Natural frequencies of thin rectangular plates clamped on contour using the Finite Element Method

L (Barboni) Haţiegan¹, C Haţiegan², G R Gillich¹, C O Hamat¹, O Vasile³ and M D Stroia²

¹Eftimie Murgu University of Resita, Department of Mechanical and Materials Engineering, Traian Vuia str., no. 1-4, 320085 Resita, Romania
²Eftimie Murgu University of Resita, Department of Electric Engineering and Informatics, Traian Vuia str., no. 1-4, 320085 Resita, Romania
³“Politehnica” University of Bucharest, Department of of Mechanics, Bucharest, Romania

E-mail: c.hatiegan@uem.ro

Abstract. This paper presents the determining of natural frequencies of plates without and with damages using the finite element method of SolidWorks program. The first thirty natural frequencies obtained for thin rectangular rectangular plates clamped on contour without and with central damages for different dimensions. The relative variation of natural frequency was determined and the obtained results by the finite element method (FEM) respectively relative variation of natural frequency, were graphically represented according to their vibration natural modes. Finally, the obtained results were compared.

1. Introduction
A plate is a solid body which has one dimension less than its other two dimensions, called the thickness of the plate and can be regarded as "materialisation" of a surface [1-7].

It will analyze a thin rectangular plate, clamped on the contour of the structural elements, where the action is perpendicular to the median plane. The reference was chosen such that the X and Y axes in the median plane and form a right system, and the Z-axis direction to have the action of gravity.

The actual study will be carried out on two types of plates having the same dimensions, so that the first plate is no damage, the second is central plate is damage.

Damages influence the behavior of dynamic and static structures, mechanical and dynamic change, such as natural frequencies, modal shape, stiffness or flexibility, the degree of damping [8].

The damage is defined as a change in a structure that negatively affects its performance. It can also be defined as a phenomenon where weakening of the structure occurs. In another way, it could be considered as a deviation from the properties of the structure of the material or its geometry, which produce unwanted vibrations, displacements and tensions [8-12].

Study of fault is very important, since a large amount of loss property and term of life can be prevented by identifying the damage. The tendency to use structures even after deterioration, the inability to make proper renovations in a timely manner, places the study of the damage in an even more important position. Studying the damage can help specialists find more effective ways to monitor structure health and even predict possible damages. Numerous algorithms for fault detection have been performed, but many factors remain to be tested until an ideal model is developed. Due to the limits of measuring devices, sensor performance, experimental errors and complexity of structures, only a few effective fault detection methods can be called [13-19].

In recent years, important research has been carried out to develop advanced techniques for obtaining non-destructive evaluation methods to determine the damage. Regular inspection and control
of engineering structures is required from the point of view of real-time damage detection, establishing the safety and reliability of the structure. Early identification of damages during the use of structures, allows scheduled maintenance, with an impact on the reduction of operating costs, or putting them back into operation and replacing the structure, to avoid breakdowns and accidents.

The development of the modern aeronautical industry, the mechanical industry and the civil industry has generated a new impetus for a more rigorous analysis of plate vibration issues and timely damage detection. The recent orientation in the development of plate theory and the detection of damages in these resistance structures is characterized by the use of performing computers using numerical methods, including the finite element method, the results comparing them with experimental data and different theoretical / analytical methods. In the literature, a large number of non-destructive control methods are treated, both local and global; these methods are not mutually exclusive, they can be used additionally [20-38].

Numeric or digital simulation uses computerized modeling and analysis to test static or dynamic behavior of stressed products under different operating conditions. Such an analysis involves, however, the assimilation of working with a specialized assisted design program as well as finite element simulation and analysis programs.

Hypotheses that underlie simulation: the system under analysis may be real or theoretical; We can build a mathematical and logical model of the analyzed system.

Performing a simulation experiment is a process that usually takes place in several stages: problem formulation; the collection and primary processing of data; formulating the simulation model; estimation of operative characteristics parameters in real data; model performance evaluation and parameter testing; building the calculation program (the simulation algorithm); program validation; planning simulation experiences; analysis of simulated data [39-42].

In this paper we determined the own frequencies of a thin rectangular plate clamped on a contour without damage, with a central damage of different sizes using the Simulation module in the SolidWorks program.

2. Analysis of the results obtained for plates with no damages clamped on all sides

The numerical method chosen for modal analysis is the finite element method (FEM) using the SolidWorks program. Modeling of the geometry to be analyzed was done in the Geometry module of the same program. To obtain its own frequencies, the static analysis module for thin rectangular plates was used under its own weight. The results obtained in this module were transferred to the modal analysis module, with the ambient temperature being 20°C. The material used in the program database, Figure 1, is steel AISI 1045, with the mechanical properties presented in Table 1.

| Table 1. Material properties for AISI 1045 steel |
|-----------------------------------------------|
| Flow Limit | Breaking Resistance | Density | Elastic modulus | Poisson Coefficient | Coefficient of thermal expansion |
| [N/mm²]    | [N/mm²]             | [kg/m³] | [N/mm²]         | [-]                  | [K⁻¹]                        |
| 530        | 625                 | 7,850   | 205             | 0.29                 | 0.0000115                    |
Figure 1. Select material from the SolidWorks database

The modal analysis determined the first 30 own frequencies, of which the first 20 own frequencies were taken into account. Meshing 3D model was made with tetrahedral finite elements.

