INTRODUCTION

The amount of global landslide disaster has been increasing as the result of human activities and exceptional changes of climate. Therefore, landslide prevention and control have become a widely concerned subject in recent years among natural disaster management. In the process of management and study of landslide disaster, different governance schemes and ideas influence on project investment impact is different. However, a good governance solution can not only stabilize the landslide, reduce investment and environmental damage, but optimize and control the effect of construction management [1]. In view of this, it is very important to adopt appropriate methods and select reasonable control scheme in the preliminary design stage of landslide control.

At present, it has made great important progress in fields of optimum selection for landslide disaster control schemes. The methods mainly include: Qualitative method, Quantitative method, Semi-qualitatively method and Semi-quantitatively method, etc. Many scholars conducted the research, and draw lots of valuable conclusions. For instance, Wang proposed that four factors could be taken into account in the landslide treatment plan in which need an abundance of engineering design experience for designers [2]. Based on landslide geologic environment, Mu selected five comparison factors to analyze the engineering geologic characteristics and stability of a landslide body from a quantitative point of view [3]. An improved AHP method based on optimum transmit matrix is presented by Xie, who used to optimize the control plans of unstable slopes [4,5]. The comparison and selection system of landslide comprehensive treatment schemes based on the factors of safety, economy and technology are established by Wang, and then the entropy weight decision method is used to quantitatively compare and select landslide control schemes [6]. Such related work is of important engineering value to evaluate the landslide prevention projects options. Throughout the evaluation theory of on research work, however, it can be noticed that many related researches are confined to take control measures mainly according to engineering experience. As a matter of fact, the optimal selection of landslide control schemes is that decision maker
makes a comprehensive evaluation of a limited number of known schemes by analyzing the decision attribute information, therefore, it can be summed up as a multi-attribute decision making problem and rank the existing schemes. Nowadays, the multiple attributes decision-making method is successfully applied in many real-life problems in engineering, finances, market analysis, management and others[7,8,9,10,11]. The selection of an optimization scheme for landslide control is a decision problem with multi-objectives and multi-properties. However, for the traditional research approach, the affecting factor of selection of an engineering disaster system is qualitatively analyzed only, as this can result in the not enough objective and scientific selecting decision. Actually, each decision object expects to minimize the deviation from the ideal object in the process of Game Decision Making [12,13,14]. As for each object is expected to be selected, which means the finer the superiority of attribute value after unified dimension, the finer the weight should be. Finally, after aggregation of decision information, the comprehensive attribute value of each decision object is maximized. Based on the above ponder, the probability dominance relation of interval numbers for a landslide scheme is put forward, and its strict theoretical reasoning is carried out in this study, and the proposed model is verified to be efficient and correct by engineering examples.

2 INTERVAL NUMBER AND PROBABILITY DOMINANCE RELATION

2.1 Interval Number

Due to the complexity of engineering disaster systems and the uncertainty of data set. As for a decision maker, who requires a basic expectation interval or the range of objective when making a plan decision, then the decision information no longer appears as a quantitative value, on the contrary, it may probably be denoted by interval numbers and fuzzy numbers. The connotation and theoretical derivation of an interval number are mentioned below [8,12].

Definition 1: Suppose
\[ \tilde{a} = [a^L, a^U] = \{x | a^L \leq x \leq a^U, a^L, a^U \in R \} \], thus \( \tilde{a} \) is described as a interval number. However, it’s equal to the same real number when \( a^L = a^U \), that is, any real number can be considered as an interval number.

The arithmetic of interval numbers has mainly in the following two points:

(1) \( \tilde{a} + \tilde{b} = [a^L + b^L, a^U + b^U] \);

(2) \( k \tilde{a} = [ka^L, ka^U] \), among them, \( k \geq 0 \), if \( k = 0 \), then \( k \tilde{a} = 0 \).

