The study of the deviations of the payload moved along the curved trajectory by the crawler crane

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Abstract. Reducing the uncontrolled deviations of the payload moved by a crawler crane with a constant boom length is a relevant objective, since it improves the operational efficiency and safety, as well as the positional accuracy of the payload at the target point. In this regard, the results of the research aimed at developing a trajectory synthesis method of two controlled coordinates of the crane, such as the swing angles of the rotary platform and boom raising which ensure a predefined trajectory of the moving payload, have been described. Furthermore, the influence of the payload movement parameters, such as the time of the prescribed payload displacement, the initial angle of the boom lift, the hoist rope length, the overall dimensions of the required payload displacement trajectory on the error of the predefined payload trajectory execution by the lifting crane has been studied. For example, we examined a predefined arc curved trajectory of the moving payload. The horizontal coordinates of the trajectory of the upper suspension point located on the boom head and providing the predefined payload trajectory were generated using the Matlab mathematical simulation model developed in the Simulink package for solving a linearized differential equation. This equation describes the deviations of a pendulum suspended payload with a horizontally moving suspension point in a separate vertically located plane of the space. The superposition of two horizontal trajectories obtained by solving the given equation ensures the payload movement along a curved trajectory predefined in the horizontal plane. The trajectory of the crane's controlled coordinates was calculated from the horizontal coordinates of the suspension point. When executing the trajectory, the examined additional errors occur. It was revealed that when the time of moving along a predefined payload trajectory exceeds 40 seconds, the residual oscillations become insignificant and payload positional accuracy at the target point increases. It suggests the limited applicability of the developed method when implemented on the rotary lifting cranes with a constant boom length. The range of the local minima errors of the predefined trajectory implementation depending on the initial inclination angle of the boom, which varies in the average values range of the boom inclination angle, has been defined.

Key-words: crawler crane, load fluctuations, curved trajectory, deviations
1. Introduction
When payloads are moving in space, the dynamics of boom cranes is described by the system of the nonlinear differential equations (DE) [1]. Crawler cranes (CC) are moving under their own power over considerable distances [2]. Moreover, unlike the pneumatic self-propelled cranes, in some cases crawler cranes with payloads can move on the ground surface [3]. If the crawler chassis of the crane is stationary, the swing of the rotary platform and lifting (lowering) the crane boom inevitably causes the payload deviations manifested as periodic deviations of the hoist rope and payload from the gravitational vertical [4]. The negative impact of uncontrolled payload oscillations on the performance indicators of the CC operating process is known and beyond doubt [5]. The operating cycle time of the crane increases [6], while the safety of the moving process decreases, since the swinging payload can touch stationary or moving objects in the working space [7]. This problem is still relevant today for various types of lifting cranes, especially bridge cranes [8], wheel mounted truck cranes [9], tower cranes [10], offshore container crane [11] and boom cranes [12].

A special feature of the DEK crawler cranes is the constant length of the boom and consequently the impossibility of moving the payload suspension point in the horizontal plane [3]. This causes additional difficulties in limiting the uncontrolled payload deviations. The solution of the latter problem can be found in a wide range of research papers that discuss different approaches. Modern approaches belong to a group of methods that synthesize a certain movement of the suspension point, which, in turn, optimizes the payload movement [13]. It involves using the shaping controllers [14], the mathematical apparatus of fuzzy logic [15], the control by means of neural networks [16], the inverse dynamics method [17] and other approaches [18].

In paper [19], a method is proposed and a mathematical model is developed to solve the problem of moving a payload by a bridge crane on a non-rigid rope suspension along any curved horizontal trajectory specified as a smoothed curve. At the same time, there are no uncontrolled pendulum payload deviations, all gravitational vertical deviations of the hoist rope can be controlled. In the mathematical model used to synthesize the trajectory of the suspension point, the principle of the direct analytical derivation of the second order linearized DE describing small angular deviations of the pendulum system «moving suspension point-payload» is applied. A first order DU is obtained and can be presented in the following form [19]:

\[ \dot{q} = -\left(\dot{x}_i + q \cdot g\right) / (L \cdot b) . \]  

(1)

where \( q \) is the gravitational vertical deviation angle of the hoist rope of the bridge crane; \( L \) is the length of the hoist rope (the length of the payload flexible suspension); \( b \) is the viscous friction coefficient reduced to the angular coordinate; \( x_i \) is the linear coordinate of the horizontal movement of the payload suspension point; \( g \) is the gravitational acceleration. The points above the variables indicate their time derivatives.

