The Two-Higgs Doublet Model and the Multiple Point Principle

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Abstract

According to the multiple point principle, Nature adjusts coupling parameters so that many vacuum states exist and each has approximately zero vacuum energy density. We apply this principle to the general two-Higgs doublet extension of the Standard Model, by requiring the existence of a large set of degenerate vacua at an energy scale much higher than the presently realized electroweak scale vacuum. It turns out that two scenarios are allowed. In the first scenario, a CP conserving Higgs potential and the absence of flavour changing neutral currents are obtained without fine-tuning. In the second scenario, the photon becomes massive in the high scale vacua. We briefly discuss the resulting phenomenology.

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1. Introduction

The success of the Standard Model (SM) strongly supports the concept of spontaneous $SU(2) \times U(1)$ symmetry breaking. The mechanism of electroweak symmetry breaking, in its minimal version, requires the introduction of a single doublet of scalar complex Higgs fields and leads to the existence of a neutral massive particle — the Higgs boson. Over the past two decades the upper [1] and lower [1]-[3] theoretical bounds on its mass have been established. Although the Higgs boson still remains elusive, the combined analysis of electroweak data indicates that its mass lies below 251 GeV with 95% confidence level [4]. Recently the experimental lower limit on the Higgs mass of 115.3 GeV was set by the unsuccessful search at LEPII [5]. The upgraded Tevatron, LHC and LC have a good chance to discover the Higgs boson in the near future.

There is, of course, no strong argument for the existence of just a single Higgs doublet, apart from simplicity. Indeed the symmetries of many models for physics beyond the SM, such as supersymmetry or the Peccei-Quinn symmetry [6], naturally introduce extra Higgs doublets with unit weak hypercharge. In this paper, we consider the application of the Multiple Point Principle to the general two Higgs doublet model, without any symmetries imposed beyond those of the SM gauge group.

The Multiple Point Principle (MPP) [7] postulates the co-existence in Nature of many phases, which are allowed by a given theory. It corresponds to the special (multiple) point on the phase diagram of the considered theory where these phases meet. At the multiple point the vacuum energy densities (the cosmological constants) of the neighbouring phases are degenerate. Thus, according to MPP, Nature fine-tunes the couplings to their values at the multiple point. We have not identified the physical mechanism underlying MPP, but it seems likely [7] that a mild form of non-locality is required, due to baby universes say [8], as in quantum gravity.

When applied to the pure SM, the MPP exhibits a remarkable agreement [9] with the top quark mass measurements. According to MPP, the renormalization group improved SM Higgs effective potential

$$V_{\text{eff}}(\phi) = -m^2(\phi)\phi^2 + \frac{\lambda(\phi)}{2} \phi^4,$$  

(1)

has two rings of minima with the same vacuum energy density [9]. The radius of the little ring is equal to the electroweak vacuum expectation value of the Higgs field $|\phi| = v = 246$ GeV. The second vacuum was assumed to be near the fundamental scale of the theory\(^1\), identified as the Planck scale $|\phi| \approx M_{Pl}$. The mass parameter $m$ in the effective potential [1] has

\(^1\)Here we assume the existence of the hierarchy $v/M_{Pl} \sim 10^{-17}$. However some of
to be of the order of the electroweak scale $v$ and is negligible compared to $M_{Pl}$. The conditions for a second degenerate minimum of $V_{eff}$ at the Planck scale then become

$$\beta_\lambda(\lambda(M_{Pl}), g_t(M_{Pl}), g_i(M_{Pl})) = \frac{d\lambda}{d\ln\phi}(M_{Pl}) = \lambda(M_{Pl}) = 0 \quad (2)$$

where $g_i(\phi)$ and $g_t(\phi)$ denote the gauge and top quark Yukawa couplings respectively. Hence, by virtue of MPP, $\lambda(M_{Pl})$ and $g_t(M_{Pl})$ are determined and one can compute quite precisely the predicted top quark (pole) and Higgs boson masses using the renormalization group flow [9]:

$$M_t = 173 \pm 5 \text{ GeV}, \quad M_H = 135 \pm 9 \text{ GeV}. \quad (3)$$

Here we study the MPP predictions for the general two Higgs doublet extension of the SM [2,11]. The structure of the general two Higgs doublet model is outlined in the next section. The MPP conditions are then formulated in section 3. Section 4 contains our conclusions.

