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Inclusive cross sections of proton-proton and proton-antiproton scattering

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Inclusive cross sections and multiplicity distributions are different for $pp$ and $p\bar{p}$

Pomeranchuk theorem:

- total cross sections
- differential elastic cross sections
- elastic cross sections

the same for $pp$ and $p\bar{p}$

It is also generally accepted that inclusive cross sections and multiplicity distributions are the same for $pp$ and $p\bar{p}$. 
Low Constituents Number Model (LCNM)

1. On the first step before the collision there is small number of constituents in initial hadrons. In every hadron this is component either with only valence quarks or with valence quarks and one additional gluon.

2. On the second step the hadrons interaction is carried out by gluon exchange between the valence quarks and initial gluons. The hadrons gain the color charge.

3. On the third step after interaction the colored hadrons move apart and when the distance between them becomes larger than the confinement radius, the lines of color electric field gather into the string. This string breaks out into secondary hadrons.

(Abramovsky, Kancheli 1980, Abramovsky, Radchenko 2009)
Three types of inelastic subprocesses

$p\bar{p}$

1 2 3

$pp$

1 2 3

gluon string

2 quark strings

3 quark strings

ISMD 2010
Results in LCNM

$\chi^2/\text{ndf} = 125/113$
- $p\ p$
- $p\bar{p}$

$\sigma_{\text{tot}} \text{ mb}$

$\ln(s)$

$53 \text{ GeV}$
$62 \text{ GeV}$

$546 \text{ GeV}$
$1800 \text{ GeV}$

$n$

$p^x$

$44.5 \text{ GeV}, \chi^2/\text{ndf} = 13/15$

$62.2 \text{ GeV}, \chi^2/\text{ndf} = 13/16$

$1800 \text{ GeV}, \chi^2/\text{ndf} = 126/115$

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Pseudorapidity density at LHC energies

\[ \left. \frac{dN_{\text{ch}}}{d\eta} \right|_{\eta=0} (14 \text{ TeV}) = 6.62 \pm 0.70 \]

- LCNM, nsd
- ALICE Coll., nsd
- CMS Coll., nsd
The importance of inclusive cross sections

- From the LCNM it follows, that multiplicity distributions are different for $pp$ and $p\bar{p}$ because of subprocess with three quark strings in $p\bar{p}$ (blue line).
- Three quark strings produce events with high multiplicity in tail of distribution.
- In order to make the difference more visible, we have to use observable $n \cdot P_n$ instead of $P_n$.
- $n \cdot P_n$ is measured independently of $P_n$ in inclusive measurements.
Inclusive cross sections in events with fixed number of particles

Topological inclusive cross section of one charged particle production

\[ \Phi \pi^{\circ} E \frac{d^3 \sigma^\text{incl}}{d^3 p} = \frac{1}{\Phi} \sum_{m=0}^{\infty} \frac{1}{m!} \int d\tau_{n-1+m} |A_{2\to n+m}|^2 \]

Total inclusive cross section and its normalization

\[ \Phi \pi^{\circ} E \frac{d^3 \sigma^\text{incl}}{d^3 p} = \sum_{n=0}^{\infty} \Phi \pi^{\circ} E \frac{d^3 \sigma^\text{incl}}{d^3 p} \quad \int d^3 p \frac{d^3 \sigma^\text{incl}}{d^3 p} = \langle n \rangle \sigma^{\text{nsd}} \]

Normalization of topological inclusive cross section

\[ \int d^3 p \frac{d^3 \sigma^\text{incl}}{d^3 p} = n \sigma_n \quad \sigma_n = \frac{1}{n!} \sum_{m=0}^{\infty} \frac{1}{m!} \int d\tau_{n+m} |A_{2\to n+m}|^2 \]

\[ \frac{1}{\sigma^{\text{nsd}}} \int d^3 p \frac{d^3 \sigma^\text{incl}}{d^3 p} = n \quad \frac{\sigma_n}{\sigma^{\text{nsd}}} = n P_n \]
Inclusive cross sections in bins

UA5 Coll. gave data in 9 bins of charged multiplicities: $2 \leq n \leq 10$, $12 \leq n \leq 20$, … $72 \leq n \leq 80$ and $n \geq 82$.

We define inclusive cross section in this bins

$$\frac{d^3\sigma^{(1)\text{ incl}}}{d^3 p} = \sum_{n=2}^{10} \frac{d^3\sigma^{\text{incl}}_n}{d^3 p}, \quad \frac{d^3\sigma^{(2)\text{ incl}}}{d^3 p} = \sum_{n=12}^{20} \frac{d^3\sigma^{\text{incl}}_n}{d^3 p}, \quad \ldots \quad \frac{d^3\sigma^{(9)\text{ incl}}}{d^3 p} = \sum_{n=82}^{\infty} \frac{d^3\sigma^{\text{incl}}_n}{d^3 p}, \quad \Rightarrow$$

$$\sum_{i=1}^{9} \frac{d^3\sigma^{(i)\text{ incl}}}{d^3 p} = \frac{d^3\sigma^{\text{incl}}}{d^3 p}$$