The coordinates of the suspension point \( x_i \) and payload \( x_l \) are connected by the geometric relationship, which in the linearized form is as follows [19]:

\[ x_l = x_i - L \cdot q . \]  

(2)

Equations (1) and (2) make it possible to synthesize the trajectory of the payload suspension point movement along a predefined trajectory of the payload in the plane and provide the sufficient solution accuracy at the small values of the angle \( q \) (to \( \pm 10^\circ \)).

The control action in equation (1) is the required payload acceleration which can be obtained by differentiating the predefined movement of the payload \( x_l \). The method described in paper [19] makes it possible to quickly, in near-real time synthesize the movement trajectory of the suspension point \( x_i(t) \) which accurately provides the predefined trajectory of the payload movement \( x_l(t) \) along the horizontal axis. The payload oscillations are examined in a separate plane. If the payload needs to be moved along a curved trajectory in space, this trajectory is represented as a superposition of two simultaneous trajectories of movement in mutually perpendicular vertical planes (in the motion planes of the bridge and bridge crane truck).
The method [19] is quite simple and does not suggest using a complex mathematical apparatus, which is typical for most other approaches. It is advisable to use its advantages for a crawler crane with a constant boom length and a rotary platform (Fig. 1). However, additional errors may occur due to the fact that in a crawler crane, the payload suspension point does not move in a horizontal plane, as in the bridge crane, but along the torus surface (in a particular case, when the boom swing point is on the rotation axis of the rotary platform it moves along the spherical surface).

![Diagram]

**Figure 1.** Self-propelled boom crawler crane with a constant length of the boom.

The present paper is devoted to the study of the applicability of the method [19] for moving the payload along a curved trajectory by a crawler crane and to determining the resulting errors of the payload positioning in the horizontal plane.

### 2. Problem statement

The initial values of the lifting angles of the boom \( q_{j0} \), CC rotary platform \( q_{s0} \) and other constant design parameters of the crane, such as the initial values of the boom length \( L_J \), hoist rope length \( L_{gk} \), viscous friction angular coefficient of the hoist rope \( b \), stiffness coefficients \( c_{qS}, c_{qJ} \) and viscous friction coefficients \( b_{qS}, b_{qJ} \) reduced to the swing angles of the platform and CC boom, payload weight \( m_{gr} \), link masses of the platform \( m_s \) and boom \( m_j \) of the crawler crane and a number of other linear dimensions of the CC links have been set. The viscous friction angular coefficient of the hoist rope was assumed to be constant for any direction of the rope deviation from the gravitational vertical.

For the predefined payload displacement time \( T_{kon} \) and payload displacement trajectory in a horizontal plane predefined as time dependencies \( x_{tred}(t); y_{tred}(t) \), the suspension point displacement trajectory is required to be synthesized in Cartesian coordinates along the corresponding horizontal axes \( x(t); y(t) \). Furthermore, the trajectory in Cartesian coordinates \( x(t), y(t) \) must be converted to the trajectory in the generalized coordinates of the rotary platform and boom of the CC: \( q_s(t), q_j(t) \). At the same time, it is assumed that the angular coordinates of the CC chassis (the swing and pitch angles of the chassis) are zero.

For example, the predefined trajectory of the payload movement has the shape of an arc and is characterized by the following dimensions: \( X_A \) is the movement along the axis \( O_0X_0 \) in the positive...
direction, \( Y \) is the movement along the axis \( O_0Y_0 \) in the positive and negative directions, as well as the values derivatives of the specified time coordinates.

