A Scatter Search Algorithm for Multi-Criteria Inventory Classification considering Multi-Objective Optimization

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Abstract

Inventory management requires thousands or millions of individual transactions each year. Classification of the items influences the results of inventory management. Traditionally, this is usually classified with considering an annual dollar usage criterion but maybe other criteria such as lead time, criticality, perishability, inventory cost, and demand type can be affected on that classification. Inventory items that have more than one criterion are discussed under multi-criteria inventory classification (MCIC) methods in the literature. In this paper, the MCIC problem is considered with two objectives as follows: (1) minimization of total inventory relevant cost and (2) minimization of the dissimilarity index. The proposed Mixed Integer Nonlinear Programming (MINLP) model of the MCIC problem is formulated using Scatter Search Algorithm (SSA). The suggested multi-objective optimization problem is solved using LP-metric, ε-constraint and weighted sum method. Pareto optimal solutions are obtained according to these different methods and selected best method by using deviation index. Scatter Search Algorithm provides high-quality solutions within reasonable computation times. The proposed model generated a Pareto frontier solution with the maximum satisfaction level and minimum distance from ideal point. Finally, the proposed model is implemented with two numerical datasets to show the performance of its efficiency and compared our results with other studies in the previous literature.

Keywords Multiple criteria analysis · ABC inventory classification · Scatter Search Algorithm · Satisfaction level · Multi-objective optimization

1 Introduction

Inventory managers have to focus on important topics by avoiding unnecessary details in order to perform the inventory related tasks efficiently. Providing a more efficient inventory management can be possible through classifying the inventories and deciding which inventory will be counted more frequently and needs close follow-up. For a company beginning inventory management studies, the first procedure is to classify items with ABC analysis. Among the aims of classification of items, there is an important place for responses to questions like how many items should be ordered according to class, how much needs to be held as stock in stores, what should the safe stock level be, how often do items need to be counted, and how often items need to be checked. ABC analysis is important to calculate the days of supply, effective in determining purchasing requirements in MRP studies with ERP programs which have begun to be used by the majority of companies (e.g., SAP, etc.). Most inventory situations involve controlling many different kinds of items, with each having a different impact on cost. The physical characteristics of items can be a major concern because of short shelf life, special storage, space, and materials handling. ABC analysis is a tool that management uses to categorize materials and components into workable classes. Items that are classified in terms of total usage value are ordered from the most valuable to the least valuable. The small A item class contains the items that have the most significant impact on the inventory investment. In contrast, the large C item class consists of items...
that usually have little consequence on the investment. In the ABC analysis, the A item class is where 80 percent of the investment is concentrated in 20 percent of the different inventory items. The A items should receive the most attention from management. Management should frequently review the inventory function of this class. The B and C item classes represent 20 percent of the investment but 80 percent of the inventory items. Management decides which items go into the A, B, and C classes. After adding any special inventory categories, management should have a good profile of the inventory investment.

Beginning with traditional ABC analysis, the MCIC problem continues to be attempted using various methods to date. The traditional ABC classification had weighted criteria applied with the Analytic Hierarchic Process (AHP) by Flores and Clay Whybark (1986) and items were listed according to the score values obtained as they began the first multi-criteria classification studies. In their study, inventory items were classified with annual dollar usage (ADU), average unit cost (ACU), criticality (CF) and lead time (LT), and they determined the weights of the criteria values by using AHP method. The score values for each item calculated with these weights were listed from largest to smallest and items were classified according to classic ABC classification percentages. Classification studies were again developed using the AHP method (Partovi and Burton 1993; Partovi and Hopton 1994).

A similar study by Ramanathan (2006) and this model were later used as the R-model in the literature. (Ramanathan 2006) used a weighted linear optimization model for classification. It was the study first attempting to solve the multi-criteria classification problem with a mathematical model after Flores et al. (1992). Ng (2007) (called hereafter Ng-model) proposed a linear optimization model similar to the R-model. Differences in the model included the automatic updating of criteria weights in the model. The decision-maker was required to enter the weights. Zhou and Fan (2007) developed an extended version of the R-model. In addition to the R-model, they calculated good index and bad index values by calculating the maximum and minimum objective function values and combined these values to obtain a composite index. Hadi-Vencheh (2010) (called with HV-model) stated the weights for each item in the Ng model did not have an effect when deciding total score and added weights to the Ng model to develop it. Park et al. (2014) developed a new model called cross-evaluation-based weighted linear optimization (CE-WLO) to find the optimal efficiency score for inventory items. Hatefi et al. (2014) presented linear optimization model that include qualitative and quantitative criteria to classify inventory items. When all these studies are examined, different classifications are obtained by applying various methods to a 47-item dataset proposed by Reid (1987).

Some studies decide the criteria weights when giving classification weights, while some studies focus on methods to minimize total inventory costs. Hadi-Vencheh and Mohammadghasemi (2011) suggested a fuzzy AHP method for MCIC problem.

In the first classification studies by Guvenir and Erel (1998), they used the genetic algorithm (GA) method. Mohammaditabar et al. (2012) proposed simulated annealing (SA) minimizing total inventory costs and dissimilarity index value. They took the total weights of the two objective functions to determine classification. Loli et al. (2014) described a model using AHP and K-means algorithms. The aim of the K-means algorithm is to minimize the distance between the centers of each cluster. Firstly, items are listed according to AHP, and then divided into classes with K-means. This method was called AHP-K in the literature. The method named AHP-K-Veto determines the class for each criterion and prevents a piece placed in a class for one or more than one criteria from being moved to another class in the final classification. Soylu and Akyol (2014) proposed determination of reference items with the assumption that criteria will have different weights in different industries. The aim of the studies was to determine classifications with minimized total classification error using linear utility and piece-wise linear function with reference items.

Chen et al. (2008) studied MCIC problem by using the case-based distance model. Kaabi et al. (2015) decided on weights with the continuous variable neighborhood search technique and then applied TOPSIS to calculate the score for each item. According to the determined score, inventory items were classified and classification was assessed according to inventory cost. Inventory cost was compared with the traditional ABC, AHP model, R-model and Mohammaditabar (SA model) in previous studies to reach the best inventory cost. Ghorabaei et al. (2015) developed a new MCIC method that is called EDAS (Evaluation based on Distance from average Solution) by using appraisal score for all inventory items. Zhang et al. (2018) proposed fuzzy clustering-means (FCM) method in the newest of all MCIC studies. They targeted increased speed with GA and local search with simulated annealing (SA). Calculating total annual inventory costs, they compared with the previous 9 studies. They found classifications achieving inventory cost with variable holding cost ($h = 0.2$ and $s = 0.5$) and ordering cost levels $\$1232.30$.

Kaabi et al. (2018) used GA to decide on criteria weights and weighted sum (WS) and TOPSIS methods to find the weighted score of items and developed a hybrid model. In their study, they made decisions according to two performance values. One of the objectives was to minimize total inventory cost, while the other was to maximize inventory cycle rate. They studied 4 different forms of GA-
WS and GA-TOPSIS for each aim separately. The best result was obtained with GA-WS. Additionally, in this study, the MCIC studies are summarized in detail as tables. (Kheybari et al. 2019) suggested goal programming for multi-objective decision analysis. The main purpose of their study is to minimize deviation from the targets. Douissa & Jabeur (2019) proposed ELECTRE III method to compute the global score of each inventory items and then applied Continuous Variable Neighborhood Search (CVNS) to estimate the required parameters. Some of the previous literature used to evaluate the performance of their algorithms with Reid dataset is summarized as in Table 1.

This paper discusses the MCIC with two objectives. The first objective is to minimize the total relevant cost including holding and setup cost. The second objective minimizes dissimilarity of the inventory items considering weighted criteria of the items. Three approaches: LP-metric, $\varepsilon$-constraint and weighted sum are proposed in this study to solve the multi-objective problem. This problem is solved by using SSA that is known as evolutionary algorithm. Two example datasets are used to show the performance of the proposed model. According to the results, SSA gives the better inventory classification results compared with the other previous studies.

The outline of this paper is organized as follows. In Sect. 2, the proposed mathematical model is defined. Scatter search algorithm is explained in Sects. 3. Computational results are presented in Sect. 4. In Sect. 5, comparative study is performed to illustrate performance of our proposed algorithm. Discussion and further research directions are concluded in Sect. 6.

