Non-Singular Gravity Without Black Holes

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“What are the everywhere regular solutions of these field equations?”

A. Einstein, Autobiographical notes, 1949.

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Abstract

A non-singular static spherically symmetric solution of the nonsymmetric gravitational and electromagnetic theory (NGET) field equations is derived, which depends on the four parameters $m, \ell^2, Q$ and $s$, where $m$ is the mass, $Q$ is the electric charge, $\ell^2$ is the NGT charge of a body and $s$ is a dimensionless constant. The electromagnetic field invariant is also singularity-free, so that it is possible to construct regular particle-like solutions in the theory. All the curvature invariants are finite, there are no null surfaces in the spacetime and there are no black holes. A new stable, superdense object (SDO) replaces black holes.
1. Introduction

After publishing his theory of gravitation in 1916, Einstein set himself the goal of finding a unified field theory of electromagnetism and gravitation. However, a more pressing issue for him was the discovery of everywhere regular solutions of such a unified field theory. He failed to discover a satisfactory unification of gravitation and electromagnetism but in his search he developed a unified field theory based on a nonsymmetric field structure. In 1979, it was proposed by one of us that the nonsymmetric field structure had nothing to do with electromagnetism but instead was a general description of the geometry of spacetime, i.e. it describes a theory of the pure gravitational field called the nonsymmetric gravitational theory (NGT).

Although Einstein’s theory has proved to be in good agreement with all the experimental tests that it has been subjected to so far, there has always been the issue of the existence of singularities in the solutions of the field equations. There exist null surfaces or event horizons in the solutions at which the red-shift becomes infinite. Although physics has learned to live with these event horizons, and the notion of cosmic censorship was invented to prevent an observer at infinity from seeing a “naked” singularity, nobody has succeeded in proving rigorously that such a cosmic censorship exists, and naked singularity solutions of Einstein’s field equations have been published. Such a naked singularity would destroy the Cauchy data on an initial space-like surface, and due to the local nature of the solution for gravitational collapse would invalidate Einstein’s theory. The big-bang singularity in the cosmological solutions of the theory have also been cause for concern among theorists, although due to the global nature of these solutions such a singularity would not vitiate the theory to the same extent as the gravitational collapse solution.
There has been much discussion recently about paradoxes that occur in connection with Hawking radiation and event horizons. An observer at spatial infinity would see infalling matter “freeze” before it can form an event horizon, whereas a freely falling observer can fall through an event horizon without difficulty, but be unable to communicate this fact to the observer at infinity. This means that spacetime is separated into two disconnected parts by a null surface and there exists no communication between the two spacetimes. A paradox arises, for these two observers completely disagree about what they see in a spacetime containing a black hole. The Hawking radiation is purely thermal and contains no information about a collapsed star. Thus, as first pointed out by Hawking\(^8\), this leads to what has been called the information loss paradox. Neither of these paradoxes has been resolved satisfactorily in spite of attempts to do so by modifying quantum mechanics and invoking an, as yet, unknown theory of quantum gravity.

An alternative way to avoid these paradoxes is to modify Einstein gravity theory (EGT), so that black holes and singularities are eliminated altogether. This cannot be done in a gravity theory which contains new degrees of freedom arising from some new conserved charge for which there exists a *smooth* limit to GR as the charge tends to zero. An attempt to avoid black hole solutions in NGT, using only solutions depending on the \(\ell^2\) charge failed for this reason\(^9\), for photons do not have an \(\ell^2\) charge and, therefore, a pure photon star is described by GR and can collapse to a black hole. Moreover, there is no reason why in every given situation the parameter \(\ell^2\) for a star should have a value that forbids collapse to a black hole. The aforementioned NGT solutions were based on a simplifying assumption about the form of the skew symmetric fields. It is only when this assumption is discarded that the true non-singular nature of NGT is revealed.

In the following, the complete non-singular solution of the NGT field equations is derived, which for non-vanishing values of a dimensionless parameter \(s\) has no null surfaces
in spacetime and the curvature invariants are finite\(^\text{10}\). For strong fields, the dependence on the parameter \(s\) is non-analytic, so that a smooth limit to Einstein’s gravitational theory does not exist. This means that there is a true absence of black holes in NGT, since even for infinitesimally small \(s\) there are no event horizons in the spacetime and no singularity at \(r = 0\). Einstein’s gravity theory is a separate theory with singularities, which exists only when it is assumed that \(g_{\mu\nu}\) and \(\Gamma^\lambda_{\mu\nu}\) are symmetric in the indices \(\mu\) and \(\nu\). For sufficiently small \(s\) and \(\ell^2\) charge, NGT agrees with all current experiments.

There are no black holes, wormholes or other exotic objects in the nonsingular NGT, and the stable superdense objects (SDO’s)\(^\text{11}\) that replace black holes for \(r \leq 2m\) will not exhibit infinite red-shifts at their surfaces. Therefore, all observers will agree on what happens when a star collapses to a SDO, and no matter can disappear into a singularity in the interior of a SDO. There will not be any Hawking radiation from the surface of a SDO; only normal radiation of both a thermal and non-thermal nature will be emitted from the surface. Stable neutron stars exist for \(s \leq 15\)\(^\text{11}\). Larger values of \(s\) will cause neutron stars to be unstable against gravitational expansion, not collapse. When the NGT \(\ell^2\) charge effects are included in the neutron star calculations\(^\text{12}\), then it is possible to counterbalance the attractive effects of \(\ell^2\) against the repulsive effects of \(s\), weakening the bounds on the coupling of NGT charge to matter, and the bound on \(s\).

