TOPOLOGY OF LUMINOUS RED GALAXIES FROM THE SLOAN DIGITAL SKY SURVEY

Yun-Young Choi1, Juhan Kim2,3, Graziano Rossi4,5,7, Sungsoo S. Kim6, and Jeong-Eun Lee6

1 Department of Astronomy and Space Science, Kyung Hee University, Gyeonggi 446-701, Korea; yy.choi@khu.ac.kr
2 Center for Advanced Computation, Korea Institute for Advanced Study, Heogiro 85, Seoul 130-722, Korea
3 School of Physics, Korea Institute for Advanced Study, Heogiro 85, Seoul 130-722, Korea
4 CEA, Centre de Saclay, Irfu/SPP, F-91191 Gif-sur-Yvette, France; graziano.rossi@cea.fr
5 Paris Center for Cosmological Physics and Laboratoire APC, Université Paris 7, F-75205 Paris, France
6 School of Space Research, Kyung Hee University, Gyeonggi 446-701, Korea

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ABSTRACT

We present measurements of the genus topology of luminous red galaxies (LRGs) from the Sloan Digital Sky Survey (SDSS) Data Release 7 catalog, with unprecedented statistical significance. To estimate the uncertainties in the measured genus, we construct 81 mock SDSS LRG surveys along the past light cone from Horizon Run 3, one of the largest N-body simulations to date, which evolved 72103 particles in a 10,815 h−1 Mpc box. After carefully modeling and removing all known systematic effects due to finite pixel size, survey boundary, radial and angular selection functions, shot noise, and galaxy biasing, we find that the observed genus amplitude reaches 272 at a 22 h−1 Mpc smoothing scale, with an uncertainty of 4.2%; the estimated error fully incorporates cosmic variance. This is the most accurate constraint on the genus amplitude to date and significantly improves on our previous results. In particular, the shape of the genus curve agrees very well with the mean topology of the SDSS LRG mock surveys in a Λ cold dark matter universe. However, comparison with simulations also shows small deviations of the observed genus curve from the theoretical expectation for Gaussian initial conditions. While these discrepancies are mainly driven by known systematic effects such as shot noise and redshift-space distortions, they do contain important cosmological information on the physical effects connected with galaxy formation, gravitational evolution, and primordial non-Gaussianity. We address the key role played by systematics on the genus curve and show how to accurately correct for their effects to recover the topology of the underlying matter. A future work will provide an interpretation of these deviations in the context of the local model of non-Gaussianity.

Key words: cosmology: observations – cosmology: theory – large-scale structure of universe – methods: data analysis – methods: numerical

Online-only material: color figures

1. INTRODUCTION

The current standard cosmological scenario, supported by observations of the cosmic microwave background and large-scale structure (LSS), appears to be consistent with the ΛCDM concordance model, in which the universe is dominated by cold dark matter (CDM) and its accelerating expansion is driven by a cosmological constant Λ or dark energy (DE). Strong support of this paradigm has been presented recently by Park et al. (2012), who demonstrated that observed high- and low-density LSSs have richness/volume and size distributions consistent with a ΛCDM universe.

In the primordial density perturbations from which halos and galaxies form are assumed to be a Gaussian random field, as predicted by inflationary theories (Guth 1981; Linde 1982; Bardeen et al. 1986); state-of-the-art data from the Wilkinson Microwave Anisotropy Probe (WMAP; Spergel et al. 2003; Komatsu et al. 2011), the Sloan Digital Sky Survey (SDSS; York et al. 2000; Stoughton et al. 2002; Abazajian et al. 2009), and the WiggleZ survey (Blake et al. 2011) still favor a ΛCDM universe. However, some claims or hints of primordial non-Gaussianity have recently appeared in the literature (Jeong & Smoot 2007; Yadav & Wandelt 2008; Komatsu et al. 2009, 2011; Slosar et al. 2008; Smidt et al. 2010) and challenged the validity of the simplest inflationary paradigm. Indeed, if detected, primordial non-Gaussianity would indicate a structure formation scenario different from the concordance cosmological model and force us to revise the physics of the very early universe, along with several aspects of LSS dynamics (but see also Hwang 2012 for a more general discussion on modern cosmology).

To this end, topology-related statistics offer a valuable benchmark for testing the underlying Gaussianity of the initial density field, since topology can be regarded as an important physical property of the matter density that can be compared with predictions of the simplest inflationary models, in which Gaussian, random-phase initial conditions are generated through quantum fluctuations of an inflation field in the early universe. In addition, topology measured at the present epoch should reflect that of the initial conditions on smoothing scales considerably larger than the correlation length, because fluctuations that are still in the linear regime maintain their initial topology (see Gott et al. 1987, who confirmed this property with N-body simulations); this fact allows one to directly test the Gaussian paradigm and permits the use of topology as a cosmic standard ruler (Park & Kim 2010).

From the theoretical side, since the pioneering work of Gott et al. (1986), a variety of analytical and numerical tools to analyze observational and simulated data for measuring topology have been developed, mainly using genus statistics to quantify the topology of isodensity contours (Hamilton et al. 1986; Gott et al. 1987, 1989; Vogeley et al. 1994; Park et al. 2005a, 2005b). In particular, the analytical prediction for the genus curve of a Gaussian field in the linear regime is well known (Gott et al. 1986), and its perturbative expression in the weakly nonlinear regime has also been obtained (Matsubara 1994);
lognormal model turned out to be a good empirical approxima-
tion in the strongly nonlinear regime (Hikage et al. 2002).
Along with analytical tools, large-volume N-body simulations
are routinely used to quantify several systematics that affect the
genus curve, such as finite pixel size, sparse sampling, peculiar-
velocity distortions in redshift space, and survey boundaries.
The ability to correct for these effects is essential, as the re-
mainding small deviations from the random-phase curve yield
important information about the physics connected with galaxy
formation, nonlinear gravitational clustering, and primordial
non-Gaussianity—if any. In fact, on smaller scales nonlinear
gravitational evolution and biased galaxy formation make the
topology of the observed galaxy distribution deviate from the
Gaussian form, even if the initial conditions were Gaussian
distributed. Using fractional volume rather than a direct density
mitigates but does not fully eliminate these nonlinear and biasing
effects (Weinberg et al. 1987; Melott et al. 1988). Ultimately, all
the secondary non-Gaussianities need to be disentangled from
the primordial contribution, and this can now be done very accu-
radately, without assuming any “a priori” model for the underlying
signal.

From the observational side, a long list of studies have been
pursued on a variety of data sets (e.g., Park et al. 1992a, 1992b,
2001, 2005a; Moore et al. 1992; Vogele et al. 1994; Rhoads
et al. 1994; Protogeros & Weinberg 1997; Canaveses et al.
1998; Hoyle et al. 2002; Hikage et al. 2002, 2003; James et al.
2007, 2009; Gott et al. 2008, 2009; Choi et al. 2010). These
focused on characterizing the three-dimensional topology and
showed that, depending on the considered scale, topology can
be useful in constraining both cosmological parameters and the
galaxy formation mechanism. For example, Park et al. (2005a)
characterized the topology of the SDSS main galaxy sample
from the New York University Value-Added Galaxy Catalog
(NYU-VAGC; Blanton et al. 2005), which has similar sky
coverage to SDSS Data Release 4 (DR4; Adelman-McCarthy
et al. 2006), and presented the first clear demonstration of
luminosity dependence of galaxy clustering topology (i.e.,
brighter galaxies show a stronger signal of meatball topology);
more recently, Gott et al. (2009) measured the three-dimensional
LSS topology of the SDSS DR4plus luminous red galaxy (LRG)
sample from the NYU-VAGC (a subsample of SDSS DR5:
Adelman-McCarthy et al. 2007) and found strong consistency
with Gaussianity of the primordial fluctuations. In the latter
case, the large available sample size allowed topology to be an
important tool for testing galaxy formation models.

Choi et al. (2010) measured the topology of the main
galaxy distribution using SDSS DR7 (Abazajian et al. 2009;
Choi et al. 2010), studied the scale-dependent topology bias,
and examined the dependence of galaxy clustering topology
on galaxy properties (i.e., luminosity, morphology, color) at
different smoothing scales. The large volume-limited sample
enabled an unprecedented measurement of the genus curve,
with an amplitude $G = 378$ at $h^{-1} \text{ Mpc}$ smoothing scale
and an estimated uncertainty of 4.8%, including all systematics
and cosmic variance. In addition, deviations of the genus curve
from Gaussianity were detected, which were interpreted to
mean that voids and superclusters are more connected and their
sizes are larger than expected for Gaussian fields. These results
were then used to test five different galaxy formation models,
which indeed are tuned to reproduce the two-point correlation
function and the luminosity function (LF) but not high-order
statistics, and significant discrepancies were found: none of the
models could reproduce all the aspects of the observed clustering
topology.

While the significant discrepancies in the SDSS DR7 galaxy
clustering topology at nonlinear scales found in Choi et al.
(2010) are mainly driven by the inaccuracy of galaxy formation
models, a cleaner problem is to consider the topology of LRGs
instead, which is the main focus of this paper. This is because
the LRG sample covers a much larger and deeper volume (i.e., it
allows one to observe topology at the largest scales), essentially
in the linear regime. In addition, LRGs are particularly useful
in refining cosmological parameters (Tegmark et al. 2006) and
are expected to play an important role in characterizing DE
through the ratio of DE pressure to energy density (see Bassett
et al. 2005; Eisenstein et al. 2005; Percival et al. 2007). A
number of previous analyses have addressed the clustering
of LRGs, especially in relation to baryon acoustic oscillation
(BAO) science (see, e.g., Ross et al. 2012 and references
therein). On the contrary, fewer studies have been based on
topology. Among those, Gott et al. (2009) measured the three-
dimensional genus topology of LRGs, using two volume-limited
samples constructed from the SDSS DR4plus sample, a dense
shallow sample with $21 h^{-1} \text{ Mpc}$ smoothing and a sparse
deep sample with $36 h^{-1} \text{ Mpc}$ smoothing. The amplitude
of the genus curve was found to reach about 167 with a 4.1% uncer-
tainty at $21 h^{-1} \text{ Mpc}$ scale. A major result of their study
was that the topology of LRGs in the SDSS agrees very well
with that of mock galaxies in a $\Lambda$CDM universe with the
same cosmological parameters: small distortions in the genus
curve, expected from nonlinear biasing and gravitational effects,
were well explained by $N$-body simulations with a subhalo-
finding technique adopted to locate LRGs. This suggests that
the formation of LRGs can be modeled well without any free fit
parameters.

