RAPIDITY GAPS IN MINIJETS PRODUCTION
AT TEVATRON ENERGIES

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ABSTRACT

Multiparton interactions modify the high energy hadronic cross section to produce minijets with a rapidity gap in the distribution of secondaries. At Tevatron energy the correction to the single scattering term is large for transverse momenta smaller than 6 GeV.

One of the topics of interest in perturbative QCD is to test the BFKL approach to semihard hadron interactions. The BFKL approach allows one to write an explicit expression for the inclusive cross section to produce minijets as a convolution of the partonic cross section with the effective parton structure functions (gluon plus 4/9 quark and antiquark structure functions) [1]:

\[
\frac{d\sigma_L}{dx_A dx_B d^2k_a d^2k_b} = f_{eff}(x_A, k_a^2) f_{eff}(x_B, k_b^2) \frac{d\hat{\sigma}_L}{d^2k_a d^2k_b}
\]  

where \(x_A, x_B\) are momentum fractions of the interacting partons and \(k_a, k_b\) are the transverse momenta of the two observed minijets. The partonic cross section is written as

\[
\frac{d\hat{\sigma}_L}{d^2k_a d^2k_b} = \left[ \frac{N_c \alpha_s}{k_a^2} \right] f(k_a, k_b, \Delta y) \left[ \frac{N_c \alpha_s}{k_b^2} \right]
\]  

where \(N_c\) is the number of colors, \(\alpha_s\) is the strong coupling constant and \(f(k_a, k_b, \Delta y)\) is the solution to the BFKL equation depending on the rapidity distance \(\Delta y\) of the two observed minijets. The partonic cross section in Eq.(2) exhibits Regge behaviour at large values of \(\Delta y\). Actually

\[
\hat{\sigma}_L \to e^{(\alpha_P - 1)\Delta y}
\]  

and the Regge intercept \(\alpha_P\) is computed in perturbative QCD. In fact the inclusive parton cross section in Eq.(2) corresponds to a parton interaction with a cut BFKL Pomeron exchange. In analogy with soft processes one may therefore consider the possibility to produce the two observed minijets without cutting the BFKL Pomeron, namely by an elastic parton-parton collision represented with an uncut BFKL Pomeron exchange[2].
The singlet contribution to elastic parton process is related to the inelastic one by the unitarity relation. The behaviour at large $\Delta y$ is therefore:

$$\hat{\sigma}_S \to e^{2(\alpha_P - 1)\Delta y}$$

and the corresponding experimental signature is a rather spectacular one. Two minijets separated by a large gap in the production of secondaries. Indeed to compare the rates of the two parton processes, inelastic and elastic, one needs to make the further request that soft interactions between spectator partons in the hadronic event are not going to fill the rapidity gap. The elastic partonic cross section needs therefore to be multiplied by the survival probability factor $\langle S^2 \rangle$, which however is expected to depend smoothly on the rapidity interval $\Delta y$ in such a way that it would provide only an overall rescaling factor to the elastic partonic cross section.

The point which we like to focus on is unitarization. In fact the relation between elastic and inelastic parton cross section is the unitarity relation applied to the parton-parton collision. If one looks rather to the the problem of unitarizing the whole semihard hadronic process one realizes that in the semihard region unitarity induces further sources of minijets other than the cut and the uncut BFKL Pomerons already considered. In fact unitarity introduces in the hadronic interaction also multiple parton collisions whose effect is to change the simple relation between semihard events with and without rapidity gap discussed above.

The argument for multiple parton collisions is the following. One may integrate the inclusive cross section for production of minijets, Eq.(1), with the cutoff $q_{t_{\text{min}}}$ which represents the lower threshold for observing a parton as a minijet in the final state. At large energy in the hadron-hadron c.m. system the integrated cross section may easily be larger with respect to the total inelastic cross section when $q_{t_{\text{min}}}$ is relatively small[4]. On the other hand the partonic cross section is in comparison still rather small. The large value of the integrated inclusive cross section is therefore the consequence of the large flux of partons in the initial state, which may give rise to an average number of partonic collisions larger than one, explaining in this way the large value of the integrated inclusive cross section[5].

