Imaging of extended objects by a negative refractive index slab

Juan Luis Garcia-Pomar$^1$ and Manuel Nieto-Vesperinas

Instituto de Ciencia de Materiales de Madrid, Consejo Superior de Investigaciones Científicas, Campus de Cantoblanco, Madrid 28049, Spain
E-mail: jlgarcia@icmm.csic.es and mnieto@icmm.csic.es

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Abstract. Using a finite element method, we numerically study the imaging of an extended object by slabs of media with negative refractive index (within an effective medium theory). We analyse the consequences of possible deviations of the refractive index of the slab from the archetypal value of $n = -1$. These variations are obtained by introducing losses in the material and also by changing the real part of $n$. In this way, we show how slight changes in the refractive index from $n = -1$ affect the resolution of the image of the extended object.

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1. Introduction

Left-handed materials (LHMs) (i.e. media with dielectric permittivity $\varepsilon$ and magnetic permeability $\mu$ being both negative) were analysed hypothetically by Veselago [1]. These media

$^1$ Author to whom any correspondence should be addressed.
would exhibit negative refractive index $n$, and this fact should produce the focusing of a source by a LHM slab of parallel faces. After the practical realization of these structures, reported by Smith and Kroll [2] and in subsequent works [3]–[5] in addition to Veselago’s predictions, it was claimed [6] that a LHM slab having $\mu = -1$ and $\varepsilon = -1$ should produce superresolution in the image of a point source, amid much debate and work on the accuracy of the statement [7]–[9]. However, all these predictions have not gone unchallenged and it has been claimed that losses will severely limit or even destroy these effects [7]–[10]. In this paper, we show by means of a numerical study by a finite element method, the lack of superresolution when we have either slight losses or deviations in the real part of the refractive index with respect to $n = -1$ in the material. In this respect, we also show the influence of the real part of $n$ in the ability of this system to attain superresolution.

2. Extended object and propagation scheme

All work done so far has been concentrated in the focusing characteristics of a slab for one or two point sources; however, a real lens should be able to provide images of extended objects with isoplanatic characteristics [11], which will be addressed in this paper.

The extended object is a TE-polarized time-harmonic electric field composed by the Fourier superposition of harmonics

$$E_z(x) = \sum_{p=1}^{15} \left[ \sin \left( \frac{p\pi x}{\ell} \right) + \cos \left( \frac{p\pi x}{\ell} \right) \right],$$

where $x$ is the horizontal coordinate and $\ell$ represents the width of the slab. Figure 1 shows the electric field modulus (the values of the electric field in contiguous peaks are in phase opposition, which is the best situation to resolve them [12]). Table 1 shows the distance between peaks at different wavelengths used. The scheme of propagation is shown in figure 2.

The extended object is at distance $z_0 = 10$ cm from the slab with a thickness $d = 2z_0$ and a width $\ell = 70$ cm. Its refractive index is $n = n_1 + in_2$ ($n_1 < 0$). We take the complex $\varepsilon = \mu = n$ to get a perfect matched surface impedance at normal incidence. $z_1$ represents the distance from the image plane to the exit face of the slab. This distance is fixed by both the real and imaginary parts of $n$ on considering the law of refraction for the phase vector $\mathbf{k}_i$ in