The main goal of this paper is to characterize the three-
dimensional genus topology of spectroscopic LRGs using the
SDSS DR7 catalog, improving on the previous results presented
by Gott et al. (2009). In particular, we strive to carefully model
and remove all known systematics that affect the observed genus
(i.e., finite pixel size, survey boundary, radial and angular selec-
tion functions, and shot noise) and estimate the uncertainties in
the measured genus accurately. This is achieved by comparing
our measurements with 81 mock SDSS LRG surveys along the
past light cone (LC) constructed from Horizon Run 3 (HR3;
Kim et al. 2011), one of the largest $N$-body simulations to date,
which evolved $7210^3$ particles in a $10,815 h^{-1} \text{ Mpc}$ box.

Our main result for the observed genus curve is shown in
Figure 1 (red solid line) and compared with a previous topology
measurement of the SDSS DR4plus data set (blue solid line; Gott
et al. 2009). The main point of the figure is to show the dramatic
increase in amplitude of the genus curve over the two different
data sets (i.e., DR4plus versus DR7), in the same redshift range
from 0.016 to 0.36 and for the same rest-frame $g$-band absolute
magnitudes of $-23.2 < M_g < -21.2$, due to the much larger
volume that is covered. In fact, we find that the genus amplitude
reaches 285 with an uncertainty of 4.0% at a $22 h^{-1} \text{ Mpc}$
Gaussian smoothing scale, including cosmic variance (the most
accurate measurement to date), while Gott et al. (2009) found
the genus curve to reach about 167 with a 4.1% uncertainty at
$21 h^{-1} \text{ Mpc}$ smoothing scale; for comparison with a different
galaxy population, Park et al. (2005a) and Choi et al. (2010)
reported uncertainties of 9.4% and 4.8% at 5 and $6 h^{-1} \text{ Mpc}$

scales, respectively, for the genus obtained from the main galaxy
sample of SDSS DR3 and DR7. We return to this measurement

$G = 378$ at $6 h^{-1} \text{ Mpc}$ smoothing scale; for comparison with a different
scale, including cosmic variance (the most
accurate measurement to date), while Gott et al. (2009) found
the genus curve to reach about 167 with a 4.1% uncertainty at
$21 h^{-1} \text{ Mpc}$ smoothing scale; for comparison with a different
galaxy population, Park et al. (2005a) and Choi et al. (2010)
reported uncertainties of 9.4% and 4.8% at 5 and $6 h^{-1} \text{ Mpc}$
scales, respectively, for the genus obtained from the main galaxy
sample of SDSS DR3 and DR7. We return to this measurement
in greater detail in Section 5, while in a forthcoming publication we will interpret the deviation of the genus curve from the expected Gaussian prediction in the context of primordial non-Gaussianity.

The paper is organized as follows. In Section 2, we present the main theoretical framework; in particular, we review the genus statistics for Gaussian fields and discuss how to extend the formalism for non-Gaussian fields. In Section 3, we describe the SDSS LRG sample used for our measurements and the methodology applied to the observational data. In Section 4, we present the HR3 N-body simulation and explain the procedure adopted to construct the 81 mock LRG surveys. In Section 5, we show our results for the LRG genus statistics, compare measurements from SDSS DR7 data and simulations, and quantify the non-Gaussian deviations of the genus curve with genus-related quantities. In Section 6, we discuss the effects of known systematics on the genus and present the genus curve after correction for systematics. We conclude in Section 7 and leave some more details on the genus curves in the appendices.

2. THEORETICAL BACKGROUND

We begin by revisiting the basic theory of the genus for Gaussian fields and by introducing some genus-related statistics. We also briefly review the formalism for describing the non-Gaussian effect on the genus curve in the weakly nonlinear regime according to second-order perturbation theory, originally derived by Matsubara (1994, 2003). We compare the theory outlined here with results from the SDSS DR7 LRG data set and with measurements from the simulated LRG sample in Section 6. The full extension to non-Gaussian LRG sample with the inclusion of primordial non-Gaussianity will be presented and discussed in a forthcoming publication.

2.1. Genus Statistics for Gaussian Random Fields

The genus is a measurement of the topology of isodensity contour surfaces in a smoothed galaxy field (Gott et al. 1987). In mathematical terms, it is defined as follows: consider a three-dimensional Gaussian random field $\rho \equiv \rho(x)$ with $x$ the spatial coordinate and measure the topology of the excursion regions where $\rho$ is equal to or above a given threshold level $\rho + \nu \sigma_0$. Here $\rho$ is the mean of the field $\rho$, $\sigma_0$ is its rms value, and $\nu = (\rho - \bar{\rho})/\sigma_0$. Denote by $M$ the space containing the set of excursion regions (i.e., a three-manifold subset) and indicate its boundaries by $\delta M$ (a two-manifold subset). For each component $S_i$ of $\delta M$, according to the Gauss–Bonnet theorem the mathematical genus satisfies the relation:

$$G_i = I - \frac{1}{2} \chi(S_i),$$

(1)

where $\chi(S_i)$ is the Euler characteristic of the surface of the three-dimensional excursion region (i.e., the integrated Gaussian curvature of the surface). Hence, the total genus of the boundary $\delta M$ becomes

$$G = \sum_i G_i = N - \frac{1}{2} \sum_i \chi(S_i) = N - \frac{1}{2} \chi(\delta M),$$

(2)

where $N$ is the number of components of $\delta M$ (see Park et al. 2013 for a full derivation of this formula).

In cosmology, the standard definition of the genus differs slightly from the mathematical one, since the genus is defined as the number of holes minus the number of isolated regions in the isodensity contour surfaces at a given threshold level $\nu$, namely,

$$G(\nu) = \text{Number of holes in contour surfaces} - \text{Number of isolated regions}.\quad (3)$$

The relation between the two definitions is simply $G = G - N$, where $N$ is as defined above. For further insights on topological invariants, and the mathematical connection between the cosmological genus and the Betti numbers for excursion sets of Gaussian random fields, we refer the reader to Park et al. (2013).

In the case of Gaussian fields, the genus per unit volume $g(\nu) = G(\nu)/V$ as a function of the density threshold level $\nu$ is known (e.g., Doroshkevich 1970; Adler 1981; Hamilton et al. 1986):

$$g(\nu) = g(0)(1 - \nu^2)\exp(-\nu^2/2).$$

(4)

The amplitude $g(0)$ is given by

$$g(0) = \frac{1}{(2\pi)^2} \left( \frac{\sigma}{\sqrt{5\sigma_0}} \right)^3,$$

(5)

while the spectral moments of the fields, $\sigma_j^2$, are computed from

$$\sigma_j^2(R_0) = \frac{1}{(2\pi)^2} \int k^{2j+1} P(k, z)dk.$$

(6)
In this relation, \( P(k, z) \) is the power spectrum smoothed on a scale \( R_G \) by a window function \( W \), where

\[
P(k, z) = P_m(k, z)W^2(kR_G)
\]

and \( P_m(k, z) \) is the matter power spectrum. In particular, in this study we adopt a Gaussian smoothing of the form \( W(kR_G) = \exp(-k^2R_G^2/2) \). Note also that for \( j = 0 \), \( \sigma_j \) is the variance of the fluctuating field, whereas \( \sigma_j \) is the variance of its derivative when \( j = 1 \).

To separate variations in topology from changes of the one-point density distribution, in this work we also measure the genus as a function of the volume fraction threshold \( v_l \) (as opposed to the direct density threshold \( v_l \)). This parameter defines the density contour surface such that the volume fraction \( v \) in the high-density region is the same as the volume fraction in a Gaussian random field contour surface having \( v = v_l \), namely,

\[
f = \frac{1}{\sqrt{2\pi}} \int_{v=0}^{\infty} \exp(-x^2/2)dx .
\]

### 2.2. Genus Statistics and Perturbation Theory

After correcting for known systematics, deviations of the observed genus curve from the Gaussian expectation (i.e., Equation (4)) are due to nonlinear gravitational evolution and non-Gaussianity of the primordial density field. A number of studies in the literature have already addressed the impact of non-Gaussianity on the genus curve (e.g., Weinberg et al. 1987; Park & Gott 1991; Park et al. 2005b). In what follows, we briefly discuss the non-Gaussian effect on the genus curve caused by nonlinear gravitational evolution in the weakly nonlinear regime (which tends to distort the Gaussian expectation for the genus statistic) in the context of second-order perturbation theory—along the lines of Matsubara (1994, 2003). More details on the non-Gaussian modifications of the genus curve will be presented by Y.-R. Kim et al. (2013, in preparation).

To first order in \( \sigma_0 \), the nonlinear correction that one must apply to the genus curve due to gravitational evolution is an odd function of the threshold \( v \). Hence, this correction causes a shift and an asymmetry between high- and low-density regions, with no change in the amplitude at \( v = 0 \).

