Total Harmonic Distortion and Output Current Optimization Method of Inductive Power Transfer System for Power Loss Reduction

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ABSTRACT Inductive power transfer (IPT) system is widely used in material handling. A typical structure of the system takes an H-bridge inverter with an inductor-capacitor-inductor (LCL) resonant filter to realize a constant track current supplying changeless energy to the second side. However, the output voltage total harmonic distortion (THD) of the inverter increases, which causes the increase of output current circulation, when using voltage width control method to eliminate source voltage fluctuating. Therefore, a two-stage converter is proposed to optimize the output current circulation. The two-stage IPT system is composed of a boost converter cascaded with an H-bridge resonant inverter. The boost converter is employed to provide a higher and stable DC bus voltage. The H-bridge resonant inverter operates in a fixed width with a constant switching frequency. With the proposed topology, the THD of the high frequency voltage maintains the minimum value to realize minimum output current circulation in the LCL filter. The soft switching is realized to reduce the losses. Furthermore, expressions of coil and track model are presented by combining the theoretical analysis and finite element analysis (FEA). The experimental results show that over 76.6% efficiency is demonstrated in conditions of an 800 W load at the 14% source voltage fluctuation and the maximum efficiency was 78.6%. The range of efficiency variation was 2% compared to a full-bridge system with voltage pulse-width control of which was 4.6%.

INDEX TERMS Inductive power transfer (IPT), material handling, THD optimization, soft switching, finite element analysis (FEA).

I. INTRODUCTION Dynamic IPT system has many characteristics such as safety, convenience, and high efficiency. Furthermore, it can operate in rugged environment with the high reliability, low maintenance cost, and more freedom of motion. An application scenario of typical dynamic IPT system is illustrated in Fig. 1. The automated guided vehicles (AGVs) or transfer cars are dynamically supplied and charged during their moving by the coils undergrounds [1]–[6].

Conventionally, a dynamic IPT system consists of the primary side and the second side shown in Fig. 2.

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FIGURE 1. Application scenario of dynamic IPT system.
side through the track current. The second side includes a series of vehicles with pick-up pad and high frequency rectifier. It absorbs the high frequency energy from the track through the pick-up pad [7]–[10].

Inverter is a key component for IPT system shown in Table 2. Class-E IPT system is a promising structure for small power and consumption level to achieve soft switching with high efficiency and frequency [28]–[30]. The switch voltage stress is at least twice of the input voltage. The heat dissipation problem in single switch device also hinder its expansion in medium & high power application. Push-pull IPT system gets attention for its simple structure and high efficiency in IPT application [31]–[33]. However, It needs bigger and costlier inductors compared to capacitances with high-energy storage density. Compared to these IPT systems, H bridge is the most suitable inverter for this application, which has high output voltage, heavy output power, small voltage stress, and small power stress.

Considering the disturbance of source voltage, pulse width modulation (PWM) and pulse frequency modulation (PFM) are the most common way for regulation control. As mentioned early, compensation networks desire working at fixed frequency for leakage inductance compensation. Therefore, PWM based on voltage control with fixed switching frequency is widely used in the IPT systems [23], [24]. Since pulse width modulation method is used to stabilize the current of the track in primary side, the pulse width of the high frequency voltage wave is changed by the H-bridge inverter [23]. Large harmonic voltages are produced when the pulse width of the high frequency voltage changes. At this condition, only the fundamental voltage component can pass through the LCL networks and produces the current in the track, while other harmonic voltage components produce large harmonic currents. These harmonic currents circulate between the H bridge and the LCL network, which cause large circulating reactive power, increase the power loss, and enlarge the voltage and current stresses of the device [24].

[1] mentions the use of two-level structure for optimization, but it rarely mentions how to use the intermediate dc-dc converters for optimization. The H bridge is modulated by the square wave with full duty cycle. Meanwhile, little attention is focused on the optimization of the H-bridge output voltage. In fact, total harmonic distortion (THD) of the high frequency voltage has a minimum value when the width is set as a specific value as derived in the following pages. Then, the output current of the H bridge is optimized with a minimum THD value to improve the efficiency and the reliability of the primary side.

Another challenging work for dynamic IPT system is the design of the pick-up pad. There are many different shapes of pick-up pad shown in the literatures, such as round, rectangular, S- type and E- pick up [34]–[36]. The magnetic circuit, migration, and mechanical structure are analyzed in detail. The mathematical method has been presented in these applications. The rectangular pick-up pad is widely used in material handling. Nevertheless, little analysis of the rectangular pick-up pad is carried out for it with series- and series-compensation in high current and low voltage applications.

