LOOKING AT THE FUNDAMENTAL PLANE THROUGH A GRAVITATIONAL LENS

G. BERTIN AND M. LOMBARDI
Dipartimento di Fisica, Universitá degli Studi di Milano, via Celoria 16, I-20133 Milan, Italy
Received 2006 March 15; accepted 2006 June 29; published 2006 August 10

ABSTRACT

We consider the fundamental plane of elliptical galaxies lensed by the gravitational field of a massive deflector (typically, a cluster of galaxies). We show that the fundamental plane relation provides a straightforward measurement of the projected mass distribution of the lens with a typical accuracy of ≈0.15 in the dimensionless column density $\kappa$. The proposed technique breaks the mass-sheet degeneracy completely and is thus expected to serve as an important complement to other lensing-based analyses. Moreover, its ability to directly measure the mass distribution on the small pencil beams that characterize the size of background galaxies may lead to crucial tests for current scenarios of structure formation.

Subject headings: dark matter — galaxies: clusters: general — galaxies: elliptical and lenticular, cD — gravitational lensing

1. INTRODUCTION

One of the most remarkable properties of early-type galaxies is the fact that their distribution in the three-dimensional parameter space of effective radius $R_e$, central velocity dispersion $\sigma_v$, and average (intrinsic) surface brightness $S_B$, within $R_e$ is extremely well localized. In particular, if $\sigma_v$ is measured in kilometers per second, $(SB)_\gamma$ in magnitudes per square arcsecond, and $R_e$ in kiloparsecs, then the parameter space is mostly filled around a locus called the “fundamental plane” (Dressler et al. 1987; Djorgovski & Davis 1987) and described by the equation

$$\log R_e = \log r_e + \log D_A(z) = \alpha \log \sigma_v + \beta (SB)_\gamma + \gamma,$$  \hfill (1)

where $\{\alpha, \beta, \gamma\}$ are three (possibly wavelength-dependent) constants, $r_e$ is the angular effective radius (taken to be measured in radians), and $D_A(z)$ is the angular diameter distance of the galaxy at redshift $z$. In general, the parameters $r_e$ and $(SB)_\gamma$ are operationally defined in terms of a best fit with the $R^{1/4}$ law (de Vaucouleurs 1948); moreover, in the evaluation of the intrinsic surface brightness $(SB)_\gamma$ from the data, a number of factors (such as cosmological dimming, k-correction, and Galactic extinction) need to be taken into account. Note that $r_e$ corresponds to the half-light radius of the galaxy only if the luminosity profile is exactly described by the de Vaucouleurs law. (Some concerns about the universality of the $R^{1/4}$ law and the structural homology of elliptical galaxies have been raised, but the small systematic deviations can be interpreted in terms of weak homology; see Bertin et al. 2002.) Usually, the central velocity dispersion $\sigma_v$ is defined as the luminosity-weighted average dispersion inside an aperture radius of $r_e/8$.

The tightness of the fundamental plane, which in the nearby universe is characterized by a relative scatter in $R_e$ of the order of 15% or below (e.g., Jørgensen et al. 1996), makes this scaling relation equivalent to the existence of a “standard rod.” Therefore, the fundamental plane has been very useful as a distance indicator (e.g., Kelson et al. 2000b; Gavazzi et al. 1999; Bender et al. 1998; see also van Albada et al. 1995). Surprisingly, this scaling relation has also been found to hold tightly at redshifts of cosmological interest, so it has allowed astronomers to investigate the evolutionary properties of early-type galaxies out to $z \approx 1$ (Treu et al. 1999; Jørgensen et al. 1999; Rusin et al. 2003a; Rusin & Kochanek 2005 [and a number of related articles]; see also the discussion reported in § 3 below).

