Do radio relics challenge diffusive shock acceleration?

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ABSTRACT

Radio relics in galaxy clusters are thought to be associated with powerful shock waves that accelerate particles via diffusive shock acceleration (DSA). Among the particles accelerated by DSA, relativistic protons should outnumber electrons by a large factor. While the relativistic electrons emit synchrotron emission detectable in the radio band, the protons interact with the thermal gas to produce gamma-rays in hadronic interactions. Using simple models for the propagation of shock waves through clusters, the distribution of thermal gas and the efficiency of DSA, we find that the resulting hadronic gamma-ray emission lies very close or above the upper limits from the Fermi data on nearby clusters. This suggests that the relative acceleration efficiency of electrons and protons is at odds with predictions from DSA. The inclusion of re-accelerated ‘fossil’ particles does not seem to solve the problem. Our study highlights a possible tension of the commonly assumed scenario for the formation of radio relics and we discuss possible solutions to the problem.

Key words: acceleration of particles – shock waves – galaxies: clusters: general – intergalactic medium – gamma-rays: galaxies: clusters.

1 INTRODUCTION

Giant radio relics are steep-spectrum radio sources that are found in the outer parts of galaxy clusters, \(\sim 0.5\)–\(3\) Mpc from their centres (e.g. Brüggen et al. 2011; Feretti et al. 2012, and references therein). They are almost exclusively found in perturbed clusters, and there is good evidence for their association with powerful shock waves triggered by mergers (Enßlin et al. 1998; Hoeft & Brüggen 2007). Despite its low efficiency at low Mach numbers, diffusive shock acceleration (DSA; Caprioli e.g. 2012; Kang & Ryu e.g. 2013, for modern reviews) has been singled out as the most likely mechanism to produce the relativistic electrons that power radio relics. According to the theory, only a tiny fraction (\(\ll 0.01\)) of the kinetic power of a shock is necessary to accelerate electrons, and the observed power-law spectra (\(s \sim 1\) where \(I(\nu) \sim \nu^{s}\) is the radio spectrum) are naturally produced by DSA (e.g. Hoeft & Brüggen 2007).

Cosmological simulations have been performed to investigate the complex intra-cluster medium (ICM) dynamics that may produce relics (e.g. Pfommer et al. 2007; Hoeft et al. 2008; Skillman et al. 2013). However, it is found that the acceleration of relativistic protons by the same shocks must also cause a significant level of hadronic gamma-ray emission (Dennison 1980; Berezinsky, Blasi & Ptuskin 1997; Blasi & Colafrancesco 1999; Pfommer et al. 2007; Aleksić et al. 2012; Vazza, Brüggen & Gheller 2013; The Fermi-LAT Collaboration et al. 2013).

In this work, we propose that a combination of radio observations that reveal the relativistic electrons and gamma-ray observations that can constrain the density of relativistic protons provides a good test for DSA at radio relics. We resort to a semi-analytical approach to estimate the total cosmic ray (CR)-energy injected by shocks over the course of their existence inside the cluster. Our results show that for several clusters with relics and Fermi data, the combination of Mach number, magnetic field strength and gas density that is required to explain the radio relics produces gamma-ray emission above the upper limits. This suggests a possible tension between the commonly assumed acceleration efficiencies in the Mach number range \(2 \leq M \leq 5\). Although the present-day gamma-ray observations are not deep enough to exclude DSA for the entire set of our clusters, this analysis favours a large acceleration efficiency (\(\xi_e \geq 10^{-4}\)–\(10^{-3}\)) at weak shocks, and a larger energy ratio between injected relativistic electrons and protons than is commonly adopted (\(K_{\nu/p} \geq 0.1\)), at odds with the standard view of shock acceleration that works seemingly well in supernova remnants.

2 THE SAMPLE

We build a sample of radio relics by combining the radio data in the catalogue by Feretti et al. (2012) and the gamma-ray data in the energy range \([0.2–100]\) GeV by the Fermi Collaboration (Ackermann et al. 2010; The Fermi-LAT Collaboration et al. 2013). This yields six objects: Abell 2163, Abell 2256, Abell 2744, Abell 754, Abell 3376 and Abell 1656 (COMA), for which the main parameters used in our calculation are shown in Table 1.

