Searching for light Dark Matter in heavy meson decays

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Abstract

Beauty and charm $e^+e^-$ factories running at resonance thresholds have unique capabilities for studies of the production of light Dark Matter particles in the decays of $B_q(D)$ meson pairs. We provide a comprehensive study of light Dark Matter production in heavy meson decays with missing energy $E$ in the final state, such as $B_q(D^0) \rightarrow \not{E}$ and $B_q(D^0) \rightarrow \gamma \not{E}$. We argue that such transitions can be studied at the current flavor factories (and future super-flavor factories) by tagging the missing-energy decays with $B(D^0)$ decays “on the other side.”

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I. INTRODUCTION

The presence of cold Dark Matter (DM) in our universe provides the most natural explanation for several observational puzzles, from the original measurement of the rotational curves \[^1\] of galaxies to the observation of background objects in the Bullet Cluster \[^2\] and spectrum features of the cosmic microwave background (CMB) fluctuations. In the conventional picture, DM accounts for the majority of mass in our Universe. However, the nature of DM is still very much a mystery, which could intimately connect astronomical observations with predictions of various elementary particle theories. Many such theories, with the notable exception of the Standard Model (SM), predict one or more stable, electrically-neutral particles in their spectrum \[^3\]. These particles could form all or part of the non-baryonic Dark Matter in the Universe.

Different models provide different assignments for DM particles’ spin and various windows for their masses and couplings to luminous matter. In the most popular models DM is a weakly interacting particle particle with mass set around the electroweak energy scale. This follows from the experimental measurements of the relic abundance \(\Omega_{DM} h^2 \sim 0.12\) by WMAP collaboration \[^4\] \[^5\].

\[
\Omega_{DM} h^2 \sim \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle^{-1} \propto \frac{M^2}{g^4} \sim 0.12,
\]

where \(M\) and \(g\) are the mass and the interaction strength associated with DM annihilation respectively. As one can see, a weakly-interacting massive particle (WIMP) with electroweak-scale mass naturally gives the result of Eq. (1). This, coupled with an observation that very light DM particles might overclose the Universe (known as the Lee-Weinberg limit \[^5\] \[^6\]), seems to exclude the possibility of the light-mass solution for DM, setting \(M_{DM} > 2 - 6\) GeV.

A detailed look at this argument reveals that those constraints could be easily avoided, so even MeV-scale particles can be good DM candidates. For instance, DM could be non-fermionic \[^6\], \[^7\], in which case the usual suppression of the DM annihilation cross-section used in setting the Lee-Weinberg limit does not hold. In addition, low energy resonances could enhance the cross-section without the need for a large coupling constant. Other solutions, which also provide low-mass candidates for DM particles, are also possible \[^8\]–\[^10\].

There are many experiments designed to search for both direct interactions of DM with the
detector and indirect evidence of DM annihilations in our or other galaxies by looking for the 
products such as gamma-rays, positrons and antiprotons. Those can in principle probe low-
mass DM. However, direct searches, performed by experiments such as DAMA and CDMS [11], 
rely on the measurement of the kinematic recoil of the nuclei in DM interactions. For cold DM 
particles, such measurements lose sensitivity with the decreasing mass of the WIMP as recoil 
energy becomes smaller [12]. Indirect experiments, such as HESS [13], are specifically tuned to 
see large energy secondaries, only possible for weak-scale WIMPs. The backgrounds for positron 
and antiproton searches by HEAT and/or PAMELA experiments [14] could be prohibitively large 
at small energies.

It is well-known that the existing $e^+e^-$ flavor factories and future super-flavor factories could 
provide the perfect opportunity to search for rare processes, especially the ones that require high 
purity of the final states. In particular, probes of rare B-decays, such as $B \rightarrow K^{(*)}\nu\bar{\nu}$, are only possible at those machines. These colliders, where $B_q(D)$ and $\bar{B}_q(D)$ are produced in charge and 
CP-correlated states, have an opportunity to tag the decaying heavy meson “on the other side,” 
which provides the charge or CP-identification of the decaying “signal” $B$ or $D$ meson. In fact, 
many CP-violating parameters at B-factories have been measured using this method [15]. It is then 
possible to perform a similar tag on the meson decaying to a pair of light DM particles or a pair of 
DM particles and a photon. The latter process might become important for some DM models as it 
eliminates helicity suppression of the final state$.^1$ Moreover, compared to $B \rightarrow K + \not{E}$ transitions, 
where $\not{E}$ is missing energy, a massless photon could provide better experimental opportunities for 
tagging without reducing the probed parameter space of the DM masses. Finally, searches for light 
DM in heavy meson decays could be more sensitive than direct detection and other experiments, 
as DM couplings to heavy quarks could be enhanced, as for example happens in Higgs portal 
models [16].

In this paper we compute branching ratios for the heavy meson states decaying into $\chi_s\bar{\chi}_s$ and 
$\chi_s\bar{\chi}_s\gamma$. Here $\chi_s$ is a DM particle of spin $s$, which appears as missing energy in a detector. The 
DM anti-particle $\bar{\chi}_s$ may or may not coincide with $\chi_s$. We shall first consider model-independent

$^1$ This is similar to the situation in leptonic decays of $B$-mesons, where the branching ratios $B(B \rightarrow \mu\nu\gamma) \approx B(B \rightarrow \mu\nu)$ and $B(B \rightarrow \nu\nu\gamma) \gg B(B \rightarrow \nu\nu)$. 

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interactions of DM particles of spin-0, spin-1/2, and spin-1 with quarks. In each case we write the most general effective Hamiltonian coupling DM particles to flavor-changing $b \to q$ (where $q = s(d)$) or $c \to u$ current and compute $B(D) \to \chi_s \overline{\chi}_s(\gamma)$ decay rates. We then consider popular models, already available in the literature, that can generate those processes.

II. FORMALISM AND THE STANDARD MODEL BACKGROUND

The computation of decay rates for two-body processes $B_q(D) \to \chi_s \overline{\chi}_s$ is a straightforward task which only requires the knowledge of appropriate $B \to$ vacuum matrix elements. We use conventional parameterization for those,

\begin{align*}
\langle 0 | \bar{b} \gamma^\mu q | B_q \rangle &= 0, \\
\langle 0 | \bar{b} q | B_q \rangle &= 0, \\
\langle 0 | \bar{b} \gamma^\mu \gamma_5 q | B_q \rangle &= i f_{B_q} P^\mu, \\
\langle 0 | \bar{b} \gamma_5 q | B_q \rangle &= -i \frac{f_{B_q} M_{B_q}^2}{m_b + m_q},
\end{align*}

where $P^\mu$ is the 4-momentum of heavy meson $B_q$. Similar formulas can be obtained for $D$-meson. In what follows we shall provide relevant derivations for $B_q$ mesons only, but report results for both $B_q$ and $D^0$-meson decays.

Before computing the relevant DM production rates, let us study the Standard Model background for the decays with missing energy realized in transitions to $\nu\overline{\nu}$ states. The Standard Model effective Hamiltonian for $B_q(D) \to \nu\overline{\nu}(\gamma)$ reads

\begin{equation}
\mathcal{H}_{\text{eff}} = \frac{4 G_F}{\sqrt{2}} \frac{\alpha}{2 \pi \sin^2 \theta_W} \sum_{l=e,\mu,\tau} \sum_k \lambda_k X_l(x_k) \left( J_{Q q}^\mu \right) \left( \bar{\nu}_L \gamma_\mu \nu_L \right),
\end{equation}

where $J_{Q q}^\mu = \bar{q}_L \gamma^\mu b_L$ for beauty, and $J_{Q q}^\mu = \bar{\nu}_L \gamma^\mu c_L$ for charm transitions, and we consider Dirac neutrinos. The functions $\lambda_k X_l(x_k)$ are relevant combinations of the Cabbibo-Kobayashi-Maskawa (CKM) factors and Inami-Lim functions. For $b \to q$ transitions these functions are overwhelmingly dominated by the top-quark contribution,

