Understanding the production of dual BEC with sympathetic cooling

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Abstract

We show, both experimentally and theoretically, that sympathetic cooling of $^{87}$Rb atoms in the $|F = 2, m_F = 2\rangle$ state by evaporatively cooled atoms in the $|F = 1, m_F = -1\rangle$ state can be precisely controlled to produce dual or single condensate in either state. We also study the thermalization rate between two species. Our model renders a quantitative account of the observed role of the overlap between the two clouds and points out that sympathetic cooling becomes inefficient when the masses are very different. Our calculation also yields an analytical expression of the thermalization rate for a single species.

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Evaporative cooling has proved to be an efficient route toward Bose-Einstein condensation (BEC). However, not all the species are eligible for evaporation. For instance, the collisions may not be sufficient \[1\] or may be forbidden as for fermions at low temperature \[2\]. In the case of a rare species, one may also want to avoid the large loss of atoms inherent to evaporative cooling. Sympathetic cooling is potentially a very good solution in those cases. It consists in putting the target sample we want to cool in thermal contact with a buffer gas that can be cooled to the desired temperature. This cooling can be done either by conventional cryogenics \[3, 4\], or by evaporative cooling as demonstrated by Myatt et al. \[5\]. These authors have produced dual BEC by sympathetic cooling of a gas of \(^{87}\text{Rb}\) atoms in the \(|F = 2, m_F = 2\rangle\) state (target gas, noted \(F = 2\) hereafter) by the use of an evaporatively cooled gas of \(^{87}\text{Rb}\) atoms in the \(|F = 1, m_F = -1\rangle\) state (buffer gas noted \(F = 1\)). In their experiment, the number of target atoms, although almost constant during the initial phase of the cooling, decreases significantly near to the condensation. In this paper, we investigate a situation similar to the one of Myatt et al., but with the number of atoms in the target gas constant in the last cooling stage, all the way down to BEC. This allows us to study quantitatively how the production of dual BEC relies on a precise control of the initial conditions. We also study the effect of the interspecies thermalization rate on sympathetic cooling efficiency, by changing the overlap between the buffer and the target gas. Our experimental observations agrees with the results of a calculation of the thermal contact between the atomic species. This calculation not only shows the crucial role of the overlap, but also why sympathetic cooling is very efficient for equal masses of the target and buffer gases. In addition, it allows us to recover in an analytic way a result previously found numerically for the thermalization rate of a single species.

Our experimental apparatus has been described in a previous publication \[6\]. After a laser cooling sequence, we transfer the \(^{87}\text{Rb}\) atoms from a dark SPOT into the magnetic trap. During the transfer, an optical pumping pulse allows us to transfer most of the atoms in \(F = 1\) (buffer), while keeping an adjustable fraction in \(F = 2\) (target). Our iron-core Ioffe-Pritchard trap operates with a high bias field \(B_0\) between 50 and 250 Gauss, and the magnetic field curvature \(C\) along the dipole direction is linked to \(B_0\) (the ratio \(B_0/C\) is fixed by construction to 1 cm\(^2\)). After adiabatic compression, the magnetic field gradient in the strong (quadrupole) directions is \(G = 1\) kG/cm. To cool down \(F = 1\), we use standard rf forced evaporation. We characterize both species simultaneously by absorption imaging.
after turning off the magnetic trap. During this release, the atoms experience a magnetic kick due to a Stern-Gerlach effect which allows us to spatially separate atoms in \( F = 1 \) and atoms in \( F = 2 \). We probe on the \( F = 2 \rightarrow F' = 3 \) transition after repumping the atoms from \( F = 1 \) to \( F = 2 \). With usual analysis, we derive the temperature of the two clouds from the image.

In a first experiment, we have studied sympathetic cooling at a bias of 56 G, with initially \( 10^8 \) atoms in \( F = 1 \) at a temperature of 300 \( \mu \)K, and various initial numbers of atoms in \( F = 2 \). We observe no loss of target atoms during the whole cooling process \([7]\). Indeed, the atoms in \( F = 2 \) see a trapping potential twice as steep as for \( F = 1 \), and as in \([8]\), very few target atoms are evaporated at the beginning of the evaporation sequence. Moreover, at our high bias field, the non linear Zeeman effect prevents rf coupling of the \(| F = 2, m_F = 2 \rangle \) trapping state to any non trapping state at the end of the evaporation \([5, 8]\). When starting with a very small \((\leq 3 \times 10^4)\) number of atoms in \( F = 2 \), we get a \( F = 1 \) condensate with typically \( 10^6 \) atoms. If we continue the evaporation, another condensate appears in \( F = 2 \) at a lower temperature. For \( 3 \times 10^4 \) atoms in \( F = 2 \), both gas condense simultaneously at a temperature of 200 nK, as shown figure \( \text{figure 1.a.} \) If we start with a larger number of target atoms (but smaller than \( 8 \times 10^4 \)), a condensate appears first in \( F = 2 \) at \( T > 200 \) nK, and then in \( F = 1 \) at \( T < 200 \) nK. For a yet larger number of target atoms, we can obtain a condensate in \( F = 2 \) and no condensate in \( F = 1 \). If the number of atoms in \( F = 2 \) is more than \( 2 \times 10^5 \), BEC is observed in neither hyperfine state.

