The 0 to \( R_1 \) cylinder radial coordinate transformation cannot be used to make cylinder layer electromagnetic cloak

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In this paper, we discover and proved that the 0 to \( R_1 \) cylinder radial coordinate transformation cannot be used to make cylinder layer electromagnetic cloak. Patent of discover, create method and proof in this paper are reserved by authors in GL Geophysical Laboratory.

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I. INTRODUCTION

First in the world, in this paper, we discover and prove that the 0 to \( R_1 \) cylinder radial coordinate transformation method cannot be used for making cylinder layer EM cloak. Because in the 0 to \( R_1 \) cylinder radial coordinate transformation, the value area of the electric wave field \( E_z(\rho, \phi, z) \) and magnetic wave field \( H_z(\rho, \phi, z) \) are invariant. After 0 to \( R_1 \) cylinder radial transformation, \( E_z(\rho, \phi, z) = E_z(\rho, \phi, z) \) and \( H_z(\rho, \phi, z) = H_z(\rho, \phi, z) \), in the physical inner cylinder boundary, \( E_z(R_1, \phi, z) \neq 0 \) and \( H_z(R_1, \phi, z) \neq 0 \). The cylinder EM wave field will propagation penetrate into the inner cylinder \( \rho < R_1, |z| < \infty \) . Therefore, the inner cylinder \( \rho < R_1, |z| < \infty \), can not be cloaked. In many published papers on EM cylinder cloak, the 0 to \( R_1 \) cylinder radial transformation is wrongfully used for their EM cylinder layer cloak.

The description of this paper are as follows. Maxwell EM equation in the cylinder coordinate in free space is presented in section 2. 0 to \( R_1 \) cylinder radial coordinate transformation can not be used to make EM cloak is proposed in section 3. Discussion and conclusion is presented in section 4.

II. MAXWELL EM EQUATION IN THE CYLINDER COORDINATE IN FREE SPACE

In this section, let the \( H_z(\rho, \phi, z) \) be magnetic field component in \( Z \) direction. We describe magnetic field equation in the cylinder coordinate free space.

\[
\frac{\partial}{\partial \rho} \left[ \frac{\partial H_z}{\partial \rho} \right] + \frac{\partial^2 H_z}{\partial \phi^2} + \rho \frac{\partial^2}{\partial z^2} H_z + k^2 \rho H_z = M_s, \tag{1}
\]

where \( \rho \) is radial coordinate, \( \phi \) is angular coordinate, \( H_z(\rho, \phi, z) \) is the magnetic wave field, \( k = \omega \sqrt{\varepsilon \mu} \) is the constant EM wave number in free space, \( \omega \) is the angular frequency. \( \varepsilon \) is the basic electric permittivity in free space, \( \mu \) is the basic magnetic permeability in free space. The magnetic source \( M_s \) is induced by the point Delta electric source in \( \vec{e}_x = (1, 0, 0) \) direction,

\[
S(r, r') = \delta(\rho - \rho_s) \delta(\phi - \phi_s) \delta(z - z_s) \vec{e}_x, \tag{2}
\]

The incident magnetic wave field \( H_{z,i}(\rho, \phi, z) \) does satisfy the Maxwell magnetic equation (1) in free space,

\[
H_{z,i}(\rho, \phi, z) = -\sin \phi \frac{\partial}{\partial \rho} g(\rho, \phi, z) \\
- \cos \phi \frac{1}{\rho} \frac{\partial}{\partial \phi} g(\rho, \phi, z), \tag{3}
\]

\[
g(\rho, \phi, z) = -\frac{1}{4\pi} \frac{e^{ik \sqrt{\rho^2 - \rho_s^2 + (z - z_s)^2}}}{\sqrt{\rho^2 - \rho_s^2 + (z - z_s)^2}}, \tag{4}
\]

\[
\sqrt{\rho^2 - \rho_s^2 + (z - z_s)^2} = \sqrt{\rho^2 - 2\rho \rho_s (\cos \phi - \phi_s) + \rho_s^2 + (z - z_s)^2}, \tag{5}
\]

\[
H_{z,i}(\rho, \phi, z) = \frac{1}{4\pi} \frac{e^{ik \sqrt{\rho^2 - \rho_s^2 + (z - z_s)^2}}}{\left( \sqrt{\rho^2 - \rho_s^2 + (z - z_s)^2} \right)^2} \\
\left( ik - \frac{1}{\sqrt{\rho^2 - \rho_s^2 + (z - z_s)^2}} \right) (\rho \sin(\phi) - \rho_s \sin(\phi_s)), \tag{6}
\]

