On the possible mixing of the electron capture and the positron emission channels in nuclear decay

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Abstract

On the basis of the idea of mixing (interaction) between the electron capture and the positron emission channels in the $\beta^+$ decay in the cases when both channels are energetically allowed, we attempt to explain oscillations of the $K$-capture rates that were possibly seen in the recent experiment.

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1. Introduction

In the papers [1], [2] the authors observed the time-dependent oscillations with the period $T_{\text{osc}}^{\text{lab}} \sim 7$ s of the electron capture rates in the allowed Gamow–Teller (GT) decays of $^{140}$Pr and $^{142}$Pm. The preliminary result for $^{122}$I [2] shows $T_{\text{osc}}^{\text{lab}} \sim 6$ s. The authors measured the decay events in a sequence of measurements, each of them was performed with a single one-electron ion. These papers were attended by the theoretical article [3], where the authors tried to explain the effect in the framework of the scheme of the neutrino oscillations. This idea became an object of a lively discussion.

Trying to explain the oscillations seen in the experiment [1] in the framework of the more-or-less standard nuclear physics, we turn our attention to another scenario. In the experiment [1] the authors observed the transition rate with respect to the electron capture (EC) only. The cases of $\beta^+$ decay were out of the “window” of observations. However, the possible coupling of the two above-mentioned channels, due to a weak interaction between them, may lead to the oscillations of the EC rate. Below we consider this possibility qualitatively.

2. Phenomenological approach

First, we remind briefly the standard picture of the neutrino oscillations in the $\beta$ decay. The neutrino born in the $\beta$ decay is the electron one, $\nu_e$. However, the state $|\nu_e\rangle$ is not the

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eigenstate of the total mass operator, thus it is not a stationary one, if there exists a mixing between the electron neutrino $\nu_e$ and muon neutrino $\nu_\mu$. In the presence of such a mixing, the eigenstates are the $|\nu_1\rangle = |\psi_1\rangle$ and $|\nu_2\rangle = |\psi_2\rangle$ ones, each of them is being a combination of $|\nu_e\rangle = |\varphi_1\rangle$ and $|\nu_\mu\rangle = |\varphi_2\rangle$, while the states $|\nu_e\rangle$ and $|\nu_\mu\rangle$ are not the stationary ones. This leads to the time oscillations of the value $\rho\langle\nu_e|\nu_e\rangle_t$ due to the transitions $\nu_e \rightarrow \nu_\mu$ and inverse ones.

Thus, in the presence of coupling we have

$$|\varphi_1\rangle = \cos \vartheta |\psi_1\rangle + \sin \vartheta |\psi_2\rangle,$$

$$|\varphi_2\rangle = -\sin \vartheta |\psi_1\rangle + \cos \vartheta |\psi_2\rangle,$$

$$|\varphi\rangle = ||M|| \cdot |\psi\rangle,$$

$$||M|| = \begin{vmatrix}
\cos \vartheta & \sin \vartheta \\
-\sin \vartheta & \cos \vartheta
\end{vmatrix},$$

(1)

where $|\psi_1\rangle$ and $|\psi_2\rangle$ are the eigenstates with the account for mixing.

The inverse transformation looks as follows:

$$|\psi\rangle = ||M^{-1}|| \cdot |\varphi\rangle,$$

$$||M^{-1}|| = \begin{vmatrix}
\cos \vartheta & -\sin \vartheta \\
\sin \vartheta & \cos \vartheta
\end{vmatrix},$$

(2)

The amplitude $A_{k_i} = \rho(\varphi_i|\varphi_k)_t$ of transformation of the state $|\varphi_i\rangle_0$ into the state $|\varphi_k\rangle_t$ is

$$A_{k_i}(t, \vartheta, \Delta) = \sum_j M_{k_j}(\vartheta)S_{jj}(t)M^{-1}_{ji}(\vartheta),$$

(3)

where

$$||S(t)|| = \begin{vmatrix}
e^{-i\Delta t/\hbar} & 0 \\
0 & e^{i\Delta t/\hbar}
\end{vmatrix},$$

(4)

is the diagonal time-evolution matrix of the stationary states (we have omitted here the insufficient common phase). Here $\Delta = (E_1 - E_2)/2$, while $E_1$ and $E_2$ are the energies of stationary states $|\psi_1\rangle$ and $|\psi_2\rangle$. Thus, the matrix $||A||$ has the form

$$||A|| = \begin{vmatrix}
\cos(\Delta t/\hbar) - i\sin(\Delta t/\hbar) \cos(2\vartheta) & i\sin(\Delta t/\hbar) \sin(2\vartheta) \\
i\sin(\Delta t/\hbar) \sin(2\vartheta) & \cos(\Delta t/\hbar) + i\sin(\Delta t/\hbar) \cos(2\vartheta)
\end{vmatrix}.$$  

(5)

So, we have

$$|A_{11}|^2 = |A_{22}|^2 = 1 - \sin^2(2\vartheta) \sin^2\left(\frac{\Delta t}{\hbar}\right),$$

$$|A_{12}|^2 = |A_{21}|^2 = \sin^2(2\vartheta) \sin^2\left(\frac{\Delta t}{\hbar}\right),$$

$$|A_{11}|^2 + |A_{12}|^2 = |A_{22}|^2 + |A_{21}|^2 = 1.$$  

(6)

For $\nu_e - \nu_\mu$ oscillations the last equation in (6) is nothing but the unitarity relation.

Here we come to the difference between the $\nu_e - \nu_\mu$ and EC $- \beta^+$ oscillations. Instead of $|\varphi_1\rangle = |\nu_e\rangle$ and $|\varphi_2\rangle = |\nu_\mu\rangle$ we have now $|\varphi_1\rangle = |EC\rangle$ and $|\varphi_2\rangle = |\beta^+\rangle$, that correspond to the
transitions $|Z, N⟩ + e^−(1s) \rightarrow |Z - 1, N + 1⟩ + ν_e$ and $|Z, N⟩ + e^−(1s) \rightarrow |Z - 1, N + 1⟩ + e^+(1s) + e^− + ν_e$ (in the last case the $1s$-electron is a spectator). The coupling between the states $|EC⟩ = |φ_1⟩$ and $|β^+⟩ = |φ_2⟩$ leads to their mixing and to the energy splitting of the corresponding eigenstates $|ψ_1⟩, |ψ_2⟩$, as well as to the time dependence of the Fermi function for the allowed Gamow–Teller operator. In the case of the $ν_e - ν_μ$ oscillations at $t = 0$, we have the electron neutrino only, while the muon neutrino appears only as a result of oscillations. In our case we have not only the depopulation of the $|EC⟩$ channel due to the $|EC⟩ → |β^+⟩$ oscillations, but also the population of this channel due to transformations $|β^+⟩ → |EC⟩$. The $|β^+⟩$ states appear not only due to the $|EC⟩ → |β^+⟩$ oscillations, but are supplementedly settled also in the $β^+$ decay. The probabilities of the electron capture and $β^+$ decay are different, therefore oscillations in both channels arise. In this way, we obtain the formulae for the transition rates in both channels:

