A Distributed Optimization Algorithm for Stochastic Optimal Control

Yuning Jiang∗ Philippe Nimmgeers∗∗ Dries Telen** Jan Van Impe∗∗ Boris Houska∗

∗ School of Information Science and Technology (SIST), ShanghaiTech University, 319 Yueyang Road, Shanghai 200031, China (e-mail: jiangyn, borish@shanghaitech.edu.cn)
** KU Leuven, Chemical Engineering Department, BioTeC+ & OPTEC, Gebroeders De Smetstraat 1, 9000 Gent, Belgium (e-mail: philippe.nimmgeers, dries.telen, jan.vanimpe@kuleuven.be)

Abstract: This paper presents a distributed non-convex optimization algorithm for solving stochastic optimal control problems to local optimality. Here, our focus is on a class of methods that approximates the probability distribution of the states of a stochastic optimal control problem with uncertain parameters by using a sigma point approach. This leads to a large but structured optimal control problem comprising a number of carefully selected uncertainty scenarios in order to enforce chance constraints. The approach achieves accuracies that are equivalent to a third order moment expansion. However, as the resulting large but structured optimal control problem is challenging to solve with existing numerical tools, this paper proposes a tailored distributed algorithm that exploits the particular structure that arises when applying the sigma point approach. The method is based on a tailored variant of the recently proposed augmented Lagrangian based alternating direction inexact Newton (ALADIN) algorithm. The approach is illustrated by the application to a benchmark case study involving a predator-prey-fishing model.

Keywords: distributed optimal control, stochastic optimal control, sigma-point approach

1. INTRODUCTION

To maintain their position in a worldwide market with increasing competition, companies need to continuously improve the design and operation of their processes to remain sustainable. Model-based optimization is a powerful tool for achieving these goals in many applications (e.g., Körkel et al. (2004), Logist et al. (2013), Mesbah et al. (2014), Amaran et al. (2016)). Uncertainty, however, is inherently present in such process models (Sahinidis (2004)). The most important sources of uncertainty are parametric uncertainty, process noise, and model-plant mismatches. Not accounting for this uncertainty possibly leads to erroneous model predictions resulting in inaccurate control actions or critical constraint violations.

In the context of optimal control different optimization approaches exist to take these uncertainties into account and guarantee sustainable operation by reducing the probability of violating critical constraints. A variety of techniques exists, depending on the available information on the parametric uncertainty distribution. For instance, in Wendt et al. (2002), Nagy and Braatz (2004) expected values for states and chance constraints are derived for a known parametric uncertainty distribution. Worst-case formulations, resulting in min-max optimization problems, are typically used when the uncertainty is fully known within a bounded set (Kurzhanski and Varaiya (2002), Houska et al. (2012)). However, a performance decrease is typically the price to pay for the increased robustness that can be achieved by these approaches (Bertsimas and Sim (2004)). Since the worst-case scenario is often too strict in practice, trade-off or backoff parameters can be used to adjust the level of robustness (Galvanin et al. (2010), Logist et al. (2011)).

The focus of the current paper is on parametric uncertainty. Here, parametric uncertainty can be directly propagated by using the knowledge on the parametric uncertainty distribution. Asprey and Macchietto (2002) presented a numerical integration over the parametric uncertainty distribution to compute the effect of the parametric uncertainty. This leads to a computationally expensive but rigorous algorithm, which has the advantage that it computes the exact moments with high numerical precision. Computationally more tractable methods are typically based on (i) a linearization or (ii) a sampling-based approach, for example Hammersley sequences (Diwekar and Kalagnanam (1997)), unscented transformation/sigma point (Julier and Uhlmann (1996)), or non-intrusive polynomial chaos expansion (Mesbah et al. (2014)). The first type of methods is based on a linear approximation of the states’ variance-covariance matrix by using a first order Taylor series approximations of the model equations (Srinivasan et al. (2003)). This type of techniques is reasonably accurate if the uncertainty is small compared to the model curvature (Nagy and
Sampling-based uncertainty propagation techniques often boil down to solving a larger system, by adding multiple repetitions of model equations, evaluated in parametric values corresponding to the different sampling points. The objective function expresses the expected value of the original objective function with or without penalization term for the variance on the objective function and is computed as a weighted sum of the original objective function evaluated at the sampling points. Chance constraints can be written in the form of a sum of the expected value and a term accounting for the confidence interval in which the constraint is not violated. The unscented transformation/sigma point approach has been exploited by Telen et al. (2015) in order to develop an approximation of the variance-covariance matrix of the objective and constraint functions for robustness with respect to parametric uncertainties and process noise. Moshah et al. (2014) proposed a stochastic model predictive control approach based on polynomial chaos expansion, see also Wiener (1938) and Xi and Karniadakis (2002).

