Nonperturbative Field Correlators in the Abelian Higgs Model

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Abstract

By making use of the duality transformation, gauge field correlators of the Abelian Higgs Model are studied in the London limit. The obtained results are in a good agreement with the dual Meissner scenario of confinement and with the Stochastic Model of QCD vacuum. The nontrivial contribution to the quartic correlator arising due to accounting for the finiteness of the coupling constant is discussed.

1. Introduction

It is commonly argued that on the phenomenological level, quark confinement in QCD could be explained in terms of the dual Meissner effect [1] (for a recent review see [2]). That is why it looks reasonable to study various properties of the Abelian Higgs Model (AHM) [3-8] (see Ref. [9] for a review), where this effect displays itself, as a simple model of confinement. In this way, in Ref. [4] the duality transformation proposed in Ref. [3] has been applied to the lattice version of AHM in the London limit in order to reformulate its partition function in terms of the integral over world-sheets of the Abrikosov-Nielsen-Olesen strings. Then the same reformulation has been performed in the continuum limit in Refs. [7] and [8] for AHM with and without monopoles, respectively.

In the present paper, we shall apply the duality transformation to the derivation of the string representations for the generating functionals of correlators of the gauge field strength tensors and of the Higgs currents in the London limit of AHM. By virtue of the obtained representations, we shall then get the bilocal correlator of the dual field strength tensors. It will be first obtained in Section 2 in a direct way, i.e. from the generating functional for the strength tensors correlators, and then in Section 3 from the correlator of two Higgs currents, by making use of the equations of motion. The asymptotic behaviours of the obtained correlator occur to be in a good agreement with the present lattice data concerning such a correlator in QCD [10]. The latter one plays an essential role in the Stochastic Vacuum Model (SVM) of QCD [11] (for a review see Refs. [2], [12], and [13]). This result supports the original conjecture of 't Hooft and Mandelstam concerning the dual Meissner mechanism of confinement.

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In Section 4, we shall consider AHM in the vicinity of the London limit, i.e. take into account the effects bringing about by the finiteness of the coupling constant. In this way, we shall see that these effects give a nontrivial contribution to the quartic correlator of the field strength tensors.

The main results of the paper will be discussed in the Conclusion.

In the Appendix, we perform a duality transformation of the generating functional for the field strengths in the London limit of AHM.

2. String Representation for the Generating Functional of the Correlators of the Gauge Field Strengths

We shall start with the following expression for the generating functional of the dual gauge field strength tensors in AHM

\[
Z[S_{\alpha\beta}] = \int |\Phi| D|\Phi| D\Phi \theta^{\text{sing}} D\theta^{\text{reg}} \exp \left\{ -\int d^4x \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} |D_\mu \Phi|^2 + \lambda \left( |\Phi|^2 - \eta^2 \right)^2 + i S_{\mu\nu} \tilde{F}_{\mu\nu} \right] \right\},
\]

where \( \Phi(x) = |\Phi(x)| e^{i\theta(x)} \) is the Higgs field, \( \theta = \theta^{\text{sing}} + \theta^{\text{reg}} \), \( D_\mu \equiv \partial_\mu - ieA_\mu \) is the covariant derivative, and \( S_{\alpha\beta} \) is a source term.

It is worth mentioning from the very beginning, that in what follows we shall be interested in the correlators of the dual field strengths, rather than in the ordinary ones. This is because our main goal will be the derivation of the two coefficient functions \( D \) and \( D_1 \), which parametrize the bilocal field strength correlator in the SVM (see Eqs. (12) and (13) below). Should we parametrize in such a way the correlator of the usual field strengths, then as it has been explained in Refs. [2] and [12], the function \( D \) is nonvanishing only in the case of AHM with monopoles (otherwise, the term with the function \( D \) violates Abelian Bianchi identities). However, this problem is absent for the bilocal correlator of the dual field strengths, in which the function \( D \) is nontrivial and by virtue of the equations of motion could be found from the correlator of two Higgs currents (see the next Section).

