On Orbifolds of (0,2) Models

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Abstract
We study orbifolds of (0,2) models, including some cases with discrete torsion. Our emphasis is on models which have a Landau-Ginzburg realization, where we describe part of the massless spectrum by computing the elliptic genus for the orbifolded theory. Somewhat surprisingly, we find simple examples of (0,2) mirror pairs that are related by a quotient action. We present a detailed description of a family of such pairs.

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1. Introduction

Mirror symmetry remains one of the more mysterious, and intriguing discoveries in string theory. While mirror symmetry has been intensively studied in (2, 2) models, the analogue of mirror symmetry in (0, 2) models is a subject largely in its infancy. What do we mean by mirror symmetry for (0, 2) models? Let us recall that for (2, 2) models, mirror symmetry is the statement that strings propagating on mirror Calabi-Yau spaces, which are generally inequivalent, give equivalent physical theories. The superconformal field theories that describe the infra-red physics for the mirror models are isomorphic. This isomorphism exchanges the moduli corresponding to Kähler deformations with those corresponding to complex structure deformations. At the level of the N=2 superconformal algebra, this map simply corresponds to changing the sign of either the left or right-moving $U(1)$ current.

For (0, 2) models, we have an additional set of moduli which are holomorphic deformations of the gauge bundle. It is natural to envision physically equivalent realizations of (0, 2) models where the role of moduli in these three classes are interchanged. An example of a duality, manifest in the Landau-Ginzburg theory, where some deformations of the gauge bundle were exchanged with complex structure deformations was described in [1]. We shall consider (0, 2) models to be mirrors if the conformal field theories to which they flow in the infra-red are related by an isomorphism changing the sign of either the left or right-moving $U(1)$ current. This definition, proposed and studied from a different approach in [2], is in complete analogy with the (2, 2) case. For compactifications on Calabi-Yau three-folds, this isomorphism exchanges the number of generations and anti-generations.

The goal of this paper is to study orbifolds of (0, 2) models, with and without discrete torsion. One of our aims in undertaking this study was to find examples of (0, 2) mirror pairs related by quotient actions. We shall find such examples, and they are remarkably simple. These models can be considered (0, 2) analogues of the (2, 2) mirror pairs described by Greene and Plesser [3]. The main examples we shall present flow to conformal field theories which have been discussed extensively in [4],[5] and [6]. For these models, there is reasonable evidence that the quotient action that we shall describe implements the desired isomorphism on the conformal field theory.

In the following section, we consider the effect of orbifolding on the spectrum of massless particles of (0, 2) orbifolds. The models that we shall consider can be obtained from the Landau-Ginzburg phase of a gauged linear sigma model [7]. In determining the spectrum, we shall not concern ourselves with issues of conformal invariance, and possible
non-perturbative obstructions to conformal invariance. Our results are only applicable to models that flow, unobstructed, to good conformal field theories. For example, see \[8\], for a discussion about possible non-perturbative obstructions. To determine the spectrum, we shall generalize the computation of the elliptic genus described in \[9\]. Section three contains some models which illustrate the effect of orbifolding on the spectrum, including several mirror pairs. We conclude in section four with some examples with non-trivial discrete torsion, and a discussion of some of the issues which arise in interpreting the quotient action in models with a Calabi-Yau phase.

2. The Spectrum of \((0,2)\) Orbifolds

2.1. The linear sigma model

The primary reason \((0,2)\) models have become accessible to study in recent times is the development of the gauged linear sigma model by Witten \[7\]. This model is a relatively tractable massive two-dimensional field theory which is believed, under suitable conditions, to flow in the infra-red to a non-trivial superconformal field theory. One of the more interesting features of the linear sigma model is its various connected vacua, or phases. At low energies, these phases appear to correspond to theories such as a non-linear sigma model, a Landau-Ginzburg orbifold, or some other more peculiar theory like a hybrid model. The linear sigma model provides a natural setting in which the relation between some of these various types of theories can be studied.

Let us begin by describing the fields in the \((0,2)\) linear sigma model. There are two sets of chiral superfields: \(\{\Phi_i | i = 1, \ldots, N_i\}\) with \(U(1)\) charges \(\omega_i\) and \(\{P_l | l = 1, \ldots, N_l\}\) with \(U(1)\) charges \(-m_l\). Furthermore, there are two sets of Fermi superfields: \(\{\Lambda^a | a = 1, \ldots, N_a\}\) with charges \(n_a\) and \(\{\Sigma^j | j = 1, \ldots, N_j\}\) with charges \(-d_j\). The superpotential of the linear \(\sigma\)-model is given by,

\[
S = \int d^2 z d\theta \left[ \Sigma^j W_j(\Phi_i) + P_l \Lambda^a F^l_a(\Phi_i) \right],
\]

where \(W_j\) and \(F^l_a\) are quasihomogenous polynomials of degree fixed by requiring charge neutrality of the action. To ensure the absence of gauge anomalies, a prerequisite for conformal invariance, we demand that the following conditions be satisfied:
\[ \sum \omega_i = \sum d_j, \]
\[ \sum n_a = \sum m_l, \]
\[ \sum d_j^2 - \sum w_i^2 = \sum m_l^2 - \sum n_a^2. \] (2.2)