The point \( O_0 \) of the origin of the stationary Cartesian coordinate system \( O_0X_0Y_0Z_0 \), in which the payload trajectory is set coincides with the initial position of the payload mass center regardless of the values of \( q_0 \) and \( q_{s_0} \) to simplify the problem.

Moreover, by implementing the synthesized trajectory of the suspension point in the CC, it is important to define the absolute linear error of the payload actual trajectory relative to its predefined trajectory.

As a higher level task, it is necessary to study the influence of a number of basic parameters values of a predefined payload trajectory and dynamic system of the CC on the absolute error by varying the given parameters values.

3. Theory

To synthesize the suspension point displacement trajectory, which ensures the movement of the payload along a predefined trajectory, a mathematical simulation model has been developed in MATLAB-based graphical programming environment for modeling dynamical systems \([20]\) on the basis of the method described in paper \([19]\) (Fig. 2). The model is aimed at solving the DE (1).

![Figure 2. Mathematical simulation model for solving the differential equation (1).](image)

Integrator blocks of the Simulink package are used for the numerical integration in this model. The result of the solution is the time dependencies of the angle \( q \) of the crane hoist rope deviations from the gravitational vertical in a separate plane, as well as the derivatives of this angle. The input effect is the time dependence of the required payload acceleration \( \ddot{x}_{ltreb}(t) \) or \( \ddot{y}_{ltreb}(t) \). The acceleration was calculated by numerical differentiating a predefined payload movement along one of the horizontal coordinate axes: \( \dot{x}_{ltreb}(t) \) or \( \dot{y}_{ltreb}(t) \) correspondingly.

To generate the horizontal coordinates values of a predefined payload trajectory, the well-known spline interpolation method was used \([21]\). We used the \textit{spapi} function of the MATLAB system, which returns a spline providing the preset values of the function and its derivatives at the nodal points of the trajectory. The movement along the axis \( O_0X_0 \) was defined by two reference points with coordinates \([0; X_0]\) (the time argument values \([0; T_{kon}]\)) with zero time derivatives at the specified points. The prescribed derivatives of each coordinate had an order from 1 to 4. The movement along the axis \( O_0Y_0 \) was defined by three reference points with coordinates \([0; Y_0; 0]\) (the time argument values \([0; T_{kon}/2; T_{kon}]\)) and with zero time derivatives at the specified points, except for the second derivative (acceleration) at the second (middle) point of the trajectory. Consequently, the required payload trajectory took the geometric shape of a curved arc in the horizontal plane.

To calculate the discrete values of the payload coordinates and acceleration (as a control variable of DE (1)) at the intermediate points between the reference points, the function \textit{fnval} of the MATLAB system was used.
After solving the differential equation (1) twice, for the time-defined trajectory of the payload movement along the axis \(O_0X_0\) and movement trajectory along the axis \(O_0Y_0\), the synthesized trajectories of the suspension point movement in the horizontal plane were used to calculate the trajectories of changes in two generalized coordinates of the crawler crane, namely the rotation angles of the platform and boom lift and can be presented in the following form:

\[
q_s = \arctg \left( \frac{y_i}{x_i} \right); \quad q_j = \arccos \left( \frac{\sqrt{x_i^2 + y_i^2} - x_{cm2}}{L_j} \right).
\]

where \(x_{cm2}\) is the constant structural dimension of the CC, the horizontal distance between the rotation axis of the platform and vertical projection of the boom rotation axis (Fig. 1). Equations (3) are derived from the elementary geometric relations between the linear and angular dimensions of the CC and are obtained under the assumption that the base chassis of the CC is located horizontally.

Furthermore, the actual trajectory of the payload movement was generated using the time dependencies \(q_s(t)\), \(q_j(t)\) by precise tracking the specified trajectory of the generalized coordinates \(q_s(t)\), \(q_j(t)\) of the CC. A simulation model of the CC, which was developed applying the Simscape Multibody expansion blocks of the MATLAB based Simulink package, was used for this purpose [22]. This expansion is intended for modeling the mechanical systems.

