Superluminal light propagation in a bi-chromatically Raman-driven and Doppler-broadened N-type 4-level atomic system

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We investigate the behavior of fast light pulse propagation in an N-type Doppler-broadened 4-level system using double Raman gain processes. This system displays novel and interesting results of two controllable pairs of the double gain lines profile with a control field. The detailed physics of the processes are explored having multiple controllable anomalous regions in the medium. In this set up, the system exhibits significant enhancement in the probing Gaussian pulse through the medium as compared with Ref. [L. J. Wang, A. Kuzmich, and A. Dogariu, Nature 406, 227(2000)]. The advance time of the retrieved Gaussian pulse is always greater than the advance time studied in the above said experiment. We analyzed that the pulse propagating through the medium with larger negative group index, $7.32 \times 10^3$, leaves the medium almost undistorted and sooner by time 76.12 ms than the pulse which leaves the medium of Wang et al. The Gaussian pulse always remains almost undistorted at output due to lossless characteristic of the medium. We also underlined the ways to suppress decoherences generated by the Doppler-broadening in the system. The limitations of the recently developed applications require to explore mechanisms for ultimate speed of a superluminal light pulse. In this connection, the proposed scheme may be helpful and can be easily adjusted with the current technology.

I. INTRODUCTION

The purpose of research on the manipulation of light group velocity from subluminal to superluminal is not only to study a novel state of matter [1] but also to explore its applications in the areas of optical memory, optical computing and optical communication [2,3]. In most of these studies coherent fields are used to control the optical properties of a medium. Consequently, this has led to many remarkable results such as enhanced nonlinear optical effects [4,5], Electromagnetically Induced Transparency (EIT) [6], Lasing Without Inversion (LWI) [7,8], ultra-slow light [9,10], storage and retrieval of optical pulses [11,12], and many others [13,14,15].

The pioneer experimental work of superluminal propagation of light pulse are credited to Chu and Wong [21] in the optical domain using a correlation technique. The credit also goes to Segard and Macke [22] for the demonstration of superluminality in microwave domain with a direct detection of the transmitted intensity of suitably shaped millimeter wave pulses through a linear molecular absorber. In both the cases the propagated pulse is less distorted. These results successfully verifies the prediction of Garrett and McCumber [23]. A remarkable control of superluminal group velocity of light in a system has also been demonstrated experimentally in Ref. [24]. However, using saturation effect the pulse advances in time through an amplifying medium is studied in Ref. [25]. More recently, gain-assisted anomalous dispersion which leads to superluminal light propagation has also been demonstrated in Refs. [26,27]. The beating of pump and probe field which generates coherent population pulsation in a medium exhibits superluminality [28].

A large number of studies exist in literature in different contexts which describe superluminal light pulse propagation, for example, see Refs. [28,29]. However, in this paper we restrict ourselves to the investigation of a viably gain-assisted atomic system for a superluminality Gaussian light pulse and its enhancement during propagation in the dispersion medium.

Wang et al. [27] demonstrated superluminal light pulse propagation for the first time using region of lossless anomalous dispersion between the two closely spaced gain lines in a double Raman gain medium. They observed that the dispersion properties of the medium can be manipulated to have negative group index for the superluminality. Moti et al. utilized this approach for temporal cloaking [30] using the concept of spatial cloaking [31,32] in the temporal domain. It is the manipulation of positive and negative group indices due to which the probe light pulse propagates around an object in such a way that creates a hole both in time and space windows. This behavior enabled us to hide a physical event from any physical observer. The future aims of these and their related experiments are to explore spatio-temporal cloaking for making history of a physical event completely hidden. However, due to the limitations of current technology, the present problem in these experiments is the less gap creation. Therefore, it requires to increase the gap to microsecond and to millisecond [33]. Obviously, we ultimately high group velocity for larger gap creation without significant distortion of the pulse shape at the
output of the medium. To the best of our knowledge, less interest has been shown by researchers in this area. Furthermore, following the experiment of temporal cloaking, Glasser et al. [41] utilized the negative group velocity of light for the measurement of images. However, in their experiment, as they discussed therein, we need an ultimate speed of the light pulse to get improve the quality of the images. According to these facts and may be many more, it is necessary to explore mechanism for an ultimate speed of light pulse with a minimum distortion of a light pulse at the output [38, 39, 50].

In this paper we propose two-paired gain assisted model for demonstration of enhancement of superluminality of a Gaussian light pulse. In this model, we initially couple two pump fields appropriately in the atomic system to have initially double Raman gain lines feature. We also couple another control field with a fourth energy level to provide a source to change the single pair of the gain lines to two controllable pairs. This behavior provides multiple controllable anomalous regions in the dispersion medium. Furthermore, we also consider the medium Doppler broadening effect in the system due to high temperature for a maximum Doppler width. This broadening sensitivity is in complete contrast with the experimental scheme of Wang et al. [27], where the induction of Doppler effect in their system creates problem of singularities and consequently converts the gain processes to absorption. In fact, their experiment, is Doppler-broadening insensitive and limits the superluminality in a laboratory along with almost undistorted pulse shape received at the output, is remarkable. This behavior of significant enhanced superluminal light pulse with its almost undistorted shape received at the output, is remarkable. Therefore, it may be of interest for researchers to demonstrate these mechanisms in a laboratory.

