Coronal Electron Density Fluctuations Inferred from Akatsuki Spacecraft Radio Observations

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Abstract  Trans-coronal radio observations were taken during the 2011 observing campaign of the Akatsuki spacecraft through superior conjunction. The observed X-band (8.4 GHz) signals exhibit frequency fluctuations (FF) that are produced by temporal variations in electron density along the radio ray path. A two-component model for interpretation of the FF is proposed: FF scales largely with acoustic wave amplitude through the inner coronal regions where the sound speed dwarfs the solar wind outflow speed, while FF in the region of solar wind acceleration is dominated by the increased density oscillation frequency on the sensing path that results from bulk advection of the plasma inhomogeneities. An estimate of fractional electron density fluctuation is obtained from the mid-corona. A radial profile of slow solar wind speed is determined in the extended corona using mass-flux continuity principles. The coronal sonic point for slow solar wind is estimated to range from 4 to 5 solar radii from the heliocenter.

Keywords  Coronal acoustic waves · Solar wind · Electron density oscillations, radio frequency fluctuations
1. Introduction

Radio occultation studies of the solar corona reveal ubiquitous frequency fluctuations that imply electron density disturbances at all heliocentric distances and heliolatitudes. The intensity and spectral form of these inferred density fluctuations are of great interest in the study of coronal turbulence and mechanisms of solar wind acceleration. Alfvén waves generated by transverse motions of magnetic structures in the photosphere propagate up into the corona, but the dissipation process to release energy for coronal heating and solar wind acceleration remains unclear (Cranmer and Winebarger, 2019; Arregui, 2015; De Moortel and Browning, 2015). One theory holds that locally generated compressive (acoustic or magneto-sonic) waves play a role in the release of Alfvén wave energy into the corona. Outgoing Alfvén waves subjected to magnetohydrodynamic (MHD) speed gradients may partially reflect and give rise to density oscillations through non-linear interactions, see e.g. (Cranmer et al., 2015; Matthaeus and Velli, 2011; Matthaeus et al., 1999). In distinction from the Alfvén waves, compressive waves do not propagate far in the corona, and thus are suitable for local coronal energy transfer and wave dissipation.

Observationally, detection of density fluctuations beyond the base of corona relies largely on transcoronal radio sensing techniques. The geometric considerations and general experimental paradigm for these methods are well documented (Tyler et al., 1981; Chashei et al., 2005; Yakovlev and Pisanko, 2018; Bird and Edenhofer, 1990; Efimov et al., 2002). Density fluctuations across the sensing radio path (line-of-sight, LOS) induce frequency fluctuations (FF) at the receiving radio telescope due to a changing index of refraction associated with the electron density disturbances. The main effects generally occur in the region straddling the point of closest solar approach on the LOS, called the proximate point. The heliocentric distance to this point is called the solar offset (SO). The FF present spectral characteristics that suggest partial or full turbulence regimes in various stages of energy cascade development, depending on the state of solar activity and the SO of the observations.

Electromagnetic wave propagation theory establishes the relation between radio frequency observed at the receiver system, $f_{\text{obs}}$, and the original transmitted frequency, $f_0$ when the radio beam encounters varying electron density (Pätzold et al., 2016; Vierinen et al., 2014; Jensen et al., 2016):

$$f_{\text{obs}} = f_0 - \frac{V_{rel}}{c} f_0 + \frac{1}{2\pi r_e \lambda} \int_{SC}^{\oplus} n_e(s,t) dS,$$

(1)

where the classical electron radius $r_e = 2.82 \times 10^{-15}$ m, $V_{rel}$ is the spacecraft radial velocity relative to the terrestrial receiving station, $n_e$ is the electron number density and $S$ is the radio sensing path length. The integration extends from the spacecraft (SC) to the Earth ($\oplus$). The second term on the right-hand side is the Doppler-shift due to radial motion of the spacecraft relative to Earth. This Doppler shift generally presents a slow trend in the radio frequency time series. After removal of the slow trend, the instantaneous frequency fluctuations, $\delta f = f_{\text{obs}} - f_0$, are attributed to the time rate of change in cumulative electron density on the sensing path (column density, $N_e$),

$$\delta f(t) = \frac{1}{2\pi r_e \lambda} \frac{d}{dt} N_e(t).$$

(2)

Although the presence of coronal FF is well established (Woo, 1978; Efimov et al., 2008, 2013; Yakovlev, 2017), physical interpretation of the findings is still being explored. Radio FF observations capture only part of the MHD wave processes. Shear Alfvén waves do not
exhibit density fluctuations so do not directly contribute to the observed FF. In contrast, magnetosonic and acoustic waves are compressive, and will produce FF in proportion to the amplitude and frequency of density oscillations on the sensing LOS.

Coronal radio FF effects are preferentially sensitive to the amplitude or frequency contributions of compressive waves and oscillations depending on the plasma flow rate relative to the speed of sound. In mid-corona, here defined as SO $1.4 - 2.5R_\odot$, streamer region plasma is subject to relative confinement and low bulk outflow speeds. Therefore it is reasonable to attribute the observed FF to compressive wave propagation across the sensing LOS. For such waves of a given frequency, the magnitude of FF will scale with density fluctuation amplitude, typically regarded as a fraction of the electron density, $n_e$. Accordingly, we expect the radial dependence of FF to trend with the electron density profile while the solar wind outflow speed is well below the speed of sound. However, in the extended corona, here defined as SO $> 2.5R_\odot$, the developing solar wind advects the spatially varying density disturbances to produce increased density oscillation frequency on the radio sensing path, and increased FF. At some point the FF enhancement due to solar wind effect on the oscillation frequency dominates over the acoustic wave amplitude contribution. Most radio FF studies have addressed the extended corona. Theoretical models based on coronal electron density 3D spatial turbulence spectrum and solar wind outflow speed have been presented in detail (Woo, 1978; Tyler et al., 1981; Armand, Efimov, and Yakovlev, 1987; Efimov et al., 2008).

Wexler et al. (2019b) developed a two-component approach in the study of coronal FF over SO $1.4 - 25R_\odot$. A composite data set consisting of MESSENGER 2009 radio occultation observations with archival Helios 1975–6 data was evaluated. In both data sets the solar activity was near minimum and the measurements were near-equatorial. The results suggested that the observed FF represented the LOS-summated density fluctuations due to acoustic (or slow radial magnetosonic) waves propagating in the mid-corona, while the effects of solar wind outflow were manifest in FF observed from the extended corona. One limitation of the study is the combination of data from different spacecraft radio measurements obtained decades apart on different equipment. This limitation is overcome in the Akatsuki spacecraft radio observing campaign of 2011 (Imamura et al., 2014; Miyamoto et al., 2014).

