Evidence for GeV Pair Halos around Low Redshift Blazars

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We report on the detection of GeV $\gamma$-ray pair halos around low redshift blazars observed by the Fermi Large Area Telescope using a number of a-priori selection criteria, including the spatial and spectral properties of the Fermi high-energy AGN-associated sources. The angular distribution of $\sim 1\, \text{GeV}$ photons around 24 stacked isolated high-synchrotron-peaked BL Lacs with redshift $z < 0.5$ shows a significant excess over that of point-like sources. A likelihood analysis yields a statistical significance $\sim 6\sigma$ for the extended emission against the point-source hypothesis, consistent with expectations for pair halos produced in the IGMF with strength $B_{\text{IGMF}} \sim 10^{-17} - 10^{-15}$ G.

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INTRODUCTION

The magnetic fields that are observed in galaxies and galaxy clusters are believed to result from the dynamo amplification of weak magnetic field seeds, whose origin remains a mystery. If these seed fields are cosmological in origin, the study of intergalactic magnetic fields (IGMFs) could provide a new window on early-universe cosmology and high-energy physics. IGMFs, deep in the voids between galaxies, provide the most accurate image of the weak primordial seed fields and they could be linked to the early stages in the evolution of the universe, perhaps in the inflationary epoch or from periods of cosmological phase transitions. Among the several methods used to study cosmological magnetic fields (see e.g. [1] for a recent review), the observation (or nondetection) of cascade emission from blazars can potentially measure very weak IGMFs. A number of blazars have been observed to emit both very-high-energy (VHE, > 100 GeV) $\gamma$-rays with ground-based $\gamma$-ray instruments and high-energy (HE, MeV/GeV) $\gamma$-rays with the Fermi Gamma-ray Space Telescope [2, 3]. Only the nearest sources have detectable TeV $\gamma$-ray emission since such high energy $\gamma$-rays cannot propagate over long distances in intergalactic space due to interactions with the extragalactic background light (EBL). Such interactions produce electron-positron pairs that are cooled by inverse Compton (IC) interactions with the Cosmic Microwave Background (CMB), ultimately leading to secondary GeV $\gamma$-ray emission from these pair cascades. Since magnetic fields deflect the electron-positron pairs changing the angular distribution of cascade emission, searches for extended GeV emission around blazars provides one of the very few avenues for constraining the IGMF.

Due to the low GeV $\gamma$-ray flux from extragalactic sources, it is hard to examine the angular extent of the photon events from a single blazar. To overcome this limitation, stacking sources has been used to make such statistical analysis feasible. Despite early hints at a signal [4], by comparing with the GeV emission from the Crab Nebula (which is essentially a point source for the Fermi Large Area Telescope (Fermi-LAT)), A. Neronov et al. [5] found no significant evidence of extended emission and argued that the apparent excess could be attributed to an underestimation of the real PSF [6]. A subsequent analysis by Ackermann et al. [7] compared an updated PSF to one hundred stacked BL Lac AGNs and found no statistically significant halo-emission.

The cascade emission from individual blazars has also been studied by modeling the intrinsic TeV spectra and adopting EBL and cosmological microwave background (CMB) models. Fitting to TeV data from, e.g., VERITAS and HESS (e.g. [8, 9]), such studies yielded detailed predictions of the halo emission. As energies increase, the pair production occurs closer to the source, reducing the angular size of the cascade. Depending on the strength of the IGMF and the redshift of the source, the highest energy emission might not be resolved by the Fermi PSF. While at lower energies (especially for the nearest sources), the emission may be too diffuse to be readily detected. From these studies it follows that only a few blazars would have cascade emission that can be statistically detected through their angular profiles.

In our study, both spectral and spatial properties of the GeV cascade emission around blazars are analyzed by using the up-to-date 5-year Fermi-LAT data. We use our knowledge of the nature of TeV sources and both the energy and redshift dependence of halo emission to identify a-priori a subset of sources most likely to show significant emission, as well as a control population where resolved halo emission is least likely to appear. Before analyzing the data, we decided to combine data from 24 isolated high-synchrotron-peaked (HSP) BL Lacs to provide the best prospects for detection and adequate photon counting statistics. We subsequently combine data from this population, and the angular distributions of the stacked source are then determined in different energy ranges and
angular bins. A likelihood ratio test (LRT) is applied for each angular bin, and the Bayesian confidence intervals for the halo parameters (the angular size and halo fraction) are evaluated. From these confidence intervals we estimate the possible range of IGMF strength and discuss the implications of our results.

**DATA PREPARATION AND SELECTION CRITERIA FOR STACKING SOURCES**

We use the Fermi-LAT Pass 7 reprocessed data through February 2014: SOURCE class front-converted photon events are binned into four GeV-energy ranges: 1-1.58, 1.58-3.16, 3.16-10, and 10-100, where logarithmically spaced bins were chosen to roughly equalize counts, prior to a detailed analysis of the data. The source candidates are selected from the AGN associated sources in the Fermi-LAT High-Energy Catalog (1FHL [3]). The region of the Galactic disk and Fermi bubbles is excluded to avoid anisotropic background emission [10].

