Effects of misalignments on thermally induced shapes of woven fabric reinforced laminates

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Abstract. Fabric misalignment leads to unsymmetry in fibre reinforced composite laminates. This unsymmetry induces out-of-plane deformation after processing. This paper evaluates the influence of fabric misalignment on the out-of-plane deformation of flat press-consolidated thermoplastic laminates. The results of this analytical and experimentally validated study shows a strong relation between deformations and length/width ratio. In particular narrow laminates show high twisting values. The results obtained are of importance for the prediction of deformations of stamp formed thermoplastic laminates.

1. Introduction
An often demanding requirement when designing and producing a fibre reinforced composite part is the control of the tolerance on the deformation of the part after processing. The source of such deformations is generally well recognised though not always understood in details, being partly originating from the inherently high anisotropic thermo-mechanical behaviour of the material, and from process induced changes in (local) fibre orientation and fraction. These two simple observations are closely linked, as a local change in fibre orientation when for example forming a composite part will lead to local unsymmetry of the laminate, leading not only to a change in thermo-mechanical properties but also to out-of-plane deformation. The control of the relation between the manufacturing process and the change in laminate morphology is therefore necessary to meet tight shape deformation tolerances. This paper is not an attempt to enhance the knowledge on this issue, but addresses the effect of misalignment of woven fabric based composite laminates on their post processing out-of-plane deformations.

Early studies on the room temperature shapes on unsymmetric cross-ply laminates showed that the curvatures measured are a function of the geometry of the laminate [1]. Not only the magnitude of the curvature does vary with the size, but also the type of shape, with even bi-stable shapes. A model was proposed based on a non-linear strain-displacement relation and a minimisation of the total potential energy. Several authors have developed the model further, in order to predict the shapes of other types of laminates [2] as well as rectangular laminates [3].

This paper uses a similar approach to evaluate the shapes of laminates having an originally symmetric lay-up, with a fibre orientation misalignment, making the results applicable for process induced fibre orientation changes. Focus is here on woven fabric based laminates. Both a model and validation are presented in this study. The next part will summarise the modelling which includes an evaluation of the thermo-mechanical properties of misaligned woven fabric laminates and the non-linear model able to predict the relation between degree of misalignment, size and deformation. The third part relates on the
material, process and measurement equipment used. The fourth part elaborates on both numerical and experimental results, followed by conclusions.

2. Analytical model
This section presents the model used to predict the out-of-plane deformations of rectangular laminates containing misaligned woven fabric layers. The model description is divided in two parts, the first one proposing a procedure to evaluate the properties of laminates having misaligned woven fabric layers, the second using these properties to predict the deformations of these laminates.

2.1. Laminate properties
Laminate basic mechanical properties are often described using the Classical Lamination Theory. The main result of the CLT is the so-called ABD matrix, a laminate stiffness matrix describing relations between deformations, i.e. strains and curvatures, and loading, i.e. forces and moments. This plane stress method is based on summing the stress-strain relation of single plies and is recognised to be well suited for unidirectional ply based laminates subjected to thermo-mechanically induced deformations of limited amplitude. The interlaced fibre structure in a woven fabric layer means that the stress-strain relation is more complex and therefore requires a dedicated approach. An existing multiscale method discretising the woven structures into simple repetitive elements [1] is used in this paper. This method was also developed to analyse the behaviour of non-orthogonal fabric with limited yarn misalignment. A full description of the method is available in [2], and couples the thermo-mechanical properties of the fibre and the matrix and the basic geometrical properties of the impregnated woven fabric to provide an ABD description of the woven fabric ply with possible misalignment considered, as well as a laminate built from these layers. The method consists of a geometrical subdivision of the woven fabric, followed by a rebuilding of the weave with its thermo elastic properties.

2.2. Laminate deformations
The out-of-plane deformations of thin laminates can be predicted using the CLT approach, though it is restricted to small deformations. To this end, a method introduced by Hyer [1] to evaluate the shape of unsymmetric cross-ply UD based laminates is used in this paper. A non-linear displacement-strain definition is used for this method, which is finally solved using a Rayleigh-Ritz approach, minimising the total potential energy. This method successfully described the shapes of square unsymmetric cross-ply laminate as a function of the laminate dimensions, which change from a saddle shape at a low side length / thickness ratio to two stable cylindrical shapes for higher side length / thickness ratio. Several authors have then subsequently extended Hyer’s theory in order to apply it to other types of arbitrary layed up unsymmetric laminates [2], as well as rectangular shaped laminates [3].

