Quark-Lepton Masses and the Neutrino Puzzle in the AGUT Model

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Abstract

We first discuss an approach to the fermion mass problem, according to which the whole of flavour mixing for quarks is determined by the mechanism responsible for generating the physical masses of the up and down quarks: the Lightest Flavour Mass Generation model. Then we consider fermion masses in the Anti-grand Unification Theory and, in particular, the neutrino mass and mixing problem.

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1 Introduction

I reviewed the general problem of the quark-lepton mass spectrum at the first Bled workshop on “What comes beyond the Standard Model” [1]. So, in this talk, I will mainly concentrate on two topics: the Lightest Flavour Mass Generation model and the Neutrino Mass and Mixing problem in the Anti-Grand Unification Theory (AGUT).

2 Lightest Flavour Mass Generation Model

A commonly accepted framework for discussing the flavour problem is based on the picture that, in the absence of flavour mixing, only the particles belonging to the third generation $t$, $b$ and $\tau$ have non-zero masses. All other masses and the mixing angles then appear as a result of the tree-level mixings of families, related to some underlying family symmetry breaking. Recently, a new mechanism of flavour mixing, which we call Lightest Family Mass Generation (LFMG), was proposed [2]. According to LFMG the whole of flavour mixing for quarks is basically determined by the mechanism responsible for generating the physical masses of the up and down quarks, $m_u$ and $m_d$ respectively. So, in the chiral symmetry limit, when $m_u$ and $m_d$ vanish, all the quark mixing angles vanish. Therefore, the masses (more precisely any of the diagonal elements of the quark and charged lepton mass matrices) of the second and third families are practically the same in the gauge (unrotated) and physical bases. The proposed flavour mixing mechanism, driven solely by the generation of the lightest family mass, could actually be realized in two generic ways.

The first basic alternative (I) is when the lightest family mass ($m_u$ or $m_d$) appears as a result of the complex flavour mixing of all three families. It “runs along the main diagonal” of the corresponding $3 \times 3$ mass matrix $M$, from the basic dominant element $M_{33}$ to the element $M_{22}$ (via a rotation in the 2-3 sub-block of $M$) and then to the primordially texture zero element $M_{11}$ (via a
rotation in the 1-2 sub-block). The direct flavour mixing of the first and third quark and lepton families is supposed to be absent or negligibly small in $M$.

The second alternative (II), on the contrary, presupposes direct flavour mixing of just the first and third families. There is no involvement of the second family in the mixing. In this case, the lightest mass appears in the primordially texture zero $M_{11}$ element “walking round the corner” (via a rotation in the 1-3 sub-block of the mass matrix $M$). Certainly, this second version of the LFMG mechanism cannot be used for both the up and the down quark families simultaneously, since mixing with the second family members is a basic part of the CKM quark mixing phenomenology (Cabibbo mixing, non-zero $V_{cb}$ element, CP violation). However, the alternative II could work for the up quark family provided that the down quarks follow the alternative I.

Here we will just consider the latter scenario.

2.1 Quark Sector

We propose that the mass matrix for the down quarks ($D = d, s, b$) is Hermitian with three texture zeros of the following alternative I form:

$$M_D = \begin{pmatrix} 0 & a_D & 0 \\ a_D^* & A_D & b_D \\ 0 & b_D^* & B_D \end{pmatrix}$$  \hspace{1cm} (1)

It is, of course, necessary to assume some hierarchy between the elements, which we take to be: $B_D \gg A_D \sim |b_D| \gg |a_D|$. The zero in the $(M_D)_{11}$ element corresponds to the commonly accepted conjecture that the lightest family masses appear as a direct result of flavour mixings. The zero in $(M_D)_{13}$ means that only minimal “nearest neighbour” interactions occur, giving a tridiagonal matrix structure.