These conditions have a simple interpretation in the Calabi-Yau phase of the model, where the ‘size’ of the space \( r >> 0 \). At low energies, the model then describes a \((0, 2)\) sigma model on weighted projective space, \( \mathbb{P}_{\omega_1, \ldots, \omega_{N_i}}[d_1, \ldots, d_{N_j}] \), with a coherent sheaf of rank \( N_a - N_l \) defined by the sequence:

\[ 0 \to V \to \bigoplus_{a=1}^{N_a} \mathcal{O}(n_a) \to \bigoplus_{l=1}^{N_l} \mathcal{O}(m_l) \to 0. \] (2.3)

For our purposes, the sheaf can be considered a vector bundle, although the distinction is important when resolving singularities [10]. The anomaly conditions (2.2) then correspond to the conditions for the vanishing of the first Chern class of the tangent and vector bundles, \( c_1(T) = c_1(V) = 0 \), and the further topological constraint, \( c_2(V) = c_2(T) \). Before proceeding, we should comment that spectator fields are generally required in these models [11], but since they are massive in the infra-red, we shall subsequently ignore them.

For the situation where \( N_l = 1 \) and \( r << 0 \), the low-energy physics is described by a Landau-Ginzburg orbifold. A phase which has been discussed in some detail in [12]. On minimizing the scalar potential, the field \( p \) gets a VEV, and the \( U(1) \) gauge symmetry is broken to a discrete \( \mathbb{Z}_m \) subgroup. The dynamics of the remaining fields is governed by a superpotential of the form,

\[ W(\Phi_i, \Lambda_a, \Sigma_j) = \sum_j \Sigma_j W_j(\Phi_i) + \sum_a \Lambda_a F_a(\Phi_i). \] (2.4)

For appropriate choices of the constraints \( W_j \) and \( F_a \), this superpotential has an isolated singularity at the origin and is quasi-homogeneous of degree one, if one assigns charges \( \omega_i/m \) to \( \Phi_i \), \( n_a/m \) to \( \Lambda_a \), and \( 1 - d_j/m \) to \( \Sigma_j \). Quasi-homogeneity implies the existence of a right-moving \( R \)-symmetry, and a left-moving \( U(1)_L \). The associated currents are denoted by \( J_R \) and \( J_L \), respectively. The charges of the various fields with respect to these \( U(1) \) currents are summarized in the following table:
Table 1: Left and right charges of the fields in the LG model.

| Field | $\phi_i$ | $\psi_i$ | $\lambda_a$ | $\sigma_j$ |
|-------|---------|---------|-------------|-------------|
| $q_L$ | $\frac{m}{m}$ | $\frac{m}{m}$ | $\frac{n_a}{m} - 1$ | $-\frac{d_j}{m}$ |
| $q_R$ | $\frac{m}{m}$ | $\frac{m}{m} - 1$ | $\frac{n_a}{m}$ | $1 - \frac{d_j}{m}$ |

Of course, the fermions, $\psi_i$, belong to the chiral superfield, $\Phi_i$, while the fermions, $\lambda_a$ and $\sigma_j$, are the lowest components of the Fermi superfields $\Lambda_a$ and $\Sigma_j$, respectively. The techniques developed in [13] can be used to compute the massless spectrum of such models. The simplifying feature of such calculations is that the right-moving supersymmetry forces the massless states to lie in the cohomology of the right-moving supercharge. At the level of cohomology, the superpotential can be set to zero, and the required calculations reduce to those in a free field theory. The cohomology of the charge can be computed in a number of ways; for instance, by running a spectral sequence.

2.2. Discrete symmetries of (0,2) LG models

We are interested in taking further orbifolds of these (0,2) Landau-Ginzburg models, in the hope of obtaining mirror pairs. Let us simplify notation somewhat, and consider a set of chiral superfields, $\{\Phi_i| i = 1, \ldots, N\}$, and a set of Fermi superfields $\{\Lambda_a| a = 1, \ldots, M = N_a + N_j\}$. The superpotential is of the form

$$W(\Phi_i, \Lambda_a) = \sum_a \Lambda_a F_a(\Phi_i).$$

Just as is familiar from (2,2) Landau-Ginzburg theories, we can consider discrete symmetries of the superpotential. For the sake of this discussion, we can consider the kinetic term for these models to be the one corresponding to a flat metric. Throughout this paper, we shall only consider symmetries which act on the fields by phases,

$$\Phi_i \rightarrow e^{2\pi i q_i} \Phi_i, \Lambda_a \rightarrow e^{-2\pi i q_a} \Lambda_a.$$  (2.6)

In particular, this excludes permutation symmetries, and non-abelian orbifolds from our discussion.

