Theoretical stellar models for old galactic clusters

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ABSTRACT
We present new evolutionary stellar models suitable for old Population I clusters, discussing both the consequences of the most recent improvements in the input physics and the effect of element diffusion within the stellar structures. Theoretical cluster isochrones are presented, covering the range of ages from 1 to 9 Gyr for the four selected choices on the metallicity Z= 0.007, 0.010, 0.015 and 0.020. Theoretical uncertainties on the efficiency of superadiabatic convection are discussed in some details. Isochrone fitting to the CM diagrams of the two well observed galactic clusters NGC2420 and M67 indicates that a mixing length parameter $\alpha = 1.9$ appears adequate for reproducing the observed color of cool giant stars. The problems in matching theoretical predictions to the observed slope of MS stars are discussed.

Key words: stars: evolution, HR diagram, open clusters and associations: general

1 INTRODUCTION
Evolutionary stellar models represent a key ingredient for any investigation about the evolution of galaxies. Theoretical predictions concerning cluster isochrones represent the only available clock for assessing the age of stellar clusters in our Galaxy and beyond, allowing a meaningful approach to the star formation history in local group galaxies. Moreover, the body of theoretical prescriptions concerning the evolutionary status of stars with different ages and chemical compositions is at the basis of the present effort for understanding the radiative properties of galaxies in terms of stellar populations. According to such evidence, it appears of obvious relevance to rely on evolutionary scenarios as detailed and reliable as possible.

In a recent paper (Cassisi et al. 1998, hereinafter Paper I) we have revisited stellar models for old Population II stars, discussing the effects of both the most recent improvements in the input physics and of element diffusion within the stellar structures. In this paper we will use the same evolutionary scenario to extend theoretical predictions to stellar metallicities and ages suitable for Population I stars. As in Paper I, we will discuss in some detail the reliability of evolutionary predictions vis-a-vis well known uncertainties of the theoretical scenario, as a useful warning against an incorrect use of theoretical predictions.

The results of selected evolutionary computations will be presented and discussed in the next section. Sections 3 will deal with a comparison with observational data for the two clusters NGC2420 and M67, taken as representatives of the “best observed” galactic clusters in selected classes of age. On this basis we will collect observational suggestions concerning a suitable choice of the mixing length parameter to be used in constructing models of cool stars, together with some warnings about the use of theoretical results. Section 4 will present and discuss cluster isochrones for selected choices about star metallicity. General conclusions are presented in the subsequent section.

2 THEORETICAL MODELS
For the sake of discussion, let us first neglect element diffusion to recall some relevant points concerning the evolutionary scenario for Population I stars. The upper panel of Figure 1 shows selected tracks in the logL, logTe diagram, as evaluated by adopting a ratio of the mixing length to the local pressure scale height $\alpha = 1.6$, and under the alternative assumptions $Z = 0.007, 0.010, 0.015$ and 0.020. Data in this figure show the remarkable similarity of tracks at fixed mass but different metallicities. As already known, an increase in the metal content moves ZAMS models toward redder colors and fainter magnitudes, whereas the difference in temperature between the ZAMS and the Red Giant Branch (RGB) decreases.

However, one knows that theory gives no firm constraints about the most appropriate value of $\alpha$, so that it appears of some relevance to bring to the light the consequences of a variation of the mixing length parameter within reasonable limits. This is shown in the lower panel of the same Figure 1, which reports evolutionary tracks given in the upper panel for $Z = 0.007$ but under the three alternative choices $\alpha = 1.0, 1.6$ or 2.2. As already known, one finds that theory gives firm predictions about the temperature of...
ZAMS stars only for stars with masses larger or of the order of 1.5 M⊙. Below this limit the uncertainty in the temperature appears roughly of the same order of magnitude of the difference produced in the upper panel of Figure 1 by the variation in metallicity. Moreover, the figure reinforces the evidence that theory gives only marginal constraints on the temperature (and thus on the color) of the Red Giant Branch (RGB), which appears as a free parameter to be adjusted in the case by tuning the assumption on α.

