Simulation of two red blood cells (RBC) collision based on granular model: case of one-dimensional collision

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Abstract. Mechanism of red blood cell (RBC) collision is still an interesting subject to study due to its importance in understanding how to count number of RBCs, e.g. in a blood test, for detecting, diagnosing, and monitoring the progression of blood disorders. In this work RBC system is simplified into a two-dimensional system and head-on one-dimensional collision is chosen for testing the system. Deformation on each cells during collision are observed.

1. Introduction
Cytoplasmic viscosity contrast between internal and external cell strongly influence cells diffusivity and trafficking that lead to the appearance of bulk viscosity of blood flow [1]. Fluid with different viscosity will flow with different velocity including the particles in the fluid, which will influence collision of red blood cell (RBC) with other cell or with blood channel wall, e.g. single-particle event is used to calculate number of RBC in a time duration, where collision of RBC with ultramicroelectrode (UME), hinders electrochemical oxidation of Fe(CN)$_6^{4-}$ in aqueous solution [2]. It is also observed that RBCs with higher internal viscosity tended to diffuse to the outer regions closer to the walls of a microfluidic channel compared to the normal ones, which is a segregation phenomenon based on cell internal viscosity [3]. Dynamics of the flow also depends on single cell dynamics, where a RBC in a simple shear flow shows the dependence of the cell dynamics on the spontaneous curvature of the membrane [4], while the curvature cell itself has many solution from then numerical simulation with Canham and Helfrich model [5]. In this work RBC curvature will follow the modified oval of Cassini [6], due to its simplicity and flexibility.

2. Model
A RBC is modeled using granular particles, which are interact with other particle in the same cell only through spring force, while with other particle in different cell through normal force, where the case of one particle in one cell could penetrate a gap between two particles in other cell could happen as in [7], instead of only collision between two particles from two different cells. All cells will have drag force from surrounding fluid and force due to pressure different from inside and outside of cell. Gravitational force and buoyant forces are assumed always having the same value, independent from cell orientation, thats make these two forces not showing in the...
model. To simplify the system we only observed two-dimensional system, where cell biconcave area is perpendicular to gravitational acceleration.

2.1. Cell curvature

Ovals of Cassini is modified by Canham to produce following form [6]

\[ y(x) = B \left[ (C^4 + 4A^2x^2)^{\frac{1}{2}} - A^2 - x^2 \right]^{\frac{1}{2}} \]  

(1)

with \( B \) is dimensionless as the modification parameter, while \( A \) dan \( C \) are in unit length.

\[ \begin{align*}
\text{Figure 1.} & \quad \text{Curvature of a RBC using modified oval of Cassini with } A = 5, \ B = 0.5, \text{ dan } C = 5.05. \\
\text{Using Equation (1) position of granular particles or grains can be obtained}
\end{align*} \]

\[ \vec{r}_i = x_i \hat{x} + y_i \hat{y}, \]  

(2)

where from Figure 2.1 it is found that \(-7.1 \leq x \leq 7.1\). In the range there can be \( N \) points and additionally two points with coordinates (-7.1, 0) and (7.1, 0).

2.2. Grains position

Suppose that a RBC is constructed from \( N \) grains, then from \( x_{\text{min}} \) to \( x_{\text{max}} \) there will be \( N/2 - 1 \) grains for upper curve and also \( N/2 - 1 \) grains for lower one. The last two are for grains at position \((x_{\text{min}}, 0)\) and \((x_{\text{max}}, 0)\). Then it can be defined that

\[ \begin{align*}
x_1 &= x_{\text{min}}, \quad y_1 = 0, \\
x_i &= \left[ x_{\text{min}} + (i - 2)\Delta x \right], \quad y_i = y(x_i), \quad i = 2, \ldots, \frac{1}{2}N, \\
x_{\frac{1}{2}N+1} &= x_{\text{max}}, \quad y_{\frac{1}{2}N+1} = 0, \\
x_i &= \left[ x_{\text{max}} + (i - \frac{1}{2}N - 2)\Delta x \right], \quad y_i = y(x_i), \quad i = \frac{1}{2}N + 2, \ldots, N,
\end{align*} \]  

(3)-(6)

with

\[ \Delta x = \frac{x_{\text{max}} - x_{\text{min}}}{\frac{1}{2}N - 2}. \]  

(7)

Substitution of Equations (3) - (7) into Equation (2) will give position vector of each grain.
2.3. Spring force
Between two grains, after all their positions are calculated using Equations (3) - (7) and (2), a normal distance can be obtained at initial time \( t = 0 \)

\[ l_{i,j} = r_{i,j}(0) = |\vec{r}_{i,j}(0)| = |\vec{r}_i(0) - \vec{r}_j(0)|, \quad (8) \]

where

\[ j = \begin{cases} 
  j + N, & j < 1, \\
  j, & 1 \leq j \leq N, \\
  j - N, & j > N,
\end{cases} \quad (9) \]

has a circular index. Two types of grains normal distance are required

\[ L^1_i = l_{i,i+1} \quad (10) \]

and

\[ L^2_i = l_{i,i+2}. \quad (11) \]

Spring force between two grains \( i \) and \( j \) acting on grain \( i \) is

\[ \vec{S}_{ij} = -k_n (r_{ij} - L^n_i) \hat{r}_{ij}, \quad (12) \]

with \( n = 1, 2 \).

