Interlayer tunneling in a non-Fermi-liquid metal

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Abstract

We study the effect of interlayer tunneling in the gauge theory describing a quasi-two-dimensional paramagnetic metal close to a second-order or weakly first-order antiferromagnetic phase boundary. In that theory, two species of fermions have opposite (rather than equal) charges with respect to the gauge field. We find that single-particle interlayer tunneling is suppressed at low energies. The effect of pair tunneling is analyzed within the \((3 - d)\) expansion. The resulting phase diagram has superconducting and non-Fermi-liquid normal phases, and so is compatible with that of the copper-oxide superconductors.
Recently, a non-trivial fixed point and the associated non-Fermi-liquid behavior have been found for a system of two-dimensional (2d) fermions interacting with a transverse gauge field \([1]–[4]\); see also \([5]–[10]\). The idea of non-Fermi-liquid metal has long been proposed \([11]\) to apply to the copper-oxide superconductors. In that case, a study of interactions between two copies (layers) of such 2d systems is important for at least two reasons. First, the real materials are three-dimensional and one would like to know if interlayer interactions destroy the fixed point. Second, interlayer tunneling of pairs of electrons, in cases when single-electron tunneling is suppressed by non-Fermi-liquid effects, was suggested as the mechanism for high-\(T_c\) superconductivity \([12, 13]\); it seems worthwhile to explore this possibility within the gauge theory framework.

A priori, one can imagine various types of charge assignments for fermions with respect to the new gauge field. The correct one, for a real system, should be determined from microscopic considerations. In the model considered in refs.\([1]–[4]\) all fermions have the same charge. Here we want to consider a different model, in which there are two species of fermions with opposite values of the charge. This charge assignment can be derived from microphysics if the antiferromagnetic (AF) correlation length extends over at least a few lattice spacings \([14, 7]\). Such situation naturally arises in systems close to a second-order or weakly first-order AF phase boundary. In the copper oxides, it is likely to occur at doping concentrations lower than those for which \(T_c\) is maximal—the "underdoped" materials; however, the model itself may remain correct even at the optimal doping concentrations. We do not have anything to add to the derivation of this model here, but concern ourselves with its phenomenology, namely, the phase diagram.

The main new element of our theory is that we outfit the model with the oppositely charged fermions with local interlayer (and in-layer) interactions. The difference in the fate of interlayer interactions between the two types of models discussed above
is indeed dramatic. In the model of refs. [1]–[4], pair tunneling is irrelevant at low energies, at least in the leading order of the approximations employed. In our model, while single-particle tunneling is suppressed at low energies by the spin gap, pair tunneling is relevant and together with the local in-layer interaction determines the phase diagram of the system.

Working in the leading order of the $\epsilon = 3 - d$ expansion [4], we find two phases: superconducting and non-Fermi-liquid normal. In the leading order, the anomalous dimension of the fermions in the normal phase coincides with that found in refs. [1, 3]; we argue that to hold to all orders, despite the presence of a non-vanishing repulsive local interaction. A phase transition with temperature between the two phases appears possible. Thus, our model, besides being to some extent motivated from the microscopic standpoint, has a phase diagram compatible with that of the copper-oxide superconductors.

The new gauge field [15] is supposed to arise as a result of spin-charge separation, with electron dissociating into two new quasiparticles. In the Néel phase on the square lattice we would have on each site of sublattice A

$$\psi_{A\sigma} = z_\sigma f_A^\dagger$$  \hspace{1cm} (1)

and on each site of sublattice B

$$\psi_{B\sigma} = \tilde{z}_\sigma f_B^\dagger,$$  \hspace{1cm} (2)

where $\psi$ is the electron operator and $\sigma$ is the spin index. The smoothly varying bosonic fields $z$ and $\tilde{z}$ represent the staggered spin variable

$$n = z^\dagger \sigma z = -\tilde{z}^\dagger \sigma \tilde{z}$$  \hspace{1cm} (3)

and therefore are related as

$$\tilde{z}^\sigma = \epsilon^{\sigma\rho} z^\rho$$  \hspace{1cm} (4)
(up to an inessential phase factor) \[14, 7\]. The redundancy in the number of components of \( z \) is the gauge redundancy, apparent in \([14] \). Because \( z \) and \( z^* \) have opposite charges with respect to the gauge field, so do \( f_A \) and \( f_B \). The fermionic fields \( f_A \) and \( f_B \) also carry the usual electric charge, the same amount for both species. In the AF phase, the new gauge symmetry is spontaneously broken by a non-zero expectation value of \( z \).

