Brief Paper

Design and implementation of an electromagnetic levitation system for active magnetic bearing wheels

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Abstract: In this study, an electromagnetic levitation system is developed as a prototype for developing active magnetic bearing wheels. The main mechanical parts of the electromagnetic levitation system consists of a rotor, a shaft, a cover and a base. A meaningful electromagnetic force, which is the minimal norm solution to an equation associated with the force and torques of the electromagnetic levitation system, is derived by using the singular value decomposition. A control system using the proportional-integral-derivative controller is developed to levitate the rotor at a target position against the force of gravity and regulate the two gimbal angles of the rotor. The numerical simulation and experimental results on the control of the electromagnetic levitation system are given to demonstrate the validity of the control design presented in this study.

1 Introduction

Spacecraft in orbit generally uses reaction wheels as an actuator for the spacecraft attitude control. As a reaction wheel, a ball bearing wheel, a magnetic bearing wheel and an active magnetic bearing wheel have mainly been used in this application. They commonly generate the control torque by spinning their wheels. First, a ball bearing wheel uses a ball bearing, which is a type of rolling element bearing using balls, to maintain the separation between the bearing races [1]. Second, a magnetic bearing wheel uses a magnetic bearing that can support a load by using magnetic levitation. Most magnetic bearing wheels use permanent magnets to carry a wheel, a control system to hold a wheel stable and power when a levitated wheel deviates from its target position [1]. Finally, an active magnetic bearing wheel levitates a rotating shaft by the principle of electromagnetic suspension. An active magnetic bearing wheel supports a wheel without physical contact and permits relative motion of a rotating shaft with low friction and no mechanical wear. An active magnetic bearing wheel generally consists of a wheel, an active magnetic bearing, an electromagnet assembly, a control system, power amplifiers to supply current to the electromagnets and gap sensors to provide the feedback required to control the position of the rotor within the gap [1].

Among three kinds of reaction wheels mentioned above, active magnetic bearing wheels generally exhibit very lower vibration than ball bearing wheels and magnetic bearing wheels. Since vibration is the critical factor for the high precision spacecraft attitude control, the active magnetic bearing wheel may be a desirable reaction wheel for the spacecraft attitude control. Among many components of active magnetic bearing wheels, the active magnetic bearing is the key component in quality assurance of active magnetic bearing wheels. Owing to the importance of the active magnetic bearing, many researchers have developed various kinds of active magnetic bearings and have studied their control methods (e.g. [2–10]): in [2], the authors designed and constructed a single degree of freedom magnetic suspension system. In [3], the authors proposed a robust fuzzy logic-based control scheme for a rotating active magnetic bearing system subject to harmonic disturbances and parameter uncertainties. In [4], the authors developed a single degree of freedom active magnetic bearing system and designed a high performance controller with $\mu$-synthesis. In [5], the authors studied the low-bias stabilisation of an active magnetic bearing subject to control-voltage saturation. In [6], the author designed an adaptive output backstepping controller to compute non-linear control currents of an active magnetic bearing system subjected to external acceleration disturbances. In [7], the author presented a methodology for the non-linear modelling and analysis of an active magnetic bearing system in the harmonic domain. In [8], the authors researched the relationship between a superconducting magnetic bearing and an active magnetic bearing in a superconducting attitude control and energy storage flywheel for spacecraft. In [9], the authors presented the application of an optimal state estimation and an optimal state feedback algorithm for the real-time control of an active magnetic bearing. In [10], the authors explored an attitude-sensing function of a magnetically suspended double-gimbal control moment gyroscope, which combines the attitude sensing and
the attitude control into a single device to achieve a compact lightweight spacecraft design.

In this paper, the author presents a novel electromagnetic levitation system that is designed and implemented as a prototype for developing active magnetic bearing wheels. The main mechanical parts of this electromagnetic levitation system consists of a rotor, a shaft, a cover and a base. Note that this electromagnetic levitation system does not include a mechanism for spinning the rotor around its rotating axis and presents a conceptual novelty compared to the previous active magnetic bearing wheels. The author derives a meaningful electromagnetic force, which is the minimal norm solution to an equation associated with the force and torques of the electromagnetic levitation system, by using the singular value decomposition [11]. The proportional-integral-derivative (PID) controller is used to levitate the rotor at a target position and regulate the two gimbal angles. The numerical simulation and experimental results on the control of the electromagnetic levitation system are given to demonstrate the control design presented in this paper.

