Nucleon Structure Functions from a Chiral Soliton*

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ABSTRACT

Nucleon structure functions are studied within the chiral soliton approach to the bosonized Nambu–Jona–Lasinio model. The valence quark approximation is employed which is justified for moderate constituent quark masses ($\sim 400\text{MeV}$) as the contribution of the valence quark level dominates the predictions of nucleon properties. As examples the unpolarized structure functions for the $\nu p$ and $\bar{\nu} p$ scattering and the structure functions entering the Gottfried sum rule are discussed. For the latter the model prediction is found to reasonably well agree with a corresponding low–scale parametrization of the empirical data.

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1. Introduction

It has been a long standing puzzle how to establish the connection between the chiral soliton picture of the baryon, which essentially views baryons as mesonic lumps, and the quark parton model, which regards baryons as composites of almost non–interacting, point–like quarks. While the former has been quite successful in describing static properties of the nucleon, the latter, being firmly established within the context of deep inelastic scattering (DIS), has been quite successful in predicting the spin average DIS nucleon structure functions. The apparent difference between models for the nucleon like the bag model [1], which have previously been employed to study structure functions [2]–[9], and soliton models is the fact that in the latter the nucleon wave–function only appears as a collectively excited (topologically) non–trivial meson configuration rather than as a product of Dirac spinors. In this letter we calculate structure functions in the Nambu–Jona–Lasinio (NJL) [10] chiral soliton model [11, 12] where the hadronic currents are formally described in terms of quark degrees of freedom which themselves are functionals of the solitonic meson fields. Since the present study is the first step towards computing nucleon structure functions from a chiral soliton we will adopt a simplifying valence quark type of approximation (to be defined after eq (20)) and leave a more complete exploration to future studies.

As in the original study [2] of structure functions for localized field configurations, the structure functions are most easily accessible when the current operator is at most quadratic in the fundamental fields and the propagation of the interpolating field can be regarded as free. Although the latter approximation is well justified in the Bjorken limit the former condition is difficult to satisfy in soliton models built from mesonic fields. In such models the soliton is a non–perturbative object involving all orders of the fundamental pion field. Hence the current operator is not confined to quadratic order. In models where mesons are fundamental fields (e.g. the Skyrme model [13, 14], the chiral quark model of ref. [15] or the chiral bag model [16]) structure functions are exceedingly difficult to obtain. In this respect the chirally invariant NJL model is advantageous because it is entirely defined in terms of quark degrees of freedom. This makes the evaluation of the required commutator (see eq (4) below) feasible. Nevertheless the quark currents become uniquely (up to regularization) defined functionals of the meson fields. The Lagrangian of the NJL model reads

\[ \mathcal{L} = \bar{q} (i\gamma / - m^0) q + 2G_{NJL} \sum_{i=0}^{3} \left( (\bar{q} \tau^i q)^2 + (\bar{q} \tau^i \gamma^5 q)^2 \right). \]  

(1)

Here \( q, \) \( m^0 \) and \( G_{NJL} \) denote the quark field, the current quark mass and a dimensionful coupling constant, respectively. Functional bosonization [17] yields the action

\[ A = \text{Tr} A \log(iD) + \frac{1}{4G_{NJL}} \int d^4x \text{ tr} \left( m^0 \left( M + M^\dagger \right) - MM^\dagger \right), \]  

(2)

\[ D = i\gamma / - \left( M + M^\dagger \right) - \gamma_5 \left( M + M^\dagger \right). \]  

(3)

The composite scalar (\( S \)) and pseudoscalar (\( P \)) meson fields are contained in \( M = S + iP \), and appear as quark–antiquark bound states. For regularization, which is indicated by the

*In the cloudy bag model the contribution of the pions to structure functions has been treated perturbatively [4, 5].
cut–off Λ, we will adopt the proper–time scheme. The free parameters of the model are the current quark mass \( m^0 \), the coupling constant \( G_{\text{NJL}} \) and the cut–off Λ. When expanding \( \mathcal{A} \) to quadratic order in \( M \) these parameters are related to the pion mass, \( m_\pi = 135\text{MeV} \) and decay constant, \( f_\pi = 93\text{MeV} \). This leaves one undetermined parameter which we choose to be the vacuum expectation value \( m = \langle M \rangle \). For apparent reasons \( m \) is called the constituent quark mass. It is related to \( m^0 \), \( G_{\text{NJL}} \) and Λ via the gap–equation, \( \text{i.e.} \) the equation of motion for the scalar field \( S \). The occurrence of this vacuum expectation value reflects the spontaneous breaking of chiral symmetry.

