\
\omega &= \arctan\left[\sqrt{(2k_D/\lambda_{ee} \gamma)^2 - 1}\right],
\end{align*}
\]

(15)

\[
\begin{align*}
a_{\alpha \beta} &= \frac{2(1/\lambda_{ee}^2 - \gamma^2/2)(1 - \gamma^2\lambda_{\alpha \beta}^2/2) + \gamma^4\lambda_{\alpha \beta}^4((2k_D/\lambda_{ee} \gamma)^2 - 1)}{(1 - \gamma^2\lambda_{\alpha \beta}^2/2)^2 + \gamma^4\lambda_{\alpha \beta}^4((2k_D/\lambda_{ee} \gamma)^2 - 1)/4}, \\
b_{\alpha \beta} &= \frac{\gamma^2(1 - \lambda_{\alpha \beta}^2/\lambda_{ee}^2)(\sqrt{(2k_D/\lambda_{ee} \gamma)^2 - 1})}{(1 - \gamma^2\lambda_{\alpha \beta}^2/2)^2 + \gamma^4\lambda_{\alpha \beta}^4((2k_D/\lambda_{ee} \gamma)^2 - 1)/4}.
\end{align*}
\]

(19)

We notice that the last term on the right-hand side of the equations (10) and (14) disappears for ion–ion and electron–electron interactions.

4. **EF of dense plasma**

The entanglement between an incident particle and a selected electron or ion in plasma is defined by the effective potential of plasma and can be quantified by the EF. Therefore, the EF is dependent on the effective potential. One can consider the relative EF by evaluation of the ratio of the EF for the effective interaction potential \(\varphi_{\alpha \beta}\) with respect to EF of the pure coulomb potential \(V_C(r) = -Z_c^2/r\) as follows

\[
R_{\alpha \beta}(k, \lambda_{\alpha \beta}, n_e, n_i) = \frac{\int_{0}^{\infty} \varphi_{\alpha \beta}(k, \lambda_{\alpha \beta}, n_e, n_i) \, dk}{\int_{0}^{2\pi} f_C(k) \, dk},
\]

(20)

Using equation (10), we can evaluate the entanglement fidelity ratio (EFR) for electron–electron interaction. In the process of calculations the measure, we use following integral relation:

\[
\int_{0}^{\infty} \exp(-C_r)\sin(kr) \, dr = \frac{k}{k^2 + C^2}.
\]

(21)

Therefore, we obtain \(R_{e-e}\) (EFR for electron–electron interaction) as follows:

\[
R_{e-e}(E, n_e, T_e, n_i, T_i) = 1 + \frac{4}{E} \left(\frac{8E/\gamma^2}{(1 + \lambda_{a}^2)^4(2E/\gamma^2 + 1)^2 + 4k_{D}^2/\lambda_{ee}^4} - 1\right).
\]

(22)

where, the dimensionless parameters are defined by:

\[
\tilde{E} = \frac{E}{Z^2 R_y}, \quad \tilde{\gamma} = a_\gamma \gamma, \quad \tilde{k}_D = a_k k_D, \quad \tilde{\lambda}_{\alpha \beta} = \frac{\lambda_{\alpha \beta}}{a_\gamma}.
\]

(23)

Also, \(a_\gamma = a_0/Z\) and \(a_0 = h^2/\mu e^2\) is Bohr radius and other parameters have their conventional meaning. In the same manner, the extraction of EFR for effective ion–ion interaction is straightforward and is given by

\[
R_{i-i}(E, n_e, T_e, n_i, T_i) = 1 + \frac{4}{E} \left(\frac{4E/\gamma^4 + E/\gamma^2 \lambda_{\alpha}^2}{(4E/\gamma^2 + 1)^2 + 4k_{D}^2/\lambda_{ee}^4} - 1\right).
\]

(24)

Calculation of the EFR for electron–ion interaction is a little more complicated in comparison with previous ones, due to the non-vanishing term in the effective potential. In this case, we obtain:

\[
R_{e-i}(E, n_e, T_e, n_i, T_i) = 1 + \frac{4}{2E F^2}.
\]

