AN ACCRETION DISK-OUTFLOW MODEL FOR HYSTERETIC STATE TRANSITION IN X-RAY BINARIES

XINWU CAO$^{1,2}$

$^1$SHAO-XMU Joint Center for Astrophysics, Shanghai Astronomical Observatory, Chinese Academy of Sciences, 80 Nandan Road, Shanghai, 200030, China; cxw@shao.ac.cn

$^2$Key Laboratory of Radio Astronomy, Chinese Academy of Sciences, 21008 Nanjing, China

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ABSTRACT

We suggest a model of the advection-dominated accretion flow (ADAF) with magnetically driven outflows to explain the hysteretic state transition observed in X-ray binaries (XRBs). The transition from a thin disk to an ADAF occurs when the mass accretion rate is below a critical value. The critical mass accretion rate for the ADAF can be estimated by equating the equilibration timescale to the accretion timescale of the ADAF, which is sensitive to its radial velocity. The radial velocity of thin disks is very small, which leads to the advection of the external field in thin disks becoming very inefficient. ADAFs are present in the low/hard states of XRBs, and their radial velocity is large compared with the thin disk. The external field can be dragged inward efficiently by the ADAF, so a strong large-scale magnetic field threading the ADAF can be formed, which may accelerate a fraction of gas in the ADAF into the outflows. Such outflows may carry away a large amount of angular momentum from the ADAF, which significantly increases the radial velocity of the ADAF. This leads to a high critical mass accretion rate, below which an ADAF with magnetic outflows can survive. Our calculations show that the critical luminosity of the ADAF with magnetic outflows can be one order of magnitude higher than that for a conventional ADAF, if the ratio of gas to magnetic pressure $\beta \sim 4$ in the disk. This can naturally explain the hysteretic state transition observed in XRBs.

Key words: accretion, accretion disks – black hole physics – X-rays: binaries

1. INTRODUCTION

Variability of X-ray binaries (XRBs) is closely related to accretion disks, which exhibits transitions between a thermal state, a substantial thermal component in the continuum spectra, and a low/hard state dominated by non-thermal emission. It is believed that the states of XRBs correspond to different accretion modes. Hot accretion flows are present in a low/hard state, while the black holes are surrounded by geometrically thin accretion disks in the thermal state of XRBs (e.g., Esin et al. 1997; Meyer et al. 2000; Zdziarski et al. 2004; Wu & Gu 2008; Belloni 2010; Cao et al. 2014; Dong & Wu 2015).

The accretion mode transition is mainly triggered by the dimensionless mass accretion rate $\dot{m} (\dot{m} = \dot{M}/M_{\text{crit}})$, which depends sensitively on the viscosity parameter $\alpha$ (see Narayan et al. 1998; Yuan & Narayan 2014, for reviews). The critical mass accretion rate $\dot{m}_{\text{crit}}$ can be estimated by equating the ion–electron equilibration timescale to the accretion timescale of an advection-dominated accretion flow (ADAF; Narayan et al. 1998). Although the detailed physics of the accretion mode transition is still unclear, it may be regulated by the processes in the accretion disk coronal systems, i.e., the evaporation of the disk and/or condensation of the hot gas in the corona (Liu et al. 1999; Meyer-Hofmeister & Meyer 1999).

