Present paper deals with the composition and modelling of compact dense astrophysical bodies under the framework of $f(R)$ gravity. The model is employed on various observed strange stars viz., SMC X-1, SAX J1808.4-3658, Swift J1818.0-1607, PSR J1614-2230 and PSR J0348+0432. Upon setting the appropriate value of dimensionless coupling parameter $\lambda$, the physical parameters such as the density, the radial and tangential pressures were obtained. Mass-Radius relation without presuming any equation of state is capable enough to accommodate all strange stars nearly having solar mass up to 2.5. The physical viability of the model is examined for all the aforementioned stars and it is found that all the regularity and stability conditions are satisfied.

Keywords Classical general relativity · Exact solutions · Relativistic stars: structure, stability and oscillations · Modified theories of gravity

1 Introduction

The phenomenon of accelerated expansion of the universe was first noticed in 1998 while observing Type Ia Supernovae. The work done by eminent researchers such as Perlmutter et al. (1999), Hawkins et al. (2003) and Eisentein et al. (2005) was instrumental in this paradigm-shifting discovery. This particular breakthrough paved the way for the existence of an exotic energy component called Dark Energy. Backed by numerous astronomical observations, dark energy has been a subject of intense research and debate in recent years. As per the conditions of the $\Lambda$CDM model, dark energy is essentially the Cosmological Constant, with the density accounting for 72% of the global energy budget of the universe. The remaining 28% is confined to normal matter in the form of galactic clusters (only 4%), with the remaining 24% is occupied by cold dark matter (CDM), the nature of which remains unclear. To explain the occurrence of dark energy, theoretical physicists have often propounded the use of modified theories of gravity brought about by making changes to the Einstein-Hilbert action integral, leading to multiple theoretical possibilities, some of which include the $f(R)$, $f(T)$, $f(T)$ and $f(R,T)$ theories of gravity, where $R$ represents the scalar curvature, $T$ represents the trace of the energy-momentum tensor, and $T$ represents torsion.

This connotes the necessity of the said cosmological constant when considering the modelling of the universe as an ensemble of stars, black holes, planets and various other entities, as well as the modelling of individual dense binaries. We use the $f(R)$ gravity model for our purpose, more specifically, the Starobinsky model, $f(R) = R + \Lambda R^2$ (Starobinsky 1980). Egeland (2007) investigated that the cosmological constant would exist due to the density of the vacuum, this is a consequence of modelling the mass and ra-
of the neutron star. To demonstrate the validity of his assumption, Egeland used the relativistic equations of hydrostatic equilibrium with the fermion equation of state (EOS). The exact solutions for physically viable theoretical models for several dense stellar systems such as binary neutron star systems, X-ray bursters, and gamma-ray bursters have been described using the aforementioned model.

For more than four decades, modelling of neutron stars based on anisotropic parameters as opposed to isotropic composition has begun to gain prominence. Ruderman (1972) and Canuto (1974) back the idea of the existence of orthogonal pressure components, \( p_r \) and \(\tilde{p}_t \), where \( p_r \) is the radial pressure component, while \( \tilde{p}_t \) signifies the tangential component. Sawyer (1972) and Sokolov (1980) hypothesized that the said anisotropy may arise as a result of the presence of a Type III-A superfluid, a solid core at the center of the star, phase transitions, or pion condensations inside a neutron star. The modelling of such stars has been carried out in an extensive fashion through the use of general relativity. The exact mathematical solutions generated from these have a wide range of astrophysical applications. The modelling proved the anisotropy of stars by separating the radial and tangential pressure components. Mak and Harko (2004) provided a class of exact solutions of Einstein’s field equations having an anisotropic source. Herrera et al. (2008) studied multiple static spherically symmetric solutions of Einstein’s field equations and discussed the physical validity and implications of such solutions. Rahaman et al. (2012) applied the Krori-Barua solutions (Krori and Barua 1975) to anisotropic compact charged stars. Sharma and Ratanpal (2013) provided a quadratic equation of state for neutron stars using the Finch-Skea ansatz (Finch and Skea 1989). Pandya et al. (2015) generated exact solutions of a generalized form of the Finch-Skea ansatz, and later on provided a way to generalize the Finch-Skea metric (Pandya et al. 2020). While compact star models have been described in substantial detail through the utilization of general relativity, further elaboration to these models and their related parameters may be propounded upon in terms of the inclusivity of dark energy based phenomena.

Paul et al. (2011) modelled compact star described by Vaidya–Tikekar metric. Maharaj and Takisa (2012) also modelled compact star having anisotropic pressure and they also took electric-charge effects into consideration. For a specific polytropic index, exact solutions to Einstein’s field equations for an anisotropic sphere admitting a polytropic EOS have been obtained by Thirukkanesh and Ragel (2012). Hossein et al. (2012) developed anisotropic star models in the presence of a varying cosmological constant. Murad and Fatema (2013) examined various charge distributions inside self-bound stars and provided analytical solutions to Einstein-Maxwell field equations in a metric ansatz suggested by Durgapal (1982). Ovalle (2017) provided the method of gravitational decoupling as a method of solving the field equations to model compact dense stars. Singh et al. (2017) gave model for compact stars whose interior admits Karmarkar Condition. Abbas et al. (2015) provided a compact star model for strange quintessence stars using the concept of \( f(R) \) gravity. Variations to the Starobinsky model of \( f(R) \) gravity has also been proposed by Astashenok et al. (2017) as a way of incorporating the more exotic forms of matter. Sharif and Waseem (2019) described an alternative method for modelling the same using gravitational decoupling. Astashenok et al. (2014) provided a way to resolve the hyperon conundrum in \( f(R) \) gravity, and provided another model (Astashenok et al. 2019), which included a way to model supermassive neutron stars in axion \( f(R) \) gravity. In all of the mathematical models proposed using modified theories of gravity, we have seen extensive use of the Krori-Barua metric.

In this paper, we investigate the usage of the Finch-Skea metric in \( f(R) \) gravity. In Sect. 2, we elaborate upon the modification of the Einstein-Hilbert action in \( f(R) \) gravity and incorporate Starobinsky’s \( f(R) \) gravity model. In Sect. 3, we match our interior metric with Schwarzschild’s exterior line element and hence, find expressions for the model constants and specify bounds on them. In Sect. 4, subsequent conditional analysis, using various parameters derived for the model, is performed. Inspection of the obtained \( M-a \) plots is given in Sect. 5 while the scope of the model and its possible applications have been highlighted in Sect. 6.

### 2 Anisotropic matter configuration in \( f(R) \) gravity

As described in the Introduction, \( f(R) \) gravity is the modification of Einstein’s general relativity. Therefore, the modification of Einstein-Hilbert action in \( f(R) \) gravity as:

\[
I = \int d^4 x \sqrt{-g} [f(R) + \mathcal{L}_{\text{matter}}].
\]

where \( g \) is the determinant of the metric tensor, \( f(R) \) is any arbitrary function of \( R \), the Ricci scalar and \( \mathcal{L} \) is the Lagrangian density. Without this modification, \( f(R) \) is just the Ricci scalar, which is a reversion to general relativity. Defining \( f(R) \) in equation (1) makes it a part of extended theories of gravity and could possibly explain the shortcomings from the Standard Cosmological Model. The unit of \( I \) (action) is the product of energy and time dimensions. On the opposite side, the Ricci scalar has \( m^{-2} \) in the \( f(R) \) expression and \( d^4 x \) changes as \( m^4 \) with the unitless space metric \( g \). We also assumed \( 8\pi G c^4 = 1 \) which is the inversely multiplicative factor in equation (1), thereby making the dimensional analysis true. Additionally, an extra term for the matter content
satisfy Karmarkar's condition (Karmarkar1948). It is a necessary condition for spacetimes because they can be embedded into 5 dimensional spacetime solutions and the Friedmann Universe are class-I members based on the extra dimensions. Schwarzschild's space-time in n+1 dimensions, which creates different classes of gravitational potential. This condition can be obtained as follows:

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T^{(\text{matter})}_{\mu\nu} + T^{(\text{curv})}_{\mu\nu} \]  

(2)

where \( T^{(\text{matter})}_{\mu\nu} \) is the matter stress energy tensor scaled by \( \frac{1}{f(R)} \) and \( T^{(\text{curv})}_{\mu\nu} \) is the benefit arising from the curvature energy density, radial and transverse pressures respectively.

