Numerical study of gluon propagators in Maximally Abelian gauge*

V.G. Bornyakov\textsuperscript{a,b,†}, M.N. Chernodub\textsuperscript{a,c,†}, F.V. Gubarev\textsuperscript{a,c,§}, S.M. Morozov\textsuperscript{a,¶}, M.I. Polikarpov\textsuperscript{a,∥}

\textsuperscript{a}ITEP, B.Cheremushkinskaya 25, Moscow,117259, Russia
\textsuperscript{b}Institute for High Energy Physics, Protvino 142284, Russia
\textsuperscript{c}Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

Propagators of diagonal and off–diagonal gluons are studied numerically in the Maximally Abelian gauge of the SU(2) lattice gauge theory. We have found the strong enhancement of the diagonal gluon in the infrared region. The enhancement factor is about 50 at the smallest available momentum, 325 MeV. We discuss also various analytical fits to the propagators.

Propagators of fundamental fields are basic quantities in quantum field theories. In non–Abelian gauge theories the infrared behaviour of the gluon propagators is believed to carry an information about the color confinement. A popular approach to the confinement is based on the dual superconductor mechanism \cite{1} which is most thoroughly studied in the Maximally Abelian (MA) gauge \cite{2} (see, e.g. \cite{3} for a review). A study of the MA gauge gluon propagator in the coordinate space \cite{4} has shown that the propagator of the off-diagonal gluons at large distances is exponentially suppressed with respect to the diagonal propagator by the effective mass about $1.2$ GeV. This property of the gluon propagators – supporting the Abelian dominance in gluodynamics \cite{5,6} – was confirmed and further explored in the momentum space in Refs. \cite{7,8}. The mass gap generation for the off-diagonal gluons was also investigated analytically \cite{9}. Below we provide numerical results supporting the Abelian dominance in terms of the gluon propagators.

We study the SU(2) pure gauge model with the Wilson action. We use the standard parameterization of SU(2) link matrices $U_{11} = \cos \varphi e^{i\theta}$ and $U_{12} = \sin \varphi e^{i\chi}$. The gauge fields are given by

$$A^a_\mu(x) = -i(U_\mu(x) - U_\mu^T(x)),$$

where $\sigma^a$ are the Pauli matrices. In terms of the link angles one gets\footnote{Note that in Ref. \cite{10} the definition of the field $A$ differs from Eq. \cite{11} by the factor of 2.}:

$$A^1_\mu(x) = 2 \sin \varphi_\mu(x) \sin \chi_\mu(x),$$
$$A^2_\mu(x) = 2 \sin \varphi_\mu(x) \cos \chi_\mu(x),$$
$$A^3_\mu(x) = 2 \cos \varphi_\mu(x) \sin \theta_\mu(x).$$

We call $A^3_\mu(x)$ the diagonal gluon field, and $A^i_\mu(x), i = 1, 2$, the off-diagonal gluon field.

The MA gauge is fixed by maximizing

$$\tilde{F}_{\text{MAG}}[\varphi] = \sum_{x,\mu} \cos 2\varphi_\mu(x).$$

This condition fixes the SU(2) gauge group up to a U(1) subgroup. In order to fix the remaining U(1) gauge symmetry we implement a generalization of the Landau gauge maximizing the functional

$$\tilde{F}_{\text{Land}}[\theta, \varphi] = \sum_{x,\mu} \cos \varphi_\mu(x) \cos \theta_\mu(x),$$

which is consistent with the definition \cite{11} of the $A^3_\mu$ field. In the continuum limit the definition \cite{12} corresponds to the standard U(1) Landau gauge.
We calculate the diagonal and off-diagonal propagators,
\[
D_{\mu\nu}^{\text{diag}}(p) = \langle A_{\mu}^{3}(k)A_{\nu}^{3}(-k) \rangle, \\
D_{\mu\nu}^{\text{off}}(p) = \langle A_{\mu}^{1}(k)A_{\nu}^{1}(-k) \rangle,
\]
respectively. Here \( A_{\mu}^{\pm} = (A_{\mu}^{1} \pm iA_{\mu}^{2})/\sqrt{2} \) and the Fourier transformed field, \( A_{\mu}(x) \), is defined as
\[
A_{\mu}(k) = L^{-4} \sum_x e^{-ikx} x^\mu A_{\mu}(x).
\]
The lattice momentum variable is \( p_{\mu} = (2/\alpha) \sin(\alpha k_{\mu}/2) \), where \( k_{\mu} = 2\pi n_{\mu}/(aL_{\mu}) \) with \( n_{\mu} = 0, ..., L_{\mu} - 1, \mu = 1, \ldots, 4 \). In terms of \( p \) the lattice propagator of a free massive scalar particle in momentum space has a familiar form, \( D(p) \propto 1/(p^2 + m^2) \). The local gauge condition corresponding to the maximization of Eq. \( 49 \) takes a simple form, \( p_{\mu}A_{\mu}^{0} = 0 \).

