Probing the dynamics of dark energy with divergence-free parametrizations: A global fit study

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The CPL parametrization is very important for investigating the property of dark energy with observational data. However, the CPL parametrization only respects the past evolution of dark energy but does not care about the future evolution of dark energy, since $w(z)$ diverges in the distant future. In a recent paper [J.Z. Ma and X. Zhang, Phys. Lett. B 699, 233 (2011)], a robust, novel parametrization for dark energy, $w(z) = w_0 + w_1(\frac{\ln(2+z)}{1+z} - \ln 2)$, has been proposed, successfully avoiding the future divergence problem in the CPL parametrization. On the other hand, an oscillating parametrization (motivated by an oscillating quintom model) can also avoid the future divergence problem. In this Letter, we use the two divergence-free parametrizations to probe the dynamics of dark energy in the whole evolutionary history. In light of the data from 7-year WMAP temperature and polarization power spectra, matter power spectrum of SDSS DR7, and SN Ia Union2 sample, we perform a full Markov Chain Monte Carlo exploration for the two dynamical dark energy models. We find that the best-fit dark energy model is a quintom model with the EOS across $-1$ during the evolution. However, though the quintom model is more favored, we find that the cosmological constant still cannot be excluded.

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I. INTRODUCTION

Since the accelerating expansion of the universe was discovered by the observations of type Ia supernovae (SN Ia) [1], dark energy, the mysterious energy budget that drives such a cosmic acceleration, has attracted lots of studies [2, 3]. The main characteristic of dark energy is encoded in the equation of state parameter (EOS), and thus the study of extracting the information of EOS by fitting with the observational data provides an important way for understanding the nature of dark energy.

Extracting the information of the EOS from the data relies on the parametrization of dark energy. Currently, beyond the simplest $\Lambda$CDM model, the Chevallier-Polarski-Linder (CPL, hereafter) parametrization $w(z) = w_0 + w_1(1 - a)$ [4], which introduces the first order Taylor's expansion in terms of the scale factor $a$, is rather popular and attracts lots of studies. The main feature of such a parametrization is that it describes the possible dynamical evolution of EOS with time. The advantage of this form is that it can be applied to fit the low redshift SN Ia data as well as the high redshift CMB data at the same time. However, as shown in our previous study [5], the EOS will get to a nonphysical value in the far future time when redshift $z$ approaches $-1$, namely, $|w(z)|$ will grow rapidly and diverge. Such a divergence feature prevents the CPL parametrization from genuinely covering the scalar-field models as well as other theoretical models.

The ultimate fate of the universe is determined by the property of dark energy: If dark energy is the cosmological constant ($w = -1$), then the fate of the universe is a de Sitter spacetime; if dark energy is phantomlike ($w < -1$), then the destiny of the universe is a doomsday (namely, the “big rip” singularity); and so on. So, it is very important to probe the dynamics of dark energy with the observational data, since the detection of the evolution of dark energy would provide the evidence of falsification of the cosmological constant, and also the ultimate fate of the universe could be foreseen. Since the CPL parametrization has the divergence problem and thus loses the prediction ability, we are interested in some other well-behaved parametrization forms for investigating the property of dark energy.

In order to keep the advantage of the CPL parametrization, and avoid its drawback at the same time, some divergence-free parameterizations have been proposed [5] in which the leading proposal is a logarithm form: $w(z) = w_0 + w_1(\frac{\ln(2+z)}{1+z} - \ln 2)$. Such a new parametrization has well behaved, bounded behavior for both high redshifts and negative redshifts. Thanks to the logarithm form in the parametrization, a finite value for $w(z)$ can be ensured, via the application of the L'Hopital's rule, in both limiting cases, $z \to \infty$ and $z \to -1$. This is the reason why a logarithm form is introduced in the new parametrization. Without doubt, such a two-parameter form of EOS can genuinely cover many scalar-field models (including quintom models with two scalar fields and/or one scalar field with high derivatives) as well as other theoretical scenarios. On the other hand, one can only justify that the EOS of dark energy is around $-1$ in the recent epoch, but for the EOS, in much earlier or far future time, there are more possibilities, and one of which is that the EOS of dark energy might exhibit oscillatory behavior during the evolution. Oscillating EOS of dark energy are widely studied [6–10], thanks to the advantage that the oscillating evolution behavior of dark energy can unify the two accelerating epochs of our universe and alleviate the so-called coincidence problem in some sense. Based on this consideration, the above new parametrization is also extended to an oscillating form. In Ref. [5], two novel parametrizations have been used to probe the dynamics of dark energy in the whole evolutionary history, and it has been proven that the divergence-free parametrizations are very successful in exploring the dynamical evolution of dark energy and have powerful prediction capability for the ultimate fate of the universe.
In this Letter, we perform a global data fitting analysis on two divergence-free parametrizations for dynamical dark energy, and present constraints on the model parameters from the current observational data, including the CMB temperature and polarization power spectra from the seven-year WMAP data, the matter power spectrum from the SDSS Data Release 7 (DR7), and SN Union2 sample. Since dark energy parameters are tightly correlated to some other cosmological parameters, for example, the matter density parameter \( \Omega_m \), the Hubble constant \( H_0 \), the spatial curvature \( \Omega_k \), the neutrino mass \( m_\nu \), and so on, it is crucial to consider a global fit procedure in the investigation of the dynamical dark energy. Also, in this procedure, the perturbation of dark energy is involved. In Ref. [5], only a preliminary analysis was performed, in which the perturbation of dark energy is absent, and the information of CMB and LSS is incomplete. In this Letter, we will perform a sophisticated analysis for the divergence-free parametrizations. The Letter is organized as follows: In Sec. II we will introduce the method and data of the global fitting procedure, and the results are presented in Sec. III, and our conclusion is given in Sec. IV.

