Inverse displacement analysis of a hyper-redundant bionic trunk-like robot

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Abstract
This article presents the inverse displacement analysis of a hyper-redundant bionic trunk-like robot. The hyper-redundant bionic trunk-like robot is constituted of some parallel mechanism modules in series. The mathematical model of the inverse displacement analysis of the hyper-redundant robot comprises a set of nonlinear consistent equations with a number of unknown variables that are more than given equations. It is impossible to find all solutions for the inverse displacement analysis since there are infinite solution possibilities. By planning the pose of distal moving platforms for robot modules, the problem of the inverse displacement analysis of the hyper-redundant robot is transformed into that of a parallel mechanism. The inverse velocity and acceleration computation are then implemented using a method of the numerical differentiation. A developed prototype of a hyper-redundant bionic trunk-like robot is presented. The approach can be applied to the hyper-redundant robot with man–machine interactive system in the unstructured environment.

Keywords
Bionic trunk-like robot, inverse displacement analysis, inverse velocity and acceleration computation, hyper-redundant

Introduction
Due to the flexible structure, a hyper-redundant robot can perform a complicated operation task in an unstructured environment occupying a smaller space with many obstacles. This kind of robot draws much attention in academia.\textsuperscript{1–4} There are mainly two types of robots designed to mimic motions of the elephant trunk or octopus tentacle, the continuum robot and hyper-redundant robot. The continuum robot\textsuperscript{5–8} usually adopts an invertebrate-type elastic structure without rigid links and joints. There are infinite degrees of freedom (DOFs) in the continuum robot. Therefore, it can bend continuously in a flexible way. The hyper-redundant robot\textsuperscript{9–13} is usually built by connecting several rigid links via actuated revolute/prismatic joints in a chain or by connecting 2- to 4-DOF parallel mechanism modules in series.\textsuperscript{9–15} Compared with the continuum robot, the hyper-redundant robot has a certain stiffness value and load capability due to the structure consists of rigid links and joints. Some prototypes of hyper-redundant robots have been developed for different potential applications.\textsuperscript{16,17}
Displacement analysis is a basic issue for the kinematics, dynamics, design, and control of the robot.

There are two types of problems for the displacement analysis of robots: forward displacement analysis\(^8\) and inverse displacement analysis.\(^{21-23}\) The forward displacement analysis is to compute the position and orientation of an end effector when variables of actuating joints are assigned. It is usually used for simulation. The inverse displacement analysis is to determine variables of actuating joints when the position and orientation of an end effector are given. The inverse displacement analysis is preliminary for the inverse kinematics and inverse dynamics analyses. The inverse displacement analysis is the basis for design, control, and motion planning of the robot. Many works have been done in the displacement analysis of robots. The forward displacement analysis of serial robots and the inverse displacement analysis of parallel robots are easy; the displacement analysis of robots mainly focuses on the inverse displacement analysis of serial robots and the forward displacement analysis of parallel robots. They are two generalized dual problems in mathematics. From the view point of the overall structure, the hyper-redundant robot is usually built by connecting several rigid links via actuated revolute/prismatic joints in a chain or by connecting parallel mechanism modules with 2- to 4-DOFs in series. Therefore, the inverse displacement analysis for the hyper-redundant robot is a challenging problem.

Solutions of the inverse displacement analysis of hyper-redundant robots are infinite since the model is a set of consistent nonlinear equations. The number of unknown joint variables is more than that of consistent nonlinear equations. This is the challenge for the inverse displacement analysis of the hyper-redundant robot. Sometimes, solutions can be achieved using an approximate search method, the search is an intensive computation process involving uncertainties and discontinuity for the inverse displacement analysis of hyper-redundant robots. It is difficult to be applied in the control of a real practice. Although there is work done in the inverse displacement analysis of 6-DOF serial robots and redundant serial robots,\(^{24,25}\) it is found that there are a few reports on the inverse displacement analysis for the hyper-redundant robot.\(^{26}\) Since it is difficult in the inverse displacement analysis for a hyper-redundant robot, some constraints can be considered in the analysis to achieve a satisfied solution. For the studied hyper-redundant bionic trunk-like robot constituted by some parallel mechanism modules, the trajectory and orientation of modules can be planned while considering the obstacle avoidance using the principle of the minimum movement. The inverse displacement analysis of the hyper-redundant robot can then be implemented by taking the advantage of the inverse displacement analysis of parallel robots.