However, the hysteretic state transition in XRBs, i.e., the transition of the low/hard state to thermal state occurs at a luminosity much higher than that for the transition from the thermal state to low/hard state (e.g., Miyamoto et al. 1995; Zhang et al. 1997; Nowak et al. 2002; Maccarone & Coppi 2003; Zdziarski et al. 2004; Yu et al. 2004; Yu & Yan 2009), is still not well understood in the frame of the above mentioned scenarios (see, e.g., Zhang 2013, for a review). In most of the previous models, the accretion mode transition is solely triggered by the dimensionless mass accretion rate (see, e.g., Narayan et al. 1998; Yuan & Narayan 2014, for reviews). In order to solve this problem, several different scenarios have been suggested (e.g., Meyer-Hofmeister et al. 2005; Balbus & Henri 2008; Petrucci et al. 2008; Begelman & Armitage 2014; Nixon & Salvesen 2014; Begelman et al. 2015). In the frame of the evaporation model, it was assumed that there are different amounts of Compton cooling or heating acting on the disk in different accretion modes, which leads to the hysteretic state transition in XRBs (Liu et al. 2005; Meyer-Hofmeister et al. 2005). Alternatively, the hysteretic state transition is assumed to be related to magnetic processes in the disks (e.g., Balbus & Henri 2008; Petrucci et al. 2008; Begelman & Armitage 2014; Li & Yan 2015). Balbus & Henri (2008) suggested that there are two different regions with $P_{m} > 1$ or $P_{m} < 1$ ($P_{m}$ is the magnetic Prandtl number) in the accretion disk. The transition radius of these two regions is regulated by the dimensionless accretion rate $\dot{m}$, and the XRB state transition is related to an unstable interface between these two regions in the disk. The critical dimensionless mass accretion rate for the accretion mode transition depends sensitively on the radial velocity of the accretion flow, which is a function of $\alpha$. Motivated by numerical simulations, the hysteretic state transition can be explained if the value of $\alpha$ is assumed to vary in different types of accretion disks, i.e., $\alpha \sim 1$ in ADAFs, while $\alpha \sim 0.01 - 0.1$ in standard thin disks (Begelman & Armitage 2014).

Magnetic fields play an important role in some of these scenarios, though the origin the fields is still unclear. One possibility is that the external generated large-scale poloidal field is dragged inward by the plasma in the disk, while the field diffuses outward simultaneously (Bisnovatyi-Kogan & Ruzmaikin 1974, 1976; van Ballegooijen 1989; Lubow et al. 1994; Ogilvie & Livio 2001). However, the advection of the field in a geometrically thin ($H/R \ll 1$) is inefficient due
to its small radial velocity. The magnetic diffusion timescale is about the same as the viscous timescale in a steady disk, which leads to a very weak radial field component at the surface of a thin disk (Lubow et al. 1994), though some specific mechanisms were suggested to alleviate the difficulty of field advection in thin disks (Spruit & Uzdensky 2005; Lovelace et al. 2009; Guilet & Ogilvie 2012, 2013; Cao & Spruit 2013). The vertical field can be dragged efficiently by an ADAF (Cao 2011), because the radial velocity of ADAFs is much larger than that of a thin accretion disk (Narayan & Yi 1994, 1995).

In this work, we suggest that outflows are driven by the magnetic field advected by the ADAF, which carry a large amount of angular momentum away from the ADAF. The radial velocity of the ADAF with magnetically driven outflows can be significantly higher than that for a conventional ADAF without magnetic outflows. This leads to a higher critical mass accretion rate of the transition from an ADAF to a thin disk. In Section 2, we estimate the critical accretion rate of the transition from an ADAF with magnetic outflows to a thin disk. The properties of such magnetically driven outflows are analyzed in Section 3. Section 4 contains the discussion of the model. The last section is a brief summary.

2. THE CRITICAL MASS ACCRETION RATE FOR ADAFS WITH MAGNETICALLY DRIVEN OUTFLOWS

There is a critical mass accretion rate $M_{\text{crit}}$, above which the hot ADAF is switched to an optically thick accretion disk (Esin et al. 1997). The hot ADAF is a two-temperature accretion flow, in which the ion temperature is significantly higher than the electron temperature. The energy of the ions is transported to electrons via Coulomb collisions (Stepney & Guilbert 1983). The critical mass accretion rate $M_{\text{crit}}$ can be estimated by equating an ion–electron equilibrium timescale to an accretion timescale of an ADAF (Narayan et al. 1998). The ion–electron equilibrium timescale

$$t_{\text{ie}} \sim \frac{u}{q_{\text{ie}}},$$

where the internal energy of the gas is

$$u = \frac{3}{2} n_e k T_i + \frac{3}{2} n_e k T_e \simeq \frac{3}{2} n_e k T_i,$$

because the internal energy of the electrons is negligible in ADAFs. The Coulomb interaction between the electrons and ions is given by (Stepney & Guilbert 1983; Narayan & Yi 1995; Zdziarski 1998)

