ANALYSIS OF HYSTERESIS EFFECT IN $p$ MODE FREQUENCY SHIFTS AND SOLAR ACTIVITY INDICES

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Abstract. Using intermediate degree $p$ mode frequency data sets for solar cycle 22, we find that the frequency shifts and magnetic indices show a “hysteresis” phenomenon. It is observed that the magnetic indices follow different paths for the ascending and descending phases of the solar cycle, the descending path always seems to follow a higher track than the ascending one. However, for the radiative indices, the paths cross each other indicating phase reversal.

1. Introduction

The solar cycle changes in the Sun’s interior are also reflected as a variation in the $p$ mode frequencies. Woodard et al. (1991) showed that the mode frequencies varied on monthly time scales and were correlated with the average magnetic flux density on the Sun in short time intervals. Bachmann and Brown (1993) pointed out that the frequency shifts were correlated differently with magnetic and radiative indices and later confirmed by Bhatnagar, Jain and Tripathy (1999). A study of solar activity indices by Donnelly (1991) has shown that certain pairs of indices follow different paths for the ascending and descending phases of the solar cycle displaying a “hysteresis” like phenomena. In some cases, these observed hysteresis patterns start to repeat over more than one solar cycle, giving evidence that this is a normal feature of solar activity. Considering seven indices of solar activity, Bachmann and White (1994) have shown that hysteresis is present among these indices during solar cycle 21 and 22. They further inferred that the hysteresis can be approximated as a hierarchy of delay times between the pair of indices. This effect has also been investigated by Özgüç and Ataç (1999) using smoothed time series of solar flare index and long duration flare index. In cosmic ray intensities in different energy ranges, the hysteresis effect is also a common feature of cosmic ray modulation (Dorman et al., 1999).

Since a pair of activity indices shows the hysteresis effect and the $p$ mode eigen frequencies are correlated with the activity indices, it is expected that the frequency shifts may also exhibit a similar hysteresis
pattern with the activity indices. Some evidence for this effect has been recently put forward by the analysis of low degree modes (Jiménez-Reyes et al., 1998), which indicates that the correlation between the mode frequencies and magnetic indices varies between the rising and falling phases of the solar activity cycle. In similar context, preliminary results for the intermediate degree modes have been discussed by Tripathy et al. (2000). In this paper, we carry out a detailed analysis of the intermediate degree modes to observe the characteristics of the hysteresis loops. We report, for the first time, that the intermediate degree $p$ mode frequency shifts show a hysteresis like phenomenon with the magnetic activity indices. Thus, we confirm that the hysteresis effect prevails not only between the various activity indices but also between the activity indices and the global oscillation modes of the Sun.

2. Observational Data Sets

Intermediate degree $p$ mode frequencies have been obtained from the Fourier Tachometer (FTACH) instrument of the High Altitude Observatory (HAO), the Big Bear Solar Observatory (BBSO), the LOWL instrument operated by HAO and the Global Oscillation Network Group (GONG) Project. These data sets covering the solar cycle 22 and part of cycle 23, between March 1986 to August 1999, are summarised in Table I. The table lists the data sets according to the source, epoch of observation and number of data sets. It may be noted that we have used the non-overlapping data sets from the GONG network.

We use four different activity indicators representing magnetic and radiative indices to study the hysteresis effect. These are: the line-of-sight photospheric magnetic flux measured from Kitt Peak (KPMI); the magnetic plage strength index (MPSI) between 10 and 100 Gauss obtained from Mount Wilson magnetograms (Ulrich, 1991); the equivalent width of HeI 10830 Å line which primarily originates from the

| Data source  | Period                  | No. of Data Sets |
|--------------|-------------------------|------------------|
| FTACH/HAO    | 17 May 1986 - 10 Nov 1990 | 18               |
| BBSO         | Mar 1986 - Sep 1990      | 4                |
| LOWL/HAO     | 26 Feb 1994 - 25 Feb 1996 | 2                |
| GONG         | 12 June 1995 - 1 Aug 1999 | 14               |
Figure 1. Variation of (a) Kitt Peak Magnetic Index and (b) Magnetic Plage Strength Index with frequency shift. It is observed for solar cycle 22 that the descending phase follows a higher track than the ascending one showing the hysteresis effect. The error-bars at the top left corner indicate 1σ values.

chromospheric material (HeI) and is obtained from Kitt Peak (Harvey, 1984); the integrated radio flux at 10.7 cm ($F_{10}$) which originates from several chromospheric-coronal sources, obtained from Solar Geophysical Data (SGD). We also include the unsmoothed International sunspot number ($R_I$) taken from SGD for completeness and comparison with earlier results.
Figure 2. Variation of (a) 10.7 cm radio flux, (b) Equivalent width of He 10830 Å line. For these indices, the descending and ascending paths cross each other indicating phase reversal. The error-bars at the top left corner indicate 1σ values.

3. Results and Discussion

The centroid frequency shifts are calculated by using the BBSO frequencies of 1988 as reference. This analysis is restricted to the spherical harmonic degree range $20 \leq \ell \leq 60$ and frequency range $2600 \mu$Hz $\leq \nu \leq 3200 \mu$Hz, due to the direct use of frequency shifts from FTACH data as given in Bachmann and Brown (1993). A mean value, together with associated error, is computed for each solar index over the same intervals corresponding to the individual frequency shifts measurements,
so simultaneous values for solar indices and mean frequency shifts are obtained.