The research objective presented in this article aims at the inverse displacement analysis of a hyper-redundant bionic trunk-like robot. The article is organized as follows. The hyper-redundant bionic trunk-like robot is explained in the second section. The pose planning for the parallel mechanism modules is provided in the third section. Inverse displacement analysis of a hyper-redundant robot is presented in the fourth section. The velocity and acceleration computation are implemented using the numerical differentiation with a case study in the fifth section. Conclusions are given in the last section.

**A hyper-redundant bionic trunk-like robot**

The hyper-redundant bionic trunk-like robot is shown in Figure 1, and its schematic diagram of the \(i\)th module is shown in Figure 2. There are some parallel mechanism modules connected in series. The parallel mechanism modules are composed of a distal moving platform connected with a proximal moving platform through four kinematic chains. The central kinematic chain is fixed on the proximal moving platform at one end and connected to the distal moving platform using a prismatic, revolute, and universal joint at the other end. Three outer kinematic chains are composed of a universal, prismatic, and spherical joint. In the prototype built, the universal joint can be replaced by two orthogonal revolute joints in order to avoid the interference and to enlarge the workspace. All the kinematic chains are driven by the prismatic joint. Due to proper constraints of the kinematic chains, each parallel mechanism module has one translational and three rotational DOFs. Considering the requirement of operation tasks, the hyper-redundant bionic trunk-like robot is
The pose of the distal moving platform in the outermost module assigned by one center point coordinate and three Euler angles

When the center point and three Euler angles of the distal moving platform in the outermost module are assigned, the coordinates of points $A_{n1}$, $A_{n2}$, and $A_{n3}$ are

$$A_{n1} = RA'_{n1} + A_{n0} \quad (5)$$
$$A_{n2} = RA'_{n2} + A_{n0} \quad (6)$$
$$A_{n3} = RA'_{n3} + A_{n0} \quad (7)$$

where

$$R = B_{10}R_{A_{n0}} = Rot(z, \phi_{n2})Rot(y, \phi_{n0})Rot(x, \phi_{n1}) \quad (8)$$

And vectors $A'_{n1}$, $A'_{n2}$, and $A'_{n3}$ are coordinates of points $A_{n1}$, $A_{n2}$, and $A_{n3}$ measured in the coordinate system $A_{n0} - u_{n}v_{n}w_{n}$.

The pose of the distal moving platform in the outermost module assigned by four non-coplanar points

When the four non-coplanar points $A_{n1}$, $A_{n2}$, $A_{n3}$, and $A_{n4}$ are assigned, the coordinate of the center point of the distal moving platform in the outermost module can be expressed as

$$A_{n0} = \frac{1}{3}(A_{n1} + A_{n2} + A_{n3}) \quad (9)$$

The unit vectors of the axes of the coordinate system $A_{n0} - u_{n}v_{n}w_{n}$ can be described in the global coordinate system $B_{10} - x_{1}y_{1}z_{1}$ as follows

$$\hat{x} = \frac{A_{n0}A_{n1}}{||A_{n0}A_{n1}||_{2}} \quad (10)$$
$$\hat{y} = \frac{A_{n3}A_{n2}}{||A_{n3}A_{n2}||_{2}} \quad (11)$$
$$\hat{z} = \frac{A_{n0}A_{n4}}{||A_{n0}A_{n4}||_{2}} \quad (12)$$

The rotation matrix for describing the coordinate system $A_{n0} - u_{n}v_{n}w_{n}$ relative to the coordinate system $B_{10} - x_{1}y_{1}z_{1}$ can be expressed as

$$R = B_{10}R_{A_{n0}} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \end{bmatrix} = Rot(z, \phi_{n2})Rot(y, \phi_{n0})Rot(x, \phi_{n1}) \quad (13)$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$
The three Euler angles can be computed as follows

(1) If \( \cos \phi_{ny} \neq 0 \Leftrightarrow \phi_{ny} \neq \pm \frac{\pi}{2} \Leftrightarrow r_{31} \neq \pm 1 \), then
\[
\phi_{nx} = \arctan(2(r_{32}, r_{33}))
\]
\[
\phi_{nz} = \arctan(2(r_{21}, r_{11}))
\]
\[
\phi_{ny} = \begin{cases} 
\arctan(-r_{31}, r_{32}/\sin \phi_{nx}) & \text{when } \sin \phi_{nx} \neq 0 \\
\arctan(-r_{31}, r_{33}/\cos \phi_{nx}) & \text{when } \sin \phi_{nx} = 0
\end{cases}
\]

