Phase coherent transport in (Ga,Mn)As

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Abstract. Quantum interference effects and resulting quantum corrections of the conductivity have been intensively studied in disordered conductors over the last few decades. The knowledge of phase coherence lengths and underlying dephasing mechanisms are crucial in understanding quantum corrections to the resistivity in different material systems. Due to the internal magnetic field and the associated breaking of time-reversal symmetry quantum interference effects in ferromagnetic materials have scarcely been explored. Below, we describe the investigation of phase coherent transport phenomena in the newly discovered ferromagnetic semiconductor (Ga,Mn)As. We explore universal conductance fluctuations in mesoscopic (Ga,Mn)As wires and rings, the Aharonov–Bohm effect in nanoscale rings and weak localization in arrays of wires, made of the ferromagnetic semiconductor material. These experiments allow to probe the phase coherence length $L_\phi$ and the spin flip length $L_{SO}$ as well as the temperature dependence of dephasing.

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1. Introduction

The discovery of the ferromagnetic III–V semiconductor materials (In,Mn)As [1] and (Ga,Mn)As [2] has generated a lot of interest as these materials combine ferromagnetic properties, typical for metals, with the versatility of semiconductors (for a review, see e.g. [3]–[7]). This allows, e.g., to control ferromagnetism by electric fields thus opening new prospects for application and fundamental research [8]. The Mn atoms in the III–V host are not only responsible for the ferromagnetism but also act as acceptors such that, at sufficiently high Mn-concentration, (Ga,Mn)As is a degenerate p-type semiconductor [9]. The ferromagnetic order of the Mn magnetic moments is mediated by holes via the Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction [10]. By now ferromagnetism in (Ga,Mn)As is well understood, allowing the prediction of Curie temperatures [11], magnetocrystalline anisotropies ([12] and references therein) as well as the anisotropic magnetoresistance (AMR) effect [13]. In this respect (Ga,Mn)As is one of the best understood ferromagnetic materials [14] and hence suitable as a model system to study quantum corrections to the conductivity in ferromagnets.

The quest to increase the Curie temperature $T_C$ in (Ga,Mn)As towards room temperature has led to a thorough investigation of the material properties (see, e.g., [15] and references therein). By annealing (Ga,Mn)As sheets or by incorporating them into sophisticated layered arrangements the Curie temperature was increased up to 173 K [16, 17] and 250 K with Mn-doping [18], respectively. Despite the high crystalline quality of the material (Ga,Mn)As is quite a disordered conductor on the verge of the metal–insulator transition (MIT). For Mn concentrations on the metallic side of the MIT the typical mean free path $l$ of the holes is a few lattice constants. Hence it was until recently an open issue whether quantum effects like Aharonov–Bohm (AB) oscillations, universal conductance fluctuations (UCFs) or weak localization (WL) can be observed in this class of materials. Accordingly the phase coherence length $L_\phi$ and the corresponding dephasing mechanisms which govern quantum mechanical interference phenomena in ferromagnetic semiconductors were unknown. Apart from being a fundamental material parameter, this information is needed to design more sophisticated layered structures from ferromagnetic semiconductors. Examples are resonant tunneling diodes or other interference devices which rely on the electrons’ wave nature. Apart from the question of the relevant phase coherence lengths, quantum corrections to the resistance-like WL are suppressed by a sufficiently strong perpendicular magnetic field $B$ [19]. Hence the question arises whether such effects can be observed at all in ferromagnets which have an intrinsic magnetic induction. Therefore, the advent of the new ferromagnetic semiconductor material
(Ga,Mn)As with significantly smaller internal field compared to conventional ferromagnets offered a new opportunity to address such questions.

Below we first describe experiments to explore UCFs in (Ga,Mn)As nanowires to get knowledge of the relevant dephasing length and the main phase breaking mechanism. We describe below periodic conductance oscillations in (Ga,Mn)As nanoscale rings to prove phase coherent transport by the AB effect. The last part of this paper describes WL experiments in arrays of (Ga,Mn)As nanowires, where aperiodic conductance fluctuations are suppressed by ensemble averaging.

2. Sample preparation and measurement technique

To explore phase coherent phenomena in (Ga,Mn)As we fabricated nanoscale wires, rings and arrays of wires, connected in parallel, from several wafers, always containing a ferromagnetic (Ga,Mn)As layer on top of a semi-insulating GaAs (100) substrate [20]. The magnetic easy axis of all investigated wafers was in plane. The relevant parameters of the samples used are listed in table 1. All investigated samples were fabricated using electron-beam-lithography (a Zeiss scanning electron microscope controlled by a nanonic pattern generator) and dry etching techniques. Ohmic contacts to the samples were made by thermal evaporation of Au and lift-off after brief in situ ion beam etching to remove the native oxide layer of (Ga,Mn)As.

The measurements were performed in a toploading dilution refrigerator (an Oxford Kelvinox TLM), equipped with a 19 T magnet, by standard four probe lock-in technique. The temperature regime accessible with this cryostat system ranges from 1 K to 15 mK. To avoid the heating of charge carriers at very low temperatures careful wiring, shielding and the use of low excitation currents (down to 10 pA) were crucial. Without such measures quantum interference effects in (Ga,Mn)As cannot be observed. The schematic experimental set-up is shown in figure 1. As we observe no saturation of the signals for the different effects (UCF, AB-oscillations and WL) at the lowest temperatures, we assume that the effective electron temperature is in equilibrium with the bath temperature even at temperatures as low as 20 mK.

