WHAT NEXT IN FLAVOUR PHYSICS AND CP VIOLATION?

John Ellis

TH Division, CERN, CH 1211 Geneva 23, Switzerland
CERN-TH/2002-339 hep-ph/0211322

ABSTRACT

The future of flavour physics and CP violation in the quark, lepton and Higgs sectors are discussed, particularly from the viewpoint of physics beyond the Standard Model, such as supersymmetry. Current issues in $B \rightarrow \pi^+\pi^-, \phi K_s$ and $D^{+\ast}\phi\pi, B_s$ physics and rare $B$ decays are reviewed. The prospects for seeing flavour and CP violation in the charged-lepton sector are discussed, using the minimal supersymmetric seesaw model as a guide. Finally, the possible consequences of CP violation in the Higgs sector are mentioned.

1 Mea Maxima Culpa

The organizers have asked me to look towards the future, rather than summarize this meeting. Unfortunately, this is just as well, because commitments at CERN prevented me from attending most of the meeting. I am very sorry
that I missed many interesting subjects, such as factorization, $J/\psi$ production at RHIC, charmonium, heavy-quark effective theory, $b\bar{b}$ production, $b$-quark fragmentation, $W \to c\bar{s}$ decay, $B \to \ell\nu\gamma$, Re$(\epsilon'/\epsilon)$, $K_S \to \gamma\gamma$, CLEO-c, LHCb light, $Z \to b\bar{b}$, $\Delta G$, $x$, $D \to \sigma, \kappa$, flavour textures, $B \to \ell^+\ell^-$, $B_s - \bar{B}_s$ mixing, $D_0 - \bar{D}_0$ mixing, $\Delta \Gamma, \Gamma$, $\tau(D_s)/\tau(D_0)$, $\tau(\Xi^+)/\tau(\Lambda^+)$, the $1D_2(\bar{b}\bar{b})$ and many more .... For these reasons, I could not in any case present a balanced summary of the meeting.

2 A Personal Point of View

There are three preferred experimental arenas for probing flavour dynamics and CP violation: the quark sector - where both are well established, the lepton sector - where flavour mixing has been seen among the neutrinos and CP violation is expected, and the Higgs sector - about which we have no direct experimental information. Reflecting my personal bias, I assume in discussing these sectors that supersymmetry will appear at some accessible energy.

In the quark sector, dare we hope that that the current triumph of the standard Kobayashi-Maskawa (KM) model in predicting correctly the value of $\sin 2\beta$ observed in $B_0 \to J/\psi K_s$ decays may be short-lived? As discussed at this meeting, the first rounds of data on $B \to \pi^+\pi^-, \phi K_s$ and $D^{*+}D^{*-}$ decay asymmetries do not agree very well with the KM model. Might one of these be a harbinger of new physics, such as supersymmetry? Answers to the tough questions are still in the future: does the unitarity triangle close, or is it a quadrangle? New tools for analyzing flavour dynamics in the quark sector await us: what will $B_s$ physics or $b \to s\gamma, s\ell^+\ell^-$ tell us?

In the neutrino sector, many questions about neutrino masses and mixing remain unanswered: is the large-mixing-angle (LMA) solar solution correct? What is the value of $\theta_{13}$? Is there a CP-violating phase $\delta$? What are the absolute values of the neutrino masses? Beyond neutrinos, in the presence of low-energy supersymmetry we may expect a new flavour frontier to open up among the charged leptons: will $\mu \to e\gamma, \tau \to e/\mu\gamma, \mu \to 3e$ and $\tau \to 3\ell$ be observable? Do the electron and muon have measurable CP-violating electric dipole moments? What is the relation to leptogenesis?

The final frontier for studies of flavour dynamics and CP violation may be the Higgs sector, which is their origin in the Standard Model (SM). In the minimal supersymmetric extension of the Standard Model (MSSM), the masses,
mixings and couplings of the physical Higgs bosons may exhibit observable
flavour- and CP-violating effects.

3 Roadmap to Physics Beyond the Standard Model

Let us first set flavour dynamics in the general context of physics beyond the
SM.

The standard list of problems beyond the SM includes those of Unification
- can one find a single simple framework for all the gauge interactions? Flavour
- why so many different types of quarks and leptons and what explains their
patterns of mixing and CP violation? and Mass - do particle masses really
originate from a Higgs boson, and if so why are they so small, where there may
be a rôle for supersymmetry? Beyond all these ‘beyonds’ there is the quest for
a Theory of Everything, capable of reconciling gravity with quantum mechanics
as well as solving all the above problems, perhaps via superstring or M theory?

At what energy scales might appear these examples of new physics? LEP
told us that they cannot appear below 100 GeV, and quantum gravity must
become strong by $10^{19}$ GeV at the latest. Within this range, we believe that
the problem of mass must be resolved at some energy below about 1 TeV, by
the discovery of a Higgs boson and/or supersymmetry. Measurements of gauge
couplings give circumstantial support to supersymmetric grand unification at
around $10^{16}$ GeV with sparticles appearing around 1 TeV. However, we have
little, if any, idea of the scale at which the flavour problem may be solved.
Perhaps only at the quantum-gravity scale $\sim 10^{19}$ GeV? perhaps at the GUT
scale $\sim 10^{16}$ GeV? perhaps at some intermediate scale, as suggested by the
seesaw model of neutrino masses? perhaps at the TeV scale? How far along
the road will we solve flavour dynamics and find the origin of CP violation?

