Modelling Bubble Lifetime of Thin Film Surfactants Solution on Fuel Spillage

Shitakha Felistus\textsuperscript{1*}, Kimathi George\textsuperscript{1} and Songa Caroline\textsuperscript{2}

\textsuperscript{1}Department of Mathematics and Applied Science, The Catholic University of Eastern Africa, Kenya.
\textsuperscript{2}School of Physics, The Catholic University of Eastern Africa, Kenya.

Authors’ contributions
This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information
DOI: 10.9734/ARJOM/2021/v17i430291
Editor(s):
(1) Dr. Sheng Zhang, Bohai University, China.

Reviewers:
(1) Noor Aniza Harun, Universiti Malaysia Terengganu, Malaysia.
(2) Om Prakash Meena, University of Delhi, India.

Complete Peer review History: http://www.sdiarticle4.com/review-history/69258

Received: 01 April 2021
Accepted: 05 June 2021
Original Research Article
Published: 09 June 2021

Abstract

Aims / Objectives: To find the lifetime of the bubble by plotting the rate of mass flow rate change against time.

Place and Duration of Study: Department of Mathematics and Applied Science, Catholic University of Eastern Africa, Nairobi, Kenya, between February 2020 and March 2021.

Methodology: The maximum lifetime of the bubble is assumed to match the time when the mass flow rate change is zero. The study also assumes the velocity of flow rate and other fluid properties at the interface of fuel-surfactant constant other than $Re$. $Re$ is varied from 0.01 to 100.

Results: The graphical plots show that for $Re \ll 1$, and $Re \gg 1$, the stability depends on diffusive viscosity and linearized convection, respectively. The simulation suggested that the bubble formed at the fuel-surfactant interface may have $Re = 1$ and its lifetime is $t_b \approx 0.28$.

Conclusion: The lifetime of surfactant depends on $Re$ while assuming other interface properties constant.

\*Corresponding author: E-mail: felistusmamaishitakha@outlook.com;
Recommendation: Future studies in the area need to consider the effect of variation in temperature, velocity, and Reynolds number in determining the lifetime of a bubble in the thin foam of the surfactant-fuel interface.

Keywords: Fuel-Surfactant interface; Bubble lifetime; interface thickness.

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16.

1 Introduction

Surfactants help in many human activities such as coating, cosmetics, environmental protection, food, and medical industry [1]. Surfactants, for instance, on fuel, allow multi-phases and formation of colloids [2]. The thin film spread specifically on fuel has numerous advantages, such as the prevention of fuel explosion. Of importance is the determination of the lifetime of the thin film given the real-world problem of fuel spillage [3]-[20].

Surfactants thin film spread on fuel is governed by surface tension [21], which spread uniformly across the surface of the fuel spill. The surface tension induces shear stresses at the fuel-air interface [22]. The stresses distribute the surfactants evenly over the surface of the fuel from low to high areas of surface tension. The variation in surface tension due to variation in stresses results in variation in spill heights, known as Marangoni flow [23, 24, 25]. Surfactants spreads are fundamental in the attainment of this phenomenon by fuel spillage. Literature on Marangoni flow are covered in a lot of existing materials [23, 24, 25, 26, 27, 28, 29].

Marangoni stresses causes non-uniform spreading hence depletion of the thin spread [30, 31]. These stresses cause the bubbles on the thin film surfactants on fuel (see Fig. 1). The spread depletion is analogous to the life of the bubble formed on the fuel due to surfactant spread on the fuel surface during spillage [32].

Cavitation is the formation and dynamic life of bubbles in or on the surface upon destabilization due to pressure difference causing stresses [33]. Bremond et al. [34], Chahine [35], Golykh [36], Kiyama et al. [37], Kyriazis et al. [38], and Reynolds [39] widely studies cavitation under various configurations like near the solid walls, and gas-liquid interface, and so on. Existing studies on the dynamics of bubbles growth and collapse in thin film or liquid are presented in the following literature [39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49]. A recent study by Li et al. [50] studied the effect of surfactant and evaporation on the thin liquid film spreading in the presence of surface acoustic waves. Li et al. [50] study is based on a theoretical model of liquid film flow in the presence of surface acoustic waves (SAWs). Li et al. [50] assumed SAWs affect insoluble surfactant and leading to evaporation on the spreading process of the partially wetting thin liquid film. However closer review of Li et al. [50] study on cavitation reveal that the study it did not consider a lifetime of bubble on thin foam or surface of fuel. Nonetheless, none of the existing studies have modeled a lifetime of bubble based on fuel-surfactant thin film layer, which has prompted the proposed study [51]-[70].

The first research on cavitation by Reynolds [39] suggested that the lifetime of a bubble depends on the boundary or surface actions of fluids, which is affected by viscosity, velocity, temperature difference, and Reynolds number. The bubble rises due to buoyancy force at the free surface leading to the ejection of aerosols into the surface, making their use dynamic in many fields [71, 72]. This tendency makes bubble formation important in fuel-surfactant thinning since its lifetime determines the time before the potential explosion [73]-[100]. Thus, the proposed paper takes an interest in
lifetime of bubbles on a thin layer between fuel and surfactant. The paper focuses on modeling the lifetime of the bubble without considering the fuel-surfactant properties. The paper considers the variation in Reynolds number in the thin film to model the life of the bubble on the surfactant-fuel interface. Modeling the lifetime of the bubble is essential in estimating the time before fuel vapor is released into the atmosphere. Thus, the proposed research seeks to inform firefighters on the time needed to prevent explosions after surfactant spread.