(2) If \( \phi_{ny} = \frac{\pi}{2} \), namely \( r_{31} = -1 \), then
\[
\phi_{nx} = \frac{\pi}{2}
\]
\[
\phi_{nz} = 0
\]
\[
\phi_{ny} = \arctan(2(r_{12}, r_{22}))
\]

(3) If \( \phi_{ny} = -\frac{\pi}{2} \), namely \( r_{31} = 1 \), then
\[
\phi_{nx} = -\frac{\pi}{2}
\]
\[
\phi_{nz} = 0
\]
\[
\phi_{ny} = \frac{\pi}{2}
\]

A rotation matrix describing the distal moving platform of modules relative to the base platform is \(^3\)
\[
B_{w} R_{A_{i0}} = Rot(z, \phi_{iz}) Rot(y, \phi_{iy}) Rot(x, \phi_{ix})
\]

The coordinate of joint \( A_{ij} \) on the distal moving platform of the \( i \)th module can be expressed as
\[
A_{i0} = \sum_{i=2}^{n} \hat{r}_{i} \left( \frac{A_{00} - A_{10}}{|A_{00} - A_{10}|} \right) + \sum_{i=2}^{n} \hat{r}_{i} \left( \frac{A_{00} - A_{10}}{|A_{00} - A_{10}|} \right)
\]
where \( l_{b0} \) is the sleeve length of the push rod mounted on the central kinematic chain of the \( i \)th module, \( \hat{r}_{i} \) is the coefficient which is chosen while considering the dimension of the module, and
\[
\sum_{i=2}^{n} \hat{r}_{i} = n - 1
\]

In order to simplify the problem, the principle of the equal distribution is adopted, so the Euler angles describing the orientations of distal moving platforms of modules can be assigned as
\[
\begin{align*}
\phi_{nx} &= \frac{i}{n} \\
\phi_{ny} &= \frac{i}{n} \\
\phi_{nz} &= \frac{i}{n}
\end{align*}
\]
ith module. Point $A_0$ is the geometric center of regular triangle $\Delta A_{11}A_{12}A_{13}$.

The coordinates of joint $B_{i+1,j}$ on the proximal moving platform of the $i$th module can be expressed as

$$B_{i+1,1} = \frac{B_0}{R_0} R_{A_0} B_{i+1,1} + A_{10}$$  \hspace{1cm} (32)

$$B_{i+1,2} = \frac{B_0}{R_0} R_{A_0} B_{i+1,2} + A_{10}$$  \hspace{1cm} (33)

$$B_{i+1,3} = \frac{B_0}{R_0} R_{A_0} B_{i+1,3} + A_{10}$$  \hspace{1cm} (34)

and

$$B_{i+1,0} = \frac{1}{3} (B_{i+1,1} + B_{i+1,2} + B_{i+1,3})$$  \hspace{1cm} (35)

where $B_{i+1,1}^t$, $B_{i+1,2}^t$, and $B_{i+1,3}^t$ are the coordinates of points $B_{i+1,1}$, $B_{i+1,2}$, and $B_{i+1,3}$, respectively, described in the coordinate system $A_{10} = u_iv_jw_k$. They are structure parameters of the $i$th module. Point $B_{2+1,0}$ is the geometric center of a regular triangle $\Delta B_{1+1}B_{1+2}B_{1+3}$.

