On the magnetotransport of 3D systems in quantizing magnetic field

M. V. Cheremisin
A.F.Ioffe Physical-Technical Institute, St.Petersburg, Russia

(Dated: November 2, 2018)

The resistivity components of 3D electron gas placed in quantizing magnetic field are calculated taking into account the correction caused by combined action of the Peltier and Seebeck thermoelectric effects. The longitudinal, transverse and the Hall magnetoconductivities exhibit familiar 1/H-period oscillations being universal functions of magnetic field and temperature.

PACS numbers: 73.50.Jt, 73.40.Hm, 73.61.Ey

The electronic transport of 3D solids subjected to strong magnetic field has been intensively studied since the observation of the Schubnikov-de Haas oscillations[1]. The most of the interest concerned the transverse (i.e. \( \perp B \)) magnetotransport. The problem was considered semiclassically[2,3] and, then, using the rigorous quantum mechanical approach[4,5]. With the help of density matrix equation the components of the conductivity tensor associated with transverse magnetotransport was obtained in Ref.[4]. Despite this progress, the incorporation of the thermal effects faced the strong difficulties[6-9]. As it was demonstrated for the first time in Ref.[10], in quantizing magnetic field it is necessary to take into account the diamagnetic surface currents. Neglecting dissipation the transverse thermal current and energy flux are given by the sum of bulk and surface components. With the diamagnetic currents accounted[10] the transport coefficients satisfies the phenomenological Einstein and Onsager relationships.

I. GENERAL FORMALISM

The main goal of the present paper concerns the peculiar thermodynamic approach[11-13] regarding 3D electron transport in strong magnetic fields. We will follow the argumentation put forward by Kirby and Laubitz [11], and then modified in Refs.[12,13] for magnetotransport in 2D electron(hole) systems. This approach allows one to account for both the Schubnikov-de Haas Oscillations and Integer Quantum Hall Effect[14] modes. Let us consider the isotropic sample subjected in strong magnetic field \( B = B_z \). Neglecting spin splitting, the 3D energy spectrum yields[15]

\[
\varepsilon_n = \hbar \omega_c (n + 1/2) + \varepsilon_z,
\]

where \( n = 0, 1, \ldots \) is the LL number, \( \omega_c = eB/mc \) is the cyclotron frequency, \( m \) is the electron effective mass. Then, \( p_z \) and \( \varepsilon_z = p_z^2/(2m) \) are the momentum and the kinetic energy respectively of an electron moving along the magnetic field. The electron motion in x-y plane is quantized, thus results in discrete Landau level set. For simplicity, we further neglect the broadening of the Landau levels. The density of states associated with the certain LL obeys the textbook result \( \Gamma = 1/2\pi l_B^2 \), where \( l_B = (\hbar eB)^{1/2} \) is the magnetic length. In the present paper we mostly interest in the strong quantum limit case, when \( \hbar \omega_c >> kT \).

In general, the macroscopic current, \( \mathbf{j} \), and the energy flux, \( \mathbf{q} \), densities are given by[16]

\[
\mathbf{j} = \hat{\sigma} (\mathbf{E} - \alpha \nabla T), \quad \mathbf{q} = (\alpha T - \zeta/e) \mathbf{j} - \hat{k} \nabla T.
\]

