The optimal strategies in supply chain under cost disruptions and asymmetric information

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Abstract. This paper aims to analysing the optimal decisions in a supply chain when the information of cost disruptions is asymmetry. The results show that the asymmetric information and cost disruptions cause a profit loss to the supply chain. The supplier should control its cost disruptions to a small scope so as to coordinate the supply chain. This paper also analysis the information value based on the decisions, and we find that it is beneficial to all the supply chain members to share information.

1. Introduction
In recent years, accompanied with financial crisis and economic recessions, as well as the rising prices, the costs of raw materials in a enterprise continue to rising, causing a disruption to the existing costs, and information asymmetry can be common seen among enterprises. Nowadays, lots of scholars have made many studies on the following questions: supply chain disruptions (Cao and Yang, 2009; Lei et al., 2016), supply chain asymmetric information and coordination of supply chain (Babich et al., 2012; Gao et al. 2007; Shen et al., 2012).

However, no one has used a revenue sharing contract to coordinate the supply chain with asymmetric disruption information. In this paper, the asymmetric information of cost disruption happens in a supply chain. We model supply chain composites of a supplier and a retailer, based on agent-principal theory. The retailer designs a menu of contracts to maximize his expected profit according to the revelation principle. The decision procedure is as follows: (1) the supplier’s cost information is observed by himself, (2) the retailer offers a menu of contracts to the retailer \(\left\{ (w, \phi) \right\} \), (3) the supplier takes the contract or leaves it, (4) if the supplier takes a contract in the menu, the retailer should buy \(Q_{(w)}\) units of product to the supplier at the wholesale price \(w\), and the ratio offer to the supplier of its profit is \(\phi\). The retailer’s decision is about how to set the menu of contracts including the wholesale price and the ratio of profit offer to the supplier.

2. Model description
We consider a two-level supply chain includes a supplier and a retailer. Retailer buys goods from the supplier with the wholesale price \(w\), and sells its products with retailing price \(p\). We define the market demand as \(Q\), and \(Q=a-bp\), \(a\) represents the market scale and \(b\) stands for the price sensitive coefficient. \(c\) means supplier’s unit production cost, we suppose the cost of the retailer is zero. \(\pi_r\), \(\pi_s\), and \(\pi\) are the retailer’s profit, the supplier’s profit and the supply chain’s profit.
respectively. Thus the profit of the supply chain is 
\[ \pi_0 = (p-c)Q = (p-c)(a-bp) \]
and it is maximized at 
\[ p^* = \frac{a+bc}{2b} \]. Consequently, the optimal order quantity of retailer is 
\[ Q^* = \frac{a-bc}{2} \], and the supply chain’s profit is 
\[ \pi = \frac{(a-bc)^2}{4b} \].

We suppose there is a cost disruption \( \Delta c \). We use the revenue sharing contract \((w, \phi)\) to study the supply chain’s optimal decisions with cost disruption based on the principal-agent theory. \( \phi \) stands for the ratio of profit distributed to the suppliers by retailers. Under symmetric information, we can express the supplier’s profit and retailer’s profit as 
\[ \pi_m = \phi pQ + (w-c)Q \] and 
\[ \pi_r = (1-\phi)pQ - wQ \] respectively. To maximize both of their profit, we can conclude that the retail price can be 
\[ w(Q) = (1-\phi) \left[ c+\Delta c+\frac{S(Q)}{Q} \right] \]
and the retailer’s and supplier’s optimal profit can be 
\[ \pi_{r1} = (1-\phi)\pi, \pi_{m1} = \phi\pi \], the sale price is equal to the price when there is no disruption, as well as the optimal production. \((p_2=p^*, Q_2=Q^*)\) there 
\[ S(Q) = \lambda_1(Q-Q^*)^+ + \lambda_2(Q^*-Q)^+ \].

