The Solar Flare Complex Network

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Abstract

We investigate the characteristics of the solar flare complex network. The limited predictability, nonlinearity, and self-organized criticality of the flares allow us to study systems of flares in the field of the complex systems. Both the occurrence time and the location of flares detected from 2006 January 1 to 2016 July 21 are used to design the growing flares network. The solar surface is divided into cells with equal areas. The cells, which include flares, are considered nodes of the network. The related links are equivalent to sympathetic flaring. The extracted features demonstrate that the network of flares follows quantitative measures of complexity. The power-law nature of the connectivity distribution with a degree exponent greater than three reveals that flares form a scale-free and small-world network. A large value for the clustering coefficient, a small characteristic path length, and a slow change of the diameter are all characteristics of the flares network. We show that the degree correlation of the flares network has the characteristics of a disassortative network. About 11% of the large energetic flares (M and X types in GOES classification) that occurred in the network hubs cover 3% of the solar surface.

Key words: Sun: flares

Supporting material: machine-readable table

1. Introduction

Since space weather is undeniably influenced by solar activities, investigation of the dynamic variations in the solar atmosphere presents an interesting field of study for researchers. Among large-scale solar phenomena, flares are influential events that release a huge amount of energy, up to $10^{27}$ J (Kane et al. 2005; Bloomfield et al. 2012), and affect space weather (Gallagher et al. 2002; Wheatland 2005). The solar corona is dynamically exposed to the effects of energetic flares (Dwivedi 2003), which frequently occur over active regions (ARs) and manifest as radiation in the extreme ultraviolet and shorter wavelengths. Generally, the accumulated energy of the freezing plasma in a twisted case of magnetic fields appears as ephemeral disturbances, while magnetic lines are reconnected, leading to flares in ARs. Solar flares directly increase the complexity of evolving magnetic fields in ARs (Priest & Forbes 2002; Aschwanden 2005). The accelerated particles of flares can cause disturbances to satellites and electrical power sources. Thus, studying the statistical properties of flares, flare simulations, and flare predictions have been the subject of many scientific articles (e.g., Alpert 2000; Parker 2000; Zhang et al. 2010; Bloomfield et al. 2012; Barnes et al. 2016; Muhamad et al. 2017). It has been accepted that these flare events are rooted in the solar interior magnetooconvection (Kosovichev & Zharkova 1998; Stein 2012).

The sudden flash of the flares generates waves within the solar atmosphere that are similar to the seismic waves produced during earthquakes. Both solar flares and earthquakes locally occur, with the intensive release of energy and momentum with temporary fluctuations in their timeseries. The energy frequency of both flares and earthquakes follows the power-law distribution (Crosby et al. 1998). To characterize the behavior of solar flares and earthquakes, commonly accepted evidence shows that both follow the same empirical laws (de Arcangelis et al. 2006). For solar flares, some of the most important laws exhibit scale invariance and self-organized criticality (de Arcangelis et al. 2008; Aschwanden 2011a; Aschwanden et al. 2016). By analogy with Omori’s law for seismic sequences, the power-law distribution is obtained for the main flares and after-flare sequences (de Arcangelis et al. 2008).

The study of complex systems requires the analysis of network theory. This helps to investigate the procedure of changes occurring in the system and to maybe extract a pattern for prediction. Therefore, to analyze the flare complex system, we employed a graph theory to construct the complex network. A network (graph) consists of nodes (vertices) and edges (links). Generally, it can be considered a simple, directed or undirected, weighted or unweighted, graph. Several networks of interest are regular, complete, scale-free, and small-world indicating the many possible physical descriptions of the system. By comparing each network property with the equivalent characteristics of the random network, first, the network type must be identified. Some characteristics (e.g., degree distribution, clustering coefficient, characteristic path length, and diameter) in the network are obtained to determine the network type. The values of these parameters help us to analyze the behavior of the system. It is normal to construct two main complex networks (i.e., scale-free and small-world networks) to conduct a survey of physical systems (Abe & Suzuki 2006; Daei et al. 2017; Rezaei et al. 2017). In a recent study, Daei et al. (2017) constructed a complex network for solar ARs. They obtained that the AR network follows regimes that govern the scale-free and small-world networks. It was shown that the probability of flare occurrence increases where ARs act as hubs all over the network.

