Evolutionary constraints on the masses of the components of HDE 226868/Cyg X-1 binary system

J. Ziolkowski

N. Copernicus Astronomical Center, ul. Bartycka 18, 00-716 Warsaw, Poland

ABSTRACT

Calculations carried out to model the evolution of HDE 226868, under different assumptions about the stellar wind mass loss rate, provide robust limits on the present mass of the star. It has to be in the range $40 \pm 5 \, M_\odot$ if the distance to the system is in the range $1.95$ to $2.35 \, kpc$ and the effective temperature of HDE 226868 in the range $30000$ to $31000 \, K$. Extending the possible intervals of these parameters to $1.8$ to $2.35 \, kpc$ and $28000$ to $32000 \, K$, one gets for the mass of the star the range $40 \pm 10 \, M_\odot$. Including into the analysis observational properties such as the profiles of the emission lines, rotational broadening of the absorption lines and the ellipsoidal light variations, one can estimate also the mass of the compact component. It has to be in the ranges $20 \pm 5 \, M_\odot$ and $13.5 \div 18 \, M_\odot$ for the cases described above. The same analysis (using the evolutionary models and the observational properties listed above) yields lower limit to the distance to the system of $\sim 2.0 \, kpc$, if the effective temperature of HDE 226868 is higher than $30000 \, K$. This limit to the distance does not depend on any photometric or astrometric considerations.

Key words: binaries: general – X-ray sources – evolution – stars: individual (Cyg X-1) – masses of the components.

1 INTRODUCTION

Cyg X-1 was the first binary system in which the presence of a black hole was suggested (Bolton 1972; Webster & Murdin 1972). For about a decade, it was the only object of that type. Many, sometimes exotic, models and scenarios were devised to avoid the presence of a black hole and to replace it with a neutron star. With the advent of subsequent black hole candidates (LMC X-3, A620–00), the motivation for such efforts substantially diminished. At present, there are no doubts about the presence of a black hole in the system. However, in spite of three decades of the investigations, there is still substantial uncertainty concerning the masses of both components. The mass function is known rather precisely. Its most recent value was given by Gies et al. (2003): $f(M_x) = 0.251 \pm 0.007 \, M_\odot$. There are also no doubts that the optical component must be close to filling its Roche lobe. Gies & Bolton (1986a, 1986b), analyzing the emission lines of the stellar wind, the rotational broadening of the absorption lines of the optical component and the photometric V-band light curve, found that the fill-out factor must be greater than 0.9 and its best value is in the range 0.95 to 1. The value of the inclination of the orbit is less certain. From the analysis mentioned above (rotational broadening of the absorption lines of the optical component and the ellipsoidal light variations in the V-band light curve), the authors estimated the inclination of the orbit to be $i = 33 \pm 5^\circ$. On the other hand, from polarimetric measurements (in three colours) Dolan & Tapia (1989) found the inclination $i = 62_{-37}^{+55} \, ^\circ$, which, while different, is not inconsistent with the value of Gies & Bolton.

Paczyński (1974) has shown that basing only on the lack of the X-ray eclipses and the fact that the optical component cannot be larger than its Roche lobe, one can obtain lower limits to the masses of both components as functions of the distance to the system. Paczyński found that the mass of the compact component has to be larger than $3.2 \, M_\odot$ if the distance is greater than $1.3 \, kpc$. He noticed also that, basing on stellar evolution consideration, the distance cannot be larger than about $2.1 \, kpc$. The distance to Cyg X-1 is not very well established, but, certainly, it is greater than $1.3 \, kpc$. Margon, Bowyer & Stone (1973) and Bregman et al. (1973) analyzed interstellar reddening in the direction of HDE 226868 and found that its distance is about $2.5 \, kpc$. Wu et al. (1982), from a study of the UV colors of HDE 226868, concluded that its distance must be greater than $1.9 \, kpc$. Bregman’s value was confirmed by Ninkov, Walker & Young (1987), who got the distance of $2.5 \pm 0.3 \, kpc$ from the equivalent width of the H$_\alpha$ line. HDE 226868 lies only $1^\circ$ from the center of NGC 6871 (the core of the Cyg OB3
association) and seems to be its member. However, the estimates of the distance to the association (or its core) gave mixed results. Crawford, Barnes & Warren (1974) used three methods (UBV photometry, the H-R diagram fitting and the equivalent width of the H$_\beta$ line) and obtained the distance to the association of about 2 kpc. Humphreys (1978) estimated the photometric distance to Cyg OB3 to be about 2.3 kpc. Janes & Adler (1982) quote in their catalogue the distance to NGC 6871 as only 1.8 kpc. The value of 1.8 kpc as the distance to Cyg OB3 was obtained again by Garmany & Stencel (1992), who used the H-R diagram fitting method. The discrepancy between the estimates of the distances to HDE 226868 and to Cyg OB3 cast the doubt on the membership of Cyg X-1 in NGC 6871. Fortunately, the extensive photometry and spectroscopy by Massey, Johnson & DeGioia-Eastwood (1995) seemed to clarify the situation. They estimated the distance to NGC 6871 as 2.14 ± 0.07 kpc, which was in reasonable agreement with the independent (H$_\alpha$ width) estimate of the distance to HDE 226868 (Ninkov et al. 1987). Let us note that a very similar value of the distance (d = 2.15 kpc) was derived also by Gies & Bolton (1986a) as a byproduct of their careful analysis of a large set of observational data for HDE 226868/Cyg X-1 binary system. Their result (see the point M$_a$ = 16 M$_\odot$ and M$_{opt}$ = 33 M$_\odot$ in their fig. 10) was obtained quite independently of any photometric considerations. We should also note, however, that while Herrero et al. (1995) do not discuss explicitly the value of the distance in their atmosphere modelling paper, their final model implies the value d ≈ 1.8 kpc. Finally, the question of Cyg X-1 membership in NGC 6871 was definitely solved in recent paper by Mirabel & Rodríguez (2003). Comparing the high precision VLBI astrometry for Cyg X-1 and Hipparcos astrometry for the members of Cyg OB3, the authors convincingly demonstrated that Cyg X-1 shares, quite precisely, common proper motion with Cyg OB3 association. The velocity of Cyg X-1 with respect to the Sun is 70 ± 3 km/s. The relative space velocity of Cyg X-1 with respect to Cyg OB3 is only 9 ± 2 km/s, which is typical of the random velocities of stars in expanding associations (Blauw 1991).

After the work by Massey et al. (1995), several more estimates of distances to both Cyg OB3 and HDE 226868 were made and the results still exhibit a substantial scatter. Malysheva (1997), using different photometry than Massey et al. (1995), obtained for the distance to Cyg OB3 again d ≈ 1.8 kpc. Dambis, Mel’nik & Rastorguev (2001) calculated the trigonometric distance to Cyg OB3 as a median value of the Hipparcos distances to the 18 individual member stars and found d = 2.3 (±1.4, ±0.6) kpc. In the case of Cyg OB3 their trigonometric distance agreed very well with the photometric distance of Blaha & Humphreys (1989), although they found that, typically, Hipparcos trigonometric distances to OB associations are by ~12% smaller than photometric distances of Blaha & Humphreys. A recent photometric distance estimate made by Mikolajewska (2002, private communication) gave the result d = 1.95 ±0.10 kpc. Lestrade et al. (1999) used the VLBI astrometry to estimate the distance to Cyg X-1 and obtained d = 1.0 ± 2.3 kpc.