The rest of this paper is organised as follows: In Section 2, a novel electromagnetic levitation system developed in this paper is introduced. In Section 3, a control system for the automatic control of the electromagnetic levitation system is presented. In Section 4, the numerical simulation and experimental results on the control of the electromagnetic levitation system are given to demonstrate the validity of the control design presented in this paper. Section 5 concludes with some remarks.

2 Electromagnetic levitation system

Fig. 1 shows the schematic of the electromagnetic levitation system developed in this paper. In this system, the cover protects the rotor, and the base supports the rotor, shaft and cover. The rotor has a radius of 100 mm and a thickness of 5 mm. The electromagnetic levitation system is designed to have four degrees of freedom and presents a conceptual novelty compared to the previous active magnetic bearing wheels. Specifically, one axis is for levitating the rotor up to 0.8 mm from the ground in the Z-axis, another axis is for rotating the rotor, and the other two axes are for gimbalizing the rotor within a small angle of ±0.2°. The four pairs of electromagnets and the four gap sensors are attached to the lower surface of the cover. The electromagnets are made by the SUS 410 ferromagnetic material, and the gap sensors are eddy current-type capacitive sensors. The power electronics, which will be explained in Section 3, generates the control currents and sends them to the coils of the four pairs of electromagnets to generate the electromagnetic forces. The electromagnetic forces are used to levitate the rotor at a target position and regulate the two gimbal angles of the rotor. The four gap sensors are used to measure the Z-axis displacement of the rotor precisely.

In Fig. 1, $E_i$ denotes the $i$th pair of electromagnets, $G_i$ denotes the $i$th gap sensor, $D_i$ is the distance from the centre of the rotor to the centre of a pair of electromagnets in the XY plane and given by $D_i = 19$ mm. Moreover, $L_{g_i}$ is the distance from the bottom surface of the $i$th gap sensor to the top surface of the rotor at the equilibrium state in the Z-axis and given by $L_{g_i} = 1$ mm, $i = 1, \ldots, 4$. $\delta L_{g_i}$, $i = 1, \ldots, 4$ are perturbations of each $L_{g_i}$ in the Z-axis, $L_{e_i}$ is the displacement from the bottom surface of the $i$th gap sensor to the top surface of the rotor in the Z-axis.

Fig. 1 Schematic of the electromagnetic levitation system

The dynamic equations of motion of the electromagnetic levitation system are given as follows

$$m\ddot{z} = F_i \quad (1)$$

$$I\ddot{\theta} = T_i \quad (2)$$

$$I\ddot{\phi} = T_i \quad (3)$$

In (1)-(3), $z$ is the displacement of the rotor in the Z-axis, $\phi$ and $\theta$ are the gimbal angles of the rotor in the X- and Y-axes, respectively, $F_i$ is the control force applied to the rotor in the Z-axis, and $T_i$ are the control torques applied to the rotor in the X- and Y-axes, respectively. Moreover, $m = 0.72$ kg is the mass of the rotor and $I = 877.367 \times 10^{-6}$ kg·m² is the inertia of the rotor for the X- and Y-axes. Note that the inertial cross products of the rotor are virtually zeros because the electromagnetic levitation system developed in this paper is manufactured by using an extremely precise machine and, thus, can be negligible when we
derive the dynamic equations of motion of the electromagnetic levitation system. Also, the control inputs \( F_i, T_i \) and \( T_y \) are independent for each signal \( z, \theta \) and \( \phi \), and their cross-influences are virtually zeros. The sensors and actuators have negligible dynamics because the sensors are only the eddy current-type capacitive sensors without dynamics and the actuators are only the four pairs of electromagnets without dynamics. In addition, we can assume that this system is a solid and rigid body without flexibilities and bending because the electromagnetic levitation system developed in this paper is made by a solid and rigid stainless steel.