Now our main hypothesis, that the second and third family diagonal mass matrix elements are practically the same in the gauge and physical quark-lepton bases, means that:

$$B_D = m_b + \delta_D \quad A_D = m_s + \delta_D'$$  \hspace{1cm} (2)

The components $\delta_D$ and $\delta_D'$ are supposed to be much less than the masses of the particles in the next lightest family, meaning:

$$|\delta_D| \ll m_s \quad |\delta_D'| \ll m_d$$  \hspace{1cm} (3)

Since the trace and determinant of the Hermitian matrix $M_D$ gives the sum and product of its eigenvalues, it follows that

$$\delta_D \simeq -m_d$$  \hspace{1cm} (4)

while $\delta_D'$ is vanishingly small and can be neglected in further considerations.
It may easily be shown that our hypothesis and related equations (2-3) are entirely equivalent to the condition that the diagonal elements \((A_D, B_D)\) of \(M_D\) are proportional to the modulus square of the off-diagonal elements \((a_D, b_D)\):

\[
\frac{A_D}{B_D} = \left| \frac{a_D}{b_D} \right|^2
\]  

(5)

Using the conservation of the trace, determinant and sum of principal minors of the Hermitian matrices \(M_D\) under unitary transformations, we are led to a complete determination of the moduli of all their elements, which can be expressed to high accuracy as follows:

\[
|M_D| = \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b - m_d \end{pmatrix}
\]  

(6)

Now the Hermitian mass matrix for the up quarks is taken to be of the following alternative II form:

\[
M_U = \begin{pmatrix} 0 & 0 & c_U \\ 0 & A_U & 0 \\ c_U^* & 0 & B_U \end{pmatrix}
\]  

(7)

The moduli of all the elements of \(M_U\) can also be readily determined in terms of the physical masses as follows:

\[
|M_U| = \begin{pmatrix} 0 & 0 & \sqrt{m_u m_t} \\ 0 & m_c & 0 \\ \sqrt{m_u m_t} & 0 & m_t - m_u \end{pmatrix}
\]  

(8)

The CKM quark mixing matrix elements can now be readily calculated by diagonalising the mass matrices \(M_D\) and \(M_U\). They are given by the following simple and compact formulae in terms of quark mass ratios:

\[
|V_{us}| = \sqrt{\frac{m_d}{m_s}} = 0.222 \pm 0.004 \quad |V_{us}|_{exp} = 0.221 \pm 0.003
\]  

(9)

\[
|V_{cb}| = \sqrt{\frac{m_d}{m_b}} = 0.038 \pm 0.004 \quad |V_{cb}|_{exp} = 0.039 \pm 0.003
\]  

(10)

\[
|V_{ub}| = \sqrt{\frac{m_u}{m_t}} = 0.0036 \pm 0.0006 \quad |V_{ub}|_{exp} = 0.0036 \pm 0.0006
\]  

(11)

As can be seen, they are in impressive agreement with the experimental values.

### 2.2 Lepton Sector

The MNS lepton mixing matrix is defined analogously to the CKM quark mixing matrix:

\[
U = U_{\nu} U_E^\dagger
\]  

(12)
Here $U_E$ and $U_\nu$ are the unitary matrices which diagonalise the charged lepton mass matrix $M_E$ and the effective neutrino mass matrix $M_\nu$ respectively. Assuming the charged lepton masses follow alternative I, like the down quarks, the LFMG model predicts the charged lepton mixing angles in the matrix $U_E$ to be:

$$\sin \theta_{e\mu} = \sqrt{\frac{m_e}{m_\mu}} \quad \sin \theta_{\mu\tau} = \sqrt{\frac{m_e}{m_\tau}} \quad \sin \theta_{e\tau} \simeq 0 \quad (13)$$

These small charged lepton mixing angles will not markedly effect atmospheric neutrino oscillations, which appear to require maximal mixing $\sin^2 2\theta_{atm} \simeq 1$. Similarly, in the case of the large mixing angle (LMA) MSW solution of the solar neutrino problem, they are essentially negligible. It follows then that the large neutrino mixings should mainly come from the $U_\nu$ matrix associated with the neutrino mass matrix.

According to the “see-saw” mechanism, the effective mass-matrix $M_\nu$ for physical neutrinos has the form

$$M_\nu = -M_N^T M_N^{-1} M_{NN} \quad (14)$$

where $M_N$ is their Dirac mass matrix, while $M_{NN}$ is the Majorana mass matrix of their right-handed components. Matsuda et al [3] have extended the alternative I LFMG texture to the Dirac $M_N$ and Majorana $M_{NN}$ matrices.