In particular, when we use a threshold rescaled by a volume fraction of the smoothed field, \( v_l \), the genus of the matter density field per unit volume, expanded to first order in mass variance \( \sigma_0 \), can be written as the sum of a Gaussian term \( g^G(v_l) \) and a non-Gaussian term \( g^{NG}(v_l) \), namely,

\[
g(v_l) = g^G(v_l) + g^{NG}(v_l).
\]

The Gaussian part is expressed as

\[
g^G(v_l) = -g(0)\exp(-v_l^2/2)H_2(v_l),
\]

while the non-Gaussian term is given by

\[
g^{NG}(v_l) = -g(0)\exp(-v_l^2/2)\times[(S^{(1)} - S^{(0)})H_3(v_l) + (S^{(2)} - S^{(0)})H_4(v_l)]\sigma_0
\]

where \( S^{(a)} \) are Hermite polynomials, and in particular, \( H_0(v_l) = 1, H_1(v_l) = v_l, H_2(v_l) = v_l^2 - 1, H_3(v_l) = v_l^3 - 3v_l, H_4(v_l) = v_l^4 - 6v_l^2 + 3 \), and \( H_2(v_l) = v_l^2 - 10v_l^2 + 15v_l \).

Also, the various \( S^{(a)}, a = 0, 1, 2 \), are skewness parameters obtained by integrating the bispectrum \( B(k_1, k_2, k_3, z) \) over \( k_1 \) and \( k_2 \) (see Equations (61)–(64) in Matsubara 2003). In addition, the bispectrum can be given in terms of the nonlinear contributions from nonlinear gravitational evolution as well as primordial non-Gaussianity—an aspect that we do not consider here (but see Appendix B in Hikage et al. 2006). Note that the non-Gaussian part \( g^{NG}(v_l) \) only appears with terms of the form \( S^{(a)} - S^{(0)} \). Assuming galaxy biasing is local and deterministic in the weakly nonlinear regime, the skewness parameters of the galaxy bispectrum, \( S^{(a)}_g \), are related to \( S^{(a)} \) with \( S^{(a)}_g = S^{(a)} / b + 3b_2 / b^2 \), where \( b \) and \( b_2 \) are bias parameters. The non-Gaussian term \( g^{NG}(v_l) \) can be approximated in the limit of \( v_l \rightarrow 0 \) to be the same as that of the unbiased mass density field in Equation (11) and hence independent of the bias parameter. This can be considered an advantage of the volume fraction threshold, as opposed to the direct density threshold (see Matsubara 2003 for more details).

### 2.3. Genus-related Statistics

The measured genus curve can be compared with predictions of the simplest inflationary models, which assume Gaussian, random-phase initial conditions (and so with Equation (4)). However, even if the initial conditions are perfectly Gaussian, small deviations from Gaussianity are expected because of systematic effects (e.g., shot noise or redshift-space distortions, RSDs) and because of the physics connected with galaxy formation, nonlinear gravitational evolution, and primordial non-Gaussianity (if any). Therefore, it is important to quantify even small departures from Gaussianity in the observed genus. This is done by parameterizing the genus curve with several derived quantities. In what follows, we consider measurements as a function of the volume fraction threshold \( v_l \) and introduce four genus-related statistics. The first is simply the best-fit genus amplitude \( G_{\text{fit}}(0) \), measured by a least-squares fit of the theoretical random-phase curve to the data considering only the range \( -1 < v_l < 1 \). In principle, its value is given by Equation (5), but the measured one is always lower because of nonlinear clustering and biasing due to coalescence of structures (Park & Gott 1991; Vogele et al. 1994; Canavezes et al. 1998; Gott et al. 2008).

The second quantity is the shift parameter \( \Delta v_{\text{T}} \), defined as

\[
\Delta v_{\text{T}} = \int_{-1}^{1} G_o(v_l)v_ldv_l / \int_{-1}^{1} G_{\text{fit}}(v_l)v_ldv_l ,
\]

where \( G_o \) and \( G_{\text{fit}} \) are the observed and best-fit Gaussian genus curves. Both are given by Equation (4), but in the former case with the observed amplitude \( G_o(0) \) and in the latter with the best-fit one, \( G_{\text{fit}}(0) \), as explained in Park et al. (1992a, 1992b). The parameter \( \Delta v_{\text{T}} \) controls the horizontal shifts of the central part of the genus curve. For a density field dominated by voids, \( \Delta v_{\text{T}} \) is positive and we say that the density field has a “bubble-like” topology. For a cluster-dominated field, \( \Delta v_{\text{T}} \) is negative and we say that the field has a “meatball-like” topology.

We then further introduce two additional quantities, \( A_C \) and \( A_V \), which measure the abundances of clusters and voids, respectively, relative to the expectations for a Gaussian random field. They are defined by the following relation:

\[
A = \int G_o(v_l)v_ldv_l / \int G_{\text{fit}}(v_l)v_ldv_l ,
\]
3. OBSERVED LRG SAMPLE : DESCRIPTION AND METHODOLOGY

In this section, we briefly describe our LRG sample obtained from SDSS DR7, along with the methodology applied to the observational data set. The genus computed from the LRG sample and its related statistics are presented in Section 5.

3.1. The SDSS DR7 LRG Sample

The SDSS is a successful ground-based survey, designed to explore the large-scale distribution of galaxies and quasars by using a dedicated 2.5 m telescope at Apache Point Observatory (see Gunn et al. 2006 for technical details). The photometric survey has imaged roughly $\pi$ steradians of the Northern Galactic Cap in five photometric bandpasses, denoted by $u$, $g$, $r$, $i$, and $z$ and centered at 3551, 4686, 6165, 7481, and 8931 Å, respectively. The imaging camera used is equipped with 54 CCDs (Fukugita et al. 1996; Gunn et al. 1998). The limiting photometric magnitudes are 22.0, 22.2, 22.2, 21.3, and 20.5 in the five bandpasses, at a signal-to-noise ratio of 5:1. The median width of the point-spread function is $1.4''$, and the rms photometric uncertainties are at the 2% level (Abazajian et al. 2004). After image processing (Lupton et al. 2001; Stoughton et al. 2002; Pier et al. 2003) and calibration (Hogg et al. 2001; Smith et al. 2002), targets are selected for spectroscopic follow-up observations. The spectra are obtained with two dual-channel, fiber-fed CCD spectrographs at a spectral resolution $\lambda/\Delta \lambda \simeq 1800$ and rms uncertainty in redshift of $\sim30$ km s$^{-1}$. Because of mechanical constraints, two fibers cannot be placed closer than 55" on the same tile. The incompleteness percentage of the spectroscopic survey reaches about 6%. The SDSS spectroscopy yields three major samples: the main galaxy sample (Strauss et al. 2002), the LRG sample (Eisenstein et al. 2001), and the quasar sample (Richards et al. 2002). In particular, the LRG sample considered here is part of the final data release of SDSS-II, identified as DR7, which yields 928,567 galaxy spectra over the legacy spectroscopic coverage of 8032 deg$^2$.

For this work, we created a volume-limited sample including 67,385 LRGs in the redshift range from 0.16 to 0.36 and for rest-frame $g$-band absolute magnitudes of $-23.2 < M_g < -21.1$, passively evolved to $z = 0.3$ (see Zehavi et al. 2005; Eisenstein et al. 2005), by using the “DR7-Full” sample of Kazin et al. (2010). $K$-corrections were applied to all the galaxies in the sample, assuming a fiducial $\Lambda$CDM model with $\Omega_m = 0.26$ and $h = 1$, not the $\Omega_m = 0.25$ that was applied to the sample by Kazin et al. (2010). To maximize the volume-to-surface ratio, we trimmed the sample as in Choi et al. (2010; see Figure 1 there for more details). Both the Southern Galactic Cap region and the Hubble Deep Field region are dropped. These cuts leave a total of 60,466 LRGs over about 2.33 sr in the survey region with an angular selection function greater than 0.6.

Figure 2 shows three-dimensional isodensity contours of the smoothed galaxy number density fields obtained from the SDSS LRG sample at $\nu_f = \pm2.0$, $\pm1.5$, and 0, for which the corresponding volume fractions are 2.3%, 6.7%, and 50%. A Gaussian smoothing is applied with $R_G = 22h^{-1}$ Mpc. As expected, the asymmetry between high- and low-density regions of the observed genus curve shown in Figure 1 is also clearly seen in this visual comparison. The low-density regions (upper left panels) tend to be more connected and filamentary than the high-density regions (upper right panels), where structures appear to be more isolated and rounder. As the volume fraction increases, the structures increase in size.

3.2. Construction of the Galaxy Mass and Number Density Fields from the Observed LRG Sample

For an arbitrary large-scale galaxy survey, the sampling of galaxies as a function of redshift is usually not uniform. Moreover, surveys are typically designed such that not only is the mean galaxy number density not constant, but the sampling in absolute magnitude is nonuniform as well. An example is shown in Figure 3, for the semi–volume-limited LRG sample. The rectangle identifies the volume-limited sample considered in this study. The high nonuniformity of the sample as a function of redshift and absolute magnitude is clearly visible.

In this situation, it would be incorrect to give a single-valued weight to galaxies in each redshift bin based only on the radial selection function; in fact, this simple scheme would overweight the galaxies that are fully sampled and underweight those that are undersampled. Galaxies with different luminosities are known to cluster differently (Park et al. 1994, 2005a; Zehavi et al. 2005; Guo et al. 2013), and therefore they should be differently weighted if the sampling varies with luminosity—to avoid the clustering mismatch. The problem becomes more serious when galaxy luminosity or mass is used as the weight to obtain the galaxy luminosity density or the mass density field, respectively. In particular, when the sampling in luminosity or mass varies with redshift, the resulting luminosity or mass density will have different mean values across different redshift bins, even if the galaxy number density is matched. Therefore, one should also consider the radial density gradient.

For the construction of our galaxy mass and number density fields from the observed LRG sample, we devised a new weighting scheme (called “LF matching”) that properly accounts for the sampling-rate variations depending on the location of the galaxy both in redshift and in absolute magnitude space, which are caused by the LRG target selection procedure. In what follows, we consider the case in which there is no evolution of the LF with redshift and briefly summarize our procedure.

1. Select a reference redshift bin and compute the reference LF in this bin, denoted $\Phi_{ef}(M_p)$. The LF determined in this (arbitrary) redshift interval will be used to match the LF in other redshift bins, as shown in Figure 4. For our study, we choose 0.16 < z < 0.20 as the reference bin; $\Phi_{ef}(M_p)$ computed in this interval is indicated by the solid line in Figure 4.