In this work the sigma point approach is applied as sampling-based stochastic optimal control technique. The size of the optimization problem arising from sampling-based reformulations increases with the number of uncertain parameters and therefore such approaches are computationally expensive. For instance, in scenario-based model predictive control, the problem size grows exponentially with the predicted horizon and number of uncertainties (Lucia et al. 2012)). Thus, for solving the large-scale structured optimization problem efficiently, it is desirable to exploit their structure. One way to achieve this is by applying a distributed optimization algorithm. Existing distributed optimization algorithms for the class of convex optimization problems are typically based on dual decomposition (Everett (1963); Necoara et al. (2009)). Another class of distributed optimization methods for convex optimization problems are based on the alternating direction method of multipliers (ADMM) (Boyd et al. 2011). However, dual decomposition methods are not applicable to nonconvex optimization problems, because a duality gap may exist. Similarly, ADMM methods are in general divergent when applied to nonconvex optimization problems, as discussed in Houska et al. (2016). A notable exception is the augmented Lagrangian based alternating direction inexact Newton (ALADIN) method, which has been proposed in (Houska et al. 2016)). ALADIN is a distributed non-convex optimization problem solver, which combines ideas from the fields of sequential quadratic programming and augmented Lagrangian algorithms.

**Contributions and Outline:** The paper starts in Section 2 with a brief review of the sigma point approach and the associated reformulation of optimal control problems with parametric uncertainties. The result of this reformulation is a structured but deterministic optimal control problem that consists of subsystems that simulate selected uncertainty scenarios. The coupling of these subsystems is due to the fact that all systems have to agree on a common control input, i.e., the coupling constraint is a consensus constraint. Moreover, the objective functions turns out to be a weighted sum that comprises additive contributing terms from all subsystems. The main contribution of this paper is a tailored variant of the original ALADIN algorithm from Houska et al. (2016), which is presented in Section 3 and which can solve the reformulated stochastic optimal control problem in a distributed way. Section 4 introduces a predator-prey-fishing benchmark case study in order to illustrate the performance of the proposed algorithm. Section 5 suggests future research directions and concludes the paper.

**Notation:** For the set of integers between two numbers $z_1$ and $z_2$ the shorthand notation $Z_{z_1}^{z_2}$ is used, i.e.,
\[ Z_{z_1}^{z_2} = \{ z \in \mathbb{Z} | z_1 \leq z \leq z_2 \} \]

**2. STOCHASTIC OPTIMAL CONTROL WITH THE SIGMA POINT APPROACH**

This paper concerns stochastic optimal control problems of the form
\[
\begin{align*}
\min_{u(t), x(t)} & \quad \mathbb{E}(M(x_{\theta}(t))) \\
\text{s.t.} & \quad \dot{x}_\theta(t) = f(x_{\theta}(t), u(t), \theta) , \ t \in [0, t_f] , \\
& \quad x_{\theta}(0) = \hat{x} , \\
& \quad u_{\text{min}} \leq u(t) \leq u_{\text{max}} , \ t \in [0, t_f].
\end{align*}
\]

Here, $[0, t_f]$ denotes a time interval and $x_{\theta}(t) \in \mathbb{R}^{n_x}$ denotes a state vector, whose evolution in time depends on an uncertain but time-invariant parameter vector $\theta \in \mathbb{R}^{n_{\theta}}$. The vector $\hat{x}$ denotes the initial states of the system. The vector $u(t) \in \mathbb{R}^{n_u}$ denotes the control inputs bounded by $u_{\text{min}}$ and $u_{\text{max}}$ and the function $M(x_{\theta}(t))$ denotes a Mayer term.

**Remark 2.1.** Notice that the following algorithmic developments can be extended for the case that (1) is augmented by single-chance state constraints, see Telen et al. (2015) for a related discussion.

