Study of FCNC mediated rare $B_s$ decays in a single universal extra dimension scenario

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Abstract

We study the rare semileptonic and radiative leptonic $B_s$ decays in the universal extra dimension model. In this scenario, with a single extra dimension, there exists only one new parameter beyond those of the standard model, which is the inverse of the compactification radius $R$. We find that with the additional contributions due to the KK modes the branching ratios of the rare $B_s$ decays are enhanced from their corresponding standard model values and the zero point of the forward backward asymmetries are shifted towards the left.
1 Introduction

Although the standard model (SM) of electroweak interaction is very successful in explaining the observed data so far, but still it is believed that there must exist some new physics beyond the SM, whose true nature is not yet well-known. Therefore, intensive search for physics beyond the SM is now being performed in various areas of particle physics. In this context the $B$ system can also be used as a complementary probe for new physics. Unfortunately, we have not been able to see any clear indication of physics beyond the SM in the currently running $B$-factories (SLAC and KEK). Nevertheless, there appears to be some kind of deviation in some $b \to s$ penguin induced transitions like the deviation in the measurement of $\sin 2\beta$ in $B_d \to \phi K_S$ and also in some related processes, polarization anomaly in $B \to \phi K^*$ and deviation of branching ratios from the SM expectation in some rare $B$ decays, etc. [1]. But, the present situation is that it is too early to substantiate or rule out the existence of new physics in the $b$-sector.

One of the important ways to look for new physics in the $b$-sector is the analysis of rare $B$ decay modes, which are induced by the flavor changing neutral current (FCNC) transitions. The FCNC transitions generally arise at the loop level in the SM, and thus provide an excellent testing ground for new physics. Therefore, it is very important to study the FCNC processes, both theoretically and experimentally, as these decays can provide a sensitive test for the investigation of the gauge structure of the SM at the loop level. Concerning the semileptonic $B$ decays, $B \to X_s l^+l^-$ ($X_s = K, K^*$, $l = e, \mu, \tau$) are a class of decays having both theoretical and experimental importance. At the quark level, these decays proceed through the FCNC transition $b \to s l^+l^-$, which occur only through loops in the SM, and therefore, these decays constitute a quite suitable tool of looking for new physics. Moreover, the dileptons present in these processes allow us to formulate many observables which can serve as a testing ground to decipher the presence of new physics. In this context, the study of the rare semileptonic $B_s$ decay modes, $B_s \to \phi l^+l^-$, which are induced by the same quark level transition, i.e., $b \to s l^+l^-$, might be worth exploring. These decay modes may provide us additional information towards the quest for the existence of new physics beyond the SM and therefore deserve serious attention, both theoretically and experimentally. Since at the quark level they are induced by the same mechanism, so we can also independently test our understanding of the quark-hadron dynamics and also study some CP violation parameters with the help of rare $B_s$ decays, apart from corroborating the finding of the $B$-meson sector. These decay modes are studied in the standard model [2] and in the two Higgs doublet
model [3]. Recently, the D0 collaboration [4] has reported a more stringent upper limit on the branching ratio of $B_s^0 \to \phi \mu^+ \mu^-$ mode as $\text{Br}(B_s^0 \to \phi \mu^+ \mu^-) < 4.1 \times 10^{-6}$.

Another sensitive process to look for new physics is the radiative dileptonic decay mode $B_s^0 \to l^+ l^- \gamma$. In contrast to the pure leptonic decays $B_s^0 \to l^+ l^-$, these modes are free from helicity suppression due to the emission of a photon in addition to the lepton pair. Thus, the branching ratios of these processes are larger than those for the pure leptonic modes despite of an additional $\alpha$ suppression. These modes are also studied in the SM [5] and some beyond the SM scenarios [6].

New physics effects manifest themselves in these rare $B_s$ decays in two different ways, either through new contribution to the Wilson coefficients or through the new structure in the effective Hamiltonian, which are absent in the SM. There are many variants of possible extensions to the SM exist in the literature but the models with extra dimensions have received considerable attention in recent years. An elegant beyond the SM scenario with extra dimensions is being considered to solve the classical problems of particle physics, mainly the hierarchy problem, gauge coupling unification, neutrino mass generation, fermion mass hierarchies etc. In fact, there are several models that exist in the literature which take into account the effect of extra dimensions and they differ from one another depending on the number of extra dimensions, the geometry of space-time, the compactification manifold, which particles can go into the extra-dimensions and which cannot etc. In the case of the scenario with Universal Extra Dimensions (UED), proposed by Appelquist-Cheng-Dobrescu (ACD) [7], all the fields are allowed to propagate in all available dimensions. In its simplest version the single extra dimension is taken to be the fifth dimension, that is, $x_5 = y$. This is in fact compactified on the orbifold $S^1/Z_2$, i.e., on a circle of radius $R$ and runs from 0 to $2\pi R$, with $y = 0$ and $y = \pi R$ are the fixed points of the orbifold. Hence a field $F(x, y)$, where $x$ denotes the usual $3 + 1$ dimension, would be a periodic function of $y$ and can be represented as

$$ F(x, y) = \sum_{n=-\infty}^{\infty} F_n(x) e^{in \cdot y}. $$

(1)

