Bragg spectroscopy of a strongly interacting Bose–Einstein condensate

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Abstract. We study Bragg spectroscopy of a strongly interacting Bose–Einstein condensate using time-dependent Hartree–Fock–Bogoliubov theory. We include approximatively the effect of the momentum-dependent scattering amplitude which is shown to be the dominant factor in determining the spectrum for large momentum Bragg scattering. The condensation of the Bragg scattered atoms is shown to alter significantly the observed excitation spectrum by creating a novel pairing channel of mobile pairs.

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1. Introduction

A strongly interacting Bose–Einstein condensate (BEC) has been a rich topic to study both theoretically and experimentally. Theoretical problems arise from the need to provide a proper description of the elementary excitations and experimental difficulties from the instability of the BEC to three-body collisions. Research on these systems offers the potential to be highly rewarding by shedding light on other strongly interacting systems, such as superfluid $^4$He.

Weakly interacting BECs, on the other hand, are generally well understood. The ground state properties are accurately described by Bogoliubov theory as the measurement of the excitation spectrum in 1999 using Bragg spectroscopy confirmed $^1$, $^2$. In Bragg spectroscopy, the condensate is excited using stimulated two-photon Bragg scattering yielding the possibility for very high momentum and energy resolution. The measured spectrum was in very good agreement with the Bogoliubov theory $^3$, $^4$. In the linear response regime, Bragg spectroscopy yields a direct measurement of the dynamic structure factor $S(q, E)$ $^1$, $^5$, where $q$ is the Bragg momentum and $E$ is the energy detuning of the Bragg scattering lasers. However, for extended pulses, in which the fraction of scattered atoms is significant, the depletion of the condensate will cause a drift in the measured spectrum. Moreover, for strongly interacting gases the linear response theory may fail also because the scattered atoms themselves can affect the energy levels of the atoms.

The Bragg spectrum of a strongly interacting BEC was measured in 2008 $^6$. The study found significant deviations from the standard Bogoliubov theory, highlighting the need for a more thorough theoretical treatment of the process.

In this work, we study the Bragg spectroscopy of a strongly interacting BEC using Hartree–Fock–Bogoliubov (HFB) theory $^7$. The theory has been widely applied to various problems, both static and time-dependent. In spite of well-known problems in the theory, such as the unphysical zero-momentum energy gap and problems with ultraviolet divergence of the gap equation, it has been able to provide correct qualitative features and good agreement with experiments $^8$–$^11$. In the present context the long-wavelength excitation gap has no effect due to the high momentum of the probing Bragg field photons. Furthermore, the ultraviolet divergence can be countered by using a two-body scattering T-matrix instead of a bare atom–atom interaction potential in the contact potential approximation. The present time-dependent theory is an extension of the one used in $^9$, adding a proper renormalization and allowing for the macroscopic occupation of the Bragg scattered states but neglecting the explicit molecule formation channel. We will show how the appearance of condensate atoms in finite momentum states leads to profound effects in the observed Bragg spectrum.

2. Hamiltonian

The system is described by the Hamiltonian

$$\hat{H}(t) = \sum_{\vec{k}} \epsilon_{\vec{k}} \hat{c}_{\vec{k}}^{\dagger} \hat{c}_{\vec{k}} + \frac{1}{2V} \sum_{\vec{k},\vec{p},\vec{q}} U_{\vec{k},\vec{p}} \hat{c}_{\vec{k}+\vec{q}/2}^{\dagger} \hat{c}_{-\vec{k}+\vec{q}/2} \hat{c}_{-\vec{p}+\vec{q}/2} \hat{c}_{\vec{p}+\vec{q}/2} + \hat{H}_{\text{Bragg}}(t),$$

where $\hat{H}_{\text{Bragg}}(t)$ is the perturbation due to the Bragg field, $\hat{c}_{\vec{k}}$, $(\hat{c}_{\vec{k}}^{\dagger})$ is the annihilation (creation) operator for a boson with momentum $\vec{k}$, $\epsilon_{\vec{k}} = (\hbar^2 k^2 / 2m)$, $V$ is the volume, and $U_{\vec{k},\vec{p}}$ is the two-body T-matrix of the atom–atom interaction.

