Low-speed streak instability in near wall turbulence with adverse pressure gradient

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Abstract. A direct numerical simulation of a turbulent channel flow with a lower curved wall is performed at Reynolds number $Re_{\tau} \approx 600$. Low-speed streak structures are extracted from the turbulent flow field and base flows formed with conditional streaks averages, superimposing the mean streamwise velocity profile, are used for linear stability analyses. The instability in the presence of a strong pressure gradient at the lower wall is shown to be of varicose type, whereas at the upper wall (with weak pressure gradient) varicose and sinuous unstable modes can coexist. The onset of the dominant instability mechanism of varicose type is shown to coincide with the strong production peaks of turbulent kinetic energy near the maximum of pressure gradient on both the curved and the flat walls. The size and shape of the counter-rotating streamwise vortices associated with the instability modes are shown to be reminiscent of the coherent vortices emerging from the streak skeletons in the direct numerical simulation.

1. Introduction

Turbulent boundary layers with adverse pressure gradient (APG) are present in many realistic internal and external flows, like flows around aerfoils, ground vehicles or turbine plates, to cite a few. The available numerical models for turbulent flow are in general based on scaling considerations for both the inner layer and for the outer region of the turbulent boundary layer. The APG however also modifies the distribution of Reynolds stresses which has been shown for instance by Skåre & Krogstad (1994) for various pressure gradients. Independently from the first peak in the inner layer, a second peak which moves away from the wall has been found and this specific behaviour of APG flow observed in numerical and experimental investigations has been compared in Shah et al. (2010) for various configurations and a large range of Reynolds numbers. It may be conjectured, that the production of strong vortices associated with the peak is connected with the breakdown of more organized turbulent flow structures, a commonly admitted scenario for the late stages of transition to turbulence in wall-bounded flow being the instability of elongated streaks (Asai et al., 2007; Jiménez & Pinelli, 1999; Waleffe, 1997). The possibility of streak instability in fully turbulent wall bounded flow for zero pressure gradient has been explored in Schoppa & Hussain (2002), who showed that only sufficiently strong streaks may become unstable. Streak instability in APG turbulent boundary layers has found less attention. In the present investigation, a direct numerical simulation for a turbulent channel flow with a lower curved wall is performed. A streaks detection procedure is applied to isolate
Pressure coefficient \( C_p = \frac{(P - P_o)}{\left(\frac{1}{2} \rho U_{max}^2\right)} \) at the lower and upper wall.

from the turbulent database averaged low-speed structures which are used, superimposing the mean velocity profile, as basic states for a stability analysis. In §2, we briefly characterize the turbulent flow under investigation and we summarize the streak extraction procedure. The stability results for the streak base flow are discussed in §3 and some conclusions are drawn in §4.

2. Characterization of the flow and streak detection

The numerical simulation procedure to solve the incompressible Navier-Stokes system is the one described in Marquillie et al. (2008), using fourth-order finite differences in the streamwise \( x \)-direction, pseudo-spectral Chebyshev collocation in the wall-normal \( y \)-direction and a spectral Fourier expansion in the (homogeneous) spanwise \( z \)-direction. The simulation domain is \( 4\pi \) in \( x \), 2 in \( y \) and \( \pi \) in \( z \), the spatial resolutions being \( 2304 \times 385 \times 576 \). The Reynolds number based on the inlet friction velocity and the half the channel height is \( Re_\tau = 617 \). The flow slightly separates at the lower curved wall and figure 1 depicts the pressure coefficient for the two walls. At the lower wall with the bump (sketched in figure 1) the adverse pressure gradient region starts at \( x = -0.2 \) (\( x = 0 \) being the bump summit) and it is shifted slightly downstream to \( x = 0.3 \) at the flat upper wall. The analysis reveals a strong production peak of turbulent kinetic energy near the maximum of the pressure gradient at both walls. To illustrate the production of intense vortices, figure 2 shows the second invariant \( Q \) of the velocity gradient tensor and strong vortices are seen to emerge at \( x \)-positions in the range of the strong pressure gradients shown in figure 1. The maximum of the turbulent energy peak at the two walls is shown in figure 3: the dominant peak at the lower wall starts at a small distance from \( x = 0 \) (the bump summit) while slightly more downstream a smoother peak can be seen for the upper wall. The aim of the present investigation being to assess, whether the emergence of the strong coherent vortices may be related to streak instability, a streak detection procedure based on a “skeletonization algorithm” (Palágyi et al., 2006) has been applied. In a first step the structure objects are identified by applying a threshold to the normalized streamwise velocity fluctuation field. As recommended by Lin et al. (2008), the value of the threshold was set to \( u' = -0.9 u'^{rms} \) with \( u'^{rms} \) the maximum over \( y \) of \( u'^{rms} \) at each streamwise position. The binary object resulting from...
Figure 2. Iso-value of the Q–criterion ($Q = \frac{1}{2}[|\Omega|^2 - |S|^2]$ with $S = \frac{1}{2}[(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T]$ and $\Omega = \frac{1}{2}[(\nabla \mathbf{u}) - (\nabla \mathbf{u})^T]$) for the whole simulation domain.