Under the parity transformation $P_5 : y \to -y$, fields which exist in the SM are even and their zero modes in the KK expansion are interpreted as ordinary SM fields. On the other hand, fields absent in the SM are odd under $P_5$, so they do not have zero modes. Compactification of the extra dimension leads to the appearance of Kaluza-Klein (KK) partners of the SM fields as well as KK modes without corresponding SM partners. Thus, in the particle spectrum of
ACD model, in addition to the ordinary particles of the SM denoted as zero mode \((n = 0)\), there are infinite towers of Kaluza-Klein modes \((n \geq 1)\). There is one such tower of each SM boson, two for each SM fermion, and for the physical scalars \(a_{(n)}^{0\pm}\), no zero mode exists. The masses of the bosonic KK modes are given as \(m_{(n)}^2 = m_0^2 + n^2/R^2\), with \(m_0\) being the mass of the zero mode. The important features of the ACD model are: (i) there is only one additional free parameter with respect to the SM, i.e., \(1/R\), the inverse of the compactification radius (ii) the KK parity is conserved, which implies that the KK modes do not contribute at tree level for the low energy interaction processes and (iii) the lightest KK particle must be stable.

The implications of physics with UED are being examined with the data from accelerator experiments, for example, from Tevatron experiments the bound on the inverse compactification radius is found to be about \(1/R \geq 300\) GeV. Analysis of the anomalous magnetic moment and \(Z \rightarrow \bar{b}b\) vertex \([8]\) also lead to the bound \(1/R \geq 300\) GeV. Possible manifestation of ACD model in the \(K_L - K_S\) mass difference, \(B^0 - \bar{B}^0\) mixing, rare decays of \(K\) and \(B\) mesons and the kaon CP violation parameter \(\varepsilon'/\varepsilon\) are comprehensively investigated in \([9]\). Exclusive \(B \rightarrow K^*\ell^+\ell^-\), \(B \rightarrow K^*\bar{\nu}\nu\) and \(B \rightarrow K^*\gamma\) decays \([10]\) and rare semileptonic \(\Lambda_b\) decays \([11]\) are also studied in the framework of the UED scenario.

In this work, we would like to study the implications of the scenario with universal extra dimensions in the semileptonic rare decay modes \(B_s^0 \rightarrow \phi \ell^+\ell^-\), and the radiative dileptonic decays \(B_s^0 \rightarrow \ell^+\ell^-\gamma\). As discussed above, since these decays are very sensitive to new physics, these studies may provide an indirect way to constrain the new physics parameter \(R\). The paper is organized as follows. In section II, we present the effective Hamiltonian for these decays in the ACD model. The decay rates and forward backward rate asymmetries for the mode \(B_s^0 \rightarrow \phi\ell^+\ell^-\) and \(B_s^0 \rightarrow \ell^+\ell^-\gamma\) are considered in section III. Section IV contains our conclusion.

2 Effective Hamiltonian

The effective Hamiltonian describing the \(b \rightarrow sl^+l^-\) transition in the SM can be written as \([12]\)

\[
H_{eff} = \frac{G_F \alpha}{\sqrt{2\pi}} V_{tb} V_{ts}^* \left[ C_9^{eff} (\bar{s} \gamma_\mu Lb)(\bar{\ell} \gamma^\mu l) + C_{10}^{eff} (\bar{s} \gamma_\mu Lb)(\bar{\ell} \gamma^\mu 5 l) - 2C_7^{eff} m_b (\bar{s} i\sigma_{\mu\nu} q^\nu Rb)(\bar{l} \gamma^\mu l) \right],
\] 

(2)
where $q$ is the momentum transferred to the lepton pair, given as $q = p_- + p_+$, with $p_-$ and $p_+$ are the momenta of the leptons $l^-$ and $l^+$ respectively. $L, R = (1 \pm \gamma_5)/2$ and $C_i$’s are the Wilson coefficients evaluated at the $b$ quark mass scale. The coefficient $C_9^{\text{eff}}$ has a perturbative part and a resonance part which comes from the long distance effects due to the conversion of the real $c\bar{c}$ into the lepton pair $l^+l^-$. Hence, $C_9^{\text{eff}}$ can be written as

$$C_9^{\text{eff}} = C_9 + Y(s) + C_9^{\text{res}},$$

where $s = q^2$ and the function $Y(s)$ denotes the perturbative part coming from one loop matrix elements of the four quark operators and is given in Ref. [13]. The long distance resonance effect is given as [14]

$$C_9^{\text{res}} = \frac{3\pi}{\alpha^2} (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \sum_{V_i = \psi, J/\psi} \frac{2m_{V_i} \Gamma(V_i \to l^+l^-)}{m_{V_i}^2 - s - i m_{V_i} \Gamma_{V_i}},$$

where the phenomenological parameter $\kappa$ is taken to be 2.3, so as to reproduce the correct branching ratio $B(B \to J/\psi K^* \to K^* l^+l^-) = B(B \to J/\psi K^*) B(J/\psi \to l^+l^-)$.

