Dynamical determination
of the Supersymmetric Higgs mass

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Abstract

Considering the supersymmetric Higgs mass (µ-parameter) as a dynamical variable to be determined by minimizing the energy, we predict its value as a function of the soft masses of the potential. We find that µ has a nonzero value close to the weak scale. This scenario offers a simultaneous solution to the doublet-triplet splitting problem and to the µ-problem. We discuss its viability in theories with gauge mediated supersymmetry breaking.
1 Introduction

In the supersymmetric standard model (MSSM), the Higgs doublets present three important features that distinguish them from the lepton and quark superfields:

a) The Higgs superfield, $H + \bar{H}$, is vector-like under the standard model (SM) group and therefore it is allowed to have a large supersymmetric mass $\mu H \bar{H}$.

b) If we grand unify the MSSM in a theory such as SU(5), the Higgs doublets cannot be embedded in a complete GUT-representation.

c) The scalar components of the Higgs doublets have to get nonzero vacuum expectation values (VEVs) to break the electroweak symmetry.

Properties (a) and (c) lead to the $\mu$-problem. If the Higgs doublets have to get nonzero VEVs, the value of $\mu$ has to be bounded from above by the weak scale. On the other hand, Higgsino searches at LEP1.5 [1] put a lower bound on $\mu$ roughly given by $|\mu| \gtrsim 50$ GeV. Due to property (a), there is, a priori, no reason to expect the value of $\mu$ to be in this small window; this is referred as the $\mu$-problem. This problem is especially severe in theories with gauge mediated supersymmetry breaking (GMSB) [2]. In these theories the supersymmetry breaking is communicated by gauge interactions from a “messenger” sector to the squarks, slepton and Higgs. Since the $\mu$-parameter cannot be induced by gauge interactions, one has $\mu = 0$ unless one enlarges the model with new interactions [2]–[5].

Property (b) leads to the doublet-triplet splitting problem. To embed the Higgs doublets in a complete SU(5)-representation, we have to introduce Higgs color triplets $H_C$ and $\bar{H}_C$ such that $\bar{5} = (\bar{H}_C, \bar{H})$ and $5 = (H_C, H)^T$. Nevertheless, the color triplets cannot be light if we do not want to have a too fast proton decay or to spoil the success of gauge coupling unification. Thus, one needs to split the $\bar{5}$ and $5$ into light Higgs doublets and heavy color triplets.

A very attractive possibility that seems to relate properties (a), (b) and (c) is to assume that the $\mu$-parameter is a dynamical variable [6]. In this case, its value is determined by the minimization conditions of the potential and one obtains that (c) leads automatically to a doublet-triplet splitting [3]. To see how this works, let us consider a SU(5)-GUT given by

$$W = \mu \bar{5}5 + \lambda' \bar{5}245,$$

where $24$ is the adjoint representation of SU(5) responsible for the breaking of SU(5) to the SM group. Its VEV is assumed to be

$$\langle 24 \rangle = M_G \text{Diag}(2, 2, 2, -3, -3), \quad M_G \simeq 10^{16} \text{ GeV}.\quad (2)$$

