Active neutrino oscillations and double beta decay characteristics with sterile neutrinos contributions

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ABSTRACT: Light sterile neutrinos contributions both for active neutrinos oscillations and neutrinoless double beta decay characteristics are estimated in the framework of the phenomenological model with three active and three sterile neutrinos assuming the Majorana nature of neutrino. Appearance and survival probabilities for active neutrinos with contributions of $eV$-sterile neutrinos in application to the short-baseline anomalies in neutrino data are obtained at the same test values of the model parameters. Modified graphical dependences for the survival probability of electron neutrinos/antineutrinos and the probability of appearance of electron neutrinos/antineutrinos in muon neutrino/antineutrino beams as functions of distance and other model parameters at different neutrino energies, and also as functions of the ratio of the distance to the neutrino energy are presented. A significant difference was found between the probability curves of the considered neutrino model and the simple sinusoidal curves of the neutrino model with one sterile neutrino. Effective electron neutrino masses for the beta decay and the neutrinoless double beta decay are estimated with account of $eV$-sterile neutrinos contributions. Besides, the two-neutrino double beta decay characteristics for selenium-82 are calculated. These findings can be used for interpretation and prediction of results of ground-based experiments on search for the sterile neutrinos as well as the neutrinoless double beta decay.

KEYWORDS: Neutrino Oscillation Anomalies at Short Distances, Sterile Neutrinos, Double Beta Decay Characteristics
1 Introduction

At present there are indications to anomalies of neutrino fluxes for some processes that cannot be explained using oscillation parameters only for three active neutrinos in the framework of the Modified Standard Model (MSM). MSM (or νSM) model is the Standard Model (SM) with three massive active neutrinos instead of SM with massless neutrinos. These anomalies include the LSND (or accelerator) anomaly (AA) \[1\text{–}4\], the reactor anomaly (RA or the reactor antineutrino anomaly RAA) \[5\text{–}9\] and the gallium (calibration) anomaly (GA) \[10\text{–}12\]. These three types of short baseline (SBL) anomalies manifest themselves at short distances (more precisely, at distances \(L\) such that the numerical value of the parameter \(\Delta m^2L/E\), where \(E\) is the neutrino energy and \(\Delta m^2\) is the characteristic difference of the neutrino mass squares, is of the order of unity). For the LSND anomaly \[1, 2\] an excess of the electron antineutrinos in beams of muon antineutrinos in comparison with the expected value according to the MSM is observed. Similar results were observed in the MiniBooNE experiments for electron neutrinos and antineutrinos \[3, 4\]. Deficit of reactor electron antineutrinos at short distances is called as RA, while the deficit of electron neutrinos from a radioactive source occurred at calibration of detectors for the SAGE and GALLEX experiments is commonly called as GA. In other words, data on SBL anomalies refer to both the inexplicable appearance of electron neutrinos or antineutrinos in beams of muon neutrinos or antineutrinos, respectively, and to the disappearance of electron neutrinos or antineutrinos at short distances. SBL anomalies may be explained by means of existence of one or two new sterile neutrinos (SNs), which do not interact directly with the SM gauge bosons. The characteristic mass scale of these light SNs is 1 eV. Now intensive searches are carried out for light SNs (or eV-sterile neutrinos). It is expected that in the
Moreover, it is necessary also to take into account the restriction on the effective neutrino mass,
considerably higher, respectively, than masses of the active neutrinos. Sensitivity of the experiments on
delimitation of the effective Majorana neutrino mass is considerably lower than for the neutrinoless double beta decay
(0νββ-decay) [31]. The study of double beta decay, both with the help of theoretical investigations and creation of
new large-scale installations is now rapidly developing. The discovery of this rare process may indicate
on Majorana’s nature of neutrino and will make it possible in this case to draw conclusions about
the neutrino mass matrix and mixing parameters.

Currently, the rarest phenomena that have been observed experimentally are reactions of two-neutrino double beta decay (2νββ-decay) with half-life time $T_{1/2} \sim 10^{19} - 10^{21}$ years [32]. The study of the 2νββ-decay allows ones to get information about the structure
of nuclei involved in the process, which is very essential for developing and testing the nuclear models for theoretical consideration of neutrinoless transitions. At the same time, two-neutrino channel is an unremovable background for 0νββ-decay and, therefore, the calculations of the corresponding differential intensities are needed for determining the sensitivity of the experiments on delimitation of the effective Majorana neutrino mass $m_{\beta\beta}$. Moreover, it is necessary also to take into account the restriction on the effective neutrino mass $m_\beta$ that can be measured in the tritium β-decay experiment KATRIN [33].

The mixing of neutrino states [25] puts into operation with the Pontecorvo–Maki–Nakagawa–Sakata matrix $U_{PMNS} \equiv U \equiv V \times P$, so that $\psi_{a}^L = U_{ai}\phi_{i}^L$, where $\psi_{a}^L$ are left chiral fields with flavor “$a$” or mass “$m_{a}$”, $a = \{e, \mu, \tau\}$ and $i = \{1, 2, 3\}$. For three active neutrinos, the matrix $V$ is expressed in the standard parametrization [30] via the mixing angles $\theta_{ij}$ and the Dirac CP phase, namely, the phase $\delta \equiv \delta_{CP}$ associated with CP violation in the lepton sector for Dirac or Majorana neutrinos, and $P = \text{diag}\{e^{i\alpha e^\beta}, e^{i\beta}\}$, where $\alpha \equiv \alpha_{CP}$ and $\beta \equiv \beta_{CP}$ are phases associated with CP violation only for Majorana neutrinos. In oscillation experiments it is impossible to measure $\alpha_{CP}$ and $\beta_{CP}$. Nonetheless, in oscillation experiments the violation of conservation laws for the lepton numbers $L_e$, $L_\mu$
and $L_\tau$ was fixed.

Using the analysis of high-precision experimental data, the values of the mixing angles and the differences of the neutrino mass squares $\Delta m_{21}^2$ and $|\Delta m_{31}^2|$ (where $\Delta m_{21}^2 = m_2^2 - m_1^2$) were found [30, 34]. CP phases $\alpha_{CP}$ and $\beta_{CP}$ are currently unknown, and also unknown is the order (hierarchy) of the neutrino mass spectrum, normal (NO) or inverse (IO). Although the $\delta_{CP}$ value is also not yet definitively determined experimentally, in a number of papers its estimate was obtained (see for example [18, 34, 35]). For the NO-case of the mass spectrum of active neutrinos (ANs), this estimate results in $\sin\delta_{CP} < 0$ and $\delta_{CP} \approx -\pi/2$. If we
take into account the restrictions on the sum of the neutrino masses from cosmological observations [36] and the results of the T2K experiment [37], then the NO-case of the neutrino mass spectrum turns out to be preferable. So in carrying out further numerical calculations, we restrict ourselves to the NO-case, assuming $\delta_{\text{CP}} = -\pi/2$.

