Soliton interaction mediated by cascaded four wave mixing with dispersive waves

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Abstract: We demonstrate that trapping of dispersive waves between two optical solitons takes place when resonant scattering of the waves on the solitons leads to nearly perfect reflections. The momentum transfer from the radiation to solitons results in their mutual attraction and a subsequent collision. The spectrum of the trapped radiation can either expand or shrink in the course of the propagation, which is controlled by arranging either collision or separation of the solitons.

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References and links
1. D.V. Skryabin and A.V. Gorbach, “Looking at a soliton through the prism of optical supercontinuum”, Rev. Mod. Phys. \textbf{82}, 1287-1299 (2010).
2. J. M. Dudley, G. Genty, and S. Coen, “Supercontinuum generation in photonic crystal fiber”, Rev. Mod. Phys \textbf{78}, 1135-1184 (2006).
3. N. Akhmediev and M. Karlsson, “Cherenkov radiation emitted by solitons in optical fibers”, Phys. Rev. A \textbf{51}, 2602-2607 (1995).
4. D.V. Skryabin, F. Luan, J.C. Knight, and P.S. Russell, “Soliton self-frequency shift cancellation in photonic crystal fibers”, Science \textbf{301}, 1705-1708 (2003).
5. A.V. Yulin, D.V. Skryabin, Optics Lett. \textbf{29}, 2411-2413 (2004).
6. D.V. Skryabin and A.V. Yulin, “Theory of generation of new frequencies by mixing of solitons and dispersive waves in optical fibers”, Phys. Rev. E \textbf{72}, 016619 (2005).
7. A. Demircan, S. Amiranshivili, and G. Steinmeyer, “Controlling light by light with an optical event horizon”, Phys. Rev. Lett. \textbf{106}, 163901 (2011).
8. A. Efimov, A.J. Taylor, P.G. Omenetto, A.V. Yulin, N.Y. Joly, F. Biancalana, D.V. Skryabin, J.C. Knight, and P.St.J. Russell, Optics Express \textbf{12}, 6499 (2004).
9. A. Efimov, A.V. Yulin, D.V. Skryabin, J. C. Knight, N. Joly, F. G. Omenetto, A. J. Taylor, and P. Russell, Phys. Rev. Lett. \textbf{95}, 213902 (2005).
10. A. Efimov and A.J. Taylor, A.V. Yulin, D.V. Skryabin, and J.C. Knight, Opt. Lett. \textbf{31}, 1624 (2006).
11. B. A. Malomed, “Potential of interaction between two- and three-dimensional solitons”, Phys. Rev. E \textbf{58}, 7928-7933 (1998).
12. F. Luan, D.V. Skryabin, A.V. Yulin, and J.C. Knight, “Energy exchange between colliding solitons in photonic crystal fibers”, Opt. Exp. \textbf{14}, 9844-9853 (2006).
13. R Driben, B. A Malomed “Suppression of crosstalk between solitons in a multi-channel split-step system”, Opt. Commun. \textbf{197}, 481-489 (2001).
14. G. Agrawal, “Nonlinear Fiber Optics” (Academic Press, New York, 2007).
Various aspects of dynamics of ultrashort pulses in photonic crystal fibers (PCFs), such as the formation of solitons, resonant radiation, supercontinuum generation, optical rogue waves and others have been the subject of intense investigations over the past decade [1,2]. One prominent theme of research in this area has been interaction of dispersive waves with solitons in strongly non-integrable cases, i.e., far from the limit of the ideal nonlinear Schrödinger equation [1]. In particular, theoretical and experimental studies reviewed in Ref. [1] have revealed novel phase-matching conditions which result in generation of new frequency components from the four-wave mixing of solitons and dispersive waves in the media with significant higher-order dispersions, and have demonstrated the crucial role of these processes in the expansion of supercontinuum spectra generated in PCFs [1,2].

