Efficient in-depth analysis and optimum design parameter estimation of MEMS capacitive pressure sensor utilizing analytical approach for square diaphragm

Sumit Kumar Jindal1 · Ishan Patel1 · Krish Sethi1 · Simrit Kaul1 · P. K. Sreekanth1 · Ajay Kumar2

Received: 21 May 2021 / Accepted: 20 April 2022 / Published online: 27 May 2022
© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2022

Abstract
Capacitive pressure sensors have become more popular as compared to piezoresistive pressure sensors as they yield superior sensitivity and lesser nonlinearity. Efficient analysis for modeling capacitive pressure sensors is thus increasingly becoming more important due to their innumerable use cases. The higher sensitivity of square diaphragm for the same side length in comparison to circular diaphragm makes it ideal for sensor design. In this work, a complete formulation for analysis of capacitive pressure sensor with the square diaphragm in normal and touch mode operation has been presented as these two modes are established operating modes for these sensors. A comprehensive study of sensor parameters like capacitance, diaphragm deflection, capacitive and mechanical sensitivity has been formulated to aid the choice of sensor characteristics. This work also focuses on the method to determine core design parameters for optimal operation. Computationally complex methods have been used in the past for analysis of square diaphragms. The analytical approach presented in this research is less complex and computationally efficient, in comparison to the finite element method. MATLAB has been used to compute and simulate results.

Keywords Capacitive pressure sensor · Computationally efficient · Optimal operation · Sensitivity · Square diaphragm · Touch mode

1 Introduction
Micro-ElectroMechanical Systems (MEMS)-based capacitive pressure sensors have a pivotal role in MEMS devices for real-world applications [1]. These sensors have been extensively used in the field of medical science, automobile industry, avionics, industrial and commercial applications [2, 3]. Thus, due to their mission critical use case these pressure sensors must be optimally designed and should be reliable if used in extreme conditions. These devices should also offer high sensitivity, low temperature drift and low noise. Initially piezoresistive sensors were used, but they were replaced by capacitive pressure sensors (CPS). The reason for this advance from piezoresistive to capacitive pressure sensors was driven by the higher sensitivity and lesser nonlinearity delivered by the CPS. Capacitive pressure sensors also have a much higher linear performance range. Reliable performance is witnessed even in high temperature conditions and harsh environments [4]. These sensors use ceramic or silicon diaphragms and can operate in two modes, the normal mode, and the touch mode.

In comparison to normal mode operation, touch mode delivers near-linear output characteristics and large over range pressures and this has led to the design of Single Touch Mode Capacitive Pressure Sensor (STMCPS) with circular diaphragm [1]. To further enhance performance, Double Touch Mode Capacitive Pressure Sensor (DTMCPS) were proposed. In this design, an additional notch was etched at the bottom of the substrate providing higher linearity and enhanced sensitivity compared to STMCPS [2, 3]. A novel upgrade to both STMCPS and DTMCPS was suggested by etching two back-to-back Touch Mode Capacitive Pressure Sensor (TMCPS)s in one substrate. This structure was...
called the Double-sided Touch Mode Capacitive Pressure Sensor (DSTMCPS) [4, 5]. This enriched design provided exceptional results for differential pressure applications but entailed increased manufacturing and design costs [6–8]. Silicon Carbide (SiC) has emerged as an appealing material for high-temperature applications owing to its excellent electrical, mechanical and chemical properties of high-thermal stability, conductivity, inertness, high critical electric field, hardness, resistance to wear and tear and mechanical robustness [4, 9]. Thus further innovations for circular diaphragms based CPS were brought about by using a sandwich structure composed of SiC and aluminum nitride (AlN) for sensor construction [10, 11].

