The criterion of differentiability of functions of quaternion variable is used as the basis of some algebraic field theory. Its necessary consequences are free Maxwell and Yang-Mills equations. The differentiability equations may be integrated in twistor variables and are reduced to algebraic ones. In the article we present bisingular solution and its topological modifications. Related EM-fields appear to be just the well-known Born solution and its modifications with singular structure of ring-like and toroidal topology. General problems of the algebrodynamical approach are discussed.

1. INTRODUCTION

Physics turns back to its basic principles. Nowadays, it claims not only to describe reality, though in a mathematically elegant and well extrapolated way. The question concerns the essence of physical laws themselves, and the existence of a general physical principle governing all natural processes.

It becomes evident that such a principle may be expressed in a purely abstract mathematical language only and manifests itself in some programme, "Code of Universe" predetermining the structure and evolution of the World. From this point of view, the laws of nature should be worked out "on paper" not in the laboratory. The true language of nature might be drastically different from that being used in modern physics, and one should then reject the correspondence principle (in the now adopted narrow sense).

The feasibility of such a paradigm certainly finds difficulty in the unreadiness of most physicists to abandon the habitual "Newton-Galilean" methods and eclectic medley of classical, quantum and geometric notions, and to make an attempt to build up physics all over again, "on the sheet of paper". More wonder is that for the present mathematics itself isn’t able to offer some really unique structure, which could naturally generate the heterogeneity and hierarchy, so inherent to the World.

Perhaps, only fractal mappings may easily reproduce the most simple of these properties, giving birth to a whole world "from Nothing". Another possibility on which we will concentrate in this paper is related to a hyper-holomorphic structure, which is a generalization
of the Cauchy-Riemann criteria in complex calculus, to some exclusive algebras of quaternionic type. Based solely on this structure it was found possible to build a non-Lagrangian field theory, where the noncommutativity of these algebras leads to nonlinearity of the generalized Cauchy-Riemann criteria making it feasible to include physical interactions in this approach.

Preliminary results and some variants of this approach, which was called algebrodynamics, were given in [3]. In the most interesting and studied case the equations defining a holomorphic function on the algebra of biquaternions $B$, are reduced to

$$d\eta = \Phi \ast dX \ast \eta,$$

where $\eta$ - 2-spinor, $\Phi$ - $2 \times 2$ matrix gauge field (for details see Sec. 2).

It turned out that system (1.1) is closely related to the fundamental equations of physics – vacuum Maxwell, Yang-Mills and Einstein equations, as well as to d’Alembert and eikonal equations [3]–[6]. Precisely, for every solution of system (1.1) these equations are satisfied identically; this itself being an astonishing fact!

It was found also that the class of singular solutions of the vacuum equations themselves is wide enough to contain solutions with not only point-like and ring-like singularities, but also toroidal, helix-like and even more complex structure of singularities. This allows us in the framework of our model to consider particles as field singularities [4, 6], and try to relate their transmutations to ”perestroikas” of singularities precisely in the sense of the catastrophe theory [2].

The most important, however, is the nonlinear and overdetermined system (1.1) itself, consistently defining both the 2-spinor $\eta(x)$ and the gauge field $\Phi(x)$. For solutions of system (1.1) the superposition principle is violated, so it serves as a ”filter” selecting those solutions of the field equations which are compatible with it and determining their evolution in time, i.e. interactions and transmutations of singularities-particles.

On the other hand and contrary to customary field theory, the overdetermined system (1.1) (8 equations for 6 functions) does not allow to fix even the initial field distribution in all $\mathbb{R}^3$. In full accordance with Einstein’s super-causality concept [1, P. 762] this ensures on the classical level ”quantization rules” for some characteristics of these particles-singularities (such as the electric charge) [3, 4, 7, 8])!

The most difficult problem is the construction of multisingular solutions and the establishment of the general form of the equation of motion for these singularities as a consequence of the field equations (1.1). After a brief survey of properties of system (1.1) (Sec. 2) and its integration in twistor variables (Sec. 3) we obtain in an algebraic way the formerly found fundamental unisingular solution (Sec. 4), present the exact bisingular solution of system (1.1) and discuss its modifications (Sec. 5). In Sec. 6 we conclude our results and point out some general questions of the algebrodynamical approach.