If we consider the four pairs of electromagnets shown in Fig. 1, the control inputs \( F_i \) in (1), \( T_i \) in (2) and \( T_y \) in (3) can be represented as follows

\[
\sum_{i=1}^4 F_{ei} = F_z + mg 
\]

(4)

\[
D_x(F_{ei} - F_{ai}) = T_x 
\]

(5)

\[
D_x(F_{ei} - F_{ai}) = T_y 
\]

(6)

In (4)–(6), \( F_{ei} \) is the electromagnetic force generated by the \( i \)th pair of electromagnets, and \( g = 9.8 \text{ m/s}^2 \) is the acceleration of gravity. Then, (4)–(6) can be written by

\[
AF_e = u + u_g 
\]

where

\[
A \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & D_x & 0 & -D_x \\ -D_x & 0 & D_x & 0 \end{bmatrix}
\]

(8)

\[ F_e \triangleq [F_{e1}, F_{e2}, F_{e3}, F_{e4}]^T, \quad u \triangleq [F_z, T_x, T_y]^T \quad \text{and} \quad u_g \triangleq [mg \ 0 \ 0]^T. \]

After we design the control inputs \( F_i, T_i \) and \( T_y \), we have to determine the four electromagnetic forces \( F_{ei}, \ i = 1, \ldots, 4 \) by (7). In this case, (7) has infinitely many solutions since it is underdetermined with the three equations in the four unknowns \( F_{ei}, \ i = 1, \ldots, 4 \). Among many solutions for (7), the author derives the minimal norm solution by using the singular value decomposition \([11]\). Specifically, let the singular value decomposition of the matrix \( A \in \mathbb{R}^{3\times 4} \) in (8) be \( U \Sigma V^\top \) and define

\[
A^+ \triangleq V \Sigma^+ U^\top
\]

(9)

where \( U \in \mathbb{R}^{3\times 3} \) and \( V \in \mathbb{R}^{4\times 4} \) are orthogonal matrices, \( \Sigma \in \mathbb{R}^{3\times 4} \), and \( A^+ \in \mathbb{R}^{3\times 4} \) and \( \Sigma^+ \in \mathbb{R}^{4\times 3} \) denote the pseudo-inverse matrices of the matrices \( A \) and \( \Sigma \), respectively. With some calculations, we obtain the following by the singular value decomposition of the matrix \( A \) in (8)

\[
V = \frac{1}{2} \begin{bmatrix} 1 & 0 & -\sqrt{2} & -1 \\ 1 & \sqrt{2} & 0 & 1 \\ 1 & 0 & \sqrt{2} & -1 \\ 1 & -\sqrt{2} & 0 & 1 \end{bmatrix}
\]

(10)

\[
\Sigma^+ = \frac{1}{2} \begin{bmatrix} 1 & 0 & \sqrt{2} & 0 \\ 0 & D_x & 0 & \sqrt{2} \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & D_x \end{bmatrix}
\]

(11)

\[
U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

(12)

In (11), \( 0_{1 \times 3} \) implies the \( 1 \times 3 \) zero matrix. Then, with \( A^+ \) of (9), we obtain

\[
F_e = A^+ (u + u_g) 
\]

(13)

Moreover, the following condition holds for any other solution \( \hat{F}_e \) to (7) \([12]\)

\[
\|F_e\|_2 < \|\hat{F}_e\|_2
\]

(14)

where \( \| \cdot \|_2 \) denotes the Euclidean norm of a vector (i.e. for a vector \( x \in \mathbb{R}^n, \|x\|_2 \triangleq \sqrt{\sum_{i=1}^n x_i^2} \)). As a result, we see that \( F_e \) of \( (13) \) is the minimal norm solution to (7), which satisfies the condition \( (14) \) for any other solution \( \hat{F}_e \) to (7). It is remarkable that the minimal norm solution \( F_e \) of \( (13) \) is a novelty in the description of this type of electromagnetic levitation systems.