3. Cost disruption with asymmetric information

Assume \( \Delta c \) is the private information of the supplier. But the retailer knows the prior probability that \( \Delta c \) is \( \overline{\Delta c} \) with probability \( \theta \) and \( \Delta c \) with probability \( 1-\theta \). So when the information is asymmetric, the retailer will provide a revenue-sharing contract \((w, \phi)\) or a contract menu \(\{(w, \phi), (w, \phi)\}\) to the supplier, and the supplier pick up one of the contracts or leave it. The retailer’s problem is aiming at designing the contracts menu including the wholesale price and the ratio of profit offer to the supplier. Denote \( w, \phi, p, Q \) is the wholesale price, the ratio of distributed profit to supplier, the sale price and the order quantities of retailer respectively when there is a cost disruption of \( \Delta c \), while \( w, \phi, p, Q \) is the wholesale price, the ratio of distributed profit to supplier, the sale price and the order quantities of retailer respectively when there is a cost disruption of \( \overline{\Delta c} \).

Under the cost disruption information asymmetry, the retailer’s optimal decision is as follows:
\[
\max_{(\pi, w) \in \{w, \phi\}} \pi_r = \theta \left[ (1-\phi)pQ - wQ \right] + (1-\theta) \left[ (1-\phi)pQ - wQ \right]
\]
\[ \text{s.t.} \quad \phi pQ + (w-c-\Delta c)Q^+ - \lambda_1(Q^*-Q)^+ + \lambda_2(Q^*-Q)^+ \geq 0 \]
(2)
\[
\phi pQ + (w-c-\Delta c)Q^+ - \lambda_1(Q^*-Q)^+ + \lambda_2(Q^*-Q)^+ \geq 0 \]
(3)
\[
\phi pQ + (w-c-\Delta c)Q^+ - \lambda_1(Q^*-Q)^+ + \lambda_2(Q^*-Q)^+ \geq \phi pQ + (w-c-\Delta c)Q^+ - \lambda_1(Q^*-Q)^+ + \lambda_2(Q^*-Q)^+ \]
(4)
\[
\phi pQ + (w-c-\Delta c)Q^+ - \lambda_1(Q^*-Q)^+ + \lambda_2(Q^*-Q)^+ \geq \phi pQ + (w-c-\Delta c)Q^+ - \lambda_1(Q^*-Q)^+ + \lambda_2(Q^*-Q)^+ \]
(5)

Eq. (2) and (3) ensure that the supplier participates the supply chain. They represent individual rationality constraints. And Eq.(4) and (5) assure that each type of the supplier has no motivation to pretend its real cost, they are incentive compatibility constraints. Let \((x)^+ = \max(x, 0)\), \(\lambda_1\) stands for the marginal cost of excess used materials cause by the over-produced quantities \( \Delta Q \), and \(\lambda_2\) is the marginal cost of wasted materials caused by less–produced quantities \( \Delta Q \).
Combine Eq. (2) and (5), we can derive
\[
\phi p Q^+ \left( w - c \Delta c \right) Q^+ - \lambda_1 \left( Q^+ \right)^+ - \lambda_2 \left( Q^+ - Q^+ \right) \geq \phi p Q\left( w - c \Delta c \right) \left( \bar{Q} - Q^+ \right)^+ - \lambda_1 \left( \bar{Q} - Q^+ \right) - \lambda_2 \left( Q^+ - Q^+ \right) \geq \bar{Q} \left( \Delta c - \Delta c \right) > 0,
\]
Then Eq. (3) established. From Eq. (2) and (3), we can get
\[
\phi p Q^+ + w Q = \left( \alpha + \Delta c \right) Q^+ + \lambda_1 \left( Q^+ - Q^+ \right)^+ + \lambda_2 \left( Q^+ - Q^+ \right)^+.
\]
From Eq. (5), we know
\[
\phi p Q^+ \left( w - c \Delta c \right) Q^+ - \lambda_1 \left( Q^+ \right)^+ - \lambda_2 \left( Q^+ - Q^+ \right) = \left( \Delta c - \Delta c \right) \bar{Q},
\]
so we can simplify the retailer’s function into
\[
\max \pi = \theta \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] + (1 - \theta) \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] + \left( \alpha - c \Delta c \right) \left( \Delta c - \Delta c \right) \bar{Q},
\]
Equation (6)

Lemma 1. We denote \( \overline{Q^+}, \overline{Q}^+ \) is the optimal quantities of Eq. (6). So we have:

1. When \( \Delta c > (1 - \theta) \Delta c \), \( \overline{Q}^+ \leq Q^+ \); or \( \overline{Q}^+ \geq Q^+ \).
2. When \( \Delta c > 0 \), \( \overline{Q}^+ \leq Q^+ \); or \( \overline{Q}^+ \geq Q^+ \).