For a given side-length, square diaphragm based CPS are more sensitive to changes in applied pressure as compared to their circular counterparts [6, 9]. In addition, square diaphragms are preferred due to ease in fabrication and better yield [12]. Previous work shows the mathematical modeling for square diaphragm CPS employs computationally expensive and complex methods [8, 13, 14]. An analytical solution for normal mode square diaphragms CPS has been illustrated in [6]. As touch mode capacitive pressure sensors offer better performance in many applications, there is a need for efficient analysis of touch mode operation. This work therefore elucidates an in-depth, step-by-step derivation of the capacitance, capacitive and mechanical sensitivity of a silicon-based CPS with square diaphragm in both normal and touch mode operation. An approach for choice of sensor dimensions for optimum design has been provided using MATLAB simulations. The analytical solution provided is based on small deflection theory and assumes linear elastic deformation of the square diaphragm. The model developed is efficient and eliminates the need for computationally exhaustive finite element method (FEM) modeling [15, 16]. FEM necessitates the use of complex software such as COMSOL, which requires a machine with significant compute and memory resources. In contrast, the proposed mathematical model can generate quick MATLAB simulations by tweaking the necessary design variables, enabling a faster analysis without using any complex simulation software. This model can be used as a solid foundation to decide on the operating range, experimental design parameters and application. It will not only save manufacturing resources but also reduce time in tuning the parameters to check results.

The paper is organized as follows: The second section deals with the general theory for analysis of square diaphragm CPS and demonstrates the different operating modes of the sensor. The third section describes the complete step-by-step approach for evaluating the capacitance, capacitive sensitivity, and mechanical sensitivity for normal and touch mode operation. The results, analysis, and comparison of various parameters, for the exact design of a MEMS pressure sensor have been discussed in Sects. 4 and 5.

## 2 Theory

A capacitive pressure curve (Fig. 1) is used to examine the behavior of square diaphragm-based CPS. Figure 1 illustrates four operation zones normal, transition, touch, and saturation mode.

During normal mode, the diaphragm does not touch the bottom electrode or the substrate which can be observed in Fig. 2. The pressure range for normal mode spans from zero to the touch point pressure. The touch point pressure is the minimum pressure that is applied so that the diaphragm just touches the electrode. When the plate touches the bottom of the cavity, a highly nonlinear relation between capacitance and pressure exists and the sensor is said to be in transition mode. The transition mode creates noise and hence is not utilized for sensor operation.

In touch mode operation of the device, the diaphragm is in contact with the bottom electrode. This is depicted in Fig. 3. A layer of dielectric material is placed over the bottom electrode to prevent short circuit. The touch mode was introduced to enhance sensor performance. In saturation mode, the diaphragm completely touches the bottom electrode as a result, very less variation in capacitance takes place and ultimately capacitance saturates to a peak value.

## 3 Analytical solution

The mathematical modeling presented in this work is based on the theory of small deflection for plate bending. The theory assumes that diaphragm deflections are small in comparison with the plate thickness.

Figure 2 illustrates a square diaphragm of thickness (h), Poisson’s ratio (ν), Young’s Modulus (E) and side length (2a). The diaphragm deflection, \( w(x, y) \) is a function of \( x \) and \( y \) co-ordinates given by [6, 16],

\[
\begin{align*}
  w(x, y) &= \frac{P}{47D} \left( \frac{(x^2 - a^2)^2 (y^2 - a^2)^2}{a^4} \right) \\
  D &= \frac{Eh^3}{12(1-\nu^2)}
\end{align*}
\]  

where \( D \) is the flexural rigidity and can be defined as resistance offered by a non-rigid structure when it is undergoing bending, or when a force couple is applied to the structure [9, 15, 16] and \( P \) is the applied pressure. Flexural rigidity can be expressed as

\[
\begin{align*}
  D &= \frac{Eh^3}{12(1-\nu^2)}
\end{align*}
\]
The central deflection for square diaphragm can be evaluated by substituting \( x = 0 \) and \( y = 0 \) in (1):

\[
w_0 = w(0, 0) = w_{\text{center}} = \frac{P_0 a^4}{47D}
\]

(3)

The touch-point pressure is defined as the minimum pressure at which the square diaphragm just touches the surface of the bottom electrode. At touch-point pressure \( (P = P_t \text{ and } w_0 = d_0) \), where \( d_0 \) is the separation between the plates at zero deflection. Touch-point pressure is evaluated as

\[
P_t = \frac{47Dd_0}{a^4}
\]