1Contrary to many methods where quaternionic calculus was based on direct linear generalizations of the Cauchy-Riemann criteria.

2Vacuum Einstein equations are satisfied only for static metrics (see Sec. 4).
2. MAIN PROPERTIES OF THE GENERATING SYSTEM

The system of equations (1.1) (hereafter called the generating system of equations, GSE) is a particular form of the differentiability criteria of a function of biquaternionic variable. A variant of noncommutative calculus corresponding to this case was set forth in details in [3, 4, 7] (references to former works are to be found therein). The multiplication (⋆) in formula (1.1) could be regarded as a matrix one, and the field Φ(x) - as a complex 2 × 2 matrix, in accordance with the well-known representation of the algebra of biquaternions B.

Under the Lorentz transformations

\[ X \rightarrow A^+ \ast X \ast A, \quad \det A = 1, \quad (2.1) \]

GSE conserves its form provided the quantities η(x) and ¯Φ(x) (a matrix adjoint to Φ(x)) behave as a 2-spinor and a 4-vector respectively:

\[ \eta \Rightarrow \bar{A} \ast \eta, \quad \bar{\Phi} \Rightarrow A^+ \ast \bar{\Phi} \ast A. \quad (2.2) \]

The GSE is also covariant under the "weak" gauge transformations, i.e. transformations of the form

\[ \eta \Rightarrow \lambda(\eta)\eta, \quad \Phi_\mu \Rightarrow \Phi_\mu + \frac{1}{2} \partial_\mu \lambda (\eta), \quad (2.3) \]

with a gauge parameter λ(η1, η2) depending on the coordinates only implicitly through components of the affected solution [4, 8].

Hence, the quantity Φ(x) may be considered as a 4-potential of a complex abelian gauge field, with dynamical restrictions which follow from commutation relations and have the form of complex self-duality conditions [3, 4]

\[ \vec{E} + i \vec{H} = 0 \quad (2.4) \]

for the C-valued fields corresponding to Φ(x). In view of relations (2.4) Maxwell equations are satisfied identically and separately for real and imaginary parts of field strengths \( \vec{E} \) and \( \vec{H} \). That is why the C-valued field Φ(x) actually describes a pair of dually-conjugate electromagnetic fields.

Moreover, the field Φ(x) admits a supplementary interpretation [1, 3]. Indeed, the left B-connection

\[ \Gamma(x) = \Phi(x) \ast dX \equiv \Gamma^0(x) + \Gamma^a(x)\sigma_a \quad (2.5) \]

in a natural way defines a strength of some complex gauge field

\[ F(x) = d\Gamma(x) - \Gamma(x) \wedge \Gamma(x). \quad (2.6) \]

The part \( \Gamma^0(x) = \Phi_\mu(x)dx^\mu \) corresponds to the above treated electromagnetic field. At the same time the traceless part \( \Gamma^a(x) \) of connection (2.3), expressed in terms of \( \Phi_\mu(x) \), due to relations (2.4) satisfies free Yang-Mills equations [4]. Observe that the electromagnetic \( F^0_{[\mu,\nu]} \) and Yang-Mills \( F^a_{[\mu,\nu]} \) strengths are related by

\[ F^a_{[\mu,\nu]}F^0_{[\mu,\nu]} = (F^0_{[\mu,\nu]})^2 \quad (2.7) \]
(summation is over the isotopic index \(a = 1, 2, 3\) only), which is different from the relation accepted in customary field theory.

Other aspects of GSE \([1, 2, 3]\), including a geometrical interpretation of connection \([2, 3]\) as a Weyl-Cartan connection of a special type, can be found in \([1, 2, 3]\).