2. Select bins in the two-dimensional plane defined by redshift versus absolute magnitude and compute the LF for
Figure 2. Three-dimensional view of the galaxy number density field from the SDSS LRG volume-limited sample, smoothed with a Gaussian filter at $R_G = 22 \, h^{-1} \text{Mpc}$ scale. Left: three representative density contours enclosing low-density regions, which occupy 2.3\% ($\nu_f = -2.0$), 6.7\% ($\nu_f = -1.5$), and 50\% ($\nu_f = 0.0$) of the sample volume, from top to bottom. Right: three density contours enclosing high-density regions, filling 2.3\% ($\nu_f = 2.0$), 6.7\% ($\nu_f = 1.5$), and 50\% ($\nu_f = 0.0$) of the sample volume from top to bottom, in symmetry with the low-density cases. Earth is located at the center of the $x$–$y$ plane shown in the figure, and the size of each axis corresponds to a scale of 200 $h^{-1}$ Mpc.

3. For each pixel of $\Lambda(z)$ above an absolute magnitude cut $M_{g,\text{cut}}$.

4. Construct the galaxy number density field, weighting each galaxy by $w(M_z, z)$, which is linearly interpolated from the array $(M_z, z)$ computed as described in the previous steps. The number density field obtained in this way will be uniform in both redshift and luminosity space (Figure 5, thick blue histogram), as opposed to one constructed by weighting each galaxy with the radial selection function alone (dotted histogram). In particular, the density field is calculated on a mesh with cubic pixels from a discrete particle distribution using the cloud-in-cell mass assignment scheme.

5. Alternatively, construct the mass-weighted halo density field from the observed galaxy sample. The galaxy mass, $M_{\text{gal}}$, should be the halo mass $M_h$ corresponding to the $g$-band galaxy luminosity $M_g$, i.e., $M_h = f(M_g)$ (see point 3 in Section 4.2 for more details). Using the LRG cumulative LF measured at the reference redshift bin and the halo cumulative mass function derived from a full cubic data snapshot of HR3 at $z = 0.2$ (which is compatible with the reference redshift), we apply the halo–galaxy one-to-one correspondence model (HGC) of Kim et al. (2008) and convert galaxy luminosities into halo masses, and vice versa.

Figure 6 shows the relation between galaxy luminosity and halo mass used to determine $M_{\text{gal}}$. The halo mass corresponding to the absolute magnitude cut, $M_{g,\text{cut}}$, is $M_{h,\text{cut}} = 10^{13.466} \, h^{-1} \, M_{\odot}$. To compute the galaxy mass
4. SIMULATED LRG SAMPLES: DESCRIPTION AND METHODOLOGY

In this section, we briefly describe the HR3 $N$-body simulation and the procedure to construct the SDSS DR7 mock LRG samples from the simulation output. We then compare numerical results and measurements from data in Section 5. The mock surveys will also be used to quantify several nonlinearities due to systematics that affect the genus curve; correcting for these effects allows one to accurately recover the topology of the underlying matter, as shown in Section 6.
4.1. The Horizon Run 3 N-body Simulation

HR3 (Kim et al. 2011) is one of the largest N-body simulations to date, made using 7210^3 = 374 billion particles, spanning a volume of (10.81h^-1\,Gpc)^3—over 8800 times the volume of the Millennium Run (Springel et al. 2005). The particle mass goes down to 2.44×10^{11}\,h^{-1}\,M_\odot, allowing resolution of galaxy-sized halos with a mean particle separation of 1.5\,h^{-1}\,Mpc. The simulation is based on a ΛCDM cosmology, with parameters fixed by the WMAP 5 yr data (Komatsu et al. 2009). The linear power spectrum used is obtained with the CAMB source, which provides a better measurement of the BAO scale. The simulation starts at which provides a better measurement of the BAO scale. The code for the run is an improved version of the Gadget code (“Grid-of-Oct-Trees-Particle-Mesh”) originally devised by Dubinski et al. (2004); a new procedure has been implemented in order to more accurately describe the particle positions using single precision.

In HR3, halos are first identified by means of a standard friends-of-friends (FOF) procedure. Then subhalos are found from of FOF halos with a subhalo-finding technique developed by Kim & Park (2006) and Kim et al. (2008). This method allows one to identify physically self-bound dark matter subhalos not tidally disrupted by larger structures at the desired epoch. In particular, LRGs are identified as the most massive dark matter subhalos. To make the comparison with observational data, we saved the particle positions and velocities along the past LC for 27 separated observers and found subhalos in the past LC surface from \( z = 0 \) to \( z = 0.7 \).

From each simulated LC, we created three mock samples using exactly the same survey mask and angular selection function as for the SDSS sample. In addition, we applied the same smoothing length as for the observational case. In total, we are able to obtain 81 nonoverlapping mock samples, thanks to the enormous volume of HR3. To this end, we note that the ability to simulate large volumes is essential (particularly for the LRG distribution), since larger volumes allow one to accurately model the true power at large scales and the corresponding power spectrum. A large box size will guarantee small statistical errors in power spectrum estimates, so that the scale of the first acoustic peak can be measured with an accuracy better than 1% and the genus curve characterized with unprecedented statistical significance.

4.2. Construction of the Mock LRG Samples

A crucial step in our analysis is the construction of realistic mock LRG samples. This requires the ability to mimic all the observational biases, such as survey boundary, radial and angular selection functions, and RSDs. To build the various simulated catalogs, 27 observers were placed in the HR3 box, each covering the redshift range \( 0 < z < 0.7 \) without overlaps; this means that the survey volumes are totally independent. In addition, the LRG mock samples are made so that they span exactly the same range in absolute magnitude as the observational sample; hence, the number of galaxies in each mock sample is nearly equal to the observed one (within 1% level accuracy). Moreover, the simulated galaxies should be observed in redshift space along the past LC with the same radial and angular selection functions as the observational sample and also with the same selection function in absolute magnitude space as in the real observation.

Figure 7 shows a three-dimensional example of the simulated LRG number density field obtained from HR3, smoothed with a Gaussian filter at \( R_G = 22\,h^{-1}\,\text{Mpc} \) scale. This plot is the equivalent of Figure 2, but now for the LRG mock samples constructed from the HR3 simulation. Again, the left panels display three representative density contours enclosing low-density regions, which respectively occupy 2.3% (\( \nu_l = -2.0; \) top), 6.7% (\( \nu_l = -1.5; \) middle), and 50.0% (\( \nu_l = 0.0; \) bottom) of the sample volume. The same thresholds, but now with positive signs and thus for high-density regions (i.e., \( \nu_l = 2.0; \) top; \( \nu_l = 1.5, \) middle; \( \nu_l = 0.0, \) bottom), are shown in the right panels.

Since we apply identical techniques to both the SDSS LRG sample and the mock surveys, we expect the results of the analysis to be identical across data sets—within statistical variations—if the simulations are correctly modeling the distribution of galaxies. In what follows, we describe in more detail how to build the SDSS DR7 mock LRG samples from the HR3 simulation output. The results from our procedure confirm that we are correctly modeling the LRG distribution (see Figures 8 and 9 below). The major steps of the construction process are summarized as follows.

1. Locate 27 observers in the HR3 simulation box and save all dark halos along the past LC of each observer during the simulation, in the redshift range \( 0 < z < 0.7 \). From each set of LC data, create three mock surveys using exactly the same survey mask and angular selection function as for the SDSS volume-limited sample.

2. Apply a proper correction to make the halo mass function uniform in redshift. In the HR3 simulation, the minimum mass of subhalos that can have LRGs with constant observed number density varies as a function of redshift (see Figure 6 of Kim et al. 2011). This leads to the relation

\[
M(z) = (8.743 \times 10^{12} h^{-1} M_\odot + 1.711 \times 10^{13} h^{-1} M_\odot) f(z)
\]

The correction one needs to apply to the halo mass at an arbitrary redshift \( z \) is then given by the ratio

\[
\frac{f(z)}{f(z_{ref})}
\]

where \( f(z_{ref}) = 1.55 \times 10^{13} h^{-1} M_\odot \) is the minimum halo mass at a median redshift of \( z_{ref} = 0.18 \) in the reference redshift bin (recall the procedure described in Section 3.2, and the chosen reference redshift interval).

3. Populate dark matter halos with galaxies using a suitable correspondence scheme. In essence, to connect galaxies with halos, one needs to make an assumption regarding the relation between galaxy luminosity and halo mass. A widely used approach is subhalo abundance matching, in which higher luminosity galaxies are assigned to higher mass halos (Kravtsov et al. 2004; Tatsitsiomi et al. 2004; Vale & Ostriker 2004; Kim et al. 2008; Conroy & Wechsler 2009; Guo et al. 2010; Behroozi et al. 2010). This scheme assumes that halos with masses above a certain threshold and a given mean number density correspond to galaxies with luminosity or mass above a certain threshold and having the same mean halo number density. For our mock samples, we apply the HGC model of Kim et al. (2008), which extends the subhalo abundance matching procedure: there is one and only one galaxy in each subhalo, and a more massive subhalo hosts a more luminous galaxy. The mapping \( M_h = f(M_h) \) is shown in Figure 6. This correspondence scheme allows us to assign a luminosity to each LRG mock galaxy and to compute galaxy masses (see also the end of Section 3.2).

4. Account for the effects of the color-dependent luminosity cut imposed by the SDSS LRG volume-limited sample.

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See http://camb.info/sources.
selection criteria, in both redshift and luminosity space, which reduces the sampling density (see again Section 3.2). To do this, we discard mock galaxies with a rejection probability given by $1/w$, where $w \equiv w(M_g, z)$ is derived from the observed sample (see Equation (14)).