\begin{equation}
\sum_k \lambda_k X_l(x_k) = V_{tq}^* V_{tb} X(x_t), \text{ with } X(x_t) = \frac{x_t}{8} \left[ \frac{x_t + 2}{x_t - 1} + \frac{3(x_t - 2)}{(x_t - 1)^2} \ln x_t \right]
\end{equation}
and \( x_t = m_t^2/M_W^2 \). Perturbative QCD corrections can be taken into account by the replacement \[ X_0(x_t) \rightarrow X_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t) \left[ 1 - \frac{\alpha_s}{3\pi} \left( \frac{\pi^2 - 25}{4} \right) \right], \] (5)

where \( X_1(x_t) \) can be found in Ref. \[17\]. They change our estimate by at most 10\%, and therefore be neglected in our analysis. For \( c \rightarrow u \) transitions we keep the contributions from both internal \( b \) and \( s \)-quarks, so

\[
\sum_k \lambda_k X^l(x_k) = V_{cs}^* V_{us} X^l(x_s) + V_{cb}^* V_{ub} X^l(x_b), \quad \text{with} \quad X^l(x_q) = \mathcal{D}(x_q, y_l)/2 \] (6)

where \( \mathcal{D}(x_q, y_l) \) is the Inami-Lim function \[18\] for \( y_l = m_l^2/m_{Vl}^2 \),

\[
\mathcal{D}(x_q, y_l) = \frac{1}{8} \frac{x_q y_l}{x_q - y_l} \left( \frac{y_l - 4}{y_l - 1} \right)^2 \log y_l + \frac{1}{8} \left[ \frac{x_q}{y_l - x_q} \left( \frac{x_q - 4}{x_q - 1} \right)^2 + 1 + \frac{3}{(x_q - 1)^2} \right] \frac{x_q \log x_q}{x_q - 1}
\] (7)

Given this, one can easily estimate branching ratios for \( B_q(D) \rightarrow \nu \bar{\nu} \) decays. One can immediately notice that the left-handed structure of the Hamiltonian should result in helicity suppression of those transitions. Assuming for neutrino masses that \( m_{\nu} \sim \sum_i m_{\nu_i} < 0.62 \text{ eV} \) \[19\], where \( m_{\nu_i} \) is the mass of one of the neutrinos, we obtain for the branching ratio

\[
\mathcal{B}(B_s \rightarrow \nu \bar{\nu}) = \frac{G_F^2 \alpha^2 f_B^2 M_B^3}{16\pi^3 \sin^4 \theta_W \Gamma_{B_s}} |V_{tb} V_{ts}^*|^2 X(x_t)^2 x_{\nu}^2 \simeq 3.07 \times 10^{-24}
\] (8)

where \( x_{\nu} = m_{\nu}/M_{B_s} \) and \( \Gamma_{B_s} = \Gamma_{B_d} = 1/\tau_B \) is the total width of the \( B_s \) meson. With \( \tau_B = 1.548 \text{ ps} \) we obtain \( \mathcal{B}(B_d \rightarrow \nu \bar{\nu}) = 1.24 \times 10^{-25} \). A similar calculation yields \( \mathcal{B}(D^0 \rightarrow \nu \bar{\nu}) = 1.1 \times 10^{-30} \). Clearly such tiny rates imply that decays of heavy mesons into neutrino-antineutrino final states in the Standard Model can be safely neglected as sources of background in the searches for DM in \( B_q(D) \)-decays. This is one of the main differences between this study and studies of DM production in \( B ightarrow K^{(\ast)} + \bar{E} \) transitions \[6\].

Helicity suppression in the final state can be overcome by adding a third particle, such as a photon, to the final state. The calculation of \( B(D) \rightarrow \nu \bar{\nu} \gamma \) has been done before \[20\], so here we
It is important to note that only one out of two form-factors is independent. Indeed, as it was shown in \([24]\),

\[
\langle \gamma(k)|\bar{b}\gamma_{\mu}q|B_{q}(k+q)\rangle = e \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} q^{\rho} k^{\sigma} \frac{f_{B}(q^{2})}{M_{B_{q}}},
\]

\[
\langle \gamma(k)|\bar{b}\gamma_{5}\gamma_{\mu}q|B_{q}(k+q)\rangle = -ie \left[ \epsilon^{*}_{\mu}(kq) - (\epsilon^{*} q) k_{\mu} \right] \frac{f_{A}(q^{2})}{M_{B_{q}}} \tag{9}
\]

\[
\langle \gamma(k)|\bar{b}\sigma_{\mu\nu}q|B_{q}(k+q)\rangle = e \frac{\epsilon_{\mu\nu\lambda\sigma}}{M_{B_{q}}^{2}} \left[ G\epsilon^{*\lambda} k^{\sigma} + H\epsilon^{*\lambda} q^{\sigma} + N(\epsilon^{*} q) q^{\lambda} k^{\sigma} \right] \tag{10}
\]

Matrix element \(\langle \gamma(k)|\bar{b}\sigma_{\mu\nu}q|B_{q}(k+q)\rangle\) can be obtained using identity \(\sigma_{\mu\nu} = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma_{\alpha\beta} \gamma_{5}\) \([21]\).

\[
G = 4g_{1}, \quad N = \frac{-4}{q^{2}} (f_{1} + g_{1}),
\]

\[
H = \frac{-4(k_{q})}{q^{2}} (f_{1} + g_{1}), \quad f_{1}(g_{1}) = \frac{f_{0}(g_{0})}{(1 - q^{2}/m_{f}^{2})^{2}} \tag{11}
\]

where \(f_{0}, g_{0}, \mu_{f}, \mu_{g}\) are known from QCD light-cone sum rules. Similar formulas hold for \(D\)-decays. It is important to note that only one out of two form-factors is independent. Indeed, as it was shown in \([22, 23]\),

\[
f_{V}(E_{\gamma}) = f_{A}(E_{\gamma}) = \frac{f_{B_{q}} M_{B_{q}}}{2E_{\gamma}} \left( -Q_{q} R_{q} + \frac{Q_{b}}{m_{b}} \right) + O \left( \frac{\Lambda_{QCD}^{2}}{E_{\gamma}^{2}} \right) = \frac{f_{B_{q}} M_{B_{q}}}{2E_{\gamma}} F_{B_{q}}, \tag{12}
\]

where \(R_{q}^{-1} \sim M_{B_{q}} - m_{b}\), and \(F_{B_{q}} = -Q_{q} R_{q} + \frac{Q_{b}}{m_{b}} \sim \frac{M_{B_{q}} Q_{b} - m_{b} (Q_{b} + Q_{q})}{m_{b} (M_{B_{q}} - m_{b})}\). \(Q_{q} = Q_{b} = +1/3\) are the electrical charges of \(q\) and \(b\)-quarks. Similar form factor can be obtained for the \(D\)-meson after a suitable redefinition of quark masses and charges. One-loop QCD corrections to the Eq. (12) can also be computed \([24]\).

The amplitude for \(B_{q}(D) \to \nu\gamma\) transition could be written as

\[
A(B_{q} \to \nu\gamma) = \frac{2eC_{1}^{SM}(x_{1})}{M_{B_{q}}} \left[ \epsilon_{\mu\rho\sigma} \epsilon^{*\nu} q^{\rho} k^{\sigma} f_{V}(q^{2}) + i \left[ \epsilon^{*}_{\mu}(kq) - (\epsilon^{*} q) k_{\mu} \right] \frac{f_{A}(q^{2})}{M_{B_{q}}} \right] \gamma^{\mu} \nu_{L}, \tag{13}
\]

where \(C_{1}^{SM}(x_{1}) = G_{F} c_{V} X_{0}(x_{1})/(2\sqrt{2} \pi \sin^{2} \theta_{W})\) and \(e\) is the electric charge. This results in the
photon spectrum and a branching ratio,

$$\frac{d\Gamma}{dE_{\gamma}}(B_q \to \nu\bar{\nu}\gamma) = \frac{4f_{B_q}^2 G_F^2 \alpha^3}{3M_{B_q}} |V_{tb}V_{td}^* X_0(x_t)|^2 \left( \frac{F_{B_q}}{4\pi^2 \sin^2 \theta_W} \right)^2 \times M_{B_q}^2 E_{\gamma}(M_{B_q} + E_{\gamma}) \sqrt{\frac{M_{B_q}(1 - 4x^2) - 2E_{\gamma}}{M_{B_q} - 2E_{\gamma}}} \right)^2,$$

$$\mathcal{B}(B_q \to \nu\bar{\nu}\gamma) = \frac{2}{\Gamma_{B_q}} f_{B_q}^2 G_F^2 \alpha^3 M_{B_q}^5 |V_{tb}V_{td}^* X_0(x_t)|^2 \left( \frac{F_{B_q}}{12\pi^2 \sin^2 \theta_W} \right)^2,$$

where we set $x_\nu = 0$. Numerically, $\mathcal{B}(B_s \to \nu\bar{\nu}\gamma) = \Gamma(B_s \to \nu\bar{\nu}\gamma)/\Gamma_{B_s} = 3.68 \times 10^{-8}$. Similar results for $B_d$ and $D^0$ mesons are $\mathcal{B}(B_d \to \nu\bar{\nu}\gamma) = 1.96 \times 10^{-9}$ and $\mathcal{B}(D^0 \to \nu\bar{\nu}\gamma) = 3.96 \times 10^{-14}$ respectively.