In the London limit, \( \lambda \to \infty \), one has \( |\Phi| \to \eta \), and Eq. (1) takes the form

\[
Z[S_{\alpha\beta}] = \int DA_\mu DA_\mu D\theta^{\text{sing}} D\theta^{\text{reg}} \exp \left\{ -\int d^4x \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{\eta^2}{2} (\partial_\mu \theta - eA_\mu)^2 + i S_{\mu\nu} \tilde{F}_{\mu\nu} \right] \right\}.
\]

Performing the duality transformation of Eq. (2) along the lines described in Refs. [3] and [5-7], we get

\[
Z[S_{\alpha\beta}] = \int DA_\mu \int Dh_{\mu\nu} \int D\xi(x) \exp \left\{ \int d^4x \left[ -\frac{1}{12\eta^2} H_{\mu\nu\lambda}^2 + i \pi h_{\mu\nu} \Sigma_{\mu\nu} - \frac{1}{4} F_{\mu\nu}^2 - i \tilde{F}_{\mu\nu} \left( \frac{e}{2} h_{\mu\nu} + S_{\mu\nu} \right) \right] \right\},
\]

2
where $\Sigma_{\mu\nu}(x) \equiv \int d\sigma_{\mu\nu}(x(\xi))\delta(x - x(\xi))$ is the vorticity tensor current defined on the world-sheet $\Sigma$ of the closed Abrikosov-Nielsen-Olesen string, $\xi = (\xi^1, \xi^2)$ is a two-dimensional coordinate, and $H_{\mu\nu\lambda} \equiv \partial_\mu h_{\nu\lambda} + \partial_\nu h_{\mu\lambda} + \partial_\lambda h_{\mu\nu}$ is a field strength tensor of an antisymmetric tensor field $h_{\mu\nu}$. The details of derivation of Eq. (3) are presented in the Appendix.

Now it is convenient to rewrite

$$
\exp \left( -\frac{1}{4} \int d^4 x F_{\mu\nu}^2 \right) = \int D G_{\mu\nu} \exp \left\{ \int d^4 x \left[ -G_{\mu\nu}^2 + i\tilde{F}_{\mu\nu} G_{\mu\nu} \right] \right\},
$$

after which $A_\mu$-integration yields

$$
\int D A_\mu \exp \left\{ -\int d^4 x \left[ \frac{1}{4} F_{\mu\nu}^2 + i\tilde{F}_{\mu\nu} \left( \frac{e}{2} h_{\mu\nu} + S_{\mu\nu} \right) \right] \right\} = \int D G_{\mu\nu} \exp \left( -\int d^4 x G_{\mu\nu}^2 \right) \left. \delta \left( \varepsilon_{\mu\nu\lambda\rho} \partial_\mu \left( G_{\lambda\rho} - \frac{e}{2} h_{\lambda\rho} - S_{\lambda\rho} \right) \right) = \right. 
$$

$$
= \int D B_\mu \exp \left[ \int d^4 x \left( \frac{e}{2} h_{\mu\nu} + S_{\mu\nu} + \partial_\mu B_\nu - \partial_\nu B_\mu \right)^2 \right]. \tag{4}
$$

Next, performing the hypergauge transformation with the function $\frac{2}{e} B_\mu$ (see e.g. Ref. [5]), we get from Eq. (4)

$$
Z \left[ S_{\alpha\beta} \right] = \exp \left( -\int d^4 x S_{\mu\nu}^2 \right) \cdot \int D x_\mu(\xi) \int D h_{\mu\nu} \exp \left\{ \int d^4 x \left[ -\frac{1}{24\eta^2} H_{\mu\nu\lambda}^2 - \frac{e^2}{4} h_{\mu\nu}^2 + ih_{\mu\nu} \left( \pi \Sigma_{\mu\nu} + ie S_{\mu\nu} \right) \right] \right\}. \tag{5}
$$

Gaussian integration over the field $h_{\mu\nu}$ in Eq. (5) leads to the following final result for the string representation of the generating functional (1)

$$
Z \left[ S_{\alpha\beta} \right] = \exp \left( -\int d^4 x S_{\mu\nu}^2 \right) \cdot \int D x_\mu(\xi) \exp \left\{ -\int d^4 x \int d^4 y \left( \pi \Sigma_{\lambda\nu}(x) + ie S_{\lambda\nu}(x) \right) D_{\lambda\nu,\mu\rho}(x - y) \left( \pi \Sigma_{\mu\rho}(y) + ie S_{\mu\rho}(y) \right) \right\}. \tag{6}
$$

It is worth mentioning, that an analogous expression for the generating functional has been for the first time obtained in terms of another fields already in the last paper in Ref. [1] and further studied in the first three papers in Ref. [2]. There it has been done by making use of the techniques quite different from the ones applied in the present paper, i.e. without casting the generating functional into the form of the integral over the Kalb-Ramond field. However, the final results presented in these papers qualitatively correspond to our Eq. (6).

