NATURAL RESOURCES AND MISSING INPUTS IN INTERNATIONAL PRODUCTIVITY COMPARISONS*

BY DAAN FREEMAN AND ROBERT INKLAAR

University of Groningen

AND

W. ERWIN DI EWERT

University of British Columbia

Standard theory for cross-country productivity comparisons assumes all countries use the same factor inputs in production. This assumption is violated when including natural resources, such as oil, gas and gold, because countries do not extract the full set of resources. In this paper we propose a solution by viewing it as a “missing goods” problem and assigning missing inputs a reservation price equal to the world resource price. We show that this has a substantial impact on relative productivity levels for countries heavily reliant on natural resources for generating their income. Under our new productivity measure, resource-rich countries are no longer uncommonly productive.

JEL Codes: E22, O13, O57

Keywords: development accounting, missing goods, natural resources, productivity measurement, reservation prices

1. INTRODUCTION

Development accounting is a popular tool that is used to establish how much of the differences in income levels across countries can be accounted for by differences in observed factor inputs—such as buildings, machinery and (skilled) workers—and how much by differences in productivity, the residual.1 This, in turn, can inform further research to explain why, for instance, investment in capital may be low or why productivity lags.2 But omission or mismeasurement of factor inputs will lead to biased measures of productivity. This has motivated researchers to expand and improve the measurement of inputs, by including additional types of intangible capital (Chen, 2018), accounting for differences in management

Note: We would like to thank two anonymous referees, participants at the General Conference of the International Association of Research in Income and Wealth (2018), the SOM PhD conference (2017) and Society for Economic Measurement conference (2017) for helpful comments.

*Correspondence to: Robert Inklaar, Groningen Growth and Development Centre, Faculty of Economics and Business, University of Groningen, Nettelbosje 2, 9747 AE Groningen, The Netherlands (r.c.inklaar@rug.nl).

1See Caselli (2005) and Hsieh and Klenow (2010) for overviews of this literature.

2See e.g. Acemoglu et al. (2019), who show that democratization increases income levels by improving investment, not by improving TFP.

© 2020 The Authors. Review of Income and Wealth published by John Wiley & Sons Ltd on behalf of International Association for Research in Income and Wealth. This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.
practices (Bloom et al., 2016) and improving estimates of human capital over the life cycle (Inklaar and Papakonstantinou, 2019; Lagakos et al., 2018). Omitted so far in these efforts is the role of natural resources, such as oil, gas, iron and gold, even though natural resources are an important source of income and wealth in many lower-income countries, as well as some (very) high-income countries (Lange et al., 2018). Inputs of subsoil assets also already fall within the asset boundary of the System of National Accounts, which means that systematically accounting for the use of these assets in production does not necessitate changes to measures of output or investment, unlike with, for instance, intangible capital.

The contribution of this paper is to propose and implement a method for incorporating natural resources as a factor of production in cross-country comparisons of productivity. We build on the work of Brandt et al. (2017) and Diewert and Fox (2016), who show how natural resources can be incorporated in a “sources of growth” framework. Many of the measurement considerations of their work, such as measures of resource rents, apply in a cross-country context. However, the extension to a cross-country setting faces a notable challenge in that countries typically extract only a few types of natural resources rather than the full set. Such missing inputs mean that relative productivity is not defined in the typical productivity comparison framework, such as that of Diewert and Morrison (1986) and Inklaar and Diewert (2016).

We propose a solution by drawing a parallel to the literature that deals with the “new goods” problem. New goods complicate inflation measurement because no price is observed in the period before the new good appears; a solution is to identify Hicksian reservation prices (Hicks, 1940), the price just high enough for demand to be zero. In the current context, we can define a producer Hicksian reservation price, which is the input price where that primary input is not used in production. Aside from the practical challenge in identifying what that price level would be, this introduces a conceptual complication in productivity measurement, because in the Diewert and Morrison (1986) framework, a Törnqvist index of primary input quantities is used. When a primary input is missing, this would then require taking the log of zero. To avoid this problem, we will treat natural resources as intermediate inputs. We illustrate this method for incorporating natural resources in international productivity comparisons for the 116 countries for which the Penn World Table (version 9.0, Feenstra et al., 2015) provides information on the input of produced and human capital and for which Lange et al. (2018) provides information on the production of natural resources—all for the year 2011.

The main unknown variable in applying this method is the reservation price for natural resources. The unit rent—defined as the resource price minus unit production cost—is the central concept, because, as Diewert and Fox (2016) show, the unit rent is equivalent to the user cost of the natural resource, i.e. the price of the input,

---

3 More specifically, we focus on what is referred to in National Accounts terminology as “subsoil assets.” Natural resources more broadly can also cover agricultural land and forests, see Lange et al. (2018).
4 See e.g. Diewert and Feenstra (2018), Redding and Weinstein (2019), Feenstra (1994) and Balk (1999).
5 That means that for our productivity computation, the value of output is defined as GDP minus resource rents and inputs consist of labour and produced capital.

© 2020 The Authors. *Review of Income and Wealth* published by John Wiley & Sons Ltd on behalf of International Association for Research in Income and Wealth
when beginning-of-period expectations are realized. Estimating a reservation price would typically require knowledge of the parameters of the (factor) demand function. But analyzing demand for natural resources simplifies this problem because—in a small open economy—the (country-level) alternative to extracting a resource is to import it at the world-market resource price. A natural value for the reservation unit rent will thus be resource price, i.e. the cost of importing the resource.