Here, we investigate the conditions of flares as a complex system using a detrended fluctuation analysis applied to the timeseries of flares, as well as their nonlinearity, limited predictability, and so on. To do this, we construct a network of 14395 flares with regard to their locations and occurrence times. Then, we compute the degree distribution of the nodes, clustering coefficient, characteristic path length, diameter, and degree correlation of the flare network.
The paper is organized as follows. In Section 2, the description of the solar flare data set is introduced. In Section 3, we survey the complexity characteristics for the solar flare system. In Section 4, the flare network is constructed. In Sections 5 and 6, we discuss the properties of the random, scale-free, small-world, and regular networks, respectively. In Section 7, we describe assortative, disassortative, and neutral networks by employing degree correlation. In Sections 8 and 9, the results and conclusions are presented, respectively.

2. Flare Data Sets

Our data set is sourced from the 14395 solar flares taken from 2006 January 1 to 2016 July 21, available at http://www.lmsal.com/solarsoft/latest_events_archive.html.

This site, which is associated with the Lockheed Martin Solar and Astrophysics Laboratory (LMSAL), provides information about the properties of solar features and updates its data center with the help of solar physics teams at National Aeronautics and Space Administration (NASA) and Stanford University. The other data center is the Solar Monitor System, which is already known as the AR Monitor (Gallagher et al. 2002). This site is supported by the National Oceanic and Atmospheric Administration (NOAA) to make solar data (e.g., solar flares, and ARs) publicly available in an updated list.

The flare information consists of an event number, EName (e.g., gev_20101114_1020), flare start, stop, and peak times, X-ray (GOES) classification (X, M, C, B, and A), event type, and position on the Sun (Table 1). The occurrence (start) times, classification types, and locations (latitude and longitude) of flares on the Sun are used to construct the network. Bad data (e.g., wrong information about locations) are removed from the analysis. Using the diff_rot function in the SunPy software, the location (longitude) of the flares is rotated with respect to 2006 January 1 (the occurrence time of the first flare in our data set). The longitudes and latitudes of the flares on the solar sphere surface are restricted to $-180^\circ$ to $180^\circ$ and $-90^\circ$ to $90^\circ$, respectively (Figure 1). The scattering of the flare positions in the solar latitudes is presented in Figure 2.

3. Do Flares Form a Complex System?

Complex system studies focus on the collective behavior of a system characterized by the relationship of elements and interactions with the environment. Many systems, in nature, economics, biology, traffic, astrophysics, and ecology, can be classified as complex (Bar-Yam 1997; Newman 2003; Humphries & Gurney 2008; Rubinov & Sporns 2010; Lotfi & Daroonh...
Some common characteristics of complex systems are emergence treatment, nonlinearity, limited predictability, and self-organized criticality (Crutchfield & Young 1988; Bar-Yam 1997; Foote 2007; MacKay 2008). In this section, we survey the complexity characteristics of the solar flare system.

During the 11 years of our flare data set, the mean daily number of flare emergence within the solar atmosphere is about 3.7. In Figure 3, the timeseries of the number of flares during 2006 January 1 to 2016 July 21 is presented. Are the large numbers of emerged flares in the timeseries related to the other large numbers? In other words, does the timeseries of the number of flares have a long-temporal correlation (self-affinity)? To address this question, we used DFA. In DFA, the value of the Hurst exponent (H) is used to explain the correlation of timeseries (Mandelbrot 1975; Peng et al. 1994; Weron 2002; Aschwanden 2013; Alipour & Safari 2015). If $H$ takes the values in the ranges of $(0.5, 1)$ and $(0, 0.5)$, we can say that the timeseries has a long-term correlation in its correlated or anti-correlated behavior, respectively. In the case of $H = 0.5$, there is an uncorrelated signal in the timeseries.

We applied DFA to the timeseries of the number of emerged flares on each day. The value of the Hurst exponent is obtained at about 0.86. This shows that the timeseries of the flares has a long-temporal correlation. This key characteristic suggests that solar flares are governed by self-organized criticality (Lu & Hamilton 1991b; Einaudi & Velli 1994; Carreras et al. 2001; Dobson et al. 2007; Alipour & Safari 2015; Barnes et al. 2016).