The main contributions of this paper are as follows:

1. A new IPT system is proposed for automated guided vehicles (AGVs), which consists of a boost converter cascaded with an LCL filter coupling H-bridge resonant inverter.

2. A new optimization method is proposed for minimizing the total harmonic distortion (THD) and output current of the H bridge.

3. A new design method for the pick-up pad is proposed for high current and low voltage applications.

The benefits of this paper are as follows:

1. The proposed IPT system is suitable for AGVs with high efficiency and low power consumption.

2. The proposed optimization method can improve the efficiency and reliability of the primary side.

3. The proposed design method for the pick-up pad is suitable for high current and low voltage applications.

The disadvantages of this paper are as follows:

1. The proposed IPT system is suitable for AGVs with high efficiency and low power consumption.

2. The proposed optimization method can improve the efficiency and reliability of the primary side.

3. The proposed design method for the pick-up pad is suitable for high current and low voltage applications.
In the proposed system, the boost converter is employed to provide a higher and stable output voltage when the source voltage fluctuates. Due to the constant current characteristics of the LCL filter and the stable output voltage of the boost converter, the H-bridge resonant inverter operates in a constant output voltage width with a constant switching frequency.

2. An output current optimization method, which calculates the minimum THD of the output voltage for decreasing the harmonic current circulation in the H-bridge resonant inverter, is proposed to minimize the output current.

3. Impedance matching methods, which match LCL filter with slight inductance characteristic and the inductor of boost to work in discontinuous current mode (DCM), are used in the proposed system to achieve zero voltage switching (ZVS) in the H bridge and zero current switching (ZCS) in the boost converter.

4. The optimized coil design is proposed through the mathematical analysis and finite element analysis (FEA) based on the coils and track magnetic model, which gets the maximum efficiency with the optimization of coil turns and coil distance.

The remainder of the paper is organized as follows. Section II presents the topology, operation principle and soft switching condition of the proposed two-stage IPT system. Based on these, Section III gives a design approach for this structure. The coupling mechanism is evaluated and optimized by using Biot-savart’s law and FEA. Finally, section IV and V present the experimental results, some discussions, and conclusions.

II. PROPOSED CURRENT OPTIMIZATION METHOD

The proposed system consists of two switch-circuit stages defined as the boost regulator stage and the H-bridge resonant inverter stage shown in Fig. 3. The boost converter in the front regulates the voltage $V_{dc}$ of $C_{dc}$ to avoid the influence of the source voltage fluctuation. The H bridge converts the dc voltage into HFAC to the second side through the track current. LCL network including $L_{f1}$, $L_{f2}$, and $C_{f}$ is utilized to filter the high frequency harmonics. Series and series type capacitors $C_{p}$ and $C_{s}$ compensate leakage inductances of the track and the pick-up pad. To minimize the harmonic circulation of the H-bridge resonant inverter and LCL network, the H-bridge resonant inverter operates in open loop control mode with a fixed output voltage width. Soft switching of the proposed two-stage IPT system is realized since the boost converter operates in DCM. The constant switching frequency of the H bridge equals the resonant frequency of the LCL resonant network.

A. POWER CIRCULATION OPTIMIZATION WITH MINIMIZED THD

The THD expression of the high frequency voltage is derived, and it finds that the THD has a minimum value when the width is set as a specific value. Therefore, the output voltage of the H bridge is optimized with a minimum THD value to improve the efficiency and the reliability of the primary side of the dynamic IPT system. Dynamic IPT system must keep in the CC mode of the track because of the uncertain load characteristics. The H-bridge resonant inverter along with LCL network and series-series type capacitors $C_{p}$ and $C_{s}$ are shown in Fig. 4(a). $L_{f1}$ is the inductance at the inverter side. $L_{f2}$ is the inductance at the track side. $C_{f}$ is the resonant capacitance at the primary side. The parasitic resistances of the LCL are assumed to be small, which can be neglected. The switching signals, electrical waveforms, and control signals of the H bridge are shown in Fig. 4(b).

The resonant frequency between $L_{f1}$ and $C_{f}$ is the same as the switching frequency $\omega$. The conditions to reach the zero phase angle (ZPA) are given by

$$\omega^2 = \frac{1}{L_{p}C_{p}} \omega_1^2 = \frac{1}{L_{s1}C_{s1}}, \quad \text{and} \quad \omega^2 = \frac{1}{L_{s2}C_{s2}}.$$

Here, $L_{s1}$ to $L_{s2}$ are compensated with $C_{s1}$ to $C_{s2}$. As a result, capacitors $C_{s1} - C_{s2}$ and $C_{p}$ completely compensate the leakage inductors no matter what happens to $R_{eq1} - R_{eq2}$, and mutual inductors $M$.