The main purpose of this paper is to consider the effects of eV-sterile neutrinos on oscillation properties of active neutrinos and neutrinoless double-beta decay characteristics in the framework of the $(0+3+3+0)$ neutrino model, which we will refer as the $(3+3)$ model, assuming neutrinos to be the Majorana type particles. In section 2 we briefly present the $(3+3)$ model with three ANs and three SNs for estimating SNs effects in the $0\nu2\beta$-decay and ANs oscillation characteristics at small distances. The propositions of this model have been outlined in the recent paper [38].

However, in contrast to the sterile neutrino mass option considered in detail in [38], where only one mass $m_4$ of sterile neutrino is of the order of 1 eV, while other two masses, $m_5$ and $m_6$, are essentially heavier (the $(3+2+1)$ model), in the present paper we consider neutrinos to be Majorana particles, and therefore it is necessary to take into account existing restrictions on the effective neutrino mass [39], together with the values of mixing angles acceptable for describing the SBL anomalies. Therefore, in this paper, as an alternative case, the masses of all three SNs are selected in the range of about 1 eV (the light SNs case or the $(3+3)$ model). It results in the qualitative difference with respect to the results obtained in [38]. In section 3 the results of detailed calculations of the ANs oscillation characteristics at small distances, taking into account the light SNs effects, are presented. Calculations were carried out at selected test values of the model parameters, using the results obtained earlier [18, 21]. We hope that the results of these calculations can help to explain the experimental data on the SBL neutrino anomalies.

In section 4, the estimations of the effective neutrino masses for the beta decay and the neutrinoless double beta decay with account of SNs contributions are performed using the $(3+3)$ model parameter values. In appendix A, the calculation of an amplitude for the $2\nu2\beta$-decay within the High-States Dominance (HSD) and Single-State Dominance (SSD) mechanisms for $^{82}\text{Se}$ are presented. In section 5 the obtained results are accentuated.

2 Some provisions of a $(3+3)$ model for active and sterile neutrinos

Below, a $(3+3)$ model is used to study the effects of SNs. This model was detailed in [38] and includes three AN $\nu_a (a = e, \mu, \tau)$ and three new neutrinos: a sterile neutrino $\nu_s$, a hidden neutrino $\nu_h$ and a dark neutrino $\nu_d$. A $6 \times 6$ mixing matrix $U_{\text{mix}}$ is used in the model framework. For the compactness of the formulas, the symbols $h_a$ and $h_{i'}$ are introduced for additional left flavor and mass fields, respectively. Above, $s$ denotes a set of indices that highlight the fields $\nu_s$, $\nu_h$ and $\nu_d$ among $h_s$, while $i'$ denotes a set of indices 4, 5 and 6. Then matrix $U_{\text{mix}}$, which establishes linkage between the flavor and mass neutrino fields, is represented as:

$$
\begin{pmatrix}
\nu_a \\
h_s
\end{pmatrix}
= U_{\text{mix}}
\begin{pmatrix}
\nu_i \\
h_{i'}
\end{pmatrix}
= 
\frac{\kappa U}{\sqrt{1 - \kappa^2}} 
\begin{pmatrix}
\sqrt{1 - \kappa^2} a & \sqrt{1 - \kappa^2} \alpha \\
\sqrt{1 - \kappa^2} b & \kappa c
\end{pmatrix}
\begin{pmatrix}
\nu_i \\
h_{i'}
\end{pmatrix}.
$$

(2.1)
Here $\varepsilon = 1 - \epsilon$, where $\epsilon$ is a small quantity, $U \equiv U_{\text{PMNS}}$, where $U_{\text{PMNS}}$ is the well-known unitary $3 \times 3$ mixing matrix of ANs ($U_{\text{PMNS}}U_{\text{PMNS}}^\dagger = I$). Moreover, $a$ and $b$ are arbitrary unitary $3 \times 3$ matrices, with $c = -b \times a$. Matrix $U_{\text{mix}}$ under these conditions is unitary. We will use the following matrices $a$ and $b$, which first were proposed in [18]:

$$
a = \begin{pmatrix}
\cos \eta_2 & \sin \eta_2 & 0 \\
-\sin \eta_2 & \cos \eta_2 & 0 \\
0 & 0 & e^{-i\nu_2}
\end{pmatrix}, \quad b = -\begin{pmatrix}
\cos \eta_1 & \sin \eta_1 & 0 \\
-\sin \eta_1 & \cos \eta_1 & 0 \\
0 & 0 & e^{-i\nu_1}
\end{pmatrix}.
$$

(2.2)

The values of mixing angles $\theta_{ij}$ of ANs for $U_{\text{PMNS}}$ matrix will be taken from the conditions $\sin^2 \theta_{12} \approx 0.297$, $\sin^2 \theta_{23} \approx 0.425$ and $\sin^2 \theta_{13} \approx 0.0215$ [30]. For additional mixing parameters concerning SNs, the following trial values have been proposed: $\kappa_1 = \kappa_2 = -\pi/2$ and $\eta_1 = 5^\circ$, while the matched values for $\eta_2$ and $\epsilon < 0.03$ will be concretized in the next Section.

Neutrino masses are given by a set of values $\{m\} = \{m_1, m_2, m_3\}$ in eV units: $m_1 \approx 0.0016$, $m_2 \approx 0.0088$ and $m_3 \approx 0.0497$ [18, 24]. In [38], it was used the value $m_4 \approx 1$ eV and the values $m_5$ and $m_6$ of the order of several keV. Such values lead to rapidly oscillating curves for the probabilities of conservation and appearance of electron neutrinos/antineutrinos that have to be averaged to make comparison with the experimental data. This situation persists until the masses $m_5$ and $m_6$ are of the order of 10 eV. For $m_5$ and $m_6$ values noticeably less than 10 eV, the curves become smoothly oscillating. In the present work, we use the values $m_4 = 1.05$ eV, $m_5 = 0.63$ eV and $m_6 = 0.27$ eV. They are consistent with the acceptable mass range found by treatment of the data relevant to SBL anomalies [40]. However, the values of $m_5$ and $m_6$ are much less than the corresponding values used in [38]. This is due to the fact that the case of Majorana neutrinos will be inspected below, for which $0\nu2\beta$-decay is possible. So, we should take into account existing restrictions on the effective Majorana neutrino mass $m_{\beta\beta}$ along with acceptable values of angles $\theta_{\mu\tau}$ and $\theta_{ee}$ taken from global processing of experimental data on SBL anomalies [15, 16, 40], as well as restrictions on the effective $\beta$-decay neutrino mass $m_\beta$ [33].