(4)

### 3.1 Normal mode capacitance analysis

For a parallel plate capacitor, capacitance \( C \) can be expressed as

\[
C = \frac{\varepsilon_0 A}{d}
\]

(5)

where \( \varepsilon_0, A \) and \( d \) are the gap permittivity, plate area and plate separation, respectively. This equation can be used
to calculate normal mode capacitance by considering an elemental area and integrating the expression over the dia-
phragm dimensions. Hence, the expression for $C$ can be
mathematically written as

$$C = \int_{y=-a}^{a} \int_{x=-a}^{a} \frac{\varepsilon_0}{(d_0 - w(x,y))} \cdot dx \cdot dy$$

(6)

Substituting the expression for $w(x, y)$ obtained in (1) in (6)

$$C = \frac{\varepsilon_0}{d_0} \int_{y=-a}^{a} \int_{x=-a}^{a} \frac{1}{(1 - \frac{P}{47D}((x^2 - a^2)(y^2 - a^2)^2))} \cdot dx \cdot dy$$

(7)

Using the binomial approximation of

$$\frac{1}{1-x} = (1 - x)^{-1} = (1 + x)$$

for $x \ll 1$ in (7)

$$C = \frac{\varepsilon_0}{d_0} \int_{y=-a}^{a} \int_{x=-a}^{a} \left(1 + \left(\frac{P}{47D}((x^2 - a^2)(y^2 - a^2)^2))\right)\right) \cdot dx \cdot dy$$

(8)

$$C = \frac{4a^2\varepsilon_0}{d_0} + \frac{\varepsilon_0}{d_0} \int_{y=-a}^{a} \int_{x=-a}^{a} \frac{P a^4}{47D} \left(\frac{1 - \frac{x^2}{a^2}}{d_0}\right)^2 \cdot dx \cdot dy$$

(9)

Assuming $C_0 = \frac{4a^2\varepsilon_0}{d_0}$, (9) is simplified as

$$C = C_0 + \frac{\varepsilon_0}{d_0} \frac{P a^4}{47D d_0} \int_{y=-a}^{a} \left(1 - \frac{y^2}{a^2}\right)^2 \cdot dy$$

$$\int_{x=-a}^{a} \left(1 - \frac{x^2}{a^2}\right)^2 \cdot dx$$

(10)

On evaluating the integral in (10), $C$ can be expressed as

$$C = C_0 + \frac{P a^4}{47D d_0} \left(\frac{64}{225}\right)$$

(11)

Substituting $p_t$ from (4) in (11), capacitance for normal mode operation is evaluated as

$$C_{\text{normal}} = C_0 + C_0 \left(\frac{64}{225}\right) \left(\frac{P}{p_t}\right)$$

(12)

### 3.2 Touch mode capacitance analysis

Touch mode contact for a square diaphragm CPS is illus-
trated in Fig. 3. Touch mode capacitance constitutes (1)
touchdown capacitance of diaphragm $C_t$ and (2) cavity
capacitance $C_{ut}$ [1]. $C_t$ can be evaluated using the concept of
parallel plate capacitance. To solve for $C_{ut}$ axial symmetry
of boundary condition approximation is used [1]. Electrical
field flux lines can be approximated as directional arcs due
to axis symmetry of the diaphragm and underlying substrate.
This is shown in Fig. 4.

According to Gauss law,

$$\rho = \bar{D} = \varepsilon \bar{E}$$

(13)

![Fig. 3 Cross-sectional view of touch mode CPS](image)

![Fig. 4 Cross-sectional view of electric flux lines between two electrodes](image)
where $\rho$ is total charge density, $D$ is electric displacement, $\varepsilon$ is permittivity of medium. Considering electric flux as directional arcs, electric field intensity is

$$E = \frac{V}{\theta L}$$

where $V$ is the applied voltage between electrodes and $\theta$ is the angle between the tangent through diaphragm. $L$ is expressed as

$$L = \frac{d}{\sin \theta}$$

Here, $d$ is the separation between the capacitor plates and is equal to $(d_0 - w(x, y))$. Elemental touch mode capacitance can be expressed as

$$dC_{\text{touch}} = dC_t + dC_{\text{ut}}$$

where, $dC_{\text{ut}}$, the elemental cavity capacitance is the function of $L$ and $\theta$ thus

$$dC_{\text{ut}} = \frac{\varepsilon_0 \cdot dA}{\theta L} = \frac{\varepsilon_0 \sin \theta \cdot dA}{\theta d} = \frac{\varepsilon_0 \sin \theta \cdot dx \cdot dy}{\theta d}$$