$$w_{EC}(t) = w_{EC}^0 \cdot |A_{11}|^2 + w_{β^+}^0 \cdot |A_{12}|^2 =$$

$$= w_{EC}^0 \left[ 1 + B \sin^2 \left( \frac{Δt}{ℏ} \right) \right], B = \frac{w_{β^+}^0 - w_{EC}^0}{w_{EC}^0} \sin^2(2θ);$$

$$w_{β^+}(t) = w_{β^+}^0 \cdot |A_{22}|^2 + w_{EC}^0 \cdot |A_{21}|^2 =$$

$$= w_{β^+}^0 \left[ 1 + D \sin^2 \left( \frac{Δt}{ℏ} \right) \right], D = \frac{w_{EC}^0 - w_{β^+}^0}{w_{β^+}^0} \sin^2(2θ). \quad (7)$$

where $w_{EC}^0$ and $w_{β^+}^0$ are the transition rates for the electron capture as well as for $β^+$ decay in the absence of mixing.

For the allowed Gamow–Teller transition we have

$$w_{EC}^0 = \frac{m_e^5 c^4}{2π^3 ℏ^2} G_A^2 B(GT; J_i → J_f) \cdot 2π^2 \sum_i Ψ_i^2(0) \left( \frac{E_ν}{m_e c^2} \right)^2,$$

$$w_{β^+}^0 = \frac{m_e^5 c^4}{2π^3 ℏ^2} G_A^2 B(GT; J_i → J_f) \cdot f(E_β, Z). \quad (8)$$

In $(8)$ $G_A$ is the effective axial vector constant in nuclei (see details in [1]), $B(GT; J_i → J_f)$ is the reduced transition probability for the Gamow–Teller operator, $f(E_β, Z)$ is the integrated Fermi function for the allowed $β^-$ decay, $E_β$ is the maximal kinetic energy of the positron in the transformation $|Z, N⟩ → |Z - 1, N + 1⟩$, $E_ν$ is the neutrino energy, while the densities of the $K$-electrons at zero $Ψ_i^2(0) (i = 1, 2, ...) that contribute into the $K$-capture rate for the allowed transitions are in $(ℏ/m_e c)^{-3}$ units. For the one-electron ion $i = 1$ only in $(8)$. We see from Eq. $(7)$ that $w_{EC}(t) + w_{β^+}(t) = w_{EC}^0 + w_{β^+}^0 = λ$, where $λ$ is the decay constant in the exponential law $e^{-λt}$, while the counting rates are

$$\frac{dn_{EC}}{dt} = w_{EC}(t) N_0 e^{-λt}, \quad \frac{dn_{β^+}}{dt} = w_{β^+}(t) N_0 e^{-λt}, \quad \frac{d(n_{EC} + n_{β^+})}{dt} = λ N_0 e^{-λt}. \quad (9)$$

The equality $w_{EC}(t) + w_{β^+}(t) = λ$ is the unitarity relation in our case. We see that the total transition rate (in both channels) does not depend on time, thus we again have the exponential law for the decay of the parent nucleus. Taking the values of nuclear masses from [5] and using
the beta-decay Tables [6] for the values of \( f(E_\beta, Z) \) and \( \Phi_2^2(0) \) (for the one-electron atom we took a half of the density of the \( K \)-shell in the neutral atom as the single-particle functions of the \( 1s \) electron are in practice the same in the one-electron and neutral atoms), we obtain for the decay of the one-electron \(^{142}\text{Pm}\) \( \frac{w^0_{\text{EC}}}{w^0_{\beta^+}} \approx 0.12 \). Thus,

\[
\begin{align*}
    w_{\text{EC}}(t) &= w^0_{\text{EC}} \left[ 1 + 7.33 \sin^2(2\vartheta) \sin^2 \left( \frac{\Delta t}{\hbar} \right) \right], \\
    w_{\beta^+}(t) &= w^0_{\beta^+} \left[ 1 - 0.88 \sin^2(2\vartheta) \sin^2 \left( \frac{\Delta t}{\hbar} \right) \right].
\end{align*}
\] (10)

Note that \( 7.33 w^0_{\text{EC}} = 0.88 w^0_{\beta^+} \). We see from (10) that the situation in \(^{142}\text{Pm}\) is favorable for oscillations in the EC channel due to its small partial width.

For the decay of the one-electron \(^{140}\text{Pr}\) we obtain \( \frac{w^0_{\text{EC}}}{w^0_{\beta^+}} \approx 0.41 \). So

\[
\begin{align*}
    w_{\text{EC}}(t) &= w^0_{\text{EC}} \left[ 1 + 1.44 \sin^2(2\vartheta) \sin^2 \left( \frac{\Delta t}{\hbar} \right) \right], \\
    w_{\beta^+}(t) &= w^0_{\beta^+} \left[ 1 - 0.59 \sin^2(2\vartheta) \sin^2 \left( \frac{\Delta t}{\hbar} \right) \right].
\end{align*}
\] (11)

