BPS monopoles and dyons in generalized BPS Lagrangian method

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Abstract. We generalize BPS Lagrangian method and rederive BPS equations for monopoles and dyons from the Lagrangian of SU(2) Maxwell-Higgs model in four dimensional spacetime. We show that in the BPS Lagrangian method the Gauss’s law constraint does not necessary to be imposed beforehand. We find that the BPS Lagrangian has non-boundary terms which in turn gives us some constraint equations. These constraint equations can be reduced by taking the standard BPS equations for Monopoles and Dyons, which turns out to be the Gauss’s law constraint in the BPS limit.

1. Introduction
Monopoles has been known as the most important and best studied solitons in three dimensional space [1]. Its existance in the classical theory of electromagnetism proved to be implausible in an evolving universe as it is shown to be constant for whole time. At the quantum level, its existance would forbid the introduction of vector potential which is one of the main ingredients in the formulation of the quantum mechanics of electrically charged particles [2]. Besides those obstacles, Dirac was able to show that the quantum mechanics of an electrically charged particle can be formulated consistently with the presence of monopoles which then lead to the infamous Dirac’s quantization condition [3]. Unfortunately, this Dirac monopole is singular near the origin and hence the mass is infinite. Furthermore, it is hard to quantize the dynamics of Dirac monopoles. Later development came when ’t Hooft and Polyakov showed that non-abelian gauge theories can have monopole solutions without singularities [4, 5]. Moreover, it was shown for a SU(2) Yang-Mills theory with Higgs fields in the adjoint representation the monopole has a analytic solution in a special limit of the theory namely BPS limit [6]. Afterwards, it was shown the analytic solution also exist for dyons; monopoles with non-zero electric charges [7]. In this BPS limit, the second-order derivative Euler-Lagrange equations are reduced to the first-order derivative Bogomolny’s equations and the energy of this monopoles (dyons) saturates the non-trivial energy bound which is proportional to topological charge of the theory [8].

Obtaining the Bogomolny’s equation for monopoles(dyons) for arbitrary theory is important in particular to derive the non-trivial energy bound and to study the topological stability of the solutions. There have been some methods developed in this directions which are the first-order formalism [9, 10], the concept of strong necessary conditions [11, 12], the on-shell method [13, 14], and the BPS Lagrangian method [15, 16]. Recently, the BPS Lagrangian method has been used to rederive the Bogomolny’ equations for monopoles and dyons in the SU(2) Yang-Mills theory.
and its Dirac-Born-Infeld extensions and derive the Bogomolny’s equations in their generalized versions [17]. However the derivations rely on particular hedgehog ansatz, or ’t Hooft-Polyakov ansatz, for the fields content. In this article we are going to show that the BPS Lagrangian method could also be used to rederive the Bogomolny’s equations of the standard $SU(2)$ Yang-Mills theory in general coordinate systems.

2. BPS Lagrangian Method in The Effective Lagrangian Description

In this section we will show how the BPS Lagrangian method works in deriving Bogomolny’s equations for monopoles and dyons in the generalized $SU(2)$ Yang-Mills-Higgs model with the following Langrangian density [17, 18]

$$L = -\frac{w(|\Phi|)}{2} \text{tr} (F_{\mu\nu}F^{\mu\nu}) + G(|\Phi|) \text{tr} (D_{\mu}\Phi D^{\mu}\Phi) - V(|\Phi|),$$  \hspace{1cm} (1)

where $w, G > 0$ and $V \geq 0$ are functions of the Higgs fields, with $|\Phi| = 2 \text{tr} \Phi^2$; $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ie [A_{\mu}, A_{\nu}]$; $D_{\mu} \equiv \partial_{\mu} - ie [A_{\mu}]$; and $\mu, \nu = 0, 1, 2, 3$ are spacetime indices with metric signature $(+ - - -)$. In terms of components, the gauge and scalar fields are

$$A_{\mu} = \frac{1}{2} \tau^a A^a_{\mu}, \quad \Phi = \frac{1}{2} \tau^a \Phi^a,$$  \hspace{1cm} (2)

with $a = 1, 2, 3$ and $\tau^a$ are the Pauli matrices.