Definition 2: Let \( \tilde{a} \) and \( \tilde{b} \) be the two interval numbers, \( \tilde{a} = [a^L, a^U] \) and \( \tilde{b} = [b^L, b^U] \) can be tenable as well. Thus, \( \tilde{l}_a = a^U - a^L \), \( \tilde{l}_b = b^U - b^L \). Therefore, the following formula is called the possibility degree of \( \tilde{a} \geq \tilde{b} \).

\[
\begin{align*}
   p(\tilde{a} \geq \tilde{b}) = & \begin{cases} 
   1, & a^U \geq b^U \\
   a^U - b^L, & a^U > b^L \land a^L < b^U \\
   0, & a^U \leq b^L 
   \end{cases} \\
\end{align*}
\]

(1)

Definition 3: Let \( \tilde{a} \) and \( \tilde{b} \) be the two interval numbers, \( \tilde{a} = [a^L, a^U] \) and \( \tilde{b} = [b^L, b^U] \) can be tenable as well. Thus, \( \tilde{l}_a = a^U - a^L \), \( \tilde{l}_b = b^U - b^L \). Therefore, the following formula is called the possibility degree of \( \tilde{a} \geq \tilde{b} \).

\[
\begin{align*}
   p(\tilde{a} \geq \tilde{b}) = & \min \{l_a + l_b, \max(a^U - b^L, 0)\} \\
\end{align*}
\]

(2)

It can be seen that Definitions 2 and 3 are equivalent to each other.

Definition 4: If \( \tilde{a} = [a^L, a^U] \), \( \tilde{b} = [b^L, b^U] \), then:

\[
\|\tilde{a} - \tilde{b}\| = |a^L - b^L| + |a^U - b^U| \\
\]

(3)

While: \( d(\tilde{a}, \tilde{b}) = \|\tilde{a} - \tilde{b}\| \) is the deviation degree between \( \tilde{a} \) with \( \tilde{b} \). Hence, we can read that the larger \( d(\tilde{a}, \tilde{b}) \), the greater the deviation degree of two interval numbers. If \( d(\tilde{a}, \tilde{b}) = 0 \), then they are always equal to each other.

Definition 5: Suppose
\[
\tilde{x}_j = \left[ x^L_j, x^U_j \right] = \left[ \max(x^L_j), \max(x^U_j) \right]. \]

Among them, \( j = (1, 2, ..., m) \) is its positive ideal point, the bigger
\( x^* \) is better.

By contrast, \( \bar{x}_j = \left[ x_{jL}^*, x_{jU}^* \right] = [\min(x_{jL}^*), \min(x_{jU}^*)] \), 
\( j = (1, 2, ..., m) \) is its negative ideal point, the smaller \( \bar{x}_j \) is better. Consequently, the ideal feature sequence that composed of ideal point can be represented as bellow: \( \bar{x}^* = (\bar{x}_1^*, \bar{x}_2^*, ..., \bar{x}_m^*) \).

2.2 Inference of Sequence Dominance Relation

Let be \( \bar{a} = [a^L, a^U], \bar{b} = [b^L, b^U] \), then the ideal feature sequence of interval numbers is \( \bar{a}^* = [c^L, c^U] \).

Such relationship is called \( \bar{a} f \bar{b} \) if the following formula is tenable:

\[
d(c^*, a) < d(c^*, b)
\]  
(4)

Inference 1: When making a positive or negative ideal decision about schemes, then we have got it, respectively.

\[
\bar{a} f \bar{b} \iff p(\bar{a} \geq \bar{b}) > \frac{1}{2} \iff a^L + a^U > b^L + b^U
\]  
(5)

\[
\bar{a} f \bar{b} \iff p(\bar{a} \leq \bar{b}) > \frac{1}{2} \iff a^L + a^U < b^L + b^U
\]  
(6)

If Eq.(7) is a positive ideal scheme, thus, the Eq.(8) bellow is established on account of ideal interval number \( c^* \).

\[
d(c^*, a) = |c^L - a^L| + |c^U - a^U|
\]  
(7)

\[
d(c^*, b) = |c^L - b^L| + |c^U - b^U|
\]  
(8)

For the Eq.(3) shown, that is:

\[
d(c^*, a) = (c^L + c^U) - (a^L + a^U),
\]  
(9)

\[
d(c^*, b) = (c^L + c^U) - (b^L + b^U)
\]

If there exists a

\[ p(\bar{a} \geq \bar{b}) = 1 \iff a^L \geq a^L \Rightarrow a^L + a^U > b^L + b^U \]  

\[ p(\bar{a} \geq \bar{b}) = 1 \iff a^L + a^U > b^L + b^U \]

Therefore, Eq. (4) is established when making a positive ideal decision about schemes.

The proof procedure of Inference 1 indicated that an equivalence relation exists between the probability measure and dominance degree of scheme attribute values [12].

