The Low Temperature Resistance in Metals without Inversion Centre

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The well known quadratic low temperature dependence of resistance in ordinary metals habitually serves as the criterium of applicability of the Landau Fermi liquid theory to the description of electron liquid in concrete material. Such type behavior is determined by momentum relaxation due to the electron-electron scattering. Here I consider this problem in the metals without inversion centre. It is shown that corresponding scattering time is practically temperature independent. However, this does not mean any non-fermi-liquid behavior.

I. INTRODUCTION

Normal metal resistivity at low temperatures is usually described by the well known formula
\[ \rho = \rho_0 + AT^2. \]
This dependence derived by Landau and Pomeranchuk \cite{1} is determined by the relaxation time due to electron-electron scattering
\[ \tau_{ee} \propto \frac{1}{T^2}, \]
which is slower than the relaxation due to electron-phonon interaction at temperatures much lower than the Debye temperature
\[ \tau_{eph} \propto \frac{1}{T^5}. \]
The Landau-Pomeranchuk’s relaxation is caused by the interaction between electrons. They, owing to the Pauli principle, can scatter with each other only in a narrow energy layer of the order of temperature near the Fermi surface. The realization of dependence (1) in a concrete material is commonly used as the direct indication of applicability of the Landau Fermi-liquid theory. And vice-versa, a deviation from this dependence stimulates the search of some physical mechanism responsible for non-fermi-liquid behavior.

In neutral Fermi liquid like normal \textsuperscript{3}He the low temperature relaxation due quasiparticle-quasiparticle scattering has the same $1/T^2$ dependence what determines the temperature dependence of viscosity, thermal conductivity \cite{2, 3} and the longitudinal spin-diffusion coefficient \cite{4}. The latter takes place even in \textsuperscript{3}He polarized by an external magnetic field $H$ where the Fermi spheres of quasiparticles with opposite spins have different radii. Nevertheless, during and after the scattering processes spin-up and spin-down quasiparticles remain at distance of the order of temperature $T$ from their Fermi surfaces.

Quite different situation is realised for the spin-diffusion in the direction perpendicular to an external magnetic field \cite{5, 6}. Here, the scattering processes involve all the states between two Fermi surfaces. And the relaxation time acquires the following temperature dependence
\[ \tau_\perp \propto \frac{1}{(2\pi T)^2 + (gH)^2}, \]
where $g$ is the \textsuperscript{3}He nuclei gyromagnetic ratio. This result has been obtained \cite{7} making use the kinetic equation for the matrix distribution function of Fermi particles \cite{5, 8, 9} in frame of Landau Fermi-liquid theory and does not originate no matter which violation of it.

Recently there was appeared a lot of interest to substances without inversion centre \cite{10}. This case the spin-orbit coupling lifts the spin degeneracy of the electron energy bands. The initial Fermi surface splits on two Fermi surfaces for the electrons characterized by the opposite helicity that is the spin projection on the direction of the momentum dependent vector of spin-orbit coupling. The difference of the Fermi momenta of these Fermi surfaces is determined by the value of spin-orbital coupling $\gamma$. Similar to the transverse spin relaxation in the spin-polarized \textsuperscript{3}He one can expect that in non-centrosymmetric metals the relaxation time due to electro-electron interaction have the following form
\[ \tau_{ee} \propto \frac{1}{(2\pi T)^2 + \gamma^2}. \]
Indeed, this is the case for the transverse spin waves attenuation \( [11] \).

Here we derive the formula for low temperature resistivity in metals without inversion centre determined by the momentum relaxation due to the electron-electron scattering. There is shown that it is also determined by equation similar to Eq.\( [10] \). The spin-orbital band splitting \( \gamma \) is directly expressed through the corresponding splitting of de Haas - van Alphen magnetization oscillation frequencies \( [12] \). Determined experimentally the typical magnitude of band splitting in many non-centrosymmetric metals is of the order of hundreds Kelvin \( [13–15] \). This is much less than the Fermi energy but comparable with the Debye temperature. Hence at temperatures lower than the Debye temperature the \( T^2 \) term in denominator of Eq.\( [10] \) is proved to be unobservable. However, this does not point on no matter which deviation from the Landau Fermi-liquid description of a concrete material.

