FILTERED MULTIPLICATIVE BASES OF RESTRICTED ENVELOPING ALGEBRAS

V. BOVDI, A. GRISHKOV, S. SICILIANO

Abstract. We study the problem of the existence of filtered multiplicative bases of a restricted enveloping algebra $u(L)$, where $L$ is a finite-dimensional and $p$-nilpotent restricted Lie algebra over a field of positive characteristic $p$.

1. Introduction and results

Let $A$ be a finite-dimensional associative algebra over a field $F$. Denote by $\text{rad}(A)$ the Jacobson radical of $A$ and let $\mathcal{B}$ be an $F$–basis of $A$. Then $\mathcal{B}$ is called a filtered multiplicative basis (f.m. basis) of $A$ if the following properties hold:

(i) for every $b_1, b_2 \in \mathcal{B}$ either $b_1 b_2 = 0$ or $b_1 b_2 \in \mathcal{B}$;
(ii) $\mathcal{B} \cap \text{rad}(A)$ is an $F$–basis of $\text{rad}(A)$.

Filtered multiplicative bases arise in the theory of representation of associative algebras and were introduced by H. Kupisch in [10]. In their celebrated paper [4] R. Bautista, P. Gabriel, A. Roiter and L. Salmeron proved that if $A$ has finite representation type (that is, there are only finitely many isomorphism classes of finite-dimensional indecomposable $A$–modules) over an algebraically closed field $F$, then $A$ has an f.m. basis.

In [9] an analogous statement was proposed for finitely spaced modules over an aggregate. (Such modules give rise to a matrix problem in which the allowed column transformations are determined by the module structure, the row transformations are arbitrary, and the number of canonical matrices is finite). This statement was subsequently proved in [13].

The problem of existence of an f.m. basis in a group algebra was posed in [4] and has been considered by several authors: see e.g. [2, 3, 5, 6, 7, 11]. In particular, it is still an open problem whether a group algebra $KG$ has an f.m. basis in the case when $F$ is a field of odd characteristic $p$ and $G$ is a nonabelian $p$-group.

Apparently, not much is known about the same problem in the setting of restricted enveloping algebras. The present paper represents a contribution in this direction. In particular, because of the analogy with the theory of finite $p$-groups, we confine our attention to the class $\mathfrak{F}_p$ of finite-dimensional and $p$-nilpotent restricted Lie algebras over a field of positive characteristic $p$. Note that under this assumption, the aforementioned result in [4] can be applied only in very special cases. Indeed, for $L \in \mathfrak{F}_p$, from [8] it follows that $u(L)$ has finite representation type if and only if $L$ is cyclic, that is, there exists an element which generates $L$ as a restricted subalgebra.

Key words and phrases. Filtered multiplicative basis, restricted enveloping algebra.
2000 Mathematics Subject Classification. Primary 16S30-17B50.

The research was supported by OTKA No.K68383, RFFI 07-01-00392A, FAPESP and CNPq(Brazil).
Theorem 1. Let $L \in \mathfrak{A}_p$ be an abelian restricted Lie algebra over a field $F$. Then $u(L)$ has a filtered multiplicative basis if and only if $L$ decomposes as a direct sum of cyclic restricted subalgebras. In particular, if $F$ is a perfect field, then $u(L)$ has a filtered multiplicative basis.

A restricted Lie algebra $L \in \mathfrak{A}_p$ is called powerful (see e.g. [15]) if $p = 2$ and $L' \subseteq L^{[p]}$ or $p > 2$ and $L' \subseteq L^{[p]}$. Here $L^{[p]}$ denotes the restricted subalgebra generated by the elements $x^{[p]}$, $x \in L$.

Theorem 2. Let $L \in \mathfrak{A}_p$ be a nonabelian restricted Lie algebra over a field $F$. If $L$ is powerful then $u(L)$ does not have a filtered multiplicative basis.

Theorem 3. Let $L \in \mathfrak{A}_p$ be a restricted Lie algebra over a field $F$. If $L$ has nilpotency class 2 and $p > 2$ then $u(L)$ does not have a filtered multiplicative basis.