Data is also divided into angular bins to provide adequate statistics. Source bins of equal solid angle are set around the direction of the source, surrounded by a larger background bin with outer boundary of 5°. To reduce systematic errors from nearby sources, we require that no nearby sources are within 2° of the stacked sources and correct for the impact of any remaining nearby sources by defining an exclusion region of radius $\theta_{\text{cut}} (=2.3^\circ)$ about these sources; we account for these exclusion regions by assuming that the signal and background effective area is reduced in proportion to the excluded solid angle. All data selection criteria (including bin widths) were designed prior to determining the signal in an effort to maximize statistics and minimize systematics, and to avoid extra trials [10]. We choose different values for the size of the source bins $\theta_{\text{in}}$, depending on energy as summarized in Table 1, which are chosen to be greater than the 95% containment angle of the PSF in the corresponding energy range [11].

| Energy (GeV) | $\theta_{\text{in}}$  |
|-------------|-----------------------|
| 1-1.58      | $2.3^\circ$           |
| 1.58-3.16   | $1.6^\circ$           |
| 3.16-10     | $1^\circ$             |
| 10-100      | $0.8^\circ$           |

Assuming that the correlation length of the IGMF $\lambda_B$ is much greater than the mean free path for inverse Compton scattering, we estimate the typical size of a pair halo to be

$$\Theta(E_\gamma, z_s, B_0) \approx 9.2 \times 10^{-4} [1 + z_\gamma(E_\gamma, z_s)]^{-2} \times \left( \frac{E_\gamma}{100\text{GeV}} \right)^{-1} \left( \frac{B_0}{10^{-16}\text{G}} \right) \left[ \frac{d_s(E_\gamma, z_s)}{d_s(z_s)} \right],$$

where $E_\gamma$ is the energy of the cascade photon observed by Fermi, $z_s$ is the observed redshift of the source, and $B_0$ is the field strength at the present epoch. To get the estimate above, we followed the discussion in A. Neronov and D. V. Semikoz [12] (see also [13]), where $z_\gamma$ is the redshift of pair production, $d_s$ and $d_s$ are the comoving mean free path for pair production and the comoving distance to the source, respectively [10]. Given the finite Fermi PSF, it is quite unlikely to detect the extended emission from high-redshift sources. For example, an IGMF of $\sim 10^{-16}\text{G}$ would result in a halo of angular radius of $\sim 1.2^\circ$ at 1GeV for a source at $z = 0.3$ (given by Eq. 1). If the same source were located at $z = 0.8$, the halo size would decrease to $\sim 0.2^\circ$, which is much smaller than the Fermi PSF at 1GeV and would appear like a point source.

Both observational and theoretical arguments lead us to expect that HSP BL Lac objects are the most likely sources of the VHE $\gamma$-rays needed to produce the GeV cascades. For example, in [14, 15], we see a strong correlation of the occurrence of a HSP energy with TeV emission. This is naturally explained if the same population of VHE electrons that produce the X-ray synchrotron radiation also produce the TeV $\gamma$-rays by IC in the source region (e.g. AGN jets). For this study, we subdivide data into Flat Spectrum Radio Quasars (FSRQs), and BL Lac objects. Since the FSRQs are typically very distant sources with lower-energy synchrotron peaks (LSP), we expect these sources to lack observable GeV pair halos, serving as a control population.

**DISTRIBUTION OF THE GEV $\gamma$-RAYS AROUND STACKED BLAZARS**

We identify 24 HSP BL Lacs with redshift $z < 0.5$ that satisfy our selection criteria and we stack their photon events, as shown in Fig. (1a) ($\gamma$-ray counts map in 1 GeV-1.58 GeV). As a control population, 26 FSRQs (with any redshift) are also selected by the same spatial criteria (no nearby sources, no sources from the region of the Galactic plane or Fermi bubbles). To visualize the difference in the data for these populations, Fig. (1b) shows the difference of the $\gamma$-ray counts in 1 GeV-1.58 GeV between the two populations. The background-counts of these two stacked sources are calculated by averaging the counts in the background bin, and then subtracted from their total counts. To make the two populations comparable, the background-subtracted counts of the stacked FSRQs are normalized to the same level as that of the stacked BL Lacs at the center. We smooth the counts maps by using a Gaussian kernel with full width at half maximum (FWHM) of $1^\circ$, and subtract the normalized FSRQs’ counts from the BL Lacs’. In Fig. (1b), the difference map shows an excess of the $\gamma$-ray emission around the stacked BL Lacs over the stacked FSRQs, consistent with an extended $\gamma$-ray emission.

As evident in past searches for pair halos, a thorough
FIG. 1. γ-ray counts maps of the stacked sources in the 1GeV-1.58GeV energy bin. The large circles show the outer edge of the detection region. (a) Counts map of the 24 stacked low-redshift HSP BL Lacs. (b) Smoothed counts difference between the stacked BL Lacs and the center-normalized stacked FSRQs. Positive values indicate the BL Lacs’ counts is greater than the FSRQs'.

FIG. 2. Angular distribution of photon events around the stacked pulsars (Crab and Geminga), the 24 BL Lacs, and the 26 FSRQs, as shown in Fig. 2. The errors give the 95% confidence intervals of getting the number of counts in each angular bin. The angular profiles for stacked pulsars agree with the up-to-date PSFs in each energy range [10]. However, the normalized angular profiles of stacked BL Lacs have a lower scaled counts per unit solid angle in the inner regions (small θ), providing evidence for extended emission since the additional counts in the extended halo reduce the scaled counts at small angles after normalization. The deficit in counts at small θ (evidence for extended emission) is more significant at lower energy ranges, consistent with the expectation that the angular extent of the halo is larger at lower energies, as indicated in Eq. 1. In contrast, the angular profiles of the stacked FSRQs are indistinguishable from our surrogate point-source data from pulsars, as shown in Fig. 2.