### 3. Twistor Variables and Integration of GSE

Equations \([1, 1]\) may be written in a 2-spinor form

\[
\nabla^{AA'} \eta^{B'} = \Phi^{B'A} \eta^{A'}, \quad A, A', B' = 1, 2,
\]

where \(\nabla^{AA'}\) stands for the derivative with respect to the spinor coordinates \(X_{AA'}\). Multiplying by an orthogonal spinor \(\eta_{A'} = \varepsilon_{A'C'} \eta^{C'}\) we exclude the potentials \(\Phi^{B'A}\) and come to the generalized nonlinear Cauchy-Riemann equations for the spinor \(\eta(x)\) components:

\[
\eta_{A'} \nabla^{AA'} \eta^{B'} = 0.
\]

The solution of \([3.2]\) may be expressed in terms of twistor variables

\[
\tau_A = X_{AA'} \eta^{A'},
\]

in an implicit with respect to \(\eta(x)\) form

\[
\Psi^{(C)}(\eta, \tau) = 0, \quad C = 1, 2,
\]

where \(\Psi^{(1)}, \Psi^{(2)}\) are two arbitrary holomorphic functions of the four complex variables \(\eta^1, \eta^2, \tau_1, \tau_2\).

Solving equations \([3.4]\) for the spinor components \(\eta^1, \eta^2\) and substituting the result in \([3.1]\) we can determine the potentials \(\Phi^{C'A}(x)\) and, consequently, the electromagnetic and Yang-Mills fields corresponding to the solution of system \([3.2]\).

Chiefly we are interested in field’s singularities: their topological structure, electric charge and evolution in time. Differentiating \([3.4]\), it is easy to show that for the caustics, where the fields blow up, we have

\[
\det \|d\Psi^{(C)}/d\eta^{B'}\| = 0.
\]

For subsequent analysis, let us note that the solution of \([3.4]\) is simplified by fixing the gauge to \(\eta^1 = 1\). Under this gauge the first of equations \([3.4]\) becomes trivial: \(\Psi^{(1)} = \eta^1 - 1 = 0\) and the remaining spinor component \(\eta^2 \equiv G(x)\) is determined algebraically from the second relation of \([3.4]\)

\[
\Psi^{(2)}(\eta^2, \tau_1, \tau_2) \equiv \Psi(G, wG + u, vG + \bar{w}) = 0,
\]

where the following notation was adopted for spinor coordinates: \(u, v = X_{11'}, X_{22'} = t \pm z, w, \bar{w} = X_{12'}, X_{21'} = x \pm iy\). The caustics equations reduce then to a simpler form

\[
\frac{d\Psi}{dG} = 0.
\]
Actually, equations (3.2) in the above gauge reduce to the well-known relations, defining a null shear-free geodesical congruence, while their solution (3.6) is given by the Kerr theorem [3, Chapter 7]. Singularities of the corresponding metric and its curvature [3] [11] [14] are determined by the caustic equation (3.7) [12] [13].

Hence the singularities of the EM, Yang-Mills and gravitational fields associated with solutions of GSE (1.1) are determined by the same relation (3.7) and therefore coincide. That’s why the here accepted interpretation of particles as common singularities of these fields seems quite natural.

4. FUNDAMENTAL UNISINGULAR SOLUTION OF GSE

A choice of a linear function $\Psi(G, \tau_1, \tau_2)$ leads to a trivial solution $G(x)$ with identically zero fields. Static axisymmetric solutions are given by functions (3.6) of the form

$$\Psi = G\tau_1 - \tau_2 - cG = 0, \quad c = \text{Const},$$

or equivalently

$$G(uG + u) - 2cG - (vG + \bar{w}) = 0. \quad (4.1)$$

The solution of the second degree equation (4.1) is

$$G(x) = \frac{\bar{w}}{(z - c) \pm \sqrt{(z - c)^2 + \rho^2}}, \quad (4.2)$$

where $z = (u - v)/2$, $\rho^2 = w\bar{w} = x^2 + y^2$. For real-valued $c$ which can be eliminated by a proper translation, solution (4.2) defines a stereographic projection $S^2 \to \mathbb{C}$, and electromagnetic field of the Coulomb type

$$E_r = \frac{q}{r^2}, \quad q = \pm 1; \quad E_\theta = E_\phi = 0, \quad (4.3)$$

with a fixed value of the electric charge (so called "algebraic charge quantization" analogous to (4.3) monopole form

$$H_r = \frac{i\mu}{r^2}, \quad \mu = q = \pm 1; \quad H_\theta = H_\phi = 0. \quad (4.4)$$

