TWO-LOOP CORRECTIONS TO THE HIGGS
BOSON MASSES IN THE MSSM

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We present a computation of the $O(\alpha t + \alpha s + \alpha^2 t)$ two–loop corrections to the MSSM Higgs masses. An appropriate use of the effective potential approach allows us to obtain simple analytical formulae, valid for arbitrary values of $m_A$ and of the mass parameters in the stop sector. In a large region of the parameter space the $O(\alpha^2 t)$ corrections are comparable to the $O(\alpha t + \alpha s)$ ones, increasing the prediction for $m_h$ by several GeV.

Talk given at the XXXVII Rencontres de Moriond,
Electroweak Interactions and Unified Theories,
Les Arcs, France, March 9–16 2002.

In the Minimal Supersymmetric Standard Model (MSSM), the Higgs sector contains two $SU(2)_L$ doublets, and the masses and the couplings of the Higgs bosons can be expressed, at the classical level, in terms of known Standard Model quantities and only two additional parameters: for example, the mass of the neutral CP–odd state, $m_A$, and the ratio of the two Higgs vacuum expectation values, $\tan \beta$. In particular, the mass $m_h$ of the lightest Higgs boson is predicted to be smaller than the mass of the $Z$ boson. However, radiative corrections modify considerably the predictions for the Higgs masses, allowing the light Higgs mass to escape the tree–level bound, and bring along additional dependences on the remaining MSSM parameters. Direct searches at LEP have already ruled out a considerable fraction of the MSSM parameter space, and the forthcoming high–energy experiments at the Tevatron and the LHC will either discover (at least) one light Higgs boson or rule out the MSSM as a viable theory for physics at the weak scale. Thus, a precise calculation of the radiative corrections to the Higgs boson masses, valid for arbitrary values of all the relevant parameters, is needed to compare reliably the MSSM predictions with the (present and future) experimental data.

In this talk, a computation of both $O(\alpha t + \alpha s)$ and $O(\alpha^2 t)$ two–loop corrections to the MSSM Higgs boson masses, based on the effective potential approach, is presented. The results of refs. extend the existing two–loop calculations to the case of arbitrary MSSM parameters, and are given as a set of simple analytical formulae, suitable for being promptly included in the experimental analyses.

We start our discussion of the corrections to the Higgs masses by recalling that in the effective potential approach the CP–odd and CP–even mass matrices are identified with the second derivatives of the effective potential, $V_{\text{eff}} = V_0 + \Delta V$, evaluated at its minimum:

\[
\left( M^2_{P\eff} \right)_{ij} \equiv \left. \frac{\partial^2 V_{\text{eff}}}{\partial P_i \partial P_j} \right|_{\min}, \quad \left( M^2_{S\eff} \right)_{ij} \equiv \left. \frac{\partial^2 V_{\text{eff}}}{\partial S_i \partial S_j} \right|_{\min}, \quad (i, j = 1, 2),
\]

where we have decomposed the Higgs fields into their VEVs plus their CP–even and CP–odd fluctuations as $H^0_i \equiv (v_i + S_i + i P_i)/\sqrt{2}$. Using the explicit expression of the tree–level potential
we express the field–dependent masses and interaction vertices that contribute to \( \Delta \) renormalized parameters: mass matrix, valid when the one–loop part of the corrections is written in terms of one–loop of the effective potential, we derive an expression for the two –loop corrections to the Higgs term of the stop mass matrix. contribution to the effective potential, can be chosen as the s top mixing angle, denoted as quark and of the stop squarks. The remaining two quantities, appearing only in the two-loop already in the one-loop contribution to the effective potenti al, are the squared masses of the top relevant for our calculation, in terms of five field–dependen t quantities. The first three, appearing respect to the CP–even and CP–odd fields, evaluated at the min imum of

\[
V \left( \frac{\phi}{v} \right) = \frac{1}{2} \left( \partial_i \phi \right) \left( \partial_j \phi^* \right) - \frac{v^2}{2} \left( H \right)^2 - \frac{v^2}{2} \left( H^* \right)^2 - \frac{v^2}{2} \left( H \right)^2 - \lambda \left( H^2 \right)^2 - \lambda \left( H^* \right)^2 - \lambda \left( H \right)^2 \left( H^* \right)^2 ,
\]

where \( v = \sqrt{\frac{\lambda}{2}} \) is the VEV of the Higgs field. The functions \( F_i \) are the wave function renormalization matrix for the Higgs fields, and

\[
F_i = \frac{\partial^2 V}{\partial \phi_i \partial \phi^*_i} \bigg|_{\min} - (-1)^{i+j} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \bigg|_{\min} ,
\]

