Optimal Strategies for Harvesting and Predator Extermination to Sustain
Plecoglossus altivelis (Ayu) Population in Stochastic River Environment

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Abstract: This paper presents a stochastic process model for optimal strategies of harvesting and predator extermination to sustain Plecoglossus altivelis (Ayu) population in stochastic river environment. Human activities, which are extermination of Phalacrocorax carbo (Great Cormorant) from the river and fishing activity to harvest P. altivelis in the river, are taken as control variables in the model. An optimal management problem to maximize the profit from harvesting P. altivelis and simultaneously to minimize the cost to exterminate P. carbo while to sustain P. altivelis, is then formulated on the basis of the dynamic programming principle. The problem to be solved mathematically reduces to approximating solutions to a Hamilton-Jacobi-Bellman equation. Sensitivity of the optimal management strategy to three critical components of the present model, which are the intrinsic growth rate, environmental noise that makes the dynamics in a river be inherently stochastic, and the two human activities, is numerically verified focusing on the case of Hii River, Japan where most of the population of P. altivelis is thought to be maintained by release of juveniles.

Keywords: Ayu; Great Cormorant; Population dynamics; Stochastic control theory;

1 Introduction

Plecoglossus altivelis (Ayu) is one of the most economically and culturally important inland fishery resources in Japan that accounts for 7.5 % of the total inland fish catch (MAFF, 2015). P. altivelis is an annual and diadromous fish species with a unique life history as reviewed in Tanaka et al. (2011). The adults spawn eggs during autumn in downstream reaches of their living river and die afterwards. Hatched larvae descend to connected coastal area of a downstream water body of the river: a sea or a brackish lake in general. They grow up to juveniles with feeding on zooplanktons till the coming spring. The juveniles migrate the river toward its midstream reaches where rock-attached algae, which are main foods of P. altivelis, are available on riverbed. They feed on the algae and aquatic insects to mature till the coming autumn. Algae-eating fish species in rivers, such as P. altivelis are common in Japanese river ecosystem, and have significant influence on diversity of aquatic species living in river environment (Iguchi, 2011, Katano et al., 2006).

Fish catch of P. altivelis in Japan has been rapidly decreasing, which is considered due to decrease of its population (Tanaka et al., 2011; Iguchi, 2011). Major causes of the population decrease would include climate and hydrological changes (Mouri et al., 2010) and artificial manipulation of river environments, such as installing transverse hydraulic structures serving as physical barriers (Takahashi et al., 2006). Another possible cause is feeding damage by Phalacrocorax carbo (Great Cormorant), which is a bird species resigning as a top predator of the river ecological systems in Japan (Yamamoto, 2008; Yamamoto, 2010). This bird species makes nesting colonies in riparian forest with less signs of human life. Total number of individuals of the bird in Japan was three thousands in 1970’s (Fukuda et al., 2002); however, its population has been rapidly increasing and eventually exceeded several tens of thousands (Yamamoto, 2008). The increase of the population triggered severe predation pressure to inland fishery resources; P. altivelis is not an exception (Yamamoto, 2008; Yamamoto, 2010). As a similar example, in European and American countries, P. carbo and its subspecies, Phalacrocorax auritus (Double-crested Cormorant) is causing significant feeding damages to fishery resources (Steffens, 2010; Sullivan, 2006). In order to overcome this situation, a management strategy to maximize the profit of total fish catch while simultaneously to minimize the excessive predation pressure from the bird should be established. Such a management strategy would also be useful for achieving sustainable management of river environment and ecology.

Population dynamics of fishery resources are inherently stochastic due to a variety of internal and external noises, such as species competitions, natural and artificial environmental changes in rivers (Doyen et al., 2012; Unami et al., 2012). The changes of temperature, discharge in a river, and constructing dams or weirs critically affect river environments (Iwasaki and Yoshimura, 2012). Such population dynamics should therefore be considered as an appropriate stochastic process. The stochastic differential equation (SDE) can effectively describe such dynamics (Øksendal, 2003). Yaegashi et al. (2015) presented a stochastic process model for population dynamics of

