Dual Learning Algorithm for Delayed Feedback in Display Advertising

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Abstract

In display advertising, predicting the conversion rate, that is, the probability that a user takes a predefined action on an advertiser’s website is fundamental in estimating the value of showing a user an advertisement. There are two troublesome difficulties in the conversion rate prediction due to the delayed feedback. First, some positive labels are not correctly observed in training data, because some conversions do not occur right after clicking the ads. Moreover, the delay mechanism is not uniform among instances; some positive feedback is much more frequently observed than the others. It is widely acknowledged that these problems cause a severe bias in the naive empirical average loss function for the conversion rate prediction. To overcome the challenges, we propose two unbiased estimators, one for the conversion rate prediction, and the other for the bias estimation. Subsequently, we propose an interactive learning algorithm named Dual Learning Algorithm for Delayed Feedback (DLA-DF) where a conversion rate predictor and a bias estimator are learned alternately. The proposed algorithm is the first of its kind to address the two major challenges in a theoretically principal way. Lastly, we conducted a simulation experiment to demonstrate that the proposed method outperforms the existing baselines and validate that the unbiased estimation approach is suitable for the delayed feedback problem.

1 Introduction

Display advertising is a way of online advertising where advertisers pay publishers for placing graphical ads, video ads, and so forth on their websites. The conventional method of selling display advertising was a direct long-term contract between advertisers and publishers. Over the last decade, selling display ads via programmatic instantaneous auction called real-time bidding (RTB) has become common in performance display advertising [17]. Advertisers have been offered several payment options, such as paying per click (CPC) and paying per conversion (CPA). CPA has become predominant since conversion has more direct effects on advertisers’ return on investment (ROI) than a click does. Especially, it is also less susceptible to notorious online fraud [14]. Therefore, we consider a CPA model after clicking on an advertisement for which advertisers pay only if a user takes a predefined action on their website. A platform that supports such performance-based payment options needs to convert advertisers’ bids to the expected price per impression (eCPM). In a CPA model, eCPM depends on the probability that a click leads to a resulted conversion, i.e., the conversion rate (CVR). To seek the optimal price to bid for each impression, accurately predicting a CVR is essential, which results in an efficient marketplace [15].
Although Click-Through Rate (CTR) prediction has been extensively studied and shown promising empirical results on benchmark datasets \([4,5,7,9]\), it is difficult to directly apply these methods to CVR prediction. This is because a predictive model should be trained on fresh data to prevent data from becoming stale and to follow the seasonal trend. There are two troublesome difficulties in using fresh data for CVR prediction. First, unlike a click event, a conversion does not always occur right after clicking a display advertisement. While the time delay between an impression and its click is normally a few seconds, the gap between a click and its conversion is a few hours or even days. Consequently, some conversions that will occur have not taken place yet at the training step, and thus, are falsely considered as negative (Positive-Unlabeled problem). The second challenge is that the missing mechanism of conversion data is missing-not-at-random (MNAR). For example, conversions are much more likely to be missing if their clicks have just happened. In addition, decisive users are much more likely to convert right after the clicks than indecisive users. Thus, the probabilities of conversions being observed correctly are not uniform among samples. In the literature of covariate shift or causal inference, it is widely recognized that the MNAR mechanism can cause sub-optimal and severely biased predictions (MNAR problem) \([8,20,21,22,23]\).

Works have already been conducted to address this delayed feedback issue. \([8]\) assumes that the delay distribution is exponential and proposes two models. One predicts the conversion, and the other predicts the delay in conversion. The two models are jointly trained via the EM algorithm or the gradient descent optimization. In addition, \([25]\) extends this study and proposes to use a non-parametric model for the estimation of the delay distribution. The work that is most related to ours is \([11]\). This study compares five different loss functions to alleviate the bias. In particular, they introduce an inverse propensity weighting estimator in causal inference and positive-unlabeled learning as two separate approaches to the delayed feedback problem. However, the delayed feedback setting contains both the positive-unlabeled and the MNAR problems as we discussed above, and a method that simultaneously solves the two challenging problems has not been proposed yet.

