Is Modified Chaplygin gas along with barotropic fluid responsible for acceleration of the Universe?

Writambhara Chakraborty and Ujjal Debnath

1Department of Mathematics, Heriitage Institute of Technology, Anandapur, Kolkata-700 107, India.
2Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.

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In this letter, we have considered a model of the universe filled with modified Chaplygin gas and another fluid (with barotropic equation of state) and its role in accelerating phase of the universe. We have assumed that the mixture of these two fluid models is valid from (i) the radiation era to $\Lambda$CDM for $-1 \leq \gamma \leq 1$ and (ii) the radiation era to quiescence model for $\gamma < -1$. For these two fluid models, the statefinder parameters describe different phase of the evolution of the universe.

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Recent measurements of redshift and luminosity-distance relations of type Ia Supernovae indicate that the expansion of the Universe is accelerating [1-5]. This implies that the pressure $p$ and the energy density $\rho$ of the Universe should violate the strong energy condition $\rho + 3p < 0$ i.e., pressure must be negative. The matter responsible for this condition to be satisfied at some stage of evolution of the universe is referred to as dark energy [6 - 8]. There are different candidates to play the role of the dark energy. The most traditional candidate is a non-vanishing cosmological constant which can also be though of as a perfect fluid satisfying the equation of state $p = -\rho$. Negative pressure leading to an accelerating Universe can also be obtained in a Chaplygin gas cosmology [9], in which the matter is taken to be a perfect fluid obeying an exotic equation of state $p = -B/\rho, (B > 0)$. The Chaplygin gas behaves as pressureless fluid for small values of the scale factor and as a cosmological constant for large values of the scale factor which tends to accelerate the expansion. Subsequently the above equation was generalized to the form $p = -B/\rho^\alpha, 0 \leq \alpha \leq 1$ [10-12] and recently it was modified to the form $p = A\rho - B/\rho^\alpha, (A > 0)$ [13, 14], which is known as Modified Chaplygin Gas. This equation of state shows a radiation era ($A = 1/3$) at one extreme and a $\Lambda$CDM model at the other extreme.

The metric of a homogeneous and isotropic universe in FRW model is

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

where $a(t)$ is the scale factor and $k (= 0, \pm 1)$ is the curvature scalar.

The Einstein field equations are (choosing $8\pi G = c = 1$)

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3}\rho$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p)$$

The energy conservation equation ($T^{\mu}_{\nu,\nu} = 0$) is

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

For modified Chaplygin gas, equation (4) yields

* writam1@yahoo.co.in
† ujjaldebnath@yahoo.com
\[ \rho = \left[ \frac{B}{1 + A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} \]  

(5)

where \( C \) is an arbitrary integration constant.

Here we consider two fluid cosmological model which besides a modified Chaplygin’s component, with equation of state (4) contains also a barotropic fluid component with equation of state \( p_1 = \gamma \rho_1 \). Normally for accelerating universe \( \gamma \) satisfies \( -1 \leq \gamma \leq 1 \). But observations state that \( \gamma \) satisfies \( -1.6 \leq \gamma \leq 1 \), i.e., \( \gamma < -1 \) corresponds to phantom model. For these two component fluids, r.h.s of equations (2) and (3), i.e., \( \rho \) and \( p \) should be replaced by \( \rho + \rho_1 \) and \( p + p_1 \) respectively. Here we have assumed the two fluid are separately conserved. For Chaplygin gas, the density has the expression given in equation (5) and for another fluid, the conservation equation gives the expression for density as

\[ \rho_1 = \frac{d}{a^{3(1+\gamma)}} \]  

(6)

where \( d \) is an integration constant.

We have described this two fluid cosmological model from the field theoretical point of view by introducing a scalar field \( \phi \) and a self-interacting potential \( V(\phi) \) with the effective Lagrangian

\[ \mathcal{L}_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]  

(7)

The analogous energy density \( \rho_\phi \) and pressure \( p_\phi \) corresponding scalar field \( \phi \) having a self-interacting potential \( V(\phi) \) are the following:

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \rho + \rho_1 = \left[ \frac{B}{1 + A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} + \frac{d}{a^{3(1+\gamma)}} \]  

(8)

and

\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) = p + p_1 = A\rho - \frac{B}{\rho^0} + \gamma \rho_1 = A \left[ \frac{B}{1 + A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} - B \left[ \frac{B}{1 + A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} + \gamma \frac{d}{a^{3(1+\gamma)}} \]  

(9)

For flat Universe \( (k = 0) \) and by the choice \( \gamma = A \), we have the expression for \( \phi \) and \( V(\phi) \):

\[ \phi = -\frac{1}{\sqrt{3(1+A)(1+\alpha)}} \int \left[ \frac{d + c \left( C + \frac{Bz}{1+A} \right)^{-\frac{1}{1+\alpha}}}{d + \left( C + \frac{Bz}{1+A} \right)^{\frac{1}{1+\alpha}}} \right] \frac{dz}{z} \]  

