Non-iterative Joint Detection-Decoding Receiver for LDPC-Coded MIMO Systems Based on SDR

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Abstract—Semi-definite relaxation (SDR) detector has been demonstrated to be successful in approaching maximum likelihood (ML) performance while the time complexity is only polynomial. We propose a new receiver jointly utilizing the forward error correction (FEC) code information in the SDR detection process. Strengthened by the code constraints, the joint SDR detector provides soft information of much improved reliability to downstream decoder and therefore outperforms existing receivers with substantial gain.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) transceiver technology represents a breakthrough in the advances of wireless communication systems. Modern wireless systems widely adopt multiple antennas, for example, the 3GPP LTE and WLAN systems [1], and further massive MIMO has been proposed for next-generation wireless systems [2]. MIMO systems can provide manifold throughput increase, or can offer reliable transmissions by spatial diversity [3]. In order to fully exploit the advantages promised by MIMO, the receiver must be able to effectively recover the transmitted information. Thus, detection and decoding remain to be one of the fundamental areas in state-of-the-art MIMO research.

It is well known that maximum likelihood (ML) detection is optimal in terms of minimum error probabilities for equally likely data sequence transmissions. However, the ML detection is NP-hard [4] and its time complexity is exponential for MIMO detection, regardless of whether exhaustive search or other search algorithms (e.g., sphere decoding) are used in data symbol detection. Aiming to reduce the high computational complexity for MIMO receivers, a number of research efforts have focused on designing near-optimal and high performance receivers. In the literature, the simplest linear receivers, such as matched filtering (MF), zero-forcing (ZF) and minimum mean squared error (MMSE), have been widely investigated. Other more reliable and more sophisticated receivers, such as successive interference cancellation (SIC) or parallel interference cancellation (PIC) receivers have also been studied. However, these receivers suffer substantial performance loss.

In recent years, various semi-definite relaxation techniques have emerged as a sub-optimum detection method that can achieve near-ML detection performance [6]. Specifically, ML detection of MIMO transmission can be formulated as least squares integer programming problem which can then be converted into an equivalent quadratic constrained quadratic program (QCQP). The QCQP can be transformed by relaxing the rank-1 constraint into a semi-definite program. With the name semi-definite relaxation (SDR), its substantial performance improvement over algorithms such as MMSE and SIC has stimulated broad research interests as seen in the works of [7], [8], [9], [10]. Several earlier works [7], [8] developed SDR detection in proposing multiuser detection for CDMA transmissions. Among them, the authors of [9] proposed an SDR-based multiuser detector for M-ary PSK signaling. Another work in [10] presented an efficient SDR implementation of blind ML detection of signals that utilize orthogonal space-time block codes. Furthermore, multiple SDR detectors of 16-QAM signaling were compared and shown to be equivalent in [11].

Although most of the aforementioned studies focused on SDR detections of uncoded transmissions, forward error correction (FEC) codes in binary field have long been integrated into data communications to effectively combat noises and co-channel interferences. Because FEC decoding takes place in the finite binary field whereas modulated symbol detection is formulated in the Euclidean space of complex field, the joint detection and decoding typically relies on the concept of iterative turbo processing. In this work, however, we present a non-iterative receiver based on SDR for joint detection and decoding. In our design, FEC codes not only are used for decoding, but also are integrated as constraints within the detection optimization formulation to develop a novel joint SDR detector [12], [13], [14], [15]. Instead of using the more traditional randomization or rank-one approximation for symbol detection, our data detection takes advantage of the last column of the optimal SDR matrix solution. When compared with the original SDR detector in [6], our integrated SDR receiver demonstrates substantial performance gain.