In Section 2., we shall present the Lagrangian density and the field equations of the nonsymmetric gravitational, electromagnetic theory (NGET). The general properties of the nonsymmetric static spherically symmetric solutions will be discussed in Section 3, while in Section 4 we analyze the non-singular nature of these solutions. We conclude with a summary of the results in Section 5.
2. NGT Lagrangian Density and Field Equations

The Lagrangian density including electromagnetism and sources, in NGT, is given by\(^4\,6\,13\,14\):

\[
\mathcal{L} = g^{\mu\nu} R_{\mu\nu} \left( W \right) + \sqrt{-g} \left[ \kappa (g^{[\mu\nu]} F_{\mu\nu})^2 - H^{\mu\nu} F_{\mu\nu} \right] + \mathcal{L}_M, \tag{2.1}
\]

where \( g^{\mu\nu} = \sqrt{-g} g^{\mu\nu} \) and \( R_{\mu\nu}(W) \) is the NGT contracted curvature tensor:

\[
R_{\mu\nu}(W) = W^\beta_{\mu\nu,\beta} - \frac{1}{2} (W^{\beta}_{\mu,\nu,\beta} + W^{\beta}_{\nu,\beta,\mu}) - W_{\alpha\nu}^\beta W_{\mu\beta}^\alpha + W_{\alpha\beta}^\gamma W_{\mu\nu}^\gamma, \tag{2.2}
\]

defined in terms of the unconstrained nonsymmetric connection:

\[
W^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \frac{2}{3} \delta^\lambda_{\mu} W_{\nu}, \tag{2.3}
\]

where \( W_\mu \equiv W^\lambda_{\mu,\lambda} = \frac{1}{2} (W^\lambda_{\mu,\lambda} - W^{\lambda}_{\lambda,\mu}) \). This equation leads to:

\[
\Gamma_\mu = \Gamma^\lambda_{\mu,\lambda} = 0. \tag{2.4}
\]

The skew tensor \( H_{\mu\nu} = -H_{\nu\mu} \) is defined in terms of the skew electromagnetic field tensor \( F_{\mu\nu} \) by the equation:

\[
g_{\sigma\beta} g^{\gamma\sigma} H_{\gamma\alpha} + g_{\alpha\sigma} g^{\sigma\gamma} H_{\beta\gamma} = 2 g_{\alpha\sigma} g^{\sigma\gamma} F_{\beta\gamma}, \tag{2.5}
\]

and \( \kappa \) is a coupling constant. The contravariant tensor \( g^{\mu\nu} \) is defined in terms of the equation:

\[
g^{\mu\nu} g_{\sigma\nu} = g^{\nu\mu} g_{\nu\sigma} = \delta^\mu_\sigma. \tag{2.6}
\]

The NGT contracted curvature tensor can be written as

\[
R_{\mu\nu}(W) = R_{\mu\nu}(\Gamma) + \frac{2}{3} W_{[\mu,\nu]}, \tag{2.7}
\]
where \( R_{\mu\nu}(\Gamma) \) is defined by

\[
R_{\mu\nu}(\Gamma) = \Gamma_{\mu\nu,\beta}^\beta - \frac{1}{2} \left( \Gamma_{(\mu\beta),\nu}^\beta + \Gamma_{(\nu\beta),\mu}^\beta \right) - \Gamma_{\alpha\nu}^\beta \Gamma_{\mu\beta}^\alpha + \Gamma_{(\alpha\beta)}^\beta \Gamma_{\mu\nu}^\alpha.
\] (2.8)

The Lagrangian density for the matter sources is given by (\( G= c=1 \)):

\[
\mathcal{L}_M = -8\pi g^{\mu\nu} T_{\mu\nu} + \frac{8\pi}{3} W_\mu S^\mu.
\] (2.9)

When only electromagnetic fields and gravitation exist in the vacuum i.e. in the absence of phenomenological matter sources, then \( S^\mu = 0 \) and the electromagnetic energy-momentum tensor is given by\(^4,13,14\):

\[
T_{\alpha\beta} = -\frac{1}{4\pi} \left[ (g_{\sigma\beta} H^{\mu\sigma} F_{\mu\alpha} - \kappa g^{[\mu\nu]} F_{\mu\nu} F_{\alpha\beta} - \frac{1}{4} g_{\alpha\beta} (H^{\mu\nu} H_{\mu\nu} - \kappa (g^{\mu\nu} F_{\mu\nu})^2) \right],
\] (2.10)

where

\[
H^{\mu\alpha} = g^{\beta\mu} g^{\gamma\alpha} H_{\beta\gamma}.
\] (2.11)

It can be proved that

\[
g^{\alpha\beta} T_{\alpha\beta} = 0.
\] (2.12)

We observe that there is a coupling term of the form \( \kappa g^{[\mu\nu]} F_{\mu\nu} \) in the Lagrangian density. However, the non-singular nature of the theory is manifest for any \( \kappa \), including \( \kappa = 0 \).

Our field equations are given by

\[
G_{\mu\nu}(W) = 8\pi T_{\mu\nu},
\] (2.13)

\[
g^{[\mu\nu]} = 0,
\] (2.14)

\[
g_{\mu\nu,\sigma} - g_{\rho\nu} \Gamma_{\mu\sigma}^\rho - g_{\mu\rho} \Gamma_{\sigma\nu}^\rho = 0,
\] (2.15)

\[
(H^{\alpha\mu} - \kappa g^{[\alpha\mu]} g^{\nu\beta} F_{\nu\beta}),_\mu = 0,
\] (2.16)
where
\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R. \]  

(2.17)

The variational principle yields for invariance under coordinate transformations the
four Bianchi identities:
\[ [g^{\alpha\nu} G_{\rho\nu}(\Gamma) + g^{\nu\alpha} G_{\nu\rho}(\Gamma)]_{,\alpha} + g^{\mu\nu}_{,\rho} G_{\mu\nu}(\Gamma) = 0. \]  

(2.18)