In the experiments described below the wires can be treated as quasi-one-dimensional (1D) in the context of phase coherent transport. The relevant parameters for the 1D treatment are the phase breaking length \( L_{\phi} \) and the thermal diffusion length \( L_T \). The phase breaking length gives the length scale an electron can travel without losing phase information. The thermal diffusion length gives the length scale on which the thermal broadening of energies around the Fermi surface leads to a smearing of the interference effects. The thermal length is given by

\[
L_T = \sqrt{\frac{\hbar D}{k_B T}}.
\]

Here \( D \) is the diffusion constant given by

\[
D = \frac{1}{3} v_F^2 \tau_P,
\]

with the Fermi velocity \( v_F \) and the momentum relaxation time \( \tau_P \). So our samples can be treated as quasi-1D as long as their width \( w \) (in our experiments ranging from 18 to 42 nm) and thickness \( t \) (ranging from 42 to 50 nm) are smaller than \( L_{\phi} \) and \( L_T \). The thermal diffusion length of our samples is \(~200\) nm at 20 mK. Below we show that the dephasing length in our samples ranges between 100 and 200 nm at 20 mK.

As we deal here with ferromagnetic material the actual value of the magnetic field inside the material requires some attention. In the experiments described below the external magnetic field is always aligned along the growth direction of the (Ga,Mn)As structures, denoted as \( z \)-direction. Taking the external magnetic field \( B_z \) along this \( z \)-direction, the magnetic field inside the (Ga,Mn)As is given by

\[
B_z' = B_z + J_z (1 - N_z),
\]

where \( J_z \) is the magnetic polarization.
Table 1. Parameters of individual wires (w), rings (r) and wire arrays (a). Length \( L \), width \( w \), thickness \( t \) and number of parallel lines \( N \) of the wire samples. The diameter \( d \) gives the inner and outer diameters of the rings. Some of the samples were annealed for time \( a \) at 200 °C. Curie temperature \( T_C \) and the hole concentration \( p \) were taken from corresponding reference samples. The hole concentration was estimated using the Hall effect. The measurements have been performed at a temperature of 300 mK and at magnetic fields up to 19 T. To remove the anomalous Hall effect from the measured Hall resistance we used a fit similar to that described in [21]. This procedure allows the determination of the hole concentration with a relatively small experimental error. The phase coherence length of the samples w1–w5, r1 and r1a were calculated using equation (1) with \( C = 0.41 \), as we measure at high-magnetic fields [22] and have spin–orbit (SO) interaction [23] explained in the text below. The phase coherence length of samples r2 and r3 were calculated using the amplitude of the AB oscillations and equation (2). For the wire arrays a1 and a2, we fitted the weak antilocalization (WAL) correction given by equation (3). The phase coherence length and the corresponding dephasing time were taken at 20 mK for all samples.

| Sample | w1 | w2 | w3 | w4 | w5 | r1 | r1a | r2 | r3 | a1 | a1a | a2 |
|--------|----|----|----|----|----|----|----|----|----|----|----|----|
| \( L \) (nm) | 100 | 200 | 300 | 800 | 370 | – | – | – | – | 7500 | 7500 | 7500 |
| \( w \) (nm) | 20 | 20 | 20 | 20 | 35 | 30 | 18 | 30 | 42 | 42 | 35 |
| \( t \) (nm) | 50 | 50 | 50 | 50 | 42 | 50 | 42 | 42 | 42 | 42 | 42 |
| \( N \) | 1 | 1 | 1 | 1 | 1 | – | – | – | – | 25 | 25 | 25 |
| \( d \) (nm) | – | – | – | – | – | 120–180 | 120–180 | 120–155 | 160–220 | – | – | – |
| \( a \) (h) | 0 | 0 | 0 | 0 | 0 | 12 | 51 | 51 | 0 | 51 | 0 |
| \( T_C \) (K) | 55 | 55 | 55 | 55 | 90 | ? | 100 | 150 | 150 | 90 | 150 | 90 |
| \( p \) (10\(^{-5}\) \( \Omega \) m) | 1.8 | 1.8 | 1.8 | 1.8 | 3.5 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 |
| \( D \) (10\(^{-5}\) m\(^2\) s\(^{-1}\)) | 4.8 | 4.8 | 4.8 | 4.8 | 2.5 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 |
| \( L_\phi \) (nm) | 110 | 95 | 90 | 155 | 135 | 125 | 160 | 130 | 130 | 150 | 190 | 160 |
| \( \tau_\phi \) (ps) | 270 | 195 | 170 | 500 | 175 | 625 | 600 | 140 | 140 | 280 | 300 | 320 |

