Electric Control of Spin-Orbit Coupling in Graphene-Based Nanostructures with Broken Rotational Symmetry

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Spin and orbital angular momenta of light are important degrees of freedom in nanophotonics which control light propagation, optical forces, and information encoding. Here, it is shown that graphene-supported plasmonic nanostructures with broken rotational symmetry provide a surprising spin to orbital angular momentum conversion, which can be continuously controlled by changing the electrochemical potential of graphene. Upon resonant illumination by a circularly polarized plane wave, a polygonal array of indium-tin-oxide nanoparticles on a graphene sheet generates the scattered field carrying electrically-tunable orbital angular momentum. This unique photonic spin-orbit interaction occurs due to the strong coupling between graphene plasmon polaritons and localized surface plasmons of the nanoparticles and leads to the controlled directional excitation of graphene plasmons. The tuneable spin-orbit conversion paves the way for high-rate information encoding in optical communications, electric steering functionalities in optical tweezers, and nanorouting of higher-dimensional entangled photon states.

1. Introduction

The uncertainty principle rules the localization of particles and has fundamental implications in many new technological developments. The large momentum components of a strongly localized wave-packet entail complex field dynamics absent in the case of a weak confinement. Many physical phenomena accompany sub-wavelength localization of photons, including the photonic spin-orbit interaction (SOI) between the polarization and spatial degrees of freedom of light, which are governed by the spin angular momentum (SAM) and orbital angular momentum (OAM).

In the paraxial regime (weak localization), SOI is usually negligible but may be augmented by inhomogeneities, anisotropy, and epsilon-near-zero materials. Non-paraxial focusing by small particles or a doped graphene enhances SOI and results in spin-dependent directionalities and optical vortices. In the case of the extreme confinement at the subwavelength scales, SOI is dominant with applications in quantum optics, high-resolution microscopy, beam shaping with planar metasurfaces, optical forces, and nanophotonics.

The physical origin of the strong SOI in the highly confined electromagnetic fields is the large longitudinal field components of the evanescent waves that produce a SAM orthogonal to the direction of propagation (transverse SAM). Under the reversal of the direction of the wave-vector, the transverse SAM changes sign and the spin-momentum locking can be regarded as an analogue of the quantum spin Hall effect (QSHE). QSHE has been observed with surface plasmon polaritons at metal/dielectric interfaces, guided waves, graphene plasmon polaritons (GPPs), and structured guided waves.

SAM to OAM conversion is among the most important SOI phenomena and enables optical micromanipulation, entanglement, and quantum protocols with higher-dimensional quantum states. Many strategies have been proposed for OAM generation, including anisotropic media, inhomogeneous liquid crystals (q-plates), metasurfaces, and whispering-gallery mode resonators. All these methods operate in the paraxial regime and generate macroscopic optical vortices with defined topological charge, ruling out the possibility of a continuous tuning of OAM with the external control signal (tuneability has only been achieved by changing the polarization state of the input optical beam). OAM to SAM conversion can take place on the nanoscale leading to subwavelength features of spin-skyrmions. SAM to OAM conversion is, however, difficult to achieve at the nanoscale as it requires spatial manipulation of the beam on subwavelength scales, and the external, continuous control of SOI is practically unexplored. At the same time, the fast encoding of OAM states is important for the development of its applications in optical communications.
Here, we describe a novel optical SOI mechanism in a hybrid system of a graphene-supported plasmonic nanostructure with broken rotational invariance and report electrically tunable SAM to OAM conversion at the nanoscale, exploiting graphene doping by an external electric bias. We also show that this process is accompanied by the emission of confined graphene plasmon polaritons, which can also be electrically controlled. We consider a polygonal array of doped indium-tin-oxide nanospheres placed near an extended graphene sheet illuminated by a plane wave carrying only SAM (Fig. 1a). The system is not invariant under the full group of rotations around the z-axis, but only with respect to the subgroup $C_N$ of discrete rotations of angles $2\pi r/N$ ($r = 0, \ldots, N - 1$). Therefore, the z-component of the radiation angular momentum is not conserved in a scattering processes. The graphene sheet lies on the plane $z = 0$ between the half spaces $z < 0$ and $z > 0$ which are filled by two transparent dielectrics of permittivities $\varepsilon_1$ and $\varepsilon_2$, respectively. The nanoparticles of radius $R$ are arranged at the vertices of a regular polygon with $N$ nanoparticles resulting in the polarisation ellipses out of the graphene plane. Due to the tilted rotation axis, the dipoles excite spin-momentum locked GPPs. The near-field interference of these circulating plasmonic fields produces a non-vanishing OAM. Changing the graphene Fermi energy tunes the value of the generated OAM, because of the spectral shift of the graphene plasmonic resonance and the resulting modification of the coupling between GPPs and LSPs. We note that, although experimentally challenging, the fabrication of the polygonal arrangement of nanoparticles should become accessible in the near future due to the present large scientific interest in graphene-nanoparticle composites (see ref. [63] and references therein).

