What can we learn from TMD measurements?¹

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Abstract. Transverse-momentum-dependent parton distribution and fragmentation functions describe the partonic structure of the nucleon in a three-dimensional momentum space. They are subjects of flourishing theoretical and experimental activity. They provide novel and intriguing information on hadronic structure, including evidence of the presence of partonic orbital angular momentum.

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TMDs is an acronym for Transverse Momentum Distributions or Transverse Momentum Dependent parton distribution functions, also called unintegrated parton distribution functions. Most of our knowledge of the inner structure of nucleons is encoded in parton distribution functions (PDFs). We introduce them to describe hard scattering processes involving nucleons. The presence of a hard probe in these processes — e.g., in DIS the photon with virtuality $Q^2$ — identifies a longitudinal direction, and a plane perpendicular to that, the transverse plane. Intuitively, standard collinear PDFs describe the probability to find in a fast-moving nucleon a parton with a specific fraction of the nucleon’s longitudinal momentum. TMDs describe also the probability that the parton has a specific transverse momentum. They are therefore a natural extension of standard PDFs from one to three dimensions in momentum space.

Although useful from the intuition point of view, the probabilistic interpretation of PDFs and TMDs has some technical problems and is not strictly needed [1]. What is essential is that PDFs and TMDs can be defined in a formally clear way, through the application of factorization theorems. They reveal crucial aspects of the dynamics of confined partons, they can be extracted from experimental data, and allow us to make prediction for hard-scattering experiments involving nucleons. In this sense, the information contained in TMDs is as important as that contained in standard PDFs.

The main difference between collinear PDFs and TMDs is that the latter do not appear in totally inclusive processes. For instance, they do not appear in totally inclusive DIS, but they are needed when semi-inclusive DIS is studied and the transverse momentum of one outgoing hadron, $P_{h\perp}$, is measured. They are necessary to describe Drell–Yan processes when the transverse momentum of the virtual photon, $q_T$, is measured.

Factorization for processes involving TMDs has been worked out explicitly at leading twist (twist 2) and one-loop order and argued to hold at all orders [2, 3]. For instance, in

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unpolarized semi-inclusive DIS we can measure the structure function $F_{UU,T}$, which in the region $P_{h\perp}^2 \ll Q^2$ can be expressed as [4]

$$F_{UU,T} = |H(x, z^{-1} \frac{\xi}{2}, \frac{1}{2}, \mu_F)|^2 \sum_a x e_a^2 \int d^2 p_T d^2 k_T d^2 l_T$$

$$\times \delta^{(2)}(p_T - k_T + l_T - P_{h\perp}/z) f^a_1(x, p_T^2; \xi, \mu_F) D^a_1(z, k_T^2; \xi, \mu_F) U(l_T^2; \mu_F). \quad (1)$$

Apart from the transverse-momentum-dependent PDFs and fragmentation functions, the formula contains the soft factor $U$, a nonperturbative and process-independent object.

For the specific case of unpolarized observables integrated over the azimuthal angle of the measured transverse momentum, the analysis is usually performed in $b$-space in the Collins–Soper–Sterman framework [5]. The region of $P_{h\perp}^2 \gg M^2$, or $b^2 \ll 1/M^2$, can be calculated perturbatively, but when $P_{h\perp}^2 \approx M^2$ a nonperturbative component has to be introduced and its parameters must be fitted to experimental data. This component is usually assumed to be a flavor-independent Gaussian [6].

At present, especially for azimuthally-dependent structure functions, phenomenological analyses are often carried out using the tree-level approximated expression

$$F_{UU,T} = \sum_a x e_a^2 \int d^2 p_T d^2 k_T \delta^{(2)}(p_T - k_T - P_{h\perp}/z) f^a_1(x, p_T^2) D^a_1(z, k_T^2). \quad (2)$$

Also in this case, the transverse-momentum dependence of the partonic functions is assumed to be a flavor-independent Gaussian [7]. The tree-level approximation and the Gaussian assumption are known to be inadequate at $P_{h\perp}^2 \gg M^2$, but they could still effectively describe the physics at $P_{h\perp}^2 \approx M^2$. Especially for low-energy experiments, this is where the bulk of the data is.

