Constraining exotic compact stars composed of bosonic and fermionic dark matter with Gravitational Wave events

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ABSTRACT

We investigate neutron star-black hole (NS-BH) merger candidates as a test for compact exotic objects. Using the events GW190814, GW200105 and GW200115 measured by the LIGO-Virgo collaboration, which represent a broad profile of the masses in the NS mass spectrum, we demonstrate the constraining power for the parameter spaces of compact stars consisting of dark matter for future measurements. We consider three possible cases of dark matter stars: self-interacting, purely bosonic or fermionic dark matter stars, stars consisting of a mixture of interacting bosonic and fermionic matter, as well as the limiting case of self-bound stars. We find that the scale of those hypothetical objects are dominated by the one of the strong interaction. The presence of fermionic dark matter requires a dark matter particle of the GeV mass scale, while the bosonic dark matter particle mass can be arbitrarily large or small. In the limiting case of a self-bound constant speed of sound parametrization, we find that the vacuum energy of those configurations has to be similar to the one of QCD.

Key words: gravitational waves – stars: neutron – (cosmology:) dark matter – equation of state – (transients:) black hole - neutron star mergers

1 INTRODUCTION

Ever since the first direct observation by the LIGO and Virgo collaborations, gravitational waves (GWs) have opened a new opportunity to further research of astronomical objects and the universe. While the first observation GW150914 (Abbott et al. 2016a) not only provided direct proof of the last remaining prediction of Albert Einstein’s theory of general relativity, the event also offered deep insight into compact objects and their merging process. For GW150914, the generally accepted explanation of the nature of the compact objects were black holes, as the calculation of the masses, final orbital separation and velocities confidently ruled out the possibility of a merger of neutron stars (Abbott et al. 2016a). However, more exotic, yet so far undetected and thus unverified compact objects, for example boson stars, could not be excluded by the detection (Abbott et al. 2016b), especially since no electromagnetic counterpart was found to coincide with the merger. This was in stark contrast to the event GW170817 (Abbott et al. 2017a), which was accompanied by a series of multi-messenger events spanning the electromagnetic spectrum and also non-electromagnetic messengers, such as high-energy cosmic rays (Abbott et al. 2017b). From this it was concluded that the most likely explanation for the merging compact objects was a system of binary neutron stars, which merged into a hypermassive neutron star remnant that collapsed within ≈ 300 milliseconds (Margil & Metzger 2017) into a black hole. This event has provided deep insight into the inner structure of neutron stars and the equation of state (EOS) describing them. A second event also theorized to be a NS-NS merger, GW190425, was observed in 2019, though it could not be ruled out that one or both of the binary components were black holes (Abbott et al. 2020). Another event, similarly exciting and groundbreaking as the former observations, was the GW190814 event where the LIGO-VIRGO collaboration (Abbott et al. 2020) reported a compact binary coalescence involving a 22.2 ± 2.43 M⊙ black hole and a compact object with a mass of 2.50 ± 2.67 M⊙ at 90% confidence level. This places this compact object in the mass gap between the currently known lightest black holes and the heaviest neutron stars with a maximum mass of about 2 M⊙ (Cromartie et al. 2019; Fonseca et al. 2016; Antoniadis et al. 2013; Demorest et al. 2010; Nieder et al. 2020; Romani et al. 2021, 2022). The source was localized at a distance of 241±41 Mpc, which is considered as one possibility to explain the absence of an electromagnetic counterpart for the spin parameters of the this event, though also the mass ratio of the binary components and the spin parameter of the resulting black hole have a strong effect on a dynamical ejecta.