Now, the variation of (1) with respect to the metric \( g_{\mu\nu} \) leads to the following field equations:

\[ L = \left( \frac{1}{2} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) F(R) - R_{\mu\nu} g_{\mu\nu} \]  

(3)

where \( F(R) \) is the function that affects the space-time continuum; like energy, the stress created by matter density, the momentum, etc. Due to \( f(R) \) gravity, we have the first term \( g_{\mu\nu}(f(R) - RF(R)) \) which is a consequence of the modified gravity model. We select the spherically symmetric metric line element as:

\[ ds^2 = e^{\mu(r)} dt^2 - e^{\nu(r)} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]  

(4)

The space-time metric describes the distance difference between two events in the four dimensions which is generalization of three dimensions. The metric is defined in the spherical symmetric coordinate system where the time \( (dt^2) \) and radial \( (dr^2) \) components have different scaling factors. These factors describe the geometry of space-time. The Ricci scalar is obtained by the metric tensor and the Ricci tensor product. For a spherically symmetric metric, the Ricci Scalar can be computed from above line-element and taken from Bhar (2019):

\[ R = \left( 2\mu''(r) + r \mu'(r)^2 - 2r \mu'(r) - 4r \nu'(r) - 4e^{\nu(r)} + 4 \right) \]  

(5)

The standard anisotropic fluid of the compact stars can be given as:

\[ T^{(m)}_{\alpha\beta} = (\rho + p_r)u_{\alpha}u_{\beta} - p_t g_{\alpha\beta} + (p_r - p_t)v_{\alpha}v_{\beta}, \]  

(6)

where \( u_{\alpha} = e^\mu \delta^{\mu}_0, v_{\alpha} = e^\nu \delta^{\nu}_0 \) and \( \rho, p_r \) and \( p_t \) correspond to matter energy density, radial and transverse pressures respectively. \( T^{(m)} \) denotes matter part of stress-energy tensor.

It can be seen that \( T^{(m)} = \text{diag}(\rho, -p_r, -p_t, -p_t) \).

Now, to create a model of a compact star, the metric potential \( e^\nu \) is assumed to be \( 1 + Cr^2 \), as suggested by Finch and Skea (Finch and Skea 1989). The metric potential presents a simple ansatz which has been widely studied in general relativity (Bhar (2015), Tikekar and Jotania (2017), Maharaj et al. (2017)) prior to its application in the study of \( f(R) \) gravity. The success of the model seen in general relativity makes it an intriguing test subject for \( f(R) \) gravity.

From equation (8), we get \( \mu'(r) = \log[(A + \frac{1}{2} Br\sqrt{r^2/C})^2] \), where, \( A \) and \( B \) are constants. Values of \( A, B \) and \( C \) are derived using the relevant boundary conditions.

Upon substituting \( \nu(r) = \log(1 + Cr^2) \) and \( \mu'(r) = \log[(A + \frac{1}{2} Br\sqrt{r^2/C})^2] \), the expression for the metric changes to:

\[ ds^2 = (A + \frac{1}{2} Br\sqrt{r^2/C})^2 dr^2 - (1 + Cr^2)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]  

(7)

\[ \sqrt{v''(r)} = -2(\mu'' + \mu'^2) + \mu'^2 + v'\mu' \]  

(8)

Solving equation (7), we get,

\[ e^\mu = (A + B) \int \sqrt{v'^2 - 1}dr \]  

(9)

Here, \( R_{2323} \neq 0 \) and \( R_{abcd} \) are the non zero components of Riemann tensor.

Using condition (6), the line element (4) can be altered to give the following differential equation (Bhar (2019)):

\[ \frac{\mu'(r)}{1 - e^\nu} = -2(\mu'' + \mu'^2) + \mu'^2 + v'\mu' \]  

(10)

The metric condition is given as follows:

\[ R_{1414} = \frac{R_{1212} R_{3434} + R_{1224} R_{1334}}{R_{2323}} \]  

(11)

The standard anisotropic fluid of the compact stars can be given as:

\[ T^{(m)}_{\alpha\beta} = (\rho + p_r)u_{\alpha}u_{\beta} - p_t g_{\alpha\beta} + (p_r - p_t)v_{\alpha}v_{\beta}, \]  

(12)

where \( u_{\alpha} = e^\mu \delta^{\mu}_0, v_{\alpha} = e^\nu \delta^{\nu}_0 \) and \( \rho, p_r \) and \( p_t \) correspond to matter energy density, radial and transverse pressures respectively. \( T^{(m)} \) denotes matter part of stress-energy tensor.

It can be seen that \( T^{(m)} = \text{diag}(\rho, -p_r, -p_t, -p_t) \).

Now, to create a model of a compact star, the metric potential \( e^\nu \) is assumed to be \( 1 + Cr^2 \), as suggested by Finch and Skea (Finch and Skea 1989). The metric potential presents a simple ansatz which has been widely studied in general relativity (Bhar (2015), Tikekar and Jotania (2017), Maharaj et al. (2017)) prior to its application in the study of \( f(R) \) gravity. The success of the model seen in general relativity makes it an intriguing test subject for \( f(R) \) gravity.

From equation (8), we get \( \mu'(r) = \log[(A + \frac{1}{2} Br\sqrt{r^2/C})^2] \), where, \( A \) and \( B \) are constants. Values of \( A, B \) and \( C \) are derived using the relevant boundary conditions.

Upon substituting \( \nu(r) = \log(1 + Cr^2) \) and \( \mu'(r) = \log[(A + \frac{1}{2} Br\sqrt{r^2/C})^2] \), the expression for the metric changes to:

\[ ds^2 = (A + \frac{1}{2} Br\sqrt{r^2/C})^2 dr^2 - (1 + Cr^2)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]  

(13)

\[ \mu'(r) = -2(\mu'' + \mu'^2) + \mu'^2 + v'\mu' \]  

(14)

Solving equation (7), we get,

\[ e^\mu = (A + B) \int \sqrt{v'^2 - 1}dr \]  

(15)

Here, \( R_{2323} \neq 0 \) and \( R_{abcd} \) are the non zero components of Riemann tensor.

Using condition (6), the line element (4) can be altered to give the following differential equation (Bhar (2019)):

\[ \frac{\mu'(r)}{1 - e^\nu} = -2(\mu'' + \mu'^2) + \mu'^2 + v'\mu' \]  

(16)

The standard anisotropic fluid of the compact stars can be given as:

\[ T^{(m)}_{\alpha\beta} = (\rho + p_r)u_{\alpha}u_{\beta} - p_t g_{\alpha\beta} + (p_r - p_t)v_{\alpha}v_{\beta}, \]  

(17)

where \( u_{\alpha} = e^\mu \delta^{\mu}_0, v_{\alpha} = e^\nu \delta^{\nu}_0 \) and \( \rho, p_r \) and \( p_t \) correspond to matter energy density, radial and transverse pressures respectively. \( T^{(m)} \) denotes matter part of stress-energy tensor.