The matter response equations are
\[ \frac{1}{2} (g_{\sigma\rho} T^{\sigma\alpha} + g_{\rho\sigma} T^{\alpha\sigma})_{,\alpha} - \frac{1}{2} g_{\alpha\beta,\rho} T^{\alpha\beta} = 0. \]  

(2.19)

For \( S^\mu \) nonzero, Eq.(2.14) becomes
\[ g^{[\mu\nu]}_{,\nu} = 4\pi S^\mu. \]  

(2.20)

If we perform a Hodge decomposition of \( g^{[\mu\nu]} \):
\[ g^{[\mu\nu]} = a^{[\mu,\nu]} + \epsilon^{\mu\nu\kappa\lambda} b_{[\kappa,\lambda]}, \]  

(2.21)

we find from (2.20) that the three degrees of freedom \( a^\mu \) are determined by the NGT charge
current \( S^\mu \). The other three components, \( b^\mu \), of \( g^{[\mu\nu]} \) are not directly coupled to the NGT
charge. It is the degrees of freedom associated with \( b^\mu \) which give rise to the non-singular
nature of NGT (referred to as NSG in Refs.[10,11]). Previous work on NGT concentrated
on the \( a^\mu \) degrees of freedom.

3. The Static Spherically Symmetric Solutions

In the case of a static spherically symmetric field, Papapetrou has derived the canonical
form of \( g_{\mu\nu} \):
\[ g_{\mu\nu} = \begin{pmatrix}
  -\alpha & 0 & 0 & w \\
  0 & -\beta & f\sin\theta & 0 \\
  0 & -f\sin\theta & -\beta\sin^2\theta & 0 \\
  -w & 0 & 0 & \gamma
\end{pmatrix}, \]  

(3.1)
where $\alpha, \beta, \gamma$ and $w$ are functions of $r$. The tensor $g^{\mu\nu}$ has the components:

\[
g^{\mu\nu} = \begin{pmatrix}
\frac{\gamma}{w^2 - \alpha \gamma} & 0 & 0 & \frac{w}{w^2 - \alpha \gamma} \\
0 & -\frac{\beta}{\beta^2 + f^2} & \frac{f \csc \theta}{\beta^2 + f^2} & 0 \\
0 & -\frac{f \csc \theta}{\beta^2 + f^2} & -\frac{\beta \csc^2 \theta}{\beta^2 + f^2} & 0 \\
-\frac{w}{w^2 - \alpha \gamma} & 0 & 0 & -\frac{\alpha}{w^2 - \alpha \gamma}
\end{pmatrix}. \tag{3.2}
\]

The electromagnetic field $F_{\mu\nu}$ is defined in terms of the potentials $A_\mu$:

\[
F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}, \tag{3.3}
\]

and it has the static components:

\[
F_{10} = E(r), \quad F_{23} = H(r) \sin \theta, \tag{3.4}
\]

all other components being zero. From (2.5) and (3.1)-(3.4), it follows that $H_{\mu\nu} = F_{\mu\nu}$ and from the equation:

\[
F_{\mu\nu,\sigma} + F_{\nu\sigma,\mu} + F_{\sigma\mu,\nu} = 0, \tag{3.5}
\]

we find that $H(r)$ is a constant, which corresponds to the magnetic charge. We shall assume in accordance with Maxwell’s theory that the magnetic charge is zero. We have

\[
H^{10} = -\frac{E}{\alpha \gamma - w^2}. \tag{3.6}
\]

The determinant of the $g_{\mu\nu}$ is given by

\[
\sqrt{-g} = \sin \theta (\alpha \gamma - w^2)^{1/2} (\beta^2 + f^2)^{1/2}. \tag{3.7}
\]

The solution to Eq. (2.14) is

\[
w^2 = \frac{\ell^4 \alpha \gamma}{\beta^2 + f^2 + \ell^4}, \tag{3.8}
\]
where $\ell^2$ is a constant of integration which is identified with the NGT charge. Eq. (2.16) has the solution:

$$E = \left( w \right) \left( \frac{Q \rho^2}{\rho^2 + \kappa^2 \ell^4} \right) = \frac{Q \rho \sqrt{\alpha \gamma - w^2}}{\rho^2 + \kappa^2 \ell^4},$$

(3.9)

where $Q$ is the electric charge of a particle and

$$\rho^2 = \beta^2 + f^2.$$  

(3.10)

For $\ell^2 = 0$, we have

$$E = \frac{Q \sqrt{\alpha \gamma}}{\rho}.$$  

(3.11)

The field equations (2.13) for the static spherically symmetric case take the form:\

$$1 + \left( \frac{f B' - \beta A'}{2\alpha} \right)' + B' \left( \beta B' + f A' \right) + \frac{1}{2} \left( \frac{f B' - \beta A'}{2\alpha} \right) \ln(\alpha \gamma U)' =$$

$$\beta \rho^2 \left( \frac{E \ell^2}{w} \right)^2 + 2\kappa \beta \left( \frac{Q \ell^2}{\rho^2 + \kappa^2 \ell^4} \right)^2,$$

(3.12)

$$c + \left( \frac{\beta B' + f A'}{2\alpha} \right)' - B' \left( \frac{f B' - \beta A'}{2\alpha} \right) + \frac{1}{2} \left( \frac{\beta B' + f A'}{2\alpha} \right) \ln(\alpha \gamma U)'$$

$$= -\frac{f}{\rho^2} \left( \frac{E \ell^2}{w} \right)^2 - 2\kappa f \left( \frac{Q \ell^2}{\rho^2 + \kappa^2 \ell^4} \right)^2,$$