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Table 1. Parameters for the radio relics and clusters considered in this paper. The data are taken from Feretti et al. (2012) (radio power, spectral index, size and cluster-centric distance of the relics) and Ackermann et al. (2010); The Fermi-LAT Collaboration et al. (2013) (gamma-ray emission, $L_X$ and $T_{500}$).

The last four columns show the values assumed for the downstream/upstream gas density, for the Mach number and for the electron acceleration efficiency necessary to match the observed radio power, in the model discussed in Section 3. In the case of A3376 there is some disagreement between the radio (Kale et al. 2012) and X-ray derived Mach numbers (Akamatsu et al. 2012). The gamma-ray emission of A2256, A754, A3376 and COMA have been derived from The Fermi-LAT Collaboration et al. (2013) after correcting for the slightly different energy band.

| Object | $z$ | $\log P_{1.4\text{GHz}}$ (W/Hz) | $R_{\text{relic}}$ (Mpc) | LLS (Mpc) | $L_X$ $(10^{44} \text{erg s}^{-1})$ | $T$ (keV) | $N_{\gamma}$ [0.2–100] (ph/$(\text{cm}^2 \text{s})$) | $n_d$ (1/cm$^3$) | $n_u$ (1/cm$^3$) | $M$ | $\xi_e$ |
|--------|----|-------------------------------|--------------------------|-----------|---------------------------------|-----------|---------------------------------|----------------|----------------|-----|---------|
| A2163  | 0.2030 | 24.32                         | 1.17                      | 0.48      | 22.73                           | 13.3      | 5.51 $\times$ 10$^{-9}$         | 6.5 $\times$ 10$^{-4}$ | 2.0 $\times$ 10$^{-4}$ | 5.5 | 2.41 $\times$ 10$^{-5}$ |
| A2256  | 0.0581 | 24.56                         | 0.44                      | 1.13      | 3.75                            | 6.6       | 2.76 $\times$ 10$^{-10}$        | 2.7 $\times$ 10$^{-3}$ | 9.1 $\times$ 10$^{-3}$ | 3.3 | 2.65 $\times$ 10$^{-6}$ |
| A754   | 0.0542 | 24.71                         | 1.56                      | 1.62      | 12.86                           | 10.1      | 2.49 $\times$ 10$^{-9}$         | 3.1 $\times$ 10$^{-4}$ | 1.0 $\times$ 10$^{-4}$ | 4.5 | 6.93 $\times$ 10$^{-5}$ |
| A744   | 0.0380 | 23.67                         | 0.5                        | 0.8       | 2.21                            | 9         | 1.05 $\times$ 10$^{-9}$         | 1.9 $\times$ 10$^{-3}$ | 8.6 $\times$ 10$^{-4}$ | 2.2 | 1.61 $\times$ 10$^{-6}$ |
| A3376W | 0.0468 | 23.88                         | 1.43                      | 0.8       | 1.08                            | 4.3       | 6.26 $\times$ 10$^{-10}$        | 1.7 $\times$ 10$^{-4}$ | 4.9 $\times$ 10$^{-5}$ | 3.3 | 6.12 $\times$ 10$^{-5}$ |
| A3376E | 0.0468 | 23.79                         | 0.52                      | 0.95      | 1.08                            | 4.3       | 6.26 $\times$ 10$^{-10}$        | 1.25 $\times$ 10$^{-3}$ | 4.2 $\times$ 10$^{-4}$ | 2.5 | 4.45 $\times$ 10$^{-5}$ |
| A1656  | 0.0231 | 23.49                         | 2.2                       | 0.85      | 3.99                            | 8.3       | 6.62 $\times$ 10$^{-10}$        | 1.0 $\times$ 10$^{-4}$ | 3.6 $\times$ 10$^{-5}$ | 3.3 | 1.63 $\times$ 10$^{-5}$ |

In addition to these objects, we include 13 double relics from the catalogue by Bonafede et al. (2012), which are not included in the set of published Fermi data by (Ackermann et al. 2010; The Fermi-LAT Collaboration et al. 2013). We included only double radio relics since they can most confidently be traced back to merger shocks. Where spectral indices were not known (A3365, A3367), we assumed a spectral index according to the observed relation between the projected largest linear size of relics, LLS, and their integrated radio spectrum, $s \sim 2 + \log_{10}(\text{LLS}/300 \text{kpc})$, as in van Weeren et al. (2009). In those clusters, where the gas temperature could not be determined from X-ray observations, we assumed a postshock temperature based on the $L_X - T_{500}$ relation (Pratt et al. 2009).

