Abstract

The Acoustomagnetoelectric Effect (AME) in Graphene Nanoribbon (GNR) was theoretically studied using the Boltzmann kinetic equation. On open circuit, the general formula for Surface Acoustomagnetoelectric field \( \vec{E}_{SAME} \) in GNR with energy dispersion \( \varepsilon(p) \) near the Fermi point was calculated. The \( E_{SAME} \) was found to depend on the magnetic strength \( \eta \), \( \alpha = \hbar \omega_q / E_g \) and the energy gap \( E_g \). The expression for \( \vec{E}_{SAME} \) was analyzed numerically for varying width of GNR, magnetic strength \( \eta \) and \( \alpha \) at different sub-bands indices \( (p_i) \). It was noted that the dependence of \( \vec{E}_{SAME} \) on the width of GNR increased to a saturation point of approximately 15Vcm\(^{-1}\) and remained constant. For \( E_{SAME} \) versus \( \eta \), the \( E_{SAME} \) increases rapidly to a maximum point and then decayed to a constant minimum value. The graph was modulated either by varying the width of GNR or the sub-band index \( p_i \) with an inversion occurring at \( p_i = 6 \). The dependence of \( E_{SAME} \)
versus $\alpha$ was analysed. The $E_{SAME}$ was constant up to a point and sharply increased asymptotically at approximately $\alpha = 1$. A 3D graph of $\vec{E}_{SAME}$ with $\eta$ and width is also presented. This study is relevant for investigating the properties of GNR.

Introduction

The study of Acustomagnetoelectric Effect (AME) in Semiconductors and its related materials have generated lot of interest recently. AME in materials such as Superlattices [1, 2, 3], Quantum Wires [4], Carbon Nanotubes [5] deals with appearance of a d.c electric field in the Hall direction when the sample is on open circuit. Studies have shown that the propagation of acoustic waves causes the transfer of energy and momentum to the conducting electrons [3]. This interaction is treated as that between the sound wave and the field which leads to a collective drift of acoustically bunched electrons. When the build up of the acoustic flux exceeds the velocity of sound it causes the formation and propagation of Acoustoelectric field [6, 7]. Other effects such as Acoustoelectric Effect (AE) [1, 2, 8], Acoustothermal Effect [9], and Acoustoconcentration Effect can occur. The AE was predicted by Grinberg and Kramer [10] for bipolar semiconductors and experimentally observed in Bismuth by Yamada [11]. By applying the sound flux ($\vec{W}$), electric current ($\vec{j}$), and magnetic fields ($\vec{H}$) perpendicularly to the sample, it is interesting to note that, with the sample opened in direction perpendicular to the Hall direction, can leads to a non-zero Acustomagnetoelectric Effect AME [12]. Mensah et. al [1] studied these effect in Superlattice in the hypersound regime, Bau et. al. [13] also, the AME of cylindrical quantum wires
as well as the AME effect in mono-polar semiconductor for both weak and quantizing field were studied [14]. Experimentally, AME has been observed in n-InSb [15], and in graphite [16].

Graphenes are the latest 2D materials recently discovered [17]. Within the low energy range ($\varepsilon < 0.5eV$), carriers in graphenes are massless relativistic particles with effective speed of $V_f \approx 10^6ms^{-1}$. One of the major limitations of graphene sheet is lack of band gap in its energy spectrum [18]. To overcome this, stripes of Graphenes called Graphene Nanoribbons (GNRs) whose characteristics are dominated by the nature of their edges (the armchair (AGNRs) and Zigzag (ZGNRs)) with well-defined width are proposed [18]. By patterning graphenes into narrow ribbons creates an energy gap where GNR behaves like semiconductor [19, 20, 21]. At the Fermi point, the energy dispersion of GNR in the presence of sound flux ($\vec{W}$), electric current ($\vec{j}$) and magnetic field ($\vec{H}$) could lead to interesting properties of acoustoelectric effect in GNR. However, Surface Acoustemagnetoelectric Effect (SAME) in GNR has not been studied. In this paper, the Boltzmann distribution function is used to study the SAME in GNR out of which the $E_{SAME}$ is calculated when the GNR is on open circuit. This paper is organised as follows: In section II, the theory of SAME in GNRs is outlined. In section III, the numerical calculations of the general expression for AME are presented; and in section IV, the results and discussion while the section V deals with the conclusion.
Theory

