Mathematical modeling of intrinsic motivation in reversal theory

– Promoting exploration for AI agents

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Abstract: The function and performance of cloud connected products such as AI speakers are continuously updated over time. Such updates are based on the user’s exploration of unknown functions. Apter’s reversal theory proposed a mental condition termed the paratelic mode in which one acts to explore the purpose and enjoy certain actions in itself. We assume that the paratelic mode motivates users to explore continuously updated functions of cloud connected products which enables them to make full use of them. In this study, we aim to create a mathematical model that can explain the paratelic mode. We propose a model that explains the condition of the paratelic mode by integrating two principal motivation theories: Apter’s reversal theory and Berlyne’s optimal arousal level (OAL). We mathematically formulate the model by applying the Bayesian information gain as an index of arousal. By analyzing the model, we predict two hypotheses: a) when OAL is low, the lower the uncertainty, and the more likely it is that the paratelic mode is achieved, and b) when OAL is high, the higher the uncertainty, the more likely it is that the paratelic mode is achieved. The experimental result of our previous study using an AI speaker supported the former hypothesis. In this study, we verify the latter hypothesis by conducting an experiment using two AI speakers with different uncertainties. The results showed that when OAL is assumed to be high, users are more likely to be in the paratelic mode for an AI speaker which was subjectively evaluated to have higher uncertainty.

Keywords: Modeling, Motivation, AI, Reversal theory, Intrinsic motivation

1. INTRODUCTION

What is the process that leads to long-term use of a product? Krippendorff stated that there are three stages between the recognition of the product and its continuous use: perception, exploration, and reliance [1]. Here, exploration determines what the product can do and how it can be used. For cloud-connected products such as AI speakers, functions and services will expand over time. In such products, it is necessary to encourage users to continuously learn about the functions and performance that are updated over time. Therefore, in this study, we focused on the motivation for continuous exploration.

We consider that motivation to explore refers to the motivation to explore an unknown purpose. Ryan and Deci defined the motivation to act for a specific purpose as extrinsic motivation, and the motivation to act for potential satisfaction rather than for a certain purpose as intrinsic motivation [2]. Building on this classification of motivation based on the existence or absence of the purpose, Apter proposed two psychological states: the goal-oriented telic mode and action-oriented paratelic mode. He proposed the reversal theory, in which the two psychological modes are reversed depending on the situation [3]. In the paratelic mode, people search for the purposes. In this study, we consider that exploration could be encouraged by encouraging users to enter the paratelic mode. The reversal theory is a qualitative model, and there is no mathematical model that can be applied to product design. In addition, the conditions for achieving the paratelic mode in product use have not been clarified.

The purpose of this study is to determine a product design that motivates users to take exploratory actions. In order to achieve this, we propose a model that mathematically explains the paratelic mode of reversal theory. We apply the knowledge obtained from model prediction to the dialog interface in AI agent-equipped products and verify its effectiveness.

2. PROPOSAL OF A MATHEMATICAL MODEL THAT INTEGRATES OPTIMAL AROUSAL LEVEL AND REVERSAL THEORY

2.1 Correspondence between optimal arousal level and reversal theory

In Berlyne’s arousal potential theory, when the horizontal axis refers to the degree of arousal obtained from novelty and complexity, and the vertical axis refers to the degree of pleasant feeling (valence), it shows an inverted U shape, as shown in Figure 1. It is supposed to form a Wundt curve [4]. Based on this model, there are no emotions that are considered neither pleasant nor unpleasant in the low arousal level, and unpleasant in the high arousal level. There is an optimal arousal level that maximizes a pleasant feeling. Hereafter, we abbreviate the optimal arousal level as OAL.
According to the reversal theory, in the telic mode (goal-oriented), low arousal gives relief, and high arousal gives anxiety. In the paratelic mode (action-oriented), people get bored when their arousal is low, and they feel excited when their arousal is high [3]. In the telic mode, arousal and the degree of pleasant (hedonic tone) feeling are inversely proportional, and in the paratelic mode, they are proportional. Depending on the situation, OAL may fall in between the two curves, and the two psychological states are reversed, as shown in Figure 2.

