H-BRANES AND CHIRAL STRINGS

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Abstract

We add a simple boundary term to the Polyakov action and construct a new class of D-branes with a single null direction. On the string world-sheet the system is described by a single quantized left-mode sector of a conformal field theory. By a Wick rotation of spacetime, we map open strings attached to null branes into chiral closed strings. We suggest that these so-called H-branes describe quantum horizons - black hole, cosmological (de-Sitter), etc. We show how one can get a space/phase space transmutation near the horizon and discuss the new features of boundary states which become squeezed states.

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1 Introduction

It was shown in the early seventies by Christodoulou [1], Penrose and Floyd [2] and Hawking [3] that the horizon area of a black hole cannot decrease. In his seminal paper [4] Bekenstein used this to identify horizon area with a black hole entropy which can be defined as the measure of information inaccessible to an exterior observer. It was also shown that the minimum increase of the black hole area has to be proportional to the square of Planck length. Later, Bardeen, Carter and Hawking found the four laws of black hole mechanics to be analogous to the usual four laws of thermodynamics [5]. This was finally realized to be the correct interpretation when Hawking found that black holes do in fact radiate energy and behave as hot objects with temperature proportional to the surface gravity [6]. Thus the mechanical laws of black holes are thermodynamical laws and one can identify the entropy of any black hole with the one quarter of the it’s horizon area (in Planck units)

\[ S_{BH} = \frac{1}{4} A \]  

(1)

The next important step was the suggestion of Bekenstein that the area \( A \) (and the mass) must be also quantized in Planck units [7]. This conjecture was based on earlier observations [1, 8] that the horizon area of a non-extremal black hole behaves as an adiabatic invariant. The minimal increase of a black hole area is an insight that quantum horizon plays the role of a phase space and one can form independent patches of equal Planck size areas, bearing in mind that each patch can locate one degree of freedom as in a usual phase space patchwork - what is called now Holographic principle [9], [10].

The idea of a discrete spectrum proposed by Bekenstein, was later discussed independently by Mukhanov [11] and one of the authors (IIK) [12]‡. Arguments used in [11] were based on entropy and further developments are discussed in [13]. In paper [12] (see also [14]) a stringy approach to quantization was suggested. It was based on a consistency of a chiral sector of a closed string moving in a Euclidean black hole background where the time coordinate is periodic. The fact that it leads to the same discrete spectrum as Bekenstein’s was an indication that chiral sectors should be relevant to the statistical counting of black hole entropy. Let us note that in the case of extremal Kerr-Newman black holes the quantization of mass follows from the quantization of electric and magnetic charges and angular momentum [15]. In recent years the discrete spectrum of quantum black holes was discussed in numerous papers, see for example the recent papers [16] and references therein. Let us also note that quantization of an area operator has been extensively discussed recently in Ashtekar’s approach to quantum gravity - see [17] and references therein.

\[ \text{‡Unfortunately at that time both VM and IIK were unaware about Bekenstein pioneering paper [7].} \]
The same paradigm about single chiral sector near horizon was later suggested by Carlip [18] first for BTZ (2+1) black hole [19, 20] and for general black holes in [21]. One starts to study the quantum version of these black holes since it is well known that (2+1)-dimensional gravity can be written as a topological Chern-Simons theory [22, 23, 24] which justify the Holographic Principle [9, 10] in 2+1 dimensions and one can say that entropy of BTZ-black holes is determined by some quantum states which are restricted to live on a (1+1)-dimensional surface. In (2+1)-dimensional gravity, the presence of a boundary is in fact necessary since the bulk by itself does not contain enough gravitational degrees of freedom to account for black hole entropy [24]. There are two natural candidates for this boundary surface, the horizon itself and spatial infinity.

Carlip considers the BTZ black hole horizon to be a true boundary where usual invariant diffeomorphisms are broken, turning "would be pure gauge of freedom" into physical ones [18]. In the Chern-Simons description of (2+1)-gravity one gets chiral sector of a WZW theory of level \( k \) which lives on a boundary. The level \( k = \frac{\sqrt{2}}{8G} \) is related to the cosmological constant \( \Lambda = -\frac{1}{l^2} \) of the AdS spacetime. In the quasi-classical approximation a conformal field theory on a boundary has the central charge \( c = 6 \). The Virasoro operator \( L_0 \) takes the form [18],

\[
L_0 \sim N - \left( \frac{\rho_+}{4G} \right)^2, \quad \rho_+^2 = 4Gl\sqrt{Ml^2 - J^2} + 4GMl^2
\]  

where \( \rho_+ \) is the value of the parameter in front of the usual \( \phi \)-periodic coordinate on the BTZ metric in Lorenz signature. Using Cardy formula [25] for number of states at a given level \( N \) in a CFT with a central charge \( c \)

\[
n(N) \sim \exp \left[ 2\pi \sqrt{\frac{cN}{6}} \right]
\]

one can get entropy \( S = \ln n(N) \). Black hole states are determined by on-shell condition \( L_0 = 0 \) which gives entropy

\[
S = \pi \sqrt{c\frac{2N}{3}} = \frac{2\pi \rho_+}{4G}
\]

in agreement with the Bekenstein-Hawking BTZ-black hole entropy. As was shown in [14] \( L_0 = 0 \) condition also leads to \( \rho_+ \) and mass quantization.

Strominger has given another approach to count the number of states of the BTZ black holes by considering the boundary as a \( (t,\phi) \) cylinder at spatial infinity [26]. The CFT at this boundary is a Liouville with central charge \( c = \frac{3}{2l^2} \) [27]. Considering the ground state of it as the AdS spacetime, Strominger has made the following identifications of Virasoro operators with the mass and angular momentum of the black hole

\[
L_0 + \bar{L}_0 = lM, \quad L_0 - \bar{L}_0 = J
\]
The correct value of the entropy is given by considering the contributions from both sectors of the theory:

\[
S = \sqrt{c\frac{L_0}{6}} + \sqrt{\bar{c}\frac{\bar{L}_0}{6}} = \frac{2\pi \rho_+}{4G}
\]  

We see that there are at least two different approaches to count the number of states (see also [21, 28, 29, 30]). The BTZ black hole example teaches us the following lesson: the contribution of states to the entropy comes from just one Virasoro algebra (a chiral sector) or from two Virasoro algebras (non-chiral sector) depending on how we choose the boundary - at the horizon or at infinity.