From the analysis of the experimental data [1] for \(^{142}\text{Pm}\) and \(^{140}\text{Pr}\) one may easily conclude on the values of the parameters \( \Delta \) and \( \vartheta \) entering Eqs. (10) and (11). The pre-exponential factor in the counting rates was defined in [1] as \( [1 + A \cos(\omega_{\text{osc}} t + \phi)] \), where \( A = 0.23(4), T^\text{lab}_{\omega_{\text{osc}}} = 2\pi/\omega_{\text{osc}} = 7.10(22) \) s for \(^{142}\text{Pm}\) and \( A = 0.18(3), T^\text{lab}_{\omega_{\text{osc}}} = 7.06(8) \) s for \(^{140}\text{Pr}\). It is better to work in the system where the \(^{142}\text{Pm}\) and \(^{140}\text{Pr}\) nuclei are at rest. Here \( T^\text{osc}_{\omega_{\text{osc}}} \approx 7/\gamma \approx 5 \) s, where \( \gamma = 1.43 \) is the corresponding Lorentz factor [2]. The phase \( \phi \) was not defined in [1] as the experimental data are absent at small values of \( t \). Our approach leads to \( \phi = \pi, A = B/(2 + B) \approx B/2 \) and \( \omega_{\text{osc}} = 2\Delta/\hbar \) (see Eq. (7)), while by using Eqs. (10), (11) and the values of \( A \) shown above we obtain \( \vartheta \approx 8^\circ, \Delta = \pm 0.407(13) \cdot 10^{-15} \) eV for \(^{142}\text{Pm}\) and \( \vartheta \approx 16^\circ, \Delta = \pm 0.410(5) \cdot 10^{-15} \) eV for \(^{140}\text{Pr}\). The patterns of oscillations based on the above-mentioned discourse are shown in Fig. 1 for the decay of \(^{142}\text{Pm}\) and in Fig. 2 for the decay of \(^{140}\text{Pr}\). Here in both cases \( w^0_{\beta^+} \) are larger than \( w^0_{\text{EC}} \). As a result, in the presence of oscillations the transition rates for the electron capture are higher than those in the absence of oscillations. At the same time, the situation is opposite for the \( \beta^+ \) decays. One can easily consider the corresponding integral effect. Let us introduce the total numbers of decays in the corresponding channels as

\[
N(\text{EC}) = N_0 \int_0^\infty w_{\text{EC}}(t) e^{-\lambda t} dt, \quad N(\beta^+) = N_0 \int_0^\infty w_{\beta^+}(t) e^{-\lambda t} dt.
\] (12)

Then, we can easily obtain

\[
\frac{N(\text{EC})}{N(\beta^+)} = \frac{w^0_{\text{EC}}}{w^0_{\beta^+}} \cdot \left\{ \frac{1 + \frac{w^0_{\beta^+} - w^0_{\text{EC}}}{2 w^0_{\text{EC}}} \sin^2(2\vartheta) \frac{1}{1 + \left( \frac{\Delta}{\hbar} \right)^2}}{1 + \frac{w^0_{\beta^+} - w^0_{\text{EC}}}{2 w^0_{\beta^+}} \sin^2(2\vartheta) \frac{1}{1 + \left( \frac{\Delta}{\hbar} \right)^2}} \right\}.
\] (13)
In cases of interest, when one can observe several oscillations at the time interval \( \tau = 1/\lambda \) we have \( \Delta/\hbar \gg \lambda \) (\( \lambda = 0.017 \) s\(^{-1} \) and \( \Delta/\hbar = 0.67 \) s\(^{-1} \) for \(^{142}\)Pm\)). Then we have

\[
N(\text{EC})/N(\beta^+) = \frac{w_{\text{EC}}^0}{w_{\beta^+}^0} \left[ \frac{1 + w_{\beta+}^0 - w_{\text{EC}}^0}{2 w_{\text{EC}}^0} \sin^2(2\vartheta) \right] \left[ 1 + \frac{w_{\text{EC}}^0 - w_{\beta+}^0}{2 w_{\beta+}^0} \sin^2(2\vartheta) \right]. \tag{14}
\]

Formula (14) can be easily obtained if we substitute \( \sin^2(\Delta t/\hbar) \) in (7) by its average value of \( 1/2 \), the averaging is over the time interval \( \delta t \) more than \( \hbar/\Delta \). This is in some sense equivalent to averaging over the ensemble of initial nuclei, that are formed during the time interval greater than \( \hbar/\Delta \), as the time counter is switched on for each nucleus in the moment of its formation.

For the decay of the one-electron \(^{140}\)Pr we have \( N(\text{EC})/N(\beta^+) = 1.34 \cdot w_{\text{EC}}^0/w_{\beta^+}^0 \). For the neutral atom of \(^{140}\)Pr the ratio \( w_{\text{EC}}^0/w_{\beta^+}^0 \) should be twice as much as for the one-electron ion, i.e. it should be equal 0.82. If we consider the electron capture from higher s-orbits, this value by using [6] is 0.97. For neutral \(^{140}\)Pr the values of \( w_{\beta+}^0 \) and \( w_{\text{EC}}^0 \) are close to each other, and we can see from the Eq.(14) that \( N(\text{EC})/N(\beta^+) \approx w_{\text{EC}}^0/w_{\beta+}^0 \cdot 1.06 \). The experimental data on the ratio of the K-capture to the \( \beta^+ \) decay of the neutral \(^{140}\)Pr are rather vague. In refs. [7] – [10], they are 0.897, 0.74, 0.9 and 0.85 correspondingly, giving the average value and the standard deviation equal to 0.846(75). This value should be compared to the theoretical value \( N(\text{EC}(1s))/N(\beta^+) = 0.82 \cdot 1.06 = 0.87 \). Thus, the accuracy of experimental data is insufficient to make definite conclusion on the enhancement of the EC(1s) rate as compared to the standard calculations, which do not include the mixing of two decay channels. For \(^{142}\)Pm with two electrons on the K shell (or for the neutral atom), formula (14) shows the increase of \( N(\text{EC})/N(\beta^+) \) to be about 16% as compared to the standard calculations (i.e. \( w_{\text{EC}}^0/w_{\beta^+}^0 \approx 0.257 \), see [11]). At the same time, the experimental value of this ratio is equal to 0.297(45) [12]. So one can see the increase of the ratio \( N(\text{EC})/N(\beta^+) \) as compared to the theoretical value \( w_{\text{EC}}^0/w_{\beta^+}^0 \) obtained without mixing of final states, the difference is in accordance with our prediction, though the experimental errors are large. The previously mentioned estimations used the values of \( \vartheta \) from the one-electron ions.

The systematics of the ratios \( (w_{\text{EC}}^0(1s)/w_{\beta^+}^0)_{\text{exp}}/(w_{\text{EC}}^0(1s)/w_{\beta^+}^0)_{\text{th}} \) shown in Fig. 3 demonstrates that they often differ from the unity up to 10%, the deviations are in both sides. Our discourse leads to a small tendency for the above-mentioned ratios to be a bit smaller than the unity at \( (w_{\text{EC}}^0/w_{\beta^+}^0)_{\text{th}} > 1 \), and to be a bit larger than the unity at \( (w_{\text{EC}}^0/w_{\beta^+}^0)_{\text{th}} < 1 \).

