Critical magnetodynamics in quantum dot arrays

V N Kondratyev 1 2
1 Nuclear Physics Department, Taras Shevchenko National University, Pr. Acad. Glushkova 2, bdg. 11, Kiev, UA-03022 Ukraine
2 Bogolubov Laboratory of Theoretical Physics, JINR, 141980, Dubna, Russia

E-mail: vkondra@univ.kiev.ua

Abstract. Arrays of coupled quantum dots are analysed by employing the randomly jumping interacting moments model including quantum fluctuations due to the discrete level structure, ferromagnetic inter-dot coupling and disorder. The respective magnetic state equation and phase diagram are found to indicate an existence of spinodal regions and critical points. In vicinity of such points magnetic induction shows jerky behaviour displayed as erratic discontinuities associated with avalanches of the dot magnetization. The model predicts some universal scaling properties for magnetic noise. On the basis of such properties we suggest and test model-independent analytical tools employed in order to specify, quantify and analyse the system with respect to the observed magnetic structure. Some specific features for arrays of dots with singlet-triplet transitions are discussed.

1. Introduction
Advances in a design of electronic devices have made it possible to reduce the size of active elements until their spatial dimensions become comparable to the de Broglie wavelength of the active electrons. Such quantum confined systems (e.g., atomic clusters, quantum dots referred for hereafter as, simply, QDs) allow, e.g., to develop new materials exhibiting a variety of characteristics going beyond traditional solid state systems and achieving possible benefits in 'figures of merits' for technological and therapeutic applications. Besides, such super-crystals are of fundamental interest for a study of interactions, transport processes and phase features at fairly wide range of various parameters, e.g., coupling strengths, densities, capacity gaps. Magnetism and magnetotransport phenomena in magnetic QDs and their arrays (e.g., arrays of iron series transition metal nano-crystals) attract recently considerable attention due to their significance for advanced electronic devices, nanoscale storage media, magnetic recording technology and advanced therapy. The magnetodynamics of realistic supercrystalline heterostructures is substantially determined by the relationship between inter-dot magnetic interaction and the disorder (see [1-3] and refs. therein). Then occurrence of self-organized (SO) criticality represents, perhaps, one of the most interesting and important phenomenon. At such a regime magnetic induction of QD arrays displays erratic stochastic discontinuities with rather wide distribution of jump amplitudes. In the present work we investigate further QD arrays paying particular attention for magnetic structure and universal scaling behaviour in magnetodynamics. The correlation properties of noise signal amplitude distributions are explored as an analytical tool for quantititative definition, description and analysis of systems at conditions of SO criticality. In sect. 2 we give brief overview of discrete spin model for realistic QD arrays and employ some methods to reveal magnetic structure at different properties of dot magnetic response. Conclusions are given in sect. 3.
2. Magnetic dot arrays with disorder
While arranged in an array the magnetic QDs with moments $m$ contribute to the overall magnetisation $P = m/V_D$ with the volume $V_D$ occupied by $i$th dot. For a case of comparable dot size and an occupied volume $V_D$ the inter-dot interaction brings, plausibly, ferromagnetic ordering [1-4]. Then, assuming a coupling of a strength $J$ between the nearest neighbour (nn) elements the respective contribution to the Hamiltonian is expressed in a form of Ising term $-J_{ij} m_i \cdot m_j$. Here the sum runs over the nn elements. Besides, inhomogeneity and disorder in the form of defects, grain boundaries, impurities lead to random crystalline anisotropy and varying interaction strengths in the super-crystalline heterostructure. Such effects can be accounted for by the random fields $h_i$. We point out also dynamical components of $h_i$ due to inexactness of the model description with nn interaction. The central limit theorem suggests, thereby, that the random fields obey the Gaussian distribution $W(h) = \exp\left(-h^2/R^2\right)/R\sqrt{\pi}$ of a width $R$, which we call the disorder. The total Hamiltonian $H$ of an array in a field $H$ can be expressed as

$$H = -\sum_i m_i h_i$$ (1)

through an interaction of the dot magnetic moment $m_i$ with local field $h_i = H(t) + J \sum_{j_{nn}} P_j + h_i$.