If the right-hand function $f$ and/or the Mayer term $M$ is nonlinear, the expected objective value is denoted by
\[
\mathbb{E}(M(x_{\theta}(t))) = \int_{\mathbb{R}^{n_{\theta}}} M(x_{\theta}(t)) \rho(\theta) \, d\theta ,
\]
which can in general only be evaluated under the assumption that the probability distribution $\rho(\theta)$ of the uncertain parameter $\theta$ is known. However, as solving the above nonlinear stochastic optimal control problem with high accuracy is in general very expensive, the sigma point approach is based on an approximation of the above problem under the assumption that (i) the mean value and the variance matrix of the random variable $\theta$ are known, and (ii) that the probability distribution $\rho$ is unimodal and symmetric. Now, a sampling-based approximation of the original stochastic optimal control problem with $n_s$ sampling points $\theta_i$ is given by
\[
\begin{align*}
\min_{u,x} & \quad \bar{M}(x(t_f)) \\
\text{s.t.} & \quad \dot{x}_i(t) = f(x_i(t), u(t), \theta_i) , \ i \in \mathbb{Z}_{0}^{n_{\theta} - 1} , \ t \in [0, t_f] \\
& \quad x_i(0) = \hat{x} , \ i \in \mathbb{Z}_{0}^{n_{\theta} - 1} , \\
& \quad u_{\text{min}} \leq u(t) \leq u_{\text{max}} , \ t \in [0, t_f]
\end{align*}
\]
approximation of the expected objective function value. The sigma point method (Julier and Uhlmann (1996)) is such a sampling-based approach, which approximates the expected objective function value as

\[ M(x(t)) = \frac{1}{n_\theta + \kappa} \left( \kappa M(x_0(t)) + \frac{1}{2} \sum_{i=1}^{2n_\theta} M(x_i(t)) \right) \]

with \( n_\theta = 2n_\theta + 1 \). The parameter \( \kappa \) depends on the considered distribution and is set to \( \kappa = 3 - n_\theta \) minimizing the mean squared error up to fourth order (Julier and Uhlmann (1996)), as we assume that the probability distribution is unimodal and symmetric.

In this sampling-based approach, the distribution of a nonlinear transformation is approximated at \( 2n_\theta + 1 \) sigma points:

\[ \theta_0 = \bar{\theta}, \quad \theta_i = \bar{\theta} + \sqrt{(n_\theta + \kappa) P_{\theta\theta_i}} \quad \text{with} \quad i = 1, \ldots, n_\theta, \quad \theta_i = \bar{\theta} - \sqrt{(n_\theta + \kappa) P_{\theta\theta_i-n_\theta}} \quad \text{with} \quad i = n_\theta + 1, \ldots, 2n_\theta. \]

with \( \bar{\theta} \) denoting the mean value of the parameter vector, \( P_{\theta\theta_i} \in \mathbb{R}^{n_\theta \times n_\theta} \) the parameter variance-covariance matrix and \( \sqrt{P_{\theta\theta_i}} \) the \( i \)-th diagonal element of the matrix square root, which can be computed by, e.g., a Cholesky decomposition (Telen et al. (2015)).

Hence, \( 2n_\theta + 1 \) model evaluations are required for the sigma point approach. Notice that, for simplicity of presentation, this paper does not discuss explicitly how to reformulate chance constraints for the process states, as the approximate evaluation of such chance constraints based on the sigma-point approach is analogous to the evaluation of the objective functions as discussed in Telen et al. (2015). In this sense, an extension of the considerations below for stochastic optimal control problems with state constraints is straightforward.

3. DISTRIBUTED STOCHASTIC OPTIMAL CONTROL WITH ALADIN

This section proposes a direct distributed method for solving (3), which is based on the optimization algorithm ALADIN (Houska et al. (2016)).

3.1 Discretization

In order to discretize problem (3) we apply a classical direct method (Bock and Plitt (1984)). Here, the control parameterization is given by

\[ u(t) \approx \sum_{k=0}^{N-1} \varphi_k(t), \quad \varphi_0, \ldots, \varphi_{N-1} : [0, t_f] \to \mathbb{R} \]

are given orthogonal functions and \( v \in \mathbb{R}^{N_{\alpha} \times n_\alpha} = [\varphi_0^\top, \ldots, \varphi_{N-1}^\top]^\top \) denotes the control coefficient vector. This definition includes the special case that we use piecewise constant control approximations (i.e., \( v_k \) constant on each interval \( k \)). Next, the notation

\[ \xi(t, v, \theta) \]

is used to denote the solution of the differential equation system at time \( t \),

\[ \frac{d\xi(t, v, \theta)}{dt} = f \left( \xi(t, v, \theta), \sum_{k=0}^{N-1} \zeta_k \varphi_k(t, \theta) \right), \quad t \in [0, t_f] \]

