Numerical simulation of the electrovortex flow in the non-inductive approximation under the influence of an external magnetic field

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Abstract. The results of a numerical study of the structure of an electro-vortex flow under an external magnetic field are presented. It was shown that an external magnetic field can lead to both an increase and a decrease in the intensity of the flux. At the same time, it is necessary to control the external magnetic field in direct current arc furnaces very carefully, since the non-optimal external magnetic field leads to the suppression of the flow in the melting bath.

1. Introduction

Among various types of magnetohydrodynamic flows of a liquid, so-called electro-vortex flows (EVF), which are formed as a result of the interaction of an electric current flowing through an electrically conductive liquid, and a self-magnetic field of the electric current, are of particular interest [1]. Studies of the electrovortex flows provide not only fundamental knowledge of the process, but also are of great importance for understanding and adequately describing the hydrodynamic mechanisms of high-current technological processes that are widespread in mechanical engineering and electrometallurgy (electroslag welding and remelting, various electric furnaces, electrolyzers, etc.) [2]. It should be noted that due to the large methodological difficulties, it is practically impossible to investigate the local thermal and hydrodynamic features of these processes in real industrial conditions, therefore, the most effective way to research is a combination of computational and experimental methods.

During the period from the 70s of the 20th century, quite a lot of computational work was carried out on the study of electrovortex currents, the complexity and setting of tasks correlated completely with the development of computing technology of the time. So we can note works where EVF are investigated in a linear formulation [3,4]. Questions related to the appearance of a spontaneous rotation of the flow are discussed in [5, 6]. Problems of instability of the surface are studied in [7]. Effects connected with the action of the external magnetic field are discussed in [8,9,10].

Now we continue the study of EVF in a liquid-metal conducting medium in a hemispherical vessel under the influence of an external axial magnetic field (MF). This form was chosen because it is quite close to the configuration of industrial arc furnaces, and it is the EVF that determine the processes of heat and mass transfer inside the furnace. EVF is extremely sensitive to external MF, so controlling the currents in a furnace using a MF is a natural method that should be used with extreme caution, since too strong MF can cause stagnant zones in the bath and cause overheating. We consider a problem with two hemispherical electrodes between which there is a conductive medium (liquid metal), and an electric current with a density W propagates from the inner electrode (small) to the outer (large) electrode.
(figure 1). The current creates MF \( B \). Thus, an electromagnetic force \( \mathbf{F} = \mathbf{J} \times \mathbf{B} \) occurs in the volume of the liquid, causing the liquid to move.

![Figure 1](image)

**Figure 1.** Scheme of the electrovortex flow with the action of the axial external MF.

If we consider an axisymmetric system without an external MF, the EVF has the form of one toroidal vortex — the upper vortex (excluding some unstable modes [8]). The action of the axial external MF causes an azimuthal rotation of the flow. In this case, due to azimuthal rotation, a secondary vortex (bottom vortex) appears, which rotates in the direction opposite to the main EVF. Strong rotation leads to the suppression of the main EVF, and also because of the interaction of the fast flow with a strong MF, a braking MHD force arises.

The present work is devoted to investigation of the electrovortex flow regimes in the wide ranges of the external MF.

### 2. Numerical method

To numerically solve the Navier-Stokes equation with added electromagnetic force, we used the finite volume method on an unstructured 2D axisymmetric grid in cylindrical coordinates (\( \partial ... / \partial \varphi = 0 \), \( U_{\varphi} \neq 0 \)) with \( k-\omega \) SST turbulence model. Area calculation area (figure 2) was a quarter circle with internal radius \( R_1 = 2.5 \text{ mm} \) (small electrode) and external radius \( R_2 = 94 \text{ mm} \) (large electrode) and consisted of 5000 quadrangular cells. We used a simple 2D axisymmetric mesh for the liquid metal region, earlier calculations showed that more complex meshes do not lead to better results [10]. The symmetry axis of the calculation area coincided with z-axis of the cylindrical coordinate system and free surface of the liquid coincided with r-axis direction. The wall boundary conditions were set on all surfaces. The physical properties of the conducting medium correspond to the indium-gallium-tin eutectic alloy (physical properties: melting point +10.5°C, \( \rho = 6482 \text{ kg/m}^3 \), \( \nu = 4.3 \times 10^{-7} \text{ m}^2/\text{s} \), \( \sigma = 3.3 \times 10^6 \text{ Sm} \)) used in the experimental setup at the JIHT RAS.
The Navier-Stokes equation has the following form:
\[
\rho \left( \frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} \right) = -\nabla p + \rho \nu \nabla^2 \mathbf{U} + \mathbf{F}_{\text{NI}},
\]
where \( \mathbf{U} \) is the velocity of the liquid, \( \rho \) is the density of the liquid, \( \nu \) is the kinematic viscosity coefficient, \( p \) is the pressure, and \( \mathbf{F}_{\text{NI}} \) is the electromagnetic force.

