Cognitive Interference Channels with Confidential Messages

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Abstract—The cognitive interference channel with confidential messages is studied. Similarly to the classical two-user interference channel, the cognitive interference channel consists of two transmitters whose signals interfere at the two receivers. It is assumed that there is a common message source (message 1) known to both transmitters, and an additional independent message source (message 2) known only to the cognitive transmitter (transmitter 2). The cognitive receiver (receiver 2) needs to decode both messages, while the non-cognitive receiver (receiver 1) should decode only the common message. Furthermore, message 2 is assumed to be a confidential message which needs to be kept as secret as possible from receiver 1, which is viewed as an eavesdropper with regard to message 2. The level of secrecy is measured by the equivocation rate. A single-letter expression for the capacity-equivocation region of the cognitive interference channel is established and is further explicitly derived for the Gaussian case. Moreover, particularizing the capacity-equivocation region to the case without a secrecy constraint, establishes a new capacity theorem for a class of interference channels, by providing a converse theorem.

I. INTRODUCTION

Interference channels model basic wireless networks, in which communication signals intended for one receiver cause interference at other receivers. Although the capacity region and the best coding schemes for the interference channel remain unknown, much progress has been made toward understanding this channel (see, e.g., [1]–[6] and the references therein).

Interference not only affects communication rates, but also raises security issues. This is because information on the original source can be extracted by other nodes that are not the intended destination (eavesdroppers) due to interference at these nodes. In certain situations it is desirable to minimize the leak of information to eavesdroppers. It is also important to evaluate the secrecy level of confidential information (which was defined in [7] as the equivocation rate for the single-user wire-tap channel) for the interference channel, and to study the reliable communication rates under a given certain level of secrecy constraint. This is what motivates the problem that we address in this paper.

We study the cognitive interference channel, with two transmitters and two receivers (see Fig. 1). Transmitter 1 knows only message 1, and transmitter 2 (the cognitive transmitter) knows both messages 1 and 2. Receiver 1 needs to decode only message 1 and receiver 2 needs to decode both messages 1 and 2. Furthermore, we also address a secrecy constraint, and assume that transmitter 2 wishes to protect message 2 (confidential message) from being known by receiver 1 (an eavesdropper) due to interference. We refer to this model as the cognitive interference channel with confidential messages.

The channel studied in this paper can model a cognitive radio (see [8]–[11]), which is a device that is introduced to wireless networks in order to improve overall performance by utilizing unused spectral resources. In the model we study, the cognitive radio is modelled by transmitter 2 which helps transmitter 1 (the primary transmitter) transmit its message. Moreover, the cognitive transmitter also transmits its own message, which should be kept confidential with respect to the non-cognitive receiver (receiver 1).

In this paper, we establish the capacity-equivocation region for the cognitive interference channel with confidential messages, which characterizes the trade-off between the achievable communication rates and the secrecy levels at the eavesdropper. We further derive the capacity-equivocation region for the Gaussian case. For the case without the secrecy constraint, the capacity-equivocation region reduces to the capacity region of the cognitive interference channel. This establishes a new capacity theorem for a class of interference channels, by providing a converse theorem.

We note that the cognitive interference channel (without secrecy constraints) was studied in [12, Theorem 5], where an achievable rate region (inner bound on the capacity region) is given. An achievable error exponent for this channel was studied in [13]. In this paper, we provide an outer bound on the capacity region that matches the inner bound given in [12, Theorem 5] and hence establish the capacity region for this channel. We also note that the channel we study is different from the channel model studied in [10], [11], [14], [15] in that receiver 2 is required to decode both messages 1 and 2. Furthermore, we also address the secrecy constraint which was not considered in [10]–[15]. We finally note that the model we study is different from the interference channel model with secrecy constraints studied in [16], in that it assumes a cognitive transmitter that knows the other transmitter’s message and a cognitive receiver that decodes...
both messages.

The paper is organized as follows. In Section II we introduce the model of the cognitive interference channel with confidential messages. Section III presents the capacity-equivocation region, and Section IV applies our results to the Gaussian case. Section V particularizes our results to the cognitive interference channel without secrecy constraints.

II. CHANNEL MODEL

Definition 1: A discrete memoryless cognitive interference channel consists of two finite channel input alphabets \( \mathcal{X}_1 \) and \( \mathcal{X}_2 \), two finite channel output alphabets \( \mathcal{Y} \) and \( \mathcal{Z} \), and a transition probability distribution \( p(y, z|x_1, x_2) \) (see Fig. 1), where \( x_1 \in \mathcal{X}_1 \) and \( x_2 \in \mathcal{X}_2 \) are channel inputs from transmitters 1 and 2, respectively, and \( y \in \mathcal{Y} \) and \( z \in \mathcal{Z} \) are channel outputs at receivers 1 and 2, respectively.

