CP, T and fundamental interactions

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Abstract

We discuss the importance of the CP (simultaneous particle-antiparticle and left-right permutation) and T (time reversal) symmetries in the context of fundamental interactions. We show that they may provide clues to go beyond the 4-D gauge interactions. We insist on the fact that T violation is not associated to a degradation (like in entropy), but simply characterized by different trajectories.

1. Introduction

Why are CP (and T) symmetries important? The main purpose of this short contribution is to try and answer this question.

There are on one side somewhat psychological reasons: on one hand, most of Nature’s laws are symmetrical under left and right (P) permutation (on the “macroscopic level, handedness seems to be linked to living organisms chemistry or behaviour). The same could be said of charge conjugation symmetry (the exchange of particles and antiparticles, which will be noted C ) were it not for the fact that antiparticles are virtually absent from our everyday observations (ignoring β+ decays and reactor antineutrinos !). In a similar way, we are used to idealize mechanics and electromagnetism as time-reversal invariant, and to blame any lack of such invariance on entropy considerations (keeping in mind that such considerations have to be added ”by hand” in the formalism, and cannot be derived directly from the local interactions).

More important are somewhat more theoretical reasons, linked to gauge theories. In this context, it can be affirmed that CP (or T) are THE natural symmetry of gauge interactions in a minimal Lagrangian (that is, in absence of scalars interactions - including fermion masses ). This is precisely the importance of studying CP and T symmetries: their breaking implies the presence something more than the part of the Standard Model of Fundamental Interactions that we know best, namely gauge theories. At the very least, CP violation probes directly the still unobserved scalar sector of the Standard Model.
One more reason for the central rôle of CP violation is linked to the matter-antimatter asymmetry of the Universe. Here we are dealing with factual evidence, and the choice to accept an elaborately tuned (specially in an inflation context) cosmological asymmetry, or new sources of CP violation.

Before closing this introduction, it is also worth to mention the various T violating processes, and to wonder if any relation exists between those different ”arrows of time”. In this context, we must mention the increase of entropy (and its associated impact on chemistry, life and death processes, but also in the context of the construction of the matter-antimatter asymmetry), the cosmological time (and the corresponding expansion of the Universe), and of course our present subject, microscopic time.

2. The evidence for CP (or T) violation”

From a pedagogical point of view, it is often difficult to single out the evidence for CP (and a fortiori for T) violation. We will try to produce below 2 clear-cut examples.

2.1. CP violation

The simplest presentation to my knowledge involves the production of $e^+$ vs $e^-$ some distance away from a hadronic collision. To speak roughly, it can be formulated as follows: ”arrange for a collision between 2 energetic particles or antiparticles, put shielding around it, and move back a sufficient distance, count the emerging $e^+\pi^-$ vs $e^-\pi^+$: there will always be a small excess of the $e^+$. The important point is that the observed excess is always in the same direction (more $e^+$), whether we start from colliding particles, or antiparticles. This process is a clear violation of C, while the integration over all directions ensures that the symmetry is not restored by P, hence establishing the CP violation.

Of course the most evident set-up for the experiment (dumping a beam of particles on a target made of ordinary matter) in itself violates CP from the start (and thus a more detailed examination of the process is then needed to establish the effect), however a perfectly equivalent experiment can be made through proton-antiproton collisions. This set-up (similar to the CP-Lear experiment, which we evoke later) is symmetrical under CP and gives the same clear conclusion: particles can be distinguished from antiparticles in absolute terms, independently of the notion of left or right.

Although this subject is of course an illustration of the very classical $K^0 - \bar{K}^0$ system, delving in some more detail will help clarify some important issues, and explain why the outcome is the same wether the initial state is a between particle-particle, particle-antiparticle, or (still a gedanken experiment) antiparticle-antiparticle.

