Generation of Entangled N-Photon States in a Two-Mode Jaynes–Cummings Model

C. Wildfeuer and D. H. Schiller
Fachbereich Physik, Universität Siegen, D-57068 Siegen, Germany

We describe a mathematical solution for the generation of entangled N-photon states in two field modes. A simple and compact solution is presented for a two-mode Jaynes–Cummings model by combining the two field modes in a way that only one of the two resulting quasi-modes enters in the interaction term. The formalism developed is then applied to calculate various generation probabilities analytically. We show that entanglement, starting from an initial field and an atom in one defined state may be obtained in a single step. We also show that entanglement may be built up in the case of an empty cavity and excited atoms whose final states are detected, as well as in the case when the final states of the initially excited atoms are not detected.

PACS numbers: 42.50.Dv, 03.65.Ud, 03.65.Fd
Keywords: Nonclassical states of the electromagnetic field; entanglement generation; two-mode Jaynes–Cummings model; algebraic solution.

I. INTRODUCTION

Entangled states are one of the building blocks in quantum information processing and non-locality tests \cite{1}. They can be used, in the case of the electromagnetic field, to improve the sensitivity of interferometric measurements \cite{2,3,4,5} and may help to overcome the classical Rayleigh diffraction limit in quantum optical lithography \cite{6}. A feasible way to generate such states is given by the atom-field interaction in the framework of one- or two-mode Jaynes–Cummings (JC) models \cite{7,8,9,10,11,12,13,14}. We let two-level atoms interact, one at a time, with two degenerate modes of a lossless cavity. Solving the corresponding JC model algebraically by an SU(2) transformation, we discuss the generation of entangled N-photon states of the general form

\[ |\Psi_N\rangle = \sum_{k=0}^{N} c_k (N-k,k), \]  

which comprises the maximally entangled Bell states

\[ |\Psi_N^+\rangle = \frac{1}{\sqrt{2}} (|N,0\rangle \pm |0,N\rangle). \]  

The field states are defined in terms of the usual two-mode Fock states \(|n_1,n_2\rangle := |n_1\rangle_1 |n_2\rangle_2\), with \(n_1\) (\(n_2\)) photons in mode one (two). The two modes have the same energy and are in resonance with the two-level atom. We solve the model algebraically by combining the two field modes into two quasi-modes of which only one enters in the interaction term, yielding an effective one-mode JC model \cite{14}. Using its known solution and the transformation between mode and quasi-mode Fock states, the generation probabilities of the entangled states are found for three different schemes.

II. ALGEBRAIC SOLUTION OF THE TWO-MODE JAYNES–CUMMINGS MODEL

The JC Hamiltonian for resonant interaction of a two-level atom \(|\langle e\rangle,|g\rangle\rangle\) with two field modes \((a_1,a_2)\) in the dipole and rotating wave approximation is given by \(H = H_0 + H_{\text{int}},\) where

\[ H_0 = \hbar \omega \left( \frac{\sigma_z+1}{2} + (a_1^\dagger a_1 + a_2^\dagger a_2) \mathbb{1} \right), \]  

\[ H_{\text{int}} = \hbar \left( \sigma^+ (g_1 a_1 + g_2 a_2) + \sigma^- (g_1^* a_1^\dagger + g_2^* a_2^\dagger) \right). \]  

Here \(\sigma_z := |e\rangle\langle e| - |g\rangle\langle g|, \sigma^+ := |e\rangle\langle g|, \sigma^- := |g\rangle\langle e|\) and \(\mathbb{1} = |e\rangle\langle e| + |g\rangle\langle g|\) are operators for the two-level atom, \(g_i\) is the coupling constant of the \(i\)th mode with the atom, and \(\hbar \omega\) is the photon energy. We introduce the quasi-mode operators

\[ A_1 = \gamma_1 a_1 + \gamma_2 a_2, \quad A_2 = -\gamma_2^* a_1 + \gamma_1^* a_2, \]  