Let us now turn to another aspect of strings in black hole background. There is an apparent inconsistencies of what two distinct observers measure in the presence of a black hole, where one is freely falling to the black hole and the other is fixed outside it [32].

First, the different rates on the clocks between these two observers from the red-shift phenomena, has consequences on the different sizes that they measure in a string falling to the horizon [31]. That is because the size of the string depends on how many excitation modes one is able to see and such number depends on the resolution time \( \epsilon \) - in Planck units - of the observation. As the string is examined with better and better time resolution it appears to slowly grow as shown by the relation,

\[
R_{\text{string}}^2 \sim \alpha' \log \frac{1}{\epsilon}
\]  

For the freely falling observer the transverse size of the string is always of the order the Planck scale since his resolution in time will not be smaller than the Planck time. However, the outside observer is able to see more excitation modes as the string approaches the horizon because of the red-shift factor on his clock. String starts to spread over the space and consequently the external observer sees a stretched horizon formed by the strings frozen near horizon. The stretched horizon is a hot place with a temperature near Hagedorn limit. Black hole - string complementarity based on this picture was discussed in [31, 34, 35].

Second, a closed string on the horizon seen by the freely falling observer can be seen as an open string by the outside fixed one, since she/he is not able to see what is inside the black hole. Susskind conjectured that the classical Bekenstein-Hawking black hole entropy arises from configurations of these open strings with the ends frozen at the horizon. Quantum corrections of the classical entropy are expected to be finite at all orders in string perturbation theory [33].

Third, the different behaviors of the string size growth indicate a possible non-commutativity of the light-cone coordinates and to the resolution of the information loss paradox [36]. Strings carry information on their oscillators and one can argue that information has spread along the stretched horizon with non-commutative light-cone coordinates,
Figure 1: Information spreading along the stretched horizon. All squares have equal area by the Heisenberg inequality

\[ [X^-, X^+] = \alpha' \]  

(8)

From this relation we can see that by taking smaller and smaller time intervals information starts to spread over the space, as shown in Figure 1.

String theory was able to give us a partial answer of how to count the states that give the one-quarter-of-the-area law by using D-brane technology on extremum black holes, with a near AdS-geometry horizon [37, 38]. The counting of states is not done by a quantum gravity describing the geometry of near horizon but by a dual theory of supergravity in the UV limit on AdS spacetime. That is the IR limit of a gauge theory that lives on D-branes [39]. In such a limit, the black hole has become smaller then the string scale and so there is no concept of a horizon in this dual description since spacetime geometry is quite fuzzy at IR scales. Thus it is difficult to understand how one can explain the universality of the Bekenstein-Hawking entropy law [40]. Such dual description is also problematic in terms of black hole unitary evolution. Figure 2 shows the embedding of the whole extremum black hole geometry in a AdS spacetime [40]. Curved rows represent the black hole time isometrics and the straight lines are time evolution in AdS spacetime with D-branes inside. From the AdS/CFT dual picture, straight lines describe time evolution of a unitary conformal field theory. However it is not obvious why black hole evolution should be unitary from the dual picture point of view. Initial configuration evolves into the black hole and forgets about the information that is in the rest of AdS spacetime. Moreover, why information cannot flow outside the black hole and go through the rest of
AdS spacetime and continue to propagate freely until it reaches infinity? Conceptually the dual picture seems not to give a *universal* description of a Bekenstein-Hawking entropy and we are still far from understanding it microscopic origin.

So far we mostly addressed an issue of black hole horizons. Another important issue is a stringy description of cosmological horizons. In the case of de-Sitter space this problem recently attracted a lot of attention - see [41] and references therein. Again we have to find where are the degrees of freedom live which contribute to the entropy of cosmological event horizon in de-Sitter space [42].

From the previous discussion we learnt one important thing. It seems that when we try to get a stringy description of a quantum horizon we either have to deal with open strings attached to this horizon or with a chiral sector of closed strings. The only common feature of these two systems is that both have one Virasoro algebra. Thinking about an open string description one can suggest a D-brane stretched along a horizon. However it will not work. As we shall demonstrate later it is impossible to get a usual separation between Neumann and Dirichlet directions. More importantly if one has open strings on a D-brane they will be open whatever analytical continuation we are going to undertake. But we need something which gives us open strings in Lorentzian signature and at the same time chiral closed strings after continuation into the Euclidean domain. One should remember that the horizon itself shrinks to a point when one passes Lorentzian signature to a Euclidean one.

In this paper we suggest a new type of branes - **H-branes**, which have one single null direction in a space with a Lorentzian signature and which naturally support chiral closed strings after analytical continuation. We shall focus our attention on the properties of these branes in flat space (leaving actual applications to black holes and cosmological
models for future publications).

The paper is organized as follows. In the next section we review the concept of a bosonic string moving in spacetime and add an extra boundary term to the Polyakov action which will lead to light-like world-sheet boundaries. In Section 3 we introduce H-branes and obtain the equations of motion that open strings must obey in Minkowski spacetime in a presence of H-brane. In Section 4 we perform a Wick rotation in both spacetime and world-sheet to obtain chiral closed strings from open strings ending on a H-brane. In section 5 an analogy between our model and a Dissipative Quantum Mechanics is discussed as well as Ishibashi states associated with our new boundary conditions. We also show that boundary state for H-brane is not a coherent but a squeezed state. In conclusion we discuss open questions to be addressed in the future.

2 Open Strings with light-like world-sheet boundaries

We start by asking the following question: is it possible to have an open string which has a Neumann condition for let say $X^+$ and a Dirichlet condition for $X^-$, where $X^\pm$ are target space light-cone coordinates?

To answer this question let’s recall how one gets boundary conditions for open strings (for more details see [44]). The starting point is the variation of the Polyakov action

$$ S_P = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma (-\gamma)^{1/2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu $$

which gives equations of motion from the bulk term plus boundary conditions from the boundary term

$$ \delta S_P = \frac{1}{4\pi\alpha'} \int d\tau d\sigma \delta X^\mu \partial_a \left( (-\gamma)^{1/2} \gamma^{ab} \partial_b X_\mu \right) - \frac{1}{4\pi\alpha'} \int_{\partial\Sigma} ds \left( \delta X^\mu t^a \partial_a X_\mu \right) $$

Boundary term is given by the second integral which is taken over the world-sheet boundary $\partial\Sigma$ and $t^a$ is a unit tangent vector.

