Can interacting dark energy solve the $H_0$ tension?

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The answer is Yes! We indeed find that interacting dark energy can alleviate the current tension on the value of the Hubble constant $H_0$ between the Cosmic Microwave Background anisotropies constraints obtained from the Planck satellite and the recent direct measurements reported by Riess et al. 2016. The combination of these two datasets points towards an evidence for a non-zero dark matter-dark energy coupling $\xi$ at more than two standard deviations, with $\xi = -0.26^{+0.16}_{-0.12}$ at 95% CL. However the $H_0$ tension is better solved when the equation of state of the interacting dark energy component is allowed to freely vary, with a phantom-like equation of state $w = -1.184 \pm 0.064$ (at 68% CL), ruling out the pure cosmological constant case, $w = -1$, again at more than two standard deviations. When Planck data are combined with external datasets, as BAO, JLA Supernovae Ia luminosity distances, cosmic shear or lensing data, we find perfect consistency with the cosmological constant scenario and no compelling evidence for a dark matter-dark energy coupling.

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I. INTRODUCTION

The recent measurements of Cosmic Microwave Background anisotropies (CMB) provided by the Planck satellite have fully confirmed the predictions of the standard cosmological model (hereafter, $\Lambda$CDM), based on cold dark matter, a cosmological constant and inflation [1]. However, when constraints on the cosmological parameters of the $\Lambda$CDM model are derived from the Planck data, some tensions appear between their values and the corresponding values obtained from independent, complementary, observables.

The most important tension concerns the value of the Hubble constant. Indeed, the latest analysis of CMB temperature and polarization data from the Planck experiment provides the constraint of $H_0 = 66.93 \pm 0.62$ km/s/Mpc at 68% CL, obtained assuming $\Lambda$CDM [2]. This is more than $3\sigma$ away from the recent direct and local determination of Riess et al. 2016 (R16, hereafter) of $H_0 = 73.24 \pm 1.74$ km/s/Mpc [3].

Another important discrepancy is present on the recovered values of the $S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$ parameter (where $\sigma_8$ is the amplitude of matter fluctuations and $\Omega_m$ is the matter density) derived independently by Planck and weak lensing surveys such as CFHTLenS [4, 5] and KiDS-450 [6]. Considering a $\Lambda$CDM scenario, the KiDS-450 result is in tension with the Planck constraint at about 2.3 standard deviations (see e.g. [7]).

Clearly, unresolved systematics can still play a key role in explaining these discrepancies, however several physical mechanisms beyond $\Lambda$CDM that could change the derived values of $H_0$ and/or $S_8$ from Planck have been proposed, either solving or alleviating the tensions with the extracted values from R16 and cosmic shear local measurements (see e.g. Refs. [7–31]).

In this paper we focus our attention on dark energy: it has indeed been shown that introducing a dark energy equation of state (constant with time) $w < -1$ not only can solve the tension on the Hubble parameter but also does it in a more efficient way than other non-standard extensions as, for example, the inclusion of extra relativistic degrees of freedom, via the $N_{eff}$ parameter [9]. At the same time, a dynamical dark energy component seems to be favoured in combined analyses of Planck and cosmic shear data (see e.g. [7]).

On top of that we should not forget that a cosmological constant clearly presents a puzzling and controversial solution to the dark energy problem, being probably the major theoretical weakness of the standard cosmological model. The possibility of having a different candidate for this component should be therefore investigated and any hints for deviations from the $\Lambda$CDM picture must be carefully scrutinized.

If the solution for the current tensions is within the dark energy sector then it is worthwhile to investigate which dark energy model is better suited for this task. Recently, several authors have considered different parameterizations of the dark energy equation of state, deriving bounds arising from a number of cosmological datasets (see e.g. [16, 32]). Here we focus on interacting dark matter-dark energy models. Indeed, a larger value for the Hubble parameter from CMB data can be obtained by including possible interactions between the dark energy and dark matter components (see e.g. [33–40]). More specifically, assuming the interacting dark energy model presented in Ref. [41], when considering the Planck 2013 data release, one gets the constraint $H_0 = 72.1_{-1.7}^{+3.2} \pm 2.3$ km/s/Mpc at 68% CL, that is significantly
larger than the constraint $H_0 = 67.3 \pm 1.2 \text{ km/s/Mpc}$ at 68\% CL obtained with the same dataset but assuming a cosmological constant and no interaction (see Ref. [42]).

It is therefore timely to investigate if the same interacting dark energy model can also solve the tension between the new Planck 2015 data release (that includes new polarization data, which significantly improve the constraining power of these measurements) and the new bound R16, that in practice reduces by $\sim 42\%$ the uncertainty of previous late-universe constraints on the Hubble constant [43].

In what follows we perform such an analysis, showing that indeed the current tension on $H_0$ can be solved by introducing an interaction between dark energy and dark matter as the one proposed by Ref. [41]. Moreover, it is also important to examine if the coupling can solve the tension more efficiently than a dark energy equation of state of the cosmological constant case and no interaction (see Ref. [42]).

Our work is organized as follows: in the following section we describe the interacting dark energy model assumed here, in Sec. III we describe the cosmological data and the analysis method and in Sec. IV we show the obtained results. Finally, we draw our conclusions in Sec. V.

