Exclusive and Inclusive Decays of the $B_c$ Meson in the Light–Front ISGW Model

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Abstract

We investigate the total decay rate of the (ground state) $B_c$ meson within the framework of the relativistic constituent quark model formulated on the light-front (LF). The exclusive semileptonic (SL) and nonleptonic (NL) beauty and charm decays of the $B_c$ meson are described through vector and axial hadronic form factors, which are calculated in terms of the overlap of the parent and daughter meson LF wave functions. The latters are derived via the Hamiltonian LF formalism using as input the update version of the ISGW model. The inclusive SL and NL partial rates are calculated within a convolution approach inspired by the partonic model and involving the same $B_c$ wave function which is used for evaluation of the exclusive modes. We predict the partial rates for 74 exclusive SL and NL channels and 43 inclusive partial rates corresponding to the underlying $\bar{b} \to \bar{c}$ and $c \to s$ quark decays. Based on our approach we find $\Gamma^{\bar{b}}(B_c) = 0.52 \pm 0.02 \; ps^{-1}$, $\Gamma^c(B_c) = 0.98 \pm 0.07 \; ps^{-1}$, where the theoretical uncertainty is dominated by the uncertainty in the choice of the threshold values at which the hadron continuum starts. For the $B_c$ lifetime we obtain $\tau_{B_c} = 0.63 \pm 0.02 \; ps$ in a good agreement with the prediction obtained using the nonrelativistic operator product expansion. We also predict decay rates for many specific weak transitions of $B_c$. In particular, for the branching fractions of the $B_{c}^{\pm} \to J/\psi \mu^+\nu_\mu$, $B_{c}^{\pm} \to J/\psi \pi^+$ and $B_{c}^{+} \to J/\psi X$ decays we obtain 1.7%, 0.1% and 13.2%, respectively.

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Weak decays of hadrons containing a heavy quark present the most direct way to determine the Cabbibo–Kobayashi–Maskawa (CKM) matrix and to test our present knowledge of the QCD confinement scale inside hadrons. Among various heavy flavor hadrons the $B_c$ meson, the bound state of the $b\bar{c}$ system with open charm and beauty, is particularly interesting. The theoretical interest in the study of the $B_c$ meson is stimulated by the experimental search at CDF and LHC. Since last year, ALEPH has a very clean signal $B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu$ with a measured $B_c$ mass $5.96^{+0.25}_{-0.19}$ GeV/$c^2$ [4]. A new preliminary OPAL analysis [2] finds two $J/\psi \pi^+$ candidates with the masses $6.29 \pm 0.17$ and $6.003 \pm 0.06$ GeV/$c^2$. Recently the CDF Collaboration reported the observation of $B_c$ in 1.8 TeV $p\bar{p}$ collisions using the CDF detector at Fermilab Tevatron [3]. The CDF results for the mass and lifetime are $M_{B_c} = 6.40 \pm 0.39 (\text{stat}) \pm 0.13 (\text{syst})$ GeV/$c^2$ and $\tau_{B_c} = 0.46^{+0.18}_{-0.16} (\text{stat}) \pm 0.03 (\text{syst})$ ps. The physics of $B_c$ mesons has stimulated much recent works on their properties, weak decays and production cross section on high energy colliders. A comprehensive analysis of the $b\bar{c}$ spectroscopy and strong and electromagnetic decays of the excited states has been given in [4].

Similarly to $D$ and $B$ mesons, the ground $b\bar{c}$ state is stable against strong or electromagnetic decay due to its flavor content and disintegrates only via weak interactions. The weak $B_c$ meson decays occur mainly through the CKM favored $b \rightarrow c$ transitions with $c$ being a spectator, and $c \rightarrow s$ transitions with $\bar{b}$ being a spectator. Weak decay properties of the ground state $b\bar{c}$ including semileptonic (SL) and various exclusive nonleptonic (NL) modes have already been discussed in refs. [3]–[5]. In refs. [3]–[7] the $B_c$ lifetime has been estimated on the basis of a modified spectator model, where the phase space for the free quark decay is modified to account for the physically accessible kinematical region [4], or the $\bar{b}$ and $c$ quark masses are reduced by the binding energy to incorporate bound state effects [8]. An estimation of the $B_c$ lifetime $\tau_{B_c}$ using a modified spectator model and information gained from the calculation of dominant exclusive modes is given in [4]. A careful analysis of the $B_c$ lifetime, performed using the nonrelativistic QCD and including the nonrelativistic corrections up to $v^2/c^2$, has been carried in [10], see also [11]. The wide range of lifetimes $\tau_{B_c}$ reported in these papers, reflects the uncertainty due to the various model assumptions on the modification of the free decay rates due to the bound state effects and the limited knowledge of the heavy quark masses.

Weak decays of charmed and bottom hadrons are particularly simple in the limit of infinite heavy quark mass, where the decay rate of a hadron $H_Q$, containing a heavy quark $Q$ is completely determined by the decay rate of the heavy quark itself. In this limit, one might expect that $\Gamma(B_c) \approx \Gamma(\bar{B}_c^0) + \Gamma(D^0)$. If this result were hold, the $B_c$ lifetime would be rather short, namely $\tau_{B_c} \approx 0.3$ ps, and $B_c$ decays would be dominated by $c \rightarrow s$ decay over $\bar{b} \rightarrow \bar{c}$ decay in the ratio of roughly 4 : 1. In reality, heavy hadrons are the bound states of heavy quarks with light constituents. The inclusion of these soft degrees of freedom generates important contributions due to the preasymptotic effects, like the binding effects and the Fermi motion of a heavy quark inside the hadron. These effects have a significant impact on the lifetime and SL branching ratios. The leading nonperturbative effect is described by a distribution function $F(x)$, ("shape function"), which arises as a result of the resummation of the heavy quark expansion [12] and has been also incorporated into phenomenological models of inclusive decays, first in refs. [13], [14], and lately in refs. [15]–[18]. The actual calculation of $F(x)$ is a difficult nonperturbative problem which in practice introduces considerable uncertainties in the evaluation of the hadron lifetime. An important advantage of the $b\bar{c}$ system is the applicability of a quark potential model treatment [4], [8], [19]. In what follows we assume that, instead of
QCD with its complicated dynamics of infinite number of degrees of freedom in the light cloud, we consider a constituent bound–state problem of a heavy quark interacting with a lighter one via a potential. Then, using the formalism of the LF relativistic quantum mechanics \([20], [21]\) it is possible to encode all the nonperturbative QCD effects in a LF quark model wave function \(\psi(x, p_\perp^2)\) of a heavy hadron. The internal motion of a heavy \(Q\)–quark inside the heavy flavor meson is described by the distribution function \(|\psi(x, p_\perp^2)|^2\), which represents the probability to find a heavy quark carrying a LF fraction \(x = p_Q^+/P_{H_Q}^+\) of the meson momentum and a transverse relative momentum squared \(p_\perp^2 = p_\perp^2\).

A priori, there is no connection between equal–time (ET) wave function \(w(k^2)\) of a constituent quark model and LF wave function \(\psi(x, p_\perp^2)\). The former depends on the center–of–mass momentum squared \(k^2 = |k|^2\), while the latter depends on the LF variables \(x\) and \(p_\perp^2\). However, there is a simple operational connection between ET and LF wave functions \([22]\). The idea is to find a mapping between the variables of the wave functions that will turn a normalized solution of the ET equation of motion into a normalized solution of the different looking LF equation of motion. That will allows us to convert the ET wave function, and all the labor behind it, into a usable LF wave function. This procedure amounts to a series of reasonable (but naive) guesses about what the solution of a relativistic theory involving confining interactions might look like.

We convert from ET to LF momenta by leaving the transverse momenta unchanged, \(p_\perp = k_\perp\), and letting

\[
k_z = (x - \frac{1}{2})M_0 + \frac{m_{sp}^2 - m_Q^2}{2M_0},
\]

where \(M_0 = \sqrt{k^2 + m_Q^2} + \sqrt{k^2 + m_{sp}^2}\) is the free mass operator with \(m_{sp}\) being the mass of the quark–spectator. Now we obtain the LF wave function from

\[
\psi(x, p_\perp^2) = \frac{\sqrt{M_0}}{4x(1-x)} \cdot \left[1 - \left(\frac{m_Q^2 - m_{sp}^2}{M_0^2}\right)^2\right] \cdot \frac{w(k^2)}{\sqrt{4\pi}}.
\]

It can be easily verified that \(\int_0^1 dx \int d^2p_\perp |\psi(x, p_\perp^2)|^2 = 1\) provided \(\int d^3k |w(k^2)|^2 = 1\). It is wave functions made kinematically relativistic in this fashion, that were used to calculate the form factors of heavy–to–heavy and heavy–to–light exclusive transitions in refs. \([20], [21]\).

A relevant feature of our approach is that both exclusive and inclusive decays are consistently treated in terms of the same heavy quark wave function \(\psi(x, p_\perp^2)\). So far, this approach has been applied only to the exclusive and inclusive partial widths of \(\bar{B}^0 [18]\), where it has been found that the overall picture is quantitatively satisfactory. In this paper we extend previous calculations to compute the lifetime and various decay branching fractions of \(B^+_c\).