FIG. 1: (a) Schematics of the atomic system. (b) Doppler-broadened system

II. MODEL AND EQUATION

We consider a 4-level double-Raman atomic-system in N-type configuration driven by two coherent pump fields, a probe and a control field, appropriately [see Fig. 1].

\[ H(t) = -\frac{\hbar}{2}(\Omega_1 e^{-i\delta_1 t} + \Omega_2 e^{-i\delta_2 t}) |a\rangle \langle d| -\frac{\hbar}{2}\Omega_3 e^{-i\delta_3 t} |b\rangle \langle c| -\frac{\hbar}{2}\Omega_4 e^{-i\Delta t} |a\rangle \langle c| + H.c. \]

where, \(\omega_1 = \omega_{ad} + \delta_1, \omega_2 = \omega_{ad} + \delta_2, \omega_3 = \omega_{bc} + \delta_3\) and \(\omega_4 = \omega_{ac} + \Delta\). In the Eq. (1) the chosen detuning parameters are \(\delta_1 = \delta_0 - \delta\), and \(\delta_2 = \delta_0 + \delta\). Further, the detuning \(\delta = \frac{\delta_2 - \delta_1}{2}\), appears for effective frequency of the two pump fields while \(\delta_0 = \delta_1 + \delta_2/2\), is the average detuning of the two pump fields from the atomic transition. Next, to find out the equations of motion we use the following general form of density matrix equation:

\[ \frac{d\rho}{dt} = -\frac{i}{\hbar}[H(t), \rho] + \Lambda\rho, \]

where, \(\Lambda\rho\) is the damping part of our system. Using the above density matrix equation and following the transformation relations for slowly and fast varying amplitudes we calculated rate equations for the proposed system. These rate equations are listed in the Appendix-A. We assume the phase factor associated with the two modes of the coherent pump fields constant following the series of papers of Wang and his co-workers [27, 12, 44].
In Ref. 27, 43, 44 they presented their theoretical results of dispersion for the constant phase. Further, in Ref. 42 they presented detailed conceptual interpretation of the phases associated with the driving fields frequencies using wave nature characteristics of the classical Maxwell’s equations for a light pulse propagating through the transparent region of the medium. Due to the negative group index and its associated negative time delay the phases of the driving fields get re-shape to have zero phase throughout the transparent medium. It gets back the same value of the phases of the associated entering pulses after the exit. These facts are discussed in detail using classical Maxwell’s equations for wave nature of a light pulse propagation before medium, in the medium and after the medium, respectively. Furthermore, the detailed analysis carried in Ref. 42 for the corresponding results of dispersion for the constant phase also agree with the experimental results reported in Ref. 42. They also given the detail procedure that they also reported the detailed analysis carried in Ref. 42 for the corresponding results of dispersion for the constant phase. Further, in Ref. 27, 43, 44 they presented their theoretical results calculated by Wang et al. Therefore, in our system if we assume \( \Gamma_{ad} = \Gamma_{bd} = 0 \), \( \Omega_3 = 0 \), \( \Delta_3 = 0 \), \( \Gamma_{ac} = \Gamma_{dc} = \Gamma_0 \), \( \delta_1 \approx \Delta, \delta_2/\delta_0, \gamma/\delta_0 \ll 1 \), \( \delta_{1,2} = 2\pi(\nu_{1,2} - \nu_{ad}) \), \( \Delta = 2\pi(\nu_p - \nu_{ac}), \nu_{ac} \approx \nu_{ad} \), we get the required results i.e.,

\[
\chi_w = \frac{M_1}{\nu_p - \nu_1 + i\gamma} + \frac{M_2}{\nu_p - \nu_2 + i\gamma},
\]

where \( M_{j=1,2} = N|\sigma_{ac}|^2\Omega_{ac,j}^2/2\pi\delta_{0}^2 \) and \( \sigma_{ac} \) is the dipole-moment between the levels \( |a \rangle \) and \( |c \rangle \) and \( \gamma = \Gamma/2\pi \). Next, we consider the Doppler frequency shifts induced by the atoms moving with a velocity \( v \), relative to the coherently driven pump fields, the control and the probe field. The configuration of the counter propagating waves of these driven fields through the atomic media is shown in the Fig. 1(b). To explore the Doppler broadening effect in the system we replace the detuning parameters such that \( \delta_1 = \delta_1 + k_1 v, \delta_2 = \delta_2 + k_2 v, \Delta = \Delta + k_3 v, \delta_3 = \delta_3 - k_3 v \), in the Eq. (15) for the Eq. (18) of susceptibility \( \chi \), where \( k_1 = k_2 = k_3 = k = k_0 \), and \( v \), is the atomic velocity of the medium. The susceptibility obtained is given by:

\[
\chi(kv) = \frac{-3iN\lambda^3\gamma}{32\pi^3} f(kv, \Delta),
\]

where \( f(kv, \Delta) = \sum_{j=1}^{4} A_j(kv, \Delta) \). The parameters \( A_{j=1-4}(kv, \Delta) \) of the above equation are listed in the Appendix-B. Now integrating \( \chi(kv) \) over the velocity distribution we have

\[
\chi(d) = \frac{1}{\sqrt{2\pi V_D}} \int_{-\infty}^{\infty} \chi(kv) e^{-\frac{(kv)^2}{2V_D^2}} dv.
\]

Here, \( V_D \) is the Doppler width and is given by \( V_D = \sqrt{\frac{K_B T J^2}{M^2}} \). Now, rewriting the above equation in the following form

\[
\chi(d) = \frac{-3iN\lambda^3\gamma}{32\sqrt{2\pi^3/2V_D}} \int_{-\infty}^{\infty} f(kv, \Delta) e^{-\frac{(kv)^2}{2V_D^2}} dv.
\]

