Mass diffusion and liner material effect in a MagLIF fusion-like plasma

F. García-Rubio1,a and J. Sanz1
E.T.S.I. Aeronáutica y del Espacio, Universidad Politécnica de Madrid, Madrid 28040, Spain.
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In this paper, liner - fuel mass diffusion and the effect of the liner material on mass ablation, energy and magnetic flux losses are studied in a MagLIF fusion-like plasma. The analysis performed in [García-Rubio and Sanz, Phys. Plasmas 24, 072710 (2017)] is extended to liner materials of an arbitrary atomic number. The liner ablates and penetrates into the hot spot, compressing and cooling down the fuel. The magnetic flux in the fuel is lost by Nernst convection through the ablated liner - fuel interface, called ablated border. Increasing the liner atomic number leads to a reduction of both energy and magnetic flux losses in the fuel for a small and moderate magnetization values. Mass diffusion is confined within a thin layer at the ablated border. Concentration gradient and baro-diffusion are the predominant mechanisms leading to liner - fuel mixing. The width of the diffusion layer may be comparable to the turbulent mixing layer resulting from the Rayleigh-Taylor instability at the ablated border. An asymptotic analysis performed for large liner atomic number $Z_2$ shows that mass ablation, energy and magnetic flux losses and liner - fuel mass diffusion scale as $1/\sqrt{Z_2}$.

I. INTRODUCTION

In the recently proposed magnetized liner inertial fusion (MagLIF) scheme, a pulsed power machine drives the implosion of a conductive cylindrical liner filled fuel that is magnetized and preheated. The advantages of magnetizing the fuel lie both on reducing heat losses and enhancing $\alpha$-particles energy deposition. The liner is typically made of low atomic number metals such as lithium, beryllium or aluminum. MagLIF concept has been scaled down in size to OMEGA laser facility, where a laser drives the implosion of a parylene-Z plastic liner, less dense than metal liners.

A relevant feature of magnetized implosions is the long time that the hot spot and the cold liner stay in contact. Understanding the effect of fuel magnetization on heat and magnetic flux losses through the hot spot - liner interface becomes essential. For this purpose, the evolution of a hot magnetized plasma (hot spot) in contact with a cold unmagnetized liner has recently been investigated in planar geometry. The problem is studied in the low Mach number and high thermal to magnetic pressure ratio $\beta$ limit, which implies isobaricity. In Ref. 8 the liner is treated as a cold dense plasma made of the same material as the fuel (deuterium), aiming to model the cryogenic fuel layer added on the inner side of the liner in high-gain MagLIF configuration. The hot spot - liner interface represents an ablation front through which the liner ablates, penetrates into the hot spot and cools it down by thermal conduction. The interface separating the ablated liner material and the fuel is referred to as ablated border. In Ref. 10 this study was revisited including finite pressure ratio $\beta$ effects.

In this paper, we extend the analysis performed in Ref. 8 to liners made of an arbitrary material, see Fig. 1. The plasma ablated into the hot spot has therefore a different atomic number $Z$ compared to the fuel. In the first part of this paper, the two plasmas are treated as immiscible, and the ablated border appears as a contact discontinuity where certain jump conditions hold.

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a)Electronic mail: fernando.garcia.rubio@upm.es
conditions must be satisfied. In the second part of this paper, we let the two plasmas diffuse and analyze liner and fuel mixing.

Diffusion of ion species in multi-component plasmas has recently gained attention as a potential mechanism explaining the observed yield anomalies in inertial confinement fusion (ICF) implosions\(^\text{11,12}\). In plasmas, such diffusion can be driven by concentration, pressure, electrostatic potential and temperature gradients. The latter three are defined as baro-diffusion, electro-diffusion and thermo-diffusion, respectively. Kagan and Tang\(^\text{13,14}\) obtained the baro and electro-diffusion ratios \(k_p\), \(k_e\) without invoking a collisional model, proving that both ratios are a thermodynamic quantity. In the authors’ words, baro and electro-diffusion are “inextricably intertwined”, and while \(k_p\) and \(k_e\) depend on the choice of the thermodynamic system (including ions only or the plasma as a whole), the overall diffusive flux stays the same regardless of this choice\(^\text{14}\). When the thermodynamic system includes the ions only, the baro-diffusion ratio \(k_p\) turns out to be identical to its counterpart in a neutral gas mix\(^\text{15}\), and the electro-diffusion ratio \(k_e\) depends on the charge-to-mass ratio of the ion species\(^\text{13}\). Thermo-diffusion, on its part, arises from the thermal force in the friction drag between different ion species and between electrons and ions. It depends on the nature of the collisions between particles. Due to the long range of the Coulomb collisions, it plays a more important role in plasmas compared to neutral gas mixtures. In addition, ion thermo-diffusion reinforces baro-diffusion in plasmas, in contrast to the neutral mixture case\(^\text{16}\).

In the work done by Simakov, Molvig and collaborators\(^\text{17-20}\), the kinetic equations of multi-component plasmas are solved by a generalized Chapman-Enskog expansion that assumes small Knudsen numbers for all species. The transport terms are obtained, including ion species diffusion, in the absence of magnetic field. In Ref. \(^\text{18}\) the electron and ion transport theories are thoroughly developed for a two ion species plasma with disparate masses. The authors solve the variational problem by expanding the density distribution in generalized Laguerre polynomials of order 3/2, following the formalism of Helander and Sigmar\(^\text{21}\). The calculation is reduced to two independent “Spitzer” problems for electrons and light ions. The results for the electron transport theory are in agreement with the results derived in Ref. \(^\text{17}\) where a generalization of Braginskii electron fluid description\(^\text{22}\) is performed for plasmas with multiple ion species, and therefore used a different set of thermodynamic forces and fluxes that lead to a different variational principle. In more recent publications\(^\text{19,20}\), Simakov and Molvig developed the ion transport theory for an unmagnetized collisional plasma with multiple ion species of arbitrary mass. The fluid equations for multi-component plasmas are closed with the expressions derived therein for individual ion species drift velocities, total ion heat flux and viscosity. They followed a generalization of the Braginskii ion fluid description, which agrees with the results previously obtained for two ions species with disparate masses in Ref. \(^\text{18}\).

We apply the theory developed in Refs. \(^\text{19,20}\) in the second part of this paper to study mass diffusion at the ablated border. Since the transport terms have only been derived for an unmagnetized plasma, we isolate the hydrodynamic problem and disregard the magnetic field evolution in this part. This paper is therefore organized as follows. In Sec. II the problem is presented and the governing equations are discussed not taking into account mass diffusion. They can be reduced to a system of two partial differential equations for temperature and magnetic field in which the solution presents a self-similar structure. In Sec. III the results are discussed and the effect of the liner material on the thermal and magnetic flux losses is studied. In Sec. IV the problem is formulated again taking into account mass diffusion. The governing equations can be reduced to a system of two independent equations for temperature and fuel concentration, whose solution also presents a self-similar nature. In Sec. V the results with mass diffusion are presented and in Sec. VI conclusions are drawn.
II. GOVERNING EQUATIONS WITHOUT MASS DIFFUSION

We consider at $t = 0$ a hot plasma medium at rest with a uniform temperature $T_0$, ion particle density $n_0$ and thermal pressure $p_0$ occupying the semi-infinite space $x > 0$. It is initially magnetized with a uniform magnetic field $\vec{B}_0 = B_0 \vec{e}_z$. The hot plasma is in contact at $x = 0$ with a cold denser unmagnetized plasma ($T \ll T_0$, $|\vec{B}| \ll B_0$) that extends towards $x < 0$, and they are in mechanical equilibrium (same pressure). Both plasmas are fully ionized and made of a different material, that is, different atomic number $Z$. They crudely represent the fuel, $Z = Z_1$, and the liner, $Z = Z_2$, respectively. Hereinafter, we consider that the fuel is made of deuterium, $Z_1 = 1$, and we take $Z_2$ as a free parameter. We let the system evolve for $t > 0$. As a consequence of thermal conduction, the liner material ablates, compresses the fuel and cools it down, while the magnetic field is subjected to convection and diffusion. The region $x > 0$, which stands for the hot spot, is therefore formed of two plasmas of different atomic number $Z$: ablated liner material and fuel, with the ablated border being the interface between them, see Fig. 1. The hot spot - liner interface, $x = 0$, represents consequently an ablation front. The liner (cold dense plasma), stays at rest and unmagnetized since its thermal conductivity, $\chi \sim T_0^{5/2}$, is low and the magnetic diffusivity, $D_m \sim T_0^{-3/2}$, is high (it can be considered as a magnetic isolant); neither heat flux nor electrical currents can take place in it.

We therefore focus on the dynamics of the plasmas in the hot spot, $x > 0$. As typically occurs in MagLIF implosions, we assume subsonic motion (low Mach number), and large thermal to magnetic pressure ratio: $\beta = 8\pi p_0 / B_0^2 \gg 1$. Due to the geometry of the problem, the only independent variables are the streamwise direction $x$ and time $t$, the plasma ion velocity is one-dimensional $\vec{v} = v(x,t) \vec{e}_x$, and the magnetic field stays perpendicular to the motion $\vec{B} = B(x,t) \vec{e}_z$. Quasi-neutrality, same ion and electron temperatures and ideal gas hypotheses are assumed, and we take the adiabatic index $\gamma = 5/3$. The evolution of the ion particle density $n$, thermal pressure $p = p_e + p_i$, temperature $T$, ion velocity $v$ and magnetic field $B$ is governed by the ion continuity, plasma momentum and energy conservation equations neglecting plasma viscosity, together with Faraday’s law (induction equation) and the equation of state, which for a small Mach number and large $\beta$ limit read

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (nv) = 0,
\]
\[ \frac{\partial p}{\partial x} = 0, \quad (2) \]

\[ \frac{1}{\gamma - 1} \frac{\partial p}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\gamma}{\gamma - 1} pv \right) = \frac{\partial}{\partial x} \left( \chi_\perp \frac{\partial T}{\partial x} \right), \quad (3) \]

\[ \frac{\partial B}{\partial t} + \frac{\partial}{\partial x} (vB) = \frac{\partial}{\partial x} \left( D_{m \perp} \frac{\partial B}{\partial x} + \frac{e^2 \beta_N T}{Ze n} \frac{\partial T}{\partial x} \right), \quad (4) \]

\[ p = (1 + Z)nT. \quad (5) \]

We use Braginskii’s expressions and notations for the transport coefficients, with a constant value equal to 7 for the Coulomb logarithm \( \lambda \). The coefficient \( \chi_\perp \) stands for electron plus ion conductivities, \( \beta_N T \) refers to the transport coefficient for the Nernst effect and \( D_{m \perp} = \alpha_\perp c^2 / 4 \pi e^2 Z^2 n^2 \) is the magnetic diffusion coefficient appearing in the Joule dissipation. The momentum equation (2) is reduced to isobaricity: thermal pressure is constant and given by the initial conditions, \( p = p_0 \). Plasma energy equation (3) is thus reduced to the balance between enthalpy convection and thermal conduction. It can be integrated once, yielding an explicit relation for the ion velocity

\[ v = \frac{\gamma - 1}{\gamma p_0} \chi_\perp \frac{\partial T}{\partial x}. \quad (6) \]

Plasma velocity is therefore proportional to the thermal conduction heat flux \( q = -\chi_\perp \partial T/\partial x \).

As discussed in Ref. 8 in the integration of the equation of energy one should add a constant of integration \( v_\infty \), which happens to be zero if the liner is modeled as a cold dense plasma. We impose zero heat flux at the liner interface since the heat conduction losses are recycled back into the hot spot via the ablated material. The position of the ablated border, denoted by \( x_b(t) \), has to be determined self-consistently. It is determined by applying the fluid surface condition

\[ \frac{dx_b}{dt} = v(x_b, t). \quad (7) \]

In this part of the paper, we consider that both plasmas are immiscible (no atomic mass diffusion), hence \( x_b(t) \) represents also a contact discontinuity. Consequently, the atomic number \( Z \) takes the value \( Z = Z_2 \) (liner material) at \( x < x_b(t) \) and \( Z = Z_1 \) (fuel) at \( x > x_b(t) \).

