**D-term Dynamical Supersymmetry Breaking Generating Split $\mathcal{N} = 2$ Gaugino Masses of Mixed Majorana-Dirac Type**

H. Itoyama$^{a,b,*}$ and Nobuhito Maru$^c$†

$^a$ Department of Mathematics and Physics, Graduate School of Science
Osaka City University and

$^b$ Osaka City University Advanced Mathematical Institute (OCAMI)
3-3-138, Sugimoto, Sumiyoshi-ku, Osaka, 558-8585, Japan

$^c$ Department of Physics, and Research and Education Center for Natural Sciences,
Keio University, Hiyoshi, Yokohama, 223-8521 JAPAN

Abstract

Under a few mild assumptions, $\mathcal{N} = 1$ supersymmetry in four dimensions is shown to be spontaneously broken in a metastable vacuum in a self-consistent Hartree-Fock approximation of BCS/NJL type to the leading order, in the gauge theory specified by the gauge kinetic function and the superpotential of adjoint chiral superfields, in particular, that possesses $\mathcal{N} = 2$ extended supersymmetry spontaneously broken to $\mathcal{N} = 1$ at tree level. We derive an explicit form of the gap equation, showing the existence of a nontrivial solution. The $\mathcal{N} = 2$ gauginos in the observable sector receive mixed Majorana-Dirac masses and are split due to both the non-vanishing $\langle D^0 \rangle$ and $\langle F^0 \rangle$ induced with $\langle D^0 \rangle$. It is argued that proper physical applications and assessment of the range of the validity of our framework are made possible by rendering the approximation into $\frac{1}{N^2}$ expansion.

*e-mail: itoyama@sci.osaka-cu.ac.jp
†e-mail: maru@phys-h.keio.ac.jp
1 introduction

Identifying the mechanism of spontaneously broken $\mathcal{N} = 1$ supersymmetry in nature has been the vital issue in theoretical particle physics for three decades. There are two order parameters in supersymmetric field theories which tell whether $\mathcal{N} = 1$ supersymmetry is spontaneously broken or not. The one is the nonvanishing vacuum expectation value (vev for short) of the $F$ term, the auxiliary field of the chiral superfield. There are mechanisms known that generate the nonvanishing $F$ term both at tree (classical)\cite{1} and quantum mechanical levels. As the nonrenormalization theorem applies \cite{2} here, the consideration beyond the lowest order in perturbation theory ought to be genuinely nonperturbative in nature. Instantons have played important roles in the development of this investigation in the past years \cite{3,4,5}. The other is the nonvanishing vev of the $D$ term, the auxiliary field of the vector superfield. There is a well-known mechanism at the tree level \cite{6} to this and the nonrenormalization theorem does not apply. In the past, there were models found, (for example, the $4 - 1$ model)\cite{7} which display dynamical supersymmetry breaking triggered by the nonperturbative effects lying in the superpotential. They generate both F-terms and D-terms.

In this letter, we will provide mechanism that provides dynamical supersymmetry breaking triggered by a nonvanishing $D$-term beyond the lowest order in perturbation theory. We will treat $\mathcal{N} = 1$ effective theory characterized by three nontrivial input functions, a Kahler potential, a gauge kinetic function and a superpotential. The vacuum of the theory at the tree level is assumed to preserve $\mathcal{N} = 1$ supersymmetry; neither $D$ term nor $F$ term is generated at the tree level. Our mechanism assumes the existence of scalar gluons in nature and, in that respect, is distinct from the previous proposals on dynamical supersymmetry breaking. Our mechanism involves condensation of composite operators and is based on the self-consistent Hartree-Fock approximation which is reminiscent of that of \cite{8,9,10} in the theory of superconductivity/chiral symmetry. (For the ideas in the past of using a NJL type of approach to assess dynamical supersymmetry breaking, gaugino condensation and the comparison with computation from instantons and symmetries, see, for instance, \cite{11}.) Such a possibility is precluded by the requirement of the perturbative renormalizability of the interactions in the lagrangian. With the advent of UV finite string models and several experimental calls beyond the standard model in particle physics, however, the mechanism in what follows can be made relevant in the effective field theory description for the energy scale up to the multi TEV that is being probed by the Large Hadron Collider. The original $D = 0$ perturbative vacuum is not lifted in our treatment and the new local minimum we find is metastable. As is anticipated from the study of the models \cite{7} mentioned, our mechanism eventually generates a nonvanishing $F$ term as well.
2 basic observation and mechanism

Let us start from a general lagrangian

\[ \mathcal{L} = \int d^4 \theta K(\Phi^a, \bar{\Phi}^a) + (gauging) + \int d^2 \theta \text{Im} \frac{1}{2} \tau_{ab}(\Phi^a) \mathcal{W}^{\alpha \alpha}_{\alpha} W^b_{\alpha} + \left( \int d^2 \theta W(\Phi^a) + \text{c.c.} \right) , \]

where \( K \) is a Kähler potential with its gauging by the gauge group understood, \( \tau_{ab}(\Phi^a) \) is a gauge kinetic superfield of the chiral superfield \( \Phi^a \) in the adjoint representation, and \( W(\Phi^a) \) is a superpotential.