in the \( z \)-direction and \( N_z \) is the corresponding demagnetization factor. In case of a 2D ferromagnetic film, \( N_z = 1 \) and internal and external fields (in the \( z \)-direction) are identical. In the case of wires, explored here, the demagnetization factors are approximated by using cigar shaped ellipsoids with their long axis along the wire axis and the short axes corresponding to wire width/thickness [24]. For aspect ratios relevant here, i.e. a wire length of order 1 \( \mu \)m (in the \( x \)-direction) and wire cross sections of about 40 nm, we obtain for the demagnetization factors \( N_x = N_y \approx 0.5 \) and \( N_z = 0 \). Hence the internal field is \( B_z' = B_z + J_z/2 \). The maximum value of \( J_z^{\text{sat}} \) is given by the saturation magnetization of (Ga,Mn)As. Maximum values of our samples’ saturation magnetization are \( \sim 30 \) emu cm\(^{-3}\) which translates in SI system into values of \( \sim 40 \) mT (see e.g. [5]). This means that the maximum difference between the externally applied field \( B_z \) and the field \( B_z' \) inside the material is about 20 mT. The difference between \( B_z' \) and \( B_z \) is practically zero at \( B_z = 0 \) and at most \( \sim 20 \) mT at the saturation field. This is only
a small correction of the external field in the sense that the difference between the internal and external fields would hardly be noticeable on the magnetic field axes displayed below. We hence use in the following that $B'_z \approx B_z = B$.

3. UCFs in (Ga,Mn)As wires and rings

One of the most straightforward methods of measuring the phase coherence length and its temperature dependence relies on measurements of UCFs in a single mesoscopic wire. UCFs stem from interference of partial electron (hole) waves, scattered in a disordered mesoscopic conductor ([25] and references therein). If the wire is smaller than $L_\phi$ in all three spatial dimensions the fluctuation amplitude $\delta G = \sqrt{\langle (G - \langle G \rangle)^2 \rangle} \approx e^2/\hbar$, where the bracket $\langle \cdots \rangle$ denotes averaging over $B$ [22]. The amplitude of these fluctuations does not get, in contrast to AB oscillations, exponentially damped once the wire length exceeds the dephasing length, but attenuated by a power law [22]:

$$\delta G_{\text{rms}} = C \frac{e^2}{\hbar} \left( \frac{L_\phi}{L} \right)^{3/2}.$$ (1)

Here, $L$ is the wire length and $C$ is a constant, with a value close to or smaller than unity, depending, e.g., on the strength of spin–orbit (SO) coupling [23] and the applied magnetic field [22]. Equation (1) describing the fluctuation amplitude of 1D conductors is applicable to extract the phase coherence length as long as $L_\phi$ is larger than the width $w$ and the thickness $t$ of the wire, and if the thermal diffusion length $L_T$ is larger than $L_\phi$ [22]: $w, t < L_\phi < L_T$. Also the temperature dependence of the dephasing length can be extracted by measuring the temperature dependence of the fluctuation amplitude as $L_\phi \propto \delta G^{2/3}$. To investigate UCFs in (Ga,Mn)As
we fabricated individual nanowires with lengths between 100 and 800 nm and measured their resistance $R$ in a perpendicularly applied magnetic field $B$. The relevant parameters of the wires, labeled w1–w4, are given in table 1. The conductance $G$ was obtained by inverting the resistance: $G = 1/R$. In figure 2(a) the conductance of three wires having lengths of 100, 200 and 300 nm, respectively, is plotted as a function of the perpendicularly applied magnetic field at 20 mK. Here, we focus on the high-field region ($B > 2$ T) where the magnetization is saturated and aligned along the external magnetic field. For all three wires aperiodic, well-reproducible conductance fluctuations are visible [26]. With increasing wire length not only the conductance of the wires decreases but also the amplitude of the conductance fluctuations drops, showing that at least the 200 and 300 nm long wires are longer than the dephasing length $L_\phi$. The fluctuation amplitude measured in the 100 nm wire is $\sim 0.5 e^2/h$. Depending on the exact value of the prefactor $C$ in equation (1) the dephasing length is close to the wire length of 100 nm at 20 mK. To investigate the temperature dependence of the dephasing length we measured the conductance fluctuations of the wires at different temperatures. The magnetoconductance trace of wire w2 is displayed in figure 2(b) for different temperatures between 20 mK and 1 K. With increasing temperature the amplitude of the conductance fluctuations is decreasing until they disappear above $\sim 200$ mK. Plotting the amplitude of the conductance fluctuations versus temperature in a log–log-plot (figure 3) gives a power law for the temperature dependence
Figure 3. Amplitude of the conductance fluctuations for the samples w1, w2, w3 and w4 (see table 1) plotted in a log–log diagram. The slope gives the temperature dependence of the conductance fluctuations: $\delta G \propto 1/T^{3/4}$. The inset shows the amplitude of the conductance fluctuations normalized by the wire length.

of the conductance fluctuations: for the 200 nm long wire w2 we obtain $\delta G \propto T^{-0.77}$. This temperature dependence is similar for all investigated wires with an exponent between $-0.77$ and $-0.81$, approximated in the following by $-3/4$. Assuming a temperature independent prefactor $C$ we arrive at the temperature dependence of the dephasing length: $L_\phi \propto 1/\sqrt{T}$. In the case that $L_T$ is smaller than $L_\phi$ the same temperature dependence of $L_\phi$ results. Instead of equation (1), $\delta G = C^e C^h L_T^{1/2}$ with $L_T = \sqrt{\hbar D/k_B T}$ has to be used [22]. Doing so, we again arrive at $L_\phi \propto 1/\sqrt{T}$. The inset of figure 3 shows the amplitude of the conductance fluctuations normalized by the wire length. As all investigated samples lie on one straight line the expected length scaling of UCF, given by equation (1), is experimentally confirmed.