2.4. Pressure force
Area inside a RBC is calculated using

\[ A = \sum_{i=1}^{N} \frac{1}{2} |(\vec{r}_i - \vec{r}_c) \times (\vec{r}_j - \vec{r}_c)| \quad (13) \]

with

\[ \vec{r}_c = \frac{1}{N} \sum_{i=1}^{N} \vec{r}_i. \quad (14) \]

Inner pressure of cell is defined as

\[ p_{\text{in}} = \gamma_P \frac{T}{A} \quad (15) \]

with \( \gamma_P \) is a proportional constant and \( T \) is absolute temperature, and then outer pressure is defined as

\[ p_{\text{out}}(t) = p_{\text{in}}(0), \quad (16) \]

which is assumed to be constant. Pressure force is calculated through

\[ \vec{P}_i = -k_P (p_{\text{in}} - p_{\text{out}}) l_{i-1,i+1} \hat{P}_i, \quad (17) \]

with

\[ \hat{P}_i = \frac{[(\vec{r}_{i+1} - \vec{r}_{i-1}) \times (\vec{r}_c - \vec{r}_i)] \times (\vec{r}_{i+1} - \vec{r}_{i-1})}{|[(\vec{r}_{i+1} - \vec{r}_{i-1}) \times (\vec{r}_c - \vec{r}_i)] \times (\vec{r}_{i+1} - \vec{r}_{i-1})|}. \quad (18) \]
2.5. Viscous force
If there is fluid velocity $\vec{v}_f$ at position of grain $i$, then the grain will have viscous force of

$$\vec{V}_i = -k_V \eta (\vec{v}_i - \vec{v}_f),$$

(19)

with $k_V$ is proportional constant and $\eta$ is fluid viscosity.

2.6. Normal force
Between two grains from different cell there will be normal force

$$\vec{N}_{ij} = -k_N \xi_{ij} \vec{r}_{ij},$$

(20)

which is acted on grain $i$, with

$$\xi_{ij} = \max \left[0, \frac{1}{2}(D_i + D_j) - r_{ij}\right]$$

(21)

is the overlap, where $D_i$ and $D_j$ are diameter of grains $i$ and $j$, respectively.

2.7. Newton’s second law of motion
All forces acted on grain $i$ can be summarized as

$$\vec{F}_i = \sum_{j=i-2}^{i+2} \vec{S}_{ij}(1 - \delta_{ij}) + \vec{P}_i + \vec{V}_i + \sum_{i'=1}^{N'} \vec{N}_{ii'},$$

(22)

where the first term stands for spring force that constructs cell membrane, the second for pressure force due to cell inner and outer pressure difference, the third for viscous force that flow the cell, and the last term for collision between two RBC cells. Using Newton’s second law of motion acceleration of grain $i$ is

$$\ddot{r}_i = \frac{1}{m_i} \vec{F}_i,$$

(23)

with $m_i$ is mass of grain $i$.

2.8. Euler method
From Equation (23) acceleration $\ddot{a}$ at time $t$ can be found. Velocity $\vec{v}_i$ at time $t + \Delta t$ can be found using Euler method

$$\vec{v}_i(t + \Delta t) = \vec{v}_i(t) + \ddot{a}_i \Delta t,$$

(24)

and also for position $\vec{r}_i$ at time $t + \Delta t$

$$\vec{r}_i(t + \Delta t) = \vec{r}_i(t) + \vec{v}_i(t) \Delta t.$$

(25)

2.9. Simulation
Simulation starts by defining iteration parameters such as initial time $t_{beg}$, final time $t_{end}$, time step $\Delta t$, grain mass $m$ and diameter $D$, grains position for the first RBC cell $\{\vec{r}_i|i = 1,..,N\}$ and for the second $\{\vec{r}_i'|i' = 1,..,N'\}$, fluid velocity $\vec{v}_f$, and also the constants, which are $k_1^S$, $k_2^S$, $\gamma_P$, $T$, $k_P$, $k_V$, $\eta$, and $k_N$. Iteration begins by initializing $t = t_{beg}$, increases $t$ by $\Delta t$ while calculating variables of motion, and ends when $t \geq t_{end}$.
Algorithm 1 Calculate total force