The eigenvalues of the neutrino Dirac mass matrix $M_N$ are taken to have a hierarchy similar to that for the charged leptons (and down quarks)

$$M_{N3} : M_{N2} : M_{N1} \simeq 1 : y^2 : y^4, \quad y \approx 0.1 \quad (15)$$

and the eigenvalues of the Majorana mass matrix $M_{NN}$ are taken to have a stronger hierarchy

$$M_{NN3} : M_{NN2} : M_{NN1} \simeq 1 : y^4 : y^6 \quad (16)$$

One then readily determines the general LFMG matrices $M_N$ and $M_{NN}$ to be of the type

$$M_N \simeq M_{N3} \begin{pmatrix} 0 & \alpha y^3 & 0 \\ \alpha y^3 & y^2 & \alpha y^2 \\ 0 & \alpha y^2 & 1 \end{pmatrix} \quad (17)$$

and

$$M_{NN} \simeq M_{NN3} \begin{pmatrix} 0 & \beta y^5 & 0 \\ \beta y^5 & y^4 & \beta y^3 \\ 0 & \beta y^3 & 1 \end{pmatrix}. \quad (18)$$

We further take an extra condition of the type

$$|\Delta - 1| \leq y^2 \quad (\Delta \equiv \alpha, \beta) \quad (19)$$
for both the order-one parameters $\alpha$ and $\beta$ contained in the matrices $M_N$ and $M_{NN}$, according to which they are supposed to be equal to unity with a few percent accuracy. Substitution in the seesaw formula \[(14)\] generates an effective physical neutrino mass matrix $M_\nu$ of the form:

$$M_\nu \simeq -\frac{M_{N3}^2}{M_{NN3}} \begin{pmatrix} 0 & y & 0 \\ y & 1 + (y - y^2)^2 & 1 - (y - y^2) \\ 0 & 1 - (y - y^2) & 1 \end{pmatrix}$$ \[(20)\]

The physical neutrino masses are then given by:

$$m_{\nu1} \simeq \frac{1}{2} - \frac{\sqrt{3}}{2} \frac{M_{N3}^2}{M_{NN3}} \cdot y,$$

$$m_{\nu2} \simeq \frac{1}{2} + \frac{\sqrt{3}}{2} \frac{M_{N3}^2}{M_{NN3}} \cdot y,$$

$$m_{\nu3} \simeq (2 - y) \frac{M_{N3}^2}{M_{NN3}}$$ \[(21)\]

The predicted values of the neutrino oscillation parameters are:

$$\sin^2 2\theta_{atm} \simeq 1, \quad \sin^2 2\theta_{sun} \simeq \frac{2}{3}, \quad U_{e3} \simeq \frac{1}{2\sqrt{2}y}, \quad \Delta m_{sun}^2 \simeq \frac{\sqrt{3}}{4} y^2 \frac{\Delta m_{atm}^2}{\Delta m_{atm}} \simeq \frac{\sqrt{3}}{4} y^2$$ \[(22)\]

in agreement with atmospheric and LMA-MSW solar neutrino oscillation data.

The proportionality condition \[(5)\] leads to the LFMG texture, is not so easy to generate from an underlying symmetry beyond the Standard Model. However Jon Chkareuli, Holger Nielsen and myself have recently shown \[(4)\] that it is possible to give a natural realisation of the LFMG texture in a local chiral $SU(3)$ family symmetry model.