Prove of Lemma 1: (1) Suppose that when \( \Delta c > (1 - \theta) \Delta c \), the optimal quantity of Eq. (6) meets the demand \( \overline{Q}^+ \geq Q^+ \), so we can simplify Eq. (6) as follows:
\[
\max \pi = \theta \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] + (1 - \theta) \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] \geq \theta \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] + (1 - \theta) \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] = \theta \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] + (1 - \theta) \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] \geq \theta \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] + (1 - \theta) \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] = \pi \left( \bar{Q}, \bar{Q}^+ \right)
\]
So when \( \Delta c > (1 - \theta) \Delta c \), max \( \pi \left( \bar{Q}, \bar{Q}^+ \right) \leq \pi \left( Q^+, Q^+ \right) \), that is to say \( \pi \left( Q^+, Q^+ \right) \) is the optimal quantity of Eq. (6), and this is contradict with the hypothesis of Lemma 1. So we can illustrate that When \( \Delta c > (1 - \theta) \Delta c \), \( \bar{Q}^+ \leq Q^+ \); or \( \bar{Q}^+ \geq Q^+ \). Similarly, we can get \( \overline{Q}^+ \leq Q^+ \) when \( \Delta c > 0 \).

(1) From Lemma 1, we know when \( \Delta c > (1 - \theta) \Delta c \), \( \overline{Q}^+ \leq Q^+ \), we can simplify Eq. (6) into the following strict concave function:
\[
\max \pi \left( \bar{Q}, \bar{Q}^+ \right) = \theta \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right] + (1 - \theta) \left[ \left( \alpha - c \Delta c \right) - \lambda_1 \left( \bar{Q} - Q^+ \right) \right],
\]
and is constrained by \( \overline{Q}^+ \leq Q^+ \).
Let \( A = \frac{1-\theta}{\theta} \frac{\Delta c - \bar{\Delta} c}{(\Delta c - \bar{\Delta} c)} \), from \( \frac{\partial \pi_1^*(\bar{Q}, Q)}{\partial Q} = 0 \), we have

when \( \bar{Q}_1^* = Q^* - \frac{b}{2} [\Delta c + \frac{1-\theta}{\theta} (\Delta c - \bar{\Delta} c) - \lambda_2] = Q^* - \frac{b}{2} [\Delta c + A - \lambda_2] \), \( \pi_1^*(\bar{Q}, Q) \) has its maximum value.

Then we will consider the constraint \( \bar{Q} \leq Q^* \) from two aspects.

**Condition 1:** when \( \Delta c \geq \theta \lambda_2 + (1-\theta) \Delta c \), \( \bar{Q}_1^* = Q^* - \frac{b}{2} [\Delta c + A - \lambda_2] \leq Q^* \), which is conformed with constraint \( \bar{Q} \leq Q^* \), so we choose \( \bar{Q}_1^* = Q^* - \frac{b}{2} [\Delta c + A - \lambda_2] \) as the retailer’s optimal quantity. Then we have the corresponding sale price \( \bar{p}_1^* = \frac{a}{2b} + \frac{c+\Delta c - \lambda_2}{2} + \frac{A}{2} \), and \( \bar{p}_1^* = \frac{c+\Delta c - \lambda_2}{2} + \frac{A}{2} \), \( \bar{p}_1^* = \frac{c+\Delta c - \lambda_2}{2} + \frac{A}{2} \).

At this moment, we have \( \bar{Q}_1^* = Q^* \), and \( \bar{Q}_1^* = \frac{c+\Delta c - \lambda_2}{2} + \frac{A}{2} \), so \( \pi_1^*(\bar{Q}, Q) \) gets its maximum value at \( Q^* \).

