Spin-independent and double-spin $\cos\phi$ asymmetries in semi-inclusive pion electroproduction

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We consider the $\cos\phi$ dependence of the longitudinal double-spin asymmetry for charged pion electroproduction in semi-inclusive deep inelastic scattering, emphasizing intrinsic transverse momentum effects. This azimuthal asymmetry allows to measure the $\cos\phi$ moments of the unpolarized and double-spin cross-section, simultaneously. The size of the asymmetry, in the approximation where all twist-3 interaction-dependent distribution and fragmentation functions are set to zero, is estimated for HERMES kinematics; both the spin-independent and the double-spin $\cos\phi$ moments are predicted to be sizable and negative.

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In the context of asymptotically free quantum chromodynamics (QCD) quarks and gluons should be produced as free particles. However, as they are not observed, non-perturbative confining effects become crucial in the amount of partons in the initial state and the formation of hadrons in the final state. Such effects, at present, cannot be calculated from first principles. They are parameterized in an effective way by introducing longitudinal and transverse (the so-called “intrinsic” transverse motion) degrees of freedom in the parton distribution and fragmentation functions. One of the most interesting consequences of non-zero intrinsic transverse momentum of partons in hadrons is the non trivial azimuthal dependence of the cross-sections for hadron production in hard scattering processes.

We focus on semi-inclusive deep inelastic scattering (DIS), $eN \to ehX$. Defining a coordinate system in the laboratory frame with the $z$ axis along the momentum transfer $q = k_1 - k_2$ between the initial and final lepton and the $x$ axis in the leptonic plane, the component of the detected hadron momentum transverse to $q$, $P_{h \perp}$, and its azimuthal orientation, $\phi$ (see Fig.1), provide interesting variables to study non-perturbative [1–3] and perturbative effects [4,5]. Recently, a particular $\cos\phi$ moment of the polarized cross-section has been considered [6] and a sizable asymmetry, in a Wandzura-Wilczek (WW) like approximation for $\pi^+$ electroproduction, was predicted.

We consider here the $\cos\phi$ azimuthal dependence of the usual longitudinal double-spin asymmetry:

$$A_{LL} = \frac{d\sigma^{++} + d\sigma^{--} - d\sigma^{+-} - d\sigma^{-+}}{d\sigma^{++} + d\sigma^{--} + d\sigma^{+-} + d\sigma^{-+}},$$

where the subscript $LL$ denotes the longitudinal polarization of the beam and target respectively and $d\sigma$ is a shorthand notation for $d\sigma^{eN\to ehX}/dx \, dy \, dz \, d^2P_{h \perp}$; the superscripts $++, -- (+-, -+)$ denote the helicity states of the beam and target respectively, corresponding to antiparallel (parallel) polarization$^1$, and $x$, $y$, and $z$, are the standard lepton-prodution variables defined as

$$x = \frac{Q^2}{2(P \cdot q)}, \quad y = \frac{P \cdot q}{P \cdot k_1}, \quad z = \frac{P \cdot P_h}{P \cdot q},$$

where $k_1$ and $P_h$ are the four-momenta of the incoming charged lepton and of the observed hadron, respectively.

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$^1$It leads to a positive $g_1(x)$. 

FIG. 1. The kinematics of semi-inclusive DIS.
As it will be shown, the most interesting consequence of this asymmetry is that it allows to determine both the spin-independent and double-spin $\cos \phi$ moments of the cross-section, simultaneously, without any dilution from geometrical acceptance of the spectrometer.

Due to parity conservation of the electromagnetic and strong interactions, Eq. (1) can be written in terms of the spin-independent ($\sigma_{UU} \equiv (d\sigma^{++} + d\sigma^{--} + d\sigma^{+-} + d\sigma^{-+})/4$) and double-spin ($\Delta\sigma_{LL} \equiv (d\sigma^{++} + d\sigma^{--} - d\sigma^{+-} - d\sigma^{-+})/2$) cross-sections for semi-inclusive DIS,

$$A_{LL} = \frac{\Delta\sigma_{LL}}{2\sigma_{UU}},$$

where, in general,

$$\Delta\sigma_{LL} = \sum_{m=1}^{\infty} \Delta\sigma_{LL}^{m} \cos\left([m-1] \cdot \phi\right),$$

$$\sigma_{UU} = \sum_{m=1}^{\infty} \sigma_{UU}^{m} \cos\left([m-1] \cdot \phi\right).$$

