Abstract

The rank-2 sector of classical 3 + 1 dimensional Ashtekar gravity is considered. It is found that the consistency of the evolution equations with the reality of the volume requires that the 3-surface of initial data is foliated by 2-surfaces tangent to degenerate triads. In turn, the degeneracy is preserved by the evolution. The 2-surfaces behave like 2 + 1 dimensional empty spacetimes with a massless complex field propagating along each of them. The results provide some evidence for the issue of evolving a non-degenerate gravitational field into a degenerate sector of Ashtekar’s phase space.

1 Introduction.

One of the fundamental aspects of the general relativity is which configuration should be interpreted as an absence of a gravitational field. Obviously, a flat metric is not the right choice. It breaks the diffeomorphism symmetry, tells the particles which curves are straight lines and which are not. From the point of view of traditional gravity, the only promising candidates are either $g_{\mu\nu} = 0$ or $g^{\mu\nu} = 0$. However, neither of them exists in the space of the solutions of the Einstein equations naturally. Ashtekar’s phase space for 3+1 gravity [1], on the other hand, contains the zero data for which both the Ashtekar connection and densitized triad vanish. It belongs to the set of data which can not be used to construct non-degenerate 3+1 metric tensors. This set is often called the degenerate sector of the 3+1 theory and its
relevance is justified by the relevance of the most degenerate, zero data. Various aspects of the degenerate sector were studied for instance by Jacobson and Romano [4], Bengtsson [2] and Reisenberger [6]. Matschull [5] showed that also degenerate Ashtekar’s data provides the corresponding 4 dimensional manifold with a causal structure. For many relativists this is yet another argument sufficient to learn more about that sector. The main motivation for our work comes from the recent paper of Jacobson [7]. He considered a general case, when the Ashtekar data defines on an initial 3-surface \( \Sigma \) a triad of colinear vector densities (instead of three linearly independent ones), which he called ‘1+1 sector of 3+1 gravity’. Jacobson solved the Einstein equations completely for that case. A general solution has the following features. The degeneracy is invariant in time and the integral curves defined by the triad and foliating \( \Sigma \) behave like ‘1+1 dimensional vacuum spacetimes with a pair of massless complex valued fields propagating along them’.

Below, we solve the Ashtekar-Einstein equations for a data of a lower degeneracy, where triad defines at a point two linearly independent vectors. The preservation of the reality by the evolution implies the existence of a foliation of \( \Sigma \) into the integral 2-surfaces tangent to a given triad. That integrability condition weakly commutes with the hamiltonian which ensures that the degeneracy is preserved. Analogously to the Jacobson’s case, the Ashtekar-Einstein equations of the 3+1 dimensional gravity make the 2-surfaces behave like 2+1 dimensional empty spacetimes with an extra massless complex field assigned to each surface and propagating along it.

In the last section we indicate relevance of Jacobson’s and the current results for some issues of the evolution of a non-degenerate gravitational field, characterise the remaining degeneracies and briefly discuss a corresponding degenerate case in the Ashtekar theory applied to the Euclidean gravity.

Some of our results have been rederived in a quite elegant way by Matschull in terms of his quasi covariant approach [9].

**Ashtekar’s theory.** The Ashtekar theory is a canonical theory on a space-time manifold \( \Sigma \times R \) where \( \Sigma \) is a three-real-surface of initial data (the ‘space’) and \( R \) is the one dimensional space of values for a time parameter. The phase space consists of the pairs of fields \( (A,E) \) where \( A \) is an algebra \( sl(2,C) \) valued one-form on \( \Sigma \) and \( E \) is an \( sl(2,C) \) valued vector density of the weight 1 defined on \( \Sigma \). Using local coordinates \( (x^a) = (x^1, x^2, x^3) \) on \( \Sigma \) and a basis \( (\tau_i) = (\tau_1, \tau_2, \tau_3) \) of \( sl(2,C) \) we write

\[
A = A_i^a \tau_i \otimes dx^a, \quad E = E^{ia} \tau_i \otimes \partial_a, \tag{1}
\]
where $A^i_a$ and $E^{ia}$ are complex valued functions defined on $\Sigma$. In $sl(2, \mathbb{C})$ we fix the standard bilinear complex valued inner product

$$k(v, w) = -2\text{tr}(vw).$$

(2)