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As already observed in [6], the momentum dependence of the scattering amplitude is the dominating effect in the high-momentum Bragg scattering and therefore we have included it in the results. A constant interaction strength $U_{k,p} = U$ would correspond to the delta function approximation of the real-space T-matrix (yielding constant $U$ in the momentum space). However, we want to include the energy dependence of the two-body T-matrix in an approximative way. For low-momentum $k$ the energy (or the momentum) dependence of the real part of the two-body T-matrix for on-shell scatterings ($k = p$) is

$$U_{k,k} \approx \frac{4\pi \hbar^2 a}{m} \text{Re} \left[ \frac{1}{1+i2ka} \right],$$

(2)

where $a$ is the scattering length. In order to satisfy the time-reversal symmetry of the scattering process (or the Hermiticity of the Hamiltonian (1)), the energy-dependent interaction strength $U_{k,p}$ must be symmetric in the exchange of $k$ and $p$. To keep the numerical solution tractable, we approximate the interaction strength by two constant interaction strengths $U_L := U_{0,0}$ and $U_H := U_{Q_B,0}$ so that $U_{k,p} = U_L$ in the low-momentum manifold $k, p \ll Q_B$ and $U_{k,p} = U_H$ in the high-momentum manifold ($k$ and/or $p \approx Q_B$). That is, the Bragg scattered condensed atoms will interact through interaction strength $U_H$ whereas the atoms in the unscattered condensate will feel $U_L$. While the quantum fluctuations and the Bragg scattering produce excitations with momentum higher than $Q_B$, the typical fraction of such atoms is at most 4% in our calculations. Thus, in order to speed up the calculation, we assume that all fluctuations interact through $U_L$.

The operator $\hat{H}_\text{Bragg}(t)$ in the rotating wave approximation can be written as

$$\hat{H}_\text{Bragg}(t) = \Omega(t) \sum_k \hat{c}_{k+Q_B}^\dagger \hat{c}_{k} + \text{H.c.},$$

(3)

where $Q_B$ is the momentum of the Bragg field and $\Omega(t)$ is the coupling between the atoms and the Bragg field. Notice that the rotating wave approximation is used here to remove the counter-rotating terms that violate the energy conservation. The energy of the Bragg field (energy difference between the photons in the two probing laser fields, $\hbar \omega_B$) is included in the coupling $\Omega(t)$. For a pulse of length $T$ we have $\Omega(t) = \Omega \Theta(T-t)e^{-i\omega_B t}$, where $\Omega$ is the strength of the coupling and $\Theta(x)$ is the Heaviside function. Using a smooth Gaussian pulse instead of the rapid switching on/off would reduce ringing oscillations in many of the results below. However, the step function is more convenient and straight-forward to implement numerically.

The HFB theory for BECs has been widely used and its properties and problems are well known [7, 12]. One important problem, especially related to spectroscopy, is the excitation gap at low momenta. This is in violation of the Hugenholtz–Pines theorem [13] and consequently the HFB theory does not yield a proper phonon spectrum. In practice, this means that the Bragg spectrum will have some pathological features when the Bragg momentum $Q_B$ is small. In that region, we can hope to gain at most a qualitatively correct picture, assuming that the effect of the excitation gap is well understood. On the other hand, for large momenta, as is the case we consider here, the HFB theory is expected to work well.

Another problem in HFB theory relevant to this work is the ultraviolet divergence of the pairing field $m_0$ when the theory is not correctly renormalized. This is a consequence of the assumption of a contact interaction potential which is unable to properly describe high-energy scattering processes. A correct result can still be obtained by employing a contact interaction approximation to the effective two-body T-matrix. However, since the calculation of the many-body T-matrix replicates the same set of diagrams, the bare two-body scattering diagrams need
Indeed, we have found that the standard procedure used in several publications [9]–[11], [14, 15] leads to instabilities by making the energies of low-momentum excitations imaginary. This regularization issue will be discussed in more detail below.