II. INTERACTING DARK ENERGY

As previously stated, we consider the interacting dark energy scenario of Refs. [41, 42], which can be parameterized as:

\[
\nabla_\mu T^{\mu}_{(dm)\nu} = Q u^{(dm)}\nu /a , \tag{1}
\]

\[
\nabla_\mu T^{\mu}_{(de)\nu} = -Q u^{(dm)}\nu /a . \tag{2}
\]

In the equations above, $T^{\mu}_{(dm)\nu}$ ($T^{\mu}_{(de)\nu}$) refers to the dark matter (dark energy) energy-momentum tensor, $Q$ is the interaction rate and $u^{(dm)}\nu$ is the four-velocity of the dark matter fluid. In order to avoid instabilities in the evolution of the linear perturbations, we restrict ourselves to the case in which $Q$ reads as:

\[
Q = \xi \mathcal{H} \rho_{de} , \tag{3}
\]

i.e. the interaction rate is proportional to the dark energy density $\rho_{de}$ via a dimensionless parameter $\xi$ (that needs to be negative) and $\mathcal{H} = \dot{a}/a$, with the dot referring to derivative respect to conformal time.

The evolution equations for the interacting background reads as [41]

\[
\dot{\rho}_{dm} + 3H\rho_{dm} = \xi \mathcal{H} \rho_{de} , \tag{4}
\]

\[
\dot{\rho}_{de} + 3H(1+w)\rho_{de} = -\xi \mathcal{H} \rho_{de} . \tag{5}
\]

The perturbation evolution, within the linear regime, and in the synchronous gauge, is given by [41]

\[
\dot{\delta}_{dm} = -(kv_{dm} + \frac{1}{2}h) + \xi \mathcal{H} \rho_{de} \right( \delta_{de} - \delta_{dm} \right) + \xi \frac{\rho_{de}}{\rho_{dm}} \left( \frac{kv_{de}}{3} + \frac{\dot{h}}{6} \right) , \tag{6}
\]

\[
\dot{\delta}_{de} = - (1+w)(kv_{de} + \frac{1}{2}h) - 3\mathcal{H}(1-w) \left[ \delta_{de} + \mathcal{H}(3(1+w) + \xi) \frac{v_{de}}{k} \right] - \mathcal{H} \left( \frac{kv_{de}}{3} + \frac{\dot{h}}{6} \right) , \tag{7}
\]

\[
\dot{v}_{dm} = -\mathcal{H}v_{dm} , \tag{8}
\]

\[
\dot{v}_{de} = 2\mathcal{H} \left( 1 + \frac{\xi}{1+w} \right) v_{de} + \frac{k}{1+w} \delta_{de} - \xi \mathcal{H} \frac{v_{dm}}{1+w} \tag{9}
\]

with $\delta_{dm,de}$ and $v_{dm,de}$ the density perturbation and the velocity of the two fluids, $v_T$ is the center of mass velocity for the total fluid and the dark energy speed of sound is $c_{s,de}^2 = 1$. The equations above include the contributions of the perturbation in the expansion rate $H = \mathcal{H}/a + \delta H$.

Following Refs. [41, 44, 45], we have considered adiabatic initial conditions for all components. It has been shown there that, if one assumes adiabatic initial conditions for all the standard cosmological fluids (photon, baryons,...), the coupled dark energy fluid will also obey adiabatic initial conditions. In the synchronous gauge and at leading order in $x = k\tau$, the initial conditions read:

\[
\delta^{in}_{de}(x) = (1 + w - 2\xi) \frac{(1+w+3\xi)}{12w^2 - 2w - 3w\xi + 7\xi - 14} \frac{-2\delta^{in}_{\gamma}(x)}{1 + w_{\gamma}} , \tag{10}
\]

\[
v^{in}_{de} = \frac{x(1+w+3\xi)}{12w^2 - 2w - 3w\xi + 7\xi - 14} \frac{-2\delta^{in}_{\gamma}(x)}{1 + w_{\gamma}} , \tag{11}
\]

where $\delta^{in}_{\gamma}(x)$ are the initial conditions for the photon density perturbations and $w_{\gamma} = 1/3$ is the equation of state.
of the photon, see Ref. [46] for the uncoupled case.

III. METHOD

The interacting dark energy scenario requires the six standard cosmological parameters of the ΛCDM plus one more parameter defined in the previous section, the coupling ξ. In particular, for the ΛCDM model, the parameters are the baryon density Ω_b, the cold dark matter density Ω_c, the reionization optical depth τ, and the ratio between the sound horizon and the angular diameter distance at decoupling θ_s. Furthermore, we consider two parameters directly related to the inflationary paradigm, that are the spectral index n_S and the logarithm of the amplitude of the primordial power spectrum, ln(10^{10} A_S). As a second step, we extend this baseline model, by adding one more parameter, a freely varying dark energy equation of state w, assumed to be constant in redshift.

