Explicit and accurate characterization of hardening-softening behavior of metals up to failure

Z H Xu¹, S Y Wang¹, L Zhan¹, H F Xi¹,²,³ and H Xiao¹,³

¹School of Mechanics & Construction Engineering and MOE Lab for Disaster Forecast & Control in Engineering, Jinan University, West Huangpu Avenue 601, 510632 Guangzhou, PR China
²State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi’an Jiaotong University, 710049 Xi’an, PR China

E-mail: xihuifeng@jnu.edu.cn (H F Xi); hxiao@jnu.edu.cn (H Xiao)

Abstract. Full and accurate characterization of metal hardening-softening effects till failure is carried out with a new explicit approach. To this purpose, a new expression for the plastic work is presented explicitly in terms of the uniaxial stress-strain function of any given form. Accordingly, the yield strength as function of the plastic work can be determined jointly from the above functions with the axial logarithmic strain as parametric variable. Using the proposed approach, any test data obtained within the entire stress-strain range with hardening and softening portions up to failure can be fitted by directly treating a suitable form of the uniaxial stress-strain function. This allows one to bypass usual tedious trial-and-error numerical procedures in dealing with nonlinear elastoplastic rate equations for identifying numerous unknown parameters. Accurate simulation results are shown with numerical examples.

1. Introduction

A comprehensive characterization of hardening and softening of metals should be based on the premise that it should accurately reproduce the uniaxial experimental data over the whole stress-strain range up to failure. It does not appear that a full treatment is available in the just-mentioned sense now. In the past decades, two approaches have been proposed toward characterizing the hardening effects of metals. With restrictions to the hardening range, these approaches either introduced certain presumed forms for the yield strength function directly [1-2] or assumed certain ad hoc formulas for the evolution rules of the yield strength [3]. In particular, simple functions of power form are usually used. Details may be found in the survey articles [4-5]. Since the history-dependent plastic work is coupled with each specific elastoplastic process and hence with all the elastoplastic constitutive equations, toward the latter purpose, tedious trial-and-error numerical procedures have to be performed in repeatedly treating nonlinear elastoplastic rate equations.

In the present study, we will propose a new and explicit approach to fully and accurately characterize metal hardening-softening effects till failure. To this goal, a new approach for determining the plastic work function via strain will be presented explicitly in terms of the uniaxial stress-strain function of any given form. Accordingly, the function of yield strength with respect to the plastic work can be obtained jointly by the just-mentioned two functions as a parametric function via the axial logarithmic strain. With this new approach, any given test data over the entire range from hardening to softening up to failure can be fitted by directly treating a suitable form of the uniaxial stress-strain function, thus
bypassing the tiresome trial-and-error numerical procedures in the forgoing. Accurate simulation results will be shown with numerical examples.

2. Finite strain elastoplastic J2-flow equations

Toward covering the whole range till failure, finite strain effects become essential and should be taken into consideration. Here, attention is directed to the self-consistent Euler rate formulation of finite elastoplasticity [6-8].

The starting point is the additive decomposition of the stretching $D$, as shown below:

$$ D = D^e + D^p, $$

(1)

In the above, $D^e$ and $D^p$ are the elastic and the plastic part, respectively. The elastic rate equation for $D^e$ is as follows:

$$ D^e = \frac{1}{2G} \dot{\varepsilon} \log + \frac{1}{E} (\text{tr} \dot{\varepsilon} \log) I, $$

(2)

where $G$, $\nu$ and $E$ are shear modulus, Poisson’s ratio, and the Young modulus, respectively; $I$ is the identity tensor; and $\dot{\varepsilon} \log$ is the co-rotation logarithmic rate of the Kirchhoff stress, i.e., $\tau = J \sigma$. Here, $J$ and $\sigma$ are the volumetric ratio and the Cauchy stress, respectively.

Moreover, the plastic rate equation, i.e., the flow rule, is of the form:

$$ D^p = \rho \frac{f}{h} \nabla^\tau, $$

(3)

where $\rho$ is the plastic index taking values 1 and 0 for the loading case and the unloading case, respectively; $f$ is the von Mises yield function, viz.

$$ f = \frac{1}{2} \text{tr} \tau^2 - \frac{1}{3} q^2, $$

(4)

Here $\tau$ is the deviatoric part of the Kirchhoff stress, $q$ is the yield strength, which is a function with respect to the plastic work $\kappa$.

$$ q = q(\kappa), $$

(5)

The latter value $\kappa$ figures in the following evolution equation:

$$ \dot{\kappa} = \tau : D^p, $$

(6)

Finally, the loading function and the plastic modulus in Equation (3) are as follows:

$$ \ddot{f} = \tau : \dot{\varepsilon}, \quad \ddot{h} = \frac{4}{9} q^3 q'(\kappa), $$

(7)

Henceforth, $\text{tr} A = A_{11} + A_{22} + A_{33}$ and $A : B = \text{tr} (AB)$.