To keep the condition of atomic motion relative to the frequencies of the driving fields more general we assumed the system parameters modifiable due to flexible environment. Therefore we consider the atomic velocity linear in the response function of the medium of the system, respectively. Finally we write the group index for our system when there is no Doppler effect as

\[
N_g = 1 + 2\pi \text{Re}[\chi] + 2\pi\omega_{ac}\text{Re}[\partial \chi/\partial \Delta],
\]

while

\[
N_g(d) = 1 + 2\pi \text{Re}[\chi(d)] + 2\pi\omega_{ac}\text{Re}[\partial \chi(d)/\partial \Delta],
\]

where \( N \) is the atomic number density of the medium. Furthermore, \( \lambda = 2\pi c/\omega_{ac} \), and the Einstein coefficient \( A_{off} = \frac{4\sigma_{ac}|^2\omega_{ac}^3}{\epsilon_0 hc} \) is equal to \( 4\gamma \).
\[
\tau_d = \frac{L(N_g - 1)}{c} \tag{10}
\]

These are our final results which will be analyzed and discussed in details in the results and discussion section. \(\tau_d\) is group delay time when its value is positive. If the value of \(\tau_d\) is negative then is called advance group delay time (\(\tau_{adv}\)).

### IV. DISTORTION MEASUREMENTS

A complex monochromatic wave-field of angular frequency \(\omega\), position \(z\) and time \(t\), is given by \(E(z,t) = \frac{1}{2}(E_0 e^{i(k_0 z - \omega t)} + c.c.)\), while its phase is \(\varphi = k(\omega)z - \omega t\).

We assume this phase of the field constant during the propagation through the medium. The group index obtained is written by

\[
N_g = n_r(\omega) + \omega \frac{\partial n_g(\omega)}{\partial \omega} \tag{11}
\]

The corresponding dispersion of the group velocity of the light pulse is:

\[
D_{c_g} = \frac{\partial}{\partial \omega}(v_g^{-1}) = \frac{1}{c}[2\frac{\partial n_g(\omega)}{\partial \omega} + \omega \frac{\partial^2 n_g(\omega)}{\partial \omega^2}] . \tag{12}
\]

Furthermore, the complex wave-number \(k(\omega)\) can be expanded via Taylor series in terms of group index as:

\[
k(\omega) = \frac{N_g^{(0)}(\omega)}{c} + \frac{1}{2!}(\omega - \omega_0)^2 \frac{1}{c} \frac{\partial N_g}{\partial \omega} |_{\omega \rightarrow \omega_0} + \frac{1}{3!} (\omega - \omega_0)^3 \frac{1}{c^2} \frac{\partial^2 N_g}{\partial \omega^2} |_{\omega \rightarrow \omega_0} + \frac{1}{4!} (\omega - \omega_0)^4 \frac{1}{c^3} \frac{\partial^3 N_g}{\partial \omega^3} |_{\omega \rightarrow \omega_0} \ldots \tag{13}
\]

where \(N_g^{(0)}\), is the group index of a medium at the central frequency \(\omega_0\). The transit time of the pulse through the medium is \(T = N_gL/c = k_1L\) and \(\Delta T = [L \partial N_g/\partial \omega] \Delta \omega\) represents the spread in transient time, for \(\Delta \omega\) being the frequency bandwidth of the pulse. Under the condition \(\Delta T < \tau_0\), there is no significant distortion in the system. We denote \(\tau_0\) for the characteristic pulse width. The distortion in the pulse propagating through the proposed medium becomes negligible, therefore \(\partial N_g/\partial \omega = 0\). Generally, it is the most favorable condition for an experiment while having the advantage of extremum of negative \(N_g\). In this connection, our proposed system is based on phenomenon which has lossless anomalous dispersion regions for the probe pulse propagation to have minimum contribution from the higher order terms in \(k(\omega)\). Satisfactorily, under these conditions the pulse shape at the output remains almost unchange while it modify the advance time \(\tau_d = \frac{L(N_g - 1)}{c}\), significantly. To developed the formalism and to study the nature of the pulse at the output we incorporate the terminology as \(n_r\) and \(N_g\) being the refractive index and the group index, respectively. Furthermore, \(\partial n_r/\partial \omega\) is related to dispersion and \(\partial N_g/\partial \omega\) describe the distortion. Obviously, the real and imaginary parts of the higher order terms of \(k_1(\omega)\) are related to the dispersion and phase distortion, and to the gain (absorption) and amplitude distortion, respectively. The output pulse \(S_{\text{out}}(\omega)\), after the propagating through the medium can be related to the input pulse \(S_{\text{in}}(\omega)\) with the transfer function \(H(\omega)\), being a convolution. The transfer function for our system is written as \(S_{\text{out}}(\omega) = H(\omega)S_{\text{in}}(\omega)\), where \(H(\omega) = e^{-ik(\omega)L}\) is the transfer function for our system. Now, we choose a Gaussian input pulse of the form

\[
S_{\text{in}}(t) = \exp[-t^2/\tau_0^2] \exp[i(\omega_0 + \xi)t], \tag{14}
\]

for \(\xi\), being the upshifted frequency of the empty cavity. The Fourier transforms of this function is then written by \(S_{\text{in}}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S_{\text{in}}(t) e^{i\omega t} dt\). The expression for the input signal is calculated as

\[
S_{\text{in}}(\omega) = \tau_0/\sqrt{2} \exp\left[-(\omega - \omega_0 - \xi)^2 \tau_0^2 / 4\right] \tag{15}
\]

