THE INFLUENCE OF MAGNETIC FIELD GEOMETRY ON THE FORMATION OF CLOSE-IN EXOPLANETS

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ABSTRACT
Approximately half of Sun-like stars harbor exoplanets packed within a radius of ~0.3 au, but the formation of these planets and why they form in only half of known systems are still not well understood. We employ a one-dimensional steady-state model to gain physical insight into the origin of these close-in exoplanets. We use Shakura & Sunyaev α values extracted from recent numerical simulations of protoplanetary disk accretion processes in which the magnitude of α, and thus the steady-state gas surface density, depend on the orientation of large-scale magnetic fields with respect to the disk’s rotation axis. Solving for the metallicity as a function of radius, we find that for fields anti-aligned with the rotation axis, the inner regions of our model disk often fall within a region of parameter space that is not suitable for planetesimal formation, whereas in the aligned case, the inner disk regions are likely to produce planetesimals through some combination of streaming instability and gravitational collapse, though the degree to which this is true depends on the assumed parameters of our model. More robustly, the aligned field case always produces higher concentrations of solids at small radii compared to the anti-aligned case. In the in situ formation model, this bimodal distribution of solid enhancement leads directly to the observed dichotomy in exoplanet orbital distances.

Key words: planetary systems – planets and satellites: formation – protoplanetary disks

1. INTRODUCTION
With the recent discovery of thousands of exoplanets, we now know that stars’ harboring of planets is not unique to our solar system. Not only are exoplanets ubiquitous throughout our galaxy, but equally commonplace is the diversity of these systems. Indeed, discoveries such as hot Jupiters and super-Earths have made our own solar system quite de

1

2. MODEL
2.1. Disk Structure
Our model consists of a one-dimensional (1D) steady-state disk comprised of both gas and solids. The steady-state gas surface density is set by the turbulent viscosity,

\[ \Sigma_g = \frac{M}{3\pi\nu} \]

where \( M \) is the steady-state mass accretion rate onto the central star, which we assume to be \( 10^{-3}M_\odot \text{ yr}^{-1} \), and the turbulent viscosity is defined via the standard Shakura & Sunyaev (1973) α parameter, \( \nu = \alpha c_s^2/\Omega \). \( c_s \) is the locally isothermal sound-speed of the gas (defined via the temperature below), and \( \Omega \) is the Keplerian angular velocity.

We take the temperature to be that of the minimum mass solar nebula model (MMSN; Hayashi 1981),

\[ T(r) = 280 \left( \frac{r}{\text{au}} \right)^{-1/2} \text{ K}. \]

We treat the solid particles, with radius \( a_p \) and mass density \( \rho_p \) as a fluid with surface density \( \Sigma_p (r, t) \). Without any particle sinks or sources, the evolution of this surface density is
is the radial flux of solids due to turbulent diffusion, and $F_{adv}$ is the radial flux of solids due to aerodynamic coupling between the solids and gas. The metallicity is $Z = \Sigma_{p}/\Sigma_{g}$, and in the limit $Z \ll 1$, the diffusive flux is written as (Clarke & Pringle 1988),
\[
F_{\text{diff}} = -D_p \Sigma_g \frac{\partial Z}{\partial r},
\]
where $D_p$ is the particle diffusivity.

We can write a form for the particle diffusivity in terms of the dimensionless stopping time of the solids, $\tau = t_{\text{stop}} \Omega$ (where $t_{\text{stop}}$ is the dimensional stopping time) and the gas diffusivity $D$ (the diffusivity that would apply to a trace gas species within the disk) as $D_p = D/(1 + \tau^2)$ (Youdin & Lithwick 2007). The gas diffusivity scales with the turbulent viscosity $\nu$, with a constant of proportionality $\xi$ that is sensitive to the structure of whatever turbulent process is giving rise to angular momentum transport (discussed more in Section 2.2); $D = \xi \nu$.

The advective radial flux $F_{adv}$ arises because of aerodynamic coupling between solid particles and the gas. For $\tau \ll 1$, the strongly coupled solids move at the same speed as the local gas, given by the accretion rate $M$ as $\nu = -M/(2\pi \Sigma_{g} \nu_k)$. For general $\tau$, there is also a component of the radial drift relative to the gas due to a mismatch between the azimuthal velocities of the particles and gas (Weidenschilling 1977). Combining these effects, we express the advective radial flux as (Takeuchi & Lin 2002),
\[
F_{adv} = \Sigma_{p} \nu_{p} = \Sigma_{p} \left( \frac{\nu_{\text{K}}}{\tau + \tau_{\text{K}}} - \eta \nu_{\text{K}} \right),
\]
where $\nu_{\text{K}}$ is the Keplerian velocity and
\[
\eta = -\frac{1}{2} \left( \frac{h}{r} \right)^2 \left[ \frac{d \ln \Sigma_{g}}{d \ln r} + (q - 3) \right],
\]
where $q$ is defined via the local radial slope of the vertical gas scale height, $h = c_{s}/\Omega$,
\[
\frac{h}{r} \propto r^{q-1}.
\]
For the assumed temperature profile, $q = 1.25$.