where \(\gamma_i := g_i/g,\) and \(g := \sqrt{|g_1|^2 + |g_2|^2}.\) Equation \(8\) defines an SU(2) transformation of the mode operators \(a_1, a_2,\) leaving the commutation relations and the number-sum operator \(a_1^\dagger a_1 + a_2^\dagger a_2 = A_1^\dagger A_1 + A_2^\dagger A_2\) invariant. The transformed Hamiltonian then reads

\[ H_0 = \hbar \omega \left( \frac{\sigma_z+1}{2} + (A_1^\dagger A_1 + A_2^\dagger A_2) \mathbb{1} \right), \]  

\[ H_{\text{int}} = \hbar g \left( \sigma^+ A_1 + \sigma^- A_1^\dagger \right), \]

representing a JC Hamiltonian for the quasi-mode \(A_1\) decoupled from the non-interacting quasi-mode \(A_2.\) Since \(H_{\text{int}}\) depends only on quasi-mode one and \([H_0,H_{\text{int}}] = 0,\) the time evolution operator \(U(t) = \exp(-iH_{\text{int}}/\hbar)\) in the interaction picture is the same as for a one-mode JC model. Expanding \(U\) in the atom basis \(|\langle e\rangle,|g\rangle\rangle\}

\[ U = U_{ee} |\langle e\rangle,|e\rangle\rangle + U_{eg} |\langle e\rangle,|g\rangle\rangle + U_{ge} |\langle g\rangle,|e\rangle\rangle + U_{gg} |\langle g\rangle,|g\rangle\rangle, \]
the matrix elements $U_{ab}(t)$ are given by \[15\]

\[
U_{ee} = \cos \left( \tau \sqrt{A_1^2 A_1 + 1} \right), \quad U_{ge} = A_1 \frac{\sin \left( \tau \sqrt{A_1^2 A_1 + 1} \right)}{i \sqrt{A_1^2 A_1 + 1}},
\]

\[
U_{eg} = \sin \left( \tau \sqrt{A_1^2 A_1 + 1} \right), \quad U_{gg} = \cos \left( \tau \sqrt{A_1^2 A_1} \right),
\]

(9)

where $\tau := g t$ is the dimensionless “interaction time”. The model can be solved in the usual way in terms of quasi-mode Fock states defined as the common eigenstates of $A_1^2 A_1$ and $A_2^2 A_2$. The complete solution is then found by giving the relation between the quasi-mode and the mode Fock states.

The quasi-mode operators $A_i, A_i^\dagger, i = 1, 2$, obey the same algebra as the mode operators $a_i, a_i^\dagger$, so that two-quasi-mode Fock states (denoted by a double-ket) can be defined by

\[
|n_1, n_2\rangle := \frac{A_1^{n_1} A_2^{n_2}}{\sqrt{n_1! n_2!}} |0, 0\rangle.
\]

(10)

To find the transformation between the two-mode Fock states $|n_1, n_2\rangle$ and the two-quasi-mode Fock states $|n_1, n_2\rangle$, we use Schwinger’s oscillator model \[16\] and introduce angular momentum states $|j, m\rangle$ and $|j, m\rangle$, where $j = (n_1 + n_2)/2$ and $m = (n_1 - n_2)/2$. In cases where $j$ is not obvious, we shall write a subindex $S$ on the state vectors to indicate the Schwinger angular momentum basis, e.g., $|2, 0\rangle = |1, 1\rangle_S$. Inserting Eq. \[8\] into Eq. \[10\] and identifying the two vacua $|0, 0\rangle$ and $|0, 0\rangle$, we obtain

\[
|j, m\rangle := \left( \gamma_1 a_1^\dagger + \gamma_2 a_2^\dagger \right)^{j+m} \left( -\gamma_2 a_1^\dagger + \gamma_1 a_2^\dagger \right)^{j-m} \frac{a_2^\dagger}{\sqrt{(j+m)! (j-m)!}} |0, 0\rangle.
\]

Expanding the products, rearranging the terms \[17\] and using the definition of the Fock basis $|n_1, n_2\rangle$ in terms of $a_1^\dagger$ and $a_2^\dagger$, we obtain the important relation between the quasi-mode and the mode Fock bases

\[
|j, m\rangle = \sum_{m' = -j}^{j} D_{m'm}^{(j)}(\bar{\phi}, \bar{\theta}, \bar{\chi}) |j, m\rangle,
\]