2.1. ’t Hooft-Polyakov Monopoles and Julia-Zee Dyons

Let us consider the hedgehog ansatz as follows:

$$\Phi^a = f(r) x^a \frac{1}{r},$$

$$A^a_0 = j(r) x^a \frac{1}{r},$$

$$A^a_i = \frac{1 - a(r)}{r} \epsilon^{aij} x^j,$$  \hspace{1cm} (3a)

where $x^a \equiv (x, y, z)$, and $x^i \equiv (x, y, z)$ as well, denotes the Cartesian coordinates. Notes that the Levi-Civita symbol $\epsilon^{aij}$ in (3) mixes the space-index and the group-index. Substituting the ansatz (3) into Lagrangian (1) we arrive at the following effective Lagrangian, in spherical coordinates,

$$L_{\text{eff}} = -G(f) \left( \frac{f'^2}{2} + \frac{a^2 f^2}{r^2} \right) + w(f) \left( \frac{j'^2}{2} + \frac{a^2 j^2}{r^2} - \frac{a'^2}{2} - \frac{(a^2 - 1)^2}{2r^2} \right) - V(f).$$  \hspace{1cm} (4)

where $' \equiv \frac{d}{dr}$ otherwise it means taking derivative over the argument. Here we have set the coupling $e = 1$ for simplicity.

Following [16], the BPS Lagrangian takes the general form:

$$L_{\text{bps}} = -Q_a \frac{a'}{r^2} - Q_f \frac{f'}{r^2} - Q_j \frac{j'}{r^2} - X_0 - X_1 a'^2 - X_2 f'^2 - X_3 j'^2.$$  \hspace{1cm} (5)

Here we have assumed all functions ($Q_a, Q_f, Q_j, X_0, X_1, X_2, X_3$) depend exciplitly on the effective fields $a, f$, and $j$. The first three terms are considered to be “boundary” terms as such

\footnote{Here we follow the notations in [17].}
\[ Q_a = \frac{\partial Q}{\partial a}, Q_f = \frac{\partial Q}{\partial f}, \text{ and } Q_j = \frac{\partial Q}{\partial j} \] with \( Q \equiv Q(a,f,j) \). The next step is rewriting \( \mathcal{L}_{eff} - \mathcal{L}_{bps} \) into the quadratic forms:

\[
\mathcal{L}_{eff} - \mathcal{L}_{bps} = \text{BPS}_a + \text{BPS}_f + \text{BPS}_j + \frac{Q_j^2}{2r^4(G-2X_2)} + \frac{Q_a^2}{4r^2(w-r^2X_1)} - \frac{Q_j^2}{2r^4(w+2X_3)} - \frac{a^4w + 2a^2\left(G f^2r^2 - w\left(j^2r^2 + 1\right)\right) + 2r^4(V - X_0) + w}{2r^4}.
\]

with

\[
\text{BPS}_a = \left(\frac{r^2X_1 - w}{r^2}\right)^2 \frac{Q_a}{2\left(r^2X_1 - w\right)},
\]

\[
\text{BPS}_f = \left(\frac{X_2 - \frac{G}{2}}{2}\right)^2 \frac{Q_f}{2r^2\left(X_2 - \frac{G}{2}\right)},
\]

\[
\text{BPS}_j = \left(\frac{w}{2} + X_3\right)^2 \frac{Q_j}{2r^2\left(\frac{w}{2} + X_3\right)}.
\]