The touch region between the diaphragm and the bottom electrode is assumed to be square in shape. Elemental touch down capacitance, $dC_t$, is expressed as

$$dC_t = \frac{\varepsilon_0 \varepsilon_i \cdot dA}{t} = \frac{\varepsilon_0 \varepsilon_i \cdot dx \cdot dy}{t}$$

The light-colored region in Fig. 5 indicates the touch region. This part contributes to $C_t$, the touch region capacitance.

$$C_t = \int_{y=b}^{b} \int_{x=a}^{a} \frac{\varepsilon_0 \varepsilon_i \cdot dx \cdot dy}{t} = \frac{\varepsilon}{t} [x]_{a}^{b} [y]_{a}^{b} = \frac{4b^2 \varepsilon_0 \varepsilon_i}{t}$$

$C_{\text{ut}}$ emerges out of the dark colored area in Fig. 5. Hence, solving for $C_{\text{ut}}$,

$$C_{\text{ut}} = 2 \int_{a}^{a} \int_{b}^{b} \frac{\varepsilon_0 \sin \theta \cdot dx \cdot dy}{\theta d} + 2 \int_{b}^{b} \int_{a}^{a} \frac{\varepsilon_0 \sin \theta \cdot dx \cdot dy}{\theta d}$$

where $b$ is half length for the touch region. From Fig. 5 it can be observed that at a point $(b, b)$ deflection of the diaphragm is $d_0$. Substituting $x = b$, $y = b$ and $w(b, b) = d_0$ in (1), $b$ can be evaluated as
\[ \begin{align*} C_1 &= \frac{2 \varepsilon_0 \sin \theta}{\theta d_0} \int_{y=-a}^{a} \int_{x=b}^{a} dx \cdot dy \\
&+ \frac{2 \varepsilon_0 \sin \theta}{\theta d_0} \int_{y=-a}^{a} \int_{x=b}^{a} \frac{P a^4}{47D} \left( \frac{1 - \frac{x}{a}}{\varepsilon_0} \right)^2 \left( \frac{1 - \frac{y}{a}}{\varepsilon_0} \right)^2 \frac{dx \cdot dy}{d_0} \tag{25} \end{align*} \]

\[ C_1 = \frac{4 a (a - b) \varepsilon_0 \sin \theta}{\theta d_0} \]
\[ + \frac{2 P \varepsilon_0 \sin \theta}{\theta d_0 p_t} \cdot \frac{16 a}{15} \left( \frac{8 a}{15} - \left( b + \frac{b^5}{5 a^4} - \frac{2 b^3}{3 a^2} \right) \right) \tag{26} \]

Similarly, solving for \( C_2 \) with the following limits, \( x = -b \) to \( b \) and \( y = b \) to \( a \) (shown in Fig. 5)

\[ C_2 = \frac{4 b (a - b) \varepsilon_0 \sin \theta}{\theta d_0} \]
\[ + \frac{2 P \varepsilon_0 \sin \theta}{\theta d_0 p_t} \cdot \frac{2}{15} \left( b + \frac{b^5}{5 a^4} + \frac{2 b^3}{3 a^2} \right) \left( \frac{8 a}{15} - \left( b + \frac{b^5}{5 a^4} - \frac{2 b^3}{3 a^2} \right) \right) \tag{27} \]

Assuming,

\[ K = \left( b + \frac{b^5}{5 a^4} - \frac{2 b^3}{3 a^2} \right) \tag{28} \]

\[ C_r = \frac{2 \varepsilon_0 \sin \theta}{\theta d_0} \tag{29} \]