To allow the comparison with the observed C-M diagrams, one has to use evolutionary tracks to predict cluster isochrones in the chosen bands of magnitudes and colors. This has been done by first producing “theoretical” isochrones in the logL, logTe diagram, to be finally transformed into C-M diagrams according to suitable assumptions about the bolometric corrections and the color temperature relations. Let us notice that the intermediate step represent the "true" evolutionary result, to be compared with the result of similar evolutionary evaluations before the intervention of further assumptions about model atmosphere computations.

Figure 2 shows a selected sample of the new theoretical isochrones, as computed for the labeled assumptions about the cluster ages, α=1.6 and Z=0.02. One may notice the mild variation of the luminosity of the clump of He burning stars for cluster ages larger than 1 Gyr, already discussed in Castellani, Chieffi & Straniero (1992, hereinafter CCS). As a consequence, for each given chemical composition and in the above quoted range of ages the difference in magnitude between the clump and the MS turn off appears a good indicator of the cluster ages and, for each age, the clump luminosity appears as a relevant standard candle for deriving the cluster distance modulus. Note that for ages lower than 1 Gyr the luminosity of the clump follows the variation of the size of the He core through and beyond the "Red Giant Transition" as already exhaustively discussed in the current literature (see, e.g., Sweigart, Greggio & Renzini 1990).

Isochrones with solar composition (Z=0.02) allow a comparison with previous CCS results, as obtained from the same code with the same assumption about the mixing length, but with "old" input physics. As a most significant comparison we will chose a cluster age large enough for red giants undergoing electron degeneracy. The comparison, as given in Figure 3 (upper panel) for the 4 Gyr isochrones, shows only marginal differences. However, the use of more recent evaluations of colors gives larger differences in the predicted CM diagrams. This is shown in the lower panel of the same Figure 3, which shows the new 4 Gyr isochrone as translated into the V, B-V diagram adopting for MS stars the empirical relations given by Alonso et al. (1996), implemented with Castelli et al. (1997a,b) model atmospheres, shifted by B-V≃0.04 to match the empirical relations at their limit of validity (log g ≃ 4, T_e ≃ 6700 K). In this way we attempted to match model atmospheres to current observational constraints, though at the cost of some marginal discontinuities in the predicted run of colors. Comparison with the isochrone presented by CCS, as given in the same lower panel, reveals -in particular- that the new transformations predict a steeper MS, with consequences that will be discussed later on in this paper.

Figure 4 discloses the effect on the isochrones of different assumptions about the mixing length. As expected on the basis of evolutionary tracks in Figure 1, one finds that cooler stars are strongly affected by the mixing length. MS and TO stars are free from such a theoretical uncertainty only for B-V smaller than B-V≃0.4. For color redder than this limit the uncertainty in color grows, reaching approxi-
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mately $\delta(B-V) \approx 0.1$ mag. in the interval $M_V = 4.0-6.0$ mag., slowly decreasing at even lower MS luminosities.

The above evolutionary evidence is a consequence of the well-known occurrence for which above a given effective temperature stellar envelopes become free of convection and, thus, not affected by the treatment of such a mechanism. Note that decreasing the MS mass, the density of the envelope increases and convection becomes less and less superadiabatic; as a consequence, the (convective) less massive MS structures are only marginally affected by assumptions about the mixing length (vandenBerg, Hartwick & Dawson 1983). This occurrence explains the decreasing shift produced by variation in the mixing length on the lower portion of the MS in Figure 4. The different dependence of RG and MS stars at a given color is again related to the density of the envelopes: RG have rarefied envelope, thus experiencing strong superadiabaticity, whereas MS stars have much denser envelopes, thus less affected by superadiabatic convection.