Then we will consider the constraint \( \bar{Q} \leq Q^* \) from two aspects.

**Condition 2:** when \( \Delta c < \theta \lambda_2 + (1-\theta) \Delta c \), \( \bar{Q}_1^* = Q^* - \frac{b}{2} [\Delta c + A - \lambda_2] \geq Q^* \), this is contradict to the constraint \( \bar{Q} \leq Q^* \) and \( \pi_1^*(\bar{Q}, Q) \) is monotone increasing in the section of \((-\infty, Q^*)\), so \( \pi_1^*(\bar{Q}, Q) \) gets its maximum value at \( Q^* \).

(2) From lemma 1, we know when \( \Delta c < (1-\theta) \Delta c \), \( \bar{Q}^* \geq Q^* \). Then we can simplify Eq (6) into the following strict concave function:

\[
\max \pi_1^*(\bar{Q}, Q) = \theta \left[ (\bar{p} - c - \Delta c) \bar{Q} - \lambda_1 (\bar{Q} - Q) \right] + (1-\theta) \left[ (\bar{p} - c - \Delta c) \bar{Q} - \lambda_1 (\bar{Q} - Q) \right] \]

, and constrained by \( \bar{Q}^* \geq Q^* \).

From \( \frac{\partial \pi_1^*(\bar{Q}, Q)}{\partial Q} = 0 \), we have when \( \bar{Q}_1^* = Q^* - \frac{b}{2} [\Delta c + A + \lambda_1] \), \( \pi_1^*(\bar{Q}, Q) \) has its maximum value.

Then we will consider the constraint \( \bar{Q} \geq Q^* \) from two aspects.

**Condition 3:** when \( \Delta c \leq - \theta \lambda_2 + (1-\theta) \Delta c \), \( \bar{Q}_1^* = Q^* - \frac{b}{2} [\Delta c + A + \lambda_1] \geq Q^* \), which is conformed with constraint \( \bar{Q} \leq Q^* \), so we choose \( \bar{Q}_1^* = Q^* - \frac{b}{2} [\Delta c + A + \lambda_1] \) as the retailer’s optimal quantity.

Then we have the corresponding sale price \( \bar{p}_1^* = \frac{a}{2b} + \frac{c+\Delta c + \lambda_1}{2} + \frac{A}{2} \), and \( \bar{p}_1^* = \frac{c+\Delta c + \lambda_1}{2} + \frac{A}{2} \), so we have the equation between \( \bar{w} \) and \( \bar{\phi} \):
\[ w_3^{**} = c + \Delta c - \Phi \left[ \frac{a}{2b} + \frac{c + \Delta c + \lambda_1}{2} + A \right] + \lambda_2 \left[ \frac{-b \Delta c - b \lambda_1 - b A}{a - bc - b \Delta c - b \lambda_1 - b A} \right] \]

**Condition 4:** when \( \Delta c \geq -\theta \lambda + (1-\theta) \Delta c \), \( Q_3^{**} = Q^* - \frac{b}{2} (\Delta c + A + \lambda_1) < Q^* \), this is contradict to the constraint \( Q^* \geq Q^* \) and \( \pi^2_1(Q, Q) \) is monotone increasing in the section of \((-\infty, Q^*)\), so \( \pi^2_1(Q, Q) \) gets its maximum value at \( Q^* \).

At this moment, we have \( Q_3^{**} = Q^* \), and \( p_1^{**} = p^* \), so \( \phi \frac{p Q^* + w Q^* = (c + \Delta c) Q}{Q} \), then \( w \) and \( \phi \) meet the following relation: \( \frac{w^*}{Q} = (c + \Delta c) - \Phi \frac{p_1^{**}}{Q} = (c + \Delta c) - \Phi \left( \frac{a + bc}{2b} \right) \).

