Grand unification of flavor mixings

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received 24 August 2012; accepted in final form 16 October 2012
published online 12 November 2012

PACS 12.15.Ff – Quark and lepton masses and mixing
PACS 14.60.Pq – Neutrino mass and mixing
PACS 14.60.St – Non-standard-model neutrinos, right-handed neutrinos, etc.

Abstract – The origin of flavor mixings in the quark and lepton sectors is still a mystery, and the structure of flavor mixings in the lepton sector seems completely different from that of the quark sector. In this letter, we point out that the flavor mixing angles in the quark and lepton sectors could be unified at a high-energy scale, when neutrinos are degenerate. It means that a minimal flavor violation at a high-energy scale can induce a rich variety of flavor mixings in the quark and lepton sectors at a low-energy scale through quantum corrections.

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The origin of flavor mixings is one of the most important mystery in elementary-particle physics. The structure of flavor mixings in the quark sector has been investigated as the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. On the other hand, neutrino oscillation experiments have revealed that the lepton sector has completely different flavor mixings, represented by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [2], in which there are one large mixing angle \( \theta_{12} \), one nearly maximal mixing angle \( \theta_{23} \) [3], and non-vanishing \( \theta_{13} \) that is pointed out by recent long baseline and reactor experiments [4]. Anyhow, both flavor structures seem completely different from each other, and this situation motivates us to pursue the origin of flavor violation.

In this letter, we will investigate the possibility that CKM and PMNS flavor mixing angles are unified at a high-energy scale. This is a kind of “grand unification of flavor mixings (GUFM)”, where a minimal flavor violation at a high-energy scale can induce a rich variety of flavor mixing structures in both the quark and the lepton sectors at a low-energy scale. A similar possibility has been studied in [5–7]. Such possibility in [5] has been realized by a radiative magnification [8]. The idea of radiative magnification has originally been proposed for the neutrino mixing angles but not for a unification of the CKM and PMNS mixing angles (see [9] for radiative magnification models). References [5–7] give some typical examples with the radiative magnification which can cause the GUFM. Reference [10] has applied the GUFM to phenomenological discussions, i.e. it has been shown that there is a correlation between lower bounds on the masses of super-particles and the upper bound on the sum of neutrino masses. Our purposes in this work are to clarify the parameter space and give some bounds on physical parameters at low energy for the realization of the GUFM rather than the construction of a high-energy model, which realizes the GUFM, and phenomenological applications. Therefore, we will take a bottom-up approach with renormalization group equations (RGEs) and experimentally observed values at low energy as input, which includes the recent update of the value of \( \theta_{13} \). Then, we will show that quantum corrections and degeneracy of neutrino masses play crucial roles for the realization of GUFM. As for the mass degeneracy, we should remind that only neutrinos can be degenerate among matter fermions. There is also a work similar to the GUFM, which is the quark-lepton similarity [11]. Reference [11] has pointed out that the PMNS matrix at a high-energy scale can be connected to the CKM matrix by a transformation. We will consider the possibility of GUFM without introducing such special transformation, i.e., the GUFM will be discussed under the RGEs with standard PDG parameterization for both CKM and PMNS matrices.

We take a setup of the minimal supersymmetric standard model (MSSM) with the Weinberg operator [12], where Yukawa interactions are given by

\[
\mathcal{L}_Y = -y_d \bar{Q_L} H d_R - y_u \bar{Q_L} H u_R - y_e \bar{L} H e_R + \kappa (H_u L)(H_u L) + h.c.
\]
Here $Q_L$ are the left-handed quarks, $u_R (d_R)$ are the right-handed up(down)-type quarks, $e_R$ are the right-handed charged leptons, $H_u (H_d)$ is the up(down)-type Higgs, and $y_\alpha (\alpha = u, d, e)$ are the corresponding Yukawa couplings, respectively. The Weinberg operator can be obtained by integrating out a heavy particle(s), for example, right-handed neutrinos with masses of order $10^{14–16}$ GeV in the type-I seesaw mechanism [13]. Thus, an effective coupling $\kappa$ is carrying mass dimension $-1$ as $\mathcal{O}(10^{14–16})$ GeV$^{-1}$. We also utilize PDG parameterization [3] for the CKM (V$_{\text{CKM}}$) and PMNS (V$_{\text{PMNS}}$) matrices as $V_{\text{CKM}} \equiv V_{\alpha L}^\dagger V_{\beta L}^\dagger$, and $V_{\text{PMNS}} \equiv V_{\alpha L}^\dagger V_{\beta L}^\dagger D_p$, respectively, where $V_{\alpha L}$ are unitary matrices diagonalizing Yukawa coupling as $V_{\alpha L}^\dagger u_L = y_{\alpha u} V_{\alpha L}$, and $D_p$ is a diagonal phase matrix as $D_p \equiv \text{Diag} \{ e^{i\epsilon_1}, e^{i\epsilon_2}, 1 \}$. The light (active) neutrinos are diagonalized as $V_{\alpha L}^\dagger \nu_L = y_{\alpha \nu}^\dagger \nu_L$ and $D_p \equiv \kappa \nu^\dagger$. Notice that two Majorana phases in $D_p$ are included in the PMNS matrix.