By virtue of the convolution theorem the output \(S_{\text{out}}(t)\), can be related to the input pulse via the transfer function using inverse Fourier transforms as \(S_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S_{\text{in}}(\omega)H(\omega)e^{i\omega t} d\omega\). Evidently, the response of the advance time is significant with the Doppler broadening and we have drastic enhancement in the superluminality of the Gaussian light pulse. In this connection, it is worthwhile to analyze the output pulse shape distortion for high enough perturbation limit. To the fourth order of the group index it is given by

\[
S_{\text{out}}(t) = \frac{\tau_0 \sqrt{c}}{\sqrt{2\pi m_1 L + c \tau_0^2}} \left[1 - \frac{i(6F_1F_2 + F_3^2)n_2L}{48cF_1^3}\right]
\times \exp\left[-\frac{c^2\tau_0^2}{4} + i(t - \frac{n_0L}{c})\omega_0\right]
\times \exp[F_2 + \frac{cF_2^2}{2(2m_1L + c\tau_0^2)}] \tag{16}
\]

\[
F_1 = \frac{2in_1L + c\tau_0^2}{4c} \tag{17}
\]

\[
F_2 = \frac{c\xi^2 - 2in_0L + 2ict}{2c} \tag{18}
\]

Where \(n_1 = \partial n_g/\partial \omega\) and \(n_2 = \partial^2 N_g/\partial \omega^2\). The nature of the output pulse after propagating thought the medium can be analyzed from the above analytical expression. Since there are multiple anomalous regions in the dispersion region therefore we provide the quantitative and graphical analysis for one of these regions for the pulse distortion measurement in the next section.
FIG. 2: Dispersion, gain, group index and group delay time against $\Delta\gamma$ such that $\gamma = 1\text{MHz}$, $\Gamma_{ad} = \Gamma_{ac} = \Gamma_{ab} = \Gamma_{cd} = \Gamma_{bd} = 2.01\gamma$, $\delta_3 = 0\gamma$, $\lambda = 586.9\text{nm}$, $\omega_{ac} = 10^3\gamma$, $\delta_1 = 30\gamma$, $\delta_2 = 50\gamma$, $\Omega_1 = 4\gamma$, $\Omega_2 = 7\gamma$, $\Omega_3 = 8\gamma$, $V_D = 0\gamma$(solid red), $V_D = 2\gamma$(dashed red), $V_D = 4\gamma$(blue dashed), $V_D = 6\gamma$(green dashed). When the intensity of control field is larger then the pump field. The gain doublet are present and the group index are decrease both in positive/negative domain with the increase of doppler width. The superlominal propagation is reduced with doppler width at two photons resonance points $\Delta = \delta_{i=1,2} = 30\gamma$, $50\gamma$ and between the two pair of the gain doublet region, $\Delta = 40\gamma$.

V. RESULTS AND DISCUSSION

We proceed with inspection of our main results presented in the Eqs. (4,7,8,9,10,14,16) for dispersion gain group index time delay and pulse shape distortions when there is no Doppler broadening effect in the system and for the system when there is its maximum effect. To investigate the soundness of the results of our proposed system, we kept checks on the main results under some approximations. First, we turn off the control field in our system which leads to the famous scheme of Wang et al. In this approximation, the results of our system are in complete agreement with the said reference under the same set of parameters and we have two closely spaced gain-peak behavior. The reduction of the analytical results to the Wang et al. scheme is satisfactory, representing the soundness of our results. Moreover, in this approximated scheme we further exclude one of two coherent pump fields, then the atomic system reduces to a single Raman gain scheme where only one gain peak exists. Further, in this set-up if we include the control field with a fourth level then this forms N-type scheme with one coherent pump field similar to the one discussed by Agarwal et al. [40]. The behavior of the gain profile...
FIG. 4: Group index group advance time and group velocity versus $\Delta$ such that $\gamma = 1MHz$, $\Gamma_{ad} = \Gamma_{ac} = \Gamma_{ab} = \Gamma_{cd} = \Gamma_{bd} = 2.01\gamma$, $\delta_3 = 0\gamma$, $\lambda = 586.9nm$, $\omega_{ac} = 10^3\gamma$, $\delta_1 = 30\gamma$, $\delta_2 = 50\gamma$, $\Omega_1 = 4\gamma$, $\Omega_2 = 7\gamma$, $\Omega_3 = 8\gamma$, $V_D = 0\gamma$(solid red), $V_D = 2\gamma$(dashed red), $V_D = 4\gamma$(blue dashed), $V_D = 6\gamma$(green dashed), $V_D = 8\gamma$(black dashed). The group index is reduce in negative domain with the increase of doppler width. The advance time is also reduce with the doppler width, but the group velocity is increase in negative domain. More negative group index is more advance time and smaller negative group velocity. Larger superlominality occur at more negative group index. The Fig[a,b,c] clearly show that the superlominality is reduced with the doppler effect.

of the single Raman gain scheme is modified in this N-type case. Here, the single gain feature is modified by the control field into two closely spaced gain-peak behavior unlike the single Raman gain scheme. The splitting of the single gain peak into two is the manifestation of the dressing of the ground state in a doublet. All these facts also agree with the results presented in Ref. [40].