STATISTICAL EVIDENCE FOR PAIR-HALO EMISSION AND ESTIMATION OF THE IGMF

To model the normalized angular profiles \( g(\theta) \), we use

\[
 g(\theta; f_{\text{halo}}, \Theta) = f_{\text{halo}}g_{\text{halo}}(\theta; \Theta) + (1 - f_{\text{halo}})g_{\text{psf}}(\theta),
\]

where \( f_{\text{halo}} \) is the fraction of the pair halo component, \( g_{\text{psf}}(\theta) \) is the PSF and \( g_{\text{halo}}(\theta; \Theta) \) is a Gaussian function of \( \theta \) in the small angle approximation convolved with the PSF. Then, the number of photon events in the \( i \)-th angular bin is estimated by

\[
 \lambda_i(f_{\text{halo}}, \Theta, \lambda_b,i, N^*) = (N^* g_i + \mu_b)\Omega_i,
\]

where \( g_i \) is the discrete value of the normalized angular distribution \( g(\theta) \) given by Eq. 2, \( N^* \) is a normalization factor, \( \mu_b \) is the assumed uniform background counts per unit solid angle, and \( \Omega_i \) is the solid angle of the \( i \)-th bin. For a given configuration of the angular bins, a set of estimators \( \{\lambda_i\} \) is a function of \( f_{\text{halo}}, \Theta, \mu_b, \) and \( N^* \).

Maximum likelihood estimation is used for the model fitting. The likelihood \( L \) is defined in the 4-dimensional space of the model parameters, \( x = (f_{\text{halo}}, \Theta, \mu_b, N^*) \), as the joint probability for a number of Poisson processes of getting a set of observed γ-ray counts in all the \( n \) angular bins \( \{N_i\} \) with \( N_{\text{bg}} \) background counts in the background bin:

\[
 L(x|\{N_i\}) = \left( \prod_{i=1}^{n} P(N_i|\lambda_i) \right) \times P(N_{\text{bg}}|\mu_b\Omega_{\text{bg}}),
\]
where \( \Omega_{\text{bg}} \) is the solid angle of the background bin. \( P(N|\lambda) \) denotes the Poisson probability of \( \lambda \) at \( N \).

To get the quantitative significance of the pair halo, we focus on the space of the two model parameters, \( f_{\text{halo}} \) and \( \Theta \). We must distinguish between two hypotheses in this space: the hypothesis of halo emission \( H_1 \) and the null hypothesis \( H_0 \), where \( H_0 \) denotes a pure point source where either \( f_{\text{halo}} = 0 \) or \( \Theta = 0 \), and for \( H_1 \), the two parameters are free. No restriction is applied on the other two parameters, \( \mu_b \) and \( N^* \). We define \( h_1(f_{\text{halo}}, \Theta) \) as a subset of \( H_1 \) for a given pair of \( f_{\text{halo}} \) and \( \Theta \). The values of the likelihood ratio \( \Lambda(f_{\text{halo}}, \Theta|N) = \sup\{L(h_1|N) : x \in h_1\}/\sup\{L(H_0|N) : x \in H_0\} \) are evaluated and displayed in two-dimensional \( (f_{\text{halo}}, \Theta) \)-space, where \( N \) denotes the set of observations \( \{N_i\} \), \( \sup \) is the supremum function.

Fig. 3 shows the likelihood ratio maps for the stacked BL Lacs (a) and the maps for the simulated point source (labeled PSF) with total number of events in each energy bin set to that of the stacked BL Lacs (b). From Fig. 3(a), we focus on the space of the two model parameters, \( f_{\text{halo}} \) and \( \Theta \). The values of the likelihood ratio \( \Lambda(f_{\text{halo}}, \Theta|N) = \sup\{L(h_1|N) : x \in h_1\}/\sup\{L(H_0|N) : x \in H_0\} \) are evaluated and displayed in two-dimensional \( (f_{\text{halo}}, \Theta) \)-space, where \( N \) denotes the set of observations \( \{N_i\} \), \( \sup \) is the supremum function. The likelihood ratio maps show peaks at non-zero \( f_{\text{halo}} \) and \( \Theta \) in the two lower energy bins (Fig. 3(b)). In the two higher energy bins \( [10] \), the highest likelihood appears close to the \( f_{\text{halo}} \) and \( \Theta \) axes (where the null model is located). The fact that the likelihood maps for the two higher energy bins are consistent with the null hypothesis matches our expectation based on the angular distribution measurements shown in Fig. 2 where no significant difference is seen between the profiles of stacked pulsars and stacked BL Lacs in the plots of the two higher energy bins. The fraction \( f_{\text{halo}} \) is shown to negatively depend on energy in Fig. 3 revealing that extended emission appears mainly at lower energies. From the distributions of the maximum values of the likelihood ratio, the pulsars are shown to appear as point sources for Fermi-LAT \( [10] \).

