Can Large \( u, d \) Quark Masses be Inferred from the \( D_s \rightarrow n\pi \) decay rate?

J. Milana  
Department of Physics, University of Maryland  
College Park, Maryland 20742, USA

S. Nussinov  
Department of Physics, University of Maryland, College Park, Maryland 20742  
and Physics Department, Tel–Aviv University, Ramat–Aviv, Tel–Aviv, ISRAEL  
(DOE/ER/40762–063, UMPP #95–011, July 24, 1995.)

The \( D_s \rightarrow n\pi \) measured branching could lead to information on \( u, d \) quark masses if it is dominated by the (helicity suppressed) \( D_s \rightarrow ud \) annihilation channel. The data would then suggest anomalously high \( m_u \approx m_d \approx 30 \text{MeV} \) masses. We find however that \( D_s \rightarrow udg \) can explain \( \sum \text{Br}(D_s \rightarrow n\pi) \) with zero up and down quark masses. We also argue that the present \( D_s \rightarrow n\pi \) data may not even convincingly imply any annihilation process whatsoever.

I. INTRODUCTION

While lepton masses are known with high precision, confinement and chiral symmetry breaking effects prevent exact determinations of the bare lagrangian masses of quarks. This is particularly true for the light \( u, d \) quarks. That \( m_u^0 + m_d^0 \neq 0 \) is determined from the pseudoscalar pions, the Nambu–Goldstone bosons. The latter get their mass only from the explicitly chiral symmetry breaking terms \( m_u^2 m_u + m_d^2 m_d \) in the lagrangian. Still, the basic relation \( \langle m_u^2 + m_d^2 \rangle = f^2 \langle m_u^2 \rangle \) leaves some ambiguity due to the unknown vacuum expectation value of the light quarks, so that only roughly can one say that \( m_u^0 + m_d^0 \approx 10 \text{MeV} \).

Lepton masses are directly determined from kinematics. In principle, inclusive rates like that of \( \tau \) decay, \( \tau \rightarrow \nu_\tau + \text{hadrons} \) can be described at high energies \( (E \gg \Lambda_{QCD}) \) via a quark process, say \( \tau \rightarrow \nu_\tau + \pi + d \) for the \( \tau \) decay into nonstrange hadrons and a \( \nu_\tau \) lepton. The effect of non–zero small \( u, d \) quark masses on the decay rate is then only \( O(\frac{m_u^2 + m_d^2}{m_\tau^2}) \) and is rather small for \( m_{u,d} \leq 10 \text{MeV} \).

There is another interesting way in which non–zero lepton masses are manifested which has to do with chirality conservation in the weak (or any other gauge) interaction. A given chirality eigenstate of a fermion with mass \( m \) and energy \( E \) coincides with the corresponding helicity state up to \( m/E \) admixture of the opposite helicity state. A well–known argument based on conservation of \( J_z \), the component of the angular momentum along the decay axis, forbids then the decay \( \pi \rightarrow l\bar{\nu} \) in the \( m_l = m_\nu = 0 \) limit. Such a decay only proceeds via the \( m/E \) “wrong helicity” components. The decay rate therefore contains an “helicity suppression factor” \( (m_u^2 + m_d^2)/m_\pi^2 \), a feature utilized in the kinematic searches proposed for finding \( \nu \) mixings in \( \pi, K \) decays.

Can such helicity suppression due to light quark masses manifest itself in hadronic physics, and in particular, can such considerations lead to an independent determination of \( m_u^2 + m_d^2 \)? A qualitative manifestation is expected in the case of glueball physics. The lowest glueball state is expected to have \( J^{PC} = 0^{++} \) and a mass in the \( 1.5–2.0 \text{GeV} \) range. The helicity arguments would then prefer \( gb \rightarrow \bar{\eta}s \) (i.e. to hadronic final states with kaons) to \( gb \rightarrow n\pi \) (i.e. n pion final states). It has been recently suggested that the decay of the \( D_s \) (the lowest lying bound state of the charm and strange quarks \( D_s = (\bar{s}s)^{0^{++}} \)) into \( n \) pions, \( D_s \rightarrow n\pi \), can be used to actually infer the \( u \) and \( d \) quark masses. This would indeed be the case if the decay is dominated by \( D_s \rightarrow ud \), Fig. (1). By analogy with the leptonic decays, one would then expect

\[
\frac{\sum \text{Br}(D_s \rightarrow n\pi)}{\text{Br}(D_s \rightarrow \mu\bar{\nu}_\mu)} \sim \frac{\text{Br}(D_s \rightarrow ud\bar{\nu}_\mu)}{\text{Br}(D_s \rightarrow \mu\bar{\nu}_\mu)} \sim 3\xi_{QCD} \frac{m_u^2 + m_d^2}{m_\mu^2}
\]

