STUDY ON DAMAGE CONSTITUTIVE MODEL OF RECYCLED AGGREGATE CONCRETE IN CODE FOR DESIGN OF CONCRETE STRUCTURES AND DEVELOPMENT IN ABAQUS

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Abstract: Based on the damage constitutive of ordinary concrete in code for Design of Concrete Structures and considering the similarity of mechanical properties between recycled aggregate concrete and ordinary concrete, a uniaxial damage constitutive of recycled concrete was proposed. At the same time, the above model was programmed as a subroutine by FORTRAN language, and the subroutine was embedded into finite element analysis software ABAQUS to simulate and analyze the uniaxial compression mechanical properties of recycled concrete. The results of finite element simulation and test are compared from the aspects of stress-strain curve and damage evolution curve. The results show that the proposed elastic-plastic damage constitutive model can effectively represent the mechanical properties of recycled aggregate concrete under uniaxial compression at different substitution rates.

1. Introduction
According to the Guardian, China produces about 2 million tons of waste concrete every year, and the trend is increasing year by year. At present, waste construction wastes are mainly treated in landfill sites, which leads to the waste of some resources and is not conducive to the sustainable development of the environment [1]. Therefore, strengthening the recycling of recycled aggregate concrete is one of the effective ways to realize the efficient utilization of resources and the sustainable development of environment. Aggregate strength is an important factor affecting concrete strength, while recycled aggregate concrete has the characteristics of "low strength, large porosity and strong water absorption" compared with natural aggregate, and the contact interface between aggregate and mortar is the weak
link in the failure of recycled aggregate concrete. Therefore, in order to ensure the reliability of recycled aggregate concrete in engineering applications, it is necessary to study the influence of different replacement rates of recycled coarse aggregate on the mechanical properties of recycled aggregate concrete, and select the replacement rates of recycled coarse aggregate according to the requirements of engineering applications on the mechanical properties of concrete.

The research on the mechanical properties of recycled aggregate concrete cannot be separated from the auxiliary support of experiments. However, in engineering applications, the experimental research on recycled aggregate concrete with different replacement rates of coarse aggregate is complicated and time-consuming. Therefore, it is of great significance to develop new research methods to better simulate the mechanical properties of recycled aggregate concrete under different substitution rates. Nowadays, the secondary development of Abaqus subroutine has become an important means for many scholars to study the mechanical properties of complex components. Shi Shaoyang [2] proposed the elasto plastic damage constitutive model of lightweight aggregate concrete based on D-P yield criterion, and wrote relevant Umat(user material subroutine), which was embedded into Abaqus to analyze the uniaxial compressive mechanical properties of lightweight aggregate concrete. The experimental results show that the material subroutine can well simulate the mechanical properties of cubic concrete and prismatic concrete under uniaxial compression. Hu Xiaobin et al. [3] wrote Umat based on uniaxial compression damage constitutive of Concrete Structure Design Code, which verified the feasibility of applying this subroutine to low-cycle reciprocating test simulation of reinforced concrete members. Based on ANSYS platform, Zhou Fen [4] verified the applicability of the prepared D-P damage model to the mechanical performance analysis of concrete under uniaxial compression and shear wall under uniaxial compression.

The study shows that the defined concrete under uniaxial compression damage constitutive subroutine into the feasibility of the finite element analysis, but the replacement ratio of recycled coarse aggregate concrete as one of the main factors influencing mechanical properties of recycled aggregate concrete, there is no research to consider the plasticity of recycled aggregate concrete under uniaxial compression constitutive model. For this reason, this article is based on code for design of concrete structures under uniaxial compression elastic damage constitutive, considering initial damage and plastic strain of two kinds of recycled aggregate concrete influence factors, established the recycled aggregate concrete elastoplastic damage constitutive model, realize the accurate simulation of mechanical properties of recycled aggregate concrete as recycled aggregate concrete in provides the theory basis for the popularization and application in engineering.

2. Recycled aggregate concrete uniaxial compression damage constitutive

Uniaxial compression of concrete includes plastic hardening and plastic softening. Under uniaxial compression, the stress-strain curve is linear before reaching the initial yield stress, then enters the plastic part with plastic hardening characteristics, and enters the plastic softening part of the stress-strain curve when the stress is greater than the limit stress. The constitutive equation of elastic damage of ordinary concrete under uniaxial compression given in Code for Design of Concrete Structures is:

\[ \sigma = (1 - d_c)E_c \varepsilon \]  

Where: is the initial elastic modulus of concrete, and is the elastic damage variable of concrete.