The paper is organized as follows. In Section 2, after a general overview of the model, we set the framework for our theoretical calculation of heavy meson lifetimes in the LF quark constituent model. Here we present general analysis focusing on calculation of inclusive SL and NL decay rates. In Section 3 we apply the model to calculate various beauty and charm decay rates of the \(B_c\) meson in parallel with those of the beauty decays of the \(\bar{B}^0\) meson. In the former case we include both weak annihilation and Pauli interference contributions to the total width. Section 4 completes the paper with a summary and conclusions. Technical details of the calculation are given in Appendix.
2 Description of the model

Without going into too many details, we present first a short outline of the method of calculation of heavy meson partial widths, which is discussed at length in [18]. We introduce the necessary definitions and describe the main steps in the calculation of exclusive and inclusive decay widths of the heavy mesons. We will make several strong assumptions to obtain a model as simple as possible. Yet, we will check that it agrees, within reasonable limits, with available data on the SL and NL $\bar{B}^0$–meson decays, as well as with other theoretical predictions. We thus feel confident that this model can be used advantageously to obtain rough estimation for other hadrons, such as $B_c$.

2.1 Kinematics

Consider the SL decay rates first. Instead of considering the exclusive modes individually we will sum (in the representative case of $\bar{B}^0$ decay) over all possible charmed final states $X_{cd}$ containing the $\bar{c}$–quark. This sum includes hadronic states with a large range of invariant mass $M_X$: $M_D \leq M_X \leq M_B$. For the bottom mesons, the $b$–quark mass $m_b$ provides a short–distance scale that leads to a large energy release into the intermediate hadronic states. Therefore the energy which flows into hadronic system $X_{cd}$ is typically much larger than the energy scale $\Lambda_{QCD}$ which characterizes the strong interactions. It will be valid over almost all of the Dalitz plot, failing only in the narrow corner region where the observed mass spectrum is dominated by the two narrow $D$ and $D^*$ peaks. Accordingly, the total SL rate of the $\bar{B}^0$ meson is represented in the following form

$$\Gamma(\bar{B}^0 \to X_{cd}\ell\nu_\ell) = \Gamma(\bar{B}^0 \to D\ell\nu_\ell) + \Gamma(\bar{B}^0 \to D^*\ell\nu_\ell) + \Gamma(\bar{B}^0 \to X'_{cd}\ell\nu_\ell),$$

(3)

where $X'_{cd}$ represents the charmed hadron continuum, including also the resonance states higher than $D$ and $D^*$. The usefulness of such an expansion rests on large energy release in the inclusive decay.

Contrary to refs. [17], [18], where the exclusive decay rates in Eq. (3) were calculated using the universal Isgur–Wise (IW) function, our calculations of the exclusive rates are not relied on the Heavy Quark Effective Theory (HQET). Instead, they use the hadronic form factors that depend on dynamics of specific channels. The relevant formulae are collected in the Appendix, see Eqs.(A.5), (A.6).

Before calculating the inclusive SL decay rates we briefly recall the necessary kinematics. The modulus square of the amplitude summed over the final hadronic states is written as

$$|M|^2 = \frac{G_F^2}{2} |V_{Q'Q}|^2 L^{\alpha\beta}W_{\alpha\beta},$$

(4)

where $V_{Q'Q}$ is the relevant CKM matrix element, $L^{\alpha\beta}$ is the leptonic tensor

$$L^{\alpha\beta} = 8[p_{\ell\alpha}p_{\nu\beta} + p_{\nu\beta}p_{\ell\alpha} - g_{\alpha\beta}(p_\ell p_{\nu\ell})] + i\epsilon^{\alpha\beta\gamma\delta} p_\gamma p_{\nu\delta},$$

(5)

with $\epsilon^{0123} = 1$, and $W_{\alpha\beta}$ is the hadronic tensor

$$W_{\alpha\beta} = (2\pi)^3 \sum_n \int \delta(P_{\bar{B}^0} - p) \langle B^0|j_{\alpha}^+(0)|n \rangle \langle n|j_{\beta}(0)|B^0 \rangle \prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3 2E_i},$$

(6)
Then the hadronic tensor \( W_{\alpha\beta} \) can be decomposed into five different Lorentz covariants:

\[
W_{\alpha\beta} = (-g_{\alpha\beta})W_1 + v_\alpha v_\beta W_2 + i\epsilon_{\alpha\beta\gamma\delta} u^\gamma v^\delta W_3 + (v_\alpha u_\beta + u_\alpha v_\beta) W_4 + u_\alpha u_\beta W_5,
\]

where \( v \) is the 4–velocity of decaying \( H_Q \), and \( u = q/M_{H_Q} \). The functions \( W_i \) depend on two invariants, \( q^2 \) and \( q_0 \), where the latter variable is related to \( M_X \) by: \( q_0 = (M_{H_Q}^2 + q^2 - M_X^2)/2M_{H_Q} \). It is convenient to scale all momenta by \( M_{H_Q} \), so that \( q^2 = M_{H_Q}^2 t, M_X^2 = M_{H_Q}^2 s \).

Then the SL width can be cast into the form

\[
\Gamma_{SL} = \frac{32}{3} \Gamma_0 |V_{Q\bar{Q}}|^2 \int_{t_{min}}^{t_{max}} dt \Phi(t, m_1^2, m_2^2) \int_{s_{min}}^{s_{max}} ds \frac{|q|}{M_{H_Q}} G(t, s),
\]

where the prefactor

\[
\Gamma_0 = \frac{G_F^2 M_{H_Q}^5}{(4\pi)^3}
\]

sets the overall scale of the rate, \( \Phi(q^2, m_1^2, m_2^2) \) is the usual triangle function scaled by \( q^2 \): \( \Phi(q^2, m_1^2, m_2^2) = \sqrt{1 - 2\lambda_+ + \lambda_-} \), with \( \lambda_\pm = (m_1^2 \pm m_2^2)/q^2, m_{1,2} \) being the lepton masses, and

\[
G(t, s) = 3t(1 + \lambda_1 - 2\lambda_2)W_1(t, s) + \left(1 + \lambda_1\right) \frac{q^2}{M_{H_Q}^2} + \frac{3}{2} \lambda_2 t \right) W_2(t, s) +
\]

\[
\frac{3}{2} \lambda_2 t (1 + t - s)W_4(t, s) + tW_5(t, s),
\]

with \( \lambda_1 = \lambda_+ - 2\lambda_2^2, \lambda_2 = \lambda_+ - \lambda_-^2 \). In Eq. (8)

\[
\frac{2|q|}{M_{H_Q}} \equiv \alpha(t, s) = \sqrt{(1 + t - s)^2 - 4t},
\]

and the limits of integrations in the \( t - s \) plane are given by

\[
s_{min} = \left(\frac{M_X^2}{M_{H_Q}}\right)^2, \quad s_{max} = 1 - \sqrt{t}, \quad t_{min} = \left(\frac{m_1 + m_2}{M_{H_Q}^2}\right)^2, \quad t_{max} = \left(1 - \frac{M_X^2}{M_{H_Q}^2}\right)^2.
\]

These expressions conclude the kinematical analysis. The next task is the calculation of the hadronic structure functions \( W_i \).

### 2.2 The LF constituent quark model approximation for \( W_{\mu\nu} \).

The theoretical treatment of inclusive SL decays of heavy-flavor mesons carries a distinct similarity to deep–inelastic lepton–nucleon scattering - this analogy has been used in refs. [17], [15] within the framework of the constituent LF quark model. The approach is based on the assumption of quark–hadron duality which means that the sum over a sufficient number of exclusive hadronic decay modes can be described in terms of partonic degrees of freedom. The standard strategy is to represent all the states higher than the ground pseudoscalar and vector states by the free–quark approximation starting from some effective continuum threshold \( M_X^{(0)} \).

