Wind Turbine Clutter Mitigation for Weather Radar by an Improved Low-Rank Matrix Recovery Method

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Abstract—Matrix completion (MC) theory has attracted much attention for its capability of recovering a low-rank matrix through its partial entries. In this paper, we investigate the novel suppression methods of wind turbine clutter (WTC) and introduce the application of MC in WTC suppression for weather radar. First, the vectors of weather signals contaminated by WTC are sequentially constructed into a low-rank snapshot matrix satisfying random under sampling, and then, the weather data can be accurately recovered by minimizing the nuclear norm in the inexact augmented Lagrangian multiplier (IALM) method. The proposed algorithm can effectively suppress not only the wind turbine clutter but also the noise, greatly improving the signal-to-noise ratio of the echo. An experimental test validates the effectiveness of the proposed MC algorithm, and its performance is superior to the widely-used multiquadric interpolation algorithm with potential engineering applications.

1. INTRODUCTION

The use of wind farms to produce electricity has increased astonishingly in recent years due to the importance of renewable energy sources. However, a typical wind farm has up to several hundred wind turbines with rotating blades reaching heights of up to a hundred feet. The dynamic clutter caused by the rotating blades of the wind turbine has become a main factor that limits the detection performance of weather radar systems [1–3]. Many scholars have delved into the WTC of Doppler characteristics in scanning and spotlight, which can be classified under two different working modes [4, 5], and several mitigation techniques have been developed to help reduce the effect of WTC on weather radars, such as those based on spatial interpolation, including multiquadric interpolation, Lagrange interpolation, and spline interpolation. These methods regard weather data as spatial data and only make use of the spatial continuity of weather data.

However, weather data are typical spatial-temporal data. The direct application of the interpolation algorithm may lead to the loss of valuable information in time dimension and affect the interpolation accuracy, and the interpolation algorithm usually selects the interpolation model and order according to experience. Therefore, its performance will be reduced when the model and order are not selected correctly. However, MC algorithm can avoid this defect. Moreover, due to the limitations of wind farm scale, fan speed, weather radar operating mode, and other practical conditions, none of the conventional clutter mitigation techniques [6–8] can simultaneously suppress WTC and recover weather information without damage. Therefore, considering the influence of radar parameters and other practical conditions, several accurate WTC mitigation schemes have been proposed. One such proposition is to use morphological component analyses (MCA) for signal separation [9], and it is a multi-source blind separation method based on sparse representation and morphological difference of signals. However,
the corresponding computation amount and complexity are also increasing. In addition, compared with MC, MCA is more complex to construct sparse dictionary matrix.

In general, we aim to incorporate the matrix completion theory [10] into WTC suppression. Therefore, this paper studies a new signal processing method of wind turbine clutter adaptive suppression. In recent years, with the deepening of research on compressed sensing theory, matrix completion has been widely used in many scientific and engineering fields, including computer vision, machine learning, radar signal processing, etc. Matrix completion is a procedure for reconstructing an unknown matrix with low-rank or approximately low-rank constraints from a sampling of its entries. According to the matrix completion technology, in our problem, if we can excise all samples contaminated by interferences and only keep the remaining data, then the WTC suppression can be cast into a matrix completion problem: Reconstruct the low-rank matrix from the sampled echo. The advantages of the new sparse recovery scheme are twofold: First, accurate recovery can be achieved with a relatively small error. Second, benefiting from noise suppression, the signal-to-noise ratio is improved. To verify the validity of the proposed approach, an IALM method for recovery of weather signals is obtained and emulated.

In this paper, the sparse optimization theory is first introduced into WTC suppression of a weather radar. According to the size of wind farm, different sparse optimization models are constructed, and a new signal processing method of adaptive wind turbine clutter suppression is studied. The above research content not only is a new engineering application of the compressed sensing sparse recovery theory in the field of weather radar WTC suppression, which solves the shortcomings of existing algorithms in wind turbine clutter suppression, but also improves and optimizes the sparse recovery mathematical model and fast iterative solution. The algorithm is the development and improvement of the compressed sensing sparse recovery theory, which can be popularized in the fields of speech, image, video, and array signal processing.

A description of the weather radar model is given in Section 2. The proposed algorithm is presented in Section 3 and Section 4. Simulation results and performance analyses are discussed in Section 5. Finally, conclusions and future work are given in Section 6.

2. WEATHER RADAR SIGNAL MODEL

In radar systems, suppose that the \( l \)th range bin contains both WTC and weather signals. The received signals in a given azimuth-range cell within the coherent integration time (CIT) can be expressed as

\[
x_l(m) = s_l(m) + c_l(m) + w_l(m) + n_l(m), \quad m = 1, \ldots, M
\]

where \( m \) denotes the slow time index, \( M \) the number of pulses in CIT, and \( s_l(m) \) the weather signal. The side lobe or even the main lobe of the radar beam will touch the ground and be blocked by hills and buildings, so the radar will receive reflection signals from these ground objects forming ground clutter \( c_l(m) \). \( w_l(m) \) is the WTC, and \( n_l(m) \) denotes the noise.

The weather signal is formed by the coherent superposition of all the scattering echoes in the \( l \)th range bin, assuming that the target is moving with a constant radial speed [11], and the weather signal can be modeled by

\[
s_l(m) = \sum_{u=1}^{U} A_u e^{j\omega_{l}(m-1)}, \quad l = 1, 2, \ldots, L, \quad m = 1, 2, \ldots, M
\]

where \( U \) denotes the number of all scattering particles, \( A_u \) the complex amplitude of each scattering particle, \( \omega_{l} = 2\pi f_d/f_r \) the time domain angular frequency, \( f_r \) the pulse repetition frequency, \( f_d \) the Doppler frequency, and \( L \) the total number of range bins.

3. CONSTRUCT A LOW-RANK MATRIX OF RANDOM SAMPLING

Matrix completion requires that the recovered matrix satisfies the strong incoherence property [12]. For any sampled data, when some array elements are unsampled or broken in the whole observing time, it means that there is no available information in the missing data, and the recovered matrix
cannot be effectively complemented. Therefore, the sampling points of array signals should be randomly distributed in the entire echo data during the sampling process.

$$
\begin{bmatrix}
1 & a^l & \ldots & a^{(m-1)l} \\
a^1 & a^{l+1} & \ldots & a^{(m-1)+1} \\
\vdots & \vdots & \ddots & \vdots \\
a^{l-1} & a^{2l-1} & \ldots & a^{M-1}
\end{bmatrix}
$$

In order to solve this problem, this paper proposes a new data recovery method, and we carry out matrix recovery for vector using a snapshot matrix. For an geometric guide vector \([a^0,a^1,\ldots,a^l,a^{l+1},\ldots,a^{M-1}]\), we construct it as a snapshot matrix as shown above. The guiding vector of the array element is originally an \(M \times 1\) dimensional vector. Then, it is rewritten as an equivalent \(l \times m\) dimensional matrix and satisfies the requirement that \(M = l \times m\).

Similarly, we can use the snapshot matrix to construct the echo signal. Firstly, for echo signal \(x_l(m), m = 1, \ldots, M\), according to the micro-Doppler characteristics of radar echoes, we can effectively detect the range bin where WTC is located. The method realizes wind turbine clutter detection by using micro-Doppler characteristics of wind turbine clutter under the condition that wind turbine and radar beams are at different positions, and it matches the estimated value of tip Doppler frequency with the theoretical value of tip Doppler frequency and detects the radar clutter of wind turbine through the matching results of the two [13]. Then, we zero the array elements containing the WTC and construct the signals under each pulse as a \(m_1 \times m_2\) dimensional snapshot matrix \(X\) in turn. In the new matrix, the high-precision completion is realized by the sparseness and strong incoherence property of snapshot matrix [14], where \(m_1 \times m_2 = L\). On the premise of known radial velocity, according to the form of the

$$
\begin{bmatrix}
(x_1(1) & x_2(1) & \ldots & x_{m_1}(1) \\
(x_1(1) & x_2(1) & \ldots & x_{m_2}(1) \\
\ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \ddots \\
\ddots & \ddots & \ddots & \ddots \\
x_{m_1}(1) & x_{m_2}(1) & \ldots & x_L(1)
\end{bmatrix}
$$

snapshot matrix above, extract a column of the weather signal matrix, and construct it as the snapshot matrix \(S \in R^{m_1 \times m_2}\). Then in order to prove the low rank of the constructed matrix, we build a square matrix \(B = S \times S^H\), and the eigendecomposition of matrix \(B\) can be written as

$$
B = V \sum_r V^T = V \begin{pmatrix} \sum_r & 0 \\ 0 & 0 \end{pmatrix} V^T\tag{3}
$$

where \(V\) is the eigenvectors arranged in columns, and \(\sum_r = \text{diag}(\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_r) \in R^{r \times m_1}\), resulting in

$$
\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0\tag{4}
$$

According to the eigendecomposition theory [15], the rank of matrix \(B\) is equal to the number of eigenvalues, i.e., \(\text{rank}(B) = r\). Moreover, due to \(r(SS^H) = r(S)\), if we suppose that \(m_1 = m_2 = 10\), from Fig. 1, it can be concluded that the rank of \(S\) is 1. Generally, the number of pulses \(M\) in CIT is far greater than \(r\); thus, we can always construct an approximate low-rank snapshot matrix \(S\) using \(s_l(m)\) in Equation (2), i.e., \(\text{rank}(S) \ll \min(m_1, m_2)\).

## 4. WEATHER SIGNALS RECOVERY USING MATRIX COMPLETION

In this section, we first define \(X, S, C, W,\) and \(N\) as the snapshot matrices with respect to the column vectors at the \(m\)th pulse of \(x_l(m), s_l(m), c_l(m), w_l(m),\) and \(n_l(m)\). In addition, we suppress the
detected ground clutter signal \[16\] before using MC to recover the weather signal \[17\]. It is known that the snapshot matrix \(S\) to be recovered is of low rank \(r\), and it has been proven that most matrices of rank \(r\) can be reconstructed by solving the convex program:

\[
\min_S \|S\|_* \quad \text{s.t.} \quad X_{ij} = S_{ij}, \ (i, j) \in \Omega \quad (5)
\]

This paper presupposes an input signal as an \(X\) matrix and a low-rank matrix as \(S\). Then, the error matrix as \(W\) is assumed from the discrepancy of \(X\) and \(S\). We can in fact recover \(S\) by solving the following constrained optimization equation \[18\]

\[
\min_S \|S\|_* \quad \text{s.t.} \quad P_\Omega(X) = P_\Omega(S + N) \quad (6)
\]

where \(\Omega\) is the space of observations; \(\|\cdot\|_*\) denotes the nuclear norm of a matrix (the sum of its singular values); \(P_\Omega : \mathbb{R}^{m \times n} \to \mathbb{R}^{m \times n}\) is a linear operator that keeps the entries in \(\Omega\) unchanged and sets those outside \(\Omega\) (i.e., in \(\bar{\Omega}\)) to be zeros, and it can be expressed as

\[
P_\Omega(X) = \begin{cases} X_{ij}, & \text{if} \ (i, j) \in \Omega \\ 0, & \text{otherwise} \end{cases} \quad (7)
\]

As \(N\) will compensate the unknown entries of \(X\), the unknown entries of \(X\) are simply set as zeros. Next, we zero the WTC elements to be suppressed in matrix \(X\). In order to solve the MC problem in Equation (7), we may apply the inexact augmented Lagrange multiplier method (IALM) \[18, 19\], and the corresponding Lagrange function can be expressed as

\[
L(S, W, N, Y, \mu) = \|S\|_* + \Re(Y, X - S - N - W) + \frac{\mu}{2}\|X - S - N - W\|_F^2 \quad (8)
\]

in which \(\Re(\cdot)\) denotes the real part operator, \(\langle X, Y \rangle = tr(X^H Y)\) the matrix inner product, \(tr(\cdot)\) the trace of the matrix, \(\mu\) a penalty factor, and \(Y\) the Lagrange multiplier matrix.

Then we can use IALM approaches for the MC problem, where for updating \(W\) the constraint \(P_\Omega(W) = 0\) should be enforced when minimizing \(L(S, W, N, Y, \mu)\), and the basic IALM iteration steps are given in Table 1.

Then, the weather matrix \(\hat{S}_m\), \(m = 1, \ldots, M\) recovered successively under all pulses, is reconstructed into the corresponding original \(L \times 1\) dimensional vectors. After arranging them, we can obtain an \(M \times L\) dimensional recovery matrix \(\hat{S}_{M \times L}\). Extract the \(l\)th row of the matrix, and the recovered weather signal is obtained: \(\hat{s}_l(m) = [\hat{s}_l(1), \hat{s}_l(2), \ldots, \hat{s}_l(M)]\).
Table 1. Matrix completion via the inexact ALM method.

| Step | Description |
|------|-------------|
| 1    | Initialization: $\mu_0 > 0$, $\rho > 1$, $\eta = 10^{-\gamma}$, $Y_0 = 0$, $W_0 = 0$, $N = 0$; |
| 2    | while $\|S_k - S_{k-1}\|_F/\|S_k\|_F \leq \eta$ |
| 3    | \( (U, \Sigma, V^H) = \text{svd}(X - N_k - W_k + \mu_k^{-1}Y_k) \); |
| 4    | $S = UT_{(\mu_k)^{-1}}[\Sigma]V^H = \begin{cases} U(1 - \frac{1}{\mu_k}\|\Sigma\|_1^{-1})\Sigma V^T, & \text{if } \|\Sigma\|_1 > \frac{1}{\mu_k} \\ 0, & \text{otherwise} \end{cases}$ |
| 5    | $W_k$ \( \hat{N}_{k+1} = P_{\Omega}(X - S_{k+1} - N_k + \mu_k^{-1}Y_k) \); |
| 6    | $N_k$ \( \hat{N}_{k+1} = P_{\Omega}(X - S_{k+1} - W_{k+1} + \mu_k^{-1}Y_k) \); |
| 7    | $Y_k$ \( \hat{N}_{k+1} = Y_k + \mu_k(X - S_{k+1} - N_{k+1} - W_{k+1}) \); |
| 8    | $\mu_k$ \( \mu_{k+1} = \rho\mu_k, k \leftarrow k + 1 \); |
| 9    | otherwise, stop; |
| 10   | Output: $S_k$. |

5. SIMULATION RESULTS AND PERFORMANCE ANALYSES

A computer simulation of the weather signal recovery is built and tested for the performance of the proposed algorithm. Simulation is performed for 64 pulses, assuming the existence of WTC in the 30th range bin; set it to zero and construct it as a snapshot matrix. Given the radar operating frequency $f_c$ of 5.5 GHz and a moderate modulation bandwidth of 6 MHz [20], the pulse repetition frequency PRF is set to 1000 Hz, while the range bin varies from 1 to 100. Firstly, the signal noise ratio (SNR) is set to 30 dB to analyze the time-domain and frequency-domain waveforms of the weather signal under the constant SNR. Then, in order to analyze the signal recovery capability under different SNRs, the SNR is set to 0 to 30 dB. Taking into account the accuracy of the test, the results below are the average of 100 independent Monte Carlo experiments.

The amplitude shown in Fig. 2 is plotted for different circumstances: the weather signal, weather signal plus noise, and weather signal recovered by MC under noise and WTC interference. When modeling the weather signal, it is simplified and assumed that its average radial velocity under each pulse is a fixed value, so the amplitude of weather signal is a constant. From Fig. 2, we can see the fluctuation of the amplitude in different situations; in order to describe this aspect quantitatively, the variance defining the dispersion degree of the signal amplitude is given by

$$Var = \frac{1}{M} \sum_{i=1}^{M} (X_i - \hat{X}_i)^2$$

The variance of the weather signal amplitude under noise interference can be calculated as $Var 1 = 6.23e - 004$, while the variance of the weather signal amplitude recovered by MC is $Var 2 = 1.86e - 004$. It is obvious that the amplitude of weather signals not contaminated by WTC fluctuates markedly under the noise disturbance, and we can see that MC greatly reduces the deviation degree of the amplitude in the presence of noise and WTC from the real value.

Figure 3 shows the Doppler spectrum of the weather signal with or without noise. In addition, we can see the Doppler spectrum of the weather signal recovered by MC, which is distributed within a certain spectrum range. When the Doppler frequency is between 300 and 400 Hz, the noise interference is not obvious, while in the sidelobe region, the amplitude of the weather signals is much higher than the truth value due to noise interference. However, after matrix completion, the peak sidelobe is reduced.
Therefore, the proposed MC algorithm can effectively suppress not only the WTC but also the noise.

Figure 4 shows the mean value of the radial velocity estimation of weather signals recovered by the MC algorithm. In the iterative process of matrix completion, the matrix to be completed only contains weather signals and part of the noise signals, and the influence of noise on the estimation accuracy of weather signals speed depends on the intensity of noise signal. When the SNR is low, the noise intensity is high, and the radial velocity estimation fluctuates greatly at low SNR and has a large deviation from the truth value. However, with the increase of SNR, the error of radial velocity estimation decreases gradually and finally converges to the truth value.

To quantitatively examine the performance of the matrix completion based on the IALM algorithm, the following root mean square error (RMSE) is defined as the performance index:

\[
\text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (S_i - \hat{S}_i)^2} / M
\]  

(10)

Figure 5 shows the curve of signal recovery error under different SNRs using the same mitigation
algorithm and analyzes the influence of the SNR of input signals on the performance of the algorithm, in which the SNR varies from 0 dB to 30 dB. It can be seen from the figure that the RMSE of matrix completion for missing data decreases with increasing SNR. Since this algorithm needs to carry out singular value decomposition for the correlation matrix, different noise factors affect the results of singular value decomposition. Therefore, as the SNR of the input signals increases, the influence of singular value decomposition and the error of matrix completion are both reduced.

Figure 6 analyzes the recovery performance of the proposed algorithm and the multi-quadratic interpolation [21] for weather signals. In order to compare the results under the same conditions, the same noise interference with SNR = 30 is adopted in this experiment. As shown in Fig. 6, the above two methods can effectively recover data. However, according to the further analysis of the denoising results in Fig. 6, it can be seen that the matrix completion has a higher output signal-to-noise ratio, and the matrix completion can effectively suppress the noise while restoring the weather signal and improve the recovery accuracy.

Figure 5. The curve of signal recovery error under different SNR.

Figure 6. The Doppler spectrum under the two algorithms.

Figure 7. The curve of signal recovery error of the two algorithms under different SNR.
Figure 7 shows the curve of signal recovery error under different SNRs using two different mitigation algorithms. It is obvious that the RMSE of the multi-quadratic interpolation also decreases with increasing SNR. However, SNR of weather signals recovered by the algorithm in this paper is smaller, and the recovery accuracy is higher, so the algorithm in this paper has a higher precision than the multiquadric interpolation.

In general, compared with the matrix completion, the multi-quadratic interpolation is more difficult to select models, and resolution loss inevitably exists in the weather echo recovered by this algorithm. In addition, the matrix completion restores the weather signal by using the space-time information [22], while the multiquadric interpolation just makes use of the time domain information, which causes the waste of prior knowledge and reduces the recovery ability.

6. CONCLUSIONS AND FUTURE WORK

In this paper, a WTC mitigation method for weather radars is presented using the improved MC recovery method. In the case of low rank, the MC algorithm reshapes the vector in a single snapshot into an equivalent low-rank matrix and effectively recovers the missing data by minimizing the nuclear norm. The proposed design approach is verified well by the simulation data, and its estimation performance is superior to the widely-used multiquadric interpolation algorithm with potential engineering applications. Increasing the efficiency and precision of MC will be further explored in future works by improving the optimization algorithm.

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