Advantages and Limitations of using Successor Features for Transfer in Reinforcement Learning

Lucas Lehnert 1  Stefanie Tellex 1  Michael L. Littman 1

Abstract

One question central to Reinforcement Learning is how to learn a feature representation that supports algorithm scaling and re-use of learned information from different tasks. Successor Features approach this problem by learning a feature representation that satisfies a temporal constraint. We present an implementation of an approach that decouples the feature representation from the reward function, making it suitable for transferring knowledge between domains. We then assess the advantages and limitations of using Successor Features for transfer.

1. Introduction

Reinforcement Learning (RL) (Kaelbling et al., 1996; Sutton & Barto, 1998) studies the problem of computing an optimal control strategy using one-step interactions sampled from an environment. For each selected action, the environment also provides a reward, a single scalar number. The goal is to compute a control strategy, also called a policy, that maximizes the cumulative reward received while interacting with the environment. One challenge in this setting is transferring knowledge about one environment to another when only the reward specification changes, but the remaining specification of the environment stays fixed. In this paper, we consider the approach presented by Barreto et al. (2016), which uses Successor Features (SF) to compute a representation of the environment that can be transferred across different reward functions. We present an implementation of this method and show that while learning a SF representation has significant benefits for transfer, it has also some fundamental limitations.

2. Background

We consider a Markov Decision Process (MDP) $M = (S, A, p, r, \gamma)$ with a finite state space $S$ and a finite action space $A$. The transition function $p$ specifies with $p(s, a, s')$ the probability of transitioning from a state $s$ to a state $s'$ when selecting an action $a$. For every such transition, the reward is specified by the reward function $r : S \times A \rightarrow \mathbb{R}$. Further, we assume a discount factor $\gamma \in [0, 1)$ that weights the tradeoffs between immediate and long term rewards.

Let $\pi$ be a policy that specifies the distribution with which actions are selected, conditioned on the state space $S$. The Q-function of this policy is defined as

$$Q^\pi(s, a) = E_\pi \left[ \sum_{t=1}^{\infty} \gamma^{t-1} r_t \mid s_1 = s, a_1 = a \right], \quad (1)$$

where the expectation $E_\pi$ is over all possible infinite length trajectories in $M$ and $r_t$ the reward at time step $t$.

Several algorithms have been developed to estimate a Q-function, however, one important question is how to represent a current Q-function estimate. For example, suppose the state space of an MDP $M$ consists of $n$ states and $m$ actions, then an estimate of the Q-function can be stored in a vector $\theta$ of dimension $mn$:

$$\theta = [Q(s_1, a_1), \ldots, Q(s_n, a_m)]^T \quad (2)$$

To compute the Q-value for a state-action pair $(s, a)$, a basis function

$$\psi : (s, a) \mapsto \psi_{s,a} \quad (3)$$

can be used, where $\psi_{s,a}$ is a one-hot bit vector of dimension $mn$. Basis functions can also be generalized to have different forms to further improve scalability of different learning algorithms (Sutton, 1996; Konidaris et al., 2011).

3. Learning Successor Features for Transfer

Dayan (1993) presented Successor Features (SFs), a particular type of basis function that represents a state as a feature vector $\psi_{s,a}$ such that under a given policy the feature representation $\psi_{s,a}$ is similar to the feature representation of its successor states. The idea originates from the Bellman...
fixed-point equation,
\[ Q^\pi(s, a) = r(s, a) + \gamma \mathbb{E}_{s', a'} \left[ Q^\pi(s', a') \right], \tag{4} \]
where \( s' \) is the sampled next state and \( a' \) is the sampled next action at state \( s' \). If the Q-function is approximated linearly, then
\[ (\psi^\pi_{s,a})^\top \theta \approx r(s, a) + \gamma \mathbb{E}_{s', a'} \left[ (\psi^\pi_{s', a'})^\top \theta \right]. \tag{5} \]
Note that, depending on the choice of basis function, (5) may not hold exactly because we only estimate a linear approximation of the true Q-function. The objective of finding a good SF representation is to find a basis function \( \psi \) such that (5) holds as exactly as possible.

Barreto et al. (2016) re-visited this approach in the context of transferring a feature representation within a set of MDPs where only the reward function varies. While various different approaches were presented to this problem (see Taylor & Stone (2009) for a survey), Barreto et al. approach this transfer problem by learning a feature representation that is descriptive of the entire set of MDPs and can be used for transfer across different reward functions.