In order to compare the prediction of hadronic emission from this second set of relics, we used recent upper limits from a collection of stacked Fermi-LAT count maps, recently performed by Huber et al. (2013) and Huber et al. (2012), which are the deepest up to date. The result of the stacking cannot be strictly used on a single-object comparison, since the properties of the single object might differ significantly from the properties of the clusters in the stacked sample. In the following, the comparison of our forecasts of hadronic emission with the stacking upper limits will rely on the assumption that the population of non-cool-core (NCC) clusters (for which the limits of Huber et al. 2013 have been derived) and one of the clusters hosting double relics is similar and affected by a similar scatter.

3 MODELLING OF RADIO RELICS

Throughout the paper, we assume that all relics trace outward propagating shocks that have swept through the cluster volume. In order to estimate the radio emission by relativistic electrons, we use the analytic model developed by Hoeft & Brüggen (2007, hereafter HB07). This model assumes an exponential cut-off in the energy distribution of electrons, determined by the balance of the acceleration rate and of the (synchrotron and Inverse Compton) cooling rate. In the downstream region, DSA is assumed to generate suprathermal electrons that follow a power law in energy. While the gas is advected with the downstream plasma, the relativistic electrons cool because of synchrotron and Inverse Compton losses. The maximum energy of the electrons is a decreasing function of the distance from the shock front, and the radio emission diminishes accordingly. The total emission in the downstream region is finally obtained by summing up all contributions from the plasma from the shock front to the distance, where the electron spectrum is too cool to allow any further radio emission at observational frequencies. The monochromatic radio power at frequency $\nu$, $P_{\nu}$, depends on the shock surface area, $S$, the downstream electron density, $n_d$, the electron acceleration efficiency, $\xi_e$, the downstream electron temperature, $T_d$, the spectral index of the radio emission, $s$, and the relic magnetic field, $B$, as in equation 32 of HB07:

$$P_{\nu} \propto S \cdot n_d \cdot \xi_e \cdot \nu^{-s/2} \cdot T_d^{1/2} \cdot B^{1+s/2} \left( \frac{B^2_{\text{CMB}} + B^2}{\Omega B^2} \right).$$

(1)

The shock surface is approximated as the square of the LLS of the relic. We fix the upstream gas density at the relic using a β-model profile for each host cluster, with $\beta = 0.75$ and the core radii scaling as $r_c = r_c,\text{Coma}(T_{\text{d,Coma}}/T_{\text{d}})^{1/2}$ (which follows from the self-similar scaling), where $r_c,\text{Coma} = 290 \text{kpc}$. Then, we fix the Mach number $M$ from the spectral index of the radio spectrum at the injection region via $M = \sqrt{\frac{\alpha}{1 - \alpha}}$ (6 is the slope of the particle energy spectra and $\delta = 2s$). From the Mach number, $M$, we compute the downstream density at the relic location using Rankine–Hugoniot jump conditions, using a ratio of specific heats $\gamma = 5/3$. Similarly, for the temperature, we assume $T_{500}$ for each cluster as the post-shock value of temperature and we derive the pre-shock temperature again using shock jump conditions.

Now, we can calculate the kinetic power dissipated at the shock, $\Phi_{\text{kin}} \propto n_v V^2_s / 2$, where $V_s = M c_s$ ($c_s \propto \sqrt{T_d}$). Next, we can compute the synchrotron power of each relic, provided we know (i) the magnetic field in the relic, $B_d$ and (ii) the electron acceleration efficiency, $\xi_e$. We explore two possible ways of proceeding.

