An Adjusted Forward Curve for Spot Rate Forecasting

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Purpose

This presentation provides an overview of the paper:

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Overview

- 3-month Libor Rate
- 3-Month Forward Curve

Model Estimation & Results
  - Detection and Adjustment of the Forward Curve Bias
  - Criteria for Model Selection of the Adjusted Forward Curve (AFC)
  - Interpreting Model Results
  - Testing: Backtesting AFC vs Forward Curve; Mean Reversion in the AFC model

AFC vs Forward Curve: COVID-19 Snapshot

Forward Curve Bias Implications

Conclusion
3-Month Libor Rate

- The LIBOR rate is the benchmark rate that major global banks (rated AA or more) lend to one another in the international interbank market for short-term loans. The LIBOR rate thus includes a component of credit risk associated with the spread between the Libor rate and the Treasury rate.

- This spread is monitored by short-term investors as an indicator of deteriorating credit market conditions. Therefore,

\[ 3 \text{ Month Libor Rate}_t = 3 \text{ Month Treasury Rate}_t + \text{ Libor Credit Spread}_t. \]

- The LIBOR rate is currently used in the U.S. for about $267 trillion of derivatives and other securities.

- Corporate debt market uses the Libor rate as the benchmark for underwriting and pricing floating debt. For example, the cost of floating debt for a corporation with credit rating of BBB is:

\[ \text{Cost of Debt for BBB rated} = 3 \text{ Month Libor Rate}_t + \text{BBB Rated Spread Over Libor}_t. \]

- The mortgage market also uses the Libor to benchmark the rate of hybrid ARM. The Libor rate, therefore, impacts household decisions to hold floating rate debt versus fixed rate debt.
3-Month Forward Curve

• Proper discounting should use the forward rate rather than the current Libor rate, as the cost of debt shall capture that interest rate associated with the maturity of the debt.

• The 3-month Libor forward curve is the market projection of the 3-month Libor rate, and is thus used to discount cash flows of financial instruments benchmarked to the Libor.

  The construction of the 3-month Libor forward curve stems from market quotes associated with swaps and future contracts. The reported curve is smoothed curve via bootstrapping so that it incorporates a discount factor for each maturity point.

  The forward curve provides yield forecasts over a 10-year or even longer time horizons.

• The forward curve is also widely used to forecast the spot rate across applications, e.g., forecasting net income from floating rate instruments.
3-Month Forward Curve (Cont..)

- The forward Libor curve reflects swap contracts in which one stream of future interest payments is exchanged for another based on a specified principal amount. Specifically, the fixing of a floating debt instrument for a maturity period, \( S - t \), is linked to the Libor curve as follows:

\[
R(t, S) = \prod_{h=0}^{s}[1 + f_{t,h}]
\]

where \( t = \) Start period of swap; and

\( f_{t,h} = \) Forward Libor rate at time \( t \) for tenor \( h \).

- The expected net gains from the swap is zero under the unbiased expectations hypothesis, i.e.,

\[
E(r_{t+h}) = f_{t,h}
\]

where \( r_{t+h} \) is the instantaneous spot rate at time \( t+h \).

- However, a rational market does not necessarily imply that the forward price equals the expected future spot rate. In particular, the term structure of the forward price introduces a liquidity premium. Ceteris paribus, this premium implies that the forward rate is larger than the expected future rate, i.e., \( E(r_{t+h}) < f_{t,h} \).

- The fixing of debt, thus, introduces a notional added cost to debt issuances in the presence of a liquidity premium.

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3-Month Forward Curve (Cont..)

- It is important to note that in over-the-counter swaps, the swap incorporates a spread that captures the differential in the credit risk of the two parties in the contract.
- The spread is usually an add-on to the Libor forward rate, and the sign of the spread depends on the positioning and rating of the two counterparties. For example,

\[
R(t, S) = \prod_{h=0}^{s} \left[ 1 + f_{t,h} + \alpha \right]
\]

where \(\alpha\) is the credit risk differential between the counterparties in the swap, and \(\alpha > 0\) (\(\alpha < 0\)) if the counterparty trading the floating debt for a fixed rate has higher credit risk than the counterparty providing the fixing.
- This implies that some of the credit spread in the Libor rate is not necessarily inherent in the forward Libor curve.
Detection and Adjustment of the Forward Curve Bias

• Fama (1984) tests the predictability of the spot price based on information provided by the futures market as follows:

\[ r_{t+h} - r_t = \alpha_h + \beta_h (f_{t,h} - r_t) + \epsilon_{t,h} \]

where the Libor rate at time \( t \) is represented by \( r_t \); the forward rate at time \( t \) for time horizon \( t+h \) is represented by \( f_{t,h} \); and \( \epsilon_{t,h} \) is the forecast error.