Now we will analyze the KK contributions to the above decay modes. Since there is no tree level contribution of KK modes in low energy processes, the new contributions, therefore, only come from the loop diagrams with internal KK modes. It should be noted here that there does not appear any new operator in the ACD model, and therefore, new effects are implemented by modifying the Wilson coefficients existing in the SM, if we neglect the contributions of the scalar fields, which are indeed very small. Thus, the modified Wilson coefficients are given as [9]

$$C_1(M_W) = \frac{11 \alpha_s(M_W)}{2} \frac{\alpha_s(M_W)}{4\pi}, \quad C_2(M_W) = 1 - \frac{11 \alpha_s(M_W)}{6} \frac{\alpha_s(M_W)}{4\pi},$$

$$C_3(M_W) = -3C_4(M_W) = C_5(M_W) = -3C_6(M_W) = -\frac{\alpha_s}{24\pi} \bar{E}(x_t, 1/R),$$

$$C_7(M_W) = -\frac{1}{2} \bar{D}'(x_t, 1/R), \quad C_8(M_W) = -\frac{1}{2} E'(x_t, 1/R).$$

The functions $\bar{E}(x_t, 1/R)$, $E'(x_t, 1/R)$ and $D(x_t, 1/R)$ are given by

$$F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n), \quad F = \bar{E}, E', D',$$

where $x_t = m_t^2/M_W^2$, $x_n = m_n^2/M_W^2$ and $m_n = n/R$. The loop functions $F_0(x_t)$ and $F_n(x_t, x_n)$ are given as [9]

$$E_0(x_t) = -\frac{2}{3} \ln x_t + \frac{x_t^2(15 - 16x_t + 4x_t^2)}{6(1-x_t)^4} \ln x_t + \frac{x_t(18 - 11x_t - x_t^2)}{12(1-x_t)^3},$$
\[ D_0'(x_t) = -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1 - x_t)^3} + \frac{x_t^2(2 - 3x_t)}{2(1 - x_t)^4} \ln x_t , \]
\[ E_0'(x_t) = \frac{-x_t(x_t^2 - 5x_t - 2)}{4(1 - x_t)^3} + \frac{3}{2} \frac{x_t^2}{(1 - x_t)^4} \ln x_t , \]
\[ E_n(x_t, x_n) = -\frac{x_t[35 + 8x_t - 19x_t^2 + 6x_n^2(10 - 9x_t + 3x_t^2) + 3x_n(53 - 58x_t + 21x_t^2)]}{36(x_t - 1)^3} \]
\[ - \frac{1}{2}(1 + x_n)(-2 + 3x_n + x_n^2) \ln \frac{x_n}{1 + x_n} \]
\[ + \frac{(1 + x_n)(-6 + 19x_t - 9x_t^2 + x_n^2(3 + x_t) + x_n(9 - 4x_t + 3x_t^2))}{6(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n} , \]
\[ D_n'(x_t, x_n) = \frac{x_t[-37 + 44x_t + 17x_t^2 + 6x_n^2(10 - 9x_t + 3x_t^2) - 3x_n(21 - 54x_t + 17x_t^2)]}{36(x_t - 1)^3} \]
\[ + \frac{x_n(2 - 7x_n + 3x_n^2)}{6} \ln \frac{x_n}{1 + x_n} \]
\[ - \frac{(-2 + x_n + 3x_t)[x_t + 3x_t^2 + x_n^2(3 + x_t) - x_n(1 + (-10 + x_t)x_t)]}{6(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n} , \]
\[ E_n'(x_t, x_n) = \frac{x_t[-17 - 8x_t + x_t^2 - 3x_n(21 - 6x_t + x_t^2) - 6x_n^2(10 - 9x_t + 3x_t^2)]}{12(x_t - 1)^3} \]
\[ - \frac{1}{2} x_n(1 + x_n)(-1 + 3x_n) \ln \frac{x_n}{1 + x_n} \]
\[ + \frac{(1 + x_n)[x_t + 3x_t^2 + x_n(3 + x_t) - x_n(1 + (-10 + x_t)x_t)]}{2(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n} . \quad (7) \]

The functions with and without \( x_n \) correspond to the KK excitation and SM contributions, respectively. The summations are carried out using the prescription presented in [9].