The probability amplitudes for describing the oscillations of neutrino flavors can be found by solving well-known equations for the neutrino propagation (see for example [18, 41]). Using these equations, analytical expressions for probabilities of different transitions between neutrino flavors in beams of neutrinos/antineutrinos with energy $E$ in vacuum can be obtained as functions of the distance $L$ from the neutrino source [29]. If $\tilde{U} \equiv U_{\text{mix}}$ is a generalized $6 \times 6$ matrix of neutrino mixing in the form of equation (2.1), then, using the notation $\Delta_{ki} \equiv \Delta m_{\alpha k}^2 L/(4E)$, we can calculate the probabilities of transitions from $\nu_\alpha$ to $\nu_{\alpha'}$ or from $\bar{\nu}_\alpha$ to $\bar{\nu}_{\alpha'}$ according to the formula

$$
P(\nu_\alpha(\bar{\nu}_\alpha) \rightarrow \nu_{\alpha'}(\bar{\nu}_{\alpha'})) = \delta_{\alpha'\alpha} - 4\sum_{i>k} \text{Re}(\bar{U}_{\alpha'i}U_{\alpha'k}) \sin^2 \Delta_{ki} \pm 2\sum_{i>k} \text{Im}(\bar{U}_{\alpha'i}U_{\alpha'k}) \sin 2\Delta_{ki},
$$

(2.3)

where the upper sign ($+$) corresponds to neutrino transitions $\nu_\alpha \rightarrow \nu_{\alpha'}$ and the lower sign ($-$) corresponds to antineutrino transitions $\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}$. Note that flavor indexes $\alpha$ and $\alpha'$ (as well as summation indices over massive states $i$ and $k$) apply to both ANs and SNs.
3 Numerical results for oscillation characteristics of active neutrinos taking into account their mixing with eV-sterile neutrinos

In this section a comparison is made between survival and appearance neutrino/antineutrino probabilities calculated in the framework of the $(3+3)$ model and similar quantities calculated in the framework of the $(3+1)$ model based on currently available data for SBL anomalies [15, 16, 40] (see figures 1, 2 and 3). First of all it refers to data on the disappearance of muon neutrinos and antineutrinos and the appearance of electron neutrinos and antineutrinos in the processes $\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$. The typical ratio of the distance traversed by the neutrino before detection to the neutrino energy is either a few meters per MeV, or one meter per several MeVs. Attempts of the simultaneous description of all the data in these processes lead to difficulties. In particular, the problems associated with different values of the excess of the output $\nu_e$ and $\overline{\nu}_e$ in the MiniBooNE experiment can be reduced under the condition of CP violation [42–44].

It is also possible to describe the reactor and gallium anomalies in the framework of the model considered by selecting the appropriate value of the parameter $\epsilon$ (see equation 2.1). It is sufficient to choose the value corresponding to the experimental data for the parameter $\kappa = 1 - \epsilon$, that naturally leads to the deficit of electron neutrinos and antineutrinos. Note that the status of RA with allowance for the recently discovered excess of the number of antineutrinos in comparison with the model calculations in the 5 MeV range and thus confirmation of the possible existence of a light SN with a mass of about 1 eV is discussed for example in Refs. [45–47].

Figure 1 shows the appearance probabilities of $\nu_e$ (upper panel) and $\overline{\nu}_e$ (lower panel) in the beams of $\nu_\mu$ and $\overline{\nu}_\mu$, respectively, as a function of the ratio of distance from the source to the neutrino energy at the coupling constant $\epsilon = 0.015$ and the parameter $\eta_2 = \pi/3$ and for the considered in this paper neutrino mass spectrum. In this case, the contribution of sterile neutrinos has the character of smooth oscillations. Furthermore, for example, already with the value of $\epsilon = 0.005$ the relative outputs of $\nu_e$ and $\overline{\nu}_e$ increase by approximately two orders of magnitude (up to $\sim 10^{-4}$) in comparison with their values for $\epsilon = 0$ ($\sim 10^{-6}$). The comparison with the experiment is provided with the help of simple $(3+1)$ model using the formula $P_{3+1}(\nu_\mu \to \nu_e) = P_{3+1}(\overline{\nu}_\mu \to \overline{\nu}_e) = \sin^2(2\theta_{\mu e}) \sin^2(1.27 \Delta m^2_{41} L/E)$, where $L$ is the distance to the detector in meters, $E$ is the neutrino energy in MeV and $\Delta m^2_{41}$ is the difference between the squared neutrino masses in eV$^2$.

In figure 2, the similar results are given for $P(\nu_e \to \nu_e) \equiv P(\overline{\nu}_e \to \overline{\nu}_e)$ for the same values of the model parameters. The new salient feature of our model curve (solid line) consists in the strong difference from the sinusoidally varying curve, which can be only obtained by the standard formula within the $(3+1)$ model for probability $P(\nu_e \to \nu_e)$, that is, by the formula $P_{3+1}(\nu_e \to \nu_e) = 1 - \sin^2(2\theta_{ee}) \sin^2(1.27 \Delta m^2_{41} L/E)$. Moreover the periods of variation for the model curves presented in figures 1 and 2 are different in spite of the same neutrino mass spectrum. So it is permissible to argue for effectively varying SN mass values in different processes. This fact can shed light on the possible obstacle in determining a precise value of a SN mass.

As for the possible description of the gallium anomaly within the $(3+3)$ model frame-
Figure 1. The probability of appearance of $\nu_e$ (top panel) and $\bar{\nu}_e$ (bottom panel) depending on the ratio of the distance $L$ from the source to the neutrino energy $E$ in the beams of $\nu_\mu$ and $\bar{\nu}_\mu$, respectively. For matrix $U_{\text{mix}}$, $\epsilon = 0.015$ and $\eta_2 = \pi/3$. The mass in square differences are $\Delta m^2_{41} = 1.1$ eV$^2$ and $\Delta m^2_{51} = 0.4$ eV$^2$. The dotted curves with circle marks that are the same in both panels and closely approximate the solid curves show the probability values calculated in the approximation of two neutrino states in the (3+1) model for $\sin^2(2\theta_{\mu e}) = 0.0007$ and $\Delta m^2_{41} = 0.7$ eV$^2$.

work, the survival probability for $\nu_e$ as a function of distance $L$ from the neutrino source for various values of the neutrino energy $E$ is shown in figure 3 for the same parameters $\epsilon = 0.015$ and $\eta_2 = \pi/3$ of the mixing matrix and for the same mass values as it was used in figures 1 and 2. In each panel of figure 3, that value of neutrino energy $E$ is selected, which is acquired by electron neutrino in the process involving artificial sources containing isotopes $^{51}\text{Cr}$, $^{37}\text{Ar}$ and $^{65}\text{Zn}$, respectively. SNs contributions have character of smooth oscillation. Moreover, these oscillations coincide in phase with the oscillations, which are obtained by the standard formula of the (3+1) model for probability $P(\nu_\mu \to \nu_e)$.

The results obtained are characteristic features of the (3+3) neutrino model with SNs contributions, which is considered in the present paper, and they can be used for interpreting both the available and coming experimental data on the SNs search.
Figure 2. The survival probabilities for $\nu_e$ ($\bar{\nu}_e$) depending on the ratio of the distance $L$ from the source to the neutrino energy $E$ in the beams of $\nu_e$ ($\bar{\nu}_e$). For the matrix $U_{\text{mix}}$, $\epsilon = 0.015$ and $\eta_2 = \pi/3$. The mass in square differences are $\Delta m^2_{41} = 1.1$ eV$^2$ and $\Delta m^2_{51} = 0.4$ eV$^2$. The dotted curve with square marks shows the probability values calculated in the approximation of two neutrino states in the $(3+1)$ model for the parameter values $\sin^2(2\theta_{ee}) = 0.0396$ and $\Delta m^2_{41} = 0.4$ eV$^2$, which were obtained by joint processing the experimental data on RAA and GA.