(1)

where a color factor and a moderate QCD enhancement factor, \( \xi \approx \alpha_s^2 \approx 1.6 \) were included. Using the experimental values \( \text{Br}(D_s \rightarrow \mu\bar{\nu}_\mu) = .6\% \) and \( \sum \text{Br}(D_s \rightarrow n\pi) \approx 1.5\% \) (the sum is actually dominated by the single mode \( D_s \rightarrow \pi^+\pi^+\pi^- \) large

\footnote{Indeed only ratio of quark masses can be extracted using chiral perturbation theory. See Ref. \cite{1} for a recent discussion.}

\footnote{The recent measurement \cite{1} of \( m_{\tau^0} \) at the \( e^+e^- \) collider at Beijing depends on the onset of the \( \tau^+\tau^- \) signal in the threshold region.}

\footnote{\textsuperscript{1}The 19770 \( 0^{++} \) glueball candidate suggested recently \cite{1} indeed most strongly prefers to decay into \( K\bar{K} \) contained states.}
$m_u \approx m_d \approx 30\text{MeV}$ mass values are inferred. The authors of Ref. [9] call attention to the severe discrepancy between these values and the more commonly accepted result that $m_u^0 + m_d^0 \approx 10\text{MeV}$ [10].

In the following we point out that (un(!))fortunately there is no QCD puzzle here. First, because the decay rate $D_s \to n\pi$ is not anomalously large and can be accounted for by gluon emission. Second, because the present data does not convincingly prove the existence of any annihilation channel with or without gluons and could still be accommodated by standard “spectator” decay, $\bar{c}s \to s + u + \bar{d} + \pi$, followed by $s\bar{s}$ annihilation. Finally, even if the analysis would have indicated unambiguously that the masses $m_u, m_d$ required to explain a large $\Gamma(D_s \to u\bar{d})$ rate are $m_u \approx m_d \approx 30\text{MeV}$, no true conflict with QCD would necessarily emerge. The point is that when explored at different energies (or momentum transfers) the $u, d$ quarks may display different masses interpolating between a “constituent” quark mass $m_c \approx 300 - 350\text{MeV}$ at low energies to $m_u^0 \sim m_d^0 \sim 10\text{MeV}$ at very large energies. Having $m_u^0 \approx m_d^0 \approx 30\text{MeV}$ at energies of $1\text{GeV} = \frac{1}{2} M_{D_s}$, would not present a genuine inconsistency: it would merely point to the importance of higher–twist effects.

We now proceed to discuss these points in more detail.

II. $D_s \to U + \bar{D} + G$ DECAY RATE

We now present our estimate for the rate $D_s \to u\bar{d}g$ using perturbative QCD (pQCD) methods. Taking that the emitted gluon hadronizes roughly 2/3 of the time into nonstrange quark pairs, this mechanism should provide a rough estimate for the total decay rate into strangeless hadronic states. The present calculation should be viewed as complementary to the earlier work of Band, Silverman, and Soni [11] which used a nonrelativistic quark model approach. We will comment at the end on the various approximations that have entered and why we consider this to be merely an order of magnitude calculation. Nevertheless, it will amply demonstrate that there is no real case to make that the decay $D_s \to n\pi$ is anomalously large. Indeed, it may in fact be smaller than expected.