Intuitively, the Q-function combines information about the reward function itself, as well as the temporal ordering of the MDP that determine which trajectories are generated. Since (6) is stated as a strict equality, the assumption is made that \( \psi \) is not too restrictive and the reward function \( r \) can be represented exactly. Using this assumption, Barreto et al. rewrite the Q-function as
\[ r(s, a) = \phi^\top s,a \psi. \tag{6} \]

For transfer, Barreto et al. present an approach that isolates the reward function from the Q-function. They define a basis function \( \phi : (s, a) \mapsto \phi_{s,a} \) to parametrize the reward function with

\[ y_{s,a,s'} = \begin{cases} \phi_{s,a} & \text{if } s' \text{ is terminal} \\ \phi_{s,a} + \gamma \mathbb{E}_{a'} \left[ \psi_{s',a'}^\pi \right] & \text{otherwise} \end{cases} \tag{10} \]

for every collected transition \((s, a, s')\). For computing this target, the SF estimate \( \psi' \) of the previous update iteration is used. Unlike Mnih et al.’s Deep Q-learning, the target \( y_{s,a,s'} \) is a vector and not a single scalar variable. For learning a SF representation, the loss objective
\[ \mathcal{L}_{SF}(\psi) = \mathbb{E}_{s,a,s'} \left[ \| \psi_{s,a} - y_{s,a,s'} \|_2^2 \right] \tag{11} \]
is used. The gradient of (11) with respect to the parameters \( \theta \) is

\[ \nabla_{\theta} \mathcal{L}_{SF}(\psi) = 2 \mathbb{E}_{s,a,s'} \left[ (\psi_{s,a} - y_{s,a,s'}) \nabla_{\theta} \psi_{s,a} \right], \tag{12} \]
which is similar to the gradient used by Deep Q-learning with the distinction that (12) is a matrix rather than a vector, and (11) is defined on the SF \( \psi^\pi \), rather than Q-values.

Algorithm 1 outlines the implemented SF learning method. Learning is stabilized by sampling a batch of transitions and using the entire batch to make a gradient descent update.
Algorithm 1 Fitted SF Learning

Initialize \( \psi, \phi, \) and \( w \).

\[
\text{loop}
\]

Collect transitions \( \tau = \{(s_t, a_t, r_t, s_{t+1})\}_{t=1}^{T+N} \) using the Q-function estimate \( Q(s, a) = (\psi_{s,a})^\top w \).

Using \( \tau \) perform gradient update on \( \mathcal{L}_T(\phi, w) \) and \( \mathcal{L}_F(\psi) \).

\[
\text{end loop}
\]

4. Experiments: Grid World

Algorithm 1 is first evaluated on a 10 \( \times \) 10 grid world navigation task with four actions: up, down, left, or right. Transitions are stochastic and with a 5% probability the agent moves sideways. Rewards are set to 1 for entering the goal cell (terminal state) in the top right corner, and otherwise a zero reward is given. Every episode is started in the bottom right corner and the discount factor is set to \( \gamma = 0.9 \).

Actions are selected using an \( \varepsilon \)-greedy policy with respect to the current Q-value estimates: with probability \( \varepsilon = 0.3 \) actions are selected uniformly at random and with probability \( 1 - \varepsilon \) the action with the highest Q-value estimate is used.

We compare our Fitted SF implementation against a Fitted Q-iteration implementation. To ensure a fair comparison, Fitted Q-iteration is identical to Fitted SF except that Fitted Q-iteration minimizes the loss objective

\[
\mathcal{L}_Q(\theta) = \mathbb{E}_{s,a,s'} \left[ ||Q(\theta(s, a) - y_{s,a,s'}||^2 \right], \tag{13}
\]

where the target is set to

\[
y_{s,a,s'} = \begin{cases} r_{s,a} & \text{if } s' \text{ is terminal} \\ r_{s,a} + \gamma V_{\theta}(s') & \text{otherwise.} \tag{14}
\end{cases}
\]

The value estimate \( V_{\theta}(s') = \max_{a'} Q_{\theta}(s', a') \) and \( Q_{\theta} \) is the Q-function estimate of the previous iteration.

In all experiments, the Q-function in Fitted Q-iteration uses a basis function tabulating the state-action space and the weight vector \( \theta \) is learned as described in (2). Further, the basis function \( \phi \) used for estimating the reward model (6) also tabulates the state-action space; that is, the reward model can always exactly represent the true reward function. The SF representation is learned as a linear transform on the tabular basis function \( \phi \):

\[
\psi_{s,a} = \Phi_{s,a}. \tag{15}
\]

Because all basis functions are chosen to be tabular, and SFs are linear in a tabular one-hot basis function, both algorithms are not constrained in their representation and can always capture the true value function, reward model, and successor features.