In Eq. (6), the propagator of the field $h_{\mu\nu}$ has the following form

$$
D_{\lambda\nu,\mu\rho}(x) \equiv D_{\lambda\nu,\mu\rho}^{(1)}(x) + D_{\lambda\nu,\mu\rho}^{(2)}(x),
$$

where
\( D_{\lambda\nu,\mu\rho}^{(1)}(x) = \frac{e\eta^2}{\delta \pi^2 |x|} K_1 \left( \delta_{\lambda\mu} \delta_{\nu\rho} - \delta_{\mu\nu} \delta_{\lambda\rho} \right), \)  
\( D_{\lambda\nu,\mu\rho}^{(2)}(x) = \frac{\eta}{4\pi^2 e x^2} \left[ \frac{K_1}{|x|} + \frac{m}{2} (K_0 + K_2) \right] \left( \delta_{\lambda\mu} \delta_{\nu\rho} - \delta_{\mu\nu} \delta_{\lambda\rho} \right) + \)
\[ + \frac{1}{2|x|} \left[ 3 \left( \frac{m^2}{4} + \frac{1}{x^2} \right) K_1 + \frac{3m}{2|x|} (K_0 + K_2) + \frac{m^2}{4} K_3 \right] \cdot \left( \delta_{\lambda\rho} x_\mu x_\nu + \delta_{\mu\nu} x_\lambda x_\rho - \delta_{\mu\lambda} x_{\rho} x_\nu - \delta_{\nu\rho} x_\mu x_\lambda \right). \]  
(8)

Here \( K_i \equiv K_i(m|x|), i = 0, 1, 2, 3, \) stand for the modified Bessel functions, and \( m \equiv e\eta \) is the mass of the gauge boson generated by the Higgs mechanism. It is worth noting that the term
\[ \int_\Sigma d\sigma_{\lambda\nu}(x) \int_\Sigma d\sigma_{\mu\rho}(y) D_{\lambda\nu,\mu\rho}^{(2)}(x-y) \]
could be rewritten as a boundary one, and therefore vanishes, since \( \Sigma \) is closed.

Let us now derive from the general form (6) of the generating functional for the correlators of the field strengths the expression for the bilocal correlator. The result reads
\[ \langle \tilde{F}_{\lambda\nu}(x) \tilde{F}_{\mu\rho}(y) \rangle = \frac{1}{\mathcal{Z}[0]} \frac{\delta^2 \mathcal{Z}[S_{\alpha\beta}]}{\delta S_{\lambda\nu}(x) \delta S_{\mu\rho}(y)|_{S_{\alpha\beta}=0}} = \]
\[ = \left( \delta_{\lambda\mu} \delta_{\nu\rho} - \delta_{\lambda\rho} \delta_{\nu\mu} \right) \delta(x-y) + 2e^2 \left[ D_{\lambda\nu,\mu\rho}(x-y) - \right. \]
\[ - 2\pi^2 \left\{ \int_\Sigma d\sigma_{\alpha\beta}(z) \int_\Sigma d\sigma_{\gamma\zeta}(u) D_{\alpha\beta,\mu\rho}(z-y) D_{\gamma\zeta,\lambda\nu}(u-x) \right\}_{x_\mu(\xi)}, \]  
(9)
where
\[ \langle \cdots \rangle_{x_\mu(\xi)} \equiv \frac{\int \mathcal{D}x_\mu(\xi) \cdots \exp \left[ -\pi^2 \int_\Sigma d\sigma_{\alpha\beta}(z) \int_\Sigma d\sigma_{\gamma\zeta}(u) D_{\alpha\beta,\gamma\zeta}(z-u) \right]}{\int \mathcal{D}x_\mu(\xi) \exp \left[ -\pi^2 \int_\Sigma d\sigma_{\alpha\beta}(z) \int_\Sigma d\sigma_{\gamma\zeta}(u) D_{\alpha\beta,\gamma\zeta}(z-u) \right]}, \]
and the term with the \( \delta \)-function on the R.H.S. of Eq. (9) corresponds to the free contribution to the correlator. According to Eqs. (7) and (8), one can see that due to the large distance asymptotics of the modified Bessel functions, \( D_{\lambda\nu,\mu\rho}(x) \) has the order of magnitude \( e^2 \eta^4 \). Therefore, the contribution of the second term in the square brackets on the R.H.S. of Eq. (9) to the bilocal correlator of the dual gauge field strengths is much smaller than the contribution arising from the first term, when
\[ e\eta^2 |\Sigma| \ll 1, \]  
(10)
where \( |\Sigma| \) stands for the area of the surface \( \Sigma \). Such an inequality holds at least in the weak coupling regime (which is in the line with the London limit) or in the case of sufficiently small \( \eta \).
Following the SVM, let us parametrize the bilocal correlator of the dual field strengths by the two Lorentz structures as follows