It can be seen that \( T^{(m)} = \text{diag}(\rho, -p_r, -p_t, -p_t) \).

Now, to create a model of a compact star, the metric potential \( e^\nu \) is assumed to be \( 1 + Cr^2 \), as suggested by Finch and Skea (Finch and Skea 1989). The metric potential presents a simple ansatz which has been widely studied in general relativity (Bhar (2015), Tikekar and Jotania (2017), Maharaj et al. (2017)) prior to its application in the study of \( f(R) \) gravity. The success of the model seen in general relativity makes it an intriguing test subject for \( f(R) \) gravity.

From equation (8), we get \( \mu'(r) = \log[(A + \frac{1}{2} Br\sqrt{r^2/C})^2] \), where, \( A \) and \( B \) are constants. Values of \( A, B \) and \( C \) are derived using the relevant boundary conditions.

Upon substituting \( \nu(r) = \log(1 + Cr^2) \) and \( \mu'(r) = \log[(A + \frac{1}{2} Br\sqrt{r^2/C})^2] \), the expression for the metric changes to:

\[ ds^2 = (A + \frac{1}{2} Br\sqrt{r^2/C})^2 dr^2 - (1 + Cr^2)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]  

(18)
As stated above to solve the Einstein field equations we need $R$ and $f(R)$. Accordingly, the Ricci Scalar for proposed Finch-Skea model (from equation (5)) can be written as:

$$R = \frac{2(2AC(Cr^2 + 3) + B\sqrt{C}(Cr^2 - 3)(Cr^2 + 2))}{(Cr^2 + 1)^2(2A + B\sqrt{C}r^2)}$$  \hspace{1cm} (11)

Thus the field equations can be modified with respect to spacetime as given in equation (4) as:

$$\rho = \frac{e^{-\nu}}{2r^2}(r^2F'\nu' + 2Fr\nu' - Fr^2e^\nu$$

$$+ Fr^2e^\nu R + 2Fe^\nu - 2r^2F'' - 4rF' - 2F)$$ \hspace{1cm} (12)

$$p_r = -\frac{e^{-\nu}}{2r^2}(-2F + 2Fe^\nu - Fr^2e^\nu$$

$$+ Fr^2e^\nu R - 2Fr\mu' - 4rF' - r^2F'\mu')$$ \hspace{1cm} (13)

$$p_t = \frac{e^{-\nu}}{4r}(2 FA\mu'' - Fr\nu\mu' + 2r\mu' F'$$

$$+ Fr\mu'^2 + 2F\mu - 2r\nu' F' + \psi(r))$$ \hspace{1cm} (14)

where $\psi(r) = -2Fv' + 2r fe^\nu - 2r F e^\nu R + 4r F'' + 4F'$.

For the early universe, there are a variety of inflationary models developed using scalar fields, which stem from super-gravity theories. (Starobinsky 1980) suggested one of the earliest inflation models which corresponds to the conformal deviation in quantum gravity, essentially accounting for the presence of the expanding universe. The $f(R)$ gravity model given by Starobinsky is therefore represented as:

$$f(R) = R + \lambda R^2$$ \hspace{1cm} (15)

where, $\lambda$ is a constant. Starobinsky proposed this constant as a way to elaborate upon the exponential growth found in the early time cosmological expansion.

To verify that the modelling of compact stars yields physically viable results in this version of modified gravity, the Starobinsky model is being tested for the same, given a certain ansatz for the metric potential.

For this model, therefore, equations (12), (13) and (14) become:

$$\rho = \frac{1}{(Cr^2 + 1)^2(2A + B\sqrt{C}r^2)^3} \cdot C(8A^3(2C\lambda((Cr^2(-C^2r^4 + Cr^2 + 45) - 69$$

$$+(Cr^2 + 3)(Cr^2 + 1)^3 + 4A^2B\sqrt{C}(3r^2(Cr^2 + 1)^3(Cr^2 + 3) - 2\lambda(3Cr^2$$

$$(Cr^2 - 7) + 32)(2Cr^2(Cr^2 + 6) - 3) + 2AB^2(2\lambda(3Cr^2(Cr^2(-3C^2r^4$$

$$+ 3Cr^2 + 55) - 303) + 316) + 108) + 3Cr^4(Cr^2 + 1)^3(Cr^2 + 3))$$

$$+ B^3\sqrt{C}r^2(-2C\lambda r^2(Cr^2(Cr^2$$

$$- 1) + 67) + 341) + 36) + Cr^4(Cr^2 + 1)^3(Cr^2 + 24\lambda))$$ \hspace{1cm} (16)

$$p_r = \frac{1}{(Cr^2 + 1)^4(2A + B\sqrt{C}r^2)^3} \cdot 8A^3C(2C\lambda(Cr^4Cr^2 + 10) + 37) - (Cr^2 + 1)^3$$

$$+ 4A^2B\sqrt{C}(2C\lambda(Cr^2(3Cr^2(Cr^2 + 10) + 103) - 64) - (Cr^2 + 1)^3(3Cr^2 - 4))$$

$$+ 2AB^2C(r^2(2C\lambda(Cr^2(3Cr^2(Cr^2 + 10) + 47) - 240) - (Cr^2 + 1)^3(3Cr^2 - 8))$$

$$- 72\lambda) + B^3C^{3/2}r^2(2\lambda(Cr^2 - 6)(Cr^2(Cr^2 + 16) + 45) + 14)$$

$$- r^2(Cr^2 + 24)(Cr^2 + 1)^3)$$ \hspace{1cm} (17)

$$p_t = \frac{1}{(Cr^2 + 1)^5(2A + B\sqrt{C}r^2)^3} \{8A^3C(-2C\lambda(Cr^2(Cr^2 + 17) + 91) - 37$$

$$- (Cr^2 + 1)^3) + 4A^2B\sqrt{C}(-2C\lambda(3Cr^2(Cr^2(Cr^2 + 13) + 55) - 85)$$

$$+ 64) - (Cr^2 - 4)(Cr^2 + 1)^3) + 2AB^2C(r^2(Cr^2 + 1)^3(Cr^2 + 8)$$

$$- 2\lambda(Cr^2(Cr^2(3Cr^2(Cr^2 + 9) + 41) - 431) + 148) + 36)) + B^3C^{3/2}r^2$$

$$(r^2(2C\lambda(44 - Cr^2(Cr^2 + 5)(C^2r^4 - 65)) + (Cr^2 + 4)(Cr^2 + 1)^3) + 24\lambda))$$ \hspace{1cm} (18)
The anisotropy measurement $\Delta = \frac{2}{r} (p_t - p_r)$ for this model is given by:

$$\Delta = \frac{1}{(Cr^2 + 1)^2} \cdot 2C^{3/2}r(8A^3 \sqrt{Cr^2 + 1})^{3} - 4C\lambda(Cr^2(Cr^2 + 13 - 4C\lambda(Cr^2(Cr^2 + 12) + 149) - 108)) + 2AB^2 \sqrt{Cr^2((Cr^2 + 1)^2(3Cr^2 - 4) - 4C\lambda(Cr^2(3Cr^2(Cr^2 + 8) - 53)) + 192) + 48) + Cr^4(Cr^2 - 2)(Cr^2 + 13))}$$

$\Delta$ represents the anisotropy of a given compact star. As described above, compact stars possess anisotropic matter distribution. Due to this, they display an inhomogeneity, manifesting as radial and tangential pressure. Sources of such anisotropy include, but are not limited to, the presence of hyperons in the core, pion condensation, or tidal deformability. The extant anisotropy inherent in such stars make them an interesting perspective, around which the current study revolves. The local anisotropy of fluid inside the star represents equivalent measurement of force. $\Delta > 0$ is a repulsive behaviour while $\Delta < 0$ is an attractive behaviour of fluid. When $p_t > p_r$, the anisotropy is said to be positive ($\Delta > 0$). At center of the star, both components of pressure become equal, meaning that the anisotropy drops to zero.