• Cochrane and Piazzesi (2005), however, indicate that lags of forward rates (in addition to contemporaneous forward rates) contain information to forecast the spot rate (1).

• Rather than modeling the changes in the spot rate for different time horizons, \( h \), as in (1), we introduce the adjusted forward curve (AFC) that models the update in the forward curve from one period to the next. That is,

\[ f_{t+1,h-1} - f_{t,h} = \rho f_{t,h} + u_{h,t} + \epsilon_{t,h} \]

where the term \( u_{h,t} \) captures systematic factors that explain forward curve movements from one period to the next, and \( \epsilon_{t,h} \) is a zero mean stochastic process.

• A direct modeling of the dynamic process of the forward curve in (2) facilitates the specification of adjustment factors to the forward curve, and it underscores the role of mean reversion (stationarity) in the nexus between the forward rate and the future spot rate.
Detection and Adjustment of the Forward Curve Bias (Cont..)

• The connection between (1) and (2) stems from solving (2) recursively, which yields an alternative model representation of the AFC that maps the forward curve to the expected future spot rate. That is,

\[ E(r_{t+h}) = (\rho + 1)^h f_{t,h} + \sum_{s=0}^{h-1} (\rho + 1)^s u_{h-s,t+s} \]

where \( r_{t+h} = f_{t+h,0} \).

• The AFC representation in (3) indicates that the cumulative forward curve bias throughout the forecasting horizon helps to explain the spot rate.

• In particular, the forward rate equals the expected spot rate only if \( \rho = 0 \) and \( u_{h,t} = 0 \).

  This indicates that the stochastic process of the forward curve is analogous to a random walk. That is, the current forward curve does not lose predictive power with the forecasting horizon.

• Conversely, if \( \rho < 0 \), then the instantaneous change in the forward curve is not fully transmitted to the future spot rate, and the effect tampers off with the forecast horizon.

  This indicates that the stochastic process of the forward curve is analogous to an autoregressive process with drift. One direct implication of stationarity is that updates to the forward curve become much less relevant to predict the spot rate in longer time horizons.
Criteria for Model Selection of the Adjusted Forward Curve

• The data used to estimate the AFC is a historical series of the forward curve from Bloomberg that spans from 1990 to the present on a quarterly basis. Estimation requires setting up the data in accordance to the definition of the dependent variable in (2), i.e., \( f_{t+1,h-1} - f_{t,h} \).

• The systematic factors to be added into the model, i.e., \( u_{h,t} \) in (2), consist of leading economic indicators. To avoid endogeneity issues in estimation, all factors are introduced in the model with a quarter lag.

• The criteria of model selection are as follows:
  ✓ The AFC model factors are statistically significant.
  ✓ The AFC model factors are supported by the economic literature as leading indicators.
  ✓ The AFC model coefficients are stable.

• To determine whether the coefficient estimates are stable, we estimate the AFC model with a filter for the extended period that encompassed a 3M Libor rate near zero (less than 1 percent). The estimate is assessed stable if the coefficient estimates satisfy the following conditions:
  ✓ Stepwise regression supports the model factors under both the filtered data set and the full sample.
  ✓ The sign of the coefficient estimates under the filtered sample does not change relative to the full sample and it is not statistically different from the full sample estimate.
Interpreting Model Results

The factors added to the model satisfy all the aforementioned conditions. These factors are the term spread of the Treasury curve, the credit spread in Baa Corporate Bond, industrial capacity utilization, and the maturity term the forward contract. The rationale of the estimated coefficients in the AFC model is as follows.

| Term spread Factor Rationale: A negative coefficient for spreads in the regression indicates that the upward bias, $E(r) < f$, correlates with the steepness of the yield curve. The steepness of the yield curve captures monetary stance and, therefore, liquidity in the system. The sign of the factor in the model thus supports the Hicks’s liquidity preference model that projects that the forward market is in constant contango (i.e., forward rate exceeds the expected spot rate). |
|---|
| Industrial Capacity Utilization Factor Rationale: The positive coefficient indicates that the downward bias, $E(r) > f$, correlates with industrial capacity utilization. This is the case as higher industry capacity utilization triggers tighter monetary stance and lower liquidity in the system. This result is thus consistent with the effect of the term structure on the bias. Industrial capacity utilization is a statistically significant factor in the AFC while the unemployment rate is not. This is not entirely surprising in the context of Gordon (1997) work that shows that capacity utilization is the main driver of the trade-off of inflation and output. |
| Credit Risk Spread Factor Rationale: The positive coefficient indicates that the downward bias, $E(r) > f$, correlates with the credit risk spread. Larger credit spreads correlate with less liquidity in the system. This result is thus consistent with the effect of the term structure on the bias. Alternatively, this result may capture a forward rate that does not fully inherit the credit risk attached to the Libor rate. |
Interpreting Model Results (Cont..)