The 3D model used is a real, rectangular plate with the following dimensions: length \( L = 950 \text{ mm} \); Width \( l = 400 \text{ mm} \) and thickness \( h = 2 \text{ mm} \).

After creating the geometry of the plate, creating the frequency study and choosing the material, you will have to go through 4 stages, namely: applying constraints, mesh, modal analysis, and visualizing the results.

Finite element discretization was achieved through the "Solid mesh" option using the following options: High for Mesh Quality, Solid Mesh for Mesh type, Standard mesh, and Curvature based Mesh for Mesher Used. Frequency determination was performed using the Simulation module in the SolidWorks program [41-44].

The following steps are used to calculate your own frequency: create the geometry of solid-state and non-damageive plates in SolidWorks; simulation mode activation; creating simulation study of frequency type and study name; of the frequency-type study option will activate Properties, where the number of calculated frequency modes is set, in this case 30 modes, Figure 2; applying restraints; finite element mesh (mesh) and calculating analysis; view results.

![Figure 1](image)

Figure 2. Clamped-clamped 1, 2, 3 and 4 – Locked in the direction Ox, Oy si Oz

Meshing 3D model was made with tetrahedral elements the average size of 2 mm finite element. Table 2 summarizes the mesh information for the studied plate.

| Type of plate / bearing / size     | Number of nodes | Number of items | Meshing time [min] |
|----------------------------------|-----------------|-----------------|--------------------|
| Clamped without damage 950x400mm | 2,315,689       | 1,352,214       | 05:11              |

To obtain the proper frequencies of the non-damageive plates, the modal analysis module of the SolidWorks finite element analysis software was used. The first 30 vibration frequencies were determined for all plates studied.
In Figure 3 presents nine vibration modes obtained by finite element simulation for a non-damageive plate with calmped-clamped support type.

Figure 3. Examples of vibration modes determined by FEM for clamped-clamped support type

Following the numerical simulation, the frequencies for the clamped plate were obtained on all sides according to the vibration modes (Tables 3 - 6, first column). The results obtained are presented numerically in Tables 3 - 6, 3rd column.

For the determination of the analytically determined own frequencies, the relation [43]:

$$\omega = \pi^2 \cdot \left[ \left( \frac{m + \frac{1}{2}}{a} \right)^2 + \left( \frac{n + \frac{1}{2}}{b} \right)^2 \right] \cdot \sqrt{\frac{D}{\rho \cdot h}}$$

(1)

where:

- $D = \frac{E h^3}{12(1-\nu^2)}$ – the bending stiffness of the plate;
- $\nu$ – cross-contractive coefficient (Poisson coefficient);
- $E$ – module of longitudinal elasticity of the plate;
- $h$ – the thickness of the plate;
- $\rho$ – density of the plate;
- $a, b$ – the length or the width of the plates;

The proprietary frequencies obtained by the FEM are compared with their analytically determined frequencies (Tables 3 - 6, column 2), comparing the percentage error with the relation [43]:

$$\epsilon = \frac{f_A - f_{\text{FEM}}}{f_A} \cdot 100 \%$$

(2)

where:

- $\epsilon$ - it represents the percent deviation or error;
- $f_A$ - the value of analytically determined own frequency;
- $f_{\text{FEM}}$ - The value of the own frequency determined by the FEM.
The values of the percentage deviation are presented in Tables 3 – 6, fourth column.

**Table 3.** Normal frequency for the clamped plate on all sides m=1 and n=1…10

| Vibration mode m-n | Analytical [Hz] | FEM [Hz] | ε [%]  |
|-------------------|-----------------|----------|--------|
| 1-1               | 78,143606       | 73,047   | 6,5221024 |
| 1-2               | 92,866676       | 87,541   | 5,73483989 |
| 1-3               | 115,45148       | 113,94   | 1,30919067 |
| 1-4               | 158,89779       | 152,73   | 3,88160842 |
| 1-5               | 213,20566       | 203,54   | 4,53349128 |
| 1-6               | 268,37512       | 265,93   | 0,91108296 |
| 1-7               | 344,40614       | 339,59   | 1,3983897 |
| 1-8               | 431,29875       | 424,31   | 1,62039653 |
| 1-9               | 529,05293       | 519,99   | 1,71304788 |
| 1-10              | 637,66868       | 626,5    | 1,75148637 |