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III. 0 TO R₁ CYLINDER RADIAL COORDINATE TRANSFORMATION CAN NOT BE USED TO MAKE EM CLOAK

A. Homogeneous magnetic wave equation in the cylinder ρ ≤ R₂ and |z| < ∞

For R₂, 0 and ρ > R₂, in the cylinder ρ ≤ R₂ and |z| < ∞, the homogeneous magnetic wave equation is

\[ \frac{\partial}{\partial \rho} \left( \rho \frac{\partial H_\rho}{\partial \rho} \right) + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial H_\rho}{\partial \rho} \right) + \frac{\partial^2 H_\rho}{\partial z^2} + k^2 \rho H_\rho = 0, \]  

(7)

On the cylinder surface boundary ρ = R₂, the magnetic wave field \( H_z = H_z(\rho, \phi, z) \) and its derivative satisfy the continuous boundary conditions

\[ H_z(R^-_2, \phi, z) = H_z(R^+_2, \phi, z), \]

(8)

\[ \frac{\partial}{\partial \rho} H_z(R^-_2, \phi, z) = \frac{\partial}{\partial \rho} H_z(R^+_2, \phi, z), \]

(9)

B. 0 to R₁ cylinder radial coordinate transformation

For R₁ > 0, R₂ > R₁, inside of the cylinder ρ ≤ R₂, the 0 to R₁ cylinder radial continuous coordinate transformation is

\[ \rho_q(r) = R_1 + Q(\rho), \quad 0 \leq \rho \leq R_2, \]

(10)

\[ \rho_q(0) = R_1, \quad Q(0) = 0, \]

(11)

\[ \rho_q(R_2) = R_2, \quad Q(R_2) = R_2 - R_1, \]

(12)

\[ \frac{\partial}{\partial \rho} \rho_q(R_2) = 1, \quad \frac{\partial}{\partial \rho} Q(R_2) = 1, \]

(13)

The inverse mapping is

\[ \rho = Q^{-1}(\rho_q - R_1), \quad R_1 \leq \rho \leq R_2, \]

(14)

C. Magnetic equation in the cylinder layer \( R_1 \leq \rho \leq R_2 \)

To substitute 0 to R₁ cylinder radial coordinate transformation (10)-(14) into (7), the magnetic wave equation (6) in the free space cylinder ρ ≤ R₂ and |z| < ∞ is translated to the following anisotropic magnetic wave equation in the cylinder layer \( R_1 \leq \rho \leq R_2, \)

\[ \frac{\partial}{\partial \rho_q} \left( \frac{\partial H_{\rho_q}}{\partial \rho_q} \right) + \frac{1}{\rho_q} \frac{\partial}{\partial \rho_q} \left( \rho_q \frac{\partial H_{\rho_q}}{\partial \rho_q} \right) + \rho_q H_z \frac{\partial^2 H_z}{\partial z^2} + k^2 \rho_q \mu_z H_z = 0, \]

(15)

Where the relative parameters induced by the 0 to R₁ cylinder radial transformation (10-14) are

\[ \varepsilon_\rho = \mu_\rho = \rho \frac{d \rho_q}{\rho_q \frac{d \rho_q}{d \rho}}, \]

(16)

\[ \varepsilon_\phi = \mu_\phi = \rho_q \frac{d \rho}{\rho \frac{d \rho_q}{d \rho}}, \]

(17)

\[ \varepsilon_z = \mu_z = \rho \frac{d \rho_q}{\rho_q \frac{d \rho_q}{d \rho}}. \]

