On the Bloch Theorem
Concerning Spontaneous Electric Current

Yoji OHASI and Tsutomu MOMOI

Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan.

(May 9, 1996)

Abstract

We study the Bloch theorem which states absence of the spontaneous current in interacting electron systems. This theorem is shown to be still applicable to the system with the magnetic field induced by the electric current. Application to the spontaneous surface current is also examined in detail. Our result excludes the possibility of the recently proposed \(d\)-wave superconductivity having the surface flow and finite total current.

KEY WORDS: Bloch theorem, spontaneous current, \(d\)-wave superconductivity, time-reversal symmetry breaking state

1 Introduction

Although the symmetry of the order parameter in high-\(T_c\) superconductivity has not been identified yet, we have some experimental evidences which indicate the \(d\)-wave superconducting state.\([1, 2, 3, 4, 5, 6, 7, 8]\) Since this type of the symmetry is qualitatively different from the conventional \(s\)-wave superconductivity, we can expect some new phenomena in high-\(T_c\) superconductivity.

Recently, some researchers pointed out the possibility of the \(d\)-wave superconductivity with broken time-reversal symmetry, which results from the \(s + id\)-wave state near the surface.\([9, 10, 11]\) In addition, it was discussed that this state has a finite electric surface current.\([12, 13]\) In this case, we should remark that this state has the following properties:

\begin{enumerate}
\item The current flows without any external field, i.e., it appears \textit{spontaneously}.
\item The total current is also finite.\([12]\)
\end{enumerate}

On the other hand, the Bloch theorem states \textit{absence of the stationary current in the ground state with no external field}.\([14, 15, 16, 17, 18]\) Then the above surface current seems inconsistent with this theorem.

Motivated by this discrepancy, we study the Bloch theorem and its application to this state. The Bloch theorem has been examined so far within the framework of the first quantization, which seems disadvantageous to superconductivity. Furthermore, the theorem in case with the magnetic field induced by the spontaneous current has not been examined in detail. For these reasons, we present the second quantized version of the Bloch theorem, which can directly

\footnote{Submitted to J. Phys. Soc. Jpn.}
cover the superconductivity, and also show its extension to the system with the induced magnetic field.

This paper is organized as follows: In §2, we examine the Bloch theorem neglecting the magnetic field induced by the current, because the spontaneous surface current is reported in this situation.\cite{12, 13} Effects of the induced magnetic field are discussed in §3, which is followed by the summary in §4.

2 Bloch Theorem

In this section, we first of all rederive the Bloch theorem from general viewpoint. Possibility of the spontaneous surface current is then discussed on the basis of this theorem.

2.1 Definitions

We start with the case of the chargeless (neutral) fermion, in which the current does not induce magnetic field. We also assume the absence of the external magnetic or electric field. Then, the Hamiltonian is given by

\[ H = \sum_{\sigma} \int dr \Psi_\sigma^\dagger(r) \left[ (\frac{\hat{p}^2}{2m} - \mu) + V(r) \right] \Psi_\sigma(r) \]

\[ + \frac{1}{2} \sum_{\sigma, \sigma'} \int dr dr' \Psi_\sigma^\dagger(r) \Psi_\sigma^\dagger(r') U(r-r') \Psi_{\sigma'}(r') \Psi_{\sigma}(r). \]  

(1)

Here \( \hat{p} \equiv \nabla / i \), while \( m \) and \( \mu \) represent the mass of an electron and the chemical potential, respectively. \( \Psi_\sigma(r) \) means the electron field operator with spin-\( \sigma \), and \( U(r-r') \) is the electron-electron interaction, in which the pairing interaction can be included. The lattice and impurity potentials are represented by \( V(r) \). (To describe the presence of the surface, we put the potential \( V(r) \) infinity outside the sample.)