The variables $(A, E)$ are canonically conjugate in the sense of the following, the only nonvanishing Poisson bracket

$$\{A^i_a(x), E^{jb}(y)\} = ik^{ij}\delta^b_a\delta(x, y).$$

(3)

A data $(A, E)$ is accompanied by Lagrange multipliers, a $-1$ weight density $N$ (the density is the lapse function), a vector field $N^a$ (the familiar shift) and an $sl(2, \mathbb{C})$ valued function $\Lambda$, all defined on $\Sigma$. The Hamiltonian is given by

$$H = C_N + C_{\bar{N}} + G_\Lambda$$

(4)

$$C_N := \int_{\Sigma} d^3x NC(A, E) := -\frac{1}{2} \int_{\Sigma} d^3x N F_{ab} E^{ib} E^{ka} c_{ijk},$$

(5)

$$C_{\bar{N}} := \int_{\Sigma} N^a C_a(A, E) := -i \int_{\Sigma} d^3x N^a F_{ab} E^{ib},$$

(6)

$$G_\Lambda := \int_{\Sigma} d^3x \Lambda_i G^i(A, E) := i \int_{\Sigma} d^3x \Lambda_i D_a E^{ia},$$

(7)

where

$$F := \frac{1}{2} F_{ab} \tau_i \otimes dx^a \wedge dx^b := dA + A \wedge A,$$

(8)

is the curvature of $A$ and $D_a$ is the standard covariant derivative

$$D_a w^i := \partial_a w^i + c^i_{jk} A^j_a w^k$$

(9)

(which applied to $E^{ia}$ ignores the $a$ index).

The constraints $C_N, C_{\bar{N}}, G_\Lambda$ generate the time evolution, diffeomorphisms in of $\Sigma$ and the Yang-Mills gauge transformations, the last being $(A, E) \mapsto (g^{-1}Ag + g^{-1}dg, g^{-1}Eg)$ where $g$ is any $SL(2,\mathbb{C})$ valued function.

Apart from the resulting constraint equations, the data $(A, E)$ is subject to the following reality conditions

$$\text{Im}(E^{ia} E^{ib}_i) = 0, \quad \text{Im}(\{E^{ia} E^{ib}_i; C_N\}) = 0.$$
As long as the matrix \((E_{ia})_{i,a=1,...,3}\) is of the rank 3 and the signature of the symmetric matrix \((E_{ia}E_{ib})_{a,b=1,...,3}\) is \((+++\) one constructs an ADM data from \((A, E)\) and the Ashtekar theory is equivalent to the Einstein gravity with the Lorentzian signature. However, the theory naturally extends to degenerate cases, when the ranks are lower then 3.

### 2 The main result.

We will be concerned now with the Ashtekar’s equations for a degenerate case characterised below. (All the considerations are local.)

**The rank 2 assumption.** What we are assuming in this section is the following degeneracy condition to be satisfied everywhere on \(\Sigma\) at the instant \(t = 0\),

\[
\text{rank}(E^a) = 2, \quad \text{and signature}(E^aE^b) = (++0). \tag{11}
\]

(See the next section for the list of the remaining cases.) In the non-degenerate case, the second matrix in (11) is related to the corresponding metric tensor on \(\Sigma\) via \(E^aE^b = \sqrt{qq^{ab}}\). However, that matrix has also its own geometric interpretation. In a non-degenerate case, the 2-area element of a 2-surface \(x^a(r, s)\) parametrised by \((r, s)\), is \(\sqrt{E^aE^b f_a f_b dr \wedge ds}\), where \(f_a := \epsilon_{abc}x^b x^c\). This interpretation, sometimes emphasised in lectures on the Ashtekar variables (see for instance [8]) extends naturally to the degenerate sector.

We fix in \(sl(2, C)\) an orthonormal basis \((\tau_i)\) such that

\[
[\tau_i, \tau_j] = \epsilon_{ijk} \tau_k. \tag{12}
\]

It follows from the first of the reality conditions (10) and from the above assumptions that there exists an \(SL(2, C)\) valued function \(g\) and two real, linearly independent vector field densities \(e_1\) and \(e_2\) defined on \(\Sigma\) such that

\[
g^{-1}Eg = \tau_1 \otimes e_1 + \tau_2 \otimes e_2. \tag{13}
\]

This observation will be usefull below.

