AN EMPIRICAL CALIBRATION OF THE MIXING-LENGTH PARAMETER $\alpha$\textsuperscript{1}

FRANCESCO R. FERRARO,\textsuperscript{2} ELENA VALENTI,\textsuperscript{2,3} OSCAR STRANIERO,\textsuperscript{4} AND LIVIA ORGILA\textsuperscript{3}

Received 2005 October 31; accepted 2005 December 27

ABSTRACT

We present an empirical calibration of the mixing-length free parameter $\alpha$ based on a homogeneous infrared database of 28 Galactic globular clusters spanning a wide metallicity range ($-2.15 < \text{[Fe/H]} < -0.2$). Empirical estimates of the red giant effective temperatures have been obtained from infrared colors. Suitable relations linking these temperatures to the cluster metallicity have been obtained and compared to theoretical predictions. An appropriate set of models for the Sun and Population II giants has been computed by using both the standard solar metallicity ($Z/X_0 = 0.0275$) and the most recently proposed value ($Z/X_0 = 0.0177$). We find that when the standard solar metallicity is adopted, a unique value of $\alpha = 2.17$ can be used to reproduce both the solar radius and the Population II red giant temperature. Conversely, when the new solar metallicity is adopted, two different values of $\alpha$ are required: $\alpha = 1.86$ to fit the solar radius and $\alpha \approx 2.0$ to fit the red giant temperatures. However, it must be noted that regardless the adopted solar reference, the $\alpha$-parameter does not show any significant dependence on metallicity.

Subject headings: globular clusters: general — stars: evolution — stars: Population II

1. INTRODUCTION

Stellar evolutionary models are common ingredients in a variety of studies addressing fundamental cosmological and astrophysical problems, such as ages, formation processes, and evolution of galaxies. However, stellar sequences need to be properly checked and calibrated before using them to derive properties of complex stellar systems. As extensively reviewed by Renzini & Fusi Pecci (1988), besides model testing, one should also take into account as a separate issue the calibration of those quantities that theoretical models are forced to parameterize because of insufficient understanding of the physical process. In this context, the lack of a rigorous theory of convection remains one of the major deficiencies in the calculations of stellar evolutionary sequences. Despite many efforts to replace the mixing-length (ML) theory of convection by less crude approximations (see Camuto & Mazzitelli 1991; Spruit 1997; Ludwig et al. 1999), the classical formulation of Böhm-Vitense (1958) continues to be universally used. The ML algorithm requires the presence of the free parameter $\alpha = l/H_p$, which represents the ratio between the mean free path of a convective element ($l$) and the pressure scale height ($H_p$); the variations of this parameter strongly affect the structure of the outer envelope (i.e., radius and temperature). In fact, this parameter determines the efficiency of energy transport by convection in the outermost layer of a star: for a given stellar luminosity, it fixes the radius of the star, and hence its temperature and colors. Evolutionary tracks must then be calibrated by comparison with stellar radii and/or temperatures derived from observations. The most obvious ML calibrator is the Sun. The ML parameter can be fixed by constraining theoretical solar models (i.e., with solar mass, age, and chemical composition) to reproduce the solar radius (see, e.g., Mazzitelli 1979; Sweigart 1983; VandenBerg 1983). The solar calibrations suggest that the parameter $\alpha$ should range from 1.5 to 2, when the classical ML algorithm is used (Cox & Giuli 1968). Observational evidence has shown that similar values can account for (1) the lower main-sequence (MS) slope of young open clusters (VandenBerg & Bridges 1984), (2) the positions in the color-magnitude diagram (CMD) of the local Population II subdwarfs (Buonanno et al. 1988), (3) the properties of well-observed binaries whose components are in widely separated evolutionary phases (VandenBerg & Hrivnak 1985), and (4) the effective temperatures of red giant branch (RGB) stars (Straniero & Chieffi 1991; Chieffi et al. 1995; VandenBerg et al. 2000; Alonso et al. 1999, 2000). However, although similar $\alpha$-values have been derived by using different ML calibrators, there is no theoretical justification that the same value of $\alpha$ should apply to any star.

In addition, Chieffi et al. (1995) emphasized that in the case of low-mass stellar models, the derived stellar radii depend on the opacity, which significantly contributes in determining the temperature gradient in their turbulent external layers. Hence, stellar models, based on different opacity tables, could require different values of $\alpha$.

A homogeneous data set of stellar temperatures at different metallicities to properly calibrate the ML parameter is urgently needed before making any further attempt to use evolutionary models to derive relevant properties of stellar populations.