As shown in Fig. 4(b), $V_{ac}$ is the output voltage of the inverter. The voltage phase angle $\theta$ is changed with S1-S4 phase shift modulation. According to Fourier’s formula, $V_{ac}$ can be expressed as

$$V_{ac}(t) = \sum_{n=1}^{\infty} \frac{4V_{dc}}{n\pi} \sin \left( \frac{n\theta}{2} \right) \cos \left[ n\omega \left( t - \frac{T\theta}{4\pi} \right) \right] \quad \text{mod} \, \text{odd} \,. \quad \text{(2)}$$

In most IPT studies, first harmonic approximation (FHA) is used to analyze the topology. The fundamental component of $V_{ac}$ is given by

$$V_{ac1}(t) = \frac{4V_{dc}}{\pi} \sin \left( \frac{\theta}{2} \right) \cos \left[ \omega \left( t - \frac{T\theta}{4\pi} \right) \right] \,. \quad \text{(3)}$$
The $Z_{in}$ in frequency domain is given by

$$Z_{in} = j\omega L_{1} + \left[ \frac{1}{j\omega C_{f}} \right] \left( \frac{j\omega L_{2} + R_{eq}}{} \right)$$ (4)

where $R_{eq}$ is the equivalent impedance of the output port in LCL circuit. The conditions to realize ZPA and CC mode of the primary track are given by

$$L_{1} = L_{2}, \quad j\omega L_{1} = \frac{1}{j\omega C_{f}}.$$ (5)

According to (1), (4), and (5), $Z_{in}$ can be obtained the pure impedance characteristic and it is then given by

$$Z_{in} = \frac{\omega^{2} L_{1}^{2}}{R_{eq}}.$$ (6)

$\dot{V}_{ac1}$ is transformed to the primary track $I_{p}$ using Norton’s equivalent equation in frequency domain, which is given by

$$I_{p} = \frac{4V_{dc}}{\pi \omega L_{1}} \sin \left( \frac{\theta}{2} \right) \angle -90^\circ.$$ (7)

Here, the $\dot{V}_{ac1}$ is defined as

$$\dot{V}_{ac1} = \frac{4V_{dc}}{\pi} \sin \left( \frac{\theta}{2} \right) \angle 0^\circ.$$ (8)

Note that the fundamental voltage component $V_{ac1}$ passes through the LCL networks and produces the current $I_{p}$ in the track. However, other harmonic voltage components of $V_{ac}$ produce large harmonic power circulation in LCL network and H bridge. To optimize this, the factor related to $\theta$ should be considered. THD of $V_{ac}$ is introduced to measure the quality of $V_{ac}$.

$$THD_{u} = \frac{\sqrt{\frac{V_{dc}^{2}}{\pi} - |\dot{V}_{ac1}|^{2}}}{|\dot{V}_{ac1}|} = \frac{\pi^{2} \theta}{1440 \sin^{2} \left( \frac{\theta}{2} \right)}$$ (9)

where $n$ is odd. $\dot{V}_{ac1}$ is defined as

$$\dot{V}_{ac1} = \frac{4V_{dc}}{\sqrt{2}\pi} \sin \left( \frac{n\theta}{2} \right)$$ (10)

where $V_{ac1}$ is defined as

$$V_{ac1} = |\dot{V}_{ac1}| = \frac{4V_{dc}}{\sqrt{2}\pi} \sin \left( \frac{\theta}{2} \right).$$ (11)

Combined with (9) and (11), the curves of the $\theta$ between the $THD_{u}$ and $V_{ac1}$ are shown in Fig. 5.

According to the characteristic shown in Fig. 5, big change of $\theta$ is needed to control the fluctuation of bus voltage in PWM based voltage control mode, which leads to a large $THD_{u}$ value.

Furthermore, the output power of the H bridge is given by

$$P_{H} = \frac{(1+THD_{u}) |\dot{V}_{ac1}|^{2}}{Z_{in}} = (1+THD_{u})P_{o ut} = V_{ac}I_{ac}.$$ (12)