Adding \( C_1 \) and \( C_2 \) and substituting (28) and (29) in the sum, \( C_{ut} \) is obtained as

\[ C_{ut} = 2 C_r (a^2 - b^2) + 2 C_r \frac{P}{p_t} \left( \frac{64 a^2}{225} - K^2 \right) \tag{30} \]

From (19) and (30), \( C_{touch} \) can be mathematically written as,

\[ C_{touch} = C_1 + C_{ut} = \frac{4 b^2 \varepsilon_0 \varepsilon_i}{t} + 2 C_r (a^2 - b^2) \]
\[ + 2 C_r \frac{P}{p_t} \left( \frac{64 a^2}{225} - K^2 \right) \tag{31} \]

### 3.3 Capacitive sensitivity for touch mode operation

Capacitive Sensitivity is defined as change in capacitance for a given change in applied pressure. It can be mathematically represented as

\[ S = \frac{dC}{dP} = \frac{dC_t}{dP} + \frac{dC_{ut}}{dP} \tag{32} \]

Substituting the values of \( C_t \) and \( C_{ut} \) in (32) from (19) and (30), respectively

\[ \frac{dC_t}{dP} = \frac{\varepsilon_0 \varepsilon_i a^2}{t} \sqrt{\frac{P_t}{P_s}} \tag{33} \]
\[ \frac{dC_{ut}}{dP} = 2 C_r \left( -\frac{a^2}{4} \sqrt{\frac{P_t}{P_s}} + \frac{1}{p_t} \left( \frac{64 a^2}{225} - K^2 \right) + \frac{P}{P_t} (-2K.K') \right) \tag{34} \]

where,

\[ K' = \frac{a^2}{8 b} \sqrt{\frac{P_t}{P_s}} \left( 1 + \frac{b^4}{a^4} - 2 \frac{b^2}{a^2} \right) \tag{35} \]

### 3.4 Mechanical sensitivity

Mechanical sensitivity is defined as the change in deflection for a given change in pressure

\[ S_{mec} = \frac{dw}{dp} \tag{36} \]

Substitute \( x = 0 \) and \( y = 0 \) in (1), \( S_{mec} \) is obtained as

\[ S_{mec} = \frac{d}{dp} \left( \frac{P}{47D a^2} \left( (-a^2)^2 (-a^2)^2 \right) \right) \tag{37} \]

\[ S_{mec} = \frac{a^4}{47 D} \tag{38} \]

### 4 Design specifications and considerations

The MATLAB simulation produced in Figs. 6, 7, 8, 9, 10, 11 is based on parameters specified in Table 1. It is evident from Table 1, that the sensor works in Normal Mode for the pressure range (0–0.023 MPa) and operates in Touch Mode for the pressure range (0.5–3 MPa). The highly nonlinear transition region between these two modes is observed in the pressure range (0.023–0.5 MPa). In order to validate and compare the results of this analytical approach, values
from the research conducted in [1] have been used and the comparison is presented in Figs. 12, 13.

5 Results and discussion

Normal and Touch mode operation are elucidated in Sect. 5.1 and 5.2, respectively. The motivation behind the choice of magnitude of the parameters involved in sensor design has been illustrated in Sects. 5.3, 5.4, 5.5 and 5.6. Comparison between the sensor performance for circular diaphragm CPS [1] and square diaphragm CPS has been detailed in Sects. 5.7 and 5.8.

5.1 Capacitance variation with pressure in normal mode

Variation in capacitance with applied pressure for normal mode operation is depicted in Fig. 6. It can be observed that the capacitance varies linearly within the pressure range (0–0.023 MPa). The maximum capacitance variation is 2.1 pF for this mode. Figure 6 also depicts touch-point pressure ($p_t$). The transition mode exists during the switch from normal mode to touch mode which can be observed in Fig. 7. Transition mode is highly nonlinear as the sensing diaphragm just starts to touch the bottom electrode. The pressure range for the transition region is (0.023–0.5 MPa).