The most general structure of both diagonal and off-diagonal propagators is
\[
D_{\mu\nu}(p) = \left( \delta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2} \right) D_{t}(p^2) + \frac{p_{\mu}p_{\nu}}{p^2} D_{t}(p^2),
\]
where \( D_{t}, t \) are the scalar functions. We have three independent formfactors, \( D_{t,t}^{\text{diag}} \) and \( D_{t,t}^{\text{off}} \), because in the Landau gauge \( D_{t}^{\text{diag}} = 0 \).

Below we present the results on 32^4 lattice at \( \beta = 2.40 \) which corresponds to the lattice spacing \( a = (1.66 \text{ GeV})^{-1} \). We generated 30 configurations which are fixed to the MA gauge by the Simulated Annealing algorithm \([10]\) with 10 randomly generated gauge copies. Then the Landau gauge is fixed by the local over-relaxation algorithm with 20 randomly generated gauge copies.

We show the gluon dressing functions, \( p^2 D_{t,t}(p^2) \), vs. momentum, \( p \), in Figure 1. The diagonal (transverse) dressing function has a relatively narrow maximum at non-zero momentum \( p^2_{\text{diag}} \approx 0.7 \text{ GeV} \). Its behavior at small momenta is qualitatively very similar to the behavior of the gluon propagator in the Landau gauge (see, e.g., \([11]\)). The longitudinal part of the off-diagonal dressing function has a wide maximum at \( p^2_{\text{off}} \approx 2 \text{ GeV} \). The transverse off-diagonal dressing function is a monotonically rising function for all available momenta.

The formfactors \( D_{t,t}^{\text{off}}(p^2) \) and \( D_{t,t}^{\text{off}}(p^2) \) coincide at small momenta. This implies that in the IR region the off-diagonal propagator is
\[
D_{\mu\nu}^{\text{off}}(p) \approx \delta_{\mu\nu} D_{t,t}^{\text{off}}(p^2), \quad p^2 \lesssim 1 \text{ GeV},
\]

Figure 1. The gluon dressing functions \( p^2 D(p^2) \).

The diagonal formfactor is dominating over the off–diagonal ones. In Figure 2 we plot the ratio
\[
R(p^2) = D_{t,t}^{\text{diag}}(p^2)/D_{t,t}^{\text{off}}(p^2),
\]
which reaches the value 50 at the smallest available momentum, \( p = 325 \text{ MeV} \), suggesting that the diagonal gluons are responsible for infrared physics. At higher momenta the suppression of the off-diagonal propagator becomes weaker.

We fit the formfactors at low momenta by the functions \([5][11][12]\):
\[
D(p^2) = \frac{Z m_{2\alpha}}{(p^2 + m^2)^{\alpha}}, \quad \text{(fit 1)},
\]
\[
D(p^2) = \frac{Z m_{2\alpha}}{p^2 + m^2}, \quad \text{(fit 2)},
\]
\[
D(p^2) = \frac{Z}{p^2 + m^2}, \quad \text{(Yukawa fit)},
\]
\[
D(p^2) = \frac{Z p^2}{p^2 + m^2}, \quad \text{(Yukawa 2 fit)},
\]
\[
D(p^2) = \frac{Z p^2}{p^2 + m^2}, \quad \text{(Gribov fit)},
\]

Figure 2. The diagonal/off–diagonal ratio \([5]\).
where \( Z, \alpha, m \) and \( \kappa \) are fitting parameters.