$$\text{RGE of } \kappa \text{ is given by [14]}$$

$$16\pi^2 \frac{d\kappa}{dt} = 6 \left[ -\frac{1}{5} g_1^2 - g_2^2 + \text{Tr} \left( y_u^T y_u \right) \right] \kappa + \left[ (y_e y_L^T) \kappa + \kappa \left( y_e y_L^T \right)^T \right],$$

(2)

where $g_1, g_2$ are gauge coupling constants. We can show the PMNS matrix at a high-energy scale, $\Lambda$, through the neutrino mass matrix at $\Lambda$, $M_\nu (\Lambda) = \kappa (\Lambda) v_u^\dagger$, by use of eq. (2). The $M_\nu (\Lambda)$ is given by $M_\nu (\Lambda) = \Lambda M_\nu (\Lambda_{\text{EW}})$ [15–18], where $\Lambda_{\text{EW}}$ is a low-energy (electroweak) scale and $I \equiv \text{Diag} \{ \sqrt{T_e}, \sqrt{T_\mu}, \sqrt{T_\tau} \}$. Here, $I_\alpha$’s ($\alpha = e, \mu, \tau$) denote quantum corrections, which are defined by $I_\alpha = \exp \left[ \frac{i}{\pi} \int_{\alpha \in \text{in} (\Lambda_{\text{EW}})} \frac{g_2^2}{T_\alpha} dt \right]$. A dominant effect of the quantum corrections comes from $y_{\nu e}$, and we define the small parameters as $\epsilon_{\nu e (\mu)} \equiv 1 - \sqrt{\frac{T_e}{T_\mu}}$. Typical values of $\epsilon_{\nu e (\mu)}$ have been given in [17], and we take a region $10^{-3} \lesssim \epsilon_{\nu e (\mu)} \lesssim 0.1$, which corresponds to $\mathcal{O}(10) \lesssim \tan \beta \lesssim \mathcal{O}(50)$ with $\kappa^2 \lesssim 10^{13}$ GeV. In the following analyses, we take a good approximation of $\epsilon \equiv \epsilon_{\nu e} = \epsilon_\mu$. Then, the $M_\nu (\Lambda)$ is given by

$$M_\nu (\Lambda) = \begin{pmatrix} m_{11} & m_{12} & m_{13}(1+\epsilon) \\ m_{21} & m_{22} & m_{23}(1+\epsilon) \\ m_{31}(1+\epsilon) & m_{32}(1+\epsilon) & m_{33}(1+2\epsilon) \end{pmatrix},$$

(3)

and

$$(M_\nu (\Lambda_{\text{EW}}))_{ij} \equiv m_{ij} = (V_{\text{PMNS}}^\dagger (\Lambda_{\text{EW}}) M_\nu^\dagger (\Lambda_{\text{EW}}) V_{\text{PMNS}}^\dagger (\Lambda_{\text{EW}}))_{ij}.$$ (4)

Note that we take a diagonal basis of the charged lepton Yukawa matrix.

Now let us investigate the effects of radiative corrections for the PMNS mixing angles. Numerical results are shown in figs. 1 and 2. We have performed scatter plots with the following input parameters.

For the mass spectra of light neutrinos, we take two types of neutrino mass ordering, normal hierarchy (NH) $m_1 < m_2 < m_3$ and inverted hierarchy (IH) $m_3 < m_1 < m_2$, since the neutrino oscillation experiments determine only two mass squared differences, $\Delta m^2_{21} \equiv |m_2^2 - m_1^2|$ and $|\Delta m^2_{32}| \equiv |m_3^2 - m_2^2|$. At the $\Lambda_{\text{EW}}$ scale, the NH case suggests

$$m_1 (\Lambda_{\text{EW}}) = \sqrt{m^2_3 (\Lambda_{\text{EW}}) - |\Delta m^2_{32}| - \Delta m^2_{21}},$$ (5)

