FeaturesMethod for Selecting VMD Parameters based on Spectrum without Modal Overlap

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Abstract: Variation Mode Decomposition (VMD) can decompose a complex set of signals into the form of a sum of multiple amplitude modulated FM eigenmode functions (IMFs). Compared with EMD decomposition, the VMD method has a strict mathematical model foundation, which further avoids modal overlap and has higher computational efficiency. Therefore, VMD has been widely used in engineering. Different from the adaptive decomposition of the signal by EMD, the VMD method needs to preset mode parameter K and the second penalty factor α in the VMD decomposition before decomposing the signals. In the case of reasonable parameter setting, VMD method shows better robustness, otherwise it will greatly affect the accuracy of decomposition results. In this paper, the influence of the two parameters on the decomposition effect is discussed by combining the correlation coefficient, kurtosis index and spectrum graph. Based on the problem that these two important parameters are difficult to select, a constraint rule and screening strategy without modal overlap is proposed. According to the parameter selection strategy, mode parameter K and the second penalty factor α are automatically selected to ensure that modal overlap does not occur. Signal decomposition shall be as detailed and accurate as possible. By decomposing the bearing data, the rationality of the parameter selection strategy is proved. A new idea is put forward for the research of VMD.

1. Introduction

Empirical Mode Decomposition (EMD) is a signal analysis method proposed by NASA's American Chinese, Nordne E. Huang et al. in 1998[1]. The complex signal can be decomposed into the sum of multiple Intrinsic Mode Functions (IMFs) by EMD[2]. In EMD, the origin signal is decomposed through residual recursion, so EMD method has mode overlap, pseudo-component and endpoint effect[3]. Variational Mode Decomposition (VMD) uses a non-recursive decomposition model[4], whose essence is the construction and solution of the variational problem. The frequency center and bandwidth of each modal function are determined by iteratively seeking the extremum of the variational model so that the frequency domain part and each component can be effectively separated. The VMD algorithm can avoid modal overlap and false components in the case of low SNR and
exhibits good noise robustness\cite{5}, VMD has been widely used in engineering and it has been many scholars’ research direction\cite{6}\cite{7}.

Different from EMD to realize the adaptive decomposition of signals, the decomposition parameters need to be preset in VMD decomposition: mode parameter $K$ and the quadratic penalty factor $\alpha$. Unsuitable $K$ and $\alpha$ have a very large impact on the decomposition effect, so adaptive selection of decomposition parameters is important. In view of the uncertainty in the selection of VMD decomposition parameters, many scholars have studied it. Zhang, X et al\cite{8} proposed a parameter adaptive VMD method based on grasshopper optimization algorithm (GOA) to analyze vibration signals. Zan T et al\cite{9} determined the value of mode number $k$ by instantaneous frequency average judgment method, and then processed the fault diagnosis signal by VMD.

2. VMD method Introduction

VMD uses a non-recursive decomposition model, whose essence is the construction and solution of the variational problem. It is assumed that the original signal is composed of modal function, and each modal function is an AM-FM signal with different center frequencies. The frequency center and bandwidth is determined by extremum of the variational model sought iteratively.

The extremum of the variational model is constructed by iterative search to determine the frequency center and bandwidth of each modal function. Thereby, the frequency domain portion of the signal and the components can be effectively separated.

Actually, the VMD decomposition process is the optimal solution to solve constrained variational problems. The constrained variational problems are as follows:

$$
\min_{\{u, k\}} \left\{ \sum_{t} \left( \left\| \left( \delta(t) + \frac{j}{\pi t} \right) \ast u(t) \right\| e^{-j\omega_k t} \right\}^2 + \sum_{t} u(t) = f \right\} \tag{1}
$$

In order to solve the above variational constraint problem, the quadratic penalty parameter $\alpha$ and the Lagrangian multiplication operator $\lambda(t)$ are introduced to transform the constrained variational problem into an unconstrained variational problem. The quadratic penalty parameter $\alpha$ makes the variable separation problem highly nonlinear and non-convex, which ensures that the signal can be accurately decomposed under Gaussian noise interference. The Lagrangian multiplication operator $\lambda(t)$ guarantees the rigor of finding the optimal solution for each IMF’s bandwidth. The augmented Lagrangian function is as follows:

$$
\ell(\{u, k\}, \lambda) = \alpha \sum_{t} \left\| \left( \delta(t) + \frac{j}{\pi t} \right) \ast u(t) \right\| e^{-j\omega_k t} + \left\| f(t) - \sum_{t} u(t) \right\|_2 + \left\langle \lambda(t), f(t) - \sum_{t} u(t) \right\rangle \tag{2}
$$

Where: * indicates convolution operation, <> indicates inner product operation.