Miyamoto et al. (2014) evaluated Akatsuki 2011 spacecraft radio frequency fluctuations over SO range $1.5 - 20.5R_\odot$. They analyzed specifically the localized spectral power enhancements, called quasi-periodic components (QPC), as identified by wavelet analysis. A range of fractional density fluctuation was reported, from 0.002 at $1.5R_\odot$ to about 0.3 at $5R_\odot$. It was suggested that the compressive waves were increasing in energy flux out to about $5R_\odot$, and related to non-linear dissipation of Alfvén waves. The model was developed with the assumption that FF resulted wholly from radially propagating acoustic waves, together with the unusual parameterization that radial length scale of the waves was scaled by the sum of sound speed and the solar wind outflow speed, and set equal to the integration length creating the LOS column density fluctuation.

Imamura et al. (2014) studied the radio signal scintillation in the Akatsuki 2011 observations. The amplitude fluctuations were evaluated for a strong-scattering model based on developed Kolmogorov-like spatial spectrum of electron density irregularities. A radial profile of derived solar wind outflow speeds was presented, showing little acceleration below SO $5R_\odot$. The authors attributed these low speeds in the inner coronal regions to the closed magnetic field structuring, and considered the contribution of propagating compressive waves to be small.

In this paper, we further evaluate the Akatsuki 2011 radio observations building on methods developed in Wexler et al. (2019b). The general power spectra of FF were assessed, without restricting the analysis to QPCs. Dual-component models are again explored, mid-coronal FF being used to estimate the fraction of plasma density fluctuation, and extended
corona FF findings used to probe solar wind bulk advection of density inhomogeneities. With basic mass flux considerations invoked, estimates of slow solar wind speed are presented, and compared with results from previous radio studies and white-light coronagraph investigations.

2. Akatsuki Radio Observations

2.1. Spacecraft radio recordings and processing

X-band (8.4 GHz) radio transmissions of the Akatsuki spacecraft were observed over 2011 6 June to 8 July (Carrington rotations 2111–2112), during superior conjunction of the spacecraft on its trajectory for eventual Venus orbital insertion. The spacecraft was about 1.7 AU from Earth. The Sun was on the rising phase of Solar Cycle 24, with a state of moderate activity sufficient to produce widely distributed white-light streamers in LASCO C2 coronagraph imaging (Figure 1). There were eight sets of observations during ingress to superior conjunction, off the western limb of the Sun over SO range 20.51 – 1.50R⊙. In egress from superior conjunction, there were seven sets of observations over SO range 1.68 – 12.20R⊙. The 8.4 GHz downlink signal of the spacecraft was received using the 64-m dish antenna at Usuda Deep Space Center of Japan. The signals were down-converted by a heterodyne system and recorded with a sampling rate of 500 kHz. Further detail on the observations are given in Imamura et al. (2014).

The spectral centroid method was applied to the raw data in consecutive one-second blocks to determine the time series of peak radio frequencies. 4000-second data frames were then evaluated. In order to obtain the FF attributed to electron density variations, the bulk frequency shift trend due to spacecraft relative radial motion (Doppler shift) and large-scale plasma structure must be removed to achieve a zero-centered, horizontal baseline. This is necessary to avoid spurious spectral power from a deviated baseline itself, especially in the lower frequencies (Song and Russell, 1999). The estimated Doppler shifts from the spacecraft orbital trajectory ephemeris gave slightly insufficient slow-trend correction for
**Figure 2** Radio frequency fluctuations at SO 1.52R⊙. Top, upper panel: zero-centered radio baseband frequency, showing the dominance of the slow-trend Doppler frequency shift. The third-order polynomial fit used to detrend the data is shown in the light green dashed line. Top, lower panel: the frequency fluctuations (FF) are readily appreciated after the detrend procedure. A shortened segment of 700 seconds is shown for clarity. In black is the running weighted 5-point smoothing procedure result. Bottom: Averaged FF power spectra, obtained from several consecutive 4000-second data frames; mean power in blue and variance in gray. The integration frequency ranges used to calculate FF variance, $\sigma^2_{FF}$ (in Hz$^2$) are shown in red horizontal bands, with the 1 – 28 mHz band shown above the spectrum, and the 0.5 – 100 mHz band shown below. Also given are the average $\sigma^2_{FF}$ and the radio wavelength-normalized RMS frequency fluctuation measure, $\sigma_{FM}$ (in Hz m$^{-1}$).

For our purposes, so the slow trend extraction (“detrend procedure”) in each data frame was accomplished by subtraction of a low-order polynomial fit, as used by Efimov et al. (1993) and Andreev et al. (1997) to evaluate Faraday rotation fluctuations. A third-order polynomial detrend in each 4000-second data frame was satisfactory to obtain the frequency fluctuation time series, $\delta f(t)$. The power spectrum of FF in each frame, denoted $|\delta \hat{f}(\nu)|^2$, was then computed. Sample time series data and spectral power results are illustrated in Figure 2.

Figure 3 shows the radial dependence of FF power spectra based on averaged 4000-second data frames. As reported by Miyamoto et al. (2014) for Akatsuki observations and in other spacecraft studies (Yakovlev and Pisanko, 2018; Chashei et al., 2005; Efimov et al., 2008; Yakovlev, 2017), the spectral form is distinctly flattened at low solar offset, and shows a break around 7 mHz. This pattern is suggestive of energy injection from low-frequency sources. The power spectra gradually assume the typical Kolmogorov negative power law form consistent with inertial subrange of developed turbulence at increased heliocentric distance. This theoretical FF log-log spectral slope is $-2/3$, based on a sensing path density variations related to an underlying 3D Kolmogorov spatial spectrum of index $-11/3$ (Yakovlev, 2002).
Figure 3  Frequency fluctuation power spectra. Akatsuki 2011 FF averaged power spectra, show a general decrease in amplitude with increasing heliocentric distance. Also the spectral form evolves from a flattened low-frequency configuration at low SO (upper curves adjacent to the horizontal line) to a negative power law form consistent with a Kolmogorov turbulence inertial sub-range at higher SO (lower curves adjacent to the dashed line, spectral slope $-2/3$). The $1–28$ mHz integration bands are shown in solid color and the extensions for the $0.5–100$ mHz band are represented in stippling.