We define the stopping time of the solids in two separate ways, depending on the location in the disk. Inside of the snowline at 2.7 au, we follow Birnstiel et al. (2009, 2012) and set the maximum $\tau$ by the fragmentation velocity, $v_{\text{frag}}$,
\[
\tau = \frac{1}{3 \alpha_{\text{mid}}^{2}} \left( \frac{v_{\text{frag}}}{c_{s}} \right)^2,
\]
where $\alpha_{\text{mid}}$ is the value of $\alpha$ at the disk midplane, where (through settling) most of the solids are concentrated. For silicates, $v_{\text{frag}} \approx 1 \text{ m s}^{-1}$ (Blum & Wurm 2008), but this velocity depends non-trivially on particle size (Blum & Wurm 2008). Here, we assume $v_{\text{frag}} = 1 \text{ m s}^{-1}$ for the fiducial case and vary it in what follows.

Outside of the snowline, we assume that particles reach millimeter sizes. Our reason for this assumption is twofold.

First, coagulation simulations have indicated that beyond the snowline, solids can grow to larger sizes, as the ice present in these solids allows for enhanced sticking and is less susceptible to fragmentation (Birnstiel et al. 2012). Second, submillimeter observations show that millimeter-size grains are prevalent at large radii in many protoplanetary disk systems (e.g., Andrews et al. 2012). Thus, the snowline serves as a natural transition from smaller particles (their exact sizes depending on $\alpha_{\text{mid}}$) to larger particles that are seen in observations, albeit at large radii. Thus, $\tau$ outside of the snowline is
\[
\tau = \frac{\pi \rho_{p} a_{p}}{2 \Sigma_{g}},
\]
where $\rho_{p} = 2 \text{ g cm}^{-3}$ and $a_{p} = 1 \text{ mm}$.

Prior to disk dispersal, the gas surface density in the inner disk evolves on a timescale of $\sim \text{Myr}$, while the local radial drift time, $r/|\nu|_{\nu}$, of millimeter- to centimeter-size particles is orders of magnitude shorter. We can thus assume a fixed gas profile, and compute the steady-state distribution of particles drifting radially through the gas from a large reservoir situated further out. Equation (3) then becomes
\[
\tau \Sigma_{g} D_{p} \frac{dZ}{dr} - \tau \Sigma_{g} \nu_{p} Z = k.
\]

The constant $k$ is the radial mass flux of solids, $k = M_{p}/(2\pi)$, which we parameterize via the ratio of solid to gas accretion rates, $M_{p}/M$. This ratio is a free parameter in our model, but we can estimate values based on taking Equation (10) in the limit of large radial distances, assuming diffusion is weak to advection at these distances (which is borne out by our calculations), and assuming $\tau \ll 1$. At large $r$, radial drift dominates over accretion with the gas (again supported by our calculations). One can then rewrite Equation (10) to calculate $M_{p}/M$ as a function of $Z$, $\tau$, $\alpha$, $\ln \Sigma_{g}/\ln r$, and $q$. The scenario we envision is a reservoir of gas at large radii, with ISM values for the metallicity ($Z = 0.01$). We take $\tau = 0.1$ and $\alpha = 10^{-3} \sim 10^{-2}$ as reasonable estimates$^2$ at large distances, and constant $\alpha$ results in $\ln \Sigma_{g}/\ln r = -1$ for our disk. For these values we find $M_{p}/M \approx 0.1$; our fiducial value is $M_{p}/M = 0.3$ and we explore $M_{p}/M = 0.1$ and $M_{p}/M = 1$ in what follows.

We numerically integrate Equation (10) to determine the metallicity as a function of radius, assuming that the concentration of solids goes to zero at the inner edge of our disk, which we take to be the dust destruction radius, $r_{\text{in}} = 0.03$ au (for the temperature structure assumed here); $Z(r_{\text{in}}) = 0$. Finally, at the snowline, we reduce $Z$ by a factor of 2, following the arguments in Lodders (2003).

2.2. Bimodal $\alpha$ from the Hall Effect

A key component to our model is the dependence of $\alpha$ on the relative orientation of the large-scale, vertical (i.e., perpendicular to the disk plane) magnetic field to the disk’s rotation axis.$^3$ Numerical simulations that include all three non-ideal
magnetohydrodynamical effects present in protoplanetary disks have shown that for \( r \lesssim 30-60 \) au, the Hall effect leads to two different states of the disk within these regions, depending on the value of \( \Omega \cdot B \) (Bai 2015; Simon et al. 2015). For aligned field and angular momentum vector (i.e., \( \Omega \cdot B > 0 \)) and for the field strengths explored here (see below), \( \alpha \sim 0.01 \) within the inner disk, whereas in the anti-aligned case (i.e., \( \Omega \cdot B < 0 \)), \( \alpha \sim 10^{-4}-10^{-3} \) (Bai 2015; Simon et al. 2015).