Over this channel, transmitters 1 and 2 jointly send one message denoted by \( W_1 \) to receivers 1 and 2, and transmitter 2 sends one message denoted by \( W_2 \) to receiver 2 and wants to keep this message as secret as possible from receiver 1. Hence the message \( W_2 \) is referred to the confidential message with respect to receiver 1.

Definition 2: A \((2^{nR_1}, 2^{nR_2}, n)\) code for the cognitive interference channel consists of the following:

- Two message sets: \( \mathcal{W}_k = \{1, 2, \ldots, 2^{nR_k}\} \) for \( k = 1, 2 \);
- Two messages: \( W_1 \) and \( W_2 \) are independent random variables that are uniformly distributed over \( \mathcal{W}_1 \) and \( \mathcal{W}_2 \), respectively;
- Two encoders: one deterministic encoder \( f_1 : \mathcal{W}_1 \to \mathcal{X}_1^n \), which maps each message \( w_1 \in \mathcal{W}_1 \) to a codeword \( x_1^n \in \mathcal{X}_1^n \); and one stochastic encoder \( f_2 : \mathcal{W}_1 \times \mathcal{W}_2 \to \mathcal{X}_2^n \), which maps each message pair \( (w_1, w_2) \in \mathcal{W}_1 \times \mathcal{W}_2 \) to a codeword \( x_2^n \in \mathcal{X}_2^n \);
- Two decoders: one is \( g_1 : \mathcal{Y}^n \to \mathcal{W}_1 \), which maps a received sequence \( y^n \) to a message \( \hat{w}_1^{(1)} \in \mathcal{W}_1 \); and the other is \( g_2 : \mathcal{Z}^n \to \mathcal{W}_1 \times \mathcal{W}_2 \), which maps a received sequence \( z^n \) to a message pair \( (\hat{w}_1^{(2)}, \hat{w}_2) \in \mathcal{W}_1 \times \mathcal{W}_2 \).

For a given code, we define the probability of error and the secrecy level of the confidential message \( W_2 \). The probability of error when the message pair \( (w_1, w_2) \) is sent is defined as

\[
P_e^{(n)}(w_1, w_2) = Pr\left\{ (\hat{w}_1^{(1)}, \hat{w}_2^{(2)}, \hat{w}_2) \neq (w_1, w_1, w_2) \right\}.
\]

and the average block probability of error is

\[
P_e^{(n)} = \frac{1}{2^{nR_1} 2^{nR_2}} \sum_{w_1=1}^{2^{nR_1}} \sum_{w_2=1}^{2^{nR_2}} P_e^{(n)}(w_1, w_2).
\]

The secrecy level of the message \( W_2 \) at receiver 1 is defined by

\[
R_{2e}^{(n)} = \frac{1}{n} H(W_2|Y^n).
\]

A rate-equivocation triple \((R_1, R_2, R_{2e})\) is said to be achievable if there exists a sequence of \( (2^{nR_1}, 2^{nR_2}, n) \) codes for \( n \geq 1 \) with the average error probability

\[
P_e^{(n)} \to 0
\]

as \( n \to \infty \) and with the equivocation rate, \( R_{2e} \), satisfying

\[
R_{2e} \leq \lim \inf_{n \to \infty} R_{2e}^{(n)}.
\]

Definition 3: The capacity-equivocation region \( \mathcal{C} \) is the closure of the union of all achievable rate-equivocation triples \((R_1, R_2, R_{2e})\).

Definition 4: The secrecy capacity region, \( C_s \), is defined by

\[
C_s = \{(R_1, R_2) : (R_1, R_2, R_{2e}) \in \mathcal{C}\},
\]

that is, the region that includes all achievable rate-pairs \((R_1, R_2)\) such that perfect secrecy is achieved for the message \( W_2 \).

III. MAIN RESULTS

We first provide an achievable rate-equivocation region for the cognitive interference channel in the following lemma.