Then the hadronic tensor \( W_{\alpha\beta} \) is given through the optical theorem by the imaginary part of
a quark box diagram describing the forward scattering amplitude in the second order in weak interactions[1]

\[ W_{\alpha\beta} = \int_0^1 \frac{dx}{x} \int d^2 p_\perp L^{(Q'Q)}_{\alpha\beta}(p_{Q'}, p_Q) \delta[(p_Q - q)^2 - m_{Q'}^2] \theta(E_{Q'}) |\psi(x, p_\perp^2)|^2, \quad (13) \]

where \( E_{Q'} \) is the energy of \( Q' \) and the tensor \( L^{(Q'Q)}_{\alpha\beta} \), describing the \( Q \to Q'W \) transitions, is defined analogously to the lepton tensor in Eq. (5):

\[ L^{(Q'Q)}_{\alpha\beta} = 4 \left[ p_{Q'} \alpha p_{Q} \beta + p_{Q'} \beta p_Q \alpha - g_{\alpha\beta}(p_{Q'} p_Q) + i e^{\alpha\beta \gamma} p_{Q'}^\gamma p_{Q}^\delta \right]. \quad (14) \]

In Eq. (14) the extra factor \( 1/2 \) corresponds to the average over the \( b \)-quark spin projections. Equation (13) incorporates all long–range QCD effects in the nonperturbative distribution function \( |\psi(x, p_\perp^2)|^2 \), whose normalization is given by \( \int_0^1 dx \int d^2 p_\perp |\psi(x, p_\perp^2)|^2 = 1 \), while hard gluon corrections can be taken into account by perturbative methods and renormalization–group techniques. In Eq. (13) the \( \delta \)-function corresponds to the decay of a \( Q \)-quark with longitudinal momentum \( xP_{H_Q} \) to a \( Q' \)-quark and has two roots in \( x \), viz.

\[ \delta[(p_Q - q)^2 - m_{Q'}^2] = \frac{\delta(x - x_+) + \delta(x - x_-)}{M_{H_Q}^2 |x_+ - x_-|}, \quad (15) \]

where

\[ x_\pm = \frac{1}{2} \left( 1 + t - s \right) \pm \sqrt{ \left( 1 + t - s \right)^2 - 4t + 4 \frac{m_{Q'}^2}{M_{H_Q}^2} }. \quad (16) \]

By the quark masses \( m_Q \) and \( m_{Q'} \), we hereafter understand the "constituent" quark masses taken from a particular constituent quark model. The root \( x_- \) is related to the contribution of the \( Z \)-graph arising from the negative energy components of the \( Q' \)-quark propagator and is prohibited by the \( \theta(E_{Q'}) \) in Eq. (13). We now substitute into the quark tensor \( L^{(Q'Q)}_{\alpha\beta} \), Eq. (14), \( p_Q = xP_{H_Q}, p_{Q'} = p_Q - q \) and use Eqs. (13) and (14) to obtain

\[ W_1 = F(x_+), \quad W_2 = 4 \frac{x_+ F(x_+)}{|x_+ - x_-|}, \quad W_3 = W_4 = -2 \frac{F(x_+)}{|x_+ - x_-|}, \quad W_5 = 0, \quad (17) \]

where

\[ F(x) = \int |\psi(x, p_\perp^2)|^2 d^2 p_\perp. \quad (18) \]

After substituting \( W_i \) from Eqs. (17) into (10) the inclusive SL decay rate, Eq. (8), is given by

\[ \Gamma_{H_{Q'} \ell\nu} = \frac{2}{3} \Gamma_0 J_{SL} |V_{Q'Q}|^2 \int_{t_{\min}}^{t_{\max}} dt \Phi(t) \int_{s_{\min}}^{s_{\max}} ds \alpha(t, s) \left[ (1 + \lambda_1) \alpha^2(t, s) \frac{x_+}{x_+ - x_-} + 3t(1 - \lambda_2^2) \right] F(x_+), \quad (19) \]

where we have inserted the factor \( J_{SL} \approx 0.9 \) representing the effect of the radiative corrections[24].

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1. In what follows we consider the \( Q \)-quark as a virtual particle of the mass \( m_{Q'}^2 = x^2 M_{H_Q}^2 \) and neglect the transverse momenta of the \( Q \)-quark. As a result, the expressions for the SL branching ratios are derived in close analogy with deep–inelastic scattering. The same result has been derived[14] using the light–cone dominance of SL inclusive \( B \)-meson decays, see also[23].
2.3 Nonleptonic decays.

The calculation of the NL decay rate closely follows the SL one. We expect $H_Q$ decays to multimeson states to proceed predominantly via the formation of a quark–antiquark state, followed by the creation of the additional $q\bar{q}$ pairs from the vacuum. The effective weak Lagrangian, e.g. for $\bar{b} \to \bar{c} u \bar{q}$ processes with $q = d, s$ is given by

$$L(\mu) = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq} (c_1 O_1 + c_2 O_2),$$

(20)

where $O_1$ and $O_2$ denote current–current operators with the color–nonsinglet and color–singlet structure, respectively:

$$O_1 = (\bar{c} \Gamma_\mu b)(\bar{q} \Gamma_\mu u), \quad O_2 = (\bar{c} \Gamma_\mu b^i)(\bar{q} j \Gamma_\mu u^i),$$

(21)

and $\Gamma_\mu = (1 - \gamma_5) \gamma_\mu$. The lepton pair is substituted by a quark pair, and the Wilson coefficients $c_i$ are the perturbative QCD corrections that describe the physics between the $W$ boson mass and the characteristic hadronic scale of the process. We shall use the values $c_1(m_b) = 1.132$, $c_2(m_b) = -0.286$; $c_1(m_c) = 1.351$, $c_2(m_c) = -0.631$,

(22)

obtained at next–to–leading order with the evolution of the running coupling constant being done at two–loop level using the normalization $\alpha_s(m_Z) = 0.118 \pm 0.003$. The inclusive NL rate is given by Eq. (19) with the substitution $\Gamma_{J_{SL} \to 3\Gamma_0 \eta}$, where

$$\eta = \frac{c^- + 1}{2N_c} + \frac{c^+ - 1}{2N_c},$$

(23)

with $c_\pm = c_1 \pm c_2$. Following ref. [18] we let $N_c \to \infty$ in Eq. (23) in which case $\eta$ is reduced to

$$\eta = \frac{1}{2}(c^+ + c^-).$$

(24)

For the exclusive two–meson NL decays $H_Q \to PP, PV, VV$, where $P$ and $V$ are the lowest–lying pseudoscalar and vector mesons, respectively, we use the BSW approach [26], [27]. There are two main ingredients in this approach:

- one assumes factorization: $<M_1 M_2 | J_\mu J_\mu | H_Q > = <M_2 | J_\mu | 0 > <M_1 | J_\mu | H_Q >$ to describe $H_Q \to M_1 M_2$ decay,

- one employs LF meson wave functions to compute $<M_1 | J_\mu | H_Q >$.

This method which lacks a firm footing for the $B$ meson, can in fact be justified in the case of $B_c$. Deviations from factorization arise from higher Fock components of the $B_c$ wave function and therefore are of higher order in the nonrelativistic expansion.

By factorizing matrix elements of 4–quark operators contained in (20), we distinguish two classes of NL decays [27] corresponding to two flavor–flow topologies relevant for our discussion: the so–called ”tree topology” (class I) and the ”color–suppressed tree topology” (class II). The class I (external decays) contains those decays where only a charged meson can be generated directly from a colour–singlet current. As before, we calculate separately the rates for the exclusive two–meson decays of the type $\bar{B}^0 \to D^+ \pi^-$, $\bar{B}^0 \to D^+ \pi^-$, $\bar{B}^0 \to D^+ \rho^-$, $\bar{B}^0 \to D^+ \rho^-$,
etc., and the rates for the decays into multimeson states like $B^0 \to X'_{d\bar{c}} \pi^-$, $B^0 \to D^+ X'_{ud}$, or $\bar{B}^0 \to X'_{d\bar{c}} + X'_{u\bar{d}}$. A second class of transitions (internal decays) consists of those decays in which the neutral meson is generated from the quark current, like $J/\psi$ in the decay $\bar{B}^0 \to \bar{K}J/\psi$. There is the third class of decays where the external and internal amplitudes interfere. These decays play important role in the case of the $D$ mesons, but are less important for $B_c$, and are absent for $\bar{B}^0$. The class I and class II two–meson amplitudes are proportional to the QCD coefficients

$$a_1 = c_1(\mu) + \frac{c_2(\mu)}{N_c}, \quad a_2 = c_2(\mu) + \frac{c_1(\mu)}{N_c},$$

respectively, with $\mu = O(m_b)$. The term proportional to $1/N_c$ arises from the Fierz reordering of operators $Q_2$ to produce quark currents to match the quark content of the hadrons in the initial and final state after adopting the factorization assumption. This well–known procedure \[27\] results in matrix elements with the right flavor quantum number but involve both color–singlet and color–octet quantum operators. We shall use the ”naive” factorization approximation in which one discards the color–octet operators \[27\]. In what follows we neglect the $1/N_c$ corrections in Eqs. (25) letting $a_1 = c_1$ and $a_2 = c_2$.

In conclusion, we list the expressions used below to calculate the NL partial widths. The exclusive two–meson decay rates are calculated using Eqs. (A.12)–(A.15) given in Appendix. The exclusive NL rates for the decays of the type $\bar{B}^0 \to D^+ X'_{ud}$ are given by Eqs. (A.7), (A.8). The inclusive NL rate of the type $\bar{B}^0 \to \pi^-(\rho^-)X_{d\bar{c}}$ are given by Eqs. (A.18), (A.19). Finally, the inclusive NL rates, like $\bar{B}^0 \to X'_{d\bar{c}} + X'_{u\bar{d}}$, have been calculated using Eq. (19), where the lepton pair is substituted by a quark pair and the prefactor $\Gamma_0$ is replaced by $3\Gamma_0 c_1^2$ or $3\Gamma_0 c_2^2$ for the external and internal inclusive decays, respectively.