$$q_{\text{ie}} = \frac{3}{2} m_e n_e \sigma_T c = \frac{k T_i - k T_e}{k T_i} \frac{k T_i}{(1/\Theta_i) K_2(1/\Theta_i)} \ln \Lambda \left[ \frac{2(\Theta_i + \Theta_e)^2 + 1}{\Theta_i + \Theta_e} K_1 \left( \frac{\Theta_i + \Theta_e}{\Theta_i \Theta_e} \right) + 2 K_0 \left( \frac{\Theta_i + \Theta_e}{\Theta_i \Theta_e} \right) \right],$$

where

$$\Theta_i = k T_i / m_e c^2, \quad \Theta_e = k T_e / m_e c^2,$$

and $\ln \Lambda = 20$ is adopted. We rewrite Equation (1) as

$$t_{\text{ie}} \sim \frac{u}{q_{\text{ie}}} \sim \frac{f(\Theta_i, \Theta_e)}{n_e \sigma_T c},$$

where

$$f(\Theta_i, \Theta_e) = \frac{m_p}{m_e} \frac{T_i}{T_e - T_e} K_2(1/\Theta_i) K_2(1/\Theta_e) \ln \Lambda \times \left[ \frac{2(\Theta_i + \Theta_e)^2 + 1}{\Theta_i + \Theta_e} K_1 \left( \frac{\Theta_i + \Theta_e}{\Theta_i \Theta_e} \right) + 2 K_0 \left( \frac{\Theta_i + \Theta_e}{\Theta_i \Theta_e} \right) \right]^{-1}.$$ 

The accretion timescale is

$$t_{\text{acc}} \sim \frac{R}{|v_R|} = \frac{R^2}{\alpha \Omega_k H^2},$$

in which the radial velocity of the ADAF,

$$v_R \simeq - \alpha c_s H \frac{R}{\Omega_k H^2},$$

has been adopted.

The ADAF is suppressed when it is accreting at $M > M_{\text{crit}}$. The value of $M_{\text{crit}}$ can be estimated with $t_{\text{ie}} = t_{\text{acc}}$, i.e.,

$$f(\Theta_i, \Theta_e) = \frac{R^2}{\alpha \Omega_k H^2}.$$ 

With Equation (8), we can estimate the critical mass accretion rate as

$$M_{\text{crit}} = -2 \pi R (2 H \rho) v_R = \alpha^2 M_{\text{crit},0},$$

where

$$M_{\text{crit},0} = 4 \pi m_p \left( \frac{H}{R} \right)^2 \frac{H^2 \Omega_k^2}{\sigma_T c} f(\Theta_i, \Theta_e)$$

is the critical mass accretion rate corresponding to the case of $\alpha = 1$.

In a similar way, we consider an ADAF with magnetically driven outflows carrying most of the angular momentum away from the accretion flow. The structure of the disk will be altered in the presence of magnetic outflows (Li & Cao 2009; Cao & Spruit 2013; Li 2014; Li & Begelman 2014). The angular momentum equation for a steady ADAF with outflows is

$$\frac{d}{dR} (2 \pi R \Sigma_\nu R^2 \Omega) = \frac{d}{dR} \left[ 2 \pi, R, \nu, \Sigma, R^2, \frac{d \Omega}{d R} \right] - 2 \pi R T_m,$$

where $T_m$ is the magnetic torque due to the outflows (we use $\nu, \Sigma, R^2$ to describe the quantities of the ADAF with outflows hereafter). Integrating Equation (11), yields

$$2 \pi R \Sigma_\nu R^2 \Omega = 2 \pi R \nu \Sigma R^2 \frac{d \Omega}{d R} - 2 \pi \int R T_m(R) dR + C,$$

where the value of the integral constant $C$ can be determined with a boundary condition on the accreting object. The magnetic torque $T_m(R)$ can be calculated if the global outflow solution is derived, which is beyond the scope of this work. For simplicity, we assume $T_m \propto R^{-\xi},$ and Equation (12) becomes

$$v_R' \simeq \frac{\nu}{R} \frac{1}{2 - \xi} \frac{T_m}{R^2 \Sigma \Omega},$$

where

$$f(\Theta_i, \Theta_e) = \frac{m_p}{m_e} \frac{T_i}{T_e - T_e} K_2(1/\Theta_i) K_2(1/\Theta_e) \ln \Lambda \times \left[ \frac{2(\Theta_i + \Theta_e)^2 + 1}{\Theta_i + \Theta_e} K_1 \left( \frac{\Theta_i + \Theta_e}{\Theta_i \Theta_e} \right) + 2 K_0 \left( \frac{\Theta_i + \Theta_e}{\Theta_i \Theta_e} \right) \right]^{-1}.$$ 