The mechanism is indeed familiar: the collision produces amongst other things neutral $K^0$ and $\bar{K}^0$ (in equal quantities) which survive long enough to emerge from the shielding. While the $K^0 - \bar{K}^0$ are eigenstates of gauge interactions, they do mix through weak interactions. The ”mass eigenstates” for free propagation are then the ”long-lived” and ”short-lived” $K^0_L, K^0_S$. Because CP violation is only a small effect (parametrized here by $\epsilon$), it is convenient to re-write these states in terms of the ”CP” even and odd eigenstates $K^0_1, K^0_2$, which, modulo a suitable choice of phase conventions, read:

$$|K^0_1> = \frac{|K^0> + |\bar{K}^0>}{\sqrt{2}}$$

$$|K^0_2> = \frac{|K^0> - |\bar{K}^0>}{\sqrt{2}}$$
\[ |K_L| = \frac{1}{\sqrt{1 + |\epsilon|^2}} (|K_0^0| + \epsilon|K_1^0|) \quad (3) \]
\[ |K_S| = \frac{1}{\sqrt{1 + |\epsilon|^2}} (|K_0^0| + \epsilon|K_1^0|) \quad (4) \]
\[ |K_L^0| = \frac{1}{\sqrt{1 + |\epsilon|^2}} (|K_0^0| + \epsilon|K_1^0|) \quad (5) \]

These equations were only introduced here to show that the \( K_0^0 \) state, for instance, proportions of \( K_0^0 \) and \( \bar{K}^0 \) differ by small amounts controlled by \( \epsilon \), the usual CP parameter more familiarly related to the ratio of 2 and 3-pion decays of the \( K_L \). Since the semi-leptonic decays are respectively (\( K^0 \) is for historical reasons defined as the \( \pi^-d \) state) \( K^0 \rightarrow e^+\nu\pi^- \) and \( \bar{K}^0 \rightarrow e^-\bar{\nu}\pi^+ \), this results in the mentioned excess of the \( e^+ \) channel, when we look only at the \( K_L \) decays (i.e. "at a sufficient distance from the production" in the above recipe). The resulting asymmetry \( \frac{\epsilon}{1 + |\epsilon|^2} \) (in those conditions is \( 2Re(\epsilon) \), somewhat less than \( 0.5 \% \)).

This is a good opportunity to discuss the relation between the manifestation of CP violation and the observation of time dependence. Indeed, although no explicit time dependence was mentioned in the discussion above, an implicit one arises from the distance between the production and the observation point. At first sight indeed the claimed excess of \( e^+ \) from \( K_0^0 \) decay at long distance could be compensated by the opposite effect for the shorter-lived \( K_S \) close to the production. There is some truth in this indeed: at the very production site, we can just consider the \( K^0 \) and \( \bar{K}^0 \), which produce equal quantities of \( e^+ \) and \( e^- \) -channel decays. However the total effect subsists even when one integrates over time (or distance) from the interaction point. This is simply due to the fact that the \( K_S \) decays much more rapidly in the \( 2\pi \) channel, hence giving the semi-leptonic mode a smaller branching ratio than in \( K_L \) (namely the branching ratio of \( K_S \rightarrow \pi^+e^-\bar{\nu} \) \( \nu \) is only \( (6.9 \pm 0.4) \times 10^{-4} \), while the corresponding decay probability for the \( K_L \) is \( (39.81 \pm 0.27)\% \)).

For this reason, the cancellation does not hold, and the very explicit CP asymmetry advocated here is observed without having to take into account the time dependence. We should also stress that this happy circumstance is due to an exceptional kinematical accident, namely the strong lifetime difference between the \( K_L \) and \( K_S \): in most other cases, timing information will be needed to exhibit and measure CP violation; this is particularly true in the \( B \) system.

We anticipate on the next section, which will show the rôle of phases to stress already here that the CP effects are linked to unremovable phases involving more than 2 states (here, the \( K, \bar{K} \) and the final states, all in communication).

2.2. \( T \) violation

The issue of \( T \) violation is indeed more complex to show. For a long time, \( T \) violation was only surmised from the fact that local Lagrangian theories obey the TCP theorem, and hence a violation of CP implies a violation of \( T \) in that context.

The obvious difficulty is that the \( T \) symmetry involves not only the reversal of \( t \) as a kinematical variable (reversing all speeds and angular momenta, for instance), but also and foremost a permutation of the initial and final states. This makes the test very difficult, and in fact impractical in the case of decays (or collisions). It is impossible indeed in practice to reconstitute the kinematical (and phase) conditions of the final particles to make them re-assemble in the original unstable one.

The situation is not completely desperate however, if we can turn to 1-particle evolution. We present 2 cases: one is currently observable, through tagging, while the other, waiting for the hypothetical observation of an electric dipole moment is for the time being a "gedanken" (but more spectacular) experiment.
We will once again take the example of the CP-Lear experiment [1] (but many B physics situations are similar). In the strong interaction production of neutral kaons, it is possible to "tag" separately \( K^0 \) and \( \bar{K}^0 \) through the accompanying particles: simultaneous production of \( K^- K^0 \) or \( K^+ \bar{K}^0 \), making use of the flavour-diagonal character of strong interactions. At the moment of decay, we have already indicated that the semi-leptonic final state \( \pi l \bar{\nu} \) allows for a similar tagging.

