Error analysis of ephemeris calibration for dual-satellite TDOA/FDOA geolocation

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Abstract: In this paper, we provide a theoretical error analysis of a satellite ephemeris calibration for the dual-satellite TDOA/FDOA geolocation method. First, the error covariance matrix for the ephemeris calibration is derived. Then, the result is incorporated into the error covariance matrix for geolocation. The derived equation is numerically evaluated to provide an intensive error analysis in time and spatial coordinates. Further, Monte Carlo simulation results are provided, and it is shown that the theoretical results coincide with the Monte Carlo simulation results.

Keywords: ephemeris, geolocation, TDOA/FDOA
Classification: Sensing

References

[1] D. P. Haworth, N. G. Smith, R. Bardelli, and T. Clement, “Interference localization for eutelsat satellites -the first European transmitter location system,” Int. J. of Sat. Comm., vol. 15, pp. 155–183, 1997. DOI:10.1002/(SICI)1099-1247(199707/08)15:4<155::AID-SAT577>3.0.CO;2-U
[2] H. Yan, J. K. Cao, and L. Chen, “Study on location accuracy of dual-satellite,” ICSP, pp. 107–110, 2010. DOI:10.1109/ICOSP.2010.5656806
[3] T. Pattison and S. I. Chou, “Sensitivity analysis of dual-satellite geolocation,” IEEE Trans. Aerosp. Electron. Syst., vol. 36, no. 1, pp. 56–71, 2000. DOI:10.1109/7.826312
[4] T. Amishima, et al., “Satellite orbit determination by time and frequency differences of arrival of multiple reference stations,” IEICE Society Conf., B-2-7, 2006.
[5] T. Amishima, et al., Japanese Patent Application, No. 2006-241903.
[6] A. Gelb, ed., “Applied Optimal Estimation,” the M.I.T. Press, Cambridge, 1974.
[7] F. R. Hoots and R. L. Roehrich, “Spacetrack Report No. 3: Models for Propagation of NORAD Element Sets,” 1988/12.

1 Introduction
In satellite communications, uplink interference from unknown emitters has become one of the major issues. In [1], the authors have proposed a method based on the dual-satellite TDOA/FDOA localization technique to locate unknown emitters.
on the earth surface. The proposed method utilizes a priori knowledge on satellite orbital information. In [2, 3], the authors have pointed out that accurate satellite ephemeris information is essential for the localization accuracy. To improve accuracy, an ephemeris calibration method is proposed in [4, 5]. As shown in Fig. 1, the authors utilize multiple known emitters and estimate the ephemerides of two satellites simultaneously. However, although this method seems promising, the authors have not provided any theoretical error bound for the calibration accuracy. Therefore, it requires a trial-and-error analysis to evaluate the resulting geolocation accuracy when the calibrated information is used. Error analysis in time and spatial coordinates has been of special interest to estimate the system design. In this paper, we provide an intensive theoretical error analysis. First, we derive the ephemeris calibration error covariance matrix. Second, we incorporate it into the geolocation error covariance matrix. By evaluating the proposed equations, we show the calibration performance both in time and spatial coordinates.

2 Ephemeris calibration error covariance matrix for TDOA/FDOA-based geolocation

2.1 Satellite ephemeris calibration

The position and velocity vectors of the two satellites at time $t_k$ are defined as $\mathbf{z}_k = [\mathbf{p}_{s1,k}^T \mathbf{v}_{s1,k}^T \mathbf{p}_{s2,k}^T \mathbf{v}_{s2,k}^T]^T$. Given the multiple known emitters, the so-called references, the TDOA/FDOA measurement model is described as follows:

$$
\tau(\mathbf{z}_k) = \frac{1}{c} \left[ \|\mathbf{p}_{s1,k} - \mathbf{p}_{0,1,k}\| - \|\mathbf{p}_{s2,k} - \mathbf{p}_{0,2,k}\| - \|\mathbf{p}_{s1,k} - \mathbf{p}_{0,1,k}\| + \|\mathbf{p}_{s2,k} - \mathbf{p}_{0,2,k}\| \right],
$$

