Decoherence from Internal Dynamics in Macroscopic Matter-Wave Interferometry

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Usually, decoherence is generated from the coupling with an outer environment. However, a macroscopic object generically possesses its own environment in itself, namely the \textit{internal} dynamics (ID)\textsuperscript{1} when only small part of the total system, e.g. its CM, is under consideration. In this paper, we would like to address a question: when and how the ID decoheres interference of a macroscopic object. We also show nontrivial expectation that the weak localization of large molecules in a disordered potential can be destroyed by the decoherence generated from the ID without any external perturbation breaking the time reversal symmetry.

Let us consider a macroscopic object consisting of \(N\) particles exposed to the external potential \(V_{ex}\). The total Hamiltonian can be written as

\[
H = \sum_{i} \frac{p_{i}^2}{2m_{i}} + V_{in}(\{x_{i}\}) + \sum_{i} V_{ex}(x_{i})
\]

\[
= \frac{p^{2}}{2M} + V_{ex}(X) + \sum_{\alpha} \frac{\mu_{\alpha}^{2}}{2m_{\alpha}} + V_{in}(\{\xi_{\alpha}\}) + \Gamma (X, \{\xi_{\alpha}\})
\]

\[
= H_{CM} + H_{in} + \Gamma,
\]

where \(V_{in}\) is an internal (or confinement) potential, \(P\) and \(X\) are a momentum and a coordinate of the CM with a mass \(M = \sum m_{i}\) respectively, while \(\mu_{\alpha}\) and \(\xi_{\alpha}\) the same quantities of the internal degrees of freedom with the reduced mass \(m_{\alpha}\). \(\Gamma\) represents the coupling between the CM and the ID. Since one finds \(V_{in}\) is determined only from the relative coordinates \(\xi_{\alpha}\), i.e. \(V_{in}(\{x_{i}\}) = V_{in}(\{\xi_{\alpha}\})\), the coupling term \(\Gamma\) depends only on the external potential \(V_{ex}\). It is easy to show that if \(V_{ex}\) does not correspond to a simple form such as constant, linear, and harmonic, the non-zero coupling \(\Gamma\) always exists\textsuperscript{1}.

The \textit{CM motion} can be entangled with \textit{its ID} when the anharmonic external potential is applied. We call such a non-trivial external potential “nonlinear” since the corresponding force is nonlinear. It is noted that existence of the external potential has nothing to do with generation of the decoherence.

In a usual two slit experiment, a macroscopic object freely flies to a screen. Therefore, no entanglement between CM and ID arises, neither does the decoherence from ID. However, one can ask what happens if the repulsive potential produced by the slits is considered. For example, the van der Waals interaction between the C\textsubscript{70}...
and the grating was assumed to correctly explain experimental results [10]. Since C_{70} molecule is too small to see the anharmonic shape of the external potential the coupling between the CM and the ID is hardly expected. One can still insist, however, that not only if the slit wall contains more rapidly varying repulsive potential but also if the size of the object becomes larger, for example by using an insulin, then the coupling term might manifest itself. In this paper we consider such situation that the coupling between CM and ID is non-negligible. It must be mentioned that we ignore all the other sources of decoherences except those originating from the ID for our discussion, but will briefly comment it. We also note that neither we derive any new formula nor estimate any values of this effect. We would like to show basic physical mechanism and possibility of the phenomena.

II. REVIEW OF DECOHERENCE

First, we briefly review the decoherence from an external environment. Figure 1(a) shows usual two path interferometry, where one of the paths interacts with a “bath” of particles, i.e. an environment. One can write an initial total wavefunction in the following way

$$\Psi(0) = \frac{1}{\sqrt{2}} [\psi_1(0, X) + \psi_2(0, X)] \otimes \chi(0, \xi),$$

where $\psi_1(2)$ and $\chi$ denote a wavefunction of a system moving along the path 1(2) and that of a bath described by variables $\xi$, respectively. Under the dynamics, which includes interaction with the bath only along the path 2, the wavefunction becomes

$$\Psi(t) = \frac{1}{\sqrt{2}} [\psi_1(t, X) \otimes \chi_1(t, \xi) + \psi_2(t, X) \otimes \chi_2(t, \xi)],$$

(3)

where $\chi_1(2)$ is a state of the bath with the system going around the path 1(2). Here, we assume the coupling with the bath is small enough to have no influence on the system, but changes only a state of the bath [17, 18].

