Entanglement in a three spin system controlled by electric and magnetic fields

Jakub Łuczak and Bogdan R Bułka

Institute of Molecular Physics, Polish Academy of Sciences, ulica M Smoluchowskiego 17, 60-179 Poznań, Poland

E-mail: jakub.luczak@ifmpan.poznan.pl

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Abstract

We study the effect of electric field and magnetic flux on spin entanglement in an artificial triangular molecule built of coherently coupled quantum dots. In a subspace of doublet states an explicit relation of concurrence with spin correlation functions and chirality is presented. The electric field modifies superexchange correlations and shifts many-electron levels (the Stark effect), as well as changing spin correlations. For some specific orientation of the electric field one can observe monogamy, for which one of the spins is separated from two others. Moreover, the Stark effect manifests itself in a different spin entanglement for small and strong electric fields. The role of magnetic flux is opposite: it leads to circulation of spin supercurrents and spin delocalization.

The last decade has seen a great interest in application of concepts from quantum information theory, for example entanglement [1], to condensed matter theory [2]. Since entanglement represents unique quantum correlations, the concept has been applied to exploration of phenomena in strongly correlated many-fermion systems in order to gain insight into the nature of quantum phase transitions. In this paper we show how entanglement is related to a spin correlation function and how it can be controlled by an external electric and magnetic field. We choose a system of three coherently coupled semiconducting quantum dots with three electrons, because it can be viewed as a realization of a three qubit system, which has recently been of great interest [3–7]. In such a system one can find two classes of truly three-partite entangled states, represented by the Greenberger–Horne–Zeilinger (GHZ) and the Werner (W) states [1, 8]. Recently there have been attempts to measure and control these states [9], as well as to apply them in logical gates [10]. In this paper we follow a scheme for universal quantum computations, proposed by Di Vincenzo et al [5] for a spin system with exchange interactions in quantum dots, in which logical qubits are encoded in the doublet subspace with \( S_z = \pm 1/2 \) (see also [6]). Recently, experimental attempts have been undertaken to use this scheme to perform coherent spin manipulations in three quantum dots [11–13].

The gate operation on the spin system in quantum dots can be performed by inhomogeneous static magnetic fields [14, 15], ac magnetic fields [16], a light-induced magnetic field in the dynamical Stark effect [17], Raman transitions [18], or using the spin–orbit interactions [19, 20] (see also [21]). However, purely electrical manipulations [22–24] enable much faster gate operations. Moreover, if these manipulations are performed [11, 24] within the scheme of Di Vincenzo et al, they can be much less sensitive to decoherence processes. In our considerations we use the electric field to manipulate the spin entanglement, but its role is different. We show that the electric field can modify superexchange coupling in the system with a triangular geometry and change the quantum correlations between the spins. The role of magnetic flux is however different: it induces spin supercurrents flowing around the triangular ring and is the main decoherence source in that kind of system [25]. One can expect that the magnetic flux acts destructively on the entanglement.

The paper is organized as follows. In section 1 we show that the concurrence, as a measure of entanglement, has an explicit relation with spin correlation functions and chirality in the triple spin system. Therefore, one can have a simple interpretation of separability, monogamy and dark spin states. Section 2 describes our system within the Hubbard model and
its canonical transformation to the Heisenberg Hamiltonian. We show that the electric field breaks the symmetry of the system and modifies exchange coupling, whereas the magnetic flux generates spin chirality. Detailed studies of the concurrence, as well as the spin correlation functions and spin chirality, are presented in section 3. For a special orientation of the electric field we find a biseparable state (monogamy). Section 4 summarizes the paper.