II. SYSTEM MODEL AND SDR DETECTION

A. Maximum-likelihood MIMO Signal Detection

Consider an $N_t$-input $N_r$-output spatial multiplexing MIMO system with memoryless channel. The baseband equivalent
model of this system at time $k$ can be expressed as

$$y_k^c = H_k^c s_k^c + n_k^c, \quad k = 1, \ldots, K,$$

(1)

where $y_k^c \in \mathbb{C}^{N_r \times 1}$ is the received signal, $H_k^c \in \mathbb{C}^{N_r \times N_t}$ denotes the MIMO channel matrix, $s_k^c \in \mathbb{C}^{N_t \times 1}$ is the transmitted signal, and $n_k^c \in \mathbb{C}^{N_r \times 1}$ is an additive Gaussian noise vector, each element of which is independent and follows $CN(0, 2\sigma_n^2)$. In fact, besides modeling the point-to-point MIMO system, Eq. (1) can be also used to model frequency-selective systems [16], multi-user systems [17], among others. The only difference lies in the structure of channel matrix $H_k^c$.

To simplify problem formulation, the complex-valued signal model can be transformed into the real field by letting

$$y_k = \begin{bmatrix} \text{Re}\{y_k^c\} \\ \text{Im}\{y_k^c\} \end{bmatrix}, \quad s_k = \begin{bmatrix} \text{Re}\{s_k^c\} \\ \text{Im}\{s_k^c\} \end{bmatrix}, \quad n_k = \begin{bmatrix} \text{Re}\{n_k^c\} \\ \text{Im}\{n_k^c\} \end{bmatrix},$$

and

$$H_k = \begin{bmatrix} \text{Re}\{H_k^c\} & -\text{Im}\{H_k^c\} \\ \text{Im}\{H_k^c\} & \text{Re}\{H_k^c\} \end{bmatrix}.$$  

Consequently, the transmission equation is given by

$$y_k = H_k s_k + n_k, \quad k = 1, \ldots, K.$$  

(2)

In this study, we choose capacity-approaching LDPC code for the purpose of forward error correction. Further, we assume the transmitted symbols are generated based on QPSK constellation, i.e., $s_k^c \in \{\pm 1 \pm j\}$ for $k = 1, \ldots, K$ and $i = 1, \ldots, N_t$. The codeword (on symbol level) is placed first along the spatial dimension and then along the temporal dimension.

Before presenting the code anchored detector, we begin with a brief review of existing SDR detector in uncoded MIMO systems for the convenience of subsequent integration. By the above assumption of Gaussian noise, it can be easily shown that the optimal ML detection is equivalent to the following discrete least squares problem

$$\min_{x_k \in \{\pm 1\}^{2N_t}} \sum_{k=1}^{K} \|y_k - H_k x_k\|^2.$$  

(3)

However, this problem is NP-hard. Brute-force solution would take exponential time (exponential in $N_t$). Sphere decoding was proposed for efficient computation of ML problem. Nonetheless, it is still exponentially complex, even on average sense [5].

B. SDR MIMO Detector

SDR can generate an approximate solution to the ML problem in polynomial time. More specifically, the time complexity is $O(N_t^{1.5})$ when a generic interior-point algorithm is used, and it can be as low as $O(N_t^{0.5})$ with a customized algorithm [6]. The trick of using SDR is to firstly turn the ML detection into a homogeneous QCQP by introducing auxiliary variables $\{t_k, k = 1, \ldots, K\}$ [6]. The ML problem can then be equivalently written as the following QCQP

$$\begin{array}{ll}
\min_{(x_k, t_k)} & \sum_{k=1}^{K} \left[ x_k^T t_k \right] \\
\text{s.t.} & t_k^2 = 1, \quad x_k^2 = 1, \quad k = 1, \ldots, K, \quad i = 1, \ldots, 2N_t,
\end{array}$$  

(4)

This QCQP is non-convex because of its quadratic equality constraints. To solve it approximately via SDR, define the rank-1 semi-definite matrix

$$X_k = \begin{bmatrix} x_k & t_k \\ t_k & t_k^T \end{bmatrix} = \begin{bmatrix} x_k^T x_k & t_k x_k \\ t_k x_k^T & t_k^2 \end{bmatrix},$$  

