Precise Linear Positioning Method Based on Transmissive Two-Stage Diffraction Grating System

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Abstract. This paper mainly studies the precise linear positioning method based on the transmissive type two-stage diffraction grating system. Starting from the analysis of the two-stage diffraction principle, the mathematical model of the two-stage grating diffraction is established, and the positioning characteristics of the differential positioning method and the modified positioning method are discussed. The simulation experiment of the linear positioning device is carried out to study the displacement characteristics. The experimental results show that the precise positioning based on the diffraction grating can obtain a positioning accuracy of ±0.4 μm.

Keywords: precision positioning, grating diffraction.

1. Introduction
As an important technology in the field of intelligent manufacturing and assembly, high-speed precision motion control technology is widely used in modern manufacturing equipment such as automotive product production lines, chip packaging equipment, and high-speed CNC machining centers. With the rapid development of the modern production line of Industry 4.0 smart manufacturing, production and processing have put forward higher requirements on the speed, acceleration, positioning accuracy and stability of the motion mechanism. The realization of the ultra-precision displacement measurement system is very critical and can be improved. Automated production efficiency is to ensure product quality and pass rate. Among them, the precise linear positioning method based on the transmissive two-stage diffraction grating system includes the modified method and the differential method [1].

2. Two-stage diffraction principle
According to physical knowledge, if a beam of light with a fixed frequency and parallel to each other irradiates the grating surface, diffraction will occur, and diffracted light of different orders will be generated. Assuming that the angle between the incident light and the negative direction of the x-axis is α, the diffracted light The angle with the positive x-axis is φ, λ represents the wavelength of the incident light, m represents the diffraction order, and P represents the grating constant, then φ can be obtained by the following formula.

\[ P(\sin \alpha \pm \sin \varphi_m) = m\lambda \]
In the formula, $\lambda$ represents the wavelength of the incident light, and $m$ represents the diffraction order and $P$ represents the grating constant.

Figure 1 shows the placement diagram of the two gratings when the two-stage grating is diffracted. Among them, G1 and G2 are two identical gratings, and the two gratings are in a parallel state.

![Grating configuration diagram](image)

**Figure 1.** Grating configuration diagram

When light parallel to G1 is irradiated with fixed frequency light, diffraction phenomenon will occur on G1 and G2. This is because the incident light will be diffracted after G1, and the emitted light of G1 is equivalent to the incident light of G2. If the distance between the two gratings is very small, the diffracted light of the first-stage grating can all be used as the incident light of the second-stage grating. The light incident on the surface of the second-stage grating will produce diffracted light of different orders. As shown in Figure 2.

![Double grating diffraction](image)

**Figure 2.** Double grating diffraction

Let $r$ be the diffraction order diffracted by the first-stage grating, and $t$ be the diffraction order diffracted by the second-stage grating, then $(r, t)$ can represent any output light diffracted by the two-stage grating. Assuming that $r$ is at most $m$, since the two gratings are almost the same, $t$ at the maximum should also be $m$. Then there are $m^2$ permutations and combinations of $(r, t)$, and each group of $(r, t)$ corresponds to a direction of emitted light, then the final diffracted light will be emitted in $m^2$ different directions.

According to the knowledge of physics, if G1 and G2 can be made almost identical, it can be regarded as a single grating with the same grating constant. Therefore, although the number of outgoing lights of the grating combination is $m^2$, if only the direction is considered, there are only outgoing light directions. The schematic diagram of the outgoing light directions is shown in Figure 3.
3. Mathematical Modeling of Two-Stage Diffraction Grating System

If the laser beam is directed toward the grating, the light transmission characteristic $T(x)$ of the grating can be expressed as:

$$
T(x) = \begin{cases} 
1 & \text{transmission area}(kP - W / 2 \leq x_0 \leq kP + W / 2) \\
0 & \text{Non-transmissive area}
\end{cases}
$$  

(1)

In formula (1), $k$ represents an integer $k=0, \pm 1, \pm 2, \ldots$

$P$ represents grating pitch (grating constant)

$W$ represents grating slit width

The formula (1) can obviously be expanded into

$$
T(x) = \sum_{n=-\infty}^{\infty} A_n \exp(i2\pi nx)
$$  

(2)
In formula (2), \( f_n \) represents spatial frequency of grating and \( f_n = 1 / P \).