So far, in the context of gravitational lensing (and mostly in the case of strong lensing), the fundamental plane has been mainly employed to characterize the properties of the deflector (see Kochanek et al. 2000), a natural approach since the lenses are often early-type galaxies. In contrast to previous applications, in this Letter we investigate the use of the fundamental plane for the background elliptical galaxies observed through a gravitational lens. Since the fundamental plane essentially provides a “standard rod” that is an absolute length scale through $R_e$, we propose to employ it to measure the magnification due to an intervening gravitational lens. This novel technique not only is expected to provide measurements of the projected mass density of the lens with relatively high signal-to-noise ratio, but also presents several unique characteristics that make it an invaluable diagnostic of the structure of gravitational lenses. The method is applicable, in principle, to a study of single lensed galaxies or, more naturally, to a small sample of lensed objects; for a given lens, it will find best application as a complement to other lensing-based investigations.

2. THE FUNDAMENTAL PLANE OF LENSED GALAXIES

The distortion on the observed images produced by gravitational lensing effects is usually described in terms of the ray-tracing function $f: \theta \rightarrow \theta'$, which gives the real position of a point source $\theta'$ given its observed position $\theta$. Gravitational lensing conserves surface brightness (Schneider et al. 1992), and consequently the observed intensity $I$ is related to the true, unlensed intensity $I'$ by the simple equation

$$I(\theta) = I'(\theta') = I'(f(\theta)).$$ \hfill (2)

If the (projected) lens mass distribution is smooth on small scales (e.g., on the typical angular size of the images of background galaxies), the ray-tracing function will also be smooth.
on these scales. As a result, it is possible to Taylor-expand to first order this function as

\[ f(\theta_0 + \delta) = f(\theta_0) + \left| \frac{\partial f}{\partial \theta} \right|_{\theta_0} \delta + O(\delta^2) \]

(3)

\[ \equiv \theta_0 + A \delta + O(\delta^2), \]

(4)

where the last line defines the source position \( \theta_0 \equiv f(\theta_0) \) and the Jacobian of the ray-tracing, \( A \). In this approximation, the quantities entering the fundamental plane are modified by the ray-tracing function in the following way:

\[ r_s \rightarrow r_s' = r_s \sqrt{|\det A|}, \]

(5)

\[ \sigma_0 \rightarrow \sigma_0' = \sigma_0, \]

(6)

\[ \langle SB \rangle_y \rightarrow \langle SB \rangle_y' = \langle SB \rangle_y. \]

(7)

The last two equations hold because the point values of both the velocity dispersion and the surface brightness are conserved by gravitational lensing, and because both values are defined \textit{intrinsically} with respect to the effective radius \( r_e \). We note that the simple transformation (5) is valid only when the effective \textit{circularized} radius \( r_s \) is defined (as usually done) as the geometric mean of the galaxy semiaxes, \( r_s = (ab)^{1/2} \). In conclusion, equation (1) refers to the \textit{unlensed} quantities \( \{r, \sigma, \langle SB \rangle, \} \). As a result, in the presence of gravitational lensing, the fundamental plane is transformed into

\[ \log r_s + \log D_s(z) = \alpha \log \sigma_0 + \beta \langle SB \rangle_y + \gamma - \frac{1}{2} \log |\det A|. \]

(8)

In this equation, all terms but the last can be derived from observations. The fundamental plane relation can thus be inverted to obtain \( |\det A| \).

3. DISCUSSION

Similarly to the case of the so-called magnification effect (e.g., Taylor et al. 1998), our technique produces an estimate of the determinant of the Jacobian matrix \( A \) of the ray-tracing. In the case of a single-screen lens, we have (see Schneider et al. 1992) \( \det A = (1 - \kappa)^2(1 - g^2) \), where \( \kappa = \Sigma / \Sigma_* \) is the lens convergence \( \Sigma \) is the mean density and \( \Sigma_* \) is the mass density and \( g \) is the modulus of the reduced shear \( (\Sigma - \Sigma_*) / \Sigma \). In the weak-lensing limit (for which the present technique is more straightforward), both \( \kappa \) and \( g \) are small quantities and the determinant reduces simply to \( \det A \approx 1 - 2\kappa \). Hence, in this important limit our technique directly provides a measurement of the lens projected mass density:

\[ 0.4343 \kappa \approx \log r_s + \log D_s(z) - \alpha \log \sigma_0 - \beta \langle SB \rangle_y - \gamma. \]

