Theory of the magnetronic laser

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Abstract

We analyze the motion of an electron in the crossed electromagnetic field of the planar magnetron. Numerically we calculate the parameters of cycloid along which electron moves. We determine the total power of radiation of an electron moving in the magnetron and the power spectrum generated by a single electron and by the system of N electrons moving coherently in the electromagnetic field of planar magnetron. We argue that for large N, and high intensity of electric and magnetic fields, the power of radiation of such magnetronic laser, MAL, can be sufficient for application in the physical, chemical, biological and medicine sciences. In medicine, the magnetronic laser, can be used for the therapy of the localized cancer tumors. The application of the MAL in CERN as an ion source for LHC is not excluded.
1 Introduction

Any of devices that produces an intensive beam of light not only of a very pure single colour but all colours is a laser. While the gas lasers and masers produce practically a single colour, the so called lasers on the free electrons such as undulators and wigglers produce all spectrum of colours. These spectra are generated in the form of so called synchrotron radiation because they are generated during the motion of electron when moving in the magnetic field.

The electrons moving inside the planar magnetron evidently produce also the synchrotron radiation because planar magnetron is a system of two metal charged plates immersed into homogenous magnetic field which is perpendicular to the homogenous electric field between plates. The planar magnetron producing photons has the same function as the free electron laser such as wiggler or undulator. So, we denote such device by word magnetronic laser, or shortly MAL. The motion of electrons is determined by the equation of motion following from electrodynamic of charges in homogenous electric and magnetic field. The produced energy of electrons is given by the Larmor formula.

History teaches us that Van de Graaff (1931), Cockcroft and Walton (1932) used the high voltage principle system for the acceleration of particles and that the next revolution in acceleration physics was caused by Lawrence (Lawrence et al., 1932) who replaced the high voltage acceleration by repeated acceleration of particles moving along the circle. Here, we use the high voltage idea with the difference, to obtain synchrotron radiation only and not to obtain high energy particles.

Although the radiation of one electron from one trajectory is very weak, the total radiation of a MAL is not weak because the process of radiation can be realized by many electrons moving along the same trajectory. There is no problem for today technology to construct MAL of high intensity electric and magnetic fields. We hope that the radiation output of such device is sufficient large for application in an physical laboratories.

We argue that MAL can be applied for instance in the therapy of cancer and it means that in the specific situations it can be used instead of Leksell gamma knife. On the other hand, the application of MAL in CERN as the ion source for LHC is not excluded.

2 The nonrelativistic motion of an electron in the field configuration of the planar magnetron

We shall identify the direction of magnetic field $H$ with $z$-axis and the direction of electric field $E$ is along the $y$-axis. Then, the nonrelativistic equation of motion of an electron is as follows:

$$m \dot{v} = eE + \frac{e}{c}(v \times H).$$

This equation can be rewritten in the separate coordinates as follows (we do not write the $z$ coordinate):

$$m \ddot{x} = \frac{e}{c} y H,$$
\[ m \ddot{y} = eE_y - \frac{e}{c} \dot{x} H. \]  

Multiplying the equation (3) by \( i \) and combining with the equation (2), we find

\[ \frac{d}{dt}(\dot{x} + i \dot{y}) + i \omega_0 (\dot{x} + i \dot{y}) = i \frac{e}{m} E_y, \]  \(4\)

where

\[ \omega_0 = eH/mc. \]  \(5\)

Function \( \dot{x} + i \dot{y} \) can be considered as the unknown, and as such is equal to the sum of the integral of the same differential equation without the right-hand term and a particular integral of the equation with the right-hand term. The first integral is \( ae^{-i\omega_0 t} \), and the second integral is a constant which can be eliminated from eq. \(4\) as \( eE_y/m\omega_0 = eE_y/H \). Thus

\[ \dot{x} + i \dot{y} = ae^{-i\omega_0 t} + \frac{cE_y}{H}. \]  \(6\)

The constant \( a \) is in general complex. So, we can write it in the form \( a = be^{i\alpha} \) where \( b \) and \( \alpha \) are real constants. We see that since \( a \) is multiplied by \( e^{-i\omega_0 t} \), we can, by a suitable choice of the time origin, give the phase \( \alpha \) any arbitrary value. We choose \( a \) to be real. Then, breaking up \( \dot{x} + i \dot{y} \) into real and imaginary parts, we find

\[ \dot{x} = a \cos \omega_0 t + \frac{cE_y}{H}, \quad \dot{y} = -a \sin \omega_0 t. \]  \(7\)

At \( t = 0 \) the velocity is along the x-axis (for \( a \neq -cE_y/H \)).

