Abstract
Quantized neural networks typically require smaller memory footprints and lower computation complexity, which is crucial for efficient deployment. However, quantization inevitably leads to a distribution divergence from the original network, which generally degrades the performance. To tackle this issue, massive efforts have been made, but most existing approaches lack statistical considerations and depend on several manual configurations. In this paper, we present an adaptive-mapping quantization method to learn an optimal latent sub-distribution that is inherent within models and smoothly approximated with a concrete Gaussian Mixture (GM). In particular, the network weights are projected in compliance with the GM-approximated sub-distribution. This sub-distribution evolves along with the weight update in a co-tuning schema guided by the direct task-objective optimization. Sufficient experiments on image classification and object detection over various modern architectures demonstrate the effectiveness, generalization property, and transfer-ability of the proposed method. Besides, an efficient deployment flow for the mobile CPU is developed, achieving up to $7.46 \times$ inference acceleration on an octa-core ARM CPU. Codes have been publicly released on Github\(^1\).

1. Introduction
Deep neural networks (DNNs) have become the de-facto method in plentiful applications, including computer vision (He et al., 2016; 2020), natural language processing (Devlin et al., 2019; Xu et al., 2020), speech recognition (Hinton et al., 2012), and robotics (Kim & Moon, 2016; Liu et al., 2016a). Although DNNs are pushing the limits of generalization performance across various areas, the growing memory and computation severely hinder the practical deployment. Abundant acceleration and compression methods have been proposed to tackle this issue, such as pruning (LeCun et al., 1989; Han et al., 2016), weight quantization (Fiesler et al., 1990; Rastegari et al., 2016), knowledge distillation (Hinton et al., 2015), efficient model design (Ma et al., 2018; Han et al., 2020; Tan et al., 2021), and even neural architecture search (Zoph et al., 2018).

Among the aforementioned schemes, weight quantization, as one of the mainstream techniques, aims to eliminate the representative redundancy via shortened bit-width. Generically, quantization can be represented as a projection $Q : W \in \mathbb{R} \rightarrow Q = \{q_0, q_1, \ldots, q_K\}$, where $W \sim D_W$ is the real-valued preimage while $Q \sim D_Q$ denotes the compressed and discrete representation. The categorical distribution $D_Q$ can be viewed as a factorized Dirac distribution for the expected precision, which inherently suffers a divergence from the preimage $D_W$. Consequently, the weight deviation after low-bit quantization generally leads to significant performance degradation.

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To address the problem mentioned above, one typical solution is straight through estimator (STE) (Bengio et al., 2013) that simulates and encourages the network discretization via modifying the backpropagation (Wu et al., 2016; Hubara et al., 2017; Choi et al., 2018; Cai et al., 2017). However, most of these approaches depend on the fixed configurations (e.g., centroids (Hubara et al., 2016; Zhou et al., 2017; Fiesler et al., 1990) and stepsize (Courbariaux et al., 2015; Gupta et al., 2015)) and it is tough to find the optimal solution. In addition, the global statistical information is not fully utilized, and thus the impact of numerical state transitions is omitted (Choi et al., 2018). Accordingly, a feasible solution is to manually set a concrete prior distribution \( D_p \) that smoothly approximates the factorized Dirac posterior \( D_q \) through variational learning and reparameterization (Jang et al., 2017; Maddison et al., 2017; Louizos et al., 2017; Ullrich et al., 2017). As shown in Fig. 1, this method aims at minimizing the KL divergence \( D_{KL}(D_p, D_q) \) during learning the task objective, which can be achieved directly by optimizing the evidence lower bound (ELBO) (Hinton & van Camp, 1993; Graves, 2011).

However, despite the fact that the differentiable learning with statistical information is adopted, it is still difficult to reach the ELBO optimum due to the large gradient variance and local optima trap. As has been pointed out by Carreira-Perpiñán & Idelbayev, there exists a gap between the prior and quantized distributions resulting from the separation of approximating the distribution and quantizing the weights in a two-step scheme. Besides, direct transfer of a manually-configured prior from one domain to another seems unappealing, since it generally requires statistical alignment training.

In this paper, we introduce Differentiable Gaussian Mixture Weight Sharing (DGMS), a novel adaptive-mapping quantization method that statistically searches for the optimal low-bit model with a latent and representative sub-distribution. Compared to prior works, the superiority of DGMS benefits from three aspects: 1) DGMS relaxes the Dirac posterior into a concrete GM approximation to estimate the optimal sub-distribution that is model-inherent but not user-specified. 2) DGMS directly minimizes the task loss via weight and sub-distribution co-tuning for task-optimal quantization. 3) We further leverage the temperature-based softmax reparameterization (Hinton et al., 2015; Jang et al., 2017) to narrow the gap between training and inference.

In summary, the main contributions of this paper can be listed as follows:

- We propose a novel quantization method DGMS that statistically explores the optimal latent low-bit sub-distribution without handcrafted settings. Distributions and weights are trainable in a self-adaptive and end-to-end fashion.
- The promising transfer ability across four domains reveals the domain-invariant character, indicating the model-inherence of the found sub-distribution.
- Extensive experiments with several classical and lightweight DNNs demonstrate the remarkable compression performance and generalizability of DGMS on both image classification and object detection.
- We also evaluate DGMS-quantized low-bit models on the mobile CPU with an efficient deployment flow that maintains the parallelism and optimizes the cache access. The results show \( 1.66 \times \sim 7.46 \times \) speedup compared to the full-precision models.