Next, we focus on the main results of our atomic system and are committed (1) to discuss the underlined changes in the physical behavior of this system as compared to the earlier related literatures (2) to discuss the subluminal and superluminal behavior of the light pulse propagating through their associated dispersion regions, (3) to discuss the induced Doppler broadening effect in the system and (4) to discuss that how the control field resolves the fundamental issue of the Wang et al. scheme where the singularities and other effects prevent the system to be Doppler broadened. This is a very important aspect of the experiment which cannot be incorporated due to singularities appeared at the denominator of the two terms of susceptibility. This fact limits the usefulness of their experiment. However, in present system this issue is resolved due to the nonsingular terms appeared at the denominator of the two terms of the susceptibility.

FIG. 5: Group index and Group delay/advanced time versus $\Omega_3$ such that $\gamma = 1MHz$, $\Gamma_{ad} = \Gamma_{ac} = \Gamma_{ab} = \Gamma_{cd} = \Gamma_{bd} = 2.01\gamma$, $\delta_3 = 0\gamma$, $\lambda = 586.9nm$, $\omega_{ac} = 10^3\gamma$, $\delta_1 = 30\gamma$, $\delta_2 = 50\gamma$, $\Omega_1 = 4\gamma$, $\Omega_2 = 7\gamma$, $\Omega_3 = 8\gamma$, $V_D = 0\gamma$(solid red), $V_D = 2\gamma$(dashed red), $V_D = 4\gamma$(blue dashed), $V_D = 6\gamma$(green dashed), $V_D = 8\gamma$(black dashed).
FIG. 6: Dispersion, gain, group index and group delay time against $\Delta$ such that $\gamma = 1 MHz$, $\Gamma_{ad} = \Gamma_{ac} = \Gamma_{ab} = \Gamma_{cd} = \Gamma_{bd} = 2.01\gamma$, $\delta_2 = 0\gamma$, $\lambda = 586.9nm$, $\omega_{ac} = 10^3\gamma$, $\delta_1 = 30\gamma$, $\delta_2 = 50\gamma$, $\Omega_1 = 4\gamma$, $\Omega_2 = 7\gamma$, $\Omega_3 = 20\gamma$, $\nu_{D} = 0\gamma$(solid red), $\nu_{D} = 2\gamma$(dashed red), $\nu_{D} = 4\gamma$(blue dashed), $\nu_{D} = 6\gamma$(green dashed), $\nu_{D} = 8\gamma$(black dashed).

In the proposed scheme, the Doppler effect is maximum for our considered appropriate orientation of the counter propagating driving fields relative to the atomics motion. The Doppler width of our system depends linearly on temperature of the cavity and density of the medium. Then, greater the temperature of the cavity or greater the density of the medium or both, the larger is the Doppler width. Subsequently, the smaller the negative group index, the minimum is the superluminal effect on the propagating Gaussian pulse. As a result, the group velocity of the superluminal Gaussian pulse can be reduce to its highest ultimate limit for this system.

The real and imaginary parts of the susceptibilities correspond to the gain and dispersion as well as corresponding group index and time delay/advance are plotted against the probe field detuning for the two cases presented in the Eqs. (4),(7)-(8,9,10) for the same set of parameters and when $\Delta = \delta_1$ or $\Delta = \delta_2$ [see Fig. 2].

The spectral profile of the gain of the system shows two pairs of the gain doublet. This behavior is completely different from the gain assisted system of Wang et al.,

FIG. 7: The intensity of a Gaussian pulse at input and output Vs the normalized time for both the increasing and decreasing function of the group index [a] $\xi = 0 MHz$[b]$\xi = 0.005 MHz$, [c] $\xi = 0.01 MHz$, $\tau_0 = 3.50\mu s$, $c = 3 \times 10^8 m/s$, $L = 0.03m$, $\omega_0 = 970\gamma$, $\Gamma_{ad} = \Gamma_{ac} = \Gamma_{ab} = \Gamma_{cd} = 2.01\gamma$, $\Gamma_{bd} = 0.01\gamma$, $\delta_1 = 0\gamma$, $\lambda = 586.9nm$, $\omega_{ac} = 10^3\gamma$, $\nu_{D} = 4\gamma$, $\delta_1 = 30\gamma$, $\delta_2 = 50\gamma$, $\Omega_1 = 4\gamma$, $\Omega_2 = 7\gamma$, $\Omega_3 = 8\gamma$
where only one pair of a doublet exists. The change in the scenarios of the gain profile for this system is important as it provides multiple anomalous regions in the dispersion medium for the superluminal pulse propagation. This kind of regions can be found in the recently reported works in the context of EIT. Further, the Doppler broadening effect of the system reduces the superluminal profile drastically. Furthermore, the control field can be used to control the anomalous regions very efficiently. The physics of these under lying changes is very interesting. The coherently driven control field splits one of the ground energy levels in a doublet. In this case the photons associated with the two coherent pump fields are added to the probe field through two paths created by the control field in the form of a two dressed-state. Thus, there are two pairs of the gain doublet. The smaller (larger) the strength of the control field, the lesser (greater) is the space between the two dressed states. Consequently, the smaller (larger) is region between the two gain doublets. Evidently, this leads to the control of the anomalous dispersion regions if and only if the detuning of the control field is small. However, when this detuning becomes large, splitting of the ground state disappears. As a result, two pairs of the doublet-gain reduce to one pair and exhibits the double Raman gain processes like the Wang et al. This behavior agrees to what we expect for large detuning where the control field interacts no more with the upper added fourth level. Consequently, two dressed ground states associated with the real ground energy level disappear. Further, the imaginary part of $\chi(d)$, shows a typical gain dip over the imaginary part of $\chi$ and the doublet is disappear with the increase of doppler width[see Fig. 3].