An example showing that Theorem 3 fails in characteristic 2 is also provided. Finally, we remark that for odd $p$ no example of noncommutative restricted enveloping algebra having an f.m. basis seems to be known.

2. Preliminaries

Let $A$ be a finite–dimensional associative algebra over a field $F$. If $\mathfrak{B}$ is an f.m. basis of $A$ then the following simple properties hold (see [14]):

(F-I) $\mathfrak{B} \cap \operatorname{rad}(A)^n$ is an $F$–basis of $\operatorname{rad}(A)^n$ for every $n \geq 1$;

(F-II) if $u, v \in \mathfrak{B} \setminus \operatorname{rad}(A)^k$ and $u \equiv v \pmod{\operatorname{rad}(A)^k}$ then $u = v$.

Let $L$ be a restricted Lie algebra over a field $F$ of characteristic $p > 0$ with a $p$–map $[p]$. We denote by $\omega(L)$ the augmentation ideal of $u(L)$, that is, the associative ideal generated by $L$ in $u(L)$. In [13], the dimension subalgebras of $L$ were defined as the restricted ideals of $L$ given by

$$\mathfrak{D}_m(L) = L \cap \omega(L)^m \quad (m \geq 1).$$

These subalgebras can be explicitly described as $\mathfrak{D}_m(L) = \sum_{i \geq m} \gamma_i(L)^{[p]}$, where $\gamma_i(L)^{[p]}$ is the restricted subalgebra of $L$ generated by the set of $p^i$–th powers of the $i$–th term of the lower central series of $L$. Note that $\mathfrak{D}_2(L)$ coincides with the Frattini restricted subalgebra $\Phi(L)$ of $L$ (cf. [12]).

It is well-known that if $L$ is finite–dimensional and $p$–nilpotent then $\omega(L)$ is nilpotent. Clearly, in this case $\omega(L)$ coincides with $\operatorname{rad}(u(L))$ and $u(L) = F \cdot 1 \oplus \omega(L)$. Consequently, if $u(L)$ has an f.m. basis $\mathfrak{B}$, then we can assume without loss of generality that $1 \in \mathfrak{B}$. For each $x \in L$, the largest subscript $m$ such that $x \in \mathfrak{D}_m(L)$ is called the height of $x$ and is denoted by $\nu(x)$. The combination of Theorem 2.1 and Theorem 2.3 from [13] gives the following.

Lemma 1. Let $L \in \mathfrak{A}_p$ be a restricted Lie algebra over a field $F$, and let $\{x_i\}_{i \in I}$ be an ordered basis of $L$ chosen such that

$$\mathfrak{D}_m(L) = \operatorname{span}_F \{x_i \mid \nu(x_i) \geq m\} \quad (m \geq 1).$$

Then for each positive integer $n$ the following statements hold:

(i) $\omega(L)^n = \operatorname{span}_F \{x \mid \nu(x) \geq n\}$, where $x = x_{i_1}^{\alpha_1} \cdots x_{i_l}^{\alpha_l}$,

$$\nu(x) = \sum_{j=1}^l \alpha_j \nu(x_{i_j}), \quad i_1 < \cdots < i_l \quad \text{and} \quad 0 \leq \alpha_j \leq p - 1.$$
From it follows at once that 

\{a restricted subalgebra. We shall prove by induction on \n
Consequently the algebra 

\n
Proof of the Theorem 1. Assume first that \( L = \bigoplus_{i=1}^{n} \langle x_i \rangle_p \). Then, by the PBW Theorem for restricted Lie algebras (see [10], Chapter 2, Theorem 5.1), we see that \( u(L) \) is isomorphic to the truncated polynomial algebra 

\[ F[X_1, \ldots, X_n]/(X_1^{e(x_1)}, \ldots, X_n^{e(x_n)}) \]

Conversely, suppose that \( u(L) \) has an f.m. basis. 