On the above-mentioned principles, let be \( A = \{a_1, a_2, ..., a_m\} \) and \( B = \{b_1, b_2, ..., b_m\} \) are the alternative sequences of interval number, \( U = \{u_1, u_2, ..., u_m\} \) is ideal sequence composed of ideal points, the aforementioned three parties require the following conditions to be satisfied: \( a_i = [a_i^L, a_i^U], b_i = [b_i^L, b_i^U] \) and \( u_i = [u_i^L, u_i^U] \). If the following formula holds, the interval number sequence A is superior to B, which can be recorded as \( A f B \):

\[
\sum_{i=1}^{m} d(a_i, u_i) < \sum_{i=1}^{m} d(b_i, u_i)
\]  
(10)

Inference 2: When making a positive or negative ideal decision about schemes, then we have got it, respectively.

\[
A f B \iff \sum_{i=1}^{m} (a_i^L + a_i^U) > \sum_{i=1}^{m} (b_i^L + b_i^U)
\]  
(11)

\[
A f B \iff \sum_{i=1}^{m} (a_i^L + a_i^U) < \sum_{i=1}^{m} (b_i^L + b_i^U)
\]  
(12)

If \( U^* = \{u_1^L, u_2^L, ..., u_m^L\} \) is an ideal sequence of positive ideal points, then:

\[
u_i^U \geq \max \{a_i^L, b_i^L\}, u_i^L \geq \max \{a_i^U, b_i^U\}
\]  
(13)

That is:

\[
\sum_{i=1}^{m} d(a_i, u_i^L) = \sum_{i=1}^{m} |a_i^L - u_i^L| + |a_i^U - u_i^U|
\]  
(14)

\[
\sum_{i=1}^{m} d(b_i, u_i^L) = \sum_{i=1}^{m} |b_i^L - u_i^L| + |b_i^U - u_i^U|
\]  
(15)

As for \( \sum_{i=1}^{m} d(a_i, u_i^*) < \sum_{i=1}^{m} d(b_i, u_i^*) \). In this thesis, the schemes can be prioritized in virtue of the size of object attribute values in the process of multi-attribute decision making.

2.3 Determining Index Weight by Using Attribute Dominance Relation

In the process of mathematical model establishment and programming, in order to reasonably confirm the weight value of each evaluation indexes, the
decision matrix $A$ can be transformed into a normalized fuzzy matrix $R = (r_{ij})_{n \times m}$ by using the following formula, $r_{ij}$ is a normalized interval number [15,16].

As for the benefit indexes, then:

$$r_{ij}^l = \frac{a_{ij}^u - a_{ij}^s}{a_{ij}^s - a_{ij}^*}, r_{ij}^U = \frac{a_{ij}^U - a_{ij}^*}{a_{ij}^s - a_{ij}^*}; \quad (16)$$

As for the cost indexes, then:

$$r_{ij}^l = \frac{a_{ij}^s - a_{ij}^U}{a_{ij}^s - a_{ij}^*}, r_{ij}^U = \frac{a_{ij}^l - a_{ij}^s}{a_{ij}^s - a_{ij}^*}. \quad (17)$$

Where: $a_{ij}^* = \max\{a_{ij}^u\}$, $a_{ij}^s = \min\{a_{ij}^l\}$ and $j = [1,2,...,m]$. The attribute dominance relation is that the probability measure of $c_j \succ c_k$, which can be shown through the following formula.

$$p(c_j \succ c_k) = 1 - \sum_{i=1}^n P(r_{ij} \leq r_{ik}^*) \quad (18)$$

The matrix is as follows:

$$P_{mn} = p(c_j \succ c_k)_{mn} \quad (19)$$

Therefore, it is necessary to weigh it from the perspective of dominance relationship and probability theory in the process of decision making [17].

$$\omega_{ij} = \sum_{j=1}^{m} p(c_j \succ c_k) / \sum_{i=1}^{n} \sum_{j=1}^{m} p(c_j \succ c_k) \quad (20)$$

### 2.4 The Detailed Process of Model

- Firstly the indicators are dealt with non-dimensional processing. Eq. (16) and Eq. (17) are used to transform them into the normalized decision-making matrix $R = (r_{ij})_{n \times m}$.