As we have said, all the formulas of this section assume that the velocity of the particle is small compared with the velocity of light. If we calculate the average value of \( x \) and \( y \), we get

\[ < \dot{x} > = \frac{cE_y}{H}, \quad < \dot{y} > = 0, \]  \(8\)

and, if we we define the drift velocity by relation

\[ v_{\text{drift}} = \frac{cE_y}{H}, \]  \(9\)

then, from the nonrelativistic condition \( v_{\text{drift}} \ll 1 \), we see that for this to be so, it is necessary in particular that the electric and magnetic intensities satisfy the following condition:

\[ \frac{E_y}{H} \ll 1, \]  \(10\)

while the absolute magnitudes of \( E \) and \( H \) can be arbitrary.

Integrating equation (7), and choosing the constant of integration so that at \( t = 0, x = y = 0 \), we obtain

\[ x = \frac{a}{\omega_0} \sin \omega_0 t + \frac{cE_y}{H} t, \quad y = \frac{a}{\omega_0} (\cos \omega_0 t - 1). \]  \(11\)
These equations define in a parametric form a trochoid. Depending on whether $a$ is larger or smaller in absolute value than the quantity $cE_y/H$, the projection of the trajectory on the plane $xy$ have different form.

If $a = -cE_y/H$, then, equation (7) becomes:

$$x = \frac{cE_y}{\omega_0 H} (\omega_0 t - \sin \omega_0 t), \quad y = \frac{cE_y}{\omega_0 H} (1 - \cos \omega_0 t).$$

(12)

These equations are the parametric equation of cycloid in the plane $xy$. The cycloid can be formed also mechanically as a curve traced out by a point on the circumference of a circle that rolls without slipping along a straight line. Constant $R = cE_y/\omega_0 H$ is equal to the radius of the rolling circle. The distance from the point with parameter $t = 0$ to the point with parameter $t = 2\pi/\omega_0$ is $2\pi R$. The drift velocity is $R\omega_0$ and it is the velocity of the center of the circle.

On the other hand we see that the average value of $\dot{x}$ is just the drift velocity $v_{\text{drift}}$ in equation (9).

The generalization to the relativistic situation can be performed using the approach of Landau et al. (1962).

The aim of this article is to investigate the radiation of electron in case that the motion is just along cycloid, i.e., the trajectory of an electron in the planar magnetron. We want to show that such configuration of fields is an experimental device which can be used as the new effective source of synchrotron radiation.

For the sake of simplicity we calculate spectrum of radiation of MAL in the system $S'$ which moves with regard to the magnetron at the drift velocity $v_{\text{drift}} = R\omega_0$. Then, all electron trajectories are circles and the radiation of this system is the synchrotron radiation. Then, we can transform the power spectrum to the system joined with the planar magnetron. However, we shall see that the spectrum is modified slightly, because the drift velocity is not relativistic in our situation.

The motion of electrons differs form the motion of electrons in wigglers and undulators where the trajectories of electrons are not cycloids and the motion is highly relativistic.

It is also possible to realize the relativistic motion of electrons in MAL. However, it needs special experimental conditions which are very expensive.