Our procedure successfully reproduces the dependence of the LRG sampling rate on luminosity and redshift in the SDSS LRG sample. This is shown in Figures 8 and 9, the counterparts of Figures 4 and 5, respectively, obtained from simulated samples. In particular, Figure 8 displays the LF computed at different redshift bins from one of our LRG mock samples. To facilitate comparison with the actual SDSS data, open circles in the figure are LF measurements derived from the SDSS LRG sample, as in Figure 4 (see Section 3.2). Clearly, the simulated results agree very well with the observational measurements in terms of matching the LF. To this end, Figure 9 shows the comoving number density of SDSS mock LRGs, averaged over all 81 mock samples, as a function of redshift (dashed line). The solid black line shows the radial distribution obtained by weighting each galaxy with $w(M_g, z)$. Thin colored lines show the results from six arbitrary mock surveys. Filled and open circles in the figure are analogous measurements derived from the SDSS LRG sample, as in Figure 5. The weighting scheme is explained in Section 3.2. Even in this case, the plot confirms the correctness of our modeling procedure: our mock samples have the same sampling rate in redshift as the observed SDSS LRG sample.

5. GENUS TOPOLOGY OF LRGs: SDSS VERSUS MOCK MEASUREMENTS

In this section, we present results for the genus measured from the SDSS DR7 LRG sample and from our LRG mock samples obtained from the HR3 ΛCDM simulation. In both cases, we compute the genus curves using the mass-weighted density field and the number density field, although further on we will only use the number density. By contrasting observational results against mock measurements that assume Gaussian initial conditions, we detect significant non-Gaussian deviations of the observed genus curve from theoretical expectations. We then quantify these discrepancies by introducing a new statistical
Figure 8. LF computed at different redshift bins from one of our LRG mock samples, as indicated in the key. Open circles represent LF measurements derived from the SDSS LRG sample in the reference redshift bin $0.16 < z < 0.20$, as in Figure 4. Clearly, the simulated results agree very well with the observational measurements.

Figure 9. Comoving number density of mock LRGs averaged over all the mock samples as a function of redshift (dashed line). The solid black line shows the radial distribution constructed by weighting each galaxy by $w(M_g, z)$. Thin colored lines show the results from six mock surveys. For comparison, the observational results are also plotted (circles). (A color version of this figure is available in the online journal.)

5.1. Genus of SDSS LRGs from the Mass-weighted Density Field and the Number Density Field

In Sections 3.2 and 4.2, we constructed the mass-weighted density field and the number density field in order to compute the genus. This is because Jee et al. (2012) found that the halo mass density has a much tighter (and simpler) relation with the underlying matter density than does the halo number density. A similar conclusion was reached by Park et al. (2010) in relation to the gravitational shear field. To illustrate, Figure 10 shows the genus curves derived from the mass density and number density fields as a function of the volume fraction $\nu_f$.

No corrections for systematics are applied yet. We smooth our density field (either the mass-weighted or the number density one) with a Gaussian filter of radius $R_G = 22 \ h^{-1} \ Mpc$ at a pixel size $p = 3.7 \ h^{-1} \ Mpc$. To minimize any nonlinearity introduced by the choice of pixel dimension, we use the smallest possible pixel size we can afford. In Figure 10, thick red lines are used for the number density field, and thin black lines are for the mass-weighted density field.

In particular, the top panel shows our measurements averaged from the 81 mock LRG samples, where we also plot the genus curve of the dark matter distribution in real space using the full simulation cube (dotted line). Differences between the halo and
The connection between galaxies and their dark matter halos is needed, as well as of the galaxy formation process in general. For simplicity, we adopt the number density instead of the mass-weighted density hereafter.

Overall, the HGC galaxy assignment scheme of Kim et al. (2008) is able to match the observed amplitudes and shapes of the corresponding genus curves well. In fact, from Table 1, one may note that the values obtained for $\Delta V_f$, $A_V$, and $A_C$ from the simulated LRG samples agree well, within the quoted uncertainties, with those measured from the observational sample. However, we detect some significant discrepancies between mock samples and observations for $A_V$ in the lower density regions beyond the integration intervals quoted in Section 2.3 (i.e., $\nu_f \leq -1$; note the difference between the red and gray lines in Figure 1). We also detect some discrepancies for the observed and predicted genus amplitudes.

To quantitatively estimate the statistical significance of these discrepancies, we use the following method. First we calculate the differences between the genus curve of each mock sample and the curve obtained by averaging all 81 mock samples; we do this at three different intervals, $-3.0 \leq \nu_f \leq -1.7$, $-0.2 \leq \nu_f \leq 0.3$, and $1.2 \leq \nu_f \leq 2.2$. We then plot the integrals of these differences as histograms (Figure 11). Finally, we place in the same plot our measurements obtained from the SDSS sample at the corresponding threshold intervals (triangles). Here we measured the genus curves from the number density fields of the samples.

As can be inferred from Figure 11 (with a significance level of 90%), departures from Gaussianity are seen near the mean-density regions and in low-density regions (i.e., in the interval $-3.0 \leq \nu_f \leq -1.7$). The radical difference between the genus curves of the observational and simulated data in low-density regions shows that topology is highly sensitive to the connectivity of voids. In the next section, we address the key role played by systematics in the genus curve, which explains some of the discrepancies, and show how to accurately correct for their effects to recover the topology of the underlying matter. In a forthcoming paper, we will provide an interpretation of the remaining deviations (i.e., after correcting for known systematics) in the context of primordial non-Gaussianity.

6. GENUS TOPOLOGY OF LRGs: SYSTEMATICS

In this section, we briefly discuss the known systematics that affect the genus curve. We then test and quantify their impact on the genus using the genus-related statistics presented in Section 2.3 with the help of mock LRG samples (Section 4.2) and show how to correct for their effects. By applying these corrections, we obtain the most accurate constraint on the genus amplitude to date, which significantly improves on our previous measurements. In particular, Figures 14 and 15 below are among the most important results of the paper.

6.1. Impact of Systematics on the Genus Curve

As anticipated in Section 2.3, even if the initial conditions are perfectly Gaussian, small deviations from Gaussianity would be expected because of systematics. Since systematics directly impact the shape and amplitude of the genus curve, it is imperative to be able to quantify and correct for their effects. This can now be done quite accurately, with the help of realistic mock catalogs such as those constructed from HR3 (Section 4.2).

Broadly speaking, systematics that cause non-Gaussian deviations in the genus curve can be divided into three main classes:
Clearly, several of the previously mentioned effects are connected, and so one needs to remove them simultaneously. In the absence of other known systematics, any eventual residual non-Gaussianity (after applying the corrections mentioned above) has to be ascribed to a primordial origin. In what follows, we discuss in particular the nonlinear gravitational evolution and the effects of galaxy bias and past LC on the genus. More details on the full modeling of systematics in topology measurements will be presented by Y.-R. Kim et al. (2013, in preparation).

6.2. Modeling and Correcting for Systematics

We perform corrections similar to those applied by Choi et al. (2010), who studied the effects of systematics on the genus computed from the nearby main galaxy sample of SDSS DR7. The overall goal is to remove the nonlinear systematics in the observed sample step by step, as well as to estimate the genus curve of the underlying matter density field using a set of mock samples.

We first consider the effect of nonlinear gravitational evolution on the genus curve, measured from our mock LRG samples, along with the effects of the survey mask and initial conditions. For this purpose, we compute the genus curves of the 81 dark matter density fields both at the initial ($z_i = 27$) and final ($z_f = 0$) redshifts; these quantities, measured in real space, are indicated by $G_{m,i}$ and $G_{m,f}$, respectively, where the index $j$ refers to the particular LC mock survey considered, $m$ stands for matter, and $r$ stands for real space. Each density field is selected within the SDSS survey mask, at a particular region in the simulation so that the evolved density field has a one-to-one correspondence with the initial density field.

Figure 12 (a) shows the effect of gravitational evolution on the genus curve in real space. The solid black line is the genus curve averaged over all 81 simulated initial matter density fields (at $z_i = 27$), while the dashed blue line is the corresponding final one, at $z_f = 0$, computed in the same way. The dotted green line is the predicted linear-theory genus relative to the entire survey volume. The SDSS survey mask is applied. The lower panel in Figure 12(a) displays the ratio $\Delta = [G_{m,i} - G_{m,f}]/G_{m,f}$ for matter, and $r$ stands for real space. Each density field is selected within the SDSS survey mask, at a particular region in the simulation so that the evolved density field has a one-to-one correspondence with the initial density field.

Figure 12 (b) instead shows the genus curve of the matter distribution in real space at $z = 0$, using the full cubic data. The solid black line is used for the full simulation cube, while the dashed blue line is obtained with the SDSS LRG survey mask applied. The ratio $\Delta_{G}$ gives information only about nonlinear gravitational evolution. We have attempted to fit the discrepancy with the second-order perturbation theory prediction of Matsubara (2003). The dotted red line in the lower panel shows the result of this fit. Line colors and styles are the same as in the upper panel. The skewness parameters from the nonlinear gravitational evolution, $S_{gr}$, are calculated by integrating the bispectrum of the matter density distribution $B_{gr}$ given in terms of the second-order correction to the density fluctuations from nonlinear gravitational clustering in the weakly non-Gaussian regime (see the equations in Section 4.2 of Matsubara 2003): $S_{gr}^{(0)} = 3.422$, $S_{gr}^{(1)} = 3.472$.

Figure 11. Statistical test to quantitatively estimate non-Gaussian discrepancies between the predicted and observed genus curves for the SDSS sample. Histograms are integrals of the differences between the genus curve of each individual mock sample and the curve obtained by averaging all 81 mock LRG samples. Three different intervals in $\nu_i$ are considered, as indicated. Triangles show measurements obtained from the SDSS sample at the corresponding threshold intervals. Discrepancies are seen near the mean-density regions and in low-density regions ($-3.0 \leq \nu_i \leq -1.7$), with a significance level of 90%. (A color version of this figure is available in the online journal.)
and $S^{(2)} = 3.695$ for the 5 yr WMAP cosmology assumed. The difference between the dotted red curve and the solid black line shows the discrepancy between the second-order perturbation theory prediction and the N-body simulation measurements.