1. Spin correlation functions and measure of entanglement

We begin defining wavefunctions for three electrons in the three qubit system. These functions can be constructed by adding a third electron to the singlet or triplet state (see [26]). Two spin subspaces may be defined: quadruplets and doublets. The quadruplets are the states with the quantum spin number $S = 3/2$ and correspond to GHZ and W states, which are well known in the literature [1, 2, 8, 27–29]. In this paper, the studies are focused on the doublet subspace with the total spin $S = 1/2$. These states were proposed for exchange-interaction universal quantum computations [5–7]. In many cases, such as the one considered in the next part of the paper, the doublet is the ground state. We assume that the system is kept coherent for a time sufficiently long in order to perform an entanglement measurement, and we fix the $z$-component of the total spin $S_z = +1/2$ in further considerations. The wavefunction can be expressed as

$$|\Psi_{D1/2}\rangle = \alpha_1|D1/2\rangle_1 + \alpha_2|D1/2\rangle_2,$$

where

$$|D1/2\rangle_1 = \frac{1}{\sqrt{2}}(c_{1\uparrow}c_{2\downarrow} - c_{2\uparrow}c_{1\downarrow})|0\rangle,$$

$$|D1/2\rangle_2 = \frac{1}{\sqrt{6}}[(c_{1\uparrow}c_{2\uparrow} + c_{1\downarrow}c_{2\downarrow})c_{1\downarrow} - 2c_{1\uparrow}c_{2\downarrow}c_{1\downarrow}]|0\rangle.$$ Symbols $c_{\sigma}^\dagger$ are creation operators of an electron with the spin $\sigma$ in the qubit $i$ acting on the vacuum $|0\rangle$. The state $|D1/2\rangle_1$ is prepared by adding a third electron to the singlet state, whereas $|D1/2\rangle_2$ can be prepared from triplet or singlet states [26]. This construction allows us to gain insight into monogamy and biseparability [30, 31].

As a measure of the entanglement we take the concurrence. In order to calculate it [33], we define a reduced density matrix of a pair of electrons in the quantum dots $i$ and $j$: $\rho_{ij} = \text{Tr}_k \rho$, where $i, j, k$ denote different quantum dot, and $\rho$ is a density matrix $\rho = |\Psi_D\rangle\langle\Psi_D|$ for the doublet subspace. Next we derive a matrix:

$$R_{ij} = \varrho_{ij}(\sigma_y \otimes \sigma_y)\varrho_{ij}^\dagger(\sigma_y \otimes \sigma_y),$$

where $\sigma_y$ is a Pauli matrix and the asterisk denotes complex conjugation of $\varrho_{ij}$. The concurrence is calculated as

$$C_{ij} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},$$

where $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_4$ are square roots of eigenvalues of $R_{ij}$ in descending order. $C_{ij}$ can take values between zero (for separate states) and one (for fully quantum entangled states). Using this definition one can calculate the concurrence for the doublet representation (1):

$$C_{12} = \frac{1}{2}|\alpha_2|\sqrt{3}\alpha_1 - \alpha_2|,$$

$$C_{13} = \frac{1}{2}|\alpha_2|\sqrt{3}\alpha_1 + \alpha_2|,$$

$$C_{23} = \frac{1}{2}|3\alpha_1^2 - \alpha_2^2|.$$ Let us now calculate spin correlation functions in the doublet subspace (1):

$$\langle S_1 \cdot S_2 \rangle = \frac{1}{2}[\sqrt{3}(\alpha_1^2 \alpha_2^2 + \alpha_1^2 \alpha_2^2) - 2|\alpha_2|^2],$$

$$\langle S_1 \cdot S_3 \rangle = \frac{1}{2}[\sqrt{3}(\alpha_1^2 \alpha_3^2 + \alpha_1^2 \alpha_3^2) - 2|\alpha_3|^2],$$

$$\langle S_2 \cdot S_3 \rangle = \frac{1}{2}(-3|\alpha_1|^2 + |\alpha_2|^2).$$

In general, the coefficients $\alpha_1$ and $\alpha_2$ can be complex, for example in the presence of the magnetic flux. We show later that in this case a spin supercurrent occurs with a nonzero value of chirality

$$\langle S_1 \cdot (S_2 \times S_3) \rangle = i\frac{\sqrt{3}}{4}(\alpha_1^2 \alpha_2^2 - \alpha_1^2 \alpha_2^2).$$

Comparing these results with the concurrence $C_{ij}$, (6)–(8), one can find, after some algebra, the following relation:

$$C_{ij} = \frac{1}{2}[\langle S_1 \cdot S_j \rangle + \langle S_1 \cdot (S_2 \times S_3) \rangle].$$

This fact allows us to propose the expectation value of spin correlation functions and chirality as an alternative measure of the entanglement.

