Quantum Poker – a pedagogical tool to learn quantum computing that is fun to play

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Abstract

Quantum computers are on the verge of becoming a commercially available reality. They represent a paradigm change to the classical computing paradigm, and the learning curve is considerably long. The creation of games is a way to ease the transition for novices. We present a game similar to the poker variant Texas hold ‘em with the intention to serve as an engaging pedagogical tool to learn the basics rules of quantum computing. The difference to the classical variant is that the community cards are replaced by a quantum register that is “randomly” initialized, and the cards for each player are replaced by quantum gates, randomly drawn from a set of available gates. Each player can create a quantum circuit with their cards, with the aim to maximize the number of 1’s that are measured in the computational basis. The basic concepts of superposition, entanglement and quantum gates are employed. We provide a proof-of-concept implementation using Qiskit \cite{qiskit}. A comparison of the results using a simulator and IBM machines is conducted, showing that error rates on contemporary quantum computers are still very high. Improvements on the error rates and error mitigation techniques are necessary, even for simple circuits, for the success of noisy intermediate scale quantum computers.

1 Introduction

Quantum computing is an emerging technology exploiting quantum mechanical phenomena – namely superposition, entanglement, and tunneling – in order to perform computation. Quantum computers have huge potential to transform society in a similar way that classical computers have, because they open up the possibility to tackle certain types of problems that are beyond the reach of classical computers. The first commercially available quantum computers are expected within the next five years, and it is expected that quantum computers will outperform their classical counterparts in some tasks within the same time period. In order to utilize the potential power of quantum computers, specialized algorithms have to be developed. These type of algorithms are fundamentally different from classical algorithms. Getting accustomed to quantum algorithms has a considerably long learning curve and requires a multidisciplinary approach. Typically, knowledge from physics, mathematics, computers science and a firm understanding from an application area such as quantum chemistry, optimization, or machine learning is required.

The New York Times estimated in October 2018\footnote{https://www.nytimes.com/2018/10/21/technology/quantum-computing-jobs-immigration-visas.html} that the global number of high-level researchers in quantum computing may be less than a thousand. The design of games that make use of the underlying rules of quantum computers are a way to attract more interest and ease the transition from classical algorithms to quantum algorithms for beginners. Some games attempt to illustrate basic quantum mechanical effects, and one might call them quantum mechanics games. Others illustrate quantum computing via qubits and quantum circuit building. It is this latter type of quantum computing game that has been developed in connection with this paper. See e.g. \cite{quantum-games} for a recent review of the subject of quantum games. There already exist a number of games in the second category, including ”Battleships with partial NOT gates”, solving puzzles by creating simple programs, and ”quantum chess”. Some previous open-source quantum computing games are available as Jupyter Notebooks in Qiskit’s Github repository of tutorials \cite{qiskit-tutorials}.

A gentle but rigorous introduction to quantum computing intended for discrete mathematicians is given in \cite{qiskit-intro}. Here we will briefly describe the basic notations used in quantum computing. The general state $|\psi\rangle$ of a quantum bit consisting of two levels, denoted by $|0\rangle$ and $|1\rangle$, can be written as

$$|\psi\rangle = c_0 |0\rangle + c_1 |1\rangle,$$

where $|c_0|^2 + |c_1|^2 = 1$, $c_0, c_1 \in \mathbb{C}$. \hfill (1)
By using the polar representation of a complex number and the fact that the state is a unit vector, one can write the general state of a two-level quantum mechanical system (qubit) as

$$|\psi_1\rangle = \cos(\theta/2) |0\rangle + e^{i\phi} \sin(\theta/2) |1\rangle,$$

where $0 \leq \theta \leq \pi$, and $0 \leq \phi < 2\pi$. \hspace{1cm} (2)