2 Model description

In this study, our aim is to find the best classification using satisfaction functions first defined by Martel and Aouni (1996) utilizing two objective functions. In this situation, we developed a model for maximizing satisfaction levels (SL) to provide convenience to decision-makers due to having two objective functions. Also, the proposed model is compared with the other multi-objective solving methods like LP-metric and $\varepsilon$-constraint. The Total Relevant Cost (TRC), which is one of the objective functions of our algorithm used in studies by Mohammaditabar et al. (2012), Ramanathan (2006), and Kaabi et al. (2018) to minimize inventory costs. However, materials with more than one criterion are effective in classification (Cohen and Ernst 1988; Flores et al. 1992). K-means clustering method (Jain 2010; Keskin and Ozkan 2013; Smet and Guzmán 2004; Sun et al. 2004) is performed to classify items with close criteria values in the same class. The second objective function is defined to minimize dissimilarity index value between items in the same class. It was noted that MCIC problems are not studies noting the choices of managers to satisfy the two objectives with together. In this study, we divided into three model’s type: minimize the TRC, minimize the dissimilarity index, and maximize satisfaction level function, respectively. There are some notations to explain the model as given as follows:

**Parameters:**

| $N$ | Number of inventory items, $n = 1, ..., N$ |
| $M$ | Number of criteria, $m = 1, ..., M$ |
| $K$ | Number of classification groups, $k = 1,2,3$ |
| $s_i$ | Ordering cost of item $i$ |
| $T_k$ | Order interval of classification class $k$ |
| $a_i$ | Total annual demand of item $i$ |
| $p_i$ | Unit purchasing price of item $i$ |
| $I_e$ | Interest rate |
| $h_i$ | Holding cost of per unit $i$ for per unit of time; $h_i = p_i*I_e$ |
| $x_{ik}$ | 1, if item $i$ and item $j$ is classified in class $k$; 0, otherwise |
| $y_{im}$ | Criteria value of item $i$ for criteria $m$ |
| $w_m$ | Weights of the $m$th criteria |

2.1 Minimize total relevant cost

The Total Relevant Cost (TRC), which is one of the objective functions of our algorithm, can be written as in Eq. (2). TRC might also be referred to as ordering cost and inventory holding cost. The larger the quantity of inventory is held, the more the inventory holding cost is incurred. Inventory holding cost incorporates storage cost, capital cost, obsolescence, damage, shrinkage, deterioration and spoilage cost, and insurance and tax costs. It is not easy to estimate the ratio of the inventory holding cost to the total value of inventory. Although various ratios are to be found in the literature, in practice, the annual holding cost is assumed to be, in accordance with the company structure, within the interval of 15–30% of the inventory investment value. Ordering cost depends on the number of orders placed in a planning period. This cost may be reduced by placing fewer orders through placing higher volume order at a time (Tersine 1994).

Model 1:

**Objective Function for TRC:**
| Author, year | Model Formulation | Objective | Criteria | Benchmark |
|-------------|-------------------|-----------|----------|-----------|
| Reid, (1987) | AHP | weight of criteria | ADU | no |
| Flores et al., (1992) | AHP | weight of criteria | ADU, ACU, LT, CF | ADU |
| Ramanathan, (2006) | DEA-like weighted linear optimization | weight of criteria | ADU, ACU, LT, CF | ADU, AHP |
| Ng, (2007) | DEA-like weighted linear optimization | weight of criteria | ADU, ACU, LT | ADU, R |
| Zhou and Fan, (2007) | DEA-like weighted linear optimization | weight of criteria | ADU, ACU, LT | R |
| Chen et al., (2008) | case-based distance model | weight of criteria | ADU, ACU, LT, CF | AHP |
| Hadi-Vencheh, (2010) | DEA-like weighted linear optimization | weight of criteria | ADU, ACU, LT, CF | Ng, ZF |
| Chen, (2012) | TOPSIS-DEA based | weight of criteria | ADU, ACU, LT, CF | RC, R, ZF, Ng, ADU |
| Mohammaditabar et al., (2012) | SA | minimize TRC and dissimilarity | ADU, ACU, LT | ADU, Zhang, AHP, R |
| Park et al., (2014) | cross-evaluation-based weighted linear optimization (CE-WLO) model, DEA-based and simulation | importance index, minimize the weights | ADU, ACU, LT | R, RC, ZF, Ng |
| Soylu and Akyol, (2014) | analytical, heuristic | minimize the total classification error | ADU, ACU, LT | AHP, R, ZF, RC |
| Lolli et al., (2014) | AHP, K-means | minimize the sum of squared distance | ADU, ACU, LT, CF | AHP, R, Ng, ZF, HV, RC, SA |
| Hatefi et al., (2014) | DEA-like weighted linear optimization | weight of criteria | ADU, ACU, LT, CF | AHP, RC, ZF |
| Kaabi et al., (2015) | TOPSIS, VNS | Euclidean distance | ADU, ACU, LT | ADU, AHP, R, SA |
| Ghorabaee et al., (2015) | Evaluation Based on Distance from Average Solution (EDAS) | Euclidean distance | ADU, ACU, LT | R, Ng, ZF, HV, RC, HT |
| Kaabi et al., (2018) | TOPSIS, WS, GA | Minimizing the Total Relevant Cost (TRC) and maximizing the Inventory Turnover Ratio (ITR) | ADU, ACU, LT | ADU, AHP, R, Ng, HV |
| Zhang et al., (2018) | GSSA -FCM (GA and SA) | minimize distance | ADU, LT, CF | AHP, R, Ng, ZF, HV, RC, FCM, AHP-K, AHP-K-Veto |
Minimize \( \sum_{i \in \text{class}(k)} S_i T_k + \frac{1}{2} \sum_{i \in \text{class}(k)} a_i h_i \) \hspace{1cm} (1)

where \( T_k \) is explained by (Chakravarty 1985), the optimal cycle length for class \( k \) can be calculated as given in Eq. (2).

\[ T_k = \sqrt{\frac{2 \sum_{i \in \text{class}(k)} S_i}{\sum_{i \in \text{class}(k)} a_i h_i}} \] \hspace{1cm} (2)

Subject to:

\[ \sum_{k=1}^{K} x_{ik} = 1, \forall i \] \hspace{1cm} (3)

\[ x_{ik} \in \{0, 1\}, \forall i, k \] \hspace{1cm} (4)

Equation (3) ensures that every item is assigned to the one class. Equation (4) defines the binary variable: equal to one if an item \( i \) is assigned to class \( k \); otherwise, zero.

### 2.2 Minimize dissimilarity index

The objective is to minimize the dissimilarity index between the items \( i \) and \( j \) of the same class in \( k \) as given in Eq. (5). \( ds_{\text{min}} \) defines the minimize dissimilarity index as shown in notations with \( ds \).

Model 2:

Objective Function for \( ds \):

\[ ds_{\text{min}} = \text{Minimize} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{K} d_{ij} x_{ik} x_{jk} \] \hspace{1cm} (5)

where \( d_{ij} \) shows that distance value between the item \( i \) and item \( j \) by considering the weights of each criteria and calculates with Eq. (6).

\[ d_{ij} = \left( \sum_{m=1}^{M} w_m (\tilde{y}_{im} - \tilde{y}_{jm}) \right)^{1/2} \] \hspace{1cm} (6)

In Eq. (6), \( \tilde{y}_{im} \) defines the normalized value of the \( ith \) item for \( mth \) criteria to covert the criteria values in a 0–1 scale because of the difference measurement values. Normalized value for each item is calculated as given in Eq. (7).

\[ \tilde{y}_{im} = \frac{y_{im} - \min_{i=1...N} \{y_{im}\}}{\max_{i=1...N} \{y_{im}\} - \min_{i=1...N} \{y_{im}\}} \] \hspace{1cm} (7)

Subject to:

Equations (3) and (4).

### 2.3 Multi-objective optimization

Multi-objective optimization is an approach applied in multi-criteria decision-making problems where it is desired to optimize more than one objective simultaneously. There is no single optimal solution for these problems, as these objective functions are in conflict with each other, that is, improving one objective function can cause the other to worsen. In this case, a number of most preferred solutions (i.e., Pareto-optimal solutions) are important for the decision maker. In this study, two-objective functions were considered, including the total relevant cost value and the dissimilarity of the items to each other. Various methods are applied for the solution of multiple objective optimization problems such as LP-metric, fuzzy satisfaction level and \( \varepsilon \)-constraint methods (Abdolazimi et al. 2021; Chu et al. 2008).
2.3.1 ε-constraint method

One of the objective functions is chosen as the main objective function \( f_1 \), and the other objective functions \( f_2, \ldots, f_n \) are considered as constraint conditions. The \( \varepsilon \) value is determined by setting intervals between the ideal and non-ideal point for the secondary objective function (Abdolazimi et al. 2021; Mohseni-Bonab et al. 2016).

\[
\min f_1(x) \\
x \in \mathbb{X} \\
f_2(x) \leq \varepsilon_2 \\
\vdots \\
f_n(x) \leq \varepsilon_n
\]  

In this study, the TRC function was chosen as the main objective function and the dissimilarity index function was considered as a constraint function. In order to calculate the \( \varepsilon \) value, firstly the lower and upper range values were decided by solving single objective function that given in Model 2. The solution values were found by running the model starting from the maximum dissimilarity index until reaching the minimum dissimilarity index by decreasing each epsilon value step by step. Equation (8) is transformed into the proposed model (shown in Eq. (9)) to solve the multiple objective problem.

\[
\begin{align*}
\text{Min } TRC &= \sum_k \left( \frac{\sum_{i \in \text{class}(k)} s_i}{T_k} + \frac{1}{2} T_k \sum_{i \in \text{class}(k)} a_i h_i \right) \\
&+ \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^K d_{ij} x_{jk} x_{jk} \leq \varepsilon_2 \\
\text{Subject to : } \\
\sum_j w_j &= 1
\end{align*}
\]  

Equations (2–4).