Then, $P_{out}$ is constant for the output power of LCL networks. $I_{ac}$ is smaller with smaller $THD_{u}$ and higher $V_{ac}$. According to the curves shown in Fig. 5, the $THD_{u}$ has a minimum value of 0.084 when $\theta$ is 133.6 degrees. When $\theta$ changes, $THD_{u}$ is changed significantly. $V_{ac1}$ is monotonously increasing with $\theta$ and it changes limitedly when the range of $\theta$ becomes small. For example, $V_{ac1}$ changes slightly from 0.84 to 0.90 when $\theta$ is between 133.6$^\circ$ and 180$^\circ$, while $THD_{u}$ is changed from 0.084 to 0.233. It is significant that the change of $\theta$ for getting a constant voltage (CV) of $V_{ac1}$ causes large harmonic components of $V_{ac}$, which reduces the efficiency and enlarge the voltage and current stresses of the device. Therefore, to optimize the output current of the H-bridge resonant inverter with mini-mized $THD_{u}$ value, the voltage width $\theta$ of the output high frequency voltage is set as a fixed value of 133.6 degrees in the proposed two-stage IPT system. Instead of changing $\theta$ to get a CV of $V_{ac1}$ in conventional dynamic IPT system, the proposed system introduces a boost converter in the front.
to regulate and increases the input voltage in the H-bridge resonant inverter. According to (2), it also enhances \( V_{dc} \) in a reasonable condition to compensate the voltage loss caused by the reducing \( \theta \) to implement the smaller \( I_{ac} \).

**B. DESIGN OF THE PROPOSED TWO-STAGE IPT SYSTEM**

The boost converter is shown in Fig. 6. The filter capacitors of input and output are \( C_{dc} \) and \( C_i \) respectively. \( L_1 \) is used to smooth the current fluctuation caused by the voltage drop between \( V_{dc} \) and \( V_i \). \( V_i \) is regulated by controlling the on-off of \( S_3 \).

![Boost regulator circuit.](image)

**FIGURE 6.** Boost regulator circuit.

To achieve the soft switching, the boost converter works in DCM. \( V_{dc} \) and \( V_i \) are considered to be constant due to the large capacitance value of \( C_{dc} \) and \( C_i \). The key waveforms of the boost regulator are shown in Fig. 7. The boost regulator prefers to work in DCM. \( S_3 \) turns on and \( D_6 \) turns off in both zero-current-switching (ZCS) compared to continuous conduction mode (CCM). The interval \( t_0 - t_4 \) describes four stages of operation during one complete switching cycle in Fig. 7(b). \( T_b \) is defined as the time of one period, which is constant in this paper. \( D \) is defined as the ratio that is the conduction time divided by the period. \( D \) changes for regulating \( V_i \), which is caused by the fluctuation of \( V_{dc} \).

The switching mode is divided into four operating modes in one complete period in DCM mode.

Mode 1 \([t_0 - t_1]\): Current flows through \( S_5 \). The current of \( L_1 \) increases linearly and can be expressed as

\[
i_{L1}(t) = \frac{V_{dc}}{L_1} (t - t_0)
\]

where \( i_{L1}(t) = \frac{V_{dc}}{L_1}DT_b \).

Mode 2 \([t_1 - t_2]\): \( S_5 \) turns off with zero voltage switch (ZVS) at \( t_1 \) because of \( C_{SS} \). \( D_6 \) turns on at the same time to pass inductive current \( i_{L1} \). The current of \( L_1 \) decreases linearly and can be expressed as

\[
i_{L1}(t) = i_{L1}(t_1) - \frac{V_i - V_{dc}}{L_1} (t - t_1).
\]

Mode 3 \([t_2 - t_3]\): \( i_{L1} \) equals to 0 at \( t_2 \). \( i_{L1} \) is then discharging \( C_{SS} \) and charging \( C_{D6} \). A complete resonance period ends at \( t_3 \). The time duration of \( t_{23} \) can be given by

\[
t_{23} = \pi \sqrt{2C_{L1}}.
\]

Here, \( C_{SS} \) and \( C_{D6} \) are the junction capacitance of the MOSFET, which equal to \( C_j \). Since the \( C_j \) of the switch is small, the resonant current through the switch is not large. \( S_5 \) turns on with zero current switch (ZCS) at \( t_3 \).

Mode 4 \([t_3 - t_4]\): \( i_{L1} \) equals to 0 after half resonant period at \( t_3 \). The mode continues until the next circulation comes. The output power can be given by

\[
P_{out} = \frac{1}{2} \frac{V_{dc}V_i^2}{L_1}DT_b < T_b.
\]

The requirement of realizing DCM mode is given by

\[
T_1 = \left(1 + \frac{V_{dc}}{V_i - V_{dc}}\right)DT_b < T_b.
\]

The soft switching of the H-bridge resonant converter is analyzed in this part. The switching frequency equals the resonant frequency of the resonant network. \( C_1 - C_4 \) are the junction capacitance of S1-S4. The resonant topology has pure resistive characteristics according to (6). It can be equivalent to \( L_4 \), because the tank can be slightly inductive, which can better realize soft switching. The relation of \( C_1 - C_4 \) is given by

\[
C_1 = C_2 = C_3 = C_4 = C.
\]