2. Hybrid Plasmonic Resonances

We begin by analyzing the collective dynamics of a polygonal array of $N$ nanoparticles placed near an extended graphene sheet illuminated by a plane wave carrying only SAM (Fig. 1a). The system is not invariant under the full group of rotations around the z-axis, but only with respect to the subgroup $C_N$ of discrete rotations of angles $2\pi r/N$ ($r = 0, \ldots, N - 1$). Therefore, the z-component of the radiation angular momentum is not conserved in a scattering processes. The graphene sheet lies on the plane $z = 0$ between the half spaces $z < 0$ and $z > 0$ which are filled by two transparent dielectrics of permittivities $\varepsilon_1$ and $\varepsilon_2$, respectively. The nanoparticles of radius $R$ are arranged at the vertices of a regular polygon with $N$ nanoparticles resulting in the polarisation ellipses out of the graphene plane. Due to the tilted rotation axis, the dipoles excite spin-momentum locked GPPs. The near-field interference of these circulating plasmonic fields produces a non-vanishing OAM. Changing the graphene Fermi energy tunes the value of the generated OAM, because of the spectral shift of the graphene plasmonic resonance and the resulting modification of the coupling between GPPs and LSPs. We note that, although experimentally challenging, the fabrication of the polygonal arrangement of nanoparticles should become accessible in the near future due to the present large scientific interest in graphene-nanoparticle composites (see ref. [63] and references therein).

Figure 1. Graphene-nanoparticle system for electrically tunable SAM to OAM conversion. a) Schematics of the system: the ITO nanospheres of radius $R$ positioned at the vertices of a regular polygon of radius $a$ on a graphene sheet. A normally incident circularly polarized plane wave feeds the nanoscale structure with SAM. The scattered field (whose amplitude is sketched in a sample situation as the colour plot) carries highly tunable OAM and the linear momentum density distribution (black arrows) swirls around the polygon axis. b,c) The dependences of (b) the imaginary part of the normalized graphene surface conductivity $\sigma$ (red surface) and the spatial spectral width $\Delta k_\perp$ of the reflected evanescent field mediating the coupling between the nanoparticle LSPs and GPPs (blue surface) on the wavelength $\lambda$ and the graphene Fermi energy $E_F$. The green dashed curves in (b) and (c) indicate the GSC threshold when the spectrum of the field generated by the nanoparticles and reflected from the graphene sheet contains the GPP wavenumber $k_{\perp}^{(\text{pla})} < \Delta k_\perp$ (grey region).
electric polarizability and \( \mathbf{E}^{\text{out}}(\omega) \) is the external field at the nanoparticle center. We calculate multiple scattering of all orders by generalizing the discrete dipole model \( \tilde{\alpha} \) to account for the interaction between the nanoparticles and the effect of a graphene substrate. The field \( \mathbf{E}^{\text{out}}(\omega) \) contains the incident field, the fields of all the other \( N - 1 \) dipoles, and the total field reflected by graphene. The dipole moments are then determined by