2.3.2 LP-metric method

This method is used to minimize the distance of the objective functions in the multi-objective model. In other words, it tries to minimize the distance of the objective function value (Eq. (10)) from the ideal point (Abdolazimi et al. 2021).

\[
\min \left[ \sum_j \left( w_j \left| \frac{f_j^* - f_j}{f_j^*} \right| \right) \right]^{1/p}
\]  

Here, \( w_j \) shows the weights the jth objective function value, and \( p \) shows the degree of emphasis on deviations \((1 < p < \infty)\). In this study, the \( p \) value was taken as 1. Weights are given so that the sum of the \( w_j \) is 1. The \( f_j^* \) value shows the optimum value that the jth objective function value can take when considered individually. Two objective functions are added in one objective function by applying Eq. (10) according to the LP-metric method and obtained Eq. (11).

\[
\min \left( w_1 \frac{TRC_{\text{min}} - TRC}{TRC_{\text{min}}} \right) + \left( w_2 \frac{ds_{\text{min}} - ds}{ds_{\text{min}}} \right)
\]  

Following equations are added as constraints functions to the proposed mathematical model. Equation (12) provides the total weights of the objective function are to be 1.

\[
\text{Subject to : } \\
w_1 + w_2 = 1; \ w_1 \text{ and } w_2 \geq 0
\]  

Equations (2–4).

2.3.3 Weighted sum approach

In the weighted sum method, a weight \( w_j \) is assigned to each objective function \( j \) and the weighted sum of the objectives is minimized (Eq. 13). Multiple objective functions are combined into a single objective function as follows:

\[
\min Z = \sum_j w_j f_j(x)
\]  

\[
\text{Subject to : } \\
\sum_j w_j = 1
\]  

In this approach, non-dominated solutions are obtained by trying different weights \( w_j \) for the objectives. The weighted sum method has been extended using the linear membership function with normalization (Chakraborty et al. 2016; Huang et al. 2006). In this case, the objective functions are normalized and take values between 0 and 1. Two objective functions are written in Eqs. (1) and (5) are the one is \( TRC \) minimization \( (TRC_{\text{min}}) \) and the second one is to minimize dissimilarity index \( (ds_{\text{min}}) \) between the criteria values. The main objective of the weighted sum approach is to maximize the satisfaction level of the two objectives by obtaining best classification results. The objective is to maximize the total satisfaction value of \( TRC \) and \( ds \) at the same time and can be written as Eq. (14). The satisfaction level for \( TRC \) functions is defined by using the \( \delta_1 \). The satisfaction level for \( ds \) functions is defined by using the \( \delta_2 \). The membership function in Eqs. (15) and (16) represents the level of satisfaction of the decision makers as a value between 0 and 1. Equation (17) ensures \( \delta_1 \) and \( \delta_2 \) are smaller than 1. Equation (18) provides the total weight is equal to 1.

Model 3:

**Multi-Objective Function:**
MaxSL = \sum_j w_j \delta_j \quad (14)

Subject to:
\delta_1 \leq (\text{TRC}^{\text{max}} - \text{TRC}^*) / (\text{TRC}^{\text{max}} - \text{TRC}^{\text{min}}) \quad (15)
\delta_2 \leq (\text{ds}^{\text{max}} - \text{ds}^*) / (\text{ds}^{\text{max}} - \text{ds}^{\text{min}}) \quad (16)
\delta_1, \delta_2 \leq 1 \quad (17)
\sum_j w_j = 1 \quad (18)

Equations (3) and (4).

The minimum value of the TRC function is found using Eqs. (1, 2, 3, 4) and calculated \delta_1 value. The minimum value of the dissimilarity index function is obtained using Eqs. (3, 4, 5, 6, 7) and then calculated \delta_2 value. Our aim is to find satisfaction values that ensure both values are at maximum levels (Fig. 1). According to Fig. 1a if the TRC minimum value is provided, the satisfaction level will be 1. If the TRC maximum value is provided, the satisfaction level will be 0. Similarly, in Fig. 1b if the ds value is at minimum levels, maximum satisfaction level will be obtained; in other words, the satisfaction level will be 1. If the ds value is at maximum level, the satisfaction level will be 0. According to Mohammaditabar et al. (2012), this model is very difficult to solve optimally since the objective functions are not linear and the value of \( x_{ik} \) is dependent on the value of \( t_k \). In this study, SSA is proposed to solve the problem of obtaining optimum or near optimum solutions in reasonable computation time because of the MINLP model.

3 Scatter search algorithm

As the developed model was an MINLP model, there was a polynomial increase in the working duration of the model as the number of items increased. As a result, to attain the optimum result in a more suitable duration, solutions were explored with SSA, one of the evolutionary algorithms. SSA was first developed by Glover (1977). This method, which is very similar to GA, has some differences to GA. One of these is that GA is a stochastic structure, while SSA makes decisions deterministically. The SSA which is an evolutionary approach derives new solutions of the reference set from one set of solutions. Unlike the GA, the SSA tries to find a solution through the smaller reference set. In the GA, he creates new solutions by applying crossover and mutation to two randomly selected solutions from the population. The SSA approach was used for scheduling problems (Fan et al. 2019; Riahi et al. 2017), for the uncapacitated facility location problem Hakli and Ortacay (2019), for the economic lot sizing problem (Khojaste Sarakhsi et al. 2016) and for the multi-objective clustering problem (Caballero et al., 2011)). SSA can basically be defined with 5 basic components (Martí et al. 2006).

1. A Diversification Generation Method (DGM) produces good and diverse individuals to create an initial population.
2. Improvement Method (IM) is applied to develop new individuals from the initial solutions.
3. Reference Set Update Method (RSM), which is included in the reference set of better individuals, removing the bad individuals from the reference set.
4. The Subset Generation Method (SGM) creates subsets from individuals in the reference set.
5. The Combination Method (CM) combines the individuals in the subset to create new individuals.
solution should be assigned to position 3 and the second element to position 6. Thus, the relevant new solution vector will be \((2, 3, 1, 2, 3, 3)\). If \(k = 4\), 
\[pv(4) = (v(4, 4), v(4, 3), v(4, 2), v(4, 1)) = pv(4, 3, 2, 6, 1, 5)\]
and will be obtained as the new vector solution \((2, 3, 3, 1, 2, 3)\). Among the new solutions are obtained in this way, 2 best solutions are chosen from the solutions with the best goal value.

### 3.2 Improvement method

Tabu Search (TS) is a widely used metaheuristic that uses some common-sense ideas to enable search process to escape from a local optimum. TS algorithm was proposed by Glover (1998). This method is a metaheuristic method to local optimization with neighborhood solutions. The basic approach prevents or punishes repetitions in the next cycle to prevent circular movements in steps leading to the final solution. Thus, by investigating new solutions, the TS algorithm guides regional heuristic searches to search for solutions which are more advanced than the regional best solution. The basic principle of the TS algorithm aiming to exceed regional optimal is based on selecting the movement with highest assessment value for each iteration of the assessment function to create the next solution. With the aim of ensuring this, a tabu list is created and the original aim of the tabu list was to prevent inversion of a previous movement rather than repetition. The tabu list has chronologic structure and used flexible memory structure. Though the TS algorithm is explained as labeling unwanted points, in practice it is used to label more attractive points. For example, if the first solution vector is \((2, 3, 3, 1, 2, 3)\) a neighboring relationship of \(n*(n-1)/2 = 6*5/2 = 15\) may form for the solution vector created from the 6 elements. As a result of swaps with neighbors, the variation ensuring best development from the target function is added to the tabu list. For example, the variation in the objective function as a result of the swap procedure with neighbors is shown below.

**Initial Trial Solution:** \((2, 3, 3, 1, 2, 3)\) \(\text{TRC} = 308.378\) $

If we swap the first element and the second element the new solution will be \((2, 3, 3, 1, 2, 3)\) \(\text{TRC} = 308.378\) $.

If we swap 4th item class and 5th item class, another new solution will be \((2, 3, 3, 1, 2, 3)\) and TRC is calculated of 306.430$. So, the 4th item class will be the tabu list and not changed in the next iteration because of the better TRC value. TS algorithm continues to get better solutions and generated new solution vectors.
3.3 Reference set update method

The reference set consisting of high quality solutions and the sum of various solutions is used to derive new solutions by applying Refset. Refset consists of two sub-sets which are \( b_1 \) and \( b_2 \), that is, \( \text{Refset} = b = b_1 + b_2 \). The best \( b_1 \) solution is selected from the first \( P \) population and these solutions are added to Refset and deleted from \( P \) (Glover et al. 2004).