\( A_n \) represents Fourier coefficients, \( n = 0, \pm 1, \pm 2 \cdots \).

\( P \) represents Grating pitch (grating constant).

By formula \( A_n = \frac{1}{P} \int_0^P T(x) \exp(-i2\pi nx) dx \), we can get \( A_n \).

As shown in Figure 5, after the grid line moves along the x axis by a distance \( \Delta x \), the light transmission characteristics will become as followed:

\[
\begin{align*}
T(x - \Delta x) &= 1 &\text{transmission area} (kP - W / 2 + \Delta x \leq x \leq kP + W / 2 + \Delta x) \\
T(x - \Delta x) &= 0 &\text{Non-transmissive area}
\end{align*}
\]

The complex form of the same transformation into Fourier series is

\[
T(x - \Delta x) = \sum_{n=-\infty}^{\infty} A_n \exp(i2\pi nx) \quad (4)
\]

In the same way, according to formula \( A'_n = \frac{1}{P} \int_{kP - W / 4 + \Delta x}^{kP + W / 4 + \Delta x} 1 \cdot \exp(-i2\pi nx) dx \), we can get \( A'_n \). Through analysis, if the laser beam of unit amplitude is directly facing the grating after \( \Delta x \) in the incident direction, referring to the transmission principle, the complex amplitude of the light in the direction opposite to the incident direction is:

\[
U(x, y, 0) = 1 \cdot T(x - \Delta x) = \sum_{n=-\infty}^{\infty} A'_n \exp(in \frac{2\pi}{P} x) 
\]

(5)

Combined with Fourier optics theory, the complex amplitude is expressed as

\[
U(x, y, 0) = \sum_{n=-\infty}^{\infty} A'_n \exp(i2\pi(f_{nx} x + f_{ny} y)) 
\]

(6)

In the formula presented above, \( f_{nx} \) and \( f_{ny} \) are the frequencies of the n-th light on the x and y axes, respectively.

Comparing formula (5) and (6), we can easily get:

\[
f_{nx} = \frac{n}{P}, \quad f_{ny} = 0
\]

(7)

4. Precision linear positioning method

Ultra-precision plane positioning is composed of two technologies: ultra-precision angular positioning and ultra-precision linear positioning. Precision linear positioning is the key to ultra-precision plane positioning, and it is also the basis for achieving three-degree-of-freedom plane positioning [2].

The principle of precise positioning is selected from laser theory. When the beam is vertically shot into the gap of two gratings arranged in parallel, a transmitted positioning signal is generated due to the diffraction of the grating, and its intensity will be constant with the relative displacement of the two
gratings. Change repeatedly in accordance with a certain rule over time. After being checked and measured by the photodiode, it is converted into an electrical signal.

Precision linear positioning methods include differential precision positioning method and modified precision positioning method.

![Figure 6. Differential grating configuration diagram](image)

### 4.1. Linear differential positioning method

The differential grating structure is shown in Figure 6. The grating structure uses two pairs of gratings, AC and BD. The AC grating is staggered by \( \frac{P}{4} \). At the same time, the BD grating is staggered by \( -\frac{P}{4} \). In this method, we get two signals \( I_1 \) and \( I_2 \) that are 180 degrees apart from each other, as shown in Figure 7.

The differential signal \( S_d \) is:

\[
S_d = I_1 - I_2
\]

Take \( S_d \) as the positioning control signal of the experiment, as shown in Figure 7(b), it is not difficult to see that the light intensity value (reflecting the displacement change) is doubled by \( S_d \), especially next to the position measurement point, the slope of \( S_d \) changes very large, a small displacement can bring a considerable increase and attenuation of light intensity [3].

The response rate of the positioning signal is effectively enhanced, and the relative displacement of the grating transformation near the position measurement point A is linearly related to the enhancement and attenuation of the light intensity, so the direction of the position offset and the size of the phase difference can be fixed.

![Figure 7. Differential signal system curve](image)
At the same time, $dS$ changes repeatedly according to a certain rule within a certain time with the displacement between the two gratings. In a fixed time of displacement, the intersection of the two sets of signals is set as the position measurement point, and the displacement difference is zero at this time.