Quotients by some of these symmetries should lead to new modular invariant (0,2) superconformal theories, and if we are fortunate, to theories whose spectra are ‘mirror’ to our starting model. Our starting model will always be one obtained from a linear sigma
model in the way we have just described. Let us introduce charges, $Q^\mu$, which generate discrete abelian symmetries of order $h^\mu$, where the index $\mu \in \{0, \ldots, P-1\}$. Alternatively, $h^\mu$ is the smallest integer such that $h^\mu q^\mu_{i,a}$ is integral for all $i$ and $a$, where the fields $\Phi_i$ and $\Lambda_a$ have charges $q^\mu_i$, $q^\mu_a$ respectively under these symmetries. One of our orbifolds should correspond to the $\mathbb{Z}_m$ discrete gauge symmetry generated in the linear sigma model. We shall set $J^0 = J_L$, the left-moving $U(1)$ current from the linear sigma model. To obtain a string vacuum, we shall, however, actually quotient by,

$$ g = e^{-\pi i Q^0} (-1)^\lambda, \quad (2.7) $$

rather than $g = e^{2\pi i Q^0}$. This $\mathbb{Z}_{2m}$ symmetry is the usual GSO projection onto integer charge states which leads to spacetime supersymmetry, and an enhancement of the gauge group to $E_6$, $SO(10)$, or $SU(5)$ depending on the rank of the vector bundle. The additional $\mathbb{Z}_2$ accounts for the number of excitations, $\lambda$, of the left-moving fermions needed for the $SO(16 - 2r)$ linearly realized gauge sector of the heterotic string. Sectors twisted by odd powers of (2.7) are left Neveu-Schwarz, while those twisted by even powers are left Ramond sectors.

We shall assume that the Landau-Ginzburg theory orbifolded by (2.7) exists, and flows to a good superconformal field theory in the infra-red. The specific examples that we shall consider have been realized as exact conformal field theories, and so this is very likely to be true. We shall then orbifold the theories further. Not all orbifold actions result in modular invariant conformal field theories. Rather, there exist conditions analogous to the constraints (2.2) which had a nice topological interpretation in the Calabi-Yau phase. For toroidal orbifolds, such conditions have been described by Vafa in [14]. We shall arrive at analogous conditions for these models by studying the modular properties of the elliptic genus. The resulting conditions are exactly of the form expected from level matching. We shall also examine the freedom to weight the various twisted sectors by a phase. This freedom, known as discrete torsion [14], should exist for (0,2) models, although it has only been explored for the subclass of models with (2,2) supersymmetry. The allowed phases are constrained if the elliptic genus is to retain good modular properties. It seems likely that these constraints are sufficient to guarantee modular invariance of the resulting orbifolded theory. They are, however, certainly necessary.
2.3. The elliptic genus of (0, 2) orbifolds

Unlike (2, 2) models where we have A and B twisted topological theories, for (0, 2) models, we only have the half-twisted model. At genus one, the partition function for the half-twisted model is known as the elliptic genus of the theory. The elliptic genus nicely encodes part of the spectrum of massless particles in the (0, 2) model. For instance, the number of generations and anti-generations can be extracted from the elliptic genus, while the remainder of the massless spectrum requires more tedious calculations [12]. Denoting the right-moving R-charge by \( F_R = \oint J_R \), the elliptic genus is defined by,

\[
Z(\tau, \nu) = \text{Tr} (-1)^{F_R} e^{2\pi i \nu Q^0} q^{L_0} e^{-\beta \bar{L}_0}, \tag{2.8}
\]

where \( q = e^{2\pi i \tau} \). It will be more convenient for us to shift \( \nu \) by \( \frac{1}{2} \) which replaces \((-1)^{F_R}\) by \((-1)^F\). The right-moving \( N=2 \) supersymmetry algebra,

\[
\{Q_-, Q_+\} = L_0, \tag{2.9}
\]

ensures that (2.8) is independent of \( \beta \). This is the outstanding feature of the elliptic genus, since it can then be computed in a perturbative (small \( \beta \)) expansion. It further ensures that the massless states are annihilated by \( \bar{L}_0 \). The trace in (2.8) is taken over states in the RR sector of the theory. Fortunately, the computation of the elliptic genus has been nicely described for (0, 2) Landau-Ginzburg theories orbifolded by the GSO projection [9]. We shall consider the generalized case of multiple orbifolds, and non-trivial discrete torsion.