The Euclidean distance value is calculated by comparing each solution in Refset1 with the solutions in \( P \). Maximum is selected from the solutions with minimum distance and this solution is added to \( b_2 \) and deleted from \( P \). This process continues for the specified size of \( b_2 \). Refset includes \( b_1 \) high quality results and \( b_2 \) diverse results. Then, the minimum distance is calculated between each \( x \) solution in \( P \)-Refset and \( y \) solutions in Refset. This calculation gives the Euclidean distance between \( x \) and \( y \) as follows:

\[
d_{\text{min}}(x) = \min_{y \in \text{Refset}} \{d(x,y)\}
\]

For example, we suppose that we have 5 solutions for \( P \) given in Fig. 3. The best two solutions in Fig. 3 are Refset1 = \{S4, S5\} as shown in Fig. 4.

Solutions \( \{S_1, S_2, S_3\} \) are the other solutions in the \( P \). The minimum distance between the solutions \( S_j \) and the solutions \( S_4 \) and \( S_5 \) in Refset1 is calculated by using Eq. (19) and the results are shown as in Fig. 5.

\[
d(x, y) = \sqrt{\sum_{i \in P} (x_i - y_i)^2}
\]

Fig. 4 The best solutions of Refset1

| \( \{S_1, S_4\} \) | 0 | 4 | 1 | 0 | 0 | 1.249 |
| \( \{S_1, S_5\} \) | 4 | 4 | 1 | 4 | 0 | 3.741 |
| \( \{S_2, S_4\} \) | 1 | 0 | 0 | 0 | 0 | 1 |
| \( \{S_2, S_5\} \) | 1 | 0 | 4 | 4 | 0 | 3 |

Fig. 5 Distance value between Refset1 and the P-Refset1

| \( S_1 \) | 1 | 1 | 2 | 1 | 3 | 1 | \$314.780 |
| \( S_2 \) | 2 | 3 | 3 | 1 | 3 | 2 | \$309.088 |
| \( S_3 \) | 3 | 3 | 2 | 3 | 1 | 2 | \$312.738 |
| \( S_4 \) | 1 | 3 | 3 | 1 | 3 | 2 | \$306.280 |
| \( S_5 \) | 3 | 3 | 1 | 3 | 3 | 2 | \$306.212 |

Fig. 3 Improved solutions

These distances are \( (2.449, 1, 2.236) \), and it appears the maximum distance is between 1st and 4th solutions. In conclusion, the 1 solution is added to refset2 and deleted from \( P \) solution space.

And update the minimum distance values. The new maximum values between the minimum distance values of 2.236 corresponding with solution 3 in Fig. 3. Finally, diverse Refset2 is as shown in Fig. 6.

3.4 Subset generation method

In this step, new solutions are tried to generate for diversification. SGM proposes to select all possible combinations of solutions in Refset1 with solutions in Refset2. For example, suppose that we have \( b_1 = b_2 = 2 \), i.e., Refset1 = \{S4, S5\} and Refset2 = \{S1, S3\}. According to the combinations of two Refset, 4 subsets will be generated as follows: \{S4, S1\}, \{S4, S3\}, \{S5, S1\}, \{S5, S3\}. 

| \( S_1 \) | 1 | 1 | 2 | 1 | 3 | 1 | \$314.780 |
| \( S_2 \) | 3 | 3 | 2 | 3 | 1 | 2 | \$312.738 |

Fig. 6 Diverse subset of Refset2

According to the combinations of two Refset, 4 subsets will be generated as follows: \{S4, S1\}, \{S4, S3\}, \{S5, S1\}, \{S5, S3\}. 

A Scatter Search Algorithm for Multi-Criteria Inventory Classification considering Multi-Objective...
3.5 Combination method

Attempts are made to obtain new solutions with the subset solutions created with this method. For this, various methods are recommended (Glover 1998). Here, the method used in the study by Caballero et al. (2011) was applied. The following methods were applied in an attempt to obtain alternative solutions. Let the selected subsets be 1st and 4th solutions. Random numbers are derived for each element. According to whether the random numbers are smaller or larger than 0.5, elements are exchanged with each other and new solutions obtained. For example, let the generated random numbers be (0.42, 0.26, 0.18, 0.03, 0.87, 0.67). As the 1st, 2nd, 3rd, and 4th elements are smaller than 0.5, they remain the same. The 5th and 6th elements change places and a new solution given below Fig. 7 is obtained.

After ending these steps, finally various stopping conditions are proposed for the termination or stopping criteria. The most widely used ones are to stop the algorithm after a specific number of generations, or after a given time period. Another way is to stop the search when the objective values for several consecutive generations do not improve.

4 Illustrative example

In this study, we apply SSA approach to get the maximum satisfaction level of classification of MCIC. A computational study is conducted by running the model for 47 inventory items used in a Hospital Respiratory Therapy Unit (Reid 1987) to evaluate the performance of the proposed SSA. We consider the ADU, ACU, and LT as criteria, in this case, to be able to compare the results with the results of other algorithms. As the CF criterion is categoric it was not included in this study, just as for the some previous studies (Chen 2012; Hadi-Vencheh and Mohamadghasemi 2011; Ng 2007; Zhou and Fan 2007). The data and normalized data values are given in Table 2.

The ADU, ACU and LT criteria weights used for classification were taken as (0.407, 0.037, 0.556) from the study by Kaabi et al. (2015). We compare our model with other MCIC studies in the literature (Traditional (ADU); Douissa & Jabeur, 2019; Kaabi et al., 2015, 2018; Lolli et al., 2014; Mohammaditabar et al., 2012; Zhang et al., 2018) that is used to benchmark dataset by Reid (1987) and also evaluate the TRC functions. To operate the model, firstly the MINLP model was programed with Lingo 2018 and tested with small size problems. The results obtained and durations were assessed in terms of the results and performance obtained with SSA. Due to the difficulty of obtaining a suitable solution in acceptable duration, the decision was made to use SSA, one of the metaheuristic methods not used in classification studies to date.

We implemented SSA with the Frontline Solver Platform (Frontline Solver 2018) and solved the problem on a computer with CPU Intel(R) Core(TM) i5-8365U CPU (1.60 GHz), memory 8 GB, Windows 10. The mathematical model was operated with 5 items, 10 items and 15 items with the Lingo 18.056 version of optimization program. Classification with 5 items reached optimum results in a very short duration. For 10 items, global optimum result took 40 min to obtain. However, for 15 items the program operation was ended at the end of 3 h and only local optimum results could be obtained. With SSA, the computational time had acceptable duration of less than 20 min.

We assume that the ordering cost is equivalent to the lead time multiplied by a fixed coefficient with 0.5 and the inventory holding cost is assumed to be 20% of the ACU. The demand is calculated by dividing annual dollar usage with the average item cost. With the aim of being able to determine the best results for solution to our multi objective model, the optimized minimum TRC value was $1098.165, and dissimilarity index was 195.026 in Model 1. When we used Model 2 for the second performance criteria of dissimilarity index, the minimum dissimilarity index was calculated as 126.522 with TRC value $1149.241. The maximum values for TRC and dissimilarity index were chosen from the largest values obtained in the other compared studies. Ideal point shows that each objective is optimized with minimum value regardless of the satisfaction of other objectives whereas at non-ideal point, each objective attains its maximum value. According to Table 3, ideal point is equal to (1098.165, 126.522).