4.2. Linear modified positioning method

Compared with linear positioning, the device of the modified method is simpler. As shown in Figure 8, only one set of gratings (A, C) is used. To obtain the inverted signal $I_1$, first check the instantaneous value $I_0$ and the maximum value of the measured positioning signal $I_{\text{max}}$ and the minimum value $I_{\text{min}}$ of the measurement positioning signal, which can be calculated by the following formula:

$$I_0 = I_{\text{max}} + I_{\text{min}} - I_1$$

After that, we can take the difference between $I_1$ and $I_0$ as the modified control signal $S_m$.

$$S_m = I_1 - I_0 = 2I_1 - (I_{\text{max}} + I_{\text{min}})$$

![Figure 8. Modified grating configuration diagram](image)

As shown in Figure 9, $I_1$ is measured by experiment and $S_m$ is calculated.

It is not difficult to obtain from the above two signals that the positioning signal value of the displacement change is quickly doubled by $S_m$. At the approaching origin, the slope of the curve of $S_m$ changes greatly.

In the modified type position measurement, the position where the correction curve is 0 is selected as the position measurement point, the displacement is 0, and the position measurement drive range is between both sides of the precise positioning point $P/2$.

![Figure 9. Modified positioning system curve](image)

5. Precision linear positioning experiment

Using the principle of laser diffraction, using the differential control position, and according to the diffraction characteristics of Fourier and grating, a mathematical model of double grating diffraction is
established. On this basis, a precise linear positioning device is adopted, including photodetection system, computer driving system, and microcomputer control system, etc. [4]. In the experiment, P=100μm, the distance between the coarse and micro-moving plates G=0.1mm, and the semiconductor laser with λ=635nm, so that the real displacement of the unit pulse after the motion signal of the high-precision device is subdivided by 10,000 can be as small as 0.7 . Due to the rapid operation of the machinery in the process of precise positioning, the control signal will contain many noise factors, the system motion fluctuates and the positioning accuracy is reduced [5]. After repeated debugging and comparison, set the time of the system's microcomputer sampling and one processing round trip to 1s, and get the influence of the displacement around the positioning point on the light intensity, horizontally and vertically. The two positioning methods have different positioning results.

5.1. Differential positioning experiment results
The positioning signal used in the experiment is $dS$, because the mechanical movement is too rapid, which causes a large proportion of noise in the drive signal, which results in poor positioning accuracy and stability of the system. The sampling and driving cycle time is limited to 1s. The differential linear position result of the horizontal position: the deviation before the position measurement is 25 μm, the computer drives the micro-moving plate to the error band in 35s, and the position measurement error band is ±80 digits. The relationship between the light intensity and the movement distance of the sagittal plane next to A is $1μm ≈ 220digits$, and the measurement accuracy of the linear position of the horizontal position is ±0.4μm.

The positioning result of the vertical position differential line: the deviation before the position measurement is 23 μm, the computer drives the micro-movement plate to the error band in 42s, the position measurement error band is ±80 digits, and the relationship between the light intensity and the movement distance of the vector plane is $1μm ≈ 205digits$, The linear position measurement accuracy of the horizontal position is ±0.4μm.

5.2. Modified positioning experiment results
The signal of linear modified method positioning is changed to $S_m$, the driving cycle time is 1s.

The result of the modified linear positioning of the horizontal position: the position deviation before the position measurement is 20 μm. It took 30s for the computer to drive the micro-moving plate to the error band. The position measurement error band is ±100 digits, the relationship between the light intensity next to A and the movement distance of the sagittal plane is $1μm ≈ 220digits$, and the linear position measurement accuracy of the horizontal position is ±0.45 μm.

Modified linear position measurement result of vertical position: the position deviation before position measurement is 21 μm, the computer drives the micro-motion stage to the error band in 45s, the positioning error band is ±100 digits, and the relationship between the light intensity and the movement distance of the sagittal plane is: $1μm ≈ 205digits$. The linear position measurement accuracy of the vertical position is ±0.49 μm.

6. Conclusions
It can be concluded that the differential device is more complex, the positioning point selects the intersection of two pairs of signals. When the light intensity of the laser source changes, the position of the positioning point is not disturbed, and it is resistant to noise. The modified method is simple and easy to reach, but has weak resistance to noise with poor stability and low accuracy. Therefore, the differential method is usually used when achieving precise positioning, and the positioning accuracy is ±0.4μm.
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