2. Related Works

It has been acknowledged that DNNs are heavily over-parameterized with significant redundancy (Denil et al., 2013), and a wide variety of approaches have been proposed for the compact representation. The straightforward solution is to design a lightweight model with handcrafted efficient blocks, such as MobileNet (Howard et al., 2017; Sandler et al., 2018; Howard et al., 2019), ShuffleNet (Zhang et al., 2018b; Ma et al., 2018) and GhostNet (Han et al., 2020), or with Neural Architecture Search (NAS) (Zoph et al., 2018; Tan & Le, 2019; Xie et al., 2019). Another attractive direction is to eliminate the inherent redundancy in existing architectures by using Knowledge Distillation (KD) (Hinton et al., 2015; Bucila et al., 2006; Zhang et al., 2021), pruning (Hagiwara, 1993; Han et al., 2015; 2016), or quantization (Fiesler et al., 1990; Rastegari et al., 2016).

In particular, quantization is widely recognized to be an efficient manner for DNN deployment on commercial products (Liao et al., 2019; Bannon et al., 2019; Jiao et al., 2020; Lin et al., 2020) profiting from the natural regularity for processing. Quantization techniques can be basically categorized into two groups: 1) discontinuous-mapping quantization based on rounding operation, and 2) continuous-mapping quantization based on adaptive reparameterization.

**Discontinuous-Mapping Quantization** One line of works employs stochastic rounding (Gupta et al., 2015; Hubara et al., 2017; Wu et al., 2018), where the mapping decisions are stochastically made among quantization intervals. Particularly, RQ (Louizos et al., 2019) is similar to our method but it employs this stochastic rounding operation for fixed quantization grids, the distribution assumption is made upon the input signal noise while we directly model the weights. In parallel, the other line of works focuses on deterministic rounding to snap the full-precision weights to the closest quantized centroids towards binary (Courbariaux et al., 2015), ternary (Marchesi et al., 1993), or power-of-two quantization (Tang & Kwan, 1993). Starting with the
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3. Methodology

In this section, we first present a reformulation of conventional network quantization and then introduce our proposed DGMS quantization method for the optimal sub-distribution searching. Finally, we describe an efficient mobile deployment flow named Q-SIMD for DGMS.

3.1. Preliminaries: Quantization Reformulation with Auxiliary Indicator

As stated in Sec. 1, a network quantizer \( Q \) maps the continuous preimage \( x \in \mathbb{R}^M \) to a discrete finite set \( Q \). For example, a symmetric uniform quantizer \( Q_U \) maps given inputs in a linear fashion:

\[
Q_U(W) = \text{sign}(W) \odot \left\{ \Delta \left[ \frac{|W|}{\Delta} + \frac{1}{2} \right], \quad |W| \leq \max(Q_U) \right\} \quad \text{where } \odot \text{ denotes the Hadamard product}
\]

where \( k \) denotes the Hadamard product, \( \Delta \) is the predetermined stepsize and \( Q_U = \{0, \pm k\Delta, k = 1, \ldots, K\} \) is the uniform centroids.

A common problem is that this discontinuous-mapping scheme may limit the representational capacity of quantization and the nearly zero derivatives of Eqn. (1) are generally useless. Hence, we employ an auxiliary quantization set \( \mathcal{Q}_A = \{\mu_0, \mu_1, \ldots, \mu_K\} \) where \( \mu_0 = 0 \) and \( \mu_1, \ldots, \mu_K \) are adaptive values. Besides, an auxiliary indicator \( \mathcal{T}^Q : \mathbb{R}^M \rightarrow \{0, 1\}^{K \times M} \) is introduced for \( W \) given any condition \( A \) (i.e., \( \mathcal{T}^Q(W) = 1 \) if \( A \) holds and zero otherwise), which has been widely used for pruning (Mallya et al., 2018; Lee et al., 2019):

\[
\mathcal{T}^Q_k(W) = \begin{cases} 1, & \text{if } W \text{ is quantized to } \mu_k \\ 0, & \text{otherwise} \end{cases}
\]

for \( k = 0, 1, \ldots, K \).

The quantization procedure can be reformulated as follows:

\[
Q_A(W) = \sum_{k=0}^{K} \mu_k \odot \mathcal{T}^Q_k(W).
\]

Here, \( Q_A(W) \in Q_A \) is the quantized low-bit representation which is defined as a one-hot linear combination of \( Q_A \) in Eqn. (3). With the foregoing reformulations, we further regard the auxiliary indicator as a probability-based decision variable to relax the quantization operations and serve for the end-to-end differentiable training.

3.2. DGMS: Differentiable Gaussian Mixture Weight Sharing

First of all, DGMS aims to search for a model-inherent sub-distribution that is defined as follows:

**Definition 1.** Given the preimage \( \mathcal{W}_W \sim \mathcal{D}_W \) and quantized data \( \mathcal{Q} \sim \mathcal{D}_Q \), the sub-distribution \( \mathcal{D}_S \) is defined as an estimation for \( \mathcal{D}_W \) (i.e., \( \mathcal{D}_S \approx \mathcal{D}_W \), where \( \approx \) denotes approximate equivalence), and under a parameter limitation \( \tau \), \( \mathcal{D}_S \) is approximately equivalent to \( \mathcal{D}_Q \) (i.e., \( \lim_{\tau \to 0} \mathcal{D}_S \approx \mathcal{D}_Q \)).