To address the two major challenges, we first propose an unbiased estimator for the loss function of interest that can be estimated from the observable data. The proposed estimator is based on the combination of the estimation methods in causal inference and positive-unlabeled learning \([19,18,8,2]\). The proposed estimator weights each observed sample using the parameter called propensity score. There is, however, a difficulty that true propensity scores are unknown in nature; thus, they have to be estimated. To accurately estimate the propensity score, we subsequently show that the unbiased propensity estimation is possible by weighing each sample using their conversion rate. Based on these observations, we propose an algorithm called Dual Learning Algorithm for Delayed Feedback (DLA-DF) motivated by the work in the field of unbiased learning-to-rank \([11]\). The proposed algorithm jointly learns CVR predictor and propensity score estimator alternately only from observed conversion data and is the first method to address the PU and MNAR problems simultaneously in a theoretically principal way.

Furthermore, to evaluate the efficacy of the counterfactual approach in the delayed feedback setting, we conducted an experiment where the delayed feedback situation is simulated. The experimental result demonstrates that the proposed algorithm outperforms the existing baselines in most cases, and the unbiased estimation approach is valid in the delayed feedback setting.

The rest of the paper is as follows. In Section 2, we formulate the delayed feedback problem. In Section 3, we explain the detail of the proposed method and provide the statistical properties of the proposed estimators. The experimental setup and results are described in Section 4. We conclude this paper with a summary of contributions and future research directions in Section 5.

# Problem Setting

In this section, we introduce some notations and formulate the delayed feedback setting.

Given a set of \(N\) units indexed by \(i\), we denote \(X_i \in \mathcal{X}\) as the feature vector for each unit \(i\). Let \(Y_i \in \mathcal{Y} = \{0, 1\}\) be a random variable representing true conversion information. If an individual \(i\) will convert then \(Y_i = 1\), otherwise, \(Y_i = 0\). In the delayed feedback setting, the true conversion variables are not fully observable because of the conversion delay. To precisely formulate this delayed feedback setting, we introduce another binary random variable \(O_i \in \{0, 1\}\). This random variable represents whether the true outcome is observed or not, which depends on the elapsed time from clicks. If \(O_i = 1\), then the conversion is observed; otherwise, the conversion is not observed. Using
these random variables, we can represent an observed outcome indicator $Y_{\text{obs}}$ as follows:

$$Y_{\text{obs}}^i = O_i \cdot Y_i$$  \hfill (1)

If we have observed the conversion of $i$, then $Y_{\text{obs}}^i = 1$, otherwise, $Y_{\text{obs}}^i = 0$. Note that the true conversion indicator $Y_i$ is not always equal to the observed conversion indicator $Y_{\text{obs}}^i$, and the conversion of $i$ is observable only when the unit will convert and the true outcome is observable (i.e., $Y_{\text{obs}}^i = 1 \iff O_i = 1 \& Y_i = 1$).

Throughout the paper, we assume that the following assumption holds.

$$Y \perp O \mid X$$  \hfill (2)

This assumption is called Unconfoundedness in the context of causal inference [8, 18, 19], which means that features affecting both $O$ and $Y$ are fully observed. From this assumption, we obtain the following equation relating the true conversion rate and the observed conversion rate.

$$P (Y_{\text{obs}}^i = 1 \mid X_i) = P (O_i \cdot Y_i = 1 \mid X_i)$$

$$= P (O_i = 1 \mid X_i) \cdot P (Y_i = 1 \mid X_i) \iff (2)$$

$$= \theta(X_i) \cdot \gamma(X_i)$$  \hfill (3)

where we denote $P (O = 1 \mid X)$ as $\theta(X)$ and $P (Y = 1 \mid X)$ as $\gamma(X)$, respectively.