(25)

where

\[
F = \frac{4((1/\lambda_{ee}^2 - 1/\lambda_{\alpha}^2)E^2/\gamma^4 + (1/\lambda_{ee}^2 - 1/\gamma^2 \lambda_{ee}^2 \lambda_{\alpha}^2 - \tilde{k}_D^2/\gamma^2 \lambda_{ee}^2)E/\gamma^2)}{(1/\lambda_{ee}^2 - \gamma^2 + \lambda_{\alpha}^2 k_D^2/\lambda_{ee}^4)((2E/\gamma^2 + 1)^2 + 4k_D^2/\lambda_{ee}^4 - 1)}
\]

(26)
it worth mentioning that, we used the effective potential which is given by equation (10). In fact, for \((2k_D/\lambda_{\text{e}}\gamma^2)^2 < 1\) all calculation steps can be done analytically. For \((2k_D/\lambda_{\text{e}}\gamma^2)^2 > 1\), the effective potential is given by equation (14) and due to the lack of an exact solution, a sort of numerical calculation is needed. In current paper, we focus on the analytical evaluations and the numerical calculation is postponed to the future work.

Figure 1 (as an example) shows that, for a dense and uniform temperature plasma, the effective potential of equation (10) (equation (14)) is valid in regions I and III. Effective potential of equation (10) is valid in regions I and III while in region II the effective potential of equation (14) is more reliable. Let us have a look to the regions I (low temperature limit) and III (high temperature limit) with more details in the following.

4.1. Low temperature limit

For an analytical consideration, we restrict ourselves to regions I and III. In region I the temperature has an upper bound and therefore, we consider the low temperature limit. EFR in this region for different electron–electron, ion–ion and ion–electron interactions is investigated diagrammatically in figure 2. It is obvious that the general behavior is the same for all cases. EFR is a monotonically decreasing function with respect to the collision energy. We compare EFR of different interactions in the figure 3, for different values of collision energy. One can observe that for a fixed collision energy, the EFR in all interactions are independent of temperature. The effective potential for small values of collision energy is more impressive and the EFR rapidly decreases by increasing the collision energy.

We consider non-isothermal dense plasmas, in the following. For fixed values of collision energy and density of...
electrons and ions as \( n_e = n_i = 2 \times 10^{23} \text{ cm}^{-3} \), calculated values of EFR for different interactions with respect to the temperature of electrons and ions are demonstrated diagrammatically in figure 4. Behavior of different interactions are the same in general and we depicted only the EFR of electron–ion interaction diagrammatically for instance. In fact, in low temperature limit, the EFR does not depend on the temperatures and it is evident that by increasing the collision energy, the EFR will be decreased.

4.2. High temperature limit

Fortunately an analytical calculation can be done for EFR in the region III where the temperature is in its lower bound. In
fact, we restrict ourselves in high temperature limit. First, we introduce a new parameter which is related to the inverse of temperature, \( b = kT^{-1} \). Therefore, the region III is identified by \( \beta \leq \beta^* \), where \( \beta^* \) is the root of the equation \( \xi(\beta) = 0 \). We notice that \( \beta = 0 \) corresponds to infinite temperature and \( \beta^* \) is related to the inverse of lower bound temperature. Also, another dimensionless parameter which is correspond to the inverse of density is defined as \( rs = a/a_B \) where, \( a = (3/4\pi n_e)^{1/3} \) is the average distance between particles and \( a_B = h^2/m_e e^2 \). It is obvious that \( rs = 0 \) denotes the infinite density.

Figure 5 shows the behavior of EFR with respect to \( \beta \) and \( rs \) for three different types of \( \alpha-\beta \) scattering for dense plasma in high temperature limit. EFR is a monotonically increasing function of \( \beta \) while it is a decreasing function of \( rs \). It seems that EFR vanishes in classical limit. Therefore, for high densities (small values of \( rs \) and near to the lower bound of temperature, the quantum effects are dominant and EFR grows up. Also, the general behavior of EFR for three different interactions are the same.

