A Way to Reopen the Window for Electroweak Baryogenesis

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Abstract

We reanalyse the sphaleron bound of electroweak baryogenesis when allowing deviations to the Friedmann equation. These modifications are well motivated in the context of brane cosmology where they appear without being in conflict with major experimental constraints on four-dimensional gravity. While suppressed at the time of nucleosynthesis, these corrections can dominate at the time of the electroweak phase transition and in certain cases provide the amount of expansion needed to freeze out the baryon asymmetry without requiring a strongly first order phase transition. The sphaleron bound is substantially weakened and can even disappear so that the constraints on the higgs and stop masses do not apply anymore. Such modification of cosmology at early times therefore opens the parameter space allowing electroweak baryogenesis which had been reduced substantially given the new bound on the higgs mass imposed by LEP. In contrast with previous attempts to turn around the sphaleron bound using alternative cosmologies, we are still considering that the electroweak phase transition takes place in a radiation dominated universe. The universe is expanding fast because of the modification of the Friedmann equation itself without the need for a scalar field and therefore evading the problem of the decay of this scalar field after the completion of the phase transition and the risk that its release of entropy dilutes the baryon asymmetry produced at the transition.

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1 Introduction

Since the appearance of the article by Kuzmin, Rubakov and Shaposhnikov in 1985 [1] who suggested that baryon number violation at high temperature in the Standard Model might have played a role in the early universe and set the basis for the theory of electroweak baryogenesis, there have been extensive studies which explored with great scrutiny the details of the mechanism and its viability both in the Standard Model and in the Minimal Supersymmetric Standard Model. The fact that the creation of the baryon asymmetry of the universe is in principle possible in the standard electroweak theory is indeed very appealing since it relies only on physics accessible at present colliders in contrast with GUT or Affleck–Dine scenarios. Electroweak physics is not though the only ingredient. Electroweak baryogenesis makes use of an intricate interplay between particle physics and cosmology which is much more involved than in the theory of nucleosynthesis. Let us summarize very briefly the main idea: In a universe with conserved vanishing $B-L$ charge and standard Friedmann–Robertson–Walker (FRW) cosmology, the baryon asymmetry of the universe (BAU) can only be produced at the electroweak phase transition (EWPT). To fulfill Sakharov’s out-of-equilibrium requirement for baryogenesis [2] the phase transition has to be first order. In that case, bubbles of non zero higgs field $vev$ nucleate from the symmetric vacuum and as they expand, particles in the plasma interact with the moving phase interface (bubble wall) in a CP violating way producing a CP asymmetric flux of particles into the symmetric phase. This CP asymmetry is converted into a baryon asymmetry by baryon number violating sphaleron processes in front of the wall. As the phase interface propagates, the baryons enter into the broken phase where baryon violation is inefficient and therefore baryon asymmetry frozen out [3].

To compute the resulting net baryon asymmetry produced during the electroweak phase transition one needs to know: (1) The details of the non trivial baryon number violating processes at high temperature (B violation is achieved thanks to sphaleron transitions interpolating between two neighboring N-vacua of the electroweak gauge theory). (2) The amount of departure from equilibrium at the electroweak phase transition. This necessitates an accurate calculation of the higgs effective potential at high temperature. (3) The generation and propagation of CP-violating particle fluxes. (1) is now well under control. The rate of B-violating sphaleron processes at high temperature has been performed accurately and checked independently using analytic perturbative and non perturbative techniques both in the broken phase [4–6] and in the symmetric phase [7, 8]. (2) has also been quite well understood. It is at this point that cosmology enters into the game when comparing B violation rate with the expansion rate of the universe. One of the strongest constraint in electroweak baryogenesis comes from the requirement that baryons produced at the bubble wall are not washed out by sphaleron processes after they enter the broken phase. Imposing that sphaleron processes are sufficiently suppressed in the broken phase at the critical temperature leads to the so-called sphaleron bound: $\langle \phi(T_C) \rangle / T_C \gtrsim 1$. $\langle \phi(T_C) \rangle$ is the order parameter of the phase transition, the $vev$ of the higgs field at the critical temperature. The derivation of this bound uses the expression of the expansion rate of the universe $H$ given by the Friedmann equation evaluated in a
radiation dominated universe. \( \langle \phi(T_c) \rangle / T_c \) on the other hand is given by the analysis of the effective scalar potential at high temperature and depends sensitively on the higgs mass so that the sphaleron bound translates into a constraint on the higgs mass. It has been shown that in the Standard Model the phase transition is not first order but just a cross over for a higgs mass above \( \approx 72 \text{ GeV} \) so that electroweak baryogenesis is excluded given the present experimental lower limit of the Standard Model higgs mass. Another reason in favour of that conclusion is related to our third point (3): CP violation in the Standard Model is not enough to account for the baryon asymmetry of the universe \[11\]. Consequently, efforts have concentrated on the Minimal Supersymmetric Standard Model which provides not only the possibility of achieving a first order phase transition but also additional sources of CP violation \[11\]. While there is a common agreement on the identification of the major CP violating sources—the complex parameters \( \mu \) and \( M_2 \) in the mass matrix of the charginos— the result of the calculation of CP violating currents on the other hand remains an open question \[12–15\]. We will not consider this issue here and assume that there are sufficient sources of CP violation in the MSSM to account for the baryon asymmetry of the universe so that the strongest constraint on electroweak baryogenesis essentially comes from the sphaleron bound.

We will therefore focus now on point (2) and on the condition \( \langle \phi(T_c) \rangle / T_c \gtrsim 1 \). \( \langle \phi(T_c) \rangle / T_c \) measures the strength of the phase transition and depends crucially on the presence of a light stop \( \tilde{t}_R \) (with small left-right mass mixing parameter \( \tilde{A}_t \)). It is difficult to compute since it is radiatively induced and perturbation theory is expected to break down at high temperature. Nevertheless, the validity of perturbation theory and results at two loop level \[16, 17\] have been confirmed by lattice calculation \[18\] which enhance \( \langle \phi(T_c) \rangle / T_c \) by \( 10–15\% \) compared with perturbative calculation. From the last results \[19\] one can infer the constraints imposed by the sphaleron bound:

\[
110 \text{ GeV} \lesssim m_h \lesssim 115 \text{ GeV} \quad \text{and} \quad 105 \text{ GeV} \lesssim m_{\tilde{t}_R} \lesssim 165 \text{ GeV} \quad (1)
\]

As the LEP bound on the higgs mass has been pushed up to \( m_h \gtrsim 115 \text{ GeV} \), the window \[1\] for electroweak baryogenesis has been seriously reduced. A large value for the higgs mass requires in particular a large stop mixing parameter \( \tilde{A}_t \). On the other hand a large \( \tilde{A}_t \) makes the phase transition weaker and leads to values of \( \langle \phi(T_c) \rangle / T_c \) in conflict with the condition of preservation of the baryon asymmetry.