2. Two Higgs doublet extension of the SM

The most general renormalizable $SU(2) \times U(1)$ gauge invariant potential of the model involving two Higgs doublets is given by

$$V_{eff}(H_1, H_2) = m_1^2(\Phi)H_1^\dagger H_1 + m_2^2(\Phi)H_2^\dagger H_2 - \left[ m_3^2(\Phi)H_1^\dagger H_2 + \text{h.c.} \right] +$$

$$\frac{\lambda_1(\Phi)}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2(\Phi)}{2}(H_2^\dagger H_2)^2 + \lambda_3(\Phi)(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(\Phi)|H_1^\dagger H_2|^2$$

$$+ \left[ \frac{\lambda_5(\Phi)}{2}(H_1^\dagger H_2)^2 + \lambda_6(\Phi)(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_7(\Phi)(H_2^\dagger H_2)(H_1^\dagger H_1) + \text{h.c.} \right] \quad (4)$$

where

$$H_n = \begin{pmatrix} \chi_n^+ \\ (H_n^0 + iA_n^0)/\sqrt{2} \end{pmatrix} \quad n = 1, 2.$$ 

It is easy to see that the number of couplings in the two Higgs doublet model (2HDM) compared with the SM grows from two to ten. Furthermore, four of them $m_3^2$, $\lambda_5$, $\lambda_6$ and $\lambda_7$ can be complex, inducing CP–violation in the Higgs sector. In what follows we suppose that the mass parameters $m_i^2$
and Higgs self-couplings $\lambda_i$ of the effective potential (4) only depend on the overall sum of the squared norms of the Higgs doublets, i.e.

$$\Phi^2 = \Phi_1^2 + \Phi_2^2, \quad \Phi_n^2 = H_n^\dagger H_n = \frac{1}{2} \left[ (H_n^0)^2 + (A_n^0)^2 \right] + |\chi_n^+|^2.$$  

The running of these couplings is described by the 2HDM renormalization group equations [12]–[13], where the renormalization scale is replaced by $\Phi$.

At the physical minimum of the scalar potential (4) the Higgs fields develop vacuum expectation values

$$< \Phi_1 > = \frac{v_1}{\sqrt{2}}, \quad < \Phi_2 > = \frac{v_2}{\sqrt{2}}$$

breaking the $SU(2) \times U(1)$ gauge symmetry and generating masses for the bosons and fermions. Here the overall Higgs norm $< \Phi > = \sqrt{v_1^2 + v_2^2} = v = 246 \text{GeV}$ is fixed by the electroweak scale. At the same time the ratio of the Higgs vacuum expectation values remains arbitrary. Hence it is convenient to introduce $\tan \beta = v_2/v_1$.

In general the Yukawa couplings of the quarks to the Higgs fields $H_1$ and $H_2$ generate phenomenologically unwanted flavour changing neutral currents, unless there is a protecting custodial symmetry [14]. Such a custodial symmetry requires the vanishing of the Higgs couplings $\lambda_6$ and $\lambda_7$. It also requires the down-type quarks to couple to just one Higgs doublet, $H_1$ (Model I) or to the second Higgs doublet $H_2$ (Model II) but not both\(^2\).

If, in addition, the Higgs coupling $\lambda_5$ vanishes, as in supersymmetric and Peccei-Quinn models, there is no CP-violation in the Higgs sector.

We emphasize that, in this paper, we do not impose any custodial symmetry but rather consider the general Higgs potential (4). Instead we require that at some high energy scale ($M_Z << \Lambda \lesssim M_{Pl}$), which we shall refer to as the MPP scale $\Lambda$, a large set of degenerate vacua allowed by the 2HDM is realized. In compliance with the MPP, these vacua and the physical one must have the same energy density. Thus the MPP implies that the couplings $\lambda_i(\Lambda)$ should be adjusted with an accuracy of order $v^2/\Lambda^2$, in order to arrange an appropriate cancellation among the quartic terms in the effective potential (4).

