EVALUATION OF DIRICHLET PROCESS GAUSSIAN MIXTURES FOR SEGMENTATION ON NOISY HYPERSPECTRAL IMAGES

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ABSTRACT

Image segmentation is a fundamental step for the interpretation of Remote Sensing Images. Clustering or segmentation methods usually precede the classification task and are used as support tools for manual labeling. The most common algorithms, such as k-means, mean-shift, and MRS, require an extra manual step to find the scale parameter. The segmentation results are severely affected if the parameters are not correctly tuned and diverge from the optimal values. Additionally, the search for the optimal scale is a costly task, as it requires a comprehensive hyper-parameter search. This paper proposes and evaluates a method for segmentation of Hyperspectral Images using the Dirichlet Process Gaussian Mixture Model. Our model can self-regulate the parameters until it finds the optimal values of scale and the number of clusters in a given dataset. The results demonstrate the potential of our method to find objects in a Hyperspectral Image while bypassing the burden of manual search of the optimal parameters. In addition, our model also produces similar results on noisy datasets, while previous research usually required a pre-processing task for noise reduction and spectral smoothing.

Keywords Hyperspectral, segmentation, DPGMM, Dirichlet Process, Gaussian Mixture Model

1 Introduction

Image segmentation is an essential step before the primary tasks such as object detection and classification. Haralick and Shapiro mentioned in their seminal paper [Haralick and Shapiro, 1985] that the clustering process can be viewed as segmentation. Several authors use these terminologies interchangeably, but it is also usual to differ on how the grouping method is done: segmentation is done on the spatial domain of the image, while clustering is done on the measurement space. Additionally, “semantic segmentation” is a classification process in which we assign object identifications to each pixel.

In the context of Hyperspectral Images (HSI) processing, spatial features such as borders and textures become less relevant due to the richness of spectral features. Analysis on HSI usually seeks for one of two tasks: object detection and materials identification [Borzov and Potaturkin, 2018], [Signoroni et al., 2019], [Heylen et al., 2014]. Semantic

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segmentation, which explores spatial features, is more prevalent for object detection, while “pixel-unmixing”, which uses spectral features, is a required step prior to the identification of materials. To keep the terminology concise, we will use the terms “Semantic Segmentation” for object detection and “Clustering” for arbitrary pixel groupings. Such terminology is also used previously as in García et al. [2022], Hamida et al. [2017], Kemker et al. [2018].

In this paper, we propose a method for HSI clustering using Dirichlet Process Gaussian Mixture Model (DPGMM). We choose this model for two main reasons: 1) investigate the separability of a hyperspectral pixel into independent signals and their respective proportions in the mix; and 2) define a model that can capture the structure and variability of HSI pixels while also being robust to the noise present in remote sensing data. Figure 1 shows the average spectral curve with the variability and noise per channel.

It has been demonstrated that GMMs can generate complex multimodal distributions that successfully capture the structure of data Görür and Rasmussen [2010], Wu and Prasad [2016], Nascimento and Bioucas-Dias [2012], and such versatility of GMMs have been explored before in earth observation, remote sensing, and hyperspectral image processing Wu and Prasad [2016], Prasad et al. [2014], Nascimento and Bioucas-Dias [2012], Zare and Gader [2008]. However, to the extent of our knowledge, the research community lacks a quantitative comparison of DPGMM against the previous and most prestigious segmentations algorithms. Also, different from existing methods, we propose a solution based on Variational Inference in a neural network setting to learn the parameters of the DPGMM.

We compute the DPGMM model by “minimizing the divergence of conjugate priors and posteriors” given the data and we use the Variational Inference (VI) method to solve the optimization problem. In order to evaluate the performance of DPGMM for clustering, we compare against existing segmentation and clustering methods recently published in the literature Dao et al. [2021]. The contribution of this research are three-fold: 1) we develop and evaluate a clustering method for HSI datasets using DPGMM; 2) we use three datasets previously explored in Dao et al. [2021] (see Section 4) and make them publicly available; and 3) we open-source our implementation of DPGMM for easier reproducibility and further exploration by the HSI and remote sensing community.

2 Related work

2.1 Clustering methods for RGB images

Clustering of pixels in RGB images has been a very active research topic since the early days of computer vision until today. Relevant work dates back from the early 80’s, such as the seminal publication of Haralick and Shapiro Haralick and Shapiro [1985], until today, with lots of impressive results from Deep Learning research, such as Minaee et al. [2021], Caron et al. [2020], Hosain and Chen [2019], Kemker et al. [2018], Hamida et al. [2017], Wang et al. [2018].

