The nature of the electroweak Higgs sector

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Abstract. In this talk I will first discuss the main features of electroweak symmetry breaking in the Standard Model and in particular I will focus on the Higgs sector responsible thereof. I will highlight the deficiencies of the Standard Model both from the experimental (Dark Matter, baryogenesis,...) and theoretical (ultraviolet sensitivity of the Higgs mass, unexplained flavor textures in the quark and lepton sectors, lack of unification of gauge couplings, strong CP problem,...) points of view which largely motivate extending it to Beyond the Standard Model scenarios. We then briefly review some of the scenarios which aim at solving some of the above problems. In particular we have discussed the minimal supersymmetric extension of the Standard Model, Little Higgs models, gauge-Higgs unification scenarios and the possibility of the Higgs living in a conformal sector (un-Higgs) as a solution to the hierarchy problem. In all cases we have focused on the electroweak sector of the models and the particular solution they offer to the Standard Model problems.

1. Introduction
The Standard Model (SM) of electroweak interactions [1] is a theory where electromagnetic and weak interactions unify in the gauge group $SU(2) \times U(1)_{Y}$ which is spontaneously broken to the electric charge gauge theory where the photon remains as the only massless interaction carrier while the other three gauge bosons $W^{\pm}$, $Z$ are massive. In the Standard Model left-handed quark and leptons come in $SU(2)$ doublets while right-handed ones come in $SU(2)$ singlets and only feel the hypercharge interactions.

A mechanism to spontaneously break the $SU(2) \times U(1)_{Y}$ symmetry and give a mass to the gauge bosons $W^{\pm}$, $Z$ and fermions was proposed in Ref. [2] where an $SU(2)$ scalar doublet was introduced whose neutral component acquires a vacuum expectation value: the scalar doublet is nowadays known as the Higgs boson. The interactions of gauge bosons (gauge interactions) and fermions (Yukawa interactions) with the Higgs field generate the desired spectrum of masses after the electroweak breaking. One of the most interesting features of the Higgs mechanism is that it unitarizes the elastic scattering of longitudinal gauge bosons whose amplitude would otherwise rise linearly with $s$ and violate the unitarity bound for values of $s$ beyond the TeV range. In the presence of the Higgs boson the theory remains unitary at any scale provided the Higgs weighs less than $\sim 1$ TeV.

The agreement of the Standard Model with high and low energy experimental data in particle physics experiments is impressive [3] and there is no serious hint of any need of physics beyond it to describe all the particle physics data. There are however some experimental and theoretical deficiencies of the Standard Model which require physics beyond it to describe and/or explain some phenomena. In particular there should be among our physical fields one which is stable enough to explain the amount of Dark Matter in the universe: there is no such candidate in the
Standard Model. Also theoretical mechanisms to explain the quantity of baryonic matter in our universe (related to the number of photons) fail inside the Standard Model, which gives another motivation to go beyond it. From the purely theoretical point of view, and since the Standard Model is an effective theory with a cutoff which can be at most $M_{\text{Planck}}$, the Higgs squared mass shows a quadratic sensitivity to the cutoff scale which makes the model completely unnatural and requires a huge fine-tuning. This is known as the hierarchy problem and it has motivated much of the physics beyond the Standard Model which exists in the literature.

There is a tension between the TeV scale (the scale at which the Standard Model is natural without the requirement of any fine tuning) and the 10 TeV scale (the scale corresponding to the non-appearance of new physics at LEP if we assume that dimension-six effective operators have coefficients of $O(1)$) which gives rise to the so-called little hierarchy problem. From the phenomenological point of view and having in mind the forthcoming LHC results it is enough to solve the little hierarchy problem and some theories have been proposed to this end. Of course if the coefficients of the dimension-six effective operators are suppressed by factors of order $10^{-2}$ then the little hierarchy problem disappears: this is the case of supersymmetric theories where those coefficients are suppressed by loop factors.

The hierarchy problem is not the only motivation to go beyond the Standard Model: there is a number of other problems which do not find an explanation within the pure Standard Model and which we hope can be explained in some more fundamental theory: why there are apparently only three generations of quarks and leptons, how can we explain the quark and lepton textures (masses and mixing angles), why the strong CP-violating parameter is so-small (strong CP-problem).... In this talk a will make a brief review of some extensions of the Standard Model and the way they aim to solve some of the problems we have just described, although I will pay particular attention to the hierarchy problem. In all cases I will concentrate on the electroweak breaking sector.

The outline of this talk is as follows. In Section 1 I will review some of the main features of the Standard Model, including the theoretical and experimental bounds and will precise the deficiencies of it we have just described. In Section 2 I will review some of the essential features of the minimal supersymmetric extension of the Standard Model and in particular the solution to the hierarchy problem, candidates to Dark Matter, gauge coupling unification and radiative electroweak breaking. In Section 3 I will cover the Little Higgs models and in particular some of their most popular scenarios as Simplest Little Higgs and Littlest Higgs models where the little hierarchy is solved by a set of global symmetries, what is called collective breaking. In Section 4 I will discuss gauge-Higgs unification models based on theories with extra dimensions. In these theories the Higgs is identified with an extra-dimensional component of a gauge boson and the Higgs mass is protected by the higher-dimensional gauge symmetry. In Section 5 I will briefly describe an approach where the Higgs belongs to a strongly coupled conformal theory (an unparticle or un-Higgs). In this case the hierarchy problem is alleviated because the dimension of the "mass" term in the Lagrangian is $4 - 2\gamma$, where $\gamma$ is a large anomalous dimension. For instance if $\gamma$ is close to 2 then the radiative corrections to the Higgs mass are close to logarithmic. Finally in Section 6 I have included my conclusions which will depend to a large extent on the forthcoming LHC results. In all cases I have described how the different theories solve the different problems and what are in turn the drawbacks they present.