We analyze this scenario by combining several cosmological probes. We consider the full temperature power spectrum provided by the Planck collaboration [47] at multipoles 2 ≤ ℓ ≤ 2500, in combination with the low-ℓ polarization power spectra in the multipoles range 2 ≤ ℓ ≤ 29. We refer to this combination as “Planck TT + lowTEB”. We also include the high multipole Planck polarization spectra [47], in the multipole range 30 ≤ ℓ ≤ 2500. We refer to this combination as “Planck TTTEEE + lowTEB”. However, we would like to remind here that this combination of datasets is considered less robust as it still under discussion due to some possible residual systematics contamination [1, 47].

Additionally to the CMB datasets described above, we consider their combination with the following cosmological measurements:

- **tau055**: We replace the “lowTEB” Planck data with a gaussian prior on the reionization optical depth τ = 0.055 ± 0.009, as obtained from the Planck HFI measurements in [2];

- **lensing**: We consider the 2015 Planck CMB lensing reconstruction power spectrum C_ℓ^φφ obtained with the CMB trispectrum analysis [48];

- **BAO**: Baryon Acoustic Oscillation measurements from the 6dFGS [49], SDSS-MGS [50], BOSS-LOWZ and CMASS-DR11 [51] surveys are also considered;

- **R16**: As previously stated, we include a gaussian prior on the Hubble constant H_0 = 73.24 ± 1.74 km/s/Mpc, by quoting the value provided with direct measurements of luminosity distances in Riess et al. [3];

- **JLA**: We also employ luminosity distance data of Supernovae type Ia from the "Joint Light-curve Analysis" derived from the SNLS and SDSS catalogs [52];

- **WL**: We add weak lensing galaxy data from the CFHTlens [4, 5] survey with the priors and conservative cuts to the data as described in Ref. [1].

The analysis is done with a modified version of the most recent publicly available Monte-Carlo Markov Chain package cosmomc [53], with a convergence diagnostic based on the Gelman and Rubin statistics. As the original code, this version implements an efficient sampling of the posterior distribution using the fast/slow parameter decorrelations [54], and it includes the support for the Planck data release 2015 Likelihood Code [47] (see http://cosmologist.info/cosmomc/).

IV. RESULTS

The constraints at 68% CL on the cosmological parameters obtained by including the interaction between dark matter and dark energy are given in Tabs. I and II for the “Planck TT + lowTEB” and the “Planck TTTEEE + lowTEB” baseline datasets, respectively. Those obtained allowing also for a freely-varying dark energy equation of state are shown in Tabs. III and IV. For comparison purposes we also quote in Tab. V the bounds on the cosmological parameters as obtained from the Planck collaboration [1] in the pure ΛCDM and wCDM context, i.e. without considering the dark matter-dark energy coupling ξ.

Notice, first of all, that the CMB-only constraints on the coupling ξ are very similar with or without the inclusion of the polarization data from Planck at higher multipoles. Tables I and II show that, in the ΛCDM + ξ scenario, CMB data only imposes a lower limit in the coupling ξ > −0.23 at 68% CL, with or without the Planck small-scale polarization data.

However, if we compare the CMB constraints on the Hubble constant in Tabs. I and II in the presence of a coupling ξ to those obtained with no interaction in Tab. V, we see that the coupling produces a shift at more than 2σ towards higher values of the Hubble constant and relaxes the error bars by a factor ~ 2. The fact that interacting dark energy alleviates the H_0 tension can also be noticed in the results depicted by the confidence level contours in the (ξ, H_0) planes in Fig. 1. The reason for that is the following: within the interacting dark matter-dark energy model explored here, the dark matter density contribution is required to be smaller, as for negative ξ the dark matter density will get an extra contribution proportional to the dark energy one. Since CMB accurately constrains Ω_c h^2, a larger value of H_0 will be required in these scenarios, which in turn, will provide a better agreement with the direct measurement of H_0 from the R16 prior. More quantitatively, notice, from the first column of Tab. II, that we obtain H_0 = 68.9 ± 1.2 km/s/Mpc for
Planck TT Planck TT Planck TT Planck TT Planck TT Planck TT Planck TT
+ lowTEB + R16 + lowTEB + BAO + lowTEB + JLA + lowTEB + WL + lowTEB + lensing + tau055
\(\Omega_b h^2\) 0.02222 ± 0.00023 0.02235 ± 0.00022 0.02221 ± 0.00020 0.02220 ± 0.00021 0.02232 ± 0.00023 0.02225 ± 0.00023 0.02207 ± 0.00021
\(\Omega_c h^2\) 0.100 ± 0.011 0.088 ± 0.013 0.104 ± 0.015 0.104 ± 0.014 0.098 ± 0.016 0.099 ± 0.017 0.098 ± 0.012
\(\tau\) 0.076 ± 0.019 0.082 ± 0.019 0.076 ± 0.018 0.076 ± 0.019 0.075 ± 0.019 0.065 ± 0.016 0.0587 ± 0.0089
\(n_s\) 0.9652 ± 0.0061 0.9696 ± 0.0059 0.9650 ± 0.0047 0.9648 ± 0.0059 0.9691 ± 0.0060 0.9675 ± 0.0059 0.9585 ± 0.0058
\(\ln(10^{10} A_s)\) 3.087 ± 0.036 3.095 ± 0.037 3.087 ± 0.035 3.087 ± 0.036 3.080 ± 0.037 3.061 ± 0.029 3.057 ± 0.018
\(H_0 [\text{km s}^{-1} \text{Mpc}^{-1}]\) 69.0 ± 1.4 70.7 ± 1.2 68.63 ± 0.8 68.6 ± 1.2 68.9 ± 1.4 69.5 ± 1.4 68.4 ± 1.4
\(\sigma_8\) 1.01 ± 0.08 1.13 ± 0.18 0.967 ± 0.05 0.964 ± 0.05 1.01 ± 0.09 1.00 ± 0.09 1.04 ± 0.12
\(\Omega_m\) 0.260 ± 0.036 0.223 ± 0.032 0.270 ± 0.038 0.249 ± 0.033 0.252 ± 0.035 0.260 ± 0.033 0.260 ± 0.031
\(\xi\) > -0.232 -0.25 ± 0.05 -0.19 > -0.184 > -0.184 > -0.17 ± 0.12 > -0.10 > -0.232 > -0.21 ± 0.09