We also introduce the current density operator

\[ \hat{J}(r) = \frac{1}{2m} \sum_{\sigma} [\Psi_\sigma^\dagger(r) \hat{p} \Psi_\sigma(r) - (\hat{p} \Psi_\sigma^\dagger(r)) \Psi_\sigma(r)]. \]

(2)

At this stage, we comment on the reason why this paper uses the second quantization in contrast to the previous papers,\cite{14, 17} which prove the Bloch theorem in the language of the first quantization. When superconductivity is examined, appropriate infinitesimal symmetry breaking field is formally necessary in order to select a special state from various superconducting states. In this case, the corresponding fictitious field Hamiltonian (\( H_{FF} \)) has a number non-conserving form, and hence it is difficult to write down within the first quantization. We thus use the second quantization in order to avoid this problem. Indeed, \( H_{FF} \) for the superconductivity is given by

\[ H_{FF} = \int dr dr' [h(r, r') \Psi_\uparrow^\dagger(r) \Psi_\downarrow^\dagger(r') + h.c.], \]

(3)

where \( h(r, r') \) is the infinitesimal symmetry breaking field. In what follows, though \( H_{FF} \) is not written explicitly in eq. (1), the symmetry breaking field is always put zero after the system size is set infinity.
2.2 Ground state

Firstly, we examine the system at \( T = 0 \). We assume that the ground state \( |\psi\rangle \) has a finite total current \( J_\psi \),

\[
J_\psi = \int dr J_\psi(r) = \int dr \langle \psi | \hat{J}(r) |\psi \rangle = \langle \psi | \sum_\sigma \int dr \Psi_\sigma(r) \frac{\hat{P}}{m} \Psi_\sigma(r) |\psi \rangle \neq 0, \tag{4}
\]

where \( J_\psi(r) \) denotes a local current. In the following discussion, we show that this assumption of finite \( J_\psi \) leads to a contradiction.

Let us consider a trial state\[^1\]

\[
|\phi\rangle \equiv \exp\{i\delta \mathbf{P} \cdot \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \Psi_\sigma(r)\} |\psi\rangle, \tag{5}
\]

and compare \( \langle \phi | H |\phi \rangle \) with the ground state energy, \( \langle \psi | H |\psi \rangle \). Physically, \( \delta \mathbf{P} \) in eq. (5) is regarded as a fluctuation applied to \( |\psi\rangle \). If \( |\psi\rangle \) is the true ground state, it must be stable for arbitrary \( \delta \mathbf{P} \), i.e., \( \langle \psi | H |\psi \rangle \leq \langle \phi | H |\phi \rangle \).

It is easily found that each of the (chemical) potential and the interaction terms gives the same expectation value in both states. On the other hand, the kinetic term gives

\[
\langle \phi | \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \frac{\hat{P}^2}{2m} \Psi_\sigma(r) |\phi \rangle = \langle \psi | \sum_\sigma \int dr \Psi_\sigma^\dagger(r) (\hat{P} + \delta \mathbf{P})^2 \Psi_\sigma(r) |\psi \rangle. \tag{6}
\]

Using eq. (4), we obtain

\[
\langle \phi | H |\phi \rangle = \langle \psi | H |\psi \rangle + \delta \mathbf{P} \cdot \mathbf{J}_\psi + \frac{N}{2m} \delta \mathbf{P}^2, \tag{7}
\]

where \( N \) represents the number of electrons,

\[
N = \langle \psi | \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \Psi_\sigma(r) |\psi \rangle. \tag{8}
\]

Now take \( \delta \mathbf{P} \) opposite to \( \mathbf{J}_\psi \) in direction and small enough so that the third term in the right hand side in eq. (7) is negligible compared with the second one. Then we find that \( |\phi\rangle \) has a lower energy than \( |\psi\rangle \),

\[
\langle \phi | H |\phi \rangle = \langle \psi | H |\psi \rangle - |\delta \mathbf{P} \cdot \mathbf{J}_\psi| < \langle \psi | H |\psi \rangle. \tag{9}
\]

Though \( |\phi\rangle \) is not always the eigenstate of \( H \), the variational principle guarantees that \( |\psi\rangle \) cannot be the ground state. This shows that the ground state of the chargeless fermions cannot have finite total current. We thus obtain the second quantized version of the Bloch theorem, which can cover the superconducting system. Moreover, we find that the theorem holds irrespective to the detail of the potential. We will use this result when the surface current is studied in section 2.4.