The decay mechanism herein being considered is shown in Fig. (2), where the operator

$$O_1 = \frac{1}{4} \bar{u}_H(x) \gamma^\mu(1 - \gamma_5)c_\beta \gamma_\mu(1 - \gamma_5)u_H,$$

(2)
is a color octet interaction. Its Wilson coefficient $C_1(\mu) \neq 0$ at scales $\mu < M_W$ due to QCD evolution. [12]

In the pQCD motivated approach we start with the lowest order Fock component of the $D_s$ meson. The decay rate then involves a perturbatively calculable hard amplitude convoluted with a nonperturbative, soft physics wavefunction, $\psi_m$, of the $D_s$. The factorization scheme advocated by Brodsky and Lepage [13] is employed, whereby the momenta of the quarks are taken as some fraction $x$ of the total momentum of the parent meson weighted by a soft physics distribution amplitude $\phi(x)$. A peaking approximation is used for $\phi_{D_s}$, wherein

$$\phi_{D_s}(x) = \frac{f_{D_s}}{2\sqrt{3}} \delta(x - \epsilon).$$

(3)
The decay constant of the $D_s$ is $f_{D_s}$ (in the convention $f_\pi = 93\text{MeV}$) and $x$ is the light cone momentum fraction carried by the light quark. The parameter $\epsilon$ in $\phi_{D_s}(x)$ is related to the difference in the masses of the $D_s$ meson and $c$–quark,

$$M_{D_s} = m_c + \Lambda_{D_s},$$

(4)
whereby $\epsilon = \Lambda_{D_s}/M_{D_s}$. Note that in a pQCD approach the mass of the strange quark, being a soft–physics momentum scale ($m_s < \Lambda_{QCD}$), is taken to be zero. The $SU_f(3)$ violating effects differentiating between the $D_s^+$ and the $D^+$ is absorbed in the parameter $\epsilon$ for each of the two mesons.

In the present context, the factorization scheme is augmented by the viability of an $\epsilon$ expansion for the decay amplitude. Only those terms in the decay amplitude that are leading in an $\epsilon$ expansion are kept. This is crucial not only because higher–order terms in the expansion are formally higher–twist, but because gauge invariance is otherwise lost. [14] Thus, only gluon emission from the strange quark is included since it contains a 1/3 arising from the strange quark’s propagator. To go to higher–order in the $\epsilon$ expansion (by e.g. including gluon emission off of the charm quark) would necessitate going beyond the leading logarithm analysis being used for $C_1(\mu)$ in order to regain gauge invariance.

The amplitude for the decay $D_s \to u\bar{d}g$ is given by

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \langle \xi, q | P_{u\bar{d}} + \epsilon_{\alpha\beta\gamma\sigma} P_{D_s}^\beta x^\sigma q^\gamma \rangle \times \bar{u}(p_1) \gamma^\mu(1 - \gamma_5)\nu(p_2),$$

(5)
where $G$ is given by

$$G = \frac{1}{6\epsilon} G_F C_1(\mu) V_{cs} V_{ud} g_s(\mu) f_{D_s},$$

(6)
$\xi$ is the polarization vector of the gluon and in which all color indices have been suppressed. The decay rate is then given by

$$\Gamma(D_s \to u\bar{d}g) = \frac{5}{648\pi^2} \frac{\alpha_s(\mu) C_s^2(\mu)}{\pi^2 \epsilon^2} (V_{cs} V_{ud}^2)^2 G_F f_{D_s}^2 M_{D_s}^3.$$  

(7)
The decay constant of the $D_s$ is obtained from the decay $D_s \to \mu\nu\mu$, whereby we get the relation that

$$\Gamma(D_s \to \mu\nu\mu) = \frac{\alpha_s(\mu) C_s^2(\mu)}{\epsilon^2} \frac{5}{2\pi} \left(\frac{M_{D_s}}{9m_{\mu}}\right)^2 \Gamma(D_s \to \mu\nu).$$

(8)
Using the measured branching fraction \[ Br(D_s \to \mu \nu) = (5.9 \pm 2.2) \times 10^{-3}, \] taking \( \epsilon \approx 1/4, \Lambda_{QCD} = 2\text{GeV}, \mu = M_{D_s}, \) and including a factor 2/3 for the hadronization of the gluon into nonstrange modes, we obtain for our estimate for \( D_s \to n\pi \) that
\[ Br(D_s \to n\pi)(\mu^2 = M_{D_s}^2) = (1.2 \pm 0.5)\%, \] in roughly good accord with the data.

