Research of the process of functioning of hierarchical multi-level complex organizational systems

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Abstract. In the work the process of increase of efficiency in the functioning of hierarchical multi-level complicated organizational systems (HMCOS) is considered and the elements of operation in the considered systems. To solve this problem the tension level of HMCOS operation is involved. Basing on the formulated principles some normative parameters of HMCOS are defined. Validation of the operation process for HMCOS is performed by the normative parameters in the certain moments of time. Considering all of the stages in HMCOS functioning it was made a conclusion that while choosing the normative parameters for the estimates of functioning of the organizational systems in order to get uniform distribution of tension between the staff it seems reasonable to use entropy model.

1. Introduction

A base for increase of efficiency in the functioning of the hierarchical multi-level complicated organizational systems (HMCOS) is performing of analysis, as well as planning of the works’ execution. Let us assume that functioning of HMCOS elements is described by the uniform tasks and procedures even under the effect of external environment. It is also necessary to account for the fact that the main task in the functioning of any HMCOS elements is attaining of the assigned goals.

The process of attaining the assigned goals should be realized on the basis of the base parameters (information) $x^d$. The leaders of HMCOS at the macro-level form the main goals that prove to be the base for the accomplishment of the planning process and validation of functioning for such systems. Average executives of HMCOS provide the process of attaining the goals at the micro-level when defining restrictions to the feasible intensity of their work. The authors assume that the process of HMCOS functioning is determinate.
2. Materials and methods

In order to provide HMCOS functioning on the basis of requirements to the normative documents it is required to standardize, systemize and optimize its basic parameters (information) determined on the basis of the normative documents and to determine criteria of functioning for this system \( H(x^H, x) \), on the set of these base parameters (information) \([1, 2]\).

The difficulties of performing this process are comprised in formation of HMCOS goals which may be probably inadequate in the process of formation. Determination of the goals, as a rule, does not depend on HMCOS structure (centralized, de-centralized and so on).

In the de-centralized structure any element possesses certain self-sufficiency during its operation while attaining its own goal. Therefore, the process of functioning of any element can be described in the local-optimal state \( W_i(x^H_i) \) on a set of admissible states \( x \).

\[
W_i(x^H_i) = \text{Arg}\max_{x_i \in X_i} f_i(x^H_i, x_i)
\]  
(1)

We consider the function for determination of efficiency in the operation of HMCOS elements \( f(x^H_i, x_i) \) the determined one. Since the models developed for HMCOS elements can not be always adequate to the goals of the elements in this system, therefore it seems reasonable to form a set: \( L(x^H_i), i=1, n \), which will determine identification of the goals at the specified normative parameters \( x^H \).

Therefore, the problem of formation of the consistent system for stimulating \( W^* \) is expressed as:

\[
W_i(x^H_i)L_i(x^H_i), i = 1, n.
\]  
(2)

Solution of the problem concerned with formation of consistent HMCOS \( W^* \) on a set of the additive systems extended when estimating operation of the additive system takes the form of,

\[
W = \{f_i(x^H_i, x_i), i = 1, n\},
\]  
(3)

\[
f_i(x^H_i, x_i) = \sum_{j=1}^{m} v_{ij}(x^H_i, x_i) x_{ij}/x^H_{ij},
\]  
(4)

where the weight functions \( v_{ij}(x^H_i, x_i) \) satisfy the restrictions:

\[
\sum_{j=1}^{m} v_{ij}(x^H_i, x_i) = 1.0 \leq v_{ij}(x^H_i, x_i) \leq 1.
\]  
(5)

Validation of the operation process for HMCOS is needed to be accomplished by the normative parameters \( x^H_i(t) \) in the moment of time \( t \). Some goals determined on the basis of normative documents prove to be basic ones with the actual parameters of \( x^H_i(t), j \in \mathcal{V}^\prime \) and analysis of the functioning of the considered systems is performed by these parameters \([1,3]\).

