MEANS AND HERMITE INTERPOLATION

ALAN HORWITZ

Abstract. Let \( m_2 < m_1 \) be two given nonnegative integers with \( n = m_1 + m_2 + 1 \). For suitably differentiable \( f \), we let \( P, Q \in \pi_n \) be the Hermite polynomial interpolants to \( f \) which satisfy \( P^{(j)}(a) = f^{(j)}(a), j = 0, 1, \ldots, m_1 \) and \( P^{(j)}(b) = f^{(j)}(b), j = 0, 1, \ldots, m_2 \), \( Q^{(j)}(a) = f^{(j)}(a), j = 0, 1, \ldots, m_2 \) and \( Q^{(j)}(b) = f^{(j)}(b), j = 0, 1, \ldots, m_1 \). Suppose that \( f \in C^{m_1 + 2}(I) \) with \( f^{(m_1 + 1)}(x) \neq 0 \) for \( x \in (a, b) \). If \( m_1 - m_2 \) is even, then there is a unique \( x_0, a < x_0 < b \), such that \( P(x_0) = Q(x_0) \). If \( m_1 - m_2 \) is odd, then there is a unique \( x_0, a < x_0 < b \), such that \( f(x_0) = \frac{1}{2} (P(x_0) + Q(x_0)) \). \( x_0 \) defines a strict, symmetric mean, which we denote by \( M_{f,m_1,m_2}(a,b) \). We prove various properties of these means. In particular, we show that \( f(x) = x^{m_1 + m_2 + 2} \) yields the arithmetic mean, \( f(x) = x^{-1} \) yields the harmonic mean, and \( f(x) = \sqrt[m_1 + m_2 + 1]{x} \) yields the geometric mean.

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