Based on the method developed in [22], the $E_{SAME}$ in GNR is calculated in this paper. Considering $ql \gg 1$ ($q$ is the acoustic wave number and $l$ is the electron mean free path), the Boltzmann transport equation given as

$$ - \left( e \vec{E} \frac{\partial f_{\vec{p}}}{\partial \vec{p}} + \Omega [\vec{h}, \vec{R}(\varepsilon)] \right) = - \frac{f_{\vec{p}} - f_0(\varepsilon_{\vec{p}})}{\tau(\varepsilon_{\vec{p}})} + \frac{\pi \xi^2 \tilde{W}}{\rho V_s^3} \left\{ [f_{\vec{p}+q} - f_{\vec{p}}] \delta(\varepsilon_{\vec{p}+q} - \varepsilon_{\vec{p}} - \hbar \omega_q) + [f_{\vec{p}-q} - f_{\vec{p}}] \delta(\varepsilon_{\vec{p}-q} - \varepsilon_{\vec{p}} + \hbar \omega_q) \right\} \quad (1)$$

is utilised. Here, $f_0(\varepsilon(\vec{p}))$ is the equilibrium distribution function, $\vec{E}$ is the constant electric field, $\omega_q = qV_s$, $\tilde{W}$ is the density of the acoustic flux, and $\vec{p}$ the characteristic quasi-momentum of the electron. The relaxation time on energy is $\tau(\varepsilon_{\vec{p}})$ and the cyclotron frequency, $\Omega = \mu H/\hbar c$ ($H$ is the magnetic field, $\mu$ is the electron mobility and $c$ is the speed of light in vacuum). The energy dispersion relation $\varepsilon(\vec{p})$ for GNRs band near the Fermi point is expressed as [18, 23]

$$ \varepsilon(\vec{p}) = \frac{E_g}{2} \sqrt{\left[ 1 + \frac{\vec{p}^2}{\hbar^2 \beta^2} \right]} \quad (2)$$

where the energy gap $E_g = 3ta_{c-c}\beta$ with $\beta$ being the quantized wave vector given as $\beta = \frac{2\pi}{a\sqrt{3}} [\frac{p_i}{N+1} - \frac{2}{3}]$, where $p_i$ is the sub-band index and $N$ is the width of the GNR. $t = 2.7eV$ is the nearest neighbour Carbon-Carbon C-C tight binding overlap energy and $a_{c-c} = 1.42\text{Å}$ is the (C-C) bond length. Multiplying the Eqn.(1) by $\vec{p} \delta(\varepsilon - \varepsilon_{\vec{p}})$ and summing over $\vec{p}$ gives the kinetic equation as

$$ \frac{\vec{R}(\varepsilon)}{\tau(\varepsilon)} + \Omega \left[ \vec{h}, \vec{R}(\varepsilon) \right] = \vec{X}(\varepsilon) + \vec{S}(\varepsilon) \quad (3)$$

where

$$ \vec{R}(\varepsilon) \equiv e \sum_{\vec{p}} \vec{p} f_{\vec{p}} \delta(\varepsilon - \varepsilon_{\vec{p}}) \quad (4)$$
\[ \vec{\Lambda} (\varepsilon) = -e \sum_{\vec{p}} \left( \vec{E}, \frac{\partial f_{\vec{p}}}{\partial \vec{p}} \right) \delta (\varepsilon - \varepsilon_{\vec{p}}) \] (5)

\[\vec{S}(\varepsilon) = \frac{\pi \varepsilon^2 \vec{W}}{\rho V_s^3} \delta (\varepsilon - \varepsilon_{\vec{p}}) \{ [f_{\vec{p}+\vec{q}} - f_{\vec{p}}] \delta (\varepsilon_{\vec{p}+\vec{q}} - \varepsilon_{\vec{p}} - \hbar \omega_{\vec{q}}) + [f_{\vec{p}-\vec{q}} - f_{\vec{p}}] \delta (\varepsilon_{\vec{p}-\vec{q}} - \varepsilon_{\vec{p}} + \hbar \omega_{\vec{q}}) \} \] (6)