The rest of the paper is organized as follows. Section 2 presents the dynamics of the model formulated and parameter transformation. The parameters were transformed into dimensionless form and contains fluid flow governing equations and interface coupling conditions for the bubble. Section 3 presents graphical simulations and results in line with the discussion of the findings. Section 4 presents findings conclusion and possible future studies.

2 Problem Formulation Model

We consider bubbles formed on thin layer of the surfactant-fuel foam (see Fig. 1). The fluids have constant density \( \rho_b \) and \( \rho_f \), velocity component \( u_{bs} \) and \( u_{bf} \), viscosity \( \nu_b \) and \( \nu_f \), where subscript \( s \) and \( f \) stand for surfactants and fuel respectively. We assume the two components of velocity \( \mathbf{U}_b \) by \( u_{xs}, u_{ys} \) and \( \mathbf{V}_f \) by \( v_{xs}, v_{ys} \) so that the fluid velocity in all the coordinates are described by

\[
\frac{dx}{dt} = u = u_b(x, y, z, t), \tag{1}
\]

then,

\[
\frac{df}{dt} = \mathbf{U}_b = \frac{d}{dt} f \left[ (u_{xs}(x, y, t), u_{ys}(x, y, t)) \right], \tag{2}
\]

where \( t \) is time. The atmospheric pressure leading to buoyancy forces are all assumed constant at \( g \). We also assume a uniform spread of surfactant over the fuel spillage. We consider the dimensional form of incompressible Navier-Stokes equations

\[
\frac{\partial \mathbf{U}_b}{\partial x_b} = 0 \tag{3a}
\]

\[
\frac{\partial \mathbf{U}_b}{\partial t} = -\frac{\partial (u_{xs} u_{ys})}{\partial x_b} + 2\nu \frac{\partial D_b}{\partial x_b} - \frac{1}{\rho} \left( \frac{\partial p_b}{\partial x_b} - s_b \right) \tag{3b}
\]

\[
= -\frac{\partial (u_{xs} u_{ys})}{\partial x_b} + \nu \frac{\partial^2 u_b}{\partial x_b^2} - \frac{1}{\rho} \left( \frac{\partial p_b}{\partial x_b} - s_b \right), \tag{3c}
\]

with the Non-dimensionalized parameters as

\[
t^* = \frac{U_b}{L} \tag{4a}
\]

\[
x_b^* = \frac{x_b}{L} \tag{4b}
\]

\[
p^* = \frac{p}{\rho u_b^2} \tag{4c}
\]

\[
\frac{1}{Fr} = \frac{gL}{u_b^2} \tag{4d}
\]

\[
u_b^* = \frac{u_b}{u_b} \tag{4e}
\]

\[
P_b = p_b^* + \frac{x_b^*}{Fr^2} s = \left[ \begin{array}{c} 0 \\ -\rho u_b g \end{array} \right] \tag{4f}
\]

\[
2\frac{\partial^2 D_b}{\partial x_b} = \frac{\partial^2 u_b}{\partial x_b^2} \tag{4g}
\]
Non-dimensionalized mass conservation equations

\[
\frac{U_b}{L_b} \frac{\partial u_b^*}{\partial x_b^*} = \frac{\partial u_b^*}{\partial x_b^*} = \frac{\partial}{\partial t} \left( \frac{U_b}{L_b} u_b^* \right) - \frac{\partial}{\partial x_b^*} \left( \frac{U_b}{L_b} \frac{\partial u_b^*}{\partial x_b^*} \right)
\]

(4h)

\[
\frac{U_b}{L_b} \frac{\partial u_b^*}{\partial x_b^*} = \frac{\partial u_b^*}{\partial x_b^*} = \frac{\partial}{\partial t} \left( \frac{U_b}{L_b} u_b^* \right) - \frac{\partial}{\partial x_b^*} \left( \frac{U_b}{L_b} \frac{\partial u_b^*}{\partial x_b^*} \right)
\]

(4i)

Atasi et al. [101] established that the largest single drainage bubble has a thickness of \( h_b \) of the thin film. The bubble decays at an exponential rate due to competition between stresses arising from drainage forces.

\[
\frac{h_b}{h_{b0}} = e^{-\frac{a_b}{r_b}},
\]

(6)

where \( h_{b0} \) is the initial film thickness, \( a_b \) is thinning rate, \( t_b \) bubble lifetime and \( r_b = \frac{a_b}{\rho g R_b} \) is the extensional flow based on viscous-gravity forces. \( \rho \) is the density at the interface, \( g \) is gravitational acceleration, and \( R_b \) is the bubble assumed radius also defined as \( R_b = \left( \frac{4}{\pi} V_b \right)^{\frac{1}{3}} \) with \( V \) being the bubble volume [102, 101]. We combine (5) with (6) to obtain bubble non-dimensionalized mass conservation equation as

\[
\frac{\partial u_b^*}{\partial t} = e^{-\frac{a_b}{r_b}} \left( H_b - \frac{\partial P_b}{\partial x_b^*} \right).
\]

