Systematic study of the system size dependence of global stopping: Role of momentum dependent interactions and symmetry energy

Sanjeev Kumar and Suneel Kumar
School of Physics and Material Science, Thapar University, Patiala-147004, Punjab (India)
E-mail: suneel.kumar@thapar.edu

Abstract. Using the isospin-dependent quantum molecular dynamical (IQMD) model, we systematically study the role of momentum dependent interactions in global stopping and analyze the effect of symmetry energy in the presence of momentum dependent interactions. For this, we simulate the reactions by varying the total mass of the system from 80 to 394 at different beam energies from 30 to 1000 MeV/nucleon over central and semi-central geometries. The study is carried in the presence of momentum dependent interactions and symmetry energy by taking into account hard equation of state. The nuclear stopping is found to be sensitive towards the momentum dependent interactions and symmetry energy at low incident energies. The momentum dependent interactions are found to weaken the finite size effects in nuclear stopping.

PACS numbers: 25.70.-z, 24.10.Lx, 21.65.Ef
1. Introduction

The heavy-ion collisions at intermediate energies have witnessed several rare phenomena such as multifragmentation, disappearance of flow, partial or complete stopping as well as sub threshold particle production [1, 2]. The recent advances in the radioactive nuclear beam (RNB) physics is providing scientific community a unique opportunity to investigate the isospin effects in heavy-ion collisions (HIC’s) [3] with respect to the above rare phenomena [1, 2]. Beside the many existing radioactive beam facilities, many more are being constructed or under planning, including the Cooling Storage Ring (CSR) facility at HIRFL in China, the Radioactive Ion Beam (RNB) factory at RIKEN in Japan, the FAIR/GSI in Germany, SPIRAL2/GANIL in France, and Facility for Rare Isotope Beam (FRIB) in the USA [4]. These facilities offer possibility to study the properties of nuclear matter or nuclei under the extreme conditions of large isospin asymmetries. Though at low incident energies, where fusion and related phenomena are dominant, systematic studies over isospin degree of freedom are available [5], no such studies are available at intermediate energies. One of the cause could be the much more complex dynamics involved at intermediate incident energies. Among various above mentioned phenomena nuclear stopping of the colliding matter has gained a lot of interest since it gives us possibility to examine the degree of thermalization or equilibration of the matter.

In a recent communication [6, 7], nuclear stopping has been explored with reference to the isospin degree of freedom. Nuclear stopping in heavy-ion collisions has been studied by the means of rapidity distribution [8] or by the asymmetry of nucleonic momentum distribution [9]. Bauer and Bertsch [9] reported that, nuclear stopping is determined by both the mean field and in-medium NN cross-sections. Different authors suggested that the degree of approaching isospin equilibration provides a mean to prove the power of nuclear stopping in HIC’s [10].

Interestingly, no systematical study is available in the literature on the effect of isospin degree of freedom via symmetry energy on nuclear stopping. This becomes much more important if one acknowledges that the effect of symmetry energy could be altered in the presence of momentum dependent interactions, which has become essential part of the interaction for any reasonable dynamical model [8, 10]. Thus, our aim is at least two folds:

- We plan to understand the role of symmetry energy in the presence of momentum dependent interactions in a systematic way.
  and
- To further examine how mass dependence alter the above findings. It is worth mentioning that the study of the mass dependence is very essential for any meaningful conclusion.

This study is done within the framework of isospin-dependent quantum molecular dynamics (IQMD) model, which is discussed in section-2. Section-3 contains the results
and summary is presented in section-4.

2. ISOSPIN-DEPENDENT QUANTUM MOLECULAR DYNAMICS (IQMD) MODEL

The isospin-dependent quantum molecular dynamics (IQMD) model treats different charge states of nucleons, deltas and pions explicitly, as inherited from the VUU model. The IQMD model has been used successfully for the analysis of a large number of observables from low to relativistic energies. The isospin degree of freedom enters into the calculations via symmetry potential, cross-sections and nucleon-nucleon interactions. The details about the elastic and inelastic cross-sections for proton-proton and neutron-neutron collisions can be found in ref. [11].