From this set-up, it is possible to compare the probabilities \( | \langle K^0 | S | K^0 \rangle |^2 \) and \( | \langle K^0 | S | \bar{K}^0 \rangle |^2 \), and check that they are indeed different (remember that other states are accessible, and will compensate those differences if needed – see below CP vs CPT). In passing, let us remark that the above transitions are NOT related by CPT, which for instance relates \( \langle K^0 | S | K^0 \rangle \) just to ...itself.

The above example relies of course on the reliability of the tagging (for instance, that no explicit CP violation takes place in the decay vertex, at least to the accuracy requested to establish the effect), and is still in some way indirect.

![Diagram](image)

Figure 1. Gedanken experiment: microscopic irreversibility in presence of an hypothetical electric dipole moment. The time-reversed trajectory is different, allowing to determine which is the "normal" and which is the "time-reversed" situation

We propose below a more spectacular, but still speculative situation. Once again it relies on the evolution of one single particle (here, the neutron). Let us assume that the electric dipole moment of the neutron has been measured. Such a dipole moment is expected at a very small level in the Standard Model (of the order of \( 10^{-32} \text{e.cm} \)), much below the current limit \( < 0.2910^{-29} \text{e.cm} \). This bound in fact already puts severe constraints on most extensions "beyond" the Standard Model (Left-Right symmetrical model, supersymmetry, ...). The fact that an electric dipole moment for an "elementary" particle (or for the ground state of an assembly of quarks, like the neutron) violates CP or T is a bit tricky. The main point is that the only direction in which the dipole moment can point (for such a particle at rest) is along the spin.

\[
\bar{d}_n = \kappa S^2_n
\] (6)
with \( \kappa \) a fixed, calculable (measurable) coefficient; (the situation is different, say for a water molecule, where other directions exist and the large dipole moment of course does not violate CP; it is also different in non-commutative theories, where extra tensors are available). The relation (6) indeed violates \( P \) and \( T \) because the first quantity \( d_m \) is an "ordinary vector" (think of the distance between the opposite charges of a dipole), it flips under \( P \) and is time invariant, while the second (as an angular momentum), behaves in the opposite way.

We have reproduced a thought experiment (see Fig 1 where such a neutron (with observable electric dipole moment) is sent through a 2-phase "Stern-Gerlach" set-up, where one stage uses an inhomogeneous magnetic field, and the other an electric one. As seen from this (gedanken) drawing, the "time-reversed" trajectory is just different from the initial one. (Note that the magnetic fields, linked to currents have been reversed, but not the electric ones).

We produced this example to illustrate one point: in this "microscopic breaking of Time invariance", no entropy comes into play: the return trajectory is in no way "degraded" with respect to the first one - it is simply different. Just as someone taking one path to go to work in the morning, and systematically a different one to return home!

3. CP or T, the natural symmetries of gauge interactions ... in 3+1 dimensions

In a way this is stating the obvious, but is probably not useless. We are indeed used to see the CP violation associated to a phase in the Cabibbo-Kobayashi-Maskawa matrix, which is closely associated to the exchange of the charged gauge bosons \( W \). This may lead to wrongly associating CP to such exchanges. Of course, there is no real paradox, and the impression arises from the fact that, producing particles with accelerators, we work spontaneously in the fermion mass-eigenstate basis.

Let us thus go back to basics. In 3+1 dimensions, the simplest fermion transforming consistently under the "proper" Lorentz group (rotations, boosts, without inclusion the reflections of the spatial or temporal coordinates) is represented by a spinor with 2 degrees of freedom. This can consist in a 2-component spinor field, i.e., 2 complex numbers (fields) (technically it is called a semi-spinor of first or second species), or in a reduction of the more familiar 4-component "Dirac spinor", either by Weyl projection (on so-called Left or Right-handed modes), or by imposing a Majorana-like condition (the equivalence of these reductions is not a general feature when working in a different number of dimensions).