Integrated measures of frequency fluctuation are used to consolidate the spectral density results into variance and RMS values. Fluctuation variance, $\sigma_{FF}^2$, is defined from the FF power spectrum

$$\sigma_{FF}^2 = \int_a^b |\hat{\delta}(\nu)|^2 d\nu \ [\text{Hz}^2] \quad (3)$$

and obtained by numerical integration over fluctuation frequency limits $[a, b]$. In theory the low-frequency limit for the spectral analysis could be set as the reciprocal of the frames length, e.g. $0.25$ mHz for a 4000-second data frame. However, the low-frequency spectral power was quite sensitive to the method of slow trend extraction. This was particularly important at higher solar offsets, where the small FF amplitudes were comparable to the detrend imperfections. Thus there is uncertainty in choice of the lower frequency limit of the power spectrum integration. There was also upper frequency practical limit, well below the Nyquist frequency of $500$ mHz, due to spectral distortion specific to the computational methods for peak-frequency determination. Given the uncertainties in frequency limits for integration, we used both a $1–28$ mHz band as reported previously (Wexler et al., 2019b) and a wider integration band, $0.5–100$ mHz. For individual 4000-second data frame, the FF variance was calculated as the mean of narrow and wide frequency band integrations, and the measurement uncertainty was estimated to be half of their difference. Groups of data were combined as mean and standard deviation in the usual manner. Power spectrum processing to obtain variance is illustrated in the lower panel of Figure 2.

To allow comparison with older radio studies using S-band ($2.3$ GHz) transmissions, the radio wavelength-normalized RMS frequency fluctuation measure, $\sigma_{FM}$ was defined as

$$\sigma_{FM} = \sqrt{\frac{\sigma_{FF}^2}{\lambda}} \ [\text{Hz m}^{-1}], \quad (4)$$

where $\lambda = 0.0357$ m for the X-band data and $\lambda = 0.1304$ m for S-band data.

2.2. Frequency Fluctuations

The Akatsuki frequency fluctuation results, as $\sigma_{FM}$, are summarized in Figure 4. Included in the figure are previously published data from MESSENGER 2009 and Helios 1975–6
Figure 4  Frequency measure fluctuation results. Akatsuki 2011 observational $\sigma_{FM}$ results shown in black, on a backdrop of previously published MESSENGER 2009 (light green) and Helios 1975–6 (light blue) results (Wexler et al., 2019b). Bursts of increased fluctuation at 12.8R$_{\odot}$ (red x’s) are attributed to passage of a CME. The dot-dashed line shows the $n_e \sqrt{r}$ curve (scaled by $10^{-12}$), which trends with propagating density oscillations, exclusive of solar wind bulk outflow effects. 

radio observations (Wexler et al., 2019b), shown in undertones. Good general agreement is found between the Akatsuki $\sigma_{FM}$ results and those of the MESSENGER–Helios composite results. Also plotted is the curve for scaled $n_e \sqrt{r}$, where $n_e$ is the electron number density as described in Section 4.1 and $r$ is the solar offset given in solar radius units. This curve shows the expected FF trend related to propagating density waves, of constant fractional amplitude, without the overlay of significant bulk solar wind flow; see Section 3. A burst of enhanced fluctuations is noted at about 12.8R$_{\odot}$, shown in red, attributed to the passage of a coronal mass ejection (CME) from the western limb of the Sun on 13 June 2011 (Miyamoto et al., 2014).

Ando et al. (2015) reported a detailed analysis of the CME event radio observations. Elevations in $\sigma_{FM}$ up to ten-fold over baseline are due to increased electron densities and steepened density gradients. Increased plasma flow speeds could also be contributory to enhanced the frequency fluctuation amplitudes. Time series data showing multiple burst-like FF events are shown in Figure 5. For rough size estimates of CME density sub-components, we consider the length to correspond to the product of the time interval of the density disturbance and the outflow speed. The density-enhanced entity crossing the sensing radio path over ~ 200 seconds in Figure 5, lower panel, at estimated outflow speed 300 km s$^{-1}$, implies a 60,000 km sub-structure. A larger structure on the order of 4R$_{\odot}$, with variable density, is implied by the irregular and decreasing FF enhancement over ~ 10,000 sec in Figure 5, upper panel.

The radial profile of RMS fluctuation $\sigma_{FM}$ in Figure 4 suggests the presence of two density-fluctuation regimes in the corona. Mid-coronal $\sigma_{FM}$ appears to exhibit a steeper dependence on radial distance than that in the extended corona. The changeover occurs around solar offset 2–3R$_{\odot}$, similar to the results in an early radio study using composite data sets by Woo (1978). We propose that models to implement Equation 2 be developed separately for the mid-corona and extended corona. For mid-coronal streamer regions, the speed of sound dominates over that of the nascent solar wind, and we expect the $d/dt$ of Equation 2 to be enacted primarily as density oscillations propagating across the LOS as acoustic waves. In contrast, as solar wind overtakes the speed of sound with increasing SO, the $d/dt$ comes to be dominated by plasma density disturbances advected across the sensing path. We now develop models to represent these two FF-generating regimes in the corona.
3. Coronal FF model development

3.1. General Considerations

The solar occultation FF measurements are sensitive to the temporal fluctuations of column density on the LOS, Equation 2. Consistent with the general Parker theory of solar wind acceleration, we contend that the inner coronal region has a low plasma outflow speed, below the speed of sound, while the forming wind accelerates to become supersonic in the extended corona. To explore the FF results in this context, we develop separate simple models for sonic-speed-dominated density oscillations and bulk outflow-dominated density variations as presenting on the effective sensing LOS. The models for these two components of FF are then used to extract information about electron density fluctuation amplitude and the radial profile of solar wind speed.