The damage degree of concrete under different stress states is also different, and the expressions of different stress states are given.

\[ d_c = \begin{cases} \frac{p_n}{n-1+x} & (x \leq 1) \\ \frac{p_r}{\alpha_r(x-1)^2 + x} & (x > 1) \end{cases} \]  

\[ p_r = \frac{f_r}{E_c\varepsilon_r} \]
\[
\begin{align*}
\eta &= \frac{E_{c} \varepsilon_{c}}{E_{c} \varepsilon_{c} - f_{c}} \quad (4) \\
x &= \frac{\varepsilon}{\varepsilon_{c}} \quad (5) \\
y &= \frac{\sigma}{f_{c}} \quad (6)
\end{align*}
\]

Where, \( \alpha \) represents the shape parameter of the descending section of the stress-strain curve of concrete, reflecting the development trend of the stress-strain curve of the descending section of concrete; \( \varepsilon_{c} \) represents the peak strain; \( f_{c} \) stands for peak stress.

A large number of experimental results show that the mechanical properties of recycled aggregate concrete are similar to those of ordinary concrete, and there will be obvious plastic deformation during loading. The above elastic damage constitutive equation of concrete under uniaxial compression did not consider the influence of plastic deformation on the mechanical properties of concrete. Therefore, based on the elastic damage constitutive equation of concrete under uniaxial compression, the elastic-plastic damage constitutive equation of recycled aggregate concrete under uniaxial compression was proposed [5], as shown in Equation (7).

\[
\sigma = (1 - D)E_{0}(\varepsilon - \varepsilon_{p}) \quad (7)
\]

In Formula (7), \( \varepsilon_{p} \) represents the plastic strain of recycled aggregate concrete, \( D \) represents the damage variable of recycled aggregate concrete, and \( \varepsilon_{0} \) represents the limit strain of the compression ratio of specimens.

It can be obtained from equations (1) and (7):

\[
D = \frac{d_{c} \varepsilon - \varepsilon_{p}}{\varepsilon - \varepsilon_{p}} \quad (8)
\]

Based on the consideration of the initial damage of recycled aggregate concrete and the damage specifications, the following damage evolution equation of recycled aggregate concrete is proposed:

\[
D = \begin{cases} 
D_{0} & 0 < x \leq x_{0} \\
\frac{d_{c} \varepsilon - \varepsilon_{p}}{\varepsilon - \varepsilon_{p}} & x > x_{0}
\end{cases} \quad (9)
\]

Among them, \( D_{0} \) represents the initial damage variable value of recycled aggregate concrete, \( x_{0} = \varepsilon_{p} / \varepsilon_{c} \).

Xiaobin Hu[6] proposed the following exponential relationship between residual plastic strain and RAC total strain according to the test results under RAC cyclic loading.

\[
\frac{\varepsilon_{p}}{\varepsilon_{u}} = m \left( \frac{\varepsilon_{u}}{\varepsilon_{c}} \right)^{n} \quad (10)
\]

In Equation (10), \( \varepsilon_{u} \) represents the strain at the unloading point. \( m \) and \( n \) are obtained by fitting the test data.

3. Umat subroutine development and validation

3.1. Umat subroutine interface

The built-in material constitutive relation in Abaqus is relatively conventional, which is not applicable to the simulation of mechanical properties of some materials, which requires the introduction of Umat subroutine. The user material subroutine can accurately define the constitutive relation of different materials to ensure the reliability of Abaqus simulation. Umat subroutines communicate with Abaqus data through the interface with Abaqus main solver [7].
3.2. Umat subroutine programming related to elastoplastic theory

The programming of user material subroutine is generally divided into four steps: the call of yield function, the judgment of elastic-plastic state, the selection of stress updating algorithm and the calculation of uniform tangent modulus. Among them, stress updating algorithm and calculation of consistent tangent modulus are the key and difficult points of user material subroutines. The stress update algorithm uses a completely implicit backward Euler integral algorithm, which has unconditional stability. The completely implicit backward Euler integral algorithm mainly includes two steps: elastic prediction step and plastic correction step. The plastic correction step is mainly used to pull the predicted elastic stress back to the plastic yield surface [8].

