Frequency ratio of Yb and Sr clocks with $5 \times 10^{-17}$ uncertainty at 150 seconds averaging time

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Transition frequencies of atoms and ions are among the most accurately accessible quantities in nature, playing important roles in pushing the frontiers of science by testing fundamental laws of physics, in addition to a wide range of applications such as satellite navigation systems. Atomic clocks based on optical transitions approach uncertainties of $10^{-18}$ (refs 1-3), where full frequency descriptions are far beyond the reach of the SI second. Direct measurements of the frequency ratios of such super clocks, on the other hand, are not subject to this limitation4-8. They can verify consistency and overall accuracy for an ensemble of super clocks, an essential step towards a redefinition of the second9. Here we report a measurement that finds the frequency ratio of neutral ytterbium and strontium clocks to an uncertainty of $5 \times 10^{-17}$ and a measurement instability as low as $4 \times 10^{-16}$ (s/√s)$^{-1/2}$. This high stability marks a 90-fold reduction in the required averaging time over a previous record-setting experiment10 that determined the ratio of Al$^+$ and Hg$^+$ single-ion clocks to an uncertainty of $5.2 \times 10^{-17}$. Enabled by the simultaneous interrogation of hundreds of atoms in optical lattice clocks10, such a highly stable ratio measurement provides a powerful probe for new physics by exploring the variation $\Delta \alpha / \alpha$ of the fine structure constant with an uncertainty reducing as $1.6 \times 10^{-15}$ (s/√s)$^{-1/2}$ (see Methods). This already improves on dedicated experiments using atomic dysprosium11 in a search for variations on a timescale of seconds12, motivated by a coupling of light bosonic dark matter13 to the electromagnetic field.

Although the excitation of an atomic transition is described by a coherent quantum evolution, the readout of the outcome finds atoms in either the ground state or the excited state, introducing quantum projection noise14 (QPN), which sets a quantum limit on clock stability that scales with atom number by $1/\sqrt{N}$. Consequently, optical lattice clocks operating with $N \gg 1$ offer a substantial advantage over single-ion clocks. Fully exploiting this excellent quantum-limited stability for a stand-alone clock relies crucially on the continued development of ultrastable clock lasers15, as the noise of the local oscillator in conjunction with the periodic interrogation of the clock transition degrades stability through the Dick effect16. This, however, can be rejected in a synchronous comparison of two clocks17,18, where shared frequency noise of the clock lasers leads to common Dick effect noise in the frequency evaluations, which can be cancelled out in determining the frequency ratio. An extension to this scheme promises interrogation times beyond the coherence time of the lasers19. Combining long interrogation times with increased atom numbers, synchronous comparisons of future optical lattice clocks may achieve a statistical uncertainty of $1 \times 10^{-18}$ within minutes of averaging time19. This will allow instant investigation not only of clock frequency ratios, but also of relativistic effects equivalent to elevation changes of a single centimetre.

We apply synchronous interrogation in determining the ratio of the clock transition frequencies $\nu_{\text{Yb}} \approx 518$ THz and $\nu_{\text{Sr}} \approx 429$ THz in neutral $^{171}\text{Yb}$ and $^{85}\text{Sr}$ using a pair of cryogenic optical lattice clocks2 designed for operation with either of the two atomic species. Figure 1 gives an overview of the experimental set-up. Laser-cooled atoms loaded into the optical lattice are spin-polarized to more than 95% purity and sideband-cooled (see Methods and Supplementary Fig. 1) to an average axial vibrational state of $n < 0.1$. We then probe the clock transitions using ultrastable clock lasers phase-locked through an optical frequency comb, as discussed below.

For clocks interrogating neutral Yb or Sr in a room-temperature environment, the largest systematic frequency shift results from the ac Stark effect caused by blackbody radiation (BBR) and significant efforts have been made to control and characterize this effect20-22. In our experiments, the atoms are transferred to a cryogenic chamber for interrogation. This shields the atoms from ambient BBR and results in a near hundred-fold reduction of the frequency shift; for Yb from $-1.278$ Hz at 300 K to $-14.2(4)$ mHz at 96 K, and for Sr from $-2.278$ Hz at 300 K to $-23.3(4)$ mHz at 95 K. With less than $10^{-18}$ uncertainty of the BBR shifts, the largest uncertainty contribution for the ratio measurement stems from the lattice light shifts in the Yb clock. Taking into account atomic hyperpolarizability23 and multipolar effects, the light shifts are modelled based on ref. 24 (see Methods and Supplementary Fig. 2 for details). We find $\nu_{\text{Yb}} = 394,798,265(9)$ MHz for the E1-magic frequency, where the electric dipole polarizabilities of ground and excited states of the clock transition are equal.