| Parameter | Planck TT Planck TT Planck TT Planck TT Planck TT Planck TT Planck TT |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|           | TTEEE + lowTEB | TTEEE + R16   | TTEEE + BAO   | TTEEE + JLA   | TTEEE + WL   | TTEEE + lensing | TTEEE + tau055 |
| \(\Omega_b h^2\) | 0.02223 ± 0.00016 | 0.02230 ± 0.00015 | 0.02223 ± 0.00014 | 0.02222 ± 0.00015 | 0.02228 ± 0.00016 | 0.02224 ± 0.00016 | 0.02214 ± 0.00015 |
| \(\Omega_c h^2\) | 0.101 ± 0.010 | 0.089 ± 0.005 | 0.104 ± 0.014 | 0.104 ± 0.014 | 0.099 ± 0.017 | 0.099 ± 0.018 | 0.097 ± 0.011 |
| \(\tau\) | 0.077 ± 0.017 | 0.081 ± 0.017 | 0.078 ± 0.016 | 0.078 ± 0.017 | 0.074 ± 0.017 | 0.062 ± 0.014 | 0.0692 ± 0.0087 |
| \(n_s\) | 0.9641 ± 0.0047 | 0.9634 ± 0.0048 | 0.9643 ± 0.0043 | 0.9643 ± 0.0047 | 0.9660 ± 0.0048 | 0.9652 ± 0.0047 | 0.9593 ± 0.0046 |
| \(\ln(10^{10} A_s)\) | 3.090 ± 0.033 | 3.095 ± 0.033 | 3.091 ± 0.032 | 3.091 ± 0.033 | 3.082 ± 0.033 | 3.056 ± 0.025 | 3.059 ± 0.018 |
| \(H_0 [\text{km s}^{-1} \text{Mpc}^{-1}]\) | 68.9 ± 1.2 | 70.2 ± 1.4 | 68.56 ± 0.8 | 68.57 ± 0.8 | 69.3 ± 1.2 | 69.2 ± 1.2 | 68.7 ± 1.4 |
| \(\sigma_8\) | 1.01 ± 0.08 | 1.14 ± 0.17 | 0.966 ± 0.05 | 0.967 ± 0.05 | 1.01 ± 0.09 | 0.96 ± 0.08 | 1.05 ± 0.12 |
| \(\Omega_m\) | 0.261 ± 0.034 | 0.227 ± 0.034 | 0.271 ± 0.039 | 0.271 ± 0.038 | 0.254 ± 0.041 | 0.254 ± 0.044 | 0.255 ± 0.030 |
| \(\xi\) | > -0.229 | -0.259 ± 0.043 | -0.309 | > -0.184 | > -0.182 | > -0.18 ± 0.13 | > -0.10 | > -0.230 | > -0.21 ± 0.08 | > -0.15 |

TABLE I: 68% CL constraints on cosmological parameters from the “Planck TT + lowTEB” baseline dataset and its further combinations with several datasets (see text). An interacting dark energy model with negative coupling \(\xi \leq 0\) is assumed. Please note that in case of the "tau055" prior the "lowTEB" dataset is not considered.

"Planck TTEEE + lowTEB" while, without interaction and for the same combination of datasets, we have \(H_0 = 67.30 \pm 0.64 \text{ km/s/Mpc},\) as can be noticed from the first column of Tab. V. This effect reduces at 2\(\sigma\) the tension of the Planck CMB anisotropy data in a \(\Lambda\)CDM framework with Riess et al. 2016 [3] (i.e. with the value \(H_0 = 73.24 \pm 1.74 \text{ km/s/Mpc}\)). It should not therefore come as a surprise that when the R16 prior on the Hubble constant is added in the analyses, a preference for a non-zero coupling appears with a significance larger than 2 standard deviations (see also Fig. 1). Indeed, the "Planck TT+lowTEB+R16" dataset (see Tab. I) gives \(\xi = -0.25 \pm 0.10 \text{ at 68% CL} (\xi = -0.25 \pm 0.17 \text{ at 95% CL})\), while the "Planck TTEEE+lowTEB+R16" dataset (see Tab. II) gives \(\xi = -0.250 \pm 0.043 \text{ at 68% CL} (\xi = -0.26 \pm 0.16 \text{ at 95% CL})\).