It is important to notice that the approach to rare radiative transitions described above works extremely well for SM neutrinos in the final state since $E_{\gamma} \gg \Lambda_{QCD}$ over most of the available phase space. It might not be the case for the DM production. In particular, for $m_{DM} \geq 2$ GeV, the photon energy is quite small and corrections to Eq. (12) could become significant. Therefore, our results obtained by using the formalism above should be corrected, for instance, using heavy meson chiral techniques.

Currently the only experimental constraints on $B_q(D^0) \to \ell E$ and $B_q(D^0) \to \gamma E$ transitions are available from $B_d$ decays [25],

$$\mathcal{B}(B_d \to \ell E) < 2.2 \times 10^{-4},$$

$$\mathcal{B}(B_d \to \ell E + \gamma) < 4.7 \times 10^{-5}.$$  (16)

One can see that while the branching ratios for the decays into $\nu\bar{\nu}\gamma$ final states are orders of magnitude larger than the corresponding decays into $\nu\bar{\nu}$ final states, they are still way beyond experimental sensitivities of currently operating detectors. Thus, we conclude that SM provides no irreducible background to studies of light DM in such decays.
III. SCALAR DARK MATTER PRODUCTION

A. Generic effective Hamiltonian and $B \to \chi_0 \overline{\chi}_0(\gamma)$ decays

Let us consider the generic case of a complex neutral scalar field $\chi_0$ describing the DM and limit our discussion to effective operators of dimensions no more than six. In this case, a generic effective Hamiltonian has a very simple form,

$$H_{\text{eff}}^{(s)} = 2 \sum_i \frac{C_i^{(s)}}{\Lambda^2} O_i,$$

where $\Lambda$ is the scale associated with the particle(s) mediating interactions between the SM and DM fields, and $C_i^{(s)}$ are the Wilson coefficients. The effective operators are

$$O_1 = m_b (\overline{b}_R q_L)(\chi_0^* \chi_0),$$
$$O_2 = m_b (\overline{b}_L q_R)(\chi_0^* \chi_0),$$
$$O_3 = (\overline{b}_L \gamma^\mu q_L)(\chi_0^* \overleftrightarrow{\partial}_\mu \chi_0),$$
$$O_4 = (\overline{b}_R \gamma^\mu q_R)(\chi_0^* \overleftrightarrow{\partial}_\mu \chi_0),$$

where $\overleftrightarrow{\partial} = (\overrightarrow{\partial} - \overleftarrow{\partial})/2$. For relevant $D$-meson decays one should substitute $m_b \rightarrow m_c$ and $b \rightarrow q$ currents with $c \rightarrow u$ currents. Operators $O_{3,4}$ disappear for DM in the form of real scalar fields. We note that while the generic form of Eq. (18) implies that the mediator of interaction between DM and the SM fields is assumed to be heavy, $M_\Lambda > m_{B_q(D)}$, it is easy to account for the light mediator by substituting $C_i^{(s)}/\Lambda^2 \rightarrow \overline{C}_i^{(s)}/(M_{B_q(D)}^2 - M_\Lambda^2)$. Clearly, a resonance enhancement of $B(D) \rightarrow \chi_0 \overline{\chi}_0$ rate is possible if for some reason the mediator’s mass happens to be close to $M_{B_q(D)}$. If observed, this resonance enhancement would be seen as anomalously large Wilson coefficients of the effective Hamiltonian of Eq. (18).

Let us first compute the $B(D) \rightarrow \chi_0 \overline{\chi}_0$ transition rate. It follows from Eq. (18) that the decay
branching ratio is

\[
B(B_q \to \chi \chi_0) = \left( \frac{C^{(s)}_1 - C^{(s)}_2}{4\pi M_{B_q} \Gamma_{B_q}} \right)^2 \left( \frac{f_{B_q} M_{B_q}^2 m_b}{\Lambda^2 (m_b + m_q)} \right)^2 \sqrt{1 - 4x^2} \]

(19)

where \( x_\chi = m_\chi / M_{B_q} \) is a rescaled DM mass. Clearly, this rate is not helicity-suppressed, so it could be quite a sensitive tool to determine DM properties at \( e^+e^- \) flavor factories. The result for a corresponding \( D \)-decay can be obtained via trivial substitution of quark masses, widths and decay constants. Computing the decay rate for various values of Dark Matter masses and comparing it with the experimental results for \( B_d \) missing energy decays [25] from Eq. (16) we get the following constraints on coupling constants:

\[
\left( \frac{C^{(s)}_1 - C^{(s)}_2}{\Lambda^2} \right)^2 \leq 2.03 \times 10^{-16} \text{ GeV}^{-4} \text{ for } m_\chi = 0 
\]

(20)

\[
\left( \frac{C^{(s)}_1 - C^{(s)}_2}{\Lambda^2} \right)^2 \leq 2.07 \times 10^{-16} \text{ GeV}^{-4} \text{ for } m_\chi = 0.1 \times M_{B_d} 
\]

(21)

\[
\left( \frac{C^{(s)}_1 - C^{(s)}_2}{\Lambda^2} \right)^2 \leq 2.22 \times 10^{-16} \text{ GeV}^{-4} \text{ for } m_\chi = 0.2 \times M_{B_d} 
\]

(22)

\[
\left( \frac{C^{(s)}_1 - C^{(s)}_2}{\Lambda^2} \right)^2 \leq 2.54 \times 10^{-16} \text{ GeV}^{-4} \text{ for } m_\chi = 0.3 \times M_{B_d} 
\]

(23)

\[
\left( \frac{C^{(s)}_1 - C^{(s)}_2}{\Lambda^2} \right)^2 \leq 3.39 \times 10^{-16} \text{ GeV}^{-4} \text{ for } m_\chi = 0.4 \times M_{B_d} 
\]

(24)

It is worth pointing out that constraints obtained here are much stricter than those in [26].

Applying the formalism described above, distribution of the photon energy and decay width of radiative decay \( B_q(D) \to \chi \chi_0 \gamma \) can be computed,

\[
\frac{d\Gamma}{dE_\gamma}(B_q \to \chi \chi_0 \gamma) = \frac{f_{B_q}^2 \alpha C^{(s)}_3 C^{(s)}_4}{3\Lambda^4} \left( \frac{F_{B_q}}{4\pi} \right)^2 \frac{2M_{B_q}^2 E_\gamma (M_{B_q} (1 - 4x^2) - 2E_\gamma)^{3/2}}{\sqrt{M_{B_q} - 2E_\gamma}} \]

(25)

\[
B(B_q \to \chi \chi_0 \gamma) = \frac{f_{B_q}^2 \alpha C^{(s)}_3 C^{(s)}_4 M_{B_q}^5}{6\Lambda^4 \Gamma_{B_q}} \left( \frac{F_{B_q}}{4\pi} \right)^2
\]

(26)
\[
\times \left( \frac{1}{6} \sqrt{1 - 4x_\chi^2} \left( 1 - 16x_\chi^2 - 12x_\chi^4 \right) - 12x_\chi^4 \log \frac{2x_\chi}{1 + \sqrt{1 - 4x_\chi^2}} \right),
\]