(9)

Before considering the advantages and the limitations of the method proposed in this Letter, we should address the issue of the statistical error associated with a single application of equation (9). Several studies of the nearby universe have shown that the fundamental plane is extremely tight. In some cases it has been claimed that the intrinsic scatter in \( R_s \) is as low as 11% (Jørgensen et al. 1993); more recent investigations find that the intrinsic scatter of the fundamental plane is approximately 15% in \( R_s \) (Jørgensen et al. 1996). Surprisingly, the scatter appears to be very small also for samples of galaxies at relatively high redshift (Bernardi et al. 2003; di Serego Alighieri et al. 2005). If we refer to the more conservative relative error of 15% in \( R_s \), we derive a relative error of 30% in \( |\det A| \) and thus an (absolute) error of \( \approx 0.15 \) in \( \kappa \). Hence, the effect considered in this Letter should be easily detected from a single observation in the central regions of massive clusters; alternatively, one could use several observations in the periphery of a cluster (see further description at the end of this section).

Another issue to be addressed is related to the evolution of the fundamental plane. So far, no clear indication of a redshift dependence of the constants \( \alpha \) and \( \beta \) has been found, while the offset of the plane \( \gamma \) depends on redshift and is typically consistent with a picture of passive galaxy evolution. These features can be easily incorporated in our study if we have good empirical knowledge of \( \gamma(z) \) for field galaxies (see, e.g., Treu et al. 2002; but see the discussion below).

The lensing of the fundamental plane presents several advantages when compared with standard gravitational lensing techniques:

1. It directly measures the projected mass density and not the shear, as typically done in weak lensing. As a result, the measurements can be interpreted in terms of clear physical quantities with no need for any further analysis.

2. Our method is not plagued by the mass-sheet degeneracy, which severely hampers lensing studies (see, e.g., Bartelmann & Schneider 2001; Bradač et al. 2004). A similar advantage would be shared by the magnification effect studied previously (Taylor et al. 1998), but its application has met with major difficulties, mainly because of its sensitivity to spurious changes in the observed density of background galaxies or to inaccurate measurements of the unlensed galaxy density (Schneider et al. 2000). In addition, a severe bias is generally introduced by the presence of bright galaxies, which tend to saturate the central regions of clusters and to significantly affect the completeness of the detection of background galaxies. In contrast, the lensing of the fundamental plane is not affected by any apparent (position-dependent) bias and should thus produce reliable measurements.

3. The technique proposed yields measurements characterized by a high signal-to-noise ratio, with a typical error on \( \kappa \) of \( \approx 0.15 \). By comparison, we note that a single galaxy usually provides a shear measurement with an accuracy of \( \approx 0.3 \) or worse (see, e.g., Bernstein & Jarvis 2002). For a typical lens \( g \approx \kappa \), that is, the reduced shear modulus and the convergence are of the same order of magnitude, but since the shear depends nonlocally on the lens mass distribution, several thousands of independent shear measurements are usually needed to obtain a good weak-lensing detection (e.g., Lombardi et al. 2005). In this respect, a small number of “lensed fundamental plane” measurements can be used together with standard lensing analyses as an efficient way to break the mass-sheet degeneracy.

4. The lensing of the fundamental plane is based on the evaluation of the lensing magnification on small pencil beams corresponding to the typical size of a background galaxy (usually a few arcseconds). These high-resolution measurements are severely undersampled and will need to be interpolated to obtain a smooth map of the lens mass distribution. Yet, the ability to obtain direct, reliable estimates of the projected mass...
distribution on small angular scales is bound to offer an invaluable diagnostic to test current structure formation scenarios (e.g., to confirm the existence of small undetected dark satellites around massive galaxies [see Kauffmann et al. 1993; Moore et al. 1999; Klypin et al. 1999] or of dark density clumps in clusters of galaxies).