3 NUMERICAL RESULTS

We are now ready to add the various contributions presented above and to estimate the $B_c$ lifetime. We first summarize the input parameters that will be used in our numerical calculations. The partial decay rates depend on the following set of parameters:

- the masses of constituent quarks building up final mesons and multime–hadrons and the masses of pseudoscalar and vector mesons; the formers are taken from the ISGW2 model \[19\]:

$$m_u = m_d = 0.33, \quad m_s = 0.55, \quad m_c = 1.82, \quad m_b = 5.2,$$

while the latters have been taken from the Particle Data Group publication \[28\]. Note that the constituent quark masses $m_b$ and $m_c$ satisfy approximately the relation $m_b = m_c + 3.4 \text{ GeV}$, which is consistent with the known formula relating the pole masses $m_{b,\text{pole}}$ and $m_{c,\text{pole}}$ in HQET;

- the meson decay constants $f_{PS}$ and $f_V$ of the pseudoscalar and vector mesons to the $W$–boson. For a pseudoscalar meson $P = (q_1 \bar{q}_2)$, we define $<0|\bar{q}_2 \gamma_\mu \gamma_5 q_1|P(p)> = i f_{PS} \gamma_\mu$. The decay constant of a vector meson $V = (q_1 \bar{q}_2)$ is defined as $<0|\bar{q}_2 \gamma_\mu \gamma_5 q_1|V> = \varepsilon_\mu m_V f_V$. The constants $f_{PS}$ and $f_V$ used in our calculations were taken from ref. \[30\]. The

\[2\] To compensate for the neglected octet operators and other non–factorizing contributions one usually treats $\xi = 1/N_c$ in Eq. (27) as a free parameter which can, in principle, be channel dependent. For a detailed discussion see refs. \[28\].
$B_c$ meson mass is known with a good accuracy from the current quark model calculations, we use $M_{B_c} = 6.3 GeV$;

- the threshold values $M_X^{(0)}$ at which the hadron continuum starts; in our calculations we have adopted two different choices: $M_X^{(0)} = M_P + m_\pi$ (case $a$) and $M_X^{(0)} = M_V + m_\pi$ (case $b$), where $M_P$ and $M_V$ are the masses of the pseudoscalar and vector ground state mesons, respectively.

For the radial wave function appearing in Eq. (2), the Gaussian ansatz of the ISGW2 model has been adopted in which the main characteristic is confinement. The oscillator parameters have been taken from [19]. Their values along with the values of the meson decay constants are collected in Table 1.

### 3.1 $B^0$ decays

First, we would like to present an overview of different $B_c$-decay mechanisms and their relative importance as obtained within the framework we are advocating. To this end, we first calculate the partial $B_0$ decay modes corresponding to various underlying quark subprocesses. In Table 2 we show the $B^0$ partial widths for two specific choices of the parameter $M_X^{(0)}$. The left set of numbers collects numerical results obtained for the threshold value for the hadron continuum $M_X^{(0)} = M_P + m_\pi$, while the right set is given for $M_X^{(0)} = M_V + m_\pi$, where in the considered case $M_P = M_D$ and $M_V = M_D$. Table 2 also shows the numbers $N_{exc}$ and $N_{inc}$ of (both external and internal) exclusive and inclusive channels, respectively, for the $\bar{b} \to \bar{c}$ decays. Our analysis incorporates 54 exclusive SL and NL $\bar{b} \to \bar{c}$ decays and 29 inclusive $\bar{b} \to \bar{c}$ decays including two baryon-antibaryon channels. The latters were calculated using the Stech approach [31]. We have also included the CKM suppressed $\bar{b} \to \bar{u}$ contributions (with $|V_{ub}/V_{cb}| = 0.08$) which slightly change the overall results. We take the vector and axial form factors for $\bar{B}^0 \to D(D^*)\ell\nu_\ell$ and $\bar{B}^0 \to \pi(\rho)\ell\nu_\ell$ transitions from ref. [27], where the fit of the form factors by the functions

$$f(q^2) = \frac{f(0)}{1 - q^2/\Lambda_1^2 - \beta \cdot q^4/\Lambda_2^4},$$

where $f = F_1$, $V$, $A_0$, $A_1$, $A_2$ have been also presented. The distribution functions $F_b(x)$ for the $\bar{B}^0$ meson and $F_b(x)$ for the $B_c$ meson are shown in Fig. 1. Note that in the later case $F_c(x) = F_b(1 - x)$. In terms of the mean value $< x > = \int_0^1 dx \ x \ F(x)$ and the variance $\sigma^2 = \int_0^1 dx \ (x - < x >)^2 \ F(x)$, one gets $< x > = 0.90, \sqrt{\sigma^2} = 0.06$ for $\bar{B}^0$ and $< x > = 0.72, \sqrt{\sigma^2} = 0.09$ for $B_c$. The difference in the values of $\sigma^2$ reflects the difference in the values of $\sqrt{< p_{\perp}^2 >} = \beta_{Hq}$ for $\bar{B}^0$ and $B_c$.

Numerical values of NL branchings depend upon estimation of QCD corrections which give contribution of order of 10% − 20%[3]. The main uncertainty ($\approx 10\%$) is related to the choice of

\[3\] Note that for the former choice there is, in principle, a danger of double counting, because e.g. the exclusive rate $B^0 \to D^0$ accounts partially for the two–particle state $D\pi$, which in our approach is included in the inclusive rate

\[4\] The treatment of radiative corrections in our phenomenological application is associated with large uncertainties. In a typical hadronic process, there are several mass scales involved: the hadron masses, the quark masses, the energy release, etc. Thus, there is an uncertainty in the choice of the “characteristic” scale $\mu$ of
In principle this is not a problem, since the products of the Wilson coefficients with the hadronic matrix elements are scale independent. However, we employ simple model estimations of the matrix elements, which do not yield an explicit scale dependence that could compensate that of the Wilson coefficients. Instead, the quark model calculations are assumed to be valid on a particular scale, typically $\mu \approx 1 - 2$ GeV.

In addition to using the measured $B^0$ lifetime one could be tempted to use the $D^0$ lifetime to eliminate the similar uncertainties for the $c \to s$ transitions. However, we refrain from this procedure, since our approach applied to charm mesons is less reliable than for $B_c$ and probably more qualitative than quantitative.
The experimental values are $R_3 = 0.95 \pm 0.24$ and $R_4 = 0.99 \pm 0.25$, again in agreement with our predictions. Similar ratios can be taken for class II amplitudes. Based on the results of our model, we expect
\[
R_5 = \frac{BR(B^0 \to \bar{K}J/\psi)}{BR(B^0 \to K^*J/\psi)} \approx 0.44 \, [0.62, \ 0.32].
\] (32)
The corresponding experimental value is $R_5 = 0.58 \pm 0.11$.

### 3.2 $B_c$ decays

The transition operators driving $B_c$ decays are the same that generate $B$ and $D$ decays. However their expectation values are evaluated for the $B_c$ wave function, rather than the $B$ and $D$ wave functions reflecting that $b \to c$ and $c \to s$ transitions proceed in a different environment. This is illustrated by the results of Table 4, where we compare various $B_c$ form factors at $q^2 = 0$ with the corresponding $B^0$ and $D^0$ form factors.

We apply the strategy outlined in the previous sections to calculate different partial rates and the lifetime of the $B_c$. The critical point with regards to this issue is charm decay. Here the energy release is not as comfortably large as it is in the case of bottom decay. As a result, our estimations of the inclusive charm decay should be more sensitive to a hadronization model. However, the inclusive charm decay contributes only 8% to the total $c \to s$ rate of $B_c$. For this reason we do not include any hadronization corrections in our calculations.

The results for the partial $B_c$ decay modes corresponding to the various underlying quark subprocesses are collected in Table 5 for $M_X^{(0)} = M_P + m_x$. As in Table 2 we present the numbers of the exclusive and inclusive channels included in the calculations. The number of the exclusive $b \to c$ decays in the $B_c$ case is less than that for $B^0$ because of the interference effects in the class $III$ amplitudes (see Table 7). For comparison we also show in Table 5 the results obtained in [10], using the nonrelativistic QCD. Viewing this comparison with due caution, regarding the model dependence and other uncertainties in the estimation of the decay modes as well as the quark mass uncertainty for the inclusive prediction, it is reassuring that the order of magnitude comes out to be consistent. Our bound state corrections are numerically a bit larger than very small effects reported in [10]. For the sum of $b \to c$ spectator contributions we obtain $\Gamma^{(b \to c)} = 0.83 \cdot \Gamma(B^0) = 0.531 \, \text{ps}^{-1}$, while for the total $c$-decay contribution one finds $\Gamma^{(c \to s)} = 0.38 \cdot \Gamma(D^0) = 0.915 \, \text{ps}^{-1}$. For the choice $M_X^{(0)} = M_V + m_x$ we obtain $\Gamma^{(b \to c)} = 0.79 \cdot \Gamma(B^0) = 0.504 \, \text{ps}^{-1}$ and $\Gamma^{(c \to s)} = 0.43 \cdot \Gamma(D^0) = 1.049 \, \text{ps}^{-1}$.