• Analogous to an econometric model with a lagged dependent variable, the AFC estimation uses $f_{t,h}$ as a model factor.

• From (2), the coefficient $(1+\rho)$ captures the effect of previous period (quarter) forecast on current forecast in the AFC.

• The AFC coefficient estimate of $\rho$ is -0.04, and it is statistically less than zero. This implies that the instantaneous change in the forward curve is not fully transmitted to the future spot rate, and the effect tampers off with the forecast horizon.

  For example, the AFC model uses 72% and 44% of the information inherent in the forward curve in a 2-year and 5-year forecast of the spot rate, respectively. In contrast, the forward curve uses 100% of the information.

• Therefore, the AFC indicates that if the forward curve is high (e.g., relative to the long run equilibrium) then the forward curve is likely to remain in contango for some time, while trending down to toward a long run equilibria.
Backtesting AFC vs forward Curve

• Clark and McCracken (2012) emphasize the important role of pseudo out-of-sample forecasts in econometric evaluation of forecasting models.

• To measure the model performance, we backtest the model for each year of the sample. The performance year used in the testing is excluded from the AFC model estimation so it is an out-of-sample test.

• We then calculate the MSE for both the forward curve and the AFC, and we determine if the differences between the MSEs are statistically significant.

• Specifically, the structure of the test is:

\[
FC.MSE_{t,Y} - AFC.MSE_{t,Y} = \sum_{h=2}^{Q4.Y} \sum_{t=Q1.Y}^{Q4.Y} \left[ \frac{(f_{t+1,h-1}-f_{t,h})^2}{120} - \frac{(f_{t+1,h-1}-f_{t,h}-\rho f_{t,h}-u_{h,t})^2}{120} \right],
\]

for \( Y = 1990 \) to \( 2018 \), where \( FC.MSE_{t,Y} \) is the mean square error associated with changes in the forward curve from one quarter to another for year \( Y \); and \( AFC.MSE_{t,Y} \) is the mean square error associated with same movements of the AFC.
Backtesting AFC vs forward Curve (Cont..)

| Year | FC. MSE_{t-1} - AFC. MSE_{t-1} | t-stat | Preferable Model |
|------|---------------------------------|--------|------------------|
| 2014 | 0.05                            | 9.50   | AFC              |
| 2008 | 0.47                            | 8.44   | AFC              |
| 2007 | 0.19                            | 7.35   | AFC              |
| 2002 | 0.19                            | 5.72   | AFC              |
| 1993 | 0.18                            | 5.54   | AFC              |
| 2013 | 0.04                            | 5.15   | AFC              |
| 1998 | 0.05                            | 5.12   | AFC              |
| 2016 | 0.03                            | 5.08   | AFC              |
| 2001 | 0.13                            | 4.72   | AFC              |
| 1991 | 0.11                            | 4.48   | AFC              |
| 1999 | 0.03                            | 4.28   | AFC              |
| 1992 | 0.24                            | 3.80   | AFC              |
| 2000 | 0.02                            | 3.78   | AFC              |
| 1995 | 0.09                            | 3.56   | AFC              |
| 2012 | 0.03                            | 3.34   | AFC              |
| 2011 | 0.06                            | 2.80   | AFC              |
| 1997 | 0.04                            | 2.57   | AFC              |
| 2004 | 0.02                            | 0.82   | Undetermined     |
| 2010 | 0.00                            | -0.20  | Undetermined     |
| 2006 | 0.00                            | -0.29  | Undetermined     |
| 2005 | 0.00                            | -0.34  | Undetermined     |
| 2003 | -0.02                           | -1.12  | Undetermined     |
| 2009 | -0.06                           | -2.15  | Forward Curve    |
| 1996 | -0.02                           | -2.48  | Forward Curve    |
| 2017 | -0.02                           | -5.09  | Forward Curve    |
| 1994 | -0.27                           | -11.51 | Forward Curve    |
| 2015 | -0.10                           | -13.94 | Forward Curve    |

- Across the 27 years in the sample period, there were 17 instances in which the MSE was statistically lower under the AFC model than under the forward curve.
- There were 5 instances in which the MSE was statistically lower under the forward curve relative to the AFC.
- And 5 instances in which the MSE error was not statistically different between the two models.
Mean Reversion in the AFC Model

• The back-testing indicates that the AFC model tends to successfully detect the sign of the bias in spot rate forecasting and, therefore, in effectively monitoring the accuracy of changes in the forward curve.

• However, the main value of the adjustment may stem from the property of mean reversion of its underlying factors.

• Specification of AFC factors as an autoregressive component reveal the presence of a mean reversion component. For example, the table below shows the share of the long run equilibria of each model factor in a 2-year and 5-year forecast horizon, respectively.