Setting \( \mathcal{L}_{eff} - \mathcal{L}_{bps} = 0 \), we obtain BPS equations \( \text{BPS}_a, \text{BPS}_f, \text{BPS}_j = 0 \) with \( r^2X_1 \neq w, 2X_2 \neq G, \text{ and } 2X_3 \neq -w \). In addition we have a constraint equation:

\[
\frac{Q_j^2}{2r^4(G-2X_2)} + \frac{Q_a^2}{4r^2(w-r^2X_1)} - \frac{Q_j^2}{2r^4(w+2X_3)} - \frac{a^4w + 2a^2\left(G f^2r^2 - w\left(j^2r^2 + 1\right)\right) + 2r^4(V - X_0) + w}{2r^4} = 0.
\]

Nontrivial solutions of the BPS equations (\( \text{BPS}_a, \text{BPS}_f, \text{BPS}_j \)) require that \( Q_a, Q_f, Q_j \neq 0 \). To solve this constraint equation, usually we expand the lhs of constraint equation (10) as a polynomial function of explicit coordinates which is only the radial coordinate \( r \) in this case. Since \( r^2X_1 \neq w \), we may multiply the constraint equation (10) with \( 4r^4 \left(r^2X_1 - w\right) \) and thus the “coefficients” for each power of \( r \) are

\[
r^0_0 : 2w\left(\frac{a^2 - 1}{w} - \frac{Q_j^2}{G - 2X_2} + \frac{Q_j^2}{w + 2X_3}\right),
\]

\[
r^2_0 : 4a^2w\left(f^2G - j^2w\right) - Q_a^2 + 2X_1\left(-\left(a^2 - 1\right)^2 w + \frac{Q_j^2}{G - 2X_2} - \frac{Q_j^2}{w + 2X_3}\right),
\]

\[
r^4_0 : 4a^2X_1\left(j^2w - f^2G\right) + 4w(V - X_0),
\]

\[
r^6_0 : 4X_1(X_0 - V).
\]

Setting all these “coefficients” to be zero will solve the constraint equation (10). From the “coefficient” (11) we obtain

\[
\frac{Q_j^2}{G - 2X_2} - \frac{Q_j^2}{w + 2X_3} = (a^2 - 1)^2 w.
\]
It then gives us solution to the “coefficient” (12) which is
\[ Q_a^2 = 4a^2w \left( f^2G - j^2w \right). \] (16)

The remaining “coefficients” imply \( X_1 = 0 \) and \( X_0(f) = V \). From (16), we may conclude that
\[ Q = (a^2 - 1)\sqrt{w(f^2G - j^2w)}. \] In addition we still have other constraint equations comming from Euler-Lagrange equations of the BPS Lagrangian (5):

for \( a \),
\[ \frac{\partial X_2}{\partial a} f^2 + \frac{\partial X_3}{\partial a} j^2 = 0 \] (17)

for \( f \),
\[ 2 \frac{d}{dr} (r^2 X_2 f') = r^2 \frac{\partial X_0}{\partial f} + r^2 \frac{\partial X_2}{\partial f} f^2 + r^2 \frac{\partial X_3}{\partial f} j^2, \] (18)

and for \( j \),
\[ 2 \frac{d}{dr} (r^2 X_3 j') = r^2 \frac{\partial X_2}{\partial f} f^2 + r^2 \frac{\partial X_3}{\partial f} j^2. \] (19)

Using the BPS equations (7), (8), and (9) the constraint equation (17) becomes
\[ \frac{\partial X_2}{\partial a} \frac{Q_f^2}{(G - 2X_2)^2} + \frac{\partial X_3}{\partial a} \frac{Q_j^2}{(w + 2X_3)^2} = 0, \] (20)

while the constraint equation (18) is now
\[
\frac{\partial}{\partial a} \left( \frac{X_2 Q_f}{G - 2X_2} \right) \frac{Q_a}{w} + \frac{\partial}{\partial f} \left( \frac{X_2 Q_f}{G - 2X_2} \right) \frac{2Q_f}{r^2(G - 2X_2)} = \frac{2Q_f}{r^2(w + 2X_3)}.
\]