We typically operate the optical lattice at $\nu_{\text{op}} = 394,798,278$ MHz, where the sensitivity to a variation of the lattice intensity is reduced by a partial cancelation of the linear and quadratic terms of the lattice-induced clock frequency shifts. For a typical trap depth $U_0 = 100 E_r$ ($\approx 10 \mu$K), where $E_r$ is the lattice photon recoil energy, our light shift model predicts a residual shift of $\Delta \nu_{\text{op}}/\nu_{\text{Yb}} = (-12 \pm 32) \times 10^{-18}$. The uncertainty is dominated by the determination of the model parameters, but also accounts for deviations from ideal lattice conditions, as might result from an intensity imbalance or a spectrally broad background in the laser emission (see Methods). The uncertainty budgets for the two clocks and the ratio measurement are given in Tables 1 and 2, respectively.

The Sr clock laser is stabilized to a 40-cm-long cavity and shows an instability of $3-5 \times 10^{-16}$ at 1 s. Its spectral characteristics are transferred to the Yb clock laser by a tight phase-lock, using an...
Figure 1 | Experimental set-up for Yb/Sr ratio measurements. a, Relevant transitions in Yb and Sr. b, Optical scheme of Yb and Sr clocks. The control systems apply frequency corrections through AOMs. PNC, phase-noise canceller. c, Generation of phase-locked clock lasers at νcl ≈ 429 THz and νcl ≈ 518 THz using an Er-fibre comb bridging the frequencies νE/2 and νE/2. Waveguide periodically poled lithium niobate crystals (WG-PPLN) double the output frequencies of intermediary external-cavity diode lasers (ECDL).

Table 1 | Corrections and uncertainty contributions for the Yb and Sr clocks.

| Systematic effect                  | Yb correction (10⁻¹⁸) | Yb uncertainty (10⁻¹⁸) | Sr correction (10⁻¹⁸) | Sr uncertainty (10⁻¹⁸) |
|-----------------------------------|-----------------------|------------------------|-----------------------|------------------------|
| Quadratic Zeeman effect           | 67.7                  | 9.8                    | 117.0                 | 1.0                    |
| BBR shift                         | 27.5                  | 0.7                    | 54.2                  | 0.9                    |
| Lattice light shift               | 8.5                   | 32.8                   | 3.5                   | 3.4                    |
| Probe light shift                 | -0.8                  | 3.2                    | 0.09                  | 0.05                   |
| Collisions                        | 0.0                   | 3.4                    | 0.9                   | 4.2                    |
| AOM chirp and switching           | 0.0                   | 1.1                    | 0.0                   | 0.2                    |
| 1st-order Doppler effect          | 0.0                   | 2.0                    | 0.0                   | 0.5                    |
| Servo error                       | 0.8                   | 1.1                    | 1.9                   | 1.6                    |
| Total                             | 103.7                 | 34.7                   | 177.6                 | 5.8                    |

Values are given in fractional units of 10⁻¹⁸ and represent averages over the complete set of contributing ratio measurements. See ref. 2 and Methods for details.

Table 2 | Ratio corrections and uncertainties.

| Contribution                  | Yb correction (10⁻¹⁸) | Yb uncertainty (10⁻¹⁸) | Sr correction (10⁻¹⁸) | Sr uncertainty (10⁻¹⁸) |
|-------------------------------|-----------------------|------------------------|-----------------------|------------------------|
| Yb systematic effects         | 103.7                 | 34.7                   | 117.0                 | 1.0                    |
| Sr systematic effects         | -177.6                | 5.8                    | 54.2                  | 0.9                    |
| Laser-to-laser link           | 0.0                   | 3.4                    | 3.5                   | 3.4                    |
| Gravitational shift           | -0.3                  | 1.1                    | 1.9                   | 1.6                    |
| Statistical uncertainty       | 0.0                   | 28.6                   | 177.6                 | 5.8                    |
| Total                         | -74.2                 | 45.6                   | 177.6                 | 5.8                    |

Values are given in fractional units of 10⁻¹⁸ and represent averages over the complete set of contributing ratio measurements. See Methods for details.