We will approach the computation of structure functions in the NJL model by first briefly reviewing the kinematics of the Bjorken limit and the NJL soliton in sections 2 and 3, respectively. In section 4 we will work out the valence quark approximation to the unpolarized structure functions. The numerical results will be presented in section 5. Finally section 6 not only serves to summarize these studies but also to propose further explorations.

### 2. Kinematics

The starting point for computing nucleon structure functions is the hadronic tensor

\[
W_{\mu\nu}^{ab}(q) = \frac{1}{4\pi} \int \! d^4 \xi \, e^{i\xi \cdot \mathbf{P}} \left[ J^{a}_\mu(\xi), J^{b \dagger}_\nu(0) \right] |N\rangle .
\] (4)

Here |\( N \rangle \) refers to the nucleon state and \( J^{a}_\mu(\xi) = \bar{q}(\xi) \gamma_\mu t_a q(\xi) \) is the hadronic vector current. In the context of weak interactions we take \( J^{a}_\mu(\xi) = \bar{q}(\xi) \gamma_\mu (1 - \gamma_5) t_a q(\xi) \). We denote by \( t_a \) (\( a = 0, \ldots, 3 \)) the flavor operators, which have to be chosen appropriately for the process under consideration. The unpolarized structure functions are related to the symmetric piece, \( W^{(S)}_{\mu\nu} = (W_{\mu\nu} + W_{\nu\mu})/2 \), which is parametrized by two scalar form factors,

\[
W^{(S)}_{\mu\nu}(q) = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1(x_{\text{Bj}}, Q^2) + \left( P_\mu - q_\mu \frac{P \cdot q}{q^2} \right) \left( P_\nu - q_\nu \frac{P \cdot q}{q^2} \right) W_2(x_{\text{Bj}}, Q^2) .
\] (5)

Here \( P_\mu \) refers to the nucleon momentum and \( Q^2 = -q^2 \). Furthermore quantities suitable to study the Bjorken scaling have been introduced: \( \nu = P \cdot q/M_N \) and \( x_{\text{Bj}} = Q^2/2M_N\nu \). Introducing the projection operators

\[
\Lambda^{\mu\nu}_i = \frac{1}{2} \left[ -g^{\mu\nu} + \frac{\eta}{M_N^2} P^\mu P^\nu \right] , \quad \Lambda^{\mu\nu}_i = \frac{1}{2} \left[ -g^{\mu\nu} + \frac{3\eta}{M_N^2} P^\mu P^\nu \right]
\] (6)

with \( \eta = 2M_N\nu x_{\text{Bj}}/(2M_N\nu x_{\text{Bj}} + \nu) \) enables one to straightforwardly extract the form factors

\[
W_i(x_{\text{Bj}}, Q^2) = \Lambda^{\mu\nu}_i W^{(S)}_{\mu\nu}(q) , \quad i = 1, 2 .
\] (7)

When discussing Bjorken scaling a slightly different definition of the form factors

\[
F_1(x_{\text{Bj}}, Q^2) = M_N W_1(x_{\text{Bj}}, Q^2) \quad \text{and} \quad F_2(x_{\text{Bj}}, Q^2) = \nu W_2(x_{\text{Bj}}, Q^2)
\] (8)

is commonly considered. The Bjorken limit corresponds to the kinematical regime

\[
q_0 = |\mathbf{q}| - M_N x_{\text{Bj}} \quad \text{with} \quad |\mathbf{q}| \rightarrow \infty .
\] (9)

\( ^\dagger \)For simplicity we omit the flavor index when not relevant.
Finally the structure functions are obtained as the Bjorken limit of the form factors

$$F_i(x_{\text{Bj}}) = \lim_{x_{\text{Bj}} \to Q^2} F_i(x_{\text{Bj}}, Q^2), \quad i = 1, 2.$$  

(10)

3. The Nucleon State in the NJL Model

As the NJL model soliton has exhaustively been discussed in a recent review article [12] we only present those features, which are relevant for the computation of the structure functions.

The chiral soliton is given by the hedgehog configuration of the meson fields

$$M_H(x) = m \exp (i \tau \cdot \hat{x} \Theta(r)).$$  

(11)

In order to compute the functional trace in eq (2) for this static configuration a Hamilton operator, $h$, is extracted from the Dirac operator (3). That is,

$$h = \alpha \cdot p + m \exp (i \gamma^5 \tau \cdot \hat{x} \Theta(r)).$$  

(12)

We denote the eigenvalues and eigenfunctions of $h$ by $\epsilon_\mu$ and $\Psi_\mu$, respectively. In the proper time regularization scheme the NJL model energy functional is found to be [11, 12]

$$E[\Theta] = \frac{N_C}{2} \epsilon_v (1 + \text{sgn}(\epsilon_v)) + \frac{N_C}{2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{4\pi s^3} \sum_\nu \exp (-s \epsilon^2_\nu) + m^2 f^2 \pi \int d^3 r (1 - \Theta(r)).$$  

(13)

with $N_C = 3$ being the number of color degrees of freedom. The subscript “v” denotes the valence quark level. This state is the distinct level bound in the soliton background with $-m < \epsilon_v < m$. The chiral angle, $\Theta(r)$, is obtained by self-consistently extremizing $E[\Theta]$ [19].

