Probing the semi-macroscopic vacuum by higher-harmonic generation under focused intense laser fields

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Abstract The invention of the laser immediately enabled the detection of nonlinear photon–matter interactions, as manifested for example by Franken et al.'s detection of second-harmonic generation.

With the recent advancement in high-power, high-energy lasers and the examples of nonlinearity studies of the laser-matter interaction by virtue of properly arranging lasers and detectors, we envision the possibility of probing nonlinearities of the photon interaction in vacuum over substantial space-time scales, compared to the microscopic scale provided by high-energy accelerators.

Specifically, we introduce the photon–photon interaction in a quasi-parallel colliding system and the detection of higher harmonics in that system. The method proposed should realize a far greater sensitivity of probing possible low-mass and weakly coupling fields that have been postulated.

With the availability of a large number of coherent photons, we suggest a scheme for the detection of higher harmonics via the averaged resonant production and decay of these postulated fields within the uncertainty of the center-of-mass energy between incoming laser photons.

The method carves out a substantial swath of new experimental parameter regimes on the coupling of these fields to photons, under appropriate laser technologies, even weaker than that of gravity in the mass range well below 1 eV.

1 Introduction

Recent astronomical observations suggest the existence of non-luminous stuff in the universe: dark matter and dark energy. Understanding their origin is one of the greatest scientific challenges of the 21st century. It may be natural to expect that even the universe is just a collection of vacua around us. Therefore, we consider a novel method to directly probe the structure of vacuum under laboratory conditions that relate to these issues. To our current knowledge no direct signature of the non-luminous stuff has ever been observed in terrestrial experiments. This is in spite of the fact that numerous advanced theories exist, including axions [1], minicharged particles [2–5], and dark energy [6–8]. The reason for this may be thought of as follows. After Rutherford’s discovery of the inner core (i.e. nucleus) of an atom being very tiny compared to the dimension of the already tiny size of the atom, the experimental search went to explore ever smaller constituents of matter and thus the thrust went for higher-energy or -momentum experiments. Theories have gone hand-in-hand with this exploration, succeeding in ever shorter-ranged interaction theories and unification of forces, as exemplified by the electroweak theory [9]. We refer to this standard and extremely successful method as the high-momentum approach. Almost all laboratory research efforts have been on this approach to date. Although successful in exploring high-energy physics, this approach is not suitable to explore energies much lower than 1 eV. These fields, which might exist in much lower domains than 1 eV, cannot strongly couple to matter, because if they did, they would have been long ago discovered in the low-energy region. Thus these fields, if they ever exist, must couple weakly. This means that we need an extremely strong driver to manifest a sufficiently strong signal overcoming this weakly coupling interaction, showing up above noise. So far such sufficiently powerful photon sources did not exist. However, this...
may be changing now with more intense lasers becoming available [10]. What we have called the high-amplitude or high-field method [11, 12] may provide an alternative path to detect such low-mass, weakly interacting fields that are spread over semi-macroscopic scales.

When the energy of the constituent is much lower than that of the probing photon, conceptually we may employ two laser beams in a co-propagating geometry. The two collinear beams produce a very low center-of-mass energy interaction at the beating frequency (being equal to the difference of the two laser frequencies) [13, 14]. This interaction could resonate with very low eigenfrequency of the constituent, should there be an eigenmode in its vicinity. Second of all, the co-propagating setup allows us to make the two beams interact over a much prolonged interaction time, thus much amplifying the nonlinearities and signals arising from these.

In order to pick up the experimental signal of a strong coupling to the long-range mode, we suggest using higher-harmonic generation. The pioneering research by Franken et al. [15] detected the nonlinearity in a quartz crystal via second-harmonic generation. Two photons in co-propagation accentuate the interaction through the quartz fields over the coherence volume in order to initiate second-harmonic generation. This process may be schematically looked upon as the case displayed in Fig. 1(a). There the quartz nonlinearities mix two forward-propagating photons (ω) to produce a photon with 2ω (and possibly another photon with frequency ≈0, also referred to as optical rectification or difference frequency mixing). The quantum electrodynamics (QED) process is illustrated by Fig. 1(b). Two incoming photons are mediated by virtual electron–positron fields and outgoing are two photons. The extreme forward-scattering amplitude with quasi-parallel incident photons is known to be largely suppressed in the QED process, as we discuss later. This is because the center-of-mass energy of the colliding two photons is too low to satisfy the relevant mass scale of the electron–positron pair.

Given these hints from analogous pictures on how to probe medium-like features of vacuum, this paper rather considers a simple photon–photon scattering process via the averaged resonant production and decay of light-mass fields within the energy uncertainty in the center-of-mass system between incoming laser photons. This gives a solid basis for the design of experiments based on intense co-propagating lasers, as much independent as possible of models on the light-mass fields in vacuum. As we discuss later in detail, the process we focus on is based on Fig. 1(c).

The QED process as illustrated in Fig. 1(b) is described by the Euler–Heisenberg effective Lagrangian [16] in the low-frequency limit of two incoming photons. This predicts that the vacuum under the influence of an electromagnetic field induces a birefringence characteristic. The ratio between the first and the second term in the Lagrangian can yield a general test to see whether the vacuum contains other effects beyond QED, examining its value at 4:7. In general, a scalar field φ and a pseudoscalar field σ in vacuum may contribute to the first and second term, respectively. Light scalar fields as candidates of dark energy have been recently intensively discussed [6, 7], while the pseudoscalar fields (axion-like-particles) may be a source of dark matter [1] and also possibly dark energy [18]. We suggested how to measure the phase shift of a probe laser across an intense electromagnetic field, based on the phase-contrast Fourier imaging [19]. However, we recognize that the test by the phase-contrast Fourier imaging is rather limited in the mass and coupling of those fields [20]. Therefore, we extend our method to search for those new types of fields by instituting co-propagating laser beams. This approach may be looked upon as in Fig. 1(c). Again two parallel photons come in, while two parallel photons come out. As we discuss later, the frequency shift in vacuum is simply explained by the strong Lorentz boost in the quasi-parallel colliding system, which causes the blue- and red-shift of photon energies emitted to the forward and backward directions, respectively. We note that our approach is similar to, but distinct from many laboratory experiments with lasers [21–30] already performed and proposed to search for those low-mass fields.
Consider more details of the effective interaction Lagrangian \( L \) as illustrated in the triangle part in Fig. 1(c), where the scalar field \( \phi \) and the pseudoscalar field \( \sigma \) couple to the electromagnetic field via quantum anomaly-type couplings [8]

\[
-L_\phi = g_\phi M_\phi^{-1} \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \phi, \\
-L_\sigma = g_\sigma M_\sigma^{-1} \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \sigma,
\]