All this extensive effort leads to the conclusion that the distance to HDE 226868/Cyg X-1 binary system cannot be significantly different from 2 kpc. Therefore, I believe that the best choice will be to follow Massey et al. (1995) and use d = 2.15 ± 0.07 (1σ error) or ± 0.2 (3σ error) kpc as the distance to Cyg X-1. As we shall see at the end of the paper, this choice will be, quite independently, supported by evolutionary considerations. The above value is used throughout the rest of this paper (except when I assume the distance to be a free parameter). In those parts of the discussion where the distance is assumed to be a free parameter, I shall consider the value 1.8 kpc as a reasonable lower limit to the distance of Cyg X-1.

2. THE MODEL INDEPENDENT LOWER LIMITS TO THE COMPONENTS MASSES

In this section, I shall repeat Paczynski’s (1974) analysis, using the present day data. This analysis is based only on solid observational facts and is, therefore, model independent. Let us remind that these solid facts include: (i) the value of the mass function, (ii) the lack of the X-ray eclipses, (iii) the obvious fact that the optical component cannot be larger than its Roche lobe, and (iii) the spectral type and the photometry of the optical component (used to derive its effective temperature and the bolometric correction).

The most recent determination of the mass function is f(M$_e$) = 0.251 ± 0.007 M$_\odot$ (Gies et al. 2003), which differs only slightly from the earlier values f(M$_e$) = 0.244 ± 0.005 M$_\odot$ (Brocksopp et al. 1999) and f(M$_e$) = 0.252 M$_\odot$ (Gies & Bolton 1982). The corresponding projected radius of the optical component orbit is a$_1$sin i = 8.36 ± 0.08 R$_\odot$. The spectral type of HDE 226868 was determined by Walborn (1973) as O9.7 Iab. This classification was confirmed by Gies & Bolton (1986a), by Ninkov et al. (1987) and by Herrero et al. (1995) and was used in most of the literature. On the other hand, Massey et al. (1995) obtained, from their new massive spectroscopy of OB associations, the somewhat earlier spectral type - ON9 Iia+. According to the calibration of effective temperatures for late O-type supergiants derived by Vacca, Garmany & Shull (1996, hereafter VGS), one has T$_{opt}$ = 32740 K for O9 Ia star and extrapolation for O9.7 Ia gives T$_e$ = 30690 K. On the other hand, Herrero et al. (1995) used atmosphere models to reproduce the observed spectrum of HDE 226868 and obtained T$_e$ = 32000 K as the best fit for the effective temperature of this star.

Both VGS and Herrero et al. calibrations were based on pure H-He models of the atmospheres that neglected the effects of metallic lines blanketing. It is well known by now, that taking into account the line blanketing leads to the lower effective temperature for a given spectral type (Martins et al. 2002, Repolust et al. 2004, Markova et al. 2004). The most recent calibration of effective temperatures for O-type supergiants (accounting for the line blanketing effects) was derived by Markova et al. (2004). Their scale gives generally lower effective temperatures than VGS scale (by up to 10 000 K for the earliest spectral types). However, for the spectral type O9.7 I, both temperature scales converge to the value T$_e$ = 30700 K. It seems that, at the present state of knowledge, the most reasonable approach would be to assume that T$_e$ = 30700 K is the best estimate of the effective temperature of HDE 226868. To take into account the uncertainty of this estimate, in further discussion I shall consider a broad interval of T$_e$ = 28000 ± 32000 K, as the
range of the possible values of the effective temperature of HDE 226868.

As the effective temperature-bolometric correction relation is not affected by the line blanketing (Martins et al. 2002), it is possible to take this relation from VGS tables. For the effective temperature 30700 K, the appropriate value of the bolometric correction is 5.39 (following Massey et al., I adopt 1.0). The corresponding unreddened colour index should be (B − V)_0 = −0.26 (Schmidt-Kaler 1982). The most recent photometry of HDE 226868 by Massey et al. (1995) gives V = 8.81 and B − V = 0.83. Therefore the reddening is 1.09 and the unreddened V magnitude is V_0 = 5.43 (following Massey et al., I adopt RV = 3.1). With the effective temperature and the bolometric correction given above, we can then express the radius and luminosity of HDE 226868 as functions of the distance:

\[ R_{\text{opt}}/R_\odot = 10.59 \text{ (d/1kpc)}, \]

\[ L_{\text{opt}}/L_\odot = 8.95 \times 10^4 \text{ (d/1kpc)}^2. \]

Following the procedure of Paczyński (1974), we can now calculate the lower limits to the masses of both components as functions of the distance to the system (under the assumptions listed at the beginning of this section). The results of these calculations are given in the first part of Table 1.

Let us notice that the changes, comparing with Paczyński’s Table 1, are mostly due to substantial change in the effective temperature (Paczyński took T_e = 25000 K) and the corresponding change in the bolometric correction for HDE 226868. Let us also notice that the effects of the changes in the effective temperature and the bolometric correction, to a large degree, cancel each other when we consider the radius of HDE 226868 and the lower limits to the masses of both components. They introduce, however, the dramatic (by a factor of ~ 2.5) increase of the luminosity of HDE 226868 (for a given distance). As a result, HDE 226868 appears to be a very bright star: its optical luminosity is \(81 \times 10^{39}\) erg/sec (by two orders of magnitude higher than the typical X-ray luminosity of Cyg X-1).

Undoubtedly, we know now the calibration of effective temperatures for O-type supergiants much better than 30 years ago. However, still, there remains some uncertainty, clearly demonstrated by the recent major revision introduced by accounting for the line blanketing effects (as discussed earlier). To investigate the effect of this uncertainty, I assumed that the effective temperature of HDE 226868 lies in the range 28000 ÷ 32000 K and calculated the corresponding lower limits to the masses of both components (using the modified versions of eqs. (1)–(2)). The results of these calculations are given in the second and the third part of Table 1. All sets of results are illustrated in Fig. 1.

Comparing the both sections of the table (or looking at the Fig. 1), we may notice that the uncertainty of the effective temperature does not affect significantly the lower limits to the masses.

Summarizing the considerations of this section, we may state that the masses of the compact and the optical components must be greater than \(~ 8\ M_\odot\) and \(~ 29\ M_\odot\), respectively (if the distance to the system is in the range 2.15 ± 0.07 kpc). Let us remind that these values are model independent and therefore very hard to contest. If one wants to be more conservative and uses the 3σ error as the uncertainty of the distance estimate, then these lower limits become \(~ 7\ M_\odot\) and \(~ 24\ M_\odot\), respectively. Even adopting the smallest value ever claimed for the distance (\(d = 1.8\) kpc), one still obtains the limits \(~ 6\ M_\odot\) and \(~ 19\ M_\odot\), respectively. One should add that our method gives only approximate estimates, since (as noted by Gies 2004, private communication) it assumes isotropic optical emission, which is not the case for tidally distorted star. The resulting inaccuracies should not be, however, significant.

