Non-adiabatic manipulation of slow-light solitons

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Abstract. We provide an exact analytic description of decelerating, stopping and reaccelerating optical solitons in atomic media in the non-adiabatic regime. Dynamical control over slow-light pulses is realized via a nonlinear interplay between the solitons and the controlling field generated by an auxiliary laser. This leads to recovery of optical information. We discuss physically interesting features of our solution, which are in good agreement with recent experiments.

Recently, significant theoretical and experimental efforts have been made to better understand the physics of light propagation in atomic vapours and Bose–Einstein condensates (BEC) whose interaction with light is well described by the nonlinear $\Lambda$-model. Partially, this attention is due to the success of research on storage and retrieval of optical information in the media [1]–[6]. Even though the linear approach to describing these effects based on the theory of electromagnetically induced transparency (EIT) [7] is developed in detail [8], modern experiments require more adequate nonlinear descriptions [6]. The linear theory of EIT assumes the probe field to be much weaker than the controlling field. To allow significant changes in the initial atomic state due to interaction with the optical pulse, here we go beyond the limits of linear theory. In the adiabatic regime, when the fields change in time very slowly, an approximate analytical solution was provided [9] and applied in the study of processes of storage and retrieval [10]. Strong nonlinearity can result in interesting new phenomena [11]. Recent experiments and numerical studies [6, 12] have shown that the adiabatic condition can be relaxed allowing for much more efficient control over the storage and retrieval of optical information.

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In this paper, we provide an exact analytical solution of the Maxwell–Bloch system of equations describing a $\Lambda$-type model in the strongly nonlinear non-adiabatic regime of the dynamics. For calculations, we use experimentally relevant parameter values specific for experiments on rubidium atoms [5, 6]. The two lower levels are, in general, non-degenerate atomic ground states denoted as $|1\rangle$ and $|2\rangle$, while $|3\rangle$ is the excited state. The medium is described by the $3 \times 3$ density matrix ($\rho$) in the interaction picture. In order to cancel the residual Doppler broadening, two optical beams are chosen to be co-propagating. The fields are described by the Rabi-frequencies ($\Omega_{a,b}$). The field $\Omega_a$ corresponds to $\sigma^-$ polarization, while the second $\Omega_b$ corresponds to $\sigma^+$ polarization. Within the slowly varying amplitude and phase approximation (SVEPA), dynamics of the atom–field system are well described by the reduced Maxwell–Bloch equations [13]:

$$\partial_t H_I = i \frac{v_0}{4} [D, \rho], \quad \partial_t \rho = i \left[ \frac{\Delta}{2} D - H_I, \rho \right].$$

(1)

Here $\zeta = z/c$, $\tau = t - z/c$, $\Delta$ is the detuning of the resonance and $v_0$ the coupling constant. The matrix $H_I = -\frac{1}{2} (\Omega_a |3\rangle \langle 1| + \Omega_b |3\rangle \langle 2|) + \text{h.c.}$ represents the interaction Hamiltonian and $D = I - 2 |3\rangle \langle 3|$ is a $3 \times 3$ diagonal matrix.

The great advantage of the system of equations (equation (1)) is that they are exactly solvable in the framework of the inverse scattering (IS) method [13]–[16]. Notice that we consider a semi-infinite $\zeta \geq 0$ active medium with a pulse of light incident at the point $\zeta = 0$. A slow-light soliton solution on a constant background field $\Omega_0$ was reported in [13, 15]. In clear distinction from the linear EIT regime, an important feature of this solution is that the background field is not separable from the soliton and exists in an intrinsically nonlinear superposition with the latter. The group velocity of the soliton depends explicitly on the field $\Omega_0$, i.e. $v_g \approx c(\Omega_0^2/2v_0)$. This expression immediately suggests a plausible conjecture that when the controlling field is switched off, the soliton stops propagating while the information borne by the soliton remains in the medium in the form of a spatially localized polarization formation, i.e. optical memory, which can be recovered later. For brevity, we refer to this formation as a ‘memory bit’.

