The Kondo effect in quantum dots at high voltage: Universality and scaling

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We examine the properties of a dc-biased quantum dot in the Coulomb blockade regime. For voltages \( V \) large compared to the Kondo temperature \( T_K \), the physics is governed by the scales \( V \) and \( \gamma \), where \( \gamma \sim V/\ln^2(V/T_K) \) is the non-equilibrium decoherence rate induced by the voltage-driven current. Based on scaling arguments, self-consistent perturbation theory and perturbative renormalization group, we argue that due to the large \( \gamma \), the system can be described by renormalized perturbation theory for \( \ln(V/T_K) \gg 1 \). However, in certain variants of the Kondo problem, two-channel Kondo physics is induced by a large voltage \( V \).

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In recent years, it became possible to observe the Kondo effect in quantum dots in the Coulomb blockade regime \[1,2\]. These systems allow to investigate, how non-equilibrium induced by external currents and bias voltages influences the Kondo physics. Similarly, the experimentally observed anomalies of the energy relaxation in strongly voltage-biased mesoscopic wires \[3,4\] have recently been shown \[5\] to be caused by scattering from magnetic impurities or two-level systems.

In equilibrium, almost all properties of the Kondo effect are well understood, and the Kondo model together with the methods used to solve it (e.g. renormalization group (RG), Bethe ansatz, conformal field theory, bosonization, density matrix RG, flow equations or slave particle techniques) have become one of the central paradigms in condensed matter theory. However, in non-equilibrium many of the above-mentioned methods fail, and despite the experimental and theoretical relevance and a substantial body of theoretical work \[5,6\], several even qualitative questions about the Kondo effect in non-equilibrium have remained controversial. Recently, Coleman et al. \[6\] claimed that the Kondo model at high voltages \( V \gg T_K \) cannot be described by (renormalized) perturbation theory (PT) but is characterized by a new two-channel Kondo fixed point (see also \[7\]). By contrast, Kaminski et al. \[8\] argue that the non-equilibrium decoherence rate \( \gamma \) destroys the Kondo effect. We will show in the following that the Kondo effect is indeed destroyed in the case of the usual Anderson model, but for certain variants of the Kondo model, where the current at high bias is suppressed, the scenario proposed in \[6\] appears to be recovered.

We model the quantum dot using the Anderson model

\[
H_A = H_0 + \varepsilon_d \sum_{\sigma} d_\sigma^\dagger d_\sigma + \sum_{\alpha \kappa \sigma} (t_{\alpha} c_{\alpha \kappa \sigma}^\dagger d_\sigma + h.c.) + U n_{d\uparrow} n_{d\downarrow}
\]

where \( H_0 = \sum_{\alpha \kappa \sigma} \varepsilon_{\alpha \kappa} c_{\alpha \kappa \sigma}^\dagger c_{\alpha \kappa \sigma} \) is the Hamiltonian of the electrons in the left and right leads, \( \alpha = L, R \), characterized by a dc bias voltage \( V \), \( \varepsilon_L/Rk = \varepsilon_k \pm V/2 \), respectively. We will consider only symmetrical dots with tunneling matrix elements \( t_{\alpha} = t \), the negative \( \varepsilon_d \) with \( |\varepsilon_d| \gg \Gamma = 2\pi N_0 t^2 \), where \( N_0 \) is the electron density of states in the leads, and the large Coloumb repulsion \( U \to \infty \) enforce the number of electrons \( n_d = \sum_{\sigma} d_\sigma^\dagger d_\sigma \) in the dot level to be approximately 1.

In this regime, the local degree of freedom of the quantum dot is a spin \( \vec{S} = \frac{1}{2} \sum_{\sigma, \sigma'} d_\sigma^\dagger \vec{\sigma}_{\sigma\sigma'} d_{\sigma'} \), where \( \vec{\sigma} \) is the vector of Pauli matrices, and the low-energy properties of \( H_A \) are well described by the two-lead Kondo (or Coqblin-Schrieffer) model,

\[
H_K = H_0 + V_0 \sum_{\sigma} (c_{L0\sigma}^\dagger + c_{R0\sigma}^\dagger) (c_{L0\sigma} + c_{R0\sigma}) + \sum_{\sigma, \sigma'} J_L \sigma^\dagger_{L0\sigma} \frac{\delta_{\sigma\sigma'}}{2} c_{L0\sigma'} + J_{LR} \sigma^\dagger_{L0\sigma} \frac{\delta_{\sigma\sigma'}}{2} c_{R0\sigma'} + (L \leftrightarrow R)
\]