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released $P.\ altivelis$ in a river considering the predation pressure from $P.\ carbo$ and the human activities, which are extermination of $P.\ carbo$ from the river and harvesting the fish by human. The model was operated to find an optimal management strategy for the population of $P.\ altivelis$ with a stochastic control theory. The problem reduces to solving a Hamilton-Jacobi-Bellman equation (HJBE), which is a time-backward and nonlinear degenerate parabolic partial differential equation (PDE) (Øksendal, 2003). HJBEs serve as powerful mathematical tools for obtaining optimal control strategy. Guo et al. (2005) used a HJBE to derive the optimal birth rate in China. Rafikov et al. (2005) solved an HJBE to find an optimal vaccination strategy for the pest. Yoshioka et al. (2015) discussed ascending behaviour of migratory fishes with a time-independent HJBE that determines the optimal swimming velocity of individual fishes. In Yaegashi et al. (2015), taking the human activities as the control variables, the optimal management strategy to maximize the total fish catch while to minimize the cost for exterminating $P.\ carbo$ was explored; however, its sensitivity; namely qualitative and quantitative change of the optimal management strategy to the critical model components, such as noise intensity, intrinsic growth rate, and human activities have not been verified in detail. Comprehending performance of the optimal management strategy is crucial for its real applications, which is a strong motivation of this paper.

The objective of this paper is to formulate the management problem for population of $P.\ altivelis$ in stochastic river environment from the viewpoint of optimal control theory and to numerically verify its sensitivity against several model parameters. In the rest of this paper, section 2 formulates the optimal management problem based on the dynamic programming principle. Section 3 numerically verifies sensitivity of the optimal management strategy. Section 4 gives conclusions of this paper and perspectives on future researches.

2 Mathematical model

2.1 Stochastic population dynamics of $P.\ altivelis$

The stochastic process model for annual population dynamics of released $P.\ altivelis$ in a river is presented in this sub-section following the previous research (Yaegashi et al., 2015). This paper assumes that the predation pressure to $P.\ altivelis$ is solely from $P.\ carbo$. The main reasons of adopting this assumption are the followings. Firstly, in Hii River system, exotic fish species that are piscivorous such as Centrarchidae (Bluegill) have been found only in the reservoirs of large dams (Hii River Fishery Cooperatives, 2015a). They have not been found in the middle and upstream reaches of the river system other than the reservoirs of dams. Secondly, although there exist other piscivorous bird species such as Ardeidae (Heron) around the river system, they have been reported to exert far less predation pressure to $P.\ altivelis$ than $P.\ carbo$ does. (Kosugi, 1960; Lekuona, 2002).

Let $\{0,T\}$ with the terminal time $T$ (day) be a period during which population dynamics of $P.\ altivelis$ in a habitat, which is a river, is considered. The initial time $t=0$ and the terminal time $t=T$ are set in spring and autumn, respectively within a year. The total biomass of $P.\ altivelis$ in this river at the time $t$ is denoted as $X_t$ (kg), which is considered as a continuous stochastic process. The initial condition $X_0$ at the time $t=0$, which is the release amount of the juveniles, is assumed to be deterministic. This is valid for the case where most part of the population of $P.\ altivelis$ in the river is introduced through intensive release events. The initial population $X_0$ is expressed as $X_0 = NW_0$ where $N$ (-) is the number of individuals released at the time $t=0$ and $W_0$ (kg) is the average weight of the juveniles at that time. The governing Itô’s SDE of the process $X_t$ is given as

$$
\text{d}X_t = \left( a(t,X_t) - l_t \right) \text{d}t + b(t,X_t) \text{d}B_t
$$

with

$$
l_t = R + k(u_t) + X_{t+T} \chi_{(t,T]} \sigma z
$$

and the growth-curve based Verhulst model (Bhowmick et al., 2014; Bhowmick and Sabyasachi, 2014)

$$
a(t,x) = R \left( 1 - \frac{X_t}{K} \right) X_t, \quad b(t,x) = \sigma X_t
$$

where $B_t$ is the 1-D standard Brownian motion (Øksendal, 2003), $R$ (1/day) is the natural mortality rate, $k(u_t)$ (1/day) is the predation pressure from $P.\ carbo$ (Iguchi et al., 2008) as a function of the control $u=u_t$ (-), which indicates the magnitude of the effort to exterminate $P.\ carbo$, $c$ (1/day) is the fishing pressure from human, and $(0<T_c<T)$ (day) is the opening time of harvesting $P.\ altivelis$, and $\chi_S(z) = \chi_S(z)$ is the indicator function such that $\chi_S(z)=1$ for $z \in S$ and $\chi_S(z)=0$ otherwise.