Iteratively updates $u_k$, $\omega_k$ and $\lambda(t)$ with the alternating direction multiplier method to solve the saddle point of the augmented Lagrangian function of equation (2), that is, the optimal solution of the constrained variational model.

3. Important parameters of the VMD method

3.1. Decomposition layer number $K$

The correlation coefficient and Kurtosis index are used to investigate the influence of the selection of the mode parameter $K$ on the decomposition effect.

(1) Correlation coefficient (COR) is an index that measures the degree of correlation between two signals\cite{10}. The correlation coefficient $pxy$ of two time series $x(n)$, $y(n)$ is defined as:
The magnitude of the correlation coefficient $p_{xy}$ measures the correlation degree between the two time series $x(n)$, $y(n)$. The correlation coefficients between the K modal components and the original signal are calculated respectively, and the correlation degree between each modal component and the original signal is measured according to the magnitude of the correlation coefficient, so as to determine the validity of each modal component.

According to the Kurtosis calculated by the Kurtosis coefficient theory, the number of fault components in the signal can be judged according to the magnitude of the value\textsuperscript{[11]} because the Kurtosis value is only affected by the impact signal in the collected signal irrelevant to other factors. The Kurtosis of the vibration signal is calculated using the following formula:

$$\text{Kurtosis} = \frac{1}{N} \sum_{i=0}^{N} \left( \frac{x_i - \bar{x}}{\delta} \right)^4 = \frac{1}{\delta^4 N} \sum_{i=0}^{N} (x_i - \bar{x})^4$$

The bearing outer ring fault test data of the Case Western Reserve University bearing data center website\textsuperscript{[12]} is used to perform VMD decomposition, and the value of the quadratic penalty factor $\alpha$ is set 5000 unchanged, and the number of decomposition layers was set in sequence $K = 3, 4, 5$. The correlation coefficient between each component and the original signal and the Kurtosis of each component under different $K$ values are calculated respectively, as shown in Table 1:

| Table 1 Correlation coefficient and Kurtosis table for each mode with different decomposition layers |
|---|---|---|---|
| | IMF1 | IMF2 | IMF3 |
| K=3 | | | |
| COR | 0.2592 | 0.5852 | 0.6070 |
| Kurtosis | 3.6864 | 3.5544 | 4.4422 |
| K=4 | | | |
| COR | 0.2130 | 0.3777 | 0.5806 | 0.6052 |
| Kurtosis | 3.7737 | 2.4018 | 3.5280 | 4.4360 |
| K=5 | | | |
| COR | 0.2059 | 0.3752 | 0.5654 | 0.4458 | 0.5392 |
| Kurtosis | 3.7787 | 2.3990 | 3.5044 | 3.5580 | 3.7263 |

In the experiments of different decomposition layers, the calculated COR and Kurtosis values show that when the mode parameter $K$ is relatively large, the signal can be decomposed in more detail and accurately, but with the increase of $K$, the Kurtosis value of each modality will become smaller, indicating that the fault information contained in each modality will decrease as the number of decomposition layers increases; On the contrary, when the value of the mode parameter $K$ is small, the Kurtosis value of each modal component becomes larger, indicating that the fault information contained in each modality is more than that of large $K$. However, if the value of the decomposition layer is too small, the signal is not fully decomposed and the fault information is implicit. Therefore, setting the appropriate mode parameter $K$ is a prerequisite for successful fault diagnosis.

### 3.2. quadratic penalty factor $\alpha$

The quadratic penalty factor called $\alpha$ is another important parameter in the VMD method. As the number $K$ of decomposition layers, the quadratic penalty factor $\alpha$ is also needed to be preset before
decomposition. The VMD decomposition effect has different effect with different \( \alpha \). When the number \( K \) is determined, the smaller the value of the quadratic penalty factor \( \alpha \) is set, the larger the bandwidth of each modal component is decomposed, and the greater the possibility of modal overlap; The larger the value of the quadratic penalty factor \( \alpha \), the smaller the bandwidth of each component is decomposed.

For the selected outer ring fault data, the decomposition decomposition layer number \( K=4 \) is set, and the quadratic penalty factor \( \alpha \) is set to 100, 2000, 5000, 8000 values respectively, and the VMD decomposition is performed. The spectrum of each modal component is decomposed by FFT as shown in Figure 1.