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where the approximations \(d\Omega/dR \approx -\Omega/R\) is adopted, and \(C\) is negligible in the region not very close to the inner disk edge (Shakura & Sunyaev 1973; Narayan & Yi 1994). In the self-similar ADAF solution, the gas pressure \(p_g \propto R^{-5/2}\) (Narayan & Yi 1995). Assuming the magnetic pressure \(B^2 \propto p_g\), we have \(B \propto R^{-5/4}\), and \(T_m \propto RB^2 \propto R^{-3/2}\). Thus, we adopt \(\xi_T = 3/2\) in all of our calculations. In this work, we use a model parameter \(f_m\) to describe the relative importance of the angular momentum removal of the disk, so the main conclusions of this work will not be altered by the precise value of \(\xi_T\) adopted. In this case, the radial velocity of the ADAF is
\[
v_R' \simeq v_R + v_{R,m},
\]
where the first term is due to the conventional turbulence in the disk, and the second term is contributed by the outflows,
\[
v_{R,m} = \frac{-2T_m}{R^2 \Sigma \Omega}.
\]
We use a parameter \(f_m\) to describe the relative importance of the outflows on the radial velocity of the ADAF,
\[
v_R' = (1 + f_m)v_R = -(1 + f_m)\alpha \Omega_k \frac{H^2}{R}.
\]
The value of \(f_m\) is determined by the properties of the ADAF and the magnetically driven outflows, which will be discussed in Section 3. The accretion timescale for the ADAF with magnetically driven outflows is
\[
t_{\text{acc}}' = \frac{R}{|v_R'|} = \frac{R^2}{\alpha \Omega_k H^2 (1 + f_m)}.\]
In a similar way as the conventional ADAF case discussed above, the value of \(\dot{M}_\text{crit}'\) can be estimated with \(t_{\text{acc}}' = t_{\text{acc}}\). Almost all gravitational energy is carried by the ions in the accretion disk, and the ions are heated to high temperatures if the energy transfer from the ions to electrons is inefficient. The ion temperature is therefore nearly virialized in ADAFs (see, e.g., Yuan & Narayan 2014, for a review). We believe that it is also the case for the ADAF with outflows accreting at the critical rate, i.e., \(\Theta_i' \approx \Theta_i\), because the ion–electron equilibration timescale is comparable to the accretion timescale for the ADAF with outflows, which is the same as the conventional ADAF case. The ion–electron equilibration timescale \(t_{\text{ie}}'\) only depends on the local properties of the gas in the ADAF, which can be estimated by the same Equation (4) for the conventional ADAF. Thus, we have
\[
\frac{f(\Theta_i, \Theta_i')}{n_i' \sigma_T c} = \frac{R^2}{\alpha \Omega_k H^2 (1 + f_m)},
\]
and the critical mass accretion rate for the ADAF with magnetic outflows is estimated as
\[
\dot{M}_\text{crit}' = -2\pi R (2 H' \rho') v_R' = \alpha^2 (1 + f_m)^2 \frac{f(\Theta_i, \Theta_i')}{f(\Theta_i, \Theta_e)} \dot{M}_\text{crit,0},
\]
where Equations (16) and (18) have been used.