3. Microscopical evaluation

Here we try to explain the above-discussed picture by using certain qualitative arguments. Suppose that there exists some additional interaction \( H_w \), which couples the EC and \( \beta^+ \) channels. First, we determine the magnitudes of the corresponding matrix elements using the values of \( \Delta \) and \( \vartheta \) shown above. These matrix elements may be easily determined from secular equation obtained in the two-level scheme with the \( |\text{EC}\rangle \) and \( |\beta^+\rangle \) as basic func-
tions. Including the interaction $H_w$, we introduce the quantities $E_{\beta} = V_{\beta^+,\beta^+} = \langle \beta^+ | H_w | \beta^+ \rangle$, $E_{EC} = V_{EC,EC} = \langle EC | H_w | EC \rangle$, as well as $V_{EC,\beta^+} = \langle EC | H_w | \beta^+ \rangle$. Then one can easily obtain

$$\delta \equiv \frac{\langle EC | H_w | EC \rangle - \langle \beta^+ | H_w | \beta^+ \rangle}{2} = \Delta \cos(2\vartheta), \quad \Delta = (E_1 - E_2)/2; \quad (15)$$

At the same time,

$$V_{EC,\beta^+} = \langle EC | H_w | \beta^+ \rangle = \Delta \sin(2\vartheta), \quad (16)$$

$V_{EC,\beta^+} = 0.11 \cdot 10^{-15}$eV for $^{142}$Pm and $V_{EC,\beta^+} = 0.22 \cdot 10^{-15}$eV for $^{140}$Pr.

We see from (15) and (16) that corresponding matrix elements are very small, of the order of $10^{-16}$eV. They may arise due to the weak interaction stipulated by both the neutral and charged weak currents. The matrix element $V_{EC,\beta^+}$ is graphically shown in Fig. 4, while the $V_{\beta^+,\beta^+}$ is represented in Fig. 5. At the same time, one can put the value of $V_{EC,EC}$ to be equal to zero, because only the higher-order diagrams contribute here.

First, consider the matrix element $V_{EC,\beta^+}$ that is shown in Fig. 4, replacing the positrons by electrons with inverse momenta. In this case we have the $\nu - e$ interaction with the matrix element that accounts for both $Z$ and $W$ bosons and looks as follows [13]:

$$M = \frac{G}{\sqrt{2}} \langle \bar{u}_{e2} | g_L \gamma_\alpha (1 + \gamma_5) + g_R \gamma_\alpha (1 - \gamma_5) | u_{e1} \rangle \langle \bar{u}_{\nu2} | \gamma^\alpha (1 + \gamma_5) | u_{\nu1} \rangle \approx \frac{G}{\sqrt{2}} \langle \bar{u}_{e2} | \gamma_\alpha + \frac{1}{2} \gamma_\alpha \gamma_5 | u_{e1} \rangle \langle \bar{u}_{\nu2} | \gamma^\alpha (1 + \gamma_5) | u_{\nu1} \rangle. \quad (17)$$

In (17), we have $g_L = \frac{1}{2} + \sin^2 \Theta_W, g_R = \sin^2 \Theta_W$. Here $\Theta_W$ is the Weinberg angle ($\sin^2 \Theta_W \approx \frac{1}{4}$), while $G = G_V / \cos \Theta_C$, where $\Theta_C$ is the Cabibbo angle ($\cos \Theta_C = 0.974$). From the experiments on the investigation of the superallowed $\beta$-transitions between the isoanalog states of nuclei, it follows [14] that the weak vector coupling constant $G_V = 1.395 \cdot 10^{-49}$erg·cm$^3$ = 87.08 eV·fm$^3$. Below we consider the contact type of the interaction between the weak currents that enter formula (17), use the plane waves for the unbounded leptons and take the wave function of the $1s$ electron (this function includes the angular part $Y_{00}$) in the non-relativistic form

$$\varphi_{1s}(r) = \frac{1}{\sqrt{\pi}} a^{3/2} e^{-ar} \quad (a = \frac{Z m_e e^2}{\hbar^2} = \alpha Z / \lambda_e = 1.154 \cdot 10^{-3}$ fm$^{-1}$ for $^{142}$Pm), \quad (18)$$

where $\lambda_e$ is the Compton wavelength of the electron. The calculations were also performed under the assumption of the uniform angular distributions of the entering fast leptons. In addition, we took into account that the process of the $\beta$ decay is mediated by left currents and the energies of positrons are rather high. Thus, we considered the positrons as having the right spirality. We also considered that the average momentum of the bounded $1s$ electron is much less than $(m_e c)$, and we neglected the corresponding contributions. Then we can schematically represent the $V_{EC,\beta^+}$ as.
\[ V_{EC, \beta^+} \sim 2G \frac{8 \sqrt{\pi}}{V_{eff}^{3/2}} \frac{a^4}{(k^2 + a^2)^{3/2}} \cdot \frac{1}{a^{3/2}} \left[ 1 - (k^2 + a^2)^{3/2} \frac{r_{max}}{2ak} e^{-ar_{max}} \sin(kr_{max} + \gamma) \right. \\
\left. + \frac{(k^2 + a^2)^2}{2ak} e^{-ar_{max}} \sin(kr_{max} + \sigma) \right], \tag{19} \]

where
\[ \gamma = \arctan \left( \frac{k}{a} \right), \quad \sigma = \arctan \left( \frac{2ak}{a^2 - k^2} \right). \]

In (19), \( V_{eff} \) is the effective volume for the leptons in the continuum, \( V_{eff} \approx \frac{1}{3} \pi r_{max}^3 \), where \( r_{max} \) is of the order of some units of \( 1/a \), \( r_{max} \sim C/a \), while \( k = \frac{1}{2} p \), where \( p = p(\nu_e, EC) - p(\nu_e, \beta^+) - p(e^+, \beta^+) \).

Note that really the spectra of both positron and neutrino, which are produced in the \( \beta^+ \) decay, are not monochromatic. Averaging the values of \( k \) over the corresponding distributions, as it was in the calculations of the integrated \( \beta \) decay Fermi function \( f(E, Z) \) and supposing the uniform angular distributions of the unbound leptons, we obtain for \(^{142}\text{Pm}\) the value \( \bar{p} \sim \sqrt{p^2} \sim 5.5 \text{ MeV}/c \), which corresponds to \( \bar{k} \sim 3.0 \cdot 10^{-2} \text{fm}^{-1} \).

We mention that the averaging of the second and third terms in the right-hand side of (19) leads to their vanishing. In any case, their contribution may be neglected by the absolute value as compared to the contribution of the first term at \( r_{max} \geq 8/a \). So, the value of \( C \sim 8 \) defines the upper limit of integration, and in this way the volume of the interaction (the normalization volume for the unbound leptons that is of the order of the volume of the neutral atom).