In a multi-qubit system it is interesting to define monogamy of entanglement. If two qubits are fully quantum entangled then they cannot be correlated with the third one [32]. In this case one uses the term monogamy, which expresses the nonshareability of entanglement. For a three qubit system monogamy can be measured by the concurrence $C_{ijk}$ between a qubit $i$ and other qubits $j, k$, which is given by the one-tangle

$$C_{ijk} = 2\sqrt{\det q_{ij}},$$

where $q_{ij} = \text{Tr}_{jk} \varrho$. For biseparable states $C_{ijk} = 0$, which means that the qubits $j$ and $k$ are fully quantum entangled whereas the qubit $i$ is separated. This quantity is related to the linear entropy $S_i(q_{ij}) = 2[1 - \text{Tr}(q_{ij}^2)] = 4\det q_{ij} = C_{ijk}^2$. Monogamy of entanglement satisfies the relation

$$C_{ijk}^2 \geq C_{ij}^2 + C_{ik}^2$$

proved by Coffman et al [33]. For our case, in the doublet subspace, we have the equality $C_{ijk}^2 = C_{ij}^2 + C_{ik}^2$, as one could expect for pure states.

2. Model of correlated spins on a triangular system of quantum dots

In this section we would like to present a specific example of a three qubit system, namely an artificial triangular molecule built of coherently coupled semiconducting quantum dots. We
show first that an external electric field and a magnetic flux can modify spin states, and later, in section 4, how these external fields influence the entanglement between spins.

Our system of three quantum dots (figure 1) is described by the Hubbard model

\[ \hat{H} = \sum_{i,\sigma} \left( \epsilon_i + Eer \cos[\theta + (i - 1)2\pi/3] \right) n_{i\sigma} + \sum_{i \neq j, \sigma} t_{ij}(e^{i\phi/3}\sigma_{i\sigma}^\dagger \sigma_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\uparrow} n_{i\downarrow}. \quad (16) \]

Here, \( \tilde{\epsilon}_i = \epsilon_i + Eer \cos[\theta + (i - 1)2\pi/3] \) corresponds to a shift of a local single electron level \( \epsilon_i \) in the electric field \( E \). The polarization energy \( E \cdot \mathbf{P} = e \sum_i \mathbf{E} \cdot \mathbf{r}_i n_{i\sigma} = Eer \sum_i \cos[\theta + (i - 1)2\pi/3] n_{i\sigma} \), where \( e \) is the electron charge, \( \mathbf{r}_i \) denotes a vector of the \( i \)-qubit position, \( \theta \) is an angle between \( \mathbf{r}_i \) and \( \mathbf{E} \) and \( n_{i\sigma} \) is an electron number operator. Later, for simplicity, we put \( \epsilon_i = 0 \) and denote \( g_E = Eer \). The second term in (16) describes chirality of electrons in the presence of the magnetic flux \( \Phi \) enclosed in the triangle. According to the Peierls scaling an electron gains during hopping a phase shift \( \phi = 2\pi \Phi/(\hbar c) \). The Coulomb onsite interaction of electrons on the quantum dots is included in the last term.