In Eq. (3), \( M_0^2 \) is the mass of the CP–odd state in the effective potential approximation, i.e. the non–vanishing eigenvalue of \( (M_F^2)_{ij}^{\text{eff}} \). Similarly, \( \overline{m}^2 = (g^2 + g'^2) v^2 / 4 \) is just the DR mass for the \( Z \) boson. Finally, \( s_\beta \equiv \sin \beta \) and \( c_\beta \equiv \cos \beta \), where \( \tan \beta = v_2 / v_1 \).

According to Eq. (4), \( (\Delta M_0^2)_{ij}^{\text{eff}} \) can be computed by taking the derivatives of \( \Delta V \) with respect to the CP–even and CP–odd fields, evaluated at the minimum of \( V_{\text{eff}} \). In this computation we express the field–dependent masses and interaction vertices that contribute to \( \Delta V \), and are relevant for our calculation, in terms of five field–dependent quantities. The first three, appearing already in the one-loop contribution to the effective potential, are the squared masses of the top quark and of the stop squarks. The remaining two quantities, appearing only in the two-loop contribution to the effective potential, can be chosen as the stop mixing angle, denoted as \( \theta_f \), and \( \cos (\varphi - \tilde{\varphi}) \), where \( \varphi \) is the phase in the complex top mass, and \( \tilde{\varphi} \) is the phase in the off–diagonal term of the stop mass matrix.

After a straightforward although lengthy application of the chain rule for the derivatives of the effective potential, we derive an expression for the two–loop corrections to the Higgs mass matrix, valid when the one–loop part of the corrections is written in terms of one–loop

\[
(\Delta M_0^2)_{ij}^{\text{eff}} = \sqrt{Z} \left( \Delta M_{0 \text{eff}}^2 \right)_{ij} \sqrt{Z} ,
\]

where \( Z \) is a function renormalization matrix for the Higgs fields, and

\[
(\Delta M_{0 \text{eff}}^2)_{11} = \frac{1}{2} h_t^2 \mu^2 s_{2\theta}^2 (F_3 + 2 \Delta_{\mu} F_3) ,
\]

\[
(\Delta M_{0 \text{eff}}^2)_{12} = h_t m_t s_{2\theta} (F_2 + \Delta_{\mu} F_2) + \frac{1}{2} h_t^2 A_t \mu s_{2\theta}^2 (F_3 + \Delta_A F_3 + \Delta_{\mu} F_3) ,
\]

\[
(\Delta M_{0 \text{eff}}^2)_{22} = 2 h_t^2 m_t^2 F_1 + 2 h_t^2 A_t m_t s_{2\theta} (F_2 + \Delta_A F_2) + \frac{1}{2} h_t^2 A_t^2 s_{2\theta}^2 (F_3 + 2 \Delta_A F_3) .
\]

In the above equations the functions \( (F_1, F_2, F_3) \) are written as \( F_i = \overline{F}_i + \Delta \overline{F}_i \), where \( \overline{F}_i \) are combinations of the derivatives of the pure two–loop effective potential whose explicit definitions are given in ref. [1]. The terms \( \Delta \overline{F}_i \) include contributions coming from the renormalization of the top and stop masses appearing in the one–loop part of the \( F_i \), as well as from the renormalization of the common factors multiplying \( F_i \), i.e. \( (h_t^2, m_t, s_{2\theta}) \). On the other hand, in each entry of \( (\Delta M_{0 \text{eff}}^2)_{ij} \) the functions \( F_i \) are multiplied by different combinations of \( \mu \) and \( A_t \). These two parameters have different \( O(\alpha_t) \) and \( O(\alpha_s) \) renormalizations that cannot be absorbed in the \( F_i \), and are then separately taken into account by the terms \( \Delta_{\mu} F_i \) and \( \Delta_A F_i \).