There has been much debate about the nature of this compact object, and some have argued that the secondary was most likely a black hole (Tsokanos et al. 2020; Godzieba et al. 2021; Tan et al. 2020). For example, (Fattoyev et al. 2020) have shown that a stiff equation of state supporting such a super-massive neutron star is inconsistent with constraints from heavy-ion collisions or from the low deformability of medium-mass stars. Sedrakian et al. (2020) examined static and fast rotating configurations of hypernuclear stars, yet found that the resulting maximal masses are well below the lower bound of the...
In this scenario, we assume that the pressure and energy density are proportional to the coupling strength \( y \) and square of the number density \( n \), i.e. \( \varepsilon \propto y^2 \cdot n^2 \), and analytically determine the limiting cases. In this case, dark matter annihilation is prevented by two conserved quantum numbers, a similar mechanism as in the Dark White Dwarf stars proposed by (Ryan & Radice 2022). In the second scenario, we assume the stars to be a mix of bosonic and fermionic dark matter, coupled by a point-like interaction. Similar to the first scenario, we recover an energy density and pressure proportional to the number density squared. After solving the Tolman-Oppenheimer-Volkoff (TOV) equation numerically, we determine bounds on the masses of the dark matter particles and coupling strength. We assume this star to be uncharged with respect to a dark charge so that the number densities of bosons and fermions are balancing each other. In the third scenario, we consider a self-bound linear equation of state, and constrain the respective parameter spaces for the necessary vacuum energy.

2 SELF-INTERACTING BOSON/FERMION STARS

It is known that an object exceeding the Roche-limit and breaking apart due to tidal disruption leaves a clearly distinct signal in the wave event, where the amplitude is affected by the number of fragments as \( 1/N \) (Maggiore 2018). Since such a signal was not observed during GW190814, GW200105 and GW200115, we can conclude that the respective secondaries did not break apart, in the following we use the Roche-radius as an estimate for the radii of the objects. From the masses of the objects, we can estimate the Roche-radius of the secondary partner by

\[
R = \frac{2GM}{c^2} \left( \frac{2M}{m} \right)^{-1/3},
\]

where \( M \) and \( m \) are the respective masses of the larger and smaller partner binary objects. This is a non-relativistic expression, and it has been numerically shown that the fully relativistic equivalent to the Roche-limit is spin dependent and considers effects from e.g. the innermost stable circular orbit (ISCO), whereas the critical radius could be above as well as below the ISCO (see (Pannarale et al. 2011) and references therein). As a working hypothesis we consider this non-relativistic approximation to investigate how restrictions of the radius constrict the possible parameter spaces of compact objects consisting of exotic dark matter. Future detections by next generation gravitational wave detectors might enable the observation of the post-merger signal, yielding information to tighten such radius constraints.

In the case of purely self-interacting fermionic or bosonic dark matter, we consider an energy density of the form

\[
\varepsilon = \varepsilon_{kin} + c \cdot n^2
\]

and a pressure

\[
P = p_{kin} + c \cdot n^2
\]

where \( n \) is the number density and \( c \) is a constant with dimensions (1/\text{mass})\(^2\). It should be noted that \( p_{kin} = 0 \) in the bosonic case. This yields a noninteracting Fermi gas in the non-relativistic limit, while in the ultra-relativistic limit it approaches the limiting causal EOS \( P = \varepsilon \), which has a speed of sound equal to the speed of light, the maximal causal value. For a derivation of this EOS, see Appendix A.

For this EOS, analytical expressions for the maximum mass and the critical radius can be found for arbitrary values for the interaction strength \( y = \sqrt{c/m_f} \), where \( m_f \) is the mass of the fermionic dark matter particle. For stars consisting of self-interacting fermionic dark
Similarly, since the crucial scaling relation for QCD-like axions is
\[ y \sim f/n \]
fermionic matter, the interaction strength
It is also interesting to note that in the case of neutrons comprising
ing hypothesis with the measured constraints for radius and mass.
consisting of these axions would not be compatible within our work-
\approx &\text{Quinn1977}) with a mass of \( m = 8.5 \) GeV and \( m_f = 0.81 \) GeV. The solid black line denotes \( f m_f = f_m \), while the black cross denotes the point where \( m_f \) is the nucleon mass and \( f_f = f_\pi \).

\begin{table}
| Event          | \( R \) [km] | \( M_{\text{max}} \) [\( M_\odot \)] | Compactness C |
|----------------|-------------|---------------------------------|-------------|
| GW190814       | 26          | \( 2.59^{+0.08}_{-0.09} \)     | 0.14        |
| GW200105       | 13          | \( 1.91^{+0.33}_{-0.24} \)     | 0.22        |
| GW200115       | 8.5         | \( 1.5^{+0.85}_{-0.29} \)      | 0.26        |
| GW191219       | 24.4        | \( 1.17^{+0.07}_{-0.06} \)     | 0.07        |
| GW200210       | 27.6        | \( 2.83^{+0.47}_{-0.42} \)     | 0.15        |