Using this model we can calculate all properties of the system numerically. In particular for three electrons we calculate the spin correlation functions as well as the concurrence and influence of the electric field as well as the magnetic flux. To understand the results we use a canonical transformation [34] of the Hubbard Hamiltonian (16) to an effective Heisenberg Hamiltonian. Taking the hopping integrals \( t_{ij} \) and the electric field \( g_E \) as small parameters with respect to the Coulomb interaction \( U \), one can get the effective Heisenberg Hamiltonian [35, 36]:

\[ \hat{H}_{\text{eff}} = \sum_{i < j} J_{ij}(S_i \cdot S_j - \frac{1}{4}) + J_x S_1 \cdot (S_2 \times S_3). \quad (17) \]

The first term describes the superexchange coupling between spins, for which the exchange parameter \( J_{ij} \) can be calculated to the third order in \( t_{ij}/U \) [35]

\[ J_{ij} = 2|t_{ij}|^2 \left( \Delta_{ij}^{-1} + \Delta_{ji}^{-1} \right) + \left( t_{ij} t_{lj} + f_{ij} t_{lj}^{\dagger} f_{ji} \right) \times \left( \Delta_{ij}^{-1} \Delta_{lk}^{-1} + \Delta_{ji}^{-1} \Delta_{lk}^{-1} + \Delta_{ij}^{-1} \Delta_{lk}^{-1} \right) - \Delta_{ij}^{-1} \Delta_{kl}^{-1} - \Delta_{ji}^{-1} \Delta_{kl}^{-1}, \quad (18) \]

where \( \Delta_{ij} = (U + \tilde{\epsilon}_i - \tilde{\epsilon}_j) \). Taking \( t_{ij} = t \) and \( g_E \ll U \) one can get an explicit form for \( J_{ij} \):

\[ J_{12} = \frac{4t^2}{U} + \frac{3t^2 g_E^2 (2 + \cos 2\theta + \sqrt{3} \sin 2\theta)}{U^3} - \frac{12t^2 \cos \phi \ g_E \cos \theta - \sqrt{3} \sin \theta}{U^3}, \quad (19) \]

\[ J_{13} = \frac{4t^2}{U} + \frac{3t^2 g_E^2 (2 + \cos 2\theta - \sqrt{3} \sin 2\theta)}{U^3} - \frac{12t^2 \cos \phi \ g_E \cos \theta + \sqrt{3} \sin \theta}{U^3}, \quad (20) \]

\[ J_{23} = \frac{4t^2}{U} + \frac{6t^2 g_E^2 (1 - \cos 2\theta)}{U^3} + \frac{24t^2 \cos \phi \ g_E \cos \theta}{U^3}. \quad (21) \]

One can see that the second term is proportional to \( g_E^2 \) and corresponds to the quadratic Stark effect. When the electric field rotates, this term leads to oscillations with the period \( \pi \). The linear Stark effect is described by the third term, which corresponds to the period of oscillations equal to \( 2\pi \). The linear term in \( J_{23} \) is always negative, whereas for \( J_{12} \) and \( J_{13} \) the linear terms can be negative. At \( \theta = 0 \) one can see that \( J_{23} \) increases linearly with \( g_E \), whereas the couplings \( J_{12} = J_{13} \) and they first decrease, and next increase quadratically for a larger \( g_E \). At \( g_E = 4t \cos \phi \) we get \( J_{12} = J_{13} = J_{23} \) and the system becomes uniform once again.

The second term in the effective Hamiltonian (17) describes chirality of electrons in the presence of the magnetic flux. The term is connected with the Aharonov–Bohm effect and with the persistent currents moving around the flux enclosed by the three quantum dot ring. The coupling parameter calculated to the third order in \( t_{ij}/U \) is given by [35]

\[ J_x = -it_{ij} t_{lm} \delta_{ij} \Delta_{ij}^{-1} \Delta_{lm}^{-1} + \Delta_{ji}^{-1} \Delta_{lj}^{-1} - \Delta_{mj}^{-1} \Delta_{mj}^{-1} \Delta_{mj}^{-1} \Delta_{mj}^{-1}, \quad (22) \]

which for \( t_{ij} = t \) and \( g_E \ll U \) simplifies to the form \( J_x = -12t^3 \sin \phi/U^2 \). This parameter depends on the electric field in higher order terms of the expansion, but we neglect them in our studies.