(11)

\[
|j, m\rangle = \sum_{m' = -j}^{j} D_{mm'}^{(j)}(\bar{\phi}, \bar{\theta}, \bar{\chi}) |j, m\rangle.
\]

(12)

Here $D_{m'm}^{(j)}(\bar{\phi}, \bar{\theta}, \bar{\chi}) = \exp[-i (m' \bar{\phi} + m \bar{\chi})] d_{m'm}^{(j)}(\bar{\theta})$ are the Wigner $D$-matrix elements of the SU(2) group \[16\, 17\], with arguments determined by $\bar{\phi} = \phi_1 - \phi_2$, $\bar{\chi} = \phi_1 + \phi_2$, $\cos(\bar{\theta}/2) := |\gamma_1|$, $\sin(\bar{\theta}/2) := |\gamma_2|$, and $\gamma_i = |\gamma_i| \exp(i \phi_i)$. It follows that the mode and quasi-mode Fock states belonging to the same total number of photons, $n_1 + n_2 = 2j$, are related by an irreducible rotation matrix of weight $j$ and with Euler angles determined solely by the interaction constants.

The action of $U_{ab}$ on the field states is easily calculated in the quasi-mode Fock basis

\[
U_{ee} (\tau) \langle j, m \rangle = \cos \left( \tau \sqrt{j + m + 1} \right) \langle j, m \rangle,
\]

\[
U_{ge} (\tau) \langle j, m \rangle = -i \sin \left( \tau \sqrt{j + m + 1} \right) \langle j + 1/2, m + 1/2 \rangle,
\]

\[
U_{eg} (\tau) \langle j, m \rangle = -i \sin \left( \tau \sqrt{j + m} \right) \langle j - 1/2, m - 1/2 \rangle,
\]

\[
U_{gg} (\tau) \langle j, m \rangle = \cos \left( \tau \sqrt{j + m} \right) \langle j, m \rangle,
\]

(13)

showing that $U_{ee}$ and $U_{gg}$ do not change the number of quasi-photons, whereas $U_{ge}$ (or $U_{eg}$) act as creation (annihilation) operators of quasi-mode one. Using Eq. \[11\] and Eq. \[12\], we find the action on the usual Fock states

\[
U_{ee} (\tau) \langle j, m \rangle = \sum_{m' = -j}^{j} C_{mm'}^{(j)} (\tau) \langle j, m' \rangle,
\]

\[
U_{ge} (\tau) \langle j, m \rangle = \sum_{m' = -j}^{j} S_{mm'}^{(j)} (\tau) \langle j + 1/2, m + 1/2 \rangle,
\]

\[
U_{eg} (\tau) \langle j, m \rangle = \sum_{m' = -j}^{j} \bar{S}_{mm'}^{(j)} (\tau) \langle j - 1/2, m - 1/2 \rangle,
\]

\[
U_{gg} (\tau) \langle j, m \rangle = \sum_{m' = -j}^{j} \bar{C}_{mm'}^{(j)} (\tau) \langle j, m' \rangle,
\]

(14)

where we have introduced the following coefficients

\[
C_{mm'}^{(j)} (\tau) = \sum_{m'' = -j}^{j} \cos \left( \tau \sqrt{j + v + 1} \right) D_{mm''}^{(j)} P_{mm''}^{(j)},
\]

\[
S_{mm'}^{(j)} (\tau) = -i \sum_{m'' = -j}^{j} \sin \left( \tau \sqrt{j + v + 1} \right) D_{mm''}^{(j+1/2)} D_{mm''}^{(j)},
\]

\[
\bar{S}_{mm'}^{(j)} (\tau) = -i \sum_{m'' = -j}^{j} \sin \left( \tau \sqrt{j + v} \right) D_{mm''}^{(j-1/2)} D_{mm''}^{(j)},
\]

\[
\bar{C}_{mm'}^{(j)} (\tau) = \sum_{m'' = -j}^{j} \cos \left( \tau \sqrt{j + v} \right) D_{mm''}^{(j)} D_{mm''}^{(j)},
\]

(15)

Given the above equations, we now have all the ingredients to calculate the time evolution of the density operator according to $\rho(t) = U(t) \rho(0) U^\dagger (t)$.