We are interested, though, in the paramagnetic phase. It is more difficult to visualize. However, the charge assignments of the fields cannot change across the phase boundary, at least as long as the correlation length stays sufficiently large. On both sides of a second-order or weakly first-order phase transition a system should be described by the same effective theory, the only difference being in the values of the parameters. Therefore, we describe the paramagnetic phase by the same theory containing two oppositely charged fermions (the index \( A, B \) now merely labels the species) and the field \( z \), interacting with the gauge field \( a_\mu \). However, the expectation value of \( z \) is now zero and all its components are massive ("paramagnetic magnons").

The Lagrangian density describing the gauge interactions of these fields in a single layer is given in refs.\([14, 7]\); our notations follow ref.\([7]\). By the same logic as above, even as we move further away from the phase boundary, the interaction of the low-energy fields—the fermions and the gauge fields—should retain its form (see Eq.\((9)\) below).

When there are two such layers, fields in each of them have their own gauge transformations. The only gauge-invariant interactions between the layers are those that correspond to tunneling of objects neutral with respect to the gauge fields. Thus, a single fermion can only tunnel accompanied by a quantum of the \( z \) field. The
corresponding lowest-dimension operators are

\[ O_{sA} = f_{1A}^\dagger f_{2A} z_{12} \]  

and similar ones for B fermions or A→B tunneling; 1,2 is the layer index. At frequencies much lower than the spin gap (which determines the mass of \( z \)) and distances much larger than the AF correlation length, the tunneling process described by (5) is suppressed. In that regime, the field \( z \) can be integrated out leaving only infrared-finite renormalizations of local interactions, such as Eqs.(6), (7) below.

Pairs of fermions can tunnel without \( z \) quanta. The pair tunneling operators are

\[ O_p = f_{1A}^\dagger f_{1B}^\dagger f_{2B} f_{2A} \]  

and its hermitean conjugate. Within the effective low-energy theory, these are viewed as local operators in the 2d space and time, analogous to the reduced BCS Hamiltonian. (For example, in the effective theory there is no objection to taking two holes at the same space point, as in (6).) The pair tunneling leads to a renormalization of the in-layer four-fermion interactions

\[ O_{r1} = f_{1A}^\dagger f_{1B}^\dagger f_{1B} f_{1A} , \quad O_{r2} = f_{2A}^\dagger f_{2B}^\dagger f_{2B} f_{2A} , \]

which we then have to include in our analysis. The particle-hole tunneling operators not involving \( z \) fields are

\[ f_{1A} f_{1A}^\dagger f_{2A}^\dagger f_{2A} , \quad f_{1A}^\dagger f_{1A} f_{2B} f_{2B} \quad \text{etc.} \]

but these will turn out to be irrelevant at small values of the corresponding coupling constants.

The physical situation we want to describe is as follows. We assume that the gauge field has become deconfining due to effects of the gapless fermions \[16\]. Because in
our model A and B fermions have opposite gauge charges, A and B particles moving with opposite momenta make parallel currents and are attracted to each other. This attraction increases their chances to be near each other and thus effectively enhances the local interactions (6), (7). We then have to understand if this enhancement can be limited by a repulsive effect of the local interactions themselves.

There may be different ways to describe the physics outlined above. Here we use the perturbation theory built with the resummed gauge propagator [5], incorporating the Landau damping, but with the usual free fermion propagator (in contrast to the 1/N method of refs.[1, 3]). The advantage of this approach is that the strongest infrared effects of the local four-fermion interactions are easy to isolate. These are the infrared-singular renormalizations coming from the BCS channel. Another useful device is the \((3 - d)\) expansion [4]. At \(d \to 3\), the infrared divergences due to the gauge interaction are also logarithmic, and the logarithms can be conveniently handled by the renormalization group. Rather than using a modified fermion propagator, the \((3 - d)\) expansion introduces an additional renormalizable parameter—the Fermi velocity. Such a trade-off is familiar from the \(\lambda \phi^4\) scalar theory: in the \((4 - d)\) expansion \(\lambda\) is a running coupling, while in the large-\(N\) expansion directly in \(d = 3\) it is not.