(3) From lemma 1 we have when \( \Delta c > 0, Q^* \leq Q^* \). Then we can simplify Eq (6) into the following strict concave function:

\[
\max \pi^1_2(Q, Q) \theta \left[ \frac{(c - \Delta c) Q - \lambda_1 (Q - Q)}{Q} \right] + (1-\theta) \left[ \frac{(p - c - \Delta c) Q - (\Delta c - \Delta c) Q - \lambda_2 (Q - Q)}{Q} \right] \]

and constrained by \( Q^* \leq Q^* \).

From \( \frac{\partial \pi^1_2(Q, Q)}{\partial Q} = 0 \), we have when \( Q_1^{**} = Q^* - \frac{b \Delta c - b \lambda_2}{2} \), \( \pi^1_2(Q, Q) \) has its maximum value.

Then we will consider the constraint \( Q^* \leq Q^* \) from two aspects.

**Condition 5:** when \( \Delta c > \lambda_2, Q_1^{**} = Q^* - \frac{b \Delta c - b \lambda_2}{2} < Q^* \), which is coincident with constraint \( Q^* \leq Q^* \), so then we choose \( Q_1^{**} = Q^* - \frac{b \Delta c - b \lambda_2}{2} \) as the retailer’s optimal quantity. Then we have the corresponding sale price \( p_1^{**} = \frac{a + bc + b \Delta c - b \lambda_2}{2b} \), and \( \phi \frac{p Q^* + w Q^* = (c + \Delta c) Q + \lambda_2 (Q - Q)^* + (\Delta c - \Delta c) Q}{Q} \), so we have the equation between \( w \) and \( \phi \):

\[
w^{**} = c + \Delta c + (\Delta c - \Delta c) \left( \frac{a + bc + b \Delta c - b \lambda_2}{a - bc - b \Delta c + \lambda_2} \right) \phi \frac{a + bc + b \Delta c - b \lambda_2}{2b} + \lambda_2 \left( \frac{b \Delta c - b \lambda_2}{a - bc - b \Delta c + \lambda_2} \right)
\]

**Condition 6:** when \( \Delta c \leq \lambda_2, Q_1^{**} = Q^* - \frac{b \Delta c - b \lambda_2}{2} \geq Q^* \), this is contradict to the constraint \( Q^* \leq Q^* \) and \( \pi^1_2(Q, Q) \) is monotone increasing in the section of \((-\infty, Q^*)\), so \( \pi^1_2(Q, Q) \) gets its maximum value at \( Q^* \).

At this moment, we have \( Q_2^{**} = Q^* \), and \( p_2^{**} = p^* \), so \( \phi \frac{p Q^* + w Q^* = (c + \Delta c) Q + \lambda_2 (Q - Q)^* + (\Delta c - \Delta c) Q}{Q} \), then \( w \) and \( \phi \) meet the following relation:

\[
w^{**} = c + \Delta c + (\Delta c - \Delta c) \left( \frac{Q^*}{Q^*} \right) - \Phi \left( \frac{a + bc}{2} \right)
\]

(4) From lemma 1, we know when \( \Delta c \leq 0, Q^* \geq Q^* \). Then we can simplify Eq (6) into the following strict concave function:
\[
\max \pi^*_1(Q) = \theta \left[ (p-c-\Delta c) Q - \lambda_1 (Q' - Q) \right] + (1-\theta) \left[ (p-c-\Delta c) Q - (\Delta c - \lambda_1) (Q - Q') \right],
\]
and constrained by \( Q^* \geq Q^* \).

From \( \frac{\partial \pi^*_1(Q)}{\partial Q} = 0 \), we have when \( Q^*_1 = \frac{b\Delta c + b\lambda_1}{2} \), \( \pi^*_1(Q, Q) \) has its maximum value.

Then we will consider the constraint \( Q^* \geq Q^* \) from two aspects.