Clustering of regular (3-channel RGB) images has been extensively studied in computer vision. Broadly speaking, the existing methods can be divided into three categories: (1) edge-based methods; (2) region-based methods; and (3) pixel-descriptors methods Hosain and Chen [2019], Dao et al. [2021], Zhou et al. [2019], Yin et al. [2015], Long et al. [2015], Wang et al. [2009], Zhang et al. [2013]. Overall, the methods (1) and (2) are directly applicable to RGB or grayscale images, although they require adaptation to work with hyperspectral images (HSI). Most of the approaches used in RGB images leverage spatial contextual information because distinctive descriptors of pixels cannot rely only
on the spectra that contain mostly color information. Therefore the extraction of spatial features such as borders and texture is crucial for RGB image classification tasks. Publications in remote sensing RGB image processing such as García et al. [2022], Hamida et al. [2017], Kampffmeyer et al. [2016], Kemker et al. [2018], Wang et al. [2018] explore Deep Learning, while some authors explore methods such as k-means Dao et al. [2021], Zhai et al. [2021], Caron et al. [2018], watershed Dao et al. [2021], Pooja and Rajesh [2015], SLIC (Simple Linear Iterative Clustering) Achanta et al. [2010], and mean-shift Dao et al. [2021], Greggio et al. [2012].

2.2 Clustering methods for HSI

Clustering for HSI can be implemented using a variety of methods for pixel descriptors. One can incorporate spatial, spectral, or the conjunction of spatial+spectral features. Several authors, such as Al-Khafaji et al. [2022], Feng et al. [2022], Li et al. [2021], Zhang et al. [2021], explore the use of spectral and spatial features combined. Despite sometimes being beneficial, the richness of features can also bring the higher computational burden, higher processing times, the Hughes phenomenon, while supervised methods also require large amounts of ground-truth data for training. Combining these issues can make the solution not viable in some applications; therefore, the usability of all the features starts to be questionable. To address these problems, some authors also explore dimensionality reduction methods in HSI clustering and classification Ahmad et al. [2019], Cahill et al. [2014], Datta et al. [2018].

For scenarios where the main task is “material identification” through pixel-unmixing, we interpret that spatial context is less relevant because a single pixel can represent several materials. Therefore the transition (borders) between objects is not evident or detectable by border-detector filters. In this context, we say that each pixel is composed of a mixture of pure endmembers\(^3\). Conversely, in very high-resolution images, every single pixel represent a unique material that might be more close to the pure endmember, therefore providing more cues about the independent signal of each material. Because the Remote Sensing-based HSI is collected at high altitudes, the image resolution is naturally low, i.e., tokenonedoteach pixel represent a combination of one or more materials.

2.3 Probability and Mixture Models-based clustering

Mixture Models have been extensively used in image classification and segmentation. Greggio et al. [2012], Nguyen and Wu [2013], Yu [2010] use GMM (Gaussian Mixture Models) for segmentation on RGB images. Shah et al. [2004] explored the Independent Component Analysis Mixture Model (ICAMM) to solve the problem of separability of endmembers. The model is implemented using the mutual information-maximization learning algorithm. However, it is assumed the mixture is a linear combination of the components. Similar to our work, Acito et al. [2003] propose a segmentation algorithm using GMM. However, it is not clear how the model is solved or trained. The number of channels is drastically reduced by simply removing the channels with higher noise. Nascimento et al Grabo et al. propose a GMM model to solve the problem of unmixing under the same assumptions of our work: Dirichlet distributions automatically enforce the sum-to-one and non-negativity constraints. The model is computed thorough the iterative expectation-maximization algorithm. The main differences in our work rely on the fact that we solve the DPGMM model through variational inference in a neural network setting.

3 Method

3.1 Dirichlet Process Gaussian Mixture Model

Consider a hyperspectral image \(X\) with \(N\) pixels: \(x_1, x_2, \ldots, x_N\), where \(x_i \in \mathbb{R}^D\). We seek to represent this image as Gaussian Mixture Model (GMM) with \(K\) components. Under this regime likelihood of \(x_i\) is

\[
\mathcal{L}(\Theta|x_i) = \sum_{j=1}^{K} \pi_j \text{Normal}(x_i|\mu_j, \Sigma_j),
\]

where Normal is the Multivariate Normal Distribution, \(\mu_j\) and \(\Sigma_j\) are the mean and variance for the \(j\)th component, respectively, and \(\pi_j\) is the fractional contribution of each component. The vector of fractional contributions \(\pi\) is a \(K\)-simplex vector, i.e., \(\sum_j \pi_j = 1\) and \(\pi_j \geq 0\). \(\mu_j \in \mathbb{R}^D\) and \(\Sigma_j \in \mathbb{R}^{D \times D}\). \(\Theta\) is a placeholder for \(\pi_j\), \(\mu_j\) and \(\Sigma_j\) for \(j \in [1, K]\).