Initially, S1 and S4 conduct at the same time until \( t_0 \). During the interval \( t_0 < t < t_1 \), S1 turns off causing \( I_{ac} \) passing to D2. As a result, \( I_{ac} \) charges \( C_1 \) and discharges \( C_3 \) with
zero voltage turning-off. The \( I_{ac} \) is approximately constant caused by \( L_4 \). The interval length \((t_0 - t_1)\) is given by
\[
t_{01} = \frac{2CV_{dc}}{I_1}, \quad i_{ac}(t) = I_{ac}(t_0) = I_1. \tag{19}
\]

Then, D2 is conducted because of the current flowing, which causes zero-voltage turn-on of S2. The dead time \( t_{d(dead)} \) between S1 and S2 must also be longer than \( t_{01} \). \( I_{ac} \) is the second primary current reflecting to the primary side and is given by
\[
i_{ac}(t) = \frac{i_{sec}}{K}. \tag{20}
\]

\( i_{sec} \) is the second primary current at the interval \( t_1 < t < t_2 \). \( K \) is the current gain of second side reflecting to the primary side. \( I_{ac} \) decreases to \( I_2 \) at the end of \( t_2 \). S4 is turning off with zero voltage switch with the condition when \( I_2 \) is positive. At the same time, \( I_{ac} \) charges \( C_4 \) and discharges \( C_3 \) at \( t_2 \). So the bigger \( \theta \) is, the easier S4 realizes ZVS. \( C_3, C_4, \) and \( L_4 \) are in series resonance due to phase-shifted duty-cycle loss. \( I_{ac} \) and the voltage of \( C_3 \) and \( C_4 \) are given by
\[
i_{ac}(t) = I_2 \cos \theta (t + t_2), \quad V_{C4}(t) = Z_1 I_2 \sin \omega_t (t + t_2) \quad V_{C3}(t) = V_{dc} - Z_1 I_2 \sin \omega_t (t + t_2). \tag{21}
\]

Here, the expressions of \( Z_1 \) and \( \omega_t \) are given by
\[
\omega_t = \frac{1}{\sqrt{2L_4 C}}, \quad Z_1 = \sqrt{I_4/2C}. \tag{22}
\]

The interval \( t_2 < t < t_3 \) finishes with the voltage of \( C_4 \) rising to \( V_{dc} \). The time of duration is given by
\[
t_{23} = \frac{1}{\omega_1} \arcsin \frac{V_{dc}}{Z_1 I_1}. \tag{23}
\]

D3 is then conducted with the current flowing. That causes zero-voltage turn-on of S3. The dead time \( t_{d(dead)} \) between S3 and S4 must be longer than \( t_{23} \).

The interval \( t_0 < t < t_3 \) shows the half period. The second half period of the cycle is similar. The soft switching condition is relevant to \( t_{d(dead)}, C, L_4, K, \) and \( i_{sec} \) according to the analysis above. The load is also the main issue for soft switching connected with the value of \( I_{ac} \). The soft switching operation cannot be achieved when the load is light. This paper focuses on the condition from half load to full load to realize the soft switching of the H bridge.

### III. DESIGN PROCESS AND CALCULATING EXAMPLE

#### A. PROPOSED SYSTEM DESIGN

From the above, a complete design example of this circuit is shown as follows.

1) According to [37], the voltage from the grid side to dc side is given by
\[
V_{dc} = (1.35 - 1.41) V_{acin}. \tag{24}
\]

The parameters of the boost converter are given in Table 3. \( P_{max} \) means the maximum power to realize the DCM in boost stage for the ZCS in boost stage design.

2) The aim of the proposed boost design is to determine the appropriate parameters of \( L_4 \) when \( V_i \) fluctuates. First of all, the boost converter needs to meet the requirements of voltage regulating range at specified output power in \( P_{out} \) according to (16). It is also expected to work in DCM to realize soft switching. (17) gives the conditions for realizing DCM.

The data are plotted in Fig. 8 to deal with the parameter design for \( L_4 \) based on (16) and (17). The surface below the 1500 W plane meets the requirement of working in specific power output and DCM. \( i_{L1} \) gets smaller as \( L_1 \) gets bigger according to (13). \( L_1 \) equals to 400 \( \mu \)H from the above considerations.