Since the two gimbal angles, \( \phi \) and \( \theta \), are very small, we can approximate \( \sin(\phi) \approx \phi \) and \( \sin(\theta) \approx \theta \) by the small angle approximation in this paper. Indeed, for the maximum gimbal angle 0.2° (\( \approx 0.0035 \text{ rad} \)), we obtain sin(0.0035) \( \approx 0.0035 \). Thus, the displacement from the bottom surface of the \( i \)th gap sensor to the top surface of the rotor in the \( z \)-axis, which is measured by the \( i \)th gap sensor, can be calculated as follows

\[
\begin{align*}
I_{g1} &= L_{g1} + \delta L_{g1} = L_{g1} - z + D_g \sin(\theta) - D_g \sin(\phi) \\
&\geq L_{g1} - z - D_g \theta - D_g \phi \\
I_{g2} &= L_{g2} + \delta L_{g2} = L_{g2} - z - D_g \sin(\theta) - D_g \sin(\phi) \\
&\geq L_{g2} - z - D_g \theta - D_g \phi \\
I_{g3} &= L_{g3} + \delta L_{g3} = L_{g3} - z - D_g \sin(\theta) + D_g \sin(\phi) \\
&\geq L_{g3} - z - D_g \theta + D_g \phi \\
I_{g4} &= L_{g4} + \delta L_{g4} = L_{g4} - z - D_g \sin(\theta) + D_g \sin(\phi) \\
&\geq L_{g4} - z - D_g \theta + D_g \phi
\end{align*}
\]

(15)

(16)

(17)

(18)

Then, from (15)–(18), the system state \( z, \phi \) and \( \theta \) can be calculated as follows

\[
\begin{align*}
z &= \frac{1}{4} \left( \sum_{i=1}^4 L_{gi} - \sum_{i=1}^4 I_{gi} \right) \\
\phi &= \frac{1}{4D_g} \left[ (I_{g1} + I_{g2} - I_{g3} - I_{g4}) \right] \\
&= (L_{g1} + L_{g2} - L_{g3} - L_{g4}) \\
\theta &= \frac{1}{4D_g} \left[ (I_{g1} + I_{g2} - I_{g3} - I_{g4}) \right] \\
&= (L_{g3} + L_{g4} - L_{g1} - L_{g2})
\end{align*}
\]

(19)

(20)

(21)
Similarly, the displacement from the bottom surface of the $i$th pair of electromagnets to the top surface of the rotor in the $Z$-axis can be calculated as follows

$$
l_i = L_{ci} + \delta L_{ci} = L_{ci} - z + D_z \sin(\theta)
$$

$$
\simeq L_{ci} - z + D_z \theta
$$

(22)

$$
l_i = L_{ci} + \delta L_{ci} = L_{ci} - z - D_z \sin(\phi)
$$

$$
\simeq L_{ci} - z - D_z \phi
$$

(23)

$$
l_i = L_{ci} + \delta L_{ci} = L_{ci} - z - D_z \sin(\theta)
$$

$$
\simeq L_{ci} - z - D_z \theta
$$

(24)

$$
l_i = L_{ci} + \delta L_{ci} = L_{ci} - z + D_z \sin(\phi)
$$

$$
\simeq L_{ci} - z + D_z \phi
$$

(25)

The electromagnetic forces in (13) are generated by the control currents supplied to the coils of the four pairs of electromagnets. By the Maxwell’s equation [13], one can obtain the following equation for the control currents supplied to the coils of the four pairs of electromagnets

$$
i_i = \frac{2l_{ci}}{n} \sqrt{\frac{F_0}{\mu_0 G}}, \quad i = 1, \ldots, 4
$$

(26)

where $i_i$ is the control current of the $i$th pair of electromagnets, $n = 240$ is the number of coil turn, $\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$ is the permeability constant of free space, $G = 50.265 \times 10^{-6} \text{m}^2$ is the cross-sectional area of a pair of electromagnets, and $F_0$, $i = 1, \ldots, 4$ and $l_{ci}$, $i = 1, \ldots, 4$ are given in (13) and (22)–(25), respectively. Note that each control current in (26) is limited by 1 A for safety.