![Figure 2](image1.png)

Figure 3. Streamwise evolution of the maximum turbulent kinetic energy peak at the two walls of the converging-diverging channel flow.

![Figure 3](image2.png)

Figure 4. Results of the detection of the low-speed streaks at the lower wall in the converging part of the domain. The skeletons are indicated with dark tubes down to $x = 1.3$ as the 3D visualization indicates that the streaks are totally destroyed further downstream.

![Figure 4](image3.png)
3. Streak base flow and stability analysis

Superimposing averaged streaks and the mean velocity profile, one recovers at each x-location along both walls a streamwise velocity $\bar{U}(y, z)$ called in the following as streak base flow. Several conditional averages of streaks have been computed and the distance of the streak’s centre from the wall proved to be an essential parameter, as shown in Marquillie et al. (2011). The regions in x where the peaks of the turbulent kinetic energy are observed (cf. figure 3) have been explored and examples of streak base flows are shown in figure 5, at $x = 1.29$ and $x = 0.22$ near the upper and lower wall, respectively. Owing to the non-homogeneity in the streamwise x-direction, local wall units are denoted conventionally with superscript + and reference wall units based on the frictional velocity at the inlet have the superscript *. The streak base flow at the upper wall proved to be particularly sensitive to the specific set of streaks considered. In figure 5, the streak base flows formed with the 20 % lowest and 20 % highest streaks are shown. While for the set with the streaks closest to the wall there is only a slightly distorted line of...
Figure 6. Streak instability growth rate $\omega_i$ at the upper wall, for conditionally averaged streak base flows, with respect to the streak’s distance from the wall, as function of the wavelength in reference wall units ($\lambda^+$) and ($\lambda^\ast$), for streak base flow at $x = 1.29$; varicose instability (left), sinuous instability (right), for set $S_1$ (20% closest streaks $\bigtriangleup$), set $S_2$ (20-40% closest streaks $\blacksquare$), set $S_3$ ($\blacklozenge$), set $S_4$ ($\blacklozenge$), set $S_5$ (20% highest streaks $\blacktriangledown$). Varicose instability for total average of streaks ($\triangle$).

inflection points with respect to $y$, for the higher streaks a second closed contour appears. For the lower wall the streak base flow with a total average of the streaks is shown and the inflection points with respect to the wall-normal coordinate are almost homogeneously distributed along the spanwise coordinate. The streak base flows using sets of conditionally averaged streaks look very similar. This is due to the inflectional mean velocity profile which appears to be the dominant contribution to the base flow at the lower wall. The streak base flow depending on the $x$-coordinate, a stability analysis assuming homogeneity of the disturbances in the streamwise direction is strictly speaking not valid. A locally parallel flow assumption in $x$ has nevertheless been made which will be shown hereafter to be indeed reliable. At each $x$ location ($y, z$) is the coordinate system in the domain normal to the wall (at the lower and the upper wall) and the velocity and pressure perturbations are

$$\mathbf{u}(x, y, z, t) = (\hat{u}(y, z), \hat{v}(y, z), \hat{w}(y, z)) e^{i(\omega t - \alpha x)}, \quad p = \bar{p}(y, z) e^{i(\omega t - \alpha x)},$$

with the complex frequency $\omega = \omega_\ast + i\omega_i$, the flow being unstable if $\omega_i > 0$ for some wavenumber $\alpha$. Linearizing the Navier-Stokes system at the base state $(\bar{U}(y, z), 0, 0)$ the perturbation modes are solution of a generalized eigenvalue problem, the system once discretized, using Chebyshev-collocation in both wall-normal $y$ direction and spanwise $z$ coordinate. Owing to the sharp gradients, high resolution with up to $300 \times 80$ collocation points had to be considered and the very large eigenvalue problems are solved using nowadays customary Arnoldi-type methods (for details, see Marquillie et al. (2011)). According to the commonly used classification, see, for instance, Asai et al. (2002), varicose modes are such that the streamwise perturbation velocity component $\hat{u}(y, z)$ is symmetric, that is $\hat{u}(y, -z) = \hat{u}(y, z)$, whereas $\hat{u}(y, z)$ for sinuous modes is anti-symmetric with $\hat{u}(y, -z) = -\hat{u}(y, z)$. Focusing on the region where the turbulent kinetic energy shown in figure 3 increases near the walls, the stability analysis has been performed for streak base flow with conditional averages of streaks. Five distinct sets have been considered, labelled $S_1$ to $S_5$, each one containing 20% of the streaks ranged according to increasing distance from the wall (with $S_1$ the 20% closest, $S_2$ the 20-40% closest etc. and $S_5$ the 20% highest streaks). The result for the upper wall at $x = 1.29$ (that is in the region of the turbulent energy increase) is shown in figure 6. Varicose and sinuous modes become unstable for the different
sets considered, the most amplified perturbations being however of various type. Note that when the total average of streaks is considered, shown as $\triangle$ on the left plot in figure 6, only varicose modes become unstable. At the lower wall no unstable sinuous modes are found: no matter what set of streaks is considered only varicose perturbations become unstable in the region of the energy peak. The detailed stability analysis is given in Marquillie et al. (2011) and in figure 7 the amplification rates $\omega_i$ are plotted as function of $x$ for the streak base flow with the total average of streaks. The wavelength has been fixed at a value associated with the highest amplification rates for the $x$-range considered. The total average streak base flow is seen to become unstable at values close to $x = 1.3$ at the upper wall and $x = 0.2$ at the lower wall, that is the instability onset coincides with the increase of the turbulent kinetic energy depicted in figure 3 for both walls. The growth rates for the streak base flow at the upper wall exhibit a local maximum, whereas at the lower wall (right plot in figure 7) they increase almost linearly in $x$. As mentioned earlier, the mean velocity profile component $U(y)$ of the streak base flow is much more visible at the lower wall. Figure 8 shows the profiles of the wall-normal derivative of $U(y)$ near the lower wall and the maximum, that is the inflection point, is seen to enter the flow domain $y > 0$ downstream the bump summit. The one-dimensional stability computations