### 3 Fermion Masses in the AGUT Model

The AGUT model is based on a non-simple extension of the Standard Model (SM) with three copies of the SM gauge group—one for each family—and, in the absence of right-handed neutrinos, one extra abelian factor: $G = SMG^3 \times U(1)_f$, where $SMG \equiv SU(3) \times SU(2) \times U(1)$. This AGUT gauge group is broken down by four Higgs fields $S$, $W$, $T$ and $\xi$ to the usual SM gauge group, identified as the diagonal subgroup of $SMG^3$. The Higgs field $S$ has a vacuum expectation value (VEV) taken to be unity in fundamental (Planck) mass units, while $W$, $T$ and $\xi$ have VEVs an order of magnitude smaller. So the pure SM is essentially valid, without supersymmetry, up to energies close to the Planck scale. The AGUT gauge group $SMG^3 \times U(1)_f$ only becomes effective near the Planck scale, where the $i$’th proto-family couples to just the $i$’th $SMG$ factor and $U(1)_f$. The $U(1)_f$ charges assigned to the quarks and leptons are determined, by anomaly cancellation constraints, to be zero for the first family and all left-handed fermions, and for the remaining right-handed states to be as follows:

$$Q_f(\tau_R) = Q_f(\mu_R) = Q_f(\tau_R) = 1 \quad Q_f(\mu_R) = Q_f(s_R) = Q_f(t_R) = -1$$ \[(23)\]
I refer to the review of the AGUT model by Holger and myself at the first Bled workshop \[5\] for more details.

The quarks and leptons are mass protected by the approximately conserved AGUT chiral gauge charges \[6\]. The quantum numbers of the Weinberg-Salam Higgs field \(\phi_{WS}\) are chosen so that the \(t\) quark mass is not suppressed, whereas the \(b\) quark and \(\tau\) lepton are suppressed. This is done by taking the four abelian charges, expressed as a charge vector \(\vec{Q} = (y_1/2, y_2/2, y_3/2, Q_f)\), for \(\phi_{WS}\) to be given by:

\[
\vec{Q}_{\phi_{WS}} = \vec{Q}_{cR} - \vec{Q}_{tL} = (0, 2/3, 0, 1) - (0, 0, 1/6, 0) = (0, 2/3, -1/6, 1)
\]  

(24)

We assume that, like the quark and lepton fields, the Higgs fields belong to singlet or fundamental representations of all the non-abelian groups. Then, by imposing the usual SM charge quantisation rule for each of the SMG factors, the non-abelian representations are determined from the weak hypercharge quantum numbers \(y_i\). The abelian quantum numbers of the other Higgs fields are chosen as follows:

\[
\vec{Q}_W = (0, -1/2, 1/2, -4/3) \quad \vec{Q}_T = (0, -1/6, 1/6, -2/3) \quad \vec{Q}_\xi = (1/6, -1/6, 0, 0) \quad \vec{Q}_S = (1/6, -1/6, 0, -1)
\]  

(25)

Since we have \(< S >= 1\) in Planck units, the Higgs field \(S\) does not suppress the fermion masses and the quantum numbers of the other Higgs fields \(W, T, \xi\) and \(\phi_{WS}\) given above are only determined modulo those of \(S\).

The effective SM Yukawa coupling matrices in this AGUT model can now be calculated in terms of the VEVs of the fields \(W, T, \xi\) and \(S\) in Planck units—up to “random” complex order unity factors multiplying all the matrix elements—for the quarks:

\[
Y_U \sim \begin{pmatrix}
W T^2 \xi^2 & WT^2 \xi & W^2 T \xi \\
WT^2 \xi^3 & W^2 T & WT \\
\xi^3 & 1 & WT
\end{pmatrix} \quad Y_D \sim \begin{pmatrix}
W (T^2 \xi^2 & WT^2 \xi & T^3 \xi \\
WT^2 \xi & W^2 T & T^3 \\
W^2 T^4 \xi & W^2 T^4 & WT
\end{pmatrix}
\]  

(27)

and the charged leptons:

\[
Y_E \sim \begin{pmatrix}
W T^2 \xi^2 & WT^2 \xi^3 & W T^4 \xi \\
WT^2 \xi^5 & W^2 T & WT^4 \xi^2 \\
W^5 T^4 \xi^3 & W^2 T^4 & WT
\end{pmatrix}
\]  

(28)

A good order of magnitude fit is then obtained \[5\] to the charged fermion masses with the following values for the Higgs field VEVs in Planck units:

\[
W = 0.179, \quad T = 0.071 \quad \xi = 0.099.
\]  

(29)

We now consider the neutrino mass matrix in the AGUT model.
4 Neutrino Mass and Mixing Problem