At the micro-level functioning of HMCOS elements with their normative parameters can be corrected by intensity of their operation.

In order to determine the needed measure of normative space \( x^H_i(t) \) it is required to trace dynamics of the change of these normative values in the moment of planning \( t - \Delta t \) due to the rate of the change of parameter \( s_{ij} (t - \Delta t), i = 1, n; j = 1, m \) for each of the element \( s_{ij} (t - \Delta t); \) now, let us assume that the rate of change for parameter \( j \) is not varied during the period of time \( \Delta t \).

The level of operation intensity (LOI) in order to attain normative \( x^H_{ij} \) is described by expression

\[
F^H_{ij}(t) = (x^H_{ij}(t) - x_{ij}(t - \Delta t))/s_{ij}(t - \Delta t)
\]  
(6)

Consider, that \( F^H_{ij} \) is a tension of functioning (TF) over the corresponding parameters. The value of LOI for \( i \)-th element by \( j \)-th parameter is determined by an expression:

\[
F_{ij}(t) = (x_{ij}(t) - x_{ij}(t - \Delta t))/s_{ij}(t - \Delta t).
\]  
(7)
Operation process is considered to be stable at the time when the values of parameters for this system are ultimate ones. It is also necessary to determine maximum function $F^*_{ij}(t)$, which should not exceed the values of LOI $\Sigma F^H_{ij}(t)$ and thus to calculate normative values basing on the expression:

$$\sum_{j=1}^{m} F_{ij}^m (t) = F^0_{ij}(t), j = 1, m; i = 1, n.$$  

where $F^0_{ij}(t)$ are the estimates of $F^*_{ij}(t)$ determined in the expert way on the basis of the level of the operational tension for the corresponding HCMOS elements in the moment of planning $t - \Delta t$.

The values of parameters $x^H(t)$ are required to be restricted either on the basis of the expert estimates or by the values of each parameter $x_{ij}$, and any of them has its maximal value $x_{ij}^{\max(t)}$ in the considered interval of time.

For the base normative values there are certain restrictions

$$\sum_{i=1}^{n} x_{ij}^H (t) = x_{ij}^d, j \in V^d.$$  

In this case the normative values of $x^H(t)$ under the formed restrictions are determined by maximization of the efficiency criterion for HCMOS which is estimated by the total value of TF:

$$F_j = \sum_{i=1}^{n} \sum_{j=1}^{m} F^H_{ij} (t).$$  

1. Target planning of HCMOS operation ensures the use of maximization of objective function under specified restrictions. If the chosen models can not adequately describe HMOS operation then it is required to determine maximal tension of the operation for HCMOS elements subject to the fact that this tension is uniformly distributed over the elements of this system.

2. Formation of the normative parameters of HMCOS is based on the principle allowing to specify maximum values of these parameters for each of HMCOS element, and in this case the staff is obliged to attain maximal normative values.

3. If an employee of HMCOS decreases his level of indexes in his work then he will need to increase his working efficiency.

4. Sometimes an employee is recommended to attain the middle values of the operation parameters of the system in his work.

It may be quite difficult to set objective function for some of HMCOS due to a great number of the main parameters, specifying the functioning of this system which prove to be parameters of the HMCOS objective function [4,5,6].

In some cases objective function is described on the basis of deviations from the normative values of parameters of HMCOS functioning, which are reasonably to be presented in the quadratic form (non-linear task) [2,3].

Determination of the maximum of objective function for HMCOS can optionally result in the increase of the functioning efficiency for this system since the statement of the optimization problem itself is rather complicated problem.

This task assumes the accomplishment of the uniform distribution between the employees of the following: functional tasks, distribution of resources, raw materials, etc. As an example, let us consider optimization distribution of the functional tasks for HMCOS employees. It is required to perform distribution of the functional tasks while accomplishment of the works $P_a$ and $P_b$ between a number of the employees $M$.