Implementing this method, we find that existing measures of comparative productivity—such as in PWT—are substantially biased in countries where natural resource rents account for a sizeable share of GDP. This is a relatively modest group of countries; for example, only 11 of the 116 countries have a resource rent share of 20 percent of GDP or more. In that group of 11 countries, the average bias in productivity levels (relative to the US) is 36 percent. If one relies on existing productivity measures, countries that have a higher resource rent share show up as more productive. Based on our new measure of productivity that accounts for inputs of natural resources, this is no longer the case. Put differently, resource-rich countries would traditionally show up as uncommonly productive, but this is the result of biased productivity measurement. Our new productivity measure thus more closely approximates a residual measure of cross-country income differences that cannot be accounted for using observable inputs.

The methodology of producer reservation prices we have introduced is relevant beyond the scope of natural resources as missing-goods problems occur in other productivity-measurement settings, too. For instance, microprocessor manufacturing is highly concentrated in a few countries and competition from low-wage countries may mean that in industries such as garment manufacturing, the products produced in high-wage countries are substantially different than in low-wage countries. This issue has so far been ignored in the cross-country industry productivity comparison literature6 and can likely be addressed using the producer reservation price tools introduced in this paper.

2. Methodology

In this section we modify the approach for productivity measurement introduced by Diewert and Morrison (1986)—and most recently presented in Inklaar and Diewert (2016)—to a setting where some of the primary input factors are not used by all production units—countries in our setting. We introduce the concept of producer reservation prices and adapt the Diewert/Morrison index-number approach to allow this concept to be implemented.

Suppose that we can observe \( K \) production units. Assume that the technology set available to unit \( k \) is the set \( S^k \) for \( k = 1, \ldots, K \). The observed \( M \)-dimensional vector of net outputs for unit \( k \) is \( y^k \equiv (y^k_1, \ldots, y^k_M) \). If \( y^k_m \) is an output that is being produced by unit \( k \), then \( y^k_m > 0 \), if it is an input used by unit \( k \) then \( y^k_m < 0 \). The primary inputs used by the production units in the sample are broken up into two

---

6See e.g. Inklaar and Timmer (2009) or Jorgenson et al. (2016).

7In the Diewert/Morrison framework, the \( S^k \) are closed, convex cones with free disposability of inputs and outputs. This setup can easily be generalized to cover not only multiple production units but also multiple periods, see Inklaar and Diewert (2016).

© 2020 The Authors. *Review of Income and Wealth* published by John Wiley & Sons Ltd on behalf of International Association for Research in Income and Wealth
groups. The Group 1 input vector for production unit \( k \) in period \( t \) is 
\[ x^k \equiv (x^k_1, \ldots, x^k_I) \succ 0_I, \]
a strictly positive vector, which means the Group 1 inputs are 
used by all production units in the productivity comparison. The Group 2 input 
vector for production unit \( k \) is 
\[ z^k \equiv (z^k_1, \ldots, z^k_J) \geq 0_J, \]
a nonnegative vector, which means the Group 2 inputs contain the missing inputs.

In the standard Diewert-Morrison formulation, we consider the value-added 
function for unit \( k \), 
\[ \pi^k(p,x,z) \] 
for \( p \succ 0_M \):

(1) 
\[ \pi^k(p,x,z) \equiv \max_y \{ p \cdot y : (y,x,z) \in S^k \} ; k = 1, \ldots, K \]

We assume that each \( \pi^k(p,x,z) \) is differentiable with respect to its arguments 
when evaluated at the data for unit \( k \). Suppose further that production 
unit \( k \) maximizes value added when facing the observed net output prices 
\( p^k \equiv (p^k_1, \ldots, p^k_M) \succ 0_M \) conditional on having available the Group 1 and 2 vectors 
of primary inputs, \( x^k \) and \( z^k \). Finally, we suppose that unit \( k \) faces the vector of 
Group 1 primary inputs, \( w^k \equiv (w^k_1, \ldots, w^k_I) \). Using Hotelling’s (1932; 594) Lemma, we 
have the following relationship between the observed net output vector \( y^k \) and the 
partial derivatives of \( \pi^k(p,x,z) \) with respect to the components of \( p^k \):

(2) 
\[ y^k = \nabla_p \pi^k(p^k,x^k,z^k) \]

Using Samuelson’s (1953, p. 10) Lemma, \(^9\) we have the following relationship 
between the observed Group 1 primary input price vector for unit \( k \), \( w^k \), and the 
partial derivatives of \( \pi^k(p,x,z) \) with respect to the components of \( x^k \):

(3) 
\[ w^k = \nabla_x \pi^k(p^k,x^k,z^k) \]

For the Group 2 primary inputs, the situation is more complex. If Group 2 
primary input \( j \) is being utilized by production unit \( k \), then let \( \omega^k_j > 0 \) be the 
(observed) price for that input. Samuelson’s Lemma can be applied to these utilized 
Group 2 inputs and so the following equations will be satisfied:

(4) 
\[ \omega^k_j = \frac{\partial \pi^k(p^k,x^k,z^k)}{\partial z^k_j} , \text{ with } j \text{ such that } z^k_j > 0 \]

Equations (2), (3) and (4) can be used as a system of estimating equations if 
the \( \pi^k \) are given specific functional forms that can be estimated. Once these estimated 
functions are available, then the Hicksian reservation price for the Group 2

---

\(^8\)We are assuming that all of the output and intermediate input prices are strictly positive. If there 
are missing outputs or missing intermediate inputs, we need to estimate positive Hicksian reservation 
prices for these missing outputs and inputs. We assume that this has been done and these positive reservation 
prices for the missing outputs and intermediate inputs are included in the strictly positive \( p \) vector. See the next section for our approach to determining the Hicksian reservation prices for the zero 
components of the \( z \) vector.