Inclusive cross sections in bins are normalized as follows

$$\int d^3 p \frac{d^3\sigma^{(i)\text{ incl}}}{d^3 p} = \sigma^{nsd} \sum_{n \text{ in bin}} n P_n = \bar{n}^{(i)\text{ incl}} \sigma^{nsd}$$
Difference in inclusive cross sections of $pp$ and $p\bar{p}$

\[ \int d^3 p \frac{d^3 \sigma^{(i) \text{ incl}}_{pp}}{d^3 p} = \int d\eta \, d^2 p_{\perp} \frac{d^3 \sigma^{(i) \text{ incl}}_{pp}}{d\eta \, d^2 p_{\perp}} = \int d\eta \, \frac{d\sigma^{(i) \text{ incl}}_{pp}}{d\eta} = \frac{n^{(i)}_{pp}}{n^{(i)}_{p\bar{p}}} \sigma^{\text{nsd}} \] (1)

\[ \int d^3 p \frac{d^3 \sigma^{(i) \text{ incl}}_{p\bar{p}}}{d^3 p} = \int d\eta \, d^2 p_{\perp} \frac{d^3 \sigma^{(i) \text{ incl}}_{p\bar{p}}}{d\eta \, d^2 p_{\perp}} = \int d\eta \, \frac{d\sigma^{(i) \text{ incl}}_{p\bar{p}}}{d\eta} = \frac{n^{(i)}_{p\bar{p}}}{n^{(i)}_{p\bar{p}}} \sigma^{\text{nsd}} \] (2)

\[ \frac{d\sigma^{(i) \text{ incl}}}{d\eta} = \int d^2 p_{\perp} \frac{d^3 \sigma^{(i) \text{ incl}}}{d\eta \, d^2 p_{\perp}} \]

Ratio of (1) to (2) gives

\[ \int d\eta \, \frac{d\sigma^{(i) \text{ incl}}_{pp}}{d\eta} = \frac{n^{(i)}_{pp}}{n^{(i)}_{p\bar{p}}} \int d\eta \, \frac{d\sigma^{(i) \text{ incl}}_{p\bar{p}}}{d\eta} \] (3)

Solution of the integral equation (3) (perhaps, the only solution)

\[ \frac{d\sigma^{(i) \text{ incl}}_{pp}}{d\eta} = \frac{n^{(i)}_{pp}}{n^{(i)}_{p\bar{p}}} \frac{d\sigma^{(i) \text{ incl}}_{p\bar{p}}}{d\eta} \] (4)
Inclusive cross sections in different bins

| Bin          | $\frac{n^{(i)}_{\bar{p}p}}{n^{(i)}_{pp}}$ |
|--------------|-----------------------------------------|
| $2 \leq n \leq 10$ | 0.76 0.01 |
| $12 \leq n \leq 20$ | 0.86 0.01 |
| $22 \leq n \leq 30$ | 0.99 0.01 |
| $32 \leq n \leq 40$ | 1.09 0.01 |
| $42 \leq n \leq 50$ | 1.10 0.01 |
| $52 \leq n \leq 60$ | 1.18 0.01 |
| $62 \leq n \leq 70$ | 1.35 0.02 |
| $72 \leq n \leq 80$ | 1.45 0.02 |
| $n \geq 82$     | 1.26 0.02 |
Inclusive cross sections summed over all bins

We obtained the numerical value of relation

\[
\frac{\frac{d\sigma^{incl}_{pp}}{d\eta}}{\frac{d\sigma^{incl}_{pp}}{d\eta}}
\]

by summing over all nine bins of multiplicity.

For \( |\eta| < 2.5 \)

\[
\frac{d\sigma^{incl}_{pp}}{d\eta} \bigg|_{|\eta| < 2.5} = 1.12 \pm 0.01
\]

This result will be used on the next slides.
Inclusive cross section with transverse momentum

From the AGK cancellation rules it follows the factorization of transverse dependence in inclusive cross section.

\[
\frac{1}{2\pi} \frac{d^2 \sigma^{incl}}{p \, d\eta \, dp} = f(p) \frac{d \sigma^{incl}}{d\eta}
\]

Returning to formulas (1) and (2) we can write down

\[
\frac{d^3 \sigma^{(i) incl}}{pp} \frac{d^3 \sigma^{(i) incl}}{pp} = \frac{\bar{n}^{(i)}}{pp} \frac{d^3 \sigma^{(i) incl}}{pp} \frac{d^3 \sigma^{(i) incl}}{pp}
\]

From this relation it can strictly be proved that

\[
f_{pp} \, \sim \, f_{p\bar{p}} \, \sim \, f_{p\bar{p}}
\]

so we can obtain

\[
\frac{1}{2\pi} \frac{d^2 \sigma^{incl}}{p \, d\eta \, dp} \bigg/ \frac{1}{2\pi} \frac{d^2 \sigma^{incl}}{pp} = \frac{d \sigma^{incl}}{pp} \bigg/ \frac{d \sigma^{incl}}{pp}
\]
Experimental evidences of difference in \( pp \) and \( p\bar{p}p \)

**ATLAS Coll.**

**ALICE Coll.**
Conclusion

• We think that ATLAS Coll. discovered a new effect – the difference in multiparticle production in $pp$ and $p\bar{p}$ interactions. ALICE Coll. confirmed this effect.

• We propose to experimentalists to measure inclusive cross section in bins with high multiplicities, where the difference will be the most evident.
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