While the Planck+R16 combined datasets clearly show evidence for a coupling, also other datasets seem to suggest this possibility. When the WL dataset is included a mild indication (slightly above one standard deviation) is indeed present. The "Planck TT+lowTEB+WL" dataset (see Tab. I) gives \(\xi = -0.17 \pm 0.12 \text{ at 68% CL},\) while the "Planck TTEEE+lowTEB+WL" dataset (see Tab. II) gives \(\xi = -0.18 \pm 0.13 \text{ at 68% CL}.\) It is worthwhile to note that when the interacting dark energy is introduced, a very large shift towards lower values of the cold dark matter density \(\Omega_c h^2\) appears and the error bars are relaxed by a factor 10, as can be seen by comparing the
FIG. 1: 68% and 95% CL in the two-dimensional ($\xi$, $H_0$) planes from the “Planck TT + lowTEB” dataset (left panel) and “Planck TTTTEEE + lowTEB” dataset (right panel) also combined with the R16 prior on the Hubble constant. Notice that the presence of a coupling $\xi$ allows for larger values for $H_0$ from Planck data. The inclusion of the R16 prior results in an indication for $\xi < 0$ with a significance above two standard deviations.

results of Tabs. I and II with those shown in Tab. V. Moreover, we find a shift of the clustering parameter $\sigma_8$ towards an higher value, compensated by a lowering of the matter density $\Omega_m$, both with relaxed error bars. In this way, we are not increasing the tension on the $S_8$ parameter between Planck CMB data and the weak lensing measurements from the CFHTLenS survey [4, 5] and KiDS-450 [6]. This can be also clearly noticed from the left panel of Fig. 2, where we plot the two-dimensional constraints on the $\Omega_m$-$\sigma_8$ plane from the “Planck TTTTEEE + lowTEB” dataset in the cases of $\xi = 0$, non-zero coupling, and combined with the WL measurements.

Also when the “tau055” prior is included a small preference for a non-zero dark matter-dark energy coupling seems to emerge. The reported constraints of $\xi = -0.21^{+0.09}_{-0.16}$ at 68% CL for the “Planck TT+tau055” and $\xi = -0.21^{+0.08}_{-0.15}$ at 68% CL for the “Planck TTTTEEE+tau055” datasets respectively could naively suggest a statistically significant detection (at more than 2 standard deviations) for a dark energy-dark matter interaction. It is however important to point out that the posterior distribution for $\xi$ is in this case highly non-gaussian. At 95% CL we have found that the tau055 prior gives no evidence for an interaction, providing a lower limit only. We obtain $\xi > -0.37$ for the “Planck TT+tau055” dataset, and $\xi > -0.38$ for the “Planck TTTTEEE+tau055” dataset, respectively, both at 95% CL. Therefore, while the tau055 prior suggests a value of $\xi < 0$, this indication is not statistically significant (only slightly above 1$\sigma$); notice this fact from the right panel of Fig. 2, where we plot the two-dimensional posteriors in the $\xi$ vs $\tau$ plane. This preference is driven by the smaller value required in interacting dark energy models for the present dark matter mass-energy density, which would itself lead to a lower value of $\tau$.

In a second step, we consider the dark energy equation of state $w$ free to vary. Indeed, if dark energy is an interacting fluid, it is expected that its equation of state differs from the canonical value in the $\Lambda$CDM scenario, $w = -1$. As we can see from the values reported in Tabs. III and IV and from Fig. 3, where we plot the two-dimensional posteriors in the $H_0$ vs $w$ plane, the inclusion of a negative coupling $\xi$ makes models with larger values of $w$ in better agreement with the CMB data. Indeed, from the “Planck TTTTEEE +lowTEB” dataset, we obtain the upper limit $w < -1.17$ at 68% CL when a negative coupling is considered (see Tab. IV), to be compared with the limit $w < -1.35$ at 68% CL obtained when we fix $\xi = 0$ (see Tab. V). This fact results even more evident by the comparison of the two-dimensional posteriors in the $H_0$ vs $w$ plane depicted in Fig. 3 from the “Planck TTTTEEE +lowTEB” dataset in the case of a negative coupling (left panel) to those with $\xi = 0$ (right panel). The CMB-only contours clearly extend to larger values of $w$ in the presence of a negative coupling. Furthermore, the posterior in this case appears as bimodal: there is a significant portion of models with $w > -1$ and $\xi < 0$ compatible with the data. The reason for this is simple: models with negative coupling mimic an effective equation of state with $w < -1$. Increasing $w$ has therefore a similar effect of decreasing $\xi$ and this enhances the comparability of models with $w > -1$ with the data. Also from Fig. 3 (and as it is well known), one can clearly notice that when a variation in $w$ is considered, the Planck constraints on the Hubble constant practically vanish. In this case we can safely include the R16 prior
on the Hubble constant, as the tension between Planck and R16 disappears, finding for this particular combination a detection of $w < -1$ at more than 2σ, obtaining $w = -1.174 \pm 0.057$ from "Planck TT + lowTEB+R16" (see Tab. III) and $w = -1.184 \pm 0.064$ from "Planck TTTEEE + lowTEB+R16" (see Tab. IV), both at 68% CL.