(7)

The pressure at the interface leading to bubble failure at time \( t \) is derived by assuming divergence of the non-dimensionalized momentum equation and invoking continuity equation, that is, \( \frac{\partial u_b^*}{\partial t} = 0 \) such that,

\[
\nabla \left( \frac{\partial u_b^*}{\partial t} - H_b + \frac{\partial P_b}{\partial x_b^*} \right) = 0,
\]

(8)

\[
-\frac{\partial}{\partial t} \left( \frac{\partial u_b^*}{\partial t} - H_b + \frac{\partial P_b}{\partial x_b^*} \right) + \frac{\partial}{\partial y_b} \left( \frac{\partial v_b^*}{\partial t} - H_{y_b} + \frac{\partial P_b}{\partial y_b} \right) = 0
\]

(9)

\[
\frac{\partial^2 P_b}{\partial t^2} + \frac{\partial^2 P_b}{\partial y^2} = \frac{\partial H_{t_b}}{\partial x_b^*} + \frac{\partial H_{y_b}}{\partial y_b^*}
\]

(10)

The discretized wall functions \( H_b \in H_{t_b}, H_{y_b} \) are Navier-Stokes equation in 2D. The vector product of the normal vector to the wall and momentum equations helps us establish the boundary conditions for the pressure. Assuming the bubble is formed due to the change in momentum equation in the fuel \( f \) and surfactant \( s \).
Fig. 1. (a) Thin film foam on fuel. (b) Microscopic view of thin film bubble of (a). (c) Extract of the largest bubble formed on the surfactant-fuel thin film with thickness $h_b$ and exponential decay postulated in (6).

If we assume a constant velocity, the momentum equations describe the homogeneous Neumann boundary conditions on the interface of the surfactant and fuel. Thus, $H_{b_s}$ describes the momentum equation on the surfactant at the interface while $H_{b_f}$ is for the fuel. The $H_{b_s}$ and $H_{b_f}$ can be described by derivatives of velocity fields as

$$H_{b_s} = \frac{\partial}{\partial x_b} \left( \frac{1}{\nu} \frac{\partial u_{b_s}}{\partial x_{b_s}} - u_{b_s}^2 \right) + \frac{\partial}{\partial y_{b_s}} \left( \frac{1}{\nu} \frac{\partial u_{b_s}}{\partial y_{b_s}} - u_{b_s} u_{b_f} \right),$$  \quad (11a)

$$H_{b_f} = \frac{\partial}{\partial x_{b_f}} \left( \frac{1}{\nu} \frac{\partial u_{b_f}}{\partial x_{b_f}} - u_{b_f}^2 \right) + \frac{\partial}{\partial y_{b_f}} \left( \frac{1}{\nu} \frac{\partial u_{b_f}}{\partial y_{b_f}} - u_{b_f} u_{b_s} \right),$$  \quad (11b)

where

$$u_{b_s} = \sin(\alpha_1 x_{b_s}) \sin(\alpha_2 y_{b_s}),$$  \quad (11c)

$$u_{b_f} = \sin(\alpha_3 x_{b_f}) \sin(\alpha_4 y_{b_f}).$$  \quad (11d)
The problem presented in (1)-(11b) present diffusion Neumann boundary conditions within the fuel-surfactant interface. Assuming that the matrix equation of continuity of mass conservation is represented by

\[ A_b p_b = \Xi_b, \]  

where

\[ \Xi_b = \frac{\gamma_p}{\gamma_c} \nabla y_b + \frac{\gamma_p}{\gamma_c} \nabla x_b, \]

\[ = H_{b_s} \nabla y_b + H_{b_f} \nabla x_b. \]  

(12b)

There is no Dirichlet boundary condition; thus, the rank of \( A_b \) is enforced to \( n^2 - 1 \) showing that (12b) is singular and an infinite number of solutions exists. In order to avoid fluctuations in bubble pressure during flow, the pressure value is assumed to be zero at the center of the bubble and maximum at the highest thickness, \( h_b \). This translates to

\[ H_{b_s} = - \alpha \sin (\alpha x_b) \cos (\alpha y_b), \]

\[ H_{b_f} = - \alpha \cos (\alpha x_b) \sin (\alpha y_b), \]

\[ P_{\text{accurate}} = \cos (\alpha x_{(b_s, b_f)}) \cos (\alpha y_{(b_s, b_f)}). \]  

(13)

In order to solve (12b) taking note of (13), we employ the explicit Euler time-series method to estimate the velocity of the bubble on the thin film assuming \( t_b = (m_b + 1) \)

\[ u((b_f, b_s) m_b + 1) = u((b_f, b_s) m_b) \nabla t_b \left( H_{b_s} - \frac{D}{\varepsilon} \right) \]

\[ v((b_f, b_s) m_b + 1) = v((b_f, b_s) m_b) \nabla t_b \left( H_{b_f} - \frac{D}{\varepsilon} \right). \]  

(14)

The stability condition of the system depends on the time step in the Euler series. Conrad, and Molnar [103], Molnar et al. [104] and Smith [105] established that stability for \( Re \ll 1 \) depends on diffusive viscosity, while \( Re \gg 1 \) stability depends on linearized convection. Therefore, the stability criterion for bubbles depends on small change in time, \( \nabla t \), small change in horizontal distance \( \nabla x \).