This result is consistent with Duffee (2013) that shows that riskless yields are (near) unit-root, but spreads are stationary.

| Factor                  | Credit Spread | Term Spread 2-year to 3-month Treasury | Term Spread 10-year to 2-year Treasury | Industrial Capacity Utilization |
|-------------------------|---------------|----------------------------------------|----------------------------------------|--------------------------------|
| Forecast Horizon        | 2-year        | 2-year                                 | 2-year                                 | 2-year                        |
|                         | 5-year        | 5-year                                 | 5-year                                 | 5-year                        |
| Share of the Long Component | 44%           | 93%                                    | 99.9%                                  | 28%                           |
|                         | 77%           |                                       |                                       | 56%                           |

• Therefore, the AFC model should improve long term forecast of the spot rate by adjusting the forward curve with liquidity and credit risk factors that tend to revert to the mean.

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Mean Reversion in the AFC Model (Cont..)

- The figures below show that the AFC evaluated in terms of long run value of the factors (i.e., historical averages of each factor) relative to the forward curve in various long-term forecasting horizons. The figures use the forward curve that originated from Jan 1990, Jan 2000, and Jan 2008.
Mean Reversion in the AFC Model (Cont..)

- The AFC model (evaluated at the long run value of its underlying factors) also appears to perform reasonably well in medium term forecasts. The figures below illustrate the differences between the AFC and the forward curve relative to the actual spot rate in a span of 4-years.

AFC vs Forward Curve Starting in 2014

AFC vs Forward Curve Starting in 2015
Mean Reversion in the AFC Model (Cont..)

• The main takeaway from the analysis is that the property of mean reversion for liquidity and credit risk factors increase the viability of the adjustment factors of the forward curve.

• Of course, implementation of more sophisticated models for forecasting the underlying factors in the AFC model are likely to produce further gains in model performance.
AFC vs Forward Curve: COVID-19 Snapshot

• We compare the projection of the 3-month Libor rate for the current COVID-19 economic environment. The AFC is first evaluated in terms of the long run values of each model factor. Secondly, the AFC is evaluated based on a calibrated AR model for each model factor.

• The table below shows that the AFC model projects a higher Libor rate than the forward curve across all time horizons. However, the current differences between the forward and AFC tend to be not very large.

| Tenor  | Forward Curve | AFC (Long Run of Factors) | AFC (Factor projected with an AR Model) |
|--------|---------------|----------------------------|----------------------------------------|
| 2-year | 0.2%          | 0.4%                       | 0.3%                                   |
| 3-year | 0.3%          | 0.6%                       | 0.4%                                   |
| 5-year | 0.7%          | 1.0%                       | 0.8%                                   |
| 7-year | 1.0%          | 1.8%                       | 1.3%                                   |
| 10-year| 1.3%          | 2.2%                       | 1.7%                                   |

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Forward Curve Bias Implications

• In normal times, the forward curve exhibits an upward bias, \( E(r) < f \). In this scenario, we show in a companion paper that fully hedging for interest rate risk is not optimal.
• In the current economic environment, the AFC model infers a downward bias in the forward curve, \( E(r) > f \). This indicates an anomalous market in which there is a negative preference for liquidity.
• In the current environment, \( E(r) > f \), we show in a companion paper that is optimal for financial institutions to fully hedge for interest rate risk.
Forward Curve Bias Implications (Cont..)

• The relatively small difference between the forward curve and the AFC suggests that the former is currently a reasonable forecast of the spot rate.

• The evolution of COVID-19 will determine future changes in the forward curve and accentuate the presence of backwardation in the Libor market through larger credit spreads. In this instance, the Fed could try to lower the forward curve by decreasing further the demand for liquidity.

• Neither the forward curve nor the AFC appear to predict a quick recovery from COVID-19.

• Large jumps in the debt to GDP ratio and associated large increases in the budget deficit may make more difficult to raise cheap funding cost in the future as demand for U.S. dollars decreases overseas.
Conclusion

• Samuelson’s original formulation of efficient markets presumes that the forward rate is the best forecast of the spot rate.
  Samuelson interprets the future price as concrete observations of the spot price anticipations, and states that examinations of past changes in the forward curve does not improve forecastability.

• The 3-month forward rate is the most commonly used floating rate to discount cash flows and it is, therefore, an intrinsic component in the book value of U.S. corporations.

• The AFC introduces other factors in addition to the forward curve in explaining the spot rate.

• The predictive power of the AFC model, however, hinges on the forecastability of the underlying factors. The testing indicates that all the AFC model factors have a mean reversion component, which make it a more effective tool for medium and long-term forecasting.

• Overall, the AFC model effectively anticipates future movements in the forward curve that tend to yield better forecasts of the future spot rate.