Anti-seismic Performance Comparison of Response Spectrum Analysis and Time History Analysis Based on Computer Big Data

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Abstract. This paper introduces the methods and principles of response spectrum analysis and time history analysis of seismic performance of structures based on computer big data. In order to discuss the common ground and the consistency of the calculated results between the two methods, the response spectrum analysis, time history analysis based on seismic level 8 synthetic seismic wave and seismic test were carried out on an IDC cabinet structure. The results showed that both the response spectrum method and the time-history analysis method reflect the effects of ground motion characteristics and structural dynamic characteristics on the seismic response of the structure in the seismic performance calculation, and the simulation results of the two methods were consistent in the response maximum value and response frequency.

Keywords: Response Spectrum Analysis, Time History Analysis, Test, Seismic Response, Response Maximum Value, Response Frequency, Big Data

1. Overview
The cabinet is an important part of internet data center(IDC), which is the main bearing unit for installing various IT equipments such as servers and memories. China has entered the era of big data, huge data value is bound to put forward higher requirements on the anti-seismic performance and reliability of IDC equipments. In order to obtain a more accurate result, a time history analysis method is needed to analyze the seismic performance of a structure. Although with the rapid development of computer, its capacity and speed are increasing, it is still difficult to carry out time-history dynamic analysis step by step[1]. In view of this situation, when it is necessary to understand the structure's response to the earthquake, response spectrum analysis is now widely used. This method is based on the linear assumption, and only a few low-order modes can be selected to obtain a satisfactory result with a small amount of computation. In this paper, by analyzing the calculation principle of response spectrum method and time history analysis method, HyperWorks series finite element softwares are used to discuss the common ground of the two methods in numerical simulation and the consistency of calculation results based on the seismic response calculation results of an IDC cabinet example and the test data.
2. Algorithm Principle

2.1. Theory of Response Spectrum Analysis

The basic principle of response spectrum analysis is using the concept of modal decomposition method to decompose the multi degree of freedom system into single degree of freedom system, and using the theory of response spectrum of single degree of freedom system to compute the modal vibration of the earthquake effect. Finally each vibration mode is used to calculate the seismic effect together according to certain rules\(^{(2,3)}\). The single point response spectrum analysis method in Hyper Works can be used for calculation. It is an engineering application of response spectrum theory. SRSS, which is the combination algorithm of square and square root, is used in the modal combination algorithm.

The maximum response on mode \(i\) is as follow:

\[ R_i = A_i \psi_i \]  

(1)

Where \(R_i\) is the maxinum response on mode \(i\); \(\psi_i\) is the mode form vector of mode \(i\); \(A_i\) is the modal coefficient of mode \(i\). \(A_i\) depends on the response spectrum value of mode \(i\) and the participation coefficient of mode \(i\). The participation coefficient is a measure of how much each mode contributes to the deformation in a particular direction.

The general form of the combined algorithm of SRSS is as follow:

\[ R_a = \left[ \sum_{i=1}^{N} (R_i)^2 \right]^{1/2} \]  

(2)

Where \(R_a\) is the overall response of the combined mode; \(N\) is the number of modes participating in the merge.

The seismic response spectrum curve reflects the characteristics of the ground motion, and the modal shape reflects the dynamic characteristics of the structure. It can be seen that the response spectrum method considers the dynamic relationship between the dynamic characteristics of the structure and the characteristics of the ground motion.

2.2. Theory of Time History Analysis

The main difference between time history analysis and response spectrum method is that the time history of earthquakes is imposed on the structure to obtain the earthquake internal force and displacement of each instantaneous reaction in the process and observe the structure under seismic action from elasticity to the inelastic phase and component crack, damage and the whole process of structure collapse\(^{(4)}\). The time-history analysis method takes the role of ground motion as a vibration process, considering not only the intensity of ground motion, but also the spectral characteristics and duration, so that the seismic response analysis of the structure is in the dynamic analysis stage, and the analysis process and results are more appropriate to the actual situation.

The central difference method is adopted to discretize the dynamic equation in the time domain and transform it into the difference scheme with respect to time. Then, according to the initial conditions, the direct integration method is used to solve a series of response values at the time step by step.

For an actual structure, after discretization by finite element method, the dynamic equation is\(^{(4)}\):

\[ [M][\ddot{u}] + [C][\dot{u}] + [K][u] = \{F(t)\} \]  

(3)

\(\{\ddot{u}\}, \{\dot{u}\}, \{u\}\) and \(\{F(t)\}\) represent acceleration, velocity, displacement and the external force vectors respectively, which are all time-dependent. \([M],[C]\) and \([K]\) are stiffness matrix, damping matrix and mass matrix respectively.

Assuming that when \(t = 0\), the displacement, velocity and acceleration are known as \(u_0, \dot{u}_0\) and
\[ \ddot{u}_0 \] respectively. And then divide the time interval into equal \( n \) parts, that is \( \Delta t = \frac{T}{n} \). The integral format we want to establish is from the known solutions at time \( 0 \), \( \Delta t \), \( 2\Delta t \), ..., \( t \) to calculate the solution of the next time step.

