Measurement-induced nonlocality in the anisotropic Heisenberg chain

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Quantum correlations are essential for quantum information processing. Measurement-induced nonlocality (MIN) which is defined based on the projective measurement is a good measure of quantum correlation, and is favored for its potential applications. We investigate here behaviors of the geometric and entropic MIN in the two-qubit Heisenberg XY chain, and reveal effects of the anisotropic parameter $\gamma$ as well as the external magnetic field $B$ on strength of them. Our results show that both $\gamma$ and $B$ can serve as efficient controlling parameters for tuning the MIN in the XY chain.

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I. INTRODUCTION

Quantum correlations plays an important role for quantum protocols which outperform those of the classical one. The typical protocols are those proposed in the literature, such as the quantum cryptography [1], teleportation [2–4], and quantum computation [5], which depend crucially on the entanglement between the considered system. While quantum entanglement has been recognized as crucial for various quantum information processing tasks, recent investigations show that it is in fact a special kind of quantum correlation measure, and there are other forms of quantum correlations which are more fundamental than that of entanglement [6]. The typical and seminal one are the concept of quantum discord proposed by Ollivier and Zurek [7], and Henderson and Vedral [8], which can exist even for the separable states, and is considered to be responsible for the power of the deterministic quantum computation with one qubit [9–11]. Due to these reasons and its fundamentals in quantum mechanics [12–15], great efforts has been devoted to this attractive field in the past few years.

The quantum discord, although is favored for its operational interpretation and possible application, is very hard to calculate [12]. To avoid this problem, Luo and Fu [17, 18] introduced a geometric description of quantum correlation which they termed as measurement-induced nonlocality (MIN). As its name implies, the MIN can also be considered as a measure of nonlocality for it is defined via the maximal global disturbance of the locally invariant measurements, and it is in some sense more general than that of the Bell-type nonlocality. Moreover, the geometric MIN can also be evaluated analytically for the two-qubit states.

Besides the geometric MIN, an entropic measure of MIN based on the von Neumann entropy was introduced recently [19], which is equivalent to that defined via the relative entropy [20]. Although its calculation is harder than that of the geometric one, it was favored for its definite physical meaning, which can be interpreted as the maximal increment of one’s uncertainty about a system after his local invariant measurements on one subsystem.

In this work, we study effects of the anisotropic parameter and the external magnetic field on the above two scenarios of MIN for a Heisenberg XY chain. The structure of this paper is arranged as follows. In Section II, we recall the concept of the geometric and entropic MIN, and the model of the XY chain we considered. Then in Section III we examine behaviors of the MIN in the XY chain with different system parameters. We summarize the main finding of this work in Section IV.

II. BASIC FORMALISM FOR MIN AND THE MODEL

Here we first recall the concept of MIN. In its original geometric quantification scheme [18], it was defined by the square of the maximum of the Schatten 2-norm between two states $\rho$ and $\Pi^A(\rho)$, which corresponds to the bipartite states before and after the projective measurements. To be explicitly, we can write it as follows

$$ N^s(\rho) = \max_{\Pi^A} ||\rho - \Pi^A(\rho)||^2, \tag{1} $$

with the maximum being taken over the local projective measurements $\Pi^A = \{\Pi^A_k\}$ on subsystem $A$, which satisfy the condition $\sum_k \Pi^A_k \rho \Pi^A_k = \rho^A$. Moreover, $||X||$ is the Schatten 2-norm where $||X||^2 = \text{Tr}(X^\dagger X)$.

For the $2 \times n$ dimensional states, solutions of the geometric MIN can be derived analytically. Here for convenience of representation, we do not list its explicit expressions, and the readers who are interested in it can resort to the Ref. [18] for more detail.