4 Milestones in CP Violation

Our progress along this road can be measured by a plethora of milestones.
Long after its discovery in the $K^0$ mass matrix via $K^0 \rightarrow \pi^+\pi^-$ decay, we have
only recently passed two important ones:

- The measurement of direct CP violation in $K^0 \rightarrow 2\pi$ decay amplitudes $\bar{\mathcal{B}}$, as long predicted in the KM model $\mathcal{B}$,
Figure 1: A global fit to the unitarity triangle, demonstrating good agreement with the measurements of \( \sin 2\beta \) by BaBar and Belle.

- Observation of CP violation elsewhere, namely in \( B^0 \to J/\psi K_s \) decay.

The latest NA48 and KTeV measurements of \( \text{Re}(\epsilon'/\epsilon) \) are now in relatively good agreement: \( (14.7 \pm 2.2) \times 10^{-4} \) and \( (20.7 \pm 2.8) \times 10^{-4} \), leading to the world average:

\[
\text{Re}(\epsilon'/\epsilon) = (16.6 \pm 1.6 \ldots 2.3) \times 10^{-4},
\]

where the first error is naive, and the second one is rescaled according to the Particle Data Group prescription. The value is consistent with theoretical calculations, but these are not very accurate, because of delicate cancellations between different non-perturbative matrix elements. Measurements of \( \sin 2\beta \) are already startlingly precise:

\[
\sin 2\beta = 0.741 \pm 0.067,
\]

and very consistent with KM expectations of mixing-induced CP violation, as seen in Fig. 1. However, the origin of the CP asymmetry in \( B^0 \to J/\psi K_s \) decay is not yet confirmed, hence the importance of the next milestone, namely:

- The measurement of direct CP violation in \( B^0 \to 2\pi \) decay amplitudes, predicted to be the angle \( \alpha (= \phi_2) \) in the KM model.

As discussed later in more detail, the search for this effect is currently the subject of some discussion. Beyond it, many other CP-violating milestones beckon:
• CP violation in other $K$ decays, such as $K^0_L \to \pi^0 \nu \bar{\nu}$ decay,
• CP violation in other $B$ decays, such as the measurement of the third unitarity angle $\gamma$,
• CP violation in $D$ decays.

As we heard at this meeting, there is no hint of CP violation in $D^0 - \bar{D}^0$ mixing \(,\) which is expected only at a very low level in the SM, making it an excellent place to look for new physics beyond it. Other places to look for new sources of CP violation include

• The neutron electric dipole moment $d_n$,
• CP violation in neutrino oscillations via the MNS phase $\delta$,
• T violation in lepton decays such as $\mu \to 3e$ and $\tau \to 3\ell$,
• The lepton electric dipole moments $d_e, d_\mu, d_\tau$.

Only after we pass some more of these milestones will we have a chance of pinning down the origin(s) of CP violation: is it due to the KM mechanism alone? or are there other contributions? perhaps due to $\theta_{QCD}$? the MNS phase? supersymmetry? or ...?

5 The Next Steps along the CP Road

5.1 Quo Vadis $B^0 \to \pi^+ \pi^-$?

As you know, this decay mode receives contributions from $b \to u\bar{u}d$ tree diagrams and $b \to s\bar{s}u$ penguin diagrams, which contain both a weak and a strong phase. The resulting CP-violating asymmetry contains two parts:

$$S_{\pi\pi} \sin(\Delta m_d \Delta t) + A_{\pi\pi} \cos(\Delta m_d \Delta t),$$

where the latter term is that due to direct CP violation. The values of $S_{\pi\pi}$ and $A_{\pi\pi}$ depend on the proportion of penguin pollution $r$ (that may be constrained by other measurements such as $B^0 \to 2\pi^0$ and $B^+ \to K_s \pi^+$) and as well the angle $\alpha$ (or $\phi_2$) that we seek to determine. As seen in Fig. 2, the first measurements by BaBar and Belle are not in good agreement, though the naive average suggests that $S_{\pi\pi} \sim -0.6, A_{\pi\pi} \sim 0.6$, which are consistent with $\phi_2$ (or $\alpha$) $\sim 110$ degrees, as also seen in Fig. 2. Naive averaging may not be adequate, however, since the Belle measurement lies outside the physical boundary: $A_{\pi\pi}^2 + S_{\pi\pi}^2 = 1$, which should be taken into account in any fit.

There has recently been much progress in calculating exclusive $B$ decay amplitudes using the QCD factorization framework, with error estimates
Figure 2: The BaBar and Belle measurements of the asymmetry parameters $S_{\pi\pi}, A_{\pi\pi}$, and their naive average, are compared with KM predictions for different values of $\phi_2 = \alpha$, the penguin pollution factor $r$ and the strong-interaction phase. Also shown is the unitarity limit $A_{\pi\pi}^2 + S_{\pi\pi}^2 = 1$.

Based on evaluations of power corrections and annihilation diagrams. This framework suggests $S_{\pi\pi} \sim -0.3$ to -0.9, consistent with the naive average shown in Fig. 3. Thus there is reason to hope that $B^0 \rightarrow \pi^+\pi^-$ decay could become a valuable check on the KM model, as soon as the experimental situation settles down.