In summary, given the last bound on the higgs mass, electroweak baryogenesis in the MSSM starts to be in bad condition. However, if it turns out that the window \[1\] is ruled out by experiment shall we definitely claim the death of electroweak baryogenesis? Adding new particle physics ingredients to enhance the phase transition would start to be arbitrary and electroweak baryogenesis would lose its main advantage over other baryogenesis mechanisms that it relies on a well motivated and well defined particle physics framework. If there is no reasonable way out on the particle physics side and if one still wants to believe that the baryon asymmetry of the universe was created at the electroweak phase transition the only way to turn around the sphaleron bound is to modify cosmology and question the common assumption made on the thermal history of the universe. There have been only a few attempts in the past to study the possibility of relaxing the sphaleron...
bound using alternative cosmologies \cite{20, 22}. An interesting discussion has been offered in \cite{21}. All these works have in common the fact that they modify the usual assumption of a radiation dominated universe by assuming the existence of a scalar field driving the fast expansion of the universe at the EWPT. One drawback with this approach is that the decay of this scalar field after the completion of the phase transition may release entropy and dilute the baryon asymmetry produced at the transition.

In this paper we take a different point of view and discuss a new cosmological solution to weaken the sphaleron bound. We still assume radiation domination at the EWPT. Instead, what we modify is the Friedmann equation itself in a way motivated by results in brane cosmology. While the usual expansion rate is recovered at late times in particular at nucleosynthesis, the expansion can be accelerated at high temperature with major consequences for electroweak baryogenesis as already suggested in \cite{24}. In section II we present the derivation of the sphaleron bound and discuss how modifications of common assumptions on the cosmological evolution could substantially weaken that bound. In section III we motivate our parametrization of deviations to Friedmann equation by reviewing generic aspects of brane cosmology and discuss under which general conditions these deviations may play a significant role at the time of the EWPT. Section IV makes some comments on the cosmology with a non standard Friedmann equation. In section V we exhibit our numerical results describing the parameter space allowing the preservation of the baryon asymmetry.

2 Derivation of the sphaleron bound

To derive the sphaleron bound, one starts with the “master equation” of electroweak baryogenesis which gives the rate at which baryon number relaxes to its equilibrium value in the presence of fermions in the hot plasma \cite{25}:

$$\frac{\partial n_{\text{com}}^B}{\partial t} = -V_B(t)n_{\text{com}}^B$$  \hspace{1cm} (2)

$$V_B(t) = M \frac{\Gamma_s}{T^3}$$ \hspace{1cm} (3)

$n_{\text{com}}^B$ is the density of baryons per comoving volume in an expanding universe, $n_{\text{com}}^B = a(t)^3n_B$, where $a(t)$ is the scale factor of the universe. $V_B(t)$ is the rate of the baryon number non conserving processes. $M$ is related to the number of fermionic degrees of freedom at thermal equilibrium which bias the free energy of the $SU(2)$ vacuum. In the Standard Model, $M = 13N_f/2$, if one assumes that all fermionic degrees of freedom (including $e_R$) are at thermal equilibrium. In the MSSM, $M = 15N_f/2$ \cite{12, 14}. The sphaleron rate, $\Gamma_s$, is the rate per unit time and unit volume of fluctuations with changing

\footnote{For exhaustiveness, let us mention another type of alternative: If the EWPT was preceded by a color and charge breaking phase, the ratio $\langle \phi(T_c) \rangle / T_c$ could be much larger than in the usual scenario. However, it has been shown that such two-stage EWPT is not cosmologically viable \cite{33}.}
of the topological number. In the broken phase, a perturbative analysis gives \[^4\]

\[
\Gamma_s \sim \eta \ T^4 \ \zeta^7 \ e^{-\zeta}
\]

where

\[
\zeta(T) = E_{sp}(T)/T \quad \text{and} \quad \eta = 2.8 \times 10^5 \left(\frac{\alpha_w}{4\pi}\right)^4 \kappa B^{-7}
\]

\(E_{sp}(T) = \sqrt{\frac{4\pi}{\alpha_w}} B \phi(T_c)\) is the energy of the sphaleron configuration \[^{29,30}\], in other words the energy barrier between two adjacent topologically non equivalent vacua of the SU(2) broken phase. \(B(\lambda/g^2)\) is evaluated numerically and is a monotonic slowly varying function of the higgs mass: In the Standard Model, \(B(0) = 1.5 \rightarrow B(\infty) = 2.7\) \[^{30,31}\]. The value of \(\kappa\) has originally been assumed to lie in the range \(10^{-4} \lesssim \kappa \lesssim 10^{-1}\) \[^4\], however, recent non perturbative calculations \[^3\] have shown that it is reasonable to use the perturbative estimate \(^4\) provided that \(\kappa \sim \mathcal{O}(1)\).

Integrating \(^2\) between the time \(t_c\) at which the electroweak phase transition takes place and some time \(t\) leads to

\[
\frac{n_{B}^{\text{com}}(t)}{n_{B}^{\text{com}}(t_c)} = e^{-\int_{t_c}^{t} V_B(t')dt'} \equiv \mathcal{S}
\]

where we have introduced the dilution factor \(\mathcal{S}\). The most efficient dilution of the baryon asymmetry takes place just after the phase transition at a temperature near the critical temperature \(T_c\) so that it is not a bad approximation (we will check it shortly) to replace the integral in \(^3\) by the value of the integrand at \(T = T_c\):

\[
\frac{n_{B}^{\text{com}}(t_c)}{n_{B}^{\text{com}}(0)} = e^{-V_B(t_c)\Delta t}
\]

where \(\Delta t\) is the time interval corresponding to a range of temperatures \(\Delta T \sim T_c - T_2 = (130 - 110)\text{ GeV}\), \(T_2\) being the temperature at which dilution is inoperative. We note that

\[
n_{B}^{\text{com}} = a^3 n_B = s_{\text{com}} \frac{n_B}{s}
\]

where \(s_{\text{com}} = a^3 s\) is the entropy per comoving volume. In a radiation dominated universe \(s \propto g_*(T) T^3\). At this stage, we need the relation between time and temperature and that is where cosmology comes in. However, we want to remain as general as possible and will not assume any specific form of the Friedmann equation for the moment. We will only assume that the universe is expanding adiabatically, \(d(\rho a^3) = -pd(a^3)\) (\(p\) is the pressure), and that the universe is radiation dominated, leading to the relationship between the energy density \(\rho\) and the scale factor of the universe \(a(t)\): \(\rho(t) = \rho_0 (a(t)/a_0)^{-4}\) where \(\rho\) is given by

\[
\rho = \frac{\pi^2}{30} g_* T^4
\]
$g_*$ counts the number of degrees of freedom at thermal equilibrium. At $T \sim 100$ GeV, $g_* = 106.75$ in the Standard Model. From (4) we get $a(t) \propto \rho(T)^{-1/4} \propto g_*^{-1/4}(T)T^{-1}$ so that $s^{\text{com}}$ scales as $s^{\text{com}} \propto g_*^{1/4}(T)$. We can rewrite the dilution as

$$ S = \frac{n_B(t)}{n_B(t_c)} \left( \frac{g_*^2(T)}{g_*^2(T_c)} \right)^{1/4} $$