Calculation was carried out in non-induction approximation (NI). Using this approximation we neglect the induced MF and the electromagnetic force has the form:
\[
\mathbf{F}_{\text{NI}} = \mathbf{J} + \sigma \mathbf{U} \times \left( \mathbf{B}_{\text{EVF}} + \mathbf{B}_{\text{ext}} \right) \times \left( \mathbf{B}_{\text{EVF}} + \mathbf{B}_{\text{ext}} \right),
\]
where \( \mathbf{J} \) is the current density, \( \mathbf{B}_{\text{EVF}} \) is the self-magnetic field, \( \mathbf{B}_{\text{ext}} \) is the external MF with the only z-component \( B_z \), and \( \sigma \) is electrical conductivity.

The current density has analytical expression in spherical coordinate system:
\[
J_r = \frac{I}{2\pi R^2}.
\]

Rewriting current density in cylindrical coordinate system we obtain:
\[
J_r = \frac{Ir}{2\pi \sqrt{(r^2 + z^2)^3}},
\]
\[
J_z = \frac{Iz}{2\pi \sqrt{(r^2 + z^2)^3}}.
\]

The MF can be found from the Maxwell’s equation:
\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}.
\]
Here the expression of the MF in spherical and cylindrical coordinate systems is:

\[ B_\theta = \frac{\mu_0 I (1 - \cos \theta)}{2 \pi R \sin \theta}, \]

\[ B_\varphi = \frac{\mu_0 I (\sqrt{r^2 + z^2} - z)}{r \sqrt{r^2 + z^2}}. \]

Accordingly the components of the electromagnetic force in the cylindrical coordinate system are:

\[ F_z = -J_z B_\varphi - \sigma U_z B_\varphi^2 - \sigma U_z B_z^2, \]

\[ F_z = J_z B_\varphi + \sigma U_\varphi B_\varphi B_z - \sigma U_z B_\varphi^2, \]

\[ F_\varphi = -(J_\varphi B_z + \sigma U_\varphi B_z^2) - \sigma U_z B_\varphi B_z. \]

The resulting expressions for the electromagnetic force were used in the Navier-Stokes equation to calculate the velocity field.

3. Results

Numerical results were obtained at electrical currents 100 A, 200 A, 400 A, 600 A, 800 A and 1000 A in the wide range of the external MF from \(5 \times 10^{-5}\) T to 1 T.

The dependence of the kinetic energy in the volume on the external MF is shown in figure 3. In cases of \(I=100\) A and \(I=200\) A with the external MF \(B_z=5 \times 10^{-5}\) T there is the system of two-toroidal-vortices. The kinetic energy rises and reaches maximum at \(B_z \sim 6 \times 10^{-3} \div 8 \times 10^{-3}\) T, after that the energy decreases, being suppressed by the external MF. In cases of \(I = 400 \div 1000\) A there is one-toroidal-vortex system \((B_z < 10^{-4}\) T or \(B_z < 2 \times 10^{-4}\) T), when the external MF rises EVF transforms to the system of two vortices and the kinetic energy reaches minimum when the second vortex appears. After that, the energy increases to maximum \((B_z \sim 10^{-2}\) T) and then (cases 100 A and 200 A) starts decreasing.

The dependence of the axial velocity on the external MF is shown in figure 4. For cases 100 A and 200 A, the zone of low axial velocity is observed in the range of the external MF \(5 \times 10^{-5} \div 5 \times 10^{-3}\) T, for cases 400 A and 600 A - \(10^{-4} \div 5 \times 10^{-3}\) T and for 800 A and 1000 A cases - \(2 \times 10^{-4} \div 5 \times 10^{-3}\) T.

The dependence of the azimuthal velocity on the external MF is shown in figures 5 and 6.

Figure 3. Dependence of the kinetic energy in the volume on the external MF.

Figure 4. Dependence of axial velocity on the external MF, \(r = 0\) mm, \(z = 10\) mm.
then decreases. The same situation holds for the case of \( r = 80 \text{ mm} \), but maximum ranges are \( B_z \sim 8 \times 10^{-3} \div 2 \times 10^{-2} \text{ T} \). At a strong external MF there are two zones: flow zone near the small electrode and flow zone near the large electrode (the flow was suppressed by the external MF).

**Conclusions**

Electrovortex flow regimes were numerically studied in a wide range of external MF. The results show that a strong axial MF leads to a decrease in the intensity of the flow in the bath. For our case, the maximum of the flow intensity is observed at \( B_z \sim 10^{-2} \text{ T} \), with a stronger axial MF, the flow intensity decreases, and the fluid moves predominantly in a limited area near the small electrode.

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