As defined above, the sub-distribution serves as a distributional bridge between the full-precision and quantized...
 representations, which is controlled by a hyper-parameter $\tau$ introduced later. In order to better understand Definition 1, we provide the theoretical discussions in Appendix B.1.

### 3.2.1. Gaussian Mixture Sub-Distribution Approximation

There are a few approaches to generate the sub-distribution $D_S$ satisfying the first condition $D_S \approx D_W$ in Definition 1, e.g., analyzing the statistical property with maximum likelihood estimation (MLE) based on a parameterized approximation. Empirically, the pretrained weights distribute like a bell-shaped Gaussian or Laplacian (Han et al., 2016; Lin et al., 2016). Thus, following Nowlan & Hinton, we assume a mixture of $K+1$ uni-modal Gaussian components as a smooth sub-distribution approximation for the full-precision network. This Gaussian Mixture (GM) approximation $GM_\varphi$ parameterized by $\varphi = \{\pi_k, \mu_k, \gamma_k\}_{k=0}^K$ (prior attributing probability, mean, standard deviation) is capable of forming a strong representation of arbitrary-shaped densities (Reynolds, 2009), revealing the statistical knowledge within the model. For efficiency and simplicity, we initialize the GM approximation with k-means algorithm, which can be viewed as a special case of expectation-maximization (EM) algorithm (Dempster et al., 1977).

### 3.2.2. Statistical Guided Weight Sharing

The magnitude-based weight salience has been early explored (Han et al., 2015; 2016), demonstrating that the larger weights tend to outweigh those with small magnitudes. Based on this observation, we naturally divide the weights into several regions using the parameterized GM. Each region is associated with a Gaussian component, where the mean value serves as a region salience for the weight sharing, i.e., the adaptive quantization set $Q_A = \{\mu_0, \mu_1, \ldots, \mu_K\}$ where $\mu_0 = 0$. Similar to Eqn. (2), here we define a region decision indicator $S^R: \mathbb{R}^M \rightarrow \{0, 1\}^{K \times M}$ for all $K+1$ regions based on the GM sub-distribution approximation. Then, for each data point $w_j$, the quantized low-bit representation $\Psi(w_j; \vartheta) \sim D_Q$ is formulated as:

$$T^S_k(w_j; \vartheta) = \begin{cases} 1, & \text{if } \arg \max_i \varphi(w_j, i) = k, \\ 0, & \text{otherwise} \end{cases}, \quad (4)$$

for $k = 0, 1, \ldots, K$;

$$\Psi(w_j; \vartheta) = \sum_{k=0}^K \mu_k \odot T^S_k(w_j; \vartheta), \quad (5)$$

where $\varphi(w_j, k) \in [0, 1]$ denotes the posterior probability $p(w_j \in k | \vartheta_k)$ of $w_j$ within the given region $k$ (parameterized by $\vartheta_k = \{\pi_k, \mu_k, \gamma_k\}$). This can be written as:

$$\varphi(w_j, k) = \frac{\exp(\pi_k \mathcal{N}(w_j | \mu_k, \gamma_k^2))}{\sum_{i=0}^K \exp(\pi_i \mathcal{N}(w_j | \mu_i, \gamma_i^2))}, \quad (6)$$

Eqn. (6) is the soft-normalized region confidence according to the GM sub-distribution. It can also be regarded as the nearest clustering based on a weighted radial basis function (RBF) kernel (Broomhead & Lowe, 1988) that calculates a soft symmetry distance between weights $w_j$ and the corresponding region salience.

### 3.2.3. Differentiable Indicator

However, recalling Eqn. (4), the hard sampling operation $\arg \max$ makes it completely non-differentiable, making it difficult to optimize with gradient descent. To tackle this issue, there exist many techniques such as Gumbel softmax (Jang et al., 2017). In this paper, we simply adopt the temperature-based softmax (Hinton et al., 2015; Maddison et al., 2017), which shifts the region confidence closer to a one-hot encoding with low temperatures, bridging the gap between the indicators during training and inference. With the introduced technique, the distributional sampling procedure is then differentiable and can be a well estimated region confidence prediction for $T^S_k(w_j; \vartheta)$:

$$\phi_k(w_j; \vartheta, \tau) = \frac{\exp(\varphi(w_j, k)/\tau)}{\sum_{i=0}^K \exp(\varphi(w_j, i)/\tau)}, \quad (7)$$

for $k = 0, 1, \ldots, K$,

where $\tau$ is set as a learnable temperature parameter that adjusts the discretization estimation level. Further, the compressed representation $\Phi_k(w_j; \vartheta, \tau) \sim D_S$ can be reformulated as follows:

$$\Phi(w_j; \vartheta, \tau) = \sum_{k=0}^K \mu_k \odot \phi_k(w_j; \vartheta, \tau). \quad (8)$$

### 3.2.4. Training and Inference

The proposed DGMS quantization can be summarized in Algorithm 1. Note that DGMS can be deployed along different granularities such as channel-wise or filter-wise. In this paper, layer-wise DGMS is explored since it is faster to train.

### 3.3. Q-SIMD: Efficient Deployment on the Mobile CPU

DGMS not only reduces the memory footprint but also speedups the inference for deployment. To evaluate the model acceleration benefiting from DGMS, we develop an efficient flow to deploy our quantized 4-bit networks on the mobile CPU. The deployment faces two challenges: 1) to fetch real weights with 4-bit indices while maintaining the parallelism (i.e., SIMD), and 2) to optimize the shifted cache bottleneck of activations after the weight compression.