(10)

and

\[ V(\phi) = A \left[ \frac{B}{1 + A} + \frac{C}{z} \right]^{\frac{1}{1+\alpha}} - B \left[ \frac{B}{1 + A} + \frac{C}{z} \right]^{\frac{1}{1+\alpha}} + \frac{(1 - A)}{z^{1+\alpha}} \frac{d}{a^{3(1+\gamma)}} \]  

(11)

where \( z = a^{3(1+A)(1+\alpha)} \).
Figs. 1 - 3 shows variation of $\phi$ and $V$ against $a$ and $\phi$ for $A(= \gamma) = 1/3$ and $\alpha = 1$ (values of other constants: $B = 1, C = 1, d = 1$).

Here we have considered $\gamma = A$ for simplicity. Although by this choice we consider the two fluids to coincide at high densities, i.e., for early Universe. Taking $\gamma = A = 0$ we consider the mixture of dust with the generalized Chaplygin gas which has been discussed in the works of Gorini et al [10, 11]. Also for the choice $\gamma = \frac{A}{3}$ both the fluids represent radiation as the density is high at radiation era.

The graphical representation of $\phi$ against $a$ and $V(\phi)$ against $a$ and $\phi$ respectively have been shown in figures 1 - 3 for $A = 1/3$ and $\alpha = 1$. From figure 1 we have seen that scalar field $\phi$ decreases when scale factor $a(t)$ increases for $A = 1/3$. In figure 2, we see that potential function $V(\phi)$ sharply decreases from extremely large value to a fixed value for $A = 1/3$. The potential function $V(\phi)$ increases to infinitely large value when scale factor $a(t)$ increases for $A = 1/3$. So the figures show how $V(\phi)$ varies with $\phi$ and $a(t)$.

Since models trying to provide a description of the cosmic acceleration are proliferating, there exists the problem of discriminating between the various contenders. To this aim Sahni et al [15] proposed a pair of parameters $\{r, s\}$, called statefinder parameters. In fact trajectories in the $\{r, s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour. The above statefinder diagnostic pair has the following form:

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r - 1}{3(q - \frac{1}{2})}$$

(12)

where $H (= \frac{\ddot{a}}{a})$ and $q (= -\frac{\dddot{a}}{\ddot{a}})$ are the Hubble parameter and the deceleration parameter respectively. The new feature of the statefinder is that it involves the third derivative of the cosmological radius. These parameters are dimensionless and allow us to characterize the properties of dark energy. Trajectories in the $\{r, s\}$ plane corresponding to different cosmological models, for example $\Lambda$CDM model diagrams correspond to the fixed point $s = 0, \ r = 1$. 
For one fluid model, these \( \{r,s\} \) can be written as

\[
    r = 1 + \frac{9}{2} \left( 1 + \frac{p}{\rho} \right) \frac{\partial p}{\partial \rho}
\]

(13)

and

\[
    s = \left( 1 + \frac{p}{\rho} \right) \frac{\partial p}{\partial \rho}
\]

(14)

For the two component fluids, equations (13) and (14) take the following form:

\[
    r = 1 + \frac{9}{2} \left( 1 + \frac{p + p_1}{\rho + \rho_1} \right) \left[ \frac{\partial p}{\partial \rho} (\rho + p) + \frac{\partial p_1}{\partial \rho_1} (\rho_1 + p_1) \right]
\]

(15)

and

\[
    s = \frac{1}{(\rho + \rho_1)} \left[ \frac{\partial p}{\partial \rho} (\rho + p) + \frac{\partial p_1}{\partial \rho_1} (\rho_1 + p_1) \right]
\]

(16)

The deceleration parameter \( q \) has the form:

\[
    q = -\frac{\ddot{a}}{a H^2} = \frac{1}{2} + \frac{3}{2} \left( \frac{p + p_1}{p + p_1} \right)
\]

(17)

For modified gas and barotropic equation states, we can set:

\[
    x = \frac{p}{\rho} = A - \frac{B}{\rho^{\alpha+1}}
\]

(18)

and

\[
    y = \frac{\rho_1}{\rho} = \frac{\frac{d}{a^{\gamma+1}}}{\frac{B}{1+A} + \frac{C}{a^{\gamma+1}}} \left[ 1 + \frac{\gamma}{A} \right]^{-\frac{1}{\gamma+1}}
\]

(19)

Thus equations (15) and (16) can be written as

\[
    r = 1 + \frac{9s}{2} \left( \frac{x + \gamma y}{1 + y} \right)
\]