(10)

$n_B/s$ measures the baryon asymmetry. The value of the asymmetry today deduced from nucleosynthesis is $1.7 \times 10^{-11} \lesssim n_B/s \lesssim 8.9 \times 10^{-11}$ so that

$$ S \sim \frac{10^{-11}}{n_B(t_c)} $$

(11)

The value of the baryon asymmetry which can be produced at the EWPT, $n_B(t_c)$, is still an open question [12–15], however, it is generically difficult to produce a large asymmetry and one can reasonably write the bound $n_B(t_c)/s(t_c) \lesssim 10^{-6}$ which means that for the baryon asymmetry not to be washed out, one demands $S \gtrsim 10^{-5}$. This leads to

$$ \ln \left( \frac{M \eta}{5 \ln 10} \right) + 7 \ln \zeta(T_c) - \zeta(T_c) + \ln(T_c \Delta t) \lesssim 0 $$

(12)

Note that equation (12) is general and does not depend on the precise form of the Friedmann equation. It only relies on the adiabaticity of the expansion and on the radiation domination assumptions. It is when replacing $\Delta t$ by its expression in terms of the temperature that we make use of a particular Friedmann equation. Let us now express this condition using the standard Friedmann equation:

$$ H = \left( \frac{\dot{a}}{a} \right) = \sqrt{\frac{\rho(T)}{3 m_{Pl}^2}} $$

(13)

$m_{Pl} = M_{Pl}/\sqrt{8\pi}$ is the reduced Planck mass in four dimensions. This leads to

$$ \rho(t) = \rho_0 \left( 1 + \sqrt{\frac{4\rho_0}{3 m_{Pl}^2} t} \right)^{-2} \approx \frac{3 m_{Pl}^2}{4 t^2} $$

(14)

where the last approximation applies if $\rho_0/\rho(t) \gg 1$ where $\rho_0$ is the energy density at some early time origin. This gives us the relation between $t$ and $T$

$$ t = \frac{3}{2\pi} \sqrt{\frac{10 m_{Pl}}{g_* T^2}} \approx 0.14 \frac{m_{Pl}}{T^2} $$

(15)

and $\Delta t = t_c(T_c^2/T_2^2 - 1) \sim 0.4 t_c$. Using (12) evaluated at $T_c$ and solving (12) numerically with $\kappa = 1/2$, $\alpha_w = 0.0336$, $B = 1.94$, $T_c = 130$ GeV, we get the sphaleron bound:

$$ \zeta(T_c) \gtrsim 45.5 \quad \rightarrow \quad \frac{\phi(T_c)}{T_c} \gtrsim 1.21 $$

(16)
where the second inequality was obtained using the fact that \( \zeta(T_c) \) is related to the order parameter \( \phi(T_c) \) of the phase transition by:

\[
\zeta(T_c) = \sqrt{\frac{4\pi}{\alpha_w}} B \frac{\phi(T_c)}{T_c}
\]  

(17)

We can now check that with a more accurate evaluation of the integral in (15), equation (12) is modified into

\[
\ln \left( \frac{2M_\eta}{5 \ln 10} \right) + 6 \ln \zeta(T_c) - \zeta(T_c) + \ln(T_c t_c) \lesssim 0
\]  

(18)

weakening the sphaleron bound slightly: \( \zeta_c \gtrsim 42.9 \rightarrow \phi_c / T_c \gtrsim 1.14. \)

### 2.1 Particle physics constraints from the sphaleron bound.

As stated in the introduction, the relation (16) is usually translated into a bound on the higgs mass. Indeed, \( \phi(T_c) / T_c \) can be computed precisely from an analysis of the effective potential of the higgs at high temperature. For instance, a simple perturbative evaluation at one loop gives (with no daisy resummation) in the Standard Model [32]

\[
\left( \frac{\phi(T_c)}{T_c} \right)_{\text{SM}} \approx \frac{2(2m^3_W + m^3_Z)}{3m^2_h \times \pi v}
\]  

(19)

Plugging this expression into (16) leads to \( m_h \lesssim 37 \text{ GeV} \). As mentioned in the introduction, more precise calculations show that in the standard Model there is no first order phase transition at all for higgs masses \( m_h \gtrsim 72 \text{ GeV} \). To make the phase transition first order, new bosonic degrees of freedom with large couplings to the higgs are needed. The MSSM precisely provides these, in particular, the right stop \( \tilde{t}_R \) with large Yukawa coupling can significantly enhance the strength of the phase transition [17, 33, 34]. Next formula (20) is an expression for \( \phi(T_c) / T_c \); it is not rigorously correct because it corresponds to a one-loop calculation and also to the ideal special case where \( m^2_U + c_s T^2 \approx 0 \) which means that the high temperature correction to the right stop mass \( c_s T^2 \) is compensated by its negative soft mass parameter \( m^2_U \). However, it is helpful to get a flavor of what controls the strength of the EWPT in the MSSM:

\[
\left( \frac{\phi(T_c)}{T_c} \right)_{\text{MSSM}} \sim \left( \frac{\phi(T_c)}{T_c} \right)_{\text{SM}} + \frac{2m^3_t}{\pi v m^2_h} \left( 1 - \frac{\tilde{A}_t}{m^2_Q} \right)^{3/2}
\]  

\[
\sim \left[ \frac{11 m_L}{m_h} \right]^2 \text{if } |\tilde{A}_t| \ll m_Q
\]

\[
I = \int_{t_c}^{T} V_B(t') dt' = 2M_\eta \int_{t_c}^{T} \int_{\zeta_c}^{\infty} \frac{d\zeta^7 e^{-\zeta}}{\zeta_2} \text{ where we treated } \phi(T) \text{ as a constant (} \phi(T) \text{ changes slowly with } T) \text{, such approximation overestimates slightly the wash out. We noted } \zeta_c = \zeta(T_c). \text{ With } \int_{\zeta_c}^{\infty} d\zeta^7 e^{-\zeta} = \sum_{n=0}^{\infty} \frac{1}{n!} \zeta^7 e^{-\zeta} \approx \zeta^7 e^{-\zeta_c} \text{ (} \zeta_c \gtrsim 10 \text{), we get } I = 2M_\eta t_c \zeta^6 e^{-\zeta_c} = 2V_B(t_c) t_c / \zeta_c.
\]
Even if this one-loop evaluation underestimates $\phi(T_c)/T_c$ compared with lattice calculation we see from this expression that the condition of preservation of the baryon asymmetry sets serious constraints on $m_h, \tilde{A}_t/m_Q$ ($m_Q$ is the soft mass parameter of the left stop $\tilde{t}_L$) as well as $m_{\tilde{U}}^2$ (and therefore $m_{\tilde{t}_R}$). It is also instructive to keep in mind the one-loop expression for the Higgs mass (in the usual limit where the mass $m_A$ of the CP-odd neutral Higgs boson is large):

$$m_h^2 = M_Z^2 \cos^2 2\beta + 3 m_t^4 \frac{m_{\tilde{t}_R}^2 m_{\tilde{t}_L}^2}{m_t^4} \log \left( \frac{m_{\tilde{t}_R}^2 m_{\tilde{t}_L}^2}{m_t^4} \right) \left[ 1 + \mathcal{O} \left( \frac{\tilde{A}_t^2}{m_Q^2} \right) \right]$$ \hspace{1cm} (21)

While small values of $\tilde{A}_t/m_Q \lesssim 250/1000$ are required for baryogenesis, large values of $\tilde{A}_t/m_Q \gtrsim 250/1000$ are required to satisfy the experimental bound on the higgs mass and are in conflict with the sphaleron bound.