$$
f(\mathbf{z}_k) = \frac{1}{\lambda} \left\{ \frac{\mathbf{v}_{s1,k}(\mathbf{p}_{s1,k} - \mathbf{p}_{s1,0})}{\|\mathbf{p}_{s1,k} - \mathbf{p}_{0,1,k}\|} - \frac{\mathbf{p}_{s1,k}^T(\mathbf{p}_{s1,k} - \mathbf{p}_{s2,k})}{\|\mathbf{p}_{s1,k} - \mathbf{p}_{0,1,k}\|} - \frac{\mathbf{v}_{s2,k}(\mathbf{p}_{s2,k} - \mathbf{p}_{s2,0})}{\|\mathbf{p}_{s2,k} - \mathbf{p}_{0,2,k}\|} - \frac{\mathbf{p}_{s2,k}^T(\mathbf{p}_{s2,k} - \mathbf{p}_{s1,k})}{\|\mathbf{p}_{s2,k} - \mathbf{p}_{0,2,k}\|} \right\}. \quad (2)
$$

Here, $\mathbf{p}_{s,k}$ is a known reference position at time $t_k$, $c$ is the speed of light, and $\lambda$ is the wavelength. Note that Eqs. (1) and (2) have the form of a differential. The reference position $\mathbf{p}_{s,0}$ will be used for geolocation as well as to cancel ephemeris errors and unknown time and frequency shifts at the satellites [1]. Further, we define the ephemeris vectors of the two satellites by $\mathbf{\xi} = [\xi_{s1}^T \xi_{s2}^T]$, and $\mathbf{\xi}_s = [\mathbf{M}_{s1} \mathbf{e}_{s1} \mathbf{A}_{s1} \omega_{s1} \Omega_{s1} i_{s1}]^T$, where $\mathbf{M}_{s1}, \mathbf{e}_{s1}, \mathbf{A}_{s1}, \omega_{s1}, \Omega_{s1}, i_{s1}$ are true anomaly, eccentricity, semi-major axis, argument of perigee, right ascension of Fig. 1. Ephemeris calibration.
the ascending node, and inclination, respectively [7]. Since \( x_k \) is a function of \( \xi \) and \( t_k \), we can express \( r(x_k) \) and \( f(x_k) \) as \( r_k(\xi) \) and \( f_k(\xi) \). Then, using the definitions above, we solve the following optimization problem [4, 5]:

\[
\hat{\xi} = \arg \min_{\xi} \sum_{k=1}^{K} (\theta_{\text{obs},k} - \theta_k(\xi))^T V^{-1}(\theta_{\text{obs},k} - \theta_k(\xi)),
\]

where \( \theta_{\text{obs},k} \) is the vector of the TDOA and FDOA observation values \( \tau_{\text{obs},k} \) and \( f_{\text{obs},k} \) at time \( t_k \), \( \theta_k(\xi) \) is the corresponding observation model, \( V \) is the error covariance matrix whose diagonal elements are the variances \( \sigma_\xi^2 \) and \( \sigma_f^2 \) of the TDOA and FDOA measurement errors, respectively.

### 2.2 Deriving the ephemeris calibration error covariance matrix

By following the method in [6] and after solving the optimization problem in Eq. (3), the ephemeris error covariance matrix \( P_\xi \) can be expressed by the following equation:

\[
P_\xi = E\left((\xi - \hat{\xi})(\xi - \hat{\xi})^T\right) = \left[ \sum_{k=1}^{K} G_{k}^T(\hat{\xi})V^{-1}G_{k}(\hat{\xi}) \right]^{-1}, \quad G_{k}(\hat{\xi}) = \left[ \nabla_\xi \tau_k(\hat{\xi})^T \nabla_\xi f_k(\hat{\xi})^T \right],
\]

