Design of Attitude Control System for Stratospheric Balloon Gondolas by Sliding Mode Control

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Abstract. This paper reports consideration of efficient design method for the azimuthal-attitude control of stratospheric balloon gondolas utilizing only a motorized pivot. To maintain sufficient robustness against variation of the model parameters, applying of sliding mode controller is tried and proposed control system is validated by numerical simulation studies.

1. Introduction

Balloon experiment is known as a powerful method for various scientific observations and engineering experiments; a large gas-balloon is used to transport a payload (gondola) equipped with observational instruments or experimental devices to stratospheric altitude. In one example, balloon-borne telescopes have been developed and used for many astronomical and planetary observations. The balloon workshop was held at NASA Glenn Research Center in 2012, and this workshop concluded that balloon platform has potential to solve many science questions in the planetary sciences field[1].

Attitude control is one of the key technologies on balloon experiments requiring to point the observational instruments on the gondola in the target direction. The dynamics of the gondola suspended from the balloon envelope via a rope has one-degree of freedom around vertical axis. Control technologies for rotating and stabilizing the gondola in azimuthal direction have been studied through the past experiments. Attitude-control methods applied in practical use can be classified into two ways: a method using a reaction wheel for actuating the gondola and a motorized pivot at top of the gondola for desaturating the reaction wheel[2,3]; other one using only a motorized pivot to twist the suspension rope and rotate the gondola[4,5]. Attitude control with only a motorized pivot has an advantage on simplifying and downsizing the gondola system. However, vibration of the gondola depending on the torsional spring characteristic of the rope affects the control performance. This characteristic unpredictably changes by atmospheric environment during the flight, so robustness for such parameter variation is essential to control-system design. Therefore, achieving high responsiveness and accuracy is difficult with this control method, and it has been applied on only some flight-experiment requiring relatively rough-accuracy attitude control.

This paper will study about a performance-improvement method of azimuthal-attitude control system with only a motorized pivot. By solving the above-mentioned problem about robust stability, this control system will be able to be applied for more balloon gondolas, and balloon experiments will be able to conduct with inexpensive cost. To substantialize enough robustness against variation of parameters, applicability of sliding mode control (SMC) will be studied. Firstly, dynamics model of the control system with a pivot will be built. After that, a control system based on SMC to realize the stability will
be considered, and effectiveness of the proposed control method will be confirmed by numerical simulations.

2. Dynamics model of the control system

Figure 1 shows the dynamics model of the balloon gondola considering only rotational motion in azimuthal direction. $\theta_b$ is the angle of the bottom of the balloon-envelope, $\theta_p$ is the angle of the pivot shaft, and $\theta_g$ is the angle of the gondola. At stratospheric altitude, the balloon envelope rotates irregularly by impact of disturbance torque. This disturbance rotation is transmitted to the gondola via the suspension rope. Azimuthal-attitude control system of the gondola must have satisfactory performance to compensate the disturbance rotation. The pivot shaft at the top of the gondola is rotated by the control torque $\tau_m$ of the motor.

Motion of the gondola in the azimuthal direction is described as:

$$J_g \ddot{\theta}_g = \tau_m, \quad (1)$$

where $J_g$ is the moment of inertia of the gondola.

Dynamics of the pivot shaft is also given as following equation by approximating the suspension rope as a torsional spring system with a damper:

$$J_p \ddot{\theta}_p + c_r (\dot{\theta}_p - \dot{\theta}_b) + k_r (\theta_p - \theta_b) = -\tau_m, \quad (2)$$

where, $k_r$ and $c_r$ are the coefficient values of torsional spring and damper, respectively.

On the many actual balloon gondolas in some previous flight missions, the actuation systems of pivot were often equipped with an angular-velocity control system for its motor. In this research, control system utilizing the angular velocity of the motor as a control input was modeled with the assumption that the motorized pivot has the angular-velocity control system.