4.1. Single Task Learning

Figure 1 compares the performance of the Fitted SF algorithm against Fitted Q-iteration. Both algorithms converge to a good solution and can perform the navigation task in few steps at the end of training. The Fitted SF algorithm converges slower, which can be explained by the fact that it has to learn a full reward model before it can form good Q-value estimates. Figure 2 shows that the Fitted SF algorithm robustly minimizes both its loss objectives.

4.2. Multi Task Learning

The Fitted SF algorithm was also tested in two transfer settings where the start and goal locations are changed periodically between a fixed set of different locations. Changing the goal location is equivalent to changing the reward function while holding the transition dynamics fixed.

Transfer with Slight Reward Changes Figure 3a compares the episode length of the Fitted Q-iteration implementation while holding the transition dynamics fixed.
Transfer with Significant Reward Changes  To further test if SFs can be used for transfer between different domains, both algorithms are evaluated again on the same grid world, but the goal location is rotated through all four corners of the grid. The start location is always the corner diagonally across the grid from the goal. Changing start and goal locations in this way causes the reward function and the optimal policy to change more significantly.

To further stabilize learning and ensure sufficient exploration, both algorithms select actions using an $\varepsilon$-greedy policy. The $\varepsilon$ probability is decayed according to the rule $\varepsilon_t = 0.9 \cdot 0.95^t + 0.1$, where $t$ is the episode index. This episode index $t$ is reset to zero after every reward function change. Ensuring sufficient exploration allows the Fitted SF algorithm to efficiently re-estimate its reward model.

Figure 4 compares the episode length of both algorithms over several repeats of the four goal locations. The ordering of the different goal locations is not changed during the experiment. One can see that the change in reward function has an impact on both algorithms, but the Fitted SF algorithm outperforms Fitted Q-iteration significantly. Table 1 compares the average episode length across all episodes and shows that our Fitted SF algorithm outperforms the Fitted Q-iteration significantly. Figure 6 shows how the loss functions of the Fitted SF algorithm evolve during the experiment. Updates were done only every 100 steps (each gradient update used a batch of 100 transitions). As expected, the reward loss $L_R$ does not seem to decrease significantly in a steady way but oscillates instead. However, the estimates seem to be good enough to achieve a significant performance difference over Fitted Q-iteration. Interestingly, the SF loss $L_{SF}$ oscillates during training between very low and high values.

Figure 5 shows a failure setting of the Fitted SF algorithm: If $\varepsilon = 0.3$ and is not annealed, only the first optimal policy and the first reward function is learned and then preserved across all subsequent changes. As a result, one can see a learning curve for the first 400 episodes and then Fitted SF hits the episode time-out of 200 steps for the next reward configuration. If a reward function similar to the first is presented to the agent again, Fitted SF solves this problem easily because it reuses the weights it has learned at the beginning of the experiment. In other words, Fitted SF is not able to transfer the solution learned in the first 400 episodes to the other tested reward functions.

5. Discussion

The goal of using SFs is to capture a feature set common to a set of MDPs and this idea seems to perform well for trans-
Advantages and Limitations of using Successor Features for Transfer in Reinforcement Learning

Figure 4. Comparison of the Fitted Q-iteration and Fitted SF algorithm when rotating every 100 episodes the goal location through all four corners of the grid. Fitted Q-iteration uses a learning rate of 0.01, Fitted SF learning uses a learning rate of 0.01 for the SF and a learning rate of 0.1 for the reward model. The episodes were capped at 4000 steps.

Table 1. Average episode length for Figure 4. The p-value of the Welch’s t-test tests if the episode lengths are significantly different.

|                   | Avg. Episode Length | p-value |
|-------------------|---------------------|---------|
| Fitted Q-iteration | 99.46 ± 10.43       |         |
| Fitted SF         | 34.50 ± 2.17        | 1.90 · 10^{-17} |

Figure 5. Episode length of Fitted SF when reward functions change every 400 episodes. The episodes were clipped at 200 steps. All other parameters are the same as in Figure 4.

Figure 6. Evolution of the Loss function for the Fitted SF algorithm. A gradient update was applied every 100 steps.

Figure 6a. Loss Objective $L_S$ (11)

Figure 6b. Loss Objective $L_R$ (9)

The fact that the SF representation has to be re-learned for each individual MDP can be seen in our experiments. In Figure 6a they contribute to the oscillations of the SF loss objective. In the failure case shown in Figure 5 the SF representation does not transfer at all and instead represents the policy that is only optimal in the current MDP.

To get a better understanding why the loss objective oscillates despite the fact that the algorithm recovers a near optimal policy quickly, consider the transfer example shown in Figure 7. In this example, the two MDPs have two actions and deterministic transitions indicated by arrows. Rewards are indicated by the arrow labels and the two MDPs only differ in reward for two specific transitions. This difference in reward causes the optimal policy for each MDP to be different: The policy $\pi_{aa}$, which only selects action $a$, is optimal in the first MDP; the policy $\pi_{ab}$, which selects action $b$ at state $\phi_2$ and action $a$ elsewhere, is optimal in the second MDP.