Various branching fractions can be also inferred from Table 5. For instance the SL branching ratio $BR(B_c \to e\nu X)$ is found to be 9.3%, i.e. $\approx 15\%$ less than $BR(B^0 \to e\nu X)$. Details of our predictions for the partial widths are presented in Table 6. In particular, we obtain
\[
\Gamma(B_c \to e\nu_e X_{cc}) = 4.1 \cdot 10^{13}|V_{cb}|^2 \text{sec}^{-1},
\] (33)
\[
\Gamma(B_c \to e\nu_s X_{cs}) = 8.5 \cdot 10^{10}|V_{cs}|^2 \text{sec}^{-1}.
\] (34)

\footnote{Note that in the heavy–quark limit the fraction ratios $R_i$, $i = 1, \ldots 4$ equal unity.}

\footnote{The sum of the other interference effects is effectively accounted for by the Pauli interference contribution $\Gamma_{PI}$, see below}

\footnote{The $b \to c$ rates have been calculated using the values of the effective CKM parameter $|V_{cb}|$ listed in Table 2 for cases $a$ and $b$, respectively.}
The corresponding results of the original ISGW2 model are [13]
\[
\Gamma(B_c \to e\nu_X) = 3.36 \cdot 10^{13} |V_{cb}|^2 \text{sec}^{-1},
\]
\[
\Gamma(B_c \to e\nu_X_{ba}) = 5.0 \cdot 10^{10} |V_{cs}|^2 \text{sec}^{-1}.
\]
The ratio of $B_c$ decay via $\bar{b} \to \bar{c}$ to $c \to s$ decay is
\[
\frac{\Gamma(B_c \to e\nu_X_{ba})}{\Gamma(B_c \to e\nu_X)} = 0.0021 \frac{|V_{cs}|^2}{|V_{bc}|^2} \approx 1.3,
\]
(37)
The SL width $\Gamma(B_c \to e\nu_X) = 0.061 \text{ps}^{-1}$ is only $\approx 20\%$ smaller than that of $\bar{B}^0$, $\Gamma(\bar{B}^0 \to e\nu_X_{bd}) = 0.074 \text{ps}^{-1}$, even though the spectator $c$ quark is no longer light. The contributions from the pseudoscalar and vector final states are $21\%$ and $44\%$, that is not too much different from the predictions of the ISGW2 model. For the SL $c \to s$ decays the recoil effects are very small due to the large daughter mass. The SL width $\Gamma(B_c \to e\nu_X_{ba}) = 0.081 \text{ps}^{-1}$ is dominated by the decays to the $B_s^0$ $(27.2\%)$ and $B_s^{0*}$ $(63.2\%)$, because the available energy is small.

In our phenomenological description we also include non–spectator contributions from weak annihilation (WA) and Pauli interference (PI) [36]. The the contribution of the annihilation channel is
\[
\Gamma_a = \sum_{i=\tau,c} \frac{G_F^2}{8\pi} |V_{cb}|^2 M_{B_c}^5 \left( \frac{f_{B_c}}{M_{B_c}} \right)^2 \left( \frac{m_i}{M_{B_c}} \right)^2 \left( 1 - \left( \frac{m_i^2}{M_{B_c}^2} \right)^2 \right)^2 \cdot \bar{c}_i,
\]
(38)
where $f_{B_c} = 0.42 \text{ Gev}$, $\bar{c}_i = 1$ for the $\tau^+\nu_\tau$ channel and $\bar{c} = (2c_+(2\mu_{red}) + c_-(2\mu_{red}))^2/3$ for the $c\bar{s}$ channel, with $\mu_{red} = m_y m_d/(m_b + m_c)$ being the reduced mass of the $b\bar{c}$ system. The result is known to two–loop accuracy [37]: $c_+ (2\mu_{red}) = 0.8$, $c_- (2\mu_{red}) = 1.5$. The leptonic constant $f_{B_c}$ has been estimated through the LF wave function $\psi(x, p_{\perp}^2)$ using Eq. (13) of ref. [36]. Note that because of the partial cancellation of weak annihilation rate and the effect of the Pauli interference diagrams, no significant uncertainty on the lifetime arises from the limited knowledge of the decay constant $f_{B_c}$. We find $\Gamma_a = 0.189 \text{ps}^{-1}$ and $\Gamma_{PI} = -0.100 \text{ps}^{-1}$. The similar value of $\Gamma_{PI}$ has been obtained in [14]. Putting everything together we obtain for the $B_c$ width $\Gamma(B_c) = \Gamma^{b\to c}(B_c) + \Gamma^{c\to s}(B_c) + \Gamma_a + \Gamma_{PI}$:
\[
\Gamma(B_c) = (0.65 \text{ ps})^{-1}, \text{ case a}; \quad \Gamma(B_c) = (0.61 \text{ ps})^{-1}, \text{ case b}.
\]
(39)
One observes a dominance of the charm decay modes over $b$–quark decays. We consider the dispersion in predicted values of $\tau_{B_c}$ as a rough measure of our theoretical uncertainty in calculation of the inclusive decay rates, therefore our final result is
\[
\tau_{B_c} = 0.63 \pm 0.02 \text{ ps}^{-1}.
\]
(40)
Note that the exclusive $\bar{b}–$ and $c$–decay rates are $0.32 \text{ps}^{-1}$ and $0.80 \text{ps}^{-1}$, i.e. $52\%$ and $87\%$ of the total $\bar{b} \to \bar{c}$ and $c \to s$ decay rates, respectively. In Tables 7,8 we compare our results for the exclusive two–body NL $B_c$ decay modes with the results obtained using the BSW and ISGW models [3] and with the results of ref. [7]. Adding the rates for twenty of these two–body decay modes one finds $0.69 \text{ps}^{-1}$ (BSW model) and $0.90 \text{ps}^{-1}$ (ISGW model) [14] and 1.15 ps$^{-1}$ in [13], to be compared with our result $0.70 \text{ps}^{-1}$ which is the sum of the rates of $0.67 \text{ps}^{-1}$ for $c \to s$ transitions and $0.03 \text{ps}^{-1}$ for $b \to c$ transitions from Table 7,8.

Finally, we note that the experimental extraction of $B_c$ signal from the hadronic background requires the reliable estimation of the branching fraction $BR(B_c \to J/\psi + X)$, because $J/\psi$ can be easily identified by its leptonic decay mode, while the experimental registration of the final states containing the $\eta_c$ or $B_s^{(*)}$ is impeded by the large hadron background. We obtain $BR(B_c \to J/\psi + \mu_\nu_\mu) = 1.7\%$, $BR(B_c \to J/\psi + \pi) = 0.1\%$, and $BR(B_c \to J/\psi + X) = 13.2\%$. 

12
4 Summary

In conclusion, we have used a relativistic constituent quark model based on the LF formalism to perform a detailed investigation of the $B_c$ meson partial widths and the lifetime of the $B_c$ meson. The hadronic form factors and the distribution function were calculated using meson wave functions derived from an effective $q\bar{q}$ interaction intended originally to describe the meson mass spectra. In this way the link between $B_c$ physics and the "spectroscopic" constituent quark models was explicitly established. For numerical estimates we employed the LF quark functions that are related to the equal–time wave functions of the ISGW2 model. In several important aspects our analysis goes beyond the quark model estimations derived previously in the literature. In addition to the frequently discussed SL and the two–meson exclusive decay modes, we have included 84 exclusive decay modes and 44 SL and NL inclusive decay modes, that increases the total $B_c$ width by $\approx 30\%$. Account of the inclusive decays leads to a considerably smaller $B_c$–lifetime, $\tau_{B_c} = 0.63 \pm 0.02$ ps, which is in fairly good agreement with the estimate obtained non–relativistic QCD, $\tau_{B_c} = (0.55 \pm 0.15)$ ps \cite{3} and also agrees with the most recent CDF measurement \cite{4} within one standard deviation.

To sum up, the LF constituent quark model makes clear predictions on the global pattern: (i) a short $B_c$ lifetime well below 1 ps and (ii) a predominance of charm over beauty decays among the NL modes.

Appendix

The vector and axial hadronic form factors relevant for weak SL decays of a pseudoscalar meson $H_Q$, containing a heavy quark $Q$, to a member of the lowest–lying multiplet of pseudoscalar or vector mesons are defined in a usual way (see e.g. \cite{10}). We denote by $P_{HQ}$, $P_{HQ'}$ and $M_{HQ}$, $M_{HQ'}$ the 4–momenta and masses of the parent and daughter meson, respectively. Then the amplitude $<P_{HQ'}|V_\alpha|P_{HQ}>$ can be expressed in terms of two form factors

$$<P_{HQ'}|V_\alpha|P_{HQ}> = (\alpha - \frac{M_{HQ}^2 - M_{HQ'}^2}{q^2} q_\alpha) F_1(q^2) + \frac{M_{HQ}^2 - M_{HQ'}^2}{q^2} q_\alpha F_0(q^2), \quad (A.1)$$

where $P = P_{HQ} + P_{HQ'}$, $q = P_{HQ} - P_{HQ'}$. There is one form factor for the amplitude $<P_{HQ'},\varepsilon|V_\alpha|P_{HQ}>$

$$<P_{HQ'},\varepsilon|V_\alpha|P_{HQ}> = \frac{2i}{M_{HQ}(1 + \zeta)} \epsilon_\alpha \beta \gamma \delta \ast \delta \varepsilon \ast \beta P_1 \ast P_2 V(q^2), \quad (A.2)$$

where $\zeta = \frac{M_{HQ}}{M_{HQ'}}$ and three independent form factors for the amplitude $<P_{HQ'},\varepsilon|A_\alpha|P_{HQ}>

$$<P_{HQ'},\varepsilon|A_\alpha|P_{HQ} > = M_{HQ}(1 + \zeta) \varepsilon_\ast \beta A_1(q^2) - \frac{(\ast q)}{M_{HQ} + M_{HQ'}} P_\alpha A_2(q^2) - 2M_{HQ'} \frac{(\ast q)}{q^2} q_\alpha A_3(q^2) + 2M_{HQ} \frac{\varepsilon \ast q}{q^2} A_0(q^2), \quad (A.3)$$