In case of imaginary values of the constant $c = ia$, $a \in \mathbb{R}$ the field singularity has a ring-like structure with radius $r = |a|$. For $r \gg |a|$ the field has a multipole structure with Coulomb-monopole main terms. This enables one to estimate the quadrupole moment of particles corresponding to solution (4.2) [6].

The Riemannian metric for solution (4.2) is of Schwarschild and Kerr type for real and imaginary values of the constant $c$ respectively [10]. In [11] it was shown that the only static empty space metric of the Kerr-Schild type whose singularities (as well as EM field singularities in our approach) are confined to a bounded region are those corresponding to solutions (4.2). It is remarkable that fields corresponding to (4.2) automatically satisfy vacuum Einstein equations as well as electrovacuum Maxwell-Einstein system of equations. Indeed, the energy-momentum tensor $T_{\mu\nu}$ for the complex self-dual EM-fields, satisfying (2.4), vanishes identically! On the other hand, Maxwell equations in the considered spaces are identical to those in flat space [10].
5. BISINGULAR SOLUTION AND ITS MODIFICATIONS

Let us consider a time-dependent axisymmetric solution, generated by a function

$$\Psi = \tau_1 \tau_2 + b^2 G = 0, \quad b = \text{Const} \in \mathbb{C}.$$  

Again, we have a second degree equation with the following solution:

$$G = \frac{-2uw}{\sigma^2 + \rho^2 + b^2 \pm \sqrt{\Delta}}, \quad \Delta \equiv (\sigma^2 + \rho^2 + b^2)^2 - 4\sigma^2 \rho^2,$$  \hspace{1cm} (5.1)

where $$\sigma^2 = uv = t^2 - z^2.$$  

The caustic for this solution is determined by a zero discriminant $$\Delta = 0,$$ which may be transformed to the following equation for singularities’ evolution:

$$\Delta = (t^2 - z^2 - \rho^2 + b^2)^2 + 4b^2 \rho^2 = 0.$$  \hspace{1cm} (5.2)

In case of real constant $$b,$$ as a solution of (5.2) we have

$$\rho = 0, \quad z = \pm \sqrt{t^2 + b^2},$$  \hspace{1cm} (5.3)

so that the field has two point-like singularities moving with constant acceleration (i.e. performing mirror *hyperbolic* motion) along $$\pm Z$$ axis (Fig.1a,b,c). Calculating the electric flux through closed surfaces surrounding each singularity we find that they possess electric charges equal in magnitude to the charge of fundamental solution and opposite to each other.

$$E_\rho = \mp \frac{8b^2 \rho z}{\Delta^{3/2}}, \quad E_z = \pm \frac{4b^2}{\Delta^{3/2}}(t^2 - z^2 + \rho^2 + b^2), \quad H_\varphi = \mp \frac{8b^2 \rho t}{\Delta^{3/2}}.$$  \hspace{1cm} (5.4)

the remaining components being identically equal to zero.

For Born solution multiple interpretations were given, and the related problem of radiation was discussed many times in literature. However, in almost all these works the second mirror charge was simply ignored [24]-[28]. In the last few years an increasing interest arose concerning the problem of radiation from a uniformly accelerated charge [20]-[22], and highly qualified works appeared proving the absence of radiation in this case [18, 19].
Our point of view is similar to that of Singal’s, though we have no opportunity to discuss here this old and confused problem. We note only that the presence of a second charge seems to be an inevitable consequence and a fundamental property of the Maxwell equations (see also [25]). The process itself (Fig.1) can be considered as a toy model for elastic scattering of two particle-like formations, interacting evidently in a non-electromagnetic way (repulsion instead of attraction!), and the charge plays the role of a field source and a conserved quantum number rather than a coupling constant.