Conversely, suppose that \( u(L) \) has an f.m. basis \( \mathfrak{B} \) with \( 1 \in \mathfrak{B} \) and put \( \mathfrak{B} = \mathfrak{B}_1 \setminus \{1\} \). Let \( n = \dim_F L/L[p] \) and \( \mathfrak{B} \backslash \omega(L)^2 = \{b_1, \ldots, b_n\} \). By Lemma 1 one has \( b_i = x_i + h_i \), where \( x_i \in L \backslash L[p] \) and \( h_i \in \omega(L)^2 \) for every \( i = 1, \ldots, n \). From [12] it follows at once that \( \{x_1, \ldots, x_n\} \) is a minimal set of generators of \( L \) as a restricted subalgebra. We shall prove by induction on \( e = \max\{i | L[p]^i \neq 0\} \) that \( L \) has a cyclic decomposition. If \( e = 1 \), then \( L = \bigoplus_{i=1}^{n} \langle x_i \rangle_p \).

Now let \( e > 1 \) and suppose that \( L \) does not decompose as a direct sum of restricted subalgebras. Since \( L = \sum_{i=1}^{n} \langle x_i \rangle_p \) and the \( p \)-map is \( p \)-semilinear, note that \( L[p]^e \) is just the vector subspace generated by the \( p^e \)-th powers of the generators \( x_i \) having exponent \( e+1 \). Therefore, without loss of generality we can assume that 

\[ e + 1 = e(x_1) = \cdots = e(x_m) \geq e(x_s), \quad (m + 1 \leq s \leq n) \]

and \( \{x_1^{[p]^e}, \ldots, x_m^{[p]^e}\} \) is an \( F \)-linearly independent set with 

\[ \text{span}_F \{x_1^{[p]^e}, \ldots, x_m^{[p]^e}\} = L^{[p]^e}. \]

In turn, we can reindex the elements \( x_{m+1}, \ldots, x_n \) so that there exists a maximal \( m \leq k < n \) such that 

\[ H = \langle x_1, \ldots, x_m, \ldots, x_k \rangle_p = \bigoplus_{i=1}^{k} \langle y_i \rangle_p \]

for suitable \( y_1, \ldots, y_k \) in \( L \) with \( y_i = x_i \) for \( i = 1, \ldots, m \). Consequently, for every \( s > k \) there exists a minimal number \( f_s \) such that 

\[ x_s^{p^f_s} = \sum_{i=1}^{k} \sum_{j=0}^{e(y_i) - 1} \mu_{1,i}^{(s)} y_i^{[p]^j} \mu_{i,j}^{(s)} (\mu_{i,j}^{(s)} \in F). \]  

(1)

Denote by \( J \) the associative ideal of \( u(L) \) generated by the elements \( b_1^{[p]^e}, \ldots, b_m^{[p]^e} \). Clearly \( J \subseteq \omega(L)^{[p]^e} \subseteq L^{[p]^e} u(L) \). Suppose by contradiction that \( J \neq L^{[p]^e} u(L) \). If \( r \) is the maximal positive integer such that 

\[ \left( L^{[p]^e} u(L) \cap \omega(L)^r \right) \setminus J \neq \emptyset, \]

then there exists \( v = x_1^{p^a_1} \cdots x_n^{p^a_n} \in \omega(L)^r \setminus J \) such that 

\[ v \equiv b_1^{p^a_1} \cdots b_m^{p^a_m} \pmod{\omega(L)^{r+1}}. \]
Consequently
\[ v - b_1^{\alpha_1} \cdots b_m^{\alpha_m} \in \left( L^{[p]^r} u(L) \cap \omega(L)^{r+1} \right) \setminus J, \]
contradicting the definition of \( r \). Therefore \( J = L^{[p]^r} u(L) \), which implies that \( u(L)/J \cong u(L/L^{[p]^r}) \). Moreover, it is easily seen that \( \mathfrak{B}_1 \cap J \) is an \( F \)-basis of \( J \), hence the elements \( b_i + J \) with \( b_i \notin J \) form an f.m. basis of \( u(L)/J \). Consequently, by induction we have that \( \mathcal{L} = L/L^{[p]^r} \) is a direct sum of restricted subalgebras.