The bubble is assumed to cease when the continuity equation is a change in flow rate, \( \nabla M_b = \frac{\partial}{\partial t_b} + \nabla (\rho u_{(b_f, b_s)}) \approx 0 \). This indicates that the Poisson equation for the velocity field is assumed to diverge freely as

\[ \frac{\partial}{\partial t_b} (\nabla \cdot u_{(b_f, b_s)}) = - \nabla m_{b_s} b_{s_f} \nabla^2 P_{(b_s, b_f)}. \]  

(15)

Equation 15 suggests a lack of local mass conservation arises due to linear growth of local continuity errors. At the time of bubble cease (bust), that is, at \( h_b \), the local conservation continuity is small but continue to grow. This means that simulations for the possible plot of local continuity against time can take several days before conclusion is drawn. In order to avoid this, the pressure must be estimated as follows:

\[ \nabla^2 P_{(b_s, b_f) m_b + 1} = \nabla H_{(b_s, b_f) m_b} + \frac{1}{\nabla t} (\nabla u_{(b_s, b_f) m_b}). \]  

(16)

3 Results

3.1 Numerical simulation and discussion

The paper focuses on modeling the lifetime of the bubble without considering the fuel-surfactant properties. The paper considers the velocity and Reynolds number of the thin film to model the life
of the bubble on the surfactant-fuel interface. The paper focuses on the effect of varying $Re$ numbers in a bubble due to surfactant and fuel thin film interface. The simulations are important in helping to estimate the time before bubble burst, which analogously fuel vapor purportedly released into the atmosphere. We assume the velocity of the surfactant-fuel thin layer is constant and has a plot of change in $\nabla M_b$ given (16), (14), and (15) considering varying $Re = [0.01, 0.1, 1, 10, 100, 1000]$ and $a_b = 0.2$ yield the graphical representations in Fig. 2a-2e.

Fig. 2a indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches $h_b$ when $Re = 0.01$. The graph indicate the rate of change of mass flow rate reduces uniformly from $t_b = 0$. The change began steeply then decelerates.

![Fig. 2. Rate of change of mass flow rate at: (a) $Re = 0.01$; (b) $Re = 0.1$; (c) $Re = 1$; (d) $Re = 10$; and (e) $Re = 100$.](image-url)
Fig. 2b indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches $h_b$ when $Re = 0.1$. The graph indicate the rate of change of mass flow rate posses an almost similar trend as that of Fig. 2a.

Fig. 2c indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches $h_b$ when $Re = 1$. The graph indicate the rate of change of mass flow rate reduces uniformly from $t_b = 0$. Unlike Fig. 2a and 2b, here, the graph indicate the $h_b$ will be reached when $t_b = 0.28s$. The observation proved the notion by [103], Molnar et al. [104] and Smith [105] that for $Re \ll 1$ the stability depends on diffusive viscosity, hence plot may take longer to obtain when thickness value becomes $h_b$.

Fig. 2d indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches $h_b$ when $Re = 10$. The graph indicate the rate of change of mass flow rate reduces uniformly from $t_b = 0$. Similarly as in Fig. 2a and 2b and unlike in Fig. 2c the graph indicate it will take longer time for thickness to reach $h_b$. The observation proved the notion by [103], Molnar et al. [104] and Smith [105] that for $Re \gg 1$ the stability depends on linearized convection, hence plot may take longer to obtain when thickness value becomes $h_b$.

4 Conclusion

Surfactants are important in many activities, such as the prevention of fuel explosion. The determination of the lifetime bubble, which depends on its thickness, is a real-world problem of fuel spillage. The proposed study aimed to establish the lifetime of the bubble. The study assumes that the rate of change of mass flow rate is proportional to the thinning rate. The study, thus, aims to find the lifetime of the bubble by plotting the rate of mass flow rate change against time. The maximum lifetime of the bubble is assumed to match the time when the mass flow rate change is zero. The study also assumes the velocity of flow rate and other fluid properties at the interface of fuel-surfactant constant other than $Re$. $Re$ is varied from 0.01 to 100.

The graphical plots suggest that for $Re \ll 1$, and $Re \gg 1$, the stability depends on diffusive viscosity and linearized convection, respectively. These findings are similar to those of [103], Molnar et al. [104] and Smith [105]. The simulation suggested that the fuel-surfactant interface may have $Re = 1$. However, since no study has modeled a lifetime of bubble based on fuel-surfactant thin film layer, the proposed study lacks no comparison of results with existing studies. Therefore, we will assume the bubble formed at the interface of fuel-surfactant thin foam has $Re = 1$ and its lifetime is $t_b \approx 0.28$. Future studies in the area need to consider the effect of variation in temperature, velocity, and Reynolds number in determining the lifetime of a bubble in the thin foam of the surfactant-fuel interface.