5.2 Quo Vadis $B \rightarrow \phi K_s$?

In the KM model, this decay is mediated by a strange gluonic penguin diagram: $b \rightarrow s + (q \rightarrow \bar{s}s)$, which has no intrinsic weak phase. Therefore, this decay should exhibit only mixing-induced CP violation, and should have the same asymmetry $\sin 2\beta$ as $B \rightarrow J/\psi K_s$. Other processes mediated by the same diagram include $B \rightarrow (\eta', K^+ K^-)K_s$, and first measurements of these decay asymmetries are consistent (within large errors) with that in $B \rightarrow J/\psi K_s$ and the KM model:

$$\eta': \ 0.76 \pm 0.36; \ K^+ K^-: \ 0.52 \pm 0.47,$$
whereas the decay asymmetry in $B \to \phi K_s$ looks rather different:

$$\phi : -0.39 \pm 0.41.$$  \hfill (5)

If this result holds up with more statistics, it would require new physics in the $b \to s + (g \to \bar s s)$ penguin diagram.

Several theoretical papers have appeared since the result (5) emerged, discussing models based on conventional $R$-conserving supersymmetry [10], $R$-violating supersymmetry [11], left-right symmetry [12] and a $Z'$ model [13].

Une affaire à suivre ....

5.3 Quo Vadis $B \to D^{*+}D^{*-}$?

The dominant diagram contributing to this process is thought to be $b \to c + (W \to \bar cd)$, with the competing penguin diagram $b \to d + (g \to \bar cc)$ thought to be rather small: $|P/T| < 0.1$. A first measurement of the CP-violating asymmetry $-\text{Im}(\lambda_+)$ that should coincide with $\sin 2\beta$ yields [14]:

$$-\text{Im}(\lambda_+) = -0.31 \pm 0.43 \pm 0.1,$$  \hfill (6)

which deviates by about 2.7 $\sigma$, nominally. However, the experimentalists caution that, with the current low statistics, the errors are not Gaussian. Une autre affaire à suivre ....

5.4 Quo Vadis $\gamma$?

There are various isospin relations between $B \to \pi K$ amplitudes that can be used to provide information about $\gamma$: e.g., the relation between those for the charged $B^+ \to \pi^0 K^+, \pi^+ K^0$, the relation between those for the neutral $B^0 \to \pi^- K^+, \pi^0 K^0$, and the mixed relation between $B^0 \to \pi^- K^+$ and $B \to \pi^+ K^0$. The charged amplitudes may be parametrized by the two quantities [15]

$$R^c, A^c_0 \equiv \frac{2B(\pi^0 K^+) \pm B(\pi^0 K^-)}{B(\pi^+ K^0) + B(\pi^- K^0)},$$  \hfill (7)

which depend on the strong tree-to-penguin ratio $r_c(\sim 0.2?)$, the electroweak tree-to-penguin ratio $q(\sim 0.7?)$, and the difference $\delta_c$ between the tree- and penguin-diagram phases.

Fig. 3 shows the current status of measurements of $R^c$ and $A^c_0$ [13]. We see from the third panel that the data prefer $\gamma > 90$ degrees, whereas the global
Figure 3: Allowed regions in the $R_c - A_0^c$ plane for charged $B \to \pi K$ decays, showing the effects of varying (a) the strong penguin pollution factor $r_c$, (b) the electroweak penguin pollution factor $q$, (c) the KM phase $\gamma$ and (d) the phase difference $\delta_c$.

KM fit shown in Fig. 3 prefers $\gamma < 90$ degrees. Again, it remains to see whether this possible discrepancy is confirmed by more data on the same decay modes, and/or on other decays such as $B^- \to D^0 K^-$, $B^0 \to D^{(*)\pm} \pi^{\pm}$, etc.

5.5 The Road Ahead for $B$ Factories?

Measurements of $\beta$ at $B$ factories are likely to attain an accuracy of $\pm 1$ degree, those of $\alpha$ may reach $\pm 5$ degrees, and those of $\gamma$ may reach $\pm 25$ degrees, which would correspond to a check of the unitarity triangle at the 15% level. It would clearly be desirable to push the experimental statistical errors down until they match the theoretical systematic errors. This provides worthwhile objectives for the subsequent generation of LHCb, BTeV and super-$B$ factory experiments.

6 The $B_s$ Road to CP Violation

There are just three neutral-meson systems where one can reasonably expect to see mixing and CP-violating effects in the SM and its plausible extensions:
Figure 4: Combined LEP/SLD/CDF results for $\Delta m_s$. The data are shown as points with error bars, the lines show the 95% C.L. curves, and the dotted curve shows the expected sensitivity $^{16}$.

Among these predictions, one may list $^{15}$:

- A large mixing parameter $x_s \equiv \Delta m_s / \Gamma_s = \mathcal{O}(20)$ - this prediction may be on the verge of being confirmed, as the compilation of present experiments on $B^0_s - \bar{B}^0_s$ mixing shown in Fig. 4 shows quite a hint of mixing with approximately the predicted value of $x_s$ $^{16}$;
- The $B^0_s - \bar{B}^0_s$ mixing phase should be very small: $\phi_s = \text{Arg}(V_{ts}^* V_{tb}) \sim -2$ degrees;
- There may be a sizeable difference in the total decay widths of the mass

the $K^0 - \bar{K}^0$ system that has been explored for many decades, the $B^0 - \bar{B}^0$ system that is now being explored at $B$ factories, and the $B^0_s - \bar{B}^0_s$ system. If we really do understand the SM and CP violation as well as we think, we can make many reliable predictions for the $B^0_s - \bar{B}^0_s$ system. Conversely, this may be a valuable laboratory for testing the SM, since any deviation from these confident predictions would be good evidence for physics beyond the SM.
eigenstates of the $B^0_s - \bar{B}^0_s$ system: $\Delta \Gamma_s/\Gamma_s \sim 10\%$.