### 3 Physical design of MAL

The expression for the instantaneous power radiated by the nonrelativistic charge $e$ undergoing an acceleration $\mathbf{a} = d\mathbf{v}/dt$ is given by the Larmor formula as follows:

$$P = \frac{2}{3} \frac{e^2}{c^3} \left( \frac{d\mathbf{v}}{dt} \right)^2 = \frac{2}{3} \frac{e^2}{m^2 c^3} \left( \frac{d\mathbf{P}}{dt} \right)^2.$$  

(13)

So we see that only electrons with undergoing big acceleration can produce sufficient power of radiation of photons. If we consider electrons in the system $S'$ with the drift velocity $v = v_{\text{drift}} = cE_y/H$, then the equations of the trajectory are as follows:

$$x = -\frac{cE_y}{\omega_0 H} \sin \omega_0 t, \quad y = \frac{cE_y}{\omega_0 H} (1 - \cos \omega_0 t).$$

(14)
They can be rewritten in the form:

\[ x^2 + \left( y - \frac{cE_y}{\omega_0 H} \right)^2 = \left( \frac{cE_y}{\omega_0 H} \right)^2, \tag{15} \]

which is evidently an equation of a circle with the radius

\[ R = \frac{cE_y}{\omega_0 H} = \left( \frac{eH}{mc} \right)^{-1} \frac{cE_y}{H} = \frac{mc^2 E_y}{eH^2}. \tag{16} \]

The corresponding tangential velocity is

\[ v_t = \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{cE_y}{H} = v_{\text{drift}}. \tag{17} \]

The corresponding acceleration is

\[ \sqrt{\ddot{x}^2 + \ddot{y}^2} = \frac{eE_y}{m}, \tag{18} \]

which corresponds to the Newton law, mass × acceleration = force = \( eE_y \), which agrees with the introducing the electrostatic intensity \( E_y \) inside the planar magnetron.

Big acceleration gives big radiation and also big tangential velocity for the given intensity of the magnetic field. However, big intensity of the magnetic field reduces the big circle to the small size.

If we describe the motion of electrons in planar magnetron in the SI system of units (Jackson, 1998) and in the system \( S' \), then, we write the following equations of motion:

\[ x = -\frac{E_y}{\omega_0 B} \sin \omega_0 t, \quad y = \frac{E_y}{\omega_0 B} (1 - \cos \omega_0 t), \tag{19} \]

where \( B \) is the magnetic induction and

\[ \omega_0 = \frac{eB}{m}. \tag{20} \]

Then, for the radius of a circle, we get

\[ R = \frac{E_y}{\omega_0 B} = \left( \frac{eB}{m} \right)^{-1} \frac{E_y}{B} = \frac{mE_y}{eB^2}. \tag{21} \]

The corresponding tangential velocity is

\[ v_t = \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{E_y}{B} = v_{\text{drift}}. \tag{22} \]

Let us determine acceleration of electron with the following parameters: \( E_y = 10^6 \text{ V m}^{-1} \), with \( e = 1.6 \times 10^{-19} \text{ C} \), and \( m = m_e = 9.1 \times 10^{-31} \text{ kg} \). We get

\[ \frac{eE_y}{m} = \frac{1.6 \times 10^{-19} \times 10^6}{9.1 \times 10^{-31}} = \frac{1.6}{9.1} \times 10^{18} \approx 1.75 \times 10^{17} \text{ m s}^{-2}. \tag{23} \]

In the SI system the radiation power is as follows (Wille, 2000):

\[ P = \frac{e^2}{6\pi\varepsilon_0 c^3} \times \left( \frac{eE_y}{m} \right)^2, \tag{24} \]
where

\[ \varepsilon_0 = 8.8 \times 10^{-12} \text{A s V}^{-1} \text{m}^{-1}. \]  \hspace{1cm} (25)

For \( E_y = 10^6 \text{ V m}^{-1} \), with \( e = 1.6 \times 10^{-19} \text{ C} \), \( c = 3 \times 10^8 \text{ m s}^{-1} \) and \( m = m_e = 9.1 \times 10^{-31} \text{ kg} \), we get using the value of acceleration determined in equation (23):

\[ P = \frac{(1.6)^2 \times (1.75)^2}{6 \times 3.14 \times 8.8 \times 27} \times (10^{-19+17})^2 \times 10^{12-24} \text{ J s}^{-1} \approx 1.75 \times 10^{-20} \text{ W}. \] \hspace{1cm} (26)

Of course this is very small radiation, however this is only for one electron, or, in other words for one elementary charge. If we prepare regime with many charges, say \( q = 10^{10}e \), then the situation will be substantially different. We obtain \( P = 1.75 \text{ W} \).