Before discussing this temperature dependence in more detail we address the conductance fluctuations in the low-field regime, excluded in the discussion of figure 2. Corresponding data taken from sample w5 are shown in figure 4. The magnetic field scale on which the conductance fluctuates is much shorter in the low-$B$ regime. The gray shaded area in figure 4 corresponds to the magnetic field range where the AMR effect is observed, i.e. the $B$-range where the magnetization is rotated from an in-plane to an out-of-plane orientation by the externally applied magnetic field. The correlation field of the conductance fluctuations is strikingly different in the low- and high-field regimes. Similar behavior was observed in previous experiments on samples with in-plane easy axis [26, 27] and ad hoc ascribed to the formation of domain walls [27]. As in the low-field regime, the magnetic configuration is continuously changed by the external field, the scattering configuration is altered and the correlation field, which can also be used to extract the phase coherence length [22, 26], is no longer a well-defined quantity. The phase coherence length extracted from the amplitude of the conductance fluctuations is, within experimental error, the same for low- and high-field fluctuations.

The $L_\phi \propto 1/\sqrt{T}$ temperature dependence of the dephasing length has been confirmed by others, also investigating UCFs in (Ga,Mn)As nanowires [27]. Such a relatively weak
Figure 4. UCFs measured in sample w5 (see table 1). An electron micrograph of sample w5 is shown in the lower left inset. The gray shaded regime corresponds to the magnetic field range where the magnetization changes direction. The upper inset shows the low-field UCFs on an expanded magnetic field scale.

temperature dependence for the dephasing length is typical for electron–electron scattering as the dominating source of dephasing. In the case of electron–phonon scattering or electron–magnon scattering one would expect a stronger temperature dependence of $L_\phi \propto 1/T^1$, . . . , $1/T^2$. At low temperatures and reduced dimension electron–electron scattering with small energy transfer, the so-called Nyquist scattering, becomes more effective, leading to $L_\phi \propto 1/T^{1/3}$ [30]. However, this temperature dependence does not describe our results correctly. A possible candidate for dephasing in (Ga,Mn)As nanowires might be critical electron–electron scattering, describing dephasing in a highly disordered metal near the MIT [31]. The corresponding phase coherence length depends, like $L_\phi \propto 1/T^{1/2}$, on temperature. This is in accord with our results, but the calculations were done for a 3D system. Hence, the detailed microscopic origin of dephasing in (Ga,Mn)As wires is still an open issue.

To probe whether electron–electron scattering is the main source of dephasing we investigated UCFs under nonequilibrium conditions, meaning that the effective electron temperature is higher than the lattice temperature. To control the effective electron temperature the applied excitation current has been varied. We assume that even for the highest excitation currents of 8 nA the lattice temperature is not increased. This current corresponds to a voltage drop of 67 $\mu$V across the sample and a heating power of $5 \times 10^{-13}$ W. The cooling power of our dilution refrigerator at 40 mK, on the other hand, is approx. 50 $\mu$W which is eight orders of magnitude higher. The sample is located in the $^3$He–$^4$He mixture, so that thermal coupling should be sufficient to keep the lattice in equilibrium with the bath temperature. Figure 5 displays UCFs in a ring with a diameter of 150 nm (sample r1a in table 1). The conductance of this ring did not exhibit periodic conductance oscillations due to the AB effect, but only aperiodic conductance fluctuations, similar to the ones observed in wires. In figure 5, the
fluctuations are successively suppressed by increasing either temperature or current. In one experiment, the bath temperature was varied from 80 to 640 mK (black lines from bottom to top), while the excitation current was kept at a value where the effective electron temperature is still in equilibrium with the bath temperature (200 pA at 80 mK, 500 pA at 160 and 320 mK, and 1 nA at 640 mK. For comparison: 1 nA corresponds here to a voltage drop $U$ of 9.5 $\mu$V across the sample, equivalent to $eU/k_B = 110$ mK). In this regime, the conductance fluctuations do not depend on the excitation current as long as the hole gas and the lattice are in equilibrium. Then we varied the excitation current, at a fixed bath temperature of 40 mK, from 1 to 8 nA (red lines from bottom to top). In that case the fluctuation amplitude depends on the excitation current. As one can see in figure 5, the black and red traces lie quite well on top of each other. Thus we arrive at the same experimental result in two ways, firstly by increasing the bath temperature and secondly by raising the excitation current. Since the excitation current cannot change the lattice temperature, as argued above, the observed dephasing cannot depend on the lattice temperature. This means that electron–phonon and electron–magnon scattering can be excluded as the source of dephasing. This is also consistent with the temperature dependence of the dephasing length, discussed above. The inset of figure 5 shows the average amplitude of the conductance fluctuations in figure 5, both as a function of the bath temperature (solid symbols) and the applied voltage in units of $eU/k_B$ (open symbols), respectively. Both traces display the same slope and lie above each other. This suggest a linear correlation of effective temperature and applied voltage. Such a linear dependence is only expected if the lattice is decoupled from the electrons [32] and if the effective electron temperature is given

\[ \delta G \propto \frac{1}{T^{3/4}}. \]
by $eU/k_B$. The dominating parameter for dephasing in the low-temperature regime is hence the effective electron temperature and electron–electron scattering is the most likely source of dephasing.