Different from $N^s(\rho)$ which measures the MIN from a geometric perspective, the entropic measure of MIN is defined as follows [19]

$$ N^v(\rho) = I(\rho) - \min_{\Pi^A} I[\Pi^A(\rho)], \tag{2} $$

where $I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho)$ denotes the quantum mutual information, and the minimum is taken over the same projective measurements as that in the definition of $N^s(\rho)$. This MIN quantifies the maximal loss of total correlations under non-disturbing local measurements, and it is equivalent to $\max_{\Pi^A} S[\Pi^A(\rho)] - S(\rho)$ because of the definition of $\Pi^A$ which leaves $\rho^A$ invariant.
The evaluation of $N^v(\rho)$ is also difficult due to the optimization processor involved. But for the special case that the bipartite states $\rho$ having nondegenerate $\rho^A$, the optimal $\Pi^A$ for obtaining the entropic MIN correspond to that of the spectral resolutions of $\rho^A = \sum_k p_k \Pi^A_k$, and thus $N^v(\rho)$ can be evaluated easily.

After review of the basic formalism for MIN, we present the model we considered in this paper. This is the usual two-spin Heisenberg XY chain with uniform external magnetic field applies along the $z$ direction. The corresponding Hamiltonian is given by [21]

$$\hat{H} = J(S_1^+ S_2^- + S_1^- S_2^+) + J\gamma(S_1^+ S_2^+ + S_1^- S_2^-) + B(S_1^z + S_2^z),$$

where $B$ denotes the strength of the external magnetic field, and $\gamma$ measures the anisotropy of the system which reduces to the XX model for $\gamma = 0$, and the Ising model for $\gamma = 1$.

After a straightforward algebra, the eigenvalues and eigenvectors of $\hat{H}$ can be derived analytically as

$$\varepsilon_{1,2} = \pm J, \varepsilon_{3,4} = \pm \eta,$$

and

$$|\psi\rangle_{1,2} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle),$$

$$|\psi\rangle_{3,4} = u^\pm(|00\rangle \pm \frac{J\gamma}{\eta \mp B}|11\rangle),$$

where $\eta = \sqrt{B^2 + J^2\gamma^2}$, and the normalization parameter $u^\pm = (\eta \mp B)/\sqrt{J^2\gamma^2 + (\eta \mp B)^2}$.

Based on the above eigenvalues and eigenvectors, we can now evaluate the MIN in the XY chain, and for this purpose, we need to introduce the notion of the thermal state $\rho = Z^{-1}\exp(-\hat{H}/k_B T)$, with $Z = \text{Tr}[\exp(-\hat{H}/k_B T)]$ being the partition function, $k_B$ is the Boltzmann constant, and $T$ denotes the temperature. When $T = 0$, $\rho$ reduces to the so-called ground state.

### III. MIN IN THE HEISENBERG XY CHAIN

We now begin our discussion about behaviors of MIN in the XY chain. To be explicitly, we will evaluate influence of the anisotropic parameter $\gamma$, strength of the external magnetic field $B$, and the temperature $T$ on MIN. We first consider the scenario of the geometric MIN. In Fig. 1 we gave three-dimensional plots of the geometric MIN $N^v(\rho)$ as functions of $B$ and $k_B T$ for various values of $\gamma$. When considering the case of ground states at absolute zero temperature, one can see clearly that for the isotropic XY chain (i.e., $\gamma = 0$, for which it is also termed as the XX chain), $N^v(\rho)$ keeps the constant value of 0.5 before the critical point $B_c = J\sqrt{1 - \gamma^2}$, and disappears suddenly when $B > B_c$. This sudden death behavior occurs only for the case of $\gamma = 0$, and for finite $\gamma$ as displayed in Fig. 1(b), $N^v(\rho)$ maintains the constant value.
0.5 before $B_c$, and decreases exponentially to zero in the infinite limit of $B$. But when the anisotropic parameter $\gamma$ exceeds a critical point $\gamma_c$, such as that displayed in Fig. 1(c) with $\gamma = 0.8$, $N^s(\rho)$ initially keeps the maximum value 0.5, and then decreases suddenly to a finite value $N^s(\rho) < 0.5$ at $B = B_c$, after which it revivals and then decreases gradually to zero. This is one of the main difference between $N^s(\rho)$ in the regions of $\gamma < \gamma_c$ and $\gamma > \gamma_c$, which implies that the anisotropic parameter $\gamma$ can serve as an efficient parameter for tuning magnitudes of the MIN. Finally, we displayed in Fig. 1(d) behaviors of $N^s(\rho)$ for the special case of $\gamma = 1$ which corresponds to the Ising chain. Different from those for $\gamma < 1$, here $N^s(\rho)$ decreases gradually from its maximum 1 to zero with increasing magnetic field $B$.