Here, \( \mathbf{E} = \nabla \zeta/e \) is the electric field, \( \mu \) and \( \zeta = \mu - e\varphi \) are the chemical and electrochemical potential respectively, \( \alpha \) is the thermopower. Then, \( \hat{\sigma} \) and \( \hat{\kappa} = LT\hat{\sigma} \) are the conductivity and thermal conductivity tensors respectively, \( L = \frac{e^2 k_T}{\sigma} \) is the Lorentz number. It will be recalled that Eq.(2) is valid for a confined-topology sample for which the diamagnetic surface currents [10] are taken into account. Both the Einstein and Onsager relationships are satisfied. The movement of the electron along the magnetic field is not quantized, thus results in Drude longitudinal component \( \sigma_z = N e^2 \tau/m \) of the conductivity tensor, where \( N \) is the 3D electron density. In what follows we assume the constant momentum relaxation time \( \tau \) in z-direction. Compared to unperturbed movement of an electron along the magnetic field, the transverse drift of electrons in x-y plane results in off-diagonal Hall components[4] \( \sigma_{yx} = -\sigma_{xy} = N e^2 \tau/m \) of the conductivity tensor. Then, in contrast to previous studies[2-5] we suggest that in strong quantum limit \( \sigma_{xx} = 0 \). Actually, the x-y plane related fraction of Eq.(2) can be viewed[7] as the current and energy dissipationless fluxes caused by electron drift in crossed fields. With the help of the above notations, the resistivity tensor \( \hat{\rho} = \hat{\sigma}^{-1} \) obeys the same symmetry, namely \( \rho_{yx} = -\rho_{xy} = 1/\sigma_{yx} \), \( \rho_{zz} = 1/\sigma_{zz} \). The crucial idea of the present paper is that a non-zero transverse magnetoconductivity \( \rho \) can, nevertheless, arise due to combined action of the Peltier and Seebeck thermoelectric effects[11-13].

II. RESULTS AND DISCUSSION

Let us discuss in details the typical experimental setup allowed to measure the Hall resistivity \( \rho_{xy} \) and transverse magnetoconductivity \( \rho \). The standard Hall-bar geometry sample (Fig.4 inset) is connected to the current source by means of two identical leads. The contacts "a" and...
"b" are ohmic. The longitudinal voltage is measured between the open ends ("e" and "d"), maintained at the ambient temperature. The sample is placed in a chamber (not shown in Fig. 1) inset) kept at the bath temperature $T_0$. Note that the electron temperature may, in principle, exceed the bath temperature if the electron-phonon coupling is weak at low temperatures. Actually, the 3D electron gas cooling can occur predominantly through the contacts of the sample and the leads connected to them. However, the heat leakage via metal leads can be disregarded similar to that reported[17] for two-dimensional electron case. Finally, we will consider the adiabatic cooling of 3D electrons.

Recall that the Peltier heat is generated by a current across the interface between two different conductors. At the contact (e.g. "a" in Fig. 1 inset), the temperature, $T_a$, electrochemical potential $\zeta$, normal components of the total current, $I$, and the total energy flux are continuous. There exists a difference $\Delta \alpha = \alpha_m - \alpha$ between the thermoelectric powers of the metal conductor and 3D sample, respectively. Then, $Q_a = I \Delta \alpha T_a$ is the amount of Peltier heat released per unit time in contact "a". For $\Delta \alpha > 0$ and current flow direction shown in Fig. 1, the contact "a" is heated and contact "b" is cooled. The contacts are at different temperatures, and $\Delta T = T_a - T_b > 0$. At small currents, the temperature gradient is small and $T_{a,b} \approx T_0$. In this case the 3D sample thermopower $\alpha$ is nearly constant, hence, one can disregard the Thompson heating $\sim IT\nabla \alpha$ in 3D bulk. Note that the amount of the Peltier heat evolved at the contacts of the sample and the leads connected to them.

We now intend to find the actually measured longitudinal voltage is $\rho^{\parallel} \nabla_x T|_{a,b} = -j \Delta \alpha T_{a,b}$. Here, we take into account that the current is known[18] to enter and leave the sample at two diagonally opposite corners (Fig. 1 inset). One can finitamente find the longitudinal temperature gradient $\nabla_x T = -\frac{\Delta \alpha}{\rho^{\parallel} \omega}$, which is linear in current. Omitting the contribution of the conductor resistances, the voltage drop, $U$, measured between the ends "e" and "d" is equal to Seebeck thermoelectromotive force $U = \int E_x dx = \int \alpha^T dt = \Delta \alpha (T_a - T_b)$, thus gives the transverse magnetoresistivity $\rho = U/jl = (\Delta \alpha)^2 \rho^{\perp}/L$.

