Eight Wave Mixing Process is suitable to make Single Photon Source for Quantum Cryptography

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Abstract. Single photon source for use in quantum cryptography should show higher order photon antibunching. We have analyzed non classical effect i.e. higher order photon antibunching in pump mode of eight-wave mixing non linear optical process using short time interaction technique and also the dependence of non classicality on the number of photons prior to interaction in non linear medium. Further we have reported that higher the order of photon antibunching, higher will be the probability to generate single photon source utilized in quantum cryptography.

Keywords: non-classicality, higher-order photon antibunching, quantum optics.

1. Introduction

It is conceivable to depict a state as a non classical state that doesn’t have any classical analogue and generally negativity of Glauber Sudarshan P function explains the non classical state [1,2]. In particular, photon antibunching and squeezing have no classical equivalent [3,4]. Higher order squeezing [5-8] is one of these non-classical phenomena that has been well investigated, while higher order photon antibunching (HOA) has not been studied precisely. Perhaps the most important prerequisite for quantum information theory is a single photon source. Photon antibunching [9-15] is a non classical feature of radiation field that may be utilized to create single photon source [16,17]. Lee proposed the notion of HOA [18], which has been seen in the trio coherent state [19] and in several nonlinear optical processes [20,21].

The interest in non-classical states have been concentrated in latest past with the advent of numerous advanced applications in quantum information processing like quantum key distribution [22], quantum cryptography [23,24], quantum teleportation [25,26], dense-coding [27,28]. As a result, investigations of non-classical characteristics of the radiation field are very significant in the circumstances of new developing subject of the quantum information theory.

Earlier the authors have studied the non classical effect i.e. photon antibunching up to second order in certain non linear optical processes [29]. In the present study, we will look into higher order photon antibunching and the dependence of the photon antibunching on photon number has also
been studied. Section 2, underscoring the criteria of non-classicality of the non linear optical system. Besides, section 3 gives a second order solution of equation of motion of an eight wave mixing non linear optical process and shows the occurrence of HOA. In section 4 we have analyzed the results obtained using graphs and section 5 underscore on end.

2. Conditions of non-classicality i.e. antibunching of a non linear optical system

2.1. Condition for Higher order photon antibunching (HOA).

“HOA models was found by Lee” [18], which is

\[ R(m,l) = \frac{\langle N_{x}^{m-1} \rangle \langle N_{x}^{l+1} \rangle}{\langle N_{x}^{m} \rangle \langle N_{x}^{l} \rangle} - 1 < 0 \]  

(1)

where the number operator is addressed by \( N \). \( \langle N^{(y)} \rangle = (N(N-1)(N-2)...........(N-y+1)) \), \( N^{(y)} \) is \( y^{th} \) factorial moment of the number operator. The two integers \((l)\) and \((m)\) fulfilling the condition \( l \leq m \leq 1 \) and addendum \( x \) connotes the specific mode. “\( m = 1 \) is choosen by Ba An” [19]. The criterion of \( l^{th} \) order photon antibunching is given as

\[ A_{x,l} = \frac{\langle N_{x}^{l+1} \rangle}{\langle N_{x}^{l} \rangle \langle N_{x} \rangle} - 1 < 0 \]  

(2)

and

\[ \langle N_{x}^{l+1} \rangle < \langle N_{x}^{l} \rangle \langle N_{x} \rangle \]  

(3)

A state that is antibunched in the higher order must be antibunched in the lower order physically. As a result, equation (3) may be simplified as follows:

\[ \langle N_{x}^{l+1} \rangle < \langle N_{x}^{l} \rangle \langle N_{x} \rangle < \langle N_{x}^{l-1} \rangle \langle N_{x} \rangle \]  

\[ < \langle N_{x}^{l-2} \rangle \langle N_{x} \rangle \]  

\[ < \langle N_{x}^{l-3} \rangle \langle N_{x} \rangle \]  

\[ < \langle N_{x}^{l-4} \rangle \langle N_{x} \rangle \]  

\[ ............\langle N_{x} \rangle^{\langle l^{th} \rangle} \]  

(4)

by disentangling equation (3) and acquiring criteria for \( l^{th} \) order photon antibunching, which is

\[ d(l) = \langle N_{x}^{l+1} \rangle - \langle N_{x}^{l} \rangle^{\langle l^{th} \rangle} < 0 \]  

(5)

Accordingly, in quantum cryptography, we can say that single photon source to be utilized must fulfill the criteria given in equation (5) of HOA.