3. Equations of motion

Using the Hamiltonian (1), we derive the Heisenberg equation of motion

\[ i\hbar \frac{d}{dt} \hat{c}_k = \epsilon_k \hat{c}_k + \sum_{q,p} U_{k+q/2,p-q/2} \hat{c}_{-k+q}^\dagger \hat{c}_{-q+p}^\dagger + \Omega(t) \hat{c}_{-\hat{Q}_0} + \Omega(t)^* \hat{c}_{\hat{Q}_0}. \]  

(4)

For condensed states, \( \hat{Q} = i\hat{Q}_B \), where \( i \) is an integer, this operator is allowed to have a nonvanishing expectation value \( \langle \hat{c}_n \hat{Q}_B \rangle =: \psi_n \) but otherwise the first nonzero terms include higher order correlators such as the normal Green’s function \( \langle \hat{c}_k^\dagger \hat{c}_\hat{k} \rangle \) and the anomalous Green’s function \( \langle \hat{c}_k \hat{c}_{-\hat{k}} \rangle \). These correlators describe both thermal and quantum fluctuations and give corrections to the Gross–Pitaevskii equation for the condensate. The equation of motion (4) produces an infinite series of higher order correlators. We truncate this series at the two operator correlator level.

Due to the Bragg field, the number of relevant mean fields is large and the resulting equation of motion is potentially involved. However, for sufficiently weak Bragg pulses, multiphoton scattering, in which the atom gets kicked twice by the Bragg field into momentum state \( \pm 2 \hat{Q}_B \), is very unlikely. In practice, we include only the condensate states with momenta \( \hat{Q}_B, 0 \) and \( \hat{0}_B \).

The equations of motion for the condensates are

\[ i\hbar \frac{d}{dt} \psi_0 = (\epsilon_0 + h_L - U_L |\psi_0|^2) \psi_0 + 2U_H (n_1 \psi_{-1} + n_1^* \psi_1) + (\Delta_0 - U_L |\psi_0|^2) \psi_0^* \\
+ U_H m_1 \psi_1^* + U_H m_{-1} \psi_{-1} + \Omega(t) \psi_0 + \Omega(t)^* \psi_1. \]  

(5)

\[ i\hbar \frac{d}{dt} \psi_1 = (\epsilon_{\hat{Q}_B} + h_H - U_H |\psi_1|^2) \psi_1 + 2U_H (n_1 \psi_0 + n_1^* \psi_2) \\
+ U_H \left( \psi_0^2 + m_0 \right) \psi_{-1} + U_H m_1 \psi_0^* + \Omega(t) \psi_0. \]  

(6)

\[ i\hbar \frac{d}{dt} \psi_{-1} = (\epsilon_{\hat{Q}_B} + h_H - U_H |\psi_{-1}|^2) \psi_{-1} + 2U_H (n_1^* \psi_0 + n_1 \psi_2) \\
+ U_H \left( \psi_0^2 + m_0 \right) \psi_1 + U_H m_{-1} \psi_0^* + \Omega(t) \psi_0. \]  

(7)

and for the fluctuations

\[ i\hbar \frac{d}{dt} \hat{c}_k = (\epsilon_k + h_L) \hat{c}_k + 2U_H \left( \delta Q \hat{c}_{k+\hat{Q}_0} + \hat{c}_{k+\hat{Q}_0} \right) + \Delta \hat{c}_{-\hat{k}}^\dagger \\
+ \Delta \hat{c}_{-\hat{k}}^\dagger \hat{c}_{-\hat{Q}_0} + \Delta \hat{c}_{-\hat{k}}^\dagger \hat{c}_{-\hat{Q}_0} + \Omega(t) \hat{c}_{-\hat{k}} \hat{Q}_0 + \Omega(t)^* \hat{c}_{\hat{Q}_0}. \]  