III. GENERATION OF ENTANGLEMENT IN ONE STEP

We start with the calculation of the probability to find at time $t$ the field state $|\Psi(t)\rangle$ in Eq. \[1\], assuming an initial field state $|\xi\rangle$ and an atom entering the cavity in either the excited or ground state. The analytical calculation is straightforward. The initial field state is expanded according to

\[
|\xi\rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} b_{n_1 n_2} |n_1, n_2\rangle = \sum_{j=0}^{\infty} \sum_{m=-j}^{j} b_{j m} |j, m\rangle,
\]

(16)
where the primed summation symbol indicates a sum over integer and half integer values of $j$. The expansion coefficients with respect to the Fock and Schwinger basis are related by $b_{j+m,j-m} = \tilde{b}_{j,m}$. The state $|\Psi_N \rangle$ is given in the Schwinger basis by

$$|\Psi_N \rangle = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} e^{\frac{i}{2} m^2} |\frac{N}{2}, m \rangle_S,$$

with density operator $\rho_{\Psi_N} = |\Psi_N \rangle \langle \Psi_N |$. From the time-evolved initial states

$$U |e; \xi \rangle = U_{ee} |e; \xi \rangle + U_{eg} |g; \xi \rangle,$$

$$U |g; \xi \rangle = U_{ge} |e; \xi \rangle + U_{gg} |g; \xi \rangle,$$

we obtain the reduced density operator of the field by tracing out the atomic degrees of freedom: $\rho_{\Psi_N}^{(a)}(t) = tr_A (U(t)|\psi \rangle \langle \psi| U^\dagger(t))$, $a = e$ or $g$. The probability to find $|\Psi_N \rangle$ at time $t$ follows from $|\rho_{\Psi_N}^{(a)}(t)| = tr (\rho_{\Psi_N}^{(a)}(t)|\rho_{\Psi_N})$ and is given by

$$\langle \rho_{\Psi_N}^{(e)} \rangle = \left| \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} \sum_{m'=\frac{-N}{2}}^{\frac{N}{2}} \tilde{b}_{e,m}^* \tilde{b}_{e,m'} e^{i \frac{m^2}{2}} \right|^2$$

$$+ \left| \sum_{m=-\frac{N-1}{2}}^{\frac{N-1}{2}} \sum_{m'=\frac{-N+1}{2}}^{\frac{N+1}{2}} \tilde{b}_{e,m}^* \tilde{b}_{e,m'} e^{i \frac{m^2}{2}} \right|^2 \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} \sum_{m'=\frac{-N}{2}}^{\frac{N}{2}} \tilde{b}_{g,m}^* \tilde{b}_{g,m'} e^{i \frac{m^2}{2}} S_{mm'}^{N-e} \left( S_{mm'}^{N-e} \right)^2$$

for the initial atom-field state $|e; \xi \rangle$, and by

$$\langle \rho_{\Psi_N}^{(g)} \rangle = \left| \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} \sum_{m'=\frac{-N}{2}}^{\frac{N}{2}} \tilde{b}_{g,m}^* \tilde{b}_{g,m'} e^{i \frac{m^2}{2}} \right|^2$$

$$+ \left| \sum_{m=-\frac{N+1}{2}}^{\frac{N+1}{2}} \sum_{m'=\frac{-N+1}{2}}^{\frac{N+1}{2}} \tilde{b}_{g,m}^* \tilde{b}_{g,m'} e^{i \frac{m^2}{2}} \right|^2 \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} \sum_{m'=\frac{-N}{2}}^{\frac{N}{2}} \tilde{b}_{e,m}^* \tilde{b}_{e,m'} e^{i \frac{m^2}{2}} S_{mm'}^{N-g} \left( S_{mm'}^{N-g} \right)^2$$

for the initial state $|g; \xi \rangle$. It follows that in order to obtain non-vanishing probabilities at time $\tau$, the initial field state must contain at least one of the Fock states from the set

$$\{|N, 0 \rangle, |N - 1, 1 \rangle, \ldots, |0, N \rangle\} \cup \{|N - 1, 0 \rangle, |N - 2, 1 \rangle, \ldots, |0, N - 1 \rangle\},$$

if the atom is initially in the excited state, or from the set

$$\{|N, 0 \rangle, |N - 1, 1 \rangle, \ldots, |0, N \rangle\} \cup \{|N + 1, 0 \rangle, |N + 1, 1 \rangle, \ldots, |0, N + 1 \rangle\},$$

if it is in the ground state.

