Manipulation of nanomechanical resonator via shaking optical frequency

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Abstract
In the usual optomechanical systems, the stability of the systems severely limits those researches of the macroscopic quantum effects. We study a usual cavity optomechanical system where the frequency of the optical mode is shaken periodically. We find that, when the optical shaking frequency is large enough, the shake of the optical mode can stabilize the system. That means we can study the macroscopic quantum effects of the mechanical resonator even in the strong coupling region where the standard optomechanical systems are always unstable. As examples, we study the ground-state cooling of the mechanical resonator and the entanglement between the optical and mechanical modes in the conventional unstable region, and the results indicate that the final mean phonon number and entanglement not only can be achieved but also can be modulated by the optical shaking parameters. Our proposal provides a method to study the macroscopic quantum effects even in conventional unstable region.

Keywords: optomechanics, frequency modulation, micromechanical resonator cooling, entanglement

1. Introduction

Cavity optomechanical system has made rapid advances in the past decades, which is mainly used to study the macroscopic quantum effects of the micromechanical resonators, such as the ground-state cooling of the mechanical resonator [1–8], mechanical squeezing [9–11], entanglement [12–15], macroscopic quantum superposition [16], etc. Owing to the unique advantages of optomechanical systems, numerous potential applications have been proposed, e.g. the ultrahigh precision metrology [17, 18], exploring the quantum–classical-mechanics boundary [19, 20], and studying the weak signal transducer. The progress of the gravitational-wave detection is a great example for the application of optomechanical systems [21].

In recent years, the periodically modulated optomechanical systems have attracted significant attention, which have been used to study various macroscopic quantum effects [22–26]. However, in those modulation proposals, most of them focus on modulating the driven laser field, which results in the first-order moments of the system operators and the effective optomechanical coupling changing periodically to achieve and study some quantum effects. On the other hand, the frequency modulated quantum systems also exhibit a rich behavior and display nonequilibrium properties that are absent in their static counterparts [27, 28], such as the phenomena of motional averaging and narrowing [29], Landau–Zener–Stückelberg–Majorana interference [30], and the formation of dressed states with the appearance of sidebands in the spectrum. However, in cavity optomechanical systems, the study of the influence coming from the frequency modulation is relatively rare to date [31–34]. In [32], the authors enhance the mechanical effects of single photons via introducing a resonance-frequency modulation to the two cavity fields in the cavity–membrane–cavity optomechanical system. We have studied the improved ground-state cooling of the
mechanical resonator via modulating both the optical and mechanical frequencies simultaneously [33]. In [35], the authors have investigated the bistability and multi-stability behaviors in optomechanical system including cross-Kerr effect. In addition, most of the previous researches about the macroscopic quantum effects are limited in the stable region of the usual optomechanical systems, which depends on the system parameters, especially the effective optomechanical coupling strength. So how to study the macroscopic quantum effects in unstable regions is of momentous significance. As far as we know, the approach of only modulating the optical frequency to study the steady macroscopic quantum effects in unstable region has not yet been reported.

In this paper, we study a usual cavity optomechanical system where the frequency of the optical mode is modulated periodically. As we all know, the stability of optomechanical systems is closely related to the effective optomechanical coupling strength. For an excessively large coupling strength, the optomechanical systems are unstable and the studying is also meaningless. However, we find that the shaking optical mode can reduce the effective optomechanical coupling strength arbitrarily when the shaking frequency is much larger than the mechanical resonator frequency, and the deeply physical mechanism can be explained by the Raman-scattering or frequency-domain pictures. The result indicates that it will be possible to study the steady quantum effects of optomechanical system even with strong coupling where the standard optomechanical systems without frequency modulation are always unstable. In order to verify the above analyses, we study the ground-state cooling of the mechanical resonator and the entanglement between the optical and mechanical modes when the effective optomechanical coupling belongs to the conventional unstable region of the standard optomechanical systems. Moreover, we also find that the final mean phonon number and the steady entanglement are related to the shaking parameters of the optical frequency, which indicates that we can manipulate the quantum effects of the mechanical resonator via changing the optical shaking parameters.