Moving $q_{t_{\text{min}}}$ towards smaller values one is therefore entering the region where multiple parton collisions are an important effect. If one continues to decrease the value of $q_{t_{\text{min}}}$ one encounters a further problem. One expects in fact that the value of $\alpha_S$ which one should use will increase at smaller values of $q_{t_{\text{min}}}$. The BFKL expressions for the inelastic and elastic parton cross sections grow as the exponential of $\alpha_S$ and on $2\alpha_S$ respectively, in such a way that the rise of the elastic parton cross section is too fast with respect to the rise of the inelastic one.

To have an indication on the values of $q_{t_{\text{min}}}$ where the problem may occur we have taken the simplest attitude. In analogy to the $s$-channel unitarization of the soft Pomeron exchange we have included in the semihard partonic interaction the exchange of two BFKL Pomerons and we have used the AGK cutting rules[6] to obtain the inelastic contributions to the cross section. The semihard partonic cross section $\hat{\sigma}_H(y)$ is therefore expressed as

$$\hat{\sigma}_H(y) = \hat{\sigma}_S(y) + (\hat{\sigma}_L(y) - 4\hat{\sigma}_S(y)) + 2\hat{\sigma}_S(y)$$

(5)
where the ‘elastic’ partonic cross section is identified with the ‘diffractive’ cut of the double BFKL Pomeron exchange contribution to the forward parton amplitude[7]. The single BFKL Pomeron exchange contributes with $\hat{\sigma}_L(y)$ and the contributions from the double BFKL Pomeron exchange, according with the AGK cutting rules, are: $\hat{\sigma}_S(y)$, the ‘diffractive’ contribution, $-4\hat{\sigma}_S(y)$, the one Pomeron cut, and $+2\hat{\sigma}_S(y)$, the two Pomeron cut. Eq.(5) allows one to define the kinematical region of applicability of the approach. Since the one BFKL cut Pomeron contribution to the cross section must be positive:

$$\left(\hat{\sigma}_L(y) - 4\hat{\sigma}_S(y)\right) > 0 \quad (6)$$

If one uses for $\alpha_s$ the expression of the running coupling constant one may obtain, from the bound in Eq.(6), a relation between $q_t^{\min}$ and the average rapidity interval of two interacting partons, which is easily translated into a relation between $q_t^{\min}$ and the hadron-hadron c.m. energy. When moving towards smaller values of $q_t^{\min}$ one can therefore distinguish three different regimes:

I- The cutoff is sizeable with respect to the typical energy available to the semihard partonic interaction. The corresponding ‘elementary’ parton interaction is small, no unitarization is needed and the semihard cross section is well described by a single partonic collision.

II- At relatively smaller values of the cutoff a single partonic interaction is still well described by the BFKL dynamics. The semihard hadronic cross section is however too large with respect to the total inelastic cross section and unitarity corrections are to be taken into account. The unitarization of the hadronic semihard cross section is achieved by taking into account multiparton interactions, namely different pairs of partons interacting independently with BFKL Pomeron exchange. Typically the different partonic interactions are localized at different points in the transverse plane, in the region of overlap of the matter distribution of the two hadrons.

III- With even smaller values of the cutoff one may still be in the regime where perturbative QCD can be used, since the value of $\alpha_s(q_t^{\min})$ is small, but the elastic parton cross section is too large with respect to the inelastic one and the ‘elementary’ parton process is not well described any more by the single BFKL Pomeron exchange. One may obtain an indication on the limits between regions II and III by testing whether the bound in Eq.(6) is satisfied.

We focus our attention on region II, where one expects to find non trivial effects from unitarization while the perturbative part of the semihard interaction is described within the BFKL approach. Although the perturbative part of the interaction is explicitly known also in region II, one is not yet in the position to write explicitly the semihard cross section. The reason is that the non perturbative input in this case is represented by the multiparton distributions[8], which are a piece of information on the hadron structure independent of the hadron structure functions of large $p_t$ physics. In fact multiparton distributions are dimensional quantities and have an explicit dependence on the multiparton correlations. The usually considered parton structure functions on the contrary can carry information only on the average number of partons. At present the only information on the multiparton distribution is an indication on the scale factor which gives the dimensionality to the multiparton distributions[9]. We take therefore the simplest attitude namely we consider
the simplest case where multiparton correlations are neglected and we express the multi-
parton distributions as Poissonians with a scale factor consistent with the experimental
indication[9]. If one in addition neglects the possibility of having semihard parton rescat-
terings in the semihard interaction, one obtains for the semihard hadronic cross section
\( \sigma_H \) the simple eikonal form:

\[
\sigma_H = \int d^2 \beta \left[ 1 - \exp(-\Phi(\beta)) \right] = \int d^2 \beta \sum_{\nu=1}^{\infty} \frac{[\Phi(\beta)]^\nu}{\nu!} \exp(-\Phi(\beta)) \tag{7}
\]

where

\[
\Phi(\beta) \equiv \Phi_S(\beta) + \Phi_P(\beta) \equiv \int_{y_m}^{y_M} dy \int_{y}^{y_M} dy' \left( \phi_S(\beta; y, y') + \phi_P(\beta; y, y') \right) \tag{8}
\]

with

\[
\phi_S, P(\beta; y, y') \equiv \int d^2 b D_A(b, x(y)) \hat{\sigma}_S, P(y - y) D_B(b - \beta, x'(y'))
\]

and \( y_M, y_m \) are the maximum and minimum rapidity values allowed by kinematics. \( D(b, x(y)) \) is the average number of partons with transverse coordinate \( b \) and fractional
momentum \( x \), which may be expressed as a function of the rapidity \( y \) of the produced
minijet. The indices \( A \) and \( B \) refer to the two interacting hadrons.

One is interested in the component of \( \sigma_H \) which represents two minijets at rapidities
\( \bar{y} \) and \( \bar{y}' \), in the central rapidity region, with associated gap \( \Delta y = \bar{y}' - \bar{y} \) in the rapidity
distribution of secondary produced gluons. To that purpose one needs to exclude in Eq.(7)
both the elastic terms, with final state minijets in the gap, and all the inelastic partonic
interactions, generated with elementary probability \( \phi_P \). The cross section to observe two
minijets at rapidities \( \bar{y} \) and \( \bar{y}' \), with the gap \( \Delta y = \bar{y}' - \bar{y} \) in the rapidity distribution of
secondaries, is therefore expressed as

\[
\frac{d\sigma_H(\Delta y)}{dyd\bar{y}'} = \int d^2 \beta \left[ \sum_{\nu=2}^{\infty} \nu(\nu - 1) \int_{y_m}^{\bar{y}} dy \phi_S(\beta; y, \bar{y}') \int_{\bar{y}'}^{y_M} dy' \phi_S(\beta; \bar{y}, y') \right. \\
\times \left. \frac{[\Phi_S(\beta, \Delta y)]^{\nu - 2}}{\nu!} \right] e^{-\Phi(\beta)} \tag{11}
\]

After summing on \( \nu \) one obtains

\[
\frac{d\sigma_H(\Delta y)}{dyd\bar{y}'} = \int d^2 \beta \left[ \phi_S(\beta; \bar{y}, \bar{y}') + \int_{y_m}^{\bar{y}} dy \phi_S(\beta; y, \bar{y}') \int_{\bar{y}'}^{y_M} dy' \phi_S(\beta; \bar{y}, y') \right] \\
\times \exp \left\{ \Phi_S(\beta, \Delta y) - \Phi_S(\beta) - \Phi_P(\beta) \right\} \tag{12}
\]
The two addenda in Eq.(12) are the single and double ‘elastic’ scattering contributions. In the single scattering term both observed minijets are produced in the same elementary partonic interaction, in the double scattering term the two minijets are generated in different partonic collisions. Both terms are multiplied by the absorption factor \( \exp\left\{ -\left( \Phi_S(\beta) - \Phi_S(\beta; \Delta y) \right) \right\} \) that removes the ‘elastic’ parton interactions which would fill the gap, actually those which produce minijets with rapidities \( y \) and \( y' \) such that \( y \leq y' \) or \( y' \leq y \). At a fixed value of \( \beta \) the cross section is multiplied by \( \exp\left\{ -\Phi_P(\beta) \right\} \) which is the probability of not having any inelastic partonic interaction in the process. One may recognize in Eq.(12) the semihard contribution to the survival probability factor \( \langle S^2(\beta) \rangle \) of ref.[3]. Actually \( \exp\left\{ -\Phi_S(\beta) - \Phi_P(\beta) \right\} \) is the probability factor of not having any semihard activity in the spectator partons.