Nucleon states possessing good spin and isospin quantum numbers are generated from the soliton by taking the zero-modes to be time dependent [14]

$$M(x, t) = A(t) M_H(x) A^\dagger(t),$$  

(14)

which introduces the collective coordinates $A(t) \in SU(2)$. The action functional is expanded [11] in the angular velocities

$$2A^\dagger(t) \dot{A}(t) = i \tau \cdot \Omega.$$  

(15)

In particular the valence quark wave–function receives a first order perturbation

$$\Psi_v(x, t) = e^{-i \epsilon_v t} A(t) \left\{ \Psi_v(x) + \frac{1}{2} \sum_{\mu \neq v} \frac{\Psi_\mu(x)}{\epsilon_v - \epsilon_\mu} \left( \mu | \tau \cdot \Omega | v \right) \right\} =: e^{-i \epsilon_v t} A(t) \psi_v(x).$$  

(16)

Here $\psi_v(x)$ refers to the spatial part of the body–fixed valence quark wave–function with the rotational corrections included. Nucleon states $|N\rangle$ are obtained by canonically quantizing the collective coordinates, $A(t)$. By construction these states live in the Hilbert space of a rigid rotator. The eigenfunctions are Wigner $D$–functions

$$\langle A | N \rangle = \frac{1}{2\pi} D_{J_3=-J_3}^{1/2} (A),$$  

(17)
with $I_3$ and $J_3$ being respectively the isospin and spin projection quantum numbers of the nucleon.

4. Unpolarized Structure Functions in the Valence Quark Approximation

The starting point for the computation of the unpolarized structure functions is the hadronic tensor in the form suitable for localized fields \[2\]

$$W_\mu^{lm}(q) = \zeta \int \frac{d^4k}{(2\pi)^4} S_{\mu\nu\rho\sigma} k^\rho \text{sgn}(k_0) \delta(k^2) \int_{-\infty}^{+\infty} dt e^{i(k_0+q_0)t} \times \int d^3x_1 \int d^3x_2 \exp[-i(k+q) \cdot (x_1-x_2)]$$

$$\times \langle N | \{ \bar{\Psi}(x_1,t)t_l t_m \gamma^\sigma \Psi(x_2,0) - \bar{\Psi}(x_2,0)t_m t_l \gamma^\sigma \Psi(x_1,t) \} | N \rangle. \quad (18)$$

Here $S_{\mu\nu\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}$ denotes the symmetric combination of $\gamma^\mu \gamma^\rho \gamma^\nu$ and $\zeta = 1(2)$ for the structure functions associated with the vector (weak) current. As explained in the preceding section the matrix element between the nucleon states ($|N\rangle$) is to be taken in the space of the collective coordinates. In deriving the expression (18) the free correlation function for the intermediate quark fields has been assumed \[2\]. This reduces the commutator $[J_\mu(x_1,t), J_\nu(x_2,0)]$ of the quark currents in the definition (4) to objects which are merely bilinear in the quark fields. In the Bjorken limit (4) the momentum, $k$, of the intermediate quark state is highly off-shell and hence not sensitive to momenta typical for the soliton configuration. Thus the use of the free correlation function is a good approximation in this kinematical regime. Accordingly, the intermediate quark states are taken to be massless, cf. eq (18). In the next step the form factors $W_i$ are extracted according to eq (7). Noting that $\Lambda_1^{\mu\nu} S_{\mu\rho\sigma} \to g_{\rho\sigma}$ and $\Lambda_2^{\mu\nu} S_{\mu\rho\sigma} \to \eta g_{\rho\sigma}$ the Callan–Gross relation follows immediately, i.e. $F_2(x_{Bj}) = 2x_{Bj} F_1(x_{Bj})$. It thus suffices to only consider the structure function $F_1(x_{Bj})$.

