Method for Parametric Shaping of Architectural Free Forms

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Abstract. The major purpose of the paper is to point out that big shape transformations of nominally flat, thin-walled folded sheets having open profiles are possible. Principal boundary conditions can be defined for the transformed sheets, by means of the number, position and curvature of roof directrices, including intermediate members. Next, we should exploit the opportunity to rationalize their natural boundary conditions. As a result, the sheets, in particular their individual folds, can perform a function of members, stiffening elements and bracings, or may only constitute a cover for roofing. The own research and created exact numerical, thin-walled, folded models for transformed sheet are used: 1) to qualitatively and quantitatively define functions provided by shell folds in the construction of a roof and even entire building, 2) to examine such wall areas, and especially their edges, which are difficult to be recognized with the help of the traditional methods, for example the ones using mechanical strain gauges. The paper proposes an innovative, consistent description of the essential shape and mechanical changes of T85 x 0.75 profiles up to 5.0 m long, which are caused by initial big shape transformations. Next, a description of the impact of the above changes on the folds under the characteristic load is presented. It is also presented a way to shape a simplified smooth model for transformed folded shell strips of sheets, connected one to another, along their longitudinal edges, which can be used in the process of engineering structure design. Based on the curvature and mutual position of roof directrices as well as the recognized geometrical and material properties, they are defined rulings, modelling longitudinal edges of shell roof folds. A sector of warped surface modelling the roof shell can be shaped on the basis of these rulings.

1. Introduction
Today parametric description and computer programming make it possible to achieve a wide range of diversified innovative building forms [1, 2]. In the present paper, the focus is on buildings with plane glass elevations and shell steel roofs made up of nominally plane sheets folded in one direction and deliberately transformed into shell shapes as a result of being spread onto roof directrices (see figure 1) [3, 4]. A method of using such a description in searching for free forms is presented. It is constantly developed by the authors [5, 6] and extended to complex free form structures. The method is aimed at intuitive shaping of primarily geometrically integrated to entire building forms, taking into account the proportions between the dimensions of their shell roofs, plane oblique elevation walls, structural systems, regular patterns on elevations and roofs, which decide of their harmonic adaptation to the man-made and natural environments (see figure 2) [7, 8]. Thus, one of the main problems in designing such architectural free forms is how to cover the respective proportions between the dimensions of the whole shape of the designed building and all its parts as well as the built up and natural environment.
Figure 1. Two different building free forms characterized by integrated shapes of elevation and roof supported by: (a) Straight directrices; (b) Curved directrices

Figure 2. Two different building free forms characterized by integrated shapes of elevation and roof: (a) Another attractive configuration; (b) Integrated with the man-made environment.

The discussion of the topic area is complemented by more complex building free forms, covered with regular shell structures and folded elevations proposed by the authors in [7-9] (see figure 3). Regular patterns on elevations also play an important functional role, figures 1-2.

Figure 3. Two different building free form structures formed from many individual free forms: (a) In a little strange but intentional regular way; (b) With folded elevation and roof shell segments arranged regularly in three-dimensional space.

2. Critical analysis of the state-of-the-art
Forced shape transformations of folded sheets were realized by Gergely, Banavalkar and Parker [10] in the 1970s to create shallow hyperbolic paraboloid roofs and their structures named hypars. The type of the sheet’s transformations was ineffective because of the necessity of using forces acting transversally
towards the fold’s longitudinal axes and reducing the widths of all shell folds to the arbitrary lengths of the adopted roof directrices. Gioncu and Petcu in [11] developed a computer program called HYCUCK aiding the process of shaping complete hypars. Davies and Bryan in [12] propose various new configurations of hypar units.

Adam Reichhart started shaping many different roof shell free forms in the 1990s. He has used corrugated shell sheeting for roofing, where all folds underwent big elastic deformations into shell shapes [13] (see figure 4). He developed a simple method for geometrical and strength shaping of such shell roofs [4]. According to his method, each roof shell sheet, or sometimes even each of its shell folds, is modelled with a central sector of a right hyperbolic paraboloid limited by a spatial quadrangle composed of four straight segments [14].

