Planar positron channeling radiation generated in an acoustic superlattice

B Azadegan\textsuperscript{1}, L Sh Grigoryan\textsuperscript{2} and W Wagner\textsuperscript{3}

\textsuperscript{1} Sabzevar Tarbiat Moslem University - P.O. 397, Sabzevar, Iran
\textsuperscript{2} NAS Institute of Applied Problems in Physics - 25 Hr. Nersessian str., 0014 Yerevan, Armenia
\textsuperscript{3} Helmholtz-Zentrum Dresden-Rossendorf - P.O.Box 510119, 01314 Dresden, Germany
E-mail: w.wagner@hzdr.de

Abstract. Planar positron channeling in an acoustic superlattice and stimulation of the emission of channeling radiation at resonance of the particle motion with the acoustic field excited in a PbTiO\textsubscript{3} single crystal is considered in the framework of classical mechanics and electrodynamics. Based on computed particle trajectories, spectral-angular distributions of channeling radiation influenced by resonant ultrasound have been simulated. The presented method explains the influence of ultrasonic waves on the intensity of channeling radiation in a rather simple and descriptive manner.

1. Introduction
Channeling radiation (CR), which has been explained as a self-contained source of X- and γ-rays in 1976 [1], is a well investigated phenomenon today (see, e.g., [2, 3, 4, 5, 6, 7]). At not too high energies (up to several tens of MeV) of the channeled light charged particles, CR emission has to be considered as a quantum process, whereas at highly relativistic energies classical mechanics and electrodynamics may be utilized for an adequate description of CR [8]. Since the decisive criterion is the number of bound states within the continuous transverse potential [9], which governs channeling, the situation is rather different for positrons and electrons at planar or axial channeling, respectively. Besides that, this number strongly depends upon the particle energy, atomic number and interatomic distances of the crystal as well [8, 10].

Furthermore, since channeling and associated CR emission is naturally controlled by the continuous potential of the considered crystallographic axis or plane of a single crystal, the idea to manipulate CR emission by means of dedicated modulation of the crystal lattice has been worked out already in the 1980s [11, 12]. Very first model calculations predicted that the intensity of CR may be increased if the oscillatory motion of the channeled particles becomes resonant with ultrasonic waves (US) excited in the crystal [13, 14]. It was suggested to apply piezoelectric single crystals in which US may be excited utilizing external electromagnetic RF-fields. Forthwith first measurements of electron CR on quartz, LiNbO\textsubscript{3} and CdS crystals were carried out at ultrarelativistic energies where the CR spectra show a smooth behaviour [15].

A consistent quantum theory of electron and positron CR at impact of US has been developed in a series of works published after year 2000 [16, 17, 18]. Based on it, systematic measurements of planar CR on quartz single crystals have been carried out at medium electron energies (14 - 32
MeV) as an important prerequisite for envisaged experimental investigations [19, 20]. Since the frequency of resonant US strongly depends upon the transition energy between bound channeling states, measurements where single CR lines are resolved were categorically necessary.

Eventually, the effect of US on planar electron CR could be verified experimentally for two different crystallographic planes of quartz [21, 22, 23]. It should be mentioned that, in agreement with preceding simulations, resonant enhancement of the CR intensity has been observed at hypersound frequencies of about 12 GHz.

In the framework of quantum theory, the mechanism of CR stimulation by resonant US has been explained by the dispersive behaviour of the eigenstates of channeled particles in the continuous planar potential [24]. Within an acoustic superlattice of certain US wavelength, the longitudinal momentum of channeled particles, which occupy corresponding transverse states, is no more an integral of motion. Hence, conservation of total energy directly leads to a quantum uncertainty [24] which appears in an intermixture of dedicated channeling states due to a resonant increase of the dispersion of transverse energy. The induced modification of the occupation of bound states finally influences the CR intensity.

At particle energies, where the classical treatment of channeling is valid (see, e.g., [25]), a description of this resonance effect is still missing. This paper for the first time offers a descriptive classical explanation of the influence of resonant US on CR. Exemplarily and for simplicity, we consider planar positron channeling in a polyatomic PbTiO$_3$ crystal. Since the piezoelectric constants of this material are much larger than those of quartz, generation of US by external electromagnetic fields should principally be more effective.

In what follows we briefly consider theoretical basics which have been applied for detailed computing of particle trajectories in an acoustic superlattice. Based on these data, spectral-angular distributions of CR have been calculated by means of classical electrodynamics. The results presented provide a simple picture how resonant US influence channeling and CR emission.