$$m_2 (\Lambda_{\text{EW}}) = \sqrt{m^2_3 (\Lambda_{\text{EW}}) - |\Delta m^2_{21}|},$$ (6)

while the IH case suggests

$$m_1 (\Lambda_{\text{EW}}) = \sqrt{m^2_3 (\Lambda_{\text{EW}}) - \Delta m^2_{21}},$$ (7)

$$m_3 (\Lambda_{\text{EW}}) = \sqrt{m^2_2 (\Lambda_{\text{EW}}) - |\Delta m^2_{21}| - \Delta m^2_{21}}.$$ (8)
We have taken $\sqrt{\Delta m_{32}^2} + \Delta m_{21} \leq m_2 < 0.1$ eV,\footnote{Note that $m_2 < 0.15$ eV, weak degenerate (0.1 eV < $m_2 < 0.15$ eV) and strong degenerate (0.15 eV < $m_2 < 0.2$ eV) cases, respectively.} which are the best-fit values of the experimentally observed neutrino mass squared differences \cite{ref}. The magnitude of 0.2 eV is consistent with cosmological bounds on the sum of neutrino masses (see, e.g., \cite{ref_2}). The mixing angles at $\Lambda_{\text{EW}}$ are taken as

$$\sin^2\theta_{12} \approx 0.355,$$

$$0.44(0.46) \leq \sin^2\theta_{23} \leq 0.57(0.58),$$

$$0.022(0.023) \leq \sin^2\theta_{13} \leq 0.029(0.030),$$

from experimental results at the 1σ level for the NH (IH) case \cite{ref}. Notice that $m_3(\Lambda_{\text{EW}})$ is a free parameter in our analyses for the NH (IH) case, and it is related to the magnitude of the degeneracy, i.e., a larger $m_3(\Lambda_{\text{EW}})$ ($m_2(\Lambda_{\text{EW}})$) stands for a stronger degeneracy. When the degeneracy becomes stronger, the mixing angles can change drastically.

The effects of quantum correction described by $\epsilon$ have been taken at $10^{-3} \leq \epsilon \leq 0.1$. In the figures, the “o” and “x” markers show a relatively small $\epsilon$ ($10^{-3} \leq \epsilon < 0.01$) and a large one ($0.01 \leq \epsilon < 0.1$), respectively. Note that $\epsilon$ is also a free parameter in our analyses, which is determined once the values of $\tan \beta$ and $\Lambda$ are fixed. The scatter plots in figs. 1 and 2 denote the PMNS mixing angles for the NH and IH cases in a typical high-energy scale of $\Lambda = 10^{14}$ GeV, respectively. We analyze separately whether all $CP$-phases are relatively large $\pi/4 \leq |\delta^l|$, $|\rho|$, $|\sigma|$ of (a)–(c)) or small $0 \leq |\delta^l|$, $|\rho|$, $|\sigma| < \pi/4$ of (d)–(f)). The CKM mixing angle at $\Lambda = 10^{14}$ GeV \cite{ref_3} is shown in each figure by a big black dot.

Fig. 2: (Colour on-line) The same figures as fig. 1 for the IH case. Red, green and blue plots show large hierarchy ($\sqrt{\Delta m_{32}^2} + \Delta m_{21} \leq m_2 < 0.1$ eV), weak degenerate (0.1 eV < $m_2 < 0.15$ eV) and strong degenerate (0.15 eV < $m_2 < 0.2$ eV) cases, respectively.

We can see that 0.005 < $\epsilon$, which corresponds to $10 < \tan \beta$, is enough for the realization of the GUFM in the NH degenerate case.

On the other hand, fig. 2 shows that the IH case cannot realize the GUFM. It is because $\theta_{23}$ becomes too large at $\Lambda = 10^{14}$ GeV. Thus, we can conclude that the strong degenerate NH mass spectrum can achieve the GUFM in a region of 0.002(0.005) < $\epsilon$ with the large (small) $CP$-phases case, which corresponds to $\tan \beta \approx 10(15)$ \cite{ref}. This situation is summarized by “CKM” in table 1. Additionally, “CKM” in table 2 shows the cases of different combinations of $CP$-phases, for instance one of three is small (large) and the others are large (small). In these cases, numerical results are not so changed, and we can conclude that the most important key for the realization of GUFM is not $CP$-phases but strong degeneracy.
have scanned over $m_3$ and $\tan \beta$, and we have successfully obtained lower bounds on $m_3$ and $\tan \beta$ for the GUFM in this work. We have also shown that the GUFM cannot be realized in the IH case. Second, there generically exist threshold effects for neutrino masses [22]. We did not take care of such effects because it was shown in [6] that the threshold corrections have negligible effects on the mixing angles, and thus the size of the effects is sufficient to have concordance between the GUFM model and experimental results of the neutrino oscillation.