Low temperature annealing of (Ga,Mn)As causes out diffusion of Mn ions which occupy interstitial lattice sites, act as double donors and couple antiferromagnetically to Mn ions on regular Ga sites. So low temperature annealing increases the carrier concentration, the Curie temperature and the total magnetic moment of the samples [16]. Has such low temperature annealing of the samples an influence on the phase coherence length? In nonmagnetic metals magnetic impurities strongly reduce the phase coherence length [33]. This poses the question whether Mn interstitials have a similar effect on the dephasing length in (Ga,Mn)As, as they increase the magnetic disorder due to their random distribution. In order to look for effects of low temperature annealing we measured the conductance of a 150 nm diameter ring at 20 mK before (r1) and after annealing (r1a). The corresponding magnetoconductance traces are shown in figure 6 (black line) and after annealing (red line). As expected, the conductance increased significantly after annealing (here by a factor of approximately 2). While the amplitude of the conductance fluctuations is $0.055 e^2/h$ before and $0.080 e^2/h$ after annealing, the corresponding dephasing length, determined by equation (1), is increased by 30% after annealing. Calculating the dephasing time $\tau_\phi = L_\phi^2/D$, we find that the dephasing time does not change after annealing. While $L_\phi$ increases, the diffusion constant $D = \frac{1}{2}v_F^2\tau_\rho$ also increases by about 75% since both the Fermi velocity $v_F$ (increased hole density) and the momentum relaxation time $\tau_\rho$ get larger. This means that the dephasing time is essentially unaffected by low temperature annealing. The change in coherence length is only due to the change of the diffusion constant. If electron–electron interaction (EEI) is the source of dephasing one would expect that $\tau_\phi$ depends on the carrier concentration. Since in our experiment the change in carrier density was only 15% the $\tau_\phi(p)$ dependence could not be resolved. The main difference of the Mn interstitials in (Ga,Mn)As compared to magnetic impurities in normal metal is the coupling strength. In normal
metals the magnetic impurities are uncoupled above the Kondo-temperature and a spin-flip process is energetically accessible for the electrons, leading to a loss of phase information \cite{34}. In (Ga,Mn)As the Mn interstitials are coupled quite strongly \cite{14} and a spin–flip process is not possible. This explains why low temperature annealing does not increase the dephasing time, but only increases the dephasing length by an increase in the diffusion constant.

SO interaction plays an important role in the valence band of (Ga,Mn)As and hence also has an effect on the analysis of UCFs. In equation \cite{1} SO interaction was not taken into account. Chandrasekhar et al \cite{23} pointed out that the presence of spin-dependent scattering affects the amplitude of the conductance fluctuations. The amplitude of the conductance fluctuations, determined by equation \cite{1}, depends on the ratio of phase coherence length \( L_\phi \) and SO lengths \( L_{SO} \). With increasing \( L_\phi/L_{SO} \) the amplitude of the UCFs gets reduced. This means that in the presence of SO interaction the phase coherence length gets larger for a given value of the UCF amplitude. To determine the corresponding factor requires knowledge of the SO length \( L_{SO} \) which can be extracted from weak-antilocalization (WAL) correction. After we have extracted \( L_{SO} \) from WL experiments, discussed below, we return to this point again.

4. AB effect in (Ga,Mn)As rings

A quite prominent manifestation of phase coherence in mesoscopic conductors is the so-called AB effect. In a ring geometry, a wave-packet is split into two partial waves propagating along the upper and lower half of the ring perimeter and interfering at the ‘exit’. The phase of the partial waves can be tuned by a perpendicular magnetic field so that, as a function of magnetic field strength, constructive and destructive interferences of the partial waves can be adjusted. At sufficiently low temperatures the conductance across the ring displays periodic oscillations with a period \( \Delta B = \Phi_0/A \), where \( A \) is the area enclosed by the ring and \( \Phi_0 \) is the flux quantum \( h/e \). Periodic AB oscillations have been observed over the last few decades in normal metals \cite{35} and in low-dimensional electron and hole systems, e.g. \cite{36,37}. However, the observation of AB oscillations in ferromagnetic ring structures is even more challenging and until quite recently it was unclear whether AB effects can be observed at all in ferromagnets. The first observation of AB effects was reported in FeNi rings in the year 2002 \cite{38}. Measuring the AB effect in (Ga,Mn)As rings requires that the phase of a wavepacket is preserved while traversing the ring. With increasing ring radius \( r \) the amplitude is damped exponentially, \( \delta G \propto \exp(-\pi r/L_\phi) \) \cite{35}. Additionally an aspect ratio close to one (ratio of inner and outer radii of the ring) is needed to avoid smearing of the interference pattern. In (Ga,Mn)As the phase coherence length is about 100 nm at 20 mK. Hence, AB oscillation should be visible in rings with diameters of about 100–200 nm. The conductances of the rings r1 and r1a, discussed above in figures 5 and 6, display no clear AB oscillations. In an attempt to resolve AB-type oscillations the ring diameter was further reduced to a diameter of only 100 nm \cite{26}. Corresponding magnetoconductance data of this ring, clearly containing a contribution of the AB effect, are published in \cite{26}. However, the associated Fourier spectrum is less clear as it displays no clear peak. One reason for this is the relatively bad aspect ratio of the ring causing a broadening of the Fourier peak. In an attempt to resolve AB oscillations we tried to increase the phase coherence length by increasing the diffusion constant of the (Ga,Mn)As material. We found the maximum diffusion constant of \( 12 \times 10^{-5} \text{m}^2 \text{s}^{-1} \) in an optimally annealed wafer containing approximately 5–6% manganese and having a Curie temperature of 150 K. From this wafer we fabricated a ring with an inner diameter of 120 nm and an outer diameter of 155 nm (sample r2 in table 1). A corresponding
Figure 7. (a) An electron micrograph of sample r3 having an average diameter of 190 nm. (b) Magnetoconductance of sample r2 in a perpendicular applied magnetic field measured at 20 mK (black line) and 100 mK (red line). The period corresponding to the AB effect is marked. (c) Magnetoconductance of sample r3 in a perpendicular applied magnetic field measured at 20 mK (black line) and 100 mK (red line). The period corresponding to the AB effect is marked. (d) Fourier transformation of the magnetoconductance at 20 mK of sample r2 (black line) and sample r3 (red line). The magnetic field interval expected for $h/e$ oscillations, evaluated by the inner and the outer diameters of the ring, is marked.