3.2.1. Yield function

The programming of Umat adopted the Mises yield function, the specific expressions of which are as follows [9].

\[
\sigma_{\text{mises}} = \sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6 (\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2)}
\] (11)

3.2.2. Plastic yield flow law

The plastic yield flow rule is used to determine the direction of the plastic strain increment generated during the uniaxial loading process of recycled aggregate concrete, so as to determine the stress change in the plastic development stage of recycled aggregate concrete. This Umat programming follows the following principles [10]:

Hydrostatic stress:

\[
\sigma_m = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}
\] (12)

For normal stress, the plastic yield flow direction is:

\[
\varepsilon_n = (\sigma_{n+1} - \sigma_m) / \sigma_{\text{mises}}
\] (13)

For shear stress, the plastic yield flow direction is:

\[
\varepsilon_{\text{mises}} = \sigma_{n+1} / \sigma_{\text{mises}}
\] (14)

For normal stress, the plastic yield flow factor is as follows:

\[
\Delta \lambda_n = \frac{3}{2} \Delta \varepsilon_p
\] (15)

For shear stress, the plastic yield flow factor is as follows:

\[
\Delta \lambda_{n+1} = 3 \Delta \varepsilon_p
\] (16)

Where, \( \Delta \varepsilon_p \) represents the equivalent plastic strain increment.

3.2.3. Stress update algorithm

The stress updating algorithm is divided into explicit integral algorithm and implicit integral algorithm. Explicit integral algorithm and implicit integral algorithm are essentially different solution algorithms. The explicit integral algorithm is a time-difference algorithm without iteration and convergence, but the time step affects the accuracy of the solution. The implicit integral algorithm is time-independent and has the characteristics of "high computational accuracy and unconditional stability". Meanwhile, it can save a lot of time for model simulation. Therefore, implicit integral algorithm is used for stress updating. The integral algorithm can be written as:

\[
\varepsilon_{n+1} = \varepsilon_n + \Delta \varepsilon
\] (17)
\[ e_{x+1} = e_x + \Delta e_{x+1} \]  
\[ e_{y+1} = e_y - \Delta e_{y+1} \]  
\[ \sigma_{x+y} = E \cdot \left( e_{x+y} - e_{x+y}^0 \right) \]  
\[ \Delta e = \Delta \epsilon \cdot \epsilon \]  
\[ E = (1 + D) E_0 \]  
\[ \begin{bmatrix} \kappa + 2G & \kappa & \kappa & 0 & 0 & 0 \\ \lambda & \kappa + 2G & \kappa & 0 & 0 & 0 \\ \lambda & \lambda & \kappa + 2G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \]  

Where, \( \Delta e \) denotes the strain increment; \( \epsilon \) denotes the rate of strain change; \( G \) denotes the shear modulus of elasticity; \( \kappa \) denotes the Rameen constant.

The stress update should also take into account the loading and unloading states, which are determined by \( d\epsilon \). As the uniaxial compression constitutive of recycled aggregate concrete is studied, \( d\epsilon < 0 \) indicates that the recycled aggregate concrete is in the loading state; otherwise, it indicates that the recycled aggregate concrete is in the unloading state. The stress update expressions of recycled aggregate concrete under loading and unloading states are as follows:

\[ \begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{13} \\ d\sigma_{23} \end{bmatrix} = -(1 - D)d\epsilon \]  
\[ \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \sigma_{11}^{\text{pre}} + d\sigma_{11} \\ \sigma_{22}^{\text{pre}} + d\sigma_{22} \\ \sigma_{33}^{\text{pre}} + d\sigma_{33} \\ 0 \\ 0 \\ 0 \end{bmatrix} \]  
\[ \begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{13} \\ d\sigma_{23} \end{bmatrix} = \begin{bmatrix} E_0(1, 1)d\epsilon_{11}^0 + E_0(1, 2)d\epsilon_{12}^0 + E_0(1, 3)d\epsilon_{13}^0 \\ E_0(2, 1)d\epsilon_{21}^0 + E_0(2, 2)d\epsilon_{22}^0 + E_0(2, 3)d\epsilon_{23}^0 \\ E_0(3, 1)d\epsilon_{31}^0 + E_0(3, 2)d\epsilon_{32}^0 + E_0(3, 3)d\epsilon_{33}^0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]  
\[ \begin{bmatrix} d\sigma_{11} \\ d\sigma_{22} \\ d\sigma_{33} \\ d\sigma_{12} \\ d\sigma_{13} \\ d\sigma_{23} \end{bmatrix} = \begin{bmatrix} E_0(1, 1)d\epsilon_{11}^0 + E_0(1, 2)d\epsilon_{12}^0 + E_0(1, 3)d\epsilon_{13}^0 \\ E_0(2, 1)d\epsilon_{21}^0 + E_0(2, 2)d\epsilon_{22}^0 + E_0(2, 3)d\epsilon_{23}^0 \\ E_0(3, 1)d\epsilon_{31}^0 + E_0(3, 2)d\epsilon_{32}^0 + E_0(3, 3)d\epsilon_{33}^0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]  