Predicting solar flares is important for space weather and communication. Several attempts have been made to predict solar flare occurrence based on flare statistics (Wheatland 2005), magnetic properties of ARs (Leka & Barnes 2003; Barnes & Leka 2008; Ahmed et al. 2013; Bobra & Couvidat 2015; Barnes et al. 2016; Raboonik et al. 2017), and cellular automaton avalanche models (Bak et al. 1987; Isliker et al. 1998, 2000; Charbonneau et al. 2001; Barabási & Bonabeau 2003; Barpi et al. 2007; Strugarek & Charbonneau 2014). The results of recent studies show that the flare system
has limited predictability. The recently developed method based on the properties of AR magnetograms can predict flares only 48 hr before the flare occurrence (e.g., Bobra & Couvidat 2015; Barnes et al. 2016).

The avalanche model of cellular automaton based on the reconnection of magnetic fields has been developed for the solar flares (Lu & Hamilton 1991a; Lu et al. 1993; Strugarek & Charbonneau 2014). This progressed model is in the category of nonlinear and self-organized critical systems (Aschwanden 2013).

The above-mentioned features (i.e., limited predictability, nonlinearity, and self-organized criticality) confirm that the solar flares build up a complex system. In the rest of this paper, the complexity properties of the flare system are investigated using the complex network approach.

4. Constructing the Solar Flare Complex Network

The occurrence time and location of the flares on the solar surface are employed to construct the growing flare graph (network). The solar spherical surface is divided into \( n \times n \) cells with equal areas considering the spherical coordinates \((\theta, \phi)\) as \( \theta_j = 4\pi R_\odot^2/n^2 \) \((i, j = 1, 2, ..., N)\), where the parameter \( R_\odot \) is the solar radius, in the same manner as in the earthquake network developed by Abe & Suzuki (2006). The angles \( \theta \) and \( \phi \) for each equal area (cell) are given by

\[
\phi_{i+1} = \phi_i + \frac{2\pi}{n}, \quad \phi_1 = -180^\circ, \quad -180^\circ < \phi < 180^\circ, \quad (1)
\]

\[
\sin(\theta_{i+1}) = \sin(\theta_i) - \frac{2}{n}, \quad \theta_1 = 0^\circ, \quad -90^\circ \leq \theta < 0^\circ, \quad (2a)
\]

\[
\sin(\theta_{i+1}) = \sin(\theta_i) + \frac{2}{n}, \quad \theta_1 = 0^\circ, \quad 0^\circ \leq \theta < 90^\circ, \quad (2b)
\]

where \( \theta \) is an angle measured from the solar equator. We construct the flare network with edges (links) and loops defined by the flare interactions. It should be noted that links and loops are representative of the correlation between sympathetic flarings (Pearce & Harrison 1990; Changxi et al. 2000; Moon et al. 2002).

Each cell is regarded as a vertex (node) if the emerged flare (s) (are) located in it (Figure 1). The edges are defined as a relation between two successive flares. If two successive flares occur in the same cell, we will have a loop. By using this approach, we can map the flare information to a growing graph. We note that the solar flare network naturally is a directed graph.

A small part of the connectivity distribution of the 12 nodes and 21 flares with ENames (e.g., /grev_20110411_2211) of the solar flare network, with loops and multiple edges, is presented in Figure 4. The nodes and edges of the flares network are shown in Figure 5. The variety and number of connections demonstrates the complexity of the flares system. Each line presents a link between two successive flares (nodes). Since there is mutual influential interaction between two hemispheres, lots of connections are made by all consecutive flares over two hemispheres (see the caption in Figure 5). A simple graph (unweighted and undirected) is obtained by removing the loops and directions, and replacing multiple edges with single links.

An important point that requires emphasis when constructing the flare network is to estimate the cell size. Here, we use an arbitrary cell size to construct the network.

Also, we converted a directed graph to an undirected one to study the small-world presentation. In other words, we use the simple graph to present an illustration for a small-world network.