| Peeks (cm) | 1–2 | 2–3 | 3–4 | 4–5 | 5–6 | 6–7 | 7–8 | 8–9 | 9–10 | 10–11 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|------|------|
| Distance $\Delta x$ | 4.4 | 4.8 | 4.3 | 5.1 | 6.7 | 6.1 | 4.3 | 4.7 | 4.4 | 4.8 |
| $\Delta x(\lambda = 3)$ | 1.47$\lambda$ | 1.60$\lambda$ | 1.43$\lambda$ | 1.7$\lambda$ | 2.23$\lambda$ | 2.03$\lambda$ | 1.43$\lambda$ | 1.57$\lambda$ | 1.47$\lambda$ | 1.60$\lambda$ |
| $\Delta x(\lambda = 6)$ | 0.73$\lambda$ | 0.80$\lambda$ | 0.72$\lambda$ | 0.85$\lambda$ | 1.12$\lambda$ | 1.02$\lambda$ | 0.72$\lambda$ | 0.78$\lambda$ | 0.73$\lambda$ | 0.80$\lambda$ |
| $\Delta x(\lambda = 9)$ | 0.49$\lambda$ | 0.53$\lambda$ | 0.48$\lambda$ | 0.57$\lambda$ | 0.74$\lambda$ | 0.68$\lambda$ | 0.48$\lambda$ | 0.52$\lambda$ | 0.49$\lambda$ | 0.53$\lambda$ |
| $\Delta x(\lambda = 12)$ | 0.37$\lambda$ | 0.40$\lambda$ | 0.36$\lambda$ | 0.42$\lambda$ | 0.56$\lambda$ | 0.51$\lambda$ | 0.36$\lambda$ | 0.39$\lambda$ | 0.37$\lambda$ | 0.40$\lambda$ |
| $\Delta x(\lambda = 15)$ | 0.29$\lambda$ | 0.32$\lambda$ | 0.29$\lambda$ | 0.34$\lambda$ | 0.45$\lambda$ | 0.41$\lambda$ | 0.29$\lambda$ | 0.31$\lambda$ | 0.29$\lambda$ | 0.32$\lambda$ |
| $\Delta x(\lambda = 18)$ | 0.24$\lambda$ | 0.26$\lambda$ | 0.24$\lambda$ | 0.28$\lambda$ | 0.37$\lambda$ | 0.34$\lambda$ | 0.24$\lambda$ | 0.26$\lambda$ | 0.24$\lambda$ | 0.26$\lambda$ |
Figure 1. Electric field modulus of the extended object represent the distance between peaks (cm).

Figure 2. Propagation scheme. The extended object is in the left boundary at distance $z_0$ from the slab. All boundaries are perfectly matched boundaries (PML).
Figure 3. Modulus of electric field of image of the extended object of figure 1 by varying the real part $n_1$ of the refractive index in the cases: (a) $\lambda = 3$ cm, (b) $\lambda = 6$ cm, (c) $\lambda = 9$ cm, (d) $\lambda = 12$ cm, (e) $\lambda = 15$ cm and (f) $\lambda = 18$ cm.
Figure 4. Theoretical calculation of $s_c$ versus imaginary part of the refractive index $n_2$ from 0.025 to 0.4 for different wavelengths.

the generally lossy slab [11]

$$\sin \theta'_t = \frac{k_i}{k_t} \sin \theta_i,$$

(2)

where $\theta_i$ and $\theta'_t$ are the incident and transmitted angle respectively in the first interface of the LHM slab, and $k_i$ being [11]

$$k_i = k_i \left\{ \frac{1}{2} \left[ \sqrt{(n_1^2 - n_2^2 - \sin^2 \theta_i)^2 + (2n_2^2)^2 + n_1^2 - n_2^2 + \sin^2 \theta_i} \right] \right\}^{1/2}.$$  

(3)

3. Results

3.1. Influence of the variation of $n_1$

Firstly, we slightly vary the real part of the refractive index of the left-handed slab from its archetypal value $n_1 = -1$ assuming its imaginary part $n_2$ to be null.

The images obtained are represented in figure 3. We can see the loss of superresolution when the real part of $n$ is moved from $n = -1$. We observe better resolution for the cases in which the refractive index is $n_1 < -1$ than in those situations with $n_1 > -1$, including a possible case of superresolution for $\lambda = 9$ cm for both $n_1 = -1.1$ and $-1.2$. 

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Figure 5. Electric modulus of the image of the extended object of figure 1 by varying the imaginary part $n_2$ of the refractive index in the cases of (a) $\lambda = 3\,\text{cm}$, (b) $\lambda = 6\,\text{cm}$, (c) $\lambda = 9\,\text{cm}$, (d) $\lambda = 12\,\text{cm}$, (e) $\lambda = 15\,\text{cm}$ and (f) $\lambda = 18\,\text{cm}$.