The capacity $K$ (kg), the intrinsic growth rate $r$ (1/day), and the noise intensity $\sigma$ (1/day$^{1/2}$) are positive parameters of the Verhulst model. The noise intensity $\sigma$ modulates natural and artificial environmental changes in a river, which are assumed to be mathematically implicitly modelled with a multiplicative noise term as in the conventional researches (Caraballo et al., 2015; Colombo and Anteneodo, 2015). Assuming that the population dynamics is limited by the released amount rather than by the environmental capacity, the parameter $K$ is expressed as (Yaegashi et al., 2015)

$$
K = mx_0 = mNW_0
$$

with a positive constant $m=O\left(10^{0.1}\right)$ (-). This assumption can be valid for a river where the population of released $P.\ altivelis$ dominates the natural counterparts and they do not saturate in the habitat. The generator $\mathcal{A}^c$ associated with the coupled stochastic process $Y_t = (t, X_t)$ conditioned on $Y_s = (s,x)$ with $s<t$ is expressed as (Øksendal, 2003)
\[
A'' = \frac{\partial}{\partial s} + (a(s,x) - l_x) \frac{\partial}{\partial x} + \frac{1}{2} \left[ b(s,x) \right]^2 \frac{\partial^2}{\partial x^2}.
\] (5)

### 2.2 Control variable and objective function

The variables \( u \) and \( c \) are taken as the controls variables in the model, which are assumed to be Markov controls (Øksendal, 2003). The admissible ranges \( U \) and \( C \) of the controls are specified as

\[
U = \{ u | 0 \leq u \leq 1 \} \quad \text{and} \quad C = \{ c | 0 \leq c \leq c_M \},
\] (6)

respectively where \( c_M \) (1/day) is the maximum fishing pressure. The objective function to be maximized

\[
v = v(s,T,X_s,u,c) \quad \text{(yen)}
\]

is assumed to represent the profit of the local fishery cooperatives. The objective function is proposed as

\[
v = \int_0^T \left\{ -f(u) + \chi_{[t \geq T]} \theta_c X_t \right\} dt - \eta X_0 \quad \text{(7)}
\]

where \( f(z \geq 0) \) (yen/day) with \( f(0) = 0 \) is an increasing function, \( \theta \) (yen/kg) is the price of matured \( P. \, altivelis \) per kilogram and \( \eta \) (yen/kg) is the cost of larval \( P. \, altivelis \) per kilogram, both of which are assumed to be constant in this paper. The first and second integrands of Eq.(7) are the total cost of operating countermeasures to exterminate \( P. \, carbo \) from the river and the total profit through the fish catch of \( P. \, altivelis \) by human, respectively. The term \( -\eta X_0 \) represents the cost to buy larval \( P. \, altivelis \) to be released at the time \( t = 0 \). The two functions \( k(u) \) and \( f(u) \) on the control variable \( u \) are simply specified as

\[
k(u) = k_0 (1 - \alpha u) \quad \text{and} \quad f(u) = \omega u,
\] (8)

respectively where \( k_0 \) (1/day) is the predation pressure from the bird without countermeasures, \( \alpha \) (-) with \( 0 < \alpha \leq 1 \) modulates efficiency to decrease the predation pressure with the control \( u \), and \( \omega \) (yen/day) modulates the cost to decrease the predation pressure (Yamamoto, 2008; Yamamoto, 2010). Eq.(7) indicates that the extinction \( (X_t = 0) \) of the population of \( P. \, altivelis \) during the period \( (0,T) \) does not lead to any profits (Fleming and Soner, 2006), which is considered as a part of the boundary conditions for the Hamilton-Jacobi-Bellman equation below. Maximizing the value function therefore leads to an optimal strategy that is economically profitable as well as ecologically sustainable.

### 2.3 Hamilton-Jacobi-Bellman equation

The ultimate goal of stochastic optimal control is to find the maximizers \( u = u^* \) and \( c = c^* \) of the value function

\[
V = V(s,x,T),
\]

which is defined as

\[
V = E^{x,x} \left[ v(s,T,X_s,u^*,c^*) \right] \quad \text{(9)}
\]

where \( E^{x,x} \left[ \right] \) is the expectation conditioned on \( X_s = x \). Application of the dynamic programming principle to Eq.(9) leads to the HJBE governing the value function \( V \) as (Øksendal, 2003)

\[
\sup_{u \in U, c \in C} \left\{ A'' V - f(u) + \chi_{[t \geq T]} \theta_c X_t \right\} = A'' V - f(u^*) + \chi_{[t \geq T]} \theta c^* x , \quad \text{(10)}
\]