The paper is organized as follows. In Section II the formula for the low temperature conductivity in non-centrosymmetric metals is derived. The solution of the kinetic equation used in this derivation is obtained in the Appendix at the end of the paper. The conclusion is given in the Section III.

II. CONDUCTIVITY

The spectrum of non-interacting electrons in a metal without inversion center is:

\[
\hat{\epsilon}(k) - \mu \hat{1} = \xi(k) \hat{1} + \gamma(k) \cdot \sigma
\]  \( (6) \)

where \( \xi(k) = \varepsilon(k) - \mu \) denotes the spin-independent part of the spectrum measured relative to the chemical potential \( \mu \), \( \hat{1} \) is the unit \( 2 \times 2 \) matrix in the spin space, \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) are the Pauli matrices. The second term in Eq. \( (6) \) describes the spin-orbit coupling whose form depends on the specific non-centrosymmetric crystal structure. The pseudovector \( \gamma(k) \) satisfies \( \gamma(-k) = -\gamma(k) \) and \( g \gamma(g^{-1}k) = \gamma(k) \), where \( g \) is any symmetry operation in the point group \( G \) of the crystal (more detailed theoretical description of non-centrosymmetric metals presented in the paper \( [10] \)). We shall work with isotropic spectrum \( \varepsilon(k) = \frac{k^2}{2m} \) and

\[
\gamma(k) = \gamma k, \tag{7}
\]

compatible with the 3D cubic crystal symmetry. Here \( \gamma \) is a constant. The calculations with anisotropic spectrum and \( \gamma(k) \) corresponding to other crystal symmetries are much more cumbersome but do not qualitatively change the results.

The eigenvalues of the matrix \( (6) \) are

\[
\xi_{\pm}(k) = \xi(k) \pm |\gamma(k)| - \mu. \tag{8}
\]

There are two Fermi surfaces determined by the equations

\[
\xi_{\pm}(k) = 0 \tag{9}
\]

with different Fermi momenta

\[
k_{F\pm} = \mp m \gamma + \sqrt{2m \mu + (m \gamma)^2} \tag{10}
\]

and common value of the Fermi velocity

\[
v_F = \frac{\partial (\varepsilon \pm \gamma k)}{\partial k} \bigg|_{k = k_{F\pm}} = \hat{k} \sqrt{\frac{2 \mu}{m} + \gamma^2}, \tag{11}
\]

here \( \hat{k} \) is the unit vector along momentum \( k \).

The matrix of equilibrium electron distribution function is

\[
\hat{n}_0 = \frac{n_+ + n_-}{2} \hat{1} + \frac{n_+ - n_-}{2|\gamma|} \gamma \cdot \sigma, \tag{12}
\]

where

\[
n_{\pm} = \frac{1}{e^{\xi_{\pm}} + 1} \tag{13}
\]

are the Fermi functions. Near the corresponding Fermi surfaces the dispersion law is

\[
\xi_{\pm} \approx v_F (k - k_{F\pm}) = \varepsilon - \mu_{\pm}, \tag{14}
\]
with
\[ \varepsilon = v_F k, \quad \mu_\pm = v_F k_F \pm, \quad \mu_+ - \mu_- = -2m v_F \gamma. \]

The matrix of non-equilibrium distribution function is the sum of the scalar and the spinor parts
\[ n = \frac{1}{2} \left[ f \hat{1} + g \cdot \sigma \right]. \]

The current density is
\[ j = e T r \int \frac{d^3 k_1}{(2\pi)^3} \frac{\partial \bar{\varepsilon}(k_1)}{\partial k_1} n = \frac{1}{2} T r \int \frac{d^3 k_1}{(2\pi)^3} \left( \frac{\partial \xi(k_1)}{\partial k_1} + \frac{\partial \gamma(k_1) \cdot \sigma}{\partial k_1} \right) (\delta f(k_1) \hat{1} + \delta g(k_1) \cdot \sigma) \]
\[ = e \int \frac{d^3 k_1}{(2\pi)^3} \left( \frac{k_1}{m} \delta f(k_1) + \gamma \delta g(k_1) \right). \]