Er–fibre frequency comb with servo bandwidths ≈ 1 MHz. The control systems for Yb and Sr then independently adjust the laser frequencies through acousto-optic modulators (AOMs). After applying an interrogation pulse to the atoms in the optical lattice, the number of atoms in the ground and excited clock states, Ng and Ne, are detected to determine the atomic excitation P = Nf/(Ng + Ne), which is used to estimate the deviation of the laser frequency from the atomic resonance and to calculate the required frequency correction.

We operate both clocks with the same cycle time of 1.5 s and simultaneously apply interrogation pulses of 200 ms duration. As a result, the atomic excitation in both clocks acquires a correlated fluctuation, as indicated in Fig. 2b. The resulting common-mode noise cancels18 for the clock frequency ratio R = νYb/νSr, which is directly extracted from the record of applied corrections and resulting atomic excitation.

Figure 2a shows the fractional statistical uncertainty of the measured ratio R, which falls as quickly as 4 × 10⁻¹⁶ (τ/s)⁻¹/₂ and reaches 1 × 10⁻¹⁷ with an averaging time below 3,000 s. This is fully competitive with the stabilities of single species comparisons21,26 that employ state-of-the-art clock lasers with instabilities of 1 × 10⁻¹⁷ at 1 s.

Judging from the evaluation of interleaved Yb measurements, we find the synchronous interrogation to yield a two-times improvement in stability compared with a measurement with uncorrelated laser noise of similar magnitude (see Methods). The results demonstrate the excess noise introduced by the comb-based phase-lock
With equal weight assigned to each measurement, we obtain an uncertainty of \(1 \times 10^{-17}\) for a measurement of \(10^4\) s improves as \(\sigma_e^2 = 4 \times 10^{-16} (\tau/s)^{1/2}\) (black dashed line), approaching the QPN limit of \(2 \times 10^{-16} (\tau/s)^{1/2}\) for \(N_{\text{eff}} = 500\) and \(N_{\text{eff}} = 1,000\) (brown line). Error bars indicate 1\(\sigma\) uncertainties, assuming white frequency noise. The stability estimated for an asynchronous measurement with similar but uncorrelated laser noise improves as \(8 \times 10^{-17} (\tau/s)^{1/2}\) (solid black line). Blue dashed line indicates the systematic ratio uncertainty of \(3.6 \times 10^{-17}\). Over a period of four months, we performed ten measurements that are used to determine the Yb/Sr frequency ratio (Fig. 3). Owing to the high short-term stability, the reproducibility of the measurements is not limited by the available averaging time, but by the uncertainties of the corrections applied for experimental parameters that differ between repeated measurements. The observed variation, with a fractional standard deviation of \(\sigma_e/R = 2.9 \times 10^{-17}\), is consistent with the systematic ratio uncertainty of \(3.6 \times 10^{-17}\). The scatter of the results may however mask extra uncertainties in the measurement (possibly resulting from changes in the effective lattice intensity experienced by the atoms, affecting the light shift, see Methods). We conservatively include this possibility by evaluating the statistical uncertainty as \(\sigma_e/R_{\text{stat.}}\) with no reduction according to the standard error of the mean. This naturally includes the smaller contribution resulting from the short-term instability. With equal weight assigned to each measurement, we find \(R = \nu_{\text{Yb}}/\nu_{\text{Sr}} = 1.207507039343377394(43)_{\text{sys}}(35)_{\text{stat.}}\) consistent with our previous measurement\(^27\). The fractional uncertainty of \(4.6 \times 10^{-17}\) represents a 30-fold reduction compared with a previous direct measurement at NMIJ, which yielded \(R_{\text{NMIJ}} = 1.2075070393433404(18)\). Our value deviates from \(R_{\text{NMIJ}}\) by 1.5 times their stated uncertainty, but is well within the uncertainty of the ratio \(R_{\text{CIPM}} = 1.2075070393433399(35)\) calculated from the transition frequencies recommended by the International Committee for Weights and Measures (CIPM)\(^28\) in 2013.

Our ratio measurement obtains an overall uncertainty that improves on the best reported value. With enhanced control of the systematic effects, the measurement stability promises a statistical uncertainty of \(1 \times 10^{-18}\) with less than two days of averaging time. Operation with larger atom numbers will further relax the QPN limit and—together with improvements in clock laser stability—allow further reductions in averaging times, making ratio measurements between optical lattice clocks a valuable tool in a search for new physics\(^11-13\) at short time scales.

### Methods

Methods and any associated references are available in the online version of the paper.