In all cases the Higgs sector is a keystone in the corresponding models. Since it is the only missing particle in the Standard Model it is the most interesting sector both from the experimental and from the theoretical point of view. Needless to say this talk would have been completely different if the Higgs would have already been discovered. For instance if a charged Higgs is found this would automatically imply that the Higgs sector has to be extended, as it happens for instance in supersymmetric theories. Depending on its mass some models would probably be easily excluded although in some cases we would need a precision machine (as e.g. a
linear collider) to disentangle the fine structure of its couplings. For instance if the Higgs is found to be heavier than the triviality bound ($\sim 190$ GeV) this would imply that new physics should appear at some scale below the Planck scale or if the Higgs is found heavier than $\sim 140$ GeV then the minimal supersymmetric extension of the Standard Model is automatically excluded. Finally if the Higgs is close to its experimental LEP limit this would imply that we are living in a metastable vacuum, although with a decay lifetime longer than the present age of the universe.

2. The Standard Model of electroweak interactions

In the Standard Model the electroweak symmetry $SU(2) \times U(1)$ is spontaneously broken to the electric charge $U(1)_{em}$ by the Higgs mechanism by which an $SU(2)_L$ doublet Higgs boson

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$  \hspace{1cm} (1)$$

is needed, with a Lagrangian

$$\mathcal{L}_{Higgs} = |D_\mu H|^2 + \mathcal{L}_Y - \frac{\lambda}{2} \left[ |H|^2 - \frac{v^2}{2} \right]^2.$$  \hspace{1cm} (2)$$

The term $|D_\mu H|^2$ gives a mass to gauge bosons $W$ and $Z$, which absorb the Goldstone bosons $H^+$ and $ImH^0$, the term $\mathcal{L}_Y$ gives a mass to SM fermions while the potential term fixes the vacuum expectation value (VEV) of the neutral component of the Higgs field to the experimental value $v = 246.22$ GeV. In particular the Higgs can “regularize” the bad UV behaviour of gauge bosons with longitudinal polarization $e^{\mu}_L \simeq p^\mu/M_V$.

2.1. Unitarity bounds

The Higgs unitarizes the scattering of longitudinal gauge bosons [4]. Using the partial wave decomposition $A = 16\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta)a_{\ell}$, $\sigma = \frac{16\pi}{\ell} \sum_{\ell} (2\ell + 1) |a_{\ell}|^2$ and the optical theorem $\sigma = \frac{4}{s} Im A(\cos \theta = 1) \Rightarrow Im(a_{\ell}) = |a_{\ell}|^2$ one gets the unitarity bound

$$|Re(a_{\ell})| \leq \frac{1}{2}$$  \hspace{1cm} (3)$$

An application of bound (3) to the elastic scattering $W_LW_L \rightarrow W_LW_L$ from the pure gauge quartic coupling gives the bound on the squared energy $s$

$$W_L \rightarrow \rightarrow \rightarrow \rightarrow W_L$$

$$A \propto g^2 \frac{s^2}{M_W^4} \Rightarrow s \leq M_W^2$$

When including the cubic gauge interactions one gets a milder bound on $s$

and when the SM Higgs is included one gets the bound on the Higgs mass Finally after including the $Z_LZ_L$ elastic scattering one gets the stronger bound $m_H \leq 780$ GeV.

2.2. Theoretical bounds

The Higgs mass $m_H^2 = 2\lambda v^2$ is an independent parameter in the SM. However radiative corrections to the quartic coupling, which are controlled by renormalization group equation (RGE)

$$8\pi^2 \frac{d\lambda}{d\log \Lambda} = 3(4\lambda^2 + 2h_i^2\lambda - h_i^2) + \ldots$$  \hspace{1cm} (4)$$

generate two bounds [5] on $m_H$ assuming that the SM is valid up to a given scale $\Lambda$: 

\[ a_0 = \frac{g^2 s}{16\pi M_W} \Rightarrow \sqrt{s} \leq 1.7 \text{ TeV} \]

\[ a_0 = \frac{g^2 m_T^2}{64\pi M_W} \Rightarrow m_H \leq 1.2 \text{ TeV} \]

• For large values of \( \lambda \) (large Higgs masses) there is a Landau pole for some value of \( \Lambda \): the triviality bound
• For small values of \( \lambda \) (small Higgs masses) the quartic coupling becomes negative for some value of \( \Lambda \): stability bound

2.2.1. Triviality bounds For large Higgs masses RGE are dominated by the \( \lambda \) coupling in the RGE (4) and \( \lambda \) increases with \( \Lambda \) approximately as

\[ \lambda(\Lambda) \simeq \frac{m_H^2}{2v^2 - \frac{3m_H^2}{2\pi^2} \log \frac{\Lambda}{v}} \]

until it reaches the Landau pole at \( \Lambda_0 \) where \( \lambda(\Lambda_0) \to \infty \)

• For fixed \( \Lambda \) there is a lower bound on the Higgs mass as \( m_H^2 \leq \frac{4\pi^2 v^2}{3 \log(\Lambda/v)} \)
• For fixed \( m_H \) there is an upper bound on \( \Lambda \) at \( \Lambda \leq v \exp(4\pi^2 v^2/3m_H^2) \)
• For \( \Lambda \to \infty \) the theory is trivial \( \lambda \to 0 \)

An accurate numerical analysis has been performed in Ref. [6] where for \( \Lambda = 10^{19} \text{ GeV} \) the upper bound

\[ m_H[\text{GeV}] < 180 \pm 4 \text{ (th.)} \pm 5 \text{ (exp.)} \]

is found where the first error is from the theoretical uncertainty and the second one from the uncertainty in the top quark mass.