Now we come to the evaluation of the matrix element \( V_{\beta^+, \beta^+} \). The corresponding diagrams are shown in Fig. 5. The diagram (a) (\( e - \nu \) interaction) is represented by the formula (17), while the \( e - e \) interaction, that is mediated by the \( Z \) boson only, is shown in the diagram (b). It is described by the matrix element

\[ M = \frac{G}{\sqrt{2}} \langle \bar{u}_{e2} | g_L \gamma_\alpha (1 + \gamma_5) + g_R \gamma_\alpha (1 - \gamma_5) | u_{e1} \rangle \times \langle \bar{u}_{e4} | g_L \gamma^\alpha (1 + \gamma_5) + g_R \gamma^\alpha (1 - \gamma_5) | u_{e3} \rangle \approx \frac{G}{\sqrt{2}} \frac{1}{4} \langle \bar{u}_{e4} | \gamma_\alpha \gamma_5 | u_{e1} \rangle \langle \bar{u}_{e4} | \gamma^\alpha \gamma_5 | u_{e3} \rangle, \tag{20} \]
as here \( g_L = -\frac{1}{2} + \sin^2 \Theta_W \) and \( g_R = \sin^2 \Theta_W \).

The formula for the matrix elements \( V_{\beta^+, \beta^+} \) of the interaction corresponding to the diagrams shown in Fig. 5 looks as follows:

\[ V_{\beta^+, \beta^+} \sim G \frac{16}{V_{eff} (k^2 + 4a^2)^2} \left[ 1 - (k^2 + 4a^2)^{3/2} \frac{r_{max}}{4ak} e^{-2ar_{max}} \sin(kr_{max} + \gamma_1) \right. \\
\left. + \frac{(k^2 + 4a^2)^2}{4ak} e^{-2ar_{max}} \sin(kr_{max} + \sigma_1) \right], \tag{21} \]
where
\[ \gamma_1 = \arctan \left( \frac{k}{2a} \right), \quad \sigma_1 = \arctan \left( \frac{4ak}{4a^2 - k^2} \right). \]

As a result, we obtain for \(^{142}\text{Pr} V_{\text{EC},\beta^+} \approx 0.056 \cdot 10^{-15} \text{ eV} \) and \( V_{\beta^+^+,\beta^+} \approx 1.4 \cdot 10^{-15} \text{ eV} \) (\( \delta \approx 0.7 \cdot 10^{-15} \) eV), which may be compared with the results of Eqs. (15) and (16), where \( V_{\text{EC},\beta^+} = 1.11 \cdot 10^{-15} \) eV and \( V_{\beta^+^+,\beta^+} \approx 1.8 \cdot 10^{-15} \) eV. Note that the mixing angle \( \vartheta \) for \(^{140}\text{Pr} (Q_{\text{EC}} \approx 3.4 \text{ MeV}) \) is larger than for \(^{142}\text{Pr} (Q_{\text{EC}} \approx 4.8 \text{ MeV}) \). This fact finds an evident explanation if we look at formula (19), where \( V_{\text{EC},\beta^+} \approx 1/(a^2 + \bar{k}^2)^2 \) (\( \vartheta \) is approximately proportional to \( V_{\text{EC},\beta^+} \) at small \( \vartheta \), while \( \bar{k} \) is larger for \(^{142}\text{Pr} \), than for \(^{140}\text{Pr} \)). At the same time, it is difficult to understand, why the periods of oscillations \( T_{\text{osc}} \) are very close to each other in the cases of decay of \(^{142}\text{Pr} \) and \(^{140}\text{Pr} \).

4. Hyperfine interaction

Here we evaluate the energy splitting of levels due to magnetic fields that exist in the accelerator as a possible source of oscillations. We also evaluate the effects of the hyperfine interaction.

The magnetic moments of the ground-state \( 1^+ \) levels of \(^{140}\text{Pr} \) and \(^{142}\text{Pr} \) are unknown by now. However, it follows from the single-particle scheme that these odd–odd nuclei have the configuration \{\( p2d_{5/2}, n2d_{5/2}; I^\pi = 1^+ \). The average value of the magnetic moment of the proton on the orbit \{\( p2d_{5/2} \) obtained from the experimental data on the proton-odd nuclei \(^{141}\text{Pr} \) and \(^{143}\text{Pm} \) is \( \sim 4.0 \mu_N (\mu_N = e\hbar/2m_N c) \), while the average value of the magnetic moment of the neutron on the orbit \{\( n2d_{5/2} \) is \( \sim 1.0 \mu_N \); this value is determined from the neutron-odd nuclei \(^{139}\text{Ce} \) and \(^{141}\text{Nd} \). The above-mentioned evaluations used the fact that the magnetic moment of the state \( |n_{\text{odd}}s = 1; J = j \rangle \) for the lowest seniority “s” does not depend on \( n_{\text{odd}} \). In the case of the two-particle configuration \( |j_1j_2I \rangle \) we have the following formula for the gyromagnetic ratio of this state:

\[
ge_I = \frac{g_1 + g_2}{2} + \frac{g_1 - g_2}{2} \cdot \frac{j_1(j_1 + 1) - j_2(j_2 + 1)}{I(I + 1)}, \quad (22)
\]

where \( g_1 \) and \( g_2 \) are the gyromagnetic ratios for the states \( |j_1 \rangle \) and \( |j_2 \rangle \). In this way we have \( \mu_I (I^\pi = 1^+; ^{140}\text{Pr}) \approx \mu_I (I^\pi = 1^+; ^{142}\text{Pm}) \approx 2.3 \mu_N \). For the the magnetic field \( H \approx 1 \text{ T} \), we obtain the magnitude of the interaction of nuclear magnetic moment with the external field equal to \( \sim \mu_I \cdot H = 0.7 \cdot 10^{-7} \text{ eV} \). At the same time, the interaction of the electron spin with the magnetic field is much stronger, \( \sim \mu_B \cdot H = 0.6 \cdot 10^{-4} \text{ eV} (\mu_B = e\hbar/2m_e c) \). However, there exists also the interaction between the electron and the nucleus spins. For an electron on the \( 1s \) orbit we have

\[
E_{I,s,F} = \frac{8}{3} \mu_B \cdot \mu_I \cdot \frac{F(F + 1) - I(I + 1) - s(s + 1)}{I} \cdot a^3, \quad (23)
\]

