Domain regime in two-dimensional disordered vortex matter

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A detailed numerical study of the real space configuration of vortices in disordered superconductors using 2D London-Langevin model is presented. The magnetic field $B$ is varied between 0 and $B_{c2}$ for various pinning strengths $\Delta$. For weak pinning, an inhomogeneous disordered vortex matter is observed, in which the topologically ordered vortex lattice survives in large domains. The majority of the dislocations in this state are confined to the grain boundaries/domain walls. Such quasi-ordered configurations are observed in the intermediate fields, and we refer it as the domain regime (DR). The DR is distinct from the low-field and the high-fields amorphous regimes which are characterized by a homogeneous distribution of defects over the entire system. Analysis of the real space configuration suggests domain wall roughening as a possible mechanism for the crossover from the DR to the high-field amorphous regime. The DR also shows a sharp crossover to the high temperature vortex liquid phase. The domain size distribution and the roughness exponent of the lattice in the DR are also calculated. The results are compared with some of the recent Bitter decoration experiments.

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INTRODUCTION

The vortex state in type-II superconductors is a paradigm for studying the effect of quenched disorder in condensated matter. Over the last decade, much of the effort has been spent on characterizing the various phases of the vortex state as a function of the magnetic field $B$ and the temperature $T$. For 3D vortex system, three phases have been identified unambiguously: the Bragg glass (BG) with quasi-long range order, the amorphous vortex glass (VG), and the vortex liquid (VL). The VG and the VL are distinguished by their superconducting and ohmic responses, respectively. Experiments in high-$T_c$ superconductors suggest that the BG phase appears in the low-$B$ and low-$T$ region whereas the VG phase occupies the high-$B$ and low-$T$ region of the $B$-$T$ phase diagram. The VL phase appears close to the upper critical field $B_{c2}(T)$.

The first detailed calculation of the real space structure of the vortex lattice in the presence of quenched impurities was carried out by Larkin and Ovchinnikov (LO). The LO theory assumes that for weak pinning, the vortex lattice is coherently pinned using 2D London-Langevin model. The vortex lattice is thus spatially ordered. The dislocations in this state are confined to the grain boundaries/domain walls. Such quasi-ordered configurations are observed in the intermediate fields, and we refer it as the domain regime (DR). The DR is distinct from the low-field and the high-fields amorphous regimes which are characterized by a homogeneous distribution of defects over the entire system. Analysis of the real space configuration suggests domain wall roughening as a possible mechanism for the crossover from the DR to the high-field amorphous regime. The DR also shows a sharp crossover to the high temperature vortex liquid phase. The domain size distribution and the roughness exponent of the lattice in the DR are also calculated. The results are compared with some of the recent Bitter decoration experiments.

In $D = 2$, the BG phase is unstable to the formation of dislocations and the positional quasi-long range order is destroyed. However, for weak pinning and at low temperatures, the unbound dislocations appear only at large length scale $\xi_D \gg R_a$, where $R_a$ is the “Random Manifold” length scale and is the distance at which positional correlation begins to decay. On length scales shorter than $\xi_D$, the topologically ordered lattice forms a quasi-Bragg glass (qBG). Such a qBG shows an exponentially sharp crossover to the high temperature VL phase, reminiscent of the “melting” transition of the pure system. A similar exponential crossover was proposed between the qBG and the VG phase as a function of $B$, or pinning strength.

The real space structure of the vortex system has been studied used neutron diffraction and Bitter decoration of vortices and Bitter decoration of vortices. The latter technique allows direct visualization of the large scale structure of the configuration and hence enables one to analyze the role of topological defects on the decay of translational order. Recent decoration experiments of NbSe$_2$ have raised some important issues concerning the nature of the disordered phase. Previous transport measurements on the same samples of NbSe$_2$ suggested an order-disorder transition on increasing $B$ (or $T$). Fasano et al. showed that the spatial configuration of vortices does not show any significant difference between the ordered and the disordered vortex phases identified in Ref. More importantly, both phases were found to be polycrystalline with dislocations form-
ing grain boundaries. Within each grain, the lattice
shows significant bond orientational order. This is in
contrast to the naive theoretical picture of the ordered
phase which expects a dislocation-free configuration, and
the disordered phase in which the distribution of the dis-
locations is expected to be homogeneous.