Since the NJL model is formulated in terms of quark degrees of freedom, quark bilinears as in eq (18) can be computed from the functional

$$\langle \bar{q}(x) \Gamma q(y) \rangle = \int D\bar{q}Dq \, \bar{q}(x) \Gamma q(y) \exp \left( i \int d^4x' \, \mathcal{L} \right)$$

$$= \frac{\delta}{i\delta \alpha(x,y)} \int D\bar{q}Dq \exp \left( i \int d^4x'd^4y' \left[ \delta^4(x-y)\mathcal{L} + \alpha(x',y') \bar{q}(x')\Gamma q(y') \right] \right) \bigg|_{\alpha(x,y) = 0}, \quad (19)$$

where $\Gamma$ is a suitable Dirac and/or isospin matrix. The introduction of the bilocal source $\alpha(x,y)$ facilitates the functional bosonization upon which eq (13) takes the form

$$\frac{\delta}{\delta \alpha(x,y)} \text{Tr}_A \log \left( \delta^4(x-y)D + \alpha(x,y)\Gamma \right) \bigg|_{\alpha(x,y) = 0}. \quad (20)$$

The operator $D$ is defined in eq (3). From this discussion it is obvious that structure functions are most easily obtained within models which can completely be formulated in terms of quark degrees of freedom where the form of the current operator is not altered by the interactions.\[1\]

\[1\] Otherwise matrix elements of operators have to be computed, which are more complicated than the bilocal quark bilinear $\bar{q}(x)\Gamma q(y)$. 

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The correlation \( \langle \bar{q}(x) \Gamma q(y) \rangle \) depends on the angle between \( x \) and \( y \). Since in general the functional (19) involves quark states of all angular momenta \( l \) a technical difficulty arises because the angular dependence has to be treated numerically. The major purpose of the present letter is to demonstrate that structure functions can be computed from a chiral soliton. With this in mind we will adopt the valence quark approximation where only quark orbital angular momenta up to \( l = 2 \) are relevant. From a physical point of view this approximation is justified at least for small constituent quark masses because in that parameter region the nucleon properties are dominated by the valence quark contribution \( [12] \). We define the valence quark approximation to the structure functions by restricting the quark configurations in (19) to the iso–rotating valence quark wave–function (16), accordingly substituting the valence quark wave–function (16) into eq (18).

When extracting the structure function \( F_I^1(x_B) \) the integrals over the time coordinate can readily be done yielding the conservation of energy for forward and backward moving intermediate quarks. Carrying out the integrals over \( k_0 \) and \( k = |k| \) yields for the isovector part of the structure function

\[
F_I^1(x_B) = \zeta N C \frac{M_N}{2\pi} \langle N | D_{3i} \int d\Omega k k^2 \left\{ \bar{\psi}_v^\dagger(p) \left( 1 + \alpha \cdot \hat{k} \right) \tau_i \bar{\psi}_v(p) \right|_{k=q_0+\epsilon_v} \text{tr} (\tau_3 t_l t_m) \\
- \bar{\psi}_v^\dagger(-p) \left( 1 + \alpha \cdot \hat{k} \right) \tau_i \bar{\psi}_v(-p) \right|_{k=q_0-\epsilon_v} \text{tr} (\tau_3 t_m t_l) \left\} |N \rangle ,
\]

where \( p = k + q \). \( N_C \) appears as a multiplicative factor because the functional trace (20) includes the color trace as well. Furthermore the Fourier transform of the valence quark wave–function

\[
\bar{\psi}_v(p) = \int \frac{d^3x}{4\pi} \psi_v(x) \exp (ip \cdot x) \quad (22)
\]

has been introduced. The isoscalar part of the structure function, \( F_I^0(x_B) \), is straightforwardly obtained from eq (21) by replacing \( D_{3i} \) with unity and omitting the isospin matrices \( \tau_i \). Note that the wave–function \( \bar{\psi}_v \) contains an implicit dependence on the collective coordinates through the angular velocity \( \Omega \), \textit{cf.} eq (16).

The dependence of the wave–function \( \bar{\psi}(\pm p) \) on the integration variable \( \hat{k} \) is only implicit. In order to carry out this integration it is most convenient to choose the external momentum along the \( z \)–axis, \textit{i.e.} \( q = qz \). In the Bjorken limit the integration variables may then be changed to \( [2] \)

\[
k^2 \, d\Omega_k = pdp \, d\Phi , \quad p = |p| ,
\]

where \( \Phi \) denotes the azimuth–angle between \( q \) and \( p \). The lower bound for the \( p \)–integral is adopted when \( k \) and \( q \) are anti–parallel: \( p^\text{min}_+ = |M_{N x_B} \mp \epsilon_v| \) for \( k = q_0 \pm \epsilon_v \), respectively. The wave–function \( \bar{\psi}(\pm p) \) acquires its dominant support for \( p \leq M_N \). Hence the integrand is different from zero only when \( q \) and \( k \) are anti–parallel and we may take \( \hat{k} = -\hat{z} \). This is nothing but the light–cone description for computing the structure functions \( [5] \). The valence quark state possesses positive parity yielding \( \bar{\psi}(-p) = \gamma_0 \bar{\psi}(p) \). We now have arrived at the
approaches which treat the nucleon as extended objects. In the context of bag type models the nucleon mass. We observe that the structure functions are well localized in the interval quark approximation is well justified. Here we assume the experimental value (940MeV) for is about 86%. This shows that the vacuum is only moderately polarized and that the valence