3 Matching conditions for the constants

3.1 Boundary conditions

Goswami et al. (2014) provided the extra matching conditions that arose when considering stellar modelling in modified theories of gravity, showing that the constraints on the thermodynamic properties and stellar structure are purely mathematical. As per the text, the Schwarzschild solution is determined to be the better choice in the exterior region when considering matching conditions in stars. Here, the solutions exist primarily in terms of the star mass and the boundary surface $r = a$. In the following section, we compare corresponding terms of our interior space-time Equation (4) to Schwarzschild’s exterior space-time metric given as:

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 \left(d\theta^2 + d\phi^2 \sin^2 \theta\right)$$

(20)
on the surface at $r = a$. The $A$, $B$, and $C$ constants are found by standard conditions for the compact stars in relativity. The required number of conditions are three because of the three unknown constants. These three standard conditions are represented by coefficients of interior metric (4) and exterior metric (20) at boundary $r = a$ as shown below,

$$e^\mu_{(r=a)} = \left(A + B \int \sqrt{e^\nu - 1} dr\right)_{(r=a)}^2 = \left(1 - \frac{2M}{r}\right)_{(r=a)}$$

(21)

$$e^\nu_{(r=a)} = \left(e^{\log(1+Cr^2)}\right)_{(r=a)} = \left(1 - \frac{2M}{r}\right)_{(r=a)}^{-1}$$

(22)

And last one is derivative of both time coefficients of interior and exterior space-time metric,

$$\left(\frac{\partial e^\mu_{(r)}}{\partial r}\right)_{(r=a)} = \left(\frac{\partial \left(1 - \frac{2M}{r}\right)}{\partial r}\right)_{(r=a)}$$

(23)

Using first two Equations (21) and (22), the final comparisons are given by,

$$\frac{1}{a^2C + 1} = 1 - \frac{2M}{a}$$

(24)

$$\left(\frac{1}{2} Ba\sqrt{a^2C + A}\right)^2 = 1 - \frac{2M}{a}$$

(25)

Using (24), we get,

$$C = \frac{-2M}{a^2(2M - a)}$$

(26)

Thus, using Equations (21)-(23) and (26), values of $A$ and $B$ can be written as:

$$A = \frac{-M}{\sqrt{a^2 - 2aM}} + \sqrt{1 - \frac{2M}{a}}$$

(27)
\[ B = \frac{\sqrt{M}}{\sqrt{2} \cdot a^3} \]  

(28)

Here, \( a \) is the star radius and \( M \) is mass of the dense star. All three unknown constants are expressed in terms of \( M \) mass of the star, and \( a \) radius of the star.

### 3.2 Analysing the bound on model constants

We use the limits obtained while gauging physical validity of the model to estimate the bounds on the model constraints. The simplest of these bounds occur in the form of the surface redshift condition and the Buchdahl condition.\(^1\) The former condition helps us find the direct bounds on \( C \), while the latter can be manipulated to derive the limits on \( A \) and \( B \).

From (49) and surface redshift condition, we have:

\[ 0 \leq -1 + \sqrt{1 + Cr^2} \leq 5 \]  

(29)

This gives us

\[ 0 \leq Cr^2 \leq 35 \]  

(30)

At the surface, we take \( r = a \), where \( a \) denotes the radius of the star. Therefore, using the surface redshift condition, we get:

\[ 0 \leq C \leq \frac{35}{a^2} \]  

(31)

Now, according to Buchdahl’s condition,

\[ 0 < \frac{M}{a} \leq \frac{4}{9} \]  

(32)

Which gives us

\[ 0 < \frac{2M}{a^2} < \frac{8}{9a} \]  

(33)

In a similar fashion,

\[ 0 < \frac{1}{-2M + a} < \frac{9}{a} \]  

(34)

Multiplying Equation (33) with (34),

\[ 0 < \frac{2M}{a^2(-2M + a)} < \frac{8}{a^2} \]  

(35)

The expression obtained in the inequality is the same as the expression for \( C \), as seen in (26). Therefore, we have another bound for \( C \), between \( 0 < C < \frac{8}{a^2} \). Similar manipulations of the Buchdahl condition yield the limits of \( A \) and \( B \) as

\[ \frac{-1}{9} < A < 1 \]  

(36)

\[ 0 < B < \frac{\sqrt{2}}{3a} \]  

(37)

\[ 0 < C < \frac{8}{a^2} \]  

(38)

Table 1 gives the values of \( A \), \( B \), and \( C \) for five different stars. Mass and Radius of SMC X–1, SAX J1808.4–3658(SS2) are given by Rawls et al. (2011) and Elebert et al. (2009). The newly discovered magnetar Swift J1818.0–1607 is also considered, as given in Esposito et al. (2020). Mass and Radius of PSR J1614–2230 are proposed in Arzoumanian et al. (2018) and Demorest et al. (2010) respectively. Similarly, Mass and Radius of PSR J0348+0432 are taken from Antoniadis et al. (2013) and Zhao et al. (2017). After putting mass and radius of the given stars, it is possible to calculate the numerical values of \( A \), \( B \), \( C \) from Equations (26)-(28), and derive the exact theoretical values of \( \rho \), \( \rho_r \) and \( \rho_t \).

### 4 Analysis of physical parameters

Many conditions based on physical parameters derived for our model are analyzed in this section which help us to verify that our model is physically legitimate.
Fig. 1 Variation of energy density $\rho$, radial pressure $p_r$, and transverse pressure $p_t$ with the radial parameter $r$ for the chosen star candidates. The values of $M$, $a$, $A$, $B$, $C$ used are mentioned in Table 1. $\lambda$ is taken to be 0.35 and $G$ and $c$ are $6.67430 \cdot 10^{-20}$ km$^3$·kg$^{-1}$·s$^{-2}$ and 299792 km·s$^{-1}$ respectively.

### 4.1 Monotone decrease condition

Figures 1a, 1b and 1c demonstrate the dependence of matter density $\rho$, radial pressure $p_r$ and transverse pressure $p_t$ on radial distance $r$, which are expressed in Equations (16)-(18).

