Some remarks about the $\beta$-delayed $\alpha$-decay of $^{16}$N

L. Buchmann,$^{1,\dagger}$ G. Ruprecht,$^1$ and C. Ruiz$^1$

$^1$TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada, V6T 2A3

Abstract

The $\beta$-delayed $\alpha$-decay of $^{16}$N has been used to restrict the E1 fraction of the ground state $\gamma$-transition in the astrophysically important $^{12}$C($\alpha,\gamma)^{16}$O reaction in several experiments including those performed at TRIUMF and several other laboratories. A review of the published measurements is given, and GEANT4 simulations and R-Matrix calculations are presented to further clarify the observed $\alpha$ spectra. A clear response-function, in the form of a low-energy tail from the scattering of $\alpha$-particles in the catcher foil is observed in these simulations for any foil thickness. Contrary to claims in the literature, the simulations show that the TRIUMF measurement and those performed at Yale and Mainz originate from the same underlying spectrum. The simulations suggests that the discrepancies between the Yale and TRIUMF final results can be attributed to incorrect deconvolution methods applied in the former case. The simulations show in general that the form (width) of the spectrum is very sensitive to the catcher foil thickness. It is concluded that the TRIUMF measurement most likely represents the currently closest approximation to the true $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N.

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$^{\dagger}$Electronic address: lothar@triumf.ca
I. INTRODUCTION

The $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N has been used to restrict the $E1$ fraction of the ground state transition in $^{12}$C($\alpha,\gamma$)$^{16}$O. This reaction has often been referred to as the “holy grail” of experimental Nuclear Astrophysics due both to the importance of the $^{12}$C($\alpha,\gamma$)$^{16}$O reaction in stellar evolution, and the experimental difficulty in measuring the cross section at low enough energies to be relevant for quasi static helium burning. Some possible assistance in determining the $E1$ ground state transition rate arises from the fact that, in the $\beta$-delayed $\alpha$-decay of $^{16}$N, the subthreshold $E_x=7.117$, $J^{\pi}=1^-\!$ state of $^{16}$O is populated with a yield of about 0.05 in the $\beta$-decay of $^{16}$N, while the unbound $E_x=9.60$ MeV, $J^{\pi}=1^-\!$ state is populated with a yield of about $10^{-5}$. This leads to a relatively stronger influence of the subthreshold state on the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N, particularly on its low energy part. As the ground state of $^{16}$N has a spin and parity of $J^{\pi}=2^-\!$, only $^{16}$O states with $J^{\pi}=1^-\!$ and $3^-\!$ will contribute to allowed $\beta \alpha$ decay of $^{16}$N, with no $J^{\pi}=3^-\!$ state being nearby the given energy range of the aforementioned $\alpha$ spectrum. Therefore the $J^{\pi}=3^-\!$ contribution to the spectrum is expected to be small, but not necessarily entirely negligible, while the relevant $J^{\pi}=1^-\!$ part is dominant.

While there is, of course, a true, underlying $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N, any measurement of the spectrum will lead to experiment-specific distortions on the spectrum, like those from detector resolution, collector foil effects and possible backgrounds. While the last of these can be removed by a correct subtraction, it is quite difficult to remove the former effects as they convolute the ideal spectrum with a usually energy-dependent response function. In this case each bin point of the measured spectrum will be mixed more or less with contributions from elsewhere in the spectrum. In general, deconvolutions are considered a difficult mathematical problem with uncertainties growing rapidly the more iteration steps are applied and typically they involve some judgment calls. We will show that minimizing the system response is the only practical solution and that all attempts of a deconvolution lead to failure.

As response functions (likely) differ in technically different experiments, experimental spectra of the $\beta$-delayed $\alpha$-decay of $^{16}$N may in principle look rather different in comparison without implying a major difference in the underlying spectrum. In a correct fitting procedure, a theoretical curve should be convoluted with an experimentally well known re-
response function, and then be compared with the data; the theoretical curve can then be varied to fit the data after convolution. The theoretical fits may then be compared among different experiments. Of course, the sensitivity and the response of an experiment should be estimated beforehand independent of the actual $^{16}$N spectrum.

There have been several measurements of the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N [1, 2, 3, 4, 5, 6, 7, 8]. These measurements will be discussed below and compared, with the measurement described in Ref. [2] serving as a benchmark. In Sec. II we will describe individual measurements and their respective methods to obtain their final results, in Sec. III we will present GEANT IV simulations of the experiments, in Sec. IV different measurements are compared to each other, in Sec. V the final conclusions will be drawn and in Sec. VI some future possibilities regarding the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N will be discussed. In particular, besides many other issues of more minor concern, it will be shown that

- the spectrum obtained at Mainz University suffers from considerable distortion effects;
- the method applied to deconvolute the spectra obtained at Yale University appears to be mathematically erroneous;
- the measurement done in Argonne National Laboratory likely suffers from an unreported background.

II. DISCUSSION OF MEASUREMENTS

A. The Mainz spectrum

In the late 1960’s to early 1970’s a group at Mainz University [36] led by H. Waffler made a series of very high statistics measurements of the $\beta$-delayed $\alpha$ spectrum of $^{16}$N [9, 10, 11] with the ultimate goal of detecting the parity-forbidden $\alpha$ decay of the $E_x=8.872$ MeV $J^\pi=2^-$ state in $^{16}$O. A gas cell containing $^{15}$N gas was bombarded with a deuteron beam, producing $^{16}$N via the (d,p) reaction. The activated gas was pumped into (one or) two detection cells, where the $\alpha$ decay was registered by (four or) eight silicon detectors mounted on the walls of the (one or) two counting cells. The silicon detectors were separated from the 6-8 Torr gas volume by 30 $\mu g/cm^2$ collodion ($C_6H_7N_{2.5}O_{10}$) foils. No coincidence requirements
FIG. 1: (color online) Comparison of the Mainz spectrum as calibrated in Ref. [6] (cross) and Ref. [2] (box); the energy is displayed in the CM system.

were placed on the events to prevent detection of α-particles possibly degraded by system response.

In 1971 H. Waffler communicated about a quarter of the events from the then available spectrum [10] to both C.A Barnes (Caltech) and F. Barker (Australian National University) (about $3.2 \times 10^7$ events). These letters contained a spectrum as counts/bin sorted by channel, and a very precise energy calibration of incompletely explained origin. The data sent were selected for the smallest $\beta$-induced tail of low energy pulses, for which the silicon detector biases were lowered to 8 V to reduce the depletion depth in the silicon. It was confirmed, however, by H. Waffler and by the position of the α peak from the singly forbidden $\beta$-decay to the $E_x=9.85$ $J^π=2^+$ $^{16}$O state in this spectrum that no correction for the $30 \mu g/cm^2$ collodion foils has been included in the calibration [12, 13]. The matter was also discussed in Ref. [14]. This spectrum, though unpublished, has been widely used in comparison to later measurements of the $\beta$-delayed α spectrum of $^{16}$N (see, e.g. Ref. [8]). In particular, in Ref. [6] a table of an energy calibrated spectrum of the Mainz data is published (Tab. A.1) without an adjustment for the foil thickness [37]. Fig. 1 shows a comparison between the two calibrations.
B. Spectra from the Yale group

1. France III et al. publications

As there is a recent publication by the Yale/U. of Conn. group about their second measurement of the $\beta$-delayed $\alpha$ spectrum of $^{16}\text{N}$, this work will be discussed here first among the two Yale measurements. However, the first measurement of this group, published in 1993, had basically the same features as the second experiment with the exception of a different data reduction from the raw ($\beta$-efficiency corrected) spectrum to the final spectrum, and, as a result, a vastly different final spectrum (Sec. IV.A.1).

In both measurements the $d^{(15}\text{N},p)^{16}\text{N}$ reaction was used for production with the $^{16}\text{N}$ escaping the deuterium target at high velocity. To catch the $^{16}\text{N}$ nuclei from the low energy part of the recoiling $^{16}\text{N}$ velocity distribution a tilted aluminum foil of 180 $\mu g/cm^2$ thickness was used. After the collection period, the catcher foil was moved in front of a silicon counter array with $\beta$-counters positioned behind the foil. A $\beta$-$\alpha$ coincidence was used to discriminate against some kinds of background events. To obtain the spectrum that was deconvoluted later, a $\beta$-efficiency correction was necessary, particularly for the higher energy $\alpha$ events where the coincident $\beta$ energies are very low. Fig. 2 shows a comparison between the measured and the $\beta$-efficiency-corrected $\alpha$ spectrum and the ratio of the two spectra. The correction rises with increasing energy to a factor of 4.5 and then falls for the highest energy point to about 2.5. Little is said in Refs. [5, 6] how this correction for the extremely low energy $\beta$ particles at the high $\alpha$-energy side was actually derived. With decreasing $\beta$ energy one would expect, in general, a smoothly rising ratio, i.e. the inverted efficiency curve, and a possible detection threshold for the $\beta$-rays.