For every of the employees $j$ his labor efficiency while the accomplishment of the works $P_a$ and $P_b$ is considered to be known.
There is a need to plan the accomplishment of works by the employees according to their functional tasks in such a way that the amount of work \( P_b - y_i \) would be equal to \( R \), while the amount of work for \( P_a - x_i \) should be maximal. This problem is solved by linear programming methods.

\[
\begin{align*}
\sum_{i=1}^{N} x_i + y_i &= \sum_{i=1}^{N} s_i, x_i, y_i \geq 0, j = 1, \ldots, M \quad (11) \\
\sum_{i=1}^{j} b_i y_i &= R \quad (12) \\
\sum_{i=1}^{j} a_i x_i &\rightarrow \text{max} \quad (13)
\end{align*}
\]

All of \( M \) employees are divided in three subsets: \( M_A \) (execute the work \( P_a \)), \( M_B \) (execute the work \( P_b \)) and \( M_0 \) (execute the works \( P_a \) and \( P_b \) at one and the same time) at the following limitations:

\[
\begin{align*}
b_j / a_j &\geq b_j / a_j, j \in M_B, \quad (14) \\
b_j / a_j &\leq b_j / a_j, j \in M_A, \quad (15)
\end{align*}
\]

HMCOS employees executing the works \( P_a \) and \( P_b \) are also the employees of the corresponding organization departments. Within these departments just the same task may be solved in order to distribute more efficiently the tasks \( P_a \) and \( P_b \). In this case it is required to use a complex approach [6,7,8].

These problems appear due to the fact that during the analysis of the functioning it is rather difficult to obtain complete information about functioning of the system. Hence, at the comprehensive description of HMCOS functioning the employees will get and have the accurate functional tasks concerning execution of their works.

Using allocation technique while solving the task it may occur that the maximum of execution of the work \( P_a \) by HMCOS employees cannot be determined. In this case the main problem is in the distribution of the functional tasks concerning execution of the work \( P_b \); so, the resources of \( S = \sum_{j=1}^{M} s_j \) (\( j \) – every of the employees is provided by the amount of work \( s_j * R / s \) in accordance with the functional tasks) are allocated between these employees.

The rest of resources required for the accomplishment of the tasks by HMCOS employees are distributed for the execution of the work \( P_a \). Such an approach is possible only when \( a_j, b_j \) are uniform, i.e. they do not change when distributing the functional tasks between employees.

When the problem has several restrictions concerning the distribution of the functional tasks between HMCOS employees, then it is necessary to use balancing adjustment technique which implements integration of the indexes by allocation method and thus ensures solving this problem in more efficient way.

Let us present an example of distribution of the functional tasks (in total number of \( \sigma \)) between HMCOS employees (\( M \) is their total number) basing on the expression:

\[
x_j \geq 0, j = 1, M, \sum_{j=1}^{M} x_j = \sigma. \quad (16)
\]

where: \( x_j \) – are functional tasks of \( j \)-th HMCOS employee.

If expression (16) is not right, then it is required to distribute the functional tasks between HMCOS employees according to the proportionally ideal distribution \( \{x_j^0\} \) and therefore expression (16) takes the form:

\[
x_j = c x_j^0, c = \sigma / \sum x_j^0. \quad (17)
\]

If HMCOS employees are divided by \( \sigma \) disjoint sets (groups) and for each element of this set is allocated the same amount of the functional tasks with their own restrictions, then it is possible to write down expression:
\[
\sum_{j \in M} x_j = \sigma_k, k = 1, e,
\]  \tag{18}

Then it is necessary to consequently use restrictions (18), each time normalizing that part of vector \( \{x_j\} \), which is involved in the next restriction. Specifying initial value for the vectors \( x_j = x_0^j \) and performing linear search of all the restrictions for each of the vectors one can consider that

\[
c_k = \frac{\sigma_k}{\sum_{j \in M_k} x_j}.
\]  \tag{19}

and thus we obtain

\[
x_i = c_k x_i, j \in M_k
\]  \tag{20}

For the expression (20) we assume that the primary values of the elements in the set (entropy or equilibrium distribution) \( \{x_j\} \) is equal to \( x_j = x_0^j \), where \( x_0^j \) is the initial value of \( x_j \); such cyclic approach is named balancing allotment technique.

