Sine and Cosine Compensators for CIC Filter
Suitable for Software Defined Radio

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Abstract
Background/Objective: Software Defined Radio (SDR) is regarded as one of the most important emerging technologies. The aim of SDR is to support different wireless standards in a single radio device. Methods/Analysis: Different wireless standard requires different sample rate for baseband processing. This can be achieved by Sample Rate Conversion (SRC) technique. Comb-Integrator-Comb (CIC) filter plays an important role in SRC. But single stage or multistage identical CIC filter cannot provide better passband and stopband characteristics. So, some compensation techniques are required to obtain better CIC filter response. Findings: This paper discusses about sine and cosine compensator. It compares the result of sine compensated and cos compensated CIC filter with uncompensated CIC filter. Application/Improvements: It also focuses on the cascade of both sine and cos compensator with CIC filter to get tradeoff between refinements of both passband and stopband characteristics.

Keywords: CIC Filter, Cosine based Compensator, Multistage CIC Filter, Sine based Compensator, Sine-Cosine based Compensator

1. Introduction
SDR requires as much functionality as possible to be programmable to emulate multiple wireless standards on a single radio device. It can be done simply by updating software without replacing the underlying hardware platform. Thus, in SDR most of the signal processing is done in digital domain. The received RF signal, digitized by ADC, has a fixed sample rate. This should be converted to proper sample rates necessary for baseband processing of different air-interfaces. This is done by decimation or sample rate decrease and interpolation or sample rate increase. In both of the cases, CIC filter plays an important role as anti-aliasing filter (in case of decimation) or anti-imaging filter (in case of interpolation).

CIC filter is a multiplier less filter which performs sample rate conversion by addition or subtraction. It consists of two sections: one is ‘Comb section’ and another is ‘Integrator section’. The transfer function of CIC filter in z domain,

\[ G_c[z] = \frac{1}{M} \sum_{k=0}^{M-1} z^{-k} = \frac{1}{N} z^{-M} \]

Where, \((1-z^{-N})\) is ‘Comb Section’ and \((1/(1-z^{-1}))\) is ‘Integrator section.’

Now, putting \(z = e^{j\omega}\) in Equation (1) the frequency response of CIC filter is obtained as,

\[ G_c(j\omega) = \frac{\sin(\omega M/2)}{M \sin(\omega/2)} e^{-j\omega (M-1)/2} \]  

However, imperfect filtering gives rise to spectral aliasing and unwanted spectral images in case of decimation and interpolation respectively. The filtering characteristics in terms of stopband attenuation of CIC filter can be modified by increasing the number of stages of CIC filter. Transfer function of multistage CIC filter is,

\[ G_c^k(z) = \left[ \frac{1 - z^{-M}}{M} \right]^k \]

\[ G_c^k(z) = \left[ \frac{1}{M} \right] \left[ \frac{1 - z^{-M}}{1 - z^{-1}} \right]^k, k = \text{No of stages} \]

As \(k\) increases, the stopband attenuation of CIC filter increases but droop in passband deteriorates significantly.

Multistage CIC filter can be used in decimation as a cascade of \(k\) integrators and \(k\) comb sections separated by a \(M\)-factor down-sampler. Another decimator block in the
next stage with decimation factor much less than the CIC
decimator will determine the particular frequency value
at which worst case aliasing will occur. It also determines
the edge frequency of the passband of interest, where
the worst case passband distortion, known as 'Passband
droop' will occur. For example, if there is a N-factor
second decimation stage in cascade with a M-factor CIC
decimation stage, the frequency of passband edge of
interest (After normalizing with respect to sampling rate
\( \omega_s = 2\pi \text{ rad or } f_s = 1 \text{ samples/second} \) is

\[
\frac{\omega_c}{\pi} = \frac{1}{NM} \tag{4}
\]

or,

\[
f_c = \frac{1}{2NM} \tag{5}
\]

Likewise, the worst case aliasing will occur at the
frequency,

\[
\frac{\omega_A}{\pi} = \frac{2}{M} - \frac{1}{NM} \tag{6}
\]

or,

\[
f_A = \frac{1}{M} - \frac{1}{2NM} \tag{7}
\]

There are several techniques to modify the magnitude
response of CIC filter. One of those is 'Sine Based
Compensation Technique'.