2. Theory

Let us consider a relativistic charged particle planar channeled in a piezoelectric single crystal, and assume that longitudinal US are excited in the crystal along the direction of channeling, $z$. In this case, the quasi-static, continuous and periodic transverse potential of the channeling plane, $U(x)$, is modulated by another periodic function, $A(z)$. Since, for a relativistic particle, $z \approx ct$ ($c$-velocity of light), the potential $V(x, z)$ is time-dependent, and the usual procedure applied to calculate the continuous planar potential, namely averaging of the atomic potential in space (over $y$ and $z$) and time (over thermal atomic vibrations), has to be modified. As shown in [26], $V(x, z)$ may approximately be written as the product $U(x)A(z)$ where

$$A(z) \cong 1 + 2 \sum_{m=1,2,\ldots} (-1)^m J_m(\omega_x a^*) \cos[m(k_x z + \varphi)]$$

(1)

does not yet depend on the crystal parameters (except, of course, the velocity of US in the crystal, $u_s$). $J_m$ denotes the Bessel function, $k_x = \omega_x / u_s$ is the wave number of US, $a^* = a \sin(\omega_s t)$ their amplitude and $\omega_x$ the frequency. The (undisturbed) periodic planar potential in the most common form reads

$$U(x) = \sum_{n=1,2,\ldots} \nu_n \exp(\imath n g x) \quad n = \ldots -1, 0, 1, 2, \ldots$$

(2)

where $\mathbf{g}$ is the reciprocal lattice vector normal to the crystallographic plane considered, and the coefficients of this Fourier series are given by

$$\nu_n = \frac{2 \pi a_0 Z_1 e^2}{V_c} \sum_j \exp[M_j(\mathbf{g})] \cdot \exp(-i \mathbf{g} \cdot \mathbf{r}_j) \sum_{i=1}^4 a_i \exp\left[-\frac{1}{4} (\frac{b_i}{4\pi^2}) (ng)^2\right]$$

(3)
Figure 1. The (110) planar potential of a PbTiO$_3$ single crystal modulated by ultrasound of $k_s = 1.282$ Å$^{-1}$.

$(a_0, Z_1, e$ and $V_c$ are the Bohr radius, atomic number, electron charge and the volume of unit cell, respectively, $r^i_j$ is the radius vector to the lattice site of atom $j$ in the unit cell, $a_i$ and $b_i$ are parameters defined in [27]). The Debye-Waller factor $M_j(\vec{g}) = g^2 \langle u^2_j \rangle / 2$ takes the one-dimensional mean-squared thermal vibration amplitude $\langle u^2_j \rangle$ of atom $j$ into account.

The continuous potential, modulated by US, of the (110) plane of a PbTiO$_3$ single crystal, calculated numerically for positron channeling, is shown in figure 1. Due to the perovskite structure of PbTiO$_3$, the undisturbed (110) potential is by no means a harmonic one (as usually assumed for analytic calculations of positron channeling, e.g., in [13]). Therefore, bound channeling states, which would result from a quantum calculation, are not equidistant. Accordingly, from a classical calculation one would expect a CR spectrum peaked at several frequencies. This situation partly resembles planar electron channeling where, as exploited in our former works [21, 22, 23], the US frequency had to be resonant to some CR transition for observing the effect of CR intensity enhancement. The classical description of CR generation is based on the calculation of the trajectory of the channeled particle (here $\vec{r}(t) = \vec{x}(t) + \vec{z}(t)$, $\vec{y}(t) \equiv 0$) obtained by solving the equations of motion

$$
\gamma m \ddot{x}(t) = -\frac{\partial V(x, z)}{\partial x}, \quad \gamma m \ddot{z}(t) = -\frac{\partial V(x, z)}{\partial z}
$$

(\gamma m - relativistic particle mass, $\gamma$ - Lorentz factor) at given initial conditions, i.e., the coordinates of the point of incidence of the particle into the crystal and the projections of its momentum for an angle of incidence, $\Theta_0$, with respect to the channeling plane

$$
x(0) = x_0 \quad z(0) = z_0 \quad p_x(0) = p \sin(\Theta_0) \quad p_z(0) = p \cos(\Theta_0).
$$

According to classical electrodynamics, the energy of CR radiated into a solid angle $d\Omega$ within
a frequency interval \((\omega, \omega + d\omega)\) is given by

\[
\frac{d^2E}{d\omega d\Omega} = \frac{e^2}{4\pi^2c} | \int_0^\tau \exp[i(\omega t - \vec{k} \cdot \vec{r})] \hat{n} \times ((\vec{n} - \vec{\beta}) \times \vec{\beta}) \frac{d\vec{r}}{(1 - \vec{\beta} \cdot \vec{n})^2} |^2
\]

where \(\vec{\beta} = \vec{r}(t)/c\) means the particle velocity, \(\vec{n}\) defines the direction of photon emission, \(\vec{k} = \omega\vec{n}/c\) denotes the wave vector of CR, and \(\tau\) is the time-of-flight of the particle through the crystal [4].

3. Trajectory calculations

In what follows we exemplarily demonstrate the output of a trajectory calculation for positrons of energy 40 MeV channeled along the (110) plane of an ultrasonic superlattice created in a PbTiO$_3$ single crystal. The equations of motion (4) have been solved numerically for the initial conditions (5): \(x_0 = 0.235 \pu{\AA}, \theta_0 = 0\). The period of transverse oscillations of a channeled particle amounts to \(T = 1.408 \times 10^{-15} \pu{s}\). As shown in figures 2 and 3, the influence of US with a wave number of \(k_s = 1.488 \times 10^{-3} \pu{\AA}^{-1}\) appears as an increase of the amplitude of transverse oscillations, \(x(t)\), and transverse velocity, \(\beta_x(t)\). Simultaneously, one observes a (coherent) decrease of the longitudinal velocity, \(\beta_z(t)\) (see figure 4). This means that US, which are resonant to the oscillatory motion of the channeled particle [13, 18], force a transfer of longitudinal momentum to transverse one. Trajectories and velocities not influenced by US are also shown in figures 2-4. They depict the results of corresponding calculations but for wave numbers different from the resonant one by \(\pm 5\% \) \((k_s = 1.571 \times 10^{-3} \pu{\AA}^{-1} \text{ and } k_s = 1.406 \times 10^{-3} \pu{\AA}^{-1})\). This finding again underlines the prediction of theory that the influence of US on CR emission is selective.