(1, 2)

where \( F_{\mu\nu} = \partial A_\mu / \partial x^\nu - \partial A_\nu / \partial x^\mu \) is the antisymmetric field strength tensor with the four-vector potential \( A_\mu \) of the electromagnetic wave and its dual tensor \( \tilde{F}_{\mu\nu} = 1/2 \epsilon^{\mu\nu\lambda\kappa} F_{\lambda\kappa} \) with the Levi-Civita symbol \( \epsilon^{\mu\nu\lambda\kappa} \). The difference \( \phi \) versus \( \sigma \) is in their allowed couplings to polarization states of two photons as we discuss later. Here \( g M^{-1} \) provides the coupling strength. \( g \) and \( M \) carry the subscripts \( \phi \) and \( \sigma \), respectively, indicating the corresponding type of fields. The dimensionless coupling \( g \) is typically proportional to the dimensionless fine structure constant \( \alpha \) for the two photons to couple to the virtual charged-particle pair in the triangle part. The effective coupling includes the large mass scale \( M \) to couple to the light field with a mass \( m \). The large \( M \) induces the weakness of the coupling via \( M^{-1} \). For example, the Newtonian constant \( G \) is expressed as \( 8\pi G = \hbar c M_p^{-2} \), where \( M_p \) is the Planckian mass of \( 10^{27} \text{ eV} \). The weakness of \( G \) is the manifestation of the large mass scale at the vertex in the triangle coupling. In what follows we omit the subscripts \( \phi \) and \( \sigma \) on the coupling \( g M^{-1} \) and the mass of the light field \( m \), unless we need to explicitly distinguish the type of the fields. We use the natural units \( \hbar = c = 1 \) throughout the subsequent sections, unless explicitly noted.

As a quite challenging case we have attempted a theoretical approach to search for an extremely light scalar field as a candidate of dark energy via the averaged resonance scattering process in [31]. Given intense laser fields in the near future, the method may provide a new window into scoping physics on the Planckian mass scale by photon interactions in a quasi-parallel incident laser beam in the laboratory. In this paper we review the essence of the approach and further develop basic formulae in Sect. 2 to apply the method to a specific experimental setup considered in Sect. 3, in order to discuss reachable limits on the coupling strength \( g M^{-1} \) and the mass \( m \) for a laser intensity attainable within the ELI project [32] in Sect. 4. In Sect. 5 we provide a perspective on how our method serves as a new type of scope to probe the semi-macroscopic vacuum, which has not been extensively investigated yet.

![Fig. 2 Definitions of kinematical variables for the suggested co-propagating photons (this figure is quoted from [31])](image-url)

**2 Quasi-parallel photon–photon interaction in low-mass and weakly coupled fields**

**2.1 Kinematics of quasi-parallel system**

As illustrated in Fig. 2, we introduce an unconventional coordinate frame, in which two photons labeled by 1 and 2 sharing the same frequency are incident nearly parallel to each other, making an angle \( \vartheta \) with the common central line along the \( z \) axis. We define the \( z-x \) plane formed by \( p_1 \) and \( p_2 \). The components of the 4-momenta of the photons are given by \( p_1 = (\omega \sin \vartheta, 0, \omega \cos \vartheta; \omega) \) and the same for \( p_2 \) but with the sign of \( \vartheta \) reversed, and \( p_3 = (\omega_3 \sin \vartheta_3, 0, \omega_3 \cos \vartheta_3; \omega_3) \) and \( p_4 \) with \( \omega_3, \vartheta_3 \) replaced by \( \omega_4, -\vartheta_4 \), respectively. The angles \( \vartheta_3 \) and \( \vartheta_4 \) are defined as shown in Fig. 2. This coordinate system can be transformed to the center-of-mass (CM) system for the head-on collision \( (\vartheta = \pi/2) \) by the Lorentz transformation with \( v/c \rightarrow 1 \) for \( \vartheta \rightarrow 0 \). Conversely, this implies that the realization of the quasi-parallel collision in the laboratory frame corresponds to the realization of an extremely low CM energy, as we see below.

In this frame one of the final photons in the forward direction along the \( z \) axis must have an upshifted frequency due to the energy-momentum conservation, independent of the physical origin of the dynamics. In the limit of \( \vartheta \rightarrow 0 \), a process of \( \omega_3 \rightarrow 2\omega \) is realized. This frequency doubling nature is an extremely valuable characteristics from the experimental point of view, as compared to the case with no frequency shift in the center-of-mass system. In addition, more importantly, it is essential to maintain a quasi-parallel nature of the incident beams in order to access resonance, as we shall stress later.
The energy-momentum conservation laws requires following relations:

\[
\begin{align*}
0 \text{-axis: } & \quad \omega_3 + \omega_4 = 2\omega, \\
\varepsilon \text{-axis: } & \quad \omega_3 \cos \theta_3 + \omega_4 \cos \theta_4 = 2\omega \cos \vartheta, \\
x \text{-axis: } & \quad \omega_3 \sin \theta_3 = \omega_4 \sin \theta_4.
\end{align*}
\]

From the conditions \(0 < \omega_{3,4} < 2\omega\), we may choose \(0 < \theta_3 < \vartheta < \theta_4 < \pi\), without loss of generality. From (3)–(5) we derive the relation

\[
\sin \theta_3 = \sin \theta_4 \frac{\cos^2 \vartheta}{1 - 2 \cos \vartheta \cos \theta_4 + \cos^2 \vartheta}.
\]

The differential elastic scattering cross section per solid angle \(d\Omega_3\) favoring the higher photon energy \(\omega_3\) is given by

\[
\frac{d\sigma}{d\Omega_3} = (8\pi \omega)^{-2} \sin^{-4} \theta \cos^2 \theta_3 |\mathcal{M}|^2,
\]

where \(\mathcal{M}\) is the invariant amplitude and

\[
\omega_3 = \frac{\omega \sin^2 \theta}{1 - \cos \theta \cos \theta_3}.
\]

Here we expect the upshifted frequency \(\omega_3 \to 2\omega\), as \(\theta_3 \to 0\) for \(\vartheta \to 0\), as mentioned before.