II. METHOD AND DATA

We consider the divergence-free parametrization for dynamical dark energy proposed in Ref. [5]:

\[
w(z) = w_0 + w_1 \left( \frac{\ln(2 + z)}{1 + z} - \ln 2 \right),
\]

where \( w_0 \) denotes the present-day value of \( w(z) \), and \( w_1 \) is another parameter characterizing the evolution of \( w(z) \). Note that a minus \( \ln 2 \) in the bracket is kept for maintaining \( w_0 \) to be the current value of \( w(z) \), and in Ref. [5] it is contrived for an easy comparison with the CPL model. Obviously, this new parametrization has well behaved, bounded behavior for both high redshifts and negative redshifts. The logarithm form in the parametrization ensures a finite value for \( w(z) \) via the application of the L'Hospital's rule, in both limiting cases, \( z \to \infty \) and \( z \to -1 \). This is the reason why we introduce a logarithm form in this parametrization. Specifically, we have \( w = w_0 - w_1 \ln 2 \) for \( z \to \infty \) and \( w = w_0 + w_1 (1 - \ln 2) \) for \( z \to -1 \). At low redshifts, this parametrization form reduces to the linear one, \( w(z) \approx w_0 + w_1 z \), where \( w_1 = -(\ln 2)w_1 \). Of course, one can also recast it at low redshifts as the CPL form, \( w(z) \approx w_0 + w_1 z / (1 + z) \), where \( w_1 = (1/2 - \ln 2)w_1 \). Therefore, it is clear to see that this parametrization exhibits well-behaved feature for the dynamical evolution of dark energy. Without question, such a two-parameter form of EOS can genuinely cover scalar-field models as well as other theoretical scenarios.

The oscillating parametrization proposed in Ref. [5] is of the form \( w(z) = w_0 + w_1 \sin(1 + z)/(1 + z) - \sin(1) \). This form has lots of advantages, as illustrated in Ref. [5]. We find that this parametrization describes the same behavior as the logarithm form (1) at low redshifts, but exhibits oscillating feature from a long term point of view. However, recent studies show that there might be oscillatory behavior within redshift range from 0 to 2 for the EOS [11], so this possibility should also be involved in our investigation. Therefore, we adopt the following oscillating parametrization form:

\[
w(z) = w_0 + w_1 \sin(A \ln(1/(1 + z))),
\]

where \( A \) is another parameter. The direct physical motivation of the parametrization (2) is from an oscillating quintom [7, 8]. Here a sine function has the advantage of exhibiting the oscillating feature of the EOS and preserving the value of EOS finite. In this Letter we set \( A = \frac{1}{\pi} \) in order to allow the EOS to cross \(-1\) more than one time within the redshift range from 0 to 2 where the SN data are most robust, as implied by the recent studies [11].

Basing on the MCMC package CosmoMC\(^1\) [12] we perform a global fitting analysis for the dynamical dark energy models parameterized above. For dynamical dark energy models, it is crucial to include dark energy perturbations [13–15]. As we know that for quintessence-like or phantom-like models, in which \( w \) does not cross the cosmological constant boundary, the perturbation of dark energy is well defined. However, when \( w \) crosses \(-1\), one is encountered with the divergence problem for perturbations of dark energy at \( w = -1 \). In order to solve this problem in the global fitting analysis, we introduce a small positive constant \( \epsilon \) to divide the full range of the allowed values of the EOS \( w \) into three parts: (i) \( w > -1 + \epsilon \), (ii) \( -1 - \epsilon \leq w \leq -1 + \epsilon \), and (iii) \( w < -1 - \epsilon \).