We use the quantified \( \alpha \) values from these numerical simulations in order to construct \( \alpha(r) \) for our model. In particular, we fit a power law to the \( \alpha \) values in the work of Simon et al. (2015), assuming a uniform-in-radius ratio of midplane gas to vertical magnetic pressure, \( \beta_z = 10^5 \). We choose this particular value because \( \beta_z = 10^5 \) provides the largest sample of \( \alpha \) values from recent simulations, and for reasonable gas surface density profiles (such as in the MMSN), the resulting \( \alpha \) values correspond to \( M = 10^{-8}M_\odot \) yr\(^{-1} \) (though here we do the opposite and assume \( M = 10^{-8}M_\odot \) yr\(^{-1} \) and then calculate \( \Sigma_z \)).

Our power-law fits to the \( \alpha \) values are given by

\[
\alpha = \begin{cases} 
6 \times 10^{-2} (r/\text{au})^{-0.87} & \Omega \cdot B > 0 \ \& \ \ r < r_{\text{outer}} \\
7.7 \times 10^{-4} (r/\text{au})^{0.43} & \Omega \cdot B < 0 \ \& \ \ r < r_{\text{outer}}, \\
3.3 \times 10^{-3} & r \geq r_{\text{outer}}
\end{cases}
\]

where \( r_{\text{outer}} = 30 \) au. The physical reason for assuming a single value for \( \alpha \) (i.e., independent of magnetic field orientation) at \( r > r_{\text{outer}} \) is the relatively weaker Hall effect at these large radii, thus removing the bimodal \( \alpha \) behavior (Bai 2015; Simon et al. 2015). We cap \( \alpha \) at 0.05; larger values do not change our results significantly but require more radial zones for integrating the solution. Finally, we set \( \alpha_{\text{mid}} = 10^{-4} \) everywhere (taken from recent simulations: Simon et al. 2013b; Bai 2015), and therefore \( \alpha \) is not allowed to go below \( \alpha_{\text{mid}} \).

Solids will be concentrated toward the midplane and thus feel a diffusivity set by \( \alpha_{\text{mid}} \). However, since these solids are swept inward by accretion, this “viscosity” component is set by \( \alpha \) as this parameterizes the accretion flow. This amounts to a definition of the ratio between diffusivity and accretion viscosity of \( \xi = \alpha_{\text{mid}}/\alpha \).

2.3. Conditions for Planetesimal Formation

We consider planetesimal formation to be possible if one of two criteria are satisfied. First, following the arguments in Youdin & Shu (2002), if the metallicity increases beyond a critical value, it is possible for gravitational collapse of particles to proceed, independent of \( \tau \) and without any interference from the Kelvin–Helmholtz instability. We use Equation (15) in Youdin & Shu (2002) to calculate a critical surface density for solids for each magnetic field orientation. This can be easily converted to a critical metallicity, \( Z_{\text{crit}} \), and we assume that if \( Z > Z_{\text{crit}} \), prompt planetesimal formation occurs.

If \( Z < Z_{\text{crit}} \), planetesimal formation is still possible when the values of \( Z \) and \( \tau \) fall within the allowed region of parameter space for the streaming instability (Youdin & Goodman 2005) to operate. This allowed space is still uncertain, but recently Carrera et al. (2015) carried out a large suite of streaming instability calculations and mapped out the allowed streaming region in \( Z-\tau \) space. In what follows, we include both planetesimal formation regions on plots of \( Z-\tau \) space and assume that if the solution enters these regions, in situ planetesimal and planet formation occurs.

3. RESULTS

Here, we run our model as described above and vary both the relative radial drift rate of solids, \( \dot{M}_r/M \), and the fragmentation velocity, \( v_{\text{frag}} \), as these are both rather uncertain parameters. We save a more in-depth parameter survey for future work.

In Figure 1, we show the fiducial case of \( \dot{M}_r/M = 0.3 \). For \( \Omega \cdot B < 0 \), the metallicity at small radii does not enter either the region of direct gravitational collapse or streaming-initiated growth. Instead, planetesimals appear to be possible at larger radii \( r \gtrsim 3 \) au. However, \( \Omega \cdot B > 0 \) shows that regions \( r \lesssim 0.3 \) au have sufficiently high metallicity such that particles can directly collapse. At larger radii, streaming-initiated growth can produce planetesimals.