In the central difference method, the velocity and acceleration vectors are discretized by the central difference.

\[
\{ \ddot{u} \}_t = \frac{1}{2\Delta t} (\{ \ddot{u} \}_{t+\Delta t} - \{ \ddot{u} \}_{t-\Delta t}) \quad (4)
\]

\[
\{ \dot{u} \}_t = \frac{1}{\Delta t} (\{ \dot{u} \}_{t+\Delta t} - 2\{ \dot{u} \}_t + \{ \dot{u} \}_{t-\Delta t}) \quad (5)
\]

The two equations above express the velocity and acceleration at time \( t \) in terms of the displacement at the adjacent time. The dynamic equation at time \( t \) is as follow:

\[
[M]\{\dot{u}\}_t + [C]\{\ddot{u}\}_t + [K]\{u\}_t = \{F\}_t \quad (6)
\]

Substitute equations (4) and (5) into equation (6) to obtain the following:

\[
\left( \frac{1}{\Delta t^2} [M] + \frac{1}{2\Delta t} [C] \right)\{u\}_{t+\Delta t} = \{F\}_t - \left( [K] - \frac{2}{\Delta t^2} [M] \right)\{u\}_t - \left( \frac{1}{\Delta t^2} [M] - \frac{1}{2\Delta t} [C] \right)\{u\}_{t-\Delta t} \quad (7)
\]

Thus, the above equation is reduced to a set of algebraic equations represented by displacements at adjacent moments. So we can solve \( \{u\}_{t+\Delta t} \).

From the perspective of computational stability, the disadvantage of the central difference method is that it is conditionally stable, that is, when the time step \( \Delta t \) is too large, the integral is unstable. So, the limit on step size is as follow:

\[
\Delta t \leq \Delta t_{cr} = \frac{l}{c} \quad (8)
\]

The \( \Delta t_{cr} \) is critical time step, \( c \) is the speed of sound in the material, \( l \) is the characteristic side length. Therefore when \( l \) is small, the \( \Delta t \) has to be small. So the calculation is very huge.

2.3. Algorithm Comparison

The advantage of the response spectrum method is that it is easy to calculate, the calculation is small and it can effectively calculate the maximum seismic response of the structure. However, the response spectrum method is only applicable to linear structures in principle, and the seismic response spectrum does not contain phase information. The calculation accuracy depends on the number of modes participating in the merging. The time-history analysis method can simulate the seismic response of the structure at each moment in the whole earthquake duration, but it requires a large amount of calculation, and the calculation value of the seismic response is largely dependent on the selection of the seismic time-history curve. The response spectrum method is in essence an algorithm of mode decomposition, and the time history analysis method is an integral algorithm, but both the two methods reflect the influence of ground motion characteristics and structural dynamic characteristics on structural seismic response. Assuming that the structural seismic response is a linear problem, and the same seismic wave is adopted, that is, the time-history curve of seismic wave corresponds to the response spectrum curve, the structural seismic response calculated by response spectrum method and time-history analysis method has a certain degree of comparability. The seismic acceleration response
spectrum [5] is shown in figure 1, and the synthetic seismic wave curve is shown in figure 2.

![Figure 1](image1.png)

**Figure 1.** The seismic acceleration response spectrum.

![Figure 2](image2.png)

**Figure 2.** The synthetic seismic wave curve.

3. **Engineering Project**

3.1. **Seismic Test Modeling**

The test was carried out on a high performance triaxial and six degrees of freedom shake table. The test object was IDC cabinet, with a counterweight of 600kg. After the seismic wave is loaded, the sensor signals of the physical prototype can be collected, including accelerations and displacements. The configuration and installation of the equipment see figure 3. The cabinet is consisted of uprights, beams, top plate, bottom plate, inner uprights and trays. The frame of the cabinet is welded, and the inner uprights and trays are connected to the frame by bolts.

![Figure 3](image3.png)

**Figure 3.** The configuration and installation of the equipment

3.2. **Finite Element Modeling**

Hyper Mesh software was used to establish the finite element model of IDC cabinet model, including
mesh division, element selection, material selection, section assignment and connection element setting, etc. The overall structure of the cabinet frame is shell element, which is divided into 220,000 units by 2D grid. Rigid units are used for welding and bolting, constrainting 6 degrees of freedom. The weight block was simulated by Mass unit and Rbody unit, and the contact relationship between the weight block and the tray was simulated by contact unit. The analytical model see Figure 4. The cabinet is made of steel and relevant parameters of the material are shown in table 1.