**Existence of a foliation tangent to** \(E^a \partial_a\). The reality of the vectors \(e_1\) and \(e_2\) in (13) shows that at each point of \(\Sigma\), \(E\) defines a real, 2-dimensional subspace of the
tangent space which is invariant upon the Yang-Mills gauge transformations. We will see that the resulting family of vector subspaces is integrable. Any given $E^{ia}$, denote by $\det E$ the determinant of the matrix $(E^{ia})_{i,a=1,2,3}$. Since $E$ is degenerate at $t = 0$, the determinant vanishes. However the Poisson bracket $\{\det E, H\}$ does not vanish in general. The reality conditions (10) imply that

$$\text{Im}(\{\det E, H\}) = 0. \quad (14)$$

The determinant is invariant with respect to the gauge transformations $E \to g^{-1}Eg$ so we can use (13) to evaluate the Poisson bracket, and derive

$$\{\det E, H\} = i\epsilon_{abc}[e_1, e_2]^a_b e_1^c. \quad (15)$$

That is, $E$ defines a foliation of $\Sigma$ into 2-sub-surfaces tangent to all the vector fields $\text{Im}(E^{ia})\partial_a$ and $\text{Re}(E^{ia})\partial_a$, $i = 1, 2, 3$.

**Propagating the degeneracy and of the integrability.** From now on let $(x^a)$ be coordinates such that

$$E^{i3} = 0, \quad i = 1, 2, 3, \quad (16)$$

(at $t = 0$). The functional $E^{i3}$ considered as a constraint commutes weakly with the hamiltonian, namely

$$\{E^{i3}, C_N\} = i\epsilon_{ijk}E^{jb}(\partial_bE^{k3} + A^m_b E^{m3}\epsilon_{kmn}) + \epsilon_{ijk}G^jE^{3k}. \quad (17)$$

Therefore, it should be preserved by the hamiltonian evolution, that is

$$E^{i3}(t) = 0 \quad (18)$$

in the coordinates $(x^a, t)$.

**Splitting of $(A, E)$ into the tangent and the transversal components.** We will use the following index notation below,

$$(x^a) = (x^B, x^3), \quad (19)$$

that is $(x^B)$ parametrise the leaves of the foliation $x^3 = \text{const}$. It is convenient now to analyse the constraints and the evolution equations from the point of view
of the splitting of \((A, E)\) into the components \((\mathring{2}A := A_i^i \tau_i \otimes dx^B, \mathring{2}E := E^j C_j \otimes \partial C)\) tangent to the leaves, and the remaining transversal component \(A_3^i\).

Those of the constraints and the Poisson brackets \(\{A_b^i, C_N\}\) and \(\{E^{ia}, C_N\}\) which neither involve \(A_3^i\) nor the transversal derivatives \(\partial_3 A^i_b, \partial_3 E^{iB}\) are
\[
\mathcal{C}(A, E) = -\frac{1}{2} \epsilon_{ijk} F_{AB}^i E^a_j E^{kB} =: \mathcal{C}^{(2)}(\mathring{2}A, \mathring{2}E),
\]
\[
\mathcal{C}_B(A, E) = -i F_{BC}^i E^C_i =: \mathcal{C}^{(2)}(\mathring{2}A, \mathring{2}E),
\]
\[
\mathcal{G}'(A, E) = i D_B E^{iB} =: \mathcal{G}^{(2)}(\mathring{2}A, \mathring{2}E).
\]

\[
\{E^{iB}, C_N\} = i \epsilon_{ijk} D_C (E^j C_N E^{kB}),
\]
\[
\{A_B^i, C_N\} = i \epsilon_{ijk} E^{jB} F_{CB}^k.
\]

All the remaining constraints and the evolution Poisson brackets are
\[
\mathcal{C}_3(A, E) = -i E^B_i (\partial_3 A^i_B - D_B A_3^i),
\]
\[
\{A_3^i, C\} = i \epsilon_{ijk} E^{jB} (D_B A_3^k - \partial_3 A_B^k).
\]

