A new system of singular integral equations for a curvilinear crack in bonded materials

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Abstract. The modified complex potentials (MCPs) functions are used to develop a new system of singular integral equations (SIEs) for a curvilinear crack in the upper part of bonded materials subjected to shear mode stress with the help of continuity conditions for resultant force and displacement functions. The unknown dislocation distribution function is mapped into a square root singularity function by using curved length coordinate method and the traction along the crack as the right hand term. The Gaussian quadrature rules were used to obtain the numerical solution for a new system of SIEs in order to compute the nondimensional stress intensity factors (SIFs) for these problems. Our results agree with those of the previous works. The findings have revealed that the nondimensional SIFs depend on the elastic constant ratio, crack geometries and the position of the cracks.

1. Introduction
In last couple of years numerous specialists utilized the complex variable function (CVF) method to explore the dependability and security of the materials dependent on the behavior of SIFs. They analyzed the crack problems in several types of materials such as crack in an infinite plane \cite{1, 2}, finite plane \cite{3, 4} and half plane \cite{5, 6}. The nondimensional SIFs was calculated by using logarithmic SIEs for a curvilinear crack problem embedded in one of bonded materials \cite{7}. The expansion collocation technique has been used to reduce bonded materials of interface crack problems to a system of SIEs with a simplified Cauchy kernel using the Fourier integral transform method \cite{8}. The relevant crack opening displacement (COD) has been used to calculate the nondimensional SIFs for bonded materials with two-dimensional interface cracks, three-dimensional penny-formed cracks, and circumferential surface cracks \cite{9}. The displacement analysis was conducted to investigate the nondimensional SIFs for interface cracks in bonded materials and the effect of Poisson’s ratio \cite{10}. The linear elastic fracture analysis for bonded materials problems of interface crack was proposed by an extended finite element method \cite{11}. The complex SIFs for bonded materials with interface cracks were calculated using a high-order extended finite element method and a novel set of material-dependent enrichment functions \cite{12}.
Chebyshev polynomials and collocation methods were used to measure the nondimensional SIFs for bonded materials with a perpendicular crack to the interface of materials [13]. By reducing the dual integral equations to an infinite system of simultaneous equations, the effect of elastic constant ratio on the axisymmetric problem for bonded materials with a penny-shaped crack was investigated [14]. The randomly targeted inclusions and networks were used to investigate several types of cracks in bonded materials such as edge, semi-infinite interface, and substrate cracks [15]. Using hypersingular integral equations (HSIEs) and an unknown COD function, nondimensional SIFs for multiple cracks in the upper plane of bonded materials subjected to shear mode stress were measured [16]. They extended their research to investigate the behavior of nondimensional SIFs for bonded materials’ upper and lower portions with multiple cracks [17]. HSIEs is used to investigate the action of nondimensional SIFs for a single crack in the upper plane of bonded materials subjected to various mechanical loads [18]. To measure the nondimensional SIFs for finite cracks perpendicular to the interface of bonded materials, the Fourier transform technique with dislocation density functions was used [19].

In the paper, considering a bonded materials containing a crack under remote shear stress on crack, a new system of SIEs is proposed to study a curvilinear crack interaction based on elementary solution. This problem is formulated into SIEs by using MCPs functions with dislocation distribution function as unknown. To validate our results, the nondimensional SIF from particular crack geometry is compared to other works. In addition, the effect of elastic constants ratio and geometry conditions on the interacting nondimensional SIFs are studied. The current discoveries may help engineers in researching the construction of the bonded materials under shear mode stress.

2. Mathematical formulation
CVF method \( \varphi(\eta) \) and \( \psi(\eta) \) for crack problems in an elasticity introduced by Muskhelishvili [20] are related to stress components \( (\sigma_x, \sigma_y, \sigma_{xy}) \), resultant force \( (X, Y) \), and displacements functions \( (u, v) \) as follows

\[
\sigma_y - \sigma_x + 2i\sigma_{xy} = 2[\eta \varphi''(\eta) + \psi'(\eta)],
\]

\[
-Y + iX = \varphi(\eta) + \eta \varphi'(\eta) + \psi(\eta),
\]

\[
2G(u + iv) = \kappa \varphi(\eta) - \eta \varphi'(\eta) - \psi(\eta),
\]

