Measuring the Higgs boson mass with transverse mass variables

Kiwoon Choi\textsuperscript{a}, Jae Sik Lee\textsuperscript{b}, and Chan Beom Park\textsuperscript{a}

\textsuperscript{a}Department of Physics, KAIST, Daejeon 305-701, Korea
\textsuperscript{b}Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan

ABSTRACT

We provide a comparative study of the Higgs boson mass measurements based on two approaches to the dileptonic decay of $W$ bosons produced by the Higgs boson decay, one using the kinematic variable $M_T^{\text{true}}$ and the other using the $M_{T2}$-assisted on-shell reconstruction of the invisible neutrino momenta. We find that these two approaches can determine the Higgs boson mass with a similar accuracy for both of the two main production mechanisms of the SM Higgs boson at the LHC, i.e. the gluon-gluon fusion and the weak vector boson fusion. We also notice that the Higgs signal distribution for the gluon-gluon fusion becomes narrower under the $M_{T2}$ cut, while the corresponding background distribution becomes flatter, indicating that one might be able to reduce the systematic uncertainties of mass measurement with an appropriate $M_{T2}$ cut.
1 Introduction

The utmost target of the LHC is to discover the Higgs boson and study its fundamental properties to establish the mechanism of electroweak symmetry breaking as the origin of particle masses [1,2]. The unsuccessful search at the LEP experiment set a lower bound on the standard model (SM) Higgs mass at 114.4 GeV (95 % C.L.) [3], while the analysis of the electroweak precision data indicates a relatively light SM Higgs boson with $m_H < \sim 185$ GeV at the 95 % confidence level [4]. Combined with the recent Tevatron search excluding $158 \text{ GeV} \leq m_H \leq 175$ GeV [5], we anticipate that the SM Higgs boson most probably lies in the mass range between 114.4 GeV and 158 GeV.

At the LHC, the Higgs boson is mainly produced by the gluon-gluon fusion (GGF) mechanism, and the second most important source is the weak vector boson fusion (VBF) process [6,7]. While the production cross section of GGF is about 10 times larger than that of VBF for the Higgs mass region $114.4 \text{ GeV} \leq m_H \leq 170$ GeV, VBF has its advantage in a kinematic structure containing two forwarding tagging jets with a large rapidity gap, which can be exploited to isolate the Higgs boson signal from backgrounds and to study the signal properties [8–10].

The specific search strategy of the Higgs boson at the LHC depends on its mass and decay pattern. According to the combined study on the expected Higgs discovery significance at ATLAS with 10 fb$^{-1}$ luminosity [1], when the Higgs boson is lighter than 130 GeV, the main search channel is GGF with the Higgs boson decaying into two tau leptons or into two photons. On the other hand, for $130 \text{ GeV} \lesssim m_H \lesssim 150$ GeV, the following three channels are available with a similar significance: (i) GGF with the Higgs boson decaying into two $Z$ bosons which subsequently decay into four charged leptons, (ii) GGF with the Higgs boson decaying into two $W$ bosons, which subsequently decay into two charged leptons and two neutrinos, and (iii) VBF with the Higgs boson decaying into two $W$ bosons, which subsequently decay into two charged leptons and two neutrinos. Finally, for $150 \text{ GeV} \lesssim m_H \lesssim 190$ GeV, Higgs boson decay into two $Z$ bosons is relatively suppressed, while channels (ii) and (iii) remain as the dominant search channels [11]. Therefore, dileptonic $W$ boson decays play a crucial role in the Higgs boson search at the LHC when the Higgs boson weighs between 130 GeV and 190 GeV. Furthermore, if one can improve the efficiency of dileptonic channels, they might play an important role even for $m_H$ lighter than 130 GeV.

In dileptonic $W$ boson decays, there are two invisible neutrinos which make a direct reconstruction of the Higgs boson mass impossible. To overcome this difficulty, one can consider various kind of transverse mass variables. A well-known example is the transverse

\footnote{The general concept of transverse mass has been introduced and studied as early as in Ref. [12]. In our work, we concentrate on the recently proposed transverse mass variables such as $M_T^{true}$, $M_{T2}$, and $m_H^{mass}.}$
mass of a $W$ boson pair in the process $H \rightarrow WW \rightarrow \ell\nu\ell'\nu'$ ($\ell, \ell' = e, \mu$ and $\nu, \nu' = \nu_e, \nu_\mu$):

$$M^2_{WW}(W) = m^2_{\ell\ell'} + m^2_{\nu\nu'} + 2 \left( \sqrt{|p^T_{\ell\ell'}|^2 + m^2_{\ell\ell'}} \sqrt{|p^T_{\nu\nu'}|^2 + m^2_{\nu\nu'}} - p^T_{\ell\ell'} \cdot p^T_{\nu\nu'} \right), \quad (1)$$

where $m_X$ and $p^X_T$ denote the invariant mass and the transverse momentum, respectively, of $X = ll', \nu\nu'$. Obviously $M_T(WW)$ is bounded by $m_H$, and this upper bound is saturated when $ll'$ and $\nu\nu'$ have the same rapidity, $\eta_{ll'} = \eta_{\nu\nu'}$, where $\eta = \frac{1}{2} \ln(E + p_L)/(E - p_L)$ for the longitudinal momentum $p_L$. Therefore, if $M_T(WW)$ is correctly reconstructed event by event, the Higgs boson mass can be read off from the endpoint value of $M_T(WW)$ distribution. However, there is an obstacle to this approach: $m_{\nu\nu'}$ cannot be experimentally determined, although $p^T_{\nu\nu'}$ can be deduced from the missing transverse momentum of the event. One way to bypass this difficulty is to simply take $m_{\nu\nu'} = m_{ll'}$ [13], and consider the distribution of

$$M_T^{\text{approx}} \equiv M_T(WW)|_{m_{\nu\nu'} = m_{ll'}}, \quad (2)$$

which would provide a good approximation to the true value of $M_T(WW)$ if the $W$ boson pair were produced at near-threshold. However, in reality, a sizable number of events are not close to such a threshold, and as a result, $M_T^{\text{approx}}$ is not bounded by $m_H$ anymore. Still, the detailed shape of its distribution has a certain correlation with $m_H$, so $M_T^{\text{approx}}$ has been widely used in the previous studies of the Higgs boson search and mass measurement [1, 8, 10].

Recently, it has been noticed that an alternative transverse mass variable [14],

$$M_T^{\text{true}} \equiv M_T(WW)|_{m_{\nu\nu'} = 0}, \quad (3)$$

may determine the Higgs boson mass more accurately than $M_T^{\text{approx}}$ does, since it is bounded by $m_H$ and thus could have a stronger correlation with $m_H$ [14, 15]. Obviously, for each event

$$M_T^{\text{true}} \leq M_T(WW) \leq m_H, \quad (4)$$

and the upper bound of $M_T^{\text{true}}$ is saturated by the events with $\eta_{ll'} = \eta_{\nu\nu'}$ and $m_{\nu\nu'} = 0$; therefore, the endpoint value of $M_T^{\text{true}}$ distribution indeed corresponds to $m_H$.

