The coronavirus disease (COVID-19) pandemic has impacted many nations around the world. Recently, new variant of this virus has been identified that have a much higher rate of transmission. Although vaccine production and distribution are currently underway, non-pharmacological interventions are still being implemented as an important and fundamental strategy to control the spread of the virus in countries around the world. To realize and forecast the transmission dynamics of this disease, mathematical models can be very effective. Various mathematical modeling methods have been proposed to investigate the transmission patterns of this new infection. In this paper, we utilized the fractional-order dynamics of COVID-19. The goal is to control the prevalence of the disease using non-pharmacological interventions. In this paper, a novel fractional-order backstepping sliding mode control (FOBSMC) is proposed for non-pharmacological decisions. Recently, new variant of this virus have been identified that have a much higher rate of transmission, so finally the effectiveness of the proposed controller in the presence of new variant of COVID-19 is investigated.

**KEYWORDS**
COVID-19, fractional-order backstepping sliding mode control, fractional-order model, non-pharmacological interventions

**JEL CLASSIFICATION**
26A33, 93D05, 92-10

1 | INTRODUCTION

In late 2019, in Wuhan, China, the latest human viral pathogen, extreme acute respiratory syndrome coronavirus-2, the source of the coronavirus disease 2019 (COVID-19) pandemic, emerged. The corona virus is spreading around the world and has brought many problems for different countries. The corona virus (COVID 19) has infected and killed many people globally, and counting.\(^1\) The virus enters humans by crossing the species barrier, continuing the infection through human-to-human transmission.\(^2\) There are now ubiquitous mathematical models of infectious disease transmission dynamics. These models play a significant role in helping to measure possible control and prevention methods for infectious diseases.\(^3-5\) The typical classical differential equations, such as the well-known and widely utilized mathematical models for the spread of infections, are the SI, SIS, SIR, SEIR, SIRD, and SEIRD models. Each variable in these models denotes the number of people in different groups. Since the discovery of COVID 2019, numerous models have been suggested to study its dynamics.\(^6-11\) According to the
first information reported in China, Zhong et al\textsuperscript{12} have developed a simple SIR model for predicting the corona virus. Given the environmental impact, Yang and Wang\textsuperscript{13} proposed a broad SEIR model for COVID-19 with variable velocity. Liang\textsuperscript{14} described the spread of the three epidemics, COVID-19, SARS, and MERS, utilizing mathematical models and found that the progress rate of COVID 19 was much higher than that of SARS and MERS. Fractional-order differential equations have recently been utilized to describe the behavior of epidemics.\textsuperscript{15–21} Fractional derivatives, besides to the current state, also depend on historical states and therefore have memory properties.\textsuperscript{16,17} The theory of control has recently emerged as one of the suggested approaches for non-pharmacological interventions.\textsuperscript{22,23} Thus, in recent years, different kinds control techniques have emerged in the literature, such as government feedback control,\textsuperscript{23} feedback linearization,\textsuperscript{22,24} and observer-based control.\textsuperscript{25} There are many techniques for designing robust controllers, such as $H_{\infty}$, fuzzy, or neural network-based techniques.\textsuperscript{26–28} In Gondim and Machado,\textsuperscript{29} the structure of the period and the quarantine rate (percentage of quarantine practices) are examined. The sliding mode controller (SMC) is also intended to design vaccination strategies\textsuperscript{30} and regulate infected individuals.\textsuperscript{31,32} The study of optimal control over the outbreak of the virus in Gondim and Machado\textsuperscript{29} and Morato et al\textsuperscript{33} has been studied. An optimal control method with the aim of reducing the socioeconomic cost function in quarantine decisions can be very efficient.\textsuperscript{34} One of the major problems with epidemics is the over usage of hospital facilities. In Berger,\textsuperscript{35} the bang–bang controller is used to manage hospital beds by knowing information such as the number of infected people. In the present paper, the fractional-order model is used. In Oud et al,\textsuperscript{36} a fractional epidemic model in the Caputo sense with the consideration of quarantine, isolation, and environmental impacts to examine the dynamics of the COVID-19 outbreak is proposed. In Chu et al,\textsuperscript{37} a fractional-order transmission model is considered to study its dynamical behavior using the real cases reported in Saudi Arabia. In Khan and Atangana,\textsuperscript{38} the mathematical modeling and dynamics of a novel corona virus (2019-nCoV) is studied. The brief details of interaction among the bats and unknown hosts, then among the peoples and the infections reservoir (seafood market), are investigated. In Khan et al,\textsuperscript{39} a new mathematical model is developed to understand COVID-19 dynamics and possible control. The model first is formulated in the integer order, and then, the Atangana–Baleanu derivative concept with a non-singular kernel is used for its generalization.