As already noticed by Chaboyer (1995) for Population II systems, one finds that the mixing length does affect the calibration of the TO magnitudes in terms of cluster ages. According to the discussion given in Paper I, the effect of mixing length becomes larger for TO points located at redder colors, i.e., when increasing either the age or the star metallicity. Figure 4 shows that such an error is rather negligible for the 3 Gyr isochrone, but it increases to about 1 Gyr for the 6 Gyr isochrone, growing up for larger ages. Thus the TO magnitude cannot supply age evaluations for old metal rich clusters more precise than this limit, until an improved treatment of convection becomes available for stellar evolutionary models.

Concerning He burning stars, the previous Figure 2 shows that for ages larger than, about, 1 Gyr, He burning occurs near the RG branch, in agreement with observational evidences. However, the beautiful theoretical constraint given by the rather constant luminosity of the clump of He burning stars, will give less firm predictions when transferred into the CM diagram, since magnitudes depend on the bolometric correction which, in turn, depends on the color of the clump and, thus, on the assumptions adopted for the mixing length parameter.

Before closing the discussion about the mixing length, let us here advise the reader that the problem is not, or at least it could not be, to find out the "right value" of the mixing length. In fact, the mixing length theory is only a rough (though useful) parametrization of the efficiency of convection, and there is no reason for constraining the mixing length parameter to be the same in stellar structures characterized not only by different stellar masses or chemical compositions, but also in different evolutionary phases of a given star. Thus the above discussion about the effects of the mixing length has to be taken as an investigation of the range of theoretical uncertainty on the various evolutionary phases, without necessarily assuming that, e.g., MS and RG should have effective temperatures corresponding to a common value of $\alpha$.

Finally, one may investigate how far the efficiency of element sedimentation affects the above results. Figure 5 shows selected isochrones for $Z=0.015$ as compared with similar computations but with element diffusion taken into account. As expected, one finds that diffusion plays a role only at larger ages, decreasing the color of the turn-off (TO), with minor influence on other evolutionary phases. It turns out that the diffusion sensitively affects only the isochrones which are already affected by much larger uncertainties in the mixing length, masking the effect of diffusion. Thus one can safely use canonical, no-diffusion models, bearing in mind the above discussed theoretical indeterminations. Note that this is not the case for old metal poor globular cluster stars, where diffusion has been efficient for much larger times (see Paper I).

3 FROM OBSERVATIONS TO THEORY AND BACK

According to the discussion given in the previous section one should feel reluctant to present a theoretical scenario based on a given assumption about the value of the mixing length parameter. In this section we will follow a different approach, comparing the evolutionary scenario depicted in the previous section with observational data, looking for more light about reasonable theoretical predictions. For accomplishing this goal, we focused our attention on two "best studied" galactic clusters, namely NGC2420 and M67, for which good C-M diagrams have been already presented in the literature. In the following we will investigate the agreement between observational data and theoretical predictions, as computed by including element diffusion, searching for observational constraints to the theory.

3.0.1 NGC 2420

Figure 6 shows the beautiful CM diagram of NGC2420 presented by Antony Twarog et al. (1990). Following these authors the cluster should have a reddening not smaller than $E(B-V) = 0.05$, with some evaluations as large as $E(B-V) = 0.14$ (Cohen 1980). Thus the color of TO stars should be lower than $B-V = 0.35$. According to the discussion given in the previous section, we note that the cluster appears young enough to have TO stars only marginally affected by the mixing length parameter. Such a fortunate occurrence decreases the degrees of freedom and made the cluster our first choice for fitting theory to observations.

According to Antony Twarog et al. (1990) metallicity estimates give for the cluster $[\text{Fe/H}] = -0.35 \pm 0.10$, but with evaluations reaching $[\text{Fe/H}] = -0.6$ (Canetra et al. 1986). We will assume $Z = 0.007$ ($[\text{Fe/H}] = -0.4$). By taking $Y = 0.23$ for old metal poor stars and $Y = 0.27$ for the Sun ($Z_c = 0.02$), a
linear interpolation on metallicity finally gives for NGC2420 Z=0.007 and Y=0.244.