After obtaining the Wilson coefficients at the scale \( M_W \), we have to run these coefficients \( C_i(M_W) \) down to the scale \( \mu \sim m_b \). The Wilson coefficients \( C_{i=1\ldots6}(\mu \sim m_b) \) at low energy can be obtained from the corresponding \( C_{i=1\ldots6}(M_W) \) ones by using the Renormalization Group (RG) equation, as discussed in Ref. [12], as

\[ C(\mu) = U_5(\mu, M_W)C(M_W) , \quad (8) \]

where \( C \) is the 6 \( \times \) 1 column vector of the Wilson coefficients and \( U_5(\mu, M_W) \) is the five-flavor 6 \( \times \) 6 evolution matrix. In the next-to-leading order (NLO), \( U_5(\mu, M_W) \) is given by

\[ U_5(\mu, M_W) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}\right)U_5^{(0)}(\mu, M_W) \left(1 - \frac{\alpha_s(M_W)}{4\pi}\right) , \quad (9) \]
where $U^{(0)}_5(\mu, M_W)$ is the leading order (LO) evolution matrix and $J$ denotes the NLO corrections to the evolution. The explicit forms of $U_5(\mu, M_W)$ and $J$ are given in Ref. [12]. For the coefficient $C_7$ we use the RG running as

$$C^{eff}_7(\mu_b) = \eta^{16}_{23} C_7(\mu_W) + \frac{8}{3} \left( \eta^{14}_{23} - \eta^{16}_{23} \right) C_8(\mu_W) + C_2(\mu_W) \sum_{i=1}^{8} h_i \eta^{a_i},$$

(10)

where $\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}$, and the coefficients $a_i$ and $h_i$ are as given in Ref. [9].

The Wilson coefficient $C_9$ in the ACD model and in the NDR scheme is

$$C_9(\mu_b) = P_{0}^{NDR} + \frac{Y(x_t, 1/R)}{\sin^2 \theta_W} - 4Z(x_t, 1/R) + P_{E}E(x_t, 1/R),$$

(11)

where $P_{0}^{NDR} = 2.60 \pm 0.25$ [9] and the last term is numerically negligible. The functions $Y(x_t, 1/R)$ and $Z(x_t, 1/R)$ are defined as:

$$Y(x_t, 1/R) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n),$$

$$Z(x_t, 1/R) = Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n),$$

(12)

with

$$Y_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t - 1)^2} \ln x_t \right],$$

$$Z_0(x_t) = \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t - 1)^3}$$

$$+ \left[ \frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t - 1)^3} - \frac{1}{9} \right] \ln x_t,$$

$$C_n(x_t, x_n) = \frac{x_t}{8(x_t - 1)^2} \left[ x_t^2 - 8x_t + 7 + (3 + 3x_t + 7x_n - x_t x_n) \ln \frac{x_t + x_n}{1 + x_n} \right].$$

(13)

Finally, the Wilson coefficient $C_{10}$, which is scale independent, is given by

$$C_{10} = -\frac{Y(x_t, 1/R)}{\sin^2 \theta_W}.$$
3 Decay rates and forward backward asymmetries

3.1 \( B_s \to \phi l^+l^- \) process

The semileptonic rare \( B \) decays are very well studied in the literature, both in the SM and in various extensions of it. Therefore, we shall simply present here the expressions for the dilepton mass spectrum and the forward backward asymmetry parameters. For the \( B_s \to \phi l^+l^- \) process, the hadronic matrix elements are given as

\[
\langle \phi(k, \epsilon) | (V - A)_\mu | B_s(P) \rangle = \epsilon_{\mu \nu \alpha \beta} \varepsilon^\nu P^\alpha k^\beta \frac{2V(q^2)}{m_B + m_\phi} - i\varepsilon^\mu_0 (m_B + m_\phi) A_1(q^2) \\
+ i(P + k)_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_\phi} + iq_\mu (\varepsilon^* q) \frac{2m_\phi}{q^2} \left[ A_3(q^2) - A_0(q^2) \right],
\]

and

\[
\langle \phi(k, \epsilon) | s\sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B(P) \rangle = i\epsilon_{\mu \nu \alpha \beta} \varepsilon^\nu P^\alpha k^\beta 2T_1(q^2) + \left[ \varepsilon^\mu_0 (m_B^2 - m_\phi^2) - (\varepsilon^\mu q)(P + k)_\mu \right] T_2(q^2) + (\varepsilon^\mu q) q_\mu - \frac{q^2}{m_B^2 - m_\phi^2} (P + k)_\mu \right] T_3(q^2),
\]

where \( V \) and \( A \) denote the vector and axial vector currents, \( A_0, A_1, A_2, A_3, V, T_1, T_2 \) and \( T_3 \) are the relevant form factors and \( m_B \) denotes the mass of \( B_s \) meson. Thus, with eqs. (2) and (16) the transition amplitude for \( B_s \to \phi l^+l^- \) is given as

\[
M(B_s \to \phi l^+l^-) = \frac{G_F \alpha}{2\sqrt{2} \pi} V_{tb} V_{ts}^* \left\{ \bar{l}_\gamma \gamma_\mu \left[ 2A_\epsilon \epsilon_{\mu \nu \alpha \beta} \varepsilon^\nu P^\alpha k^\beta + iB \varepsilon^\mu_0 \right] \\
- iC(P + k)_\mu (\varepsilon^* q) - iD(\varepsilon^* q) q_\mu \right\} + \bar{l}_\gamma \gamma_\mu \gamma_5 \left[ 2E \epsilon_{\mu \nu \alpha \beta} \varepsilon^\nu k^\alpha P^\beta \right. \\
\left. + iF \varepsilon^\mu_0 - iG(\varepsilon^* q)(P + k)_\mu - iH(\varepsilon^* q) q_\mu \right\},
\]