where \( \eta = x + iy \), \( G \) is shear modulus, \( \kappa = (3 - \nu)/(1 + \nu) \) for plane stress, \( \kappa = 3 - 4\nu \) for plane strain and \( \nu \) is Poisson’s ratio. The derivation of resultant force function (2) with respect to \( \eta \) gives

\[
d\eta \left( -Y + iX \right) = \varphi'(\eta) + \varphi'(\eta) + d\eta \left( \eta \varphi''(\eta) + \psi'(\eta) \right) = N + iT
\]

where the traction along the segment \( \eta, \eta + d\eta \) is in terms of normal \( (N) \) and tangential \( (T) \) components. CVF for a crack \( L \) in an infinite plane modeled by the dislocation distribution function \( g'(\tau) \) are denoted by [21]

\[
\varphi(\eta) = \frac{1}{2\pi} \int_L g'(\tau) d\tau, \quad \psi(\eta) = \frac{1}{2\pi} \int_L \frac{1}{(\tau - \eta)^2} \left( (\tau - \eta)g'(\tau)d\tau - \tau g'(\tau)d\tau \right),
\]

where \( g'(\tau) \) is defined by

\[
g'(\tau) = -\frac{2Gi}{(\kappa + 1)} \frac{d}{d\tau} \left( (u(\tau) + iv(\tau))^+ - (u(\tau) + iv(\tau))^+ \right), \quad \tau \in L
\]

\((u(\tau) + iv(\tau))^+ \) and \((u(\tau) + iv(\tau))^+ \) denote the displacements at point \( \tau \) of the upper and lower crack faces, respectively.
Figure 1. Upper part of bonded materials with a curvilinear crack subjected to shear mode stress.

Consider the upper part of bonded materials with a curvilinear crack subjected to shear mode stress as defined in figure 1. The condition for strain components in the upper, \( \varepsilon_{x_1} \), and lower parts, \( \varepsilon_{x_2} \), of bonded materials in terms of Young’s modulus of elasticity in the upper, \( E_1 = 2G_1(1 + v_1) \) and lower parts, \( E_2 = 2G_2(1 + v_2) \), consider only shear mode stress, is

\[
\varepsilon_{x_1} = \varepsilon_{x_2}, \quad \frac{1}{E_1} \sigma_{x_1}^\infty = \frac{1}{E_2} \sigma_{x_2}^\infty. \tag{7}
\]

The MCPs functions for the upper part of bonded materials with a crack involve principal \((\varphi_{1p}(\eta), \psi_{1p}(\eta))\) and complementary parts \((\varphi_{1c}(\eta), \psi_{1c}(\eta))\) defined as

\[
\varphi_{1}(\eta) = \varphi_{1p}(\eta) + \varphi_{1c}(\eta), \quad \psi_{1}(\eta) = \psi_{1p}(\eta) + \psi_{1c}(\eta). \tag{8}
\]

Whereas for a crack in the lower part, the CVF is denoted by \( \varphi_{2}(\eta) \) and \( \psi_{2}(\eta) \). Note that the principal parts of CVF is equal to the complex variable functions for a crack in an infinite plane.

The resultant force (2) and displacement (3) continuity conditions are described as follows

\[
\left( \varphi_{1}(\tau) + \tau \varphi'_{1}(\tau) + \psi_{1}(\tau) \right)^+ = \left( \varphi_{2}(\tau) + \tau \varphi'_{2}(\tau) + \psi_{2}(\tau) \right)^-, \tag{9}
\]

\[
G_2 \left( \kappa_1 \varphi_{1}(\tau) - \tau \varphi'_{1}(\tau) - \psi_{1}(\tau) \right)^+ = G_1 \left( \kappa_2 \varphi_{2}(\tau) - \tau \varphi'_{2}(\tau) - \psi_{2}(\tau) \right)^-, \tag{10}
\]

where \( \tau \in L \). Applying equation (8) into equations (9) and (10), the expressions obtained are as follows

\[
\varphi_{1c}(\eta) = \Gamma_1 \left( \eta \varphi'_{1p}(\eta) + \psi_{1p}(\eta) \right), \quad \eta \in S_1 + L_b, \tag{11}
\]

\[
\psi_{1c}(\eta) = \Gamma_2 \varphi_{1p}(\eta) - \Gamma_1 \left( \eta \varphi'_{1p}(\eta) + \eta^2 \varphi''_{1p}(\eta) + \eta \psi'_{1p}(\eta) \right), \quad \eta \in S_1 + L_b, \tag{12}
\]