Here, \( \delta f(k_1) \) and \( \delta g(k_1) \) are the deviations of scalar and spinor distribution functions from the equilibrium distribution function. The derivation of them from the kinetic equation is presented in the Appendix. They are
\[ \delta f(k_1) = -\frac{4e}{\pi W_0 m^3 I_1(\varepsilon_1)} E \cdot v_1 \frac{\partial (n_1 + n_{1-})}{\partial \varepsilon_1}, \]
\[ \delta g(k_1) = -\frac{4e}{\pi m^3 [W_0 I_1(\varepsilon_1) + W_0 I_2]} \left[ E \cdot v_1 \frac{\partial (n_1 - n_{1-})}{\partial \varepsilon_1} k_1 + \frac{(n_1 + n_{1-})}{|k_1|} (E - (E \cdot k_1) k_1) \right]. \]

where the integrals \( I_1(\varepsilon_1), I_2 \) are given by Eqs. (A20) and (A29). Substituting these expressions in Eq. (17) we obtain
\[ j = \frac{4e^2}{3\pi m^3} \left\{ v_F \frac{N_{0+} k_F + N_{0-} k_F}{m W_0 I_1(\mu_+)} \right\} E, \]
\[ + \gamma \left[ \frac{N_{0+} v_F}{W_0 I_1(\mu_+) + W_0 I_2} - \frac{N_{0-} v_F}{W_0 I_1(\mu_-) + W_0 I_2} - 2 \int \frac{d^3 k_1}{(2\pi)^3} k_1 \frac{n_1 + n_{1-}}{W_0 I_1(\varepsilon_1) + W_0 I_2} \right] E, \]

where \( N_{0\pm} = \sqrt{2m^3 \mu_\pm / 2\pi^2} \) are the density of states at the corresponding Fermi momentum. Thus, the conductivity is
\[ \sigma = \frac{4e^2}{3\pi m^3} \left\{ v_F \frac{N_{0+} k_F + N_{0-} k_F}{m W_0 I_1(\mu_+)} \right\} E, \]
\[ + \gamma \left[ \frac{N_{0+} v_F}{W_0 I_1(\mu_+) + W_0 I_2} - \frac{N_{0-} v_F}{W_0 I_1(\mu_-) + W_0 I_2} - 2 \int \frac{d^3 k_1}{(2\pi)^3} k_1 \frac{n_1 + n_{1-}}{W_0 I_1(\varepsilon_1) + W_0 I_2} \right] E. \]

In the absence of spin-orbit interaction \( \gamma = 0, I_1(\mu_+ = \mu_-) = (\pi T)^2, I_2 = 0 \) and we come to the Landau-Pomeranchuk temperature dependence of conductivity
\[ \sigma = \frac{8e^2 v_F^2 N_0}{3\pi^3 m^3 W_0} \frac{1}{T^2}. \]

On the other hand at finite Fermi radii splitting \( |\mu_+ - \mu_-| >> T \) the integrals
\[ I_1(\mu_+) = I_1(\mu_-) \propto I_2 \propto (\mu_+ - \mu_-)^2 \]
are practically temperature independent and the conductivity is determined by the temperature independent Eq. (21).

**III. CONCLUSION**

We have derived the low temperature conductivity in the metals without inversion centre determined by the momentum relaxation due to electron-electron scattering. Unlike to the centre-symmetric metals the conductivity in the materials with the space parity violation proved to be temperature independent. The physical reason for this behavior is that in non-centrosymmetric metals the processes of relaxation are determined by all the electronic states in between the Fermi surfaces split by the spin-orbit coupling. Thus, the calculations performed here in frame of the simplest isotropic Fermi-liquid model are still qualitatively valid for the metals with any crystalline symmetry.
Appendix A: Kinetic equation

