Binding energy of a holographic deuteron and tritium in anti-de-Sitter space/conformal field theory (AdS/CFT)

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In the large ’t Hooft coupling limit, the hadronic size of baryon is small and nucleon-nucleon potential is obtained from massless pseudo-scalar exchanges and an infinite tower of spin one mesons exchanges. In this paper we use the holographic nucleon-nucleon interaction and obtain the corresponding potential and binding energy for deuteron and tritium nuclei. The obtained potentials are repulsive at short distances and clearly become zero by increasing distance as we expected.

Key words: AdS/CFT Correspondence; QCD Holography; Nucleon-Meson Interaction; Nucleon-Nucleon Potential; Binding Energy.

PACS numbers:

I. INTRODUCTION

One application of AdS/CFT duality is in low energy hadron dynamics [1] that is referred as holographic QCD or AdS/QCD. This method express two related issues from opposite directions, one from string theory [2,3] and the other from low energy chiral effective field theory of mesons and baryons[4,5]. From the string theory viewpoint, what extent the gravity theory in the bulk sector in a controlled weak coupling limit, is interested that is called, via duality, the strongly coupled dynamics of QCD. On the other hand, from the low energy chiral effective field theory outlook the purpose is whether holographic QCD can make clear predictions on processes which are difficult to study by QCD proper.

There are many remarkable of this type examples [6-11] such as chiral dynamics of hadrons, in particular baryons at low energy that are typically of strong-coupling QCD and very difficult to obtain by QCD techniques. Between the holographic models of QCD suggested recently, the Sakai and Sugimoto (SS) model [2] is one of the most interesting and realistic model, because of accurate results of this model. For example the predicted results from SS model on glueball spectrum of pure QCD are in a good agreement with lattice simulation[12]. Also this model successfully described baryons and their interactions with mesons[2,13,14]. This is a $D_4 - D_8$ model that involves a large number of colors, $N_c$ large ’t Hooft coupling $\lambda$ and quenching of fermions.

The holographic baryon in the $D_4 - D_8$ model is same as the skyrmion of chiral perturbation theory. Actually ‘holographic baryon’ is a direct uplift of skyrmion in the holographic picture, but considering baryons as solitons is not a proper way to obtain the nucleon-nucleon interaction. It is arises because finding a suitable configuration is impossible for such complicated solitons.

In order to study baryons interaction in large distances, where the inter-baryon distance is large compared with the size of the baryons, it is a good approximation to consider baryons as a point-like particles. In this case, the interactions can be all ascribed by exchange of light particles such as mesons and one can find the baryon-baryon interaction with the Feynman diagrams using cubic interaction vertices including baryon currents and light mesons[15]. Fortunately from the $D_4 - D_8$ holographic QCD model, all nucleon-meson coupling constants, at least in large $\lambda N_c$, is obtained [13]. Also some of these coupling constants such as the axial coupling to pions $g_A$ and vector meson couplings $g_{\rho NN}$ and $g_{\omega NN}$ are good agreed with experimental data. Recently the nucleon-nucleon potential in holographic picture, is studied using the meson exchange method [16].

In this paper we are going to calculate the binding energy of light nuclei such as deuteron and tritium nuclei using the AdS/QCD. So we use the $D_4 - D_8$ model, nucleon-meson interaction and nucleon-nucleon potential. Then we consider the exchange of pions, isospin singlet vector mesons, isospin triplet vector mesons and triplet axial-vector mesons in this potential. The minimum of the nuclei potential is considered as nuclei binding energy. Finally we obtain the radius of these nuclei in the holographic picture.

II. A $D_4 - D_8$ HOLOGRAPHIC QCD

In this model $N_c$ stack of $D_4$-branes and $N_f D_8$-branes are considered in the background of TypeIIA superstring [2] and the flavor symmetries of the quark sector are embedded into a $U(N_f)$ gauge symmetry in $R^{1+5} \times I$. By restricting to the modes that are localized near the origin of the fifth direction, which