When a variation on $w$ is included, the previous hint for $\xi < 0$ from the Planck+R16 dataset is still present but relaxed. In fact, there is still a mild indication at about 1σ for $\xi < 0$ from the "Planck TT +R16" dataset that persists when the Planck polarization data is included: we obtain $\xi = -0.17 \pm 0.09$ from "Planck TT+lowTEB+R16" (see Tab. III) and $\xi = -0.16 \pm 0.06$ from "Planck TTTEEE+lowTEB+R16" (see Tab. IV), both at 68% CL. As we can see from Fig. 3 the contour plots in the case of "Planck TTTEEE+lowTEB+R16" (left panel) fully confirm the preference for $w < -1$ but also appear as slightly bimodal. The reason is that, for stability reasons, we have not considered models with positive coupling $\xi$ that would have been degenerate with models with $w < -1$. Once again, introducing models with negative coupling increases the compatibility with the data of models in which $w > -1$, however in case of the R16 prior this is not enough to prevent an indication for $w < -1$ at more than 95% CL.

When either BAO or JLA measurements are added in the analyses the indication for $w < -1$ disappears and a cosmological constant is now consistent with the data within two standard deviations. It is interesting to note, from Tab. III, that in the case of "Planck TT+lowTEB+BAO" and "Planck TTTEEE+lowTEB+BAO" datasets one gets the constraint $\xi = -0.26 \pm 0.09$ at 68% CL, apparently suggesting the evidence for a dark matter-dark energy coupling at more than 2σ. This is probably due to the small tension present between the Planck and BAO data. We have found however that also in this case the posterior for $\xi$ is highly non-gaussian and that the indication for a coupling from this dataset is only slightly larger than one standard deviation. Indeed, if we consider the two standard deviation constraint we obtain only a lower limit ($\xi > -0.426$ at 95% CL). Since a negative coupling is degenerate with models in which $w > -1$, we find that, when the BAO or JLA datasets are included, the constraints on $w$ are shifted towards larger values with respect to the case with no coupling, hinting to $w > -1$ at more than one standard deviation. Indeed, we get $w = -0.918 \pm 0.076$ from "Planck TT+lowTEB+BAO" (see Tab. III) and $w = -0.934 \pm 0.074$ from "Planck TTTEEE+lowTEB+BAO" (see Tab. IV), both at 68% CL to be compared to the value $w = -1.030 \pm 0.070$ from "Planck TTTEEE+lowTEB+BAO" at 68% CL but with no coupling (see Tab. V). Similarly, we get $w = -0.932 \pm 0.067$ from "Planck TT+lowTEB+JLA" (see Tab. III) and $w = -0.934 \pm 0.064$ from "Planck TTTEEE+lowTEB+JLA" (see Tab. IV) at 68% CL, to be compared to the value $w = -1.034 \pm 0.053$ obtained from "Planck TTTEEE+lowTEB+JLA" at 68% CL but with $\xi = 0$ (see Tab. V).

### Table 1

| $\Omega_m$ | $\sigma_8$ |
|---|---|
| 0.20 | 0.90 |
| 0.24 | 1.05 |
| 0.28 | 1.20 |
| 0.32 | --- |

### Table 2

| $\xi$ | $\tau$ |
|---|---|
| 0.04 | 0.04 |
| 0.08 | 0.08 |
| 0.12 | 0.12 |

### Figure 2

- **Left panel:** 68% and 95% CL in the two-dimensional ($\Omega_m$, $\sigma_8$) plane from the "Planck TT+lowTEB" dataset. Notice that the coupling allows for smaller values for $\sigma_8$ and larger values for $\Omega_m$, relaxing the Planck bounds on the $\sigma_8$ parameter and mildly alleviating the tension with the $S_8$ values measured by cosmic shear surveys as CFHTLenS and KiDS-450.
- **Right panel:** 68% and 95% CL in the two-dimensional ($\xi$, $\tau$) plane from the "Planck TTTEEE," dataset, and also combined with the "tau055" prior on the reionization optical depth. Notice that the "tau055" prior affects only marginally the constraints on $\xi$, resulting in a $\sim 1\sigma$ indication for $\xi < 0$ (after marginalization over $\tau$).

![Contour plots](image_url)
FIG. 3: Left panel: 68% and 95% CL in the two-dimensional \((w, H_0)\) plane from the combination of “Planck TTTEEE + lowTEB” measurements (grey contours), “Planck TTTEEE + lowTEB+ R16” (red contours) and “Planck TTTEEE + lowTEB+ BAO” (blue contours), for an interacting dark matter-dark energy scenario. Right panel: As in the left panel but with \(\xi = 0\).

Within the \(w\)CDM + \(\xi\) scenario, the shift towards lower values of the cold dark matter density \(\Omega_c h^2\) is incremented, as can be noticed by comparing the results in Tab. IV with those in Tab. V, and therefore the tension between the Planck values and the weak lensing estimations of the \(S_8\) parameter gets alleviated. When the WL dataset is included one gets an indication for a negative coupling and \(w < -1\) at slightly more than one standard deviation.

