An Argument for Nonminimal Higgs Coupling to Gravity

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The coupling of gravity to a scalar field raises a number of interesting questions of principle since the usual minimal coupling obtained by replacing ordinary derivatives with covariant derivatives is not available — they are the same operation on scalar fields. Conformal couplings in the Lagrangian proportional to $\phi^2R$ have been suggested before, usually to maintain conformal invariance for massless scalar fields, but at the cost of breaking the equivalence principle. Here we give intuitive arguments for the appearance of such a term due to fluctuations of scalar particles about their classical world lines. Remarkably, these arguments give precisely a correction of the form required to maintain conformal invariance. We also show that such a term would naturally be expected for the Higgs field in the Standard Model, making a perhaps surprising connection between weak-scale physics and gravity. The nonminimal coupling, whether induced by quantum corrections or already present as a bare term, can be constrained from measurements of the Higgs width assuming the Higgs particle is to be detected.

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INTRODUCTION

The coupling of scalar fields to gravity is a matter of some subtlety. The usual minimal coupling obtained from a flat spacetime Lagrangian by replacing ordinary derivatives with covariant derivatives is not available since both derivatives are the same when acting once on scalar fields. The associated geometrical picture is quite clear. Curvature at a point $x$ is measured classically by parallel-transporting an object with a Lorentz (e.g. a vector) index around an infinitesimal closed path, say the parallelogram spanned by $dx^\mu$ and $dx^\nu$ in a tangent plane at $x$. The curvature tensor $R^\alpha_{\beta\mu\nu}$ associates an infinitesimal rotation matrix $R^\alpha_{\beta\mu\nu}dx^\mu/dx^\nu$ to that plane which represents the fact that the object comes back to $x$ rotated. A scalar test particle, classically at least, will not have any way of indicating how much it has been rotated, so it cannot detect (or couple to) curvature via a covariant derivative.

For a scalar field $\phi$ a nonminimal coupling $\xi\phi^2R$ is often introduced via discussions of the conformal anomaly [1]. Within this picture one starts with a massless scalar field, which is conformally invariant, and finds that a coupling of the form $\xi\phi^2R$ is required so that the trace of the stress-energy tensor $T^\mu_\mu$ is zero. For a general scalar field one expects $T^\mu_\mu$ should be zero when the mass of the corresponding particle is zero.

For a $(+++)$ signature metric, a massive scalar field then obeys the equation:

$$-\hbar^2D^\mu D_\mu \phi + m^2c^2\phi + \xi\hbar^2R\phi = 0,$$

(1)

which can be derived from a Lagrangian density

$$\mathcal{L} = \frac{1}{2}\left(-\hbar^2g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - m^2c^2\phi^2 - \xi\hbar^2R\phi^2\right).$$

(2)

which has $T^\mu_\mu \to 0$ as $m \to 0$ if $\xi = 1/6$. While there has been some debate in the literature (see especially the last reference cited in [1]) over the years, it seems generally accepted that for $T^\mu_\mu = 0$ and conformal invariance to be maintained, one must choose $\xi = 1/6$.

An alternative point of view, which we take here, is to calculate what quantum corrections one would expect to the Klein-Gordon equation for a massless scalar particle. Here we present a heuristic derivation which we feel has the virtue that it makes the nature of the corrections and the naturalness of a non-minimal coupling quite intuitive.

Start with the massless Klein-Gordon equation in momentum space in obvious notation:

$$[E^2 - c^2(p_x^2 + p_y^2 + p_z^2)]\phi = 0.$$  

(3)

On shell we can (even if only locally, but this is a local equation) choose the particle to go along the $z$ axis so that $E^2 - c^2p_z^2 = 0$ and $p_x = p_y = 0$. This is the classical path about which we want to find fluctuations in a space with some curvature.

To leading order in curvature (and without derivative couplings to curvature) we can take the curvature to be represented by a scalar 4-curvature $R$ which is not zero. Any other terms proportional to the curvature tensor with uncontracted indices (like the Ricci tensor) are ruled out by general covariance since $\phi$ is a scalar.

Now consider the particle path fluctuations in the $x$ and $y$ directions (the directions which are off the classical path and can be modelled as small perturbations — we discard the rather drastic large fluctuation from the $x = ct$ path to the $x = -ct$ path which we would effectively have as antiparticles anyway). Note that fluctuations off the classical path are fluctuations off the light
cone, so one might well expect a breaking of naive conformal invariance associated with the particle classically following light-like null paths only.