Importantly for the present work, the back action of the Cherenkov radiation [1,3,4] and of the four-wave mixing [5,6,7,8,9,10] processes on solitons has also been considered and revealed plausible applications for the control of the soliton carrier frequencies and group velocities. In this context, a natural problem to consider is how multiple re-scattering of dispersive waves on two or several well separated solitons can be used to mediate interactions between them, and what spectral and temporal-domain effects can be predicted in this case. While the short-range soliton interaction through the overlapping soliton tails has been reported in numerous papers, see, e.g. Refs. [12][13][14] and references therein, the long-range soliton-soliton interaction via dispersive optical waves or polariton waves in the material has not yet attracted the same degree of interest. Nevertheless, some important results on this topic have been already published — in particular, the interaction between solitons through the radiation in the integrable limit [15], binding of solitons through the dispersive waves generated by fourth-order dispersion [16], and interactions between solitons mediated by acoustic waves [17]. More recently, it has been noticed that fission of N-solitons in the presence of strong higher-order dispersion leads at a later stages of the supercontinuum development to collisions between solitons [18][19][20][21][22]. The impact of a dispersive wave present in the space between colliding solitons on the random changes of the soliton amplitude and frequency has been studied in details in Refs. [20], formation of bound multi-peak soliton states through this mechanism has been reported in Ref. [18], and fusion of several soliton into a single high-amplitude pulse was reported too [21]. From the results presented in Ref. [18][19][20][21], one can conclude that dispersive waves induce attraction between solitons. However, the above-mentioned works studied the problem under conditions typical for the supercontinuum generation, which involves many poorly controlled factors. Thus, presently there is no clear understanding of the basic mechanisms driving the interaction between well-separated solitons, mediated by the dispersive ra-
radiation. Investigation of such mechanisms by means of analytical and numerical methods is a subject of the present work. We also investigate spectral reshaping of the dispersive radiation interacting with two solitons, which reveals some interesting opportunities for spectral control.

As the model we use a normalized form of the generalized NLS equation, which includes the third-order dispersion (TOD), with respective coefficient β₃, and the Raman effect:

\[ i \partial_t A + \frac{1}{2} \partial_x^2 A - i \frac{\beta_3}{6} \partial_x^3 A + (1 - \theta)|A|^2 A + A \int_{-\infty}^{+\infty} R(t')|A(t - t')|^2 dt' = 0, \]  

(1)

where the response function, with Raman constant θ, delay times τ₁ and τ₂, and step function \( \Theta \), is

\[ R = \theta \frac{\tau_1^2 + \tau_2^2}{\tau_1^2 \tau_2} \Theta(t) \exp \left( -\frac{t}{\tau_1} \right) \sin \left( \frac{t}{\tau_1} \right). \]

The analysis presented here pertains to all widths of interacting solitons. The Raman term affects the dynamics only in the case of subpicosecond pulses, and it is considered only in simulations displayed below in Fig. 3.

To understand the interaction of radiation with two solitons, we first need to recapitulate results for the interaction of a single dispersive wave packet with a single soliton. The black curve in Fig. 1 shows the dispersion of linear waves, \( A \sim e^{ikx - i\delta t} \), \( k(\delta) = -1/2\delta^2 + (\beta_3/6)\delta^3 \), where \( k \) and \( \delta \) are, respectively, shifts of the propagation constants and frequencies. Assuming that the soliton has zero frequency detuning, we have for its wavenumber \( k_s(\delta) = q \), where \( q \) is the soliton parameter (half the peak power, see the Fig. 1 caption). For given frequency \( \delta_i \) of the incident wave packet, see Fig. 1(a), one can derive the phase-matching (resonance) condition for the frequency of the reflected wave, \( \delta_r \), as shown in Fig. 1(a) [1]. For the resonances due to the four-wave scattering of the dispersive wave on the soliton-induced shift of the refractive index (the only resonances relevant here), we have

\[ k(\delta_r) = k_s(\delta_r) - |k_0(\delta_r) - k(\delta_i)|, \]

(2)

two intersection points in Fig. 1 corresponding to roots of this equation. Taking into regard that the initial soliton momentum is zero and neglecting the Raman effect, we can apply the momentum conservation: \( M_i = M_f + M_r \), where \( M = \text{Im} \{ \int_{-\infty}^{+\infty} A \partial_t A^* dt \} \), and the weak transmitted
wave is disregarded. It is also natural to assume that the number of photons in the soliton, $Q_s$, does not change upon the collision. Then, after a straightforward algebra, the conservation law can be transformed into the form of $\delta_i |k'_i||L = \delta_r Q_s + \delta_r |k'_r||L$, where $L$ and $I$ are the spatial lengths and intensities of the incident and reflected wave packets (which are approximately equal for both packets), and $k' \equiv dk(\delta)/d\delta$. Transforming this conservation law into a differential form (for $L \rightarrow 0$), and adopting $|k'_i| \simeq |k'_r|$ (see Fig. 1), we find

$$\partial_z \delta_s \simeq (\delta_i - \delta_r) I |k'_r|/Q_s.$$  (3)

As follows from Eq. (3), for $\delta_i - \delta_r > 0$ ($\delta_i - \delta_r < 0$) the soliton’s frequency increases (decreases) resulting, respectively, in the acceleration (deceleration) of the soliton. Swapping the frequencies of the incident and reflected waves means sending the incident radiation pulse either ahead of or behind of the soliton, while its trajectory in the $(t,z)$-plane deviates towards the collision side in either case, see Figs. 1(b,c).