where \( c_{L/R0\sigma} = \sum_{k} c_{L/Rk\sigma} \). For an Anderson model with symmetrical coupling to the leads, one obtains \( J_L = J_R = J_{LR} = J_{RL} = 4V_0 = 2t^2/\varepsilon_d \equiv J \). For sufficiently small \( J \), the potential scattering term \( V_0 \) can be neglected and, as will be seen, the equilibrium and the non-equilibrium physics of the Kondo model is completely universal, characterized by a single scale, the Kondo temperature \( T_K = D \sqrt{\sum_{k} e^{1/(2N_0 J_k)}} \), where \( D \) is a high-energy cutoff. The precise formula for the prefactor of \( T_K \) depends on details of the model. However, for \( T_K, T, V \ll D \) relevant physical quantities like the conductance \( G \) are universal, \( G = G(V/T_K, T/T_K) \) and do not depend on details of the original Hamiltonian.

In the first part of the paper, we investigate in detail the Anderson model in the Kondo regime at high voltages using the so-called non-crossing approximation (NCA). In the second part we will use the insight gained from this analysis to study a heuristic version of poor man’s scaling in non-equilibrium for a Kondo model with \( J_{LR} < J_{L/R} \).

To derive NCA, one first rewrites \( H_A \) in the limit \( U \to \infty \) using a so-called pseudo-fermion \( f_{\sigma} \) and a spin-less slave boson \( b \) with \( d_{\sigma} = b f_{\sigma} \), subject to the constraint \( Q = \sum_{\alpha} f_{\alpha}^2 f_{\sigma} + b^2 b = 1 \). The Anderson model then takes the form \( H_A = H_0 + \varepsilon_d b f_{\sigma} + \sum_{\alpha, \sigma} (V_{a_{\alpha}} f_{\alpha}^\dagger f_{\sigma} + h.c.) \).
In this language, the NCA is just the lowest-order self-
consistent PT in $t_o$, where the constraint $Q = 1$ is taken
into account exactly. While the NCA fails to describe the
low-energy Fermi liquid fixed point in the Kondo regime
correctly [10], it gives reliable results (with errors of the
order of 10%) in equilibrium for temperatures down to a
fraction of $T_K$. As a self-consistent and conserving ap-
proximation, it also displays the correct scaling behavior
and reproduces the relevant energy scales.

While the NCA equations in non-equilibrium have
been solved by many groups [13], we are not aware of any
careful analysis of the relevant scales at high bias voltage,
which is central for a qualitative understanding of the
non-equilibrium Kondo effect. Generally, the NCA equa-
tions have to be solved numerically; however, in the limit
of extremely high voltage, $ln V / T_K \gg 1$ (but $V \ll D$), an
analytical solution is possible: the problem is in the weak
coupling regime. Finite $V$ induces an inelastic spin relax-
ation or decoherence rate. Since in NCA the spin density
induces an inelastic spin relax-
ation or decoherence rate. Since in NCA the spin density
is just a convolution of the pseudo-fermion propagators,
this rate is given by $2Im\Sigma_f(0) = 2\gamma$, with $\Sigma_f$ the
pseudo-fermion self-energy. We start by calculating the retarded
self-energy $\Sigma_f^R(\omega)$ of the boson, using the fact that (as
shown below) the spectral function of the pseudo-fermion is
a sharp peak of width $\gamma \ll V$. Throughout we consider
the low temperature limit, $T = 0$, and obtain $\text{Im} \Sigma_f^R(\omega) \approx -\pi J N_0 |\epsilon_d| (f^g_R(-\omega) + f^g_L(-\omega))$, where $f^g_R/L$ are the Fermi
functions in the left and right leads, broadened by $\gamma$. The
step-function in $\text{Im} \Sigma_f^R(\omega)$ leads to logarithmic contribu-
tions to $\text{Re} \Sigma_f^R(\omega)$, cut off by $\gamma$ and the band width $D$. Us-
ing relations like $1 - 2N_0 J \ln |D/|\omega|| = 2N_0 J \ln |\omega|/|T_K|$ one obtains for the real part of the boson propagator
$G^R_b(\omega)$, for $ln V / T_K \gg 1$,
\begin{equation}
N_0 J^{NCA}_0(\omega) = 2N_0 e^2 \text{Re} G^R_b(\omega) \\
\approx \frac{1}{\ln \left(\frac{\omega - V/2}{T_K} + \frac{\gamma}{T_K}\right) + \ln \left(\frac{\omega + V/2}{T_K} + \frac{\gamma}{T_K}\right)}.
\end{equation}