We also comment on a correlative mixing pattern, $\theta_{12} + \theta_{23} + \theta_{13} = \pi/2$, at a low-energy scale. Even when we change value of $\theta_{13}$ as keeping the relation $\theta_{12} + \theta_{23} + \theta_{13} = \pi/2$ within the experimentally allowed values, the main results given in table 1 are not changed.

Finally, although it has nothing to do with the GUFM, we comment on bi-maximal ($\sin^2 \theta_{12} = \sin^2 \theta_{23} = 1/2$ and $\sin^2 \theta_{13} = 0$) and tri-bimaximal ($\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{23} = 1/2$, and $\sin^2 \theta_{13} = 0$) mixing angles just for reference, which are shown by big black triangles in figs. 1 and 2.

We can find regions where all the PMNS mixing angles at $\Lambda = 10^{14}$ GeV are close to the bi-maximal and tri-bimaximal [23] points in figs. 1(a)–(c) and figs. 2(a)–(c). For the BM mixing, $0.0015(0.002) \lesssim \epsilon$ is required for the NH (IH) case, which corresponds to $\tan \beta \approx 8(10)$ [17]. And, the BM mixing cannot be realized in small CP-phases cases both for the NH and IH cases. In different combinations of CP-phases, the BM cannot be realized unless $\pi/4 \lesssim |\rho|, |\sigma|$ for both the NH and IH cases. These mean the largeness of $|\rho|$ and $|\sigma|$ is important for the realization of BM at the high-energy scale (see table 2). On the other hand, the TBM mixing angles at high energy are allowed for all cases (NH/IH and large/small CP-phases (see table 2)). All figures show that the TBM is easy to be realized at the high-energy scale with relatively small quantum effects ("$\epsilon$" marker), since the TBM fits the PMNS mixing angles well at the low-energy scale.

We have investigated whether the GUFM is possible or not in the framework of the MSSM. We have found the GUFM is really possible when the neutrino has a degenerated NH spectrum with $0.1 \text{ eV} \lesssim m_3$ through the quantum corrections, $0.005 \lesssim \epsilon (10 \lesssim \tan \beta)$. We have also investigated the possibility that BM and TBM mixing angles are realized at the high-energy scale.

Table 1: This is the summary of the main results. $\circ (\times)$ means that the corresponding mixing angles can (not) be realized at a high-energy scale.

|  | NH ($m_1 < m_2 < m_3$) |  |
|---|---|---|
| $\pi/4 \leq |\delta'|, |\rho|, |\sigma| \leq \pi/4$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\delta'|, |\rho|, |\sigma| \leq \pi/4$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\rho|, |\sigma| \leq \pi/4$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\delta'|, |\rho|, |\sigma| \leq \pi/4$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\rho|, |\sigma| \leq \pi/4$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\delta'|, |\rho|, |\sigma| \leq \pi/4$ | $\circ$ | $\times$ |

Table 2: The realizations of CKM, BM, and TBM in cases of various combinations of CP-phases for the NH (IH) case.

|  | CKM | BM | TBM |
|---|---|---|---|
| $\pi/4 \leq |\delta'|, |\rho|, 0 \leq |\sigma| < \pi/4$ | $\circ (\times)$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\delta'|, |\rho|, 0 \leq |\sigma| < \pi/4$ | $\circ (\times)$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\rho|, 0 \leq |\delta'|, 0 \leq |\sigma| < \pi/4$ | $\circ (\times)$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\delta'|, 0 \leq |\rho|, 0 \leq |\sigma| < \pi/4$ | $\circ (\times)$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\rho|, 0 \leq |\delta'|, 0 \leq |\sigma| < \pi/4$ | $\circ (\times)$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\sigma|, 0 \leq |\delta'|, 0 \leq |\rho| < \pi/4$ | $\circ (\times)$ | $\circ$ | $\times$ |
| $\pi/4 \leq |\delta'|, 0 \leq |\rho|, 0 \leq |\sigma| < \pi/4$ | $\circ (\times)$ | $\circ$ | $\times$ |

** This work is partially supported by Scientific Grant by Ministry of Education and Science, Nos. 00293803, 20244028, 21244036, and 23340070. The work of RT is supported by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists.

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