\(^2\)Similarly the leptons are required to only couple to one Higgs doublet, usually chosen to be the same as the down-type quarks. However there are variations of Models I and II, in which the leptons couple to $H_2$ rather than to $H_1$.  

4
3. Implementation of the MPP in the 2HDM

In this section, we aim to determine a large set of minima of the 2HDM scalar potential with almost vanishing energy density, which may exist at the MPP high energy scale $\Lambda$ where the mass terms in the potential can be neglected. The most general vacuum configuration takes the form:

$$< H_1 > = \Phi_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad < H_2 > = \Phi_2 \begin{pmatrix} \sin \theta \\ \cos \theta e^{i\omega} \end{pmatrix},$$

where $\Phi_1^2 + \Phi_2^2 = \Lambda^2$. Here, the gauge is fixed so that only the real part of the lower component of $H_1$ gets a vacuum expectation value.

We now consider the conditions that must be satisfied in order that minima of $V_{eff}$ should exist for all possible values of the phase $\omega$. The $\omega$-dependent part of the potential takes the form:

$$V_\omega = \frac{\lambda_5(\Phi)}{2} \Phi_1^2 \Phi_2^2 \cos^2 \theta e^{2i\omega} + \left[ \lambda_6(\Phi) \Phi_1^3 \Phi_2 + \lambda_7(\Phi) \Phi_1 \Phi_2^3 \right] \cos \theta e^{i\omega} + h.c. \quad (7)$$

In order that $V_\omega$ should become independent of $\omega$ at the MPP scale minima, we require that the coefficients of $e^{i\omega}$ and $e^{2i\omega}$ in (7) both vanish at $\Phi = \Lambda$. Similarly for minima to exist for all values of $\omega$, we require the derivatives:

$$\frac{\partial V_\omega}{\partial \Phi_1} = \left[ \lambda_5(\Phi) \Phi_1 \Phi_2^2 + \beta \lambda_5(\Phi) \frac{\Phi_1^4 \Phi_2}{2 \Phi_2^2} \right] \cos^2 \theta e^{2i\omega} +$$

$$\left[ 3 \lambda_6 \Phi_1^3 \Phi_2 + \beta \lambda_6 \frac{\Phi_1^4 \Phi_2}{\Phi_2} + \lambda_7 \Phi_1^3 + \beta \lambda_7 \frac{\Phi_1^2 \Phi_2^3}{\Phi_2} \right] \cos \theta e^{i\omega} + h.c. \quad (8)$$

and

$$\frac{\partial V_\omega}{\partial \Phi_2} = \left[ \lambda_5(\Phi) \Phi_1^2 \Phi_2 + \beta \lambda_5(\Phi) \frac{\Phi_1 \Phi_2^4}{2 \Phi_2^2} \right] \cos^2 \theta e^{2i\omega} +$$

$$\left[ \lambda_6 \Phi_1^3 + \beta \lambda_6 \frac{\Phi_1^4 \Phi_2}{\Phi_2} + 3 \lambda_7 \Phi_1^3 \Phi_2 + \beta \lambda_7 \frac{\Phi_1^2 \Phi_2^3}{\Phi_2} \right] \cos \theta e^{i\omega} + h.c. \quad (9)$$

to be independent of $\omega$ at $\Phi = \Lambda$. Here $\beta \lambda_5(\Phi) = \frac{d \lambda_5}{d \ln \Phi}(\Phi)$ is the renormalisation group beta function for the Higgs self-coupling $\lambda_i(\Phi)$.

