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**Determination of Parameters of Selected Mathematical Models of Arc in Circuits with Actual Energy Sources**

**Abstract:** The article provides general justification concerned with the performance of simulation tests of processes in circuits. The simulation tests aimed to identify the effectiveness related to the use of modified integral methods used in the experimental determination of parameters of arc mathematical models. In addition, the article specifies conditions for the performance of experiments, i.e. involving the numerical or physical elimination of near-electrode arc voltage drops. The research described in the article also involved the modification of integral dependences into HL-type (hyperbolic-linear) parameters of the Mayr and Pentegov models by providing the above-named dependences with corrective functions. The performance of simulations of processes in circuits with arcs and with actual energy sources (with a variable parameter, i.e. internal inductance) generating sinusoidal current or voltage waves, resulted in the obtaining of diagrams of the family of correction factor function. Those of the above-named functions which significantly differed from unity were subjected to approximation.

**Keywords:** electric arc, Mayr model, Pentegov model, integral methods

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**Introduction**

Since the beginning of the 20th century, the development of the practical use of electric arc has been accompanied by the development of theoretical and experimental tests of plasma in numerous areas of physics, electrical engineering, mechanics, chemistry, materials engineering etc. The adoption of appropriate simplifying assumptions enabled the creation of various mathematical models of electric arc. The above-named assumptions entail various ranges of the use of arc models when modelling electric processes in various operating states of a vast range of electric equipment and appliances. The scientific and engineering value of these models depends not only on the precise description of primary energy transformations in the plasma column, but also on the ease and availability of methods enabling the experimental determination of parameters and functions of mathematical models. In turn, the quality of the above-named parameters and functions affects the quality (e.g. stability) of the functioning of macromodels in simulation programmes capable of describing processes occurring in circuits with arcs. The development of measurement...
and computational techniques was accompanied by advances in the development of new methods enabling the determination of traditional parameters and new mathematical models. Previously known graphic methods [1] have been replaced with numerical methods, yet without abandoning the highly subjective selection of fragments of waveforms in time and characteristic points on such waveforms [2, 3]. Newly developed measurement methods described in publications [4-6] are used for the experimental determination of parameters of selected mathematical models of arc with sinusoidal or rectangular current excitation (internal admittance of such a source $Y_w = 0 \, S$) and with relatively low levels of disturbance both in the supply source and in the arc column. The use of values of spectral or integral (mean and root-mean-square) waveforms of current, power and voltage in the column enables the automatic determination of parameters of mathematical models, which, to a significant extent, eliminates the subjective selection of data necessary in analysis.

In industrial practice and in power engineering, the burning of electric arc occurs under conditions of supply by sources of electric energy having various, usually non-linear characteristics. Because of economic and technological reasons, sources of current having characteristics in some fragments close to ideal are used to power arcs of relatively low power (e.g. in welding engineering). High-power arcs, used in electrometallurgy, chemistry and ecology as well as arcs accompanying joining processes and failures in power engineering are usually powered by actual linear (current or equivalent voltage) energy sources [7].

The selection of types of supply sources depends on the shape of static and dynamic characteristics of given arcs. Because of the fact that in most cases the above-named characteristics are more or less steeply declined, to ensure discharging stability it is usually sufficient to apply an actual source close to the ideal current source [8]. In some cases, voltage-current characteristics within the high-current range may be rising (depending on the size and shape of electrodes as well as on the type and pressure of gas etc.) In such situations, the stable burning of arc requires the use of an actual source close to the ideal voltage source.

The article describes an attempted extension of usability ranges of known analytical dependences [5] enabling the determination of arc mathematical models to include cases of the use of actual voltage ($Z_w = X_L = \omega L > 0 \, \Omega$) and current ($Y_w = 1/Z_w < \infty \, S$) sources generating sinusoidal waveforms only in conditions without loading by arc (voltage source – no-load state, current source – short circuit state). To this end, it was necessary to apply appropriate correction functions depending on parameters of source internal impedance (admittance).