We can extend this theorem to the system with magnetic impurities as well. Consider

\[
H_{\text{mag}} = \sum_{\sigma, \sigma'} \int dr \sum_i u_i(r - \mathbf{R}_i) \mathbf{S}_i \cdot \Psi_\sigma^\dagger(r) \sigma_{\sigma, \sigma'} \Psi_{\sigma'}(r), \tag{10}
\]

where \( u_i \) is the potential of an impurity at \( \mathbf{R}_i \) with spin \( \mathbf{S}_i \), and \( \sigma \) represents the Pauli matrix. Then because of

\[
[H_{\text{mag}}, \exp\{i\mathbf{P} \cdot \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \mathbf{r} \Psi_\sigma(r)\}] = 0, \tag{11}
\]

we obtain \( \langle \phi | H_{\text{mag}} |\phi \rangle = \langle \psi | H_{\text{mag}} |\psi \rangle \); the Bloch theorem is still valid even if magnetic impurities are present.
2.3 Finite temperature

In the next step, we examine the thermodynamic stability of the spontaneous current. Introduce an orthonormal complete set of eigenstates \( \{|\psi_i\}\) and define the density matrix \( \hat{\rho}_\psi \) which gives the lowest free energy at \( T \):

\[
\hat{\rho}_\psi \equiv \sum_i |\psi_i\rangle w_i \langle \psi_i|.
\] (12)

Here \( w_i \) is the statistical weight which satisfies \( \sum_i w_i = 1 \) and \( 0 \leq w_i \leq 1 \). (In many cases, the weight \( w_i \) is set as \( e^{-E_i/T}/\text{Tr}[e^{-H/T}] \), where \( E_i \) represents the energy of the eigenstate \( |\psi_i\rangle \).)

We again assume that \( \hat{\rho}_\psi \) gives a finite total current, \( J_\psi \):

\[
J_\psi = \int dr J_\psi(r) = \int dr \text{Tr}[\hat{\rho}_\psi \hat{J}(r)] = \text{Tr}[\hat{\rho}_\psi \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \frac{\hat{P}_m}{m} \Psi_\sigma(r)] \neq 0.
\] (13)

In the following, we show that \( \hat{\rho}_\psi \) is always accompanied by another density matrix which gives a lower free energy, i.e., \( \hat{\rho}_\psi \) cannot describe the most stable state.

We consider a trial density matrix created by the operation of the exponential operator in eq. (2):

\[
\hat{\rho}_\phi \equiv \sum_i |\phi_i\rangle w_i \langle \phi_i|,
\] (14)

where

\[
|\phi_i\rangle \equiv \exp\{i\delta \mathbf{P} \cdot \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \mathbf{J}(r) \Psi_\sigma(r)\} |\psi_i\rangle.
\] (15)

It is found that \( \{|\phi_i\rangle\} \) also forms an orthonormal complete set.

Since \( \hat{\rho}_\psi \) and \( \hat{\rho}_\phi \) have the same statistical weight, the entropy is also the same for these states,

\[
S_\phi \equiv -\text{Tr}[\hat{\rho}_\phi \log \hat{\rho}_\phi] = -\sum_i w_i \log w_i = -\text{Tr}[\hat{\rho}_\psi \log \hat{\rho}_\psi] \equiv S_\psi.
\] (16)

On the other hand, the expectation values of the energy of \( \hat{\rho}_\phi \) (\( E_\phi = \text{Tr}[\hat{\rho}_\phi H] \)) and \( \hat{\rho}_\psi \) (\( E_\psi = \text{Tr}[\hat{\rho}_\psi H] \)) are relating to each other in the form

\[
E_\phi = \sum_i w_i \langle \phi_i | H | \phi_i \rangle = \sum_i w_i \langle \psi_i | H | \psi_i \rangle + \sum_i w_i \langle \psi_i | \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \left( \frac{\delta \mathbf{P} \cdot \hat{\mathbf{P}}}{m} + \frac{\delta \mathbf{P}^2}{2m} \right) \Psi_\sigma(r) | \psi_i \rangle
\]

\[
= E_\psi + \delta \mathbf{P} \cdot \mathbf{J}_\psi + \frac{N}{2m} \delta \mathbf{P}^2.
\] (17)

From eqs. (14) and (17), we can compare the free energy for \( \hat{\rho}_\phi \) (\( \equiv F_\phi \)) with that for \( \hat{\rho}_\psi \) (\( \equiv F_\psi \)):

\[
F_\phi = E_\phi - TS_\phi = F_\psi + \delta \mathbf{P} \cdot \mathbf{J}_\psi + \frac{N}{2m} \delta \mathbf{P}^2.
\] (18)

For a \( \delta \mathbf{P} \) being small so that the term with \( O(\delta \mathbf{P}^2) \) in eq. (18) can be neglected and having an opposite direction to \( \mathbf{J}_\psi \), the free energy of the trial state, \( F_\phi \), is smaller than \( F_\psi \) for arbitrary statistical weights. (Even if we put \( \hat{\rho}_\psi \) the Gibbs state which must give the lowest free energy, \( F_\psi > F_\phi \) holds.) In conclusion, the state with a finite total current is not thermodynamically stable.