The lower limits, presented in Table 1 and Fig. 1, become the actual values if the fill-out factor is equal 1.0 and the inclination of the orbit corresponds to the grazing eclipse orientation. The fill-out factor cannot be much different from 1.0 (Gies & Bolton 1986a,b). However, the inclination is the source of larger uncertainty. If the inclination is close to 62° (as suggested by the polarimetry, Dolan & Tapia 1989), then we are close to the grazing eclipse situation and the masses are close to the lower limits obtained in this section, i.e. \(~ 8±9

### Table 1. Parameters of the components of HDE 226868/Cyg X-1 binary system as functions of the assumed distance d for three values of the effective temperature of the optical component.

| d [kpc] | R_{opt} [R_\odot] | M_{opt,min} [M_\odot] | M_{x,min} [M_\odot] | \log(L/L_\odot) |
|--------|-----------------|----------------------|----------------------|-----------------|
|        |                 |                      |                      |                 |
| T_e = 30700 K |
| 1.80   | 19.7            | 22.3                 | 6.6                  | 5.73            |
| 1.95   | 21.3            | 27.6                 | 7.8                  | 5.40            |
| 2.08   | 22.8            | 32.9                 | 8.7                  | 5.46            |
| 2.15   | 23.5            | 35.9                 | 9.2                  | 5.49            |
| 2.22   | 24.3            | 39.2                 | 9.1                  | 5.51            |
| 2.35   | 25.7            | 45.7                 | 10.8                 | 5.56            |
| 2.50   | 27.3            | 53.9                 | 12.0                 | 5.62            |
| 2.70   | 29.5            | 66.3                 | 13.7                 | 5.68            |
| T_e = 28000 K |
| 1.80   | 18.7            | 19.5                 | 6.3                  | 5.50            |
| 1.95   | 20.3            | 24.1                 | 7.2                  | 5.59            |
| 2.08   | 21.6            | 28.7                 | 8.0                  | 5.64            |
| 2.15   | 22.4            | 31.4                 | 8.5                  | 5.67            |
| 2.22   | 23.1            | 34.2                 | 8.9                  | 5.70            |
| 2.35   | 24.4            | 39.9                 | 9.9                  | 5.75            |
| 2.50   | 26.0            | 47.1                 | 11.0                 | 5.80            |
| 2.70   | 28.1            | 57.9                 | 12.5                 | 5.87            |
| T_e = 32000 K |
| 1.80   | 19.7            | 22.3                 | 6.6                  | 5.52            |
| 1.95   | 21.3            | 27.6                 | 7.8                  | 5.50            |
| 2.08   | 22.8            | 32.9                 | 8.7                  | 5.46            |
| 2.15   | 23.5            | 35.9                 | 9.2                  | 5.49            |
| 2.22   | 24.3            | 39.2                 | 9.1                  | 5.51            |
| 2.35   | 25.7            | 45.7                 | 10.8                 | 5.56            |
| 2.50   | 27.3            | 53.9                 | 12.0                 | 5.62            |
| 2.70   | 29.5            | 66.3                 | 13.7                 | 5.68            |
The broken vertical lines indicate \( \pm \) luminosity must be \( M \). The lower limits to the masses of the optical (A) and compact (B) components as functions of the assumed distance, \( d \). Thick solid lines correspond to the most likely value of the effective temperature of HDE 226868 (\( T_e = 30 \text{ 700 K} \)). The broken lines and the dash-dotted lines correspond to the effective temperatures 28 000 K and 32 000 K, respectively. Thin solid vertical line indicates the most likely value of the distance (\( d = 2.15 \text{ kpc} \)). The broken vertical lines indicate \( \pm 1\sigma \) errors in the distance estimate. The \( \pm 3\sigma \) error range corresponds to the distance interval 1.95 \( \div \) 2.35 kpc.

\[ M_\odot \text{ and } 30 \div 36 M_\odot, \text{ respectively. If (as it seems more likely} \] see the discussion in Section 5), the inclination is rather close to \( 33^\circ \) (advocated by Gies & Bolton 1986a and still not excluded by polarimetry), then the masses are substantially higher and probably close to the values suggested by Gies & Bolton (16 \( \pm \) 5 \( M_\odot \) and 33 \( \pm \) 9 \( M_\odot \), respectively). I shall return to this point in further discussion.

Figure 1. The lower limits to the masses of the optical (A) and compact (B) components as functions of the assumed distance, \( d \). Thick solid lines correspond to the most likely value of the effective temperature of HDE 226868 (\( T_e = 30 \text{ 700 K} \)). The broken lines and the dash-dotted lines correspond to the effective temperatures 28 000 K and 32 000 K, respectively. Thin solid vertical line indicates the most likely value of the distance (\( d = 2.15 \text{ kpc} \)). The broken vertical lines indicate \( \pm 1\sigma \) errors in the distance estimate. The \( \pm 3\sigma \) error range corresponds to the distance interval 1.95 \( \div \) 2.35 kpc.

\[ M_\odot \text{ and } 30 \div 36 M_\odot, \text{ respectively. If (as it seems more likely} \] see the discussion in Section 5), the inclination is rather close to \( 33^\circ \) (advocated by Gies & Bolton 1986a and still not excluded by polarimetry), then the masses are substantially higher and probably close to the values suggested by Gies & Bolton (16 \( \pm \) 5 \( M_\odot \) and 33 \( \pm \) 9 \( M_\odot \), respectively). I shall return to this point in further discussion.

3 THE EVOLUTIONARY STATUS OF HDE 226868

Quite independent constraints on the masses may be obtained from analysis of the evolutionary status of the optical component. First, let us note that we know relatively well its luminosity. At the distance of 2.15 \( \pm \) 0.07 kpc, the luminosity must be \( M_{\text{bol}} \approx -9.29 \pm 0.07 \) (equivalent to \( \log(L/L_\odot) = 5.62 \pm 0.03 \) in Table 1). Including the uncertainty of the effective temperature and assuming \( 3\sigma \) error in the distance estimate, the range for the luminosity becomes \( M_{\text{bol}} \approx -8.8 \div -9.6 \) (equivalent to \( \log(L/L_\odot) = 5.40 \div 5.75 \) in Table 1). The real range is somewhat wider because one should include also the errors in \( E_{B-V} \) (about \( \pm 0.05 \)) and \( R_V \) (about \( \pm 0.1 \)) estimates. The final range becomes therefore \( M_{\text{bol}} \approx -8.5 \div -9.9 \). One may note that this range corresponds to \( M_V \approx -5.7 \div -6.7 \), which agrees quite well with \( M_V \approx -6.5 \pm 0.2 \) obtained by Ninkov et al. (1987) from the equivalent width of the \( H_\alpha \) line.