In this model, baryons are represented by Gaussian-shaped density distributions

\[ f_i(\vec{r}, \vec{p}, t) = \frac{1}{\pi^{3/2} \hbar^2} \cdot e^{-\frac{(\vec{r}-\vec{r}_i(t))^2}{2}} \cdot F \cdot e^{-\frac{(\vec{p}-\vec{p}_i(t))^2}{2}}. \]  

(1)

Nucleons are initialized in a sphere with radius \( R = 1.12A^{1/3} \) fm, in accordance with the liquid drop model. Each nucleon occupies a volume of \( h^3 \), so that phase space is uniformly filled. The initial momenta are randomly chosen between 0 and Fermi momentum \( p_F \). The nucleons of target and projectile interact via two and three-body Skyrme forces, Yukawa potential and momentum-dependent interactions. These interactions are similar as used in the molecular dynamical models like quantum molecular dynamics (QMD) and relativistic QMD. The isospin degree of freedom is treated explicitly by employing a symmetry potential and explicit Coulomb forces between protons of colliding target and projectile. This helps in achieving correct distribution of protons and neutrons within nucleus.

The hadrons propagate using Hamilton equations of motion:

\[ \frac{dr_i}{dt} = \frac{d\langle H \rangle}{dp_i}; \quad \frac{dp_i}{dt} = -\frac{d\langle H \rangle}{dr_i}, \]  

(2)

with

\[ \langle H \rangle = \langle T \rangle + \langle V \rangle = \sum \frac{p_i^2}{2m_i} + \sum \int f_i(\vec{r}, \vec{p}, t)V^{ij}(\vec{r}', \vec{r}) \times f_j(\vec{r}', \vec{p}', t)d\vec{r}'d\vec{p}'d\vec{p}d\vec{r}. \]  

(3)

The baryon-baryon potential \( V^{ij} \), in the above relation, reads as:

\[ V^{ij}(\vec{r}' - \vec{r}) = V_{\text{Skyrme}}^{ij} + V_{\text{Yukawa}}^{ij} + V_{\text{Coul}}^{ij} + V_{\text{mdi}}^{ij} + V_{\text{sym}}^{ij} \]

\[ = \left( t_1\delta(\vec{r}' - \vec{r}) + t_2\delta(\vec{r}' - \vec{r})\rho^{-1} \left( \frac{\vec{r}' + \vec{r}}{2} \right) \right) + t_3\exp\left(\frac{1}{\mu} |\vec{r}' - \vec{r}| \right) + Z_iZ_je^2 \frac{Z_iZ_je^2}{|\vec{r}' - \vec{r}|}. \]
Here $Z_i$ and $Z_j$ denote the charges of $i^{th}$ and $j^{th}$ baryon, and $T_{3i}, T_{3j}$ are their respective $T_3$ components (i.e. 1/2 for protons and -1/2 for neutrons). Meson potential consists of Coulomb interaction only. The parameters $\mu$ and $t_1,...,t_6$ are adjusted to the real part of the nucleonic optical potential. For the density dependence of the nucleon optical potential, standard Skyrme-type parameterization is employed. As is evident, we choose symmetry energy that depends linearly on the baryon density.

The binary nucleon-nucleon collisions are included by employing collision term of well known VUU-BUU equation. The binary collisions are done stochastically, in a similar way as are done in all transport models. During the propagation, two nucleons are supposed to suffer a binary collision if the distance between their centroids

$$|r_i - r_j| \leq \sqrt{\frac{\sigma_{\text{tot}}}{\pi}}, \sigma_{\text{tot}} = \sigma(\sqrt{s}, \text{type}),$$

"type" denotes the ingoing collision partners (N-N, N-$\Delta$, N-$\pi$, ...). In addition, Pauli blocking (of the final state) of baryons is taken into account by checking the phase space densities in the final states. The final phase space fractions $P_1$ and $P_2$ which are already occupied by other nucleons are determined for each of the scattering baryons. The collision is then blocked with probability

$$P_{\text{block}} = 1 - (1 - P_1)(1 - P_2).$$

Delta decays are checked in an analogous fashion with respect to the phase space of the resulting nucleons. Recently, several studies have been devoted to pin down the strength of the NN cross-section[8].