We now intend to find the actually measured longitudinal resistivity taking into account the Peltier-Seebeck thermoelectric effects. Since the longitudinal movement of an electron unaffected by magnetic field, we make use of the result[11] valid for $B = 0$. The longitudinal resistivity of 3D electron gas acquire the correction $\Delta \rho^{xx} = (\Delta \alpha)^2 \rho^{xx}/L$. Finally, we summarize the actually measured values of the 3D electron gas resistivities as it follows

$$\rho^{xx} = \frac{B}{N c \epsilon}, \quad \rho = \sigma \rho^{xx}, \quad \rho^{zz} = (1 + s) \frac{\rho^{xy}}{\omega \epsilon T}, \quad (3)$$

where we take into account that for the actual case of the metal leads $s = (\Delta \alpha)^2/L \simeq \alpha^2/L$. Remind that in the strong quantum limit, the thermopower of dissipationless 3D electron gas is a universal thermodynamic quantity proportional to the entropy per electron[10]:

$$\alpha = -\frac{S}{eN}, \quad \tag{4}$$

where $S = -\left( \frac{\partial H}{\partial T} \right)_{\mu,B}$ is the 3D electron entropy, $N = \left( \frac{\partial H}{\partial \mu} \right)_{T,B}$ is the 3D electron density, $\Omega = -2kT \cdot \Gamma \sum_{n,p} \ln(1 + \exp((\mu - \varepsilon_n)/kT))$ is the thermodynamic potential, which accounts the spin degeneracy.

It is instructive to discuss the applicability of the Gibbs statistics formalism for actual 3D electrons case. The make use of thermodynamic potential presumes a variable number of 3D particles while the chemical potential is assumed to be a constant. We argue that this method is not prohibited since the chemical potential of 3D sample, respectively. Then, $N_a = I \Delta \alpha T_a$ is the amount of Peltier heat released per unit time in contact "a". For $\Delta \alpha > 0$ and current flow direction shown in Fig. 1, the contact "a" is heated and contact "b" is cooled. The contacts are at different temperatures, and $\Delta T = T_a - T_b > 0$.
purely discrete, thus leads to quantized Hall resistivity \( \rho_{yx} \), with the longitudinal resistivity \( \rho \) vanishing within the Hall plateaus[14].

We now make an attempt to compare our results with experimental observations. As expected, the longitudinal, transverse and the Hall components of magnetoresistivity exhibit familiar 1/H-period in-phase oscillations[20] associated with discrete LL energy spectrum. However, in contrast to our predictions the behavior of the longitudinal and the transverse magnetoresistivity components[20] becomes uncorrelated at high fields when the only few LLs are filled. The possible reason for the above discrepancy could be, for example, the constant momentum relaxation time, i.e. \( \tau \neq \tau(B) \), adopted in our simple model. For typical n-InSb sample \( n_0 = 1.2 \cdot 10^{16} \text{cm}^{-3} \) used in Ref.[20] we obtain \( \mu = 170 \text{K} \) and, then \( \rho_c = 0.1 \text{Ohm-cm} \). At filling \( \nu = 3/2 \) our zero spin-split approach provides(see Fig.1) the transverse magnetoresistivity amplitude \( \rho \simeq 0.002 \text{Ohm-cm} \) at liquid helium temperature \( \xi = 0.025 \). This value is consistent with that ~ 0.015 Ohm-cm observed experimentally[20] at \( B = 2.3 \text{T} \).

III. CONCLUSIONS

In conclusion, we have calculated the longitudinal, transverse and the Hall magnetoresistivities of a 3D solid placed in quantizing magnetic taking into account the contribution caused by combined action of Peltier and Seebeck thermoelectric effects. The transport coefficients demonstrate familiar 1/H-period oscillations being universal functions of the filling factor. The amplitude of the transverse resistivity is consistent with that observed in experiment.