Let us introduce a vector notation, \( \vec{q}_a, \vec{q}_i, \) and \( \vec{Q} \) for the charges of our discrete symmetries. It is also convenient to introduce a tensor of charges,

\[
R^{\mu\nu} = \sum_{a=1}^{M} q_a^\mu q_a^\nu - \sum_{i=1}^{N} q_i^\mu q_i^\nu, \tag{2.10}
\]

and a vector:

\[
r^\mu = \sum_{a=1}^{M} q_a^\mu - \sum_{i=1}^{N} q_i^\mu. \tag{2.11}
\]

The elliptic genus can be expressed in terms of a Jacobi theta function. Let us recall that,

\[
\vartheta_1(\tau, \nu) = ie^{-\pi i \nu} \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{1}{2}(n-\frac{1}{2})^2} e^{2\pi i n \nu}, \tag{2.12}
\]

where

\[
\vartheta_1(\tau, \nu) = ie^{-\pi i \nu} \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{1}{2}(n-\frac{1}{2})^2} e^{2\pi i n \nu}, \tag{2.12}
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and

\[
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\]

and

\[
\vartheta_1(\tau, \nu) = ie^{-\pi i \nu} \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{1}{2}(n-\frac{1}{2})^2} e^{2\pi i n \nu}, \tag{2.12}
\]
and that \( \vartheta_1(\tau, \nu) \) has the following nice modular properties,

\[
\begin{align*}
\vartheta_1(\tau + 1, \nu) &= e^{\frac{\pi i}{4}} \vartheta_1(\tau, \nu) \\
\vartheta_1\left(\frac{-1}{\tau}, \frac{\nu}{\tau}\right) &= (-i\tau)^{\frac{1}{2}} e^{\frac{\pi i \nu^2}{\tau}} \vartheta_1(\tau, \nu),
\end{align*}
\]

and obeys the double quasi-periodicity relation:

\[
\vartheta_1(\tau, \nu + n\tau + m) = (-1)^{n+m} e^{-\pi i (n^2 \tau + 2n\nu)} \vartheta_1(\tau, \nu).
\]

To include the effect of further quotients, let us consider the elliptic genus with the boundary condition in time twisted by the generators of the additional discrete symmetries,

\[
Z(\tau, \nu, \vec{z}) = \text{Tr} \left( -1 \right)^F e^{2\pi i\nu Q^0} e^{2\pi i\vec{z} \vec{Q}} q^{L_0} e^{-\vec{b} \bar{L}_0},
\]

where \( z^\mu \) is integral for all \( \mu \in \{0, \ldots, P-1\} \). Note that \( \nu \) is a continuous parameter in this expression. The elliptic genus can be described in terms of the following function \([9]\),

\[
\begin{align*}
\bar{\vartheta}_0(\tau, \nu, \vec{z}) &= \eta(\tau)^{N-M} \prod_a \vartheta_1(\tau, q^0_a \nu + \vec{q}_a \vec{z}) \\
&\quad \times \prod_i \vartheta_1(\tau, q^0_i \nu + \vec{q}_i \vec{z}),
\end{align*}
\]

where in the untwisted sector, we set \( \vec{z} = 0 \). This function is well-defined for arbitrary values of \( \vec{z} \) and \( \nu \). The untwisted sector does not mix with the twisted sectors under modular transformations, but obeys the relations:

\[
\begin{align*}
\bar{\vartheta}_0(\tau + 1, \nu, 0) &= e^{\frac{\pi i}{6} (M-N)} \bar{\vartheta}_0(\tau, \nu, 0) \\
\bar{\vartheta}_0\left(\frac{-1}{\tau}, \frac{\nu}{\tau}, 0\right) &= e^{\pi i R^{00} \frac{\nu^2}{\tau}} \bar{\vartheta}_0(\tau, \nu, 0) \\
\bar{\vartheta}_0(\tau, \nu + \gamma \tau + \delta, 0) &= (-1)^{\nu(\gamma + \delta)} e^{-\pi i R^{00} (\gamma^2 \tau + 2\nu \gamma)} \bar{\vartheta}_0(\tau, \nu, 0),
\end{align*}
\]

where \( \gamma \) and \( \delta \) are integral multiples of \( h^0 \). The elliptic genus for the sectors twisted in the time direction are easy to determine since the twisting is generated by just inserting the charge operator into the trace:

\[
\bar{\beta}_0(\tau, \nu, 0) = \bar{\vartheta}_0(\tau, \nu, \vec{\beta}), \quad \vec{\beta} \in \mathbb{Z}^P.
\]

There is no arbitrariness in the way these sectors contribute to the orbifold partition function. The remaining twisted sectors are determined, to within a phase ambiguity, by
demanding that the twisted orbifold partition function make sense under modular transformations. For instance, the sectors twisted in space are determined by those twisted in time by noting:

\[ \vec{0} \vec{\alpha}(\tau, 0, 0) = \vec{\alpha} \vec{0}(-\frac{1}{\tau}, 0, 0) \]

\[ = \vec{0} \vec{\alpha}(-\frac{1}{\tau}, 0, \vec{\alpha}) \]

\[ = e^{\pi i \vec{\alpha} R \vec{\alpha}} \vec{0} \vec{\alpha}(\tau, 0, \tau \vec{\alpha}). \]