(5)

and for notational convenience, denote the cost matrix by

$$C_k = \begin{bmatrix} H_k^T H_k & H_k^T y_k \\ -y_k^T H_k & \|y_k\|^2 \end{bmatrix}.$$  

(6)

Using the property of trace $v^T Qv = \text{tr}(v^T Qv) = \text{tr}(Qvv^T)$, the QCQP in Eq. (4) can be relaxed to SDR by removing the rank-1 constraint on $X_k$. Therefore, the SDR formulation is

$$\begin{array}{ll}
\min_{X_k} & \sum_{k=1}^{K} \text{tr}(C_k X_k) \\
\text{s.t.} & \text{tr}(A_i X_k) = 1, \quad k = 1, \ldots, K, \quad i = 1, \ldots, 2N_t + 1, \quad X_k \succeq 0, \quad k = 1, \ldots, K,
\end{array}$$  

(7)

where $A_i$ is a zero matrix except that the $i$-th position on the diagonal is 1, so $A_i$ is used for extracting the $i$-th element on the diagonal of $X_k$. It is noted that $A_{i,k} \equiv A_{i,k}, \forall k$; thus, the index $k$ is omitted for $A_{i,k}$ in Eq. (7). Finally, we would like to point out that the SDR problems formulated in most papers are targeted at a single time snapshot, since their system of interest is uncoded. Here, for subsequent integration of code information, we consider a total of $K$ snapshots that can accommodate an FEC codeword.

III. FEC Codes in Joint SDR Receiver Formulation

If MIMO detector can provide more accurate information to downstream decoder, an improved decoding performance can be expected. With this goal in mind, we propose to use FEC code information when performing detection.

A. FEC Code Anchoring

Consider an $(N_c, K_c)$ LDPC code. Let $\mathcal{M}$ and $\mathcal{N}$ be the index set of check nodes and variable nodes of the parity check matrix, respectively, i.e., $\mathcal{M} = \{1, \ldots, N_c - K_c\}$ and $\mathcal{N} = \{1, \ldots, N_c\}$. Denote the neighbor set of the $m$-th check node as $\mathcal{N}_m$ and let $\mathcal{S} \triangleq \{\mathcal{F} | \mathcal{F} \subseteq \mathcal{N}_m \text{ with } |\mathcal{F}| \text{ odd}\}$. Then one characterization of fundamental polytope is captured by the following forbidden set (FS) constraints [18]

$$\sum_{n \in \mathcal{F}} f_n - \sum_{n \in \mathcal{N}_m \setminus \mathcal{F}} f_n \leq |\mathcal{F}|-1, \quad \forall m \in \mathcal{M}, \forall \mathcal{F} \in \mathcal{S}$$  

(8)

plus the box constraints for bit variables

$$0 \leq f_n \leq 1, \quad \forall n \in \mathcal{N}.$$  

(9)
\[
\min_{\{x_k,f_n\}} \sum_{k=1}^{K} \text{tr}(C_kX_k) \\
\text{s.t.} \quad \text{tr}(A,X_k) = 1, \, X_k \succeq 0, \quad k = 1, \ldots, K; \quad i = 1, \ldots, 2N_t + 1, \\
\text{tr}(B,X_k) = 1 - 2f_{2N_t(k-1)+2i-1}, \quad k = 1, \ldots, K; \quad i = 1, \ldots, N_t, \\
\text{tr}(B_{i+N_t},X_k) = 1 - 2f_{2N_t(k-1)+2i}, \quad k = 1, \ldots, K; \quad i = 1, \ldots, N_t, \\
\sum_{n \in F} f_n - \sum_{n \in N_m \setminus F} f_n \leq |F| - 1, \quad \forall m \in \mathcal{M}, \forall F \in \mathcal{S}; \\
0 \leq f_n \leq 1, \quad \forall n \in \mathcal{N}.
\]

Recall that the bits \( \{f_n\} \) are mapped by modulators into transmitted data symbols in \( x_k \). It is important to note that the parity check inequalities \([3]\) can help to tighten our detection solution of \( x_k \) by explicitly forbidding the bad configurations of \( x_k \) that are inconsistent with FEC codewords. Thus, a joint detection and decoding algorithm can take advantage of these linear constraints by integrating them within the SDR problem formulation.