![Fig. 1. Cognitive interference channel with confidential messages](image-url)
Lemma 1: The following region is achievable for the cognitive interference channel with confidential messages:
\[
\mathcal{R} = \bigcup_{(R_1, R_2, R_{2e}) : \begin{align*}R_1 & \geq 0, R_2 \geq 0, R_{2e} \geq 0, R_{21} \geq 0, R_{22} \geq 0 \\
R_2 & = R_1 + R_{22} \\
R_1 + R_{21} & \leq I(U, X_1 ; Y) \\
R_{21} + R_{22} & \leq I(U, X_2 ; Z|U, X_1) \\
R_1 + R_{2e} & \leq I(U, X_1 , X_2 ; Z) - I(X_2 ; Y|U, X_1) \end{align*}}
\]

(5)

Proof: (outline) We briefly outline the idea of the achievable scheme in the following. The details of the proof and the computation of the equivocation rate are omitted and can be found in [17]. The message $W_2$ is split into two components, $W_{21}$ and $W_{22}$, with rates indicated by $R_{21}$ and $R_{22}$, respectively, in (5). Receiver 1 decodes both $W_1$ and $W_{21}$, and receiver 2 decodes $W_1$, $W_{21}$ and $W_{22}$. Since $W_{21}$ is decoded and fully known at receiver 1, $W_{21}$ does not contribute to the secrecy level of $W_2$ at receiver 1 (the eavesdropper). Hence, only $W_{22}$ may be hidden from the eavesdropper.

We now present our main result in the following theorem.

Theorem 1: For the cognitive interference channel with confidential messages, the capacity-equivocation region is given by
\[
\mathcal{C} = \bigcup_{(R_1, R_2, R_{2e}) : \begin{align*}R_1 & \geq 0, R_2 \geq 0, R_{2e} \geq 0 \\
R_1 & \leq I(U, X_1 ; Y) \\
R_2 & \leq I(U, V ; Z|X_1) \\
R_1 + R_2 & \leq \min\{I(U, X_1 ; Y), I(U, X_1 ; Z)\} + I(V; Z|U, X_1) \end{align*}}
\]

(6)

Proof: (outline) To establish the achievability part of Theorem 1, we first note that if we define a random variable $V$ that satisfies the Markov chain condition $(X_1, U) \leftrightarrow V \leftrightarrow X_2$, and change $X_2$ to be $V$ in $\mathcal{R}$ given in Lemma 1, the resulting region is also achievable. This follows by prefixing one discrete memoryless channel with the input $V$ and the transition probability $p(x_2|v)$ to transmitter 2 (similarly to [18, Lemma 4]). For this new achievable region, we apply Fourier-Motzkin elimination (see, e.g., [19]) to eliminate $R_{21}$ and $R_{22}$ from the bounds and then obtain the region $\mathcal{C}$ in Theorem 1.

The proof of the converse part of Theorem 1 is omitted and can be found in [17].

The capacity-equivocation region provided in Theorem 1 in (6) can also be written in the following equivalent form.

Corollary 1: The following region is equivalent to the region in Theorem 1 in (6) and provides a simpler form of the capacity-equivocation region for the cognitive interference channel with confidential messages:
\[
\mathcal{C} = \bigcup_{(R_1, R_2, R_{2e}) : \begin{align*}R_1 & \geq 0, R_2 \geq 0, R_{2e} \geq 0 \\
R_1 & \leq \min\{I(U, X_1 ; Y), I(U, X_1 ; Z)\} + I(V; Z|U, X_1) \end{align*}}
\]

(7)

Remark 1: The capacity-equivocation region of the cognitive interference channel with confidential messages given in (7) reduces to the capacity-equivocation region of the broadcast channel with confidential messages given in [18, Theorem 1] when setting $X_1 = \emptyset$.

For the case of perfect secrecy, we obtain the following secrecy capacity region based on Corollary 1.

Corollary 2: The secrecy capacity region of the cognitive interference channel with confidential messages is given by:
\[
\mathcal{C}_s = \bigcup_{p(x_1, v)p(x_2|v)p(y, z|x_1, x_2) : \begin{align*}(R_1, R_2, R_{2e}) : R_1 & \geq 0, R_2 \geq 0, R_{2e} \geq 0 \\
R_1 & \leq \min\{I(U, X_1 ; Y), I(U, X_1 ; Z)\} + I(V; Z|U, X_1) \end{align*}}
\]

(8)

We next present the capacity-equivocation region for two classes of degraded cognitive interference channels, which will be useful when we study the Gaussian case.