(2.19)

There is a basic check on the \(\tau\)-dependent phase factor in this expression. If we twist by the identity operator, we had better recover the expression for the elliptic genus in the untwisted sector! Therefore, let us take \(\vec{\alpha}\) to be a lattice vector \(\vec{\gamma}\), where \(\gamma^\mu\) is an integral multiple of \(h^\mu\). We then find that,

\[ \vec{0} \vec{\gamma}(\tau, 0, 0) = e^{\pi i \vec{\gamma} R \vec{\gamma}} \vec{0} \vec{\gamma}(\tau, 0, \tau \vec{\gamma}) \]

\[ = (-1)^{\vec{\gamma} \vec{\gamma}} \vec{0} \vec{\gamma}(\tau, 0, 0), \]

which is the desired result up to a \(\tau\)-independent phase factor. With similar motivation for the general twisted sector, we define:

\[ \vec{\beta} \vec{\alpha}(\tau, \nu, 0) = e^{\pi i \vec{\alpha} R \vec{\beta}} e^{\pi i (\vec{\alpha} R \vec{\alpha} + 2 \alpha^\mu R^\mu \nu)} \vec{0} \vec{\alpha}(\tau, \nu, \vec{\alpha} \tau + \vec{\beta}), \quad \vec{\alpha}, \vec{\beta} \in \mathbb{Z}^P. \]

(2.21)

For later convenience, we have added a phase, \(e^{\pi i \vec{\alpha} R \vec{\beta}}\), to the definition (2.21). The freedom to weight the contribution from different twisted sectors with a phase has been explored in (2,2) Landau-Ginzburg theories \([15\, [16]\), and a mirror pair involving discrete torsion has been described in \([17]\). So let us multiply the contribution from each twisted sector by a phase, \(\epsilon(\vec{\alpha}, \vec{\beta})\), where this phase includes the freedom to add non-trivial discrete torsion. Under modular transformations, the contribution to the orbifold partition function from these sectors then has the following behavior,

\[ \epsilon(\vec{\alpha}, \vec{\beta}) \vec{\beta} \vec{\alpha}(\tau + 1, \nu, 0) = e^{\pi i (M^\mu - N)} \epsilon(\vec{\alpha}, \vec{\beta}) \vec{\alpha} + \vec{\beta} \vec{\alpha}(\tau, \nu, 0) \]

\[ \epsilon(\vec{\alpha}, \vec{\beta}) \vec{\beta} \vec{\alpha}(\tau + 1, 0, \nu) = e^{\pi i R_{00}} \epsilon(\vec{\alpha}, \vec{\beta}) \vec{\alpha} + \vec{\beta} \vec{\alpha}(\tau, \nu, 0). \]

(2.22)
Sectors related by modular transformations should appear with the same phase in the orbifold partition function, which implies that,

\[
\epsilon(\vec{\alpha}, \vec{\beta}) = \epsilon(\vec{\alpha}, \vec{\alpha} + \vec{\beta})
\]

\[
\epsilon(\vec{\alpha}, \vec{\beta}) = \epsilon(\vec{\beta}, -\vec{\alpha}).
\]

We can now consider a consistency requirement, which is essentially level matching. We demand that shifting the twist operators in the space and time directions by the identity operator leave the elliptic genus for the twisted sector invariant,

\[
\epsilon(\vec{\alpha} + \vec{\gamma}, \vec{\beta} + \vec{\delta}) = \epsilon(\vec{\alpha}, \vec{\beta}) e^{-\pi i (\vec{r} \cdot \vec{\gamma} + \vec{\delta} - \vec{\gamma} R \vec{\delta} + \vec{\gamma} R \vec{\beta} - \vec{\alpha} R \vec{\delta})}.
\]

where \(\gamma^\mu, \delta^\mu\) are integral multiples of \(h^\mu\). This condition implies the following constraint on the phases,

\[
\epsilon(\vec{\alpha} + \vec{\gamma}, \vec{\beta} + \vec{\delta}) = \epsilon(\vec{\alpha}, \vec{\beta}) e^{-\pi i \vec{r} \cdot (\vec{\gamma} + \vec{\delta}) - \vec{\gamma} R \vec{\delta} + \vec{\gamma} R \vec{\beta} - \vec{\alpha} R \vec{\delta}).
\]

This condition provides us with strong constraints on the possible charge vectors. We shall take the phase factors to be of the general form:

\[
\epsilon(\vec{\alpha}, \vec{\beta}) = e^{\pi i \vec{w} \cdot (\vec{\alpha} + \vec{\beta})} e^{\pi i \vec{a} \cdot Q \vec{\beta}}.
\]