**Table 4.** Normal frequency for the clamped plate on all sides for modes m=2 and n=1…9

| Vibration mode m-n | Analytical [Hz] | FEM [Hz] | ε [%]  |
|-------------------|-----------------|----------|--------|
| 2-1               | 203,67575       | 193,99   | 4,75547531 |
| 2-2               | 225,39891       | 208,78   | 7,37311019 |
| 2-3               | 257,98363       | 234,18   | 9,22679862 |
| 2-4               | 301,42993       | 279,78   | 7,18240886 |
| 2-5               | 345,73781       | 318,91   | 7,75958233 |
| 2-6               | 410,90726       | 378,66   | 7,84781948 |
| 2-7               | 466,93829       | 449,92   | 3,64465506 |
| 2-8               | 563,8309        | 532,56   | 5,54614868 |
| 2-9               | 681,58507       | 626,45   | 8,08924262 |

**Table 5.** Normal frequency for the clamped plate on all sides m=4 and n=1…4

| Vibration mode m-n | Analytical [Hz] | FEM [Hz] | ε [%]  |
|-------------------|-----------------|----------|--------|
| 4-1               | 632,53827       | 617,59   | 2,36321986 |
| 4-2               | 654,26142       | 632,81   | 3,27872305 |
| 4-3               | 686,84615       | 658,33   | 4,15175218 |
| 4-4               | 730,29245       | 694,4    | 4,91480502 |
Table 6. Normal frequency for the clamped plate on all sides m=3 and n=1…7

| Vibration mode m-n | Analytical [Hz] | FEM [Hz] | ε [%] |
|--------------------|-----------------|----------|-------|
| 3-1                | 387,47398       | 375,67   | 3,04639295 |
| 3-2                | 409,19713       | 390,69   | 4,52279076  |
| 3-3                | 441,78185       | 416,07   | 5,82003312  |
| 3-4                | 485,22816       | 452,15   | 6,81703222  |
| 3-5                | 539,53603       | 499,21   | 7,4742052   |
| 3-6                | 604,70549       | 557,51   | 7,80470672  |
| 3-7                | 680,73651       | 627,11   | 7,87771909  |

In Figures 4, 6, 8 and 10 it’s present the frequency variation according to the vibration modes for the analyzed cases and in Figures 5, 7, 9 and 11 the percentage deviation of the frequency from the analytically determined frequency is plotted, depending on the vibration modes for these cases.

![Figure 4. Frequency variation according to vibration modes for m=1 and n=1÷10](image1)

![Figure 5. Percentage frequency deviation for m=1 and n=1÷10](image2)

![Figure 6. Frequency variation according to vibration modes for m=2 and n=1÷9](image3)

![Figure 7. Percentage frequency deviation for m=2 and n=1÷9](image4)
Input frequency values for the clamped plate on all sides \(m=3 \text{ and } n=1...7\)

![Input frequency values for the clamped plate on all sides \(m=3 \text{ and } n=1...7\)](image)

**Figure 8.** Frequency variation according to vibration modes for \(m=3\) and \(n=1\div 7\)

Deviation \(m=3\) and \(n=1...7\)

![Deviation \(m=3\) and \(n=1...7\)](image)

**Figure 9.** Percentage frequency deviation for \(m=3\) and \(n=1\div 7\)

Input frequency values for the clamped plate on all sides \(m=4 \text{ and } n=1...4\)

![Input frequency values for the clamped plate on all sides \(m=4 \text{ and } n=1...4\)](image)

**Figure 10.** Frequency variation according to vibration modes for \(m=4\) and \(n=1\div 4\)

Deviation \(m=4\) and \(n=1...4\)

![Deviation \(m=4\) and \(n=1...4\)](image)

**Figure 11.** Percentage frequency deviation for \(m=4\) and \(n=1\div 4\)

From the results presented in Figures 5, 7, 9, 11 and Tables 3 - 6 it is observed that the maximum error is 9.3% for mode \(m=2\) and \(n=3\) it is also noted that larger deviations occur in mode \(m=2\).

3. **Analysis of the results obtained for clamped plates on all sides with longitudinal central damage**

For modal analysis of damageive plates, the Finite Element Method (FEM) was also used using the SolidWorks program. Modeling of the geometry to be analyzed was done in the Geometry module of the same program.

To obtain its own frequencies, the static analysis module for thin rectangular plates was used under its own weight. The results obtained in this module were transferred to the modal analysis module. By modal analysis the first 30 frequencies were determined. Meshing 3D model was made with 2 mm thick tetrahedron finite elements (Table 7), the material used in the program database being identical to the one used in the previous subchapter.