Using the Heisenberg model (17) one can derive many physical quantities analytically, in particular energy for the quadruplet and doublet states presented in section 1. For |GHZ⟩ and |W⟩ state one has the energy

\[ E_{\text{GHZ}} = E_{\text{W}} = 0. \quad (23) \]

These states are independent of the electric field because the spins cannot be transferred between quantum dots (due
to the Pauli exclusion principle). In the doublet subspace \([|D_{1/2}\rangle_1, |D_{1/2}\rangle_2]^T\) the effective Hamiltonian is expressed as

\[
H_{\text{eff}} = \begin{bmatrix} -\frac{1}{4}(J_{12} + 4J_{23} + J_{31}) & \frac{\sqrt{3}}{4}(J_{31} - J_{12} + iJ_x) \\ \frac{\sqrt{3}}{4}(J_{31} - J_{12} - iJ_x) & -\frac{3}{4}(J_{12} + J_{31}) \end{bmatrix}.
\]

The eigenenergies are

\[
E_{\pm} = -\frac{1}{2}(J_{12} + J_{23} + J_{31}) \pm \frac{\Delta}{2},
\]

where \(\Delta = [3J_2^2/4 + J_2^2 + J_3^2 - J_{12}J_{13} - J_{12}J_{23} - J_{13}J_{23}]^{1/2}\). The corresponding eigenfunctions may be written as

\[
|\Psi_{\pm}\rangle = \frac{z^{\pm}}{\sqrt{1 + |z|^2}} |D_{1/2}\rangle_1 + \frac{1}{\sqrt{1 + |z|^2}} |D_{1/2}\rangle_2,
\]

where \(z^\pm = (J_{12} - 2J_{23} + J_{13} \pm 2\Delta)/[\sqrt{3}(J_{31} - J_{12} - iJ_x)]\).

For the considered case the positions \(E_{\pm}^D\) of doublet states are always below \(E_{\text{GHZ}}\) and \(E_W\) for the GHZ and W-states. One can define an energy gap between the doublet and the quadruplet states—compare (23) and (25):

\[
E_{\text{gap}} = E_{\text{GHZ}} - E_{\pm}^D.
\]

The energy gap is of the order \(4r^2/U\), the electric field and the magnetic flux are taken as the perturbation parameters and cannot close the gap between these two subspaces. In the further consideration we restrict ourselves to the subspace with the \(z\)-component of the total spin \(S_z = +1/2\), which means that the system is under a moderate magnetic field \(\Delta < \mu_B H_z < (E_{\text{GHZ}} - E_{\pm}^D)/2\). Here \(\mu_B\) is the Bohr magneton.

### 3. Influence of external fields on entanglement

We calculated the spin–spin correlation functions \((S_i \cdot S_j)\) and the concurrence \(C_{ij}\) and \(C_{ijk}\) for the Hubbard model (16) as well as for the effective Heisenberg model (17). The results for both the models were the same, within relative accuracy better than 0.01, for the parameters used below. Therefore our analysis is performed for the Heisenberg model (17), which is much simpler.

#### 3.1. Role of electric field

Let us first study how entanglement can be modified by an electric field only (in the absence of the magnetic field, i.e. for \(\phi = 0\) and \(J_x = 0\)). The electric field breaks the symmetry of the system and induces electric polarization (although its value is small due to the strong onsite repulsion \(U\)). The electric field also changes the concurrence \(C_{ij}\), \(C_{ijk}\) and the spin correlation function \((S_i \cdot S_j)\). Figure 2 shows these quantities for the ground state as a function of the angle \(\theta\) of the electric field with respect to the axes of the system. The left panels are plotted for a small electric field \(g_E = 1\); the right one is for a large electric field \(g_E = 6\). The other parameters are \(U = 20\), \(e_1 = 0\) and \(t_y = t = 1\).