(3.13)

$$-A'' + \frac{1}{2} (\ln \alpha)' A' - \frac{1}{2} [(A')^2 + (B')^2] - \frac{1}{2} \ln(\gamma U)'' + \frac{1}{4} \ln(\gamma U)' \ln \left( \frac{\alpha}{\gamma U} \right)'$$

$$= -\frac{\alpha}{\rho^2} \left( \frac{E \ell^2}{w} \right)^2 + 2\kappa \alpha \left( \frac{Q \ell^2}{\rho^2 + \kappa^2 \ell^4} \right)^2,$$

(3.14)

$$\frac{\gamma}{2\alpha} [(1 - U)](A')^2 + (B')^2 + \frac{1}{2} (\ln \gamma)' \ln \left( \frac{\gamma \rho^2}{\alpha} \right)'$$

$$+ \frac{1}{2} (\ln U)' \ln \left( \frac{\gamma \rho^4}{\alpha^2} \right)' + \frac{1}{2} \frac{U - 1}{2} [(\ln U)'' + \ln(\gamma U)'^2]$$

$$= \frac{\gamma}{\rho^2} \left( \frac{E \ell^2}{w} \right)^2 - 2\kappa \gamma \left( \frac{Q \ell^2}{\rho^2 + \kappa^2 \ell^4} \right)^2.$$  

(3.15)
Here, $A$, $B$ and $U$ are defined by

$$A = \ln \rho, \quad B = \tan^{-1}\left(\frac{\beta}{f}\right),$$

(3.16)

$$U = \frac{\rho^2}{\ell^4 + \rho^2} = 1 - \frac{w^2}{\alpha\gamma},$$

(3.17)

and $A' = \partial A/\partial r$.

It is convenient to use the notation\textsuperscript{15–18}:

$$x = \frac{\rho^2}{\alpha}, \quad y = \gamma U, \quad \exp(q) = \exp(A + iB) = f + i\beta.$$  

(3.18)

Then the field equations can be written in the form:

$$2A'' - (A')^2 + (B')^2 + A'\ln(x/y) = 0,$$

(3.19)

$$(\ln y)'' + \frac{1}{2}(\ln y)'\ln(xy)' = \frac{2}{x}F,$$

(3.20)

$$q'' + \frac{1}{2}q'\ln(xy)' + 2(i + c)\frac{\exp(q)}{x} = \left(\frac{\exp(q)}{x}\right)G - \frac{2}{x}F,$$

(3.21)

where

$$G = -8\kappa\exp(-q)\left[\frac{Q\ell^2}{\rho^2 + \kappa^2\ell^4}\right]^2,$$

(3.22)

$$F = \left(\frac{E\ell^2}{w}\right)^2 - 2\kappa^2\left[\frac{Q\ell^2}{\rho^2 + \kappa^2\ell^4}\right]^2.$$

(3.23)

Let us define:

$$\lambda(r) = (y')^2\left(\frac{x}{y}\right).$$

(3.24)

Then, Eq.(3.21) can be written as

$$2\frac{d^2p}{dz^2}\lambda + \frac{d\lambda}{dz}\frac{dp}{dz} + 4(i + c)\exp(p) = 2G\exp(p),$$

(3.25)
where \( q + z = p, \) \( z = \ln y \) and \( c \) is a constant. An integral of (3.26) is given by

\[
\left( \frac{dp}{dz} \right)^2 \lambda + 4(i + c)\exp(p) = \int 2G \frac{d\exp(p)}{dz} dz + c_1,
\]

where \( c_1 \) is a complex constant. We require that

\[
\frac{d\lambda}{dz} = 4\exp(z)F = 2\text{Re} \left[ \int \exp(p) \frac{dG}{dz} \right] - (\text{Re} \ c_1) + \lambda.
\]

(3.27)

We shall consider the solution for which \( c_1 = \lambda_0(1 + is) \) where \( \lambda_0 \) and \( s \) are real constants. We choose as a further boundary condition that

\[
f \to f_0 \text{ as } r \to \infty.
\]

(3.28)

To guarantee that we obtain the Reissner-Nordstrom solution\(^{19}\):

\[
\gamma = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}, \quad \alpha = \left( 1 - \frac{2m}{r} + \frac{Q^2}{r^2} \right)^{-1}, \quad \beta = r^2,
\]

(3.29)

when \( f = \ell^2 = 0 \), we require that \( c = 0 \).

When \( Q = 0 \), we obtain the Vanstone solution\(^{18}\):

\[
f + i\beta = \left[ \frac{i\lambda_0}{4y} \right] (1 + is)\text{csch}^2[\sqrt{1 + is/2\ln y}],
\]

(3.30)

\[
\gamma = \left( \frac{\ell^4 + f^2 + \beta^2}{f^2 + \beta^2} \right)y, \quad \alpha = \frac{(y')^2(f^2 + \beta^2)}{y(4yQ^2 + \lambda_0)},
\]

(3.31)

\[
w = \frac{\ell^2(y')}{\sqrt{4yQ^2 + \lambda_0}},
\]

(3.32)

where \( \lambda_0 = 4m^2 \) and \( y \) is an arbitrary function of \( r \).

For \( Q \neq 0 \) and \( \ell^2 = 0 \), Eq.(3.23) gives

\[
G = 0, \quad F = Q^2 = \text{const.}
\]

(3.33)
and (3.28) leads to
\[ \lambda = 4\exp(z), \quad (\text{Re } c_1) = \lambda_0, \quad (3.34) \]

From (3.26), we get
\[ \frac{2}{\sqrt{c_1}} \arcsinh \left[ \sqrt{\frac{i c_1}{4}} \exp\left( -\frac{p}{2} \right) \right] = \int \frac{dy}{y(4yQ^2 + \lambda_0)^{1/2}}. \quad (3.35) \]

Solutions to the NGET field equations have been found by Mann\textsuperscript{14}, in the cases \( B' = 0 \) and \( \ell^2 = 0 \). We shall only be concerned here with the latter solution. We must choose realistic boundary conditions. For \( r \to \infty \) these must include:
\[ \alpha \to 1, \quad \gamma \to 1, \quad \beta \to r^2. \quad (3.36) \]