The only alternative to reopen the window for electroweak baryogenesis is to weaken the sphaleron bound by modifying the last term in the left hand side of inequality (18). For instance, to allow higgs masses in the range $115 \text{ GeV} \lesssim m_h \lesssim 118 \text{ GeV}$ with large $\tilde{A}_t/m_Q \sim 700/1000$ and unconstrained right stop mass (i.e. positive $m_{\tilde{U}}^2$ values) one typically gets $\phi(T_c)/T_c \sim 0.3$ corresponding to $\zeta(T_c) \sim 11.3$. Therefore one would need to weaken the sphaleron bound by a factor 4.

### 2.2 Evading the sphaleron bound.

Let us find out what value for the expansion rate would be needed at a temperature $T_c \sim 130 \text{ GeV}$ to preserve the baryon asymmetry and be consistent with the larger window for the higgs and stop masses as presented below i.e. when using the value $\zeta(T_c) \sim 11$. Replacing $\zeta(T_c) \sim 11$ into (12) leads to

$$\ln (T_c \Delta t) \lesssim 10.1$$ \hspace{1cm} (22)

In a radiation dominated era with the relation (15) we get

$$\ln \left( \frac{0.4T}{2H} \right) \lesssim 10.1 \rightarrow H \gtrsim 10^{-3} \text{GeV}$$ \hspace{1cm} (23)

while we have $H \sim 3 \times 10^{-14} \text{ GeV}$ so $H$ is too small by 11 orders of magnitude at the EWPT.

Earlier proposals to weaken the sphaleron bound have been to relax the standard assumption of radiation domination at the EWPT and assume instead a universe dominated by the energy density of a scalar field [21,22]. The main observational constraint is that such domination must terminate by the nucleosynthesis epoch. In [21,22], it was shown that the energy density in a kinetic mode of a scalar field (which scales like $a(t)^{-6}$, faster than in the radiation case $a(t)^{-4}$) can significantly weaken the sphaleron bound and that

\footnote{An ultimate alternative to both satisfy the bound on the higgs mass and be consistent with small values of $\tilde{A}_t/m_Q$ would be to push the left stop mass to 2 or 3 TeV instead of 1 TeV.}
such a phase of “kinetion” is required to evade large entropy release diluting the asymmetry. In [22] it was argued that a universe dominated by the energy density of a more conventional scalar field (not necessarily in a kinetic mode) could also evade the erasure condition if the reheat temperature was as low as \( T_r \sim \text{MeV} \). However, the baryogenesis mechanism would have to be much more efficient than in the standard case thus standard electroweak baryogenesis does not work in models with low reheat temperature and one would need to implement an Affleck–Dine mechanism for instance.

In this paper, we do not get rid of the radiation domination assumption and therefore do not have to assume any particular behaviour of a scalar field driving the expansion of the universe before nucleosynthesis. Our point of view is quite different. We are instead allowing deviations to the Friedmann equation itself at early times and study its consequences for electroweak baryogenesis. Let us assume for example that before nucleosynthesis the expansion rate behaved like

\[
H \sim \frac{\rho(T)}{M^3} \tag{24}
\]

where \( M \) is some scale to be discussed later. Such behaviour may \textit{a priori} seem \textit{ad hoc} but will be motivated in the next section. For a radiation dominated universe we get

\[
H \sim \frac{\pi^2 g_* T^4}{30 M^3} \tag{25}
\]

leading to

\[
\rho(t) \sim \rho_0 \left(1 + \frac{4 \rho_0 t}{M^3}\right)^{-1} \tag{26}
\]

and the relation between time and temperature is now

\[
t \sim \left(1 - \left(\frac{g_* T}{g_* (T_0) T_0}\right)^4\right) \frac{15}{2 \pi^2 g_* T^4} \frac{M^3}{T^4} \approx \frac{15}{2 \pi^2 g_*} \frac{M^3}{T^4} \tag{27}
\]

so that \( \Delta t = t_c \left(\frac{T_c^4}{T^4} - 1\right) \sim 0.95 t_c \). Equation (22) now translates into \( M \lesssim 20 \text{ TeV} \). Moreover, for \( M \lesssim 15 \text{ TeV} \), the condition (12) for the preservation of the baryon asymmetry is always satisfied whatever the value of \( \zeta \). Before concluding that the expansion rate (24) can successfully lower the sphaleron bound or even lead to the disappearance of that bound for sufficiently low scale \( M \) we have to check two more things: 1) that sphalerons are at equilibrium in the symmetric phase \textit{i.e.} that baryon number violation is still completely efficient outside the bubble, 2) that electroweak interactions are at equilibrium as well so that usual techniques to compute the baryon asymmetry continue to apply even in such rapidly expanding universe. 2) is actually automatically satisfied when 1) is satisfied since the sphaleron processes are the slowliest ones, being suppressed by a high power of

\[4\text{Weaker bounds on } M \text{ are obtained when evaluating more properly the integral (1) in the same way as equation (18) was obtained.}\]
α_w; so we have to compare the rate of baryon number violation in the symmetric phase \( \nu_B(t)^{sym} \) with the expansion rate \( H \) and require:

\[
\mathcal{M} \Gamma^{sym} > H T^3
\]

(28)

According to latest calculations [8]:

\[
\Gamma^{sym} = (25.4 \pm 2) \alpha_w^5 T^4
\]

(29)

Using the standard expansion rate of the universe one obtains from (28) the condition \( T < 7 \times 10^{12} \text{ GeV} \) for sphaleron to be at equilibrium in the symmetric phase. On the other hand, for \( M \sim 20 \text{ TeV} \) using the expansion rate (24) we get \( T < 180 \text{ GeV} \). This temperature is now close but still above the critical temperature. It is actually sensitive to the exact prefactor in (23) as well as the value of the numerical factor in the right hand side of equation (24) (which we took \( \mathcal{O}(1) \) in a first attempt). So a more accurate analysis will be needed to determine which precise values of \( M \) and the prefactor in \( H \) are consistent with sphalerons at equilibrium in the symmetric phase.