To determine the frequentist statistical significance of the result in Fig. 3(a), we perform a LRT by evaluating the test statistic \( TS = 2\log \Lambda \), where \( \Lambda = \sup\{L(H_1|N) : x \in H_1\}/\sup\{L(H_0|N) : x \in H_0\} \) is given by the maximum value in the figure. In our study, we simulated the distribution of the TS by using a Monte Carlo method based on the null hypothesis. The result shows that if the stacked source appears to be a point source given by the Fermi PSF, the significance (probability) of getting an observation of the stacked BL Lacs in 1 GeV-1.58 GeV is equivalent to the significance (probability) of getting a normal distributed sample at \( \sim 6\sigma \) \( [10] \). Alternatively, we calculate the Bayes factors \( [18, 19] \) to test the extended-emission hypothesis \( H_1 \) for a given value of \( \Theta \) against the null hypothesis \( H_0 \) \( [10] \). We find that in the 1 GeV-3.16 GeV, the common logarithm of the Bayes factors \( \log_{10}(B_{10}) > 1 \), showing strong evidence \( [18] \) for the hypothesis of extended emission against the null hypothesis. While at higher energies, the logarithm of the Bayes factors \( \log_{10}(B_{10}) < 0 \), in favor of the null hypothesis. From the Bayes factors, we obtain the values of \( \Theta \) given by the most likely hypothesis: \( \sim 1.2^\circ \) in the first energy range and \( \sim 1.4^\circ \) in the second energy range. Recalling Eq. 1 using the average redshift of the stacked BL Lacs \( \langle z \rangle \approx 0.23 \), the strength of IGMF is conservatively estimated to be in the range of \( B_{10}\text{IGMF} \sim 10^{-17} - 10^{-15}\text{G} \). These values are larger than the lower bound derived from observations of 1ES0347-121 in \( [8] \) and consistent with the results in \( [4, 20] \). Moreover, the typical size of the pair halo we obtain at 1 GeV is \( \sim 1.2^\circ \). Then, the energy dependence of the bending angle from Eq. 1 gives a corresponding typical halo size at 10 GeV of \( \sim 0.05^\circ \), which is consistent with the results above given by the Bayes factors that the stacked source appears to be point like at higher energies.

**DISCUSSION**

In this study we presented an analysis of the angular distribution of \( \gamma \)-rays from a subset of sources selected
a-priori to minimize systematics and maximize chances of finding spatially resolved halo emission. Most of the 2FGL sources have nearby sources (within 2°), which will contaminate the stacked angular profiles. Previous studies restricted the energy range to be greater than 1 GeV to limit the contamination. However, this criterion is only valid in stacking the brightest sources and analyzing their angular photon-distribution. While HSP BL lacs are the most likely halo sources, they are not the brightest sources for Fermi-LAT. Moreover, the containment angle of the PSF at 1 GeV is ~ 1°, large enough to still allow contamination from nearby sources for many of these AGNs.

Unlike previous studies, we consider the possibility of larger halos (Θ > 2°). Note that for the entire map, the modeled source counts leaking into the background region fall below one standard deviation of the background counts ~ √N_{bg}. Hence we are justified neglecting spillover into the background region. We choose not to use the Fermi diffuse background models in this study, because the empirical model could contain contributions from extended sources and other assumptions about the angular distribution of the emission from local sources.

Both f_halo and Θ depend on the energy. In this study, we find evidence of pair halos with a resolved angular extent at lower energies. Such results agree with the fact that lower energy electrons are deviated by larger angles for a given IGMF strength (e.g. [13]). Also, the observed pattern of the decreasing f_halo with energy (see Fig. 3) is consistent with previous simulations of source and halo spectra (e.g. [20]). The fraction f_halo for a single source could potentially be studied via its TeV-GeV spectrum by using numerical methods (e.g. [8]), but the limited statistics for any single source make this difficult, even with the complete 5-year Fermi data set.

Finally, given the limitations of the stacking-source method, only an average range of the IGMFs can be recovered. In particular, we potentially underestimate the strength of the IGMF, because our method is insensitive to very large pair halos, whose photon fluxes are too extended to be measured statistically. Since the maximum angular search window is limited by source confusion and other experimental factors, we can not provide as strong a constraint on the maximum allowed angular extent of the GeV γ-ray emission and the maximum field strength as we can on the minimum angular extent and field strength. Thus, the estimation of IGMFs in this study is still marginally consistent with the results from Tashiro et al. [21], in which the strength of the helical component of the IGMF is given as ~ 10^{-14}G by analyzing the Fermi extragalactic diffuse background.

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Supplemental Material

DETAILS OF DATA SELECTION

The angular distribution of the GeV emission is measured by counting the number of photon events in angular bins chosen as depicted in Figure S-1(a): source bins of equal solid angle are set around the direction of the source, surrounded by a larger background bin. Before measuring the angular distribution of the $\gamma$-ray emission around each source, the following assumptions are made: 1) The photon events are distributed with azimuthal symmetry with respect to the direction of the source; 2) The $\gamma$-ray flux in the background bin is assumed to be dominated by a diffuse background and can therefore be used to estimate the background in the source bins; 3) The background $\gamma$-ray flux is uniformly distributed in the detection region within all the angular bins.