5. Random and Scale-free Networks

A graph—consisting of vertices and edges—is a geometrical representation of a network. In general, graphs can be classified as directed, undirected, weighted, and unweighted graphs, depending on their vertices and edges. A graph is called undirected if the links are bi-directional. A graph with different numbers labeled to links is known as a weighted network. An unweighted graph is a weighted one when all the weights are set to one. Every node is not in a relationship with itself; in other words, the elements lying on the main diagonal of the matrix take the value zero. In the complex network approach, the topological properties (local and global scales) taken from the related graph lie on the adjacency matrix (Cormen et al. 2001; Steen 2010). The simplest way to study the network is using the properties extracted from the adjacency matrix \( A \). The adjacency matrix for a network with \( N \) nodes is a square matrix of order \( N \). The adjacency matrix for a directed network with \( N \) nodes is defined as \( A_{ij} = 1 \), if node \( j \) is linked to node \( i \) \((i, j = 1, 2, 3, ..., N)\); the component \( A_{ij} \) equals 0 if there is no link between the \( j \)th node toward the \( i \)th node. For a weighted network, the value of \( A_{ij} \) can take an arbitrary value \( A_{ij} = W_{ij} \). For undirected networks, the adjacency matrix is symmetric (i.e., \( A_{ij} = A_{ji} \) and \( A_{ii} = 0 \)). The degree of the \( i \)th node \( k_i \) in an undirected network that can be extracted from the adjacency matrix is

\[
k_i = \sum_{j=1}^{N} A_{ij} = \sum_{i=1}^{N} A_{ij}.
\]

For a directed network, we have

\[
k_i^{in} = \sum_{j=1}^{N} A_{ij}, \quad k_i^{out} = \sum_{i=1}^{N} A_{ij},
\]

where \( k_i^{in} \) and \( k_i^{out} \) are the incoming and outgoing degrees of the node \( i \). The degree of the \( i \)th node is obtained as

\[
k_i = k_i^{in} + k_i^{out}.
\]

To describe a network, the average of the nodes, \( \langle k \rangle \), plays a key role. The average degree can be written as

\[
\langle k \rangle = \frac{2L}{N},
\]

where \( L \) is the number of links.

The several known and applicable networks are random, scale-free, complete, regular, and small-world. These networks are distinguishable from each other by their degree distributions. Degree distribution is an important characteristic of complex networks. A random network is constructed by \( N \) labeled nodes where each pair is linked with the same probability \( P \). Two ways to generate a random network with \( N \) nodes, \( L \) edges, and a probability \( P \) were explained by Gilbert (1959) and Erdős & Rényi (1960). For a random network, the degree distribution follows a Poisson distribution.
The degree distribution of a scale-free network is characterized by a power-law distribution, $P(k) \sim k^{-\gamma}$, where $\gamma$ is a positive constant called the degree exponent.

The basic difference between a random and a scale-free network appears in the hubs (high-$k$ region). For example, on the internet, which is a scale-free network with approximately $10^{12}$ nodes (e.g., https://venturebeat.com/2013/03/01/ or https://googleblog.blogspot.com/2008/07/), the probability of having a node with $k = 100$ is about $P(100) \approx 10^{-94}$ in a Poisson distribution; meanwhile, it is about $P(100) \approx 10^{-4}$ in a power-law distribution. In a random network, the average degree $\langle k \rangle$ is comparable with lots of degrees. In a random network, the difference between two degrees is of the order of $\langle k \rangle$, which results in: (a) the degree of nodes being comparable with average degree $\langle k \rangle$ and (b) highly connected nodes (hubs) are not possible. These points are the keys to distinguishing a random network from a scale-free network. In a random network, a hub is
effectively forbidden, whereas in a scale-free network, a hub is absolutely necessary.

For a scale-free network, there is a limit on the degree of the largest hub. The upper limit on the degrees of the largest hub is called the cutoff maximum degree \( k_{\text{cut}} \) or the natural cutoff of the degree distribution. The degree exponent with a natural cutoff for a scale-free network is estimated as (Dorogovtsev & Mendes 2002)

\[
\gamma_{\text{est}} \approx 1 + \frac{\ln N}{\ln k_{\text{cut}}},
\]

where \( N \) is the number of nodes. Following Equation (9), if \( \gamma \) takes sufficiently high values, scale-free and random networks are hardly distinguishable. It seems that distinguishing the power-law distribution from the Poisson distribution is crucial. If the ratio of \( k_{\text{max}}/(k) \) is large enough, the network can be categorized in a group of scale-free networks. In this case, the parameter \( k_{\text{max}} \) is the node with the highest degree.