where \( I = 1 \) and \( s = 1/2 \) are spins of the nucleus and electron respectively, while \( F = I \pm 1/2 \) is the total spin of the one-electron ion. The magnitude of (23) is equal to \( 0.3 \cdot [F(F + 1) - 11/4] \)}
eV that is much larger than the interactions of magnetic moments of the electron and of the nucleus with the magnetic field. Thus, the two spins are strictly coupled to each other. By using formula (22) we find for \( F = 3/2 \) the value of \( \mu_{F=3/2} = \mu_I (I = 1^+) + \mu_B \approx \mu_B \) while for \( F = 1/2 \) we obtain \( \mu_{J=1/2} = 2/3 \mu_I (I = 1^+) - 2/3 \mu_B \approx -2/3 \mu_B \). So the corresponding energy splitting due to the magnetic field is of the order of \( 10^{-4} \) eV, which is much greater than characteristic magnitude of the effect \(( \sim 10^{-16} )\) eV seen in the experiment [1]. However, we mention here the paper [15], where it was shown that under certain conditions one may expect modulation of the EC decay rate due to the resonance multiphoton transitions between the magnetic substates of the ground \( F = 1/2 \) state of \(^{140}\text{Pr}^{58+} \), or \(^{142}\text{Pm}^{60+} \).

Another source of splitting in the one-electron ion may be the weak interaction in the neutral channel between the electron and the nucleus. Neglecting the spin structure of this interaction, we obtain the evaluation of its strength, being equal by the order of magnitude to \( \Psi_s (0) \cdot G_{TV} \sim 10^{-7} \) eV, that is also much greater, than the value of \( \Delta \), seen in the experiment.

We mention here that, to our opinion, the \( \nu_e - \nu_\mu \) and \( \nu_\mu - \nu_\tau \) oscillations, that explain the experiments showing the suppression of the Solar neutrino and reactor antineutrino, as well as the atmospheric muon neutrino fluxes, do not refer to the results of [1]. The above-mentioned experiments correspond to the mass differences \( \Delta (m^2)_{e,\mu} \sim 10^{-4} \) eV\(^2\) (\( \Delta m_{e,\mu} \sim 10^{-2} \) eV) and \( \Delta m_{\mu,\tau} \sim 10^{-1} \) eV, these numbers are larger in 14 – 15 orders of magnitude than the value of \( \Delta \) observed in [1].

5. Two-electron atoms

Here, we consider the difference between the \( K \)-capture rates in the one-electron and the two-electron ions of \(^{142}\text{Pm}\) and \(^{140}\text{Pr}\). For the one-electron ions we have the transition between the initial state \( |I_i \rangle = 1, s_e = 1/2; F_i \rangle \) and the final state \( |I_f \rangle = 0, s_\nu = 1/2; F_f \rangle \). As the Hamilton operator conserves the total angular momentum, we have \( F_i = F_f = 1/2 \). For the Gamow-Teller transition in the \( \beta^+ \) and the EC channels we have

\[
\hat{H}_{int} = G_A \left( \sum_i \sigma_L (i) \tau^+_L (i) \right) \cdot \left( \sum_k \sigma_N (k) \tau^-_N (k) \right),
\]

(24)

where the summations over \( \text{“} i \text{”} \) and \( \text{“} k \text{”} \) refer to electrons and nucleons, correspondingly, while \( \tau^\pm \) are the operators that change the charge of a particle by one. By using the standard Racah algebra [16] we obtain for the transition matrix element \( M \)

\[
M = \langle I_f = 0, s_\nu = 1/2; 1/2 \vert \hat{H}_{int} \vert I_i = 1, s_e = 1/2; 1/2 \rangle = \langle I_f = 0, s_\nu = 1/2; 1/2 \vert W (1/2, 1/2, 1, 1 \text{;} 0, 1) (1/2 \vert \sigma \vert 1/2 \rangle \langle I_f = 0 \vert || (\sum_k \sigma_N (k) \tau^-_N (k)) || I_i = 1 \rangle
\]

\[
= -G_A \langle I_f = 0 \vert \hat{m} \text{GT} \rangle \vert I_i = 1 \rangle \psi_{1s} (0),
\]

where \( \psi_{1s} (0) \) is the upper component of the single-particle electron wave function at zero. Thus,

\[
|M|^2 = G_A^2 \psi_{1s}^2 (0) \langle I_f = 0 \vert \hat{m} \text{GT} \rangle \langle I_i = 1 \rangle^2 \equiv 3 \cdot G_A^2 \psi_{1s}^2 (0) B_{\text{GT}} (1^+ \rightarrow 0^+) ,
\]

(26)
as

\[ B_{\text{GT}}(I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \langle I_f = 0 | \hat{m}(\text{GT}) | I_i = 1 \rangle^2. \]  

(27)

At the same time, the initial state may have the value of the total spin \( F \) equal to both 1/2 and 3/2, while only the transition from the \( F = 1/2 \) really happens. Thus, we should multiply the value \(|M|^2\) which defines the transition rate and is given by (26) by the factor 1/3. This factor was considered in Eq. (8), where \( \Psi_i(0) = \psi_{1s}^2(0) = \frac{1}{4\pi} |g_{1s}(0)|^2 \).

In the case of the two-electron atom we have the state \(|I_i = 1, (s_e = 1/2)^2J = 0; F = 1\rangle\) as the initial one, while the final state is \(|I_f = 0, (s_e = 1/2, s_\nu = 1/2)J = 1; F = 1\rangle\). By considering the lepton system we should obligatory account for the antisymmetrization between the remaining electron and the neutrino, as we have the process where these leptons transform into each other. As a result, we have

\[ |M|^2 = 2 \cdot G_A^2 \psi_{1s}^2(0) B_{\text{GT}}(1^+ \rightarrow 0^+). \]  

(28)

In this way, we obtain formula (8) for the two-electron atom, where \( \Psi_i(0) = 2 \psi_{1s}^2(0) = 2 \frac{1}{4\pi} |g_{1s}(0)|^2 \).