In the first case, we fix the magnetic field and derive the acceleration efficiency of electrons by equating $P_{\nu}$ to the observed radio power. We then set the CR-proton acceleration efficiency to $\eta = \xi_e \eta_{\text{C}/\text{P}}$, where $\eta_{\text{C}/\text{P}}$ is the electron-to-proton ratio. The power in CRs is then given by $\Phi_{\text{CR}} = n_v \Phi_{\text{kin}}$. The estimate of $K_{\text{C}/\text{P}}$ is highly uncertain and presently cannot be derived from first principles. A value of $K_{\text{C}/\text{P}} \sim 0.01$ is commonly assumed for the Milky Way.

\footnote{We note that, in agreement with most of the literature on DSA of electrons, we include in our definition of the electron acceleration efficiency the dependence on $M$ in equation (1), i.e. $\xi_e = \xi_{e,0} \Psi(M)$, where $\xi_{e,0} = 0.05$ and $\Psi(M)$ is given in equation 31 of HB07.}
since about \( \sim 1 \) percent of the observed Galactic flux of CRs with \( E \approx \) GeV is due to electrons (e.g. Schlickeiser 2002). On the other hand, the modelling of the spectra of young supernova remnants generally requires \( K_{ep} \sim 10^{-3} - 10^{-5} \) to fit the observational data (e.g. Berezhko, Ksenofontov & Völk 2009; Morlino, Amato & Blasi 2009; Ellison et al. 2010; Fukui et al. 2012). In our computation here, we set the electron to proton ratio to \( K_{ep} = 0.01 \), which is the most conservative assumption since it already minimizes the acceleration of CR protons for a given amount of CR electrons. As we will see in the next sections, already in this case we tend to predict too large values of CR proton acceleration in several systems and, therefore, the adoption of lower (and perhaps more realistic) values of \( K_{ep} \) only exacerbates the problem.

In a second model, we will assume that \( \eta \) depends on \( M \) according to the results of Kang & Ryu (2013), who studied the dissipation of energy into CRs as a function of the sonic Mach number and also included the magnetic field amplification by streaming instabilities excited by CRs (see Kang & Ryu 2013, for more details). The additional re-accelerating of pre-existing CRs can be included by using modified versions of \( \eta(M) \) that depend on the ratio of the non-thermal to thermal energy densities, \( \epsilon \), in the upstream region. To this end, we use a fit to their fig. 3. The electron acceleration efficiency is given by \( \xi_e(M) = \eta(M)K_{ep} \). Now, \( B_0 \) is a free parameter, which we fix by matching the observed radio power given by equation (1), where all the other parameters have been fixed as before and as given in Table 1. However, unlike in the previous model, we fix the Mach number at the current relics to \( M = 2 \), which is a conservatively low value. This then yields the corresponding \( \eta(M) \) given a value of \( \epsilon \). In this study, we tested \( \epsilon = 0.01 \) and \( \epsilon = 0.05 \), i.e. in the range of what found in our previous cosmological simulations (Vazza et al. 2012a) and of the order of the maximum allowed budget of CRs by the most recent upper limits (The Fermi-LAT Collaboration et al. 2013). Because of their large lifetime (\( \sim 10^{10} \) yr), the pre-existing CR-protons could come from previous internal and external shocks. In contrast, the pre-existing CR-electrons must be produced locally, e.g. by turbulent re-acceleration or hadronic collisions, because of their short radiative lifetime (\( < 10^5 \) yr for a \( B \sim \mu G \) magnetic field).

As in Kang & Ryu (2013), we assume that the blend of several populations of pre-existing CR-electrons are characterized by an energy spectrum \( \delta_e = (48 + 1)/3 \), where \( \delta \) is the energy spectrum derived at the relic as before. This follows from the fact that the general effect of shock re-acceleration is that of generating particle spectra that are flatter than the spectrum of the seed particles if the re-acceleration Mach number is strong, while conserving the slope of the seed energy distribution if the re-acceleration Mach number is weaker. In both cases, shock re-acceleration increases the normalization of the distribution more than what simple adiabatic compression does (Blasi 2004; Markevitch et al. 2005). In order to predict the gamma-ray emission throughout the cluster, we must make some assumptions about the properties of the shock as they travel through the cluster. We assume that the shock surface scales with cluster-centric distance, \( r \), as \( \delta_s = S_0(r/\text{Rel}})^{2} \) (i.e. the lateral extent of the shock surface is set to LLS of the relic, which decreases with \( \propto r \) inwards). The total energy dissipated in CRs is given by