The subscripts $U$ and $L$ stand for unpolarized and longitudinally polarized beam and target. At sub-leading ($1/Q$) order one has non-vanishing spin-dependent and double-spin $\cos \phi$ asymmetries: they originate from non-perturbative effects, both kinematical [1,3,6] and dynamical [2], and from perturbative [4] effects. A $\cos 2\phi$ asymmetry only appears at order $1/Q^2$ [1,7], unless one allows for time-reversal odd structure functions [8]. We do not consider such contributions here. Then, up to sub-leading order $1/Q$, Eq.(2) can be rewritten as

$$A_{LL} = \frac{\Delta\sigma_{LL}^1/2\sigma_{UU}^1 + \langle \cos \phi \rangle_{LL} \cdot \cos \phi}{1 + 2 \langle \cos \phi \rangle_{UU} \cdot \cos \phi},$$

where $\langle \cos \phi \rangle_{UU}$ and $\langle \cos \phi \rangle_{LL}$ are the unpolarized and double polarized $\cos \phi$ moments, respectively:

$$\langle \cos \phi \rangle_{UU} \equiv \frac{\int d\phi \cdot \sigma_{UU} \cdot \cos \phi}{\int d\phi \cdot \sigma} = \frac{\sigma_{UU}^2}{2\sigma_{UU}^1},$$

$$\langle \cos \phi \rangle_{LL} \equiv \frac{\int d\phi \cdot \Delta\sigma_{LL} \cdot \cos \phi}{\int d\phi \cdot \sigma} = \frac{\Delta\sigma_{LL}^2}{2\sigma_{UU}^1}. \tag{7}$$

Eq. (5) shows how a measurement of the $\cos \phi$ dependence of $A_{LL}$ allows a determination of the moments (6) and (7) ($\Delta\sigma_{LL}^1$ and $\sigma_{UU}^1$ are given by the usual collinear partonic expressions). In order to see whether or not such a $\cos \phi$ dependence is significant and detectable, we give an estimate of $A_{LL}(\cos \phi)$ at HERMES energies. To this purpose we need to evaluate the $\cos \phi$-moments and we do it by proceeding in the same way as in Ref. [6], i.e. we use the approximation that all interaction-dependent functions are equal to zero. The explicit formulas corresponding to this approximation are given in Ref. [6].

Let us consider the detector acceptance effects in the $\cos \phi$ weighted $A_{(LL)_{ab}}^{\cos \phi}$ asymmetry defined as [6]

$$A_{(LL)_{ab}}^{\cos \phi} = \int d\phi \cos \phi \cdot \frac{\Delta\sigma_{LL}(\phi)}{\int d\phi \cdot \sigma_{UU}(\phi)}.$$ \tag{8}

Here, the measured polarized (unpolarized) cross section for semi-inclusive DIS is the product of $\Delta\sigma_{LL} \cdot \epsilon(\phi)$, where $\epsilon(\phi)$ is the detector acceptance. It can be expanded in Fourier series,

$$\epsilon(\phi) = C_0 + \sum_{m=1}^{\infty} \left[ C_m \cos(m \cdot \phi) + D_m \sin(m \cdot \phi) \right]. \tag{9}$$

Using the Eqs.(3) and (4) and integrating over $\phi$ one obtains (up to sub-leading $1/Q$ order)

$$A_{(LL)_{ab}}^{\cos \phi} = \frac{4C_1 \cdot \Delta\sigma_{LL}^1 + C_0 \cdot \Delta\sigma_{LL}^2 + \frac{1}{2}C_2 \cdot \Delta\sigma_{LL}^2}{2C_0 \cdot \sigma_{UU} + C_1 \cdot \sigma_{UU}^1}. \tag{10}$$

So, the asymmetry clearly depends on the detector acceptance. On the contrary, the acceptance effects, being independent of beam and target helicities, cancel out in the ratio which gives the cross section azimuthal asymmetry of Eq.(1). It is worth noticing that the $\phi$-integrated asymmetry $A_{LL}$ is also sensitive to the acceptance, which may affect the determination and interpretation of $A_1$ in semi-inclusive DIS (let us denote it by $A_1^h$):

$$A_1^h \equiv A_{LL} = \frac{\int d\phi \cdot \Delta\sigma_{LL} \cdot \epsilon(\phi)}{\int d\phi \cdot \sigma_{UU} \cdot \epsilon(\phi)}, \tag{11}$$

which after the integration over $\phi$ becomes (again up to sub-leading $1/Q$ order),

$$A_1^h = \frac{\Delta\sigma_{LL}^1/\sigma_{UU}^1 + \frac{\zeta_0}{\zeta_0^2} \cdot \langle \cos \phi \rangle_{LL}}{1 + \frac{\zeta_0}{\zeta_0^2} \cdot \langle \cos \phi \rangle_{UU}}. \tag{12}$$

Therefore, aside from their own physical interest, the $\cos \phi$ moments must be carefully determined in order to take into account their effects on the $\phi$-integrated semi-inclusive asymmetries $A_1^h$.