Assumptions 2 and 3 indicate that we cannot distinguish the extended halo emission from the background emission in the background bin, even if the real flux of the halo component in the background region was large. Assumptions 2 and 3 are valid if we select the data outside the region of the Galactic plane and Fermi bubbles, and the sources are isolated so that the angular distribution of photon flux in the source region is not influenced by other nearby sources. The region of the Galactic disk and Fermi bubbles is excluded to avoid anisotropic background emission, as shown in Figure S-2. Moreover, [referring to Figure S-1(b)], we exclude sources for which the distance to their nearest source, $\theta_d$, falls below some cut value, $\theta_{cut}$, which is the minimum allowed angular distance from the candidate source to its nearby sources. If assumptions 2 and 3 are satisfied for both the candidate source and its nearby source with the same values of $\theta_{in}$ and $\theta_{out}$, the ideal uncontaminated angular distribution in the source bins of a candidate source requires $\theta_d > \theta_{in} + \theta_{cut}$ and $\theta_{cut} \geq \theta_{in}$. However, this proved to be an overly restrictive criterion, resulting in very few viable sources and limited statistics in the stacked photon counts. Instead, we choose $\theta_d > \theta_{cut} \geq \theta_{in}$ to ensure that the center of the source region is not contaminated, but require an additional solid angle exclusion region for cases where nearby sources overlap the background bin. From assumptions 2 and 3, we can correct for the impact of the nearby sources by defining an exclusion region of radius $\theta_{cut}$ about these sources, and accounting for these exclusion regions by assuming that the signal and background effective area is reduced in proportion to the excluded solid angle. All data selection criteria (including bin widths) were designed prior to determining the signal in an effort to maximize statistics and minimize systematics, and to avoid extra trials.

We use this simple method of cutting close sources, because more sophisticated methods of modeling nearby sources (e.g. using likelihood analysis) have built-in assumptions about the number of nearby sources, the spectrum of these sources, and the angular distribution of $\gamma$-ray emission, introducing additional trials and other potential biases that are very difficult to accurately quantify. We carefully determine the values of $\theta_{in}$, $\theta_{out}$, and $\theta_{cut}$ to define a conservative
exclusion region based on the PSF for even a very soft-spectrum source. We choose $\theta_{cut} = 2.3^\circ$, $\theta_{out} = 5^\circ$, and different values for $\theta_{in}$ depending on energy as summarized in Table I (see main text), which are chosen to be greater than the 95% containment angle of the PSF in the corresponding energy range [2]. For a given source candidate, we determine its nearby sources from Fermi Source Catalogues-2FGL [3] and 1FHL [1], decide whether it meets the isolated-source criteria, and, if so, cut the patches around its nearby sources within $\theta_{cut}$. The detection region of each source covers $\sim 10^\circ$ diameter in which the exposure of the Fermi-LAT cannot be neglected. We correct the counts per unit solid angle by using the Fermi exposure maps, and for each source, it is subsequently calibrated to the exposure level at the center of the detection region where the source is located. The calibrated counts are stacked, as illustrated in Figure S-1(c).

Angular Size of Pair Halos

Following the discussion in A. Neronov and D. V. Semikoz [4] (see also [5]), we derive the typical angular size of a pair halo as a function of the observed energy of cascade photons $E_\gamma$, the typical redshift of the source $z_s$, and the IGMF strength $B_0$ at the present epoch, given by

$$
\Theta(E_\gamma, z_s, B_0) \approx 9.2 \times 10^{-4} [1 + z_\gamma(E_\gamma, z_s)]^{-2} \left( \frac{E_\gamma}{100\text{GeV}} \right)^{-1} \left( \frac{B_0}{10^{-16}\text{G}} \right)^{-1} [d_{\gamma}(E_\gamma, z_s)]^{-1} [d_s(z_s)]^{-1},
$$

(S-1)

The basic geometry of propagation of the direct and cascade $\gamma$-rays from the source to the observer (see Fig. 3 in [4]) gives the typical opening angle of the cascade emission, within the small angle approximation, as

$$
\Theta = \frac{d_\gamma}{d_s} \delta,
$$

(S-2)

where $d_\gamma$ and $d_s$ are the commoving mean free path for pair production and the commoving distance to the source, respectively, and $\delta$ is the deflection angle of electron-positron pairs by the IGMF. Assuming that the correlation length of the IGMF $\lambda_B$ is much greater than the mean free path for IC scattering $D_e$, $\delta$ can be estimated as the ratio of $D_e$ and the Larmor radius of electron $R_L$ in the magnetic field, which are given by

$$
D_e \approx 10^{21} (1 + z_\gamma)^{-4} \left( \frac{E_e}{10^{16}\text{eV}} \right)^{-1} \text{m} \approx 2 \times 10^{21} (1 + z_\gamma)^{-4} \left( \frac{E_{\gamma_0}}{10^{16}\text{eV}} \right)^{-1} \text{m},
$$

(S-3)

$$
R_L \approx \frac{E_e}{ceB} \approx \frac{E_{\gamma_0}}{2ceB_0(1 + z_\gamma)} \approx 1.67 \times 10^{24} (1 + z_\gamma)^{-2} \left( \frac{E_{\gamma_0}}{10^{16}\text{eV}} \right) \left( \frac{B_0}{10^{-16}\text{G}} \right)^{-1} \text{m}.
$$