This combination plays the role of an effective (frequency
dependent) exchange coupling $J_{\text{eff}}$. Remarkably, the
perturbative expression Eq. (3) would develop a pole close
to $\omega = \pm V/2$ if $\gamma < T^* = \sqrt{T_K^2 + (V/2)^2} - V/2 \approx T_K^2 / V$.
The breakdown scale $T^*$ of PT has also been discussed in
[4th Ref. of [1]] and [13]. It indicates that Eq. (3) is only
valid for $T^* < \gamma < V$. Indeed, this criterion is fulfilled
(see Fig. 3), as one finds within NCA
\begin{equation}
\gamma \approx \frac{\pi}{8 \ln \frac{\pi}{T_K}} \left[ 1 + \frac{2}{\ln \frac{\pi}{T_K}} + O \left( \frac{\ln \frac{\pi}{T_K}}{\ln^2 \frac{\pi}{T_K}} \right) \right].
\end{equation}

For the conductance in units of the conductance quantum
$G_0 = 2e^2 / (2\pi h)$ we obtain for $\ln V / T_K \gg 1$
\begin{equation}
\frac{G^{NCA}}{G_0} \approx \frac{\pi^2}{4 \ln^2 \frac{V}{T_K}} \left[ 1 + \frac{2}{\ln \frac{V}{T_K}} + O \left( \frac{\ln \frac{V}{T_K}}{\ln^2 \frac{V}{T_K}} \right) \right].
\end{equation}

Numerical results for smaller voltages down to $V < T_K$
are shown in Fig. 3 and display universal behavior over
the complete range of voltages and over several orders of
magnitude in $T_K$. Despite the fact that for high voltages,
$\ln V / T_K \gg 1$, one stays in the weak coupling regime, the
prefactors of $\gamma^{NCA}$ and $G^{NCA}$ are not exact, since the
NCA for the Anderson model treats the potential scattering
$V_0$ and the Kondo coupling $J$ incorrectly on equal footing.
It is not difficult to obtain the correct asymptotic prefactors
by calculating $\gamma$ and $G$ in leading order PT in $J$ for the Kondo model Eq. (4) (with $V_0 = 0$) and by replacing $J$ by $1/(2 \ln V / T_K)$. This corrects the
leading term of the NCA results Eqs. (4), (5) by a prefactor
$3/4$. It is, however, important to stress, that the
asymptotic result in the limit $\ln V / T_K \to \infty$ is almost
useless as, due to the logarithmic dependence (Eq. (4)),
sub-leading corrections are very large (e.g. still 10% for $V / T_K = 10^6$).

In the limit of large $V$, the scale $\gamma$ influences quanti-
ties like the conductance, where all electrons in an energy
window $V$ contribute, only slightly. The situation is dif-
ferent for the local spin susceptibility on the quantum
dot, $\chi = 1/\gamma$, or the spectral function $A_{\omega}(\omega)$ of the
electron on the quantum dot. $A_{\omega}(\omega)$ calculated numerically
within NCA is shown in Fig. 3. Like many groups before,
we obtain two well defined peaks at voltages $\pm V/2$. In the
limit $\ln V / T_K \to \infty$ we find approximately
$A^{NCA}_{\omega}(\omega) \approx (\pi^2 / \Gamma) \left[ N_0 G^{NCA}_0(\omega) \right]^2$, with large but universal sub-
leading corrections and a non-universal, (almost) con-
stant potential scattering background of $O(\Gamma / \varepsilon^2_0)$. NCA
incorrectly treats potential and spin flip scattering on
equal footing and, thus, overestimates the asymmetry of
the peaks w.r.t. $\omega \leftrightarrow -\omega$ in the small $J$ limit. This can be seen from an analysis of the Schrieffer-Wolff transfor-
FIG. 2. Spectral function $A_d^{NCA}(\omega)$ for various voltages $V$, each calculated in NCA at two values of $T_K$ (solid and dashed lines) differing by a factor of 10. The NCA strongly overestimates the anisotropy of the peaks. Inset: asymptotic behavior of $A_d^{NCA}(\omega)$ in the Kondo scaling limit. Squares: numerical NCA result; solid line: asymptotic expression for $\ln(V/T_K) \to \infty$.