Acknowledgement

The research in this work was supported by the Technical University of Kenya (TU-K).
Competing Interests

Authors have declared that no competing interests exist.

References

[1] Liu Yingxin, Jessop Philip G, Cunningham Michael, Eckert Charles A, Liotta Charles L. Switchable surfactants. Science. 2006;313(5789):958-960.
[2] Marliere Claire, Creton Benoit, Oukhemanou Fanny, Wartenberg Nicolas, Courtaud Tiphaine, Fénéron Christophe, Betoulle Stéphane, Defiolle Denis, Mougin Pascal, others. Impact of live crude oil composition on optimal salinity of a surfactant formulation. SPE EOR Conference at Oil and Gas West Asia; 2016.
[3] Fingas Merv. Introduction to spill modeling. Oil Spill Science and Technology. 2011;187-200.
[4] Schubert Gerald, Turcotte Donald Lawson, Olson Peter. Mantle convection in the Earth and planets. Cambridge University Press; 2001.
[5] Pugh RJ. Foaming, foam films, antifoaming and defoaming. Advances in Colloid and Interface Science. 1996;64:67-142.
[6] Lopez JM, Hirsa A. Dynamics of surfactant-influenced gas/liquid interfaces. Second International Conference on CFD in the Minerals and Process Industries, CSIRO, Melbourne, Australia. 1999;6-8.
[7] Tackley Paul J. Mantle geochemical geodynamics. Treatise on Geophysics. 2007;7:437-505.
[8] Myers Drew, others. Surfaces, interfaces, and colloids. Wiley New York. 1999;415.
[9] Thin films with high surface tension. SIAM Review. 1998;40(3):441-462.
[10] Farajzadeh Rouhollah, Andrianov Alexey, Krastev Rumen, Hirasaki GJ, Rossen William Richard. Advances in Colloid and Interface Science. 2012;183:1-13.
[11] Amaral Maria Helena, das Neves José, Oliveira Ângela Z, Bahia M Fernanda. Foamability of detergent solutions prepared with different types of surfactants and waters. Journal of surfactants and Detergents. 2008;11(4):275-278.
[12] Chesters Allan K, Buzhikov Ivan B. Effect of insoluble surfactants on drainage and rupture of a film between drops interacting under a constant force. Journal of Colloid and Interface Science. 2000;230(2):229-243.
[13] Li Daming, Tang Xingchen, Li Yanqing, Wang Xiao, Zhang Hongqiang. Mathematical Modeling of Marine Oil Spills in the Luanjiangou District, near the Port of Yantai. Discrete Dynamics in Nature and Society; 2018.
[14] Basu S, Nandakumar K, Masliyah Jacob H. A study of oil displacement on model surfaces. Journal of Colloid and Interface Science. 1996;182(1):82-94.
[15] Palmer Harvey J, Berg John C. Hydrodynamic stability of surfactant solutions heated from below. Journal of Fluid Mechanics. 1972;51(2):385-402.
[16] Noskov BA, Loglio G. Dynamic surface elasticity of surfactant solutions. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 1998;143(2-3):167-183.
[17] Bhat Farooq Ahmad, Samanta, Arghya. Linear stability analysis of a surfactant-laden shear-imposed falling film. Physics of Fluids. 2019;31(5):054103.
[18] Fletcher Thomas Hobbs. From Love Canal to environmental justice: the politics of hazardous waste on the Canada-US border. Broadview Press Peterborough; 2003.
[19] Blawzdziewicz Jerzy, Cristini Vittorio, Loewenberg Michael. Near-contact motion of surfactant-covered spherical drops: ionic surfactant. Journal of colloid and interface science. 1999;211(2):355-366.

[20] Booty MR, Siegel M. A hybrid numerical method for interfacial fluid flow with soluble surfactant. Journal of Computational Physics. 2010;229(10):3864-3883.

[21] Lu Jiakai, Corvalan Carlos M, Chew YM John, Huang Jen-Yi. Coalescence of small bubbles with surfactants. Chemical Engineering Science. 2019;196:493-500.

[22] Glassman Irvin, Dryer Frederick L. Flame spreading across liquid fuels. Fire Safety Journal. 1981;3(2):123-138.

[23] Nikolov Alex D, Wasan Darsh T, Chengara Anoop, Koczo Kalman, Policello George A, Kolossvary Istvan. Superspreading Driven by Marangoni Flow. 2002;96(1-3):325-338.

[24] Xu Xuefeng, Luo Jianbin. Marangoni flow in an evaporating water droplet. Applied Physics Letters. 2007;91(12):124102.

[25] Namura Kyoko, Sono Toshiki, Kumar Samir, Nakajima Kaoru, Suzuki Motofumi. Thermochromic visualization of the heated region around a microbubble during Marangoni flow generation. Journal of Nanophotonics. 2021;15(1):016007.

[26] Li Yanshen, Diddens Christian, Prosperetti Andrea, Lohse Detlef. Marangoni instability of a drop in a stably stratified liquid. Physical Review Letters. 2021;126(12):124502.

[27] Ahmadian-Yazdi Mohammad-Reza, Eslamian Morteza. Effect of marangoni convection on the perovskite thin liquid film deposition. Langmuir. 2021;37(8):2596-2606.