electron micrograph is shown in figure 7(a). The magnetoconductance of this ring is displayed in figure 7(b). At a temperature of 20 mK well pronounced periodic oscillations emerge from the background with a period $\Delta B = 220–370$ mT expected for the ring geometry. Also in the Fourier spectra a clear peak is visible at approximately $3.7 \, \text{T}^{-1}$ (see figure 7(d), black trace). The aperiodic conductance fluctuations in the magnetoconductance arise, as in wires, from interference effects in the individual ring arms. These fluctuations are superimposed on the periodic AB conductance oscillations. From the amplitude of the oscillations we can estimate a
phase coherence length of $l_\phi \approx 130\,\text{nm}$ by using [35]:

$$\delta G = \frac{e^2}{\hbar} \frac{L_T}{L_\phi} \exp(-\pi r/L_\phi).$$

(2)

By increasing the temperature to 100 mK these periodic oscillations disappear. This strong sensitivity to temperature is a consequence of the temperature dependence of the phase coherence length and the exponential damping of the oscillations. To prove the dependence of the period on the ring diameter, i.e. the enclosed magnetic flux, we fabricated a ring, labeled r3, with an average diameter of 190 nm. Magnetococonductance data of this ring in a perpendicular applied magnetic field are shown in figure 7(c). At 20 mK periodic AB oscillations are again visible but the amplitude is much smaller than the ones observed in the smaller ring. This is expected from the exponential suppression. Here the value of $L_\phi$ is again $\sim 130\,\text{nm}$. The Fourier spectra in figure 7(d) (red trace) show a peak at approximately $6.8\,\text{T}^{-1}$ which is in very good agreement with the expected value between $4.8$ and $9.1\,\text{T}^{-1}$. While the lower value corresponds to the value of the inner diameter, the higher one is extracted from the flux through the outer diameter. Also in this ring the AB oscillations are gone at 100 mK. A contribution of the first harmonic of the AB effect, with a period of $\hbar/2e$, could not be observed in any of the rings investigated. The value of the dephasing length extracted from the AB oscillations is $\sim 130\,\text{nm}$ for both samples (samples r2 and r3) and thus consistent with the value extracted from the UCFs.

5. WL and WAL in (Ga,Mn)As wire arrays

The effect of WL [19] belongs to the class of so-called time-reversed interference effects. Scattered partial waves of particles on time-reversed closed paths interfere constructively, causing an enhanced probability of backscattering which decreases the conductance. A perpendicular magnetic field destroys the constructive interference and hence the quantum mechanical correction to the conductivity and the resistance returns within a characteristic magnetic field scale towards the Drude value. The resulting negative magnetoresistance is the characteristic hallmark of WL. As the maximum area of the closed loops which contribute to WL is limited by the phase coherence length, fits of the WL magnetoresistance provide another means to extract the phase coherence length. In the presence of SO interaction, the spin part of the wavefunction needs also to be taken into account. The two partial waves on time-reversed closed paths experience a spin rotation in opposite directions causing (partially) destructive interference [19]. So, SO interaction leads to reduced backscattering and reverses the sign of the WL, hence is called WAL. A typical signature of WAL is a double dip in the magnetoconductance trace [19]. As a sufficient strong magnetic field removes time reversal symmetry, the question arises, whether WL can be observed at all in a ferromagnet. In conventional ferromagnets several experimental works explored this problem [39]–[42], but a direct signature of WL was not found. The ferromagnetic semiconductor (Ga,Mn)As sheds new light on this question, as the internal magnetic field in ferromagnetic semiconductors is quite weak compared to conventional ferromagnets. To search for WL in the ferromagnetic semiconductor (Ga,Mn)As we fabricated arrays of wires connected in parallel (see table 1,

\footnote{The conductance shown in figures 7(b) and (c) is the conductance of the ring and the leads up to the voltage probe. Equation (2) describes only the conductance oscillation of the ring. This issue has to be taken into account when calculating the phase coherence length.}

New Journal of Physics 10 (2008) 055016 (http://www.njp.org/)
samples a1 and a2) and measured their resistance in a perpendicular applied magnetic field. By measuring an array of geometrically identical wires UCF get suppressed by ensemble averaging. Also here the conductance was obtained by inverting the resistance, $G = 1/R$. In figure 8(a), an electron micrograph of sample a1 containing 25 nanowires in parallel is shown. The magnetoconductance$^3$ of sample a1 is shown in figure 8(b) for temperatures ranging from 600 mK down to 20 mK. We first start with the description of the dominant features observed in experiment. The conductance maximum around zero field is due to the so-called AMR (marked AMR and gray shaded in figure 8(b)). Without applied magnetic field the magnetization of the (Ga,Mn)As wires lies in-plane. For such in-plane magnetization the conductance is higher than