\[
E_{\text{CR}} = \int_{r_{\text{core}}}^{r_{\text{cluster}}} \eta \Phi_{\text{rms}}(r) dr,
\]

where \( dr = dr/V_c \) and \( r_c \) is the cluster core radius. The shock surface and strength will vary with radius, and so will \( \eta(M) \). For the scaling of \( M \) with \( r \), we inverted the relation between the radio spectral index and the injection Mach number, using the results of van Weeren et al. (2009), who observed flatter spectral indices of radio relics increasing with the distance from the cluster centre and approximate it as \( M = M_0(r/R_{\text{Rel}})^{1/2} \). Although this observed relation has large uncertainties, such gentle functional dependence with radius is in line with cosmological simulations (e.g. Vazza et al. 2010, fig. 15).

The lower integration limit in the equation for \( E_{\text{CR}} \) is the core radius, \( r_c \), meaning that the shocks are assumed to be launched only outside of the cluster core, which is also supported by cosmological numerical simulations (Vazza et al. 2012b; Skillman et al. 2013).

Finally, we neglect the presence of gas clumping and of the enhanced gas density along the merger axis, which would only increase the hadronic emission, since \( \epsilon \propto n^2 \).

We compute the hadronic gamma-ray emission from the CR-protons as in Pfrommer & Enßlin (2004) and Donnert et al. (2010), and assume that the momentum spectrum of protons follows a power law with a slope \( \delta_p \approx \delta \), and a threshold proton energy \( E_{\text{min}} = 780 \text{ MeV} \). The spectral index is fixed to the radio spectral index (e.g. Kang, Ryu & Jones 2012). Unlike in Pfrommer & Enßlin (2004) and Donnert et al. 2010, we use the parametrization of the proton–proton cross-section given by Kelnner, Aharonian & Bugayov (2006).

4 RESULTS

4.1 Fixed magnetic field

In our first model, we fix the magnetic field for each relic and derive the values of \( \xi_e \) and \( \eta = \xi_e/K_{ep} \) that are necessary to match the observed radio power. To be conservative, we choose \( B_0 = 7 \mu G \), which is the largest values of the magnetic field inferred for the radio relic in CIZA J2242.8+5301 (van Weeren et al. 2010). By doing so, we are minimizing the electron energy required to match the observed radio power, and hence the CR-proton energy used to compute the hadronic emission from the host clusters.

The first panel in Fig. 1 shows the result of this test for our sample, and compare it to the gamma-ray limits from Ackermann et al. (2010), The Fermi-LAT Collaboration et al. (2013) and Huber et al. (2013), within the energy range [0.2-100] GeV. The Fermi data cover the cluster volume inside \( R_{500} \) of clusters, which is about one order of magnitude larger than the volume swept up by our shocks, and include the core region of clusters which we exclude. Hence, any additional source of CRs within the ICM would enhance the total hadronic emission predicted by our model. In the double relic systems, the radio and the gamma-ray fluxes are added and the swept-up volume is doubled.

No gamma-ray fluxes above the Fermi limits are predicted for our relics, if we compare with the single-object data provided by Ackermann et al. (2010) and The Fermi-LAT Collaboration et al. (2013). However, for about one half of the sample, we predict levels of hadronic emission that are larger than the upper limits of the stacking analysis by Huber et al. (2013), considering a range of possible proton energy spectra (\( \delta = 2.0 - 2.8 \)). These limits have been derived for a sample of 32 NCC clusters, and they cannot be used on a single-object basis. Should clusters with radio relics have an intrinsically larger gamma-ray emission than NCC clusters, the stacking analysis of the first population would lead to a higher upper limit. At this level, we can only rely on the assumption that the two populations are similar in their content of CRs, which is reasonable following the scenario in which cluster mergers are both responsible for the release of non-thermal energy and for the disappearance of cool cores within clusters. With this caveat, we suggest that the hint of a tension is shown by this test, since many clusters hosting double relics have a predicted gamma-ray emission \( \sim 5 - 10 \) times above the available upper limits from the stacking of the NCCs.
Figure 1. Top panel: synchrotron power at 1.4 GHz and estimated hadronic gamma-ray fluxes for our sample (blue squares). We fix $B_d = 7 \, \mu G$ and we constrain $\xi_e$ by matching the observed radio power. The grey arrows show the Fermi upper limits for single sources (Ackermann et al. 2010), while the vertical grey line shows the result of the stacking by Huber et al. (2013) for $\delta = 2.0$ or $\delta = 2.8$. The bottom panel shows the values of $\xi_e$ for each single relic of the sample. The additional grey line shows the expectation from DSA, as in fig. 3 of Kang & Ryu (2013).