Let us choose space-time light-cone coordinates:

$$ X^\pm = \frac{1}{\sqrt{2}} (X \pm T) $$

and choosing the usual world-sheet metric in conformal gauge with world-sheet boundaries as space-like lines. The the boundary conditions are,

$$ \delta X^+ \partial_a X^- = 0 $$
$$ \delta X^- \partial_a X^+ = 0 $$
$$ \delta X^i \partial_\sigma X^i = 0 $$

(12)
where \( i = 2, 3, \ldots, 25 \) is an index of the transverse coordinates (in this paper we only consider bosonic strings).

From the first line of (12), we see that if we choose the Dirichlet condition for \( X^+ \), then \( \partial_{\sigma} X^+ \neq 0 \). The from the second line we are forced to choose also a Dirichlet condition for \( X^- \), since \( X^+ \) don’t obey a Neumann condition. By the same argument, if we choose the Neumann condition for \( X^- \), then we will end also with a Neumann condition for \( X^+ \). The reason is very simple - to have either Dirichlet or Neumann condition for let say \( X^+ \) we must have it for both \( X \) and \( T \), but then we shall get the same condition for \( X^- \). On transverse coordinates there are no such restrictions since boundary conditions don’t have a cross structure as in \(+, -\) case. We can choose for each transverse coordinates, either Dirichlet or Neumann conditions. Thus, even if we know that the endpoints of an open string move at speed of light, the previous argument show us that is not possible to attach both ends on a single null direction. It seems a D-brane with a single null direction is impossible.

Is it really so?

### 2.1 World-sheet with light-like boundaries

We are going to introduce a new boundary term which will correspond to a new type of world-sheet boundaries for open strings. To be more precise we shall get a light-like boundary which is the key element to get a brane with a single null direction. Let us start from postulating an extra boundary term

\[
S_B = \frac{1}{8\pi\alpha'} \int_{\partial\Sigma} ds \left( X^+ t^a \partial_a X^- - X^- t^a \partial_a X^+ \right) \tag{13}
\]

where the integral is taken over the world-sheet boundary and \( t^a \) is a unit tangent vector. This term is Weyl and diffeomorphism invariant so one can add it to the Polyakov action. Let us choose a coordinate system such that the variable \( \tau \) parameterizes the world-sheet boundary \( \partial\Sigma \). The infinitesimal proper distance \( ds \) is defined by \( ds = (-\gamma)^{1/4} d\tau \).

Since under an infinitesimal Weyl transformation the world-sheet metric is changed as \( \delta_W \gamma_{ab} = 2\gamma_{ab} \delta \omega \) we see that \( ds \) is changed in the following manner \( \delta_W ds = ds \delta \omega \) where we have kept the coordinate \( \tau \) unchanged. The tangent vector \( t^a \) on the extra boundary term is changed by a Weyl transformation since we have to keep it as a unit vector

\[
\delta_W \left( \frac{t^a}{|t|} \right) = -\frac{t^a}{|t|} \delta \omega. \tag{14}
\]

The partial derivatives are not changed by the Weyl transformation since they only depends on the world-sheet coordinates \( \sigma^a \):
\[ \delta W \frac{\partial}{\partial \sigma^a} = 0 \] (15)

The proper distance ds and the unit tangent vector \( t^a \) are changed by the same magnitude but with different signs so the total contribution to the infinitesimal Weyl transformation on the extra boundary term is null

\[ \delta W S_B = 0 \] (16)

In a similar way one can show that this boundary term is invariant under diffeomorphisms. However, the boundary term is no longer spacetime Poincare invariant as it depends on the explicit null spacetime coordinates. As we shall demonstrate, this arises from the presence of a (null) brane. The total action is

\[ S_{P+B} = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma (-\gamma)^{1/2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu - \beta \frac{4\pi\alpha'}{\pi} \int_{\partial \Sigma} ds X^+ t^a \partial_a X^- \] (17)

where we integrated the boundary term by parts and introduced a free parameter \( \beta \). Both the world-sheet and the target space have Lorenz signature.

The action has a local Weyl\( \otimes \)Diff symmetry and and one can impose a three gauge conditions on our world-sheet metric. We will work in light-cone gauge for the world-sheet time coordinate and impose two more conditions defined as follows [44]:

\[ X^+ = \tau, \quad \partial_\sigma \gamma_{\sigma \sigma} = 0, \quad \gamma = -1. \] (18)

The procedure to choose the above conditions is discussed in details in §1.3 of [44] where one defines the world-sheet coordinate \( \sigma \) by constructing it from one boundary \( \sigma = 0 \) to another one \( \sigma = \pi \) using an invariant length \( dl = \gamma_{\sigma \sigma} (-\gamma)^{-1/2} d\sigma \). Under the above conditions and by splitting \( X^- \) into mean value

\[ \bar{X}^- (\tau) = \frac{1}{\ell} \int_0^\ell d\sigma X^- (\tau, \sigma), \] (19)

and \( Y^- (\tau, \sigma) = X^- (\tau, \sigma) - \bar{X}^- (\tau) \) one can see that \( Y^- \) acts as a Lagrange multiplier constraining \( \partial_\sigma \gamma_{\tau \sigma} \) to vanish. The presence of the new boundary term in the action can be seen as a deformation of the original Lagrangian by boundary vertex operators

\[ L_{P+B} = L_P + \beta \frac{4\pi\alpha'}{\pi} \left( X^+ \partial_\tau X^- \big|_{\sigma=0} + X^+ \partial_\tau X^- \big|_{\sigma=\pi} \right) \] (20)

but this deformation does not change the abovementioned constraint \( \partial_\sigma \gamma_{\tau \sigma} = 0 \). But open string boundary conditions are modified:

\[ \gamma_{\tau \sigma} \partial_\tau X^+ - \gamma_{\tau \sigma} \partial_\sigma X^+ - \beta \partial_\tau X^+ = 0 \]
\[ \gamma_{\tau \sigma} \partial_\tau X^- - \gamma_{\tau \sigma} \partial_\sigma X^- + \beta \partial_\tau X^- = 0 \]
\[ \gamma_{\tau \sigma} \partial_\tau X^i - \gamma_{\tau \sigma} \partial_\sigma X^i = 0 \] (21)
where we have used $ds = d\tau$. The different signs between the first two lines comes from integration by parts when we take the variation of the boundary term. Using the light-cone gauge choice $X^+ = \tau$ one gets

