Charge effect and finite ’t Hooft coupling correction on drag force and Jet Quenching Parameter

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Abstract
The effects of charge and finite ’t Hooft coupling correction on drag force and jet quenching parameter are investigated. To study charge effect and finite ’t Hooft coupling correction, we consider Maxwell charge and Gauss-Bonnet terms, respectively. The background is Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity. It is shown that these corrections affect drag force and jet quenching parameter. We find an analytic solution of drag force in this background which depends on Gauss-Bonnet coupling and charge. We set Gauss-Bonnet coupling to be zero and find drag force in the case of Reissner-Nordström-AdS background. Also we discuss the relaxation time of a moving heavy quark in this gravity background.
I. INTRODUCTION

The experiments of Relativistic Heavy Ion Collisions (RHIC) have produced a strongly-coupled quark-gluon plasma (QGP) [1]. The AdS/CFT correspondence [2–5] has yielded many important insights into the dynamics of strongly-coupled gauge theories. It has been used to investigate hydrodynamical transport quantities in various interesting strongly-coupled gauge theories where perturbation theory is not applicable. Methods based on AdS/CFT relate gravity in $AdS_5$ space to the conformal field theory on the 4-dimensional boundary. It was shown that an $AdS$ space with a black brane is dual to conformal field theory at finite temperature. In the framework of $AdS/CFT$, an external quark is represented as a string dangling from the boundary of $AdS_5$-Schwarzschild and a dynamical quark is represented as a string ending on flavor D7-brane and extending down to some finite radius in $AdS$ black brane background. Then one can calculate energy loss of quarks to the surrounding strongly-coupled plasma.

One of the interesting properties of the strongly-coupled plasma at RHIC is jet quenching of partons produced with high transverse momentum. The jet quenching parameter controls the description of relativistic partons and it is possible to employ the gauge/gravity duality and determine this quantity at the finite temperature theories. There has been the $AdS/CFT$ calculation of jet quenching parameter [6–15] and the drag coefficient which describes the energy loss for heavy quarks in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [16–20, 22, 23].

The universality of the ratio of shear viscosity $\eta$ to entropy density $s$ [25–28] for all gauge theories with Einstein gravity dual raised the tantalizing prospect of a connection between string theory and RHIC. The results were obtained for a class of gauge theories whose holographic duals are dictated by classical Einstein gravity. But string theory contains higher derivative corrections from stringy or quantum effects, such corrections correspond to $1/\lambda$ and $1/N$ corrections. In the case of $\mathcal{N} = 4$ super Yang-Mills theory, the dual corresponds to type IIB string theory on $AdS_5 \times S^5$ background. The leading order corrections in $1/\lambda$ arises from stringy corrections to the low energy effective action of type IIB supergravity, $\alpha'^3 R^4$.

Recently, $\frac{2}{\pi}$ has been studied for a class of CFTs in flat space with Gauss-Bonnet gravity [29–34]. They compute the effect of $R^2$ corrections to the gravitational action in AdS space and show that the conjecture lower bound on the $\frac{2}{\pi}$ can be violated. It was shown that in the Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity, $\frac{2}{\pi}$ bound is violated [34], and the Maxwell charge slightly reduces the deviation. Regarding this study and motivated by the vastness of the string landscape, one can explore the modification of jet quenching parameter and drag force on a moving heavy quark in the strongly-coupled plasma. In general, we don’t know about forms of higher derivative corrections in string theory, but it is known that due to string landscape one expects that generic corrections can occur.

In this paper, we investigate the corrections on jet quenching parameter and drag force on a moving heavy quark in the Super Yang-Mills plasma using the $AdS/CFT$ correspondence. We investigate the effects of charge and finite 't Hooft coupling correction on drag force and jet quenching parameter. The finite 't Hooft coupling correction can be considered as stringy correction from $AdS/CFT$. To study this correction, we consider Gauss-Bonnet terms. The effect of the charge is considered by adding Maxwell charge. As it has been proposed in [43], one can consider charge either as the R-charge or baryon charge. In the latter case, one can interpret charge effect as the effect of finite baryon density. The background is Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity. It is shown that these corrections affect drag force and jet quenching parameter. We find an analytic solution of drag force in this background which depends on Gauss-Bonnet coupling and charge. We set Gauss-Bonnet coupling to be...
zero and find drag force in the case of Reissner-Nordstrom-AdS background, too.