The merits described above come with some price to pay. First, we note that the study of the fundamental plane at intermediate and high redshifts is challenging, especially for the low-density environments considered in this Letter. In particular, the precise evolution of $\gamma$ with redshift, $\gamma(z)$, is still under debate. For example, Kochanek et al. (2000), van Dokkum et al. (2001), van Dokkum & Ellis (2003), and Rusin et al. (2003b) report a relatively slow evolution of $\gamma(z)$, while Treu et al. (2002) and Gebhardt et al. (2003) claim a somewhat faster evolution. Unfortunately, an error in the estimate of the fundamental plane intercept $\gamma(z)$ will directly affect the estimate of the lensing magnification and, thus, of the lens dimensionless mass density $\kappa$. Currently, the determination of the derivative of $\gamma$ with redshift ranges from $\gamma'(z) = 1.35 \pm 0.15$ (Treu et al. 2002) to $\gamma'(z) = 1.00 \pm 0.12$ (van Dokkum & Ellis 2003); hence, these two extremes would lead to a systematic uncertainty of $\approx 0.1$ for $\gamma$ at $z = 0.8$. On the other hand, since several independent studies (including those mentioned above) have shown that the thickness of the fundamental plane does not increase significantly with redshift, the differences reported must be related to systematic effects rather than to pure statistical uncertainties. As a result, it is natural to expect that in the near future it will be possible to reduce this error source and thus to base the study that we propose on a quite reliable estimate of $\gamma(z)$ for field galaxies.

The point just discussed reminds us that the acquisition of the relevant fundamental plane data for distant early-type galaxies is nontrivial. Therefore, observations will necessarily be limited to relatively bright objects (currently $R < 23$ mag). In addition, elliptical galaxies represent only a fraction (approximately 30%) of all galaxies, and their relative number decreases with redshift. In conclusion, all these difficulties suggest that the present application of the method proposed here should be viable only for low- to mid-redshift lensing clusters of galaxies, for which the probability of identifying background early-type galaxies typically observed through a lens. These considerations suggest that the lensing of the fundamental plane would be best employed in conjunction with other lensing analyses. In fact, ground-based weak-lensing observations are observationally challenging and can be applied to the limited number of early-type galaxies typically observed through a lens. These considerations suggest that the Tully-Fisher scaling relation is often used as a diagnostic to test current structure formation scenarios. In fact, ground-based weak-lensing observations are based on approximately 25 galaxies arcmin $^{-2}$, with each galaxy typically giving an estimate of the local shear with 0.4 error. Hence, by averaging the various shear measurements over a square arcminute, an error of $\approx 0.08$ on the shear can be achieved. This value should be compared with the $\approx 0.1$ error expected on the convergence $\kappa$ from our method alone, or with the $\approx 0.06$ error from a combined analysis. In addition, as noted earlier in this Letter, a combined analysis would break the mass-sheet degeneracy of weak lensing.

4. CONCLUSIONS AND PROSPECTS

In this Letter we have considered the application of the fundamental plane of elliptical galaxies, as equivalent to a "standard candle," to investigate the mass distribution of gravitational lenses from their magnification of the intrinsic scale length $R_g$. We have shown that the proposed technique can provide column density measurements on pencil beams with an accuracy of 0.15 in the reduced dimensionless mass density $\kappa$.

