Correlation between Io's lead angle and the satellite's magnetic footprint

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Abstract This research studies the nature of auroral feature on Jupiter, especially which connects to one of its satellite, Io. Jupiter has a large magnetosphere, as a result of strong magnetic field strength. This magnetosphere corotates with Jupiter and extends over all of Galilean satellites. The interaction between Jupiter's rotating magnetic field and Io causes plasma particles to flow along the magnetic field lines in directions toward both north and south hemispheres. Some particles will penetrate into Jupiter's ionosphere and collide with atmosphere particles, leading to aurora emission, at the position of Io's auroral footprint. Io is surrounded along its path, by a cloud of plasma particles with high density, which is called Io torus. This torus enhances the effect of bending magnetic field lines when they pass Io and result in inaccuracy of the prediction of longitudinal position of Io footprint. This shift of longitudinal prediction can be mapped to the shifted position of Io, which is called lead angle. Our objective is finding the relation between all three parameters, which are magnetic field strength, Io’s footprint brightness and lead angle at the same footprint position or the same Io’s longitude. We use VIPAL magnetic field model to trace along the magnetic field line and to find magnetic field strength at any given position. This tool is vital for determination of the relation between magnetic field strength, Io footprint brightness and lead angle.

1. Introduction
Jupiter is the biggest planet in our solar system both in mass and volume, $1,898 \times 10^{24}$ kg and $1.431 \times 10^{15}$ km$^3$ respectively [1]. It is beneficial to study the consequences from Jupiter's massive mass and size, which is much bigger than other planets.

Aurora is the phenomenon that occur in the planet that has sufficient atmosphere and strong internal magnetic field. It is electromagnetic wave emission from collision of high-energy charged particles with planet's atmospheric particles [2]. In this research, we focus on Io's auroral emission, which is caused by auroral particles that are originated from the vicinity of Io.

In Jupiter magnetosphere, magnetic field lines form as an enormous sphere that corotates with Jupiter. In order to determine the magnetosphere's structure, magnetic field models are created to simulate Jupiter's magnetic field. We use VIPAL magnetic field model [3] as our tool in this project. The name, VIPAL, refers to Voyager, Io footprint, Pioneer observations and modelling of the lowest orders.
of the magnetic anomaly and corrects for the longitudinal position of the magnetic field line mapping to Io's orbit.

The magnetic field strength varies between the magnetic field lines of Jupiter. Magnetic field is strong at the poles and weak around equator plane. Particles move from the equator which has the weakest magnetic field strength to the polar region, which has the strongest magnetic field strength. Experiencing stronger magnetic field strength along the path, some particles do not have enough parallel velocity to penetrate into ionosphere. Therefore they are reflected at the mirror point. This effect is called magnetic mirror. The magnetic field strength at Io's footprint position should cause lower footprint brightness due to this effect.

Due to high volcanic activity on Io, the plasma torus is formed around Io's orbit. The effect from Io torus can be defined as equatorial lead angle. Equatorial lead angle is the different of two Io longitudes, both corresponding to the same footprint but with and without Io torus. Because Io position in plasma sheet is inconsistent along the torus, the lead angle in each longitude will also be varied according to the changing of magnetic field bending in the torus. The lead angle therefore should have positive relation with footprint brightness due to the longer travel time in torus.

2. Methods

2.1. Tracing along the magnetic field line
Initially, the position of Io was put into VIPAL model. Next the model will give the magnetic field strength in three spherical coordinates, under assumption of static process. Because charge particles flow in the magnetic field direction, therefore from the magnetic field strength in each coordinate can be used to calculate the magnetic field direction at Io position. Accordingly, the magnetic field vector was altered to move a little to a new position. At new position, the new magnetic field direction was calculated and moved to the next position by a constant distance. Iterating this process, we continuously trace along the magnetic field line until we reach the end point in Jupiter's ionosphere, which is the position of Io footprint. Furthermore, we completed the footpath by repeating the process with various positions of Io. The magnetic field direction of Jupiter can be traced from Io to either north or south poles. Therefore, if we trace one magnetic field direction, and reach south hemisphere footprint, tracing opposite direction would reach the north hemisphere footprint.