\(^9\)See also Diewert (1974; p. 140) for a proof of the result.
inputs $j$ that are missing for unit $k$ are determined from equation (4), i.e. the price at which demand for input $j$ in unit $k$ equals zero.

Now it would seem that we can simply apply the Diewert-Morrison exact index number method for estimating productivity differences between any two units using observed prices and quantities for all net outputs and primary inputs that are being used along with the estimated Hicksian reservation prices for the primary inputs that are not being used; i.e. use reservation prices for the inputs that are missing. However, when implementing the Diewert-Morrison methodology, the standard assumption is that the production function is translog, so that relative net outputs can be computed using Törnqvist-Theil price indexes and relative factor inputs using Törnqvist-Theil quantity indexes. Yet when inputs are missing, this would require taking the logarithm of a zero quantity.

A solution to this problem is to shift the Group 2 primary inputs in the intermediate input category; i.e., treat the Group 2 inputs as negative net outputs. This leads us to define the following \textit{modified value-added function}, $\alpha^k(p, \omega, x)$, for unit $k$ where the net output price vectors $p$ is strictly positive and the input price vector $\omega$ is also strictly positive:

$$
\alpha^k(p, \omega, x) \equiv \max_{y \succeq z \in S^k} \left\{ p \cdot y - \omega \cdot z : (y, x, z) \in S^k \right\}; k = 1, \ldots, K
$$

The productivity concept for \textit{modified value added} will be relative to the Group 1 primary inputs $x$ instead of the whole range of primary inputs.

Using Hotelling’s Lemma, we have the following relationship between the observed net output vector for unit $k$, $y^k$, and the partial derivatives of $\alpha^k(p^k, \omega^k, x^k)$ with respect to the components of $p^k$:

$$
y^k = \nabla_p \alpha^k(p^k, \omega^k, x^k)
$$

Using Samuelson’s Lemma, we have the following relationship between the observed Group 1 primary input price vector for unit $k$, $w^k$, and the partial derivatives of $\alpha^k(p^k, \omega^k, x^k)$ with respect to the components of $x^k$:

$$
w^k = \nabla_x \alpha^k(p^k, \omega^k, x^k)
$$

For the Group 2 primary inputs, we can again distinguish two situations. If Group 2 primary input $j$ is being utilized by production unit $k$, then, as before, $\omega_j^k > 0$ is the observed price for that input and Hotelling’s Lemma can be applied and the following equations will be satisfied:

$$
-z_j^k = \frac{\partial \alpha^k(p^k, \omega^k, x^k)}{\partial \omega_j}, \text{ with } j \text{ such that } z_j^k > 0
$$
For a missing Group 2 input, i.e. \( z^k_j = 0 \), the corresponding price \( o^k_j \) is a reservation price, which is not observed but could be estimated using the following equation:

\[
(9) \quad 0 = \frac{\partial \alpha^k(p^k, o^k, x^k)}{\partial o_j}, \text{ with } j \text{ such that } z^k_j = 0
\]

Rather than explicitly solving equation (9), we will instead choose an approximation to the reservation price. Our main argument will be that the alternative to extracting and processing a natural resource domestically will be to buy it on the world market and pay the world market price to import the metal, oil or gas—the next section discusses this approximation in more detail.

Given reservation prices, we follow Diewert and Morrison (1986) and Inklaar and Diewert (2016) and compare productivity across countries. In doing so, we assume that the modified value-added function of equation (5) has a translog functional form with constant returns to scale and constant parameters on the second-order terms. Under that assumption, we can use Törnqvist-Theil indexes to construct output, input and productivity indexes.

Define the value of each net output as \( v^k_m \equiv p^k_m y^k_m \) for each unit \( k \) and net output \( m = 1, \ldots, M \). Likewise, the value of each input in Group 1 is \( V^k_i \equiv w^k_i x^k_i \), for each input factor \( i = 1, \ldots, I \). The value of each input in Group 2 is \( c^k_j = o^k_j z^k_j \) for each input \( j = 1, \ldots, J \). Having defined these values, the share of each net output (input factor) in the value of total country net outputs (input factors) can be defined as:

\[
(10) \quad s^k_m \equiv \frac{v^k_m}{v^k - c^k}
\]

\[
(11) \quad \sigma^k_j \equiv \frac{c^k_j}{v^k - c^k}
\]

\[
(12) \quad S^k_i \equiv \frac{V^k_i}{V^k}
\]

where \( v^k \equiv \sum_{m=1}^{M} v^k_m \), \( c^k \equiv \sum_{j=1}^{J} c^k_j \) and \( V^k \equiv \sum_{i=1}^{I} V^k_i \) are the total value of net outputs and input factors for each country \( k \). Since we are implementing the modified value-added function of equation (5), the Group 2 inputs, which include missing inputs, are treated as part of net output and thus enter in the denominator with a negative sign. By construction, we ensure that \( v^k - c^k \equiv V^k \) to be consistent with the assumption of constant returns to scale. Next define the cross-country arithmetic averages of the shares in equations (10)-(12) as \( s_m = \frac{1}{K} \sum_{k=1}^{K} s^k_m \), \( \sigma_j = \frac{1}{K} \sum_{k=1}^{K} \sigma^k_j \) and
$S_j = \frac{1}{K} \sum_{k=1}^{K} S^k_j$. These average shares, as well as the average prices and quantities, will allow for base-country invariant comparisons of output, inputs and productivity.