As explained above, our technique has several advantages with respect to standard lensing methods, but it is also observationally challenging and can be applied to the limited number of early-type galaxies typically observed through a lens. These considerations suggest that the Tully-Fisher scaling relation is often used as a diagnostic to test current structure formation scenarios. In fact, ground-based weak-lensing observations are based on approximately 25 galaxies arcmin $^{-2}$, with each galaxy typically giving an estimate of the local shear with 0.4 error. Hence, by averaging the various shear measurements over a square arcminute, an error of $\approx 0.08$ on the shear can be achieved. This value should be compared with the $\approx 0.1$ error expected on the convergence $\kappa$ from our method alone, or with the $\approx 0.06$ error from a combined analysis. In addition, as noted earlier in this Letter, a combined analysis would break the mass-sheet degeneracy of weak lensing.

In principle, an analogous technique could be based on the existence of a "standard candle," as provided by the Tully-Fisher relation for late-type galaxies, $L \propto V^p$ (Tully & Fisher 1977), where $L$ is the total absolute luminosity of the galaxy, $V$ is its maximun rotational velocity, and $p$ is a (wavelength-dependent) constant. Similarly to the fundamental plane, the Tully-Fisher scaling relation is often used as a distance estimator or to analyze the evolutionary properties of spiral galaxies. In the nearby universe, the measured scatter in $L$ can be as small as 20% (e.g., Pierce & Tully 1988; Willick 1990; Raychaudhury et al. 1997; Verheijen 2001; see Strauss & Willick 1995 for a review), suggesting that the Tully-Fisher relation might be used to measure with a similar accuracy the lens magnification $|\det A|$ and would thus provide a direct
estimate of \( \kappa \) with an rms error as small as 0.1. The practical use of the Tully-Fisher relation as a distance estimator has been limited to low redshifts. Recently, it has been possible to investigate the Tully-Fisher relation at relatively high redshifts (e.g., Bamford et al. 2006; Böhm et al. 2004). Unfortunately, a rather large scatter is observed in these studies (approximately 0.8–1 mag), which severely limits applications of the type proposed in this Letter on the basis of the fundamental plane. Nevertheless, if the observed scatter is not (totally) intrinsic, a rather large scatter is observed in these studies (approximately 0.8–1 mag), which severely limits applications of the type proposed in this Letter on the basis of the fundamental plane.

Type Ia supernovae (SNe Ia) also appear to be very good (calibrated) “standard candles” (Riess et al. 1996; Branch 1998), and thus they could be suitable for the lensing analysis proposed in this Letter. In fact, such a possibility has already been considered by various authors (see, e.g., Holz 2001; Goobar et al. 2002; Oguri et al. 2003; Oguri & Kawano 2003), especially in relation to the prospects offered by the Supernova Acceleration Probe (SNAP). This satellite is expected to provide approximately eight multiply imaged supernovae per year within its 20 fixed, 1 deg\(^2\) fields (Holz 2001); of these, approximately two are expected to be Type Ia SNe for which the absolute magnification can be measured. Since the probability of observing a strong lensing effect on a randomly selected area of the sky (the strong-lensing optical depth) is \( \tau \approx 0.001 \) for sources out to redshift 1.5, lensing applications of SNe will be limited to the study of the few clusters for which serendipitous observations become available.

We wish to thank Piero Rosati for several helpful and stimulating discussions. This work was partly supported by MIUR (Cofin-2004).

REFERENCES

Bamford, S. P., Aragón-Salamanca, A., & Milvang-Jensen, B. 2006, MNRAS, 368, 308

Bartelmann, M., & Schneider, P. 2001, Phys. Rep., 340, 291

Bender, R., Saglia, R. P., Ziegler, B., Belloni, P., Greggio, L., Hopp, U., & Bruzual, G. 1998, ApJ, 493, 529

Bernardi, M., et al. 2003, AJ, 125, 1866

Bernstein, G. M., & Jarvis, M. 2002, AJ, 123, 583

Bertin, G., Ciotti, L., & Del Principe, M. 2002, A&A, 386, 149

Böhm, A., et al. 2004, A&A, 420, 97

Bradač, M., Lombardi, M., & Schneider, P. 2004, A&A, 424, 13

Branch, D. 1998, ARA&A, 36, 17

Broadhurst, T., et al. 2005, ApJ, 621, 53
de Vaucouleurs, G. 1948, Ann. d’Astrophys., 11, 247