Second, let us consider the evolutionary phase of HDE 226868. Even without the specific evolutionary calculations, it is relatively straightforward to argue that it must be a core hydrogen burning configuration. It comes from the fact that HDE 226868 must be in a relatively stable phase of its evolution. We observe no measurable variations of the orbital period of the system. The \( 3\sigma \) upper limit for the relative change of the orbital period \( |d \ln P/d t| \) is \( \approx 2 \div 3 \times 10^{-3} \) yr\(^{-1} \) (Gies & Bolton 1982). This implies that the present evolutionary timescale of HDE 226868 must be substantially longer than \( \approx 3 \div 5 \times 10^4 \) yr. This excludes the possibility of the post-main sequence evolution, as the expected evolutionary time scale is, in that case, of the order of \( 10^5 \) yr. Ziolkowski (1977), investigating the post-main sequence evolution of a 25 \( M_\odot \) star (of similar radius but smaller luminosity than HDE 226868) approaching its Roche lobe, found that its radius was increasing from the fill-out factor equal 0.95 to 1.00 in only \( \approx 600 \) yr. After filling the Roche lobe, it took only \( \approx 200 \) yr for the mass transfer rate to exceed the Eddington limit by 3 orders of magnitude. For HDE 226868, which is more luminous and, probably, more massive (35 \( \div \) 45 \( M_\odot \), as we shall see later), the relevant time scales should be even shorter. If HDE 226868 were expanding that fast (5\% radius increase in less than 600 yr), we should observe the noticeable rise of its stellar wind strength and of the X-ray luminosity of the system (both parameters are sensitive to the fill-out factor if it is close to 1). Moreover, the chance of observing the system during such a short time window (just few hundred years, before it gets extinguished as an X-ray source due to hyper-Eddington accretion) is very small. Therefore, we have to assume that HDE 226868 (similarly as the optical components of other massive X-ray binaries - see Ziolkowski 1977) must still burn hydrogen in its core.

Exactly the same conclusion may be obtained directly as a unique outcome of the numerical modelling of the evolution of HDE 226868. This topic will be discussed in the next section.

4 THE EVOLUTIONARY CALCULATIONS FOR HDE 226868

4.1 The general description

I computed evolutionary tracks for core hydrogen burning phase of stars with the initial masses in the range 40 \( \div \) 80 \( M_\odot \). The Warsaw evolutionary code developed by Bohdan Paczyński and Maciek Kozlowski and kept updated by Ryszard Sienkiewicz was used. The initial chemical composition \( X=0.7 \) and \( Z=0.03 \) is adopted. The opacity tables incorporating OPAL opacities (Iglesias & Rogers 1996) as well as molecular and grain opacities (Alexander & Ferguson 1994) were used. The nuclear reaction rates are those
of Bahcall & Pinsonneault (1995). The equation of state used was Livermore Laboratory OPAL (Rogers, Svenson & Iglesias 1996). I neglected the semiconvective mixing, as it is not important during the evolutionary phase considered (most of the models of interest had central hydrogen content $X_c \gtrsim 0.2$). Similarly any overshooting at the border of the convective core was neglected (it is even less important).

The calculations were carried out under the assumption that the evolution starts from the homogeneous configurations. It means that the consequences of the fact that some of the matter of the star, possibly dumped from the progenitor of the present black hole, could have somewhat modified chemical composition, were neglected. It means also, that the consequences of the fact that some nuclear evolution (hydrogen burning) could, possibly, take place while the mass of the star was smaller (prior to the mass transfer) were neglected as well. It seems that both simplifications do not alter significantly the outcome of the evolutionary calculations. I shall return to this point in later discussion.

4.2 The stellar wind mass loss

The most uncertain element of the calculations of the early evolution of massive stars is the mass loss due to stellar wind. The uncertainty of the estimate of its rate is the single most important factor influencing the outcome of the calculations. The observations seem to indicate that there is a substantial scatter of the mass loss rates (typically, by a factor of two, but sometimes this factor can reach up to five) among the stars of similar luminosities and effective temperatures (see e.g. de Jager, Nieuwenhuijzen & Van Der Hucht 1988). I decided to use the prescription given by Nieuwenhuijzen & de Jager (1990) in the form of the formula derived by Hurley, Pols & Tout (1999, hereafter HPT). Bearing in mind that the formula gives the mass loss rate estimate with the accuracy that is probably not better than within a factor of two, I introduced the multiplying factor $f_{SW}$ applied to HPT formula. For each initial mass of the star three evolutionary sequences were calculated with the value of the parameter $f_{SW}$ equal, in sequence, 0.5, 1 and 2. In this way, the uncertainty of the theory of evolution could be, hopefully, taken into account. It might be interesting to note, at this point, the astonishingly good agreement between the HPT formula and the observational determination of the mass loss rate for HDE 226868: for the most likely values of the parameters of the star, the HPT value agrees with the observed one ($\dot{M} = -2.6 \times 10^{-6} \, M_\odot/yr$, Gies et al. 2003, see below) to an accuracy of about 5% (this is, definitely, better than the precision of both the HPT formula and of the observational estimate).

I should remind at this point that the supergiant components in some high mass X-ray binaries are significantly undermassive for their luminosities (Ziółkowski 1977). In some systems, like Cen X-3, this undermassivness is very serious and requires much stronger mass loss than the normal stellar wind (Ziółkowski 1978). However, most likely, it is not the case for HDE 226868. Its parameters may be fully explained by the evolution with the normal stellar wind (as will be demonstrated in the further discussion).

4.3 The evolutionary tracks

Some of the obtained evolutionary tracks in the H-R diagram are shown in Fig. 2. The careful reader might notice that the luminosities of my models are by about 0.1 dex ($\sim 25\%$) smaller than the evolutionary tracks of Schaller al. (1992). This difference should be attributed mainly to the fact that I used later edition of the opacity tables containing higher values of the opacities (the opacity tables always evolve in the direction of the growing opacity – never the other way). The correctness of the new opacities was confirmed by stellar pulsation calculations – see e.g. Pamyatnykh (1999). Part of the luminosity difference is due to the fact, that I used higher metallicity ($Z = 0.03$ instead of $Z = 0.02$). And, finally, part of the luminosity difference (all factors work in the same direction) is due to the fact, that my stellar winds are stronger (again due to higher metallicity).

As may be seen from a quick look at Fig. 2, the initial (zero age main sequence) mass of HDE 226868 had to be in the range 35 to 55 $M_\odot$. The masses of the models corresponding to the present day state of HDE 226868 (inside the 3$\sigma$ parallelogram) are in the range 32.4 to 50.5 $M_\odot$. The central hydrogen content in these models is between 0.126 and 0.265 and their evolutionary age (since the beginning of the central hydrogen burning) is between 2.7 and 4.3 $\times 10^6$ yr.