(8)
The Hartree shift of low-momentum atoms is $h_L = 2U_L|\psi_0|^2 + 2U_Ln_0 + 2U_H|\psi_1|^2 + 2U_H|\psi_\perp|^2$, where $n_0 = \sum_k(c_k^\dagger c_k)$ is the fraction of atoms in the excitations, and $h_H = 2U_Hn$, where $n$ is the total density of the gas. The off-diagonal fluctuation density (or the density modulations of the fluctuations) $n_1 = \sum_k\langle \hat{c}_k^\dagger \rho_k \hat{c}_k \rangle$ and $\delta_1 = U_H(\psi_0^\dagger \psi_1 + \psi_\perp^\dagger \psi_\perp + n_1)$. The mean-field pairing fields are defined as $\Delta_0 = (U_L\psi_0^\dagger \psi_0 + U_H 2\psi_1^\dagger \psi_\perp + U_L m_0)$ and $\Delta_\pm = U_H(2\psi_0^\dagger \psi_\pm + m_\pm)$, where $m_\pm = \sum_k\langle \hat{c}_k^\dagger \rho_k \hat{c}_\mp \rangle$. Notice that the first three equations of motion (5)–(7) are mean-field condensates but the last one (8) is an equation of motion for a fluctuation operator. From the fluctuation operator, we form equations of motion for the fluctuation fields $\langle \hat{c}_k^\dagger \rho \rangle$ and $\langle \hat{c}_k^\dagger \rho \rangle$. All anomalous fluctuation fields $m_0$ and $m_±$ are ultraviolet divergent and need to be regularized.

Instead of coupling the mobile condensate $\psi_0$ directly into the excitations $\langle \hat{c}_k^\dagger \rho \rangle$, the Bragg field $\Omega$ provides the coupling into mobile condensate states $\psi_{\pm}$. Initially, these mobile condensate states are empty but the Bragg pulse will break the translational symmetry of the condensate by rotating the condensed atoms into a superposition of the zero-momentum condensate state and the mobile condensate states. The two-body scattering processes couple these atoms into excitations, eventually leading into dephasing of the single-particle density matrix and, in principle, to fragmentation into separate condensates $\psi_0$ and $\psi_{\pm}$. As will be shown below, this coupling is relatively strong as it leads into rapid decay of the excited condensates. However, the present theory is unable to describe the dephasing process.

The set of equations of motion above shows that the Bragg field also creates new excitation fields, such as $\langle \hat{c}_k^\dagger \rho \rangle$ and $\langle \hat{c}_k^\dagger \rho \rangle$. In the calculations below, we have included the equations of motion for all these fields in addition to the standard excitation fields $\langle \hat{c}_k^\dagger \rho \rangle$ and $\langle \hat{c}_k^\dagger \rho \rangle$. It is these mean fields and the condensate fields $\psi_0$, $\psi_{\pm}$ that we propagate in real time in our theory. The initial state is obtained by solving the static HFB theory and the propagation amounts to a fully self-consistent solution of the time-dependent HFB theory. As our analysis below shows, all these various mean fields are needed for a full description of even relatively weak Bragg pulses in which only a small fraction of atoms is excited by the field. In particular, the backward scattered condensate $\psi_\perp$ is important in order to obtain the proper Bogoliubov spectrum in the weakly interacting limit.

In all the numerical calculations below, we have considered a uniform $^{85}$Rb condensate of density $10^{14}$ cm$^{-3}$.

4. Regularization of the ultraviolet divergence

Before studying the Bragg spectroscopy any further, we will address some issues related to the regularization of the anomalous fluctuation fields. Using Matsubara Green’s functions, the anomalous fluctuation field $m_0$ can be written as

$$m_0 = \frac{1}{\beta} \sum_k G(\vec{K})G_0(-\vec{K}),$$

where $G_0(\vec{K})$ is the bare Bose Green’s function and $G(\vec{K})$ is the dressed Green’s function obtained for some self-energy $\Sigma$. The four-vector $\vec{K} = (i\omega, \vec{k})$ consists of a Matsubara frequency $i\omega$ and a three-dimensional momentum $\vec{k}$. The regular HFB theory gives, and is given by,

$$G(\vec{K}) = u_k^2 \frac{1}{i\omega - E_k} - v_k^2 \frac{1}{i\omega + E_k},$$
where \( u_k^2, v_k^2 = \frac{1}{2}(\epsilon_k' / E_k \pm 1) \), \( \epsilon_k' = \epsilon_k + 2Un - \mu \), \( E_k = \sqrt{\epsilon_k^2 + \Delta^2} \) and \( \Delta = U|\psi_0|^2 + Um_0 \). However, the important point is that the bare Green’s function needs to include the Hartree shifts, i.e.