Equation (1) can be rewritten using the equation (2) as:

$$\ddot{\theta}_g = \frac{1}{J_g} \tau_m = -\frac{J_p}{J_g} \ddot{\theta}_p - \frac{c_r}{J_g} (\dot{\theta}_p - \dot{\theta}_b) - \frac{k_r}{J_g} (\theta_p - \theta_b) \quad (3)$$

Relational expression between $\theta_p$, $\theta_g$, and the angle of the pivot motor $\theta_m$ is given as:

$$\theta_p = \theta_g - \theta_m \quad (4)$$

Following expression can be obtained by inserting the $\theta_p$ expressed in the equation (4) into the equation (3):

$$\ddot{\theta}_g = -\frac{J_p}{J_g} (\ddot{\theta}_g - \ddot{\theta}_m) - \frac{c_r}{J_g} (\dot{\theta}_g - \dot{\theta}_m - \dot{\theta}_b) - \frac{k_r}{J_g} (\theta_g - \theta_m - \theta_b) \quad (5)$$

Figure 1. Dynamics model of the azimuthal-control system.
In general, the moment of inertia of the pivot $J_p$ is extremely smaller than it of the gondola. By approximation as $J_p/J_g \approx 0$, equation (5) can be represented as:

$$\ddot{\theta}_g = -\frac{c_r}{J_g}(\dot{\theta}_g - \dot{\theta}_m) - \frac{k_r}{J_g}(\theta_g - \theta_m) + \frac{c_r}{J_g}\dot{\theta}_b + \frac{k_r}{J_g}\theta_b$$  \hspace{1cm} (6)

Through these processes, the control system using $\dot{\theta}_m$ as a control input can be defined. In this research, responsiveness of the motor driving system was assumed as:

$$\dot{\theta}_m = -\alpha\dot{\theta}_m + \alpha\dot{\theta}_m^*,$$  \hspace{1cm} (7)

where, $\dot{\theta}_m^*$ is a target value of $\dot{\theta}_m$, and $\alpha$ represents characteristic of the motor driving system.

From equation (6) and equation (7) are written in the state equation as:

$$\dot{x} = Ax + Bu + Dd,$$
$$y = Cx,$$

where,

$$x = [\theta_g, \dot{\theta}_g, \theta_m, \dot{\theta}_m]^T, \quad u = \dot{\theta}_m^*, \quad d = [\theta_b, \dot{\theta}_b]^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_r}{J_g} & -\frac{c_r}{J_g} & \frac{k_r}{J_g} & \frac{c_r}{J_g} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$C = [1 \ 0 \ 0 \ 0], \quad D = \begin{bmatrix} 0 & 0 \\ \frac{k_r}{J_g} & \frac{c_r}{J_g} \end{bmatrix}. \hspace{1cm} (9)$$

3. Define of Parameter Conditions and Control Requirement

In this chapter, parameter conditions of the dynamics model and limitation of the control actuator will be defined for design and verification of the control system.

3.1. Nominal model

Nominal parameter values of the dynamics model were defined as:

$$J_g = 100 \text{ kgm}^2, k_r = 1.0 \text{ Nm/rad}, c_r = 0.2 \text{ Nm/(rad/s)}, \alpha = 10.$$  

These values were determined referring some previous studies.

3.2. Parameter variation

The pivot applies torque to suspension rope and twist it; it also induces torsional vibration in azimuthal direction. Therefore, characteristic variation of the suspension rope expressed as $k_r$ and $c_r$ extremely affects control stability and performance. Control system must be designed considering such parameter variation to keep enough stability and desirable responsiveness.

In this research, performance validation of the designed control system was performed in three cases, as follows:

- Case-1: $k_r = 1.0 \text{ Nm/rad}, c_r = 1.0 \text{ Nm/(rad/s)} \cdots$ conditions of the ground test.
- Case-2: $k_r = 0.2 \text{ Nm/rad}, c_r = 0.2 \text{ Nm/(rad/s)} \cdots$ conditions of the flight with a low $k_r$.
- Case-3: $k_r = 2.4 \text{ Nm/rad}, c_r = 0.2 \text{ Nm/(rad/s)} \cdots$ conditions of the flight with a high $k_r$.

In the actual developments of balloon-gondola system, ground tests are often conducted to evaluate the performance of the control system; the balloon gondola is suspended from a crane via a short suspension rope. Case-1 represents conditions in the ground test. Damping coefficient $c_r$ in the ground test is usually much larger than it in the flight is. Parameter values of Case-1 were set referring previous development of the gondola system[6]. Case-2 and Case-3 represent flight situations with parameter
variation of $k_r$. The suspension rope tends to become rigid because of low-temperature environment at the flight altitude. On the other hand, torsional spring coefficient varies by total weight of suspended gondola. Robustness for variation of $k_r$ is necessary to conduct azimuthal-attitude control successfully under various situations. Here, variational-range of $k_r$ were defined as Case-2 and Case-3 based on the previous study of azimuthal-control system[4].