The parameters of the pandemic model are always associated with unwanted uncertainties because the parameters of disease spread and transmission rates, mortality, and contact rates between different people in the community are not constant and may change depending on social conditions and disease variations. Also, new variants of COVID-19 have different parameters and behavior. So a robust control method is necessary for control of COVID-19 outbreak.

The combination of backstepping method and sliding mode control (BSMC) can effectively deal with the problems caused by parameters uncertainties in the model of COVID-19 disease and can improve the robustness of the control system. Also using the long-term memory properties of fractional-order operator in control signal design can enhance the robustness against model uncertainty. So a novel fractional-order backstepping sliding mode control (FOBSMC) is proposed to deal with parameter uncertainty in model of COVID-19 disease. Also, the fractional-order model of COVID-19 is used to verify the effectiveness of the proposed method. Some contributions and advantages of this paper are as follows:

- The new nonlinear fractional-order SEIARM model for COVID19 is utilized to design the proposed controller.
- A novel FOBSMC is proposed.
- The performance of the proposed FOBSMC and the conventional BSMC are investigated against the new variants of COVID-19 (parameter uncertainty).
- The simulation results of the proposed method guarantee that the beds of hospital intensive care unit will never reach saturation level, even if the disease transmission rate increases by 10%.
- The proposed controller can be designed for each country according to the population and hospital facilities of that country.
- Stability of the proposed method is theoretically proved.

2 | BASICS OF THE FRACTIONAL CALCULUS

Fractional calculus is the generalized form of the integer calculation. In this approach, the fractional-order integration and differentiation are defined by the fundamental operator of $D_{a}^{\alpha}$ ($\alpha \in R$), in which $a$ and $t$ are the bounds of the operation. The continuous fractional-order integral and differential operators are defined as follows\textsuperscript{40}: 

Basically, three definitions are presented for fractional calculations as follows: Grünwald–Letnikov (GL), Riemann–Liouville (RL), and Caputo. GL derivative definition of $\alpha$ order is defined as follows:

\[
aD^\alpha_t f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^\lfloor (t-a)/h \rfloor (-1)^j \binom{\alpha}{j} f(t-jh)
\]  

where $h$ is time increment, $\lfloor (t-a)/h \rfloor$ is the integer part and

\[
\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}
\]  

where $\Gamma$ is the gamma function. RL derivative definition of $\alpha$ order is defined as follows:

\[
\frac{\text{RL}}{a} D^\alpha_t f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f(\tau) d\tau}{a(t-\tau)^{\alpha+n+1}}, n-1 < \alpha < n
\]

where $n$ is the first integer number bigger than $\alpha$. Caputo derivative definition of $\alpha$ order is defined as follows:

\[
\frac{\text{C}}{a} D^\alpha_t f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{\alpha-n+1} f^n(\tau) d\tau, n-1 < \alpha < n
\]

To define the Laplace operator $S$ in the fractional calculus, the integer-order transfer function is first approximated. Based on the Oustaloup approximation, the following transfer function is defined in the frequency domain:

\[
H(S) = S^\alpha, \alpha > 0
\]

Then, Equation (6) can be approximated by Equation (7).

\[
H(S) \approx K \prod_{n=-N}^{N} \frac{1 + \left(\frac{S/\omega_{z,n}}{\omega_{b}}\right)^{n\alpha+1/2^{n+1}}}{1 + \left(\frac{S/\omega_{p,n}}{\omega_{b}}\right)^{n\alpha+1/2^{n+1}}}, \alpha > 0
\]

where $K$ is the obtained gain from two sides of Equation (7) with a unit (1 rad/s), and the number of poles and zeros is $2N+1$, and $\omega_{z,n}$ and $\omega_{p,n}$ are defined as follows.