$$\gamma_{\tau \sigma} = \beta \quad \text{at} \quad \sigma = 0, \pi.$$  \hspace{1cm} (22)

and since $\partial_\sigma \gamma_{\tau \sigma} = 0$ we have $\gamma_{\tau \sigma} = \beta$ everywhere. By the condition $\gamma = -1$, we must have at each point that $\gamma_{\tau \tau} \gamma_{\sigma \sigma} = \beta^2 - 1$. When $\beta = 0$ we have the usual world-sheet metric with time-like boundaries for open strings. However, for $\beta = 1$ we have the condition

$$\gamma_{\tau \tau} \gamma_{\sigma \sigma} = 0$$  \hspace{1cm} (23)

and the metric takes either the form

$$ds^2 = \gamma_{\tau \tau} d\tau^2 + 2d\tau d\sigma$$  \hspace{1cm} (24)

for the case $\gamma_{\sigma \sigma} = 0$ or

$$ds^2 = \gamma_{\sigma \sigma} d\sigma^2 + 2d\tau d\sigma$$  \hspace{1cm} (25)

for $\gamma_{\tau \tau} = 0$. One can see that when $\beta = 1$ there are two type of boundaries (let us remind that along the boundary $d\sigma = 0$)

$$ds^2 = \gamma_{\tau \tau} d\tau^2 \quad \text{for} \quad \gamma_{\sigma \sigma} = 0 \quad \text{or},
\quad ds^2 = 0 \quad \text{for} \quad \gamma_{\tau \tau} = 0$$  \hspace{1cm} (26)

In the first case, boundaries are still time-like but in the second case we have a totally new type of light-like boundaries with the metric

$$\gamma_{ab} = \begin{bmatrix} 0 & 1 \\ 1 & \gamma_{\sigma \sigma}(\tau) \end{bmatrix}$$  \hspace{1cm} (27)

In the rest of the paper we shall work with this metric and with world-sheets with light-like boundaries.

Let us find the equations of motion for transverse coordinates. The Lagrangian is reduced to

$$L = -\ell \frac{1}{2\pi \alpha'} \gamma_{\sigma \sigma} \partial_\tau \bar{X}^- (\tau) 
+ \frac{1}{4\pi \alpha'} \int_0^\ell d\sigma \left[ \gamma_{\sigma \sigma} \partial_\tau X^i \partial_\tau X^i + 2(\partial_\sigma Y^- - \partial_\tau X^i \partial_\sigma X^i) \right]$$  \hspace{1cm} (28)

and the conjugate momenta are:

$$p_- = -p^+ = \frac{\delta L}{\delta (\partial_\tau \bar{X}^-)} = -\ell \frac{1}{2\pi \alpha'} \gamma_{\sigma \sigma}(\tau)$$
$$\Pi^i = \frac{\delta L}{\delta (\partial_\tau X^i)} = \frac{p^+}{\ell} \partial_\tau X^i - \frac{1}{2\pi \alpha'} \partial_\sigma X^i$$  \hspace{1cm} (29)
The Hamiltonian is the following:

\[
H = p_\tau \partial_\tau \bar{X}^- + \int^\ell_0 d\sigma \Pi_i \partial_\sigma X^i - L \\
= \frac{\ell}{2\pi \alpha'} \int^\ell_0 d\sigma \left[ \pi \alpha' \Pi^i \Pi_i + \Pi^i \partial_\sigma X^i + \frac{1}{4\pi \alpha'} \partial_\sigma X^i \partial_\sigma X^i \right] \\
- \frac{1}{2\pi \alpha'} \int^\ell_0 d\sigma \partial_\sigma Y^-
\]  

(30)

from where we take the equations of motion

\[
\partial_\tau p^+ = \frac{\delta H}{\delta x^-} = 0 \\
\partial_\tau \Pi^i = -\frac{\delta H}{\delta X^i} = \frac{\ell}{2\pi \alpha'} \left[ \partial_\sigma \Pi^i + \frac{1}{2\pi \alpha'} \partial^2_\sigma X^i \right]
\]  

(31)

with \( p^+ \) a conserved quantity. From the second line we have a wave equation

\[
\partial_\sigma \partial_\tau X^i = \frac{\pi \alpha' p^+}{\ell} \partial^2_\sigma X^i
\]  

(32)

It is convenient to take \( p^+ = 0 \) (\( \gamma_{\sigma \sigma} = 0 \)) and consider the \( \tau \) and \( \sigma \) coordinates as light-like

\[
\tau = x^+, \quad \sigma = x^-
\]  

(33)

so one can write the wave equation in a usual form. The world-sheet metric takes now the form \( ds^2 = dx^- dx^+ \) with boundaries at \( x^- = 0 \) and \( x^- = \pi \). The world-sheet of such configuration is drawn in Figure 3. In this limit the Lagrangian takes the form,
\[ L = \frac{1}{2\pi\alpha'} \int_0^\ell dx^- \left[ \partial_- Y^- - \partial_+ X^i \partial_- X^i \right] \]  

(34)

where the partial derivatives refers to the world-sheet coordinates \( x^- \) and \( x^+ \). Since the momentum \( \Pi^i \) is

\[ \Pi^i = -\frac{1}{2\pi\alpha'} \partial_- X^i \]  

(35)

we can see that the Hamiltonian takes unusual form

\[ H = -\frac{1}{2\pi\alpha'} \int_0^\ell dx^- \partial_- Y^- \]  

(36)

Let us note that in the limit \( p^+ \to 0 \) all transverse oscillators are frozen.