Given that the smallest numeric data type in current ARMv8 is BYTE, simply employing the 4-bit indices wastes
Algorithm 1 Model compression using Differentiable Gaussian Mixture Weight Sharing.

Input: Training dataset \( \mathcal{D} = \{ \mathcal{X}, \mathcal{Y} \} \) like ImageNet and DNN \( \mathcal{F} \) of \( L \) layers with full-precision weights \( \mathcal{W} = \{ \mathbf{w}^l \}_{l=1}^L \), GM component number \( K + 1 \) and initial temperature \( \{ \tau^l \}_{l=1}^L \).

1: Initialization
2: for \( \ell = 1 \) to \( L \) do
3: \( \mathcal{R} \leftarrow \{ \mathbf{w}^l : \mathbf{w}^l \in \text{region } j \}_{k=0}^K \}; \{ \text{initial region generation with } k\text{-means} \}
4: \( \min_{k=0}^K (|\hat{\mu}_k|) \leftarrow 0 \), \( \theta^l \leftarrow \left\{ \begin{array}{l} \hat{\mu}_k, \hat{\pi}_k \leftarrow |\mathcal{R}_k|/|\mathcal{W}|; \hat{\gamma}_k \leftarrow \sqrt{\frac{\sum_{j=1}^{|\mathcal{W}|} (\mathbf{w}^l - \hat{\mu}_k)^2}{|\mathcal{W}| - 1}} \\ k=0 \end{array} \right. \)
5: end for
6: \( \Theta \leftarrow \{ \phi^l, \theta^l \}_{l=1}^L \);
7: Training
8: while not converged do
9: for \( \ell = 1 \) to \( K \) do
10: \( \phi^l_k(\mathbf{w}^l; \theta^l, \tau^l) \leftarrow \exp \left( \frac{\phi(\mathbf{w}^l, k)}{\tau^l} \right) \cdot \sum_{i=0}^{K-1} \exp \left( \frac{\phi(\mathbf{w}^l, i)}{\tau^l} \right) \).
11: Eqn. (6) and Eqn. (7);
12: end for
13: \( \Phi(\mathbf{w}^l; \theta^l, \tau^l) \leftarrow \sum_{k=0}^K \phi^l_k(\mathbf{w}^l; \theta^l, \tau^l), \)
14: Eqn. (8);
15: end for
16: Evaluate with task loss, e.g., Cross Entropy \( \mathcal{L}_{\text{CE}}(\mathcal{F}(\mathcal{X}; \Phi(\mathcal{W}; \Theta)), \mathcal{Y}) \);
17: Backpropagation and update \{\mathcal{W}, \Theta\} with the stochastic gradient descent;
18: Inference
19: \( \hat{\mathcal{Y}} \leftarrow \Phi(\mathcal{W}; \Theta) \), Eqn. (4) and Eqn. (5);
20: Output prediction \( \hat{\mathcal{Y}} \) with quantized neural network: \( \hat{\mathcal{Y}} = \mathcal{F}(\mathcal{X}; \hat{\mathcal{W}}) \).

4. Experiments

4.1. Experimental Setup

4.1.1. Datasets and Models

To demonstrate the effectiveness and generalization ability of DGMS, we evaluate our method on image classification and object detection. Classification experiments are conducted on CIFAR-10 (Krizhevsky, 2009) and ImageNet (Deng et al., 2009). PASCAL VOC (Everingham et al., 2015) dataset is used as the detection benchmark. We use VOC2007 plus VOC2012 trainval for training and evaluate on VOC2007 test. We principally select a series of lightweight models to achieve further compression, which is more significant for an advanced quantization technique at present. For CIFAR-10, we evaluate DGMS on ResNet-20, ResNet-32, ResNet-56 (He et al., 2016) and VGG-Small (Zhang et al., 2018a) (a small-sized variant of VGG-Net (Simonyan & Zisserman, 2015)). For ImageNet, we use ResNet-18 and ResNet-50 for evaluations (He et al., 2016). Besides, we evaluate DGMS on lightweight architectures including MobileNetV2 (Sandler et al., 2018) and two NAS-based MnasNet-A1 (Tan et al., 2019) and ProxylessNAS-Mobile (Cai et al., 2019). The experiments on PASCAL VOC are performed on SSDLite, a publicly available lite version of SSD (Liu et al., 2016b). Here, we use SSDLite-MB2 and SSDLite-MB3 with MobileNetV2 (Sandler et al., 2018) and MobileNetV3 (Howard et al., 2019) as the backbone models, respectively.

4.1.2. Deployment

We extend the TVM framework (Chen et al., 2018) to support the Q-SIMD flow introduced in Sec. 3.3. Our evaluation is performed on the octa-core ARM CPU in Qualcomm 888 (Samsung S21). We conduct various image classification and object detection models quantized by DGMS to evaluate their runtime using Android TVM RPC tool. The baseline results are obtained under full-precision models using the primitive optimal TVM scheduling.