We have derived explicit analytical formulae for the \( O(\alpha_t \alpha_s) \) and \( O(\alpha_t^2) \) parts of the functions \( F_i \), obtained under the assumption that the \( O(\alpha_t) \) parts are expressed in terms of DR quantities.
In ref. we presented the complete formulae for the $O(\alpha_t \alpha_s)$ corrections, valid for arbitrary choices of all the input parameters. On the other hand, the general formulae for the $O(\alpha_t^2)$ corrections are quite long, thus we made them available upon request in the form of a computer code. In ref., we presented explicit analytical formulae valid for $m_Q = m_U = M_S$, in which $O(m_t/M_S)$ corrections are neglected, but $m_A$ is left arbitrary.

In the phenomenological analyses of the MSSM Higgs sector, it may be useful to express or results in terms of input parameters given in a different renormalization scheme, which will be indicated generically as $R$. To obtain the $O(\alpha_t \alpha_s + \alpha_t^2)$ correction in the $R$ scheme one has just to shift the parameters appearing in the one–loop term, i.e. $x^{\text{DR}} = x^{\text{R}} + \delta x$, where $x$ is a generic parameter. This induces additional two–loop contributions that can be absorbed in a redefinition of the functions $F_i$ and $\Delta F_i$ entering Eqs. (8)–(10). Explicit formulae for the transition from the $\overline{\text{DR}}$ scheme to a generic $R$ scheme are given in refs. (3).

The parameters entering the one–loop part of the corrections can be identified as $(m_t, m_{t_1}, m_{t_2}, \theta, \tan \beta)$. Some of them are related by the equality

$$\sin 2\theta = \frac{2m_t (A_t + \mu \cot \beta)}{m_{t_1}^2 - m_{t_2}^2}.$$  

(9)

We choose to express our result identifying the stop masses $m_{t_1}$ and the top mass $m_t$ with the corresponding pole masses. For the electroweak symmetry–breaking parameter $\mu$, we use the value obtained in terms of the precisely known muon decay constant $G_\mu$. These choices specify $\delta m_{t_1}^2$, $\delta m_t$ and $\delta \mu$ in terms of the finite parts of stop, top and W–boson self–energies respectively. For the stop mixing angle, several OS prescriptions are present in the literature. We find suitable for our $O(\alpha_t \alpha_s + \alpha_t^2)$ calculation the following ‘symmetrical’ definition

$$\delta \theta = \frac{1}{2} \frac{\Pi_{12}(m_{t_1}^2) + \Pi_{12}(m_{t_2}^2)}{m_{t_1}^2 - m_{t_2}^2},$$  

(10)

where $\Pi_{12}(p^2)$ stands for the finite part of the off–diagonal stop self–energy.

It is not clear what meaning should be assigned to an OS definition of the remaining quantities, i.e. $(A_t, \mu, \tan \beta)$. For instance, they could be related to specific physical amplitudes. However, given our present ignorance of any supersymmetric effect, such a choice does not seem particularly useful. Then, we find simpler to assign the quantities $\mu$ and $\tan \beta$ in the $\overline{\text{DR}}$ scheme, at a reference scale $Q_0 = 175$ GeV, that we choose near the present central value of the top quark mass. In this framework, we are going to take $\delta \mu = \delta \beta = 0$, and we treat $A_t$ as a derived quantity, obtained through Eq. (11).

Our effective potential calculation is equivalent to the evaluation of the Higgs self–energies in the limit of vanishing external momentum. A diagrammatic computation of the two–loop $O(\alpha_t \alpha_s)$ contributions to the Higgs boson self–energies at zero external momentum has been performed in ref. (4). Analytical formulae, valid in the simplified case of degenerate soft stop masses and zero mixing (with $\mu = A_t = 0$), have been presented in the first paper of ref. (5). For arbitrary values of the top and stop parameters, however, the complete analytical result of ref. (5) is far too long to be explicitly presented, and is only available as a computer code. We have checked that, in the case of zero mixing and degenerate stop masses, our results coincide with those of ref. (5). Moreover, after taking into account the difference in the definitions of the OS renormalized angle $\theta$, we find perfect agreement with the numerical results of ref. (5) for arbitrary values of all the input parameters.