In $\mathcal{N} = 4$ SYM plasma, the gravity dual is type IIB superstring theory and the inverse ’t Hooft coupling correction to the jet quenching parameter and drag force has been found in [12, 42]. It is known that the curvature-squared corrections are not the first higher derivative corrections in type IIB superstring theory. These corrections on drag force have been studied in [41]. The effects of finite-coupling corrections and charge are calculated in [18]. We produce the result of $\mathcal{N} = 4$ super Yang-Mills plasma case in (18). We also derive drag force in the case of Reissner-Nordström-AdS black brane solution. In this case, we have considered only charge effects to the drag force.

The article is organized as follows. In section 2, we introduce the action which describes the Gauss-Bonnet term and $U(1)$ gauge field. Then we calculate drag force and study different limits of it. In the next section we calculate jet quenching parameter and study the effect of the corrections on this parameter. Finally, in the last section we discuss our results together with possible extensions for future work.

II. CHARGE EFFECT AND FINITE COUPLING CORRECTIONS ON THE DRAG FORCE

We study the gauge theory at finite temperature $T$ and assume the geometry has a black hole. In the gauge theory side, an external quark can be introduced by a string that has a single end point at the boundary and extends down to the horizon. We study the black holes with higher derivative curvature in Anti-de Sitter space. In five dimensions, the most general theory of gravity with quadratic powers of curvature is Einstein-Gauss-Bonnet (EGB) theory. The exact solutions and thermodynamic properties of the black branes in the Gauss-Bonnet gravity have been discussed in [37–39]. The authors in [29, 31, 33, 34] showed that for a class of CFTs with Gauss-Bonnet gravity dual, the ratio of shear viscosity to entropy density could violate the conjectured viscosity bound. We try to understand more about the drag force on a moving heavy quark in the boundary gauge theory by string trailing in the Gauss-Bonnet gravity.

To study the effects of charge and finite ’t Hooft coupling, we consider the Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity [40]. The following action in 5 dimensions describes the Gauss-Bonnet term and $U(1)$ gauge field

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \Lambda + \lambda_{GB} \left( R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right) - 4\pi G_5 F_{\mu\nu}F^{\mu\nu} \right], \quad (1)$$

where $\lambda_{GB}$ is Gauss-Bonnet coupling constant and the negative cosmological constant is related to radius of AdS space by $\Lambda = -\frac{12}{R^2}$. We also know from the standard Maxwell action that the field strength is given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The charged black brane solution in 5-dim is

$$ds^2 = -N^2 R^{-2} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + \frac{r^2}{R^2} d\vec{x}^2, \quad (2)$$

where

$$f(r) = \frac{1}{2\lambda_{GB}} \left( 1 - \sqrt{1 - 4\lambda_{GB} \left( 1 - \frac{mR^2}{r^4} + \frac{q^2 R^2}{r^6} \right)} \right). \quad (3)$$
In our coordinates, $r$ denotes the radial coordinate of the black hole geometry and $t, \vec{x}$ label the directions along the boundary at the spatial infinity. In these coordinates the event horizon is located at $f(r_h) = 0$ where $r_h = r_+$ is the largest root and it can be found by solving this equation. The boundary is located at infinity and the geometry is asymptotically AdS with radius R. The electric field strength for the Maxwell charge $q$ is $F \equiv \frac{q^2}{4\pi \ell^2} dt \wedge dr$. The gravitational mass of $M$ and the charge $Q$ are expressed as $M = \frac{3V_5 R^2 m}{16\pi G_5}$, $Q = \frac{q^2}{12\pi}$. The constant $N^2$ is arbitrary which specifies the speed of light of the boundary gauge theory and we choose it to be unity. As a result at the boundary, where $r \to \infty$, 

$$f(r) \to \frac{1}{N^2}, \quad N^2 = \frac{1}{2} \left( 1 + \sqrt{1 - 4\lambda_{GB}} \right).$$