\[
\begin{align*}
\langle \bar{F}_{\lambda\nu}(x)\bar{F}_{\mu\rho}(0) \rangle = & \left( \delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\lambda\rho}\delta_{\nu\mu} \right) D(x^2) + \\
& + \frac{1}{2} \left[ \partial_{\lambda} \left( x_\mu \delta_{\nu\rho} - x_\rho \delta_{\nu\mu} \right) + \partial_{\nu} \left( x_\rho \delta_{\lambda\mu} - x_\mu \delta_{\lambda\rho} \right) \right] D_1(x^2). 
\end{align*}
\]

(11)

In Eq. (11), the division into such Lorentz structures is chosen in such a way that one can due to the Stokes theorem rewrite the double surface integral of the structure with the function \(D_1\) as a boundary one, whereas this is impossible to do for the structure with the function \(D\). Then in the approximation (10), by virtue of Eqs. (7) and (8), we arrive at the following values of the functions \(D\) and \(D_1\)

\[
D(x^2) = \frac{m^3}{4\pi^2} \frac{K_1}{|x|} 
\]

and

\[
D_1(x^2) = \frac{m}{2\pi^2 x^2} \left[ \frac{K_1}{|x|} + \frac{m}{2} \left( K_0 + K_2 \right) \right].
\]

(13)

In Eq. (12), we have neglected the free \(\delta\)-function type contribution. The asymptotic behaviours of the coefficient functions (12) and (13) at \(|x| \ll \frac{1}{m}\) and \(|x| \gg \frac{1}{m}\) read

\[
D \longrightarrow \frac{m^2}{4\pi^2 x^2},
\]

(14)

\[
D_1 \longrightarrow \frac{1}{\pi^2 |x|^4}
\]

and

\[
D \longrightarrow \frac{m^4}{4\sqrt{2\pi} \frac{4}{\sqrt{2}} \left( m |x| \right)^{\frac{3}{2}} e^{-m|x|}}
\]

(16)

\[
D_1 \longrightarrow \frac{m^4}{2\sqrt{2\pi} \frac{4}{\sqrt{2}} \left( m |x| \right)^{\frac{5}{2}} e^{-m|x|}}
\]

(17)

respectively.

One can now see that according to the lattice data [10] the asymptotic behaviours (14)-(17) are very similar to the ones of the functions \(D\) and \(D_1\), which parametrize the gauge-invariant bilocal correlator of gluonic field strengths in the SVM approach to QCD. In particular, at large distances both functions decay exponentially, and the function \(D\) is much larger then the function \(D_1\), whereas at small distances the function \(D_1\) behaves as \(\frac{1}{|x|}\) and is much larger than the function \(D\) in this limit, which is in the line with the SVM. This similarity in the large- and short distance asymptotic behaviours of the functions \(D\) and \(D_1\), which parametrize the bilocal correlator of the dual field strengths in AHM and the gauge-invariant correlator in QCD, supports the original conjecture by ’t Hooft and Mandelstam concerning the dual Meissner nature of confinement.
3. String Representation for the Generating Functional of the Higgs Currents Correlators