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$^3$ In samples a1 and a1a, the two voltage leads were fabricated on opposite sites. So, also the Hall conductance was measured. To remove the Hall conductance from the data only the symmetric part of the trace was taken. As this leads to the same result as seen for sample a2, where the voltage leads are on the same side, the physics is not changed by this procedure.
Figure 9. Magnetoconductance of sample a1 measured at different temperatures and normalized to the conductance value at $B = 3$ T. The magnetic field range where the magnetization is rotated from in-plane to out-of-plane is gray-shaded.

for an out-of-plane oriented magnetization [13]. Hence in the low-$B$ region the magnetization is rotated from in-plane to out-of-plane and causes the negative magnetoconductance. This magnetic field region is highlighted by gray-shading. At higher fields the positive slope of $G$ (marked NMR in figure 8(b)) is ascribed to an increase of magnetic order [43], or to 3D WL [44]. At temperatures above $\sim 65$ mK the different $G(B)$ traces are shifted without a noticeable change of their shape and the AMR height scales linearly with the background conductance. This is shown in figure 9, where the different $G(B)$ traces are normalized to their $G(3$ T) value. For temperatures between 65 and 600 mK all traces lie on top of each other. This means that the conductivity at different temperatures differs only by a constant factor $\alpha$, i.e. $G(T, B) = \alpha(T)G(B)$. The decreasing conductance with decreasing temperature displayed in figure 8(b) is due to EEI. In contrast to the one-electron interference of scattered waves which causes WL, EEI results from interference of EEI amplitudes corresponding to successive electron–electron scattering events in disordered systems [45, 46]. In contrast to WL, EEI is independent of magnetic field. In 1D samples the conductance decrease due to EEI follows a $1/\sqrt{T}$ dependence [45]. This dependence is shown for the three investigated wire arrays in figure 10(a). For all three wire samples the conductivity decrease follows such a $1/\sqrt{T}$ dependence. The effect of EEIs in 1D and 2D (Ga,Mn)As is discussed in more detail in [47]. Here, we only note that EEI is responsible for the shift of the traces at different temperatures in figure 8(b). At $T < 65$ mK two down cusps start to appear in the magnetoconductance of sample a1 in figure 8(b) at approximately $\pm 400$ mT and become a prominent feature at 20 mK. This is also seen in figure 9, where the different $G(B)$ are normalized to their $G(3T)$ value. While the high-temperature traces all lie on top of each other, the low-temperature traces show two downward cusps with a size becoming comparable to the AMR at 20 mK. To separate this novel effect from the other transport contributions we have to subtract the high-temperature background containing AMR and NMR. Here, we assume that the background scales like at
Figure 10. (a) Conductivity change of the samples a1, a1a and a2 relative to 50 mK, taken at $B = 0$ (solid symbols) and $B = 3$ T (open symbols). The solid lines are the best linear fits for the samples a1 and a2. (b) WL contribution of the samples a1, a1a and a2 obtained after subtracting the 120 mK trace as background conductance. To compare the different samples the total $\Delta G$ was divided by the number of parallel wires. Again the gray-shaded $B$-range corresponds to the regime where the samples's magnetization follows the external field and changes direction. The red lines are best fits to equation (3), discussed in the text. The fit parameters were $L_\phi = 190$ nm and $L_{SO} = 113$ nm for sample a1a, $L_\phi = 160$ nm and $L_{SO} = 93$ nm for sample a2 and $L_\phi = 150$ nm and $L_{SO} = 93$ nm for sample a1.

temperatures above 50 mK, i.e. that $\Delta G = G(20 \text{ mK}) - \alpha G(120 \text{ mK})$. Here we have chosen the conductance at 120 mK as reference conductance to account for the background. The factor $\alpha$ takes the linear scaling of AMR and NMR with the conductance value into account. We notice
that the choice of the reference trace we subtract, has an indiscernible influence on the result as the high-temperature traces all lie on top of each other when normalized. The resulting traces which we ascribe to the WL correction are shown in figure 10(b) for all three samples (a1, a1a and a2 in table 1). All three \( \Delta G \) traces in figure 10(b) display a local conductance maxima at \( B = 0 \) and two conductance minima at \( B = \pm 400 \) mT. Such \( \Delta G(B) \) shape is typical of the effect of WAL in systems with SO interaction. To extract the characteristic lengths from the data we utilize existing theory. The correction due to WAL in a quasi-1D system is given by [33, 48]:

\[
\Delta G = g_s \frac{e^2}{h} \left[ \frac{1}{2L} \left( \frac{1}{L_\phi^2} + \frac{1}{3 L_\phi^4 L_H^4} \right)^{1/2} - \frac{3}{2L} \left( \frac{1}{L_\phi^2} + \frac{4}{3 L_{SO}^2} + \frac{1}{3 L_H^4} \right)^{1/2} \right],
\]