About 235,000 events were produced from the analysis of the time versus energy spectrum. The raw $\alpha$ spectrum ($\beta$-efficiency corrected, 285,000 events) from this analysis (displayed in Ref. [5], Fig. 5.5) is shown in Fig. 3 in comparison with the published TRIUMF spectrum. It is quite obvious from Fig. 3 that the spectrum of Ref. [5, 6] is both shifted and broadened compared to the TRIUMF spectrum. Also the low energy plateau is about an order of magnitude higher than in the TRIUMF measurement. This is clearly an effect of the very thick collector foil compared with the one used at TRIUMF (10 $\mu g/cm^2$ of carbon). In principle, a careful simulation of the spectrum in Ref. [5, 6] might have produced information
about the underlying $\beta$-delayed $\alpha$ spectrum, though likely with a large uncertainty and considerable dependence on the model used, see Sec. III C 2. However, in Refs. 5, 6 a different approach to the data reduction was chosen, described as follows:

"The measured spectral line shape was corrected for distortions caused by the variability of our time and energy resolution, see Fig. 5. For a spectrum constant in energy the yield measured at each point in that spectrum is directly
FIG. 3: (color online) Comparison of the final TRIUMF [2] (bars) and the $\beta$-efficiency corrected $\alpha$-spectrum of Ref. [5] (crosses). The spectrum of Ref. [5] has been normalized to the height of the TRIUMF spectrum. The abscissa represents the $\alpha$-energy in the laboratory.

Note that no reference to an experiment is given, in which a similar deconvolution method has been used.

The word ‘resolution’ in regard to the energy is used here in a somewhat unusual way, as it clearly labels the energy-dependent energy loss through the aluminum foil. The same is true for the ‘time resolution’ which is just the time of flight between the collector foil and the $\alpha$-particle detector, and not the experimental spread of this time-of-flight signal. The final spectrum shown in Refs. [4, 5, 6] is indeed derived by division by this ‘energy resolution’ and this ‘time resolution’. Questions of dimensionality, i.e. divisions by energy and time, and the obvious renormalization that must have been performed to conserve the total number of counts are not discussed in these references.

The claim that an experimental convolution, i.e. the result of a bad resolution, can be undone by dividing by an arithmetic function[39] that is independent of the final spectrum is not correct. Such a general claim can be easily refuted by a counter-example. In many cases in experimental physics a $\delta$-like function (e.g. a narrow line) is folded in first order due to
FIG. 4: (color online) A Monte Carlo simulation (1000000 events) of the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N as found in Ref. [2] (bar points) exposed to the energy loss as described in Refs. [5, 6] (cross points), and a comparison to the France III raw spectrum (asterisk points), appropriately matched in height.

Statistical processes in the event collection by a Gaussian convolution. No division by a finite number will recover the initial $\delta$-like function from the Gaussian measured. The number of events will stay conserved by the convolution. If only a fraction of the Gaussian function is integrated, the number of counts will, of course, change depending on the integration interval. Such a change is neither linear, nor will the total number of counts ever be exceeded, however far one may choose the integration interval. A reasonable integration interval for such a peak will, of course, depend on the resolution, but is not identical to it. In fact, by this method of division, for a $\delta$-shaped input function, a $\delta$-shaped output function results as the input only needs to be multiplied by the constant resolution. I.e. no degradation or convolution of such a line will result. It may be argued that the situation is different for a continuous spectrum, where, indeed, the resolution may be energy dependent. However, that would make the response function dependent on the spectral form, which is not expected from a reasonable experiment.

It is straightforward to simulate, how the bare energy loss from the Yale catcher foil will influence the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N. The results of the Monte Carlo simulations are shown in Fig. 4. It is obvious that the energy loss effect gives a good description of the high energy side of the France III spectrum but misses further response effects on the low energy side (see Sec. [III]). Therefore there is more to the response function than the simple
stopping power effect (see Sec III C 1).

The main contribution to the deconvolution by division in Ref. [6], already shown to be incorrect, comes from the time of flight resolution as displayed in their Fig. 5. as it shows a steep energy dependence. This use of the time-of-flight spectrum has no physical justification, as (i) the time-of-flight measurement does not influence the pulse height in the energy spectrum; (ii) the time-of-flight through the foil is about 50 fs for a 1 MeV α-particle, therefore the implantation (start) position in the catcher foil has nothing to do with the observed time-of-flight. The increase in the time of flight for lower energy α-particles should follow \( \frac{1}{\sqrt{E}} \) which is clearly exceeded in Fig. 5 of Ref. [6]. However, as the spectrum discussed in Refs. [4, 5, 6] is not derived from the time-of-flight information, but from the pulse height in the silicon detectors, the time-of-flight, not influenced by the catcher foil thickness, has nothing to do with the response to the foil, but only with some kinds of background discrimination applied. For example, events degraded in the silicon detectors will be removed by the time-of-flight information.

Besides a correction by division in the yield, the position of the energy points (initial bins) has also been changed, i.e. shifted to higher energies. Of course, a correct deconvolution does cause a shift and yield a change of channels simultaneously, see Sec. III C 2. This is claimed to be done in the following way [6]:

‘The effective energy of the emerging α-particles for each data point was calculated using the expected variation of the yield over the energy width of the catcher foil for each slice. [...] Note that due to fast variation in the yield, the effective α-particle energy is not the one due to α-particles emitted from the center of the catcher foil.’

This remark appears to suggest that a preconceived knowledge of the final spectrum has been used to derive these corrections. It can now be checked, how far the energy shifts used in Refs. [4, 6] are consistent with the spectra shown in Fig. 4. To calculate the effective energy \( E_{\text{eff}} \) we solve numerically the integral

\[
E_{\text{eff}} = \frac{\int_{E-\Delta(E)}^{E} E' w(E') dE'}{\int_{E-\Delta(E)}^{E} w(E') dE'}
\]

for the effective energy. Here \( E \) is the initial energy, \( \Delta \) the foil thickness, and \( w(E) \) the β-delayed α-distribution of \(^{16}\text{N}\) using \( R \)-matrix theory applied to the TRIUMF spectrum.
FIG. 5: (color online) Expected energy shift for the foil used in Refs. [5, 6] (line) using Eq. 4. The original and the shifted spectrum are shown in Fig. ???. In comparison, the energy shift used in Refs. [5, 6] is shown here (bars). The energy is in the laboratory system.

For simplification, the minor change in stopping power over the integration integral is being ignored. To calculate the energy shift, we subtract the effective energy from the initial energy. Fig. 5 displays the energy shift found here and the one used in Refs. [5, 6]. Compared to the Monte Carlo calculations, taking also the energy dependence of the stopping power into account, the curve shown in Fig. 5 slightly underestimates the energy shift encountered. While the absolute position of these curves on the abscissa may be disputed, it is clear that the energy dependence given by the calculation shown differs greatly from the approximately linear relation used in Refs. [5, 6]. The use of the France III final spectrum as input would change little, if employed in the simulations. In fact, it is the quickly changing yield over the energy loss range in the foil of the $\beta$-delayed $\alpha$ spectrum that causes these quite distinct changes in the energy correction. It is apparent that the energy correction used in Refs. [5, 6] is equivalent to taking the center of the catcher foil employing the nearly linear energy dependence of the stopping power despite the opposite claim in the above quotation. This leads on the low energy side of the spectrum to an underestimate of the necessary energy correction in Refs. [5, 6], while the correction is considerably overdone on the high energy side. Most importantly, calculating ‘the effective $\alpha$-particle energy’ requires a knowledge of
the true distribution beforehand.

2. Discussion of Z. Zhao et al.

As mentioned above, in a measurement previous to Refs. [5, 6] a \( \beta \)-delayed \( \alpha \)-spectrum was obtained at Yale University by largely the same group [3]. The experimental method was approximately identical to the one described for France III, except that the \( \alpha \) collection system was changed from upstream to downstream of the target. Therefore, a spectrum very similar to the one shown for France III was collected (See Fig. 1a of Ref. [3]). However, the deconvolution method was different. As this measurement has not been indicated as incorrect or superseded in the later publications by this group and has been used for comparison in a recent publication [8], it will be discussed in the subsequent section.

In Ref. [3] an \( R \)-matrix description of the spectrum measured is given, following the parametrization of Ref. [16]. Beyond this, the authors present in their Fig. 1b the “\( \ell = 3 \) component of our fit” and conclude that “only with the introduction of the small \( \ell = 3 \) component was it possible to reproduce the line shape of the unfolded spectrum”. It has since been stated [17] that the \( \ell = 3 \) component of Ref. [3] is not the result of a fit, but simply “ad-hoc”, i.e. that it is an invented assumption by the authors without any basis in theory [41].

To see, however, the results of a consistent analysis of the data of Ref. [3] we have analyzed those data using a full \( R \)-matrix fit, following the description of Ref. [2]. For the best fit a value of \( \chi^2/\text{point} = 0.17 \) was found for the data of Ref. [3]. This unexpectedly low value is attributed mainly to the data points of the main peak of Fig. 1(b) of [3], which show smaller fluctuations than those of Fig. 1(a), a result not normally expected from the deconvolution of an experimental spectrum (see Sec. III C 2). To further understand these findings, the deconvolution process based on the thesis of Z. Zhao [15] will be subsequently discussed and the likely reason for the low \( \chi^2_{\beta} \) presented.

The first step of the data reduction, after a \( \beta \)-efficiency correction (Sec. II B 1), was to correct for the energy loss in the target thickness. For this, it appears that the maximum of the experimental spectrum was shifted to match the maximum of the Mainz spectrum without further corrections.