\[
\frac{1}{\alpha} \mathbf{p}^{(i)} = \sum_{j=0}^{N-1} \tilde{H}^{(i,j)} \mathbf{p}^{(j)} = \left[ e^{i\omega t_r} r_{\text{GR}}(0) + e^{-i\omega t_r} \right] \mathbf{E}_i \mathbf{e}_i, \tag{1}
\]

where \( \tilde{H}^{(i,j)} \) is the interaction matrix and \( r_{\text{GR}}(0) \) is the reflectivity of the graphene sheet for the normally incident (transverse-electric) plane waves (details in the Supporting Information). Using the dipole moments from Equation (1), we find the scattered field from its angular spectrum representation. The 3N hybrid plasmonic resonances of the graphene-nanoparticle system, corresponding to the 3N components of the coupled nanoparticle dipoles, can be found from Equation (1), assuming the incident plane wave is absent \( (\mathbf{E}_0 = 0) \). This hybridization is due to the coupling between nanoparticle LSPs and GPPs. The spatial spectrum of the reflected field is dominated by the evanescent waves with an asymptotic tail \( e^{-\Delta_0 R} \), where \( 2R \) is the total distance traveled by the field from the polygon plane \( (z = -R) \) to graphene \( (z = 0) \) and back. The coupling to GPP occurs when the spectrum of the reflected field scattered by the nanoparticles contains a GPP wavenumber \( k_{\text{GPP}}^{(0)} < \Delta k_0 \) (Figure 1c).

The invariance of the graphene-nanoparticles system under the group \( C_\text{iv} \) implies that the 3N dimensional dipoles space breaks into \( N \) tri-dimensional invariant subspaces each carrying an irreducible representation of the group (see Supporting Information). The impinging plane wave \( \mathbf{E}_0 \mathbf{e}_z \) selects only one of these invariant subspaces that characterize the unique relationship between the dipole moments \( (q = 0, \ldots, N - 1) \):

\[
\mathbf{p}^{(q)} = e^{\frac{2\pi}{N} q R_z} \mathbf{p}^{(0)}, \tag{2}
\]

where \( R_z \) is the dyadic operator which rotates a vector around the \( z \)-axis by an angle \( 2\pi q/N \). The \( q \)-th dipole coincides with the \( 0 \)-th dipole rotated by the angle \( 2\pi q/N \), multiplied by the phase factor \( e^{\frac{2\pi}{N} q} \), in accordance with the Bloch theorem for discrete rotational symmetry. This symmetry enables to cast Equation (1) as a single equation for the \( 0 \)-th dipole moment

\[
\mathbf{p}^{(0)} \equiv 4\pi\varepsilon_0 R_z \left( \tilde{a}_z \mathbf{e}_z + \tilde{a}_{x,y} \mathbf{e}_{x,y} + \tilde{a}_e \mathbf{e}_e \right) \mathbf{E}_0, \tag{3}
\]

where \( \mathbf{e}_e = \mathbf{e}_z \) is the right-hand circular unit vector and \( \tilde{a}_z, \tilde{a}_{x,y}, \) and \( \tilde{a}_e \) are the dimensionless generalized dipole polarizabilities in the circular basis.

In the numerical calculations, we consider \( \varepsilon_1 = 2.0136, \varepsilon_2 = 2.0851, R = 30 \text{ nm}, \) and \( a = \frac{1}{2} a_{\text{gr}} \) corresponding to the polygon edge of \( 3R \). We model the ITO permittivity with the Drude model \( \varepsilon_0(\omega) = 1 - \frac{\omega_p^2}{\omega_0^2 + \omega^2} \). The plasma frequency \( \omega_p \) can be tuned in such a way that the particles resonate at the vacuum wavelength \( \lambda_o = 2\pi c/\omega_0 \) (i.e., \( \varepsilon_0(\omega_o) = -2\varepsilon_1 + 0.1 \)) in order to achieve strong coupling.