In the linear approximation where \( f_{\vec{p}} \rightarrow f_0 (\varepsilon_{\vec{p}}) \) with \( \vec{p} \rightarrow -\vec{p} \), \( f_{\vec{p}} \equiv f_0 (\varepsilon_{\vec{p}}) = f_0 (\varepsilon_{-\vec{p}}) \), Eqn.(5) and Eqn.(6) can be respectively expressed to

\[ \vec{\Lambda} (\varepsilon) = \vec{E} \left( \frac{2h^2 \beta^2}{\hbar q} \alpha - \frac{\hbar q}{2} \right) \frac{\partial f_0}{\partial \varepsilon} \delta (\varepsilon - \varepsilon_{\vec{p}}) \] (7)

\[ \vec{S}(\varepsilon) = \frac{2\pi \vec{W}}{\rho V_s \alpha \Gamma_0} \left( \frac{2h^2 \beta^2}{\hbar q} \alpha - \frac{\hbar q}{2} \right) \frac{\Theta (1 - \alpha^2)}{\sqrt{1 - \alpha^2} f_0 (\varepsilon)} \frac{1}{\partial \varepsilon} \delta (\varepsilon - \varepsilon_{\vec{p}}) \] (8)

with \( \alpha = \hbar \omega_{\vec{q}} / E_g \), \( \Gamma_0 = (E_g^2 \alpha^2 / 2V_s^2) f_0 (\varepsilon) \) and \( \Theta \) is the Heaviside step function given as

\[ \Theta (1 - \alpha^2) = \begin{cases} 1 & \text{if } (1 - \alpha^2) > 0 \\ 0 & \text{if } (1 - \alpha^2) < 0 \end{cases} \]

Substituting Eqn.(7) and Eqn.(8) into Eqn.(3) and solving for \( \vec{R}(\varepsilon) \) gives

\[ \vec{R}(\varepsilon) = \{ \frac{2\pi \vec{W}}{\rho V_s \alpha \Gamma_0} \left( \frac{2h^2 \beta^2}{\hbar q} \alpha - \frac{\hbar q}{2} \right) \frac{\Theta (1 - \alpha^2)}{\sqrt{1 - \alpha^2} f_0 (\varepsilon)} \frac{1}{\partial \varepsilon} \delta (\varepsilon - \varepsilon_{\vec{p}}) \times \{ \vec{W} \tau (\varepsilon) + \Omega [\vec{h}, \vec{W}] \tau (\varepsilon)^2 + \Omega^2 \vec{h} (\vec{h}, \vec{W}) \tau (\varepsilon)^3 \} + \left( \frac{2h^2 \beta^2}{\hbar q} \alpha - \frac{\hbar q}{2} \right) \frac{\partial f_0}{\partial \varepsilon} \times \{ \vec{E} \tau (\varepsilon) + \Omega [\vec{h}, \vec{E}] \tau (\varepsilon)^2 + \Omega^2 \tau (\varepsilon)^3 \vec{h} (\vec{h}, \vec{E}) \} \} \} \{ 1 + \Omega^2 \tau (\varepsilon)^2 \}^{-1} \delta (\varepsilon - \varepsilon_{\vec{p}}) \} \times \]