The bilinears made of the \( \mathcal{N} = 1 \) gaugino \( \lambda^a \) and the matter fermion \( \psi^a \), which are referred to as \( \mathcal{N} = 2 \) gauginos in this letter, are obtained from the second and the third line of eq.(1):

\[ -\frac{1}{2} \left( \lambda^a, \psi^a \right) \begin{pmatrix} 0 & -\sqrt{2} \tau_{abc} D^b \\ \frac{\sqrt{2}}{4} \tau_{abc} D^b & \partial_a \partial_c W \end{pmatrix} \left( \begin{array}{c} \lambda^c \\ \psi^c \end{array} \right) + \text{c.c.}, \]

where \( \tau_{abc} \equiv \partial_c \tau_{ab} \). Note that the nonvanishing value of \( \tau_{abc} \) ensures the coupling of the auxiliary \( D^a \) field to the fermionic bilinears while there is no bosonic counterpart in the lagrangian.

Let us assume that \( \tau_{ab} \) is obtained as the second derivatives of a trace function \( f(\Phi^a) \). The nonvanishing vevs are \( \langle \tau_{0aa} \rangle \). The holomorphic and nonvanishing part of the mass matrix is

\[ M_{F,a} \equiv \begin{pmatrix} 0 & -\sqrt{2} \langle \tau_{0aa} D^0 \rangle \\ -\frac{\sqrt{2}}{4} \langle \tau_{0aa} D^0 \rangle & \langle \partial_a \partial_a W \rangle \end{pmatrix} \]

to each generator. We see that, upon diagonalization, this has two unequal and nonvanishing eigenvalues provided \( \langle D^0 \rangle \neq 0 \) and \( \langle \partial_a \partial_a W \rangle \neq 0 \). In this case, the \( \mathcal{N} = 2 \) gauginos receive masses of mixed Majorana-Dirac type and are split. This observation generalizes the proposal of [12] where the masses are of a pure Dirac type and the \( \mathcal{N} = 2 \) gauginos are degenerate while the supersymmetry is broken.

The value of \( \langle D^0 \rangle \) is well-determined quantum mechanically in a self-consistent Hartree-Fock approximation as long as the fluctuations are small. To one-loop order off-shell, it is given by the stationary condition to the part of the effective potential which contains \( D^a \)

\[ V^{(D)} + V_{\text{c.t.}} + V_{1-\text{loop}}, \]

where \( V^{(D)} = -\frac{1}{2} g_{ab} D^a D^b \), \( g_{ab} = \text{Im} \tau_{ab} \), \( V_{1-\text{loop}} \) is the one-loop contribution and \( V_{\text{c.t.}} \) is a counterterm which we prepare with regard to the lagrangian. Note that equation of motion for \( D^0 \) tells us that the condensation of the Dirac bilinears is responsible for the nonvanishing order parameter: \( \langle D^0 \rangle = -\frac{1}{2\sqrt{2}} g^{00} \left( \tau_{0cd} \psi^d \lambda^c + \bar{\tau}_{0cd} \bar{\psi}^d \bar{\lambda}^c \right) \). In fact, the stationary condition of eq. [11] with respect to the auxiliary field \( \langle D^0 \rangle \) is nothing but the well-known gap equation of the theory on-shell which contains four-fermi interactions.
In order to make our dynamical framework more concrete and to reduce the number of input functions to one, we temporarily impose $\mathcal{N} = 2$ supersymmetry on the abelian \cite{13} and nonabelian \cite{14} action. For definiteness, we take the gauge group to be $U(N)$ which are unbroken although our results are applicable to a variety of product gauge groups which contain an overall $U(1)$ and which are broken in the vacua specified by the tree level potential.