From the reduced density matrix the interference term is given as

$$\text{Re} \left[ \psi_1^*(t, X) \psi_2(t, X) \int d\xi \chi_1^*(t, \xi) \chi_2(t, \xi) \right].$$

(4)

The physical meaning of Eq. (4) is obvious. The first term contains usual information of the interference pattern of the system going through the two paths. The second term corresponds to the visibility which measures the decoherence. Eq. (4) allows one to interpret the reduction of the interference in terms of a reduction in the overlap of the bath states for the two paths. Stern, Aharony and Imry [17] argue that one can make the identification

$$\langle e^{i\phi} \rangle = \int d\xi \chi_1^*(t, \xi) \chi_2(t, \xi),$$

(5)

where $\langle e^{i\phi} \rangle \equiv \int d\xi \chi^*(0, \xi)e^{i\phi(\xi)}\chi(0, \xi)$, and $\phi(\xi) = -\int dt V_I(t, X_2(t), \xi)$. Here $V_I$ denotes the interaction potential between the system and the environment in the interaction picture, i.e. $V_I \equiv \exp(iH_{env}t)V(X_2, \xi)\exp(-iH_{env}t)$ in which $H_{env}$ and $V$ represent the Hamiltonian of the environment and its interaction with the system, respectively. Eq. (5) implies that the reduction of the interference can also be ascribed to the accumulating phase uncertainty of the system on the interacting path being subject to an uncertain potential. In this sense decoherence is often referred to as “dephasing”.

III. DECOHERENCE FROM INTERNAL DYNAMICS

A. Two slit interferometry

Now let us consider two path interferometry of a macroscopic object with its complicated ID. The CM and the ID now play roles of a system and an environment, respectively. We take into account the case that the object is moving freely as shown in Fig. 1(b), so that there is no entanglement between the CM and the ID. The motion of the CM is easily described by a plane wave, $\psi_\ell(t, X)$. The CM of the object then shows perfect coherence since its CM dynamics is completely isolated. When one increases the path length of one of the two paths by amount of $l$, clear interference pattern is expected from the term
\[ \psi_1^*(t, X) \psi_2(t, X) = \psi_0^*(t + \Delta t, X) \psi_0(t, X) \] between two CM states, where \( \Delta t = l/v \) (\( v \) is the velocity of the CM).

The final wavefunction is given as

\[ \Psi(t) = \frac{1}{\sqrt{2}} \left[ \psi_0(t + \Delta t, X) + \psi_0(t, X) \right] \otimes \chi(t, \xi). \] (6)

The time delay \( \Delta t \) is only applicable to the CM since the ID independently evolves in time. The overlap of the ID always yields that of the same states, i.e., complete coherence. Without entanglement with the ID the decoherence of the CM cannot be generated.

Let us consider now the case an object goes through nonlinear external potentials along the paths as shown in Fig. 1(c). We assume these nonlinear potentials are equivalent for the two paths. In a two slit interferometry, in general, the two slits are made to have the same geometry. Even though the CM motion is entangled with the ID during the passage through such nonlinear region, this does not generate any decoherence. The reason is that the evolution of the ID is always equal for the two paths, so that \( \chi_1(t, \xi) = \chi_2(t, \xi) \), i.e., \( \int d\xi \chi_1(t, \xi) \chi_2(t, \xi) = 1 \). The entanglement with internal environment makes the phases of each of the partial waves of the CM uncertain in viewpoint of Eq. (6), but does not alter the relative phase. In Fig. 1(d), we introduce additional delay for one of the paths. When the delay is given after the passage through the nonlinear region, the situation exactly corresponds to the case described in Fig. 1(b). The only difference is that one starts not with an initial ID state \( \chi_1(0, \xi) = \chi_2(0, \xi) \) but with \( \chi_1(t_0, \xi) = \chi_2(t_0, \xi) \), where \( t_0 \) is the time when the object departures from the nonlinear region. When the delay is given before, one can see that the situation is also the same as the case shown in Fig. 1(b) by considering the argument related to Eq. (6).

It should be noted that in usual decoherence generated from an outer environment it is not easy to find the case that the two paths have the same environment. If the decoherence occurs mainly by interacting with other particles, the system going through two paths accumulates different random phases from scattering with different particles of different states. The system going through the two paths thus see different environments. This is the reason why the decoherence from an outer environment has been dealt with the setup shown in Fig. 1(a). One example that the same environment is applied to the two interfering waves is the interaction of an interfering electron with zero-point (or vacuum) fluctuation, where the electron does not decohere [13, 20, 21].