Before ending this section, we briefly note Bohm’s study for a finite current at \( T \neq 0 \). In his proof, the off-diagonal matrix elements of \( H \) for \( \{|\phi_i\rangle\} \) are dropped by using the properties
that (i) they are $O(\delta P)$ and (ii) from the perturbative analysis, they are found to affect the energy at most $O(\delta P^2)$. However, if there is degeneracy in the diagonal elements and, for example, $\langle \phi_i | H | \phi_i \rangle = \langle \phi_j | H | \phi_j \rangle$ ($i \neq j$) is realized, $(i,j)$-element may affect the energy in $O(\delta P)$. Hence this off-diagonal element cannot be neglected within the perturbation theory. On the other hand, since our proof does not rely on the perturbation theory, we do not meet this problem.

2.4 Application to the spontaneous surface current

Recently, the spontaneous surface current has been discussed in unconventional superconductivity. We examine this problem on the basis of the Bloch theorem.

Let us consider a semi-infinite system as shown in Fig. 1: Presence of the surface is described by putting $V(x \leq 0) = \infty$. The surface may have a roughness. At the end edges of the system, we take the open boundary condition, so that we can choose arbitrary small $\delta P$. (The system size is finally put infinity.)

Suppose the presence of the spontaneous surface current as shown in Fig. 2(a). Then, what does the Bloch theorem state about this situation? The answer is that there must be another counter current inside the system which cancels the surface one as a whole (Fig. 2(b)). Indeed, we can find an example of this situation in the domain wall problem in unconventional superconductivity.[19, 20] In this example, the local surface currents exist near both the sides of the wall. They, however, flow toward opposite directions to each other, thereby satisfying the Bloch theorem. At this stage, we emphasize that the presence of the counter flow purely results from the Bloch theorem and is irrespective to the origin of the surface current.

In the next step, we discuss some specific systems in which the spontaneous surface currents are proposed.

(1) As noted in the introduction, the surface current which is also finite in total was proposed in a $d$-wave superconductivity with broken time-reversal symmetry near the surface.[12, 13] Since the detail of the state, e.g., whether the time-reversal symmetry is broken or not, does not affect the Bloch theorem, we immediately find that it is not stable at all temperatures.

(2) Sigrist et. al. also reported the surface current in an unconventional superconductivity with broken time-reversal symmetry.[13, 21] They showed that the surface flow is canceled by the screening current inside the system resulting from the Meissner effect. Apparently, their results do not contradict the Bloch theorem. (Exactly speaking, the Meissner effect needs the charge of electron, which is neglected in this section. In §3, the Bloch theorem is shown to be still valid in the presence of the charge.) However, we have to note that the Meissner effect is not intrinsic origin of the back flow. The counter current has to appear even for the chargeless electrons where the Meissner effect does not work; the role of the Meissner effect is merely determining the distribution of the local current so as to exclude the magnetic field from the superconductor.

Before going to the next section, we comment on the boundary condition at the end edges of the system. One may think that there is not stationary current under the open boundary condition (isolated system); however, when the thermodynamic limit is taken with appropriate symmetry breaking field, we can safely reach the bulk system in which the stationary current can flow. Furthermore, it is expected that the detail of the boundary condition does not affect
the presence of the surface current. Consequently, although we cannot examine the cases with arbitrary boundary conditions, the statement that the surface current state is not stable is believed to describe the property of the real system.

2.5 Possibility of the spontaneous circulating current

We examine the possibility of the circulating flow. In a real system, a circuit as shown in Fig. 3(a) is necessary in order to maintain the current. In this case, the total current passing through the line ”A” in Fig. 3(a) is zero, which looks escaping from the Bloch theorem. Then, is the spontaneous current possible when the circuit is taken into account?