\[
\omega_{z,n} = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{n\alpha+1/2^{n+1}}
\]

\[
\omega_{p,n} = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{n\alpha+1/2^{n+1}}
\]

where $\omega_b$ and $\omega_h$ are the estimation bounds of the upper and lower frequency, respectively. Generally, the following relation is satisfied: $\omega_b\omega_h = 1$ and $K = \omega_h^2$. The definition of fractional integral is as follows:

\[
aD^\alpha_t = \begin{cases} 
\frac{d^n}{dt^n} & \text{Re}(\alpha) > 0 \\
1 & \text{Re}(\alpha) = 0 \\
\int_a^t (dr)^\alpha & \text{Re}(\alpha) < 0
\end{cases}
\]
\[ I^{\alpha}_{a} f(t) = \frac{1}{\Gamma(\alpha)} \int_{a}^{t} (t - \tau)^{\alpha - 1} f(\tau) d\tau = D^{\alpha}_{a} f(t) \]  

**Theorem 1.** If the function \( x(t) \in \mathbb{R} \) is a continuous and real derivative function and \( \mu = \frac{m}{n} \geq 1 \), which is \( m > 0 \) odd numbers and \( n \in \mathbb{N}^+ \), then Equation (11) for \( t \geq a \) is established:

\[ \zeta \mathcal{D}_{a} f^{\alpha}(t) \leq \mu \mathcal{D}_{a} f^{\alpha-1}(t) \zeta \mathcal{D}_{a} f(t), \forall \alpha \in (0, 1) \]  

**Proof:** Details of proof of Theorem 1 are provided in Dai and Chen.\(^{41}\)

**Theorem 2.** Assume a nonlinear fractional-order system \( \zeta \mathcal{D}_{a} x(t) = f(x, t) \) with equilibrium point \( x = 0 \) and the domain \( \mathbb{D} \subset \mathbb{R}^n \) which also includes the origin. Consider \( V(x(t), t) : [0, \infty) \times \mathbb{D} \to \mathbb{R} \) as a derivative and continuous function that is relative to the variable \( x \) Lipschitz and holds in conditions (12) and (13):

\[ \alpha_1 \| x \|^a \leq V(x(t), t) \leq \alpha_2 \| x \|^a \]  

\[ \zeta \mathcal{D}_{a} V(x(t), t) \leq - \alpha_3 \| x \|^a \]  

If \( \alpha \in (0, 1) \), \( x \in \mathbb{D} \), \( t \geq 0 \) and the constants \( \alpha_1, \alpha_2, \alpha_3, a, b \) are selected positively and arbitrarily, then \( x = 0 \) is a Mittag–Leffler stable and if the relations (12) and (13) generally held in the space \( \mathbb{R}^n \), the system is a global Mittag–Leffler stable.

**Proof:** Details of proof for Theorem 2 are provided in reference Li et al.\(^{42}\)

### 3 | MATHEMATICAL MODEL

In this section, dynamic equations using the Caputo fractional-order derivative will be briefly defined as Equation (14). In this fractional-order model, the total human population \( N(t) \) can be divided into five categories: Susceptible \( S(t) \), Exposed \( E(t) \), Infected (symptomatic) \( I(t) \), Asymptomatic infected \( A(t) \), and the recovered people \( R(t) \) and \( M(t) \) represents COVID-19 in a reservoir or place or seafood market. The fractional-order equations of COVID-19 disease are defined as follows:\(^{43}\)

\[ \zeta \mathcal{D}_{a}^{\alpha} S(t) = \Lambda - \mu S(t) - \lambda(t)S(t)(1 - u(t)) \]  

\[ \zeta \mathcal{D}_{a}^{\alpha} E(t) = \lambda(t)S(t)(1 - u(t)) - K_1 E(t) \]  

\[ \zeta \mathcal{D}_{a}^{\alpha} I(t) = (1 - \theta)\omega E(t) - K_2 I(t) \]  

\[ \zeta \mathcal{D}_{a}^{\alpha} A(t) = \theta \rho E(t) - K_3 A(t) \]  

\[ \zeta \mathcal{D}_{a}^{\alpha} R(t) = \tau_a A(t) + \tau I(t) - \mu R(t) \]  

\[ \zeta \mathcal{D}_{a}^{\alpha} M(t) = \phi I(t) + \sigma A(t) - \nu M(t) \]  