The system of governing equations (1), (4) and (6) has to be completed with the appropriate boundary conditions. We impose that far from the liner, \( x \to \infty \), the plasma recovers the initial fuel state \( n = n_0, T = T_0 \) and \( B = B_0 \). At the interface with the liner, \( x = 0 \), we require that the plasma temperature and magnetic field be equal to the temperature and magnetic field values at the liner, that is \( T/T_0 \to 0 \) and \( B/B_0 \to 0 \).

In addition to the boundary conditions, we demand that temperature, magnetic field and ion velocity be continuous at the ablated border. The latter condition implies continuity of the heat flux \( q \). According to Eq. (5), the ion particle density is discontinuous, and it satisfies the relation

\[ (Z_2 + 1) n(x^-, t) = (Z_1 + 1) n(x^+, t). \quad (8) \]
We also introduce the mass density as ρ = m_in, with m_i being the ion mass. For simplicity, we consider that in both fuel and liner materials, the mass number doubles the atomic number, and we take the ion mass m_i to be related to the proton mass m_p through m_i = 2Zm_p. The former jump condition turns into

$$\frac{Z_2 + 1}{Z_2} \rho (x_b^-, t) = \frac{Z_1 + 1}{Z_1} \rho (x_b^+, t).$$

(9)

Notice that for Z_1 = 1, n(x_b^-, t) ≤ n(x_b^+, t) but ρ (x_b^-, t) ≥ ρ (x_b^+, t).

Finally, the induction equation (4) forces the sum of the Joule dissipation plus the Nernst term \(D_{m,\perp} \partial B/\partial x + (c\beta_0^q T / Zen) \partial T/\partial x\) to be continuous through the ablated border.

### A. Normalization and self-similarity

The ion particle density, temperature and magnetic field in the hot spot are normalized with their initial value in the fuel: σ = n/n_0, θ = T/T_0 and φ = B/B_0. Ion density is related to temperature through the equation of state \(\mathcal{E}\), yielding σ = (Z_1 + 1) / (Z + 1) θ. The ion mass density is likewise normalized \(\overline{\rho}\) = ρ/ρ_0, with ρ_0 = 2Z_1m_pn_0. The thermal conductivity coefficient is normalized with the unmagnetized electron conductivity, \(\chi_{\perp} = K T_0^{3/2} \beta^q T / e Z n_0 (x_e; Z)\), where

$$\overline{K} = \frac{Z n \tau_e \gamma_0}{m_e T^{3/2}} = \frac{3 \gamma_0}{4 \sqrt{2} \pi m_e e^4 \lambda Z}$$

(10)

is the constant factor in Spitzer conductivity, and \(\tau_e, m_e\) are the electron collision time and mass, respectively. The thermoelectric transport coefficient is proportional to the thermal conductivity coefficient \(c\beta_0^q T / Zen\) [1] \(\chi_{\perp} / \gamma_0 \mathcal{E}_n (x_e; Z)\), while the magnetic diffusion coefficient is written as \(D_{m,\perp} = D T_0^{3/2} \beta^q Z \mathcal{E}_d (x_e; Z)\), with

$$\overline{D} = \frac{c^2 m_e T^{3/2} \alpha_0}{4 \pi e^2 Z n \tau_e} = \frac{c^2 \sqrt{2} m_e e^2 \alpha_0 Z}{3 \sqrt{\pi}}$$

(11)

being a diffusivity constant. Notice that the conductivity and diffusivity constants are functions of the atomic number: \(\overline{K} (Z), \overline{D} (Z)\). The terms \(\mathcal{P}_c, \mathcal{P}_n\) and \(\mathcal{P}_d\) account for the effect of magnetization on the transport coefficients, see Fig. 2. They are rational functions of the electron Hall parameter (electron cyclotron frequency times the electron collision time) \(x_e = \omega_e \tau_e = (eB/m_e c) \tau_e\) and the atomic number Z, and read

$$\mathcal{P}_c (x_e; Z) = \frac{\gamma_0 \gamma' \overline{x_e^2} + \gamma_0' \Delta_e}{\gamma_0 \Delta_e} + \frac{1}{Z^3} \sqrt{\frac{2 m_e}{m_i}} \frac{2 \overline{x_e^2} + 2.645}{\gamma_0 \Delta_i},$$

$$\mathcal{P}_n (x_e; Z) = \frac{Z + 1}{Z} \frac{\gamma}{\gamma - 1} \times$$

$$\frac{\beta_0'' \overline{x_e^2} + \beta_0'}{\gamma_0' \overline{x_e^2} + \gamma_0'} \sqrt{\frac{2 m_e \Delta_e}{m_i \Delta_i} (2 \overline{x_e^2} + 2.645)},$$

$$\mathcal{P}_d (x_e; Z) = \frac{1 - \alpha' \overline{x_e^2} + \alpha'}{\Delta_e} \frac{\Delta_e}{\alpha_0},$$

with \(\Delta_e = x_e^4 + \delta_1 x_e^2 + \delta_0\) and \(\Delta_i = x_i^4 + 2.70 x_i^2 + 0.677\), and \(x_i = \omega_i \tau_i = x_e \sqrt{2 m_e / m_i Z}\) standing for the ion magnetization. Finally, the coefficients \(\alpha_0, \alpha_0', \gamma_0'\) along with others are functions of the atomic number and are given in Braginskii [22] for Z = 1, 2, 3, 4 and Z → ∞.
In an unmagnetized plasma, $x_e \ll 1$, thermal conduction is mainly due to electrons because of their small mass and $\mathcal{P}_c \approx 1$. However, the electrons get magnetized for lower magnetic field intensities compared to the ions ($x_e = 1$ with respect to $x_i = 1 \Rightarrow x_e = Z \sqrt{m_i/2m_e} \approx 43Z^{3/2}$), and when electron conduction is suppressed due to magnetization, the ions carry the heat transport. For large magnetization values, $x_e > Z \sqrt{m_i/2m_e}$, the ions also get magnetized and the thermal conductivity decreases asymptotically as $\mathcal{P}_c \sim x_e^{-2}$.

The electron Hall parameter anywhere in the hot spot, $x_e$, can be written as a function of its initial value in the fuel, the atomic number $Z$ and the dimensionless temperature and magnetic field profiles

$$x_e = x_{e0} \frac{Z + 1}{Z^2} \frac{Z_1^2}{Z_1 + 1} \phi \theta^{5/2}. \quad (12)$$

We define the magnetic Lewis number $L_{em}$ as the ratio between the thermal and magnetic diffusivities in the fuel, defined, on their part, as

$$\kappa_0 \equiv \frac{(\gamma - 1) \dot{K} (Z_1) T_0^{7/2}}{\gamma p_0} \approx \frac{40,500}{\lambda} \frac{T_0^{5/2}}{\rho_0} \text{ cm}^2/\text{sec} \quad (13)$$

and

$$\nu_{m0} \equiv \dot{D} (Z_1) T_0^{-3/2} \approx \frac{13.33\lambda}{T_0^{1/2}} \text{ cm}^2/\text{sec}, \quad (14)$$

respectively. It reads

$$L_{em} \equiv \frac{\kappa_0}{\nu_{m0}} \approx \frac{3038 T_0^4}{\lambda^2 \rho_0}. \quad (15)$$
In the practical formulas here and below, the temperature is expressed in keV and the mass density in g/cc. The magnetic Lewis number can also be expressed in terms of the pressure ratio $\beta$ and the initial electron Hall parameter $x_{e0}$,

$$\text{Le}_m \equiv \frac{\kappa_0}{\nu_{m0}} = \frac{\gamma - 1}{2\gamma} \frac{Z_1^2}{(Z_1 + 1)^2} \frac{\gamma_0(Z_1)}{\alpha_0(Z_1)} \beta x_{e0}^2 \approx 0.31 \beta x_{e0}^2.$$  \hfill (16)

The initial value problem proposed lacks a characteristic length and velocity, and the ablated border position $x_b(t)$ is an eigenvalue of the problem. Therefore, the solution to (1), (4) and (6) is sought under the form of a self-similar diffusive wave\textsuperscript{[25]}, as done in Refs. \textsuperscript{[7]} and \textsuperscript{[8]}. We introduce, consequently, the independent self-similar variable $\eta \geq 0$

$$\eta = \frac{x}{\sqrt{\kappa_0 t}}.$$  \hfill (17)

Consistently, the plasma ion velocity is scaled self-similarly

$$V(\eta) = 2\sqrt{\frac{t}{\kappa_0}} v(x,t),$$  \hfill (18)

which, by the use of the equation of energy (6), can be linked to temperature and magnetic field profiles as

$$V = 2\bar{K}_R \bar{\rho}_c \theta^{5/2} \frac{d\theta}{d\eta}.$$  \hfill (19)

Here, $\bar{K}_R$ stands for the Spitzer conductivity constant normalized with its value in the fuel,

$$\bar{K}_R(Z) = \frac{\bar{K}(Z)}{\bar{K}(Z_1)} = \frac{\gamma_0(Z)}{\gamma_0(Z_1)} \frac{Z_1}{Z}.$$  \hfill (20)

The ion continuity equation (1) is rewritten as

$$(V - \eta) \frac{d\sigma}{d\eta} + \sigma \frac{dV}{d\eta} = 0,$$  \hfill (21)

which, by the use of the equation of state and inserting the expression for $V$, becomes

$$\eta \frac{d\theta}{d\eta} + 2\theta^2 \frac{d}{d\eta} \left( \bar{K}_R \bar{\rho}_c \theta^{3/2} \frac{d\theta}{d\eta} \right) = 0.$$  \hfill (22)

The induction equation (4) likewise normalized gives:

$$-\eta \frac{d\phi}{d\eta} + 2 \frac{d}{d\eta} \left[ \bar{K}_R \bar{\rho}_c (1 - \mathcal{P}_n) \theta^{5/2} \frac{d\theta}{d\eta} \right] \right] = \frac{2}{\text{Le}_m} \frac{d}{d\eta} \left( \bar{D}_R \bar{\rho}_d \frac{d\phi}{d\eta} \right).$$  \hfill (23)

Similarly, $\bar{D}_R$ is the diffusivity constant normalized with its value in the fuel,

$$\bar{D}_R(Z) = \frac{\bar{D}(Z)}{D(Z_1)} = \frac{\alpha_0(Z) Z}{\alpha_0(Z_1) Z_1}.$$  \hfill (24)

The conductivity ratio $\bar{K}_R(Z)$ is a decreasing function of $Z$, while $\bar{D}_R(Z)$ is an increasing function of $Z$. 
Two opposite effects take part in the convection term in Eq. (23): magnetic field convection by the plasma motion, in the opposite direction to the conduction heat flux, and convection due to the Nernst term, in the same direction. The balance between both effects is given by the term $1 - \mathcal{P}_n$. There is a threshold magnetization value $x_{\text{cth}}$ below which $\mathcal{P}_n > 1$, the Nernst term is predominant and the magnetic field is convected towards the liner. In the regions where $x_e > x_{\text{cth}}$, the Nernst terms is less effective and the magnetic field is convected by the plasma towards the center of the hot spot. The value of $x_{\text{cth}}$ increases with $Z$, see Fig. 2(b). It takes the value $x_{\text{cth}} = 4.37$ for $Z = 1$, while $\mathcal{P}_n$ is always greater than 1 for $Z \to \infty$.