**Determination of Parameters of Selected Mathematical Models of the Column of Arc Powered by the Ideal Current Source of Sinusoidal Waveform**

The initial stage of experimental tests and the modelling of electric arc involved the determination of values of near-electrode voltage drops, usually using various direct or indirect methods [9]. Once the values of the above-named voltage drops are known, it is possible to eliminate their effect on plasma column characteristics. For this purpose, an appropriate electronic compensator can be used [5]. In cases of measurement systems including a computer, it is comfortable to use the computational method of the compensation of near-electrode voltage drops and obtain a momentary voltage drop only on the column of arc. In cases of symmetric arcs, near-electrode voltage drops do not depend on the direction of current flow ($U_{K1} = U_{K2}$, $U_{A1} = U_{A2}$) and $U_{AK1} = U_{K1} + U_{A1} = U_{K2} + U_{A2}$. The drop of voltage on the column of such arc amounts to

$$U_{col} = u - U_{AK} \, \text{sgn} \, i$$
If arc is asymmetric, \((U_{K1} \neq U_{K2}, U_{A1} \neq U_{A2})\) and \(U_{AK1} = U_{K1} + U_{A1} \neq U_{AK2} = U_{K2} + U_{A2}\). As a result, the drop of voltage on the column of such arc amounts to

\[
u_{col} = \begin{cases} u - U_{AK1}, & \text{if } i \geq 0 \\ u + U_{AK2}, & \text{if } i < 0 \end{cases}
\]

(2),

where \(U_{AK}\) – sum of near-electrode voltage drops in a circuit with symmetric arc; \(U_{AK1}, U_{AK2}\) – sums of near-electrode voltage drops in a circuit with asymmetric arc, depending on the direction of current flow; \(u\) – momentary value of arc voltage. Value \(u_{col}\) constitutes the basing for calculating:

- mean value of voltage on the column of arc \(U_{av}\)

\[
U_{av} = \frac{1}{T} \int_{-T/2}^{+T/2} u_{col} dt
\]

(3),

- root-mean-square voltage on the column of arc \(U_{rms}\)

\[
U_{rms} = \sqrt{\frac{1}{T} \int_{-T/2}^{+T/2} u_{col}^2 dt}
\]

(4),

- active power of the column \(P_{col}\)

\[
P_{col} = \frac{1}{T} \int_{-T/2}^{+T/2} u_{col} idt
\]

(5),

- mean value of the column resistance \(R_{col}\)

\[
R_{col} = \frac{1}{T} \int_{-T/2}^{+T/2} u_{col} \frac{idt}{i}
\]

(6),

Pentegov-HL has a voltage-current static hyperbolic-linear characteristic [6, 10, 11]. As a result, the Pentegov-HL model is capable of approximating characteristics of selected arcs in their entire ranges and can be particularly useful when modelling arcs in gases under high pressure and burning between pointed electrodes [12].

Table 1 contains formulas enabling the identification of their parameters on the basis of measurements in a circuit with the source of sinusoidal current [5].

| Model         | Equation of model | Static characteristic | Parameters identified using the integral method |
|---------------|-------------------|-----------------------|-----------------------------------------------|
| Mayr (7)      | \(\theta_p \frac{dg}{dt} + g = i^2\) \(P_{col}\) \(U_{col} = \frac{P_{M}}{I}\) | \(U_{st}\) \(U_{col} = \frac{P_{M}}{I}\) | \(\theta_M = \frac{1}{2\omega \left(\frac{U_{rms} I_{rms}}{P_{col}}\right)^4 - 1}\) |
| Pentegov-HL (8) | \(\theta_p \frac{di_p^2}{dt} + i_p^2 = i^2\) | \(u = R_p (i_p^2) = U_{st} \frac{R_p}{I}\) | \(\theta_R = \frac{1}{2\omega \left(\frac{U_{rms} I_{rms}}{R_{col}}\right)^2}\) |

In the Mayr model the following designations were used: \(U_{col}\) – static voltage on the column of arc; \(I\) – DC; \(g\) – electric conductance; \(P_{M}\) – constant value of the Mayr model power; \(\theta_M\) – time constant of the Mayr model, \(U_{rms}\) – root-mean-square voltage on the column of arc; \(I_{rms}\) – root-mean-square current; \(P_{col}\) – mean value of the column temporary power (active power). The determination of the Mayr model parameters involves the use of root-mean-square current \(I_{rms}\), root-mean-square voltage \(U_{rms}\) and the mean value of momentary electric power \(P_{col}\).

