Single weighted laterally constrained inversion of airborne transient electromagnetic data with sharp boundary

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Abstract. Traditional laterally constrained inversion (LCI) can quickly and efficiently invert large geophysical data with minimizing the difference between adjacent model parameters. However when processing the data with large boundary differences, the inversion generates the result whose the boundaries are poorly fitness with the actual underground geology. In order to overcome this problem, some specific measurement stations of original LCI has been adjusted to weaken the horizontal continuity constraint of the horizontal direction near the sharp boundary. The simulation results show that the determinations of station and weights value depend on the thickness or depth change at sharp boundary. In this paper, we employed single weighted laterally constrained inversion (SWLCI) method on the model with sharp boundary. The result showed that SWLCI can improve both the recovering of sharp boundary and the stability of inversion.

1. Introduction
In recent years, airborne electromagnetic has developed fast. As an efficient geophysical exploration method, it has been widely applied to mining, oil, gas, underground water and geothermal resources exploration. According to the observation field, airborne electromagnetic method can be divided into frequency domain method and time domain method. As a time-domain method, Airborne transient electromagnetic (ATEM) has the advantages of high resolution and deeper prospecting depth [1].

Like other geophysical inversion problems, the inversion of airborne electromagnetic data is also an ill-posed problem. Due to the huge amount of airborne electromagnetic data, one-dimensional inversion is often used in practical applications. Laterally constrained inversion (LCI) [2], as a quasi-2D inversion method, divides two-dimensional data into one-dimensional data which is arranged along the survey lines. Then 1D inversion can be implemented, at the same time, making the parameter difference between adjacent lines as small as possible. Subsequently, [3] applied the LCI method to 3D transient electromagnetic data inversion and got a good result. (Siemon et al.) applied the LCI to the processing of frequency domain airborne electromagnetic data[4], and (Vallée and Smith) first used LCI to process the time domain airborne electromagnetic data, both of them showed good results [5-6]. Yin et al. (2016) proposed weighted LCI to realize the smoothness constraint proportion of different parameters [7].
Vignoli et al. [9] proposed a sharp spatially constrained inversion via regularization and improved the ability to recover the sharp boundary of quasi-3D data inversion [8].

For the ATEM data with sharp boundary, the constraint of horizontal continuity will make the result poorly match the real subsurface structure, especially for the surface close to the sharp boundary. Considering this problem, here we describe a novel weighting algorithm to decrease the constraint among the measuring points above the sharp boundary. We test this method on a synthetic model and compared the result with that obtained by the single point leastsquare inversion and conventional LCI. The result shows that, in the appropriate conditions, SWLICI can both improve the recovering of sharp boundary and the stability of inversion.

2. Methodology

2.1. ATEM 1D forward modeling

In this paper, the layered model is used for forward modeling. The central loop device is used. The x and y axes are located on the horizontal surface of the earth, the z axis takes downward direction as positive, the center of loop is directly above the origin of the Cartesian coordinate system. Under quasi-static conditions, the expression of the vertical component of frequency domain magnetic field intensity at the center of the circular loop is

\[ H_z = \frac{Ia}{2} \int_0^\infty \left[ e^{-\lambda(z+h)} + r_{te} e^{\lambda(z-h)} \right] \lambda J_1(\lambda a) d\lambda, \] (1)

where, \( I \) is the magnitude of the transmitter step current, \( a \) is the radius of the transmitter coil, \( z \) is the coordinate of the receiving point position, \( h \) is the height of the transmitter coil from the ground \((h = -z \geq 0)\), \( J_1 \) is the first-order Bessel function, and \( r_{te} \) is the reflection coefficient.

The 47-point hankel filter coefficient proposed by [4] is used to calculate the frequency domain response, and the 250-point sinusoidal filter coefficient proposed by Wang (2004) was used to calculate the time domain response [9-10]. The expression of time domain response is

\[ \frac{dh(t)}{dt} = -\frac{1}{t} \sqrt{2 \sum_{n=-\infty}^{\infty} \text{Im} \left[ H_z \left( \frac{e^{i\lambda}}{t} \right) \right]} W_n, \] (2)

In which, \( t \) is the time, \( H_z \) is the frequency domain response function, \( n \) is the number of filtering coefficients, \( \Delta \) is the sampling interval, and \( W_n \) is the filtering coefficient.