Without introducing new physics below the AGUT scale, the effective light neutrino mass matrix $M_\nu$ is generated by tree level diagrams involving the exchange of two Weinberg-Salam Higgs tadpoles and the appropriate combination of $W$, $T$, $\xi$ and $S$ Higgs tadpoles. In this way we obtain:

$$M_\nu \simeq \frac{(\phi_{WS})^2}{M_{Pl}^2} \begin{pmatrix} W^2 \xi^4 T^4 & W^2 \xi T^4 & W^2 \xi^3 T \\ W^2 \xi T^4 & W T^5 & W^2 T \\ W^2 \xi^3 T & W^2 T & W^2 T^2 \xi^2 \end{pmatrix}. \quad (30)$$

The off-diagonal element $(M_\nu)_{23} = (M_\nu)_{32}$ dominates the matrix, giving large $\nu_\tau - \nu_\mu$ mixing with the following two neutrino masses and mixing angle:

$$m_2 \sim m_3 \sim \frac{(\phi_{WS})^2}{M_{Planck}} W^2 T \sin^2 2\theta_{\mu\tau} \simeq 1 \quad (31)$$

Although the large mixing angle $\sin^2 2\theta_{\mu\tau}$ is suitable for atmospheric neutrino oscillations, there are two problems associated with the neutrino masses. Firstly the ratio of neutrino mass squared differences $\Delta m^2_{23}/\Delta m^2_{12} \sim 2T\xi^2 \sim 1.4 \times 10^{-3}$, whereas the small mixing angle (SMA) MSW solution to the solar neutrino problem requires $\Delta m^2_{23}/\Delta m^2_{12} \sim 10^{-2}$. Secondly the predicted overall absolute mass scale for the neutrinos $(\phi_{WS})^2/M_{Planck} \sim 3 \times 10^{-6}$ eV is far too small.

We conclude it is necessary to introduce a new mass scale into the AGUT model. Two ways have been suggested of obtaining realistic neutrino masses and mixings in the AGUT model:

1. By extending the AGUT Higgs spectrum to include a weak isotriplet Higgs field $\Delta$ with SM weak hypercharge $y/2 = -1$ and a VEV $(\Delta^0) \sim 1$ eV; also a new Higgs field $\psi$ giving large $\mu - \tau$ mixing in the charged lepton Yukawa coupling matrix $Y_E$ is required.

2. By including right-handed neutrinos and extending the AGUT gauge group to $G_{\text{extended}} = (SMG \times U(1)_{B-L})^3$; also two new Higgs fields $\phi_{B-L}$ and $\chi$ are introduced to provide a see-saw mass scale and structure to the Majorana right-handed neutrino mass matrix.

Yasutaka Takanishi reported on the second approach \cite{7} at the workshop; so I will report on the first approach \cite{8} here. We must therefore consider the introduction of a new Higgs field $\psi$, which can yield large mixing from the charged lepton mass matrix without adversely affecting the quark mass matrices. With the following choice of charges for the $\psi$ field

$$\vec{Q}_\psi = 3\vec{Q}_\xi + \vec{Q}_W + 4\vec{Q}_T = \left( \frac{1}{2}, -\frac{5}{3}, \frac{7}{6}, -4 \right), \quad (32)$$

we obtain new expressions for the quark Yukawa matrices:

$$Y_U = \begin{pmatrix} W T^2 \xi^2 & W T^2 \xi & W^2 T \xi \\ W^2 T \xi^3 & W T^2 & W^2 T \\ \xi^3 & 1 & W T \end{pmatrix}, \quad Y_D = \begin{pmatrix} W T^2 \xi^2 & W T^2 \xi & T^3 \xi \\ W^2 T \xi & W T^2 & T^3 \\ W^2 T \psi & W^2 T \xi \psi & W T \end{pmatrix} \quad (33)$$
and the charged lepton Yukawa matrix:

\[
Y_E = \begin{pmatrix}
WT^2\xi^2 & W^2T^2\psi & \xi^4\psi \\
W^4T\xi^2T\psi & WT^2 & \xi\psi \\
W^3\xi^2T\psi & W^2T\xi\psi & WT
\end{pmatrix}.
\] (34)