Corollary 3: If the cognitive interference channel satisfies the following degradedness condition
\[
\begin{align*}
p(y, z|x_1, x_2) & = p(y|x_1, x_2)p(z|y, x_1).
\end{align*}
\]

(9)

then the capacity-equivocation region is given by
\[
\mathcal{C}_{deg1} = \bigcup_{p(x_1, x_2)p(y, z|x_1, x_2) : \begin{align*}(R_1, R_2, 0) : R_1 & \geq 0, R_2 \geq 0 \\
R_1 + R_2 & \leq \min\{I(X_1 , X_2 ; Y), I(X_1 , X_2 ; Z)\} \end{align*}}
\]

(10)

Proof: (outline) The achievability follows from (6) given in Theorem 1 by setting $U = V = X_2$. The proof of the converse part is omitted and can be found in [17].
**Corollary 4:** If the cognitive interference channel satisfies the following degradedness condition
\[ p(y, z|x_1, x_2) = p(z|x_1, x_2)p(y|x, x_1), \]  
then the capacity-equivocation region is given by
\[ C_{deg2} = \bigcup_{(R_1, R_2, R_{2e}) :} \left\{ \begin{array}{l}
R_1 \geq 0, R_2 \geq 0, R_{2e} \geq 0 \\
R_1 \leq \min \{ I(U, X_1; Y), I(U, X_1; Z) \} \\
R_2 \leq I(X_2; Z|U, X_1) \\
R_{2e} \leq R_2 \\
R_{2e} \leq I(X_2; Z|U, X_1) - I(X_2; Y|U, X_1) \end{array} \right\} \]  
(12)

**Proof:** (outline) The achievability follows from (7) by setting \( V = X_2 \) and observing that the sum-rate bound in (7) is equal to the sum of the two bounds on the individual rates in (12) and that \( I(X_2; Z|U, X_1) \leq I(U, X_2; Z|X_1) \). The proof of the converse part is omitted and can be found in [17].

Corollary 3 (and similarly, Corollary 4) continues to hold also for a stochastically degraded channel, i.e., a channel \( p(y, z|x_1, x_2) \) whose conditional marginal distributions \( p(y|x_1, x_2) \) and \( p(z|x_1, x_2) \) are the same as those of a channel satisfying the degradedness condition (9) (and correspondingly (11) for Corollary 4).

We note that while achieving the capacity-equivocation region for the general cognitive interference channel with confidential messages requires application of a rate splitting scheme (described in the proof for Lemma 1), it is unnecessary in the degraded channel cases (11), (9).

**IV. GAUSSIAN COGNITIVE INTERFERENCE CHANNEL WITH CONFIDENTIAL MESSAGES**

In this section, we consider the Gaussian cognitive interference channel. The channel outputs at receivers 1 and 2 at time instant \( i \) are given, respectively, by
\[ Y_i = X_{1,i} + aX_{2,i} + N_{1,i} \]
\[ Z_i = bX_{1,i} + X_{2,i} + N_{2,i} \]  
(13)

where \( \{N_{1,i}\}_{i=1}^{\infty} \) and \( \{N_{2,i}\}_{i=1}^{\infty} \) are independent i.i.d. Gaussian processes, and \( a \) and \( b \) are real constants. Without loss of generality, we set \( Var(N_{1,i}) = Var(N_{2,i}) = 1 \). We assume that the transmitters are subject to the following power constraints
\[ \frac{1}{n} \sum_{i=1}^{n} X_{1,i}^2 \leq P_1 \quad \text{and} \quad \frac{1}{n} \sum_{i=1}^{n} X_{2,i}^2 \leq P_2. \]  
(14)

We consider the cases with \( |a| \geq 1 \) and \( |a| < 1 \), separately. For the case when \( |a| \geq 1 \), we have the following theorem on the capacity-equivocation region.

**Theorem 2:** For the Gaussian cognitive interference channel with confidential messages, if \( |a| \geq 1 \), then the capacity-equivocation region is given by
\[ C = \bigcup_{-1 \leq \rho \leq 1} \left\{ \begin{array}{l}
(R_1, R_2, 0) : \\
R_1 \geq 0, R_2 \geq 0 \\
R_2 \leq \frac{1}{2} \log \left( 1 + (1 - \rho^2)P_2 \right) \\
R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + b^2P_1 + P_2 + 2b\rho \sqrt{P_1P_2} \right) \\
R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + P_1 + a^2P_2 + 2a\rho \sqrt{P_1P_2} \right) \end{array} \right\} \]  
(15)

From Theorem 2 we observe that no secrecy can be achieved whenever \( |a| \geq 1 \), i.e., \( R_{2e} = 0 \). This is because when \( |a| \geq 1 \) and \( X_1 \) is given, receiver 2’s output \( Z \) is degraded with regard to receiver 1’s output \( Y \), and hence receiver 1 can obtain any information that receiver 2 obtains.