### 3 Control system

A control system is designed and implemented to levitate the rotor up to 0.8 mm from the ground against the force of gravity and regulate the two gimbal angles of the rotor within $\pm 0.2^\circ$. The control system developed in this paper is actually run at 1 kHz.

The control flow of the control system for the automatic control of the electromagnetic levitation system is as follows: first, after the gap sensor electronics reads each output of the four gap sensors, it converts each output to voltage and sends each voltage to the analog multiplexer. The analogue-to-digital converter reads each voltage by sequentially selecting each channel of the analog multiplexer and digitises each voltage. After the digital signal processor, which is the TMS320C32 chip made by Texas Instruments company, receives each digital value of voltage, it calculates the displacement of the rotor, the two gimbal angles of the rotor, the displacements from the bottom surfaces of the four pairs of electromagnets to the top surface of the rotor, the control inputs designed by a user, the four electromagnetic forces and the control currents supplied to the coils of the four pairs of electromagnets and sends each digital value of the control currents to the digital-to-analog converter. The digital-to-analogue converter converts each digital value of the control currents to voltage and sends each voltage to the power electronics. Then, the power electronics generates each control current and sends it to each coil of the four pairs of electromagnets. Each control current is limited by 1 A for safety. In the power electronics, an electronic feedback circuit for regulating each control current is designed and implemented. From the result, the electromagnetic forces are generated by the control currents supplied to the coils of the four pairs of electromagnets and used to levitate the rotor at a target position and regulate the two gimbal angles of the rotor.

### 4 Numerical simulation and experimental results

This section shows the numerical simulation and experimental results on the control of the electromagnetic levitation system.

The following discretised PID controller is used to control the displacement of the rotor in (19) and the two gimbal angles of the rotor in (20) and (21) because this PID controller is easy for implementation in a computer and useful in practical applications

$$u(t) = K_p x_i(t) + K_i \sum_{i=0}^{t-1} x_i(i) + K_d \frac{x_i(t) - x_i(t-1)}{T_s}
$$

(27)

where $T_s$ is the sampling time and given by $T_s = 1 \text{ms}$, $x_i = x_i - x$ is the error between the system state $x \triangleq [z \phi \theta]^T$ and the target position state $z_i \triangleq [z \phi \theta]^T$ and $K_p \triangleq \text{diag}[K_{p_z}, K_{p_\phi}, K_{p_\theta}], K_d \triangleq \text{diag}[K_{d_z}, K_{d_\phi}, K_{d_\theta}]$ denote the $3 \times 3$ diagonal positive definite matrices.

The target position and target gimbal angles of the rotor are set to be $z_i = 0.3 \text{mm}$, $\phi_i = 0^\circ$ and $\theta_i = 0^\circ$, respectively. The update rate of the control system is set to be 1 kHz. The author initially decides the feedback gains of the PID controller in (27) that can achieve the control objective by adopting the well-known Ziegler–Nichols method [14], which is very useful to select the control gains of a PID-type controller for complex dynamic systems in practice, and then finely tunes the feedback gains of the PID controller in (27) by an experiment in order to obtain a desired control performance. Also, the anti-windup compensator is used to make the overshoot, caused by windup, as small as possible. According to the Ziegler–Nichols method [14], first, $K_i = K_d = \text{diag}[0, 0, 0]$ are set, and the proportional gain $K_p$ is then increased until the system just oscillates. The proportional gain is then multiplied by 0.6, and the integral and derivative gains are calculated as $K_p = 0.6 K_m, K_i = K_i (\omega_m / \pi)$ and $K_d = K_d (0.25 \pi / \omega_m)$ where $K_m \triangleq \text{diag}[K_{m_z}, K_{m_\phi}, K_{m_\theta}]$ denotes the $3 \times 3$ diagonal positive definite matrix with the gain elements at which the proportional system oscillates, and $\omega_m \triangleq \text{diag}[\omega_{m_z}, \omega_{m_\phi}, \omega_{m_\theta}]$ denotes the $3 \times 3$ diagonal positive definite matrix with the oscillation frequency elements. As a result, the feedback gains of the PID controller in (27) are chosen as follows: $K_p = 1000, K_i = K_d = 20, K_m = 5000, K_i = 8.33, K_d = 100$ and $K_d = K_d = 0.013$. As shown in (26), as the electromagnetic force is proportional to the inverse of the square of the position, the linearised model of the electromagnetic levitation system includes typically a feedback of the position. This leads to the necessity of a derivative feedback loop in the controller for stabilising the electromagnetic levitation system. For this reason, the application of the Ziegler–Nichols method to this electromagnetic levitation system is not straightforward. Note that the relatively small derivative feedback gains are chosen because there are visible noises in the experimental data and the problem of noisy
signals makes the use of large derivative action undesirable. Consequently, we see that the dominant feedback gains in this PID controller are $K_p$ and $K_i$, and, thus, one can obtain $K_m \approx 1666.67$, $K_{mg} = K_{mg} \approx 33.33$, $\omega_{mg} \approx 15.71$ rad/s and $\omega_{mg} = \omega_{mg} \approx 1.31$ rad/s from the definition of $K_p$ and $K_i$ in the Ziegler–Nichols method.