The gravitational energy dissipation rate from the unit area of the disk is
\[
Q_+ = \frac{1}{2} \nabla^2 \left( R \frac{d\Omega}{dR} \right)^2,
\]
where \(\nu = \alpha c_s H\). Substituting Equations (7) and (9) into Equation (20), we derive the energy dissipation rate of a conventional ADAF accreting at the critical rate as
\[
Q_+ = \frac{1}{4\pi} \left( R \frac{d\Omega}{dR} \right)^2 M_{\text{crit}} \equiv \frac{1}{4\pi} \left( R \frac{d\Omega}{dR} \right)^2 \alpha^2 M_{\text{crit,0}}.
\]
For the ADAF with magnetically driven outflows, we substitute Equations (16) and (19) into Equation (20), and the gravitational energy dissipation rate is available,
\[
Q_+ = \frac{1}{4\pi} \left( R \frac{d\Omega}{dR} \right)^2 \frac{M_{\text{crit}}'}{(1 + f_m)}
\]
\[
= \frac{1}{4\pi} \left( R \frac{d\Omega}{dR} \right)^2 \frac{f(\Theta_i, \Theta_i')}{f(\Theta_i, \Theta_e)} (1 + f_m) \alpha^2 M_{\text{crit,0}}.
\]
For an ADAF accreting at the critical rates, the luminosity is
\[
L_{\text{ADAF}} \sim \int 2\pi R Q_+ dR,
\]
since the advection is not important for the ADAFs accreting at a critical rate (e.g., Narayan et al. 1998; Yuan & Narayan 2014).

The local density of an ADAF with magnetic outflows accreting at the critical rate is the same as that of a conventional ADAF accreting at \((1 + f_m)^2 \frac{M}{\Sigma} \frac{f(\Theta_i, \Theta_i')}{f(\Theta_i, \Theta_e)}\) with a large viscosity parameter of \((1 + f_m)\alpha\), while the energy dissipation rate of the ADAF with magnetic outflows is only \(\sim (1 + f_m) \frac{f(\Theta_i, \Theta_i')}{f(\Theta_i, \Theta_e)}\) times this conventional ADAF. It means that the electron temperature of the ADAF with outflows is lower than that of this conventional ADAF, i.e., \(\Theta_e' < \Theta_e\), which leads to \(f(\Theta_i', \Theta_i') > f(\Theta_i, \Theta_e)\) \((\Theta_i' \sim \Theta_i)\). In this work, we conservatively adopt the lower limits of \(\dot{M}_{\text{crit}}'\) and \(Q_+\) for the ADAF with outflows in our discussion, i.e.,

\[
\dot{M}_{\text{crit}}' = -2\pi R (2 H') v_R' \simeq \alpha^2 (1 + f_m)^2 \dot{M}_{\text{crit,0}}.
\]
and
\[
Q_+ \simeq \frac{1}{4\pi} \left( R \frac{d\Omega}{dR} \right)^2 (1 + f_m) \alpha^2 M_{\text{crit,0}}.
\]
This means that \(L_{\text{ADAF}} \sim (1 + f_m) L_{\text{ADAF}}\), i.e., the ADAF with magnetic outflows is \(\sim f_m \) times more luminous than the conventional ADAF accreting at the critical rate. The critical mass accretion rate for the ADAF with magnetic outflows is \(\sim (1 + f_m)^2\) times that for a conventional ADAF. Only \(\sim (1 + f_m)\) of the kinetic energy is dissipated in the accretion flow. The luminosity of the ADAF with magnetic outflows is not proportional to its mass accretion rate because \(\sim f_m / (1 + f_m)\) of the kinetic energy of the disk is tapped to accelerate the outflows.
The cooling timescale of an ADAF is estimated by

\[ t_{\text{cool}} \sim \frac{2H_u}{F_{\text{rad}}}, \]  

(26)

where \( F_{\text{rad}} \) is the radiation flux from the unit surface area of the disk. The radiation flux \( F_{\text{rad}} \) is related to the gravitational energy rate \( Q_+ \) by

\[ F_{\text{rad}} = (1 - f_{\text{adv}}) Q_+, \]  

(27)

where \( f_{\text{adv}} \) is the fraction of the dissipated energy advected in the flow. Equation (26) can be rewritten as

\[ t_{\text{cool}} \sim \frac{2H_u}{(1 - f_{\text{adv}}) Q_+} \sim \frac{3}{\alpha \Omega_k f_{\text{f1}}^2 (1 - f_{\text{adv}})}, \]  