(4)

Beyond $\lambda_{GB} \leq \frac{1}{4}$ there is no vacuum AdS solution and one cannot have a conformal field theory at the boundary. Casuality leads to new bounds on $\lambda_{GB}$. The authors in [31, 35] found that in five dimensions $\lambda_{GB}$ is less than .09 and when dimensions of space-time go up, casuality restricts the value of $\lambda_{GB}$ in the region $\lambda \leq 0.25$.

The temperature of the hot plasma is given by the Hawking temperature of the black hole

$$T = \frac{N r_h}{2\pi R^2} \left( 2 - \frac{q^2 R^2}{r_h^2} \right) = \frac{N r_h}{2\pi R^2} (2 - \tilde{q}) = \frac{1}{2} T_0 N (2 - \tilde{q}) .$$

(5)

where $\tilde{q} = \frac{q^2 R^2}{r_h^2}$ and $T_0$ is the temperature of AdS black hole solution without any corrections. In this limit $\tilde{q} \to 2$, the black brane approaches to the extremal case. We introduce a new dimensionless coordinate $u = \frac{r_h}{r}$, the five-dimensional metric (2) is deformed as 

$$ds^2 = - \frac{N^2 f(u) r_h^2}{R^2 u} dt^2 + \frac{R^2 du^2}{4w^2 f(u)} + \frac{r_h^2}{R^2 u} d\vec{x}^2 ,$$

(6)

where

$$f(u) = \frac{1}{2\lambda_{GB}} \left( 1 - \sqrt{1 - 4\lambda_{GB} (1 - u) (1 + u - \tilde{q} u^2)} \right) .$$

(7)

Now in this coordinate, the event horizon is located at $u = 1$, while $u = 0$ is located at the boundary of the AdS space.

The relevant string dynamics is captured by the Nambu-Goto action

$$S = -\frac{1}{2\pi \alpha'} \int d\tau d\sigma \sqrt{-\det g_{ab}} ,$$

(8)

where the coordinates $(\sigma, \tau)$ parameterize the induced metric $g_{ab}$ on the string world-sheet and $X^\mu(\sigma, \tau)$ is a map from the string world-sheet into the space-time. Defining $\dot{X} = \partial_\tau X$, $X' = \partial_\sigma X$, and $V \cdot W = V^\mu W^\nu G_{\mu\nu}$ where $G_{\mu\nu}$ is the AdS black brane metric (2), we have

$$- \det g_{ab} = (\dot{X} \cdot X')^2 - (X')^2 (\dot{X})^2 .$$

(9)

One can make the static choice $\sigma = u$, $\tau = t$ and following [18, 19] focus on the dual configuration of the external quark moving in the $x^1$ direction on the plasma. The string in this case, trails behind its boundary endpoint as it moves at constant speed $v$ in the $x^1$ direction

$$x^1(u, t) = vt + \xi(u), \quad x^2 = 0, \quad x^3 = 0 .$$

(10)
given this, one can find the lagrangian as follows

\[ L = \sqrt{-\det g_{ab}} = r_h \sqrt{\frac{N^2}{4u^3} - \frac{v^2}{4u^3 f(u)} + \frac{f(u) N^2 r_h^2}{R^4 u^2} \xi'^2}, \tag{11} \]

The equation of motion for \( \xi \) implies that \( \frac{\partial L}{\partial \xi'} \) is a constant. One can name this constant as \( \Pi_{\xi} \) and it can be found from \( \text{(11)} \). We solve the relation for \( \xi' \), the result is

\[ \xi'^2 = \frac{R^4 \Pi_{\xi}^2}{4 u f(u) N^2 r_h^2} \left( \frac{N^2 - \frac{v^2}{f(u)}}{\frac{R^4 u^2}{f(u) N^2 r_h^2} - \Pi_{\xi}^2} \right). \tag{12} \]