The stress update under the unloading state of recycled aggregate concrete also adopts the expression form of increment, as shown in Equation (26).

\[ 3.2.3.1 \text{Yield surface stress return algorithm} \]

The yield surface stress return algorithm is divided into two steps: elastic stress prediction and plastic correction. If the equivalent elastic predicted stress is less than or equal to the yield stress, then the elastic predicted stress is the real stress state of the material. On the contrary, the material is considered to have entered a plastic state, and the predicted elastic stress needs to be pulled back to the plastic yield.
surface until the error of the predicted equivalent elastic stress and the yield stress is within an acceptable
range, as shown in FIG. 1. Meanwhile, the elastic strain state variables and plastic strain state variables
need to be updated.

\[ \text{RHS} = \sigma_{\text{mises}} - 3Gd\bar{\varepsilon}_p - \sigma_s \]  

(27)

If \( \text{RHS} \leq \text{TOL} \), It is considered that recycled aggregate concrete enters into plastic yield state. Where,
\( \text{TOL} \) stands for allowable error.

Since this study focuses on the uniaxial compression performance of recycled aggregate concrete, in
order to simplify the program, it is only necessary to update the elastic strain state variables and plastic
strain state variables in the uniaxial compression direction of recycled aggregate concrete. The updating
principle of strain state variables and plastic strain state variables follows Equations (17) and (18)
respectively.

\[
\sigma_{n+1}^{\text{trial}} = \sigma_n + \frac{p_n \, n^2 \, x^{n-1}}{(n-1 + x^n) \, \varepsilon_{c,e}} \quad (x \leq 1) \\
\sigma_{n+1}^{\text{trial}} = \sigma_n + \frac{p_n \, [2a_c \, (x-1) + 1]}{\varepsilon_{c,r} \, \alpha_c \, (x-1)^2 + x} \quad (x > 1)
\]

(28)

\[
\frac{\partial D}{\partial \varepsilon_e} = \frac{\partial d}{\partial \varepsilon_e} + \frac{\partial d}{\partial \varepsilon_p} \left( \varepsilon_p - \varepsilon \right) + \frac{(d_e - 1) \, \varepsilon_p}{\varepsilon_e^2}
\]

(29)

\[
\frac{\partial \sigma}{\partial \varepsilon_e} = \frac{\partial D}{\partial \varepsilon_e} \left( \varepsilon_p - \varepsilon \right) + (1 - D) \, E_0
\]

(30)

**3.2.3.2 Uniform tangent modulus**

The premise of stress updating accurately is to select the appropriate consistent tangent modulus. The
uniform tangent modulus, also known as the Jacobian matrix, is defined as \( \partial \sigma / \partial \varepsilon \). The accurate
definition of uniform tangent modulus can not only avoid pseudo loading and pseudo unloading
problems caused by continuous elastic-plastic tangent modulus, but also ensure the convergence rate of
Abaqus main program. Based on the damage constitutive equation of recycled aggregate concrete under
uniaxial compression proposed in formula (7) above, the uniform tangent modulus is deduced. Since
this study only focuses on the uniaxial compression state of recycled aggregate concrete, and the
recycled aggregate concrete is regarded as isotropic material for the purpose of simplifying the research
and analysis, the damage variables and consistent tangent modulus of the 1d stress state of recycled
aggregate concrete are only needed to be studied.

\[
\frac{\partial d}{\partial \varepsilon_e} = \frac{p_n \, n^2 \, x^{n-1}}{(n-1 + x^n) \, \varepsilon_{c,e}} \quad (x \leq 1) \\
\frac{\partial d}{\partial \varepsilon_p} = \frac{p_n \, [2a_c \, (x-1) + 1]}{\varepsilon_{c,r} \, \alpha_c \, (x-1)^2 + x} \quad (x > 1)
\]