In Figure 12 the longitudinal central damage plate is clamped in the contour, using modal analysis with the finite element method.
Figure 12. Type of support for plate with longitudinal central damage

Table 7. Meshing information

| No. | Type of plate / bearing / size                        | Number of nodes | Number of items | Meshing time [min] |
|-----|-------------------------------------------------------|-----------------|-----------------|--------------------|
| 1   | No damage                                             | 2,489,930       | 1,476,647       | 04:42              |
| 2   | With longitudinal central damage 5.3 × 24.8 mm       | 2,477,378       | 1,437,411       | 04:35              |
| 3   | With longitudinal central damage 5.8 × 48.8 mm       | 2,517,888       | 1,467,061       | 04:38              |
| 4   | With longitudinal central damage 6.05 × 75.2 mm      | 2,510,268       | 1,460,333       | 05:25              |
| 5   | With longitudinal central damage 4.9 × 101.6 mm      | 2,763,438       | 1,648,846       | 05:25              |

In Figure 13 there are ten modes of vibration obtained by finite element simulation for a plate with longitudinal central damage clamped on the contour.

Following the numerical simulation, the actual frequencies for a real clamped plate on the contour were obtained, according to their own vibration modes with a longitudinal central damage having the dimensions: 5.3 × 24.8 mm; 5.8 × 48.8 mm; 6.05 × 75.2 mm; 4.9 × 101.6 mm. The results obtained are presented numerically in Tables 8-11 and the percent deviation values calculated by the formula (2) are presented in Tables 12 - 15, for each analyzed case.

Figure 13. Examples of vibration modes for the central fault plate 50×50 mm
### Table 8. Normal frequency for the clamped plate with central damage \( m=1 \) and \( n=1 \ldots 10 \)

| Vibration mode \( m-n \) | No damage | Damage 5,3×24,8 mm | Frequency [Hz] | Damage 5,8×48,8 mm | Damage 6,05×75,2 mm | Damage 4,9×101,6 mm |
|--------------------------|-----------|---------------------|----------------|---------------------|---------------------|---------------------|
| 1-1                      | 73,047    | 72,903              | 72,497         | 71,846              | 70,922              |
| 1-2                      | 87,541    | 87,54               | 87,535         | 87,529              | 87,491              |
| 1-3                      | 113,94    | 113,81              | 113,49         | 113,01              | 112,45              |
| 1-4                      | 152,73    | 152,73              | 152,71         | 152,68              | 152,57              |
| 1-5                      | 203,54    | 203,38              | 203,08         | 202,67              | 202,25              |
| 1-6                      | 265,93    | 265,92              | 265,86         | 265,79              | 265,57              |
| 1-7                      | 339,59    | 339,38              | 339,03         | 338,47              | 337,69              |
| 1-8                      | 424,31    | 424,27              | 424,16         | 423,99              | 423,7               |
| 1-9                      | 519,99    | 519,75              | 519,51         | 519,33              | 519,31              |
| 1-10                     | 626,5     | 626,47              | 626,32         | 625,97              | 625,67              |

### Table 9. Normal frequency for the clamped plate on all sides with central damage \( m=2 \) and \( n=1 \ldots 9 \)

| Vibration mode \( m-n \) | No damage | Damage 5,3×24,8 mm | Frequency [Hz] | Damage 5,8×48,8 mm | Damage 6,05×75,2 mm | Damage 4,9×101,6 mm |
|--------------------------|-----------|---------------------|----------------|---------------------|---------------------|---------------------|
| 2-1                      | 193,99    | 193,96              | 193,82         | 193,39              | 192,51              |
| 2-2                      | 208,78    | 208,75              | 208,71         | 208,66              | 208,63              |
| 2-3                      | 234,18    | 234,15              | 233,98         | 233,42              | 232,2               |
| 2-4                      | 270,78    | 270,68              | 270,57         | 270,46              | 270,34              |
| 2-5                      | 318,91    | 318,88              | 318,64         | 317,8               | 315,86              |
| 2-6                      | 378,66    | 378,53              | 378,37         | 378,21              | 377,89              |
| 2-7                      | 449,92    | 449,87              | 449,54         | 448,25              | 445,01              |
| 2-8                      | 532,56    | 532,42              | 532,23         | 532,04              | 531,06              |
| 2-9                      | 626,45    | 626,26              | 626,03         | 625,14              | 624,36              |