We are interested in a string that stretches from the boundary to the horizon. In such a string, \( \xi'^2 \) remains positive everywhere on the string. Hence both numerator and denominator change sign at the same point and with this condition, one can find the constant of motion \( \Pi_{\xi} \) in terms of the critical value of \( u_c \) as follows

\[ \Pi_{\xi} = \frac{v r_h^2}{R^2 u_c}, \tag{13} \]

Numerator and denominator in \( \text{(12)} \) change sign at \( u_c \) and it can be found by solving this equation

\[ \frac{1}{2\lambda_{GB}} \left( 1 - \sqrt{1 - 4\lambda_{GB} (1 - u_c) (1 + u_c - \tilde{q} u_c^2)} \right) - \frac{v^2}{N^2} = 0. \tag{14} \]

The drag force that is experienced by the heavy quark is calculated by the current density for momentum along \( x^1 \) direction. After straightforward calculations, the drag force is easily simplified in terms of \( \Pi_{\xi} \)

\[ F = -\frac{1}{2\pi \alpha'} \Pi_{\xi}. \tag{15} \]

Plugging these relations into the drag force leads to the following result

\[ F(\lambda_{GB}, q) = -\frac{r_h^2}{2\pi \alpha' R^2} \left( \frac{v (24 N^4)^{\frac{1}{3}} (N^4 - N^2 v^2 + \lambda_{GB} v^4) (1 + \tilde{q}) + 2 \tilde{q} v \tilde{y}}{6^{\frac{4}{3}} N^{\frac{4}{3}} (N^4 - N^2 v^2 + \lambda_{GB} v^4) \tilde{y}} \right), \tag{16} \]

where \( \tilde{y} \) is a function of coupling constant of Gauss-Bonnet, charge and velocity of heavy quark as

\[ \tilde{y} = -9 \tilde{q} \left( N^4 - N^2 v^2 + \lambda_{GB} v^4 \right)^2 + \sqrt{3} \sqrt{(N^4 - N^2 v^2 + \lambda_{GB} v^4)^3 \left( 27 \tilde{q}^2 \lambda_{GB} v^4 - 27 \tilde{q}^2 N^2 v^2 - (\tilde{q} - 2)^2 (1 + 4 \tilde{q}) N^4 \right)}. \tag{17} \]

As sequence, drag force depends on the coupling constant of Gauss-Bonnet \( \lambda_{GB} \) and Maxwell charge \( q \). One can express the drag force in \( \text{(16)} \) in terms of gauge theory parameters. Notice that \( \tilde{q} = \frac{R^2}{r_h^2} q^2 \) then one should use \( \text{(5)} \) to write \( r_h \) in terms of the temperature of the hot plasma.

It would be interesting to discuss the extremal case, too. For \( \tilde{q} = 2 \), the temperature of the plasma is zero and the black hole becomes an extremal one with a double zero at the horizon. We note that in the extremal case, \( \tilde{q} = 2 \) the drag force is regular \( \text{[24]} \).
A. special limits of drag force

In this section, we will discuss different limits of drag force. Drag force in these limits can be calculated directly from the related action. Here, we derive results from the drag force in (16).

First of all, let us consider the case of $\lambda_{GB} \to 0$ and $q \to 0$ in (16). In this limit, one does not consider any correction in the action (1). The drag force $F(\lambda_{GB} \to 0, q \to 0)$ is nothing but the drag force in the case of $\mathcal{N} = 4$ strongly-coupled SYM plasma $F_{\mathcal{N}=4}$. The authors of [18, 19] have obtained

$$F_{\mathcal{N}=4} = -\left( \frac{\pi \sqrt{\lambda T^2_0}}{2} \right) \frac{v}{\sqrt{1 - v^2}}.$$  

(18)

where $\lambda$ is ’t Hooft coupling.