The price level for modified value added is a Törnqvist-Theil index of net output prices and Group 2 input prices—either observed or reservation prices:

$$\ln p^k = \sum_{m=1}^{M} \frac{1}{2} (s_{m} + s^k_{m}) \ln \left( \frac{p^k_{m}}{p_{m}} \right) - \sum_{j=1}^{J} \frac{1}{2} (\sigma_j + \sigma^j_k) \ln \left( \frac{\omega^k_{j}}{\omega_{m}} \right)$$

Here $\ln p_{m} \equiv \frac{1}{K} \sum_{k=1}^{K} \ln p^k_{m}$ and $\ln \omega_{m} \equiv \frac{1}{K} \sum_{k=1}^{K} \ln \omega^k_{m}$. Prices in equation (13) are expressed relative to a (hypothetical) “average” country. In further analysis, it is common to express the price level of equation (13) with respect to a reference country, such as the United States, i.e. $P^k / P^{USA}$. Given the price level from equation (13), real modified value added $Y^k$ is equal to:

$$Y^k = (v^k - c^k) / P^k$$

The computation of real factor inputs is broadly analogous, but rather than an aggregate of relative prices, these are computed as a weighted average of relative quantities:

$$\ln X^k = \sum_{i=1}^{I} \frac{1}{2} (S_{i} + S^k_{i}) \ln \left( \frac{x^k_{i}}{x_{i}} \right)$$

Here $\ln x_{i} \equiv \frac{1}{K} \sum_{k=1}^{K} \ln x^k_{i}$. The productivity level of unit $k$ is then the ratio of equations (14) and (15):

$$\Gamma^k = \frac{Y^k}{X^k}$$

To prepare for our empirical illustration, it is helpful to contrast the productivity measure in equation (16) with the measure that is currently used in the Penn World Table (PWT), see Feenstra et al. (2015). That productivity measure is based on the same Diewert-Morrison theoretical framework, but omits Group 2 inputs, the components of $z$:10

$$\tilde{\Gamma}^k = \frac{\tilde{Y}^k}{\tilde{X}^k} = \left[ \frac{v^k / \exp \left( \sum_{m} \frac{1}{2} (\tilde{s}_{m} + \tilde{s}^k_{m}) \ln \left( \frac{\tilde{p}_{m}}{p_{m}} \right) \right) }{\exp \left( \sum_{i} \frac{1}{2} (\tilde{s}^k_{i}) \ln \left( \frac{x^k_{i}}{x_{i}} \right) \right) } \right]$$

10The PWT productivity measure $CTFP$ is based on a bilateral comparison with the United States rather than a multilateral comparison. For a clearer comparison, our biased, PWT-type measure will be the multilateral productivity measure defined in equation (17).
This omission has several implications. Equation (11), the second term on the right-hand side of equation (13), and the $c^k$ in equation (14) drop out. Furthermore, the shares in equation (10) are redefined to add up to one without input costs $c$, label these $\tilde{S}^k_m$. More subtly, the adding-up constraint of nominal net output and input costs changes to $v^k \equiv \tilde{V}^k$. In practice, some input costs are readily observable, think of labor compensation of workers. That leaves the cost of produced capital, which will be assigned the residual of total input costs after subtracting the readily-observable input costs, so $c^k$ is added to the costs of produced capital. This leads to an overstatement of the produced-capital share so some of the $\tilde{S}^k = v^k / \tilde{V}^k$ will be too large, and some will be too small.

It is thus clear that $\tilde{\Gamma}^k$ from equation (17) is biased relative to the true $\Gamma^k$ from equation (16). The size of the bias will depend on the on the importance of Group 2 income in nominal value added, $\sum_j \sigma_j^k$, in the countries under comparison. Where $\sum_j \sigma_j^k$ is small, the second term on the right-hand side of equation (13) can be small (depending on the $\sigma_j$), real value added in equation (14) will be similar with or without $c^k$ and $\tilde{S}^k_j \approx \tilde{S}^k_j$, so equation (15) based on either set of cost shares will be very similar. If $\sum_j \sigma_j^k$ is not small, there will be a bias in real value added in equation (14), from both the numerator $(v^k - c^k)$ and the denominator $P^k$. The bias in real input levels from equation (15) need not be large. If the $x^k_i$ between two countries under comparison are similar, the bias in cost shares will not have a large effect on overall real input levels.

The direction of the bias depends on the reference country. In our results, we will use the United States as the reference country, which means that productivity in any country that relies less on natural resources than the United States, i.e. $\sum_j \sigma_j^k < \sum_j \sigma_j^{USA}$, will tend to be biased downwards when ignoring natural resources and it will typically be biased upwards when $\sum_j \sigma_j^k > \sum_j \sigma_j^{USA}$. Yet given that both real output and real inputs in equation (17) differ from those in equation (16), the direction of the bias cannot be predicted with certainty from the $\sum_j \sigma_j^k$ s.