4.2. Image Classification Results

4.2.1. CIFAR-10 Results

As shown in Table 1, we compare the proposed DGMS on CIFAR-10 to LQNets (Zhang et al., 2018a) which is trained with Quantization Error Minimization (QEM), and TTQ (Zhu et al., 2017) which is trained with task-objective optimization only (see discussions in Sec. B.2). From the experimental results, we observe that DGMS consistently achieves a promising model sparsity at the same bit-width with a more lightweight representation while maintaining the performance. For example, the 2-bit ResNet-20 architecture that is extremely tiny can still achieve 44.44% pa-
Table 1: Comparison across different quantization methods on CIFAR-10. #Params: the number of non-zero model parameters, Bits: weights quantization bit-width, TOO: task-objective optimization only. Compared methods are TTQ (Zhu et al., 2017) and LQNets (Zhang et al., 2018a).

| Model          | Method | TOO | #Params | Bits | Top-1 Acc. |
|----------------|--------|-----|---------|------|------------|
| ResNet-20      | Our FP32 | N/A | 0.27M   | FP32 | 93.00%     |
|                | LQNets  | ×   | 0.27M   | 3    | 92.00%     |
|                |         | ✓   | 0.20M   | 3    | 92.84%     |
| ResNet-32      | Our FP32 | N/A | 0.46M   | FP32 | 94.27%     |
|                | TTQ     | ✓   | 0.46M   | 2    | 92.37%     |
| ResNet-56      | Our FP32 | N/A | 0.85M   | FP32 | 94.61%     |
| VGG-Small      | LQNets  | ×   | 4.66M   | 3    | 93.80%     |
|                |         | ✓   | 3.12M   | 3    | 94.46%     |
|                | LQNets  | ×   | 4.66M   | 2    | 93.80%     |
|                |         | ✓   | 2.24M   | 2    | 94.36%     |

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Table 2: Comparison across different quantization-only and joint quantization-pruning methods on ImageNet. P+Q: joint pruning-quantization methods. Compared methods are LQNets (Zhang et al., 2018a), AutoQ (Lou et al., 2019), HAQ (Wang et al., 2020), TQT (Jain et al., 2020), BB (van Baalen et al., 2020), CLIP-Q (Tung & Mori, 2020), ANNC (Yang et al., 2020), UNIQ (Baskin et al., 2021) and HAWQ (Yao et al., 2021).

| Model      | Method | P+Q | W/A | Top-1 Acc. | Δ Acc. |
|------------|--------|-----|-----|------------|-------|
| ResNet-18  | Our FP32 | ×   | 4/32 | 74.59% | N/A   |
|            | LQNets  | ×   | 4/32 | 74.00% | +0.40%|
|            | Ours    | ✓   | 4/32 | 74.00% | +0.40%|
| ResNet-50  | BB      | ✓   | 4/8  | 67.60% | -0.88%|
|            | UNIQ    | ×   | 4/8  | 67.02% | -2.86%|
|            | HAQ     | ×   | 4/8  | 70.19% | -1.52%|
|            | Ours    | ✓   | 4/8  | 70.19% | +0.46%|
| ResNet-50  | AutoQ   | ×   | 4/4  | 73.45% | -1.33%|
|            | HAQ     | ×   | 4/4  | 68.45% | -3.02%|
|            | Ours    | ×   | 4/4  | 68.95% | -0.81%|
| MobileNetV2| Our FP32 | ×   | 4/32 | 71.88% | N/A   |
|            | HAQ     | ×   | 4/32 | 71.88% | -0.01%|
|            | Ours    | ×   | 4/32 | 71.88% | +0.13%|
| MnasNet-A1 | BB      | ×   | 3/32 | 73.70% | +0.60%|
|            | UNIQ    | ×   | 3/32 | 73.74% | -0.85%|
|            | HAQ     | ×   | 3/32 | 75.91% | -0.24%|
|            | Ours    | ×   | 3/32 | 75.91% | +0.13%|
| MnasNet-A1 | TQT     | ×   | 4/8  | 75.19% | -2.33%|
|            | UNIQ    | ×   | 4/8  | 74.37% | -6.65%|
|            | HAQ     | ×   | 4/8  | 74.79% | -0.52%|
|            | Ours    | ×   | 4/8  | 76.22% | +0.07%|
| MnasNet-A1 | AutoQ   | ×   | 4/4  | 74.23% | -2.37%|
|            | HAQ     | ×   | 4/4  | 74.23% | -3.48%|
|            | Ours    | ×   | 4/4  | 75.05% | -1.10%|

4.2.2. ImageNet Results

In Table 2, we show the classification performance of compressed models on the ImageNet benchmark, compared to several strong baselines including LQNets (Zhang et al., 2018a), HAQ (Wang et al., 2020), TQT (Jain et al., 2020), AutoQ (Lou et al., 2019), UNIQ (Baskin et al., 2021), HAWQ V3 (Yao et al., 2021) and joint pruning-quantization methods including BB (van Baalen et al., 2020), CLIP-Q (Tung & Mori, 2020) and ANNC (Yang et al., 2020). Note that we adopt full-precision models from torchvision by 0.7.0 and torch. Since this work focuses on weights quantization, the weights are quantized with DGMS and activations are quantized in a post-training fashion following Li et al.. Table 2 shows that for the 4-bit quantization setting, our DGMS-quantized ResNet-18 and ResNet-50 achieve even higher classification accuracy than the full-precision models. Even though lightweight NAS-based models are challenging for compression, the efficient MnasNet-A1 and ProxylessNAS-Mobile quantized by our DGMS lead to a negligible accuracy loss. As a result, DGMS presents a competitive or better classification accuracy with consistently decent CRs in comparison to both quantization-only and joint pruning-quantization model compression methods.
### 4.3. Object Detection Results

To demonstrate the generalization ability of our proposed DGMS, we quantize the lightweight detectors SSDLite-MBV2 and SSDLite-MBV3 to 4-bit models on PASCAL VOC benchmark. Note that our method can be deployed on these detectors directly with no additional modification efforts. As displayed in Table 3, we observe 10.12× and 15.17× compression rates respectively for SSD-MBV2 and -MBV3 while retaining the detection accuracy. For some objects like motor-bike (mbike), our quantized detectors can achieve more accurate perception and we argue that the degradation comes from the lack of training samples and the intrinsic detection difficulty for tiny objects.