\[ \xi(0, v, \theta) = \bar{x}, \]

in dependence on the control parameterization vector \( v \) and the parameter \( \theta \). The function \( \xi \) can be evaluated numerically by using a Runge-Kutta integrator (Burden and Faires (2011)). A discrete-time approximation of (3) is now given by

\[ \min_{v} w_0 M(\xi(t_N, v, \theta_0)) + \sum_{i=1}^{2n_\theta} w_i M(\xi(t_N, v, \theta_i)) \]

s.t. \( u_{\min} \leq v_k \leq u_{\max}, \quad k \in Z_0^{N-1} \]

with weights \( w_0 = \frac{\kappa}{n_\theta + \kappa}, \quad w_i = \frac{1}{2(n_\theta + \kappa)}, \quad i = 1, \ldots, 2n_\theta \).

3.2 Problem Reformulation

In order to be able to exploit the particular structure of the above discrete-time optimal control problem (5), a reformulation of this problem is introduced in this section. Notice that for \( n_\theta > 3 \), the weight \( \kappa \) of the first summand in the objective of (5) becomes negative. For the case that \( M \) is convex, such negative weights are unfortunate, as they flip the convexity properties of the objective summands. In addition, our empirical numerical observations indicate that it is—not only for convex but also for mildly non-convex problems—advisable to reformulate the problem differently depending on whether \( n_\theta \leq 3 \) or \( n_\theta > 3 \).

1) For \( n_\theta \leq 3 \) we write problem (5) in the equivalent lifted form

\[ \min_{v_i} w_0 M(\xi(t_N, v_0, \theta_0)) + \sum_{i=1}^{2n_\theta} w_i M(\xi(t_N, v_i, \theta_i)) \]

s.t. \( v_i = v_{i+1}, \quad i \in Z_0^{2n_\theta-1} \]

where \( v = [\zeta_0, \ldots, \zeta_{N-1}]^\top \in \mathbb{R}^N \) denotes the decoupled control inputs. The coupled affine constraints, \( v_i = v_{i+1} \) for all \( i = 0, \ldots, 2n_\theta-1 \), enforce consensus between the decoupled subsystems.

2) Similarly, for \( n_\theta > 3 \), we write problem (5) in the equivalent form

\[ \min_{v_i} \sum_{i=1}^{2n_\theta} \left( w_0 \frac{M(\xi(t_N, v_i, \theta_0))}{2n_\theta} + w_i M(\xi(t_N, v_i, \theta_i)) \right) \]

s.t. \( v_i = v_{i+1}, \quad i \in Z_0^{2n_\theta-1} \]

Notice that this problem comprises only \( 2n_\theta \) decoupled subsystems, since every of the agents keeps its own copy and the nominal evaluation. This is in contrast to problem (6), which is based on \( 2n_\theta + 1 \) decoupled subsystems.
In order to introduce a uniform notation for both situations, we denote the number of decoupled systems by
\[ m = \begin{cases} 
2n_\theta + 1 & \text{if } n_\theta \leq 3 \\
2n_\theta & \text{if } n_\theta > 3.
\end{cases} \] (8)

Based on this, the decoupled objectives can be written in the form
\[ \Phi_i(v) := \begin{cases} 
 w_{i-1}M(\xi(t_N, v, \theta_{i-1})) & \text{if } n_\theta \leq 3 \\
 \frac{w_0}{2n_\theta} M(\xi(t_N, v, \theta_0)) + w_i M(\xi_i(t_N, v, \theta_i)) & \text{if } n_\theta > 3
\end{cases} \]
for all \( i = 1, \ldots, m \). By using this notation the distributed optimization problem formulation can for both situations be written in the form
\[
\min_{v_i} \sum_{i=1}^{m} \Phi_i(v_i) \quad \text{s.t.} \quad \begin{cases} 
 u_{\min} \leq v_{i,k} \leq u_{\max}, & k \in \mathbb{Z}_0^{N-1} \\
v_i = v_{i+1} \quad & \lambda_i, \quad i \in \mathbb{Z}_1^{m-1}.
\end{cases} \] (9)

Here, \( \lambda_i \in \mathbb{R}^{n_\theta} \) denotes the multiplier of i-th coupled affine equality constraints.