Following the same procedure, as to the constraint equation (10), implies \( X_0 \) is a constant that can be set to zero; \( \frac{X_3 Q_j}{w + 2X_3} \) is independent of \( a \); and
\[
\frac{\partial X_2}{\partial f} \frac{Q_f^2}{(G - 2X_2)^2} + X_2 \frac{\partial}{\partial f} \left( \frac{Q_f}{G - 2X_2} \right)^2 = \frac{\partial X_3}{\partial j} \frac{Q_j^2}{(w + 2X_3)^2} + X_3 \frac{\partial}{\partial j} \left( \frac{Q_j}{w + 2X_3} \right)^2.
\]

Similarly for the constraint equation (19) we obtain that \( \frac{X_3 Q_j}{w + 2X_3} \) is independent of \( a \) and
\[
\frac{\partial X_3}{\partial j} \frac{Q_j^2}{(w + 2X_3)^2} + X_3 \frac{\partial}{\partial j} \left( \frac{Q_j}{w + 2X_3} \right)^2 = \frac{\partial X_2}{\partial j} \frac{Q_j^2}{(G - 2X_2)^2} + X_2 \frac{\partial}{\partial j} \left( \frac{Q_j}{w + 2X_3} \right)^2.
\]

If we take \( X_0 = 0 \), the equations (18) and (19) are symmetric under \( f \leftrightarrow j \) and \( X_2 \leftrightarrow X_3 \). This may suggest that \( f \) and \( j \) are related, along with \( X_2 \) and \( X_3 \). Suppose that \( j \equiv j(f) \) then substituting this into equations (18) and (19) they both will be identical if only if \( j \propto f \) and \( X_2 \propto X_3 \).

2.2. The standard BPS Dyon

The standard BPS Dyon as discussed in [17] is given by setting \( j = \beta f \), with \( \beta \) is a real constant.

In this case we take \( Q_j = X_3 = 0 \) in the BPS Lagrangian (5) as such
\[
\mathcal{L}_{eff} - \mathcal{L}_{bps} = BPSa + BPSf + \frac{Q_a^2}{4r^2(w - r^2X_1)} + \frac{Q_j^2}{2r^4(G + \beta^2(-w) - 2X_2)} - \frac{2a^2f^2r^2(G - \beta^2w) + (a^2 - 1)^2 w}{2r^4} - V + X_0,
\] (24)
with BPS equations

\[
BPS_a = \frac{r^2 X_1 - w}{r^2} \left( a' + \frac{Q_a}{2(r^2 X_1 - w)} \right)^2, \tag{25}
\]

\[
BPS_f = \frac{2X_2 - G + \beta^2 w}{2} \left( f' + \frac{Q_f}{r^2 (2X_2 - G + \beta^2 w)} \right)^2. \tag{26}
\]

The constraint equations are now

\[
0 = 2w \left( (a^2 - 1)^2 w - \frac{Q_j^2}{G - \beta^2 w - 2X_2} \right), \tag{27}
\]

\[
0 = 4a^2 f^2 w (G - \beta^2 w) - Q_a^2 + 2X_1 \left( -(a^2 - 1)^2 w + \frac{Q_j^2}{G - \beta^2 w - 2X_2} - \frac{Q_j^2}{w + 2X_3} \right), \tag{28}
\]

\[
0 = 4a^2 f^2 X_1 (\beta^2 w - G) + 4w(V - X_0), \tag{29}
\]

\[
0 = 4X_1(X_0 - V). \tag{30}
\]

The only nontrivial solutions of these constraint equations are \(X_0 = V = 0, X_1 = 0,\) and \(X_2 = 0\) which are solutions for the standard BPS dyons [17] and hence give zero pressure.