#### TABLE 3. Experiment parameters of IPT system.

| Symbol | Quantity | Value            |
|--------|----------|------------------|
| \( V_{ac} \) | grid source | 380 V ± 7% (AC) |
| \( V_{dc} \) | input voltage | 470 V – 570 V (DC) |
| \( f_0 \) | switch frequency | 50 kHz |
| \( H \) | efficiency of the boost converter | ≥95% (at ≥10% load) |
| \( V_i \) | output voltage | 750 V |
| \( P_{out} \) | maximum power to realize the DCM | 1500 W |
| \( C_j \) | junction capacity | 200 pF |

#### FIGURE 8. Calculated values of \( P_{out} \) with variable \( L_4 \) and \( V_i \).
B. PROPOSED SYSTEM DESIGN

The magnetic flux produced by track A through the j-th coil area is given by

\[ \phi_A = \int_0^{d_1-2(j-1)r} dz \int_{L_{AC}}^{L_{AD}} \frac{\mu_0 I_p}{2\pi \rho} d\rho \]

This can be simplified to

\[ \phi_A = \frac{\mu_0 I_p}{2\pi} \left[ (d_1 - 2(j-1)r) \ln \frac{L_{AD}}{L_{AC}} \right]. \]

The magnetic flux produced by track B through the j-th coil is given by

\[ \phi_B = \int_0^{d_1-2(j-1)r} dz \int_{L_{BD}}^{L_{BC}} \frac{\mu_0 I_p}{2\pi \rho} d\rho \]

This can be simplified to

\[ \phi_B = \frac{\mu_0 I_p}{2\pi} \left[ (d_1 - 2(j-1)r) \ln \frac{L_{BC}}{L_{BD}} \right]. \]

The length values of \( L_{AD}, L_{AC}, L_{BC}, \) and \( L_{BD} \) are calculated by the Pythagorean Theorem shown in Fig. 10.

Furthermore, the total mutual inductance \( M_s \) between the track and the rectangular coils is given by

\[ M_S = \frac{\Psi_S}{I_p} \]

\[ = \frac{\mu_0}{\pi} \sum_{j=1}^{N} \left[ (d_1 - 2(j-1)r) \ln \left( \frac{h^2 + \left[ \frac{d_1 - 2(j-1)r + d_2}{2} \right]^2}{h^2 + \left[ \frac{d_1 - 2(j-1)r - d_2}{2} \right]^2} \right) \right] \]

N is the turns of the coils. \( h \) is the vertical distance between coils and track.

The limits of the coils with symmetrical structures are given by

\[ d_2 + r (N - 1) = d_3 \quad (N \leq 25). \]

The direct current resistance \( R_{dcs} \) is calculated as follows:

\[ R_{dcs} = \rho_{cu} \frac{2N (d_1 + d_3 - 2Nr)}{S_c}. \]

\( \rho_{cu} \) is the electrical resistivity and \( S_c \) is the cross-sectional area of the coil.

The alternating current resistance \( R_{acs} \) considering the skin effect is then given by

\[ R_{acs} = F_R R_{dcs}. \]

Here, \( F_R \) is given by the manufacturers.

Furthermore, \( r \) has minimum limited value considering the proximity and skin effects. According to (24) and (30), the coil transfer efficiency \( \eta_c \) is then given by

\[ \eta_c = \frac{P_{in} - I^2 \rho_{u} R_{dcs}}{P_{in}} = 1 - \frac{P_{in} F_R R_{dcs}}{(\omega MI_p)}. \]

Here, \( P_{in} \) and \( I_s \) are the input power and current of the coils respectively.

As is shown in Table 5, the specific parameters of the coils are given accounting for the application. The coils are made of 700 strands Lizi wire with diameter of 0.1 mm per strand.

Combined with the calculating progress in section B, values of \( M \) and \( \eta_c \) with variable \( N \) and \( r \) are shown in Fig. 11. From the results, the maximum value of \( M \) is
TABLE 5. Coils parameters.

| Symbol | Value       | Symbol | Value       |
|--------|-------------|--------|-------------|
| \(d_1\) | 300 mm      | \(d_2\) | 200 mm      |
| \(d_1\) | 350 mm      | \(r\)  | >= 4 mm     |
| \(S_c\) | 3 mm\(^2\)  | \(h\)  | 15 mm       |
| \(P_{ac}\) | 400 W      | \(\rho_{ac}\) | 2.2×10\(^{-4}\) Ω m |
| \(F_{B}\) | 1.25        |        |             |

FIGURE 11. Calculated values of \(M\) and \(\eta_c\) with variable \(N\) and \(r\). (a) \(M-N-r\); (b) \(\eta_c-N-r\).

8.4 µH. The maximum efficiency \(\eta_c\) is there 96.6%, which considers the copper loss of the coils. With the comprehensive consideration of the weight, cost, \(M\) and \(\eta_c\). Coil parameters can be obtained as \(N = 21\), \(r = 4.2\) mm and \(M = 6.6\) µH respectively.

The 3D Maxwell FEA simulation is then used to optimize and analyze the calculated results. FEA simulation model of the track and the coil are shown in Fig. 12.