The definition of quark TMDs is [1, 3] (taking the example of the fully unpolarized distribution of a quark with flavor $a$)

$$f^a_1(x, p_T^2; \xi, \mu_F) = \int \frac{d^2 p_T d^2 k_T}{(2\pi)^4} e^{ip_T \xi} \langle P | \bar{\psi}^a(0) \gamma^+ \mathcal{L}(\pm \infty, 0) \gamma^\alpha \mathcal{L}(\pm \infty, \xi) \psi^a(\xi) | P \rangle \bigg|_{\xi^+=0}. \quad (3)$$

The Wilson lines, $\mathcal{L}$, guarantee the gauge invariance of the TMDs. They depend on the gauge vector $v$ and contain also components at infinity running in the transverse direction. A remarkable property of TMDs is that the detailed shape of the Wilson line is process-dependent. This immediately leads to the conclusion that TMDs are not universal. However, the situation is not hopeless and the predictive power of TMD factorization is not completely destroyed, for the following reasons

- For transverse-momentum-dependent fragmentation functions, the shape of the Wilson line appears to have no influence on physical observables [8].
- In SIDIS and Drell–Yan, the difference between the Wilson line consists in a simple direction reversal and leads to calculable effects, namely a simple sign reversal of all T-odd TMDs [9].
- In hadron–hadron collisions to hadrons, standard universality cannot be applied. It is however conceivable that only a manageable number of TMDs with distinct Wilson lines are needed, preserving part of the predictive power of the formalism [10].
TABLE 1. Twist-2 transverse-momentum-dependent distribution functions. The U,L,T correspond to unpolarized, longitudinally polarized and transversely polarized nucleons (rows) and quarks (columns). Functions in boldface survive transverse momentum integration. Functions in gray cells are T-odd.

- If we consider specific transverse-momentum-weighted observables instead of unintegrated observables, it should be possible to obtain factorized expressions in terms of transverse moments of TMDs multiplied by calculable, process-dependent factors [11].

Our understanding of TMDs and their extraction from data has made giant steps in the last years, thanks to new theoretical ideas and experimental measurements. In the near future, more experimental data are expected from HERMES, COMPASS, BELLE and JLab.

When the spin of the nucleon and that of the quark are taken into account, eight twist-2 functions can be introduced. They are listed in Tab. 1. As with collinear PDFs, extracting TMDs calls for global fits to semi-inclusive DIS, Drell–Yan, and $e^+e^-$-annihilation data. Care has to be taken when considering the peculiar universality properties of TMDs. At the moment, we have some information only about the two functions in the first column of the table: $f_1$ (unpolarized function) and $f_{1T}$ (Sivers function).

Ultimately, the knowledge of TMDs should allow us to build tomographic images of the inner structure of the nucleon in momentum space, complementary to the impact-parameter space tomography that can be achieved by studying generalized parton distribution functions (GPDs). An example of tomographical images of the nucleon based on a model calculation of TMDs [12] is presented in Fig. 1.

TMDs measurements should allow us to address some intriguing questions, e.g.,

- Are there differences between the TMDs of different quark flavors (and of gluons)? We know that collinear PDFs are different, not only in normalization, but also in shape. We can expect that also the transverse momentum distribution is different. See Ref. [13] for an example of an experimental analysis of this issue.
- How does the transverse momentum dependence change with $x$? Such a dependence has already been introduced to describe data at low $x$ [6].
- Does the transverse momentum dependence of fragmentation functions change for different quark flavors and different produced hadrons?
- Are there reasons to abandon a Gaussian ansatz? We know that this assumption fails at high transverse momentum, but there are no compelling reasons to take a Gaussian shape even for the low-transverse-momentum, nonperturbative region.
FIGURE 1. Momentum-space tomographic “images” of the up quarks in a nucleon obtained from a model calculation of TMDs [12]. The circle with the arrow indicates the nucleon and its spin orientation. The distortion in the lower panels is due to the Sivers function. In the future, it should be possible to reconstruct these images from experimental data.