3. Data

To measure productivity, as laid out in equations (10)-(16), requires data on values of net output and inputs, relative prices, and quantities. Although the general measurement framework applies for any type of units, our focus is on comparing country productivity levels and we want to compare resource-intensive and non-resource-intensive countries as well as countries at different income levels. Thus, the starting point for the dataset is the Penn World Table (PWT), version 9.0, by Feenstra et al. (2015). The dataset we compile for the analysis in this paper is for the year 2011, the latest year for which direct observations on GDP prices are available, based in large part on World Bank (2015). For 2011, complete data can be compiled for 116 countries, including most of the resource-intensive countries, such as oil- and gas-rich countries in the Middle East, but also mineral-rich countries such as Mauritania and Mongolia.
From PWT, the value of net output $v_k$ equals nominal GDP. The price level of net output, the aggregate over $m$ of $p_{nk}$, is the purchasing power parity (PPP) for GDP from PWT. This variable is not constructed as in the first term of equation (13) because the available price data are for final expenditure items, rather than for industry net outputs. Yet at the level of GDP, total final expenditure (consumption plus investment plus export minus imports) equals total net output $v_k$ and Feenstra et al. (2015) detail how the price measurement in PWT arrives at the same conceptual outcome as net output in the Dievert-Morrison framework.

Factor inputs $x$ consist of labor and produced capital, so $I = 2$. PWT contains information on the share of labor income in GDP $(V_{k1}/v_k)$\textsuperscript{11}. The income flowing to owners of produced capital will be determined as a residual, $V_{k2} = V_k - V_{k1}$. The quantity of labor input, $x_{k1}$ is based on PWT and measured as total hours worked by all persons engaged, adjusted for differences in educational attainment. The educational attainment adjustment follows Caselli (2005) and is based on the average years of schooling in a country and the (Mincerian) return to education in terms of higher wages. Data on average hours worked is not available from PWT for all countries; where these data are missing, we assume the cross-country mean of average hours worked. The quantity of produced capital, $x_{k2}$ is also from PWT. This measure is constructed based on investment by type of asset, adding up to gross fixed capital formation from the National Accounts. Nine asset types are distinguished and the perpetual inventory method with asset-specific depreciation rates is used to construct capital stocks.\textsuperscript{12} The current-cost capital stocks are converted to real stocks using a (current-cost capital stock) weighted average of asset-specific PPPs for investment products, from the same data underlying the GDP PPPs. We follow Feenstra et al. (2015) in this measurement, which is certainly subject to caveats,\textsuperscript{13} but we focus on the bias in measured productivity from not including natural resources, leaving constant the measurement of other factors.

The source of data on natural resources is Lange et al. (2018), whose data cover 15 subsoil assets, consisting of 10 mineral assets and 5 energy assets. The quantity of inputs used, $z_{jk}$ is equal to the production of each asset\textsuperscript{14} and Table 1 shows for each of the 15 assets how many countries show positive production levels. Mining of gold, gas and oil are relatively widespread, taking place in 70–74 of the 116 countries, while the other assets are produced in a minority of countries. The median number of assets produced by a country is 4 and 10 of the countries

\textsuperscript{11}Labour share data are missing for the United Arab Emirates, but due to its resource-intensity, we add it to the dataset assuming $S^{e L}_{k1}=0.5$, which is comparable to countries in the region. The labour share for Togo in PWT is 85 percent and its resource rent share in GDP is 20 percent, implying negative shares for produced capital. We set $S^{e L}_{k2}=0.95$ to reflect its high labour intensity, while ensuring positive income to produced capital.

\textsuperscript{12}The nine assets are computers, communication equipment, other machinery, transport equipment, residential buildings, other structures, software, other intellectual property products and cultivated assets. Land and inventories are not covered.

\textsuperscript{13}It would, for instance, be preferable to include the nine capital assets as separate factor inputs rather than an aggregated stock; see e.g. Inklaar and Timmer (2009).

\textsuperscript{14}This production is measured in terms of (tons of) metal or coal, not in terms of ore mined, i.e. production includes the processing of ore. For oil, the production is in barrels of crude oil. For gas, it is in terajoules.
have no subsoil asset production at all. This clearly illustrates the missing input problem that our method sets out to address.

The Lange et al. (2018) data also provide information on the $\omega_j^k$, the input prices. In line with Brandt et al. (2017), the relevant input price for each subsoil asset is taken as the unit rent, the price earned selling the mineral or energy products minus the production costs. As Diewert and Fox (2016) demonstrate, the unit rent is equivalent to the rental price of natural resources when beginning-of-period expectations are realized, thereby providing a price concept that corresponds with standard production theory.

The Lange et al. (2018) data are the only comprehensive global source on resource production, price and production costs, but it is important to note that the available basic data differ by variable, see Lange et al. (2018, Table A1, p. 214). Production statistics are typically available by country from the International Energy Agency, but other sources are also employed. For many resources, a world resource price is provided in the database, which can be justified from the perspective that these are mostly homogenous products, so price variation should be limited. Unit production costs are typically not available for every country and resource type but instead sources are described as, for example, “country-specific case studies from various sources; assumed to be representative for the region.” This suggests that the largest weakness of this sources is that the data likely understate the variation in unit rents, but it is unclear whether that would lead to a systematic bias in the productivity measures.

The formal criterion for determining the reservation unit rents is that equation (9) should be satisfied, i.e. the reservation unit rent should be such that the optimal $z^* = 0$. Estimating factor demand and deriving the unit rent by setting demand equal to zero would entail substantial econometric complications, so instead, we proceed with the following reasoning. Factor demand for natural resources is not about whether, say, oil is used in a country, but instead whether oil is extracted in a country. When extracted, the price for that input is equal to the unit rent (when beginning-of-period expectations are realized). The alternative to extraction is importing the mineral or fuel, in which case the price is the resource price on the world market (abstracting from trade costs).