The current research uses a displacement field description based on the one proposed by Dano & Hyer [2], though adapted for rectangular laminates. The out-of-plane deformations are evaluated by minimising the potential energy, based on a process-induced thermal loading. An expression for the potential energy, adapted in order to use the stiffness characteristics as described in 2.1 is given as

$$\Pi = \int_{-L_x}^{L_x} \int_{-L_y}^{L_y} \left( \frac{1}{2} \{ \varepsilon^0 \}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \{ \varepsilon^0 \} - \{ \varepsilon^0 \}^T \{ N^{Th} \} - \{ M^{Th} \} \right) dx \; dy$$

(1)

where $L_x$ and $L_y$ respectively denote the length and width of the laminate, $\{ \varepsilon^0 \}$ and $\{ \kappa \}$ the mid-plane strain and curvature vectors which contain all unknowns to be evaluated. $[A]$, $[B]$ and $[D]$ are the extensional stiffness matrix, the coupling matrix and the bending stiffness matrix of the laminate, $\{ N^{Th} \}$ and $\{ M^{Th} \}$ the process-induced thermal force and moment vectors. Both laminate stiffness matrices and thermal loading vectors are calculated specifically for woven fabrics using the method shortly described in 2.1.
The model uses a non-linear strain-displacement relation for the in-plane strains, also known as the von Karman description, where the approximation for the displacement field contains the unknown to be solved, meant at being applied for arbitrary lay-up and rectangular thin laminates. The in-plane displacements $u^0$ and $v^0$ and the out-of-plane displacement $w^0$ are defined as follows, based on work by Dano and Hyer [2], introducing a set of 14 unknowns.

\[
\begin{align*}
    u^0(x,y) &= s_1x + s_2y + s_3x^2 + s_4x^2y + s_5xy^2 + s_6y^3 \\
    v^0(x,y) &= s_2x + s_7y + s_8x^3 + s_9x^2y + s_{10}xy^2 + s_{11}y^3 \\
    w^0(x,y) &= \frac{1}{2}(ax^2 + by^2 + cxy)
\end{align*}
\]  

(2)

Strains are derived from these expression according to the von Karman strain relations

\[
\begin{align*}
    \varepsilon_x &= \varepsilon_x^0 + z\kappa_x^0 = \frac{\partial u^0}{\partial x} + \frac{1}{2} \left( \frac{\partial w^0}{\partial x} \right)^2 - z \left( \frac{\partial^2 w^0}{\partial x^2} \right) \\
    &= s_1 + 3s_3x^2 + 2s_4xy + 3s_5y^2 + \frac{(2ax + cy)^2}{8} - za \\
    \varepsilon_y &= \varepsilon_y^0 + z\kappa_y^0 = \frac{\partial v^0}{\partial y} + \frac{1}{2} \left( \frac{\partial w^0}{\partial y} \right)^2 - z \left( \frac{\partial^2 w^0}{\partial y^2} \right) \\
    &= s_7 + 3s_{11}y^2 + 2s_{10}xy + 3s_9x^2 + \frac{(2by + cx)^2}{8} - zb \\
    \varepsilon_{xy} &= \varepsilon_{xy}^0 + z\kappa_{xy}^0 = \frac{1}{2} \left( \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w^0}{\partial x} \frac{\partial w^0}{\partial y} \right) - 2z \left( \frac{\partial^2 w^0}{\partial x\partial y} \right) \\
    &= \frac{1}{2} \left( 2s_2 + \left( s_4 + 3s_8 + \frac{ac}{2} \right)x^2 + \left( 3s_6 + s_{10} + \frac{bc}{2} \right)y^2 \right) + \left( 2s_5 + 2s_9 + ab + \frac{c^2}{4} \right)xy - zc
\end{align*}
\]  

(3)

where $\varepsilon$ are the in-plane strains, the superscript $^0$ denotes the mid-plane, $\kappa$ are the curvatures, $s_i$ and $a$, $b$, $c$ are the unknown to be solved, the last three corresponding to the three curvatures.  