We can also easily determine the radius of circle in the system \( S' \). The corresponding formula in the SI system of units is given by equation (21): \( R = mE_y/eB^2 \), where \( B \) is the magnetic induction. For \( B = 1 \text{T} \), we get:

\[ R = \frac{9.1}{1.6} \times 10^{-31+6+19} \approx 5.7 \times 10^{-6} \text{ m}. \] \hspace{1cm} (27)

It follows from the last formula, that the radius of circle is very small and it means that the external observer sees the radiating “straight line”, which is parallel with the \( x \)-axis. Every point of this “straight line” radiates, however the enhancement of radiation is only at the direction of “straight line”.

If we are interested also in the value of the drift velocity for our parameters of the MAL, then we get:

\[ v_{drift} = \frac{E}{B} = 10^6 \text{ m s}^{-1}. \] \hspace{1cm} (28)

So we see, that we have chosen the parameters in a such way that the drift velocity is substantially smaller than the velocity of light.

The quantity \( \omega_0 \) in the considered situation with \( B = 1 \text{T} \) is determined as follows:

\[ \omega_0 = \frac{eB}{m} = \frac{1.6}{9.1} \times 10^{12} \text{ s}^{-1} \approx 1.7 \times 10^{11} \text{ s}^{-1}. \] \hspace{1cm} (29)

For current which is formed by charges moving along the trajectory, we can derive simple formula using the physical ingredients in the preceding text.

\[ J = e(N/L)v_{drift} = e(1/2\pi R)v_{drift} = 1.6 \times 2.8 \times 10^{-9} \text{ A} \approx 4.5 \times 10^{-9} \text{ A}. \] \hspace{1cm} (30)

We see, that the current generated by the high-intensity field with only one electron at the arc of the cycloid is very small. However for \( e \rightarrow 10^{10}e \), we obtain \( J = 45 \text{ A} \).

If the distribution of the synchrotron is \( P(\lambda) \), then, the maximal intensity of radiation is is for the following \( \lambda = \lambda_{max} \) (Schwinger, 1949):

\[ \lambda_{max} \approx (4\pi/3)R \left( \frac{mc^2}{W} \right)^3, \] \hspace{1cm} (31)

where \( R \) is the radius of the circle and \( W \) is the energy of electron with the rest mass \( m \) which moves along the circle.
If we assume that MAL works at above conditions then, with the electron rest energy $mc^2 = 0.5 \text{ MeV}$, and $W \approx 0.5 \text{ MeV}$, we get:

$$\lambda_{\text{max}} \approx \left(\frac{4\pi}{3}\right)R \approx 23.9 \times 10^{-6} \text{ m} = 23.9 \mu\text{m}, \quad B = 1\text{T},$$  \hspace{1cm} (32)

which is the infrared wavelength. For $B = 5\text{T}$, we obtain

$$\lambda_{\text{max}} \approx \left(\frac{4\pi}{3}\right)R \approx 960 \text{ nm}, \quad B = 5\text{T}.$$  \hspace{1cm} (33)

However, the synchrotron radiation is generated in the form of the all wavelength and it means also for $\lambda < \lambda_{\text{max}}$. It means that the planar magnetron generates also the Roentgen radiation. Of course, the intensity of the Roentgen radiation of one electron is very weak. However, in case with many electrons, it can be very strong and it differs from the laser radiation, which is produced in the optical frequencies. So, the practical application of the planar magnetron as a source of the Roentgen, or, synchrotron radiation is possible.