Using this method maximization of (18) is implemented (weighted entropy):

\[
\sum_{j=1}^{M} x_j \ast \ln \left( \frac{x_0^j}{x_j} \right) \rightarrow \max
\]  \tag{21}

Equilibrium distribution must demonstrate the following properties [6,9]:

1). Expressions (10) and (20) determine the entropy allotment. It depends on \( x_0^j \), as well as on the right parts of restrictions (18); moreover, it is an inner point of polyhedron of the admissible solutions.

2). Multiplying of vector for the right parts of (18) and vector \( \{x_0^j\} \) by scalar value results in multiplying of allotment by the same scalar value.

3). Fixation of variable \( x_{j_1} \) at the level of \( x_{j_1}^* \) does not lead to the change of allotment proportions, calculated for the rest of \( j \in M \).

For the equilibrium distribution of tension and with the specified criterion of planning it is possible to use the entropy models [9] and by estimating the value of the total tension with the use of expression

\[
F = F_1 + \sum_{i=1}^{n} \sum_{j=1}^{m} F_{ij}^H(t) \ln \left( \frac{\Delta t/F_{ij}^{H}(t)}{} \right).
\]  \tag{22}

where \( \{F_{ij}^H(t)\} \) is the entropy function of the distribution for intensity of the functional tasks between HMCOS employees, and \( F \) is the entropy function.

\[
F = \sum_{i=1}^{n} \sum_{j=1}^{m} F_{ij}^H(t) \ln \left( e\Delta t/F_{ij}^{H}(t) \right),
\]  \tag{23}

Since the gradient of \( F \) is equal to zero in the point \( F_{ij}(t)=\Delta t, j=1,m, i=1,n \), then while maximization (23) a uniform distribution of intensity levels \( F_i^0(i) \) is obtained.

The problem of maximization for the entropy function (23) with restrictions (8):

\[
\sum_{j=1}^{m} F_{ij}^H(t) = F_0^0(t), i = 1, n,
\]

\[
\sum_{i=1}^{n} x_{ij}^H(t) = x_{ij}^d, j \in Z^d,
\]

which for the main variables can be represented by expression:

\[
\sum_{i=1}^{n} F_{ij}^H(t) = \frac{x_{ij}^d - \sum_{i=1}^{n} x_{ij}(t-\Delta t)}{x_{ij}(t-\Delta t)}, j \in V^d,
\]  \tag{24}

and so it is possible to use efficient algorithms for solution by the balancing adjustment of variables over the restrictions [4,7].
3. Conclusion
Assessment of the functioning for the organizational systems (OS) is a basis for stimulating of the employees’ work that is a base of the integrated mechanism of organization functioning. The estimate of efficiency of OS functioning is required to coordinate with the planned normative values of the organization.

When choosing the normative values for the estimation of the functioning efficiency it is required to account for the capabilities of OS employees while executing those normative units. To do this it is possible to use simply determined characteristic of dynamics in the change of the values of parameters; it is just a tension that is a time interval necessary for attaining of the normative value which is determined for each organization in the individual manner.

While choosing the normative values for the estimate of OS functioning in order to get the uniform distribution between the employees it is reasonable to use the entropy model. When distributing of the works between the employees it is necessary to choose normative values basing on the principles of planning and following uniform distribution of tension for the employees.

Integrated function for estimating of the efficiency of OS functioning should determine the level of accomplishment for the normative values of the functioning parameters. For the developing OS the leadership of organization should increase efficiency of functioning by determination of the ultimate tension for the employees.

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