It is readily verified (unless $\cos \theta = 0$) that the vanishing of the coefficients of $e^{i\omega}$ and $e^{2i\omega}$ in Eqs. (7) - (9) leads to the conditions:

$$\lambda_5(\Lambda) = \lambda_6(\Lambda) = \lambda_7(\Lambda) = 0 \quad (10)$$

and

$$\beta \lambda_5(\Lambda) = \beta \lambda_6 \Phi_1^2 + \beta \lambda_7 \Phi_2^2 = 0. \quad (11)$$
When $\lambda_5 = \lambda_6 = \lambda_7 = 0$, the Higgs potential manifests an extra Peccei-Quinn-like $U(1)$ symmetry, and the only non-vanishing contributions to the beta functions $\beta_{\lambda_5}$, $\beta_{\lambda_6}$ and $\beta_{\lambda_7}$ arise from the Yukawa couplings to the fermion sector. We shall consider just the third generation fermions here and neglect the smaller Yukawa couplings from the first two generations. An obvious method of ensuring that $\beta_{\lambda_5}$, $\beta_{\lambda_6}$ and $\beta_{\lambda_7}$ also vanish, and thereby satisfy Eq. (11), is to extend the $U(1)$ symmetry to the fermion sector at the MPP scale. In other words the Yukawa couplings at the MPP scale can be taken to be of the 2HDM Model I or Model II form discussed in section 2. This is illustrated by the explicit expression for $\beta_{\lambda_5}$ (in a notation where we have re-defined the Higgs doublets so that the top quark only couples to $H_2$ at the MPP scale):

$$\beta_{\lambda_5}\left(\lambda_5 = \lambda_6 = \lambda_7 = 0, \Lambda\right) = -\frac{1}{(4\pi)^2} \left[12h_b^2(\Lambda)g_b^2(\Lambda) + 4h_\tau^2(\Lambda)g_\tau^2(\Lambda)\right].$$

Here $h_b$ and $g_b$ are the couplings of $H_1$ and $H_2$ to the $b$-quark, while $h_\tau$ and $g_\tau$ are the corresponding couplings of the Higgs doublets to the $\tau$-lepton. For definiteness, we have chosen a phase convention in which $h_t$, $h_b$, $h_\tau$ and $g_b$ are real and $g_\tau$ is complex. The beta function (12) vanishes when

\begin{align*}
(I) \quad h_b(\Lambda) &= h_\tau(\Lambda) = 0; \\
(II) \quad g_b(\Lambda) &= g_\tau(\Lambda) = 0;
\end{align*}

\begin{align*}
(III) \quad h_b(\Lambda) &= g_\tau(\Lambda) = 0; \\
(IV) \quad g_b(\Lambda) &= h_\tau(\Lambda) = 0.
\end{align*}

(13)

corresponding to the 2HDM Model I and Model II Yukawa couplings and their leptonic variations.

An alternative method of solving the MPP conditions (10, 11), without a Peccei-Quinn-like $U(1)$ symmetry, is for the $b$ and $\tau$ contributions to cancel in Eq. (12) with $g_\tau$ being imaginary. However the manifold of such MPP solutions in the space of coupling constants is of the same dimension as that of the Peccei-Quinn-like solutions. Hence no fine-tuning is required to obtain one of the Peccei-Quinn-like MPP solutions, as they are just as abundant as MPP solutions without such a $U(1)$ symmetry. We shall therefore concentrate on the phenomenologically favoured MPP solutions, having the 2HDM Model I or Model II Yukawa couplings.

The Peccei-Quinn-like $U(1)$ custodial symmetry of the Higgs and Yukawa sector implies that

$$\beta_{\lambda_5}(\Lambda) = \beta_{\lambda_6}(\Lambda) = \beta_{\lambda_7}(\Lambda) = 0.$$  

(14)

It then follows from Eqs. (10) and (11) that the renormalization group evolution does not generate any $U(1)$ custodial symmetry breaking couplings below the MPP scale, where we thus have:

$$\lambda_5(\Phi) = \lambda_6(\Phi) = \lambda_7(\Phi) = 0.$$  

(15)
Figure 1: The running of $\lambda_1$ and $\lambda_2$, as a function of $\log[\Phi^2/M_{Pl}^2]$, below $M_{Pl}$ for $\lambda_i(M_{Pl}) = 0$, $m_t(M_t) = 165$ GeV and $\alpha_3(M_Z) = 0.117$. The renormalization group flow is plotted for $\tan \beta = 2$. The solid and dashed lines correspond to $\lambda_1$ and $\lambda_2$ respectively.