where \( 0 < \alpha < 1 \) and \( u \) is the control signal. The system model parameters are given in Table 1. The basic conditions are relevant

\[ S(t_0) = S_0 \geq 0, \ E(t_0) = E_0 \geq 0, \ I(t_0) = I_0 \geq 0 \]  

\[ A(t_0) = A_0 \geq 0, \ R(t_0) = R_0 \geq 0, \ M(t_0) = M_0 \geq 0 \]
The parameters are defined for simplicity as follows:

\[
\lambda(t) = \frac{\eta(I(t) + \psi A(t))}{N} + \eta_w M(t)
\]

\[
K_1 = (\theta \rho + (1 - \theta) \omega + \mu)
\]

\[
K_2 = (\tau + \mu)
\]

\[
K_3 = (\tau_a + \mu)
\]

(16)

The values in Table 1 are for the population of China and \(N(0) = 1.4\) billion.

| Parameter | Description | Value |
|-----------|-------------|-------|
| \(\Lambda\) | The recruitment rate (per day) | \(\mu \times N(0)\) |
| \(\eta\) | Contact rate | 0.134 |
| \(\mu\) | Natural mortality rate (per year) | \(1/(76.79 \times 365)\) |
| \(\psi\) | Transmissibility multiple | 0.0002 |
| \(\eta_w\) | Disease transmission coefficient | 0.0000000080082 |
| \(\theta\) | The proportion of asymptomatic infection | 0.41003 |
| \(\omega\) | Incubation period of \(I\) | 0.0000213 |
| \(\rho\) | Incubation period of \(A\) | 0.480322 |
| \(\tau\) | Rate of recovery of \(I\) | 0.000033 |
| \(\tau_a\) | Rate of recovery of \(A\) | 0.59 |
| \(q\) | Transmission of the virus to \(M\) by \(I\) | 0.0101 |
| \(\sigma\) | Transmission of the virus to \(M\) by \(A\) | 0.1314 |
| \(\nu\) | Removal rate of virus from \(M\) | 0.5 |

4 | STATEMENT OF THE PROBLEM

The disease has a high rate of transmission. If in a specific and short time, the number of infected people in the community exceeds the maximum health facilities in a community; the community will suffer adverse consequences such as saturation of hospital beds, reduced efficiency of medical staff, and public dissatisfaction with the lack of health facilities. As a result, the number of deaths may increase dramatically. Therefore, in different countries, different strategies have been implemented to reduce the rate of disease transmission so that medical facilities are not saturated for a certain period of time. The following constraints can be set accordingly:

\[
I(t) \leq I_{\text{max}}
\]

(17)

where \(I_{\text{max}} = 6.87e6\) indicates the maximum level of health facilities, which determines the number of beds in the hospital intensive care unit. This amount has been announced by the health authorities of each country. The goal is to control the prevalence of the disease in order to not saturate the number of hospital intensive care beds. Therefore, the goal is to control the number of patients. In fact, the number of patients should not exceed the \(I_{\text{max}}\). The basic and important parameter for deciding on non-pharmacological interventions is \(u(t)\) in (14). \(u(t)\) is based on percentage, which can be called the percentage of quarantine and social constraints. However, despite the challenges of quarantine in most countries, it is important to consider how society will be affected by any quarantine rate. In fact, the fixed quarantine rate has its limitations, because if the quarantine rate is high, it will be difficult to implement due to the economic costs incurred during the quarantine, both by the community and the authorities. Also, high-rate quarantine works only for a limited time, because the country is practically shut down after several months of quarantine, and irreparable damage
will be done to society. On the other hand, quarantine at a low rate makes the disease prevalence and has virtually no effect on reducing the prevalence of the disease. Also, the parameters of the pandemic model are always associated with unwanted uncertainties, because the parameters of disease spread and transmission rates, mortality, and contact rates between different people in the community are not constant and may change depending on social conditions and disease variations. Therefore, the quarantine rate can be considered variable over time to decrease or increase if needed. The parameters of the pandemic model are always associated with unwanted uncertainties because the parameters of disease spread and transmission rates, mortality, and contact rates between different people in the community are not constant and may change depending on social conditions and disease variations. Also, new variants of COVID-19 have different parameters and behavior. So a robust control method is necessary for control of COVID-19 outbreak. Hence, in the next section, a novel fractional-order robust method is proposed.