Now, we will investigate the drag force in two following special limits.

- In the case of $\lambda_{GB} \neq 0$, $q \to 0$.

In this limit, the action (1) is reduced to the Gauss-Bonnet action without Maxwell action. It corresponds to the effect of finite-coupling corrections to the drag force on a moving heavy quark in the Super Yang-Mills plasma. These corrections are related to curvature-squared corrections in the corresponding gravity dual [41, 42]. We name this case as $F(\lambda_{GB}) = F(\lambda_{GB}, q \to 0)$. One can find drag force in this limit from (16) and derive this result

$$F(\lambda_{GB}) = -\left( \frac{\pi \sqrt{\lambda T^2_{GB}}}{2} \right) \frac{v}{\sqrt{N^4 - N^2 v^2 + \lambda_{GB} v^4}}.$$  

(19)

where $T_{GB}$ is the Hawking temperature of Gauss-Bonnet black hole and is defined as

$$T_{GB} = \frac{N r_h}{\pi R^2}.$$  

(20)

As it is seen from Eq. (19), the drag force depends on the Gauss-Bonnet coupling constant. It is worse to mention that our result is exactly the same as ones which are obtained in [41, 42].

- In the case of $q \neq 0$ and $\lambda_{GB} \to 0$.

In this case, we consider only the Maxwell action in (1) where it has the Reissner-Nordström-AdS black brane solution. This is the first calculation of drag force in this background. One can follow the definition of drag force in (15) and derive the following result from the Reissner-Nordström-AdS black brane solution.

Applying this limit, drag force is found as

$$F(\bar{q}) = -\frac{r_h^2}{2\pi \alpha' R^2} v$$
The relevant Wilson loop is a rectangle with large extension $L^-$ in the $x^-$ direction and small extension $L$ along the $x^2$ direction. Then, $\hat{q}$ is given by the large $L^+$ behavior of the Wilson loop

$$W = e^{-\frac{1}{4\sqrt{2}\hat{q}}L^+L^2}.$$  

We choose the static gauge in which

$$\tau = x^- , \sigma = x^2.$$  

By assuming a profile of $u = u(\sigma)$ and substituting induced metric of fundamental string into the Nambu-Goto action (8), one obtains

$$S = \frac{L^-}{\sqrt{2\pi\alpha'}} \int_0^L d\sigma \frac{r_h}{Ru} \sqrt{(1 - f(u))N^2} \left( \frac{r_h^2}{R^2} - \frac{u'^2R^2}{4uf(u)} \right).$$  

where $u' = \partial_\sigma u$. Regarding $\sigma$ as time, this point that the lagrangian is independent of time implies that the energy is conserved. The boundary condition for bulk coordinate $r$ is

$$r \left( \pm \frac{L}{2} \right) = \infty.$$  

and such an embedding preserves a symmetry \( r(\sigma) = r(-\sigma) \). This symmetry implies that \( r'(-\sigma) = 0 \) and one concludes that at \( \sigma = 0 \) the string world sheet must reach the horizon, \( r(\sigma = 0) = r_0 \). Now from (26), one can find the transverse width of Wilson loop, \( L \).

We must subtract from action \( S \), the self-interaction of isolated quark and antiquark, which corresponds to the Nambu-Goto action of two strings that extend from boundary \( (u \to 0) \) to the horizon \( (u = 1) \). This action is \( S_0 \) and one can calculate it. The resulting \( S_I = S - S_0 \) is the extremal action in the definition of jet quenching parameter in [6]. There are some points about the extremal action in [14, 15].

We follow the approach [6] and calculate jet quenching parameter. In this way, we are interested in the expectation value of thermal Wilson loop as \( L \to 0 \). After applying this limit on \( S \), the leading order term is canceled by subtracting \( S_0 \) in \( S_I \). Considering metric in (6), \( S_I \) can be found as the following

\[
S_I = \frac{r_0^3 L^2}{4 \sqrt{2} R^4 \pi \alpha'} I^{-1},
\]

where \( I \) is

\[
I = \int_0^1 du \frac{u}{\sqrt{f(u)(1 - f(u))}}.
\]

and \( f(u) \) was defined in (7).