![Figure 4. Two complex roof structures composed of many intentionally transformed corrugated shell sheetings spread on: (a) Curved directrices; (b) Straight directrices](image)

Jacek Abramczyk assumed that the great freedom in shaping diversified transformed shell forms for roofing (see figure 5), can be used to integrate and increase the attractiveness of the entire building forms, figure 3. Consequently, he decided to incline and fold elevation walls to the vertical depending on the shape of shell roof and entire building [8, 9], figures 1, 2. He noticed that there is interdependence between the efficiency of the roof sheeting transformation and the location of its contraction on the length of each roof fold in the shell roof [3]. The relation strongly influences the attractiveness of the entire form and the integrity of the shape of the roof and elevation [15, 16].

![Figure 5. Two experimental corrugated shell sheetings spread on: (a) Curved directrices; (b) Straight directrices](image)

Aleksandra Prokopska conducted multivariate interdisciplinary analyses of consistent morphological systems that can be designed in harmony with the natural or man-made environments [17, 18]. As a result of these analyses she presented a wide range of theoretical solutions and their application in the architectural practice [19]. The solutions involve many interdisciplinary topics needed to develop experience in shaping attractive different architectural free forms and their visualizations [20, 21].
Some main principles of shaping complete and compound innovative free forms presented in [16, 21] are the result of the cooperation between Aleksandra Prokopska and Jacek Abramczyk, see figure 6. On the basis of these principles they invented a preliminary version of a method for parametric shaping of the above-mentioned particular forms, figure 7.

![Figure 6](image)

**Figure 6.** Two experimental corrugated shell sheeting’s spread on: (a) Curved directrices; (b) Straight directrices

The method is aimed at intuitive shaping of architectural free forms, developed by Jacek Abramczyk in [6]. In the present paper, it is extended to the shaping of a regular pattern on elevations to improve the free form visual attraction [22].

![Figure 7](image)

**Figure 7.** Visualization of a complex free form with folded elevations and segmented shell roof

The proposed type of free form buildings requires innovative structural systems transferring loads from steel roofs [4, 23] and glass elevations [24, 25] to the foundations.

### 3. Aims and scope of the paper

The main aim is to present the author’s method that gives the designer the possibility of searching for architectural free forms of buildings and using his intuitive 3D reasoning. The method is dedicated to shaping innovative shapes of buildings roofed with effectively transformed corrugated sheeting. The effective shape transformations are detailed defined in [3]. They consist in providing each fold in shell sheeting freedom of transversal increments in width and height during their adapting to the respective supporting conditions in order to reduce the initial stresses caused by the effective transformations. In such a way, it is eliminated the effect of mutual influence of adjacent folds in the sheeting or necessity to provide additional, forced shape transformation adjusting the incorrectly calculated fold width to the designed spacing of the support points at fold’s ends.
The parametric description exploited by the method is based on some specific proportions between the dimensions of the roof, elevations and the entire free form as well as the pitch of the elevation pattern. The method is aided by the authors’ computer programs written in the AutoCAD programming language. The programs are constantly upgraded and their description goes beyond the scope of this paper.

4. Concept of the research

The specific objective of the project taken by the authors is to help the user, i.e. designer, to find fine proportions between variables used in the quantitative description of the followings. Overall shape of the free form has to be formed. Essential dimensions of various parts of elements of the free form including columns, girders, elevation facets and roof shell segments have to be created. Patterns on the elevation walls or roof shell structure should be made. Such a description ought to enable the designer to integrate the general shape of the created free form and adapt it to the man-made environment.

Firstly, the method requires adopting values of the parameters describing transversal dimensions of the created free form. They constitute measurements of a sum of four auxiliary rectangles \( \Delta \) \( i = 1 \) to 4, figure 8. There are eight parameters: \( a_1 = |C_1C_2|, b_1 = |C_2C_3|, c_1 = |C_3C_6|, d_1 = |C_4C_5|, a_2 = |C_1C_{15}|, b_2 = |C_{15}C_{17}|, c_2 = |C_6C_{11}| \) and \( d_2 = |C_{10}C_{11}| \) describing the sum.