5 PROPOSED CONTROL STRATEGY AND STABILITY ANALYSIS

Designing a robust controller can be very effective in reducing the prevalence of the disease. The controller can overcome the condition in Equation (17). In this section, a novel FOBSMC will be suggested to control the prevalence of the disease. Based on the above discussion, the error can be defined as Equation (18)

\[ e(t) = I_{\text{max}} - I(t) \] (18)

Then, the fractional derivative of error with respect to time can be given by (19)

\[ ^c_0 D_t^\alpha e(t) = ^c_0 D_t^\alpha I_{\text{max}} - ^c_0 D_t^\alpha I(t) \] (19)

where \(^c_0 D_t^\alpha I_{\text{max}}\) will go to zero. Define the stabilizing function

\[ \zeta = \delta e(t) \] (20)

where \(\delta > 0\) is a stabilizing coefficient. An auxiliary error variable will be defined as Equation (21)

\[ e_1(t) = ^c_0 D_t^\alpha e(t) + \zeta \] (21)

The first Lyapunov function is defined as follows:

\[ V(t) = \frac{1}{2} e(t)^2 \] (22)

By taking \(^c_0 D_t^\alpha (. )\) form both side of Equation (22), we can obtain

\[ ^c_0 D_t^\alpha V(t) \leq e(t)^2 \] (23)

A fractional sliding surface is proposed as Equation (24)

\[ \sigma(t) = C_1 e(t) + C_2 e_1(t) + C_3 e_1(t)^{-\alpha} \] (24)

where, \(C_1, C_2, C_3, C_4 > 0\) are the controller parameters and have constant value. By taking fractional derivative of Equation (24), we have

\[ ^c_0 D_t^\alpha \sigma(t) = C_1 ^c_0 D_t^\alpha e(t) + C_2 ^c_0 D_t^{2\alpha} e(t) + C_3 e_1(t) \] (25)

A second augmented Lyapunov function is defined as Equation (26)
\[ V_1(t) = V(t) + \frac{1}{2} \sigma(t)^2 \] (26)

According to Theorem 1 and by taking fractional derivative of Equation (26) and utilizing Equation (23), we have
\[ \begin{align*}
\mathcal{D}_{t}^{\alpha} V_1(t) & \leq \mathcal{D}_{t}^{\alpha} V(t) + \sigma(t) \mathcal{D}_{t}^{\alpha} \mathcal{D}_{t}^{\alpha} V(t) + \sigma(t) \mathcal{D}_{t}^{\alpha} \sigma(t) \\
& = e(t)e(t) - \mathcal{D}_{t}^{\alpha} \mathcal{D}_{t}^{\alpha} V(t) + \sigma(t) \mathcal{D}_{t}^{\alpha} \sigma(t) \\
& = e(t)e(t) - e(t)e(t) - \delta e(t)^2 + \sigma(t) \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) \\
& = -\delta e(t)^2 + \sigma(t) \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) \\
& = -\delta e(t)^2 + \mathcal{D}_{t}^{\alpha} \mathcal{D}_{t}^{\alpha} \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) \\
& = -\delta e(t)^2 + \mathcal{D}_{t}^{\alpha} \mathcal{D}_{t}^{\alpha} \left( \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) - C_4 \sigma(t) \right) \leq 0
\end{align*} \] (29)

By substituting Equation (14) in Equation (29), the control law \( u(t) \) is obtained by the proposed FOBSMC as
\[ u(t) = 1 + \frac{1}{\lambda(t) S(t)} \left( (C_2 (1 - \theta) \omega)^{-1} \left( -\frac{e(t)}{\sigma(t)} \mathcal{D}_{t}^{\alpha} e(t) - C_1 \mathcal{D}_{t}^{\alpha} e(t) - C_3 e_1(t) - C_2 \mathcal{D}_{t}^{2\alpha} I(t) - \frac{\sigma(t)}{\sigma(t)} \right) - K_1 E(t) \right) \] (30)

where \( C_4 > 0 \) is the controller parameter and has constant value, and \( K_1, K_2 \) are defined in Equation (16), and the other parameters and its value are described in Table 1. The proposed control signal (30) is applied to the fractional-order model of COVID-19 disease in Equation (14) to obtain the simulation results for the proposed FOBSMC method.