Hence, we find that perturbation theory considerably disagrees with the numerical simulation result at the smoothing scale of $R = 22 h^{-1}$ Mpc. The gravitational evolution produces a negative shift and decreases the genus amplitude by $\sim 3\%$.

The top part of Table 2 lists the genus-related statistics of the samples used in Figure 12 to quantify the deviations of the genus curves from the Gaussian expectation due to the mentioned systematic effects.

We next consider the effect of galaxy (halo) biasing on the genus, along with shot noise, past LC gradient, and RSDs. To this end, we created 81 mock samples from the snapshot halo full cubic data at $z = 0$ in exactly the same way as for the past LC of the LRG mock samples. We then compared the genus curve averaged over all the halo density fields, $G_{m,\Delta}(z_f)$, where here $h$ stands for halo, with the one obtained by averaging the 81 matter density fields at the same redshift, $G_{m,\Delta}(z_f)$. The results are displayed in Figure 13(a), with the solid black line for the genus obtained from the averaged matter density field and the dashed blue line for the one obtained from an average of the halo density field in real space. Similarly to Figure 12, the middle panel of Figure 13(a) displays the quantity $\Delta$ defined above, which shows that the dark matter density field and the halo number density have very different topology at the $22 h^{-1}$ Mpc smoothing scale. Note however that the genus curve here includes shot noise due to discrete sampling of the galaxy density field, as well as the galaxy biasing effect. Their combined effect has been presented by Hikage et al. (2001, 2003) and Park et al. (2005b). As can be seen from the scatter between the two curves, the combined effect of galaxy biasing and shot noise yields significantly larger non-Gaussianities than the nonlinear gravitational evolution effect. The bottom panel shows the combined effect. In particular, the combined effect increases the genus amplitude and strongly alters the skewness of the genus curve (see the value of $G_{m,\Delta}(z_f)$ in Table 2; shifted toward meatball topology, more percolated and thus larger void structures, and a larger number of isolated clusters compared with those from the matter density fields, $G_{m,\Delta}(z_f)$).

Past LC effects and RSDs are quantified in Figure 13 (b). This is achieved by comparing the genus curve measured from our 81

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**Figure 12.** Impact of systematics on the genus curve: cosmic variance, SDSS survey mask, initial conditions of the simulations, and gravitational evolution. A Gaussian smoothing length of $R = 22 h^{-1}$ Mpc is applied. (a) The solid black line shows the genus curve averaged over all 81 initial matter density fields at $z = 27$; in real space. The dashed blue line is the corresponding final one, at $z = 0$, computed similarly. The SDSS survey mask is applied. The dotted green line shows the genus curve averaged over all 81 past light cone mock galaxy density fields in real space at the initial epoch.

(b) Genus curve of the matter distribution in real space at $z = 0$, computed similarly. The SDSS survey mask is applied. The dotted green curve is the predicted linear-theory genus relative to the entire survey volume. The lower panel displays the ratio $\Delta = [G(v_f) - G_{\text{LIN}}(v_f)]/G_{\text{LIN}}(v_f = 0)$ as a function of the volume threshold $v_f$ and for the two different redshifts considered. (b) Genus curve of the matter distribution in real space at $z = 0$, using the full cubic data. The solid black line is used for the full simulation cube, while the dashed blue line is obtained with the SDSS LRG survey mask applied. The dotted red line is the second-order perturbation theory prediction of Matsubara (2003). The difference between the red and black lines shows the discrepancy between the second-order perturbation theory prediction and N-body simulations.

**Table 2**

| Genus-related Statistics for the Samples Used in Figures 12 and 13 |
|-----------------|--------|-----|-----|-----|
| $G_{m,\Delta}(z_f)^a$ | 285.1 | $-0.009$ | 0.97 | 1.05 |
| $G_{m,\Delta}(z_f)^b$ | 288.0 | $-0.007$ | 0.95 | 1.05 |
| $G_{m,\Delta}(z_f)^c$ | 297.1 | 0.003 | 0.99 | 1.10 |
| $G_{m,\Delta}(z_f)^d$ | 310.1 | $-0.044$ | 0.80 | 1.12 |
| $G_{m,\Delta}(z_f)^e$ | 307.0 | $-0.057$ | 0.79 | 1.13 |
| $G_{m,\Delta}(z_f)^f$ | 299.0 | $-0.058$ | 0.79 | 1.14 |

**Notes.**

a: Genus value of the $z = 0$ snapshot matter distribution in real space using the full cubic data.

b: Genus value averaged over all 81 dark matter density fields in real space at the initial epoch $z = 27$.

c: Genus value averaged over all 81 dark matter density fields in real space at the final epoch $z = 0$.

d: Genus value averaged over all 81 halo density fields in real space at the final epoch.

e: Genus value averaged over all 81 past light cone mock galaxy density fields in real space.

f: Genus value averaged over all 81 past light cone mock galaxy density fields in redshift space.
The Astrophysical Journal Supplement Series, 209:19 (20pp), 2013 December

**Figure 13.** Impact of systematics on the genus curve: galaxy (halo) biasing, shot noise, past LC gradient, and RSDs. A Gaussian smoothing length of the halo density field. Similarly to Figure 12, the middle panel displays the number density have different topology. In the bottom panel, $G_{\text{sys}}$ reaches 2% (solid black line in the bottom panel), which is nearly as significant as the effect of gravitational evolution (see the solid line in the middle panel of Figure 13(a)), while RSDs decrease the amplitude of the genus curve but do not alter its shape significantly.

With the help of our mock samples, we are able to understand and quantify the nonlinear systematics involved in the observational sample. In particular, we find that a considerable portion of the nonlinearity comes from the combined effects of galaxy biasing and shot noise. We can remove the nonlinearities due to those systematics and thus estimate the genus curve of the underlying matter density field from the observed sample. The correction term that should apply to the observed data, derived from the $j$th mock sample, is given by

$$\Delta G_{\text{sys}} = G_{h,j}^{\text{LC}} - G_{\text{m},i}^{\text{z}_f}.$$  \hspace{1cm} (15)

This correction removes all the systematic effects previously mentioned, including RSDs, survey mask, shot noise, and galaxy biasing, from the observed genus curve $G_{\text{obs}}$; clearly, the underlying assumption is that our HGC assignment scheme is able to correctly model the relation between galaxies in our sample and the underlying halos.

The genus of the observed (underlying) matter density field at $z = 0$ in real space, reconstructed by applying the correction term, is calculated as follows:

$$G_{\text{o},m,i} = G_{\text{o}} - \Delta G_{\text{pix}} - \sum_{j=1}^{N} (\Delta G_{\text{sys}}^{j} - \Delta G_{\text{sim}}^{j}) / N,$$  \hspace{1cm} (16)

where now $N$ is the number of mock surveys. We also included the correction for finite pixel size, $\Delta G_{\text{pix}}$, and one for the bias of the $j$th mock sample due to the cosmic variation in the initial conditions of the HR3 simulation, $\Delta G_{\text{sim}}^{j} = G_{\text{m},i}^{\text{z}_f} - G_{\text{LIN}}$. The correction for the effect of finite pixel size is given by

$$\Delta G_{\text{pix}} = A_{\text{exp}} \left[ \frac{v_f^2}{2} \right] \times \left[ a H_0 + b H_1(v_f) + c H_2(v_f) + d H_3(v_f) \right] p^2 / R_G^2,$$  \hspace{1cm} (17)

where $A_{\text{exp}}$ is the genus amplitude calculated from the variance and derivative (i.e., $\sigma_0$, $\sigma_1$) of the matter density field on a mesh with vanishing pixel effect (where $p \approx 0$) and the coefficients of each Hermite polynomial, $a$, $b$, $c$, and $d$, are 0.04794, 0.02337, 0.33146, and 0.03843, respectively. For the density field smoothed with a Gaussian filter $R_G = 22 h^{-1}$ Mpc and pixel size $p = 3.7 h^{-1}$ Mpc, this effect can be as large as about 1% and should be taken into account. Full details of the modeling of the pixel effect will be presented by Y.-R. Kim et al. (2013, in preparation).

### 6.3. Genus Curve after Corrections for Systematics

Finally, we are able to obtain the genus curve of the underlying matter density field from the SDSS LRG sample with the effects of shot noise, galaxy bias, RSDs, survey boundary, and finite pixel size all corrected. Our final result contains only the nonlinearity produced by nonlinear gravitational evolution and a possible primordial non-Gaussian component (if any). Figure 14 shows the observed genus curve before the corrections for past LC mock samples in real space, $G_{h,r}^{\text{LC}}$, constructed as explained in Section 4.2, with the averaged one obtained from the halo density fields, $G_{h,r}^{\text{z}_f}$. Similarly, we can quantify the effect of RSDs by computing the difference between $G_{h,r}^{\text{LC}}$ and the average genus curve measured from the 81 past LC mock samples, but now in redshift space, $G_{h,r}^{\text{LC}}$, (see also Table 2). All these curves are displayed in Figure 13(b). Again, the lower panels clearly describe these effects: basically, the systematic effect introduced by the past LC on the genus curve reaches 2% (solid black line in the bottom panel), which is nearly as significant as the effect of gravitational evolution (see the solid line in the middle panel of Figure 13(a)), while RSDs decrease the amplitude of the genus curve but do not alter its shape significantly.
 systemsatics (open circles) and after applying those corrections (thick gray line with 1 σ error bars). In the same figure, we also display the genus curve averaged over all the mock surveys after applying the same systematics correction (thin black line) and the linear-theory prediction (dashed green line). Overall, the shape of the SDSS LRG genus curve agrees very well with the mean topology of the HR3 mock surveys in a ΛCDM universe.