To solve the multi-objective problem, three different methods, LP-metric, $\varepsilon$-constraint and weighted sum, explained in Sect. 2.3, were applied. During the implementation of the LP-metric and weighted sum method, different combinations were chosen such that the weight
| Item no | ADU ($)  | ACU ($)  | LT (week) | Normalized ADU | Normalized ACU | Normalized LT |
|---------|----------|----------|-----------|----------------|----------------|---------------|
| 1       | 5840.64  | 49.92    | 2         | 1.000          | 0.219          | 0.167         |
| 2       | 5670.00  | 210.00   | 5         | 0.971          | 1.000          | 0.667         |
| 3       | 5037.12  | 23.76    | 4         | 0.862          | 0.091          | 0.500         |
| 4       | 4769.56  | 27.73    | 1         | 0.816          | 0.110          | 0.000         |
| 5       | 3478.80  | 57.98    | 3         | 0.594          | 0.258          | 0.333         |
| 6       | 2936.67  | 31.24    | 3         | 0.501          | 0.127          | 0.333         |
| 7       | 2820.00  | 28.20    | 3         | 0.481          | 0.113          | 0.333         |
| 8       | 2640.00  | 55.00    | 4         | 0.450          | 0.243          | 0.500         |
| 9       | 2423.52  | 73.44    | 6         | 0.412          | 0.333          | 0.833         |
| 10      | 2407.50  | 160.50   | 4         | 0.410          | 0.758          | 0.500         |
| 11      | 1075.20  | 5.12     | 2         | 0.181          | 0.000          | 0.667         |
| 12      | 1043.50  | 20.87    | 5         | 0.175          | 0.077          | 0.667         |
| 13      | 1038.00  | 86.50    | 7         | 0.174          | 0.397          | 1.000         |
| 14      | 883.20   | 110.40   | 5         | 0.148          | 0.514          | 0.667         |
| 15      | 854.40   | 71.20    | 3         | 0.143          | 0.323          | 0.333         |
| 16      | 810.00   | 45.00    | 3         | 0.135          | 0.195          | 0.333         |
| 17      | 703.68   | 14.66    | 4         | 0.117          | 0.047          | 0.500         |
| 18      | 594.00   | 49.50    | 6         | 0.098          | 0.217          | 0.833         |
| 19      | 570.00   | 47.50    | 5         | 0.094          | 0.207          | 0.667         |
| 20      | 467.60   | 58.45    | 4         | 0.076          | 0.260          | 0.500         |
| 21      | 463.60   | 24.40    | 4         | 0.075          | 0.094          | 0.500         |
| 22      | 455.00   | 65.00    | 4         | 0.074          | 0.292          | 0.500         |
| 23      | 432.50   | 86.50    | 4         | 0.070          | 0.397          | 0.500         |
| 24      | 398.40   | 33.20    | 3         | 0.064          | 0.137          | 0.333         |
| 25      | 370.50   | 37.05    | 1         | 0.059          | 0.156          | 0.000         |
| 26      | 338.40   | 33.84    | 3         | 0.054          | 0.140          | 0.333         |
| 27      | 336.12   | 84.03    | 1         | 0.053          | 0.385          | 0.000         |
| 28      | 313.60   | 78.40    | 6         | 0.050          | 0.358          | 0.833         |
| 29      | 268.68   | 134.34   | 7         | 0.042          | 0.631          | 1.000         |
| 30      | 224.00   | 56.00    | 1         | 0.034          | 0.248          | 0.000         |
| 31      | 216.00   | 72.00    | 5         | 0.033          | 0.326          | 0.667         |
| 32      | 212.08   | 53.02    | 2         | 0.032          | 0.234          | 0.167         |
| 33      | 197.92   | 49.48    | 5         | 0.030          | 0.217          | 0.667         |
| 34      | 190.89   | 7.07     | 7         | 0.028          | 0.010          | 1.000         |
| 35      | 181.80   | 60.60    | 3         | 0.027          | 0.271          | 0.333         |
| 36      | 163.28   | 40.82    | 3         | 0.024          | 0.174          | 0.333         |
| 37      | 150.00   | 30.00    | 5         | 0.021          | 0.121          | 0.667         |
| 38      | 134.80   | 67.40    | 3         | 0.019          | 0.304          | 0.333         |
| 39      | 119.20   | 59.60    | 5         | 0.016          | 0.266          | 0.667         |
| 40      | 103.36   | 51.68    | 6         | 0.013          | 0.227          | 0.833         |
| 41      | 79.20    | 19.80    | 2         | 0.009          | 0.072          | 0.167         |
| 42      | 75.40    | 37.70    | 2         | 0.009          | 0.159          | 0.167         |
| 43      | 59.78    | 29.89    | 5         | 0.006          | 0.121          | 0.667         |
| 44      | 48.30    | 48.30    | 3         | 0.004          | 0.211          | 0.333         |
| 45      | 34.40    | 34.40    | 7         | 0.002          | 0.143          | 1.000         |
| 46      | 28.80    | 28.80    | 3         | 0.001          | 0.116          | 0.333         |
| 47      | 25.38    | 8.46     | 5         | 0.000          | 0.016          | 0.667         |
values of the objective functions were between 0 and 1 and their sum was 1. The $\varepsilon$-constraint method TRC objective function was chosen as the main objective and the dissimilarity function was added to the constraint conditions. The mathematical model was run by decreasing the $\varepsilon$ value by 4.68 at each step from the dissimilarity index selected as maximum 267.040 to the ideal point by 126.522. Equations (15) and (16) were rearranged as follows:

$$\delta_1 = \begin{cases} 
1 & \text{if } 0 \leq \delta_1 < 1098.165 \\
(1367.586 - \text{TRC}^*)/(1367.586 - 1098.165) & \text{if } 1098.165 \leq \delta_1 < 1367.586 \\
0 & \text{if } \delta_1 \geq 1367.586 
\end{cases}$$

(20)

$$\delta_2 = \begin{cases} 
1 & \text{if } 0 \leq \delta_2 < 126.522 \\
(267.040 - \text{ds}^*)/(267.040 - 126.522) & \text{if } 126.522 \leq \delta_2 < 267.040 \\
0 & \text{if } \delta_2 \geq 267.040 
\end{cases}$$

(21)

Equation (20) fully meets the targeted TRC in situations when the TRC is lower than $1098.165$ and satisfaction level will be 1. If TRC is larger than $1367.586$, it will be an unwanted cost for management and satisfaction will be 0. Values between these two values will be calculated with the equation in Eq. (20). Equation (21) shows that if the targeted dissimilarity index is lower than 126.522, the satisfaction level will be 1. If the dissimilarity index is higher than 267.040, the classification will have an unwanted distance value and satisfaction level will be 0. Values between these two values will be calculated with Eq. (21).

In this study, multi-criteria and multi-objective inventory item classification problem is discussed. The problem was solved using the Frontline Solver Platform with the SSA approach, which is an evolutionary algorithm. Among the solutions obtained by three different methods, dominated and non-dominated solutions were obtained. Among these solutions, non-dominated solutions show pareto frontier solutions. In Fig. 8, pareto frontier solutions and the best solutions obtained by three different methods are shown.

Meanwhile, the important aspects for decision makers is to decide which solution is the best solution. Different methods such as TOPSIS, VIKOR, min–max, deviation index can be used to decide on the best solution (Kumar et al. 2016; Mohseni-Bonab et al. 2016). In this study, the deviation index method given below was used to decide on the best solution among the obtained solutions:

$$\text{deviation index} = \sqrt{\sum_{j=1}^{n} (f_j - f_j^{\text{ideal}})^2} + \sqrt{\sum_{j=1}^{n} (f_j - f_j^{\text{non-ideal}})^2}$$

(22)

Lowest value of deviation index indicates that the solution is near the ideal point and far from the non-ideal point. The last column in Table 4 shows the deviation index values of the optimal solutions obtained by using the LP-metric, $\varepsilon$-constraint and weighted sum method.

The minimum value of deviation index is 0.193, which signifies that TRC and dissimilarity index function selected by weighted sum method is more efficient. In Fig. 8, Point A presents the solution point obtained by $\varepsilon$-constraint method. LP-metric method suggests point B. Weighted sum method provides point C. Different weights of objectives that varies between 0 and 1 were tested while weighted sum method is applying. The best solution was obtained with the weights taken as 0.6 for the TRC function and 0.4 for the dissimilarity index. The results were obtained with the weighted sum method using different weights are given in Table 5.

Proposed SSA is computed when the stopping criteria reached for desired solving time applied in order to find the best value of the satisfaction level. The change in the solution values by CPU time (sec) is shown in Fig. 9.

In order to analyze the performance of the proposed algorithms, the statistical analysis was applied in MINITAB 21.1. The statistical analysis based on TRC and dissimilarity index was performed, and the results were shown.