These experimental observations can be explained with a simple model based on an energy budget. We assume that both species are always thermalized (see last part of this paper), and that the number \( N_2 \) of target atoms is constant. The total energy of the \( N_1 \) buffer and \( N_2 \) target atoms in harmonic traps is therefore \( E = 3(N_1 + N_2)k_B T \). The buffer gas is evaporatively cooled with an energy cutoff \( \eta k_B T \). We denote \( dN_1 \) the elementary (negative) variation of the number of buffer atoms. The corresponding (negative) energy variation is \( dE = dN_1(\eta + 1)k_B T \). After rethermalization at a lower temperature \( T + dT \), the total energy becomes \( E + dE = 3(N_1 + dN_1 + N_2)k_B (T + dT) \). Keeping only the first order, we have \( dT/T = \alpha dN_1/(N_1 + N_2) \) with \( \alpha = (\eta - 2)/3 \). Assuming that \( \eta \) is constant, we get:

$$ T = T^{\text{min}} \left( \frac{N_1}{N_2} + 1 \right)^{\alpha} \quad \text{with} \quad T^{\text{min}} = T^{\text{ini}} \left( \frac{N_2}{N_1^{\text{ini}}} \right)^{\alpha}. \quad (1) $$
In this formula, $T_{\text{ini}}$ and $N_{1\text{ini}}^1$ are the initial values, $N_2$ is constant and has been neglected with respect to $N_{1\text{ini}}^1$ in the expression of $T_{\text{min}}$ (as the temperature has to be reduced by several orders of magnitude, the initial fraction of atoms in the target gas must be very small). The temperature $T_{\text{min}}$ is reached when all the buffer atoms have been evaporated ($N_1 = 0$). To understand the different regimes experimentally observed, we first study the evolution of the phase space densities $D_1$ of the buffer and $D_2$ of the target gas, in harmonic traps of mean frequencies $\omega_1$ and $\omega_2$, during the evaporation. Equation 1 allows us to express $D_1$ and $D_2$ as functions of $N_1$ only. Figure 2 represents the evolution of $D_1$ and $D_2$ during the evaporation (while $N_1$ is decreasing). We observe that the phase space density of the target gas $D_2$ always increases to reach its maximum value

$$D_2^{\text{max}} = \frac{2.17}{N_2^{(3\alpha-1)}} \left( \frac{N_{1\text{ini}}^\alpha}{\hbar \omega_2} \right)^3$$

at the very end of the evaporation: this is because $N_2$ is constant and the temperature always decreases. We also observe that the buffer gas phase space density $D_1$ first increases to a maximum $D_1^{\text{max}}$ that we write

$$D_1^{\text{max}} = D_2^{\text{max}} \left( \frac{\omega_1}{\omega_2} \right)^3 \frac{(3\alpha - 1)^{(3\alpha-1)}}{(3\alpha)^{3\alpha}},$$

and then decreases to zero. At the intersection of the two curves, the phase space densities $D_1$ and $D_2$ have the same value

$$D^{(=)} = D_2^{\text{max}} \left( 1 + \left( \frac{\omega_2}{\omega_1} \right)^3 \right)^{-3\alpha}.$$  

We note that a modification of the various parameters only changes the relative position of the two curves, in particular the ordering of $D^{(=)}$, $D_1^{\text{max}}$ and $D_2^{\text{max}}$. This is what determines the outcome of sympathetic cooling.