### 5.1 Stability proof of closed-loop system

Consider the second augmented Lyapunov function in Equation (26) and according to Equations (26–29), and by using the fractional-order model of COVID in Equation (14), one can obtain
\[ \begin{align*}
\mathcal{D}_{t}^{\alpha} V_1(t) & = e(t) \mathcal{D}_{t}^{\alpha} e(t) + \sigma(t) \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) \\
& = e(t) \mathcal{D}_{t}^{\alpha} e(t) + \sigma(t) \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) \\
& = \mathcal{D}_{t}^{\alpha} \mathcal{D}_{t}^{\alpha} \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) - C_4 \sigma(t) \leq 0
\end{align*} \] (31)

By applying the control signal, we have Equation (32)
\[ \begin{align*}
\mathcal{D}_{t}^{\alpha} V_1(t) & = e(t) \mathcal{D}_{t}^{\alpha} e(t) + \sigma(t) \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) \\
& = e(t) \mathcal{D}_{t}^{\alpha} e(t) + \sigma(t) \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) \\
& = \mathcal{D}_{t}^{\alpha} \mathcal{D}_{t}^{\alpha} \left( C_1 \mathcal{D}_{t}^{\alpha} e(t) + C_2 \mathcal{D}_{t}^{2\alpha} e(t) + C_3 e_1(t) \right) - C_4 \sigma(t) \leq 0
\end{align*} \] (32)

By simplifying Equation (32) and using Theorem 2 results:
\[ \begin{align*}
\mathcal{D}_{t}^{\alpha} V_1(t) & \leq -C_4 |\sigma(t)| \leq 0
\end{align*} \] (33)

The stability of the closed-loop system is guaranteed by Equation (33).
6 | SIMULATION RESULTS

The results in this article are based on statistics from China, and the controller parameters are $\delta = 1.1, C_1 = 10, C_2 = 8, C_{3,4} = 5$ and $\alpha = 0.8$. In this section, the results will be examined in four parts: (1) performance survey of model with open-loop controller; (2) performance survey of model with the proposed controller; (3) comparison between open-loop controller, BSMC, and FOBSMC; and (4) performance survey in the presence of new variants of COVID-19.

6.1 | Performance survey of model with open-loop controller

The percentage of quarantine and social constraints (non-pharmacological interventions, i.e., $u(t)$) can directly effect on the disease prevalence. However, despite the challenges of quarantine in most countries, it is important to consider how society will be affected by any quarantine rate. In fact, the fixed quarantine rate has its limitations, because if the quarantine rate is high (close to 100%), it will be difficult to implement due to the economic costs incurred during the quarantine, both by the community and the authorities. Also, high-rate quarantine works only for a limited time.

![Figure 1](https://wileyonlinelibrary.com)  
**Figure 1** Performance of susceptible behavior [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 2](https://wileyonlinelibrary.com)  
**Figure 2** Performance of exposed behavior [Colour figure can be viewed at wileyonlinelibrary.com]
because the country is practically shut down after several months of quarantine and irreparable damage will be done to society. On the other hand, quarantine at a low rate (close to 0%) makes the disease prevalence and has virtually no effect on reducing the prevalence of the disease.

In this section, we examined the behavior of the outbreak model with open-loop controller. In this case, the control signal is assumed to be \( u(t) = [0 \ 0.2 \ 0.4 \ 0.5 \ 1] \). The behavior of fractional model dynamic is examined and compared in terms of 0%, 20%, 40%, 50%, and 100% quarantine (non-pharmacological interventions, i.e., \( u(t) \)).

The performance of susceptible and exposed behavior is shown in Figures 1 and 2. As shown by increasing the amount of input signal (increasing the quarantine limit), the number of susceptible individuals increases. It can be concluded that fewer people have been infected with this disease. In the beginning, when the disease is spreading, everyone in a society is susceptible. As quarantine increases in a community, communication between individuals decreases, which naturally reduces disease transmission, and it can be concluded that the number of susceptible increases with increasing quarantine because fewer people became infected. As restrictions increase, the number of exposed people also decreases (Figure 2).

Figures 3 and 4 show symptomatic and asymptomatic patients. Reducing the rate of disease transmission through place or seafood markets is shown in Figure 5.