### Table 3: The results of the individual objective functions

| Objectives | TRC ($) | Dissimilarity index |
|------------|---------|---------------------|
| Min TRC    | 1098.165| 195.026             |
| Min ds     | 1149.241| 126.522             |

Bold values show the ideal point of TRC and ds objectives.
According to the p-value of ANOVA, there is significant difference between the mean of the three algorithms results by considering the confidence interval of 95%. According to Tukey’s analysis, there is a significant difference between the results of the algorithms.

| Optimization Algorithm | Multi-objective method | Objectives | Deviation index |
|-------------------------|------------------------|------------|-----------------|
| SSA                     | LP-metric              | 1138.54    | 126.57          | 0.203           |
| e-constraint            |                        | 1149.24    | 126.52          | 0.249           |
| Weighted sum            |                        | 1133.06    | 136.49          | 0.193           |

Single objective optimization for minimize total relevant cost:

| Objectives | Deviation index |
|------------|-----------------|
| 1098.17    | 195.02          | 0.338       |

Single objective optimization for minimize dissimilarity index:

| Objectives | Deviation index |
|------------|-----------------|
| 1149.24    | 126.52          | 0.249       |

Table 5 The results of the weighted sum method with different weights of the objective functions

| Weights | Objectives | Deviation index |
|---------|------------|-----------------|
| w_{TRC} | w_{ds}    | TRC ($) | ds |
| 0       | 1          | 1149.24 | 126.52 | 0.250 |
| 0.1     | 0.9        | 1149.24 | 126.52 | 0.250 |
| 0.2     | 0.8        | 1149.24 | 126.52 | 0.250 |
| 0.3     | 0.7        | 1138.54 | 129.57 | 0.207 |
| 0.4     | 0.6        | 1139.29 | 130.16 | 0.211 |
| 0.5     | 0.5        | 1138.54 | 129.57 | 0.207 |
| 0.6     | 0.4        | **1133.06** | **136.49** | **0.193** |
| 0.7     | 0.3        | 1100.38 | 187.70 | 0.310 |
| 0.8     | 0.2        | 1099.96 | 189.04 | 0.315 |
| 0.9     | 0.1        | 1099.96 | 189.04 | 0.315 |
| 1       | 0          | 1098.16 | 195.02 | 0.338 |

Bold values show minimum value of deviation index and ideal weights of TRC and ds objectives.
similarity between the weighted sum and the $\varepsilon$-constraint method on minimizing the TRC function (Fig. 10a), but the LP-metric method has a significant difference from other methods shown in Fig. 10b. For the dissimilarity index objective function, the results obtained with the $\varepsilon$-constraint method have a significant difference from other methods. Smallest values are searched for both objective functions, and Fig. 11 shows that we can find these smallest values with the weighted sum method.

The first experimental test in this study was applied with the weights of criteria as $w_{ADU} = 0.407$, $w_{ACU} = 0.037$ and $w_{LT} = 0.556$ by using Reid’s dataset. According to the deviation index given in Table 4, the weighted sum method was chosen as the best method. After decided the best method for the multi-objective problem solving, the SSA algorithm was applied with different criterion weights. The smallest objective solutions were obtained for TRC and dissimilarity index by using 0.6 and 0.4 weights. In Table 4, first row weights show the weights of criteria obtained in the (Kaabi et al. 2015) study. The second row weights were obtained by the ELIII-CVNS method in (Douissa and Jabeur 2019) study and higher importance was given to the ADU criterion. In the third set of experiments, the condition that all criteria have equal importance was tested. Experimental set number 7 shows the criteria weights obtained in the (Lolli et al. 2014) study. Experimental sets 8 and 9 were taken from the (Kaabi et al. 2018) study. By changing the places of the weights in the second experimental set, the 4th and 6th experimental sets were obtained. The solutions obtained from this analysis are given in Table 7.

According to Table 7, the weights of the ADU, ACU and LT criteria, which provide the lowest deviation index value, were calculated as 0.691, 0.183 and 0.126, respectively. This solution value is equal to 0.0243 Euclidean distance from the ideal point. The table of differences with other studies was prepared with the criteria weights (0.691, 0.183 and 0.126) that were decided last. The optimal result computed for the MCIC problem is compared with the results obtained by previous studies that used Reid’s dataset in Table 8. The results obtained with single objective function based on minimum TRC and minimum dissimilarity index are computed separately and placed in 8th and 9th columns of Table 8. 10th column of Table 8 presents the SSA results for multi-objective function. Last
column of Table 8 indicates GA results for multi-objective optimization, which is one of the most widely used evolutionary algorithms.

In Table 9, the results of SSA by using the (0.691, 0.183, 0.126) criteria weights are shown and the results of other studies. By applying the SSA, the satisfaction level values of the TRC and the dissimilarity index were found to be 0.984 and 0.996, respectively. TRC value was calculated as $1102.539 and dissimilarity index was calculated as 127.132. According to these objective function values the total satisfaction level was obtained 0.989. SSA provided better inventory relevant cost and dissimilarity index than the previous studies. In the study of Douissa & Jabeur (2019), the TRC value is $1122,779 as a result of the classification obtained by the ELIII-CVNS method, while the cost obtained in this study is calculated as $1102,539 with two objectives. It is seen that the dissimilarity index within classes was obtained by the SSA method applied in this study. On the other hand, GA was also applied on the proposed model to solve the multi-objective problem and the solution had better results than the previous studies. However, SSA still provided the best solution out of all.

Figure 12 shows the graph of the TRC and dissimilarity index values obtained from our proposed algorithm and the other comparison studies. Minimum TRC and dissimilarity index are calculated by our proposed SSA approach. Weight values are very important to calculate the dissimilarity index. According to the weight value, the item class is to change based on the most important criteria. In our study, satisfaction level was recommended ensuring both objective functions approach best values and solutions were found with the SSA approach. In our study, there were 10 items in class A, 19 items in class B and 18 items in class C. According to Table 10, the highest similarity to the proposed model was observed for the study of traditional ABC analysis and Douissa & Jabeur (Douissa and Jabeur 2019). For example, items 25, 26, 27, 30, and 32 were classified as class C by the Douissa & Jabeur’s model. However, it was classified as class B in our proposed model. If we classify these items in class B, TRC value will be 48.73. But, if we assign this item in class C, TRC value will be 63.48. Thus, the total TRC value will be decrease by percentage of %23.24.

We applied three models to get the best satisfaction level of two objectives. The first model is to minimize the TRC. The second is to minimize the dissimilarity index. The third one is maximization the satisfaction level of the two functions. It is shown in Table 8 that item 9 is classified Class B for the first model. Item 9 has 57.94 TRC value, while has dissimilarity index 5.97. But the same item is assigned to Class A in the second model and the TRC values and dissimilarity index changes as 60.66 and 2.95, respectively. Finally, item 9 is assigned to Class A because of the big decreasing in dissimilarity index by 50.58% even though TRC function increased by 4.69%.

We applied our model with using the SSA and GA approach for single objective and then for two objectives simultaneously. Figure 13. shows that the classification results for four models. It can be seen this figure, there is not big changes in the A class items distribution, but B and C class have very big changes in each other.

5 Experimental results with Liu et al.’s dataset

Liu et al.’s dataset (Liu et al. 2016) contains 63 inventory items used in the manufacturer of sports equipment in China. In this problem, four criteria were evaluated, namely the annual usage (ADU), average unit cost (ACU), lead time (LT) and turnover ratio (TR). Douissa and Jabeur (2019) studied with the same dataset using the Electre III-CVNS method and compared the results with the findings in the study of Liu et al. (Liu et al. 2016). Douissa and

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**Table 7** The results of the SSA with different criteria weight values

| #  | Weights (ADU, ACU, LT) | Objectives | Deviation index |
|----|------------------------|------------|----------------|
|    |                        | TRC ($)    | ds             |
| 1  | (0.407, 0.037, 0.556)  | 1133.06    | 136.49         | 0.1925         |
| 2  | **(0.691, 0.183, 0.126)** | **1102.54** | **127.13**     | **0.0243**     |
| 3  | (0.333, 0.333, 0.333)  | 1133.98    | 138.54         | 0.2012         |
| 4  | (0.183, 0.691, 0.126)  | 1133.98    | 134.32         | 0.1926         |
| 5  | (0.125, 0.750, 0.125)  | 1100.38    | 156.41         | 0.1605         |
| 6  | (0.126, 0.183, 0.691)  | 1138.40    | 145.20         | 0.2380         |
| 7  | (0.558, 0.320, 0.122)  | 1099.96    | 137.56         | 0.0615         |
| 8  | (0.41, 0.17, 0.42)     | 1138.54    | 131.70         | 0.2094         |
| 9  | (0.5, 0.03, 0.47)      | 1133.06    | 130.37         | 0.1823         |

Bold values show minimum value of deviation index with ideal weights of criterias (ADU, ACU, LT)
Jabeur (2019) demonstrated a better classification solution with lower total relevant cost in comparison to the other studies. In this paper, the dataset of Liu et al. (2016) was used to test the proposed SSA method. Unlike the Reid’s dataset, 4 criteria and 63 items are handled. In the experimental test, which is considered in this section, the same criteria weights as Liu et al.’s study were used ($w_{\text{ADU}} = 0.2$, $w_{\text{ACU}} = 0.23$, $w_{\text{LT}} = 0.3$, $w_{\text{TR}} = 0.27$). Two objective functions were considered in accordance with the method we followed in the previous section, and the weighted sum method was run by applying the mathematical model SSA using 0.6 and 0.4 weight values.

Proposed classification results and the results of other studies are summarized in Table 11. TRC is equal to $58,669.31$ with the proposed method in this study when only a single objective function to minimize the TRC is considered. It is indicated that with the multi-objective optimization method, TRC equals $60,006.66$, and this is also shown to be the smallest result. Also, the smallest dissimilarity index shown in the last row of Table 11 is calculated by using our suggested model. The maximum satisfaction level of two objectives equals 0.84 and gives the best result. When only a single objective function is considered, the TRC will be decreased by 4.8% according to the ELIII-CVNS method and dissimilarity index will be decreased by 44.1%.