Table 1. Calculated Roche Radius \( R \) and Maximum Mass \( M \) for the events GW190814, GW200105 and GW201015

\begin{figure}
Figure 1. Exclusion plot for interaction strength \( f = m/y \) and particle mass \( m \) assuming \( M_{\text{max}} > 2.5 M_\odot \) and \( R_{\text{crit}} < 26 \) km. The allowed region for the fermionic case has a minimum at \( f = 0.16 \) GeV and \( m_f = 8.1 \) GeV. The solid black line denotes \( f m_f = f_m \), while the black cross denotes the point where \( m_f \) is the nucleon mass and \( f_f = f_\pi \).

\section{3 COMPACT STARS CONSISTING OF BOSON-FERMION DARK MATTER}

We now consider uncharged objects consisting of a mixture of bosonic and fermionic dark matter. Similar configurations are realized in nature in the form of white dwarfs, where the nuclei are comprising the role of the bosonic matter, while electrons are fulfilling the part of fermionic matter, since the contribution of the lattice of nuclei in the interior has a negligible effect on the EOS. Another example of such a configuration are pion stars proposed by (Brandt et al. 2018), where a charged condensate of pions and a gas of charged leptons and neutrinos fulfill the roles of bosonic and fermionic matter, respectively. We will assume those dark matter stars to be globally uncharged stars while the number of fermions and bosons are equal, which requires the existence of a dual electromagnetism mediating between the two particles via dark photons. One such proposed and well motivated mechanism is the one of \textit{mirror dark matter} (Berezhiani et al. 1996; Chacko et al. 2006), for example.

Similarly to section 2, we assume a dimensionless energy density and pressure proportional to the square of the interaction strength and number density:
\[ e = e_{\text{boson}} + e_{\text{fermion}} + y^2 n_B^2 \]
where \( y \) is the interaction strength and \( n_B \) is the number density of bosons. Such an equation of state can be motivated using thermodynamic relations (see Appendix A), resulting in an EOS that can be rewritten in terms of the squares of the number densities of fermions and bosons. The assumption of charge-neutrality requires the number densities of fermionic and bosonic dark matter to be equal, i.e. \( n_F = n_B \).

Solving the Tolman-Oppenheimer-Volkoff equation and assuming \( M_{\text{max}} > 2.6M_\odot \) and \( R_{\text{crit}} < 26 \) km in the case of GW190814, we find solutions for the coupling strength \( f \) and the respective dark matter particle masses (see Figure 2). One can see that the coupling strength \( y \) is constrained to a range of \([2.5, 22.5]\). Interestingly, a five-fold increase in \( y \) can result in a change of two orders of magnitudes in the respective masses. Also, one of the two dark matter particle masses is always of the order of the GeV-scale, regardless of the coupling strength. This is in stark contrast to the findings in section 2, where the bosonic masses can be arbitrarily small.

In the case of GW200105 and its high compactness, a similar behaviour can be seen (see Figure 3), however, the allowed parameter space is much more restricted compared to GW190814. For masses of the bosonic particle to be smaller than \( 10^4 \) MeV, the interaction strength is constrained to values in the range of \( y \approx [10, 20] \). In the case of the even more compact GW200115 secondary component, no possible solutions in the parameter space can be found.

## 4 SELF-BOUND STARS

For a self-bound linear equation of state of the form \( p = s \cdot (e - \epsilon_0) \), the maximum mass and critical radius depend on the speed of sound \( s = c_s^2 = \frac{\partial p}{\partial e} \). This equation of state is related to the MIT bag model with a bag-constant \( \epsilon_0 = 4B \) and \( s = 1/3 \). Note that the defining feature of a self-bound star is that the pressure vanishes at non-vanishing energy density. With this EOS, one finds (Narain et al. 2006; Agnihotri et al. 2009)

\[
M_{\text{max}} = 2.57 M_\odot \left( \frac{\epsilon_{nm}}{\epsilon_0} \right)^{1/2} \quad s = 1/3
\]

\[
M_{\text{max}} = 4.23 M_\odot \left( \frac{\epsilon_{nm}}{\epsilon_0} \right)^{1/2} \quad s = 1
\]

where \( \epsilon_{nm} = 140 \) MeV fm\(^{-3} \) is the energy density of saturated nuclear matter. Given a maximum mass, these expressions analytically provide an upper bound for \( \epsilon_0 \). In the case of a minimum radius, the TOV-equations have to be solved numerically to constrain \( \epsilon_0 \) from below (the results are shown in Tables 2, 3 and Figure 4.)

