Measuring $\Omega$ with galaxy streaming velocities

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The mean pairwise velocity of galaxies, $v_{12}$ has traditionally been estimated from the redshift space galaxy correlation function. This method is notorious for being highly sensitive to the assumed model of the pairwise velocity dispersion. Here we propose an alternative method to estimate $v_{12}$ directly from peculiar velocity samples, which contain redshift-independent distances as well as galaxy redshifts. This method can provide an estimate of $\Omega^0.6\sigma_8^2$ for a range of $\sigma_8$ where $\Omega$ is the cosmological density parameter, while $\sigma_8$ is the standard normalization for the power spectrum of density fluctuations. We demonstrate how to measure this quantity from realistic catalogues and identify the main sources of bias and errors.

1 A model of $v_{12}(r)$

In this presentation we report on the the possibility of using the “mean tendency of well-separated galaxies to approach each other” to measure the cosmological density parameter, $\Omega$. The statistic we consider is the mean relative pairwise velocity of galaxies, $v_{12}$. It was introduced in the context of the BBGKY theory describing the dynamical evolution of a collection of particles interacting through gravity. In this discrete picture, $\vec{v}_{12}$ is defined as the mean value of the peculiar velocity difference of a particle pair at separation $\vec{r}$. In the fluid limit, its analogue is the pair-density weighted relative velocity $\vec{v}_{12}(r)$.

\[
\vec{v}_{12}(r) = \langle \vec{v}_1 - \vec{v}_2 \rangle = \frac{\langle (\vec{v}_1 - \vec{v}_2)(1 + \delta_1)(1 + \delta_2) \rangle}{1 + \xi(r)},
\]

where $\vec{v}_A$ and $\delta_A = \rho_A/\langle \rho \rangle - 1$ are the peculiar velocity and fractional density contrast of matter at a point $\vec{r}_A$, $r = |\vec{r}_1 - \vec{r}_2|$, and $\xi(r) = \langle \delta_1 \delta_2 \rangle$ is the two-point correlation function. The pair-weighted average, $\langle \cdots \rangle_p$, differs from simple spatial averaging, $\langle \cdots \rangle$, by the weighting factor $\rho_1 \rho_2 (\rho_1 \rho_2)^{-1}$, proportional to the number-density of particle pairs. In a recent letter, one of us has shown...
that an excellent approximation to $v_{12}$ is given by

$$v_{12}(r) = -\frac{2}{3} H r f \tilde{\xi}(r)[1 + \alpha \bar{\xi}(r)],$$  

(2)

$$\bar{\xi}(r) = \frac{3}{r^3} \int_0^r \xi(x) x^2 dx \equiv \tilde{\bar{\xi}}(r)[1 + \bar{\xi}(r)].$$  

(3)

Here $\alpha$ is a parameter, which depends on the logarithmic slope of $\xi(r)$, while $f = d \ln D/d \ln a$, with $D(a)$ being the standard linear growing mode solution and $a$ – the cosmological expansion factor. Finally, $H = 100 \, h \, \text{km s}^{-1} \, \text{Mpc}$ is the present value of the Hubble constant. This approximate solution of the pair conservation equation, accurately reproduces results of high resolution N-body simulations in the entire dynamical range.\[3]

If we restrict ourselves to $r = 10 h^{-1} \, \text{Mpc}$, one can use the APM catalogue of galaxies\[3] for an estimate of galaxy correlation function, $\xi = (r/r - 0)^{-\gamma}$; the slope at the separation considered is $\gamma = 1.75 \pm 0.1$ (the errors we quote are conservative). One obtains then

$$v_{12}(10 h^{-1} \, \text{Mpc}) = -605 \sigma_8^2 \Omega^{0.6} (1 + 0.43 \sigma_8^2)/(1 + 0.38 \sigma_8^2)^2 \, \text{km/s}. \quad (4)$$

The above relation shows that at $r = 10 h^{-1} \, \text{Mpc}$, $v_{12}$ is almost entirely determined by the values of two parameters: $\sigma_8$ and $\Omega$. It is only weakly dependent on $\gamma$. The uncertainties in the observed $\gamma$ lead to an error in Eq. (4) of less than 10% for $\sigma_8 \leq 1$.

2 The estimator

Since we observe only the line-of-sight component of the peculiar velocity, $s_A = \hat{r}_A \cdot \tilde{v}_A/r \equiv \hat{r}_A \cdot \tilde{v}_A$, rather than the full three-dimensional velocity $\tilde{v}_A$, it is not possible to compute $v_{12}$ directly. Instead, we propose to use the mean difference between radial velocities of a pair of galaxies, $(s_1 - s_2)_r = v_{12} \hat{r} \cdot (\hat{r}_1 + \hat{r}_2)/2$, where $\hat{r} = \hat{r}_1 - \hat{r}_2$. To estimate $v_{12}$, we use the simplest least squares techniques, which minimizes the quantity $\chi^2(r) = \sum_{A,B} \left( (s_A - s_B) - p_{AB} \tilde{v}_{12}(r)/2 \right)^2$, where $p_{AB} \equiv \hat{r} \cdot (\hat{r}_A + \hat{r}_B)$ and the sum is over all pairs at fixed separation $r = |\hat{r}_A - \hat{r}_B|$. The condition $\partial \chi^2 / \partial \tilde{v}_{12} = 0$ implies

$$\tilde{v}_{12}(r) = \frac{2 \sum (s_A - s_B) p_{AB}}{\sum p_{AB}^2}.$$  

(5)