Bohm proved for a large ring that the ground state must be in zero total angular momentum. Recently, Vignale extended Bohm’s proof to systems with toroidal geometry. The outline of Bohm’s proof is as follows: Consider a large but thin superconducting ring with the width $L \ll R$ (Fig. 3(b)). The ground state $|\psi\rangle$ is assumed to have a finite total angular momentum as a result of the circulating current. The trial state is

$$|\phi\rangle \equiv \exp\{in \sum_{\sigma} \int dr \Psi_{\sigma}^\dagger(r) \sigma \Psi_{\sigma}(r)\}|\psi\rangle. \tag{19}$$

Because of the single-value condition of $|\phi\rangle$, $n$ must be an integer. We find

$$\langle \phi|H|\phi\rangle = \langle \psi|H|\psi\rangle + \langle \psi| \sum_{\sigma} \int dr \Psi_{\sigma}^\dagger(r) \frac{n \hat{L}_z}{m r^2} \Psi_{\sigma}(r)|\psi\rangle$$

$$+ \langle \psi| \sum_{\sigma} \int dr \Psi_{\sigma}^\dagger(r) \frac{n^2}{2m r^2} \Psi_{\sigma}(r)|\psi\rangle, \tag{20}$$

where $\hat{L}_z$ is the $z$-component of the angular momentum operator. When the total current is $J_\psi$, the angular momentum is estimated as $\langle L_z \rangle \simeq mR J_\psi$. Then the second and third terms in eq. (20) are evaluated as $n J_\psi / R$ and $n^2 N / (2m R^2)$, respectively. Now, let us examine the case with $J_\psi = O(R)$. Because of $N = O(R)$, the third term in eq. (20) becomes negligible compared with the second one for large $R$. Thus, when we choose $n$ so as to be $n J_\psi < 0$, it is found that $|\phi\rangle$ has a lower energy than the assumed ground state. Consequently, the state that has a finite angular momentum due to a circulating current with $O(R)$ cannot be the ground state.

In conclusion, even if the circuit is taken into account, the spontaneous current which is proportional to the size along the circuit is not realized in the bulk system.

3 Extension of the Bloch Theorem to the Case with Induced Magnetic Field

In this section, we examine effects of the charge. Since the real electron has the charge $e$, the electric current induces magnetic field. Then, one may expect that the spontaneous current can be stabilized by the induced magnetic field. This section is devoted to study this possibility.
3.1 Definitions

We summarize the definitions and equations used in this section. The Hamiltonian and the current density operator are given by

\[
H = \sum_\sigma \int dr \Psi_\sigma^\dagger(r) \left[ \left( \hat{p} - eA(r) \right)^2 / (2m) \right] - V(r) \Psi_\sigma(r) + \frac{1}{2} \sum_{\sigma, \sigma'} \int dr dr' \Psi_\sigma^\dagger(r) \Psi_{\sigma'}^\dagger(r') U(r - r') \Psi_{\sigma'}(r') \Psi_\sigma(r)
\]  

(21)

and

\[
\hat{J}(r) = \frac{e}{2m} \sum_\sigma \left[ \hat{p} \Psi_\sigma^\dagger(r) \Psi_\sigma^\dagger(r) - (\hat{p}) \Psi_\sigma^\dagger(r) \Psi_\sigma(r) \right] - \frac{e^2}{m} A(r) \sum_\sigma \Psi^\dagger_\sigma(r) \Psi_\sigma(r).
\]  

(22)

Here \( A(r) \) represents the vector potential. (We treat the vector potential classically in this paper.) We also define the spin magnetization operator

\[
\hat{M}_S(r) \equiv -g \mu_B \sum_{\sigma \sigma'} \Psi^\dagger_{\sigma'}(r) \frac{1}{2} \sigma_{\sigma \sigma'} \Psi_\sigma(r),
\]  

(23)

where \( g \) and \( \mu_B \) represent the \( g \)-value and the Bohr magneton, respectively.

The vector potential \( A(r) \) originates from the local spin magnetization \( M_S(r) = \langle \hat{M}_S(r) \rangle \) and the electric current \( J(r) \). The magnetic flux density \( B(r) \) and the magnetic field \( H(r) \) satisfy

\[
\nabla \times A(r) = B(r) = \mu_0 H(r) + M_S(r)
\]  

(24)

and the Maxwell equation

\[
\nabla \times H(r) = J(r).
\]  

(25)

In eq. (24), \( \mu_0 \) denotes the magnetic permeability of the vacuum. Under the Coulomb gauge, \( \nabla \cdot A(r) = 0 \), we find, from eqs. (24) and (25),

\[
\Delta A(r) = -\mu_0 J(r) - \nabla \times M_S(r).
\]  

(26)

Once the wavefunction \( (T = 0) \) or the density matrix \( (T \neq 0) \) is specified, \( A(r) \) is determined by eq. (26) with appropriate boundary conditions.