The theory we work with is given by the lagrangian

$$L_{U(N)} = \text{Im} \left[ \int d^4\theta \text{Tr} e^{a\theta} \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} + \int d^2\theta \frac{1}{2} \frac{\partial^2 \mathcal{F}(\Phi)}{\partial \Phi^a \partial \Phi^b} W^{a\alpha} W^{b}_\alpha \right] + \left( \int d^2\theta W(\Phi) + \text{c.c.} \right), \quad (5)$$

where the superpotential is

$$W(\Phi) = \text{Tr} \left( 2e\Phi + m \frac{\partial \mathcal{F}(\Phi)}{\partial \Phi} \right). \quad (6)$$

The superpotential consists of the electric and magnetic Fayet-Iliopoulos terms whose representation is obtained by pointing these two three dimensional vectors in a particular direction under the rigid $SU(2)_R$ rotation \cite{15}. The bare action reads

$$S_{\text{bare}} = S[\mathcal{F}] + S_{\text{c.t.}}, \quad (7)$$

and

$$S[\mathcal{F}] = \int d^4x L_{U(N)}, \quad S_{\text{c.t.}} = S[\mathcal{F} = (\Lambda/2)(\Phi^0)^2]. \quad (8)$$

Here, we have introduced the supersymmetric counterterm $S_{\text{c.t.}} = S[\mathcal{F} = (\Lambda/2)(\Phi^0)^2]$ which conforms to the form of the action. The counterterm plays a specific role in our treatment of isolating UV infinities at the one-loop effective potential and the gap equation in later section. More explicitly, the counterterm to the one-loop effective potential is written as

$$V_{\text{c.t.}} = -\text{Im} \frac{\Lambda}{2} \int d^2\theta W^{a\alpha} W^{b}_\alpha = -\text{Im} \frac{\Lambda}{2}(D^0)^2, \quad (9)$$

where $\Lambda$ is a complex parameter that we have prepared in order to cancel the infinity we will come across.

The effective potential up to one-loop order is

$$V = V^{(D)} + V^{(\text{sup})} + V_{\text{1-loop}} + V_{\text{c.t.}}, \quad (10)$$

where $V^{(\text{sup})} = g^{ab} \partial_a W \partial_b W$ and $\partial_a W = \sqrt{2N}(e\delta^0_a + m F_{0a})$. At the tree level, $\langle D^a \rangle = 0$ and the vacuum condition is $\partial_a V^{(\text{sup})} = 0$. There are two vacua and the positive definiteness of the Kähler metric selects one. We will assume $\frac{\text{Im}}{m} e < 0$ for definiteness, so that the condition is $e\delta^0_c + m F_{0c} = 0$. The second supersymmetry is spontaneously broken at the tree level \cite{13, 14}. For the spectrum of this theory at the tree level in various phases, see \cite{16}.
3 effective action up to one-loop and subtraction of UV infinity

Coming back to the mass matrix eq. (3) and the issue, we now compute the two eigenvalues \( \Lambda^{(\pm)}_a \) for each \( a \) to be

\[
\Lambda^{(\pm)}_a = m_a \lambda^{(\pm)}, \quad \lambda^{(\pm)} = \frac{1}{2} \left( 1 \pm \sqrt{1 + \Delta^2} \right), \quad \Delta^2 = \frac{(D^0)^2}{4Nm^2},
\]

where \( m_a \equiv \sqrt{2Nm} \langle g^{aa} F^{0a} \rangle \) is the dimension-two parameter of the magnetic Fayet-Iliopoulos term and \( |m_a|^2 \) are the masses of the scalar gluons which are receiving experimental attentions.

It is now clear that the entire contribution to the 1PI vertex function \( i\Gamma_{1\text{-loop}} \) is

\[
\int d^4x \sum_a |m_a|^4 \int d^4\ell \frac{\mu}{(2\pi)^4} \ln \left[ \frac{(\lambda^{(+)2} - \ell^2 - i\epsilon)(\lambda^{(-)2} - \ell^2 - i\epsilon)}{(1 - \ell^2 - i\epsilon)(-\ell^2 - i\epsilon)} \right]. \tag{12}
\]

There is a variety of methods available which regulate and evaluate this expression. Here we adopt one which first handles the determinant by the integral \( \log a/b = -\int_0^\infty dt \left( e^{-at} - e^{-bt} \right)/t \) and subsequently continues the power of \( t \) to the number designating the spacetime dimension \( d \). Let us proceed this way. The one-loop contribution to the effective potential is expressed as

\[
\sum_a |m_a|^4 V_{1\text{-loop}} = \frac{1}{16\pi^2} \int_0^\infty \frac{dt}{t^3} \left( e^{-\lambda^{(+)2}t} + e^{-\lambda^{(-)2}t} - e^{-t} - 1 \right). \tag{13}
\]

Continuing \( 3 \to 1 + d/2 \) in the case of \( d \) dimensional integral for regularization, we obtain

\[
\frac{1}{32\pi^2} \left[ A(d) \left( \Delta^2 + \frac{1}{8}\Delta^4 \right) - \lambda^{(+4)}\log \lambda^{(+2)} - \lambda^{(-4)}\log \lambda^{(-2)} \right], \tag{14}
\]

where

\[
A(d) = 3/4 - \gamma + \frac{1}{2 - d/2}, \tag{15}\]

Now the part of the one-loop effective potential which contains \( \Delta \) reads

\[
V_{1\text{-loop}}^{(D)} = V^{(D)} + V_{\text{c.t.}} + V_{1\text{-loop}} = \sum_a |m_a|^4 \left[ -\beta\Delta^2 - \Lambda_{\text{res}}\Delta^2 \right. \\
+ \frac{1}{32\pi^2} \left\{ A(d) \left( \Delta^2 + \frac{1}{8}\Delta^4 \right) - \lambda^{(+4)}\log \lambda^{(+2)} - \lambda^{(-4)}\log \lambda^{(-2)} \right\} \right], \tag{16}
\]

\(^1\)This is equivalent to the dimensional reduction as opposed to the dimensional regularization scheme in the Feynman diagram language. In the latter, the spinor traces are also evaluated in \( d \) dimensions while in the former, they are in four dimensions and supersymmetry is preserved at least at one-loop \([17]\).
where \( \beta = \frac{(g_{00})^2 2 N m^2}{\sum_a |m_a|^4} \), \( \Lambda_{\text{res}} = \frac{(\text{Im} \Lambda)^2 2 N m^2}{\sum_a |m_a|^4} \).

The procedure preserves supersymmetry as both the regularization and the counterterm \( V_{\text{c.t.}} \) are supersymmetric. In eq. (16), however, the parameter \( \text{Im} \Lambda \) (or \( \Lambda_{\text{res}} \)) is still unrelated to our one-loop computation and hence the resulting infinity lying in \( A(d) \). We can relate these and absorb the infinity into \( \text{Im} \Lambda \) (or \( \Lambda_{\text{res}} \)) by imposing one (renormalization) condition. We adopt the following one:

\[
\frac{1}{\sum_a |m_a|^4} \left. \frac{\partial^2 V_{1\text{-loop}}^{(D)}}{\partial \Delta^2} \right|_{\Delta=0} = 2c,
\]

where \( c \) is a fixed non-universal number. \( \Lambda_{\text{res}} \) is now expressible in terms of \( A(d), c, \beta \) as

\[
\Lambda_{\text{res}} = \Lambda_{\text{res}}(d) = -\beta - \left( c + \frac{1}{64 \pi^2} \right) + \frac{1}{32 \pi^2} A(d).
\]

Our final expression for \( V_{1\text{-loop}}^{(D)} \) is

\[
\frac{1}{\sum_a |m_a|^4} V_{1\text{-loop}}^{(D)} = \left( c + \frac{1}{64 \pi^2} \right) \Delta^2 + \Lambda_{\text{res}}'(d) \Delta^4 - \frac{1}{32 \pi^2} \left( \lambda^{(+)}^4 \log \lambda^{(+)}^2 + \lambda^{(-)}^4 \log \lambda^{(-)}^2 \right)
\]

where \( \Lambda_{\text{res}}'(d) \equiv c + \beta + \Lambda_{\text{res}}(d) + \frac{1}{64 \pi^2} \). By a change of parametrization of the potential from \( \Lambda_{\text{res}} \) to \( c \), we have been able to isolate the original infinity into the coefficient of \( \Delta^4 \), which is regarded as the overall scale of the effective potential and the gap equation. A similar treatment is seen, for instance, in [18] Our treatment differs from the original treatment of the NJL model [10] on spontaneous chiral symmetry breaking which proceeds solely on the bare theory without counterterms and introduces the relativistic cutoff. We consider this difference as the difference of physics aiming at: in our case, the UV physics which may underlie our model is set by the prepotential function.