As disclosed by the same Figure 6, the fitting is not only possible, but it appears remarkably good, since it reproduces to a high degree of accuracy the shape of H burning stars near and beyond the overall contraction phases. In this way one would derive $E(B-V) \sim 0.16$, $DM \sim 12.4$, $\alpha = 1.9$ and a cluster age of about 1.5 Gyr, in reasonable agreement with previous CCS results, as obtained for a solar composition. Comparison with the isochrones for the ages 1.25 and 1.75 Gyr, as given in the same figure, shows that the formal error in age should not exceed 0.1 Gyr! Of course, reddening depends on the adopted relation between temperatures and colors. Adopting only Castelli et al. (1997) models, one would derive a reddening smaller by 0.05, thus $E(B-V) \sim 0.11$. Similarly, the distance modulus depends on the amount of bolometric corrections which, however, appears much less model dependent than colors do.

One has finally to notice a disagreement between the theoretical and the observed location of the lower portion of the main sequence, as due to the evident inadequacy of either stellar models or color temperature relations. The agreement would be improved adopting for these models smaller values of the mixing length, which means to assume external convection to be less efficient in MS stars than expected adopting a common mixing length ($\alpha = 1.9$) for both MS and RG stars. However, Figure 7 shows that low masses MS stars are less and less affected by the mixing length. Thus it appears difficult to reconcile theory with observations only by tuning the mixing length. As we will discuss in the case of M67, we suspect that such a disagreement is due to the inadequacy of adopted color-temperature relations.

Theoretical predictions can be submitted to a further independent test. As largely discussed in CCS, in relatively young clusters the distribution of stars in the advanced evolutionary phases is a rather sensitive function of the cluster age. Thus beyond the agreement between the CM diagram loci, one can test the theoretical distribution vis-a-vis the observed distribution. This has been done by computing, with a Monte Carlo technique, the synthetic CM diagram of the cluster, with the same number of off main sequence stars as observed. Theoretical predictions, as shown in Figure 8, middle panel, appear in reasonable agreement with observations, giving further support to the predicted cluster age. The lower panel finally compares the synthetic cluster with the theoretical isochrone, giving light on the effect of binary stars on the topology of the overall contraction gap we will discuss in the final section.

3.1 M67

Figure 9 shows the CM diagram of M67 presented by Montgomery, Marschall & Janes (1993). According to these authors, the cluster is characterized by a metallicity [Fe/H] = -0.05. Correspondingly we will refer to an evolutionary scenario as computed for $Z=0.015$. Reddening evaluations range in the interval $E(B-V) = 0.03 - 0.10$ (Antony Twarog et al. 1990); in particular Cohen (1980) analyzed absorption lines of sodium of the interstellar gas giving $E(B-V) = 0.09$.

Inspection of Figure 9 discloses that now all the cluster stars lie in a region affected by the uncertainty about the mixing length. Moreover, the MS of the cluster is now largely in the range of colors where we already found a mismatch between theory and observation. The same figure 9 shows that the most luminous portion of cluster stars can be reasonably fitted by theory for an age of 3.25 Gyr, as...
tions appear in increasing disagreement with the calibration that the temperature-color relation is perhaps the weak link (Di Benedetto, private communication). This suggests to us regarded only as an exercise, whereas a reliable investigation theoretical fittings as given in Figure 9 should probably be re-
problem in using evolutionary theory. In this context, the-
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fitting of the RG colors induced the same authors to con-
ready noticed the good agreement between theory and ob-
a much firmer assessment of this problem of cluster evolutionary parameters should probably wait for
幸运ly, more recent predictions of model atmospheres are moving theory away from observations.
In this respect one has to notice that recent empirical cal-
bribations of V-K colors in terms of stellar temperature, as
given by stellar interferometry, have revealed that the results of recent model atmospheres given by Gratton et al. (1996)
are supported by observation only at temperatures around 7000 K, whereas at lower temperature theoretical predic-
tions appear in increasing disagreement with the calibration (Di Benedetto, private communication). This suggests to us
that the temperature-color relation is perhaps the weak link in the connection theory-observation. According to these ev-
idences, we are inclined to conclude that the transformation of temperatures into colors is probably the most relevant problem in using evolutionary theory. In this context, the-
etical fittings as given in Figure 9 should probably be regarded only as an exercise, whereas a reliable investigation of cluster evolutionary parameters should probably wait for a much firmer assessment of this problem.
Let us finally notice that Montgomery et al. (1993) by fitting CCS isochrones to the same observational data already noticed the good agreement between theory and observations in the MS and TO regions. However, the missed fitting of the RG colors induced the same authors to conclude that "the current generation of theoretical isochrones cannot be fit to the observed sequence within the observational errors". Even tough one cannot disagree with such a statement, we wish here to stress once again that the RG colors is largely matter of a cosmetic adjustment of theoretical results, as we have done in this section.