We observe that Eqs. (25) and (26) do not depend on \( C^{(s)}_{1,2} \). This can be most easily seen from the fact that \( B_q(D) \rightarrow \gamma \) form factors of scalar and pseudoscalar currents are zero, as follows from Eq. (9). Computing decay rates for various values of Dark Matter mass we are able to restrict DM properties based on experimental constraints on \( B_d \) decays with missing energy given in Eq. (16):

\[
\frac{C^{(s)}_3 C^{(s)}_4}{\Lambda^2} \leq 1.55 \times 10^{-12} \text{ GeV}^{-4} \text{ for } m = 0
\]
\[
\frac{C^{(s)}_3 C^{(s)}_4}{\Lambda^2} \leq 1.86 \times 10^{-12} \text{ GeV}^{-4} \text{ for } m = 0.1 \times M_{B_d}
\]
\[
\frac{C^{(s)}_3 C^{(s)}_4}{\Lambda^2} \leq 3.20 \times 10^{-12} \text{ GeV}^{-4} \text{ for } m = 0.2 \times M_{B_d}
\]
\[
\frac{C^{(s)}_3 C^{(s)}_4}{\Lambda^2} \leq 9.06 \times 10^{-12} \text{ GeV}^{-4} \text{ for } m = 0.3 \times M_{B_d}
\]
\[
\frac{C^{(s)}_3 C^{(s)}_4}{\Lambda^2} \leq 7.44 \times 10^{-11} \text{ GeV}^{-4} \text{ for } m = 0.4 \times M_{B_d}
\]

Note that Eqs. (25) and (26) depend on \( C_3 \) and \( C_4 \), while Eq. (19) only on \( C_1 \) and \( C_2 \). Since the models with self-conjugated DM scalar fields only contain operators \( O_1 \) and \( O_2 \), \( B_q(D) \rightarrow \chi_0\chi_0(\gamma) \) transitions could be used to test the structure of the scalar DM sector.

B. Production rates in particular models with scalar DM

In this section we apply the techniques described above for the most general effective Hamiltonian for DM particles interacting with the SM fields to particular model implementations of scalar DM, already available in the literature. The list of models considered below is by no means exhaustive.
1. **Minimal and next-to-minimal Scalar Dark Matter models**

The simplest possible model for scalar DM involves a real scalar field $\chi_0 \equiv S$ coupled to the SM particles through the exchange of Higgs boson $H$ (see also [28]). This is also a very constrained model, where the only two new parameters are the mass parameter $m_0$ of the scalar DM particle $S$ and the Higgs-scalar coupling $\lambda$. Nevertheless, it is possible to have light DM in this model even though it might require some degree of fine-tuning. The SM Lagrangian is modified by

\[- L_S = \frac{\lambda S^4}{4} + \frac{m_0^2}{2} S^2 + \lambda S^2 H^\dagger H\]

\[= \frac{\lambda S^4}{4} + \frac{1}{2} (m_0^2 + \lambda v_{EW}^2) S^2 + \lambda v_{EW} S^2 h + \frac{\lambda}{2} S^2 h^2 \] (28)

where $H$ is the Standard Model Higgs doublet, $v_{EW} = 246$ GeV is the Higgs vacuum expectation value and $h$ is the corresponding physical Higgs boson. We require $S$ to satisfy $S \to -S$ to make it a good Dark Matter candidate. The scalar DM particle can be made light by requiring cancellations between the terms defining its mass, $m^2 = m_0^2 + \lambda v_{EW}^2$.

The transition $B \to SS$ occurs in the minimal model as a one-loop process, and since mediating Higgs boson is much heavier than other particles involved in the process, it can be integrated out. The resulting effective Hamiltonian reads

\[\mathcal{H}_{eff}^{(s)} = \frac{3\lambda g_w^2 V_{ts} V_{tb}^* S x_t m_b}{64 M_H^2 \pi^2} (\tilde{b}_L q_R) S^2, \] (29)

which implies that $C_{1,3,4}^{(s)} = 0$, $C_2^{(s)} = 3\lambda g_w^2 V_{ts} V_{tb}^* S x_t / 128 \pi^2$, and $\Lambda = M_H$. Thus, from Eq. (19), the branching ratio for the $B \to SS$ decay in this model is

\[\mathcal{B}(B_q \to SS) = \left[\frac{3g_w^2 V_{ts} V_{tb}^* S x_t m_b}{128 \pi^2}\right]^2 \frac{\sqrt{1 - 4x_S^2}}{16\pi M_B \Gamma_{B_q}} \left(\frac{\lambda^2}{M_H^4}\right) \left(\frac{f_{B_q} M_{B_q}^2}{m_b + m_q}\right)^2, \] (30)

where $x_S = m_S / m_{B_q}$. Note that this rate depends not only on the mass of $S$ but also on the parameter $\kappa = \lambda^2 / M_H^4$. This parameter also drives the calculation of the relic density of $S$ [27],

\[\sigma_{ann} v_{rel} = \frac{8\nu_{EW}^2 \lambda^2}{M_H^2} \times \lim_{m_{h^*} \to 2m_S} \frac{\Gamma_{h^* X}}{m_{h^*}^2}, \] (31)
where $\Gamma_{h^*X}$ is the rate for the decay $h^* \to X$ for a virtual Higgs with $M_H \sim 2m_S$. We can, therefore, fix $\kappa$ from the relic density calculation. This gives for the branching ratios of $B_q$ and $D$-decays,

$$
\mathcal{B}(B_s \to SS) \approx \left(4.5 \times 10^5 \, \text{GeV}^4 \right) \times \frac{\lambda^2}{M_H^2} \sqrt{1 - 4x_S^2} \quad (32) \\
\mathcal{B}(B_d \to SS) \approx \left(1.3 \times 10^4 \, \text{GeV}^4 \right) \times \frac{\lambda^2}{M_H^4} \sqrt{1 - 4x_S^2} \quad (33) \\
\mathcal{B}(D^0 \to SS) \approx \left(2.9 \times 10^{-6} \, \text{GeV}^4 \right) \times \frac{\lambda^2}{M_H^4} \sqrt{1 - 4x_S^2} \quad (34)
$$

We require the branching ratios to be smaller than the current experimental upper bound [25] for the missing energy decay given in Eq. (16). With this we are able to put the following restriction onto the parameters of this model:

$$
\left(\frac{\lambda}{M_H^2}\right)^2 \sqrt{1 - 4x_S^2} \leq 1.68 \times 10^{-7}. \quad (35)
$$

We present the resulting branching ratios as a function of $m_{\chi_0}$ in Fig. 1. Comparing the above branching ratio with the available experimental data we can put constraints on the parameters of this model, which we present in Fig. 2. For the particular values of Dark Matter particles mass
FIG. 2: (a) allowed values of the DM-Higgs coupling \( \lambda \) as a function of \( x = m_S/M_B \) (below the curves) for the Higgs masses of 110 GeV (red), 120 GeV (green), and 150 GeV (blue). (b) Allowed values of the Higgs mass in GeV (above the curves) for \( \lambda = 0.1 \) (red), 1 (green), and 5 (blue) as a function of \( x = m_S/M_B \).

we get

\[
\left| \frac{\lambda}{M_H^2} \right| \leq 8.2 \times 10^{-4} \text{ GeV}^{-2} \text{ for } m_S = 0
\]

\[
\left| \frac{\lambda}{M_H^2} \right| \leq 8.3 \times 10^{-4} \text{ GeV}^{-2} \text{ for } m_S = 0.1 \times M_B
\]

\[
\left| \frac{\lambda}{M_H^2} \right| \leq 8.6 \times 10^{-4} \text{ GeV}^{-2} \text{ for } m_S = 0.2 \times M_B
\]

\[
\left| \frac{\lambda}{M_H^2} \right| \leq 9.2 \times 10^{-4} \text{ GeV}^{-2} \text{ for } m_S = 0.3 \times M_B
\]

\[
\left| \frac{\lambda}{M_H^2} \right| \leq 1.1 \times 10^{-3} \text{ GeV}^{-2} \text{ for } m_S = 0.1 \times M_B
\]

The minimal scalar model described above can be made less restricted if we introduce another mediator for DM-SM interactions, which should somewhat alleviate the fine-tuning present in the minimal model \[27\]. This can be done in a variety of ways. The simplest one is to introduce another Higgs-like field \( U \),

\[
-\mathcal{L}_S = \frac{\lambda_S}{4} S^4 + \frac{m_0^2}{2} S^2 + (\mu_1 U + \mu_2 U^2) S^2 + V(U) + \eta U^2 H^\dagger H
\]

\[
= \frac{m_s^2}{2} S^2 + \frac{m_u^2}{2} u^2 + \mu u S^2 + \eta v_{EW} u h + \ldots,
\]
where we only display mass and relevant interaction terms; ellipses stands for other terms in the Lagrangian that are irrelevant for this discussion.