Substituting equation (3) into (1) gives an algebraic expression of the total potential energy. The deformed shape of a laminate corresponds to a state of minimal potential energy strain energy. This leads to a set of 14 algebraic nonlinear equations

\[
F_i = \frac{\partial \Pi}{\partial c_i} = 0; \quad c_i = s_1, s_2, ..., s_{11}, a, b, c
\]  

(4)

The 11 first equations can be reorganised and substituted in the remaining equations leading to 3 nonlinear equations to be solved for the 3 curvatures. The equations are solved for each combination of length and width using a Newton-Raphson scheme. All solutions of the system are found by running the scheme using strategically chosen starting values of the unknowns. Stability is checked by examining the second variation of the potential energy with respect to the unknown coefficients.

### 3. Material and experimental methods

This section presents the experiments performed to validate the model presented in the second chapter. The first part reports on the material used and its relevant properties. The second part describes
production of the laminates with misalignments, while the third part summarises the setup used to measure the curvatures of the laminates.

3.1. Material properties
The laminates considered in this study are based on a generic 280 g/m² Toray T300 carbon 5HS woven fabric having a width of 500 mm and used in combination with a Sabic PolyEtherImide Ultem 1000 polymer in 50 µm thick film form. The PEI matrix is amorphous and has a glass transition temperature \( T_g \) of 210°C. The thermo-mechanical properties of the constituents and the basic weave description are necessary to evaluate the basic properties of the (misaligned) layer and are given in respectively table 1 and 2. The properties of the fabric concern in particular nominal values as the warp and weft count \( C_{x,y} \), the layer thickness \( h_{lay} \), and the layer fibre volume fraction \( v_{f,lay} \). Besides, values measured on micrographs of cross section of orthogonal cross-ply laminates are the undulation \( U_{x,y} \) and the yarn fibre volume fraction \( v_{f,yarn} \). Details for the definition of these parameters can be found in [2]. A sensitivity analysis of the mentioned parameters on the curvature prediction was performed. Though the details of this analysis are omitted in this paper, it is worth adding that the model is particularly sensitive to variation in matrix properties as well as transverse properties of the fibre. The matrix properties are generally well known, in contrast to the transverse fibre properties. The elasticity modulus \( E_{f22} \) varies from 10 to 40 GPa in the literature, while the coefficient of thermal expansion \( \alpha_{f2} \) varies from 5 to 15 µε/°C. These bounds are used when comparing the measured curvatures to the predictions in the fourth part of this paper.

| Toray T300 carbon fibre [5] | Sabic PEI Ultem 1000 |
|-----------------------------|----------------------|
| \( E_{f11} \) [GPa] | 231 | \( E_m \) [GPa] | 3.3 |
| \( E_{f22} \) [GPa] | 40 | \( E_m \) [GPa] | 3.3 |
| \( G_{f12} \) [GPa] | 24 | \( G_m \) [GPa] | 1.2 |
| \( G_{f23} \) [GPa] | 14.4 | - | - |
| \( \nu_{f2} \) [-] | 0.26 | \( \nu_m \) [-] | 0.36 |
| \( \nu_{f1} \) [-] | 0.39 | - | - |
| \( \alpha_{f1} \) [µε/°C] | -0.4 | \( \alpha_{f12} \) [µε/°C] | 56 |
| \( \alpha_{f2} \) [µε/°C] | 5.6 | - | - |
| \( \rho_f \) [kg/m³] | 1760 | \( \rho_m \) [kg/m³] | 1270 |
| \( T_g \) [°C] | - | \( T_g \) [°C] | 217 |

| Table 2. Nominal (left) and measured (right) fabric specification. |
|-----------------------------|----------------------|
| \( C_x \) [1/m] | 700 | \( U_x \) [1/m] | 0.67 |
| \( C_y \) [1/m] | 710 | \( U_y \) [1/m] | 0.67 |
| \( h_{lay} \) [mm] | 0.31 | \( V_{f,yarn} \) [%] | 68 |
| \( v_{f,lay} \) [%] | 50 | - | - |
| \( v_{f,yarn} \) [%] | 68 | - | - |
3.2. Laminate production

The misalignment is performed by shearing the dry woven fabric and laminating the composite plate by a film stacking technique. It is chosen in this work to fix the warp direction of the woven fabric, while varying the weft direction. This results in an anti-symmetric \( [(0, 90-\theta)\times(90+\theta, 0)\times]\) layup, where \( 0 \) corresponds to the warp direction. The misalignment angle \( \theta \) was given values of 5°, 10° and 15°, while \( N \) was given values of 1, 2 and 3. A total of 9 test laminates was therefore produced for the validation.