In this way we have naturally obtained the absence of flavour changing neutral currents and a CP conserving Higgs potential from the MPP requirement that vacua at the MPP scale should be degenerate with respect to the phase $\omega$.

We now consider whether we can impose further MPP conditions on the couplings. The simplest way to ensure that the quartic part of the effective potential vanishes for any vacuum configuration $6)$ at the MPP scale is to impose the condition that all the self-couplings should vanish there:

$$\lambda_1(\Lambda) = \lambda_2(\Lambda) = \lambda_3(\Lambda) = \lambda_4(\Lambda) = \lambda_5(\Lambda) = \lambda_6(\Lambda) = \lambda_7(\Lambda) = 0.$$  \hspace{1cm} (16)

However, further investigation reveals that the configurations $6)$ do not correspond to minima of the effective potential in this case. This can be shown by consideration of the 2HDM renormalization group equations for the quartic couplings $12)-13).$ The detailed results depend on the choice of Model I or Model II Yukawa couplings, but they are qualitatively similar. So, for convenience, we shall concentrate on the Model II couplings here.

For moderate values of $\tan \beta$ the Higgs self–coupling $\lambda_1$ becomes negative just below the MPP scale (see Fig. 1). The renormalization group running of $\lambda_2$ exhibits the opposite behaviour, because of the large and negative top quark contribution to the corresponding beta–function. This
Figure 2: The running of $\lambda_1$, $\lambda_2$ and $\tilde{\lambda}$, as a function of $\log[\Phi^2/M_{Pl}^2]$, below $M_{Pl}$ for $\lambda_i(M_{Pl}) = 0$, $m_t(M_t) = 165$ GeV and $\alpha_3(M_Z) = 0.117$. The renormalization group flow is plotted for $\tan \beta = 50$. The solid, dashed and dash–dotted lines correspond to $\lambda_1$, $\lambda_2$ and $\tilde{\lambda}$ respectively.

means that $V_{eff}$ does not have a minimum at the MPP scale and, just below it, there is a huge negative energy density ($V_{eff} \sim -\Lambda^4$) where $<\Phi_2> = 0$ and $<\Phi_1> \lesssim \Lambda$.

The renormalization group flow of $\lambda_1$ changes at very large $\tan \beta$ (see Fig. 2). The absolute value of the $b$–quark and $\tau$–lepton contribution to $\beta_{\lambda_1}$, although negligible at moderate values of $\tan \beta$, grows with increasing $\tan \beta$. At $\tan \beta \sim m_t(M_t)/m_b(M_t)$ their negative contribution to the beta function of $\lambda_1$ prevails over the positive contributions coming from loops containing Higgs and gauge bosons. The negative sign of $\beta_{\lambda_1}$ results in $\lambda_1(\Phi) > 0$ if the overall Higgs norm $\Phi$ is less than $\Lambda$. However the positive sign of $\lambda_1$ does not ensure the stability of the vacua (6). Substituting the vacuum configuration (6) into the quartic part of the 2HDM scalar potential, and using Eq. (15), one finds for any $\Phi$ below the MPP scale:

$$V(H_1, H_2) \approx \frac{1}{2} \left( \sqrt{\lambda_1(\Phi)\Phi_1^2} - \sqrt{\lambda_2(\Phi)\Phi_2^2} \right)^2 + \left( \sqrt{\lambda_1(\Phi)\lambda_2(\Phi)} + \lambda_3(\Phi) + \lambda_4(\Phi) \cos^2 \theta \right) \Phi_1^2\Phi_2^2. \tag{17}$$