**Proof:** (outline) The achievability follows from (10) given in Corollary 4 by computing the mutual information terms with \( (X_1, X_2) \) that are zero-mean jointly Gaussian with \( E[X_1^2] = P_1 \), \( E[X_2^2] = P_2 \), and \( E[X_1X_2] = \rho \sqrt{P_1P_2} \).

The converse follows by applying the bounds in the converse proof for Theorem 1 (see [17]) to the Gaussian case. The power constraints (14) translate to upper bounds on the second moments of \( X_1 \) and \( X_2 \), i.e., \( Var(X_1) \leq P_1 \) and \( Var(X_2) \leq P_2 \). The proof also applies the degradedness property in this case, i.e., \( Z \) is degraded with regard to \( Y \) if \( X_1 \) is given. The details of the proof are omitted and can be found in [17].

In Figure 2 the capacity region of the Gaussian cognitive interference channel is shown for \( P_1 = P_2 = 1 \), \( b = 3 \), and \( a > 1 \). This is because for the chosen parameters \( P_1 = P_2 = 1 \) and \( b = 3 \), if \( a \geq 3 \), receiver 1 always decodes \( W_1 \) if receiver 2 decodes this message. Hence receiver 2 is the bottleneck receiver.

We now show the capacity-equivocation region when \( |a| < 1 \).
Theorem 3: For the Gaussian cognitive interference channel with confidential messages, if \(|a| < 1\), the capacity-equivocation region is given by

\[
C = \bigcup_{-1 \leq \rho \leq 1, 0 \leq \beta \leq 1} \{ (R_1, R_2, R_{2e}) : \begin{align*}
R_1 &\geq 0, R_2 \geq 0, R_{2e} \geq 0 \\
R_1 &\leq \frac{1}{2} \log \left( 1 + \frac{P_1 + \rho^2 P_2 + 2 \rho \alpha \sqrt{\beta P_1 P_2}}{1 + (1 - \rho^2) P_2} \right) \\
R_2 &\leq \frac{1}{2} \log \left( 1 + \frac{P_2 + \rho^2 P_1 + 2 \rho \beta \sqrt{\beta P_1 P_2}}{1 + (1 - \rho^2) P_2} \right) \\
R_{2e} &\leq \frac{1}{2} \log \left( 1 + (1 - \rho^2) P_2 \right) \\
0 &\leq R_{2e} \leq R_2 \\
R_1 + R_2 &\leq \min \{ I(U, X_1; Y), I(U, X_1; Z) \} + I(X_2; Z | U, X_1) \}
\} \tag{16}
\]

We note that the equivocation rate in this case can be positive. This is because when \(|a| < 1\), receiver 1’s output \(Y\) is degraded with regard to receiver 2’s output \(Z\) if \(X_1\) is given. Hence receiver 2 may be able to receive some information that receiver 1 cannot obtain. Note also, in \((16)\), if \(a > 0, b > 0\), then \(\beta = 1\).

Proof: (outline) The achievability follows from \((12)\) given in Corollary 4 by setting \(V = X_2\) and computing the mutual information terms with \((U, X_1, X_2)\) having the following joint distribution:

\[
X_1 \sim N(0, P_1), U = \rho \sqrt{\frac{P_2}{P_1}} + U', \text{ and } X_2 = U + X_2'
\]

where \(U' \sim N(0, \beta \rho^2 P_2)\) and \(X_2' \sim N(0, (1 - \rho^2) P_2)\), and \(X_1, U', X_2'\) are independent.

The converse follows by applying the bounds in the converse proof for Corollary 4 (see [17]) to the Gaussian case and applying the entropy power inequality. The proof also applies the degradedness property in this case, i.e., \(Y\) is degraded with regard to \(Z\) if \(X_1\) is given. The details of the proof are omitted in this paper and can be found in [17].

In Figure 3 the capacity region and the secrecy capacity region of the Gaussian cognitive interference channel are shown for \(P_1 = P_2 = 1, b = 1\) and \(a = 0.5, 0.8\) and 1. It can be seen that as \(a\) increases, receiver 1 decodes more information about \(W_2\) via rate splitting. While this helps receiver 1 improve \(R_1\) by interference cancellation, it causes the equivocation rate \(R_{2e}\) to decrease due to more leakage of \(W_2\) to receiver 1. When \(a = 1\), receiver 1 decodes everything that receiver 2 decodes, and hence \(R_{2e} = 0\), which is consistent with Theorem 4.