A 280 g/m² T300 carbon 5HS woven fabric having a width of 500 mm is used in combination with a PolyEtherImide (PEI, Ultem 1000, 50 µm) in film form to layup the laminates. The PEI matrix is amorphous and has a glass transition temperature \( T_g \) of 210 °C.

The lay-up of a \( [(0,90-\theta)/(90+\theta,0)] \) laminate is shown in Figure 1. The glass selvage used during the weaving of the fabric is taken as a reference for the warp direction when shearing the fabrics. One side parallel to the warp yarns is fixed to prevent the fabric from shifting. Next, the opposite side is skewed to the desired misalignment angle. The resulting weft misalignment angle is checked using a protractor. A predefined amount of the 50 µm thick PEI film is added to the layup to end up with a nominal fibre volume fraction \( v_f \) of 50% in case the fabric is orthogonal. The orientation of the fabric is locked by ultrasonic spot welding the sides of the fabric and matrix layers.

![Figure 1](image)

**Figure 1.** Illustration of the film stacking of a \( [(0,90-\theta)/(90+\theta,0)] \) T300 3k 5HS - PEI laminate.

The 0 direction and the x-coordinate corresponds to the warp direction.

The laminate with misalignments are produced using a classical press consolidation process. Release agent (Marbo) coated caul sheets are used as a mould material. The laminates are consolidated at 350°C and 2 MPa pressure on a Fontijne press. The consolidation state of the laminates is controlled with ultrasonic scanning. The resulting fibre volume fraction \( v_f \) measured based on the laminate thickness varies from 52 to 54 % depending on the misalignment. A higher value than the nominal 50 % is due to some flow of matrix during the process.

3.3. Set-up and procedure to measure the curvatures

A set-up was built in order to measure the curvatures of the misaligned laminates. The principle was to measure the out-of-place \( (z) \) deflection of the laminate while scanning the laminate in the x and y coordinates. The laminate was measured in a vertical position in order to prevent deflection due to the mass of the laminate. As illustrated in Figure 2, the scanning in x and y direction was performed with respectively a manually operated linear slider and a fine screw driven linear motion system. The x and y coordinates were monitored with respectively a Linear Variable Displacement Transducer and a micrometer. The \( (z) \) deflection was measured with a 20mm range non-contact deflection transducer based on a laser and a Position Sensing Device (Micro-Epsilon NCDT).
Figure 2. Set-up for the measurement of the laminate deflection.

The surface to be measured was coated with a solvent free carbon spray, which reduced the scattering of the laser by the composite surface. The accuracy of the system was evaluated at 2% by measuring a well-defined cylinder covered with the same coating. The precision of the system was evaluated by measuring a laminate 5 times, leading to a coefficient of variation of 0.1%. The curvatures were derived by fitting a second order polynomial through the xyz data.

In order to evaluate the effect of the length to width ratio on the deformation of the laminate, it was chosen to fix the length of the laminates (x-direction in Figure 2) and vary the width (y direction). This was performed by cutting the 450 mm x 450 mm laminate in different width, with increments of 50 mm down to a width of 50 mm, and with increments of 10 mm down to a width of 20 mm.

4. Results
The numerical results are first presented, followed by the experimental results.

4.1. Numerical results
The main result of this research is a relation between the length / width aspect ratio and the curvatures of the misaligned laminates. The loading taken into consideration for the system proposed is restricted to a $\Delta T$ from $T_g$ to room temperature. Effects from tool laminate interaction are disregarded in these results. A typical plot of the three relevant curvatures for a 2 layers $[(0, 90-\theta)/(90+\theta, 0)]$ laminate for misalignments $\theta = 5$, 10, 15° having a fixed length $L_x$ of 500 mm is shown in Figure 3. The prediction on these laminate all show bi-stable shapes (B-C and B-D in Figure 3 and in corresponding Figure 4) for width having a value larger than a so-called bifurcation point (B). A third stable shape is found below the bifurcation point, i.e. at low width, as a twisted beam (Shape AB). It is worth adding that the solution at $L_y=0$ mm corresponds to the solution obtained with the CLT.