In figure 7, a comparison between EFR of different interactions for fixed density and certain value of collision energy in isothermal plasma indicates that \( R_{ii} = R_{ei} > R_{\alpha-\beta} \) in high temperatures. By decreasing the temperature, EFR of all interactions tends to an identical saturated value. Also, at very high temperature or classical limit, only EFR of electron–ion interaction becomes zero.

Variation of EFR with respect to \( \beta \) and \( E \) is depicted in the figure 6. \( R_{\alpha-\beta} \) and \( R_{ei} \) have completely the same behavior. Also, the general behavior of \( R_{ei} \) is very similar to other

\[ R_{\alpha-\beta} \]

\[ R_{ei} \]

\[ R_{ei} \]

\[ R_{ei} \]
situations. It is obvious that by decreasing the temperature (increasing $T$) EFR of all interactions grows up. Also, in small values of collision energy, the effective potential is more impressive and the EFR decreases by increasing the collision energy. Figure 8 shows the EFR of two-temperature plasma with respect to electron and ion temperatures for electron–electron and ion–ion interactions with fixed values of collision energy and plasma density. Here, both parameters, i.e. $R_{ee}$ and $R_{ii}$, show completely the same behavior. Decreasing the temperatures leads the system into quantum regime and thus, the EFR grows up. In order to show the roles of electron and ion temperatures more clearly, the EFR of electron–ion interaction with respect to temperatures has been represented in figure 9 with a suitable contour plot. It is obvious that in very high temperatures (classical limit for the temperature of both species) EFR vanishes as expected. However, EFR is more sensitive to electron temperature in comparison with ion temperature. By decreasing the electron temperature EFR starts to grow.

5. Cross section and transport cross section

Now let us consider some aspects of scattering theory when it happens in a non-ideal non-isothermal dense plasma. As it is known, almost every pieces of information about the structure of nuclei and particles in nuclear and particle physics are come from the data related to the particle-particle scattering. The scattering cross section, transport cross section and differential cross section are some characteristic parameters which are going to be calculated here. The differential cross section is defined as follows [51, 56]

$$\frac{d\sigma}{d\Omega} = |f_{\alpha\beta}(\mathbf{k}', \mathbf{k})|^2,$$

where, $f_{\alpha\beta}(\mathbf{k}', \mathbf{k})$ is scattering amplitude of $\alpha$–$\beta$ particles, $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of incident and scattered waves, respectively. We restrict ourselves to the first order Born approximation and using equation (10) we find the scattering amplitude as follows

$$f_{\alpha\beta}^{(1)}(\theta) = -\frac{i\mu}{2\pi\hbar^2} \int d^3r e^{i(k-k')\mathbf{r}} \varphi_{\alpha\beta}(r'),$$

$$= \frac{2\mu Z_{\alpha} Z_{\beta} e^2}{\hbar^2} \sum_{n=1}^{3} \frac{(-1)^n X_n}{Y_n^2 + 2k^2(1 - \cos \theta)},$$

(28)
where, scattering angle $\theta$ is the angle between $\mathbf{k}$ and $\mathbf{k}'$, $Y_1 = B$, $Y_2 = A$, $Y_3 = 1/\lambda_{a3}$ and
\[
X_1 = \frac{1}{\sqrt{1 - (2k_D/\lambda_{e3})^2}} \frac{1}{1 - B^2 \lambda_{a3}^2} , \\
X_2 = \frac{1}{\sqrt{1 - (2k_D/\lambda_{e3})^2}} \frac{1}{1 - A^2 \lambda_{a3}^2} , \\
X_3 = \frac{\delta_{a3} - 1}{C_{a3} + 1} .
\]
Then the first order total cross section for $\alpha-\beta$ collisions will be obtained by integrating the first order differential cross section $d\sigma^{(1)} / d\Omega = |f^{(1)}(\theta)|^2$ as following
\[
\sigma^{(1)} = \frac{4\pi \mu^2 Z_e^2 Z_i^2 e^4}{h^4 k^2} \sum_{m=1}^{3} \sum_{n=1}^{3} \frac{(-1)^{m+n} X_m X_n}{(Y_m^2 - Y_n^2)} \times \ln \left( \frac{1 + 4k^2 / Y_m^2}{1 + 4k^2 / Y_n^2} \right) .
\]