Figure 3. Fits (9–10, 12–13) of the transverse diagonal formfactor at low momenta.

| fit         | \( m, \text{GeV} \) | \( \alpha \) or \( \kappa \) | \( Z \) |
|-------------|---------------------|-------------------------|--------|
| fit 1       | 0.73(2)             | 0.92(3)                 | 16.9(4) |
| fit 2       | 0.58(2)             | 0.49(5)                 | 8.5(2)  |
| Gribov      | 0.33(1)             | -                       | 4.58(5) |
| Yukawa 2    | 0.50(2)             | 0.19(3)                 | 8.3(3)  |

Table 1

The best parameters of the low momentum fits (9–10, 12–13) of the diagonal propagator.

Here we concentrate on the fits of the diagonal propagator, shown in Figure 3. The best fit parameters are presented in Table 1. The three parameter fits (9, 10, 12) are working well in a low momentum region (\( p_{\text{max}} \leq 0.8, 0.4, 0.9 \) GeV, respectively). The corresponding curves are almost indistinguishable from each other. The mass parameter \( m \) do not coincide in these fits, and the difference between its values is about 30%. We have also applied the two-parameter fits given by Yukawa (11) and Gribov (13) functions. The Gribov fit is working well in the region \( p_{\text{max}} \leq 0.9 \) GeV. The Yukawa fit of the diagonal propagator does not work at all (we get \( \chi^2/d.o.f. \sim 6 \) for fits in \( p_{\text{max}} < 1 \) GeV region).

Summarizing, our results obtained in the MA gauge of \( SU(2) \) gluodynamics clearly show that in the infrared region the propagator of the diagonal gluon is much larger (about 50 times at lowest available momentum) than the propagator of the off-diagonal gluon. Thus the colored objects at large distances interact mainly due to an exchange by the diagonal gluons in agreement with the Abelian dominance [5,6]. The diagonal propagator has qualitatively the same momentum dependence as the gluon propagator in Landau gauge. All infrared fits suggest that the diagonal propagator contain massive parameters, although it is not of the Yukawa form. Further results on the propagators in the MA gauge can be found in Ref. [8].

REFERENCES

1. G. ’t Hooft, in High Energy Physics, ed. A. Zichichi, EPS International Conference, Palermo (1975); S. Mandelstam, Phys. Rept. 23 (1976) 245.
2. A. S. Kronfeld et al, Phys. Lett. B 198 (1987) 516; Nucl. Phys. B 293 (1987) 461.
3. T. Suzuki, Nucl. Phys. Proc. Suppl. 30 (1993) 176; M. N. Chernodub, M. I. Polikarpov, in "Confinement, duality, and nonperturbative aspects of QCD", Ed. by P. van Baal, Plenum Press, p. 387, hep-th/9710205. R.W. Haymaker, Phys. Rept. 315 (1999) 153.
4. K. Amemiya and H. Suganuma, Phys. Rev. D 60 (1999) 114509.
5. Z. F. Ezawa and A. Iwazaki, Phys. Rev. D 25 (1982) 2681.
6. T. Suzuki and I. Yotsuyanagi, Phys. Rev. D 42 (1990) 4257.
7. V. G. Bornyakov, S. M. Morozov and M. I. Polikarpov, hep-lat/0209031.
8. V. G. Bornyakov et al, Phys. Lett. B 559 (2003) 214.
9. K. I. Kondo, Phys. Rev. D 58 (1998) 105019; Phys. Lett. B 514 (2001) 335; M. Schaden, hep-th/9909011. K. I. Kondo, T. Shinohara, Phys. Lett. B 491 (2000) 263; U. Ellwanger, N. Wschebor, Int.J.Mod.Phys. A 18 (2003) 1595; D. Dudal, H. Verschelde, J. Phys. A 36 (2003) 8507.
10. G. S. Bali et al, Phys. Rev. D 54 (1996) 2863.
11. D. B. Leinweber et al., Phys. Rev. D 60 (1999) 094507.
12. M. N. Chernodub, E. M. Ilgenfritz, A. Schiller, Phys. Rev. Lett. 88 (2002) 231601; Phys. Lett. B 555 (2003) 206; Phys. Rev. D67 (2003) 034502.