In the Pentegov-HL, the parameters of power \(P_{MR}\) and resistance \(R_p\) are determined using the integral method, root-mean-square voltage and current as well as the mean values of power and resistance (Table 1).
Determination of Parameters of Mathematical Models of the Column of Arc Powered by Actual Energy Sources

In simulated electrical processes in a circuit with actual supply sources, the value of current flowing through the macromodel of arc depends, among other things, on voltage $U_{AK}$. In cases of short arc (having a low value of voltage drop on the column) the use of an electronic compensator \cite{5} significantly changes the conditions of arc burning. For this reason, the programme-based compensation of voltage $U_{AK}$ should be preferred. In turn, long arc corresponds to high values of voltage drop on the column. As a result, $U_{AK}$ is usually ignored. However, even in the above-presented case the use of programme-based compensation may improve the accuracy of the determination of arc model parameters.

In spite of the use of an energy source, the current of which in the short circuit state is sinusoidal, after being loaded with electric arc having a non-linear conductance and connected in parallel to the linear admittance of the source, both the voltage and current of arc become distorted. Likewise, in spite of the use of an energy source, the voltage of which in the short circuit state is sinusoidal, after being loaded with a serial branch containing electric arc having non-linear resistance and linear source impedance, both the voltage and current of arc become distorted, which, in turn, infringes on the assumptions of the analytical methods, the use of which enabled the obtainment of dependences (7) and (8).

Table 2 contains formulas used to determine correction functions and parameters of selected mathematical models of arc. Their values were identified in a simulation manner as this method enables the precise setting of arc and supply

$$THD_I = \sqrt{\frac{\sum_{k=2}^{\infty} I_k^2}{I_i}}$$

$$THD_U = \sqrt{\frac{\sum_{k=2}^{\infty} U_k^2}{U_i}}$$

where $I_k$ – numbers of current harmonics; $U_k$ – numbers of voltage harmonics. Particularly important is the low value of $THD_I$ as it enables the satisfaction of the assumption of analytical methods used to obtain the primary dependences (7) and (8).

The first stage of simulation tests involved the use of an actual current source generating a sinusoidal waveform having frequency $f = 50$ Hz, root-mean-square short circuit current $I_{sr}$ and parallel inductive susceptance $B_L = (\omega L)^{-1}$ formed by ideal inductance $L$. This is equivalent to the actual voltage source having the difference of potentials on terminals in the no-load state $E_{sr} = jX_L L_{sr} = j\omega L L_{sr}$ and serial reactance $X_L = \omega L$. The tests involved the gradual decrease in the value of parallel inductance until the loss of the stability of processes in the parallel (arc termination) circuit.

The second stage of the simulation tests involved the use of an actual voltage source generating a sinusoidal waveform having frequency $f = 50$ Hz, root-mean-square no-load voltage $E_{sr}$ and serial inductive reactance $X_L = \omega L$ formed by ideal inductance $L$. Such a serial branch is equivalent to the actual current source having short circuit current efficiency $I_{sr} = -j B_L E_{sr} = -j (\omega L)^{-1} E_{sr}$ connected with parallel susceptance $B_L = (\omega L)^{-1}$. The tests involved the gradual decrease in the value of serial inductance until the obtainment of zero or until the loss of the stability of processes in the serial (arc termination) circuit.

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source characteristics without the influence of external disturbance factors.

Figures 1 and 2 present the results of the tests involving processes taking place in the circuit with arc powered by the actual current source. As can be seen, the power of arc only slightly affects correction functions located close to one another. Functions $F_{PM}(L,\theta_{MZ},P_{MZ})$ change only slightly and in the range under consideration their values can be adopted as amounting to 1. The family of function $F_{\theta M}$ can be approximated using the following power dependence

$$F_{\theta M}(L,\theta_{MZ},P_{MZ})=F_{\theta M}(L)=0.806L^{0.0406}$$

(13), where $L$ – inductance in mH. In cases of inductance $L > 100$ mH, the above-named function is close to the horizontal line and its value can be adopted as amounting to 1.

It is possible to observe a certain correlation between the ranges of changes in $L$ of the usability of the approximation of correction functions using a value of 1 and the ranges of quasi-horizontal diagrams of total harmonic distortion, where $THD_{IM}$ values are below 10%, whereas $THD_{UM}$ values are above 75%.