The term \(e^{\pi i \vec{a} \cdot Q \vec{\beta}}\) automatically satisfies the factorization constraint which arises at genus two [14]. These conditions imply that \(w^\mu\) must be integral, and that the following conditions must be met:

\[
Q^{\mu \nu} + Q^{\nu \mu} \in 2\mathbb{Z},
\]

\[
w^\mu + Q^\mu \mu \in 2\mathbb{Z},
\]

\[
(w^\mu - r^\mu) h^\mu = 0 \mod 2,
\]

\[
(Q^{\mu \nu} + R^{\mu \nu}) h^\nu = 0 \mod 2,
\]

for any \(\mu, \nu \in \{0, \ldots, P - 1\}\). In turn, these conditions provide constraints on \(r^\mu\) and \(R^{\mu \nu}\), depending on whether \(h^\mu\) is even:

\[
r^\mu h^\mu \in 2\mathbb{Z},
\]

\[
R^{\mu \mu} h^\mu \in 2\mathbb{Z},
\]

or odd,

\[
r^\mu h^\mu \in \mathbb{Z},
\]

\[
R^{\mu \mu} h^\mu \in \mathbb{Z}.
\]
These conditions are very similar to those described in [18]. The final set of constraints we need to consider relate $r^\mu$ to $R^{0\mu}$. In the case of a single quotient, the condition,

$$r^0 = R^{00},$$

(2.30)
can be seen to arise by requiring reasonable behavior of the twisted partition function under the spectral flow acting on the left $U(1)$ current, $J^0$, and the energy-momentum tensor $T$ [19]. Condition (2.30) can also be seen to arise from the requirement that the anomalies in the operator product expansion (OPE) of $T$ and $J_R$ with $J^0$ vanish. To study the general case of multiple quotients, let us momentarily restrict to those discrete symmetries which can be promoted to continuous symmetries of the theory. In this case, the superpotential satisfies,

$$W(e^{-2\pi i \theta q^\mu} \Lambda_a, e^{2\pi i \theta q^\mu} \Phi_i) = W(\Lambda_a, \Phi_i),$$

(2.31)
for any $\theta$. Let us further restrict our discussion to the level of $\bar{Q}+$-cohomology [20], where we can define a $U(1)$ current for this symmetry:

$$J^\mu = -\sum q^\mu_a \lambda_a \bar{\lambda}_a - \sum q^\mu_i (\partial - \phi) \phi_i,$$

(2.32)
which is well-defined in $\bar{Q}+$-cohomology. Requiring that the OPE between $T$ and $J^\mu$ be non-anomalous yields,

$$r^\mu = R^{0\mu}.$$  

(2.33)
However, since we are actually considering only discrete symmetries, it seems reasonable to impose the weaker condition:

$$(r^\mu - R^{0\mu}) h^\mu \in \mathbb{Z},$$

(2.34)
together with $r^0 = R^{00}$. We shall restrict our discussion to orbifolds of models which satisfy the constraint (2.34). Finally, we can define the orbifold partition function,

$$Z_{orb}(\tau, \nu) = \frac{1}{\prod h^\mu} \sum_{\alpha^0, \beta^0=0}^{h^0-1} \ldots \sum_{\alpha^{P-1}, \beta^{P-1}=0}^{h^{P-1}-1} \epsilon(\bar{\alpha}, \bar{\beta}) \beta_{\bar{\alpha}}(\tau, \nu, 0),$$

(2.35)
which transforms under modular transformations according to (2.17).
In order to obtain information about the massless spectra, we need only study the leading terms in an expansion of (2.35) in powers of \( q \). Let \( y = e^{2 \pi i \nu} \), then, following the definition in [9], the \( \chi_y \) genus is given by,

\[
\chi_y = \lim_{q \to 0} (i)^{N-M} q^{N-M} y^{\frac{1}{2}} Z_{orb}(q, y).
\]  

(2.36)

We shall denote the contribution to \( \chi_y \) from a twisted sector \( \bar{\alpha} \) by \( \chi_{\bar{\alpha}} \). The contribution from each twisted sector is determined in terms of the function,

\[
f_{\bar{\alpha}}(\bar{z}) = (-1)^{\bar{\alpha}} e^{2 \pi i \bar{z} \bar{Q}_{\bar{\alpha}}} q_{\bar{\alpha}} \prod_a (-1)^{[\bar{\alpha} \bar{q}_a]} (1 - e^{2 \pi i \bar{z} \bar{q}_a}) q^{\{\bar{\alpha} \bar{q}_a\}} (1 - e^{-2 \pi i \bar{z} \bar{q}_a}) q^{1-\{\bar{\alpha} \bar{q}_a\}},
\]  

(2.37)

where \( \chi_{\bar{\alpha}} \) is given by expanding \( f_{\bar{\alpha}}(\bar{z}) \) in powers of \( q \), and retaining terms of the form \( q^0 e^{-2 \pi i (\bar{\sigma} + \bar{n})} \), where \( \bar{n} \in \mathbb{Z}^P \) and \( \bar{\sigma} = \frac{1}{2} \bar{\bar{\sigma}} + \frac{1}{2} \bar{\bar{\alpha}} (Q - R) \). Finally, we set \( z_1 = \ldots = z_{P-1} = 0 \).