which is a time-backward and nonlinear degenerate parabolic partial differential equation (PDE) that has to be equipped with appropriate terminal and boundary conditions for well-posedness. A straightforward calculation leads to the optimal controls \( u^* \) and \( c^* \) via the value function \( V \) as

\[
u^* = 1 \left( k_0 \alpha \gamma - \omega \geq 0 \right), \quad u^* = 0 \quad \text{(Otherwise)}
\] (11)

\[
c^* = c_M \left( \theta \delta - \gamma \geq 0 \right), \quad c^* = 0 \quad \text{(Otherwise)},
\] (12)

respectively, where \( \gamma = x \partial V / \partial x \). Both of the optimal controls are the bang-bang type. The domain of the population of \( P. \, altivelis \) \( x \) is set as \( \Omega = (0,L) \) with \( L > 0 \) (kg) determined later. Substituting \( u = u^* \) and \( c = c^* \) into Eq.(10) fully specifies the HJBE in the domain \( \Omega \). The terminal condition for the HJBE is set as \( V \mid _{t = T} = 0 \) and the boundary conditions as \( V \mid _{x=0} = 0 \) and \( \partial V / \partial x \mid _{x=L} = 0 \). Solving the HJBE yields the optimal controls \( u^* \) and \( c^* \) in the rectangular spatio-temporal domain \( (0,T) \times (0,L) \), which is performed numerically in this paper.

### 2.4 Indices for verifying the management strategy

Once the optimal controls \( u^* \) and \( c^* \) are computed in the spatio-temporal domain \( (0,T) \times (0,L) \), indices to verify the optimal management strategy can be obtained considering the link between SDEs and terminal and boundary value problems of PDEs (Øksendal, 2003). The index

\[
J = E^{x,x} \left[ \int_0^T g(X_t) dt \right]
\] (13)

with an univariate function \( g \) solves the Kolmogorov’s backward equation

\[
A'' J + g = 0
\] (14)

subject to appropriate terminal and boundary conditions.
Hereafter, the notation $J_0 = J(0,X_0)$ is used for the sake of brevity. This index represents the expectation of the temporal integration of $g(X)$ from $t = s$ to $t = T$. The indices considered in this paper are $J_{0,\text{cum}}, J_{0,\text{pr}}$, and $J_{0,\text{tot}}$, which are the total cost to exterminate $P. \text{carbo}$ (yen), the total weight of predated $P. \text{altivelis}$ (kg), and the total weight of caught $P. \text{altivelis}$ by human (yen), respectively. They are calculated through Eq.(14) with $g(X)$ specified as $f(u'), \chi_{(0,T)} c^X$, and $k(u') X$, respectively. In addition, $V(0,X,T)$ is denoted as $J_{0,\text{cum}}$, which is the total profit during $(0,T)$ with $X_0$ subject to the optimal controls $u'$ and $c'$.

3 Numerical computation

3.1 Study area

Parameter values of the present mathematical model are estimated from the data collected around Hii River, San-in area, Japan. Most part of population of $P. \text{altivelis}$ are introduced through release events under the initiative of Hii River Fishery Cooperatives. The total length of the mainstream and the catchment area of the river are 153 (km) and 2,070 (km$^2$), respectively (MLIT, 2015). Two downstream brackish lakes named Lake Shinji and Lake Nakaumi from upstream, are connected to the river. Hatched larvae of $P. \text{altivelis}$ are thought to descend to Lake Shinji; however, it is not known where they survive during winter. According to the Hii River Fishery Cooperatives, the population of $P. \text{altivelis}$ ascending the river during spring has been significantly decreasing (Hii River Fishery Cooperatives, 2015b). Juveniles of $P. \text{altivelis}$ are released in its midstream reaches during May. The grown fishes spawn in downstream reaches of the river during October to November.

3.2 Computational conditions

Table 1 summarizes the parameter values determined based on the information from Hii River Fishery Cooperatives, (2015b). On the other parameters, the natural mortality rate $R$ is estimated as $4.6 \times 10^{-3}$ (1/day) (Miyaji et al., 1963). The value of $\alpha$ is estimated as 0.50 (Yaegashi et al., 2015). The maximum fishing pressure $c_m$ is set as 0.01 (1/day) following the previous research (Yaegashi et al., 2015) and it has preliminary been checked that this parameter does not significantly affect quality of the value function and the controls than the other parameters. Assuming that Eq.(1) with $k = c_m = 0$ has a non-trivial and non-singular probability density for large $T$ (Grigoriu, 2014), the range of the noise intensity $\sigma$ is set as $0.04 \leq \sigma \leq 0.32$ (1/day$^{1/2}$). The value of the parameter $k_0$ is estimated from an exact solution to a deterministic counterpart of Eq.(1) with $c_m = 0$ and given $X_0$ for each computation (Yaegashi et al., 2015). Based on the weights of matured and released $P. \text{altivelis}$ (Yaegashi et al., 2015; Hii River Fishery Cooperatives, 2015b; Murayama et al., 2010), the ranges of the growth rate $r$ and the release amount $X_0$ are set as $6.1 \times 10^{-2} \leq r \leq 7.5 \times 10^{-2}$ (1/day) and as $5.0 \times 10^3 \leq X_0 \leq 4.0 \times 10^3$ (kg), respectively.