The classification result evaluated the percentage of similarity with other methods, and it is shown in Table 12. According to Table 11, only 16 items have the same classification for the three methods (ELIII-CVNS, Liu et al., SSA two obj.). The similarity between the ELIII-CVNS and SSA with two objectives is 63.49% according to Table 12. For example, Item 5 was assigned to Class B in Douissa & Jabeur’s study and Class C in Liu et al.’s study. In the models proposed in this study, the 5th item is calculated as Class A if the objective is to minimize the TRC or maximize the satisfaction level, Class B if the objective is to minimize dissimilarity index. The inventory cost of 5th item will be $1142.51$ if it is taken as Class B, whereas, if it is taken as Class A, the cost will be $979.03$.

### Table 8 Comparison results for previous studies and new proposed model

| Item No | [11] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] | [11] |
|---------|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|-----|
| 1       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 2       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 3       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 4       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 5       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 6       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 7       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 8       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 9       | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 10      | A    | A   | A   | A   | A   | A   | A   | A   | A   | A   | A    | A   |
| 11      | B    | A   | B   | C   | A   | B   | A   | C   | B   | A   | A    | A   |
| 12      | B    | B   | B   | B   | C   | B   | C   | B   | B   | B   | A    | A   |
| 13      | B    | B   | B   | A   | C   | A   | B   | B   | C   | B   | A    | A   |
| 14      | B    | B   | B   | B   | A   | B   | B   | C   | B   | B   | A    | A   |
| 15      | B    | B   | B   | C   | C   | B   | B   | B   | B   | B   | A    | A   |
| 16      | B    | B   | B   | C   | C   | C   | B   | B   | B   | B   | A    | A   |
| 17      | B    | B   | B   | C   | C   | B   | C   | B   | B   | B   | A    | A   |
| 18      | B    | B   | B   | C   | C   | B   | C   | B   | B   | B   | A    | A   |
| 19      | B    | B   | B   | B   | C   | B   | C   | B   | B   | B   | A    | A   |
| 20      | B    | B   | B   | B   | C   | C   | C   | B   | B   | B   | A    | A   |
| 21      | B    | B   | B   | C   | C   | B   | B   | B   | B   | B   | A    | A   |
| 22      | B    | B   | B   | C   | C   | B   | B   | B   | B   | B   | A    | A   |
| 23      | B    | B   | B   | C   | C   | A   | B   | B   | B   | B   | A    | A   |
| 24      | B    | B   | B   | C   | C   | C   | B   | B   | B   | B   | A    | A   |
| 25      | C    | C   | C   | C   | C   | C   | B   | A   | B   | B   | A    | A   |
| 26      | C    | C   | C   | C   | C   | C   | B   | B   | B   | B   | A    | A   |
| 27      | C    | C   | C   | C   | C   | A   | B   | C   | B   | A   | A    | A   |
| 28      | C    | C   | B   | B   | C   | A   | C   | C   | C   | C   | A    | A   |
| 29      | C    | C   | B   | C   | C   | A   | C   | C   | C   | C   | A    | A   |
| 30      | C    | C   | C   | C   | B   | C   | C   | C   | C   | C   | A    | A   |
| 31      | C    | B   | B   | B   | C   | C   | C   | B   | C   | C   | A    | A   |
| 32      | C    | B   | B   | C   | C   | C   | C   | B   | B   | B   | A    | A   |
| 33      | C    | C   | C   | B   | C   | C   | C   | C   | C   | C   | A    | A   |
| 34      | C    | C   | C   | B   | C   | C   | C   | C   | C   | C   | A    | A   |
| 35      | C    | C   | C   | C   | B   | C   | C   | C   | C   | C   | A    | A   |
| 36      | C    | B   | B   | C   | C   | C   | C   | B   | C   | C   | A    | A   |
| 37      | C    | C   | B   | C   | C   | C   | C   | B   | C   | C   | A    | A   |
| 38      | C    | B   | B   | C   | C   | C   | C   | B   | C   | C   | A    | A   |
| 39      | C    | C   | C   | C   | C   | C   | C   | C   | C   | C   | A    | A   |
| 40      | C    | C   | C   | C   | C   | C   | C   | C   | C   | C   | A    | A   |
| 41      | C    | C   | C   | C   | C   | C   | C   | C   | C   | C   | A    | A   |
| 42      | C    | C   | C   | C   | C   | C   | C   | C   | C   | C   | A    | A   |
| 43      | C    | C   | C   | C   | C   | C   | C   | C   | C   | C   | A    | A   |
| 44      | C    | C   | C   | C   | C   | C   | C   | C   | C   | C   | A    | A   |
| 45      | C    | C   | B   | B   | C   | C   | C   | C   | C   | C   | A    | A   |
| 46      | C    | C   | C   | C   | C   | C   | C   | C   | C   | C   | A    | A   |

Jabeur (2019) demonstrated a better classification solution with lower total relevant cost in comparison to the other studies. In this paper, the dataset of Liu et al. (2016) was used to test the proposed SSA method. Unlike the Reid’s dataset, 4 criteria and 63 items are handled. In the experimental test, which is considered in this section, the same criteria weights as Liu et al.’s study were used ($w_{\text{ADU}} = 0.2$, $w_{\text{ACU}} = 0.23$, $w_{\text{LT}} = 0.3$, $w_{\text{TR}} = 0.27$). Two objective functions were considered in accordance with the method we followed in the previous section, and the weighted sum method was run by applying the mathematical model SSA using 0.6 and 0.4 weight values.
In the case of Class C, the inventory cost increases to $1533.74. On the other hand, the dissimilarity index increases from 9.01 to 13.72 in Class B and from 9.01 to 11.59 in class C.

### 6 Conclusion

The importance of inventory management problems gradually increases in today’s economic and hard competitive conditions. Due to the increasing product range and growing alternative supply channels, reaching the desired product at desired time is one of the most important factors in obtaining customer satisfaction. The main step of the inventory management is started with the classification of the inventory items that consider multi-criteria values. We have proposed a very efficient algorithm for helping inventory manager and company directors on how to classify the items about the inventory management.

In this paper, MCIC problem is formulated with the objectives of minimizing total inventory cost and dissimilarity index. The proposed model is developed as MINLP
model. We offered three models to success the desired best results. The first model is to minimize the TRC function that includes the inventory holding cost and the ordering cost. The second model is to minimize the dissimilarity index. The aim of the second model is to assign inventory items that have the same criteria values to the same class. In inventory item classification, not only costs but also the same item types may need to be evaluated together. This

### Table 10 Percentage of similarity results with other methods

|                  | Traditional ABC | Mohammaditabar et al | Lolli et al. AHP-K veto | Kaabi et al. (2015) | Zhang et al |
|------------------|-----------------|-----------------------|--------------------------|---------------------|-------------|
|                  | A    | B    | C    | A    | B    | C    | A    | B    | C    | A    | B    | C    | A    | B    | C    | A    | B    | C    |
| Proposed SSA-two obj | 10   | 10   | 1    | 1    | 8    | 1    | 9    | 1    | 4    | 5    | 1    |
| B                | 14   | 5    | 1    | 14   | 4    | 15   | 4    | 1    | 5    | 13   | 19   |
| C                | 18   | 3    | 15   | 8    | 10   | 17   | 11   | 18   |       |       |      |
| Similarity number| 42   | 39   | 26   | 25   | 22   |      |      |      |       |       |      |
| Similarity %     | 89.36| 82.97| 55.32| 53.19| 46.80|      |      |      |       |       |      |

| Proposed SSA-two obj | 3    | 2    | 5    | 10   | 10   | 10   | 9    | 1    | 10   |
| Douissa & Jabeur    | 5    | 6    | 8    | 14   | 5    | 2    | 16   | 1    | 1    |
| GA-two obj          | 2    | 6    | 10   | 18   | 3    | 15   | 18   | 1    | 9    |
| similarity number    | 19   | 42   | 41   |      | 45   |      |      | 28   |
| similarity %         | 40.42| 89.36| 87.23|      | 95.74|      |      | 59.57|