Figure 3. The $\nu_e$ survival probability depending on the distance $L$ from the source, in the capacity of which are the nuclei of various elements with different neutrino energy, for the case of the mixing matrix $U_{\text{mix}}$ with $\epsilon = 0.015$ and $\eta_2 = \pi/3$. The differences of the squared masses are $\Delta m^2_{41} = 1.1$ eV$^2$ and $\Delta m^2_{51} = 0.4$ eV$^2$. The dotted curves with square marks show the probability values calculated in the approximation of two neutrino states of a $(3+1)$ model for parameter values $\sin^2(2\theta_{ee}) = 0.0396$ and $\Delta m^2_{41} = 0.4$ eV$^2$ taken from global processing of experimental data on GA and RAA [15, 16, 40].
4 Contributions of eV-sterile neutrinos to the probability of beta decay and neutrinoless double beta decay

If the resolution of the experimental setup does not allow to distinguish between the masses of massive neutrinos involved in beta decay, then in this case the weighted mass of the electron neutrino is used, which is called as effective neutrino mass $m_\beta$ [33]:

$$m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2 .$$

(4.1)

We will use as $\{m_i\}$ the set of masses suggested in the previous section along with the matrix elements $U_{ei}$ used there. The value of $m_\beta$ calculated in this way is 0.131 eV. Note that for now this value cannot be detected in the current KATRIN experiment, for which the achievable lower bound of the effective mass of electron neutrino is estimated as 0.2 eV (at 90% CL) [48]. Furthermore, the value of $m_\beta$ is about 0.01 eV with only three ANs and without SNs contribution.

To calculate the probability of neutrinoless double beta decay due to exchange by light massive Majorana neutrinos, it is necessary to take into account their contributions with allowance for the Majorana phases [49]:

$$m_{\beta\beta} = \left| \sum_i U_{ei}^2 m_i \right| .$$

(4.2)

Since the values of the Majorana phases are still not known, we estimate the maximum value of the effective Majorana mass of an electron neutrino using the set $\{m_i\}$ proposed in the previous section along with the same matrix elements $U_{ei}$ from there. Then the value of $m_{\beta\beta}$ is equal to 0.027 eV. This value is approximately half of the currently most constraining upper limit on effective Majorana mass of electron neutrino, which is obtained in the KamLAND-Zen experiment with $^{136}$Xe (0.061 eV) [39]. Notice that $m_{\beta\beta}$ is equal to 0.0026 eV when only contribution of ANs is taken into account.

For the half-life with respect to the neutrinoless decay, the expression takes place as

$$T_{1/2}^{0\nu} = \frac{m_e^2}{G^{0\nu} g_1^4 |M^{0\nu m_{\beta\beta}}|^2} .$$

(4.3)

Here $G^{0\nu}$ is the phase-space factor, $M^{0\nu}$ is the nuclear transition matrix element, and $m_e$ is the electron mass.

Now there is an intensive searches for $0\nu2\beta$-decay and plans are underway for a number of large-scale experiments such as SuperNEMO, LEGEND, EXO, CUPID-0, etc. $^{82}$Se is the one of perspective stable isotopes for these investigations, which is used in recent CUPID-0 and SuperNEMO projects. So, it is of great interest to perform calculations of double beta-decay characteristics of $^{82}$Se. Namely for this isotope, $2\nu2\beta$-decay for the first time was observed in a direct experiment in 1987. In NEMO-3 experiment, where 0.93 kg of $^{82}$Se were used as the $2\beta$-source [50], the total and differential intensities for the two-neutrino channel were measured and low bound for half-decay time of neutrinoless transition was determined: $T_{1/2}^{0\nu} > 2.5 \times 10^{23}$ years. In CUPID-0 setup effectively 5.53 kg of $^{82}$Se are involved in data recording, and the following result was obtained [51–53]: $T_{1/2}^{0\nu} > 2.4 \times 10^{24}$ years. Phase
factor calculations [54] and existing models of nuclear structure [55] lead to the following conclusions for $^{82}\text{Se}$: $G^{0\nu} = 10.16 \times 10^{-15}$ year$^{-1}$, $|M^{0\nu}|_{\text{min}} = 2.64$, $|M^{0\nu}|_{\text{max}} = 4.64$. Taking into account the above estimate of $m_{\beta\beta}$, we can calculate the values of $T^{0\nu}\nu_{1/2}$ for the stable isotope $^{82}\text{Se}$: $\{T^{0\nu}_{1/2}\}_{\text{min}} = 6.17 \times 10^{26}$ years, $\{T^{0\nu}_{1/2}\}_{\text{max}} = 1.96 \times 10^{27}$ years. Thus, it is necessary to increase significantly the sensitivity of the experiment on search for neutrinoless double beta decay of $^{82}\text{Se}$. The new large-scale installation SuperNEMO [56], where it is planned to use up to 100 kg of $^{82}\text{Se}$, serves this purpose. As noted above, for evaluation of sensitivity of the planned experiment, it is necessary to determine precisely the energy spectrum of electrons arising in a two-neutrino channel that will be done in appendix A.

5 Discussion and conclusions

In this paper, the light SNs contributions both to ANs oscillations at short distances and neutrinoless double beta decay characteristics are estimated in the framework of the phenomenological (3+3) neutrino model. The properties of these characteristics at the test values of the model parameters are numerically investigated. All calculations were performed for the case of a normal hierarchy of the mass spectrum of active neutrinos with allowance for possible violation of the CP invariance in the lepton sector and for the value $-\pi/2$ for the Dirac CP phase in the $U_{\text{PMNS}}$ matrix. In section 3, graphical dependences of the probabilities of appearance and disappearance of electron neutrinos and antineutrinos pertaining to both acceleration and reactor/gallium anomalies, respectively, are exhibited as a functions of the ratio of the distance from the neutrino source to the neutrino energy. Significant difference of the probability curves of our (3+3) neutrino model from simple sinusoidal curves of the (3+1) neutrino model, which is clearly seen in figures 1, 2 and 3 is one of the main results of the present work. Such behavior of the survival and appearance probabilities of electron neutrinos and antineutrinos may be a cause of difficulties concerning with a valid confirmation of neutrino anomalies at short distances. Besides, the considered (3+3) neutrino model gives possibility to describe the data of all SBL neutrino anomalies with the same model parameters, that is with the same mixing parameters and the mass values pertaining to the eV-sterile neutrinos, that cannot be reached in the scope of the simple (3+1) neutrino model. So, a global self-consistent description of the neutrino data for all SBL anomalies on the basis of the simple (3+1) model of neutrino may not be possible.

The results obtained make it possible to interpret the experimental data on oscillations of neutrinos and antineutrinos that admit the existence of LNSD and MiniBooNE anomalies, as well as the reactor antineutrino anomaly together with the gallium anomaly. The most promising test for searching light SNs is, as noted above, a verification of the existence of a gallium anomaly. In any case the experimental data on SBL anomalies should be processed separately for the every anomaly and also separately for neutrino and antineutrino data for acceleration anomaly.