3. Determination of the yield strength via parametric variable

We first derive an direct equation of the plastic work. Let the function $\tau = \varphi (h)$ represents the uniaxial stress-strain curve, with $\tau$ the uniaxial stress and $h$ the uniaxial logarithmic strain, respectively. Then, from the reduced forms of Equations (1-6) in the uniaxial case, we obtain the following expression:
\[
\kappa = \int_{h_0}^{h} \phi(h) dh - \frac{1}{2E} (\phi(h)^2 - q_0^2) \quad (\equiv \psi(h)),
\]

(8)

As such, two equations of the uniaxial logarithmic strain \( h \) are available, namely

\[
\begin{cases}
q = \phi(h) \\
\kappa = \psi(h)
\end{cases}
\]

(9)

Here, the new idea is as follows: with the uniaxial logarithmic strain \( h \) as the parameter variable, the above two supply two parameter-variable equations which uniquely determine the yield strength \( q = q(\kappa) \) as a function with respect to the plastic work \( \kappa \). With such a function for the yield strength, the elastoplastic equations given can automatically simulate any given test data for the uniaxial stress-strain responses, whenever the uniaxial stress-strain function \( \tau = \varphi(h) \) is chosen to fit such data. This will be done below.

4. Stress-strain function for hardening-softening features till failure

Next, we present the stress-strain function \( \tau = \varphi(h) \) that can represent the hardening-softening features over the whole range till failure as follows:

\[
\phi(h) = \frac{1}{2} q_0 \left( \left[ 1 + (a + d_0 h) \tanh [m(h-h_0)] \right] \left[ 1 - \tanh [n(h-h_t)] \right] \right),
\]

(10)

where \( h_0 = q_0/E \) with the initial yield strength \( q_0 \). In the above, the three parameters \( a, d_0 \) and \( m \) characterize hardening effects, while the parameter \( n \) represent softening effects up to failure. Details are explained as follows: parameter \( a \) specifies the maximum stress, parameter \( m \) controls the hardening effect at the initial yield stage, parameters \( m \) and \( d_0 \) characterize the subsequent hardening effect, and, finally, parameters \( n \) and \( h_t \) represent softening effects up to failure.

Appropriate values of these parameters can be determined in such a manner that stress-strain curve over the whole range till failure is fitted, as will be illustrated by numerical examples given below.

5. Numerical examples

Test data for UFG Cu under the conditions of 8 and 16 passes, CG Cu and SC Cu under the condition of 1 pass given in literature [9] are taken into consideration. The value for the Young modulus is \( E = 320 \text{GPa} \). Other parameter values are given in Table 1. The numerical results are depicted in figures 1 and 3. The curves for the yield strength were constructed via Equation(9) and depicted in figures 2 and 4.

| Experiment sample | \( q_0 \) (MPa) | \( a \)  | \( d_0 \) | \( m \)  | \( h_0 \) | \( n \)  | \( h_t \) |
|-------------------|----------------|--------|--------|--------|--------|--------|--------|
| UFG Cu, 8 passes  | 310            | 0.635  | -5.35  | 90     | 0.0097 | 90     | 0.1235 |
| UFG Cu, 16 passes | 282            | 0.56   | -1.15  | 120    | 0.107  | 30     | 0.155  |
| CG Cu, 1 pass     | 241            | 0.39   | -2.5   | 160    | 0.0078 | 45     | 0.14   |
| SC Cu, 1 pass     | 275            | 0.09   | -7.2   | 1.85   | 0.0145 | 30     | 0.2    |

Table 1. Parameter values for experiments.
Figure 1. Simulation results compared to experimental data on UFG Cu of 8 passes in [9].

Figure 2. The yield strength with respect to plastic work for ultrafine grained Cu of 8 passes.

Figure 3. Simulation results compared to experimental data in [9].

Figure 4. The yield strength with respect to plastic work for SC CU, CG CU, and UFG CU.

6. Summary
As seen in figures 1 and 3, an accurate simulation was achieved with the proposed approach. The parameter values are obtained by directly fitting the stress-strain function in Equation (10) to the stress-strain data. Also, the yield strength curve in figures 2 and 4 was constructed using two formulas in Equation (9) with the uniaxial strain serving as the parametric variable. This made it possible to avoid cumbersome numerical procedures in handling elastoplastic rate equations.

In general, the stress-strain function $\tau = \varphi(h)$ in Equations (8-9) may be given by a suitable interpolating function precisely fitting the available stress-strain data. As a result, any data for the whole hardening-softening range up to failure may be fitted without involving any adjustable parameters. This perspective will be explored in our follow-up studies.
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