[28] Sauleda Madeline L, Chu Henry CW, Tilton Robert D, Garo Stephen. Surfactant driven marangoni spreading in the presence of predeposited insoluble surfactant monolayers. Langmuir; 2021.

[29] Pesach Dora, Marmur Abraham. Marangoni effects in the spreading of liquid mixtures on a solid. Langmuir. 1987;3(4):519-524.

[30] Wei Hsien-Hung. Marangoni-enhanced capillary wetting in surfactant-driven superspreading. Journal of Fluid Mechanics. 2018;855:181-209.

[31] Lee KS, Ivanova N, Starov VM, Hilal N, Dutschk V. Kinetics of wetting and spreading by aqueous surfactant solutions. Advances in Colloid and Interface Science. 2008;144(1-2):54-65.

[32] Zhong Yu, Rogers RE. Surfactant effects on gas hydrate formation. Chemical Engineering Science. 2000;55(19):4175-4187.

[33] Bai Lixin, Yan Jiuchun, Zeng Zhijie, Ma Yuhang. Cavitation in thin liquid layer: A review. Ultrasound Sonochemistry. 2020;66:105092.

[34] Bremond Nicolas, Arora Manish, Ohl Claus-Dieter, Lohse Detlef. Cavitation on surfaces. Journal of Physics: Condensed Matter. 2005;17(45):S3603.

[35] Chahine Georges L. Modeling of cavitation dynamics and interaction with material. Advanced Experimental and Numerical Techniques for Cavitation Erosion Prediction. 2014;123-161.

[36] Golykh Roman N. Theoretical and experimental study of cavitation dispersing in “liquid-solid” system for revelation of optimum influence modes. American Journal of Engineering Research. 2016;11:159.

[37] Kiyama, Akihito and Tagawa, Yoshiyuki and Ando, Keita and Kameda, Masaharu. Effects of a water hammer and cavitation on jet formation in a test tube. Journal of Fluid Mechanics. 2016;787:224-236.

[38] Kyriazis Nikolaos, Koulovounis Phoevos, Gavaises Manolis. Modelling cavitation during drop impact on solid surfaces. Advances in Colloid and Interface Science. 2018;260:46-64.
[39] Reynolds Osborne. IV. On the theory of lubrication and its application to Mr. Beauchamp tower’s experiments, including an experimental determination of the viscosity of olive oil. Philosophical transactions of the Royal Society of London. 1886;177:157-234.

[40] Dear JP, Field JE. A study of the collapse of arrays of cavities. Journal of Fluid Mechanics. 1988;190:409-425.

[41] Bourne NK, Field JE. Shock-induced collapse of single cavities in liquids. Journal of Fluid Mechanics. 1992;244:225-240.

[42] Bourne NK. On the collapse of cavities. Shock Waves. 2002;11(6):447-455.

[43] Cui Jianying, Hamilton Mark F, Wilson Preston S, Zabolotskaya Evgenia A. Bubble pulsations between parallel plates. The Journal of the Acoustical Society of America. 2006;119(4):2067-2072.

[44] Azam Fahad Ibn, Karri Badarinath, Ohl Siew-Wan, Klaseboer Evert, Khoo Boo Cheong. Dynamics of an oscillating bubble in a narrow gap. Physical Review E. 2013;88(4):043006.

[45] Mohammadzadeh Milad, Li Fenfang, Ohl Claus-Dieter. Shearing flow from transient bubble oscillations in narrow gaps. Physical Review Fluids. 2017;2(1):014301.

[46] Quinto-Su Pedro A, Lim Kang Y, Ohl Claus-Dieter. Cavitation bubble dynamics in microfluidic gaps of variable height. Physical Review E. 2009;80(4):047301.

[47] Gonzalez-Avila Silvestre Roberto, van Blokland Anne Charlotte, Zeng Qingyun, Ohl Claus-Dieter. Jetting and shear stress enhancement from cavitation bubbles collapsing in a narrow gap. Journal of Fluid Mechanics. 2020;884.

[48] Hsiao C-T, Choi J-K, Singh S, Chahine GL, Hay TA, Ilinskii Yu A, Zabolotskaya EA, Hamilton MF, Sankin G, Yuan F, others. Modelling single-and tandem-bubble dynamics between two parallel plates for biomedical applications. Journal of Fluid Mechanics. 2013;716.

[49] Kuvshinov GI, Prokhorenko PP, Dezhkunov NV, Kuvshinov VI. Collapse of a cavitation bubble between two solid walls. International Journal of Heat and Mass Transfer. 1982;25(3):381-387.

[50] Li Chunxi, Shi Zhixian, Xiao Han, Ye Xuemin. Effect of surfactant and evaporation on the thin liquid film spreading in the presence of surface acoustic waves. Physics of Fluids. 2020;32(6):062106.

[51] Boomkamp PAM, Boersma BJ, Miesen RHM, Beijnon GV. A Chebyshev collocation method for solving two-phase flow stability problems. Journal of computational Physics. 1997;132(2):191-200.

[52] Breward CJW, Howell PD. Straining flow of a micellar surfactant solution. European Journal of Applied Mathematics. 2004;511-531.