The slope of dispersion in these gain dip regions are normal for both $\chi(d)$ when the strenght of control field is smaller then the pump field, which shows slow light propagation. The group indices in these dispersion regions are positive and enhances with the Doppler-broadened system. Interestingly, the gain increase significantly with the Doppler broadening effect between the two gain peaks around the two photons resonance condition i.e., $\Delta = \delta_1$ or $\Delta = \delta_2$, there is anomalous region for both the $\chi(d)$, and $\chi$. Furthermore, the anomalous dispersion associated with $\chi(d)$, is less pronounced and steeper than the dispersion of $\chi$. The group index close to the resonance condition i.e., $\Delta = 30\gamma$, $\Delta = 50\gamma$, in the anomalous dispersion region reduces for the Doppler-broadened system and we have $N_g = -2.2284 \times 10^6$ and $N_g(d) = -1.09624 \times 10^6$. Their corresponding group velocities are given by $v_g = -c/2.2284 \times 10^6 = -134.626m/s$ and $v_g = -c/1.09624 \times 10^6 = -273.664m/s$, respectively. The negative group advance time for $v_g = -c/2.2284 \times 10^6$, is $-22.28\mu s$, and for $v_g = -c/1 \times 10^6$, is $-10.96\mu s$, across the $3cm$, length of the dispersion medium if the doppler width is $V_D = 2\gamma$. The pulse at $v_g$ is slow down than the pulse of $v_g$. The reduce advance time is $-11.32\mu s$[see Fig. 2[c,d]] further increase of doppler width reduce the advance time rapidly. The plots traced for group index group advance time and group velocity against probe detuning as a increasing function of doppler width around the point $\Delta = 40\gamma$ are also shown in Fig 4. The group index is reduce in negative domain when the doppler width is increase stepwise from $0\gamma$, $2\gamma$, $4\gamma$, $6\gamma$, $8\gamma$, $\gamma$. The group index varies stepwise from $N_g = -1.78 \times 10^6$, $-1.79 \times 10^6$, $-1.73 \times 10^6$, $-1.47 \times 10^6$, $-1.20 \times 10^6$ while the group velocity varies from $v_g = [-168.29, -167.20, -172.62, -203.42, -249.95]m/s$, and the group advance time varies from $t_g = [-17.82, -17.94, -17.37\mu s, -14.74, -12.002\mu s$ as shown in Fig4[a,b,c]. Fig5 show variation of group index with control field at different values of doppler widths. At high intensity of the control field $\Omega = 20\gamma$, the region between the two pair of gain doublet is disapear and simply three gain dipth are observe. The gain dipth is reduce with the increase of doppler width. The slope of dispersion in these gain depth region are normal and reverse manipulated with the doppler width. The group index in these regions are positive and reduce doppler width,which enhance the group velocity in positive domain. There is large time delay in the absent of doppler effect. The delay time is reduce with doppler effect Fig6[a,b,c,d].

Furthermore, it is noteworthy that a larger negative group index is not a necessary and sufficient condition for practical application. It is necessary to choose a system with minimum possible losses (gain) along with its best negative group index characteristics. The scheme of Wang et al. due to its very less gain at the anomalous region of the medium is responsible for its successful demonstration in a laboratory. Therefore, a scheme having significant gain at anomalous region may distort the output pulse and may lead to unpractical condition. Obviously, the response of the advance time is significant with the Doppler broadening effect and we have the drastic enhancement in the superluminality of the pulse. In this connection, it is worthwhile to analyze the output pulse shape distortion for high enough perturbation limit. The analytical results are provided in the text to the fourth order of the group index for the output Gaussian pulse. The Gaussian pulse shape is presented in the time domain by transforming the angular-frequency dependent input Gaussian pulse using transfer function of the medium as a convolution. In this way we provide the output pulse in time domain to compare with the input Gaussian pulse and almost no distortion is seen. This behavior confirms the characteristics similar to the Wang et al. model for Doppler-free and Doppler-broadened systems and obeys an ideal behavior. Consequently, the imbedded Doppler effect which is responsible for the drastic increase of the group index is still practical. The anomalous regions are always there in the mid of each pair of the gain lines with the proviend of good quality characteristics for the pulse shape at the output of the medium. Quantitatively, we choose one of the lossless anomalous regions appears at $\Delta = 30\gamma$ and the atomic transition frequency as $\omega_{ac} = 1000\gamma$. Furthermore, we also consider the central frequency of the probe field at
\[ \Delta = 30\gamma \text{, as } \omega_0 = 970\gamma. \] The detuning of the probe field is written by \[ \Delta = \omega_{ac} - \omega, \text{ where } \omega = 2\pi \nu_p. \] The values of group indices for the two cases in the lossless region are \( n_0 = -2.22 \times 10^6, \rho_{0d} = -1.95 \times 10^6. \) However, the first order derivatives of group indices are calculated as \( n_1 = 2.65 \times 10^7, n_{1d} = -3.46 \times 10^7 \) and , the second order derivatives of group indices are \( n_2 = 1.26 \times 10^9, n_{2d} = -4.54 \times 10^8. \) In these cases the real parts correspond to the amplitude distortion while the imaginary parts correspond to the phase distortion of the propagating Gaussian pulse through our proposed medium.