Furthermore, we have used the abbreviation \( \{ x \} = x - [x] \) in (2.37). The fractionalized charges and energies in the twisted sectors are given by the formulae:

\[
\bar{Q}_{\bar{\alpha}} = \sum_a \bar{q}_a (\bar{\alpha} \bar{q}_a - [\bar{\alpha} \bar{q}_a] - \frac{1}{2}) - \sum_i \bar{q}_i (\bar{\alpha} \bar{q}_i - [\bar{\alpha} \bar{q}_i] - \frac{1}{2}),
\]

\[
E_{\bar{\alpha}} = \frac{1}{2} \sum_a (\bar{\alpha} \bar{q}_a - [\bar{\alpha} \bar{q}_a] - 1)(\bar{\alpha} \bar{q}_a - [\bar{\alpha} \bar{q}_a]) - \frac{1}{2} \sum_i (\bar{\alpha} \bar{q}_i - [\bar{\alpha} \bar{q}_i] - 1)(\bar{\alpha} \bar{q}_i - [\bar{\alpha} \bar{q}_i]).
\]  

(2.38)

To serve as the internal sector of a heterotic string compactification, we need to impose further constraints on the allowed models. Firstly, to combine the internal sector of the theory with the space-time and gauge sectors, we require that in all the twisted sectors, \( Q_{L\bar{\alpha}} - Q_{R\bar{\alpha}} \in \mathbb{Z} \), where we have denoted the right-moving charge in the twisted sector \( \bar{\alpha} \) by \( Q_{R\bar{\alpha}} \). We find that:

\[
Q_{R\bar{\alpha}} = \sum_a q_{Ra} (\bar{\alpha} \bar{q}_a - [\bar{\alpha} \bar{q}_a] - \frac{1}{2}) - \sum_i q_{Ri} (\bar{\alpha} \bar{q}_i - [\bar{\alpha} \bar{q}_i] - \frac{1}{2}),
\]  

(2.39)

which implies that,

\[
\sum_a \bar{q}_a - \sum_i \bar{q}_i \in \mathbb{Z}^P.
\]  

(2.40)

In addition, we require that our orbifold actions preserve the vacuum from the untwisted sector. Among the problems that occur if this state is projected out is the lack of enough
gauginos to obtain an $E_6$, $SO(10)$ or $SU(5)$ enhanced gauge group. To preserve the vacuum, we require that

$$\vec{w} = \left( \sum_a \vec{q}_a - \sum_i \vec{q}_i \right) \mod 2. \quad (2.41)$$

Lastly, we want our canonical projection onto states with left-moving charge $q_L = \frac{1}{2} r^0 \mod \mathbb{Z}$. This requirement leads to the condition,

$$\sum_\mu \alpha^\mu (Q^\mu - R^\mu) \in 2\mathbb{Z} \quad (2.42)$$

for every twisted sector, $\vec{\alpha}$. This determines $Q^\mu$ in terms of $R^\mu \mod 2$.

### 3. Mirror Pairs

Our first example is the $(0,2)$ model with the following field content, and $U(1)$ charge assignments.

| Field | $\Phi^{1,2,3,4}$ | $\Phi^{5,6}$ | $\lambda^{1,\ldots,7}$ |
|-------|------------------|--------------|------------------------|
| $ql$  | $\frac{1}{5}$    | $\frac{2}{5}$| $-\frac{4}{5}$        |
| $qr$  | $\frac{1}{5}$    | $\frac{2}{5}$| $\frac{1}{5}$         |

**Table 2:** Left and right charges of the fields in the LG theory.

This model has been studied in previous work [4]. The relevant geometry in the Calabi-Yau phase corresponds to the vector-bundle defined by the exact sequence,

$$0 \to V \to \bigoplus_{a=1}^5 \mathcal{O}(1) \to \mathcal{O}(5) \to 0, \quad (3.1)$$

over the threefold configuration $\mathbb{P}_{(1,1,1,1,2,2)}[4,4]$. In the Landau-Ginzburg phase, the massless sector contains $N_{16} = 80$ chiral multiplets which transform in the spinor representation of $SO(10)$. There are no states transforming in the conjugate spinor representation, and there are $N_{10} = 74$ chiral multiplets which transform in the vector representation. The description of the conformal field theory for this model is known, and the exactly solvable point corresponds to a Landau-Ginzburg theory with superpotential [4],

$$W = \sum_{i=1}^4 \Lambda_i \Phi_i^4 + \Lambda_5 \Phi_5^2 + \Lambda_6 \Phi_6^2 + \Lambda_7 \Phi_5 \Phi_6. \quad (3.2)$$

