On the vertex corrections in antiferromagnetic spin fluctuation theories

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We argue that recent calculations by Amin and Stamp (PRL 77, 301 (1996)) overestimate the strength of the vertex corrections in the spin-fermion model for cuprates. We clarify the physical origin of the apparent discrepancy between their results and earlier calculations. We also comment on the relative sign of the vertex correction.

In a recent publication Amin and Stamp computed the leading order correction to the spin-fermion vertex in the nearly antiferromagnetic Fermi-liquid model. They considered a region near optimal doping in which the Fermi surface is large, and precursors of the spin-density-wave state have not been formed yet. They argued that the leading vertex correction is rather large ($\Delta g/g \sim 1$) for realistic values of the parameters of the electron dispersion and the spin fluctuation spectrum. This result is in clear disagreement with previous calculations which reported a much smaller amplitude of the correction. Whether vertex corrections are large or not is relevant to the validity of the Eliashberg-type calculations of the superconducting transition temperature in cuprates.

In this letter, we clarify the origin of the discrepancy between Ref. [1] and earlier results and show that Ref. [1] strongly overestimates the amplitude of the vertex correction near optimal doping.

The spin-fermion model describes fermions coupled to spin fluctuations by

\[ \mathcal{H}_{\text{ spin-f}} = g \sum_{k,q} c_{k,q}^\dagger \sigma_{\alpha,\beta} c_{k+q,\beta} S_{-q}. \]

Here $g$ is the coupling constant, and $\sigma_i$ are the Pauli matrices. It is assumed that the Fermi-liquid description is valid, i.e., near the Fermi surface, the electronic Green’s function behaves as $G(k,\omega_n) = Z/(i\omega_n - \epsilon_k)$, where $\epsilon_k = \epsilon_k - \mu$, and $Z \leq 1$ is a positive constant.

The dispersion near the Fermi surface is given by $\epsilon_k = -2t\cos k_x \cos k_y - 4t' \cos k_x \cos k_y$. Spin fluctuations are described by a dynamical spin susceptibility which is assumed to be strongly peaked near the antiferromagnetic momentum $Q = (\pi, \pi)$, and to behave at low energies as $\chi(q,\omega_n) = \chi_0/(1 + \xi^2 q^2 + i|\omega_n|/\omega_{sf})$. Here $\xi = q - Q$, $\xi$ is the magnetic correlation length, and $\omega_{sf} \propto \xi^{-2}$ is a typical spin fluctuation frequency which is much smaller than any other energy scales in the problem due to the proximity to antiferromagnetism. Amin and Stamp computed the vertex corrections for the points on the Fermi surface which are connected by $Q$ (the so-called “hot spots”), and also for $k$ along the Brillouin zone diagonal. They argued that the relative correction is larger for the hot spots, a result we do not dispute. For the vertex correction at the hot spot they obtained assuming that fermions are free particles $\Delta g/g = -g^2 \chi Q^2 \omega_{sf} / 4\pi^2 \mu_0^2 I(k_h)$ where $I(k_h)$ is a dimensionless quantity. It turns out that $I(k_h)$ is rather large which implies a large amplitude of the correction. Amin and Stamp claim that the origin of the discrepancy between their and earlier results lies in their more accurate numerical evaluation of $I(k_h)$. We disagree with their explanation and will argue that the reason for their discrepancy with earlier results in fact has a physical rather than a numerical origin.

We independently computed the vertex corrections analytically and numerically along the same lines as described above. The analytical expression for $\Delta g/g$ is given by

\[ \frac{\Delta g}{g} = -\frac{g^2 Z^2 \chi Q^2 \omega_{sf}}{4\pi^2 \mu_0^2} \times \left[ \text{Re} \int_0^\pi d\phi \log\left]\frac{\sin(\phi/2)}{\cos(\phi) + \cos \phi_0}\right]\log\left]\frac{\sin(\phi/2)}{\delta^2} + O(\delta^2) \right] \]

where $\delta = c_{sw}^2/(2\gamma \nu \xi) \equiv \xi \omega_{sf} / \nu \ll 1$, and $\nu$ and $\phi_0$ define the fermionic dispersion around a hot spot: $\nu = v(k - k_h) \cos(\phi)$ and $\nu + \phi_0 = v(k - k_h) \cos(\phi + \phi_0)$. The angle $\phi_0$ is the angle between the normals to the Fermi surface at the hot spots (see the Fig).

We emphasize that Eq. (2) contains a term which logarithmically depends on $\delta$ and a term independent of $\delta$. Previous analytical calculations of the vertex correction restricted with the $|\log(\delta)|$ term only. We will see that for parameters relevant to cuprates both terms nearly equally contribute to the vertex renormalization.