Among the interesting $B^0_s$ decay modes, let us mention $B^0_s \rightarrow J/\psi \phi$, whose CP-violating asymmetry should be

$$A_{CP} \simeq \sin(\Delta m_s t) \sin \phi_s,$$

and hence very small in the SM. This makes it a good place for new contributions to $B^0_s - \bar{B}^0_s$ mixing, as might occur in supersymmetry, for example.

Another interesting decay mode is $B^0_s \rightarrow D^{\pm} K^\mp$, whose CP-violating asymmetry should be proportional to $\phi_s + \gamma$, and hence could (within the SM) be a good way to measure $\gamma$.

There are currently no plans to try to accumulate large samples of $B^0_s$ mesons at the operating $B$ factories, so $B^0_s$ physics may be left as the hunting preserve of the hadronic experiments LHCb and BTeV.

7 The Supersymmetric Flavour and CP Problems

In the supersymmetric limit, flavour mixing in the MSSM is identical to that in the SM, but supersymmetry must be broken. It is commonly thought that this occurs via gaugino masses $M_a$, scalar mass-squared parameters $(m^0_{ij})$ and trilinear couplings $A_{ijk}$. The gaugino mass parameters might have CP-violating phases that could show up in electric dipole moments and/or the Higgs sector, as discussed later. The big questions concerning $(m^0_{ij})$ and $A_{ijk}$ are whether they are universal, or at least can be diagonalized in the same basis as the quark and lepton flavours, and whether they contain extra CP-violating phases. Is the super-CKM mixing of squarks the same as the KM mixing of quarks? If not, how does it differ, and why?

Three generic classes of options can be distinguished:

- Minimal flavour violation, in which the $(m^0_{ij})$ and $A_{ijk}$ are universal at the GUT scale, being renormalized at lower energies by the Yukawa couplings $\lambda_{ij}$, and resulting in a super-CKM mixing pattern that is related to, and derivable from, the conventional CKM mixing;

- Extra supersymmetric loop effects, that may in general be parameterized as quark mass insertions $(\delta^L_{ij} (d,u))_{LL,RR}$;

- Extra tree-level effects, as could arise from generic $R$-violating interactions.
Quite frankly, fundamental theory provides no clear guidance which option Nature might have chosen. On the other hand, the observed suppressions of flavour-changing neutral interactions put severe constraints on $R$-violating models, which will not be discussed further here. These constraints certainly favour models with minimal flavour violation, although the best one can do phenomenologically, in a model-independent way, is to set upper bounds on the insertions $(\delta_{ij})_{LL,RR}$, as exemplified in Fig. 5.

If supersymmetric flavour violation is indeed minimal, one expects the squarks to be approximately degenerate, apart from the $\tilde{t}$ and possibly the $\tilde{b}$. These loopholes open up interesting opportunities in $B$ physics. For example, there could be significant supersymmetric contributions to the mass differences $\Delta m_d$ and $\Delta m_s$, though not to the ratio $\Delta m_d/\Delta m_s$. These would generate knock-on effects in the global unitarity triangle fits and $B_s$ physics. Rare $B$ decays already provide interesting upper limits on supersymmetric flavour violation and opportunities for the future, as we discuss next.

8 Rare $B$ Decays

This is a very rich area, and just a few examples are given here.

- $b \to s\gamma$ decay: This process may receive significant contributions from
the exchanges of charged Higgs bosons $H^\pm$ and chargino partners of the $W^\pm$ and $H^\pm$, and the fact the observed decay rate agrees within errors with the SM provides important constraints on the MSSM parameters, as seen in Fig. 6. In time, one could hope to measure a CP-violating asymmetry in $b \rightarrow s\gamma$ decays, and verify whether $\sin 2\beta$ measured at the loop level coincides with the value measured in the $J/\psi K_s$ decay mode.

- $b \rightarrow s\ell^+\ell^-$ decay: The case where the $s$ quark yields a $K$ meson has been observed, but not where it yields an excited state $K^*$. There could in principle be supersymmetric effects on the total decay rate, on the $\ell^+\ell^-$ mass spectrum, as seen in Fig 7(a), and on the forward-backward asymmetry $A_{FB}$, as seen in Fig 7(b). Again, the question whether $\sin 2\beta(\text{loop})$ coincides with $\sin 2\beta(J/\psi K_s)$ can be posed.

- $b \rightarrow s\bar{\nu}\nu$ decay: This process can also be calculated reliably in the SM,
and observation of $B \to X_s \pm$ nothing would be interesting for constraining extensions of the SM.

- $B \to \mu^+ \mu^-$ decay: This can receive important supersymmetric corrections \cite{24}, in particular for larger values of $\tan \beta$.
- $B \to \tau^+ \tau^-$ decay: This offers some prospects for studying CP violation in the MSSM \cite{25}.
- $B \to \tau^\pm, \mu^\pm e^\mp \mu^\pm e^\mp$ decays: These could in principle provide interesting windows on flavour violation in the lepton sector \cite{26}, which is the subject of the next section.