where \( g_s \) is the spin degeneracy, \( L_{SO} \) is the SO scattering length describing the strength of SO interaction, and \( L_H = \sqrt{h/eB} \) is the magnetic length. Equation (3) is valid as long as the quasi-1D assumption is justified: \( w, t < L_\phi, L_\phi \ll L \). With the \( L_\phi \) value extracted from previous experiments, discussed above, this is justified for \( T = 20 \) mK and \( |B| < 400 \) mT. As the valence band in ferromagnetic (Ga,Mn)As is split, the holes are highly (but not fully) spin polarized [49]. To account for spin polarization, we approximated \( g_s \) either by 1 (fully spin polarized) or by 2 (spin degenerate) as an adjustable parameter. As fits with \( g_s = 2 \) cannot describe the experimental results we resort to \( g_s = 1 \) below. (The fit with \( g_s = 2 \) for sample a1 is shown in [50].) Corresponding fits are presented in figure 10(b) as red lines. The fit parameters are \( L_\phi = 150 \) nm and \( L_{SO} = 93 \) nm for sample a1, \( L_\phi = 190 \) nm and \( L_{SO} = 113 \) nm for sample a1a, and \( L_\phi = 160 \) nm and \( L_{SO} = 93 \) nm for sample a2. While the fits are in good agreement with experiment for \( |B| < 400 \) mT, they are less satisfying at higher fields. The WL or WAL correction is, as a function of increasing \( B \), more abruptly suppressed than expected from theory. There is a striking correlation with the magnetic field dependence of the AMR effect. The magnetic field region where the AMR occurs is highlighted by gray shading in figures 8(b), 9 and 10(b). Within this \( B \)-field range, the magnetization is rotated from in-plane to out-of-plane. Once the magnetization is out-of-plane, the WL correction drops quickly. At the same magnetic field, the magnetic length matches wire width and thickness, \( L_H \sim w, t \). Hence, the discrepancy between fit and experiment might be associated with dimensional crossover (1D–3D), if \( |B| \) exceeds 400 mT and equation (3) might be inapplicable. We further note that neither the field-dependent change of the magnetization direction nor the 3/2-spin of the involved hole states was taken into account as the theory was developed for spin-1/2 electrons in disordered metals. Especially, the latter could add a number of additional interference diagrams not yet treated theoretically.

While WAL has already been observed in nonmagnetic p-type (Al,Ga)As/GaAs quantum wells [51], the observation in a ferromagnet came as a surprise. A recent theory suggests that the process leading to WAL is totally suppressed in a ferromagnet due to the spin-splitting which excludes a contribution of the so-called singlet Cooperon diagram, responsible for WAL [52]. This theory was calculated for a quite strong ferromagnet with relatively high mean free path. This is not the case for (Ga,Mn)As and the observation of WAL in (Ga,Mn)As is not excluded by this theory [53, 54].

Also in this experiment we measured the magnetoconductance of one sample before and after annealing (samples a1 and a1a) and compared the resulting phase coherence length. Here, the change of the phase coherence length was 27%. Again, this change can be ascribed to a
change of the diffusion constant, while the relevant dephasing time stayed nearly unchanged (see table 1). This underlines that the Mn interstitials do not cause dephasing.

With knowledge of $L_{SO}$ we can now give a more precise value of the dephasing length $L_\phi$ extracted from the UCF measurement discussed in section 3. The dephasing length is dependent on the fluctuation amplitude $\delta G_{rms}$, the wire length $L$ and a prefactor $C$ by $\delta G_{rms} = C\frac{e^2}{\hbar} \left( \frac{L}{T} \right)^{3/2}$. To determine the prefactor $C$, knowledge of the ratio of $L_\phi/L_{SO}$ is necessary [23]. For a ratio $L_\phi/L_{SO} \approx 1.5$ (taking a SO scattering length of $\sim 100$ nm and a dephasing length of $\sim 150$ nm) we get a prefactor of $C = 0.58$ [23]. As we also deal with a relatively high-magnetic field ($B = 2$ T, ..., 15 T) where no WL can be observed, the prefactor is reduced by a factor of $\sqrt{2}$. This factor is due to the absence of the Cooperon term at high $B$ [22]. Taking this into account we obtain a prefactor of $C \approx 0.41$. At 20 mK this leads to dephasing lengths for the samples w1, w2, w3, w4, w5, r1 and r1a ranging from 90 to 160 nm (see table 1). These values are in very good agreement with the ones extracted from the amplitude of the AB oscillations ($L_\phi \approx 130$ nm) and the ones obtained by fitting the weak localization correction ($L_\phi = 150$ to 190 nm).

6. Conclusion

We investigated phase coherent transport in the ferromagnetic semiconductor (Ga,Mn)As by measuring UCFs, AB oscillations and WL. All three methods reveal a phase breaking length of 90–190 nm at 20 mK. By investigating UCFs at different temperatures we found a temperature dependence of $L_\phi \propto 1/T^{1/2}$. As the main source of dephasing, electron–electron scattering or in our case hole–hole scattering was identified by investigating the temperature and excitation current dependence of the UCFs. This is consistent with the quite weak temperature dependence of $L_\phi$. The Mn interstitials do not cause dephasing; low-temperature annealing increases the dephasing length only due to an increase of the diffusion constant, while the corresponding dephasing time stays unchanged. The magnetotransport in arrays of wires is modified by WL at temperatures below 50 mK, showing that time-reversed interference effects can be observed in a ferromagnet. The existence of WAL shows that SO interaction is affecting the transport in (Ga,Mn)As. The corresponding SO scattering length is always shorter than $L_\phi$ and of order $\sim 100$ nm.

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