To obtain evolutionary models which satisfactorily reproduce the present day state of HDE 226868, one has to match not only the luminosity and the effective temperature but also the rate of the stellar wind mass loss. The first observational determination of this parameter for HDE 226868 was done by Hutchings (1976) who, analyzing the visual spectrographic data, got the value $\dot{M} = -2.5 \times 10^{-6} \, M_\odot/yr$. Persi et al. (1980) estimated the rate of the mass outflow from the infrared emission of the expanding circumstellar envelope and got the value $\dot{M} = -3.5 \times 10^{-6} \, M_\odot/yr$. More recent estimate of Herrero et al. (1995) was based on the fits of the $H_\beta$ profile and led to the conclusion that the mass loss rate lies between 2 and $6 \times 10^{-6} \, M_\odot/yr$. They noted that the inaccuracy of the fits is, probably, due to the fact that the stellar wind from HDE 226868 is focused towards the black hole (for the discussion of the "focused" wind model see Gies & Bolton 1986 and Miller et al. 2002). In their test modelling of the atmosphere of HDE 226868, Herrero et al. were using the value $\dot{M} = -4 \times 10^{-6} \, M_\odot/yr$ and for their final model they chose $\dot{M} = -3 \times 10^{-6} \, M_\odot/yr$. The most recent estimate by Gies et al. (2003) gave the value $\dot{M} = -2.6 \times 10^{-6} \, M_\odot/yr$ for the low/hard state (which is a typical state of Cyg X-1) and $\dot{M} = -2.0 \times 10^{-6} \, M_\odot/yr$ for the high/soft state (which is less frequent in this source). Taking all this into account, I assumed that the observed rate of mass outflow from HDE 226868 is $\dot{M} = -2.6 \times 10^{-6} \, M_\odot/yr$ with an error by a factor of about 2, i.e. I assumed that the observed rate of mass outflow is between 1.3 and $5.2 \times 10^{-6} \, M_\odot/yr$. Let me remind that our theoretical evolutionary models were using mass loss rates in the range 0.5 to 2 times the rate given by HPT. Altogether, it means, that the model for which the mass loss rate calculated with the HPT formula would be by a factor of up to four smaller or by a factor of up to four larger than the nominal observational value ($\dot{M} = -2.6 \times 10^{-6} \, M_\odot/yr$) is still considered to be an acceptable match. This is, probably, more than sufficient.
allowance for the uncertainty of the mass loss rates. The parameters of some of these models are given in Table 2. The positions of the acceptable models in the mass–luminosity diagram (for the most likely value of the effective temperature $T_e = 30700$ K) are shown in the Fig. 3. Since the luminosity of a model of HDE 226868 can be (for an assumed effective temperature) directly translated into the distance to the system (see Table 1), the Fig. 3 can be also considered as the mass–distance diagram. The corresponding distance scale is shown on this picture. The line corresponding to the lower mass limit–distance relation obtained in section 2 (see Table 1 and Fig. 1) is also shown. One may notice that the deduced mass of HDE 226868 depends mainly on the assumed distance to the binary system, and only very weakly on the assumptions about the mass loss rates. For the assumed effective temperature ($T_e = 30700$ K) this mass has to be in the range $35 \div 45$ M$_\odot$ (if the distance to the system is in the range 1.8 to 2.35 kpc). One may also conclude (with the help of the lower mass limits derived in Section 2) that the distance to the system cannot be larger than $\sim 2.5$ kpc (still for the assumed effective temperature).

Similar diagrams as Fig. 3 may be constructed for other possible values of the effective temperature of HDE 226868. Qualitatively, they look similar to the Fig. 3. For the effective temperature $T_e = 28000$ K the mass of the star has to be in the range $29 \div 35$ M$_\odot$ and the upper limit for the distance to the system is $\sim 2.1$ kpc. For the effective temperature $T_e = 32000$ K the mass of the star has to be in the range $37 \div 50$ M$_\odot$ and the upper limit for the distance to the system is $\sim 2.7$ kpc.

5 DISCUSSION

5.1 The mass of HDE 226868

The obvious and fairly strong conclusion derived from the evolutionary calculations is that HDE 226868 had to be quite massive in the past and is still very massive at present. To put it very briefly, it has to be very massive because it is
Masses of the components of HDE 226868/Cyg X-1

Figure 3. The positions of the evolutionary models of HDE 226868 (with an assumed effective temperature $T_e = 30700$ K) in the mass–luminosity and mass–distance diagram ("mass" means here the present mass of the star). The scale of the distances (in kpc) is shown with the thin broken vertical lines (the most likely value of the distance is $2.15 \pm 0.2$ kpc, $3\sigma$ error). The triangles, circles and squares correspond to the models with the stellar wind mass loss rates calculated with the HPT formula multiplied by the factors 0.5, 1 and 2, respectively. Only the models with the acceptable rates of mass loss (in the range $0.25 \div 4$ times the nominal observed value, $\dot{M} = -2.6 \times 10^{-6}$ $M_\odot$/yr) are shown. The solid line (without circles) describes the relation between the distance and the lower limit for the mass of HDE 226868 obtained in Section 2. The location of crossing of this relation by the sequences of the evolutionary models indicates that the distance to the binary system cannot be larger than $\sim 2.5$ kpc (for the assumed effective temperature). The oval indicates the range of the models of the optical star for which viable models of the binary system could be constructed (see the discussion in the Section 5.2).

very bright (while still burning hydrogen in the core). For the most likely range of parameters (30000 to 31000 K for the effective temperature and 2.08 to 2.22 kpc for the distance) the present mass of HDE 2268668 has to be in the range 37 to 44 $M_\odot$. Increasing the interval of the possible values of the distance to 1.95 to 2.35 kpc increases this range only slightly (to 33.5 $\div$ 47 $M_\odot$). Extending the interval of the possible values of the effective temperatures to 28000 to 32000 K leads to the range of possible masses 31 $\div$ 50 $M_\odot$. Finally, extending the range of the possible distances down to 1.8 kpc (the smallest value quoted in the literature) results in the final range of the possible masses of HDE 226868: 29 $\div$ 50 $M_\odot$. The value of 29 $M_\odot$ seems to be a firm lower limit for the present mass of HDE 226868.

One may ask whether it is possible to construct substantially less massive evolutionary models by taking more liberal estimate of the observational uncertainties or accounting for some simplifications of our models. The answer is negative. Decreasing the effective temperature of HDE 226868 would result in lower luminosities and so would lead to less massive configurations. However, the value $T_e = 28000$ K, taken as the lower limit to the effective temperature in the above consideration is probably already too low (the true value is probably in the range 30000 to 31000 K). Decreasing the minimum distance from 1.95 to 1.8 kpc decreased the minimum possible masses of the models by only $\sim 2 M_\odot$, as was mentioned above. Let us now consider the possible consequences of some simplifications of our evolutionary calculations. I neglected the possible dumping of matter on HDE 226868 from the progenitor of Cyg X-1 (and so "rejuvenation" of the star) during the first mass loss/exchange phase. Most likely, there was no significant accretion on HDE 226868 because any substantial mass transfer in such a close binary and in so early stage of evolution (when both stars
had no distinct cores) would lead to the rapid built-up of a common envelope and the coalescence of both components. So, most likely, the first mass transfer was mainly the mass loss from the system. If there were no serious accretion, then the nuclear evolution prior to the mass transfer is properly accounted for in our calculations. If (which is rather unlikely) there was a substantial mass dumping on HDE 226868, then the nuclear evolution before the mass transfer is negligible because the substantially less massive star was evolving much more slowly (large mass gain essentially resets the evolutionary clock to ZAMS).

One may wonder whether smaller metallicity ($Z = 0.02$ instead of $Z = 0.03$) would not be a better choice for HDE 226868. To check the possible effects of such alternative choice, I calculated several evolutionary tracks for stars with $Z = 0.02$. As might be expected, the changes in the results were small. For the models reproducing the "best" parameters of HDE 226868, the masses decreased from about 41.7 $M_\odot$ to about 39.9 $M_\odot$ (only about 4%).