where the parameters \( A, B, \cdots H \) are given as [3]

\[
A = C_9^{eff} \frac{V(q^2)}{m_B + m_\phi} + 4 \frac{m_b}{q^2} C_7^{eff} T_1(q^2), \\
B = (m_B + m_\phi) \left( C_9^{eff} A_1(q^2) + 4 \frac{m_b}{q^2} (m_B - m_\phi) C_7^{eff} T_2(q^2) \right), \\
C = C_9^{eff} \frac{A_2(q^2)}{m_B + m_\phi} + 4 \frac{m_b}{q^2} C_7^{eff} \left( T_2(q^2) + \frac{q^2}{m_B^2 - m_\phi^2} T_3(q^2) \right), \\
D = 2C_9^{eff} \frac{m_\phi}{q^2} \left( A_3(q^2) - A_0(q^2) \right) - 4C_7^{eff} \frac{m_b}{q^2} T_3(q^2), \\
E = C_{10} \frac{V(q^2)}{m_B + m_\phi}, \quad F = C_{10}(m_B + m_\phi) A_1(q^2),
\]

8
Another observable is the lepton forward backward asymmetry ($A_{FB}$), which is also a very powerful tool for looking new physics. The position of the zero value of $A_{FB}$ is very sensitive to the presence of new physics. The normalized forward-backward asymmetry is defined as

$$A_{FB}(s) = \frac{\int_0^1 \frac{d^2\Gamma}{ds d\cos \theta} d\cos \theta - \int_{-1}^0 \frac{d^2\Gamma}{|s\cos \theta|} d\cos \theta}{\int_0^1 \frac{d^2\Gamma}{ds d\cos \theta} d\cos \theta + \int_{-1}^0 \frac{d^2\Gamma}{|s\cos \theta|} d\cos \theta}$$  

$$= \frac{G_F^2 \alpha^2 |V_{tb}V_{ts}^*|^2}{2 \lambda m_B} \frac{8 m_B^2 \lambda v_l^2 s (|Re[B^*E^*] + Re[A^*F^*]|)}{d\Gamma/ds},$$

where $\theta$ is the angle between the directions of $l^+$ and $B_s$ in the rest frame of the lepton pair.

For numerical evaluation we use the form factors calculated in the LCSR [15] approach, where the $q^2$ dependence of various form factors are given by simple fits as

$$f(q^2) = \frac{r_2}{1 - q^2/m_f^2}, \quad \text{for } A_1, T_2$$

$$f(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_f^2}, \quad \text{for } V, A_0, T_1$$

$$f(q^2) = \frac{r_1}{1 - q^2/m_f^2} + \frac{r_2}{(1 - q^2/m_f^2)^2}, \quad \text{for } A_2, \bar{T}_3.$$
The values of the parameters $r_1$, $r_2$, $m_R$ and $m_{fit}$ are taken from [15]. The uncertainties occur in these fitted results are also given in [15]. The form factors $A_3$ and $T_3$ are given as

$$A_3(q^2) = \frac{m_B + m_V}{2m_\phi} A_1(q^2) - \frac{m_B - m_\phi}{2m_\phi} A_2(q^2),$$

$$T_3(q^2) = \frac{m_B^2 - m_\phi^2}{q^2} \left( \tilde{T}_3(q^2) - T_2(q^2) \right).$$

The particle masses, lifetime of $B_s$ meson and the weak mixing angle as $\sin^2 \theta_W$ are taken from [16]. The quark masses (in GeV) used are $m_b=4.6$, $m_c=1.5$ and the CKM matrix elements as $V_{tb}V_{ts}^* = 0.04$.

In order to see the effect due to the uncertainties in the form factors, we plot the differential branching ratio (19) and the forward backward asymmetry (21) for $B_s \rightarrow \phi \mu^+\mu^-$ in Figure-1. It can be seen from the figure that due to the uncertainties in the form factors these distributions deviate slightly from their corresponding central values in the low $s$ region. However, in the large $s$ region these deviations are highly suppressed and the zero position in the forward backward asymmetry is insensitive to these uncertainties. Therefore, we will not consider the effect of these uncertainties for the differential decay distributions and the forward backward asymmetries but we will incorporate their effects in the total decay rates.

![Figure 1: The variation of the differential branching ratio $d\text{Br}/ds$ (in units of $10^{-7}$) and the forward backward asymmetry in the SM with $s$ (in GeV$^2$) for the process $B_s \rightarrow \phi \mu^+\mu^-$. The solid line denotes the result using the central values of the form factors and the dashed lines represent the effects due to the uncertainties in these parameters.](image-url)

Next, to see the effect of the UED, we plot the differential decay rates (19) and the forward backward asymmetries (21) for $B_s \rightarrow \phi l^+l^-$ against $s$, as depicted in Figures 2 and
3. It can be seen from these figures that there is considerable enhancement in the decay rates due to the KK contributions for $1/R = 200$ GeV. The zero position of the forward backward asymmetry $A_{FB}$ shifts towards the left due to the NP effect. This shifting is also more prominent for $1/R = 200$ GeV. Therefore, its experimental determination would constrain the parameter $1/R$.