Here $u$ denotes the excitation around vev of $U$, and $\mu$ and $\eta$ are parameters with values of the order of electroweak scale. As far as the studies of DM production in heavy flavor decays are concerned, extended models of this class are equivalent to the minimal model after suitable redefinition of parameters [27]. Performing such redefinitions, we obtain

$$B(B_u \to SS) \approx (2.1 \times 10^{-4}) \times \frac{\eta^2\mu^2}{M_U^4} \sqrt{1 - 4x_S^2},$$

$$B(B_d \to SS) \approx (6.3 \times 10^{-6}) \times \frac{\eta^2\mu^2}{M_U^4} \sqrt{1 - 4x_S^2},$$

$$B(D^0 \to SS) \approx (1.38 \times 10^{-14}) \times \frac{\eta^2\mu^2}{M_U^4} \sqrt{1 - 4x_S^2},$$

(38)

where $M_U$ is the mass of the Higgs-like field $U$ of Eq. (37). In the results above, the mass of the Higgs boson was fixed at $M_h = 120$ GeV. Since the $S$-field is a real scalar field in both the minimal and the extended models, these models do not give rise to the radiative decay $B_q \to SS\gamma$.

2. Dark Matter with two Higgs doublets (2HDM)

In this subsection we consider a singlet scalar WIMP $S$ that interacts with two Higgs doublets, $H_u$ and $H_d$ [6, 29],

$$-\mathcal{L} = \frac{m_0^2}{2} S^2 + \lambda_1 S^2 (|H^0_d|^2 + |H^0_u|^2) + \lambda_2 S^2 (|H^+_d|^2 + |H^+_u|^2) + \lambda_3 S^2 (H^-_d H^+_u - H^-_u H^+_d).$$

(39)

We shall assume that $\lambda_1 \gg \lambda_2$, as the opposite limit gives results that are not different from the minimal scalar model considered above. The contribution of $\lambda_3$ is suppressed because of the cancelation of two diagrams, as explained in [6].

Calculating the effective Hamiltonian results in the following expressions of the Wilson coefficients,

$$C^{(s)}_{2} = C^{(s)}_{1} = \frac{\lambda_1 g_w^2 V_{ts} V^*_{tb} x_t (1 - a_t + a_t \log a_t)}{128 \pi^2 (1 - a_t)^2}$$

and $\Lambda = M_H$, (40)

where $a_q = (m_q/M_H)^2$. As in the previous subsection, no decay into the dark matter with photon
possible within the framework of this model. However, decay into a pair of dark matter particles is possible

\[ B(B_s \rightarrow SS) \approx (0.73 \times 10^2 \text{ GeV}^4) \times \lambda_1^2 \sqrt{1 - 4x_S^2} \left( \frac{a_t \log a_t - a_t + 1}{M_H^2(1 - a_t)^2} \right)^2, \]

\[ B(B_d \rightarrow SS) \approx (2.1 \text{ GeV}^4) \times \lambda_1^2 \sqrt{1 - 4x_S^2} \left( \frac{a_t \log a_t - a_t + 1}{M_H^2(1 - a_t)^2} \right)^2, \]

\[ B(D^0 \rightarrow SS) \approx (5.0 \times 10^2 \text{ GeV}^4) \times \lambda_1^2 \sqrt{1 - 4x_S^2} \left( \sum_{q=b,s,d} V_{uq} V^*_{cq} \frac{a_q \log a_q - a_q + 1}{M_H^2(1 - a_q)^2} \right)^2. \]  

Eqs. (41) can be used for constraining parameters of this model in \( B_q \rightarrow SS \) transitions.

IV. FERMIONIC DARK MATTER PRODUCTION

A. Generic effective Hamiltonian and \( B_{d(s)} \rightarrow \chi_{1/2} \bar{\chi}_{1/2} (\gamma) \) decays

Let us now consider a generic case of fermionic Dark Matter production. It is possible that the DM particles have half-integral spin; so many New Physics models, including Minimal Supersymmetric Standard Model (MSSM), have fermionic DM candidates. Most of those models, however, naturally assign rather large masses to their DM candidates. Nevertheless, either after some fine-tuning of the relevant parameters or after introducing a light DM-SM mediator, relatively light DM particles are still possible. Let us consider their production in the decays of heavy mesons. Once again, limiting ourselves to the operators of dimension of no more than six, a relevant effective Hamiltonian reads

\[ H_{eff}^{(f)} = \frac{4}{\Lambda^2} \sum_i C_i^{(f)} Q_i, \]

where \( C_i \)'s are relevant Wilson coefficient and \( \Lambda \) represents the mass scale relevant for DM-quark interactions (e.g. mediator mass). In general, there are twelve possible effective operators,

\[ Q_1 = (\bar{b}_L \gamma_{\mu} s_L)(\bar{\chi}_{1/2 L} \gamma^\mu \chi_{1/2 L}), \quad Q_2 = (\bar{b}_L \gamma_{\mu} s_L)(\bar{\chi}_{1/2 R} \gamma^\mu \chi_{1/2 R}), \]
where the Dark Matter fermion $\chi_{1/2}$ can be either of Dirac or Majorana type. The latter choice leads to some simplification of the basis. All needed matrix elements have been given in Eq. (2). Note that the matrix elements of the tensor operators vanish,

$$\langle 0|\bar{\sigma}^{\mu\nu}P_{L,R}q|B_q\rangle = 0.$$  

(44)

For relevant $D$-meson decays one should substitute $m_b \rightarrow m_c$ and $b \rightarrow q$ currents with $c \rightarrow u$ currents. Using the Hamiltonian of Eq. (43) we get for the branching ratio of $B_q \rightarrow \bar{\chi}_{1/2}\chi_{1/2}$,

$$B(B_q \rightarrow \bar{\chi}_{1/2}\chi_{1/2}) = \frac{f_{B_q}^2 M_{B_q}^3}{16\pi \Gamma_{B_q} \Lambda^2} \sqrt{1 - 4x^2} \left[ C_{57} C_{68} \frac{4M_{B_q}^2 x^2}{(m_b + m_q)^2} - (C_{57}^2 + C_{68}^2) \frac{M_{B_q}^2 (2x^2 - 1)}{(m_b + m_q)^2} \right] - 2\tilde{C}_{1-8} \frac{x^2 M_{B_q}}{m_b + m_q} + 2(C_{13} + C_{24})^2 x^2 \right],$$

(45)

where we employed short-hand notations for the combinations of Wilson coefficients $C_{ij} = C_{i}^{(f)} - C_{j}^{(f)}$, and $\tilde{C}_{1-8} = C_{13} C_{57} + C_{24} C_{57} + C_{13} C_{68} + C_{24} C_{68}$. Due to its larger mass chirality suppression for the GeV-scale Dark Matter is not as severe as for neutrinos, even for purely left-handed interactions. The obtained result leads to model-independent constraints on the Wilson coefficients of Eq. (12), which is based on experimental data for missing energy decays of $B_d$ meson (see, e.g. Eq. (16)). They can be found in Table I. The results presented there can be used to constrain parameters of particular models of fermionic Dark Matter considered below.