The results show a large dependence of the misalignment angle $\theta$ on the curvature amplitude. This dependence can be explained by the reduction in bending stiffness in the weft direction with increasing misalignment, leading to an increase in curvature. The same reduction in stiffness also leads to larger curvatures in the $\kappa_y$ dominant shape (around a factor 2 higher than the curvature in $\kappa_x$ the dominant shape. The bifurcation point or length, determining a change from stable twisted shape at low width to
bistable shapes at higher width also depends on the misalignment. A low value of the $\kappa_{xy}$ curvature is predicted for high width/length, while it increases in a non-linear way from the bifurcation point for low width/length ratio, leading to high twisting curvature at low width. Interesting from an engineering point of view is that the width at which the twist becomes significant, i.e. at point ‘B’ in figure 3, is independent on the amount of misalignment.

![Curvature plots](image)

**Figure 3.** Curvature plots of $[(0, 90-\theta)/(90+\theta, 0)]$ for three values of the misalignment $\theta$ and a fixed $L_x=500mm$

![Basic curved shapes](image)

**Figure 4.** Basic curved shapes corresponding to Figure 3.

The influence of the laminate thickness on the deformation is illustrated with Figure 5, where the curvatures for a 4 layers laminate $[(0, 90-\theta)/2/(90+\theta, 0)_2]$ are given. As expected, the behaviour is similar to that of a 2 layers laminate, while the global level of curvature decreases. The bifurcation length changes most significantly. The model also predicts that for small misalignments, a large range of aspect ratios exists where all curvatures stay at a low level, i.e. $\kappa_x = \kappa_y = 0, \kappa_{xy} < 0.5 /m$. This trend is extended for thicker laminates, where the bifurcation length exceeds the chosen length $L_x$ of 500 mm for a 6 layer laminate.
Figure 5. Curvature plots of \([(0, 90-\theta)/(90+\theta, 0)\)] for various misalignment \(\theta\).

4.2. Experimental results

Experimental results are presented on similar types of figures as used for the numerical results. Figure 6 shows the curvatures of a 2 layers \([(0, 75)/(105, 0)]\) laminate. Both numerical (in continuous lines) and experimental (discrete points) are presented, as well as light-gray continuous lines showing the potential variation on the model following the uncertainty on the fibre transverse properties \(E_{f2}\) and \(\alpha_{f2}\) values as discussed in the material section. The experimentally measured curvatures are shown to be generally lower than the predicted ones. The prediction of the bifurcation length seems to overestimate the measured bifurcation, though the change from a single torsional curvature to a bistable behaviour as described by the bifurcation length is probably not as discontinuous as the model suggests. Besides, the effect of depicted uncertainty on the fibre transverse properties is such that the measurements are within the modelled boundaries.

Results on the effect of both misalignment and thickness follow the same trends as described on 4.1, and are best described by the lower bound. An example is given in Figure 7 for the thickest laminate considered and a misalignment of 5°. In this case the laminate does only show a twisting curvature with the size considered in this study.
Figure 6. Curvature plots of $[(0, 75)/(105, 0)]$ with numerical and experimental results.

Figure 7. Curvature plots of $[(0, 85)/3/(95, 0)/3]$ with numerical and experimental results.
5. Conclusions
A numerical procedure is presented to quantify the effect of misalignment on the process induced deformation of fibre reinforced laminates. The procedure, specifically proposed for woven fabric based laminates, is based on two steps. The first provides the basic thermo mechanical properties of the misaligned ply based on a geometrical description and a multiscale approach. The second part concerns a model able to predict the thermally induced shapes of unsymmetric laminates using a non-linear definition of the in-plane strain-displacement relation. A validation is performed on a generic carbon 5H satin reinforced thermoplastic laminate.

The results show out-of-plane deformations as a function of the dimensions of the laminate. Large square laminates tend to show a bi-stable bending curvature. This deformation changes to a twisted curvature for rectangular laminates. The dimensions at which this change takes, also called the bifurcation point, is shown to depend on both the misalignment amplitude as the thickness of the laminate. Last but not least the amount of twist increases in a non-linear way towards narrow laminates. This makes it in general difficult to produce narrow laminates with a low twisting curvature.

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