The kinetic equation for the matrix of nonequilibrium distribution function of electrons derived by V.P.Silin [8] is

\[
\frac{\partial n_1}{\partial t} - i[\hat{\varepsilon}_1, n_1] + \frac{1}{2} \left( \frac{\partial \hat{\varepsilon}_1}{\partial r} \frac{\partial n_1}{\partial r} + \frac{\partial \hat{\varepsilon}_1}{\partial r} \frac{\partial n_1}{\partial r} \right) - \frac{1}{2} \left( \frac{\partial \hat{\varepsilon}_1}{\partial r} \frac{\partial n_1}{\partial r} + \frac{\partial \hat{\varepsilon}_1}{\partial r} \frac{\partial n_1}{\partial r} \right) = \hat{I},
\]

where \([\hat{\varepsilon}_1, n_1]\) is the commutator of \(\hat{\varepsilon}_1 = \hat{\varepsilon}(k_1)\) and \(n_1 = \hat{n}(k_1)\). We put \(\hbar = 1\). The collision integral [9] including the umklapp processes of scattering is

\[
\hat{I} = 2\pi \int d^3k \frac{d^3k''}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \sum \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \delta \left( k_1 + k_2 - k'' - \frac{2\pi m}{a} \right) \hat{F},
\]

where \(\frac{2\pi m}{a}\) is a vector of reciprocal lattice,

\[
\hat{F} = \frac{1}{2} W_1 \left\{ [n_1', (\hat{\varepsilon}_1 - n_1)] + Tr((\hat{\varepsilon}_2 - n_2)n'' - [(\hat{\varepsilon}_1 - n_1'), n_1] + Tr(\hat{n}_2 (\hat{\varepsilon}_1 - n_1]) \right\} + \frac{1}{2} W_2 \left\{ [n_2'(\hat{\varepsilon}_2 - n_1)\hat{n}_1', (\hat{\varepsilon}_1 - n_1)] + [(\hat{\varepsilon}_1 - n_1')\hat{n}_2(\hat{\varepsilon}_1 - n_1'), n_1] \right\}.
\]

Here \([\hat{A}, \hat{B}]_+\) means the anticommutator, and the following designations \(\hat{n}' = \hat{n}(k')\), \(\varepsilon' = \varepsilon(k')\) etc are introduced. In the isotropic Fermi liquid like \(^3\)He \(W_1 = |V([k_1 - k'])|^2\), \(W_2 = -V([k_1 - k'])V([k_1 - k''])\) are expressed through the Fourier transform of the quasiparticles potential of interaction. The latter in concrete metal is unknown and due to charge screening we put them as the constants: \(W_1 = W_0/2\), \(W_2 = -W_0/2\).

The matrix of equilibrium electron distribution function is

\[
\hat{n}_0 = \hat{P}_+ n_+ + \hat{P}_- n_-
\]

where

\[
\hat{P}_\pm (k) = \frac{1}{2} \pm \frac{\gamma(k) \cdot \sigma}{2|\gamma(k)|}
\]

are the projection operators such that \(\hat{P}_\pm^2 = \hat{P}_\pm\), \(\hat{P}_+ \hat{P}_- = \hat{P}_- \hat{P}_+ = 0\), \(\hat{P}_+ + \hat{P}_- = 1\). The substitution of equilibrium distribution Eq. [A1] causes the vanishing collision integral. Indeed, substituting Eq. [A1] to Eq. [A3] we obtain

\[
\hat{F} = \frac{1}{4} W_0 \sum_{\lambda, \mu, \nu, \tau} \left\{ [n_1'(1 - n_1\mu)(1 - n_2\nu)n'_\lambda - (1 - n_1'\mu)n_1\mu n_2\nu(1 - n_2')\nu] \left( \hat{P}_\lambda \hat{P}_{1\mu} + \hat{P}_{1\mu} \hat{P}_\lambda \right) Tr(\hat{P}_{2\nu} \hat{P}_{1\tau}) 
- [n'_\lambda(1 - n_2\mu)n''_\lambda(1 - n_1\tau) - (1 - n'_\lambda) n_2\mu(1 - n''_1\mu)] \left( \hat{P}_{1\mu} \hat{P}_{2\nu} \hat{P}_{1\tau} \hat{P}_\lambda + \hat{P}_\lambda \hat{P}_{1\mu} \hat{P}_{2\nu} \hat{P}_{1\tau} \right) \right\}
\]

The substitution of this expression to the Eq. [A2] yields zero because the combination in square parenthesis \([n_1'(1 - n_1\mu)(1 - n_2\nu)n''_\lambda - (1 - n_1'\mu)n_1\mu n_2\nu(1 - n_2')\nu]\) is equal to zero at arbitrary \(\lambda, \mu, \nu, \tau = \pm\) due to the energy conservation \(\varepsilon_1 + \varepsilon_2 = \varepsilon' + \varepsilon''\).