Note that the logarithmic cusps of $A_d(\omega)$ have an additional, small rounding of $O(\gamma)$ compared to Eq. (3), but for large voltage, the half width at half-maximum of the peaks is not given by $\gamma$ but by $\sqrt{\gamma V}/2 \approx 0.3V/\ln(V/T_K) > \gamma$ (see Fig. 2).

Our analysis of the Kondo model suggests, that qualitatively different behavior can be expected, if the non-equilibrium relaxation rate $\gamma$ is sufficiently small, $\gamma < T_K^2/V$. Since a non-zero $\gamma$ requires finite current, e.g. within bare PT $\gamma \propto J_{LR}^2 V$, it is therefore interesting to study the Kondo model Eq. (2) for $J_{LR} \ll J_L J_R$, using ideas from perturbative RG. Such a model cannot be derived from a simple Anderson model but may arise in more complicated situations. Not much is known about how the concepts of fixed points and renormalization group can be applied to a non-equilibrium situation (see, however, [1]). The problem is that in the presence of a finite bias voltage, many physical quantities like the conductance are not determined by low-energy excitations even at $T = 0$, since all states with energies of order of the applied voltage $V$ contribute. Therefore, a controlled perturbative RG must probably be formulated for the full frequency-dependent vertices in Keldish space. We will not try to develop such a method here but propose to use a heuristic version of poor-man’s scaling adapted to the present situation. As usual, we investigate, how coupling constants change, when the cutoff $\Lambda$ of the theory is modified. As long as the cutoff is large compared to the voltage, we expect that the usual poor-man’s scaling equations hold. For the model with $N_0 J_L = N_0 J_R = g_d$ and $N_0 J_{LR} = g_{LR}$ and $V \ll \Lambda$ one obtains [14]

$$\frac{dg_d}{d\ln \Lambda} = -(g_d^2 + g_{LR}^2), \quad \frac{dg_{LR}}{d\ln \Lambda} = -2g_d g_{LR}.$$  

These are the RG equations of a channel-asymmetric two-channel Kondo model, where the even and odd channels couple to the spin with coupling constants $g_e = g_d + g_{LR}$ and $g_o = g_d - g_{LR} \leq g_o$. Note that for the Anderson model, the odd channel decouples and $g_o = 0$. Two parameters, $T_K$ and $\alpha$, determine the physics of the channel-asymmetric two-channel Kondo model,

$$T_K = D e^{\gamma/(g_d + g_{LR})}, \quad \alpha = \frac{(g_d - g_{LR})(g_d + g_{LR})}{2g_{LR}}.$$  

where $T_K$ is defined by $g_e(T_K) = 1$. The dimensionless number $\alpha$ is the natural parameter to characterize the channel anisotropy, since it is invariant under the perturbative RG flow Eq. (1), i.e. $\alpha(\Lambda) = \alpha_0 = const.$ for $\Lambda > V$. If higher orders of $g$ are included in Eqs. (6), the prefactor of $T_K$, Eq. (8), changes and the definition of the RG invariant $\alpha$ has to be slightly adjusted (a dimensionless invariant characterizing the flow will exist even in higher orders). For the usual one-channel Kondo effect or the Anderson model Eq. (3), $\alpha_0 = 0$, while for $\alpha_0 \to \infty$ the model is just the well-known channel-symmetric two-channel Kondo model. We will therefore investigate, how $\alpha$ will change for $\Lambda < V$ in order to determine if the system flows towards a two-channel fixed point, that has been proposed by Wen [13] and Coleman et al. [14]. For $V \gg T_K$, Eq. (9) is valid down to $\Lambda = V$ and we obtain

$$g_d(V) = \frac{1}{2} \left( \ln \frac{V}{T_K} \right)^{-1} + \left[ \frac{1}{\alpha_0} + \ln \frac{V}{T_K} \right]^{-1}$$  

(8)

$$g_{LR}(V) = \frac{1}{2} \left( \ln \frac{V}{T_K} \right)^{-1} - \left[ \frac{1}{\alpha_0} + \ln \frac{V}{T_K} \right]^{-1}.$$  

(9)

For $\Lambda < V$, the calculation of the RG flow is less obvious. Some of the logarithmically diverging vertex corrections of $J$ are cut off by the voltage $V$, changing the RG flow to

$$\frac{dg_d}{d\ln \Lambda} = -g_d^2, \quad \frac{dg_{LR}}{d\ln \Lambda} = 0,$$  

(10)

in complete agreement with the analysis of Coleman et al. [14]. However, all remaining logarithmic contributions are cut off by the decoherence rate $\gamma$ as it is evident, e.g., from our analysis of NCA. Thus, Eqs. (10) are only valid for $\gamma < \Lambda < V$.