Below, we discuss an exactly solvable, though realistic, case describing controlled preparation, manipulation and readout of slow-light solitons in atomic vapours and BEC. To make parameters dimensionless, we measure the time in units of optical pulse length, $t_p = 1 \mu s$ typical for the experiments on the slow-light phenomena in rubidium vapours [5]. The retarded time ($\tau$) is measured in microseconds and the Rabi frequencies are normalized to MHz. The spatial coordinate is normalized to the spatial length of the pulse slowed down in the medium, i.e. $l_p = v_p t_p \approx c(\Omega_0^2/2v_0)t_p$. In experiments with rubidium, the controlling background field is of the order of a few megahertz. We choose $\Omega_0 = 3$ as a representative value. This corresponds to the group velocity of several metres per second, depending on the density of the atoms. We take the group velocity to be $10^{-7}c$, so the pulse spatial length is $30 \mu m$ and $\zeta$ is normalized to $10^{-13}$ seconds. Then, in the dimensionless units, the coupling constant $v_0 = \Omega_0^2/2 = 4.5$.

We consider the following scenario for the dynamics (see figure 1). Before $\tau = 0$, we create in the medium a slow-light soliton, and assume it is propagating on the constant background $\Omega_0$. We then slow down the soliton by switching off the laser source of the background field. We assume an exponential decay of the background field with a decay constant $\alpha$, i.e. $\Omega_0 e^{-\alpha t}$. At a certain moment of time, say $T_1 = 4/\alpha$, the field becomes negligible. Therefore, we cut-off the exponential tail and approximate it by zero. At this step the soliton is completely stopped. The position where the soliton stops depends on the decay constant $\alpha$ and on the moment when we switch the laser off. The information borne by the soliton is stored in the form of a spatially

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localized polarization. This formation can live a relatively long time in atomic vapours or BEC [6].

At the moment \( T \), we restore the slow-light soliton by abruptly switching on the laser. The whole dynamics are divided into four time intervals,

\[
\bigcup_{i=0}^{3} D_i = (-\infty, 0] \cup [0, T_1] \cup (T_1, T] \cup (T, \infty).
\]

The time dependence of the intensity of the background field at entrance into the medium is given in figure 1.

Before the soliton enters the medium, the physical system is assumed to be prepared as

\[
\Omega_a^{(0)} = 0, \quad \Omega_b^{(0)} = \Omega(\tau), \quad |\psi_{at}\rangle\langle\psi_{at}| = |1\rangle\langle 1|.
\]

(2)

Note that the state \( |1\rangle \) is a dark state for the controlling field \( \Omega(\tau) \). This means that the atoms do not interact with the field created by the auxiliary laser. The configuration equation (2) above corresponds to a typical experimental setup (see e.g. [1, 5]). The function \( \Omega(\tau) \) models switching the controlling field off and on again. This function reads

\[
\Omega(\tau) = \Omega_0[\Theta(-\tau) + e^{-\alpha \tau}(\Theta(\tau) - \Theta(\tau - T_1)) + \Theta(\tau - T)]
\]

(cf figure 1). Here \( \Theta(\cdot) \) is the Heaviside step function. For the state equation (2), we solve exactly the nonlinear system equations (1) as well as the auxiliary scattering problem underlying its complete integrability. The latter result is the cornerstone of analytical progress achieved in this paper. This result allows us to mount a soliton on the background equation (2) using the Darboux–Bäcklund transformation [13]. The one-soliton solution corresponding to the time-dependent background equation (2) reads as

\[
\tilde{\Omega}_a = \frac{(\lambda^* - \lambda)w(\tau, \lambda)}{\sqrt{1 + |w(\tau, \lambda)|^2}} e^{i\tilde{\phi}_s} \text{sech} \tilde{\phi}_s, \quad \tilde{\Omega}_b = \frac{(\lambda - \lambda^*)w(\tau, \lambda)}{1 + |w(\tau, \lambda)|^2} e^{i\tilde{\phi}_s} \text{sech} \tilde{\phi}_s - \Omega(\tau),
\]

(3)

with the atomic state \( \tilde{\rho} = |\tilde{\psi}_{at}\rangle\langle\tilde{\psi}_{at}| \), where

\[
|\tilde{\psi}_{at}\rangle = \frac{\text{Re}\lambda - \Delta - i\text{Im}\lambda \tanh \tilde{\phi}_s}{|\lambda - \Delta|} |1\rangle + \frac{\tilde{\tilde{\Omega}}_a}{2|\lambda - \Delta| |w(\tau, \lambda)|^2} |2\rangle - \frac{\tilde{\tilde{\Omega}}_a}{2|\lambda - \Delta|} |3\rangle.
\]