Inserting (2) in (3), we obtain

$$W = (\mu + 2\lambda'M_G)\bar{H}C H + (\mu - 3\lambda'M_G)\bar{H}H,$$

and the potential for the scalar components is given by

$$V = |\mu + 2\lambda'M_G|^2(|H_C|^2 + |\bar{H}_C|^2) + |\mu - 3\lambda'M_G|^2(|H|^2 + |\bar{H}|^2) + V_{soft} + D\text{-terms}, \quad (4)$$
where $V_{soft}$ includes the terms that softly break supersymmetry. From eq. (4) we can see that for values of $\mu$ different from $-2\lambda'M_G$ or $3\lambda'M_G$, the Higgs doublets and color triplets are very heavy and forced to get zero VEVs. The potential (4) at this minimum will then be zero. On the other hand, for $\mu = 3\lambda'M_G$ the Higgs doublets are light and their VEVs are determined by the low-energy MSSM potential. If at low-energies $H$ and $\tilde{H}$ get VEVs of order of their soft masses (of $O(m_Z)$), the potential at the minimum has a value smaller than zero. Thus, this vacuum is energetically favored. The Higgs color triplets at this vacuum are very heavy ($M_{HC} = 5\lambda'M_G$) in agreement with gauge coupling unification and proton decay limits. There could be a third possibility with $\mu = -2\lambda'M_G$ and light Higgs color triplets. This case is however energetically disfavored because the soft masses of $H_C$ and $\tilde{H}_C$ tend to be positive at low-energy due to the SU(3) strong coupling (like the squark soft masses) forcing zero VEVs for the color triplets.

Here we will assume that $\mu$ is a dynamical variable and calculate the value of $\mu$ by minimizing the low-energy effective potential (including the soft supersymmetry breaking terms). We will show that a local minimum exists where the supersymmetric Higgs mass is of $O(m_Z)$. This minimum is stable under gravity corrections if supersymmetry is broken at low-energies $\sim 10^5$ GeV. Thus, this scenario can solve simultaneously the doublet-triplet splitting problem and the $\mu$-problem.

2 The dynamical value of $\mu$

Let us promote the $\mu$-parameter to a superfield

$$\mu \rightarrow \lambda S,$$

where $S$ is a SM singlet superfield and $\lambda$ is its coupling to $H\bar{H}$. Since we are only interested in the vacuum where the Higgs doublets are light and the Higgs color triplets are heavy, we expand $\lambda S$ around $3\lambda'M_G$. This means making the replacement $S \rightarrow S + 3\lambda'M_G/\lambda$ in the superpotential (3). The low-energy effective potential for the neutral scalars is given by,

$$V = V_{SUSY} + V_{soft},$$

where

$$V_{SUSY} = |\lambda S|^2(|H|^2 + |\bar{H}|^2) + |\lambda H\bar{H}|^2 + \frac{g^2 + g'^2}{8}(|H|^2 - |\bar{H}|^2)^2,$$

and

$$V_{soft} = m_H^2|H|^2 + m_{\bar{H}}^2|\bar{H}|^2 + m_S^2|\lambda S|^2 - (B\lambda S H H + h.c.).$$

The origin of the soft terms will be discussed in the next section. Considering the limit $\lambda \ll 1$ (as we will see, in this limit the experimental constraints are always satisfied), we have that the potential (3) has a stationary value for

$$\frac{1}{2}m_Z^2 = \frac{m_H^2 - m_{\bar{H}}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2,$$
\[
\sin 2\beta = \frac{2B\mu}{m_W^2 + m_H^2 + 2\mu^2},
\]
(10)
\[
\mu = \frac{m_W^2 B \sin 2\beta}{2m_W^2 + g^2 m_S^2}.
\]
(11)

where \(m_W^2 = \frac{g^2}{2}(\langle H \rangle^2 + \langle \bar{H} \rangle^2)\), \(\tan \beta = \langle H \rangle / \langle \bar{H} \rangle\) and \(\mu = \lambda \langle S \rangle\). Eqs. (9) and (10) are the usual minimization conditions of the MSSM. Notice that we have an extra condition [eq. (11)] coming from the stationarity of the potential with respect to the new variable \(S\). We still have to guarantee that eqs. (9)-(11) lead to a (at least, local) minimum of the potential. This means that the scalar mass matrices must have positive eigenvalues. While charged and pseudoscalar Higgs masses turn out to be always positive, we find that the condition of positive masses for the real part of the neutral scalars is very restrictive. The sign of the determinant of the scalar mass matrix is given by the quantity

\[
\text{Det} \mathcal{M}^2 \propto \left[ 1 + \frac{x^2}{\cos^2 2\beta} \left( 1 + \frac{B^2}{m_Z^2} \right) - \frac{x^3}{\cos^2 2\beta} \right],
\]
(12)

where \(x \equiv \frac{g^2 m_S^2}{2m_W^2}\). Requiring \(\text{Det} \mathcal{M}^2 > 0\), we obtain a bound on \(x\). This bound is approximately given by

\[
|x| < \left| \frac{m_Z \cos 2\beta}{\sqrt{m_Z^2 + B^2}} \right|.
\]
(13)