3.3. **Limitation of the control device**

Limitation value of the rotational speed of the motor was set as:

$$|\theta_m| \leq 40 \text{ deg/s}$$

4. **Study of the control system design utilizing an optimal-servo controller**

Firstly, applying an optimal servo control on the azimuthal-attitude control system was considered. In this chapter, design process of the controller will be introduced briefly, and its validation by numerical simulation will be presented.

4.1. **Control law**

The optimal-servo controller to regulate the azimuthal angle $\theta_g$ according to the target value $\theta_g^*$ was designed. An augmented system with an integral element is defined as:

$$\begin{align*}
    e &= \theta_g^* - \theta_g, \\
    \omega &= \int_0^t e \, dt, \\
    \ddot{\theta} &= u - u_\infty, \\
    \ddot{x} &= \omega - \omega_\infty, \\
    \begin{bmatrix} \dot{x} \\ \dot{\omega} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ \omega \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \ddot{u}, \\
    e &= \begin{bmatrix} -C & 0 \end{bmatrix} \begin{bmatrix} x \\ \omega \end{bmatrix},
\end{align*}$$

(11)

where, $x_\infty$, $u_\infty$ and $\omega_\infty$ are steady-state value of $x$, $u$, and $\omega$, respectively. Servo controller can be constructed by design an optimal regulator using this augmented system.

A performance index $J_a$ including design parameters $Q_{11}$, $Q_{22}$, $R_e$ is represented as:

$$J_a = \int_0^\infty \left( e(t)^T Q_{11} e(t) + \ddot{\theta}(t)^T Q_{22} \ddot{\theta}(t) + \ddot{\theta}(t)^T R_e \ddot{\theta}(t) \right) dt$$

(12)

Optimal servo controller can be designed by solving $\ddot{u}$ which minimize the performance index $J_a$. The concrete derivation process is introduced in the reference[7].

4.2. **Numerical simulation**

Numerical-simulation studies were performed to validate the designed control system in four cases; Nominal case and Case-1~3 shown in previous chapter. In these studies, target values of azimuthal-attitude control were set as 30 degrees with reference to the initial position, and rotational status of the balloon envelop $\theta_b$ and $\theta_b$ were set as zero. Numerical-simulation environment was built as a continuous-time system. Design parameters $Q_{11}$, $Q_{22}$ and $R_e$ must be tuned properly through trial and error process. Here, these parameters were tuned as $Q_{11} = 50$, $Q_{22} = 5$, and $R_e = 5$ in order to achieve higher responsiveness as much as possible with avoiding output saturation of the motor.

Firstly, controller was designed using parameter values of the ground test environment shown as Case-1. Figure 2 shows the simulation results (Simulation-1). Left one and right one indicates the history of the gondola azimuthal angle, and rotational speed of the motor, respectively. In this simulation, attitude control of Case-2 became unstable. At the flight altitude, control becomes easily unstable because of low damping coefficient of gondola dynamics caused by low density of the atmosphere. Therefore, it is possible that controllers optimized for ground-test condition induce undesirable response in the flight. Next, controller was designed applying the model parameters of nominal model. Figure 3 shows the simulation result (Simulation-2). The controller could not stabilize attitude control of Case-2. As this result indicates, a low torsional spring coefficient (it means that low rigidity of the suspension
A series of simulation studies confirmed that there is difficulty to achieve sufficient robustness against parameter variation with only the optimal-servo control.

5. Study of applying a sliding-mode controller

This chapter will describe a validation about design of azimuthal-attitude control system with applying a sliding-mode control (SMC)[8]; it is one of non-linear control methods which has a potential to achieve enough robustness against errors of model parameters of the control system.

5.1. Control law
In this research, SMC for the augmented system shown as in equation (11) was considered. SMC system includes two control inputs; a liner control input for obtaining desirable control characteristics on a switching hyperplane, and a non-liner control input for keeping the system state on the switching surface. An equivalent control input $u_{eq}$ in sliding mode state can be represented with a switching hyperplane matrix $S$ as:

$$\sigma = Sx,$$

$$u_{eq} = -(SB)^{-1}(SAx + Sqr),$$  

(13)

where, $\sigma$ is a switching function. Here, in order to design $S$, a deriving method giving an arbitrary stability margin was applied. The matrix $S$ can be determined by solving a Riccati equation shown as:

$$PA' + A'^TP - PBB'TP + Q = 0, \quad S = B'TP,$$

$$A' = A + \varepsilon I, \quad \varepsilon \geq 0,$$

(14)

where, $Q$ was set as $Q = I$. $\varepsilon$ is a design parameter indicating an arbitrary stability margin. Control input $u$ of SMC can be expressed with the liner input $u_l$ and non-liner input $u_{nl}$ as:

$$u = u_l + u_{nl} = -(SB)^{-1}(SAx + Sqr) - k(SB)^{-1}\frac{\sigma}{|\sigma|},$$