![Figure 2](image_url)

**Figure 2.** Dependence of the concurrence \(C_{ij}\), \(C_{ijk}\) and the spin correlation functions \((S_i \cdot S_j)\) on the angle \(\theta\) of the electric field calculated for the ground state. The (red) dashed curves represent \(C_{12}, C_{1(23)}\) and \((S_1 \cdot S_2)\), the (blue) solid curves \(C_{23}, C_{2(13)}\) and \((S_2 \cdot S_3)\), and the (black) dotted curves \(C_{13}, C_{3(12)}\) and \((S_1 \cdot S_3)\). The left panel is plotted for a small electric field \(g_E = 1\); the right one is for a large electric field \(g_E = 6\). The other parameters are \(U = 20\), \(e_1 = 0\) and \(t_y = t = 1\).
Figure 3. Plots of the spin correlation function $\langle S_1 \cdot S_3 \rangle$, the chirality $\langle C_z \rangle$ and the concurrence $C_{1(23)}$ in the plane $\theta$ and $\phi = 2\pi \Phi/(hc/e)$ for a small electric field $g_E = 2$ and the other parameters the same as in figure 2.

3.2. Role of magnetic flux

In the presence of magnetic flux ($\phi \neq 0$) the chirality of the spin system (described by the last term in (17)) becomes relevant [36–42]. Recently Hsieh et al [42] proposed to use chirality for quantum computations. Their quantum circuits are based on qubits encoded in chirality of electron spin complexes in systems of triangular quantum dot molecules. The magnetic flux removes degeneracy between the states with different orbital momenta and leads to spin supercurrents circulating around the triangle [37, 38]. We take the expectation value of the operator [41]

$$C_z = (4/\sqrt{3})S_1 \cdot (S_2 \times S_3)$$

as a measure of chirality [37]. In the basis of the doublets (1) and using (26) one gets the expectation value

$$\langle C_z \rangle = i(\alpha_1^* \alpha_2^* - \alpha_1^* \alpha_2) = -\sqrt{3}J_i$$

in the ground state. This means that chirality depends on the splitting $\Delta$ between the doublets which can be controlled by the electric field as well.

In the absence of the electric field (when all exchange couplings are equal, $J_{ij} = J$) the expectation value $\langle C_z \rangle = \pm 1$, because the supercurrent is $\propto \sin \phi$ and circulates clockwise or anticlockwise (for $0 < \phi < \pi$ or $\pi < \phi < 2\pi$, respectively). For this case the spins are delocalized and the expectation values of the spin correlation functions $\langle S_i \cdot S_j \rangle$ and $C_{i(jk)}$ describe maximal spin mixing.

In a general situation, in the presence of both the fields, one can expect a competition between localization and delocalization of spins. These processes should manifest themselves in the spin correlation function $\langle S_i \cdot S_j \rangle$ (in $C_{ij}$, $C_{i(jk)}$) as well as in the expectation value of chirality $\langle C_z \rangle$. Figures 3 and 4 present plots for $\langle S_1 \cdot S_3 \rangle$, $\langle C_z \rangle$ and $C_{1(23)}$ in the $\theta$–$\phi$ plane for a small and large electric field, respectively. Close to $\phi = 0$ and $\pi$ the spin supercurrent ($\propto \sin \phi$) is