Both component models are based on simple idealized geometry based on stacked slabs of plasma crossing perpendicularly to the sensing LOS path as shown in Figure 6. The length

Figure 5  Burst in FF, 2011 June 13 observations, after detrend procedures. Akatsuki 2011 observational results, upper: 20 000 seconds; lower: first 2000-second segment. These data are the time series of FF corresponding to red x’s data in Figure 4.
Generalized stacked slab model of density fluctuations crossing the radio sensing line of sight (LOS). Each slab carries electron density oscillation of transverse length scale $L_{LOS}$ across the LOS. The effective integration path $R$ is set equal to the solar offset. The acoustic waves propagate with frequency $v_c$, while radial-like density oscillations of length scale $L_{RAD}$ are advected across the LOS by bulk solar wind outflow of speed $V_{SW}$ to produce LOS density oscillations of frequency $V_{SW}/L_{RAD}$.

scale of the slab along the LOS is designated $L_{LOS}$. The temporal fluctuation of electron density $n_e(t)$ occurring on one such slab contributes frequency fluctuation on the LOS as

$$\delta f(t) = \frac{1}{2\pi} r_c \lambda \frac{d}{dt} n_e(t)L_{LOS}. \quad (5)$$

The fluctuation variance, $\sigma^2_{FF}$, obtained from power spectral integration (Equation 3), will scale with $L^2_{LOS}$ for a single slab. Total variance for the effective LOS is determined as sum of the individual slab variances. The effective integration length is generally set equal to the radial distance $R$ to the LOS point of closest solar approach, that is, equal to the solar offset. The number of slabs stacked on the effective integration path is $R/L_{LOS}$ and the total variance will scale with $L_{LOS}R$.

The slab thickness, $L_{LOS}$, also referred to as the correlation scale, has been used in various contexts. Hollweg et al. (1982), Hollweg, Cranmer, and Chandran (2010) for example, interpret the correlation scale as the effective spacing between magnetic flux tubes, as pertinent for estimates of transverse magnetic field oscillations to explain coronal Faraday rotation fluctuations. Here we interpret the correlation scale more generally, as a length along the LOS that exhibits the representative density oscillation crossing the sensing path, without assuming this is identical to the width of a white-light radial striation or a magnetic flux tube. The white-light striated streamer structures generally expand approximately radially. However, polarized white-light structure brightness relates to the electron number density, whereas the radio FF scales with rate of time-varying electron density. The density fluctuations are ubiquitous feature of the corona. See for example, the intriguing Figure 12 of DeForest et al. (2018), which shows coronal brightness variations at all azimuths, without a clear radial expansion of fluctuation scales or obvious segmentation into flux tubes. Here we use $L_{LOS}$ in the sense of a plasma slab thickness containing the density variation, whether manifested physically as acoustic waves or part of a turbulent cascade. There could more than one of such density-fluctuation slabs in the corresponding white-light streamer element.

In addition to the slab thickness scale, each model requires implementation of a fluctuation frequency, corresponding to the $d/dt$ in Equation 5, and an amplitude of the density fluctuations. The frequency implementation is carried out as rest-frame density waves in...
the mid-corona, and as advected spatial density oscillations in the extended corona. In both cases, the RMS density fluctuation is considered a fraction \( \epsilon \) of the mean electron density for a given SO, \( n_e(r) \).

### 3.2. Acoustic Wave Density Oscillations

Compressive waves propagating longitudinally at the speed of sound in a plasma slab will produce FF of the same frequency when the LOS and the density waves are in a common reference frame. This condition is approximately achieved in the inner mid-corona of streamer regions, where a typical acoustic wave speed of about 150 km s\(^{-1}\) is well above the LOS sky-projected speed due to spacecraft motion, and the solar wind outflow speed. The fluctuating electron density is treated as an oscillation of the form \( \delta n_e(t) = \delta n_e e^{-i \omega t} \), with angular frequency \( \omega \), following the method of Wexler et al. (2019b). Since the time derivative of the oscillation scales as \( \omega \delta n_e \), the power spectrum of electron density fluctuations, \( |\delta n_e(\omega)|^2 \), is related to the power spectrum of observed frequency fluctuations for a single slab as

\[
|\delta \hat{f}(\omega)|^2 = \frac{1}{4\pi^2} r_e^2 \lambda^2 \omega^2 |\delta n_e(\omega)|^2 L_{LOS}^2. \tag{6}
\]

Converting to the radio wavelength-normalized frequency measure fluctuation power spectrum, \( |\delta \hat{f}_m(\nu)|^2 \), with fluctuation frequency \( \nu \) as s\(^{-1}\), and multiplying by number of plasma slabs, \( R/L_{LOS} \), the relation between the power spectra of observational frequency measure fluctuations and the inferred underlying electron density fluctuations becomes

\[
|\delta \hat{f}_m(\nu)|^2 = r_e^2 \nu^2 |\delta n_e(\nu)|^2 L_{LOS} R \tag{7}
\]

in units Hz\(^2\)/m\(^2\).

In the acoustic wave regime, electron density fluctuation variance, \( \sigma_{n_e}^2 \) is obtained by numerical integration for frequency range \( [a, b] \) as

\[
\sigma_{n_e}^2 = \frac{1}{r_e^2 L_{LOS} R} \int_a^b \frac{|\delta \hat{f}_m(\nu)|^2}{\nu^2} d\nu. \tag{8}
\]

The RMS electron density fluctuation, \( \sigma_{n_e} \) is considered proportional to the underlying plasma electron number density, \( n_e \). When a suitable electron density model is applied, the fractional electron density fluctuation amplitude is found,

\[
\epsilon = \frac{\sigma_{n_e}}{n_e}. \tag{9}
\]

Note that the integrated measures already incorporate the power spectral shape, spectral index etc. The electron density variance is a \( 1/\nu^2 \)-scaled expression of the observed frequency fluctuations, and as such, is sensitive to the power-spectral organization. Using Equations 3 and 8, Equation 7 can be rewritten in terms of variances,

\[
\sigma_{FM(\nu)}^2 = r_e^2 \nu_c^2 \sigma_{n_e}^2 L_{LOS} R, \tag{10}
\]

where a “scaling frequency”, \( \nu_c \) (as s\(^{-1}\)), is observationally defined from the frequency fluctuation measure power spectra as

\[
\nu_c^2 = \frac{\int_a^b |\delta \hat{f}_m|^2 d\nu}{\int_a^b |\delta \hat{f}_m|^2 \nu^{-2} d\nu}. \tag{11}
\]
The scaling frequency is essentially implementing the $d/dt$ in Equation 5, when formatting in fluctuation variances. It is considered applicable directly for SO $< 1.6R_{\odot}$, where the speed of sound is much greater than solar wind outflow speed. Using the integrated power spectra from the data over SO range $1.50 - 1.57R_{\odot}$, $v_c = 0.0030 \pm 0.0003 \text{ s}^{-1}$.