The Higgs scalar potential (17) attains its minimal value for $\cos \theta = 0$ if $\lambda_4 > 0$ or $\cos \theta = \pm 1$ when $\lambda_4 < 0$. For these values of $\cos \theta$, the scalar
potential can be written as

$$V_{\text{eff}}(H_1, H_2) \approx \frac{1}{2} \left( \sqrt{\lambda_1(\Phi)} \Phi_1^2 - \sqrt{\lambda_2(\Phi)} \Phi_2^2 \right)^2 + \tilde{\lambda}(\Phi) \Phi_1^2 \Phi_2^2,$$

(18)

where

$$\tilde{\lambda}(\Phi) = \sqrt{\lambda_1(\Phi) \lambda_2(\Phi)} + \lambda_3(\Phi) + \min\{0, \lambda_4(\Phi)\}.$$

If at some intermediate scale the combination of the Higgs self–couplings $\tilde{\lambda}(\Phi)$ is less than zero, then there exists a minimum with negative energy density that causes the instability of the vacua at the electroweak and MPP scales. Otherwise the Higgs effective potential is positive definite and the considered vacua are stable.

In Fig. 2 the Higgs self–couplings $\lambda_1(\Phi)$ and $\lambda_2(\Phi)$, as well as $\tilde{\lambda}(\Phi)$, are plotted as a function of $\Phi$ for a very large value of $\tan \beta$. It is clear that the vacuum stability conditions, i.e.

$$\lambda_1(\Phi) \gtrsim 0, \quad \lambda_2(\Phi) \gtrsim 0, \quad \tilde{\lambda}(\Phi) \gtrsim 0$$

(19)

are not fulfilled simultaneously. The value of $\tilde{\lambda}(\Phi)$ becomes negative for $\Phi < \Lambda$. So we conclude that indeed the conditions (16) can not provide a self–consistent realization of the MPP in the 2HDM.

At the next stage it is worth relaxing the conditions (16), by permitting $\lambda_1(\Lambda)$, $\lambda_2(\Lambda)$ and $\lambda_3(\Lambda)$ to take on non–zero values. In order to avoid a huge and negative vacuum energy density in the global minimum of the 2HDM effective potential that precludes the implementation of MPP, the vacuum stability conditions (19) should be satisfied for any $\Phi$ in the interval: $v \lesssim \Phi \lesssim \Lambda$. In this case both terms in the quartic part of the scalar potential (17) are positive. In order to achieve degeneracy of the vacua at the electroweak and MPP scales, they must go to zero separately at the scale $\Lambda$. Since $\lambda_4(\Lambda)$ is still taken to be zero, the second term in Eq. (17) vanishes when

$$\lambda_3(\Lambda) \simeq -\sqrt{\lambda_1(\Lambda) \lambda_2(\Lambda)}.$$

(20)

For finite values of $\lambda_1(\Lambda)$ and $\lambda_2(\Lambda)$ the first term in the quartic part of the scalar potential can also be eliminated by the appropriate choice of Higgs vacuum expectation values:

$$\Phi_1 = \Lambda \cos \gamma, \quad \Phi_2 = \Lambda \sin \gamma, \quad \tan \gamma = \left( \frac{\lambda_1(\Lambda)}{\lambda_2(\Lambda)} \right)^{1/4}.$$

(21)

The sum of the quartic terms in $V_{\text{eff}}(H_1, H_2)$ then tend to zero at the MPP scale independently of the angle $\theta$ and the phase $\omega$. 

9
Nevertheless the situation is not as promising as it first appears, since again we can show it does not correspond to a local minimum of $V_{\text{eff}}$ at $\Phi = \Lambda$, in which all partial derivatives of the 2HDM scalar potential go to zero. The degeneracy of the vacua, parameterized by Eqs. (6) and (21), implies that the following derivatives
\[
\frac{\partial V_{\text{eff}}(H_1, H_2)}{\partial \Phi_i} \propto \frac{1}{2} \beta_{\lambda_1} \tan^{-2} \gamma + \frac{1}{2} \beta_{\lambda_2} \tan^2 \gamma + \beta_{\lambda_3} + \beta_{\lambda_4} \cos^2 \theta.
\]
should vanish at the MPP scale for any choice of $\theta$ and $\omega$. In order for these derivatives to be independent of $\theta$, we require $\beta_{\lambda_4}(\Lambda) = 0$. However, for $\lambda_4(\Lambda) = 0$, this requirement is in conflict with the form of the beta function:
\[
\beta_{\lambda_4}(\Lambda) = \frac{1}{(4\pi)^2} \left[ 3g_2^2(\Lambda)g_1^2(\Lambda) + 12h_1^2(\Lambda)h_6^2(\Lambda) \right]
\]
which is strictly positive. Thus our attempt to adapt the MPP idea to the 2HDM with $\lambda_4(\Lambda) = 0$ fails. Then we have two MPP scenarios.

When $\lambda_4(\Lambda) < 0$ (the first scenario), a self-consistent implementation of the MPP can only be obtained if $\lambda_1(\Lambda), \lambda_2(\Lambda)$ and $\lambda_3(\Lambda)$ have non-zero values. Then $\cos \theta = \pm 1$ near the MPP scale minima, where the Higgs effective potential takes the form (18). In order to ensure the vanishing of $V_{\text{eff}}$ at the MPP scale with an accuracy of order $v^2/\Lambda^2$, the combination of Higgs self-couplings $\tilde{\lambda}(\Lambda)$ must go to zero. Furthermore, if the 2HDM effective potential is to possess a set of local minima at the MPP scale, the derivative of $\tilde{\lambda}(\Phi)$ must vanish when $\Phi = \Lambda$ and hence $\beta_{\tilde{\lambda}}(\Lambda) = 0$. In this first scenario, the following set of MPP scale vacua
\[
<H_1> = \left( \begin{array}{c} 0 \\ \Phi_1 \end{array} \right), \quad <H_2> = \left( \begin{array}{c} 0 \\ \Phi_2 e^{i\omega} \end{array} \right)
\]
have the same energy density for any $\omega$. The ratio of the Higgs field norms $\Phi_1$ and $\Phi_2$ in Eq. (24) is defined by the equations for the extrema of the 2HDM scalar potential, whose solution is given by Eq. (21). In the minima (24) the photon remains massless and electric charge is conserved.

In the second scenario $\lambda_4(\Lambda) > 0$ the parameter $\cos \theta$ tends to zero. We note that our general derivation of the MPP conditions (10, 11), and the consequent $U(1)$ custodial symmetry without fine-tuning, breaks down in this case, since the Higgs potential does not depend on the phase $\omega$ near its minimum (where $\cos \theta = 0$). If $\lambda_4(\Lambda) - |\lambda_5(\Lambda)| > 0$ and $\lambda_1(\Lambda) = \lambda_2(\Lambda) = \lambda_3(\Lambda) = \lambda_6(\Lambda) = \lambda_7(\Lambda) = 0$ the following set of vacua
\[
<H_1> = \left( \begin{array}{c} 0 \\ \Phi_1 \end{array} \right), \quad <H_2> = \left( \begin{array}{c} \Phi_2 \\ 0 \end{array} \right)
\]
10
are degenerate for any $\Phi_1$ and $\Phi_2$ satisfying $\Phi_1^2 + \Phi_2^2 = \Lambda^2$. In order to ensure the existence of the minima given by Eq. (25), the conditions for extrema must be fulfilled which lead to $\beta_{\lambda_1}(\Lambda) = \beta_{\lambda_2}(\Lambda) = \beta_{\lambda_3}(\Lambda) = 0$. The cancellation of different contributions to these $\beta$-functions only becomes possible for large values of the Yukawa couplings at the MPP scale (corresponding to large $\tan \beta$ in Model II). The resulting vacuum energy density vanishes because $\cos \theta$ goes to zero in these vacua. At the set of minima (25) the $SU(2) \times U(1)$ gauge symmetry is broken completely and the photon gains a mass of the order of $\Lambda$. This is not in conflict with phenomenology, since an MPP scale minimum is not presently realised in Nature. However, on phenomenological grounds, we prefer the first scenario, although it is consistent to impose an ad hoc $Z_2$ custodial symmetry on the second scenario which we will discuss separately elsewhere.