Table 2. Mathematical models of the electric arc column and formulas enabling the measurement-based identification of their parameters [6] ($\theta_{MZ}, \theta_{RZ}$ – preset values of time constants; $P_{MZ}, P_{RZ}$ – preset power value; $R_{PZ}$ – preset resistance value)

| Model | Parameters of arc powered by the actual energy source identified using the integral method | Correction functions $F(Z)$ |
|-------|--------------------------------------------------------------------------------------------|----------------------------|
| Mayr (11) | $\theta_{MZ} = \frac{F_{PM}(Z)}{2\omega \sqrt{\left(\frac{U_{rms}I_{rms}}{P_{col}}\right)^4} - 1}$ | $F_{M}(Z) = \theta_{MZ} 2\omega \sqrt{\left(\frac{U_{rms}I_{rms}}{P_{col}}\right)^4} - 1$ |
| | $P_{MZ} = F_{PM}(Z)\frac{P_{col}}{P_{col}}$ | $F_{PM}(Z) = \frac{P_{MZ}}{P_{col}}$ |
| Pentegov-HL (12) | $\theta_{RZ} = \frac{F_{MR}(Z)}{2\omega \sqrt{\left(\frac{(R_{col} - R_{p})U_{rms}I_{rms}}{P_{MR}}\right)^2} - 1}$ | $F_{MR}(Z) = \theta_{RZ} 2\omega \sqrt{\left(\frac{(R_{col} - R_{p})U_{rms}I_{rms}}{P_{MR}}\right)^2} - 1$ |
| | $P_{RZ} = F_{MR}(Z)\left(\frac{U_{rms}I_{rms}}{P_{col}}\right)^2 - P_{col}$ | $F_{MR}(Z) = P_{RZ} \frac{R_{col}U_{rms}^2 - P_{col}}{R_{col}U_{rms}^2 - P_{col}}$ |
| | $R_{PZ} = F_{RP}(Z)\left(\frac{R_{col}P_{col}U_{rms} - P_{col}^2}{R_{col}U_{rms}^2 - P_{col}}\right)$ | $F_{RP}(Z) = R_{PZ} \frac{R_{col}U_{rms}^2 - P_{col}}{R_{col}P_{col}U_{rms} - P_{col}}$ |

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| | $P_{MZ} = F_{PM}(Z)\frac{P_{col}}{P_{col}}$ | $F_{PM}(Z) = \frac{P_{MZ}}{P_{col}}$ |
| Pentegov-HL (12) | $\theta_{RZ} = \frac{F_{MR}(Z)}{2\omega \sqrt{\left(\frac{(R_{col} - R_{p})U_{rms}I_{rms}}{P_{MR}}\right)^2} - 1}$ | $F_{MR}(Z) = \theta_{RZ} 2\omega \sqrt{\left(\frac{(R_{col} - R_{p})U_{rms}I_{rms}}{P_{MR}}\right)^2} - 1$ |
| | $P_{RZ} = F_{MR}(Z)\left(\frac{U_{rms}I_{rms}}{P_{col}}\right)^2 - P_{col}$ | $F_{MR}(Z) = P_{RZ} \frac{R_{col}U_{rms}^2 - P_{col}}{R_{col}U_{rms}^2 - P_{col}}$ |
| | $R_{PZ} = F_{RP}(Z)\left(\frac{R_{col}P_{col}U_{rms} - P_{col}^2}{R_{col}U_{rms}^2 - P_{col}}\right)$ | $F_{RP}(Z) = R_{PZ} \frac{R_{col}U_{rms}^2 - P_{col}}{R_{col}P_{col}U_{rms} - P_{col}}$ |

Fig. 1. Diagrams of the correction functions for the determination of the parameters of the Mayr model of arc powered by the actual current source ($I_{zr} = 5$ A) of variable susceptance ($B_{L} = (\omega L)^{-1}$): a) correction functions of the Mayr time constant $F_{M}(L)$; b) correction functions of the Mayr power $F_{PM}(L)$; (1: preset power of the column $P_{MZ} = 50$ W; 2: $P_{MZ} = 100$ W; $\theta_{MZ} = 1 \times 10^{-3}$ s)
are close to unity. The change in the arc damping value affects the variability of correction functions $F_{\theta R}$. Figure 3b corresponds to the approximation of the family of functions

\[ F_{\theta R}(L, \theta_{RZ}, P_{RZ}, R_{PZ}) \approx F_{\theta R}(L) = 1.017L^{-0.00363} \] (14),

where $L$ – inductance in mH.