(4)
This ensures that the physical variables such as the wave-function and field amplitudes evolve λ-form, specific to each time region and spatially localized polarization. At the time the solution in the region continuously. The values of the data for the region are of piecewise form, specific to each time region $D_i$. For clarity, we organize elements of the solution corresponding to different time regions in Table 1. We use an auxiliary function $w_0 = \Omega_0/(\alpha + \sqrt{\lambda^2 + \Omega_0^2})$, the index of Bessel functions is defined as $\gamma = (\alpha + i\lambda)/(2\alpha)$. The values $C_i$ of the constant $C$ for each time region $D_i$ are specified in the rightmost column of the table, the moment of time $T$ is chosen as in figure 1, i.e. $T = 4/\alpha + 3$. Note that in the table $w_2 = w_1(\infty, \lambda)$ and $z_2 = z_1(\infty, \lambda)$. Therefore, the solution in the region $D_2$ is parametrized by the asymptotic values of the data for the region $D_1$ corresponding to the absence of cut-off of the exponentially vanishing tail. The region $D_2$ describes the phase when the slow-light soliton is stopped: the fields vanish, while the information borne by the soliton is stored in the medium in the form of a spatially localized polarization. At the time $T$, the laser is instantly turned on again. The stored localized polarization then generates a moving slow-light soliton. This process is described by the solution in the region $D_3$. Except for the moment $T_1$, the functions $w$ and $z$ are continuous in $\tau$. This ensures that the physical variables such as the wave-function and field amplitudes evolve continuously.

We demonstrate typical dynamics of the intensity of the fields in figures 1 and 2. In figure 1 the decaying shock wave, whose front has an exponential profile, propagates with the speed of light, reaches the slow-light soliton and stops it. An intense and narrow peak is developing in the background field when the shock wave hits the soliton (dotted curve). This peak signifies a transfer of energy from the soliton to the background field. After the auxiliary laser has been switched on again, another step-like shock wave reaches the localized polarization formed in the medium by the incoming soliton, and retrieves stored information in the form of a new slow-light soliton. A narrow and deep depression in the intensity plot means now the energy transfer is in the opposite direction, i.e. from the background field to the restored soliton. The dynamics of the field $\Omega_2$ are plotted in figure 2. The contour plot shows that in the process of rapid deceleration, the solitonic trail profiles end sharply. Notice that the characteristics of the restored pulse, i.e. the

| $D_i$ | $\Omega_i$ | $w(\tau, \lambda)$ | $z(\tau, \lambda)$ | $C_i$ |
|------|---------|-------------|-------------|------|
| $D_0$ | $\Omega_0$ | $w_0$ | $\frac{1}{2} \Omega_0 w_0 \tau$ | $0$ |
| $D_1$ | $\Omega_0 e^{-\gamma \tau}$ | $-\alpha \gamma \tau + \ln \frac{C J_{\gamma,1}(-\frac{\Omega_0}{\gamma}) + J_{\gamma,1}(-\frac{\Omega_0}{\gamma})}{C J_{\gamma,1}(-\frac{\Omega_0}{\gamma}) + J_{\gamma,1}(-\frac{\Omega_0}{\gamma})}$ | $C_1 = C_1$ |
| $D_2$ | $0$ | $0$ | $0$ |
| $D_3$ | $\Omega_0 \tan \left( \frac{1}{2} \sqrt{\lambda^2 + \lambda_0^2} (\tau - T) \right)$ | $\frac{1}{\lambda \tan \left( \frac{1}{2} \sqrt{\lambda^2 + \lambda_0^2} (\tau - T) \right)} \left[ e^{-i \frac{1}{2} \sqrt{\lambda^2 + \lambda_0^2} (\tau - T)} - e^{-i \frac{1}{2} \sqrt{\lambda^2 + \lambda_0^2} (\tau - T)} \right]$ | $\Omega_0^2 + 2 \xi_0 (\lambda - \sqrt{\lambda^2 + \lambda_0^2}) \Omega_0^2$ |

Here, $\tilde{\phi}_s = \phi_0 + \frac{\nu_0 \xi}{\lambda} \frac{1}{2} \Im \frac{1}{\lambda - \Delta} + \Re [z(\tau, \lambda)] + \ln \sqrt{1 + |w(\tau, \lambda)|^2}$, $\tilde{\theta}_s = \theta_0 - \frac{\nu_0 \xi}{\lambda} \frac{1}{2} \Im \frac{1}{\lambda - \Delta} + \Im [z(\tau, \lambda)]$. Where $\lambda$ is an arbitrary complex parameter. The functions $w(\tau, \lambda)$ and $z(\tau, \lambda)$ are of piecewise form, specific to each time region $D_i$.
Figure 2. Contour plot of the intensity of $\tilde{\Omega}_a$. We choose $\lambda = -4.1i$ and zero detuning, $\Delta = 0$. The break-up area in between the two solitonic trails manifests the creation of a standing memory bit in the medium.

width and group velocity, are very close to those of the input signal existing in the medium before the stopping.