We specifically focus on the situation where $D^{(=)}$ is reached before $D_1^{\text{max}}$ (this happens if $3\alpha - 1 > (\omega_1/\omega_2)^3$), which corresponds to the case of figure 2 and to our experimental parameters. The target gas can condense only if $D_2^{\text{max}} > 2.612$, and in the same way, the buffer gas can condense only if $D_1^{\text{max}} > 2.612$. If these two conditions are satisfied, the buffer condenses first if $D^{(=)} > 2.612$ otherwise the target condenses first. These three conditions can be written as conditions on the number $N_2$ of target atoms. This leads to define three critical numbers $N_2^a$, $N_2^b$ and $N_2^c$ of target atoms, for which $D^{(=)}$, $D_1^{\text{max}}$ and...
$D^\text{max}_2$ are respectively equal to 2.612. Although each of these numbers depend on all initial conditions, the ratios $N^a_2/N^b_2$ and $N^b_2/N^c_2$ are independent of the initial values $N^{\text{ini}}_1$ and $T^{\text{ini}}$ because $D^\text{max}_1/D^\text{max}_2$ and $D(\text{,}/D^\text{max}_2$ do not depend on the initial conditions (see eq. 3 and 4) and because $D^\text{max}_2 \propto N^{(1-3\alpha)}_2$ (see eq. 2). These ratios are represented in a diagram of the energy cutoff $\eta = 3\alpha + 2$ versus the number of target atoms $N_2/N^c_2$ on figure 3. We can identify four regions on this diagram [12], corresponding to the different regimes of sympathetic cooling. From the experiment, we measure $N^a_2 = 3 \times 10^4$, $N^b_2 = 8 \times 10^4$ and $N^c_2 = 2 \times 10^5$ which is in good agreement with the calculated values for a typical value of the evaporation parameter $\eta \simeq 6.5$ (see fig. 3).

In a second experiment, we have studied the role of thermalization in sympathetic cooling by changing the bias field from 56 G to 207 G. This reduces the vertical oscillation frequencies $\omega_{Fz} = \left\{ (-1)^F m_F \mu_B(G^2/B_0 - C)/2M \right\}^{1/2}$ where $M$ is the mass of the atom and $\mu_B$ the Bohr magneton, and therefore increases the relative gravitational sag $\Delta = g/\omega^2_{1z} - g/\omega^2_{2z}$ between the two atomic clouds [4]. Figure 4b represent successive absorption images while decreasing the final frequency of the evaporation ramp, at a bias field of 207 G ($\Delta = 26 \mu m$). We initially observe sympathetic cooling of the target gas until a temperature of about 400 nK is reached. Then sympathetic cooling stops. This is in contrast with the first experiment at a bias field of 56 G, where the relative sag is smaller ($\Delta = 7 \mu m$), and sympathetic cooling works all the way down to BEC of the two species (see fig. 4a).

To render a quantitative account of these observations, we study how the thermal contact between the two clouds evolves during the cooling. For that, we calculate the energy exchange rate $W$ via interspecies elastic collisions. We assume Maxwell-Boltzman distribution at temperatures $T_1$ for the buffer and $T_2$ for the target gas (we neglect the cut-off due to possible evaporation). Introducing the notations $v_G$ for the center of mass velocity, $v$ and $v'$ for the relative velocities respectively before and after the collision, the energy received in the laboratory frame by a buffer atom during the collision is $\frac{M}{2} v_G \cdot (v - v')$. At low temperature we only consider $s$-wave scattering and the term $v_G \cdot v'$ averages out to zero. After integration over positions and velocities, we get $W = k_B(T_2 - T_1) \Gamma$, where $\Gamma$ is the number of interspecies collisions per unit of time, found equal to

$$\Gamma = \frac{N_1 N_2}{\pi^2 \rho_x \rho_y \rho_z} \times \sigma_{12} \times V \times \exp\left(-\frac{\Delta^2}{2 \rho_z^2}\right).$$
In equation 5, $\sigma_{12}$ is the interspecies elastic cross-section (taken independent of the temperature), $V = \sqrt{k_B(T_1 + T_2)/M}$ is the RMS sum of thermal velocities, and $\rho_z = \sqrt{k_B/M(T_1 + T_2)/M}$ is the RMS sum of the vertical sizes of the clouds (with similar expressions for $\rho_x$ and $\rho_y$).

The exponential term in (5) describes the overlap of the two clouds. Thus the energy exchange rate $W$ will become vanishingly small if $\rho_z$ becomes smaller than the gravitational sag $\Delta$. We can now analyze the role of the thermal contact in the experiments. At a bias field of 207 G (sag $\Delta = 26 \mu m$), and high temperature (above 400 nK), the effect of the sag is negligible and the clouds are thermalized. During the cooling, the sizes of the clouds and therefore $\rho_z$ decreases. At the temperature of $T_1 \simeq T_2 \simeq 400$ nK where we observed that sympathetic cooling stops, we have $\rho_z = 12 \mu m$ and the exponential term in equation 5 takes the value 0.1, so that the energy exchange rate is reduced by an order of magnitude. In contrast, at a bias field of 56 G, where $\Delta = 7 \mu m$, the parameter $\rho_z$ at the temperature of condensation 300 nK, ($\rho_z \simeq 8 \mu m$) is still bigger than $\Delta$, the collision rate $\Gamma$ is only reduced by 30 % because of the overlap, and sympathetic cooling works until reaching BEC.