Working in the conformal Newtonian gauge, the perturbations of dark energy can be described by

\[
\dot{\delta} = -(1 + w)(\theta - 3\Phi) - 3\mathcal{H}(c_s^2 - w)\delta, \tag{3}
\]

\[
\dot{\theta} = -3\mathcal{H}(1 - 3w)\theta - \frac{\dot{w}}{1 + w}\theta + k^2\left(\frac{c_s^2 - w}{1 + w} + \Psi\right). \tag{4}
\]

Neglecting the entropy perturbation, for the regions (i) and (ii), the EOS does not cross \(-1\) and the perturbation is well defined by solving Eqs. (3) and (4). For the case (ii), the perturbation of energy density \( \delta \) and divergence of velocity \( \theta \), and the derivatives of \( \delta \) and \( \theta \) are finite and continuous for the realistic dark energy models. However for the perturbations of the parameterizations, there is clearly a divergence. In our analysis for such a regime, we match the perturbations in region (ii) to the regions (i) and (iii) at the boundary and set \( \dot{\delta} = 0 \) and \( \dot{\theta} = 0 \). In our numerical calculations we limit the range to be \( |\Delta w| = \epsilon | < 10^{-4} \) and find our method to be a very good approximation to the multi-field dark energy model. More detailed treatments can be found in Ref. [16].

Our most general parameter space vector is:

\[
P \equiv (w_0, \omega_\gamma, \Theta, \tau, w_0, w_1, \Omega_b, n_s, A_s, c_s^2), \tag{5}
\]

where \( \omega_\gamma \equiv \Omega_\gamma h^2 \) and \( \omega_s \equiv \Omega_sh^2 \), with \( \Omega_b \) and \( \Omega_s \) the physical baryon and cold dark matter densities relative to the critical density, \( \Omega_k \) is the spatial curvature satisfying \( \Omega_b + \Omega_c + \Omega_k = 1 \). \( \Theta \) is the ratio (multiplied by 100) of the sound horizon to the angular diameter distance at decoupling, \( \tau \) is the

\(^1\) Available at: http://cosmologist.info/cosmomc/.
optical depth to re-ionization, \(w_0\) and \(w_1\) are the parameters of dark energy EOS given by Eqs. (1) and (2), \(A_s\) and \(n_s\) are the amplitude and the spectral index of the primordial scalar perturbation power spectrum, and \(c_s\) is the sound speed of dark energy. For the pivot scale we set \(k_0 = 0.05\,\text{Mpc}^{-1}\). Note that we have assumed purely adiabatic initial conditions.

In the computation of the CMB, we include the 7-year WMAP temperature and polarization power spectra [17] with the routine for computing the likelihood supplied by the WMAP team.\(^2\) For the large scale structure (LSS) information, we use the matter power spectrum data from SDSS DR7 [18]. The supernova data we use are the recently released “Union2” sample of 557 data [19]. In the calculation of the likelihood from SN we marginalize over the relevant nuisance parameter [20].

Furthermore, we make use of the Hubble Space Telescope (HST) measurement of the Hubble constant \(H_0 \equiv 100h\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}\) by a Gaussian likelihood function centered around \(h = 0.738\) and with a standard deviation \(\sigma = 0.024\) [19].

### III. NUMERICAL RESULTS

In this section we present our global fitting results. In Table I we list the 1\(\sigma\) constraint results on the dark energy models. We have compared the results with and without the inclusion of the systematic errors of SN Union2 sample. By including the systematic errors, the constraints on cosmological parameters become a little bit relaxed, which can be seen by comparing the error bars listed in the table for the two cases. Note that the fit results listed in Table I are the mean of the likelihood.

Since our aim is to probe the dynamics of dark energy, we should try to avoid other indirect factors weakening the observational limits on the EOS. Thus, in our analysis we have assumed a flat universe, \(\Omega_k = 0\), consistent with the inflationary cosmology. Moreover, the sound speed of dark energy is also fixed in our analysis. In the framework of the linear perturbation theory, besides the EOS of dark energy, the dark energy perturbations can also be characterized by the sound speed, \(c_s^2 = \delta P_{de}/\delta\rho_{de}\). The sound speed of dark energy might affect the evolution of perturbations, and might leave signatures on the CMB power spectrum [21]. However, it has been shown that the constraints on the dark energy sound speed \(c_s^2\) in dynamical dark energy models are still very weak, since the current observational data are still not accurate enough [22].