In section 4, estimates of the effective masses $m_{\beta}$ and $m_{\beta\beta}$ of the electron neutrino taking into account the sterile neutrino contributions are obtained, which can be tested in experiments on $\beta$-decay and $0\nu2\beta$-decay, in particular, in the KATRIN, SuperNEMO
and CUPID experiments. Besides, an analysis of the energy structure of $2\nu2\beta$-decay of $^{82}\text{Se}$ made in appendix A results in the conclusion that this process can be considered as a virtual two-step transition, which connects the initial and final states through the first $1^+$-state of the intermediate nucleus (that is the Single State Dominance mechanism – SSD). For $^{82}\text{Se}$, this state is excited $1^+_1$-state of $^{82}\text{Br}$ with $E_2 = 75$ keV, since the quantum number of the ground state of bromine-82 is $J^\pi = 5^-$. In this paper, we obtain estimates of the half-life times for neutrinoless and two-neutrino double beta decays of $^{82}\text{Se}$, respectively, in section 4 and appendix A. Measurement of the half-life time $T^{2\nu2\beta}_{1/2}$ enables to calculate theoretically the value of a nuclear matrix element $M^F_1 = \langle 0_f^+ || \beta^- || 1_f^+ \rangle$, which has not yet been determined experimentally from the recharge reaction. In addition, the calculated distribution in energy of one electron will give possibility to establish, how it is planned during the project SuperNEMO, what mechanism, SSD or HSD, is valid for a two-neutrino channel (see figure 4). This is important both for calculating an unrecoverable background and for evaluating the sensitivity of the experiment on search of the neutrinoless double beta decay of isotope $^{82}\text{Se}$.

A The amplitude of two-neutrino double beta decay of selenium-82

Let us carry out the theoretical calculations of the total and differential intensities of $2\nu2\beta$-decay of $^{82}\text{Se}$. To calculate the intensity of two-neutrino transitions one needs to perform summation over all possible $1^+$-states of the intermediate nucleus [57, 58]. For this, it is necessary to know the values of modules and phases of matrix elements $\langle 0_f^+ || \beta^- || 1_f^+ \rangle$ and $\langle 1_N^+ || \beta^- || 0_i^+ \rangle$, with $\beta^- = \sigma \tau^-$, which are involved in the expression for $T^{2\nu2\beta}_{1/2}$:

$$
T^{2\nu2\beta}_{1/2} \left( 0_i^+ \rightarrow 0_f^+ \right) = \frac{G^2_{\beta\beta} A^{A^1}_{\beta\beta}}{32\pi^2 \ln 2} \int_{m_e}^{T+m_e} d\varepsilon_1 \int_{m_e}^{T+2m_e-\varepsilon_1} d\varepsilon_2 \int_0^{T+2m_e-\varepsilon_1-\varepsilon_2} d\omega_1 \times 
F(Z_f, \varepsilon_1) F(Z_f, \varepsilon_2) p_1 \varepsilon_1 p_2 \varepsilon_2 \varepsilon_1^2 \varepsilon_2^2 \omega_1 \omega_2^2 A_{0_f^+}. 
$$

(A.1)

The expression for $A_{0_f^+}$ is as follows:

$$
4A_{0_f^+} = \left| \sum_N \langle 0_f^+ \| \beta^- \| 1_N^+ \rangle \langle 1_N^+ \| \beta^- \| 0_i^+ \rangle (K_N + L_N) \right|^2 + 
\frac{1}{3} \sum_N \langle 0_f^+ \| \beta^- \| 1_N^+ \rangle \langle 1_N^+ \| \beta^- \| 0_i^+ \rangle (K_N - L_N) \right|^2.
$$

(A.2)

Here $p_1$, $p_2$ and $\varepsilon_1$, $\varepsilon_2$ are, respectively, momentums and energies of the electrons, $\omega_1$ and $\omega_2 = T + 2m_e - \varepsilon_1 - \varepsilon_2 - \omega_1$ are antineutrino energies, $T = E_i - E_f - 2m_e = Q_{\beta\beta}$ is the total kinetic energy of leptons in final state, and $E_i (E_f)$ is the mass of the parent (daughter) nucleus. Then, $F(Z_f, \varepsilon)$ is Coulomb factor taking into account the influence of electrostatic field of the nucleus to the emitted electrons, while the quantities $K_N$ and $L_N$ contain the
energy denominators of the second-order perturbation theory:

\[
K_N = \frac{1}{\mu_N + (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)/2} + \frac{1}{\mu_N - (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)/2}, \tag{A.3a}
\]

\[
L_N = \frac{1}{\mu_N + (\varepsilon_1 + \omega_2 - \varepsilon_2 - \omega_2)/2} + \frac{1}{\mu_N - (\varepsilon_1 + \omega_2 - \varepsilon_2 - \omega_1)/2}, \tag{A.3b}
\]

with \( \mu_N = E_N - (E_i + E_f)/2 \) and \( E_N \) the energy of the \( N \)-th \( 1^+ \)-state of intermediate nucleus.

Calculation of nuclear matrix elements \( \langle 0^+_N \| \hat{\beta}^- \| 1^+_N \rangle \) and \( \langle 1^+_N \| \hat{\beta}^- \| 0^+_N \rangle \) is a very difficult theoretical problem [59]. At the same time, it can be assumed that for some isotopes the main contribution in the sums over \( N \) in the expression (A.2) is the contribution of the ground state of the intermediate nucleus in the case if this state has the quantum number \( J^\pi = 1^+ \). This two-neutrino double beta decay mechanism meets the hypothesis of dominance of the ground state of the intermediate nucleus (SSD mechanism – Single State Dominance [60, 61]). This situation occurs for \(^{100}\text{Mo}\), where the \( 2\nu\beta\beta \)-transition can be considered with a good accuracy as a two-step process, which links the initial \((^{100}\text{Mo})\) and final \((^{100}\text{Ru})\) states of the process via the ground \( 1^+ \)-state of the intermediate nucleus \(^{100}\text{Tc}\).

Nuclear matrix elements \( M^I_\beta = \langle 1^+_{g.s.} \| \hat{\beta}^- \| 0^+_N \rangle \) and \( M^F_\beta = \langle 0^+_N \| \hat{\beta}^- \| 1^+_{g.s.} \rangle \) can be found from the force transition values \( f_I \) for the electron capture or single beta decay process. Here \( f_I \) is the product of the phase factor by the half-life time of the corresponding single \( \beta \)-process. So,

\[
M^I_\beta = \frac{1}{g_A} \sqrt{\frac{3D}{f_{\beta EC}}}, \quad M^F_\beta = \frac{1}{g_A} \sqrt{\frac{3D}{f_{\beta}^2}}, \tag{A.4}
\]

where \( g_A = 1.27561, \ D = \frac{2m_\beta \ln 2}{G^2 F^2 m_e^2} = 6288.6 \text{ s} \), \( G_\beta = G_F \cos \theta_C, G_F = 1.166378 \times 10^{-5} \text{ GeV}^{-2} \), and \( \cos \theta_C = 0.97425 \).