5.2 Capacitance variation with pressure in touch mode

Figure 7 indicates capacitance variation with applied pressure for touch mode operation within the pressure range (0.5–3 MPa). Touch mode can be further divided into two sub-regions (1) linear and (2) saturation. The linear region forms the basis of touch mode sensor operation, as it can be accurately calibrated in terms of pressure. In the saturation region, the diaphragm completely touches the bottom electrode, hence no further change in capacitance is observed.

Table 1 Details of parameters used and the chosen design values

| Parameters                                      | Design values                  |
|------------------------------------------------|--------------------------------|
| Young’s modulus ($E$)                          | $170 \times 10^9 \text{N/m}^2$ |
| Diaphragm thickness ($h$)                      | $10 \times 10^{-6} \text{m}$   |
| Half-length of diaphragm ($a$)                  | $300 \times 10^{-6} \text{m}$  |
| Poisson’s ratio for silicon ($\nu$)             | 0.28                           |
| Permittivity of vacuum ($\varepsilon$)         | $8.854 \times 10^{-2} \text{F/m}$ |
| Dielectric constant of SiO$_2$ ($\varepsilon_i$) | 3.7                             |
| Dielectric constant of cavity/air ($\varepsilon_0$) | 1                               |
| Gap depth ($d_0$)                              | $2 \times 10^{-6} \text{m}$    |
| Thickness of insulation layer ($t$)             | $0.1 \times 10^{-6} \text{m}$  |
| Pressure range in normal mode                  | 0–0.023 MPa                    |
| Pressure range in transition mode              | 0.023–0.5 MPa                  |
| Pressure range in touch mode                   | 0.5–3 MPa                      |

Fig. 6 Capacitance variation with pressure in normal mode
Figures 6 and 7 indicate that the capacitance developed in touch mode is larger than that in the normal mode of operation. Higher sensitivity and ability to withstand large overload pressure are indicators of touch mode’s superior performance.
Fig. 9  Capacitance sensitivity vs gap-depth in touch mode

![Graph showing capacitance sensitivity vs gap-depth in touch mode.](image)

Fig. 10  Capacitance sensitivity vs applied pressure in touch mode

![Graph showing capacitance sensitivity vs applied pressure in touch mode.](image)
**Fig. 11** Variation of mechanical sensitivity with diaphragm thickness

![Graph showing the variation of mechanical sensitivity with diaphragm thickness.](image)

**Fig. 12** Comparative plot between square diaphragm CPS and circular diaphragm CPS (Normal Mode)

![Graph comparing the capacitance for square and circular diaphragms under applied pressure.](image)
5.3 Variation in capacitive sensitivity with half-length of diaphragm for touch mode operation

The capacitive sensitivity plays a crucial role in determining the performance of a sensor. For \( P = 1.0 \) MPa, Fig. 8 examines the capacitive sensitivity variation with half-length of diaphragm for touch mode operation. It indicates a negligible variation in capacitive sensitivity for \((0–250 \text{ m})\) with a sudden increase after \((300 \text{ m})\). Even though capacitive sensitivity is further enhanced with increase in half-length of diaphragm beyond \(300 \text{ m}\), there is a reduction in the linear operating range of the sensor. Considering the trade-off between sensitivity and linear range, \(a = 300 \text{ m}\) has been selected for the sensor design as it offers higher sensitivity without compromising the linear operating range.

5.4 Variation in capacitive sensitivity with gap depth for touch mode operation

For \( P = 1.0 \) MPa, Fig. 9 illustrates the capacitive sensitivity variation with gap-depth for touch mode operation. A significant decrease in capacitive sensitivity is observed after a gap depth of \(0.5 \mu\text{m}\). The design presented in this paper has a gap depth of \(2 \mu\text{m}\) which acts as the limiting value for optimum sensor operation.

5.5 Variation in capacitive sensitivity with applied pressure for touch mode operation

Variation of capacitive sensitivity with applied pressure for touch mode operation is presented in Fig. 10. Capacitive sensitivity decreases with increase in applied pressure and the variation with pressure is almost constant after 3 MPa. In the pressure range between \((0.023 \text{ MPa} \text{ and } 0.5 \text{ MPa})\), the value of captative sensitivity is considerably large, and this demonstrates the high nonlinearity of the transition region. As depicted in Fig. 10, linear operating range for this design is \((0.5–3 \text{ MPa})\) and the sensor enters the saturation region after 3 MPa where the capacitive sensitivity is significantly reduced. Figure 7 also supports these observations.