Using Eq. (28) and the relation \( \alpha' = \frac{R^2}{\sqrt{\lambda}} \), the jet quenching parameter is

\[
\hat{q} = \frac{2 r_0^3 \sqrt{\lambda}}{R^6 \pi} I^{-1}.
\]

Let us study the behavior of jet quenching parameter in terms of Gauss-Bonnet coupling constant and charge corrections.

First, we consider there are no charges in (30). In this case, \( I_{GB} \) is defined as

\[
I_{GB} = \int_0^1 du \frac{u}{\sqrt{h(u)(1 - h(u))}}.
\]

where

\[
h(u) = \frac{1}{2 \lambda_{GB}} \left(1 - \sqrt{1 - 4 \lambda_{GB}(1 - u^2)}\right).
\]

Using the above equation, we have plotted \( I^{-1}_{GB} \) versus \( \lambda_{GB} \) in the left plot of figure 1. Based on this result, one finds that the jet quenching parameter is enhanced due to the Gauss-Bonnet corrections with positive \( \lambda_{GB} \), while \( \hat{q} \) decreases with negative \( \lambda_{GB} \). It is worse to mention that our results are in good agreement with [15].

Now, we discuss the jet quenching parameter at the presence of finite charge and no Gauss-Bonnet corrections. In this case, one finds

\[
I_{\text{charge}} = \int_0^1 du \frac{u}{\sqrt{g(u)(1 - g(u))}},
\]

where

\[
g(u) = (1 - u) \left(1 + u - \hat{q} u^2 \right).
\]
FIG. 1: The left figure shows $I^{-1}_{GB}$ versus $\lambda_{GB}$. In this figure, we consider only Gauss-Bonnet corrections. The figure on the right shows $I^{-1}_{\text{charge}}$ versus $\tilde{q}$. In this case we consider only the effect of the charge.

We have also plotted $I^{-1}_{\text{charge}}$ versus $\tilde{q}$ in the right plot of figure 1. Based on this result, one can find the jet quenching parameter. As it is obviously seen from this plot, there is a maximum value for $I^{-1}_{\text{charge}}$ at $\tilde{q}_c = 0.750$. This critical value of charge can be found by studying the derivative of (33). In other words, $I^{-1}_{\text{charge}}$ is increased with $\tilde{q}$ till $\tilde{q}_c$ and then it is decreased and goes to zero at maximum value of charge $\tilde{q} = 2$.

If we set charge and Gauss-Bonnet coupling corrections to be zero, one finds that

$$I_0 = \int_0^1 du \sqrt{\frac{1}{u(1-u^2)}} = \frac{2\pi^{\frac{3}{2}} \Gamma[\frac{5}{4}]\Gamma[\frac{3}{4}]}{\Gamma[\frac{7}{4}]}.$$  \hspace{1cm} (35)

This case is related to $\mathcal{N} = 4$ SYM theory. It is straightforward to derive Jet quenching parameter as

$$\hat{q}_{\mathcal{N}=4} = \frac{\pi^{\frac{3}{2}} \Gamma[\frac{5}{4}]\Gamma[\frac{3}{4}]}{\Gamma[\frac{7}{4}]} \sqrt{\lambda} T_0^3.$$  \hspace{1cm} (36)

where $T_0$ is the temperature of the SYM $\mathcal{N} = 4$ plasma. This result is exactly the same as the result in [6].

One can discuss the jet quenching parameter at different temperatures in the experimentally relevant range. In this case, there are different temperature matching schemes in [20].

IV. DISCUSSION

In this paper, we have computed the drag force on a moving heavy quark and the jet quenching parameter in a field theory dual to gravity with Gauss-Bonnet terms. In addition to previous publications here a Maxwell charge has been added. Also the jet quenching parameter in this background is new.