\[
G_0(\vec{K}) = \frac{1}{i\omega - \epsilon_k}.
\]

(11)

Regularization can now be done self-consistently by removing the free particle diagrams where the free particle propagator is given by the same energy shifted Green’s function \( G_0 \). Thus we remove from \( m_0 \) the term

\[
m^{2B}_0 = \frac{1}{\beta} \sum_k G_0(\vec{K})G_0(-\vec{K}).
\]

(12)

This regularization differs from the standard scheme used in the literature \([9]–[11], [14, 15]\) by the energy shift \( 2Un - \mu \) in the bare Green’s function \( G_0(\vec{K}) \). Neglecting the energy shift, i.e. replacing \( \epsilon_k' \) by \( \epsilon_k \) in the regularizing term \( m^{2B}_0 \) will make the regularizing part larger than the initial anomalous term \( |m_0| < |m^{2B}_0| \). This has the unfortunate effect of turning the regularized anomalous pairing field positive. The low-momentum energy gap in the HFB spectrum \( \sqrt{-4Un_0m_0} \) then becomes imaginary and the low-energy excitations turn unstable. However, the imaginary energies are small and the corresponding lifetimes are long, so that the instability can easily go unnoticed. In addition, instabilities apply only to the very lowest energy excitations requiring a dense grid in the momentum space in order to play a role.

Including a constant energy shift \( \delta \) in the bare Green’s function does not affect the two-body scatterings but guarantees \( m_0 < 0 \) if \( \delta \) is large enough. Indeed, choosing \( \delta = 2Un - \mu \), as in (11), is enough to guarantee positivity of \( m_0 \). However, this would require an iterative solution of \( \delta \) as the chemical potential \( \mu \) depends on \( \delta \). Here we approximate \( \delta = Un \) but the results are relatively insensitive to the actual choice of \( \delta \) as long as it is large enough. Regularization of the mobile pairing fields \( m_{\pm 1} \) proceeds in an identical manner.

5. Bragg spectroscopy in the linear response limit

We will first study the Bragg spectroscopy of a weakly interacting Bose gas or the effect of a very short Bragg pulse. In this regime, we can assume that finite momentum pairing and density fields, \( m_{\pm 1} \) and \( n_1 \), are empty or that the corresponding energy shifts are small, \( Un_1 \ll Un \ll \epsilon_{Q_0} \). The equations of motion of the condensates are now (assuming that the populations of the excited condensate states \( \psi_1 \) and \( \psi_{-1} \) are likewise small)

\[
i\hbar \frac{d}{dt} \psi_0 = \left( \epsilon_0 + h_L - U_L|\psi_0|^2 \right) \psi_0 + U_Lm_0\psi_0^* + \Omega(t)\psi_{-1} + \Omega(t)^*\psi_1,
\]

(13)

\[
i\hbar \frac{d}{dt} \psi_1 = \left( \epsilon_{Q_0} + h_H \right) \psi_1 + U_H \left( \psi_0^2 + m_0 \right) \psi_{-1} + \Omega(t)\psi_0
\]

(14)

and

\[
i\hbar \frac{d}{dt} \psi_{-1} = \left( \epsilon_{Q_0} + h_H \right) \psi_{-1} + U_H \left( \psi_0^2 + m_0 \right) \psi_1 + \Omega(t)^*\psi_0.
\]

(15)

