Federated Online Sparse Decision Making

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Abstract

This paper presents a novel federated linear contextual bandits model, where individual clients face different K-armed stochastic bandits with high-dimensional decision context and coupled through common global parameters. By leveraging the sparsity structure of the linear reward, a collaborative algorithm named Fedego Lasso is proposed to cope with the heterogeneity across clients without exchanging local decision context vectors or raw reward data. Fedego Lasso relies on a novel multi-client teamwork-selfish bandit policy design, and achieves near-optimal regrets for shared parameter cases with logarithmic communication costs. In addition, a new conceptual tool called federated-egocentric policies is introduced to delineate exploration-exploitation trade-off. Experiments demonstrate the effectiveness of the proposed algorithms on both synthetic and real-world datasets.

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1 Introduction

Federated learning (FL) (McMahan et al., 2017; Konečný et al., 2016) is a new distributed machine learning paradigm in which multiple clients learn a shared machine learning model cooperatively while maintaining all training data on local database. FL differs from traditional centralized machine learning in the following ways (Kairouz et al., 2019). (1) Heterogeneous local datasets. The local datasets, which are generated from clients’ respective regions, are likely drawn from non-independent and identically distributed distributions. (2) Local data privacy security. FL protects local data privacy security by only sharing model updates instead of the raw data and design communication architecture. (3) Communication efficiency. The communication cost scales with the number of clients and increase risk of indirect data leakage. It is critical to minimize the communication cost while maintaining the learning accuracy.

While the primary focus of cutting-edge FL is on supervised learning, a few academics have lately begun to adapt FL to the multi-armed bandits (MAB) framework (Lai and Robbins, 1985; Auer et al., 2002; Bubeck and Cesa-Bianchi, 2012; Agrawal and Goyal, 2012, 2013b). In the canonical scenario of MAB, a player selects one arm from a set of arms to play at each decision step. If an arm is played, it will provide a reward based on its underlying distribution, which is unknown to the player. Using all of the prior observations, the player must pick which arm to pull at each decision step towards maximizing the cumulative reward. In the nutshell, MAB is an online learning model that naturally reflects the inherent exploration-exploitation trade-off in many sequential decision-making tasks.

A variety of applications, including recommender systems and clinical trials, naturally inspire the extension of FL to the MAB paradigm. The sequential decision making in those applications involves several clients (small business owners, hospitals) and is distributed by nature. While conventional MAB models require that the learning agent has immediate access to the sequentially generated data, in the new world of FL, local datasets can be hosted and processed at the clients, lowering communication load and potentially safeguarding data
privacy and preventing data leakage. However, in these practical applications, the bless of modern economy digitization leads the curse of dimensionality: typically only small number of high-dimensional decision context are decisive.

Despite the potential benefits of FL, the MAB paradigm’s sequential decision making and bandit feedback impose distinct constraints on the design of FL algorithms. In contrast to supervised learning, which requires the storage of static datasets in advance, the MAB paradigm generates data sequentially as decisions are made, actions are taken, and observations are obtained. To optimize cumulative reward while lowering learning regret, sophisticated client behavior coordination is required. Further, disparities in client reward distributions further complicate and impede the coordination process. Meanwhile, privacy and communication limits impose significant impediments to efficient information sharing and aggregation between local clients and the central server. Additionally, high-dimensional decision context confounds learning agent with vague boundary between signal and noise.

In this work, we attempt to overcome those issues by building federated linear contextual bandits architectures with high-dimensional decision context. This particular issue is driven by the following exemplary applications: (1) Personalized content recommendation: For content (arm) recommendation in e-commerce, user engagement (reward) depends on the profile of a user (decision context). The central server may deploy a recommender system on each small business owner (client) to personalize recommendations without knowing the personal profile or behavior of the user. (2) Personalized dosage searching and Collaborative medical imaging. For treatment (arm) recommendations in e-hospital, clinical response (reward) depends on the profile of a patient (decision context). The central server may deploy a clinical decision support system on each hospital (client) to personalize recommendations without compromising the patient profile or medical records.

In those applications, the reward of pulling the same arm at different clients follows different distributions dependent on the context as in contextual bandits (Auer, 2002; Langford and Zhang, 2007). Conventional linear contextual bandits (Li et al., 2010; Agrawal and Goyal,
is defined with respect to a single player, where the time-varying context can be interpreted as different incoming decision contexts. In contrast, we consider a multi-client version of sparse linear reward model with time-varying high-dimensional decision contexts (Bastani and Bayati, 2020; Wang and Cheng, 2020). Such a model automatically takes local dataset heterogeneity into account and is elaborated at Section 2.2.A.

1.1 Main contributions

We deliver a novel architecture for federated sparse online decision making and associated algorithms. Following that, we highlight our major contributions.

(i) **Fedego Lasso Algorithm.** Federated-egocentric Lasso algorithm (Fedego Lasso) exploits Lasso regression to learn sparse local reward models and to inform future decision without compromising local data privacy. In particular, it enables each client to compute a personalized, low-dimensional local reward model, which we refer to as the client private Lasso, that utilizes client’s unique decisions, actions and observations of local data.

(ii) **Convergence Rate and Regret bound.** We establish convergence results of three type of Lasso estimates implemented in the proposed Fedego Lasso algorithms (Lemma 1, 2 and 3). Such convergence guarantees is not trivial given the non-i.i.d. properties of dataset collected during online decision making process (See Remark 1). With these convergence rate results, we establish a theoretical regret upper bound (Theorem 1) for the algorithm performance of Fedego Lasso.

(iii) **Empirical Results.** Through a combination of synthetic and real datasets (PharmGKB, Medical MNIST) we show the benefits of Fedego Lasso in (a) benefits of federation architecture and (b) benefits of recruiting more clients on further regret reduction and faster error rate convergence in real-world tasks including personalize dosage searching and medical image labeling. (See Section 4.)

**Benefits of Fedego Lasso.** We list benefits of Fedego Lasso over standard linear contextual bandit learning (that learns a single reward model with no central server to share
Figure 1: Teamwork-Selfish Sampling. In the teamwork stage (Blue), all clients pull prescribed arms, no matter the current decision context, to form collaborate exploration for a quick recovery of arm reward parameters. In the selfish stage (Red), all clients pull the arm with highest estimated reward for current decision context to form egocentric exploitation for maximizing their own cumulative reward.

and no high dimensional decision context).

(I) *More efficient, effective and secure decision making procedure.* By sharing the online-learned Lasso regression, each client can make more efficient and effective local updates at each communication round. Such update is beneficial in committing its own individual decision making. Besides, our federation architecture reduce the risks of indirect data leakage or inverse decision rule recovery. This is unlike standard linear contextual bandit learning where in a heterogeneous setting requires several rounds of model updates to recover oracle policy, and thus *hurts* performance.

(II) *Provable federation architecture for online decision making with high-dimensional decision context.* Existing works on linear contextual bandit-based online decision making with high-dimensional decision context focus on the effect of sparsity-inspired methods on explore-exploit trade-off. Current state of related literature does not have work designing federation architecture to further facilitate explore-exploit trade-off in bandit learning. To our knowledge, this is the first provable federation architecture for online decision making with high-dimensional decision context that demonstrates the benefit of cooperation.
1.2 Related work

**Federated bandits.** Recent works have called attention to the topic of federated bandits. Shi et al. (2021) examine efficient client-server communication and coordination methods for federated MAB with personalization, using heterogeneous reward distributions at local clients. Zhu et al. (2021); Li et al. (2020); Dubey and Pentland (2020) discuss how to protect local data privacy in federated bandits via differential privacy. Our work advances these prior efforts by relaxing low dimension reward model assumptions to the case with high-dimensional decision context.

**Lasso bandits.** Bastani and Bayati (2020) first introduced linear contextual bandit with high-dimensional covariate and proposed the Lasso bandit algorithm. Follow-up works such as Wang et al. (2018) and Wang and Cheng (2020) improved the regret bounds and extended it to different problems. In contrast, another line of literature Kim and Paik (2019); Hao et al. (2020); Oh et al. (2021); Li et al. (2021a) proposed different style of algorithms for solving high dimensional bandit problems. However, these efforts consider a completely different bandit problems where $K$ contexts are observed at each round and only one underlying parameter $\beta$ is present for all arms. The results in these works are not directly comparable with Bastani and Bayati (2020). Our works advances the setting in Bastani and Bayati (2020) by designing federation architecture to ensure local data privacy security while delineating explore-exploit tradeoff in online decision making with high-dimensional decision context.

1.3 Notations and basic problem formulation

**Federated sparse online decision making problems.** We investigate a federated linear contextual bandits scenario in which $M$ clients are pulling the same set of $K$ items (arms), represented by $[K] := 1, 2, \ldots, K$. Each client $m \in [M]$ at each time $t \in [T]$ pulls an arm $k \in [K]$ based on historical information. The expected reward of pulling arm $k$ at decision context $x$ is the inner product $\langle x, \beta_k \rangle$ between $x$ and the reward parameter $\beta_k$. Additionally, there is a sparsity parameter $s_0 \in [d]$, defined as the smallest integer such that for all $k \in [K]$,
we have $\|\beta_k\|_0 \leq s_0$.

**Regret of bandit algorithms $\pi$.** A bandit algorithm $\pi$ of client $m$ pulls arm $\pi_t^{(m)}$ at decision step $t$. The objective is to design bandit algorithms $\pi$ that minimizes the expected cumulative regret among all clients, defined as:

$$\text{Regret}(T) = \mathbb{E} \left[ \sum_{m=1}^{M} \sum_{t=1}^{T} \left( \langle x_t^{(m)}, \beta_{k_t^{(m)},*} - \beta_{\pi_t^{(m)}} \rangle \right) \right]$$

where $k_t^{(m),*} \in [K]$ is a decision context-specific optimal arm at $t$ for client $m$ such that :

$$\forall l \neq k_t^{(m),*}, \langle x_t^{(m)}, \beta_{k_t^{(m)},*} \rangle \geq \langle x_t^{(m)}, \beta_l \rangle.$$

**Definition 1.** Given a dataset $\mathcal{D} = \{(X, Y)\}$ where $Y$ is a $|\mathcal{D}|$-dimension response vector and $X$ is a $|\mathcal{D}| \times p$ decision context matrix from the dataset $\mathcal{D}$. The Lasso regression estimator with regularization level $\lambda \geq 0$ is defined as

$$\hat{\beta}(\mathcal{D}, \lambda) \equiv \arg \min_{\beta} \left\{ \|Y - X\beta\|_2^2 / |\mathcal{D}| + \lambda \|\beta\|_1 \right\}.$$  \hspace{1cm} (1)

**Remark 1.** (Non-i.i.d. properties of dataset $\mathcal{D}$.) Due to the nature of online decision making problem, the Lasso estimate is trained on the dataset $\mathcal{D}$ that cannot satisfy typical distributional (or i.i.d.) assumptions in the literature of federated learning (Huang et al., 2021). The key reason is that the decision at certain time steps are depending on previous history, leading to dependency between decision and historical data. Therefore, standard convergence theory is not applicable for analyzing Lasso trained on the dataset $\mathcal{D}$. Fortunately, it is still possible to analyze and design the statistical properties of the trained Lasso estimate if we carefully design the bandit policy to coordinate the exploration across clients and exploitation within clients during whole online decision making process. See Figure 1 and Section 3.1 for bandit sampling design and Section 3.2 for formal convergence results of trained Lasso estimates.

## 2 Federated sparse online decision making

This section elaborates a horizontal federated learning (as defined in Yang et al. (2019)) version of bandit problems with high-dimensional decision context and linear arm rewards
(as introduced by Bastani and Bayati (2020) and others). Section 2.1 identifies the three main challenges arise from such federated contextual bandits. Section 2.2 established our framework that resolved the challenges.

2.1 Challenges from federated contextual bandits

A. Model local dataset heterogeneity. The first challenge arises from the restriction on uploading to the central server only the local-estimated parameters $\{\hat{\beta}_k\}_{k=1}^K$. Although this is not an issue for stochastic MAB since the parameters $\{\beta_k\}_{k=1}^K$ are scalars, it does create major challenges in aggregating the local estimates into a "global model". This is due to the fact that, under the linear reward structure, locally received rewards $\{r_t\}_{t=1}^T$ consist solely of the projection of the parameter $\beta_k$ along the decision context direction $\{x_t^{(m)}\}_{t=1}^T$; the portion of information lying outside range $\{x_t^{(m)}\}_{t=1}^T$ is not captured in $\{r_t\}_{t=1}^T$. Thus, by utilizing $\{r_t\}_{t=1}^T$, the locally estimated $\beta_k$, denoted as $\hat{\beta}_k$, cannot provide any information of $\beta_k$ beyond range $\{x_t^{(m)}\}_{t=1}^T$. Since $\{x_t^{(m)}\}_{t=1}^T$ are different for the same arm $k$, the locally estimated $\{\hat{\beta}_k^{(m)}\}_{m=1}^M$ are essentially lying in different subspaces. Accordingly, while aggregating local models to generate the global model, the central server must take this geometric structure into account.

B. Tighten local data and decision rule privacy security. The second challenge arises from the data leakage risk during federation process. Indeed, the main objective of federated bandit is to build reward models and decision rules based on datasets that are isolated across multiple clients while preventing indirect data leakage. In our scenario, leakage of Lasso estimate updates may actually leak important data information when exposed together with raw data. An ideal federated bandit learning architecture ensure there is of low risk on indirect data leakage. Thus, the precise information of the client decision contexts $\{\{x_t^{(m)}\}_{t=1}^T\}_{m=1}^M$ are retained from the central server. As a side effect, the server may lack an accurate assessment of the uncertainty associated with the local Lasso estimates at each client, or of the degree to which coordination might benefit specific clients. Unfortunately,
this would make efficient and effective coordination much more complex.

C. Coordinate efficient central communication. The third challenge arises from explore-exploit dilemma in bandit learning. The geometric structure of local rewards adds complexity to the exploration coordination of local client actions. Heuristically, in order to facilitate client \( m \) to estimate the expected reward by pulling arm \( k \), it is enough to obtain an accurate projection of actual parameter \( \beta_k \) on range \( \{ x_{(m)} T \}_{t=1}^T \); any component of true parameter \( \beta_k \) lying outside this subspace is unnecessary. Thereby, if two clients \( m_1 \) and \( m_2 \) have \( \{ x_{(m_1)} T \}_{t=1}^T \) and \( \{ x_{(m_2)} T \}_{t=1}^T \) orthogonal to each other, sharing the local estimates \( \hat{\beta}_{k}^{(m_1)} \) and \( \hat{\beta}_{k}^{(m_2)} \) does not benefit the other client augment her own local estimation. Instead, if \( \{ x_{(m_1)} T \}_{t=1}^T \) and \( \{ x_{(m_2)} T \}_{t=1}^T \) are completely aligned with each other, \( \hat{\beta}_{k}^{(m_1)} \) and \( \hat{\beta}_{k}^{(m_1)} \) can be integrated directly to augment the local estimation accuracy of both. With \( M \) possible subspaces spanned by \( \{ \{ x_{(m)} T \}_{t=1}^T \}_{m=1}^M \), it is quite possible that different clients retrieve different amounts of relevant information through information sharing provided by the central server. Consequently, in order to minimize total regret, it is extremely important to tactfully coordinate the bandit policies of clients.
2.2 Resolution to federated sparse bandit challenges

A. Clients and local bandit models. To account for the intrinsic correlation between rewards associated with different clients pulling the same arm, we assume the local bandit model of client \( m \) that the expected reward of pulling arm \( k \) is a linear function of the decision context \( x_t \); formally,

\[
r(x_t) \equiv \langle \beta_k, x_t \rangle + \epsilon_t^{(m)}.
\]  

(2)

The parameter \( \beta_k \in \mathbb{R}^d \) is a constant but unknown vector for each arm \( k \in [K] \). The noise process \( \{\epsilon_t^{(m)}\}_{t=1}^T \) is assumed to be a martingale difference noise. At each decision step \( t \), the noise \( \epsilon_t^{(m)} \) is drawn independently from a mean zero \( \sigma \)-sub-Gaussian distribution (that is, \( \mathbb{E}[\exp(\lambda \epsilon_t^{(m)})] \leq \exp(\sigma^2 \lambda^2/2) \) for all real \( \lambda \) and \( \{(\epsilon_t^{(m)}, F_t^{(m)})\}_{t\in[T]} \) forms a martingale difference sequence (that is, \( E[\epsilon_t^{(m)} | F_{t-1}] = 0 \)). See Section A for essential statistical regularity assumptions for analyzing federated contextual bandit algorithms.

Clients have distinct reward distribution under local bandit model (2), for the same arm based on their decision context sequence \( \{x_t^{(m)}\}_{t=1}^T \). Such a linear model naturally depicts the heterogeneity of data distributions at the clients, while allowing for prospective cooperation among clients due to the common parameters \( \{\beta_k\}_{k=1}^K \). Thus, the local bandit model (2) resolves the local dataset heterogeneity challenge in Section 2.1.A.

B. Federation and decision rule hiding. Our proposal to resolve local data privacy dilemma identified at Section 2.1.B is via Teamwork-Selfish sampling policy (Figure 1), established at Section 3.1. To protect local data privacy, we design federation architecture and hide clients’ decision rules from central server. Our approach, which in spirit is an application of the doubling trick Besson and Kaufmann (2018), do not require the agents to share their datasets, neither collected from Teamwork mode nor Selfish mode, to the server. The only information to share is the Lasso estimates based on datasets collected during Teamwork mode at certain pre-specified decision points. Thus, the Teamwork-Selfish bandit policy (Figure 1) resolves the local data privacy security challenges in Section 2.1.B.

C. Communication via horizontal federation. Our proposal to resolve explore-
exploit dilemma identified at Section 2.1.C is via client-center communication protocol (Figure 2), established at Section 3.3. To delineate the explore-exploit dilemma, each client trains two Lasso estimates, one from Teamwork dataset (for exploration) and the other from Teamwork-Selfish aggregation dataset (for exploitation). Our strategy is to introduce two different modes for agent. The two modes are Teamwork mode and Selfish mode. Clients only upload their Lasso estimates at the end of the Teamwork mode. The communication cost is \( \log(T/Kq) \). Our approach, which in spirit is an applications of horizontal federated learning architecture Yang et al. (2019), coordinates efficient communication between clients and center to build reward model and achieve near-optimal regret performance. In horizontal federated-learning system, \( M \) clients with same online decision making task collaboratively learn a model with the help of server. Consequently, the communication protocol (Figure 2) resolves the local data privacy security challenges in Section 2.1.C.

Algorithm 1: Fedego Lasso: client \( m \)

1. **Input:** Decision horizon \( T \), number of arms \( K \), optimality gap \( h^{(m)} \), Teamwork stage \( T \).
2. **for** \( t = 1 \) **to** \( T \) **do**
   3. Observe the decision context \( x^{(m)}_t \)
   4. **if** \( t \in T \) (Teamwork mode) **then**
      5. \( \pi(x^{(m)}_t) = \text{ColExplore}(x^{(m)}_t, t) \)
   6. **end**
   7. **if** \( t \notin T \) (Selfish mode) **then**
      8. \( \hat{K} = \text{FedScreen}(x^{(m)}_t, \{\hat{\beta}_{k,t}^{(m)}\}_{k=1}^K, h^{(m)}) \)
      9. \( \pi(x^{(m)}_t) = \text{EgoCommit}(\hat{K}, \{\hat{\beta}_{k,t}^{(m)}\}_{k=1}^K) \)
   10. **end**
11. Pull the arm \( \pi(x^{(m)}_t) \), receive reward \( r^{(m)}_t \).
12. **end**
13. **Output:** Cumulative reward \( R(T) = \sum_{t=1}^T r^{(m)}_t \).

3 Fedego Lasso algorithms

Algorithm 1 presents our solutions, the Fedego Lasso bandit algorithms, to the federated online decision making problems established at Section 2. There are 3 key components
in our design of Fedego Lasso algorithms: the teamwork-selfish bandit sampling strategy (Figure 1), the clients-central server communication protocol (Figure 2) and local data privacy preserving. Section 3.1 establishes the teamwork-selfish bandit sampling strategy to implement sparsity-award collaborate exploration as solutions to the challenge 2.1.A. Section 3.2 illustrates how Fedego Lasso protect local clients privacy as solutions to the challenge 2.1.B. Section 3.3 establishes the communication protocol of resulting Lasso estimates between clients and central server towards optimal regret performance of online decision making task as solutions to the challenge 2.1.C.

3.1 Teamwork-Selfish bandit sampling

This section presents the three key elements implementing the teamwork-selfish bandit sampling strategy (Figure 1): sampling mode, resulting datasets and their Lasso estimates.

Sampling modes: Teamwork and Selfish. Every local client agent has two sampling modes: Teamwork mode (Blue block in Figure 1) and Selfish mode (Red block in Figure 1). In the Teamwork mode, all clients run collaborate exploration, aiming at a quick recover of the arm reward parameter set $\{\beta_k\}_{k=1}^K$. In Selfish mode, clients run egocentric exploitation individually, aiming at an optimal decision for their own current decision context. Such alternating sampling is designed to guarantee the convergence of Lasso estimate while optimizing algorithm performance.

Datasets: teamwork set $\mathcal{T}$ and selfish set $\mathcal{S}$. Every local client agent maintains two datasets: teamwork dataset $\mathcal{T}$ and selfish dataset $\mathcal{S}$. In general, datasets $\mathcal{T}^{(m)}$ and $\mathcal{S}^{(m)}$ collect samples from Teamwork and Selfish mode respective for the client $m$. In particular, notations $\mathcal{T}^{(m)}_{[t],k}$ and $\mathcal{S}^{(m)}_{[t],k}$ denote the teamwork and selfish dataset respectively from pulling arm $k$ during decision period $[t] = \{1, 2, \cdots, t\}$. Such maintenance is to separate the data source. Technically, the teamwork set’s decision is public due to agreement of all clients. The selfish set’s decision is private and only accessible by the client itself. Theoretically, the teamwork set’s samples are independently distributed since in which the decisions are
independent of the previous history, while in selfish set the samples are dependent since the decisions are dependent on the history.

**Estimates:** teamwork Lasso $\hat{\beta}^\sharp$ and private Lasso $\hat{\beta}^\flat$. Every local client agent maintains two Lasso estimates: client teamwork Lasso and local private Lasso. In principle, the client teamwork Lasso $\hat{\beta}^\sharp = \hat{\beta}(T)$ is from running Lasso regression (Definition 1) on the Teamwork dataset; in contrast, local private Lasso $\hat{\beta}^\flat = \hat{\beta}(T \cup S)$ is trained on the aggregation dataset of Teamwork and Selfish set. Such maintenance is to separate federation and decision making. In Fedego Lasso algorithm, all local clients upload their client teamwork Lasso to central server at the end of each Teamwork mode to facilitate federation. At each decision step in Selfish mode, all clients commit final decisions based on local private Lasso estimate.

### 3.2 Decision rules in Teamwork and Selfish mode

This section presents the three key subroutines implemented in the Fedego Lasso bandit algorithms (Algorithm 1): collaborate exploration (Algorithm 2), federated screening (Algorithm 3), and egocentric commitment (Algorithm 4).

**A. ColExplore: Collaborate exploration**

**Algorithm 2: ColExplore($x,t$)**

1. **Input:** decision context $x$, decision step $t$;
2. **Output:** $\pi(x) \equiv t \mod K$;

**Planning of Teamwork stage $T$.** Each block in Figure 1 contains $Kq$ decision steps. The planning of teamwork stage follows doubling trick: $T = \bigcup_{n=1}^{\lceil \log_2(T/Kq) \rceil} T_n$, where $T_n = [(2^n - 1)Kq + 1 : 2^n Kq]$ is the $n$th Teamwork stage.

**Client teamwork Lasso.** The client teamwork Lasso for client $m$ at time step $t$ is defined by running Lasso regression (Definition 1) in the teamwork dataset $T^{(m)}_{[t],k}$, formally,

$$
\hat{\beta}^\sharp_{k,t}^{(m)} \equiv \hat{\beta}(T^{(m)}_{[t],k}, \lambda_1^{(m)}).
$$
Lemma 1. For all arms $k \in [K]$, the client teamwork Lasso estimate (3) satisfies

$$\mathbb{P}\left(\|\hat{\beta}^{\sharp}_{k,t} - \beta_k^{(m)}\|_1 > h^{(m)}/4x_{\max}\right) \leq 5/t^4$$

if $\lambda^{(m)}_1 = (\phi_0^{(m)})^2 p_k^{(m)} h^{(m)} / (64 s_0 x_{\max})$ and $t \geq (Kq)^2$.

Lemma 1 justifies the contribution of collaborate exploration to teamwork Lasso convergence.

B. FedScreen: Federated screening.

At each step in Selfish mode, the client screen available arms with central federated Lasso.

| Algorithm 3: FedScreen($x,\{\hat{\beta}_k\}_{k=1}^K, h$) |
|----------------------------------------------------------|
| 1. **Input:** decision context $x$, federated estimates $\{\hat{\beta}_k\}_{k=1}^K$, optimality gap $h$ |
| 2. **Output:** the candidate set $\hat{K}(x) \equiv \{k \in A : \langle x, \hat{\beta}_k \rangle \geq \max_{l \in A} \langle x, \hat{\beta}_l \rangle - h/2\}$ |

Central federated Lasso. After receiving all clients’ teamwork Lasso estimates, the central server performs federation by computing the central federated Lasso. The central federated Lasso for arm $k$ at decision step $t$ is defined as the average of all client Teamwork Lasso estimates; formally

$$\hat{\beta}^{\sharp}_{k,t} \equiv \frac{1}{M} \sum_{m=1}^{M} \hat{\beta}^{\sharp,(m)}_{k,t}. \quad (4)$$

Remark 2. Note that the central federated Lasso is the average of teamwork Lasso estimates, which are trained on i.i.d. samples, and therefore has convergence guarantees. Such analytical advantage is due to our policy design that all agents explore the efficacy of pre-specified arms during Teamwork mode (Blue block in Figure 1).

Lemma 2. For all arms $k \in [K]$, if $t \geq (Kq)^2$, the central federated Lasso estimate (4) satisfies

$$\mathbb{P}\left(\|\hat{\beta}^{\sharp}_{k,t} - \beta_k\|_1 > h^{(m)}/4x_{\max}\right) \leq 5M/t^4.$$ 

C. EgoCommit: Egocentric commitment
Given the resulted set of optimal arm candidates \( \hat{K}(x^m_t) \) for decision context \( x^m_t \), the client agent \( m \) commits to the arm with the highest estimated expected reward estimated by the private egocentric Lasso (5).

**Algorithm 4: EgoCommit(\( \hat{K}(x), \{\hat{\beta}_k\}_{k=1}^K \))**

1. **Input:** candidate set \( \hat{K}(x) \), decision context \( x \), private estimates \( \{\hat{\beta}_k\}_{k=1}^K \)
2. **Output:** \( \pi(x) \equiv \arg\max_{k\in\hat{K}(x)} \langle x, \hat{\beta}_k \rangle \)

**Private egoistic Lasso.** The private egoistic Lasso for client \( m \) at time step \( t \) is defined by running Lasso regression (Definition 1) in the teamwork-selfish aggregate dataset \( T_{[t],k}^{(m)} \cup S_{[t],k}^{(m)} \); formally,

\[
\hat{\beta}_{k,t}^{(m)} \equiv \hat{\beta}\left(T_{[t],k}^{(m)} \cup S_{[t],k}^{(m)}, \lambda_{2,t}^{(m)}\right).
\]

(5)

**Lemma 3.** For all optimal arms \( k \in \mathbb{A}_{opt}^{(m)} \), if \( t \geq C_5 \), the private egocentric Lasso estimate (5) satisfies

\[
P(\|\hat{\beta}_{k,t}^{(m)} - \beta_k^{(m)}\|_1 > 16\sqrt{(\log t + \log d)/(p^2C_1(\phi_0) t)})
\leq 2(t^{-1} + \exp(-p^2C_2(\phi_0^{(m)})^2/32 \cdot t))
\]

if \( \lambda_{2,t}^{(m)} = \left[(\phi_0^{(m)})^2 / (2s_0)\right] \sqrt{\log(dt) / (p^2C_1(\phi_0) t)} \).

**Remark 3.** Non-trivial convergence guarantees of private egoistic Lasso. The private egoistic Lasso (5) is trained on an aggregated non-i.i.d. dataset \( T_{[t],k}^{(m)} \cup S_{[t],k}^{(m)} \). The non-i.i.d. property of the aggregated dataset is due to it is a mixture of i.i.d. (teamwork dataset \( T_{[t],k}^{(m)} \)) and non-i.i.d. (selfish dataset \( S_{[t],k}^{(m)} \)) data set. Such non-i.i.d property roots non-trivial convergence guarantee for the private egoistic Lasso.

### 3.3 Communication between clients and central server

Figure 2 presents the three key events in the clients-central server communication protocol: upload, federation and broadcast. In the following we discuss the contribution of such horizontal federated learning architecture to bandit learning and local data privacy security.

**Upload: Clients upload local teamwork Lasso to server.** At the end of each round of the Teamwork mode, all clients upload their current client Teamwork Lasso estimates (3)
to the central server to facilitate the federation. The local teamwork Lasso have difference convergence level due to local data heterogeneity. The collaborate exploration between clients in Teamwork mode ensuring decisions of a client will not compromise to the other client. Such advantage is from our design of Teamwork-Selfish sampling strategy, all clients agree to do experiment for collaborate exploration during Teamwork mode.

**Federation:** Server compute central federated Lasso. The central sever do averaging to mitigate statistical error of received teamwork Lasso estimates. The outcome estimate is called central federated Lasso (defined at (4)). The federation event helps security by ensuring internal potential privacy leakage from the central server since clients do not upload any dataset but only their local teamwork Lasso estimates.

**Broadcast:** Server broadcast federated Lasso to clients. After federation, the central server broadcasts the central federated Lasso (4) to clients. The central federated Lasso helps client to do valid screening procedure to exclude sub-optimal arms and find out the candidates of optimal arms. Such screening procedure is termed *federated screening* and elaborated at Algorithm 3. The broadcast event helps security by the fact that one client cannot infer other clients’ decisions based on the federated Lasso, since the final decision is based on private egocentric Lasso (defined at 5) as in Algorithm 4. Such a feature is special in our architecture; it helps us avoid internal potential privacy leakage from one client to the other.
3.4 Regret guarantee

The following theorem bounds the regret of Fedego Lasso bandit algorithm (Algorithm 13).

**Theorem 1.** The regret of Fedego Lasso bandit algorithms $\pi$ over decision horizon $[T]$ satisfies

$$
\text{Regret}_\pi(T) \leq M \left\{ [C_3] (\log T)^2 + \left[ 2b x_{\text{max}} K (6q + 2) + C_3 \log d \right] \log T + [2b x_{\text{max}} (C_5 + K (1 + 4C_4))] \right\} = O \left( MK s_0^2 \sigma^2 [\log T + \log d]^2 \right)
$$
4 Empirical results

While the theoretical regret analysis (Theorem 1) provides worst-case guarantees for Fedego Lasso, we now examine their performance on a variety of tasks empirically. We implement our algorithms to a variety of jobs. This examination is conducted using both synthetic and real-world data. The synthetic data enables us to manage the learning problem’s characteristics for internal validity, while the real-world data serves as a data point for external validity.

A. Experiment setup. We compare Fedego Lasso bandit with the vanilla Lasso bandit algorithm in (Bastani and Bayati, 2020) on the three datasets. We run each algorithm for 10 independent trials on each dataset and plot the average results with two standard deviations. Additional experimental details are provided in Appendix B.

(a). Synthetic Data. The sparse parameters $\{\beta^{(m)}_k\}^K_{k=1}$ for each client is generated from randomly sampled support, and then the nonzero parameter values are generated from the uniform distribution on $[0,1]$. The decision context are drawn randomly from a multivariate normal distribution.

(b). Real data: PharmGKB. The first real-world task is personalized dosage searching. The Pharmacogenomics Knowledge Base (PharmGKB) dataset was used by Bastani and Bayati (2020) to support the superiority of Lasso bandit over the other bandit algorithms. We inherit their settings and investigate the error rates of the two algorithms when giving warfarin dosages based on patient-level decision context such as demographics, diagnosis, and medications.

(c). Real data: Medical MNIST. The second real-world task is to collaborate classification of Medical MNIST images. To extract the useful features vectors from the medical images, we first train a fully-connected neural network on the dataset till it reaches good training and testing accuracies. At each round of the bandit problem, an image is sampled from the dataset and fed into the neural network. The covariates for the bandit algorithms are the output of an intermediate layer of the neural network and thus it should contain useful information of the image. The instantaneous regret is defined by whether the image is correctly classified.
B. Benefits of federation architecture. Figure 3 supports the benefits of federation architecture enjoyed by the proposed Fedego Lasso over the vanilla Lasso bandit in Bastani and Bayati (2020). In synthetic data scenario, Fedego Lasso enjoys substantial regret reduction in Figure 3.(a). In the task of personalized dosage searching, Fedego Lasso enjoys faster error rate convergence in Figure 3.(b). In the task of medical image labeling, Fedego Lasso again enjoys substantial regret reduction in Figure 3.(c). These empirical evidences supports the benefits of federation architecture in online sparse decision making.

C. Benefits of large number of clients. Figure 4 supports the benefit of having large number of clients in federated bandit learning. In the synthetic data and the task of medical image labeling, Fedego Lasso enjoys further regret reduction by recruiting more clients in bandit learning, as supported by Figure 4.(a) and (c). In the task of personalized dosage searching, Fedego Lasso with more clients enjoy faster error rate convergence, as in Figure 4.(b).
5 Discussion and Conclusion

In this work, we have established a novel architecture federated linear contextual bandits model with high-dimensional decision context. Such architecture delivers a unified federated bandit framework that resolves local dataset heterogeneity, local data and decision rule privacy security and explore-exploit tradeoff simultaneously. The associated algorithm Fedego Lasso utilizes the sparse structure of local bandit models to recover the global parameters and inform efficient federated exploration with effective egocentric exploitation. Theoretical analysis indicates that Fedego Lasso achieves near-optimal regret in the order of $O((\log T)^2)$ with a communication cost in the order of $O(\log_2(T/Kq))$.

A possible future direction for further advancing the $O((\log T)^2)$ regret into order $O(\log T)$ is to replace Lasso regression in Fedego with Minimax Concave Penalized (MCP) penalty, although whether this rate matches the theoretical lower bound remains unknown. See Wang et al. (2018) for the technique of MCP penalty for high-dimensional linear contextual bandit model.
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A Statistical Regularity Conditions

In section A, we list essential statistical regularity assumptions towards convergence rate analysis of Lasso estimation (Definition 1) and regret analysis of algorithms. We note that these assumptions are standard in the literature of Lasso bandits (Bastani and Bayati, 2020).

**Assumption 1.** For a client m, there exists a constant \( C_0^{(m)} \geq 0 \) such that for any two different arms \( k_1 \neq k_2 \) in \( \mathbb{A} \), the decision context distribution \( X \) satisfy \( \mathbb{P}(|\langle X, \beta_{k_1}^{(m)} - \beta_{k_2}^{(m)} \rangle| \in (0, \kappa)) \leq C_0^{(m)} \kappa \) for given \( \kappa > 0 \).

Assumption 1 is referred to Margin Condition in the classification literature (Tsybakov et al., 2004) and is introduced in multi-armed linear bandit literature to ensure only a small fraction of features can be drawn near the classification boundary \( \{x : \langle x, \beta_{k_1} - \beta_{k_2} \rangle = 0 \} \) in which efficacy of both treatments are almost equivalent (Rusmevichientong and Tsitsiklis, 2010; Wang and Cheng, 2020).

**Assumption 2.** For a client m, there exists an optimality gap constant \( h^{(m)} \) and two mutually exclusive arm subsets \( \mathbb{A}_{\text{opt}}^{(m)} \) and \( \mathbb{A}_{\text{sub}}^{(m)} \) with \( |K| = \mathbb{A}^{(m)} = \mathbb{A}_{\text{opt}}^{(m)} \cup \mathbb{A}_{\text{sub}}^{(m)} \) such that (a) For each action \( k \) in \( \mathbb{A}_{\text{sub}}^{(m)} \), it holds for every decision context \( x \in \mathcal{X} \) that \( \langle \beta_k, x \rangle < \max_{a \in \mathbb{A} \setminus \{k\}} \langle \beta_a, x \rangle - h^{(m)} \). and (b) For each action \( k \) in \( \mathbb{A}_{\text{opt}}^{(m)} \), there exists a constant \( p^{(m)}_* > 0 \) of minimal sampling probability such that \( \min_{k \in \mathbb{A}_{\text{opt}}^{(m)}} \mathbb{P}(X \in U^{(m)}_k) \geq p^{(m)}_* \) where \( U^{(m)}_k \equiv \{x \in \mathcal{X} : \langle \beta_k, x \rangle > \max_{a \in \mathbb{A} \setminus \{k\}} \langle \beta_a, x \rangle - h^{(m)} \} \).

Assumption 2 is referred to the Action Optimality Condition and is required in the literature Rusmevichientong and Tsitsiklis (2010); Bastani and Bayati (2020); Wang and Cheng (2020). The purpose of Assumption 2 is to separate available arms into an optimal subset \( \mathbb{A}_{\text{opt}}^{(m)} \) and a suboptimal subset \( \mathbb{A}_{\text{sub}}^{(m)} \) such that (a) all sub-optimal actions are strictly sub-optimal for every decision context and (b) every optimal treatment \( a \in \mathbb{A}_{\text{opt}}^{(m)} \) is strictly optimal for some decision context (denoted by the set \( U^{(m)}_k \) at Assumption 2).

**Assumption 3.** For a client m, there exists a constant \( \phi_0^{(m)} > 0 \) such that for each optimal arm \( k \in \mathbb{A}_{\text{opt}}^{(m)} \), its population covariance matrix \( \Sigma_k \equiv E[XX^\top | X \in U_a] \) belongs to the compatibility set with respect to the true parameter \( \beta_k \). That is, \( \Sigma_a \in \mathcal{C}(I, \phi_0^{(m)}) \), where

\[
\mathcal{C}(I, \phi) \equiv \{M \in \mathbb{R}_{\geq 0}^{p \times p} \mid \forall v \in \mathbb{R}^p \text{ such that } \|v_I\|_1 \leq 3\|v_I\|_1 \text{, we have } \|v_I\|^2 \leq |I|v^\top Mv/\phi^2 \}
\]

Assumption 3 is referred to as the Compatibility Condition in high-dimensional statistics literature Bühlmann and Van De Geer (2011). The purpose of Assumption 3 is to ensure that the Lasso estimate trained on samples \( X \in U_w \) converges to the true parameter \( \beta_w \) with high probability as the number of samples grows to infinity.
B Experiment Details and Additional Experiments

B.1 Synthetic Data

For the baseline case, we set \( d = 100, K = 5, M = 5 \) and \( s = 5 \), where \( s \) is the sparsity level of the parameters that should satisfy \( s \ll d \). The \( \beta \) of each arm for each client is generated by first choosing the possibly nonzero positions for all clients. In our case we randomly choose 10 positions for each arm and the 10 positions are shared across the different clients to represent that the bandit environment faced by different clients are similar (but not exactly the same). We then choose the \( s \) non-zero elements randomly within the 10 positions of each arm for each client, and then generate the values from a uniform distribution on \([0, 1]\). Note that the nonzero values of each arm across different clients are different, and the positions are different too. The covariates of each client are generated from a standard normal distributions independently.

We provide some additional experimental results for different choices of \( d, K, M \) in Figure ?? for readers interested in the effect of changing these hyper-parameters in our synthetic data.

B.2 PharmGKB

We utilize the publicly available code for PharmGKB at https://github.com/chuchro3/Warfarin for the processing of the dataset. Specifically, the covariates are generated from either the value in the dataset if it is numerical or from a bag-of-words of all the categories if categorical. Then the covariates are transformed into one-hot encodings and thus the vector is very high-dimensional \((d = 5528)\). We follow Bastani and Bayati (2020) and classify the different dosages of Warfarin into three classes. \([0, 3] \) is the low-dosage class, \([3, 7] \) is the moderate dosage class, and \([7, \infty] \) is the high-dosage class. The reward is defined to be 1 if the correct dosage class is chosen and 0 if not, and we add standard Gaussian noise to the reward. The error rate (in Figure 3) is defined by the number of wrong classifications divided by the total number of classifications.

B.3 Medical MNIST

Neural Network Setting. The fully-connected neural network used to train on the Medical MNIST dataset has the architecture shown in Table 1, where input size is the size of the vectorized image, which is 10800 in our case and output size is the number of classes, which is 6. The dataset was randomly split into a training set and a testing set in the ratio of 9:1. The images are pre-processed with random rotation, random horizontal flipping, resizing, center cropping and normalization. The training batch size was 32. The optimizer was SGD with learning rate 0.01 and no momentum or weight decay. The neural network was trained on the training set for 10 epochs and evaluated on the testing set. The final testing classification accuracy was around 99%.

Bandit Setting. The number of arms for each agent is equal to the number of classes (6) in this problem. We use the output of the last ReLU layer (i.e., before the last layer) as
the feature vector for bandit problems, which means that the feature dimension is 500. At each round, The regret is defined to be whether the feature vector is correctly classified, i.e., the reward is 0 if the chosen arm is different from the class of the image and 1 if they are the same. Standard Gaussian noise is added to the reward received by the bandit algorithms.

Table 1: Architecture of the neural network

| Layer   | Description                  |
|---------|------------------------------|
| inLayer | Linear Layer(input size, 4000) |
|         | ReLU()                       |
| hidden1 | Linear Layer(4000, 2000)     |
|         | ReLU()                       |
| hidden2 | Linear Layer(2000, 1000)     |
|         | ReLU()                       |
| hidden3 | Linear Layer(1000, 500)      |
|         | ReLU()                       |
| outLayer| Linear Layer(500, output size) |