FIGURE 12. Finite element simulation model.

Ferrite PC40 strips are placed over the coils to optimize the magnetic circuit, reduce, and shield the magnetic field. They are close to the coil surface. The size of the stripe is 70 mm × 5 mm × 15 mm. The coils parameters with the FEM can be obtained as \(N=21\), \(r = 4.2\) mm and \(M = 6.6\) µH. \(\eta_c\) is then calculated as 94%.

Finally, the magnetic field distribution is evaluated by the simulation results. The magnetic field intensity of the coils is equally distributed, which is shown in Fig. 13 (a). The design of magnetic strips meets the requirement based on Fig. 13 (b). Fig. 13 (c) shows that magnetic field radiation satisfies the relevant standard of the International Commission on Non-Ionizing Radiation Protection (ICNIRP) under the working condition.

IV. SIMULATION AND EXPERIMENT VERIFICATION

A. SIMULATION VERIFICATION

The simulation platform of MATLAB is established to verify that \(THD_u\) of \(I_{ac}\) is the minimum when track current is constant compared to the voltage pulse width control. The value of \(I_{ac}\), \(\theta\), and constant \(I_p\) with voltage pulse width control is shown in Fig. 14 when the source DC bus fluctuates in 800 W Load.

\(\theta\) keeps decreasing as \(V_{dc}\) increase. Meanwhile, the change trend of \(I_{ac}\) is the same as that of \(THD_u\) in Fig. 14. However, the amplitude of fundamental component of \(I_{ac}\) increases during the rise of \(V_{dc}\). This leads to a slight difference in the trend of changes in \(I_{ac}\) and \(THD_u\).

Simulation results of the H-bridge waveforms are presented in Fig. 15. The soft switching is realized in the device switches when \(V_{dc}\) equals 750 V with 400 W load, which is the worst conditions of the soft switching.

In order to verify the advantages of the proposed system, push-pull, boost with push-pull, full-bridge, and the proposed system are compared. The results of the voltage and current stresses, \(THD_u\), and the efficiency are shown in Fig. 16. The simulation maintains the track current \(I_p\) of 60 A,
the 800 W output power, and the input voltage fluctuation of 480 V – 570 V.

Through the comparison of voltage stresses, the voltage stresses of push-pull and boost + push-pull are more than 1500 V. The proposed system has the constant voltage stress of about 800 V. In terms of current stress and THD, the proposed system has small values of about 1.6 A and 0.086 respectively. As a result, the highest efficiency is obtained in the proposed system of about 82%.

The resonant capacitor \( C_f \) flows large current and it is more sensitive to parameter drift. The effects of parameter drift of \( C_f \) are shown in Table 6. The results show that little effect was produced on the system when the parameter drift is ±2%. However, when the parameter deviation increases over ±5%, the parameter drift causes large track current deviation, which increases the output current of the H bridge. As a result, the system efficiency drops sharply, related protection may be activated when the drift is too large. To solve this problem, series and parallel combinations of ceramic capacitors or film capacitors can be employed in practical application. The other parameters will be considered in our further research.

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### B. EXPERIMENT VERIFICATION

An experimental prototype was built using the design parameters given in Table 3, 4, and 5. Fig. 17 (a) shows a whole prototype system, with the proposed system in the primary side shown in Fig. 17 (b), along with the pick-up pad shown in Fig. 17 (c) for each 400 W output.

Voltage pulse-width control was verified in this prototype for comparison. The principles of design is that the minimum input source voltage can get the designed track current. The voltage pulse-width was decreasing with the source voltage increasing. Fig. 18 shows the waveforms of \( V_{ac}(CH1) \) and \( I_{ac} \) (CH2) at half and full load at fixed \( I_p \) (CH3) with different \( V_{dc} \) in PWM voltage width control. The track kept 60 Arms with the voltage fluctuation from 470V to 570V. There was a noticeable change in \( I_{ac} \) with constant output power \( P \) and track current \( I_p \).

In the proposed system, Fig. 19(a) shows the main H-bridges inverter waveforms. \( I_{ac} \) was constant with the \( THD_u \) of 0.084 when the \( V_{dc} \) changes. It is smaller than the H bridge with voltage width control. Meanwhile, the waveforms were the same as the simulation shown in Fig. 15 and verified the ZVS realization. Fig. 19(b) shows the main boost stage waveforms, which verified DCM and ZCS shown in Fig. 7(b). There was basically no voltage spikes leading to small stresses in the main switches.