Fig. 2 shows the control flow diagram of the electromagnetic levitation system: first, after measuring the displacements from the bottom surfaces of the four gap sensors to the top surface of the rotor, the displacement of the rotor and the two gimbal angles of the rotor are calculated by (19) and (20) and (21), respectively. The PID controller in (27) makes the control input $u \triangleq [F_z, T_x, T_y]^T$. Then, the four electromagnetic forces and the displacements from the bottom surfaces of the four pairs of electromagnets to the top surface of the rotor are calculated by (13) and the (22)–(25), respectively. After the control currents are calculated by (26), they pass through the current limiters, which limit each control current by 1 A, and are supplied to the coils of the four pairs of electromagnets by the power electronics.

In the numerical simulation, it will be demonstrated that the electromagnetic force $F_z$ of (13) satisfies the condition...
Fig. 4 Time histories of the control currents, which are obtained by the numerical simulation

Fig. 5 Time histories of the Euclidean norm of the electromagnetic force, which are obtained by the numerical simulation
of (14) with respect to the following electromagnetic force \( \mathbf{F}_e \), which is another solution to (7):

\[
\mathbf{\hat{F}}_e = \begin{bmatrix} \frac{1}{2} \mathbf{F}_z + \frac{1}{2} (\mathbf{T}_i - \mathbf{T}_s) \mathbf{I} \mathbf{D}_e^{-1} - \frac{1}{2} \mathbf{mg} \\ \mathbf{T}_s \mathbf{D}_e + \mathbf{mg} \\ \frac{1}{2} \mathbf{F}_z - \frac{1}{2} (\mathbf{T}_i + \mathbf{T}_s) \mathbf{I} \mathbf{D}_e^{-1} - \frac{1}{2} \mathbf{mg} \end{bmatrix}
\]

\[\triangleq \begin{bmatrix} \mathbf{\hat{F}}_{e_1} \\ \mathbf{\hat{F}}_{e_2} \\ \mathbf{\hat{F}}_{e_3} \\ \mathbf{\hat{F}}_{e_4} \end{bmatrix} \tag{28}\]