![Figure 3](image-url)  
**Figure 3** Performance of infected (symptomatic) behavior [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 4](image-url)  
**Figure 4** Performance of infected (asymptomatic) behavior [Colour figure can be viewed at wileyonlinelibrary.com]
Naturally, increasing the restrictions will reduce the number of symptomatic and asymptomatic patients, but the main and important point is that increasing the restrictions can cause adverse consequences such as social and economic crises and so on.

In open-loop systems, no information from the output will affect the amount of input, and in any case, without measuring the output of the model, decisions are made to adopt quarantine, and the amount of input is constant, that does not lead to good performance. The percentage of 0% to 100% quarantines by trial and error has been implemented on the system model, which according to the results, 50% was the best results for open-loop controllers.

6.2 Performance survey of model with the proposed controller

In the closed-loop system according to the amount of $I_{\text{max}}$, the proposed controller produces the proper control law. The performance of the proposed method (FOBSMC) is shown in Figures 6–10. To obtain Figures 6 and 7, our proposed
control signal in Equation (30) is applied to the fractional-order model of COVID-19 disease in Equation (14). In this paper, the proposed FOBSMC method is compared with the conventional BSMC method. The proposed method has more precise and better performance as shown in Figures 6–10. This method has been able to reduce the level of exposed people and patients in the community. The susceptible people are shown in Figure 6. At the beginning of the outbreak, all people in a society are considered susceptible. Reducing the susceptible population to zero means removing people from the category of susceptible and being divided into exposed-infected groups with symptoms-asymptomatic infections, and so forth. The proposed method (FOBSMC) puts a higher percentage of people in the category of susceptible, which means less infect of people in the community (Figure 6). Fractional-order value $\alpha = 0.8$ is utilized.

The purpose of this article is to manage hospital intensive care beds. The amount of hospital beds is announced by governments ($I_{\text{max}}$). The proposed controller defines the best control law according to this $I_{\text{max}}$. The exposed people are shown in Figure 7. The increase and decrease of exposed people are directly related to the number of infected people.
Reducing the number of exposed people in society has been the most important non-pharmacy intervention strategy of countries. As shown in Figure 7, the proposed method has the lower level of exposed individuals, which in turn reduces the number of infected individuals. Also it can be seen from Figure 7 that the slope and the trend of increasing the number of exposed people is small, which makes it possible to use the hospital facilities without its saturated or overuse. The fewer exposed people naturally means fewer people are affected. However, the BSMC method has reached the exposed population more rapidly and to a higher threshold, which can disrupt the management of the intensive care unit.

In Figure 8, the proposed FOBSMC has a very good performance and has reduced the number of infected people in the community much more than BSMC. This gentle increase process allows the health facilities to not reach their saturation level. On the other hand, gentle growth prevents the accumulation of many patients in a certain period of time; hence, medical staff will not lose their energy for service work and can be most efficient. It can be concluded that the control of the prevalence of the disease and the management of the intensive care unit has been done better with the

![Figure 9](wileyonlinelibrary.com)  
**Figure 9** Infected (asymptomatic) behavior comparison with BSMC and the proposed FOBSMC [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 10](wileyonlinelibrary.com)  
**Figure 10** Place or seafood markets behavior comparison with BSMC and the proposed FOBSMC [Colour figure can be viewed at wileyonlinelibrary.com]
The number of asymptomatic infected individuals is shown in Figure 9. A decrease in the number of asymptomatic infected people indicates a decrease in exposed people. Naturally, if the number of exposed people decreases, people in the community will have less contact with each other and the rate of disease transmission between people will decrease. As shown in Figure 9, the proposed method has a lower level of infected (asymptomatic) people that can greatly help control the disease outbreak. Figure 10 shows the rate of disease transmission through contaminated seafood. Food contamination in the community will decrease when infected people decrease.

6.3 Comparison between open-loop controller, BSMC, and FOBSMC

In this paper, a novel FOBSMC technique is proposed for COVID-19 outbreak by utilizing the fractional-order model dynamic of COVID-19. For a better comparison between open-loop controller, BSMC, and FOBSMC, all the results are shown in Figures 11–15. Results show that the proposed method has been very effective and prevents the saturation of health facilities usage.
Recently, new variant of this virus has been identified that have a much higher rate of transmission. This section contains simulations which illustrates the effectiveness of the proposed controller in the presence of new variants of COVID-19 as a parameters uncertainty in model. The actual parameters of the model are given by contact rate $U = 0.134$, disease transmission coefficient $U_w = 0.0000000080082$, while the nominal parameters are given by contact rate $U = 0.1474$, disease transmission coefficient $U_w = 8.8090e - 09$.