Figure 9. The color magnitude diagram presented by Montgomery et al. (1993) for the galactic cluster M67 and the best fit with the 3.25 Gyr isochrone.

In this context, we have already noticed that new computations appear in satisfactory agreement with previous CCS evaluation. Degl’Innocenti & Marconi (1998) have also shown that the new theoretical 4 Gyr isochrone already presented in the upper panel of Figure 2 appears also in good agreement with similar predictions by Bertelli et al. (1994). Thus theory appears rather solid in predicting the location in the theoretical HR diagram of similar stars, with only a minor influence of the updated physics inputs. Curiously enough, one finds that these isochrones were already used to give a good fitting of the M67 MS not only by CCS but also by Carraro et al. (1996). If we add the evidence that CCS were also nicely fitting the MS of NGC2420, one is driven to conclude that the fitting of these MS is largely a matter of the adopted color-temperature relations and that, unfortunately, more recent predictions of model atmospheres are moving theory away from observations.

As already discussed, the computations of the synthetic clusters can give an independent indication at least of the compatibility of the theoretical scenario one is dealing with. Figure 10 shows that assuming 10% of binary stars one finds a satisfactory agreement of theoretical predictions with observational data.

Figure 10. The observed CM diagram for M67 (upper panel) compared with the synthetic cluster for the labeled values of chemical composition and age aiming to reproduce M67 taking into account the contribution of 10% of binaries, without (middle panel) and with superimposed the corresponding theoretical cluster isochrone (bottom panel). Observational data are not dered-

Figure 11. Cluster isochrones in the observational plane for the labeled assumptions about the chemical composition and for α=1.9. Isochrones are computed including diffusion of helium and heavy elements. Ages as labeled. Theoretical isochrones are translated in the observational plane by adopting the empirical relations given by Alonso et al. (1996), implemented with Castelli et al. (1997a,b) model atmospheres, as described in the text.

As already discussed, data for all the computed isochrones are available at the anonymous ftp at astr18pi.difi.unipi.it (/pub/open), where we give theoretical (logL, log Te) isochrones together with V, B-V and V-I magnitudes as predicted according to the already quoted match between Alonso et al. (1996) and Castelli et al. (1997a,b) evaluations. Bearing in mind the caveat concerning theoretical colors, let us now discuss some theoretical predictions of general relevance.

Figure 11. Cluster isochrones in the observational plane for

Figure 12 gives the bottom luminosity of the clump of He burning stars as a function of the cluster age in the interval 1-6 Gyr and for the five explored metallicities. For each

4 CLUSTER ISOCHRONES

The discussion given in the previous sections has shown that α=1.9 appears the adequate (cosmetic) choice to produce cluster isochrones for old open clusters, all over the range of metallicity Z=0.007, 0.015, at least. According to such a result, Figure 11 shows cluster isochrones computed under the above quoted assumptions and for four selected metallicities. Data for all the computed isochrones are available at the anonymous ftp at astr18pi.difi.unipi.it (/pub/open), where we give theoretical (logL, log Te) isochrones together with V, B-V and V-I magnitudes as predicted according to the already quoted match between Alonso et al. (1996) and Castelli et al. (1997a,b) evaluations. Bearing in mind the caveat concerning theoretical colors, let us now discuss some theoretical predictions of general relevance.