In a constant electric field the stationary kinetic equation acquires the form

\[
\left( eE \cdot \frac{\partial}{\partial k_1} \right) \hat{n}_1 = \hat{I}
\]

The matrix Fermi distribution function is convenient to represent as the sum of the scalar and the spinor parts

\[
\hat{n} = \frac{1}{2} \left[ f \hat{1} + g \cdot \sigma \right]
\]

The kinetic equations equations for this functions are

\[
\left( eE \cdot \frac{\partial}{\partial k_1} \right) f_1 = Tr\hat{I},
\]

\[
\left( eE \cdot \frac{\partial}{\partial k_1} \right) g_1 = Tr(\sigma \hat{I}),
\]
where

\[ \text{Tr} \hat{I} = \frac{\pi}{2} W_0 \int d^3 k' \frac{d^3 k''}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \sum_m (\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \delta \left( k_1 + k_2 - k' - k'' - \frac{2\pi m}{a} \right) \times \]

\[ \left\{ \left[ f'' - \frac{1}{2} (f_1 f' + g_1 \cdot g') \right] \left[ f'' - \frac{1}{2} (f_2 f'' + g_2 \cdot g'') \right] - \left[ f_1 - \frac{1}{2} (f_1 f' + g_1 \cdot g') \right] \left[ f_2 - \frac{1}{2} (f_2 f'' + g_2 \cdot g'') \right] \right\}, \quad (A11) \]

\[ \text{Tr} (\sigma \hat{I}) = \frac{\pi}{2} W_0 \int d^3 k' \frac{d^3 k''}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \sum_m (\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \delta \left( k_1 + k_2 - k' - k'' - \frac{2\pi m}{a} \right) \times \]

\[ \left\{ \left[ f'' - \frac{1}{2} (f_2 f'' + g_2 \cdot g'') \right] \left[ g'' - \frac{1}{2} (f'' g_2 + f_2 g'') \right] - \left[ f_2 - \frac{1}{2} (f_2 f'' + g_2 \cdot g'') \right] \left[ g_2 - \frac{1}{2} (f'' g_2 + f_2 g'') \right] \right\}, \quad (A12) \]

In process of derivation of these equations there was important to keep all the integrations in the collision integral what allows demonstrate vanishing of its imaginary part due to symmetry \( k' \leftrightarrow k'' \). Now, integrating over \( d^3 k' \) we liquidate the \( \delta \)-function of momenta.

Following usual linearization procedure we substitute in the left hand sides of Eqs. \((A9), (A10)\) the equilibrium distribution \( \text{Eq. (12)} \) and in the right hand side we leave only the terms linear in deviation from equilibrium distribution. Thus, we obtain the linear integral equations for the functions:

\[ \delta f = f - f_0 = f - (n_+ + n_-), \quad \delta g = g - \frac{\gamma}{|\gamma|} (n_+ - n_-). \quad (A13) \]

To establish the parametrical dependence of relaxation time we don’t need, however, solve this difficult problem. For this purpose it is enough to keep only the terms with \( \delta f_1 \) and \( \delta g_1 \), neglecting other terms containing \( \delta f'', \delta g'' \) etc. This is a sort of relaxation time approximation. Thus, we come to the following equations

\[ \text{eE} \cdot \text{v}_1 \frac{\partial (n_{1+} + n_{1-})}{\partial \varepsilon_1} = -\frac{\pi}{4} W_0 \int \frac{dk''}{(2\pi)^3} \frac{dk_2}{(2\pi)^3} \delta (\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \times \]