In this paper we present an empirical calibration of the $\alpha$-parameter, based on accurate estimates of the RGB effective temperature at fixed luminosity levels for a homogeneous sample of Galactic globular clusters (GCs) with different metallicities, observed by our group in the last 10 years (see, e.g., Ferraro et al. 2000; Valenti et al. 2004a, 2004c, 2005; Sollima et al. 2004 and references therein). This infrared data set is the ideal tool to properly calibrate the $\alpha$-parameter for low-mass stellar models and to study its possible dependence on metallicity.

2. THE MIXING LENGTH

One of the main sources of uncertainty in stellar models concerns the efficiency of convection. In the external layer of a red giant star, the convective heat transfer may substantially depart from the adiabatic regime, in which any internal excess of heat
is efficiently transported outward by the ascending convective bubbles. In the framework of the standard mixing-length theory, the convective efficiency is tuned by changing the $\alpha$-parameter. Large values of $\alpha$ (i.e., larger than 2.5) correspond to a very efficient convection. In this case, the temperature gradient is close to the minimum allowed value (the adiabatic one), and the predicted red giant temperatures are larger. On the other hand, for lower values of the $\alpha$-parameter, the internal heat excess is only partially removed by convection, and the radiative energy transport plays an important role. This is the case for GC RGB stars, where the stellar effective temperature is sensitive to the radiative opacity. For this reason, models computed with the same $\alpha$-value, but larger opacity, produce lower effective temperatures. Hence, a proper variation of $\alpha$ may counterbalance a variation of low-temperature opacities (Chieffi et al. 1995).

The $\alpha$-parameter is usually calibrated by forcing stellar models of the Sun to fit the solar radius, which is known with high precision. However, the envelope of a red giant has rather different physical characteristics with respect to the solar envelope. A direct comparison of empirical temperatures of red giant stars with the corresponding model predictions provides an independent calibration of $\alpha$ and allows us to measure the efficiency of the convective energy transport in very different physical conditions.

### 3. THE EMPIRICAL DATABASE

The homogeneous data set of fiducial RGB ridge lines presented here is based on high-quality $J$, $H$, and $K$ photometry of 28 Galactic GCs published by our group in the last few years (Ferraro et al. 2000; Valenti et al. 2004a, 2004c, 2005; Sollima et al. 2004; Origlia et al. 2005b). The data were obtained at European Southern Observatory (ESO) La Silla Observatory (Chile) using IRAC2 at the ESO/MPM (Max-Planck-Institut) 2.2 m telescope and SoFI (Son of ISAAC) at NOT (New Technology Telescope)/ESO IR cameras and at the Telescopio Nazionale Galileo with ARNICA (Arcetri Near-Infrared Camera) in several observing runs. On average, the central $\sim 20$ arcmin$^2$ of each cluster have been mapped, allowing us to sample a significant fraction of the total cluster light (typically $\approx 70\%-90\%$). For all the program clusters, the same data reduction procedure has been applied (see Valenti et al. 2004a for more details); the instrumental magnitudes have been calibrated into the 2MASS (Two Micron All Sky Survey) photometric and astrometric system, allowing us to build the largest homogeneous IR database of GCs ever obtained.$^5$

For all the clusters listed in Table 1, the observations were deep enough to properly sample the entire RGB extension, from the base (typically 2–3 mag below the horizontal branch [Hb]) up to the RGB tip, thus allowing a complete study of the RGB morphological features and a clear definition of the mean ridge line (see, e.g., Figs. 1 and 2 of Valenti et al. 2004a; see also Valenti et al. 2005).

---

$^5$ The photometric catalogs are available in electronic form at the CDS (Centre de Données Astronomiques de Strasbourg) World Wide Web site.
The detailed procedure followed to obtain the RGB fiducial ridge lines of the clusters and to transform them into the absolute plane can be found in Ferraro et al. (2000). Since for the aim of this study the homogeneity of the data set is a crucial issue, we adopt the distance scale established by Ferraro et al. (1999) based on an empirical measurement of the zero-age HB level in a sample of 61 Galactic GCs. Note that the Ferraro et al. (1999) distance scale has been adopted by Ferraro et al. (2000) and Valenti et al. (2004b) in order to perform a detailed comparison of the observed RGB bump and tip luminosity with theoretical expectations, finding an excellent agreement. However, it is worth of noticing that the assumption of a different distance scale has a little effect on the derived temperatures. In fact, even a significant difference in the adopted distance moduli of ±0.1 mag would produce only a difference of ±30 K in the derived temperatures.