We use below the Weyl representation, and take as example a "Left" projection. Assuming a massless fermion, the solutions of the Dirac equation can be developed as usual in plane waves, with the operators \( a \) describing the destruction of a particle. For simplicity, we have used a sum over the various modes (an integral over the continuous variables is understood). As seen below, the sum includes both positive and negative-energy states, and the helicity (\( \lambda \) is the projection of the spin on the direction of motion) is opposite to the sign of the energy, namely the \( \Psi_L \) field describes a positive energy particle of negative helicity (L) and a negative energy particle of positive helicity (R).

\[
\Psi_L(x, t) = \sum_{\vec{p}', \omega > 0} \frac{e^{-ipx}}{\sqrt{2\omega}} a_{\vec{p}', \lambda} \psi_{\lambda -} u(\omega, \vec{p}', \lambda -) + \sum_{\vec{p}', \omega < 0} \frac{e^{-ipx}}{\sqrt{2\omega}} a_{\vec{p}', \lambda_+} \psi_{\lambda +} u(-\omega, \vec{p}', \lambda +) \tag{7}
\]

We make the next step explicitly to avoid confusion between C and CP operations. To avoid negative-energy particles, we proceed along the usual "trick" of replacing the destruction operator of the negative-energy mode (energy \( -\omega \)), \( a_{-\omega, \vec{p}', \lambda} \) by the creation operator \( b^*_{-\omega, -\vec{p}', \lambda} \). In this operation, we are forced (to conserve energy-momentum and angular momentum) to flip the 4 components of the 4-momentum, and the spin; as a result, the helicity \( \lambda_+ \) (product of spin by 3-momentum) is not affected in the process.
For the same reason, other conserved quantities, like the fermion number or electric charge are also flipped. The new state introduced is thus an antiparticle with Right helicity, namely the CP (and not the yet inexistant C conjugate) of the initial fermion. Summation of dummy variables hides the process somewhat, but we finally reach:

\[
\Psi_L(x,t) = \sum_{p^0=\omega, \vec{p}} \left\{ a_{\omega, \vec{p}} \lambda_- e^{-ipx} u(\omega, \vec{p}, \lambda_-) + b_{\omega, \vec{p}}^+ \lambda_+ e^{ipx} v(\omega, \vec{p}, \lambda_+) \right\}
\]  

(8)

With this result in hand, it is straightforward to check that gauge interactions between fermions automatically respect CP (remember, we have not introduced masses yet). We will take the significant example of a charged gauge boson interacting with two fermions:

\[
g W^\mu \overline{\psi} L \gamma_\mu \psi L + h.c.
\]  

(9)

Once this equation is expanded in terms of the creation and destruction operators (as in eq.(8)), it is obvious that the second term is just the CP conjugate of the first. CP invariance results since the coefficients of both terms are identical (note that we did not need to introduce the charge or CP conjugation matrix C: it is only needed if we want to face the case of Majorana masses - a possibility in the leptonic sector, which will not be considered further here).

This is sufficient to claim a very important result, which is actually independent of the number of spatial dimensions (this will be used later): gauge interactions alone (i.e. in absence of scalar interactions, which include the mass terms) respect CP as a symmetry. In 3+1 dimensions, neither C nor P are granted for gauge interactions, but CP of course is. Note that even the triangular anomalies don’t take exception to this: in the massless case, the effective T and CP violating term \( \theta \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \) can always be rotated away.

We now remind very briefly how CP violation is introduced in the phenomenological Cabibbo-Kobayashi-Maskawa way before evoking in the next section possible more fundamental sources.

The above argument about CP invariance does not apply to the scalar couplings (in practice, Yukawa couplings since raw mass terms are not allowed in the Standard Model). Consider indeed the Yukawa couplings: (\( \Phi_a \) are Brout-Englert-Higgs scalar doublets – we allow for several of them, labeled by the index \( a \), while \( \Psi_{Lj} \) is a left-handed quark doublet field associated to the fermion family \( j \), and \( u_{Ri} \) similarly describes the right-handed ”up” quarks, labeled by \( i \) for (u,c,t) )

\[
\lambda_{aij} u_{Ri} \Phi_a + \Psi_{Lj}\psi_{Ld} + h.c.
\]  

(10)

Here indeed, the matrix \( \lambda \) is in general complex, and distinct from its complex conjugate. Furthermore, relative phases between the vacuum expectation values in case of more than one scalar fields can add another source of CP violation. (the latter case will be referred to as ”spontaneous CP violation” if the Lagrangian is otherwise CP invariant, and the only breaking originates in the relative phases of vacuum expectation values).