Our computation has been so far with regard to the \( \mathcal{N} = 2 \) action (eq. (5)) but it is easy to see that the computation and the final conclusion eq. (19) are valid with regard to the more general \( \mathcal{N} = 1 \) action (eq. (1)) as well. The difference between these two cases is absorbed in the redefinition of \( \Delta^2 \): \( \Delta^2 \equiv \frac{(D_0^a)^2}{4 N m^2} \Rightarrow \Delta^2 \equiv \frac{(D_0^a)^2}{a} \), with \( a \) a positive number. The formulae in this general setting are

\[
\Lambda^\pm_a = \frac{1}{2} m_a \left[ 1 \pm \sqrt{1 + \Delta^2} \right], \quad m_a \equiv \langle g^{aa} \partial_a \partial_a W \rangle, \quad \Delta^2 \equiv \frac{\langle \tau_{0aa} D_0^a \rangle^2}{2 \langle \partial_a \partial_a W \rangle^2}.
\]

(20)
4 gap equation and nontrivial solution

The gap equation is nothing but the stationary condition of eq. (19) with respect to $\Delta$:

$$0 = \frac{\partial V_{1-\text{loop}}}{\partial \Delta} = \Delta \left[ 2 \left( c + \frac{1}{64\pi^2} \right) + \frac{\Lambda_{\text{res}}}{2} \Delta^2 - \frac{1}{32\pi^2} \frac{1}{\sqrt{1 + \Delta^2}} \left\{ (\lambda^+)^3 (2 \log(\lambda^+)^2 + 1) - (\lambda^-)^3 (2 \log(\lambda^-) + 1) \right\} \right].$$

The gap equation has a trivial solution $\Delta = 0$ which corresponds to the vacuum of unbroken supersymmetry. Now our interest is whether the nontrivial solution $\Delta \neq 0$ exists or not. The gap equation is transcendental, and we first solve it approximately. To begin with, consider the case where the D-term VEV is very small, $\Delta^2 \ll 1$. Noting that $\lambda^+ \simeq 1, \lambda^- \simeq 0$ in this case, the gap equation can be approximated as

$$2 \left( c + \frac{1}{64\pi^2} \right) + \frac{\Lambda_{\text{res}}}{2} \Delta^2 \simeq \frac{1}{32\pi^2} \left( 1 + \frac{5}{4} \Delta^2 \right).$$

(22)

If $c > 0$, then we have no solution of (22) because of $\Lambda_{\text{res}}$. If $c < 0$, then we have a solution $\Delta^2 \simeq -4c/(\Lambda_{\text{res}} - \frac{5}{2} \cdot \frac{1}{32\pi^2})$.

Next, consider the case where the D-term VEV is very large, $\Delta^2 \gg 1$. Noting that $\lambda^{(\pm)} \simeq \pm \Delta/2$ in this case, the gap equation can be approximated as

$$c + \frac{1}{64\pi^2} + \Lambda_{\text{res}} \left( \frac{\Delta}{2} \right)^2 \simeq \frac{1}{32\pi^2} \left( \frac{\Delta}{2} \right)^2 \log \left( \frac{\Delta}{2} \right)^2,$$

which has a unique nontrivial solution $\Delta \neq 0$ if $\Lambda_{\text{res}} > 0$.

We have also checked the existence of the nontrivial solution to the gap equation numerically. In Figure 1, the quantity $\partial V_{1-\text{loop}}/(\Delta \partial \Delta)$ is plotted as a function of $\Delta$. A unique zero point is found, which implies the existence of a nontrivial solution to the gap equation. Although

![Figure 1: The plot of the quantity $\partial V_{1-\text{loop}}/(\Delta \partial \Delta)$ as a function of $\Delta$. A particular set of parameters $c + \frac{1}{64\pi^2} = 1, \Lambda_{\text{res}}/8 = 0.001$ is chosen as an illustration.](image-url)
a particular set of parameters is chosen in the Figure 1 to illustrate this, we have checked the existence of a nontrivial solution in a wide range of parameters. Therefore, we conclude that SUSY is broken by the dynamically generated VEV of D-term in a self-consistent Hartree-Fock approximation.

Note that the value of \( D^0 \) being a maximum of \( V^{(D)}_{1\text{-}\text{loop}} \) is not uncommon in supersymmetric field theories: in fact, at the tree level, \( V^{(D)}_{1\text{-}\text{loop}} \) has a maximum at \( D^a = 0 \) in our case. The stability criterion is given for the entire effective potential with respect to the field space of the scalar vevs and the vacuum we have found is a local minimum. Following a discussion on the estimation of the lifetime of the metastable vacuum in [19], one can show that our metastable SUSY breaking vacuum can be made long-lived by taking the parameter \( m/\Lambda^2 \) to be sufficiently small: an estimate of its decay rate can be given, based on eqs. (32) and (33) in section 6. Space does not permit us to go into the detail here.