![Figure 1. Spectrogram of each modality with different \( \alpha \) values](image)

The Figure 1 shows that when the value of \( \alpha \) is too large or too small, the modal overlap phenomenon is obvious and the decomposition effect is not very good. Therefore, it is very important to select the appropriate value of quadratic penalty factor \( \alpha \) in the VMD decomposition process. It is verified by experiments that the quadratic penalty factor \( \alpha \) does not have only a suitable certain value, but a suitable value interval. That is, the value of the quadratic penalty factor \( \alpha \) in the suitable interval has little effect on the decomposition effect.

4. VMD parameter constraint rules

It is found from experiments that if the mode parameter \( K \) is set to a large value, no matter how the quadratic penalty factor \( \alpha \) changes, the modal overlap problem cannot be avoided in the \( K \) modalities decomposed; and when the mode parameter \( K \) is set small, the decomposition is incomplete and the decomposition accuracy is decreased. At this time, no matter how the quadratic penalty factor \( \alpha \) changes, there is no meaning for decomposition.

In order to smoothly extract the fault information and ensure that the fault information is decomposed without dispersing, it is hoped that the fault features are prominent and obvious, and the signal is accurately decomposed. That is, it is hoped that the mode parameter \( K \) is as large as possible without modal overlap with a suitable \( \alpha \). According to it, a constraint rule of \( K \) and \( \alpha \) can be set: It is possible to find the quadratic penalty factor \( \alpha \) so that the \( K \) components that are decomposed without modal overlap. The maximum \( K \) and the corresponding \( \alpha \) are the appropriate value of decomposition layers and a suitable quadratic penalty factor.

It is determined by the following formula that the VMD decomposes whether each mode is overlapped:
\[
\sum_{i=1}^{K} \sum_{j=1}^{K} |I_k(i) \cdot I_k(j)| = 0 \quad (i \neq j)
\]  

Where \( K \) denotes the value of modal decomposed by VMD, \( I_k(i) \) and \( I_k(j) \) denote the \( i \)-th and \( j \)-th modal components in the same decomposition, and "\( \cdot \)" denotes the inner product operation of the \( i \)-th and \( j \)-th modals. If the equation (5) is satisfied, it means that no overlap occurs in the \( K \) modal decomposed by the VMD.

5. VMD parameter screening strategy

The screening strategy is based on the constraint rules of \( K \) and \( \alpha \) in equation (6) as follows:

\[
X_m = \text{Sort(Smooth(FFT)}(I_k))
\]

\[
X_m(1) < X_m(2) < \cdots < X_m(k)
\]

\[
\frac{Y_m}{\delta} \rightarrow Ljx, \frac{Y_m}{\delta} \rightarrow Rjx \quad (Ljx < Rjx)
\]

\[
Ljx(n + 1) - Rjx(n) \quad k \in \{3, 9\} \quad \alpha \in \{1000, 8000\}
\]

\[
\min[Ljx(n + 1) - Rjx(n)] > 0 \quad n \in (1, k)
\]

Where \( I_k \) represents the \( k \) components of the signal that are decomposed by the VMD; \( X_m \) represents the peak abscissa of the IMF component spectrum processed by Smooth filtering, \( Y_m \) represents the ordinate; \( Ljx \) represents the abscissa of the left punctuation found in the IMF component spectrum after smooth filtering, \( Rjx \) represents the abscissa of the right punctuation.

The specific implementation steps of the constraint rules and screening strategies of two important parameters \( K \) and \( \alpha \) in VMD decomposition are as follows:

1. \( K \) starts from 9 and decreases one by one; \( \alpha \) starts from 1000 and reaches 8000, and each increments by 500;

2. Perform the Smooth filter processing on the frequency domain waveform of the \( K \) IMFs, and treat the frequency domain waveform abstraction as a normal distribution curve, and find the peak value of the peak, and record the peak coordinates as \((X_m, Y_m)\);

3. setting a threshold \( \delta \), the parameter \( \eta \) defines as peak value \( Y_m \) divided by \( \delta \), in this case \( \delta \) is 20;

\[
\eta = \frac{Y_m}{\delta}
\]

(4) Taking the value of \( \eta \) as the ordinate, find the two coordinate points corresponding to this ordinate in the frequency domain curve. According to the size of the abscissa of the two coordinate points, the two coordinate points are respectively recorded as a left judgement and right judgement.