One might also worry that in a fastly expanding universe it becomes difficult to nucleate bubbles and that the nucleation temperature \( T_n \) might be quite different from the critical temperature \( T_c \) at which the effective higgs potential has two degenerate minima. However, it has been shown [21] that the bubble nucleation temperature depends only weakly on the expansion rate of the universe so that \( T_c \sim T_n \) remains valid even when varying the expansion rate \( H \) over orders of magnitude. Consequently, the velocity of the phase interface may be subject to minor changes only. The same remark applies for other quantities which determine the baryon asymmetry generated like the wall thickness and perturbations in thermal population densities of particles at the vicinity of the wall.

From this rough analysis we remark that one can substantially weaken the sphaleron bound using the expansion rate (24) as far as the scale \( M \) is low enough. Let us now study in which context this kind of behaviour might arise and how much freedom we have with respect to observational constraints.

\section{Two motivations to study deviations from Friedmann equation}

\subsection{No probe of FRW cosmology before nucleosynthesis}

Friedmann’s equation relates the expansion rate \( H \) of the universe with its energy density \( \rho(T) \) at a given temperature \( T \). It is derived from the 00 component of Einstein’s equations in four dimensions, applied to the Robertson–Walker metric

\[
ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)
\]

(30)
where \( a(t) \) is the scale factor and \( k \) the curvature parameter and reads:

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{\rho(T)}{3m_{pl}^2} - \frac{k}{a^2} \tag{31}
\]

Equation (31) is a key ingredient of the standard model of cosmology, it serves as a basis to compute freeze-out temperatures and the density of relic particles in the universe. One of its major test comes from the success of the theory of primordial nucleosynthesis. However, there is no observational test of the Friedmann equation at times earlier than the primordial nucleosynthesis epoch characterized by an energy density \( \rho \sim 0.1 \text{ MeV}^4 \), and it is legitimate to question the extrapolation of FRW cosmological evolution before that epoch. Therefore, one could, from an empirical point of view, allow deviations to the expansion rate of the universe of the form:

\[
H^2 = \frac{\rho(T)}{3m_{pl}^2} (1 + \alpha(T)) \tag{32}
\]

where \( \alpha(T) \) is a monotonic increasing function of temperature with a constrained value at nucleosynthesis, \( \alpha(T = T_{\text{nuc}}) \lesssim 1/10 \) \[35,36\]. The origin of these deviations may come from modifications to the Robertson–Walker ansatz (30) by for example questioning the homogeneity and isotropy assumptions or it may come from allowing modifications to general relativity itself as suggested by string theory. The existence of extra dimensions may induce corrections to the standard four-dimensional gravity, which are undetectable at low energy but might have played a significant role in the early universe.

\[\text{2) Deviations to Friedmann equation arise in higher dimensional theories of gravity}\]

The parametrization (32) is actually motivated by results obtained in the context of brane cosmology. Any higher dimensional theory of gravity which reproduces 4d Einstein gravity is expected to reproduce Friedmann–Robertson–Walker cosmology at least at late times. One famous example is the 5d Randall–Sundrum (RS) model \[37\] where the warped non factorizable geometry allows 4d Einstein gravity to be recovered on the 3-brane. For instance, the effective non-relativistic gravitation force between two masses \( m_1 \) and \( m_2 \) on the brane obeys the usual Newton’s potential plus some corrections scaling like \( O(r^{-3}) \):

\[
V(r) = V_{\text{Newton}}(r) \left( 1 + \frac{1}{k^2 r^2} \right), \quad V_{\text{Newton}}(r) = \frac{m_1 m_2}{m_{pl}^2 m r} \tag{33}
\]

where \( k \) is the inverse of the AdS radius of the RS geometry. The validity of the Newton’s formula has been checked down to distances \( r \lesssim 200 \mu m \) so that sets the constraint

\[
k \gtrsim 10^{-14} \text{GeV} \tag{34}
\]

The cosmology of the RS model has been studied by perturbing the static solution of the 5d Einstein equation and adding some energy density on the brane responsible for the

\[\text{\footnote{In the rest of the paper we will set } k = 0.}\]
expansion \cite{34,38}. It was shown that the expansion rate of the universe is of the form (32) with
\[
\alpha(T) = \mathcal{O}\left(\frac{\rho(T)m_{Pl}^2}{M^6}\right)
\]
where $M$ is the five dimensional Planck mass. It is remarkable that such a modified Friedmann equation is not in conflict with primordial nucleosynthesis provided that $M \gtrsim 10$ TeV while it leads to drastic changes at early times for which $\alpha(T)$ dominates and therefore $H^2 \propto \rho^2$ instead of the conventional behaviour $H^2 \propto \rho$ \cite{39,40}. It turns out that a more severe constraint on $M$ comes from (34) which can be translated into $M \gtrsim 4 \times 10^3$ TeV by using the relation between the 5d and 4d Planck mass in the RS model $M^3 = kM_{Pl}^2$. Such bound on $M$ is in conflict with our aim to weaken the sphaleron bound.\footnote{Also, if one wants to use the RS geometry to solve the hierarchy problem then two branes are needed and $M \sim m_{Pl}$ which is much too large to evade the sphaleron bound.} However, the result (32) with (35) can be generalized to any codimension one brane universe (for instance with vanishing tension and vanishing cosmological constant in the bulk i.e. for a relatively small warping) with compact stabilized fifth dimension. It has been extensively discussed in \cite{41} that provided the 55 component of Einstein equation has been used to stabilize the radion one generically recovers FRW cosmology for any codimension one brane model.\footnote{In brane-world geometry one can find solutions to Einstein’s equations where our four common dimensions only are expanding while the fifth extra dimension can be stabilized. This is to be contrasted with Kaluza–Klein cosmology where the energy density is uniformly distributed in all dimensions and leads to the expansion of the compact fifth dimension as well, consequently to a problematic time-dependant Newton ‘constant’.} Let us consider the particular case of an Arkani-Hamed–Dimopoulos–Dvali type of geometry \cite{42}. As is well-known, in the case with one extra dimension the fundamental Planck scale $M$ cannot be lowered to the TeV scale. This would lead to major modifications of gravity at observable distances. Therefore, if we want $M \sim 10–50$ TeV as required to lower the sphaleron bound we need at least more than one large extra dimension. So our point is to extrapolate the behaviour (32) to brane models with a higher number $n$ of extra dimensions where $M$ can be lowered to a scale of order $\mathcal{O}(10$ TeV). We do not know how the Friedmann equation looks like in the general case with $n > 1$. While brane cosmology has been the subject of intense investigation in the last two years it is still at its infancy. Most studies have focused on the 5d case and also were essentially concerned with the recovery of the FRW cosmology at low energy. An important point to stress is that the result (32,33) is obtained first once the radion is stabilized and second in a perturbative approach by linearizing Einstein equations with respect to the perturbation $\delta a(t, y)$ (in $a(t, y) = a(y)_{RS} + \delta a(t, y)$) and $\rho$. What we are interested in in this paper is not the recovery of FRW but precisely the regime where this perturbative expansion breaks down and the nature of the modification to the usual Friedmann equation. A specific feature of brane cosmology in 5d are the $\rho^2$ term corrections. They are somehow a consequence of the junction conditions on the brane \cite{39} which are derived by identifying the singularity of the second derivative of the metric $a''/a$ with the singularity of the energy-momentum tensor. To determine the
junction conditions in the higher dimensional case one would need to solve equations of the type $\Delta a(y_1, ..., y_n)/a \sim \rho \delta^n(y_1, ..., y_n)$ where $\Delta$ is the laplacian in $n$ dimensions. Being ignorant about the non static solutions of Einstein equations in the $n > 1$ case we will assume that $\rho^2$ terms still arise in Friedmann equation and are suppressed by $M^6$ as in the 5d case so that they dominate over the usual term at early times and in particular at the time of the EWPT. We will therefore use the expression