Traditionally, the physical interpretation of superluminality by virtual reshaping is now not reasonable \([43, 53, 56]\) with the explanation of amplification of the front edge with the relatively absorption of its tail. In the Wang et al. experiment the band-width of the probe pulse is chosen very smaller than the separation of the two gain peaks. In the pulse distortion measurement we kept in our system the probe pulse bandwidth much smaller than gain lines separation to avoid resonances with the Raman transitions frequencies for the probe pulse. Consequently, there is no amplification of the front edge of the pulse. Moreover, the average time of stay of an atom inside the volume of the Raman probe beam is also shorter than FWHM of the probe pulse. This means that both the front edge and the tail would be amplified if the atoms of the medium amplify the probe pulse. However, this is not in accord with the earlier claims and even with our displayed results. Obviously, the superluminal light propagation arises due to the anomalous dispersion region created by the two nearby Raman gain resonances. In fact, if the gain becomes large, its effect appears as a compression of the pulse \([43]\). This fact is very clearly observable from graphical analysis of our pulse distortion measurement as shown in the Fig. 7. The detail analysis reveals that the pulse distortion measurement fully agree with the Wang et al. studies even with the fourth order perturbation limit. Evidently, in our system it is shown that pulse shape is preserved over the very small region in between the gain lines for some specific value of the upshift frequency. The shifting of this upshift frequency toward the either gain line results in compression or enhancing the pulse shape of the input probe field corresponding to an increase or decrease in the gain, respectively. In fact it is the consideration of probe pulse-width much smaller than the gain lines separation which results in the transparency over a very narrow domain of the upshifting frequency. The movement of the shifting frequency to either sides of the gain line results in the suppression and enhancement of the shape of the probe pulse and are the essence of the pulse distortion as shown in Fig. 7. Unlikely, this behavior does not agree with the earlier interpretation and is more likely with Wang et al. presented results and interpretation. Satisfactorily, we are providing analytical results to the literature where this behavior can be predicted and proceeded beyond the literature.

VI. SUMMARY

We proposed an N-shaped 4-level atomic system driven by two pump fields, a control and a probe field appropriately. The proposed scheme displays interesting and novel results of the two pairs Raman gain processes having various advantages than the double Raman gain scheme of Wang’s et al. Generally, the present system consists of two controllable pairs of double Raman gain peaks with control of the control field. However, the Doppler broadening effect induced by the control field in the system enhances significant enhancement in the superluminality of the probe Gaussian light pulse. Consequently, under experimentally feasible parameters the advanced time is shorter by 76.12\(ms\) than the advance time of Wang et al Doppler insensitive experiment. The proposed two-paired gain system is lossless with the extra advantages of Doppler sensitivity and multiple controllable lossless anomalous regions while having almost undistorted output pulse. In fact, making an object hidden in space (time) from a physical observer \([33, 50, 52, 54]\), is a laboratory reality which requires negative group indices \([27]\) with almost undistorted pulse shape at the output of the medium. Similarly, superluminal light pulse can also be used for imaging \([41]\). However, due to limitation of the current technology, it is appealing to explore mechanisms to create the hole for cloaking to microsecond and millisecond and to improve a best quality image measurement \([38, 39, 50]\). In this connection, the present scheme may provide this ground to get improve the applied aspects of the superluminality and is easily adjustable with the current technology.

VII. APPENDIX-A

The dynamical equations obtained from Eq. (2) for slowing varying amplitudes are given by

\[ \tilde{\rho}_{aa} = \frac{i}{2}(\Omega_1 + \Omega_2) \cos \phi + i(\Omega_1 - \Omega_2) \sin \phi] \tilde{\rho}_{da} \]
\[ \quad - \frac{i}{2}[(\Omega_1^2 + \Omega_2^2) \cos(\phi) - i(\Omega_1^2 - \Omega_2^2) \sin(\phi)] \tilde{\rho}_{ad} \]
\[ + \frac{i}{2} \Omega_p \tilde{\rho}_{ca} - \frac{i}{2} \Omega_p^* \tilde{\rho}_{ac} - (\Gamma_{ad} + \Gamma_{ac}) \tilde{\rho}_{aa}, \quad (19) \]

\[ \tilde{\rho}_{bb} = - (\Gamma_{bc} + \Gamma_{bd}) \tilde{\rho}_{bb} + \frac{i}{2} (\Omega_3 \tilde{\rho}_{cb} - \Omega_3^* \tilde{\rho}_{bc}), \quad (20) \]

\[ \tilde{\rho}_{cc} = \frac{i}{2} \Omega_3^* \tilde{\rho}_{bc} - \frac{i}{2} \Omega_3 \tilde{\rho}_{cb} + \frac{i}{2} \Omega_p (\tilde{\rho}_{ac} - \tilde{\rho}_{ca}), \quad (21) \]

\[ \tilde{\rho}_{bc} = (i \delta_3 - \Gamma_{bc}) \tilde{\rho}_{bc} + \frac{i}{2} \Omega_3 (\tilde{\rho}_{cc} - \tilde{\rho}_{bb}) - \frac{i}{2} \Omega_p \tilde{\rho}_{ba}, \quad (22) \]
\[ \dot{\rho}_{ad} = \frac{i}{2} (\Omega_1 + \Omega_2) \cos \phi + i(\Omega_1 - \Omega_2) \sin \phi \dot{\rho}_{ad} \]
\[ - \frac{i}{2} (\Omega_1 + \Omega_2) \cos \phi + i(\Omega_1 - \Omega_2) \sin \phi \dot{\rho}_{da} \]
\[ + \Gamma_{ad} \rho_{aa} + \Gamma_{bd} \rho_{ab}, \] (23)