There is another widely discussed transverse mass variable, $M_{T2}$, which is defined for a generic event with two identical invisible particles [16] and was applied recently to the mass measurement of supersymmetric particles [16, 17]. For the Higgs boson event $H \rightarrow WW \rightarrow \ell(p)\nu(k)\ell'(q)\nu'(l)$, $M_{T2}$ is given by

$$M_{T2} \equiv \min_{k_T + l_T = p_T} \left[ \max \left\{ M_T^{(1)}, M_T^{(2)} \right\} \right], \quad (5)$$

where $M_T^{(1)}$ and $M_T^{(2)}$ are the transverse masses of the two leptonically decaying $W$ bosons:

$$\left( M_T^{(1)} \right)^2 = 2 \left( |p_T| |k_T| - p_T \cdot k_T \right), \quad \left( M_T^{(2)} \right)^2 = 2 \left( |q_T| |l_T| - q_T \cdot l_T \right). \quad (6)$$
This $\mathcal{M}_{T2}$ has an endpoint at $m_H/2$ when $m_H \leq 2m_W$ [18], suggesting that $\mathcal{M}_{T2}$ also might be useful for the Higgs boson mass measurement.

In fact, with $\mathcal{M}_{T2}$ and additional on-shell constraints, one can approximately reconstruct the invisible particle 4-momenta in each event [19]. This method of “$\mathcal{M}_{T2}$-assisted-on-shell (MAOS) reconstruction” of invisible particle momenta has been applied to the Higgs boson event $H \rightarrow WW \rightarrow \ell(p)\nu(k)\ell'(q)\nu'(l)\ [18]$ in order to examine the invariant mass variable

$$\left( m_{H}^{\text{maos}} \right)^2 \equiv \left( p + q + k_{\text{maos}} + l_{\text{maos}} \right)^2 ,$$

(7)

where $k_{\text{maos}}$ and $l_{\text{maos}}$ are the reconstructed neutrino 4-momenta. It was then argued that the distribution of this MAOS Higgs mass exhibits a peak at the true Higgs boson mass. Furthermore, the reconstructed MAOS momenta $k_{\text{maos}}$ and $l_{\text{maos}}$ are closer to the true neutrino momenta for the events near the upper endpoint of $\mathcal{M}_{T2}$. As a result, the $m_{H}^{\text{maos}}$ distribution can exhibit even a narrow resonance peak at $m_H$ when a strong $\mathcal{M}_{T2}$ cut is employed. These observations suggest that $m_{H}^{\text{maos}}$ might provide a powerful tool to probe the Higgs boson mass through dileptonic $W$ boson decays.

Since it has been claimed that various kinematic variables are useful for the Higgs boson mass measurement, it would be instructive to perform a comparative study of the efficiencies of those variables in the LHC environment. In this paper, we wish to perform such a comparative study for $\mathcal{M}_{T\text{true}}$, $\mathcal{M}_{T2}$, and $m_{H}^{\text{maos}}$, and explore the possibility to improve the efficiency of the analysis. This paper is organized as follows. In Sec. 2 we discuss the distinctive features of $\mathcal{M}_{T\text{true}}$, $\mathcal{M}_{T2}$, and $m_{H}^{\text{maos}}$ for dileptonic $W$ boson decays, mainly focusing on the possible correlations between these three kinematic variables. From the discussion in Sec. 2 it is evident that $\mathcal{M}_{T2}$ is less efficient than $\mathcal{M}_{T\text{true}}$ as a main observable for the Higgs boson search. However, $\mathcal{M}_{T2}$ might be a useful cut variable in the analysis using $\mathcal{M}_{T\text{true}}$ or $m_{H}^{\text{maos}}$ as the main observable. As for the comparison of $\mathcal{M}_{T\text{true}}$ and $m_{H}^{\text{maos}}$, it is not obvious which one is more efficient since each variable has its own virtue and weak point. We thus perform in Secs. 3 and 4 a detailed comparative study of $\mathcal{M}_{T\text{true}}$ and $m_{H}^{\text{maos}}$. Our results show that $m_{H}^{\text{maos}}$ and $\mathcal{M}_{T\text{true}}$ will eventually have a similar efficiency for both the GGF and VBF Higgs productions at the LHC. In this study, we also examine the possible role of $\mathcal{M}_{T2}$ as a cut variable, and find that the Higgs signal distribution for the GGF channel becomes narrower under the $\mathcal{M}_{T2}$ cut, while the background distribution becomes flatter. Although an estimate of any systematic uncertainty is beyond the scope of this paper, such behavior of the signal/background distributions under the $\mathcal{M}_{T2}$ cut indicates that one might be able to reduce the systematic errors in the GGF channel with an appropriate $\mathcal{M}_{T2}$ cut. We summarize our conclusions in Sec. 5.
2 Distinctive features of the transverse mass variables

Unlike the Higgs boson decay into two photons or two tau leptons,[1] the Higgs boson mass cannot be reconstructed in the event \( H \to WW^{(*)} \to \ell(p)\nu(k)\ell'(q)\nu'(l) \) due to the missing neutrinos. One then often considers the transverse mass of the \( W \) pair, \( M_T(WW) \), defined in Eq. (1), as the main observable to probe the Higgs signal event \([1, 8, 13, 14]\). If one could determine the correct value of \( m_{\nu\nu'} \) event by event, the resulting distribution of \( M_T(WW) \) would have an upper endpoint at \( m_H \). However, \( m_{\nu\nu'} \) is not available, in general, and there have been two proposals to fix the unknown \( m_{\nu\nu'} \). The first one is to choose \( m_{\nu\nu'} = m_{\ell\ell'} \) \([1, 8, 13]\), and consider the distribution of \( M_{T\text{approx}}^\equiv M_T(WW)|_{m_{\nu\nu'}=m_{\ell\ell'}} \). This has been motivated by the observation that, for a Higgs boson mass close to \( 2m_W \), the \( W \) bosons are produced at near-threshold and almost at rest in their center-of-mass frame, for which \( m_{\nu\nu'}^{\text{true}} = m_{\ell\ell'} \), where \( m_{\nu\nu'}^{\text{true}} \) is the true value of \( m_{\nu\nu'} \) in the event. Although this threshold approximation is not necessarily suitable for generic events and therefore \( M_{T\text{approx}} \) is not strictly bounded from above by \( m_H \), still the shape and range of its distribution can provide information on the Higgs boson mass. The second proposal \([14]\) is deduced from the simple relation

\[
M_T^{\text{true}} \equiv M_T(WW)|_{m_{\nu\nu'}=0} \leq M_T(WW)|_{m_{\nu\nu'}=m_{\nu\nu'}^{\text{true}}} \leq m_H. \tag{9}
\]

It was reported that one can obtain a more accurate Higgs boson mass by using \( M_T^{\text{true}} \) rather than using \( M_{T\text{approx}} \) \([14, 15]\).