The problem is therefore reduced to solving the equations (22) and (23), which form a system of two ordinary differential equations of fourth order for the normalized temperature and magnetic field profiles $\theta(\eta)$, $\phi(\eta)$. They are coupled through the dependence of the thermal conductivity on the magnetization and need to be completed with the previously established boundary conditions:

$$\theta(0) = 0, \quad \theta(\eta \to \infty) = 1,$$

$$\phi(0) = 0, \quad \phi(\eta \to \infty) = 1.$$ (25) (26)

The normalized position of the ablated border, $\eta_b = x_b/\sqrt{\kappa_0 t}$, has to be solved self-consistently. It is determined by Eq. (7), which transforms into

$$\eta_b = V(\eta_b).$$ (27)

The eigenvalue $\eta_b$ is thus proportional to the heat flux at the ablated border. Notice that, according to Eq. (21), the ion velocity has a stationary point there. The electron Hall parameter, Eq. (12), is discontinuous, and its value to the right $x_e^+ = x_e(\eta_b^+)$ is larger than its value to the left $x_e^- = x_e(\eta_b^-)$, related by

$$x_e^+ = x_e^- \frac{Z_2^2}{Z_2 + 1} \frac{Z_1 + 1}{Z_1^2}.$$ (28)

The continuity of the heat flux and the Joule plus Nernst terms establishes the following conditions for the derivatives of the temperature and magnetic field

$$\frac{d\theta}{d\eta} \bigg|_{\eta_b^+} = \tilde{K}_R(Z_2) \frac{\mathcal{P}_n^-}{\mathcal{P}_c^+} \frac{d\theta}{d\eta} \bigg|_{\eta_b^-},$$

$$\frac{d\phi}{d\eta} \bigg|_{\eta_b^+} = -\frac{\theta_b}{\mathcal{P}_d^+} \tilde{K}_R(Z_2) \frac{\mathcal{P}_n^-}{\mathcal{P}_c^+} \frac{d\theta}{d\eta} \bigg|_{\eta_b^-} \left(\mathcal{P}_n^- - \mathcal{P}_n^+\right) \phi_b +$$

$$\frac{\mathcal{D}_R(Z_2)}{\mathcal{P}_d^+} \frac{\mathcal{P}_n^-}{\mathcal{P}_d^+} \frac{d\phi}{d\eta} \bigg|_{\eta_b^-},$$ (29) (30)

where $\theta_b$, $\phi_b$ refer to their value at the ablated border, and the superscripts $^{+/-}$ on the transport polynomials refer to their evaluation to the left $(x_e^+; Z_2)$ and to the right $(x_e^-; Z_1)$ side of the ablated border, respectively. Since Le is typically large in MagLIF implosions, the first term in the right-hand side of Eq. (30) is usually predominant. It is therefore the variation of the Nernst convection velocity with $Z$, that is, the factor $(\mathcal{P}_n^- - \mathcal{P}_n^+)$, that originates a steep thin layer at the ablated border where the magnetic field value changes drastically.

The system of governing equations consists thereby of Eqs. (22), (23), together with boundary conditions (25), (26) and jump conditions (29), (30). It depends on three free
parameters: the atomic number of the liner material $Z_2$, the magnetic Lewis number $Le_m$ and the electron Hall parameter of the unperturbed plasma $x_e$. Typically, high magnetic Lewis numbers, $Le_m \sim 100-10^6$, are attained throughout a MagLIF implosion. Notice that the system of governing equations derived in Ref. 8 is recovered if we take $Z_2 = Z_1 = 1$. Our final aim is to study mass ablation, thermal energy and magnetic flux losses as a function of these parameters.

B. Mass ablation, thermal energy and magnetic flux losses

An asymptotic analysis of the Eqs. (22) and (23) performed for $\eta \ll 1$ reveals that the temperature and magnetic field profiles close to the liner take the form

$$\theta|_{\eta \ll 1} = s_\theta \eta^{2/5}, \quad \phi|_{\eta \ll 1} = Le_ms_\phi \eta^{8/5}. \quad (31)$$

The parameters $s_\theta$ and $s_\phi$ depend on $Le_m$, $x_e$ and $Z_2$, and are obtained by solving the complete problem with the boundary conditions far from the liner.

We define mass ablation $m$ and magnetic flux losses $\Phi$ per unit area in the hot spot as

$$m = \int_0^{x_b} \rho dx, \quad (32)$$

$$\Phi = \int_0^\infty (B_0 - B) dx, \quad (33)$$

Other quantities of interest are the thermal energy losses $E$ and magnetic flux losses $\tilde{\Phi}$ in the fuel, defined as

$$E = \int_0^\infty \frac{p_0}{\gamma - 1} dx - \int_{x_b}^\infty \frac{p_0}{\gamma - 1} dx = \frac{p_0}{\gamma - 1} x_b, \quad (34)$$

$$\tilde{\Phi} = \int_0^\infty B_0 dx - \int_{x_b}^\infty B dx. \quad (35)$$

Following a similar analysis as performed in Ref. 8 these quantities are related to $s_\theta$, $s_\phi$ and $\eta_b$ as

$$\frac{m}{\rho_0 \sqrt{\kappa_0 t}} = \frac{Z_2}{Z_1} \frac{Z_1 + 1}{Z_2 + 1} \tilde{K}_R (Z_2) \frac{4}{5} s_\theta^{5/2}, \quad (36)$$

$$\frac{\Phi}{B_0 \sqrt{\kappa_0 t}} = \tilde{D}_R (Z_2) \frac{16}{5} s_\phi^{3/2}, \quad (37)$$

$$\frac{(\gamma - 1) E}{p_0 \sqrt{\kappa_0 t}} = \eta_b, \quad (38)$$

$$\frac{\tilde{\Phi}}{B_0 \sqrt{\kappa_0 t}} = 2s_b, \quad (39)$$

with

$$s_b = \left( \tilde{K}_R P_n \Pi_c \theta^{5/2} \frac{d\theta}{d\eta} \phi + \frac{P_d \tilde{D}_R}{Le_m \theta^{3/2}} \frac{d\phi}{d\eta} \right) |_{\eta = \eta_b}. \quad (40)$$

The magnetic flux in the fuel is thereby lost due to Nernst convection and magnetic diffusion through the ablated border, the former being predominant if $Le_m$ is large. The fuel thermal energy is lost due to thermal conduction at the ablated border, proportional to $\eta_b$, and is inverted into heating the ablated liner material.
III. ANALYSIS AND RESULTS WITHOUT MASS DIFFUSION

Numerical computations of the governing equations are plotted in Fig. 3. Temperature and magnetic field profiles for the unmagnetized case and $L_e = 10^2$ are compared in Fig. 3(a) for two different liner materials, $Z_2 = 1$ (deuterium) and $Z_2 = 4$ (beryllium). It can be seen that the plasma temperature at the hot spot is significantly higher in the second case. The magnetic field, convected by the Nernst term into the ablated material, is pushed against the liner and diffuses in a thin layer adjacent to it. When the liner is made of beryllium, the magnetic field experiments an abrupt decrease at the ablated border, $\eta_b = 0.303$; and as a consequence, the peak close to the liner attains lower values.

According to the jump condition (30), this decrease corresponds to the dependence of the Nernst convection velocity on $Z$ through the polynomial $P_n(x_e; Z)$. As seen in Fig. 2(b), $P_n(x_e; 4) > P_n(x_e; 1)$ for any $x_e$, hence the Nernst convection velocity is stronger in the ablated material. In order to conserve the amount of magnetic field convected through the ablated border, $\phi$ is diffused in a thin magnetic diffusion layer at the fuel side and its value is reduced.

In Fig. 3(b), mass density, magnetic field and magnetization profiles are shown for $L_e = 10^6$, $x_{e0} = 50$ and $Z_2 = 4$. In the magnetized region, the Nernst term is suppressed and the magnetic filed moves frozen into the plasma. Since the ablated material pushes the fuel inwards and compresses it, the magnetic field presents a second peak at the right side of the ablated border. At the left side, the electron Hall parameter is relatively small and Nernst convection dominates. Again, a magnetic diffusion layer takes place at the ablated border, $\eta_b = 0.08$. In this position, both the magnetization and mass density present a discontinuity, given by Eqs. (28) and (41). The ablated border moves with negative acceleration $g = d^2x_b/dr^2 = -\eta_b \sqrt{\eta_b / 4t^{3/2}} < 0$, which implies that the light side of the ablated border (fuel) is pushing backwards the heavy side (liner material). This magnetohydrodynamic structure is susceptible to be Rayleigh-Taylor unstable, and would deserve a more detailed study in a forthcoming paper.

A. Magnetic diffusion layer at the ablated border

In order to obtain the structure of the diffusion layer at the fuel side of the ablated border, we expand the self-similar variable as $\eta = \eta_b + \epsilon r$, with $\epsilon \ll 1$, $r > 0$. We assume $d\phi/dr \sim O(1)$ in this layer and $\theta = \theta_b + O(\epsilon)$, $V = \eta_b + O(\epsilon^2)$. Inserting these ansätze into the induction equation (33) yields to the leading order

$$
\frac{d}{dr} (P_n \phi) = -\frac{2}{\epsilon \eta_b \theta_b^{3/2}} \frac{d}{dr} \left( P_n \frac{d\phi}{dr} \right), \tag{41}
$$

where the electron Hall parameter in the transport polynomials shall be written as $x_e = x_{e0} \theta_b^{3/2} \phi$. We obtain then the characteristic width of this layer

$$
\epsilon = \frac{2}{P_n (0; Z_1) L_e \eta_b \theta_b^{3/2}}, \tag{42}
$$

which scales with the magnetic Lewis number as $\epsilon \sim O(1/L_e)$. This layer is thinner than the diffusion layer adjacent to the liner, which scales as $O \left( 1/\sqrt{L_e} \right)$. Equation (41) stands for a nonlinear equation of second order for $\phi$ that must be complemented with the boundary conditions: $\phi (r = 0) = \phi_b$ and the magnetic field derivative given by Eq. (50). An analytical solution can be obtained in the unmagnetized limit, $x_{e0} \ll 1$, in which case Eq. (41) takes the form $d^2\phi/dr^2 + d\phi/dr = 0$, and the solution reads

$$
\phi (r) = \phi_b \left[ \frac{P_n^+ - P_n^-}{P_n^-} (1 - e^{-r}) + 1 \right]. \tag{43}
$$
FIG. 3. (a) Temperature $\theta$ and magnetic field $\phi$ profiles for $L_{em} = 10^4$, $x_{e0} = 0$ and $Z_2 = 1$ (gray) and $Z_2 = 4$ (black). The magnetic field diffusion layer at $\eta = \eta_b$ is compared to the analytic expression Eq. (43), which is shown in dashed lines in gray. (b) Mass density $\rho$, magnetic field $\phi$ and Electron Hall parameter $x_e$ profiles for $L_{em} = 10^6$, $x_{e0} = 50$ and $Z_2 = 4$.

This analytical expression is compared to the numerical solution in Fig. 3(a), showing good agreement. Evaluating this profile far from the layer, $r \to \infty$, allows to obtain the magnetic field jump across it ($\phi^r$ to the right and $\phi^l = \phi_b$ to the left), yielding:

$$\frac{\phi^r}{\phi_b} = \frac{P_n^-}{P_n^+} > 0. \quad (44)$$

### B. Analytic solution for large $Z_2$

In order to shed more light on the effect of the liner material, we solve the governing equations (22), (23) in the limit of large liner atomic number $Z_2$ and $x_{e0} \ll 1$. When $Z_2$ is large, the conductivity constant $\bar{K}$ in the ablated material is reduced, while the diffusivity constant $\bar{D}$ is increased. In this limit, we can write

$$\bar{K}_R (Z_2 \gg 1) = \frac{3.94}{Z_2} \equiv \epsilon_K, \quad (45)$$

$$\bar{D}_R (Z_2 \gg 1) = 0.57Z_2 \equiv \frac{\alpha}{\epsilon_K}, \quad (46)$$

with $\epsilon_K \ll 1$ and $\alpha \equiv \alpha_0 (\infty) \gamma_0 (\infty)/\alpha_0 (1) \gamma_0 (1) \approx 2.27$. When $x_{e0} \ll 1$, the continuity equation is uncoupled from the induction equation, and can be solved independently. The transport polynomials take the value $P_c (0; Z) \approx 1$, $P_d (0; Z) = 1$, and $P_n (0; 1) \approx 1.24$, $P_n (0; \infty) \approx 1.83$. 
1. Temperature profile

In the ablated liner material, the small parameter $\epsilon_K$ can be absorbed in the continuity equation by scaling the independent variable as $\eta = \sqrt{\epsilon_K} s$. Letting $\theta_i$ be the temperature profile in this region, the continuity equation reads

$$s \frac{d\theta_i}{ds} + 2\theta_i^2 \frac{d}{ds} \left( \theta_i^{3/2} \frac{d\theta_i}{ds} \right) = 0. \tag{47}$$

The temperature profile close to the liner takes the form $\theta_i(s \ll 1) = s_b' s^{2/5}$, with $s_b' = s_b \epsilon_K^{1/5}$. The solution has to satisfy the jump conditions at the ablated border. Scaling its position as $s_b \equiv \eta_b / \sqrt{\epsilon_K}$, it is determined by Eq. (27), which transforms into

$$2\theta_i^{5/2} \frac{d\theta_i}{ds} \bigg|_{s_b} = s_b. \tag{48}$$

At this position, the temperature must be continuous, $\theta_i(s_b) = \theta_f(\eta_b)$, being $\theta_f(\eta)$ the temperature profile in the fuel region. The temperature derivatives are linked through the jump condition [29], giving $d\theta_f/d\eta|_{\eta_b} = \sqrt{\epsilon_K} \theta_b'$, where $\theta_b' \equiv d\theta_i/ds|_{s_b}$. If we assume that $d\theta_i/ds \sim O(1)$, then the temperature derivative in the fuel is small of order $\sqrt{\epsilon_K}$, and the solution will not differ much from $\theta_f \approx 1$. Therefore, to the leading order, we can impose that $\theta_i(s_b) = 1$. Consequently, the temperature profile at the ablated material $\theta_i$ can be solved independently of the fuel region in a small $\epsilon_K$ limit. The solution yields $s_b' = 1.21$, $s_b = 0.81 = 2\theta_b'$.