In this Section, we shall rederive the coefficient function \( D \) in the bilocal correlator of the dual field strength tensors from the string representation for the generating functional of correlators of the Higgs currents. In the London limit, such a generating functional reads

\[
\hat{Z}[J] = \int D A_\mu D \theta^{\text{sing}} \cdot D \theta^{\text{reg}} \cdot \exp \left\{ \int d^4 x \left[ -\frac{1}{4} F_{\mu\nu}^2 - \frac{\eta^2}{2} (\partial_\mu \theta - e A_\mu)^2 + J_\mu j_\mu \right] \right\},
\]

where \( j_\mu \equiv -e\eta^2(\partial_\mu \theta - e A_\mu) \) is the Higgs current in this limit.

Performing the same duality transformation as the one used in Section 2, we get the following string representation for \( \hat{Z}[J] \)

\[
\hat{Z}[J] = \exp \left[ \frac{m^2}{2} \int d^4 x J^2(x) \right] \int D x_\mu (\xi) \exp \left[ -\pi^2 \int_\Sigma d\sigma_{\alpha\beta} (z) \int_\Sigma d\sigma_{\gamma\delta} (u) D_{\alpha\beta,\gamma\delta} (z - u) \right] \cdot \exp \left\{ e \epsilon_{\lambda\nu\alpha\beta} \int d^4 x d^4 y \left[ \frac{e^2}{4} \epsilon_{\mu\rho\gamma\sigma} \left( \frac{\partial^2}{\partial x_\alpha \partial y_\gamma} D_{\lambda\nu,\mu\rho} (x - y) \right) J_\beta (x) J_\delta (y) + \pi \Sigma_{\mu\rho} (y) \left( \frac{\partial}{\partial x_\alpha} D_{\lambda\nu,\mu\rho} (x - y) \right) J_\beta (x) \right\}. \tag{18}
\]

Varying Eq. (18) twice w.r.t. \( J_\mu \), setting then \( J_\mu \) equal to zero, and dividing the result by \( \hat{Z}[0] \), we arrive at the following expression for the correlator of two Higgs currents

\[
\langle j_\beta (x) j_\sigma (y) \rangle = m^2 \delta_{\beta\sigma} \delta (x - y) + e^2 \epsilon_{\lambda\nu\alpha\beta} \epsilon_{\mu\rho\gamma\sigma} \left[ -\frac{1}{2} \frac{\partial^2}{\partial x_\alpha \partial y_\gamma} D_{\lambda\nu,\mu\rho} (x - y) + \pi \Sigma_{\mu\rho} (y) \frac{\partial}{\partial x_\alpha} D_{\lambda\nu,\mu\rho} (x - y) \right] \int_\Sigma d\sigma_{\delta\zeta} (z) \int_\Sigma d\sigma_{\chi\varphi} (u) \left( \frac{\partial}{\partial x_\alpha} D_{\lambda\nu,\delta\zeta} (x - z) \right) \left( \frac{\partial}{\partial y_\gamma} D_{\mu\rho,\chi\varphi} (y - u) \right) \right]_{x_\mu (\xi)}. \tag{19}
\]

It is easy to see that the contribution of the term (8) to the R.H.S. of Eq. (19) vanishes, whereas the contribution of Eq. (7) to the second term in the square brackets on the R.H.S. of Eq. (19) could be disregarded w.r.t. its contribution to the first term, if the inequality (10) holds. Within this approximation, making use of the equation

\[
\langle j_\beta (x) j_\sigma (y) \rangle = \left( \frac{\partial^2}{\partial x_\mu \partial y_\mu} \delta_{\beta\sigma} - \frac{\partial^2}{\partial x_\beta \partial y_\sigma} \right) D \left( (x - y)^2 \right), \tag{20}
\]

which follows from the equation of motion, we get from Eqs. (19), (20), and (7) the following value of the function \( D \)

\[
D (x^2) = \frac{m^3 K_1 (m \mid x)}{4\pi^2 \mid x \mid}, \tag{21}
\]

\(^1\)An analogous equation for the dual fields has been first presented in Ref. [2]. For the first time, the fact that the function \( D \) becomes nonvanishing in QED with magnetic monopoles has been mentioned in Ref. [12].
where we have not again presented the free contribution. One can see that Eq. (21) coincides with the value of the function $D$ given by Eq. (12) obtained from the string representation for the partition function of correlators of the dual gauge field strengths. Notice also, that only the function $D$ could be obtained from the correlator (19) due to the independence of the latter of the function $D_1$.