(S-4)

where we assumed that the pair produced electron/positron has half the energy of the initial photon, $E_e \approx E_{\gamma_0}/2$. We related the magnetic field at the time of the pair production and IC scattering, $B$, to the magnetic field today, $B_0$,
assuming that it is only affected by the $(1 + z_{\gamma\gamma})^2$ redshift evolution. The energy of cascade $\gamma$-rays after IC scattering, when they are observed on Earth, is given by

$$E_\gamma = \frac{4}{3}(1 + z_{\gamma\gamma})^{-1} \epsilon_{\text{CMB}} \left( \frac{E_\gamma}{m_e c^2} \right)^2.$$  \hfill (S-5)

Using the evolution of the CMB photons, $\epsilon_{\text{CMB}} = 6 \times 10^{-4}(1 + z_{\gamma\gamma})\text{eV}$, we obtain

$$E_\gamma = \frac{77\text{GeV}}{(10\text{TeV})^2}.$$  \hfill (S-6)

Hence, the deflection angle is given by

$$\delta = \frac{D_\gamma}{R_L} \approx 9.2 \times 10^{-4}(1 + z_{\gamma\gamma})^{-2} \left( \frac{E_\gamma}{100\text{GeV}} \right)^{-1} \left( \frac{B_0}{10^{-16}\text{G}} \right).$$  \hfill (S-7)

Eq. (S-1) can be obtained by substituting Eq. (S-7) into Eq. (S-2).

Assuming a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe, if a TeV photon is emitted at time $t_s$ from the AGN and pair produces on an EBL photon at time $t_{\gamma\gamma}$, the comoving distances $d_\gamma$ and $d_s$ are given by

$$d_\gamma = \int_{t_{\gamma\gamma}}^{t_s} \frac{c}{a(t)} \, dt,$$  \hfill (S-8)

$$d_s = \int_0^{t_s} \frac{c}{a(t)} \, dt,$$  \hfill (S-9)

where $a(t)$ is the scale factor at time $t$. For the redshifts of interest, we can take the universe as made of matter and cosmological constant only. Using the change of variables

$$\frac{dt}{dz} = \frac{1}{H(1 + z)},$$  \hfill (S-10)

we can express the comoving distance (Eq. (S-8) and (S-9)) in terms of the redshift of emission and pair production, $z_s$ and $z_{\gamma\gamma}$. Hence,

$$d_\gamma = \frac{c}{a_0 H_0} \int_{z_s}^{z_{\gamma\gamma}} \frac{dz}{\sqrt{\Omega_M (1 + z)^3 + \Omega_\Lambda}},$$  \hfill (S-11)

$$d_s = \frac{c}{a_0 H_0} \int_0^{z_s} \frac{dz}{\sqrt{\Omega_M (1 + z)^3 + \Omega_\Lambda}},$$  \hfill (S-12)

where $a(z) = a_0/(1 + z)$, and the Hubble parameter is given by $H = H_0 \sqrt{\Omega_M (1 + z)^3 + \Omega_\Lambda}$ for a flat matter-FLRW universe.

Note that $z_{\gamma\gamma}$ in Eq. (S-1) and Eq. (S-11) cannot be detected directly. Taking the expression of the mean free path (which assumes a redshift dependence of the EBL number density $\propto (1 + z)^{-2}$) from [4]:

$$D_\gamma \approx 80 \frac{\kappa}{(1 + z)^2} \left( \frac{E_{70}}{10\text{TeV}} \right)^{-1} \text{Mpc} \approx 80 \frac{\kappa}{(1 + z)^2} \left( \frac{E_\gamma}{77\text{GeV}} \right)^{-1/2} \text{Mpc},$$  \hfill (S-13)

where $\kappa \sim 1$ accounts for the EBL model uncertainties, and Eq. (S-13) is used to express the mean free path $D_\gamma$ in terms of $E_\gamma$. The optical depth of the $\gamma$-ray propagating from the source grows as

$$\frac{d\tau}{dt} = \frac{c}{D_\gamma(E_\gamma, z)}.$$  \hfill (S-14)

The time of pair production corresponds to when $\tau$ reaches 1, and can be found implicitly from

$$\int_{t_{\gamma\gamma}}^{t_s} \frac{c dt}{D_\gamma(E_\gamma, z)} = \int_{z_s}^{z_{\gamma\gamma}} \frac{c dz}{D_\gamma(E_\gamma, z)} = 1.$$  \hfill (S-15)
Eq. S-15 allows us to solve for \( z_{\gamma\gamma} (E_\gamma, z_s) \) implicitly in terms of \( E_\gamma \) and \( z_s \). We can then obtain \( d_s (z_{\gamma\gamma}, z_s) \) and \( d_s (z_s) \) from Eq. S-11 and Eq. S-12 and find the angular size of pair halos \( \Theta \) as a function of \( E_\gamma, z_s, \) and \( B_0 \) from Eq. S-1.

For large \( z_s \) and \( E_\gamma \), one can assume that \( z_{\gamma\gamma} \approx z_s \) \([4, 5]\), leading to

\[
\Theta \approx 1.5 \times 10^{-5} (1 + z_s)^{-3} \left( \frac{E_\gamma}{100 \text{GeV}} \right)^{-3/2} \left( \frac{B_0}{10^{-16} \text{G}} \right) \left( \int_0^{z_s} \frac{dz}{\sqrt{\Omega_M (1 + z)^3 + \Omega_\Lambda}} \right)^{-1}.
\]

(S-16)

However, this assumption is not true in general. In particular, it overestimates the angular extent of the pair halos around low redshift sources. Hence, we do not take this assumption when using Eq. S-16 to estimate \( B_0 \) from the most likely values of \( \Theta \).