For $N = 1$ the set of contributing atom-field states according to Eq. (20) is

$$\{|e; 1, 0 \rangle, |e; 0, 1 \rangle\} \cup \{|e; 0, 0 \rangle\}$$

and according to Eq. (21)

$$\{|g; 1, 0 \rangle, |g; 0, 1 \rangle\} \cup \{|g; 2, 0 \rangle, |g; 1, 1 \rangle, |g; 0, 2 \rangle\}.$$
Detecting the atom in the ground state leaves the field in the state \( |\chi_1 \rangle = K_1 ( -i |\sin (\tau_1 ) \rangle |\frac{1}{2}, \frac{1}{2} \rangle )_S \), where \( K_1 = |\sin (\tau_1 ) |^{-1} \exp (i\alpha_1 ) \) is a normalization constant. By choosing the phase \( \alpha_1 \) appropriately, the factor entering the normalized state may be set equal to one, yielding the state \(|\frac{1}{2}, \frac{1}{2} \rangle \rangle_S \). Proceeding this way the field state obtained after \( N \) conditional steps is simply given by

\[
|\chi_N \rangle = \left| \frac{1}{2}, \frac{1}{2} \right\rangle \rangle_S = \sum_{k=0}^{N} D_{N-k, k}^{(2, 2)} \left( \Phi, \theta, \chi \right) |N-k, k \rangle , \tag {26}
\]

where we have used Eq. (11) and Fock-state notation on the r.h.s. This is precisely a state of the form given in Eq. (11) with coefficients determined by the Wigner rotation matrix elements. Since these elements depend solely on the coupling constants, the generated entangled state is sensitive to their magnitudes and phases. The state in Eq. (26) corresponds to the quasi-mode Fock state \( |N, 0 \rangle \), implying that each conditional step generates one photon in quasi-mode one. The generation probabilities of the states \( |\Psi_N \rangle \) and \( |\Psi_N^\perp \rangle \) after \( N \) conditional steps are given by

\[
|\langle \Psi_N | \chi_N \rangle |^2 = \sum_{m=-N/2}^{N/2} \left| \frac{\varepsilon_m^N}{2} D_{m, 0}^{(2, 2)} \right|^2 , \tag {27}
\]

\[
|\langle \Psi_N^\perp | \chi_N \rangle |^2 = \left| \frac{1}{2} \right|^2 \left| D_{N/2, 0}^{(2, 2)} \right|^2 \tag {28}
\]

We shall show that the probability to detect the atoms \( N \) times consecutively in the ground state is a rapidly decaying function of \( N \). But, as discussed below, it is not essential to rely on this assumption. Actually, it is sufficient to detect them in a sequence of \( n (\geq N) \) steps \( N \) times in the ground state.

V. NON-CONDITIONAL GENERATION

In the following we consider a non-conditional scheme. We start with an empty cavity and send a sequence of excited atoms through it without detecting their final states. The reduced density operator of the field after the passage of the first atom (interaction time \( \tau_1 \)) is given by

\[
\rho_F^{(1)} (\tau_1 ) = \cos^2 (\tau_1 ) |0, 0 \rangle \langle 0, 0 | + \sin^2 (\tau_1 ) \left| |\frac{1}{2}, \frac{1}{2} \rangle \rangle_S \langle \frac{1}{2}, \frac{1}{2} | \right|
\]

and serves as the “initial” field configuration for the second excited atom. Proceeding this way, the reduced density operator of the field after \( n \) steps turns out to be of the form

\[
\rho_F^{(n)} (\{\tau_n \}) = \sum_{j=0}^{n-1} \rho_F^{(j)} (\{\tau_n \}) |j, j \rangle \langle j, j | , \tag {29}
\]