To have a quantitative indication on the boundaries of the kinematical regions, we have worked out a numerical example. The input is the average number of partons \( D(b, x) \) and the ‘elementary’ partonic cross sections \( \hat{\sigma}_{L,S} \). We have factorized \( D(b, x) \) as \( f_{\text{eff}}(x) \times F(b) \), where \( f_{\text{eff}}(x) \) is the effective structure function and \( F(b) \) is a gaussian, normalized to one and such as to give for the double scattering term a scale factor consistent with the experimental indication of \( \sigma_{\text{eff}} \)[9]. In our numerical example we have chosen \( \sigma_{\text{eff}} = 20\,\text{mb} \) and as a scale factor for the structure functions we have taken \( q_{\text{min}}^2/2 \).

\( \alpha_S \) is a free parameter in the BFKL approach, it is not a running coupling constant, one expects however that the value of \( \alpha_S \) which one should use is not too different from the value of the running \( \alpha_S \) at the scale of the typical momentum transferred in the process. We have chosen as a value of \( \alpha_S \) the value of the running coupling computed with \( q_{\text{min}}^2/2 \) as a scale factor. The values of the semihard cross section \( \sigma_H \), as expressed in Eq.(7), which we obtain with this input are consistent with the experimental values published by UA1[10].

An estimate of the boundaries between the regions I, II and III discussed above is shown in fig.1. The boundary between region I and II is obtained by requiring that the unitarized expression for \( \sigma_H \) is 20% smaller with respect to the single scattering term. The boundary between regions II and III is obtained by saturating on the average, namely after integrating with the structure functions, the bound in Eq.(6). The effect on the cross section to produce minijets with rapidity gap is shown in fig.2, where we plot the cross section in Eq.(12) divided by the survival probability factor \( \exp\left\{ -\Phi_S(\beta) - \Phi_P(\beta) \right\} \). The process is \( p\bar{p} \) at \( \sqrt{s} = 1.8\,\text{TeV} \) and \( q_{\text{min}}^2 = 5\,\text{GeV} \). The continuous curve is obtained by using the value \( \sigma_{\text{eff}} = 20\,\text{mb} \) as a input and the dashed curve is obtained with the value \( \sigma_{\text{eff}} = 12\,\text{mb} \). The dotted curve is the result of the single scattering term alone.

As it is shown in fig.2 the effect of unitarization on the behaviour of the cross section is large. In the actual case the main modification to the dependence on \( \Delta y \) is due to the presence of multiple elastic parton scatterings whose effect on the cross section is twofold. A different dependence on \( \Delta y \), with respect to the single scattering term, is induced by the contribution of the process where the two observed minijets originate in different elastic partonic interactions, the second term in Eq.(12). A second source for the different dependence on \( \Delta y \) is the correction induced by multiple elastic scatterings to the survival probability factor. In fact not all underlying hadron activity needs to be excluded. Elastic parton scatterings which produce minijets outside the gap are allowed and the corresponding contribution to the cross section depends on \( \Delta y \). The effect of the inelastic
semihard partonic interactions is, on the contrary, factorized at fixed impact parameter $\beta$ and independent on $\Delta y$. The main effect of the inelastic partonic processes is to contribute to the survival probability $\langle S^2 \rangle$ of ref.[3] rather than modifying the dependence on $\Delta y$.

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Figure captions

Fig.1- The three different kinematical regions which characterize semihard hadronic interactions. I: only the single partonic collision, described by a single BFKL Pomeron exchange, is relevant; II: multiparton collisions are to be taken into account, each partonic interaction is however well described by single BFKL Pomeron exchange; III: the single BFKL Pomeron exchange is not any more an adequate description of the single parton interaction.

Fig.2- Cross section for production of minijets with rapidity gap as a function of the gap $\Delta y$. The process is $p\bar{p}$ at $\sqrt{s} = 1.8 TeV$ and $q_t^{min} = 5 GeV$. The dotted curve is the single scattering term contribution without unitarity corrections; the continuous and dashed curves include all disconnected multiple parton collisions. The continuous curve is obtained with $\sigma_{eff} = 20 mb$ and the dashed curve with $\sigma_{eff} = 12 mb$. 
Fig. 1
Fig. 2