In the case of GW190814, the required values of \( \epsilon_0 \) to find solutions are of the order of the standard MIT bag value \( \approx 57.5 \) MeV fm\(^{-3} \).
As one can see, the necessary \( \varepsilon_0 \) is in the range of the QCD scale. This suggests that such an object would be dominated by interactions of a physical scale comparable to QCD interactions, and that weak interactions do not play a significant role in such self-bound stars, regardless of their actual microscopic composition. This is however in stark contrast to GW200105 and GW200115, which require higher values for \( \varepsilon_0 \) to fulfill the imposed conditions. Especially in the case of GW200115 for \( s=1 \), the required values for \( \varepsilon_0 \) can be as high as \( 1 \text{ GeV fm}^{-3} \). This is still in the typical range of QCD energy densities, as e.g. the one of the QCD crossover transition. However, it is not possible to find values for \( \varepsilon_0 \) that can fit all signals for a given fixed speed of sound. Hence, self-bound stars could generally be ruled out in our adopted scenario.

5 CONCLUSIONS

In this work we investigated NS-BH merger candidates to constrain compact exotic object mergers, by assuming as a working hypothesis that these objects have radii no larger than the Roche-radius. This allows us to demonstrate the constraining power of future radius measurements of merger events for exotic compact objects comprised of dark matter. We studied for this purpose generic models of self-interacting fermion and boson stars. We have shown that in all the studied cases, the characteristic scales necessary to explain GW190814 are very close to the QCD-scale, suggesting that weak interactions do not play a significant role in hypothetical dark matter stars with such high masses and large radii. Additionally, while in the case of self-interacting stars the bosonic dark matter particle mass can be arbitrary, the fermionic dark matter particle mass is required to be in the GeV range. Interestingly, in the case of a mixture of bosonic and fermionic dark matter one of both particle masses is also always in the GeV scale. This implies that if the secondary object in GW190814 was comprised of dark matter with a dark photon mechanism, then the possible dark QCD scale must be similar to the QCD scale. In the limiting case of a self-bound linear equation of state, we find again that the vacuum energy needs to be of the scale of QCD. However, in the case of masses and radii which are common for neutron stars, the applied models are either constrained to very small parameter spaces, or allow for no solutions at all, as demonstrated in the case of the very compact object of GW200115.
This paper has been typeset from a TEX/LATeX document. We arrive at equation 9.

\[ F_{\text{Fermion}} + y_1^2 \cdot n_F + y_2^2 \cdot n_B. \]  

and for the bosonic particle it only consists of a mass term

\[ u_{\text{kin}} = m \cdot n_B. \]  

Thus, we find for the chemical potentials

\[ \mu_F = \frac{\partial U}{\partial N_F} = \frac{\partial u}{\partial n_F} + 2 \cdot y_1^2 \cdot n_F \]  

\[ \mu_B = \frac{\partial U}{\partial N_B} = \frac{\partial u}{\partial n_B} + 2 \cdot y_2^2 \cdot n_B. \]  

By making use of the relation \( p = \sum_i n_i \cdot \mu_i - u \) we find the total pressure, given as

\[ P_{\text{total}} = p_{\text{kin}} + P = p_{\text{Fermion}} + n_{\text{Fermion}}^2 + y_1^2 \cdot n_F + y_2^2 \cdot n_B. \]  

Since \( p_{\text{Fermion}} = 0 \) and setting \( y_1^2 + y_2^2 = y^2 \) as well as \( n_F = n_B \) we arrive at equation 9.

\[ P = p^2_{\text{kin}} + y^2 \cdot n_B^2. \]  

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