![Figure 1: The optimal arousal level (OAL) in arousal potential theory](image1)

**Figure 1:** The optimal arousal level (OAL) in arousal potential theory

![Figure 2: Telic mode and Paratelic mode in reversal theory](image2)

**Figure 2:** Telic mode and Paratelic mode in reversal theory

In this study, we considered that the monotonically increasing part of Berlyne's inverted U-shaped curve corresponds to Apter's paratelic mode, and the decreasing part corresponds to the telic mode. However, the idea that the low arousal part of one inverted U-shaped curve is the paratelic mode and the high-arousal part is telic mode differs from that of Apter's theory. Apter stated that even with the same arousal, there are different psychological states: relief and boredom where arousal is low, and anxiety and excitement where the arousal is high. Therefore, we consider that the OAL of Berlyne shifts in the horizontal axis depending on the situation, as shown in Figure 3. In the part surrounded by the square frame in Figure 3, both telic and paratelic modes can be achieved even with the same arousal. Our theory explains both theories in a unified manner. We verify this unified model using a mathematical analysis.

![Figure 3: The proposed unified model of OAL and reversal theory](image3)

**Figure 3:** The proposed unified model of OAL and reversal theory

### 2.2 Mathematical model of information gain corresponding to arousal

Yanagisawa, the second author, proposed a perceptual model based on Bayes' theorem and experimentally verified this model [5]. Yanagisawa et al. further proposed that the arousal level of emotions is formulated using information gain (Bayesian surprise or KL divergence from posterior to prior) [6], which refers to the amount of information acquired after the experience of the event [7]. Assuming a normal distribution for the prior and posterior distributions of the Bayesian model, the information gain between these can be expressed by the function shown in Eq. (1).

\[
G = KL(prior|posterior) = \frac{1}{2} \left( \frac{s_p}{s_p + s_i} \right) \delta^2 + \log \frac{s_p + s_i}{s_i} - \frac{s_p}{s_p + s_i}
\]  

where \( \delta \) is the difference between the expected value of the prior distribution and the likelihood function. \( \delta \) represents the difference between the prior prediction and the actual stimulus, which is called the prediction error. \( s_p \) represents the uncertainty of expectation and the difficulty of making predictions. \( s_i \) is the variation of data, that is, the disturbance (noise) mixed with the sensory stimulus. Eq. (1) shows that the information gain is a function of three parameters \( \delta, s_p \) and \( s_i \). We use Eq. (1) as a mathematical formulation of arousal.

### 3. HYPOTHESES OBTAINED FROM ANALYSIS OF MATHEMATICAL MODEL

#### 3.1 Analysis of the relationship between information gain and prediction error

A previous study analyzed how the relationship between information gain and prediction error changes depending on the two states of uncertainty [7]. Consider two situations: one when the uncertainty is high (\( s_p \), 1), and one when the uncertainty is low (\( s_p \), 2). The noise (\( s_i \)) is
fixed to one value, assuming that the variation can be controlled depending on the experimental system. When the product of the two uncertainties is larger than the square of the noise \((s_p1 \times s_p2 > s_i^2)\), if the prediction error becomes large, reversal occurs.

### 3.2 Hypotheses obtained from model analysis

Consider the case where uncertainty is greater than the disturbance \((s_p1 \times s_p2 > s_i^2)\). In this case, from the analysis of 3.1, the relationship of the information gain under two uncertainties is reversed as the prediction error increases (shown in Figure 4 and 5). In our model, OAL signifies optimal information gain. Now, consider two states, a state where OAL is low (Wundt curve on the left in Figure 3) and a state where OAL is high (Wundt curve on the right in Figure 3). In these two states, the OALs (information gain at which valence peaks) are \(G_l\) and \(G_h\), respectively.