Choosing \( \beta = r^2 \) so that the radial coordinate satisfies: \( (r\text{-coordinate})=(\text{proper circumference})/2\pi \), we find that
\[ y = \gamma = \exp(\nu), \quad \alpha = \frac{(\gamma')^2(f^2 + r^4)}{\gamma(4\gamma Q^2 + \lambda_0)} \quad (3.37) \]
\[ f = \left( \frac{\lambda_0}{2\gamma} \right)(\cosh\psi_a - \cos\psi_b)^{-2}[s(1 - \cosh\psi_a\cos\psi_b) + \sinh\psi_a\sin\psi_b], \quad (3.38) \]
where
\[ \psi_a = 2a \left( \text{arcsinh} \sqrt{\frac{\lambda_0}{4Q^2}} - \text{arcsinh} \sqrt{\frac{\lambda_0}{4Q^2\gamma}} \right), \quad \psi_b = \psi_a(a \leftrightarrow b), \quad (3.39) \]
and
\[ a = \sqrt{\frac{\sqrt{1 + s^2} + 1}{2}}, \quad b = \sqrt{\frac{\sqrt{1 + s^2} - 1}{2}}. \quad (3.40) \]
Moreover, we have
\[ \lambda_0 = 4(m^2 - Q^2). \quad (3.41) \]

The function \( \nu \) is given implicitly by
\[ 2\exp(\nu)(\cosh\psi_a - \cos\psi_b)^2\frac{f^2}{\lambda_0} = [\sinh\psi_a\sin\psi_b - (1 - \cosh\psi_a\cos\psi_b)]. \quad (3.42) \]
For $Q = \ell^2 = 0$, we recover the Wymann solution:\(^{16}\)

\[
\gamma = \exp(\nu),
\]

\[
\alpha = m^2(\nu')^2 \exp(-\nu)(1 + s^2) (\cosh(\alpha) - \cos(\nu))^{-2},
\]

\[
f = [2m^2 \exp(-\nu)(\sinh(\alpha)\sin(b\nu) + s(1 - \cosh(\alpha)\cos(\nu)))(\cosh(\alpha) - \cos(\nu))^{-2},
\]

where now $\nu$ is implicitly determined by the equation:

\[
\exp(\nu)(\cosh(\alpha) - \cos(\nu))^2 \frac{r^2}{2m^2} = \cosh(\alpha)\cos(b\nu) - 1 + s\sinh(\alpha)\sin(b\nu).
\]

4. Analysis of the Non-Singular Solutions

We shall be interested in the branch of multiple solutions for $\nu$ in Eqs. (3.42) and (3.46), which matches onto the unique solution for large $r$. Thus, we are interested in the unique inversions of (3.42) and (3.46) which yield an asymptotically flat spacetime. We are only able to invert (3.43) analytically for $r/m < 1, r/Q < 1$ and $2m/r < 1, Q/r < 1$ and we must resort to numerical methods to establish the intermediate behavior.

We find for $2m/r < 1, Q/r < 1$ and $0 < sm^2/r^2 < 1$ that the metric takes the near Reissner-Nordstrom form:

\[
\gamma = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} + \frac{s^2(m^2 - Q^2)^2}{15r^4} \left( \frac{m}{r} + \frac{4m^2 - Q^2}{r^2} + \ldots \right),
\]

\[
\alpha = \left[ 1 - \frac{2m}{r} + \frac{Q^2}{r^2} + \frac{s^2(m^2 - Q^2)^2}{9r^4} \left( 2 + \frac{7m}{r} + \ldots \right) \right]^{-1},
\]

\[
f = s(m^2 - Q^2) \left[ \frac{1}{3} + \frac{2m}{3r} + \frac{6m^2 - Q^2}{5r^2} + \ldots \right],
\]

where the higher order terms in $m/r$ and $Q/r$ include higher powers of $s$ also. We observe that for small enough $s$ the NGT corrections to the Reissner-Nordstrom solution and
for $Q = 0$ to the Schwarzschild solution, can be made arbitrarily close to experimental predictions of Einstein-Maxwell theory and EGT.

We can develop expansions near the origin where $r/m < 1, r/Q < 1$ and $0 < s < 1, Q/m < 1$. Similar expansions exist for $-1 < s < 0$. The leading terms are:

$$\gamma = \gamma_0 + \mathcal{O}\left(\left(\frac{r}{m}\right)^2\right)$$

$$\alpha = \frac{4}{s^2} \left(1 + \frac{1}{2} \frac{Q^2}{m^2}\right) \exp\left(-\frac{\pi}{s} - 2 - \frac{\pi s}{8}\right) \left(\frac{r}{m}\right)^2 + \mathcal{O}\left(\left(\frac{r}{m}\right)^4\right).$$

$$f = m^2 \left(4 - \frac{s\pi}{2} + s^2 - \frac{Q^2}{m^2}\right) + \mathcal{O}\left(\left(\frac{r}{m}\right)^2\right),$$

where $\gamma_0$ is given by

$$\gamma_0 = \left(1 - \frac{1}{2} \frac{Q^2}{m^2}\right) \exp\left(-\frac{\pi}{s} - 2 - \frac{\pi s}{8}\right).$$

For $r$ near zero, $0 < s < 1$ and $(m - Q)$ small, we get

$$\gamma = \gamma_0 + \mathcal{O}\left(\left(\frac{r}{m}\right)^2\right),$$

$$\alpha = \frac{4\gamma_0}{s^2} \frac{r^2}{m^2 - Q^2} + \mathcal{O}\left(\left(\frac{r}{m}\right)^4\right),$$

$$f = Q^2 \left(1 - \frac{\pi s}{8} + \frac{s^2}{4} (1 - \ln 2) + \ldots\right) + \mathcal{O}\left(\left(\frac{r}{m}\right)^2\right).$$