\[ F_I^0(x_{Bj}) = \zeta \left( F_+^I(x_{Bj}) - F_-^I(x_{Bj}) \right) \]

\[ F_{\pm}^I(x_{Bj}) = N_C \frac{M_N}{2\pi} \langle N | \int_{M_N|x_\pm|}^\infty dp \int_0^{2\pi} d\Phi \ \tilde{\psi}_v^\dagger(p_\pm)(1 \mp \alpha_3) \tilde{\psi}_v(p_\pm)|N\rangle \text{tr}[t_1 t_3] \]

\[ F_{\pm}^I(x_{Bj}) = N_C \frac{M_N}{2\pi} \langle N | D_{3\Omega} \int_{M_N|x_\pm|}^\infty dp \int_0^{2\pi} d\Phi \ \times \tilde{\psi}_v^\dagger(p_\pm)\tau_i (1 \mp \alpha_3) \tilde{\psi}_v(p_\pm)|N\rangle \text{tr} \left[ \tau_3 \left( \frac{t_1 t_3}{t_m t_1} \right) \right] , \]

where \( x_\pm = x_{Bj} \pm \epsilon_v/M_N \) and \( \cos(\Theta_p^\pm) = M_N x_\pm/p \). The polar–angle, \( \Theta_p^\pm \), between \( p_\pm \) and \( q \) is fixed for a given value of the Bjorken parameter, \( x_{Bj} \). Hence the wave–function depends implicitly on \( x_{Bj} \) because \( \tilde{\psi}_v(p_\pm) = \tilde{\psi}_v(p, \Theta_p^\pm, \Phi) \).

Turning to the evaluation of the nucleon matrix elements defined above we first note that the Fourier–transform of the wave–function is easily obtained because the angular parts are tensor spherical harmonics in both coordinate and momentum spaces. Hence, only the radial part requires numerical treatment. Performing straightforwardly the azimuthal integrations in eqs (24) and (25) reveals that the isoscalar part, \( F_1^I=0 \), depends solely on the classical part of the valence quark wave–function, \( \Psi_0 \). Thus \( F_1^I=0 \) is identical for all nucleon states. On the other hand the isovector part, \( F_1^I=1 \), is linear in the angular velocity, \( \Omega \). Since the \( z \)–direction is distinct the collective quantities appear as combinations of \( D_{33} \Omega_3 \) and \( D_{3i} \Omega_i \). When quantizing the collective coordinates these combinations are substituted by the nucleon spin operator yielding

\[ \langle N | D_{33} \Omega_3 | N \rangle = -\frac{I_3}{3\alpha^2} \quad \text{and} \quad \langle N | D_{3i} \Omega_i | N \rangle = -\frac{I_3}{\alpha^2} . \]

Here \( I_3 = \pm(1/2) \) is the nucleon isospin projection and \( \alpha^2 \) refers to the moment of inertia of the soliton. For consistency we constrain it to the valence quark contribution, \( \alpha_v^2 \), cf. eq (31). The isovector part is obviously proportional to the isospin projection but independent of the spin projection, as expected for unpolarized structure functions. It is convenient to define structure functions \( f_{\pm}^I(x_{Bj}) \) with the nucleon matrix elements already computed via

\[ F_{\pm}^0(x_{Bj}) = f_{\pm}^0(x_{Bj}) \quad \text{and} \quad F_{\pm}^1(x_{Bj}) = 2I_3 f_{\pm}^1(x_{Bj}) . \]

5. Results

In figure 1 we display the unpolarized structure functions \( f_{\pm}^{0,1} \) for a constituent quark mass of \( m = 350 \text{MeV} \). In that case the valence quark contribution to the moment of inertia is about 86%. This shows that the vacuum is only moderately polarized and that the valence quark approximation is well justified. Here we assume the experimental value (940MeV) for the nucleon mass. We observe that the structure functions are well localized in the interval \( 0 \leq x_{Bj} \leq 1 \). The result that the structure functions slightly exceed \( x_{Bj} = 1 \) is common to approaches which treat the nucleon as extended objects. In the context of bag type models
Figure 1: The unpolarized structure functions $f_{0,1}^0$ as functions of the Bjorken variable $x_{Bj}$.

Various projection techniques have been proposed [20, 6, 9] to remedy this problem. This, however, is not the central issue of this paper.