Such an interpretation can be confirmed by considering our modifications of the Born solution corresponding to complex values of the constant $b^2 = \alpha + i\beta, \alpha, \beta \in \mathbb{R}$ in (5.1). In this case (supposing $\alpha, \beta \neq 0$) we have for singularities the following relations:

$$\rho = \sqrt{\frac{\sqrt{\alpha^2 + \beta^2} - \alpha}{2}}, \quad z = \pm \sqrt{t^2 + \frac{\alpha + \sqrt{\alpha^2 + \beta^2}}{2}}$$

Hence, here again the singularities perform hyperbolic motion along $\pm Z$ axis but have a ring-like structure with fixed radii $\rho$ (5.5). In the same way as in the case of real constant $b$ we find again that these ring-like formations have opposite charges equal in magnitude to the fundamental one.

The case of pure imaginary content $b = ia, a \in \mathbb{R}$ requires special consideration. From (5.2) we have the following relation for caustic:

$$z^2 + (\rho \pm a)^2 = t^2 \quad (5.6)$$

This, at first sight, simple equation describes a rather nontrivial evolution of singularities. At $t = 0$ we have a ring of radius $|a|$ lying in the $xy$–plane (Fig.2a), which at $t > 0$ turns into a torus with a radius expanding at the speed of light (Fig.2b) (on these figures we represent a cross-section along the axis of symmetry $Z$). At $t = |a|$ the torus fills its gap (Fig.2c) and at the same time a second singularity given by the opposite sign in (5.6) occurs.

At $t > |a|$ the torus keeps on expanding, penetrating through itself. After this the singularity has a form of two toroidal "bridges", connecting their common points with coordinates

$$\rho = 0, \quad z = \pm \sqrt{t^2 - a^2} \quad (5.7)$$

(the corresponding cross-section is represented on Fig.2d).

\footnote{Let us note, however, that the effective charge \cite{17} $\sqrt{q^2 + (i\mu)^2}$, as for the unisingular solution (Compare with \cite{13}) is identically equal to zero, formally justifying the absence of EM-interaction!}

\footnote{As before we deal here with solutions of the ordinary real Maxwell equations.}
At \( t \gg |a| \) the law of motion of these singularities approaches that of the Born solution \((5.3)\). However, their velocity falls from \( \infty \) to the velocity of light. This makes the process to look like a run-away of two *tachion-like* formations. In the interval between \( t = -\infty \) and \( t = 0 \) we have an inverse process of accelerated rapprochement of these tachions until their *annihilation* at \( t = -|a| \), with formation of a toroidal *resonance* structure and its subsequent degeneration into a singular ring of radius \(|a|\) at \( t = 0 \). At this instant the electric lines configuration undergoes topological changes since the winding number could be seen to change sign.

Observe that asymptotically at \(|t| \gg |a|\) the field of the tachion solution coincides with that of Born solution for all directions determined by small azimuths \( \theta \ll \pi/2 \) (i.e. far from directions to singular bridges). This observation allows us to consider these tachion-like singularities \((5.7)\) as ”quasi-charges”, with charge magnitude equal again to that of the fundamental solution.

The metric corresponding to this solution of GSE is of the Kerr-Schild type,

\[ g_{\mu\nu} = \eta_{\mu\nu} + he^3_{\mu} e^3_{\nu} \tag{5.8} \]

where \( e^3 = du + Gdw + \bar{G}d\bar{w} + \ddot{G}dv \) is a null tetrad 1-form.

For real constants \( b \) in analogy with \([14, 15]\) we have a real metric with the function

\[ h = \frac{m\partial_u \bar{G}}{\bar{\tau}_2(\bar{\tau}_2 G - \bar{\tau}_1)^2}, \quad m \equiv m(\bar{\tau}_1) \tag{5.9} \]

satisfying the Einstein field equations for a null radiation matter tensor \( T_{\mu\nu} = P_{33} e^3_{\mu} e^3_{\nu} \), where \( P_{33} \) is given by

\[ P_{33} = 2(\dot{m}(\bar{\tau}_2 G - \bar{\tau}_1)\bar{\tau}_1 + 3m(\bar{\tau}_1 + G\bar{\tau}_2))(\bar{\tau}_2 \partial_u \bar{G})^2, \quad \dot{m} = \frac{dm}{d\bar{\tau}_1}. \tag{5.10} \]

Note that the shear-free geodesic congruence defined by the function \( G \) in \((5.1)\) has zero twist (i.e. pure expansion). Solution \((5.1)\) describes a curved space filled with null radiation. The EM-field may be considered as a test field, since by the virtue of complex self-duality \((2.4)\) it does not contribute to the energy-momentum tensor.