\footnote{In what follows we suppress the indeces 1 and 2 (corresponding to the meson containing a quark–spectator and to the one generated from a $W$ current, respectively) of $\zeta$ everywhere except in Eq. (A.1).}
where \( A_0(0) = A_3(0) \) and the form factor \( A_3(q^2) \) is given by the linear combination

\[
A_3(q^2) = \frac{1}{2\zeta} \left( (1 + \zeta)A_1(q^2) - (1 - \zeta)A_2(q^2) \right). \tag{A.4}
\]

We view the form factors as functions of the dimensionless variable \( y = v_{HQ} \cdot v_{HQ'} \), where \( P_{HQ} = M_{HQ} \cdot v_{HQ}, \ P_{HQ'} = M_{HQ'} \cdot v_{HQ'}, \) and \( q^2 = M^2_{HQ} + M^2_{HQ'} - 2M_{HQ}M_{HQ'}y^2 \). The differential rates for a decay of a pseudoscalar particle into another pseudoscalar particle and lepton or quark pair is given by

\[
\frac{d\Gamma}{dy} = \frac{16}{3} \Gamma_0 c^2 |V_{Q'Q}|^2 |V_{q_1q_2}|^2 \zeta^4 \Phi(q^2, m^2_1, m^2_2)J \cdot \\
\sqrt{y^2 - 1}[1 - \frac{y^2}{2} + 3\lambda_2(\zeta^{-2} - 1)F_0^2(y)] \tag{A.5}
\]

where \( V_{Q'Q} \) and \( V_{q_1q_2} \) are the corresponding CKM matrix elements and the factor \( J \) takes into account the perturbative QCD corrections. For the SL decays \( c^2 = 1, \ V_{q_1q_2} = 1, \) and \( J \approx 0.9; \) for the external (internal) NL decays \( c^2 = 3c^2_1 (3c^2_2) \) and \( J = 1. \) The differential transition rate to a vector particle is

\[
\frac{d\Gamma}{dy} = 4\Gamma_0 c^2 |V_{Q'Q}|^2 |V_{q_1q_2}|^2 \zeta^2 \Phi(q^2, m^2_1, m^2_2)J \cdot \\
\sqrt{y^2 - 1}[1 - \frac{y^2}{2}H^2(y) + 6\lambda_2\zeta^2(y^2 - 1)A_0^2(y)], \tag{A.6}
\]

where

\[
H^2(y) = \frac{t}{M^2_{HQ}}(H^2_+ (y) + H^2_-(y) + H^2_0(y)), \tag{A.7}
\]

and the helicity amplitudes are given by

\[
H_\pm(y) = M_{HQ} \left( (1 + \zeta)A_1(y) \mp 2\sqrt{y^2 - 1} \frac{\zeta}{1 + \zeta} V(y) \right), \tag{A.8}
\]

\[
H_0(y) = \frac{M^2_{HQ}}{\sqrt{q^2}} \left( (y - \zeta)(1 + \zeta)A_1(y) - 2(y^2 - 1)\frac{\zeta}{1 + \zeta} A_2(y) \right) \tag{A.9}
\]

The maximal value of \( y \) is \((M^2_{HQ} + M^2_{HQ'})/(2M_{HQ}M_{HQ'})\), corresponding to \( q^2 = 0. \)

Recall that in the case of the heavy–to–heavy transitions \( H_Q \to H_{Q'} \) the heavy quark symmetry implies simple relations between various form factors:

\[
F_1(y) = V(y) = A_0(y) = \frac{1 + \zeta}{2\sqrt{\zeta}} \xi_{IW}(y), \tag{A.10}
\]

\[
F_0(y) = A_1(y) = \frac{2\sqrt{\zeta}}{1 + \zeta} \frac{y + 1}{2} \xi_{IW}(y), \tag{A.11}
\]

where \( \xi_{IW}(y) \) is the Isgur–Wise function. For the realistic case of finite heavy–quark masses, these relations are modified by corrections that break heavy quark symmetry. These corrections

\footnote{Although we are using the variable \( v_1 \cdot v_2 \) we are not treating the daughter quark as heavy.}
have been analyzed for $B^0$ and $D$ decays using the LF technique [20], [21]. We use the same technique to calculate various form factors for the $B_c$ decays. The values of these form factors at $q^2 = 0$ have been already given in Table 4.

The NL two–body partial widths are given by expressions

\[
\Gamma_{P_2P_1} = \frac{c^2}{2} \cdot \frac{\Gamma_0|V_{Q'Q}|^2|V_{q_1q_2}|^2 F_{PS}^2}{M_{H_Q}} \cdot (1 - \zeta^2)^2 \alpha F_0^2(\zeta^2),
\]

(A.12)

\[
\Gamma_{V_2P_1} = \frac{c^2}{2} \cdot \frac{\Gamma_0|V_{Q'Q}|^2|V_{q_1q_2}|^2 F_V^2}{M_{H_Q}} \cdot \alpha^3 F_1^2(\zeta^2),
\]

(A.13)

\[
\Gamma_{P_2V_1} = \frac{c^2}{2} \cdot \frac{\Gamma_0|V_{Q'Q}|^2|V_{q_1q_2}|^2 F_{PS}^2}{M_{H_Q}} \cdot \alpha^3 A_0^2(\zeta^2),
\]

(A.14)

\[
\Gamma_{V_2V_1} = \frac{c^2}{2} \cdot \frac{\Gamma_0|V_{Q'Q}|^2|V_{q_1q_2}|^2 F_V^2}{M_{H_Q}} \cdot \alpha H^2(\zeta^2),
\]

(A.15)

where

\[
F_{PS} = \frac{2\pi f_{PS}}{M_{H_Q}}, \quad F_V = \frac{2\pi f_V}{M_{H_Q}},
\]

(A.16)

\[
\alpha \equiv \alpha(\zeta_1^2, \zeta_2^2) = \sqrt{(1 + \zeta_1^2 - \zeta_2^2)^2 - 4\zeta_1^2},
\]

(A.17)

and $c = a_1$ for the external two–body decays, $c = a_2$ for the internal two–body decays.

Finally, the partial widths of inclusive NL decays in which the final state contains a charged or neutral meson directly generated by a color–singlet current are given by

\[
\frac{d\Gamma_{P_X}}{ds} = \frac{\Gamma_0 c^2 F_{PS}^2}{4} |V_{Q'Q}|^2 |V_{q_1q_2}|^2 \cdot \left[ (1 + s - \zeta^2)^2 W_2 - 4\zeta^2 W_1 + 4\zeta^2 (1 + s - \zeta^2) W_4 \right] \alpha(\zeta^2, s)
\]

(A.18)

\[
\frac{d\Gamma_{V_X}}{ds} = \frac{\Gamma_0 c^2 F_V^2}{4} |V_{Q'Q}|^2 |V_{q_1q_2}|^2 \cdot \left[ \alpha^2 (\zeta^2, s) W_2 + 12\zeta^2 W_1 \right] \alpha(\zeta^2, s)
\]

(A.19)

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Table 1. The oscillator parameters $\beta$ for various mesons [19] and the meson constants $f_{PS}, f_V$ [30] used in the calculations.

| Meson | $\pi$ | $K$ | $\eta_c$ | $D$ | $D_s$ | $B^0$ | $B_s$ | $B_c$ | $\rho$ | $K^*$ | $J/\psi$ | $D^*$ | $D^*_s$ |
|-------|------|-----|---------|-----|-------|-------|-------|-------|-------|-------|---------|-------|-------|
| $\beta$ | 0.41 | 0.44 | 0.88 | 0.45 | 0.56 | 0.43 | 0.54 | 0.92 | 0.30 | 0.33 | 0.62 | 0.38 | 0.44 |
| $f_{PS}$ | 0.13 | 0.16 | 0.38 | 0.21 | 0.23 | - | - | 0.42 | | | | | |
| $f_V$ | | | | | | | | | 0.20 | 0.21 | 0.41 | 0.20 | 0.27 |

Table 2. The inclusive partial widths of the $\bar{B}^0$ meson for the different choices of the continuum threshold $M_X^{(0)}$ (in units $|V_{bc}|^2/[0.039]^2 \text{ ps}^{-1}$). We use the values of the CKM parameters $|V_{sc}| = 0.974$, $|V_{dc}| = |V_{us}| = 0.221$. The CKM matrix element $|V_{cb}|$ is calculated using the experimental value of $\Gamma_{\text{tot}}(\bar{B}^0) = 0.641 \text{ ps}^{-1}$. Case $a$: $M_X^{(0)} = M_V + m_\pi$, case $b$: $M_X^{(0)} = M_P + m_\pi$.