The energy of the electron system is obtained as the expectation value of eq. (21). To obtain the total energy, we have to take the magnetic field energy also into account, which is given by

\[
E_M = \frac{1}{2} \int dr \hat{H}(r) \cdot B(r) = \frac{1}{2} \int dr \hat{A}(r) \cdot J(r).
\]  

(27)

In the last part of eq. (27), we have put the surface integration of \( H(r) \times A(r) \) equal to zero by taking the surface much larger than the sample. (When the sample is isolated, the current must circulate in it, which looks like a magnetic dipole moment at a place far away from the sample. Then, since the corresponding vector potential and magnetic field behave as \( \sim r^{-2} \) and \( r^{-3} \), respectively \((r: \) distance from the sample\), the surface integration of their product disappears for an infinitely large surface.\[22\])

Exactly speaking, there is the Zeeman energy by the spin magnetization

\[
E_Z = -\int dr M_S(r) \cdot H(r).
\]  

(28)

However, the following analysis does not treat this effect. We will give a brief discussion about this point later. (Note that the Zeeman energy by the orbital magnetization due to the spontaneous current is included in eq. (21) through the vector potential.)
3.2 Zero temperature

Let us start with the case at $T = 0$. As discussed in §2, we introduce $|\psi\rangle$, which has the local current, $J_\psi(r)$, the total current, $J$, the induced magnetic field described by $A(r)$ and the local magnetization, $M_{S\psi}(r)$. We also introduce $|\phi\rangle$ defined by eq. (2) with the local current, $J_\phi(r) = J_\psi(r) + \delta J(r)$, the vector potential, $A_\phi(r) = A_\psi(r) + \delta A(r)$ and the local magnetization, $M_{S\phi}(r)$. Since the exponential factor in eq. (2) does not change spins of electrons, the magnetization should be the same between the two states, $|\psi\rangle$ and $|\phi\rangle$. Then, from eq. (23) for the sets $(A_\psi(r), J_\psi(r), M_{S\psi}(r))$ and $(A_\phi(r), J_\phi(r), M_{S\phi}(r))$, one finds

$$\triangle \delta A(r) = -\mu_0 \delta J(r), \tag{29}$$

where the explicit form of $\delta J(r)$ is derived from the expectation value of eq. (22) to be

$$\delta J(r) = \frac{e}{m} \langle \psi | \sum_\sigma \Psi^\dagger_\sigma(r) \Psi_\sigma(r) | \psi \rangle [\delta P - e\delta A(r)]. \tag{30}$$

The term with $O(\delta P^2)$ is dropped in eq. (30). Substituting eq. (31) into eq. (29) and putting $\delta A(r) \equiv \delta P_\sigma(r)$, we find that the equation of $\sigma(r)$ is independent of $\delta P$: $\delta A(r)$ and $\delta J(r)$ are therefore estimated as $O(\delta P)$. Keeping it in mind, we henceforth drop terms with $O(\delta P^2)$, by putting $\delta P$ small enough so that such terms can be neglected compared with the ones with $O(\delta P)$.

Difference between $|\psi\rangle$ and $|\phi\rangle$ in the field energy up to $O(\delta P)$ is

$$E_{M\phi} \equiv \frac{1}{2} \int dr A_\phi(r) \cdot J_\phi(r) = \frac{1}{2} \int dr A_\psi(r) \cdot J_\psi(r) + \int dr \delta A(r) \cdot J_\psi(r) \equiv E_{M\psi} + \delta E_M, \tag{31}$$

where we have put the surface integrations of $\delta A(r) \times (\nabla \times A_\psi(r))$ and $A_\psi(r) \times (\nabla \times A(r))$ equal to zero as noted before. Substitution of $J_\psi(r)$ into $\delta E_M$ gives

$$\delta E_M = \langle \psi | \int dr \delta A(r) \cdot \frac{e}{m} \sum_\sigma \Psi^\dagger_\sigma(r) [\hat{p} - eA_\psi(r)] \Psi_\sigma(r) | \psi \rangle. \tag{32}$$

where we have used the constraint $\nabla \cdot \delta A = \nabla \cdot [A_\phi(r) - A_\psi(r)] = 0$.

