Hybrid Impedance Control of Robot Manipulators based on Generalized Momentum

Kaining Li, Xianmin Zhang and Yanjiang Huang

School of Mechanical and Automotive Engineering, South China University of Technology, Guangzhou 510640, China

* Email: mehuangyj@scut.edu.cn

Abstract. In the hybrid impedance control strategy, the task space is divided into position control subspace and force control subspace, and the constraint direction in the manipulator task space is controlled by torque-based impedance control, while the free movement direction is controlled by position-based impedance control. The hybrid impedance control has strong force tracking characteristics in the direction of force control and strong flexibility in the direction of position control. In this paper, we use the generalized momentum observer instead of the torque sensor to estimate the torque exerted by the external environment on the robot. Simulation results are shown for a three-link SCARA robot.

1. Introduction

The manipulator impedance control realizes the interaction between manipulator and environment through the dynamic relationship between position and environmental forces [1]. By specifying the relationship between contact forces and position, it ensures that the robot operates in a restricted environment while maintaining the appropriate contact forces. Impedance control is divided into two different control structures based on torque and position, torque-based impedance control adjusts the contact force and displacement of the end-effector by controlling the joint driving torque [2], while position-based impedance control adjusts the position/velocity of the end-effector according to the contact force deviation between the robot and the environment [3]. Hybrid impedance control [4] combines the above two control strategies, achieving accurate force control and position control with good robustness. In [5] and [6], hybrid impedance control strategy is designed and proved accurately and robustly. There are still some main issues being studied on the impedance control, such as the capability of position tracking and force tracking [7]. Moreover, the external contact force of the manipulator in contact with the environment is usually measured by a six-dimensional force sensor [8]. If without the sensor, external contact torque can also be obtained by means of a generalized momentum observer [9-10].

In this paper, we provide a hybrid impedance control strategy which has precise force tracking characteristics in the direction of torque-based impedance control and strong position tracking in the direction of position-based impedance control. Generalized momentum observer is used instead of torque sensor to estimate external contact torque.
2. Robot dynamics and properties
In this section, we summarize the dynamics model of joint space and the relevant properties of robot. The torque of the external environment on the manipulator is taken into account and the torque is estimated by using a generalized momentum observer.

2.1. Robot dynamics model
The robot dynamic model is expressed by the following equation:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) + \tau_f(\dot{\theta}) = \tau + \tau_e \tag{1}$$

where $\theta$, $\dot{\theta}$ and $\ddot{\theta}$ are the joint angle, joint angular velocity and joint angular acceleration, respectively. $M(\theta)$ is the inertia matrix, $C(\theta, \dot{\theta})\dot{\theta}$ is the centrifugal and Coriolis vector, $G(\theta)$ is the gravity vector, $\tau_f(\dot{\theta})$ is torque of joint friction vector, $\tau$ is the torque applied to the robot by the actuators, and $\tau_e$ is the torque of the external environment on the manipulation.

The robot dynamic equation satisfies the following two characteristics:
1) $M(\theta)$ is the symmetric and positive-definite matrix;
2) $M(\theta) - 2C(\theta, \dot{\theta})$ is the skew-symmetry matrix, which is also equivalent to the identity

$$\dot{M}(\theta) = C(\theta, \dot{\theta}) + C^T(\theta, \dot{\theta}). \tag{2}$$

2.2. Generalized momentum observer
The generalized momentum of the robot is defined as

$$p = M(\theta)\dot{\theta}. \tag{3}$$

Combining equations (1) and (2), the derivative of equation (3) is

$$\dot{p} = C^T(\theta, \dot{\theta})\dot{\theta} - G(\theta) - \tau_f(\dot{\theta}) + \tau + \tau_e. \tag{4}$$

Define the residual vector $r$ as

$$r = K\left[p(t) - p(0) - \int_0^t (C^T\dot{\theta} - G - \tau_f + \tau + r)ds\right] \tag{5}$$

with $r(0) = 0$, where $K$ is a diagonal gain matrix and satisfies $K > 0$, $p(t)$ is the robot generalized momentum at time $t \geq 0$, as defined in equation (3). The vector $r$ can be calculated from the measured $(\theta, \dot{\theta})$ and the commanded joint torque $\tau$. The advantage of this method is that the inverse of the inertial matrix and the joint acceleration are not required [11].