### 6. Small-world and Regular Networks

We computed the values of the clustering coefficient, characteristic path length, and diameter parameters of the network to describe a small-world network. The clustering coefficient is a key parameter for studying most of the networks. In graph theory, the clustering coefficient represents the tendency of neighbors to cluster around each other in an undirected simple graph (Watts & Strogatz 1998). Mathematically, it is defined as

\[
c_i = \frac{2t_i}{k_i(k_i - 1)},
\]

where \( c_i \) and \( k_i \) are the local clustering coefficient and the number of neighbors, respectively. The parameter \( t_i \) is the number of edges linked between the neighbors of the \( i \)-th vertex. Indeed, \( k_i(k_i - 1)/2 \) is the maximum number of links that could exist between the neighbors. The clustering coefficient is given by

\[
C = \frac{1}{N} \sum_{i=1}^{N} c_i,
\]

where \( N \) is the network size. The values defined for the clustering coefficient of a complete graph (all nodes have connections with each other) \( C_{\text{comp}} \) and a random graph \( C_{\text{rand}} \) are unity and much smaller than unity, respectively. In network science, the regular network is a network where all nodes have the same degrees. The clustering coefficients for random and regular network are respectively given by (Barabási & Albert 2002; Fortunato et al. 2009)

\[
C_{\text{rand}} \approx \frac{\langle k \rangle}{N}, \quad (12)
\]

\[
C_{\text{reg}} = \frac{3\langle k \rangle - 1}{4\langle k \rangle - 2}. \quad (13)
\]

The clustering coefficient for most of the networks depends on the degree of nodes. For a random and a regular network, the clustering coefficient is not related to the degree of nodes. One way to distinguish a random network from a scale-free one is by using the average local clustering coefficient of the nodes with the same degree, which is called the \( C(k) \) function. The function \( C(k) \) for a random network is constant for all degrees of the nodes (Equation (12)).

The path in a connected graph (e.g., flares network) is a finite sequence of edges defined for every two connected vertices. Sometimes, there are several paths for each pair. The average shortest path \( d_{ij} \) between all pairs of nodes is an important parameter for analyzing the network. The average shortest paths for all pairs is called the characteristic path length \( \Lambda \) and is defined as

\[
\Lambda = \frac{1}{N(N-1)} \sum_{i<j}^{N} d_{ij}. \quad (14)
\]

The characteristic path lengths of random and regular networks are respectively expressed as (Boccaletti et al. 2006; Fortunato et al. 2009)

\[
\Lambda_{\text{rand}} \sim \frac{\ln N}{\ln \langle k \rangle - 1}, \quad (15)
\]

\[
\Lambda_{\text{reg}} \sim \frac{N}{2\langle k \rangle}. \quad (16)
\]

The other key parameter in the constructed network is the longest path length or network diameter \( D \).

As explained, in a simple graph, a path is an edge that connects vertices. The average path length of a random graph is smaller than that defined for a regular graph \( \Lambda_{\text{reg}} > \Lambda_{\text{rand}} \). In addition, the clustering coefficient of the regular graph is larger than that assigned for its equivalent random graph \( C_{\text{reg}} \approx C_{\text{rand}} \). In the small-world networks, a typical path between two arbitrary nodes is peculiarly short. In comparing \( C, C_{\text{rand}} \), and \( C_{\text{reg}} \) with the same network size (the same number of nodes, links, and equal average degree of nodes), the clustering coefficient of the small-world network takes greater and lesser coefficients than those defined for random and regular networks, respectively (i.e., \( C_{\text{reg}} > C > C_{\text{rand}} \) (Watts & Strogatz 1998)). For the small-world networks, there is a relation between \( N \) and \( \Lambda \) as follows (Cohen & Havlin 2003; Bollobás & Riordan 2004):

\[
\Lambda \sim \log N. \quad (17)
\]

The degree exponent is extracted from the power-law distribution to give a better description of a network. If the degree exponent of the scale-free network takes a value greater than three, the network is a small-world one (Cohen & Havlin 2003).