### Table 10. Normal frequency for the clamped plate on all sides with central damage \( m=3 \) and \( n=1 \ldots 7 \)

| Vibration mode \( m-n \) | No damage | Damage 5,3×24,8 mm | Frequency [Hz] | Damage 5,8×48,8 mm | Damage 6,05×75,2 mm | Damage 4,9×101,6 mm |
|--------------------------|-----------|---------------------|----------------|---------------------|---------------------|---------------------|
| 3-1                      | 375,67    | 374,56              | 371,64         | 366,91              | 359,92              |
| 3-2                      | 390,69    | 390,68              | 390,65         | 390,6               | 390,39              |
| 3-3                      | 416,07    | 415,12              | 413            | 410,37              | 407,87              |
| 3-4                      | 452,15    | 452,11              | 452            | 451,86              | 451,18              |
| 3-5                      | 499,21    | 498,38              | 496,55         | 494,18              | 492,24              |
| 3-6                      | 557,51    | 557,44              | 557,23         | 556,95              | 555,77              |
| 3-7                      | 627,11    | 626,41              | 624,93         | 622,95              | 621,74              |
### Table 11. Normal frequency for the clamped plate on all sides with central damage m=4 and n=1…4

| Vibration mode m-n | Frequency [Hz] |
|--------------------|----------------|
| No damage          |                |
| Damage 5.3×24.8 mm |                |
| Damage 5.8×48.8 mm |                |
| Damage 6.05×75.2 mm|                |
| Damage 4.9×101.6 mm|                |

| Damage 5.3×24.8 mm | 617,59 | 617,31 | 615,73 | 608,99 | 583,47 |
| Damage 5.8×48.8 mm | 632,81 | 632,78 | 632,73 | 632,66 | 632,49 |
| Damage 6.05×75.2 mm| 658,33 | 658,06 | 656,69 | 652,47 | 645,58 |
| Damage 4.9×101.6 mm| 694,4  | 694,23 | 694,13 | 693,84 | 693,06 |

### Table 12. Deviation normal frequency for the clamped plate on all sides with a longitudinal central damage m=1 and n=1…10

| Vibration mode m-n | Damage 5.3×24.8 mm | Damage 5.8×48.8 mm | Damage 6.05×75.2 mm | Damage 4.9×101.6 mm |
|--------------------|--------------------|--------------------|---------------------|--------------------|
| 1-1                | 0,197133           | 0,7529             | 1,64417             | 2,909086           |
| 1-2                | 0,001142           | 0,0069             | 0,013708            | 0,057116           |
| 1-3                | 0,114095           | 0,3949             | 0,816219            | 1,307706           |
| 1-4                | 0                  | 0,0131             | 0,032738            | 0,10476            |
| 1-5                | 0,078609           | 0,226              | 0,427434            | 0,633782           |
| 1-6                | 0,00376            | 0,0263             | 0,052645            | 0,135374           |
| 1-7                | 0,061839           | 0,1649             | 0,329809            | 0,559498           |
| 1-8                | 0,009427           | 0,0354             | 0,075417            | 0,143763           |
| 1-9                | 0,046155           | 0,0923             | 0,126926            | 0,130772           |
| 1-10               | 0,004789           | 0,0287             | 0,084597            | 0,132482           |

### Table 13. Deviation normal frequency for the clamped plate on all sides with a longitudinal central damage m=2 and n=1…9

| Vibration mode m-n | Damage 5.3×24.8 mm | Damage 5.8×48.8 mm | Damage 6.05×75.2 mm | Damage 4.9×101.6 mm |
|--------------------|--------------------|--------------------|---------------------|--------------------|
| 2-1                | 0,015465           | 0,0876             | 0,309294            | 0,762926           |
| 2-2                | 0,014369           | 0,0335             | 0,057477            | 0,071846           |
| 2-3                | 0,012811           | 0,0854             | 0,324537            | 0,845503           |
| 2-4                | 0,03693            | 0,0776             | 0,118177            | 0,162494           |
| 2-5                | 0,009407           | 0,0847             | 0,348061            | 0,956383           |
| 2-6                | 0,034332           | 0,0766             | 0,11884             | 0,203349           |
| 2-7                | 0,011113           | 0,0845             | 0,371177            | 1,091305           |
| 2-8                | 0,026288           | 0,062              | 0,097642            | 0,281658           |
| 2-9                | 0,03033            | 0,067              | 0,209115            | 0,333626           |
Table 14. Deviation normal frequency for the clamped plate on all sides with a longitudinal central damage m=3 and n=1…7

| Vibration mode m-n | Damage 5,3×24,8 mm | Damage 5,8×48,8mm | Damage 6,05×75,2mm | Damage 4,9×101,6mm |
|-------------------|--------------------|-------------------|--------------------|--------------------|
| 3-1               | 0,295472           | 1,0728            | 2,331834           | 4,192509           |
| 3-2               | 0,00256            | 0,0102            | 0,023036           | 0,076787           |
| 3-3               | 0,228327           | 0,7379            | 1,369962           | 1,970822           |
| 3-4               | 0,008847           | 0,0332            | 0,064138           | 0,214531           |
| 3-5               | 0,166263           | 0,5328            | 1,007592           | 1,396206           |
| 3-6               | 0,012556           | 0,0502            | 0,100447           | 0,312102           |
| 3-7               | 0,111623           | 0,3476            | 0,66336            | 0,856309           |