IV. APPENDIX

Let us examine the charge relaxation and screening for 3D solids placed in magnetic field. These effects can be described by continuity equation

\[
\frac{\partial \rho_c}{\partial t} + \text{div} \mathbf{j} = 0. \tag{6}
\]

In quantizing magnetic field the non-dissipative transverse current specified by Eq. (2) can be re-written[21] as it follows \( \mathbf{j} = -\frac{e}{m} \mathbf{\Omega} \cdot \mathbf{B} \mathbf{\hat{c}} \). Thus, the charge relaxation is absent since \( \text{div} \mathbf{j} \equiv 0 \), therefore \( \partial \rho_c / \partial t = 0 \). The retardation effects, if accounted, could result in slow charge relaxation \( \sim (\alpha_{yz}/c)^2 \) analogous to that in 2D electron gas case[22]. We argue that the transverse length of the Debye screening could be comparable with the macroscopic( i.e. sample) length scale. We conclude that in quantizing magnetic field the electron plasma neutrality may be violated in 3D sample bulk.

FIG. 1: Dimensionless Hall resistivity \( \rho_{yx} \), transverse magnetoresistivity \( \rho(\text{both scaled by } \rho_c) \) and the longitudinal resistivity \( \rho_{zz}/\rho_{zz}(0) \) vs magnetic field \( B \sim \nu^{-1} \) for certain value of the dimensionless temperature \( \xi = 0.025 \). The dashed(dotted) line represents the classical result for Hall and longitudinal resistivity respectively. Inset: the experimental observations. As expected, the longitudinal, transverse and the Hall components of magnetoresistivity exhibit familiar 1/H-period in-phase oscillations[20] associated with discrete LL energy spectrum.
[1] L.V. Schubnikov and W.I. de Haas, Commun. Kemerlingh Onnes Lab, 207, 207a, 210a, 210b (1930)
[2] S. Titeica, Ann. Phys., 22, 128, (1935)
[3] B. Davidov and N. Pomeranchuk, ZhETF, 9, 1924 (1939)
[4] E.N. Adams and T.D. Holdstein, J.Phys. Chem. Sol., 10, 254 (1959)
[5] R. Kubo, H. Hasegawa and N. Hashitsume, J. Phys. Soc. Japan, 14, 56 (1959)
[6] A.I. Anselm and B.M. Askerov, Sov. Phys. Solid State, 2, 2330, 1960; ibid 3, 3668 (1961)
[7] P.S. Zyryanov and V.P. Silin, Sov. Phys. JETP, 19, 366 (1964)
[8] A.I. Akhiezer, V.G. Bar' yakhtart and S.V. Peletinskii, Sov. Phys. JETP, 21, 136 (1965)
[9] S.V. Peletinskii and V.G. Bar' yakhtart, Sov. Phys. Solid State, 7, 446 (1965)
[10] Yu.N. Obraztsov, Sov. Phys. Solid State, 6, 331, 1964 ibid 7, 573 (1965)
[11] G.G.M. Kirby and M.J. Laubitz, Metrologia, 9, 103 (1973)
[12] M.V. Cheremisin, Sov. Phys. JETP, 92, 357 (2001)
[13] M.V. Cheremisin, Physica E, 28, 393, 2005; ibid 64, 15 (2014)
[14] K. von Klitzing, G. Dorda and M. Pepper, Phys. Rev. Lett., 45, 494 (1980)
[15] L. Landau, Zs. Phys., 64, 629 (1930)
[16] L.D. Landau, E.M. Lifshits, Electrodynamics of Continuous Media, Pergamon, New York, (1966)
[17] A. Mittal et al., Physica B, 194-196, 167 (1994)
[18] A. H. Thompson and G.S. Kino, J. Appl. Phys. 41, 3064 (1970)
[19] I.M. Lifshitz and A.M. Kosevich, Sov. Phys. JETP, 29, 730 (1955)
[20] F. A. Egorov and S.S. Murzin, Sov. Phys. JETP, 67, 1045 (1988)
[21] F.S. Zyryanov, Fizika Tverdogo Tela, 6, 3562 (1964)
[22] A.O. Govorov and A.V. Chaplik, Sov. Phys. JETP, 68, 1143 (1989)