As the images of \( y_1, \ldots, y_k \) are \( F \)-linearly independent in \( \mathcal{L}/\Phi(\mathcal{L}) \), from \([12]\) it follows that there exists a restricted subalgebra \( P \) of \( L \) with \( L^{[p]^r} \subseteq P \) such that
\[ \mathcal{L} = H/L^{[p]^r} \oplus P/L^{[p]^r}. \]
As a consequence, for every \( s > k \) we have \( x_s = v_s + w_s \) (mod \( L^{[p]^r} \)) with \( v_s \in H \) and \( w_s \in P \) and, moreover, it follows from \([1]\) that \( w_s^{[p]^{fs}} \in L^{[p]^s} \).

One has
\[ v_s = \sum_{i=1}^{k} \sum_{j=0}^{e(y_i) - 1} k_{i,j}^{(s)} b_i^{[p]^j}, \quad (k_{i,j}^{(s)} \in F). \]

Since \( x_s^{[p]^{fs}} \equiv v_s^{[p]^{fs}} \) (mod \( L^{[p]^r} \)), we conclude that \( \mu_{i,j}^{(s)} \in F^{p^{fs}} \) provided \( j < e \). We claim that for every \( 1 \leq i \leq k \) the coefficient \( \mu_{i,e}^{(s)} \) is also in \( F^{p^{fs}} \). Indeed, write
\[ w_s = \sum_{b \in \mathfrak{B}} \lambda_b b \quad \text{(for suitable \( \lambda_b \in F \)).} \]

Then, as \( \mathfrak{B} \) is a filtered multiplicative basis of \( u(L) \), it follows that
\[ w_s^{[p]^{fs}} = \sum_{b \in \mathcal{C}} \mu_b^{[p]^{fs}} b^{p^{fs}}. \]

where \( \mathcal{C} \) is a subset of \( \mathfrak{B} \) and the \( \mu_b \)'s are nonzero elements of \( F \). Moreover, since \( w_s^{[p]^{fs}} \in L^{[p]^s} \) we have
\[ w_s^{[p]^{fs}} = \sum_{i=1}^{m} \alpha_i b_i^{[p]^s}. \]

As \( \mathfrak{B} \) is a filtered \( F \)-basis of \( u(L) \), by comparing \([1]\) and \([2]\) we conclude that for every \( i = 1, 2, \ldots, m \) there exists \( \beta_i \in F \) such that \( \alpha_i = \beta_i^{p^{fs}} \). At this stage, the relations \([1]\) and \([2]\) allows to conclude that for every \( 1 \leq i \leq k \) one has
\[ \mu_{i,e}^{(s)} = \left( k_{i,e}^{(s)} \right)^{p^{fs}} + \beta_i^{p^{fs}} \in F^{p^{fs}}, \]
as desiderate (here \( \beta_{m+1}, \ldots, \beta_k = 0 \)).

Now, by \([1]\) and the above discussion we have \( x_s^{[p]^{fs}} = z^{p^{fs}} \) for some \( z \in H \). Therefore \( (x_s - z)^{p^{fs}} = 0 \) and then the minimality of \( f_s \) forces \( (x_s - z)_p \cap H = 0 \). This contradicts the definition of \( k \), yielding the claim.

Finally, if \( F \) is perfect, then \( L \) decomposes as a direct sum of cyclic restricted subalgebras (see e.g. \([1]\), Chapter 4, Theorem in Section 3.1). The proof is done. \( \Box \)
Unlike group algebras, a commutative restricted enveloping algebra need not have an f.m. basis. Indeed, we have the following

**Example.** Let $F$ be a field of positive characteristic $p$ containing an element $\alpha$ which is not a $p$-th root in $F$. Consider the abelian restricted Lie algebra
\[
L_{\alpha} = Fx + Fy + Fz
\]
with $x^p = \alpha z$, $y^p = z$, and $z^p = 0$. Suppose that $u(L_{\alpha})$ has an f.m. basis. By Theorem 1, $L_{\alpha}$ is a direct sum of cyclic restricted subalgebras. Since $L_{\alpha}^p \neq 0$ and $L_{\alpha}^{[p]} = 0$, we have $L_{\alpha} = \langle a \rangle_p \oplus \langle b \rangle_p$ with $c(a) = 2$ and $c(b) = 1$.