2.2. Finding Equatorial Lead Angle
First we acquire footprint positions from tracing model, in this research we calculated 3600 points in 360 degrees. Therefore we have 0.1 degree precision in our calculation. To find the lead angle at specific Io position, first we located the modeled footprint which has the closest distance to the observed footprint corresponding to given Io position. From given Io's footprint, we acquire the associated modeled longitude of Io. The modeled longitude was subtracted by actual longitude of Io to obtain the lead angle. Finally the process was iterated to find the lead angles from other longitudes of Io.

3. Results and Discussion

3.1. Correlation between lead angle and footprint brightness
The relation between Io's footprint brightness in kilorayleighs ($1 \text{kR} = 10^9 \text{photon cm}^{-2}\text{s}^{-1} \text{into} 4\pi\text{steradians} [2]$) according to previous work [4] and Io’s lead angles from both hemispheres, based on the observed data and tracing model, are shown in figure 1 and 2.
Figure 1 shows the relation and correlation coefficients between lead angle and observed Io's footprint brightness in (a) North pole and (b) South pole.

Figure 2 shows relation and correlations coefficients between lead angle and Io's footprint brightness based on VIPAL magnetic field model for footprint in (a) North pole and (b) South pole.

Based on Hubble space telescope's observations of Jupiter auroral region in 2007, Io's footprint brightness variation [4] was detected. Accordingly, results in Figure 1 and Figure 2 are constrained by the same observed Io's longitudes. Therefore, the data may not span over all 360 degrees due to the limited number of observations. We have 2 sets of lead angle data, one from the observed data and one from VIPAL magnetic field model [3]. Both calculations refer to the same observed brightness and Io's longitudes. We obtained the correlation coefficients between lead angle and footprint brightness in each hemisphere from 3 different correlation formulas. All three correlations are Pearson product moment, Spearman's rank-order and Kendall's Tau [5], as shown in figure 1 and Figure 2. For observation data in both hemispheres, all three correlation coefficients suggest that Io's footprint brightness and lead angle have a positive relation between each other, although, the correlation coefficients are very low. From lead angle analysis by previous work [3], all three correlation coefficients suggest the same positive relation as our observed lead angle data, but with higher correlation coefficients in all three types and in both hemispheres. We can conclude from the correlation coefficients between lead angle and footprint brightness that they have slight positive relation to each other. These correlation coefficients support our hypothesis that the more lead angle means the further distance of bending magnetic field line along the Io torus. Hence more of picked-up auroral particles could be the reason for brighter auroral emission.

3.2. Correlation between footprint brightness and magnetic field strength
The relation between observed footprint brightness and magnetic field strength in both hemispheres is shown in figure 3.

**Figure 3** shows relation and correlation coefficients between magnetic field strength and observed Io’s footprint brightness.

Io’s footprint brightness and magnetic field strength at the footprint in Figure 3 are constrained by the same positions on Jupiter’s surface. That is the position of observed footprint for the corresponding position of Io. We find correlation coefficients between footprint brightness and magnetic field strength in each hemisphere from 3 different correlation coefficients, which are Pearson product moment, Spearman’s rank-order and Kendall’s Tao as shown in Figure 3. In the north hemisphere, all correlation coefficients suggest positive relation between footprint brightness and magnetic field strength at footprint position. However, all three values indicate that the relation between these two are very low, the maximum value is Spearman rank-order is 0.297. In the south, all three correlation coefficients have the opposite trend in comparison to those of northern footprint. Pearson and Spearman correlation suggest very strong negative relation, -0.736 and -0.771 respectively. Kendall correlation suggest moderate negative value of -0.568. With disagreement between 2 sets of data, we conclude that the magnetic field and footprint brightness have low correlation between each other. These correlation coefficients suggest that the magnetic mirror have no significant effect on Io footprint brightness.

4. Conclusions
From different correlation coefficients in four data sets, we conclude that lead angle and Io’s footprint brightness has a positive relation to each other. On the other hand, correlation coefficients between footprint brightness and magnetic field strength show disagreement of correlation coefficient between both hemispheres. One explanation could be the negative relationship between magnetic field strength and the number of precipitating auroral particles. It is clear that, while magnetic field strength and magnetic field configuration can play a controlling role in the footprint brightness, other influences should be taken into consideration for detail analysis.

5. Acknowledgments
We would like to thank Sebastien Hess at LESIA, Observatoire de Paris, France, for the use of VIPAL magnetic field model. Additional support was from Thailand Research Fund grant TRG5880091.

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