### 4.4. Deployment Results on the Mobile CPU

Table 4 compares the inference performance of the Q-SIMD flow with primitive TVM scheduling on the mobile CPU. It is observed that a 4.34-to-7.46× speedup is achieved for ResNet-18 and ResNet-50, respectively. Lightweight DNNs have scant room because separable convolutions are widely used in models with less weight reuse probability and reduced computation. Despite this, a 1.66-to-1.74× speedup is achieved by our method in three lightweight models.

### 5. Discussions

#### 5.1. Gaussian Mixture Evolution

For DGMS, the quantization mapping is adaptively learned with the GM evolution during training. In order to further understand this weight-distribution co-tuning schema, as shown in Fig. 3, we visualize the weight redistribution process of 2-bit quantization (i.e., $K = 3$) with DGMS and the learning curve of ResNet-20 on CIFAR-10. Fig. 3(a) shows the pretrained weight distribution with the initial GM densities plotted. Fig. 3(b) and (c) respectively illustrate the distributions of weights after DGMS and quantization. We can observe that the proposed DGMS is capable of tuning the GM according to the continuous distribution. Besides, the model-inherent statistical information is modeled and the weights are accordingly redistributed. Furthermore, the weights projected by DGMS serve as a fine hard-quantized estimation, which manages to narrow the gap between the original and compressed representations. Fig. 3(d) plots the validation performance and the weight deviation measured by mean square error (MSE) of the uncompressed and quantized weights. Note that no such quantization error is used for DGMS training, but it can be seen that the quantization
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5.2. Domain Invariance

In Table 5, we evaluate the transfer ability with 4-bit ResNet-18 across ImageNet, CUB200-2011 (Welinder et al., 2010), Stanford Cars (Krause et al., 2013), and FGVC Aircraft (Maji et al., 2013). We directly transfer the parameterized sub-distribution from the source domain to perform DGMS quantization and report the Top-1 accuracy (%) without any or with only one-epoch tuning. It is noteworthy that the zero epoch results are zero-shot quantization results on the target dataset via domain transfer. From the table, we can observe that except for the large-scale ImageNet dataset, all the models achieve competitive performances compared to full-precision models with zero training. This is because the sub-distribution learning has converged enough for the three medium-scale datasets, but it is not adapted to ImageNet with a reparable gap. Therefore, after only one epoch of fine-tuning with DGMS, the models are able to match the performance of uncompressed or vanilla DGMS-quantized models (trained for 60 epochs). This demonstrates the domain-invariant character of the sub-distribution, which is a model-inherent representation with superior generalization performance and training efficiency. Beyond that, this also provides a novel angle orthogonal to existing methods for zero-shot model quantization, i.e., through domain generalization of adaptive quantization parameters.

6. Conclusions

In this paper, we introduce a novel quantization method named DGMS to automatically find a latent task-optimal low-bit sub-distribution, forming a distributional bridge between full-precision and quantized representations. Weights are projected based on the GM-parameterized sub-distribution approximation, which evolves with weights in a joint fashion by directly optimizing the task-objective. As a result, DGMS adaptively quantizes the DNNs with a found task-optimal sub-distribution and achieves negligible performance loss with high compression rates. Extensive experiments on both image classification and detection on over-parameterized and lightweight DNNs have been conducted, demonstrating a competitive compression performance compared to other state-of-the-art methods and the transferability of the obtained model-inherent sub-distributions across different domains. To date, the formulation and optimization direction remain open problems in the quantization community. Among numerous existing solutions, we propose to view the optimal quantization configuration searching in a different way as to find the task-optimal sub-distribution. We also provide a great potential to solve zero-shot quantization through domain generalization, and we hope our work could spur future related researches. In the future, we would like to explore DGMS with a bimodal distribution (e.g., arcsine distribution) for binary DNNs, and it would be intriguing to solve the task-optimal sub-distribution searching problem with techniques like RL or NAS.

Table 5: Top-1 accuracy of 4-bit quantized ResNet-18 across domains. DGMS without (w/o) transfer is the vanilla DGMS using GM initialization method introduced in Algorithm 1, and with (w/) transfer denotes GM initialization with sub-distribution found in the source domain.

| Target      | ImageNet | CUB200-2011 | Stanford Cars | FGVC Aircraft |
|-------------|----------|-------------|---------------|---------------|
|             |          |             |               |               |
| Full-Model  | 69.76%   | 78.68%      | 86.58%        | 80.77%        |
| w/o transfer (4-bit DGMS) | 70.25%   | 77.90%      | 86.39%        | 80.41%        |
| w/ transfer (4-bit DGMS) |          |             |               |               |
| ZERO-SHOT   | 34.69%   | 62.31%      | 73.53%        | 74.29%        |
| ONE-EPOCH   | 68.37%   | 69.13%      | 77.70%        | 77.50%        |

error spontaneously declines with accuracy boosting.
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A. Details of Q-SIMD

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Fig. 4 shows the detailed illustration of our Q-SIMD flow. DGMS firstly quantizes a full-precision model to a low-bit model of 2-bit or 4-bit weights. Because the real values of weights are still FP32, the compact model is stored as a low-bit index tensor with the corresponding codebook. Naturally, when executing the multiply-add operations, the real weights are loaded through indirect addressing with indices. However, the current ISA does not support 4-bit or 2-bit operations and requires at least 8-bit alignment. Additionally, the indirect addressing is irregular and thus damages the SIMD operations.