Figure 12 gives the bottom luminosity of the clump of He burning stars as a function of the cluster age in the interval 1-6 Gyr and for the five explored metallicities. For each
Figure 12. The bottom luminosity of the clump of He burning stars (from the isochrones of Figure 11) as a function of the cluster age and metallicity.

Figure 13. The predicted magnitude of the He clump (from the isochrones of Figure 11) as a function of the metallicity $Z$ for the various explored cluster ages.

Given metallicity, the figure discloses the good constancy of this parameter, which should allow the use of He burning stars as useful standard candles.

However, one finds that for ages larger than 2 Gyr, the luminosity of the clump is slowly decreasing when the age increases. Since the clump is in the meantime becoming redder, the bolometric correction increases, increasing the variation in magnitudes. This is shown in Figure 13, where we report the predicted (bottom) magnitudes of the clump as a function of the metallicity $Z$ for selected assumptions about the cluster ages. One finds that the linear relation:

$$M_v = 1.59 + 0.59 \log Z$$  \hspace{1cm} (1)

reproduces the theoretical results over the whole range of ages within 0.1 mag.

As another relevant prediction, Figure 14 gives the difference in magnitude between the clump and the top luminosity of the H burning sequence (a parameter already introduced in CCS) as a function of the cluster ages for the chosen metallicities.

5 CONCLUSIONS

Galactic stellar clusters have covered a central role in the progress of stellar evolutionary theories, giving direct evidences for the evolution of metal rich, Population I stars for a fairly large variety of cluster ages and metallicities. However, throughout this paper we have shown that the situation is far from being completely satisfactory. Theoretical uncertainties on the efficiency of external convection play a major role in shading a disturbing degree of freedom in relevant theoretical predictions. Indetermination on both the temperature-color relations and in the actual cluster reddening add further problems into this scenario.

Figure 14. The predicted difference in magnitude between the He clump and the top of the H burning sequence (from the isochrones of Figure 11) as a function of the cluster age and for the labeled assumptions about the cluster metallicity.

In this paper we have attempted an empirical calibration of the theory, reaching what we regard as a satisfactory agreement between theory and observation. It remains the disturbing discordance of the MS slope at the larger explored B-V value, whose origin is far from firmly established. Further improvements in color-temperature relations and in the determination of cluster reddening are possible and expected. However external convection keep being the main outstanding problem. In this sense new approaches to the theory of convection, as the one presented by Canuto & Mazzitelli (1992) or Lydon, Fox & Sofia (1993), should be carefully tested to well studied galactic clusters and, if necessary, improved in the hope of reaching a real knowledge of such a fundamental ingredient of stellar evolutionary theories.

As a final point we note that that the predicted luminosity of He burning clumps is directly correlated with the size of the He cores at the end of central H burning. Both NGC2420 and M67 are predicted with Red Giants undergoing electron degeneracy. The good correspondence between observations and theory found in the previous section supports current evaluations of the physical mechanisms affecting these structures. Note that in this case, the predicted He core are only marginally affected by the possible efficiency of convective overshooting, which however should modulate the shape of the CM diagram in the Turn Off region.

In this context DeMarque, Sarajedini & Guo (1994) have recently proposed an alternative approach to constrain the efficiency of overshooting, as based on the topology of the CM diagram in the region of the overall contraction gap. In our feeling that approach, tough ingenious and theoretically well founded, suffers of some limitations mainly due to the occurrence of binary stars within the gap (see bottom panels in the previous Figures 8 and 10), preventing from firm observational constraints on the suggested CM parameters.