Reddening values have been taken from Harris (1996) for all the clusters but the most extinguished ones toward the bulge direction, for which a differential method based on the comparison of CMDs and luminosity functions (LFs) with those of a reference cluster of similar metallicity has been applied (see Valenti et al. 2004a for more details). In order to use this empirical data set for the calibration of theoretical models, the mean ridge lines in the $(M_{\text{Bol}}, \log(T_e))$ observational plane must be transformed into the $(M_{\text{Bol}}, \log(T_e))$ theoretical one. In doing this we adopted the bolometric correction $(B_C)$ and the temperature scale computed by Montegriffo et al. (1998). These empirical relations have been specifically calibrated on Population II giants in Galactic GCs (see also Alonso et al. 1999). Figure 1 shows the fiducial RGB ridge lines for the 28 Galactic GCs in our sample in the $(M_{\text{Bol}}, \log(T_e))$ theoretical plane.

3.1. The Metallicity Scale

As widely discussed in Ferraro et al. (1999, 2000), the location of the RGB in color (i.e., in temperature) strongly depends on the cluster metal content. Indeed, the effective temperature of a red giant star decreases when the total mass fraction of heavy elements ($Z$) increases, mainly because of the larger opacity. Actually, electron donors, like Fe and $\alpha$-elements, provide the free electrons needed to form $H^-$ ions, which are the most important opacity source in the cool red giant envelope. Thus, stars with similar $Z$, but different distributions of heavy elements, should have similar RGB temperatures, if the total amount of electron donors is similar. This is the case, for example, of scaled solar and $\alpha$-enhanced mixtures with the same $Z$ (Salaris et al. 1993). On the other hand, the He content ($Y$) and the mass ($M$) only slightly affect the RGB effective temperature: a similar variation of the effective temperature $\delta \log T_e \approx -0.005$ is indeed obtained for masses increasing from 0.8 to 0.9 $M_\odot$ or for $Y$ increasing from 0.245 to 0.30. Hence, the correct parameterization of the RGB location does require precise knowledge of the so-called global metallicity, which takes also into account the contribution of the $\alpha$-elements, in particular the $[\text{Mg + Si + Fe}]$ abundance mixture, rather than relying on the $[\text{Fe}/H]$ abundance alone (Straniero & Chieffi 1991; Salaris & Cassisi 1996).

In our previous works (Ferraro et al. 2000; Valenti et al. 2004a, 2004c, 2005) we have computed the global metallicity $[M/H]$ from the Carretta & Gratton (1997, hereafter CG97) scale ($[\text{Fe/H}_{\text{CG97}}]$) by adopting an enhancement factor of the $\alpha$-elements that linearly decreases to zero for metal-rich clusters with $[\text{Fe/H}_{\text{CG97}}] > -1$. However, there is now a growing body of evidence that this trend is not applicable to the bulge clusters. In fact, the most recent high-resolution spectroscopic observations of both bulge cluster and field giants (Rich & McWilliam 2000; Carretta et al. 2001; Origlia et al. 2002, 2005a, 2005b; Origlia & Rich 2004; Zoccali et al. 2004; Rich & Origlia 2005) suggest an $\alpha$-enhancement up to solar metallicity. Hence, here we have adopted a constant $\alpha$-enhancement ($\alpha/\text{Fe} \sim 0.3$ dex) over the entire $-2.2 < [\text{Fe}/H_{\text{CG97}}] < -0.1$ range of metallicities. For the NGC 6553 and NGC 6528 clusters, which represent the metal-rich extremes of our entire database, we use the updated values inferred from high-resolution IR spectroscopy by Origlia et al. (2002, 2005a; see also Carretta & Gratton 1997; Carretta et al. 2001; Melendez et al. 2003; Zoccali et al. 2004). The adopted metallicities for all the program clusters are listed in column (3) of Table 1.

4. RESULTS AND DISCUSSION

We used the CMD plotted in Figure 1 to measure the RGB effective temperatures at fixed bolometric magnitudes, namely, $M_{\text{Bol}} = -3, -2$, and $-1$. The derived values are listed in Table 1 together with a formal uncertainty of 50–100 K and are plotted in Figure 2 as a function of the cluster global metallicity ($[M/H]$). Well-defined quadratic relations best fit the observed data with a very small rms ($\Delta \log T_{\text{eff}} < 0.01$). These relations can serve as calibrators for the current and future generations of theoretical models for Population II stars. In the following, we use them to calibrate the $\alpha$-parameter for the stellar evolution code described by Straniero et al. (1997, hereafter SCL97), which adopts the latest input physics and the opacity from OPAL above log $T_{\text{eff}} = 3.75$, and from Alexander & Ferguson (1994) for log $T_{\text{eff}} < 3.75$.  