| Decay mode | $N_{\text{exc}}$ | $N_{\text{inc}}$ | $a$ | $b$ |
|------------|-----------------|-----------------|-----|-----|
| $b \rightarrow \bar{c} + e + \nu_e$ | 2 | 1 | 0.074 | 0.082 |
| $b \rightarrow \bar{c} + \mu + \nu_\mu$ | 2 | 1 | 0.074 | 0.081 |
| $b \rightarrow \bar{c} + \tau + \nu_\tau$ | 2 | 1 | 0.015 | 0.017 |
| $b \rightarrow \bar{c} + u + d$ | 12 | 6 | 0.313 | 0.358 |
| $b \rightarrow \bar{c} + c + \bar{s}$ | 12 | 6 | 0.123 | 0.135 |
| $b \rightarrow \bar{c} + u + \bar{s}$ | 12 | 6 | 0.016 | 0.018 |
| $b \rightarrow \bar{c} + c + d$ | 12 | 6 | 0.006 | 0.006 |
| $\bar{B}^0 \rightarrow N\Lambda_c, \Lambda_c \Xi_c$ | 2 | | 0.023 | 0.023 |
| $b \rightarrow \bar{u}$ | 54 | 29 | 0.007 | 0.008 |
| $\Sigma_{\text{tot}}$ | 108 | 58 | 0.652 | 0.728 |
| $|V_{bc}|$ | | | 0.0386 | 0.0366 |
| $Br_{SL}$ | | | 11.35% | 11.26% |
| $n_c$ | | | 1.198 | 1.194 |
Table 3. The two–meson branching fractions of $B^0$ (in %).

| Decay mode          | This work | ref. [33] | ref. [34] | Exp. data [34] |
|---------------------|-----------|-----------|-----------|----------------|
| **Class I**         |           |           |           |                |
| $\bar{B}^0 \to D^+ \pi^-$ | $0.34a_1^2$ | $0.257a_1^2$ | $318a_1^2$ | $0.29 \pm 0.05$ |
| $\bar{B}^0 \to D^+ \rho^-$ | $0.78a_1^2$ | $0.643a_1^2$ | $0.778a_1^2$ | $0.81 \pm 0.17$ |
| $\bar{B}^0 \to D^{*+} \pi^-$ | $0.32a_1^2$ | $0.247a_1^2$ | $0.296a_1^2$ | $0.25 \pm 0.05$ |
| $\bar{B}^0 \to D^{*+} \rho^-$ | $0.83a_1^2$ | $0.727a_1^2$ | $0.870a_1^2$ | $0.70 \pm 0.16$ |
| $\bar{B}^0 \to D^+ D_s^-$ | $1.05a_1^2$ | $0.879a_1^2$ | $1.004a_1^2$ | $0.82 \pm 0.36$ |
| $\bar{B}^0 \to D^+ D_s^{*+}$ | $0.92a_1^2$ | $0.817a_1^2$ | $0.830a_1^2$ | $0.95 \pm 0.45$ |
| $\bar{B}^0 \to D^{*+} D_s^-$ | $0.66a_1^2$ | $0.597a_1^2$ | $0.603a_1^2$ | $0.85 \pm 0.33$ |
| $\bar{B}^0 \to D^{*+} D_s^{*+}$ | $2.32a_1^2$ | $2.097a_1^2$ | $2.414a_1^2$ | $1.85 \pm 0.72$ |
| $\bar{B}^0 \to D^+ K^-$ | $0.028a_1^2$ | $0.020a_1^2$ | $0.037a_1^2$ |                |
| $\bar{B}^0 \to D^- K^{*-}$ | $0.045a_1^2$ | $0.035a_1^2$ | $0.041a_1^2$ |                |
| $\bar{B}^0 \to D^{*+} K^-$ | $0.015a_1^2$ | $0.019a_1^2$ | $0.022a_1^2$ |                |
| $\bar{B}^0 \to D^{*+} K^{*-}$ | $0.033a_1^2$ | $0.042a_1^2$ | $0.049a_1^2$ |                |
| **Class II**        |           |           |           |                |
| $\bar{B}^0 \to \pi^0 D^0$ | $0.28a_2^2$ | $0.164a_2^2$ | $0.084a_2^2$ | $< 0.033$      |
| $\bar{B}^0 \to \pi^0 D^{*0}$ | $0.21a_2^2$ | $0.230a_2^2$ | $0.116a_2^2$ | $< 0.055$      |
| $\bar{B}^0 \to \rho^0 D^0$ | $0.12a_2^2$ | $0.111a_2^2$ | $0.078a_2^2$ | $< 0.055$      |
| $\bar{B}^0 \to \rho^0 D^{*0}$ | $0.21a_2^2$ | $0.240a_2^2$ | $0.199a_2^2$ | $< 0.117$      |
| $\bar{B}^0 \to K^0 + \eta_c$ | $0.81a_2^2$ |           |           |                |
| $\bar{B}^0 \to K^0 J/\psi$ | $0.64a_2^2$ | $2.262a_2^2$ | $0.800a_2^2$ | $0.075 \pm 0.021$ |
| $\bar{B}^0 \to K^{*0} \eta_c$ | $0.41a_2^2$ |           |           |                |
| $\bar{B}^0 \to K^{*0} J/\psi$ | $1.46a_2^2$ | $3.645a_2^2$ | $2.518a_2^2$ | $0.151 \pm 0.091$ |
Table 4. Form factors for the external $B_c^+$-decays at $q^2 = 0$. In parentheses are given the values of corresponding form factors for the decays of $\bar{B}^0$ or $D^0$.

| Decay mode $B_c \to \eta_c, J/\psi$ | $B_c \to B_s, B_s^*$ | $B_c \to D^0, D^{*0}$ | $B_c \to B^0, B^{*0}$ |
|-------------------------------------|----------------------|----------------------|----------------------|
| $F_1(0)$                            | 0.622 (0.683)        | 0.564 (0.780)        | 0.089 (0.293)        | 0.362 (0.681) |
| $A_0(0)$                            | 0.477 (0.678)        | 0.447 (0.730)        | 0.050 (0.214)        | 0.282 (0.600) |
| $A_1(0)$                            | 0.447 (0.623)        | 0.453 (0.633)        | 0.045 (0.170)        | 0.290 (0.502) |
| $A_2(0)$                            | 0.390 (0.556)        | 0.522 (0.464)        | 0.040 (0.155)        | 0.370 (0.366) |
| $V(0)$                              | 0.621 (0.683)        | 2.17 (0.777)         | 0.073 (0.216)        | 1.50 (0.663)  |

Table 5. Inclusive $B_c$ partial rates (in units of $ps^{-1}$).

| Decay mode | $N_{exc}$ | $N_{inc}$ | This work | ref. [10] |
|------------|-----------|-----------|-----------|-----------|
| $b \to \bar{c} + e + \nu_e$       | 2         | 1         | 0.061     | 0.075     |
| $b \to \bar{c} + \mu + \nu_\mu$   | 2         | 1         | 0.061     | 0.075     |
| $b \to \bar{c} + \tau + \nu_\tau$ | 2         | 1         | 0.013     | 0.018     |
| $b \to \bar{c} + u + d$            | 12        | 6         | 0.259     | 0.310     |
| $b \to \bar{c} + c + \bar{s}$      | 8         | 6         | 0.102     | 0.137     |
| $b \to \bar{c} + u + \bar{s}$      | 12        | 6         | 0.013     | –         |
| $b \to \bar{c} + c + d$            | 8         | 6         | 0.006     | –         |
| $B_c \to \Sigma_c \Sigma_c$        | 2         |           | 0.011     | –         |
| $b \to \bar{u}$                     | 46        | 29        | 0.005     | –         |
| $\Sigma^{b}_{tot}$                   | 92        | 58        | 0.531     | 0.615     |
| $c \to s + e + \nu_e$               | 2         | 1         | 0.081     | 0.162     |
| $c \to s + \mu + \nu_\mu$          | 2         | 1         | 0.077     | 0.162     |
| $c \to s + u + d$                    | 12        | 6         | 0.698     | 0.905     |
| $c \to s + u + \bar{s}$             | 12        | 6         | 0.023     | –         |
| $c \to d$                           | 28        | 14        | 0.036     | –         |
| $\Sigma^{c}_{tot}$                   | 56        | 28        | 0.915     | 1.229     |
| $bc \to \tau + \nu_\tau$           |           |           | 0.052     | 0.056     |
| $bc \to c + \bar{s}$                |           |           | 0.137     | 0.138     |
| PI                                    |           |           | $\approx -0.100$ | -0.124   |
| $\Sigma_{tot}$                       | 148       | 86        | 1.535     | 1.914     |
Table 6. SL and NL external partial rates for $\bar{b} \rightarrow \bar{c}$ and $c \rightarrow s$ transitions (in units $10^{-3}ps^{-1}$). The values of the CKM parameters used in the calculations are the same as in Table 2. The values of the QCD two–meson amplitudes are $a_1 = 1.0$, $a_2 = -0.3$ and $a_1 = c_1(m_c)$, $a_2 = c_2(m_c)$ for the $\bar{b} \rightarrow \bar{c}$ and $c \rightarrow s$ transitions, respectively.