From Eq. S-1, it is obvious that \( \Theta \propto B_0 \). The \( z_s \) and \( E_\gamma \) dependence of \( \Theta \), however, is not explicit. Figure S-3 shows the sensitivity of \( \Theta \) to various model parameters assuming an IGMF of \( B_0 = 10^{-16} \text{G} \) and using Eq. S-1. The \( z_s \) dependence of \( \Theta \) for \( E_\gamma = 1 \text{GeV} \) is shown in Figure S-3(a), and the \( E_\gamma \) dependence of \( \Theta \) for \( z_s = 0.2 \) is shown in Figure S-3(b). From Figure S-3(a), we find it is quite unlikely to detect the extended emission from high-redshift sources, supporting our selection criteria for stacking sources based on redshift. We can also find, from Figure S-3(b), that lower energy electrons are deflected by larger angles, consistent with the results we have obtained in this study (as discussed in the main text).

**FIG. S-3.** Redshift and energy dependence of the angular size of pair halos produced by IGMF of strength \( B_0 = 10^{-16} \text{G} \), given by Eq. S-1. (a) The \( z_s \) dependence of the typical angular size \( \Theta \) of pair halos at \( E_\gamma = 1 \text{GeV} \). (b) The \( E_\gamma \) dependence of the typical angular size \( \Theta \) of pair halos from AGNs with redshift \( z_s = 0.2 \).

**DETAILS OF THE MAXIMUM LIKELIHOOD ANALYSIS**

We choose the Crab and Geminga pulsars as our calibration sources since they are effective point sources for Fermi-LAT \([6, 7]\). Figure S-4 shows the angular distribution of photon events around the stacked pulsars and the angular distribution of PSFs calculated for the same observation times and the observed spectrum.

In our study, we evaluate the likelihood in the two-dimensional space of \( f_{\text{halo}} \) and \( \Theta \). The likelihood ratio maps show peaks at non-zero \( f_{\text{halo}} \) and \( \Theta \) in the two lower energy bins, and the likelihood maps for the two higher energy bins are consistent with the null hypothesis matches our expectation based on the angular distribution measurements, as shown in Figure S-5(a). Furthermore, we calculate the likelihood maps for the simulated point source (labeled PSF) with total number of events in each energy bin set to that of the stacked BL Lacs [Figure S-5(b)], as well as the likelihood maps for the stacked pulsars [Figure S-6(a)] and the corresponding simulated point source [Figure S-6(b)]. From the distributions of the maximum likelihood and the values of the likelihood ratio, the pulsars are shown to appear as point sources for Fermi-LAT.

**LIKELIHOOD RATIO TEST OF THE PAIR HALOS**

A classical likelihood ratio test (LRT) applied to this problem is potentially inaccurate since the probability distribution of the test statistic (TS) is nontrivial. Wilks theorem gives a useful approximation that a TS \( = 2 \ln \Lambda \) (the
FIG. S-4. Angular distribution of photon events around the stacked pulsars (error bars) and the angular distribution of PSFs calculated for the same observation times and the observed spectrum (diamonds): vertical errors are the 95% confidence intervals; horizontal errors show the size of angular bins.

FIG. S-5. Likelihood ratio maps. Colors show the ratio of the likelihood of extended-emission hypothesis to that of the null hypothesis (the PSF). (a) Likelihood ratio maps for stacked BL Lacs; (b) Likelihood ratio maps for a point source with angular distribution given by the PSF with total number of events in each energy bin set equal to that of the stacked BL Lacs.

likelihood ratio Λ as defined in the main text) will appear to be $\chi^2$-distributed. However, the theorem is only valid under certain conditions including restrictions on the sample population and the formulation of the hypotheses to be tested $\Lambda$. In this study, the set of the model parameters $x = (f_{\text{halo}}, \Theta, \mu_b, N^*)$ is not open and the null hypothesis is defined on the boundaries of the domain. In such a case, we cannot directly apply Wilks theorem to determine the distribution of the TS $\Lambda$. In the study of $\Lambda$, a Monte Carlo method is used to check the distribution of TS and a factor of $1/2$ is found in the resulting $\chi^2$-distribution because the null hypothesis stands on the symmetric boundaries of the parameter space. We also apply the Monte Carlo method to determine the distribution of the TS. We find that the probability distribution of the non-zero TS values lays between the distributions of $\chi^2_1$ and $\chi^2_2$, and the population of non-zero values is just half of the total, as shown in Figure S-7.

In our study, the maximum likelihood ratio gives the TS of $\sim 40$ [as shown in Fig. 3(a) in the main text], corresponding to a p-value of $\sim 10^{-9} - 10^{-10}$, which is constrained by the $\chi^2_1$ and $\chi^2_2$ distributions. This p-value
FIG. S-6. Likelihood ratio maps. Colors show the ratio of the likelihood of extended-emission hypothesis to that of the null hypothesis (the PSF). (a) Likelihood ratio maps for stacked pulsars; (b) Likelihood ratio maps for a point source with angular distribution given by the PSF with total number of events in each energy bin set equal to that of the stacked pulsars.

indicates a significance which is equivalent to the probability of getting a normal-distributed sample at \( \sim 6\sigma \). Note that we calculate this frequentist significance to supplement our more rigorous Bayesian analysis and to allow readers, unfamiliar with Bayesian statistics, to assess the strength of the evidence in more familiar terms.