Therefore, in our analysis, we have treated the dark energy as a scalar-field model (multi-fields or single field with high derivative) and set \(c_s^2\) to be 1. Of course, one can also take \(c_s^2\) as a parameter, but the fit results would not be affected by this treatment [22].

In the dynamical dark energy model where the EOS of dark energy is parameterized by the logarithm form (1), we get the best-fit results (including the systematic errors of Union2 sample): \(\omega_0 = 0.0225, \omega_c = 0.114, h = 0.683, \tau = 0.0877, w_0 = -1.089, w_1 = -1.552, n_s = 0.969\) and \(10^3A_s = 2.207\), which are consistent with the results of 7-year WMAP [17]. Note that here the results are maximum likelihood values. The panel (a) of Fig. 1 shows the joint two-dimensional marginalized constraint on the parameters \(w_0\) and \(w_1\) for the logarithm parametrization (1). The contours show the 68% and 95% confidence levels (CL) for the cases without the systematic errors of SN (color shaded regions and red solid lines) and with the systematic errors of SN (unshaded regions and black dashed lines). We find that the best-fit dark energy model is a quintom model [23], whose \(w(z)\) crosses the cosmological constant boundary \(w = -1\) during the evolution. With the current observational data, the variance of \(w_0\) and \(w_1\) we get are still large; the 95% constraints on \(w_0\) and \(w_1\) are \(-1.154 < w_0 < 0.771\) and \(-1.682 < w_1 < 4.251\), which can also be seen in the panel (a) of Fig. 1. This result implies that though the dynamical dark energy models are mildly favored, the current data cannot distinguish different dark energy models decisively. With the fitting results in hand, we can reconstruct the evolution of the EOS of dark energy, \(w(z)\). The reconstructed result for the logarithm form (1) is shown in the panel (a) of Fig. 2. The red solid line is plotted with the best fit values, while the color shaded region represents the 1\(\sigma\) limit. From this figure, we can directly see that although the quintom model is more favored, the cosmological constant boundary (\(\Lambda\)CDM model), however, still cannot be excluded.

Our results are consistent with Ref. [5], in which the constraints are given by using the data combination of the WMAP distance prior and BAO information instead of the full CMB temperature and polarization power spectra and LSS matter power spectrum. Though the WMAP distance prior, including \(R, \lambda_s\) and \(z_s\), encoding the information of background cosmic distances, can be applied to investigate dark energy models and can greatly simplify the numerical calculations in determining cosmological parameters, it was found that the prior is somewhat cosmological model dependent and the utilization of this prior may lose some of the CMB information [17]. For example, the distance prior does not capture the information on the growth of structure probed by the late-time ISW effect. As a result, the dark energy constraints derived from the distance prior are similar to, but weaken than, those derived from the full analysis. Therefore, in this Letter, in order to improve the analysis in Ref. [5], we present the full Markov Chain Monte Carlo exploration of this model.

Next, we discuss the dynamical dark energy model with the EOS of dark energy parameterized by the oscillatory form (2). For this model, we get the best-fit results: \(\omega_0 = 0.0225, \omega_c = 0.112, \alpha = 0.697, \tau = 0.0878, w_0 = -1.089, w_1 = -1.553, n_s = 0.969\) and \(10^3A_s = 2.207\). We find that the EOS of dark energy that has an oscillating behavior can also fit the data well. The 2\(\sigma\) CL constraints on \(w_0\) and \(w_1\) in this dark energy model are \(-1.149 < w_0 < -0.810\) and \(-0.192 < w_1 < 0.357\). We show the two-dimensional marginalized constraint on \(w_0\) and \(w_1\) for this model in the panel (b) of Fig. 1. The reconstructed evolution behavior of the EOS of dark energy, \(w(z)\), is shown in the panel (b) of Fig. 2. It is indicated that
TABLE I: Constraints on the dark energy EOS and some background parameters from the observations.