\[ \left\{ \left[ f_0 f'' \left( 1 - \frac{1}{2} f_0 \right) \right] \left[ 1 - \frac{1}{2} f_0 \right] \left( 1 - \frac{1}{2} f'' \right) \right\} \delta f_1 \text{g}_0 \cdot \delta f_1 - \text{g}_0 \cdot \delta f_1 + (f''_0 - f''_0) \text{g}_0 \cdot \delta f_1 \]

\[ -\left( \text{g}_0' - \text{g}_2 \right) \cdot \text{g}_0 \delta f_1 - \left( 2 - f''_0 - f''_0 \right) \text{g}_0 \cdot \delta g_1 - \left( f''_0 - f''_0 \right) \text{g}_0' \delta g_1 \} \quad (A14) \]

\[ \text{eE} \cdot \text{v}_1 \frac{\partial (n_{1+} - n_{1-})}{\partial \varepsilon_1} \frac{k_1}{|k_1|} + \frac{(n_{1+} - n_{1-})}{|k_1|} \left( \text{eE} - \frac{(\text{E} \cdot \text{k}_1)k_1}{k_1^2} \right) = -\frac{\pi}{4} W_0 \int \frac{dk''}{(2\pi)^3} \frac{dk_2}{(2\pi)^3} \delta (\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \times \]

\[ \left\{ \left[ f''_0 f'' \left( 1 - \frac{1}{2} f_0 \right) \right] \left[ 1 - \frac{1}{2} f'' \right] \right\} \delta g_1 - \text{g}_0 \cdot \delta g_1 + (f''_0 - f''_0) \text{g}_0' \delta f_1 \]

\[ -\left( \text{g}_0' - \text{g}_2 \right) f'' f_1 - \left( 2 \text{g}_0 - f''_0 \text{g}_0 - 2 \text{g}_0' \right) \delta f_1 - \left( \text{g}_0 - \text{g}_0' \right) \cdot \text{g}_0 \delta f_1 \} \quad (A15) \]

Several terms in Eqs. \((A14), (A15)\) vanishes at integration over directions of momenta. Other sub-integrant terms like \( \propto (f'' - f_2)^2 g'' \cdot \delta g_1 \) cancel out due to antisymmetry in respect of interchange of arguments \( \varepsilon_2 \leftrightarrow \varepsilon'' \). We obtain

\[ \text{eE} \cdot \text{v}_1 \frac{\partial (n_{1+} + n_{1-})}{\partial \varepsilon_1} = -\frac{\pi}{4} W_0 \int \frac{dk''}{(2\pi)^3} \frac{dk_2}{(2\pi)^3} \delta (\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \times \]

\[ \left\{ f''_0 f'' \left( 1 - \frac{1}{2} f_0 \right) \right\} \left( 1 - \frac{1}{2} f'' \right) \right\} \delta f_1 \] \quad (A16)

\[ \text{eE} \cdot \text{v}_1 \frac{\partial (n_{1+} - n_{1-})}{\partial \varepsilon_1} \frac{k_1}{|k_1|} + \frac{(n_{1+} - n_{1-})}{|k_1|} \left( \text{eE} - \frac{(\text{E} \cdot \text{k}_1)k_1}{k_1^2} \right) = -\frac{\pi}{4} W_0 \int \frac{dk''}{(2\pi)^3} \frac{dk_2}{(2\pi)^3} \delta (\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'') \times \]

\[ \left\{ f''_0 f'' \left( 1 - \frac{1}{2} f_0 \right) \right\} \left( 1 - \frac{1}{2} f'' \right) \right\} \delta g_1 \] \quad (A17)
Following the procedure of Ref.3, reproduced in Ref.17 in somewhat different manner, we reexpress the integration over \( k' \) and \( k_2 \) as

\[
d^\epsilon k'' d^\epsilon k_2 \approx \frac{m^3}{2 \cos(\theta/2)} d\varepsilon'' d\varepsilon_2 d\varepsilon' \sin \theta d\phi d\phi_2. \tag{A18}
\]