It is worth to remind here that the impact of resonant US in a quantum treatment of channeling appeared as a large increase of the dispersion of corresponding eigenvalues. Hence, the quantum uncertainty explored in [24] also reflects a resonant coupling of longitudinal and transverse momentum of the channeled particle. Since the effect is discrete, its verification for planar
Figure 3. Transverse velocity $\beta_x(t) = \dot{x}/c$ of positrons channeled along the (110) plane of a PbTiO$_3$ crystal at impact of resonant and non-resonant ultrasonic waves (see text).

Figure 4. Longitudinal velocity $\beta_z(t) = \dot{z}/c$ of positrons channeled along the (110) plane of a PbTiO$_3$ crystal at impact of resonant and non-resonant ultrasonic waves (see text).

electron channeling [21, 22, 23] succeeded only for selected CR transitions at appropriate tuning of the US frequency.

4. Stimulation of CR emission by ultrasound
The radiation emitted by channeled positrons moving through the PbTiO$_3$ crystal on oscillating trajectories as found in Sect. 3 has been calculated according to (6). The corresponding spectral-angular distributions of forward emitted CR are shown in figure 5. At impact of US resonant with
Figure 5. Spectral-angular distributions of forward emitted CR for positrons of energy 40 MeV impinging with $\Theta_0 = 0$ at $x_0 = 0.235 \text{Å}$ and channeled along the (110) plane of a 10 $\mu$m thick PbTiO$_3$ crystal at impact of resonant (larger amplitude) and non-resonant ultrasonic waves.

Figure 6. Spectral-angular distributions of forward emitted CR for positrons of energy 40 MeV channeled along the (110) plane of a 10 $\mu$m thick PbTiO$_3$ crystal at impact of US with wave number $k_s = 1.488 \times 10^{-3} \text{ Å}^{-1}$ (larger amplitude) compared to the CR spectrum without US (smaller amplitude).
Figure 7. Spectral-angular distributions of forward emitted CR for positrons of energy 40 MeV channeled along the (110) plane of a 10 μm thick PbTiO$_3$ crystal at impact of US with wave number $k_s = 1.282 \times 10^{-3}$ Å$^{-1}$ (larger amplitude) compared to the CR spectrum without US (smaller amplitude).

the oscillatory motion of channeled particles one observes an enhancement of the CR intensity (upper curve in figure 5) at a photon energy of $E_{CR} = 2\gamma^2 \hbar 2\pi/T$. Non-resonant US does not influence CR emission (lower curve in figure 5).

 Naturally, variation of the initial conditions results in varied trajectories. Therefore, averaging over relevant initial parameters is necessary to simulate the residual CR spectrum. For a parallel positron beam ($\Theta_0 = 0$) one would observe CR spectra as shown in figure 6, where the upper spectrum reveals the influence of US with wave number $k_s = 1.488 \times 10^{-3}$ Å$^{-1}$, and the lower one depicts CR emission without US. The spectra are characterised by two maxima (CR "lines"). From comparison with figure 5 one may conclude that the relative influence of US with given $k_s$ on the CR intensity is larger for the smaller CR peak at about 35 keV. A second example calculated for $k_s = 1.282 \times 10^{-3}$ Å$^{-1}$ is shown in figure 7. In this case, US mainly stimulate the intensity of the dominant first CR peak at about 30 keV.

5. Summary

This work presents first classical trajectory calculations for channeling in an ultrasonic superlattice. As may easily be realised, resonant ultrasound influences the transverse amplitude of the oscillatory motion as well as the transverse and longitudinal velocities of the channeled particle. Its longitudinal momentum is no more a conserved quantity. A resonant ultrasonic field changes the CR intensity as has been demonstrated by simulations of CR spectra for planar positron channeling in a PbTiO$_3$ single crystal. The possible magnitude of CR stimulation is, of course, subject to experimental investigations.

Due to the unharmonicity of the continuous potential of the crystallographic (110) plane of PbTiO$_3$, features found for positron channeling resemble planar electron channeling. Up to now, stimulation of CR emission by ultrasonic waves has only been proved for electron channeling in piezoelectric quartz crystals. Suitable positron beams were not available. However, efforts
to modify the positron beam line of the Beam Test Facility at INFN LNF Frascati, Italy, are reported in [28] recently. The verification of the observed effect of CR intensity enhancement by resonant ultrasonic waves on other crystals and/or beams of relativistic charged particles is, therefore, of principal interest.

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