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Author contributions

H.K. initiated and coordinated the experiments. M.T. and I.U. characterized and operated the Sr clock, N.N. and T.O. the Yb clock. N.O. maintained and operated the frequency comb. All authors contributed to the experimental set-ups, discussed the results and commented on the manuscript.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to H.K.

Competing financial interests

The authors declare no competing financial interests.
Methods

Operation of the Yb clock. Fermionic 173Yb (I = 1/2) atoms are decelerated from a thermal beam that contains both Sr and Yb atoms by a Zeeman slower and trapped in a magneto-optical trap (MOT), operating on the 1S0 − 1P1 transition at 399 nm. A second cooling stage in a MOT on the 1S0 − 1P1 transition at 556 nm reduces the atomic temperature to 15 µK. The optical lattice is kept at full intensity during loading. It provides a trap depth U0,Δk = 10 µK or U0,Δk = 100 µK, with the lattice photon recoil energy eL = (h/2Δk)2/2 m, where Δk = 759 nm is the lattice laser wavelength, and m is the atomic mass.

Around 500 atoms at a temperature of 3 µK remain in the lattice after turning off the MOT beams. Changing the frequency of one of the two independent lattice beams creates a moving lattice that transports the atoms by a chosen distance of typically 18–20 mm into the cryogenic chamber located below the MOT region (see Fig. 1). Outside the 60 ms transport interval, the relative phase of the lattice beams is actively stabilized to fix the lattice antinodes relative to a reference mirror2. To interact with the atoms inside the cryogenic chamber, extra coaxial laser beams (Fig. 1b) are superposed on the optical lattice with the same linear polarization. Spin-polarization is realized by optical pumping on the 1S0 = (F = 1/2) − 1P1 (F = 2/2) transition, where a bias magnetic field of 70 mT separates the Zeeman components by 900 kHz and allows the transition (mF = 1/2 to mF = 1/2) to be independently addressed. Two extra beams are used for axial sideband cooling. The first beam excites the red sideband of the 1S0 − 1P1 clock transition at 578 nm with a Rabi frequency of G0 = 2π × 10 Hz. The second beam is resonant with the 1P0 (F = 2/2) − 1P2 (F = 1/2) transition at 1,388 nm and returns the atoms to the ground state. Simultaneous operation of all three beams accumulates spin-polarized atoms in the dark n = 0 axial vibrational state over a period of 100 ms (see Supplementary Fig. 1). We typically find >95% of atoms in the desired spin state and an average fractional uncertainty of less than 10−16.

Ratio determination from recorded steering data. The control systems store the AOM frequencies Δfl(n)/fl(n) and resulting atomic excitation P. The excitation is used to calculate the error signal Δν, corresponding to a search for a long-term drift of the applied clock laser frequency from the HWHM point of the Rabi lineshape. The subharmonics of the clock laser frequencies, vfl/2 and vfl/3, are used to create beat signals with the comb lines with indices m0 and m0. The clock frequency ratio r = vfl/m0 can be extracted from recorded data as

\[ r = \frac{r_{\text{fl}}(\Delta m/2)}{r_{\text{fl}}(\Delta m/2)} = \frac{r_{\text{fl}}(\Delta m/2)}{r_{\text{fl}}(\Delta m/2)} \]  

where \( r = v_{\text{fl}}/m_0 \) and \( r_{\text{fl}} \) and \( r_{\text{fl}} \) are the comb repetition rate and carrier-envelope offset frequency. The frequency offsets \( \Delta \nu_{\text{fl}} = \Delta \nu_{\text{fl}}(\Delta m/2) + \Delta \nu_{\text{fl}}(\Delta m/2) + \Delta \nu_{\text{fl}}(\Delta m/2) + \Delta \nu_{\text{fl}}(\Delta m/2) \) include the AOM frequency \( \Delta \nu_{\text{fl}}(\Delta m/2) \) corrected for the frequency step \( \Delta \nu_{\text{fl}}(\Delta m/2) \) applied to the probe and right low slopes of the two Zeeman components. All constant frequency offsets are included in \( \Delta \nu_{\text{fl}}(\Delta m/2) \), whereas \( \Delta \nu_{\text{fl}}(\Delta m/2) \) corrects for systematic shifts. The division by two accounts for the use of two combined subharmonics. During a ratio measurement, \( \Delta \nu_{\text{fl}}(\Delta m/2) \) is stabilized to a reference frequency. Instead of directly measuring \( \Delta \nu_{\text{fl}}(\Delta m/2) \), we substitute \( m_0 r_{\text{fl}} + 2 \Delta \nu_{\text{fl}}(\Delta m/2) = \nu_{\text{fl}}/2 \), which is known to a fractional uncertainty of less than 10−16.