In this paper, we analyze in detail the real space con-
figuration of the disordered phase using numerical sim-
ulation of a 2D vortex system at \( T = 0 \). The magnetic
field \( B \) is varied over a wide range for various values of
the pinning strength \( \Delta \). The real space configuration
shows that for the intermediate field range, the system
shows inhomogeneous disordering. The majority of the
dislocations are confined to the grain boundaries which
forms the domain wall between regions of ordered lat-
tice. The domain size and its distribution is dependent
on \( B \) and \( \Delta \). We refer the intermediate fields in which
the vortex state is quasi-ordered as the domain regime
(DR). The DR is distinct from the amorphous regime at
low-fields and high-fields, where the defects appear at a
length scale \( \sim a_0 \) (lattice constant) and its distribution
is homogeneous over the entire system. Analysis of the
real space images suggests domain wall roughening as a
possible mechanism for the crossover between the DR
and the high-fields amorphous regime. We also obtained
the roughening exponent \( \zeta \) of the vortex lattice in the do-
main regime. Finite temperature simulation shows that
the domain regime undergoes a sharp crossover to the
high-\( T \) liquid phase, which is reminiscent of the thermal
melting in the pure vortex system.

The paper is organized as follows: in section II, we
discuss the simulation approach in detail. The results
and the analysis of the real space configuration are pre-
seated in section III, followed by conclusions in section IV.

**SIMULATION METHOD**

We consider a 2D cross-section perpendicular to the mag-
netic field \( B = Bz \) of a bulk type-II superconductor
in the mixed state. Within London's approximation, the vortex
can be considered as a point particle with the dy-
namics governed by an overdamped equation of motion

\[
\frac{dr_i}{dt} = -\sum_{j \neq i} \nabla U^v(r_i - r_j) - \sum_k \nabla U^p(r_i - R_k) + F_{ext} + \zeta(t).
\]

Here, \( \eta \) is the flux-flow viscosity. On the left hand
side, the first term represents the inter-vortex interac-
tion \( U^v(r) = \frac{\phi_0^2}{8\pi\lambda x^2}K_0(\tilde{r}/\lambda) \), where \( K_0 \) is the zeroth-
order Bessel function, and \( \tilde{r} = (r^2 + 2\xi^2)^{1/2} \). \( \phi_0 \) is the
flux quantum, and the \( \lambda \) and \( \xi \) are the penetra-
tion depth and the coherence length of the supercon-
ductor, respectively. This form of the inter-vortex in-
teraction includes the finite core size of the vortex.

The second term represents vortex pinning by parabolic
potential wells, where \( U^p(r) = U_0(\frac{r^2}{\lambda^2} - 1) \) for \( r < r_p \),
and 0 otherwise. The pinning centers are randomly lo-
cated at positions \( R_k \) in the simulation box. The third
term \( F_{ext} = -\frac{1}{\eta} \int \sigma \dot{z} \) is the Lorentz force experi-
exed by the vortex due to the transport current density \( J \).
The thermal noise is represented by \( \zeta \) with \( \langle \zeta_i(t) \rangle = 0 \),
and \( \langle \zeta_i(t)\zeta_i(t') \rangle = 2k_BT\eta\delta_{ij}\delta_{pp'}\delta(t-t') \), where \( T \)
is the temperature, \( k_B \) is the Boltzmann constant, and
\( p, p' = x, y \). The length is in units of \( \lambda(B = 0, T = 0) = \lambda_0 \),
and the temperature \( T \) is in units of \( \lambda_0 f_0/k_B \), where
\( f_0 = \frac{\phi_0^2}{8\pi\lambda_0^2} \). The current density \( J \) and the velocity \( v \)
of the vortices are in units of \( cf_0/\phi_0 \) and \( f_0/\eta \), respectively.
Also, the prefactor for the pinning potential \( U_0 \) is scaled
by \( f_0/\lambda_0 \).