Here, we have

$$\gamma_0 = \left(\frac{m^2 - Q^2}{Q^2}\right) \exp\left(-\frac{\pi}{s} - 2\right) + \mathcal{O}\left(\left(\frac{r}{m}\right)^2\right).$$

The above results (4.4)–(4.11) clearly illustrate the non-analytic nature of the NGET solution in the limit $s \to 0$ in the limit of strong gravitational fields. Numerical results have confirmed that all the expansions given above produce excellent approximations to the exact solution in their respective regions of validity.
When $Q = 0$, we find for $2m/r < 1$ and $0 < sm^2/r^2 < 1$ that the metric takes the near-Schwarzschild form:

$$\gamma = 1 - \frac{2m}{r} + \frac{s^2m^5}{15r^5} + \frac{4s^2m^6}{15r^6} + \ldots,$$

(4.12)

$$\alpha = \left(1 - \frac{2m}{r} + \frac{2s^2m^4}{9r^4} + \frac{7s^2m^5}{9r^5} + \ldots\right)^{-1},$$

(4.13)

$$f = \frac{2sm^2}{3} + \frac{2m^3}{3r} + \frac{6sm^4}{5r^2} + \ldots.$$  

(4.14)

Near $r = 0$ we can develop expansions where $r/m < 1$ and $0 < |s| < 1$. The leading terms are

$$\gamma = \gamma_0 + \frac{\gamma_0(1 + \mathcal{O}(s^2))}{2|s|} \left(\frac{r}{m}\right)^2 + \mathcal{O}\left(\left(\frac{r}{m}\right)^4\right),$$

(4.15)

$$\alpha = \frac{4\gamma_0(1 + \mathcal{O}(s^2))}{s^2} \left(\frac{r}{m}\right)^2 + \mathcal{O}\left(\left(\frac{r}{m}\right)^4\right),$$

(4.16)

$$f = m^2 \left(4 - \frac{|s|\pi}{2} + s|s| + \mathcal{O}(s^3)\right) + \frac{|s| + s^2\pi/8 + \mathcal{O}(s^3)}{4} r^2 + \mathcal{O}(r^4),$$

(4.17)

$$\gamma_0 = \exp\left(-\frac{\pi + 2s}{|s|} + \mathcal{O}(s)\right)\ldots$$

(4.18)

As in the electrically charged case, these solutions clearly illustrate the non-analytic nature of the limit $s \to 0$ in the strong gravitational field regime.

The solution for the extremal case $Q = m$ cannot be obtained from the above expansions, but must be derived from another branch of the non-singular solution given by:

$$f = \frac{s}{2\gamma}(1 - \cosh\psi\cos\psi)(\cosh\psi - \cos\psi)^{-2},$$

(4.19)

$$2\gamma r^2 = ssinh\psi\sin\psi(\cosh\psi - \cos\psi)^{-2},$$

(4.20)

where

$$\psi = \sqrt{\frac{s}{2}} \left(\frac{1}{\sqrt{\gamma}} - 1\right).$$

(4.21)
Near \( r = 0 \) we get
\[
\gamma = \frac{s}{(\sqrt{s} + \pi \sqrt{2})^2} + \mathcal{O}\left(\left(\frac{r}{Q}\right)^2\right), \tag{4.22}
\]
\[
\alpha = \frac{8(\cosh\pi + 1)}{(\cosh\pi - 1)(\sqrt{s} + \pi \sqrt{2})^2} \left(\frac{r}{Q}\right)^2 + \mathcal{O}\left(\left(\frac{r}{Q}\right)^4\right). \tag{4.23}
\]
\[
f = \frac{Q^2(\sqrt{s} + \pi \sqrt{2})^2}{2(1 + \cosh\pi)} + \mathcal{O}(r^2). \tag{4.24}
\]
Note that (4.22), (4.23) and (4.24) are exact for any \( s \).

Let us consider the electric field obtained from (3.11). We have
\[
f^2 + r^4 = \lambda_0 \exp(-2\nu)(1 + s^2)(\cosh\psi_a - \cos\psi_b)^{-2}, \tag{4.25}
\]
and
\[
\alpha\gamma = \frac{(\gamma')^2(f^2 + r^4)}{4[m^2 + (\gamma - 1)Q^2]}. \tag{4.26}
\]
This leads to the result:
\[
E_r = \frac{Q\gamma'}{2(m^2 + (\gamma - 1)Q^2)^{1/2}}. \tag{4.27}
\]
Using (3.4) and (3.6), we can calculate the invariant quantity:
\[
\Phi = F^\mu\nu F_{\mu\nu}.
\]
We find that
\[
\Phi = -\frac{2Q^2}{r^4 + f^2}, \tag{4.28}
\]
which is finite at \( r = 0 \). Moreover, if we define the dual tensor: \( ^*F^\mu\nu = \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \), then it follows that the other electromagnetic invariant, \( ^*F^\mu\nu F_{\mu\nu} \) equals zero. The electric field invariant \( \Phi \) is finite in the presence of the nonsymmetric gravitational field. In fact, when any gauge field is incorporated into NGT, the resulting combined field theory invariants will be made finite at \( r = 0 \), because of the geometrical properties of NGT at \( r = 0 \).
It should be stressed at this stage that the general solution with $\ell^2 \neq 0$ is non-singular everywhere in spacetime. Our results derived for $\ell^2 = 0$ can be extended to the general Vanstone or Mann solution, depending in a non-analytic way on the dimensionless parameter $s$.