The goal of this paper is to obtain a hypothesis $f : \mathcal{X} \to (0, 1)$ that well predicts the true conversion rate $\gamma(\cdot)$. To achieve this goal, we define the ideal loss function that should be optimized to obtain a well-performing hypothesis as follows.

**Definition 1.** (Ideal loss function for conversion rate prediction) The ideal loss function to be optimized to obtain a hypothesis $f$ is defined as follows.

$$\mathcal{L}_{\text{cvr}}(f) = \mathbb{E}_{(X,Y)} \left[ Y \delta^{(1)}(f) + (1 - Y)\delta^{(0)}(f) \right]$$

$$= \mathbb{E}_X \left[ \gamma(X)\delta^{(1)}(f) + (1 - \gamma(X))\delta^{(0)}(f) \right]$$  \hfill (4)

where the functions $\delta^{(1)}(f)$ and $\delta^{(0)}(f)$ characterize the loss function. For example, when these functions are defined as follows, then Eq. (4) is called the binary cross entropy loss.

$$\delta^{(1)}(f) = - \log(f(X)), \ \delta^{(0)}(f) = - \log(1 - f(X))$$

The ideal loss function in Eq. (4) is defined using the true conversion indicator $Y$. In standard supervised machine learning setting, model parameters of $f$ are obtained by the empirical risk minimization framework as follows [16].

$$\min_{f \in \mathcal{F}} \hat{\mathcal{L}}_{\text{cvr}}(f) = \frac{1}{N} \sum_{i=1}^{N} \left[ Y_i \delta_i^{(1)}(f) + (1 - Y_i)\delta_i^{(0)}(f) \right]$$

where $\mathcal{F}$ is a space of real-valued functions called the hypothesis space. Here, $\hat{\mathcal{L}}_{\text{cvr}}(f)$ is the empirical average estimator for the ideal loss function and is unbiased (i.e., $\mathbb{E}[\hat{\mathcal{L}}_{\text{cvr}}(f)] = \mathcal{L}_{\text{cvr}}(f)$). However, in the delayed feedback setting, the true conversion indicators $\{Y_i\}$ are unobserved, and thus, the empirical risk minimization is infeasible in nature. Therefore, the critical component of the delayed feedback problem is to estimate the ideal loss function using only observable variables.

### 3 Proposed Method

In this section, we propose a dual learning framework inspired by a learning procedure of unbiased learning-to-rank [1]. The proposed framework treats CVR and propensity score estimation problems as a dual problem and jointly learns two predictors only from observed conversions.
3.1 Unbiased Conversion Rate Prediction

First, we formally define the propensity score for the delayed feedback setting as follows.

**Definition 2.** (Propensity score) The propensity score for the delayed feedback setting is defined as

\[
\theta(X) = P(O = 1 \mid X) = P(Y^{\text{obs}} = 1 \mid Y = 1, X)
\]  

(5)

The propensity score for the delayed feedback setting is interpreted as the probability of each conversion being correctly observed. In the context of causal inference, the unbiased estimator for the causal effects of treatments can be derived by weighting each sample by the inverse of the propensity score \[10\] [18] [8]. However, in the delayed feedback setting, observation indicators \( \{O_i\} \) are unobserved, and the inverse propensity score (IPS) estimation technique cannot be applied directly. Thus, we combine the IPS estimator with the estimation technique in the field of positive-unlabeled learning \[6\] [2] and define the IPS estimator for the delayed feedback setting as follows.

**Definition 3.** (IPS estimator for the delay feedback setting) Given propensity scores, the IPS estimator for the ideal loss function in Eq. (4) is defined as

\[
\hat{L}_{\text{IPS}}(f) = \frac{1}{N} \sum_{i=1}^{N} Y_i^{\text{obs}} \left( \frac{1}{\theta(X_i)} \delta_i^{(1)}(f) + \left( 1 - \frac{1}{\theta(X_i)} \right) \delta_i^{(0)}(f) \right) + (1 - Y_i^{\text{obs}}) \delta_i^{(0)}(f)
\]

(6)

The following proposition formally proves that the IPS estimator is unbiased against the ideal loss function.