As we can see from the charged lepton matrix we will indeed have large mixing if \(\langle \psi \rangle = O(0.1)\), so that \((Y_E)_{23} \sim (Y_E)_{33}\). In the following discussion we shall take \(\psi\) to have a vacuum expectation value of \(\langle \psi \rangle = O(0.1)\) for definiteness. The effect of the field \(\psi\) on the charged fermion masses is then small, since the elements involving \(\psi\) do not make any significant contribution to the determinant, or the sum of the minors, or the mass matrices, or the trace of the squares \(YY^\dagger\) of the Yukawa matrices. The mixings of the quarks is essentially unaffected by the terms involving \(\psi\), and the only significant effect is on the mixing matrix \(U_E\), which is now given by:

\[
U_E \sim \begin{pmatrix}
1 & \frac{\xi\psi^2X}{\xi^2} & \frac{\xi^3}{X} \\
-W\psi & \frac{W}{\xi^2} & \frac{\xi}{X} \\
\frac{X^2}{\xi^2} & -1 & \frac{W}{\xi^2}
\end{pmatrix} \sim \begin{pmatrix}
1 & 0.021 & 6.4 \times 10^{-4} \\
-0.016 & 0.75 & 0.66 \\
0.014 & -0.66 & 0.75
\end{pmatrix}
\] (35)

where \(X = \sqrt{1 + W^2T^2/\xi^2\psi^2} \sim 1.51\) This gives the large mixing required, \(\sin^2 2\theta_{atm} \sim 1\), for the atmospheric neutrino oscillations.

We can further obtain a solution with vacuum oscillations for the solar neutrinos by choosing appropriate charges for the isotriplet Higgs field \(\Delta\). We require a large off-diagonal \((1,2)\) element for the neutrino mass matrix and hence we choose the charges on \(\Delta\) to be

\[
\bar{Q}_\Delta = (-\frac{1}{2}, -1, \frac{1}{2}, \frac{5}{3})
\] (36)

We then obtain the neutrino mass matrix,

\[
M_\nu \sim \langle \Delta^0 \rangle \begin{pmatrix}
W\xi^6 & W\xi^3 & T\xi^2\psi \\
W\xi^3 & W & T\xi\psi \\
T\xi^2\psi & T\xi\psi & T^2\xi\psi
\end{pmatrix}.
\] (37)

This has the eigenvalues,

\[
m_1 \sim \langle \Delta^0 \rangle \left(-T\xi^2\psi + \frac{T^2\xi\psi}{2}\right), \quad m_2 \sim \langle \Delta^0 \rangle W, \quad m_3 \sim \langle \Delta^0 \rangle \left(T\xi^2\psi + \frac{T^2\xi\psi}{2}\right)
\] (38)

where the splitting between \(m_1\) and \(m_2\) comes from the mass matrix element \((M_\nu)_{33}\). The neutrino mixing matrix is then given by,

\[
U_\nu \sim \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{\xi^3}{\sqrt{2}} & \frac{\xi^3}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{\xi^3}{\sqrt{2}} & \frac{\xi^3}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{\xi^3}{\sqrt{2}} & \frac{\xi^3}{\sqrt{2}}
\end{pmatrix}
\] (39)
Hence, using $U_E$ from eqn. [35], we have the lepton mixing matrix $U = U_E^\dagger U_{\nu}$:

$$
U \sim \begin{pmatrix}
\frac{1}{\sqrt{2}} \left(1 + \frac{T}{\xi}\right) & -W\xi & \frac{1}{\sqrt{2}} \left(1 - \frac{T}{\xi}\right) \\
\frac{1}{\sqrt{2}} \left(1 - \frac{T}{\xi}\right) & \frac{W}{\xi}\psi X & \frac{1}{\sqrt{2}} \left(1 + \frac{T}{\xi}\right) \\
-W^T (1 - \frac{T}{\xi}) & \frac{1}{X} & \frac{W}{\xi}\psi X
\end{pmatrix} \sim \begin{pmatrix}
0.83 & -0.016 & 0.58 \\
0.38 & 0.75 & -0.55 \\
-0.43 & 0.66 & 0.62
\end{pmatrix},
$$

(40)

which, as we can see, has large electron neutrino mixing, as we require for a vacuum oscillation solution to the solar neutrino problem.