Independent of the energy shift, the number of counts in each bin has also undergone a
data reduction procedure. As stated in Ref. [3], the final result is expected to be similar to that of Ref. [1], which is referred to as the “zero target thickness” spectrum [42]. Since the spectrum measured was far from agreeing with the one of Ref. [1], the authors chose to complete the unfolding of the data by finding a response function which is to be determined “semi-empirically” [3].

A response function, \( G(E, E - E') \times R(E) \), was then constructed by using it to convolute the so-called “zero target thickness” spectrum to yield the experimental Yale spectrum exactly. The first factor, the function \( G(E, E - E') \) corresponds to the simple folding over the catcher thickness, examples of such a folding have been shown above (e.g. Sec. II B 1, Fig. 4). Because this folding is still inadequate, a further correction factor, \( R(E) \), the second factor above, was required. This multiplicative correction factor \( R(E) \) was then determined precisely by taking the ratio of the experimental curve to the curve convoluted with \( G(E, E - E') \). Note that in this procedure, \( G(E, E - E') \) does not have to be realistic, as \( R(E) \) takes into account all shortcomings inherent in the process. In fact, \( G(E, E - E') \) can be set to 1.0 at all energies with the same final result. Again a simple arithmetic function is used to perform a deconvolution and the response function is not independent of the actual \( \alpha \)-spectrum.

The procedure described represents the ‘semi-empirical’ determination of the response function. The circular logic of this data reduction is obvious: if the “folding” of the “zero thickness spectrum”, i.e. the spectrum of Ref. [1], with \( G(E, E - E') \times R(E) \) yields exactly the experimental spectrum of Ref. [3], then the “unfolding” of the spectrum of Ref. [3] with \( G(E, E - E') \times R(E) \) will yield exactly the spectrum of Ref. [1]. Expressed differently: the data of of Ref. [3] are a reproduction of those of Ref. [1] within the energy region given in Ref. [1], albeit with larger error bars [43] and fewer total counts. This explains the major part of the low \( \chi^2/\text{point}=0.17 \) in the total \( R \)-matrix fit to these data.

It may be argued that the identity derived above was intended and that the procedure was designed specifically to provide a means of extrapolating the correction factor \( R(E) \) down to the low energy region, and that data points above \( E_\alpha=1500 \) keV are thus irrelevant to the analysis. While such a method would remain unphysical, there is no evidence anywhere in Refs. [3, 15] to indicate such a possibility. Factually in any serious \( R \)-matrix analysis, these points at higher energy are of utmost importance. As displayed in their Fig. 1b, the \( R \)-matrix analysis in Ref. [3] suggests indeed the use of the entire spectrum.
FIG. 6: The function $R(E)$ as derived in Ref. 3 versus center-of-mass energies for the lower energy part of the spectrum.

The extrapolation of $R(E)$ to lower energies is not described in Ref. 3; in Ref. 15 the following quotation can be found (Sec. 4.1, p.87):

‘Consequently, the unfolding procedure carries small uncertainty down to 1.5 MeV and we further extrapolate it down to 1.1 MeV. Since $R(E)$ reflects a tail from higher energies, the extrapolation is such that $R(E)$ below 1.45 MeV is a constant value of 3.6.’

With this statement the function $R(E)$ for the low energy part of the spectrum is shown in Fig. 6. It is clear that this choice of an arbitrarily constant function $R(E)$ after a steep increase in the “known” region is unjustified, even, if the deconvolution procedure was correct. More reasonably $R(E)$ should follow some functional form which is essentially unknown below $E=1.45$ MeV; and could, in principle, even increase or decrease rapidly with increasing energy. However, $R(E)$ is not a response function of the system by any conventional use of the word [44], but an arithmetic divider relying on preconceived knowledge. Thus $R(E)$ has no experimental basis.

Applying the constant $R(E)$ to the initial spectrum shows immediately that the low energy points are still above the final points, so that some different kind of ‘deconvolution’ step must have been involved. A suggestion of what was really done is given in the following quotation from Ref. 15 (p.87):
'In fig. 43 we show the convolution of the theoretical curve corresponding to $S_{E1}=95$ keV-barn with the response function, together with the experimental data.'

Thus, the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N as presented in Ref. [3] is, in the high energy region, a close reproduction of the data of Ref. [1], while for the low energy points it closely follows a purely theoretical curve. It does not come as any surprise that the final result of Ref. [3] is indeed $S_{E1}(300)=95$ keV b.

It is also noteworthy that the fractional errors of the low energy data points of Fig. 1(b) [3] are no larger than those of Fig. 1(a) in spite of the major unfolding procedure required. If Fig. 1(b) is converted to counts, some of the lowest energy points have about 20 counts/channel for which statistical errors of at least $\pm25$-30\% are expected, even excluding errors from the unfolding procedure. However, e.g., for the energy point at 1.448 MeV, the yield is 20.2±2.5, i.e. with an error 0.56 of the expected square root of the number of counts. If realistic errors were taken into account, the errors on $R(E)$ should be added in quadrature with those resulting from statistics, when the data reduction is done.

There has been a suggestion that the errors in the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N of Ref. [3] are correlated. It is, however, abundantly clear from the above procedure that the final errors should be uncorrelated, if the initial experimental errors are uncorrelated. The unfolding procedure is achieved simply by division of correction factors, a procedure that does not induce correlations between data points. There has, however, been a claim in Ref. [15] that the errors of the data points are correlated as the detector resolution is about 50 keV, while the catcher thickness is up to 250 keV. Such a claim would be incorrect, unless some procedure had been employed which moves events among the channels after the data acquisition. There is no evidence of such data shuffling anywhere in Refs. [3, 15] as each point in the spectrum is divided by an individual number labeled as the response correction. The low $\chi^2$/point found in the $R$-matrix fits is completely explained by the fact that the $\beta$-delayed $\alpha$-decay spectrum of $^{16}$N of Ref. [3] is basically a reproduction of the high-statistics spectrum of Ref. [1] over most of the energy range, combined with a normalization to a theoretical curve for the lower energy data.
C. The Seattle spectrum

In 1993/94 a measurement of the $\beta$-delayed $\alpha$-spectrum of $^{16}\text{N}$ was started at the University of Washington in Seattle. While this measurement was never published by those who carried out the measurement, the data were, however, distributed for comparison and made available for publication by other groups. Particularly the spectrum is published in tabular form in Ref. \cite{6}. The only, rather incomplete, information available about the experiment is from two annual research reports of 1994 \cite{7} and 1995 \cite{18}.

The experiment used a Ti$^{15}\text{N}$ target bombarded by a deuterium beam to produce $^{16}\text{N}$ which was partially captured in carbon foils of 10 or 20 $\mu\text{g/cm}^2$ thickness. The activity was then transfered to a position between two silicon detectors, and $\alpha$ and $^{12}\text{C}$ particles were observed in coincidence, as in Ref. \cite{2}. It is not clear whether the distributed spectrum was made with a 10 or 20 $\mu\text{g/cm}^2$ thickness, or, if both spectra were combined. Other unknown information includes the method of energy calibration and the resolution of the detectors. It is, however, of interest to note the following remark in Ref. \cite{7}:

‘In Fig. 1.2-1, we show the two dimensional histogram of carbon energy versus alpha-particle energy. The contribution of the broad 1$^-$ state at 9.6 MeV in $^{16}\text{O}$ is the dominant feature of the histogram, while the true low energy events are visible in a curved group roughly along the diagonal line.’

Certainly, in the measurements of Refs. \cite{2,8} which used approximately 10 $\mu\text{g}$ carbon collector foils, the pulse height relation between $^{12}\text{C}$ and $\alpha$ events was strictly linear. The curvature may suggest that more mass per unit area than stipulated in Ref. \cite{7}, possibly caused by carbon deposition during bombardment, was present in the collector foil, or that the data acquisition had some other problem. It is not clear what measures, if any, were taken to prevent carbon build-up on the foils, or if the foil thickness was otherwise monitored. In any case, accurate energy calibration of a curved group in the two dimensional spectrum appears difficult. The method of energy calibration was, however, not reported (see Sec. \text{IVA2}). No coincidence efficiency measurement is described.

The actual geometry, details such as beam spot size and detector distances, of the Seattle experiment are not known. Therefore only a simulation using very generic features can be performed, see Sec. \text{III D}.
D. The Argonne measurement

Recently a paper was published [8] in which a measurement of the $\beta$-delayed $\alpha$-spectrum of $^{16}$N at Argonne National Laboratory is described. In this experiment, a fast $^{16}$N beam produced by the $d(^{15}$N,p)$^{16}$N reaction was slowed down in a gas cell and captured onto a target carbon foil with a claimed beam spot of 5 mm. Within 15 s, the collected activity was rotated into an ionization chamber where both the $\alpha$-particle and the $^{12}$C recoil were detected. A similar ratio cut as in Ref. [2] has been applied, eliminating degraded target and detector events. Because of the particular geometry, events hitting the target frame were also visible in the two dimensional pulse height spectra and were eliminated. 220,000 events were published. The energy calibration used the $^{10}$B(n,$\alpha$)$^{7}$Li and the $^{6}$Li(n,$\alpha$)t reactions, with the lowest energy calibration line at 1.472 MeV $\alpha$ energy. The electronic coincidence efficiency was checked with a pulser; however, no check of the coincidence efficiency for the full system has been presented. The energy resolution is quoted with 40 keV at 1.4 MeV; no response function of the detector is reported.