**Proposition 1.** (Unbiasedness of the IPS estimator) The IPS estimator in Eq. (6) is unbiased against the ideal loss function in Eq. (4).

\[
\mathbb{E} \left[ \hat{L}_{\text{IPS}}(f) \right] = \mathcal{L}_{\text{cvr}}(f)
\]

Proof.

\[
\mathbb{E} \left[ \hat{L}_{\text{IPS}}(f) \right]
= \mathbb{E} \left[ \frac{1}{N} \sum_{i=1}^{N} Y_i^{\text{obs}} \left( \frac{1}{\theta(X_i)} \delta_i^{(1)}(f) + \left( 1 - \frac{1}{\theta(X_i)} \right) \delta_i^{(0)}(f) \right) + (1 - Y_i^{\text{obs}}) \delta_i^{(0)}(f) \mid X \right]
= \mathbb{E} \left[ \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_{Y^{\text{obs}}} \left[ Y_i^{\text{obs}} \mid X \right] \left( \frac{1}{\theta(X_i)} \delta_i^{(1)}(f) + \left( 1 - \frac{1}{\theta(X_i)} \right) \delta_i^{(0)}(f) \right) + (1 - \mathbb{E}_{Y^{\text{obs}}} \left[ Y_i^{\text{obs}} \mid X \right]) \delta_i^{(0)}(f) \right]
= \mathbb{E} \left[ \frac{1}{N} \sum_{i=1}^{N} \theta(X_i) \cdot \gamma(X_i) \left( \frac{1}{\theta(X_i)} \delta_i^{(1)}(f) + \left( 1 - \frac{1}{\theta(X_i)} \right) \delta_i^{(0)}(f) \right) + (1 - \theta(X_i) \cdot \gamma(X_i)) \delta_i^{(0)}(f) \right]
= \frac{1}{N} \sum_{i=1}^{N} \mathbb{E} \left[ \gamma(X_i) \delta_i^{(1)}(f) + (1 - \gamma(X_i)) \delta_i^{(0)}(f) \right]
= \mathcal{L}_{\text{cvr}}(f)
\]

Proposition 1 validates that the proposed IPS estimator is unbiased against the ideal loss, and thus, the unbiased training of the true conversion rate predictor is feasible in the delayed feedback setting.

3.2 Unbiased Propensity Estimation

The unbiasedness stated in Proposition 1 is desirable to obtain a conversion rate predictor, but is dependent on the availability of the true propensity score. In general, estimation of the propensity score in the IPS estimator can be formulated as the classification problem \[24\] [13], but, the indicator variables \( \{O_i\} \) are never observable in the delayed feedback setting. Therefore, we propose a method to unbiasedly estimate the propensity score from observed conversion indicators.

We first define the ideal loss function for the propensity estimation as follows.
Definition 4. (Ideal loss function for the propensity estimation) The ideal loss function for the propensity estimation is defined as follows.

\[
\mathcal{L}_\text{propensity}(g) = \mathbb{E}_{(X,y)} \left[ O \delta_i^{(1)}(g) + (1 - O) \delta_i^{(0)}(g) \right] \\
= \mathbb{E}_{X} \left[ \theta(X) \delta_i^{(1)}(g) + (1 - \theta(X)) \delta_i^{(0)}(g) \right]
\]  

(7)

where \( g : \mathcal{X} \rightarrow (0, 1) \) is a hypothesis that estimates the propensity score.

Then we define the Inverse Conversion Rate (ICVR) estimator having the same structure as the IPS estimator below.