Therefore, the state of a single qubit can be geometrically represented by a 2-sphere, called Bloch sphere, see Figure 1. The state of two qubits is given by

$$|\psi_2\rangle = c_{00} |00\rangle + c_{01} |01\rangle + c_{10} |10\rangle + c_{11} |11\rangle,$$

where $|c_{00}|^2 + |c_{01}|^2 + |c_{10}|^2 + |c_{11}|^2 = 1$, \hspace{1cm} (3)

and the general state of a quantum register with $n$ qubits can be expressed as

$$|\psi\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle,$$

where $\sum_{i=0}^{2^n-1} |c_i|^2 = 1$, \hspace{1cm} (4)

where \{0, \ldots, 2^n - 1\} is a basis of the Hilbert space $\mathbb{C}^{2^n}$. Certain operations, often called gates, can be applied to a quantum register. Mathematically these operations are given by unitary matrices in $\mathbb{C}^{2^n \times 2^n}$. Examples are the Hadamard gate $H$, the NOT gate $X$, and the controlled not gate $CX$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \hspace{0.5cm} X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \hspace{0.5cm} CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. $$

Information on the coefficients $c_i$ of a state $|\psi\rangle$ is not directly accessible, and can only be obtained by an operation called measurement. As an example, the laws of quantum mechanics dictate that the probability of $|\psi_2\rangle$ to collapse to the state $01$ upon measurement is given by $|c_{01}|^2$.

## 2 Quantum Poker Rules

The classical Texas hold'em round involves five community cards on the table shared by all the players, while each player holds 2 unique cards in their own hand. The small and big blind bets are placed at the start of each round, relative to a rotating dealer position. The community cards are gradually revealed, the first three - called "the flop" - are revealed simultaneously, and the last two called "the turn" and "the river" are revealed one at a time. The players take turns betting at the start of each round. In order to stay in the game one has to match or increase the highest bet currently in play. Otherwise, one can fold and forfeit the chance to win the pot, i.e., the sum of money that players wager during a single hand or game. After all five cards have been revealed there is a final round of betting before the players can choose to either show their hand in the hope of winning or folding it and forfeiting the pot. The goal is to combine up to five cards from both your own hand and the table to form the best poker hand\(^2\) out of all the players. The player or players with the best hand win(s) the pot.

The quantum poker game considered here draws inspiration from Texas hold 'em poker and shares the structure. The betting rounds in the two games are identical, but the cards are replaced by other objects. Community cards are

\(^2\)See for example the Wikipedia entry of best poker hands
In this example there are four players: Niels, Richard, Werner, and Max. The first game has Niels as small blind and Richard as big blind, meaning they have already money in the pot. The starting screen of the game is shown in Figure 2, except that Werners hand is shown as he has pressed the "Show Hand" button. Pressing the "Show Hand" button again will hide Werners hand again. In the rightmost column on the right-hand side, we see the gates that were available in the "deck" from which the players were dealt. For now, there are no qubits on the table because the flop is first shown after the first round of betting. A player chooses to either check/call or fold by pressing the corresponding button, or raise the current highest bid by entering a self-determined amount into the text box and hitting the enter button during their turn.

As Werner has not placed any bets and is not too optimistic about his cards, he chooses to fold. The turn then passes to Max, who also starts by looking at his cards. As he is more optimistic about his chances than Werner, he chooses to check the current bet. Niels and Richard also decide to check and they stay in the game.

The flop is then revealed. Niels goes first on this turn and checks. Richard thinks he has good gates on his hand to

3 An Example Game

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### Table 1: List of useful quantum operations for quantum poker. Note that $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ and that global phase and normalization have been ignored as is proper for a projective Hilbert space.

| Operation | State |
|-----------|-------|
| $X$ | $|0\rangle$, $|1\rangle$ |
| $H$ | $|+\rangle$, $|-\rangle$ |
| $Z$ | $|0\rangle$, $|1\rangle$ |