### 3. BPS Lagrangian Method in General Formalism

The previous example only concerned on particular ansatz of the fields. Let us now consider more general case without taking any a priori ansatz. Defining \(E_i \equiv F_{0i}\) and \(B_i \equiv \frac{1}{2} \epsilon_{ijk} F_{jk},\) the Lagrangian density (1) for standard SU(2 Yang-Mills-Higgs model, with \(G = w = 1,\) can be written as

\[
\mathcal{L} = \text{tr} \left( E_i \right)^2 - \text{tr} \left( B_i \right)^2 + \text{tr} \left( D_0 \Phi \right)^2 - \text{tr} \left( D_i \Phi \right)^2 - V, \tag{31}
\]

with \(i = 1, 2, 3\) is the spatial indices.

Using the Bogomoly’s trick [8], one can obtain the well-known BPS equations for monopoles and dyons by squaring the energy density,

\[
E_i = \pm \sin \alpha \ D_i \Phi, \quad B_i = \pm \cos \alpha \ D_i \Phi, \quad D_0 \Phi = 0. \tag{32}
\]

However there is one constraint equation that must be considered, in addition to those BPS equations, in order to satisfy the Euler-Lagrange equations, and that is the Gauss’s law constraint,

\[
D_i F_{0i} = i e \ [\Phi, D_0 \Phi], \tag{33}
\]

which is essentially Euler-Lagrange equation for the gauge scalar potential \(A_0.\) Here, we assume the existence of the Bogomoly’s equations (32) and find what would be the BPS Lagrangian density. We then show that the Gauss’s law constraint comes form the existence of “non-boundary” terms in the BPS Lagrangian density. What we mean by “non-boundary” terms are terms in the BPS Lagrangian density that give non-trivial Euler-Lagrange equation. Generally, we write the BPS equations for monopoles and dyons as

\[
E_i = \alpha D_i \Phi, \quad B_i = \beta D_i \Phi, \quad D_0 \Phi = 0, \quad V = 0, \tag{34}
\]

with \(\alpha\) and \(\beta\) are arbitrary constants. In this case the Lagrangian density can be rewritten as

\[
\mathcal{L} = \text{tr} \left( E_i - \alpha D_i \Phi \right)^2 - \text{tr} \left( B_i - \beta D_i \Phi \right)^2 + \text{tr} \left( D_0 \Phi \right)^2 - V + \mathcal{L}_{BPS}, \tag{35}
\]
where the BPS Lagrangian is given by
\[
L_{BPS} = -2\beta \text{tr} \left( B_i D_i \Phi \right) + 2\alpha \text{tr} \left( E_i D_i \Phi \right) - \left( \alpha^2 - \beta^2 + 1 \right) \text{tr} \left( D_i \Phi \right)^2.
\] (36)

One can show that Euler-Lagrange equation of the first term in the BPS Lagrangian density above is trivial using the Bianchi identity \( D_i B_i = 0 \) and a relation \( \left[ D_i, D_j \right] \Phi = -ie [F_{ij}, \Phi] \), and hence it is indeed a boundary term. The remaining terms turn out to be “non-boundary” terms in which their Euler-Lagrange equations: for \( \Phi \),
\[
\alpha D_i F_{0i} - \left( \alpha^2 - \beta^2 + 1 \right) D_i D_i \Phi = 0;
\] (37)
for \( A_i \),
\[
\alpha \left( D_0 D_i \Phi - ie [F_{0i}, \Phi] \right) = ie \left( \alpha^2 - \beta^2 + 1 \right) [\Phi, D_i \Phi];
\] (38)
and for \( A_0 \),
\[
\alpha D_i D_i \Phi = 0.
\] (39)

The equations (37), (38), and (39) are additional constraint equations, beside the BPS equations (34), that must be considered in finding solutions for BPS monopoles and dyons. It seems that in total we have more equations than the number of fields to be solved. Now let see what happens to these additional constraint equations in a BPS limit, in which the BPS equations (34) are satisfied, and they now become respectively
\[
\left( 1 - \beta^2 \right) D_i D_i \Phi = 0,
\] (40)
\[
\left( 1 - \alpha^2 - \beta^2 \right) \left[ D_i \Phi, \Phi \right] = 0,
\] (41)
\[
\alpha D_i D_i \Phi = 0,
\] (42)

where we have used \( [D_0, D_i] \Phi = -ie [F_{0i}, \Phi] \). We could reduce the number of constraint equations by setting \( \alpha^2 + \beta^2 = 1 \) in which now there is only one constraint equation
\[
\alpha^2 D_i D_i \Phi = 0.
\] (43)