Definition 5. (ICVR estimator) Given conversion rates, the ICVR estimator for the ideal loss function in Eq. (7) is defined as

\[
\hat{\mathcal{L}}_{\text{ICVR}}(g) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_Y \left[ \frac{1}{\gamma(X)} \delta_i^{(1)}(g) + \left(1 - \frac{1}{\gamma(X)} \right) \delta_i^{(0)}(g) \right] + \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_{y^{\text{obs}}} \left[ \frac{1}{\gamma(X)} \delta_i^{(1)}(g) + (1 - \frac{1}{\gamma(X)}) \delta_i^{(0)}(g) \right]
\]  

(8)

Following the same logic flow in Proposition 1, the next proposition proves that the ICVR estimator is unbiased against the ideal loss function for the propensity estimation.

Proposition 2. (Unbiasedness of ICVR estimator) The ICVR estimator in Eq. (8) is unbiased against the ideal loss function in Eq. (7).

\[
\mathbb{E} \left[ \hat{\mathcal{L}}_{\text{ICVR}}(g) \right] = \mathcal{L}_\text{propensity}(g)
\]

Proof.

\[
\mathbb{E} \left[ \hat{\mathcal{L}}_{\text{ICVR}}(g) \right] \\
= \mathbb{E} \left[ \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_Y \left[ \frac{1}{\gamma(X)} \delta_i^{(1)}(g) + \left(1 - \frac{1}{\gamma(X)} \right) \delta_i^{(0)}(g) \right] + \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_{y^{\text{obs}}} \left[ \frac{1}{\gamma(X)} \delta_i^{(1)}(g) + (1 - \frac{1}{\gamma(X)}) \delta_i^{(0)}(g) \right] \right]
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} \mathbb{E} \left[ \frac{1}{\gamma(X)} \delta_i^{(1)}(g) + (1 - \gamma(X)) \delta_i^{(0)}(g) \right]
\]

\[
= \mathcal{L}_\text{propensity}(g)
\]

\( \square \)

Proposition 2 indicates that the proposed ICVR estimator is unbiased against the ideal loss for the propensity estimation, and thus, a better conversion rate predictor leads to a well-performing propensity estimator and vice versa. Based on these unbiased estimators, we propose Dual Learning Algorithm for Delayed Feedback that jointly trains the propensity estimator and the conversion rate predictor with observed conversion data.

3.3 Dual Learning Algorithm for Delayed Feedback

Here we state the proposed DLA-DF algorithm. The basic idea behind the algorithm is to estimate the propensity score and the true conversion rate using the unbiased estimators simultaneously.

First, given a propensity estimator \( g_\phi \) parameterized by \( \phi \), the loss function to derive the parameter of a conversion rate predictor \( f \) is given below.

\[
\hat{\mathcal{L}}_{\text{IPS}}(f | g_\phi) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_{y^{\text{obs}}} \left[ \frac{1}{g_\phi(X)} \delta_i^{(1)}(f) + \left(1 - \frac{1}{g_\phi(X)} \right) \delta_i^{(0)}(f) \right] + \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}_{y^{\text{obs}}} \left[ \frac{1}{\gamma(X)} \delta_i^{(1)}(f) + (1 - \gamma(X)) \delta_i^{(0)}(f) \right]
\]  

(9)

5
Algorithm 1 Dual Learning Algorithm for Delayed Feedback (DLA-DF)

Input: training data $D = \{X_i, Y_i^{\text{obs}}\}_{i=1}^N$; mini-batch size $m$; learning rate $\eta$.
Output: model parameters $\psi$ and $\phi$.
1: Initialize parameters with random weights $\psi, \phi$.
2: repeat
3: Sample mini-batch $\{X_j, Y_j^{\text{obs}}\}_{j=1}^m$ from $D$.
4: Compute $\hat{L}_{\text{IPS}}(f_\psi \mid g_\phi)$ in Eq. (9) with fixed $\phi$.
5: Update parameter by $\psi \leftarrow \psi + \eta \cdot \nabla_\psi \hat{L}_{\text{IPS}}(f_\psi \mid g_\phi)$.
6: Compute $\hat{L}_{\text{ICVR}}(g_\phi \mid f_\psi)$ in Eq. (10) with fixed $\psi$.
7: Update parameter by $\phi \leftarrow \phi + \eta \cdot \nabla_\phi \hat{L}_{\text{ICVR}}(g_\phi \mid f_\psi)$.
8: until convergence;
9: return $\psi, \phi$.