5 finding an expansion parameter

The idea of the Hartree-Fock approximation is that the one-loop contributions become sufficiently large and start competing against the tree ones, eventually developing into a new vacuum. In fact, the gap equation has been obtained by matching these two. To make understanding on the validity of the approximation and proper applications to observables, it is desirable to find an expansion parameter. Let us regard the nonvanishing matrix elements of \( \mathcal{F}, g \) and \( W \) and their derivatives to be \( \mathcal{O}(N^2) \). All three terms in the action can be rescaled to have \( N^2 \) in front and \( 1/N^2 \) may replace the original loop expansion parameter \( \hbar \), becoming a new expansion parameter. Here, we will just demonstrate that both the first and the second terms of the gap equation are \( \mathcal{O}(N^2) \) and the present approximation is justified in the large \( N^2 \) limit.

For that purpose, we get back to recast the gap equation into another form with integrations over loop momenta. Recall that the part of the one-loop effective action (1PI vertex functional) which contains the auxiliary field \( D^0 \) is

\[
i\Gamma^{(D)}(\langle \phi_a \rangle, \langle \bar{\phi}_a \rangle, D^0) = \frac{i}{2} \int d^4x g_{00}(D^0)^2 + \frac{1}{2} \sum_a \int d^4x \int \frac{d^4k}{(2\pi)^4} \ln \det \mathcal{P}_a(k, M_F, \bar{M}_F) \tag{24}
\]

where \( \mathcal{P}_a = \gamma^\mu k_\mu - \mathcal{M}_{F,a}, \quad \mathcal{M}_{F,a} = \begin{pmatrix} M_{F,a} & 0 \\ 0 & \bar{M}_{F,a} \end{pmatrix} \). After diagonalizing \( \mathcal{M}_{F,a} \), we obtain

\[
\mathcal{P}_{a,(\text{diag})} = \begin{pmatrix} \gamma \cdot k - |m_a| \lambda^+ & 0 \\ 0 & \gamma \cdot k - |m_a| \lambda^- \end{pmatrix} \tag{25}
\]
\[ m_a = \langle g^{aa} \partial_a \partial_a W \rangle. \] The gap equation reads
\[
0 = i \langle g_{00} \rangle - \frac{1}{2} \sum_a |m_a| \int \frac{d^4 k}{(2\pi)^4} \text{tr} \left[ \frac{1}{D_{a,(diag)}^0 D^0} \frac{\partial}{\partial D^0} \begin{pmatrix} \lambda^+ & 0 \\ 0 & \lambda^- \end{pmatrix} \right].
\] (26)

Here, the inclusion of the contribution from the counterterm at \( \langle g_{00} \rangle \) is understood. After some calculations, we obtain
\[
0 = i \langle g_{00} \rangle - \frac{1}{2} \sum_a \langle g^{aa} F_{aa} \rangle \int \frac{d^4 k}{(2\pi)^4} \frac{k^2 - |m_a|^2 \Delta^2}{(k^2 - |m_a|^2 \lambda^+)^2 (k^2 - |m_a|^2 \lambda^-)^2}. 
\] (27)

In the unbroken phase of the U(\( N \)) gauge group, \( \langle g_{aa,0} \rangle = \langle g_{00,0} \rangle, \langle F_{0aa} \rangle = \langle F_{000} \rangle, \langle \partial_a \partial_a W \rangle = \langle \partial_0 \partial_0 W \rangle \), so that \( m_a = m_0 \). Eq. (27) is actually
\[
0 = i \langle g_{00} \rangle - \frac{N^2}{2} \langle g^{00} \rangle \langle F_{000} \rangle \int \frac{d^4 k}{(2\pi)^4} \frac{k^2 - |m_0|^2 \Delta^2}{(k^2 - |m_0|^2 \lambda^+)^2 (k^2 - |m_0|^2 \lambda^-)^2}. 
\] (28)

The second term is \( O(N^2) \), which is interpreted that an index loop \( a = 0 \cdots N^2 - 1 \) circulates together with the loop momenta.

## 6 Non-vanishing F term induced by \( \langle D^0 \rangle \neq 0 \) and its implications in fermion masses

The above analysis demonstrates that dynamical supersymmetry breaking of the \( \mathcal{N} = 1 \) supersymmetry of the theory given by eq.(11) or eq. (5) is triggered by the non-vanishing value of the \( \langle D \rangle \) term to the leading order in the present approximation. For proper physical assessments and applications to observables, this is not sufficient however. As is already discussed in the introduction, the nonvanishing VEV of F term leads in principle to that of D term and the converse can also be true. Let us see how this takes place in the present situation. For illustration, we will discuss its implications in the fermion mass spectrum.