In the fuel region, the continuity equation reads

$$\eta \frac{d\theta_f}{d\eta} + 2\theta_f^2 \frac{d}{d\eta} \left( \theta_f^{3/2} \frac{d\theta_f}{d\eta} \right) = 0. \tag{49}$$

Since the temperature derivatives are small, we expand the solution as $\theta_f = 1 - \sqrt{\epsilon_K} \theta_{f1} + O(\sqrt{\epsilon_K})$. Inserting this ansatz in the previous equation and retaining the leading order terms allows to obtain the first correction, giving $\theta_{f1} = C[1 - \text{Erf}(\eta/2)]$. The constant of integration $C$ is obtained by applying the jump condition [29], yielding $C = \sqrt{\pi} \theta_b' \approx 0.72$. The ion velocity and the Nernst velocity, proportional to the temperature derivative, are also small of order $O(\sqrt{\epsilon_K})$.

From these results, we infer that the ablated mass and energy losses are written as

$$\frac{m}{\rho_0 \sqrt{\kappa_0 l}} = \frac{Z_1 + 1 + 4 \phi_0 s^5/2}{5} \sqrt{\epsilon_K} \approx \frac{5.09}{\sqrt{Z_2}}, \tag{50}$$

$$\frac{(\gamma - 1) E}{\rho_0 \sqrt{\kappa_0 l}} = s_b \sqrt{\epsilon_K} \approx \frac{1.61}{\sqrt{Z_2}}. \tag{51}$$

To conclude, in a large $Z_2$ limit, the ablated border steps back, the temperature profile is almost constant in the fuel and decreases in the thin ablated liner layer whose width scales as $O(1/\sqrt{Z_2})$, and both mass ablation and energy losses are reduced when $Z_2$ increases.

2. Magnetic field profile

We assume, as typically occurs in MagLIF, large $\text{Le}_m$. Consequently, diffusion can be initially neglected in the fuel. The magnetic field is convected by the plasma motion and the Nernst term, being the latter predominant in the unmagnetized case. We introduce $\delta_l = \mathcal{P}_n(0; \infty) - 1 \approx 0.83$ and $\delta_f = \mathcal{P}_n(0; 1) - 1 \approx 0.24$. The equation governing the magnetic field in the fuel, $\phi_f$, reads then

$$\eta \frac{d\phi_f}{d\eta} + 2\delta_f \sqrt{\epsilon_K} \theta_b' \frac{d}{d\eta} \left( e^{-\eta^2/4} \phi_f \right) = 0. \tag{51}$$
The solution satisfying $\phi_1(\infty) = 1$, retaining terms up to $O(\sqrt{c_K})$, gives $\phi_f = 1 - \sqrt{c_K} \pi \delta f / \theta_b [1 - \text{Erf}(\eta/2)]$. The magnetic field is barely perturbed since the convection velocity is small, of order $O(\sqrt{c_K})$, and only the leading order $\phi_f \approx 1$ will be retained. Close to the ablated border, the thin diffusion layer described in Subsec. III A takes place, its width given by Eq. (42): $\epsilon \approx O (1/L_{eM} \sqrt{c_K})$, and the magnetic field in this layer drops from $\phi_f \approx 1$ to $\phi_b = (1 + \delta_f) / (1 + \delta_t) = 0.68$.

In the ablated plasma liner, the induction equation shall be rewritten as

$$-s \frac{d \phi_t}{ds} - 2 \delta_t \frac{d \theta_t}{ds} \left( \theta_t^{5/2} \frac{d \phi_t}{ds} \right) = \frac{2 \alpha}{L_{em} \epsilon_K^2} \frac{d}{ds} \left( \theta^{-3/2} \frac{d \phi_t}{ds} \right),$$

(52)

where we assume $d \phi_t / ds \sim O(1)$. It can be seen that the magnetic Lewis number appropriate for this region is $L_{ei} = L_{em} \epsilon_K^2 / \alpha$. As discussed at the end of this section, we will assume that $L_{ei}$ is large too. Consequently, diffusion can be neglected in the main part of the ablated material region, while it is confined to a thin sub-layer close to the liner. Letting $\phi_{lo}$ denote the leading order solution (outer solution) of Eq. (52), and $w = 2 \theta_t^{5/2} \frac{d \theta_t}{ds}$, the former satisfies

$$\frac{d \phi_{lo}}{ds} = -\delta_t \phi_{lo} \frac{ dw / ds }{ s + \delta_t w },$$

(53)

together with $\phi_{lo}(s) = \phi_b$. Solving this equation allows to obtain the shape of this profile close to the liner $\phi_{lo}(s \ll 1) = s \phi_o s^{-2/5}$, with $\phi_o = 0.28$. A thin sub-layer must then take place where the magnetic field diffuses and drops to its value at the liner, $\phi_l(0) = 0$. In order to obtain the magnetic field in this sub-layer (inner solution $\phi_{li}$), we expand Eq. (52) in the variable $s = \epsilon B z$, with $\epsilon B \ll 1$, and assume $d \phi_{li} / ds \sim O(1)$. Requiring diffusion to be important gives the width of this boundary layer

$$\epsilon_B = \sqrt{\frac{5}{2 \delta_t s_\theta L_{ei}}},$$

(54)

while the resulting equation for $\phi_{li}$ is

$$\frac{d}{dz} \left( z^{2/5} \phi_{li} \right) + \frac{d}{dz} \left( \frac{1}{z^{3/5}} \frac{d \phi_{li}}{dz} \right) = 0.$$

(55)

The solution satisfying the boundary condition $\phi_{li}(0) = 0$ is

$$\phi_{li} = Q \left[ \int_0^z z^{3/5} \exp \left( \frac{z^2}{2} \right) dz \right] \exp \left( -\frac{z^2}{2} \right),$$

(56)

where the constant of integration $Q$ is obtained by matching the inner solution $\phi_{li}(z)$ for $z \to \infty$ with the outer $\phi_{lo}(s)$ for $s \to 0$, yielding $Q = s \phi_o (2 \delta_t s_\theta^5 L_{ei} / 5)^{1/5}$. The solution in the ablated material region can be therefore expressed by an inner-outer composite expansion.

$$\phi_l(s) = \phi_{lo}(s / \epsilon_B) + \phi_{li}(s) = s \phi_o \delta_t s_\theta s^{-2/5}.$$

The parameter $s_\theta$ yields, in this double limit large $Z_2$ and large $L_{ei}$, $s_\theta = \epsilon_K^{6/5} s \phi_o \delta_t s_\theta^{5/4} / 4 \alpha \approx 0.066 \epsilon_K^{6/5}$. From this analysis, we can derive the magnetic flux losses in the hot spot and in the fuel, reading

$$\frac{\Phi}{B_0 \sqrt{\kappa_0 t}} = 4 \frac{\delta_t s \phi_o s_\theta^{7/2}}{\sqrt{2} \sqrt{c_K}} \approx 0.72 \sqrt{Z_2},$$

(57)

$$\frac{\hat{\Phi}}{B_0 \sqrt{\kappa_0 t}} = 2 (1 + \delta_t) \theta_b \phi_b \sqrt{c_K} \approx 2.01 \sqrt{Z_2},$$

(58)

In the large $L_{ei}$ limit, the main contribution to the magnetic flux losses in the fuel corresponds to the magnetic field convection through the ablated border due to the Nernst
velocity, that is, the first term in the right-hand side of Eq. (40). The magnetic flux conservation both in the fuel and in the whole hot spot (fuel plus ablated material) is improved when \( Z_2 \) increases, following the same power law. This implies that the 64% \[ = 100 \times \left( 1 - \Phi / \tilde{\Phi} \right) \] of the magnetic flux losses in the fuel are spent into magnetizing the ablated material, while the remaining 36% is lost through dissipation at the liner.

It is interesting to check the validity of this limit in realistic MagLIF conditions. We have assumed that the ablated liner material is fully ionized. The last ionization energy of aluminum, \( Z_2 = 13 \), corresponds to 2.3 keV, which is attained by the end of the implosion. At this stage, the magnetic Lewis number is large \[ \text{Le}_l \sim \text{Le}_m/Z_2^2 \gg 1 \] is well satisfied.

C. Results for mass ablation, thermal energy and magnetic flux losses

In Fig. 4, the integral quantities mass ablation, energy losses and magnetic flux losses are shown as a function of the atomic number of the liner \( Z_2 \) for \( \text{Le}_m = 10^9 \) and \( x_{e0} = 0 \). In order to obtain the integral quantities for any \( Z_2 \), we have interpolated the transport coefficients given by Braginskii\[22\] using splines of order 3, and taking \( Z^{-1} \) as the interpolation variable. The conservation of both the thermal energy and the magnetic flux in the fuel is improved when \( Z_2 \) increases. On the contrary, mass ablation and magnetic flux losses in the hot spot attain a maximum for \( Z_2 = 4 \), \( Z_2 = 12 \), respectively, where they take the value 1.12 and 0.084. Although large \( Z_2 \) values are physically meaningless, they are plotted to check the accuracy of the asymptotic laws derived in Subsec. III B, which are also shown in the figure.

As stated in Subsec. III B, increasing \( Z_2 \) lowers the thermal conductivity in the ablated material and the ion and Nernst velocities in all the hot spot. Consequently, heat flux at the ablated border is reduced, and less magnetic field is convected through it by the Nernst term. Both \( \mathcal{E} \) and \( \Phi \) decrease then with \( Z_2 \). One would expect the same trend for \( m \) and \( \Phi \), as the convection velocity is reduced and less magnetic field is accumulated at the liner. However, when \( Z_2 \) increases, the ions coming from the liner are heavier, and the magnetic Lewis number at the ablated liner, \( \text{Le}_l \sim \text{Le}_m/Z_2^2 \), is reduced, that is, the plasma becomes less conductive. These effects enhance \( m \) and \( \Phi \), respectively, and are predominant over the reduction of convection velocity for small \( Z_2 \), while the latter becomes more important for large \( Z_2 \), and therefore \( m \) and \( \Phi \) present a maximum.

The integral quantities mass ablation, thermal energy and magnetic flux losses as a function of the fuel initial magnetization \( x_{e0} \) and for different liner materials (lithium \( Z_2 = 3 \),...
Mass diffusion and liner material effect in a MagLIF fusion-like plasma

\[ Z_2 = 1 \]
\[ Z_2 = 3 \]
\[ Z_2 = 4 \]
\[ Z_2 = 13 \]

\[ \frac{m}{\rho_0 \sqrt{\kappa_0 t}} \]

\[ \eta_b \]

**FIG. 5.** Normalized mass ablation \( m/\rho_0 \sqrt{\kappa_0 t} \) (a) and thermal energy losses \( (\gamma - 1) E/\rho_0 \sqrt{\kappa_0 t} = \eta_b \) (b) for different liner materials. The curves keep a constant \( \beta = 1000 \) when the magnetization is increased. In (b), the difference in percentage between the \( Z_2 = 1 \) and \( Z_2 = 4 \), \( Z_2 = 13 \) cases is plotted, computed as \( 100 \times \left( \eta_b|_{Z_2=4} - \eta_b|_{Z_2=1} \right) / \eta_b|_{Z_2=1} \) and \( 100 \times \left( \eta_b|_{Z_2=13} - \eta_b|_{Z_2=1} \right) / \eta_b|_{Z_2=1} \), respectively.

beryllium \( Z_2 = 4 \) and aluminum \( Z_2 = 13 \) are shown in Figs. 5 and 6. The curves are computed keeping a large \( \beta = 1000 \). In the same figures, asymptotic laws for large \( x_{e0} \) are plotted, some of which were obtained in Ref. 8 and adjusted by numerical fitting.