---

6 For the 2003, updated version see http://physwww.mcmaster.ca/%7Eharris/mwgc.dat.

7 The difference in effective temperature may be somewhat larger at high metallicity (Kim et al. 2002).
As a first step, we have used this code to compute a standard solar model and tune the $\alpha$-parameter in order to reproduce the solar radius. In doing this, we followed the procedure described in Chieffi et al. (1995).

An important consideration concerning the adopted solar abundances is worth noting. The Anders & Grevesse (1989, hereafter AG89) values [giving a total metallicity $(Z/X)_0 = 0.0275$] have been widely used for years. However, more recently significantly lower abundances have been proposed by Lodders (2003, hereafter L03; $(Z/X)_0 = 0.0177$) and by Asplund et al. (2005; $(Z/X)_0 = 0.0165$), both using CNO abundances from three-dimensional model atmospheres (see also Allende Prieto et al. 2002). By adopting the AG89 and the L03 solar compositions, we have found that the best SCL97 models to reproduce the solar radius require $\alpha = 2.17$ and $\alpha = 1.86$, respectively.

Then, by using the same code (SCL97) and the same input physics, we have computed a set of RGB models for low-mass Population II stars. Of course, in order to compare the empirical effective temperatures with those predicted by stellar models, we need the absolute abundance of metals, namely $Z$. The relation between these two quantities is

$$[\text{M/H}] = \log (Z/X) - \log (Z/X)_0. $$

Here $X = 1 - Y - Z$ is the mass fraction of H in the envelope of our star/model. Thus, the absolute amount of heavy elements in a star with known [M/H] substantially depends on the $Z/X$ ratio in the solar photosphere $[(Z/X)_0]$. Hence, for example, for $[\text{M/H}] = -1.5$ one obtains $Z/X = 0.00056$ with the latest solar abundances by L03, or $Z/X = 0.00087$ with the widely adopted AG89 ones.

We have computed a set of RGB models with fixed mass ($M = 0.8 \, M_\odot$). The abundance $Z$ has been varied from 0.0001 to 0.02. In our analysis, if not explicitly specified, we adopt $Y = 0.245$, a value in agreement with the primordial He abundance (Cassisi et al. 2003). A 10%–20% uncertainty in $X$ (or $Y$) induces a variation of $Z$ that is always smaller than the 1 $\sigma$ error in the [M/H] empirical estimates. The initial heavy-element distribution is a scaled solar distribution, using both the AG89 and L03 references.

Figure 3 reports the results. By using the AG89 solar reference, the measured RGB temperatures are well reproduced by the $\alpha = 2.17$ curve, the same value obtained by best fitting the solar radius. Conversely, by using the L03 solar reference, the $\alpha$-value adopted to reproduce the Sun ($\alpha = 1.86$) systematically underestimates the giant temperatures, while the model with $\alpha = 2.17$ systematically overestimates them. An accurate inspection of Figure 3 suggests that the observational data are best reproduced by a model with an intermediate value of the ML parameter ($\alpha \approx 2$).

Hence, from the comparison of the empirical data with theoretical predictions, we find that by adopting the “old” solar abundances by AG89 both the solar and the Population II red giant structure (regardless of the metallicity) can be nicely reproduced with a single value of the ML parameter ($\alpha = 2.17$). Conversely, different values of $\alpha$ for the Sun (1.86) and the giant stars (2.0) are required when the L03 solar abundances are used. Note that the curve with $\alpha = 2.0$ for the L03 solar metallicity is not plotted in Figure 3, since it is nearly coincident with the one obtained with $\alpha = 1.86$ for the AG89 solar reference (solid line in Fig. 3).

As already stated in § 1, in principle, there is no theoretical reason that the same value of $\alpha$ should apply to any star; hence, the two results are formally equally possible.

However, there are other issues that should be considered and discussed. For example, it must be noted that the empirical abundances for the clusters listed in Table 1, have been derived using one-dimensional model atmospheres and one-dimensional solar references such as AG89 or subsequent updates. Conversely, the L03 solar references are partially based on abundances derived from three-dimensional models. Hence, the results using the L03 solar references have to be taken with caution, since only when both a complete set of solar abundances and a reanalysis of the
empirical stellar abundances, fully based on three-dimensional model atmospheres (see, e.g., Kucinskas et al. 2005) are available can firm conclusions be drawn.