Table 15. Deviation normal frequency for the clamped plate on all sides with a longitudinal central damage m=4 and n=1…4

| Vibration mode m-n | Damage 5,3×24,8 mm | Damage 5,8×48,8mm | Damage 6,05×75,2mm | Damage 4,9×101,6mm |
|-------------------|--------------------|-------------------|--------------------|--------------------|
| 4-1               | 0,045338           | 0,3012            | 1,39251            | 5,524701           |
| 4-2               | 0,004741           | 0,0126            | 0,023704           | 0,050568           |
| 4-3               | 0,041013           | 0,2491            | 0,890131           | 1,936719           |
| 4-4               | 0,024482           | 0,0389            | 0,080645           | 0,192972           |

Figure 14. Frequency variation according to vibration modes for m=1 and n=1÷10

Figure 15. Frequency variation according to vibration modes for m=2 and n=1÷9
Normal frequency depending on its shape modes for the plate with central transverse damage (m=3 and n=1...7)

Figure 16. Frequency variation according to vibration modes for m=3 and n=1÷7

Normal frequency depending on its shape modes for the plate with central transverse damage (m=4 and n=1...4)

Figure 17. Frequency variation according to vibration modes for m=4 and n=1÷4

Percentage frequency deviation for m=1 and n=1...10

Figure 18. The relative frequency variation depending on the modes vibration for m=1 and n=1÷10

Percentage frequency deviation for m=2 and n=1...9

Figure 19. The relative frequency variation depending on the modes vibration for m=2 and n=1÷9
Percentage frequency deviation for \( m=3 \) and \( n=1...7 \)

**Figure 20.** The relative frequency variation depending on the modes vibration for \( m=3 \) and \( n=1\cdots7 \)

Percentage frequency deviation for \( m=4 \) and \( n=1...4 \)

**Figure 21.** Relative frequency variation according to vibration modes for \( m=4 \) and \( n=1\cdots4 \)

From the results presented in Figures 18 - 21 and tables 12 - 15 it is noted that the maximum error is 5.52% for the \( m = 4 \) mode and \( n = 1 \) for the defective plate of \( 4.9 \times 101.6 \) mm it is also noted that larger deviations occur in the defect plate of \( 4.9 \times 101.6 \) mm.

4. **Conclusions**

From the modal analysis regarding the determination of the own frequencies and the mass participation coefficients, the following conclusions can be made:

- The meshing in finite element for all the analyzed plates was made using the same solid mesh type, resulting in total numbers of nodes and different elements for each type of plate being analyzed;
- For each analyzed plate, the value of natural frequencies gradually increases from the first mode to the last mode of vibration;
- Frequency values of the plain back plates are in all cases higher than those of the plain plates that are lengthened in width and free in length; This confirms that a flat and flawless plate has the highest values compared to other types of plates studied by the authors.

Analysis of defective contour clamped plates in a wide variety of sizes, positions, and plate orientation has highlighted the fact that each fault has its own significant signature by the relative frequency variation for a reasonable number of vibration modes \( m \) and \( n \).

In the case of small size defects, these "signatures" are very close to those obtained in the case of bars with bigger defects, i.e., when changing the defect sides. In order to increase them, disturbances are introduced in the "signature" due to the essential mass change and the modal distribution.

In order to obtain credible results, an analysis of the influence of the mean sizes of the elements used in the modeling was made and chosen for various applications that lead to sufficiently precise results.
References

[1] Haţiegan C, Nedeloni M D, Tufoi M, Protocil C and Răduca M 2013 Modal Analysis Of Natural Frequencies And Mass Participation Coefficients Of Simply Supported Thin Plates With Damages, *Constanţa Maritime University Annals* XIV(19) 115-120

[2] Haţiegan C, Tufoi M, Răduca E, Protocil C and Mituleţu C 2013 Modal Analysis Through Solidworks Software Of Clamped Thin Plates With Damages, *Constanţa Maritime University Annals* XIV(19) 121-124

[3] Haţiegan C, Gillich G R, Gillich N, Tufoi M and Răduca M 2013 Modal Analysis of Thin Plates with Damage Simply Supported on all Edges, *Analele Universităţii "Eftimie Murgu"-Fascicula de Inginerie* XXI(1) 117-126

[4] Haţiegan C, Gillich G R, Răduca E, Nedeloni M D and Cîndea L 2013 Equation of Motion and Determining the Vibration Mode Shapes of a Rectangular Thin Plate Simply Supported on Contour Using MATLAB, *Analele Universităţii "Eftimie Murgu"-Fascicula de Inginerie* XXI(1) 127-138

[5] Tufoi M, Gillich G R, Praisach Z I, Ntakpe J L and Haţiegan C 2014 An Analysis of the Dynamic Behavior of Circular Plates from a Damage Detection Perspective, *Romanian Journal of Acoustics & Vibration* X(1) 41-46