where the coefficients \( \rho_F^{(j)} (\{\tau_n \}) \) are given recursively by

\[
\rho_0^{(n)} = \cos^2 (\tau_n) \rho_0^{(n-1)} ,
\]

\[
\rho_j^{(n)} = \cos^2 (\tau_n \sqrt{2j+1}) \rho_j^{(n-1)} + \sin^2 (\tau_n \sqrt{2j}) \rho_{j+1}^{(n-1)} ,
\]

\[
\rho_{n/2}^{(n)} = \sin^2 (\tau_n \sqrt{n}) \rho_{n/2}^{(n-1)} , \tag {30}
\]

for \( 1/2 \leq j \leq (n-1)/2 \) and \( \rho_0^{(0)} = 1 \). The argument \( \{\tau_n \} \) stands for all interaction times \( \{\tau_1, \ldots, \tau_n \} \) of the \( n \) steps. Equation (29), which is obviously true for \( n = 1 \) and \( n = 2 \) (see Eq. (26)), can be proven by induction.

The coefficients \( \rho_j^{(n)} \) in Eq. (29) are the probabilities to find the field after \( n \) non-conditional steps in the quasi-mode state \(|j, j \rangle \rangle_S \). In particular \( \rho_0^{(n)} = \cos^2 (\tau_n) \cos^2 (\tau_2 \sqrt{2}) \ldots \cos^2 (\tau_n \sqrt{n}) \) and \( \rho_{n/2}^{(n)} = \sin^2 (\tau_n) \sin^2 (\tau_2 \sqrt{2}) \ldots \sin^2 (\tau_n \sqrt{n}) \) correspond to the cases where in \( n \) steps the initially excited atoms emerge \( n \) times in the excited and ground state, respectively. The intermediate \( \rho_j^{(n)} \)'s correspond to the cases where the \( n \) atoms emerge \( 2j \) times in the ground state and \( n-2j \) times in the excited state, irrespective of the order of appearance. The coefficient \( \rho_j^{(n)} \) consists of a sum of \( \binom{n}{j} \) terms, each of which corresponds to a particular sequence of \(|g \rangle \)'s and \(|e \rangle \)'s contributing, respectively, a sine squared and cosine squared factor. There are altogether \( 2^n \) terms in Eq. (29). All this is easily seen by giving \( \rho_F^{(2)} \) as an example:

\[
\rho_F^{(2)} = \cos^2 \tau_1 \cos^2 \tau_2 |0, 0 \rangle \langle 0, 0 | + (\cos^2 \tau_1 \sin^2 \tau_2 + \sin^2 \tau_2 \cos^2 (\tau_2 \sqrt{2}) ) \left| |\frac{1}{2}, \frac{1}{2} \rangle \rangle_S \langle \frac{1}{2}, \frac{1}{2} | \right| + \sin^2 \tau_1 \sin^2 (\tau_2 \sqrt{2}) \left| |1, 1 \rangle \rangle_S \langle 1, 1 | \right| , \tag {31}
\]

Here the four terms correspond to the final state sequences \((e,e), (e,g), (g,e), \) and \((g,g)\).

The states \(|\Psi_N \rangle \) and \(|\Psi_N^\perp \rangle \) are generated in a non-conditional \( n \)-step process with probabilities

\[
\langle \rho_{\Psi_N} \rangle = \rho_F^{(n)} \sum_{m=-N/2}^{N/2} \varepsilon_m^N D_{m, 0}^{(2, 2)} \right|^2 , \tag {32}
\]

\[
\langle \rho_{\Psi_N^\perp} \rangle = \frac{1}{2} \left( \rho_F^{(n)} \right)_{\text{even}} \left| D_{N/2, 0}^{(2, 2)} \right|^2 + \frac{1}{2} \left( \rho_F^{(n)} \right)_{\text{odd}} \left| D_{N/2, 0}^{(2, 2)} \right|^2 , \tag {33}
\]

which are the conditional probabilities found before, multiplied by the probability \( \rho_{N/2}^{(n)} \). Here the interaction times must be chosen such that \( \rho_{n/2}^{(n)} \neq 0 \), which amounts to control the \( n \) parameters \( \{\tau_1, \ldots, \tau_n \} \). The state \(|\Psi_N \rangle \) can be generated in a minimum number of \( n = N \) steps with probability \( \rho_{N/2}^{(N)} \) if, however, is a rapidly decaying function of \( N \).