Short-term stability during Yb/Sr comparisons and interleaved evaluation.

In a simple model of the Dick effect contribution to the short-term stability based on ref. 39, we treat the Sr clock laser as dominated by cavity thermal noise, with a constant clock transition frequency of \( \nu_0 = 3 \times 10^{-16} \) and the link between clock lasers as a source of white phase noise chosen to reproduce \( \sigma_0^{\text{eff}}(\tau) = 4 \times 10^{-16} (\tau/s)^{1/2} \). The model predicts an instability of \( 4 \times 10^{-15} (\tau/s)^{1/2} \) for an interleaved measurement in the Yb clock with a cycle time of 3 s, in agreement with experimental results under optimal conditions. For a comparison of Yb and Sr clocks with a combined instability of \( 4 \times 10^{-16} (\tau/s)^{1/2} \) and the link between clock lasers as well-suited to synchronous interrogation due to the similar clock frequencies, and further noise reduction in the phase-lock between the lasers will allow even greater stability.

Uncertainty budget of the Yb clock. The following sections discuss the uncertainty contributions stated in Table 1 for the Yb clock.

Quadratomic Zeeman shift. The alternating interrogation of the \( m_0 = 3/2 \) components provides a continuous measurement of the magnetic field. We use the coefficients reported in ref. 40 to correct for the resulting quadratomic Zeeman shift.

BBR shift. A detailed study of the residual BBR frequency shifts for atoms inside the cryogenic chamber and their uncertainty was performed for Sr (ref. 2). As the contribution from the dynamic polarizability is insignificant at \( T = 96 \) K, the lower dc polarizability of Yb compared with Sr leads to a further reduction of shift and uncertainty. Precise experimental polarizability coefficients are available in refs. 22,41.

As the lattice waist is located between the loading position and the entrance aperture of the cryogenic chamber, the trap depth falls with increased transport distance inside the chamber. To avoid excessive atom loss in the Yb clock, where the temperature of the trapped atoms is higher than for Sr, atoms are interrogated at a chosen position located 5–7 mm inside the chamber, as opposed to 9 mm for Sr. The increased effect of room-temperature BBR leaking into the chamber is included in the uncertainty contribution of 7 \( \times 10^{-16} \).
the sideband spectra and the adopted value of \( d \), we employ a Monte-Carlo method for our analysis. In this procedure, fitting is repeated for different initializations of these parameters and the RMS deviation of the obtained coefficients is added to their uncertainties in quadrature. The effects of collisional shifts on the light shift evaluation are also included at this stage.

Overall, we find \( a = 0.021(6) \times 10^{-15} \) Hz MHz\(^{-1} \) and \( b = -0.68(71) \times 10^{-16} \) Hz as well as \( \nu_{\text{bg}} = 394.798,263(9) \) MHz. Typical parameter values are \( c = 0.72(5) \) and \( n = 0.08(8) \) when sideband cooling is applied.

**Lattice light impurity.** The optical lattice is generated by the output of a Ti:Sapphire laser, filtered with a volume Bragg grating (VBG) with a FWHM of 40 GHz. The lattice frequency is determined by the difference between the frequencies of the optical front surfaces black-coated to suppress reflections above the noise floor of the spectrometer analyser, and a monitoring Fabry–Perot etalon shows no spurious frequency components during normal operation.

Although we do not expect a frequency shift, we perform a check by deliberately detuning the centre frequency of the VBG by \( f - \nu_{\text{bg}} \). The optical lattice remains in a state of no spontaneous emission above the noise floor of the spectrometer analyser, and a monitoring Fabry–Perot etalon shows no spurious frequency components during normal operation.

Running wave frequency shifts. The multipolar polarizability assumed in our light shift model contributes a frequency shift of \( \Delta \nu = 1.4(2.8) \times 10^{-18} \) for Sr, where the instabilities have been determined from the Allan deviation.

The effect of the servo error on the measured frequency ratio is corrected shot-to-shot by including in the error bars of the experimental data, the uncertainties of the comparison of the two clocks.