Recently, $\frac{2}{\lambda}$ has been studied for a class of CFTs in flat space with Gauss-Bonnet gravity [29-34]. They compute the effect of $R^2$ corrections to the gravitational action in AdS space
and show that the conjecture lower bound on the $\frac{\eta}{s}$ can be violated. Also it was shown that in the Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity, $\frac{\eta}{s}$ bound is violated [34]. Actually the Maxwell charge slightly reduces the deviation. As long as the universality of $\frac{\eta}{s}$ bound is not settled, a detailed phenomenology of theories with gravity dual is interesting and might reveal the common conceptual reasons for violating or obeying the viscosity bound.

In this study, we have considered two corrections on the drag force and jet quenching parameter; finite coupling correction and charge effect. To study finite coupling correction, we considered Gauss-Bonnet terms. The effect of the charge is considered by adding Maxwell charge where one can interpret charge effect as the effect of finite baryon density [43]. The background is Reissner-Nordström-AdS black brane solution in Gauss-Bonnet gravity.

It is shown that these corrections affect drag force and jet quenching parameter. We found an analytic solution of drag force in this background which depends on Gauss-Bonnet coupling and charge. We also derived drag force in the case of Reissner-Nordström-AdS black brane solution. In this case, we have considered only charge effects to the drag force.

From phenomenology point of view, we discuss relaxation time and diffusion coefficient of a non-relativistic heavy quark. Relaxation time and diffusion coefficient of a non-relativistic heavy quark have been studied in [18, 20, 21]. The effect of curvature-squared corrections on these quantities has been studied in [41]. We discuss the finite coupling corrections on thermal transport properties of a moving heavy quark in gauss Bonnet gravity. The drag force has been calculated in (37), we use this expression and find relaxation time and diffusion coefficient of a non-relativistic heavy quark.

The drag force in the Gauss-Bonnet background is given by

$$F(\lambda_{GB}) = -\frac{\pi}{2} \sqrt{\lambda} T^2_{GB} \frac{v}{\sqrt{N^2(N^2 - v^2) + \lambda_{GB}v^4}}. \quad (37)$$

we follow the approach in [41]. Because of the small velocity of non-relativistic heavy quark, one can neglect the squared-velocity term in (37) and momentum of heavy quark is given by $p = m v$. It is clear the momentum of heavy quark fall off exponentially

$$p(t) = p(0) e^{-\frac{t}{t_{\lambda_{GB}}}}, \quad t_{\lambda_{GB}} = \frac{m \left(1 + \sqrt{1 - 4\lambda_{GB}}\right)}{\pi \sqrt{\lambda} T^2_{GB}}. \quad (38)$$

where $t_{\lambda_{GB}}$ is relaxation time of heavy quark in Gauss-Bonnet gravity.

One can derive the result of $N = 4$ case by considering $\lambda_{GB} = 0$

$$t_{N=4} = \frac{2m}{\pi T^2 \sqrt{\lambda}}. \quad (39)$$

This result has been discussed in [18, 20].

The diffusion coefficient is related to the temperature of the plasma $T$, the heavy quark mass $m$ and the relaxation time $t_D$ as $D = \frac{T}{m} t_D$. It is straightforward to obtain the diffusion coefficient in the case of $N = 4$ SYM plasma [18, 20]

$$D_{N=4} = \frac{2}{\pi T \sqrt{g^2_{YM} N}}. \quad (40)$$
This result has been achieved with a different approach in [21], for non-relativistic heavy quarks. Using the above approach, one can obtain the diffusion coefficient of a moving heavy quark in Gauss-Bonnet gravity, too. Also the extremal case can be considered to calculate drag force and jet quenching parameter. To clarify this case, one finds from Eq. (5) that $\tilde{q} = 2$ corresponds to $T = 0$. As a result, one should consider a moving heavy quark in finite charge plasma at zero temperature. In this case drag force is finite [24].

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