We use the reduced magnetic field \( B = B/\lambda_2 \), where
the upper critical field \( \lambda_2 = \frac{\phi_0}{\eta \delta_{ct}} \) and \( \xi_0 = \xi(B = 0, T = 0) \). The \( B \) is calculated from
the lattice constant \( a_0 = (\frac{\phi_0^2}{\eta \delta_{ct}})^{1/2}(\frac{1}{\xi_0}) \). The Ginzburg-Landau parameter
\( \kappa = \frac{1}{\lambda} \) is an input to the simulation. The magnetic field
dependence of the length scales \( \lambda \) and \( \xi \) follows the rela-
tion \( \lambda(b) = f(b)\lambda_0 \) and \( \xi(b) = f(b)\xi_0 \), respectively. The
renormalization factor \( f(b) = (1-b^2)^{-\frac{1}{4}} \). This form
of the renormalization factor is similar to the tempera-
ture dependence of \( \xi \) and \( \lambda \) in the Ginzburg-Landau
theory with \( T/T_c \) replaced by \( (B/\lambda_2) \). Similar
form of the renormalization factor for \( \lambda \) have been used in
Ref. The parameters used in the simulation are
\( \kappa = 10 \) and \( \lambda_0 = 1000 \) A, which are close to the values
for the low-\( T_c \) superconductors, particularly NbSe\(_2\). Peri-
odic boundary conditions are imposed in both directions,
and the minimum image convention is followed. The
magnetic field \( b \) is varied by changing the size of the sim-
ulation box, keeping the number of vortices \( N_v = 4096 \)
fixed. Simulations were also performed using \( N_v \) be-
tween 800-1200, and for some parameters \( N_v = 6400 \)
was used to check for the finite size effects. The \( U_0 \) is
distributed randomly between \( \Delta \pm 0.01 \), where \( \Delta = \langle U_0 \rangle \).

The range of the pinning potential \( r_p = \xi_0 \). In this pa-
per, we present results for pin density \( n_p = 2.315/\lambda_0^2 \).
For \( T = 0 \), Eq.(1) is time integrated by the predictor-
corrector scheme, and the finite temperature simulation
is performed using Heun’s method. The simulation
at high vortex densities requires long computational time
and parallel algorithms were employed to reduce the run
time. Details of the implementation of the parallel algo-
rints can be found in Ref.

The real space configuration is characterized by the
(topological defect density \( n_d/\lambda_0^2 \) (number of defects per
unit area of the simulation box). Below, we also use the
defect fraction \( f_d \), which is defined as the number of
defects per vortex. The defects are defined as vortices
with coordination number other than 6 and are identified.
by Delaunay triangulation of the real space position of the vortices. In 2D systems, the vortices with coordination number 5 and 7 are disclinations. A 5-disclination and a 7-disclination separated by a distance $a_0$ forms a bound pair which is an edge dislocation. Over most of the field range, the fraction of free disclinations is negligibly small and the majority of the defects are edge dislocations. Hence, the defect density $n_d$ is approximately twice the dislocation density in the system. The hexatic order in the system is quantified by the six-fold orientational order parameter $\Psi_6 = |\sum_{ij} e^{i\theta_{ij}}|$, where $\theta_{ij}$ is the angle between the nearest neighbor vortex relative to a reference axis.

The simulation is performed by two different methods. In the first method, we start with a perfect vortex lattice and the driving current $I(\propto J)$ is reduced to 0 from a value much greater than the depinning current $I_c$. This is referred as the current annealing (CA) method. In the second method, the conventional thermal annealing (TA) is applied wherein the temperature $T$ is reduced to 0 in small steps from the high temperature liquid phase (also known as simulated annealing). Experimentally, the TA is equivalent to the field cooling procedure. We have shown previously that the configuration obtained by CA is stable to small perturbations compared to the configuration obtained by TA. This is also supported by experiments, which show that the field cooled state is unstable to small driving force $I \ll I_c$ and a stable configuration is obtained when the system is brought to rest after driven with $I \gg I_c$. The two methods, CA and TA, are compared in section III. B.