The diagrams of the functions of the total harmonic distortion in current waveforms $THD_{IM}$ (a) and voltage waveforms $THD_{UM}$ (b) of arc described by the Mayr model in the circuit powered by the actual voltage source (1: preset power of the column $P_{MZ} = 50$ W; 2: $P_{MZ} = 100$ W; $\theta_{MZ} = 1 \times 10^{-3}$ s) $THD_{ip}$ (Fig. 4a, b) reveal that an increase in source inductance is accompanied by a decrease in the above-named functions, which corresponds to the system being powered by sinusoidal current. The diagrams of the total harmonic distortion in voltage waveforms $THD_{UP}$ (Fig. 4c) reveal that the value of the THD depends on the power of arc. Within the range of low inductance ($L < 100$ mH) the above-named values are the highest if power is low, whereas within the range of high inductance, the values are the highest if power is high. A similar dependence can be observed in relation to higher values of time constant (Fig. 4d). Unfortunately, the shapes of the $THD_{IP}$ and $THD_{UP}$ diagrams cannot be used as indications identifying the ranges of the usability of previously adopted approximations, yet

\[ F_{\theta R}(L, \theta_{RZ}, P_{RZ}, R_{PZ}) \approx F_{\theta R}(L) = 1.017L^{-0.00363} \] (14),

\[ \omega L \]

\[ R_{PP} \]

\[ F_{\phi R}(L, \theta_{RZ}, P_{RZ}, R_{PZ}) \approx F_{\phi R}(L) = 1.017L^{-0.00363} \] (14),

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\[ R_{PP} \]
the previous analysis of the diagrams presented in Figure 3 reveals that the above-named need of identification does not exist in relation to \( L > 50 \, \text{mH} \).

Figure 5 presents the diagrams of the correction functions used to determine the parameters of the Mayr model of arc powered by the actual voltage source. In spite of significant changes in power, the above-named functions are located close to one another. Functions \( F_{\theta M} \) change significantly, particularly within the range of low inductance values (below 20 mH). The family of the functions can be approximated using the following dependence

\[
F_{\theta M}(L, \theta_{RZ}, P_{RZ}) = F_{\theta M}(L) = 1.23L^{0.061},
\]

where \( L \) – inductance in mH.

In turn, the variability of functions \( F_{PM} \) is not significant and for that reason their values can be adopted as amounting to 1.

The diagrams of the functions of the total harmonic distortion in current waveforms \( \text{THD}_{IP} \) and voltage waveforms \( \text{THD}_{UP} \) (Fig. 6a, b) reveal that their low values are present only within the limited range of changes in the source inductance \( (40-90) \, \text{mH} \), where the values of \( \text{THD}_{IP} \) are below 10%, and those of \( \text{THD}_{UP} \) are below 80%. Particularly important is the lower inductance limit as this value corresponds to the lowest value of \( F_{\theta M} \) obtained in the simulations.

![Fig. 4. Diagrams of the functions of the total harmonic distortion in current waveforms \( \text{THD}_{IP} \) (a, b) and voltage waveforms \( \text{THD}_{UM} \) (c, d) of arc described by the Pentegov-HL model in the circuit powered by the actual voltage source (1 – preset power of the column \( P_{MZ} = 60 \, \text{W}; 2 – P_{MZ} = 80 \, \text{W}; 3 – P_{MZ} = 100 \, \text{W}; R_{MZ} = 3 \, \Omega \) )](image)

![Fig. 5. Diagrams of the correction functions for the determination of the parameters of the Pentegov-HL model of arc powered by the actual voltage source (\( E_z = 100 \, \text{V} \) ) of variable impedance (inductance \( X_L = \omega L \) ): a) correction functions of the Mayr time constant \( F_{\theta M}(L) \); b) correction functions of the Mayr power \( F_{PM}(L) \) (1 – preset power of the column \( P_{MZ} = 50 \, \text{W}; 2 – P_{MZ} = 100 \, \text{W}; \theta_{MZ} = 1 \cdot 10^{-3} \, \text{s} \) )](image)