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REFERENCES
Alonso, A., Arribas, S. & Martinez-Rogers, C. 1996, A&A, 313, 873
Anthony-Twarog, B. J., Twarog, B. A., Kaluzny, J. & Shara, M. M. 1990, AJ, 99, 1504
Bertelli G., Bressan A., Chiosi C., Fagotto F., Nasi E. 1994, A&AS 106, 275
Brocato, E., Castellani, V. & Romaniello M. 1998, in preparation
Canterna, R., Geisler, D., Harris, H. C., Olszewski, E. & Sommer, R. 1986, AJ, 92, 79
Canuto V. & Mazzitelli I. 1992, ApJ 389, 724
Caputo F. & DeSantis, R. 1992, AJ, 104, 253
Carraro G., Girardi L., Bressan A. Chiosi C. 1996, A&A 305, 849
Cassisi, S., Castellani, V., Degl’Innocenti, S., Weiss, A., 1998, A&AS, in press
Castellani, V., Chieffi, S., & Straniero, O. 1992, ApJS, 78, 517
Castelli, F., Gratton, R. G. & Kurucz R. L. 1997a, A&A, 318, 841
Castelli, F., Gratton, R. G. & Kurucz R. L. 1997b, A&A, 324, 432
Chaboyer B. 1995, ApJ, 444, L9
Cohen, J. G. 1980, ApJ, 241, 981
Demarque P., Sarajedini A. & Guo X.-J. 1994, ApJ 426, 165
Gratton R.G., Carretta E., Castelli F. 1996, A&A 314, 191
Lydon T.J., Fox P.A., Sofia S. 1993, ApJ 413, 390
Montgomery, K. A., Marschall, L. A. & Janes, K. A. 1993, AJ, 106, 181
Sweigart A., Greggio L. & Renzini A. 1990, ApJ 364, 527
vandenBorg D.A., Hartwick F.D.A. & Dawson P. 1983, ApJ 266, 747
Z = 0.02  Y = 0.27
Age = 4 Gyr

Present work

CCS92

Z = 0.02  Y = 0.27
Age = 4 Gyr

Present work

CCS92
$Z = 0.015$
$Y = 0.27$
$\alpha = 1.6$

Age [Gyr] :
3
4
5
6
8
10
12

--- standard
--- diffusion
NGC 2420

$DM = 12.4 \ E(B-V) = 0.16$

Age =

1.25, 1.5, 1.75 Gyr
$Z = 0.007 \quad Y = 0.27$

Age = 1.5 Gyr

- $\alpha = 1.6$
- $\alpha = 1.9$
NGC 2420

$Z = 0.007\quad Y = 0.244$

$\alpha = 1.9$

Age = 1.5 Gyr

30% of binaries

$M_v$ vs $B-V$
M67

DM=9.9  E(B-V)=0.09

Z=0.015  Y=0.27

Age= 3.25 Gyr
Age [Gyr]

$\log L/L_0$

$Y = 0.27 \quad \alpha = 1.9$

$Z = 0.004$

$Z = 0.007$

$Z = 0.01$

$Z = 0.015$

$Z = 0.02$
$Y=0.27 \quad \alpha = 1.9$

$M_v$

$\log Z$

$\alpha = 1.9$

$Y=0.27$

$t=1$ Gyr

$t=2$

$t=3$

$t=4$

$t=5$

$t=6$

diffusion
\[ \Delta M_{V}(\text{TO-HB}) \]

![Graph showing the relationship between Age [Gyr] and \( \Delta M_{V}(\text{TO-HB}) \), with markers for different metallicity (Z) values: Z=0.004, Z=0.007, Z=0.01, Z=0.015, Z=0.02. The graph shows a trend of increasing \( \Delta M_{V}(\text{TO-HB}) \) with increasing age, and the text "diffusion" is indicated within the graph. The metallicity values are marked on the graph with corresponding symbols.]

- \( Y=0.27 \)
- \( \alpha=1.9 \)