The above expression is a sum over positive quantities and so is stable. This estimator is appropriate to be applied to a point process which will sample an
underlying continuous distribution. The sampling is quantified in terms of the selection function, $\phi(\vec{r})$. The continuum limit of Eq. 5 is then

$$\tilde{v}_{12}(r) = \frac{2 \int dm_1 \, dm_2 \, \Phi_{12}(s_1 - s_2)p_{12}}{\int dm_1 \, dm_2 \, \Phi_{12}p_{12}^2},\tag{6}$$

with $dm_A = \rho_A \, d^3\vec{r}_A$, and a two-point selection function given by $\Phi_{12} = \delta_D(|\vec{r}_1 - \vec{r}_2| - r) \, \phi(\vec{r}_1) \, \phi(\vec{r}_2)$, where $\delta_D$ is the Dirac delta function. For ease of notation we shall denote the denominator in Eq. 6 by $W(r)$. If we take the ensemble average of Eq. 6 we find that $\langle \tilde{v}_{12}(r) \rangle = v_{12}(r)$. Note that, unlike the estimators for the velocity correlation tensor proposed in 6, the ensemble average of the estimator is $v_{12}(r)$ independent of the selection function. For an isotropic selection function this estimator is insensitive to systematic effects such as a bulk flow, large scale shear and small scale random velocities (as one might expect from virialized objects).

3 Biases and errors

How unbiased is this estimator? We have applied our statistic to mock catalogues extracted from N-body simulations of a dust-filled universe with $\Omega = 1$ and $P(k) \propto 1/k$.

In Figure 1(a) we plot $\tilde{v}_{12}(r)$ with one standard deviation calculated with 20 mock catalogues extracted with a deep selection function. Each catalogue has a different observation position within the simulation volume and so an average over this set should resemble a true ensemble average. The mean is consistent with what one would expect from a direct calculation with Eq. 2 (which is plotted in Figure 1(a) as a solid line). We have also performed this analysis without collapsing the cores; the results changed by very little.

We repeat this calculation for a set of 9 catalogues all constructed from the same observation point using a deep (Figure 1b) or shallow (Figure 1c) selection function to randomly sample a fraction of galaxies within the simulation box. The variance in $\tilde{v}_{12}(r)$ is now solely due to finite sampling (“shot noise”); for catalogues with 2000 to 3000 galaxies we expect the variance to be $\sqrt{2}-\sqrt{3}$ times larger. We find that a shallow selection function changes the functional form, or slope, of the mean, making it a more sharply decreasing function of $r$ than the ensemble average. It is therefore crucial when analyzing a catalogue to restrict oneself to scales much smaller than the effective cutoff scale of the selection function.

Errors in distance measurements will naturally affect our results; the best estimators use empirical correlations between intrinsic properties of the galaxies and luminosities and lead to log-normal errors in the estimated distance of
around 20%. These errors will naturally lead to biases in cosmological estimators involving distance measurements and peculiar velocities and are generically called Malmquist bias. We shall model our errors assuming a Tully–Fisher law which resembles that inferred from the Mark III catalogue. To correct for Malmquist bias we use the prescription put forward in

In Figure 2(a), we plot the results for the uncorrected simulations; Malmquist errors systematically lower the values of \( \bar{v}_{12} \) on small scales while enhancing its amplitude on large scales (where the effect should be more dominant). However in Figure 2(b) we show that with the correction for general Malmquist errors to the distance estimator, it is possible to overcome this discrepancy. The 1-\( \sigma \) errors now encompass the true \( \bar{v}_{12} \) over a wide range of scales.

4 Conclusions

In this contribution we report on a recent proposal to estimate the mean pairwise streaming velocities of galaxies directly from peculiar velocity samples. We identified three possible sources of systematic errors in estimates of \( v_{12} \) made directly from radial peculiar velocities of galaxies. We also found ways
Figure 2: $v_{12}(r)$ and its variance evaluated from 100 mock catalogues with errors (described in the text) and the full selection function. The solid points are the $\tilde{v}_{12}$ of the error-free simulation seen from the same observation point, the solid line is the mean and dashed lines are the 1σ. a) uncorrected distances; b) distances corrected for Malmquist bias of reducing these errors; these techniques were successfully tested with mock catalogues. The potential sources of errors and their proposed solutions can be summarized as follows.

1) On the theoretical front, assuming a linear theory model of $v_{12}(r)$ at $r \approx 10h^{-1}$ Mpc can introduce a considerable systematic error in the resulting estimate of $\sigma^2 \Omega^{0.6}$. For example, if $\sigma_s = 1$ using the linear prediction for $v_{12}$ at $r = 10h^{-1}$ Mpc would introduce a 25% systematic error (see eq. [4]). We solve this problem by using a nonlinear expression for $v_{12}$.

2) On the observational front, a shallow selection function induces a large covariance between $\tilde{v}_{12}$ on different scales. This must be taken into consideration by measuring $\tilde{v}_{12}(r)$ only on sufficiently small scales. A rule of thumb is that for estimating $\tilde{v}_{12}$ at $10h^{-1}$ Mpc, the selection function should be reasonably homogeneous out to at least $30h^{-1}$ Mpc.

3) Finally, care must be taken with generalized Malmquist bias due to log-normal distance errors; these induce a systematic error in $\tilde{v}_{12}$. We have shown that, under certain assumptions about selection and measurement errors, the method of Landy & Szalay for corrected distance estimates allows one to recover the true $\tilde{v}_{12}$. Naturally, this particular correction must be addressed on a case-by-case basis, given that different data sets will have different selection
criteria and correlations between galaxy position and measurement errors.

In a future publication we shall analyze the Mark III and the SFI catalogues of galaxies with this in mind.

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