Although the expression for FF variance does not overtly require knowledge of the power law spectral index,\(^1\) $\alpha$, inertial range power-law characteristics of the FF are incorporated into $v_c$. Whereas $v_c$ is obtained from integrated observational data as per Equation 11, it can also be shown in terms of idealized power spectra of form $\nu^{-\alpha}$ and frequency integration limits $\nu_a$, $\nu_b$ to be (see Wexler et al., 2019b)

$$v_c = \sqrt{\alpha + 1 \left( \frac{\nu_b^{-\alpha} - \nu_a^{-\alpha}}{\nu_a^{-\alpha-1} - \nu_b^{-\alpha-1}} \right)}.$$ \hspace{1cm} (12)

Observationally determined $v_c$ thus gives a sense of the effective spectral index. The classic Kolmogorov spectral index for FF, $\alpha = 2/3$ (Yakovlev, 2002), corresponds to $v_c = 0.0032$ Hz in the idealized case.

In the acoustic wave regime, we assume that Equation 10 is operative with $v_c$, $\epsilon$ and $L_{LOS}$, or at least the product $v_c\epsilon\sqrt{L_{LOS}}$, remaining constant during the observation interval. Expressing the integration length $R$ as parameter $r$ in units of solar radius ($r = R/R_{\odot}$, $R_{\odot} = 696000$ km), the acoustic wave contribution to the observed FF in the inner mid-coronal region should scale as

$$\sigma_{FM} = Kn_c\sqrt{r},$$ \hspace{1cm} (13)

where $K$ represents the consolidation of constants. Parameter assignments are addressed in Section 4.1.

In the extended corona, Equation 10 must be reframed to account for solar wind outflow speed. Higher wind speed advects density variations of given length scale more quickly across the sensing radio LOS; the higher frequencies of density fluctuation on the sensing radio path result in higher observational $\sigma_{FM}$.

### 3.3. Advected density disturbances regime

Beyond SO $\sim 2.5R_{\odot}$, the effect of solar wind outflow dominates the FF by advecting spatial fluctuations of electron density across the sensing LOS. For a given length scale of spatial density variations carried across the sensing path at outflow speed $V_{SW}$, we set

$$\frac{d}{dt} = V_{SW} \cdot \nabla$$ \hspace{1cm} (14)

in Equation 4. Allowing that the radial configuration of the system near the point of closest solar approach on the LOS is approximated by horizontal flow, with density variations of characteristic length scale $L_{RAD}$ crossing perpendicularly to the sensing LOS, we set $\nabla \sim 1/L_{RAD}$ so the time derivative expressed as the equivalent frequency is simply $V_{SW}/L_{RAD}$.

In the solar wind acceleration region, the RMS frequency measure fluctuation becomes

$$\sigma_{FM} = r e V_{SW} L_{RAD} n_e \epsilon \sqrt{L_{LOS} R}.$$ \hspace{1cm} (15)

\(^1\)Positive power law index notation is used, giving the spectral power in proportion to $\nu^{-\alpha}$. \hspace{1cm}
Considering the simple mass flux relation for outflow in a radially expanding system,

$$n_e V_{SW} = \frac{K_1}{r^2},$$

(16)

with constant \( K_1 \), substitution into Equation 15 yields the working model

$$\sigma_{FM} = K_2 r^{-3/2},$$

(17)

where \( K_2 = K_1 r_e \epsilon \sqrt{L_{LOS} R_\odot} \). Thus in the slow solar wind acceleration region of the corona, the model predicts an \( r^{-3/2} \) radial dependence of the frequency fluctuations.

The combination of Equations 10 and 17 constitutes the two-term model. In the next section the model is completed by specification of parameters, and fitting to the experimental data.

4. Analysis of the two-component model

4.1. Assignment of Parameters

We first consider the expression to represent the FF due to propagating density waves, Equation 13. An electron number density model is needed to establish the constant \( K \). A number of coronal electron density models are reviewed in Bird and Edenhofer (1990). Also see Wexler et al. (2019a). Streamer regions are known to have higher electron density than coronal hole regions of similar altitude. In our observation window, the Sun was in a moderately active state. Accordingly the streamers were fairly broadly distributed about the solar surface (Figure 1). Since the observations sampled both polar and lower heliolatitude regions, we explored available regional electron density data for the 2011 Sun.

The radio observations of Mercier and Chambe (2015) were taken over 2004–2011. We used their 2011 inferred electron density results over SO 1.4–1.6\( R_\odot \) from both polar and equatorial regions. Zucca et al. (2014) deduced electron densities from Type II radio bursts related to CME-induced shocks. Their modeled mid-coronal results over SO 1.4–2.4\( R_\odot \) for active regions and coronal holes were considered in our model development. The CME study by Susino, Bemporad, and Mancuso (2015) included dynamic radio spectra and analysis of SOHO/LASCO C2 white-light coronagraphs. The coronal electron density over SO 2–6\( R_\odot \) was determined by the method of polarized brightness inversion and plotted by heliolatitude and heliocentric distance. Their pre-CME event density results for 4 June 2011 were kindly made available to us. The data were binned into results bands for equatorial (0–30°), mid-latitudes (30–60°) and the polar (60–90°) regions.

In this work we used the Mercier–Hollweg electron density formula as described in Wexler et al. (2019b):

$$n_e(r) = \left[ \frac{65}{r^{5.94}} + \frac{0.768}{(r - 1)^{2.25}} \right] \times 10^{12}$$

(18)

in m\(^{-3}\). This model is coplotted with the 2011 data, and comparison plots of the classic Allen–Baumbach model (Allen, 1947; Bird and Edenhofer, 1990), and an extended corona model from Edenhofer et al. (1977), in Figure 7.

The Mercier–Hollweg density model matches fairly well the 2011 polar regions densities for SO < 1.5\( R_\odot \). For the extended corona, the Mercier–Hollweg model seems reasonable from 4\( R_\odot \), compared to Susino, Bemporad, and Mancuso (2015) findings and the
conventional models of Allen–Baumbach and Edenhofer. The most uncertain portion of the Mercier–Hollweg model is over $2–4R_\odot$, given the high variability in low- to mid-latitude densities. The density model trends in the high end of this range. With these limitations noted, the Mercier–Hollweg composite electron density model (Equation 18) appeared satisfactory for our purposes. Ideally, a coordinated study of white-light coronagraphs mapped to concurrent radio observations is desirable in future work.