The personal cards come from a set of available quantum logic gates which can be applied to the qubits. This set consists of operations acting on one or two qubits. In our implementation, the set consists of the CX gate. The action of each gate applied to the $(|1\rangle, |0\rangle)$- and $(|+\rangle, |-\rangle)$-basis is shown in Table 1. The final score of each player is the number of qubits measured to the state $|1\rangle$ in the computational basis at the end of the round. The winner(s) of each pot is/are the player(s) with the highest score.
use later, and he chooses to bet 10 by entering '10' into the text box and pressing enter. This is shown in Figure 3. From Figure 3 we can also see that qubit 2 has a 50% chance of being measured to $|−⟩$ if measured in the $(|+⟩, |−⟩)$-basis, and a 50% chance of being measured to be $|1⟩$ in the $(|0⟩, |1⟩)$-basis. As in this implementation, all the coefficients in the expansion into computational basis states are real-valued, one can surmise that qubit 2 is entangled with at least one of the other qubits which have yet to be revealed.

Both Niels and Max choose to call Richard’s bet. After the turn is revealed, all three remaining players check, and the river is revealed. By pressing the 'Bell2'-button, and then clicking on qubits 2 and 5, we can see the probability of measuring each of the four Bell states if we were to measure in a two-qubit basis, as shown in Figure 3. From the result in the bottom right corner we see that qubit 2 and 5 are in the state $|00⟩ − |11⟩$, when not considering global phase and normalization. They can thus be disentangled by operating on them with a CX gate. All three players choose to check also in the last round of betting. In the last phase all three of them have to use their gates on the qubits, if they want to have a chance to win.

As Richard was the last to place a bet or raise, he goes first. He chooses to apply a CX gate to the entangled pair to disentangle it into a $|0⟩$ and a $|−⟩$. He then applies a ZH gate to qubit 3 and 5, turning both into $|+⟩$-states.

This gives him one qubit in state $|1⟩$ and four qubits in single-particle states with a 50% chance of measuring 1 each. After he is finished he presses the 'End' button to end his turn. It is then Max’s turn. He applies the X gate to qubit 3, then ZH to qubit 4 before using CX on qubit 4 with qubit 3 as control. This gives him three qubits in a single-particle state of $|1⟩$, and two qubits in an entangled state such that either both measure to 1 or 0. Finally Niels applies an X gate to qubit 3 and an H gate to qubit 4, giving him the same chance of succeeding as Max. Then the qubits are measured, giving Niels and Max a score of 5 and Richard a score of 3. Thus Niels and Max split the pot, winning 30 each, i.e. 10 more than they bet.

After closing the window, a new round will start, the position of the dealer and the blinds moves in a clockwise direction. If a player is left with no money at the end of a round, they are eliminated, and the game continues until only one player has money left.

4 Implementation Details

The implementation is based on the Qiskit package for Python from IBM [1]. This provides the framework required to simulate a quantum circuit on a classical machine, as well as the possibility to execute quantum circuits on real quantum
Figure 5: The probability distribution of Max’s qubits for 8192 executions from the example game in the computational basis for different circuits which in principle give the same end state as Max’s. The original circuit produced by the game is shown in red, the circuit using the smallest number of SWAPs in green, and the circuit using the highest number of SWAPs in blue, both configurations is given in equation (5). Note that, for readability, the state labels have been corrected to show them as if the qubits had not been permuted. Furthermore, the states are labeled by Qiskit’s convention, i.e., 01101 and 11111, are the only ones that would have appeared (with equal probability) on a system without noise.

The connectivity of the qubits and error rates of operations on single qubits and between different qubits on IBMQX2 are shown in Figure 6a. It shows that CX gates are more error-prone than single-qubit gates. If an operation between two qubits that are not connected has to be executed, one needs to apply SWAP gates. SWAP gates rely on a combination of three CX gates. The number of the necessary SWAP operations depends on the connectivity graph of the qubits and the circuit to be executed. Minimizing the number of SWAP gates can thus offer a way to improve the accuracy of an execution. The problem of finding the optimal initial ordering of qubits is in general NP-complete. However, the poker game only has 5 qubits, so there are only 5! = 120 different possible initial configurations. We use a brute force method for finding the best and worst initial configurations, under the constraint that SWAP gates are only applied right before a CX gate that does not conform to the connectivity of the qubits as shown in Figure 6a and only a maximum of one such SWAP gate is applied before each CX gate. The initial configurations are then ordered according to the number of SWAP gates that are necessary, and the best- and worst-case are defined as those initial configurations needing the fewest and most SWAP gates respectively. Note that the qubits need not be swapped back after a CX gate, rather the rest of the circuit is modified to the new configuration. The number of different initial configurations of qubits per number of required SWAP gates is given in the table shown in Figure 6c.