![Figure 2](image1.png)

**Figure 2:** The variation of the differential branching ratio $d\text{Br}/ds$ (in units of $10^{-7}$) and the forward backward asymmetry with $s$ (in GeV$^2$) for the process $B_s \rightarrow \phi \mu^+ \mu^-$. The solid line denotes the SM result, the dashed line represents the contribution from UED model with $1/R = 200$ GeV and the long-dashed line for $1/R=500$ GeV.

![Figure 3](image2.png)

**Figure 3:** Same as Figure-2 for the process $B_s \rightarrow \phi \tau^+ \tau^-$

We now proceed to calculate the total decay rate for $B_s \rightarrow \phi l^+ l^-$. It should be noted that the long distance contributions arise from the real $\bar{c}c$ resonances with the dominant contributions coming from the low lying resonances $J/\psi$ and $\psi'$. In order to minimize the hadronic uncertainties it is necessary to eliminate the backgrounds coming from the resonance
regions. This can be done by using the following cuts for $B_s \to \phi \mu^+ \mu^-$ process as [17]

$$2.9 \text{ GeV} < m_{\mu^+ \mu^-} < 3.3 \text{ GeV}, \quad \text{and} \quad 3.6 \text{ GeV} < m_{\mu^+ \mu^-} < 3.8 \text{ GeV},$$

and for $B_s \to \phi \tau^+ \tau^-$ process $m_{\psi'} - 0.08 < m_{\tau^+ \tau^-} < m_{\psi'} + 0.08$.

Using these veto windows we obtain the branching ratios for the semileptonic rare $B_s$ decays which are given in Table-1. It is seen from the table that the branching ratios obtained in the ACD model are enhanced from the corresponding SM values for lower $1/R$ value. The variation of the total decay rates with $1/R$ are depicted in Figure-4. Thus, observation of these decay modes can be used to constrain the compactification radius $R$. As seen from the figure for large $1/R$ the standard model results are recovered.

### 3.2 $B_{s}^{0}\rightarrow l^{+}l^{-}\gamma$ process

Now let us consider the radiative dileptonic decay modes $B_s \to l^+l^-\gamma$, which are also very sensitive to the existence of new physics beyond the SM. Due to the presence of the photon
Table 1: The branching ratios for various rare semileptonic (in units of $10^{-7}$) and radiative leptonic (in units of $10^{-9}$) $B_s$ decays, where $\text{Br}_\text{SM}$ represents the SM branching ratio and $\text{Br}|_{1/R=200}$ is the branching ratio with KK contributions for $1/R = 200$ GeV and $\text{Br}|_{1/R=500}$ for $1/R = 500$ GeV. The errors are due to the uncertainties in the form factors.

| Decay process | $\text{Br}_\text{SM}$ | $\text{Br}|_{1/R=200}$ | $\text{Br}|_{1/R=500}$ | Expt.[4] |
|---------------|-------------------------|-------------------------|-------------------------|---------|
| $B_s \to \phi \mu^+ \mu^-$ | 15.29 ± 0.95 | 20.82 ± 1.06 | 16.01 ± 0.99 | < 41 |
| $B_s \to \phi \tau^+ \tau^-$ | 0.87±0.02 | 1.53 ± 0.03 | 0.98±0.025 | - |
| $B_s \to \mu^+ \mu^- \gamma$ | 3.05±0.50 | 4.45±0.69 | 3.27±0.53 | - |
| $B_s \to \tau^+ \tau^- \gamma$ | 13.93±0.09 | 24.93±1.14 | 15.77 ± 0.10 | - |

in the final state, these decay modes are free from helicity suppression, but they are further suppressed by a factor of $\alpha$. However, in spite of this $\alpha$ suppression, the radiative leptonic decays $B_s \to l^+ l^- \gamma$, $l = (\mu, \tau)$ have comparable decay rates to that of purely leptonic ones.

The matrix element for the decay $B_s \to l^+ l^- \gamma$ can be obtained from that of the $B_s \to l^+ l^-$ one by attaching the photon line to any of the charged external fermion lines. In order to calculate the amplitude, when the photon is radiated from the initial fermions (structure dependent (SD) part), we need to evaluate the matrix elements of the quark currents present in (2) between the emitted photon and the initial $B_s$ meson. These matrix elements can be obtained by considering the transition of a $B_s$ meson to a virtual photon with momentum $k$. In this case the form factors depend on two variables, i.e., $k^2$ (the photon virtuality) and the square of momentum transfer $q^2 = (p_B - k)^2$. By imposing gauge invariance, one can obtain several relations among the form factors at $k^2 = 0$. These relations can be used to reduce the number of independent form factors for the transition of the $B_s$ meson to a real photon. Thus, the matrix elements for $B_s \to \gamma$ transition, induced by vector, axial-vector, tensor and pseudo-tensor currents can be parametrized as [18]