The partial current density is given as

\[ \vec{j} = - \int_{0}^{\infty} \vec{R}(\varepsilon) d\varepsilon \] (10)
With \( H = \left( \frac{2\hbar^2j^2}{\hbar q} - \hbar q \right) \), substituting Eqn.(9) into Eqn.(10) yields

\[
\vec{j} = \frac{HT_0 \Theta (1 - \alpha^2)}{\rho V_s \alpha \sqrt{1 - \alpha^2}} \left\{ \tau(\varepsilon) \left[ \frac{\tau(\varepsilon)}{1 + \Omega^2 \tau(\varepsilon)^2} \right] \vec{W} + \Omega \left[ \left\{ \frac{\tau(\varepsilon)^2}{1 + \Omega^2 \tau(\varepsilon)^2} \right\} \vec{W} \right] + \Omega^2 \left[ \left\{ \frac{\tau(\varepsilon)^3}{1 + \Omega^2 \tau(\varepsilon)^2} \right\} \vec{W} \right] \right\} + \vec{E}
\]

In Eqn.(11), the following averages were used

\[
\langle \ldots \rangle = - \int_0^\infty (\ldots) \frac{d\varepsilon}{\varepsilon}
\]

\[
\langle \langle \ldots \rangle \rangle = - \frac{2\pi}{f_0(\varepsilon)} \int_0^\infty (\ldots) \frac{d\varepsilon}{\varepsilon}
\]

Where \( f_0 = \left[ 1 - \epsilon \exp \left( - \frac{1}{kT} (\varepsilon - \varepsilon_F) \right) \right]^{-1} \) is the Fermi-Dirac distribution function.

The Eqn. (11) can further be simplified with the following substitution \( g = 1/1 + \Omega^2 \tau(\varepsilon)^2 \), \( \gamma_k \equiv \langle g \tau(\varepsilon)^k \rangle \), and \( \eta \equiv \langle g \tau(\varepsilon)^k \rangle \) where \( k = 1, 2, 3 \). This yields

\[
\vec{j} = \frac{HT_0 \Theta (1 - \alpha^2)}{\rho V_s \alpha \sqrt{1 - \alpha^2}} \left\{ \eta_1 \vec{W} + \Omega \eta_2 \vec{W} + \Omega^2 \eta_3 \vec{W} \right\} + \Omega \left\{ \gamma_1 \vec{E} + \gamma_2 \Omega \vec{E} + \Omega^2 \gamma_3 \vec{E} \right\}
\]

With the sample opened in all directions (\( j = 0 \)), and ignoring higher powers of \( \Omega \) gives

\[
\gamma_1 \vec{E}_x - \gamma_2 \Omega \vec{E}_y = - \gamma_1 \vec{E}_x
\]

\[
\gamma_2 \Omega \vec{E}_x + \gamma_2 \Omega \vec{E}_y = - \gamma_2 \Omega \vec{E}_y
\]

where \( E_\alpha = \frac{\Gamma_\alpha \Theta (1 - \alpha^2)}{\rho S \alpha \sqrt{1 - \alpha^2}} \). Making the \( \vec{E}_y \) the subject of the equation yields

\[
\vec{E}_y = \vec{E}_x \Omega \left\{ \left( \frac{\eta_1 \gamma_2 - \eta_2 \gamma_1}{\gamma_1^2 + \gamma_2^2 \Omega^2} \right) \right\}
\]
substituting the expressions for $\eta_1, \eta_2, \gamma_1, \gamma_2$ into Eqn. (15), with $\vec{E}_y = \vec{E}_{SAME}$ gives

$$\vec{E}_{SAME} = \bar{E}_0 \Omega \left\{ \frac{\langle \tau(\epsilon)^2 \rangle}{1 + \Omega^2 \tau(\epsilon)^2} \left( \frac{\langle \tau(\epsilon)^2 \rangle}{1 + \Omega^2 \tau(\epsilon)^2} \right)^2 \right\}$$

(16)