Let $b = k_1 x + k_2 y + k_3 z$, $k_i \in F$. Since $\alpha \notin F^p$, we get $0 = b^p = (k_1^p \alpha + k_2^p)z$, so $k_1 = k_2 = 0$ and $0 \neq a^p \in Fz = \langle b \rangle_p$, a contradiction.

**Lemma 2.** Let $A$ be a finite-dimensional nilpotent associative algebra over a field $F$. Suppose that $A$ has a minimal set of generators $\{u_1, \ldots, u_n\}$ such that:

(i) $[u_i, u_j] \in A^3$ for every $i, j = 1, \ldots, n$;
(ii) $u_iu_j \notin A^3$ for every $i, j = 1, \ldots, n$;
(iii) $\text{span}_F\{u_iu_j | 1 \leq i < j \leq n\} \cap \text{span}_F\{u_i^3 | i = 1, \ldots, n\} \subseteq A^3$.

Then $A$ has no f.m. basis.

**Proof.** By contradiction, assume that there exists an f.m. basis $B$ of $A$. Clearly, we have $\dim_F A/A^2 = n$ and, by property (F-I), $B \setminus \omega(L)$ is a minimal set of generators of $A$ as an associative algebra. Write $B \setminus A^2 = \{b_1, \ldots, b_n\}$. Obviously
\[
b_k \equiv \sum_{i=1}^n \alpha_{ki}u_i \pmod{A^2}, \quad (\alpha_{ki} \in F)
\]
and the determinant of the matrix $M = (\alpha_{ki})$ is not zero. Now
\[
b_r b_s \equiv \sum_{i=1}^n \alpha_{ri}\alpha_{si}u_i^2 + \sum_{i, j=1 \atop i < j}^n (\alpha_{ri}\alpha_{sj} + \alpha_{rj}\alpha_{si})u_iu_j - \sum_{i, j=1 \atop i < j}^n \alpha_{rj}\alpha_{si}[u_i, u_j] \pmod{A^3}.
\]
By assumption (i) of the statement we have that $[u_i, u_j] \equiv 0 \pmod{A^3}$, so
\[
b_r b_s \equiv \sum_{i=1}^n \alpha_{ri}\alpha_{si}u_i^2 + \sum_{i, j=1 \atop i < j}^n (\alpha_{ri}\alpha_{sj} + \alpha_{rj}\alpha_{si})u_iu_j \pmod{A^3}.
\]
Suppose $b_r b_s \in A^3$ for some $r, s$. Because of (3) and the assumptions (ii) and (iii) of the statement we have $\alpha_{ri}\alpha_{si} = 0$ and $\alpha_{ri}\alpha_{sj} + \alpha_{si}\alpha_{rj} = 0$ for every $i, j$. It follows that $\alpha_{ri}\alpha_{sj} - \alpha_{si}\alpha_{rj} = 0$. Consequently, all of the order two minors formed by the $k$-th and $s$-th lines of the matrix $M$ are zero, which is impossible because $\det M \neq 0$. Hence $b_r b_s, b_r b_s \notin \omega(L)^3$ and $b_r b_s \equiv b_s b_r \pmod{\omega(L)^3}$ for every $r, s$. By property (F-II) of the f.m. bases we conclude that $b_r b_s = b_s b_r$. Thus $A$ is a commutative algebra, a contradiction. □
Proof of the Theorem 2. Let $S$ be a minimal set of generators of $L$ as a restricted Lie algebra. Then, as $L$ is powerful, by Lemma 1 we conclude that $S$ is a minimal set of the nilpotent associative algebra $\omega(L)$ satisfying the hypotheses of Lemma 2 and the claim follows. □