In Figs. 2 and 3, the unpolarized and doubly polarized $\cos \phi$ moments defined by Eqs. (6) and (7) for $\pi^+$ production on a proton target are shown as functions of $x$ for three different values of the mean squared transverse momentum $\langle p_T^2 \rangle$ of the initial parton.
The curves are calculated by integrating over HERMES kinematical ranges, corresponding to 1 GeV \( \lesssim Q^2 \lesssim 15 \) GeV\(^2\), 4.5 GeV \( \lesssim E_x \lesssim 13.5 \) GeV, 0.2 \( \lesssim z \lesssim 0.7\), and 0.2 \( \lesssim y \lesssim 0.8\) and taking \( \langle P_{h\perp} \rangle = 0.4 \) GeV as input [9]. We use \( Q^2\)-independent parameterizations for the distribution, \( f_1(x)\), \( g_1(x)\) [10], and fragmentation, \( D_1(x)\) [11], functions, as well as the recent sets of LO distribution [12,13] and fragmentation [14] functions. From Fig. 2 one can see that the result for spin-independent azimuthal asymmetry is insensitive to the input parameterizations choice due to the cancellation of effects in the \\
ratio defining \( \langle \cos\phi \rangle_{LL} \), while double-spin asymmetry (dash-dotted curves in Fig. 3) displays essential changes at \( x < 0.2\). Note that the change in behavior of the curves at \( x \approx 0.36\) is only due to the integrations of chosen kinematical ranges.

The figures show that, within WW approximation, both spin-independent and double-spin \( \cos\phi \) moments are negative and large enough in magnitude to be measurable (in particular the spin-independent one) at HERMES. The results for \( \pi^- \) are similar and only slightly smaller. The numerical results for \( \langle \cos\phi \rangle_{LL} \) given in Fig. 3 and of \( A_{LL}^{\cos\phi} \) given in Fig. 2 of Ref. [6] are consistent if one takes into account that \( \langle \cos\phi \rangle_{LL} \approx \langle P_{h\perp} \rangle \cdot A_{LL}^{\cos\phi} \), the choice of the different \( \langle P_{h\perp} \rangle \), and the linear dependence on \( \langle p_T^2 \rangle \). The “kinematical” contribution to \( \langle \cos\phi \rangle_{LL} \) coming from the transverse component of the target polarization is small [6] and was not taken into account.

Our estimates also indicate that \( \cos\phi \)-moments are very sensitive to the \( \langle p_T^2 \rangle \) value. This underlines the importance of having a reliable value of \( \langle p_T^2 \rangle \), at least for the considered kinematical regions. On the other hand, the measurement of the moments, via Eq. (5), may provide an estimate of \( \langle p_T^2 \rangle \).
the averages generally depend on. This does not allow to consider the \( \cos \phi \) asymmetry consistently at different kinematical conditions and obtain some constraints on mean transverse momenta from available data \cite{16,17}. In the considered kinematical regions, \( \langle z^2 \rangle \) is small \((\sim 0.2)\) and then its contribution is strongly suppressed. Then \( k_T \) gives the main contribution to \( \langle p^T_{h \perp} \rangle \). Using the values \( \langle k_T^2 \rangle = (0.44)^2 \text{GeV}^2 \) \cite{18} and \( \langle p^2_T \rangle = (0.7)^2 \text{GeV}^2 \), the corresponding values of \( \langle p^2_{h \perp} \rangle \) are 0.24, 0.265, and 0.29 \text{GeV}^2, respectively; they are reasonable for HERMES kinematics. It is worth noticing that these values of the parameters lead to the same results for \( \cos \phi \) moments if one integrates over \( p_{h \perp} \) assuming a Gaussian transverse momentum dependence of distribution and fragmentation functions.

In Fig. 4 the \( A_{LL} \) asymmetry of Eq.(5) for \( \pi^+ \) production on a proton target is presented as a function of the azimuthal angle \( \phi \) at \( \langle p^2_T \rangle = (0.7)^2 \text{GeV}^2 \), and for different values of \( x \). The strong dependence of the magnitudes of the double-spin \( \cos \phi \) and \( \phi \)-independent asymmetries on the input parameterizations at low \( x \)-region leads to the changes of \( A_{LL} \) asymmetry shown in Fig. 4. As it is seen, the asymmetry is large and well detectable experimentally. Hence, it can provide the simultaneous measurement of the unpolarized and double polarized \( \cos \phi \) moments.

In conclusion, we have considered effects related to parton intrinsic motion, which, in semi-inclusive DIS processes, manifest as azimuthal dependences of cross-sections; in particular, we have examined the \( \cos \phi \) dependence of the double longitudinal spin asymmetry for charged pion electroproduction. This dependence has been shown to be measurable; it depends on and allows the simultaneous determination of the spin-independent and double-spin \( \cos \phi \)-moments of the cross-section.

The sizes of these moments, in the approximation where all twist-3 interaction-dependent distribution and fragmentation functions are set to zero, is estimated for HERMES kinematical configurations; they turn out to be significantly large in magnitude and negative. The results also indicate a great sensitivity to the choice of the partons intrinsic average transverse momentum; they might provide a way of access and an estimate for the value of \( \langle p^2_T \rangle \).

The proposed asymmetry is measurable in running or planned experiments at HERMES, COMPASS, JLAB (upgraded) and may answer the question of the importance of twist-3 contributions in semi-inclusive DIS as well as provide some information on the \( p_T \) behavior of the structure functions \( f_1(x) \) and \( g_1(x) \).

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