There remains a rather remote possibility that the chemical composition of HDE 226868 is not normal, i.e. that it contains less hydrogen than a normal Population I star. E.g., Herrero et al. (1995) concluded, from fitting models of the atmosphere, that helium might be overabundant by a factor of about two. The evolutionary calculations presented here indicate that the stellar wind mass loss from HDE 226868 was not severe enough for hydrogen depleted layers to show up on the surface. However, if a large amount of matter with significantly decreased hydrogen content was dumped on HDE 226868 from the progenitor of Cyg X-1, then the star might contain less hydrogen than the assumed value of 0.7. The lack of the CNO anomalies (Dearborn 1977) testifies against this being the case. However, to check the consequences of such (rather unlikely) situation, I calculated several evolutionary sequences for the stars with the initial chemical composition $X = 0.5$, $Z = 0.03$ (this corresponds to an overabundance of helium by a factor of two). As could be easily expected, the models with similar luminosities were now less massive, but the change was not very substantial. For the "best fit" models, the masses decreased from about 41.7 $M_\odot$ to about 35.3 $M_\odot$ (only about 15%). However, I would like to stress again, that there are no good reasons to expect such abnormal hydrogen content in HDE 226868.

The only way to produce an evolutionary model with the mass $\lesssim 20 M_\odot$ is to decrease drastically the distance to the system. I constructed one evolutionary track that produced a 19.5 $M_\odot$ configuration at the effective temperature 30700 K with the mass loss rate $\dot{M} \approx -2.5 \times 10^{-6}$ (the initial mass of the star was 32.5 $M_\odot$). However, the price was very high: the distance had to be decreased to $\sim 0.9$ kpc and the rate of the mass loss during the evolution had to be multiplied by a factor of $\sim 20$ with respect to the HPT formula. Any of these conditions, even taken separately, seems extremely unlikely.

There is a clear conflict between the above considerations and the mass estimate given by Herrero et al. (1995). They used atmosphere models to reproduce the observed spectrum of HDE 226868. Then, they used their fit to determine the effective temperature, the surface gravity, the radius and the helium abundance of the star. Their mass estimate ($M \sim 18 \pm 4 M_\odot$) is a consequence of the surface gravity and the radius determination.

I see no solution of this conflict. I may only repeat that the evolutionary calculations for such an early evolutionary phase are very robust and rather difficult to dispute (it is, essentially, the mass-luminosity relation for the main sequence stars). I may also add that the evolutionary results presented above are roughly consistent with the model of Gies & Bolton (1986a), based on an extensive analysis of the large and diversified collection of the observational data.

### 5.2 The Mass of Cyg X-1

In the previous section we concluded that the present mass of HDE 226868 has to be in the range of $29 \div 50 M_\odot$ (assuming the widest possible ranges of the distances and the effective temperatures). The mass of its companion, black hole Cyg X-1, does not come out directly from the evolutionary calculations. However, it can be calculated (with the help of some observational constraints), once we select a chosen evolutionary model of HDE 226868. Once the selection is made, we know the mass $M_{\text{opt}}$ and the radius $R_{\text{opt}}$ of the optical component. We can also calculate the distance, with the help of eq. (1) or (2) (or the corresponding expressions for other effective temperatures). Subsequently, we can use two equations to solve for the inclination of the orbit $i$ and the mass ratio $q = M_{\text{opt}}/M_\odot$. One of these equations makes use of the mass function,

$$f(M_\odot) = M_{\text{opt}}\sin^3 i/[q(1 + q)^2].$$

The other relates the radius of the star to the size of the orbit,

$$R_{\text{opt}} = R_{\text{RL}} \times f_{\text{RL}} =$$
$$= f_{\text{RL}}(0.38 + 0.2 \log q)A =$$
$$= f_{\text{RL}}(0.38 + 0.2 \log q)R_L(1 + q),$$

where $R_{\text{RL}}$ is the radius of the Roche lobe around HDE 226868, $f_{\text{RL}}$ is the fill-out factor ($f_{\text{RL}} = R_{\text{opt}}/R_{\text{RL}}$), $A$ is the orbital separation of the binary components and $R_L$ is the radius of the orbit of HDE 226868 (see section 2).

Inserting the observational data, eqs. (3)–(4) can be written as:

$$M_{\text{opt}}\sin^3 i/[q(1 + q)^2] = 0.251,$$

$$R_{\text{opt}} = f_{\text{RL}}(0.38 + 0.2 \log q)(1 + q) \times 8.36/\sin i.$$

Once a given evolutionary model of HDE 226868 is selected from the grid of the acceptable models and a value of the parameter $f_{\text{RL}}$ is assumed, the eqs. (5)–(6) can be solved for $i$ and $q$. Knowing $q$ we can immediately calculate also the mass of compact component $M_\bullet$. In principle, this procedure can be applied to any combination of the evolutionary model and of the value of $f_{\text{RL}}$. In fact, however, not every evolutionary model of HDE 226868 (acceptable if we consider the optical component alone) permits the construction of a consistent model of the binary system. This is because of the observational constraints on the value of the inclination $i$ and, especially, because of the strong observational constraints on the value of the fill-out factor.
$f_{RL}$. As demonstrated by Gies & Bolton (1986a,b), in order to explain quantitatively the He emission lines produced in the stellar wind from HDE 226868, the fill-out factor $f_{RL}$ has to be larger than 0.9 and, most likely, not smaller than 0.95 (perhaps the best value would be around 0.98). On the other hand, Gies & Bolton demonstrated that observed rotational broadening of the photospheric absorption lines of HDE 226868 and the observed amplitude of the ellipsoidal light variations impose substantial constraints on the values of the mass ratio, the fill-out factor and the inclination.

The rotational broadening depends mainly on the mass ratio and the measured broadening indicates the mass ratio in the range $\sim 2 \div 2.5$. The amplitude of the ellipsoidal light variations is determined mainly by the fill-out factor and the inclination. For the assumed value of $f_{RL}$ equal 0.9, 0.95 and 1, the resulting inclination is $\sim 38^\circ$, $\sim 33^\circ$ and $\sim 28^\circ$, respectively.

Constructing models of the binary system to be consistent with observations, I assumed that for any adopted value of $f_{RL}$, the calculated inclination should be within $\pm 5^\circ$ of the corresponding values quoted above. I started with different models of the optical component, acceptable from the point of view of the stellar evolution, as described in section 4.3. Then, I assumed the value of $f_{RL}$ equal 0.95 and solved eqs. (5)–(6) to find $q, i$ and $M_\ast$. Subsequently, I tried higher values of $f_{RL}$. The higher values of $f_{RL}$ produced the solutions with lower (in many cases unacceptably low) values of the inclination. The dependence of $q, i$ and $M_\ast$ on the assumed value of $f_{RL}$ may be seen from many sequences of binary models, presented in Tab. 2. For all sequences included in the table, I present only two limiting models for the lowest and the highest value of $f_{RL}$ for which an acceptable model could still be obtained. If there is only one entry for a given sequence (as is the case for $M_{opt} = 39.67$ $M_\odot$ and $M_{opt} = 47.37$ $M_\odot$), it means that only one value of $f_{RL}$ produced an acceptable solution (the value of $f_{RL}$ was varied with the increment of 0.01).