2.2 Dynamics of two-photon interaction via resonance

We consider the scattering amplitudes only for the case when light-mass fields are exchanged via resonance (s-channel amplitude). The resonance decay rate of the low-mass field with the mass \(m\) into two photons is expressed as

\[
\Gamma = (16\pi)^{-1} (gM^{-1})^2 m^3.
\]

The low-mass field is exchanged between the pairs \((p_1, p_2)\) and \((p_3, p_4)\), thus giving the squared four-momentum of the field

\[
q^2_\xi = (p_1 + p_2)^2 = 2\omega^2 (\cos 2\vartheta - 1)
\]

with the metric convention \((+++\ldots)\) for the definition of the four momenta. \(q_\xi\) corresponds to the CMS energy of the photon–photon collision.

With the polarization vectors given by \(\mathbf{e}^{(\beta)}_i\), where \(i = 1, \ldots, 4\) are the photon labels, whereas \(\beta = 1, 2\) are for the kind of linear polarization as depicted in Fig. 2, we summarize the non-zero invariant amplitudes for scalar field exchanges

\[
\mathcal{M}_{1111} = \mathcal{M}_{2222} = -\mathcal{M}_{1122} = -\mathcal{M}_{2211},
\]

and for pseudoscalar field exchanges

\[
\mathcal{M}_{1212} = \mathcal{M}_{1221} = -\mathcal{M}_{2112} = -\mathcal{M}_{2121},
\]

where the first two digits in the subscripts correspond to the states of the linear polarization of the incoming two photons 1 and 2, respectively, and the last two correspond to those of the outgoing two photons 3 and 4, respectively, as illustrated in Fig. 2.

We focus on one of these non-zero amplitudes by denoting it as \(\mathcal{M}\):

\[
\mathcal{M} = -\left(gM^{-1}\right)^2 \frac{\omega^4 (\cos 2\vartheta - 1)^2}{2\omega^2 (\cos 2\vartheta - 1) + m^2},
\]

where the denominator, denoted by \(\mathcal{D}\) in the following, is the low-mass field propagator. We note that \(q^2_\xi\) in (10) is time-like. We then make the replacement

\[
m^2 \to (m-i\Gamma)^2 \approx m^2 - 2im\Gamma.
\]

Substituting this into the denominator in (13) and expanding around \(m\), we obtain

\[
\mathcal{D} \approx -2(1 - \cos 2\vartheta) (\chi + ia), \quad \text{with } \chi = \omega^2 - \omega^2_\xi,
\]

where

\[
\omega^2_\xi = \frac{m^2}{1 - \cos 2\vartheta}, \quad a = \frac{m\Gamma}{1 - \cos 2\vartheta}.
\]

From (9) and (16), \(a\) is also expressed as

\[
a = \frac{\omega^2_\xi}{8\pi} \left( \frac{gm}{M} \right)^2,
\]

which explicitly shows the proportionality to \(M^{-2}\). We then finally obtain the expression for the squared amplitude as

\[
|\mathcal{M}|^2 \approx (4\pi)^2 \frac{a^2}{\chi^2 + a^2}.
\]

As for the off-resonance case \(\chi \gg a\), \(|\mathcal{M}|^2\) is largely suppressed due to the factor \(a^2 \propto M^{-4}\) for the case of a small coupling \(M^{-1}\). On the other hand, if experiments take the limit of \(\omega \to \omega_r\), \(|\mathcal{M}|^2 \to (4\pi)^2\) is realized from (18) ideally. This is independent of the smallness of the factor \(M^{-4}\), as expected from the off-resonance case or equivalently from the square of (13). This is the most important feature arising from the resonance that overcomes the weak coupling stemming from the large relevant mass scale such as \(M = M_p\). However, we are then confronted with an extremely narrow width \(a\) for e.g. \(gm \ll 1\ eV, M \sim M_p = 10^{27} \ eV\) and \(\omega_r \sim 1\ eV\). We now discuss how to overcome this difficulty.
2.3 Enhancement by averaging including resonance

In conventional high-energy collisions, the beam momenta are implicitly supposed to be their mean values. This is because the momentum spread, or the uncertainty expected from the de Broglie wavelength of the relativistic particle, is negligibly small compared to the relevant momentum exchanges in the interaction that experiments are interested in. Resonance searches in such experiments must adjust $\chi$ in (15) and (18) such that the mean $\chi$ is close enough to the peak location within $\pm a$. On the other hand, in the case of co-propagating laser beams aiming at the detection of extremely small momentum exchanges via the resonance, the situation is quite different due to the nature of the incident waves. This is because the uncertainty included in the initial photon momenta is much larger than the relevant energy scale of the resonance, leading to the condition $|\chi| \gg a$. In this case the squared scattering amplitude must be integrated over the possible uncertainties on the incident wave function on an event-by-event basis. As we will explain below, the uncertainty in the incident photon momenta is related to the uncertainty in $\chi$ via the uncertainty in the incident angle $\vartheta$.

Therefore, it is instructive to consider the feature of the integral of the resonance function of the Breit–Wigner (BW) formula [33] from $\chi_-$ to $\chi_+$

$$I = \int_{\chi_-}^{\chi_+} \frac{d\chi}{\chi^2 + a^2} = a \tan \left( \frac{\chi}{a} \right) \bigg|_{\chi_-}^{\chi_+},$$

(19)

where $I = a \pi /2$ and $I = a \pi$ for $\chi_+ = -\chi_- = a$ and $\chi_+ = -\chi_- = \infty$, respectively. This indicates that the value of the integral is proportional to $a$, i.e., $M^{-2}$ from (17). The value ranges for the finite and infinite integrals over only a factor of two. From this fact, we expect that the integral enhances the squared scattering amplitude by a factor of $M^2$ compared to the non-resonant interaction proportional to $M^{-4}$ from (13), as long as the peak is contained within the experimental resolution on $\chi$, i.e., the condition $\chi_+ > a$ and $\chi_- < -a$ is satisfied. This implies that experiments in the co-propagating laser beam configuration need no efforts to adjust $\chi$ close to the extremely narrow resonance region, thanks to the subsequent huge enhancement by the integral over the wide range on $\chi$. Meanwhile, it is difficult to identify the exact location of the resonance mass within the wide gate on $\chi$. Since we are interested in having a sensitivity to an extremely weak coupling such as gravity $M = M_\rho$, the enhancement of the squared amplitude is more crucial than finding the exact location of the resonance masses. As we discuss in the following sections, however, we may be able to provide a crude estimate on the order of the mass scale of the resonance even in such a situation.