Two permutations of the qubits yielding a best- and worst-case configuration are

\[
\sigma_{\text{best}} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 3 \end{bmatrix} = (34),
\]

\[
\sigma_{\text{worst}} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 2 & 0 & 4 & 3 & 1 \end{bmatrix} = (0241).
\]
(a) The connectivity of the quantum computer IBMQX2\cite{6}. The error rates are representative, but vary with each calibration. The error depends on which qubits are involved.

\[
q_0 : |0\rangle X •
q_0 : |0\rangle X •
q_0 : |0\rangle • X •
q_0 : |0\rangle H Z Z H
q_0 : |0\rangle H Z •
\]

\[
c_0 : 0
c_0 : 0
c_0 : 0
c_0 : 0
c_0 : 0
\]

(b) The circuit resulting from Max’s game. The gates HZZH in the dotted box have been removed before execution since they are equivalent to the identity operator.

Figure 6: Circuits with multi-qubit operations that involve qubits that are not connected need to execute SWAP gates. SWAP gates involve three CNOT operations. Since the multi-qubit CNOT error rate is roughly a factor of 10 larger than the single-qubit U3 error rate, one can try to minimize the number of SWAP operations needed in order to minimize noise in the results.

(c) For the given circuit, there are 5! = 120 different possible permutations, leading to the necessity of executing 1, 2 or 3 extra SWAP operations, due to non-connected qubits. There are only 8 optimal permutations.

| Number of SWAPs | Number of configurations |
|-----------------|--------------------------|
| 1               | 8                        |
| 2               | 56                       |
| 3               | 56                       |

These qubit permutations were used to make the best- and worst-case circuits executed to obtain the data shown in Figure 5. It should be noted that the ordering of qubits in the initial circuit needs only one SWAP gate to execute the circuit. However, the SWAP gate was not specified when the circuit was sent to IBMQX2. In addition, the original circuit requires one more CX gate to be applied opposite to the directionality of the CX gates of IBMQX2 than the best-case circuit, which in turn requires four extra single-qubit gates.

From Figure 5 we see that the best-case circuit is better than both the original circuit and the worst-case circuit, as is to be expected. However, all three circuits give poor results, and even the best-case circuit gives a 68% chance of measuring a state which should not be included in the result.

5 Conclusion

We have presented a game intended to serve as a pedagogical tool for learning the basic rules of quantum computers. The aim was to make it a fun experience in order to avoid a shortage in experts when quantum computers become commercialized. The quantum poker game works well on classical computers, because it requires only 5 qubits. However, on contemporary quantum computers the use of multiple error-prone CX gates, gives a large error in the output state of the circuits. Even after minimizing the number of SWAP (CX) gates used, our tests gave only about a 32% chance of measuring a valid end state for the given circuit. Thus the game shows that current quantum computers need error mitigation techniques, see e.g., \cite{7}, in order to improve the accuracy of their results.

At the moment this game is available in Python and as a Jupyter Notebook at \cite{8}. To make the threshold for acquiring the game lower, it could be made available on smart phones as well. There are also several other possible variants and extensions of the game. Examples include generating a completely random initial state, the possibility of a Bloch sphere representation of single-particle states, and revealing community gates to be used on a set of qubits shown from the outset instead of revealing qubits. Finally, one could switch the role of gates and qubits. The players would choose certain qubits from their hand on which to act with chosen community gates after the betting rounds. The same goal of measuring as many 1’s as possible could be kept.
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