After diagonalization of the mass matrix, the Left- and Right-handed charged quarks currents in the new basis read:(\( K^{L(R)} \) being the Kobayashi-Maskawa mixing matrix)

\[
J_{\pm,L(R)}^\mu = \overline{u}_L(R) \gamma_\mu K_{Lj}^{\mu} d_j L(R)
\]  

(11)

In principle, each of these mixing matrices contains \( n(n+1)/2 \) phases, where \( n \) is the number of fermion families; however as is well-known, not all the phases are observable. It is thus possible to redefine the left-handed fermion fields to reduce the number of \( K^L \) phases to \( (n-1)(n-2)/2 \), at the cost of a rotation of the right-handed fields to keep the mass matrix real; therefore the count of phases in \( K^R \) in general
stays maximal. It is only because the Standard Model only gauges the left-handed currents (hence $K^R$ can be ignored) that the famous result that 3 families are needed for CP violation holds. In the leptonic sector, the same count is valid, except in the case where neutrinos are Majorana particles (the PMNS matrix): in that case, the right-handed fields cannot be used to make the neutrino mass matrix real, and (n-1) additional phases must be included.

4. Which object should we call ”Antiparticle”

Before moving to more speculative ground, we may pause for a question of nomenclature. What object should we call "antiparticle"? As long as only quantum electrodynamics was involved, the definition did not matter much, since, as far as particles interact only with the photons, both the $C$ and $CP$ conjugates are present (for the $e_L^-$, respectively the $e_R^+$ and the $e_R^R$). In the framework of the Standard Model however, the fields appearing don’t respect the $C$ symmetry, as we have seen, and the $e_L^-$ and $e_R^+$ belong to completely different fields, with different $SU(2)$ properties. For the neutrino, we don’t even know if the $\nu_R^R$ state exists at all. It may thus be expedient (and some other articles in this issue have quite independently taken the same view) to call "antiparticle" the $CP$ conjugate state to the particle. This does not imply any confusion, as one can keep the appellations "C-conjugate", "P-conjugate" whenever one needs to refer specifically to these objects (nowadays much less frequently).

5. Towards a fundamental origin of CP violation

The Kobayashi-Maskawa approach describes correctly the known CP violation, and the result in this latter respect is quite impressive; still, this appears more as a successful parametrization than a fundamental understanding.

The main difficulty with CP violation is that, if we look for a more fundamental theory to avoid the Yukawa couplings (and ideally the arbitrary scalars), the most logical choice is to turn to gauge interactions, which, as we have seen find in CP their natural symmetry. This is precisely one of the reasons why we consider that CP violation is a key to understanding physics beyond the standard model, and a challenge which must be faced in any fundamental approach.

We would like to hint here at 2 possibilities. The first one takes place in the usual 3+1-dimensional context, in so-called ”dynamical symmetry breaking”. In such theories, the usual scalars are replaced by bound states of fermions, in a ”pure gauge” context (bound for instance through a new extra-strong interaction, often referred to as ”technicolor”). In such a case, the only solution is to have these effective scalars develop several vacuum expectation values (condensates), whose relative phases then lead to ”spontaneous CP violation”, as mentioned about eq. (10).

One other interesting possibility is to have the CP violation originate from a pure gauge theory in more than 3+1 dimensions. The principle can be explained quite simply, by turning back to the gauge interaction, but this time in 4 or 5 spatial dimensions. The spinors used are now (at least) 4-component ones, and contain (as usual Dirac spinors) both the Left- and Right-handed fermions (in terms of a 3+1 reduction):

$$\bar{\psi}_M \Gamma^M \psi$$

(12)

We distinguish now between the usual 3+1 dimensions (called $\mu$ and the remaining ones: $M = \mu, 4, 5$, and assume the remaining ones are compactified (either on a torus with a radius $R$, such that $1/R >>$
the accessible energy – say several TeV, or on a compact orbifold). It is then useful to single out the inaccessible components, and one observes terms like (again in 3+1-dimensions language)

$$\overline{\psi}_L (i\gamma_5 W^4 + W^5) \psi_R$$

(13)

Such terms correspond to pseudoscalar and scalar interactions respectively, and can generate complex mass terms. To be more accurate, we must keep in mind that the quantities above are in fact gauge-dependent; however the integration over the extra dimensions takes care of this, and brings in a new quantity in the equation, namely the integral (or flux) of the gauge field along the extra dimensions.