In the absence of the Bragg coupling \( \Omega(t) = 0 \), the equations for the excited condensates \( \psi_1 \) and \( \psi_{-1} \) can be solved by the Bogoliubov transformation, showing that the energies of the
excited condensates do indeed match the energies of the corresponding excitations of momenta ±\( Q_B \) in the HFB spectrum. Due to the quantum depletion, the states with momenta ±\( Q_B \) can already be initially populated by a few atoms even though these atoms do not constitute a condensate. Linear response theory (see, for example [14]) that does not allow for the condensation of these excited states is valid as long as the number of scattered atoms does not exceed the initial population of these excited states. In this regime, the Bragg spectrum will yield a measurement of the dynamic structure factor \( S(Q_B, \hbar \omega_B) \) [5] and the feedback from the scattered atoms has no effect on the measured spectrum even with a strongly interacting gas. However, this transition regime is left already when the fraction of scattered atoms exceeds \( 10^{-3} \). Even though the number of scattered atoms is still very tiny, the broken symmetry of the ±\( Q_B \) states will have a significant effect on the energies of these scattered states. Notice, however, that the energy levels of all other states are unaffected. For a weakly interacting gas, the above equations still hold even for longer pulses because the effect of these mobile fields on the energy levels is very small. However, in that case one needs to take into account the depletion of the initial condensate and the corresponding drift in the resonant energy.

Notice that, in addition to the Bragg field, the zero-momentum pairing field \( \Delta_0 \) acts as a source for the two condensates \( \psi_{\pm 1} \) even though it cannot level out the difference in the populations due to momentum conservation (i.e. a zero-momentum pair of atoms can be turned into a pair \( \psi_1 \psi_{-1} \) which conserves momentum but also the population difference in the two states). This stability of the population difference between the \( \psi_1 \) and \( \psi_{-1} \) condensates underlines the importance of the off-diagonal fields \( \Delta_{\pm 1} \) and \( n_1 \) for the decay of the multiply condensed state. Indeed, it is only through these fields that the system finally decays into an equilibrium state. However, this decay process is slow in a weakly interacting gas, and can therefore be neglected in most cases.

6. Bragg spectroscopy of a strongly interacting BEC

For strong interactions, and for experimentally relevant fractions of scattered atoms, the off-diagonal fields are needed for the decay processes and for the correct excitation spectrum. This effectively takes the theory beyond the linear response because the scattered atoms will have a feedback loop into the energy levels of the atoms and hence into further scattering processes of atoms. The presence of additional pairing fields \( \Delta_{\pm 1} \) allows the atoms to lower their energies even further by pair formation and this is reflected also in the Bragg spectrum.

In all the numerical results shown here we have used \( \hbar^2 Q_B^2 / 2m \approx \hbar 15.4 \text{ kHz} \), \( \Omega \approx \hbar 1 \text{ kHz} \) and the length of the pulse was approximately 0.1 ms. The results are relatively insensitive to the strength of the Bragg coupling \( \Omega \) and the pulse length as long as the coupling strength is not too strong (leading into very broad spectral peaks) and the fraction of scattered atoms is not too large (leading into strong depletion of the initial condensate). We have assumed uniform density of \( 10^{14} \text{ cm}^{-3} \) and the various interaction strengths \( na^3 \) are shown separately for each set of data in figures 3–5.

Figure 1 shows the total momentum of the system as a function of time during and after the Bragg pulse. Notice that the number of atoms \( N \) is conserved throughout the simulation. The total momentum is not quite conserved due to a finite momentum cutoff in the summations. However, the error can be made arbitrarily small by increasing the momentum grid size. Figure 2 shows the decay of the excited condensates after (and during) the Bragg pulse. The decay lifetime scales as \( \hbar / Un \), showing that the effect of the mobile pairing field is large even though the anomalous fields \( m_{\pm 1} \) are small.

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Figure 1. The total momentum $P$ as a function of time increases rapidly when the Bragg field is switched on at $t = 0$ and becomes constant when the probing field is switched off at $t = 0.1$ ms. Here we have used a Gaussian pulse envelope, yielding a smooth time dependence of the total momentum. Here $na^3 \approx 0.003$.

Figure 2. Evolution of the excited condensates as a function of time. The atoms in the excited condensates decay into ordinary excitations following roughly an exponential decay with lifetime of $\hbar/U_n$. After the Bragg pulse (of length 0.1 ms) the momentum of the excited condensates is transferred to the excitations. The $|\psi_{-1}|^2$ line is hard to see due to small values of the field. Here $na^3 \approx 0.003$.