For thermal states with finite temperatures, one may expects that the MIN $N^s(\rho)$ may be decreased by increasing the temperature, and this is indeed the case for weak magnetic fields. But there is a peculiar phenomenon needs to be pay attention to. As can be observed from Fig. 1(a) and (b), the MIN may be increased by increasing temperature in the region of relative strong external magnetic field. This shows again the combined effects of temperature and the anisotropic parameter on the geometric MIN.

From the above discussion, one can see that the geometric MIN $N^s(\rho)$ is sensitive to the parameter $\gamma$ and $B$. We now turn to investigate behaviors of the entropic MIN in the XY chain. The exemplified plots are displayed in Fig. 2 where the entropic MIN $N^v(\rho)$ are plotted as functions of magnetic field $B$ and the scaled temperature $k_B T$ with different anisotropic parameters $\gamma$. From these plots one can observe that for relative small values of $\gamma$, e.g., the case of $\gamma = 0$ as shown in Fig. 2(a) and $\gamma = 0.5$ as shown in Fig. 2(b), $N^v(\rho)$ exhibit similar behaviors as those for $N^s(\rho)$ with the same parameters [see, Fig. 1(a) and (b)], and the only difference is that the constant value for $N^v(\rho)$ is twice that of $N^s(\rho)$. For large values of $\gamma$, however, $N^v(\rho)$ and $N^s(\rho)$ exhibit completely different behaviors, which is more evident in the low temperature region. As can be seen from Fig. 2(c) and (d), $N^v(\rho)$ is an non-increasing function of $B$ for any fixed $k_B T$, and there are no revival of $N^v(\rho)$, even at zero absolute temperature. This is the first difference between geometric and entropic description of MIN in the Heisenberg XY chain. Besides this, one can also note that the MIN $N^v(\rho)$ can be increased by increasing temperature in the whole region of $\gamma$ with appropriate chosen magnetic fields, and this constitutes another main difference between $N^v(\rho)$ and $N^s(\rho)$ in describing the nonlocal property of the Heisenberg XY chain.

Before ending this section, we would like to point out that here the different behaviors between $N^v(\rho)$ and $N^s(\rho)$ for the Heisenberg XY chain is caused by the different metric adopted for defining the MIN. This reveals the relativity of different quantum correlation measures in quantifying a specific quantum system \cite{14}, and the searching of an universal scheme for quantifying correlations remains an open problem.
IV. SUMMARY AND DISCUSSION

To summarize, we have studied the influence of the anisotropic parameter $\gamma$ and strength of the external magnetic field $B$ on behaviors of the MIN. Through detailed calculation and comparison, we showed that both of them can serve as efficient parameters for controlling magnitudes of MIN in the Heisenberg XY chain. We also showed that the geometric measure of MIN and the entropic measure of MIN may correspond to completely different behaviors of nonlocal correlations, and we interpret this phenomenon as the different metric adopted in their respective definition.

As the MIN is a well-defined measure of nonlocal correlations, and investigations of its properties in different quantum systems will provide us with useful information which includes its controlling methods, its potential applications and other aspects related to the fundamental problem of quantum mechanics, we hope our work presented here may shed light on understanding MIN for the quantum spin system, and stimulate related investigations in this field.

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