The paper is organized as follows: in section 2, we derive the linearized Hamiltonian of the optomechanical system with frequency modulation and explain the physical mechanism through the Raman-scattering and frequency-domain pictures. In section 3, we give the correlation matrix of the system operators. In section 4, we first study the ground-state cooling of the mechanical resonator with strong optomechanical coupling in the presence of the frequency modulation and discuss the effect of modulation parameters on the final mean phonon number. In section 5, we calculate the entanglement between the optical and mechanical modes by utilizing the logarithmic negativity and give the influence of different system parameters, respectively. Finally, a conclusion is given in section 6.

### 2. System and Hamiltonian

We consider a usual optomechanical system—including one mechanical resonator and one optical mode—in which the frequency of optical mode is cosine modulated. In the rotating frame at the driven laser frequency \( \omega_l \), the Hamiltonian of the system is given as (\( \hbar = 1 \))

\[
\begin{align*}
\hat{H} &= (\Delta_c + \xi \nu \cos(\nu t))a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) \\
&+ (Ea^\dagger + E^* a),
\end{align*}
\]

(1)

where \( a (b) \) and \( a^\dagger (b^\dagger) \) represent the annihilation and creation operators for optical (mechanical) mode with frequency \( \omega_c (\omega_m) \), respectively. \( \Delta_c = \omega_c - \omega_l \) is the original cavity laser detuning, \( \xi \) is the normalized modulation amplitude, \( \nu \) is the modulation frequency, \( g \) is the single photon optomechanical coupling strength, and \( E = \sqrt{2\kappa P}/(\hbar \omega) \) is the amplitude of the driven laser, where \( \kappa \) is the decay rate of the optical mode and \( P \) is the power of the driven laser. In the presence of frequency modulation, using the usual linearization approach (see appendix), e.g., \( a = \alpha + \delta a \) and \( b = \beta + \delta b \), we can derive the linearized Hamiltonian

\[
\begin{align*}
H_L &= [\Delta'_c + \xi \nu \cos(\nu t)]\delta a^\dagger \delta a + \omega_m b^\dagger b \beta \\
&- (G \delta a^\dagger + G^* \delta a)(bb^\dagger + \delta b),
\end{align*}
\]

(2)

where \( \Delta'_c = \Delta_c - \beta (\beta^* + G) \) and \( G = g \alpha \) is linearized optomechanical coupling strength. To show the effect from shaking the optical frequency clearly, we perform the rotating transformation defined by

\[
\begin{align*}
V_2 &= \mathcal{T}\exp \left\{ -i \int_0^t d\tau [\Delta'_c + \xi \nu \cos(\nu t)]\delta a^\dagger \delta a \\
&+ \omega_m b^\dagger b \beta \beta \right\} \\
&= \exp \left\{ -i [\Delta'_c t + \xi \nu \sin(\nu t)]\delta a^\dagger \delta a - i \omega_m t b b^\dagger \beta \beta \right\},
\end{align*}
\]

(3)

where \( \mathcal{T} \) denotes the time ordering operator. In the rotating frame defined by the transformation operator \( V_2 \), the transformed Hamiltonian is derived as

\[
\begin{align*}
\hat{H}_{LM} &= V_2^\dagger \hat{H}_L \hat{V}_2 - i \mathcal{V}_2 \mathcal{V}_2^\dagger \\
&= -G \delta a^\dagger b b^\dagger e^{i(\Delta'_c + \omega_m + \xi \nu \sin(\nu t))} \\
&+ \delta a^\dagger b b^\dagger e^{i(\Delta'_c - \omega_l + \xi \nu \sin(\nu t))} + \text{h.c.} \\
&= - \sum_{k=-\infty}^{\infty} [G J_k(\xi) \delta a^\dagger b b^\dagger e^{i(\Delta'_c + \omega_m + k \nu t)} + \text{h.c.}] \\
&+ G J_k(\xi) \delta a^\dagger b b^\dagger e^{i(\Delta'_c + \omega_m + k \nu t)} + \text{h.c.}.
\end{align*}
\]