I classified, as acceptable, the models satisfying the following criteria:

1. $d = 1.8 \div 2.35$ kpc
2. $T_e = 28000 \div 32000$ K
3. $-\dot{M} = 1.3 \div 5.2 \times 10^{-6}$ $M_\odot$/yr
4. $f_{RL} \geq 0.95$
5. $i = 28^\circ + (1 - f_{RL}) \times 100^\circ \pm 5^\circ$ (this corresponds to $i \approx 28 \div 38^\circ$ for $f_{RL} = 0.95$ and $i \approx 23 \div 33^\circ$ for $f_{RL} = 1.00$, as advocated by Gies & Bolton 1986a).

Parameters of selected acceptable models are given in the second part of Table 2. For each assumed value of the effective temperature, I present two sequences of models corresponding to the lowest and the highest current mass of the optical component. The exception to this rule is the sequence for $T_e = 30700$ K and $M_{opt} = 41.67$ $M_\odot$ ($d = 2.15$ kpc). The initial evolutionary parameters ($M_0$ and $f_{SW}$) of the optical component model for this sequence were adjusted so as to obtain the perfect fit with the "best" values of the observational parameters of HDE 226868 (effective temperature, luminosity and the rate of stellar wind mass loss). All models from this sequence (for $f_{RL} = 0.96 \div 1.00$) are successful from the point of view of our criteria.

There are several conclusions that can be drawn from the collection of the obtained models of the binary system (only some of these models are shown in Table 2). The first concerns the mass of the compact component. Assuming the widest possible intervals of the distances and the effective temperatures: 1.8 to 2.35 kpc and 28000 to 32000 K, the mass of the compact component must be in the range $13.5 \div 28.5$ $M_\odot$. For the most likely intervals of these parameters: 1.95 to 2.35 kpc and 30000 to 31000 K, this range narrows to $15.5 \div 25$ $M_\odot$. The second conclusion is related to the distance to the binary system. It appears that for a given (assumed) effective temperature, consistent models are possible only for some, relatively narrow, interval of distances (the distances out of this interval would require unacceptably low or unacceptably high values of the inclination). For the effective temperatures equal to 28, 30, 30.7, 31 and $32 \times 10^3$ K, the corresponding intervals are $1.80 \div 1.88$, $1.98 \div 2.15$, $2.04 \div 2.22$, $2.07 \div 2.27$ and $2.21 \div 2.35$ kpc, respectively. It is certainly encouraging that the most likely range of the effective temperatures (30000 to 31000 K) requires the distance interval ($1.98 \div 2.27$ kpc) that almost exactly coincides with the independent estimate of the most likely distance range ($1.95 \div 2.35$ kpc). The obtained relation between the distance and the effective temperature of HDE 226868 means that, if in the future the distance will be known more precisely (e.g. from the future astrometric space missions), then it will be possible to set constraints on the effective temperature. For example, if it is found that the distance to the system is greater than 2.1 kpc, it would mean that the effective temperature of HDE 226868 has to be higher than 30000 K. Also the opposite is true. If it is found (e.g. from the better models of the atmospheres) that the effective temperature of HDE 226868 is greater than 30000 K, it would mean that the distance to the system must be larger than 2.0 kpc (and this conclusion would not depend on any photometric or astrometric considerations).

Finally, the third conclusion confirms the earlier results (based on the evolutionary calculations alone) limiting the present mass of HDE 226868 to the range $29 \div 50$ $M_\odot$ and the initial mass to the range $33.5 \div 55$ $M_\odot$.

Let us note that for the black hole mass in the range $15 \div 25$ $M_\odot$, the state transitions occur in Cyg X-1 at the luminosity level equal 0.025 to 0.015 of the Eddington luminosity (assuming $d = 2.15$ kpc and using the flux values given by Zdziarski et al. 2002). These values lie in the range 0.007 to 0.03 found for other X-ray binaries (Maccarone 2003).

To summarize this section, the evolutionary considerations provide much stronger lower limits to the masses of both components than the model independent analysis, presented in section 2. In addition, they yield some independent constraints on the distance to the system.

6 THE POSSIBLE EVOLUTIONARY SCENARIO

Let us speculate a little bit about the possible evolutionary past of our binary system. It had to start as a very massive system (the initial primary had to complete its evolution by the time the secondary (HDE 226868) reached its present evolutionary state i.e. in less than $\sim 3 \div 4 \times 10^9$ yr). The initial masses of the components were probably $\sim 80 \div 100$ and $\sim 40 \div 50$ $M_\odot$. The massive primary (the progenitor of a black hole) was shedding the mass mainly in the form of the stellar wind (there was probably little, if any, mass transfer
to the companion). The mass of the primary decreased, at the end of its evolution (before the collapse), to $\sim 15 \div 25 M_\odot$. The low spatial velocity of Cyg X-1 with respect to its parental association indicates, as argued by Mirabel & Rodríguez (2003), that the final collapse to a black hole proceeded with very little ($\lesssim 1 M_\odot$) mass ejection. It could be, even, a prompt collapse with no accompanying supernova explosion at all ("formation of a black hole in the dark").

The secondary (the progenitor of HDE 226868) was also losing mass in the form of the stellar wind. This implies, in turn, that the initial orbital period was very tight. One cannot exclude that the initial masses were smaller, the initial orbital period longer and the system very tight. The above evolutionary history is only one of the possible scenarios. As noted by Gies (2004, private communication), the above scenario implies that about half of the total initial mass of the binary system was lost in the form of stellar wind. This implies, in turn, that the initial orbital period was $\lesssim 3$ days, which means that the system was very tight. One cannot exclude that the initial masses were smaller, the initial orbital period longer and the system passed through the mass transfer and the common envelope phase.

7 CONCLUSIONS

(1) The calculations modelling the evolution of HDE 226868, under different assumptions about the stellar wind mass loss rate, provide robust limits on the present mass of the star. For the most likely intervals of the values of the distance and of the effective temperature: 1.95 to 2.35 kpc and 30000 to 31000 K, the mass of HDE226868 is $40 \pm 5 M_\odot$. Extending the intervals of these parameters to 1.8 to 2.35 kpc and 28000 to 32000 K, one obtains the mass of the star in the range $29 \div 50 M_\odot$.

(2) Including the additional constraints resulting from the observed properties of the binary system HDE 226868/Cyg X-1, one can estimate the mass of the black hole component. For the most likely values of the parameters mentioned in the item (1) above this mass is $20 \pm 5 M_\odot$. For the extended intervals of the parameters the mass is in the range of $13.5 \div 29 M_\odot$.

(3) The distance to the binary system has to be in the ranges $1.80 \pm 0.18$, $1.98 \pm 0.25$, $2.04 \pm 0.22$, $2.07 \pm 0.27$ and $2.21 \pm 0.25$ kpc, for the effective temperature of HDE 226868 equal to 28, 30, 30.7, 31 and $32 \times 10^5 K$, correspondingly.

ACKNOWLEDGEMENTS

I would like to thank A. Zdziarski for careful reading of the manuscript and for many helpful comments and stimulating discussions. I would like also to thank D. Gies who, acting as a referee, made several comments and suggestions which helped to improve this paper. This work was partially supported by the State Committee for Scientific Research grants No 1 P03D 018 27 and No PBZ KBN 054/P03/2001.