For the event \( H \to WW^{(*)} \to \ell(p)\nu(k)\ell'(q)\nu'(l) \), one may exploit the event variable \( M_{T2} \) which has been designed for a general situation with two invisible particles (with the same mass) in the final state \([16]\). The variable is defined as

\[
M_{T2} \equiv \min_{k_T+1_T=p_T} \left[ \max \left\{ M_T^{(1)}, M_T^{(2)} \right\} \right], \tag{10}
\]

where \( M_T^{(1)} \) and \( M_T^{(2)} \) are the transverse masses of the two leptonically decaying \( W \) bosons:

\[
\left( M_T^{(1)} \right)^2 = 2 (|p_T||k_T| - p_T \cdot k_T), \quad \left( M_T^{(2)} \right)^2 = 2 (|q_T||l_T| - q_T \cdot l_T). \tag{11}
\]

For a specific set of events for which the Higgs boson has vanishing transverse momentum, \( p_T = -p_T - q_T \), one can use the following analytic expression of \( M_{T2} \):

\[
M_{T2}^2 = 2 (|p_T||q_T| + p_T \cdot q_T). \tag{12}
\]

† In the di-tau channel, each tau decays to hadron (or lepton) + a neutrino(s). Although the final state neutrinos make a full reconstruction of the event impossible, it is known that the di-tau invariant mass can be determined well in collinear approximation \([20]\).
to find the upper bound

$$M_{T_2}^2 \leq (|p_T| + |q_T|)^2 \leq \frac{m_H^2}{4}. \quad (13)$$

Although the above relation is derived for the specific event set with vanishing Higgs transverse momentum, it is likely that the same upper bound applies for generic Higgs boson events with nonvanishing Higgs boson transverse momentum. We found through numerical analysis that it is indeed the case and the inequality $M_{T_2} \leq m_H/2$ remains true for generic dileptonic Higgs boson events $H \to WW^{(*)} \to \ell\nu\ell'\nu'$. [See the later discussion of the inequality between $M_{T_2}$ and $M_{T_{true}}$ in Eq. (21).] On the other hand, in the case that both $W$ bosons are on shell, $M_{T_2}$ is bounded by $m_W$. Combining this with (13), one may find that the $M_{T_2}$ of the Higgs boson decay is bounded by $m_{W}$.

$$M_{T_2}^{max} = \begin{cases} 
m_H/2 & \text{for } m_H \leq 2m_W \\
m_W & \text{for } m_H \geq 2m_W, \end{cases} \quad (14)$$

implying that $M_{T_2}$ can provide information on the Higgs boson mass through its endpoint.

Apart from $M_{T_2}^{approx}$, $M_{T_{true}}$, and $M_{T_2}$ in dileptonic $W$ boson decays, there has been a proposal to reconstruct an invariant mass by using the MAOS reconstruction of the two neutrino momenta [18]. In MAOS reconstruction, the transverse momenta are defined as the ones that determine the value of $M_{T_2}$, i.e.

$$M_{T_2}(p_T, q_T, p_T') = M_T^{(1)}(p_T, k_{T_0}^{maos}) = M_T^{(2)}(q_T, l_{T_0}^{maos} = p_T - k_{T_0}^{maos}), \quad (15)$$

while the longitudinal and energy components are obtained from the following constraints:

$$(k_{T_0}^{maos})^2 = (l_{T_0}^{maos})^2 = 0, \quad (p + k_{T_0}^{maos})^2 = (q + l_{T_0}^{maos})^2 = M_{T_2}^2. \quad (16)$$

It was noticed that the MAOS 4-momenta determined as the solutions of (15) and (16) provide a reasonable approximation to the true neutrino 4-momenta, and they become closer to the true momenta for the near-endpoint events in the $M_{T_2}$ distribution [18 19]. Then the distribution of the following invariant mass ($\equiv$ MAOS Higgs boson mass),

$$(m_{H_{maos}}^2)^2 \equiv (p + q + k_{maos} + l_{maos})^2 \quad (17)$$

exhibits a peak at the true Higgs boson mass, and the peak shape becomes narrower if one imposes an event cut selecting the events near the endpoint of $M_{T_2}$. Although such an $M_{T_2}$ cut can cause the $m_{H_{maos}}^2$ distribution to have a resonance peak at $m_H$ and also might eliminate some of the backgrounds, it can also sacrifice the number of the Higgs boson events.
signal, thus worsening the statistical significance of the mass measurement. Therefore for a given luminosity a careful analysis is required to see if the efficiency of the Higgs mass measurement can be improved by a proper $M_{T2}$ cut.

To proceed, let us consider a specific set of events for which the $W$ boson pair has a small transverse momentum: $p_T^{WW} \approx 0$. Although it does not cover the full event set, this subset of events reveal some of the essential features of the kinematic variables under discussion. For those events with small $p_T^{WW}$, we have

$$p_T = -(p_T + q_T),$$

and then the following simple expressions of $M_{T2}$ and MAOS momenta are available [18,19]:

$$k_T^{\text{maos}} = -q_T, \quad k_L^{\text{maos}} = \frac{|k_T^{\text{maos}}|}{|p_T|} p_L,$$

$$l_T^{\text{maos}} = -p_T, \quad l_L^{\text{maos}} = \frac{|l_T^{\text{maos}}|}{|q_T|} q_L,$$

$$M_{T2}^2 = 2 (|p_T||q_T| + p_T \cdot q_T). \quad (18)$$

With these expressions of $M_{T2}$ and MAOS momenta, it is straightforward to find

$$\left( m_H^{\text{maos}} \right)^2 = \left( 2 + \frac{|p_T|}{|q_T|} + \frac{|q_T|}{|p_T|} \right) \left( 2 (|p_T||q_T| + p_T \cdot q_T) + m_{\ell\ell'}^2 \right),$$

$$M_T^{\text{true}} = |p_T + q_T| + \sqrt{|p_T + q_T|^2 + m_{\ell\ell'}^2}. \quad (19)$$

Since

$$M_{T2}^2 = 2 (|p_T||q_T| + p_T \cdot q_T) \leq |p_T + q_T|^2,$$

$$m_{\ell\ell'}^2 = 2 (|p| |q| - p \cdot q) \geq 0, \quad (20)$$

both $m_H^{\text{maos}}$ and $M_T^{\text{true}}$ are bounded below by $2M_{T2}$, and therefore

$$2M_{T2} \leq M_T^{\text{true}} \leq m_H, \quad 2M_{T2} \leq m_H^{\text{maos}}. \quad (21)$$