We refer for this model as randomly jumping interacting moments (RJIM) model [2,3].

![Figure 1](image)

**Figure 1.** Normalized avalanche size distributions $D(S)/D(1)$ from numerical simulations when applying the RJIM model for simple cubic lattice of a size $(30)^3$. Results for sub-critical disorder $R=1$ are shown by solid circles, the vicinity of critical point $R=1.8$ are indicated by solid squares, while open circles correspond to over-critical value $R=3$. Dotted line displays power law size dependence with an exponent $t=1.6$. 

The International Conference on Theoretical Physics ‘Dubna-Nano2008’
Journal of Physics: Conference Series 129 (2008) 012013
doi:10.1088/1742-6596/129/1/012013
2.1. Simulations of the magnetodynamics

The numerical modelling are performed here for an array of dots with single discontinuity in magnetic moment response, cf. e.g. [2,3] and refs. therein. When the local field $b_i$ of some $i$-th lattice element changes sign the moment $m_i$ jumps step-wise from $m$ to $-m$, i.e., $m_i = m \text{sign}(b_i)$. Due to the ferromagnetic interaction a jumping moment can cause some of the nearest neighbours to jump, which may in turn trigger some of their neighbours, and so on, generating, thereby, (re)magnetizing avalanche. As a consequence some sharp stepwise discontinuity arises on magnetization curves. It is worthy to notice that such sharp changes of magnetic induction result in a release of magnetic energy similarly to the well-known Barkhausen effect. Evidently, the energy values of noise signals are proportional to avalanche sizes. We perform the simulations for simple cubic lattice of a size $(30)^3$ while assuming an adiabatic time dependence of the external magnetic field $H$ and a moment jump amplitude corresponding to a relation $|m|/V_0=1$. The inclusive size distribution $D(S)$ presented in Fig. 1 gives the number of avalanches with $S$ jumped dots and provides the first clue on system behaviour. One sees a clear transition from the `U' shape distribution at small disorders to an abrupt exponential suppression of large size avalanches at large disorders. At transitional values $R$ the distribution shows behaviour close to the power law dependence with an exponent $t=1.6$ providing, thereby, a signal for self-organized criticality.

![Magnetization (Panel A) and magnetic susceptibility (Panel B) for an array of non-interacting QDs with single jump at B=0 and disorder R.](image)

2.2. Mean-field approximation

The basic features of the nonequilibrium system corresponding to the Hamiltonian Eq. (1) can be analysed by employing the mean-field approach, in which one assumes equal interaction strength between QDs. In this picture all the $\Pi$ moments in an array act as nearest neighbours with the coupling of a strength $J_\Pi = J/\Pi$. The local field in Eq. (1) can be then simplified to the form $b_i^{mf} = H(t) + JP + h_i$ with an averaged over a sample magnetization $P = \sum_i P_i / \Pi$. We see, therefore, that random fields can be viewed as mean-field fluctuations (cf. e.g., [2,3,5]).

In the thermodynamic limit $\Pi \to \infty$ we calculate the magnetic state equation (MSE) $P = \int dhW(h)m(b)$ and find for the magnetic susceptibility

$$\chi = -dP/dH = \chi_{NI} [1 - J \chi_{NI}]$$

(2)
with $c_{NI}=W(b)$ representing the susceptibility of an array with vanishing interdot interaction (i.e. $J \to 0$). Here we account for a single (re)magnetizing jump. The negatively defined susceptibility Eq. (2) yields spinodal region for an array. Figure 2 shows the dependence of magnetization $P$ and susceptibility $\chi_{NI}$ on the disorder $R$ and magnetic field $b=H+JP$. The instability regions correspond to the condition $\chi_{NI} \geq J^{-1}$ and they are indicated on $\{b,R\}$-plane by respective contour lines as is shown in Fig. 2b. For the dots with single jump it turns out that such simplified treatment already reflects most of the essential features of the long-length scale behavior of the system in finite dimensions, e.g., the model exhibits hysteresis, while the external field changes sign (cf. sect. 2.2). With increasing disorder we observe a continuous transition from the case of the macroscopic discontinuities in hysteresis loop to the almost smooth magnetization curve, cf. Fig. 2a. The former conditions of small disorders correspond to an avalanche spanning almost entire sample while at large $R$ an array favours to evolve through the series of magnetic jumps of the smallest size. At transitional values of the disorder the system displays the power law distributions of the noise intensities (i.e. avalanches) and an universal behavior providing, thereby, criticality signals.