REFERENCES

Alexander D.R., Ferguson J.W., 1994, ApJ, 437, 789
Bahcall J.N., Pinsonneault M.H., 1995, Rev. Mod. Phys., 67, 781
Blaauw A., 1991, in Lada C.J., Kylafis N.D., eds, NATO ASIC Proc. 342, The Physics of Stars Formation and Early Stellar Evolution, Kluwer, Dordrecht, Netherlands, p. 125
Blaha C., Humphreys R.M., 1989, AJ, 98, 1598
Bolton C.T., 1972, Nature, 235, 271
Bregman J., Butler D., Kemper E., Koski A., Kraft R.P., Stone R.P.S., 1973, ApJ, 185, L117
Brooksopp C., Tarasov A.E., Lyuty V.M., Roche P., 1999, A&A, 343, 861
Crawford D.L., Barnes J.V., Warren, W.H., 1974, AJ, 79, 623
Dambis A.K., Mel’nik A.M., Rastorguev, A.S., 2001, Astr. Lett., 27, 58
Dearborn D.S.P., 1977, Astrophysical Lett., 19, 15
De Jager C., Nieuwenhuijzen H., Van Der Hucht, K.A., 1988, A&A, 257, 259
Dolan J.F., Tapia, S., 1989, ApJ, 344, 830
Garmany C.D., Stencel, R.E., 1992, A&A, 94, 211
Gies D.R., Bolton, C.T., 1982, ApJ, 260, 240
Gies D.R., Bolton, C.T., 1986a, ApJ, 304, 371
Gies D.R., Bolton, C.T., 1986b, ApJ, 304, 389
Gies D.R., Bolton, C.T., Thomson J.R., Huang W., McSwain M.V., Riddle R.L., Wang Z., Wiita P.J., Wingert D.W., Cik B., Kiss LL., 2003, ApJ, 583, 424
Herrero A., Kudritzki R.P., Gabler R., Vilchez J.M., Gabler A., 1995, A&A, 297, 556
Humphreys R.M., 1978, ApJS, 38, 309
Hurley J.R., Pols O.R., Tout, C.A., 2000, MNRAS, 315, 543 (HPT)
Hutchings J.B., 1976, ApJ, 203, 438
Iglesias C.A., Rogers, F.J., 1996, ApJ, 464, 943
Janes K., Adler, D., 1982, ApJS 49, L113
Lehndre J.-F., Preston R.A., Jones D.J., Philips R.B., Rogers A.E.E., Titus M.A., Rijo M.J., Gobuzda D.C., 1999, A&A, 344, 1014
Maccarone T.J., 2003, A&A, 409, 697
Malysheva L.K., 1997, Astr. Lett., 23, 585
Margon B., Bowyer S., Stone R.P.S., 1973, ApJ, 185, L113
Markova N., Puls J., Repolust T., Markov H., 2004, A&A, 413, 693
Martins F., Schaerer D., Hillier D.J., 2002, A&A, 382, 999
Massey P., Johnson K.E., DeGioia-Eastwood K., 1995, ApJ, 454, 151
Miller J.M., Wojdowski P., Schultz N.S., Marshall H.L., Fabian A.C., Remillard R.A., Wijers R., Lewin W.H.G., 2002, astro-ph 0208463
Mirabel I.F., Rodríguez I., 2003, Science, 300, 1119
Nieuwenhuijzen H., de Jager C., 1990, A&A, 231, 134
Ninkov Z., Walker G.A.H., Yang S., 1987, ApJ, 321, 425
Paczynski B., 1974, A&A, 34, 161
Pamyatnykh A.A., 1999, Acta Astr., 49, 119
### Table 2. Parameters of the selected evolutionary models of the binary system HDE 226868/Cyg X-1.

| $T_e$ [10^4 K] | $M_{opt}$ [M⊙] | $d$ [kpc] | $M_0$ [M⊙] | $f_{SW}$ | $R_{opt}$ [R⊙] | log$L$ [L⊙] | $\dot{M}$ [10^{-6} M⊙/yr] | $f_{RL}$ | $i$ [°] | $M_x$ [M⊙] |
|---------------|-----------------|----------|-------------|---------|-----------------|------------|------------------------|--------|-------|-------------|
|   30.7        | 2.15            |          | 22.77       | 5.617   | 2.60            |            | \geq 0.95              | 33     |       |             |
| -2.7,+1.3     | ±0.2            |          | ±2.3        | ±0.13   | -1.3,+2.6       |            |                        |        |       |             |

### Observational parameters

|   30.7 | 2.15 | 22.77 | 5.617 | 2.60 | \geq 0.95 | 33 |
|-------|------|-------|-------|------|----------|----|
| -2.7,+1.3 | ±0.2 | ±2.3   | ±0.13 | -1.3,+2.6 |         |    |

### Acceptable models

|   28  | 29.30 | 1.80 | 33.5 | 2 | 19.69 | 5.330 | 1.84 | 0.95 | 34.6 | 13.6 |
|------|-------|------|------|---|------|-------|------|------|------|------|
| 28   | 30.31 | 1.88 | 35   | 2 | 20.53 | 5.367 | 2.12 | 0.98 | 33.9 | 14.2 |
| 30   | 36.74 | 1.98 | 40   | 1 | 21.10 | 5.511 | 1.68 | 0.95 | 30.0 | 18.3 |
| 30   | 39.53 | 2.15 | 44.3 | 1.17 | 22.88 | 5.581 | 2.60 | 0.99 | 33.9 | 16.6 |
| 30.7 | 39.67 | 2.04 | 43.5 | 1 | 21.63 | 5.572 | 2.07 | 0.95 | 28.9 | 19.9 |
| 30.7 | 41.67 | 2.15 | 46.3 | 1.05 | 22.77 | 5.617 | 2.60 | 0.96 | 36.1 | 16.0 |

### NOTES:

1. $M_0$ denotes the initial (ZAMS) mass of the optical component, $f_{SW}$ denotes the multiplying factor applied to HPT formula; other symbols have their usual meanings.

2. The bold face entries correspond to the "best fit" models (see the text).

---

Persi P., Ferrari-Toniolo M., Grasdalen G.L., Spada G., 1980, A&A, 92, 238
Repolust T., Puls J., Herrero A., 2004, A&A, 415, 349
Rogers F.J., Svenson F.J., Iglesias C.A., 1996, ApJ, 456, 902
Schaller G., Schaerer D., Meynet G., Maeder A., 1992, A&A Suppl., 96, 269
Schmidt-Kaler Th., 1982, in Schaifers K., Vogt H.H., eds, Landolt-Bornstein, Numerical Data and Functional Relationships in Science and Technology, New Series, Springer-Verlag, Berlin, Group VI, Vol. 2b, p. 31
Vacca W.D., Garmany C.D., Shull J.M., 1996, ApJ, 460, 914 (VGS)
Walborn N.R., 1973, ApJ, 179, L123
Webster B.L., Murdin P., 1972, Nature, 235, 37
Wu C.-C., Eaton J.A., Holm A.V., Milgrom M., Hammerschlag-Hensberge G., 1982, PASP, 94, 149
Zdziarski A.A., Poutanen J., Paciesas W.S., Wen L., 2002, ApJ, 578, 357
Ziolkowski J., 1977, Eight Texas Symposium on Relativistic Astrophysics, Annals New York Academy Sci., 302, 47
Ziolkowski J., 1978, in Zytkow A.N., ed., Nonstationary Evolution of Close Binaries, Polish Scientific Publishers, Warsaw, p. 29