Although the above inequalities between $M_{T2}$, $M_T^{\text{true}}$ and $m_H^{\text{maos}}$ could be derived analytically only for the events with $p_T^{WW} = 0$, we find numerically that they hold true also for the events with $p_T^{WW} \neq 0$. In Fig. 1 (left panel), we show the distributions of these three kinematic variables for the true Higgs mass $m_H = 160$ GeV. Here we use the Higgs boson events produced by the GGF process at the LHC, while incorporating the detector effects with the fast detector simulation program PGS4 [21]. Figure 1 (left panel) shows, first of all, that both $M_T^{\text{true}}$ and $2M_{T2}$ have a common upper endpoint at $m_H = 160$ GeV, as we have anticipated in the above discussion. However, the $M_T^{\text{true}}$ distribution is significantly narrower than the $2M_{T2}$ distribution, as implied by the first inequality in Eq. (21). This
indicates that $M_T^{\text{true}}$ is more sensitive to $m_H$ than $M_{T2}$, and thus is a more efficient variable to determine $m_H$. If one compares $M_T^{\text{true}}$ to $m_H^{\text{maos}}$, Fig. 1 (left panel) shows that $m_H^{\text{maos}}$ has a peak at $m_H$, while $M_T^{\text{true}}$ has an endpoint at $m_H$. With this feature, one might expect that $m_H^{\text{maos}}$ is more sensitive to $m_H$ than $M_T^{\text{true}}$. However the $M_T^{\text{true}}$ distribution around $m_H$ is significantly narrower than the $m_H^{\text{maos}}$ distribution, and thus one needs a detailed analysis to see which of $M_T^{\text{true}}$ and $m_H^{\text{maos}}$ is more efficient for the determination of $m_H$. In the next two sections, we perform such an analysis for both the GGF Higgs production (Sec. 3) and the VBF Higgs production (Sec. 4) at the LHC.

3 Measuring the Higgs boson mass in GGF

In this section, we consider the SM Higgs boson production at the LHC through the GGF process and its decay into two leptonically decaying $W$ bosons. To investigate the experimental performance of $m_H^{\text{maos}}$ and $M_T^{\text{true}}$, we use the PYTHIA6.4 event generator [22] at the proton-proton center-of-mass energy of 14 TeV. The generated events have been further processed through the fast detector simulation program PGS4 [21] to incorporate the detector effects with reasonable efficiencies and fake rates. The dominant background comes from the continuum $q\bar{q}, gg \rightarrow WW \rightarrow l\nu l'\nu'$ process, and we also include the $t\bar{t}$ background in which the two top quarks decay into a pair of $W$ bosons and two $b$ jets. For the details of the MC event samples of the SM Higgs boson signal and the two main backgrounds, and also the employed event selection cuts, we refer to Ref. [18].

In Fig. 1 we show the $m_H^{\text{maos}}$ and $M_T^{\text{true}}$ distributions of the signal events for $m_H = 160$ GeV without (left panel) and with (right panel) the $M_{T2}$ cut: $M_{T2} > (M_{T2})^{\text{cut}} = 60$ GeV. We use the same event samples at the detector level as in Ref. [18] but do not apply the event selection cuts other than the $M_{T2}$ cut. First we note that the $M_T^{\text{true}}$ distribution is bounded above by $m_H = 160$ GeV, while the $m_H^{\text{maos}}$ distribution is peaked around it, as noticed in the discussion of the previous section. Under an $M_{T2}$ cut, both distributions are bounded from below by $2(M_{T2})^{\text{cut}} = 120$ GeV with a rather good approximation, which can be understood by the inequalities in (21). Incidentally, in the $m_H^{\text{maos}}$ distribution, the region above $m_H$ is less sensitive to the $M_{T2}$ cut than the region below $m_H$. We also see clearly that the $M_T^{\text{true}}$ distribution is narrower than the $m_H^{\text{maos}}$ distribution as implied by Eq. (21).

Next, we proceed to perform the template likelihood fitting to the $M_T^{\text{true}}$ and $m_H^{\text{maos}}$ distributions. Here, a template means a simulated distribution with a trial Higgs mass which, in general, is different from the nominal one used to generate the data. For each pseudoexperiment distribution with the nominal Higgs mass $m_H$, 11 templates are generated with the trial Higgs masses between $m_H - 10$ GeV and $m_H + 10$ GeV, in steps of 2 GeV. For example, in each frame of Fig. 2, the solid line shows the template distribution
The Higgs boson mass is taken to be 160 GeV, and we use the same event samples as in Ref. [18].

when the trial Higgs mass is the same as the nominal one. Each template is normalized to the corresponding pseudoexperiment distribution. For the definitions of the likelihood between a pseudoexperiment data distribution and a template, and also for more on the Higgs masses and 1-σ errors obtained by fitting the log likelihood distributions, we again refer to Ref. [18].

Empirically, the template likelihood fitting, which is used to estimate the efficiency of the Higgs mass determination in this work, provides a better result when the width of the distribution is narrower. On the other hand, compared to the peak of the distribution which is generally easier to determine, the endpoints are more vulnerable to detector smearing, backgrounds and low statistics. Even though the peak of the $M_T^{\text{true}}$ distribution does not have a strong correlation with the input Higgs mass like that of the $m_H^{\text{maos}}$ distribution, the overall shape has a definite correlation with the Higgs boson mass and, moreover, its narrower width might result in a comparable sensitivity to $m_H^{\text{maos}}$. The problem is the shape of the background distribution which could smear the edge structure of $M_T^{\text{true}}$. In Fig. 2 we have shown the $m_H^{\text{maos}}$ (left panels) and $M_T^{\text{true}}$ (right panels) distributions for the Higgs signal and the backgrounds. We use the same detector-level event samples as in Ref. [18], and all the event selection cuts are applied.

Our MC study shows that, generically, the background distributions for both $M_T^{\text{true}}$...
Figure 2: The $m_{H}^{\text{maos}}$ (left frames) and the $M_{T}^{\text{true}}$ (right frames) distributions at 10 fb$^{-1}$ luminosity. In each frame, the shaded region represents the backgrounds ($t\bar{t}$ and $WW$) and the event selection cuts without (upper frames) and with (lower frames) the $M_{T2}$ cut ($M_{T2} > 66$ GeV) imposed. The Higgs boson mass is taken to be 160 GeV, and we again refer to Ref. [18] for the details.

and $m_{H}^{\text{maos}}$ become flatter if we impose a stronger $M_{T2}$ cut. On the other hand, a stronger $M_{T2}$ cut causes the signal distributions to have a narrower shape (see Fig. 1). This behavior of the signal and background under the $M_{T2}$ cut suggests that one might be able to improve
Figure 3: The bands showing the 1-σ deviation of the statistical error for the Higgs boson mass determined by the $m_H^{\text{maos}}$ and $M_T^{\text{true}}$ distributions in the GGF process $gg \to H \to WW^{(*)} \to ll\nu\nu'$. The dots and lines denote the Higgs boson mass obtained by the likelihood fit to the distributions. We use the same event sample and the cut procedure as in Ref. [18], where the right (left) panel is the result with (without) an optimal $M_{T2}$ cut, chosen differently for different values of $m_H$.