RESULTS AND DISCUSSIONS

Zero temperature simulation

In this section, we analyze the zero-temperature configurations obtained by the current annealing method. The system is slowly brought to rest across the depinning current for each value of magnetic field $b$. In the absence of thermal fluctuations, the vortex configuration is determined by the balance between the long range elastic force and the pinning force. We first show the real space images of the configuration as the magnetic field $b$ is increased.

Real space configuration

Fig.1 shows the Delaunay triangulation in a region of the simulation box for various values of the magnetic field. The pinning strength $\Delta = 0.02$, and $N_v = 4096$, except for $b = 0.1$ for which $N_v = 900$. At small fields $b \lesssim 0.1$, the defect distribution is homogeneous over the entire system and the configuration is amorphous. The defect fraction (number of defects per vortex) $f_d > 0.35$ at these low fields.

With increasing $b$, small regions of ordered lattice start appearing. This can be seen for $b = 0.1$ in which ordered lattice is formed in regions less than $3-4a_0$ wide. For $b \gtrsim 0.2$, the defect distribution becomes inhomogeneous. The dislocations come closer to form a network of grain boundaries across the system. For $b = 0.4$ and $b = 0.5$, we find that $\approx 90\%$ of the dislocations in the system are confined to the grain boundaries whereas $\approx 10\%$ of the dislocations are free within the domains. We also find that $\approx 5\%$ of the dislocations within the grain boundaries unbind into disclinations, which occurs generally at the intersection of the grain boundaries. Though the free disclinations are absent in the system, it does not lead to a long range hexatic order in the system. For $b = 0.6$, we find $\Psi_6 \approx 0.14$, and for other values of the field $\Psi_6 < 0.05$ (for a perfect vortex lattice, $\Psi_6 = 1$). The small value of $\Psi_6$ is caused by the random orientation of the domains which destroys the long range orientational order.

We call the intermediate field range in which the system breaks into regions of ordered lattice the domain regime (DR). The domain regime is configurationally distinct from the conventional picture of a disordered state.
In recent experiments, the changes in the real space configuration of vortices were studied in weak pinning NbSe$_2$ samples across the order-disorder transition by the Bitter decoration technique [25, 26]. The order-disorder transition was previously identified in transport measurements and have been speculated to underlie the peak effect in the critical current density [27]. The decoration images show that the vortices form large ordered domains. The domains are separated by domain walls, which are defined by chains of dislocations. This domain formation is present throughout the B-T plane (below the melting line), hence the authors summarized their findings as the “absence of amorphous vortex matter”. Fasano et al. found that $\approx$85-90% of the defects are in the grain boundaries, whereas the remaining defects are isolated dislocations. All of these findings are consistent with our numerical findings and estimates in the intermediate field range for $\Delta = 0.02$.

**Domain size distribution**

A useful quantity to characterize the DR is the distribution of the domain size $N(s_d)$, where the area of the domain $s_d$ is in units of $a_0^2$. Unlike in lattice models, extracting $N(s_d)$ in models with continuous symmetry is not straightforward. The lattice vectors can change continuously from domain to neighboring domain without nucleating defects, which makes it difficult to define the domain wall. In many cases, the domain walls, which are formed by the grain boundaries are not closed. Analy-
The distribution is relatively narrow with few large characterizing a single parameter, e.g., its half-width. small and large fields, the histogram can be adequately parametrized by the exponent \( \zeta > 0 \) (the roughness exponent)

\[
W(r) = \frac{1}{2} \left[ u(r) - u(0) \right]^2,
\]

where the overbar represents the average over quenched impurities. The \( u(r) \) is the displacement of the vortex relative to its position in the perfect lattice. The positional order parameter correlations \( C_G(r) \) can be expressed in terms of \( W(r) \) as

\[
C_G(r) = e^{-G^2 W(r)/2},
\]

where \( G \) is one of the reciprocal lattice vectors. For the crystalline state, \( W(r) = 0 \) and \( C_G(r) = 1 \). The effect of the quenched impurities is to increase \( W(r) \) and hence reduce the positional order parameter correlations of the lattice. The structure factor at \( G \), measured in the neutron scattering experiments, is related to the Fourier transform of \( C_G(r) \).