To address this problem mentioned above, we introduce an extended codebook with coupled or fourfold values for 4-bit and 2-bit quantization, respectively. In this way, the number of codebook entries is increased to 256, and then the index bit-width is changed to be 8-bit. With the ARM NEON SIMD instructions (e.g., float32x2_t vldl_f32 and float32x4_t vldlq_f32), we can avoid bit wasting for low-bit alignment and maintain the parallel data loading for SIMD operations.

Furthermore, given that we mainly focus on weight quantization, the bottleneck of memory access turns to be activations. Therefore, we reduce the activation reloading via output-reuse (OR) and input-reuse (IR) scheduling. For the inner-loops, we keep the outputs local in the L1 cache to avoid reloading the partial sums. For the outer-loops, we put the oh and ow dimensions to reuse inputs across different output channels.

The aforementioned operation is registered as a customized operator in TVM (Chen et al., 2018). With the Android TVM RPC tool, the quantized models can be conveniently deployed on the ARM CPU and evaluated for inference time.

B. Further Discussions

B.1. Sub-Distribution

In this section, we discuss about sub-distribution introduced in Definition 1, which is essential and serves for the moral of this paper.

B.1.1. Statistical Approximation

The concept “approximate equivalence”, denoted as \( \approx \), is statistical and it means the divergence between the estimated and the real data distribution does not exceed a small error \( \epsilon \).
B.1.2. Full-Precision Distribution Estimation

The full-precision distribution estimation is the first condition in Definition 1, i.e., $D_S \approx D_W$. As stated before, the GM sub-distribution is initialized by MLE method, and MLE can model the statistical feature of data samples based on a parameterized approximation (Dempster et al., 1977). Hence, the parameterized GM is approximately equivalent to the data’s real distribution which can not be precisely measured but can be statistically estimated (Reynolds, 2009).

B.1.3. Categorical Distribution Approaching

The quantization distribution approaching is the second condition in Definition 1, i.e., $\lim_{\tau \to 0} D_S \approx D_Q$. This means after hard quantization, the weights are re-distributed to discrete values according to the categorical distribution $D_Q$ that originates from the concrete GM. Formally, given full-precision weight $w \sim D_W$, $\Phi(w) \sim D_S$ as the statistically shared weights according to the GM model and $\Psi(w) \sim D_Q$ as the quantized value. Assuming that $k = \arg \max_i \varphi(w, i)$, then we have $\lim_{\tau \to 0} \varphi_k(w) = 1$ and $\lim_{\tau \to 0} \varphi_j(w) = 0$ if $j \neq k$, hence it can be easily proved that $\lim_{\tau \to 0} \Phi(w) = \Psi(w)$ (notations taken from Sec. 3.2).

B.2. MDL View on DGMS

Supposing that we have the training dataset $D = \{X, Y\} = \{x_i, y_i\}_{i=1}^N$ and neural network $F$ with weights $\mathcal{W}$, DGMS directly optimizes the objective with gradient descent:

$$\min_{\mathcal{W}, \vartheta, \tau} \mathcal{L}^E = - \log p(Y|X, \Phi(\mathcal{W}; \vartheta, \tau)), \quad (9)$$

where the Gaussian Mixture is parameterized by $\vartheta = \{\pi_k, \mu_k, \gamma_k\}_{k=1}^K$ and weights are smoothly quantized by the differentiable indicator defined in Eqn. (8).

From the minimum description length (MDL) view in information theory (Shannon, 1948; Grünwald, 2007), $\mathcal{L}^E$ is the lower bound of the information cost expectation to fit the data (Rissanen, 1978; 1986). In contrast, the previous works (Jang et al., 2017; Maddison et al., 2017; Ullrich et al., 2017; Roth & Pernkopf, 2020) optimize the evidence lower bound (ELBO) to train DNNs with smoothed stochastic weights from an estimated categorical prior distribution (e.g. Gaussian Mixture). The objective can be written with the error cost $\mathcal{L}^E$ plus the complexity cost $\mathcal{L}^C$, and for DGMS this can be given as follows:

$$\min_{\mathcal{W}, \vartheta, \tau} \underbrace{- \log p(Y|X, \Phi(\mathcal{W}; \vartheta))}_{\mathcal{L}^E} + \underbrace{\mathcal{L}^C}_{\mathcal{L}^C} - \log p(\Phi(\mathcal{W}, \vartheta, \tau)) + H\left(q_\vartheta(\mathcal{W})\right), \quad (10)$$

where $\mathcal{L}^C$ amounts to the Kullback-Leibler divergence between the prior distribution and the Bayesian posterior distribution $q_\vartheta(\mathcal{W})$, and $H$ is the entropy term which can be approximately reduced to a constant (Ullrich et al., 2017). In this paper, the complexity cost $\mathcal{L}^C$ is dropped while the weights and the parameterized GM co-adapt for redistribution.

C. Details of Experiments

C.1. Datasets

CIFAR-10  The CIFAR-10 dataset (Krizhevsky, 2009) consists of 60,000 $32 \times 32$ tiny images for 10 classes object recognition, with 6,000 images per class.