\[
\langle \gamma(k, \varepsilon) | \bar{s} \gamma_\mu \gamma_5 b | B_s(p_B) \rangle = \left[ \frac{\varepsilon^\ast (p_B \cdot k) - (\varepsilon^* \cdot p_B) k_\mu}{m_B} \right] \frac{F_A}{m_B}, \\
\langle \gamma(k, \varepsilon) | \bar{s} \gamma_\mu b | B_s(p_B) \rangle = \varepsilon_{\mu \rho \beta} \varepsilon_{\ast \nu} p_B^\rho k_\beta \frac{F_V}{m_B}, \\
\langle \gamma(k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q^\ast \gamma_5 b | B_s(p_B) \rangle = \varepsilon_{\mu} (p_B \cdot k) - (\varepsilon^* \cdot p_B) k_\mu \left[ \frac{F_{TA}}{m_B} \right], \\
\langle \gamma(k, \varepsilon) | \bar{s} \sigma_{\mu \nu} q^\ast b | B_s(p_B) \rangle = \varepsilon_{\mu \rho \beta} \varepsilon_{\ast \nu} p_B^\rho k_\beta F_{TV},
\]

(24)
where $\epsilon$ and $k$ are the polarization vector and the four-momentum of photon, $p_B$ is the momentum of initial $B_s$ meson and $F_i$’s are the various form factors.

Thus, the matrix element describing the SD part takes the form

$$\mathcal{M}_{SD} = \frac{\alpha^3 G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^* \left\{ \epsilon^{\mu\nu\alpha\beta} F^\nu_{P_B} k^\beta \left( A_1 \bar{l}\gamma^\mu l + A_2 \bar{l}\gamma^\mu\gamma_5 l \right) ight\} ,$$

where

$$A_1 = 2C_7 \frac{m_b}{q^2} F_{TV} + C_9 \frac{F_V}{m_B} , \quad A_2 = C_{10} \frac{F_V}{m_B} ,$$

$$B_1 = -2C_7 \frac{m_b}{q^2} F_{TA} - C_9 \frac{F_A}{m_B} , \quad B_2 = -C_{10} \frac{F_A}{m_B} .$$

The form factors $F_V$ and $F_A$ have been calculated within the dispersion approach [19]. The $q^2$ dependence of the form factors are given as [18]

$$F(E_\gamma) = \frac{\beta f_{B_s} m_B}{\Delta + E_\gamma} ,$$

where $E_\gamma$ is the photon energy, which is related to the momentum transfer $q^2$ as

$$E_\gamma = \frac{m_B}{2} \left( 1 - \frac{q^2}{m_B^2} \right) .$$

The values of the parameters $\beta$ and $\Delta$ are given in Table-2. The same ansatz (27) has also been assumed for the form factors $F_{TA}$ and $F_{TV}$. The decay constant of the $B_s$ meson is not yet well known because the pure leptonic decays of $B_s$ meson (i.e., $B_s \to l^+l^-$) from which it could be extracted are highly suppressed in the SM as they occur at one one-loop level. Using QCD Sum rule approach, its value is found to be $f_{B_s} = 236 \pm 30$ MeV [20] and $f_{B_s} = 244 \pm 21$ MeV [21]. The Lattice QCD result gives $f_{B_s} = 242 \pm 9 \pm 34$ MeV [22]. Therefore, in this analysis we use the value $f_{B_s} = 240$ MeV.

When the photon is radiated from the outgoing lepton pairs, the internal bremsstrahlung (IB) part, the matrix element is given as

$$\mathcal{M}_{IB} = \frac{\alpha^3 G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^* f_{B_s} m_l C_{10} \left[ \bar{l} \left( \frac{\not{p}_B \not{p}_l}{p_+ \cdot k} - \frac{\not{p}_B \not{p}_l}{p_- \cdot k} \right) \gamma_5 l \right] .$$

Thus, the total matrix element for the $B_s \to l^+l^-\gamma$ process is given as

$$\mathcal{M} = \mathcal{M}_{SD} + \mathcal{M}_{IB} .$$
Table 2: The parameters for $B \rightarrow \gamma$ form factors.

| Parameter | $F_V$ | $F_{TV}$ | $F_A$ | $F_{TA}$ |
|-----------|-------|---------|-------|---------|
| $\beta$(GeV$^{-1}$) | 0.28 | 0.30 | 0.26 | 0.33 |
| $\Delta$(GeV) | 0.04 | 0.04 | 0.30 | 0.30 |

The differential decay width of the $B \rightarrow l^+l^-\gamma$ process, in the rest frame of $B_s$ meson is given as

$$
\frac{d\Gamma}{ds} = \frac{G_F^2 \alpha^3}{2^{10} \pi^4} |V_{tb}V_{ts}^*|^2 m_B^3 \Delta_1,
$$

where

$$
\Delta_1 = \frac{4}{3} m_B^2 (1 - \hat{s})^2 v_l \left( (\hat{s} + 2 r_l) (|A_1|^2 + |B_1|^2) + (\hat{s} - 4 r_l) (|A_2|^2 + |B_2|^2) \right)
- 64 \frac{f_{B_s}^2}{m_{B_s}^2} \frac{r_l}{1 - \hat{s}} C_{10}^2 \left( (4 r_l - \hat{s}^2 - 1) \ln \frac{1 + v_l}{1 - v_l} + 2 \hat{s} v_l \right)
- 32 r_l (1 - \hat{s})^2 f_{B_s} \Re(C_{10} A_1^*),
$$

with $s = q^2$, $\hat{s} = s/m_B^2$, $r_l = m_l^2/m_B^2$, $v_l = \sqrt{1 - 4m_l^2/q^2}$. The physical region of $s$ is $4m_l^2 \leq s \leq m_B^2$.