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure7a.png}
\includegraphics[width=0.45\textwidth]{figure7b.png}
\caption{Growth rate $\omega_i$ of the varicose mode at fixed wavelengths $\lambda^*$ for the unconditionally averaged streak base flow at different $x$-locations. Left: upper wall, $\lambda^* \approx 300$. Right: lower wall, $\lambda^* \approx 160$, for the streak base flow ($\times$); $+$: Growth rates for the one-dimensional mean velocity profiles; $\blacksquare$: Growth rates for the one-dimensional mean velocity profiles with $\nu_T(y)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure8.png}
\caption{Profiles of the wall-normal derivative of the mean streamwise velocity at several streamwise positions at the lower wall.}
\end{figure}
for \( U(y) \), that is the Orr-Sommerfeld type analysis, are superimposed in figure 7 to the streak base flow results and the inflectional profile is seen to become indeed unstable. The stability results for this profile using an eddy-viscosity \( \nu_T(y) \) (del Alamo & Jiménez, 2006) are shown as well. It is seen, that the three curves are only slightly shifted which indicates that at the lower wall the inflectional mean velocity profile contributes significantly to the instability. At the upper wall however, with the weaker pressure gradient, the mean velocity profile proved to be stable. A typical perturbation structure for the varicose instability near the lower wall is depicted in figure 9 (upper left plot), the vortex structure shown being counter-rotating. Three-dimensional structures have been extracted from the direct numerical simulation database: the individual streak skeleton near the lower wall is shown in figure 9 (upper right plot) and a hairpin-type streamwise vortex is seen to emerge at \( x \approx 0.78 \). Interestingly, the size of the structure is comparable to that of the almost neutral perturbation structure, which according to the stability analysis starts to increase at \( x \approx 0.22 \). One may conjecture, that according to the convective stability behaviour the perturbation must experience some growth before it becomes visible in the simulation results. The lower plot of figure 9 shows that the sudden formation of hairpin vortices is indeed a recurring event along the spanwise direction for \( 0.4 \leq x \leq 1.4 \).

4. Conclusion

Previous investigations (Asai et al., 2007; Schoppa & Hussain, 2002) have shown, that for zero-pressure-gradient turbulent channel flow, a threshold amplitude is necessary for streaks to become unstable and the sinuous instability scenario seems to prevail. For the present APG
wall turbulence however base flows formed with the total average of streaks extracted from the simulation data become unstable with respect to varicose modes, precisely in the region of the observed kinetic energy increase at both walls. There is no definite averaged streak base model, the type and strength of instability depending, in particular for the upper-wall weak pressure-gradient flow, on the specific set of streaks considered. In this respect, the streak’s distance from the wall appears as an essential parameter. Of prime importance for the type of instability are the more or less homogeneously (with respect to the spanwise coordinate) distributed inflection points in the wall-normal coordinate for the streak base flow. While at the upper wall (with a weaker pressure gradient) the stability results are dominated by the streak’s contribution to the base flow, for the strong adverse pressure gradient near the lower curved wall the mean velocity component in the streak base flow also contributed to the sudden onset of the instability. One may conjecture, whether the reported results are generic for adverse-pressure wall turbulence. Note that characteristic turbulent energy peaks seem to be a general feature in the presence of APG (cf. Shah et al. (2010)). To connect this behaviour to streak breakdown will certainly be interpreted in the light of turbulence modeling. Indeed, the mean turbulent velocity gradients are at the heart of Reynolds-averaged Navier-Stokes modeling, which is unlikely to be reliable in the presence of turbulence production peaks as a result of a streak instability mechanism.

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