For $Q = 0$ and $s < 1$ the maximum red-shift is between $r = 0$ and $r = \infty$, and is given by

$$z = \exp\left(\frac{\pi}{2|s|} + \frac{s}{|s|} + \mathcal{O}(s)\right) - 1.$$  \hspace{1cm} (4.29)

A timelike Killing vector at spatial infinity remains timelike throughout the spacetime, which means that our solutions are free of event horizons and do not possess black holes.

The NGT curvature invariants such as the generalized Kretschmann scalar are finite. This follows from the fact that the Mann, Vanstone and Wyman solutions share the same form of expansion near $r = 0$:

$$\gamma = \gamma_0 + \gamma_2 r^2 + ..., \hspace{1cm} (4.30)$$

$$\alpha = \alpha_2 r^2 + \alpha_4 r^4 + ..., \hspace{1cm} (4.31)$$

$$f = f_0 + f_2 r^2 + .... \hspace{1cm} (4.32)$$

Because the NGET solution takes this form a calculation shows that all the curvature invariants are finite. For example, the generalized Kretschmann invariant is given near $r = 0$ by

$$K = R_{\mu\nu\kappa\lambda} R_{\mu\nu\kappa\lambda} = -\frac{4}{f_0^4\alpha_2^4\gamma_0^4} \left[ \frac{\alpha_2^4 f_0^2 \gamma_0^4 - \gamma_2^4 \alpha_2^2 \gamma_0^2 f_0^4 - f_2^4 \alpha_2^2 \gamma_0^4}{f_0^4\alpha_2^4\gamma_0^4} \right] + \mathcal{O}(r^2). \hspace{1cm} (4.33)$$

For the case $Q = 0$, we find to leading order in $s$:

$$K = \frac{\exp(2\pi/s + 4)}{16m^4}, \hspace{1cm} r = 0, \hspace{1cm} (4.34)$$
\[ \frac{48m^2}{r^6}, \quad r \to \infty. \quad (4.35) \]

We note that the singularity caused by the vanishing of \( \alpha(r) \) at \( r = 0 \) is a \textit{coordinate} singularity, which can be removed by transforming to another coordinate frame of reference. The curvature invariants do not, of course, contain any coordinate singularities.

We can transform the standard line element:

\[ ds^2 = \gamma dt^2 - \alpha dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (4.36) \]

to a line element which is regular near \( r = 0 \) by the transformation:

\[ \bar{r} = \frac{r^2}{m}. \quad (4.37) \]

We obtain for \( Q = 0 \):

\[ ds^2 = \left[ \gamma_0 + \mathcal{O}\left(\frac{\bar{r}}{m}\right) \right] dt^2 - \left[ \frac{\gamma_0(1 + \mathcal{O}(s^2))}{s^2} + \mathcal{O}\left(\frac{\bar{r}}{m}\right) \right] d\bar{r}^2 - m\bar{r}(d\theta^2 + \sin^2 \theta d\phi^2). \quad (4.38) \]

In this coordinate system, a radially directed photon near \( \bar{r} = 0 \) has the finite coordinate velocity:

\[ \frac{d\bar{r}}{dt} = s + \mathcal{O}(s^3). \quad (4.39) \]

Let us now consider the proper volume obtained from (3.7):

\[ V_p = \int \sqrt{-g} dr d\theta d\phi. \quad (4.40) \]

By using (3.7), we get for \( Q = \ell^2 = 0 \) near \( r = 0 \):

\[ \sqrt{-g} = \frac{4}{3} \exp(-\pi/s - 2) r \sin \theta, \quad (4.41) \]

and

\[ V_p = \frac{16\pi r^2 m}{s} \exp(-\pi/s - 2). \quad (4.42) \]
In comparison, EGT gives
\[ V_p = \frac{4\pi r^3}{3}. \] (4.43)

The surface area and circumference of a body in the non-singular NGT solution are given by \( S = \text{Area} = 4\pi r^2 \) and \( \text{Circumference} = 2\pi r \), respectively, which is the same for the Schwarzschild solution in EGT. The curvature invariants are proportional to \((S/V_p)^2\) and they are finite constants at \( r = 0 \). The \( V_p \) scales as the surface area for small \( r \) which demonstrates that a unique geometry exists at the origin. It is this unique geometry which renders all fields finite at \( r = 0 \). The proper volume near \( r = 0 \) in NGT is infinitely larger than in EGT, and it is by this mechanism that infinite energy is avoided.

The equations of motion of a test particle, in NGT, have been derived from the matter response equations (2.19)\(^5\):
\[
\frac{du^\mu}{d\tau} + \left\{ \frac{\mu}{\alpha\beta} \right\} u^\alpha u^\beta = \ell_t^2 K^{\mu\nu} u^\nu + \frac{e}{m_t} F^{\mu\nu} u^\nu, \tag{4.44}
\]
where \( u^\mu = dx^\mu/d\tau \), \( \ell_t^2, m_t \) and \( e \) are the test particle NGT charge, mass and electric charge, respectively, and
\[
K^{\mu\nu} = \frac{1}{2} \gamma^{(\mu\rho)} R_{[\rho\nu]}(\Gamma). \tag{4.45}
\]
Moreover,
\[
\left\{ \frac{\lambda}{\alpha\beta} \right\} = \frac{1}{2} \gamma^{(\lambda\rho)} \left( g(\mu\rho),\nu + g(\rho\nu),\mu + g(\mu\nu),\rho \right), \tag{4.46}
\]
and
\[
\gamma^{(\lambda\rho)} g(\lambda\sigma) = \delta^\sigma_\nu. \tag{4.47}
\]

We have \( R_{[10]}(\Gamma) = R_{[23]}(\Gamma) = 0 \) for \( \ell^2 = 0 \) and for \( Q = 0 \), it follows from (4.41) that test particles follow geodesics as in EGT:
\[
\frac{du^\mu}{d\tau} + \left\{ \frac{\mu}{\alpha\beta} \right\} u^\alpha u^\beta = 0. \tag{4.48}
\]
For the non-singular solutions of NGT the spacetime is geodesically complete.