Calculation of the expectation value of the Hamiltonian is performed in the same manner as in §2. Noting that $A_\psi(r)$ ($A_\phi(r)$) has to be used in eq. (21) in evaluating $\langle \psi | H | \psi \rangle$ ($\langle \phi | H | \phi \rangle$), we obtain

$$\langle \phi | H | \phi \rangle = \langle \psi | H | \psi \rangle + \delta P \cdot \langle \psi | \int dr \frac{1}{m} \sum_\sigma \Psi^\dagger_\sigma(r) [\hat{p} - eA_\psi(r)] \Psi_\sigma(r) | \psi \rangle - \delta E_M \tag{33}$$

where

$$J_\psi = \langle \psi | \sum_\sigma \int dr \Psi^\dagger_\sigma(r) \frac{e}{m} [\hat{p} - eA_\psi(r)] \Psi_\sigma(r) | \psi \rangle. \tag{34}$$

From eqs. (31) and (33), the relation between the total energy of $|\psi\rangle$ ($E_\psi$) and that of $|\phi\rangle$ ($E_\phi$) is

$$E_\phi = E_{M\phi} + \langle \phi | H | \phi \rangle = E_\psi + \frac{1}{e} \delta P \cdot J_\psi. \tag{35}$$
When $\delta P$ is put opposite to $J_\psi$ in direction and small enough so that dropped terms with $O(\delta P^2)$ are less important in comparison with the second one in eq. (35), $|\phi\rangle$ always has a lower energy than $|\psi\rangle$; this means that any state with a finite total electric current is always accompanied by a state with a lower energy, and it cannot be the ground state.

The present theory does not deny the possibility that the Zeeman energy given by eq. (28) stabilizes the spontaneous current state with finite net current. When this kind of mechanism really works, however, the system has a finite local spin magnetization, which should be observed experimentally.

### 3.3 Finite Temperature

Discussion at finite temperature in the presence of the induced magnetic field is almost the same as that in section 2.3. The difference is that $E_\phi$ and $E_\psi$ are now given by

\[
\begin{align*}
E_\psi &= \sum_i w_i \left[ \langle \psi_i | H(\mathbf{A}\psi_i) | \psi_i \rangle + E_{M\psi_i} \right], \\
E_\phi &= \sum_i w_i \left[ \langle \phi_i | H(\mathbf{A}\phi_i) | \phi_i \rangle + E_{M\phi_i} \right].
\end{align*}
\]

(36)

where $H(\mathbf{A}\psi_i(\phi_i))$ means that $\mathbf{A}\psi_i(\phi_i)(r)$ is used in eq. (21). Each term in eq. (36) can be evaluated in the same way as in section 3.2, and we again reach eq. (17), where $J_\psi$ in the present case is given by

\[
J_\psi = \sum_i w_i \langle \psi_i | \sum_\sigma \int d\mathbf{r} \Psi_\sigma^\dagger(r) \frac{e}{m} [\mathbf{p} - e\mathbf{A}\psi(r)] \Psi_\sigma(r) | \psi_i \rangle.
\]

(37)

We thus arrive at the conclusion that the spontaneous current which is finite in total cannot be stable thermodynamically even if the magnetic field induced by the current is taken into account, except for the case that coupling between the spin magnetization and the magnetic field stabilizes this state.

### 4 Summary

In this paper, we have examined second quantized version of the Bloch theorem, which can explicitly cover the superconductivity, and applied it to the recently proposed spontaneous surface current. We have also extended the theorem to systems with (1) the magnetic field by the spontaneous current and (2) magnetic impurities.

Now, we summarize the results in this paper.

(1) Any state with finite spontaneous total current is not stable at all temperatures, even if the magnetic field induced by the current is taken into account. This statement is correct unless the Zeeman energy by the spin magnetization stabilizes this state.

(2) The recently proposed spontaneous surface current, which is also finite in total, is not stable at all temperatures. (Note that the Bloch theorem does not exclude the possibility of the surface current itself.) If a state with surface current is realized, the theorem states the existence of another counter current inside the system, which cancels the surface one as a whole.