(4)

The above derivation needs the Jacobi–Anger expansions: \( e^{i \xi \sin(\nu t)} = \sum_{k=-\infty}^{\infty} J_k(\xi) e^{i k \nu t} \), where \( J_k(\xi) \) is the Bessel function of the first kind with \( k \) being an integer. The physics process of equation (4) can be explained via the Raman-scattering picture figure 1(a) or the frequency-domain picture figure 1(b). Under the red detuning sideband resonant condition (\( \Delta'_c = \omega_m \)), \([n, m]\) denotes the state of \( n \) photons and \( m \) phonons in the displaced frame, the red arrow represents the beam-splitter interaction with the coupling strength \( G J_1(\xi) \) and detuning \( k \nu \), the blue arrow represents the two-mode squeezing interaction with coupling strength \( G J_1(\xi) \) and detuning \( 2\omega_m + k \nu \), the black arrow represents the driven laser, and the yellow arrows represent the leakage of the
optical mode. In Figure 1(b), the red (blue) peaks represent the beam-splitter (two-mode squeezing) interactions corresponding to different sidebands, respectively, which are discrete and have been separated with modulation frequency \( \nu \) space. We can see that the nearest resonant sideband \( (k = k_0) \) of all is the leading order in optomechanical interactions and the effective coupling strength \( G J_{k,k}(\xi) \) is modulated by parameter \( \xi \), independently. Moreover, if the modulation frequency \( \nu \) is large enough, the other sidebands will be negligible due to far from the resonator condition. Then the Hamiltonian (4) can be reduced to the standard optomechanical Hamiltonian with effective coupling \( G J_{k,k}(\xi) \), which is modified by the Bessel function of the first kind \( J_{k,k}(\xi) \) and infers that we can study the quantum effects of the optomechanical system via changing the optical shaking parameters.

3. Correlation matrix of the system

To study the dynamics and quantum effects of the system, we utilize the correlation matrix of the system operators as main processing method. Firstly, it is convenient to define the quadrature components of optical and mechanical modes, i.e.

\[
\begin{align*}
x &= (\alpha a^\dagger + \alpha^* a)/\sqrt{2}, \\
y &= i(\beta a^\dagger - \beta^* a)/\sqrt{2}, \\
q &= (\delta b^\dagger + \delta^* b)/\sqrt{2}, \\
p &= i(\delta b^\dagger - \delta^* b)/\sqrt{2}.
\end{align*}
\]

Similarly, the corresponding noise quadratures are given by

\[
\begin{align*}
x_n &= (a_n^\dagger + a_n)/\sqrt{2}, \\
y_n &= i(a_n^\dagger - a_n)/\sqrt{2}, \\
q_n &= (b_n^\dagger + b_n)/\sqrt{2}, \\
p_n &= i(b_n^\dagger - b_n)/\sqrt{2}.
\end{align*}
\]

Based on equation (A4), we derive and give the dynamical equation of those quadrature components in the compact form, i.e.

\[
u_j A \dot{u}_j = V_j + VA_j + D_j,
\]

where the diagonal diffusion matrix \( D \) is related to the noise correlations and given by

\[
D = \text{diag}[\kappa/2, \kappa/2, \gamma(2\omega_{\text{in}} + 1)/2, \gamma(2\omega_{\text{in}} + 1)/2].
\]

4. Sideband cooling beyond weak coupling limit

In the conventional optomechanical system, the ground-state cooling utilizing the red sideband \( (\Delta' = \omega_{\text{in}}) \) self-cooling method cannot be achieved when the optomechanical coupling is too large [38–42], where the system is unstable (see figure 2) and the dynamical evolution of the mean phonon number is divergent. Such as, the threshold of optomechanical coupling strength is almost \( |G| = 0.5\omega_{\text{in}} \) when the system is satisfied with the red sideband resonant condition \( \Delta' = \omega_{\text{in}} \). It indicates that the system is unstable when the optomechanical coupling strength is larger than \( 0.5\omega_{\text{in}} \) at this time. However, in the presence of frequency modulation, we prove that the ground-state cooling is a possible task even with arbitrarily
large optomechanical coupling and the final mean phonon number is related to the parameters of shaking optical mode.

To explain the physical mechanism of shaking optical mode in mathematics, we simplify equation (4) by using the rotating wave approximation (RWA). For large modulation frequency \(\nu \gg [\omega_m, G_0(\xi)]\), the RWA method is feasible for the Hamiltonian in equation (4). Ignoring those sidebands with large detuning and returning to the original rotating frame, the reduced Hamiltonian is rewritten as

\[
H_{\text{RWA}} = \Delta_c' \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G_0(\xi)(\delta a^\dagger \delta b^\dagger + \delta b^\dagger \delta b) + \text{h.c.},
\]

which is the same to the conventional optomechanical Hamiltonian excepting the modified coupling strength \(G_0(\xi)\).

Based on the property of the Bessel function (see the insert in figure 3(b)), we find that the effective optomechanical coupling \(G_0(\xi)\) can be arbitrarily reduced by choosing different \(\xi\), which indicates that the ground-state cooling can be achieved even in arbitrarily strong coupling region. The final mean phonon number can be calculated through the mathematical expression

\[
\langle \delta b^\dagger \delta b \rangle = \frac{1}{2} ((q^2) + \langle p^2 \rangle - 1),
\]

where the result can be obtained by solving equation (6) numerically, as shown in figure 3. Here, we have assumed that the mean phonon number of the initial system equals the thermal phonon number \(\langle \delta b^\dagger \delta b \rangle (t = 0) = n_\theta\). It is worth noting that the system can be stable with those parameters.
5. Manipulating entanglement between the optical and mechanical modes

The entanglement about nanomechanical resonator has been investigated widely in various systems and proposals [12–14, 25, 43, 44]. However, most of those studies are all relied on the stable condition which can be indicated as the limit of the optomechanical coupling strength, as shown in figure 2. Through the above analysis, we have found that the shaking optical frequency can reduce the effective coupling strength of the optomechanical system arbitrarily. In our proposed cavity optomechanical system, therefore, the manipulation of steady entanglement about macroscopic mechanical resonator can also be achieved even in the strong coupling region where the standard optomechanical systems are always unstable.

In order to estimate the entanglement between the optical and mechanical modes, we utilize the logarithmic negativity $E_N$ as the quantity to measure the entanglement, which can be calculated via the definition

$$E_N = \max[0, -\ln 2 \eta^-],$$

where $\eta^- = 2^{-1/2} \left( \Sigma(V) - [\Sigma(V)^2 - 4 \det V]^{1/2} \right)^{1/2}$, with $\Sigma(V) = \det A + \det B - 2 \det C$, and the correlation matrix $V$ by the $2 \times 2$ block matrices is given as

$$V = \begin{pmatrix} A & C^T \\ C & B \end{pmatrix}.$$  

Therefore, the optical mode and mechanical mode are said to be entangled ($E_N > 0$) if and only if $\eta^- < 1/2$, which is equivalent to the Simon’s necessary and sufficient non-positive partial transpose criteria [12, 45].

The relationships between logarithmic negativity $E_N$ and modulation amplitude $\xi$ with different parameters are shown in figure 4, e.g. corresponding to different modulation frequencies, optomechanical couplings, effective detunings, and temperatures, respectively. In figure 4(a), we first show $E_N$ changing with $\xi$ for different modulation frequencies. One can see that $E_N$ has a sharp decline at the zero point of $J_0(\xi)$ due to the too weak coupling of the nearest resonant sideband, but it does not vanish due to the existence of those non-RWA terms (other sidebands). Then we study the change of entanglement with different optomechanical couplings and show the result in figure 4(b). We find that the system with frequency modulation is stable and the entanglement is existent even in the conventional unstable region of the standard optomechanical systems. However, the adjustable range of $\xi$ decreases gradually with the increase of optomechanical coupling $G$. That is because the larger coupling needs the smaller $J_0(\xi)$ vanishing the optomechanical coupling and results in the failure for ground-state cooling.