| model          | data                      | $\Omega_{de}$ | $w_0$   | $w_1$ | $H_0$ (km s$^{-1}$ Mpc$^{-1}$) |
|----------------|---------------------------|---------------|---------|-------|--------------------------------|
| Log Union2 (w/sys)+WMAP7+LSS | 0.726$^{+0.006}_{-0.006}$ | $-0.951^{+0.010}_{-0.010}$ | 0.975$^{+0.006}_{-0.006}$ | 70.461$^{+2.146}_{-2.142}$ |
| Log Union2 (w/o sys)+WMAP7+LSS | 0.729$^{+0.006}_{-0.006}$ | $-0.952^{+0.010}_{-0.010}$ | 1.106$^{+1.056}_{-1.054}$ | 70.879$^{+2.256}_{-2.254}$ |
| Osc Union2 (w/sys)+WMAP7+LSS | 0.720$^{+0.012}_{-0.012}$ | $-0.995^{+0.010}_{-0.010}$ | 0.0935$^{+0.0082}_{-0.0081}$ | 69.490$^{+1.781}_{-1.781}$ |
| Osc Union2 (w/o sys)+WMAP7+LSS | 0.721$^{+0.015}_{-0.015}$ | $-0.964^{+0.012}_{-0.012}$ | 0.0673$^{+0.010}_{-0.010}$ | 69.542$^{+1.343}_{-1.343}$ |

FIG. 1: Joint two-dimensional marginalized constraint on the parameters $w_0$ and $w_1$ for (a) the logarithm parametrization (1) and (b) the oscillating parametrization (2). The contours show the 68% and 95% CL from WMAP+SDSS+SN, for the cases without the systematic errors of SN (color shaded regions and red solid lines) and with the systematic errors of SN (unshaded regions and black dashed lines).

an oscillating quintom model is more favored, whose EOS crosses $-1$ more than one time within the redshift range from 0 to 2. According to the best-fit result, for the future evolution, the EOS of this model will experience the $-1$ crossing for many times. However, one can also see that at the $1\sigma$ level, the cosmological constant is still a good fit.

IV. SUMMARY AND DISCUSSION

In this Letter, we have performed a global fit study on two divergence-free parametrizations for dark energy. It is known that the frequently used CPL parametrization actually can only describe the past evolution history of dark energy but cannot genuinely depict the future evolution of dark energy owing to the divergence of $w(z)$ as $z$ approaches $-1$. Such a divergence feature forces the CPL parametrization to lose its prediction capability for the fate of the universe and to fail in providing a complete evolution history for the dark energy. Consequently, the CPL model cannot genuinely cover scalar-field models as well as other dark energy theoretical models. In Ref. [5], a robust parametrization form, $w(z) = w_0 + w_1 (\ln(2 + z)^{-1} - \ln 2)$, was proposed, which is divergence-free and has well-behaved feature for the EOS of dark energy in all the evolution stages of the universe.

This parametrization, without doubt, could cover many dark-energy theoretical models. Obviously, according to this parametrization, quintom models with two scalar fields and/or with one field with high derivatives can be successfully reconstructed. Another example can be provided by the holographic dark energy model [24]. The holographic dark energy model arises from the holographic principle of quantum gravity. Its EOS satisfies $w(z) = -1/3 - 2/(3c) \sqrt{\Omega_{de}(z)}$, where $c$ is a phenomenological parameter determining the dynamical evolution of the dark energy, and $\Omega_{de}(z)$ satisfies a differential equation [25]. In this model, if $c < 1$, the dark energy will behave like a quintom, i.e., the EOS crosses $-1$ during the evolution [26]. We find that the holographic evolution can be roughly mimicked by the logarithm form parametrization, provided that $w_0$ and $w_1$ are around $-1$. All in all, this parametrization can be used to reconstruct many dark-energy theoretical models, and can be used to probe the dynamics of dark energy in light of the observational data.

In Ref. [5], the logarithm parametrization (1) has been used to probe the dynamics of dark energy in the whole evolutionary history. However, it should be pointed out that only a preliminary analysis was performed in Ref. [5] because the data sets in the analysis are the WMAP distance prior and BAO information instead of the full CMB temperature and polarization power spectra and LSS matter power spectrum. Such an analysis might lose some of the CMB and LSS information. In this Letter, we have improved the analysis by im-
implementing a full Markov Chain Monte Carlo exploration of this model. The result is consistent with that of Ref. [5]. We found that the best-fit dark energy model is a quintom model with the EOS $w(z)$ across $-1$ during the evolution. However, while the quintom model is more favored, the cosmological constant still cannot be excluded.

We also explored the possibility that the EOS may oscillate and cross $-1$ many times during the evolution. We used the oscillating parametrization (2) that is also divergence-free to probe the evolution of dark energy. Though the motivation of this parametrization is not as robust as the form (1), it can fit the data well. The result shows that it is indeed possible that $w(z)$ crosses $-1$ for many times during the whole evolution history.

We believe that it is fairly important to use some divergence-free parametrizations to probe the dynamical evolution of dark energy. We have shown that the logarithm form (1) is a good proposal and it has been proven to be very successful in exploring the property of dark energy. We suggest that this parametrization should be further investigated.

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