**Figure 3.** Mean avalanche size versus the linear size of the biggest avalanche in units of array length. Results of RJIM model for $(30)^3$ simple cubic lattice at various disorders are shown by dots while dashed-dotted line joins respective average values for each disorder. Solid line displays the prediction of the mean-field approximation in the thermodynamic limit. The difference between the average number of induced jumps and 1, $d = J \chi_{NI} - 1$, provides a measure for a vicinity of SO criticality. Since at such conditions the mean number of induced jumps shows an extremum approaching 1, the mean linear size $l_b$ of the biggest avalanche $S_b$ can be estimated as $l_b^{eff} = (1+d)/2$ of the total linear size, cf. [3].
Since within the mean-field treatment an average number of the moments induced to jump by a single jumping moment is site independent the avalanche size distribution \( D(S) \) is related to the probability \( Q(S) \) of triggering \( S \) consequent jumps as \( D_{mf}(S) = Q(S)/S \). Since any one from \( S \) moments can switch on the jump series. For \( S \ll \Pi \) we can employ the Poissonian probability \( Q(S) = \exp\{-S(1+d)\}/S! \). For large avalanche size \( S \gg 1 \) we incorporate the Stirling formula \( S! = [S/e]^S \sqrt{2\pi S} \). Then in the vicinity of critical conditions \( d \to 0 \) (see above) we expand \( \ln[1+d] = d + d^2/2 + ... \) and obtain the noise size distribution in the form

\[
D_{mf}(S) \sim S^{-3/2} \exp\{-Sd^2/2\}
\]

Making use of the analytical form Eq. (3) we analyze some analytical tools which might be employed in order to specify and analyze magnetic systems with respect to SO criticality. Similarly to methods of high energy physics (cf., e.g., [6,7] and refs. therein) the correlations of avalanche size distribution might provide the criticality signals and a tool specifying and quantifying respective phenomena. For certain \( i^{th} \) (re)magnetization event we define the mean noise signal

\[
\langle p \rangle = \sum S D(S)/\sum D(S) = (\Pi - S_b)/(N_{tot} - 1)
\]

where the sum runs over the avalanche sizes \( S \) excluding the biggest one, the quantity \( N_{tot} \) gives the total number of noisy jumps, i.e. avalanches. Substituting Eq. (3) into Eq. (4) the mean avalanche size, i.e. mean value for magnetic emission signals, is evaluated to be \( \langle p \rangle_{mf} = |d|^{-1} + \text{const}(d) \), and diverges at critical conditions, i.e. \( d \to 0 \), in the thermodynamic limit \( \Pi \to \infty \).

**Figure 4.** Magnetization (Panel A) and magnetic susceptibility (Panel B) for an array of non-interacting QDs with singlet-triplet transition at \( B_{st} = 1 \) and disorder \( R \).

If the system undergoes a critical behaviour being a precursor of SO criticality in some particular (re)magnetization events, strong correlations will appear in magnetic noise. For instance, in case of magnetodynamics we can study correlations between the strongest signal (i.e., the largest avalanche \( S_b \)) and the mean signal value \( \langle p \rangle \) for remaining avalanches in this particular event, e.g., from numerical model or experimental data. These correlations are similar to the Campi scatter plots [6,7].

In Fig. 3 we plot the mean avalanche size versus the length of largest avalanche for certain event. The data in Fig. 3 were obtained from RJIM simulations assuming various disorders as partially presented in Fig. 1. In the figure we can clearly distinguish two branches corresponding to under-critical, i.e., large size of the biggest jump amplitude and small mean value, and overcritical, i.e. small size of the biggest avalanche and small average values. The right branch consists mainly of events with small disorders, while the left branch originates from events having large \( R \). The set of two branches meet in the critical region. The results of the mean field approximation in the thermodynamic limit are in a reasonable agreement with numerical data for overcritical disorders. At sub-critical conditions the mean field approach reproduces only qualitatively the RJIM model simulations.
2.3. Dots with singlet-triplet transition