III. GEANT4 SIMULATIONS OF $\beta$-DELAYED $\alpha$-DECAY SPECTRA OF $^{16}$N

A. Introduction

For extended Monte-Carlo (MC) simulations, the GEANT4 [19] toolkit has been used. It provides excellent possibilities for the definition of complex geometries and particle tracking therein, but lacks in support for processes often relevant for nuclear physics. In particular, there is no process for Coulomb scattering of ions at low energies, and the energy loss tables are not up-to-date. The built-in multiple scattering process is only useful for high-energy light particles scattered on heavy ions, neglecting the energy transferred to the recoil particle. Therefore, a custom-made Coulomb scattering process has been developed and added to GEANT4 to test primarily ion propagation in ionization chambers. This process has been applied here.

GEANT4 asks in each process for the mean free path, and takes the smallest of these values, or the distance to the next geometrical boundary, applies all "continuous" processes (only the energy loss in our case), and finally all "discrete" processes (only Coulomb scattering in our case). The custom-made Coulomb scattering process as well as the energy loss
process are described in the following.

1. **The Coulomb scattering process**

   The code is based on a paper from Möller, Pospiech, and Schrieder [20], describing the implementation of a Monte Carlo (MC) simulation for a chain of single scatterings on a screened Thomas-Fermi potential. Using reduced energy and reduced cross-section quantities the problem can be simplified to the integration of one scalar function (see. Ref. [21] for the tabulated function) for all charges, masses, energies, and angles. In this approach, the total cross section is a free parameter; the larger the cross-section, the more scattering will be calculated, and the result will be more accurate for small scattering angles. The physical limit is the half-distance $r_0$ between the atoms of the material the particle is moving in, $r_0 = 0.5 n^{-1/3}$, where $n$ is the atomic density of the material. A lower limit is given by the screening radius $a = 0.8853 a_B/\sqrt{Z_1^{2/3} + Z_2^{2/3}}$, with $a_B$ as Bohr radius. The reduced total cross-section $J_{\text{tot}}$ is defined as $J_{\text{tot}} = \sigma/\pi a^2$. For the calculations here we used values of $J_{\text{tot}} = 1-10$.

   For each scattering, the change of direction and energy is calculated and the recoil ion is generated if is above a threshold (typically 1 keV). GEANT4 also tracks the recoils, so they obey the same processes as defined for the primary ions.

2. **The energy loss process**

   The stopping power calculation in GEANT4 is based on the Ziegler parameterization [23] for stopping of protons in all matter, and on the effective charge model to scale the energy loss from protons to higher charged ions. However, different sets (tables) of coefficients can be selected based on different evaluations. The default set is based on the ICRU 49 report [24] but available are also tables from Ziegler 1977, Ziegler 1985, and SRIM 2000. For the ICRU and Ziegler tables two variants can be selected, based on helium stopping powers and on proton stopping powers. For the calculations here, we used the ICRU-49 table based on He stoppings but we also compared the results with calculations based on other tables. The differences are not significant.
FIG. 7: (color online) GEANT4 geometry used to simulate the Mainz experiment \cite{11}. The figure shows the gas cell and the outside detectors. $\alpha$-particles and $^{12}$C nuclei are multiple scattered by the gas, the collodion foil, and in the gold layer of the detector surface.

FIG. 8: (color online) GEANT4 simulations and comparison with the Mainz spectrum. The simulated spectrum is shown with the x-points, the simulated, shifted (63 keV) spectrum is shown by the cross points (+), and the experimental Mainz spectrum by the star (*) symbol. The latter two spectra overlap almost perfectly. The energy is in the laboratory system.

B. The Mainz spectrum

The exemplary description of the Mainz experiment presented in Ref. \cite{11} allows one to simulate this measurement with a high degree of confidence. The geometry used in the GEANT4 simulation is shown in Fig. \ref{fig:geant4}. The fiducial volume, i.e., the volume from where $\alpha$ particles can reach the detectors is identical to the one of Ref. \cite{11}.

The results of the simulation are shown in Fig. \ref{fig:simulation}. The simulations (using the TRIUMF
initial distribution) show that the Mainz spectrum would be shifted downward by about 60 keV, if an intrinsic calibration had not been applied. Applying such a shift shows that the experimental and the simulated Mainz spectrum show excellent agreement. The shift and broadening relative to the TRIUMF spectrum confirms that any notion of a ‘zero mass spectrum’ attributed to the Mainz measurement in Ref. [3] is not justified.

C. The Yale spectra

1. GEANT4 Simulations of the Yale spectra

The information available about the Yale experiments was used to simulate them. A $^{16}\text{N}$ source was uniformly distributed through a 180μg/cm$^2$ aluminum foil, at first without any extension in area. All angles $\theta$ were simulated, where $\theta$ is the initial emission angle of the $\alpha$ particle relative to the foil. (Zero degrees is perpendicular to the foil.) The spectrum derived from the TRIUMF R-matrix fit [2] was used in most studies for the original distribution, excluding thus the detector resolution in the measured TRIUMF spectrum. Also the final spectrum of Ref. [6] was employed once. The result of our kind of simulation is equivalent to a convolution of the initial spectrum and always leads to a ‘broader’ spectrum. Thus it is pointless to use as an initial spectrum one which is already ‘broader’ or of the same width, as no agreement will result. Fig. 9 shows a comparison between the original input spectrum, the spectrum subjected to a thick catcher as in Fig. 4 and the GEANT4 calculations for a detector opening angle of 10°. It is obvious that the GEANT4 simulated spectrum agrees very well with the previously simulated spectrum (Fig. 4) that includes only catcher thickness effects. However, at the low energy side, a tail develops caused by small to large angle scattering obscuring the physical low energy plateau of the $^{16}\text{N}$ spectrum and eventually leading to many very low energy events. Those would typically be below the detection threshold, or obscured by $\beta$ rays from the abundant $^{16}\text{N}$ decays.

The spectrum deteriorates with increasing opening angle of the detectors. This is shown in Fig. 10 displaying different angle ranges. As can be seen, there is a target thickness effect by projection, making the main peak wider as $\theta$ increases. In addition, the Coulomb scattering becomes more and more significant, until at high angles, it obscures the underlying spectrum completely. The total counts in the spectra of Fig. 10 scale with sin $\theta$ until at
FIG. 9: (color online) Comparisons of the original spectrum (bars) used in the simulation (TRIUMF), a spectrum with energy loss only applied (asterisks), and one with full scattering (crosses) from GEANT4 for a point source extending down to zero energy. The energy is for $\alpha$-particles in the laboratory system.

FIG. 10: (color online) The GEANT4 simulated spectrum for different angular ranges of $\theta$, as indicated by the labels for the Yale conditions.

angles close to $90^\circ$ the foil itself obscures detection.

In a subsequent step, we have simulated the geometry of the France III experiment \cite{5, 6}, and, as the geometry is similar, the Zhao experiment \cite{3}, including the extended catcher foil and the large-area silicon detector array. We have derived both single energy response functions for each of the energy points in the France III spectrum, as well as an integrated spectrum from the TRIUMF distribution. Fig. \ref{fig:11} shows the single energy response function
FIG. 11: Response function using the Yale setup for a monoenergetic α-source at $E_\alpha=1.71$ MeV showing the energy loss range (plateau), a short range tail (down to about 1.2 MeV), and a long range tail with a many very low energy events. The energy is in the laboratory system.

FIG. 12: (color online) GEANT4 simulation using the Yale set-up for an α-spectrum corresponding to the TRIUMF distribution. For $E_\alpha=1.71$ MeV, close to the main peak of the spectrum. As is visible, the response function shows the typical broadening by the catcher foil energy loss (plateau) with a short range tail going down to about 1.2 MeV, followed by a long range tail leading to many very low energy degraded events.

In Fig. 12 a simulation is shown which uses the geometry of the Yale experiments and the TRIUMF α-particle energy distribution. No detector energy resolution has been included into the simulations which may partially explain the slightly wider than simulated France
FIG. 13: (color online) Spectral response by response functions taken at single energy points (see Fig. 11) employing the Yale set-up for an α-spectrum corresponding to the TRIUMF spectrum (bar) and the final France III spectrum (asterisks) in comparison with the France III β-corrected (cross) and β-uncorrected spectra (square). The simulation spectra have been folded with a 55 keV Gaussian detector response [5].

III spectrum in the high energy region. We also suggest that the β-efficiency correction in the high energy region of Ref. [6] is somewhat overdone (see Fig. 2). The simulation tail is slightly higher than the measured one. This might be corrected by assuming a nonisotropic or nonhomogeneous distribution of $^{16}$N in the catcher foil, but there is no information about such a possible distribution.

As discussed above, response functions for each energy in the France III spectrum were calculated. From these similar spectra to that shown in Fig. 12 can be derived for both the TRIUMF and the France III final spectrum. These are shown in Fig. 13 in comparison with the France III raw spectrum and the non-β-efficiency corrected spectrum (see also Fig. 2). As in Fig. 12 Fig. 13 shows the simulated spectrum with the TRIUMF initial spectrum to be between the β-corrected and β-uncorrected spectra in the high energy region, while the France III spectral input leads to points above the corrected spectrum. However, the agreement with the France III spectrum at the low energy side, where β-efficiency corrections are negligible, is excellent for the case of the TRIUMF spectral input, while for the France III final spectrum as input there is a considerable discrepancy. The simulations also demonstrate that the six lowest energy points in the France III raw spectrum are not related to the plateau in the spectrum at low energies found at TRIUMF, but to the tail in
the response function mainly caused by Coulomb scattering in the thick catcher foil.