6. Conclusions

In conclusion, we have studied a usual cavity optomechanical system where the frequency of the optical cavity is modulated in the form of cosine. By using the usual linearization approach and transforming the system to the interaction picture, we explain the physical mechanism via the Raman scattering and frequency-domain pictures. We find that the frequency modulation of the optical mode can reduce the optomechanical coupling strength arbitrarily when the modulation frequency is large enough. Therefore, it is possible to study the steady macroscopic quantum phenomenon even in the strong coupling region where the standard cavity optomechanical system is usually unstable. Different from [33], the physical mechanism of stabilizing the cavity optomechanical system is to reduce the effective optomechanical coupling strength, not to eliminate the two-mode squeezing interaction. We first study the ground-state cooling of the mechanical resonator by numerically solving the correlation matrix of the system and discuss the effect of modulation...
parameters on the final mean phonon number. The result shows that, in the presence of the frequency modulation, the mechanical ground-state cooling is not limited to the conventional stability boundary (small optomechanical coupling strength) and the final mean phonon number can also be manipulated by the modulation parameters. In addition, we also study the effect of modulation parameters on the steady entanglement between the optical and mechanical modes which is estimated by the logarithmic negativity. We find that, in the presence of frequency modulation, the steady entanglement is existent even with the very large optomechanical coupling. Our proposal would open up the possibility for studying and manipulating the macroscopic quantum effects in the strong coupling region and not limited by the stability of the system.

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Appendix. Linearizing the system Hamiltonian in the presence of frequency modulation

In the presence of frequency modulation, the system Hamiltonian of the standard cavity optomechanical system reads

\[ H = [\omega_c + \xi \nu \cos(\omega t)]a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) + (E e^{i\omega_l t} + E^* e^{-i\omega_l t}) \]

\[ + (E d^\dagger e^{-i\omega_l t} + E^* a e^{i\omega_l t}) \]

where \(a (b)\) and \(a^\dagger (b^\dagger)\) represent the annihilation and creation operators for optical (mechanical) mode with the corresponding frequency \(\omega_c (\omega_m)\), respectively. The parameter \(g\) is the single photon optomechanical coupling rate. The parameters \(E\) and \(\omega_l\) are the driving amplitude and frequency, respectively. By performing a rotating transformation defined by \(V_i = \exp[-i\omega_l t d^\dagger a]\), the transformed Hamiltonian

\[ \tilde{H} = \tilde{H}_0 + \xi \nu \tilde{H}_\text{mod} \]

\[ \tilde{H}_0 = [\omega_c + \xi \nu \cos(\omega t)]a^\dagger a + \omega_m b^\dagger b - g a^\dagger a \]

\[ + (E d^\dagger e^{-i\omega_l t} + E^* a e^{i\omega_l t}) \]

\[ + (E d^\dagger e^{i\omega_l t} + E^* a e^{-i\omega_l t}) \]

where \(\tilde{H}_\text{mod}\) is the modulation Hamiltonian.
where $\Delta_\chi = \omega_\chi - \omega_\Delta$ is the cavity laser detuning. The quantum Langevin equations of the system are given by

$$\dot{a} = -i[\Delta_\chi + \xi \nu \cos(\nu t)]a - \frac{k}{2}a - iEa + E^*a,$$

$$\dot{b} = -i\omega_\beta b - \frac{\gamma}{2}b + ig\nu a - \gamma^*b,$$  \hspace{1cm} (A3)

where $k$ and $\gamma$ are the decay rate of optical cavity and the damping rate of mechanical resonator, respectively. $a_{in}$ and $b_{in}$ are the corresponding noise operators. Under the strongly coherent laser driving, we can apply a displacement transformation to linearize equation (A3), i.e. $a = \alpha + \delta a$ and $b = \beta + \delta b$, where $\alpha$ and $\beta$ are $c$-numbers representing the displacement mean values of the cavity and mechanical resonator modes. $\delta a$ and $\delta b$ are the operators relating to the quantum fluctuations of the cavity and mechanical resonator modes. Equation (A3) can be separated to two different sets of equations, one for the mean values, and the other for the fluctuations, which are given by