(18)

on the cylinder surface boundary ρ = R₂ the magnetic wave field solution \( H_z = H_z(\rho, \phi, z) \) of equation (15) and its derivative satisfy the following continuous boundary conditions

\[ H_z(R^-_2, \phi, z) = H_z(R^+_2, \phi, z), \]

(19)

\[ \frac{1}{\varepsilon_\phi} \frac{\partial}{\partial \rho_q} H_z(R^-_2, \phi, z) = \frac{\partial}{\partial \rho_q} H_z(R^+_2, \phi, z), \]

(20)

D. 0 to R₁ cylinder radial coordinate transformation can not be used to make cylinder layer EM cloak

**Theorem 1**: Suppose that the magnetic wave \( H_z(\rho_q, \phi, z) \) does satisfy the magnetic equation (15) with relative anisotropic EM parameters (16-18), and satisfy the field and derivative boundary conditions (19) and (20) that is necessary for no scattering from the cylinder ρ ≤ R₂, |z| < ∞, then \( H_z(\rho_q, \phi, z) \) has the analytic express as follows:

\[ H_z(\rho_q, \phi, z) = \frac{1}{\varepsilon_\phi} \left( \frac{i k}{\sqrt{|Q^{-1}(\rho_q - R_1)|^2 + (z - \rho s)^2}} \right) \left( \frac{1}{\sqrt{|Q^{-1}(\rho_q - R_1)|^2 + (z - \rho s)^2}} \right) \]

\[ \left( Q^{-1}(\rho_q - R_1) \sin(\phi) - \rho s \sin(\phi s) \right) , \]

(21)

Proof: Substitute the analytic magnetic wave field \( H_z(\rho_q, \phi, z) \) in (21) into the equation (15), by inverse transformation, \( H_z(\rho_q, \phi, z) \) is put back to \( H_z(\rho, \phi, z) \) in (6) which is solution of (7). Therefore, \( H_z(\rho_q, \phi, z) \) does satisfy the acoustic equation (15) and does satisfy the no scattering boundary condition (19) and (20). The theorem is proved. In next paper, we will use GLHUA analytical expand method to prove the theorem.

**Theorem 2**: Suppose that the relative electric permittivity and magnetic permeability are induced by 0 to
The magnetic wave does satisfy the necessary no scattering boundary condition on the outer and inner boundary. The magnetic field $H_z(\rho, \phi, z)$ is propagation penetrate into the inner cylinder $\rho \leq R_1$, $|z| < \infty$. The inner cylinder $\rho \leq R_1$, $|z| < \infty$. can not be cloaked.

Proof: When the point is on the inner cylinder surface boundary $\rho_q = R_1$, and $|z| < \infty$. Substitute $\rho_q = R_1$ into (21), we have

$$H_z(\rho_q, \phi, z)|_{\rho_q = R_1} = \left\{ \begin{array}{l} \frac{1}{4\pi} e^{ik\sqrt{\rho_q^2 + (z-z_0)^2}} \\ R_1 \left( i k - \frac{1}{\sqrt{|\rho - R_1 - \rho_q^2 + (z-z_0)^2|}} \right) \end{array} \right.$$

and the following boundary conditions on the $\rho = R_1$, that is necessary no scattering from inner cylinder $\rho \leq R_1$, $|z| < \infty$

$$H_z(R_1, z) = -\frac{i}{2\pi} \rho_s \sin(\phi_s) \frac{e^{ik\sqrt{|\rho^2 + (z-z_0)^2|}}}{\sqrt{|\rho^2 + (z-z_0)^2|}}$$

(29)

And

$$\frac{\partial}{\partial \rho} H_z(\rho, z)|_{\rho = R_1} = 0$$

(30)

We expand the boundary value of the magnetic wave, $H_z(R_1, z)$ in (29) as

$$H_z(R_1, z) = \int_0^\infty h_g(R_1, k_z) \cos(k_z z) dk_z$$

(31)