In general, the simulations show that the measurements at TRIUMF and Yale are in reasonable agreement (for both France III and Zhao), while the final France III spectrum is in disagreement with the original spectrum.

In the subsequent section, we investigate, by applying a deconvolution method described in literature, the degree to which useful information can be extracted from the Yale measurements.

2. Deconvolution of the Yale spectra

Deconvolution of experimental information is, in general, a difficult problem. There are several techniques known, all of them iterative, but not convergent. In the case of the $\beta$-delayed $\alpha$-decay of $^{16}$N, we are dealing with a spectrum, i.e. a finite number of discrete, positive data points, something we also expect as the final solution. For such a problem the Gold algorithm of deconvolution is most appropriate \cite{25,46}. The data analysis frame ‘ROOT’ \cite{26} developed at CERN provides an implementation of the Gold deconvolution algorithm that we have applied here, in particular by the TSpec::Unfolding() function. For the response function, necessary to be known in the deconvolution, the GEANT 4 simulated spectra for each of the raw Yale data points, as discussed in Sec. III C1 Fig. 11 are employed.

As the deconvolution in the Gold algorithm is an iterative process, the number of iterations itself is a free parameter to be chosen by judgment as the process, in general, is not very well convergent. For the initial iterations the spectrum gets narrower and narrower while the low energy tail gets smaller. We find only a mild convergence for the high statistics points at the maximum of the $\beta$-delayed $\alpha$ decay distribution of $^{16}$N for an increasing number of iterations starting at about 20 iterations, while the low count regions at both the low and the high energy sides of the $^{16}$N spectrum start to fluctuate with an increasing number of iterations. The width of the main peak converges to the TRIUMF width or to a value slightly below. In Fig. 14 deconvoluted spectra obtained by the Gold algorithm for increasing iteration steps are shown.

From this deconvolution process applied here it can be concluded that, without any preconceived knowledge of the shape of the $\beta$-delayed $\alpha$ spectrum of $^{16}$N, only the presence of
FIG. 14: (color online) Deconvoluted spectra according to the Gold algorithm for increasing numbers of iteration steps (n=1-1000) as indicated by the labels on the dashed lines. The France III final spectrum is shown as the boxes and the TRIUMF R-matrix fit as straight line. The energy is in the laboratory system.

The major peak with some reasonable width can be concluded, while any other information is obscured by the experimental procedure chosen in the Yale measurements. Poor energy resolution cannot be reversed.

D. Simulations of the Seattle and TRIUMF experiments

Using the available information, we attempted to simulate the Seattle experiment. As this spectrum has used $^{12}$C+α coincidences it was first simulated, how, despite the ratio cut [47], the spectral form changes with increasing target foil thickness. What happens for a 10µg/cm$^2$ C foil in comparison to a 100 µg/cm$^2$ C foil is shown in Fig. [15] The spectrum of the 100 µg/cm$^2$ C foil has been shifted higher in energy to match that of the 10 µg/cm$^2$ C foil. In both simulations, the spectra use the TRIUMF input. It is clear that while tail events stay relatively well preserved, there is a short range tail component that is not easily removed by the ratio cut. Therefore also the method of coincidence ratio cuts calls for as thin foils as possible.

In Fig. [16] a comparison between the TRIUMF based simulated spectrum based on a
FIG. 15: (color online) Comparison of a simulated spectrum using a 10 $\mu g/cm^2$ C foil (x) and a 100 $\mu g/cm^2$ C foil (bar). The energy is in the laboratory system.

FIG. 16: (color online) Comparison of a simulated spectrum using a 20 $\mu g/cm^2$ C foil (cross) and the Seattle spectrum (bar). The energy is in the laboratory system.

20$\mu g/cm^2$ C foil and the Seattle spectrum is shown. A slight energy adjustment, corresponding to the energy loss in the foil has been done. Both the high and the low energy side of the Seattle spectrum are wider than in this comparison, see also the discussion in Sec. [IV.A.2] regarding energy calibration. It may be noted that approximate agreement on the low energy side can be obtained with a foil thickness of approximately 50 $\mu g/cm^2$. However, no possible foil thickness in the simulation can reproduce the high energy side of the spectrum from the TRIUMF input.

As seen for the simulations to the Seattle data a 10 $\mu g/cm^2$ C foil introduces little
FIG. 17: (color online) Simulated two dimensional energy versus energy spectrum for a gas chamber with 150 Torr as described for the Argonne experiment. The asymmetry in regard to the 45° axis originates from the asymmetric collector foil mounting in reference to the foil supporting frame. The energies are in the laboratory frame.

disturbance to the β-delayed α spectrum of 16N. We have used the TRIUMF geometry and input to simulate the TRIUMF data as well. The agreement is good after folding with the approximately 30 keV energy resolution of the detectors.

E. Simulations of the Argonne experiment

Following the description of the experiment [8], the spectra have been simulated in GEANT4 based on the TRIUMF spectrum as previously. No 16N isotopes in gaseous form and no noise, as reported in Ref. [8], have been included. The two-dimensional energy versus energy spectrum from the simulations is shown in Fig. 17. Particles were tracked through the P10 gas (90% argon, 10% CH4) to complete stop either in the gas or a boundary. The reported partial energy collection for high energy events for ion chamber pressures of 150 Torr [8] are confirmed.

In Fig. 18 the comparison between the simulated GEANT4 spectrum and the spectrum of the Argonne work is shown. In the simulated spectrum a condition (cut) for maximum
A pressure of 195 Torr is used. The energy is the $\alpha$-energy in the laboratory in keV.

carbon energy (channel) as well as a ratio condition were applied. As the numerical value for the latter is not reported in Ref. [8], we applied one similar to Ref. [2]. The figure shows reasonable agreement on the low energy side of the main peak, with a slight excess of counts in the simulated spectrum around $E_{\alpha}=1500$ keV and $E_{\alpha}=1$ MeV, see also the discussion in Sec. IV A 4. The high energy side of the spectrum is definitely wider, but no detector resolution effects have been included in the simulation. The simulation demonstrates also that measurements using ionization chambers have potential problems with the response tail from the target, as this limit seems to be reached in the Argonne measurement. In conclusion: the Argonne measurement is consistent with the TRIUMF one at least for the region of $E_{\alpha}=1.1$ to 1.3 MeV. For further discussion, see Sec. IV A 4.

IV. COMPARISON AND R-MATRIX FITS TO DIFFERENT DATA

A. Fits and comparison of spectra

Independent of their credibility all of the $\beta$-delayed $\alpha$ spectra of $^{16}$N can be subjected to $R$ matrix fits. In particular, this can show how much the apparent differences between the spectra influences the deduced S-factor $S_{E1}(300)$. Only fits with an interaction radius of $a=5.5$ fm and with the $\beta$ feeding factors $A_{23}=0$ and $A_{33}=0$ for the $f$-wave (as in Ref. [2]; also see Ref. [2] for $R$-matrix notation) were performed, as it is not the goal of this paper
FIG. 19: (color online) Comparison of the final $\beta$-delayed $\alpha$-spectra of Ref. [4, 5, 6] (bars) and Ref. [3] (crosses) with both spectra normalized in height to the one of Ref. [2]. The energy axis is the center-of-mass-system. Also shown are $R$-matrix fits to these spectra.

to produce a final combined result with a variation of all parameters. However, it must be stressed again that the results derived from meaningless spectra are meaningless.

Comparisons of the final experimental spectra are as long meaningless as all of the measurements have different energy resolutions and different $^{16}$N source thicknesses. In principle, only unconvoluted $R$-matrix fits, with convolutions applied in the fittings, should be compared, as clearly a spectrum with a detector energy resolution of 10 keV will be different from one with a resolution of 100 keV, even, if the detector responses would be simply Gaussian over many orders of magnitude.

1. **Comparison between the France III and the Zhao spectra**

   In Refs. [4, 5, 6] no comparison between the $^{16}$N spectrum presented there and the one of Ref. [3] is given, even though differences with the spectrum of Ref. [2] are pointed out with considerable detail. Both spectra [3, 6], together with R-matrix fits to them, are shown in Fig. 19. Several observations may be made: the two spectra do not agree, not only on the low energy side of the main peak, but also on the high energy side even though they both originate from the same initial distribution. The quality of the $R$-matrix fits for both spectra is more than excellent, as already noted above for the case of Ref. [3]. Because both spectra have gone through extensive, non-justified, data reduction, it is also unclear,
FIG. 20: (color online) Comparison of the final $\beta$-delayed $\alpha$-spectra of Ref. [7] (crosses) and Ref. [2] (bars) with the Seattle spectrum normalized to that of Ref. [2] at the peak. Energies are in the center-of-mass system. Also shown are $R$-matrix fits to these spectra.

how the detector resolutions should be treated. E.g. for the Mainz spectrum [11], a detector resolution of 12-18 keV (FWHM) at 1.5 MeV is quoted, with the full system response leading to about 40 keV at 1.2 MeV $\alpha$ energy, while for Ref. [15] 50 keV is given at an unspecified energy. However, as discussed above, the spectrum of Ref. [3] has been normalized to that of the Mainz group. Indeed, a low energy resolution (20 keV or less) fits the spectrum of Ref. [3] best, particularly around the maximum. On the other hand, the broad spectrum of Refs. [4, 5, 6] is well fitted with the quoted experimental detector resolution of 55 keV. The particular fits shown in Fig. 19 correspond to $S_{E1}(300)$=79.6 keV b for Refs. [4, 5, 6] for a bad resolution fit and $S_{E1}(300)$=101 keV b for Ref. [3] for a low resolution fit. Ref. [3] suggests $S_{E1}(300)$=95 keV b, see also Sec. II B 2. No value of $S_{E1}(300)$ has been suggested for the France III [6] measurement.