3. Results and Discussion

The global stopping in heavy-ion collisions has been studied with the help of many different variables. In earlier studies, one used to relate the rapidity distribution with global stopping. The rapidity distribution can be defined as [13, 14]:

$$Y(i) = \frac{1}{2} \ln \frac{E(i) + p_z(i)}{E(i) - p_z(i)},$$

where $E(i)$ and $p_z(i)$ are, respectively, the total energy and longitudinal momentum of $i^{th}$ particle. For a complete stopping, one expects a single Gaussian shape. Obviously, narrow Gaussian indicate better thermalization compared to broader Gaussian.

The second possibility to probe the degree of stopping is the anisotropy ratio ($R$) [6]:

$$R = \frac{2 \left( \sum_i |p_\perp(i)| \right)}{\pi \left( \sum_i |p_\parallel(i)| \right)},$$

where, summation runs over all nucleons. The transverse and longitudinal momenta are $p_\perp(i) = \sqrt{p_x^2(i) + p_y^2(i)}$ and $p_\parallel(i) = p_z(i)$, respectively. Naturally, for a complete
For the present study, simulations were carried out for the reactions $^{20}\text{Ca}^{40} + ^{20}\text{Ca}^{40}$, $^{28}\text{Ni}^{58} + ^{28}\text{Ni}^{58}$, $^{41}\text{Nb}^{93} + ^{41}\text{Nb}^{93}$, $^{54}\text{Xe}^{131} + ^{54}\text{Xe}^{131}$ and $^{79}\text{Au}^{197} + ^{79}\text{Au}^{197}$ at different beam energies ranging between 30 and 1000 MeV/nucleon at central and semi-peripheral geometries. The incident energy of 30 MeV/nucleon is the lowest limit for any semi-classical model. Below this incident energy, quantum effects as well as Pauli blocking need to be redefined. A hard (H) and hard momentum dependent (HM) equation of state (EOS) has been employed with symmetry energy $E_{\text{sym}} = 0$ and 32 MeV. The corresponding values of the symmetry energy are indicated as subscript ($H_0$, $H_{32}$, $HM_0$ and $HM_{32}$). It is worth mentioning that global stopping is insensitive toward the nature of static equation of state. Our present attempt is to perform a theoretical study with wider variation in the value of symmetry energy and to see how much it can affect the heavy-ion dynamics. Though MDI destabilizes the nuclei, a careful analysis is made by Puri et al. \[15\] and found that upto 200 fm/c, emission of the nucleons with momentum dependent interactions is quite small.

Let's start with the aspect of rapidity distribution as an indicator for nuclear stopping. As discussed earlier, nuclear stopping is a phenomena which originates from the participant zone. To study the nuclear stopping in term of rapidity distribution in Fig.\[1\], we display the rapidity distribution of free particles (which originates from the participant zone) for different forms of the hard equation of state ($H_0$, $H_{32}$, $HM_0$ and $HM_{32}$). We see that free particles emitted in the central collisions form a narrow Gaussian for the heavier system as well as at higher incident energy (say $E = 400$ MeV/nucleon). It is evident from here that nuclear stopping is dominating with increase in size of the system as well as at higher incident energy (say $E = 400$ MeV/nucleon). With increase in incident energy, this behavior is not expected to be universal. It is clear from the Ref.\[16\] that maximum stopping is observed around 400 MeV/nucleon. This study is done in detail in term of anisotropy ratio $R$ in the Figs. \[2\] and \[3\] of letter and found to be in supportive nature with Ref.\[16\]. On the other hand, nuclear stopping in term of rapidity distribution of protons is found to be weakly sensitive towards symmetry energy and momentum dependent interactions. This is due to the reason that there are not only free particles which originates from the participant zone. The other candidate which originates from this zone are the light charged particles (LCP’s) ($2 \leq A \leq 4$).