6. CONCLUSION. INTERACTIONS & TRANSMUTATIONS OF PARTICLES IN BIQUATERNIONIC FIELD DYNAMICS

The reduction of GSE \((1.4)\) to the algebraic solution of the shear-free geodesic congruence equation \((3.6)\) (or to equations \((3.4)\) in the arbitrary gauge) allows us in a fairly simple way to generate complex solutions of the GSE and corresponding to them solutions of the Maxwell and (C-valued) Yang-Mills equations. Until now other solutions among them an electrically neutral ”photon-like” one with helix-like singularity structure have been studied (V.N. Trishin). A wide class of singular solutions of the free Maxwell equations was given in \([23]\), however they still have to be adjusted with the basic GSE.

For more complicated distributions, their most interesting characteristic - the structure of singularity and its evolution in time - may be obtained without explicitly solving equation
In fact, excluding the main function \( G(x) \) from (3.6) and (3.7) immediately results in the equation of singularity. There is a variety of such complex singularities which will be published elsewhere.

Nevertheless, the main two tasks naturally arising in our approach - the complete classification of singularities and derivation of their equations of motion - remain unsolved, requiring probably more sophisticated mathematics. In view of the overdetermined GSE the Cauchy initial data can be specified for a 2-dimensional surface rather than in all 3-space, since evolution in the third coordinate as well as in time are determined by system (1.1) itself.

As to the classification problem of singularities and the related problem of their admissible transmutations, it probably (at least for vacuum Maxwell equations) will be solved soon in the frame of general catastrophe theory \[2\]. The identification of singularities with elementary particles automatically comes to mind. However physicists are not used to such an approach, since there, singularities usually emerge as divergences of some integral characteristics (energy etc). But in the considered here non-Lagrangian approach this difficulty may be fictitious, since quantum numbers as well as the equations of motion for these singularities are determined by solutions of field equations themselves.

Conceptually, we deal here with a new approach to nonlinear electrodynamics, where instead of modifying Maxwell equations themselves we regard them as necessary consequences of some overdetermined nonlinear preodynamics.

Thus, behind a compact and following from a unique general principle (hyper-holomorphy of field functions) GSE (1.1) we have an entire world of structures close in many aspects to the real one. After appropriate interpretation of mathematical variables arising, the language of the theory appears to be much similar (but not identical!) to that used in customary field theory.

Moreover, some facts (such as quantization of electric charge), may be described in the framework of GSE (1.1) much more naturally and adequately, than in conventional field-theoretical schemes. On the other hand, the true potentialities of GSE (1.1) in describing the structure and interaction of particles still have to be clarified. Certainly, we don’t regard GSE (1.1) as an ultimate mathematical structure coding the ”Theory of Everything”. However, even its exclusive properties already obtained make this system a suitable and impressive demonstration of basic principles of algebroadynamical approach.

Note that this scheme admits natural generalizations to algebras of higher dimensions as well as to ”local algebras” with structure ”constants” depending on space points \[3, Chapter 2\] and to nonassociative algebras like octonions. All these generalizations seem to be promising.

The authors are indebted to Yu. S. Vladimirov, Tz. I. Gutzunayev, D. P. Zhelobenko and Yu. P. Rybakov for useful advice and interest exhibited. One of us (V.K.) is grateful to A. V. Aminova, D. A. Kalinin and other organizers of the school-seminar ”Volga-10” for cordial reception, and to its participants, personally to V. G. Bagrov and A. P. Shirokov - for stimulating discussions.

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