**Numerical analysis and Discussions**

The general equation for $\vec{E}_{SAME}$ in Eqn. (16) is simplified as

$$\vec{E}_{SAME} = \frac{E_g \bar{W} \bar{h} \omega q \Theta(1 - \alpha^2)}{2 \rho V_s^3} \sqrt{1 - \alpha^2} \left\{ F_{(-1/2, \eta^2)} F_{(-3/2, \eta^2)} - F_{(0, \eta^2)} F_{(-2, \eta^2)} \right\} \times \left\{ \frac{3\sqrt{\pi}}{4} F_{(-1/2, \eta^2)}^2 + \frac{9\pi}{16} \eta^2 F_{(0, \eta^2)}^2 \right\}^{-2}$$

(17)

with $F_{m,n} = \int_0^\infty \frac{x^m}{1 + \Omega^2(\epsilon)^2 x^n} \frac{\partial f_0(\epsilon)}{\partial x} dx$. From Eqn. (17), the $\vec{E}_{SAME}$ is a function of the following parameters: magnetic strength ($\eta = \Omega \tau$); $\alpha$; and the energy gap $E_g = 3ta^c - c\beta$. The $E_g$ depends on the quantized wave vector $\beta$. The parameters used in the numerical calculations are $\tau = 10^{-12}s$, $\omega_q = 10^{10}s^{-1}$, $S = 5 \times 10^3 ms^{-1}$, $q = 2.23 \times 10^6cm$. In analysing the Eqn. (17), the condition ($(1 - \alpha^2) > 0$) was considered. Figure 1a, shows the occurrence of $\vec{E}_{SAME}$ at various sub-bands for increasing width of the GNR. The $\vec{E}_{SAME}$ increases to a saturation value of approximately 15Vcm$^{-1}$ and remains constant. In Figure 1b, the $\vec{E}_{SAME}$ was plotted against $\alpha$ at sub-bands $p_i = 1, 2, 3$. The $\vec{E}_{SAME}$ was constant to a point and then increased asymptotically at approximately $\alpha = 1$. For $p_i = 6$, there is an inversion of the graph. Figure 2 (a and b) shows the dependence of $\vec{E}_{SAME}$ against $\eta$ by varying either the width of GNR or the sub-band index ($p_i$). In both graphs, the $\vec{E}_{SAME}$ increases to a maximum point and decays to a constant value. The graph is modulated
Figure 1: (a) The $\tilde{E}_{SAME}$ versus width for $p = 1, 3, 5$, (b) The $\tilde{E}_{SAME}$ versus $\alpha$ for $p = 1, 3, 4, 6$.

Figure 2: (a) Dependence of $\tilde{E}_{SAME}$ versus the width (7-GNR) at different sub-bands, (b) Dependence of $\tilde{E}_{SAME}$ on $\eta$ for $N = 7, 9, 12$.

either by varying the width of GNR or the sub-band index. In Figure 2a, increasing $p_i$, decreases the $\tilde{E}_{SAME}$ and eventually invert at $p_i = 6$ whilst the $\tilde{E}_{SAME}$ increases for varying width of GNR (see Figure 2b). A 3D graph of $\tilde{E}_{SAME}$ versus $\eta$ at $p_i = 1$ and width at $p_i = 6$ are presented (see Figure 3 a and b).
Conclusions

From the Boltzmann kinetic equation, the general formula for Acoustic-magnetoelectric field ($E_{SAME}$) in Graphene Nanoribbon (GNR) is derived using the energy dispersion $\varepsilon(p)$ near the Fermi point. The $E_{SAME}$ is analysed numerically at various sub-bands for parameters including the width of the GNR, the magnetic strength $\eta$ and $\alpha$. The graphs of $E_{SAME}$ against these parameters are presented and analysed. For $E_{SAME}$ against the width of GNR and $\alpha$, it was observed that $E_{SAME}$ increases to a saturation value of 15Vcm$^{-1}$ and remains constant but asymptotically increase at approximately $\alpha = 1$. For a graph of $E_{SAME}$ against $\eta$ or $\alpha$, the $E_{SAME}$ varies for increases in the sub-band but invert at $p_i = 6$.

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