2.2.2. Stability bounds For small Higgs masses the RGE (4) are dominated by the term \( h_T^2 \) and \( \lambda \) decreases with \( \Lambda \) approximately as

\[ \lambda(\Lambda) \simeq \lambda(v) - \frac{3}{8\pi^2} h_T^4 \log \frac{\Lambda}{v} \]

• When \( \lambda(\Lambda) < 0 \) the potential is unbounded from below
• For fixed \( \Lambda \) there is a lower bound on the Higgs mass \( m_H^2 \geq \frac{3h_T^2 m_T^2}{2\pi^2} \log \frac{\Lambda}{v} \)
• For fixed \( m_H \) there is an upper bound on \( \Lambda \) at \( \Lambda \leq v \exp(2\pi^2 m_H^2/3h_T^2 m_T^2) \)
Numerical analysis for $\Lambda = 10^{19}$ GeV provides the lower bound on the Higgs mass given by [7, 8]:

$$m_{H}^{\text{stab}}[\text{GeV}] > 130.2 + 2.88 \frac{m_t - 173}{1.5} - 1.42 \frac{\alpha_s(M_Z) - 0.118}{0.002} \pm 3 \text{ (th.)} \quad (8)$$

where the last error is an estimate of the uncertainty in the theoretical calculation.

2.2.3. Metastability bounds  Stability bounds on the Higgs mass are based upon imposing that the electroweak minimum to be the true (lowest or unique) minimum of the theory. However strictly speaking this condition, although sufficient for the consistency of the theory is by no means necessary. In fact the situation where the electroweak minimum is a false one but the decay rate into the (unphysical) true vacuum is slower than the expansion rate of the universe at the corresponding (zero or finite) temperature should provide the necessary condition for the consistency of the electroweak theory. To study the tunneling rate one need to consider the SM at finite temperature and compute the bounce solution extrapolating between the false and true minima. This has been considered in Ref. [9] where the lower bound on the Higgs mass was computed as a function of the scale $\Lambda$. For $\Lambda = 10^{19}$ GeV one obtains the numerical result [9]

$$m_{H}^{\text{meta}}[\text{GeV}] > 121.9 + 3.50 \frac{m_t - 173}{1.5} - 1.61 \frac{\alpha_s(M_Z) - 0.118}{0.002} \pm 3 \text{ (th.)} \quad (9)$$

Given the most recent D0 and CDF combined results on the top mass [10]

$$m_t = 173.1 \pm 0.6 \text{ (stat.)} \pm 1.1 \text{ (syst.)} \text{ GeV} \quad (10)$$

and the value world average value of the strong coupling [11]

$$\alpha_s(M_Z) = 0.1176(20) \quad (11)$$

Eqs. (8) and (9) provide the bounds

$$m_{H}^{\text{stab}} > 130.2 \pm 4.3 \text{ (exp.)} \pm 3 \text{ (th.)} \text{ GeV}, \quad m_{H}^{\text{meta}} > 122 \pm 5.1 \text{ (exp.)} \pm 3 \text{ (th.)} \text{ GeV} \quad (12)$$

2.3. Experimental bounds

Present experimental bounds on the Higgs mass are based on direct searches of the Higgs boson at LEP and Tevatron, and on the comparison on the SM predictions of electroweak observables with the corresponding experimental data at LEP and low energy experiments, the so-called indirect searches.

2.3.1. Indirect searches The Higgs mass enters the quantum corrections of electroweak observables, in particular through the $\rho = 1 + \Delta \rho = 1 + T$ and $S$ parameters where [12]

$$T = \frac{\Pi_{33}(0) - \Pi_{+-}}{M_W^2} \approx \frac{3G_F}{8\pi^2\sqrt{2}} \left[ m_t^2 - (M_Z^2 - M_W^2) \log \frac{m_H^2}{M_Z^2} \right] \quad (13)$$

$$S \propto \Pi'_{33}(0) \approx \frac{1}{6\pi} \log \frac{m_H}{M_Z} \quad (14)$$

Figure 1 shows the $\Delta \chi^2$ curve derived from high-$Q^2$ precision electroweak measurements [13], performed at LEP and by SLD, CDF, and D0, as a function of the Higgs-boson mass, assuming the Standard Model to be the correct theory of nature. The preferred value for its mass, corresponding to the minimum of the curve, is at 90 GeV, with an experimental uncertainty of +36 and -27 GeV. This result is only little affected by the low-$Q^2$ results such as the NuTeV measurement discussed above. The precision electroweak measurements tell us that the mass of the Standard-Model Higgs boson is lower than about 163 GeV (one-sided 95 % CL upper limit derived from $\Delta \chi^2 = 2.7$ for the blue band). This limit increases to 191 GeV when including the LEP-2 direct search limit of 114 GeV shown in yellow.
2.3.2. Direct searches  The Higgs boson is also searched for directly. The LEP Higgs Working Group combines the results on the direct searches for the Higgs boson at LEP. Non-observation of the Higgs at LEP-2 in the process $e^+e^- \to ZH$ imposes the direct lower bound [14]

$$m_H > 114.4 \text{ GeV} @ 95\% \text{ CL}$$  \hspace{1cm} (15)