The consideration above leads us to parametrize the squared scattering amplitude:

$$|\mathcal{M}|^2 = \int_{-\infty}^{+\infty} \rho(p_1, p_2) |\mathcal{M}|^2 d\chi = (4\pi)^2 \int_{-\infty}^{+\infty} \rho(p_1, p_2) \frac{a^2}{\chi^2(p_1, p_2) + a^2} d\chi, \quad (20)$$

where $\rho(p_1, p_2)$ is the normalized probability distribution to supply nominal combinations $(p_1, p_2)$ from the incident laser fields. The real part $\chi(p_1, p_2)$ is indirectly specified by $(p_1, p_2)$ via the incident angle $\vartheta$ between the incident two photons. This parametrization expresses averaging over possible combinations of $p_1$ and $p_2$. We note that $(p_1, p_2)$ are not a priori the observed momenta, but just a nominal specification among the possible initial momenta. In other words, $\rho(p_1, p_2)$ is not a statistical weight on the discrete momenta after the contraction of the wavepacket of each photon state. This implies that an infinite statistics is not necessary to obtain the continuous nature. Rather, we need this treatment even for a two-photon state as long as the source of photons is not a perfect plane wave. This allows for a continuous integral on $\chi$ via the continuous combination of $\rho(p_1, p_2)$ in (20). As long as the probability weight $\rho(p_1, p_2)$ is close enough to unity around the resonance peak, the enhancement discussed with (19) is guaranteed. This is the essence of our main strategy of the co-propagating configuration in order to overcome the difficulty due to the narrow resonance width $a$.

In order to design experiments, we start from the resonance condition, the first of (16), by assuming $\vartheta \ll 1$.

$$m \sim 2\vartheta \omega_r, \quad (21)$$

where the subscript $r$ in both angle and energy refers to a state satisfying a resonance condition. We note that the product $2\vartheta \omega_r$, corresponds to the CM energy of the incident two photons. This indicates that experiments have two adjustable handles for a given mass scale or the CM energy. We emphasize that we can lower the CM energy by several orders of magnitude by only introducing smaller values of $\vartheta$ with a fixed $\omega$. This should be contrasted to high-energy colliders, where a large effort is needed to increase the CM energy by an order of magnitude. This advantage is also supported from a technical point of view, since scanning the incident angle $\vartheta$ should be much easier than scanning the energy $\omega$ of the resonance. We point out that the resonance condition in (21) is not just given at one point, but rather in a hyperbolic band in the $\vartheta - \omega$ plane, with a finite resolution with $\delta\vartheta$ of the incident angle $\vartheta$. This implies that the deviation $\delta\omega$ from the resonance energy $\omega_r$ can satisfy the same resonance condition with a different $\vartheta$ within $\pm \delta\vartheta$. As far as $\delta\omega/\omega_r \ll \delta\vartheta/\vartheta_r$ is satisfied in the setup, we can ignore the
effect of $\delta \omega$. This is in fact the case, as can be seen in the following discussions. Therefore, we can take the attitude that we fix the incident energy at the optical frequency $\omega = \omega_{\text{opt}}$ and scan $m$ by changing $\vartheta$ around $\vartheta_r$, where $\omega_{\text{opt}}$ and $\vartheta_r$ satisfy the resonance condition based on (21)

$$\omega_{\text{opt}}^2 = \frac{m^2}{(4\vartheta_r^2)}.$$  \hspace{1cm} (22)

From (21) and (22) we see that detecting lower-mass fields requires smaller values of $\vartheta_r$, namely the parallelism of incident photons. In this case the variation on $\vartheta$ leads to the variation on $\chi$. This is expressed by the following relation based on the second part of (15)

$$\chi(\vartheta) = \omega^2_{\text{opt}} = \frac{m^2}{4\vartheta^2} = \omega^2_{\text{opt}}(1 - e^{-2}),$$  \hspace{1cm} (23)

where $\varepsilon \equiv \vartheta/\vartheta_r$ in the unit of $\vartheta_r$ is introduced.

We now discuss the average of the squared amplitude $|\mathcal{M}|^2$ over the possible uncertainty on the incident angle $\vartheta$.

$$|\mathcal{M}|^2 = \int_{0}^{\pi/2} \rho(\vartheta)|\mathcal{M}|^2 d\vartheta$$

$$= \int_{0}^{\pi/2} \rho(\varepsilon)|\mathcal{M}|^2 d\vartheta_r d\varepsilon,$$  \hspace{1cm} (24)

where $\rho(\vartheta)$ is a probability distribution function normalized between 0 and $\pi/2$ as a function of the continuous uncertainty on $\vartheta$ from arbitrarily chosen two-photon combinations within a laser pulse. The incident angle $\vartheta$ is re-expressed with $\varepsilon$ by $d\vartheta = \vartheta_r d\varepsilon$ for the second part of (24). From the relation (23), we find $\varepsilon = (1 - x)^{-1/2}$ and $d\varepsilon = 1/(2\omega^2_{\text{opt}})\varepsilon^3 dx$. Equation (24) is then re-expressed with $\chi$

$$|\mathcal{M}|^2 = (4\pi)^2 \frac{\vartheta_r}{2\omega^2_{\text{opt}} a}$$

$$\times \int_{-\infty}^{1-(2\vartheta_r/\pi)^2} \rho((1 - \chi)^{-1/2}) \frac{a^2}{(1-\chi)^{3/2}} \frac{d\chi}{\chi^2 + a^2} d\xi,$$  \hspace{1cm} (25)

where (18) is substituted. This equation is the exact representation of (20), starting from the uncertainty on the incident angle $\vartheta$, if the upper limit of the integral range is regarded as large enough compared to $a$. Let us define $x \equiv a\xi$ to explicitly discuss the structure of the integral kernel in units of the width $a$ of BW. With $\xi$, (25) is further re-expressed as

$$|\mathcal{M}|^2 = (4\pi)^2 \frac{\vartheta_r}{2\omega^2_{\text{opt}} a}$$

$$\times \int_{-\infty}^{a^{-1}(1-(2\vartheta_r/\pi)^2)} \rho((1 - a\xi)^{-1/2}) \frac{1}{(1-a\xi)^{3/2}} \frac{1}{\xi^2 + 1} d\xi,$$  \hspace{1cm} (26)

where the first factor of the integral kernel corresponds to a normalized weight function and the second is BW with a width of unity. This expression explicitly shows the enhancement by the factor of $a$, implying the proportionality to $M^{-2}$ based on (17). As long as $\rho$ is a monotonic function, the weight function in front of BW can be close to unity for small $\xi$ because of $a\xi \ll 1$. With such a weight we expect that the value of the integral may be close to that of BW, as we discuss with a specific weight function in the following section.

