Investigation of directivity of the nanoantenna by its inherent resonant states

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Abstract. We present a quasinormal mode (QNM) approach for modeling the nanoantenna and describe the response of localized dielectric cylinder resonators. The inherent resonant states of the dielectric cylinder nanocavity are investigated for modified and reduced geometrical symmetry. We find some modes contributing mainly to the directivity and have a high-quality factor. The variation of the eigenmodes with cylinder height and substrate refractive index has also been investigated.

Nanoantennas are of great importance for the development of wireless communication systems for future photonic chips \cite{1,2} as semiconductor chips are currently limited by the low data transfer rate due to high temperature rising \cite{3}. As the electromagnetic fields of surface-plasmon-polariton (SPP) waves are highly localized to dielectric/metal interfaces, SPP-wave-based photonic chips can overcome the limitation \cite{4}. However, a metal nanostructure possesses high ohmic losses which limit the propagation distance \cite{5}. Therefore, integrating all-dielectric optical nanoantennas in a photonic chip is one of the most promising for the integrated platform.

All-dielectric nanoantennas usually consist of several dielectric nanoparticles of various shapes, e.g., nanospheres \cite{6}, nanoblocks \cite{7}, nanodisks \cite{8}, nanocubes \cite{9}, and V-shaped nanoparticles \cite{10}. Forward and backward scattering characteristics of individual nanoparticles can be controlled by engineering the electric and magnetic resonances \cite{11-13}. A dielectric cylinder resonator has been studied extensively both theoretically and experimentally to investigate its optical properties. Several studies examine the multipole decomposition of the dielectric cylinder \cite{14–16} and its effect on optical emission. Cylinders can be optimized to enhance scattering by constructively overlapping several multipoles \cite{17}, to transfer power from lower-order multipoles to higher ones, and reproduce the quasi-bound state in the continuum (BICs) \cite{18}, or to suppress scattering as traditionally known as the anapole state \cite{19}. However, few studies only analyze the inherent resonant states of a dielectric cylinder nanocavity and its influence on the directivity in order to obtain superdirective nanoantenna.

With this motivation, we investigate the inherent resonant states of a dielectric cylinder nanocavity by employing the quasinormal modes (QNMs) method and reveal how the QNMs contribute to the directivity. First, the effect of the height of the nanocylinder and refractive index of the substrate on the complex eigenfrequency will be determined together with the contributions of major modes to the directivity. Next, we determine how the directivity of a Si-cylinder nanoantenna is tailored when the
nanostructure height is varying. Finally, we demonstrate the nanoantenna with substrate providing very high directivity.

1. Inherent resonant states of a dielectric nanocylinder
We consider a geometry consisting of a Si-dielectric cylinder ($n = 3.6$) with height $h_{cyt}$ and radius $r_{cyt}$. For all the numerical results presented hereinafter, we choose the radius of the cylinder $r_{cyt} = 100$ nm and the cylinder is shined by x-polarized light.

We identify the inherent resonant mode of the dielectric nanocylinder by using the quasinormal mode (QNM) expansion. The natural response of resonance modes of the plasmonic and dielectric resonators is called QNMs. The QNMs are the time-harmonic solution of Maxwell’s equations. It is used to describe the resonances of the non-conservative system which is not possible by normal modes. The eigenfrequency $\omega_m$ of the QNMs is complex with the quality factor $Q_m = \frac{1}{2} \text{Re}(\omega_m)/\text{Im}(\omega_m)$. The electric field $E_m$ and eigenfrequency $\omega_m$ of the $m^{th}$ QNM are determined by COMSOL Multiphysics with its eigenmode solver [20, 21]. Once the $E_m$ and $\omega_m$ are known, we determine the scattered field $E_s(r, \omega)$ due to interaction of light with dielectric cylinder resonator by the QNM expansion of the scattered field, i.e.

$$E_s(r, \omega) = \sum_m \alpha_m(\omega) E_m(r),$$

where $\alpha_m(\omega)$ is the excitation coefficient, which depends on the incident fields. More details to determine the $\alpha_m(\omega)$ are given elsewhere [21].