Towards the end of the LEP running in the year 2000, tantalising hints for the direct observation of the Higgs signal, corresponding to a Higgs-boson mass around 116 GeV, have been detected; a mass value well compatible with the constraints derived from the precision electroweak measurements [15]

Finally, based on the decay channel $H \to WW^* \to \ell\nu\ell\nu$ the Tevatron has just provided an exclusion window for the Higgs mass in the range [16]

$$160 \text{ GeV} < m_H < 170 \text{ GeV} @ 95\% \text{ CL}$$  \hspace{1cm} (16)

Therefore if we combine direct and indirect searches bounds there are two allowed windows at 95\% CL

$$115 \text{ GeV} < m_h < 160 \text{ GeV}, \quad \text{and} \quad 170 \text{ GeV} < m_h < 191 \text{ GeV}.$$  \hspace{1cm} (17)

Of course the above figures are not the final ones since Tevatron is accumulating data very efficiently and will improve very quickly on the excluded range. On the other hand were we taking only 90\% CL the upper Tevatron limit essentially overlaps with the LEP lower bound and the only allowed window will reduce to $115 \text{ GeV} < m_h < 155 \text{ GeV}$.

2.4. Standard Model drawbacks  
In spite of the amazing agreement with experimental data we believe the SM is not the ultimate theory because it presents a number of drawbacks:

- Big Hierarchy problem [17]: The Higgs mass is sensitive to the UV physics. Quantum corrections are quadratically sensitive to the cutoff $\Lambda$ as

$$\Delta m_H^2(F, B) = \frac{n_{F, B} g_{F, B}^2}{16\pi^2} \Lambda^2$$  \hspace{1cm} (18)
In other words they are not protected by any symmetry which is enhanced when $m_H = 0$. On the contrary fermions masses

$$\Delta m_F \propto \frac{m_F}{16\pi^2} \log \Lambda$$

are protected by chiral symmetry for $m_F = 0$.

- There is no candidate to Dark Matter inside the SM.
- There is no gauge coupling unification.
- There is no explanation for the lepton and quark Yukawa textures, i.e. masses and mixing angles [11].
- There is a strong CP-problem: an axion required [18].
- There is no explanation for the baryon asymmetry in the SM. Even if all the required ingredients are present (B violation, C and CP violation, and out-of-equilibrium in the first order phase transition) they fail to quantitatively reproduce the observed amount of baryon-to-photon ratio [19].

In particular the leading quantum correction to the Higgs mass parameter is expected to come from the top sector as

$$\Delta m_H^2 = -\frac{3h^2_t}{8\pi^2}\Lambda^2$$

In the absence of tuning this implies a lower bound on the cutoff scale as

$$\Lambda < 600\, GeV \left(\frac{m_H}{200\, GeV}\right)$$

Why did LEP not detect any deviation from the SM predictions? This is known as the LEP paradox [20]. In particular one can parametrize the new possible effects as non-renormalizable ($d = 6$) operators

$$L_{\text{eff}} = \frac{c_i}{\Lambda^2} (\bar{e}\gamma^\mu e)^2 + \ldots$$

If $c_i = O(1)$ then

$$\Lambda > 10 \, TeV.$$  

The tension between the bounds in Eqs. (21) and (23) is known as little hierarchy problem.

### 3. The minimal supersymmetric extension of the Standard Model

The big hierarchy problem can be solved in particular by introducing supersymmetry [21]. Here we will concentrate on the minimal supersymmetric extension of the SM (MSSM).

In particular the Higgs sector of the MSSM [22] is an extended one with two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}_{1/2}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}_{1/2}$$

- After the Higgs mechanism $\langle H_1^0 \rangle = v_1$, $\langle H_2^0 \rangle = v_2$, $\tan \beta = v_2/v_1$ there are five physical Higgs fields left: two scalar ($h, H$), one pseudoscalar ($A$) and two charged ($H^\pm$).
- Supersymmetry has to be broken, e.g. by embedding the MSSM into a local supersymmetry.
- The Higgs spectrum is determined by two free parameters: $m_A$ and $\tan \beta$.

$$m_{H^\pm} = m_A^2 + M_W^2, \quad m_{h,H}^2 = \frac{1}{2} \left[ m_A^2 + M_Z^2 \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \right]$$

where $m_A$ is the mass of the pseudoscalar Higgs. We will briefly describe how in the MSSM some of the SM drawbacks are solved:
3.1. The big hierarchy problem
- Because quantum corrections to the Higgs mass from bosonic and fermionic loops have opposite signs there is a cancellation between supersymmetric partners. Supersymmetry protects the Higgs mass.
- When supersymmetry is broken by soft terms the supersymmetric cancellation holds up to supersymmetry breaking terms. The quadratic divergences are still absent.
- The hierarchy problem is technically solved by the non-renormalization theorems of supersymmetry [23].

3.2. Dark Matter
There is a natural candidate for Cold Dark Mater in the MSSM: the lightest neutralino, provided that $R$-parity is unbroken [24].