### 3 H-Branes

Let us study an open string on this new world-sheet with light-like boundaries induced by the special boundary term we have introduced. Taking the variation of the Polyakov action

\[ \delta S_P = \frac{1}{4\pi\alpha'} \int_{\Sigma} dx^- dx^+ \delta X^\mu \partial_- \partial_+ X^\mu - \frac{1}{4\pi\alpha'} \int_{\partial\Sigma} dx^+ \delta X^\mu \partial_+ X^\mu \]  

(37)

and adding the variation of the boundary term

\[ \delta S_b = -\frac{1}{4\pi\alpha'} \int_{\partial\Sigma} dx^+ \left[ \delta X^+ \partial_+ X^- + X^+ \partial_+ \delta X^- \right] \]

\[ = -\frac{1}{4\pi\alpha'} \int_{\partial\Sigma} dx^+ \left[ \delta X^+ \partial_+ X^- - \delta X^- \partial_+ X^+ \right] \]  

(38)

one can see that the total contribution involving target space light-cone coordinates is

\[ + \delta X^+ \partial_+ X^- + \delta X^- \partial_+ X^+ \quad \text{from Polyakov action} \]

\[ + \delta X^+ \partial_+ X^- - \delta X^- \partial_+ X^+ \quad \text{from extra boundary term} \]

\[ = 2\delta X^+ \partial_+ X^- \]  

(39)

Thus we end with just one boundary condition on the light-cone coordinates

\[ \delta X^+ \partial_+ X^- \mid_{\partial\Sigma} = 0 \]  

(40)

At the same time nothing changed for the transverse coordinates \( X_i, i = 2, 3, ..., 25 \)

\[ \delta X^i \partial_+ X^i \mid_{\partial\Sigma} = 0 \]  

(41)

From the boundary condition (40) we are free to choose either \( \delta X^+ = 0 \) or \( \partial_+ X^- = 0 \) - contrary to what we have had for open strings with time-like boundaries. Thus we
can have the endpoints of the open string fixed in a single null $X^+$ direction with no boundary condition for $X^-$ or vice-versa. This defines a new type of branes which we shall call \textbf{H-brane}. §

Equations of motion for $X^\pm$ are

$$\partial_- \partial_+ X^\pm = 0 \quad (42)$$

have general solutions

$$X^-(x^-, x^+) = X^+_L(x^-) + X^-_R(x^+)$$

$$X^+(x^-, x^+) = X^+_L(x^-) + X^+_R(x^+) \quad (43)$$

The second term of each light-cone coordinate is not restricted to discrete spectrum (we do not insist on periodicity in $x^+$)

$$X^-_R(x^+) = X^-_R(0) + p^-_R x^+ + (4\pi\alpha')^{1/2} \int \left[ \frac{a^-_n}{n} \cos(nx^+) + \frac{b^-_n}{n} \sin(nx^+) \right] dn$$

$$X^+_R(x^+) = X^+_R(0) + p^+_R x^+ + (4\pi\alpha')^{1/2} \int \left[ \frac{a^+_n}{n} \cos(nx^+) + \frac{b^+_n}{n} \sin(nx^+) \right] dn \quad (44)$$

Let us take the Dirichlet boundary condition for $X^-$

$$\partial_+ X^- = 0 \quad (45)$$

This leads to constraints for the oscillators as well as for the momentum $p^-$

$$\int a^-_n \sin(nx^+) dn - \int b^-_n \cos(nx^+) dn = 0$$

$$p^-_R = 0 \quad (46)$$

at $x^- = 0$ and $x^- = \pi$. Since the condition must be valid at any light-like time $x^+$, the oscillators $a^-_n$ and $b^-_n$ must all vanish, as well as the $p^-$ momentum. Inserting the values of $X^\mu_R(0)$ into $X^\mu_L(x^-)$, we have the solution

$$X^-(x^-, x^+) = X^+_L(x^-), \quad X^i(x^-, x^+) = X^i_L(x^-)$$

$$X^+(x^-, x^+) = X^+_L(x^-) + p^+_L x^+ + (4\pi\alpha')^{1/2} \int \left[ \frac{a^+_n}{n} \cos(nx^+) + \frac{b^+_n}{n} \sin(nx^+) \right] dn \quad (47)$$

§ A second choice was \textit{N-brane}, where \textit{N} stands for null, but we prefer to call it \textit{H-brane} stressing the natural relation to horizons. Let us note that nevertheless there is some information about null structure in the letter \textit{H} also - if you read it in Russian (Russian \textit{H} = Latin \textit{N}).
The light-cone coordinates at the endpoints of the open string have some fixed values for \(x^- = 0\) and \(x^- = \pi\). Note that the Dirichlet boundary condition \(\partial_+ X^- |_{\partial \Sigma} = 0\) restricts the endpoints of an open string to be fixed at the \(X^-\) light-cone coordinate

\[
\partial_+ X^- |_{\partial \Sigma} = 0 \Rightarrow \delta X^- |_{\partial \Sigma} = 0
\]

(48)

The same is true for the transverse coordinates, as we have the same equations of motion and boundary conditions. We just don’t have any boundary condition for \(X^+\). Thus, we end up with a string that has its endpoints frozen along the null \(X^-\) direction as well as in all the transverse directions, but is allowed to move along the \(X^+\) direction. Of course one can swap \(X^-\) and \(X^+\) in this construction.

The solution \(X^-(x^-, x^+) = X^-_L(x^-)\) admits open strings attached to a single H-brane as well as an open string with endpoints attached to two different H-branes and there are two cases:

\[
\begin{align*}
X^-_1(x^- = 0) &= X^-_1(x^- = \pi) \\
X^-_2(x^- = 0) &= X^-_2(x^- = \pi) + D^- 
\end{align*}
\]

(49)

with \(D^-\) a proper distance along the null direction \(X^-\) between the two H-branes, as Figure 4 shows. In terms of world-sheet picture, oscillating modes evolve parallel to the light-like boundaries without any reflection on it. Because the energy-momentum tensor \(T_{ab}\) has only one non-zero component

\[
T_{--} = -\partial_+ X^\mu \partial_- X_\mu, \quad T_{+-} = 0, \quad T_{++} = 0
\]

(50)

there is only one nontrivial Virasoro algebra with generators:

\[
L_m = \int dx^- e^{2imx^-} T_{--}
\]

(51)

We see that not only left and right modes decouple from each other, but we only have a single chiral sector.
3.1 Quantization of the String Modes

The general expansion for $X^\mu(x^-, x^+)$ can be written as:

$$X^\mu(x^-, x^+) = X^\mu(0) + p_L^\mu x^- + p_R^\mu x^+ + i(4\pi\alpha')^{1/2} \left[ \sum_{n \neq 0} \frac{\alpha_n^\mu}{n} e^{inx^-} + \int \frac{\beta_n^\mu}{n} e^{inx^+} dn \right]$$

(52)

where due to a Dirichlet boundary condition for $\mu = -i$ there is no $x^+$ dependence for $X^-$ and $X^i$, i.e. all $\beta_n^{-,i} = 0$. At the same time there are no any boundary conditions for $X^+$ and because of this one can forget about $\beta_n^+$ oscillators. Quantization of the string modes is done through the well known equal time commutation relations between density momentum and the spacetime coordinates

$$[\Pi^\mu(x^-, x^+), X^\nu(x'^-, x^+)] = i\eta^{\mu\nu} \delta(x^- - x'^-)$$

(53)

where $x^+$ is the world-sheet time coordinate. Simple calculations give

$$\Pi^\mu = \frac{1}{4\pi \alpha'} \partial_- X^\mu = \frac{1}{4\pi \alpha'} \left[ p_L^\mu - \sum_{n \neq 0} \alpha_n e^{inx^-} \right]$$