The technique which we use for the computation of $\Gamma(B_q(D) \rightarrow \chi_{1/2}\bar{\chi}_{1/2})$ is very similar to the one used for the radiative decay of heavy meson into scalar DM particles discussed above. The hadronic part of the matrix element remains the same, we only modify the part that describes
While there are many models of light fermionic DM that employ operator \( s \) involving charged scalars, these lead to no contribution to this decay. Therefore, we chose not to provide a closed analytic expression for \( B_9 \rightarrow \chi_{1/2} \bar{\chi}_{1/2} \) here due to overall bulkiness of the resulting expression. An interested reader can perform numerical integration of Eq. (47) for a particular model, if needed.
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$x$ & $C_1/\Lambda^2$, GeV$^{-2}$ & $C_2/\Lambda^2$, GeV$^{-2}$ & $C_3/\Lambda^2$, GeV$^{-2}$ & $C_4/\Lambda^2$, GeV$^{-2}$ \\
\hline
0 & $6.3 \times 10^{-7}$ & $6.3 \times 10^{-7}$ & $6.3 \times 10^{-7}$ & $6.3 \times 10^{-7}$ \\
0.1 & $7.0 \times 10^{-7}$ & $7.0 \times 10^{-7}$ & $7.0 \times 10^{-7}$ & $7.0 \times 10^{-7}$ \\
0.2 & $9.2 \times 10^{-7}$ & $9.2 \times 10^{-7}$ & $9.2 \times 10^{-7}$ & $9.2 \times 10^{-7}$ \\
0.3 & $1.5 \times 10^{-6}$ & $1.5 \times 10^{-6}$ & $1.5 \times 10^{-6}$ & $1.5 \times 10^{-6}$ \\
0.4 & $3.4 \times 10^{-6}$ & $3.4 \times 10^{-6}$ & $3.4 \times 10^{-6}$ & $3.4 \times 10^{-6}$ \\
\hline
\end{tabular}
\caption{Constraints (upper limits) on the Wilson coefficients of operators of Eq. (43) from the $B_q \to \chi_{1/2}\chi_{1/2}\gamma$ transition. Note that operators $Q_5 - Q_8$ give no contribution to this decay.}
\end{table}

Integrating Eq. (46) over the photon energy analytically we obtain

$$B_{1-8}(B_q \to \chi_{1/2}\chi_{1/2}\gamma) = \frac{F_{B_q}^2 f_{B_q}^2 M_{B_q}^2 \alpha}{144 \pi^2 \sqrt{1 - 4x_\chi^2 \Lambda^2}} \times \left[(C_1^2 + C_2^2 + C_3^2 + C_4^2) Y(x_\chi) + \frac{9}{2} (C_1 C_2 + C_3 C_4) Z(x_\chi)\right], \quad (48)$$

where the factors $Y(x_\chi)$ and $Z(x_\chi)$ are defined as

$$Y(x_\chi) = 1 - 2x_\chi^2 + 3x_\chi^2 (3 - 6x_\chi^2 + 4x_\chi^4) \sqrt{1 - 4x_\chi^2} \log \left(\frac{2x_\chi}{1 + \sqrt{1 - 4x_\chi^2}}\right) - 11x_\chi^4 + 12x_\chi^6,$$

$$Z(x_\chi) = x_\chi^2 \left(1 + 2x_\chi^2 + 8x_\chi^2 (1 - x_\chi^2) \sqrt{1 - 4x_\chi^2} \log \left(\frac{2x_\chi}{1 + \sqrt{1 - 4x_\chi^2}}\right) + 8x_\chi^4\right). \quad (49)$$

This equation can be used to place constraints on the individual Wilson coefficients of Eq. (43). They can be found in Table II. Both Eq. (45) and Eq. (48) can now be used to constrain the parameters of the particular models of fermionic DM.

**B. Production rates in particular models with fermionic DM**

1. **Models with hidden valleys**

It was pointed out in [30] that there could be light particles called $v$-quarks interacting with Standard Model sector via heavy mediator $Z'$. In the simplest $v$-Model, a $SU(n_v) \times U(1)$ gauge
group with couplings $g'$ and $g_v$ is added to the Standard Model\textsuperscript{2}. The $U(1)$ symmetry is broken by vacuum expectation value of the scalar field $\langle \phi \rangle$, giving $Z'$ a mass of $\sim 1 - 6$ TeV. The latter can mix with Standard Model $Z$ via kinetic mixing $k F^{\mu \nu} F'_{\mu \nu}$. In this model the role of Dark Matter is played by the $v$-quarks ($\chi_{1/2} \equiv v$).

The model corresponds to the following set of parameters for the decay of $B_s$ meson (for decays of $B_d$ and $D^0$ parameters will be similar):

$$C_1 = \frac{G_F k g' M Z M' Z' \alpha}{2 g_w \sqrt{2} \sin^2 \theta_W} V_{tb} V_{ts}^* X(x), \quad \text{and} \quad \Lambda = M_{Z'} \quad (50)$$

where $k$ is the kinetic mixing parameter, $g'$ is a gauge coupling of the $Z'$ and $v$-quarks, and $M_{Z'}$ is the mass of the heavy mediator. The rest of the Wilson coefficients $C_i$ are zero. Thus, from Eq. \textsuperscript{15},

$$B(B_s \to v \bar{v}) \approx (1.76 \text{ GeV}^2) x_v^2 \sqrt{1 - 4 x_v^2} \left( \frac{g' k}{M_{Z'}} \right)^2 \quad (51)$$

where $x_v = m_v / M_{B_q}$. The corresponding results for $B_d$ and $D^0$ decays are

$$B(B_d \to v \bar{v}) \approx (4.68 \times 10^{-2} \text{ GeV}^2) x_v^2 \sqrt{1 - 4 x_v^2} \left( \frac{g' k}{M_{Z'}} \right)^2, \quad (52)$$

and

$$B(D^0 \to v \bar{v}) \approx (2.68 \times 10^{-8} \text{ GeV}^2) x_v^2 \sqrt{1 - 4 x_v^2} \left( \frac{g' k}{M_{Z'}} \right)^2, \quad (53)$$

respectively. The corresponding expression for the decay into two $v$-quarks and photon can be obtained by defining

$$C_1 = \frac{G_F k g' M Z M' Z' \alpha}{2 g \sqrt{2} \sin^2 \theta_W} V_{tb} V_{ts}^* X(x) \frac{e}{3}, \quad \text{and} \quad \Lambda = M_{Z'} \quad (54)$$

We present our results in Fig. 3(a) in order to extract the dependence on DM mass. The analytic

\textsuperscript{2} The $g'$ coupling constant introduced here is not to be confused with the SM hypercharge coupling constant.
FIG. 3: (a) $B(B_d \rightarrow vv)$ as a function of $x = m_{v}/M_{B_d}$ evaluated at $g' = 1$, $k = 1$ and $M_{Z'} = 1$ TeV; (b) Allowed values of the $M_{Z'}$ mass in GeV (above the curves) for $g_1k = 1$ (black), 0.1 (red), and 10 (green) as a function of $x = m_{v}/M_{B_d}$. Solid lines represent the constraints from the 2-body, and the dashed ones – from the 3 body (radiative) decay. As one can see, the constraints on the mass of $Z'$ are very loose.

results for the branching ratios can be well approximated by the following formulas,

$$B(B_s \rightarrow v\bar{v}\gamma) \approx (2.76 \times 10^{-4} \text{ GeV}^2) \frac{g_1^2k^2}{M_{Z'}^2} \times \frac{Y(x_v)}{\sqrt{1 - 4x_v^2}}$$  \hspace{1cm} (55)

for the branching ratio of $B_s$ radiative decay and

$$B(B_d \rightarrow v\bar{v}\gamma) \approx (9.07 \times 10^{-6} \text{ GeV}^2) \frac{g_1^2k^2}{M_{Z'}^2} \times \frac{Y(x_v)}{\sqrt{1 - 4x_v^2}}$$  \hspace{1cm} (56)

$$B(D^0 \rightarrow v\bar{v}\gamma) \approx (3.68 \times 10^{-12} \text{ GeV}^2) \frac{g_1^2k^2}{M_{Z'}^2} \times \frac{Y(x_v)}{\sqrt{1 - 4x_v^2}}$$  \hspace{1cm} (57)

for $B_d$ and $D^0$ decays, respectively. The structure function $Y(x)$ appearing in this equation was defined in Eq. \((49)\).