[6] Haţiegan C, Gillich G R, Răduca M, Budai A M, Muntean F and Răduca E 2012 Finite Element nalysis Of Natural Frequencies And Mass Participation Coefficients For Thin Plates With Defects, *Scientific Bulletin of “Politehnica” University of Timişoara* 57(71)2 43-53

[7] Leissa A W 1969 *Vibration of Shells*, NASA SP-288, Washington, DC

[8] Tufoi M, Gillich G R, Mituleţu I C and Hatiegan C 2015 An Analysis of the Dynamic Behavior of Rectangular Plates from a Damage Detection Approach, *Romanian Journal of Acoustics & Vibration* XII(2) 146-150

[9] Tufoi M, Haţiegan C, Vasile and O Gillich G R 2013 Dynamic Analysis of Thin Plates with Defects by Experimental and FEM Methods, *Romanian Journal of Acoustics and Vibration* X(2) 83-88

[10] Tufoi M, Gillich G R, Haţiegan C, Mituleţu I C and Vasile I 2014 Effects of Structural Defects on Dynamic Behavior at Sandwich Composite Beams- Part I-Theoretical Approach, *Analele Universităţii "Eftimie Murgu"-Fascicula de Inginerie* XXI(1) 327-334

[11] Tufoi M, Gillich G, Haţiegan C, Mituleţu I C and Korka Z I 2014 Effects of Structural Defects on Dynamic Behavior at Sandwich Composite Beams- Part II- FEM Analysis, *Analele Universităţii "Eftimie Murgu"-Fascicula de Inginerie* XXI(1) 335-344

[12] Nicknam A, Hosseini M H and Bagheri A 2011 Damage detection and denoising in two-dimensional structures using curvelet transform by wrapping method, *Arch Appl Mech* 81 1915-1924

[13] Choi S, Park S and Stubbs N 2005 Nondestructive damage detection in structures using changes in compliance, *International Journal of Solids and Structures* 42(15) 4494-4513

[14] Choi S H 2002 Development of Nondestructive Damage Detection Algorithms for Plate Structures, Strength of Materials, *Journal of Civil Engineering* 6(4) 495-501

[15] Eftekhari S A and Jafari A A 2013 A simple and accurate Ritz formulation for free vibration of thick rectangular and skew plates with general boundary conditions, *Acta Mech* 224 193-209

[16] Nicknam A and Hosseini M H 2012 Structural damage localization and evaluation based on modal data via a new evolutionary algorithm, *Arch Appl Mech* 82 191-203

[17] Haţiegan C, Nedeloni M D, Popescu C, Tufoi M, Pădureanu I and Rudolf C 2015 Comparative Study through Modal Analysis of Thin Trapeze Shape Plates Clamped on Contour without and with Damages, *Analele Universităţii "Eftimie Murgu"-Fascicula de Inginerie*, Anul XXII(2) 148-161

[18] Nedeloni M D, Haţiegan C, Muntean F, Jurcu M, Terfăloagă I M and Magheţ P 2015 A Modal Analysis Study of Thin Parallelogram Plates Clamped on the Rim with and without Defects Using the SolidWorks Program, *Analele Universităţii "Eftimie Murgu"-Fascicula de Inginerie* XXII(2) 273-288

[19] Gillich G R, Praisach Z I, Bobos D and Hatiegan C 2013 Assessment of corrosion damages with important loss of mass and influences on the natural frequencies of bending vibration modes,
Acoustics & Vibration of Mechanical Structures, Book Series: Applied Mechanics and Materials, vol. 430, pp 95-100

Haţiegan C, Gillich G R, Popovici G and (Barboni) Haţiegan L 2017 The influence of temperature changes on the eigen - frequencies and mode shapes for rectangular plates clamped on the contour, International Conference Knowledge-Based Organization, Vol. 23, Issue 3, pp 168-173

Bratu P 2000 Vibration of elastic systems, Technical Publishing House, Bucharest, Romania

Buzdugan G, Mihăilescu E and Radeş M 1979 Măsurarea vibraţiilor, Ed. Academiei, Bucuresti, Romania

Elshafey A A, Marzouk H and Haddara M R 2011 Experimental Damage Identification Using Modified Mode Shape Difference, J. Marine Sci. Appl. 10 150-155

Feng G Q, Li G, Liu Z H, Niu H L and Li F C 2010 Numerically Simulating the Sandwich Plate System Structures, J. Marine Sci. Appl. 9 286-291

Elishakoff I and Sternberg A 1980 Vibration of Rectangular Plates with Edge-Beams, Acta Mechanica 36 195-212

Bratu P 2001 Structural requirements imposed to vibration systems, International Journal of Acoustics and Vibration 5(2) 84-92

Monfared A H 2012 Numerical Simulation of Welding Distortion in Thin Plates, Journal of Engineering Physics and Thermophysics 85(1) 187-194