The remaining issue is how to further cope in experiments with the problem of still very small values of $M^{-2}$, although much larger than $M^{-4}$. First, this can be solved by the $\sin^{-4} \vartheta$ behavior of the cross section in (7), which arises from the phase volume factor and the flux factor in the quasi-parallel two-photon interaction. For an extremely light mass, this factor gains a large number due to the small $\vartheta_r$. Second, the intense laser fields can provide a large luminosity and the intensity of the signal is proportional to the square of the intensity of the laser in the limited case of an incoherent two-photon interaction. We have three ingredients or knobs: the $M^2$ enhancement by the weighted BW integral, the $\vartheta_r$ dependence and the growth of the laser intensity. By marshalling these knobs, we expect to increase the detectability for undiscovered low-mass fields in vacuum, which have evaded from our grasp to date.

In the following sections we consider experimental realizations with $\omega_r \sim 1$ eV (optical laser), aiming at the mass range as low as possible. We then plug explicit weight functions into (26), based on the suggested experimental setup. By combining (26) and (7), we discuss reachable mass-coupling limits for a given laser intensity attainable in near future experiments.

### 3 Second-harmonic detection in the quasi-parallel system

We emphasized the importance of the photon–photon interaction in a quasi-parallel system or a small-angle setup, in order to enhance the signal due to low-mass constituents. A simple way is to explore the mass range $m < \pi \omega$ by using two independent laser beams with a small incident angle. We then directly measure the resonance curve in (21) by scanning both $\vartheta$ and $\omega$ to quantitatively observe the nature of the resonance curve. For the much smaller mass scale, or equivalently smaller incident angle, however, we must take into account the beam spread in the diffraction limit. This determines the controllable smallest incident angle, or the mass scales of the light fields, we look for. We consider here the case of a single focused laser beam, in order to provide the simplest basis to quantify reachable mass-coupling limits for a given set of experimental parameters. We concentrate on
the detection near the second-harmonic, rather than the laser frequency itself, to enhance the sensitivity of detecting the photon–photon interaction.

The conceptual experimental setup with a single-beam focusing geometry is illustrated in Fig. 3. Incident photons from a Gaussian laser pulse with linear polarization are focused by an ideal lens into the diffraction limit. Quasi-simultaneously, incident and outgoing photons are drawn only for the scalar exchange with a scattering amplitude [11, 12] as an example.

In what follows we prove formulae to evaluate the accessible limit on the mass of a pseudo-scalar field exchange. We note that we can choose the type of fields we search for by setting the initial polarization state. In the case of single-beam focusing, the search for a scalar field is easier, because we do not have to mix the two polarization states as in the case of a pseudoscalar field.

Furthermore, the selection of the rotated final state 22 can enhance the signal-to-background ratio for the scalar field exchange, respectively. We note that we can choose the type of fields we search for by setting the initial polarization state. In the case of single-beam focusing, the search for a scalar field is easier, because we do not have to mix the two polarization states as in the case of a pseudoscalar field. In what follows we provide formulae to evaluate the accessible limit on the mass-coupling defined in (2) for a given laser intensity, in the case that we detect a double-frequency photon per laser shot.

The Gaussian profile is a basic constraint in typical laser fields, where the aperture of a lasing material has a finite size in the transverse area. The solution of the electromagnetic field propagation in vacuum with a Gaussian profile in the transverse plane with respect to the propagation direction z is well-known [34]. The electric field component in spatial coordinates (x, y, z) is expressed as

$$ E(x, y, z) \propto \frac{w_0}{w(z)} \exp \left\{ -i(kz - H(z)) \right\} - r^2 \left( \frac{1}{w(z)^2} + \frac{ik}{2R(z)} \right), $$

where $k = 2\pi/\lambda$, $r = \sqrt{x^2 + y^2}$, $w_0$ is the minimum waist, which cannot be smaller than $\lambda$ due to the diffraction limit, and other definitions are

$$ w(z)^2 = w_0^2 \left( 1 + \frac{z^2}{\varepsilon R^2} \right), $$

$$ R = z \left( 1 + \frac{zR^2}{\varepsilon^2} \right), $$

$$ H(z) = \tan^{-1} \left( \frac{z}{z_R} \right), $$

$$ z_R \equiv \frac{\pi w_0^2}{\lambda}. $$

Based on the Gaussian laser parameters above, we now estimate the effective luminosity $\mathcal{L}$ over the propagation volume of the laser pulse. We now restore the physical dimensions of $\hbar$ and $c$ in this section, unless explicitly noted. Consider a Gaussian laser pulse with duration time $\tau$, with the speed of light $c$ and an average number of photons $\bar{N}$ per pulse. The exchange of a low-mass field may take place anywhere within the volume defined by the transverse area of the Gaussian laser times the focal length $f$ before reaching the focal point. We first consider the effective number of photons $N_{\text{int}}$ during an interaction with the time scale of $\Delta t$. As a result of the interaction we observe a frequency-doubled photon in the laboratory frame. The momentum transfer of $\sim \hbar \omega/c$ between photons defines the minimum interaction time scale from the uncertainty principle by

$$ \Delta t > 2\pi \omega^{-1}. $$

The effective number of photons during $\Delta t$ is expressed as

$$ N_{\text{int}} = \frac{\Delta t}{\tau} \bar{N}. $$

Making the pulse duration $\tau \sim \Delta t$ maximizes the instantaneous luminosity. Suppose a point $z$ along the laser propagation axis. The instantaneous luminosity at the point $z$ is defined as

$$ \mathcal{L}(z) = \frac{C(N_{\text{int}}, 2)}{\pi w(z)^2} \sim \frac{N_{\text{int}}^2}{2\pi w_0^4} \frac{z_R}{z^2 + z_R^2}. $$
where $C(N_{\text{int}}, 2)$ denotes a combinatorics to choose two photons amongst a large number of photons available within the time scale $\Delta t$, and the expression $w^2(z)$ in (28) is substituted to obtain the second part with the approximation for the combinatorics. We then consider the averaged instantaneous luminosity $\bar{L}$ over the focal length $f$:

$$\bar{L} = f^{-1} \int_0^f L(z) \, dz \sim \frac{N_{\text{int}}^2}{2\pi f w_0^2} z_R \tan^{-1}(f/z_R)$$

$$= \frac{N_{\text{int}}^2}{2f\lambda} \tan^{-1}(f/z_R). \quad (35)$$

The number of effective bunches $b$ is related with $f$ as

$$b = \frac{f}{c\Delta t}. \quad (36)$$

The effective luminosity $\mathcal{L}$ over the propagation volume of the laser pulse is finally expressed as

$$\mathcal{L} = b\bar{L}$$

$$= \frac{f}{c\Delta t} \left( \frac{N_{\text{int}}^2}{2f\lambda} \right) \tan^{-1}(f/z_R)$$

$$= \frac{\Delta t}{\tau} \left( \frac{\bar{N}^2}{2c\tau\lambda} \right) \tan^{-1}(f/z_R), \quad (37)$$

where (33) is substituted in the last step.

The minimum beam waist $w_0$ at $z = 0$ varies with the experimental conditions, the focal length $f$, and the diameter of the incident beam $d$,

$$w_0 \sim \frac{2f}{\pi d\lambda}, \quad (38)$$

where the approximation is valid when $f\lambda \ll (\pi/4)d^2$. The uncertainty on the incident angle between two light waves is then expected to be

$$\Delta \theta \sim \frac{\lambda}{\pi w_0} \sim \frac{d}{2f}. \quad (39)$$

We see that $\Delta \theta$ is controlled via $w_0$ by choosing suitable values for $f$ and $d$ in experiments.

The possible uncertainty on the incident angle $\theta$ affects the average of the squared amplitude $|\mathcal{M}|^2$ as shown in the first part of (24). In order to obtain an approximation close enough to reality, we plug the following step function into (24):

$$\rho(\theta) = \begin{cases} 1/\Delta \theta & \text{for } 0 < \theta \leq \Delta \theta \\ 0 & \text{for } \Delta \theta < \theta \leq \pi/2 \end{cases} \quad (40)$$

which is normalized to the physically possible range $0 < \theta \leq \pi/2$. By substituting (40) into (26), we obtain

$$|\mathcal{M}|^2 = \frac{(4\pi)^2}{2\omega_{\text{opt}}^2} \frac{\theta_r}{\Delta \theta} a$$

where the first factor of the integral kernel corresponds to the weight function and the second is the Breit–Wigner function (BW) with a width of unity. The weight function of the kernel is close to unity for small $\xi$, due to the smallness of $a$ in (17). Therefore, the value of the integral in (41) is almost equivalent to that of BW [35]. This is because the monotonic positive weight function approaches zero as $\xi \to -\infty$ more rapidly than the pure BW, whereas the pure BW suppresses the increase of the weight function close to zero at $\xi \to -a^{-1}[1 - (\theta_r/\Delta \theta)^2]$ for $\Delta \theta < 1$. We then approximate (41) as the integrated BW over $\pm \infty$ as follows:

$$|\mathcal{M}|^2 \sim \frac{(4\pi)^2}{2\omega_{\text{opt}}^2} \frac{\theta_r}{\Delta \theta} a\pi. \quad (42)$$

Let us introduce the integrated form of the cross section in (7) over the solid angle $2\pi \sin \theta_1 \, d\theta_1$ up to $\theta_3$, which is equivalent to the lower limit of the frequency $w_3/\omega \equiv 2 - \hbar$ though the relation of (8) by specifying the frequency deviation $0 < \hbar \ll 1$ from the exact $2\omega$:

$$\bar{\sigma}(h) = \frac{|\mathcal{M}|^2}{(8\pi \omega)^2 \sin^{-4} \theta} \int_0^\theta \left( \frac{\omega_3}{\omega} \right)^2 2\pi \sin \theta_3 \, d\theta_3$$

$$\sim \pi (h/2)(8\pi \omega)^{-2} \theta^{-2} |\mathcal{M}|^2, \quad (43)$$

where $\theta \ll 1$ is used for the approximation in the last step.

With $a$ in (17) and $|\mathcal{M}|^2$ in (42), the weighted cross section in (43) is finally expressed as

$$\bar{\sigma}(h) \sim \frac{\pi^2}{32} \left( \frac{2\pi}{\lambda} \right) \left( \frac{\theta_r}{\Delta \theta} \right)^2 \left( \frac{g m}{M} \right)^2 \theta_r^{-2} \hbar, \quad (44)$$

where the approximation $\theta_r \ll 1$ is also taken into account.

Multiplying (37) by (44), we obtain the integrated yield $\mathcal{Y}$ above the lowest frequency $w_3$ specified by $\hbar$ per laser pulse focusing by

$$\mathcal{Y} = \mathcal{L}\bar{\sigma}(h)$$

$$= \frac{1}{256} \frac{\Delta t}{\tau} \frac{\lambda}{c\tau} \tan^{-1}(f/z_R) \left( \frac{\theta_r}{\Delta \theta} \right) \left( \frac{g m}{M} \right)^2 \theta_r^{-2} h \bar{N}^2. \quad (45)$$

There are several experimental knobs to affect the observable events in (45). If we choose $\tau \sim \Delta t \sim \lambda/c$, resulting in $c\tau \sim \lambda$, we can maximize the effective luminosity. From (39), the reduction of $\Delta \theta$ or increasing $w_0$ enhances the yield in the case of very low-mass particle exchange. From (31) and (38), we express $f/z_R$ as

$$f/z_R \sim \frac{\pi d}{4f\lambda}. \quad (46)$$
From this relation, a shorter focal length enlarges \( f/z_R \). This introduces a slight increase for the factor \( \tan^{-1}(f/z_R) \), though its effect is tempered by the nature of \( \tan^{-1} \).