where \( E\left[ . \right] \) is the expectation operator and \( \hat{\xi} \) is the estimated value. The detailed expression of \( G_{k}(\hat{\xi}) \) is omitted due to space limitation and can be found in [5]. The error covariance matrix \( P_{\xi_{\text{int}}} \) of the position-velocity vector \( \chi_k \) can be defined as follows:

\[
P_{\xi_{\text{int}}} = E\left((\chi_k - \chi_k(\hat{\xi}))(\chi_k - \chi_k(\hat{\xi}))^T\right) = G_{\xi \rightarrow \chi_k} P_{\xi_{\text{int}}} G_{\chi_k \rightarrow \xi_{\text{int}}},
\]

\[
G_{\xi \rightarrow \chi_k} = \left[ \nabla_\xi x_{i1,k}(\hat{\xi}) \cdots \nabla_\xi x_{i\cdot,k}(\hat{\xi}) \quad \nabla_\xi v_{r_{i1,k}}(\hat{\xi}) \cdots \nabla_\xi v_{r_{i\cdot,k}}(\hat{\xi}) \right].
\]

A detailed expression of Eq. (7) is described in [5]. Using Eqs. (6) and (7), the error equations \( \text{ERR}_{s_{\text{pos}},k} \) and \( \text{ERR}_{s_{\text{vel}},k} \) for positions and velocities of the two satellites \( i \) at time \( t_k \) are obtained as follows:

\[
\text{ERR}_{s_{\text{pos}},k} = \sqrt{\sum_{n=6(i-1)+1}^{6(i-1)+3} P_{\xi_{\text{int}}}(n,n)}, \quad \text{ERR}_{s_{\text{vel}},k} = \sqrt{\sum_{n=6(i-1)+4}^{6(i-1)+6} P_{\xi_{\text{int}}}(n,n)}.
\]

### 2.3 Incorporating ephemeris calibration error covariance matrix into geolocation error covariance matrix

In the following, we omit the time index \( k \) because the geolocation requires only a single set of TDOA and FDOA and no multiple sets as we have seen for the ephemeris calibration. The position vector of the unknown emitter location \( p_{\text{int}} \) is estimated using the position and velocity vectors \( p_{i} \) and \( v_{i} \) of the two adjacent satellites and by simultaneously solving the following equations with the constraint that the unknown emitter is located on the earth surface:

\[
r(p_{\text{int}}, Z) = \frac{1}{2} \left\{ \| p_{\text{int}} - p_{0} \| - \| p_{\text{int}} - p_{2} \| - \| p_{0} - p_{1} \| + \| p_{0} - p_{2} \| \right\},
\]

\[
f(p_{\text{int}}, Z) = \frac{1}{2} \left\{ \| v_{1}(p_{\text{int}} - p_{0}) \| - \| v_{2}(p_{\text{int}} - p_{2}) \| - \| v_{1}(p_{0} - p_{1}) \| + \| v_{2}(p_{0} - p_{2}) \| \right\}.
\]
Note that (9) and (10) are both functions of $p_{\text{int}}$ and $\chi$ since the sensitivity function is derived from both parameters. Further, we define the error deviation of the satellite positions and velocities by $\delta \chi$. Denote the sensitivity function from $\delta \chi$ to the deviations in $r$ and $\tau$, by $\delta r$ and $\delta f$. Further, we denote the gradient of $\chi$ by $\nabla \chi$.

The first-order expansion is obtained by the following equations:

$$\delta r(p_{\text{int}}, \chi) = [\nabla_x^r U(p_{\text{int}}, \chi)^T - \nabla_x U(p_{\text{int}}, \chi)^T] \delta \chi \simeq F_1(p_{\text{int}}, \chi) \delta \chi,$$

$$\delta f(p_{\text{int}}, \chi) = [\nabla_x f U(p_{\text{int}}, \chi)^T - \nabla_x f U(p_{\text{int}}, \chi)^T] \delta \chi \simeq F_2(p_{\text{int}}, \chi) \delta \chi,$$

where $F_1(p_{\text{int}}, \chi)$ and $F_2(p_{\text{int}}, \chi)$ are the matrix function of $p_{\text{int}}$ and $\chi$.