In our recent communication\[17\], these LCP’s are shown to be more sensitive towards symmetry energy due to pairing nature and also good indicator for nuclear stopping.

For more meaningful, we will check the sensitivity of symmetry energy and momentum dependent interactions on nuclear stopping in term of anisotropy ratio $R$, which is defined in detail in the above paragraph. In Fig. \[2\] we display the time evolution of the anisotropy ratio $R$ for the central collisions of $^{20}\text{Ca}^{40} + ^{20}\text{Ca}^{40}$ (left panel) and $^{79}\text{Au}^{197} + ^{79}\text{Au}^{197}$ (right panel). The incident energies of 30, 50, 400 and 1000 MeV/nucleon are employed using the four different possibilities of hard equation of state ($H_0$, $H_{32}$, $HM_0$ and $HM_{32}$). Interestingly, the anisotropy ratio $R$, though, is insensitive

stopping, $R$ should be close to unity.
towards the symmetry energy, shows appreciable effect for the momentum dependent interactions. At high enough incident energy, both effects (of momentum dependent interactions as well of symmetry energy) wash away. For further test, we simulated two reactions (i) $^{20}Ca^{40} + ^{20}Ca^{40}$ and kept the same $N/Z$ ratio by taking $^{100}X^{200} + ^{100}X^{200}$ reaction. In other case (ii) we took neutron rich reaction $^{79}Au^{197} + ^{79}Au^{197}$ and simulated the reactions of $^{16}X^{40} + ^{16}X^{40}$ by keeping $N/Z$ ratio again same. In both the cases, the above trend holds good. Therefore, indicating that the above behavior is universal. Even rapidity cuts to mid-rapidity region do not yield different results. Below the beam energy of 50 MeV/nucleon, collision dynamics is governed by the mean field. Therefore, interactions involving isospin particles like nn, np, pp dominate the outcome and hence symmetry effects are visible.

Figure 1. The rapidity distribution $\frac{dN}{dY}$ as a function of reduced rapidity for free nucleons for the reactions of $^{20}Ca^{40} + ^{20}Ca^{40}$, and $^{79}Au^{197} + ^{79}Au^{197}$ for four different possibilities of hard equation of state. The top and bottom panels are at $E = 50$ and 400 MeV/nucleon.
As discussed earlier, $R$ approaches to 1 at low incident energies, indicating the isotropic nucleon momentum distribution of the whole composite system. The behavior at 30 MeV/nucleon is little different due to the fact that binary collisions do not play any role and mean field will take larger time to thermalize the colliding nuclei. As beam energy increases above the certain energy, $R$ starts decreasing from 1 towards the lower values, indicating partial transparency. This value of the beam energy, above which $R$ starts decreasing, depends on the size of the system. In our observations for the reaction of $^{79}Au^{197} + ^{79}Au^{197}$, it is close to 400 MeV/nucleon. This finding is similar to the one reported by W. Reisdorf et al. [16]. The value of $R > 1$, can be explained by the preponderance of momentum perpendicular to beam direction [18]. This is true for all equations of state. It is also seen that relaxation time decreases with the increase in the beam energy, while, increases with the increase in the mass of the colliding system. It
Figure 3. The final state anisotropy ratio $R$ as a function of composite mass of the system $A_{\text{tot}}$ for different possibilities of hard equation of state (discussed in text). The left and right panels are at central and semi-peripheral geometries. The panels labeled with a, b, c and d are at $E = 30$, 50, 400 and 1000 MeV/nucleon, respectively. All the curves are fitted with power law shows that higher beam energies and lighter systems lead to more violent NN collisions and faster dissipation. This is consistent with the isospin equilibrium process as shown by Li et.al. \[3\].