**Pose planning for distal moving platform of modules with an obstacle**

Since distal moving platforms of modules can not translate along the $x$-axis and $y$-axis, the center point of the moving platform of the first module can be assigned the same as that of the pattern without any obstacle

$$A_{10} = \begin{bmatrix} 0 & 0 & l_{b10} + \lambda_{11} l_{a10} \end{bmatrix}^T$$  \hspace{1cm} (36)

In the pattern without any obstacle, the center points of distal moving platforms from the second to the $n$th modules are

There are following two cases: (1) There is an obstacle on the line from the center point of the moving platform of the first module to the center point of the distal moving platform of the outermost module. (2) The sum of the minimum lengths from the second to the $n$th modules is larger than the line length from the center point of the moving platform of the first module to the center point of the distal moving platform of the outermost module. The center points of distal moving platforms from the second to the $n$th modules should move along the directions of $A_{10}A_{13}$ or its opposite direction with $r_{d0}^b$, then the center points of the distal moving platforms are

$$A_{10}^b = A_{10} \pm r_{d0}^b \times \frac{A_{10}A_{13}}{\|A_{10}A_{13}\|}$$  \hspace{1cm} (38)

where $r_{d0}^b$ is the scope along the direction of $A_{10}A_{13}$ or its opposite direction that the distal moving platform of modules should move to avoid the obstacle on the line from the center point of moving platform of the first module to the center point of distal moving platform of the outermost module. In the practical application, the absolute value of $r_{d0}^b$ should be as small as possible and it is measured or defined in the workspace.
The rotation matrix describing the distal moving platform of modules relative to the base platform is

$$B_{ij}^n R_{A_{ij}} = \text{Rot}(z, \phi_{ij}^n) \text{Rot}(y, \phi_y) \text{Rot}(x, \phi_x)$$  \hspace{1cm} (42)

Therefore, in the pattern with an obstacle, coordinates of joints $A_{ij}$ on the distal moving platform of the $i$th module can be expressed as

$$A_{1i} = B_{ij}^n R_{A_{ij}} A_{1i}^0 + A_{ij}^0$$  \hspace{1cm} (43)

$$A_{2i} = B_{ij}^n R_{A_{ij}} A_{2i}^0 + A_{ij}^0$$  \hspace{1cm} (44)

$$A_{3i} = B_{ij}^n R_{A_{ij}} A_{3i}^0 + A_{ij}^0$$  \hspace{1cm} (45)

and

$$A_{4i} = \frac{1}{3} (A_{1i} + A_{2i} + A_{3i})$$  \hspace{1cm} (46)

The coordinates of joints $B_{i+1,j}$ on the proximal moving platform of the $i+$1th module are

$$B_{i+1,1} = B_{ij}^n R_{A_{ij}} B_{i+1,1}^0 + A_{ij}^0$$  \hspace{1cm} (47)

$$B_{i+1,2} = B_{ij}^n R_{A_{ij}} B_{i+1,2}^0 + A_{ij}^0$$  \hspace{1cm} (48)

$$B_{i+1,3} = B_{ij}^n R_{A_{ij}} B_{i+1,3}^0 + A_{ij}^0$$  \hspace{1cm} (49)

and

$$B_{i+1,0} = \frac{1}{3} (B_{i+1,1} + B_{i+1,2} + B_{i+1,3})$$  \hspace{1cm} (50)

### Inverse kinematics analysis

The actuating joint variable associated with the $j$th kinematic chain of the $i$th module can be written as

$$q_{ij} = \|B_{ij} A_{ij}\|_{i=1.2} \quad j=0,1,2,3$$  \hspace{1cm} (51)

where $q_{ij}$ is the length of the $j$th kinematic chain of the $i$th module. The efficient solution should satisfy

$$l_{bij} \leq q_{ij} \leq l_{bij} + l_{ajj} \quad i=1.2 \ldots n \quad j=0,1,2,3$$  \hspace{1cm} (52)

where $l_{bij}$ and $l_{ajj}$ are the length of the sleeve and the length of the push rod, respectively, on the $j$th kinematic chain of the $i$th module. If the actuating joint variable is not satisfied with equation (52), it is not a reliable solution. The poses of distal moving platforms of modules should be replanned.

For a motion period $T$, the velocity and acceleration of the actuating joint of the $j$th kinematic chain of the $i$th module can be obtained by the numerical differentiation as

$$\dot{q}_{ij}(t_k) = \frac{q_{ij}(t_k + \Delta t) - q_{ij}(t_k)}{\Delta t}$$  \hspace{1cm} (53)

$$\ddot{q}_{ij}(t_k) = \frac{\dot{q}_{ij}(t_k + \Delta t) - \dot{q}_{ij}(t_k)}{\Delta t}$$  \hspace{1cm} (54)

| $i$  | $B_0$ | $B_1$ | $B_2$ | $B_3$ |
|-----|------|------|------|------|
| 1   | 0.000000 | 0.215000 | -0.107500 | -0.107500 |
| 2   | 0.000000 | 0.000000 | 0.186195 | -0.186195 |
| 3   | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

where

$$\Delta t = \frac{T}{\text{num}}$$  \hspace{1cm} (55)

$\Delta t$ should be small enough. And $\text{num}$ is the number of the intervals, in general

$$\text{num} = 2^n$$  \hspace{1cm} (56)

where $n$ is a positive integer.