However, a comparison with the simulations also shows small deviations of the observed genus curve from the theoretical expectation for Gaussian initial conditions. Figure 15 quantifies these deviations, showing the difference between the genus curves for the SDSS (thick red line)—after the correction for systematic effects—and the simulated dark matter distribution at z = 0 (solid black line), as well as the linear analytical predictions $G_{\text{LIN}}$ (normalized by the maximum value of $G_{\text{LIN}}$). The shaded area indicates the 1 σ limits calculated from the 81 HR3 halo mock surveys with the same correction applied. The solid gray line shows the average deviation from the mock surveys. The dashed line shows the second-order perturbation expectation at the median redshift $z = 0.28668$ of the SDSS sample given by Matsubara (2003); clearly, the underlying matter distribution at the present epoch needs additional terms when compared with the second-order perturbation expectation.

Second-order perturbation formula:

$$G^{\text{NG}}(\nu_t) = -G(0)\exp\left(-\nu_t^2/2\right) \times \left[ \sigma_0 \left[ (S_{gr}^{(1)} - S_{gr}^{(0)}) H_3(\nu_t) + (S_{gr}^{(2)} - S_{gr}^{(0)}) H_1(\nu_t) \right] \right.$$  

$$+ \sigma_r^2 \left[ A_0 H_0(\nu_t) + A_2 H_2(\nu_t) \right] \right],$$  

(18)

where $G(0) = 298.5$. Second-order perturbation theory predicts the power spectrum $P(k, z)$ in Equation (6) as follows: $P(k, z) = P_{\text{LIN}}(k, z) W^2(kR) + O(\sigma_r^4)$, where $P_{\text{LIN}}(k, z)$ is the linear power spectrum and $\sigma_0$ (up to the lowest order) is 0.15381 at the median redshift $z = 0.28668$ of the SDSS sample. The additional coefficients $A_0$ and $A_2$ are $-0.2226$ and $1.4702$, respectively. Table 3 lists the genus-related statistics for all the genus measurements relative to Figure 15.

Finally, in Figure 16 we present the genus-related statistics for the previous genus curves; this is helpful in order to understand systematic biases. The statistics for the random-phase perturbation expectation at the median redshift $z = 0.28668$ of the SDSS sample given by Matsubara (2003) depending on $H_0$ and $H_2$. To this end, the dotted line is a fit to the simulation using the following new perturbation theory cannot model gravitational evolution effects properly. On the contrary, those effects are modeled well if we add extra terms in the second-order perturbation formula of Matsubara (2003) depending on $H_0$ and $H_2$. To this end, the dotted line is a fit to the simulation using the following new perturbation theory:

$$G^{\text{NG}}(\nu_t) = -G(0)\exp\left(-\nu_t^2/2\right) \times \left[ \sigma_0 \left[ (S_{gr}^{(1)} - S_{gr}^{(0)}) H_3(\nu_t) + (S_{gr}^{(2)} - S_{gr}^{(0)}) H_1(\nu_t) \right] \right.$$  

$$+ \sigma_r^2 \left[ A_0 H_0(\nu_t) + A_2 H_2(\nu_t) \right] \right],$$  

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where $G(0) = 298.5$. Second-order perturbation theory predicts the power spectrum $P(k, z)$ in Equation (6) as follows: $P(k, z) = P_{\text{LIN}}(k, z) W^2(kR) + O(\sigma_r^4)$, where $P_{\text{LIN}}(k, z)$ is the linear power spectrum and $\sigma_0$ (up to the lowest order) is 0.15381 at the median redshift $z = 0.28668$ of the SDSS sample. The additional coefficients $A_0$ and $A_2$ are $-0.2226$ and $1.4702$, respectively. Table 3 lists the genus-related statistics for all the genus measurements relative to Figure 15.

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$$+ \sigma_r^2 \left[ A_0 H_0(\nu_t) + A_2 H_2(\nu_t) \right] \right],$$  

(18)

where $G(0) = 298.5$. Second-order perturbation theory predicts the power spectrum $P(k, z)$ in Equation (6) as follows: $P(k, z) = P_{\text{LIN}}(k, z) W^2(kR) + O(\sigma_r^4)$, where $P_{\text{LIN}}(k, z)$ is the linear power spectrum and $\sigma_0$ (up to the lowest order) is 0.15381 at the median redshift $z = 0.28668$ of the SDSS sample. The additional coefficients $A_0$ and $A_2$ are $-0.2226$ and $1.4702$, respectively. Table 3 lists the genus-related statistics for all the genus measurements relative to Figure 15.

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$$+ \sigma_r^2 \left[ A_0 H_0(\nu_t) + A_2 H_2(\nu_t) \right] \right],$$  

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where $G(0) = 298.5$. Second-order perturbation theory predicts the power spectrum $P(k, z)$ in Equation (6) as follows: $P(k, z) = P_{\text{LIN}}(k, z) W^2(kR) + O(\sigma_r^4)$, where $P_{\text{LIN}}(k, z)$ is the linear power spectrum and $\sigma_0$ (up to the lowest order) is 0.15381 at the median redshift $z = 0.28668$ of the SDSS sample. The additional coefficients $A_0$ and $A_2$ are $-0.2226$ and $1.4702$, respectively. Table 3 lists the genus-related statistics for all the genus measurements relative to Figure 15.
Figure 16. Genus-related statistics for the genus curves in Figure 14. Thick plus signs are the statistics for the random-phase fluctuations. Squares and circles are the distributions of the LRG density and derived matter density, respectively. Triangles are statistics from the real-space genus curve of the dark matter particle distribution at the initial epoch, which includes only the contribution of primordial non-Gaussianity. Filled and open symbols represent the observed and simulated cases, respectively.

Table 3
Genus-related Statistics for the Genus Curves in Figure 15

| Genus       | G_{\text{fit}} | \Delta \nu | A_V | A_C |
|-------------|----------------|------------|-----|-----|
| Observation |                |            |     |     |
| G_{\text{fit}} | 285.2          | 0.047      | 0.79| 1.22|
| G_{\text{fit,}\nu | 271.7          | 0.007      | 0.96| 1.13|
| G_{0,\nu | 280.8          | 0.017      | 1.00| 1.08|
| Simulation  |                |            |     |     |
| G_M         | 299.0 \pm 11.5 | -0.058 \pm 0.018 | 0.79 \pm 0.06 | 1.14 \pm 0.06 |
| G_{\text{fit,}\nu} | 285.5          | -0.007    | 0.95| 1.05|
| G_{\text{fit,}\nu} | 294.5          | 0.003     | 0.99| 1.00|

Notes. G_s is the genus value of the SDSS LRG sample, and G_M is that averaged over all 81 light-cone LRG mock samples, \(G_{\text{fit,}\nu}\). G_{\text{fit,}\nu} and G_{\text{fit,}\nu} are genus values of the underlying dark matter distributions derived from G_{\text{fit,}\nu} and G_{\text{fit,}\nu}, respectively, after corrections for systematics. G_{\text{fit,}\nu} and G_{\text{fit,}\nu} are real-space genus values of the dark matter distribution at the initial epoch of the simulation, including only the contribution of primordial non-Gaussianity, for both the observation and simulations.

The difference between circles and triangles indicates the effect of nonlinear gravitational evolution. While the statistics of the initial matter density field of the ΛCDM N-body simulation with primordial Gaussianity (open triangles) are nearly the same as those of the random-phase curve (as expected), the statistics of the initial matter density field obtained from the LRG sample (filled triangles) still show deviations from the Gaussian expectation; \(A_V\) and \(A_C\) are within about 1 \(\sigma\) of the Gaussian values, and the amplitude and \(A_C\) show a relatively large difference between the observation and the Gaussian prediction.

7. CONCLUSIONS

We have presented measurements of the genus topology of LRGs from the SDSS DR7 catalog with unprecedented statistical significance. We created a volume-limited sample in the redshift range \(0.16 < z < 0.36\) with rest-frame g-band absolute magnitudes of \(-23.2 < M_g < -21.1\) using the DR7-Full sample of Kazin et al. (2010) and then imposed additional cuts as in Choi et al. (2010), leaving a total of 60,466 LRGs over about 2.33 sr. We constructed the galaxy mass and number density fields from the observed LRG sample using a novel technique—“LF matching”—outlined in Section 3 and computed the observed genus curve. We also produced 81 independent mock LRG samples from the HR3 simulation (Kim et al. 2011), one of the largest N-body simulations currently available, which evolved 7210^3 particles in a 10,815 h^{-1} Mpc box. The construction of simulated LRG catalogs required several subtle steps, explained in Section 4. In particular, we adopted the halo–galaxy one-to-one monotonic correspondence model of Kim et al. (2008) to populate dark matter halos with galaxies and identified LRGs as the most massive subhalos.

Thanks to the unprecedented volume of HR3, we were able to carefully model and study all known systematics that affect the genus curve, such as finite pixel size, survey boundary mask, radial and angular selection functions, past LC gradient, initial conditions of the simulations, shot noise, cosmic variance, RSDs, and galaxy biasing. Upon removal of all known systematics, our final genus curve (Figure 14, Section 6.3) contains only the nonlinearity produced by nonlinear gravitational evolution and a possible primordial non-Gaussian component (if any). In particular, we find that the observed genus amplitude reaches 285 with an uncertainty of 4.0%, including cosmic variance (before correction for systematics); this is the most accurate constraint on the genus amplitude to date and significantly improves on the previous measurement by Gott et al. (2009).

Overall, the shape of the observed genus curve agrees very well with the mean topology of the SDSS LRG mock surveys in a ΛCDM universe, and this should be considered a success of our large-volume N-body simulation, as well as of our procedure for constructing mock LRG samples from HR3 (see Figure 1).

However, a comparison with simulations also shows small but significant deviations of the observed genus curve from the theoretical expectation for Gaussian initial conditions: Figures 15 and 16 show these deviations explicitly. We used genus-derived statistics (Sections 2.3, and 5.2, and Tables 1–3) to estimate and quantify departures from Gaussianity of the genus curve. While a consistent part of the non-Gaussian deviations is caused by systematics, and mainly driven by shot noise and biasing, removing these effects on the genus curve still leaves some discrepancies from the Gaussian expectations. This can be attributed to the nonlinearity produced by gravitational evolution, in addition to a possible primordial non-Gaussian component. We investigated here the role of nonlinear gravitational evolution on the genus curve, while in a forthcoming publication we will provide an interpretation of the remaining deviations in the context of primordial non-Gaussianity. In particular, in this study we found that the second-order perturbation theory prediction of Matsubara (2003) disagrees significantly with the genus curve measured from the gravitationally evolved matter density field.
in the HR3 simulation (Figure 15, solid line). On the contrary, the nonlinear gravitational evolution effects are modeled well if we add extra terms in the second-order perturbation formula of Matsubara; to this end, we also provided a new second-order perturbation formula (Equation (18)) that better fits our results.