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is topologically an interval, we can arrive to the four dimensional low energy physics. Also the massless part of the theory is pure $U(N_c)$ Yang-Mills theory because the fermions are given anti-periodic boundary condition. In the large $N_c$ limit, the $D_4$ branes dynamics is dual to a closed string theory in the curved background with flux in accordance with general AdS/CFT idea. In the large ’t Hooft coupling limit ($\lambda = g_Y^2 N_c >> 1$) and neglecting the gravitational backreaction from the $D_8$ branes the metric is \cite{17}

\[ ds^2 = \left( \frac{U}{R} \right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu + f(U) dt^2 \]
\[ + \left( \frac{R^3}{U} \right)^{3/2} \left( \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right), \]
\[ (1) \]

where $R^3 = \pi g_s N c l_s^4$ and $f(U) = 1 - U_{KK}^3 / U^3$. The coordinate $\tau$ is compactified as $\tau = \tau + \delta \tau$ with $\delta \tau = 4 \pi R^3 / (3 U_{KK}^2)$.

The effective action on a $D_8$ brane embedding in $D_4$ background has the following form

\[ S_{D8} = \mu_8 \int d^9 x e^{-\phi} \sqrt{-\det (g_{MN} + 2 \pi \alpha' F_{MN})} + \mu_8 \int C_3 \wedge \text{Tr} e^{2 \pi \alpha' F}, \]
\[ (2) \]

with $\mu_8 = \frac{2 \pi}{(2 \ell_s)^4}$. By introducing the conformal coordinate $w$ instead of holographic coordinate $U$ as

\[ w = \int_{U_{KK}}^{U} \frac{R^{3/2} dU'}{\sqrt{U^3 - U_{KK}^3}}, \]
\[ (3) \]

the noncompact 5D part of the metric is conformally flat, then the induced metric on $D_8$ brane has the following form

\[ g_{s+1} = \frac{R^{3/2}(w)}{U^{3/2}} (dw^2 + \eta_{\mu\nu} dx^\mu dx^\nu) + \frac{R^{3/2}}{U^{1/2}(w)} d\Omega_4^2, \]
\[ (4) \]

where $U_{KK} = 2/9 g_Y^2 N_c M_{KK} l_s^4$. $M_{KK}$, $\lambda$ and $N_c$ determine all the physical scales such as the QCD scale and the pion decay constant.

In the low energy limit, the worldvolume dynamics of the multi-$D_8$ brane system gives the following Yang-Mills action with a Chern-Simons term as

\[ \frac{1}{4} \int_{S^4} e^{-\phi} V_{S^4} \frac{1}{2 \pi (2 \ell_s)^5} \text{tr} F_{mn} F_{mn} \]
\[ + \frac{N_c}{24 \pi^2} \int_{S^4} d \omega_5(A), \]
\[ (5) \]

where $V_{S^4}$ is the $S^4$ volume while the dilaton is

\[ e^{-\phi} = \frac{1}{g_s} \left( \frac{R}{U} \right)^{3/4}, \]
\[ (6) \]

and $d \omega_5(A) = \text{tr} F^3$.

III. BARYON HOLOGRAPHY AND NUCLEON-NUCLEON POTENTIAL

Witten introduced a $D_4$ brane wrapping the compact $S^4$ as a baryon vertex on the 5D space-time \cite{18}. It is shown that a $D_4$ brane wrapping $S^4$ looks like an object with electric charge with respect to the gauge field on $D_8$ and it is possible to say that $D_4$ brane spread inside $D_8$ brane as an instanton. The size of this instanton is determined by minimizing its total energy \cite{13,14}, which is combined mass and coulomb energy,

\[ \rho_{\text{baryon}} \sim \frac{9.6}{M_{KK} \sqrt{g_Y N}}. \]
\[ (7) \]

Thus, in the large ’t Hooft coupling limit, instantonic baryon is a small object in 5 dimension and baryon can be considered as a point-like quantum field in 5D. In consequence, there should couplings between this quantum field and the 5 dimensional gauge field moreover the standard Dirac kinetic and a position dependent mass term \cite{19}.

The action involving the baryon field and the gauge field in the conformal coordinate $(x^\mu, w)$ is written as

\[ \int d^4 x dw \left[ - i \bar{\gamma}^m D_m B - i m_b(w) B \bar{B} + g_5(w) \rho_{\text{baryon}}^2 \bar{B} \gamma^{mn} F_{mn} B \right] - \int d^4 x dw \frac{1}{4 e^2(w)} \text{tr} F_{mn} F^{mn}, \]
\[ (8) \]

$g_5(w)$ is an unknown function of $w$ that only evaluated at $w = 0$, $g_5(0) = 2 \pi^2 / 3$.