The experimental data below SO $1.6R_\odot$, applied to Equation 13 with the above density model, determines $K = 1.013 \times 10^{-11}$. From Equations 9, 10, and 13

$$9.69 \times 10^{-12} = r_e v_e \epsilon \sqrt{L_{LOS} R_\odot}.$$  

(19)

Applying the observational result $v_e = 0.0030$ s$^{-1}$ from Section 3.2, the fractional density fluctuation, $\epsilon$, is found once the slab thickness, $L_{LOS}$, is specified.

The physical interpretation of $L_{LOS}$ as a slab thickness or correlation scale, is still an uncertain matter. Traditionally it is considered to be related to magnetic flux tube spacing, inversely proportional to the square root of background magnetic field strength, e.g. Spruit (1981). This concept seems especially suitable in the lower to middle corona, where the low plasma beta indicates magnetic dominance to coronal structuring could channel the waves and turbulent phenomena. In this report we use $L_{LOS}$ to signify the transverse length scale for a slab passing time-varying electron density across the sensing path. We use the magnetic structuring concept to help set initial scales, but depart from conventional approaches in that we do not demand that the density fluctuation increase in size as the magnetic structure radially expands in the extended corona.

Using the empirical formula of Hollweg, Cranmer, and Chandran (2010), $L_{LOS} = 4900$ km at SO $1.53R_\odot$. Cranmer used a correlation scale range of 4900–6700 km for SO $1.2–1.4R_\odot$ in early work (Cranmer and van Ballegooijen, 2005), then later revised the estimates to 2800–4300 km (Cranmer and van Ballegooijen, 2012). Estimates of density slab width are also obtained from imaging data. AIA 171 Å studies of density striations in the Comet Lovejoy tail indicated density structures with spacing of 4000–5000 km at SO $1.3R_\odot$ (Raymond et al., 2014). Radial density striations of about 5000 km width are noted in 171 Å EIT images of polar plumes up to about $1.2R_\odot$ (Ofman, Nakariakov, and DeForest, 1999).

Taking the working value for the correlation scale in the mid-corona to be 5000 km, we determine the fractional electron density fluctuation from the data over SO $1.50–1.57R_\odot$.
as $\epsilon = 0.019 \pm 0.002$. Miyamoto et al. (2014) provided an alternative interpretation for the same Akatsuki FF data used in the present study, estimating a fractional fluctuation range of 0.002 – 0.025 up to 2R⊙, increasing to a maximum of 0.3 by 5R⊙, then spanning 0.02 – 0.2 at higher helioaltitudes. Their method scaled the column density fluctuation to the sum of an estimated fixed sound speed and previously determined estimates of solar wind speed. Our study, in contrast, does not use the solar wind outflow speed to set the transverse length scale.

Wexler et al. (2019b) found $\epsilon = 0.017 \pm 0.002$ in the mid-corona using MESSENGER FF observations. The present value also compares favorably with 0.013 – 0.016 ± 0.001 estimates determined independently by radio burst methods (Mohan et al., 2019) for a limited mid-corona SO range ∼1.40 – 1.47R⊙. Mugundhan, Hariharan, and Ramesh (2017), studying type IIIb solar radio bursts, reported a fractional density fluctuations of 0.006 ± 0.002 over 1.6 – 2.2R⊙. They found the fluctuation fraction to increase in the extended corona following a power law $r^{0.33}$ such that a value of 0.01 is predicted by SO 3R⊙ and 0.02 by 20R⊙. Ofman, Nakariakov, and DeForest (1999) found fractional densities ranging 0.04 – 0.12 over SO range 1.02 – 1.17R⊙ in plumes over polar coronal holes. Hollweg, Cranmer, and Chandran (2010), evaluating a wave-heating model based on Helios data, showed $\epsilon \sim 0.023 – 0.031$ at SO 2R⊙, and rising an order of magnitude by SO 20R⊙. Krupar et al. (2018) evaluated density fluctuations from type III radio bursts measured by STEREO spacecraft, and more recently Krupar et al. (2020) gave results from the Parker Solar Probe first and second perihelia. Both studies revealed fractional density fluctuation of 0.06 – 0.07 in the extended corona. Interestingly, they presented a negative power law expression for the radial dependence of $\epsilon$, proportional to $r^{-0.55}$. Given the present uncertainly in the values and radial dependence of $\epsilon$, we maintain a conservative stance and use a fixed value, $\epsilon = 0.019$, for the remainder of this study. This is a minimum value, since radial elongation of the density fluctuation could be associated with an increased value of $\epsilon$, as discussed at the end of this section.

In the extended corona, the density disturbances advected across the LOS with the solar wind bulk flow are predicted to dominate the FF. After subtraction of the small acoustic wave contribution, the $\sigma_{FM}$ data beyond SO 2.5R⊙ were fitted using Equation 17 to find $K_2 = 49.7$. The CME-related FF burst around SO 12.8R⊙, shown by red symbols in Figures 4 and 6, was omitted from this fit. The final two-term fit is

$$\sigma_{FM} = 9.69 \times 10^{-12} n_e \sqrt{r} + 49.7 r^{-3/2}. \quad (20)$$

The summary curve for this two-term model is shown as the thin dark solid line in Figure 8. The first term dominates in the inner mid-corona, and potentially gives information to estimate the fractional fluctuation of electron density, as shown above. The second term incorporates information on solar wind outflow speed. Combining Equations 15 and 17, and using the above specification for $K_2$, we solve for solar wind speed as

$$V_{SW}(r) = \frac{49.7 L_{RAD}}{r_e n_e r^2 \sqrt{L_{LOS} R_⊙}}. \quad (21)$$

Values for $L_{LOS}$ and $L_{RAD}$ in the extended corona are needed to obtain the solar wind speed estimates. Here we assign $L_{LOS} = 10000$ km as a fixed value in the 2.5 – 20R⊙ SO range. This value is suggested by enhanced white-light solar eclipse images (Rušin et al., 2010) showing grouped streamer density structures of about that width at about 2R⊙, which maintain a reasonably constant dimension over the next solar radius or two as viewed on the sky plane. Also, using Hollweg’s model for the correlation scale (Hollweg, Cranmer, and...
Figure 8 Frequency measure fluctuation results. The extended corona findings scale generally with $r^{-3/2}$, while $\sigma_{FM}$ scales initially with $n_e \sqrt{r}$. The combined fit curve is shown as a dark thin solid line. Bursts believed related to CME activity at 12.8R⊙ (red x’s) deviate from the trend line of resting state frequency fluctuations.