Figure 7 presents the diagrams of the correction functions corresponding to the Pentegov-HL model of arc powered by the actual voltage source. Also in this case, in spite of
significant changes in the preset power of the macromodel $P_{RZ}$, the diagrams of the correction functions of damping constant $F_{θR}$, power $F_{MR}$ and resistance $F_{RP}$ are characterised by a relatively low scatter and form narrow bands. Functions $F_{θR}$ are characterised by significant changes, particularly in cases of high time constants. Based on Figure 7b it is possible to approximate the above-named family using the following dependence

$$F_{θR}(L, θ_{RZ}, P_{RZ}, R_{PZ}) \approx F_{θR}(L) = 1 - 2.22 \cdot 10^{-4}L$$

(16),

In turn, the values of correction functions $F_{MR}$ and $F_{RP}$ are close to unity and, for this reason, in interference conditions the use of approximation may not be justified.

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**Fig. 6.** Diagrams of the functions of the total harmonic distortion in current waveforms $THD_{IM}$ (a) and voltage waveforms $THD_{UM}$ (b) of arc described by the Pentegov-HL model in the circuit powered by the actual voltage source (1 – preset power of the column $P_{MZ} = 50$ W; 2 – $P_{MZ} = 100$ W; $θ_{MZ} = 1 \cdot 10^{-3}$ s)

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**Fig. 7.** Diagrams of the correction functions for the determination of the parameters of the Pentegov-HL model of arc powered by the actual voltage source ($E_z = 200$ V) of variable impedance (inductance $X_L = \omega L$): a), b) correction functions of time constant $F_{θR}(L)$; c), d) correction functions of power $F_{MR}(L)$; e), f) resistance $F_{RP}(L)$ (1 – preset power of the column $P_{RZ} = 60$ W; 2 – $P_{RZ} = 80$ W; 3 – $P_{RZ} = 100$ W; $R_{PZ} = 3$ Ω)
The diagrams of the functions of the total harmonic distortion in current waveforms $THD_{IP}$ and voltage waveforms $THD_{UP}$ (Fig. 8) reveal that their low values are present only within the limited range of changes in the source inductance (150-250 mH), where the values of $THD_{IP}$ are below 20%, and those of $THD_{UP}$ below 150%. However, the diagrams are not correlated with the per-unit values of the correction functions, hence their usability is very low.

In the physical test conditions, electric arc is accompanied by interference, the level of which depends on numerous factors. The above-named factors are external in nature, affect the column or electrode and have the form of mechanical (gasodynamic) or electromagnetic disturbance, leading to changes in the column length and in the heat dissipation conditions. The sources of supply may also provide current or voltage waveforms deformed in relation to sinusoidal waveforms. When undertaking various stabilising actions, it is possible, to some extent, to reduce effects disturbing measurements of arc characteristics. For this reason, the use of correction functions, the values of which only slightly differ from unity, is not always justified, particularly when testing free arcs or in industrial disturbance conditions. Based on the shapes of the correction functions, it is possible to state that only in terms of the Mayr model (Fig. 1a and Fig. 5a) the correction functions of the time constant are characterised by significant variability.

Fig. 8. Diagrams of the functions of the total harmonic distortion in current waveforms $THD_{IP}$ (a, b) and voltage waveforms $THD_{UP}$ (c, d) of arc described by the Pentegov-HL model in the circuit powered by the actual voltage source (1 – preset power of the column $P_{RZ} = 60$ W; 2 – $P_{RZ} = 80$ W; 3 – $P_{RZ} = 100$ W; $R_{PZ} = 3 \Omega$)

**Conclusions**

1. The integral method used when determining the electric arc Mayr model parameters, developed in relation to arc powered by the ideal current source, can also be directly used in cases of arc powered by actual energy (current and voltage) sources characterised by high values of internal impedance (inductance $L > 50$ mH). If inductance is lower, it is justified to use the corrective function of time constant $F_{BM}$.

2. The integral method used when determining the electric arc Pentegov model HL parameters, developed in relation to arc powered by the ideal current source, can also be used in cases of arc powered by actual energy (current and voltage) sources. Only in cases of high damping function values it is justified to use the corrective function of time constant $F_{BR}$. 

3. The power of arc (depending primarily on the arc column length) and the damping function (depending on the type of gas) do not significantly affect the position of corrective functions.

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