3.3. Gauge coupling unification
Consistently with LEP measurements and if superparticles are at $3.4$. Electroweak breaking
\begin{align}
M_{\text{GUT}} &= \frac{2\pi}{\alpha_{\text{GUT}}} \left[ \frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} \right], \\
\frac{1}{\alpha_{\text{GUT}}} &= \frac{1}{\alpha_2(M_Z)} - \frac{b_2}{b_1-b_2} \left( \frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} \right)
\end{align}
where $g_2$ and $g_1 = \sqrt{3}/3$ $g'$ are the $SU(2)$ and $U(1)$ gauge couplings normalized to unify in $SU(5)$, and $(b_1, b_2, b_3) = (33/5, 1, -3)$ the $\beta$-function coefficients for the MSSM. After using the input values
\begin{align}
\frac{1}{\alpha_1(M_Z)} &= 58.98, \\
\frac{1}{\alpha_2(M_Z)} &= 29.57, \\
\frac{1}{\alpha_3(M_Z)} &= 8.40 \pm 0.14
\end{align}
one gets a unification scale $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV and a unification coupling $\alpha_{\text{GUT}} \sim 1/24$.

3.4. Electroweak breaking
If soft breaking parameters are generated at $M_{\text{GUT}}$ a tachyonic mass can be triggered by RGE at the weak scale. To understand this phenomenon it is sufficient to write the RGEs for the Higgs mass terms $m_{1,2}^2$,
\begin{align}
\frac{d m_1^2}{d \log Q} &= \frac{1}{16\pi^2} \left\{ -6g^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 \right\}, \\
\frac{d m_2^2}{d \log Q} &= \frac{1}{16\pi^2} \left\{ -6g^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + 6h_1^2 (m_Q^2 + m_U^2 + m_D^2 + A_U^2) \right\}
\end{align}
where $M_{1,2}$ are the corresponding Majorana masses for electroweak gauginos, $m_{Q,U}^2$ the soft breaking masses for the corresponding squark fields, $A_t$ is the trilinear soft-breaking which corresponds to the top Yukawa term in the superpotential and we have neglected the bottom Yukawa coupling, which is a reasonable assumption for not vary large values of $\tan \beta$. Notice that since $H_1$ couples to the bottom quark and we have neglected its Yukawa coupling the evolution of $m_1^2$ is governed only by gauge interactions which make it to increase when the renormalization scale goes down from $M_{\text{GUT}}$ to the electroweak scale. However since $H_2$ couples to the top quark and its corresponding contribution in the RGE is dominating that of the gauge couplings and has an opposite sign, the evolution of $m_2^2$ makes it to decrease when the renormalization scale goes down from $M_{\text{GUT}}$ to the electroweak scale and eventually becomes negative at the low scale which signals electroweak breaking induced by radiative corrections.
3.5. Stability/triviality problems

- The stability ($\lambda < 0$) and triviality/Landau pole ($\lambda \to \infty$) problems are solved because of the supersymmetric relation

$$\lambda = \frac{1}{8} (g^2 + g'^2)$$  \hspace{1cm} (29)

- Thus because the gauge couplings remain perturbative (and positive) up to $M_{GUT}$ there is no stability and/or triviality problem in the MSSM.

- As a consequence the Higgs mass in the MSSM (unlike in the SM) is not a free parameter. In particular for the SM-like Higgs one can write in the one-loop approximation the following expression for its physical squared mass:

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \left[ \log \frac{m_t^2}{m_t^2} + \frac{A_t^2}{M_S^2} \left( 1 - \frac{A_t^2}{12M_S^2} \right) \right]$$  \hspace{1cm} (30)

where $M_S$ is the common supersymmetry breaking soft mass for squarks and $A_t$ the trilinear soft-breaking which corresponds to the top Yukawa term in the superpotential, although much more accurate expressions are computed analytically and numerically in the literature [3].

- Therefore the Higgs mass is a prediction in a supersymmetric theory which implies strong theoretical constraints on the MSSM.

3.6. Theoretical constraints

The Higgs mass is a prediction in the MSSM:

- At the tree level there exists the absolute bound $m_h^2 \leq M_Z^2$ which would already be ruled out by the LEP Higgs direct searches at CERN [14].

- However at one-loop there is an important contribution controlled by the top/stop sector

$$\Delta m_h^2 = \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \left[ \log \frac{m_t^2}{m_t^2} + \frac{A_t^2}{M_S^2} \left( 1 - \frac{A_t^2}{12M_S^2} \right) \right].$$  \hspace{1cm} (31)

- Even if the one-loop contribution can be larger than the tree-level the validity of perturbation theory is not spoiled.

- There is nevertheless a little fine-tuning problem: in order to satisfy the experimental bounds on the Higgs mass a stop around the TeV scale is needed which produces a $\sim 1\%$ fine-tuning in the determination of the $Z$-mass.

There is a very strong upper bound on the Higgs boson mass in the MSSM. A numerical analysis for the light Higgs including RGE re-summation was performed in Ref. [25, 26] as it is shown in Figs. 2, 3 and 4 as a function of $M_S$, $A_t$ and $m_A$ respectively:

The mass of the heavy eigenstate is plotted in Fig. 5

3.7. Drawbacks of the MSSM

There are a number of drawbacks in the MSSM, some of them have already been mentioned. I will briefly enumerate some of them

- There is little fine tuning $\sim 1\%$ fine-tuning since the present bounds on the Higgs mass put lower bounds on the mass of third generation squarks.