Since the four-dimensional low energy physics is found by restricting to the modes that are localized near the origin of fifth direction $w$, the physical 4D nucleons would arise as the lowest eigenmodes of the 5D baryon along $w$ coordinate. Thus the five-dimensional action, equation (8), must to be reduced to four dimensions. It can be done by applying the mode expansion for the baryon field and the gauge field and plugging these to the baryon action.

On one hand, the gauge field $A_\mu$, in the $A_5 = 0$ gauge, has the following mode expansion

\[ A_\mu(x, w) = i a_{\mu}(x) \psi_0(w) + i b_{\mu}(x) + \sum_n a_{\mu}^{(n)}(x) \psi^{(n)}(w). \]
\[ (9) \]
The eigenmode analysis was done by Sakai and Sugimoto in [2] previously. We only note that \( \psi_{(2k+1)}(w) \) is even, while \( \psi_{(2k)}(w) \) is odd under \( w \rightarrow -w \), corresponding to vector and axial-vector mesons respectively. Also the eigenfunctions \( \psi_{(n)} \) obey the following equation according to [2]

\[
-K^{-1/3} \partial_w (K^{1/3} \partial_w \psi_{(n)}) = (U^2_{KK} M^2_{KK}) \lambda_n \psi_{(n)}. \tag{10}
\]

where \( K = (V_{LR})^3 \). Also they satisfy the orthonormality condition

\[
\int dw \frac{e^{-\Phi} V_{SS}}{4 \pi (2 \pi \lambda_0)} \psi_{(n)}(w)^* \psi_{(m)}(w) = \delta_{nm}. \tag{11}
\]

We have to solve (10) with the normalization condition given by (11) to find the eigenfunction \( \psi_{(n)} \). These equations solved numerically by means of a shooting method. The corresponding computations are given in [2] in details.

On the other hand, the nucleon field can be expanded as \( B_{L,R}(x^\mu,w) = B_{L,R}(x^\mu) f_{L,R}(w) \) where \( \gamma^\mu B_{L,R} = \pm B_{L,R} \) are 4D chiral components. \( f_{L,R}(w) \) are profile functions that satisfy the following conditions in the interval \( w \in [-w_{max}, w_{max}] \)

\[
\partial_w f_L(w) + m_B f_L(w) = m_B f_R(w),
\]

\[
-\partial_w f_R(w) + m_B f_R(w) = m_B f_L(w). \tag{12}
\]

The eigenvalue, \( m_B \) is the mass of the nucleon mode \( B(x) \) where 4D Dirac field for the nucleon is \( B = \left( \begin{array}{c} B_L \\ B_R \end{array} \right) \) and the eigenfunctions \( f_{L,R}(w) \) are normalized as

\[
\int_{-w_{max}}^{w_{max}} dw |f_L(w)|^2 = \int_{-w_{max}}^{w_{max}} dw |f_R(w)|^2 = 1. \tag{13}
\]

Using the properties \( f_L(w) = \pm f_R(-w) \) as well as \( \psi_0(w) \) and \( \psi_n(w) \) under \( w \rightarrow -w \) and by plugging into the mode expansion of gauge field, the 4D effective action is achieved for nucleon

\[
\int dx^4 (-i \bar{B} \gamma^\mu \partial_\mu B - im_B \bar{B} B + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{axial}}), \tag{14}
\]

where the nucleon coupling to vector mesons, \( \mathcal{L}_{\text{vector}} \), and axial mesons, \( \mathcal{L}_{\text{axial}} \) are

\[
\mathcal{L}_{\text{vector}} = -i \bar{B} \gamma^\mu \beta_\mu B - \sum_{k \geq 0} g^{(k)}_V \bar{B} \gamma^\mu a_{(2k+1)}^{(k)} B,
\]

\[
\mathcal{L}_{\text{axial}} = -\frac{i g_A}{2} \bar{B} \gamma^\mu \gamma^5 \alpha_\mu B - \sum_{k \geq 1} g^{(k)}_A \bar{B} \gamma^\mu \gamma^5 a_{(2k)}^{(k)} B, \tag{15}
\]

where various coupling constants, \( g^{(k)}_A \) as well as the pion-nucleon axial coupling, \( g_A \) are calculated by suitable wave-function overlap integrals. These coupling constants are studied in [13] with all the specifics. Finally, in general, the one boson exchange nucleon-nucleon potential is written as [16]