(54)

The non-vanishing canonical commutation relations follows straightforwardly

$$[p_L^\mu, X^\nu(0)] = i\eta^{\mu\nu}$$

$$[\alpha_n^\mu, \alpha_m^\nu] = m\eta^{\mu\nu} \delta_{m+n,0}$$

(55)

with all the other vanishing, including $\beta_n^+$ with itself

$$[\beta_+^\mu, \beta_+^\nu] = 0$$

(56)

Let us make usual identifications

$$\alpha_n^\mu = \frac{1}{\sqrt{n}} \alpha_n^\mu \quad \alpha_n^\mu = \frac{1}{\sqrt{n}} \alpha_n^\mu \quad \text{for } n > 0$$

(57)

and notice that we have only left-moving quantized oscillators. We thus only have the quantized left sector of the Virasoro algebra.

4 Chiral closed strings from open strings: Lorenz versus Euclidean picture

In this section, we will make a correspondence between open strings and chiral closed strings. That is done by making a Wick rotation on the world-sheet as well as in target-space. As we saw the coordinate $x^+$ is a light-cone time coordinate and one could try
to make a Wick rotation along $x^+$. This leaves $x^-$ unchanged. However this is not a consistent continuation. Let us look at the world-sheet metric

$$ds^2 = dx^-dx^+$$

(58)

where the boundaries are $x^- = 0$ and $x^- = \pi$. We cannot get real metric by rotating only $x^+$, so the only way is continue a world-sheet time coordinate $\tau \rightarrow i\tau$ and not on the light-like time $x^+$. Because $\tau = x^+ - x^-$ and $\sigma = x^+ + x^-$, we immediately get the following relation between $x^-$ and $x^+$

$$x^- = x^+ *$$

(59)

which means that we get ordinary complex coordinates

$$x^- = z$$
$$x^+ = \bar{z}$$

(60)

An interesting fact is that the previous light-like boundaries of a world-sheet with Lorenz signature are now mapped to points on a complex plan:

$$x^- = 0 \rightarrow z = \bar{z} = 0$$
$$x^- = \pi \rightarrow z = \bar{z} = \pi$$

(61)

We end up with no boundaries on complex worksheet and so our open strings are mapped into chiral closed strings. With a Euclidean signature world-sheet, our action is written in the complex form

$$S_P + S_B = \frac{1}{2\pi\alpha'} \int_{\Sigma} dzd\bar{z}G_{\mu\nu}\partial_zX^\mu\partial_{\bar{z}}X^\nu + V(0) - V(\pi)$$

(62)

where the boundary term disappeared due to the simple fact that boundaries shrinked into points. Instead of boundary terms we have vertex operators located at these points. The operator at point $a$ is given by

$$V(a) = \frac{i}{8\pi\alpha'} \oint dz (X^*\partial_zX - X\partial_zX^*)$$

(63)

where the contour integral is taken over an infinitesimal contour around $a$. Since our variables are complex, the fields $X^\pm$ must also be complex and so we need to carry a Wick rotation also in target space-time $T \rightarrow iT$ and one finds that space-time light-cone coordinates are complex conjugate to each other

$$X = X^- = R - iT = (R + iT)^* = (X^+)^*$$

(64)

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As we see, all the unquantized oscillators $\beta_n^+$ and momentum $p^+$ of the light-cone coordinate $X^+$ vanishes after the spacetime Wick rotation. Since world-sheet boundaries have also disappear, we arrive to a closed string configuration with only left moving oscillator modes. Strings frozen to H-branes are thus mapped by a Wick rotation to closed chiral strings as we only have one sector of the Virasoro algebra in both cases.

Let us briefly discuss what happens when we have non-trivial space-time metric in our model. The action takes the form

$$S_{P+B} = -\frac{1}{4\pi\alpha'}\int_{\Sigma} dx^- dx^+ G_{\alpha\beta} \gamma^{ab} \partial_a X^\alpha \partial_b X^\beta - \frac{1}{4\pi\alpha'}\int_{\partial\Sigma} dx^+ G_{+-} X^+ \partial_x^+ X^-$$

and the world-sheet metric will be $ds^2 = dx^- dx^+$. It is easy to write equations of motion

$$\partial_{x^-} \partial_{x^+} X^\mu + \Gamma^\mu_{\alpha\beta} \partial_{x^-} X^\alpha \partial_{x^+} X^\beta = 0$$

and the boundary conditions

$$\begin{align*}
[2G_{ij} \partial_{x^+} X^j + G_{+-,i} X^+ \partial_{x^+} X^-] \delta X^i &= 0 \\
\left[ G_{+-,i} X^+ \partial_{x^+} X^- + G_{+-,i} X^+ \partial_{x^+} X^i \right] \delta X^- &= 0 \\
\left[ G_{+-,i} X^+ \partial_{x^+} X^- + G_{+-,i} X^+ \partial_{x^+} X^i \right] \delta X^- &= 0
\end{align*}$$

The indices (i,j) denote the transverse spacetime coordinates where now explicit write partial derivatives with respect to the world-sheet coordinates (denoted by $\partial_{x^\pm}$) and to the spacetime light-like coordinates (denoted by $\partial_+,\partial_-$). We now impose Dirichlet boundary condition for the light-cone coordinate $X^-$ as well as for the transverse coordinates $X^i$. It is important to note that the chiral solutions for both $X^-$ and $X^i$ coordinates are still valid in a case of curved space-time. Even if is not the most general solution, the consequence of such a particular solution is that the relation between Dirichlet boundary condition on world-sheet and the presence of H-brane in curved space-time is hold:

$$\begin{align*}
\partial_+ X^- |_{\partial\Sigma} = 0 &\Rightarrow \delta X^- |_{\partial\Sigma} = 0 \\
\partial_+ X^i |_{\partial\Sigma} = 0 &\Rightarrow \delta X^i |_{\partial\Sigma} = 0
\end{align*}$$

Moreover, we have no boundary conditions for the $X^+$ light-cone coordinate and so open strings frozen on H-brane can oscillate outside the brane which can be interpreted as quantum fluctuations of H-branes.