The roughness of an elastic medium is parameterized by the exponent \( \zeta \), which is defined as \( W(r) \sim r^{2\zeta} \). In the flat phase of the medium \( \zeta < 0 \), and in the rough phase \( \zeta > 0 \) (the \( \zeta \) gives logarithmic roughening with \( W(r) \sim \ln r \)). For a 2D vortex system, there are three length scales which emerge in various theories depending upon the displacement \( u(r) \):
(1) $r < R_c$: In the collective pinning theory, $R_c$ represents the size of the region in which the vortex lattice is coherently pinned by the impurities. More precisely, $R_c$ is the length scale at which the displacement $u(r = R_c) \sim \xi$. $R_c$ is obtained by minimizing the total energy (elastic energy + pinning energy) and is given by

$$R_c \approx \frac{C_{gb} \xi}{f n_p^{1/2}}. \quad (3)$$

The $C_{gb}$ is the shear modulus of the vortex lattice and the average pinning force $f \sim \Delta/r_p$, where $r_p$ is the range of the pinning potential as defined in Section II. For the $K_0(r/\lambda)$ potential, the field dependence of the shear modulus have been derived and is given as $C_{gb} \approx \frac{\kappa B}{(4\pi \lambda)^2} (1 - b)^2$. In dimensionless units, the $R_c$ becomes

$$\frac{R_c}{a_0} \approx \frac{1}{(2\pi)^{3/2} f n_p^{1/2}} b^{3/2} (1 - b)^2. \quad (4)$$

The $R_c$ increases with $b$ and attains a maximum before decreasing as $b \to 1$.

The $r < R_c$ regime is often referred as the random force (RF) regime. The roughness exponent in this regime is given by $\zeta = \frac{4 - D}{2}$ for a $D$-dimensional system. Thus, $\zeta_{RF}^D = 1$ and $\zeta_{RF}^D = 0.5$ for 2D and 3D systems, respectively.

(2) $R_c < r < R_a$: Beyond $R_c$, the displacement $u(r)$ continues to grow but with smaller exponent. $R_a$ defines the length scale at which the positional correlation begins to decay, i.e., the displacement $u(r = R_a) \sim a_0$. Between $R_c$ and $R_a$, the system is in the random manifold (RM) regime. In this regime, the roughness exponent have been obtained using a Flory type argument which gives $\zeta_{RM}^D = 1$. A more refined scaling argument gives $\zeta_{RM}^D = 0.4$. For weak pinning, the length scales $R_a$ and $R_c$ are related by $R_a \sim R_c(a_0/\xi)^{1/\zeta_{RM}}$.

(3) $R_a < r < \xi_D$: Beyond $R_a$, $W(r)$ grows as $W(r) \sim \ln^2(r)$ as derived through a variational approach and confirmed by replica symmetric RG, assuming the lack of dislocations at these scales. This growth form holds up to the length scale $\xi_D$, at which unbound dislocations appear. For weak pinning, $\xi_D \gg R_a$.

$$\xi_D \sim R_a \exp \left[ c \left( \frac{1}{8} - \sigma_0 \right) \ln \left( \frac{R_a}{a_0} \right) \right]. \quad (5)$$

where $c$ is a temperature dependent numerical constant and $\sigma_0$ is the impurity strength. For $R_a \gg a_0$ and low temperatures, $\xi_D$ can become exceedingly large and the system appears similar to the BG phase in 3D.

(4) $\xi_D < r$: Beyond $\xi_D$, unbound dislocations lead to exponential decay of the positional correlation and the system is disordered.