The domain of the population of $P. \text{altivelis}$ $x$ is set as $\Omega = (0,6.0 \times 10^3)$ (kg) and is uniformly discretized into a mesh with 300 elements and 301 nodes. The time increment for temporal integration is 0.01 (day). Eqs.(10) and (14) are solved with the finite element scheme that has already been extensively verified through numerical and mathematical analysis (Yoshioka et al., 2014). Increasing computational resolution in space and time does not significantly affect the computational results presented below.

| $T$ (day) | $T_c$ (day) | $m$ | $W_0$ (kg) | $\eta$ (yen/kg) | $\theta$ (yen/kg) |
|----------|------------|-----|------------|----------------|-----------------|
| 180      | 90         | 5.4 | $9.4 \times 10^3$ | $4.4 \times 10^3$ | $4.0 \times 10^3$ |

3.3 Demonstrative optimal management strategy

The optimal management strategy is numerically computed for a set of parameters as a demonstrative example. The parameters $r$, $\sigma$, and $X_0$ are set as $6.8 \times 10^{-2}$ (1/day), $0.18$ (1/day$^{1/2}$), and $2,250$ (kg), respectively. Figure 1(a) through 1(c) plot the computed value function $V$ and the optimal controls $u'$ and $c'$, respectively. Figure 1(a) shows that the value function $V$ does not have spurious oscillations, indicating that the numerical scheme can handle the HJB. Figure 1(b) shows that the optimal strategy for extermination is to intensively reduce the predation pressure mainly after the opening time $T_c$ except the terminal time $T$, and before the opening time $T_c$, for the case of small biomass of $P. \text{altivelis}$. Figure 1(c) indicates that optimal strategy is to harvest $P. \text{altivelis}$ after the opening time $T_c$ except for small population $x$.

3.4 Numerical verification of sensitivity

Sensitivity of the optimal management strategy is verified with the indices $J_0$ for assumed ranges of the parameters $r$, $\sigma$, and $X_0$. Sub-sections 3.4.1, 3.4.2, and 3.4.3 verify the sensitivity on $r-\sigma$, $\sigma-X_0$, and $r-X_0$ planes, respectively. Practically, these parameters can be manipulated as follows. The release amount $X_0$ can be determined by fishery cooperatives. The growth rate can be increased by removal of a physical barrier whose upstream reaches have rich rock algae serving as staple food of $P. \text{altivelis}$. The noise level $\sigma$ would be difficult to manipulate because it modulates a number of internal and external stochasticity. Exploring manipulation method for this parameter is beyond the scope of this paper, which is an important topic to be addressed in future researches.

3.4.1 Sensitivity on the parameters $r$ and $\sigma$

Sensitivity of the optimal management strategy on the growth rate $r$ and the noise intensity $\sigma$ with the release
amount $X_0 = 1.7 \times 10^5$ (kg) is verified. Figure 2(a) through 2(d) plot the computed $J_{0,pr}$, $J_{0,cau}$, $J_{0,cos}$, and $J_{0,tot}$ in the $r - \sigma$ plane, respectively. Figure 2 shows that $J_{0,pr}$, $J_{0,cau}$, and $J_{0,tot}$ monotonically increase as $r$ increases. This is considered due to that in average $X_j$ increases as $r$ increases, which consequently leads to the increase of $J_{0,cos}$, $J_{0,pr}$, and $J_{0,tot}$. Figure 2 also shows that they monotonically decrease as $\sigma$ increases. A heuristic consideration based on simplified models (Grigoriu, 2014) infers that increase of $\sigma$ increases stochastic fluctuations of the population dynamics, which in general results in higher probability of the population extinction. According to Figure 2(d), $J_{0,tot}$ is maximized in the plane for low $\sigma$ and high $r$, and it becomes negative for large $\sigma$. The index $J_{0,tot}$ more sensitively responds to $\sigma$ than $r$ for the range of the parameters. The computational results imply that to decrease $\sigma$ than $r$ more effectively gain profit. Being different from the three indices that monotonically response to $r$ and $\sigma$, the index $J_{0,cos}$ is not monotonic as shown in Figure 2(c); it is maximized at $(r, \sigma) = (6.1 \times 10^{-2}, 0.16)$. It has been checked that the extreme point in the plane moves toward the lower left corner of the panel area as the maximum fishing pressure $c_{mf}$ increases.

