Frustration phenomena in Josephson junction arrays on a dice lattice

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Abstract

AC magnetoimpedance measurements performed on proximity-effect coupled Josephson junction arrays on a dice lattice reveal unconventional behaviour resulting from the interplay between the frustration $f$ created by the applied magnetic field and the particular geometry of the system. While the inverse magnetoinductance exhibits prominent peaks at $f = 1/3$ and at $f = 1/6$ (and weaker structures at $f = 1/9, 2/9, 1/12, ...$) reflecting vortex states with a high degree of superconducting phase coherence, the deep minimum at $f = 1/2$ points to a state in which the phase coherence is strongly suppressed. These observations are discussed at the light of recent theoretical work in which the concept of accidental degeneracy plays a central role.

Key words: Josephson junctions arrays, frustration, accidental degeneracy

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Two-dimensional Josephson junction arrays (JJAs) exposed to a transverse magnetic field provide the opportunity to study the influence of a tunable level of frustration in systems with a variety of geometries ranging from periodic to random structures, including quasiperiodic and fractal lattices [1]. Such systems are usually considered as a physical realization of the frustrated classical XY model [2], where the degree of frustration is governed by a parameter $f$ expressing the magnetic flux threading an elementary cell, $\Phi_{\text{cell}}$, of the array in units of the superconducting flux quantum $\phi_0: f = \Phi_{\text{cell}}/\phi_0$.

In this contribution we study the interplay of frustration and geometry in proximity-effect coupled JJAs on an unconventional lattice, the dice lattice shown in Fig.1, by measuring the complex sheet impedance $Z(T, \omega, f) = R + i\omega L$ of the system with a SQUID-operated two-coil mutual inductance technique [3]. The inverse sheet inductance $L^{-1} = \text{Im}[Z]/\omega$, which is proportional to the areal superfluid density, measures the degree of superconducting phase coherence in the sample, and the sheet resistance $R$ reflects dissipative processes. Inverse magnetoinductance $L^{-1}(f)$ and magnetore-
Fig. 1. SEM picture of a portion of a JJA on a dice lattice. The superconducting lead islands are Josephson-coupled by an underlying Cu layer (dark ground plane). The elementary cell is rhombic in shape (dashed line) and has a side $a = 8 \mu m$.

The prominent peaks appearing in $L^{-1}(f)$ at $f = 1/3$ and $f = 1/6$ point to vortex states with a high degree of superconducting phase coherence, and are robust against thermal fluctuations: for instance, the height of the structure in $L^{-1}(f)$ at $f = 1/3$ changes only by a factor two in the temperature range covered by the data of Fig.2. Weaker peaks in $L^{-1}(f)$ at $f = 1/9, 2/9, 1/12$ are also a manifestation of vortex states with an appreciable degree of phase coherence, however more vulnerable to thermal fluctuations. This is easily understood if one realizes that the corresponding periodic ground states consist of unit cells larger than those for $f = 1/3$ and $f = 1/6$, thereby implying superconducting phase coherence to extend at larger length scales and, therefore, to be less robust against thermal fluctuations.

The resistance $R(f)$ isotherms are shown in Fig.2, $\tau = k_B T/J(T)$ being the relevant reduced temperature expressed in terms of the Josephson coupling energy.

In striking contrast with the behaviour at $f = 1/3, 1/6, 1/9...$, the deep minimum in $L^{-1}(f)$ at full frustration ($f = 1/2$) indicates a strong suppression of the phase coherence, hence a state quite vulnerable to thermal fluctuations: in the temperature range of the data shown in Fig.2, the strength of the dip changes by at least two orders of magnitude. As expected, the $R(f)$ curves show absolute maxima at $f = 1/2$ and local minima at $f = 1/3, 2/9, 1/6, 1/9, 1/12...$, corresponding, respectively, to dips and peaks in $L^{-1}(f)$. 

Fig. 2. Magnetoresistance isotherms (top) and inverse magnetoinductance isotherms (bottom) for a JJA on a dice lattice measured at an excitation frequency $\omega/2\pi = 7.03Hz$. 