Proof of the Theorem 3. Suppose, by contradiction, that $u(L)$ has an f.m. basis $\mathfrak{B}_1$ with $1 \in \mathfrak{B}_1$, so that $\mathfrak{B} = \mathfrak{B}_1 \setminus \{1\}$ is an f.m. basis of $\omega(L) = \mathfrak{rad}(u(L))$. Put $n = \text{dim}_F \mathfrak{D}_1(L)/\mathfrak{D}_2(L)$ and write $\mathfrak{B} \omega(L)^2 = \{b_1, \ldots, b_n\}$. Consider an $F$–basis $B$ of $L$ as in the statement of Lemma 1 and let $u_1, \ldots, u_n$ be the elements of $B$ having height 1. Thus, by Lemma 1(ii), the set $\{u_j + \omega(L)^2| j = 1, \ldots, n\}$ forms an $F$–basis of $\omega(L)/\omega(L)^2$. Then, for every $k = 1, \ldots, n$ there exist $\alpha_{k1}, \ldots, \alpha_{kn} \in F$ such that

$$b_k = \sum_{i=1}^n \alpha_{ki} u_i \quad (\text{mod } \omega(L)^2), \quad (k = 1, \ldots, n).$$

Set $\bar{u}_k = \sum_{i=1}^n \alpha_{ki} u_i$. Plainly, $\{\bar{u}_1, \ldots, \bar{u}_n\}$ is an $F$–linearly independent set which generates $L$ as a restricted subalgebra.

Now, if $L$ is powerful then, by Theorem 2 $u(L)$ cannot have any f.m. basis, a contradiction. Therefore $L' \not\subseteq L^{[p]}$ and so there exist $1 \leq r < s \leq n$ such that the element $c_{rs} = [\bar{u}_r, \bar{u}_s]$ is not in $L^{[p]}$. Since $L$ is nilpotent of class 2, we have that

$$\mathfrak{D}_2(L) = L' + L^{[p]} \supset L^{[p]} = \mathfrak{D}_3(L),$$

hence $c_{rs}$ has height two. Furthermore, one has

$$b_s^2 \equiv \bar{u}_r^2 \quad (\text{mod } \omega(L)^3); \quad b_s b_r \equiv \bar{u}_r \bar{u}_s - c_{rs} \quad (\text{mod } \omega(L)^3).$$

Since $L$ is nilpotent of class 2, it follows that

$$b_r b_s^2 \equiv \bar{u}_r \bar{u}_s^2 \quad (\text{mod } \omega(L)^4);$$
$$b_s u_r b_s \equiv \bar{u}_r \bar{u}_s^2 - \bar{u}_s c_{rs} \quad (\text{mod } \omega(L)^4);$$
$$b_r^2 b_s \equiv \bar{u}_r \bar{u}_s^2 - [\bar{u}_r, \bar{u}_s] = \bar{u}_r \bar{u}_s^2 - 2\bar{u}_s c_{rs} \quad (\text{mod } \omega(L)^4).$$

Therefore the elements

$$v_1 = b_r b_s^2, \quad v_2 = b_r^2 b_s \quad \text{and} \quad v_3 = b_r b_s b_s$$

are $F$–linearly dependent modulo $\omega(L)^4$. In view of property (F-I),

$$(\mathfrak{B} \cap \omega(L)^3) \omega(L)^4$$

is an $F$–basis for $\omega(L)^3$ modulo $\omega(L)^4$. Consequently, it follows that either $v_i \in \omega(L)^4$ for some $i \in \{1, 2, 3\}$ or $v_j \equiv v_k (\text{mod } \omega(L)^4)$ for some $j, k \in \{1, 2, 3\}$. In each case we have a contradiction to Lemma 1 and the proof is complete. □

We remark that the previous result fails without the assumption on the characteristic of the ground field. Indeed, let $F$ be a field of characteristic 2 and consider the restricted Lie algebra $L = Fa + Fb +Fc$ with $[a, b] = c, [a, c] = [b, c] = 0$, and $a[2] = b[2] = c[2] = 0$. Then it is straightforward to show that

$$\{1, a, b, ab + c, ac, bc, abc\}$$

is an f.m. basis of $u(L)$.