Here \( \theta \) is the angle between \( k_1 \) and \( k_2 \), \( \phi \) is the azimuthal angle of \( k_2 \) around direction \( k_1 \), and \( \phi_2 \) is the angle between the planes \((k_1, k_2)\) and \((k', k'')\). Due to the Fermi surfaces separation this formula is valid with in an accuracy of the terms of the order of \( \gamma k F / \mu \). Taking into account the \( \delta \)-function of energies we can integrate over \( \varepsilon' \) and using the equality \( \varepsilon' = \varepsilon_1 + \varepsilon_2 - \varepsilon'' \), we come to the equations

\[
eE \cdot v_1 \frac{\partial (n_1 + n_{-1})}{\partial \varepsilon_1} = -\frac{\pi}{4} \tilde{W}_0 m^3 \int d\varepsilon'' d\varepsilon_2 \left\{ f''_{0} f_{0}^{*} \left(1 - \frac{1}{2} f_{02} \right) + \left(1 - \frac{1}{2} f'_{0} \right) \left(1 - \frac{1}{2} f''_{0} \right) 2 f_{02} \right\} \delta f_1, \tag{A19}
\]

\[
eE \cdot v_1 \frac{\partial (n_{1} - n_{-1})}{\partial \varepsilon_1} k_1 \frac{(n_{1} - n_{-1})}{|k_1|} + (n_{1} - n_{-1}) \left( \frac{eE \cdot k_1}{k_1^2} \right) = -\frac{\pi}{4} \tilde{W}_0 m^3 \int d\varepsilon'' d\varepsilon_2 \left\{ f''_{0} f_{0}^{*} \left(1 - \frac{1}{2} f_{02} \right) + \left(1 - \frac{1}{2} f'_{0} \right) \left(1 - \frac{1}{2} f''_{0} \right) 2 f_{02} \right\}
\]

\[-\frac{\pi}{4} \tilde{W}_0 m^3 \int d\varepsilon'' d\varepsilon_2 \int \frac{\cos^2 \theta d\theta d\phi d\phi_2}{(2\pi)^6} g_{0}^{*} g_{0}^{*} \delta g_{1}, \tag{A20}
\]

where \( \tilde{W}_0 = W_0 \cos \frac{\theta d\theta d\phi d\phi_2}{(2\pi)^6} \). These equations contain the integral

\[
I_1(\varepsilon_1) = \int d\varepsilon'' d\varepsilon_2 \left\{ f''_{0} f_{0}^{*} \left(1 - \frac{1}{2} f_{02} \right) + \left(1 - \frac{1}{2} f'_{0} \right) \left(1 - \frac{1}{2} f''_{0} \right) 2 f_{02} \right\}
\]

\[
= \frac{1}{2} \sum_{\lambda, \mu, \nu} \int d\varepsilon'' d\varepsilon_2 \left\{ n_{\lambda} n_{\mu}^{*} (1 - n_{2\nu}) + (1 - n_{\lambda}) (1 - n_{\mu}^{*}) n_{2\nu} \right\}, \tag{A21}
\]

where, the indices \( \lambda, \mu, \nu \) = \( \pm \).

Let us calculate one particular term in this sum

\[
\frac{1}{2} \int d\varepsilon'' d\varepsilon_2 n_{\lambda} n^{*}_{\mu} (1 - n_{2\nu}) = \frac{1}{2} T^2 \int dxdy \frac{1}{e^{x+y} + 1} \frac{1}{e^{-x} + 1} \frac{1}{1 + e^{-y}} \frac{1}{1 + e^{-x}} = \frac{1}{2} T^2 \frac{\pi^2 + t^2_+}{2} n(t_+). \tag{A22}
\]