[53] Karapetisas George, Craster Richard V, Matar Omar K. On surfactant-enhanced spreading and superspreading of liquid drops on solid surfaces. Journal of Fluid Mechanics. 2011;670(5).

[54] Craster RV, Matar OK, Papageorgiou DT. Breakup of surfactant-laden jets above the critical micelle concentration. Journal of Fluid Mechanics. 2009;629:195-219.

[55] Edmonstone BD, Craster RV, Matar OK. Surfactant-induced fingering phenomena beyond the critical micelle concentration. Journal of Fluid Mechanics. 2006;564:105.

[56] Shaw DJ. Introduction to surface and colloid chemistry. Butterworth-Heinemann: London; 1992.

[57] Milliken WJ, Stone Howard A, Leal LG. The effect of surfactant on the transient motion of Newtonian drops. Physics of Fluids A: Fluid Dynamics. 1993;5(1):67-79.

[58] Ji Wei, Setterwall Fredrik. On the instabilities of vertical falling liquid films in the presence of surface-active solute. Journal of Fluid Mechanics. 1994;278:297-323.
[59] Karapetsas George, Bontozoglou Vasilis. The primary instability of falling films in the presence of soluble surfactants. J. Fluid Mech. 2016;729:123-150.

[60] Karapetsas George, Bontozoglou Vasilis. The role of surfactants on the mechanism of the long-wave instability in liquid film flows. J. Fluid Mech. 2016;741:139-155.

[61] Sun ZF, Fahmy M. Onset of Rayleigh-Benard-Marangoni convection in gas-liquid mass transfer with two-phase flow: Theory. Industrial Engineering Chemistry Research. 2006;45(9):3293-3302.

[62] You Xue-Yi, Zhang Le-Dao, Zheng Jing-Ru. Marangoni instability of immiscible liquid-liquid stratified flow with a planar interface in the presence of interfacial mass transfer. Journal of the Taiwan Institute of Chemical Engineers. 2014;45(3):772-779.

[63] Zaisha Mao, Ping Lu, Zhang Guangji, Chao Yang. Chinese Journal of Chemical Engineering. 2008;16(2):161-170.

[64] Chu Kang-Ching, Hu Ssu-Wei, Tsao Heng-Kwong, Sheng Yu-Jane. Strong competition between adsorption and aggregation of surfactant in nanoscale systems. Journal of Colloid and Interface Science. 2019;553:674-681.

[65] Mohamed Abdelhalim IA, Sultan Abdullah S, Hussein Ibenlwaleed A, Al-Muntasheri Ghaithan A. Influence of surfactant structure on the stability of water-in-oil emulsions under high-temperature high-salinity conditions. Journal of Chemistry; 2017.

[66] Rangel-Yagui Carlota O, Pessoa Jr, Adalberto, Tavares L Costa. Micellar solubilization of drugs. J. Pharm. Pharm. Sci. 2005;8(2):147-163.

[67] Chang Chien-Hsiang, Franses Elias I. Adsorption dynamics of surfactants at the air/water interface: a critical review of mathematical models, data, and mechanisms. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 1995;100:1-45.

[68] Rosen Milton J, Hua Xi Yuan, Zhu Zhen Huo. Dynamic surface tension of aqueous surfactant solutions: IV relationship to foaming. Surfactants in Solution. Springer. 1991;315-327.

[69] Yih Chia-Shun. Instability due to viscosity stratification. Journal of Fluid Mechanics. 1967;27(2):337-352.

[70] Renardy Yuriko. Instability at the interface between two shearing fluids in a channel. The Physics of Fluids. 1985;28(12):3441-3443.

[71] Manga M, Stone HA. Collective hydrodynamics of deformable drops and bubbles in dilute low Reynolds number suspensions. Oceanographic Literature Review. 1996;7(43):661.

[72] Gonnermann Helge M, Manga Michael. The fluid mechanics inside a volcano. Annu. Rev. Fluid Mech. 2007;39:321-356.

[73] Frenkel Alexander L, Halpern David. Stokes-flow instability due to interfacial surfactant. Physics of Fluids. 2002;14(7):L45–L48.

[74] Halpern D, Grotberg JB. Dynamics and transport of a localized soluble surfactant on a thin film. Journal of Fluid Mechanics. 1992;237:1-11.

[75] Pozrikidis C. Effect of inertia on the Marangoni instability of two-layer channel flow, Part I: numerical simulations. Journal of Engineering Mathematics. 2004;50(2-3):311-327.

[76] Pozrikidis C. Instability of multi-layer channel and film flows. Advances in Applied Mechanics. 2004:40:179-239.

[77] Kalogirou Anna, Papageorgiou Demetrios T. Nonlinear dynamics of surfactant-laden two-fluid Couette flows in the presence of inertia. Journal of Fluid Mechanics. 2016;802:5-36.

[78] Kalogirou Anna, Blyth Mark. The role of soluble surfactants in the linear stability of two-layer flow in a channel. Journal of Fluid Mechanics. 2019;873:18-48.
[79] Kalogirou, Anna and Blyth, Mark G. Nonlinear dynamics of two-layer channel flow with soluble surfactant below or above the critical micelle concentration. Journal of Fluid Mechanics. 2020;900.