Figure 4 shows the range of telic and paratelic modes when the optimal information gain is low. From the intersection of \(G_l\) and information gain, the prediction error at the boundary between the telic and paratelic modes is larger when the uncertainty is low \((\delta_1 < \delta_2)\). On the other hand, as shown in Figure 5, in the case of \(G_h\), where OAL is high, there is a larger prediction error for higher uncertainty at the boundary between the telic and paratelic modes \((\delta_n1 > \delta_n2)\). Keeping the paratelic mode for a larger prediction error means that it is easy to enter the paratelic mode.

From the above discussion, we propose the following two hypotheses depending on the location of the OAL:

(a) When people are in a state of high OAL \((G_h)\), they are more likely to be in the paratelic mode with higher uncertainty.

(b) When people are in a state of low OAL \((G_l)\), they are more likely to be in the paratelic mode with lower uncertainty.

The experimental results of our previous study supported hypothesis (a) using AI speakers [8]. These findings were not based on the hypothesis derived from the proposed mathematical model. However, if hypothesis (b) is confirmed by operating OAL in higher conditions than those of the previous study [8], the prediction by the unified model can be verified.

### 4. VERIFICATION OF THE PROPOSED MODEL

#### 4.1 Method

To verify the hypothesis derived from the analysis of the proposed model, we conducted an experiment using two AI speakers, as in the previous study [8]. We controlled the parameters by asking the experiment participants to ask prepared questions for two AI speakers as learning. The speaker with low uncertainty always answers easy questions correctly and always answers difficult questions incorrectly. On the other hand, the other speaker with high uncertainty sometimes answers easy questions incorrectly and difficult questions correctly.

According to Figure 6, the range of the paratelic mode when the same information gain (arousal) is obtained is wider when the OAL is at a higher position \((G_h)\) than when it is at a lower position \((G_l)\). We consider that OAL would increase when the user was involved in the product in the paratelic mode. We then prepared questions that would allow the participants to engage in the paratelic mode for both conditions with low and high uncertainty. We confirmed the effect of learning by subjectively evaluating whether AI agent participants felt paratelic or telic motivation in a questionnaire.

#### 4.2 Results

Figure 7 shows the average value of how much 20 people were motivated. The degree of those in the paratelic mode is 0.975 out of a maximum of 5 in the condition with high uncertainty, and the degree of telic mode was 0.525 in the condition with low uncertainty. There was a significantly higher degree of paratelic mode for higher levels of uncertainty.
5. DISCUSSION

In hypothesis (a), if OAL is assumed to be low, people are more likely to go into the paratelic mode in situations with low uncertainty. The results of our previous study verified this hypothesis [8]. Apter proposed a notion of protective frame as a condition for promoting the paratelic mode [9]. The protective frame is a theory that states that if safety is guaranteed, a person will be in a psychological state that seeks more danger and excitement, that is, the paratelic mode. In state (a), we consider that the ease of making predictions for a product becomes a protective frame for the safety and credit of the product. Therefore, it is easy to enter the paratelic mode.

In hypothesis (b), when OAL is assumed to be high, people are the more likely to go into the paratelic mode in situations with larger uncertainty. In the subjective evaluation of this experiment, we found a significant difference in the fact that AI speakers with high uncertainty promote more paratelic motivation. This finding supports the prediction of the model. According to Figure 6, when OAL is high, there is a wide range of paratelic modes when the same information gain is obtained. We consider that the user is already involved in the product and the paratelic psychological state. In state (b), they trust the relationship with the product itself and develop a protective frame. Therefore, we consider that the more difficult it is to predict the behavior of a product, the more curious people will be about it and the more motivated they will be to explore the product.

6. CONCLUSION

In this study, we proposed a mathematical model that explains motivational states based on theories such as the reversal theory and arousal potential theory. We derived two hypotheses (shown in 3.2) by analyzing the mathematical model, using information gain as the arousal level. Our previous study supported the first hypothesis [8]. We conducted an experiment to verify the second hypothesis that when OAL is high, the higher the uncertainty, and the more likely it is to be in the paratelic mode. We found a significant difference in that participants had more paratelic motivation for AI speakers with higher uncertainty. Based on the findings of the previous study and the subjective evaluation of this study, we verified the hypothesis based on the model and its predictions.

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