2. Comparison between the Seattle and the TRIUMF spectra

In Fig. 20 a comparison between the Seattle [7] and the TRIUMF spectra are shown [2]. Obviously, the two spectra disagree, as the Seattle spectrum is wider both in the low and the high energy regions of the TRIUMF spectrum. However, as the method of energy calibration for the Seattle spectrum is unknown to us, it may be noted that a simple linear recalibration would give a nearly perfect match between the two spectra ($E_{new} = 0.95*E_{old} + 0.118$ MeV).
Note the following quotation from Ref. [18]:

‘Our results are consistent with the previous measurements at TRIUMF [2] and Yale [3,48].’

This suggests that the Seattle group does not consider the differences between those spectra as important. In fact, the difference between these spectra and the one of France III is the most pronounced.

The detector resolution in the Seattle experiment is not published. In principle, a larger energy resolution can achieve agreement between the TRIUMF and the Seattle spectra. We have varied that resolution and find an optimum value at about 30 keV with $S_{E_1}(300)=97.1$ keV b. However, fitted simultaneously with the radiative $E1$ ground state data, a smaller resolution (20 keV) is preferred with $S_{E_1}(300)=95.7$ keV b. For larger resolutions than 30 keV, $S_{E_1}(300)$ declines with a decrease in fit quality. No $S_{E_1}(300)$ has been suggested by the Seattle group.

3. **Comparison of the Mainz and TRIUMF spectra**

A comparison between the Mainz single count spectrum and the TRIUMF coincidence spectrum is shown in Fig. 15 of Ref. [2]. It has never been claimed the two both spectra agree. However, it was found and pointed out in Ref. [2] that the TRIUMF singles spectrum, before application of the coincidence cuts, agrees well with the Mainz spectrum. The difference with the coincidence spectrum is largely from the coincidence ratio cuts applied to remove degraded $\alpha$ events found in the coincidence spectrum. In the simulations of Sec. III B it has indeed be shown that the Mainz (single detector) spectrum is fully consistent with the TRIUMF one.

4. **Comparison between the TRIUMF and Argonne spectrum**

In Ref. [8] a comparison between the low energy part of the TRIUMF spectrum and the Argonne spectrum as well as with the Z. Zhao spectrum [3], is presented. Better agreement of the Argonne spectrum with the Mainz and Z. Zhao spectrum is claimed than with the TRIUMF spectrum. The comparison is in principle invalid as the TRIUMF combined detector and $\beta$-$\nu$ recoil resolution is $30\pm5$ keV, while a resolution of $40$ keV (no error) at
FIG. 21: (color online) Comparison of the $\beta$-delayed $\alpha$-spectra of $^{16}$N Ref. [2] (bars) and Ref. [8] (cross).

1472 keV is claimed in Ref. [8]; combined with the $\beta$-$\nu$ recoil resolution that averages to 43 keV [49]. Fig. 21 shows a comparison with the TRIUMF spectrum [50].

We make the following observations: As described, the region of the Argonne spectrum from about $E_\alpha = 1.05$ MeV to about 1.3 MeV is above the TRIUMF spectrum. However, there are other differences: (i) the high energy points of the Argonne spectrum are above the TRIUMF spectrum starting at about $E_\alpha = 1900$ keV. We attribute this to the poorer detector resolution in the Argonne experiment (see arguments above for non-compatibility of spectra) and also $^{18}$N background effects. (ii) Also in the region near $E_\alpha = 1.5$ MeV, the Argonne data are above the TRIUMF data. In that region, the continuation of the response function that cannot be removed by the ratio cut in the TRIUMF spectrum, has been subtracted, see Fig. 10 of Ref. [2]. No removal of a similar response function in the Argonne spectrum, nor of their additional background from $^{16}$N in the gas phase have been reported. We also note that points in the Argonne spectrum from about $E_\alpha = 850$ keV to 1050 keV are systematically higher than the TRIUMF points. This, however, can be resolved by different normalizations as the main peaks do not need to match perfectly.

Most interesting are the two points at about $E_\alpha = 1074$ and 1094 keV. In Fig. 22 we show the previous Figure 21 with the scaled $\beta$-delayed $^{18}$N spectrum of Ref. [28]. No particular adjustment in the energy scale of either spectrum has been done. The agreement with the main peak of the $^{18}$N spectrum for these two points is remarkable. The lower energy point is about 3.6$\sigma$ above its expected value and the higher energy point 2.6$\sigma$. The statistical
deviation has been determined from R-matrix fits excluding this region, see Sec. IV B. The random chance for such a peak at the energy of the $^{18}$N main peak is about $3 \times 10^{-6}$. This strongly indicates a background of $^{18}$N in the beam. While it is argued in Ref. [8] that the combination of $^{15}$N+d cannot produce any $^{17,18}$N, fusion evaporation reactions of the $^{15}$N beam with carbon or other light elements at windows or collimators can produce both isotopes.

While $^{18}$N is a relatively trivial contamination (see Sec. IV C 2) showing mainly up as one narrow peak and being removable by decay time cuts, $^{17}$N is not. The $\beta$-delayed $\alpha$ spectrum of $^{17}$N [29] has no narrow peak, and the half life (4.2 s) is near that of $^{16}$N (7.1 s). Removing both the likely $^{18}$N background and an $^{17}$N background of roughly the same intensity leads to the spectrum shown in Fig. 23. The spectrum derived by such a background subtraction agrees well both on the plateau and in the region above up to $E_\alpha=1300$ keV with the TRIUMF spectrum. The low yield point at about $E_\alpha=1054$ keV is statistically consistent with the TRIUMF spectrum. In its calculation, the method of the subtraction, and details like matching energy calibrations and different detector resolutions that are involved pose rather subtle problems that are not worth improving at this stage.
FIG. 23: (color online) Comparison of the $\beta$-delayed $\alpha$-spectra of $^{16}\text{N}$ of Ref. [8] (asterisk) with the one with $^{17,18}\text{N}$ background removed (cross), as described in the text. In addition, the TRIUMF spectrum is shown.

B. $R$-matrix fits to the Argonne spectrum

The Argonne spectrum has been fitted at different interaction radii $a$. In general, we find the $S$-factors $S_{E1}(300)$ to be smaller than given in Ref. [8]. However, as our spectrum is read out from a paper, we encounter large deviations from the fit at the peak of the $^{16}\text{N}$ distribution due to the difficulties of reading out the spectrum there. While it is possible to adjust the points to lower the least squares sum considerably, this may lead to biased fits. In Fig. 24 we show a fit done with $a=5.5$ fm. For the resolution, we chose 57 keV in the center-of-mass, very close to the optimum (minimum $\chi^2$) setting. For this particular fit $S_{E1}(300)$ is found to be 68 keV. As it is not described in Ref. [8] what constitutes a statistical error, we cannot derive those. In addition, it is not mentioned what fit parameters have been used, and what degrees of freedom are included in the partial least squares parameters.

The Argonne group has put great emphasis on the region between $E_\alpha=1.0$ to 1.3 MeV showing a deviation from the TRIUMF data. To see the relevance of this region, the data points there have been taken out of the fit. Fig. 25 shows the fit in the region of interest with the data used and not used in the fit. Obviously, when those data are excluded, the fit runs close to the TRIUMF data. For the same exercise with the TRIUMF data the fit does not change significantly.

The deviation of those points identified as likely $^{18}\text{N}$ events was taken from the fit that
FIG. 24: (color online) $R$-matrix fits to the Argonne spectrum with $a=5.5$ fm. Shown are the spectrum (bar), the resolution convoluted fit (long dash), the unconvoluted total fit (short dash), the p-wave component (dotted) and the f-wave component (dash-dotted). The energy is in the center of mass system.

FIG. 25: (color online) $R$-matrix fits to the Argonne spectrum with $a=5.5$ fm for the energy region displayed. Shown are the spectrum (bar), the resolution convoluted fit (long dash), and the fit, with those data excluded (short dash). The energy is the center of mass energy.

had those events removed, see Sec. IV A 4.
FIG. 26: (color online) Comparison of the $\beta$-delayed $\alpha$-spectra of $^{16}$N Ref. [2] (bar) and Ref. [1], (as energy-calibrated in Ref. [6]) (asterisks) with the spectrum where previously subtracted events from the $\beta$-delayed $\alpha$-decay of $^{17}$N have been added back (cross). [Only the relevant low energy region is shown.] The energy is the center-of-mass energy.