[80] Kalogirou Anna. Instability of two-layer film flows due to the interacting effects of surfactants, inertia, and gravity. Physics of Fluids. 2018;30(3):030707.

[81] Bassom, Andrew P and Blyth, MG and Papageorgiou, DT. Nonlinear development of two-layer Couette–Poiseuille flow in the presence of surfactant. Physics of Fluids. 2010;22(10): 102102.

[82] Frenkel Alexander L, Halpern David. Strongly nonlinear nature of interfacial-surfactant instability of Couette flow. arXiv Preprint Nlin; 2006.

[83] Wei Hsien-Hung. On the flow-induced Marangoni instability due to the presence of surfactant. Journal of Fluid Mechanics. 2005;544:173.

[84] Blyth MG, Pozrikidis C. Effect of inertia on the Marangoni instability of two-layer channel flow, Part II: normal-mode analysis. Journal of Engineering Mathematics. 2004;50(2-3):329-341.

[85] Halpern David, Frenkel Alexander L. Destabilization of a creeping flow by interfacial surfactant: linear theory extended to all wavenumbers. Journal of Fluid Mechanics. 2003;485:191-220.

[86] Hooper AP, Boyd WGC. Shear-flow instability due to a wall and a viscosity discontinuity at the interface. Journal of Fluid Mechanics. 1987;179:201-225.

[87] Yiantsios Stergios G, Higgins Brian G. Linear stability of plane Poiseuille flow of two superposed fluids. The Physics of Fluids. 1988;31(11):32253238.

[88] Song Qing, Couzis Alexander, Somasundaran Ponniseril, Maldarelli Charles. A transport model for the adsorption of surfactant from micelle solutions onto a clean air/water interface in the limit of rapid aggregate disassembly relative to diffusion and supporting dynamic tension experiments. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 2006;282:162-182.

[89] Craster Richard V, Matar Omar K. Dynamics and stability of thin liquid films. Reviews of Modern Physics. 2009;81(3):1131.

[90] De Sourav, Malik Susanta, Ghosh Aniruddha, Saha Rumpa, Saha Bidyut. A review on natural surfactants. RSC Advances. 2015;5(81):65757-65767.

[91] Grotberg JB. Pulmonary flow and transport phenomena. Annual Review of Fluid Mechanics. 1994;26(2):529-571.

[92] Chesters Allan K, Bazhlekov Ivan B. Effect of insoluble surfactants on drainage and rupture of a film between drops interacting under a constant force. Journal of Colloid and Interface Science. 2000;230(2):229-243.

[93] Li Daming, Tang Xingchen, Li Yanqing, Wang Xiao, Zhang Hongqiang. Mathematical Modeling of Marine Oil Spills in the Luanjiakou District, near the Port of Yantai. Discrete Dynamics in Nature and Society; 2018.

[94] Basu S, Nandakumar K, Masliyah Jacob H. A study of oil displacement on model surfaces. Journal of colloid and interface science. 1996;182(1):82-94.

[95] Palmer Harvey J, Berg John C. Hydrodynamic stability of surfactant solutions heated from below. Journal of Fluid Mechanics. 1972;51(2):385-402.

[96] Noskov BA, Loglio G. Dynamic surface elasticity of surfactant solutions. Colloids and Surfaces A: Physicochemical and Engineering Aspects. 1998;143(2-3):167183.

[97] Bhat Farooq Ahmad, Samanta Arghya. Linear stability analysis of a surfactant-laden shear-imposed falling film. Physics of Fluids. 2019;31(5):054103.

[98] Fletcher Thomas Hobbs. From Love Canal to environmental justice: the politics of hazardous waste on the Canada-US border. Broadview Press Peterborough; 2003.
[99] Bławzdziewicz Jerzy, Cristini Vittorio, Loewenberg Michael. Near-contact motion of surfactant-covered spherical drops: ionic surfactant. Journal of Colloid and Interface Science. 1999;211(2):355-366.

[100] Picardo, Jason R and Radhakrishna, TG and Pushpavanam, S. Solutal Marangoni instability in layered two-phase flows. Journal of Fluid Mechanics. 2016;793:280-315.

[101] Atasi Omer, Legendre Dominique, Haut Benoit, Zenit Roberto, Scheid Benoit. Lifetime of surface bubbles in surfactant solutions. Langmuir. 2020;36(27):7749-7764.

[102] Breward CJK, Howell PD. The drainage of a foam lamella. Journal of Fluid Mechanics. 2002;379-406.

[103] Conrad CP, Molnar P. Convective instability of a boundary layer with temperature-and strain-rate-dependent viscosity in terms of ‘available buoyancy’. Geophysical Journal International. 1999;139(1):51-68.

[104] Molnar Peter, Houseman Gregory A, Conrad Clinton P. Rayleigh-Taylor instability and convective thinning of mechanically thickened lithosphere: effects of non-linear viscosity decreasing exponentially with depth and of horizontal shortening of the layer. Geophysical Journal International. 1998;133(3):568-584.

[105] Smith KA. On convective instability induced by surface-tension gradients. Journal of Fluid Mechanics. 1966;24(2):401-414.

© 2021 Felistus et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sciarticle4.com/review-history/69258