The derivative of these expressions with respect to the radial parameter $r$ will give:

\[
\frac{dp}{dr} = \frac{1}{(Cr^2 + 1)^6(2A + B\sqrt{Cr^2})^4} \left[ \frac{2C^2r(Cr^2 + 5)}{(Cr^2 + 1)^3} \right] \left[ 8C^{3/2} \lambda r (16A^4C^{3/2}(Cr^2)

\[(Cr^2(Cr^2 - 3) - 89) + 195) + 32A^3BC(Cr^2(Cr^2(Cr^2 - 3) - 77)

+ 211) - 66) + 8A^2B^2\sqrt{C}(Cr^2(Cr^2(3Cr^2(Cr^2 - 3) - 179) + 689)

- 532) - 128) + 8AB^3(Cr^2(Cr^2(Cr^2(Cr^2 - 3) - 25) + 279)

- 353) - 202) - 39) + B^4\sqrt{Cr^2}(Cr^2(Cr^2(Cr^2(Cr^2 - 3) + 135)

+ 819) + 108) - 24) - 12)) \right]
\]

(39)

\[
\frac{dp_r}{dr} = \frac{1}{(Cr^2 + 1)^5(2A + B\sqrt{Cr^2})^4} \cdot \left[ 2Cr((Cr^2 + 1)^3(2A + B\sqrt{Cr^2})^2(4A^2C

+ 4AB\sqrt{C}(Cr^2 - 2) + B^2(C \times r^2(Cr^2 - 8) - 4)) + \lambda(-64A^4C^2(Cr^2

\]
This condition suggests that these derivatives should be in the negative quadrant, thereby connoting that the density and pressure parameters will decrease with an increase in the magnitude of the radial coordinates. This is known as the monotone decrease of parameters $\rho$, $p_r$, and $p_t$.

$$\frac{dp_t}{dr} \leq 0, \quad \frac{dp_r}{dr} \leq 0, \quad \text{and} \quad \frac{dp_t}{dr} \leq 0$$

Figures 2a, 2b and 2c represent evolution of Equations (39)-(41) with $r$. It can be seen that density, radial pressure and transverse pressure gradients are less than zero, in the range $0 \leq r \leq a$, for the chosen stars. This indicates that the maximum values of density and pressure will be located at $r = 0$.

### 4.2 Central anisotropy condition

The radial pressure and tangential pressure become equal at the center of the star. That means the anisotropy representing the difference between radial and tangential pressure becomes zero. This condition should be satisfied to avoid collapse at the center Pandya et al. (2020). The condition is given by,

$$\Delta(r=0) = 0$$

This $\Delta$ expression, given in Equation (19), is plotted in Fig. 3 for the five stars. It can be noticed that at $r = 0$, $\Delta = 0$. This makes our model satisfy the given condition and hence, can be said to be well behaved.

Also, another aspect that can be reckoned from Fig. 3 is that $\Delta > 0$ for all the stellar configurations, in the range $0 < r \leq a$. This marks the presence of a repulsive force, which acts as a counter phenomenon against gravitational collapse.

### 4.3 Energy conditions

The energy bounds and strength of gravitational field are analyzed by several energy conditions and it is necessary to satisfy them. The validity of these energy conditions is vital for a physically reasonable energy-momentum tensor. The strength of gravitational field is described by energy conditions and this gravitational field should be enough to sustain the star’s stability. The energy conditions for an anisotropic fluid are defined by the following relations,

$$\text{NEC} : \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0 \quad (42)$$

$$\text{WEC} : \quad \rho \geq 0, \quad \rho - p_r \geq 0, \quad \rho - p_t \geq 0 \quad (43)$$

$$\text{SEC} : \quad \rho - p_r - 2p_t \geq 0 \quad (44)$$

$$\text{DEC} : \quad \rho > |p_r|, \quad \rho > |p_t| \quad (45)$$

Here NEC, WEC, SEC, and DEC stand for the null energy, weak energy, strong energy, and dominant energy conditions respectively. The NEC is seen to be satisfied because the profiles of density, tangential, and radial pressure are in positive behaviour. WEC and SEC are described in Figs. 4a-4c. It is clear from the figures that conditions are appropriately satisfied as they exhibit positive values for the five chosen realistic stars. This also proves that $\rho$ is greater.
Fig. 2 Variation of energy density gradient $\frac{d\rho}{dr}$, radial pressure gradient $\frac{dp_r}{dr}$ and transverse pressure gradient $\frac{dp_t}{dr}$ with the radial parameter $r$ for the chosen star candidates. The values of $M$, $a$, $A$, $B$, $C$ used are mentioned in Table 1. $\lambda$ is taken to be 0.35 and $G$ and $c$ are $6.67430 \times 10^{-20}$ km$^3$·kg$^{-1}$·s$^{-2}$ and 299792 km·s$^{-1}$ respectively.

Fig. 3 Variation of Anisotropy profile ($\Delta = p_t - p_r$) with the radial parameter $r$ for the chosen star candidates. The values of $M$, $a$, $A$, $B$, $C$ used are mentioned in Table 1. $\lambda$ is taken to be 0.35 and $G$ and $c$ are $6.67430 \times 10^{-20}$ km$^3$·kg$^{-1}$·s$^{-2}$ and 299792 km·s$^{-1}$ respectively.
Fig. 4 Variation of strong energy and weak energy conditions with the radial parameter \( r \) for the chosen star candidates. The values of \( M, a, A, B, C \) used are mentioned in Table 1. \( \lambda \) is taken to be 0.35 and \( G \) and \( c \) are $6.67430 \times 10^{-20}$ km$^{-3}$ · kg$^{-1}$ · s$^{-2}$ and 299792 km · s$^{-1}$ respectively.

Table 2 Central and Surface Densities and Strong Energy Condition at \( r = a \) and \( r = 0 \) for chosen stars:

| Star Name       | \( M (M_\odot) \) | \( a \) (km) | \( \rho_c \) (MeV fm$^{-3}$) | \( \rho_s \) (MeV fm$^{-3}$) | \( (\rho - p_r - 2p_t)_{r=a} \) (MeV fm$^{-3}$) | \( (\rho - p_r - 2p_t)_{r=0} \) (MeV fm$^{-3}$) |
|-----------------|-------------------|--------------|--------------------------|--------------------------|---------------------------------|---------------------------------|
| SMC X–1         | 1.04              | 8.301        | 727.704                   | 363.67                   | 702.5087                        | 337.804755                     |
| SAX J1808.4–3658(SS2) | 0.9               | 7.951        | 679.422                   | 352.452                   | 640.7078                        | 313.7378                       |
| Swift J1818.0–1607 | 2.0               | 12.5         | 504.598                   | 187.192                   | 324.1968                        | 143.5138                       |
| PSR J1614–2230   | 1.908             | 13           | 400.283                   | 164.814                   | 282.812                         | 130.692                        |
| PSR J0348+0432   | 2.01              | 13           | 439.164                   | 169.865                   | 295.502                         | 132.069                        |

Table 3 Red-shift and Adiabatic Index at \( r = a \) and \( r = 0 \) for chosen stars:

| Star Name       | \( M (M_\odot) \) | \( a \) (km) | \( M/a \) Surface red-shift | \( Z_s \) Adiabatic-Index |
|-----------------|-------------------|--------------|-----------------------------|---------------------------|
| SMC X–1         | 1.04              | 8.301        | 0.18                        | 0.262045                  |
| SAX J1808.4–3658(SS2) | 0.9               | 7.951        | 0.17                        | 0.213592                  |
| Swift J1818.0–1607 | 2.0               | 12.5         | 0.24                        | 0.380493                  |
| PSR J1614–2230   | 1.908             | 13           | 0.21                        | 0.331505                  |
| PSR J0348+0432   | 2.01              | 13           | 0.23                        | 0.359922                  |
Table 4 Buchdahl Ratio and Herrera’s Condition at $r = 0$ and $r = a$ for chosen stars:

| Star Name         | $M$ ($M_\odot$) | $a$ ($M_\odot$) | $\Gamma_r$ (km) | $(\upsilon^2 - \upsilon_r^2)_{r=0}$ | $(\upsilon^2 - \upsilon_r^2)_{r=a}$ |
|-------------------|-----------------|-----------------|-----------------|--------------------------------------|--------------------------------------|
| SMC X–1           | 1.04            | 8.301           | 2.55384         | -0.0115                              | -0.0819                              |
| SAX J1808.4–3658(SS2) | 0.9             | 7.951           | 2.66197         | -0.0182                              | -0.0845                              |
| Swift J1181.0-1607 | 2.0             | 12.5            | 1.94174         | -0.0330                              | -0.0578                              |
| PSR J1614–2230     | 1.908           | 13              | 1.98721         | -0.0451                              | -0.0612                              |
| PSR J0348+0432     | 2.01            | 13              | 1.9423          | -0.0399                              | -0.0586                              |

4.5 Gravitational redshift

The gravitational redshift $Z_g$ is formulated as:

$$Z_g = -1 + e^{\mu} \quad (46)$$

$$Z_g = -1 + \frac{1}{A + \frac{1}{2} B \sqrt{C r^2}} \quad (47)$$

$Z_g$ should be a decreasing function of the radial parameter $r$ for our model to be well behaved. This is evident from Fig. 5a.