(15)

where, $u_l$ is equal to $u_{eq}$, and $u_{nl}$ is a non-liner control input. If a Lyapunov function $V$ achieving $\sigma \to 0$ is defined as:

$$V = \frac{1}{2} \sigma^2,$$

(16)

the system can reach the sliding mode by using the control gain $k$ which is satisfied with sufficient condition of $\dot{V} < 0$. In this research, a smoothing function was applied on the non-liner input for reduction of chattering of the control input as:

$$u_{nl} = k(SB)^{-1}\frac{\sigma}{|\sigma| + \eta},$$

(17)

where, $\eta$ is a parameter to adjust smoothing effect.

5.2. Numerical simulation

SMC for the azimuthal attitude control system was designed according to the process described in previous section. Parameter values of Case-3 were used as condition of the control system design. The stability margin $\varepsilon$ were chosen as 0.06 through trial and error to realize same followability of response as the designed control system in previous chapter. Gain value $k$ of the non-liner control was set to 50 to satisfy reaching condition of sliding mode against change of the model parameter. Smoothing parameter $\eta$ was set to 0.1.

Numerical-simulation study was performed to evaluate control characteristics of the designed control system. Target value of the control was set as 30 degrees, and rotational status of the balloon envelop $\theta_n$ and $\theta_b$ were set as zero in this simulation. Figure 5 presents the results of the simulation (Simulation-4) indicating the gondola azimuthal angle, rotational speed of the motor, and control inputs of SMC. As Figure 5 shows, azimuthal angle of the gondola converged successfully to the target value in all cases. The rotational speed of the motor also maintained enough stability in all cases. As the graph of $u_{nl}$ in Figure 5 shows, it was found that non-liner control functioned properly to compensate influences of the change of model parameters.

From these results, it was confirmed that SMC method is effective for azimuthal attitude control of the balloon-gondola which has variation of the parameter, and sufficient robustness can be realized certainly.

6. Conclusion
This paper reported consideration about the design of the azimuthal-attitude control system for stratospheric balloons using only a motorized pivot.

Firstly, the overview of the azimuthal-attitude control system was presented, and the derivation process of the numerical model was described. In this research, numerical model of the control system using angular velocity of the motor as a control input was constructed. Parameter conditions and control requirements for design of the control were defined based on the actual balloon-gondola systems. Characteristics variation of the suspension rope is predicted to influence the control performance seriously. Therefore, some simulation cases with different characteristic values of the suspension rope were defined considering the ground-test situation and flight environment.

Next, azimuthal-attitude controllers were designed and verified. Optimal-servo control was applied on the control system, and its control performance was confirmed through some numerical-simulation studies. This study revealed that it is difficult to maintain robustness against change of the parameters with only an optimal-servo controller. After that, applying SMC was considered. Designed control system with SMC was also validated by numerical-simulation study. It was confirmed that SMC realize enough robustness for variation of the characteristics of the suspension rope. From this research, the azimuthal-control system with SMC could be proposed, and the basic information about its availability was obtained.

References
[1] Kremic T, Hibbits K, Young E, Landis R, Noll K and Baines K 2013 Proc. IEEE. Aerosp. Conf. 1-8.
[2] Chingcuanco A, Lubin P, Meinhold P and Tomizuka M 1990 Dyn. Sys. Meas. Control. 112(4) 703-10.
[3] Lecinski A, Card G, Knöller M and Hardy B 2017 J. Astro. Instru. 6(2) 1-15.
[4] Bando N, Fuke H, Shoji Y, Doetinchem P, Hailey C, Sakai S and Hashimoto T 2015 Aerosp. Tech. Japan., JSASS 14 59-65.
[5] Matko D, Yajima N and Hinada M 1996 Rep.ISAS. 665 1-53.
[6] Shoji Y, Taguchi M, Nakano T, Maeda A, Imai M, Gouda Y, Watanabe M, Takahashi Y, Sakamoto Y and Yoshida K 2016 Trans. JSASS. 14(ists30) 95-102.
[7] Ikeda M and Suda N 1988 Trans. SICE. 24(1) 40-46.
[8] Nonami K and Den K 1994 *Sliding Mode Control* (Tokyo: Corona Publishing Co., Ltd.)