(20)

and

\[
    s = \frac{(1 + x)\{A(1 + \alpha) - \alpha x\} + \gamma(1 + \gamma)y}{x + \gamma y}
\]

(21)

with

\[
    y = \left[ \frac{d[(1+\alpha)(1+A)] B^{\gamma-A} (1 + \gamma)^{1+\gamma}}{C^{1+\gamma}(1 + A)^{1+\gamma}(A - x)^{\gamma-A}} \right]^{(1+\alpha)[1+A]}
\]

(22)

Now for cosmic acceleration \( q < 0 \), we have \( x + \gamma < -\frac{1}{3} \), since \( y > 0 \). Here \( y \) represents the ratio of the energy density of the barotropic fluid to that of Chaplygin gas and \( x, \gamma \) represent the ratios of fluid pressure to energy density for barotropic fluid and Chaplygin gas respectively. Since \( \gamma \) is a constant, we can assign different values to this barotropic index. But for cosmic acceleration we see that at least one of the fluids must generate negative pressure. Also for \( \gamma > \frac{2}{3} \), we must have \( x < -1 \), which is not possible physically as the Chaplygin gas can explain the evolution of the Universe only to ΛCDM model [14]. Here for different values of \( \gamma \) we have different scenarios, viz, (i) For \( \gamma = \frac{1}{3} \), we have \( x < -\frac{2}{3} \), i.e., we have cosmic acceleration without having the barotropic fluid to violate the energy condition. Here the Chaplygin
Fig. 4 - 7 show the variation of $s$ against $r$ for different values of $\gamma = 1/3$, 0, $-1$, $-1.5$ respectively and for $\alpha (= 0.5, 1), A = 1/3$. Fig. 8 shows the variation of $s$ against $r$ for different values of $\gamma = 1/3$, 0, $-1$, $-1.5$ and for $\alpha (= 1), A = 1/3$.

Gas represents the dark energy and the barotropic fluid represents the dark matter. (ii) $\gamma = 0$ and $x < -\frac{1}{3}$ also give cosmic acceleration. Here the Chaplygin gas violates the energy condition whereas the other fluid represents dust. This particular choice can also represent the present era, i.e., $q = -\frac{1}{2}$ provided $x < -\frac{2}{3}$. Like the previous case here also the Chaplygin gas represents the dark energy and the dust represents the dark matter. (iii) For $\gamma = -1$ we get cosmic acceleration without having the Chaplygin gas to violate the energy condition. This model can also represent the present epoch as for $q = -\frac{1}{2}$ and $\gamma = -1$ we have $x < \frac{1}{3}$. Here the barotropic fluid represents the dark energy and most likely behaves as the cosmological constant and itself is enough to generate cosmic acceleration. Here the Chaplygin gas represents the dark energy or the dark matter according as the value of $x$. (iv) For $\gamma < -1$ this fluid represents the phantom model whereas the Chaplygin gas represents the dark energy or the dark matter according as the value of $x$. 
The above four cases have been considered taking some particular values of $\gamma$. We have discussed the possibility of both the fluids to represent dark energy or dark matter. For $\frac{2}{3} \geq \gamma > -\frac{1}{3}$ the Chaplygin gas represents the dark energy and the barotropic fluid represents the dark matter. For $-\frac{1}{3} \geq \gamma \geq -1$ the barotropic fluid represents the dark energy. In this case the Chaplygin gas can represent both dark energy or dark matter depending on the values of the other parameters and the the ratio of the energy densities of the two fluids. For $\gamma < -1$ the model represents phantom energy.

From the equations (20) and (21) we can not written the relationship between $r$ and $s$ in closed form. Thus the relation between the parameters $r$ and $s$ in $\{r, s\}$ plane for different choices of other parameters are plotted in figures 4 - 8. The figures 4 - 7 shows the variation of $s$ against $r$ for different values of $\gamma = 1/3, 0, -1, -1.5$ respectively and for $\alpha (= 0.5, 1), A = 1/3$. Fig. 8 shows the variation of $s$ against $r$ for different values of $\gamma = 1/3, 0, -1, -1.5$ and for $\alpha (= 1), A = 1/3$. Thus the figures 4 - 6 represent the evolution of the universe starting from the radiation era to the $\Lambda$CDM model for $\gamma = 1/3, 0$ and the figure 7 represents the evolution of the universe starting from the radiation era to the quiescence model for $\gamma = -1.5$. Thus $\gamma$ plays an active role for the various stages of the evolution of the universe. If we choose the arbitrary constant $d$ is equal to zero, we recover the model of Modified Chaplygin gas [14]. If $A$ and the barotropic index $\gamma$ are chosen to be zero, we get back to the results of the works of Gorini et al [10].

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