Let us note that this picture has a direct relation to the mass spectrum of a Schwarzshild black hole. After Euclidean continuation, open strings attached to H-branes become chiral closed strings that wound around the horizon (see Fig.5), where now the complex time coordinate is to be identified as an angular variable with periodicity of the inverse
Figure 5: Open strings attached to H-branes in Lorenz picture becomes chiral closed string in Euclidean picture

temperature $\beta$ of the black hole. In the case of a Schwarzshild black hole with mass $M$ the identification is,

$$iX^0 \sim iX^0 + \beta$$

where in units of Planck mass $\beta$ is related to the horizon radius

$$\beta = 2\pi R_+ = \frac{4\pi M}{M_P^2}$$

It was shown in [12] (see also [14]) that chiral sector can exist only for quantized black holes with the discrete spectrum $R_+^2 = n$, i.e, the Schwarzshild black hole mass spectrum $M = M_P\sqrt{n}$. which is precisely the spectrum we have discussed in the introduction.

5 $X^-, X^+$ as a Phase Space: non-commutative geometry from H-branes without $B_{\mu\nu}$ field and squeezed boundary states.

In this section, we shall demonstrate that $X^+$ and $X^-$ light-cone coordinates can be treated as a canonical pair in a phase space. To see it we shall study the dynamics of open string endpoints frozen along H-brane in the $X^+$ - direction. We shall follow an analogy between open strings with extra boundary action and a dissipative quantum mechanics (DQM) [45] discussed in [46, 47, 48, 49].
By the very nature of the extra boundary term in curved spacetime,

\[ S_B = \frac{1}{8\pi\alpha'} \int_{\partial\Sigma} dx^+ G_{--} \left( X^+ \partial_x X^- - X^- \partial_x X^+ \right) \]  

(71)

it resemble to an action describing a massless charged particle in a constant magnetic field. Such term is familiar in string theory where the boundary action is given by the Wilson line and non-commutative geometry can arise in configurations where the NS-NS \( B_{\mu\nu} \) field is large (see for example [50] and references therein). In our situation, the role played by a background magnetic field is played by the \(+\) components of the spacetime metric \( G_{--} \). In this paper we only discuss flat space-time (like Rindler), the general case will be discussed in a future publication.

Let us again introduce parameter \( \beta \) in our action

\[ S_\beta = S_P + \beta S_B \]  

(72)

If we start from \( \beta = 0 \), we have the Polyakov action describing free strings in Minkowski space-time with the world-sheet as the usual one with time-like boundaries. Ordinary open strings carry both left and right oscillating modes and any boundary state \( |B\rangle \) must satisfy the condition which relates Virasoro generators from both sectors

\[ (L_n - \bar{L}_{-n}) |B\rangle = 0 \]  

(73)

This condition means that the boundary does not destroy conformal symmetry and there is no energy flux through the boundary [51]. For a boundary state associated with the Neumann boundary condition we have [46],

\[ |B\rangle_N = \exp \left( - \sum_{m=1}^{+\infty} \frac{1}{m} \alpha_{-m} \cdot \tilde{\alpha}_{-m} \right) |0\rangle \]  

(74)

and for a state associated to a Dirichlet boundary condition we have

\[ |B\rangle_D = \exp \left( + \sum_{m=1}^{+\infty} \frac{1}{m} \alpha_{-m} \cdot \tilde{\alpha}_{-m} \right) |0\rangle \]  

(75)

All these states belongs to a Hilbert space \( \mathcal{H}_L \otimes \mathcal{H}_R \) where the vacuum carries both left and right vacuum sectors,

\[ |0\rangle = |0\rangle_L \otimes |0\rangle_R \]  

(76)

Let us consider the Neumann state, for example. We see that by increasing \( \beta \) the initial Neumann boundary conditions for light-cone coordinates are replaced by

\[ \partial_\sigma X^+ + \beta \partial_\sigma X^+ = 0 \]
\[ \partial_\sigma X^- - \beta \partial_\sigma X^- = 0 \]  

(77)
We are still using world-sheet metric $ds^2 = -d\tau^2 + d\sigma^2$ with time-like boundaries. By introducing the parameter $\beta$ we arrive at a coherent string state that is analogous to a path integral in the DQM system in a magnetic field \cite{17,48,49} where the parameter $\beta$ is proportional to the magnetic field. The two point function was calculated in \cite{48,49} and in the case of real valued light-cone target-space coordinates $X^\pm$ one gets in the limit $\beta \rightarrow 1$

$$\langle 0|X^+(\tau, \sigma)X^-(\tau', \sigma)|0\rangle = -\frac{a}{a^2 + b^2}\log(\tau - \tau')^2\delta_{--} - \frac{\pi b}{a^2 + b^2}\text{sign}(\tau - \tau')\epsilon_{--} \quad (78)$$

where $a^{-1} = 2\pi\alpha'$ and $b^{-1} = 8\pi^2\alpha'$.

The first term on right hand side measures the delocalization of the string shape. In condensed matter physics literature the quantity that measures a delocalization is called mobility. The logarithmic growth is a transition between two extreme limits: long-time behaviour is bound by a constant or it grows without limit. The coefficient in front of the logarithm is the value of the critical mobility.

The second term can be interpreted as a Hall effect for our target space-time fields. However, in a conformal field theory, one can compute commutators of operators from the short distance behaviour of T-product (famous BJL relation, for recent discussion see for example \cite{52}, \cite{50} and references therein) and after simple calculations one gets

$$[X^+(\tau), X^-(\tau')] = T\left(X^+(\tau)X^-(\tau') - X^+(\tau)X^-(\tau')\right) = \Theta\epsilon_{--} \quad (79)$$

that is, the boundary term we introduced to construct $H$-brane at the same time tells us that $X^+, X^-$ can be seen as coordinates in a non-commutative spacetime, with non-commutativity parameter $\Theta = \frac{\pi b}{a^2 + b^2}$.