The evolution of the hybrid plasmonic resonances with the graphene Fermi energy shows a clear graphene strong coupling (GSC) threshold (Figure 2). Below the threshold, graphene is not resonant and the dipole excitation is not affected by the weak field reflected from graphene. In this region, the longitudinal polarizability \( \tilde{\alpha}_z \) is negligible because the dipole-dipole interaction fields are in plane with the graphene sheet. Furthermore, the polarizability \( \tilde{\alpha}_e \) produced by the dipole-dipole direct interaction between the nanoparticles is also small, because the impinging CLP field only interacts with the polarizability \( \tilde{\alpha}_z \) of the nanoparticles. When the wavelength is close to \( \lambda_o \), both \( \tilde{\alpha}_z \) and \( \tilde{\alpha}_e \) are large owing to the LSP excitation. In this case, the hybridization due to the dipole-dipole interaction in the nanoparticle polygon produces the splitting of the LSP resonance into two peaks, clearly observed when the distance between the nanoparticles becomes smaller as in the case of \( N = 13 \) polygon (Figure 2d,e). The impact of graphene on the dipole excitation process dramatically increases above the GSC threshold (Figure 2f). In this regime, a significant longitudinal polarizability \( \tilde{\alpha}_z \) arises from the excitation of GPPs with a giant \( z \)-component of the electric field, which orients the dipoles in the longitudinal direction. Resonance hybridization is particularly important for the wavelengths close to the nanoparticle LSP wavelength \( \lambda_o \). The dependence of the hybrid resonances on the number of nanoparticles in the polygon shows that as \( N \) increases, the dipole-dipole interaction splitting of the nanoparticle resonance increases, because the distance between the nanoparticles decreases (Figure 2g). The quality factors of hybridized modes also increase with the number of nanoparticles.

3. Tunable SAM to OAM Conversion

As discussed in the above section, the incident LCP plane wave generally excites dipoles at the nanoparticles which are not purely left circularly polarized owing to their mutual interaction and the evanescent coupling with the graphene sheet. Due to their polygonal arrangement with the \( C_\text{iv} \) symmetry, these coupled dipoles radiate fields with non-trivial chirality. The densities of the energy, linear and angular momenta of the field in the substrate \( (z > 0) \) can be calculated as

\[
U = \frac{1}{4} \left( \varepsilon_0 \varepsilon_1 |E|^2 + \mu_0 |H|^2 \right),
\]

\[
\mathbf{P} = [||\mathbf{p}| + \frac{1}{2} \mathbf{V} \times [||\mathbf{S}|| \equiv \mathbf{P}^\ast + \mathbf{P}^\circ, \tag{4}
\]

\[
J = [||\mathbf{L}| + ||\mathbf{S}||],
\]

where \( \mathbf{p} = \frac{1}{2} \mathbf{V}, \mathbf{L} = \mathbf{r} \times \mathbf{p}, \) and \( \mathbf{S} = \varepsilon_0 \varepsilon_1 \mathbf{e}_z \mathbf{e}_w \mathbf{e}_w \) are the linear and angular momenta and spin operators, respectively, and we used the notation

\[
[||\mathbf{O}||] = \frac{1}{4\varepsilon_0} \text{Re} \left( \varepsilon_0 \varepsilon_1^* \mathbf{O}_j \mathbf{O}_j^* + \frac{\mu_0}{\varepsilon_1} \mathbf{H}_j^* \mathbf{H}_j \right).
\]

The linear momentum density \( \mathbf{P} = \frac{1}{2\varepsilon_0} \text{Re}(\mathbf{E} \times \mathbf{H}^\ast) \) is the well known Belinfante’s decomposition, \( [||\mathbf{O}||] \) where the orbital contribution \( \mathbf{P}^\circ = [||\mathbf{p}||] \) is equal to the canonical momentum density,
proportional to the local phase gradient. The spin contribution \( P^s = \frac{1}{2} V \times |\langle \hat{S} | \rangle | \) corresponds to a transverse spin of evanescent waves.\(^{14}\) In order to quantify the SAM to OAM conversion, we consider the quantity (introducing \( r_z = xe_x + ye_y \))

\[
\langle J_z \rangle = \frac{\omega}{2} \int d^2 r_z \frac{\langle J_z \rangle}{U(r_z)}
\]

which is a dimensionless average value of the longitudinal angular momentum density \( J_z \) on the plane \( z = 0^\circ \). From Equation (4), it follows

\[
\langle J_z \rangle = \frac{r_z \times P^s + |\langle \hat{S}_z \rangle|}{U(r_z)}
\]

which implies that the OAM contribution is entirely due to the orbital part of the linear momentum density.\(^{14}\) Since the eigenvalues of \( \hat{S}_z \) are \( \pm 1 \) and \( 0 \), the SAM contribution is bounded and satisfies the inequality \( |\langle \hat{S}_z \rangle| \leq 1 \). Correspondingly, when \( |\langle J_z \rangle| > 0 \), the OAM contribution in the left-hand side of Equation (7) is necessarily not vanishing. Below the GSC threshold (Figure 3a), \( \langle J_z \rangle \approx 1 \) and the angular momentum of the scattered field is dominated by the SAM retained from the input. However, when the GSC is achieved, \( \langle J_z \rangle \) is significantly enhanced \( |\langle J_z \rangle| \approx 6 \). The most relevant spectral region for the OAM generation is around the nanoparticle resonance \( \lambda_s = 5 \mu m \). The SAM to OAM conversion is associated with the OAM generated by the nanostructure-graphene system during the scattering process and the orientation of the induced dipoles. It is now clear that the generated OAM is also tunable as \( \langle J_z \rangle \) dramatically depends on \( E_F \), which can be electrically controlled via a gate voltage applied to graphene.

To illustrate such a tunability, we consider three specific configurations as indicated in Figure 3a: i) a generic non-resonant configuration with \( \langle J_z \rangle \approx 1.01 \), and two configurations corresponding to ii) the maximum \( \langle J_z \rangle \approx 5.72 \) and iii) the minimum \( \langle J_z \rangle \approx -6.14 \). The dependence of \( \langle J_z \rangle \) on \( E_F \) is particularly interesting near the resonant wavelengths, which shows that the external gate voltage can continuously tune the scattered angular momentum. The weights of the total angular momentum eigenstates generated by the polygonal array depend on the dipole orientation \( P^s \). Owing to the \( C_N \) symmetry, only eigenstates with eigenvalues \( m = nN + 1 \) are excited, where \( n \) is an integer (see Supporting Information). The average angular momentum can be expressed as a weighted average of the eigenvalues

\[
\langle J_z \rangle = \sum_{n=-\infty}^{\infty} (nN + 1)\hat{w}_n,
\]

where \( \hat{w}_n \) is the normalized distribution of angular momentum eigenvalues. As we can see from the distributions of the eigenvalues \( m = nN + 1 \), \( \hat{w}_n \) goes rapidly to zero as \( |n| \) increases. In the nonresonant configuration (Figure 3b), the distribution is nearly symmetrical with respect to \( m = 1 \) (\( n = 0 \)) and the average is very close to 1. In the resonant configuration (Figure 3c,d), the distributions are not symmetric around \( m = 1 \) and for \( \langle J_z \rangle \approx 6 \) is...
The orbital linearmomentum \( \vec{P} \) five lobes, spatially overlapping the position of the nanoparticles. The angular momentum exchange upon scattering is absent. The situation is completely different in the resonant configuration (Figure 4b,c). The electric field magnitude is an order of magnitude larger than the input (due to the plasmonic field enhancement), and the pattern has the shape of a ratchet wheel with a marked 2D chiral trait. The vector fields \( \vec{P} \) swirl around the z-axis and their positive and negative swirling directions produce positive and negative OAM contributions \((\vec{r}_z \times \vec{P})\) in Equation (7)). This is in agreement with the sign of the average angular momentum in each case. Because the orbital linear momentum density \( \vec{P} \) produces a mechanical force on an absorbing particle, these results suggest a novel way of manipulating the motion and rotation of nanoparticles through electric gating of a graphene substrate.

The SAM to OAM conversion mechanism has a simple and clear physical interpretation in terms of unidirectional excitation of GPPs with defined rotation direction. The transmitted angular momentum depends on the dipole moments of the nanoparticles forming the polygon, which are all rotated doppelgangers of \( \vec{r} \) with a geometric phase factor (Equation (2)). The time-dependent components dominate the eigenvalues, while for \( \langle J_z \rangle \approx -6 \), the most important are the eigenvalues \( m = -9 \) and \( m = -4 \).

4. Excitation of GPPs with Defined Rotation Direction

It is interesting to note that during the observed SAM-to-OAM conversion, the scattered angular momentum can have opposite sign with respect the incoming SAM: the scattering can flip chirality of light. In order to examine this effect in more detail, we evaluate the electric field and the transverse orbital linear momentum density in the plane \( z = 0^+ \), \( |E| = |E_{\hat{r}}|/E_0 \) and \( \vec{P}_z = [P_{z\hat{r}}(\vec{r},\omega)\hat{r} + P_{z\hat{r}}(\vec{r},\omega)\hat{r}]^2/P_0 \), normalized by the impinging field magnitude \( |E_0| \) and linear momentum density \( P_0 = 1/2 \pi v_0 |E_0|^2 \) respectively. The spatial shapes of these real quantities are invariant under discrete rotations (by angles \( 2\pi/\sqrt{N} \)) around the z-axis. In the non-resonant case (Figure 4a), the scattered electric field nearly coincides with the incoming field, with the exception of five lobes, spatially overlapping the position of the nanoparticles. The orbital linear momentum \( \vec{P}_z \) spreads out of each lobe and is negligible everywhere else, therefore, yielding a very small OAM contribution \( (\vec{r}_z \times \vec{P}_z) \) in Equation (7). Accordingly, \( \langle J_z \rangle \approx 1 \) and the angular momentum exchange upon scattering is absent. The situation is completely different in the resonant configurations (Figure 4b,c). The electric field magnitude is an order of magnitude larger than the input (due to the plasmonic field enhancement), and the pattern has the shape of a ratchet wheel with a marked 2D chiral trait. The vector fields \( \vec{P}_z \) swirl around the z-axis and their positive and negative swirling directions produce positive and negative OAM contributions \((\vec{r}_z \times \vec{P}_z)\) in Equation (7)). This is in agreement with the sign of the average angular momentum in each case. Because the orbital linear momentum density \( \vec{P} \) produces a mechanical force on an absorbing particle, these results suggest a novel way of manipulating the motion and rotation of nanoparticles through electric gating of a graphene substrate.

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The spatial distributions around the polygon ($r_\perp < 3 a$) of the normalized electric field magnitude $|\vec{E}(\tau)| = |E/E_0|$ and transverse orbital linear momentum density $\vec{P}_o = P_\perp / P_0$ in the plane $z = 0^+$ under the conditions indicated by the coloured disks in Figure 3. In the non-resonant conditions (a), the electric field does not show an overall spatial chirality, $\vec{P}_o$ does not circulate around the $z$-axis, and there is no net angular momentum exchange upon scattering. In the resonant conditions (b) and (c), the electric field patterns have a marked spatial 2D chiral trait. The vector fields $\vec{P}_o$ swirl around the $z$-axis and produce OAM contributions $\langle r_\perp \times P_\perp \rangle$, which agree with the signs of the average angular momenta $\langle J_z \rangle$. In the non-resonant configuration (d), the polarization ellipses are parallel to the graphene sheet and the dipoles do not excite GPPs. In the resonant configurations (e) and (f), the dipole spins precess with inward or outward orientation toward the polygon centre and excite the GPPs traveling in anticlockwise or clockwise azimuthal directions, respectively. Their interference results in the GPP field on the graphene plane clockwise or anticlockwise rotating around the $z$-axis, which generate the OAM carried by the scattered field.

The dipole moment $p(t) = \text{Re}[p^{(0)}e^{-i\omega t}]$ sweeps a polarization ellipse whose spatial orientation and rotation direction are given by the time-independent angular normal vector

$$n = \mathbf{p} \times \frac{\mathbf{dp}}{dt} = \frac{i \omega}{2} \mathbf{p}^{(0)} \times \mathbf{p}^{(0)*},$$

as shown in Figure 4 (Dipole kinematics). In the strong coupling regime, we obtain (see Supporting Information)

$$\langle J_z \rangle = 1 - \frac{\gamma}{\omega} n_z$$

with $\gamma > 0$. Equation (10) states that the orientation of the polarization ellipse of $p^{(0)}$ determines the scattered angular momentum, which increases (decreases) the incident light SAM if the polarization ellipse is negatively (positively) oriented with respect to the $x$-axis. The polarization ellipse of the $q$-th dipole coincides with the 0-th ellipse rotated by an angle $2\pi q/N$. If $n_z < 0$, all the normal vectors point toward the $z$-axis and the opposite for $n_z > 0$ (Equation (2)). In the non-resonant configuration (Figure 4d), the polarization ellipses are parallel to the graphene sheet as the dipole $p^{(0)}$ is nearly left polarized with $n_z \approx 0$ and unitary angular momentum. The dipoles rotating parallel to the graphene sheet provide negligible excitation of GPPs. This changes in the resonant configurations for $n_z < 0$ and $\langle J_z \rangle > 0$ (Figure 4e) and $n_z > 0$ and $\langle J_z \rangle < 0$ (Figure 4f). Equation (10) supports the interpretation of the SAM to OAM conversion in terms of the GPP excitation.\(^{[18]}\) For the configuration in Figure 4e, each dipole rotates with the normal vector $\mathbf{n}$ pointing toward the $z$-axis and excites respective GPP, which travels in the positive azimuthal direction (yellow arrows in Figure 4). The interference of all these azimuthally traveling GPPs produces a scattered electromagnetic field rotating around the $z$-axis in the positive direction, which is responsible for the positive OAM of the scattered field. The reversed situation occurs in the configuration of Figure 4f, where the normal vectors $\mathbf{n}$ point in opposite direction of the $z$-axis, and the excited GPP travels in the opposite direction to the previous case.
5. Discrete Radial Emission of GPPs

The nanophotonic mechanism supporting the SAM to OAM conversion is also responsible for another effect of shaping GPPs beams excited by the nanospheres outside the polygon central region (although it is not connected to SAM-OAM exchange). In the non-resonant configuration (Figure 5a), the scattered electric field is nearly equal to the incident one and the transverse orbital linear momentum density is vanishing. In the resonant case (Figure 5b,c), the electric field is enhanced within specific circular sectors over which the orbital linear momentum density is almost radial. These circular sectors are located around the polygon apothem lines, along the direction from the nanospheres to the polygon center (Figure 5b) and from the center to nanoparticles (Figure 5c). The circular sectors, as the whole region $r_\perp > 3a$, provide a vanishing OAM contribution since $r_\perp \times \mathbf{P}_\perp$ $\neq$ 0 everywhere, so that the effect is independent of the above described angular momentum exchange. A detailed analysis of the transverse asymptotic field behavior (see Supporting Information) indicates that each circular section hosts the GPP excited by an individual nanosphere and traveling along the corresponding polygon apothem. Such discrete radial emission of GPPs can be understood in terms of unidirectional excitation and interference of graphene plasmon modes produced by the rotating dipoles. Since $\mathbf{p}^{(0)} = p^{(0)}_r \mathbf{e}_r + p^{(0)}_\phi \mathbf{e}_\phi + p^{(0)}_z \mathbf{e}_z$, where $\mathbf{e}_r$, $\mathbf{e}_\phi$, and $\mathbf{e}_z$ are the cylindrical coordinates unit vectors, each polarization ellipse has both azimuthal and radial projections swept by the rotating dipole of complex amplitudes $p^{(0)}_r \mathbf{e}_r + p^{(0)}_\phi \mathbf{e}_\phi$ and $p^{(0)}_z \mathbf{e}_z$, respectively (Figure 5). The azimuthal projections of the ellipses are responsible for the excitation of the azimuthally propagating GPPs. These GPPs have finite transverse width in the radial direction, and the azimuthal projections of the ellipses play no role in the region $r_\perp > 3a$. On the other hand, the radial projections of the rotating dipoles are able to excite GPPs along the polygon apothems. While in the non-resonant configuration (Figure 5d) GPP excitation is negligible, in the resonant case (Figure 5e), the polarization ellipses are inward inclined and this implies that the radial projections of the rotating dipoles launch GPPs along the polygon apothems in the direction from each nanoparticle toward the polygon center. Conversely, in the case of Figure 5f, the polarization ellipses are outward inclined and accordingly the radial projections of the rotating dipoles launch GPPs propagating along the direction from the polygon center toward each nanoparticle.