\[
\varphi_{2}(\eta) = (1 + \Gamma_2) \varphi_{1p}(\eta), \quad \eta \in S_2 + L_b, \tag{13}
\]

\[
\psi_{2}(\eta) = (\Gamma_1 - \Gamma_2) \eta \varphi'_{1p}(\eta) + (1 + \Gamma_1) \psi_{1p}(\eta), \quad \eta \in S_2 + L_b. \tag{14}
\]
where \( \varphi_{1p}(\eta) = \varphi_{1p}(\bar{\eta}) \), \( L_b \) is boundary of bonded materials, \( S_1 \) and \( S_2 \) are upper and lower parts of bonded materials, respectively, and \( \Gamma_1 \) and \( \Gamma_2 \) are bi-elastic constants denoted as

\[
\Gamma_1 = \frac{G_2 - G_1}{G_1 + \kappa_1 G_2}, \quad \Gamma_2 = \frac{\kappa_1 G_2 - \kappa_2 G_1}{G_2 + \kappa_2 G_1}.
\]

In order to develop a new system of SIEs for the upper part of bonded materials with a curvilinear crack (Figure 1), we need to define two traction components which are \( (N(\tau_0) + iT(\tau_0))_{1p} \) and \( (N(\tau_0) + iT(\tau_0))_{1c} \) for the principal and complementary parts, respectively. To obtain the principle part \( (N(\tau_0) + iT(\tau_0))_{1p} \), substituting equation (5) into (4), then imposing point \( \eta \) closer to \( \tau_0 \) on the crack and setting \( d\eta/d\tau \) into \( d\tau_0/d\tau \) gives

\[
(N(\tau_0) + iT(\tau_0))_{1p} = \frac{1}{\pi} \int_L \frac{g'(\tau)d\tau}{\tau - \tau_0} + \frac{1}{2\pi} \int_L A_1(\tau, \tau_0)g'(\tau)d\tau + \frac{1}{2\pi} \int_L A_2(\tau, \tau_0)\bar{g}'(\tau)d\bar{\tau} \quad (15)
\]

where

\[
A_1(\tau, \tau_0) = \frac{-1}{\tau - \tau_0} + \frac{1}{\tau - \tau_0} \frac{d\bar{\tau}_0}{d\tau_0}, \quad A_2(\tau, \tau_0) = \frac{1}{\tau - \tau_0} - \frac{\tau - \tau_0}{(\tau - \tau_0)^2} \frac{d\bar{\tau}_0}{d\tau_0}.
\]

Whereas to obtain the complementary part \( (N(\tau_0) + iT(\tau_0))_{1c} \), substituting equations (11) and (12) into (4) and applying equation (5), then imposing point \( \eta \) closer to \( \tau_0 \) on the crack and setting \( d\eta/d\tau \) into \( d\tau_0/d\tau \) gives

\[
(N(\tau_0) + iT(\tau_0))_{1c} = \frac{1}{2\pi} \int_L A_3(\tau, \tau_0)g'(\tau)d\tau + \frac{1}{2\pi} \int_L A_4(\tau, \tau_0)\bar{g}'(\tau)d\bar{\tau} \quad (16)
\]

where

\[
A_3(\tau, \tau_0) = \Gamma_1 \left[ \frac{1}{\tau - \tau_0} + \frac{\tau - \bar{\tau}}{(\tau - \tau_0)^2} + \frac{1}{\tau - \tau_0} \frac{d\bar{\tau}_0}{d\tau_0} \left( 2\tau_0(\tau - \bar{\tau}) + \frac{\tau - 3\bar{\tau}_0}{(\tau - \tau_0)^3} + \frac{2}{(\tau - \tau_0)^3} - \frac{1}{\tau - \tau_0} \right) \right] + \Gamma_2 \frac{1}{\tau - \tau_0} \frac{d\bar{\tau}_0}{d\tau_0},
\]

\[
A_4(\tau, \tau_0) = \Gamma_1 \left( \frac{1}{\tau - \tau_0} + \frac{\tau - \bar{\tau}}{(\tau - \tau_0)^2} + \frac{\tau_0 - \tau}{(\tau - \tau_0)^2} \frac{d\bar{\tau}_0}{d\tau_0} \right).
\]