The efficiency of the Higgs mass measurement with an appropriate $M_{T2}$ cut, particularly reduce the systematic uncertainties associated with various origins such as the detector energy resolution, fit routines and poorly estimated backgrounds. As for statistical error, one needs a detailed analysis, as the improvement due to better shapes of signal and background distributions can be compensated by the reduced statistics. Indeed, for an integrated luminosity $\sim 10 \text{ fb}^{-1}$, we find (Fig. 3) that there is no appreciable improvement of the statistical error gained by the $M_{T2}$ cut, although the situation might be different at higher luminosity.

In Fig. 3 we show the Higgs boson masses and the 1-σ (statistical) errors obtained by the likelihood fit to the $m_H^{\text{maos}}$ and $M_T^{\text{true}}$ distributions in the GGF process $gg \to H \to WW^{(*)} \to ll\nu\nu'$. We see that the reconstructed mass is rather close to the input one. For the sake of comparison, we have not employed an $M_{T2}$ cut in the left frame since the cut was not included in the original suggestion of the $M_T^{\text{true}}$ variable [14]; however, it is employed in the right frame. We observe that the efficiency of $m_H^{\text{maos}}$ is comparable to that of $M_T^{\text{true}}$ up to systematic errors, which are not considered in our analysis. In particular, when the $M_{T2}$ cut is employed, it is hard to say which one shows a better efficiency. Still, we note that $m_H^{\text{maos}}$ could be a better choice when the distribution is spoiled by some unknown
backgrounds beyond the SM, since the peak is less vulnerable to unknown backgrounds than the endpoint.

4 Measuring the Higgs boson mass in VBF

In this section, we turn to the VBF process, which is the second most important production channel of the SM Higgs boson at the LHC. The characteristic feature of the process is the existence of the two forward tagging jets with suppressed hadronic activity between them and the central Higgs decay products. Thanks to the exchanges of the colorless vector bosons in the process, these features lead to a fairly clean environment with well-isolated signal events in a low background. Therefore, the VBF process is useful to measure properties of the Higgs boson in a hadron collider environment.

The same variables introduced in the GGF process could be used to determine the Higgs boson mass in VBF, since the tagging jets provide additional information without touching the decay of the Higgs boson itself. To investigate the experimental performance of the MAOS Higgs mass at the LHC through the VBF process, we have generated MC event samples of the signal $H + 2j$ events and the main $t\bar{t}$ background events by PYTHIA6.4 [22]. The subleading background, i.e. electroweak $WW + 2j$ event showing a similar characteristic, has been generated using MadGraph/MadEvent [23]. The generated events have been further processed with the fast detector simulation program PGS4 [21], which approximates an ATLAS or CMS-like detector. The PGS4 program uses a cone algorithm for jet reconstruction, with a default value of the cone size $\Delta R = 0.5$, where $\Delta R$ is a separation in the azimuthal angle and pseudorapidity plane. We note that the $b$-jet tagging efficiency is introduced as a function of the jet transverse energy and pseudorapidity, with a typical value of about 50% in the central region for the high energy jets.

Proceeding in a similar way as in Ref. [8], we have imposed the following basic selection cuts on the Higgs signal and the backgrounds:

- Preselection cuts
- $b$-jet veto
- Tag-jet conditions (forward jet tagging, leptons between jets, central jet veto)
- $\gamma^*/Z +$ jets, $Z \to \tau\tau$ rejection

\[^{12}\text{The pseudorapidity is defined as } \eta = -\ln \tan(\theta/2) \text{ for the angle } \theta \text{ between the particle momentum and the beam axis. Note that the pseudorapidity for a massless particle is equal to the rapidity defined as } \eta = \frac{1}{2}\ln(E + p_L)/(E - p_L).\]
Since the final state consists of two hard jets, two charged leptons and significant missing transverse energy, the preselection cuts should be the minimal conditions to be imposed. Specifically, we require at least two jets with $p_T > 20$ GeV and $|\eta| < 4.8$, and two isolated, opposite-sign leptons ($e$ or $\mu$) with $p_T > 15$ GeV, and $|\not{p}_T| > 30$ GeV. The b-jet veto is used to exclude the main $t\bar{t}$ background which contains two b jets. The tag-jet conditions should reflect the characteristic features of the VBF process: two forward tagging jets with suppressed hadronic activity between them and central Higgs decay products. In this work, we define the tagging jets as the two highest $p_T$ jets in the event while rejecting the event if they are in the same hemisphere. Explicitly, we have imposed the following tag-jet conditions.

- Forward jet tagging: the two hardest jets are required to be in opposite hemispheres ($\eta_{j_1} \times \eta_{j_2} < 0$) and to have a pseudorapidity separation $|\Delta \eta_{j_1, j_2}| > 3$.
- Leptons between jets: both leptons are required to be between the tag jets in pseudorapidity.
- Central jet veto: the event is rejected if it contains any extra jet (in addition to the two forward tagging jets) with $p_T > 20$ GeV in the pseudorapidity region $|\eta| < 3.2$.

The last basic selection cut is to reject the Drell-Yan background involving the variable [see Eq. (1)]:

$$m_{ll\nu}^{\text{UT}} \equiv M_T(WW)|_{m_{ll}=m_{\nu\nu}=0}. \quad (22)$$

Though the Drell-Yan background has been neglected in this work, we have included the rejection cut to see its effect on the signal and the other background events under consideration. For the $Z \to \tau\tau$ rejection, the di-tau invariant mass is constructed using the collinear approximation, rejecting the events with $|M_{\tau\tau} - m_Z| < 25$ GeV.

It is well known that the spin-zero nature of the Higgs boson makes two charged leptons come out in the same direction, making its opening angle in the Higgs signal smaller than that in the background; see Fig. 4. To incorporate this spin information, in addition to the basic selections, we have further required

$$\Delta \Phi_{ll} < \Delta \Phi_{ll}^{\text{cut}}, \quad \Delta \eta_{ll} < \Delta \eta_{ll}^{\text{cut}}, \quad (23)$$

where $\Delta \Phi_{ll}$ and $\Delta \eta_{ll}$ are the transverse opening angle and pseudorapidity gap between the charged leptons.

In the literature, the dilepton invariant mass $M_{ll}$ and the dijet invariant mass $M_{jj}$ for two tagging jets are also used for the event cut. In Fig. 5, we show the $M_{ll}$ and $M_{jj}$ distributions for both the Higgs signal and the $t\bar{t}$ background. As in the GGF process discussed in the previous section, one might employ $M_{T2}$ as a cut variable in the VBF process also. In the Higgs signal with the two tagging jets and also in the main $t\bar{t}$ background with two
Figure 4: Transverse opening angle $\Delta \Phi_{ll}$ (left panel) and pseudorapidity gap $\Delta \eta_{ll}$ (right panel) between the charged leptons in the VBF process ($m_H = 160$ GeV) and the background events. The preselection cuts are used for the event selection.

Figure 5: The $M_{ll}$ (left panel) and $M_{jj}$ (right panel) distributions of the signal and the $t\bar{t}$ background, after applying the preselection cuts. $m_H = 160$ GeV is taken for the signal.
Figure 6: The $M_{T2}^{\text{sub}}$ (left panel) and $M_{T2}^{\text{full}}$ (right panel) distributions of the VBF process ($m_H = 160$ GeV) and the $t\bar{t}$ background, after applying the preselection cuts.

$b$ jets, one can construct two sorts of $M_{T2}$—one using the transverse masses of the $W$-pair system and the other using the $W$-pair + two jets system:

$$M_{T2}^{\text{sub}} \equiv \min_{kT+lT=\not{p}_T} \left[ \max \left\{ M_{T}^{\text{sub}(1)}, M_{T}^{\text{sub}(2)} \right\} \right],$$

$$M_{T2}^{\text{full}} \equiv \min_{kT+lT=\not{p}_T} \left[ \max \left\{ M_{T}^{\text{full}(1)}, M_{T}^{\text{full}(2)} \right\} \right],$$

(24)

where $M_{T2}^{\text{sub}(i)}$ and $M_{T2}^{\text{full}(i)}$ are the transverse masses of $W^{(i)} \rightarrow l^{(i)}\nu^{(i)}$ and $j^{(i)}W^{(i)} \rightarrow j^{(i)}l^{(i)}\nu^{(i)}$, respectively. The $M_{T2}^{\text{sub}}$ variable is defined without any combinatorial ambiguity and bounded from above by $\min(m_W, m_H/2)$ [see Eq. (14)] in the signal and by $m_W$ in the background. On the other hand, there is a combinatorial uncertainty in constructing $M_{T2}^{\text{full}}$ since, in each selected event, there are two possibilities in associating two jets with two charged leptons. Between the two possibilities, we choose the combination giving the smaller $M_{T2}^{\text{full}}$. Then, for the $t\bar{t}$ background, $M_{T2}^{\text{full}}$ is bounded from above by $m_t$ at parton level, which suggests that $M_{T2}^{\text{full}}$ can be used as a cut variable to eliminate the $t\bar{t}$ backgrounds by requiring $M_{T2}^{\text{full}} > m_t$. For the Higgs signal events, tagging jets tend to have a large $M_{jj}$, which has been used to select the signals by requiring $M_{jj} > M_{jj}^{\text{cut}}$. In Fig. [5] we show the $M_{T2}^{\text{sub}}$ (left panel) and $M_{T2}^{\text{full}}$ (right panel) distributions for the signal and the $t\bar{t}$ background at detector level. In the right panel of Fig. [6], we observe that the $M_{T2}^{\text{full}}$ distribution for $t\bar{t}$ backgrounds has a non-negligible tail above $m_t$, which arises from the fact that one (or both) of the tagging dijets used for the construction of $M_{T2}^{\text{full}}$ is not the $b$ jet from the top quark decay, but a hard jet coming from the initial state radiation or an erroneously
Table 1: Cut flows (in fb) for $m_H = 160$ GeV.

| Selection cuts                          | VBF $H \rightarrow WW$ | $t\bar{t}$ | EW $WW + \text{jets}$ | $S/B$ |
|----------------------------------------|-------------------------|-----------|-----------------------|------|
| Preselection                           | 33.8                    | 10910.0   | 15.9                  | 0.0031|
| $b$-jet veto                           | 32.8                    | 5193.1    | 15.5                  | 0.0063|
| Forward jet tagging                    | 22.5                    | 542.1     | 6.3                   | 0.041 |
| Leptons between jets                   | 20.9                    | 347.3     | 5.6                   | 0.059 |
| Central jet veto                       | 16.8                    | 166.9     | 3.5                   | 0.099 |
| $m_T^{ll\nu} > 30$ GeV, $Z \rightarrow \tau\tau$ rej. | 15.1                     | 138.5     | 2.7                   | 0.11  |
| $M_{ll} < 85$ GeV                      | 14.9                    | 62.0      | 0.8                   | 0.24  |
| $M_{jj} > 550$ GeV                     | 9.3                     | 8.0       | 0.7                   | 1.07  |
| $\Delta\Phi_{ll} < 1.6, \Delta\eta_{ll} < 1.5$ | 8.4                      | 5.5       | 0.5                   | 1.39  |
| $M_{T2}^{full} > 160$ GeV              | 8.9                     | 7.1       | 0.7                   | 1.14  |
| $M_{T2}^{sub} > 60$ GeV                | 3.6                     | 1.7       | 0.3                   | 1.80  |
| $\Delta\Phi_{ll} < 1.6, \Delta\eta_{ll} < 1.5$ | 3.6                      | 1.6       | 0.2                   | 2.00  |

Table 2: Cut flows (in fb) for $m_H = 140$ GeV.

| Selection cuts                          | VBF $H \rightarrow WW$ | $t\bar{t}$ | EW $WW + \text{jets}$ | $S/B$ |
|----------------------------------------|-------------------------|-----------|-----------------------|------|
| Preselection                           | 17.6                    | 10910.0   | 15.9                  | 0.0016|
| $b$-jet veto                           | 17.3                    | 5193.1    | 15.5                  | 0.0033|
| Forward jet tagging                    | 12.2                    | 542.1     | 6.3                   | 0.022 |
| Leptons between jets                   | 11.6                    | 347.3     | 5.6                   | 0.033 |
| Central jet veto                       | 9.8                     | 166.9     | 3.5                   | 0.058 |
| $m_T^{ll\nu} > 30$ GeV, $Z \rightarrow \tau\tau$ rej. | 8.5                      | 138.5     | 2.7                   | 0.060 |
| $M_{ll} < 85$ GeV                      | 8.5                     | 62.0      | 0.8                   | 0.14  |
| $M_{jj} > 550$ GeV                     | 5.7                     | 8.0       | 0.7                   | 0.66  |
| $\Delta\Phi_{ll} < 1.7, \Delta\eta_{ll} < 1.6$ | 5.4                      | 5.5       | 0.5                   | 0.89  |
| $M_{T2}^{full} > 140$ GeV              | 5.5                     | 7.5       | 0.7                   | 0.67  |
| $M_{T2}^{sub} > 45$ GeV                | 2.1                     | 3.7       | 0.5                   | 0.50  |
| $M_{T2}^{sub} < 72$ GeV                | 2.1                     | 3.1       | 0.3                   | 0.62  |
| $\Delta\Phi_{ll} < 1.7, \Delta\eta_{ll} < 1.6$ | 2.1                      | 2.6       | 0.3                   | 0.72  |

reconstructed jet. With this, comparing to $M_{jj}$ in Fig. 3, we see that $M_{T2}^{full}$ can be as efficient as $M_{jj}$ in discriminating the Higgs signal from the $t\bar{t}$ background. On the other hand, Figs. 4 and 5 show that both the Higgs signal and the $t\bar{t}$ background have a similar shape of $M_{T2}^{sub}$ distribution, so $M_{T2}^{sub}$ is not so useful as a cut variable.
Table 3: The Higgs boson masses and the 1-σ statistical errors obtained by the likelihood fit to the \( m_{H}^{\text{maos}} \) distributions in the VBF processes at 10 fb\(^{-1}\).