\(^3\)Endmember is the spectral signature of a single material
We seek to estimate $\Theta$ by minimizing the overall negative log-likelihood

$$-\log L(\Theta|X) = -\sum_{i=1}^{N} \log L(\Theta|x_i).$$

(2)

Note that parameters $\pi_j$ and $\Sigma_j$ take special forms: values $\pi_j$ must meet sum-to-one and non-negativity constraints and $\Sigma_j$ are covariance matrices and these must be symmetric and semi-positive definite. This suggests that it is not sufficient simply minimize the loss in Eq. (2). We also need to define auxiliary losses or regularizing terms to constrain these parameters appropriately. We achieve this by defining priors for these parameters. Samples drawn from a Dirichlet distribution satisfy the $K$-simplex nature of vector $\pi = \{\pi_1, \pi_2, \cdots, \pi_K\}$. Therefore, let

$$\pi \sim \text{Dirichlet}\left(\frac{\alpha_1, \cdots, \alpha_K}{K}\right),$$

where

$$\alpha_j \sim \text{InverseGamma}(1, 1).$$

Additionally, we assume

$$\mu_j \sim \text{Normal}(0, 1).$$

For more details on our choice of priors, please refer to work by Görür et al. [2010], Görür and Rasmussen [2010], Deisenroth [2020], Gelman [1995], and Mathal and Moschopoulos [1992].

It is possible to solve the minimization problem defined in Eq. (2) within a variational inference setting, e.g., by defining a Kullback-Leibler divergence loss using the priors on $\pi_j$, $\mu_j$, and $\Sigma_j$. We also tried this approach first; however, we noticed slow and numerically unstable convergence behavior. Instead we use the priors to construct negative log-likelihood values for $\pi_j$, $\mu_j$, and $\Sigma_j$. We found that this approach works better in practice. Putting it all together, parameters $\Theta$ of the Gaussian Mixture Model are estimated by minimizing the following loss term

$$l(X; \Theta) = -\log L(\Theta|X)$$

(3)

$$-\log p_{\pi}(\pi|\cdot)$$

(4)

$$-\log p_{\mu}(\mu|\cdot)$$

(5)

$$-\log p_{\Sigma}(\Sigma|\cdot),$$

(6)

where $p_{\pi}$, $p_{\mu}$ and $p_{\Sigma}$ are priors defined above.

At inference time, $x_i$ is classified into one of $K$ clusters as follows

$$c_i = \arg \max_{j \in [1,K]} \text{Normal}(x_i|\mu_j, \Sigma_j),$$

(7)

where $c_i$ denotes the cluster for pixel $x_i$.

### 3.2 Segmentation Metrics

Once we have a trained DPGMM model, we run the segmentation algorithm, i.e., inference mode, of the DPGMM, on the three datasets (Section 3.2). As also explored in the previous work of Dao et al. [2021] where the authors compared the results against a comprehensive set of scales and different algorithms, we evaluate our method using the commonly used metrics OS (over-segmentation), US (under-segmentation), and ED (Euclidean distance between OS and US).

$$\text{OS}_{i,j} = 1 - \frac{\text{area}(r_i \cap s_j)}{\text{area}(r_i)}$$

(8)

$$\text{US}_{i,j} = 1 - \frac{\text{area}(r_i \cap s_j)}{\text{area}(s_j)}$$

(9)

$$\text{ED}_{i,j} = \sqrt{\frac{\text{US}_{i,j}^2 + \text{OS}_{i,j}^2}{2}}$$

(10)
where $r_i \in R$ is the area (in pixels) of the ground-truth polygon $i$ (as depicted in Figure 2), $s_j \in S$ is the area of segment $j$ computed by the algorithms.

However, due to the capability of DPGMM to automatically “find” the number of clusters, we compare only with the optimal scale selected in the work of Dao et al. [2021], in which, the scales were manually selected using the ROC (Rate of change) curves of variance.