Summing equations (15) and (16) yields a new system of SIEs for the upper part of bonded materials with a curvilinear crack

\[
(N(\tau_0) + iT(\tau_0)) = (N(\tau_0) + iT(\tau_0))_{1p} + (N(\tau_0) + iT(\tau_0))_{1c} = \frac{1}{\pi} \int_L \frac{g'(\tau)d\tau}{\tau - \tau_0} + \frac{1}{2\pi} \int_L B_1(\tau, \tau_0)g'(\tau)d\tau + \frac{1}{2\pi} \int_L B_2(\tau, \tau_0)\bar{g}'(\tau)d\bar{\tau}, \quad (17)
\]

where

\[
B_1(\tau, \tau_0) = A_1(\tau, \tau_0) + A_3(\tau, \tau_0), \quad B_2(\tau, \tau_0) = A_2(\tau, \tau_0) + A_4(\tau, \tau_0).
\]

Note that the first integral in equation (17) with the minus sign represents the singular integral and must be denoted as a finite part integral [1]. If \( G_1 = G_2 \), then \( \Gamma_1 = \Gamma_2 = 0 \) and equation (17) reduce to the SIEs for a crack in an infinite plane [21]. Whereas, if \( G_2 = 0 \), then \( \Gamma_1 = \Gamma_2 = -1 \) and equation (17) reduce to the SIEs for a crack in a half plane elasticity [6].

In order to solve the new systems of SIEs (17), we map the function \( g'(\tau) \) on a real axis \( s \) with an interval \( 2\alpha \) using the curved length coordinate function as follows

\[
g'(\tau)|_{\tau=\tau(s)} = \frac{H(s)}{\sqrt{a^2 - s^2}}, \quad \text{where } H(s) = H_1(s) + iH_2(s). \quad (18)
\]
Then we apply the quadrature formulas introduced by Mayrhofer and Fischer [22]. In order to investigate the behavior of nondimensional SIFs for the upper part of bonded materials with a curvilinear crack subjected to shear mode stress (Figure 1), we define the SIFs at the crack tip $T_j$ as follows

$$K_{T_j} = (K_1 - iK_2)_{T_j} = \sqrt{2\pi} \lim_{\tau \to T_j} \sqrt{|\tau - \tau_j|} g'_1(\tau_1), \quad j = 1, 2,$$

$$= \sqrt{2\pi} \lim_{s \to T_j} \sqrt{|s - s_T|} \left[ \frac{-s_1 H_1(s_1)}{\sqrt{a_1^2 - s_1^2}} e^{-i\theta T_j} \right],$$

$$= \sqrt{a_1^2 \pi F_{T_j}}, \quad \text{(19)}$$

where

$$F_{T_j} = H_1(-a_1) e^{-i\theta T_j} = F_{1T_j} + iF_{2T_j}.$$ $F_{1T_j}$ and $F_{2T_j}$ are the Mode I and Mode II nondimensional SIFs at crack tip $T_j$, respectively. The amplitude of the standard stress singularity and the amplitude of the shear stress singularity, respectively, are expressed by $F_{1T_j}$ and $F_{2T_j}$. If the estimate of nondimensional SIFs is more notable than or equal to the estimation of critical nondimensional SIFs, the crack will spread [23]. As the estimation of nondimensional SIFs increases, the materials’ strength is becoming more weak [24].

3. Numerical results

A benchmark example is presented in this section to validate the proposed approach for nondimensional SIFs extraction. We compared our numerical results with a curvilinear crack in an elastic half plane investigated by Elfakakhre et al. [6] in table 1. It shows the nondimensional SIFs for the upper part of bonded materials with a curvilinear crack subjected to shear mode stress when $h = R$ and $G_2 = 0$. As can be seen, our results are comparable with those of Elfakakhre et al. [6]. We noticed that $F_1$ at crack tip $T_1$ is equal to $F_1$ at crack tip $T_2$. However, due to the equivalence of the stress acting at crack tips, it resulted in the value of $F_2$ at crack tip $T_1$ is equal to the negative of $F_2$ at crack tip $T_2$.