| \( m_{H} \) (GeV) | 130  | 140  | 150  | 160  | 170  | 180  | 190  |
|-------------------|------|------|------|------|------|------|------|
| Fitted value (GeV) | 132.5| 140.7| 150.8| 161.0| 170.6| 183.4| 188.6|
| 1-σ error (GeV)   | 7.5  | 7.0  | 4.8  | 3.7  | 3.8  | 6.2  | 6.8  |

With the above observations, in our event selection we have applied \( M_{ll} \) and \( M_{jj} \) cuts after the basic selection cuts:

\[
M_{ll} < M_{ll}^{\text{cut}}, \quad M_{jj} > M_{jj}^{\text{cut}}. \tag{25}
\]

Then, more refined \( M_{T2} \) cuts have been introduced to reduce the number of background events further. In Tables 1 and 2, we show how the cross sections of the signal and backgrounds change under each selection cut for \( m_{H} = 160 \) GeV and 140 GeV, respectively.

We note that one may also use the upper cut on the \( M_{\text{sub}T2} \) when \( m_{H} \leq 2m_{W} \); see the third set of cuts in Table 2. This is because the signal distribution is bounded above by \( m_{H}/2 \) in this case. This upper cut cannot be applied when \( m_{H} \geq 2m_{W} \), as the \( M_{\text{sub}T2} \) distributions of both the signal and the background have an equal maximum, \( m_{W} \).

In Fig. 7, we show the \( m_{H}^{\text{maos}} \) distributions for the two nominal values of the Higgs boson mass: \( m_{H} = 140 \) and 160 GeV. Here \( (m_{H}^{\text{maos}})^{2} = (p_{1} + k_{1}^{\text{maos}} + p_{2} + k_{2}^{\text{maos}})^{2} \) for \( W(1)W(2) \rightarrow l^{(1)}(p_{1})\nu^{(1)}(k_{1})l^{(2)}(p_{2})\nu^{(2)}(k_{2}) \), where the transverse components of \( k_{i}^{\text{maos}} \) (\( i = 1, 2 \)) are used to determine \( M_{\text{sub}T2}^{\text{cut}} \), and the longitudinal and energy components are fixed by the constraints \( (p_{1} + k_{1}^{\text{maos}})^{2} = (p_{2} + k_{2}^{\text{maos}})^{2} = (M_{\text{sub}T2}^{\text{cut}})^{2} \) and \( (k_{1}^{\text{maos}})^{2} = (k_{2}^{\text{maos}})^{2} = 0 \). We observe that the \( m_{H}^{\text{maos}} \) distribution has a clear peak at the true (nominal) Higgs mass. With the \( m_{H}^{\text{maos}} \) distribution constructed as above, we performed a template fitting to determine the Higgs boson mass as in Sec. 3, the GGF case. The estimated Higgs masses, together with the 1-σ errors obtained by fitting the log likelihood distributions for various Higgs masses, are listed in Table 3. The 1-σ deviated value is defined as the one that increases \( -\ln L \) by 1/2 [24]. We again see that the reconstructed mass is quite close to the input Higgs boson mass.

For a comparison of the efficiency of \( m_{H}^{\text{maos}} \) to that of \( M_{T}^{\text{true}} \), we also perform a likelihood fit of the \( M_{T}^{\text{true}} \) distribution with the same event sets, including the backgrounds. In Fig. 8, we show the \( M_{T}^{\text{true}} \) template distributions (solid lines) when the trial Higgs mass is 140 GeV (left panel) and 160 GeV (right panel). The dots with error bars represent the \( M_{T}^{\text{true}} \) distributions for the pseudoexperiment data with the same nominal Higgs masses.

---

\( \text{¶} \)The background bump appearing around \( m_{H}^{\text{maos}} \gtrsim 200 \) GeV is an accidental consequence of our analysis for the VBF case. We observe that it becomes less significant like as in the GGF case for a different choice of the bin size.
Figure 7: The $m_{H}^{\text{mass}}$ distributions for the pseudoexperiment data (dots) and the template (solid line) for $m_{H} = 140$ (left) and 160 GeV (right) at 10 fb$^{-1}$. The lightly and thickly shaded regions represent the $t\bar{t}$ and $WW + 2j$ backgrounds, respectively.

Figure 8: The $M_{T}^{\text{true}}$ distributions for the pseudoexperiment data (dots) and the template (solid line) for $m_{H} = 140$ (left) and 160 GeV (right) at 10 fb$^{-1}$ luminosity. The lightly and thickly shaded regions represent the $t\bar{t}$ and $WW + 2j$ backgrounds, respectively.
We show our results on the Higgs boson masses and the 1-σ errors in the left frame of Fig. 9; these results are obtained using the likelihood fit to the $m_{H}^{\text{maos}}$ and $M_{T}^{\text{true}}$ distributions in the VBF process $qq \rightarrow qqH$ with $H \rightarrow WW^{(*)} \rightarrow l\nu l'\nu'$. We observe that the efficiencies of the two methods are comparable to each other, and both of them provide a good accuracy for the determination of the Higgs mass. Furthermore, to see the role of the $M_{T2}$ cuts in the measurement, we repeat the likelihood fitting of the distributions after applying the same event selection cuts except the $M_{T2}$ cuts; see the right frame of Fig. 9. We find that the final result does not change much since the purity of the event samples is already high enough before imposing the $M_{T2}$ cuts and also the number of signals is relatively smaller than that of the GGF process. Although $M_{T}^{\text{true}}$ and $m_{H}^{\text{maos}}$ have a similar efficiency for the VBF process, we again note that in a situation where there are unknown backgrounds beyond the SM which might smear the endpoint of the $M_{T}^{\text{true}}$ distribution, the $m_{H}^{\text{maos}}$ could be a better choice since the peak is less vulnerable to unknown backgrounds than the endpoint.
5 Conclusions

We performed a comparative study of the Higgs boson mass measurements based on two kinematic observables, $M_T^{\text{true}}$, and $m_{H}^{\text{maos}}$ in dileptonic decays of a $W$ boson pair. For this, we first discussed some features of $M_T^{\text{true}}$ and $M_{T2}$, and also of the MAOS reconstruction of the neutrino momenta in dileptonic $W$ boson decays and the associated invariant mass observable $m_{H}^{\text{maos}}$. It is found that $M_T^{\text{true}}$ and $m_{H}^{\text{maos}}$ can determine the Higgs boson mass in the range $130 \text{ GeV} \leq m_H \leq 190 \text{ GeV}$ with a similar accuracy for both of the two main production mechanisms of the SM Higgs boson at the LHC, i.e. GGF and VBF, up to systematic errors which are not considered in our analysis. Still, it should be noted that the $m_{H}^{\text{maos}}$ distribution has a peak at $m_H$, while the $M_T^{\text{true}}$ distribution has an endpoint, and thus $m_{H}^{\text{maos}}$ can be a better observable when there are some additional backgrounds due to new physics beyond the SM since it is likely that the peak is less vulnerable to unknown backgrounds than the endpoint. One might consider an approach using both $m_{H}^{\text{maos}}$ and $M_T^{\text{true}}$ together to extract maximal information on the Higgs boson mass from experimental data. However, our results in Sec. [2] imply that these two variables have a rather strong correlation, and therefore it is not likely that such an approach significantly improves the accuracy of the Higgs mass measurement.

Our study suggests that $M_{T2}$ can be a useful cut variable in the GGF process, as the signal distribution becomes narrower while the background distribution becomes flatter under an $M_{T2}$ cut. Although systematic errors are not considered in our analysis, such behavior of the signal and background distributions might be useful, particularly for reducing various systematic uncertainties in the real analysis of experimental data. In our analysis of the VBF process, $M_{T2}$ is not a particularly useful cut variable compared to others such as the dijet and dilepton invariant masses.

Acknowledgements

We thank W. S. Cho for useful discussions. This work was supported by the KRF Grants funded by the Korean Government (No. KRF-2008-314-C00064 and No. KRF-2007-341-C00010) and the KOSEF Grant funded by the Korean Government (No. 2009-0080844).

References

[1] G. Aad et al. [The ATLAS Collaboration], arXiv:0901.0512

[2] G. L. Bayatian et al. [CMS Collaboration], J. Phys. G 34, 995 (2007).
[3] R. Barate et al. [The LEP Working Group for Higgs boson searches, ALEPH, DELPHI, L3, and OPAL Collaborations], Phys. Lett. B 565, 61 (2003) arXiv:hep-ex/0306033.

[4] ALEPH, CDF, D0, DELPHI, L3, OPAL, SLD Collaborations, arXiv:0811.4682.

[5] CDF Collaboration and D0 Collaboration, arXiv:0903.4001; CDF Collaboration and D0 Collaboration, arXiv:0911.3930; The TEVNPH Working Group of the CDF and D0 Collaborations, arXiv:1007.4587.

[6] M. Spira, Fortsch. Phys. 46, 203 (1998) arXiv:hep-ph/9705337.

[7] For a recent review, see A. Djouadi, Phys. Rept. 457, 1 (2008) arXiv:hep-ph/0503172.

[8] S. Asai et al., Eur. Phys. J. C 32S2, s19 (2003) arXiv:hep-ph/0402254.

[9] C. Ruwiedel, N. Wermes, and M. Schumacher, Eur. Phys. J. C 51, 385 (2007).

[10] E. Yazgan et al., Eur. Phys. J. C 53, 329 (2008) arXiv:0706.1898.

[11] V. D. Barger, G. Bhattacharya, T. Han, and B. A. Kniehl, Phys. Rev. D 43, 779 (1991); M. Dittmar, and H. K. Dreiner, Phys. Rev. D 55, 167 (1997) arXiv:hep-ph/9608317.

[12] W. L. van Neerven, J. A. M. Vermaseren, and K. J. F. Gaemers, NIKHEF-H/82-20; J. Smith, W. L. van Neerven, and J. A. M. Vermaseren, Phys. Rev. Lett. 50, 1738 (1983); V. D. Barger, A. D. Martin, and R. J. N. Phillips, Z. Phys. C 21, 99 (1983).

[13] D. L. Rainwater and D. Zeppenfeld, Phys. Rev. D 60, 113004 (1999) [Erratum-ibid. D 61, 099901 (2000)] arXiv:hep-ph/9906218.

[14] A. J. Barr, B. Gripaios, and C. G. Lester, JHEP 0907 (2009) 072 arXiv:0902.4864.

[15] See S. Dawson and T. Plehn in P. Nath et al, Nucl. Phys. B, Proc. Suppl. 200-202 (2010) 185, arXiv:1001.2693.

[16] C. G. Lester and D. J. Summers, Phys. Lett. B 463, 99 (1999) arXiv:hep-ph/9906349; A. Barr, C. Lester, and P. Stephens, J. Phys. G 29, 2343 (2003) arXiv:hep-ph/0304226.

[17] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, Phys. Rev. Lett. 100, 171801 (2008) arXiv:0709.0288; B. Gripaios, JHEP 0802 (2008) 053 arXiv:0709.2740; A. J. Barr, B. Gripaios, and C. G. Lester, JHEP 0802 (2008) 014 arXiv:0711.4008; W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, JHEP 0802 (2008) 035 arXiv:0711.4526; M. M. Nojiri, Y. Shimizu, S. Okada, and K. Kawagoe, JHEP
(2008) 035 [arXiv:0802.2412]; M. Burns, K. Kong, K. T. Matchev, and M. Park, JHEP 0903 (2009) 143 [arXiv:0810.5576]; A. J. Barr and C. Gwenlan, Phys. Rev. D 80, 074007 (2009) [arXiv:0907.2713].

[18] K. Choi, S. Choi, J. S. Lee, and C. B. Park, Phys. Rev. D 80, 073010 (2009) [arXiv:0908.0079].

[19] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, Phys. Rev. D 79 031701, (2009) [arXiv:0810.4853].

[20] R. K. Ellis, I. Hinchliffe, M. Soldate, and J. J. van der Bij, Nucl. Phys. B297, 221 (1988).

[21] J. Conway, http://www.physics.ucdavis.edu/~conway/research/software/pgs/pgs4-general.htm.

[22] T. Sjostrand, P. Eden, C. Friberg, L. Lonnblad, G. Miu, S. Mrenna, and E. Norrbin, Comput. Phys. Commun. 135 (2001) 238 [arXiv:hep-ph/0010017]; T. Sjostrand, S. Mrenna, and P. Skands, JHEP 0605 (2006) 026 [arXiv:hep-ph/0603175].

[23] J. Alwall, P. Demin, S. d. Visscher, R. Frederix, M. Herquet, F. Maltoni, T. Plehn, D. L. Rainwater, and T. Stelzer, JHEP 0709 (2007) 028 [arXiv:0706.2334].

[24] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).