Notice that coded bits \( \{f_n\} \) are in fact binary. Hence, the box constraint of \([9]\) is a relaxation of the binary constraints. In fact, if variables \( f_n \)'s are forced to be only 0's and 1's (binary), then the constraints \([8]\) will be equivalent to the original binary parity-check constraints. To see this, if parity check node \( m \) fails to hold, there must be a subset of variable nodes \( \mathcal{F} \subseteq \mathcal{N}_m \) of odd cardinality such that all nodes in \( \mathcal{F} \) have the value 1 and all those in \( \mathcal{N}_m \setminus \mathcal{F} \) have value 0. Clearly, the corresponding parity inequality in \([8]\) would forbid such outcome.

### B. Symbol-to-Bit Mapping

To anchor the FS constraints into the SDR formulation in Eq. \([7]\), we need to connect the bit variables \( f_n \)'s with the data vectors \( x_k \)'s or the matrix variables \( X_k \)'s.

As stated in \([6]\), if \((x_k^T,t_k^T)\) is an optimal solution to \([7]\), then the final solution should be \( t_k^T x_k^T \), where \( t_k^T \) controls the sign of the symbol. In fact, Eq. \([5]\) shows that the first \( 2N_t \) elements of last column or last row are exactly \( t_k^T x_k^T \). We also note that the first \( N_t \) elements correspond to the real parts of the transmitted symbols and the next \( N_t \) elements correspond to the imaginary parts. Hence, for QPSK modulation, the mapping constraints for time instant \( k = 1, \ldots, K \) are simply as follows

\[
\text{tr}(B,X_k) = 1 - 2f_{2N_t(k-1)+2i-1}, \quad i = 1, \ldots, N_t, \\
\text{tr}(B_{i+N_t},X_k) = 1 - 2f_{2N_t(k-1)+2i}, \quad i = 1, \ldots, N_t,
\]

where \( B \) is a selection matrix designed to extract the \( i \)-th element on the last column/row of \( X_k \) (except last element):

\[
B_i = \begin{bmatrix}
0 & \ldots & \ldots & \ldots & 1/2 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & 0 & 1/2 & \ddots & \vdots \\
1/2 & \ldots & 1/2 & \ldots & 0
\end{bmatrix}, \quad 1 \leq i \leq 2N_t.
\]

The non-zero entry of \( B \) is the \( i \)-th element on the last column. For the same reason as that of \( A_i \), the index \( k \) is omitted in \( B_i \). Moreover, note the subtle difference that \( A_i \) is defined for \( 1 \leq i \leq 2N_t + 1 \) while \( B_i \) is defined for \( 1 \leq i \leq 2N_t \).

### C. Joint ML-SDR Receiver

Having defined the necessary notations and constraints, a joint ML-SDR detector can be formulated as the optimization problem in Eq. \([12]\) for QPSK modulation. For higher order QAM beyond QPSK, the necessary changes for our joint SDR receiver include the relaxed box constraints for diagonal elements \([11]\) and the symbol-to-bit mapping constraints. We refer interested readers to the works \([17],[19],[20],[15]\) for the details of mapping higher order QAM constraints.

Recall that the matrix \( X_k \succeq 0 \) is a relaxation of the rank one matrix \( X_k = \begin{bmatrix} x_k^T & t_k^T \end{bmatrix} \).

After obtaining the optimal solution \( \{X_k\} \) of the SDR, one must determine the final detected symbol values in \( x_k \). Traditionally, one “standard” approach to retrieve the final solution is via Gaussian randomization that views \( X_k \) as the covariance matrix of \( x_k \), and another method is to apply rank-one approximation of \( X_k \) \([6]\).