(28)

where \( c_s = H \Omega_k, f_{\text{f1}} = \Omega / \Omega_k, \) Equation (2), and the approximation

\[ Q_+ = \frac{1}{2} \nu \Sigma \left( \frac{R d \Omega}{d R} \right)^2 \sim \frac{1}{2} \alpha c_s H \Sigma \Omega^2 \]  

(29)

is used. We note that the cooling timescale of the flow only depends on the local properties of the ADAF, which is not explicitly relevant to the magnetic outflows. This means that Equation (28) is valid either for a conventional ADAF or an ADAF with magnetic outflows.

The advection of the external magnetic field is balanced by the magnetic diffusion in a steady accretion flow, and therefore the magnetic diffusion timescale is comparable to the accretion timescale of the flow, i.e., \( t_{\text{diff}} \sim t_{\text{acc}} \) (Lubow et al. 1994; Cao & Spruit 2013). For the magnetic field of an ADAF with magnetic outflows, its magnetic diffusion timescale is

\[ t'_{\text{diff}} \sim t'_{\text{acc}} \sim \frac{R}{|v_R|} = \frac{R^2}{\alpha \Omega_k H^2 (1 + f_m)}. \]  

(30)

The duration of the transition from an ADAF to a thin disk is at the order of the cooling timescale of the ADAF (see Equation (28)). After such an accretion mode transition, the strong magnetic field of the ADAF decays at the magnetic diffusion timescale (see Equation (30)). For ADAFs accreting at critical rates, \( H/R \sim 1, f_{\text{f1}} \lesssim 1, \) and \( f_{\text{adv}} \sim 0 \) (Yuan & Narayan 2014), the magnetic diffusion timescale,

\[ t'_{\text{diff,ADAF}} = f_{\text{f1}}^2 (1 - f_{\text{adv}}) \left( \frac{R}{H} \right)^2 \frac{t'_{\text{cool}}}{3 (1 + f_m)} \sim \frac{t'_{\text{cool}}}{3 (1 + f_m)}, \]  

(31)

is much shorter than the cooling timescale. The magnetic diffusion timescale of a thin/slim disk is

\[ t_{\text{diff,SD}} \sim \frac{R H_{\text{SD}} \kappa_0}{\eta} = \frac{R \kappa_0}{\alpha c_s P_m}, \]  

(32)

where \( H_{\text{SD}} \) is the half-thickness of the disk, \( \kappa_0 = B_0 B_{\text{gas}}/r \) is the inclination of the field line at the disk surface, \( \gamma \) is the magnetic diffusivity, and the magnetic Prandtl number is \( P_m = \eta / \nu = \eta / \alpha c_s H. \) Compared to the cooling timescale of the ADAF with outflows accreting at the critical rate, we have

\[ t_{\text{diff,SD}} \sim \frac{1}{3} \left( \frac{H_{\text{SD}}}{R} \right)^{-1} \kappa_0 P_m^{-1} t'_{\text{cool}}. \]  

(33)

The critical mass accretion rate can be close to the Eddington rate provided \( \alpha \sim 0.1 \) and \( f_m \sim 10 \) (see Equation (24)), and therefore the ADAF with outflows may transit to a slim disk. The relative half-thickness \( H_{\text{SD}}/R \sim 0.5 \) for a slim disk accreting at the Eddington rate (see, e.g., Li et al. 2010). For the typical values of the parameters, \( P_m \sim 1 \) and \( \kappa_0 \sim 1 \) (see Cao & Spruit 2013 for a detailed discussion, and references therein), we estimate the magnetic diffusion timescale of a slim disk accreting at the Eddington rate, \( t_{\text{diff,SD}} \sim t'_{\text{cool}} \) (see Equation (33)).