We can now use eqs. (11) and (12), and infer the values of \(\mu\) that lead to \(\text{Det} \mathcal{M}^2 > 0\) as a function of \(\tan \beta\) and \(B\). In Fig. 1 we plot the allowed area of the plane \(\mu-\tan \beta\) for different values of \(B\). This area is well approximate by values of \(\mu\) inside the interval

\[
\mu = \frac{1}{2} B \sin 2\beta \left( 1 \pm \frac{m_Z \cos 2\beta}{\sqrt{m_Z^2 + B^2}} \right)^{-1},
\]
(14)

that can be obtained using eqs. (11) and (13). As \(B\) increases, \(\mu\) approaches to the central value

\[
\mu \simeq \frac{1}{2} B \sin 2\beta,
\]
(15)

that we plot in Fig. 1 as a dashed line. This is our prediction for \(\mu\). We see that in order to have large values of \(\mu\), we need large values for \(B\) and/or small values for \(\tan \beta\). For example, a \(|\mu| \gtrsim 50\) GeV such that Higgsinos escape from LEP1.5 detection \([1]\) requires \(\tan \beta \lesssim 3.5, 5, 6\) for \(B \simeq 100, 150, 200\) GeV.

In the limit \(m_Z \ll B\), we can use eqs. (11) and (13) to write \(\mu\) as a function of the parameters of the MSSM:

\[
\mu^2 \simeq \frac{1}{2} \left( B^2 - m_H^2 - m_{\bar{H}}^2 \right).
\]
(16)

Requiring \(0 \leq \sin^2 2\beta \leq 1\), we obtain that \(B^2\) has to lay in the window

\[
2(m_H^2 + m_{\bar{H}}^2) \gtrsim B^2 \gtrsim (m_H^2 + m_{\bar{H}}^2).
\]
(17)

\(^1\)Except for the region \(B < m_Z\) and \(\cos 2\beta \simeq -1\). In this region, however, we find \(|\mu| \lesssim 50\) GeV.
Figure 1: Allowed region (in white) of the plane $\mu$–$\tan \beta$ for different values of $B$. The dashed lines correspond to $\mu = B \sin 2\beta/2$.

### 3 Origin of the soft breaking terms

Supersymmetry is usually assumed to be broken in a “hidden” sector. The supersymmetry breaking is transmitted from the hidden to the observable sector by either gravity or gauge interactions. In both scenarios soft terms like those in eq. (8) are induced and are proportional to $F/M \simeq O(m_Z)$ where $\sqrt{F}$ is the scale of supersymmetry breaking in the hidden sector and $M$ is the messenger mass in GMSB models, or the Planck mass ($M_P$) if gravity mediates the supersymmetry breaking. In a model with a dynamical $\mu$, however, there are two possible extra soft-terms that can be induced [7]:

$$V'_{\text{soft}} = m_{12}^2 \bar{H}H + \rho \lambda S + h.c.$$  \hspace{1cm} (18)

The origin of these terms is different from that of eq. (8); they turn out not to be proportional to $F/M$ and can destabilize the $m_Z - M_G$ hierarchy [7]. Here we will study the origin of these extra terms and the constraints on the scale $\sqrt{F}$ derived from the requirement $\rho^{1/3}, m_{12} \lesssim m_Z$. 