Figure 3 shows the Bragg spectrum for a Bragg pulse exciting roughly 10% of atoms. Here and in the rest of the figures Bragg detuning $\hbar \nu_B$ refers to the Bragg field energy offset from the free particle resonance $\hbar \nu = \hbar \omega_B - (\hbar^2 Q_B^2 / 2m)$. The figure shows two peaks corresponding to the forward and backward scattering and the shift from the free particle line with increasing...
interaction strength. The line shape also changes from a Lorentzian (for weakly interacting gas) into an asymmetric peak (for strongly interacting gas) because quantum fluctuations are also affected by the Bragg field and the corresponding atoms have the Bragg resonance at higher detuning than the condensate atoms (because the excitation spectrum is a concave function of momentum $k$). For finite temperatures, the asymmetricity of the peak would be even more pronounced as the condensate fraction is reduced.

From spectra such as shown in figure 3, we determine the position of the peak maximum and figure 4 shows how this evolves as a function of pulse length. For very short pulses the linewidth is very large due to the lifetime broadening and the two peaks in the spectra overlap. Thus, the position of the peak maximum depends strongly on the pulse length. Once the pulse is long enough, so that the linewidth of the pulse is less than the distance between the backward and forward scattering peaks, the Bragg resonance can be resolved. There is still some slow drift in the peak position due to the depletion of the initial condensate, causing some error in the determination of the resonance energy. As long as the total fraction of excited atoms is small enough (less than 10%), the Bragg spectrum is insensitive to the strength of the Bragg coupling. Therefore, in order to resolve the Bragg resonance well, one can use very weak pulses. Furthermore, a Gaussian Bragg pulse would reduce the initial oscillations in the spectrum peak position. In an actual experimental setup, the maximum length of the pulse is limited due to the finite size of the system and the inhomogenous trapping potential. Figure 4 also shows how well the time-dependent HFB theory reproduces the resonance energy of the corresponding static theory when the mobile pairing fields are neglected.

Figure 5 shows the Bragg resonance energy as a function of the interaction strength. At strong interactions, the resonance drops far from the mean-field shift line. Notice that the Bogoliubov line includes the effect of $k$-dependent scattering length and it agrees surprisingly
Figure 4. The position of the Bragg spectrum maximum as a function of the Bragg pulse length. For very short pulses, the lifetime broadening mixes the backward and forward scattering peaks. For slightly longer pulses the peaks separate and the resonance energy can be resolved. The gradual drift in the peak position for long pulses is caused by the depletion of the initial condensate. Here the interaction strength is $n a^3 = 0.003$ and plots shown are both with and without the mobile pairing fields $\Delta_{\pm 1}$, and the static HFB result (the straight horizontal line). The top x-axis shows the approximate fractions of excited atoms for the corresponding pulse length.

well with the HFB theory even though the Bogoliubov line assumes a linear response theory while the HFB line includes the depletion of the condensate. Without the $k$-dependent scattering length, the Bogoliubov line would be close to the mean-field shift line. The mean-field shift and the Bogoliubov line are calculated using a static theory but the HFB lines are from the present time-dependent theory. The presence of the mobile pairing fields drops the resonance energy even further.

Neglecting the effect due to the mobile pairing fields $\Delta_{\pm 1}$, the effect of the $n a^3$ corrections introduced by the HFB theory (as compared to the Bogoliubov result) is very small despite the much larger effect on the chemical potential (roughly 10% for $n a^3 = 0.015$). Interestingly, in the $n a^3$ expansion of the corrections to the Bogoliubov theory, the leading order term in the chemical potential (the Lee–Huang–Yang (LHY) correction $[16, 17]$) is 74%. However, the next higher-order correction $[18]$ is roughly 150% showing that the $n a^3$ expansion is breaking down. The different treatment of the LHY terms may be the reason for the discrepancy between our HFB results and the Beliaev theory $[19]$ based results in $[6]$. The HFB theory does actually agree very well with the gapless HFB–Popov theory for large Bragg momenta, and therefore we expect that the long-wavelength excitation gap does not play a role here.