In the non-conditional scheme all field states \(|j, j \rangle \rangle_S = |j, j \rangle \rangle_S \) for \( j = 0, 1, 2, \ldots, n/2 \) are produced, Eq. (29). On the contrary, in the conditional scheme only the entangled \( N \)-photon state \(|N, 0 \rangle \rangle_S \), Eq. (26), is generated, if in \( n \) steps \( N \) atoms are detected in the ground state. To produce \(|\Psi_N \rangle \) it is, therefore, not crucial that the atoms have been detected \( N \) times consecutively in their ground state. Any sequence of ground and excited states containing \( N \) times the ground state will do it. Finally, we note that there is a particular choice of the interaction time of the \( j \)th atom, given by \( \tau_j = \pi/(2\sqrt{2}) \) for which both, the conditional and non-conditional scheme give (with probability one) the same entangled state \(|N, 0 \rangle \rangle_S \) in Eq. (26).
VI. CONCLUSION

To conclude, we have solved the two-mode JC model algebraically by reducing it to an effective one quasi-mode JC model. The mode and quasi-mode picture are unitarily related by an SU(2) transformation. The solution found is used to discuss three different schemes for the generation of entangled states of the two field modes. To generate entangled \(N\)-photon states in a single step the initial state must contain at least \(N - 1\) photons and an excited atom. Starting from the vacuum we need at least \(n \geq N\) steps to produce pure (mixed) field states in the conditional (non-conditional) scheme presented.

Acknowledgments

C.W. acknowledges helpful discussions with H.D. Dahmen and thanks R.J. Glauber and H. Walther for their interest and encouragement expressed at ICAP 2002.

[1] D. Bouwmeester, A. Ekert, and A. Zeilinger (Eds.), The Physics of Quantum Information (Springer Verlag, Berlin, 2000).
[2] B. Yurke, S.L. McCall, and J.R. Klauder, Phys. Rev. A 33, 4033 (1986).
[3] M. Hillery and L. Mlodinow, Phys. Rev. A 48, 1548 (1993).
[4] C. Brif and A. Mann, Phys. Rev. A 54, 4505 (1996).
[5] J.P. Dowling, Phys. Rev. A 57, 4736 (1998).
[6] A.N. Boto, P. Kok, D.S. Abrams, S.L. Braunstein, C.P. Williams and J.P. Dowling, Phys. Rev. Lett. 85, 2733 (2000).
[7] B. Deb, G. Gangopadhyay, and D.S. Ray, Phys. Rev. A 51, 2651 (1995).
[8] M. Ikram, S.-Y. Zhu, and M.S. Zubairy, Opt. Commun. 184, 417 (2000).
[9] A. Rauschenbeutel, P. Bertet, S. Osnaghi, G. Nogues, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. A 64, 050301 (2001).
[10] E. Solano, G.S. Agarwal, and H. Walther, quant-ph/0202071.
[11] J. Fiurášek, Phys. Rev. A 65, 053818 (2002).
[12] R.G. Unanyan and M. Fleischhauer, Phys. Rev. A 66, 032109 (2002).
[13] K. Vogel, V.M. Akulin, and W.P. Schleich, Phys. Rev. Lett. 71, 1816 (1993).
[14] A. Quattropani, Phys. kondens. Materie 5, 318 (1966); S.M. Dutra and P.L. Knight, Phys. Rev. A 49, 1506 (1994); G. Benivegna and A. Messina, J. Phys. A 27, L453 (1994); J. Mod. Optics 41, 907 (1994).
[15] M.O. Scully and M.S. Zubairy, Quantum Optics (Cambridge University Press, 2001).
[16] J.J. Sakurai, Modern Quantum Mechanics (Addison-Wesley, Reading Massachusetts, 1994).
[17] E.P. Wigner, Group Theory (Academic Press, New York, 1971).