The required mutual location of these rectangles \( \Delta \) has to be adopted in a very specific manner, that is they are coplanar and each of these rectangles has to have one edge contained in the z-axis. In addition, rectangles \( \Delta_1 \) and \( \Delta_4 \) have to possess common vertex \( C_1 \).

Secondly, a plane reference polygon \( Pr \) is created as a result of the connection of the chosen rectangle’s vertices by means of straight segments. The vertices are as follows: \( C_1, C_2, C_5, C_8, C_9, C_{11}, C_{15}, C_{16} \), see figure 8(b). The proportions between the areas of triangles \( C_5C_8C_9 \) and \( C_5C_{16}C_{11} \) as well as between the areas of the plane quadrangles \( C_1C_2C_8C_9 \) and \( C_1C_9C_{16}C_{15} \) of \( Pr \) express the analogous relations between respective parts of the roof and elevations of the shaped free form in an approximate manner under the adopted direction of observation. Thus, the shape integration of roof and elevations are estimated visually.
Thirdly, a reference tetrahedron $\Gamma$ is obtained as a result of an extension of $P_r$ into a spatial figure, see figure 9. Thus, a few lines perpendicular to $P_r$’s plane have to be drawn. Vertices $E_2$, $E_5$, $E_{15}$, $E_{16}$, $F_2$, $F_8$, $F_{11}$ and $F_{15}$ of $\Gamma$ can be found as the points of the intersection of the above-mentioned lines and planes $\gamma_1$ and $\gamma_3$ adopted as the planes of two gable walls of $\Gamma$.

A large variety of free forms is obtained by changing two collections of parameters. These include the proportions between the length of $\Delta_i$’s edges expressed by eight parameters, and the locations and inclination of planes $\gamma(E_2, E_5, E_{15})$ and $\gamma(F_2, F_8, F_{15})$ to the vertical represented by four parameters describing the location of $\gamma_1$ and $\gamma_3$ towards the plane $(x, z)$, where $\gamma_1$ and $\gamma_3$ are parallel to $x$-axis, see figure 9(b). The reference polygon $P_r$ is extended into a spatial shape $\Gamma$, where the distances between the vertices of $P_r$ and the vertices of $\Gamma$ have been strictly defined by $\gamma_1$ and $\gamma_3$.

A closed spatial quadrangle $E_5F_8F_{11}E_{16}$ is created in four planes of $\Gamma$. The quadrangle represents the eaves of the roof shell. One pair of the opposite segments of the eaves line produces two roof directrices, which may be straight, as here, or curvilinear, whereas two other opposite segments of the eaves are straight as referring to two straight edges of shell roof sheeting. On the basis of the above-mentioned skew directrices contained in the opposite planes of the reference tetrahedron, a warped shell roof is determined. The roof surface limits the tetrahedron $\Gamma$ from the top.

Finally, an elevation pattern is formed by means of two families of straight lines creating two bundles of lines, figure 1, including they may be drawn at constant distances or intersect at one point. The roof thickness and cantilever protruding out of the elevation plane affect some main proportions characterizing the free form, its attractiveness and internal integration.

![Figure 9](a) $\gamma$ modelling one gable plane; (b) $\gamma$ modelling the opposite gable plane

5. Searching for attractive shape proportions

A way of adopting proportions between the parameters described in the previous section is proposed in [6]. He presents an application of the method for searching diversified attractive building free forms, where the number of variables is limited. In the simplest case, he proposes to adopt one independent variable and to relate the other parameters to this variable, for example to $a = a_1 = a_2$. Following the method, the diagram presented in figure 10 is discussed in what follows.
Figure 10. Parallelogram limited by lines $p_a$ and $p_k$ and expressing fine proportions between $a$, $b$ and $c$ covering attractive shapes

The basic proportions adopted for $\Gamma$ employed in previous section, figure 10, are: $b_1 = 2a_1$, $c_1 = 2.5a_1$, $d_1 = 1.5a_1$, $k_1 = |C_2F_2| = a_1$, $tg(\zeta) = 5/7$, where $a_1 = a = 5000.0$ mm, $\zeta_1$ – measure of the angle between $\gamma$ and $(x, z)$, $\zeta_2$ – measure of the angle between $\gamma$ and $(x, z)$, $a_2 = 0.75a_1$, $b_2 = 0.75b_1$, $c_2 = 0.75c_1$, $d_2 = 0.75d_1$, $k_2 = k_1$ and $\zeta_2 = \zeta_1$. This configuration of $\Gamma$ is represented by point $P_w$ in figure 10. Here $P_w$ is drawn near line $p_k$, which ensures that the proportions used will meet the expectations related to the visual attractiveness of the designed free form and can lead to fine integration with the man-made surroundings.