$$\dot{\alpha} = -i[\Delta'_\chi + \xi \nu \cos(\nu t)]\alpha - \frac{k}{2}\alpha - iE,$$

$$\dot{\beta} = -i\omega_\beta \beta - \frac{\gamma}{2}\beta + Ig\alpha^2,$$

$$\dot{\delta a} = -i[\Delta'_\chi + \xi \nu \cos(\nu t)]\delta a - \frac{k}{2}\delta a + iG(\delta b^2 + \delta b) - \sqrt{k}a_{in},$$

$$\dot{\delta b} = -i\omega_\beta \delta b - \frac{\gamma}{2}\delta b + iG\delta a^2 + iG^*\delta a - \sqrt{\gamma}b_{in},$$  \hspace{1cm} (A4)

where $\Delta'_\chi = \Delta_\chi - g(\beta^* + \beta)$ is the effective detuning modified by optomechanical coupling, $G = ga$ is linearized optomechanical coupling strength, and we have neglected the nonlinear terms $iga(\delta b^2 + \delta b)$ and $iga^2\delta a$ due the strong coherent driving conditions. The noise operators satisfy the following autocorrelation functions

$$\langle a_{in}(t)a_{in}^*(t') \rangle = \delta(t - t'),$$

$$\langle b_{in}(t)b_{in}^*(t') \rangle = (n_{in} + 1)\delta(t - t'),$$  \hspace{1cm} (A5)

where $n_{in} = \langle \text{exp}[\kappa_\Delta/\kappa_\Delta(T) - 1]\rangle$ is the mean thermal excitation number of the mechanical resonator at temperature $T$, $k_B$ is the Boltzmann constant. Then we can obtain the linearized Hamiltonian (see equation (2) in the main text) in the presence of frequency modulation.

**References**

[1] Mancini S, Vitali D and Tombesi P 1998 Optomechanical cooling of a macroscopic oscillator by homodyne feedback Phys. Rev. Lett. 80 688

[2] Wilson D J, Sudhir V, Piro N, Schilling R, Ghadimi A and Kippenberg T J 2015 Measurement-based control of a mechanical oscillator at its thermal decoherence rate Nature 524 325

[3] Wilson-Rae I, Nooshi N, Zwerver W and Kippenberg T J 2007 Theory of ground state cooling of a mechanical oscillator using dynamical backaction Phys. Rev. Lett. 99 093901

[4] Wilson-Rae I, Nooshi N, Dobrindt J, Kippenberg T J and Zwerver W 2008 Cavity-assisted backaction cooling of mechanical resonators New J. Phys. 10 095007

[5] Marquardt F, Chen J P, Clerk A A and Girvin S M 2007 Quantum theory of cavity-assisted sideband cooling of mechanical motion Phys. Rev. Lett. 99 093902

[6] Liu Y C, Xiao Y F, Luan X and Wong C W 2013 Dynamic dissipative cooling of a mechanical resonator in strong coupling optomechanics Phys. Rev. Lett. 110 153606

[7] Clark J B, Lecocq F, Simmonds R W, Aumentado J and Teufel J D 2017 Sideband cooling beyond the quantum backaction limit with squeezed light Nature 541 191

[8] Rossi M, Kralj N, Zippilli S, Natali R, Borrielli A, Pandraud G, Serra E, Giuseppe G and Vitali D 2017 Enhancing sideband cooling by feedback-controlled light Phys. Rev. Lett. 119 123603

[9] Wollman E E, Lei C U, Weinstein A J, Suh J, Kronwald A, Marquardt F, Clerk A A and Schwab K C 2015 Quantum squeezing of motion in a mechanical resonator Science 349 952