For instance in 4+1 dimensions, the "Hosotani loop" is definitely a way to induce CP violation in an otherwise CP conserving gauge theory, through the term [2]:

$$\overline{\psi}_L \int (i\gamma_5 W^4) dx^4 \psi_R$$

(14)

In this context, the Hosotani terms appear as some extra elements (similar to boundary conditions, bringing extra parameters) in the theory, and can actually break both CP and the initial gauge group.

Introducing such an approach however requires to consider larger gauge groups. [2]

6. CP versus TCP, and the Matter-Antimatter asymmetry

If we apply the TCP invariance to the special case of the survival of a single particle $X$, we see the matrix element is identical to that of the antiparticle $\bar{X}$

$$< X | S | X > = < \bar{X} | S | \bar{X} >$$

A comparison to the familiar expression:

$$< X | S | X > = e^{-i(m-d\Gamma/2)(t-t_0)}$$

establishes that particle and antiparticle have both equal masses and equal "total decay width" (the inverse of the lifetime) $\Gamma = 1/\tau$. This equality of the lifetimes makes it impossible to imagine a cosmological scheme where the initially equal numbers of particles and antiparticles produced in connection with gravity decay at different speeds.

It still allows however for the creation of a matter-antimatter imbalance. The reason is that TCP only constrains the total decay probabilities to be equal, but not the partial ones. In other terms, $X$ and $\bar{X}$ live the same time, but can suffer different deaths...

More explicitly, let consider a particle $X$ with only the 2 decay processes $X \rightarrow a, X \rightarrow b$, and the charge conjugate processes, $\bar{X} \rightarrow \bar{a}, \bar{X} \rightarrow \bar{b}$. Let us adopt the notation: ($f$ is any of the final states)

$$A_{X \rightarrow f} = < f | S | X >$$

for the amplitude, while we use $P$ for the transition probability: $P_{X \rightarrow f}$.

Summing over all possible decay channels $f$, TCP implies

$$\sum_f P_{X \rightarrow f} = \sum_f P_{\bar{X} \rightarrow \bar{f}}$$

but does not imply

$$P_{X \rightarrow a} = P_{\bar{X} \rightarrow \bar{a}}$$

as long as the difference is compensated by other decay channels!
In particular, if the channels $a$ and $b$ have different baryon (or lepton) number, a net baryon (lepton number) is created.

It should be clear however that whatever mechanism allowing for a difference between $P_{X \rightarrow a}$ and $P_{\bar{X} \rightarrow \bar{a}}$ must do so in such a way that the total lifetimes are kept equal, namely, the calculation leading to this difference must in some way know of the existence (and physical availability) of the other channels. In the calculation, this indeed typically occurs through higher-order contributions, and in the present case, it requires that the $a$ final state can be reached either directly, or by re-scattering $X \rightarrow b; b \rightarrow a$. The fact that the $b$ channel must be kinematically accessible is traduced in the following graphical illustration by the presence of a non-vanishing "unitarity cut".

It is not our purpose to go here into the details of baryo- or lepto-genesis, but it may be useful to notice that here again, a 3-state scheme comes into play, allowing for physical (unremovable) phases between the direct $X \rightarrow a$ and the indirect $X \rightarrow b; b \rightarrow a$, a situation we already found in the Kaon system (direct $K^0$ decay, or via $K^0 - \bar{K}^0$ transition).

We just close by reminding that for a baryo-or leptogenesis scheme to be successful, new mechanisms for CP violation must be found, as the usual Cabibbo-Kobayashi-Maskawa is notoriously insufficient for this purpose.

Acknowledgements

This work was supported in part by the French Community of Belgium (IISN), and by the Belgian Federal Policy Office (IAP VI/11).

Remark

Much of the material presented above is covered (usually from a different perspective) in textbooks; we just mention a few extra references dealing more precisely with the above approach.

References

[1] A. Angelopoulos et al. [CLEO Collaboration], Phys. Lett. B 444 (1998) 43.

[2] N. Cosme, J. M. Frère and L. Lopez Honorez, Phys. Rev. D 68 (2003) 096001 [arXiv:hep-ph/0207024]; N. Cosme and J. M. Frère, Phys. Rev. D 69, 036003 (2004) [arXiv:hep-ph/0303037].