Figure 2. Punctuation selection diagram
(5) Sort K components according to peak-to-peak abscissa Xm of each component.

Figure 3. Schematic diagram of component sorting

(6) Subtract the abscissa of the component right punctuation by the horizontal coordinate of the left component of the next component, so as to judge the modal overlap occurs according to the positive or negative difference.

Figure 4. Schematic diagram of overlap decision

In order to ensure that the fault information is enough obvious in each component, the decompose the mode parameter K should be set as large as possible. Based on experiment, K is set from 9 to 1; The value of the quadratic penalty factor $\alpha$ does not need a certain value but a suitable interval from 1000 to 8000. When the constraint rules are satisfied, all modal can be decomposed without overlap.

6. Example analysis

The bearing fault test data is from the Case Western Reserve University bearing data center website. During the experiment, the motor speed is 1750r/min, the signal sampling frequency is 12000kHz, and the sampling points are 12000.

Inner ring failure frequency:

$$f_i = \frac{N}{2} f_r \left[ 1 + \frac{d}{D} \cos \alpha \right] \quad (8)$$

Outer ring fault frequency:

$$f_o = \frac{N}{2} f_r \left[ 1 - \frac{d}{D} \cos \alpha \right] \quad (9)$$
Where N represents the number of steel balls, d represents the diameter of the steel ball, D represents the pitch diameter of the ball set, α represents the contact angle, and fr represents the fundamental frequency of the rotating shaft. Among the selected fault data, N=9, d=8mm, D=40mm, α=0°, fb=29.95Hz. According to the eq.(8) and eq.(9), when the rotating shaft speed is 1750r/min, the fault characteristic frequency of bearing outer ring is about 107.8Hz, and the fault characteristic frequency of the inner ring of the bearing is about 161.7Hz.

The VMD method is used to decompose and process the bearing signal including the outer ring fault. After screening, the mode parameter K of decomposition layers conforming to the set constraint rule is 5, and the quadratic penalty factor α is 4500. The decomposition result spectrum and envelope spectrum are as shown:

![Figure 5. Outer Cycle Fault Decomposition Results](image)

The Spectrogram in Figure 5 shows that the five modalities that have been decomposed without overlap and satisfied with the decomposition requirements. The envelope spectrum of the IMF5 component is shown in the figure:

![Figure 6. IMF5 component envelope spectrum](image)

Figure 6 shows that the fundamental frequency of the rotating shaft is 29.95 Hz, the outer ring fault characteristic frequency of 107.8 Hz and its multiple spectral lines are clearly displayed.

The VMD method is used to decompose and process the bearing signal including the inner ring fault. After screening, the number K of decomposition layers conforming to the set constraint rule is 4, and the quadratic penalty factor α is 2000. The decomposition result spectrum and envelope spectrum are as shown:
Figure 7. Inner Cycle Fault Decomposition Results

The spectrogram in Figure 7 shows that the four modalities that have been decomposed without overlap and meet the decomposition requirements. The envelope spectrum of the IMF3 component is shown in the figure:

Figure 8. IMF3 component envelope spectrum

Figure 8 shows that the fundamental frequency of the rotating shaft is 29.95 Hz, the characteristic frequency of the inner ring fault is 161.7 Hz and its multiple spectral lines are clearly displayed.

From the decomposition experiments of the inner and outer ring faults, the $K$ and $\alpha$ selected according to the constraint rules are used as the VMD decomposition parameters, and the decomposed results are satisfied with the requirements in the actual data analysis. It demonstrates the feasibility of formulating VMD parameter constraint rules and screening strategies are reasonable.

7. Conclusion

In this paper, the influence of two important mode parameter $K$ and the quadratic penalty factor $\alpha$ on the decomposition effect in VMD decomposition is discussed. The larger the mode parameter $K$, the more detailed the signal is decomposed but modal will overlap. The quadratic penalty factor $\alpha$ as long as it is within a reasonable range, the decomposition effect can be guaranteed. According to the frequency domain wave characteristics, two parameters constraint rules and screening strategies are formulated.

In the process of VMD decomposition, according to the parameter selection strategy, the maximum number of mode parameter $K$ and the quadratic penalty factor $\alpha$ are selected as decomposition parameters to ensure that the $K$ components decomposed by the signal are not overlapped when the
decomposition is as detailed as possible. The experimental bearing signal decomposition experiments show that the parameter screening strategy is reasonable.

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