### Table 1

| Mineral assets | # of countries | Energy assets | # of countries |
|----------------|----------------|---------------|----------------|
| Bauxite        | 22             | Brown coal    | 29             |
| Copper         | 39             | Coking coal   | 22             |
| Gold           | 74             | Thermal coal  | 41             |
| Iron ore       | 42             | Gas           | 71             |
| Lead           | 33             | Oil           | 73             |
| Nickel         | 19             |               |                |
| Phosphate      | 34             |               |                |
| Silver         | 51             |               |                |
| Tin            | 16             |               |                |
| Zinc           | 39             |               |                |

*Notes: The total number of countries is 116.*
The production sector in country $k$ has a choice between importing one unit of, say, metal $j$ at the world price of $R_j$ per unit of metal or extracting the mineral from a domestic mine and refining it into metal. The per-unit metal cost of the mining and processing of ore $j$ is, say, $C^k_j$ per unit of final metal. If $C^k_j$ is larger than the corresponding world (import) price $R_j$, then no ore will be mined. If $C^k_j$ is less than the world (import) price $R_j$, then the ore will be mined, and the rent earned by the production sector of country $k$ will be $u^k_j = R_j - C^k_j$. In the limit, no ore of type $j$ will be mined by country $k$ if $C^k_j = R_j$. Thus, in this case, ore will not be mined and the Hicksian producer reservation price for the natural resource input will be $R_j$.

This argument leads us to define the factor price for ore type $j$ in country $k$, $\omega^k_j$, as follows:

$$
\omega^k_j = \begin{cases} 
  u^k_j = R_j - C^k_j & \text{if } R_j > C^k_j \\
  R_j & \text{if } R_j \leq C^k_j
\end{cases}
$$

where $u^k_j$ is the unit rent, $R_j$ is the world price of resource $j$ and $C^k_j$ is the cost of extracting resource $j$ in country $k$. Since $C^k_j$ is not observed when production of a resource equals zero, we operationalize equation (17) as $\omega^k_j = \min \left( u^k_j, R_j \right)$.$^{15}$

4. Results

Given factor prices and resource production, we can compute resource rents as a share of GDP, $\omega^k_j / v^k$, to illustrate the importance of natural resources. This provides a first indication of the number of countries where we would expect to see a notable bias in their productivity level when omitting natural resources. In Table 2 we group countries by their resource rent share in GDP and this shows that 50 of

| Range                  | # of countries |
|------------------------|----------------|
| <1%                    | 50             |
| ≥1% and <5%            | 29             |
| ≥5% and <10%           | 12             |
| ≥10% and <20%          | 14             |
| ≥20%                   | 11             |

Notes: The total number of countries is 116.

TABLE 2
CROSS-COUNTRY DISTRIBUTION OF RESOURCE RENTS AS A SHARE OF GDP

For most resources, only a single world price is given in the Lange et al. (2018) data. For gas, prices differ by location, between $4037 and $6518 per Terajoule and for oil prices differ by type of benchmark, between $86 and $107 per barrel. In countries with zero production of these assets, we set the world price equal to the maximum observed resource price to reflect (especially in the case of gas) that non-producing countries will face higher prices due to transhipment fees for pipelines or from shipping facilities, such as LNG plants. Productivity levels when selecting the minimum price level instead of the maximum are very similar.

© 2020 The Authors. *Review of Income and Wealth* published by John Wiley & Sons Ltd on behalf of International Association for Research in Income and Wealth
### TABLE 3
Comparing productivity measures for the top-11 resource-intensive countries

| Resource rent share (% of GDP) | Productivity level (USA=1) |
|-------------------------------|-----------------------------|
|                               | Including natural resources, $\Gamma^k$ | Excluding natural resources, $\bar{\Gamma}^k$ | Bias, $\bar{\Gamma}^k / \Gamma^k - 1$ |
| Kuwait                        | 61                           | 1.15                                      | 1.80                                      | 0.57                                      |
| Iraq                          | 58                           | 0.86                                      | 1.24                                      | 0.45                                      |
| Saudi Arabia                  | 50                           | 0.97                                      | 1.11                                      | 0.15                                      |
| Mauritania                    | 47                           | 0.35                                      | 0.59                                      | 0.70                                      |
| Gabon                         | 40                           | 0.60                                      | 0.89                                      | 0.48                                      |
| Qatar                         | 38                           | 1.15                                      | 1.44                                      | 0.25                                      |
| United Arab Emirates          | 28                           | 0.80                                      | 0.91                                      | 0.13                                      |
| Iran                          | 26                           | 0.87                                      | 0.95                                      | 0.10                                      |
| Kazakhstan                    | 25                           | 0.64                                      | 0.86                                      | 0.33                                      |
| Mongolia                      | 24                           | 0.35                                      | 0.42                                      | 0.18                                      |
| Togo                          | 20                           | 0.12                                      | 0.16                                      | 0.31                                      |
| Average                       | 38                           | 0.69                                      | 0.94                                      | 0.36                                      |

**Notes:** Included are all countries where natural resource rents contribute 20 percent or more to GDP. The resource rent share does not match the data in the World Development Indicators, because our set of natural resources only covers subsoil assets and excludes agricultural land and forests under cultivation.
the 116 countries have a share of less than one percent of GDP. Even the United States, despite all attention to the boom in shale oil and gas, falls in the “less than 1%” group. For 11 countries, the share of resource rents is higher than 20 percent of GDP and these include high-income oil-rich countries in the Middle East such as Iran, Iraq, Kuwait, Saudi Arabia and Qatar, but also resource-dependent countries with much lower income levels such as Mauritania, Togo and Mongolia (see Table 3). Another way of illustrating the concentration of resource rents is to note that the top-11 countries in terms of (nominal) resource rents earned account for 80 percent of global resource rents and the bottom-80 countries account for less than 4 percent.