V. CONCLUSIONS

We have explored the well-known Hubble constant \(H_0\) tension between the current estimates from late-time universe data of Riess et al. 2016, which indicates a value of \(H_0 = 73.24 \pm 1.74\) km/s/Mpc, and the Planck Cosmic Microwave Background (CMB) measurement of \(H_0 = 66.93 \pm 0.62\) km/s/Mpc, (both at 68% CL), in the context of interacting dark matter-dark energy scenarios. Such a coupling could affect the value of the present matter energy density \(\Omega_m\). Therefore, if within an interacting model \(\Omega_m\) is smaller, a larger value of \(H_0\) would be required in order to satisfy the peaks structure of CMB observations, which accurately determine the value of \(\Omega_m h^2\). We find that for one of the most interesting and viable coupled dark matter-dark energy scenarios in the literature, in which the exchanged energy rate is negative (i.e. the energy flows from the dark matter system to the dark energy one) and proportional to the dark energy density, the existing 3\(\sigma\) \(H_0\) tension is alleviated. In addition, when combining CMB measurements with the Hubble constant prior from Riess et al. 2016, a preference for a non-zero coupling appears with a significance larger than 2\(\sigma\). However, it is certainly not unexpected that in interacting scenarios the dark energy equation of state differs from its canonical value within the \(\Lambda\)CDM picture, i.e., is different from \(w = -1\). We have therefore considered as well such a possibility, finding that, when the dark energy equation of state \(w\) is also a free parameter, the Hubble constant tension gets strongly alleviated, obtaining \(\sim 3\sigma\) evidence for a phantom-like \((w < -1)\) dark energy fluid when combining CMB and Riess et al. 2016 measurements. However, when other datasets, as BAO or Supernovae Ia luminosity distances from JLA, are also included in our numerical analyses, a good consistency with a pure \(\Lambda\)CDM cosmological scenario is found with models with negative coupling and \(w > -1\) suggested at slightly more than one standard deviation.

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Planck TT Planck TT Planck TT Planck TT Planck TT Planck TT Planck TT
+ lowTEB + lowTEB + R16 + lowTEB + BAO + lowTEB + JLA + lowTEB + WL + lowTEB + lensing + tau055

| Parameter | Planck TT | Planck TT | Planck TT | Planck TT | Planck TT | Planck TT | Planck TT |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \(\Omega_b h^2\) | 0.02226 ± 0.00023 | 0.02223 ± 0.00022 | 0.02228 ± 0.00022 | 0.02223 ± 0.00023 | 0.02235 ± 0.00023 | 0.02228 ± 0.00023 | 0.02211 ± 0.00022 |
| \(\Omega_c h^2\) | 0.100 ± 0.015 | 0.100 ± 0.015 | 0.088 ± 0.012 | 0.089 ± 0.012 | 0.101 ± 0.014 | 0.099 ± 0.017 | 0.099 ± 0.020 |
| \(\tau\) | 0.075 ± 0.020 | 0.075 ± 0.019 | 0.079 ± 0.0019 | 0.076 ± 0.020 | 0.073 ± 0.020 | 0.060 ± 0.018 | 0.0580 ± 0.0087 |
| \(n_s\) | 0.9657 ± 0.0062 | 0.9648 ± 0.0061 | 0.9672 ± 0.0056 | 0.9656 ± 0.0062 | 0.9692 ± 0.0060 | 0.9682 ± 0.0061 | 0.9589 ± 0.0059 |
| \(\ln(10^{10}A_s)\) | 3.082 ± 0.038 | 3.082 ± 0.036 | 3.091 ± 0.037 | 3.086 ± 0.038 | 3.076 ± 0.038 | 3.050 ± 0.033 | 3.055 ± 0.018 |
| \(H_0\) [\(\text{km s}^{-1} \text{Mpc}^{-1}\)] | 74.2 ± 1.8 | 74.2 ± 1.8 | 74.2 ± 1.8 | 74.2 ± 1.8 | 74.2 ± 1.8 | 74.2 ± 1.8 | 74.2 ± 1.8 |
| \(\sigma_8\) | 1.12 ± 0.05 | 1.06 ± 0.15 | 1.08 ± 0.15 | 1.09 ± 0.15 | 1.12 ± 0.14 | 1.10 ± 0.12 | 1.11 ± 0.18 |
| \(\Omega_m\) | 0.190 ± 0.022 | 0.223 ± 0.025 | 0.243 ± 0.027 | 0.242 ± 0.023 | 0.244 ± 0.023 | 0.174 ± 0.017 | 0.190 ± 0.028 |
| \(\xi\) | > -0.194 | > -0.194 | > -0.194 | > -0.194 | > -0.194 | > -0.194 | > -0.194 |
| \(w\) | -1.39 ± 0.24 | -1.174 ± 0.057 | -0.918 ± 0.075 | -0.932 ± 0.067 | -1.48 ± 0.20 | -1.29 ± 0.33 | -1.29 ± 0.38 |