Further, we denote the gradient of $\chi$ by $\nabla \chi$.

The coordinate transformation from the two-dimensional earth surface coordinates $x_{\text{surf}} - y_{\text{surf}}$ to earth-centered earth-fixed coordinates [7] is defined by $B(p_{\text{int}})$. Then, Eq. (17) can be expressed as follows:
\[ \delta v(p_{int}, x) = G^T(p_{int}, x)B(p_{int})\delta x_{int,surf}, \]  
(20)

where \( \delta x_{int,surf} \) is a deviation in position on the earth surface. With \( P_{int,surf} = E\{\delta x_{int,surf} \delta x_{int,surf}^T\} \), Eq. (16), and Eq. (21), the geolocation error covariance matrix \( P_{int,surf} \) becomes:

\[ P_{int,surf} = ((G^T(p_{int}, x)B(p_{int})))^T R_v^{-1}(p_{int}, x)(G^T(p_{int}, x)B(p_{int}))^{-1}. \]  
(21)

The error equation for geolocation is obtained as follows:

\[ ERR_{int,surf} \doteq \sqrt{P_{int,surf}(1,1) + P_{int,surf}(2,2)}. \]  
(22)

As shown in Eq. (16), the ephemeris error covariance matrix is incorporated in the error equation for geolocation.

3 Errors analysis of ephemeris calibration and geolocation

3.1 Simulation setting

In this section, we numerically evaluate theoretical error bounds in time and spatial coordinates. We assume two geostationary satellites located at 158 E and 162 E, respectively. The orbital elements are obtained via NORAD resources [7]. The TDOA and FDOA accuracy is 1 µs and 1 mHz, respectively. The reference locations are Tokyo, Kobe, Sapporo, and Fukuoka whose uplink frequencies are 14 GHz. Tokyo is set as \( p_{ro} \). The measurement interval and the measurement time length are 10 min and 24 h, respectively. The calibrated orbital elements are used for computing the orbit for geolocation at a specified time. For error analysis in time coordinates, we fix the location of the unknown emitter at Niigata, and the geolocation is conducted every 20 min. Both theoretical and Monte Carlo simulations of 1000 trials are evaluated. For error analysis in spatial coordinates, we fix the orbital time at 12 h and evaluate the theoretical error for the region around Japan and its surrounding seas.

3.2 Simulation results

Figs. 2(a)–(c) show the calibration and geolocation errors versus time. (a) and (b) show the #1 and #2 satellites’ position and velocity errors, and (c) the geolocation error. In (d) and (f), the theoretical geolocation error map is shown. (d) shows the error map without calibration and (e) with calibration. In (f), the difference between with and without calibration is shown.

From (a) and (b), we first note that the theoretical and Monte Carlo results are in good agreement. Therefore, the derived equation can be applied to evaluate calibration and geolocation performances. From (c), we notice periodic peaks every 12 h. This is because the TDOA and FDOA lines align parallel to each other during this points in time. The resulting locations cause a large error along the two lines. Similar results are discussed in [1].

Comparing (d) and (e), we confirm that by using the calibration results, the error has been reduced at all mentioned locations on earth. Further, the closer the reference station, the more the geolocation error is reduced. This is due to the satellite ephemeris-error-canceling effect by using known reference emitters. Regarding (f), it is worthwhile to mention that the farther the region from the reference position, the larger the difference in geolocation error. This implies that
in regions with a marginal canceling effect, the ephemeris error causes a large geolocation error. Therefore, calibration is crucial in these regions.

4 Conclusion

In this paper, we have provided a mathematical foundation to predetermine the performance of the dual-satellite geolocation system. First, we have derived the ephemeris calibration error covariance matrix and incorporated it into the geolocation error covariance matrix. Based on the theoretical result, we showed the ephemeris calibration performance both in time and spatial coordinates. The results coincide with the Monte Carlo simulation results.