In this simulation model (Fig. 3), several basic mechanical expansion blocks Simscape Multibody, such as Solid blocks (which describe solid bodies), Joint blocks (rotary joints with one degree of freedom for the platform and boom swings and rotary joints with two degrees of freedom for swinging the hoist rope with the payload) and Rigid Transform blocks (blocks of constant shifts and swings of the local coordinate systems of the model) were used. The coordinates of the payload center in the coordinate system \(O_0X_0Y_0Z_0\) were recorded in the model using the virtual displacement measurement sensor block Transform Sensor.

Figure 3. Simulation model for the precise trajectory tracking of a rotary platform and boom of a self-propelled crawler crane displacement.

Thus, using the CC simulation model, the payload displacement trajectory \(x_{out}(t)\), \(y_{out}(t)\) which was taken as the actual one was stored in the memory of the working space of the MATLAB system.

The absolute error at any time was defined as the Cartesian distance between the target and the actual point of the payload trajectory in the horizontal plane and is calculated according to the following formula:
The maximum value $\Delta_{\text{max}}^{\Sigma}$ of the absolute error vector $\Delta$ calculated by (4) for the trajectory under consideration within the time values $[0; T_{\text{kon}}]$ (i.e., within the time limit when the suspension point is mobile) was taken as one of the criteria for its estimation.

Another criterion for estimating the same trajectory was the maximum value $\Delta_{\text{max}}^{r}$ of the absolute error vector elements $\Delta$ calculated for the residual payload deviations around the final equilibrium position and is defined by the formula:

$$\Delta = \sqrt{(x_{\text{out}} - x_{\text{in}})^2 + (y_{\text{out}} - y_{\text{in}})^2}. \quad (5)$$

According to equation (5), the values of the vector were defined for the time interval $t \geq T_{\text{kon}}$, i.e. when the suspension point of the payload stops in the final position. Then, the element with the maximum value of $\Delta_{\text{max}}^{r}$ was also selected from this vector.

### 4. Experimental results

Figure 4 shows an example of some results of the simulation carried out according to the method described above. The main parameters of the CC and working process of moving payload accepted the following default values: $q_{j0}=1$ rad, $q_{s0}=0$ rad, $L_{j}=12$ m, $L_{gk}=10$ m, $b=100$ N·m/(rad/s), $c_{pq}=10000000$ N·m/rad, $c_{qj}=10000000$ N·m/rad, $b_{qs}=1000000$ N·m/(rad/s), $b_{qj}=1000000$ N·m/(rad/s), $m_{g}=1000$ kg, $m_{j}=1680$ kg, $x_{cm} = 0.7$ m, $T_{\text{kon}}=40$ s, $X_{a}=3$ m, $Y_{a}=2$ m, $y_{\text{out}}(T_{\text{kon}}/2)=-0.03$ m/s$^2$.

This combination of parameter values is accepted as the central point when varying the values of the experimental factors one by one. The variable factors and their variation ranges are as follows: $T_{\text{kon}}$ is from 25 to 55 s, $q_{j0}$ is from 0.72667 to 1.52667 rad, $L_{gk}$ is from 6.258 to 14 m, $X_{a}$ is from 2.0968 to 5 m, $Y_{a}$ is from 1.0968 to 4 m.