We first compare our analytical (Eq. (2)) and numerical results with the ones obtained by Amin and Stamp for the same set of parameters, namely $t = 0.25 eV$, $t' = -0.45 t$, $|\mu| = 1.46 t$, $\omega_{sf} = 14 meV$, $\xi = 2.5 a$, $g = 0.64 eV$ and $Z = 1$. They found $\Delta g/g \approx -0.7$ while Eq. (2) yields $\Delta g/g \approx 0.55$, and numerical computations yield $\Delta g/g \approx 0.4$ (for this set of parameters $\Phi_0 = 1.78$ and $\delta = 0.27$). We see that apart from the sign difference which we discuss later, our results and the ones by Amin...
and Stamp are in close agreement which implies that a more accurate evaluation of $I(k_b)$ cannot be the main reason for the discrepancy mentioned above. The actual reason for the discrepancy with earlier work lies in the fact that Amin and Stamp considered $\omega_{sf}$ as an input parameter, whose value can be inferred from NMR experiments, and simultaneously treated fermions as free particles with $Z = 1$. We will show that these two assumptions are likely to be incompatible.

The point is that near optimal doping, the key source for the spin damping is the interaction with fermions (the damping due to this interaction is much stronger than the one due to a direct spin-spin exchange). Since the damping term in the spin susceptibility is related to the imaginary part of the particle-hole bubble at momentum transfer $Q$ and low frequencies, the fermions which contribute to $\omega_{sf}$ are located in the vicinity of the hot spots. One can then compute the damping within the spin-fermion model of Eq.(1) which, we remind, is valid between the normals to the Fermi surface at hot spots (dashed lines). For clarification we omitted the parts of the Fermi surface in the second and fourth quadrant.

Alternatively, one can consider fermions with arbitrary $Z$, compute $\omega_{sf}$ using Eq.(3), and substitute the result into Eq.(2). Doing this, we obtain neglecting terms of $O(\delta^2)$

$$\frac{\Delta g}{g} = -\frac{|\sin \phi_0|}{4\pi^2} \text{Re} \int_0^\pi d\phi \frac{\log[\sin(\phi/2)]}{\cos \phi + \cos \phi_0} \log \sin(\phi/2) \delta^2$$

(4)

This is the way the vertex correction was obtained in [1]. For the set of parameters chosen, we indeed recover the same result. Similar calculations for $\phi_0 = \pi$ have been performed in Ref. [2] which also yielded a very small value of $\Delta g/g$.

We now address two technical points. The first one is the sign of the vertex correction. We have already mentioned that the sign obtained in earlier calculations is opposite to the one obtained by Amin and Stamp. A simple way to check the sign of the correction is to consider the limit where both the coupling constant and the chemical potential are much larger than the fermionic bandwidth, and $|\mu| \approx g/2$. In this limit the electronic structure develops precursors of the spin-density-wave state, as two of us have recently demonstrated explicitly. Accordingly, the renormalized spin-fermion vertex should be much smaller than the bare one due to a “near” Ward identity (our argument here parallels the one recently displayed by Schrieffer [3]). Meanwhile, a simple examination of Eq.(6) in Ref. [3] shows that at large $\mu$, the vertex correction is positive, i.e., $\Delta g/g \rightarrow +1$ rather than tending to $-1$ which is necessary to obtain the physically motivated strong reduction of the vertex.

The second point concerns the computations near the edge of the antiferromagnetic instability, when the correlation length is very large. It follows from Eq.(2) that as $\delta \propto \omega_{sf} \xi \propto \xi^4$ tends to zero, $\Delta g$ diverges logarithmically. In this situation, higher-order corrections are indeed relevant. It has been shown in [2,4] that the logarithms sum up to a power law, and the full vertex takes the form

$$g_{\text{tot}} = g \left( \frac{\xi}{\alpha} \right)^\beta$$

(5)

where

$$\beta = \frac{\sin \phi_0}{2\pi^2} \text{Re} \int_0^\pi d\phi \frac{\log[\sin(\phi/2)]}{\cos \phi + \cos \phi_0}$$

(6)

As we already discussed, however, $\beta$ is negative and numerically quite small. We thank D. Pines and P.C.E. Stamp for useful conversations. The work by A. Ch. and D. M. has been supported by the NSF DMR 9629839. A. Ch. is an A.P. Sloan Fellow. P.M. acknowledges support from the National High Magnetic Field Laboratory and the State of Florida.
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