| SIF     | $15^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $75^\circ$ | $90^\circ$ | $105^\circ$ | $120^\circ$ |
|---------|------------|------------|------------|------------|------------|------------|------------|------------|
| $F_{1T_1}$ | 0.4484     | 0.4012     | 0.3448     | 0.2772     | 0.2046     | 0.1354     | 0.0785     | 0.0217     |
| $F_{1T_2}$ | 0.4459     | 0.4048     | 0.3450     | 0.2756     | 0.2042     | 0.1356     | 0.0732     | 0.0204     |
| $F_{2T_1}$ | 0.2301     | 0.2393     | 0.3184     | 0.4021     | 0.4328     | 0.4668     | 0.4843     | 0.5105     |
| $F_{2T_2}$ | 0.2310     | 0.2984     | 0.3553     | 0.4004     | 0.4353     | 0.4636     | 0.4891     | 0.5165     |
| $F_{1T_1}$ | 0.4484     | 0.4012     | 0.3448     | 0.2772     | 0.2046     | 0.1354     | 0.0785     | 0.0217     |
| $F_{1T_2}$ | -0.2301    | -0.2393    | -0.3184    | -0.4021    | -0.4328    | -0.4668    | -0.4843    | -0.5105    |

*Current study

**[6]

Figures 2 and 3 show the nondimensional SIFs against $h$ for different values of $G_2/G_1$ when $\alpha = 90^\circ$ and $R = 1$ as depicted in figure 1. It is found that, the Mode I nondimensional SIFs, $F_1$ at the crack tip $T_1$ is equal to $F_1$ at crack tip $T_2$ as shows in figure 2. As $G_2/G_1$ increases, $F_1$ decreases at all crack tips. Whereas, as $h$ increases, $F_1$ decreases for $G_2/G_1 < 1.0$, increases
Figure 2. $F_1$ at the crack tips $T_1$ and $T_2$ when $\alpha = 90^\circ$, $R = 1$ and $h$ varies.

Figure 3. $F_2$ at the crack tips $T_1$ and $T_2$ when $\alpha = 90^\circ$, $R = 1$ and $h$ varies.

Figure 4. $F_1$ at the crack tips $T_1$ and $T_2$ when $R = 2h$ and $\alpha$ varies.

Figure 5. $F_2$ at the crack tips $T_1$ and $T_2$ when $R = 2h$ and $\alpha$ varies.

for $G_2/G_1 > 1.0$ and constant for $G_2/G_1 = 1.0$ at all crack tips. It can be seen that, the Mode II nondimensional SIFs, $F_2$ at the crack tip $T_1$ is equal to the negative of $F_2$ at crack tip $T_2$ as shows in Figure 3. As $G_2/G_1$ increases, $F_2$ decreases at crack tip $T_1$ and increases at tip $T_2$ for $h > 1.5$. Whereas, as $h$ increases, $F_2$ increases at crack tip $T_1$ and decreases at tip $T_2$. These results show that the strength of the materials depends on the values of $G_2/G_1$ and $h$.

The nondimensional SIFs against $\alpha$ for different values of $G_2/G_1$ when $R = 2h$ are displayed in figures 4 and 5. It portrays that $F_1$ at the crack tip $T_1$ is equal to $F_1$ at crack tip $T_2$ as shows in figure 4. As $G_2/G_1$ increases, $F_1$ decreases at all crack tips. Whereas, as $\alpha$ increases, $F_1$ increases at all crack tips. It is observed that $F_2$ at the crack tip $T_1$ is equal to the negative of $F_2$ at crack tip $T_2$ as shows in figure 5. As $G_2/G_1$ increases, $F_2$ increases at crack tip $T_1$ and decreases at tip $T_2$. Whereas, as $\alpha$ increases, $F_2$ increases at crack tip $T_1$ and decreases at tip $T_2$. 


for $\alpha < 50^\circ$. These results enable us to conclude that the materials are more stable as $G_2/G_1$ increases and $\alpha$ decreases.

4. Conclusions
A system of SIEs was proposed to study the bonded materials with a curvilinear crack subjected to shear mode stress. This system was formulated by using the MCPs function and the continuity conditions of resultant force and displacement with the dislocation distribution function as the unknown. The system of SIEs is reduced to a crack in half plane and infinite plane for $G_2 = 0$ and $G_1 = G_2$, respectively. Numerical results and graphical representations have been used to demonstrate the large impact of the elastic constant ratio $G_2/G_1$ and crack geometry conditions on nondimensional SIFs. Because of the geometric of the cracks, the proportionality of the stress acting at the crack tips resulting in equal nondimensional SIF results. The strength of the materials depend on the estimation of $G_2/G_1$ and geometry conditions.

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