In summary, the main achievements of this paper are as follows.

1. We measured the genus amplitude from the SDSS LRG volume-limited sample and found it to reach 285 with an uncertainty of 4.0% including cosmic variance; this is the most accurate constraint on the genus amplitude to date and significantly improves on the results of Gott et al. (2009).

2. The overall shape of the observed genus curve agrees very well with the mean topology of the SDSS LRG mock surveys in a ΛCDM universe, confirming the correctness of our large-volume N-body simulation and procedure to construct mock LRG samples. This should also be seen as strong support of the ΛCDM paradigm, similar to that recently presented by Park et al. (2012), who were able to show that observed high- and low-density LSSs have richness/volume and size distributions consistent with a ΛCDM universe.

3. Thanks to our unprecedentedly large volume simulation (HR3), we gained excellent control of the numerous systematics that affect the genus curve, ranging from observational to statistical or cosmological effects that introduce non-Gaussianities in the genus shape. We were able to successfully model and remove all known systematics.

4. We proposed a new method to construct the galaxy mass and number density fields from the observed LRG sample (the “LF matching” technique) and an accurate procedure to construct LRG mock samples.

5. We demonstrated that second-order perturbation theory (Hikage et al. 2002; Matsubara 2003) cannot model the genus curve measured from the gravitationally evolved matter density field in the HR3 simulation and provided a new fitting formula that adds extra terms to the original second-order perturbation expression and better matches our results.

After removing all known systematics and modeling the nonlinear gravitational evolution more accurately, we are still left with some additional non-Gaussian signal. Clearly, since we were able for the first time to isolate and quantify a non-Gaussian contribution directly from the observed genus curve that is not due to systematics, and argued that even an improved second-order perturbation theory cannot explain all the non-Gaussian discrepancies, our next step is to interpret those deviations in the context of a local $f_{NL}$-type model (Salopek & Bond 1990; Gangui et al. 1994; Verde et al. 2000; Komatsu & Spergel 2001). This will allow us to constrain the standard non-Gaussianity parameter $f_{NL}$ directly from topological measurements of the LSS.

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APPENDIX A

TABLES OF GENUS CURVES

In this appendix, we provide tables of the genus curves for the reader who wishes to use galaxy clustering topology. Table 4...
contains the mean genus values measured from number density fields and mass-weighted density fields of the 81 past LC mock samples, and the genus values of the SDSS LRGs, as a function of both volume fraction threshold level ($v_f$) and direct density threshold level ($v$), as measured in Sections 5.1 and Appendix B (see also the corresponding genus curves, plotted in Figures 10 and 17). We additionally provide similar quantities for the dark matter particle distribution in the ΛCDM model at the current epoch, $G_m$, for comparison. The observed genus values ($G_o$ and $G_{o,m}$) before and after the correction for systematic effects as a function of volume fraction threshold levels, plotted in Figure 14, are listed in Table 5. $G_M$ and $G_{M,m}$ are the mean genus values before and after the applied corrections, measured from the 81 LC mock LRG samples in the same way as done for the SDSS sample. Electronic versions of these tables are available from the authors upon request.
Table 5
Genus Values at a Given Threshold Level for the Samples Used in Figures 14 and 16

| $v_t$ | $G_M$ | $G_{M,10}^v = 0$ | $G_{M,10}^v$ | $G_{M,10}^v$ |
|-------|-------|-----------------|--------------|--------------|
| −2.5  | −37.6 | −41.2           | −45.8        | −61.7 ± 8.4  |
| −2.4  | −44.2 | −48.3           | −52.9        | −71.1 ± 9.1  |
| −2.3  | −66.6 | −70.1           | −74.3        | −80.1 ± 8.6  |
| −2.2  | −71.5 | −76.4           | −82.2        | −88.7 ± 9.6  |
| −2.1  | −87.9 | −94.3           | −99.6        | −98.4 ± 10.0 |
| −2.0  | −91.9 | −98.7           | −105.5       | −104.6 ± 9.7 |
| −1.9  | −97.5 | −107.1          | −115.0       | −108.1 ± 9.8 |
| −1.8  | −101.4| −115.0          | −123.6       | −108.9 ± 10.0|
| −1.7  | −97.4 | −112.7          | −123.8       | −107.1 ± 10.7|
| −1.6  | −99.7 | −119.9          | −130.7       | −100.9 ± 10.2|
| −1.5  | −98.7 | −116.3          | −124.1       | −91.1 ± 10.2 |
| −1.4  | −82.9 | −100.5          | −108.2       | −77.3 ± 10.8 |
| −1.3  | −61.7 | −86.4           | −97.8        | −55.7 ± 12.1 |
| −1.2  | −38.5 | −59.9           | −63.1        | −31.8 ± 12.1 |
| −1.1  | −25.4 | −55.7           | −59.6        | −2.3 ± 11.9  |
| −1.0  | 17.8  | −12.6           | −14.0        | 30.4 ± 14.0 |
| −0.9  | 49.3  | 15.1            | 14.9         | 68.9 ± 13.7 |
| −0.8  | 93.6  | 61.9            | 57.0         | 106.5 ± 13.8 |
| −0.7  | 125.3 | 100.8           | 103.2        | 143.6 ± 13.7 |
| −0.6  | 152.6 | 124.3           | 126.7        | 183.5 ± 15.0 |
| −0.5  | 212.1 | 194.9           | 205.4        | 215.2 ± 14.4 |
| −0.4  | 250.9 | 222.9           | 225.7        | 247.2 ± 14.5 |
| −0.3  | 275.8 | 257.4           | 271.7        | 271.5 ± 18.1 |
| −0.2  | 265.8 | 249.1           | 254.2        | 287.2 ± 16.8 |
| −0.1  | 280.9 | 265.5           | 271.0        | 297.5 ± 16.3 |
| 0.0   | 281.1 | 268.0           | 275.5        | 298.2 ± 17.1 |
| 0.1   | 264.5 | 250.8           | 250.0        | 289.9 ± 16.1 |
| 0.2   | 250.0 | 244.4           | 248.9        | 271.6 ± 16.0 |
| 0.3   | 225.9 | 225.3           | 234.7        | 248.1 ± 16.2 |
| 0.4   | 211.1 | 214.1           | 223.4        | 218.0 ± 16.3 |
| 0.5   | 193.0 | 197.8           | 212.0        | 183.0 ± 14.4 |
| 0.6   | 148.6 | 159.2           | 169.5        | 140.5 ± 15.2 |
| 0.7   | 100.9 | 107.7           | 111.8        | 98.0 ± 14.7 |
| 0.8   | 45.9  | 59.8            | 64.1         | 55.8 ± 14.1 |
| 0.9   | 10.3  | 30.1            | 41.8         | 14.5 ± 13.9 |
| 1.0   | −24.5 | −7.9            | −2.9         | −23.0 ± 12.5 |
| 1.1   | −56.6 | −39.9           | −33.7        | −56.9 ± 12.7 |
| 1.2   | −89.6 | −71.9           | −69.5        | −86.6 ± 11.8 |
| 1.3   | −113.5| −92.7           | −90.7        | −112.3 ± 12.2|
| 1.4   | −138.8| −116.6          | −114.0       | −129.7 ± 12.5|
| 1.5   | −152.8| −133.3          | −133.4       | −142.4 ± 11.0|
| 1.6   | −147.9| −130.4          | −127.8       | −148.2 ± 11.8|
| 1.7   | −166.3| −152.2          | −152.2       | −148.9 ± 10.1|
| 1.8   | −146.0| −130.4          | −128.5       | −146.3 ± 10.1|
| 1.9   | −130.8| −115.2          | −113.0       | −140.4 ± 10.1|
| 2.0   | −132.4| −123.1          | −122.7       | −131.1 ± 10.7|
| 2.1   | −120.9| −108.8          | −108.5       | −122.8 ± 10.5|
| 2.2   | −110.9| −101.4          | −102.9       | −109.9 ± 9.8 |
| 2.3   | −94.5 | −85.9           | −86.6        | −97.6 ± 9.3  |
| 2.4   | −84.8 | −78.4           | −78.4        | −85.2 ± 9.1  |
| 2.5   | −76.1 | −71.1           | −70.6        | −72.0 ± 8.0  |

Notes. $G_0$ is the genus value of the SDSS LRG sample, and $G_M$ is that averaged over all 81 light-cone LRG mock samples, $G_{M,10}^v$ is the genus value of the underlying dark matter distributions derived from $G_0$ and $G_M$, respectively, after corrections for systematics. $G_{M,10}^v$ are real-space genus values of the dark matter distribution at the initial epoch of the simulation, including only the contribution of primordial non-Gaussianity, for both the observation and simulations. $G_{M,10}^v$ is the genus value for the matter distribution in real space at $z = 0$, using the full cubic data.

APPENDIX B

THRESHOLD DENSITY VERSUS VOLUME FRACTION

Figure 17 shows the SDSS and mock genus curves, plotted versus the direct density threshold $v_t$. These curves deviate from the Gaussian predictions more than those obtained using the volume fraction $v_t$ shown in Figure 1. Table 4 from Appendix A lists the genus values as a function of $v_t$. By inspection of these values, we conclude that the direct density threshold parameterization is more sensitive to skewness in the density.
probability distribution. However, given the vanishing genus and its large dispersion at threshold levels below $\nu \approx -1.7$ (see Figure 18), it is not appropriate to use this genus curve to inspect the non-Gaussian deviation of the observational sample from perturbation theory at the smoothing scale adopted.

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