Clearly a number of approximations have been made to obtain this result. There is for example considerable dependence on the scale \( \mu \) in the above, leading to roughly a tripling of the predicted branching fraction if one takes \( \mu = M_{D_s}/2. \) It is also not unreasonable to expect sizable corrections to the \( \epsilon \) expansion, which, although found to work quite well \[ 13 \] in the case of the two body decays of the \( B \) meson \( (\epsilon \beta \leq .1) \), may in fact be marginal in the present case. Factors of two or so corrections are hence certainly quite likely. Nevertheless this estimate, as well as that to be found in Ref. \[ 14 \] obtained by different means, indicates that at the moment there is no glaring problem in the decay rate for \( D_s \to n\pi \).

III. POSSIBLE GENERATION OF \( D_s \to N\pi \) VIA SPECTATOR DIAGRAMS

The spectator diagram, Fig. (3), driven by the operator \( O_2 \) in \( H_{eff}, \)
\[ O_2 = \frac{1}{4} s_c \gamma^\mu(1 - \gamma_5) c_u d_\beta \gamma_\mu(1 - \gamma_5) u_\beta, \] is the leading decay mechanism of the charm quark \( (C_2 \approx 1.25) \) and in the presence of the spectator \( s \) quark found in the \( D_s \) meson, leads to a 4 quark state. Note that the fact that the \( d, u \) quark system (as well as separately the \( s\bar{s} \) pair) form a color singlet, distinguishes these diagrams from the annihilation mechanisms of Section (II). These features are nicely reflected in the final hadronic systems containing mesons with an \( s\bar{s} \) component:
\[ Br(D_s \to \phi\pi) = 3.5\%, \quad Br(D_s \to \phi\rho^+) = 6.5\%, \]
\[ Br(D_s \to \eta\rho^+) = 10.0\%, \quad Br(D_s \to \eta'\rho^+) = 12.0\%, \]
\[ Br(D_s \to \eta'\pi^+) = 4.7\% \] (11)
or strange meson pairs (thru color rearrangement in final state by soft gluon exchanges)
\[ Br(D_s \to K\bar{K}) = 3.5\%, \quad Br(D_s \to K^*\bar{K}) = 7.5\%, \]
\[ Br(D_s \to K^*\bar{K}^*) = 5.6\%. \] (12)

Along with \( \sim 20\% \) semileptonic \( (\epsilon\nu_\mu X, \mu\nu_\mu X) \) branchings, \( \sim 7\% \) purely leptonic \( (0.6\% \mu_\mu \text{ and } (m_\tau/m_\mu)^2 (1 - m_\tau^2/M_{D_s}^2)^2 \) times as much \( D_s \to \tau\nu_\tau, \) and some genuine multibody decays, this could roughly account for the full \( D_s \) decay. Thus from this particular viewpoint, there is no need for a strong \( D_s \) annihilation channel. \[ \] It is furthermore conceivable that some final state interactions will admix a few percent nonstrange final states and hence account for the observed \( D_s \to n\pi \) rate even without invoking \( D_s \) annihilations.

If the \( D_s \to n\pi \) decays are indeed dominated by \( D_s \to u\bar{d}, \) the pattern of these decay is rather puzzling.

1. \( e^+e^- \to \text{hadrons proceeding via } e^+e^- \to q\bar{q} \text{ produces at } W = 2\text{GeV} \text{ an average multiplicity of } \approx 5. \)
2. Why does the \( u\bar{d} \) system in \( D_s \) annihilation predominantly hadronize into a 3\( \pi \) system?

Both these puzzles are readily explained if \( D_s \to \pi^+\pi^-\pi^- \) originates from \( D_s \to (X^0)_{ss} + \pi^+ \) generated via the standard spectator decay diagram. The \( (X^0)_{ss} \) should be a resonance with a strong \( ss \) component which decays into \( \pi^+\pi^- \) with a substantial rate (but not into \( \pi^+\pi^-\pi^- \)). These requirements are all satisfied by the \( f_{980} \) state. Amusingly, the original experiment \[ 16 \] did find evidence for substantial \( D_s \to f_{980} + \pi^+ \) decay, accounting roughly for 30\% of \( D_s \to \pi^+\pi^-\pi^- \). If the width of the \( f_{980} \) is much larger than the \( \Gamma \approx 50\text{MeV} \) implied in the fit in this experiment, \( D_s \to f_{980} + \pi^+ \) could conceivably become dominant! Hopefully this issue will be soon settled by the new Fermilab experiment.