The relationships between the characteristic path length \( \Lambda \) and the degree exponent \( \gamma \) can be expressed as (Cohen & Havlin 2003; Bollobás & Riordan 2004)

\[
\Lambda = \begin{cases} 
\text{Constant} & \text{if } \gamma = 2, \\
\frac{\ln\ln(N)}{\ln(\gamma - 1)} & \text{if } 2 < \gamma < 3, \\
\frac{\ln N}{\ln(\ln(N))} & \text{if } \gamma = 3, \\
\ln N & \text{if } \gamma > 3.
\end{cases}
\]

In the case of \( \gamma = 2 \) (anomalous regime), the average path length has no relation to \( N \). In this regime, when the system size increases, the hub with the highest degree grows linearly. If \( \gamma \) ranges between two and three (ultra-small-world), the characteristic path length is proportional to \( \ln(\ln(N)) \). It has a
considerably slower regime than the $\ln(N)$, which is determined for random networks. When $\gamma = 3$ (critical point), the characteristic path length takes values slightly smaller than that obtained for the random network because of the presence of $\ln(\ln(N))$. Finally, in the case of $\gamma > 3$ (small-world), the hubs do not have a meaningful influence on the characteristic path length (Bollobás & Riordan 2004).

7. Assortative, Disassortative, and Neutral Networks

Degree correlations are indicative of the relation between the degrees of nodes that are linked to each other. Using the adjacency matrix $(A)$, the average degree of the neighbors $(k_{nn})$ for the $i$th node is given by

$$k_{nn}(i) = \frac{1}{k_i} \sum_{j} A_{ij} k_j. \quad (18)$$

The degree correlation function for nodes with degree $k$ is obtained as

$$k_{nn}(k) = \frac{1}{N_{k \geq k}} \sum_{i \geq k} k_{nn}(k), \quad (19)$$

where $N_k$ is the number of nodes with the degree $k$. The degree correlation function has the following relation (Pastor-Satorras et al. 2001)

$$k_{nn}(k) \propto k^{\mu}, \quad (20)$$

where the parameter $\mu$ is a correlation exponent. For assortative networks, the correlation exponent is positive ($\mu > 0$) and for disassortative networks, the correlation exponent is negative ($\mu < 0$). In the case of $\mu = 0$, $k_{nn}(k)$ is independent of $k$. In a such a case, no correlation is found in the network (neutral network). In the assortative networks, hubs tend to connect to other hubs. Thus, in this kind of network, the nodes with approximately the same degree have a tendency to connect with each other. Indeed, in assortative/disassortative networks, the parameter $k_{nn}(k)$ increases (decreases) with increasing $k$.

8. Results

We constructed the flare complex network using the positions and occurrence times of 14395 flares. On the basis of solar differential rotation, the positions (longitudes and latitudes) on the solar sphere were rotated with respect to the position of the first flare (2006 January 1). We divided the solar surface into cells with equal areas, as presented in Figure 1. The number of cells ($n^2$) ranged between 1936 and 7744. The birth positions of the flares are set to assigned cells. The filling factor of nodes ($N/n^2$) over the solar surface varies from 0.59 to 0.45 (Table 2). As seen in Figure 2, when the aggregation of the number of flares in one of the solar hemispheres increases over several years, it decreases in the other hemisphere. During the years 2006–2009, the number of flares in the southern hemisphere is noticeably greater than that in the northern hemisphere. In the vicinity of the southern pole (latitudes $<-80$), a smaller number of flares was detected. About 47% and 53% of the flares occurred at the northern and southern solar hemispheres, respectively. The DFA method is applied to the timeseries of the occurrence flares and the result of this analysis is 0.85. As noted, if the value of the Hurst exponent is ranged in (0.5 1), there is a long-temporal correlation over the timeseries.

The probability distribution function (PDF) for the degree of nodes is shown in Figure 6. Aschwanden (2015) showed that the thresholded power-law distribution is a suitable function for describing the solar and stellar flare size (energy) distributions. The thresholded power-law function is given by

$$p(k) \propto (k + k_0)^{-\gamma}, \quad (21)$$

where $k_0$ and $\gamma$ are the thresholded value and the power-law exponent. In the fitting process, we used the key steps as prescribed by Aschwanden (2015). The uncertainty of the power-law exponent is $\sigma_{\gamma} = \gamma/\sqrt{n}$ Aschwanden (2011b). As we see in the figure, the values of the degree exponent for the different network sizes are greater than three.