**The equations on \((\mathring{2}A, \mathring{2}E)\).** The equations (20, 21, 22) and (24, 23) come down to a family of equations labelled by values of the coordinate \(x^3\). The fields subject to them are only \((\mathring{2}A, \mathring{2}E)\). Obviously, the components \(F_{BC}^i\) of the curvature of \(A\) set up the curvature of \(\mathring{2}A\) defined on each leaf,
\[
2d \mathring{2}A + \mathring{2}A \wedge \mathring{2}A = \frac{1}{2} F_{BC}^i \tau_i \otimes dx^B \wedge dx^C.
\]

Let us fix \(x^3\) and study the 2+1 dimensional theory of the data \((\mathring{2}A, \mathring{2}E)\) restricted to the corresponding leaf \(\Sigma_{x^3}\) resulting from the equations (20, 21, 22) and (24, 23).

The Poisson bracket (3) induces the following bracket on \(\Sigma_{x^3}\),
\[
\{A_B^i, E^{jC}\}^{(2)} = i k_{ij} \delta^C_B \delta^{(2)}(x, y).
\]

Consider a hamiltonian
\[
H^{(2)}(\mathring{2}A, \mathring{2}E, N, N^A, \Lambda^i) := \int_{\Sigma_{x^3}} d^2x (NC^{(2)} + N^B C_B^{(2)} + \Lambda_i \mathcal{G}^{(2)i}).
\]
It is easy to see that as long as we consider only the shifts which preserve the leaves, that is such that

$$N^3 = 0,$$

then

$$\{A^i_B, H\} = \{A^i_B, H^{(2)}\}^{(2)}, \quad \{E^{iB}, H\} = \{E^{iB}, H^{(2)}\}^{(2)}.$$  

The hamiltonian $H^{(2)}$ algebraically coincides with the Ashtekar’s hamiltonian for the 2 + 1 vacuum gravity \cite{1}. The only difference is, that to define the real 2 + 1 gravity the fields $(A^i_B, E^j_C)$ should satisfy the following reality condition for every space index $B$,

$$A^i_B \tau_i, iE^{ijB} \tau_j \in so(2, 1)$$  

where by $so(2, 1) \subset sl(2, C)$ we mean the real Lie subalgebra spanned by the generators $(i\tau_1, i\tau_2, \tau_3)$,

$$so(2, 1) = \text{span}(i\tau_1, i\tau_2, \tau_3).$$

**The 3+1 theory reality conditions.** Apart from the constraints $C^{(2)}$, $C^{(2)}_B$, and $G^{(2)}_i$ the fields $(A^i_B, E^j)$ are restricted by the reality conditions \cite{10}. From the equation \cite{13} resulting from the first of the reality conditions, we conclude that there does exist a Yang-Mills gauge transformation $E \rightarrow g^{-1}Eg$ such that \cite{32} is obeyed by $E$. Let us restrict ourselves to such gauge, that is assume that $E$ satisfies \cite{32}. Clearly, if $A^i_B$ also happens to satisfy \cite{32} then $\text{Im}(\{E^{iA}E^j_i, H\}) = 0$. Indeed, if we use $iA^2E$ as a variable, then all the factors $i$ get absorbed in \cite{23, 24}, and the only operations used on the $so(2, 1)$ valued fields are the derivatives and commutators.

Thus

$$\{iE^{iB}, C_N\} \in so(2, 1)$$

itself as long as $A^i_B$ satisfies \cite{32}. It is not hard to show that $A^i_B \tau_i \in so(2, 1)$ is also a necessary condition for the 3 + 1 reality conditions \cite{14} to hold, provided the Gauss constraint is satisfied. To see that momentarily, notice, that it follows from \cite{13} that there exists a Yang-Mills gauge transformation generated by some $so(2, 1)$ valued function and coordinates $(x^A, x^3)$, such that (dropping the primes)