3. THE MAGNETICALLY DRIVEN OUTFLOWS

For an ADAF with magnetically driven outflows, the magnetic torque exerted by the outflows in unit area of the disk surface is

\[ T_m = \frac{B_0 B_z^2}{2\pi}, \]  

(34)

where \( B_0^z \) is the azimuthal component of the large-scale magnetic field at the disk surface. The radial velocity of the ADAF is

\[ v'_R = v_R + v_{R,m} = -\alpha c_s H \frac{R}{R} \frac{2T_m}{\varpi R \Omega} = -\alpha c_s H \frac{B_0 B_z^2}{\pi \varpi \Omega} = v_R \left( 1 + \frac{B_0 B_z^2}{\pi \varpi \Omega \alpha c_s H} \right), \]  

(35)

which can be rewritten as

\[ v'_R = v_R \left( 1 + \frac{\xi_\phi}{H \beta \Omega_{f1}} \right), \]  

(36)

where the dimensionless quantities are defined as \( \xi_\phi = -B_0^z / B_z, \) \( \beta = 8 \pi p_{\text{gas}} / B_z^2, \) and \( H = H/R. \) Comparison of Equations (16) and (36) leads to

\[ f_m = \frac{4\xi_\phi}{H \beta \Omega_{f1}}. \]  

(37)

The ratio \( \xi_\phi \lesssim 1 \) (see the detailed discussion in Livio et al. 1999). For ADAFs, \( H \sim 1 \) and \( f_{\text{f1}} \lesssim 1, \) so we can estimate the value of \( f_m \) as \( f_m \sim 4 \beta / \alpha. \) For a typical value of \( \alpha = 0.1, \) \( f_m \) can be as large as \( \sim 10 \) with magnetic field strength of \( \beta \sim 4. \)

4. DISCUSSION

It is known that the radial velocity of the disk \( v_R \propto (H/R)^2, \) and therefore \( v_R \) is very small for thin disks. The field advection in the thin accretion disk is quite inefficient, and therefore the strength field threading the disk is very weak (Lubow et al. 1994). When the mass accretion rate decreases to a rate below \( \sim \dot{M}_{\text{crit}}, \) the thin disk transits to an ADAF. The ADAF is hot, and its radial velocity is large compared to the thin disk. In the initial ADAF state, the magnetic field of the ADAF is very weak, which is similar to the thin disk. Due to the large radial velocity of the ADAF, the weak external field can be dragged inward efficiently to form a strong field at the order of accretion timescale (Cao 2011), which may drive a fraction of gas from the ADAF to form outflows. Such outflows may carry away a large amount of angular momentum from the ADAF, which increases the radial velocity of the ADAF significantly. The critical mass accretion rate for the ADAF with magnetic
outflows is estimated by equating the equilibration timescale to the accretion timescale of the ADAF (Narayan et al. 1998). The accretion timescale of the ADAF with magnetic outflows is much lower than that of a conventional ADAF due to its large radial velocity. This leads to a high critical mass accretion rate, below which an ADAF with magnetic outflows can survive (see Section 2 for the detailed calculations).

Due to the mass loss in the outflows, the mass accretion rate of the ADAF decreases with decreasing radius $R$. The mass-loss rate in the outflows is regulated by the magnetic field configuration/strength and the disk properties (e.g., the density and temperature of the gas in the ADAF). The present analysis on the critical accretion rate does not depend on the detailed properties of the outflows (e.g., the mass-loss rate in the outflows). In this work, we focus on the local disk properties, and the mass accretion rate measured at a certain radius is considered in our model. The properties of the outflow can be derived with the magnetic outflow solution if suitable boundary conditions are provided (e.g., Cao & Spruit 1994; Cao 2014), which is beyond the scope of this work.

In the low/hard state of the XRB, the ADAF with outflows will be suppressed when the mass accretion rate increases above $\dot{M}_\text{crit}^f$. The duration of the transition of an ADAF to a thin disk is at the order of the cooling timescale $t_\text{cool}^f$ of the ADAF (see Equation (28)). In the accretion mode transition, the magnetic field will decay at the magnetic diffusion timescale $t_\text{diff}$ (see Equation (30)). We find that $t_\text{diff}$ is much shorter than $t_\text{cool}^f$ of the ADAFs accreting at the critical rates (see Equation (31)), while the diffusion timescale of the thin disk transited from the ADAF with outflows is comparable with the cooling timescale $t_\text{cool}$. This means that the strong field of the ADAF diffuses away rapidly during its transition to a thin/slim disk, which implies that the field of the thin/slim disk is too weak to accelerate outflows, though some mechanisms have been suggested to solve strong field formation difficulty in thin disks (Spruit & Uzdensky 2005; Lovelace et al. 2009; Guilet & Ogilvie 2012, 2013; Cao & Spruit 2013).