- Furthermore Eq. (18) and Eq. (19) are used to solve $R = (r_{ij})_{n \times m}$, thus the probability dominance degree $p(C_j \succ C_k)$ and probability dominance matrix $P_{mn}$ can be obtained. From above-mentioned calculation, we can further obtain the weight values of each evaluation $\omega_{ij}$ indexes by Eq. (20).

- And then the weighted normalized matrix and comprehensive attribute values are constructed as follows.

$$z_j(\omega) = \sum_{j=1}^{m} r_{ij}^* \omega_j \quad (21)$$

Thus, the probability measure that the dominance degree of schemes $x_j$ is better than that of $x_i$ can be expressed as:

$$p(c_j \succ c_k) = p(\sum_{i=1}^{m} r_{ij}^* \omega_j \geq \sum_{i=1}^{m} r_{ik}^* \omega_j) \quad (22)$$

That is:

$$P_{mn} = p(x_j \geq x_k)_{mn} \quad (23)$$

- Finally, the dominance probability measures of decision-making schemes are solved by Eq. (22) and Eq. (23), which is collated and sorted accordingly [18,19].

### 3 CASE STUDY

A landslide-dam under study locates at the upstream of a planning town area, and its safety is very important for developing tourism, engineering construction and the safety of life and property in the lower reach region. The main axis direction of the landslide body is from southwest to northeast, whose size is 1000m long, 600m wide and 80m in height. Among them, the total volume and angle of a landslide body are $611.6 \times 10^4 m^3$, 10 degrees, respectively. It belongs to large-scale soil accumulation landslide, and its landsat image map and engineering layout map are shown below (Fig.1) [20,21].

Owing to the complexity of an engineering project, the front of landslide needs to be excavated to a certain extent (5-20m). The excavation datum elevations of the north, middle and south areas of the landslide during the project construction are 928m, 932m and 921m, respectively, which will directly affect the stability of original landslide. Therefore, to ensure the smooth progress of project quality standards. As proved by the investigation done by the author, a total of eight technologists and disaster experts are invited to make decision and consultation for these slope remediation problems. In this thesis, the alternative control schemes are put forward in the preliminary design stage of a project are as follows [22,23]. Subsequently, the choose of control indexes is very important in stability analysis and treatment of the landslide. Based on this, we selected the reasonable, feasible and effective control indexes for landslide schemes on account of the characteristics of hazards, prevention measures, etc. Mainly include: Total Project Investment $U_1$, Construction Difficulty $U_2$, Construction Risk $U_3$ and Construction Effect $U_4$. Among them, $U_1$, $U_2$, and $U_3$ are a kind of cost index as mentioned above, conversely, $U_3$ is a kind of benefit index (as shown in Table 1). This paper proposes a method of attribute dominance relation for obtaining the attribute weights of
the decision-making matrix in multi-attribute decision-making problem, in which attribute weights are unknown completely and the attribute values are in the forms of interval numbers. Moreover, the decision priority results of deviation maximization algorithm [24,25] and inference 2 are selected to verify and analysis the usefulness and accuracy of the proposed model.

![Landsat image map of a landslide](image1)

Figure 1 - Design and Construction of Landslide Treatment

- **Scheme S1**: Lattice slope protection + bamboo living water + intercepting drain and drainage ditch
- **Scheme S2**: Surface-drainage + local clearing + intercepting drain and drainage ditch
- **Scheme S3**: Landscape type anti-slide pile + intercepting drain and drainage ditch
- **Scheme S4**: Surface-drainage + Load reducing foot + monitoring and warning
- **Scheme S5**: Local clearing + cantilever anti-slide column + surface-drainage
- **Scheme S6**: Lattice ditch slope protection + underground-drainage + cantilever anti-slide column
- **Scheme S7**: Anchor cable anti-slide pile + cantilever anti-slide column + surface-drainage
- **Scheme S8**: Cantilever anti-slide pile + cantilever anti-slide column + surface-drainage

![Regional terrain plan of a landslide](image2)