The terms of eq. (18) can be generated from different sources depending on the underlying theory at high energies:

a) In supergravity theories with flat Kähler metric, there are contributions to $\rho$ and $m_{12}$ arising when we shift the singlet, $S \to S + 3\lambda M_G/\lambda$ (see below eq. (5)), in the gravity-induced soft supersymmetry breaking terms:

\[
m^2_{3/2}|S|^2 \to \frac{3}{5\lambda}m^2_{3/2}M_{HC}S + \ldots ,
\]

\[
m_{3/2}(\lambda[1 + \epsilon]S - 3\lambda M_G)\bar{H}H \to \frac{3}{5}\epsilon m_{3/2}M_{HC}\bar{H}H + \ldots ,
\]

(19)

where $M_{HC} = 5\lambda M_G$ and $m_{3/2} = F/\sqrt{3}M_P$ is the gravitino mass. $\epsilon$ parametrizes deviations from proportionality between the superpotential (3) and the trilinear soft terms. Even if exact proportionality holds at $M_P$ ($\epsilon = 0$), it will not hold at $M_G$ due to loop effects. Thus, $\epsilon \simeq 1/(4\pi)^2 \simeq 10^{-2}$. The stability of the weak scale requires (from eq. (19)) $3\epsilon m_{3/2}M_{HC} \lesssim m^2_Z$ that leads, for $M_{HC} \sim 10^{15} - 10^{16}$ GeV, to a bound on $\sqrt{F}$:

\[
\sqrt{F} \lesssim 20 - 60 \text{ TeV} .
\]

(20)

This constraint can be relaxed if $m_{3/2} \ll F/M_P$ like in no-scale models [8], or can disappear if the MSSM is not embedded in a grand-unified theory (in such a case the singlet $S$ does not get a VEV of $O(M_G)$ and the contributions of eq. (19) do not arise).

b) In supergravity theories with nonminimal Kähler metric, one can have operators like

\[
\frac{1}{M_P} \int d^4\theta SXX^\dagger ,
\]

(21)

where $X$ denotes the superfield (in the hidden sector) that breaks supersymmetry. Once supersymmetry is broken, $\langle X \rangle = \theta^2 F$, the above operator generates a tadpole contribution given by

\[
\rho \simeq \frac{F^2}{\lambda M_P} .
\]

(22)

Requiring $\rho \lesssim (100 \text{ GeV})^3$, we obtain a bound on the supersymmetry breaking scale:

\[
\sqrt{\frac{F}{\sqrt{\lambda}}} \lesssim 10^6 \text{ GeV} .
\]

(23)

The contribution to $m_{12}$ from the operator (21) is zero (unless the scalar component of $X$ gets a VEV).

c) There are also nongravitational contributions to the tadpole term coming from loops of Higgs color triplets. These contributions can be understood as arising from the operator $\frac{1}{M_{HC}} \int d^4\theta SXX^\dagger$ induced when the heavy Higgs color triplets are integrated out at the one-loop level. This gives

\[
\rho \simeq \frac{1}{16\pi^2} \frac{M^2 m^2_{HC}}{M_{HC}} ,
\]

(24)
where \( m_{HC} \) is the color triplet soft mass and \( M \) is the messenger scale. If we impose \( \rho \lesssim (100 \text{ GeV})^3 \), we get an upper bound on \( M \):

\[
M \lesssim 10^{10} \text{ GeV},
\]

for \( M_{HC} \simeq 10^{16} \text{ GeV} \) and \( m_{HC} \simeq 100 \text{ GeV} \). There are also contributions to \( m_{12}^2 \) coming from loops of color triplets but they are small for \( M \lesssim 10^{10} \).