\[
V_n + V_q + \sum_{k=1}^{\infty} V^q_{(k)} + \sum_{k=1}^{\infty} V^a_{(k)} + \sum_{k=1}^{\infty} V^f_{(k)}, \tag{16}
\]

that is a sum of the pseudo-scalar, vector and axial vector mesons exchange terms respectively. But only following four classes of these couplings have a leading contribution in nucleon-nucleon potential

\[
\frac{g_\pi NN M_{KK}}{2 m_N} \sim \frac{\tilde{g}_q NN M_{KK}}{2 m_N} \sim \frac{g_a NN M_{KK}}{2 m_N} \tag{17}
\]

In the \( D_4 - D_8 \) holography model, the pion mass is zero then one pion exchange potential (OPEP) in this sense has the following form

\[
V_\pi = \frac{1}{4 \pi} \left( \frac{g_\pi NN M_{KK}}{2 m_N} \right)^2 \frac{1}{M_{KK}^2} S_{12} \vec{r}_1 \cdot \vec{r}_2. \tag{18}
\]

Also, the holographic potentials for the isospin singlet vector mesons, \( \omega^{(k)} \), isospin triplet vector mesons, \( \rho^{(k)} \) and the triplet axial-vector mesons, \( a^{(k)} \) are

\[
V^{\omega(k)} = \frac{1}{4 \pi} \left( \frac{g_\omega NN M_{KK}}{2 m_N} \right)^2 m^{\omega(k)} \ y_0 (m^{\omega(k)} r), \tag{19}
\]

and

\[
V^{\rho(k)} \simeq \frac{1}{4 \pi} \left( \frac{\tilde{g}_{\rho NN} M_{KK}}{2 m_N} \right)^2 \frac{m^{\rho(k)}^3}{3 M_{KK}^2} \times \right. \left. [2 y_0(m^{\rho(k)} r) \vec{\sigma}_1 \cdot \vec{\sigma}_2 - y_2(m^{\rho(k)} r) S_{12}(\vec{r})] \vec{r}_1 \cdot \vec{r}_2, \tag{20}
\]

\[
V^{a(k)} \simeq \frac{1}{4 \pi} \left( \frac{g_{a NN} M_{KK}}{3} \right) \frac{m^{a(k)}}{3} \times \right. \left. [-2 y_0(m^{a(k)} r) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + y_2(m^{a(k)} r) S_{12}(\vec{r})] \vec{r}_1 \cdot \vec{r}_2, \tag{21}
\]

respectively.

In the above equations, level \( p \) is determined by distance scale and

\[
y_0(x) = \frac{e^{-x}}{x}, \quad y_2(x) = \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{x}. \tag{23}
\]

The massess of all mesons are of order \( M_{KK} \) and \( m^{\rho(k)} = m^{\omega(k)} < m^{a(k)} \).

In general, for large limited \( \lambda \), in the smallest distance, \( 1/\sqrt{\lambda M_{KK}} \), the one meson exchange potential is satisfied. Also, \( \rho \simeq \lambda/10 \) is an acceptable value for this potential.

In the large \( \lambda N_c \) limit, the coupling constants are given by [13]

\[
\frac{g_\pi NN M_{KK}}{2 m_N} \approx 8.43 \sqrt{\frac{N_c}{\lambda}}. \]
\( g_{\omega N N} \cong \sqrt{2.33 \pi^3} \hat{\psi}\left(2k - 1\right)(0) \sqrt{N_c} \sqrt{\frac{N_c}{\lambda}}, \)

\[ \hat{g}_{\rho N N} \cong \sqrt{2 \pi^3} \hat{\psi}\left(2k - 1\right)(0) \sqrt{\frac{N_c}{\lambda}}, \]

\[ g_{\sigma N N} \cong \sqrt{2 \pi^3} \hat{\psi}'\left(2k\right)(0) \sqrt{\frac{N_c}{\lambda}} = \chi_k \sqrt{\frac{N_c}{\lambda}}. \]  

(24)

where the coefficients, \( \xi_k, \zeta_k, \chi_k \), are calculated using the \( \hat{\psi} \) values by numerical methods. The value of these coefficients, calculated in [16], are listed in Table I.