$$H = \sigma \frac{\rho}{M^3}$$

where $\sigma$ parametrizes our ignorance on the exact numerical factor; we will focus on a typical range of values $[0.1 - 1]$.

4 Some comments on the cosmology with non standard Friedmann equation

Note that the transition between the standard and non standard cosmologies would take place when $\sqrt{\rho}/\sqrt{3}m_{Pl} \sim \sigma \rho/M^3$. In a radiation dominated era this leads to

$$T_{\text{trans}} \sim \left( \frac{M^3}{m_{Pl} \pi \sigma \sqrt{g_*}} \right)^{1/2}$$

(37)

With $\sigma = 1/5$, $M = 20$ TeV we get $T_{\text{trans}} \sim 1.4$ MeV i.e. $\rho \sim 135$ MeV$^4 \sim 1350 \rho_{\text{nuclei}}$. Therefore, transition to conventional cosmology takes place slightly before nucleosynthesis.

While it is reasonable to expect that the highest temperature in the universe $T_{\text{reheat}}$ was $O(M)$ it has been argued that in theories with low fundamental Planck scale $T_{\text{reheat}}$ has to be much lower ($\lesssim 1$ GeV) to be compatible with observational constraints [43,44]. This is due to the presence of light modes other than standard model particles which can propagate in extra dimensions, in particular the graviton, and can affect substantially the energy density of the radiation after the decay of the inflaton on our brane. There are different independent kinds of bounds on $T_{\text{reheat}}$. They come from requiring that 1) the energy density of Kaluza–Klein gravitons does not modify the expansion rate of the universe at the time of nucleosynthesis, 2) the decay of KK gravitons into photons does not generate distortions in the 2.7K photon background radiation, 3) the energy density of KK gravitons does not lead to overclosure of the universe, 4) the cooling of the energy density of the radiation on our brane is dominated by the expansion and not the escape of gravitons into the bulk. All these constraints are based on the assumption that the bulk is empty (and static) and gravitons can only decay on our brane. Therefore they can be evaded if assuming that graviton can decay on other brane than ours. Also, a toroidal geometry is assumed for extra dimensions. Other choices of geometry lead to different spectra of KK modes which lead to weaker bounds on $T_{\text{reheat}}$ [45] and the occurrence of the EWPT in a radiation dominated era is no more questioned.

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8For other discussions on the reheating temperature in non conventional brane cosmologies see also [46].
It is interesting however to come back on condition 4). In our previous analysis of subsection 2.2, the relation between time and temperature was derived assuming that cooling is due to the expansion of the universe so that $\rho(t) \sim a(t)^{-4}$ and $\dot{\rho}|_{\exp} = -4H\rho$. The cooling due to the escape of gravitons into the extra dimensions was evaluated in [43]:

$$\dot{\rho}|_{\text{esc}} \sim -T^{n+7}/M^{n+2}$$  \hspace{1cm} (38)

Requiring that the cooling by escape of graviton is less than ten percents leads to $H\rho \gtrsim 5T^{n+7}/2M^{n+2}$. For a standard cosmology this gives

$$T^{n+1} \approx \frac{2}{5\sqrt{3}} \left( \frac{\pi^2 g_*}{30} \right)^{3/2} \frac{M^{n+2}}{m_{Pl}}$$  \hspace{1cm} (39)

If $M = 10$ TeV we obtain $T \lesssim 13$ GeV for $n = 2$. On the other hand using the expansion rate (36) we get

$$T^{n-1} \lesssim \frac{2}{5} \sigma \left( \frac{\pi^2 g_*}{30} \right)^2 M^{n-1}$$  \hspace{1cm} (40)

So, even if we were to assume the validity of (38), at any temperature below $M$, the cooling is well dominated by the expansion.

Let us finish this section with the evaluation on the lower bound on $M$ obtained by requiring that the expansion rate (36) should be suppressed at the time of nucleosynthesis when $\rho \sim 0.1$ MeV$^4$. Requiring $\alpha(T) = 3\sigma^2 \rho m_{Pl}^2/M^6 \lesssim 1/10$ leads to

$$M \gtrsim \sigma^{1/3} 16.1 \text{ TeV}$$  \hspace{1cm} (41)

so that for $\sigma = 1$, $M \gtrsim 16.1$ TeV, for $\sigma = 1/5$, $M \gtrsim 9.4$ TeV and for $\sigma = 1/10$, $M \gtrsim 7.4$ TeV.

In the next section we refine the rough analysis of subsection 2.2. Our aim is to study the parameter space leading to a significant weakening of the sphaleron bound. There are three parameters to work with: The higher dimensional Planck mass $M$, the factor $\sigma$ appearing in the expansion rate (46) and $\kappa$ the prefactor in the sphaleron rate (5). We will limit our study to values of $\kappa$ in the range $[0.1 - 1]$ since non perturbative calculations of the sphaleron rate indicate $\kappa \gtrsim 0.1$.

5 Condition to preserve the baryon asymmetry using non conventional cosmology

Let us now analyse in more details the condition $S \gtrsim 10^{-5}$ expressing the preservation of the baryon asymmetry produced at the EWPT. The integral in (6) is now $I \approx 4M\eta T_{c} \zeta e^{-\nu} \zeta \zeta (T_{c}) - \zeta (T_{c}) + \ln(T_{c} t_{c}) \lesssim 0$ leading to

$$\ln \left( \frac{4M \eta}{5 \ln 10} \right) + 6 \ln \zeta (T_{c}) - \zeta (T_{c}) + \ln(T_{c} t_{c}) \lesssim 0$$  \hspace{1cm} (42)
which is very similar to ([13]). It can be rewritten as

\[ f\left(\frac{\phi_c}{T_c}, M, \kappa, \sigma\right) \lesssim 0 \]  \tag{43}

where

\[ f\left(\frac{\phi_c}{T_c}, M, \kappa, \sigma\right) = 3.06 + \ln\left(\frac{\kappa M^3}{\sigma T_c^3}\right) + 6 \ln\left(\frac{\phi_c}{T_c}\right) - 37.52 \frac{\phi_c}{T_c} \]  \tag{44}