Thus, setting two identical well-separated solitons and sending a pulse between them with any frequency from the shaded interval in Fig. 1(a), effective attraction between the solitons is induced by multiple scattering events between the pulse and the solitons, see Fig. 2(a). It is important to note that the spectrum of radiation trapped between the solitons evolves in the course of the propagation, and can be controlled by the initial choice of the soliton frequencies. If the two solitons are not identical, then, after two consecutive scattering events, the radiation comes back to the soliton with an altered frequency, resulting in spectral evolution of the radiation. In the case shown in Fig. 2, the soliton frequencies drift in the opposite directions due to interaction with the dispersive waves, which, in turn, makes the frequency of the dispersive wave varying too, see Fig. 2(b). Changing the intensity of the dispersive pulse, one can control the distance at which the solitons eventually collide due to the attraction, see Fig. 2(c). This control technique may be used if one needs to produce a high-intensity pulse as a result of the collision. Since the dispersion law in the soliton frequency range is quasi-parabolic, the soliton group velocity is proportional to its frequency. Hence $\partial_z \delta_s$ in the left hand side of Eq. (3) is in fact the soliton acceleration or the second derivative of its temporal coordinate and therefore the effective interaction force $F_{int}$ induced by the radiation pulse and acting between the solitons.
is proportional to the intensity of the latter, \( I \). Accordingly, the distance to collision, \( z_c \), should naturally scale as \( 1/\sqrt{F_{\text{int}}} \sim 1/\sqrt{I} \), see Fig. 2(c). We would like to notice here that the effect in question is not sensitive to the exact shape of the dispersive wave envelope.

Fig. 3. The attraction and collision induced by the random wave field trapped between the solitons. (a) The intensity evolution in the \((z,t)\)-plane for the solitons with zero frequency and \( q = 8 \). (b,c) Initial \((z = 0)\) and intermediate \((z = 15)\) time-frequency diagrams showing the solitons and dispersive waves. Density plots in (b,c) show XFROG function calculated as \(|\int_{-\infty}^{+\infty} A \exp(-it^2/w_r^2) \exp(-i\delta t) dt|\) with \( w_r = 2 \). The parameters are \( \beta_3 = -0.025 \), \( \theta = 0.18 \), \( \tau_1 = 0.061 \), and \( \tau_2 = 0.16 \), cf. Ref. [1].

Moreover, in fact it is not necessary to use a dispersive pulse with a sharply defined frequency and shape to generate the radiation-mediated attraction between solitons. Instead, one can take a random field with the spectrum filling a sufficiently broad frequency interval in the range of normal group-velocity dispersion, see Fig. 1(a). In this case, multiple waves experience cascaded scattering events with solitons, resulting in their mutual attraction, as shown in Fig. 3 (in the presence of the Raman effect). Panel 3(a) shows the evolution in the course of the propagation in the fiber, while 3(b,c) show XFROG diagrams illustrating spectral-temporal characteristics of the field at the input and at an intermediate propagation distance. Adding the Raman effects (along with the self-steepening) into Eq. (1) bends the solitons’ trajectories but does alter the nature and outcome of the interaction. This conclusion agrees with the previously reported studies of the radiation-induced soliton collisions in the full-scale supercontinuum modeling and experiments [18,19,20,21].

Fig. 4. The narrowing of the radiation spectrum trapped between two separating solitons. The initial solitons are taken with \( q = 12.5 \) and frequencies \( \delta = 1.5 \), \( \bar{\delta} = 0 \). (a) The dispersion diagram showing the scattering cascade. (b,c) The evolution in the temporal and frequency domains. In this case, \( \beta_3 = -0.01 \).

By choosing initial soliton frequencies so that they separate in the course of the propagation, we have found that, in this case, the dispersive waves do not strongly affect the motion of the solitons, see Fig. 4. On the other hand, applying the resonance condition in panel (a), we find
that the frequencies of the dispersive pulse scattered on the right and left solitons get closer, implying that the spectrum narrows, as is indeed seen in panel (c).

On the other hand, spectral expansion of the radiation is observed if the solitons are initially moving towards each other, see Fig. 5. Panel (a) shows how the resonant scattering modifies the frequency of the dispersive pulse, and the modification of the spectrum with the propagation distance is shown in (c).

![Fig. 5. The same as in Fig. 4, but for expansion of the radiation spectrum trapped between two separating solitons. The initial solitons are taken with $q = 18$ and frequencies $\delta = -0.3$, $\delta = 0.3$.](image)

Summarizing, we have studied effects resulting from the resonant multiple scattering of dispersive radiation on a pair of well separated solitons in optical fibers with strong TOD. We have demonstrated in the numerical form and explained analytically the effect of the soliton attraction mediated by the multiply scattered radiation, the spectral reshaping of the radiation trapped between the two solitons has been investigated too.

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