**TABLE I: Numerical results for \( \hat{\psi}(2k-1)(0), \hat{\psi}'(2k)(0), \xi_k, \zeta_k \) and \( \chi_k \) for spin one mesons interacting with nucleons[16].**

| \( k \) | \( \hat{\psi}(2k-1)(0) \) | \( \xi_k \) | \( \zeta_k \) | \( \hat{\psi}'(2k)(0) \) | \( \chi_k \) |
|---|---|---|---|---|---|
| 1 | 0.5973 | 24.44 | 8.925 | 0.629 | 9.40 |
| 2 | 0.5450 | 22.30 | 8.143 | 1.10 | 16.4 |
| 3 | 0.5328 | 21.81 | 7.961 | 1.56 | 23.3 |
| 4 | 0.5288 | 21.64 | 7.901 | 2.02 | 30.1 |
| 5 | 0.5270 | 21.57 | 7.874 | 2.47 | 36.9 |
| 6 | 0.5261 | 21.52 | 7.860 | 2.93 | 43.8 |
| 7 | 0.5255 | 21.50 | 7.852 | 3.38 | 50.5 |
| 8 | 0.5251 | 21.48 | 7.846 | 3.83 | 57.3 |
| 9 | 0.5249 | 21.48 | 7.843 | 4.29 | 64.1 |
| 10 | 0.5247 | 21.47 | 7.840 | 4.74 | 70.9 |

**IV. THE BINDING ENERGY OF DEUTERON AND TRITIUM**

Here we aim to calculate the binding energy of deuteron and tritium nuclei using the holographic nucleon-nucleon potential represented in section 3. To calculate the binding energy of deuteron, the following potential is considered

\[ V_{\text{deuteron}}^{\text{holo}} = V_C + (V_T^\sigma \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^S S_{12}) \vec{r}_1 \cdot \vec{r}_2. \]  

(25)

where

\[ V_C = \sum_{k=1}^{10} \frac{1}{4\pi} \left( g_{\omega(k) N N} \right)^2 \left( m_{\omega(k)} y_0(m_{\omega(k)} r) m, \right) \]  

(26)

\[ V_T^\sigma = \sum_{k=1}^{10} \frac{1}{4\pi} \left( \hat{g}_{\rho(k) N N} M_{KK} \right)^2 \frac{m_{\rho(k)}}{3 M_{KK}^2} \left[ -2 y_0 \left( m_{\rho(k)} r \right) \right] \]  

and,

\[ V_T^S = \frac{1}{4\pi} \left( \hat{g}_{\sigma N N} M_{KK} \right)^2 \frac{1}{M_{KK}^2}. \]  

(27)

The values of couplings constants for different amount of \( k \), along with the mass of the vector and axial vector mesons (in unit of \( M_{KK} \) and for large \( N_c \)) are presented in the table II. In these computations, we choose \( \lambda = 400 \), \( m_N = 0.55GeV \) and \( N_c = 3 \) for realistic QCD.

**TABLE II: Numerical results for masses and coupling constants for spin one mesons interacting with nucleons in the large \( N_c \) limit. We choose \( \lambda = 400 \), \( m_N = 0.55GeV \) and \( N_c = 3 \) for realistic QCD.**

| \( k \) | \( m_{\omega(k)} \) | \( m_{\rho(k)} \) | \( \hat{g}_{\rho(k) N N} \) | \( \hat{g}_{\sigma(k) N N} \) | \( g_{\omega(k) N N} \) |
|---|---|---|---|---|---|
| 1 | 0.818 | 1.25 | 2.1165 | 0.7055 | 0.8140 |
| 2 | 1.69 | 2.13 | 1.9312 | 0.6437 | 1.4202 |
| 3 | 2.57 | 3.00 | 1.8888 | 0.6296 | 2.0178 |
| 4 | 3.44 | 3.87 | 1.8740 | 0.6246 | 2.6067 |
| 5 | 4.30 | 4.73 | 1.8680 | 0.6226 | 3.1956 |
| 6 | 5.17 | 5.59 | 1.8636 | 0.6212 | 3.7931 |
| 7 | 6.03 | 6.46 | 1.8619 | 0.6206 | 4.3734 |
| 8 | 6.89 | 7.32 | 1.8602 | 0.6200 | 4.9623 |
| 9 | 7.75 | 8.19 | 1.8602 | 0.6200 | 5.5512 |
| 10 | 8.62 | 9.05 | 1.8593 | 0.6197 | 6.1401 |