The results displayed in figure 1 are the central issue of this calculation and it is of great interest to compare them with the available data. In this context we consider the structure functions for electron nucleon scattering. The associated isospin matrices are $t_a t_b = t_b t_a = (5 + 3\tau_3)/18$ yielding

$$F_{1}^{eN} = \frac{1}{9} \left( 5(f_+^0 - f_+^0) - 6I_3(f_+^1 - f_+^1) \right) = \frac{1}{2x_{Bj}} F_{2}^{eN}, \quad (28)$$

where the second equation results from the Callan–Gross relation. As for all effective low-energy models of the nucleon, the predicted results are at a scale lower than the experimental data. In order to carry out a sensible comparison either the model results have to be evolved upward or the QCD renormalization group equations have to be used to extract structure functions at a low-renormalization point. The latter procedure has been employed in ref [21] to make available a low-scale parametrization of the empirical data on $F_{2}^{eN}$. From figure 2 we observe that the NJL model prediction for $F_{2}^{ep} - F_{2}^{en}$ reproduces the gross features of this parametrization although the maximal value of the prediction is a bit too large. On the other hand the low-scale value are more enhanced at small $x_{Bj}$. To illustrate the origin of the bump at $x_{Bj} \sim 0$ we have also included the low-scale parametrization with the $\alpha_s$-corrections omitted, cf. eq (7) of ref [21]. These are actually the starting point for computing the low-scale parametrization. When including the $\alpha_s$-corrections the integral (24) is forced to remain unchanged. As the $\alpha_2$-corrections shift the structure functions to

§These authors also provide a low scale parametrization of quark distribution functions. However, these refer to perturbatively interacting partons. Distributions for the NJL-model constituent quarks could in principle be extracted from eqs. (24)–(25). It is important to stress that these distributions may not be compared to those of ref [21] because the associated quarks fields are different in nature.
Figure 2: The valence quark approximation to the unpolarized structure functions as functions of the Bjorken variable $x_{Bj}$. Left panel: The prediction on the Gottfried sum for two values of the constituent quark mass $m$. We compare with the low–scale parametrization of ref [21]. Right panel: $F_{1}^{ep}$ and $F_{1}^{en}$ for $m = 450$MeV.

larger $x_{Bj}$ the (artificial) bumb at $x_{Bj} \sim 0$ emerges. As an aside we would like to mention that the agreement between the NJL–model predictions and the parametrized structure functions is better when the $\alpha_{s}$–corrections are omitted. This indicates that a fine–tuning of the low–scale momentum might improve the agreement even more.

With regard to the vacuum contribution it should be emphasized that it will not simply add to the valence quark piece because when computing the isovector structure functions we have substituted $\alpha_{s}^{2} < \alpha^{2}$ in eq (26). For $m = 400(450)$MeV we find $\alpha_{s}^{2}/\alpha^{2} = 78(72)$%. We observe that for the combination $F_{2}^{ep} - F_{2}^{en}$ the agreement with the parametrization [21] improves as $m$ increases. We conjecture that this feature survives when the vacuum contribution is included because the moment of inertia enters the denominator of $F_{2}^{ep} - F_{2}^{en}$. Furthermore the integral

$$S_{G} = \int_{0}^{\infty} \frac{dx_{Bj}}{x_{Bj}} (F_{2}^{ep} - F_{2}^{en}) \rightarrow 2 \int_{0}^{\infty} dx_{Bj} (F_{1}^{ep} - F_{1}^{en}) = 0.29 \ (0.27) \ (29)$$

agrees reasonably well with the empirical value $S_{G} = 0.235 \pm 0.026$ [22] for the Gottfried sum rule. In particular the deviation from the naïve value $1/3$ [23] is in the correct direction.

For the weak scattering precesses $\nu p$ and $\bar{\nu} p$ we demand $t_{a}t_{b} = (1 \pm \tau_{3})/2$ yielding the linear combinations

$$F_{1}^{ep} = 2 \left( f_{0}^{0} - f_{0}^{+} + f_{+}^{+} + f_{+}^{1} \right) \quad \text{and} \quad F_{1}^{en} = 2 \left( f_{0}^{0} - f_{0}^{-} - f_{+}^{-} - f_{-}^{1} \right) \ (30)$$

which are also plotted in figure 2. Although our wave–functions (cf. section 3) are quite different from those in the bag model the shape of the structure functions is similar. In
particular the structure functions $F_{1}^{p,p}$ do not vanish at $x_{Bj} = 0$ in both models. Despite that we essentially take only one quark eigenstate into account, we find a clear smearing of the structure functions. This shows that relativistic effects, i.e. a sizable lower component of the valence quark wave–function, play a significant role. These effects also cause the maximum of the structure to be shifted from $\epsilon_{v}/M \approx 0.26$ to about 0.37. As in the bag model calculation of ref [2] we find that $F_{1}^{p,p}$ is negative in the vicinity of $x_{Bj} = 0$. This appears to be linked to the omission of the vacuum states when computing the hadronic tensor (18).