The nominal parameters do not match with the actual ones. Moreover, there is no information on their possible bounds. Though, regardless of these uncertainties, the designed controller would be able to achieve the control goal.

The performance of BSMC and FOBSMC controllers with parameters uncertainty is compared and shown in Figures 16–18. Figure 16 shows the performance of the two controllers against parameters uncertainty for the exposed population. The proposed controller has shown good behavior and has made small changes in increasing the number of exposed people, which shows that the proposed FOBSMC has more robust performance against parameters
uncertainty. On the other hand, the BSMC_Un (BSMC in the presence of uncertainty) has increased the number of exposed people, which can increase the disease in the community.

As shown in Figure 17, the proposed controller has the lowest number of patients. On the other hand, the rate of disease increase was slower than the BSMC method. As we know, if the number of patients people increases,
significantly over a period of time, the health care system will face challenges such as saturating of hospital beds, reducing the efficiency of medical staff, and so on. According to Figure 17, the proposed controller has shown a satisfactory behavior. A 10% increase in the rate of disease transmission and contact of people in a community can pose a major challenge in the management of the health care system, but nevertheless, the proposed controller has shown very good behavior and has been robust to parameter uncertainty. The proposed controller behavior in the presence of uncertainty ensures that the controller is robust and prevents the saturation of hospital beds.

One of the biggest challenges of the corona virus is its incubation period. The incubation period means that the sufferer is unaware that he or she has the disease. From the time the disease enters the body until the person has symptoms, it takes a period of time. This group of infected people who do not know about their disease can be included in the category of asymptomatic infected people. The implementation of social restrictions such as quarantine and school closures should be decided in such a way as to reduce the contact of asymptomatic infected people in the community so that the disease is not transmitted from person to person and this cycle of human transmission is stopped. As shown in Figure 18, the performance of the proposed controller was very good in the presence of a 10% increase in disease transmission and human contact, and its changes compared to BSMC-Un were very small, which according to the results we can ensure the controller is robust. To show the superiority of the proposed method, the amount of area below the plot diagram is shown in the Table 2.

Results show the proposed method has a smaller numerical value than BSMC, meaning that fewer people get infected with FOBSMC method.

### 7 CONCLUSION

The parameters of the pandemic model are always associated with unwanted uncertainties. Also, new variants of COVID-19 have different parameters and behavior. So a novel robust control method is proposed by combination of fractional calculus, backstepping and sliding mode control (FOBSMC). The proposed controller is applied to a fractional-order model of COVID-19. At first open-loop controller for the percentage of 0% to 100% quarantines by trial and error has been implemented on the system model, which according to the results, 50% was the best results for open-loop controller. But the fixed quarantine rate has its limitations and difficulty for implementation; also, the model parameters are always associated with unwanted uncertainties. So the robust proposed method (FOBSMC) is applied.
Simulation results show that the rate of disease increase was slower by proposed FOSMC than the BSMC method; the proposed controller has the lowest number of infected people, and also, it has the lower level of exposed individuals. The fewer exposed people naturally means fewer people are affected. Also, the slope and the trend of increasing the number of exposed people with FOSMC is small, which makes it possible to use the hospital facilities without its saturated or overuse. Finally, the performance of the proposed controller in the presence of new variants of COVID-19 is compared with BSMC and open-loop controller. Simulation results ensure the proposed FOBSMC has more robust performance against parameters uncertainty and prevents the saturation of hospital intensive care beds.

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**CONFLICT OF INTERESTS**

This work does not have any conflicts of interest.

**AUTHOR CONTRIBUTIONS**

Amir Veisi: Conceptualization; data curation; formal analysis; funding acquisition; investigation; methodology; resources; software; validation; visualization. Hadi Delavari: Conceptualization; data curation; formal analysis; funding acquisition; investigation; methodology; project administration; resources; software; supervision; validation; visualization.

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