We have obtained the length scale and the roughness exponent of the vortex lattice in the DR. The relative displacement correlation $W(r)$ was calculated using the following procedure. First, a crystalline state with the lattice constant corresponding to the value of $b$ is constructed using one of the vortex coordinates $r_0$ in the real lattice as the origin. The mean square displacement between the perfect lattice and the underlying real lattice is then minimized by varying the orientation of the perfect lattice relative to the real lattice. The $u(r)$ is then computed for each of the vortices. This procedure is repeated for different $r_0$'s, and the $W(r)$ is computed by averaging over all $r_0$'s.

Fig. 6 shows the plot of $W(r)/a_0^2$ for $b = 0.6$ and $\Delta = 0.01$ for $N_v = 6400$. The inset shows the same for $N_v = 4096$. The $W(r) \sim r^{\zeta}$ where the exponent $\zeta$ is shown for the random force (RF) regime and the random manifold (RM) regime.
\[ W(r) \approx 0.12a_0^3 \] for \( b = 0 \). From Fig.6, we find that \( W(r) \) flattens at \( \approx 0.1 \) at \( r \approx 13a_0 \) for \( N_v = 6400 \), which suggests that \( R_a \approx 13a_0 \) (for \( N_v = 4096, R_a \approx 18a_0 \)). Beyond \( R_a \), the growth of \( W(r) \) slows down considerably which indicates the appearance of the asymptotic regime. Within the qBG theory, \( W(r) \) is expected to grow as \( \ln^2(r) \) in the asymptotic regime. This behavior unfortunately could not be verified due to insufficient range of data points. In sum, we identify the \( r \approx 10a_0 - 5a_0 \) as the RF regime, the \( r \approx 5a_0 - 15a_0 \) as the RM regime, and \( r > 15a_0 \) as the asymptotic regime.

The value of \( \zeta \approx 0.65 - 0.72 \) obtained from the simulation in the RF regime is smaller than the theoretical prediction for \( \zeta_{RF} = 1 \). We speculate that the interaction \( K_0(r/\lambda) \) between the vortices in 2D increases the stiffening of the vortex lattice at short distances which leads to weaker roughening. In the RM regime, the exponent \( \zeta \approx 0.40 - 0.42 \) is in good agreement with the value of 0.4 expected from the scaling argument. Using \( \zeta_{RM} = 0.4 \), the value of \( R_a \approx 15a_0 \) is much smaller than the value \( R_a \approx R_c(a_0/\xi)^{1/2}\kappa a_0 \). This is possibly related to the large magnetic field \( b = 0.6 \) used in obtaining \( W(r) \). For this field, \( a_0 \) is comparable to \( \xi \), and \( R_c \) is large compared to smaller magnetic fields. The RM regime is expected to disappear for \( a_0 = \xi \), and have been shown in the case of 3D system.

An interesting outcome of the above analysis of \( W(r) \) is that the average domain size \( R_d \gg R_c \), and hence, the collective pinning theory cannot account for the appearance of domains in the intermediate fields. The asymptotic regime in \( W(r) \) suggests that the qBG theory is qualitatively correct. Within the qBG theory, the distribution of dislocations beyond the length scale \( \xi_l \) is expected to be homogeneous, unlike the grain boundary formation observed in our simulation. One possible way to account for the grain boundary formation is to consider the long range interaction between the dislocations. Since the interaction between the dislocations is anisotropic, for some values of dislocation density, the grain boundaries may lead to a lower energy state. This is also supported from a recent work on dipole systems. At low densities, the dipoles exhibit a gaseous phase, and their distribution is roughly homogeneous. At higher densities the phase is characterized by dipoles forming chains or strings. Since the dislocations of the vortex lattice are in fact dipoles of disclinations, these results are quite analogous to our identification of a domain regime in the vortex matter.