If \( \alpha \neq 0 \) then the constraint equation is \( D_i D_i \Phi = 0 \). Plugging it into the BPS equations (37) implies \( D_i F_{0i} = 0 \) which is the Gauss’s law constraint in the BPS limit, with \( D_0 \Phi = 0 \). If \( \alpha = 0 \) then \( \beta^2 = 1 \) and there is no constraint equation, but then the BPS equations (34) imply \( E_i = F_{0i} = 0 \) which automatically satisfies the Gauss’s law constraint in the BPS limit.

3.1. Stress-Energy-Momentum Tensor

The Lagrangian density (1) has the following stress-energy-momentum tensor:
\[
T_{\mu\nu} = 2G \text{tr} \left( D_\mu \Phi D_\nu \Phi \right) - 2w \text{tr} \left( F_{\lambda\mu} F^\lambda_{\nu} \right) - \eta_{\mu\nu} L.
\] (44)

For the case of standard \( SU(2) \) Yang-Mills, in which \( G, w = 1 \), the energy density in the BPS limit is
\[
T_{00} = 2 \text{tr} \left( D_i \Phi D_i \Phi \right),
\] (45)
while the momentum densities are all zero, \( T_{0i} = 0 \). The off-diagonal components of stress-tensor are
\[
T_{0i} = 2 \text{tr} \left( D_0 \Phi D_i \Phi \right) - 2 \text{tr} \left( F_{0\mu} F_i^\mu \right) = 2 \text{tr} \left( D_0 \Phi D_i \Phi \right) + 2\epsilon_{ijk} \text{tr} \left( E_j B_k \right).
\] (46)

\(^2\) We may take \( [D_i \Phi, \Phi] = 0 \), but it would not help us in reducing the number of constraint equations and of the arbitrary constants. Furthermore it would also restrict solutions for \( \Phi \).
In the BPS limit, they are given by
\[ T_{0i} = 2\epsilon_{ijk}\alpha\beta \text{tr} (D_j\Phi D_k\Phi) = 0, \quad (47) \]
while for the diagonal components\(^3\)
\[ T_{ii} = 2(1 - \alpha^2) \text{tr} (D_i\Phi D_i\Phi) + 2\beta^2 \text{tr} (D_j\Phi D_j\Phi) + (\alpha^2 - \beta^2 - 1) \text{tr} (D_i\Phi D_i\Phi + D_j\Phi D_j\Phi) = 0, \quad (48) \]
with index \(i\) is fixed and \(j \neq i\). So clearly the known BPS Monopoles and Dyons have zero stress-tensor.

4. Remarks
We have shown that the Bogomolny’s equations for monopoles(dyons) in the \(SU(2)\) Yang-Mills-Higgs theory, with particular hedgehog ansatz in spherical coordinates, correspond to BPS Lagrangian (5) with only boundary terms in the BPS Lagrangian method. We may try other possibility by allowing \(X_0, X_1, X_2, X_3\) in (5) to depend explicitly on coordinate \(r\) in general. However, this will bring more complications in the calculation. For more general coordinate systems, we found that the BPS Lagrangian contains non-boundary terms in addition to the boundary terms. The Euler-Lagrange equations of the non-boundary terms give an additional constraint equation which we may consider the constraint equation (43) as the Gauss’s law constraint in the BPS limit. Nevertheless, the existance of these non-boundary terms do not imply the stress-tensor to be non-zero in contradiction with the case of BPS Vortices in Maxwell-Higgs theory in two spatial dimension [16].

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\(^3\) Recall that in the BPS limit \(\alpha^2 + \beta^2 = 1\).