3. Eight wave mixing process

The above process can be used to examine HOA in such a manner that three photons of frequency \( \omega_{1} \) are absorbed with four photons of frequency \( \omega_{2} \) and one photon of frequency \( \omega_{3} \) being expelled. For this form, the Hamiltonian is given as

\[ H = \omega_{1} a^{\dagger} a + \omega_{2} b^{\dagger} b + \omega_{3} c^{\dagger} c + g(a^{\dagger}b^{\dagger}c^{\dagger}+a^{\dagger}b^{\dagger}c^{\dagger}) \]  

(6)

where coupling constant is addressed by \( g \), annihilation (creation) operators are \( a \) \( a^{\dagger} \), \( b \) \( b^{\dagger} \), \( c \) \( c^{\dagger} \), respectively. \( A = a \exp i\omega_{1}t \), \( B = b \exp i\omega_{2}t \), \( C = c \exp i\omega_{3}t \) are the gradually varying operators wavering at frequencies \( \omega_{1} \), \( \omega_{2} \) and \( \omega_{3} \).

3.1. Time evolution of pump mode \( A \).

Time evolution of operator \( A \) in pump mode is described as
\[
\frac{dA}{dt} = \frac{\partial A}{\partial t} + i[H, A]
\]
we have got
\[
\dot{A} = -3igA^2B^4C
\]
(8)
\[
\dot{B} = -4igA^3B^{-3}C^6
\]
(9)
and
\[
\dot{C} = -igA^4B^{-4}
\]
(10)
by applying short time approximation, expanding \( A(t) \) in accordance with Taylor series and acquiring the terms up to second order as
\[
A(t) = A - 3igtA^2B^4C + \frac{3}{2}g^2t^2(6A^2A^2N_b^4N_c^4 + 6AN_b^4N_c^4 - 16A^2A^2N_b^3N_c^3 - 72A^2A^3N_b^2N_c^3 - 96A^2A^3N_bN_c^4 - 24A^2A^3N_b^4N_c^4 - 16A^2A^3N_b^3N_c^3 - 72A^2A^3N_b^2N_c^3 - 96A^2A^3N_bN_c^4 - 24A^2A^3N_c^4)
\]
(11)
using equation (11), number operator \( N_A(t) \) is given as
\[
N_A(t) = A^A - 3igt(A^3B^4C - A^3B^{-4}C^4) + 3g^2t^2(9A^2A^2N_b^4N_c^4 + 18A^2A^4N_b^4N_c^4 + 6B^4B^{-4}N_c^4 - 16A^3A^3N_b^3N_c^3 - 72A^3A^3N_b^2N_c^3 - 96A^3A^3N_bN_c^4 - 24A^3A^3N_b^4N_c^4 - 72A^3A^3N_b^3N_c^3 - 96A^3A^3N_b^2N_c^3 - 24A^3A^3N_c^4)
\]
(12)
To examine photon antibunching, at first we consider a quantum state \( |\psi \rangle \) as
\[
|\psi \rangle = |\alpha \rangle_A |0 \rangle_B |0 \rangle_C
\]
(13)
where A mode is in coherent state i.e. \( |\alpha \rangle \) and B and C mode are in vacuum state \( |0 \rangle \).
using equations (12) and (13) expectation value of photons number in pump mode A might be communicated as
\[
\langle N_A(t) \rangle = |\alpha|^2 - 72g^2t^2|\alpha|^6
\]
(14)
where \( A|\alpha \rangle = \alpha |\alpha \rangle \). And value of \( < N_A^4(t) > \) and \( < N_A^5(t) > \) is given as
\[
\langle N_A^4(t) \rangle = |\alpha|^8 - 288g^2t^2(|\alpha|^4 + 3|\alpha|^2 + 2|\alpha|^6)
\]
(15)
and
\[
\langle N_A^5(t) \rangle = |\alpha|^{10} - 360g^2t^2(|\alpha|^4 + 4|\alpha|^2 + 4|\alpha|^{10})
\]
(16)
now using equations (14-16) in equation (5), a direct however strenuous calculation yields
\[
d_A(3) = -576g^2t^2(|\alpha|^6 + |\alpha|^8)
\]
(17)
and
\[
d_A(4) = -288g^2t^2(2|\alpha|^4 + 3|\alpha|^{10})
\]
(18)
equations (17) and (18) fulfill the criteria of higher order photon antibunching.
Moreover, to examine higher order photon antibunching, we have now taken the initial state \( |\psi \rangle \) as
\[
|\psi \rangle = |0 \rangle_A |\beta \rangle_B |0 \rangle_C
\]
(19)
where A and C mode are in vacuum state i.e. \( |0 \rangle \) and B mode is in quantum state \( |\beta \rangle \).
presently taking the expectation values of $N_A(t)$, $N_A^4(t)$ and $N_A^5(t)$ in pump mode A corresponding to condition (19) is