**Defect density**

The three field regimes discussed in the context of the real space configuration can also be inferred from the behavior of the defect density \( n_d(b) \). Fig.7(a) shows \( n_d(b) \) for \( \Delta = 0.02 \) and \( N_v = 4096 \). The behavior for smaller system sizes (\( N_v = 800 - 1200 \)) is also shown on the same plot. The \( n_d(b) \) increases linearly in the low field amorphous regime. Above a crossover field \( b_1 \approx 0.1 \), \( n_d(b) \) flattens and becomes weakly field dependent in the DR. For \( b \geq 0.6 \), \( n_d(b) \) increases rapidly, and above \( b_h \approx 0.8 \), the system crosses over to the high field amorphous regime. It is possible to define a length scale \( L_d \approx n_d^{-1/2} \) as the nominal average defect separation. For \( b = 0.6 \) (domain regime) \( L_d \approx 3a_0 \), which is much smaller than even \( R_c \) and does not correspond to any feature in the real space configuration, and reflects the highly inhomogeneous nature of the defect distribution in the domain regime. On the other hand, in the high-field amorphous regime \( L_d \approx a_0 \), which is also the distance between the defects, thus reflecting homogeneity of the distribution of defects.

The \( n_d(b) \) in Fig.7(a) shows strong similarity with the experimental observation in 2D system of magnetic bubbles. In Ref.14, the intermediate regime was interpreted as the hexatic phase and the high field amorphous phase as the isotropic liquid phase. Later simulation also suggested a \( T = 0 \) dislocation unbinding transition driven by disorder. As discussed above, the presence of domain walls (grain boundaries) in our system suppresses the long range orientational order. This rules out the possibility of a transition between the hexatic phase and the isotropic liquid phase as the underlying reason for the rapid increase in \( n_d(b) \). However, a rapid crossover, similar to that predicted between the qBG at low temperatures and vortex liquid at high temperatures, is still possible between the DR and the high field amorphous regime, especially at weaker pinning where the domain size \( R_d \) is large. For smaller system size (\( N_v = 800 - 1200 \), a topologically ordered phase appears in the intermediate field range in which \( n_d = 0 \) (see Fig.7(a)). This is a finite size effect, which reflects the sensitivity of the DR to the system size \( L \). For \( L < \xi_d \), the DR can appear as
a topologically ordered state free of dislocations. This implies that for a given system size, there exists a critical $\Delta_c$ below which dislocations are not favored. This is observed in the simulation, as shown in Fig.7(b) where the defect fraction $f_d(\Delta)$ (number of defects per vortex) is plotted for $b = 0.6$. For the smaller system $N_v = 900$, $f_d$ goes to zero at $\Delta_c = 0.03$, and the system exhibits an ordered phase for $\Delta < \Delta_c$. Increasing $N_v$ to 4096 reduces $\Delta_c$ to 0.01, and in the asymptotic limit $N_v \to \infty$ (hence, $L \to \infty$), we expect $\Delta_c \to 0$. In the DR, $f_d$ does not increase continuously with increasing $L N_{v^{1/2}}$ but shows a sharp jump from the dislocation-free state to the domain state at a characteristic system size as shown in the inset of Fig.7(b).

**Crossover from DR to high-field amorphous regime**

As discussed above, the $n_d(b)$ shows a rapid crossover from the DR to the high field amorphous regime. To understand the mechanism for this sharp crossover, we identified the domains and the domain walls between $b = 0.5$ and $b = 0.8$ for $\Delta = 0.02$. For the intermediate fields $b \approx 0.5-0.6$, the grain boundaries are generally smooth and $n_d(b)$ is weakly field dependent. For $b \gtrsim 0.6$, the rapid increase in $n_d(b)$ occurs within the domain walls. Consequently, the domain wall length increases, which is accommodated through enhanced roughening of the domain walls. This is evident from Fig.8(b). The increase in the roughening also facilitates the unbinding of the dislocations into free disclinations and subsequently drives the crossover into the VG state. In such a scenario, we conjecture that domain walls undergo disorder driven roughening transition at the crossover between the DR and the high field VG. It would be of interest to obtain the domain wall roughening exponent across the crossover regime.