Tufoi M, Gillich G R, Haţiegan, Gillich N and Lorenz P 2015 Some Aspects Regarding the Transition from Beam to Plate Behavior of Vibrating Structures, Romanian Journal of Acoustics & Vibration XII(1) 62-68

Nedeloni M D, Haţiegan C, Vasile O, Hamat C O, Fânică C and Gillich N 2015 Numerical Study Regarding the Influence of Material Components for a Booster - Ultrasonic Horn Assembly on the Natural Frequency, Romanian Journal of Acoustics & Vibration XII (2) 155-160

Haţiegan C, Pădureanu I, Jurcu M, Nedeloni M D, Hamat C O, Chioncel C P, Trocaru S, Vasile O, Bădescu O, McIluic D, (Filip) Nedeloni L, Băra A and (Barboni) Haţiegan L 2016 Vibration analysis of a hydro generator for different operating regimes, IOP Conf. Ser.: Mater. Sci. Eng. 163 012030

Iancu V, Protocil C, Gillich N, Haţiegan C and Gillich G R 2016 The Influence of the Number of Finite Elements upon the Accuracy of the Results Obtained Using Discrete Models, Analele Universității "Eftimie Murgu"-Fascicula de Inginerie XX(3) 189-198

Hamat C O, Nedeloni M D, Haţiegan C, Ciubotariu R C and Pădureanu I 2015 Cavitation Erosion Research On C45 Carbon Steel. Part I: Multiple Tests Of 180 Minutes, Annals of „Constantin Brâncuși” University of Târgu Jiu 3 127-132

Nedeloni M D, Haţiegan C, Vasile O, Hamat C O, Fânică C and Gillich N 2015 Numerical study regarding the influence of material components for a booster - ultrasonic horn assembly on the natural frequency, Annals of „Constantin Brâncuși” University of Târgu Jiu 3 100-105

Muntean F, Haţiegan C, Popescu C, Gillich E V and Iancu V 2015 Study Regarding The Determining Of The Natural Frequencies And Modal Shapes Of The Column Type Structures With Additional Mass, Annals of „Constantin Brâncuși” University of Târgu Jiu 3 52-55

Nedeloni M D, Haţiegan C, (Filip) Nedeloni L, (Barboni) Haţiegan L, Birtărescu E, Băra A and Pellac A 2016 Research on the cavitation erosion resistance of the x3crni13-4 stainless steel used to manufacture the runner blades of kaplan turbines, Annals of Constantin Brăncuși University of Târgu Jiu 3 70-75

Pop C, Hatiegan C, Vlase T, Raduca M, Gillich G R, Vasile O and Raduca E 2014 Fermionic oscillators and their connection with the isokinetic temperature, Romanian Reports in Physics 66(3) 716-722

Haţiegan C, Molnar M, Trocaru S, Pădureanu I, Jurcu M R and Ilie F 2016 Modeling and Simulation of Thermal Analysis of a Teflon Coated Plate, International Conference Knowledge-Based Organization, Vol. 22, Issue 3, pp 639-643
[38] Nicolina P, Popov D and Hațiegan C 2013 Some Theoretical Observations Concerning the Reverberation Time for the Case of a Harmonic Emitting Source, *Analele Universității "Eftimie Murgu", Fascicula de Inginerie* XX(2) 251-258

[39] Hațiegan C, Gillich E V, Vasile O, Nedeloni M D and Pădureanu I 2015 Finite Element Analysis of thin plates clamped on the rim of different geometric forms. Part I: Simulating the Vibration Mode Shapes and Natural Frequencies, *Romanian Journal of Acoustics & Vibration* XII(1) 69-74

[40] Hațiegan C, Gillich E V, Vasile O, Nedeloni M D, Jurcu M and Magheți P 2015 Finite Element Analysis of thin plates clamped on the rim of different geometric forms. Part II: The Absolute and Relative Variation of Natural Frequencies, *Romanian Journal of Acoustics & Vibration* XII(1) 81-86

[41] Tufoi M, Hațiegan C, Gillich G R, Protocsil C and Negru I 2013 Frequency Changes in Thin Rectangular Plates due to Geometrical Discontinuities. Part I: Finite Element Analysis, *Analele Universității "Eftimie Murgu"-Fascicula de Inginerie* XX(3) 221-232

[42] Tufoi M, Hațiegan C, Gillich G R and Vasile O 2013 Frequency Changes in Thin Rectangular Plates due to Geometrical Discontinuities. Part II: Frequency Shift Interpretation, *Analele Universității "Eftimie Murgu"-Fascicula de Inginerie* XX(3) 233-244

[43] Hațiegan C 2013 *Identification of Defects in Thin Elastic Plates by Means of Modal Analysis*, University "Eftimie Murgu" of Resita, Doctoral Thesis

[44] Hațiegan C and Suciu L 2010 *Fizică tehnologică-Teorie și aplicații*, Editura „Eftimie Murgu” Reșița, Orizonturi Tehnice