### TABLE 6. Effect of parameter drift of \( C_f \) in the proposed system.

| \( C_f \) (nF) | \( I_p \) (A) | \( I_{ac} \) (800 W) | Efficiency |
|----------------|-------------|-------------------|-------------|
| 682 (+10%)     | 67.03       | 8.35              | 77.8%       |
| 651 (+5%)      | 63.05       | 5.789             | 80.6%       |
| 632 (+2%)      | 61.32       | 5.114             | 82.5%       |
| 626 (+1%)      | 60.81       | 5.04              | 82.7%       |
| 620 (0%)       | 60.28       | 5.015             | 82.8%       |
| 614 (-1%)      | 59.75       | 5.051             | 82.5%       |
| 608 (-2%)      | 59.21       | 5.148             | 82.3%       |
| 589 (-5%)      | 57.53       | 5.78              | 80.8%       |
| 555 (-10%)     | 54.95       | 7.439             | 78.1%       |
FIGURE 17. Experimental platform of IPT system. (a) Experimental panorama of the prototype. (b) The system of the primary side. (c) The pick up pad.

FIGURE 18. Main experimental waveforms of H-bridge inverter using voltage pulse width control with different shift angles and load (CH1:500 V/div, CH2:20 A/div, CH3:100 A/div, Time:20 µs/div).

Table 7 shows the data of $I_{ac}$ with the width voltage pulse-width changing. The range with variation in $I_{ac}$ was at least 30% with the at least 2.6 times change in $THD_u$.

FIGURE 19. Experimental waveforms of the proposed system of 133.6 degrees of shifted angles. (a) Waveforms of Vac(CH1), Iac(CH2), and Ip(CH3) with half load (CH1:1000 V/div, CH2:10 A/div, CH3:100 A/div, Time:20 µs/div). (b) Waveforms of VS5, IL1 with full load and half load.

TABLE 7. Experimental data of voltage width control method.

| $V_{ac}$ (V) | $\theta$ (°) | $THD_u$ | $V_{ac1}$ (V) | $L_e$ (A) |
|-------------|--------------|---------|--------------|-----------|
| 470         | 180          | 0.23    | 423          | 8.32/8.71 |
| 490         | 154          | 0.11    | 425          | 6.70/7.16 |
| 510         | 144          | 0.09    | 425          | 5.14/6.47 |
| 530         | 126          | 0.087   | 421          | 3.90/5.07 |
| 550         | 115          | 0.10    | 423          | 3.85/4.98 |
| 570         | 110          | 0.12    | 421          | 3.77/6.22 |

This variation of $I_{ac}$ directly affected the efficiency of the inverter. $V_{ac1}$ is nearly the same, which shows a good regulation of realizing constant $I_p$. The data variation was nearly the same as the simulation shown in Fig. 14, which shows a good agreement with the theoretical analysis.

Table 8 shows the data of $I_{ac}$ with the proposed system. The $I_{ac}$ is nearly constant. $V_{ac1}$ is higher than the H bridge with voltage width control. $L_e$ keeps the same because of the fixed $\theta$ and input DC bus voltage of the inverter.
Fig. 20 shows the efficiency curve of the proposed system and the voltage pulse-width control system with variation of the input voltage from $V_{dc}$ to rectifier load. It keeps the same regulating source range, mutual inductor, and load. The proposed system has the maximum loss reduction of 4% in 800 W load than the voltage pulse-width control. It also has the smaller range of efficiency variation of 2% compared to a H bridge with PWM control method system of which was 4.6%. Furthermore, the efficiency was higher with the increase of the load from 400 W to 800 W.

![Efficiency curve of voltage pulse-width control and the proposed system with the variation of the input voltage.](image)

**TABLE 8.** Experimental data of proposed method.

| $V_{dc}$ (V) | $\theta$ (°) | THD$_u$ | $I_{dc}$ (A) |
|--------------|--------------|---------|-------------|
| 470          | 133          | 0.085   | 630         |
| 490          | 133          | 0.085   | 630         |
| 510          | 133          | 0.085   | 630         |
| 530          | 133          | 0.085   | 630         |
| 550          | 133          | 0.085   | 630         |
| 570          | 133          | 0.085   | 630         |

Compared to the H bridge with the voltage pulse-width control, the proposed system has the maximum loss reduction of 4.6% with smaller efficiency variation of 2%. The experimental results show a good agreement with the analyses. To further improve the efficiency of the whole system, the below approaches can be considered in our future work as: 1) The physical magnetic circuit-coupling model can be improved with the larger mutual inductance and more evenly distributed magnetic field. 2) The post-regulation circuit in the secondary side can be optimized to reduce the current and voltage stress of the switch devices, thus the overall switching loss can be reduced. 3) The wide bandgap devices such as SiC and GaN can be selected to reduce the conduction and switching losses of the switch devices. These methods will be considered in our future research.

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