We have also run our model with \( v_{\text{frag}} = 0.5 \) m s\(^{-1} \) and \( v_{\text{frag}} = 2 \) m s\(^{-1} \); the output of these calculations is shown in Figure 2. In all cases, the \( \Omega \cdot B > 0 \) solution allows for
planetesimal formation. For $\Omega \cdot B < 0$ and $v_{\text{frag}} = 0.5 \text{ m s}^{-1}$, the metallicity is larger than the critical value for collapse, whereas for the same field geometry and $v_{\text{frag}} = 2 \text{ m s}^{-1}$, the solution does not enter the region of planetesimal formation.

The second parameter we explore is $M_p/M$, and as explained above, we run our model for $M_p/M = 0.1$ and 1 (shown in Figure 3), in addition to the fiducial case (Figure 1). For the lowest flux case, neither magnetic field orientation enters the direct collapse regime, though the $\Omega \cdot B > 0$ case becomes unstable to the streaming instability between approximately 1 and 10 au. The higher values of $M_p/M$ again lead to $\Omega \cdot B > 0$, allowing for planetesimals through either direct collapse or streaming instability. In the model with $M_p/M = 1$, the $\Omega \cdot B < 0$ case is unstable to direct collapse at small radii, but enters the streaming unstable regime beyond $\sim 1$ au. We do not expect $M_p/M$ to be significantly less than 0.1, as a value of $M_p/M = 0.01$ would equate to co-accretion of solids with the gas flow at large distances from the star; a simple calculation reveals that radial drift strongly dominates accretion at these radii.

Finally, we tested the robustness of the fiducial case to uncertainties in the gas surface density profile using a different set of $\alpha$ values (i.e., those from Bai 2015). We found no qualitative difference.

While the solutions fall into different regions of the parameter space, depending on the exact values for the parameters, the metallicity always reaches higher values for the $\Omega \cdot B > 0$ configuration compared to the $\Omega \cdot B < 0$ configuration.

4. DISCUSSION

We have shown that whether or not a protoplanetary disk forms planets within the region $r < 0.3$ au depends on the orientation of any large-scale vertical magnetic field threading the disk with respect to the disk’s rotation axis. The key ingredient is the dependence of the accretion stress on the product $\Omega \cdot B$, as mediated by the Hall effect. When $\Omega \cdot B > 0$, the resulting $\alpha$ values are enhanced and in a steady-state, this equates to a lower gas surface density. The opposite is true for $\Omega \cdot B < 0$; weaker $\alpha$ values lead to higher gas surface densities.

It is worth pointing out that in the $\Omega \cdot B < 0$ case, the gas surface density depends more steeply on radius (see Figure 4) and in fact is reasonably close to the MMSN profile (which is particularly intriguing since our own solar system lacks tightly packed, close-in planets, in agreement with the $\Omega \cdot B < 0$ case). Despite a larger radial drift velocity induced by this steeper profile, the shallower gas density profile still attains higher solid concentration. The reasons for this are twofold. First, with higher $\alpha$ values, the co-accretion of the solids with
the gas dominates over radial drift, ensuring that (given a sufficiently high $M_p/M$) there will always be a relatively large number of solids at small radii, regardless of the value of $\tau$ (see Equation (5)). Second, the gas surface density is lower for $\cdot W > B_0$, which means it is easier to attain relatively large $Z$.

In most of the cases explored here, $\cdot W > 0$ leads to a sufficiently high metallicity for $r < 0.3$ au to cause direct gravitational collapse into planetesimals, though the degree to which this is true depends on the value of $M_p/M$. Furthermore, the inner regions of the disk with $\cdot W < 0$ fall within the region of parameter space where planetesimal formation is difficult, though this result is even less robust, as it depends on both $M_p/M$ and $v_{\text{frag}}$ (and possibly other parameters not explored here). What is robust, however, is that for $\cdot W > 0$, the metallicity at small radii is enhanced, sometimes quite significantly, compared to $\cdot W < 0$. Higher metallicity leads to more planetesimals and planetesimals of larger sizes (J. B. Simon et al. 2016, in preparation). Thus, even if parameters are such that both magnetic field orientations can lead to planetesimals at small radii, there remains a bimodal distribution in the number and sizes of these planetesimals.

Encouragingly, pebble accretion depends on the size of the accreting planetesimals (K. Kretke, private communication), and while one would need planet accretion and dynamical models to strengthen the link between our results and exoplanet observations, the magnetic field geometry may very well be the key to explaining the dichotomy in exoplanet orbital configurations.

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Figure 4. Gas surface density as a function of radius for $\cdot W > 0$ (black, solid) and $\cdot W < 0$ (blue, dashed). Also shown is the MMSN profile (red, dotted–dashed). As denoted on the plot, co-accretion of solids with the gas flow dominates the inner regions of the disk for $\cdot W > 0$, whereas drift dominates in the $\cdot W < 0$ case.