**Algorithm 1** ALADIN variant for solving Problem (9)

**Input:** Initial guess primal and dual variable \((z, \lambda)\).

**Output:** Optimal control vector \(v^*\).

**Repeat:**

1. Solve the decoupled NLPs:
\[
\min_{v_i} \Phi_i(v_i) - \lambda_{i-1} v_i + \lambda_i^T v_i + \frac{\rho}{2} ||v_i - z_i||^2 
\text{s.t.} 
\begin{cases} 
 u_{\min} \leq v_{i,k} \leq u_{\max}, & k \in \mathbb{Z}_0^{N-1} \\
 v_i = v_{i+1} \quad & \lambda_i, \quad i \in \mathbb{Z}_1^{m-1}.
\end{cases}
\] (10)

Here, \( \lambda_0 = 0 \) and \( \lambda_m = 0 \), respectively.

2. If \( \sum_{i=1}^{m} ||v_i - z_i|| \leq \epsilon \) and \( \sum_{i=1}^{m-1} ||v_i - v_{i+1}|| \leq \epsilon \), terminate with \( v^* = v_1 \).

3. Solve the coupled QP:
\[
\min_{\Delta v_i, s_i} \sum_{i=1}^{m} \left\{ \frac{1}{2} \Delta v_i^T H_i \Delta v_i + g_i^T \Delta v_i \right\}
+ \sum_{i=1}^{m-1} \left\{ \lambda_i^T s_i + \frac{\mu}{2} ||s_i||^2 \right\} 
\text{s.t.} 
\begin{cases} 
 0 = \Delta v_{i,k}, & k \in \mathcal{A}(v_i), \quad i \in \mathbb{Z}_1^m \\
s_i = v_i + \Delta v_i - v_{i+1} + \Delta v_{i+1} \quad & \lambda_i^{QP}, \\
 \text{with } H_i = -\nabla^2 \Phi_i(v_i) \text{ and } g_i = -\nabla \Phi_i(v_i).
\end{cases}
\] (11)

4. Update the iterates \( z_i^+ = v_i + \Delta v_i, i \in \mathbb{Z}_1^m \) and \( \lambda_i^+ = \lambda_i^{QP}, i \in \mathbb{Z}_1^{m-1} \).

3.3 Complete Parallelizable Scheme

This section focusses on an application of ALADIN for solving (9). In Algorithm 1 the main primal and dual iterates of ALADIN are denoted by \( z \in \mathbb{R}^{mNn_\theta} \) and \( \lambda \in \mathbb{R}^{(m-1)Nn_\theta} \). Notice that if the uncertainty is small, the dual variables \( \lambda \) can be expected to be close to 0. Thus, in this case, \( \lambda = 0 \) can be expected to be a good initialization point, which has also been confirmed empirically by our case study. In practice, the primal optimization variables \( z \) correspond to a discretization of a physical control input, whose initialization depends on the application.

Algorithm 1 has two main steps run in a loop, Parallel Step (1) and Consensus Step (3): in Step (1) decoupled NLPs (11) are solved in parallel. Notice that the ALADIN iterates \( z \) and \( \lambda \) are parameters of the decoupled NLPs, which change from iteration to iteration. In Step (3) a coupled QP is solved. This consensus QP (11) is constructed based on the local solutions from the decoupled NLPs. Here, \( \mathcal{A}(v_i) \) denotes the set of indices that correspond to active control bounds, and the slack variables \( s_i \in \mathbb{R}^{Nn_\theta} \) ensure that the QP (11) remains feasible as discussed in Houska et al. (2016). The primal-dual optimal solution \( (\Delta v_i, \lambda_i^{QP}) \) is used to update the iterates of the algorithm (Step (4)).

In addition, the algorithm terminates if the residuals of the coupling constraints and the difference between the primal solution of the local NLPs and the coupled QP are below a certain tolerance (Houska et al. (2016)). A local convergence proof as well as more advanced globalization routines for Algorithm 1 can be found in Houska et al. (2016).

4. NUMERICAL CASE STUDY

4.1 Model

This section analyzes a two-species predator-prey-fishing model (Sager (2013)),
\[
\frac{dx_1(t)}{dt} = x_1(t) - \theta x_2(t)x_2(t) - \theta x_1(t)u_1(t) 
\] (13)
\[
\frac{dx_2(t)}{dt} = -x_2(t) + \theta x_1(t)x_2(t) - \theta x_2(t)u_1(t) 
\] (14)
\[
\frac{dx_3(t)}{dt} = (x_1(t) - 1)^2 + (x_2(t) - 1)^2, 
\] (15)

where \( x_1 \) and \( x_2 \) denote the scaled population densities of a prey and a predator species as a function of time. The second term in the state equations (13)-(14) accounts for the change of population density by contact between the prey and predator population. The effect of fishing on the prey’s and predator’s population densities is formulated as the last term in Equations (13)-(14). In this case it is assumed that fishing will have more impact on the prey than on the predator’s biomass (since a bigger predator fish is more difficult to catch).