Next, given a conversion rate predictor $f_\psi$, parameterized by $\psi$, the loss function to derive the parameter of a propensity estimator $g$ is given below.

$$
\hat{L}_{\text{ICVR}}(g \mid f_\psi) = \frac{1}{N} \sum_{i=1}^N \frac{Y_i^{\text{obs}}}{f_\psi(X_i)} \left( \frac{1}{f_\psi(X_i)} \delta_i^{(1)}(g) + \left( 1 - \frac{1}{f_\psi(X_i)} \right) \delta_i^{(0)}(g) \right) + (1 - Y_i^{\text{obs}}) \delta_i^{(0)}(g).
$$

(10)

The detailed procedure of the proposed DLA-DF is described in Algorithm 1.

### 3.4 Statistical Properties

In this subsection, we state some statistical properties of the proposed estimators. Note that the formal proofs can be found in the supplementary material.

**Theorem 1.** (Variance) Given sets of independent random variables $\{(Y_i^{\text{obs}}, O_i, X_i)\}$, propensity scores $\{\theta(X_i)\}$, and a conversion rate predictor $f_\psi$, the variance of the IPS estimator is

$$
\mathbb{V} \left( \hat{L}_{\text{IPS}}(f) \right) = \frac{1}{N^2} \sum_{i=1}^N \gamma(X_i) \left( \frac{1}{\hat{\theta}(X_i)} - \gamma(X_i) \right) \left( \delta_i^{(1)}(f) - \delta_i^{(0)}(f) \right)^2.
$$

Replacing $\gamma$, $\theta$, $\delta_i^{(1)}(f)$, and $\delta_i^{(0)}(f)$ for $\gamma$, $\theta$, $\delta_i^{(1)}(g)$, and $\delta_i^{(0)}(g)$ provides the variance of the ICVR estimator.

**Proposition 3.** (Estimation Error Tail Bound) Under the same assumption as in Theorem 1, for any $\eta \in (0, 1)$, the following inequality holds with a probability of at least $1 - \eta$

$$
\left| \hat{L}_{\text{ICVR}}(f) - \hat{L}_{\text{IPS}}(f) \right| \leq \frac{1}{N} \sqrt{\frac{1}{2} \log \frac{2 N}{\eta} \sum_{i=1}^N \left( \frac{\delta_i^{(1)}(f) - \delta_i^{(0)}(f)}{\theta(X_i)} \right)^2}.
$$

Replacing $\gamma$, $\delta_i^{(1)}(f)$, and $\delta_i^{(0)}(f)$ for $\theta$, $\delta_i^{(1)}(g)$, and $\delta_i^{(0)}(g)$ provides the estimation error tail bound of the ICVR estimator.

The RHS of both the variance and the estimation error tail bound depending on the inverse of the propensity scores. Thus, these bounds can be loose, especially when there exists a severe delay. Moreover, the analysis implies that applying a variance reduction technique to the unbiased estimators might improve the statistical property of the estimator by reducing its variance at the cost of introducing some bias. From the implications above, we utilized the following non-negative estimator inspired by the work in positive-unlabeled learning [10] as follows:

**Definition 6.** (Non-negative estimator) When propensity scores and a constant $\beta \geq 0$ are given, then the non-negative estimator is defined as

$$
\hat{L}_{\text{nn}}(f) = \frac{1}{N} \sum_{i=1}^N \max \left\{ Y_i^{\text{obs}} \left( \frac{1}{\theta(X_i)} \delta_i^{(1)}(f) + \left( 1 - \frac{1}{\theta(X_i)} \right) \delta_i^{(0)}(f) \right) + (1 - Y_i^{\text{obs}}) \delta_i^{(0)}(f), -\beta \right\}.
$$

(11)
A larger value of $\beta \geq 0$ reduces a variance of the estimator at the cost of introducing some bias. We explore the effect of different values of the hyperparameter $\beta$ on the performance of the conversion rate predictor.