C. Background subtraction in the TRIUMF spectrum

It has been suggested [6, 7, 51] that the final TRIUMF result is questionable beyond the errors quoted, because a background of events from the $\beta$-delayed $\alpha$-decay of both $^{17}$N [29] and $^{18}$N [28] has been subtracted to obtain the final spectrum. While the total number of these subtracted events is only 2130 compared to 1,026,500 total events after subtraction, it has been implied that those subtractions, if reversed, would (i) reconcile the differences between different measurements, and (ii) cause changes larger than the assigned errors given to the final S-factor $S_{E1}(300)$ quoted in Ref. [2]. We show below that these arguments are without merit, and could have been checked by the authors of Refs. [6, 7] themselves, as both the total number of events subtracted as well as the shape of these backgrounds are given in Ref. [2], particularly in Fig. 10.

1. The subtraction of the $\beta$-delayed $\alpha$-spectrum of $^{17}$N

In Fig. 26 we compare the final TRIUMF $^{16}$N spectrum, the TRIUMF $^{16}$N spectrum with all $^{17}$N events added back, and the Mainz spectrum (as energy calibrated by the Yale group [6]). Clearly, the effect of the subtraction of the $^{17}$N spectrum from the TRIUMF spectrum is
minor, and does not produce any reasonable identity with the Mainz single event spectrum. The S-factor at 300 keV is found to be $S_{E1}(300) = 82.2$ keV b with the $^{17}$N events added back to be compared with $S_{E1}(300) = 79 \pm 21$ keV b with the $^{17}$N events subtracted \cite{2}. Note that in Ref. \cite{2} an error of $\pm 5$ keV b is given for the systematic uncertainty in the R-matrix fits resulting from the $^{17}$N event subtraction. It is therefore concluded that a claim of significant uncertainties resulting from the subtraction of $^{17}$N events is irrelevant.

2. The subtraction of the $\beta$-delayed $\alpha$-spectrum of $^{18}$N

The $\beta$-delayed $\alpha$ emitter $^{18}$N has a half life of 0.63 s. As reported in Ref. \cite{2}, the implantation time for the radioactive $^{16}$N beam was 3 s with 0.25 s moving time between three subsequent decay stations. While there is small but clear evidence of the low energy 1.081 MeV peak from $^{18}$N in the spectra from the first detector station, this peak is no longer visible in the subsequent two stations as expected from the short $^{18}$N half life. The four later spectra were then used to remove the $^{18}$N contamination from the $^{16}$N spectra. This information has been given previously in Ref. \cite{2}.

The low energy $^{18}$N peak allows for an easy scaling of the $^{18}$N subtraction. In Fig. \ref{fig:27} we compare the TRIUMF $^{16}$N spectrum, the TRIUMF $^{16}$N spectrum with all $^{18}$N events added back and the Mainz spectrum, as calibrated by the Yale group \cite{6}. Again, it is clear that agreement with the Mainz spectrum is not significantly improved by adding $^{18}$N events back. The fit to the added-back spectrum has a relatively poor quality as it cannot accommodate the narrow $^{18}$N peak. However, leaving the $^{18}$N events in the spectrum leads to an $S_{E1}(300) = 74.2$ keV b, still well within the quoted error of Ref. \cite{2}, i.e. $79 \pm 21$ keV b. Note, however, that the $^{18}$N $\alpha$ peak in the $^{16}$N spectrum provides an excellent intrinsic $\alpha$-particle energy calibration with a precision of about 1 keV in this energy region.

V. CONCLUSION

In Table I we give an overview over the $^{16}$N decay experiments discussed here. Discussing the rows of Tab. I: Aside from the TRIUMF experiment, all others use the $^{15}$N(d,p)$^{16}$N reaction for $^{16}$N production. However, only the Mainz experiment \cite{1}, the TRIUMF and the Argonne one thoroughly separated the region of $^{16}$N production from the detection region.
FIG. 27: (color online) Comparison of the $\beta$-delayed $\alpha$-spectra of $^{16}$N Ref. [2] (bars) and Ref. [1] (asterisks), (energy calibrated in Ref. [6]), with the TRIUMF spectrum to which all events from the $\beta$-delayed $\alpha$-decay of $^{18}$N have been added back (cross). [Only the relevant low energy region is shown.] The energy is in the center-of-mass system.

| Table I: Overview of the $\beta$-delayed $\alpha$ decay experiments discussed in the article. |
|---------------------------------------------------------------|
| Experiment | Mainz [1] | TRIUMF [2] | Yale 1 [3] | Seattle [4] | Yale 2 [6] | Argonne [8] |
| $^{16}$N production | $^{15}$N(d,p) | Isotope Separator | $^{15}$N(d,p) | $^{15}$N(d,p) | $^{15}$N(d,p) | $^{15}$N(d,p) |
| $^{16}$N implantation speed | low | low | high | high | high | high |
| mass/composition | $30$ $\mu$g/cm$^2$C$_6$H$_7$N$_2$.5O$_{10}$ | $180$ $\mu$g/cm$^2$ Al | $10/20$ $\mu$g/cm$^2$ C | $180$ $\mu$g/cm$^2$ Al | $17$ $\mu$g/cm$^2$ C |
| of $^{16}$N catcher | +0.1.5 cm 6-8 Torr $^{15}$N$_2$ gas | eqv. 0-17 $\mu$g/cm$^2$ N |
| detectors | $<$35 $\mu$m Si | 10.4-15.8 $\mu$m Si | 50 $\mu$m Si | 15-20 $\mu$m Si | 50 $\mu$m Si | ion chamber |
| background (measured) | $\beta$-tail | $^{18}$N, $^{17}$N | unknown | $\beta$-\alpha | $\alpha$-$^{12}$C | $\beta$-\alpha |
| degraded event suppression | none | $\alpha$-$^{12}$C | $\beta$-$\alpha$ | $\alpha$-$^{12}$C | $\beta$-$\alpha$ | $\alpha$-$^{12}$C |
| efficiency corrections | none | $\beta$ | unknown | $\beta$ | unknown | none |
| energy calibration | $^{10}$B(n,\alpha) | $^{18}$N, $^{20}$Na | $^{10}$B(n,\alpha) | unknown | $^{10}$B(n,\alpha) | $^{10}$B(n,\alpha) |
| deconvolution applied | none | none | division (see text) | unknown | division (see text) | none |

While care may have been taken to avoid $^{16}$N hitting e.g. the catcher foil frames in the Yale, Seattle and Argonne experiments, a deep implantation of $^{16}$N ions has not been excluded at the level of 1/10,000 leading to potentially degraded $\alpha$ events.

While the Mainz spectrum has been dubbed a ‘zero thickness spectrum’ in Ref. [3], it is certainly not the one that uses the least amount of material to hold the $^{16}$N. As the GEANT4 calculations show, the combination of an extended volume source and the foils in front of the detectors leads to some broadening. In contrast, the $10$ $\mu$g/cm$^2$ C foils in the TRIUMF experiment slowly decreased in thickness in the course of the implantations, until a hole was produced by the stable beam at mass A=30, and the four active $^{16}$N collector foils had to be replaced. Spectra collected at different degrees of foil degradation showed no
difference. From Ref. [11] it is indeed known that for the Mainz spectrum the accumulated mass traversed by the $\alpha$-particles leads to an increase from about 15 keV detector resolution to 40 keV total resolution. The Mainz spectrum also allowed a relatively large angular range of collection, leading to a slightly increased range in matter. As far as the Seattle spectrum is concerned, it is not clear whether the spectrum that has been distributed is from a 10 or 20 $\mu$g/cm$^2$ foil. It also can not be excluded that additional carbon build up was involved. Of course, the catcher foils used by the Yale group are much the thickest and, as shown above, this leads to heavily distorted spectra dominated for low energies by scattering.

As far as detectors are concerned, all groups but one use silicon based detectors, with the thinnest detectors, i.e. those with the least $\beta$ energy losses and thus $\beta$ response used by the TRIUMF group. The thickness of the gas layer used to stop the $\alpha$-particles in the Argonne experiment is comparable with the TRIUMF detectors.

Both, in the Mainz and the TRIUMF spectra, backgrounds have been subtracted. This issue is extensively discussed above for the TRIUMF spectrum (Sec. IV C). In fact, for the Mainz spectrum, the $\beta$ induced background from the $^{16}$N decay is quite significant at low energies.

As the primary goal of the Mainz experiment was not to determine the shape of the $^{16}$N spectrum exactly, but to search for the narrow line of the parity-forbidden $\alpha$-decay of $^{16}$O at 1.283 MeV of $\alpha$ energy, no measures were taken to check for degraded events. However, in all other experiments, some measures for increased discrimination against degraded or otherwise unwanted events were taken. In the case of the Yale experiments $\beta$-$\alpha$ coincidences were employed. Those, in principle, eliminate events from the $\beta$ tail of the $\alpha$-detector and from events, where $\alpha$ particles hitting the detector do not deposit the full charge. However, no means exists in this method to eliminate events that suffer from some degradation of $\alpha$-particle energies in the thick aluminum foil. This is consistent with the observations and simulations about these measurements discussed above. As the detection of very low energy $\beta$-particles is required for high energy $\alpha$-particles, extensive $\beta$ efficiency corrections were necessary, in addition, to derive the two Yale spectra.