To illustrate the impact of accounting for the input of natural resources on measured productivity, we compute the bias in the relative productivity measure based on equations (16) and (17), 
$$
\text{Bias} \equiv \frac{\tilde{\Gamma}^k}{\Gamma^k} - 1. 
$$
In Figure 1, we plot this bias measure against the share of natural resource rents in GDP. As discussed in the methodology section, the bias will tend to be negative when a country’s resource rent share is smaller than in the United States, the reference country, and Figure 1 shows that is the case for all countries with smaller resource rent shares. Of the 67 countries with larger resource rent shares, 29 also show a negative bias though it is typically smaller in size. Most notable in this figure are the countries with high resource rent shares, in the 20 percent or higher group from Table 2. The bias in relative productivity exceeds 10 percent and even reaches more than 50 percent in Iraq, Mauritania and Kuwait.

Table 3 shows the results for these resource-intensive countries in more detail, listing the 11 countries in descending order of the resource rent share. The subsequent columns show the newly-developed productivity measure $\Gamma^k$ (see equation
(16)) that incorporates natural resources, the measure $\bar{F}^k$ that excludes natural resources (equation (17)) and the measure of bias charted in Figure 1 is shown in the final column of the table. The table shows that the impact on productivity levels is particularly striking in countries where productivity levels exceeded those in the United States when not accounting for natural resources: Kuwait’s productivity level decreases from 180 percent to 115 percent of the US level, Iraq’s from 124 to 86, Saudi Arabia from 111 to 97 and Qatar’s from 144 to 115. More broadly, the average bias across these countries is 36 percent, a substantial adjustment.

As Figure 1 demonstrated, the impact on productivity of accounting for the input of natural resources is most notable for the 11 countries highlighted in that figure and shown in Table 3 and more modest effects for the other 105 countries. A consequence is that the inclusion of natural resource has a very limited impact on the broader discussion of development accounting. Development accounting assess the degree to which we can account for income differences through measured inputs of human, produced and (now) natural resource capital. This degree can be established, for instance, by regressing (log) productivity levels on (log) income levels. If the slope coefficient of that regression decreases when extending the set of inputs, more of the variation in income levels has been accounted for. Using our results, we find a slope coefficient of 0.335 for both of the productivity measures. Another view on this is that the bias in measured productivity is higher in countries with a higher resource rent share (see Figure 1) and the correlation between the resource rent share and (log) GDP per capita is only 0.07.
Given that more general result, Figure 2, shows how the position of the 11 most resource-intensive countries changes when accounting for natural resources. The red solid dots show the productivity levels excluding natural resources and the solid blue dots the productivity levels including natural resources. Not only do all 11 solid dots move down, but in 7 of the 11 cases, they move closer to the regression line. This is a more general result: in the regression of productivity levels, excluding natural resources, on income levels, the residuals show a correlation of 0.44 with the resource rent share. In the regression with productivity levels including natural resources, this correlation is −0.03. If we focus on the 105 countries with lower levels of resource rent shares, the correlation decreases from 0.25 to −0.04. Put differently, resource-rich countries used to be uncommonly productive, but after accounting for inputs of natural resources, that is no longer the case.

5. Conclusions

The measurement discussion associated with “missing goods” has typically concentrated on consumer inflation. Prices of newly-introduced products often decline, but the biggest decline may occur at the point of introduction as a previously unobtainable product is suddenly within reach of consumers. To determine the reservation price of this new good, one would typically need to resort to specifying consumer preferences and estimating consumer demand. In this paper, we analyzed a “missing goods” problem in the setting of cross-country productivity measurement. Mining and processing of subsoil assets such as oil, iron and gold does not occur in every single country, and for good reason: often mining will be a more expensive option than buying the metals or barrels of oil on the world market. This is an attractive feature of this particular case, since it allows us to identify the world price as the reservation price for the input of the natural resource.

Following this argument and adapting the standard cross-country productivity methods to incorporate natural resources, we find that traditional productivity measures—such as included in the Penn World Table—are severely biased for countries that rely heavily on inputs of natural resources, such as Qatar and Saudi Arabia, but also Mauritania and Togo. For these resource-intensive countries, the average bias was 36 percent. More generally, traditional productivity measures suggest that more resource-intensive countries are more productive. We show that this is no longer the case with our newly-developed productivity measure. This measure should thus be seen as a superior alternative to traditional productivity measures that omit natural resources from the set of inputs.

More broadly, this paper has focused on the fact that missing-goods problems can also occur in the context of productivity measurement instead of solely being a consumer inflation problem. In measuring prices over time, the aggregate effect of the missing-good problems can be limited as a new good is initially not consumed very intensively. Yet in comparing productivity across countries, there can be many countries where the good in question is not missing and we have shown that the bias from ignoring this problem can be substantial. We can think of several other situations where this may occur. For instance, some countries may have started investing in computers and software much sooner than others. But this
problem may be most severe for the output side of productivity accounts. For example, the dominant producer of microprocessor units is Intel and the firm only operates wafer fabrication sites in the United States, Ireland, Israel and China,16 while other countries specialize in different types of semiconductors. This problem can also occur in lower-tech industries, such as garments. High-wage countries still retain a garment industry and this industry may survive by focusing on higher-quality products but also by focusing on different products than firms in low-wage countries. These settings would also be amenable to the logic we employed in choosing reservation prices for natural resources, i.e. use the import price of the non-produced product as the reservation price. We hope this producer reservation price methodology can serve as a useful new tool for productivity researchers faced with missing-goods situations.