When the fuel is unmagnetized, low \( x_{e0} \), the mass and energy losses follow the same trend observed in Fig. 4. The ablated mass is enhanced for lithium and beryllium, it attains a maximum, and then decreases for higher \( Z_2 \) (aluminum). The energy losses, on their part, decrease monotonically with the atomic number of the liner. Magnetizing the fuel reduces the mass ablated and improves thermal insulation. The effect of \( Z_2 \) on thermal insulation becomes less important when the fuel is magnetized. It could even be inverted when the electron Hall parameter exceeds a certain threshold, see Fig. 5(b), but it barely enhances the energy losses by less than 5%.

In Fig. 6, the magnetic flux losses in the hot spot and in the fuel are depicted. They are shown for \( x_{e0} > 1 \), since the magnetic Lewis number becomes relatively small when the magnetization is further reduced keeping a constant \( \beta = 1000 \), and is out of the range of application to MagLIF. Nevertheless, the arguments derived for the unmagnetized case still apply for moderate \( x_{e0} \). It can be seen that the magnetic flux losses in the hot spot are enhanced when \( Z_2 \) is increased up to the aluminum value, as the ablated material becomes more diffusive. This effect is preserved for all values of \( x_{e0} \). However, the magnetic flux conservation in the fuel, Fig. 6(b), follows a different trend. It is improved with \( Z_2 \) for moderate magnetizations due to the reduction of the Nernst velocity; but this effect diminishes when the magnetization is increased. It can even be inverted for a liner made of beryllium for \( x_{e0} > 15 \), degrading the magnetic flux conservation by less than 10% compared to a liner made of dense deuterium.
FIG. 6. Normalized magnetic flux losses computed in: (a) the hot spot ($x > 0$), $\Phi/B_0\sqrt{n_0t}$ and (b) the fuel ($x > x_b$), $\Phi/B_0\sqrt{n_0t}$, for different liner materials. The curves keep a constant $\beta = 1000$ when the magnetization is increased. In (b), the difference in percentage between the $Z_2 = 1$ and $Z_2 = 4$, $Z_2 = 13$ cases is plotted, computed as $100 \times \left( \frac{\Phi_{Z_2=4}}{\Phi_{Z_2=1}} - \frac{\Phi_{Z_2=13}}{\Phi_{Z_2=1}} \right) / \Phi_{Z_2=1}$, respectively.

IV. GOVERNING EQUATIONS WITH MASS DIFFUSION

We formulate now the problem sketched in Fig. 1 taking into account mass diffusion through the ablated border. The hot spot is modeled as a two ion species plasma composed by deuterium (fuel) and liner material, and we assume that every ion species is fully ionized. The hydrodynamic description of a multiple species plasma, including a closed expression for the transport terms, is derived in Refs. [19] and [20]. We follow the notation therein, therefore the fuel (light species) is labeled with the subscript 1, the liner material (heavy species) with 2 and the electrons with $e$. We name $n_k$, with $k = \{1, 2, e\}$, the particle number density, $\rho_k$ is the mass density, with $\rho_k = m_k n_k$, and $m_k \approx 2Z_k m_p$ when $k$ refers to an ion species, being $Z_k$ its atomic number and $m_p$ the proton mass. We assume same temperature for electrons and ions $T_e = T_i = T$. The partial pressure is $p_k = n_k T$, and $\vec{v}_k$ refers to the flow velocity. We define the plasma density $\rho = \sum_k \rho_k \approx \rho_1 + \rho_2$, the total ion number density $n_i = n_1 + n_2$, the total ion pressure $p_i = p_1 + p_2 = n_i T$ and the total plasma pressure $p = p_i + p_e$. Note that in Refs. [19] and [20], $p$ denotes the total ion pressure, yet, we choose to use $p$ for the total plasma pressure to be consistent with the formulation in the first part of this paper, which at the same time inherits the original formulation in Ref. [8]. The plasma ion velocity is defined as $\vec{v} = \left( \rho_1 \vec{v}_1 + \rho_2 \vec{v}_2 \right) / \rho$, and consequently the electron and ion drift velocities are expressed as $\vec{u}_k = \vec{v}_k - \vec{v}$. Again, although $\vec{v}$ refers to the plasma ion velocity and $\vec{v}_k$ to the drift velocities in Refs. [19] and [20], we choose to swap them and denote the plasma ion velocity as $\vec{v}$ to keep consistent with the first part of this paper. Finally, $x_k = n_k / n_i$ is the number density fraction of the ion species $k$, such
that \( x_1 + x_2 = 1; \ y = \rho_1/\rho \) is the mass concentration of the fuel and \( 1 - y \) is the mass concentration of the liner material.

As commented in the introduction, we have to restrict the fuel liner mixing analysis to the unmagnetized plasma case, where the plasma motion is uncoupled from the magnetic field evolution. Studying mass diffusion in a magnetized plasma would require first to extend the transport theory of multi-component plasmas to take into account the effect of magnetic fields, which is out of the scope of this paper. In the absence of magnetic field, the Ampère’s law, \( \vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t \), drives the electric field \( \vec{E} \) to the value set by the simultaneity condition \( \vec{j} = 0 \), with \( \vec{j} = e (Z_1 n_1 \vec{u}_1 + Z_2 n_2 \vec{u}_2 - n_e \vec{u}_e) \) being the plasma current. As a consequence of the planar geometry and the absence of currents, the electron and ions velocities as well as the rest of the velocities introduced only present a streamwise component, and will be treated as scalars: \( v, \ n \) and \( u_k \).

The evolution of the plasma density \( \rho \), pressure \( p \), velocity \( v \) and fuel concentration \( y \) in the hot spot, \( x > 0 \), is given by the total ion continuity, plasma momentum conservation, energy conservation and fuel continuity equations, which in planar geometry and low Mach and high \( \beta \) limit yield

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) = 0, \quad (59)
\]

\[
\frac{\partial p}{\partial x} = 0, \quad (60)
\]

\[
\frac{3}{2} \frac{\partial p}{\partial t} + \frac{\partial}{\partial x} \left[ \frac{5}{2} \rho v + q_e + q_i \right] + \frac{5}{2} (p_1 u_1 + p_2 u_2 + p_e u_e) = E \cdot \vec{j} \approx 0, \quad (61)
\]

\[
\rho \frac{\partial y}{\partial t} + \rho v \frac{\partial y}{\partial x} = -\frac{\partial}{\partial x} (\rho u_1), \quad (62)
\]

where \( q_e \) and \( q_i \) are the electron and ion heat fluxes, respectively. The momentum conservation equation \( (60) \) reduces to isobaricity, \( p = p_0 \). In addition to these equations, we impose plasma quasi-neutrality, \( n_e = Z_1 n_1 + Z_2 n_2 \). Besides, from the definition of plasma ion velocity, we have \( \rho_1 u_1 + \rho_2 u_2 = 0 \), and the aforementioned ambipolarity condition gives \( Z_1 n_1 u_1 + Z_2 n_2 u_2 - n_e u_e = 0 \). Using \( n_1 = \rho_1/m_1 = \rho y/m_1 \), any ion species or electron densities and drift velocities can be expressed in terms of \( \rho \), \( y \) and \( u_1 \), uniquely. The equation of state can therefore be written as

\[
p = p_0 = \rho T \left[ (1 + Z_1) \frac{y}{m_1} + (1 + Z_2) \frac{1 - y}{m_2} \right]. \quad (63)
\]

With the isobaric assumption, plasma energy equation \( (61) \) can be integrated once, giving an explicit expression for the plasma velocity

\[
v = -\frac{2}{5p_0} \left[ q_e + q_i + \frac{5}{2} (p_1 u_1 + p_2 u_2 + p_e u_e) \right]. \quad (64)
\]

Expressing the plasma density \( \rho \) in terms of \( T \) and \( y \) by means of Eq. \( (63) \), and making use of Eq. \( (64) \), we can reduce the governing equations to a system of two equations, \( (59) \) and \( (62) \), for \( T(x, t) \) and \( y(x, t) \). To close this system, we require the expressions for the electron and ion heat fluxes, \( q_e \) and \( q_i \), and the fuel drift velocity \( u_1 \). These relations are given in Refs. [19] and [20], and its derivation is briefly summarized in the Appendix A. We make therefore use of Eqs. \( (A7), (A18) \) and \( (A16) \) to relate \( q_e, q_i \) and \( u_1 \) to \( T \) and \( y \). Notice that the order of the system of differential equations is four. As boundary conditions, we impose that far from the liner, \( x \to \infty \), we recover the initial state, \( T = T_0 \) and \( y = 0 \), while at the liner - hot spot interface, \( x = 0 \), we have \( T = 0 \) and \( y = 0 \).
A. Normalization and self-similarity

Similarly to Subsec. II A, we normalize temperature and plasma density with their initial value in the fuel: $\theta = T/T_0$ and $\varrho = \rho/\rho_0$. Again, the solution is sought under the form of a self-similar diffusive wave. We introduce the independent self-similar variable $\eta = x/\sqrt{\kappa_0 t}$, with

$$\kappa_0 \equiv \frac{2 \bar{K}_e (Z_1) T_0^{5/2}}{5 \rho_0} \approx 40,500 \frac{T_0^{5/2}}{\log \Lambda_{ee} \rho_0} \text{ cm}^2/\sec,$$  \hspace{1cm} (65)

being the thermal diffusivity as likewise defined in Eq. (13). Consistently, the plasma velocity and the fuel drift velocities are scaled as

$$V (\eta) = 2 \left( \frac{t}{\kappa_0} v (x,t) \right), \quad U_1 (\eta) = 2 \left( \frac{t}{\kappa_0} u_1 (x,t) \right).$$  \hspace{1cm} (66)

The governing equations (59) and (62) are rewritten as

$$(V - \eta) \frac{d \varrho}{d \eta} + \varrho \frac{d V}{d \eta} = 0,$$  \hspace{1cm} (67)

$$\varrho (V - \eta) \frac{d y}{d \eta} = - \frac{d}{d \eta} \left( \varrho y U_1 \right).$$  \hspace{1cm} (68)

The dimensionless density $\varrho$ can be related to $\theta$ and $y$ through the equation of state (63), yielding

$$\varrho = \frac{1}{\theta} \left( y + \frac{Z_2 + 1 - y}{Z_1 + 1} \right),$$  \hspace{1cm} (69)

with $\mu = m_2/m_1$. The dimensionless plasma velocity is obtained from Eq. (64), reading

$$V = 2 \left[ \frac{\bar{K}_e (Z_{\text{eff}})}{\bar{K}_e (Z_1)} \right] \varrho^{5/2} \frac{d \theta}{d \eta} - f_{\nu} \frac{y}{(Z_1 + 1) y + (Z_2 + 1) \frac{1 - y}{\mu}} V_1,$$  \hspace{1cm} (70)

with $f_{\nu} = \left[ 2 \bar{D}_T / (\Delta_{11} x_1 + Z_1 + 1 - (Z_2 + 1) / \mu) \right]$. Finally, the normalized fuel drift velocity is recovered from Eq. (A16), and gives

$$U_1 = - \frac{2}{\bar{L}e} \frac{\theta^{7/2}}{y} \left[ \left( D_c + D_p \right) \frac{d y}{d \eta} + \left( D_Tc + D_Ti \right) \frac{d \log \theta}{d \eta} \right],$$  \hspace{1cm} (71)

where Le stands for the Lewis number, typically defined in mass transfer problems as the ratio between thermal and mass diffusivities, $\kappa_0$ and $\nu_0$, respectively. The latter corresponds to the characteristic value of the coefficient relating fuel diffusion velocity and fuel mass concentration gradient in Eq. (A16), and reads

$$\nu_0 \equiv \frac{2 n_1}{\nu_1 T^{3/2} Z_1 + 1} \frac{\Delta_{11} (0) T_0^{5/2}}{\rho_0} \approx \frac{1.493}{\log \Lambda_{11}} \frac{P \rho}{Z_2^2} \frac{Z_2 + 1 T_0^{5/2}}{\rho_0} \text{ cm}^2/\sec,$$  \hspace{1cm} (72)
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with $P_\mu$ being an increasing function of the ion mass ratio $\mu$, given in Eq. (A22). It ranges from 0.595 for $\mu = 1$ to 1.20 for $\mu \to \infty$.