2.2. Data and model space

The data space has \( L \) time gates observation data, suppose the model has \( K \) reference node points, the data vector can be expressed as

\[ d_{obs} = [d_{i1}, d_{i2},...,d_{IL}] \] (3)

\[ d_{obs} = [d_{obs1},d_{obs2},...,d_{obsK}] \] (4)

Model space has \( N \) layers, which is consist of \( K \times (2N-1) \) parameters,

\[ m_i = [\rho_{i1}, \rho_{i2},...,\rho_{iN}, t_{i1}, t_{i2},...,t_{iN-1}] \] (5)

\[ M = [m_1, m_2, ..., m_K] \] (6)

where \( \rho \) denotes the apparent resistivity, \( t \) denotes the thickness.

2.3. Single weighted laterally constrained inversion

Based on the established practice of linearized approximation by the first term of the Taylor expansion

\[ d_{obs} \approx f(m_0) + J(m - m_0) + e \] (7)

Formula (7) can be simplified as

\[ \Delta d_{obs} = J \Delta m + e \] (8)

where \( J \) denotes Jacobian matrix.
\[ J = \frac{\partial d_{\text{obs}}}{\partial \lg(m)} = m \frac{\partial d_{\text{obs}}}{\partial m} \]  \tag{9}

LCI aims at making the parameter difference between adjacent points as small as possible. This condition can be added to the objective function as a constraint term. Formally, the constraint is applied to the model space as

\[ \Delta m = R \Delta m + e_r \]  \tag{10}

where \( \Delta m = -\Delta \), Constraint matrix R is consist of 1 and -1 for the model parameters and 0 for all other places

\[
R = \begin{bmatrix}
1 & 0 & \ldots & 0 & -1 & 0 & \ldots & 0 & 0 \\
0 & 1 & \ldots & 0 & -1 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & -1
\end{bmatrix}
\]  \tag{11}

From the constraint matrix \( R \), it can be seen that when processing the data with sharp boundary, the constraint on horizontal continuity will make the result over-smooth. Here we propose single weighted LCI. From the view of geology significance, the parameter difference (thickness or depth) close to the sharp boundary will be larger than that close to other surface. As a result, during the iteration, the parameter difference value of the measuring points at the sharp boundary will be larger than others. Utilizing this difference to locate the measuring points on the sharp boundary, we can find the constraint factor corresponding to the measuring points. We multiplied it by a smaller weight to weaken the constraint of horizontal continuity on this measuring points. The weight expression is as the follows:

\[
W(k,:) \begin{cases}
1, & \text{if } R(k,:)m \geq \text{threshold} \\
\frac{1}{\text{threshold}}, & \text{if else}
\end{cases}
\]  \tag{12}

where the threshold can be the thickness or depth difference value close to the actual sharp boundary, and \( k \) denotes the measuring point position above the sharp boundary.

The whole inversion expression can be wrote as

\[
\begin{bmatrix}
\Delta d_{\text{obs}} \\
\Delta r
\end{bmatrix} = \begin{bmatrix} J \\ WR \end{bmatrix} \Delta m + \begin{bmatrix} e \\ e_r \end{bmatrix}
\]  \tag{13}

Formula (12) can be wrote compactly as

\[ \Delta d = J' \Delta m + e' . \]  \tag{14}

We utilize damped least square method to minimize the error and apply SVD on Jacobian, the modification of model can be estimated by

\[ \Delta m = V(L^2 + \delta^2 I)^{-1} U^T d \]  \tag{15}

3. Result
In this part, we will demonstrate the SWLCI result, and compare it with the result of single point damped leastsquare inversion and LCI without weighting. We set the radius of transmitter coil to 15 meters, the transmitter and receiver elevation is 30m on average and the transmit current is 100A. Each signal contains 30 time gates.

We designed a three layers synthetic model as shown in Figure 1. The resistivity of first layer is 10Ω·m, the second and third have the resistivity of 100Ω·m and 20Ω·m, respectively. The thickness of first layers is 20m, the second layer interface fluctuates along the measuring line. There exists 101 measure points along the profile. The distance between neighbor points is 5m. We obtain the measured data using 1D forward modeling, and 3% Gaussian noise is added to the simulated responses.
Compared with the single point leastsquare inversion, LCI obviously improved the boundary laterally continuity. Figure 3 shows the result using LCI without weighting, it can be seen that two depression boundaries match poorly with the real model because of the laterally continuity constraint. Figure 4 shows the inversion result of SWLCI. It can be seen that these two depression boundaries has been improved while maintain the horizontal continuity of other boundary.
4. Conclusions
Compared with conventional damped least square inversion, laterally constrained inversion can both recover the resistivity and thickness of the model well and improve the surface horizontal continuity efficiently. Besides, it can decrease the multiplicity of inversion. To overcome the shortcomings of laterally constrained inversion for processing the data with sharp boundary, we propose single weighted laterally constrained inversion to improve the inversion quality of the sharp boundary. From the inversion results, we are confirmed that our method can both recover the sharp boundary well and improve the inversion stability.

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