6. Searching for attractive regular patterns on elevation walls

Regular patterns on elevations together with general shapes and proportions between the main parts of the free forms like elevations, roofs and structural systems play an important role in improving the visual attractiveness of these forms and their harmonic integration with man-made environments. Therefore, the designing manner such patterns by means of visible elements supporting elevation glass plates or filling the areas between these plates is presented below, see figures 11-13. However, the members of the structural system may also be the visible elements forming the regular pattern.

Moreover, the patterns used here are reduced to only one type formed from five strips of glass plates close to the vertical and five inclined strips close to the horizontal. These strips are divided by oblique columns and spandrel beams, figure 11. The presented pattern forms a network of quadrilateral plane glass “cells” that are a result of dividing each elevation wall edge at five mutually congruent segments.

The thickness of a roof together with the elevation part unfilled with glass beneath the roof create a strip whose height is also important in the process of architectural shaping the elevations of great attractiveness, as seen in figure 11. The constant division of elevation edges may produce a fine impression of harmony and integration of all elements and the entire free form due to the compatible changes of: the height of each window row together with the change of the elevation height and the width of each window column together with the change of the elevation width.

A visualization of two different free forms characterized by a constant height of all horizontal strips and situated at a square between a few high building is presented in figure 12. Their reference polygons and polyhedrons are z-axis symmetrical. An additional goal may be achieved if the length of the cantilever and thickness of the roof depend upon the width and height of the elevation rows and columns. Similarly, the inclination, height and width changes of windows rows and columns can depend on the shapes and elevation patterns of the adjacent buildings to adapt the designed free form to the man-made environments.
Figure 11. Constant height of the window row ends along the elevation wall edges and constant width of the subsequent windows columns in the horizontal direction: (a) scheme of an elevation wall; (b) model of a free form

Figure 12. Two free forms characterized by constant height of all horizontal strips and situated at a square between a few higher buildings

Figure 13. Linear variation of the height of window row ends along elevation wall edges and linear variation of the width of subsequent windows columns in the horizontal direction: (a) scheme of an elevation wall; (b) model of a free form
Variable division of elevation edges along the ground and eaves can also be satisfying and may lead to further improvement of the attractiveness of architectural free forms. Consequently, a very interesting effect is obtained when the height of windows rows is increased in conformity with increasing the height of the whole elevation wall, here from the right side to the left one. This effect is also achieved when the width of window columns is increased together with increasing the level of the eaves, as presented in figure 13.

7. Conclusions

The usage of spatial reasoning in searching for attractive building free forms of the considered type is a special philosophy, art and science of architecture design. The innovative method of architectural design was elaborated by the authors to shape attractive buildings characterized by free shell roofs integrated with plane elevations inclined to the vertical, where users are eager to spend time and feel good. It is based on 3D intuitive reasoning. Creating the proposed auxiliary flat reference polygons expressing some crucial proportions between the dimensions of roofs and elevations, and next extending each polygon into a reference spatial tetrahedron require such an ability of intuitive spatial reasoning and creating compound shapes in three-dimensional space.

The presented innovative author’s method employs the parametric description of these proposed forms, which enabled the authors to write a few preliminary computer programs in the AutoLISP language of AutoCAD programming. Computer applications written by Jacek Abramczyk facilitate the process of searching for attractive consistent building free forms. In addition, the programs enable designers to adapt their forms and elevation patterns to built environments. To simplify the procedures realized by the method, a number of independent variables can be freely reduced depending on the expected complexity, attractiveness and innovative nature of the designed architectural free form.

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