### 4 Hyperspectral datasets

We used three high spatial resolution hyperspectral images for the studies presented in this paper (Figure 2). These images were captured using the Micro-HyperSpec III sensor (from Headwall Photonics Inc., USA) mounted at the bottom of a helicopter. The images were captured during the daytime at 10:30 am on August 20, 2017. The original images with 325 bands were resampled to obtain 301 bands from 400 nm to 1000 nm with an interval of 2 nm. Raw images were converted to at-sensor radiance using HyperSpec III software.
The images were also atmospherically corrected to surface reflectance using the empirical line calibration method [Dao et al. 2019] with field spectral reflectance measured by FieldSpec 3 spectroradiometer from Malvern Panalytical, Malvern, United Kingdom. These images represent 1) urban, 2) transitional suburban, and 3) forests landcover types. These three landcover types cover a large fraction of use cases for hyperspectral imagery; urban and sub-urban images are often used for city planning and land use analysis. Forest images are typically used for forest management, ecological monitoring, and vegetation analysis. The overlaid polygons in Figure 2 depict the annotated regions for which ground-truth pixel labels are available. Figure 2 (second row, left) shows the hyperspectral image collected in an urban-rural transitional area. We refer to this image as the “Suburban” dataset. It was captured around the Bolton area in southern Ontario and covers an area between 43°52′32″ and 43°53′04″ in latitude and −79°44′15″ and −79°43′34″ in longitude. This region consists of various land cover types, such as rooftops, asphalt roads, swimming pools, ponds, grassland, shrubs, and urban forest. The image also contains regions that are in shadows. The image resolution is 0.3 square meters, and the covered area is around 41,182 square meters.

Figure 2 (second row, middle) shows the hyperspectral image collected in a residential urban area, also around the Bolton region in southern Ontario. We refer to this image as the “Urban” dataset. It contains rooftops, under-construction residences, roads, and lawns landcover types. The dataset also exhibits regions that are in shadows. This image covers the area between 43°45′30″ and 43°45′43″ in latitude and −79°50′06″ and −79°49′51″ in longitude. The image resolution is 0.3 square meters, and the area after removing background pixels is around 59,834 square meters.

Figure 2 (second row, right) shows the hyperspectral dataset collected in a natural forest located at a biological site of the University of Toronto in the King City region in southern Ontario. We refer to this dataset as the “Forest” dataset. It covers the area between 44°01′58″ and 44°02′04″ in latitude and −79°32′06″ and −79°31′55″ in longitude. The image resolution is 0.3 square meters, and the area after removing background pixels is around 43,084 square meters.

5 Experiments and Results

The mathematical abstraction of DPGMM allows for an infinite number of classes. However, we limited the maximum number of clusters \( \max(K) = 5 \) in our experimental set because the number of categories present in our datasets is at most 4. Being an unsupervised model, we trained the model using the entire datasets to learn the parameters of the distribution \( \mathcal{L}(x; \pi, \mu, \sigma) \). Once trained, the model estimates the likelihood of a new pixel belonging to any of the clusters.

We used the OS, US and ED (Equations 8, 9, and 10 respectively) for a quantitative measurement of the segmentation. The smaller the values, the better the quality of segmentation.

The existing labeled samples used to measure the segmentation results provide one class per pixel instead of a mix of materials. Therefore we can fairly compare with previous segmentation methods that look assigns a single class per pixel.

Figure 3 shows, side-by-side, the segments found by DPGMM (right) and the boundaries of the segments overlaid on the RGB image (left). We can observe from Figure 3 that the Forest dataset presented the most challenge in classification due to a large number of shadows which present small values of reflectance, therefore less distinctive.

Despite excelling in the segmentation metrics and eventually winning state-of-the-art at some instances, DPGMM is not the winner method for all datasets and classes investigated in this research. However, the overall and qualitative results (Figure 1) demonstrate that DPGMM can capture the structure of the pixel spectra while also providing extra information about the mixed materials in the pixel spectra, and also showcases that this method can be successfully applied in a multinomial classification task as the next step in a pixel-unmixing pipeline.

5.1 Execution time

We also compared the execution time of the DPGMM algorithm to k-means, mean-shift, and watershed. We did not include the values of the MRS (Multi-resolution segmentation) algorithm due to the considerably larger execution time, as demonstrated by [Dao et al. 2021]. The runtime for inference on DPGMM is two orders of magnitude faster than the other algorithms, as observed in Figure 5.1 and Table 1. The training time is not considered because it is a one-time-only task and does not affect the prediction time. Once the model is trained, the model can be reused several times for inference.
6 Conclusion

We developed a segmentation algorithm based on the Dirichlet Process Gaussian Mixture Model that automatically finds the scale (or the number of clusters) on spectral features of Hyperspectral Images. We compared our results with the most common and recent techniques found in the literature of HSI segmentation. Our results demonstrate that our method is comparable to the state-of-the-art while also allowing to bypass search for an optimal scale. While the previous methods require higher runtimes and the evaluation of several parameters and scales, the algorithm based on Dirichlet Process can find the near-optimal parameters. The qualitative results showed in Figure 3 also indicate that the DPGMM model is able to capture the structure of the data to identify meaningful segments, which also opens a window to further extensions of this work in the realm of pixel-unmixing.

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Figure 3: Segmentation results using DPGMM algorithm. Left column: objects; right column: boundaries of the segments. From top to bottom: Suburban, Urban, and Forest HSI datasets.
Figure 4:
Figure 5: 

Figure 6: