We present estimates of the turbulent energy cascade rate, derived from a Hall-MHD third-order law. We compute the contribution from the Hall term and the MHD term to the energy flux. We use MMS data accumulated in the magnetosheath and the solar wind, and compare the results with previously established simulation results. We find that in observation, the MHD contribution is dominant at inertial scales, as in the simulations, but the Hall term becomes significant in observations at larger scales than in the simulations. Possible reasons are offered for this unanticipated result.

Fully developed turbulence is characterized by scale-invariant energy transfer across the inertial range of length scales. In the solar wind, planetary magnetospheres, and other turbulent astrophysical plasmas, large
scale fluctuation energy is transferred across scales and dissipated at kinetic scales. This turbulent energy cascade and dissipation have important effects in space and astrophysical systems, representing an important source for plasma heating [2] and acceleration of energetic particles.

In homogeneous fluid turbulence, the Kolmogorov-Yaglom law [3, 4] quantifies the mean energy dissipation rate in terms of third-order structure functions. This law is extended to the case of plasmas, in the incompressible magnetohydrodynamic (MHD) description, by Politano and Pouquet [5, 6]. This MHD theory accounts for the incompressive channel of the energy cascade. For plasmas with small density fluctuations, such as the cases presented here, incompressive transfer is expected to contribute the bulk of the total energy transfer [7, 9].

In the rapidly streaming solar wind (at mean speed \( \langle V \rangle \)), the Taylor hypothesis [10] \((r = t \langle V \rangle)\) permits conversion of space \((r)\) and time \((t)\) arguments. Then the Politano-Pouquet law prescribes the linear scaling of the mixed, third-order moment

\[
Y(\ell) \equiv \langle \Delta \psi(t) |\Delta \mathbf{v}|^2 + |\Delta \mathbf{b}|^2 \rangle - 2 \langle \Delta \psi(\Delta t) \rangle = \frac{4}{3} \epsilon \ell \tag{1}
\]

where \(\Delta\) indicates an increment, e.g., \(\Delta \psi(t, \Delta t) = \psi(t + \Delta t) - \psi(t)\) for a generic field \(\psi\) and \(\ell = \Delta t \langle V \rangle\). In MHD, we compute increments of the plasma velocity \(\mathbf{v}\) or the magnetic field, \(\mathbf{b} = \mathbf{B}/\sqrt{4\pi \rho}\) (in Alfvén units, mass density \(\rho\)) using a temporal scale \(\Delta t\). The subscript \(t\) indicates longitudinal components, and \(\epsilon\) is the mean energy transfer rate.

Assuming stationary and homogeneous turbulence [11], Eq. (1) enables estimation of the fluid-scale energy transfer rate. The Politano-Pouquet law, in its isotropic form, has been verified in solar wind studies [12–15], and more recently in the terrestrial magnetosheath [8, 16] and magnetospheric boundary layer [17]. The cascade rate measured this way accounts well for observed solar wind heating [18–20]. The presence of a significant mean magnetic field in the solar wind leads to an expectation of spectral anisotropy [21]. However, anisotropic form of the Politano-Pouquet law gives heating rates fairly close to that obtained from the isotropic scaling law [22, 23].

The single-fluid MHD phenomenology is only suitable for the larger scale fluid regime. At smaller scales, near the ion gyro-radius \((\rho_i)\) or ion-inertial length \((d_i)\), the nature of the cascade is expected to change. For example, the magnetic field should remain “frozen in” the electron fluid at velocity \(\mathbf{v}_e\), rather than frozen into the plasma at the \((\sim \text{proton})\) velocity \(\mathbf{v}\). Near the kinetic scales, to first-order approximation, kinetic physics can be partially included via the Hall electric field in the fluid model [21]. Accordingly, employing incompressible Hall MHD, a scaling law analogous to its MHD counterpart, can be derived to obtain the energy cascade flux at the scale of interest [25, 27]. In the Hall MHD formulation, the third-order moment scaling law includes the additional Hall term

\[
H(\ell) \equiv \langle 2\Delta \mathbf{b}(\Delta \mathbf{b} \cdot \Delta \mathbf{j}) - \Delta j_l |\Delta \mathbf{b}|^2 \rangle. \tag{2}
\]

Hellinger et al. [26] derive the Hall contribution to \(Y\) as \(H\), neglecting an additional contribution equal to \(-H/2\) [27], so that complete scaling law reads:

\[
Y + \frac{1}{2} H = -\frac{4}{3} \epsilon \ell. \tag{3}
\]

Here, \(\mathbf{j}\) is the electric current density in Alfvén units: \(\mathbf{j} = \mathbf{v} - \mathbf{v}_e\); where, \(\mathbf{v}\) is the proton velocity and \(\mathbf{v}_e\) is the electron velocity. When the displacement current is neglected, this is equivalent to \(\mathbf{j} = \nabla \times \mathbf{b}\).

For weakly-collisional plasmas, such as the interplanetary medium and the planetary magnetospheres, the Hall-MHD third-order scaling law provides a better estimate of the energy-transfer rate near the kinetic scales, compared to MHD. The linear scaling, Eq. (4), has been recently tested using hybrid-kinetic numerical simulations [26, 27]. Analysis shows that the Hall-MHD-generalized flux becomes dominant at small scales, continuing a cascade further into the sub-proton range. However, the energy-cascade flux decreases near the kinetic scale, even after including the contribution from the Hall-term. This decrease is stronger in high-\(\beta\) plasma where the Hall contribution becomes sub-dominant.

Here we study the Hall-MHD third-order law using data from the Magnetospheric Multiscale (MMS) spacecraft in the interplanetary solar wind and in the terrestrial magnetosheath. We compare the results with the analysis of two-dimensional hybrid-kinetic numerical simulations [26].

MMS [28] provides high-resolution multi-point measurements, offering a unique opportunity to address the cascade problem using in-situ plasma observations. The Fast Plasma Investigation (FPI) [29] instrument measures proton and electron moments every 150ms and 30ms, respectively. The Flux-Gate Magnetometer (FGM) [30] measures the vector magnetic field with 128Hz resolution.

To study the Hall MHD third-order law, we use burst mode data accumulated in two distinct turbulent plasma environments. The first one is an hour-long solar wind (SW) interval on 2017-11-26 from 21:09:03 to 22:09:03 UTC, far from the Earth’s bow shock. We do not find any signature of reflected ions from the bow shock, so the solar wind interval can be considered as “pristine.” Due to limitations of FPI in the solar wind, some systematic uncertainties remain in moments such as temperature. Therefore, we cross-check the average parameters of the selected interval (table I) with Wind Faraday Cups (FC) in the Wind spacecraft’s Solar Wind Experiment (SWE) [31] and Magnetic Field Investigation (MFI) [32] data. The average density, velocity, and magnetic field values are in good agreement. However, significant discrepancy
where rms fluctuation amplitude is defined as $\delta B$. From 00:45:53 to 00:49:43 UTC. Here, the sampled interval in the terrestrial magnetosheath (MSH) on

deoted as $\delta B/\langle B \rangle$, $\delta \rho/\langle \rho \rangle$, $M_i$, $\langle V \rangle$, $V_A$, $d_i$, and $L_{corr}$ for the two

| Interval | $\beta_i$ | $|\langle B \rangle|$ (nT) | $\delta B/|\langle B \rangle|$ | $\delta \rho/\langle \rho \rangle$ | $M_i$ | $\langle V \rangle$ (km s$^{-1}$) | $V_A$ (km s$^{-1}$) | $d_i$ (km) | $L_{corr}$ (km) |
|----------|-----------|--------------------------|--------------------------|--------------------------|-------|--------------------------|--------------------------|-------|--------------------------|
| SW       | 0.4       | 7.4                      | 0.3                      | 0.08                     | 0.33  | 330                      | 51                        | 75    | $11 \times 10^4$        |
| MSH      | 13        | 13.1                     | 1.9                      | 0.11                     | 0.8   | 135                      | 45                        | 17    | 425                      |

$\delta B/\langle B \rangle$, ratio of root-mean-squared (rms) fluctuation to the average magnetic field $\delta B/|\langle B \rangle|$, where rms fluctuation amplitude is defined as $\delta B = \sqrt{\langle (B(t) - |\langle B \rangle|)^2 \rangle}$, ratio of rms fluctuation to the average mass density $\delta \rho/\langle \rho \rangle$, turbulent Mach number $M_i = \delta v/\nu_{th}$, average flow speed $|\langle V \rangle|$, Alfvén speed $V_A$, inertial length $d_i$, and the correlation length $L_{corr}$ for the two selected intervals. The correlation lengths have been estimated in the following way. We calculate the correlation tensors using the Blackman-Tukey method with subtraction of the local mean field and fit an exponential function to the trace of the correlation tensors to estimate, $\tau_{corr}$, the correlation time, defined such that $R(\tau_{corr}) = 1/e$. Finally, the Taylor hypothesis is used to convert the correlation time to correlation length, $L_{corr} = |\langle V \rangle|\tau_{corr}$. From Table I we note that for the chosen intervals, the flow speed is larger than the Alfvén speed, indicating that Taylor hypothesis is expected to be valid. These MMS intervals are typical — other solar wind and magnetosheath samples produce qualitatively similar results.

Figure 1 illustrates the power spectral density (PSD) of the magnetic field fluctuations for the two chosen intervals, plotted against $kd_i$. Here, $k$ is the wavenumber, estimated assuming Taylor hypothesis: $k \simeq 2\pi f/|\langle V \rangle|$. The level of fluctuations is considerably higher in the magnetosheath interval than in the solar wind. Both spectra exhibit Kolmogorov-like $-5/3$ scaling in the inertial range, followed by a steepening near $kd_i = 1$. However, the solar-wind spectrum has a significantly broader bandwidth of inertial range, representative of a larger and higher Reynolds number system, compared to the magnetosheath interval.

Having shown that the two chosen intervals exhibit extended, inertial-range Kolmogorov spectra, we compute the energy flux from Eq. (4). The required electron and proton moments provided by FPI in the solar wind are processed using the method described in [37] to exclude instrumental artifacts in the solar wind. The analyses are averaged over the four MMS spacecraft. FPI particle current densities are not sufficiently accurate for the chosen intervals. Therefore, we use the curlometer-based [36] current for the Hall term.

Figure 6 shows the scaling of the third-order structure functions, decomposed into the MHD ($-Y$) and Hall MHD ($-H/2$) terms from Eq. (4), with spatial lag in units of $d_i$. Only parts of the structure functions that lie well above the instrumental noise level are plotted here. The MHD structure function, $-Y$, is plotted in green line with round symbols. The Hall MHD contribution, $-H/2$, is plotted in red line with square symbols. Additionally, the sum of the two terms, $-(Y + H/2)$, is plotted in black, dashed line. A roughly linear scaling is observed in the inertial range for both samples. The vertical dotted line represents proton-gyroradius $\rho_i$ and the vertical solid line denotes the estimated correlation length $L_{corr}$. In both samples, the MHD component $-Y$ shows better scaling in the inertial range, where it is dominant with respect to the Hall term $-H/2$. The latter has more defined scaling at scales near or below $d_i$, where its contribution to the energy transfer becomes of the same order as for the MHD term. In order to compare the above results with numerical simulations, we use two different runs of the two-dimensional version of

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
Interval & $\beta_i$ & $|\langle B \rangle|$ (nT) & $\delta B/|\langle B \rangle|$ & $\delta \rho/\langle \rho \rangle$ & $M_i$ & $|\langle V \rangle|$ (km s$^{-1}$) & $V_A$ (km s$^{-1}$) & $d_i$ (km) & $L_{corr}$ (km) \\
\hline
SW       & 0.4       & 7.4                      & 0.3                      & 0.08                     & 0.33  & 330                      & 51                        & 75    & $11 \times 10^4$        \\
MSH      & 13        & 13.1                     & 1.9                      & 0.11                     & 0.8   & 135                      & 45                        & 17    & 425                      \\
\hline
\end{tabular}
\caption{Some Plasma Parameters of the Selected Intervals. SW $\equiv$ Solar Wind, MSH $\equiv$ Magnetosheath.}
\end{table}
FIG. 2. MHD (−Y) and Hall (−H/2) structure function from generalized third-order law (Eq. 4) from MMS data. Top: solar wind, β_i = 0.4. Bottom: Magnetosheath, β_i = 13. The proton gyro-radius, ρ_i, is shown as a dotted, vertical line and the correlation length, L_corr, is shown as a solid vertical line. A linear scaling is shown for reference.

FIG. 3. MHD (−Y) and Hall (−H/2) structure function from generalized third-order law (Eq. (4)) from 2D hybrid-kinetic simulations. Top panel: β_i = 0.5. Bottom panel: β_i = 4. The dotted vertical line indicates the ion gyroradius, ρ_i, and the solid vertical line indicates the correlation length L_corr. A linear scaling is shown for reference.

The hybrid code CAMELIA, where ions are described by a particle-in-cell model whereas electrons are a massless, charge-neutral fluid. The two runs have initially isotropic protons and β_i = 0.5 and 4, chosen to probe variations of β comparable to the contrast in the solar wind and magnetosheath plasma properties. The simulation box has the size 256d_i × 256d_i for both runs. An out-of-plane uniform magnetic field is imposed, and the system is perturbed with an isotropic 2-D spectrum of random-phased modes, with relative rms amplitude 0.25, linear Alfvén polarization and vanishing correlation between v and b. For more details, see Ref. 26.

Figure 3 shows the Hall MHD third-order law [4] [27], in Alfvén speed units, in a format similar to figure 6. The thin, green line plots the MHD term, −Y. The Hall contribution, −H/2, is plotted in thick, red line. The transition between MHD and ion (or Hall) scales, occurs roughly at the ion gyroradius ρ_i which is indicated by the dotted vertical line. The correlation length is about 10 d_i for both runs and it is indicated by solid black vertical line. The linear scaling is poorly defined, as typical in hybrid numerical simulations, due to the limited range of computed scales. However, there is reasonable level of qualitative agreement with the observations. In particular, in the low-β simulation the Hall term, −H/2, becomes relevant closer to the transition scale more prominently than in the high-β case. Conversely the dominance of the MHD contribution is established more dramatically at larger scales in the lower β solar wind and lower beta simulation. Note that these include a corrected Hall-term contribution relative to earlier results 26.

While the observational results behave qualitatively similar to the simulations near the kinetic scales and at larger scales, the comparison in the sub-proton range of scales is less clear. Unlike in the numerical simulations, in the observations, the Hall-contributed cascade does not dominate over the MHD contribution, rather the two contributions become of similar order. Still, in all cases we can confirm that the Hall physics becomes important for the energy transfer at subproton scales. However, both simulation and in-situ observations have implicit limita-
tions that may provide possible explanation for the apparent (if somewhat subtle) differences.

The magnetosheath is a smaller system, and exhibits a significantly narrower range between kinetic scale (either \( \rho_i \) or \( d_i \)) and the correlation scale. It is possible that the small separation of scales does not allow the two contributions to the energy flux to be sufficiently distinct \[39\]. Further, there are enhanced current sheets and reconnection, transient structures advecting from the bow shock into the magnetosheath, making it a more complicated system than the pristine solar wind. Deviation from strict homogeneity and incompressibility may play also a role. A notable feature is that, in both magnetosheath and solar wind cases, the Hall and standard-MHD cascade contributions (Fig. 6) become comparable at a few \( d_i \), at nearly the scales where the corresponding kinetic range modifications to the spectra begin to be seen (Fig. 1). In the simulations, the more dramatic crossover of Hall and MHD effects occurs at moderately smaller sub-\( d_i \) scales.

At the same time, the hybrid-kinetic simulations are two-dimensional and admit rather low Reynolds number values; both of which may potentially alter the nature of energy cascade. Additionally, the hybrid simulations ignore the kinetic effects of electrons. With the current computational ability, three-dimensional hybrid simulations would be severely limited in Reynolds number, even more so in full kinetic simulations. So, a direct comparison with in-situ observations are not feasible at this point.

Finally, Table II reports the approximate values of the inertial-range energy-transfer rate obtained for the two chosen intervals from the Hall MHD scaling law (Eq. (4)). The second column denotes the total energy transfer rate from the Hall MHD law: \( \epsilon_{\text{inertial}} = -3(9Y + H/2)/4\ell \). The magnetosheath-energy decay rate is about three orders of magnitude larger than the interplanetary solar wind \[8, 16, 17\]. The final column is a rough estimate of the global energy decay rate, at the energy-containing scale, obtained from a von Kármán-Taylor \[3, 10\] phenomenology (see \[16\] and the supplementary material). The von Kármán estimates are close to the inertial-scale ones from the third-order law. In evaluating these comparisons, it is important to recall that the third-order laws generally ignore all dissipation processes, and that the von Kármán phenomenology contains a proportionality constant which is subject to small variations based on the plasma parameters \[41, 42\]. So, these estimates are approximate, at best.

Understanding how collisionless plasmas dissipate remains a topic of central importance in space physics, astrophysics, and laboratory plasma. In the recent years, it has become increasingly recognized that MHD description must be refined to clearly make connection with kinetic plasma dissipation. The present results provide a step towards understanding this problem. Based on the unprecedented capabilities of the MMS mission instrumentation, the findings of this paper confirms the applicability of the Hall-modified third order order laws, as similar, but not identical behavior is seen in the transition to kinetic effects near proton scales, in the observations, say 1 to 10 \( d_i \) (or \( \rho_i \)), and in the simulations, at scales just smaller than \( d_i \) or \( \rho_i \). Clarification of these subtle differences awaits investigations with more advanced simulations and observational data, when available in the future.

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### SUPPLEMENTAL MATERIAL

In this supplemental material, we present additional material which supplement the letter.

Figure 4 shows the time series of the FGM-measured magnetic field, the FPI/DIS measured ion velocity, and the FPI/DES measured electron density for the selected solar wind interval.

We plot the the magnetic field, the ion velocity, and the electron density for the chosen magnetosheath sample in figure 5.

We recall that the generalized Hall-MHD third-order law in three-dimension is given by \[25, 27\]

\[
Y + \frac{1}{2} H = -\frac{4}{3} \ell, \tag{4}
\]

and in two dimensional system, like the simulations presented here, the 3 in the denominator is replace by 2.

For a more quantitative comparison between the simulation and MMS observation results, some type of normalization to the energy cascade rate, extracted from the Hall-MHD generalized third-order law, is in order. A natural choice of normalization is to compensate the
FIG. 4. Time series plot of magnetic field (top panel), ion velocity (middle panel), and electron density (bottom panel) in GSE coordinate for the solar wind interval.

FIG. 5. Time series plot of magnetic field (top panel), ion velocity (middle panel), and electron density (bottom panel) in GSE coordinate for the magnetosheath interval.
inertial-range energy flux with some form of the global decay rate. If the inertial-range transfer rate, derived from equation 4, yields the average energy loss rate in the system, the normalized value is expected to be close to unity.

In hybrid-kinetic simulations, the energy decay rate can be calculated exactly, from the resistive heating:

$$\epsilon = -\frac{\partial E}{\partial t} = \nu (\nabla \cdot \nabla) + \eta (\nabla b \cdot \nabla b), \quad (5)$$

where, $\nu$ is the viscosity and $\eta$ is the resistivity. The same cannot be done for the MMS observations, since the viscosity and resistivity in weakly-collisional plasma are not defined. However, a straightforward application of a von Kármán decay phenomenology, generalized to the viscosity and resistivity in weakly-collisional plasma where, $\nu$ and $\eta$, gives a reasonable estimate of the global energy transfer in these length scales.

Although the horizontal scaling is poorly defined here, implying a scale-invariant energy transfer in these length scales.

The right two panels in figure 6 plot the results obtained from the simulation data [26], but now normalized to the resistive heating rate, obtained from equation 5. Although the horizontal scaling is poorly defined here, compared to the observations, the values of the normalized energy flux are close to the MMS results.
FIG. 6. Energy cascade rate derived from the generalized Hall-MHD third-order law, normalized to global decay rate.

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