Combining equations (4) and (5), the resulting dynamics of $r$ is

$$\dot{r} = K(\tau_e - r). \tag{6}$$

And then, a transfer function for the residual dynamics can be written as

$$\frac{r_i(s)}{\tau_{e,i}(s)} = \frac{K_i}{K_i + s}, \quad i = 1, \ldots, n, \tag{7}$$

where $i$ is the serial number of links. It can be seen that $r$ tracks $\tau_e$ in the form of a first-order system, and $r(0) = \tau_e(0)$ at steady state. So we can use $r$ as an estimate of the external torque, meaning

$$K \to \infty \quad \Rightarrow \quad r \approx \tau_e. \tag{8}$$
For the compactness of the formula, we denote \( \gamma(\theta, \dot{\theta}) = C^T (\theta, \dot{\theta}) \dot{\theta} - G(\theta) - \tau_f(\dot{\theta}) \) in Figure 1 that shows the block diagram of the observer. It’s worth noting that equation (8) is the ideal result if the model is correct. In fact, due to the inaccuracy of the robot dynamic model and parameters, there is an error between the contact force and the actual external force. For the knowledge of model and parameter identification to improve model accuracy, please refer to the literature [12], we will not repeat them here.

\[ \tau = \gamma(\theta, \dot{\theta}) + \tau_c \]

Figure 1. The block diagram of the generalized momentum observer.

3. Robot impedance control strategy

In this section, we will show how to use the external contact force information provided by the residual vector (5) to design an impedance control scheme to ensure that the movement of the contact point along the obstacle surface and the approximate adjustment of the contact force. The performance of the proposed control scheme will indeed depend on the accuracy of the "measurement" of joint contact torque provided by the residual \( \tau_r \), that is, on the accuracy of the model and on the system affecting the input and measurement noise.

3.1. Joint space position-based impedance control

For simplicity, let us denote \( h(\theta, \dot{\theta}) = C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) + \tau_f(\dot{\theta}) \), so that (1) can be rewritten as

\[ M(\theta) \ddot{\theta} + h(\theta, \dot{\theta}) = \tau + \tau_c. \]  

In the joint space, the desired dynamic behavior of the robot after applying position-based impedance control can be expressed as

\[ M_d(\ddot{\theta}_d - \ddot{\theta}) + B_d(\dot{\theta}_d - \dot{\theta}) + K_d(\theta_d - \theta) = \tau_c \]  

where \( M_d, B_d, \) and \( K_d \) are symmetrical matrices that denote the desired inertia, damping, and stiffness matrices, respectively. The vector \( \theta_d \) denotes the desired joint angle. The equation (9) is arranged as

\[ \ddot{\theta} = \ddot{\theta}_d - M_d^{-1}[\tau_c - B_d(\dot{\theta}_d - \dot{\theta}) - K_d(\theta_d - \theta)]. \]  

The resulting control law after combining the equations (9) and (11) becomes

\[ \tau = M(\theta)(\ddot{\theta}_d - \ddot{\theta}) - M_d^{-1}[\tau_c - B_d(\dot{\theta}_d - \dot{\theta}) - K_d(\theta_d - \theta)] + h(\theta, \dot{\theta}) - \tau_c. \]  

3.2. Joint space torque-based impedance control

In terms of force control direction of the mechanical arm, the mechanical arm is equivalent to a mass-damping system, and its stiffness is set to 0, thus achieving accurate force control. For the torque-based impedance control case, the desired relationship between \( \theta \) and \( \tau_c \) can be expressed as the impedance function

\[ M_d(\ddot{\theta}_d - \ddot{\theta}) + B_d(\dot{\theta}_d - \dot{\theta}) = \tau_c - \tau_i \]  

(13)
where $\tau_d$ is the desired force. Form (9) and (13), the resulting control law can be deduced as

$$\tau = M(\theta)(\ddot{\theta} - M^{-1}_d[\tau_d - \tau_e - B_d(\dot{\theta} - \dot{\theta})]) + h(\theta, \dot{\theta}) - \tau_e. \quad (14)$$