$$E = e^\sigma(\tau_1 \partial_1 + \tau_2 \partial_2),$$

\footnote{An easy way to derive this fact is to start with the Ashtekar action of the 3 + 1 dimensional gravity and next restrict it to the case when the spacetime is the orthogonal product of a spacelike 1 dimensional manifold with a 2+1 manifold.}
σ being a real function defined on Σ. It is enough to prove that upon this gauge choice \( A_B^i \tau_i \in \text{so}(2, 1) \). Now, \( \mathcal{D}_B E^{iB} = 0 \) implies that
\[
\text{Im}(A_B^3) = 0, \quad \text{and} \quad A_2^1 = A_2^1,
\]
whereas the reality conditions (10) read
\[
0 = \text{Re}(A_2^2) = \text{Re}(A_1^1) = \text{Re}(A_2^1),
\]
which completes the proof that the one form \( \mathcal{A} \) takes values in \( \text{so}(2, 1) \).

Concluding, the reality conditions of the \( 3 + 1 \) theory and the constraints \( \mathcal{C}, \mathcal{C}_B, \mathcal{G}^i \) get reduced to the constraints \( \mathcal{C}^{(2)}, \mathcal{C}_B^{(2)}, \mathcal{G}^{(2)i} \) and the reality conditions of the \( 2 + 1 \) theory given by \( (2A, 2E) \) on each leaf \( \Sigma_{x_3} \) separately. Moreover, the evolution generated by the hamiltonian of the \( 3 + 1 \) theory coincides with the evolution equations of the vacuum \( 2 + 1 \) gravity. Hence, we have solved completely the equations given by (20,21,22) and (24, 23).

**The equations for the field** \( A_3^i \). Contrary to the equations solved above, the equations on \( A_3^i \) (that is the remaining equations (25, 26) contain also the transversal derivatives \( \partial_3 A_B^i \). One might conclude, that there is some interaction between the leaves. However, the scalar and diffeomorphism constraint equations \( \mathcal{C} = 0, \mathcal{C}_B = 0 \) (and the rank assumptions (11)) imply that
\[
F_{BC}^i = 0.
\]
Therefore, we can chose a Yang-Mills gauge in such that
\[
A_B^i = 0. \tag{39}
\]
In such a gauge, all the transversal derivatives disappear from the equations, and the remaining equations (25, 26) read (for a given \( E^{iB} \) and unknown complex valued \( A_3^i \))
\[
\partial_B(A_3^i E^{iB}) = 0, \tag{40}
\]
\[
\frac{\partial}{\partial t} A_3^i = i\epsilon_{ijk} E^{jB} \partial_B A_3^k, \tag{41}
\]
(if we use a basis \( (i\tau_1, i\tau_2, \tau_3) \) and multiply the first equation by \( i \) then the coefficients become real.) To make that equations really simple, let us apply our knowledge
about the solutions $(A^2E)$ of the vacuum $2 + 1$ gravity equations. Since they are just given by a slicing of the 3 dimensional Minkowski spacetime, there exists a choice of coordinates $(x'^A, x'^3, t')$ in $\Sigma \times R$ (possibly defining a different slice $\Sigma'_{t'=\text{const}}$) such that (dropping the primes)

$$A'_A = 0, \quad E = (\tau_1 \partial_1 + \tau_2 \partial_2). \quad (42)$$

Then a general solution to the equations (25,26) is given by a complex valued potential $\lambda$ defined on $\Sigma \times R$ which satisfies the wave equation

$$\lambda_{tt} - \lambda_{11} - \lambda_{22} = 0, \quad (43)$$

via

$$A^1_3 = -\lambda_2, \quad A^2_3 = \lambda_1, \quad A^3_3 = -i\lambda_t. \quad (44)$$