4. $1/\lambda$-Corrections to the Quartic Correlator of the Dual Field Strength Tensors

In this Section, we shall demonstrate that accounting for the finiteness of the coupling constant $\lambda$ in AHM leads to a nontrivial contribution to the quartic correlator of the dual field strength tensors. Let us notice from the very beginning that we shall not be interested in free contributions to the correlators and therefore in what follows consider only the correlators of the fields defined in different space-time points. In the London limit, within the approximation (10), the dominant contribution to the correlator

\[ \langle \tilde{F}_{\mu_1 \nu_1}(x_1) \tilde{F}_{\mu_2 \nu_2}(x_2) \tilde{F}_{\mu_3 \nu_3}(x_3) \tilde{F}_{\mu_4 \nu_4}(x_4) \rangle, \quad x_1 \neq x_2 \neq x_3 \neq x_4, \]

comes from the four-fold variation of the following term in the expansion of the generating functional (6)

\[ \frac{e^4}{2} \int d^4 x d^4 y d^4 z d^4 u S_{\mu \nu}(x) D_{\mu \lambda \rho}(x - y) S_{\lambda \rho}(y) S_{\alpha \beta}(z) D_{\alpha \beta \gamma \zeta}(z - u) S_{\gamma \zeta}(u). \]  

The $4! = 24$ terms containing all possible combinations of indices and arguments, which one gets during this variation, are obvious and we shall not list them here for shortness.

Let us now consider AHM in the vicinity of the London limit. To this end, we shall expand the radial part of the Higgs field, $|\Phi(x)|$, in Eq. (1) as $|\Phi(x)| = \eta + \tau \psi(x)$, where $\tau \equiv \frac{1}{\lambda} \to 0$, and $\psi(x)$ is an arbitrary quantum fluctuation. Correspondingly, in what follows we shall keep only the terms linear in $\tau$. Then neglecting the Jacobian emerging during the change of the integration variables, $|\Phi(x)| \to \psi(x)$, which will be eventually cancelled after division of the final expression for the field correlator by $Z[0]$, we arrive at the new expression for the generating functional (1), which reads

\[ Z[S_{\alpha \beta}] = \int \mathcal{D} \psi \mathcal{D} A_\mu \mathcal{D} \theta^\text{sing} \mathcal{D} \theta^\text{reg} \exp \left\{ - \int d^4 x \left[ \frac{1}{4} F_{\mu \nu}^2 + \frac{\tau^2}{2} (\partial_\mu \psi)^2 + 4 \tau \eta^2 \psi^2 + \eta \left( \frac{\eta}{2} + \tau \psi \right) (\partial_\mu \theta - e A_\mu)^2 + i S_{\mu \nu} \tilde{F}_{\mu \nu} \right] \right\}. \]  

Following the same steps which led from Eq. (1) to Eq. (6), we get from Eq. (23) an additional weight factor in the integral over string world-sheets standing in Eq. (6). This weight factor emerges due to the $\psi$-integration and reads

\[ \exp \left\{ \frac{M}{288 \pi^2 \eta^6} \int d^4 x d^4 y H_{\mu \alpha \beta \gamma}^\text{extr}(x) K_1(M|x - y|) \frac{H_{\mu \alpha \beta \gamma}^\text{extr}(y)}{|x - y|} \right\}. \]  