Our calculations show that the critical luminosity of the ADAF with magnetic outflows can be one order of magnitude higher than that for a conventional ADAF, if the ratio of gas to magnetic pressure $\beta \approx 4$ in the disk for $\alpha = 0.1$ (see Section 3 for the detailed analyses). This is roughly consistent with the strength of the field advected in the inner region of a conventional ADAF (Cao 2011). The magnetic field advected in the ADAF could be stronger, if the angular momentum carried by the outflows is properly considered, which leads to higher radial velocity. This is also consistent with relativistic jets always being associated with the low/hard state in X-ray binaries, while jets are switched off in the thermal state (Fender et al. 1999, 2003, 2004; Corbel et al. 2001; Gallo et al. 2003; Fender & Belloni 2004) because jets are believed to be related to strong magnetic fields (Blandford & Znajek 1977; Blandford & Payne 1982). The viscosity parameter $\alpha = 0.1$ corresponds to $m_\text{crit} \sim 0.01$, which is roughly consistent with those derived from the transition from the thermal state to the low/hard state in some XRBs.

Our results can be compared to those of the MHD simulations on accretion disks (e.g., Igumenshchev et al. 2003; Beckwith et al. 2009; Penna et al. 2010; Tchekhovskoy et al. 2011; McKinney et al. 2012; Narayan et al. 2012). The magnetic pressure can dominate over the gas pressure in the inner region of the disk, and it becomes a magnetic arrested disk (Igumenshchev et al. 2003). For a given external field, the ADAF is magnetically arrested only if the accretion rate is lower than a certain rate (Cao 2011). In our model, the ADAF with outflows is accreting at the critical rate, and it may not be in magnetic arrested state. The field strength of the ADAF with outflows considered in this work is much lower than that of the magnetic arrested case. The gas to magnetic pressure ratio $\beta \geq 4$ required in our analysis should be reasonable. The MHD simulations of magnetized ADAFs show that a large fraction of gas goes into the outflows, and the radial velocity of such ADAFs can be more than one order of magnitude higher than that of a self-similar ADAF (see Narayan et al. 2012, for the details). In order to explain the observed hysteretic state transitions in XRBs, the radial velocity of an ADAF with outflows being up to $\sim 10$ times higher than that of a conventional ADAF is required in some extreme state transition cases. This is consistent with the numerical simulation in Narayan et al. (2012).

The radial velocity of a conventional ADAF is

$$v_R \simeq -c_t \alpha v_K,$$

where $v_K$ is the Keplerian velocity at $R$ and $c_t$ is a function of the disk parameters (Narayan & Yi 1995). Our calculation indicates that $c_t$ is in the range of $\sim 0.4 - 0.6$. If a typical $\alpha = 0.1$ is adopted, the radial velocity of the ADAF with magnetic outflows can be as high as about half of the free-fall velocity when $(1 + f_\text{m}) \sim 10$. The critical accretion rate will be close to the Eddington rate. The ADAF with such extreme properties needs to be verified by the global ADAF solution with magnetic outflows in the future.

5. SUMMARY

In the thermal state, the XRB contains a thin disk. The thin disk transits to an ADAF perhaps due to the evaporation of the disk (Liu et al. 1999; Meyer-Hofmeister & Meyer 1999), when the mass accretion rate declines below the critical rate. In the low/hard state, the external weak field of the gas from the companion star can be dragged inward efficiently in the ADAF, which may drive outflows and increase the radial velocity of the ADAF significantly. Thus, the ADAF with magnetic outflows can survive at a higher accretion rate than a conventional ADAF. With an increasing mass accretion rate, the transition from an ADAF to a thin disk occurs at a higher luminosity than that from the thin disk to an ADAF. The ADAF-outflow model suggested in this work can naturally explain the hysteretic state transition observed in XRBs.

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