2. **Right-handed massive neutrinos as a Fermionic Dark Matter**

Massive right-handed neutrinos appear naturally in left-right symmetric models (see for example \([31]\)). The see-saw mechanism is used to get light left-handed neutrinos and massive right-handed ones. The coupling of the massive neutrino to the SM fields in this case is mediated by a
right-handed gauge boson with mass in the TeV range. In this section \( \chi_{1/2} \equiv \nu_R \).

\[ H_{\text{eff}} = \frac{4G_F^{(R)}}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \sum_k \lambda_k X(x_k) \left( J_{q q}^\mu \right) \left( \overline{\nu}_R \gamma_\mu \nu_R \right), \]  

(58)

where \( J_{q q}^\mu = q_R \gamma_\mu b_R \) for beauty and \( J_{q q}^\mu = \overline{u}_R \gamma_\mu c_R \) for charm transitions. The functions \( \lambda_k X(x_k) \) are the combinations of the Cabbibo-Kobayashi-Maskawa (CKM) factors and Inami-Lim functions. \( G_F^{(R)} \) is defined similarly to the usual Fermi constant,

\[ \frac{G_F^{(R)}}{\sqrt{2}} = \frac{g^2}{8M_W^2}, \]  

(59)

which implies that

\[ C_4 = \frac{g^2}{8} \frac{\alpha}{2\pi \sin^2 \theta_W}. \]  

(60)

Following the procedure described above, we obtain the following results for decay branching ratios,

\[ \mathcal{B}(B_s \to \nu_R \overline{\nu}_R) \approx \frac{3.6 \times 10^3 \text{ GeV}^4}{M_W^4} \frac{\overline{x_v^2 \sqrt{1 - 4x_v^2}}}{}, \]  

(61)

\[ \mathcal{B}(B_s \to \nu_R \overline{\nu}_R \gamma) \approx \frac{0.57 \text{ GeV}^4}{M_W^4} \times Y(x_\nu), \]  

(62)

\[ \mathcal{B}(B_d \to \nu_R \overline{\nu}_R) \approx \frac{10^2 \text{ GeV}^4}{M_W^4} \frac{x_v^2 \sqrt{1 - 4x_v^2}}{,} \]  

(63)

\[ \mathcal{B}(B_d \to \nu_R \overline{\nu}_R \gamma) \approx \frac{1.9 \times 10^{-2} \text{ GeV}^4}{M_W^4} \times Y(x_\nu), \]  

(64)

\[ \mathcal{B}(D^0 \to \nu_R \overline{\nu}_R) \approx \frac{5.6 \times 10^{-5} \text{ GeV}^4}{M_W^4} \frac{x_v^2 \sqrt{1 - 4x_v^2}}{,} \]  

(65)

\[ \mathcal{B}(D^0 \to \nu_R \overline{\nu}_R \gamma) \approx \frac{7.6 \times 10^{-9} \text{ GeV}^4}{M_W^4} \times Y(x_\nu), \]  

(66)

where \( Y(x) \) is defined in Eq. \( (49) \). These results are also presented in Fig. \( 4 \).
FIG. 4: (a) $\mathcal{B}(B_d \to \nu_R \bar{\nu}_R)$ as a function of $x = m_{\nu_R}/M_{B_d}$ evaluated at $M_{W_R} = 1$ TeV, (b) Allowed values of the $M_{W_R}$ mass in GeV (above the curves) as a function of $x = m_{\nu_R}/M_{B_d}$. Solid lines represent the constraints from the 2-body, and the dashed ones – from the 3 body (radiative) decay. As one can see, the constraints on the mass of $W_R$ are very loose.

C. Majorana fermions

Majorana particles $\chi_{1/2} \equiv \chi$ often appear in many models of physics beyond the Standard Model. For generic studies of decays of heavy mesons to Majorana DM particles we can also use Lagrangian of Eq. (43). The resulting formulas, however, will be simplified due to the known properties of Majorana fermions [32],

$$\bar{\chi}\gamma_\mu\chi = 0,$$
$$\bar{\chi}\sigma^{\mu\nu}\chi = 0.$$

Taking into account the conditions of Eq. (67), we can obtain the branching ratio for $B_q \to \chi\chi$ decay,

$$\mathcal{B}(B_q \to \chi\chi) = \frac{f_{B_q}^2 M_{B_q}^5}{16\pi \Gamma_{B_q}(m_b + m_q)^2 \Lambda^2} \sqrt{1 - 4x^2} \left[ C_{57}^2 + C_{68}^2 - 2x^2(C_{57} - C_{68})^2 \right]. \quad (67)$$

The photon energy distribution in $B_q \to \chi\chi\gamma$ decay reads

$$\frac{d\Gamma}{dE_\gamma} = \frac{f_{B_q}^2 F_{B_q}^2 \alpha M_{B_q}^2 E_\gamma}{48\pi^2 \Lambda^2} \sqrt{M_{B_q}(1 - 4x^2) - 2E_\gamma} \times \left( C_{12}^2 + C_{34}^2 \right) (M_{B_q}(1 + 2x^2) + E_\gamma), \quad (68)$$
which can be integrated over to obtain the branching fraction

\[ B(B_q \to \chi \chi \gamma) = \frac{f_{B_q}^2 F_{B_q}^2 \alpha M_B^5}{1152 \pi^2 \Lambda^2} (C_{12}^2 + C_{34}^2) \times \]

\[ \left( 36x_\chi^2 \log \frac{2x_\chi}{\sqrt{1 - 4x_\chi^2} + 1} + (4 + 17x_\chi^2 + 6x_\chi^4) \sqrt{1 - 4x_\chi^2} \right). \]

As an example, we consider a realization of the fermionic dark matter scenario proposed in [6]. In this model the Majorana fermion coupled to a higgs-higgsino pair is considered. It must be noted that by “higgsino” we mean a fermionic field with the same quantum numbers as a Higgs field. We, however, do not place any supersymmetric requirements on the coupling constants. With that,

\[ -L_f = \frac{M}{2} \bar{\psi} \psi + \mu \bar{\tilde{H}}_d \tilde{H}_u + \lambda_d \bar{\psi} \tilde{H}_d \tilde{H}_d + \lambda_u \bar{\psi} \tilde{H}_u \tilde{H}_u, \]

where \( M \ll \mu, \lambda_u v_u \). The Dark Matter candidate is the lightest mass eigenstate, which we define as

\[ \chi = -\psi \cos \theta + \tilde{H}_d \sin \theta, \quad \sin^2 \theta = \frac{\sqrt{\lambda_d^2 v_u^2 + \mu^2}}{\sqrt{\lambda_u^2 v_u^2 + \mu^2}}, \]

\[ m_1 = M \left( 1 - \frac{\lambda_u^2 v_u^2}{\lambda_u^2 v_u^2 + \mu^2} \right). \]

We are thus led to the following effective Lagrangian,

\[ \mathcal{L}_{\text{eff}} = \frac{1}{2} V_{ts} V_{ts}^* \tan \beta \left( \frac{\lambda_d \lambda_u v_u \mu}{\lambda_u^2 v_u^2 + \mu^2} \right) \frac{m_b a_t \ln a_t}{1 - a_t} \left( \tilde{b}_{LR} \tilde{\chi} \right), \]

where \( a_t = m_t^2/M_h^2 \) and \( \tan \beta = v_u/v_d \). Matching this Lagrangian to Eqs. [42, 43], we observe that \( C_5 = C_6 \), and the remaining coefficients \( C_i = 0 \). In addition,

\[ C_5 = C_6 = \frac{V_{ts} V_{tb}^* \tan b}{(16\pi)^2 v_{sm}^3} \left( \frac{\lambda_d \lambda_u v_u \mu}{\lambda_u^2 v_u^2 + \mu^2} \right) \frac{m_b m_b^2 \ln a_t}{1 - a_t}, \quad \text{and} \quad \Lambda = M_h. \]

Taking into account Eq. [69] we conclude that no decay into \( \chi \chi \gamma \) is possible in this particular
FIG. 5: (a) \( \mathcal{B}(B_d \to \chi \bar{\chi}) \) as a function of \( x = m_\chi / M_{B_d} \). The following numerical values were used: \( \kappa = (\lambda_d \lambda_u v_u \mu) / (\lambda_u^2 v_u^2 + \mu^2) = 1 \), \( \tan \beta = 10 \), \( M_h = 102 \) GeV (b) Allowed values of the \( \kappa \) (above the curves) for the values of \( \tan \beta = 1 \) (red), 10 (green), 100 (blue), and 1000 (purple) while mass of Higgs boson was fixed at \( M_h = 120 \) GeV as a function of \( x = m_\chi / M_{B_d} \).

model. However, a simpler decay into \( \chi \bar{\chi} \) is possible,

\[
\mathcal{B}(B_s \to \chi \chi) \approx 1.47 \times 10^{-10} \sqrt{1 - 4x^2_{\chi}} \left( \frac{\log^2(a_t)}{(1 - a_t)^2} \left( \frac{\tan(\beta)v_u \lambda_d \lambda_u \mu}{(v_u^2 \lambda_u^2 + \mu^2)} \right)^2 \right),
\]

(73)

\[
\mathcal{B}(B_d \to \chi \chi) \approx 4.16 \times 10^{-12} \sqrt{1 - 4x^2_{\chi}} \left( \frac{\log^2(a_t)}{(1 - a_t)^2} \left( \frac{\tan(\beta)v_u \lambda_d \lambda_u \mu}{(v_u^2 \lambda_u^2 + \mu^2)} \right)^2 \right),
\]

(74)

\[
\mathcal{B}(D^0 \to \chi \chi) \approx 1.81 \times 10^{-11} \sqrt{1 - 4x^2_{\chi}} \left( \frac{\tan(\beta)v_u \lambda_d \lambda_u \mu}{(v_u^2 \lambda_u^2 + \mu^2)} \sum_{q=b, s, d} V_{cq} V^{*}_{uq} \frac{a_q \log(a_q)}{(1 - a_q)} \right)^2,
\]

(75)

where \( a_q = (m_q / M_H)^2 \) and \( x_{\chi} = m_\chi / M_{B_q} \). These results can be used to constrain the parameters of this model.