Now we consider the dots exhibiting sharp step-vice anomalies for magnetic moments at non-zero field, cf., e.g., [2,3,8]. Well-known examples are represented by the singlet-triplet transitions (cf., e.g., [9,10] and refs. therein) in semiconductor QDs and carbon nanotubes under an influence of magnetic field of a strength $B_{st}$. The dot magnetic moment then reads $m=\text{sign}(b) \cdot (|b|- B_{st})$ with step function $\theta(x)$. As is seen in Fig. 4b for the case of multiple jumps in dot magnetization the array with weak coupling experience well isolated spinodal regions. The respective magnetodynamics is, therefore, quite similar to the case of a single jump. However, for strong coupling the region of instability is very spread so that we observe a confined isolated stability domain at small disorders.

3. Summary and outlook

The inter- and intra-dot structure effects in magnetodynamics of QD arrays were analysed. As is shown the arrays display jerky magnetodynamics with sharp discontinuities in the magnetization process caused by sharp step-vice anomalies for dot magnetic moments in external field in conjunction with ferromagnetic inter-dot coupling. For the description of such noisy magnetodynamics we employ the RJIM model [2,3] accounting for quantum fluctuations due to the discrete level structure, inter-dot coupling and disorder. The magnetization discontinuities can be attributed to avalanche propagations of moment jumps. Magnetic state equation and phase diagram of dot arrays is demonstrated to exhibit spinodal regions on $\{\text{disorder, magnetic field}\}$-plane and the critical points. Exploring correlations of noise amplitudes represents then convenient analytical tool for quantitative definition, description and study of self-organized criticality in magnetic QD assemblies. On an example of 3-dimensional supercrystals we show strong correlations in avalanche size distributions. The linear size of the biggest avalanche makes about half of system length, while mean size attains maximum and diverges in thermodynamic limit. The tools might be employed to quantify a roughness and a disorder in magnetic dot arrays, which are of great importance for advanced electronic devices, nanoscale storage media and magnetic recording technology. In addition, ligand (e.g. oleic acid) stabilized nanocrystals of iron series transition metals with enhanced magnetic moments represent promising candidates for the magnetically responsive component of macro-molecule beads, significant for advanced therapy.

Arrays of dots with singlet-triplet transitions are predicted to display some specific features. In a case of weak coupling such arrays exhibit the two well-separated instability regions surrounding the positions of the dot singlet-triplet magnetic anomalies. With increasing coupling we observe further structure modification, plausibly, of bifurcation type. At strong coupling the instability region become wide while the stable regime arises as a narrow island at small disorders. Further analysis based on numerical simulations within RJIM model and implications of analytical tools considered in this paper will allow to specify and study quantitatively this phenomenon and shed a light on the origin and properties of the transition.

Acknowledgments. The author thanks JINR (Dubna, Russia) for the warm hospitality and the financial support.

References
[1] Kondratyev V N and Lutz H O 1998 Phys. Rev. Lett. 81 4508; 1999 Eur. Phys. J. D 9 483
[2] Kondratyev V N 2002 Phys. Rev. Lett. 88 221101; 2002 JAERI-Research 2001–057
[3] Kondratyev V N 2006 Phys. Lett. A 354 217
[4] Hutten A et al. 2004 J. Biotechnology 112 47
[5] Kondratyev V N 1993 Phys. Lett. A 179 209; 1994 ibid. 190 465; 1996 Z. Phys. B 99 473
[6] Kondratyev V N 1997 AIP Conf. Proc. 416 447
[7] Kondratyev V N, Lutz H O and Ayik S 1997 J. Chem. Phys. 106 7766
[8] Kondratyev V N 2002 J. Nucl. Sci. Technol. 1 Suppl. 2. 550; 2002 J. Nucl. Radiochem. Sci. 3 205
[9] Giuliani D, Jouault D and Tagliacozzo A 2001 Phys. Rev. B 63 125318
[10] Moniyama S, Fuse T, Suzuki M, Aoyagi Y and Ishibashi K 2005 Phys. Rev. Lett. 94 186806