The forward backward asymmetry is given as

$$
A_{FB} = \frac{1}{\Delta_1} \left[ \frac{2 m_B^2 \hat{s}(1 - \hat{s})^3 v_l^2}{\hat{s}} \Re(A_1^* B_2 + B_1^* A_2) \right.
+ 32 f_{B_s} r_l (1 - \hat{s})^2 \ln \left( \frac{4 r_l}{\hat{s}} \right) \Re(C_{10} B_2^*) \right].
$$

Again to visualize the effects due to the uncertainties in the form factors (assuming it to at the level of 10%) we first plot the differential decay distribution (31), and the forward backward asymmetry (33) for $B_s \rightarrow \mu^+\mu^-\gamma$ in Figure-5. From the figure it can be seen that these uncertainties can affect only the differential decay rate but not the forward backward asymmetry. Furthermore, it is found that these effects are significant only for $B_s \rightarrow \mu^+\mu^-\gamma$ and not for $B_s \rightarrow \tau^+\tau^-\gamma$. Next, we would like to see the effect of UED in these differential distributions. For this purpose, we consider only the central values of the form factors, $f_{B_s} = 0.24$ GeV, $\alpha = 1/128$ and plot plot the dilepton mass spectrum (31), and the forward backward asymmetries (33) for $B_s \rightarrow l^+l^-\gamma$ decays which are shown in Figures-6 and 7.
these figures, we see that the branching ratio for $B_s \to l^+l^-\gamma$ enhanced significantly from their corresponding SM values. However, the forward backward asymmetry is reduced slightly from the corresponding SM value for the $B_s \to \mu^+\mu^-\gamma$ process and there is a backward shifting of the zero position. For $B_s \to \tau^+\tau^-\gamma$ process there is no significant change in the asymmetry distribution.

![Figure 5](image_url)

Figure 5: The differential branching ratio (31) (in units of $10^{-9}$) and the forward backward asymmetry ($A_{FB}$) for the process $B_s \to \mu^+\mu^-\gamma$ in the standard model where the solid line represents the central value and the dashed lines represent the uncertainties due to the form factors.

To obtain the branching ratios it is necessary to eliminate the background due to the resonances $J/\psi(\psi')$ with $J/\psi(\psi') \to l^+l^-$. We use the following veto windows to eliminate these backgrounds

$$B_s \to \mu^+\mu^-\gamma : \quad m_{J/\psi} - 0.02 < m_{\mu^+\mu^-} < m_{J/\psi} + 0.02;$$

$$B_s \to \tau^+\tau^-\gamma : \quad m_{\psi'} - 0.02 < m_{\tau^+\tau^-} < m_{\psi'} + 0.02.$$

Furthermore, it should be noted that the $|M_{IB}|^2$ has infrared singularity due to the emission of soft photon. Therefore, to obtain the branching ratio, we impose a cut on the photon energy, which will correspond to the experimental cut imposed on the minimum energy for the detectable photon. Requiring the photon energy to be larger than 25 MeV, i.e., $E_\gamma \geq \delta m_{B_s}/2$, which corresponds to $s \leq m^2_{B_s}(1 - \delta)$, and therefore, we set the cut $\delta \geq 0.01$.

Thus, with the above defined veto windows and the infrared cutoff parameter, we obtain the branching ratios as shown in Table-1 which are enhanced from their SM values. It should be mentioned that the $B^0_s \to \tau^+\tau^-\gamma$ could be observable in the Run II of Tevatron.
4 Conclusion

It is now widely believed that the scenario with extra dimensions is a strong contender to reveal physics beyond the SM and has received considerable attention in the literature. In view of this anticipation it is also worthwhile to study its implications in the b-sector. In this paper, therefore, we have studied the rare semileptonic decay mode $B_s \rightarrow \phi \ell^+\ell^-$ and the radiative leptonic $B_s \rightarrow \ell^+\ell^-\gamma$ in the model with a single extra dimension. The branching ratios for various decay modes are found to be larger than their corresponding SM values but they lie within the present experimental upper limit. Furthermore, the zero point of the forward backward asymmetries for the decays under consideration are shifted to the left and the change is found to be sensitive to the inverse compactification radius. In future, with more intensive data, the UED scenario will be subjected to more stringent tests and in turn will enrich us with a better understanding of the flavor sector.

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Figure 7: Same as Figure-7 for the process $B_s \to \tau^+\tau^-\gamma$. The additional long-dashed line is for $1/R=500$ GeV.

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