[10] Wang D Y, Bai C H, Wang H F, Zhu A D and Zhang S 2016 Steady-state mechanical squeezing in a hybrid atom-optomechanical system with a highly dissipative cavity Sci. Rep. 6 24421

[11] Lu X Y, Liao J Q, Tian L and Nori F 2015 Steady-state mechanical squeezing in an optomechanical system via dufting nonlinearity Phys. Rev. A 91 013834

[12] Vitali D, Gigan S, Ferreira A, Böhm H R, Tombesi P, Guerreiro A, Vedral V, Zeilinger A and Aspelmeyer M 2007 Optomechanical entanglement between a movable mirror and a cavity field Phys. Rev. Lett. 98 030405

[13] Paternostro M, Vitali D, Gigan S, Kim M S, Brukner C, Eisert J and Aspelmeyer M 2007 Creating and probing multipartite entanglement with ground state cooling light Phys. Rev. Lett. 99 250401

[14] Bai C H, Wang D Y, Wang H F, Zhu A D and Zhang S 2017 Classical-to-quantum transition behavior between two oscillators separated in space under the action of optomechanical interaction Sci. Rep. 7 2545

[15] Neumeier L and Chang D E 2018 Exploring unresolved sideband, optomechanical strong coupling using a single atom coupled to a cavity New J. Phys. 20 083004

[16] Liao J Q and Tian L 2016 Macroscopic quantum superposition in cavity optomechanics Phys. Rev. Lett. 116 163602

[17] Caves C M, Thorne K S, Drever R W P, Sandberg V D and Zimmermann M 1980 On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle Rev. Mod. Phys. 52 341

[18] Bocko M F and Onofrio R 1996 On the measurement of a weak classical force coupled to a harmonic oscillator: experimental progress Rev. Mod. Phys. 68 755

[19] Mancini S, Giovannetti V, Vitali D and Tombesi P 2002 Entangling macroscopic oscillators exploiting radiation pressure Phys. Rev. Lett. 88 120401

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DeJesus E X and Kaufman C 1987 Routh-hurwitz criterion in the examination of eigenvalues of a system of nonlinear ordinary differential equations Phys. Rev. A 35 5288

[20] Wei L F, Liu Y X, Sun C P and Nori F 2006 Probing tiny motions of nanomechanical resonators: Classical or quantum mechanical? Phys. Rev. Lett. 97 237201

[21] Abbott B P et al 2016 Observation of gravitational waves from a binary black hole merger Phys. Rev. Lett. 116 061102

[22] Mari A and Eisert J 2009 Gently modulating optomechanical systems Phys. Rev. Lett. 103 213603

[23] Liao J Q and Law C K 2011 Parametric generation of quadrature squeezing of mirrors in cavity optomechanics Phys. Rev. A 83 033820

[24] Farace A and Giovannetti V 2012 Enhancing quantum effects via periodic modulations in optomechanical systems Phys. Rev. A 86 013820

[25] Chen R X, Shen L T, Yang Z B, Wu H Z and Zheng S B 2014 Enhancement of entanglement in distant mechanical vibrations via modulation in a coupled optomechanical system Phys. Rev. A 80 023843

[26] Yin T S, Liu X Y, Zheng L L, Wang M, Li S and Wu Y 2017 Nonlinear effects in modulated quantum optomechanics Phys. Rev. A 95 053861

[27] Huang J F, Liao J Q, Tian L and Kuang L M 2017 Manipulating counter-rotating interactions in the quantum rabi model via modulation of the transition frequency of the two-level system Phys. Rev. A 96 043849

[28] Silveri M P, Tuorila J A, Thuneberg E V and Paraoanu G S 2017 Quantum systems under frequency modulation Rep. Prog. Phys. 80 056002

[29] Anderson P W 1954 A mathematical model for the narrowing of spectral lines by exchange or motion J. Phys. Soc. Japan 9 316–39

[30] Shevchenko S, Ashhab S and Nori F 2010 Landau-zener-stückelberg interferometry Phys. Rep. 492 1