**Table 1 – Decision Matrix of the Index Attributes**

| Schemes | Total Project Investment $U_1$ | Construction Difficulty $U_2$ | Construction Risk $U_3$ | Construction Effect $U_4$ |
|---------|--------------------------------|-------------------------------|------------------------|--------------------------|
| $S_1$   | [6700,7000]                    | [0.20,0.25]                   | [0.25,0.30]            | [0.10,0.20]              |
| $S_2$   | [5900,6200]                    | [0.23,0.28]                   | [0.20,0.25]            | [0.12,0.22]              |
| $S_3$   | [5900,6200]                    | [0.24,0.29]                   | [0.21,0.26]            | [0.12,0.22]              |
| $S_4$   | [5700,6000]                    | [0.18,0.23]                   | [0.25,0.30]            | [0.15,0.25]              |
| $S_5$   | [5400,5700]                    | [0.16,0.21]                   | [0.18,0.23]            | [0.14,0.24]              |
| $S_6$   | [7200,7500]                    | [0.22,0.27]                   | [0.15,0.20]            | [0.16,0.26]              |
| $S_7$   | [6200,6500]                    | [0.19,0.24]                   | [0.23,0.28]            | [0.14,0.24]              |
| $S_8$   | [6400,6700]                    | [0.20,0.25]                   | [0.16,0.21]            | [0.15,0.25]              |

Firstly, the indicators are dealt with non-dimensional processing, Eq.(16) and Eq.(17) are used to transform them into the normalized decision-making matrix $\tilde{R} = (\tilde{r}_{ij})_{n \times m}$, as shown in Table 2.
Table 2–Decision Information Table after Normalized

| Schemes | \(U_1\) | \(U_2\) | \(U_3\) | \(U_4\) |
|---------|---------|---------|---------|---------|
| \(S_1\) | [0.6190,0.7619] | [0.3077,0.6923] | [0.6667,1.0000] | [0.3750,1.0000] |
| \(S_2\) | [0.2381,0.3810] | [0.0769,0.4615] | [0.3333,0.6667] | [0.2500,0.8750] |
| \(S_3\) | [0.2381,0.3810] | [0.0000,0.3846] | [0.4000,0.7333] | [0.2500,0.8750] |
| \(S_4\) | [0.1429,0.2857] | [0.4615,0.8462] | [0.6667,1.0000] | [0.0625,0.6875] |
| \(S_5\) | [0.0000,0.1429] | [0.6154,1.0000] | [0.2000,0.5333] | [0.1250,0.7500] |
| \(S_6\) | [0.8571,1.0000] | [0.1538,0.5385] | [0.0000,0.3333] | [0.0000,0.6250] |
| \(S_7\) | [0.3810,0.5238] | [0.3846,0.7692] | [0.5333,0.8667] | [0.1250,0.7500] |
| \(S_8\) | [0.4762,0.6190] | [0.3077,0.6923] | [0.0667,0.4000] | [0.0625,0.6875] |

Then the probability dominance matrix \(P_{max}\) and weight values \(\omega_j\) of each evaluation indexes can be obtained by Eq.(18-20), which is shown below.

\[
P_{max} = p(x_j \geq x_k) = \begin{bmatrix}
0.5000 & 0.5017 & 0.2875 & 0.4254 \\
0.4983 & 0.5000 & 0.4268 & 0.5119 \\
0.7125 & 0.5732 & 0.5000 & 0.5587 \\
0.5746 & 0.4881 & 0.4413 & 0.5000
\end{bmatrix}
\]

\[
\omega_j = \sum_{i=1}^{n} p(c_j \neq c_i) / \sum_{i=1}^{n} \sum_{j=k}^{m} p(c_j \neq c_i)
\]

\[
= (0.2024, 0.2395, 0.3074, 0.2507)
\]

Next, the weighted normalized matrix and weighted synthetic attribute values are calculated by Eq. (21) and shown in Table 3.

Table 3–The Weighted Decision Information Table

| Schemes | \(U_1\) | \(U_2\) | \(U_3\) | \(U_4\) |
|---------|---------|---------|---------|---------|
| \(S_1\) | [0.1253,0.1542] | [0.0737,0.1658] | [0.2049,0.3074] | [0.0940,0.2507] |
| \(S_2\) | [0.0482,0.0771] | [0.0184,0.1105] | [0.1025,0.2049] | [0.0627,0.2193] |
| \(S_3\) | [0.0482,0.0771] | [0.0000,0.0921] | [0.1230,0.2254] | [0.0627,0.2193] |
| \(S_4\) | [0.0289,0.0578] | [0.1105,0.2026] | [0.2049,0.3074] | [0.0157,0.1723] |
| \(S_5\) | [0.0000,0.0289] | [0.1474,0.2395] | [0.0615,0.1639] | [0.0313,0.1880] |
| \(S_6\) | [0.1735,0.2024] | [0.0368,0.1290] | [0.0000,0.1025] | [0.0000,0.1567] |
| \(S_7\) | [0.0771,0.1060] | [0.0921,0.1842] | [0.1639,0.2664] | [0.0313,0.1880] |
| \(S_8\) | [0.0964,0.1253] | [0.0737,0.1658] | [0.0205,0.1230] | [0.0157,0.1723] |