Diego, J. M., Sandvik, H. B., Protopapas, P., Tegmark, M., Benítez, N., & Broadhurst, T. 2005, MNRAS, 362, 1247
di Serego Alighieri, S., et al. 2005, A&A, 442, 125

Djorgovski, S., & Davis, M. 1987, ApJ, 313, 59

Dressler, A., Lynden-Bell, D., Burstein, D., Davies, R. L., Faber, S. M., Terlevich, R. J., & Wegner, G. 1987, ApJ, 313, 42

Fukugita, M., Shimasaku, K., & Ichikawa, T. 1995, PASP, 107, 945

Gavazzi, G., Boselli, A., Scodellio, M., Pierini, D., & Belsole, E. 1999, MNRAS, 304, 595

Gebhardt, K., et al. 2003, ApJ, 597, 239

Glazebrook, K., Ellis, R., Santiago, B., & Griffiths, R. 1995, MNRAS, 275, L19

Goobar, A., Mörtsell, E., Amanullah, R., & Nugent, P. 2002, A&A, 393, 25

Holz, D. E. 2001, ApJ, 556, L71

Jørgensen, I., Franx, M., Hjorth, J., & van Dokkum, P. G. 1999, MNRAS, 308, 833

Jørgensen, I., Franx, M., & Kjærgaard, P. 1993, ApJ, 411, 34

Kauffmann, G., White, S. D. M., & Guiderdoni, B. 1993, MNRAS, 264, 201

Kelson, D. D., Illingworth, G. D., van Dokkum, P. G., & Franx, M. 2000a, ApJ, 531, 137

Kelson, D. D., et al. 2000b, ApJ, 529, 768

Klypin, A., Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, ApJ, 522, 82

Kochanek, C. S., et al. 2000, ApJ, 543, 131

Lombardi, M., et al. 2005, ApJ, 623, 42

Moore, B., Ghigna, S., Governato, F., Lake, G., Quinn, T., Stadel, J., & Tozzi, P. 1999, ApJ, 524, L19

Oguri, M., & Kawano, Y. 2003, MNRAS, 338, L25

Oguri, M., Suto, Y., & Turner, E. L. 2003, ApJ, 583, 584

Pierce, M. J., & Tully, R. B. 1988, ApJ, 330, 579

Raychaudhury, S., von Braun, K., Bernstein, G. M., & Gualachakura, P. 1997, AJ, 113, 2046

Riess, A. G., Press, W. H., & Kirshner, R. P. 1996, ApJ, 473, 88

Rusin, D., & Kochanek, C. S. 2005, ApJ, 623, 666

Rusin, D., Kochanek, C. S., & Keeton, C. R. 2003a, ApJ, 595, 29

Rusin, D., et al. 2003b, ApJ, 587, 143

Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses (Berlin: Springer)

Schneider, P., King, L., & Erben, T. 2000, A&A, 353, 41

Strauss, M. A., & Willick, J. A. 1995, Phys. Rep., 261, 271

Taylor, A. N., Dye, S., Broadhurst, T. J., Benítez, N., & van Kampen, E. 1998, ApJ, 501, 539

Treu, T., Stiavelli, M., Bertin, G., Casertano, S., & Møller, P. 2001, MNRAS, 326, 237

Treu, T., Stiavelli, M., Casertano, S., Møller, P., & Bertin, G. 1999, MNRAS, 308, 1037

Taylor, A. N., Dye, S., Broadhurst, T. J., Benítez, N., & van Kampen, E. 1998, ApJ, 501, 539

Verheijen, M. A. W. 2001, ApJ, 563, 694

Willick, J. A. 1990, ApJ, 351, L5