REFERENCES

Acemoglu, D., S. Naidu, P. Restrepo, and J. A. Robinson, “Democracy Does Cause Growth,” *Journal of Political Economy*, 127, 47–100, 2019.

Balk, B. M., *On Curing the CPI’s Substitution and New Goods Bias*, Department of Statistical Methods, Statistics Netherlands, Mimeograph, 1999.

Bloom, N., R. Sadun, and J. van Reenen, “Management as a Technology?” Working Paper, No. 16-133, Harvard Business School, Boston, MA, 2016.

Brandt, N., P. Schreyer, and V. Zipperer, “Productivity Measurement with Natural Capital,” *Review of Income and Wealth*, 63, S7–S21, 2017.

Caselli, F., Accounting for Cross-Country Income Differences in P. Aghion and S. N. Durlauf (eds), *Handbook of Economic Growth, Volume 1A*, Elsevier, 679–741, 2005.

Chen, W., “Cross-Country Income Differences Revisited: Accounting for the Role of Intangible Capital,” *Review of Income and Wealth*, 64, 626–648, 2018.

Diewert, W. E., “Applications of Duality Theory,” in M. D. Intriligator and D. A. Kendrick, (ed.), *Frontiers of Quantitative Economics, Volume 2*. North-Holland, Amsterdam, 106–171, 1974.

Diewert, W. E. and K. J. Fox, “The User Cost of Nonrenewable Resources and Green Accounting,” Discussion Paper 16-01, The University of British Columbia, Vancouver, 2016.

Diewert, W. E. and R. C. Feenstra, “Estimating the Benefits and Costs of New and Disappearing Products,” Discussion Paper 18-05, The University of British Columbia, Vancouver, 2018.

Diewert, W. E. and C. J. Morrison, “Adjusting Output and Productivity Indexes for Changes in the Terms of Trade,” *Economic Journal*, 96, 659–679, 1986.

Feenstra, R. C., “New Product Varieties and the Measurement of International Prices,” *American Economic Review*, 84, 157–177, 1994.

Feenstra, R. C., R. Inklaar, and M. P. Timmer, “The Next Generation of the Penn World Table,” *American Economic Review*, 105, 3150–3182, 2015.

Hotelling, H., “Edgeworth’s Taxation Paradox and the Nature of Demand and Supply Functions,” *Journal of Political Economy*, 40, 577–616, 1932.

Hicks, J. R., “The Valuation of the Social Income,” *Economica*, 7, 105–140, 1940.

Hsieh, C.-T., and P. J. Klenow, “Development Accounting,” *American Economic Journal: Macroeconomics*, 2, 207–223, 2010.

Inklaar, R. and W. E. Diewert, “Measuring Industry Productivity and Cross-Country Convergence,” *Journal of Econometrics*, 191, 426–433, 2016.

Inklaar, R. and M. Papakonstantinou, “Vintage Effects in Human capital: Europe Versus the United States,” *Review of Income and Wealth*, 2019. forthcoming, doi: 10.1111/roiw.12382.

Inklaar, R., and M. P. Timmer, “Productivity Convergence Across Industries and Countries: The Importance of Theory-Based Measurement,” *Macroeconomic Dynamics*, 13, 218–240, 2009.

Jorgenson, D. W., K. Nomura, and J. Samuels, “A Half Century of Trans-Pacific Competition: Price Level Indices and Productivity Gaps for Japanese and U.S. Industries, 1955–2012” in D.

16As documented at https://www.intel.com/content/www/us/en/architecture-and-technology/global-manufacturing.html, accessed on November 7, 2018.

© 2020 The Authors. *Review of Income and Wealth* published by John Wiley & Sons Ltd on behalf of International Association for Research in Income and Wealth
W. Jorgenson, K. Fukao, and M. P. Timmer, (eds), *The World Economy – Growth or Stagnation?* Cambridge University Press, Cambridge, Chapter 13, 469–507, 2016.

Lagakos, D., B. Moll, T. Porzio, N. Qian, and T. Schoellman, “Life Cycle Wage Growth Across Countries,” *Journal of Political Economy*, 126(2), 797–849, 2018.

Lange, G.-M., Q. Wodon, and K. Carey (eds), *The Changing Wealth of Nations 2018*, World Bank, Washington DC, 2018.

Redding, S. and D. Weinstein, “Measuring Aggregate Price Indexes with Taste Shocks: Theory and Evidence for CES Preferences,” *Quarterly Journal of Economics*, 2019, forthcoming.

Samuelson, P. A., “Prices of Factors and Goods in General Equilibrium,” *Review of Economic Studies*, 21, 1–20, 1953.

World Bank, *Purchasing Power Parities and the Real Size of World Economies: A Comprehensive Report of the 2011 International Comparison Program*. World Bank, Washington, DC, 2015.

**Supporting Information**

Additional supporting information may be found in the online version of this article at the publisher’s web site:

**Appendix**

**TABLE A.1:** Comparing productivity measures for 2011