We also stress that such GPPs radial emission is strongly affected by the Fermi level in terms of efficiency (in Figure 5a there is no emission) and directionality (in Figure 5b,c, GPPs travel along opposite directions of the polygon apothem lines). This implies that the beaming of the radial GPPs can be tuned by gated electric signals, in the same way as the OAM of the scattered light described above.

Figure 5. Radial excitation of GPPs. a–c) The spatial distributions around the polygon ($3a < r_\perp < 12a$) of the normalized electric field magnitude $|\mathbf{E}^{(\perp)}| = |\mathbf{E}/E_0|$ and transverse orbital linear momentum density $\mathbf{P}^{(\perp)}_\perp = \mathbf{P}_\perp / P_0$ in the plane $z = 0^\circ$ under the conditions indicated by the coloured disks in Figure 3 (cf. Figure 4). In the non-resonant conditions (a), the GPPs excitation is negligible. In the resonant conditions (b) and (c), each nanoparticle is responsible for the excitation of a single GPP propagating along the corresponding polygon apothem. d–f) Pictoral representation of the corresponding dipole polarization ellipses. The GPPs are excited by the radial projections of the dipolar moments. In the resonant conditions (e) and (f), the radial projections of the dipole moments rotate in clockwise or anticlockwise directions in the radial plane, respectively. Accordingly, in (e), the GPPs are launched from the nanoparticles toward the polygon center, whereas in (f), away from it.
6. Conclusions
We have described electrically tunable SAM to OAM conversion at the nanoscale using a plasmonic nanoparticle arrangement with broken rotational invariance placed on a graphene substrate. A polygonal array of ITO nanospheres deposited on top of a graphene sheet generates scattered light carrying net OAM under illumination by a plane wave carrying only SAM. The effect originates from the inclined induced dipoles of the nanoparticles due to the strong coupling between LSPs of the nanoparticles and graphene plasmons and, therefore, resonant and strongly depends on the graphene Fermi energy. This opens up a possibility to continuously tune the generated OAM via electric biasing of the graphene sheet. The novel nanophotonic mechanism supporting this effect results from the interplay of two physical mechanisms. First, the hybridization of the LSPs of the nanoparticle and GPPs entails the excitation of the nanoparticle dipoles whose rotation plane is not parallel to the graphene sheet. Second, the inclined rotating dipoles excite the GPPs traveling in the azimuthal direction around the polygon centre on the graphene so that their interference produces a field distribution with a net OAM. The same spin-orbit interaction results also in a controlled emission of the graphene plasmons radially propagating along the polygon apothem lines. The amplitude and propagation direction of these strongly confined plasmonic beams can also be controlled by electrically doping the graphene layer.

The presented results may find applications in tunable rotational optical tweezing since the angular momentum tunability would imply that the mechanical momentum exerted on probe particles could be controlled by the graphene electric bias. In plasmon routing, they provide a means for effectively launching, on the graphene sheet, strongly confined polaritons with the propagation direction depending on the graphene Fermi energy. In addition, the proposed hybrid graphene-plasmonic nanostructure offers a rich new framework for studying many new physical phenomena driven by the strong field localization, and unexpected couplings with non-trivial plasmonic resonances and electromagnetic field topologies. The described effect may trigger many possibilities for the precise control of information and beam propagation at the nanoscale in 2D materials and may also be extended to nonlinear and quantum regimes. The developed approach opens up new avenues for manipulating optical OAM at the nanoscale and the development of integrated devices for optical manipulation, information processing, communication, and classical and quantum computing at the nanoscale.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

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