Here

\[
x = \frac{\varepsilon_2 - \mu_+}{T}, \quad y = \frac{\varepsilon'' - \mu_+}{T}, \quad t_+ = \frac{\varepsilon_1 - \mu_+}{T}, \tag{A23}
\]

and

\[
n(t_+) = \frac{1}{e^{t_+} + 1} \tag{A24}
\]

is the Fermi distribution function. Similar integration in all the other terms yields

\[
I_1(\varepsilon_1) = \frac{1}{2} \sum_{\lambda, \mu, \nu} \int d\varepsilon'' d\varepsilon_2 \left\{ n_{\lambda} n_{\mu}^{*} (1 - n_{2\nu}) + (1 - n_{\lambda}^{*} 0 (1 - n_{\mu}^{*}) n_{2\nu} \right\} = \frac{1}{4} T^2 \left\{ [\pi^2 + t^2_+] n(t_+) + [\pi^2 + (t_+ - \kappa)^2] n(t_+ - \kappa) + [\pi^2 + (t_+ + \kappa)^2] n(t_+ + \kappa) + [\pi^2 + t^2_+] n(t_-) \right\}
\]

\[
+ [\pi^2 + t^2_+] n(t_-) + [\pi^2 + (t_- - \kappa)^2] n(t_- - \kappa) + [\pi^2 + (t_- + \kappa)^2] n(t_- + \kappa) + [\pi^2 + t^2_+] n(t_-) \}
\]

\[
+ \left( t_{\pm} \rightarrow -t_{\pm}, \quad \kappa \rightarrow -\kappa \right) \right\} = \frac{1}{4} T^2 \left\{ [\pi^2 + t^2_+] + [\pi^2 + (t_+ - \kappa)^2] + [\pi^2 + (t_+ + \kappa)^2] + [\pi^2 + t^2_+] \right\}
\]

\[
+ [\pi^2 + t^2_+] + [\pi^2 + (t_- - \kappa)^2] + [\pi^2 + (t_- + \kappa)^2] + [\pi^2 + t^2_+] \right\}, \tag{A25}
\]
where we used the notation $\kappa = (\mu_+ - \mu_-)/T$. Finally we obtain

$$I_1(\varepsilon_1) = \frac{1}{2} \left( 2(\pi T)^2 + (\varepsilon_1 - \mu_+)^2 + (\varepsilon_1 - \mu_-)^2 + (\mu_+ - \mu_-)^2 \right).$$

(A26)

Thus, the deviation of the scalar part of distribution function from its equilibrium value is

$$\delta f(k_1) = -\frac{4e}{\pi W_0 n^3 I_1(\varepsilon_1)} \mathbf{E} \cdot \mathbf{v}_1 \frac{\partial(n_{1+} + n_{1-})}{\partial \varepsilon_1} \tag{A27}$$

The second integral in Eq. (A20) is

$$I_2 = \int d\varepsilon'' d\varepsilon d\phi \frac{\cos \theta d\phi d\phi'}{(2\pi)^3} \mathbf{g}_0 \cdot \mathbf{g}_0'' = \int d\varepsilon'' d\varepsilon d\phi \frac{\cos \theta d\phi d\phi'}{(2\pi)^3} \mathbf{g}_0 = \int d\varepsilon'' d\varepsilon d\phi \frac{\cos \theta d\phi d\phi'}{(2\pi)^3} \mathbf{g}_0 = \frac{1}{|\mathbf{k}_1|} (n_{0+} - n_{0-}) (n_{0'} - n_{0''}). \tag{A28}$$

The analytic calculation of this integral is impossible. However, at $\mu^+ \rightarrow \mu^-$ it tends to zero as $\propto (\mu^+ - \mu^-)^2$. We will treat this integral as energy and momentum independent constant

$$I_2 \approx (\mu^+ - \mu^-)^2. \tag{A29}$$

Thus, the deviation of the spinor part of distribution function from its equilibrium value is

$$\delta g(k_1) = -\frac{4e}{\pi m^3 [\tilde{W}_0 I_1(\varepsilon_1) + W_0 I_2]} \left[ \mathbf{E} \cdot \mathbf{v}_1 \frac{\partial(n_{1+} - n_{1-})}{\partial \varepsilon_1} \hat{k}_1 + \frac{(n_{1+} - n_{1-})}{|\hat{k}_1|} \left( \mathbf{E} - (\mathbf{E} \cdot \hat{k}_1) \hat{k}_1 \right) \right]. \tag{A30}$$

where $\hat{k}_1 = \frac{k_1}{|k_1|}$.

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