**Condition 7:** when \( \Delta c \leq -\lambda_1 \), \( Q_1^* = \frac{b\Delta c + b\lambda_1}{2} \geq Q^* \), which is coincident with constraint \( Q^* \geq Q^* \), so then we choose \( Q_1^* = \frac{b\Delta c + b\lambda_1}{2} \) as the retailer’s optimal quantity. Then we have the corresponding sales price \( p_1^* = \frac{a + bc + b\Delta c + b\lambda_1}{2b} \)

and \( \phi p + w = (c + \Delta c) Q + \lambda_1 (Q' - Q) + (\Delta c - \lambda_1) \frac{Q}{Q} \), so we have the equation between \( w \) and \( \phi \):

\[
w^* = c + \Delta c + (\Delta c - \lambda_1) \left( \frac{Q}{2} \right) - \frac{a + bc + b\Delta c + b\lambda_1}{2b} \phi \left( \frac{-b\Delta c - b\lambda_1}{a - bc - b\Delta c - b\lambda_1} \right)
\]

**Condition 8:** when \( \Delta c > -\lambda_1 \), \( Q_1^* = \frac{b\Delta c - b\lambda_2}{2} < Q^* \), this is contradict to the constraint \( Q^* \geq Q^* \), and \( \pi^*_1(Q, Q) \) is monotone increasing in the section of \((-\infty, Q^*)\), so \( \pi^*_1(Q, Q) \) gets its maximum value at \( Q^* \).

At this moment, we have the \( Q_4^* = Q^* \) and \( p_4^* = p^* \), so

\[
w_4^* = c + \Delta c + (\Delta c - \lambda_1) \left( \frac{Q^*}{Q} \right) - \phi \left( \frac{a + bc}{2} \right).
\]

We summarize the former conclusions as follows:

**Theorem 1.** Let \( \left\{ (w, \phi), (w, \phi) \right\} \) represent the contracts menu under cost disruption with information asymmetry. We have

1. When \( \Delta c > \lambda_2 \) and \( \Delta c > \lambda_2 \), then

\[
Q^* = Q^* - \frac{\Delta c}{2} \left[ - \frac{a +bc + \lambda_1}{2} \right]
\]

\[
p^* = \frac{a}{2(b^2 + bc + b\lambda_1)} \frac{1}{2}
\]

\[
w^* = c + \Delta c + \lambda_1 \left( \frac{Q^*}{Q} \right) - \phi \left( \frac{a + bc + \lambda_1}{2(b^2 + bc + b\lambda_1)} \right)
\]
When \(-\theta \lambda_1 + (1-\theta) \Delta c \leq \Delta c \leq \theta \lambda_2 + (1-\theta) \Delta c\) and \(-\lambda_1 \leq \Delta c \leq \lambda_2\), then

\[
Q^* = \frac{a-bc}{2} \cdot \frac{\Delta c + b\lambda_i}{2} \cdot \frac{\lambda_i}{2} + A
\]

\[
p^* = a + b \Delta c + a \Delta c + A
\]

\[
w^* = c + \Delta c \cdot \phi \left(a + b \Delta c + a \Delta c + A \right) + \lambda_i \left(\frac{b \Delta c + b \lambda_i}{a-bc-b \Delta c-b \lambda_i}\right)
\]

(2) When \(-\theta \lambda_1 + (1-\theta) \Delta c \leq \Delta c \leq \theta \lambda_2 + (1-\theta) \Delta c\) and \(-\lambda_1 \leq \Delta c \leq \lambda_2\), then

\[
Q^* = \frac{a-bc}{2} \cdot \frac{\Delta c + b\lambda_i}{2} \cdot \frac{\lambda_i}{2} + A
\]

\[
p^* = a + b \Delta c + a \Delta c + A
\]

\[
w^* = c + \Delta c \cdot \phi \left(a + b \Delta c + a \Delta c + A \right) + \lambda_i \left(\frac{b \Delta c + b \lambda_i}{a-bc-b \Delta c-b \lambda_i}\right)
\]

(3) When \(-\theta \lambda_1 + (1-\theta) \Delta c < \Delta c < \lambda_1\), then

\[
Q^* = \frac{a-bc}{2} \cdot \frac{\Delta c + b\lambda_i}{2} \cdot \frac{\lambda_i}{2} + A
\]

\[
p^* = a + b \Delta c + a \Delta c + A
\]

\[
w^* = c + \Delta c \cdot \phi \left(a + b \Delta c + a \Delta c + A \right) + \lambda_i \left(\frac{b \Delta c + b \lambda_i}{a-bc-b \Delta c-b \lambda_i}\right)
\]