Figure 1 shows the variation in mean magnetic field values represented by KPMI and MPSI as a function of frequency shifts. For solar cycle 22, both the indices display a hysteresis pattern by following different paths for the ascending and descending phases. It is also observed that the descending path follows a higher track than the ascending one, which is a normal characteristic of hysteresis in magnetic materials. Figure 2 shows similar plots for radiative indices represented by $F_{10}$ and He I. Here, the ascending and descending paths do not exhibit hysteresis patterns, instead the paths cross each other and indicate phase reversal. Similarly, $R_I$ does not reveal any hysteresis effect and appears to follow the radiative indices.

The presence or absence of the hysteresis effect in different activity indices is further confirmed by carrying out the Spearman’s rank correlation analysis between these indices and mean frequency shifts. The correlation coefficients ($r_s$) for the ascending, descending and the complete solar cycle 22 are summarised in Table II. In all the cases, $F_{10}$ has the maximum correlation while KPMI has the minimum correlation. It is evident that the radiative indices have a better rank correlation than the magnetic field indices represented by KPMI and MPSI while $R_I$ is in close agreement with the radiative indices. This behaviour of the sunspot number needs to be examined more closely and is outside the scope of this paper.

The trend of variation of frequency shifts with activity indices for cycle 23 (upto August 1999) is also shown in Figures 1a and 2a. It is apparent that the solar activity level for the ascending phase of cycle 23 is higher than cycle 22. However, any conclusion regarding the hysteresis effect for the current cycle 23 would require consistent data sets for the complete cycle e.g., from GONG network.
Table III. Values of the parameter \( \oint \Delta \nu \) for different activity indices for cycle 22

| Activity Index | \( \oint \Delta \nu \) (nHz) |
|----------------|-----------------------------|
| KPMI           | 110 ± 8.56                  |
| MPSI           | 60 ± 8.39                   |
| \( R_I \)      | 20 ± 8.36                   |
| \( F_{10} \)   | -10 ± 8.93                  |
| HeI            | -20 ± 8.43                  |

We have further evaluated the parameter \( \oint \Delta \nu \) (Table III) which represents the mean frequency difference between the descending and ascending phases of solar activity cycle. The methodology for calculating this parameter is the same as that adopted by Jiménez-Reyes et al. (1998). In brief, first we omitted the saturation part of the diagrams which are obviously at low and high activity. Next, the area enclosed by the closed path was calculated and then divided by the value of the common scanned range of activity. Thus we obtain \( \oint \Delta \nu \) for each diagram together with the associated error considering the propagation of the individual errors throughout the process.

We find from Table III that the value of the parameter \( \oint \Delta \nu \) is fairly significant for the magnetic indices KPMI and MPSI while for radiative indices the values are nearly zero. On the other hand, \( R_I \) has a small positive value indicating an intermediate behaviour.

Observationally, it now appears that the \( p \) mode frequency shifts and the activity indices exhibit hysteresis phenomenon. A preliminary interpretation for the hysteresis like effect between a pair of activity indices has been proposed by Bachmann and White (1994) on the basis of delay times behind the leading activity index. This argument is tested by plotting \( F_{10} \) as a function of both KPMI and \( R_I \) (Figure 3) for the solar cycle 22. The \( F_{10} \) and KPMI show a conspicuous hysteresis loop while this effect is marginally seen between \( F_{10} \) and \( R_I \). This suggests a long time delay between the activity pairs; KPMI – \( F_{10} \) and \( R_I \). This is supported by the widely accepted fact that the surface magnetic fields precede in time to the appearance of the sunspots.

The hysteresis phenomenon observed in the case of the global oscillation modes can also be interpreted as a time delay between the activity indices and the mode frequencies. Thus, the hysteresis effect
Figure 3. Monthly mean plot of (a) $F_{10}$ versus KPMI and (b) $F_{10}$ versus $R_I$ for solar cycle 22. It may be noted that the hysteresis effect is much pronounced for the pair of indices $F_{10}$ and KPMI as compared to the pair $F_{10}$ and $R_I$. The points in these plots represent the monthly mean of the activity indices on which a running mean of 12 months is applied.

seen between the surface magnetic fields (KPMI and MPSI) and the mode frequencies is attributed to the long delay times between them. This is also manifested in the weak correlations between these magnetic indices and the $p$ mode frequency shifts. On the other hand, the insignificant hysteresis effect observed between the frequency shifts with the sunspots and the radiative indices denotes small time lag. Thus, we be-
lieve that the phenomenon of hysteresis may provide an explanation as to why the radiative indices have better correlation with the frequency shift than the magnetic indices as was earlier pointed out by Bachmann and Brown (1993) and Bhatnagar, Jain and Tripathy (1999).

In summary, we find that the intermediate degree frequencies of solar cycle 22 show a “hysteresis” phenomenon with the magnetic field indices whereas no such effect is seen in the radiative indices. However, more consistent frequency data sets for the current solar cycle would provide a better plausible explanation of the hysteresis shapes between the activity indices and structural parameters such as $p$ mode frequencies.

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