### 3.4.2 Sensitivity on the parameters $\sigma$ and $X_0$

Sensitivity of the strategy on the noise intensity $\sigma$ and the release amount $X_0$ with the growth rate $r = 7.1 \times 10^{-2}$ (1/day) is examined. Figure 3(a) through 3(d) plot...
computed $J_{0,pr}$, $J_{0,cos}$, $J_{0,cos}$, and $J_{0,lot}$ in the $\sigma - X_0$ plane, respectively. Figure 3 shows that increase of $X_0$ in the plane increase $J_{0,pr}$, $J_{0,cos}$, and $J_{0,lot}$ for small $\sigma$. On the other hand, increase of $X_0$ doesn’t significantly affect them for large $\sigma$. Figure 3 also shows that all the indices monotonically decrease as $\sigma$ increases. The decrease of the $J_{0,cos}$ is the most significant among them. Figure 3(d) shows that the total profit $J_{0,lot}$ is negative for large $\sigma$ or small $X_0$.

3.4.3 Sensitivity on the parameters $r$ and $X_0$

Sensitivity of the strategy on the growth rate $r$ and the release amount $X_0$ with the noise intensity $\sigma = 0.24$ (1/day$^{1/2}$) is examined. Figure 4(a) through 4(d) plot computed $J_{0,pr}$, $J_{0,cos}$, $J_{0,cos}$, and $J_{0,lot}$ in the $r - X_0$ plane, respectively. Increase of $r$ in the plane doesn’t increase $J_{0,pr}$, $J_{0,cos}$ and $J_{0,lot}$ for small $X_0$. On the other hand, for large $X_0$, increase of $r$ leads to their significant increase. For the index $J_{0,cos}$, increase of $X_0$ induces its rapid increase. Figure 4(d) shows that the total profit $J_{0,lot}$ is negative for small $r$ or $X_0$.

4 Conclusions

A mathematical model for population dynamics of $P. altivelis$ in stochastic river environment was presented. In addition, an optimal management strategy to maximize the profit from harvesting $P. altivelis$ and simultaneously to minimize the cost to exterminate $P. carbo$ while to sustain $P. altivelis$ was formulated. Finding the optimal management strategy was ultimately reduced to approximating solutions to a HJBE derived on the basis of the dynamic programming principle. Sensitivity of the optimal management strategy of the population dynamics on the critical parameters related to the growth rate, environmental noise, and the release amount was verified focusing on the case of Hii River, San-in area, Japan. The obtained results quantified sensitivity of the strategy and its monotone and non-monotone responses to the parameters were identified. The results are useful for assessing reliability of the model in real applications, which would help lead to sustainable management of river environment and ecology.

Currently, weekly observations of spatio-temporal fluctuation of hydrological characteristics, such as water temperature and turbidity, at multiple stations along Hii River are being carried out by the authors and union members of Hii River Fishery Cooperatives, which would help revealing the relationship between hydrological characteristics and environmental noise level $\sigma$. Such field observations and biological and ecological considerations would lead to developing a more realistic mathematical model considering hydrological characteristics of the river. Future research will extend the present mathematical model to investigate population dynamics of $P. altivelis$ in partially fragmented rivers by transverse hydraulic structures where multiple habitats exist. This extension can be achieved considering population dynamics in fragmented habitats and migrations of the population among them as in Dubois et al. (2015). Finding effective linkages between the population dynamics models and the local swimming behavior models (Yoshioka et al., 2015) is another important research topic to be addressed, which would lead to better comprehensions of both of the models.

The HJBEs are highly nonlinear PDEs whose solution behavior cannot be easily analyzed. Approaching the HJBEs from the view point of mathematics by itself is therefore an important research topic, which would open a possibility to develop an innovative management strategy collaborating science and engineering.

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JOURNAL OF RAINWATER CATCHMENT SYSTEMS/VOL.22 NO.1 2016 ■ 13