If the SSD hypothesis is valid under the condition that the quantum number of the ground state of the intermediate nucleus is \( J^\pi = 1^+ \), in this case the intensity of the two-neutrino transition is determined only by the intensities of single beta processes, which are characterized by factors \( f_{\beta^-} \) and \( f_{\beta EC} \), and is independent of \( G_\beta \) and \( g_A \) [62]:

\[
T^{(2\nu)}_{1/2}(0^+ \rightarrow 0^+) = \frac{16\pi^2 f_{\beta EC} f_{\beta^-}}{3 \ln 2 (\lambda C/\varepsilon)} H(T, 0^+) = 2.997 \times 10^{14} \text{ years} s \frac{1 \text{ log } f_{\beta EC} + \text{ log } f_{\beta^-}}{H(T, 0^+)}, \tag{A.5}
\]

where

\[
H(T, 0^+) = \int_1^{T+1} d\varepsilon_1 \int_1^{T+2-\varepsilon_1} d\varepsilon_2 \int_0^{T+1-\varepsilon_1-\varepsilon_2} d\omega_1 \times F(Z_f, \varepsilon_1) F(Z_f, \varepsilon_2) p_1 \varepsilon_1 p_2 \varepsilon_2 \omega_1^2 \omega_2^2 (K^2 + KL + L^2). \tag{A.6}
\]

The value of \( \log f_{\beta^-} \) is well established from the beta decay of \(^{100}\text{Tc}\) and is equal to 4.59 that corresponds to \( M^F_\beta = 0.546 \). Definition of \( \log f_{\beta EC} \) from experiments on the study of electron capture in \(^{100}\text{Tc}\) is a difficult experimental task. Currently the most accurate value of \( \log f_{\beta EC} \) for electron capture in \(^{100}\text{Tc}\) was obtained in [63], namely, \( \log f_{\beta EC} = 4.29^{+0.08}_{-0.07} \).
When calculating the half-life time for a $2\nu 2\beta$ transition, often it is assumed that the kinetic energies of the emitted leptons are approximately equal [57, 58, 64]. Then $K \approx L \approx 2/\mu$. This situation is equivalent by dominance in the expression for $T_{1/2}^{2\nu 2\beta}$ of contributions related to the intermediate nucleus states with high excitation energy (HSD mechanism – High States Dominance). As has been shown in [65], such an approach, when the dependence of $K$ and $L$ on the lepton energies is neglected, leads to overestimating the theoretical value of $T_{1/2}^{2\nu 2\beta}$. In the case of $0^+ \rightarrow 0^+_{g.s.}$ transition in $^{100}\text{Mo}$, the effect is about 25%. Taking into account the dependence of $K$ and $L$ on lepton energies on the basis of the SSD mechanism allows one to obtain differential intensity in energy of one electron $P(\varepsilon) = d\ln I/d\varepsilon$ for the $2\nu 2\beta$-decay of the isotope $^{100}\text{Mo}$ [66, 67], which matches the NEMO-3 data [68].

![Figure 4](image.png)

**Figure 4.** Distribution in energy of one electron in $2\nu 2\beta$-decay of $^{82}\text{Se}$ for HSD (dotted curve 1) and SSD (solid curve 2) mechanisms.

In the case of double beta decay of $^{82}\text{Se}$, the ground state of the intermediate nucleus $^{82}\text{Br}_{g.s.}(5^-)$ has a mass of 423 keV less than the mass of initial nucleus $^{82}\text{Se}$. Virtual Gamov–Teller transition is possible through the first excited $1^+_1$-state of the nucleus $^{82}\text{Br}$ with $E_x = 75$ keV. Accordingly, $M(^{82}\text{Se}) - M(^{82}\text{Br}^*, 1^+_1) = 348$ keV, and there is reason to believe that the SSD mechanism will be realised for $2\nu 2\beta$-decay of $^{82}\text{Se}$.

The excited state of bromine-$82$ ($^{82}\text{Br}^*, 1^+_1$) with $E_x = 75$ keV was found in the experiment on charge-exchange reaction $^{82}\text{Se}(^3\text{He},t)^{82}\text{Br}$ [69], and this state is characterized by large Gamov-Teller transition force $B(GT) = 0.338$. It should be noted that the overlying excited $1^+_1$-states of bromine-$82$ with $E_x < 2$ MeV correspond to the transition forces by an order of magnitude smaller. Then, basing on the SSD hypothesis, the contribution of $1^+_1$-state of the intermediate nucleus $^{82}\text{Br}$ in the sum over $N$ in the expression (A.2) should only be taken into account. Alternatively, if transition proceeds through many higher intermediate excited states, then higher-state dominance mechanism (HSD) governs $2\nu 2\beta$-decay of $^{82}\text{Se}$. The choice of the model is the question of physical interest, for it affects differential intensities in two-neutrino channel, and consequently, background estimations. In favor of SSD approach indicate the results of measurements of the intensity distribution in the energy of one electron, which were carried out with the NEMO-3 setup [50]. Also,
investigations performed in CUPID-0 experiment show, that SSD gives better description of the total electron energy distribution, than HSD [53]. NEMO-3 and future SuperNEMO detectors, composed of a tracker and calorimeter, have capability of reconstructing of full topology of $\beta\beta$ events. Thus a precise high-statistic study of single-electron energy distribution, sensitive to nuclear mechanism, can be used to distinguish between the two theoretical approaches [50].

Nuclear matrix element $M_1^I = \langle 1^+ || \hat{\beta}^- || 0^+ \rangle$ is determined from the value of the Gamov-Teller force $B(GT) = 0.338$ [69] according to the relation $|M(GT)|^2 = B(GT)$ that is valid for $0^+ \rightarrow 0^+ 2\nu 2\beta$-transition. Thus, $M_1^I = 0.581$. It will be possible to determine the value of matrix element $M_1^F = \langle 0^+_f || \hat{\beta}^- || 1^+_1 \rangle$ from the study of the recharging reaction $^{82}\text{Kr}(d,^2\text{He})^{82}\text{Br}$, which has not yet been completed. However, one can find $M_1^F$ with using the equations (A.1) and (A.2) on the base of the $T_{1/2}^{2\nu 2\beta}$ value obtained during the NEMO-3 experiment. In the case of SSD mechanism of $2\nu 2\beta$-decay of $^{82}\text{Se}$, $T_{1/2}^{2\nu 2\beta} = 9.39 \times 10^{19}$ years [50]. The corresponding value of $M_1^F$ is 0.23, with $B(GT^+) = 0.0529$.

In figure 4, the distributions in the energy of one electron corresponding to HSD and SSD mechanisms are depicted. It is certainly interesting to compare obtained theoretical distributions with the measurement results that will be carried out in SuperNEMO experiment.