We will explicitly show in a subsequent paper that the surface current state really has higher energy than the one with zero total current. [23]
Acknowledgements

We wish to thank Professor S. Takada for pointing out the importance of the Bloch theorem, reading this manuscript and giving us various comments. Thanks are due to Mr. M. Matsumoto for kindly teaching us his theory at the seminar in University of Tsukuba. One of the authors (Y.O.) would like to thank Professors K. Kubo, T. Soda, Dr. D. Hirashima and Mr. T. Mutou for discussions and comments. He is also indebted to Mr. H. Ikeda for sending him ref.18.

References

[1] T. Imai, T. Shimizu, H. Yasuoka, Y. Ueda and K. Kosuge: J. Phys. Soc. Jpn. 57 (1988) 2280.

[2] P. C. Hammel, M. Takigawa, R. H. Heffner, Z. Fisk and K. C. Ott: Phys. Rev. Lett. 63 (1989) 1992.

[3] Y. Kitaoka, S. Ohsugi, K. Ishida and K. Asayama: Physica C170 (1990) 189.

[4] Y. Ito, H. Yasuoka, Y. Fujiwara, Y. Ueda, T. Machi, I. Tomeno, K. Tai, N. Koshizuka and S. Tanaka: J. Phys. Soc. Jpn. 61 (1992) 1287.

[5] D. A. Wollman, D. J. Van Harlingen, W. C. Lee, D. M. Ginsberg and A. J. Legget: Phys. Rev. Lett. 71 (1993) 2134; D. A. Wollman, D. J. Van Harlingen, J. Giapintzakis and D. M. Ginsberg: ibid. 74 (1995) 797; D. J. Van Harlingen: Rev. Mod. Phys. 67 (1995) 515.

[6] C. C. Tsuei, J. R. Kirtley, C. C. Chi, L. S. Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun and M. B. Ketchen: Phys. Rev. Lett. 73 (1994) 593.

[7] I. Iguchi and Z. Wen: Phys. Rev. B49 (1994) 12388.

[8] A. Mathai, Y. Gim, R. C. Black, A. Amar and F. C. Wellstood: Phys. Rev. Lett. 74 (1995) 4523.

[9] K. Kuboki and M. Sigrist: J. Phys. Soc. Jpn. 65 (1996) 361.

[10] M. Sigrist, D. B. Bailey and R. B. Laughlin: Phys. Rev. Lett. 74 (1995) 3249.

[11] M. Matsumoto and H. Shiba: J. Phys. Soc. Jpn. 64 (1995) 3384.

[12] M. Matsumoto and H. Shiba: J. Phys. Soc. Jpn. 64 (1995) 4867.

[13] M. Matsumoto and H. Shiba, preprint.

[14] D. Bohm: Phys. Rev. 75 (1949) 502.

[15] H. G. Smith and J. O. Wilhelm, Rev. Mod. Phys. 7 (1935) 266.

[16] S. Takada and T. Izuyama: Prog. Theor. Phys. 41 (1969) 635.

[17] A. Haug: *Theoretical Solid State Physics* (Pergamon Press 1972) Vol.1 Chap.II.A.
[18] L. Brillouin: Proc. Roy. Soc. A

[19] M. Sigrist, T. M. Rice and K. Ueda: Phys. Rev. Lett. 63 (1989) 1727.

[20] M. Sigrist and K. Ueda: Rev. Mod. Phys. 63 (1991) 239.

[21] G. Vignale: Phys. Rev. B51 (1995) 2612.

[22] H. Takahashi: Electro-Magnetism (Shokabo 1959) Chap.4, [in Japanese].

[23] Y. Ohashi and T. Momoi: in preparation.
Figure Captions

Fig. 1: Schematic picture of the semi-infinite system. $V(r) = 0$ in the right hand side in the figure and $V(r) = \infty$ in the shaded region. In the figure, $z$-axis is perpendicular to the surface of the paper.

Fig. 2: (a) Assumed ground state, $|\psi\rangle$, which has a finite surface current. In the figure and also in the following figures, thick arrows represent the electric current. (b) A possible surface current state having a counter flow inside the system which cancels the former in total.

Fig. 3: (a) Circuit of spontaneous current. (b) A large ring with the width $L$ and the radius $R \gg L$. 
Fig. 1
Fig. 2
Fig. 3