Let us briefly comment on the Adler sum rule. Note that in the Bjorken limit, where the Callan–Gross relation is satisfied, the Adler and Bjorken sum rules are equivalent. It is an easy matter of exercise to verify that

$$\int_{0}^{\infty} dx \left[ f_{+}^{1}(x) + f_{-}^{1}(x) \right] = -\frac{N_{C}}{4\alpha^{2}} \sum_{\mu} \frac{|\langle \mu | \tau_{3} | v \rangle|^{2}}{\epsilon_{\mu} - \epsilon_{v}} = -\frac{\alpha^{2}}{2\alpha^{2}} \epsilon_{v}. \quad (31)$$

Thus the Adler sum rule is satisfied once we assign the moment of inertia to its valence quark contribution, $\alpha^{2}_{v}$. It is obvious that this sum rule will be recovered without this restriction when the contribution of the polarized vacuum is included in the evaluation of the functional trace (20). The Adler sum rule also serves as a test for our numerical treatment. It furthermore manifests the parton model interpretation because adopting unity as the upper boundary of the integral (31) saturates this sum rule already by 99% for the parameters used here. We should mention that the momentum sum rule is not satisfied in the valence quark approximation. The analytical proof of the momentum sum rule involves the classical equation of motion for the chiral field. As the polarized vacuum contributes to this equation it is obvious that including only the valence quark level in the calculation of the structure functions violates this sum rule. Numerically, however, this violation is small. For example, for $m = 450$MeV we are missing about 20%. This number decreases with the constituent mass and may be interpreted as the momentum carried by the polarized vacuum.

6. Summary and Outlook

The present study is intended as the first step towards clarifying the connection between the chiral soliton picture of the nucleon and the quark parton description. This has also to be regarded as an attempt to combine the phenomenologically successful concept of chiral symmetry with the quark parton model. For this purpose we have presented a first calculation of nucleon structure functions in the Bjorken limit from a chirally symmetric model. The mean field quark wave–functions in the background of a chiral soliton represent a non–trivial coupling of spin and isospin to the so–called grand spin. Baryon states possessing good spin and isospin are subsequently generated by cranking the soliton, see eq (17). As a consequence there are rotational corrections to the mean field predictions of the structure functions. These corrections contribute to the isovector part of the unpolarized structure functions and are mandatory to reproduce the Adler sum rule. This form of the nucleon

\*This consistency check requires one to use the soliton mass ($\sim 1.2$GeV) rather than the experimental value for the nucleon mass.
wave–function constitutes a major difference to quark models which are not based on a non–trivial chiral field as e.g. the bag model where baryons are described as direct products of spin and isospin eigenstates. In order to establish the connection between the two pictures we have (as a first step) restricted ourselves to the valence quark approximation. The results are in reasonable agreement with bag model results \[\text{(2)}\] (which does not include the Dirac sea either) and the empirical value of the Gottfried sum rule. Also the low–scale parametrization of the combination \(F_{2}^{ep} - F_{2}^{en}\), which enters this sum rule, is satisfactorily reproduced.

This encourages further studies in various directions. It is obvious that for the full computation of the structure functions the polarization of the vacuum quark states has to be incorporated. Although for small constituent quark masses the soliton is dominated by the valence quark configuration it will be very interesting to have available direct access to the sea quarks. The regularization of the functional trace \(\text{(20)}\) will be rather involved. Fortunately the Adler and momentum sum rules may be employed to perform consistency checks. This study will illuminate whether (and how) the vacuum contribution violates the identity \(f_{+}^{I}(0) = f_{-}^{I}(0)\), which is also observed in the bag model \[\text{(2)}\]. Within the parton model picture the sea quarks cause a violation of this relation.

Of special interest are the polarized structure functions which are to be extracted from the anti–symmetric part of the hadronic tensor \(W_{\mu\nu}^{(A)} = (W_{\mu\nu} - W_{\nu\mu})/2\). The smallness of the first moment of the associated flavor singlet structure function is known as the proton spin puzzle. Since almost all chiral soliton models provide a reasonable explanation of this puzzle the computation of the entire structure function will provide further understanding how the nucleon is built up from its constituents.

One wonders whether the functional trace \(\text{(20)}\) has a suitable interpretation in chiral models with mesons as the fundamental fields. Although it is possible to identify quark bilinear quantities in such soliton models via saturation of the Ward–identities, the analogues of the quark bilinears are always local. Hence models with fundamental meson fields contributing to the currents seem to be less tractable for calculating structure functions. In this respect an expansion of the functional trace \(\text{(20)}\) in derivatives of the chiral field might provide an effective operator suitable to compute structure functions in purely mesonic soliton models. However, first investigations along this line have given disappointing results \[\text{(24)}\]. This might be related to the failure of the gradient expansion in the soliton sector.