This last expression has been obtained by replacing \( t_c \) in (42) by its expression (27) and that is what makes the difference with the conventional cosmology. In figure 1 we plotted, for different values of \( M \), \( f\left(\frac{\phi_c}{T_c}, M, \kappa, \sigma\right) \) as a function of \( \phi_c/T_c \) in the case \( \sigma = 1, \kappa = 0.2 \). We find that for \( M \lesssim 23.1 \) TeV, \( f \) is always negative which means that the condition of preservation of the baryon asymmetry is always satisfied whatever the value for \( \phi_c/T_c \). Therefore, if \( \sigma = 1 \) and \( M \) is in the range \([16.1 - 23.1]\) TeV there is no sphaleron bound. For larger values of \( M \), \( f \) becomes positive for certain values of \( \phi_c/T_c \) and therefore there is a sphaleron bound. Notice that \( M = 23.1 \) TeV corresponds to a discontinuity in the bound on \( \phi_c/T_c \). From no constraint on \( \phi_c/T_c \) we jump to the constraint \( \phi_c/T_c > 0.16 \).

In figure 2, we summarize the status of the sphaleron bound in different regions of the parameter space \((M, \kappa)\) for the two cases \( \sigma = 1 \) and \( \sigma = 1/10 \). The horizontal line corresponds to the lower bound on \( M \) dictated by compatibility with nucleosynthesis. Another type of lower bound on \( M \) is obtained by requiring that sphalerons are at thermal equilibrium in the symmetric phase (see equation (28)) at \( T_c \). It leads to the condition \( M > 0.1 T_c \sigma^{1/3} \). For \( T_c = 130 \) GeV we get \( M > 14.3 \sigma^{1/3} \) TeV, which is slightly weaker than the bound obtained from nucleosynthesis. Anyhow, it is interesting to point out that independently from the nucleosynthesis constraint, what electroweak baryogenesis singles out is a window of a few tens of TeV for \( M \). The upper bound corresponds to an expansion rate which is too small to freeze the baryon asymmetry in the broken phase at the EWPT while the lower bound corresponds to an expansion rate which is so large that sphalerons become out of equilibrium in the symmetric phase as well. The upper curves corresponds to \( \phi_c/T_c = 0.3 \) which we presented in section 2 as the value obtained for higgs masses as large as 115 GeV and stop masses no more constrained to be lighter than the top mass. The lower curve delimits the regions where there is a sphaleron bound at least \( \phi_c/T_c \gtrsim 0.15 \), and no sphaleron bound at all.

We should add a last clarifying remark here. The main formula we have made use of in our analysis is equation (4) which gives the rate of baryon number violation in the broken phase. One should keep in mind that this formula is valid in a relatively narrow temperature interval \( M_W(T) \ll T \ll E_{sp}(T) \). As stated in [24, 27], for \( E_{sp}/T \lesssim 1 \) i.e. for \( \phi/T \lesssim 0.1 \), configurations other than the sphaleron contribute to the probability of making a transition with changing baryon number. Consequently, we can no longer apply the analysis of decay of metastable states [17] with allowance for only the saddle point (i.e. sphaleron solution) of the energy functional. As \( \phi/T \) reaches zero, the sphaleron rate is expected to approach its value in the symmetric phase and the turnover around \( \phi/T \sim 0.15 \) in the graph of figure 1 looses meaning. This however does not alter the validity of our qualitative results. Moreover, in the range \( \phi/T \sim 0.2 - 0.4 \), formula (4)
leads to an overestimation of the value of the sphaleron rate (it slightly overpasses the value in the symmetric phase) which means that by using that formula in that regime we have slightly underestimated the freeze out.

6 Conclusion

In summary, we studied the condition for preservation of the baryon asymmetry produced at the EWPT when questioning the usual assumption on the thermal history of the universe. To weaken the sphaleron bound by a factor 3 or 4 as required to allow higgs masses as large as 115 GeV as well as a wider window of the MSSM parameter space, the expansion rate of the universe at the EWPT has to be increased dramatically (by ten orders of magnitude at least) in order to freeze out the sphaleron processes in the $SU(2)$ broken phase. On the other hand, the expansion rate should not be larger than a critical value above which sphalerons start to be inoperative in the symmetric phase as well. It turns out that the the expansion rate inspired by brane cosmology possesses these properties for $M$ in the approximate range 15 – 50 TeV depending on the precise values of the parameters $\kappa$ and $\sigma$. It would be interesting to check that the Friedmann equation we have postulated effectively arise in generic brane models and more generally to study deviations to Friedmann equation before the nucleosynthesis epoch in the spirit of which do not necessarily rely on the existence of extra dimensions.

The possibility we have investigated is one among others. For instance, we are still assuming radiation domination. One could imagine various ways to modify the cosmology at the EWPT. Also, in our work, $M$ is the fundamental Planck scale, but it could correspond to a different scale. For example, the authors of found that the $\rho^2$ corrections in Friedmann equation could be suppressed by powers of $\Lambda_W$ (in a matter dominated era) where $\Lambda_W$ is a scale associated with the mass of the radion. Note that we are considering a regime where gravity becomes essentially higher dimensional and where radion modes could play a role. One thing to care about is that decay of these modes will not dilute the baryon asymmetry.

Finally, we have focused here on the “out of equilibrium” condition of electroweak baryogenesis. There is still much activity in the electroweak baryogenesis community to determine whether there are enough CP violation sources in the MSSM to account for the amount of baryon asymmetry observed today ($n_B/s \sim 10^{-11}$).

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9One could also exploit the fact that for small values of $M$, sphalerons are frozen even in the symmetric phase and cannot erase any preexisting baryon asymmetry produced by mechanisms which do not make use of a first order phase transition.
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References

[1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155, 36 (1985).
[2] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, 32 (1967) [JETP Lett. 5, 24 (1967)].
[3] This mechanism was first worked out in: A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B 245, 561 (1990); Nucl. Phys. B 349, 727 (1991); Nucl. Phys. B 373, 453 (1992); Phys. Lett. B 336, 41 (1994) [arXiv:hep-ph/9406345].
[4] L. Carson, X. Li, L. D. McLerran and R. T. Wang, Phys. Rev. D 42, 2127 (1990).
[5] J. Baacke and S. Junker, Phys. Rev. D 49, 2055 (1994) [arXiv:hep-ph/9308314].
[6] G. D. Moore, Phys. Lett. B 439, 357 (1998) [arXiv:hep-ph/9801204].
[7] G. D. Moore, [arXiv:hep-ph/9902464].
[8] D. Bodeker, G. D. Moore and K. Rummukainen, Nucl. Phys. Proc. Suppl. 83, 583 (2000) [arXiv:hep-lat/9909054].
[9] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Phys. Rev. Lett. 77, 2887 (1996) [arXiv:hep-ph/9605288].
[10] M. B. Gavela, P. Hernandez, J. Orloff and O. Pene, arXiv:hep-ph/9407403.
[11] First analyses of EW baryogenesis in the MSSM can be found in: S. Myint, Phys. Lett. B 287, 325 (1992) [arXiv:hep-ph/9206260]; A. G. Cohen and A. E. Nelson, Phys. Lett. B 297, 111 (1992) [arXiv:hep-ph/9209245]; P. Huet and A. E. Nelson, Phys. Rev. D 53, 4578 (1996) [arXiv:hep-ph/9506477].
[12] M. Carena, J. M. Moreno, M. Quiros, M. Seco and C. E. Wagner, Nucl. Phys. B 599, 158 (2001) [arXiv:hep-ph/0011055].
[13] S. J. Huber, P. John and M. G. Schmidt, Eur. Phys. J. C 20, 695 (2001) [arXiv:hep-ph/0101249].