\subsection{B. Quantification of decoherence from internal dynamics}

In the above discussion it has been shown that it is not easy to see the decoherence generating from the ID with usual simple geometry of interferometry. The only way to observe the decoherence from the coupling with ID is that the ID’s should see different nonlinear interactions for each path. Without loss of generality this situation can be represented as the case that only one path contains the nonlinear region. The overlap of the ID can then be written as

\[ \langle \chi(0, \xi) | e^{iH_{in} \Delta t} \hat{T} \exp \left[ -i \int_0^{\Delta t} dt \left( H_{in} + \Gamma \right) \right] | \chi(0, \xi) \rangle, \] (7)

where \( \Delta t \) and \( \hat{T} \) denote the interaction time within the nonlinear potential and the time ordering operator, respectively. Asymmetric geometry of interferometry is sometimes useful, for example, measurement of phase of the transmission coefficient through a quantum dot [22], where a quantum dot is plugged into one of the arms of an Aharonov-Bohm ring. From the Aharonov-Bohm oscillation one can determine the phase shift of electron passing through the quantum dot.

The quantity given in Eq. (7) is known as so called fidelity [23]. The decay of such a quantity determines the decoherence rate. One important remark is that even for the ID with a few degrees of freedom the fidelity decays exponentially if its dynamics is chaotic [21, 25]. It opens possibility that the decoherence can occur in molecules consisting of even several atoms from entanglement with its ID [21] (See also [24, 27, 28, 29, 30, 31] for decoherence generated from chaos) in a certain special condition. When a single coherent state, a minimal wavepacket, is chosen as an initial state of ID, one can expect golden rule or Lyapunov decay depending on the strength of the coupling [32], in which the fidelity decay does not much depend on the initial condition for a given energy as far as the completely chaotic dynamics is concerned for the ID. This situation corresponds to the ID governed by rather small number of degrees of freedom at low temperature. As we mentioned in the beginning, however, for small molecules it is hard to expect the coupling between the CM and the ID. To see this effect in the systems with small degrees of freedom we need something different from usual molecules.

For rather bigger systems, which we are interested in, it is not easy to directly calculate how much the interaction between the CM and the ID make an influence on the state of the ID because the internal degrees of freedom consist of many particles. First let us consider the zero temperature case. If the interaction is strong enough to generate any kind of elementary excitations such as phonons, charge density waves, magnons for magnetic systems, and so on, then the CM will lose its coherence completely. Nothing happens for the system going through the path 1 in Fig. 1 while the state of the ID through the path 2 in Fig. 1 is changed from the ground state to the excited state of the elementary excitation. By checking the state of the ID one can see which path the system go through. It is nothing but a complete decoherence. In this case it is crucial to know whether the excitation is gapless or not. At finite temperature the situation is much more complicated. In the beginning we assumed that there is no other sources of decoherences
reflecting from random scatterers in a disordered system. (b) Information thus flows from the CM to the ID. Following outer environment, the ID plays a role of an observer. does not know the difference between an internal and an from the object to outside. Since a system (the CM) only within a macroscopic object itself. In this sense no world. The decoherence is generated from entanglement system including CM and ID is isolated from the external want to calculate it in this paper, but only point out the theoretical technique once the time dependent interac-
tion $\Gamma$ is known. Surely it is still hard task. We do not
to calculate it this paper, but only point out the possible existence of the decoherence generating from the coupling with the ID.

Another remark on Eq. (1) is its physical interpreta-
tion in terms of quantum measurement. Here, the total system including CM and ID is isolated from the external world. The decoherence is generated from entanglement only within a macroscopic object itself. In this sense no external observer exists. No information is transferred from the object to outside. Since a system (the CM) does not know the difference between an internal and an outer environment, the ID plays a role of an observer. Information thus flows from the CM to the ID. Following Zurek’s argument [3], one can say the ID is watching the CM.