$$\langle N_A(t) \rangle_\beta = 0 \quad \text{(20)}$$
$$\langle N_A^4(t) \rangle_\beta = 0 \quad \text{(21)}$$
$$\langle N_A^5(t) \rangle_\beta = 0 \quad \text{(22)}$$

along these lines third and fourth order photon antibunching in mode A as for $|0\rangle|\beta\rangle|0\rangle$ is given as

$$d_A(3)_\beta = 0 \quad \text{(23)}$$
$$d_A(4)_\beta = 0 \quad \text{(24)}$$

Additionally we can contemplate the non classical effect (HOA) by assuming a quantum state $|\psi\rangle$ as

$$|\psi\rangle = |0\rangle_A |0\rangle_B |\gamma\rangle_C \quad \text{(25)}$$

where A and B mode are in vacuum state $|0\rangle$ and C mode is in quantum state i.e. $|\gamma\rangle$.

now taking expectation values of $N_A(t)$, $N_A^4(t)$ and $N_A^5(t)$ in pump mode A in relation to condition (25) is

$$\langle N_A(t) \rangle_\gamma = 0 \quad \text{(26)}$$
$$\langle N_A^4(t) \rangle_\gamma = 0 \quad \text{(27)}$$
$$\langle N_A^5(t) \rangle_\gamma = 0 \quad \text{(28)}$$

Therefore third and fourth order photon antibunching in mode A with respect to $|0\rangle|0\rangle|\gamma\rangle$ is given as

$$d_A(3)_\gamma = 0 \quad \text{(29)}$$
$$d_A(4)_\gamma = 0 \quad \text{(30)}$$

from equations (23, 24) and (29, 30) we can say that neither third order photon antibunching nor fourth order photon antibunching is existing in mode A with respect to $|0\rangle|\beta\rangle|0\rangle$ and $|0\rangle|0\rangle|\gamma\rangle$ respectively.

4. Result
The presence of third and fourth order photon antibunching is shown in equations (17) and (18) respectively. Taking $g^2t^2 = 10^{-4}$, the variations of third order photon antibunching as shown in equation (17) and fourth order photon antibunching as shown in equation (18) as a function of photons number in mode A i.e. $|\alpha|^2$ are shown in figures 1 and 2. It is obvious from figures that HOA increases non linearly with $|\alpha|^2$. This affirms that the antibunched state are related with the photons number in pump mode A i.e. $|\alpha|^2$. Further it is evident from the figures that non classicality increases as we go towards the higher order of photon antibunching.
Figure 1. Dependence of third order photon antibunching $d(3)$ on $|\alpha|^2$ (taking $g^2 = 10^{-4}$).

Figure 2. Dependence of fourth order photon antibunching $d(4)$ on $|\alpha|^2$ (taking $g^2 = 10^{-4}$).

5. Conclusion
We have reported non classicality in terms of HOA in pump mode of eight wave mixing non linear optical process. We have found that third and fourth order photon antibunching straightforwardly depends on the number of pump photons existing in the system and non classicality of a system increases as we move towards the higher order of photon antibunching. Thus we can conclude that fourth order photon antibunching is more suitable to generate single photon source utilized in quantum cryptography as compared to lower order photon antibunching.

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