Following Equation (9), if we use $k_{\text{max}}$ instead of $k_{\text{cut}}$, the estimated power-law exponent ($\gamma_{\text{est}}$) will be in good agreement with the values given in Table 2 (Columns 8 and 9). The ratio of the maximum to the average degree of nodes ($k_{\text{max}}/\langle k \rangle$) in the flare network for different network sizes is obtained to be greater than 3.5 (Table 2, Column 7). This indicates that the flare network is not a random network.

In Figure 7, two “flare belts” ($-29 \text{ latitudes} < -4$ and $1 \text{ latitudes} < 29$) are exhibited. As seen, we found that more than 65% of the flares were only generated at 15% of the solar surface. The positions of the 118 hubs (high-connectivity regions) are demonstrated in Figure 8. About 3% of the solar surface is assigned to regions consisting of hubs and about 11% of the generated flares were located at these positions. The occurrence rates of the flares (M and X) are three times as much as those computed for the hubs. In Figure 9, the degree correlation $k_{nn}(k)$ versus the degree of nodes for different network sizes is presented. The negative value obtained for the slope of the fitted straight line shows that the network is disassortative. Similar behavior was found for the “arxiv.org” network (Lee et al. 2006).

The average of the clustering coefficient for the same degree of nodes $C(k)$ is presented in Figure 10. The values of the power-law exponent ($\alpha \approx 0.5$) are approximately constant for different sizes of the networks. The power-law behavior of $C(k) \sim k^{-\alpha}$ ensures that the flare network is a scale-free network. In some scale-free networks (e.g., the internet, semantic web, etc.), the probability of achieving a new link to a new node increases by increasing the connectivity of a node (Barabási & Albert 2002; Dorogovtsev & Mendes 2003;
Figure 6. PDFs for the degree distributions of the flare networks are plotted in a log–log scale for the network sizes: (a) 1137, (b) 2018, (c) 2681, and (d) 3487. The degree exponents for the power-law fits for different network sizes are found to be greater than three.

Figure 7. Two “flare belts” (−29 <latitude < −4 and 1 <latitude < 29) are shown. We see that these two belts cover more than 65% of the flares generated at 15% of the solar surface. The probability of large flares (M and X) occurring over these regions determined by the belts is about twice as much as that in other regions.
This is a generic property of hierarchical networks. An explanation of hierarchical networks is given by Lee et al. (2006). They showed that the power-law exponent of $C(k)$ remains approximately constant for the scale-free networks with degree exponents that fall in the range of 3–5 (See Figure 9 therein). The clustering coefficient of the
hubs for the flare network takes small values. By decreasing the
degrees of nodes, the clustering coefficient increases.

As shown in Figure 11, the clustering coefficient of the
constructed network ($C$) and its equivalent random network
($C_{\text{rand}}$) are presented. When the cell size is small (i.e.,
the network resolution increases), the ratio of the flare clustering
coefficient to the random one ($C/C_{\text{rand}}$) takes larger values (see
Table 3 and Figure 11). This means that the flare network
becomes completely distinguishable from its equivalent
random network. In Figure 12, the behavior of the characteristic path length versus the network size is displayed. The characteristic path length has a logarithmic relation with
the network size: $\Lambda \sim 2.58 \log(N)$. Furthermore, when the
network size grows from 1137 to 3487, the diameter of the
flare network changes slightly from 10 to 14 (Table 3, Column 8).

Figure 10. Average of the clustering coefficient for nodes (circles) with the same degree in the undirected flare network is presented. The power-law fits (solid lines) for different network sizes ($N$), (a) 1137, (b) 2018, (c) 2681, and (d) 3487, are presented.

Figure 11. Behavior of the clustering coefficient vs. both the size of the flare network (circle) and its equivalent random network (square) is shown. The ratio of the $C/C_{\text{rand}}$ (triangle) becomes larger when the cell size decreases or the network resolution increases.
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Figure 12. Characteristic path length of the flares networks vs. the network size \( N \) and a fitted straight line as \( \Lambda \sim 2.58 \log(N) \) are displayed.