Figure 4a shows an example of the time dependences of the predefined payload trajectory horizontal coordinates and their accelerations for the moving time of the payload suspension point $T_{\text{kon}}=20$ s. The other parameters took the default values given above.
Figure 4. The results of the simulation: (a) is the required payload coordinates and their accelerations; (b) is the required payload accelerations and corresponding accelerations of the suspension point in the horizontal plane; (c) is the controlled generalized coordinates of the crane; (d) is the required and actual payload trajectories, the suspension point trajectory (plan view, the example at $T_{kon}=20 \text{ s}$); (e) is the required and actual payload trajectory, the suspension point trajectory (plan view, the example at $T_{kon}=40 \text{ s}$); (f)–(j) are the maximum absolute errors of the payload trajectory when implemented on the crane depending on the crane and trajectory parameters.
Figure 4b shows the payload suspension point accelerations in the horizontal plane which have been synthesized using the simulation model for the same trajectory in addition to the second coordinate derivatives of the given payload trajectory to solve the differential equation (1) (Fig. 2). Besides, Fig. 4c demonstrates the time dependences of the controlled generalized coordinates of the CC defined by equation (3) for the same trajectory, while Fig. 4d demonstrates a plan view (from above) of the predefined payload trajectory $[x_{l_{\text{treb}}}, y_{l_{\text{treb}}}]$, the synthesized trajectory of the suspension point (boom head) $[x_t, y_t]$, and actual payload trajectory $[x_{\text{out}}, y_{\text{out}}]$ at $T_{kon} = 20$ s. For comparison, Fig. 4e shows all the trajectories mentioned for Fig. 4d, but for the displacement time of $T_{kon} = 40$ s, providing that all other variables remain constant (by default). Furthermore, Fig. 4 f-j shows the maximum absolute errors of the entire payload trajectory $\Delta_{\text{max}\Sigma}$ and residual absolute errors $\Delta_{\text{max}r}$ implemented on the crane depending on the moving time $T_{kon}$ (Fig. 4f), initial angle of the boom lift $q_0$ (Fig. 4g), hoist rope length $L_{gk}$ (Fig. 4h) and overall dimensions of the predefined payload trajectory arc $X_A$ (Fig. 4i) and $Y_A$ (Fig. 4j).

5. Results discussion
Comparing the graphs of the actual payload trajectories represented in Fig. 4d and Fig. 4e, as well as the analysis of the functional dependencies shown in Fig. 4f reveals that with an increase in the displacement time of the suspension point over 40 s, the residual absolute error $\Delta_{\text{max}r}$ decreases to the insignificant values (less than 0.01 m). At the same time, when the displacement time of the suspension point increases to the maximum value (55 s) in the considered range, the maximum absolute error on the entire trajectory $\Delta_{\text{max}\Sigma}$ does not decrease below 0.06 m. Changing the initial value of the boom inclination angle $q_0$ causes the local minima of the maximum absolute error of the entire trajectory $\Delta_{\text{max}\Sigma}$ and the residual absolute error $\Delta_{\text{max}r}$ in the range $q_0$ from 1.1 to 1.2 rad. As the length of the hoist rope $L_{gk}$, as well as the overall dimensions of the arc of the given payload trajectory $X_A$ and $Y_A$ are increased, the errors $\Delta_{\text{max}\Sigma}$ and $\Delta_{\text{max}r}$ are also monotonically increased.

6. Conclusions
The conducted studies have shown that the method of generating the payload suspension upper point trajectory on a flexible rope suspension, which has been developed for the bridge cranes, is also possible to be used for the rotary cranes with a constant boom length. The method is based on the sequential solution of the linearized differential equation of the payload deviations in the plane for two directions of movement in the space. The advantage of the method is the simplicity and low computational complexity, without the use of a sophisticated mathematical tools. However, a curved trajectory of the suspension point in the horizontal plane is generated. Then it is transferred into the displacement trajectory of two controlled coordinates of the crane, such as the rotation angle of the rotary platform and the angle of the boom lift by means of the geometric relationships. The implementation of the obtained time dependences of the controlled crane coordinates by using the developed simulation model makes it possible to obtain the actual trajectory of the payload displacement in the horizontal plane, which differs from the predefined one. The resulting errors depend on the dimensions of the predefined payload trajectory, moving time, initial angle of the boom inclination, length of the hoist rope, and other parameters of the crane and process. With an increase in the displacement time of the crane moving parts over 40 s, the residual error of the horizontal payload coordinates at the end point (the residual oscillations of the payload) becomes insignificant. The results obtained could be of interest to the researchers involved in investigating the issues of automation of payload displacement control by means of the self-propelled crawler cranes. A future research direction is related to the improvement and adaptation of the method used for the rotary cranes with a constant boom length in order to increase the accuracy of the method.

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