Input: \((\vec{r}_i| i \in [1,N])\), \((\vec{r}'_i| i \in [1,N'])\)

Output: \((\vec{F}_i| i \in [1,N])\)

1: function FORCE\((\{\vec{r}\}, \{\vec{r}'\})\)
2: for \(i = 1\) to \(N\) do
3: \(\vec{F}_i \leftarrow 0\) \(\triangleright\) Initial value
4: for \(j = -2\) to 2 do
5: \(\vec{F}_i \leftarrow \vec{F}_i + \vec{S}_{ij}(1 - \delta_{ij})\) \(\triangleright\) Spring force (12)
6: end for
7: \(\vec{F}_i \leftarrow \vec{F}_i + \vec{P}_i\) \(\triangleright\) Pressure force (17)
8: \(\vec{F}_i \leftarrow \vec{F}_i + \vec{V}_i\) \(\triangleright\) Viscous force (19)
9: for \(i' = 1\) to \(N'\) do
10: \(\vec{F}_i \leftarrow \vec{F}_i + \vec{N}_{ii'}\) \(\triangleright\) Normal force (20)
11: end for
12: end for
13: end function

How to calculate total force is given in Algorithm 1, while implementation of Euler method to get motion variables is shown in Algorithm 2.

Algorithm 2 Perform Euler method

Input: \((\vec{F}_i, \vec{v}_i, \vec{r}_i| i \in [1,N])\)

Output: \((\vec{v}_i, \vec{r}_i| i \in [1,N])\)

1: function EULER\((\{\vec{F}\}, \{\vec{v}\}, \{\vec{r}\})\)
2: for \(i = 1\) to \(N\) do
3: \(\vec{a}_i \leftarrow \vec{F}_i/m\) \(\triangleright\) Acceleration (23)
4: \(\vec{v}_i \leftarrow \vec{v}_i + \vec{a}_i \Delta t\) \(\triangleright\) Velocity (24)
5: \(\vec{r}_i \leftarrow \vec{r}_i + \vec{v}_i \Delta t\) \(\triangleright\) Position (25)
6: end for
7: end function

And for the whole program is given in Algorithm 3, where the previous two algorithms, Algorithms 1 and 2, are used.

Algorithm 3 Collision of two RBCs

Input: \((\vec{r}_i(t)| i = 1,..,N), (\vec{r}'_i(t)| i' = 1,..,N')\) at \(t = t_{beg}\)

Output: \((\vec{r}_i(t)| i = 1,..,N), (\vec{r}'_i(t)| i' = 1,..,N')\) at \(t = t_{end}\)

1: \(t \leftarrow t_{beg}\)
2: while \(t \leq t_{end}\) do
3: \(\{\vec{F}\} \leftarrow \text{FORCE}(\{\vec{r}\}, \{\vec{r}'\})\)
4: \(\{\vec{F}'\} \leftarrow \text{FORCE}(\{\vec{r}'\}, \{\vec{r}\})\)
5: \(\{\vec{v}'\}, \{\vec{r}'\} \leftarrow \text{EULER}(\{\vec{F}\}, \{\vec{v}\}, \{\vec{r}\})\)
6: \(\{\vec{v}'\}, \{\vec{r}'\} \leftarrow \text{EULER}(\{\vec{F}'\}, \{\vec{v}'\}, \{\vec{r}'\})\)
7: \(t \leftarrow t + \Delta t\)
8: end while

Notice that Algorithms 1 and 2 return value in the form of array (and also two arrays) of vector, where in the implementation could be slightly different.
3. Results and discussion

Following parameters are used in the simulation: $D = 0.1$, $\rho = 1000$, $N = 100$, $k_S^0 = 500$, $k_1^2 = 100$, $k_2^2 = 100$, $\gamma_P = 1$, $k_P = 1$, $\eta = 1$, $v_f = 0$, $T = 300$, $K_N = 10^4$, $t_{\text{beg}} = 0$, $t_{\text{end}} = 4$, $\Delta t = 10^{-3}$, $T_{\text{data}} = 0.1$.

![Graphs](image)

**Figure 2.** Collision of two RBCs: (a) center of mass position, (b) center of mass velocity, and (c) area inside each cell (right).

From Figure 2(a) collision occurs at simulation time $t = 0.5$, which indicated by turning back of motion in $x - t$ chart. This is also supported Figure 2(b), where at that time velocity of each RBC change sign from previous step. Figure 2(c) shows evolution of area inside RBCs, where higher value indicates cell elastic potentil energy due to its structure and also pressure different with surrounding fluid.

4. Conclusion

Collision between two two-dimensional RBCs based on ovals of Cassini modified by Canham has been simulated, which is limited only to one-dimensional collision. It shows that cell deformation plays important role in transforming initial kinetic energy into elastic energy, which is later dissipated through viscous interaction with surrounding fluid.

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