It may be interesting to note some of the statistics that may be provided by present and forthcoming experiments. For $B \to K^* \gamma$, we may expect 6,000 events at the $B$ factories, 25,000 at LHCb or BTeV, and 120,000 at a super-$B$ factory. The corresponding numbers for $B \to X_s \mu^\pm \mu^-$ are 120, 4,500 and 6,000, respectively, whilst for $B \to X_s \bar{\nu} \nu$ they are 8, 0 and 160, respectively. There is ample justification for another generation of $B$ experiments even beyond LHCb and BTeV.

9 Neutrino Flavour Violation

There is no good reason why either the total lepton number $L$ or the individual lepton flavours $L_{e,\mu,\tau}$ should be conserved. We have learnt that the only
conserved quantum numbers are those associated with exact gauge symmetries, just as the conservation of electromagnetic charge is associated with $U(1)$ gauge invariance. On the other hand, there is no exact gauge symmetry associated with any of the lepton numbers.

Moreover, neutrinos have been seen to oscillate between their different flavours \cite{27, 28), showing that the separate lepton flavours $L_{e,\mu,\tau}$ are indeed not conserved, though the conservation of total lepton number $L$ is still an open question. The observation of such oscillations strongly suggests that the neutrinos have different masses. Again, massless particles are generally associated with exact gauge symmetries, e.g., the photon with the $U(1)$ symmetry of the Standard Model, and the gluons with its $SU(3)$ symmetry. In the absence of any leptonic gauge symmetry, non-zero lepton masses are to be expected, in general.

The conservation of lepton number is an accidental symmetry of the renormalizable terms in the Standard Model lagrangian. However, one could easily add to the Standard Model non-renormalizable terms that would generate neutrino masses, even without introducing a ‘right-handed’ neutrino field. For example, a non-renormalizable term of the form \cite{29)

$$\frac{1}{M}\nu H \cdot \nu H,$$

where $M$ is some large mass beyond the scale of the Standard Model, would generate a neutrino mass term:

$$m_{\nu} \cdot \nu : m_{\nu} = \frac{\langle 0 | H | 0 \rangle^2}{M}.$$  \hspace{1cm} (10)

Of course, a non-renormalizable interaction such as (9) seems unlikely to be fundamental, and one should like to understand the origin of the large mass scale $M$.

The minimal renormalizable model of neutrino masses requires the introduction of weak-singlet ‘right-handed’ neutrinos $N$. These will in general couple to the conventional weak-doublet left-handed neutrinos via Yukawa couplings $Y_\nu$ that yield Dirac masses $m_D \sim m_W$. In addition, these ‘right-handed’ neutrinos $N$ can couple to themselves via Majorana masses $M$ that may be $\gg m_W$, since they do not require electroweak symmetry breaking. Combining
the two types of mass term, one obtains the seesaw mass matrix (10):

$$(\nu_L, N) \begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N \end{pmatrix},$$

(11)

where each of the entries should be understood as a matrix in generation space.

This seesaw model can accommodate the neutrino mixing seen experimentally, and naturally explains the small differences in the masses-squared of the light neutrinos. By itself, it would lead to unobservably small transitions between the different charged-lepton flavours. However, supersymmetry may enhance greatly the rates for processes violating the different charged-lepton flavours, rendering them potentially observable, as we discuss below.

The effective mass matrix for light neutrinos in the seesaw model may be written as:

$$M_\nu = Y_\nu^T M_D Y_\nu \sin^2 \beta$$

(12)

where we have used the relation $m_D = Y_\nu v [\sin \beta]$ with $v \equiv \langle 0 | H | 0 \rangle$, and the factors of $\sin \beta$ appear in the supersymmetric version of the seesaw model. Diagonalizing the neutrino mass matrix (12) and the charged-lepton masses introduces in general a mismatch between the mass and flavour eigenstates (11):

$$V_{MNS} = V_\ell V_\nu^\dagger,$$

(13)

which is reminiscent of the way the CKM matrix appears in the quark sector (2):

$$V_{CKM} = V_d V_u^\dagger,$$

(14)

though the difference in the ways the quark and neutrino masses (11) arise may give us some hope that the patterns of neutrino and quark mixing, $V_{MNS}$ and $V_{CKM}$, could be somewhat different.

The MNS matrix describing neutrino oscillations can be written in the form

$$V = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & c_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} e^{-i\delta} \end{pmatrix},$$

(15)

and there are in addition two CP-violating phases that are not observable in neutrino oscillations, but appear in neutrinoless double-$\beta$ decay.

The first matrix factor in (15) is measurable in solar neutrino experiments, and the recent data from SNO (28) and Super-Kamiokande (32) prefer quite
strongly the large-mixing-angle (LMA) solution to the solar neutrino problem with $\Delta m_{12}^2 \sim 6 \times 10^{-5}$ eV$^2$ and large but non-maximal mixing: $\theta_{12} \sim 30^\circ$. The validity or otherwise of the LMA solution is expected to be settled quite soon by the KamLAND experiment. The second matrix factor in (15) is measurable in atmospheric neutrino experiments, and the data from Super-Kamiokande in particular $[27]$ favour maximal mixing of atmospheric neutrinos: $\theta_{23} \sim 45^\circ$ and $\Delta m_{23}^2 \sim 2.5 \times 10^{-3}$ eV$^2$. However, the third matrix factor in (15) is basically unknown, with experiments such as Chooz $[33]$ and Super-Kamiokande only establishing upper limits on $\theta_{13}$, and a fortiori providing no information on the CP-violating phase $\delta$.