In models where gravity mediates the supersymmetry breaking \( (M \simeq M_P \text{ and } \sqrt{F} \simeq 10^{10} \text{ GeV}) \) the bounds (20), (23) or (25) are not fulfilled and the mechanism described in the previous section cannot be operative \[7\]. On the other hand, in GMSB models with low-energy supersymmetry breaking, \( M \simeq \sqrt{F} \simeq 10^5 \text{ GeV} \), these bounds are satisfied. Furthermore, in these theories the soft mass of \( S \) is one-loop factor suppressed with respect to the soft masses of the Higgs doublets

\[
m_S^2 \simeq \frac{1}{4\pi^2}(m_H^2 + m_{H}^2 + B^2) \ln \frac{m_Z}{M},
\]

and the constraint (13) can be also satisfied. Nevertheless, in the minimal GMSB model the \( B \)-parameter at the messenger scale is also a one-loop factor smaller than the other soft masses. This implies a small \( \mu \)-parameter (for \( B \sim 10 \text{ GeV} \), we find \( \mu \lesssim 15 \text{ GeV} \)). A possible way out is to have \( \rho \simeq (100 \text{ GeV})^3 \). In this case

\[
\mu \simeq \frac{g^2 \rho}{2m_W^2 + g^2 m_S^2},
\]

and we can have \( \mu \sim 100 \text{ GeV} \) even in the minimal GMSB model. Although this possibility could be viable, we do not see any reason why \( \rho = O(m_Z^3) \). A more interesting possibility is to consider GMSB models with messenger-matter mixing \[4\] or with messenger-Higgs mixing \[2\]. In these models a large value of \( B \) can be obtained \[2\]. For example, the coupling \( yHQ\bar{D}_M \) where \( Q \) and \( D_M \) denote the ordinary quark and messenger superfield respectively, would generate a \( B \)-parameter at the one-loop level given by

\[
B = \frac{3y^2 F}{16\pi^2 M}.
\]

Surprisingly, the contribution to the soft masses of the Higgs arising from \( yHQ\bar{D}_M \) is comparable, for \( F/M^2 \lesssim 0.1 \) \[4\], to the universal two-loop contribution due to the cancellation of the leading term of \( O(F^2/M^2) \) \[2, 4\]. In these GMSB models \( B \) comes out to be of the same order of the other soft masses and a \( \mu \)-parameter from eq. (14) can be larger than 50 GeV. Considering that a messenger-matter mixing can also avoid some cosmological problems present in GMSB theories \[4\], we find this scenario very attractive. This is the simplest mechanism to generate a \( \mu \neq 0 \).

## 4 The light spectrum and fine-tuning criteria

In the limit that \( \rho \) and \( m_{12} \) are smaller than the weak scale, the potential \( \Theta \) has an approximate extra U(1) symmetry under which \( S \) transforms nontrivially. There is a pseudo-Goldstone boson
associated with the spontaneous breaking of this U(1) and its mass is given by

\[ m_{PG} \simeq \lambda \sqrt{\frac{\rho}{\mu} + \frac{m_W^2}{\mu^2} \sin 2\beta} \cdot m_{12}^2. \]  

(29)

Depending on the origin of \( \rho \) and \( m_{12} \) [(a), (b) or (c) in the previous section], \( m_{PG} \) is given by

\[ m_{PG} \sim \begin{cases} 
100 \left( \frac{\lambda}{0.1} \right) \left( \frac{\sqrt{F}}{10^5 \text{ GeV}} \right) \text{ GeV}, \\
100 \left( \frac{\lambda}{0.1} \right) \left( \frac{F}{10^5 \text{ GeV}^2} \right) \text{ MeV}, \\
0.1 \left( \frac{\lambda}{0.1} \right) \left( \frac{M}{10^5 \text{ GeV}} \right) \text{ MeV}, 
\end{cases} \] 