**Finite temperature simulation**

In this section, we compare the current annealing method with the conventional simulated annealing method, as it is well known that different sample preparation techniques can result in the vortex system not reaching its equilibrium configuration. In the latter method, the temperature $T$ is reduced from the high temperature liquid phase slowly so as reach thermal equilibrium at each value of $T$. This method is commonly used to search for the ground state of disordered systems.

For the thermal annealing, the system was equilibrated for $5 \times 10^4 - 1 \times 10^5$ time steps before averaging over a similar time scale to calculate the defect fraction $f_d(T)$. The number of vortices $N_v = 900$ and $\Delta = 0.03$. For this system size, the CA method gives a topologically ordered phase for $b$ between 0.6 and 0.75. Fig.9(a) shows $f_d(T)$ for various values of the magnetic field. As the temperature is lowered, for $b = 0.2$ $f_d(T)$ decreases monotonically to a finite value with $\frac{df_d}{dT}$ slowly varying. There is no evidence of a transition as a function of the temperature. With increasing $b$, the slow freezing is replaced by a sharp decrease in $f_d(T)$ at a particular temperature $T_m$, similar to the equilibrium melting transition. For $b = 0.65$, $f_d(T)$ at $T_m$ decreases by $\approx 76\%$ of the value above $T_m$. For $b > 0.8$, the melting-like transition is again replaced by slow freezing of the high temperature liquid phase.

In Fig.9(b), $f_d(b)$ at $T = 0$ obtained by TA is compared with that obtained by CA. At intermediate fields, the thermally annealed samples exhibit the presence of dislocations already at these smaller systems sizes. As described above, the current annealing method requires larger systems sizes to correctly display this same phenomenon. Otherwise, the two curves track each other very closely over most of the field range, including the low field slow decay of $f_d(b)$ and the rapid rise at high fields. For the intermediate fields, the $T = 0$ configuration obtained from TA also shows grain boundary formation, similar to that observed from the CA method.

**CONCLUSION**

We have presented a detailed numerical analysis of the real space configuration of $2D$ vortex system in the pres-
ence of quenched impurities. For weak pinning, the disordered state in the intermediate field range is inhomogeneous. The majority of the dislocations in this state are confined to grain boundaries, which form domain walls between regions of topologically ordered vortex lattice. There are no free disclinations in the system. This state is referred as the domain state and the intermediate field range as the domain regime.

The domain size distribution \( N(s_d) \) was calculated in the domain regime. \( N(s_d) \) shows a broad distribution with a large weight in the tail region at intermediate fields. Therefore, more than one length scale is required to properly characterize the domain size distribution in the domain regime. With increasing \( b \), the distribution becomes narrow and the peak shifts toward the origin. For weak pinning, the size of the domains can become exceedingly large.

The domain regime is bounded by an amorphous regime at low fields and high fields. The defects in the amorphous regime are separated by the smallest length scale \( \sim a_0 \) and show homogeneous distribution unlike the grain boundary formation in the domain regime. The domain regime shows rapid crossover into the high field amorphous regime. From the changes in the configuration, we identified the roughening of the domain walls as the plausible mechanism driving the rapid crossover.

The relative displacement correlation \( W(r) \) in the domain state was also calculated for weak pinning. Three distinct regimes were observed: a random force regime, a random manifold regime and the asymptotic regime. Crossover from random force regime to the random manifold regime is found to occur at \( R_c \sim 4 - 5 a_0 \). The value of \( R_c \) agrees with that obtained from the collective pinning theory. The roughness exponent \( \zeta \) in the random manifold regime is found to \( \approx 0.40 \), within the range of various theoretical predictions.

The observation of random manifold and asymptotic regimes within the domains for weak pinning suggests that the vortex lattice is correctly described by the qBG idea, though the exact form of the \( W(r) \) could not be ascertained. At length scales greater than the domain size, the appearance of the domain wall formed by dislocations is not captured by the quasi-Bragg glass theory. Therefore, it remains to be seen whether besides the domain regime the 2D vortex matter supports a quasi-Bragg glass where the dislocations are homogeneously distributed.

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