The qualitative features of figure 5 agree with the experimental data in $[6]$. However, since the present study has been done for a uniform gas, this work should be extended to a trapped gas using local density approximation for proper comparison with the experimental results.

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Figure 5. The Bragg resonance detuning $\nu_B$ as a function of the interaction strength for various theories. Shown are linear response results using mean-field shift spectrum $U_n$ and the Bogoliubov spectrum, and the spectra using the present theory both with and without the mobile pairing fields $\Delta_{\pm 1}$. The Bogoliubov line is roughly the same as the $\sqrt{8\pi na^3}$ line in figure 1 of [6] (the densities in the two plots are different).

7. Summary

To summarize, we have studied the Bragg spectroscopy of a strongly interacting BEC using time-dependent HFB theory. Taking into account the momentum dependence of the scattering amplitude, the theory is in qualitative agreement with the experiment done on the strongly interacting Rb-85 condensate. The most surprising effect comes from the creation of mobile pairing fields and the subsequent change in the excitation spectrum. While the present experimental results cannot confirm this effect, it should be more visible in the low-momentum excitation spectrum. Another interesting question would be the relation to fragmented condensates [20]. Because of the coherence of the Bragg field, the different condensates at momenta $n\vec{Q}_B$ have a well-defined phase difference and these can therefore be understood as a single condensate in a superposition of the states $\psi_n$. The coupling to the fluctuations should, in principle, dephase the condensate, leading into a fragmented state. However, the present formalism is unable to describe the dephasing of the excited condensates because these fields are described only by complex numbers instead of density matrices. The Bragg scattering induced mobile pairing fields also have an interesting connection to the FFLO-type pairing [21, 22] in polarized Fermi gases that would be worth further research.

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References

[1] Stenger J, Inouye S, Chikkatur A P, Stamper-Kurn D M, Pritchard D E and Ketterle W 1999 Phys. Rev. Lett. 82 4569
[2] Stamper-Kurn D M, Chikkatur A P, Görlitz A, Inouye S, Gupta S, Pritchard D E and Ketterle W 1999 Phys. Rev. Lett. 83 2876
[3] Steinhauer J, Ozeri R, Katz N and Davidson N 2002 Phys. Rev. Lett. 88 120407
[4] Tozzo C and Dalfóvo F 2003 New J. Phys. 5 54
[5] Zambelli F, Pitaevskii L, Stamper-Kurn D M and Stringari S 2000 Phys. Rev. A 61 063608
[6] Papp S B, Pino J M, Wild R J, Ronen S, Wieman C E, Jin D S and Cornell E A 2008 Phys. Rev. Lett. 101 135301
[7] Griffin A 1996 Phys. Rev. B 53 9341
[8] Holland M, Park J and Walser R 2001 Phys. Rev. Lett. 86 1915
[9] Kokkelmans S J J M F and Holland M J 2002 Phys. Rev. Lett. 89 180401
[10] Milstein J N, Menotti C and Holland M J 2003 New J. Phys. 5 52
[11] Wüster S, Hope J J and Savage C M 2005 Phys. Rev. A 71 033604
[12] Shi H and Griffin A 1998 Phys. Rep. 304 1
[13] Hugenholtz N M and Pines D 1959 Phys. Rev. 116 489
[14] Giorgini S 2000 Phys. Rev. A 61 063615
[15] Kokkelmans S J J M F, Milstein J N, Chiofalo M L, Walser R and Holland M J 2002 Phys. Rev. A 65 053617
[16] Lee T D, Huang K and Yang C N 1957 Phys. Rev. 106 1135
[17] Lee T D and Yang C N 1957 Phys. Rev. 88 1119
[18] Tsun Wu T 1959 Phys. Rev. 115 1390
[19] Beliaev S T 1958 Sov. Phys. JETP 34 299
[20] Mueller E J, Ho T-L, Ueda M and Baym G 2006 Phys. Rev. A 74 033612
[21] Fulde P and Ferrel R A 1964 Phys. Rev. A 135 550
[22] Larkin A I and Ovchinnikov Yu N 1965 Sov. Phys.—JETP 20 762