4 Experimental Results

In this section, we provide an empirical comparison of the proposed method and the baseline methods using synthetic dataset.

4.1 Experimental Setup

**Synthetic data generation procedure:** We created a synthetic dataset simulating the delayed feedback setting in display advertisement to evaluate the performance of the methods for the delayed feedback problem. The components of the synthetic data are as follows:

1. The number of click events $N$, and the number of features observed for each event $p$. We set $N = 100,000$ and $p = 10$, respectively.
2. The distribution $D_X$ of the feature vectors $X$. We drew odd-numbered features independently from a Gaussian distribution with a standard deviation of $0.25$. In contrast, we drew even-numbered features independently from a Bernoulli distribution with the parameter of $0.5$.
3. The training period $E$, which can be varied depending on the experimental condition.
4. The timestamps of clicks $ts_{\text{click}}$ sampled from the uniform distribution.
5. The delay between the click and the conversion $D$. Following the probabilistic model used in [3], we assumed that the distribution of the delay is exponential.
6. Coefficient vectors generating the true conversion rate $W_{\text{cvr}} \in \mathbb{R}^p$ and the parameter of the exponential distribution $W_{\text{expo}} \in \mathbb{R}^p$. Both coefficient vectors were sampled from a Gaussian distribution with a standard deviation of $\sigma = 0.01$.

To summarize, our data generation model is,

$$X_i \sim D_X, \quad W_{\text{cvr}} \sim \mathcal{N}(0, \sigma^2 I), \quad W_{\text{expo}} \sim \mathcal{N}(0, \sigma^2 I)$$
$$\gamma(X_i) = \text{sigmoid}(W_{\text{cvr}}X_i), \quad \lambda(X_i) = \exp(W_{\text{expo}}X_i)$$
$$ts_{\text{click}}_i \sim \text{Unif}(0, E), \quad D_i \sim \text{Expo}(\lambda(X_i))$$
$$O_i = 1\{ts_{\text{click}}_i + D_i \leq E\}, \quad Y_i \sim \text{Bern}(\gamma(X_i)), \quad Y_{\text{obs}}^i = O_i \cdot Y_i$$

where $ts_{\text{click}}_i + D_i$ represents $i$'s timestamp of conversion. If this timestamp exceeds the end of the training period $E$, this conversion is not correctly observed, (i.e., $O_i = 0$).

In the experiments, we used 25 randomly sampled realizations.

**Baselines and the proposed method:** We compared the performance of the following methods.

- **Oracle:** The logistic regression model trained with the true conversion data that is, in reality, unobservable. The performance of the oracle model is the best achievable prediction performance.
- **Naive:** The logistic regression model naively trained with the observed conversion data.
- **nnPU:** The logistic regression model trained with the following loss function.

$$\mathcal{L}_{\text{nnPU}}(f) = \frac{1}{N} \sum_{i=1}^{N} \max \left\{ Y_{i}^{\text{obs}} \left( \delta_i^{(1)}(f) - \delta_i^{(0)}(f) \right) + (1 - Y_{i}^{\text{obs}}) \delta_i^{(0)}(f), 0 \right\}$$

This is the positive-unlabeled loss was proposed in [12]. We applied the non-negative variance reduction technique to this loss function for ensuring fair comparison.
- **Delayed Feedback Model (DFM):** This model was proposed in [3], and we implemented it in the Tensorflow environment. We obtained the model parameters of DFM following the joint optimization procedure described in Section 4.2 of [3].
• nnDLA-DF: This is our proposed method. We used the logistic regression model for both conversion rate predictor ($f$) and propensity estimator ($g$). Both estimators were trained with the non-negative loss function defined in Eq. (11).