A calculation of both $O(\alpha_t \alpha_s)$ and $O(\alpha_t^2)$ two–loop corrections to the lightest Higgs boson mass $m_h$, based on the formalism of the effective potential, has been presented in refs. (6). In these papers, however, the dependence of the stop masses and mixing angles on the fields $P_i$ (the CP–odd components of the neutral Higgs fields) is not taken into consideration. Therefore,
the $O(\alpha_t\alpha_s + \alpha_s^2)$ corrections to the parameter $m_A$ are not evaluated. If one wants to relate the input parameters to measurable quantities, the computation is applicable only in the limit $m_A \gg m_Z$, in which $m_h$ is nearly independent of $m_A$. Moreover, the results of refs. 6, 7 and 20 for $m_h$ are available in numerical form, and simple analytical formulae are provided only in the limit of universal soft stop masses (degenerate with $m_A$ and $m_{\tilde{g}}$) much larger than the top quark mass. We have verified that, in such limit, our analytical results for $m_h$ agree with those of refs. 6, 7, 20.

In the analysis of ref. 20 and some ‘leading logarithmic’ $O(\alpha_s^2)$ corrections, obtained by renormalization group methods, have indeed been added to $(M_S^2)_{22}$. However, when comparing our complete $O(\alpha_t^2)$ result with the renormalization group result, it turns out that, for some choices of the SUSY parameters, the leading logarithmic corrections amount only to a fraction of the full ones. Fig. 1 shows $m_h$ as a function of $m_A$, for $\tan \beta = 2$ or 20 and $X_t^{\text{OS}} = 0$ or 2 TeV. The other input parameters are chosen as $m_t = 175$ GeV, $m_Q^{\text{OS}} = m_U^{\text{OS}} = 1$ TeV, $\mu = 200$ GeV, $m_{\tilde{g}} = 800$ GeV. The cases with $X_t^{\text{OS}} = 0$ and $X_t^{\text{OS}} = 2$ TeV correspond, respectively, to the so-called ‘no–mixing’ and ‘$m_{\tilde{h}}–\max$’ benchmark scenarios considered in the experimental analyses. The curves in Fig. 1 are the two–loop corrected Higgs mass at $O(\alpha_t\alpha_s)$ (short–dashed line), at $O(\alpha_t\alpha_s + \alpha_s^2)$ (dash–dotted line), and at $O(\alpha_t\alpha_s + \alpha_s^2)$ including our full computation (solid line). The one–loop result for $m_h$ is also shown for comparison (long–dashed line). It can be seen from the figure that the $O(\alpha_t\alpha_s)$ corrections are in general large, reducing the one–loop result for $m_h$ by 10–20 GeV. On the other hand, the $O(\alpha_s^2)$ corrections tend to increase $m_h$: for small stop mixing they are generally small (less than 2–3 GeV), whereas for large stop mixing they can reach 7–8 GeV, i.e. a non–negligible fraction of the $O(\alpha_t\alpha_s)$ ones. Moreover, it appears that the ‘non–leading’ corrections included in our full result are always comparable in size with the ‘leading logarithmic’ ones, and are even more significant than the latter in the case of large stop mixing. For $m_A \gg m_{\tilde{g}}$, we find good numerical agreement with the value of $m_h$ obtained from the approximate formulae of ref. 20.

Figure 1: The mass $m_h$ as a function of $m_A$, for $\tan \beta = 2$ or 20 and $X_t^{\text{OS}} = 0$ or 2 TeV. The other parameters are $m_Q^{\text{OS}} = m_U^{\text{OS}} = 1$ TeV, $\mu = 200$ GeV, $m_{\tilde{g}} = 800$ GeV. The meaning of the curves is explained in the text.

\( a \)To facilitate the comparison with the existing analyses of the two–loop corrected Higgs masses, we make use of the unphysical parameters \( m_Q^{\text{OS}}, m_U^{\text{OS}}, X_t^{\text{OS}} \) that can be derived by rotating the diagonal matrix of the OS stop masses by an angle $\theta_t$. 

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In conclusion, refs. 1, 2 contain the most advanced calculation (so far) of the two–loop corrections to the MSSM neutral Higgs boson masses. Our work should lead to a more accurate interpretation of the experimental searches for the neutral MSSM Higgs bosons at LEP, the Tevatron, the LHC and other possible future colliders. The importance of the new $\mathcal{O}(\alpha^2 t)$ effects we have computed will increase further when the top quark mass is measured more precisely. Then, we can hope for the next step, the discovery of supersymmetric particles and supersymmetric Higgs bosons.

Acknowledgments

The author thanks the organizers of the XXXVII Rencontres de Moriond for the pleasant atmosphere in which the talk was given. This work was supported in part by the European Union under the contracts HPRN-CT-2000-00149 (Collider Physics) and HPRN-CT-2000-00148 (Across the Energy Frontier).

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