5.6 Mechanical sensitivity

Figure 11 demonstrates the variation of mechanical sensitivity with diaphragm thickness. Increase in diaphragm thickness reduces mechanical sensitivity due to reduced diaphragm deflection. It can be concluded that for the given design parameters (Table 1), if the thickness is increased beyond \(5 \mu\text{m}\), the mechanical sensitivity drops by a significant amount. Therefore, the diaphragm thickness is constrained to \(5 \mu\text{m}\) [1].

5.7 Comparative analysis between square and circular diaphragms in normal mode operation

Figure 12 presents the capacitance variation with applied pressure in normal mode operation for both square and circular diaphragm CPS. It can be inferred that the capacitance, capacitive sensitivity, and full pressure range of normal mode operation is higher for square diaphragms.

5.8 Comparative analysis between square and circular diaphragms in touch mode operation

Figure 13 illustrates the capacitance variation with applied pressure in touch mode operation for both square and circular diaphragm CPS. A square diaphragm occupies a larger area in comparison to a circular diaphragm with the same half-length \((r = a, \text{ where } r \text{ is the radius of the circle and } 2a, \text{ is the square’s side length})\). As an increase in the area leads to an increase in capacitance \((C = \varepsilon A/d)\), hence higher capacitance and sensitivity are achievable with a square diaphragm CPS which is substantiated by Fig. 13. The results obtained and the interpretations have been summarized in Table 2 (Table 3; Fig. 14).

| Diaphragm Geometry | Capacitance developed (pF) for applied pressure of 1.2 MPa | Capacitance developed (pF) for applied pressure of 1.8 MPa | Comment on sensitivity |
|--------------------|----------------------------------------------------------|----------------------------------------------------------|------------------------|
| Circular [1]       | 1.33                                                     | 1.56                                                     | For the same pressure range, the capacitance change is higher for square diaphragm CPS. Hence it can be interpreted that the sensitivity of Square diaphragm CPS is higher than that of Circular diaphragm CPS |
| Square [Proposed work] | 17.8                                                     | 20.7                                                     |                        |
Table 3  Comparative table for finite element modelling method and analytical model method

| Simulation method | Deflection in μm for applied pressure of 0.6 MPa | Execution time (s) for the same model | Comments |
|-------------------|------------------------------------------------|--------------------------------------|----------|
| FEM method        | 2.06                                           | 13.8                                 | For the same pressure value, the execution time for the FEM Method is higher as compared to the analytical method. It can be observed that the deflection values calculated are similar. The execution time for the FEM Method is not as high as the number of meshes created were less. This will increase if number of meshes created is higher, resulting in an even longer execution time which is not the case for the analytical method |
| Analytical method | 2                                              | 1.6                                  |          |

Fig. 13  Comparative plot between square diaphragm CPS and circular diaphragm CPS (Touch Mode)

Fig. 14  FEM Analysis for square diaphragm CPS for an applied pressure of 0.6 MPa
6 Conclusions

This study presents a novel analytical model for the determination of normal and touch mode operation of a capacitive pressure sensor with a clamped square diaphragm. Accurate analysis of these modes of operation is central to evaluating the operation range and stability of capacitive pressure transducers. Derived parameters like capacitive and mechanical sensitivity provide constructive insights for efficient sensor design. A justification for the sensor dimensions for optimum operation has been provided to further validate the presented analysis. Finite element method enables accurate prediction and modeling of normal and touch mode capacitance. However, the resource and time complexity of these tools make their usage less pragmatic. This analytical method offers significant ease of computation and clearly examines the higher sensitivity resulting out of the square geometry of the diaphragm. It also aims to provide a fast analysis model for prototyping the sensor to circumvent the need for complex simulation software. The examined theory and simulated results fully validate the superiority of square diaphragms in terms of ease of manufacturing, higher sensitivity, increased capacitance, and robustness.