The Lewis number is then

$$Le \equiv \frac{\kappa_0}{\nu_0} = \frac{1}{5\sqrt{2}} \frac{\gamma_0 (Z_1) Z_1^2}{Z_2 + 1} \frac{1}{\Delta'_{11} (0)} \left( \frac{m_1}{m_e} \right) \approx 27 \frac{Z_2^2}{P_\mu Z_1 + 1}. \quad (73)$$

Notice that it only depends on the liner atomic number, and its value is shown in Table I for typical liner materials. The Lewis number is relatively large and becomes larger for higher $Z_2$, which implies that mass transport is less effective than heat transport. The motion of the plasma in the hot spot is therefore governed by the heat wave, and mass diffusion is confined within a thin layer placed at the ablated border.

The terms $D_c$, $D_p$, $D_{Te}$, and $D_{Ti}$ are the dimensionless coefficients for concentration gradient diffusion, baro-diffusion, electron thermo-diffusion and ion thermo-diffusion, respectively,

$$D_c \equiv \frac{\Delta'_{11} (y)}{\Delta'_{11} (0)} \left( \frac{1 - y + \frac{Z_1 + 1}{Z_2 + 1} \mu y}{y (1 - y + \mu y)^2} \right), \quad (74)$$

$$D_p \equiv D_c y (1 - y) \times \frac{\mu (Z_2 - Z_1)^2}{Z_2 (Z_2 + 1) \left( 1 - y + \frac{Z_1}{Z_2} \mu y \right) \left( 1 - y + \frac{Z_1 + 1}{Z_2 + 1} \mu y \right)}, \quad (75)$$

$$D_{Te} \equiv \frac{\Delta'_{11} (y)}{\Delta'_{11} (0)} y (1 - y) \left( 1 - y + \frac{Z_1 + 1}{Z_2 + 1} \mu y \right) \times \frac{Z_1}{Z_2} \frac{Z_2 - Z_1}{(1 - y + \mu y) \left( 1 - y + \frac{Z_1^2}{Z_2^2} \mu y \right)} \beta_0 (x_1), \quad (76)$$

$$D_{Ti} \equiv \frac{\Delta'_{11} (y)}{\Delta'_{11} (0)} \mu \left( 1 - y + \frac{Z_1 + 1}{Z_2 + 1} \mu y \right). \quad (77)$$

Notice that, as a consequence of isobaricity, both the electron and ion pressure gradient terms in Eq. (A13) can be expressed in terms of fuel mass concentration gradients. The sum of both effects is accounted for in $D_p$, and is denoted hereinafter as baro-diffusion.

As will be explained in Subsec. V A concentration gradient diffusion and baro-diffusion are predominant for large Lewis numbers. Both coefficients are plotted in Fig. 7 for increasing $Z_2$. The former decreases with $Z_2$, while the latter increases. Baro-diffusion is zero in both pure liner ($y = 0$) and fuel ($y = 1$) limits, and it dominates over concentration gradient diffusion in most part of the layer when the liner atomic number is large. As can be seen in Fig. 7(b), the sum of both coefficients approaches $D_c + D_p = 1 - y$ when $Z_2$ is large.

The system of normalized governing equations consists therefore of Eqs. (67) and (68), with density given by Eq. (69), plasma velocity by Eq. (70) and fuel drift velocity by Eq. (71). It only depends on the free parameter $Le$, or, equivalently, the liner atomic number.
FIG. 7. Concentration gradient diffusion coefficient, $D_c$, and baro-diffusion coefficient, $D_p$, for $Z_2 = 3$ (lithium), $Z_2 = 4$ (beryllium) and $Z_2 = 13$ (aluminum), plotted separately in (a) and summed in (b).

V. RESULTS WITH MASS DIFFUSION

The numerical resolution of the normalized governing equations is shown in Fig. 8. Temperature, velocity and mass diffusion rate profiles are plotted in Fig. 8(a) for deuterium fuel and beryllium liner. Temperature and velocity profiles are compared to the solution neglecting mass diffusion explained in the first part of this paper. Both solutions should be identical for an infinite Lewis number. It can be seen that, although the Lewis number for beryllium is not significantly large, $Le = 72$, the profiles are remarkably similar. This suggests that, in a MagLIF context, the hydrodynamic motion would not need to be solved self-consistently with the mass diffusion problem. It can rather be solved independently (immiscible plasmas), and mass diffusion be computed afterwards yielding a similar result. This argument is reinforced by the results shown in Table I where the position of the ablated border and mass ablation are computed with and without mass diffusion, showing good agreement. The fuel mass diffusion rate, $-\rho y U_1$, is relatively small and has a maximum at the ablated border.

The most significant difference between the diffusion and no diffusion solutions lies in the plasma velocity profile, which presents a bump at the ablated border. It is there where liner and fuel materials are in contact and mass diffusion takes place. As a consequence of the strong fuel concentration variations, pressure inhomogeneities arise, which locally accelerate the plasma. The plasma velocity is modified and convects fuel material in order to restore isobaricity.

Liner mass concentration profiles with and without mass diffusion are plotted in Fig. 8(b). Without diffusion, the liner concentration drops from 1 to 0 at the ablated border. When mass diffusion is taken into account, the ablated liner material penetrates into the
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| Liner material | Lithium | Beryllium | Aluminum |
|----------------|---------|-----------|-----------|
| Le             | 85      | 111       | 323       |
| $\eta_b|_{\text{Diff.}}$ | 0.325   | 0.313     | 0.249     |
| $\eta_b|_{\text{No diff.}}$ | 0.315   | 0.304     | 0.245     |
| $\frac{m}{\rho_0\sqrt{\kappa_0}t}|_{\text{Diff.}}$ | 1.15    | 1.16      | 0.973     |
| $\frac{m}{\rho_0\sqrt{\kappa_0}t}|_{\text{No diff.}}$ | 1.12    | 1.13      | 0.951     |
| $\frac{m_{lf}}{\rho_0\sqrt{\kappa_0}t}|_{\text{B.L.}}$ | 0.0471  | 0.0422    | 0.0272    |
| $100 \times \frac{m_{lf}}{m}$ | 4.09%   | 3.64%     | 2.80%     |
| $\epsilon_d$ | 0.0753  | 0.0682    | 0.0471    |
| $\frac{h_D}{gt^2}$ | 0.926   | 0.871     | 0.756     |

TABLE I. Numerical values for deuterium fuel, $Z_1 = 1$, and liners made of lithium, $Z_2 = 3$, beryllium, $Z_2 = 4$, and aluminum, $Z_2 = 13$.

fuel a relatively short distance compared to the characteristic thermal length, given by $\eta_b$. In Fig. 8(c) fuel mass concentration profiles are shown for different liner materials. It can be seen that the width of the diffusion layer shrinks when $Z_2$ increases, as the Lewis number becomes higher.

A. Mass diffusion boundary layer

When the Lewis number is large, mass diffusion is confined within a sharp boundary layer placed at the ablated border. We recall that the position of the ablated border is obtained by Eq. (27) and that velocity has null derivative there.

In order to obtain the structure of the thin diffusive layer, we expand the independent variable as $\eta = \eta_b + \epsilon_d s$, with $\epsilon_d \ll 1$. The fuel concentration varies from 0 to 1 in this region, hence we assume $dy/ds \sim O(1)$. Since the layer is thin, the temperature does not vary significantly in it, and we make the isothermal approximation $\theta \approx \theta_b$, where $\theta_b$ stands for the temperature value at the ablated border. This hypothesis is consistent with the analysis of a diffusing gas-metal interface in a thermonuclear plasma made in Ref.
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FIG. 8. (a) Normalized temperature $\theta$, velocity $V$ and fuel mass diffusion rate $-\rho y U_1$ for a liner made of beryllium, depicted in solid gray lines. Temperature and velocity profiles are compared to the solution without mass diffusion, plotted in dashed black lines. (b) Beryllium mass concentration profile, $\rho_2/\rho = 1 - y$, in solid gray lines compared to the solution without mass diffusion, plotted in dashed lines. (c) Fuel mass concentration $y$ for different liner materials. The black dashed line is the fuel mass concentration profile for aluminum liner obtained from the boundary layer model derived in Subsec. V A. The position of the ablated border $\eta_b$ is marked in gray dashed lines.

where the authors assumed both isobaric and isothermal conditions. Taking the first derivative in the equation of state (69), it can be obtained that $d\rho/\rho \sim dy/ds \sim O(1)$; that is, density variation is important in this layer. According to Eq. (67), this variation forces the first derivative of the velocity, $dV/d\eta$, to be of order unity as soon as we move far from the ablated border while still being inside the layer. Consequently, we expand $V \approx \eta_b + \epsilon dV_c(s)$, with $dV_c/ds \sim O(1)$. In the isothermal approximation, and defining
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$D_y \equiv D_c + D_p$, the fuel drift velocity simplifies to

$$U_1 = -\frac{2\theta_b^{7/2} D_y}{Le} \frac{dy}{y} \frac{dy}{dx}. \quad (78)$$

Notice that only concentration gradients and baro-diffusion drive mass mixing in the layer, while thermo-diffusion is negligible due to the smallness of the temperature derivatives compared to the fuel concentration derivatives. Inserting these expressions into the diffusion equation (68), and forcing the mass diffusion term to be important, we can obtain the width of the diffusion layer

$$\epsilon_d = \sqrt{\frac{2\theta_b^{7/2}}{Le}}. \quad (79)$$

The fuel drift velocity is therefore of order $O \left( \frac{1}{\sqrt{Le}} \right)$,

$$U_1 = -\sqrt{\frac{2\theta_b^{7/2}}{Le}} \frac{D_y}{y} \frac{dy}{dx}. \quad (80)$$

Defining $a \equiv \mu (Z_1 + 1) / (Z_2 + 1) - 1$, the continuity and diffusion equations (67), (68) take the form

$$\begin{align*}
(V_c - s) \frac{a \frac{dy}{1 + ay}}{ds} &= \frac{dV_c}{ds}, \\
(V_c - s) \frac{1}{1 + ay} \frac{dy}{ds} &= \frac{d}{ds} \left( \frac{D_y}{1 + ay} \frac{dy}{ds} \right).
\end{align*} \quad (81)$$

As boundary conditions, we impose $y(s \to -\infty) = 0$, $y(s \to \infty) = 1$, and $V_c(s = 0) = 0$. These equations can be combined, yielding

$$\left[ a \left( \frac{D_y}{1 + ay} \frac{dy}{ds} - \zeta \right) - s \right] \frac{1}{1 + ay} \frac{dy}{ds} = \frac{d}{ds} \left( \frac{D_y}{1 + ay} \frac{dy}{ds} \right), \quad (83)$$

where

$$\zeta = \left. \frac{D_y}{1 + ay} \frac{dy}{ds} \right|_{s=0} \quad (84)$$

is an eigenvalue that has to be obtained self-consistently with $y(s)$ applying the former three boundary conditions. This eigenvalue is related to the fuel mass diffusion rate at the ablation front,

$$-\rho_y U_1 \big|_{s=0} = \sqrt{\frac{2\theta_b^{7/2}}{Le}} (a + 1) \zeta. \quad (85)$$

Finally, the first order correction to velocity can be obtained $a posteriori$ from Eq. (81) as

$$V_c = a \left( \frac{D_y}{1 + ay} \frac{dy}{ds} - \zeta \right). \quad (86)$$

The structure of the layer for an aluminum liner is plotted in Fig. 9. Since large Lewis number values imply large $Z_2$, we use in this section the analytic expressions (A21) derived for a large $\mu$ ratio for the transport coefficients. It can be seen that the profiles are not
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![Profiles in the boundary layer model. Fuel mass concentration, $y$, liner material mass concentration, $1 - y$, and first order correction for velocity, $V_c$, for aluminum liner ($Z_2 = 13$).](image)

symmetric. The asymmetry is introduced by the coefficient $D_y$, plotted in Fig. 4(b). The fuel mass concentration profile presents a well defined wave front with clean fuel to the right at $s \approx 1$, and decreases exponentially to the left of the ablated border, where the fuel diffuses into the ablated material. Consequently, the parameter $\epsilon_d$ can be chosen as an accurate characterization of the width of the diffusion layer. The first order velocity correction $V_c$ corresponds to the bump appearing in the velocity profile in Fig. 8(a). The eigenvalue $\zeta$ gives {0.25, 0.24, 0.22, 0.21} for $Z_2 = \{3, 4, 13, \infty\}$, respectively. Although not shown here, the profiles keep a similar structure for different $Z_2$. Particularly, the wave front becomes a sharp corner when $Z_2 \to \infty$. The existence of a clear wave front was also discussed in the analysis made in Ref. 28. Comparisons of the boundary layer model with the complete solution is shown in Fig. 8(b), showing good agreement.