| $\bar{b} \rightarrow \bar{c}l\nu_l$ | $c \rightarrow s l\nu_l$ |
|----------------------------------|-------------------------|
| $\eta_c$  | $J/\psi$  | $X_{c\bar{c}}$ | $B_s$ | $B_s^*$ | $X_{bs}$ |
| $e\nu_e$  | 13.05     | 26.6          | 20.6  | 22.0   | 51.2    | 7.35   |
| $\mu\nu_\mu$ | 13.02   | 26.5          | 20.6  | 21.1   | 48.2    | 6.34   |
| $\tau\nu_\tau$ | 4.37    | 7.53          | 1.81  | –      | –       | –      |

| $\bar{b} \rightarrow \bar{c}ud$ | $b \rightarrow \bar{c}u\bar{s}$ |
|---------------------------------|-------------------------------|
| $\pi^+$  | $\rho^+$  | $X_{ud}$  | $K^+$ | $K^{*+}$ | $X_{us}$ |
| $\eta_c$  | 2.25     | 5.15     | 46.4  | 0.178   | 0.289    | 2.30   |
| $J/\psi$  | 1.27     | 3.57     | 95.9  | 0.096   | 0.213    | 4.61   |
| $X_{c\bar{c}}$ | 5.71    | 13.3     | 65.9  | 0.329   | 0.751    | 2.99   |

| $b \rightarrow \bar{c}cs$ | $b \rightarrow \bar{c}cd$ |
|--------------------------|--------------------------|
| $D_s$   | $D_s^*$   | $X_{cs}$  | $D^+$ | $D^{*+}$ | $X_{cd}$ |
| $\eta_c$  | 8.03     | 6.68     | 10.8  | 0.344   | 0.198    | 0.690  |
| $J/\psi$  | 3.28     | 13.2     | 17.5  | 0.146   | 0.354    | 1.16   |
| $X_{c\bar{c}}$ | 6.79    | 14.0     | 2.68  | 0.426   | 0.440    | 0.245  |

| $c \rightarrow sud$ | $c \rightarrow ssu$ |
|---------------------|---------------------|
| $\pi^+$  | $\rho^+$  | $X_{ud}$  | $K^+$ | $K^{*+}$ | $X_{us}$ |
| $B_s$   | 96.4     | 65.4     | 0.180 | 8.06    | 0.432    | –      |
| $B_s^*$ | 55.0     | 340      | –     | 3.22    | 0.213    | –      |
| $X_{bs}$ | 54.5     | –        | –     | 1.56    | –        | –      |
Table 7. Two-meson $b \to c$ rates of $B^+_c$ (in $10^{-3} \text{ps}^{-1}$).

| Decay mode          | This work    | ref. [6] | ref. [7] |
|---------------------|--------------|----------|----------|
| **Class I**         |              |          |          |
| $B^+_c \to \eta_c \pi^+$ | $2.23a_1^2$  | $1.83a_1^2$ | $3.14a_1^2$ |
| $B^+_c \to \eta_c \rho^+$ | $5.09a_1^2$  | $4.32a_1^2$ | $8.33a_1^2$ |
| $B^+_c \to J/\psi \pi^+$ | $1.25a_1^2$  | $1.92a_1^2$ | $3.00a_1^2$ |
| $B^+_c \to J/\psi \rho^+$ | $3.53a_1^2$  | $5.42a_1^2$ | $9.04a_1^2$ |
| $B^+_c \to \eta_c K^+$      | $0.23a_1^2$  | $0.14a_1^2$ | $0.245a_1^2$ |
| $B^+_c \to \eta_c K^{**}$   | $0.36a_1^2$  | $0.22a_1^2$ | $0.435a_1^2$ |
| $B^+_c \to J/\psi K^+$      | $0.12a_1^2$  | $0.14a_1^2$ | $0.231a_1^2$ |
| $B^+_c \to J/\psi K^{**}$   | $0.27a_1^2$  | $0.28a_1^2$ | $0.492a_1^2$ |
| **Class II**        |              |          |          |
| $B^+_c \to D^+ D^0$     | $4.13a_2^2$  |          | $1.01a_2^2$ |
| $B^+_c \to D^+ D^{*0}$   | $3.19a_2^2$  |          | $1.06a_2^2$ |
| $B^+_c \to D^{**} D^0$   | $1.31a_2^2$  |          | $0.992a_2^2$ |
| $B^+_c \to D^{**} D^{*0}$ | $2.01a_2^2$  |          | $1.64a_2^2$ |
| **Class III**       |              |          |          |
| $B^+_c \to \eta_c D_s^{+}$ | $(3.19a_1 + 4.19a_2)^2$ | $(1.39a_1 + 2.44a_2)^2$ |
| $B^+_c \to \eta_c D_s^{**}$ | $(2.91a_1 + 2.45a_2)^2$ | $(1.28a_1 + 2.34a_2)^2$ |
| $B^+_c \to J/\psi D_s^{+}$ | $(2.04a_1 + 3.60a_2)^2$ | $(1.26a_1 + 2.40a_2)^2$ |
| $B^+_c \to J/\psi D_s^{**}$ | $(4.09a_1 + 4.79a_2)^2$ |          |          |
| $B^+_c \to \eta_c D^+$    | $(0.58a_1 + 0.90a_2)^2$ | $(0.24a_1 + 0.54a_2)^2$ |
| $B^+_c \to \eta_c D^{**}$  | $(0.45a_1 + 0.81a_2)^2$ | $(0.22a_1 + 0.53a_2)^2$ |
| $B^+_c \to J/\psi D^+$    | $(0.37a_1 + 0.54a_2)^2$ | $(0.22a_1 + 0.54a_2)^2$ |
| $B^+_c \to J/\psi D^{**}$  | $(0.59a_1 + 0.98a_2)^2$ |          |          |
Table 8. Two-meson $c \to s$ rates of $B_c^+$ (in $10^{-3}ps^{-1}$).

| Decay Mode         | This work | ref.[8] | ref.[9] | ref.[10] |
|--------------------|-----------|---------|---------|---------|
| Class I            |           |         |         |         |
| $B_c^+ \to B_s \pi^+$ | 52.86$a_1^2$ | 47.2$a_1^2$ | 66.8$a_1^2$ | 88.7$a_1^2$ |
| $B_c^+ \to B_s \rho^+$ | 35.88$a_1^2$ | 19.0$a_1^2$ | 30.7$a_1^2$ | 68.1$a_1^2$ |
| $B_c^+ \to B_s^{*\pi^+}$ | 30.14$a_1^2$ | 38.9$a_1^2$ | 52.7$a_1^2$ | 78.4$a_1^2$ |
| $B_c^+ \to B_s^{*\rho^+}$ | 186.46$a_1^2$ | 175.6$a_1^2$ | 231.1$a_1^2$ | 228$a_1^2$ |
| $B_c^+ \to B_0 \pi^+$ | 2.30$a_1^2$ | 1.47$a_1^2$ | 2.87$a_1^2$ | 5.01$a_1^2$ |
| $B_c^+ \to B_0 \rho^+$ | 2.93$a_1^2$ | 1.43$a_1^2$ | 3.25$a_1^2$ | 9.07$a_1^2$ |
| $B_c^+ \to B_0^{*\pi^+}$ | 1.19$a_1^2$ | 2.40$a_1^2$ | 1.94$a_1^2$ | 4.41$a_1^2$ |
| $B_c^+ \to B_0^{*\rho^+}$ | 10.3$a_1^2$ | 13.4$a_1^2$ | 13.5$a_1^2$ | 18.07$a_1^2$ |
| Class II           |           |         |         |         |
| $B_c^+ \to B^+ K^{*0}$ | 36.54$a_2^2$ | 42.8$a_2^2$ | 93.3$a_2^2$ | 147$a_2^2$ |
| $B_c^+ \to B^+ K^{*0}$ | 20.93$a_2^2$ | 15.2$a_2^2$ | 36.6$a_2^2$ | 104$a_2^2$ |
| $B_c^+ \to B^{*+} K^{0}$ | 13.55$a_2^2$ | 47.1$a_2^2$ | 43.0$a_2^2$ | 111$a_2^2$ |
| $B_c^+ \to B^{*+} \bar{K}^{*0}$ | 125.10$a_2^2$ | 223.5$a_2^2$ | 248.9$a_2^2$ | 214$a_2^2$ |
| $B_c^+ \to B^+ \pi^{0}$ | 1.56$a_2^2$ | 0.73$a_2^2$ | 1.44$a_2^2$ | 2.51$a_2^2$ |
| $B_c^+ \to B^+ \rho^{0}$ | 1.95$a_2^2$ | 0.71$a_2^2$ | 1.63$a_2^2$ | 4.53$a_2^2$ |
| $B_c^+ \to B^{*+} \pi^{0}$ | 0.80$a_2^2$ | 1.20$a_2^2$ | 0.97$a_2^2$ | 2.20$a_2^2$ |
| $B_c^+ \to B^{*+} \rho^{0}$ | 6.93$a_2^2$ | 6.70$a_2^2$ | 6.73$a_2^2$ | 9.05$a_2^2$ |
Figure 1: Distribution functions $F_b(x)$ for $B^0$ and $B_c$ mesons.