\[ \dot{\rho}_{ad} = \frac{i}{2} (\Omega_1 + \Omega_2) \cos \phi + i(\Omega_1 - \Omega_2) \sin \phi (\dot{\rho}_{ad} - \dot{\rho}_{aa}) \]
\[ + \left[ \frac{i}{2} (\delta_1 + \delta_2 - \Gamma_{ad}) \right] \dot{\rho}_{ad} + \frac{i}{2} \Omega_p \dot{\rho}_{cd}, \] (24)

\[ \dot{\rho}_{ac} = (i \Delta - \Gamma_{ac}) \rho_{ac} + \frac{i}{2} \Omega_p (\rho_{cc} - \rho_{aa}) - \frac{i}{2} \Omega_3 \rho_{ab} \]
\[ + \frac{i}{2} (\Omega_1 + \Omega_2) \cos \phi \]
\[ + i(\Omega_1 - \Omega_2) \sin \phi \dot{\rho}_{dc}, \] (25)

\[ \dot{\rho}_{dc} = \frac{i}{2} (\Omega_1^* + \Omega_2^*) \cos \phi - i(\Omega_1^* - \Omega_2^*) \sin \phi \dot{\rho}_{ac} \]
\[ - \frac{i}{2} \Omega_3 \rho_{db} + \frac{i}{2} \left[ i (2 \Delta - \delta_1 - \delta_2) - 2 \Gamma_{cd} \right] \rho_{dc} \]
\[ - \frac{i}{2} \Omega_p \dot{\rho}_{da}, \] (26)

\[ \dot{\rho}_{ab} = \frac{i}{2} (\Omega_1 + \Omega_2) \cos \phi + i(\Omega_1 - \Omega_2) \sin \phi \dot{\rho}_{db} \]
\[ + \frac{i}{2} \Omega_p \dot{\rho}_{bc} - \frac{i}{2} \Omega_3 \rho_{ac} + (i(\Delta - \delta_3) - \Gamma_{ab}) \rho_{ab}, \] (27)

\[ \dot{\rho}_{bd} = \frac{1}{2} \left[ i (2 \delta_3 + \delta_1 + \delta_2 - 2 \Delta) - 2 \Gamma_{bd} \right] \rho_{bd} \]
\[ - \frac{i}{2} (\Omega_1 + \Omega_2) \cos \phi + i(\Omega_1 - \Omega_2) \sin \phi \dot{\rho}_{ba} \]
\[ + \frac{i}{2} \Omega_3 \rho_{ct}, \] (28)

In the above equations the phase \( \phi = \pi (\nu_2 - \nu_1) t \) is associated with the frequency difference of the two coherent driving pump fields of the system.

VIII. APPENDIX-B

The expressions \( P_{j=1-4}(\Delta) \) of the Eq. (3) are listed below
\[ P_{1,2}(\Delta) = \frac{|\Omega_1| |K_1| (\Gamma_{ab} - i(\Delta - \delta_1 - \delta_3)) - \frac{\Omega_2^2}{4}}{M(\Delta)(\Gamma_{ad} + \Gamma_{ac})(\Gamma_{ad}^2 + \delta_{1,2}^2)}, \] (29)
and
\[ P_{3,4}(\Delta) = \frac{|\Omega_2| |K_1| (\Gamma_{ab} - i(\Delta - \delta_2 - \delta_3)) - \frac{\Omega_2^2}{4}}{M(\Delta)(\Gamma_{ad} + \Gamma_{bd})(\Gamma_{bd}^2 + \delta_{1,2}^2)}, \] (31)
respectively, where
\[ M(\Delta) = (\Gamma_{ac} - i\Delta)(\Gamma_{ab} - i(\Delta - \delta_3)) + \frac{\Omega_2^2}{4}, \] (32)

However, in Eq. (6) \( A_i \) for \( i = 1 - 4 \) are given by
\[ A_{1,2}(\Delta) = \frac{2 \Gamma_{ad} (\Gamma_{ac} - i(2 \nu + \Delta - \delta_3)) |\Omega_{1.2}|^2}{(\Gamma_{ad} + \Gamma_{ac})(\Gamma_{ad}^2 + (\nu + \delta_{1,2})^2)[\beta + \frac{\Omega_2^2}{4}]}, \] (33)
and
\[ A_{3,4}(\Delta) = \frac{[\alpha (\Gamma_{bd} - i(\nu + \Delta - \delta_3))] - \frac{\Omega_2^2}{4}|\Omega_2^2}{(\Gamma_{ad} + \Gamma_{bd})(\Gamma_{bd}^2 + (\nu + \delta_{1,2})^2)[\beta + \frac{\Omega_2^2}{4}]}, \] (34)
respectively, where
\[ B_{1,2}(\Delta) = [(\Gamma_{cd} - i(\Delta - \delta_{1,2}))(\Gamma_{bd} - i(\nu + \Delta - \delta_1 - \delta_2 - \delta_3))] + \frac{\Omega_2^2}{4} \],
\[ T_{1,2} = (\Gamma_{cd} - i(\Delta - \delta_{1,2}))(\Gamma_{bd} - i(\Delta - \delta_1 - \delta_2 - \delta_3)), \]
\[ \beta = (\Gamma_{ac} - i(\nu + \Delta))(\Gamma_{ab} - i(2 \nu + \Delta - \delta_3)), \]
\[ K_1 = [\Gamma_{ab} - i(\Delta - \delta_3)], \]
and
\[ \alpha = (\Gamma_{ab} - i(2 \nu + \Delta - \delta_3)). \]

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