As a short summary, we make the most important note from the experimental point of view based on this conceptual design. The condition \( \vartheta_r/\Delta \vartheta = 1 \) maximizes the chance to search for a resonance, while \( \vartheta_r/\Delta \vartheta > 1 \) results in a huge suppression of the cross section by \( M^{-4} \) \((\hbar = c = 1)\) as we discussed. This is because the resonance peak is out of the region covered by \( \Delta \vartheta \). This parameter corresponds to a sharp cut-off of the cross section. Therefore, controlling \( \Delta \vartheta \) via the relations of (39), (38), and (46) can provide an experimental way to define the mass range that we exclude, if no signal is found. From the resonance condition in (21), we evaluate the cut-off on the mass range we can exclude by this conceptual design as follows \((\hbar = c = 1)\):

\[
m_{\text{cut}} \equiv 2 \Delta \vartheta \omega_{\text{opt}} = \frac{d}{f} \omega_{\text{opt}}, \tag{47}\]

where (39) was substituted. We note that the measurement is still sensitive to the mass range even below this cut-off, however, we cannot state where the mass is located below this limit. Therefore, if we cannot find any symptoms below this cut-off, we can exclude the entire mass range below this cut-off. On the other hand, if something is found, we must make effort to lower the cut-off by introducing a longer focal length and a smaller beam diameter in order to determine the location of the mass at which the signal disappears.

### 4 Sensitivity enhancement by high-intensity lasers

As a demonstration we now discuss how much high-intensity lasers improve the accessibility to the weak coupling domain for the following reference case, based on a quasi-parallel colliding system.

First, we note that \( \Delta t \) in (32) is the resolvable minimum time scale. As long as we discuss an extremely low-mass field, the interaction time scale may be over \( \hbar/mc^2 \). In the case of low mass below 1 eV, the interaction time scale may be much longer than \( 2\pi \omega_{\text{opt}}^{-1} \). On the other hand, photon–photon interactions must occur within a laser pulse duration time \( \tau \). This fact implies that we should assume \( \Delta t/\tau = 1 \) even if \( \Delta t \gg \tau \). Since the beam waist of focused laser becomes smaller as it approaches the diffraction limit, a short-pulse laser with the duration time close to \( 2\pi \omega_{\text{opt}}^{-1} \) maximizes the effective luminosity per laser shot. Therefore, we assume this condition in the following discussion.

We are now ready to estimate the sensitivity to the coupling \( g/M \) for a given average number of photons \( \bar{N} \) in the laser pulse, in the case that \( Y \) is found to be zero per laser shot based on (45) for the basic experimental parameters discussed above: \( \omega_{\text{opt}} \sim 1 \text{ eV}, \tau \sim 10 \text{ fs}, \Delta t/\tau = 1, d \sim 1 \text{ m} \), and \( h = 0.1 \) to require the frequency close to \( 2\omega_{\text{opt}} \). Figure 4 shows accessible lower bounds on \( g/M \) as a function of \( m \) via the search for higher-harmonic generation by focusing a single laser shot. The blue dotted and solid lines indicate cases for \( \bar{N} = 2 \text{ J} \) and \( \bar{N} = 20 \text{ kJ} \), respectively. The upper and lower lines on each line style correspond to the case for \( f = 1 \text{ m} \) and \( f = 1 \text{ km} \), respectively. As we discussed with (47), the cut-off values on mass are lowered by introducing longer focal lengths. The sky-blue band shows the resolvable mass range by gradually changing focal lengths from \( 1 \text{ m} \) to \( 1 \text{ km} \) in which we can determine a mass based on the disappearance of higher-harmonic signals from the appearance state. The black shaded area is the excluded region by conventional laser-driven experiments so called “Light Shining through a Wall (LSW)” [36]. In LSW a laser pulse propagates under a static magnetic field \( B \) and a low-mass particle \( \sigma \) is assumed to be produced via two-photon coupling to the particle (\( \sigma-B \)-laser coupling). Since \( \sigma \) is expected to couple with matter only weakly, \( \sigma \) can penetrate a wall which prevents penetrations of laser photons on the other hand. The particle may couple again to \( B \)-field placed over the wall resulting in a photon in the induced decay process.

Our arguments so far have been based on the approach in which photons both in initial and final states are treated incoherently, giving an observable yield proportional to \( \bar{N}^2 |M|^2 \). It is worth noting, however, that the intrinsic nature of laser fields may further improve the sensitivity to even smaller couplings as weak as the gravitational coupling \( g/M_P \sim 10^{-20} \text{ GeV}^{-1} \) with \( g \sim \alpha = 1/137 \) which is indicated by the magenta line in Fig. 4. Degenerated photons in a laser beam can induce the decay of resonantly produced low-mass fields into a degenerated final state as in the case of the static magnetic field to induce the decay of low-mass particle used for LSW. If this is the case, compared to their spontaneous decays in vacuum, we can expect an enhancement factor \( \sim \bar{N}_{\text{ind}}^{1/2} \) in the scattering amplitude, caused by the creation operator to the degenerated state, where \( \bar{N}_{\text{ind}} \) is the number of degenerated photons involved in the induced laser field [31, 37]. For example, if we limit \( h \) in (43) to a narrow range around \( 0.5 \) in the spontaneous decay process, a phase space decaying into \( \omega \sim 1.5\omega_{\text{opt}} \) and \( \omega_3 \sim 0.5\omega_{\text{opt}} \) can be selected. If we induce the \( \omega_3 \sim 0.5\omega_{\text{opt}} \) by the degenerated state, the probability to emit \( \omega_3 \sim 1.5\omega_{\text{opt}} \) photons is enhanced accordingly. If we rescale all photon frequencies by a factor of two, what is suggested here is that we simply look for higher-harmonic generation via the following process: \( 2\omega + 2\omega \rightarrow 1\omega + 3\omega \), where \( 1\omega \) is assigned as the induced field. In this case the yield of the \( 3\omega \) wave should follow \( \bar{N}^2 N_{\text{ind}} |M|^2 \). Similar ideas based on the classical treatment are discussed in [38, 39]. The red lines in Fig. 4 show the lower bounds when \( \bar{N} = N_{\text{ind}} \) is assumed. The conditions and conventions are same as those for blue
lines except this additional induced field. It is a bit of a surprise that it is not impossible to reach weaker coupling domains even beyond the gravitational coupling in a scattering experiment by accumulating statistics over a reasonable time period, if we could introduce the induced $1\omega$ waves into the same geometry as $2\omega$ to make them focus at a time. We note that this method is also applicable to ns pulses, typical for MJ-class laser facilities, as long as the energy per pulse is larger than that of short laser pulses which may compensate the longer time duration by the $N^2 N_{\text{ind}}$ dependence of the higher-harmonic yield. The actual coupling limit must be evaluated, eventually based on the statistics of the background of higher harmonics.