Figure 4. Plots of $\langle S_1 \cdot S_3 \rangle$, $\langle C_z \rangle$ and $C_{1(23)}$ in the $\theta$–$\phi$ plane for a large electric field $g_E = 6$ and the other parameters the same as in figure 2.
small and the Stark effect dominates. In the other regions the magnetic flux becomes relevant and \( \langle C_1 \rangle \) reaches its extremal value \( \pm 1 \) (see the plateau with small oscillations caused by the electric field in figure 3(b)). According to (28), \( \langle C_1 \rangle \) is inversely proportional to the energy gap \( \Delta \) between the doublet states and reaches maximal values at the symmetry points \( \theta = 0, 2\pi/3 \) and \( 4\pi/3 \) (see also figure 4(b)). The functions \( \langle S_1 \cdot S_3 \rangle \) and \( C_1(2\theta) \) are very sensitive to symmetry breaking caused by the electric field. They show changes when the electric field becomes larger \( g_E > 4/\cos \phi \) (compare figures 3 and 4). In this case the quadratic Stark effect is more relevant than in the previous analysis (section 3.1). The magnetic flux reduces the linear component, and the quadratic Stark effect dominates. If \( g_E \) increases, the spins become more localized and the amplitude of the Stark oscillations, seen in \( \langle S_1 \cdot S_3 \rangle \) and \( C_1(2\theta) \), increases. The localization process is monotonic; we could not observe any drastic changes—in contrast to the situation in the region close to \( \phi = 0 \) and \( \pi \) when the spin correlation functions and the concurrence drastically change their characteristics in large electric fields.

4. Conclusions

Summarizing, we have shown that entanglement can be controlled by the electric field in a three spin system of three coherently coupled quantum dots with a triangular geometry. The studies were focused on bipartite entanglement in the subspace of doublets with \( S_z = \pm 1/2 \), for which the concurrence was related to the spin correlation function and the spin chirality, equation (13). This relation was exemplified for the Hubbard model and its canonical transformation to the effective Heisenberg model. The superexchange coupling exhibits a linear and quadratic dependence on the electric field (the linear and quadratic spin Stark effect), which manifests itself in different periods of oscillations of the concurrence and the spin correlation functions when the electric field changes its orientation. The competition between these two Stark effects leads to different characteristics of the concurrence for a small and large electric field. For a special field orientation we found a biseparable state, for which one of the spins is separated from two others that are fully quantum entangled (monogamy). For small fields one should direct the field precisely toward the quantum dot \( (\theta_0 = 0, 2\pi/3 \) and \( 4\pi/3 ) \) to get the spin separation at this dot. In the case of large electric field its orientation \( \theta_0 \) should be different; the angle \( \theta_0 \) depends on the strength of the field and it should be directed toward one of the opposite quantum dots.

We also considered an influence of spin chirality on entanglement. The magnetic flux \( \Phi \) induces circulation of spin supercurrents and leads to spin delocalization. The bipartite concurrence becomes uniform. For small electric fields the spins are delocalized, the Stark effect can be only seen in a very narrow range of \( \Phi \). For larger fields the Stark effect becomes more visible; the concurrence and the spin correlations exhibit oscillations as a function of the angle \( \theta \) of the electric field. Analyzing the oscillations one can see how the magnetic flux modifies relative contributions of the linear and quadratic Stark effects to entanglement.

Our studies can be also related to recent experiments on coherent spin manipulation in three quantum dots [11–13]. We showed that the scheme proposed by Di Vincenzo et al [5], with logical qubits encoded in the doublet subspace \( |D_{1/2}\rangle_1 \) and \( |D_{1/2}\rangle_2 \), can be realized due to the spin Stark effect, in which the electric field changes spin entanglement. The ground state is a superposition of the doublet states, which can be controlled by relative orientation of the electric field. Therefore, this effect can be used for preparation of a proper initial quantum state and control of a logical operation.

Let us point out that the scheme of Di Vincenzo et al is different than operations between quadruplet and doublet states performed by Gaudreau et al [13] in the experiment in triple quantum dots and between the singlet and triplet states in double quantum dots [15, 16, 21, 22], where passages were accompanied by reorientation of nuclear spins in quantum dots. Within the scheme of Di Vincenzo et al the dynamical passages are performed only between the doublet states (without nuclear spins), and total spin \( S = 1/2 \) as well as its \( z \)-component \( S_z = 1/2 \) are conserved.

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