By considering the process of possible time oscillations of the \( K \)-capture rate in the neutral atoms (here we consider the two-electron atoms) one should also take into account the many-body effects and the Pauli principle. These effects can reveal themselves both in variation of the energy shifts and in variation of the mixing amplitude. Here, the two-electron wave function \(|(1s)^2J = 0\rangle\) looks as

\[ |(1s_{1/2})^2J = 0\rangle = \varphi_{1s}(\textbf{r}_1)\varphi_{1s}(\textbf{r}_2) \frac{\chi_{1/2}(1)\chi_{-1/2}(2) - \chi_{1/2}(2)\chi_{-1/2}(1)}{\sqrt{2}}. \]  

(29)

If we consider the energy shift in the \( \beta^+ \)-channel and average over the directions of the electron and the neutrino, then the diagonal matrix element of the interaction increases by two as compared to the case of one-electron ion shown in Fig. 5, i.e. the value of \( \Delta \) increases by two. At the same time, the matrix element of mixing becomes equal to \( \frac{1}{\sqrt{2}} (V_{\text{EC,}\beta^+}(m_{1s} = 1/2) - V_{\text{EC,}\beta^+}(m_{1s} = -1/2)) \), where \( V_{\text{EC,}\beta^+}(m_{1s}) \) are the matrix elements shown in the Fig. 4. As we do not have the selected axis and average over the directions of the particles, both these matrix elements are equal to each other. As a result, the mixing between the \( \beta^+ \) and the EC channels is absent (\( \vartheta = 0 \)), and thus the oscillations disappear. If we adopt this assertion, we conclude that all filled \( (ns) \) shells do not contribute into the oscillation effect. The electron structures for the neutral atoms of \(^{142}\text{Pm}\) and \(^{140}\text{Pr}\) are the \((4f_{5/2})^5(6s_{1/2})^2\) and \((4f_{5/2})^3(6s_{1/2})^2\) ones correspondingly (we show only the electrons above the Xe core). Thus, only the electrons with \( \ell \neq 0 \) can contribute, their possible contribution is negligibly small. We mention here the paper \([13]\) where it was shown that that the spectra of the bound-state \( \gamma \)-quanta following the radiative electron capture are different in the cases of one and two-electron ions, this difference is also due to the Pauli principle.

Here we indicate the analogy of the \( \beta^+/\text{EC} \) decays with the decays of \( K \)-mesons. In the last case, due to the second order weak interaction that does not conserve the strangeness \( S \), the real eigenstates are not the \(|K^0\rangle\) and \(|\bar{K}^0\rangle\), but the \(|K^0\rangle = (|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2} \) and

\[ |K^0\rangle = (|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2} \]  

as
\[ |K^0_2\rangle = (|K^0_1\rangle - |\bar{K}^0_\bar{1}\rangle)/\sqrt{2} \] ones. At the same time, if we neglect the CP-violation, the \(|K^0_2\rangle\) meson, due to the structure of its wave function does not decay via the 2\(\pi\) mode (only via the 3\(\pi\) one), and is a long-lived particle, \(|K^0_2\rangle = |K_L\rangle\), while \(|K^0_1\rangle = |K_S\rangle\) is a short-lived one. Thus, in the case of \(K\)-mesons the \(|K^0_2\rangle\)-meson is the long-lived one, while in our case the mixing between the two channels is close to zero due to the Pauli principle. The states \(|K^0\rangle\) and \(|\bar{K}^0\rangle\) oscillate in time, while the total decay rate (in both channels) is the sum of the two exponents \[17\]. It is appropriate here to notice the difference with the oscillations of \(\pi\) and \(\pi\) mesons due to the structure of their wave function does not decay via the 3\(\pi\) mode (only via the 2\(\pi\) one), while the energy shift between the \(|K^0\rangle\) and \(|\bar{K}^0\rangle\) mesons is due only to the non-diagonal mixing. The experimental data show, that in our case the mixing angle between the EC and the \(\beta^+\) channels is 3 – 6 times less. Thus, we are obliged to introduce the very small additional energy shift, \(V_{\beta^+}\). In this regard we have the situation intermediate between the oscillations of \(K\)-mesons and the neutrino oscillations.

Here we mention the experimental paper \[19\], performed with the ensemble of neutral atoms of \(^{142}\text{Pm}\) arising as a result of the reaction \(^{124}\text{Sn} (^{23}\text{Na}, 5n)^{142}\text{Pm}\) in a sequence of short irradiation bursts. The duration of each burst was much less than the period of the expected oscillations, while the interval between the bursts was much more than the half-life of the initial \(1^+\) state of \(^{142}\text{Pm}\). The best fit corresponds to amplitude \(A = 0.0145(74)\), while \(T_{\text{osc}} = 3.18\) s (in the system where the \(^{142}\text{Pm}\) nucleus is at rest; here the results \[1\] for the one-electron ions are \(A = 0.23\) and \(T_{\text{osc}} = 5\) s). Thus, we performed model calculations that correspond to the duration of the irradiation burst equal to 0.5 s, as it was in \[19\]. The counting rate \(N(t, \tau)\), see Eq. (30) is normalized in such a way, that \(N(t \to 0, \tau \to 0) \to 1\).

\[
N(t, \tau) = \frac{1}{(1 - A) \tau} \int_{-\tau}^{0} \exp(-\lambda(t - t')) \cdot \left[ 1 + A \cos(\omega(t - t') + \pi) \right] dt' = \frac{\exp(-\lambda t)}{(1 - A) \tau} \times \left[ \frac{1 - \exp(-\lambda \tau)}{\lambda} + \frac{A}{\sqrt{\omega^2 + \lambda^2}} \left[ \cos(\omega t + \pi + \psi) - \cos(\omega(t + \tau) + \pi + \psi) \exp(-\lambda \tau) \right] \right] ;
\]

\[
\psi = \arctan(\omega / \lambda). \tag{30}
\]

The pattern of oscillations of the \(K\)-capture rate by \(^{142}\text{Pm}\) is shown in the Fig. 6, for different values of the entering parameters. For the one-electron ion we have the same diagram, as in Fig. 1, i.e. the interval \(\tau = 0.5\) s is small, and the oscillation picture is not washed away. If we adopt the values of \(\Delta\) and \(\vartheta\) in the two-electron ion the same as for the one-electron ion, the amplitude of oscillations \(A\) attenuates due to the decrease of the factor \(\frac{w^0_{\beta^+} - w^0_{EC}}{w^0_{EC}}\), see Eq. (7), while the frequency of oscillations remains the same, \(\omega_2 = \omega_1\). If we increase the value of \(\Delta\) by two (\(\Delta_2 = 2\Delta_1\)) by remaining the value of the coupling matrix element the same as in the one-electron ion, the frequency of oscillations also increases by two (\(\omega_2 = 2\omega_1\)), while their amplitude \(A\) further decreases (as the mixing angle \(\vartheta \sim V_{EC,\beta^+}/\Delta\) decreases). If, under \(\Delta_2 = 2\Delta_1\), we decrease the coupling matrix element, we have \(\omega_2 = 2\omega_1\), while the amplitude of oscillations \(A\) decreases still more, and we approach to the exponential decay law.