4.2 Results

Here we report the results of the experiment.

First, Figure 1-(b) shows the log-loss on the test sets relative to the performance of the oracle model. Note that the values of the training period $E$ were set as 0.5, 1, 2, 3, 4, 5 (days). A smaller value of $E$ introduces smaller propensities, as shown in Figure 1-(a), and this leads to a larger bias in observed data. The results show that the proposed method significantly outperformed the other methods when $E = 1, 2, 3$. In contrast, the benefit of DLA-DF was slight when $E = 0.5, 4, 5$, but, it was not largely outperformed by the other method in all the settings, which suggests the stable prediction performance of the proposed algorithm.

Figure 1-(c) shows the effect of the hyperparameter $\beta$ in Eq. (11), which controls the bias-variance trade-off of estimators for ideal loss functions. $\beta$ were set as $\{10^{-3}, 5 \times 10^{-3}, \ldots, 1\}$. A small $\beta$ reduces the variance of the estimator at the cost of introducing bias, in contrast, a large $\beta$ has little bias, but the variance can be huge. The figure shows the performance of the DLA-DF algorithm with different values of $\beta$ relative to $\beta = 0$. It was observed that it is possible to improve the log loss by approximately 2.5% by setting $\beta$ as 0.1 or 0.5 by appropriately controlling the bias-variance trade-off.

5 Conclusion

In this study, we explored the delayed feedback problem where the true conversion indicators are not fully observable due to the conversion delay. To address the problem, we developed a framework called Dual Learning Algorithm for Delayed Feedback that trains a conversion rate predictor and a propensity estimator alternately. In the empirical evaluation using the synthetic data, the proposed algorithm generally outperformed the existing baseline methods with respect to the log-loss. The results also suggest the benefit of the unbiased estimation approach in the delayed feedback setting.

Important future research directions are the theoretical analysis of the variance reduction technique and the empirical comparison using real-world datasets.

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A.2 Hoeffding’s Inequality

Lemma 1. (Hoeffdings Inequality) Independent bounded random variables $Z_1, ..., Z_n$ that take values in intervals of sizes $\zeta_1, ..., \zeta_n$ satisfy the following inequality for any $\epsilon > 0$.

$$
\mathbb{P} \left( \sum_{i=1}^{n} Z_i - \mathbb{E} \left[ \sum_{i=1}^{n} Z_i \right] \geq \epsilon \right) \leq 2 \exp \left( \frac{-2\epsilon^2}{\sum_{i=1}^{n} \zeta_i^2} \right)
$$

(13)
A.3 Proof of Proposition 3

Proof. For each data, as for a random variable $Z_i$ defined in Eq. (12), the following hold.

$$
P\left(Z_i = \delta_i^{(1)} \right) + \left( 1 - \frac{1}{\theta(X_i)} \right) \delta_i^{(0)} = \theta(X_i) \gamma(X_i)$$

$$
P\left(Z_i = \delta_i^{(0)} \right) = 1 - \theta(X_i) \gamma(X_i)$$

Thus, $Z_i$ takes values in the following interval,

$$\rho_i = \delta_i^{(1)} \theta(X_i) + \left( 1 - \frac{1}{\theta(X_i)} \right) \delta_i^{(0)} = \delta_i^{(1)} - \delta_i^{(0)}$$

Using the above interval, the following inequality holds by applying Lemma 1,

$$
P \left( \left| L_{cv} - \hat{L}_{IPS} \right| \geq \epsilon \right) \leq 2 \exp\left( -\frac{2\epsilon^2 N^2}{\sum \rho_i^2} \right) \tag{14}$$

Putting the LHS of Eq. (14) as $\delta$ and solving it for $\epsilon$ completes the proof. \qed