We expand the magnetic wave solution of (28), $H_z(\rho, z)$ as

$$H_z(\rho, z) = \int_0^\infty h(\rho, k_z) \cos(k_z z) dk_z$$

(32)

For any $k_z \geq 0$, $h(\rho, k_z)$ does satisfy the following Bessel equation

$$\frac{\partial}{\partial \rho} \rho \frac{\partial h}{\partial \rho} - \rho k_z^2 h + k^2 \rho h = 0$$

(33)

$$h(R_1, k_z) = h_g(R_1, k_z)$$

(34)

$$\frac{\partial}{\partial \rho} h(\rho, k_z)|_{\rho = R_1} = 0$$

(35)

For $0 \leq k_z \leq k$, the solution of (33)-(35), $h(\rho, k_z)$ is

$$h(\rho, k_z) = \frac{1}{2} \sqrt{k^2 - k_z^2} R_1 h_g(R_1, k_z) \left( N_0 \left( R_1 \sqrt{k^2 - k_z^2} \right) J_0 \left( \rho \sqrt{k^2 - k_z^2} \right) - J_0 \left( R_1 \sqrt{k^2 - k_z^2} \right) N_0 \left( \rho \sqrt{k^2 - k_z^2} \right) \right)$$

(36)

For $k_z \geq k$, the solution of (33)-(35), $h(\rho, k_z)$ is

$$h(\rho, k_z) = -\sqrt{k_z^2 - k^2} \sqrt{R_1} h_g(R_1, k_z) \left( K_0 \left( R_1 \sqrt{k_z^2 - k^2} \right) I_0 \left( \rho \sqrt{k^2 - k_z^2} \right) - I_0 \left( R_1 \sqrt{k_z^2 - k^2} \right) K_0 \left( \rho \sqrt{k^2 - k_z^2} \right) \right)$$

(37)

Summary, the magnetic wave solution of (28) C(30) in the inner cylinder,

$$H_z(\rho, z)$$

is

$$H(\rho, z) = \frac{1}{2} \left( I_0 \left( R_1 \sqrt{k^2 - k_z^2} \right) J_0 \left( \rho \sqrt{k^2 - k_z^2} \right) \right)$$

(38)
The theorem 2 is proved. The magnetic field $H_2(\rho, \phi, z)$ is propagation penetrate into the inner cylinder $\rho \leq R_1$, $|z| < \infty$. The inner cylinder $\rho \leq R_1$, $|z| < \infty$, can not be cloaked.

Similarly, The electric wave propagation penetrate into the inner cylinder, The inner cylinder $\rho \leq R_1$, $|z| < \infty$, can not be cloaked.

IV. DISCUSSION AND CONCLUSION

In many published paper[4], authors proposed 0 to $R_1$ cylinder radial linear transformation for cylinder layer EM cloak,however the EM wave propagation penetrate into their inner cylinder $\rho \leq R_1$, $|z| < \infty$. Their inner cylinder can not be cloaked. The 0 to $R_1$ cylinder radial coordinate transformation can not be used to make cylinder layer electromagnetic cloak.

That necessary no scattering boundary conditions induced that there is unbounded spherical symmetry magnetic wave $H(\rho, z)$ propagation in the inner cylinder $\rho < R_1$. If the inner magnetic wave $H(\rho, z)$ meet the some EM scattering object in inner sphere, for example a fly, the EM scattering wave will propagation go out outside of the cylinder $\rho \leq R_2$. The cylinder $\rho \leq R_2$ will be detected. Also, the linear 0 to $R_1$ cylinder transformation makes the infinite and exceeding light speed propagation. In next papers, we will propose a novel transformation [8] for EM cylinder cloak that will overcome the difficult on the EM wave propagation penetrate into the inner cylinder $\rho < R_1$. It is totally different from transformation cylinder cloak,our GLHUA double layer cylinder EM invisible cloak by [1,2,3-5,6,7,8] and GLHUA seismic double cloak and their exact analytical wave propagation will overcome all difficulties in 0 to $R_1$ transformation cylinder cloak.

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