2. Sine based Compensation
Filter

Sine-based compensator is introduced for improving the
passband characteristic of the CIC filter. The magnitude
response of sine based filter compensates the passband
droop of the CIC filter. The transfer function of the sine
based filter\(^{11,12}\) is given as,

\[
G_s(\omega) = e^{(j\omega M)} [1 + 2^{-b} \sin^2 (M\omega/2)]
\]

\[
|G_s(\omega)| = |1 + 2^{-b} \sin^2 (M\omega/2)|
\]

Using the well known relation \( \sin 2\alpha = \frac{(1 - \cos 2\alpha)}{2} \),
Equation (8) becomes,

\[
G_s(\omega) = e^{(j\omega M)} [1 + 2^{-b} (1-\cos (M\omega))/2]
\]

\[
= e^{(j\omega M)} [1 + 2^{-b-1} \{1-\cos (M\omega)\}]
\]

\[
= e^{-j\omega M} \left[ 1 + 2^{-(b+1)} \left\{ 1 - \left( \frac{e^{j\omega M} + e^{-j\omega M}}{2} \right) \right\} \right]
\]

\[
= e^{-j\omega M} + 2^{-(b+1)} e^{-j\omega M} - 2^{-b} e^{-j(2\omega)} - 2^{-b} e^{-2j\omega M}
\]

Now, putting \( z = e^{j\omega} \) in Equation (10),

\[
G_{sin}(z^M) = -2^{-b+2}[1 - (2^{b+2} + 2)z^{-M} + z^{-2M}] \tag{9}
\]

\[
= A[1 + Bz^{-M} + z^{-2M}] \tag{10}
\]

Where, \( A = -2^{-(b+2)}, \quad B = (2^{b+2} + 2) \)

The two principal properties of the filter of Equation
(12) are:

- The transfer function is a function of \( z^M \). It can be
applied at a lower rate after down sampling by making
use of the multi rate function.
- The compensator filter has the scaling factor \( A \) and
one coefficient value \( B \). Both of them are realized
using additions and shifts. Therefore, the sine
based compensation filter can be implemented as a
multiplier-less filter.

In a sine based compensator, a sine filter in connected
in series with a multistage CIC filter as shown in Figure
1. Using Equation (3) and (10) the transfer function of
the two stage cascaded sine compensated CIC decimation
filter can be obtained as expressed in Equation (12)

\[
G_{sin,comp}(z) = G^c(z) G_{sin}(z)
\]

\[
= \left[ \frac{1}{M} \left( \frac{1 - z^{-M}}{1 - z^{-2}} \right) \right] \left[ A[1 + Bz^{-M} + z^{-2M}] \right] \tag{12}
\]

In the above equation, the parameter \( k \) i.e., numbers
of stages controls the stopband characteristic. Parameter
\( b \) compensates the passband droop of the magnitude
response of the sine based compensation filter. So we try to
find the optimum pair of \( k \) and \( b \) for better performance.

Figure 1. Sin compensated CIC filter.

Figure 2(a), 2(b) and 2(c) illustrates the passband
droops of the two stage sine compensated CIC filter of
Equation (12) for \( M = 10, 48 \) and \( 64 \) respectively and \( N = 4, k = 6 \) which are constant for all three cases. Now, for
different values of the parameter \( b \), it can be noticed that
the \( b = 0 \) provides the optimum compensation for all the
three cases of \( M = 10, M = 48 \) and \( M = 64 \). Thus, it can be
said that, \( b \) does not depend on decimation factor \( M \) but
it depends on the value of \( k \) i.e., number of stages of CIC
filter (In the example of Figure 2. \( k = 6 \)).
Similarly for different values of $k$ optimum value of $b$ is found for which the pass band droop of the sine-compensated CIC filter will be minimum. It is shown in Table 1. For a given value of $k$ and its corresponding optimum value of $b$ it can be seen that the values of decimation factor does not significantly affect the passband droop and worst case alias rejection. It is seen from Figure 3(a) and 3(b). These two graphs present passband droop and worst case aliasing of sine compensated CIC filter for distinct optimal pairs of $k$ and $b$ for several values of $M$ ranging from $M = 10$ to $M = 100$. It can be seen that passband droop remains within the range $-0.131$ dB to $-0.1364$ dB for $k = 2$, $b = 2$; $-0.05271$ dB to $-0.05934$ dB for $k = 3$, $b = 1$; $0.07622$ dB to $0.06534$ dB for $k = 5$, $b = 0$; $-0.146$ dB to $-0.1592$ dB for $k = 6$, $b = 0$ and $0.009534$ dB to $-0.01257$ dB for $k = 10$, $b = -1$. Similarly, worst case aliasing lies within the range $-33.69$ dB to $-33.98$ dB for $k = 2$, $b = 2$; $-50.39$ dB to $-50.82$ dB for $k = 3$, $b = 1$; $-83.82$ dB to $-84.44$ dB for $k = 5$, $b = 0$; $-100.8$ dB to $-101.6$ dB for $k = 6$, $b = 0$ and $-167.8$ dB to $-171.1$ dB for $k = 10$, $b = -1$.