References

[1] LSND Collaboration, C. Athanassopoulos et al., Evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations from the LSND experiment at the Los Alamos meson physics facility, Phys. Rev. Lett. 77 (1996) 3082 [nucl-ex/9605003].
[2] LSND Collaboration, A. Aguilar et al., Evidence for neutrino oscillations from the observation of $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam, Phys. Rev. D 64 (2001) 112007 [hep-ex/0104049].
[3] MiniBooNE Collaboration, A.A. Aguilar-Arevalo et al., Improved search for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations in the MiniBooNE experiment, Phys. Rev. Lett. 110 (2013) 161801 [arXiv:1303.2588].
[4] MiniBooNE Collaboration, A.A. Aguilar-Arevalo et al., Significant excess of electronlike events in the MiniBooNE short-baseline neutrino experiment, Phys. Rev. Lett. 121 (2018) 221801 [arXiv:1805.12028].
[5] Th.A. Mueller et al., Improved predictions of reactor antineutrino spectra, Phys. Rev. C 83 (2011) 054615 [arXiv:1101.2663].
[6] G. Mention, M. Fechner, Th. Lasserre, Th.A. Mueller, D. Lhuillier, M. Cribier and A. Letourneau, Reactor antineutrino anomaly, Phys. Rev. D 83 (2011) 073006 [arXiv:1101.2755].
[7] P. Huber, Determination of antineutrino spectra from nuclear reactors, Phys. Rev. C 84 (2011) 024617 [Erratum ibid. C 85 (2012) 029901] [arXiv:1106.0687].
[8] I. Alekseev et al., Search for sterile neutrinos at the DANSS experiment, Phys. Lett. B 787 (2018) 56 [arXiv:1804.04046].
[10] SAGE Collaboration, J.N. Abdurashitov et al., *Measurement of the solar neutrino capture rate with gallium metal. III. Results for the 2002-2007 data-taking period*, Phys. Rev. C 80 (2009) 015807 [arXiv:0901.2200].

[11] F. Kaether, W. Hampel, G. Heusser, J. Kiko and T. Kirsten, *Reanalysis of the Gallex solar neutrino flux and source experiments*, Phys. Lett. B 685 (2010) 47 [arXiv:1001.2731].

[12] C. Giunti, M. Laveder, Y.F. Li and H.W. Long, *Pragmatic view of short-baseline neutrino oscillations*, Phys. Rev. D 88 (2013) 073008 [arXiv:1308.5288].

[13] K.N. Abazajian et al., *Light sterile neutrinos: a white paper*, arXiv:1204.5379.

[14] V. Barinov, B. Cleveland, V. Gavrin, D. Gorbunov and T. Ibragimova, *Revised neutrino-gallium cross section and prospects of BEST in resolving the gallium anomaly*, Phys. Rev. D 97 (2018) 073001 [arXiv:1710.06326].

[15] A. Diaz, C.A. Argüelles, G.H. Collin, J.M. Conrad and M.H. Shaevitz, *Where are we with light sterile neutrinos*, arXiv:1906.00045.

[16] S. Böser, C. Buck, C. Giunti, et al., *Status of light sterile neutrino searches*, Prog. Part. Nucl. Phys. (2019) doi: https://doi.org/10.1016/j.ppnp.2019.103736 [arXiv:1906.01739].

[17] V.I. Lyashuk, *Intensive electron antineutrino source with well defined hard spectrum on the base of nuclear reactor and 8-lithium transfer. The promising experiment for sterile neutrinos search*, JHEP 06 (2019) 135 [arXiv:1809.05949].

[18] V.V. Khruschov, S.V. Fomichev and O.A. Titov, *Oscillation properties of active and sterile neutrinos and neutrino anomalies at short distances*, Phys. Atom. Nucl. 79 (2016) 708 [arXiv:1612.06544].

[19] L. Canetti, M. Drewes and M. Shaposhnikov, *Sterile neutrinos as the origin of dark and baryonic matter*, Phys. Rev. Lett. 110 (2013) 061801 [arXiv:1204.3902].

[20] J.M. Conrad, C.M. Ignarra, G. Karagiorgi, M.H. Shaevitz and J. Spitz, *Sterile neutrino fits to short-baseline neutrino oscillation measurements*, Adv. High Energy Phys. 2013 (2013) 163897 [arXiv:1207.4765].

[21] V.V. Khruschov and S.V. Fomichev, *Oscillation characteristics of neutrino in the model with three sterile neutrinos for analysis of the anomalies on small distances*, Phys. Part. Nucl. 48 (2017) 990.

[22] V.V. Khruschov, A.V. Yudin, D.K. Nadyozhin and S.V. Fomichev, *Enhancement of the sterile neutrino yield at high density and increasing neutronization of matter*, Astron. Lett. 41 (2015) 260 [arXiv:1412.6262].

[23] M.L. Warren, G.J. Mathews, M. Meixner, J. Hidaka and T. Kajino, *Impact of sterile neutrino dark matter on core-collapse supernovae*, Int. J. Mod. Phys. A 31 (2016) 1650137 [arXiv:1603.05503].

[24] A.V. Yudin, D.K. Nadyozhin, V.V. Khruschov and S.V. Fomichev, *Neutrino fluxes from a core-collapse supernova in a model with three sterile neutrinos*, Astron. Lett. 42 (2016) 800 [arXiv:1701.04713].

[25] S.M. Bilenky and B.M. Pontekorvo, *Lepton mixing and neutrino oscillations*, Sov. Physics-Uspekhi 20 (1977) 776.

[26] T. Schwetz, M. Tórtola and J.W.F. Valle, *Global neutrino data and recent reactor fluxes: the
status of three-flavour oscillation parameters, New J. Phys. 13 (2011) 063004 [arXiv:1103.0734].

[27] J. Kopp, P.A.N. Machado, M. Maltoni and T. Schwetz, Sterile neutrino oscillations: the global picture, JHEP 05 (2013) 050 [arXiv:1303.3011].

[28] S. Gariazzo, C. Giunti, M. Laveder and Y.F. Li, Updated global $3+1$ analysis of short-baseline neutrino oscillations, JHEP 06 (2017) 135 [arXiv:1703.00860].

[29] S.M. Bilenky, Some comments on high precision study of neutrino oscillations, Phys. Part. Nucl. Lett. 12 (2015) 453 [arXiv:1502.06158].

[30] Particle Data Group, M. Tanabashi et al., Review of particle physics, Phys. Rev. D 98 (2018) 030001.

[31] W. Rodejohann, Neutrino-less double beta decay and particle physics, Int. J. Mod. Phys. E 20 (2011) 1833 [arXiv:1106.1334].

[32] A.S. Barabash, Average and recommended half-life values for two-neutrino double beta decay: Upgrade-2019, AIP Conf. Proc. 2165, (2019) 020002 [arXiv:1311.2421].

[33] KATRIN Collaboration, M. Aker et al., Improved upper limit on the neutrino mass from a direct kinematic method by KATRIN, Phys. Rev. Lett. 123, (2019) 221802 [arXiv:1909.06048].

[34] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler and T. Schwetz, Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity, JHEP 01 (2017) 087 [arXiv:1611.01514].

[35] S.T. Petcov, I. Girardi and A.V. Titov, Predictions for the Dirac CP violation phase in the neutrino mixing matrix, Int. J. Mod. Phys. A 30 (2015) 1530035 [arXiv:1504.02402].

[36] S. Wang, Y.-F. Wang and D.-M. Xia, Constraints on the sum of neutrino masses using cosmological data including the latest extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample, Chin. Phys. C 42 (2018) 065103 [arXiv:1707.00588].

[37] The T2K Collaboration, K. Abe et al., Measurement of neutrino and antineutrino oscillations by the T2K experiment including a new additional sample of $\nu_e$ interactions at the far detector, Phys. Rev. D 96 (2017) 092006 [Erratum ibid. D 98 (2018) 019902] [arXiv:1707.01048].