In Eq. (24), $M \equiv 2\sqrt{2} \lambda \eta$ is the mass of the Higgs field, and $H_{\mu \alpha \beta \gamma}^\text{extr}$ stands for the strength tensor of the saddle-point value of the field $h_{\mu \nu}$ following from Eq. (5). Taking into account that $\partial_\mu \Sigma_{\mu \nu} = 0$, since $\Sigma$ is a closed surface, we get the following result for this saddle-point value
\[ h_{\lambda\nu}^{\text{extr.}}(x) = \frac{m}{2\pi^2} \int d^4y \frac{K_1(m|x-y|)}{|x-y|} \left\{ \eta^2 [i\pi \Sigma_{\lambda\nu}(y) - eS_{\lambda\nu}(y)] + \right. \\
+ \left. \frac{1}{e} \partial_\mu [\partial_\nu S_{\lambda\rho}(y) - \partial_\lambda S_{\nu\rho}(y)] \right\}. \] (25)

Let us prove that the terms with the derivatives of \( S_{\mu\nu} \) on the R.H.S. of Eq. (25) yield zero. Due to the Hodge decomposition theorem (see e.g. [14]), \( S_{\mu\nu} \) could be always represented in the form

\[ S_{\mu\nu} = \partial_\mu K_\nu - \partial_\nu K_\mu + \partial_\alpha L_{\alpha\mu\nu}, \]

where \( K_\mu \) and \( L_{\alpha\mu\nu} \) stand for some vector and an antisymmetric rank-3 tensor respectively. In the product \( S_{\mu\nu} \tilde{F}_{\mu\nu} \), the contribution of the vector \( K_\mu \) obviously vanishes due to the partial integration, and we are left with the representation of \( S_{\mu\nu} \) of the form \( S_{\mu\nu} = \partial_\alpha L_{\alpha\mu\nu} \). Substituting it into Eq. (25) we see that the terms with the derivatives of \( S_{\mu\nu} \) on the R.H.S. of this equation vanish.

Finally, upon substitution of the rest of Eq. (25) into Eq. (24), we arrive at the following additional term in the expansion of the generating functional, whose contribution to the quartic correlator in the approximation (10) is dominant

\[ e^2 \frac{M^2 m^2}{32\pi^2} \int d^4xd^4yd^4vd^4wd^4w \frac{K_1(M|x-y|)}{|x-y|} D_1 ((x-u)^2) D_1 ((x-v)^2) D_1 ((y-z)^2) \times \\
\cdot D_1 ((y-w)^2) (x-u)_\mu(y-z)_\alpha [ (x-v)_\mu(y-w)_\alpha S_{\nu\lambda}(u)S_{\nu\lambda}(v)S_{\beta\gamma}(z)S_{\beta\gamma}(w) + \\
+4(x-v)_\lambda(y-w)_\gamma S_{\nu\lambda}(u)S_{\nu\lambda}(v)S_{\beta\gamma}(z)S_{\alpha\beta}(w) + \\
+4(x-v)_\mu(y-w)_\gamma S_{\nu\lambda}(u)S_{\nu\lambda}(v)S_{\beta\gamma}(z)S_{\alpha\beta}(w) ]. \] (26)

In Eq. (26), \( D_1 \) stands for the coefficient function (13) entering the bilocal correlator in the London limit. Even without explicit writing down \( 3 \cdot 4! = 72 \) terms following from Eq. (26) after its four-fold variation, we see that the resulting quartic correlator differs from the one of the London limit, which could be obtained from Eq. (22). The most crucial difference is due to the presence of the Higgs boson exchange in Eq. (26), which is absent in Eq. (22).

The other important outcome of Eq. (26) is that the leading correction to the quartic correlator, which arises due to the finiteness of \( \lambda \), could be completely described in terms of the coefficient function (13), if the inequality (10) holds.

5. Conclusion

In the present paper, we have studied dual gauge field strength correlators in AHM both in the London limit and beyond. To this end in Section 2, the generating functional for these correlators in the London limit has been cast into the form (6) of the integral over world-sheets of the Abrikosov-Nielsen-Olesen strings. In approximation (10), this generating functional yielded the values of the two coefficient functions, \( D \) and \( D_1 \), which parametrize the bilocal correlator and play a key role in SVM of QCD. These functions are given by Eqs. (12) and (13), and there asymptotic behaviours (14)-(17) at small and large distances are found to be in a good
agreement with the known lattice data [10] concerning the corresponding behaviours in QCD. This fact together with SVM supports the conjecture of 't Hooft and Mandelstam about the dual Meissner nature of confinement. In Section 3, by making use of the equations of motion, we have rederived the coefficient function $D$ from the string representation for the correlator of two Higgs currents (19). In Section 4, we have studied AHM in the vicinity of the London limit and demonstrated that there appears a nontrivial contribution to the quartic correlator of the dual field strengths. This contribution is due to accounting for the finiteness of the Higgs boson mass, which leads to intermediate exchanges by this boson in the quartic correlator. Besides that, we have demonstrated that according to Eq. (26), in approximation (10), the quartic correlator near the London limit could be described only in terms of the function $D_1$, given by Eq. (13).