In the Argonne, Seattle and TRIUMF experiments $\alpha$-$^{12}$C coincidences with subsequent pulse height ratio cuts were used. That, in principle, removes $\beta$-tail events, a good fraction of degraded detector events and events scattered in the collection foil. As our simulations show, a low level of degraded response events will remain depending on catcher foil thickness.
While we do not know the method of energy calibration for the Seattle experiment, the Argonne, Yale and Mainz experiments employed the \(^{10}\text{B}(n,\alpha)^{7}\text{Li}\) reaction for their energy calibration. The reaction results in two \(\alpha\)-particle groups, at \(E_{\alpha}=1472\) and 1777 keV, i.e. close together and close to the \(^{16}\text{N}\) main peak. One also does obtain two lower energy \(^{7}\text{Li}\) groups. However, those are less suitable for calibration, as different pulse height defects or other differences for the differences in the detection of the \(^{7}\text{Li}\) ions are difficult to take into account. Therefore, no low energy calibration is available in these experiments. It should be noted that the method of energy calibration is rather uncr itical for the Yale experiments as they are shifted upward by about 200 keV and use an invalid deconvolution method. In addition, Argonne used the \(^{6}\text{Li}(\alpha,n)t\) reaction for a high-energy calibration point. The Argonne measurement claims a smaller systematic error from their energy calibration (5\%) than the TRIUMF one (10\%) [30].

While not having employed it as a calibration in the TRIUMF experiment, we have observed that the actual position of the \(\alpha\) peaks from \(^{10}\text{B}(n,\alpha)^{7}\text{Li}\) was not consistent with the calibration obtained from \(^{18}\text{N}\) and \(^{20}\text{Na}\). Indeed the calibration with \(^{10}\text{B}(n,\alpha)^{7}\text{Li}\) was found to be dependent on the direction of the incident neutron flux, suggesting that only incomplete neutron thermalization was achieved. Complete thermalization may be hard to prove without an accurate measurement of the neutron spectrum at the position of the \(^{10}\text{B}\) target.

It is our opinion that the best energy calibration comes from the narrow decay lines of well known \(\beta\)-delayed \(\alpha\)-emitters that can be used without modifying the experimental set-up.

Besides the Yale measurements, no other measurement relies on deconvolutions. It has been demonstrated above that both methods applied are incorrect. It also has been shown that both original measurements are consistent with the TRIUMF data when compared with simulations and, for the case of France III [6], inconsistent with their own ‘deconvoluted’ spectrum.

The TRIUMF spectrum, among all measurements, shows the narrowest main peak. It has been shown above that the energy calibration used for the derivation of this spectrum is the best one available, and that the shape of the TRIUMF spectrum is little influenced by background subtractions. In principle, besides resolution broadening, also an energy dependent efficiency can lead to a broadening, and more likely narrowing (efficiency dropping
off at low energies) of the spectrum. However, it is shown experimentally in Ref. [2] that the coincidence detection efficiency stays constant over the energy range covered in the measurement. None of the other measurements considers this point. As far as broadening by target and detector effects is concerned to the authors of this article there is no reasonable physical response function known that would transform the wider spectra of the other measurements into a narrower one, as measured at TRIUMF. But there are many well known transformations that can produce a wider spectrum from a narrow one.

VI. OUTLOOK

We have presented fits to several spectra in this article. However, either the derivation of those spectra is in doubt, even if one does not take systematic errors into account, as for the Yale spectra, or the spectra are not completely published, as for the Argonne spectrum. Therefore any derivation of a common S-factor $S_{E1}(300)$ would lead to questionable results. We therefore reiterate the value of $S_{E1}(300)$ derived in Ref. [2], i.e. $S_{E1}(300)=79\pm21$ keV b including systematic errors. A new evaluation of the entire reaction rate of $^{12}$C($^{\alpha},\gamma$)$^{16}$O is in work [31] taking new information from other measurements into account, particularly phasen shift and radiative capture data.

While other partial cross sections of $^{12}$C($^{\alpha},\gamma$)$^{16}$O have relative uncertainties exceeding those of the $E1$ ground state transition, in the long run, it certainly would be desirable to improve the value derived in Ref. [2] to a relative error of less than 10%. While statistics plays some role, experimental limitations and systematic errors are the real limits.

Some numbers from other measurements are part of the fit to the $\beta$-delayed $\alpha$-spectrum of $^{16}$N. These are the energy of the subthreshold $1^-$ state, its radiative width, and the $\beta$-decay branching ratio into the $E_x=9.6$ MeV $1^-$ state in $^{16}$O. While the former numbers appear reasonably well known, there is some uncertainty about the latter branching ratio. The compilation of Ref. [32] quotes $1.20\times10^{-5}\pm0.05$ for this branching ratio citing Ref. [33] as the origin of this number. However, Ref. [33] is a theoretical work quoting Ref. [34], another theoretical work. Ref. [34] quotes Ref. [10] (an early publication of the Mainz group) which gives a branching ratio of $1.19\times10^{-5}\pm0.10$. Both theoretical works do not quote an experimental error, so the origin of the error in Ref. [32] is unknown to us. However, Ref. [11], p. 327, gives a revised value of $1.13\times10^{-5}\pm0.08$ for the branching
ratio into the E_x = 9.6 MeV state of 16O, a not insignificant deviation. Therefore a revised measurement with improved errors of this branching ratio is desirable.

Among the errors inherent to the measurement is the energy calibration of the spectrum. Ref. [2] quotes a systematic error of 10 keV for S_{E1}(300), while Ref. [30] quotes only 5 keV with a more indirect measurement. This error can certainly be improved.

Part of the uncertainties in the fits originates from the presence of both a p- as well as an f-wave in the β-delayed α-spectrum of 16N. Higher statistics data at very low energies (E_α < 600 keV) would be very desirable in this context as they originate only from p-wave decay, while due to penetrability reasons the f wave drops off faster and at higher energies. However, this α-energy region is difficult to access, first because of possible β^- background in the detectors that eventually may be removed, but more important second because of a possible response of high energy α events into this region. Our simulations have shown (Secs. III D, III E) that indeed the most advanced experiments using carbon foils of about 10 μg/cm^2 and coincidence techniques are close to this response limit. Foils with much less mass seem to be impractical. As an alternative 16N could be trapped in electromagnetic or opto-magnetic traps with no substrates for the 16N involved. Because of trapping efficiencies and likely difficult geometries such an experiment will require high yields of 16N. In this case careful simulations of the next order of response like those originating from the detectors are necessary. Another possibility, in principle, is to measure the β-α correlations in the decay of polarized 16N that would differentiate the data between the p- and the f-wave in the region of about and above E_α = 1 MeV, see Ref. [16].

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[35] We use the expressions ‘convolution’ and ‘folding’ (and their opposites) with the same meaning. However, in literature often a distinction between ‘folding’ with a response function $G(E - E')$
and ‘convolution’ with a response function $G(E, E - E')$ is made.

36. We label the experiments subsequently by the laboratory where they took place.

37. There is an additional error in Table A.1 as the points at 1468 and 1482 keV have the same number of counts. The correct number of counts of the point at 1468 keV should be 3785.

38. [...] notes here and in other quotations an omission of text for clarification.

39. Of course, if the final result is known, it is possible to construct a response function $G(E, E' - E) = \frac{f_i(E)}{f_f(E)} \delta(E' - E)$ with $f_i(E)$ the initially measured spectrum and $f_f(E)$ the ‘unfolded’ spectrum, see Sec. II B 2. Such a function, however, is not independent of the initial ($\alpha$) spectrum.

40. Highlighted here.

41. It may be noted that any attempt to find an R-matrix solution based on three states for the $f$-wave presented in Ref. 3 leads to no acceptable result.

42. The Mainz measurement certainly employed a larger catcher thickness than the TRIUMF one, but none of the experiment had zero target thickness, see Sec. III B.

43. There is no description how the errors were propagated, but it appears, they also went from original to final by the same division.

44. A response function is defined by the energy dependent function (spectrum) derived from the propagation of events from a monoenergetic ($\alpha$) source through the experimental apparatus.

45. Emphasized here.

46. In their introduction the authors of Ref. 25 make the following statement about deconvolution: ‘From the numerical point of view, deconvolution belongs to one of the most critical problems. It is a so-called ill-posed problem, which means that many different functions solve the convolution equation within the allowed error bounds. The estimates of the solution are extremely sensitive to errors in the input data. Very frequently the noise present in the input data causes enormous oscillations in the results after deconvolution.’

47. We call a ‘ratio cut’ a condition on the spectra where only a limited range of ratios of apparent $\alpha$ and $^{12}$C events is considered to be valid. If there were no resolution and pulse height issues, the energetic ratio between $\alpha$- and $^{12}$C events would be exactly 3. For a description and application of such a condition, see Ref. 2.

48. Citations as quoted here.

49. In the $R$-matrix fits of Ref. 8 it is not mentioned, if the fits do include a convolution by the
detector resolution. $\beta$-$\nu$ recoil effects are not mentioned in Ref. [8] and for that matter in any publication of $^{16}$N spectra, except the TRIUMF one [2].

[50] As the Argonne group has not agreed to make the digitized spectrum publicly available [27], we have read it out from the publication. Minor inconsistencies particularly for the high count points, are therefore possible. Errors are taken as the square root of the counts, as apparently has been done for the low count points, where errors can be read out.

[51] Quoting from Ref. [6]: 'The disagreement could quite possibly be due to over subtraction of $^{18}$N contamination in the TRIUMF data. [...] The disagreement in the region of the interference minimum around 1.4 MeV is sufficient to change the f-wave component and leads to imprecise determination of the p-wave contribution to the $^{16}$N spectrum.' Obviously an imprecision beyond the errors presented by the TRIUMF experimental group is meant.