![Figure 4. The analytical model](image)

### Table 1. Material parameters

| Material | Type | σ_y (MPa) | UTS (MPa) | ε_UTS | Density (t/mm³) | Modulus of elasticity (MPa) | Poisson's ratio |
|----------|------|-----------|-----------|--------|-----------------|---------------------------|----------------|
| Steel    | Q235 | 274       | 366       | 0.22   | 7.85e-9         | 2.0e5                     | 0.3            |

Remark: σ_y: material yield strength; UTS: The maximum tensile stress which is obtained from the engineering stress-engineering strain curve. ε_UTS: The engineering strain value at the maximum tensile stress.

3.3. Calculation Process

Calculation process of the response spectrum analysis is as follows: (1) static analysis to obtain the prestress under gravity; (2) modal analysis; (3) single point response spectrum analysis; (4) modal expansion; (5) modal combination; (6) result post-processing. The first 10 modes are extracted from modal analysis. In the single-point response spectrum analysis, the acceleration response spectrum is selected. The excitation direction is X direction of the model. Both the structure damping ratio and response spectrum damping ratio are 0.03.

Calculation process of the time history analysis is as follows: (1) selection of initializing time step; (2) application of load conditions; (3) element processing; (4) centralized mass matrix generation; (5) contact processing; (6) application of velocity conditions; (7) displacement calculation; (8) acceleration calculation; (9) stress and strain calculation; (10) internal force calculation; (11) speed
update; (12) configuration update; (13) time t update; (14) determine whether t is equal to the termination time, otherwise return to the calculation; (15) finish.

4. Comparison of Results
In response spectrum analysis, rigid units are built at the four mounting holes at the bottom of the cabinet, and six degrees of freedom are restrained. In time history analysis, rigid units are built at the four mounting holes at the bottom of the cabinet releasing the translational freedom of seismic wave loading direction. The seismic response displacement at the top and bottom of the main frame of the cabinet (node 1 and node 2 in Figure 4) were extracted from the calculation results.

4.1. Comparison of Relative Displacement Results
In response spectrum analysis, the total response displacement of node 1 is 36.59mm, and the displacement cloud diagram is shown in Figure 5. The response displacement of node 1 and node 2 at each moment are extracted from the calculation results of time-history analysis, and the response displacement curves are drawn to obtain the response relative displacement peak, as shown in Figure 6. The calculation process of relative displacement is to subtract the displacement response value of two nodes at each moment. The response displacement obtained by the two calculation methods was compared with the actual displacement obtained by the test (Figure 7). The actual displacement was used as a reference to calculate the respective errors, as shown in table 2.

Figure 5. Displacement of node 1.

Figure 6. Obtain the response relative displacement peak.
Figure 7. The actual displacement obtained by the test.

Table 2. Respective errors

| Item                   | Displacement (mm) | Errors reference to test (%) |
|------------------------|-------------------|------------------------------|
| Response spectrum analysis | 36.6             | 3.6                          |
| Time history analysis   | 33.0             | 6.5                          |
| Test                   | 35.3             | \                           |

It can be seen from the table that the total response displacement calculated by the response spectrum method is close to the response displacement peak value calculated by the time-history analysis method, and the simulation results of the two are consistent. Both the response spectrum method and the time-history analysis method take into account the dynamic characteristics of the structure and the characteristics of the ground motion[6], so they are consistent in the calculation of the maximum response value.

4.2. Comparison of Response Frequency

According to the mode superposition principle, the dynamic total response of the structure is the superposition of the mode responses of each order of the structure, and the mode that causes the resonance of the structure contributes the most. The modal coefficients and modal vectors of each mode are extracted from the calculation results of response spectrum method, and the response values of each mode are calculated. The main response frequencies of structural resonance are determined according to the calculated response values. The response values of each mode are the product of the modal coefficients and the mode vectors. In the calculation results of time-history analysis, Fourier transform (hereinafter referred to as FFT) is applied to the extracted time-domain response curve to draw the spectral characteristic curve of the structural response, as shown in Figure 8, and the main response frequency of the structural resonance is determined. In this paper, the first-order frequencies are compared, and the natural frequencies collected in the test are used as the reference for error calculation. The results are shown in table 3.

Figure 8. The spectral characteristic curve of the structural response.

Table 3. The main response frequency of the structural resonance.

| Item                                      | Frequency (Hz) | Errors reference to test (%) |
|-------------------------------------------|----------------|------------------------------|
| First order frequency participating in the calculation of response spectrum analysis | 4.55           | 6                            |
| Time history analysis displacement result FFT processing result | 4.102         | 4.5                          |
| First order frequency of the test         | 4.297          | \                           |

It can be seen that the main response frequencies determined by the response spectrum method and
the time-history analysis method are consistent with the actual values. The results show the resonance between the seismic load and the structure.

5. Conclusion

According to the comparison results, both the response spectrum method and the time-history analysis method reflect the influence of ground motion characteristics and structural dynamic characteristics on the seismic response of structures in the seismic performance calculation. If the response spectrum used in the response spectrum analysis is the response spectrum generated by the seismic wave used in the time-history analysis, and the analysis is limited to the elastic phase, the two are consistent in the response maximum and the main response frequencies.

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