6. Acknowledgments

The author is deeply grateful to Drs. N. Brambilla, M.N. Chernodub, H. Dorn, Chr. Preitschopf, A. Vairo, and especially to Profs. H.G. Dosch, M.I. Polikarpov, and Yu.A. Simonov for useful discussions. He would also like to thank Prof. H. Kleinert for bringing his attention to some papers from Refs. [1] and [2], the theory group of the Quantum Field Theory Department of the Institute of Physics of the Humboldt University of Berlin for kind hospitality, and Graduiertenkolleg Elementarteilchenphysik for financial support.

Appendix. Derivation of Eq. (3).

In this Appendix, we shall present some details of derivation of Eq. (3). Firstly, one can linearize the term $(\partial_\mu \theta - eA_\mu)^2$ on the R.H.S. of Eq. (2) and carry out the integral over $\theta^{\text{reg.}}$ as follows

$$\int D\theta^{\text{reg.}} \exp \left\{ -\frac{\eta^2}{2} \int d^4x (\partial_\mu \theta - eA_\mu)^2 \right\} =$$

$$= \int DC_\mu D\theta^{\text{reg.}} \exp \left\{ \int d^4x \left[ -\frac{1}{2\eta^2} C_\mu^2 + iC_\mu (\partial_\mu \theta - eA_\mu) \right] \right\} =$$

$$= \int DC_\mu \delta (\partial_\mu C_\mu) \exp \left\{ \int d^4x \left[ -\frac{1}{2\eta^2} C_\mu^2 + iC_\mu \left( \partial_\mu \theta^{\text{sing.}} - eA_\mu \right) \right] \right\}. \quad (A.1)$$

The constraint $\partial_\mu C_\mu = 0$ could be uniquely resolved by representing $C_\mu$ in the form $C_\mu = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} \partial_\nu h_{\lambda\rho}$, where $h_{\lambda\rho}$ stands for an antisymmetric tensor field. Notice, that the number of degrees of freedom during such a replacement is conserved, since both of the fields $C_\mu$ and $h_{\mu\nu}$ have three independent components.

Now, taking into account that $\varepsilon_{\mu\nu\lambda\rho} \partial_\lambda \partial_\rho \theta^{\text{sing.}}(x) = 2\pi \Sigma_{\mu\nu}(x)$ [3], we get from Eq. (A.1)

$$\int D\theta^{\text{sing.}} D\theta^{\text{reg.}} \exp \left\{ -\frac{\eta^2}{2} \int d^4x (\partial_\mu \theta - eA_\mu)^2 \right\} =$$

$$= \int Dh_{\mu\nu} Dx_\mu(\xi) \exp \left\{ \int d^4x \left[ -\frac{1}{12\eta^2} H_{\mu\nu\lambda}^2 + i\pi h_{\mu\nu} \Sigma_{\mu\nu} - \frac{ie}{2} \varepsilon_{\mu\nu\lambda\rho} A_\mu \partial_\nu h_{\lambda\rho} \right] \right\}. \quad (A.2)$$
In derivation of Eq. (A.2), we have replaced $D\theta^{\text{sing.}}$ by $D_x \mu(\xi)$ (since the surface $\Sigma$ parametrized by $x_\mu(\xi)$ is just the surface, at which the field $\theta$ is singular) and for simplicity have not taken into account the Jacobian arising during such a change of the integration variable.

Finally, adding the term $-iS_{\mu\nu}\tilde{F}_{\mu\nu}$ to the argument of the exponent standing on the R.H.S. of Eq. (A.2), we arrive at Eq. (3).

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\footnote{For the case when the surface $\Sigma$ has a spherical topology, this Jacobian has been calculated in Ref. [7].}