The planar magnetron is composed from the cathode and anode. If the cathode is cold, then after application the voltage the emission of electrons occurs accidentally from the arbitrary points of the cathode. In order to establish only one starting point of the emission of electrons, we connect cathode with the prismatical, or conical protrusion with the vertex between anode and cathode. Then, the electrons are sucked from the vertex to the anode and move along the trajectory of the constant geometrical form. Then, the planar magnetron as a source of synchrotron radiation works.

In case of the thermal cathode, the initial velocities of electrons are determined by the statistical law. The consequence of it is, that many different cycloids are generated and the coherence of motion is broken.

4 The spectral function of the radiation of relativistic electrons

Hitherto, we considered the nonrelativistic electrons. Now, we shall work with the relativistic electrons for which it is elaborated the Schwinger formalism (source theory) which enables to perform calculations in a simple and brilliant form. We shall derive the power spectrum formula of the synchrotron radiation generated by the motion of an electron moving in a planar magnetron. We follow Schwinger et al. (1976), but we will follow also the author articles (Pardy, 1994, 1997).

Let us remark, that source theory methods (Schwinger, 1970, 1973; Dittrich, 1978) was initially constructed for a description of high-energy particle physics experiments. It was found that the original formulation simplifies the calculations in electrodynamics and gravity, where the interactions are mediated by the photon and graviton, respectively. It simplifies particularly the calculations with radiative corrections (Dittrich, 1978; Pardy, 1994).

The basic formula of the Schwinger source theory is the so called vacuum to vacuum amplitude:

$$\langle 0_+|0_- \rangle = e^{\frac{i}{\hbar}W},$$  \hspace{1cm} (34)
where, for the case of the electromagnetic field in the medium, the action \( W \) is given by

\[
W = \frac{1}{2} c^2 \int (dx)(dx') J^\mu(x) D_{+\mu\nu}(x - x') J^\nu(x'),
\]

(35)

where

\[
D_{+\mu\nu} = \frac{\mu}{c} [g^{\mu\nu} + (1 - n^{-2}) \beta^\mu \beta^\nu] D_{+}(x - x'),
\]

(36)

and \( \beta^\mu \equiv (1, 0), J^\mu \equiv (c\bar{\rho}, J) \) is the conserved current, \( \mu \) is the magnetic permeability of the medium, \( \epsilon \) is the dielectric constant of the medium, and \( n = \sqrt{\epsilon\mu} \) is the index of refraction of the medium. Function \( D_{+} \) is defined as follows (Schwinger et al., 1976):

\[
D_{+}(x - x') = \frac{i}{4\pi^2 c} \int_0^\infty d\omega \frac{\sin \frac{\omega}{c} |x - x'|}{|x - x'|} e^{-i\omega|t - t'|}.
\]

(37)

The probability of the persistence of vacuum follows from the vacuum amplitude (34) in the following form:

\[
|\langle 0_+ | 0_- \rangle|^2 = e^{-\frac{2}{\hbar} \text{Im} W},
\]

(38)

where \( \text{Im} W \) is the basis for the definition of the spectral function \( P(\omega, t) \) as follows:

\[
-\frac{2}{\hbar} \text{Im} W \overset{d}{=} - \int dt d\omega \frac{P(\omega, t)}{\hbar \omega}.
\]

(39)

Now, if we insert equation (35) into eq. (39), we get, after extracting \( P(\omega, t) \), the following general expression for this spectral function:

\[
P(\omega, t) = -\frac{\omega}{4\pi^2 n^2} \int dx dx' dt' \left[ \frac{\sin \frac{\omega}{c} |x - x'|}{|x - x'|} \right] \times \cos[\omega(t - t')] [\varrho(x, t) \varrho(x', t') - \frac{n^2}{c^2} J(x, t) \cdot J(x', t')].
\]

(40)

Formula (40) was obtained also by Schwinger from the classical electrodynamics without using source theory methods (Schwinger, 1945, 1949).

5 Radiation of N electrons moving coherently in a planar magnetron

We determine the power spectral formula in the system \( S' \) moving with the drift velocity \( v_{\text{drift}} = cE_y/H \) with regard to magnetron. In this case motion of electron is not cycloid but the circle and we can repeat some formulae which were used in the previous work of author (Pardy, 2000; 2002). Such approach is meaningful because we can return to the system \( S \) using the formulas connecting frequencies and distribution in the system \( S \) with frequencies and photon distribution in the system \( S' \) (Landau et al., 1962).