The deuteron nucleus consist of one proton and one neutron, thus by superselection rules we have

\[ S_{12} = 2, \quad \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, \quad \vec{\tau}_1 \cdot \vec{\tau}_2 = -3. \]  

(29)

The deuteron potential for the large \( N_c \) with \( p = 10 \) is calculated and has been shown in figure 1. As it is clear from this figure, the deuteron potential has a minimum point at \( 4.41M_{KK} \). For distance, \( r \) less than \( r_{min} \) potential increases rapidly and become infinity at \( r = 0 \) as expected. The minimum value of potential is \(-1.9645M_{KK} N_c/4\pi\lambda\). So the binding energy of deuteron is obtained roughly \(-2.204MeV\) that is consistent with the experimental nuclear data. Also, tritium consist of three nucleons, two neutrons and one proton, so we suppose the following form for the tritium potential

\[ V_{\text{Tritium}}^{\text{holo}} = V_{12} + V_{13} + V_{23} \]

\[ = V_C + (V_T^\sigma \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T^S S_{12}) \vec{r}_1 \cdot \vec{r}_2 \]

\[ + V_C + (V_T^\sigma \vec{\sigma}_1 \cdot \vec{\sigma}_3 + V_T^S S_{12}) \vec{r}_1 \cdot \vec{r}_3 \]

\[ + V_C + (V_T^\sigma \vec{\sigma}_2 \cdot \vec{\sigma}_3 + V_T^S S_{23}) \vec{r}_2 \cdot \vec{r}_3. \]  

(30)

The superselection rules for this three-nucleon systems implies that

\[ S_{12} = 2, \quad \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 1, \quad \vec{\tau}_1 \cdot \vec{\tau}_2 = -3 \]

\[ S_{13} = 0, \quad \vec{\sigma}_1 \cdot \vec{\sigma}_3 = -3, \quad \vec{\tau}_1 \cdot \vec{\tau}_3 = -3 \]

\[ S_{23} = 0, \quad \vec{\sigma}_2 \cdot \vec{\sigma}_3 = -3, \quad \vec{\tau}_2 \cdot \vec{\tau}_3 = 1. \]  

(31)
The holographic potential of tritium in terms of $M_{KK}r$ has been shown in figure 2. This potential also has a minimum that occurs in $7.46 M_{KK}$. The value of potential in its minimum is $-0.617 M_{KK} N_c / 4\pi \lambda$, so the binding energy of tritium is equal to $-1.034 \, MeV$. This figure also shows the repulsive behavior at short distances.

FIG. 1: The deuteron potential in large $\lambda N_c$ limit and $p=10$. The horizontal axis is $r M_{KK}$, while the potential energy along the vertical axis is in unit of $M_{KK} N_c / 4\pi \lambda$.

FIG. 2: The Tritium potential in large $\lambda N_c$ limit and $p=10$. The horizontal axis is $r M_{KK}$, while the potential energy along the vertical axis is in unit of $M_{KK} N_c / 4\pi \lambda$.

This method may be improved to obtain binding energies of heavier nuclei by considering exchange of heavier mesons.

V. CONCLUSION

In this investigation we calculated the deuteron and tritium binding energy using the QCD holography model. Here we used the nucleon-nucleon interaction in the $D_4 - D_8$ model in the base of one-boson exchange picture. This potential involves only the exchanges of pions, isospin singlet mesons, isospin triplet mesons and triplet axial-vector mesons. We selected the $\lambda = 400$ and at least 10 terms of infinite tower of spin one mesons are considered.

We depicted the deuteron and tritium potential in terms of $M_{KK}r$ and in unit of $M_{KK} N_c / 4\pi \lambda$. As it is indicated in figures 1 and 2, these potentials have repulsive behavior at short distances and became roughly zero at large $M_{KK}r$. The deuteron potential have a shallow minimum in depth $\sim -13.84 M_{KK} N_c / \lambda$ around the $r M_{KK} = 4.41$. The tritium potential too, have a more shallow minimum around the $r M_{KK} = 7.46$ with depth $\sim -4.35 M_{KK} N_c / \lambda$. Thus by using this information the binding energies of deuteron and tritium nuclei approximated by $-2.204 \, MeV$ and $-1.039 \, MeV$, respectively.
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