4.6 Surface redshift

The surface redshift $Z_s$ is another essential bound for testing the physical acceptability of the model (Böhmer and Harko 2006). Associated with the metric coefficient, the surface redshift is given by:

$$Z_s = -1 + e^{-\nu} \quad (48)$$

$$Z_s = -1 + \sqrt{1 + Cr^2} \quad (49)$$

$Z_s$ must be finite for $0 \leq Z_s \leq 5$ (Böhmer and Harko 2006). As shown in Fig. 5b, our model satisfies this condition. The expression of the surface redshift is given in Equation (49). In Table 4 we have given the values of surface redshift for our chosen realistic stars at $r = a$. It is seen that the values obtained conform with the conditions provided with the values being less than 5. This provides increased credibility to the claim of our model being stable.

Another interesting property in the purview of the gravitational redshift as described in Fig. 5a emerges with respect to the hyperon connotations in maximal neutron star masses. PSR J0348+0432, when compared to the magnetar Swift J1808.0-1607, has similar mass and radius orientations. However, the gravitational redshift of the former is considerably lower than that of the latter. According to a treatise detailing the hyperon coupling constants and their relation with gravitational redshift in maximal mass neutron stars, the presence of the coupling constant for hyperons $\Xi$ reduces the gravitational redshift for neutron stars of similar masses (Chao 2019), suggesting the presence of hyperon structures inside PSR J0348+0432, a hypothesis which has been proposed in the paper, and has been verified through the graph presented in the current paper.

**Fig. 5** Gravitational redshift and surface redshift are shown over here for the chosen star candidates. The values of $M$, $a$, $A$, $B$, $C$ used are mentioned in Table 1. $\lambda$ is taken to be 0.35 and $G$ and $c$ are $6.67430 \times 10^{-20}$ km$^3$ kg$^{-1}$ s$^{-2}$ and 299792 km s$^{-1}$ respectively than $p_r$ and $p_t$. DEC is combination of NEC and WEC conditions. Table 2 shows the SEC calculated for the selected stars at $r = 0$ and $r = a$.

4.4 Buchdahl’s condition

As per Buchdahl (1979), the mass radius relationship for all realistic stellar objects must obey the inequality $\frac{M}{a} \leq \frac{4}{3}$. According to Table 3, we can see that our model satisfies this inequality for all the five stars.
4.7 Stability conditional analysis

The stability analysis needs certain parameters to analyze the model. The parameters are written exhaustively in this Section.

(1) Herrera’s Cracking Condition

This includes radial and tangential sound speeds for the analysis. These are denoted by \( v_r \) and \( v_\tau \), respectively. The square of radial and transverse speeds are given as:

\[
v_r^2 = \frac{dp_r}{d\rho} = [2Cr((Cr^2 + 1)^3(2A + B\sqrt{Cr^2})^2(4A^2C + 4AB\sqrt{C}(Cr^2 - 2) + B^2(Cr^2(Cr^2 - 8) - 4)) + \lambda(64A^4C^2(Cr^2(Cr^2 + 14) + 69) - 128A^3BC^{3/2}(Cr^2(Cr^2 + 14) + 166) - 31 - 32A^2B^2C(Cr^2(3Cr^2(Cr^2 + 14) + 163) - 264) - 24) - 32AB\sqrt{C}(Cr^2(Cr^2(Cr^2 + 14) + 18) - 219) - 59) - 3) + 4B^4Cr^2(Cr^2(640 - Cr^2(Cr^2 - 107)) + 380) + 84)))/(((Cr^2 + 1)^3(2A + B\sqrt{Cr^2})^2)((8\sqrt{3}/2\lambda r(16A^4C^{3/2}(Cr^2(Cr^2 - 3) - 89) + 195) + 32A^3BC(Cr^2(Cr^2(Cr^2 - 3) - 77) + 211) - 66) + 8A^2B^2\sqrt{C} \times (Cr^2(Cr^2(3Cr^2(Cr^2 - 3) - 179) + 689) - 532) - 128) + 8AB^3(Cr^2(Cr^2(Cr^2 \times (Cr^2(Cr^2 - 3) - 25) + 279) - 353) - 202) - 39) + B^4\sqrt{C}r^2(Cr^2(Cr^2(Cr^2(Cr^2 - 3) + 135) + 819) + 108) - 24) - 12)])/(((Cr^2 + 1)^3(2A + B\sqrt{Cr^2})^2) - \frac{2C^2r(Cr^2 + 5)}{(Cr^2 + 1)^3})]
\]

\[
v_\tau^2 = \frac{dp_\tau}{d\rho} = [4Cr((Cr^2 + 1)^3(2A + B\sqrt{Cr^2})^2(4A^2C + 2AB\sqrt{C}(Cr^2 - 3) - B^2(Cr^2(Cr^2 + 6) + 2)) + \lambda(32A^4C^2(Cr^2(Cr^2 + 24) + 165) - 138) + 64A^3BC^{3/2}(Cr^2(Cr^2(Cr^2 + 21) + 129) - 183) + 58) + 16A^2B^2C(Cr^2(Cr^2(Cr^2 - 17) + 241) - 759) + 364) + 56) + 16AB^3\sqrt{C}(Cr^2(Cr^2(Cr^2(Cr^2 + 1)C(Cr^2 - 11) - 372) + 181) + 78) + 15) + 2B^4Cr^2(Cr^2(Cr^2(Cr^2 - 9)C(Cr^2 - 15) - 780) - 132) - 64) - 12)))/(((Cr^2 + 1)^3(2A + B\sqrt{Cr^2})^2 (2A + B\sqrt{Cr^2})^2)/(8\sqrt{3}/2\lambda r(16A^4C^{3/2}(Cr^2(Cr^2 - 3) - 89) + 195) + 32A^3BC)(Cr^2(Cr^2(Cr^2 - 3) + 211) - 66) + 8A^2B^2\sqrt{C}(Cr^2(Cr^2(3Cr^2(Cr^2 - 3) - 179) + 689) - 532) - 128) + 8AB^3(Cr^2(Cr^2(Cr^2(Cr^2 - 3) - 25) + 279) - 353) - 202) - 39) + B^4\sqrt{C}r^2(Cr^2(Cr^2(Cr^2(Cr^2 - 3) + 135) + 819) + 108) - 24) - 12)))/(((Cr^2 + 1)^3(2A + B\sqrt{Cr^2})^2(2A + B\sqrt{Cr^2})^2) - \frac{2C^2r(Cr^2 + 5)}{(Cr^2 + 1)^3})]
\]
From Equations (50) and (51), we have the expression for Herrera’s condition as:

\[
\nu_t^2 - \nu_r^2 = \left( (C r^2 + 1)^3 (2A + B \sqrt{C} r^2)^2 (4A^2 \sqrt{C} (C r^2 - 1) + 4AB (C r^2 - 1)^2 \\
+ B^2 C^3/2 r^4 (C r^2 - 5) + 4\lambda (-16A^4 C^3/2 (C r^2 (C r^2 (2C r^2 + 39) + 248)
- 69) - 32A^3 B C (C r^2 (C r^2 (2C r^2 + 18) + 209) - 148) + 27)
- 16A^2 B^2 \sqrt{C} (C r^2 (C r^2 (2C r^2 (2C r^2 + 27) + 43) - 573) - 97) + 16)
+ 12) + B^{4 \sqrt{C} r^2 (C r^2 (C r^2 (C r^2 (2C r^2 - 2C r^2 + 21)) + 1527)
+ 1152) + 278) + 96)) (16A^4 C^3/2 (C r^2 (C r^2 - 89) + 195)
- 32A^3 B C (C r^2 (C r^2 (C r^2 - 77) + 211) - 66) - 8A^2 B^2 \sqrt{C} (C r^2
(C r^2 (3C r^2 (C r^2 - 179) + 689) - 532) - 128) - 8AB^3 (C r^2 (C r^2
(C r^2 (C r^2 - 25) + 279) - 353) - 202) - 39) - B^{4 \sqrt{C} r^2}
(C r^2 (C r^2 (C r^2 - 3) + 135) + 819) + 108 - 24) - 12))
+ \sqrt{C} (C r^2 + 1)^3 (2A + B \sqrt{C} r^2)^2 < 0 \tag{52}
\]

Herrera et al. (2004) provided the cracking condition for a stable anisotropic compact star that results when equilibrium is disturbed which may be consequence of local anisotropy. This condition is constituted in terms of \(\nu_r\) and \(\nu_t\).

Herrera’s condition is given by,

\[
|\nu_t^2 - \nu_r^2| \leq 1
\]

Using Herrera’s condition (Herrera et al. 2004), the stability of the star is extensively verified with Fig. 7a. Taking this condition into consideration, Abreu et al. (2007) demonstrated that parts of the sphere where \(-1 \leq \nu_t^2 - \nu_r^2 \leq 0\) are potentially stable, while the parts where \(0 < \nu_t^2 - \nu_r^2 \leq 1\) are potentially unstable when \(\rho_t\) is not zero within this sphere. It is noticeable from Table 4 that all the chosen star candidates for our model are compatible with the former condition and hence our model is potentially stable.

Also, the Equations (50)-(52) represent the square of the radial and tangential sound speed. Based on these, the causality condition can be given by,

\[
0 \leq \sqrt{\frac{d\rho_t}{d\rho}} \leq 1 \quad \text{and} \quad 0 \leq \sqrt{\frac{d\rho_r}{d\rho}} \leq 1
\]

Here \(\sqrt{\frac{d\rho_t}{d\rho}}\) and \(\sqrt{\frac{d\rho_r}{d\rho}}\) are \(\nu_r\) and \(\nu_t\). The range of these two parameters are described in the Figs. 6a-6b. It is apparent that candidate stars lie between 0 and 1. This verified the stability of the stars using Causality condition.

These conditions are needed to be satisfied. The probability of the gravitational collapse is much higher if these conditions are not compatible with the model.

(2) Relativistic Adiabatic Index

The tangential and radial adiabatic index are expressed as \(\Gamma_t = \frac{\rho_t}{\rho_r} \cdot \frac{dp_r}{d\rho} \) and \(\Gamma_t = \frac{\rho_t}{\rho_r} \cdot \frac{dp_t}{d\rho}\). The adiabatic indices in terms of the radial and tangential components are required to verify the model. This is essential to the modelling process, because these expressions are related to the stability of the star. The higher the stability of the star, the higher are its adiabatic indices.

Due to the complexity of the expressions of the two conditions, we have carried out the graphical analysis, and subsequently used it to verify the model.

The condition is,

\[
\Gamma_t > \frac{4}{3}
\]

The pictorial representation of condition is shown in the Fig. 7b with reasonable conciliation. It shows that the profile increases exponentially after certain point but at center, the values are greater than 1.333 as per the condition. This is also evident from Table 3.
Variation of radial sound speed $\upsilon_r$ and transverse sound speed $\upsilon_t$ with the radial parameter $r$ for the chosen star candidates. The values of $M$, $a$, $A$, $B$, $C$ used are mentioned in Table 1. $\lambda$ is taken to be 0.35 and $G$ and $c$ are 6.67430 $\cdot$ $10^{-20}$ km$^3$ $\cdot$ kg$^{-1}$$\cdot$s$^{-2}$ and 299792 km$\cdot$s$^{-1}$ respectively.

### 4.8 Equation of state

In the Krori-Barua metric, as shown by Abbas et al. (2015), the EOS has a linear behaviour. A similar trend can be observed in the current model, wherein, it can be elicited from Figs. 8a and 8b that EOS exhibits nearly linear behaviour for all the five stars. The graph is plotted between $\rho$ versus $\rho_r$, which demonstrates the linear nature of the EOS.

The graphical analysis of stars of larger masses leads to intriguing trends being observed in the density and pressure gradients, wherein the curves are much shallower in Figs. 2a, 2b and 2c for the stars Swift J1808.0-1607, PSR J1614-2230 and PSR J0348+0432, when compared with the lower mass stars (SMC X-1 and SAX J1808.4-3685). Larger radii for the higher mass stars coupled with lower values of density and pressure parameters also indicates a decrease in the anisotropy. This decrease in pressure anisotropy also indicates softer equations of state. As shown in Figs. 8a and 8b, this can be verified by the fact that the EOS obtained is more linear than low mass stars.

### 5 Analysis of the $M$-$a$ plots

It is of imperative consequence to calculate the permissible masses for compact star models generated by various theories of gravity, lest the model admits masses of higher capacity and provides spurious values of the density and the pressure parameters, thereby leading to incorrect physical analyses of the stars in question. The $M$-$a$ plot provides the maximum and minimum possible mass at a given surface density $\rho_s$. Here, we have obtained $M$-$a$ for the Starobinsky model of $f(R)$ gravity. As described in the sections above, using the Starobinsky model $\lambda = 0.35$, we compared the results of $M$-$a$ curve for different surface densities for the given metric. The $M$-$a$ profiles for both cases $\rho = 360$ MeV$\cdot$fm$^{-3}$ and $\rho = 450$ MeV$\cdot$fm$^{-3}$ have been provided in Fig. 9. As can be seen from Fig. 1a, both of the density values considered here fall well within the range of the central densities of the stars described in Table 2. This indicates that the values are...
Table 5 Maximum and minimum mass results for \( \lambda = 0.35 \) at surface density of \( \rho_s = 360 \) and \( \rho_s = 450 \text{ MeV} \cdot \text{fm}^{-3} \)

| \( \rho_s \) (MeV \cdot fm\(^{-3} \)) | Maximum Mass (\( M_\odot \)) | Minimum Mass (\( M_\odot \)) |
|----------------------------------|--------------------------|--------------------------|
| 360                             | \( M = 2.66 \)           | \( M = 0.155 \)          |
| 450                             | \( M = 2.36 \)           | \( M = 0.20 \)           |

valid, and are taken in consideration with the values that one may expect while studying similar compact stars (However, it must be noted that the values can be increased or decreased to study the effects that may occur at threshold densities for these stars). Due to the complexity of the expressions, the usage of discrete values to obtain the \( M-a \) plot was considered an appropriate measure. To compare the variations in the maximum and minimum masses which can be attained by stars at particular surface density, we described a tabular data for the outcome of our model. As shown below, the table elaborates upon the results of the \( M-a \) plot:

In Fig. 8 and Table 5, it is observed that for \( \rho_s = 360 \text{ MeV} \cdot \text{fm}^{-3} \), the neutron star masses reach a maximum mass of 2.66 \( M_\odot \). When compared to the maximum mass of stars at \( \rho_s = 450 \text{ MeV} \cdot \text{fm}^{-3} \), we find that the mass of the star in question peaks at 2.36 \( M_\odot \). When we observe the minimum masses, however, we see a different scenario. The minimum mass is higher for 450 MeV \cdot fm\(^{-3} \) surface density and lower for 360 MeV \cdot fm\(^{-3} \). The minimum masses at \( \rho_s = 450 \text{ and } 360 \text{ MeV} \cdot \text{fm}^{-3} \) are 0.20 and 0.155 \( M_\odot \) respectively. However, it is also observed that for the lower masses and similar radii, the masses for \( \rho_s = 450 \text{ MeV} \cdot \text{fm}^{-3} \) exceeds, in value, the mass at \( \rho_s = 360 \text{ MeV} \cdot \text{fm}^{-3} \). This indicates that the range of the aforementioned masses becomes progressively restricted at higher densities.

6 Discussion

Using the Finch-Skea ansatz, we provided a physically viable, analytical model of anisotropic compact celestial bodies. The solutions obtained supports the choice of Finch-Skea metric as a suitable candidate for modelling in \( f(R) \) gravity. Using this model we can also infer the presence of hyperons in neutron stars with higher than usual maximal masses, specifically for the stars PSR J0438+0432 and PSR J1614-2230. The model verifies the presence of hyperon structures in this star through two primary factors: the presence of a soft EOS due to the linear nature of the pressure-density relation, and the relation between the presence of hyperon structures and their effect on gravitational redshift for compact stars of larger masses. Another factor which makes the Finch-Skea model a viable candidate for compact star modelling in \( f(R) \) gravity is the fact that the model has been proven to be valid for a wide range of stars. This model is also valid for magnetars which is demonstrated here by evaluating it for newly discovered Swift J1818.0–1607 (Esposito et al. 2020). The model also provides a generalized algebraic solution to the constants \( A, B \) and \( C \) in the Finch-Skea metric, and makes it suitable for use in further modelling endeavours. The pressures, as obtained above, can be described as combined forces, arising from myriad phenomena such as neutron-hyperon interactions, phase transitions, electromagnetic interactions, etc. The paper and its concomitant EOS can also be utilised to provide further knowledge about the quark-gluon interactions at the center of strange stars, which may provide information and variables which are necessary to glean more reasons behind a star’s inherent anisotropy. Another standout feature of the paper is that the \( M-a \) plots were obtained solely through the usage of the analytical solutions, without the EOS as a pre-requisite towards obtaining the \( M-a \) relations of the plot. This indicates that the \( M-a \) plots hold true.
for any EOS of linear nature, which, in turn, has been proven through Figs. 8a and 8b. A physically viable model for neutron stars has therefore been suitably obtained through the application of \( f(R) \) gravity on the Finch-Skea metric.

**Acknowledgements**  The authors are grateful to Pandit Deendayal Petroleum University, Gandinagar, India, and the Ludwig-Maximilians Universität, München, Germany, for providing a strong platform to discuss, ruminate upon and bring the manuscript to a physical fruition.

**Conflict of interest**  The authors declare that they have no conflicts of interest.

**Publisher’s Note**  Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

**References**

Abbas, G., et al.: Astrophys. Space Sci. 358(2), 26 (2015)
Abreu, H., et al.: Class. Quantum Gravity 24, 4631 (2007)
Antoniadis, J., et al.: (2013). arXiv:1304.6875
Arzoumanian, Z., et al.: Astrophys. J. 235, 37 (2018)
Astashenok, A.V., et al.: Phys. Rev. D 89, 103509 (2014)
Astashenok, A.V., et al.: Class. Quantum Gravity 34, 20 (2017)
Astashenok, A.V., et al.: (2019). arXiv:2001.08504
Bhar, P.: Eur. Phys. J. C 79, 138 (2019)
Böhmer, C.G., Harko, T.: Class. Quantum Gravity 23, 6479 (2006)
Buchdahl, H.A.: Acta Phys. Pol. 10, 673 (1979)
Canuto, V.: Annu. Rev. Astron. Astrophys. 12, 167 (1974)
Demorest, P., et al.: (2010). arXiv:1010.5788
Durgapal, M.C.: J. Phys. A, Math. Gen. 15, 2637 (1982)
Egeland, E.: In: Compact Stars. Trondheim, Norway (2007)
Eisentein, D.J., et al.: Astrophys. J. 633, 560 (2005)
Elebert, p., et al.: Mon. Not. R. Astron. Soc. 395, 884 (2009)
Esposito, P., et al.: Astrophys. J. 896, 2 (2020)
Finch, M.R., Skea, J.E.F.: Class. Quantum Gravity 6(4), 467 (1989)
Goswami, R., et al.: Phys. Rev. D 90, 084011 (2014)
Hawkins, E., et al.: Mon. Not. R. Astron. Soc. 346, 78 (2003)
Herrera, L., Ospino, J., di Prisco, A.: Phys. Rev. D 77, 027502 (2008)
Herrera, L., et al.: Phys. Rev. D 69, 084026 (2004)
Hossein, S.K.M., et al.: Int. J. Mod. Phys. D 21, 1250088 (2012)
Karmarkar, K.R.: Proc. Indian Acad. Sci. A 27, 56 (1948)
Krš, K.D., Barua, J.: J. Phys. A, Math. Gen. 8, 508 (1975)
Maharaj, S.D., Takisa, P.M.: Gen. Relativ. Gravit. 44, 1419 (2012)
Mak, M.K., Harko, T.: Int. J. Mod. Phys. D 13, 149 (2004)
Murad, M.H., Fatema, S.: Int. J. Theor. Phys. 52, 4342–4359 (2013)
Ovalle, J.: Phys. Rev. D 95, 104019 (2017)
Pandya, D.M., Thomas, V.O., Sharma, R.S.: Astrophys. Space Sci. 356(2), 173 (2015)
Pandya, D.M., Thakore, B., Goti, R., Rank, J.P., Shah, S.: Astrophys. Space Sci. 365(2), 10 (2020)
Paul, B.C., et al.: Mod. Phys. Lett. A 26, 575 (2011)
Perlmutter, S., et al.: Astrophys. J. 517, 565 (1999)
Rahaman, R., et al.: Eur. Phys. J. C 72, 2071 (2012)
Rawls, M.L., et al.: Astrophys. J. 730, 25 (2011)
Ruderman, R.: Astron. Astrophys. 10, 427 (1972)
Sawyer, R.F.: Phys. Rev. Lett. 29(6), 382 (1972)
Sharif, M., Waseem, A.: Ann. Phys. 405, 14–28 (2019)
Singh, K.N., et al.: Chin. Phys. C 41, 015103 (2017)
Sokolov, A.I.: Sov. Phys. JETP 25(4), 575 (1980)
Starobinsky, A.A.: Phys. Lett. 91(1), 1–102 (1980)
Thirukkanesh, S., Ragel, F.S.: Pramana J. Phys. 78, 687 (2012)
Zhao, X., et al.: (2017). arXiv:1712.05894
Bhar, P.: Astrophys. Space Sci., 41, 359 (2015)
Maharaj, S.D., et al.: Int. J. Mod. Phys. D 3, 26 (2017)
Tikkar, R., Jotania, K.: Pramana J. Phys. 397(406), 68 (2017)