Figure 2. Pass band droop of sin compensated CIC filter for different values of $b$ when. (a) $M = 10$. (b) $M = 48$. (c) $M = 64$.

Figure 3. Graph for. (a) Passband droop. (b) Worst case aliasing for different optimum pair of k-b for several values of M.

Table 1. Optimum value of parameter $b$ corresponding to $k$ for obtaining minimum passband droop

| Value of parameter $k$ | Optimum value of parameter $b$ corresponding to $k$ for minimum passband droop |
|------------------------|--------------------------------------------------------------------------------|
| 1,2                    | 2                                                                              |
| 3,4                    | 1                                                                              |
| 5,6,7,8,9              | 0                                                                              |
| 10-18                  | -1                                                                             |
| 19,20,....             | -2                                                                             |
Example 1: Design a decimation filter with a total down sampling factor of 32 and having an attenuation at worst case aliasing point of at least 220 dB.

Solution: Let, the CIC decimator in the first stage has a decimation factor $M = 8$ and in the next stage decimation factor is $N = 4$ so that total decimation factor be $M \times N = 32$. So, from Equation (6) we calculate that worst case aliasing will occur at frequency

$$\frac{\omega_A}{\pi} = \frac{2}{8} - \frac{1}{32} = 0.21875.$$ 

A single stage CIC decimator provides 16.94 dB attenuation at this worst case aliasing point. So to provide minimum 220 dB attenuation at this worst case aliasing point $\frac{220}{16.94} = 12.997 \approx 13$ identical CIC filter should be cascaded. A thirteen stage uncompensated CIC filter provides passband droop at frequency $\frac{\omega_c}{\pi} = \frac{1}{32} = 0.03125$ obtained from Equation (4) and the value of passband droop will be -2.872 dB. Again, for $k = 13$, optimal value of $b$ will be -1 (From Table 1) for which minimum passband droop can be obtained. Now, designing a sine compensated CIC filter considering these specifications a graph can be obtained as shown in Figure 4(a). Its passband zoom and stopband zoom is shown in Figure 4(b) and 4(c) respectively. From Figure 4(b) passband droop of sine compensated CIC filter can be obtained as -0.7261 dB which is much closer to 0dB or ideal Low Pass Filter (LPF) passband characteristics compared to uncompensated CIC filter. But in sine compensated 13 stage CIC filter the attenuation at worst case aliasing point becomes -218.1 dB which is slightly less than that of uncompensated CIC filter (-220.2 dB).

So another compensator known as ‘Cosine Based Compensation Technique’ is used to improve the overall stopband attenuation of a sine compensated CIC filter.

3. Cosine based Compensation Filter

A cosine filter is cascaded in series with uncompensated or sine compensated CIC filter for improvement of the overall stop-band characteristics of it. Impulse response of cosine based CIC filter is given as follows:

$$G_{\text{cos}}(z) = \frac{(1 + z^{-L})}{2}$$  \hspace{1cm} (13)

Putting $z = e^{j\omega}$ in Equation (13) the frequency response of CIC filter is obtained as,

$$G_{\text{cos}}(z) = \frac{1 + e^{-j\omega L}}{2}$$  \hspace{1cm} (14) 

$$= e^{-j\omega L/2} \left( \frac{e^{j\omega L/2} + e^{-j\omega L/2}}{2} \right)$$ 

$$= e^{-j\omega L/2} \left[ \cos \left( \frac{\omega L}{2} \right) \right]$$  \hspace{1cm} (15)
So, this filter has a magnitude response in cosine form. From Equations (2) and (13) we obtain the transfer function of the cos compensated CIC filter as,

\[ G_{\text{coscomp}}(z) = G_{\text{cs}}^L(z) G_{\text{cos}}(z) \]  

(16)

The responses of the cos compensated CIC filters are shown in Figure 5(a), 5(b) and 5(c) for \( L = 2, 3, \) and 4 respectively.