We will apply the formula (40) to the N-body system with the same charged particles which moves along the same circles with diameters \( R \) with the centres at points 0, 2R, 4R, ..., 2(N − 1)R, where \( N \) is the natural number.
The general approach involves the consideration of parameters of medium and the possibility of involving the Čerenkov electromagnetic radiation which is generated by a fast-moving charged particle in a medium when its speed is faster than the speed of light in this medium. It seems that in our case of MAL this radiation represents only the academical interest, nevertheless the future application cannot be excluded. So, the connection with the experimental observation of this radiation by Čerenkov (1934) and theoretical interpretation by Tamm and Frank (1937) in the framework of classical electrodynamics is possible as the generalization of theory of MAL. A source theory description by Schwinger et al. (1976) in the zero-temperature regime and the at the finite-temperature regime by Pardy (1989, 1995) can be also considered as meaningful.

So we write for the charge density $\rho$ and for the current density $J$ of the N-body system:

$$\rho(x, t) = e \sum_{i=1}^{i=N} \delta(x - x_i(t))$$  \hfill (41)

and

$$J(x, t) = e \sum_{i=1}^{i=N} v_i(t) \delta(x - x_i(t)),$$  \hfill (42)

where $x_i(t)$ is the trajectory of electrons in MAL. In order to be in the formal identity with the Schwinger approach, we perform elementary transformation of variables in eq. (12) which has no influence on the spectrum of emitted photons. In other words we make the following transitions:

$$S \rightarrow S'; \quad \omega_0 \rightarrow -\omega_0; \quad y \rightarrow y - R; \quad \omega_0 t \rightarrow \omega_0 t + \frac{\pi}{2}. \hfill (43)$$

Then, we can write $x_i(t)$ in eqs. (41) and (42) in the form:

$$x_i(t) = ia + x(t) = ia + R(i \cos(\omega_0 t) + j \sin(\omega_0 t)), \quad i = 1, 2, 3, ..., N; \quad a = (2R, 0, 0). \hfill (44)$$

From the physical situation follows with ($H = |\mathbf{H}|, W =$ energy of a particle)

$$v_i = dx_i/dt = dx/dt = v(t), \quad \omega_0 = v/R, \quad R = \frac{\beta W}{eH}, \quad \beta = v/c, \quad v = |v|. \hfill (45)$$

After insertion of eqs. (41) and (42) into eq. (40), and after some mathematical operations we get

$$P(\omega', t) = -\frac{\omega'}{4\pi^2 n^2} \int_{-\infty}^{\infty} dt' \cos \omega'(t - t') \left[ 1 - \frac{v(t) \cdot v(t')}{c^2 n^2} \right] \times$$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\sin \frac{\omega'}{n} |x_i(t) - x_j(t')|}{|x_i(t) - x_j(t')|}. \hfill (46)$$

Using $t' = t + \tau$, we get with $x(t) = R(i \cos(\omega_0 t) + j \sin(\omega_0 t)$:
\[ \mathbf{x}_i(t) - \mathbf{x}_j(t') = (i - j)\mathbf{a} + \mathbf{x}(t) - \mathbf{x}(t + \tau) = (i - j)\mathbf{a} + \mathbf{A}, \]  
(47)

where \( \mathbf{A} \triangleq \mathbf{x}(t) - \mathbf{x}(t + \tau) \).