### 3.3. Hybrid impedance control strategy based on r-observer

Based on the above ideas, a switching matrix $S$ is introduced into the hybrid impedance control. This matrix, a diagonal matrix with elements either 1 or 0, is used to separate the torque-based and position-based impedance control. Combining (12) and (14) with the switching matrix, the desired dynamics behavior of the system can be described as

$$M_d(\ddot{\theta} - \dot{\theta}) + B_d(\dot{\theta} - \dot{\theta}) + SK_d(\theta_d - \theta) = (I - S)\tau_d - \tau_e \quad (15)$$

where $I$ is the unit matrix. Then we can deduce the following control law:

$$\tau = M(\theta)(\ddot{\theta} - M^{-1}_d[(I - S)\tau_d - \tau_e - B_d(\dot{\theta} - \dot{\theta}) - SK_d(\theta_d - \theta)]) + h(\theta, \dot{\theta}) - \tau_e. \quad (16)$$

The method of generalized momentum observer is used to obtain the external contact torque instead of a torque sensor. The symbol $\hat{\tau}_e$ represents an estimate of the external torque $\tau_e$. As shown in Figure 2, the hybrid impedance controller receives torque error between the desire torque and the contact torque of the environment on the robot manipulation and then sends the position $\theta$ to the robot servo controller to modify the joint position in order to maintain a constant torque.

![Figure 2. The implementation of the hybrid impedance controller based on r-observer.](image)

### 4. Simulation

To verify the proposed hybrid impedance control strategy, a series of simulation studies are conducted and presented in this section. The simulation platform is based on Matlab/Simulink which mainly consists of the following parts: the hybrid impedance controller, robot dynamics model, and the generalized momentum observer.

![Figure 3. The model of SCARA robot.](image)
4.1. Simulation setup
In the simulation, a SCARA robot [13], as shown Figure 3, is used as an object to be controlled. This robot has three rotational degrees of freedom in the horizontal direction and one in the vertical direction in general. Since the 3rd-axis rotating joint and the 4th-axis prismatic joint are coupled, and the 4th-axis prismatic joint can be independently realized without accompanying the rotation of the 3rd joint, the 3rd joint and the 4th joint are merged as a combined link 3 in modelling, holding the joint type as a prismatic joint and keeping the 3rd joint locked during simulation experiment. Table 1 shows the SCARA robot model parameters which including the mass, length and inertia of each manipulation.

| Axle | $m$ (kg) | $L$ (m) | $I$ (kg·m$^2$) |
|------|----------|---------|----------------|
| 1    | 2.4312   | 0.25    | 32.1907        |
| 2    | 3.7860   | 0.25    | 33.7615        |
| 3    | 0.5552   | 0.14    | —              |

In the hybrid impedance controller, the first two planar rotating joints of the SCARA robot execute the position-based impedance control strategy while the 3rd vertical prismatic joint executes the torque-based impedance control strategy by choosing the elements of the matrix $S$, namely, $S = \text{diag}(1,1,0)$.

To verify the tracking error performance of the hybrid impedance control, the control parameters need to be set reasonably. The value of diagonal gain matrix $K$ needs to be large enough, so set it to $K = \text{diag}(2000,2000,1600)$. The hybrid impedance controller parameters $M_d = \text{diag}(5,5,2)$, $B_d = \text{diag}(50,50,30)$, $K_d = \text{diag}(500,500,0)$ are selected when the two rotating joints execute position tracking and the 3rd joint executes force tracking. The sample time $t$ in simulation is set to 5 s. The input of position tracking is $\theta_1 = \theta_2 = 1 - \cos(\pi t)$ rad while the desired tracking force is $\tau_d = 10$ N at steady state. Moreover, we take the viscous and coulomb friction into consideration:

$$\tau_f = f_v \dot{\theta} + f_c \text{sign}(\dot{\theta})$$  \hspace{1cm} (17)

where $f_v = \text{diag}(0.5,0.2,0.1)$ and $f_c = \text{diag}(1.5,0.5,0.3)$.

4.2. Simulation results
Figure 4 and Figure 5 show the joint driving torque of joint 1 and 2 when executing the above position tracking in the hybrid impedance controller. As shown in the simulation results, the torque of the robot will fluctuate slightly during start-up because of the friction and so on. This is a major source of tracking error.

![Figure 4](image1.png)  \hspace{1cm} ![Figure 5](image2.png)

Figure 4. The driving torque of joint 1.  \hspace{1cm} Figure 5. The driving torque of joint 2.