Figure 4. PCosine compensated thirteen stage filter for \( M = 8, N = 4, k = 13 \) a) \( L = 2 \) b) \( L = 3 \) c) \( L = 4 \).

4. Sine-Cosine based Compensator

It can be noticed that the proper choice of \( b \) and \( L \) and cascade of sin filter and cos filter with CIC filter provides the trade-off between the improvement of passband and the stopband characteristics of a CIC filter.

\[ G_{\text{sin,coscomp}}(z) = G_{\text{cs}}^L(z) G_{\text{sin}}^2(z) G_{\text{cos}}^3(z) \]  

(17)

The higher the values of parameters, \( k_1 \) and \( k_3 \), the stopband response improves while higher value of \( k_2 \) improves the passband droop.

In Example 1 instead of using only a sine compensated CIC filter if a sine and cos compensated CIC filter is used, improved response can be obtained. In this case, \( k_1 = 13, k_2 = k_3 = 1, b = -1 \) and \( L = 4 \) is chosen. The response is shown in Figure 6. In this case passband droop is -0.8054 dB and worst case aliasing is -232.2 dB. That means a tradeoff between refinement of passband droop and stop band attenuation is provided in comparison to uncompensated CIC filter or only sine or only cos compensated CIC filter.

Figure 6. Both sine-cosine compensated CIC filter for \( M = 8, N = 4, k_1 = 13, k_2 = k_3 = 1, b = -1, L = 4 \).

5. Conclusion

SDR tries to implement different wireless standards in a single radio device simply by updating some software but without replacing the underlying hardware platform. This can be done by digital signal processing. Synchronization should be maintained between the sample rate of digital
signal coming out from Analog to Digital Converter (ADC) and that required for baseband processing of different wireless standards. This can be done by either sample rate decrease (Decimation) or sample rate increase (Interpolation). In both of the cases, CIC filter is important as anti-imaging (Interpolation) or anti-aliasing (Decimation) filter. But a single CIC filter cannot provide optimum passband and stopband characteristics. Multistage identical CIC filter improves stopband but passband response decreases monotonically with increasing number of stages. So, different compensator techniques are introduced to solve this problem. One of them is sine compensator which improves the passband character of multistage CIC but degrades stopband attenuation. This again can be compensated using cosine compensator. So applying both sine and cos compensator in cascade with CIC filter provides a tradeoff between the optimal values of passband droop and stopband attenuation. Thus provides an acceptable result.

6. References

1. Tribble AC. The software defined radio: Fact and fiction. IEEE Radio and Wireless Symposium; Orlando, FL. 2008 Jan 22-24. p. 5–8.
2. Hentschel T, Fettweis G. The digital front-end – bridge between RF and baseband-processing. In: Tuttlebee WHW, editor. Dresden University of Technology, Chapter 6. Software defined radio: Enabling technologies.
3. Shafer O, Buck. Discrete-time signal processing. Prentice Hall Inc; 2000.
4. Hogenauer EB. An economical class of digital filters for decimation and interpolation. IEEE Trans on Acoustics, Speech, and Signal Processing. 1981 Apr; 29(2):155–62.
5. Singh C, Patterh MS, Sharma S. Design of programmable digital down converter for WiMax. Indian Journal of Science and Technology. 2009; 2(3).
6. Abbas M. On the implementation of integer and non-integer sampling rate conversion. Linköping, Sweden: LiU-Tryck; 2012.
7. Milic L. Multirate filtering for digital signal processing: MATLAB applications. IGI Global; 2009.
8. Singh A, Singhal P, Ratan R. Multistage implementation of multirate CIC filter. Indian Journal of Science and Technology. 2011 Aug; 4(8).
9. Jovanovic D, Mitra. On design of CIC decimation filter with improved response. IEEE ISCCSP; Malta. 2008 Mar 12-14.
10. Kwentus A, Willson AJr. Application of filter sharpening to cascaded integrator-comb decimation filters. IEEE Trans on Signal Processing. 1997 Feb; 45(2):457–67.
11. Jovanovic D, Mitra. A new two-stage sharpened comb decimator. IEEE Transactions on Circuits and Systems: Regular papers. 2005 Jul; 52(7):1414–20.
12. Jovanovic D, Harrist. Design of CIC compensator filter in a digital IF Receiver. IEEE ISCIT; Lao. 2008 Oct 21-23. p. 638–43.