When $\beta = 1$ we get $H$-brane. Using the same method as in \cite{13} we can construct a boundary state associated with $H$-brane where instead of (refoldLn) we have chiral constraint $L_n|H\rangle = 0$

To find $|H\rangle$ we shall use the fact that $\partial_+ X^\mu = 0$ for $\mu = -, i$. and

$$X^\mu(x^-, x^+) = X^\mu(0) + p^\mu_L x^- + i(4\pi\alpha')^{1/2} \sum_{n \neq 0} \frac{\alpha_n}{n} e^{inx^-} = X^\mu(0) + i(4\pi\alpha')^{1/2} \sum_{n > 0} \frac{1}{\sqrt{n}} X^\mu_n \quad (80)$$

where

$$X^\mu_n = a_n^\mu e^{inx^-} - a_n^\dagger \epsilon^{-inx^-} \quad (81)$$

Let us consider states

$$|X\rangle = \exp\left(-\frac{1}{4}(X|X) - \frac{1}{2}(a^\dagger|a^\dagger) - (X|a^\dagger)\right)|0\rangle_L \quad (82)$$
where we use the same notation as [46]

\[(X|X) = \sum_{\mu=0}^{25} \sum_{m=1}^{\infty} X^\mu_m X^\mu_m \] (83)

These states satisfy the eigenvalue condition

\[(-a_n^+ + a_n - X_n)|X\rangle = 0 \] (84)

as well as the completeness relation

\[\int_{-\infty}^{+\infty} dX |X\rangle \langle X| = 1 \] (85)

Taking \(H\)-brane at \(x^- = 0\) the boundary state associated with it is given by

\[|H\rangle = \int \mathcal{D}X |X\rangle \]

\[= \int \mathcal{D}X \exp \left( -\left( \frac{1}{2} X + a^+ \right)^2 \right) \cdot |0\rangle_L \] (86)

and after simple gaussian integration one gets the boundary state

\[|H\rangle = \exp \left( \frac{1}{2} \sum_{m=1}^{\infty} \frac{1}{m} \alpha_{-m} \cdot \alpha_{-m} \right) |0\rangle_L \] (87)

where \(\alpha_{-n} = \sqrt{na^+_n}\). We stress that in spite of similarity between the previous three states, \(|B\rangle_N, |B\rangle_D\) and \(|H\rangle\) we see that this last state is:

- a chiral state
- a squeezed state

Those two properties are to be compare with the analogous properties of the first two boundary states, that are non-chiral and coherent states. Squeezed quantum states known for a long time in quantum optics and measurement theory (for a review of squeezed states see, for example [53]). The simplest one-mode squeezed state is parameterized by the two parameters \(r\) and \(\phi\) and can be obtained by acting on the vacuum with the unitary squeezing operator \(S(\xi)\)

\[|\xi\rangle = S(\xi)|0\rangle = \exp \left[ \frac{1}{2} \left( \xi a^2 - \xi (a^+)^2 \right) \right] \] (88)

where \(\xi = r \exp(i\phi)\) is the squeezing parameter. The mean number of quanta in the squeezed state is \(\bar{N} = \sinh^2 r\). To see this let us note that using the squeezing operator
S one can make the Bogolyubov transformation \( b = SaS^\dagger, \quad b^\dagger = SaS^\dagger \) and after some algebra one gets:

\[
\begin{align*}
  b &= \cosh ra + \exp(i\phi) \sinh ra^\dagger, \quad b^\dagger = \exp(-i\phi) \sinh ra + \cosh ra^\dagger \\
  a &= \cosh rb - \exp(i\phi) \sinh rb^\dagger, \quad a^\dagger = -\exp(-i\phi) \sinh ra + \cosh ra^\dagger
\end{align*}
\]

(89)

The new operator \( b \) is the annihilation operator for the squeezed state

\[
|\xi > = bS|0 > = S aS^\dagger S|0 > = Sa|0 > = 0
\]

(90)

Then it is easy to see that

\[
\bar{N} = < \xi | a^\dagger a | \xi > = \sinh^2 r < \xi | b b^\dagger | \xi > = \sinh^2 r
\]

(91)

After some algebra (see for example [54] and references therein) one can write the normalized squeezed state as

\[
|\alpha > = (1 - |\alpha|^2)^{1/4} \exp\left(\frac{\alpha}{2} (a^\dagger)^2\right) |0 >
\]

(92)

In ours case \( \alpha = 1 \) and our state can not be normalized, but it is interesting that it is just at the border between normalizable and non-normalizable states. A squeezed state is a minimum uncertainty state as well as a coherent state. Contrary to coherent states which have minimal quantum uncertainty for both conjugate variables for the squeezed state one can get large uncertainty for one variable while the other is “squeezed” to keep the product fixed. It seems that this interesting property is related to the very nature of localization of open strings on H-brane. It is also interesting that in this state we have only pairs of open string oscillators. More detailed analysis of this state will be given in separate publication.

Let us note that one can get a density matrix starting from dynamics in both sectors I and III and then mapping the wave function

\[
|\Psi > = \sum_m A_m |m\>_L \otimes |m\>_R
\]

(93)

into density matrix

\[
\rho = \sum_m A_m |m\>_R \langle m|_R
\]

(94)

as been suggested in [53]. It is quite interesting that besides squeezed state we have discussed it may be another candidate for a quantum state of H-brane, but in this case it will have non-zero entropy

\[
S = -Tr \rho \ln \rho = -\sum_m A_m \ln A_m
\]

(95)

\[
\text{when this paper was prepared for publication we became aware about recent paper [56] in which density matrix from wave function on maximally extended eternal AdS black hole was discussed}
\]
We hope to return to this issue in the future publication and see if the entropy of H-brane in a mixed state can explain entropy of quantum horizon.

6 Conclusion

In this paper we suggested a simple model where open string endpoints are fixed in a single null direction. We conjectured that this gives us a new class of branes - $H$-branes. The excitations of H-branes are chiral open strings. These strings are not just Schild null-strings [57], where all points of a string (and not only its endpoints) travel at the speed of light. The fact that H-branes are associated with a single chiral sector on world-sheet gives us a totally new boundary conditions. It is interesting to analyze in more details relations between these conditions and usual boundary conditions for boosted D-branes [51, 43] and also to recently introduced A-branes and B-branes [58, 59].

We conjecture that H-branes play important role in stringy description of quantum horizons, such as black hole or cosmological horizons. $H$-branes has a remarkable property that after analytical continuation one gets chiral closed strings in near-horizon Euclidean geometry, which have been introduced some time ago [12, 14]. At the same time they may give us a space/phase space transmutation and non-commutativity of light-cone coordinates. It will be quite natural to conjecture that $H$-branes ultimately related to Bekenstein-Hawking entropy. These and related questions will be discussed in future publications.

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