With the initial system state given by \( z_{ini} = 0 \text{ mm} \), \( \dot{z}_{ini} = 0 \text{ mm/s} \), \( \phi_{ini} = \theta_{ini} = 0^\circ \) and \( \dot{\phi}_{ini} = \dot{\theta}_{ini} = 0^\circ/\text{s} \), Figs. 3–5 show the numerical simulation results on the control of the electromagnetic levitation system using \( \mathbf{F}_e \) of (13) and \( \mathbf{\hat{F}}_e \) of (28). From the numerical simulation results shown in Fig. 3, we see that the rotor reaches the target position \( z = 0.3 \text{ mm} \), and the control forces using \( \mathbf{F}_e \) of (13) and \( \mathbf{\hat{F}}_e \) of (28) show the same behaviours, respectively, because the difference between these numerical simulations is only using the different electromagnetic forces. Note that the trajectories of two gimbal angles and control torques are all zeros and, thus, omitted in Fig. 3. As shown in Fig. 4, each control current using \( \mathbf{F}_e \) of (13) reaches the same value because \( F_{e_i}, i = 1, \ldots, 4 \) of (13) become 0.25 mg and \( I_{e_i}, i = 1, \ldots, 4 \) of (22)–(25) become \( L_{e_i} - z = 0.3 \text{ mm} \), \( i = 1, \ldots, 4 \), in the steady-state region. On the other hand, the control currents of \( i_1 \) and \( i_2 \) using \( \mathbf{\hat{F}}_{e_1} \) and \( \mathbf{\hat{F}}_{e_2} \) of (28), respectively, reach the same value, and the control currents of \( i_3 \) and \( i_4 \) using \( \mathbf{\hat{F}}_{e_3} \) and \( \mathbf{\hat{F}}_{e_4} \) of (28), respectively, reach the same value because \( \mathbf{\hat{F}}_{e_1} \) and \( \mathbf{\hat{F}}_{e_2} \) of (28) become mg, \( \mathbf{\hat{F}}_{e_3} \) and \( \mathbf{\hat{F}}_{e_4} \) of (28) become \( -0.5 \text{ mg} \), and \( I_{e_i}, i = 1, \ldots, 4 \) of (22)–(25) become \( L_{e_i} - z = 0.3 \text{ mm} \), \( i = 1, \ldots, 4 \), in the steady-state region. From the numerical simulation results shown in Fig. 5, we see that the Euclidean norm of \( F_{e_i} \) of (13) is about 3.162 times smaller than that of \( \mathbf{\hat{F}}_e \) of (28) in the steady-state region because the Euclidean norms of \( F_{e_i} \) of (13) and \( \mathbf{\hat{F}}_e \) of (28) become 0.5 mg = 3.528 N and \( \sqrt{2} \times 0.5 \text{ mg} = 11.157 \text{ N} \) in the steady-state region, respectively. Therefore, the numerical simulation results shown in Fig. 5 illustrate that \( F_{e_i} \) of (13) satisfies the condition of (14) with respect to \( \mathbf{\hat{F}}_e \) of (28). In addition, the numerical simulation results shown in Fig. 5 imply that the electromagnetic levitation system using \( F_{e_i} \) of (13) consumes much less electric power, which is required to control the electromagnetic

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**Fig. 6** Time histories of the system state of the electromagnetic levitation system using the electromagnetic force \( F_{e_i} \) of (13), which are obtained by the experiment (a human hand presses down hard on the rotor to the ground at about 20 s, and it is removed from the rotor momentarily)
levitation system, than that using $\hat{F}_e$ of (28) because the electric power is proportional to the control currents supplied to the coils of the four pairs of electromagnets and the control currents are proportional to the electromagnetic forces by (26).