V. VECTOR DARK MATTER PRODUCTION. GENERIC EFFECTIVE HAMILTONIAN AND \( B_q(D^0) \to \chi_1 \chi_1 \) DECAYS

Vector DM is a quite popular concept in non-supersymmetric solutions of the hierarchy problem. In particular, it can be encountered in models with Universal Extra Dimensions (UED), little Higgs models with T-parity, and some variations of Randall-Sundrum models. All of the proposed models that the authors are aware of involve weak-scale DM particles. This however, does not preclude the existence of the low mass vector DM.

Let us consider a generic case of a vector field \( \chi_1^\mu \) describing Dark Matter. This DM particle
could be either a gauge boson, corresponding to some abelian or non-abelian gauge symmetry broken at some higher scale, or some composite state. The only assumption that we shall make is that \( \chi_1 \) is odd under some \( Z_2 \)-type discrete symmetry, \( \chi_1^\mu \rightarrow -\chi_1^\mu \). This condition results in the pair-production of DM particles.

We shall limit our discussion to the effective operators of the dimension no more than six. Since no gauge symmetry related to \( \chi_1^\mu \) is present at the scale \( m_Q \), the most general effective Hamiltonian should be built out of the vector field \( \chi_1^\mu \) and its field strength tensor \( \chi_1^{\mu\nu} \). In this case, an effective Hamiltonian has a very simple form,

\[
\mathcal{H}_{\text{eff}}^{(v)} = \sum_i \frac{C_i^{(v)}}{\Lambda^2} O_i, \tag{76}
\]

where \( \Lambda \) is the scale associated with the mass of the particle mediating interactions between the SM and DM fields, and \( C_i^{(V)} \) are the Wilson coefficients. The effective operators are

\[
\begin{align*}
O_1 &= m_b (\bar{b}_L q_R) \chi_1^\mu \chi_1^\mu, \\
O_2 &= m_b (\bar{b}_R q_L) \chi_1^\mu \chi_1^\mu, \\
O_3 &= (\bar{b}_L \gamma_\mu q_L) \chi_1^{\mu\nu} \chi_1^\nu, \\
O_4 &= (\bar{b}_R \gamma_\mu q_R) \chi_1^{\mu\nu} \chi_1^\nu, \\
O_5 &= (\bar{b}_L \gamma_\mu q_L) \chi_1^{\mu\nu} \chi_1^\nu, \\
O_6 &= (\bar{b}_R \gamma_\mu q_R) \chi_1^{\mu\nu} \chi_1^\nu,
\end{align*}
\tag{77}
\]

where \( \tilde{\chi}_1^{\mu\nu} = (1/2) \epsilon^{\mu\alpha\beta} \chi_{1\alpha\beta} \) and \( q = s, d \). As before, the Hamiltonian relevant for charmed meson decays can be obtained by the proper substitution of \( b \rightarrow q \) current with \( c \rightarrow u \) current.

The \( B_q(D) \rightarrow \chi_1 \chi_1 \) transition rate can be computed using Eq. (77). Using the form-factors defined in Eq. (2), we obtain

\[
B(B_q \rightarrow \chi_1 \chi_1) = \frac{f_B^2 \lambda^2 m_\phi^2}{256(m_b + m_q)^2 \pi x_\chi^4 \Gamma_{B_q} \Lambda^4} \left[ C_{12}^2 \left( 1 - 4x_\chi^2 + 12x_\chi^4 \right) + (m_b + m_q)^2 \left( 8C_{56}^2 \left( 1 - 4x_\chi^2 \right) + 3C_{34}^2 \right) x_\chi^4 \right.
\]

\[
+ 2C_{12}C_{34}(m_b + m_q) \left( 1 + 2x_\chi^2 \right) x_\chi^2 \bigg], \tag{78}
\]

where \( C_{ik} = C_i^{(v)} - C_k^{(v)} \) and \( x_{\text{DM}} = m_\chi/m_{B_q} \). It is necessary to point out that Eq. (78) is divergent.
TABLE III: Constraints (upper limits) on the Wilson coefficients of operators of Eq. (77) from the $B_q \to \chi_1 \chi_1$ transition.

| $x_\chi$ | $C_1/\Lambda^2$, GeV$^{-2}$ | $C_2/\Lambda^2$, GeV$^{-2}$ | $C_3/\Lambda^2$, GeV$^{-2}$ | $C_4/\Lambda^2$, GeV$^{-2}$ | $C_5/\Lambda^2$, GeV$^{-2}$ | $C_6/\Lambda^2$, GeV$^{-2}$ |
|----------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 0        | 0                           | 0                           | $1.4 \times 10^{-8}$        | $1.4 \times 10^{-8}$        | $8.9 \times 10^{-9}$        | $8.9 \times 10^{-9}$        |
| 0.1      | $1.2 \times 10^{-9}$        | $5.1 \times 10^{-9}$        | $1.5 \times 10^{-8}$        | $1.5 \times 10^{-8}$        | $9.1 \times 10^{-9}$        | $9.1 \times 10^{-9}$        |
| 0.2      | $1.3 \times 10^{-8}$        | $1.3 \times 10^{-8}$        | $1.6 \times 10^{-8}$        | $1.6 \times 10^{-8}$        | $1.0 \times 10^{-8}$        | $1.0 \times 10^{-8}$        |
| 0.3      | $2.9 \times 10^{-8}$        | $2.9 \times 10^{-8}$        | $1.9 \times 10^{-8}$        | $1.9 \times 10^{-8}$        | $1.2 \times 10^{-8}$        | $1.2 \times 10^{-8}$        |

at $m_\chi = 0$, which is related to the fact that operators in Eq. (77) contributing to the effective Lagrangian are not gauge invariant. Thus, for the case of massless DM the upper limit on the Wilson coefficients $C_1^{(v)}$ and $C_2^{(v)}$ is zero (see Table III).

Using Eq. (78), we can place general constraints on the Wilson coefficients of the effective Hamiltonian describing interactions of vector DM with quarks (see Eq. (76)). They are presented in Table III.

We are not aware of particular models of light DM with spin-1 particles and masses $m_\chi < 3$ GeV.

VI. CONCLUSIONS

We have argued that missing energy decays of the heavy mesons - $B_d$, $B_s$ and $D^0$ - provide an important way to probe different properties of Dark Matter. Consideration of different decay modes - two body decays, radiative and light meson + DM decays - restricts different regions of the Dark Matter parameter space. Combined constraints obtained from different decay modes of various heavy mesons provide indispensable probe of physics beyond the Standard Model in general and the nature of the Dark Matter in particular. For instance, observation of $B_q(D^0) \to \gamma E$, but non-observation of $B_q(D^0) \to E$ transitions directly point to non-self-conjugated nature of scalar DM.

We reported general constraints on the Wilson coefficients of the effective operators describing interactions of DM with quarks (see Tables I-III). Restrictions obtained in our paper are much stricter than constraints from single decay modes. Our results combined with constraints from astrophysical observables (for example [26]), direct detection of Dark Matter and invisible decays

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of heavy hadrons\cite{33} could provide a full set of tools needed to test (or rule out) the models of light Dark Matter.

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