We now calculate the half-width of the polarization flip written into the medium after the soliton is completely stopped. It reads as

$$W_s = 4c \ln(2 + \sqrt{3}) \frac{|\Delta - \lambda|^2}{v_0 |\text{Im}(\lambda)|}. \quad (5)$$

It is important to note that the width (equation (5)) of the optical memory formation does not depend on the rate $\alpha$. In other words, the width of the memory bit is not sensitive to how rapidly, i.e. non-adiabatically, the controlling field is switched off. This leads to an important conclusion. Indeed, through the variation of the experimentally adjustable parameter $\alpha$, it is possible to control the location of the memory bit, while its characteristic size remains intact. Dutton et al [6] have already reported that when the switching is made quickly compared to the natural lifetime of the upper level, the adiabatic assumptions are no longer valid but, remarkably, the quality of the storage is not reduced in the non-adiabatic regime. Our analytical result is in excellent agreement with this observation.

The group velocity of the slow-light soliton reads as

$$\frac{v_g}{c} = \frac{|w(\tau, \lambda)|^2}{\frac{1}{2}|\Delta - \lambda|^2 + |w(\tau, \lambda)|^2}. \quad (6)$$

Notice that in the case of a constant background field, i.e. in the case $\alpha = 0$, the conventional expressions for the slow-light soliton [13] along with the expression for the group velocity, the main motivational quantity for this paper, can be readily recovered from equations (3) and (6).
Figure 3. Population of level $|2\rangle$. Here, $\lambda = -4.1i$ and $\Delta = 0$. The time interval $1 \leq t \leq 4$ corresponds to a standing localized polarization flip. Compare with experimental results in figure 1 of [5].

The distance $L_s(\alpha)$ that the slow-light soliton will propagate from the moment when the laser is switched off until the full stop is

$$L_s(\alpha) = \frac{2c|\Delta - \lambda|^2}{\nu_0 \Im(\lambda)} \phi_{s, \alpha}^{r=\infty}_{t=0}$$

$$= \frac{2c|\Delta - \lambda|^2}{\nu_0 |\Im(\lambda)|} \left[ \ln \sqrt{1 + |w_0|^2} - \Re[z(\infty, \lambda)] \right]. \quad (7)$$

It is evident from our solution (equation (7)) that the soliton possesses some inertia or, in other words, a momentum of motion. Indeed, even if the controlling field is switched off instantly (notice that $\lim_{\alpha \to \infty} \Re z(\infty, \lambda) = 0$), the soliton will still propagate over some finite distance after the shock wave of the vanishing field, propagating with the speed of light, has reached the soliton.

In figure 3, we show the dynamics of localized polarization corresponding to soliton dynamics as shown in figure 2. The plot is remarkably similar to the corresponding figure in [5] describing recent experiments. The central part of the plot in figure 3 shows the standing memory bit imprinted by the slow-light soliton. Note that in the presence of the soliton, the population flip from level $|1\rangle$ to $|2\rangle$ in the centre of the peak is almost complete. This property of the solution manifests the major distinction between the strongly nonlinear regime considered in our paper and the linear EIT theory. As was pointed out earlier [6, 10, 12], the regime of two fields being comparable in magnitude opens up new avenues for an effective control over superposition of two lower states of the atoms. On changing the parameters of the slow-light soliton, we can coherently drive the system to access any point on the Bloch sphere, which describes the lower levels. For zero detuning, the solution discussed here shows that when the field vanishes the maximum population of the second level reaches unity. Using equation (4), it is also not difficult to estimate the maximum population of the level $|2\rangle$ for finite detuning after the soliton was completely stopped: $|\lambda|^2/|\Delta - \lambda|^2$. Notice that only a small fraction of the total population is
located in the upper level $|3\rangle$ and provides for some atom–field interactions. This population is proportional to $|\Omega_0|^2$. Numerical studies of the Maxwell–Bloch equations with relaxation terms included [6, 10] show that for experimentally feasible group velocities of the slow–light pulses, i.e. when the maximum intensity of the controlling field is not very high, the pulses are stable against relaxation from the level $|3\rangle$. Here we consider the same range of parameters. Therefore, the destructive influence of relaxation on our solution (equation (3)) is negligible.

To conclude, in this paper we constructed a realistic, exactly solvable model of manipulation, i.e. preparation, control and readout of optical memory bits in the strongly nonlinear, and more importantly, non-adiabatic regime.

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