Acknowledgements Not applicable.

Author contributions All authors contributed to the study conception and design. Material preparation, simulation and analysis were performed by all the authors together. The first draft of the manuscript was written by Mr. Sumit Kumar Jindal and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Funding Not applicable.

Data availability Not applicable.

Declarations

Conflict of interest Not applicable.

Ethics approval and consent to participate Not applicable.

Consent for publication Yes.

Human and animal rights Not applicable.

Informed consent Not applicable.

References

1. Jindal, S.K., Mahajan, A., Raghuwanshi, S.K.: A complete analytical model for clamped edge circular diaphragm non-touch and touch mode capacitive pressure sensor. Microsyst. Technol. 22(5), 1143–1150 (2016)
2. Jindal, S.K., Raghuvanshi, S.K.: Capacitance and sensitivity calculation of double touch mode capacitive pressure sensor: theoretical modelling and simulation. Microsyst. Technol. 23(1), 135–142 (2017)
3. Jindal, S.K., Varma, M.A., Thukral, D.: Comprehensive assessment of mems double touch mode capacitive pressure sensor on utilization of sic film as primary sensing element: mathematical modeling and numerical simulation. Microelectron. J. 73, 30–36 (2018)
4. Varma, M.A., Thukral, D., Jindal, S.K.: Investigation of the influence of double-sided diaphragm on performance of capacitance and sensitivity of touch mode capacitive pressure sensor: numerical modeling and simulation forecasting. J. Comput. Electron. 16(3), 987–994 (2017)
5. Molla-Alipour, M., Ganji, B.A.: Analytical analysis of mems capacitive pressure sensor with circular diaphragm under dynamic load using differential transformation method (dtm). Acta Mech. Solida Sin. 28(4), 400–408 (2015)
6. Rochus, V., Wang, B., Tilmans, H.A.C., Chaudhuri, A.R., Helin, P., Severi, S., Rottenberg, X.: Fast analytical design of mems capacitive pressure sensors with sealed cavities. Mechatronics 40, 244–250 (2016)
7. Kang, M.-C., Rim, C.-S., Pak, Y.-T., Kim, W.-M.: A simple analysis to improve linearity of touch mode capacitive pressure sensor by modifying shape of fixed electrode. Sens. Actuators A 263, 300–304 (2017)
8. Jindal, S.K., Raghuvanshi, S.K.: A complete analytical model for circular diaphragm pressure sensor with freely supported edge. Microsyst. Technol. 21(5), 1073–1079 (2015)
9. Ganji, B.A., Shams, N.M: Modeling of capacitance and sensitivity of a mems pressure sensor with clamped square diaphragm (2013)
10. Jindal, S.K., Mahajan, A., Raghuvanshi, S.K.: Reliable before-fabrication forecasting of normal and touch mode mems capacitive pressure sensor: modeling and simulation. J. Micro Nanolithogr. MEMS MOEMS 16(4), 045001 (2017)
11. Tai-Ran, H.: Mems and Microsystems: Design and Manufacture. McGraw Hill, New York (2002)
12. Sharma, A., Mittal, M., Saini, D.: Virtual fabrication, 3d electromechanical simulation and system level modeling of mems capacitive pressure sensor for tire pressure monitoring system. Sens. Transducers 143(8), 106 (2012)
13. Lee, M.K., Eom, J., Choi, B.: Numerical analysis of touch mode capacitive pressure sensor using graphical user interface. In: Gaol, F.L. (ed.) Recent Progress in Data Engineering and Internet Technology, pp. 371–377. Springer, Berlin (2013)
14. Bergqvist, J.: Finite-element modelling and characterization of a silicon condenser microphone with a highly perforated backplate. Sens. Actuators A 39(3), 191–200 (1993)
15. Bao, M.: Analysis and Design Principles of MEMS Devices. Elsevier, Amsterdam (2005)
16. Timoshenko, S., Woinowsky-Krieger, S., et al.: Theory of Plates and Shells, vol. 2. McGraw-Hill, New York (1959)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.