IV. HIGHER TWIST EFFECTS: RUNNING, NON-PERTURBATIVE \( U, D \) QUARK MASSES

The effects of spontaneous chiral symmetry breaking \( (S\chi SB) \) are often parametrized as giving the original, almost massless \( u, d \) quarks substantial “constituent” masses, \( m_u^c \approx m_d^c \approx 350\text{MeV}. \) Viewing the lagrangian quarks as bare point-like quanta, the “constituent quarks” are then the corresponding “quasi–particles” obtained by attaching a cloud of gluons and \( q\bar{q} \) pairs. This cloud endows the quarks with their constituent mass.

This physical picture suggests that when probed with some momentum transfer or some energy \( E \) the quarks

\[ 5 \] Though such an annihilation channel was originally invoked in Ref. \[ 14 \] as in the \( D^0 \) decay in order to explain the shorter lifetime \( \tau_{D_s} < \tau_{D^0}. \)

\[ 6 \] Note however that such a suppression is natural in the mechanism considered in Section (II), \( D_s \to u\bar{d}g, \) where one expects a leading hadron formed from each of the three outgoing partons.
will exhibit some effective mass \( m_{\text{eff}}^q (E) \). Roughly speaking, only some “inner core” of radius \( r \approx 1/E \) of the “cloud” around the constituent quark will be probed in this case and only those chiral symmetry breaking effects which are generated on distance scales \( r \leq r \) will become manifest.

The \( m_{\text{eff}}^q (E) \) curve interpolates between \( m_{\text{eff}}^q (E) = m_q^0 \) at large energies and \( m_q^\ast \) at small \( E \). Its actual shape depends on the QCD dynamics underlying the \( S \chi \)SB. If \( S \chi \)SB is due to the same mechanism which generates confinement \( ^7 \), then the constituent quarks would have hadronic sizes and \( m_{\text{eff}}^q (E) \) would be expected to abruptly decrease as \( E \) is increased beyond \( 0.3 - 0.5 \text{GeV} \). If on the other hand, \( S \chi \)SB is, to a large extent due to gauge field configurations of small distance scales, then the \( m_{\text{eff}}^q (E) \) curve would be much flatter. A remnant “constituent” mass of say \( m_{\text{eff}}^q \approx m_{\text{eff}}^q \approx 30 - 40 \text{MeV} \) might then conceivably linger at \( E = \frac{1}{2} M_D \), and manifest in \( D_\pm \rightarrow u d \) annihilation. Such high values would of course have important repercussions, indicating that nonperturbative effects in the \( 1 - 2 \text{GeV} \) region are much more than previously believed. \( ^7 \) Still no true conflict with the \( m_q^0 = 10 \text{MeV} \) determination follows, although in view of the previous two sections, this observation is mainly of academic interest.

**ACKNOWLEDGEMENTS**

One of the authors, S. Nussinov, would like to acknowledge most helpful discussions with H. J. Lipkin. This work was supported in part by DOE Grant DOE-FG02-93ER-40762.

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\( ^{\dagger\dagger} \) Thus for example, such higher–twist effects would reduce the \( \Gamma(\tau \rightarrow \nu_\tau ud) \approx \Gamma(\tau \rightarrow \nu_\tau + \text{hadrons}) \) rate by \( \sim 16 m_q^0 m_\tau^\ast \approx 1 \pm \frac{1}{2} \% \) (phase space reduction) for \( m = 30 - 60 \text{MeV} \). This exceeds the accuracy of the experimental determination of \( R_\tau \equiv \Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})/\Gamma(\tau \rightarrow \nu_\tau e^- \bar{\nu}_e) \) and the QCD perturbative estimates. For a recent interesting synthesis of several QCD results, including \( \tau \) decay, see Ref. [8].

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FIG. 1. The annihilation diagram for $D_s \to u\bar{d}$. The operator $O_2$ is defined in Section (III). This process, as well as additional gluon emission from the outgoing light quarks, is helicity suppressed.

FIG. 2. The leading contribution for $D_s \to u\bar{d}g$. Emission from the charm quark is subleading. Its inclusion would require decomposing the operator $O_1$ to restore gauge invariance.

FIG. 3. The dominant decay mechanism of the $D_s$. 