The error of the numerical computation should satisfy

$$|\dot{q}_{ij}(t_k)_{\text{num}} - \dot{q}_{ij}(t_k)_{\text{exact}}| \leq e$$  \hspace{1cm} (57)

$$|\ddot{q}_{ij}(t_k)_{\text{num}} - \ddot{q}_{ij}(t_k)_{\text{exact}}| \leq e$$  \hspace{1cm} (58)

where $e$ is the maximum allowable numerical error and $n$ is the positive integer selected for the numerical differentiation.

It should also be pointed out that the solutions of the inverse velocity and acceleration computation are infinite for the hyper-redundant robot since the unknown variables are much more than equations that can be formed. The equations (53) and (54) are the solutions obtained by taking the derivative of displacement with respect to time.

### Case study

In this section, a numerical example is provided to show the inverse displacement analysis for a hyper-redundant bionic trunk-like robot with six modules. The simulation was implemented using the MATLAB R2014a software. Parameters of hyper-redundant bionic trunk-like robot are given in Tables 1 and 2. The length of sleeves and the length of push rods are $l_{bij} = 0.2m$ and $l_{ajj} = 0.2m$, respectively.

The pose of the distal moving platform of the outermost module is assigned, as shown in Table 3.

In the pattern without any obstacle, the solution of the inverse displacement analysis of the hyper-redundant bionic trunk-like robot is presented in Table 4. And

$$Q_i = [q_{i0} \quad q_{i1} \quad q_{i2} \quad q_{i3}]^T$$  \hspace{1cm} (59)

is the vector of actuating joint variable of the $i$th module.
| Table 2. The coordinate of joint $A_i$ on the distal moving platform of the $i$th module described in the coordinate system $A_0 - u_{i}v_{i}w_{i}$ (m). |
|---------------------------------------------------------------|
| $i = 1, 3, 5$                                               |
| $A_0$ | $A_1$ | $A_2$ | $A_3$ |
| 0.000000 | 0.090000 | -0.045000 | -0.045000 |
| 0.000000 | 0.000000 | 0.077942 | -0.077942 |
| 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| $i = 2, 4, 6$                                               |
| $A_0$ | $A_1$ | $A_2$ | $A_3$ |
| 0.000000 | 0.045000 | -0.090000 | 0.045000 |
| 0.000000 | 0.077942 | 0.000000 | -0.077942 |
| 0.000000 | 0.000000 | 0.000000 | 0.000000 |

| Table 3. The pose of the distal moving platform of the outermost module. |
|---------------------------------------------------------------|
| $\phi_x$ | $\phi_y$ | $\phi_z$ | $x$ (m) | $y$ (m) | $z$ (m) |
| 15° | 15° | 1.4 |

| Table 4. The solution of the inverse displacement analysis without any obstacle (m). |
|---------------------------------------------------------------|
| $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ |
| 0.300000 | 0.224054 | 0.224054 | 0.224054 | 0.224054 |
| 0.318139 | 0.248665 | 0.252906 | 0.227972 | 0.269600 |
| 0.335321 | 0.314257 | 0.236738 | 0.303801 | 0.211619 |
| 0.322592 | 0.296406 | 0.279562 | 0.325934 |

| Figure 3. The configuration of the hyper-redundant bionic trunk-like robot without any obstacle. |

The configuration of the hyper-redundant bionic trunk-like robot without any obstacle is shown in Figure 3.