Using geometrical representation of vector \( \mathbf{x}(t) \), we get:

\[ |\mathbf{A}| = \sqrt{R^2 + R^2 - 2RR \cos(\omega_0 \tau)} = 2R \left| \sin \left( \frac{\omega_0 \tau}{2} \right) \right|, \]  
(48)

and

\[ \mathbf{v}(t) \cdot \mathbf{v}(t + \tau) = \omega_0^2 R^2 \cos \omega_0 \tau. \]  
(49)

The absolute values of the \( \mathbf{x} \) differences are as follows:

\[ |x_i(t) - x_j(t')| = \sqrt{\left( (i - j)^2 a^2 + 2(i - j) \mathbf{a} \cdot \mathbf{A} + A^2 \right)^2}. \]  
(50)

Now, using the definition of \( \mathbf{a} \equiv (2R, 0, 0) \) and \( |\mathbf{A}| \), we get

\[ |x_i(t) - x_j(t')| = \sqrt{\left( (i - j)^2 4R^2 + 8R^2(i - j) \cos \theta + 4R^2 \sin^2 \frac{\omega_0 \tau}{2} \right)^2}, \]  
(51)

where \( \theta \) is an angle between \( \mathbf{a} \) and \( \mathbf{A} \).

Now, we are prepared to write the power spectrum formula for emission of photons by the planar magnetron with \( N \) electrons. We have all ingredients for determination of the final formula and we can follow the way of author article (Pardy, 2002). However, we see that the last formula is very complicated and it means that the calculation of the power spectrum will be not easy. So, in such a situation, we can use some approximation. One possible approximation is to consider the contributions of terms with \( i = j \). Then we get:

\[ P_N \approx NP, \]  
(52)

where \( P(\omega') \) was expressed by Schwinger et al. (1976) in the form

\[ P(\omega') = \sum_{l=1}^{\infty} \delta(\omega' - l\omega_0) P_l, \]  
(53)

where

\[ P_l(\omega', t) = N \frac{e^2}{\pi n^2} \frac{\omega'^2 \mu \omega_0}{v} \left( 2n^2 \beta^2 J_l'(2ln\beta) - (1 - n^2 \beta^2) \int_0^{2ln\beta} dx J_{2l}(x) \right), \]  
(54)

where \( N \) is the number of arcs forming the trajectory of electrons with one electron at one arc.

Our aim is to apply the last formula to the situation of MAL, where electrons move in a vacuum. In this case we can put \( \mu = 1, n = 1 \). At the same time, with regard to the equation (52), we get for \( N \) electrons:

\[ P_{(N \text{ electrons})} = N \frac{e^2}{\pi} \frac{\omega'^2 \omega_0}{v} \left( 2\beta^2 J_1'(2l\beta) - (1 - \beta^2) \int_0^{2\beta} dx J_{2l}(x) \right). \]  
(55)

If we take the idea that the discrete spectrum parametrized by number \( l \) is effectively continuous for \( l \gg 1 \), then, in such a case there is an relation (Schwinger, 1949):
\[ P(\omega') = P_{(\omega'_{\omega_0})} \left( \frac{1}{\omega_0} \right). \] (56)

Formulas (55), (56) concern the situation with electrons moving along the circular trajectory. In other words, we work in the system which moves with regard to the magnetron at the drift velocity \( v_{\text{drift}} \). In the case that we are in the system joined with the magnetron, all wave lengths are shifted to the blue edge because of the Doppler effect. If we consider only photons moving along the \( x \)-axis, then, we can use the formula for the Doppler shift of the following form (Rohlf, 1994):

\[ \lambda = \lambda' \sqrt{\frac{1 - \beta}{1 + \beta}}. \] (57)

because we move toward the photon emission.

For the drift velocity \( v_{\text{drift}} = E_y / B \), with the electric field \( 10^6 \text{ V/m} \) and magnetic field \( 1 \text{T} \), we get \( \lambda / \lambda' \approx 1 - \beta \approx 0.997 \), for \( \beta = 10^6 / (3 \times 10^8) \approx 3.3 \times 10^{-3} \ll 1 \). It represents practically no shift, because of the small drift velocity.

The radiative corrections obviously influence the spectrum (Schwinger, 1970 and Pardy, 1994). Determination of this phenomenon is not the interest of this article.