A major instrumental background for the frequency shifted radiation is in principle expected to be higher-harmonic generation (HHG) from the final focusing optical element and the one to reflect HHG to the photon detectors. The dominant source of HHG may be the interface between the residual gas and the surface of the optical element, where the centrosymmetry is maximally broken. Even from the maximal estimate $\sim 10^{13}$ W/cm$^2$ for a typical damage threshold which is a much lower intensity compared to HHG due to relativistic motion of surface electrons, we expect a negligible amount of $10^{-10}$ for the case of second-harmonic photons from a 1 m$^2$ aperture size with a 10 fs irradiation, if the optical components are housed in a vacuum containing $10^{10}$ atoms/cm$^3$ ($\sim 10^{-5}$ Pa) [40]. The confirmation of a negligible contribution of the background from higher harmonics is a crucial subject for the present concept.

As a dominant physical background we expect the lowest-order QED photon–photon scattering with a forward cross section $\sim (\alpha^2/m_e^4)\omega^6\vartheta^4$ [41]. This turns out to be much smaller than (44), due to the specific behavior with respect to the incident angle $\vartheta$. This indicates that the lowest-order QED contribution is negligible.

If there will be no signal in the single-beam focusing, we only have to update the condition such that it satisfies $\Delta\vartheta > \vartheta_r$ for heavier masses by increasing $\Delta\vartheta$. In such a heavier mass region, however, the two-beam crossing geometry relaxes the constraints on the optical design such as the focal length. In either case the single-beam focusing setup considered in this paper provides a basis to define the mass-coupling limit as well as the necessary beam intensity as we have demonstrated here.

5 Conclusion

We have suggested an approach to probe the nature of vacuum with intense lasers. The resonance search for extremely light-mass fields via higher-harmonic generation has been explored by focusing high-intensity laser beams. This is similar to the idea already developed to probe matter.

In this method we take note of the nonlinearities of vacuum that are speculated to wait for our sensitive detection of an extraordinarily feeble signal. In order to detect weak nonlinearities, we need to spectacularly enhance the signal. The large leap in enhancing these signals is garnered by the
Probing the semi-macroscopic vacuum by higher-harmonic generation under focused intense laser fields

Fig. 5 Experimental domains of various approaches to probe matter and vacuum as a function of the system size vs. the energy density. Selected systems are LC: electron–positron collision in the center-of-mass energy \( E_{\text{cm}} = 1 \text{ TeV} \) at the future linear collider [42], assuming the electron size \( 10^{-18} \text{ cm} \) which is the upper limit of the electron radius obtained in high-energy collider experiments; LHC: proton–proton collision in \( E_{\text{cm}} = 14 \text{ TeV} \) at the Large Hadron Collider [42]; RHIC: gold–gold collision in \( E_{\text{cm}} = 200 \text{ GeV per nucleon pair} \) at the Relativistic Heavy Ion Collider [42]; ELI: an optical laser pulse expected in the ELI project [32]; Schwinger: the Schwinger limit [44], Sun, the Milky Way Galaxy and the cosmic horizon with \( \Omega_{\text{tot}} \sim 1.0 \) and \( h \equiv H_0/100 \text{ [km/s/Mpc]} \sim 0.7 \) [43]. The energy density axis is qualitatively interpreted as the inverse of the force range or the mass scale \( m \) of the exchanged force, because the mean free path becomes shorter in higher density states, as long as the coupling to matter is not weak. On the other hand, the coupling strength to matter \( g M^{-1} \) as defined in (1) qualitatively reflects the necessary size of matter or vacuum in order to make the interaction manifest. The arrow to the higher energy density toward the Schwinger limit is the direction to probe nonlinear QED interactions and also towards an understanding of the non-perturbative nature of the intense field. The arrows directing to the lower energy density region indicate the extensible domain by using co-propagating intense laser fields, since the sensitive mass range is below \( \sim 1 \text{ eV} \) covering mass scale relevant for dark energy and the coupling may be probed to a scale as weak as gravitational coupling for lighter mass scales. The energy density in this direction depends on the context. In the context of the scalar field as a candidate of dark energy in [6, 7, 31], the energy density should be close to that of the cosmic horizon.

Combination of (i) the rapid development of the intense laser technology and its adoption here; (ii) the employment of our suggested scheme allows for an enhanced interaction with the pursued fields. The former element (i) may be brought in, for example, by an intense optical laser beyond 1 kJ. For the latter factor (ii) we have suggested a method to include a resonance with co-propagating photons for the exploration of possible new low-mass fields, with the aid of induced laser fields to promote the decay into a degenerate vacuum state.

With the detection of higher-harmonic generation in the co-propagating setup, we should be able to survey a large sweep of the energy domain of the intermediating vacuum fields. If and when we pick up some signal in one particular energy range, perhaps we can zoom in to this specific energy (and thus wavelength) of photons by arranging the various knobs, such as the crossing angle and the (long) beating wavelength of the electromagnetic waves. By setting up a specific resonance cavity, we may be able to further increase the sensitivity and more deeply study their properties.

Given a highly intense optical laser beyond 1 kJ per fs-pulse duration in the near future, the realization of these suggestions may become an exciting challenge for future experiments investigating the physics of the vacuum. We might be able to reach coupling strengths as weak as the gravitational strength by utilizing induced laser fields.

Figure 5 illustrates the experimental domains of various approaches to probe matter and vacuum, in terms of the system size as a function of the energy density. The energy density axis is qualitatively interpreted as the inverse of the force range or the mass scale \( m \) of the exchanged force in (13), because the mean free path becomes shorter in higher-density states as long as the coupling to matter is relatively strong. On the other hand, the coupling to matter \( g M^{-1} \) in (13) or (1) qualitatively reflects the necessary size of matter or vacuum in order to make the interaction visible.

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The Galileo-type telescope observes gravitational phenomena. These are on the extremely weak coupling scale of $M_F^{-1}$ with zero mass exchange. High-energy particle colliders, the Rutherford-type microscopes, focus on particle generation phenomena. These are due to strong couplings with heavy mass exchanges within the fm scale. There is a huge gap between these two approaches. In other words, the region of weak couplings with finite but light-mass exchanges have hardly been probed so far. It is quite natural to start exploring if there exist important pieces of the puzzle of nature in these domains. These explorations might grant us deeper understanding of the nature of vacuum such as dark energy \cite{45, 46}. The progress of modern physics has been simply driven by those two types of experimental approaches. The proposed method with high-intensity lasers probes the semi-macroscopic vacuum compared to particle physics and on a much smaller scale of vacuum compared to cosmology. Provided such semi-macroscopic vacuum scope, we increase our observational window into a new parameter regime of the vacuum.

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