In the pattern with an obstacle, the solution of the inverse displacement analysis of the hyper-redundant bionic trunk-like robot is shown in Table 5. And $r_{d_i}^b$ are assigned as

$r_{d_1}^b = 0.00$m, $r_{d_2}^b = 0.075$m, $r_{d_3}^b = 0.10$m, $r_{d_4}^b = 0.10$m, $r_{d_5}^b = 0.055$m, $r_{d_6}^b = 0.00$m

\[ \begin{align*}
Q_1 &= 0.300000, Q_2 = 0.235007, Q_3 = 0.217354, Q_4 = 0.216363, Q_5 = 0.232104, Q_6 = 0.243881 \\
0.318139 &> 0.249368, Q_7 = 0.234891, Q_8 = 0.216272, Q_9 = 0.296479, Q_{10} = 0.235413 \\
0.335321 &> 0.359225, Q_{11} = 0.232791, Q_{12} = 0.298840, Q_{13} = 0.205850, Q_{14} = 0.274595 \\
0.322592 &> 0.268899, Q_{15} = 0.282709, Q_{16} = 0.322891, Q_{17} = 0.280641, Q_{18} = 0.387206
\end{align*} \]

\[ x = r \sin \theta \cos \phi \\
y = r \sin \theta \sin \phi \\
z = l_{d10} + \frac{1}{2} l_{d10} + r \cos \theta \]

\[ \begin{align*}
\phi_x &= \phi_{x \text{down}} + \frac{a_{\phi \text{max}} T^2}{7.513188} (35 \tau^4 - 84 \tau^3 + 70 \tau^2 - 20 \tau) \\
\phi_y &= \phi_{y \text{down}} + \frac{a_{\phi \text{max}} T^2}{7.513188} (35 \tau^4 - 84 \tau^3 + 70 \tau^2 - 20 \tau) \\
\phi_z &= \phi_{z \text{down}} + \frac{a_{\phi \text{max}} T^2}{7.513188} (35 \tau^4 - 84 \tau^3 + 70 \tau^2 - 20 \tau)
\end{align*} \]

(61)

where

\[ \begin{align*}
\theta &= \theta_{\text{down}} + \frac{a_{\theta \text{max}} T^2}{7.513188} (35 \tau^4 - 84 \tau^3 + 70 \tau^2 - 20 \tau) \\
\varphi &= \varphi_{\text{down}} + \frac{a_{\varphi \text{max}} T^2}{7.513188} (35 \tau^4 - 84 \tau^3 + 70 \tau^2 - 20 \tau)
\end{align*} \]

(62)
Figure 5. Variations of position of (a) module 1, (b) module 2, (c) module 3, (d) module 4, (e) module 5, and (f) module 6.

Figure 6. Variations of velocity of (a) module 1, (b) module 2, (c) module 3, (d) module 4, (e) module 5, and (f) module 6.
Variations of the displacement, velocity, and acceleration of the actuating joints are shown in Figures 5 to 7, respectively. The \( r_0 \) assigned as equation (60), \( e=10^{-4} \), and \( n=12 \) are adopted in the computation.

A prototype of a hyper-redundant bionic trunk-like robot is developed and shown in Figures 8 and 9. The presented work on the inverse kinematics analysis has been used in the control design and motion planning. The approach can be applied to the hyper-redundant robot with man–machine interactive system in the unstructured environment, especially for the remote operation. It can also be adopted to the software to implement the inverse velocity and acceleration computation for the robot.
Conclusions

The model of the inverse displacement analysis of a hyper-redundant robot is a set of nonlinear consistent equations having infinite solutions. This article presents the inverse displacement analysis of a hyper-redundant bionic trunk-like robot based on the pose planning of distal moving platforms of parallel mechanism modules. The principle of the equal distribution is employed in the pose planning of distal moving platforms of modules. The geometrical sizes of modules are also considered when positions of center points of distal moving platforms of modules are planned. The poses planning of distal moving platforms of modules are implemented in the pattern both with an obstacle and without any obstacle. By planning the pose of distal moving platforms of modules, the inverse displacement analysis of the hyper-redundant bionic trunk-like robot is transformed into the inverse displacement analysis of modules. Therefore, the problem of the inverse displacement analysis of the hyper-redundant bionic trunk-like robot is transformed into that of a parallel mechanism. The inverse velocity and acceleration computation are implemented by use of numerical differentiation. A prototype of a hyper-redundant bionic trunk-like robot is developed. The presented inverse kinematics analysis has been used in the control design and motion planning. The approach can be applied to the hyper-redundant robot with man–machine interactive system in the unstructured environment. It can also be adopted to the software to implement the inverse velocity and acceleration computation for the robot.

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