A significant discrepancy between theory and observation has been already noted when the “new” L03 solar abundance was used to compute standard solar models. Indeed, the predicted depth of the solar convective zone, as obtained by adopting the new CNO abundances from Asplund (2003), is significantly larger than the one derived from the analysis of the helioseismic data (Bahcall et al. 2003). The bad news is even worse, because an excellent reproduction of the seismic data was previously found by adopting the “old” AG89 solar abundances. Bahcall et al. (2005) invoke a significant increase of the solar Ne (about 10^3 times larger) to solve the solar convective zone problem. A recent analysis of X-ray spectra of nearby stars (Drake & Testa 2005) seems to confirm this expectation. Note that such a substantial increase of the solar Ne abundance would also affect the relation between [M/H] and Z and, in turn, the calibration of the mixing length.

While the issue of the firm determination of the absolute solar abundance will be addressed by future spectroscopic works, the major result presented here is that regardless of the adopted solar reference, there is not any clear evidence, within the errors, of a significant dependence of the mixing length parameter on the stellar metallicity (see also Palmieri et al. 2002). This is a somewhat surprising result, since the classical formulation of the mixing-length theory (Böhm-Vitense 1958) is a quite naive approximation of the mean free path of the convective bubbles based on very simple assumptions. The results shown here suggest that in spite of these crude approximations and the very basic assumptions, the efficiency of the convective energy transport parameterized by α is mainly controlled by the local value of $H_p$, and it does not require any extra dependence on the metallicity of the environment, once an appropriate set of opacity coefficients have taken into account in the calculation of the stellar model.

The financial support by the Ministero dell’Istruzione, Università e Ricerca (MIUR) is kindly acknowledged. This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and Infrared Processing and Analysis Center/California Institute of Technology, founded by the National Aeronautics and Space Administration and the National Science Foundation.

REFERENCES

Alexander, D. R., & Ferguson, J. W. 1994, ApJ, 437, 879
Allende Prieto, C., Lambert, D. L., & Asplund, M. 2002, ApJ, 573, L137
Alonso, A., Salaris, M., Arribas, S., Martínez-Roger, C., & Asensio Ramos, A. 1999, A&AS, 140, 261
Álvarez, L., & García-Reinaldos, R. 2000, A&A, 355, 1060
Anders, E., & Grevesse, N. 1989, Geochim. Cosmochim. Acta, 53, 197 (AG89)
Asplund, M. 2002, in ASP Conf. Ser. 304, CNO in the Universe, ed. T. G. Barnes III & F. N. Bash (San Francisco: ASP), 25
Bahcall, J. N., Gonzalez-Garcia, M. C., & Pena-Garay, C. 2003, Phys. Rev. Lett., 90, 131301
Bahcall, J. N., Sarbani, B., & Serenelli, A. M. 2005, ApJ, 631, 1281
Böhm-Vitense, E. 1958, Z. Astrophys., 46, 108
Buonanno, R., Corsi, C. E., & Fusi Pecci, F. 1988, in IAU Symp. 126, The Harlow-Shapley Symposium on Globular Cluster Systems in Galaxies, ed. J. E. Grindlay & A. G. D. Philip (Dordrecht: Kluwer), 635
Canuto, V. M., & Mazzitelli, I. 1991, ApJ, 370, 295
Carretta, E., Cohen, J., Gratton, R. G., & Beir, B. 2001, AJ, 122, 1469
Carretta, E., & Gratton, R. 1997, A&AS, 121, 95 (CG97)
Cassisi, S., Salaris, M., & Irwin, A. W. 2003, ApJ, 588, 862
Chieffi, A., Straniero, O., & Salaris, M. 1999, ApJ, 445, L39
Cox, J. P., & Giuli, R. T. 1968, Principles of Stellar Structure (New York: Gordon & Breach)
Drake, J. J., & Testa, P. 2005, Nature, 436, 525
Ferraro, F. R., Messineo, M., Fusi Pecci, F., De Palo, A. M., Chieffi, A., & Limongi, M. 1999, AJ, 118, 1738
Ferraro, F. R., Montegriffo, P., Origlia, L., & Fusi Pecci, F. 2000, AJ, 119, 1282
Harris, W. E. 1996, AJ, 112, 1487
Kim, Y.-C., Demarque, P., Yi, S. K., & Alexander, D. R. 2002, ApJS, 143, 499
Kucinskas, A., et al. 2005, A&A, 442, 281
Lodders, K. 2003, ApJ, 591, 1220 (L03)