Acknowledgement. The authors are grateful to the referee for pointing out a problem in the original proof of Theorem 1.
References

[1] Y. Bahturin, A. A. Mikhalev, V. M. Petrogradsky, and M. V. Zaicev. Infinite-dimensional Lie superalgebras, volume 7 of de Gruyter Expositions in Mathematics. Walter de Gruyter & Co., Berlin, 1992.
[2] Z. Balogh. On existing of filtered multiplicative bases in group algebras. Acta Math. Acad. Paedagog. Nyházi. (N.S.), 20(1):11–30 (electronic), 2004.
[3] Z. Balogh. Further results on a filtered multiplicative basis of group algebras. Math. Commun., 12(2):229–238, 2007.
[4] R. Bautista, P. Gabriel, A. V. Roĭter, and L. Salmerón. Representation-finite algebras and multiplicative bases. Invent. Math., 81(2):217–285, 1985.
[5] V. Bovdi. On a filtered multiplicative basis of group algebras. Arch. Math. (Basel), 74(2):81–88, 2000.
[6] V. Bovdi. On a filtered multiplicative bases of group algebras. II. Algebr. Represent. Theory, 6(3):353–368, 2003.
[7] V. Bovdi. Erratum to “On a filtered multiplicative basis...” [Arch. Math. (Basel) 74(2)(2008)81–88]. Arch. Math. (Basel), pages 1–5, 2010.
[8] J. Feldvoss and H. Strade. Restricted Lie algebras with bounded cohomology and related classes of algebras. Manuscripta Math., 74(1):47–67, 1992.
[9] P. Gabriel and A. V. Roĭter. Representations of finite-dimensional algebras. In Algebra, VIII, volume 73 of Encyclopaedia Math. Sci., pages 1–177. Springer, Berlin, 1992. With a chapter by B. Keller.
[10] H. Kupisch. Symmetrische Algebren mit endlich vielen unzerlegbaren Darstellungen. I. J. Reine Angew. Math., 219:1–25, 1965.
[11] P. Landrock and G. Michler. Block structure of the smallest Janko group. Math. Ann., 232(3):205–238, 1978.
[12] M. Lincoln and D. Towers. Frattini theory for restricted Lie algebras. Arch. Math. (Basel), 45(5):451–457, 1985.
[13] D. M. Riley and A. Shalev. Restricted Lie algebras and their envelopes. Canad. J. Math., 47(1):146–164, 1995.
[14] A. V. Roĭter and V. V. Sergeichuk. Existence of a multiplicative basis for a finitely spaced module over an aggregate. Ukrainian Math. J., 46(5):604–617 (1995), 1994.
[15] S. Siciliano and T. Weigel. On powerful and p-central restricted Lie algebras. Bull. Austral. Math. Soc., 75(1):27–44, 2007.
[16] H. Strade and R. Farnsteiner. Modular Lie algebras and their representations, volume 116 of Monographs and Textbooks in Pure and Applied Mathematics. Marcel Dekker Inc., New York, 1988.

VICTOR BOVDI,
University of Debrecen, H–4010 Debrecen, P.O. BOX 12, Hungary
E-mail address: vbodvi@math.klte.hu

ALEXANDER GRISHKOV
IME, USP, Rua do Matao, 1010 – CIDADE UNIVERSITÁRIA, CEP 05508-090, SAO PAULO, BRAZIL
E-mail address: shuragri@gmail.com

SALVATORE SICILIANO,
Dipartimento di Matematica “E. De Giorgi”, Università del Salento, Via Provinciale Lecce–Arnesano, 73100–LECCE, ITALY
E-mail address: salvatore.siciliano@unile.it