We thus consider the Lagrangian

\[
L = \sum_{l=1,2} \left[ -\frac{1}{4\tilde{e}^2}(\partial_\mu a_{l\nu} - \partial_\nu a_{l\mu})^2 + F_l^\dagger \left( i\partial_t + \sigma_3 a_{l0} + \frac{1}{2m}(\nabla - i\sigma_3 \mathbf{a}_l)^2 + \zeta \right) F_l \right] - g(\mathcal{O}_{r1} + \mathcal{O}_{r2}) - g'(\mathcal{O}_p + \mathcal{O}^\dagger_p), \tag{9}
\]

where \(l\) is the layer index, \(\tilde{e}^2\) is the gauge coupling (which is not renormalized by the gauge interaction [9]), \(F_l = (f_{lA}, f_{lB})^T\), \(\sigma_3\) is the third Pauli matrix, and \(\zeta\) is the chemical potential. We follow the evolution of three couplings: \(g, g'\) and \(\alpha \equiv \tilde{e}^2 v_F / 4\pi\) where \(v_F\) is the Fermi velocity. The one-loop renormalization of these couplings in
d → 3 dimensions is
\begin{align*}
g(\mu) &= g_b + \frac{6g_b\alpha_b}{\epsilon^2\pi\mu^\epsilon/3} - c_b(g_b^2 + g_b^{'2}) \ln \frac{\Lambda}{\mu} \\
g'(\mu) &= g'_b + \frac{6g'_b\alpha_b}{\epsilon^2\pi\mu^\epsilon/3} - 2c_b g_b g'_b \ln \frac{\Lambda}{\mu} \\
\alpha(\mu) &= \alpha_b - \frac{\alpha_b^2}{\epsilon\pi\mu^\epsilon/3}
\end{align*}

(10)

(11)

(12)

where we have used the sharp infrared cutoff at frequency \(\mu\); the subscript \(b\) denotes the bare parameters; \(\epsilon = 3 - d\), \(c_b = p_F^2/2\pi^2 v_{Fb}\), and we do not write terms suppressed by powers of the ultraviolet cutoff \(\Lambda\). The second terms on the right-hand sides of (10), (11) come from the gauge interaction in the BCS channel, while the last terms from the BCS diagrams with the local four-fermion vertices. To reiterate the important point, because in our model \(f_A\) and \(f_B\) have opposite gauge charges, the renormalization of \(g\), \(g'\) due to the gauge interaction is of the opposite sign compared to the model of refs.\[1\]–\[4\] and makes small \(g\), \(g'\) grow in the infrared. Eqs.(10), (11) do not contain corrections due to the renormalization of the fields \(f\). These are of the order \(\epsilon\) compared to the second terms on the right-hand sides of (10), (11) and therefore are subleading in the \((3 - d)\) expansion.

We have found that the one-loop correction to four-fermion interactions from the gauge interaction in the \(2p_F\) channel does not contain an infrared divergence in our perturbation theory at \(d \to 3\). Due to the renormalization of the fields, the particle-hole operators (8) are then irrelevant as long as the bare values of couplings with which they are added to the Lagrangian (4) are sufficiently small.

Eqs.(10)–(12) also show that the naive infrared dimensions of \(g\), \(g'\) are zero, while that of \(\alpha\) is \(\epsilon/3\) (not \(\epsilon\), its mass dimension, as stated in ref.[4]). So, we introduce
\[\lambda(\mu) = \frac{\alpha(\mu)}{\mu^\epsilon/3}.\]

(13)

It is also convenient to introduce \(u = cg\) and \(u' = cg'\) instead of \(g\) and \(g'\). Notice
that $c$ contains renormalizable quantity $v_F$ but the difference between $c(\mu)$ and $c_0$ is inessential to the leading order of the $\epsilon$ expansion. The one-loop beta-functions are

$$\frac{du}{d \ln \mu} = - \frac{2u\lambda}{\epsilon\pi} + u^2 + u'^2$$

(14)

$$\frac{du'}{d \ln \mu} = - \frac{2u'\lambda}{\epsilon\pi} + 2uu'$$

(15)

$$\frac{d\lambda}{d \ln \mu} = - \frac{\epsilon}{3} \lambda + \frac{\lambda^2}{3\pi}.$$  

(16)

Note that the first two of these are singular at $d \to 3$ (renormalization directly at $d = 3$ would produce $\ln^2 \mu$).