**TABLE IV:** 68% CL constraints on cosmological parameters from the “Planck TT + lowTEB” baseline dataset and its further combinations with several datasets (see text). An interacting dark energy model with negative coupling \(\xi \leq 0\) and a dark energy equation of state \(w\) are assumed. Please note that in case of the “tau055” prior the “lowTEB” dataset is not considered.

| Parameter | Planck TT | Planck TT | Planck TT | Planck TT | Planck TT | Planck TT | Planck TT |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \(\Omega_b h^2\) | 0.02225 ± 0.00016 | 0.02223 ± 0.00016 | 0.02226 ± 0.00015 | 0.02224 ± 0.00016 | 0.02231 ± 0.00016 | 0.02224 ± 0.00016 | 0.02218 ± 0.00015 |
| \(\Omega_c h^2\) | 0.100 ± 0.013 | 0.101 ± 0.013 | 0.090 ± 0.013 | 0.090 ± 0.011 | 0.102 ± 0.013 | 0.099 ± 0.014 | 0.099 ± 0.019 |
| \(\tau\) | 0.074 ± 0.017 | 0.075 ± 0.017 | 0.078 ± 0.016 | 0.078 ± 0.017 | 0.070 ± 0.017 | 0.054 ± 0.015 | 0.0597 ± 0.0089 |
| \(n_s\) | 0.964 ± 0.0048 | 0.9636 ± 0.0045 | 0.9651 ± 0.0044 | 0.9644 ± 0.0049 | 0.9660 ± 0.0047 | 0.9650 ± 0.0048 | 0.9601 ± 0.0046 |
| \(\ln(10^{10}A_s)\) | 3.082 ± 0.033 | 3.085 ± 0.033 | 3.090 ± 0.034 | 3.091 ± 0.033 | 3.072 ± 0.033 | 3.049 ± 0.027 | 3.058 ± 0.018 |
| \(H_0\) [\(\text{km s}^{-1} \text{Mpc}^{-1}\)] | 74.2 ± 1.7 | 74.2 ± 1.7 | 74.2 ± 1.7 | 74.2 ± 1.7 | 74.2 ± 1.7 | 74.2 ± 1.7 | 74.2 ± 1.7 |
| \(\sigma_8\) | 1.13 ± 0.09 | 1.05 ± 0.14 | 1.08 ± 0.14 | 1.09 ± 0.15 | 1.13 ± 0.15 | 1.10 ± 0.13 | 1.11 ± 0.17 |
| \(\Omega_m\) | 0.188 ± 0.023 | 0.224 ± 0.026 | 0.244 ± 0.027 | 0.244 ± 0.028 | 0.173 ± 0.017 | 0.201 ± 0.029 | 0.203 ± 0.029 |
| \(\xi\) | -0.16 ± 0.14 | -0.16 ± 0.11 | -0.29 ± 0.09 | -0.351 | > -0.351 | > -0.351 | > -0.351 |
| \(w\) | -1.41 ± 0.24 | -1.184 ± 0.064 | -0.934 ± 0.071 | -0.934 ± 0.064 | -1.52 ± 0.20 | -1.28 ± 0.35 | -1.33 ± 0.35 |

**TABLE III:** 68% CL constraints on cosmological parameters from the “Planck TT + lowTEB” baseline dataset and its further combinations with several datasets (see text). An interacting dark energy model with negative coupling \(\xi \leq 0\) and a dark energy equation of state \(w\) are assumed. Please note that in case of the “tau055” prior the “lowTEB” dataset is not considered.

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TABLE V: 68% CL constraints on cosmological parameters from the “Planck TTTEEE + lowTEB” baseline dataset and its combination with external datasets assuming ΛCDM and wCDM models in the absence of an interaction, i.e. Ω = 0.

| Parameter | Planck TTTEEE + lowTEB | Planck TTTEEE + lowTEB + RAO | Planck TTTEEE + lowTEB + JLA | Planck TTTEEE + lowTEB + lensing | Planck TTTEEE + lowTEB + RAO + lowTEB + JLA + lowTEB + lensing |
|-----------|------------------------|------------------------------|-------------------------------|----------------------------------|-------------------------------------------------------------|
| $\Omega_b h^2$ | 0.02226 ± 0.00015 | 0.02229 ± 0.00014 | 0.02227 ± 0.00015 | 0.02228 ± 0.00016 | 0.02225 ± 0.00016 |
| $\Omega_c h^2$ | 0.1198 ± 0.0014 | 0.1196 ± 0.0014 | 0.1193 ± 0.0014 | 0.1196 ± 0.0015 | 0.1196 ± 0.0014 |
| $n_s$ | 0.9646 ± 0.0047 | 0.9661 ± 0.0041 | 0.9650 ± 0.0047 | 0.9652 ± 0.0048 | 0.9649 ± 0.0047 |
| $\sigma_8$ | 0.831 ± 0.013 | 0.832 ± 0.013 | 0.831 ± 0.013 | 0.8416 ± 0.0088 | 0.831 ± 0.012 |
| $\Omega_{m} h^2$ | 0.3152 ± 0.0089 | 0.3119 ± 0.0065 | 0.3142 ± 0.0089 | 0.3122 ± 0.0049 | 0.307 ± 0.017 |
| $w$ | -1 | -1 | -1 | -1 | -1 |

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