[14] J. M. Cline, M. Joyce and K. Kainulainen, JHEP 0007, 018 (2000) [arXiv:hep-ph/0006119].

[15] J. M. Cline, M. Joyce and K. Kainulainen, [arXiv:hep-ph/0110031].

[16] J. R. Espinosa, Nucl. Phys. B 475, 273 (1996) [arXiv:hep-ph/9604320]. B. de Carlos and J. R. Espinosa, Nucl. Phys. B 503, 24 (1997) [arXiv:hep-ph/9703212].

[17] M. Carena, M. Quiros and C. E. Wagner, Nucl. Phys. B 524, 3 (1998) [arXiv:hep-ph/9710401].

[18] F. Csikor, Z. Fodor, P. Hegedus, A. Jakovac, S. D. Katz and A. Piroth, Phys. Rev. Lett. 85, 932 (2000) [arXiv:hep-ph/0001087]. M. Laine and K. Rummukainen, Nucl. Phys. B 597, 23 (2001) [arXiv:hep-lat/0009025].

[19] M. Quiros, Nucl. Phys. Proc. Suppl. 101, 401 (2001) [arXiv:hep-ph/0101230].

[20] M. Joyce, Phys. Rev. D 55, 1875 (1997) [arXiv:hep-ph/9606223]. T. Prokopec, Phys. Lett. B 483, 1 (2000) [arXiv:hep-ph/0002181].

[21] M. Joyce and T. Prokopec, Phys. Rev. D 57, 6022 (1998) [arXiv:hep-ph/9709320].

[22] S. Davidson, M. Losada and A. Riotto, Phys. Rev. Lett. 84, 4284 (2000) [arXiv:hep-ph/0001301].

[23] J. M. Cline, G. D. Moore and G. Servant, Phys. Rev. D 60, 105035 (1999) [arXiv:hep-ph/9902220].

[24] J. M. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. 83, 4245 (1999) [arXiv:hep-ph/9906523].

[25] A. I. Bochkarev and M. E. Shaposhnikov, Mod. Phys. Lett. A 2, 417 (1987).

[26] P. Arnold and L. D. McLerran, Phys. Rev. D 36, 581 (1987).

[27] S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B 308 (1988) 885.

[28] L. Carson and L. D. McLerran, Phys. Rev. D 41, 647 (1990).

[29] R. F. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. D 10, 4138 (1974).

[30] N. S. Manton, Phys. Rev. D 28, 2019 (1983). F. R. Klinkhamer and N. S. Manton, Phys. Rev. D 30, 2212 (1984).

[31] J. Kunz, B. Kleihaus and Y. Brihaye, Phys. Rev. D 46, 3587 (1992). Y. Brihaye and J. Kunz, Phys. Rev. D 48, 3884 (1993) [arXiv:hep-ph/9304256]. J. M. Moreno, D. H. Oaknin and M. Quiros, Nucl. Phys. B 483, 267 (1997) [arXiv:hep-ph/9605387].
[32] M. Quiros, arXiv:hep-ph/9901312.

[33] M. Carena, M. Quiros and C. E. Wagner, Phys. Lett. B 380, 81 (1996) [arXiv:hep-ph/9603420].

[34] J. M. Cline and G. D. Moore, Phys. Rev. Lett. 81, 3315 (1998) [arXiv:hep-ph/9806354].

[35] K. A. Olive, G. Steigman and T. P. Walker, Phys. Rept. 333, 389 (2000) arXiv:astro-ph/9905320.

[36] S. M. Carroll and M. Kaplinghat, arXiv:astro-ph/0108002.

[37] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-th/9905221]. L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999) [arXiv:hep-th/9906064].

[38] C. Csaki, M. Graesser, C. Kolda and J. Terning, Phys. Lett. B 462, 34 (1999) arXiv:hep-ph/9905013.

[39] P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B 565, 269 (2000) [arXiv:hep-th/9905012].

[40] D. J. Chung and K. Freese, Phys. Rev. D 61, 023511 (2000) [arXiv:hep-ph/9906542].

[41] C. Csaki, M. Graesser, L. Randall and J. Terning, Phys. Rev. D 62, 045015 (2000) [arXiv:hep-ph/9911403].

[42] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) arXiv:hep-ph/9803317.

[43] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D 59, 086004 (1999) arXiv:hep-ph/9807344.

[44] K. Benakli and S. Davidson, Phys. Rev. D 60, 025004 (1999) [arXiv:hep-ph/9810280]. L. J. Hall and D. R. Smith, Phys. Rev. D 60, 085008 (1999) [arXiv:hep-ph/9904267]. S. Hannestad, Phys. Rev. D 64, 023515 (2001) [arXiv:hep-ph/0102290].

[45] N. Kaloper, J. March-Russell, G. D. Starkman and M. Trodden, Phys. Rev. Lett. 85, 928 (2000) [arXiv:hep-ph/0002001].

[46] R. Allahverdi, A. Mazumdar and A. Perez-Lorenzana, Phys. Lett. B 516, 431 (2001) [arXiv:hep-ph/0105123]. A. Mazumdar, Nucl. Phys. B 597, 561 (2001) [arXiv:hep-ph/0008087]. A. Mazumdar, Phys. Rev. D 64, 027304 (2001) [arXiv:hep-ph/0007269].

[47] J. S. Langer, Annals Phys. 54 (1969) 258. I. Affleck, Phys. Rev. Lett. 46, 388 (1981).
Figure 1: $f(\phi_c/T_c, M, \kappa, \sigma)$ expresses the condition of preservation of the baryon asymmetry. For negative values of $f$, baryon asymmetry is preserved. This leads to a bound on the value of $\phi_c/T_c$ for $M \gtrsim 23$ TeV which depends on the fundamental Planck scale. As stated at the end of section 5, one should not pay attention to what happens to the graph for $\phi_c/T_c \lesssim 0.15$. The turnover is an artifact of the breakdown of the approximation leading to formula (4).
Figure 2: Sphaleron bound in the parameter space $\kappa, M$. In Region I, there is no condition at all on $\phi_c/T_c$. In region II, $0.16 \leq \phi_c/T_c \leq 0.3$. In region III, $\phi_c/T_c > 0.3$. 