[31] Piergentili P, Catalini L, Bawaj M, Zippilli S, Malossi N, Natali R, Vitali D and Giuseppe G D 2018 Two-membrane cavity optomechanics New J. Phys. 20 083024

[32] Liao J Q, Law C K, Kuang L M and Nori F 2015 Enhancement of mechanical effects of single photons in modulated two-mode optomechanics Phys. Rev. A 92 013822

[33] Wang D Y, Bai C H, Liu S, Zhang S and Wang H F 2018 Optomechanical cooling beyond the quantum backaction limit with frequency modulation Phys. Rev. A 98 023816

[34] Li J, Xuereb A, Malossi N and Vitali D 2016 Cavity mode frequencies and strong optomechanical coupling in two-membrane cavity optomechanics J. Opt. 18 084001

[35] Xiong W, Jin D Y, Qiu Y, Lam C H and You J Q 2016 Cross-kerr effect on an optomechanical system Phys. Rev. A 93 023844

[36] Teufel J D, Donner T, Li D, Harlow J W, Allman M S, Cicak K, Sirois A J, Whittaker J D, Lehner K W and Simmonds R W 2011 Sideband cooling of micromechanical motion to the quantum ground state Nature 475 359

[37] Yin Z Q 2009 Phase noise and laser-cooling limits of optomechanical oscillators Phys. Rev. A 80 033823

[38] Yin Z Q, Li T and Feng M 2011 Three-dimensional cooling and detection of a nanosphere with a single cavity Phys. Rev. A 83 013816

[39] Karuza M, Molinelli C, Galassi M, Biancofiore C, Natali R, Tombesi P, Giuseppe G D and Vitali D 2012 Optomechanical sideband cooling of a thin membrane within a cavity New J. Phys. 14 095015

[40] Liu Y C, Shen Y F, Gong Q and Xiao Y F 2014 Optimal limits of cavity optomechanical cooling in the strong-coupling regime Phys. Rev. A 89 053821

[41] Liu Y M, Bai C H, Wang D Y, Wang T, Zheng M H, Wang H F, Zhu A D and Zhang S 2018 Ground-state cooling of rotating mirror in double-laguerre-gaussian-cavity with atomic ensemble Opt. Express 26 6143

[42] Hofer S G, Wieczorek W, Aspelmeyer M and Hammerer K 2011 Quantum entanglement and teleportation in pulsed cavity optomechanics Phys. Rev. A 84 053827

[43] Akram U, Munro W, Nemoto K and Milburn G J 2012 Photon–phonon entanglement in coupled optomechanical arrays Phys. Rev. A 86 042306

[44] Simon R 2000 Peres-horodecki separability criterion for continuous variable systems Phys. Rev. Lett. 84 2726

[45] Schlosser A, Rivière R, Anetsberger G, Arcizet O and Kippenberg T J 2008 Resolved-sideband cooling of a micromechanical oscillatorNat. Phys. 4 415

[46] Rocheleau T, Ndumuk T, Macklin C, Hertzberg J B, Clerk A A and Schwab K C 2009 Preparation and detection of a mechanical resonator near the ground state of motion Nature 463 72

[47] Ockeloen-Korppi C F, Damskugg E, Pirkkalainen J M, Asjad M, Clerk A A, Maserl F, Woolley M J and Sillanpää M A 2018 Stabilized entanglement of massive mechanical oscillators Nature 556 478

[48] Clark J B, Lecoq F, Simmonds R W, Aumentado J, Teufel J D and Schwab K C 2016 Observation of strong radiation pressure forces from squeezed light on a mechanical oscillator Nat. Phys. 12 683

[49] Riedinger R, Hong S, Nort R A, Slater J A, Shang J, Krause A G, Anant V, Aspelmeyer M and Gröblacher S 2016 Non-classical correlations between single photons and phonons from a mechanical oscillator Nature 530 313

[50] Pauw F G, Fedorov A, Harmans C J P M and Mooij J E 2009 Tuning the gap of a superconducting flux qubit Phys. Rev. Lett. 102 090501