Such better resolution in cases with $n_1 < -1$ is due to the larger effective aperture and to the smaller distance of the position of the focus image $z_1$ from the exit interface of the LHM slab.
3.2. Influence of the variation of \( n_2 \)

Next, we fix the value of \( n_1 \) to \(-1\) and vary the imaginary part \( n_2 \). Thus, we introduce losses in the LHM slab. Under these circumstances, in [13] a resolution

\[
\Delta y = \frac{\lambda}{2s_c},
\]

was predicted, where \( s_c \) is

\[
s_c = \left\{ \left[ \frac{\lambda}{4\pi z_0} \ln \left( \frac{2}{n_2} \right) \right]^2 + 1 \right\}^{1/2}.
\]

Figure 4 shows for different wavelengths the better resolution obtained from equations (4) and (5), as \( z_0/\lambda \) becomes larger.

Consequently, we carry out the following numerical simulations of the imaging process whose results are shown in figure 5.

We observe that for \( \lambda = 3 \text{ cm} \), the image manifests a loss of resolution of the peaks 2 and 4 for \( n_2 = 0.050 \) and of the peak 7 when \( n_2 = 0.250 \). For \( \lambda = 6 \text{ cm} \), the image looses the resolution of the peaks 2 and 4 for \( n_2 = 0.050 \), the peak 7 when \( n_2 = 0.125 \) and of the peaks 9 and 11 for \( n_2 = 0.200 \). In the case \( \lambda = 9 \text{ cm} \), the peaks 2, 4, 7 and 9 involving superresolution are lost with \( n_2 = 0.100 \), this being in agreement with equations (4) and (5), since in this case \( s_c = 1.0228 \) and \( \Delta y = 0.49\lambda \). For the cases with \( \lambda > z_0 \), most of the peaks involving superresolution are lost even with very small losses in the LHM slab.

4. Conclusions

We have proposed a model for the formation of the image of an extended object if a LHM slab is acting as a real lens, since then it should work in a regime of isoplanatism. We have analysed the influence of the variation of the real part of \( n \) from \(-1\), showing how a resolution of the extended object worsens with deviations from \( n = -1 \). Also we have shown in the cases with \( n_1 < -1 \), a better resolution than in those cases with \( n_1 > -1 \).

On the other hand, since the existence of losses in metallic metamaterials (LHM) is unavoidable, we must take into account the imaginary part of \( n \) in the image-formation process. We have studied these effects and shown how this absorption has severe effects on the superresolution of an extended object by means of an LHM slab. In this way, we have shown agreement of the numerical simulations with a previously proposed theory.

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References

[1] Veselago V G 1968 Sov. Phys.—Usp. 10 509
[2] Smith D R and Kroll N 2000 Phys. Rev. Lett. 84 4184
[3] Shelby R A, Smith D R, Nemat-Nasser S C and Schultz S 2001 Appl. Phys. Lett. 78 489
[4] Parazzoli C G, Greegor R B, Li K, Koltenbah B E C and Tanielian M 2003 Phys. Rev. Lett. 90 107401
[5] Houck A A, Brock J B and Chuang I L 2003 Phys. Rev. Lett. 90 137401
[6] Pendry J B 2000 Phys. Rev. Lett. 85 3966
[7] Garcia N and Nieto-Vesperinas M 2002 Phys. Rev. Lett. 88 207403
[8] Kuo C H and Ye Z 2004 Phys. Rev. E 70 026608
[9] Pokrovsky A L and Efros A L 2002 Phys. Rev. Lett. 89 093901
[10] Valanju P M, Walser R M and Valanju A P 2002 Phys. Rev. Lett. 88 187401
[11] Garcia-Pomar J L and Nieto-Vesperinas M 2004 Opt. Express 12 2081
[12] Goodman J W 1968 Introduction to Fourier Optics (New York: McGraw-Hill)
[13] Nieto-Vesperinas M 2004 J. Opt. Soc. Am. A 21 491