- There is a large number ($\sim 10^2$) of free parameters. This number includes all different supersymmetry breaking parameters: Scalar masses and trilinear couplings and gaugino Majorana masses.
Figure 2. $m_h$ vs. $M_S$ for $m_A \sim 1$ TeV. Different curves correspond to: (a,b) $\tan \beta = 15$ $A_t/M_{SUSY} = (\sqrt{6},0)$; (c,d) $\tan \beta = 2$. The figure is taken from Ref. [25].

Figure 3. $m_h$ vs. $A_t$ for $M_S \sim m_A \sim 1$ TeV. The cases (a,b,c,d) correspond to the parameters of Fig. 2. The figure is taken from Ref. [25].

• There is a big uncertainty in the mechanism of supersymmetry breaking. Different possibilities are:
  
  – **Gravity mediation** [27]:
    * It is a universal mechanism solving the $\mu/B\mu$ problem [28].
    * Its minimal version (known as mSUGRA) reduces the number of free parameters to a few.
    * They are the so-called supergravity models.
  
  – **Gauge mediation** [29]:
    * It is flavor blind.
    * It has $\mu/B\mu$ problems.
    * The gravitino is the LSP.
  
  – **Anomaly mediation** [30]:
    * A very universal and predictive mechanism but it predicts tachyonic sleptons.

• In supersymmetric theories there is the so-called supersymmetric flavor problem: supersymmetric partners can create FCNC and CP violating operators. As a consequence gravity mediation has to be subdominant ($\sim 0.1\%$ of gauge mediation).
Figure 4. $m_h$ Vs. $m_A$ for $M_S \sim 1$ TeV. The cases (a,b,c,d) correspond to the parameters of Fig. 2. The figure is taken from Ref. [25].

Figure 5. $m_H$ Vs. $m_A$ for $M_S \sim 1$ TeV. The cases (a,b,c,d) correspond to the parameters of Fig. 2. The figure is taken from Ref. [25].

4. Little Higgs
Little Higgs (LH) models [31]-[35] aim to solve the Little Hierarchy problem. In particular the symmetry that protects the (little) hierarchy is a global symmetry of which the Higgs is an approximate (pseudo) Goldstone boson. They are inspired from low energy hadronic physics: there $\pi^{\pm 0}$ are Goldstone bosons associated to the spontaneous breaking $SU(2)_L \times SU(2)_R \rightarrow SU(2)_I$. Similarly the Higgs is the Goldstone boson of a global symmetry $G_0 \rightarrow H_0$. It is in the coset space $H \in G_0/H_0$. The symmetry $H \rightarrow H + c$ is broken (in particular) by Yukawa interactions which lead to

$$m_H^2 \sim \frac{\alpha_t}{4\pi} \Lambda^2$$

which can solve the LEP paradox. In short LH models are clever constructions to avoid the appearance of the lowest order contribution to $m_H^2$. They are based on the so-called collective breaking mechanism.
4.1. Collective breaking

The mass of a Higgs pseudo-Goldstone boson from the different couplings $\alpha_i$ that break the Goldstone symmetry is \[ m_H^2 = \left( c_i \frac{\alpha_i}{4\pi} + c_{ij} \frac{\alpha_i\alpha_j}{(4\pi)^2} \right) \Lambda^2 \] (33)

where the coefficients are controlled by selection rules. If the Goldstone symmetry is restored when any single coupling $\alpha_i = 0$ then in order to totally destroy the Goldstone symmetry one requires the combined effect [collective breaking] of at least two non-zero couplings, which leads to

\[ \Rightarrow m_H^2 \sim \left( \frac{\alpha}{4\pi} \right)^2 \Lambda^2 \Rightarrow \Lambda \sim 10 \text{ TeV} \] (34)

This is the way LH models solve the LEP paradox/Little Hierarchy problem.

4.2. General structure

In LH models there is a global group $G_g$ which spontaneously breaks to a subgroup $H_g$ at a scale $f \sim 1 \text{ TeV}$ and the theory becomes strong at the scale $\Lambda \sim 4\pi f \sim 10 \text{ TeV}$. The scales are similar to $\Lambda_{QCD}$ and $f_\pi$ in QCD. The subgroup $G_l \subset G_g$ is gauged: $G_l \supset SU(2) \times U(1)$. The combination of spontaneous and collective breaking makes: $G_l \rightarrow SU(2) \times U(1)$ leaving heavy vector bosons and fermions with masses

\[ M_{\text{Heavy}} \sim g f \sim 1 \text{ TeV}. \] (35)

The Higgs boson is part of the Goldstone multiplet which parametrizes the coset space $G_g/H_g$

| Model    | $G_g$    | $H_g$   | $G_l$                      |
|----------|----------|---------|----------------------------|
| Littlest | $SU(5)$  | $SO(5)$ | $[SU(2) \times U(1)]^2$   |
| Simplest | $SU(3)^2$| $SU(2)^2$| $SU(3) \times U(1)$       |

The Higgs boson arises as a pseudo-Goldstone boson from the spontaneous breaking $SU(3) \rightarrow SU(2) \times U(1)$

The gauge structure is also enlarged as it is shown in Table 1. For instance in the Littlest model [32]

- $SU(5) \rightarrow SO(5)$ which leaves 24-10=14 Goldstone bosons.
• 4 Goldstone bosons are absorbed by the broken gauge group.
• The 10 remaining Goldstone bosons leave 4 Higgs doublets and 6 Higgs triplet.