In the infrared, $\lambda$ runs to the stable fixed point \([4]\), $\lambda = \epsilon\pi$, which describes the same non-Fermi-liquid behavior as the modified fermion propagator of refs.\([1, 3]\) and, probably, the fixed point of ref.\([2]\). Substituting the fixed point value of $\lambda$ into Eqs.\((14),(15)\), we obtain the equations for infrared flows, $t = -\ln \mu$,

$$\frac{du}{dt} = 2u - u^2 - u'^2$$  

(17)

$$\frac{du'}{dt} = 2u' - 2uu'.$$

(18)

The resulting flows (which can be found analytically) are plotted in Fig.1. The flows are symmetric under $u' \to -u'$, so we assume $u'$ non-negative. There is a critical line at $u = u'$. If $u_b > u_b'$, the couplings run to the infrared stable fixed point $u = 2$, $u' = 0$. This represents a non-Fermi-liquid normal state with repulsive in-layer four-fermion interaction. The interlayer electric charge transport in this state, either by single or pair tunneling, is suppressed. If $u_b < u_b'$, the couplings run without limit, so we expect that the system generates a mass gap. Because the couplings run to negative values of $u$ (attraction), we expect that this phase is superconducting. It should be similar in properties to that of ref.\([13]\). Because the mass gap cuts off the infrared renormalization, the single-particle and the particle-hole tunneling operators are partially recovered in this state.
The microscopic mechanism of superconductivity may be purely electronic (in which case \( u_b \) is the Coulomb pseudopotential usually known as \( \mu \)), or a value of \( u_b \) smaller than \( u'_b \) may be achieved by a partial compensation of the Coulomb repulsion by some attractive effect, due to phonons, spin fluctuations etc.

By analogy with the BCS theory, we expect that the critical line (in the BCS case, point) moves with temperature so that more points in the \((u, u')\) plane fall in the domain of attraction of the normal phase. For a given point \((u_b, u'_b)\), the temperature at which the line passes through that point is the critical temperature of the superconducting to normal transition.

Because of the singularity at \( \epsilon \to 0 \) in the beta-functions (14), (15), the first, linear in \( u, u' \) terms in (17), (18) are not proportional to \( \epsilon \), and the fixed point value of \( u \) is not suppressed by \( \epsilon \). This does not mean a breakdown of the \( \epsilon \) expansion. All the terms on the right-hand sides of Eqs.(14), (15) arise from the exceptional, BCS ladder diagrams, which only contribute to the beta-functions at one loop (cf. a similar discussion of the \( 1/N \) expansion in ref.[1].) In particular, the usual argument from the Fermi-liquid theory shows that there will be no terms of higher order in \( u, u' \) not accompanied by powers of \( \tilde{e}^2 \) in any of the three beta-functions. The renormalization of \( \lambda \) is restricted even further. The renormalization of \( v_F \) is determined from the fermion self-energy. Because in the \((3-d)\) expansion the gauge interaction adds only logarithmic singularities, it dressing vertices and internal lines of infrared-convergent Fermi-liquid graphs cannot turn them into divergent ones. Corrections to the gauge vertex are suppressed at low energies in \( d > 2 \) [3]. Then, the only self-energy graphs that produce an infinite renormalization of \( v_F \) are those in which a gauge line encloses the rest of the graph. These are summed up by the one-loop beta-function (16); the finite renormalizations replace \( v_{Fb} \) with some \( v^*_F \) in the bare coupling \( \lambda_b \). By appealing again to the Fermi-liquid theory, we argue that fermions with local interactions can
be integrated out with essentially the same effect as those without it \[^1\] , so that
the gauge coupling $\tilde{e}^2$ is still not renormalized. Hence, we argue that the beta-
function \[^{\text{[16]}}\) for $\lambda$ is in fact exact. The fact that the fixed point value of $u$ does
not vanish at $d \rightarrow 3$ suggests the existence of a non-Fermi-liquid normal state in
more isotropic three-dimensional metals close to an AF phase boundary, in which the
quasiparticle weight vanishes logarithmically at small energies but the four-fermion
repulsion remains constant.

In summary, we have studied the effect of interlayer tunneling in the 2d gauge
theory with two oppositely charged species of fermions, arising as a low-energy energy
description of quasi-2d paramagnetic metals close to an AF phase boundary. In the
leading order of the $(3-d)$ expansion, we have found two phases, superconducting and
non-Fermi-liquid normal, suggesting that our theory may apply to the copper-oxide
superconductors.

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B. Kim and X.-G. Wen, Phys. Rev. B 50, 8078 (1994). In the superconducting phase, where the fermions have a gap, confinement of charges may be restored (this question requires further study) but in any case we expect the confinement radius to be much larger than the coherence length of superconductivity.
FIGURE CAPTION

FIG. 1. Renormalization-group flows at the fixed point value of $\lambda$ for the model in text. The arrows point towards the infrared.
Figure 1