An experiment on the control of the electromagnetic levitation system is performed with the control system presented in Section 3. The experimental results on the control of the electromagnetic levitation system using the electromagnetic forces $F_{ei}, i = 1, \ldots, 4$ of (13) are shown in Figs. 6–9. As shown in Figs. 6 and 7, the PID controller designed in this paper successfully levitates the rotor at the target position $z_t = 0.3$ mm against the force of gravity with well regulating the two gimbal angles. The reason why the numerical simulation results shown in Fig. 3 are a little bit different to the experimental results shown in Figs. 6 and 7 is that the real initial system state may be different to the initial system state assumed in the numerical simulation and there may exist unmodelled dynamics, uncertain nonlinearities and external disturbances in the electromagnetic levitation system. Figs. 8 and 9 show the trajectories of control currents and Euclidean norm of $F_e$ of (13), respectively. If we compare the numerical simulation results shown in Figs. 4 and 5 with the experimental results shown in Figs. 8 and 9, we see that the trajectories of control currents and Euclidean norm of $F_e$ of (13) obtained by the experiment move around the values obtained by the numerical simulation. The reason why the control torques and the Euclidean norm of $F_e$ of (13) are a little bit increasing in Figs. 7 and 9 is that there exists an unknown friction force physically between the rotor and the shaft when the rotor is levitated and the two gimbal angles are regulated. As explained in Section 1, the existence of such a friction force is the inherent property of active magnetic bearing wheels. However, since each control current is limited by 1 A, the control torques as well as the Euclidean norm of $F_e$ of (13) are physically limited in the end. In addition, since the electromagnetic force is physically generated by the control current, we see that the electromagnetic force will be saturated if the corresponding control current is saturated. Since the control currents are limited for safety, the stability of the electromagnetic levitation system can be discussed in the set $\mathcal{A} = \{i \in \mathbb{R}^4\mid i_i \leq i_{max}, i = 1, \ldots, 4\}$ where $i_{max}$ denotes the maximum value of control current. In order to guarantee the stability of the electromagnetic levitation system in the set $\mathcal{A}$, $i_{max}$ should be chosen carefully such that the generated electromagnetic force is able to levitate the rotor at a target position and regulate the two gimbal angles of the rotor with overcoming the friction force between the rotor and the shaft. Therefore we see that the maximum value of control current plays an important role of guaranteeing the stability of the electromagnetic levitation system in the set $\mathcal{A}$ and, thus, this value should be adapted according to a given control purpose in practical applications.

In practical applications, it may happen that an external disturbance is applied to the electromagnetic levitation system, and this has a bad influence on the control of the position and two gimbal angles of the rotor. In order to simulate such an external disturbance in the experiment, a human hand presses down hard on the rotor to the ground at about 20 s, and it is removed from the rotor momentarily. This is a kind of severe external disturbance because a

![Fig. 7](image-url)  
*Fig. 7 Time histories of the control inputs of the electromagnetic levitation system using the electromagnetic force $F_e$ of (13), which are obtained by the experiment (a human hand presses down hard on the rotor to the ground at about 20 s, and it is removed from the rotor momentarily)*
Fig. 8  Time histories of the control currents of the electromagnetic levitation system using the electromagnetic force $F_e$ of (13), which are obtained by the experiment (a human hand presses down hard on the rotor to the ground at about 20 s, and it is removed from the rotor momentarily)

Fig. 9  Time histories of the Euclidean norm of the electromagnetic force $F_e$ of (13), which are obtained by the experiment
general external disturbance such as an external impact and an external vibration may cause the rotor to move slightly. As shown in Fig. 6, the system state becomes a steady state within 2 s after removing the external disturbance from the rotor. Also, the experimental results shown in Fig. 8 demonstrate that each control current increases to resist the external disturbance as the external disturbance applied to the rotor increases. This is a desirable feature of the PID controller designed in this paper because it implies more aggressive control action when the system state is far away from the equilibrium state. Consequently, this experiment illustrates that the control system developed in this paper is capable of resisting the external disturbance and recovering the system state from an unstable state caused by the external disturbance.

5 Conclusions

The electromagnetic levitation system developed in this paper is a prototype for developing active magnetic bearing wheels. A control system, which mainly consists of a gap sensor electronics, an analogue multiplexer, an analogue-to-digital converter, a digital signal processor, a digital-to-analogue converter and a power electronics, was developed to control the position and two gimbal angles of the rotor. The experimental results on the control of the electromagnetic levitation system demonstrated that the control system developed in this paper can levitate the rotor at a target position against the force of gravity and regulate the two gimbal angles. It was observed that there were little bits of differences between the numerical simulation and experimental results because the real initial system state may be different to the initial system state assumed in the numerical simulation and there may exist unmodelled dynamics, uncertain non-linearities and external disturbances including an unknown friction force between the rotor and the shaft in the electromagnetic levitation system. Further research topic is the refinement of the electromagnetic levitation system for the development of active magnetic bearing wheels. For this research, the electromagnetic levitation system may be modified such that the rotor is levitated in the radial direction and the rotor is spined around its rotating axis without physical contact.

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