**Servo error.** Both control systems contain algorithms to determine and predict the drift of the clock lasers. For a combined dataset of all ratio measurements, we find that the mean in-loop error signal \( \delta \nu \), extracted from the detected atomic excitation, reaches \( -0.0(1.1) \times 10^{-14} \) for Yb and \( -1.9(1.6) \times 10^{-14} \) for Sr, where the instabilities have been determined from the Allan deviation.

**Extra uncertainties for ratio measurements.** The following sections describe extra uncertainties in the comparison of the two clocks.

**Laser-to-laser link uncertainty.** The frequency comb was calibrated at AIST/NMIJ, where the frequency instability of a near-identical comb was evaluated\(^{45} \) as approximately \( 2.3 \times 10^{-18} \) for \( r < 10^{3} \) s and falling to below \( 10^{-18} \) for \( r > 10^{4} \) s. As we operate the comb in an environment temperature-controlled to 0.1 K, we expect similar stability.

Phase instabilities from variations in the optical path lengths between the clock lasers and the comb also contribute here. To avoid including the unstabilized lengths of the optical fibres of the WGP, the systematic dependencies of the comb repetition rate and the Yb clock laser use the residual fundamental light transmitted through the crystal, where the frequency doubling process ensures a stable phase relation with the generated harmonic output (Fig. 1). The fibres connecting the clock lasers to the comb are equipped with phase-noise cancellation systems\(^{46} \). Based on the measurements of the loop-back improvements described above, which include the contribution of the residual unstabilized optical path (approximately 2 m between Yb and Sr systems), we adopt an uncertainty of \( 2 \times 10^{-18} \).

The stability of the RF reference affects the ratio measurements through the last term of equation (1), particularly the reference frequencies for the beat signals used in the phase lock as well as the phase noise cancellation, entering through \( \nu_{\text{enn}} \). For the first four measurements we include an uncertainty contribution of \( 7 \times 10^{-19} \). The applied frequency offsets were then modified to minimize \( (\nu_{\text{CI}} - \nu_{\text{Sr}})/2 - r_{\text{CI}} + \nu_{\text{Sr}}/2 \), reducing this uncertainty to \( 3 \times 10^{-19} \) for measurements from June onwards. The average uncertainty due to RF instability is \( 4.4 \times 10^{-18} \) over all evaluated measurements, for a total fractional uncertainty of \( 5.4 \times 10^{-18} \) resulting from the laser-to-laser link.

**Gravitational frequency shift.** Both clocks share the same construction and are installed on the same optical table. The stated gravitational shift correction of \( -0.3(2) \times 10^{-18} \) accounts for a transport distance that is \( 2-4 \) mm less for Yb than for Sr, with a positioning uncertainty of 1 mm for each clock.

**Reproducibility of ratio measurements.** The scatter of the individual ratio measurements does not allow us to exclude the presence of extra uncertainties. A possible cause for such an uncertainty is an excess variation in the effective lattice intensity experienced by the Yb atoms, introducing a larger light shift uncertainty than expected. This may occur partly due to a breakdown of our simplified model: We evaluate the averaged lattice light intensity for each measurement based on the observed sideband spectra (see Supplementary Fig. 1) based on a model\(^{42} \) that assumes close-to-harmonic potential, with vibrational states occupied according to a Boltzmann distribution. However, the real situation is far more complex. Although...
the axial lattice trap depth is about 10 µK, the gravitational sag due to the lattice tilt of 15° (see also ‘First-order Doppler shift’ section) reduces the effective radial potential (supported by the beam radius of w ≈ 70 µm) to a depth of 5 µK for ¹⁷¹Yb atoms. The radial temperature, extracted from the model mentioned above to be 3 µK, then implies a truncated Boltzmann distribution and indicates a significant fraction of atoms sampling the anharmonic part of the radial potential. Both of these are beyond the reach of the model, and moreover, the resulting deviation may critically depend on the overlap of the independent lattice beams. For ⁸⁷Sr on the other hand, the atomic temperature reaches 1/10 of the lattice potential depth of ≈ 10 µK due to more effective Doppler cooling, resulting in a much more stable confinement.

If we assume that there is no extra instability, an extrapolation of the short-term instability to the full measurement time yields a statistical uncertainty of 2.4 × 10⁻¹⁸, for an overall ratio uncertainty of 3.6 × 10⁻¹⁷.

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Erratum: Frequency ratio of Yb and Sr clocks with $5 \times 10^{-17}$ uncertainty at 150 seconds averaging time

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In the version of this Letter originally published online, in Table 2, the column heading ‘$^{171}$Yb’ was mistakenly included. This has now been corrected in all versions of the Letter.