(4) When \(-\theta \lambda_2 + (1-\theta) \Delta c > \lambda_1\) and \(-\lambda_1 \leq \Delta c \leq \lambda_2\), then

\[
Q^* = \frac{a-bc}{2} \cdot \frac{\Delta c + b\lambda_i}{2} \cdot \frac{\lambda_i}{2} + A
\]

\[
p^* = a + b \Delta c + a \Delta c + A
\]

\[
w^* = c + \Delta c \cdot \phi \left(a + b \Delta c + a \Delta c + A \right) + \lambda_i \left(\frac{b \Delta c + b \lambda_i}{a-bc-b \Delta c-b \lambda_i}\right)
\]
\[
\begin{align*}
Q^n &= Q^* \\
p^n &= p^* = \frac{a + bc}{2b}
\end{align*}
\]
\[
\begin{align*}
w^n &= c + \Delta c + (\Delta c - \Delta c) \left[ \frac{a - bc - b \Delta c - b A + b \lambda_2}{a - bc} \right] \cdot \frac{a + bc}{2b}
\end{align*}
\]
(5) When $\Delta c > \theta \lambda_2 + (1 - \theta) \Delta c$ and $\Delta c < -\lambda_1$, then
\[
\begin{align*}
Q^n &= Q^* - \frac{b}{2} \left[ a + c - \Delta c - \lambda_2 \right] \\
p^n &= \frac{a}{2b} \left[ a + c + \Delta c - A + \lambda_2 \right] \\
w^n &= c + \Delta c - \phi \frac{Q^n - Q^n}{Q^n} + \lambda_2 \left( \frac{b \Delta c + b A - b \lambda_2}{a - bc - b \Delta c - b A - b \lambda_2} \right)
\end{align*}
\]
\[
\begin{align*}
Q^n &= Q^* \cdot \frac{b \Delta c + b \lambda_1}{2} \\
p^n &= a + bc + b \Delta c + b \lambda_1 \\
w^n &= c + \Delta c + (\Delta c - \Delta c) \left[ \frac{a - bc - b \Delta c - b A + b \lambda_2}{a - bc - b \Delta c - b A - b \lambda_2} \right] \cdot \frac{a + bc + b \Delta c + b \lambda_1}{2b} + \lambda_1 \left( \frac{-b \Delta c - b \lambda_1}{a - bc - b \Delta c - b \lambda_1} \right)
\end{align*}
\]
(6) When $-\theta \lambda_1 + (1 - \theta) \Delta c \leq \Delta c \leq \theta \lambda_1 + (1 - \theta) \Delta c$ and $\Delta c < -\lambda_1$, then
\[
\begin{align*}
Q^n &= Q^* = \frac{a - bc}{2} \\
p^n &= p^* = \frac{a + bc}{2b} \\
w^n &= c + \Delta c - \phi \frac{Q^n - Q^n}{Q^n} = c + \Delta c - \phi \left( \frac{a + bc}{2b} \right)
\end{align*}
\]
\[
\begin{align*}
Q^n &= Q^* \cdot \frac{b \Delta c + b \lambda_1}{2} \\
p^n &= a + bc + b \Delta c + b \lambda_1 \\
w^n &= c + \Delta c + (\Delta c - \Delta c) \left[ \frac{a - bc - b \Delta c - b A + b \lambda_2}{a - bc - b \Delta c - b A - b \lambda_2} \right] \cdot \frac{a + bc + b \Delta c + b \lambda_1}{2b} + \lambda_1 \left( \frac{-b \Delta c - b \lambda_1}{a - bc - b \Delta c - b \lambda_1} \right)
\end{align*}
\]
We can see that when $-\theta \lambda_1 + (1 - \theta) \Delta c < \Delta c < \theta \lambda_1 + (1 - \theta) \Delta c$ or $-\lambda_1 < \Delta c < \lambda_1$, the optimal order quantity has some robustness, the supplier will choose to produce the original quantity of goods. And when $\Delta c \leq -\theta \lambda_1 + (1 - \theta) \Delta c$ or $\Delta c \leq -\lambda_1$, the supplier will choose to increase the amount of goods to be produced, and when $\Delta c \geq \theta \lambda_2 + (1 - \theta) \Delta c$ or $\Delta c > \lambda_2$, the supplier will decrease the amount.
4. The value of cost disruption information
When there is a $\Delta c$ disruption to the original cost $c$, the supplier can earn nothing, while the disruption is $\Delta c$, the value of information to supplier is $\theta \Delta \pi + (1-\theta) \overline{Q (\Delta c - \Delta c)}$, and this is so-called information rent given by the retailer. Similarly, the retailer’s information value and the supply chain’s information value are $\theta \Delta \pi$ and $\theta \Delta \pi$ respectively. We can also derive from the former context that the revenue-sharing contract can coordinate the supply chain when the information is symmetric, but can not achieve the same effect when the information is asymmetric, that is to say, the supply chain has efficiency loss. We denote $\Delta \pi$ as the supply chain’s profit loss, then we have