**Bayes Factors of the Pair Halo Detection**

As mentioned above, we cannot provide an analytical function for the distribution of the TS values. Instead, we constrain the statistical significance by using two \( \chi^2 \)-distribution functions enveloping the simulated TS distribution. Alternatively, as suggested by [8], we can test the statistical significance of the extended emission by evaluating the Bayes factors [10].

The information about the IGMF is contained in the extended emission. Here we focus on the model factor \( \Theta \), and seek to get the quantitative significant range of its values for the stacked BL Lacs. We introduce a hypothesis of extended emission for a given \( \Theta^* \), \( H_1(\Theta^*) \), which is defined as a subset of \( H_1 \) (see the main text) for \( \Theta = \Theta^* \) with all possible values of \( f_{\text{halo}} \). The Bayes factors of \( H_1(\Theta^*) \) against the null hypothesis \( H_0 \) are given by:

\[
B_{10}(\Theta^*) = \frac{L_B(H_1|N)}{L_B(H_0|N)},
\]

where \( L_B \) is the Bayesian likelihood function. Applying Bayes theorem, the likelihood function is given by the Bayesian probability, which is obtained by integrating (not maximizing) over the parameter space [10]. Hence, for a hypothesis \( H_j(H_j = H_0, H_1) \),

\[
L_B(H_j|N) = \int dx \mathcal{P}(N|\lambda(x), H_j)\pi(x|H_j),
\]

where \( N \) is the set of observed counts \( \{N_i\} \) in the angular bins, \( x = (f_{\text{halo}}, \Theta, \mu_b, N^*) \) is the set of model parameters, and \( \lambda \) is the set of Poisson estimators \( \{\lambda_i\} \) given by the halo model. The posterior density in Eq. S-18 is given by
FIG. S-7. Probability distribution of the test statistic (TS), as shown by the crosses. Dashed line and the dot-dashed line are the chi-squared distributions, bounding the points (crosses) determined by Monte Carlo calculation.

the joint probability for a set of Poisson processes in the $n$ angular bins:

$$P(N|\lambda(x), H_j) = \prod_{i=1}^{n} P(N_i|\lambda_i),$$  \hspace{1cm} (S-19)

where $P(N|\lambda)$ denotes the Poisson distribution of $\lambda$ at $N$. The prior density $\pi(x|H_j)$ is the probability density for getting a set of model parameters $x$ with a hypothesis $H_j$. The total number of counts $N_{tot}$ in the detection region with angular size $\Omega_{tot}$ and the background counts $N_{bg}$ in the background bin with angular size $\Omega_{bg}$ are measured over a long-term observation in large angular regions (much larger than any single source bin). These measurements can help us to evaluate $\pi(x|H_j)$ if we assume that they are prior measurements: $N^* + \mu_b \cdot \Omega_{tot}$ is an estimation of $N_{tot}$, and $\mu_b \cdot \Omega_{bg}$ is an estimation of $N_{bg}$. The measurements of $N_{tot}$ and $N_{bg}$ can also be treated as Poisson experiments. Hence,

$$\pi(x|H_1) = P(N_{tot}|N^* + \mu_b \cdot \Omega_{tot})P(N_{bg}|\mu_b \cdot \Omega_{bg})\delta(\Theta - \Theta^*),$$  \hspace{1cm} (S-20)

$$\pi(x|H_0) = P(N_{tot}|N^* + \mu_b \cdot \Omega_{tot})P(N_{bg}|\mu_b \cdot \Omega_{bg})\delta(\Theta - 0),$$  \hspace{1cm} (S-21)

From Eq. S-19-S-21, the Bayesian likelihood in Eq. S-18 can be rewritten as

$$L_B(H_j|N) = \int_0^1 df_{halo} \int_{0}^{\infty} d\Theta \int_0^\infty d\mu_b \int_0^\infty dN^* \left( \prod_{i=1}^{n} P(N_i|\lambda_i(f_{halo}, \Theta, \mu_b, N^*)) \right) \times P(N_{tot}|N^* + \mu_b \cdot \Omega_{tot})P(N_{bg}|\mu_b \cdot \Omega_{bg})\delta(\Theta - \Theta^*).$$  \hspace{1cm} (S-22)

$L_B(H_0|N)$ is then just a special case of Eq. S-22 when $\Theta^* = 0$. The Bayes factors are calculated as a function of $\Theta^*$, as shown in Figure S-8. There is strong evidence against the null hypothesis $H_0$ if $\log_{10}(B_{10}) > 10$. We find that
in 1 GeV-3.16 GeV, the hypotheses of extended emission are strongly favored against the null hypothesis. While in the higher energy bins, the Bayes factors are smaller than 0, which provides evidence in favor of the null hypothesis.

![Graph showing probability distribution of the test statistic (TS)](image)

**FIG. S-8.** Probability distribution of the test statistic (TS), as shown by the crosses. Dashed line and the dot-dashed line are the chi-squared distributions, bounding the points (crosses) determined by Monte Carlo calculation.

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