6. Conditions of non-idealness and quantum limit for plasma

Finally, in order to evaluate the medium state from the point view of the applicability of using the mentioned effective potential for the case of dense quantum non-ideal plasma limit, let us calculate some characteristic parameters. For the case $n_e = 2 \times 10^{23} \text{cm}^{-3}$ the average separation of electrons or ions inside the plasma is $a = (4\pi n_e / 3)^{-1/3} \approx 1.06a_0$ and therefore $r_s = (a / d) \approx 2$. For the quantum plasmas the characteristic length $a$ should be comparable or less than the de Broglie wavelength $\frac{\hbar}{\sqrt{mk_B}}$, that is 1.71$A_0$ and 0.67$A_0$ for the low and high temperatures of $T = 3 \times 10^4 \text{K}$ and $T = 1.97 \times 10^5 \text{K}$ for electrons, respectively, which confirms the quantum condition for both cases. For the non-ideal plasma gas, the average kinetic energy of plasma species should be comparable or less than any interactional energy of particles. For a completely ionized plasma, the dominant interactional energy is Coulomb energy between charged particles, therefore the parameter of idealness of plasma or coupling parameter can be defined as the ratio of Coulomb energy to the kinetic one as $\Gamma_{a3} = \frac{Z_e Z_i e^2}{k_B T_e a_{a3}}$, $a_{a3}$ is the average distance between sorts of $\alpha$ and $\beta$. For non-isothermal plasma the coupling parameters for different species are as following
\[
\Gamma_{ee} = \frac{e^2}{a K_B T_e}, \Gamma_{ii} = \frac{Z_i^2 e^2}{a K_B T_i} \left( \frac{n_i}{n_e} \right)^{1/3}, \Gamma_{ei} = \frac{Z_i e^2}{a K_B T_e} .
\]
when the electron and ion density is $n_0 = 2 \times 10^{23} \text{cm}^{-3}$, calculations show that for the high temperature isothermal case of $T_e = T_i = 1.97 \times 10^5 \text{K}$ the coupling parameter is $\Gamma_{ee} = \Gamma_{ii} = \Gamma_{ei} = 0.8$ while for the low temperature case of $T_e = T_i = 3 \times 10^4 \text{K}$, it is $\Gamma_{ee} = \Gamma_{ii} = \Gamma_{ei} = 5.24$ which confirms the non-idealness of plasma for both cases.

7. Conclusions

Entanglement of quantum states is extensively investigated in different areas of physics such as spin chains in condensed matter, non inertial frames in relativistic quantum information, entanglement generation due to expansion of universe in cosmology and Lorentz violation in high energy physics. Entanglement of an incident particle with constituents of a dense plasma, especially in quantum regime is an interesting problem, which is mainly investigated by effective potential models.

The EFR of various effective potentials in different kinds of plasmas has been investigated. We considered a non ideal dense plasma and studied the influence of effective potential on the EFR. Our consideration is restricted to special circumstances in order to obtain analytical relations. Two different regions which satisfy the circumstances have been investigated.

We showed that in region I or low temperature limit, the EFR is independent of temperature and it is a monotonically increasing function of the collision energy, while the EFR in the region II does not change by temperature variations. However, in region III or high temperature limit, an explicit dependency to temperature is evident and the EFR decreases by increasing the temperature of particles. We showed that only the EFR of electron–ion interaction vanishes in infinite temperature limit while it has non-zero value for electron–electron and ion–ion interactions.

For a two-temperature dense plasma model with fixed density and collision energy, decreasing both temperatures leads to growing of EFR for all kinds of interactions. Of course, the EFR of electron–ion interaction is more sensitive to electron temperature in comparison with the ion temperature. As an additional work, some essential parameters in the scattering of $\alpha–\beta$ species of degenerate non-ideal non-isothermal plasma is calculated. Moreover, the conditions under which we should consider plasma as a non-ideal degenerate gas is discussed.

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