Table 3
The Properties of the Small-world Extracted from the Complex Flare Networks

| \( N \) | \( C_{\text{reg}} \) | \( C_{\text{rand}} \) | \( C \) | \( C / C_{\text{rand}} \) | \( \Lambda_{\text{reg}} \) | \( \Lambda_{\text{rand}} \) | \( \Lambda \) | \( \log(N)/\Lambda \) | \( D \) |
|-------|----------------|----------------|--------|----------------|---------------|---------------|--------|----------------|-------|
| 1137  | 0.75           | 0.0223         | 0.0692 | 3.11           | 26.33         | 2.33          | 2.99   | 1.02           | 10    |
| 2018  | 0.75           | 0.0071         | 0.0398 | 5.63           | 70.72         | 2.94          | 3.56   | 0.93           | 11    |
| 2681  | 0.75           | 0.0040         | 0.0341 | 8.50           | 124.84        | 3.47          | 3.97   | 0.86           | 13    |
| 3487  | 0.75           | 0.0024         | 0.0247 | 10.43          | 211.19        | 4.12          | 4.23   | 0.80           | 14    |

9. Conclusions
In this work, the characteristics of the solar flares network are studied to extract the laws governing flare occurrence over the solar surface. To do this, the complex network is constructed using a flare data set (including positions and occurrence times) recorded during 2006 January 1 to 2016 July 21. Since the system of flares is a limited system that is predictable, self-organized with long-temporal correlation, nonlinear, and scale-free, we conclude that the flare system is a complex one. We construct the complex network of the flare system using their positions and occurrence times on the solar surface in the same way that Abe & Suzuki (2006) proposed to construct earthquake networks. We divided the solar surface into cells with equal areas where the number of cells increases from 1936 to 7744. Because the lengths of cells along the solar latitudes is non-uniform (Equation (2)) and the recorded positions of the flares are in degree form (integer), constructing a network with small cell sizes (<1°) is crucial with the present data. By increasing the spatial resolution of the flare positions, designing a larger flare network is possible.

The power-law nature of the PDF degree confirms that the flare network is a scale-free network. At the positions of the network hubs, the flaring probability is higher than that at other nodes. We found out that over the flare networks, hubs do not have a tendency to form links with the other hubs. There is a tendency to create a link between small degrees of nodes and hubs. Our results show that the probability of the occurrence of large flares (M and X) over regions generating flares covering only 15% of the solar surface is about twice as much as that in other regions. Also, we found that the flares occurring over one of the hemispheres have a certain effect on flares that occur in the other hemisphere.

Our results show that the flare network is not a random network because the degree distribution does not follow the Poisson distribution. In the flare network, there are several special nodes with large values of degree (large \( k \)) where the nodes become hubs characterizing the scale-free network. The degree exponents of the nodes for undirected, incoming, and outgoing networks are the same.

Furthermore, the ratio of \( k_{\text{max}}/k \) ensures that the flare network is scale-free, thus hubs are naturally generated. Also, the power-law behavior of degrees with \( \gamma > 3 \) indicates that all flares networks construct a small-world network (Cohen & Havlin 2003).

Since the degree correlation exponents take the negative values, the flares network is categorized in the group of disassortative networks. We found that in the flare networks, the hubs are not correlated to the other hubs; they are only correlated with nodes including smaller degrees. In other words, although some of the hubs are neighbors on the solar surface, they do not tend to interact directly with each other.

Computing the filling factors of hubs in the different temporal ranges of our data set shows that the hubs always cover about 3% of the solar surface. The scale-free and small-world behavior of flares confirms that there is a characteristic universality in the solar flare system.

Given the low resolution (spatial, temporal, and energy band) of early solar instruments, the lack of full-covering of the solar surface by telescopes, and the computational algorithmic errors for the identification of small events, the number of low-energy flares (A type) with certain positions is thinly populated.
in the solar flare data set. Furthermore, the number of high-energy flares (X type) intrinsically occurs at a lower rate. Although the flare data set provides parameters for constructing a flare network, it is not yet adequate for investigating the time evolution of the system.

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