Since in the perturbative regime of the RG the bare coupling constant $N_0 J_{LR}$ is replaced by the renormalized one, $g_{LR}$, we find

$$\gamma \sim V g_{LR}^2(V),$$  

(11)

which is $V/[2 \ln(V/T_K)^2]$ for $\alpha_0 \ll 1/\ln(V/T_K)$ and
Kondo effect, as their decoherence rate $\gamma \approx \text{equilibrium even at}$ high voltages, because relaxation processes allowed in non-model at high voltages $T^{*}$ regime for $T < T^{*}$. $\alpha > 1$ indicates that this regime is dominated by two-channel physics.

$V/[4\alpha_0^2 \ln(V/T_K)]$ for $\alpha_0 \gg 1/\ln(V/T_K)$. (The precise prefactor is irrelevant for our discussion). If we assume for the moment that $\gamma$ is small, we find that $g_d$ flows to strong coupling at a scale $T^*$ defined by $g_d(T^*) = 1$,

$$T^* \approx T_K \left(\frac{T_K}{V}\right)^{1/[1+2\alpha_0 \ln(V/T_K)]}.$$  \hspace{1cm} (12)

For $\alpha_0 = 0$ this scale coincides with the one introduced in [4], where the effects of $\gamma$ have been neglected. The system will, however, only flow to strong coupling if $\gamma > T^*$. For the usual Kondo- or Anderson model with $\alpha_0 = 0$, $\gamma$ is always larger than $T^*$ for $V > T_K$ (as $V/T_K > \ln(V/T_K)$), and we therefore conclude in contradiction to Ref. [4] that in the symmetrical Kondo model there is no strong coupling regime for $V > T_K$. The situation is, however, different in the asymmetric model with $\alpha_0 \geq 1/2$ (Fig. 3). Here the ratio $\gamma/T^* \approx (V/T_K)/[(4\alpha_0^2 \ln^2(V/T_K)]$ is small for $V \ll V^* \approx T_K (4\alpha_0^2 \ln^2[4\alpha_0^2 \ln^2[4\alpha_0^2 \ln^2[\ldots]])$, e.g. $V^* \approx 6 \cdot 10^4 T_K$ for $\alpha_0 = 1$. What is the nature of this strong coupling regime which is reached for $T_K < V < V^*$ and $\alpha_0 > 1/2$? Insight into this question can be gained from a calculation of $\alpha(\Lambda = T^*)$, defined in Eq. (5). Note that in the regime $\gamma < \Lambda < V$, $\alpha$ is not invariant under the RG flow Eqs. (10). We obtain

$$\alpha^* \equiv \alpha(T^*) \approx \ln \frac{V}{T_K} \left(1 + \alpha_0 \ln \frac{V}{T_K} \right) \approx \alpha_0 \ln^2 \frac{V}{T_K}.$$  \hspace{1cm} (13)

Obviously, $\alpha$ is strongly enhanced by the voltage (e.g. $\ln^2[10^3] \approx 50$). Since for $\alpha \rightarrow \infty$ the system maps to a two-channel Kondo problem, we conclude that for $\alpha_0 > 1/2$ and $T_K < V < V^*$ the system will likely be dominated by two-channel physics over a large regime.

In this paper, we have shown that the usual Kondo model at high voltages $T_K \ll V < D$ is a weak coupling problem because relaxation processes allowed in non-equilibrium even at $T = 0$ destroy the building up of the Kondo effect, as their decoherence rate $\gamma \gg T^* \approx T_K^2/V$.

Nevertheless, bare perturbation theory cannot be applied and even the leading order of renormalized perturbation theory does not give precise results in the experimentally accessible regime due to large sub-leading corrections. In variants of the Kondo model with $J_{LR} < J_L, J_R$, a large voltage can, however, induce a qualitatively new behavior reminiscent of two-channel Kondo physics.

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