We also have the mass hierarchy,

$$
\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \sim 2 \frac{T^3 \xi^3 \psi^2}{W^2} \sim 3 \times 10^{-7}.
$$

(41)

Hence, if we then take $\langle \Delta^0 \rangle \sim 0.2$ eV, so that we have an overall mass scale suitable for the atmospheric neutrino problem, then we will also have,

$$
\Delta m_{13}^2 \sim 2 \langle \Delta^0 \rangle^2 T^3 \xi^3 \psi^2 \sim 3 \times 10^{-10} \text{eV}^2.
$$

(42)

With such a hierarchy of $\Delta m^2$s we effectively have two-neutrino oscillations for the solar neutrinos, with the mixing angle given by,

$$
\sin^2 2\theta_{\text{sun}} = \frac{4}{U_{\nu}^2 E_{\nu}^2} \sim 0.9.
$$

(43)

So, we have the ‘just-so’ vacuum oscillation solution to the solar neutrino problem with large electron neutrino mixing. We remark that $U_{\nu 2} = -0.016$ satisfies the CHOOZ electron neutrino survival probability bound ($U_{\nu 2}$ is the relevant mixing matrix element, since $\Delta m_{12}^2 \sim \Delta m_{23}^2 \gg \Delta m_{13}^2$).

It is also possible to obtain a small mixing angle SMA-MSW solution to the solar neutrino problem, with a different choice of charges for $\Delta$:

$$
\tilde{Q}_\Delta = (-\frac{1}{2}, -\frac{2}{3}, -\frac{1}{6}, 0)
$$

(44)

which gives the quasi-diagonal neutrino mass matrix

$$
M_\nu \sim \langle \Delta^0 \rangle \begin{pmatrix}
W^4 T^2 \xi^2 \psi^2 & W T^2 \xi^3 & T^3 \xi^2 \psi \\
W T^2 \xi^3 & W T^2 & W T \xi^2 \\
T^3 \xi^2 \psi & W T \xi^2 & \xi \psi
\end{pmatrix}.
$$

(45)

The mixing matrix $U_\nu$ for this mass matrix is given by,

$$
U_\nu \sim \begin{pmatrix}
1 & \frac{\xi^3}{\xi} & -\frac{T^3 \xi}{W T \psi} \\
-\xi^3 & 1 & \frac{W T \xi}{\psi} \\
-T^3 \xi & -\frac{W T \xi}{\psi} & 1
\end{pmatrix}.
$$

(46)

Thus we obtain the lepton mixing matrix:

$$
U = U_E^\dagger U_\nu \sim \begin{pmatrix}
\frac{1}{\xi \psi^2 X} & -\frac{W \psi}{\xi} & \frac{\xi^2}{\psi} \\
\frac{W}{\xi \psi X} & \frac{1}{\xi} & -\frac{1}{X} \\
-\frac{\xi^3 \psi}{X} & \frac{W}{\xi \psi X} & \frac{1}{X}
\end{pmatrix} \sim \begin{pmatrix}
0.021 & 0.75 & -0.66 \\
6 \times 10^{-4} & 0.66 & 0.75
\end{pmatrix}.
$$

(47)
Taking \( \langle \Delta^0 \rangle \sim 3 \text{ eV} \), we then obtain suitable masses and mixings for the solution of both the solar and atmospheric neutrino problems:

\[
\sin^2 2\theta_{atm} \sim 1 \Delta m^{2}_{23} \sim 1 \times 10^{-3} \text{eV}^2 \quad \sin^2 2\theta_{sun} \sim 1 \Delta m^{2}_{12} \sim 6 \times 10^{-6} \text{eV}^2
\]

We did not manage to find an LMA-MSW solution, which is favoured by the latest solar neutrino data from Sudbury and SuperKamiokande, using this approach. However, during this workshop, Holger, Yasutaka and I constructed a promising LMA-MSW solution \[9\] using the extended version of the AGUT model with right-handed neutrinos and the usual see-saw mechanism.

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