\[
z_j(\omega) = \sum_{j=1}^{m} r_{ij} \omega_j = ([0.4979,0.8781], [0.2318,0.6119], [0.2338,0.6140], [0.3601,0.7402], [0.2402,0.6204], ...)
\]

\[= [0.2104,0.5905], [0.3645,0.7447], [0.2062,0.5864] \]

Subsequently, the dominance matrix of decision schemes \(P_{max}\) also can be calculated as bellow.

\[
P_{max} = \begin{bmatrix}
0.5000 & 0.8501 & 0.8474 & 0.6814 & 0.8390 & 0.8782 & 0.6755 & 0.8836 \\
0.1499 & 0.5000 & 0.4973 & 0.3313 & 0.4889 & 0.5281 & 0.3254 & 0.5335 \\
0.1526 & 0.5027 & 0.5000 & 0.3340 & 0.4916 & 0.5309 & 0.3281 & 0.5363 \\
0.3186 & 0.6687 & 0.6660 & 0.5000 & 0.6576 & 0.6969 & 0.4941 & 0.7023 \\
0.1610 & 0.5111 & 0.5084 & 0.3424 & 0.5000 & 0.5392 & 0.3365 & 0.5446 \\
0.1218 & 0.4719 & 0.4691 & 0.3031 & 0.4608 & 0.5000 & 0.2973 & 0.5054 \\
0.3245 & 0.6746 & 0.6719 & 0.5059 & 0.6635 & 0.7027 & 0.5000 & 0.7082 \\
0.1164 & 0.4665 & 0.4637 & 0.2977 & 0.4554 & 0.4946 & 0.2918 & 0.5000
\end{bmatrix}
\]

Therefore, the schemes are optimized and sorted by comprehensive probability dominance as follows:
The ranking result of control schemes is obtained:
\[ S_1 \succ S_5 \succ S_4 \succ S_7 \succ S_3 \succ S_2 \succ S_6 \succ S_8 \].

On the basis of conclusion and its comprehensive attribute values in inference 2, then:
\[ \sum_{i=1}^{4} x_{10i}^L + x_{10i}^U = 1.3761 \]
\[ \sum_{i=1}^{4} x_{20i}^L + x_{20i}^U = 0.8437 \]
\[ \sum_{i=1}^{4} x_{30i}^L + x_{30i}^U = 0.8478 \]
\[ \sum_{i=1}^{4} x_{40i}^L + x_{40i}^U = 1.1003 \]
\[ \sum_{i=1}^{4} x_{50i}^L + x_{50i}^U = 0.8606 \]
\[ \sum_{i=1}^{4} x_{60i}^L + x_{60i}^U = 0.8009 \]
\[ \sum_{i=1}^{4} x_{70i}^L + x_{70i}^U = 1.1092 \]

Therefore, the control schemes are sorted by above inference 2 as follows:
\[ S_1 \succ S_3 \succ S_7 \succ S_4 \succ S_5 \succ S_2 \succ S_6 \succ S_8 \]

To validate and compare the proposed model, this thesis uses the Deviation Maximization Algorithm to empower the various indicators of the control schemes, its corresponding weighting formula is as bellow.
\[ \omega_j = \frac{\sum_{i=1}^{n} \sum_{k=1}^{n} d(r_{ij}^*, r_{kj}^*)}{\sum_{j=1}^{m} \sum_{k=1}^{n} d(r_{ij}^*, r_{kj}^*)}, j \in M. \]

In this method, the weight value of control schemes is calculated by the above formula, that is:
\[ \omega_{c_1} = 0.3183 \quad \omega_{c_2} = 0.2386 \]
\[ \omega_{c_3} = 0.3009 \quad \omega_{c_4} = 0.1421 \]

And then the data in table 3 is weighted with the result of upper simulation. Thus, the weighted synthetic attribute values are obtained as follows.