Gluonic effects are known to significantly contribute to the structure functions, they may even cause some of them to be singular\[\parallel\] at \(x_{Bj} = 0\). Such singularities will not appear in the soliton model calculation (neither do they in the bag model calculation \[\text{(8)}\]). Hence a further study of the structure functions may provide some insight how to effectively incorporate gluonic degrees of freedom in NJL–type models.

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\[\parallel\]For example, the twist three spin average structure function \(e(x_{Bj})\) receives a pomeron contribution which behaves like \(x_{Bj}^{-2}\) \[\text{(8)}\].
References

[1] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D9 (1974) 3471; A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D10 (1974) 2599; For a review see: A. W. Thomas, Adv. in Nucl. Phys. 13 (1982) 1.

[2] R. L. Jaffe, Phys. Rev. D11 (1975) 1953; R. L. Jaffe and A. Patrascioiu, Phys. Rev. D12 (1975) 1314.

[3] A. I. Signal and A. W. Thomas, Phys. Lett. B211 (1988) 481.

[4] V. Sanjose and V. Vento, Phys. Lett. B225 (1988) 15; Nucl. Phys. A501 (1989) 672.

[5] R. L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552.

[6] A. W. Schreiber, A. I. Signal, and A. W. Thomas, Phys. Rev. D44 (1991) 2653.

[7] A. W. Schreiber, P. J. Mulders, A. I. Signal, and A. W. Thomas, Phys. Rev. D45 (1992) 3069.

[8] R. L. Jaffe and X. Ji, Nucl. Phys. B375 (1992) 527; For a review see: R. L. Jaffe, Spin, Twist and Hadron Structure in Deep Inelastic Processes, hep-ph/9602236.

[9] X. Song and J. S. McCarthy, Phys. Rev. D49 (1994) 3169.

[10] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; 124 (1961) 246.

[11] H. Reinhardt, Nucl. Phys. A503 (1989) 825.

[12] R. Alkofer, H. Reinhardt, and H. Weigel, Phys. Rep. 265 (1996) 139.

[13] T. H. R. Skyrme, Proc. R. Soc. 127 (1961) 260; For reviews see: G. Holzwarth and B. Schesinger, Rep. Prog. Phys. 49 (1986) 825; I. Zahed and G. E. Brown, Phys. Rep. 142 (1986) 481.

[14] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. B228 (1983) 552.

[15] M. Birse and M. K. Banerjee, Phys. Rev. D31 (1985) 118; S. Kahana, G. Ripka, and V. Soni, Nucl. Phys. A415 (1984) 351; P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D40 (1988) 1571; For a review see: M. K. Banerjee, W. Broniowski, and T. D. Cohen, A Chiral Quark Soliton Model in Chiral Solitons, K. F. Lui (ed.), p. 255.

[16] G. E. Brown and M. Rho, Phys. Lett. B82 (1979) 177; V. Vento, M. Rho, E. M. Nyman, J. H. Jun, and G. E. Brown, Nucl. Phys. A345 (1980) 413. For a review see: A. Hosaka and H. Toki, Chiral Bag Model for the Nucleon, preprint Dec. 1995.

[17] D. Ebert and H. Reinhardt, Nucl. Phys. B271 (1986) 188.

[18] J. Schwinger, Phys. Rev. 82 (1951) 664.

[19] H. Reinhardt and R. Wünsch, Phys. Lett. B215 (1988) 577; B 230 (1989) 93; T. Meißner, F. Grümmer and K. Goeke, Phys. Lett. B227 (1989) 296; R. Alkofer, Phys. Lett. B236 (1990) 310.

[20] R. L. Jaffe and G. G. Ross, Phys. Lett. 93B (1980) 313; R. L. Jaffe, Ann. Phys. (NY) 132 (1981) 32; X. M. Wang, X. Song and P. C. Yin, Phys. Lett. 140B (1984) 413; C. J. Benesh and G. A. Miller, Phys. Rev. D36 (1987) 1344; D38 (1988) 48; A. I. Signal and A. W. Thomas, Phys. Rev. D40 (1989) 2832.

[21] M. Glück, E. Reya, and A. Vogt, Z. Phys. C67 (1995) 433.

[22] M. Arneodo et al. (NMC), Phys. Rev. D50 (1994) R1.

[23] K. Gottfried, Phys. Rev. Lett. 18 (1967) 1174.

[24] R. L. Jaffe, private communication.