Figure 9 Relative solar wind speeds (arbitrary scaling), based on Equation 21. The top curve is the result for both $L_{LOS}$ and $L_{RAD}$ being scaled as $r^{0.918}$ according to Hollweg’s formula. The middle curve has both parameters fixed at 10 000 km. The lower curve scales $L_{LOS}$ as $r^{0.918}$ but leaves $L_{RAD}$ constant. Each curve includes the $1/n_e r^2$ factor of Equation 21.

Chandran, 2010), $L_{LOS} = 3.35 \times 10^6 r^{0.918}$ in meters, yields about 10 000 km at SO 3.5R⊙. The question is whether the correlation scale, or thickness of the density variation slab, enlarges with increasing SO, as if it were tied to magnetically controlled flux tube width. Similarly, we consider whether the radial density variation scale, $L_{RAD}$ increases with SO. Let us consider the ratio $L_{RAD}/\sqrt{L_{LOS}}$ in Equation 21 evaluated with variables scaled in accordance with Hollweg’s formula versus constant 10 000 km in the extended corona, with fractional fluctuation parameter $\epsilon$ held constant.

The relative solar wind speeds are shown in Figure 9. The most realistic radial profile for relative speed is obtained from the ratio $L_{RAD}/\sqrt{L_{LOS}}$ being held constant (the blue curve), meaning the wind speed scales with $1/(n_e r^2)$. Note, however, that the length scales are not required to be fixed, only the ratio. The other two test cases, involving the flux tube expansion formula, give less promising results. With $L_{LOS}$ expanding and $L_{RAD}$ fixed (the black curve) result in loss of solar wind acceleration above $\sim 5R_{\odot}$. For the case $L_{LOS} = L_{RAD}$ set to scale with $r^{0.918}$ (red curve), the wind accelerates excessively at the higher heliocentric distances. These extreme cases could be compensated by increasing or decreasing $\epsilon$, respec-
tively. If $L_{RAD}$ expands as say, $r^{0.918}$, while $L_{LOS}$ is held constant, the wind acceleration is unrealistically large and sustained, so that is not a likely scenario.

For this work we tentatively apply the simple case of fixed values in the extended corona: $\epsilon = 0.019$, $L_{LOS} = 10,000$ km, and $L_{RAD} = 10,000$ km. Even if the individual variables are different, or are found to change with a radial dependence, we believe the ratio $L_{RAD}/(\epsilon \sqrt{L_{LOS}}) = 0.16$ is useful as a starting point in studying the slow solar wind speed profile using radio FF. Radially elongated density oscillations, $L_{RAD} > L_{LOS}$, are possible; $\epsilon$ would increase in proportion to $L_{RAD}$ for a given $\sigma_{FM}$. Studies combining concurrent observational techniques will be needed to constrain the individual parameters with confidence.

How can we reconcile $L_{LOS} = L_{RAD}$ when white-light coronagraphs so clearly indicate elongated density structures, like outgoing blobs (Sheeley et al., 1997) and other anisotropic density entities moving outward in the corona, e.g. DeForest et al. (2018)? Since the radio frequency is perturbed by the time rate of change in density, not the density itself, a moving bar-like density structure with tapered ends would appear elongated in the white light, while the radio FF responses will occur only as the tapered ends of the density bar cross the sensing path. $L_{LOS}$ signifies the transverse scale over which a given density perturbation applies, i.e. the bar width, and $L_{RAD}$ indicates the radial scale of the local density change region, i.e. the tapered ends. It is not unreasonable for these two quantities to be of similar value in the density bar structure. Thus radial density elongation is not excluded generally by our tentative $L_{LOS} = L_{RAD}$ condition. In the case of outflowing bar-like density structures, an aspect ratio of, say 10:1, would result in only a tenth the number of slab elements contributing FF on the sensing LOS at any given time. The observed $\sigma_{FM}$ would then imply increased density fluctuation RMS amplitude on the remaining slabs by a factor of $\sqrt{10}$, yielding $\epsilon = 0.06$ for our data. This value is consistent with recent results from studies of solar radio bursts using STEREO (Krupar et al., 2018) and Parker Solar Probe spacecraft (Krupar et al., 2020) observations.

### 4.2. Outflow speed of the solar wind

Frequency fluctuations in the extended corona encode solar wind outflow speed information. In the context of Equation 21, condensed with the parameter assignments above, the solar wind speed is found as

$$V_{SW} = \frac{1.11 \times 10^{17}}{n_e r^2},$$

in m s$^{-1}$. In this formulation, the idealized $\sigma_{FM}$ is already incorporated via the power law fit, Equation 17, and electron density model reflects wind acceleration when its negative power law exponent is greater than 2. Additionally, the observational $\sigma_{FM}$ data may be used to calculate the wind speed estimates based on Equation 15:

$$V_{SW} = \frac{2.24 \times 10^{15} \sigma_{FM}}{n_e \sqrt{r}},$$

where again $L_{RAD} = 10,000$ km, $L_{LOS} = 10,000$ km and $\epsilon = 0.019$ are considered constant over the SO range under consideration.

The results are shown together with solar wind speed estimates for other studies in Figure 10. Our solar wind outflow speeds deduced from trans-coronal radio FF observations are generally consistent with the speeds of white-light density blobs reported by Sheeley et al.
Figure 10  Solar wind outflow speeds, combined results of multiple studies. White-light blob speeds of Sheeley et al. (1997) shown as approximate scatter range, in gray. The optical flow speeds from DeForest et al. (2018) are shown in green. The outflow speeds based on our $r^{-3/2}$ model are plotted in the blue dashed line, and the inferred speeds using $\sigma_{FM}$ in Equation 23 are shown with blue circles. Recent radio work by Wexler et al. (2019b) is shown in brown and two-station frequency fluctuation speeds by Efimov et al. (2018) are shown with stars. The radio scintillation results of Imamura et al. (2014) are plotted with squares. The sonic point appears to lie between 4–5R$_{\odot}$.