V. IMPLICATION TO COGNITIVE INTERFERENCE CHANNELS

Sections 4 and 5 study the cognitive interference channel with confidential messages. If we do not consider the secrecy constraint, i.e., message \(W_2\) need not be confidential from receiver 1, the capacity-equivocation region given in Theorem 1 reduces to the capacity region of the corresponding cognitive interference channel without secrecy constraints.

Theorem 4: The capacity region of the cognitive interference channel is given by

\[
C = \bigcup_{p(u, x_1, x_2) p(y, z | x_1, x_2)} \{ (R_1, R_2) : \begin{align*}
R_1 &\geq 0, R_2 \geq 0 \\
R_1 &\leq I(U, X_1; Y) \\
R_2 &\leq I(X_2; Z | X_1) \\
R_1 + R_2 &\leq \min \{ I(U, X_1; Y), I(U, X_1; Z) \} + I(X_2; Z | U, X_1) \}
\} \tag{18}
\]

Proof: (outline) From Corollary 4 we deduce that the capacity region of the cognitive interference channel is given by \((18)\) with an additional bound \(R_1 \leq I(U, X_1; Z)\). This is done by setting \(R_{2e} = 0\) and because the remaining bounds do not decrease if one sets \(V = X_2\) due to the Markov chain relationship \(V \rightarrow (X_1, X_2) \rightarrow (Y, Z)\). We further show that the bound \(R_1 \leq I(U, X_1; Z)\) is, in fact, redundant.

We note that the region \((18)\) was given as an achievable rate region (inner bound on the capacity region) in [12, Theorem 5]. The converse proof [17] that we have given to show the more general result of Theorem 1 provides a converse to establish that the region \((18)\) is, in fact, the capacity region.

We note that another achievable region for the cognitive interference channel (without secrecy constraints) was reported in [13], which is included within the larger achievable region in [12, Theorem 5].

The cognitive interference channel includes a few classical channels as special cases.

Remark 2: The cognitive interference channel reduces to the broadcast channel with degraded message sets studied in [20] if we set \(X_1 = \phi\). Under this condition, it is easy to see that the capacity region of cognitive interference channel given in Theorem 4 reduces to the capacity region of the broadcast channel with degraded message sets given in [18,
It is easy to see that the preceding region is maximized by setting $\tilde{U} = X_2$, and hence the first bound is not necessary. The resulting region is the capacity region of the multiple access channel with degraded message sets given in [21] (see also [22]) if we set $Y = Z$. In this case, the region given in (19) becomes

$$C = \left\{ \left( R_1, R_2 \right) : \begin{array}{l} R_1 \geq 0, R_2 \geq 0, \hfill \\
R_1 \leq I(U, X_1; Z), \hfill \\
R_2 \leq I(X_2; Z|X_1), \hfill \\
R_1 + R_2 \leq I(X_1, X_2; Z) \end{array} \right\}$$

It is easy to see that the preceding region is maximized by setting $\tilde{U} = X_2$, and hence the first bound is not necessary. The resulting region is the capacity region of the multiple access channel with degraded message sets given in [21] (see also [22]).

Corollary 5: By setting $R_{2c} = 0$, Theorems 2 and 3 respectively reduce to the capacity regions in the cases $|a| \geq 1$ and $|a| < 1$ for the Gaussian cognitive interference channel without secrecy constraints, where the cognitive receiver is required to reliably decode both messages.

VI. CONCLUSIONS

In this paper we have presented a single-letter characterization for the capacity-equivocation region of the cognitive interference channel with confidential messages. The capacity-achieving random scheme is based on superposition coding, rate-splitting and stochastic encoding. We have further specialized the expression for the capacity-equivocation region to several cases: (a) perfect secrecy, that is, the secrecy-capacity region; (b) no secrecy constraints, i.e., a new capacity theorem for the cognitive interference channel; (c) degraded channel in which given the first channel input, the observation available to the receiver that decodes both messages is a degraded version of the observation available to the eavesdropping receiver; and (d) degraded channel in which given the first channel input, the observation available to the eavesdropping receiver is a degraded version of the observation available to the other receiver. We have further applied the results to the Gaussian cognitive interference channel with confidential messages, which falls under cases (c) or (d), and have explicitly characterized the capacity-equivocation region in closed form expressions.

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