The one-loop quadratic divergence from the top quark

$$\Delta M_{H}^2 \sim -\frac{\alpha_t}{4\pi} \Lambda^2$$

is cancelled by that from the $T$ quark. The one-loop quadratic divergence from the $W$ gauge boson

$$\Delta M_{H}^2 \sim \frac{\alpha_W}{4\pi} \Lambda^2$$

is cancelled by that from the $W_H$ gauge boson.

4.3. Electroweak breaking
It is triggered by the $t-T$ sector analogously to the MSSM:

$$\Delta m_{H}^2 = -\frac{3}{8\pi^2} h_t^2 m_T^2 \log \frac{\Lambda}{m_T}$$

Since $\Delta m_{H}^2 \sim m_T^2$ electroweak breaking requires some tuning of at least 5% as in the MSSM.

4.4. Dark Matter
In the Littlest LH models [32] one can introduce a $T$-parity such that SM particles (extra particles) are T-even(T-odd) [35]. In this case the lightest T-odd gauge boson is a candidate to DM.

4.5. Electroweak precision tests
In LH models with T-parity the latter forbids the mixing between T-odd and T-even gauge bosons leading naturally to $S = 0$.

5. Gauge-Higgs unification
Up to now we have explored two symmetries protecting the Higgs from quadratic divergences: supersymmetry and a global symmetry. In higher dimensional theories there is another symmetry which could do the job: a gauge symmetry.

5.1. General structure
The gauge bosons of a higher dimensional gauge symmetry have the Lorentz decomposition as

$$A^A_M = A^A_\mu, \quad A^A_i, \quad [\mu = 0, \ldots, 3, \ i = 1, \ldots, d]$$

where $A^A_\mu$ are gauge bosons in four dimensions and $A^A_i$ are scalar in the adjoint representation. They can play the role of Higgs bosons and for that reason these kinds of models are known as gauge-Higgs unification models (GHU).

For the orbifold constructions we need to compactify extra dimensions in an orbifold: e.g. for $d = 1$ ($A_\mu, A_5$) on $S^1/Z_2$ [37].

The orbifold group has to act non trivially on the group generators such that:

$$A^A_\mu = E^A_\mu (even), \quad O^A_\mu (odd)$$

$$A^A_5 = O^A_5 (odd), \quad E^A_5 (even)$$
Only even fields have zero modes $E_{\text{even}}^{(n)}$, $n = 0, 1, 2,\ldots$ while odd field have only non zero modes $O_{\text{odd}}^{(n)}$, $n = 1, 2,\ldots$

The Higgs mechanism acts for all non-zero modes as

\[
(A^\mu_{\text{massless}} + A^\mu_{\text{odd}})^{(n\neq 0)} = A^\mu_{\text{(n=0) massive}}
\]

\[
(A^a_{\text{massless}} + A^a_{\text{odd}})^{(n\neq 0)} = A^a_{\text{(n=0) massive}}
\]

while the zero modes $A^\mu_{(n=0)}$, $A^a_{(n=0)}$ remain massless and can acquire a vacuum expectation value by the Hosotani mechanism [38].

To get a doublet out of an adjoint one has to make a careful orbifold breaking. In particular one has to enlarge the gauge group since the SM Higgs is not in the adjoint representation of $SU(2) \times U(1)$. For instance $SU(3) \rightarrow SU(2) \times U(1)$ is achieved by the orbifold action $A^\mu(-y) = U A^\mu(y) U^\dagger$, $A^3(-y) = -U A^3(y) U^\dagger$ with

\[
U = \text{diag}(-1, -1, +1)
\]

which breaks $SU(3)$ into $SU(2) \times U(1)$. In particular the gauge bosons are defined by the matrix

\[
\begin{pmatrix}
E^3_\mu + E^8_\mu/\sqrt{3} & E^2_\mu - iE^5_\mu & O^4_\mu - iO^5_\mu \\
E^3_\mu + iE^8_\mu & -E^3_\mu + E^8_\mu/\sqrt{3} & O^6_\mu - iO^9_\mu \\
O^3_\mu + iO^8_\mu & O^6_\mu + iO^7_\mu & -2E^8_\mu/\sqrt{3}
\end{pmatrix}
\]

while the Higgs bosons are defined by the matrix

\[
\begin{pmatrix}
O^3_3 + O^8_3/\sqrt{3} & O^2_3 - iO^5_3 & E^4_3 - iE^5_3 \\
O^3_3 + iO^8_3 & -O^3_3 + O^8_3/\sqrt{3} & E^6_3 - iE^9_3 \\
E^3_3 + iE^8_3 & E^6_3 + iE^9_3 & -2O^8_3/\sqrt{3}
\end{pmatrix}
\]

The Higgs mass is protected from quadratic divergences in the bulk of the extra dimension by the five-dimensional gauge symmetry. However the orbifold has two fixed points at $y = 0, \pi R$ (where $R$ is the radius of the fifth dimension) which are singular and four-dimensional. The Higgs mass is protected from quadratic divergences at the fixed points by the shift symmetry (inherited from the five-dimensional gauge invariance) $\delta A_3 = \partial_3 A_3$ [39].

Now since the space is compactified there can be finite contributions to the $E^8_3$ mass proportional to $1/R$. They can break the $SU(2) \times U(1)$ symmetry by the mechanism called Hosotani breaking [38]. The diagrams contributing to the mass squared of $A_3^3$ are [40] which give to the Higgs field a mass

\[
m^2_A = \frac{3g^2}{32\pi^4R^2}\zeta(3)[3C_2(G) - 4T(R)N_f]
\]

where $N_f$ is the number of fermion families. So depending on the gauge and matter content a tachyonic mass will appear and the electroweak breaking will proceed by radiative corrections.