### Fig. 13 The classification of items with different objective functions

8802 I. Saracoglu

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Table 11  ABC classifications results obtained from SSA and comparison of the different studies with using Liu’s et al. dataset

| Item | AUC   | ARMBU  | LT | TR | ELIII-CVNS (Douissa and Jabeur 2019) | Liu et al. (2016) | Proposed model |
|------|-------|--------|----|----|------------------------------------|-------------------|----------------|
|      |       |        |    |    | SSA TRC min | SSA dis min | SSA two obj |
| 1    | 210.39| 413,692.2 | 18 | 0.487 | C | B | C |
| 2    | 70.92 | 363,303.1  | 29 | 1.870 | A | B | A |
| 3    | 125.24| 452,711.6  | 12 | 1.653 | B | A | A |
| 4    | 26.72 | 391,531.3  | 16 | 1.848 | B | C | A |
| 5    | 81.98 | 164,125.6  | 24 | 1.204 | B | A | A |
| 6    | 164.18| 101,627    | 12 | 0.162 | C | C | C |
| 7    | 219.19| 327,849.6  | 5  | 2.994 | B | B | B |
| 8    | 190.53| 55,345.3   | 18 | 0.130 | C | C | C |
| 9    | 202.96| 443,096.1  | 18 | 0.378 | C | B | C |
| 10   | 119.58| 294,066.8  | 25 | 1.374 | B | A | A |
| 11   | 71.21 | 390,861.5  | 30 | 2.047 | A | B | A |
| 12   | 113.36| 298,718.7  | 11 | 1.211 | C | B | B |
| 13   | 71.83 | 88,071.9   | 1  | 2.646 | C | B | A |
| 14   | 89.35 | 41,150.6   | 2  | 1.019 | C | B | C |
| 15   | 153.28| 414,547.2  | 14 | 1.032 | C | B | C |
| 16   | 103.63| 77,582     | 30 | 1.836 | B | C | C |
| 17   | 230.34| 31,681.6   | 2  | 2.822 | B | A | B |
| 18   | 80.27 | 295,351.8  | 11 | 0.531 | C | C | C |
| 19   | 187.75| 233,313.8  | 6  | 0.353 | C | C | C |
| 20   | 51.35 | 231,721.6  | 26 | 1.376 | B | A | A |
| 21   | 75.92 | 454,758.8  | 20 | 2.929 | A | A | A |
| 22   | 67.16 | 80,556.6   | 27 | 2.381 | C | A | A |
| 23   | 173.29| 397,196.1  | 23 | 0.471 | B | B | C |
| 24   | 41.71 | 336,693    | 15 | 2.370 | B | A | B |
| 25   | 132.89| 459,578.7  | 3  | 2.152 | C | B | B |
| 26   | 50.33 | 281,313.1  | 16 | 0.917 | C | B | C |
| 27   | 39.29 | 101,493.5  | 13 | 0.431 | C | B | C |
| 28   | 189.24| 128,298.4  | 18 | 2.979 | B | A | A |
| 29   | 228.69| 311,478.1  | 29 | 1.819 | A | A | A |
| 30   | 54.94 | 188,630.7  | 29 | 1.540 | B | C | B |
| 31   | 42.17 | 180,117    | 30 | 1.135 | B | C | A |
| 32   | 199.8 | 15,296.7   | 16 | 0.799 | C | B | C |
| 33   | 152.85| 383,919.9  | 22 | 1.790 | B | A | A |
| 34   | 193.37| 119,454.6  | 5  | 0.324 | C | C | C |
| 35   | 138.47| 333,290.6  | 6  | 1.650 | C | B | B |
| 36   | 73.4  | 374,496.8  | 28 | 0.619 | C | C | C |
| 37   | 147.65| 364,491.1  | 28 | 1.666 | A | A | B |
| 38   | 40.93 | 407,329.5  | 3  | 1.856 | C | C | B |
| 39   | 92.86 | 370,301.3  | 21 | 0.107 | C | B | C |
| 40   | 225.49| 322,614.6  | 6  | 2.548 | B | B | B |
| 41   | 102.61| 50,402.1   | 25 | 0.609 | B | A | C |
| 42   | 207.53| 499,699.9  | 10 | 0.692 | C | B | C |
| 43   | 243.36| 209,629.8  | 25 | 1.072 | B | A | A |
| 44   | 140.26| 38,914.1   | 25 | 0.109 | C | A | C |
| 45   | 170.96| 370,885.2  | 10 | 2.002 | B | B | A |
| 46   | 136.45| 499,854.8  | 7  | 0.376 | C | B | C |
objective can be very important for the operations of warehouse. In the third model, multi-objective optimization problem is constructed to maximize satisfaction level of the minimization of TRC and dissimilarity index. The SSA approach is applied to solve the model because of the complexity of solving time. A SSA has been proposed and illustrated in this paper for classifying inventory items with multiple criteria and multiple objectives. Three different methods are used to solve the multi-objective optimization problem. These are LP-metric, $\epsilon$-constraint and weighted sum methods. To find the best result, deviation indexes are calculated for each solution. We found that there is a significant difference between the methods according to the statistical analysis. The smallest deviation index is obtained using the weighted sum method. This method supplied the maximum satisfaction level for two objectives. After selecting the multi-objective method, different criteria weights are examined to find the better result. The proposed model is compared with seven previously published studies to show the performance. These seven studies were selected because they all used Reid’s dataset and calculated the Total Relevant Cost. It can be clearly seen that the proposed SSA algorithm for the MCIC problem produced more effective results than the other

| Item | AUC   | ARMBU  | LT    | TR    | ELIII-CVNS (Douissa and Jabeur 2019) | Liu et al. (2016) | Proposed model |
|------|-------|--------|-------|-------|-------------------------------------|-------------------|----------------|
|      |       |        |       |       | SSA TRC min | SSA dis min | SSA two obj |
| 47   | 187.27| 274,935.6 | 5     | 2.512 | B         | B       | A            | B |
| 48   | 160.84| 296,976.8 | 28    | 2.108 | A         | A       | B            | A |
| 49   | 36.47 | 78,051.4  | 22    | 0.882 | B         | C       | A            | B |
| 50   | 209.5 | 318,688.5 | 4     | 2.259 | C         | B       | B            | B |
| 51   | 32.53 | 273,490.9 | 20    | 2.507 | B         | C       | A            | A |
| 52   | 171.64| 142,923   | 5     | 0.815 | C         | C       | B            | C |
| 53   | 235.08| 329,205.1 | 3     | 1.574 | B         | B       | C            | C |
| 54   | 113.84| 497,119.6 | 23    | 0.030 | C         | B       | C            | C |
| 55   | 27.85 | 255,434   | 6     | 2.227 | C         | B       | C            | C |
| 56   | 15.4  | 103,414   | 1     | 2.902 | C         | C       | B            | B |
| 57   | 194.05| 316,586   | 1     | 2.928 | B         | C       | A            | B |
| 58   | 0.68  | 341,859.6 | 13    | 1.991 | B         | C       | B            | B |
| 59   | 151.69| 228,109   | 18    | 1.259 | B         | B       | A            | B |
| 60   | 89.23 | 43,136.7  | 2     | 0.025 | C         | C       | C            | C |
| 61   | 98.65 | 495,254.8 | 1     | 1.089 | C         | B       | C            | C |
| 62   | 99.15 | 20,635.4  | 18    | 2.069 | B         | C       | A            | A |
| 63   | 104.84| 370,919.4 | 11    | 1.214 | C         | C       | B            | B |
| TRC ($) | 61,642.32 | 65,759.56 | 58,669.31 | 64,455.87 | 60,006.66 |
| ds   | 659.602| 756.807 | 478.142 | 368.757 | 432.487 |
| $\delta_1$ of TRC | 0.644 | 0.151 | 1.000 | 0.307 | 0.840 |
| $\delta_2$ of ds  | 0.275 | 0.033 | 0.727 | 1.000 | 0.841 |
| Total satisfaction level (SL) | 0.496 | 0.104 | 0.891 | 0.584 | 0.840 |

| Table 12 Percentage of similarity between the SSA two objective results and other compared studies |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Methods                        | SSA-two obj | ELIII-CVNS | Liu et al | SSA-TRC | SSA-ds |
| SSA-two obj                    | 1             | 63.49       | 39.68      | 74.60      | 61.9   |
| ELIII-CVNS                     | 1             | 50.79       | 47.61      | 55.55      | 36.5   |
| Liu et al                      | 1             | 33.33       | 33.33      | 36.5       | 1      |
| SSA-TRC                        | 1             | 53.96       | 53.96      | 36.5       | 1      |
| SSA-ds                         | 1             | 53.96       | 53.96      | 36.5       | 1      |
algorithms. Additionally, SSA algorithm provided the better result when compared to the GA, which is also another evolutionary algorithm. In order to evaluate our suggested method, we tested another problem found in the literature (Liu et al. 2016) that had 63 items and different four criteria. Better solutions are also obtained by applying SSA in the multi-objective MCIC problem in a reasonable time.

In future, the algorithm can be used to outperform existing methods for different models of classification of items. The proposed classification model can be applied on the large inventory items that holding in the warehouse for example pharmaceutical items, spare parts, electronic components, and electromechanical items, etc. Studies are performed with fuzzy along with different inventory policies such as (s, S) and (Q, r). These studies may be developed and tested with different real-life problems and types of criteria. Mathematical model can be converted to the linear programming model to solve the problem with the exact optimization method in a reasonable time. Other inventory performance measures, for example safety stock cost, shortage cost, service level can be considered to solve the MCIC problem.

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