6 The use of MAL at the therapy of cancer

We know from the medicine literature (Concise Medical Dictionary, 2003), that the cancer is any malignant tumor, including carcinoma and sarcoma. It arises from the abnormal and uncontrolled division of cells that then invade and destroy the surrounding tissues. Cancer tumor is composed from cancer cells which we call metastases and which are spread by blood stream in the lymphatic channels, thus setting up secondary tumors at sites distant from the original tumor. There are many causative factors of cancer, such as cigarette smoking, radiation, viruses and so on. In more than half of all cancers a gene called p53 is deleted or impaired. Its normal function is to prevent the uncontrolled division of cells. Treatment of cancer depends on the type of tumor, the site of the primary tumor and the extend of spread.

We know, (Concise Medical Dictionary, 2003) that the usual methods of the cancer therapy are chirurgical, biochemical and radiological. Radiotherapy, or, therapeutic radiology is the treatment of desease with penetrating radiation, such as X-rays, beta rays, gamma rays, proton beams, pion beams and so on. Specially, high energy protons have ideal characteristics for treating deep-seated tumors (Jones et al., 2001). The rays are usually produced by machines, or by the certain radioactive isotopes. Beams of radiation may be directed at a deseased object from a distance of radioactive material. Well known technique is Leksell gamma knife (Leksell, 1951). At present time it uses approximatelly 200 sources of the gamma rays where gamma rays are produced by radionuclides \(^{60}\text{Co}\). Many forms of cancer are destroyed by this type of radiotherapy.

At present time the successful method is to treat cancer by the synchrotron radiation generated by synchrotrons, betatrons and microtrons. We proposed in this article, the planar magnetron laser, MAL, as the new medicine element for treating cancer. The size
of the planar magnetron is in no case so large as the synchrotron and it means the cancer tumors can be negated ambulantly. So, the cancer therapy by MAL is hopeful.

7 Discussion

The generation of the synchrotron radiation by the planar magnetron, forms the analogue of the wiggler and undulator generation of radiation (Winick, 1987). However, while the wiggler and undulator radiators needs the high-energy electrons from additional accelerator, the planar magnetron works with electrons which are accelerated by own magnetic and electric fields. The acceleration depends on the intensity of electric field, and, the curvature of trajectory depends also on the magnetic field. The spectrum of the magnetron radiation in system $S'$ moving with the drift velocity $v_{drift} = cE_y/H$, is the spectrum of the synchrotron radiation of N electrons moving along the circles of the same radius. According to the special theory of relativity, there is only the magnetic field in the system $S'$. The MAL can work with the single electron on the trajectory, or, with many electrons on one trajectory or many trajectories. In case of many electrons the radiation can be very intensive. The merit of the MAL is in small size with regard to large synchrotrons. On the other hand the opening angle of radiation is not so small as in the synchrotron because electrons in MAL are nonrelativistic. The small opening angle is possible only for high energy electrons. According to Winick (1987), if an electron is given a total energy 5 GeV, the opening angle over which synchrotron radiation is emitted is only 0.0001 radian, or about 0.006 degree. This can be regarded as a beam with the nearly parallel rays. This is practically the same as the laser beam situation. The wavelength of the magnetronic photons is from zero to infinity. If we want to produce maximal energy of photons at the very short length of photons, then we must apply in MAL very high magnetic field and very high voltage. If we use high-frequency high-voltage alternate current generated by Tesla transformer, then we must also alternate the magnetic field in order to keep the motion of electrons in one direction. This can be easily realized, because the $x(t)$ is invariant with regard the simultaneous alternating the electric and magnetic field, as we can see:

$$
\frac{cE_y}{H} \rightarrow c\frac{(-E_y)}{(-H)}, \quad \omega_0 \rightarrow -\omega_0.
$$

(58)

The variable $y$ is not invariant with regard to the transformation (58). It is $y(t) \rightarrow -y(t)$. However, the prismatical protrusion causes that only trajectories with the positive electric and magnetic fields are realized.

From formula (32) follows that MAL produces all wave lengths including synchrotron radiation and visible light. Using optical filters we can leave only the short wave lengths. Of course, then, the total applied radiation is decreased. In conclusion, we hope, that MAL will play fundamental role in all branches of science.
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