Another hint for tension with DSA results is shown in the second panel in Fig. 1, where the values of $\xi_e$ needed to match the observed radio power at 1.4 GHz are shown assuming $B_d = 7 \, \mu G$. In the same figure, we show the predicted dependence of $\xi_e$ on Mach numbers according to the numerical study by Kang & Ryu (2010). For about half of the relics of our sample, the required value of $\xi_e$ is large compared to the theoretical expectation of DSA for weak shocks, $M \leq 3$. The tension can be even larger if magnetic fields are lower than the high 7 $\mu G$ considered here.

Our result for weak shocks confirms the result of recent studies that highlighted the importance of shock re-acceleration of CRs in weak shock regime, in order to explain observations of several relics (Kang et al. 2012; Pinzke, Oh & Pfommer 2013). However, rather high values of $\xi_e$ even for stronger shocks seem necessary for $M \geq 3$ relics, adding odds with the expectation of DSA theory. In the next section, we investigate how the inclusion of shock re-acceleration can change this picture.

4.2 Variable magnetic field and shock re-acceleration

In our second model, we parametrize the acceleration efficiency of CRs assuming a functional dependence on the Mach number, $\eta(M)$, predicted by DSA following Kang & Ryu (2013) (or using a fixed value), and we constrain the magnetic field by matching the observed synchrotron emission.

As a first simple case, we impose a constant value for all Mach numbers and explored the cases $\eta = 0.1$, $10^{-2}$ and $10^{-3}$, see Fig. 2. If a low acceleration efficiency is assumed, the predicted hadronic gamma-emission is below the Fermi limits in most cases, but in half of the objects the required magnetic field becomes unrealistically large $\gg 10 \, \mu G$ (up to $\sim 10^3 \mu G$ in a few cases), at odds with observations (Feretti et al. 2012). On the other hand, if we assume a higher acceleration efficiency the synchrotron emission at relics is generally recovered using a value of the magnetic field consistent with observations, yet the hadronic emission from the downstream exceeds the Fermi limits.

For weak shocks ($M \leq 3$–4) the re-acceleration of ‘fossil’ electrons (Markevitch et al. 2005; Kang et al. 2012; Pinzke et al. 2013)
in supernova remnant shocks (e.g. Caprioli 2012). As a result, the CR-proton acceleration efficiency is also large, and for a significant fraction of objects this yields hadronic gamma-ray fluxes above the upper limits from the stacking of NCC cluster by Fermi. A complementary approach (Section 4.2) of assuming a functional dependence for the acceleration of CRs (as in Kang & Ryu 2013) and constraining the required value of $B_0$ at relics yields other problems: either the magnetic field values are unrealistically large ($\gg 10 \mu G$), as in the direct injection model, or the predicted hadronic gamma-ray emission violates the Fermi upper limits for several objects.

Relaxing any of the assumptions made in our modelling (e.g. absence of clumping, only moderate enhancement of the cluster profile along the merger direction, absence of any other source of CRs but the shock, neglect of the adiabatic compression of CRs, exclusion of the core regions for the launch of shocks, etc.) would increase, in some cases drastically, the predicted hadronic emission and thus exacerbate the tension with the standard prediction of DSA.