One can see that the pattern of oscillations of the \(K\)-capture rate in neutral \(^{142}\text{Pm}\) shown by us before is in a qualitative agreement with the result of \[19\].
6. Conclusion

In this paper, in the framework of the hypothesis of mixing between the electron capture and $\beta^+$ channels we tried to explain the oscillations of the $K$-electron capture rate that were presumably seen in the recent experiments. Such a mixing leads to a small variation of the $EC/\beta^+$ ratio in the decay of the one-electron ions and to an even much smaller variation of this ratio for the ensemble of neutral atoms as compared to standard calculations. The precision of both the available experimental data as well as of the theoretical calculations of this ratio is not sufficient to make conclusions on this subject.

According to our hypothesis, the time oscillations of the electron capture rate should be strongly hindered if one makes an experiment analogous to [1] but with the two-electron ions, or neutral atoms of $^{140}$Pr, $^{142}$Pm, or $^{122}$I. This statement is confirmed by the results of [19]. The most direct way to check the hypothesis is to observe the time-antiphase oscillations in the $\beta^+$-decay branch in the decay of one-electron $^{140}$Pr, where one can expect the amplitude of oscillations $A$ of about 0.08, this amplitude is 0.03 in $^{142}$Pm, as the effect of the $\beta^+$ oscillations increases by the decrease of $w(\beta^+)/w(EC)$, i.e. by the decrease of $Q_{\beta}$. The preliminary experimental data [2] relating to the $\beta^+$ branch in the decay of the one-electron ion of $^{142}$Pm give the result $A = 0.03(3)$.

Our approach for calculation of the oscillation parameters is rather simplified, especially the introduction of the effective interaction volume for the unbounded leptons. Actually, unbound leptons may leave the atom before the interaction. One important remark is in place here. It was noted by [20] that the Darmstadt effect can not arise due to the interaction in the final state. Our approach is not the study of the final state interaction. Really, the effect arises due to the interference of the two possible paths of evolution: the direct $K$-capture, and the population of the $K$-capture channel through the intermediate $\beta^+$-decay state. As a result, the quantum beatings arise. As the mixing matrix element is very small, the period of these beatings is very large. The more detailed analysis of this effect should be the subject of a separate investigation.

This paper was the subject of numerous discussions with my colleagues, particularly relating to the mechanism of a possible mixing. However, the time scale of the effect, if it really exists, denotes the weak interaction between the objects of large dimension being the only source of the necessary energy splitting.

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Fig. 1 Counting rates for the electron capture and the $\beta^+$ decay for the one-electron ion $^{142}$Pm in the presence of the weak coupling between the two decay channels; $T_{1/2} = 40.5$ s, $T_{osc} = 4.96$ s (in the system, where the $^{142}$Pm$^{+60}$ ions are at rest). The counting rate in the $\beta^+$ channel at $t = 0$ is adopted to be unity.
Fig. 2 Counting rates for the electron capture and $\beta^+$ decay for the one-electron ion $^{140}$Pr in the presence of the weak coupling between the two decay channels; $T_{1/2} = 3.39$ min, $T_{osc} = 4.94$ s (in the system, where the $^{140}$Pr$^{+58}$ ions are at rest). The counting rate in the $\beta^+$ channel at $t = 0$ is adopted to be unity.
Experimental data versus theoretical ratios of the electron capture to the $\beta^+$ decay rates.

**Fig. 3** Experimental data versus theoretical ratios of the $w_{\text{EC}}^0(1s)/w_{\beta^+}^0$ in neutral atoms as a function of $(w_{\text{EC}}^0(1s))_{\text{th}}/(w_{\beta^+}^0)_{\text{th}}$. Only the allowed Gamow–Teller transitions are shown here. The ratio $Z(\text{mix})$ is calculated by using the mixing parameters from the one-electron ions. The decrease of the mixing angle $\vartheta$ in the two-electron or neutral atoms leads to the tendency $Z(\text{mix}) \rightarrow 1$. The notations are as follows:

1. $^{140}\text{Eu}$ ($1^+ \rightarrow 0^+$, 1.51 s);
2. $^{48}\text{Sc}$ ($2^+ \rightarrow 2^+$, 3.07 h);
3. $^{91}\text{Mo}$ ($9/2^+ \rightarrow 9/2^+$, 15.49 min);
4. $^{22}\text{Na}$ ($3^+ \rightarrow 2^+$, 2.60 y);
5. $^{142}\text{Pm}$ ($1^+ \rightarrow 0^+$, 40.5 s);
6. $^{61}\text{Cu}$ ($3/2^- \rightarrow 3/2^-$, 3.33 h);
7. $^{134}\text{La}$ ($1^+ \rightarrow 0^+$, 6.45 min);
8. $^{48}\text{V}$ ($4^+ \rightarrow 4^+$, 15.97 d);
9. $^{140}\text{Pr}$ ($1^+ \rightarrow 0^+$, 3.39 min);
10. $^{143}\text{Sm}$ ($3/2^+ \rightarrow 5/2^+$, 8.75 min);
11. $^{120}\text{Sb}$ ($1^+ \rightarrow 0^+$, 15.89 min);
12. $^{52}\text{Mn}$ ($6^+ \rightarrow 6^+$, 5.59 d);
13. $^{64}\text{Cu}$ ($1^+ \rightarrow 0^+$, 12.70 h);
14. $^{89}\text{Zr}$ ($9/2^+ \rightarrow 9/2^+$, 78.41 h);
15. $^{89}\text{Zr}$ ($1/2^- \rightarrow 3/2^-$, 4.16 min);
16. $^{116}\text{Sb}$ ($8^- \rightarrow 7^-$, 60.30 min);
17. $^{58}\text{Co}$ ($2^+ \rightarrow 2^+$, 70.86 d);
18. $^{65}\text{Zn}$ ($5/2^- \rightarrow 3/2^-$, 244.06 d);
19. $^{141}\text{Nd}$ ($3/2^+ \rightarrow 5/2^+$, 6.45 min);
20. $^{107}\text{Cd}$ ($5/2^+ \rightarrow 7/2^+$, 6.50 h).
Fig. 4 The diagrams demonstrating possible coupling between the electron capture and $\beta^+$ channels.

Fig. 5 The diagram showing the energy shift in the $\beta^+$ channel. The exchange by the $\gamma$-quantum in the diagram (b) is not considered, as the corresponding effect is included in the Coulomb functions of the charged leptons.
Fig. 6 Model calculation of the decay law relative to the electron capture for the ensemble of one-electron ions or the neutral atoms of $^{142}$Pm as a function of the entering parameters. The $^{142}$Pm nuclei are supposed to be produced in the irradiation bursts with duration $\tau = 0.5$ s. The time reading begins just after the termination of the burst.