In the vacuum of nonvanishing \( \Delta \), the VEV’s of the scalar fields get in fact shifted and another order parameter \( \langle F^0 \rangle \) becomes nonvanishing as well as a result of the vacuum condition. The entire effective potential up to one-loop to be extremized is
\[
V = g^{ab} \partial_a W \partial_b W - \frac{1}{2} g_{ab} D^a D^b + V_{1\text{-loop}} + V_{\text{c.t.}}.
\] (29)

Let \( \delta \) be a holomorphic variation of \( V \) with respect to the scalar fields. In the unbroken phase of the vacua of \( \Delta \neq 0 \),
\[
\langle \delta V \rangle = -\langle \delta \phi^0 \rangle \left\{ \langle \partial_0 g_{00} \rangle \langle F^0 \rangle^2 + \langle \partial_0 \partial_0 W \rangle \langle F^0 \rangle + \frac{1}{2} \langle \partial_0 g_{00} \rangle \langle D^0 \rangle^2 - \langle \partial_0 V_{1\text{-loop}} \rangle \right\}.
\] (30)
For the sake of simplicity, we assume here that the fourth prepotential derivatives vanish. Then
\[ \langle \partial_0 V_{1-\text{loop}} \rangle = -2 \langle g^{00} \rangle \langle \partial_0 g_{00} \rangle \langle V_{1-\text{loop}} \rangle. \] (31)

The vacuum condition \( \langle \delta V \rangle = 0 \) reads
\[ |\langle F^0 \rangle|^2 + \frac{m_0}{\langle g^{00} \partial_0 g_{00} \rangle} \langle F^0 \rangle + \frac{1}{2} \langle D^0 \rangle^2 + 2 \langle g^{00} \rangle \langle V_{1-\text{loop}} \rangle = 0. \] (32)

Combining with \( \langle \delta V \rangle = 0 \), we further obtain
\[ \frac{m_0}{\langle g^{00} \partial_0 g_{00} \rangle} \langle F^0 \rangle = \frac{m_0^*}{\langle g^{00} \partial_0 g_{00} \rangle} \langle F^0 \rangle. \] (33)

Eqs. (32), (33) determine the value of non-vanishing F term triggered by the non-vanishing D term.

One implication of this phenomenon is that it is no longer true that the masses of the \( SU(N) \) fermions to the leading order are determined by the second derivative of the superpotential and the nonvanishing \( \langle D^0 \rangle \): the determinations require the non-vanishing value of \( \langle F^0 \rangle \) as well. In fact, the holomorphic part of the fermion mass matrix to the leading order is
\[ \mathcal{L}_{\text{mass}}^{(\text{holo})} = -\frac{1}{2} \langle g_{0a,a} \rangle \langle F^0 \rangle \psi^a \phi^a + \frac{i}{4} \langle F_{0aa} \rangle \langle F^0 \rangle \lambda^a \lambda^a - \frac{1}{2} \langle \partial_a \partial_a W \rangle \psi^a \psi^a + \frac{\sqrt{2}}{4} \langle F_{0aa} \rangle \psi^a \lambda^a \langle D^0 \rangle \]
\[ \equiv -\frac{1}{2} \sum_{a=1}^{N^2-1} \Psi(x)^a \right| \mathcal{M}_{a,a} \Psi^a(x), \quad \Psi^a(x) = \begin{pmatrix} \lambda^a(x) \\ \psi^a(x) \end{pmatrix}. \] (34)

As for gaugino and matter fermions in the \( U(1) \) sector, the hidden sector where the Nambu-Goldstone fermion resides, the index loop circulates in the one-loop self energy part as well, which is, in addition to the above contributions, regarded as the leading contribution to the mass matrix. The massless fermion ensured by the theorem is an admixture of \( \lambda^0 \) and \( \psi^0 \).

7 application

Finally, let us touch upon an application of our dynamical mechanism to supersymmetric particle physics phenomenology. The MSSM (minimal supersymmetric standard model) lagrangian symbolically reads \( \mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge-matter}} \). Here the three terms represent respectively the part containing the vector superfields of the gauge supermultiplets, the one containing the chiral superfields of the matter supermultiplets and the coupling of these two types of superfields. Clearly, the simplest and the most conservative application is to extend just the first term to the type of actions discussed in this letter, so that we do not introduce the mirror
fermions of the standard model which make theory non-chiral and which tend to endanger the asymptotic freedom of QCD, giving rise to a Landau pole. As for the gauge group of this term, we take, for instance, the product of a hidden gauge group and the standard model (SM) gauge group $G' \times G_{SM}$ and $G'$ includes the overall $U(1)$ responsible for the dynamical supersymmetry breaking. Due to the non-Lie algebraic nature of the third prepotential derivatives $F_{abc}$ or $\tau_{abc}$, we do not really need messenger superfields.