The phase $\delta$ could in principle be measured by comparing the oscillation probabilities for neutrinos and antineutrinos as seen in Fig. 8 $[34]$. This is possible only if $\Delta m_{12}^2$ and $s_{12}$ are large enough - as now suggested by the success of the LMA solution to the solar neutrino problem, and if $s_{13}$ is large enough - which remains an open question.

![Figure 8: A simultaneous fit to $\theta_{13}$ and $\delta$, using a neutrino-factory beam with different baselines and detector techniques $[34]$, may enable the CP-violating phase $\delta$ to be extracted.](image)

The effective low-energy mass matrix for the light neutrinos contains 9 parameters, 3 mass eigenvalues, 3 real mixing angles and 3 CP-violating phases.
However, these are not all the parameters in the minimal seesaw model. In fact, this model has a total of 18 parameters \(^{35,36}\). The remaining 9 associated with the heavy-neutrino sector may be measurable via their renormalization effects on soft supersymmetry-breaking parameters, as we discuss below. The total number of CP-violating parameters is 6, including the MNS phase \(\delta\), the two Majorana phases relevant to neutrinoless double-\(\beta\) decay, and three extra phases that play a key rôle in leptogenesis, as we discuss later.

10 Flavour and CP Violation for Charged Leptons

Assuming that the soft supersymmetry-breaking parameters put it at the GUT scale are universal, and working in the leading-logarithmic approximation with degenerate heavy singlet neutrinos, one finds the following radiative corrections to the soft supersymmetry-breaking terms for sleptons:

\[
(\delta m^2_L)_{ij} = -\frac{1}{8\pi^2} \left(3m_0^2 + A_0^2\right) (Y_\nu^\dagger Y_\nu)_{ij} \operatorname{Ln} \left(\frac{M_{\text{GUT}}}{M}\right),
\]

\[
(\delta A_\ell)_{ij} = -\frac{1}{8\pi^2} A_0 (Y_{\nu_i}^\dagger Y_\nu)_{ij} \operatorname{Ln} \left(\frac{M_{\text{GUT}}}{M}\right).
\]

(16)

The non-universality of the corrections (16) leads to processes that violate the different charged lepton numbers, such as \(\mu \rightarrow e\gamma\), \(\tau \rightarrow \mu\gamma\), \(\tau \rightarrow e\gamma\), \(\mu N \rightarrow eN, \mu \rightarrow 3e\), \(\tau \rightarrow 3e, e2\mu, \mu2e\) and \(3 \mu \rightarrow 3e, e2\mu, \mu2e\). Fig. 9(a) shows that the branching ratio for \(\mu \rightarrow e\gamma\) could be close to the present experimental upper limit, and Figs. 9(b) and (c) makes the same point for the decays \(\tau \rightarrow \mu\gamma\) and \(\tau \rightarrow e\gamma\), respectively \(^{38}\).

The electric dipole moments of the electron and muon depend sensitively on the non-degeneracy of the heavy singlet neutrinos \(^{37}\). As seen in Fig. 10, they could take values as large as \(d_e \sim 3 \times 10^{-30}\) e.cm and \(d_\mu \sim 10^{-27}\) e.cm, to be compared with the present experimental upper limits of \(d_e < 1.6 \times 10^{-27}\) e.cm \(^{38}\) and \(d_\mu < 10^{-18}\) e.cm \(^{40}\). An ongoing series of experiments might be able to reach \(d_e \sim 3 \times 10^{-30}\) e.cm, and a type of solid-state experiment that might be sensitive to \(d_e \sim 10^{-33}\) e.cm has been proposed \(^{41}\). Also, \(d_\mu \sim 10^{-24}\) e.cm might be accessible with the PRISM experiment proposed for the JHF \(^{12}\), and \(d_\mu \sim 5 \times 10^{-26}\) e.cm might be attainable at the front end of a neutrino factory \(^{13}\).
11 Leptogenesis

One of the favoured scenarios for baryogenesis is first to generate a lepton asymmetry via CP-violating decays of heavy singlet neutrinos, which is then recycled into a baryon asymmetry via non-perturbative electroweak interactions \[14\]. The CP asymmetry in this leptogenesis scenario is related to the product \( Y_\nu Y_\nu^\dagger \). The total decay rate of a heavy neutrino \( N_i \) may be written in the form

\[
\Gamma_i = \frac{1}{8\pi} (Y_\nu Y_\nu^\dagger)_i M_i, \tag{17}
\]
and one-loop CP-violating diagrams involving the exchange of heavy neutrino $N_j$ would generate an asymmetry in $N_i$ decay of the form:

$$
\epsilon_{ij} = \frac{1}{8\pi} \frac{1}{\left(Y_\nu Y_\nu^\dagger\right)_{ii}} \text{Im} \left(\left(Y_\nu Y_\nu^\dagger\right)_{ij}\right)^2 \frac{f(M_j)}{f(M_i)} , \quad (18)
$$

where $f(M_j/M_i)$ is a known kinematic function.

The relevant combination $Y_\nu Y_\nu^\dagger$ is independent of $V_{MNS}$ and hence of the light neutrino mixing angles and CP-violating phases. The basic reason for this is that one makes a unitary sum over all the light lepton species in evaluating the decay asymmetry $\epsilon_{ij}$ (18). Fig. 11 shows explicitly that one can generate a lepton asymmetry even if the MNS phase $\delta$ vanishes.

Figure 11: Heavy singlet neutrino decay may exhibit a CP-violating asymmetry, leading to leptogenesis and hence baryogenesis, even if the neutrino oscillation phase $\delta$ vanishes (45).

In general, one may formulate the following strategy for calculating leptogenesis in terms of laboratory observables:

- Measure the neutrino oscillation phase $\delta$ and the Majorana phases,
- Measure observables related to the renormalization of soft supersymmetry-breaking parameters, that are functions of $\delta$, the Majorana and leptogenesis
phases,

- Extract the effects of the known values of $\delta$ and the Majorana phases, and thereby isolate the leptogenesis parameters.

12 CP Violation in the MSSM Higgs Sector

A popular alternative scenario for baryogenesis has been to generate a quark asymmetry at the electroweak scale [46]. This requires a breakdown of thermal equilibrium, necessitating a first-order electroweak phase transition. This is impossible in the SM, since LEP tells us that the Higgs boson weighs more than 114.4 GeV, whereas a first-order electroweak phase transition is possible only if $m_H < 70$ GeV [47]. Generating a first-order phase transition would require extra light scalar bosons, as could be provided in supersymmetry, if the lighter $\tilde{t}$ is very light. This scenario would also require more CP violation than is present in the SM.

Indeed, two extra CP-violating phases appear in the MSSM, even if the soft supersymmetry-breaking parameters are universal at the input GUT scale, as assumed here. These can be taken as the (supposedly common) phases of the trilinear soft supersymmetry-breaking parameters $\text{Arg}(A_{t,b})$ and the phase of the gluino mass $\text{Arg}(m_{\tilde{g}})$. These generate mixing between the ‘scalar’ and ‘pseudoscalar’ MSSM Higgs bosons: at the one-loop level

$$\delta m^2_{SP} \sim \frac{m_t^4 \mu \text{Im}A_t}{v^2 32\pi^2 m^2_{\text{susy}}} + \cdots,$$

and a dependence on $\text{Arg}(m_{\tilde{g}})$ appears at the two-loop level.

In the presence of CP violation, it is convenient to parametrize the MSSM Higgs sector in terms of $m_{H^+}$ and $\tan\beta$. As seen in Fig. [12], there may be level crossing between the two lightest neutral Higgs bosons, and the lightest Higgs $H_1$ may have a suppressed coupling in the process $e^+e^- \rightarrow Z + H_1$. In this case, it could be that there exists a light Higgs boson lurking below the lower limit established by LEP in the SM. The prospects that experiments at hadron colliders may be able to plug this hole are discussed in [49].

The phenomenology of CP-violating Higgs bosons is very rich, and only its surface has been scratched. A $\mu^+\mu^-$ collider - either at the energy of the lightest Higgs boson, or close to the nearby masses of the second and third neutral Higgs bosons - may be necessary one day to unravel this physics [50].
Figure 12: In the MSSM with maximal CP violation in the Higgs sector, (a) there may be level-crossing between the lightest and second-lightest Higgs bosons, and (b) the lightest Higgs boson may have a small coupling in the process $e^+e^- \rightarrow Z + H$.

13 Some Answers

At a round-table discussion earlier this week, some central questions were raised, to which I would like to provide some personal answers.

- **Q:** What is the rôle of flavour studies in providing clues about new physics?
  
  A: They may cast light on the darkest corners of supersymmetry, namely its flavour and CP problems.

- **Q:** What are the implications of CP studies for our understanding of baryogenesis?
  
  A: Standard Model CP violation is inadequate for the task, but CP violation in either the lepton or Higgs sector could do the job. Both may be tested in future experiments.

- **Q:** What are the implications of lepton mixing for unification and phenomenology?
  
  A: It provides a direct window on physics at the GUT scale, and could open up a whole new arena for experiments on decays that violate the charged lepton flavours, such as $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, $\tau \rightarrow e\gamma$ and many more.

Flavour physics and CP violation surely have a long and glorious future!
References

1. J. R. Batley et al. [NA48 Collaboration], Phys. Lett. B 544, 97 (2002) [arXiv:hep-ex/0208009]; A. Alavi-Harati et al. [KTeV Collaboration], arXiv:hep-ex/0208007.

2. J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B 109, 213 (1976).

3. K. Abe et al. [Belle Collaboration], Phys. Rev. D 66, 071102 (2002) [arXiv:hep-ex/0208025]; B. Aubert et al. [BABAR Collaboration], [arXiv:hep-ex/0207042.

4. A. Stocchi, arXiv:hep-ph/0211243.

5. K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 89 (2002) 071801; B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0207055.

6. T. E. Browder, arXiv:hep-ex/0210012.

7. U. Egede [BABAR Collaboration], arXiv:hep-ex/0210060; A. Petrov, R. Ray, talks at this meeting, obtainable from http://frontierscience.lnf.infn.it/2002/IWFS.html.

8. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245 (2001) [arXiv:hep-ph/0104110].

9. M. D. Sokoloff, talk at this meeting, available from http://frontierscience.lnf.infn.it/2002/IWFS.html.

10. L. Silvestrini, arXiv:hep-ph/0210031; and references therein.

11. A. Datta, Phys. Rev. D 66, 071702 (2002) [arXiv:hep-ph/0208012]; B. Dutta, C. S. Kim and S. Oh, arXiv:hep-ph/0208226.

12. M. Raidal, arXiv:hep-ph/0208091.

13. G. Hiller, Phys. Rev. D 66, 071502 (2002) [arXiv:hep-ph/0207356].

14. B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0207072.

15. R. Fleischer, arXiv:hep-ph/0210323; and references therein.
16. Working group on B oscillations, \texttt{http://lepbosc.web.cern.ch/LEPBOSC/}.

17. N. Harnew, talk at this meeting, available from \texttt{http://frontierscience.lnf.infn.it/2002/IWFS.html}. The homepage of this experiment is: \texttt{http://lhcb.web.cern.ch/lhcb/}.

18. D. Christian, talk at this meeting, available from \texttt{http://frontierscience.lnf.infn.it/2002/IWFS.html}.

19. A. Masiero and O. Vives, New J. Phys. \textbf{4} (2002) 4; and references therein.

20. A. Ali, \texttt{arXiv:hep-ph/0201120}.

21. C. Degrassi, P. Gambino and G. F. Giudice, JHEP \textbf{0012} (2000) 009 \texttt{[arXiv:hep-ph/0009337]}; M. Carena, D. Garcia, U. Nierste and C. E. Wagner, Phys. Lett. B \textbf{499} (2001) 141 \texttt{[arXiv:hep-ph/0010003]}; D. A. Demir and K. A. Olive, Phys. Rev. D \textbf{65} (2002) 034007 \texttt{[arXiv:hep-ph/0107329]}.

22. J. Ellis, T. Falk, K. A. Olive and Y. Santos, \texttt{arXiv:hep-ph/0210207}; and references therein.

23. A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D \textbf{61}, 074024 (2000) \texttt{[arXiv:hep-ph/9910221]}.

24. A. Dedes, H. K. Dreiner and U. Nierste, Phys. Rev. Lett. \textbf{87}, 251804 (2001) \texttt{arXiv:hep-ph/0108037}.

25. A. Dedes and A. Pilaftsis, \texttt{arXiv:hep-ph/0209306}.

26. A. Dedes, J. R. Ellis and M. Raidal, \texttt{arXiv:hep-ph/0209207}.

27. Y. Fukuda \textit{et al.} [Super-Kamiokande Collaboration], Phys. Rev. Lett. \textbf{81}, 1562 (1998) \texttt{arXiv:hep-ex/9807003}.

28. Q. R. Ahmad \textit{et al.} [SNO Collaboration], Phys. Rev. Lett. \textbf{89}, 011301 (2002) \texttt{arXiv:nucl-ex/0204008}; Phys. Rev. Lett. \textbf{89}, 011302 (2002) \texttt{arXiv:nucl-ex/0204009}.

29. R. Barbieri, J. R. Ellis and M. K. Gaillard, Phys. Lett. B \textbf{90}, 249 (1980).
30. M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (edited by A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

31. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

32. S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Lett. B 539, 179 (2002) [arXiv:hep-ex/0205075].

33. M. Apollonio et al. [CHOOZ Collaboration], Phys. Lett. B 466, 415 (1999) [arXiv:hep-ex/9907037].

34. A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena and S. Rigolin, Nucl. Phys. B 579, 17 (2000) [Erratum-ibid. B 593, 731 (2001)] [arXiv:hep-ph/0002108].

35. J. A. Casas and A. Ibarra, Nucl. Phys. B 618, 171 (2001) [arXiv:hep-ph/0103065].

36. J. R. Ellis, J. Hisano, S. Lola and M. Raidal, Nucl. Phys. B 621, 208 (2002) [arXiv:hep-ph/0109123].

37. J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B 528, 86 (2002) [arXiv:hep-ph/0111323].

38. J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, [arXiv:hep-ph/0206110].

39. B. C. Regan, E. D. Commins, C. J. Schmidt and D. DeMille, Phys. Rev. Lett. 88 (2002) 071805; B. E. Sauer, talk at this meeting, available from http://frontierscience.lnf.infn.it/2002/IWFS.html.

40. H. N. Brown et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 86, 2227 (2001) [arXiv:hep-ex/0102017].

41. S. K. Lamoreaux, [arXiv:nucl-ex/0109014].
42. M. Furusaka et al., JAERI/KEK Joint Project Proposal *The Joint Project for High-Intensity Proton Accelerators*, KEK-REPORT-99-4, JAERI-TECH-99-056.

43. J. Äystö et al., *Physics with Low-Energy Muons at a Neutrino Factory Complex*, CERN-TH/2001-231, [hep-ph/0109217](http://arxiv.org/abs/hep-ph/0109217) and references therein.

44. M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

45. J. R. Ellis and M. Raidal, Nucl. Phys. B **643**, 229 (2002) [arXiv:hep-ph/0206174](http://arxiv.org/abs/hep-ph/0206174).

46. See, for example: G. R. Farrar and M. E. Shaposhnikov, Phys. Rev. D **50**, 774 (1994) [arXiv:hep-ph/9305273](http://arxiv.org/abs/hep-ph/9305273).

47. M. Laine, [arXiv:hep-ph/0010273](http://arxiv.org/abs/hep-ph/0010273).

48. M. Carena, J. R. Ellis, A. Pilaftsis and C. E. Wagner, Phys. Lett. B **495**, 155 (2000) [arXiv:hep-ph/0009212](http://arxiv.org/abs/hep-ph/0009212).

49. M. Carena, J. R. Ellis, S. Mrenna, A. Pilaftsis and C. E. Wagner, CERN-TH/2002-299, in preparation.

50. C. Blochinger et al., [arXiv:hep-ph/0202199](http://arxiv.org/abs/hep-ph/0202199) and references therein.