ImageNet  The ImageNet (ILSVRC 2012) dataset (Deng et al., 2009) is a challenging large-scale dataset for image classification, containing 1.3M training samples and 50K test images with 1,000 object classes for recognition.

PASCAL VOC  The PASCAL VOC dataset (Everingham et al., 2015; 2010) is widely used as both semantic segmentation and object detection benchmarks. For object detection, VOC2007 trainval and VOC2012 trainval respectively contain 5,011 images and 11,540 images for training (16,551 images in all) and VOC07 test contains 4,952 images for evaluation. PASCAL VOC involves 21 classes including the a background class for detection and segmentation.
CUB200-2011  The CUB200-2011 dataset (Welinder et al., 2010) is a medium-scale dataset for fine-grained classification task, which contains 11,788 images in all for 200 bird species (5,994 samples for training and 5,794 images for validation).

Stanford Cars  The Stanford Cars dataset (Krause et al., 2013) contains a total of 16,185 car images of 196 categories, of which 8,144 samples are used for training and 8,041 images are used for validation.

FGVC Aircraft  The FGVC Aircraft dataset (Maji et al., 2013) consists of 10,000 images for 100 aircraft variants, the training set involves 6,667 samples and validation set involves 3,333 images.

C.2. Implementation Details

Optimization  We set the batch size as 128 on CIFAR-10 and 256 on ImageNet for all the models. SGD with 0.9 momentum and $5 \times 10^{-4}$ weight-decay is used during training. All the models are trained for 350 epochs and 60 epochs respectively on CIFAR-10 and ImageNet. The max learning rate is set to $2 \times 10^{-5}$ ($1 \times 10^{-5}$ for 2-bit ResNet-50) using one-cycle scheduler (Smith & Topin, 2019) and the initial temperature parameter $\tau$ in Eqn. (7) is 0.01 for all the experiments. The training configurations for transferability experiments on the medium-scale datasets (CUB200-2011, Stanford Cars and FGVC Aircraft) are the same as ImageNet. For PASCAL VOC detection, the batch size is 32 with $1 \times 10^{-5}$ learning rate and we employ the cosine scheduler. The detectors are trained for only 10 epochs. With regard to the data augmentation, we simply adopt random crop and horizontal flip for image classification and the same augmentation operations as that in public source\(^2\) for object detection.

Implementation  The experiments are implemented with PyTorch 1.6 (Paszke et al., 2019) on PH402 SKU 200 with 32G memory GPU devices. All the layers except the first and the last are quantized, which has become a common practice for model quantization. Similarly for detectors, we perform the quantization on all layers except for the first layer of the model and the last layer of the classification head and regression head.

D. Further Ablation Study

Table 6: Ablation study on initialization. The results are Top-1 classification accuracy (%) on CIFAR-10.

| Model     | Bits | Std. Init. | Empirical Initialization |
|------------|------|------------|--------------------------|
|            |      |            | $\gamma = 0.010$ | $\gamma = 0.005$ | $\gamma = 0.001$ |
| ResNet-20  | 2    | 89.26      | 92.13       | 90.88       | 91.30       |
|            | 3    | 92.16      | 92.84       | 92.54       | 92.34       |
| VGG-Small  | 2    | 92.55      | 94.36       | 94.40       | 94.41       |
|            | 3    | 94.30      | 94.46       | 94.56       | 94.52       |

D.0.1. Initialization

For the GM initialization, we adopt k-means which can be simply and efficiently implemented on GPU devices\(^3\). In the CIFAR-10 experiments, we test two types of initialization for the parameter $\gamma$, including standard deviation initialization and empirical initialization. As a result, we empirically find that the calculation of standard deviation $\gamma$ in Algorithm 1 may not be optimal but is simple and effective compared to the empirical initialization (see Table 6).

D.0.2. Temperature

As has been stated in Sec. 3.2, the temperature hyper-parameter $\tau$ controls the discretization estimation level. And when $\tau$ approaches zero, the estimated representation is approximately equivalent to the quantized one. This hyper-parameter can be set as a fixed or learnable parameter with a predefined initialized value. In Table 7, we test 3-bit VGG-Small on CIFAR-10 with different temperature initializations. It can be observed that a relatively lower temperature is more recommended.

\(^2\)https://github.com/qfgaohao/pytorch-ssd
\(^3\)https://github.com/subhadarship/kmeans_pytorch
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Table 7: Ablation study on temperature hyper-parameter $\tau$. The results are Top-1 classification accuracy (%) with 3-bit VGG-Small on CIFAR-10.

| Temperature Type | Initialization Value | $\tau=0.100$ | $\tau=0.010$ | $\tau=0.001$ |
|------------------|----------------------|--------------|--------------|--------------|
| Fixed            | 93.57                | 94.56        | 94.27        |
| Learned          | 93.54                | 94.64        | 94.04        |

to bridge the quantization gap, and a fixed temperature can achieve competitive performances compared to an adaptively learned temperature.

**E. Broader Impact**

The rapid development of deep learning brings huge convenience and advanced productivity to our society, together with a higher daily cost (e.g., carbon footprint). The proposed DGMS is a novel quantization method to address this issue, and it can be utilized to compress a variety of DNNs. In real-life, this technique provides an eco-friendly and energy-efficient deployment solution for deep learning models, profiting from lower power consumption and less CO$_2$ emission. Though DGMS facilitates the AI development on edge devices like mobile phones, if without legal restraint, it will potentially lead to the infringement of image rights and personal privacy.