C. Weak localization in disordered systems

Finally, let us consider one interesting example, weak localization in disordered systems [32]. Two time reversal paths multiply reflecting from the random scatterers leads to the localization of a wavefunction due to their constructive interference as shown in Fig. 2(a). Such localization is fragile for both the decoherence and the per-
turbation breaking time reversal symmetry. A random potential generates complicated and chaotic dynamics, which can give rise to the entanglement between CM and ID. At first glance the decoherence from the ID is hardly expected to arise since the total system has time reversal symmetry. One might think that the state of the ID of the CM rotating clockwise must be the same as that of counter-clockwise. It can be shown, however, that the coupling to the ID can destroy the weak localization by generating the decoherence from the ID.

To prove the appearance of the decoherence let us consider the overlap between two ID, namely $\chi_1$ and $\chi_2$, of the time reversal paths of CM after a round trip along a closed loop. Since we are interested only in the ID, the influence from its coupling to the CM can be regarded as a time dependent external perturbation, i.e. $\Gamma(X_{1(2)}(t), \xi) \equiv \Gamma_{1(2)}(t, \xi)$. Note $\Gamma_{1(2)}(t + \tau, \xi) = \Gamma_{1(2)}(t, \xi)$, where $\tau$ denotes the duration time taken for a round trip around the closed loop. The final state $\chi_{1(2)}(\tau)$ is then given as

$$\chi_{1(2)}(\tau, \xi) = \hat{T} \exp \left\{ -i \int_0^\tau dt \left[ H_{in}(\xi) + \Gamma(X_{1(2)}(t), \xi) \right] \right\} \chi_0.$$  

(8)

Even though $X_1(t) = X_2(\tau - t)$ holds, i.e. $\Gamma_1(t, \xi) = \Gamma_2(\tau - t, \xi)$, one finds $\chi_1(\tau) \neq \chi_2(\tau)$ due to existence of the time ordering operator $\hat{T}$. To make it more clear, let us consider a simple example: three scatterers well localized in space as shown in Fig. 2(b). During free propagation between two scatterers the CM is decoupled from the ID. We assume the process of collision with the scatterers is short enough to be described by delta-function in time. The interaction term $\Gamma_1$ for the clockwise propagation can then be given as

$$\Gamma_1(t, \xi) = f_1(\xi)\delta(t - \tau_1) + f_2(\xi)\delta(t - \tau_2),$$  

(9)

where $\tau_1$ and $\tau_2$ are the collision times upon the first and the second scatterer, respectively, and $0 < \tau_1 < \tau_2 < \tau$. After one round trip, the states of the ID for the clock-
wise and the counter-clockwise $\chi_1$ and $\chi_2$ are respectively given as

$$\chi_1(\tau) = U_{3K_2U_2K_1U_1}\chi_0,$$

$$\chi_2(\tau) = U_{1K_1U_2K_2U_3}\chi_0,$$  

(10)

where by using the eigenstates of $H_0$ ($H_0 |i\rangle = E_i |i\rangle$) one obtains $U_{n,ij} = \exp[-iE_i\Delta t_n]|ij\rangle$, and $K_{n,ij} = \langle ij | \exp[-i\phi_n(\xi)]|ji\rangle$. Here, $\Delta t_1 = \tau_1$, $\Delta t_2 = \tau_2 - \tau_1$, and $\Delta t_3 = \tau - \tau_2$. It is obvious that in general $U_3K_2U_2K_1U_1 \neq U_1K_1U_2K_2U_3$ since $K_1$ and $K_2$ are not

FIG. 2: (a) Two time reversal paths, $\chi_1$ and $\chi_2$, multiply reflecting from random scatterers in a disordered system. (b) Simple example with three scatterers labeled by 0, 1, and 2. $U_n$ denotes an unitary operator freely propagating between two scatterers.
diagonal. Consequently one can find that in general $\langle \chi_1(\tau) | \chi_2(\tau) \rangle < 1$. The weak localization of the CM of a macroscopic object in disordered potentials can be destroyed due to the coupling to the ID.

IV. SUMMARY

In summary, we have investigated the decoherence generated from the internal dynamics of a macroscopic object. In a usual setup of two slit interferometry, it is hard to expect the appearance of such decoherence. Only asymmetric geometry of the interfering paths containing anharmonic external potential allows one to observe the decoherence from the internal dynamics. Such decoherence can then be measured by the fidelity given in Eq. (7). In this case, the internal degrees of freedom of a macroscopic object are watching its center of mass motion. The weak localization of the center of mass motion of a macroscopic object in disordered potentials can also be destroyed by such decoherence without any external perturbation breaking time reversal symmetry.

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