The analysis of this layer presents slight differences compared to the aforementioned study of a diffusing gas-metal interface in a thermonuclear plasma, Ref. 28. Although the expressions for the diffusion coefficients are the same (an equivalent notation taken from Ref. 18 is used therein), and the interface structure is also self-similar, the authors solved the mass diffusion problem without taking into account the hydrodynamic motion induced in the plasma. The governing equation of their diffusion layer, Eq. (9) in Ref. 28, is equivalent to Eq. (82) in this paper setting $V_c = 0$. We find that similar results are obtained when the hydrodynamic motion of the plasma is not taken into account. Particularly, the eigenvalue $\zeta$ is only modified by 1%.

B. Liner material and fuel mixing

We can characterize the fuel pollution by the amount of liner material that diffuses into it, defined as

$$ m_{lf} = \int_{x_b}^{\infty} \rho_2 dx = \int_{x_b}^{\infty} \rho (1 - y) dx. \quad (87) $$

Taking the time derivative and making use of the diffusion equation (62), we obtain

$$ \frac{dm_{lf}}{dt} = -\rho y u_1|_{x_b} = -\frac{1}{2} \rho_0 \sqrt{\frac{k_0}{t}} \rho y U_1|_{\eta_b}, \quad (88) $$

that is, fuel pollution is given by the mass diffusion rate at the ablated border, where it is maximum. If we integrate this expression in time, we have

$$ \frac{m_{lf}}{\rho_0 \sqrt{k_0 t}} = -\rho y U_1|_{\eta_b}. \quad (89) $$
We can use the boundary layer model, Eq. (85), to further develop this expression in a large Le limit,

\[
\frac{m_{lf}}{\rho_0 \sqrt{\kappa_0 t}} \bigg|_{\text{B.L.}} = \sqrt{\frac{2\theta_b^{5/2}}{\text{Le}}} \frac{Z_1 + 1}{Z_2 + 1} \zeta.
\] (90)

The liner material diffused into the fuel scales therefore as the inverse of the square root of the Lewis number. It is computed in Table I for different liner materials comparing the exact result, Eq. (89), with the boundary layer model, (90), giving similar results.

We can also compute the fuel mass diffused into the ablated liner,

\[
m_{fl} = \int_0^b \rho_1 dx,
\]
where \[\int_0^b \rho_1 dx = \int_0^b \rho (1 - y) dx,
\] (91)

Performing a similar manipulation, we can express this quantity as

\[
\frac{m}{\rho_0 \sqrt{\kappa_0 t}} = \mu \frac{Z_1 + 1}{Z_2 + 1} \frac{K_e (Z_2)}{K_e (Z_1)} \frac{\sqrt{\zeta}}{\theta_b},
\] (92)

where \[\theta_b\] is related to the temperature profile close to the liner as defined in Eq. (31). Note that this expression for mass ablation is equivalent to its counterpart in the problem without diffusion, Eq. (36).

It is interesting to compute the percentage of mass ablation that pollutes the liner, \[100 \times \frac{m_{lf}}{m},\] shown in Table I. It decreases from 4.09% for lithium liner to 2.80% for aluminum liner.

As commented in Sec. III, the steep negative density gradient taking place at the ablated border may lead to a Rayleigh-Taylor instability (RTI), causing liner and fuel mixing. In order to compare fuel pollution caused by mass diffusion (microscopic mixing) to the one due to the turbulent motion following the RTI (macroscopic mixing), we roughly estimate the ratio \[h_D/h_T\], where \[h_D = \epsilon_d \sqrt{\kappa_0 t}\] is the width of the diffusion layer, and \[h_T \propto gt^2\] is the width of the turbulent mixing layer. The constant of proportionality depends on the particularities of the problem, and ranges from 0.03 – 0.07. We have then

\[
\frac{h_D}{gt^2} = \frac{4\epsilon_d}{\theta_b},
\] (93)

independently of time. This ratio is computed in Table I. It is of order unity and decreases slightly with \[Z_2\], which indicates that fuel pollution due to diffusion is as important as pollution due to turbulent mixing.

Finally, we can derive straightforward results in the limit \[Z_2 \to \infty\]. The Lewis number, given by Eq. (73), can be taken as \[\text{Le} \approx 23Z_2\]. Since it is large, we can assume that the hydrodynamic motion is uncoupled from the mass diffusion problem, hence we take the results derived in Subsec. III B for the position of the ablated border and mass ablation, and we assume \[\theta_b \approx 1\]. Consequently, we can derive

\[
\frac{m_{lf}}{\rho_0 \sqrt{\kappa_0 t}} \approx \frac{0.12}{\sqrt{Z_2}},
\] (94)

\[
\epsilon_d \approx \frac{0.29}{\sqrt{Z_2}}.
\] (95)

Which gives \[100 \times m/m_{lf} \approx 2.36\%\] and \[h_D/gt^2 \approx 0.72\], independently of \[Z_2\].
VI. CONCLUSIONS

The effect of the liner material on mass ablation, energy and magnetic flux losses and liner - fuel mass diffusion have been studied in a MagLIF fusion-like plasma. The self-similar evolution of a hot magnetized fuel plasma in contact with a cold dense unmagnetized liner plasma has been thoroughly described.

In the first part of the paper, mass diffusion at the liner - fuel interface (ablated border) has been neglected. The problem is governed by heat conduction, and the fuel energy is lost in heating up the ablated liner material. The magnetic field in the fuel is convected by the Nernst velocity towards the liner. It penetrates into the ablated liner and diffuses in a thin layer close to the ablation front. The ablated border is a contact discontinuity, and the plasma density at the liner side is greater than at the fuel side. This configuration is Rayleigh-Taylor unstable as the ablated border penetrates into the hot spot while being decelerated by the light fuel. The magnetic field diffuses in a thin layer placed at the fuel side of the ablated border and decreases its value to compensate the variation of the Nernst velocity with the atomic number $Z$. The width of this diffusive layer scales with the inverse of the magnetic Lewis number $L_{em}$, being therefore thinner than the diffusive layer at the ablation front, which scales with the inverse of the square root of $L_{em}$.

For moderate and small magnetization levels, both thermal energy and magnetic flux losses in the fuel decrease with the liner atomic number $Z_2$, while mass ablation and magnetic flux losses in the hot spot, composed by the ablated liner and fuel regions, present a maximum for $Z_2 = 4$ and $Z_2 = 12$, respectively. An asymptotic analysis performed in the large $Z_2$ limit shows that the four quantities scale as $1/\sqrt{Z_2}$.

In the second part of this paper, mass diffusion is taken into account, but only the unmagnetized limit has been studied. The problem is governed by the Lewis number $Le$, which only depends on the fuel and liner atomic numbers and is an increasing function of the latter. In a MagLIF context, it is typically large, hence heat conduction governs the evolution of the problem, with mass diffusion being confined within a thin layer placed at the ablated border. The width of this layer scales with the inverse of the square root of $Le$. Among the mechanisms giving rise to mass diffusion in a plasma, classical diffusion due to concentration gradients is predominant for moderate $Z_2$, while baro-diffusion becomes the most important for large $Z_2$. Since the diffusion layer is thin and the temperature does not change notably inside, thermo-diffusion results in a minor effect. The temperature and velocity profiles are similar to the solution without mass diffusion, and the mass diffusion problem can therefore be solved \textit{a posteriori}.

The amount of liner material that diffuses into the fuel scales as $1/\sqrt{Z_2}$. This mass represents a small percentage of the total liner mass ablated into the hot spot. The percentage decreases with $Z_2$ but attains a minimum of $2.36\%$ that cannot be further reduced. Straightforward estimations suggest that the liner material diffused pollutes the fuel in a layer that is comparable to the turbulent mixing layer induced by the Rayleigh-Taylor instability that can develop at the ablated border. This indicates that liner fuel mixing by microscopic motion (diffusion) may therefore be as important as mixing due to macroscopic motion (hydrodynamic instabilities leading to turbulence).

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Appendix A: Review of transport coefficients

In this Appendix, we summarize the theory already developed in Refs. 17–20 leading to the derivation of the electron and ion heat fluxes, \( q_e \) and \( q_i \), respectively, and the drift velocity \( u_1 \) in a two ion species plasma.

We first introduce the the electron-electron collision frequency

\[
\nu_{ee} = \frac{4\sqrt{2\pi n_e e^4 \log \Lambda_{ee}}}{3 m_e^{1/2} T_e^{3/2}}, \tag{A1}
\]

the electron-ion collision frequency

\[
\nu_{em} = \frac{4\sqrt{2\pi m_n Z_m^2 e^4 \log \Lambda_{em}}}{3 m_e^{1/2} T_e^{3/2}}, \tag{A2}
\]

and the ion-ion collision frequency

\[
\nu_{mn} = \frac{4\sqrt{\pi n_e Z_m^2 Z_n^2 e^4 \log \Lambda_{mn}}}{3 m_m^{1/2} T_m^{3/2}}. \tag{A3}
\]

Through all the appendix, the subscripts \( m \) and \( n \) only refer to ion species. We assume the same value for the Coulomb logarithms \( \log \Lambda_{ee} = \log \Lambda_{em} = \log \Lambda_{mn} \).

The electron heat flux \( q_e \) for a plasma with multiple ion species was independently obtained in Refs. 17 and 18 using a different variational principle, but yielding the same result

\[
q_e = -\frac{\gamma_0 p_e}{m_e (\nu_{e1} + \nu_{e2})} \nabla T_e. \tag{A4}
\]

The coefficient \( \gamma_0 \) is a function of the effective ion charge \( Z_{\text{eff}} \), defined as

\[
Z_{\text{eff}} \equiv \frac{\nu_{e1} + \nu_{e2}}{\nu_{ee}} = \frac{Z_1^2 n_1 + Z_2^2 n_2}{Z_1 n_1 + Z_2 n_2} \geq 1, \tag{A5}
\]

and reads

\[
\gamma_0 (Z_{\text{eff}}) = \frac{25 Z_{\text{eff}} (5, 299, 888 Z_{\text{eff}}^3 + 21, 559, 755\sqrt{2} Z_{\text{eff}}^2 + 17, 831, 746 Z_{\text{eff}} + 1, 272, 672 \sqrt{2})}{4 (2, 447, 104 Z_{\text{eff}}^4 + 17, 445, 571 \sqrt{2} Z_{\text{eff}}^3 + 57, 670, 090 Z_{\text{eff}}^2 + 16, 033, 384 \sqrt{2} Z_{\text{eff}} + 2, 013, 696)}. \tag{A6}
\]

The electron heat flux can be written into a Spitzer form as

\[
q_e = -\bar{K}_e (Z_{\text{eff}}) T^{5/2} \frac{\partial T}{\partial x}. \tag{A7}
\]

with the Spitzer coefficient for the electron heat flux defined as

\[
\bar{K}_e (Z_{\text{eff}}) = \frac{3 \gamma_0 (Z_{\text{eff}})}{4\sqrt{2\pi m_e e^4 \log \Lambda_{ee} Z_{\text{eff}}}}. \tag{A8}
\]

Note that this expression is the same as its counterpart for a single ion species plasma, Eq. (10), substituting \( Z \) for \( Z_{\text{eff}} \).