A slice in the x-y plane through the 4-sphere of radius \( r \) has curvature \( 1/r \) as does one through the x-y plane. Associate a momentum due to being localized by that curvature of \( p = h/r \). A more rigorous derivation will be presented in a later paper\[^1\]. Going back to the Klein-Gordon equation we now have:

\[
[(E/c)^2 - p_z^2 - (h/r)^2 - (h/r)^2] \phi = 0
\]

or, restoring the fact that the \( z \)-axis was arbitrary,

\[
[(E/c)^2 - p_z^2 - 2(h/r)^2] \phi = 0.
\]

Now we want to connect \( r \) to the scalar curvature \( R \).

\( R \) is the sum of sectional curvatures in each plane with each orientation counted separately\[^2\]. For example, in 2 dimensions, \( R = K(x \wedge y) + K(y \wedge x) = 2/r^2 \) which gives the usual factor of 2 from elementary differential geometry. Here we have 12 planes (6 for the usual differential and rotation planes times 2 for 2 orientations) so that 12/r^2 = R or 1/r^2 = R/12. Using Eq. (5)

\[
\left( (E/c)^2 - p_z^2 - \frac{R}{6} h^2 \right) \phi = 0
\]

or, in general

\[
\left( -\hbar^2 D^\sigma D_\sigma + \frac{1}{6} \hbar^2 R \right) \phi = 0
\]

Now we want to connect \( \xi \) to the scalar curvature \( R \).

\( \xi \) is its vacuum expectation value. Making the usual expansion about \( \phi = \frac{v+H}{\sqrt{2}} \) where \( H \) is the experimentally detectable Higgs field, and expanding the term \( \xi R \phi \) then gives the terms:

\[
L_{\text{Higgs}} = -g^\mu\nu \partial_\mu \phi^4 \phi_\nu - \lambda \left( \phi^4 \phi - \frac{v^2}{2} \right)^2 - \xi \phi^4 \phi R.
\]

The first term can be interpreted as a (finite) renormalization of Newton’s constant, the second as a correction to the mass of the physical Higgs particle, and the last term as a coupling of the Higgs boson to gravitons which contributes to the Higgs decay width.

We leave a discussion of the effects of fluctuations due to other fields on the parameter \( \xi \) to another paper\[^1\]. Note that if for any reason one believes that \( R \) is in fact large (for example due to rolled up extra dimensions which would contribute large sectional curvatures to \( R \)) then \( \xi \) could become very large as well. There are also clearly interesting contributions of nonminimal couplings of the kind described here to cosmological models involving scalars, a topic to which we plan to return in a later publication.

A NONZERO BARE \( \xi \)?

Once one has acknowledged the fact that a term of the form \( \xi R \phi^2 \) in the Lagrangian will generally be induced by quantum effects, it is by no means obvious that one can rule out a bare term of almost any value. The term is in many ways analogous to a magnetic moment term, involving as it does a direct coupling to a curvature – here to gravitational curvature, while in the electromagnetic case, to the curvature \( F_{\mu\nu} \) of the \( U(1) \) connection that describes electromagnetism. Nonminimal couplings for electromagnetism are ruled out on the ground of renormalizability, but such arguments are not available in theories with Einstein gravity. Feynman, in his lectures on gravitation^3 discusses such a coupling and points out that the classical value is essentially arbitrary. Since we have not yet had any experimental access to any fundamental scalar fields, it is interesting to ask what such a term would apply for the Higgs field of the Standard Model.

THE HIGGS BOSON

In the Standard Model, the scalar field used to give masses to particles is actually, before the field takes its vacuum expectation value, a massless scalar field with a quartic self-coupling. Neglecting coupling to other fields, but including the nonminimal scalar coupling to gravity, we have

\[
L_{\text{Higgs}} = -\frac{\phi^4}{2} \partial_\mu \phi^4 \partial^\mu \phi - \frac{v^2}{2} \left( \phi^4 \phi - \frac{v^2}{2} \right)^2 - \xi \phi^4 \phi R.
\]

\[^1\] See, for example, N. D. Birrell and P.C. W. Davies, “Quantum Fields in Curved Space” (Cambridge University Press, 1982); S. A. Fulling, “Aspects of Quantum
Field Theory in Curved Space-Time”, London Mathematical Society Student Texts 17 (Cambridge University Press, 1989); R. M. Wald, “Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics” (The University of Chicago Press, 1994); L. Parker and D. Toms, “Quantum Field Theory in Curved Spacetime” (Cambridge University Press, 2009); F. Bastianelli and P. van Nieuwenhuizen, “Path Integrals and Anomalies in Curved Space” (Cambridge University Press, 2009).

[2] See, for example, T. Frankel, “The Geometry of Physics” (Cambridge University Press, 1997).

[3] R. P. Feynman et al., “Feynman Lectures On Gravitation” (Frontiers in Physics, 2002).

[4] in preparation