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- $T = 4.6K (\tau = 0.08)$
- $T = 4.7K (\tau = 0.10)$
- $T = 4.8K (\tau = 0.12)$

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In order to understand this unusual behaviour, we compare the occurrence of (i) vortex ordering and (ii) superconducting phase coherence at $f = 1/2$ and $f = 1/3$, the phase transitions (i) and (ii) being driven by different topological excitations.

Allowing for the formation of zero-energy domain walls (DWs), the ground state (GS) at full frustration ($f = 1/2$) exhibits a well developed accidental (i.e. not related to symmetry) degeneracy [4]. Within the framework of the uniformly frustrated XY model, this degeneracy is removed by considering the free-energy difference (due to small-amplitude thermal fluctuations) between ground states corresponding to different periodic vortex patterns [5]. This order-from-disorder mechanism is so weak that a phase transition, rather than a genuine phase transition, is expected, the crossover temperature $\tau_\omega$ depending on the time scale $1/\omega$ of our measurements. The Arrhenius plot of Fig.3 is clearly consistent with this interpretation. The crossover temperature $\tau_\omega$ separates the frequency-independent exponential behaviour of $R(\tau)$ characteristic of a vortex liquid at $\tau > \tau_\omega$ from the frequency-dependent frozen-liquid regime at $\tau < \tau_\omega$ [6].

![Fig. 3. Sheet resistance versus inverse reduced temperature measured over a wide frequency range at full frustration ($f = 1/2$).](image)
It should be noticed that our interpretation of the $f = 1/2$-anomaly in JJAs on a dice lattice relies on the idea of accidental degeneracy and is therefore fundamentally different from the Ginzburg-Landau mean-field treatment [7] invoked in Ref.[8] to explain the depression of $T_c$ and of the critical current in fully frustrated superconducting wire networks on the same lattice. The existence of an analogous accidental degeneracy in such systems has been demonstrated in Ref.[9].

Within the framework of the uniformly frustrated XY model, at $f = 1/3$ the accidental degeneracy of the GS is so well developed that the vortex pattern is predicted [10] to remain disordered at very low temperature. Nevertheless, at low temperatures the system is characterized by a non vanishing helicity modulus, a behaviour consistent with the presence of a pronounced superfluid peak in $L^{-1}(f)$ at $f = 1/3$ (see Fig.2). At low temperature, superconducting phase coherence is stabilized by pairs of bound fractional vortices and antivortices carrying a half-integer topological charge [10]. Thus, the superconducting-to-normal phase transition at $f = 1/3$ is expected to be completely analogous to the Berezinskii-Kosterlitz-Thouless (BKT) transition of the unfrustrated ($f = 0$) system, and will be discussed in detail in a subsequent paper. A uniformly frustrated JJA on a dice lattice at $f = 1/3$ is therefore a unique example of 2D superconductor where quasi-long-range phase coherence coexists, at any temperature, below the BKT transition, with a disordered vortex pattern due to the proliferation of domain walls.

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References

[1] P. Martinoli and Ch. Leemann, J. Low Temp. Phys. 118, 699 (2000), and references cited therein

[2] S. Teitel and C. Jayaprakash, Phys. Rev. B 27, 598 (1983)

[3] B. Jeanneret, J.L. Gavilano, G.-A. Racine, Ch. Leemann, and P. Martinoli, Appl. Phys. Lett. 55, 2336 (1989)

[4] S.E. Korshunov, Phys. Rev. B 63, 134503 (2001)

[5] S.E. Korshunov, Phys. Rev. B 71, 174501 (2005)

[6] M. Calame, S.E. Korshunov, Ch. Leemann, and P. Martinoli, Phys. Rev. Lett. 86, 3630 (2001)

[7] J. Vidal, R. Mosseri, and B. Douçot, Phys. Rev. Lett. 81, 5888 (1998)

[8] C.C. Abilio, P. Butaud, Th. Fournier, B. Pannetier, J. Vidal, S. Tedesco, and B. Dalzotto, Phys. Rev. Lett. 83, 5102 (1999)
[9] S.E. Korshunov and B. Douçot, Phys. Rev. B 70, 134507 (2004)

[10] S.E. Korshunov, Phys. Rev. Lett. 94, 087001 (2005)