This result also agrees with the first transfer experiment shown in Figure 3. Because the reward function and optimal policy is only changed slightly, the SF representations corresponding to each optimal policy and reward function are likely to be very similar. As a result, the algorithm can adjust to the new reward function very quickly. Barreto et al. also presented empirical results using a variation of Generalized Value Iteration (Sutton & Barto, 1998) on a version of Puddle World (Sutton, 1996) where the location of the puddle changed slightly. Their experiment, which shows a significant performance boost by transferring a SF representation, is similar to slight reward change test case because the changes in the reward function did not cause a drastic change in the optimal policy.
Advantages and Limitations of using Successor Features for Transfer in Reinforcement Learning

6. Conclusion

The presented empirical results demonstrate an interesting advantage and dis-advantage of transferring SFs between MDPs that only differ in reward function. While we were able to show a significant performance boost by using this approach, we also highlighted that the learned feature representation is dependent on the policy they are learned for. Hence, SF representations are an unsuitable choice in this context because one is typically interested in transferring knowledge between tasks with different optimal policies.

The fact that transferring a SF representation between tasks gives a significant boost in learning speed also suggests that learning a transferrable feature representation might be an interesting direction to pursue. However, such a feature representation needs to be independent of the task’s optimal policy.

References

Abadi, Martin, Agarwal, Ashish, Barham, Paul, Brevdo, Eugene, Chen, Zhifeng, Citro, Craig, Corrado, Greg S., Davis, Andy, Dean, Jeffrey, Devin, Matthieu, Ghemawat, Sanjay, Goodfellow, Ian, Harp, Andrew, Irving, Geoffrey, Isard, Michael, Jia, Yangqing, Jozefowicz, Rafal, Kaiser, Luke, Kudlur, Manjunath, Levenberg, Josh, Mané, Dan, Monga, Rajat, Moore, Sherry, Murray, Derek, Olah, Chris, Schuster, Mike, Shlens, Jonathon, Steiner, Benoit, Sutskever, Ilya, Talwar, Kunal, Tucker, Paul, Vanhoucke, Vincent, Vasudevan, Vijay, Viégas, Fernanda, Vinyals, Oriol, Warden, Pete, Wattenberg, Martin, Wicke, Martin, Yu, Yuan, and Zheng, Xiaoqiang. TensorFlow: Large-scale machine learning on heterogeneous systems, 2015. URL http://tensorflow.org/. Software available from tensorflow.org.

Antos, András, Szepesvári, Csaba, and Munos, Rémi. Learning near-optimal policies with bellman-residual minimization based fitted policy iteration and a single sample path. In International Conference on Computational Learning Theory, pp. 574–588. Springer, 2006.

Barreto, André, Munos, Rémi, Schaul, Tom, and Silver, David. Successor features for transfer in reinforcement learning. CoRR, abs/1606.05312, 2016. URL http://arxiv.org/abs/1606.05312.

Dayan, Peter. Improving generalization for temporal difference learning: The successor representation. Neural Computation, 5(4):613–624, 1993.

Kaelbling, Leslie Pack, Littman, Michael L, and Moore, Andrew W. Reinforcement learning: A survey. Journal of artificial intelligence research, 4:237–285, 1996.

Konidaris, George, Osentoski, Sarah, and Thomas, Philip. Value function approximation in reinforcement learning using the fourier basis. Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence, pp. pages 380–385, August 2011.

Mnih, Volodymyr, Kavukcuoglu, Koray, Silver, David, Rusu, Andrei A, Veness, Joel, Bellemare, Marc G, Graves, Alex, Riedmiller, Martin, Fidjeland, Andreas K, Ostrovski, Georg, et al. Human-level control through deep reinforcement learning. Nature, 518(7540):529–533, 2015.

Sutton, Richard S. Generalization in reinforcement learning: Successful examples using sparse coarse coding. Advances in neural information processing systems, pp. 1038–1044, 1996.

Sutton, Richard S. and Barto, Andrew G. Reinforcement Learning: An Introduction. A Bradford Book. MIT Press, Cambridge, MA, 1 edition, 1998.

Taylor, Matthew E. and Stone, Peter. Transfer learning for reinforcement learning domains: A survey. Journal of Machine Learning Research, 10(1):1633–1685, 2009.

Zhang, Jingwei, Springenberg, Jost Tobias, Boedecker, Joschka, and Burgard, Wolffram. Deep reinforcement learning with successor features for navigation across similar environments. arXiv preprint arXiv:1612.05533, 2016.