The position tracking trajectories of the first two joints are shown in Figure 6 (a) and Figure 7 (a), respectively. Meanwhile, the corresponding performance of the position tracking error is drawn in Figure 6 (b) and Figure 7 (b). It is observed that the position tracking errors are under one degree. Thus, the hybrid impedance controller performs well at the first two rotating joints.
Figure 6. The position tracking performance of joint 1.

Figure 7. The position tracking performance of joint 2.

The force tracking performance on a plane trajectory, a slope trajectory and a sine trajectory of joint 3 is shown Figure 8, Figure 9 and Figure 10, respectively. Namely, execute force tracking control under different command trajectory $\theta_0 = 0.05$, $\theta_o = 0.02t$, and $\theta_j = 0.05\sin(0.4\pi t)$. Figures 8 (a) - 10 (a) show that the actual trajectory tracks the command trajectory on a plane, on a slope and in sine working conditions, respectively, while Figures 8 (b) - 10 (b) show that the actual contact force tracks the desired contact force on a plane, on a slope and in sine working conditions, severally. In the simulation, the desired tracking force is set to 10 N at steady state, but we change the direction of the desired force at 2.5 seconds to better demonstrate the force tracking performance of the hybrid impedance controller.

It can be seen that the force tracking performance is brilliant for the third joint when given the corresponding position command. At the moment of starting and intermediate desired force change direction, the actual force will reach steady state within 0.3 seconds.

Figure 8. The force tracking performance on a plane trajectory of joint 3.
Figure 9. The force tracking performance on a slope trajectory of joint 3.

Figure 10. The force tracking performance on a sine trajectory of joint 3.

5. Conclusion
With the emergence of interactive robot, the importance of robot contact operation becomes more and more prominent. A hybrid impedance controller based on generalized momentum is proposed for dynamic contact force tracking and non-contact axis position tracking. Thus, it can be determined that position and force can be controlled simultaneously by impedance control.

The above simulation shows that the hybrid impedance control strategy has good tracking performance. Besides, by estimating the external force on the manipulation, we don’t need to know the environmental stiffness and also need no force sensors.

Acknowledgments
This work was supported by the National Nature Science Foundation of China (Grant Nos. 51820105007 and U1501247).

References
[1] Hogan N 1985 Impedance control: An approach to manipulation: Part I—Theory J. Dyn. Sys., Meas., Control 107 1–7
[2] Bonitz R C and Hsia T C 1996 Internal force-based impedance control for cooperating manipulators IEEE Trans. on Robotics and Automation 12 78–89
[3] Yuan J, Qian Y, Gao L, Yuan Z and Wan W 2019 Position-based impedance force controller with sensorless force estimation Assembly Automation 39 489–496
[4] Cao H, Chen X, He Y and Zhao X 2019 Dynamic adaptive hybrid impedance control for dynamic contact force tracking in uncertain environments IEEE Access 7 83162–74
[5] Anderson R J and Spong M W 1988 Hybrid impedance control of robotic manipulators IEEE Journal on Robotics and Automation 4 549–556
[6] Liu G J and Goldenberg A A 1991 Proc. IEEE Int. Conf. on Robotics and Automation (Sacramento: IEEE) pp 287–292
[7] Duan J, Gan Y, Chen M and Dai X 2018 Adaptive variable impedance control for dynamic contact force tracking in uncertain environment Robotics and Autonomous Systems 102 54–65
[8] Keemink A Q, van der Kooij H and Stienen A H 2018 Admittance control for physical human–robot interaction The International Journal of Robotics Research 37 1421–44
[9] De Luca A and Mattone R 2005 Proc. IEEE Int. Conf. on Robotics and Automation (Barcelona: IEEE) pp 999–1004
[10] Wahrburg A, Morara E, Cesari G, Matthias B and Ding H 2015 IEEE Int. Conf. on Automation Science and Engineering (Gothenburg: IEEE)
[11] De Luca A, Albu-Schaffer A, Haddadin S and Hirzinger G 2006 Proc. IEEE Int. Conf. on Intelligent Robots and Systems (Beijing: IEEE) pp 1623–30
[12] Afrough M and Hanieh A A 2019 Identification of dynamic parameters and friction coefficients Journal of Intelligent & Robotic Systems 94 3–13
[13] Zhang Y, Qiu Z and Zhang X 2019 Calibration method for hand-eye system with rotation and translation couplings Applied optics 58 5375–87