## 3 Conclusions.

### Summary.

We have solved locally the Ashtekar-Einstein equations for a general case of a data $(A, E)$ which satisfies the degeneracy conditions (11). The reality conditions of the $3 + 1$ dimensional theory with the Lorentzian signature imply that $E(t = 0)$ defines in $\Sigma$ a two dimensional foliation. That integrability property expressed by (16) propagates due to the weak comutation relation (17). Locally, in an appropriate Yang-Mills gauge (39) whose existence is guarantied by the constraints the theory of the sector (11) comes down to a family of independent theories leaving on the leaves of the foliation. For every leaf, the corresponding theory is the vacuum $2 + 1$ dimensional gravity (28, 29) (of the $(- + +)$ signature) plus a complex valued field $A^i_3$ subject to a constraint equation (40) and an evolution equation (41). A general $(A, E)$ may be described in the following way. There exist coordinates $(x^A, x^3)$ on $\Sigma$ such that (18) holds and, for each value of $x^3$, the components $(A^i_B, E^{ij}, N, N^A, \Lambda^i(x^A, x^3, t))$ define on $\Sigma_{x^3=\text{const}} \times R$ a $2 + 1$ dimensional vacuum gravitational field via Ashtekar’s anzatz. There exist coordinates $(x'^A, x'^3, t')$ in $\Sigma \times R$, such that all the Lagrange multipliers vanish and $(A, E)(x'^A, x'^3, t')$ is given by (42-44).

It has been natural to restrict the diffeomorphisms of $\Sigma$ to the ones preserving the leaves of the foliation, that is such that $N^3 = 0$. However, the choice $E^{33} = 0$ implies only $N^3 = N^3(x^3)$. So there is still a gauge symmetry which mixes the theories living on different leaves.
As indicated at the beginning, all our results are local.

The other degenerate sectors. It is worth to list all possible kinds of degeneracy which can potentially occur in Ashtekar’s theory for the Lorentzian signature. If the rank of \((E^i a)\) is maximal then so is the rank of the ‘2-area matrix’ \(E_i^a E^b\). However, since \(E\) is complex valued, in general the rank of the 2-area matrix is lower equal the rank of \((E^i a)\). If we restrict ourselves to semi-positive definite case of the 2-area matrix, the possible cases are \((0, 0), (1, 0), (1, 1), (2, 1), (2, 2), (3, 3)\), where the numbers indicate the ranks of the triad matrix and the 2-area matrix respectively. The cases which are known thus far are \((3, 3), (0, 0), (1, 1)\) and \((2, 2)\). An example of the \((1, 0)\) case is \(E = (\tau_1 + i\tau_2) \otimes X\) and an example of the \((2, 1)\) case is \(E = (\tau_1 + i\tau_2) \otimes X + \tau_3 Y\), where \(X, Y\) are complex vector field densities on \(\Sigma\). Some relevance of these cases is indicated below.

Relevance for the issues of the time evolution of a classical gravitational field. Shortly after introducing the Ashtekar variables it was conjectured that may be the time evolution can carry a nondegenerate data through a degenerate sector of Ashtekar’s phase space back to the non-degenerate sector (of either the Lorentzian or perhaps Euclidean gravity). However no evidence proving or disproving that conjecture has been provided. The recent results on the 1 + 1 sector \([7]\) and the current work show that a non-degenerate data of the Lorentzian gravity can not evolve into the rank \((1, 1)\) or \((2, 2)\) sector.

It is worth noting that the arguments used above do not continue to be true in the Euclidean case.

Degenerate sectors of the Euclidean gravity: similarities and differences. The Ashtekar theory applies also to the Euclidean gravity. All one has to do, is to remove the \(i\) factor from the Poisson brackets, and assume that all the fields are real. This reduces the number of possible degeneracies to the cases \((0, 0), (1, 1), (2, 2)\) and \((3, 3)\). Let us discuss briefly the \((2, 2)\) case. However at each point of \(\Sigma\), \(E\) defines again a 2-dimensional subspace of the tangent space, the obtained distribution may be non-integrable. Indeed, now there is no \(i\) factor in the right hand side of \((13)\). So, locally, we have two cases: either \((i)\) \(E\) defines a foliation of \(\Sigma\) into 2-submanifolds (by an assumption), or \((ii)\) the commutators of vector fields \(E^i a \partial_a\) generate at each point a third, linearly independent vector field. In the case \((i)\), the results of the Sec 2 summarised above still apply modulo removing the \(i\)
from the Poisson brackets and few changes of signes (this time we don’t have to prove the reality, since everything is real by definition). In the case (ii), on the other hand, the calculation analogous to that of eq. (13) shows that $\frac{d}{dt}\det E \neq 0$ at $t \neq 0$. This proves, that for a sufficiently small $t$, the evolution carries our $(A, E)$ out of the $(2, 2)$ sector into a non-degenerate data.

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