(a) In Theorem 1(1), when $\Delta c > \theta \lambda_2 + (1-\theta) \Delta c$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\Delta \pi = \frac{b(4+\Delta c - \lambda_2)^2}{4}$; and when $\Delta c > \lambda_2$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\Delta \pi = \frac{b(\lambda_2 - \Delta c)^2}{4}$.

(b) In Theorem 1(2), $\pi^* = \pi^* = \pi^*$, $\Delta \pi = 0$.

(c) In Theorem 1(3), when $\Delta c \leq -\theta \lambda_1 + (1-\theta) \Delta c$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\pi^* = \frac{(a-bc)^2 - b^2(A+\Delta c + \lambda_1)^2}{4b}$, $\Delta \pi = \pi^* - \pi^* = \frac{b(A+\Delta c + \lambda_1)^2}{4}$; and when $\Delta c < -\lambda_1$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\pi^* = \frac{(a-bc)^2}{4b}$, $\Delta \pi = \pi^* - \pi^* = \frac{b(\lambda_1 + \Delta c)^2}{4}$.

(d) In Theorem 1(4), when $\Delta c > \theta \lambda_1 + (1-\theta) \Delta c$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\pi^* = \frac{(a-bc)^2}{4b}$, $\Delta \pi = \frac{b(4+\Delta c - \lambda_2)^2}{4}$, when $-\lambda_1 < \Delta c < \lambda_2$ , $\pi^* = \pi^*$, $\Delta \pi = 0$.

(e) In Theorem 1(5), when $\Delta c > \theta \lambda_2 + (1-\theta) \Delta c$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\pi^* = \frac{(a-bc)^2}{4b}$, $\Delta \pi = \frac{b(4+\Delta c - \lambda_2)^2}{4}$, when $\Delta c < -\lambda_1$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\pi^* = \frac{(a-bc)^2}{4b}$, $\Delta \pi = \pi^* - \pi^* = \frac{b(\lambda_1 + \Delta c)^2}{4}$.

In Theorem 1(6), when $-\theta \lambda_1 + (1-\theta) \Delta c < \Delta c < \theta \lambda_2 + (1-\theta) \Delta c$ , $\pi^* = \pi^*$, $\Delta \pi = 0$; when $\Delta c < -\lambda_1$ , $\pi^* = \frac{(a-bc)^2}{4b}$, $\pi^* = \frac{(a-bc)^2}{4b}$, $\Delta \pi = \pi^* - \pi^* = \frac{b(\lambda_1 + \Delta c)^2}{4}$.

5. Conclusion
This paper aims to analyzing the effects of asymmetric cost disruption on supply chain members and the value of the information. We find that the retailer has an efficiency loss, and this is owing to he or she has to pay for the information given by the suppler. This paper also shows that only the cost disruption is very mild the supply chain can be coordinated by the revenue sharing contract, or the supply chain’s profit is lower than that under symmetric information.
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