The objective in this case study is to determine a fishing strategy that brings the system to a steady state which avoids economical problems by a disequilibrium in prey and biomass population. The state \( x_3(t) \) has been added as an auxiliary state for the implementation of this objective function (i.e., a tracking performance), given by
Fig. 1. First row: One uncertain parameter case. Second row: Four uncertain parameters case. 

\[ M(x(t_f)) = x_3(t_f). \] The initial state \( \hat{x} = [0.5, 0.7, 0.0]^T \) is assumed to be given.

The dynamic optimization problem is discretized with a single shooting discretization scheme in which the controls are discretized in 30 piecewise constant intervals. The algorithm has been implemented in Julia-0.4.6.

4.2 Numerical Results

Table 1. Result for the expected objective value \( \mathbb{E}(x_3(t_f)) \) evaluated by using a Monte-Carlo simulation method. The first column lists the expected value for the robustly optimized control, while the second column list the corresponding values for a nominally optimal control input.

| Case | Robust(ALADIN) | Nominal |
|------|----------------|---------|
| \( \theta_1 \) | 5.298809409801407 | 10.24945693661716 |
| \( \theta_1, \theta_2 \) | 9.602055127066297 | 13.849070128010021 |
| \( \theta_1, \theta_2, \theta_3 \) | 20.12555661408897 | 28.81785929837138 |
| \( \theta_1, \theta_2, \theta_3, \theta_4 \) | 69.07748436984379 | 79.82808160807117 |

Four different scenarios have been considered: one uncertain parameter \( \theta_1 \), two uncertain parameters \( \theta_1, \theta_2 \), three uncertain parameters \( \theta_1, \theta_2, \theta_3 \), and four uncertain parameters \( \theta_1, \theta_2, \theta_3, \theta_4 \). Since the true parameter values are intrinsically unknown, the uncertain parameters are assumed to be normally distributed with the expected parameter values equal to \( \bar{\theta} = [1.0, 1.0, 0.4, 0.2]^T \) and a relative standard deviation of 30%.

In order to compare the robust control generated by the sigma point method and the nominal control generated by solving the certainty-equivalent OCP, Monte Carlo simulations have been implemented for all cases, each involving 500 random sampling points. Table 1 shows the expected value of \( x_3(t_f) \) based on nominal control and robust control indicating that the robust control strategy leads to significantly improved performance, as expected.

The first row of Figure 1 shows numerical results for the case that one parameter is uncertain, while the second row shows the same but for the case that four parameters take random values. In detail, the column (a) shows the variances of the states, \( \text{Var}(x_1) \) (dashed line) and \( \text{Var}(x_2) \) (dotted line), the nominal and robust control for different scenarios. For the robust fishing strategy the population densities vary less, which appears to correspond to better control performance. The column (b) shows the robust control profiles obtained from ALADIN (red dotted line) and SQP (black dashed line). The blue solid line shows the nominal control profiles. In this example, ALADIN finds the same local minimizer as SQP. Finally, the column (c) shows the progress of ALADIN (red asterisks) versus SQP (black circles) by plotting the norm of the difference of the current iterate to the local minimizer, \( \|v - v^*\|_\infty \) (\( v = z_1 \) for ALADIN), in dependence on the number of iterations. At least for this case study, ALADIN has been found to converge considerably faster than SQP and this result appears to be relatively independent of how much uncertainty is added. One might explain this observation by the fact that ALADIN exploits the distributed structure of the reformulated stochastic optimal control problem in a better way than SQP does.

5. CONCLUSIONS AND FUTURE WORK

This paper has reviewed an approximate reformulation of stochastic optimal control problems based on a sigma point approach. Here, the main contribution is an exploitation of the particular structure of the resulting optimization...
problem by using a variant of the distributed optimization method ALADIN. By analyzing a predator-prey-fishing benchmark case study, it has been found that the proposed ALADIN variant succeeds in solving the reformulated stochastic optimal control problem faster and with fewer iterations than more traditional SQP solvers. Future work will investigate applications of ALADIN to large-scale stochastic optimization problems.

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