The conditions

$$\begin{cases}
\lambda_5(\Lambda) = \lambda_6(\Lambda) = \lambda_7(\Lambda) = \beta_{\lambda_5}(\Lambda) = \beta_{\lambda_6}(\Lambda) = \beta_{\lambda_7}(\Lambda) = 0, \\
\tilde{\lambda}(\Lambda) = \beta_{\tilde{\lambda}}(\Lambda) = 0,
\end{cases} \tag{26}$$

leading to the appearance of the degenerate vacua (24) in our preferred scenario, should be identified with the MPP conditions analogous to those of Eq. (2). The conditions (26) have to be supplemented by the vacuum stability requirements (19), which must be valid everywhere from the electroweak to the MPP scale. Any failure of either the conditions (26) or the inequalities (19) prevents a consistent realization of the MPP in the 2HDM.

We have made a detailed numerical analysis of these MPP constraints on the Higgs spectrum in the 2HDM for high energy scales ranging from $\Lambda = M_{Pl}$ down to $\Lambda = 10$ TeV, which we shall report elsewhere. In the large $\tan \beta$ limit, the allowed range of the Higgs self–couplings is severely constrained by the MPP conditions (26) and vacuum stability requirements (19). As a consequence, using the Model II Yukawa couplings and the lower limit on the charged scalar mass deduced from the non-observation of $B \to X_s \gamma$ decay, the Higgs spectrum exhibits a hierarchical structure for most of the large $\tan \beta$ ($\tan \beta \gtrsim 2$) region. While the heavy scalar, pseudoscalar and charged Higgs particles are nearly degenerate with a mass greater than 300 GeV, the mass of the SM–like Higgs boson $m_h$ does not exceed 180 GeV for any scale $\Lambda \gtrsim 10$ TeV. The bounds on $m_h$ become stronger as the MPP scale is increased. For $\Lambda = M_{Pl}$ and large values of $\tan \beta$ we find: $m_h = 137 \pm 12$ GeV. However, for very large $\tan \beta \simeq m_t(M_t)/m_b(M_t)$ the MPP restrictions on the Higgs self–couplings and the lightest Higgs scalar mass turn out to be substantially relaxed, due to a loosening of the allowed upper limit on $\lambda_2(\Lambda)$. In particular the upper bound on $m_h$ is increased to 180 GeV for $\Lambda = M_{Pl}$ and very large $\tan \beta$. 

11
4. Conclusion

We have studied the constraints imposed by the Multiple Point Principle on the general two Higgs doublet model, by requiring the existence of a large number of vacua at a high energy scale $\Lambda$ which are degenerate with the electroweak scale vacuum. The MPP conditions at the scale $\Lambda$, derived in our preferred scenario with the vacua (24), are summarized in Eq. (26). In addition the vacuum stability conditions (19) must be satisfied. The MPP conditions in the first line of Eq. (26) give CP invariance of the Higgs potential and the presence of a softly broken (by the $m_3^2 H_1^\dagger H_2$ term in $V_{\text{eff}}$) $Z_2$ symmetry of the usual type responsible for the absence of flavour changing neutral currents without fine-tuning. The $Z_2$ invariance of the 2HDM Lagrangian is not spoiled by the renormalization group flow. This means that the MPP provides an alternative mechanism for the suppression of flavour changing neutral currents in the 2HDM.

In addition the MPP conditions in the second line of Eq. (26) provide two relationships between the non-zero Higgs self-couplings, at the scale $\Lambda$, which can in principle be checked when the masses and couplings of the Higgs bosons are measured at future colliders. It is interesting to remark that these relationships are satisfied identically in the minimal supersymmetric standard model at all high energy scales, where the soft SUSY breaking terms can be neglected.

In conclusion we have constructed a new simple MPP inspired non-supersymmetric two Higgs doublet extension of the SM.

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