It will be further interesting to see whether the above findings have mass dependence or not. This is particular important since the role of the momentum dependent interactions and the symmetry energy depends on the size of the system. For this, we display in Fig. 3 the anisotropy ratio $R$ as a function of the composite mass of the system ($A_{\text{tot}} = A_T + A_P$) at different beam energies ranging from 30 to 1000 MeV/nucleon. The left panel represents the results at central geometry, while, right panel is at semi-peripheral geometry.

Our findings are:

- The anisotropy ratio $R$ increases with the composite mass of the system. This is true for all incident energies and impact parameters. This dependence becomes
weaker as one moves from central to semi-peripheral geometry. It is due to the fact that nuclear stopping is governed by the participant zone only. This is further supported by the fact that at higher incident energies e.g. $E = 1000$ MeV/nucleon, the stopping is almost independent of the composite mass of the system at semi-peripheral geometry and almost 50% decrease is observed in the nuclear stopping as compared to central geometry.

- The effect of the symmetry energy is visible below the Fermi energy. Same conclusion was also reported in Fig. 2. The effect of the symmetry energy diminishes in the absence of momentum dependent interactions. Moreover, this effect weakens at semi-peripheral geometries.

- The importance of momentum dependent interactions is also visible in the nuclear stopping. This effect decreases as one moves from the low to higher incident energy and from central to peripheral geometry. At the higher incident energies e.g. at $E = 1000$ MeV/nucleon, the anisotropy ratio is independent of the equation of state and symmetry energy. It is worth mentioning that the inclusion of momentum dependent interactions is found to suppress the binary collisions and as a result is found to affect the sub threshold particle production as well as disappearance of collective flow [1, 8].

4. Summary

Using the IQMD model, we have studied the nuclear stopping for isospin effects. For this, we simulated the reactions of $^{20}\text{Ca}^{40} + ^{20}\text{Ca}^{40}$, $^{28}\text{Ni}^{58} + ^{28}\text{Ni}^{58}$, $^{41}\text{Nb}^{93} + ^{41}\text{Nb}^{93}$, $^{54}\text{Xe}^{131} + ^{54}\text{Xe}^{131}$ and $^{79}\text{Au}^{197} + ^{79}\text{Au}^{197}$ in the presence of momentum dependent interactions and symmetry energy. The role of symmetry energy at low incident energy gets enhanced in the presence of momentum dependent interactions. Further, we can conclude that maximum stopping is obtained for the heavier systems at low incident energies in central collisions in the absence of momentum dependent interactions implying that momentum dependent interaction suppresses the nuclear stopping.

Acknowledgment

This work is supported by the grant no. 03(1062)06/EMR-II, from the Council of Scientific and Industrial Research (CSIR) New Delhi, govt. of India.
References

[1] Puri R K et al. 1994 Nucl. Phys. A 575 733; Puri R K et al. 1996 Phys. Rev. C 54 R28; Puri R K et al. 2000 J. Comp. Phys. 162 245; Kumar S, Kumar S and Puri R K 2010 Phys. Rev. C 81, 014611; Kumar S and Kumar S 2010 Pramana J. of Physics P-8343 (in press); Vermani Y K, Dhawan J K, Goyal S, Puri R K and Aichelin J 2010 J. of Phys. G: Nucl. and Part. 37 015105.

[2] Gossiaux P B et al. 1997 Nucl. Phys. A 619 379; Kumar S et al. 1998 Phys. Rev. C 58 3494; Fuchs C et al. 1996 J. Phys. G: Nucl and Part. 22 131; Vermani Y K et al. 2009 Eur. Phys. Lett. 85 62001.

[3] Li B A et al. 1998 Int. J. Mod. Phys. E 7 147; Toro M D et al. 1999 Prog. Nucl. Part. Phys. 42 125.

[4] Zhan W et al. 2006 Int. J. Mod. Phys. E 15 1941; Yano Y 2007 ”The RIKEN RI Beam Factory Project: A Status Report” Nucl. Inst. Meth. B 261 1009; [http://www.gsi.de/fair/index_e.html http://www.ganinfo.in2p3.fr/research/developments/spiral2; Whitepapers of the 2007 NSAC Long Range Plan Town Meeting, Jan., 2007, Chicago, [http://dnp.aps.org.