Figure.1  Yield surface stress return algorithm

4. Validation of Umat subroutines
4.1. The numerical simulation results were compared with the experimental results

This subroutine was verified and analyzed by quoting Xiaobin Hu et al. [6] experimental data of uniaxial compressive mechanical properties of recycled coarse aggregate concrete with different substitution rates. The main material parameters of the relevant recycled aggregate concrete tests used for finite element simulation are shown in Table 1.

| TYPE       | \( f_{uc} \)/MPa | \( f_{cr} \)/MPa | \( \varepsilon_{cr} \)  | \( E_0 \)/GPa | \( E_s \)/GPa |
|------------|------------------|------------------|--------------------------|--------------|--------------|
| NAC        | 31.14            | 24.59            | 0.00196                  | 28.85        | 12.55        |
| RAC-25     | 32.48            | 26.81            | 0.00211                  | 28.76        | 12.71        |
| RAC-50     | 29.22            | 24.19            | 0.00209                  | 27.12        | 11.57        |
| RAC-70     | 37.28            | 27.53            | 0.00223                  | 28.00        | 12.35        |
| RAC-100    | 38.05            | 27.84            | 0.00241                  | 26.12        | 11.55        |

Where, \( f_{uc} \) represents the compressive strength of the concrete cube; \( f_{cr} \), represents the compressive strength of concrete under monotone loading; \( \varepsilon_{cr} \), represents the peak strain of concrete under monotone loading; \( E_0 \), represents the elastic modulus of concrete under monotone loading; \( E_s \), represents the secant modulus of concrete under monotone loading.

4.1.1. Uniaxial loading stress-strain curve

FIG. 2 shows the variation trend of stress and strain of recycled aggregate concrete under uniaxial loading under different substitution rates. The stress-strain curve of recycled aggregate concrete is similar to that of ordinary concrete under uniaxial loading, showing a development trend of "the stress first increases with the increase of strain, and then decreases with the increase of strain" [11]. According to the simulation curve and test results of the finite element software, there is a deviation between the finite element simulation and the test data before the peak stress, which is mainly due to the simplified approximation treatment of the constitutive model. The simulation results of the stress-strain curve before the bending point in the falling section of recycled aggregate concrete are better than the experimental data. The decline after the inflection point located above test curve model curve, this is mainly because of recycled aggregate concrete in the process of test more than a certain critical value, the rapid growth of the damage cause of recycled aggregate concrete mechanics performance is falling fast, selection of model to describe mechanical properties of recycled aggregate concrete also has certain deviation [12]. In general, the peak stress of recycled aggregate concrete decreases with the increase of the replacement rate of recycled aggregate, but the test results are contrary to this, which can be interpreted as the fact that there are many micro-cracks in recycled aggregate, thus increasing the friction between the aggregate and enhancing the compressive strength of recycled aggregate concrete.
5. Conclusion
In this paper, based on the uniaxial compression elastic damage constitutive of Concrete Structure Design Code and considering the replacement rate of recycled aggregate, a uniaxial compression damage constitutive model of recycled aggregate concrete was proposed. Umat subroutine embedded finite element analysis software Abaqus was used to simulate and analyze the uniaxial compression mechanical properties of recycled aggregate concrete. The main conclusions are as follows:

- The replacement rate of recycled aggregate concrete has a significant impact on the mechanical properties of recycled aggregate concrete. According to the test data, the model parameters of recycled aggregate concrete with different replacement rates are determined, and a relatively accurate uniaxial compression constitutive equation of recycled aggregate concrete is established.
• Stress update of Umat subroutine programming adopts completely implicit backward Euler mapping algorithm, which has high calculation accuracy and unconditional stability. The stress-strain curve of recycled aggregate concrete simulated by embedding Umat subroutine into Abaqus software has a high degree of fitting with the test results, which proves the accuracy of the proposed constitutive equation for the damage of recycled aggregate concrete under uniaxial compression.

• The Umat subroutine can better reflect the mechanical properties of recycled aggregate concrete under different substitution rates, so as to provide accurate material subroutine for the application of recycled aggregate concrete in practical engineering and reduce the deviation between simulation and practice. At the same time, it lays a good theoretical foundation for the extensive application of recycled aggregate concrete in engineering structures.

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