Expressions for the ion drift velocities \( u_m \) and the ion heat flux \( q_i \) have been derived in Ref. 19 for an unmagnetized plasma with multiple ion species, and particularized for a two ion species plasma in Ref. 20. As stated in these references, \( u_m \) and \( q_i \) are identified...
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as thermodynamic fluxes that can be related to their conjugate thermodynamic forces $d_m$, $\partial \log T_i / \partial x$, respectively, through a symmetric transport matrix

$$
\begin{pmatrix}
  u_1 \\
  u_2 \\
  q_i/p_i
\end{pmatrix}
= -
\begin{pmatrix}
  \Delta_{11} & \Delta_{12} & D_{T1} \\
  \Delta_{21} & \Delta_{22} & D_{T2} \\
  D_{T1} & D_{T2} & \kappa_i/n_i
\end{pmatrix}
\begin{pmatrix}
  d_1 \\
  d_2 \\
  \partial \log T_i / \partial x
\end{pmatrix},
$$

(A9)

with $\Delta_{12} = \Delta_{21}$ being the generalized diffusion coefficients, $k_i$ being the ion heat conduction coefficient and $D_{Tm}$ being the thermo-diffusion coefficients (the relation between $u_m$ and $\partial \log T_i / \partial x$ is often referred to as the Ludwig-Soret effect and reciprocal relation between $q_i$ and $d_m$ is referred to as the Dufour effect).

The thermodynamic force $d_m$ stands for the imbalance between the fluid inertial force on the ion species $m$ ($\rho_m Dv_i / Dt$, with $D/Dt$ being the substantial derivative) and the kinetic forces acting on it. The diffusion fluxes arise to relax this imbalance, the origin of which lies in the collision between ion species. It can be written as

$$
d_m = \frac{1}{p_i} \left\{ \frac{\partial p_m}{\partial x} - Z_m n_m E - F_{me} \right\} - \frac{\rho_m}{\rho} \left( \frac{\partial p_i}{\partial x} - e n_e E - \sum_m F_{me} \right),
$$

(A10)

with $E$ being the electric field and $F_{em} = -F_{me} = -(\beta_0 n_e \nu_{em} / \sum_m \nu_{em}) \partial T_e / \partial x$ being the electron collisional friction force with the ion species $m$, such that $\sum_m F_{em} = -\beta_0 n_e \partial T_e / \partial x$. The coefficient $\beta_0$ is a function of the effective ion charge,

$$
\beta_0(Z_{\text{eff}}) = \frac{30}{Z_{\text{eff}}} \left( \frac{1}{11} Z_{\text{eff}}^2 + \frac{1}{15} \sqrt{2} \right) / \left( \frac{1}{27} Z_{\text{eff}}^2 + \frac{3}{604} \sqrt{2} Z_{\text{eff}} + \frac{288}{27} \right).
$$

The electric field can be obtained from the electron momentum equation which, after neglecting electrons inertia and making use of the ambipolarity condition, reads

$$
e n_e E = -\frac{\partial p_e}{\partial x} - \beta_0 n_e \frac{\partial T_e}{\partial x}.
$$

(A11)

Inserting these relations into Eq. (A10), and taking into account that $p_m = x_m p_i$, we can obtain more convenient expressions for $d_m$

$$
d_m = \frac{\partial x_m}{\partial x} + \left( x_m - \frac{\rho_m}{\rho} \right) \frac{\partial \log p_i}{\partial x}
+ \left( \frac{\rho_m}{\rho} - \frac{Z_m n_m}{n_e} \right) \frac{n_e e E}{n T_i}
+ \left( \frac{Z_m n_m}{n_e} - \frac{\rho_m}{\rho} \right) \frac{1}{p_i} \frac{\partial p_e}{\partial x}
+ \left( \frac{Z_m n_m}{n_e} - \frac{Z_m^2 n_m}{Z_1^2 n_1 + Z_2^2 n_2} \right) \frac{n_e \beta_0}{n T_i} \frac{\partial T_e}{\partial x},
$$

(A12)

or, equivalently, using Eq. (A11),

$$
d_m = \frac{\partial x_m}{\partial x} + \left( x_m - \frac{\rho_m}{\rho} \right) \frac{\partial \log p_i}{\partial x}
+ \left( \frac{Z_m n_m}{n_e} - \frac{\rho_m}{\rho} \right) \frac{1}{p_i} \frac{\partial p_e}{\partial x}
+ \left( \frac{Z_m n_m}{n_e} - \frac{Z_m^2 n_m}{Z_1^2 n_1 + Z_2^2 n_2} \right) \frac{n_e \beta_0}{n T_i} \frac{\partial T_e}{\partial x}.
$$

(A13)
Notice that, although the original expression for $d_m$, Eq. (A10), is the same as Eq. (20) in Ref. [19] the more convenient expression (A13) differs from its counterpart in the same reference, Eq. (25) therein, in the coefficient $n_e/n$ multiplying the temperature derivative. We suspect that this factor has been dropped by mistake, since Eq. (A13) agrees with the definition of the same thermodynamic force in Eq. (125) in Ref. [18].

The system (A9) can be reduced noticing that

$$
\sum_m d_m = 0,
$$

yielding

$$
\begin{pmatrix}
  u_1 \\
  q_i/p_i
\end{pmatrix} =
- \begin{pmatrix}
  \Delta'_{11} & D_{T1} \\
  D'_{T1} & \kappa_i/n_i
\end{pmatrix}
\begin{pmatrix}
  d_1 \\
  \partial \log T_i/\partial x
\end{pmatrix},
$$

(A14)

where $\Delta'_{11} \equiv \Delta_{11} - \Delta_{12}$, $D'_{T1} \equiv D_{T1} - D_{T2}$. As done in Ref. [20] it is convenient to normalized the coefficients in Eq. (A14) as

$$
\Delta'_{11} = \frac{2T}{m_1 \nu_{11}} \hat{\Delta}_{11},
D_{T1} = \frac{2T}{m_1 \nu_{11}} \hat{D}_{T1},
$$

$$
D'_{T1} = \frac{2T}{m_1 \nu_{11}} \hat{D}'_{T1},
\kappa_i = \frac{2n_i T}{m_1 \nu_{11}} \hat{\kappa}_i.
$$

(A15)

The fuel drift velocity can then be written as

$$
u_1 = - \frac{2T}{m_1 \nu_{11}} \Delta'_{11} \left( d_1 + \frac{\hat{D}'_{T1}}{\Delta'_{11}} \frac{\partial \log T_i}{\partial x} \right),
$$

(A16)

and we use this relation to express the ion heat flux in a more convenient way

$$
q_i = - \frac{2p_i T}{m_1 \nu_{11}} \left( \hat{\kappa}_i - \frac{\hat{D}'_{T1}}{\Delta'_{11}} \frac{\partial \log T_i}{\partial x} \right) + \frac{\hat{D}'_{T1}}{\Delta'_{11}} p_1 u_1,
$$

(A17)

which can therefore be written as a Spitzer term plus the contribution due to mass diffusion

$$
q_i = - \bar{K}_i T^{5/2} \frac{\partial T}{\partial x} + \frac{\hat{D}'_{T1}}{\Delta'_{11}} x_1 p_1 u_1.
$$

(A18)

The Spitzer coefficient for the ion heat flux is

$$
\bar{K}_i (v) \equiv \frac{3}{2 \sqrt{\pi} Z_i e^4 \log \Lambda_{11} / \sqrt{m_1}} \left( \hat{\kappa}_i - \frac{\hat{D}'_{T1}}{\Delta'_{11}} \right).
$$

(A19)

A straightforward estimation of the electron and ion conduction coefficients gives $\bar{K}_i/\bar{K}_e \sim \sqrt{m_e/m_p} \ll 1$, which implies that $q_i$ is smaller than $q_e$, as happens in an unmagnetized single ion species plasma. Nevertheless, ion heat flux is retained in this analysis.

In Eq. (A19), every mechanism contributing to ion diffusion in an unmagnetized plasma can be identified. The first term of $d_1$, Eq. (A12), represents diffusion due to concentration gradients, the second one stands for ion baro-diffusion, the third one is the electro-diffusion, and the fourth one represents thermo-diffusion due to electron temperature gradients. The Ludwig-Soret effect in Eq. (A16) stands for thermo-diffusion driven by ion temperature gradients. Noticing that the fuel number density fraction $x_1$ can be related to the fuel mass concentration $y$ through

$$
x_1 = \frac{y}{y + \frac{1 - y}{\mu}}.
$$

(A20)
we can obtain the same baro and electro-diffusion ratios, $k_p$ and $k_E$, as derived by Kagan and Tang\textsuperscript{23} in Eqs. (26) and (33) therein, without need to specify the transport coefficients of Eq. (A14). This confirms the statement made in the same reference that the baro and electro-diffusion ratios can be calculated uniquely by thermodynamic means, independently of the nature of collisions. Expressing $d_m$ by Eq. (A12) would correspond to choosing the ion mixture as the thermodynamic system, hence only the ion pressure appears and the electric field has to be taken into account. However, using Eq. (A13) corresponds to choosing the plasma as a whole, then both ion and electron pressure take place, and the electric field does not appear explicitly as the plasma is quasi-neutral. Hereinafter, Eq. (A13) will be used for $d_l$, since all the terms can be directly related to temperature $T$ and fuel mass concentration $y$.

The evaluation of the normalized coefficients in Eq. (A15) requires to solve a linear system of 6 equations for every value of the fuel concentration $y$, as explained in Sec. III in Ref. \textsuperscript{20}. However, analytical solutions were derived in the same reference for a mixture of ion species with disparate masses, $\mu \gg 1$. Defining $\zeta = Z_2^2 \log \Lambda_{12}/Z_1^2 \log \Lambda_{11} = Z_2^2/Z_1^2$, $v \equiv \nu_{12}/\nu_{11} = (n_2/n_1) \zeta = (1-y)\zeta/\mu y$, they read

\begin{align}
\hat{\Delta}_{11}^t(y) &= \frac{\mu (\zeta+v)}{\zeta (\zeta+\mu)} \frac{217\sqrt{2}v^2 + 1208v + 288\sqrt{2}}{16 (16v^2 + 61\sqrt{2}v + 72)}, \\
\hat{D}_{11}(y) &= \frac{v_{\mu}}{\zeta + \mu} \frac{15 (11\sqrt{2}v + 30)}{8 (16v^2 + 61\sqrt{2}v + 72)}, \\
\hat{\Delta}_{11}^T(y) &= \frac{15 (11\sqrt{2}v + 30)}{8 (16v^2 + 61\sqrt{2}v + 72)}, \\
\hat{\bar{\Delta}}_t(y) &= \frac{25 (26\sqrt{2}v + 45)}{8 (16v^2 + 61\sqrt{2}v + 72)} + \\
&\quad \frac{125v (35\sqrt{2} + 9\sqrt{2}v)}{4\zeta (5.250 + 1.375\sqrt{2}v + 144\mu v^2)}.
\end{align}

In this paper, we use the evaluation of the coefficients for any $\mu$ rather than the analytical expressions derived for large $\mu$, with the exception of the boundary layer model in Subsec. V.A. Finally, we solved the linear system in the particular case $y = 0$ to obtain $\hat{\Delta}_{11}^t(y = 0)$ for any $\mu$, required in the definition of the Lewis number, Eq. (73). It gives $\hat{\Delta}_{11}^t(0) = P_\mu/\zeta$, with

\begin{align}
P_\mu &= \sqrt{\frac{\mu + 1}{\mu} 434\mu^6 + 3,912\mu^5 + 17,071\mu^4 + 33,152\mu^3 + 46,764\mu^2 + 38,080\mu + 21,000} \\
&\quad \frac{16\sqrt{2} (16\mu^6 + 192\mu^5 + 1,064\mu^4 + 3,136\mu^3 + 5,058\mu^2 + 4,760\mu + 2,625).}
\end{align}

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