(1997) and the more recent white-light coronagraphic results by DeForest et al. (2018). Allowing for a fractional error of 0.25 in both $L_{LOS}$ and $L_{RAD}$, and the 0.10 fractional error in $\epsilon$, the net fractional error in deduced $V_{SW}$ is 0.37, shown as blue stippling in Figure 10. This error band estimate is roughly similar to the spread in speeds from Sheeley et al. (1997), which is shown as the broad gray band in Figure 10. Prior radio FF studies (Wexler et al., 2019b; Efimov et al., 2018) also show generally comparable results. Radio scintillation analysis of this Akatsuki 2011 data set was reported by Imamura et al. (2014). The extended corona outflow speed results from their work are included on the plot. A representative sound speed curve from Wexler et al. (2019b) is coplotted, indicating the sonic point to be around 4–5R$_{\odot}$.

Adveected density disturbances dominate the FF when $V_{SW}/L_{RAD} > v_c$. Using $L_{RAD} = 10000$ km and $v_c = 0.003$ s$^{-1}$, the expected solar wind speed at the crossover into the advection-dominated regime is 30 km s$^{-1}$, well inside the sonic point and near the center of the smooth bend in the $\sigma_{FM}$ data around SO 2R$_{\odot}$ seen in Figure 8.

The mass-flux $r^{-3/2}$ model above gives remarkably close results to speeds obtained in white-light imaging. Other radio FF data sets show radial dependencies differing from the $-3/2$ power law (Efimov et al., 2008, 2013). Such differing results suggest departure from strict $r^2$ scaling of the coronal expansion regionally. A supraradial expansion, say, between polar open fields and streamers, would produce scaling greater than 3/2 index, while confined magnetic structures may show subradial expansion with the index $< 3/2$. More generally, scrutiny of the model leads to the intuition that the electrons and ions are not strictly confined to the “slabs”; some charged particles must be spreading beyond the idealized slab confines to conform with established electron density power laws on radial distance. These matters warrant further study of coronal magnetic structure, for example using Carrington rotation-specific 3D coronal models and high-resolution coronagraphs.
All FF models include scaling parameters requiring independent determination or estimation, e.g. the radial dependence of electron number density and a density disturbance length scale. In our simplified approach, a slab width (the LOS correlation scale), and an observationally determined scaling frequency are used without explicit reliance on the spectral index. In contrast, the spatial turbulence spectrum models are highly sensitive to the choice of spectral index, and still require density variation length scaling, such as that provided by an outer scale of turbulence (Bird et al., 2002; Efimov et al., 2004; Yakovlev, 2017).

The work presented here relies on a number of assumptions. The main model is constructed as simple system of stacked plasma slabs, each presenting a temporal sinusoidal density variation across the sensing LOS. The variance on the LOS adds linearly with the number of such slabs on the effective integration path. Slab thickness, as the physical interpretation of the LOS correlation length, was set to 10,000 km and kept constant over the extended corona SO range studied. This differed from prior studies which equated correlation scale to magnetic flux tube spacing, e.g. Hollweg et al. (1982), or to a single slab equal that varies with the radial density length scale (Miyamoto et al., 2014). In our model of RMS frequency fluctuation, an increasing integration length is enacted by the factor \( \sqrt{R} \), effectively incorporating more of the density fluctuation elements as the heliocentric distance to the LOS proximate point increases. The width scale of spatial density variation, \( L_{LOS} \), was set equal to the radial density variation scale, \( L_{RAD} \), but that equality is by no means a certainty. Radial elongations of the density fluctuation are possible with the model, with concordant changes in \( \epsilon \) and/or \( L_{LOS} \) implied. Further studies are needed to distinguish these variables observationally.

5. Conclusions

The Akatsuki spacecraft trans-coronal radio observations provided detailed frequency fluctuation (FF) information. In a stacked slab model of the corona, coronal electron density fluctuations were inferred directly from the FF in the inner mid-corona below SO 1.6R\(_\odot\). Interpreted as propagating compressive waves, FF data in that region constrain the RMS amplitude of electron density fluctuation. Acceleration of solar wind also produces FF, by advecting electron density inhomogeneities across the sensing LOS. The solar wind effect dominates the observed FF in the extended corona, SO > 2.5R\(_\odot\), and can be used to help characterize the solar wind acceleration.

In this study, the fractional density fluctuation, \( \epsilon \), was found to be 0.019 ± 0.002 in the mid-corona, and assumed to remain at about that level in the extended corona. In contrast, Miyamoto et al. (2014) interpreted these same FF data to show increasing amplitude of density fluctuations, up to about 0.3 by SO 5R\(_\odot\). Their model was different in that the length scales were linked to the sum of solar wind and acoustic speeds. In addition, only quasi-periodic wave trains were analyzed rather than a broader band of the FF power spectrum as done here. Our model assumed the amplitude of the density variation was essentially constant, but that the increasing solar wind speed increased the rate of change in spatial density across the sensing path, thereby increasing the FF variance in accordance with Equation 4.

The main conclusions are:

i) Spacecraft X-band radio frequency fluctuations probe the solar corona down into the mid-corona (1.4 – 2.5R\(_\odot\)) and provide information on electron density fluctuations.

ii) The heliocentric radial dependence of frequency measure fluctuations for observations taken through mid- and extended coronal regions is analyzed using a two-term model.
iii) Mid-coronal FF, prominently appearing despite very low solar wind speed, initially scales with $n_e \sqrt{r}$. The FF effect is attributed to acoustic or slow magnetosonic waves propagating across the radio ray path.

iv) Solar wind speed is incorporated into the second term of the coronal FF model. The RMS frequency fluctuation scales as a $-3/2$ power law index on radial distance, based on the assumed local coronal expansion following $r^2$. Other data sets may provide evidence of non-radial local coronal expansion, potentially providing insight into variations of regional magnetic structuring.

v) Using mass-flux principles, the radial profile of the solar wind acceleration may be deduced from the ratio of observed FF to electron density, subject to the assumption of a stable ratio of fractional density fluctuation and scaling lengths.

Frequency fluctuation analysis of trans-coronal radio observations should be useful in the continued study of coronal density waves and solar wind acceleration. Despite uncertainties in model parameters used here, a solar wind acceleration profile similar to those obtained by different techniques is an encouraging indicator of the method’s potential. Parker Solar Probe instrumentation is providing important new in-situ coronal data that should lead to better parameter constraints and refined models. Also, the inclusion of magnetic structuring from 3D coronal models and density analysis in high-resolution coronagraphs will enable more precise rendering of solar wind acceleration pathways and improved scaling of coronal structure.

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