The last item of the list is connected also to another fundamental issue that makes TMDs interesting, i.e., the observation of partonic orbital angular momentum. In non-relativistic quantum mechanics, it is well known that wavefunctions with orbital angular momentum vanish at zero momentum. This is a general statement independent of the specific potential in which the wavefunction is computed. This feature is reflected also in TMDs: contributions from partons with nonzero angular momentum have to vanish at zero transverse momentum (and therefore cannot be described by a simple Gaussian). In general, a downturn of a TMD going to zero transverse momentum signals the presence of nonzero orbital angular momentum. While this effects could barely be visible in unpolarized TMDs, certain combinations of polarized TMDs could filter out more clearly the configurations with nonzero orbital angular momentum. Fig. (2) shows an example of this phenomenon, using a model calculation for illustration purposes.

Apart from the details of their shape, all the TMDs that are not boldface in Tab. 1 vanish in the absence of orbital angular momentum due to angular momentum conservation. Measuring any one of them to be nonzero is already an unmistakable indication of the presence of partonic orbital angular momentum. We know already from other sources (in particular the measurement of nucleons’ anomalous magnetic moments) that partonic orbital angular momentum is not zero, however TMDs have the advantage that
FIGURE 2. An illustration of how the presence of orbital angular momentum can influence the shape of TMDs. The model calculation shows different combinations of the $f_1$ and $g_1$ TMDs for $u$ and $d$ quark at $x = 0.02$. The downturns for $p_T^2 \to 0$ are due to the presence of orbital angular momentum.

they can be flavor-separated and that they are $x$ dependent. Thus, they allow us to say if orbital angular momentum is present for each quark flavor and for gluons, and at each value of $x$.

If stating that a fraction of partons have nonzero orbital angular momentum is relatively simple, it is not easy to make a quantitative estimate of the net partonic orbital angular momentum using TMDs. Any statement in this direction is bound to be model-dependent. Generally speaking, TMDs have to be computed in a model and the parameters of the model have to be fixed to reproduce the TMDs extracted from data. Then, the total orbital angular momentum can be computed in the model. Unfortunately, it is possible that two models describe the data equally well, but give two different values for the total orbital angular momentum.

As an example of a procedure of this kind, let us take the measurement of the Sivers function. We know that the proper way to measure the quark total angular momentum is by measuring the combination [14]

$$ J^a = \int_0^1 dx \left( H^a(x,0,0) + E^a(x,0,0) \right), $$

where the definition of the generalized parton distribution $E$ in terms of light-cone wavefunctions is

$$ E(x,0,0) = \lim_{q_T \to 0} \left( -\frac{1}{q_x - iq_y} \int \frac{d^2 p_T}{16\pi^2} \left[ \psi_+^+(x,p_T) \psi_-^- (x,p_T + (1-x)q_T) 
+ \psi_-^-(x,p_T) \psi_+^+ (x,p_T + (1-x)q_T) \right] \right). $$

On the other hand, the definition of the Sivers function in terms of light-cone wavefunctions can be written as

$$ f_{1T}^+ (x,p_T) = \frac{1}{16\pi^3} \text{Im} \left[ \psi_+^+(x,p_T) \psi_+^+ (x,p_T) + \psi_-^- (x,p_T) \psi_-^- (x,p_T) \right]. $$

In spite of the similarities between the two expressions and the fact that the same light-cone wavefunctions are involved, in general there is no straightforward connection
between the Sivers function and the GPD $E$ [15]. Nevertheless, in a certain class of spectator models it turns out that

$$f_{1T}^a(x) = -L(x)E^a(x,0,0).$$

Exploiting this very simple relation and using for illustration purposes the results of the Sivers function fit from Ref. [16] we obtain

$$
\frac{E^a(x,0,0)}{E^u(x,0,0)} = \frac{f_{1T}^a(x)}{f_{1T}^u(x)} = \frac{A_d}{A_u} \frac{f_1^a(x)}{f_1^u(x)},
$$

where

$$A_d/A_u = -1.8, \quad A_s/A_u = -1.1, \quad A_s/A_u = 1.3, \quad A_s/A_u = -4.8.$$ (9)

Although assumption-based, the above analysis shows that the measurement of the Sivers function can be used to give interesting constraints on the GPD $E$ and ultimately on the amount of total orbital angular momentum for each flavor.

In summary, TMDs open new dimensions in the exploration of the partonic structure of the nucleon. They require challenging extensions of the standard formalism used for collinear parton distribution functions, leading us to a deeper understanding of QCD. Among other things, they give evidence of the presence of partonic orbital angular momentum and, with model assumptions, they can help constraining its size.

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