5.2. Drawbacks of Gauge-Higgs unification models

There are a number of difficulties with this (otherwise very nice) scenario:

- In more than five dimensions a (quadratically divergent) tadpole localized at the fixed points $F_{ij}$ is generated by radiative corrections [41] while the quartic Higgs coupling is sizeable and generated by the term $F_{ij}^2$ in the bulk [42].
Figure 6. The one-loop diagrams contributing to mass and wave function renormalization

Figure 7. One loop tadpole diagrams

- In five dimensions there is no localized tadpole but there is neither a tree-level quartic coupling which means difficulties with too small a Higgs mass.
- It is difficult to have a theory with the correct prediction for the weak angle [extra $U(1)'s$ are usually required [42]]
- Fermion masses are difficult to accommodate since they come from gauge couplings: in particular the top quark use to be too light.
- The compactification scale is usually too small in conflict with electroweak precision tests.
- The theory has a very low cutoff after which it becomes non-perturbative.

5.3. Ways out

Some of these difficulties can be alleviated by embedding GHU in a warped (Randall-Sundrum) five-dimensional space time [43].
- Warped models are valid up to scales of order $M_{GUT}$ or $M_{Planck}$ and thus they can unify.
- The Higgs is holographic, i.e. it is localized towards the infra-red (IR) brane [at higher scales it is composite].
- Fermion masses can be implemented by means of their localization, by means of five-dimensional masses.
- The top quark (to get a big mass) has to be localized as the Higgs. So it is also holographic.
- Electroweak precision tests as well as corrections to the $Zb\bar{b}$ vertex lead to KK-masses in the $2.5 - 4$ TeV, which imply $\sim 1\%$ fine-tuning for the Higgs mass (similar to the MSSM).
- These models are the modern version of technicolor theories: they make use of the AdS/CFT correspondence for calculability.
6. Conformal Higgs
A different approach to the hierarchy problem can reside in the fact that the Higgs belongs to an (approximately) conformal sector and interacts with the rest of the SM by local (effective) interactions. This way of approaching the phenomenology of a conformal sector has been called unparticles by Georgi [44].

6.1. Unparticles
Recently Georgi [44] has introduced a new way of studying conformal sectors with a fixed point at the scale \( \Lambda \), that couple to the Standard Model. Fields in a conformal theory can acquire large anomalous dimensions \( \gamma \) and modify the scaling dimension \( d \) of the field.

If the conformal symmetry is broken at a scale \( m_g \), which provides a continuum of states above the mass gap \( m_g \) the propagator for a scalar particle can be described as

\[
\Delta(p) \propto \frac{1}{(-p^2 + m_g^2 - i\epsilon)^{1-\gamma}}
\]  
(47)

and the usual particle propagator is reached for the case \( \gamma = 0 \).

6.2. Un-Higgs
Making a step forward along the previous direction one can speculate with the idea that the Higgs is an object of a conformal theory (unparticle) with a fixed point at a scale \( \Lambda \) and a scaling dimension \( d = 1 + \gamma \), where \( \gamma \) is the anomalous dimension: an un-Higgs [45]. The un-Higgs is coupled to the SM fields by Yukawa interactions

\[
\mathcal{L} = h_t \frac{1}{\Lambda^\gamma} H^\dagger \bar{q}_L t_R + h.c
\]  
(48)

For \( \gamma > 0 \) the operator is irrelevant and does not take the conformal theory out of the fixed point. The conformal symmetry should be broken at a scale \( m_g \) which is related to the VEV of the un-Higgs, \( v^d \): it can be triggered by SM top-loop effects [46].

6.3. The fine-tuning problem
In the un-Higgs approach the Higgs mass term is given by

\[
m_H^2 (1-\gamma) |H|^2.
\]  
(49)

The radiative corrections induced by the top Yukawa coupling are

\[
\delta m^2_H (1-\gamma) = \frac{3h_t^2}{8\pi^2} \Lambda^{2(1-\gamma)}.
\]  
(50)

Then the sensitivity of the Higgs mass to radiative corrections is

\[
1 + \frac{3h_t^2}{8\pi^2} \left( \frac{\Lambda^2}{m_H^2} \right)^{1-\gamma}.
\]  
(51)

- For \( \gamma = 0 \) it is the usual sensitivity appearing from quadratic divergences.
- For \( \gamma \to 1 \) the sensitivity is tiny for any value of \( \Lambda \).
- For instance for \( \gamma = 0.7 \) one can push \( \Lambda = 10 \) TeV without much tuning.
7. Conclusions

Although the last word will be for the LHC one can spell out a few possibilities that can arise depending on the forthcoming experimental data.

- One possibility is that the theory below $M_{Planck}$ is just the Standard Model: in that case we should try to find other solutions to the hierarchy problem, as e.g. an anthropic solution/landscape solution.
- If the Higgs is light ($m_H < 135$ GeV) then an excellent candidate is the MSSM although supersymmetric particles should show up eventually at LHC.
- If the Higgs is heavy then other particles should appear to restore agreement with present electroweak precision tests.
- If there is no Higgs at all other resonances should appear to restore unitarity in $WW$ scattering.
- Even if the Higgs is found we will (probably) need a linear collider for Higgs precision physics

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