However, a more convenient way is to directly use the first \( 2N_t \) elements in the last column of \( X_k \). If hard-input hard-output decoding algorithm (such as bit flipping) is used, we can first quantize \( t_k^T x_k^T \) into binary values before feeding them to the FEC decoder for error correction. On the other hand, for soft-input soft-output decoder such as sum-product algorithm (SPA), log-likelihood ratio (LLR) can be generated from the unquantized \( t_k^T x_k^T \).

### IV. Simulation Results

In the simulation tests, a MIMO system with \( N_t = 4 \) and \( N_r = 4 \) is assumed. The MIMO channel coefficients are assumed to be ergodic Rayleigh fading. QPSK modulation is used and a regular \((256,128)\) LDPC code with column weight \( 3 \) is employed.

In this section, we will demonstrate the power of code anchoring. We term the formulation in Eq. \([7]\) as disjoint ML-SDR, while that in Eq. \([12]\) as joint ML-SDR. With the optimal
BER solution \( \{X_k^+\} \), there are several approaches to retrieve the final solution \( \hat{s}_k \):

- **Rank-1 approximation**: Perform eigen-decomposition on \( X_k^+ \) to obtain the largest eigenvalue \( \epsilon_k \) and its corresponding eigenvector \( v_k \). The final solution \( \hat{s}_k = \sqrt{\epsilon_k} v_k [1 : 2N_t] \times v_k [2N_t + 1] \).

- **Direct approach**: The final solution is retrieved from the last column of \( X_k \), i.e., \( \hat{s}_k = X_k [1 : 2N_t, 2N_t + 1] \).

- **Randomization**: Generate \( v_k \sim \mathcal{CN}(0, X_k) \) for a certain number of trials, and pick the one that results in smallest cost value. Note that when evaluating the cost value, the elements of \( v_k \) are quantized to \( \{-1, +1\} \).

We caution that, among the methods mentioned above, randomization is not suitable for soft decoding, because the magnitudes of the randomized symbols do not reflect the actual reliability level. Therefore, in the following, we will only consider rank-1 approximation and direct method, the BER curves of which are shown in Fig. 1 and Fig. 2 respectively. In the performance evaluation, we consider 1) hard decision on symbols, 2) bit flipping (BF) decoding and 3) SPA decoding. In some sense, hard decision shows the “pure” gain by incorporating code constraints. BF is a hard decoding algorithm that performs moderately and SPA using LLR is the best. If we compare the SPA curves within each figure, the SNR gain is around 2 dB at BER = 1e-4. For other curves, the gains are even larger. On the other hand, if we compare the curves across the two figures, their performances are quite similar. Therefore, we do not need an extra eigen-decomposition; the direct approach is just as good.

Moreover, we compare ZF and MMSE against the SDR receivers in Fig. 3. All BER curves are shown after SPA decoding. It is clear that ZF and MMSE receivers are far worse than the disjoint SDR, let alone joint SDR. Given that ZF and MMSE are \( O(N_t^3) \) complexity and SDR receiver is \( O(N_t^{3.5}) \) complexity, the performance gap is quite large given the relatively small difference in complexity. In addition, the performance of the exponential-complex ML receiver is plotted. Here we use a soft-output ML detector [21] and then feed the LLRs to SPA decoder. It is seen that the BER performances of ML and joint SDR are very close, even though joint SDR is polynomial-complex.

**V. Conclusion**

This work introduces joint SDR detectors integrated with code constraints for MIMO systems. The joint ML-SDR detector takes advantage of FEC code information in the detection procedure, and it demonstrates significant performance gain compared to the SDR receiver without code constraints. In current stage, this joint receiver works well with short-to-medium length FEC code. However, since the computation capability is ever increasing, this design should be able to accommodate longer codes. In the meantime, we would like to conduct complexity reduction of the joint receiver in future works [22]. It is also interesting to investigate the robust receiver’s performance against RF imperfections, such as I/Q...
imbalance and phase noise \[23\]. Moreover, joint design of precoder \[24\] and receiver would be a good topic to pursue.

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