(30)

where we have used eq. (13), eq. (22) and eq. (24) respectively. In the first case, the pseudo-Goldstone is very heavy and can easily escape detection \(^2\). In the second and third case of eq. (30) such a light particle with axion-like couplings is excluded by the LEP experiment if \( \lambda \sim 1 \). Nevertheless, we have the freedom to reduce \( \lambda \) and decouple the pseudo-Goldstone from matter without modifying the above prediction on \( \mu \) (notice that eqs. (9)-(11) do not depend on \( \lambda \)). In the limit of small \( \lambda \), the full supermultiplet \( S \) is in fact light (the scalar and fermion component have masses \( \sqrt{2\lambda m_W}/g \) and \( 2\lambda^2 m_W^2 \sin 2\beta/(g^2 \mu) \) respectively) but it is also almost decoupled from matter. Constraints from Z-decays require \( |\mu| \lesssim 0.1 \). Searches for axion-like particles in hadron collisions \(^4\) put the bound \( \lambda \lesssim 10^{-2} \), but this only applies for \( m_{PG} \lesssim 200 \text{ MeV} \). Astrophysical constraints are more severe and imply \( \lambda \lesssim 10^{-7} \). These, however, can be evaded if \( m_{PG} \gtrsim 1 \text{ MeV} \) that can be easily satisfied.

Let us now turn to the fine-tuning criteria. It can be seen from eq. (4) that if the soft masses of the Higgs are much larger than \( m_Z \), the \( \mu \)-parameter has to be fine-tuned

\[ \mu^2 \simeq \frac{m_H^2 - m_H^2 \tan^2 \beta}{\tan^2 \beta - 1}, \] 

(31)

in order to have the right value of \( m_Z^2 \). Since \( \mu \) and the soft masses are independent parameters in the MSSM, such a fine-tuning is unnatural. The degree of fine-tuning can be estimated as \( ^2 \)

\[ \Delta m_H^2 = \frac{m_H^2}{m_Z^2} \frac{\partial m_Z^2}{\partial m_H^2} \sim \frac{m_H^2}{m_Z^2}, \] 

(32)

that can be used to put upper bounds on the soft masses \( ^2 \). In our model the \( \mu \)-parameter is a dynamical variable that adjusts itself in order to minimize the energy; one may then think that no fine tuning at all is needed even if soft masses are large. However, for \( B \gg m_Z \), we see from eq. (14) that \( \mu \) is forced to approach its asymptotic value \( \mu = B \sin 2\beta/2 \). If this equality holds, we need to fine-tune the potential parameters to satisfy also (9) and (10); this situation is in fact equivalent to the MSSM one. We can quantify the amount of fine-tuning which is needed in our model when \( B \sim m_H \gg m_Z \) by following a procedure similar to the MSSM one. Using eqs. (9)-(11) and (13), we obtain

\[ \Delta m_H^2 \sim \frac{m_H}{m_Z}. \] 

(33)

\(^2\)In this case \( \lambda \) could be of \( O(1) \). We have checked that the effect of a \( \lambda \sim 1 \) is to slightly enlarge the allowed regions of Fig. 1 for \( \tan \beta \) close to 1.
We see that the fine-tuning scales linearly with the ratio $m_H/m_Z$ instead of quadratically as in the MSSM. This implies less fine-tuning to have the electroweak scale smaller than the sparticle masses. Nevertheless, we have to stress that as $B$ increases, we need $m_S^2$ to decrease (see eq. (13)). This could be unnatural if $m_S^2$ is tied to the Higgs doublet soft masses such as in eq. (26). To address this question properly, one needs to specify the details of the mechanism that generates the soft breaking terms; this is beyond the scope of this paper.

5 Conclusions

We have proposed a scenario where the supersymmetric Higgs mass ($\mu$-parameter) is dynamically determined. This has allowed to calculate $\mu$ as a function of the soft breaking terms of the potential and then reduce the parameters of the MSSM. We have found that $\mu$ gets a weak scale value close to $B \sin 2\beta/2$. Thus, this scenario provides a solution to the $\mu$-problem. If the MSSM is embedded in a GUT, this scenario solves automatically the doublet-triplet splitting problem. Our mechanism is operative in models with low-energy supersymmetry breaking scale such as in GMSB theories. In such theories we can obtain a realistic $\mu$-parameter. We have also shown that naturalness constraints on soft masses seem to be less stringent than in the usual MSSM.

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