**Figure 1.** Variation of the quasinormal modes (QNMs) of a Si-cylinder nanoparticle with; (a) the height of the cylinder $h_{cyt} \in [100, 400]$ nm and (b) refractive index of the substrate $n_{sub} \in [1.0, 4.0]$; when the radius $r_{cyt} = 100$ nm. Arrows show how the complex eigenfrequency moves with the increase in $h_{cyt}$ and $n_{sub}$. The colors of the dots show the directivity of the cylinder nanoantenna at that QNMs frequency. Three main calculated modes that contribute most in the scattering are numbered. Other modes (blue dots) are background modes that do not contribute to the scattering and directivity; (c) E-field profile inside the cylinder for three most relevant mode when $h_{cyt} = 200$ nm and $n_{sub} = \{1, 1.6\}$.
Figures 1 (a) and (b) show the variation of the QNM modes eigenfrequencies with cylinder height ($h_{cyl}$) and refractive index ($n_{sub}$) of the substrate, respectively. Arrows in the figures describe how the eigenmodes change with an increase in $h_{cyl}$ and $n_{sub}$. Further, the color of the dots in the figures shows the directivity of the nanoantenna at that eigenfrequency. It can be observed from Fig. 1(a), as the height of the cylinders increases, Re ($\omega_m$) increases, i.e. $Q_m$ increases too, which enables the electromagnetic fields are trapped for sufficiently long times in a small space. Similarly, $Q_m$ decreases when $n_{sub}$ increases (see Fig. 1(b)). Furthermore, Modes I, II, and III have high directivity and contribute mostly to the directional scattering. Other modes (with blue dots) do not significantly contribute to the directivity and we call them background modes. Figure 1(b) also shows the directivity of the nanoresonator increases, when $n_{sub}$ increases. The electric field profile inside the cylinder for Mode I, II and III are shown in Fig. 1(c) when $h_{cyl} = 200$ nm and $n_{sub} = \{1, 1.5\}$.

The directivity of the Si-cylinder nanoantenna is investigated numerically based on the COMSOL by the finite-element method. Figure 2(a) shows the directivity of the nanoantenna in the visible spectrum $\lambda \in [450, 800]$ nm, when the height of the cylinder varies in the range $h_{cyl} \in [140, 400]$ nm. We observe that high directivity is obtained when the height of the cylinder is also high. Maximum directivity 6.1 is obtained at the wavelength $\lambda = 450$ nm when $h_{cyl} = 400$ nm. Further, when we introduce a substrate, indirectly we are reducing the structure overall symmetry, which results in higher directivity as can be seen by Fig. 2(b). By introducing the asymmetricity, significant higher modes are excited (as can be seen from Fig. 1(b)), which, in its turn, results in higher directivity of scattering. In Fig. 2(b) it is clearly seen that directivity increases sharply when the nanocylinder is placed on a substrate.

![Figure 2](image)

**Figure 2.** The dependency of the directivity on wavelength and (a) the height of the cylinder ($h_{cyl}$); (b) substrate refractive index ($n_{sub}$); $h_{cyl}$ is fixed to be 200 nm.

2. **Conclusion**

In summary, we have investigated the directivity of all-dielectric nanoantenna with its inherent resonant states by using the QNMs method. We demonstrated that the directivity of a nanoantenna can be controlled by the height of the cylinder and the refractive index of the substrate. The strong enhancement of the directivity has been observed when we break the symmetry by introducing the substrate. We showed directivity and eigenmodes frequency evolution of the dielectric nanoantenna by QNMs expansion method and found a directivity with the increase of the substrate refractive index. We showed three different QNMs contributing most to the directivity and other modes called background modes do not contribute.

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