Outside of cluster cores, the CR-energy is only marginally subject to hadronic and Coulomb losses, and CRs do not move from their radius of injection, which is reasonable if the CRs are frozen into the gas; owing to the tangled distribution of magnetic fields in the ICM, which makes the diffusion time long [i.e. $\tau \sim 2 \times 10^8 yr (R/\text{Mpc})^2 (E/\text{GeV})^{-1/3}$] for a constant $B = 1 \mu G$ magnetic field and assuming Bohm diffusion, e.g. Berezinsky et al. 1997. In the case that CRs can sink further into the cluster atmosphere, they would get adiabatically compressed by the increasing ambient pressure. This effect would increase the CR energy density in the innermost regions and was not included in our simulations. We offer a few possible solutions to this problem:

(i) ‘Geometrical’ arguments: the shocks are launched only in the outer parts, very close to where the relics are presently observed. This is not in accord with what most simulations predict, even though there are cases where the radio relic appears to be very close to the cluster centre (e.g. A2256).

(ii) The Mach numbers inferred from the radio spectrum are systematically overestimated. This would reduce the production of gamma-ray emission in some systems (A3376, A2744), but require a much higher value for $B$. If particles are accelerated directly from the thermal pool and the assumptions of the test-particle regime hold, then the radio-derived Mach number should be consistent with the Mach number derived from X-ray data. In fact, this is the case for all the other merger shocks confirmed at radio relics, except for the relic in 1RXS J0603.3+4214 (Ogrean et al. 2013). In addition, this cannot solve the problems for objects that are already believed to host weak shocks, such as A2256, A754 and Coma.

(iii) The ICM is filled with fossil CR electrons, but not with fossil CR-protons. In this case, shocks can energize the CR electron population without producing hadronic gamma-ray emission above the constraints from Fermi. It is difficult to imagine such a scenario since most acceleration mechanisms in the ICM accelerate CR-protons at least as much as electrons.

(iv) The spectra of pre-existing CR electrons are significantly different from a power law increasing at low energies and the bulk of the population is already highly relativistic ($\gamma \sim 10^5$). In this case, it is conceivable that weak shocks can efficiently energize the electrons, without pushing the CR proton budget above the levels implied by Fermi. Again, it is not trivial to find circumstances under which the relativistic electrons are protected against expedient radiative losses.

(v) In principle, fast streaming of CRs can progressively deplete the downstream region of the shock and reduce the amount of

5 CONCLUSIONS

In this study, we modelled a sample of observed radio relics using DSA and computed the resulting acceleration of CR-protons. We point out several issues with the standard explanation for radio relics: First (Section 4.1), if the magnetic field in the relics is fixed to a large value ($B_0 = 7 \mu G$), the observed radio power demands a high acceleration efficiency for electrons ($\xi_e \sim 10^{-4} - 10^{-5}$). In several cases, i.e. $\sim 10^{-10^2}$ higher than what is typically attained

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**Figure 3.** Synchrotron power at 1.4 GHz and estimated hadronic gamma-ray emission for three models of DSA with an increasing level of re-accelerated CRs (Section 4.2). The magnetic field is derived by matching the observed radio power of real relics, and with an additional big square mark the relics for which the solution yields $B \geq 10 \mu G$. The gamma-ray limits are as in Fig. 1.

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Do radio relics challenge DSA?
hadronic emission (e.g. Enßlin et al. 2011). However, the validity of this scenario is controversial (e.g. Donnert 2013; Wiener, Oh & Guo 2013) and this mechanism has been suggested to be maximally efficient in radio-quiet clusters. Instead, about one half of the objects in our sample also have a central radio halo, at odds with the expectation of this model.

(vi) All models discussed in this paper assume a similar dependence between the acceleration efficiency of CR electrons and protons and the Mach number. However, recent studies using particle-in-cell simulations suggested the possibility of an efficient acceleration of electrons in quasi-perpendicular shocks by oblique whistler waves excited in the shock foot (Riquelme & Spitkovsky 2011). Moreover, it has been shown that the acceleration of electrons and ions might require different magnetic orientations and that the two species can be accelerated in different regions of the shock (Gargaté & Spitkovsky 2012). In order to reconcile the observed relic synchrotron emission and the non-detection of hadronic emission, an assumed oblique magnetic field at shocks would have to lead to an efficient acceleration of electrons but not of protons. However, owing to the present theoretical and numerical difficulties in investigating the process of particle acceleration at the ‘microphysical’ scales, little can be said about the validity of this scenario in galaxy clusters.

Certainly, future deeper limits from the Fermi satellite or other gamma-ray telescopes will greatly enhance the significance of the limits we derived here.

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