[38] V.V. Khruschov and S.V. Fomichev, Sterile neutrinos influence on oscillation characteristics of active neutrinos at short distances in the generalized model of neutrino mixing, Int. J. Mod. Phys. A 34 (2019) 1950175 [arXiv:1806.05922].

[39] S.T. Petkov, The nature of the neutrino (Dirac/Majorana) and double beta decay with or without neutrinos, arXiv:1910.09331.

[40] M. Dentler, Á. Hernández-Cabezudo, J. Kopp, P. Machado, M. Maltoni, I. Martinez-Soler and T. Schwetz, Updated global analysis of neutrino oscillations in the presence of eV-scale sterile neutrinos, JHEP 08 (2018) 10 [arXiv:1803.10661].

[41] M. Blennow and A.Yu. Smirnov, Neutrino propagation in matter, Adv. High Energy Phys. 2013 (2013) 972485 [arXiv:1306.2903].

[42] M. Maltoni and T. Schwetz, Sterile neutrino oscillations after first MiniBooNE results, Phys. Rev. D 76 (2007) 093005 [arXiv:0705.0107].

[43] S. Palomares-Ruiz, S. Pascoli and T. Schwetz, Explaining LSND by a decaying sterile neutrino, JHEP 09 (2005) 048 [hep-ph/0505216].
[44] G. Karagiorgi, A. Aguilar-Arevalo, J.M. Conrad, M.H. Shaevitz, K. Whisnant, M. Sorel and V. Barger, Leptonic CP violation studies at MiniBooNE in the $(3+2)$ sterile neutrino oscillation hypothesis, Phys. Rev. D 75 (2007) 013011 [Erratum ibid. D 80 (2009) 099902] [hep-ph/0609177].

[45] M. Dentler, Á. Hernández-Cabezudo, J. Kopp, M. Maltoni and T. Schwetz, Sterile neutrinos or flux uncertainties? - Status of the reactor anti-neutrino anomaly, JHEP 11 (2017) 099 [arXiv:1709.04294].

[46] M. Danilov, Searches for sterile neutrinos at very short baseline reactor experiments, J. Phys.: Conf. Ser. 1390 (2019) 012049 [arXiv:1812.04085].

[47] S. Gariazzo, C. Giunti, M. Laveder and Y.F. Li, Model-independent $\bar{\nu}_e$ short-baseline oscillations from reactor spectral ratios, Phys. Lett. B 782 (2018) 13 [arXiv:1801.06467].

[48] M. Aker et al., First operation of the KATRIN experiment with tritium, arXiv:1909.06069.

[49] GERDA collaboration, M. Agostini et al., Probing Majorana neutrinos with double-$\beta$ decay, Science 365 (2019) 1445 [arXiv:1909.02726].

[50] R. Arnold et al., Final results on $^{82}$Se double beta decay to the ground state of $^{82}$Kr from the NEMO-3 experiment, Eur. Phys. J. C 78 (2018) 821 [arXiv:1806.05553].

[51] O. Azzolini et al., First result on the neutrinoless double-$\beta$ decay of $^{82}$Se with CUPID-0, Phys. Rev. Lett. 120 (2018) 232502 [arXiv:1802.07791].

[52] O. Azzolini et al., Final result of CUPID-0 Phase-I in the search for the $^{82}$Se neutrinoless double-$\beta$ decay, Phys. Rev. Lett. 123 (2019) 032501 [arXiv:1906.05001].

[53] O. Azzolini et al., Evidence of single state dominance in the two-neutrino double-$\beta$ decay of $^{82}$Se with CUPID-0, Phys. Rev. Lett. 123 (2019) 262501 [arXiv:1909.03397].

[54] J. Kotila and F. Iachello, Phase-space factors for double-$\beta$ decay, Phys. Rev. C 85 (2012) 034316 [arXiv:1209.5722].

[55] J. Barea, J. Kotila and F. Iachello, $0\nu\beta\beta$ and $2\nu\beta\beta$ nuclear matrix elements in the interacting boson model with isospin restoration, Phys. Rev. C 91 (2015) 034304 [arXiv:1506.08530].

[56] R. Arnold et al., Probing new physics models of neutrinoless double beta decay with SuperNEMO, Eur. Phys. J. C 70 (2010) 927 [arXiv:1005.1241].

[57] M. Doi, T. Kotani and E. Takasugi, Double beta decay and Majorana neutrino, Progr. Theor. Phys. Suppl. 83 (1985) 1.

[58] J. Suhonen and O. Civitarese, Weak-interaction and nuclear-structure aspects of nuclear double beta decay, Phys. Rep. 300 (1998) 123.

[59] A. Faessler and F. Šimkovic, Double beta decay, J. Phys. G 24 (1998) 2139 [hep-ph/9901215].

[60] J. Abad, A. Morales, R. Núñez-Lagos and A.F. Pacheco, An estimation of the rates of (two-neutrino) double beta decay and related processes, Journal de Physique Colloques 45 (1984) C3-147.

[61] O. Civitarese and J. Suhonen, Is the single-state dominance realized in double-$\beta$-decay transitions?, Phys. Rev. C 58 (1998) 1535.

[62] S.V. Semenov, F. Šimkovic and P. Domin, The single state dominance in $2\nu/3\nu$-decay transitions to excited $0^+$ and $2^+$ final states, Part. Nucl. Lett. No. 6[109] (2001) 26.
[63] S.K.L. Sjue et al., *Electron-capture branch of $^{100}$Tc and tests of nuclear wave functions for double-$\beta$ decays*, Phys. Rev. C **78** (2008) 064317 [arXiv:0809.3757].

[64] W.C. Haxton and G.J. Stephenson, Jr., *Double beta decay*, Progr. Part. Nucl. Phys. **12** (1984) 409.

[65] S.V. Semenov, F. Šimkovic, V.V. Khruschev and P. Domin, *Contribution of the lowest $1^+$ intermediate state to the $2\nu/3\beta$-decay amplitude*, Phys. Atom. Nucl. **63** (2000) 1196.

[66] F. Šimkovic, P. Domin and S.V. Semenov, *The single state dominance hypothesis and the two-neutrino double beta decay of $^{100}$Mo*, J. Phys. G **27** (2001) 2233 [nucl-th/0006084].

[67] P. Domin, S. Kovalenko, F. Šimkovic and S.V. Semenov, *Neutrino accompanied $\beta^+\beta^\pm$, $\beta^+/EC$ and EC/EC processes within single state dominance hypothesis*, Nucl. Phys. A **753** (2005) 337 [nucl-th/0411002].

[68] Yu.A. Shitov (on behalf of NEMO Collaboration), *Double-beta-decay experiment NEMO-3: Preliminary results of phase I (2003-2004)*, Phys. Atom. Nucl. **69** (2006) 2090.

[69] D. Frekers et al., *High energy-resolution measurement of the $^{82}$Se($^3$He,t)$^{82}$Br reaction for double-$\beta$ decay and for solar neutrinos*, Phys. Rev. C **94** (2016) 014614.