The supersymmetry breaking which originates from this sector is then transmitted to the rest of the theory by higher order loop corrections. Once the gaugino masses are generated by our dynamical mechanism, the sfermion masses $m_{s\ell}^2$ are generated in the next loop order and take the following form:

$$m_{s\ell}^2 \sim \frac{C_i(R)\alpha_i}{\pi} m(\Delta)^2 \log \left( \frac{(m_a)^2}{m(\Delta)^2} \right).$$

(35)

Here $m(\Delta)_i$ ($i = SU(3), SU(2), U(1)$) are the gaugino masses of $G_{SM}$ (they correspond with $\Lambda_a^{(-)}$ in eq. (11)), $C_i(R)$ is the quadratic Casimir of representation $R$. This is a general feature common to gaugino mediation, in particular, the proposal of [12] and applies here as well. These scalar masses are positive and flavor-blind and are free from the supersymmetric flavor and CP problems. Furthermore, the generation mechanism of these masses themselves is insensitive to the UV scale [12] and the window to the multi-TEV scale is confined to the sector whose dynamics is discussed in this letter.

The authors’ research is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture, Japan (23540316 (H. I.), 21244036 (N. M.)) and by Keio Gijuku Academic Development Funds (N. M.).

References

[1] L. O’Raifeartaigh, Nucl. Phys. B 96, 331 (1975).

[2] M. T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B 159, 429 (1979).

[3] E. Witten, Nucl. Phys. B 188, 513 (1981); Nucl. Phys. B 202, 253 (1982).

[4] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B 241, 493 (1984); Phys. Lett. B 140, 59 (1984); Nucl. Phys. B 256, 557 (1985).

[5] Y. Meurice and G. Veneziano, Phys. Lett. B 141, 69 (1984); D. Amati, K. Konishi, Y. Meurice, G. C. Rossi and G. Veneziano, Phys. Rept. 162 (1988) 169.
[6] P. Fayet and J. Iliopoulos, Phys. Lett. B 51 (1974) 461; P. Fayet, Nucl. Phys. B 90 (1975) 104.

[7] M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D53 (1996) 2658-2669, hep-ph/9507378; L. M. Carpenter, P. J. Fox and D. E. Kaplan, hep-ph/0503093.

[8] J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

[9] Y. Nambu, Phys. Rev. Lett. 4 (1960) 380; Phys. Rev. 117, 648 (1960).

[10] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; Phys. Rev. 124 (1961) 246.

[11] A. de la Macorra and G. G. Ross, Nucl. Phys. B404, 321 (1993), hep-ph/9210219; G. Faisel, D. W. Jung and O. C. W. Kong, JHEP 1201 (2012) 164, arXiv:1108.0214; W. Buchmuller and S. T. Love, Nucl. Phys. B204 (1982) 213.

[12] P. J. Fox, A. E. Nelson and N. Weiner, JHEP 0208, 035 (2002).

[13] I. Antoniadis, H. Partouche and T. R. Taylor, Phys. Lett. B 372, 83 (1996).

[14] K. Fujiwara, H. Itoyama and M. Sakaguchi, Prog. Theor. Phys. 113 (2005) 429; arXiv:hep-th/0410132, SUSY 04, Tsukuba June 2004;

[15] H. Itoyama, K. Maruyoshi and S. Minato Nucl. Phys. B 830 (2010) 1; K. Fujiwara, H. Itoyama and M. Sakaguchi, Nucl. Phys. B 740 (2006) 58.

[16] K. Fujiwara, H. Itoyama and M. Sakaguchi, Nucl. Phys. B 723 (2005) 33; Prog. Theor. Phys. Suppl. 164 (2007) 125; AIP Conf. Proc. 903 (2007) 521 arXiv:hep-th/0611284.

[17] W. Siegel, Phys. Lett. 84B(1979)193.

[18] S. Coleman, E. Weinberg, Phys. Rev. D7(1973)1888; H. Itoyama, Prog. Theor. Phys. 64(1980)1886

[19] K. A. Intriligator, N. Seiberg and D. Shih, JHEP 0604, 021 (2006).