According to the basic calculating procedure of deviation maximum decision model. By comparing the weighted synthetic attribute values of Eq. (1),
the dominance matrix of decision schemes \( P_{mca} \) can be obtained as follows.

Forasmuch as the relevance theorem and definition of an interval number. It can be seen that the above probability matrix \( P_{mca} \) is a fuzzy complementary matrix. Thereby, the ranking vector \( v = (v_1, v_2, ..., v_n) \), \( i \in N \) of probability matrix can be obtained by the transfer algorithm of sorted matrix.

Among them:
\[ v_i = \frac{1}{n(n-1)} \left( \sum_{j=1}^{n} p_{ij} + \frac{n}{2} - 1 \right) \]

Furthermore, we gained a ranking vector \( v \) of probability matrix \( P_{mca} \) as bellow:
\[ v = (0.1705, 0.1061, 0.1065, 0.1357, 0.1050, 0.1235, 0.1414, 0.1113) \]

Then the control schemes are sorted by above ranking vector as follows:
\[ S_1 \succ S_7 \succ S_4 \succ S_6 \succ S_8 \succ S_3 \succ S_2 \succ S_5 \]

In summary, on the basic of above theoretical analysis, the simulation results are compared as follows (Table4).
Table 4—Comparison of Several Decision Model for Control Schemes

| Schemes | Deviation Maximum Decision Model | Alternative Ranking | Inference 2 | Alternative Ranking | Probability Dominance Decision Model | Alternative Ranking |
|---------|----------------------------------|---------------------|-------------|---------------------|--------------------------------------|---------------------|
| S1      | 0.1705                           | 1                   | 1.3761      | 1                   | 0.8079                              | 1                   |
| S2      | 0.1061                           | 7                   | 0.8437      | 6                   | 0.4078                              | 6                   |
| S3      | 0.1065                           | 6                   | 0.8478      | 5                   | 0.4109                              | 5                   |
| S4      | 0.1357                           | 3                   | 1.1003      | 3                   | 0.6006                              | 3                   |
| S5      | 0.1050                           | 8                   | 0.8606      | 4                   | 0.4205                              | 4                   |
| S6      | 0.1235                           | 4                   | 0.8009      | 7                   | 0.3756                              | 7                   |
| S7      | 0.1414                           | 2                   | 1.1092      | 2                   | 0.6073                              | 2                   |
| S8      | 0.1113                           | 5                   | 0.7927      | 8                   | 0.3694                              | 8                   |

![Comparison Diagram of Landslide Control Schemes for Three Models](image)

Hence we can read that the ranking between the proposed model and inference 2 are roughly the same (Fig.2). However, the ranking of schemes $S_2$, $S_3$, $S_5$, $S_6$, and $S_8$ in the deviation maximum model are different from the others (Table 4). This is mainly due to the fact that ignored the importance of a scheme attributes themselves. An important theme to notice: The optimal schemes of the three models are all uniform ($S_1$: Lattice slope protection + bamboo living water + intercepting drain and drainage ditch). As a matter of fact, the scheme $S_1$ recommended within this paper has not been adopted during the treatment of the central and northern landslide areas. The phenomenon of unreasonable excavation and soil extraction at the foot of slope eventually leads to large deformation in the connecting area between the north and middle of the project. Additionally, the inappropriate construction procedure at the foot of the slope in the north landslide area leads to that the stability of landslide becomes worse. And then the rationality and validity of the proposed scheme $S_1$ is once again proved in this study. In this respect, this paper proposed an interval probability dominance relation algorithm compared with game theory, and its strict theoretical deduction and proof reasoning are also carried out in this procedure. Simultaneously, the inversion result is obtained for inference 2 with eight schemes, and then simulation results show that both the proposed model and algorithm are reasonable and feasible.

4 CONCLUSIONS

With the requirements increasing, the management works for landslide are becoming complicated and professional more and more. Especially for the complex decision-making problems, the traditional decision algorithms and models have always had some limitations, which leads to the overall deci-
sion-making errors in scheme decision. Based on the above ponder, this article mainly proposed an interval probability dominance relation algorithm compared with game theory, and the algorithmic inversion is also carried out in this dissertation. Through the application and analysis of engineering examples, it is indicated that the optimum selection of alternatives can be achieved by the ranking of dominance relations. Finally, the simulation results illustrated the effectiveness of the proposed model.

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