Starting from the interspecies energy exchange rate $W$, and using the total energy conservation, we can derive evolution equations for the temperatures $T_1$ and $T_2$ of each species taken separately at thermal equilibrium. Denoting $\Delta T = T_1 - T_2$, and $T = (T_1 + T_2)/2$, we obtain an interspecies thermalization rate, which, for equal trapping frequencies ($\omega_1 = \omega_2 = \omega$), takes the simple form

$$\frac{1}{\tau} = -\frac{1}{\Delta T} \frac{d \Delta T}{dt} = \frac{(N_1 + N_2) \omega^3 \sigma_{12} M}{3k_B T} \frac{2 \pi^2}{M^2}.$$  

(6)

This result is remarkable, by analogy with the individual thermalization rate of a single species $\tau^{-1} \simeq \gamma/3$ [3], where $\gamma = N\omega^3\sigma M/2\pi^2k_BT$ is the average elastic collision rate in an harmonic trap. Assuming identical values for the elastic cross sections $\sigma_1$, $\sigma_2$, and $\sigma_{12}$ [4], we then have $\tau^{-1} \simeq \tau_1^{-1} + \tau_2^{-1}$ where $\tau_1^{-1}$ and $\tau_2^{-1}$ are the thermalization rates of each species considered separately. This means that thermalization between the two species happens faster than thermalization of each species. So, if the evaporation ramp is adapted to cooling of the buffer alone, interspecies thermalization will be efficient.

If the masses are different, equation 6 remains valid at lower order in $\Delta T/T$, if we replace $M$ by $\mathcal{M} = \frac{8(M_1M_2)^2}{(M_1+M_2)^2}$, which is symmetric in the masses $M_1$ of the buffer and $M_2$ of the target. The equivalent mass $\mathcal{M}$ is maximum for $M_1 = M_2$ as in our experiment, and decreases for uneven masses (for significant different masses, $\mathcal{M}$ is much smaller than the smallest mass).
It is interesting to note that if equation 6 is applied to any arbitrary partition in a single species, where \( N_1 \) and \( N_2 = N - N_1 \) atoms are taken at different temperatures, then the thermalization rate between the two subsamples is exactly equal to \( \gamma/3 \), in agreement with the numerical simulations of \[13\]. To our knowledge, such an analytical result had not been published.

To conclude, we have first presented a simple analysis allowing one to quantitatively predict the outcome of sympathetic cooling, in the case of no losses in the target. We have then obtained an expression of the thermalization rate between the buffer and the target, that quantitatively describes the effect of the overlap between the two clouds. Moreover, when the overlap is good, the interspecies thermalization rate is the same of the intraspecies thermalization rate, and sympathetic cooling is very efficient \[2, 15\]. In contrast, we predict a strongly diminished interspecies rethermalization rate for very different masses of the buffer and target. Our approach allows to derive an analytical expression of the intraspecies thermalization rate in surprisingly good agreement with previous numerical calculation. Although our modeling is very simple and does not take into account quantum statistics effects near condensation \[16\], it is a useful tool, that can easily be generalized, to plan or analyze experiments using sympathetic cooling.

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FIG. 1: Sympathetic cooling of $^{87}$Rb atoms in $|F = 2, m_F = 2\rangle$ (upper images) by evaporatively cooled atoms in $|F = 1, m_F = -1\rangle$ (lower images): effect of the thermal contact. a) Small sag (7 $\mu$m) between the two clouds: sympathetic cooling yields BEC for both species. b) Larger sag (26 $\mu$m): sympathetic cooling stops after the third snapshot (400 nK) when decreasing final rf frequency.

FIG. 2: Phase space densities of the buffer (solid line) and the target gas (dashed) as a function of the evaporation progress, characterized by the ratio $N_1/N_1^{\text{ini}}$ of remaining buffer atoms. This plot corresponds to the case $3\alpha - 1 > (\omega_1/\omega_2)^3$. The horizontal line is the critical value 2.612 above which BEC occurs. Here, the target and then the buffer will condense.

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[10] The factor of 2.17, related to the quantum statistics effect, is valid close to degeneracy.

[11] This is true for $3\alpha > 1$, which is the usual condition for the phase space density to increase during evaporation of a single specie.

[12] In addition, there is a small region corresponding to $(\omega_1/\omega_2)^3 > 3\alpha - 1 > 0$, where a condensate appears in the buffer gas, then desapears and finally the target gas reaches degeneracy. This case is relevant if $\omega_1 > \omega_2$.

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FIG. 3: Outcome of sympathetic cooling as a function of the number of target atoms $N_2/N_2^c$. The evaporation parameter $\eta$ is the ratio of the energy cutoff to the thermal energy $k_B T$. Trap parameters are fixed to $\omega_2/\omega_1 = \sqrt{2}$. The lines represent the critical number ratios $N_2^a/N_2^c$, $N_2^b/N_2^c$ and $N_2^c/N_2^c$. They separate regions where either dual, single or no BEC can be formed. Black dots indicate the critical number found in our experiment.