In EGT, it follows from the Hawking-Penrose theorem that when a star collapses it forms a trapped surface and this leads inevitably to a singularity at \( r = 0 \), provided the density \( \rho \) and the pressure \( p \) satisfy \( \rho + 3p > 0 \). In NGT, the presence of repulsive forces for \( |s| > 0 \) prevents the formation of singularities, trapped surfaces and event horizons, while the positivity condition: \( \rho + 3p > 0 \) is satisfied at all times. Thus, even an infinitesimally small value of \( s \) leads to a non-singular theory of gravity free of black holes, which satisfies all known experimental observation in gravity. It would be tempting to say that nature would prefer such a rational description of spacetime compared to EGT, in which singularities occur and observable quantities like densities and red-shifts become infinite at points in spacetime.

Let us calculate the energy density of a charged body between \( r = 0 \) and \( r = \infty \). We have

\[
T^0_0 = \frac{Q^2}{8\pi(r^4 + f^2)}. \tag{4.49}
\]

Integrating this over a volume in spherical polar coordinates gives

\[
\mathcal{E} = \int \sqrt{-g} T^0_0 dr d\theta d\phi = \frac{Q^2}{2} \int_0^\infty \frac{(\alpha\gamma)^{1/2}}{(r^4 + f^2)^{1/2}} dr
\]

\[
= \frac{Q^2}{2} \int_0^\infty \frac{y'}{2} \frac{dr}{(4yQ^2 + \lambda_0)^{1/2}} = \frac{1}{4} \left[ (4Q^2 + \lambda_0)^{1/2} - (4\gamma_0 Q^2 + \lambda_0)^{1/2} \right]. \tag{4.50}
\]

For the case when \( \lambda_0 = 4(m^2 - Q^2) \), we find that

\[
\mathcal{E} = \frac{1}{2} \left\{ m - \left[ m^2 - Q^2(1 - \gamma_0) \right]^{1/2} \right\}. \tag{4.51}
\]

For \( m >> Q \), this becomes

\[
\mathcal{E} = \frac{Q^2}{2m} (1 - \gamma_0) + \mathcal{O}\left( \left( \frac{Q}{m} \right)^2 \right). \tag{4.52}
\]
For the extremal case $Q = m$, we get

$$\mathcal{E} = Q \left[ \frac{\pi}{\sqrt{2}(\sqrt{2\pi} + \sqrt{s})} \right].$$  \hspace{1cm} (4.53)

We have obtained the remarkable result that the total electromagnetic field energy for a spherically symmetric charged particle is finite. In the past, it was suggested that electrons had a finite size in order to overcome the problem of divergences in field theory. This proposal suffered from the problem that the Coulomb repulsive force would blow the particle apart. Here the finiteness of the particle energy is not achieved by giving it a finite size, but by increasing the proper volume near $r = 0$. A correct description of an electron must be based on a quantum field theory, so we cannot expect a classical NGT description of the electron to be realistic. In natural units $Q \gg m$ for the electron, whereas in our solution $Q \leq m$.

5. Conclusions

We have analyzed solutions to the NGET and NGT field equations, which are non-singular everywhere in spacetime. The curvature invariants are finite and there are no event horizons, which means that there are no black hole solutions in the theory. There will exist stable, super dense objects that replace black holes, which have large red-shifts at their surfaces. These SDO’s will radiate thermal and non-thermal radiation; they will not radiate Hawking radiation and there will not be an information loss problem as with black holes. This has profound implications for our understanding of quantum gravity, for there is no need for a fundamental change in quantum mechanics to resolve the problem of predictability associated with black holes and Hawking radiation in EGT, because spacetime is no longer separated into two disconnected regions described by two different Hilbert spaces.
The problem of gravitational collapse is presently being investigated\textsuperscript{21}, together with the singularity problem in early Universe cosmology. A preliminary analysis reveals that there will be a non-zero acceleration of the coordinate scale factor $R(r, t)$ as a collapsing star approaches $r = 0$, yielding a non-singular final state of collapse. Similar conclusions can be drawn about the non-singular nature of the cosmological solution at $t = 0$.

It is clear that in the light of the success of constructing a consistent non-singular theory of gravity without black holes, in the form of non-singular NGET and NGT, the whole notion that black holes exist in nature must be critically reconsidered. It is, of course, difficult to observationally distinguish a SDO from a black hole. The SDO is kept stable by the repulsive skew symmetric forces which supplement the standard matter pressures when the SDO reaches extremely high densities. In the case of active galactic nuclei, that have been purported to be large black holes\textsuperscript{22}, the event horizon at $r = 2m$ would be deep within the interior of the galaxy. Only an unambiguous detection of a null surface at $r = 2m$ would clearly settle the issue, but such a null surface is hidden from the view of the observer. The same is true of black hole candidates such as Cygnus X-1\textsuperscript{23} for which the event horizon is hidden from view by Newtonian-like accretion disks. The estimated mass of the unseen companion of Cyg X-1 is $M_x \sim 10 - 16 M_\odot$, which is too large to be a neutron star. But the criterion used to identify the dark companion with a black hole is based on EGT and the choice of an equation of state for matter at or greater than nuclear densities. Studies have shown that an equation of state derived from alternative exotic types of matter, such as soliton stars\textsuperscript{24}, or an equation of state based on effective field theories of bulk nuclear matter\textsuperscript{25}, can lead to stable compact objects with masses in excess of $10^6 M_\odot$. At present, there is no known unique signature that distinguishes such EGT objects from an SDO in NGT.
It is imperative that new attempts be made to experimentally settle the question of whether genuine event horizons exist in nature, because an unambiguous answer will have profound implications for our understanding of the nature of the geometry of spacetime.

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