[5] Arora R et al. 2000 Eur. Phys. J. A 8 103; Puri R K et al. 1991 Phys. Rev. C 43 315; Puri R K et al. 1998 Eur. Phys. J A 3 277; 1992 Phys. Rev. C 45 1837; 1992 J. Phys. G: Nucl. and Part. 18 903; 1989 Eur. Phys. Lett. 9 767; Puri R K et al. 2005 Eur. Phys. J. A 23 429; Malik S S et al. 1989 Pramana J. Phys. 32 419; Dutt I and Puri R K 2010 Phys. Rev. C 81 047601; ibid. 81 (in press) nuc-th/1004.0493; ibid. 81 (in press); ibid. 81 (in press)

[6] Liu J Y et al. 2001 Phys. Rev. Lett. 86 975; Feng L Q and Xia L Z 2002 Chin. Phys. Lett. 19 321; Liu J Y, Guo W J, Xing Y Z, Li X G and Gao Y Y 2004 Phys. Rev. C 70 034610.

[7] Li B A, Danielewicz P and Lynch W G 2005 Phys. Rev. C 71 054603; Luo X F et al. 2007 ibid. 76 044902; Luo X F, Shao M, Dong X and Li C, 2008 Phys. Rev. C 78 031901.

[8] Khoa D T et al. 1992 Nucl. Phys. A 592 39; Huang S W et al. 1993 Phys. Lett. B 298 41; Batko G et al. 1994 J. Phys. G: Nucl. and Part. 20 461; Singh J et al. 2000 Phys. Rev. C 62 044617; Sood A D et al. 2004 Phys. Rev. C 70 034611; Kumar S et al. 1998 Phys. Rev. C 58 1618, Huang S W et al. 1993 Prog. Part. Nucl. Phys. 30 105; Vermani Y K et al. 2009 J. Phys. G: Nucl. and Part. 36 105103; Sood A D et al. 2004 Phys. Lett. B 594 260; Lehmann E et al. 1996 Z. Phys. A 355 55; Kumar S et al. 2008 Phys. Rev. C 78 064602.

[9] Bauer W 1998 Phys. Rev. Lett. 81 2534; Bertsch G F, Brown G E, Koch V and Li B A 1998 Nucl. Phys. A 490 745.

[10] Bass A et al. 1994 in GSI Annu. Report p.66; Li B A and Yennello S J 1995 Phys. Rev. C 52 R1746; Johnston H et al. 1996 Phys. Lett. B 371 186; Yennello S J et al. 1994 Phys. Lett. B 321 15.

[11] Hartnack C et al. 1998 Eur. Phys. J. A 1 151.

[12] Lehmann E et al. 1995 Phys. Rev. C 51 2113; 1993 Prog. Part. Nucl. Phys. 30 219.

[13] Dhawan J K, Dhiman N, Sood A D and Puri R K 2006 Phys. Rev. C 74 057901 and the refrences within; Goasiaux P B and Aichelin J 1997 Phys. Rev. C 56 2109.

[14] Wong C Y 1994 Introduction to High-Energy Heavy-Ion Collisions (World Scientific, Singapore).

[15] Singh J, Kumar S and Puri R K 2001 Phys. Rev. C 63 054603; Sood A D and Puri R K 2006 Eur. Phys. J. A 30 571; Kumar S and Puri R K 1999 Phys. Rev. C 60 054607; Vermani Y K et al. 2009 Phys. Rev. C 79 064613; Sood A K and Puri R K 2009 Phys. Rev. C 79 064618.

[16] Reisdorf W et al. 2004 Phys. Rev. Lett. 92 232301.

[17] Kumar S., Kumar S and Puri R. K. 2010 Phys. Rev. C 81 014601.

[18] Renforolt R E et al. 1984 Phys. Rev. Lett. 53 763.