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Let us begin with some general observations. Clearly, the superpotential is invariant under the discrete symmetry given by:

$$\phi_1 \rightarrow e^{\frac{2\pi i}{n}} \phi_1, \quad \phi_2 \rightarrow e^{-\frac{2\pi i}{n}} \phi_2$$
$$\lambda_1 \rightarrow e^{-2\pi i \frac{4}{n}} \lambda_1, \quad \lambda_2 \rightarrow e^{2\pi i \frac{4}{n}} \lambda_2.$$

(3.3)

The charges of the various fields with respect to this new symmetry are therefore,

$$(\vec{q}_i; \vec{q}_a) = \left( \frac{1}{n}, \frac{1}{n}, 0, 0, 0, 0; \frac{4}{n}, \frac{4}{n}, 0, 0, 0, 0 \right).$$

(3.4)

In the subsequent discussion, we will use this shorthand notation to denote the action of the discrete symmetries on the fields. Calculating $r^\mu$ and $R^{\mu\nu}$ gives,

$$\vec{r} = (4, 0), \quad R = \begin{pmatrix} 4 & 0 \\ 0 & \frac{30}{n^2} \end{pmatrix}.$$

(3.5)

The choice of $n$ is strongly constrained by the conditions (2.28) and (2.29), leaving only $n = 3, 5, 15$ as allowed possibilities. This example demonstrates how restrictive the previous conditions really are. Note, that unlike the corresponding (2, 2) quintic, we are not forced to consider $\mathbb{Z}_5$ actions only.

Now, let us consider a $\mathbb{Z}_5$ discrete symmetry, which we denote $G_{1,2}$, that is given by:

$$\phi_1 \rightarrow e^{\frac{2\pi i}{5}} \phi_1, \quad \phi_2 \rightarrow e^{-\frac{2\pi i}{5}} \phi_2$$
$$\lambda_1 \rightarrow e^{\frac{2\pi i}{5}} \lambda_1, \quad \lambda_2 \rightarrow e^{-\frac{2\pi i}{5}} \lambda_2.$$

(3.6)

The charges of the various fields with respect to this new symmetry are therefore,

$$(\vec{q}_i; \vec{q}_a) = \left( \frac{1}{5}, -\frac{1}{5}, 0, 0, 0, 0; -\frac{1}{5}, \frac{1}{5}, 0, 0, 0, 0 \right),$$

(3.7)

where,

$$\vec{r} = (4, 0), \quad R = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}.$$

(3.8)

This choice of charges satisfies the constraints described in the previous section with trivial discrete torsion. All the examples in this section will have trivial discrete torsion.

The result of applying the general formula for the $\chi_y$ genus from the previous section to this example is summarized in the following table. The table is arranged so that the $(l, a)$ entry gives the $\chi_y$ genus for the $(l, a)$ twisted sector, where $l$ and $a$ denote the twisted sectors of the $G_{1,2}$ quotient and the GSO quotient, respectively.
Summing up the contributions from all the twisted sectors gives the result:

\[ \chi_y = 40(y - y^3). \]  

(3.9)

To obtain the number of generations and anti-generations, we only have to count the positive and negative \( y^1 \) terms in table 3 separately. Therefore, we see that there are \( N_{16} = 42 \) generations and \( \bar{N}_{16} = 2 \) anti-generations in this model. The number of \( 10' \)s that arise from the \( a = 0 \) sectors can not be extracted from the \( \chi_y \) genus. Rather, a detailed analysis of the cohomology is needed to reveal that \((a, l) = (0, 0)\) contains \( N_{10} = 18 \) vectors, and each of the other \((a, l) = (0, l)\) sectors contain \( N_{10} = 4 \) vectors. Thus, we end up with \( N_{10} = 42 \) vectors. As expected from the discussion in [2], this model has the same spectrum as the \( (0,2) \) model defined by the following bundle and Calabi-Yau data,

\[ V(3, 4, 4, 4, 5; 20) \rightarrow \mathbb{P}_{3,4,4,5,8,8}[16, 16]. \]  

(3.10)

This data should be viewed as input information for a \((0,2)\) linear sigma model, whose Calabi-Yau phase is described by this geometric information.

We can carry out a similar analysis by quotienting our starting model by another \( \mathbb{Z}_5 \) action, denoted \( G_{2,3} \), given by:

\[ (\bar{q}_i; \bar{q}_a) = \left( 0, \frac{1}{5}, -\frac{1}{5}, 0, 0, 0; 0, -\frac{1}{5}, \frac{1}{5}, 0, 0, 0 \right). \]  

(3.11)

The resulting spectrum then agrees exactly with the model described by,

\[ V(13, 15, 16, 16, 20; 80) \rightarrow \mathbb{P}_{13,15,16,20,32,32}[64, 64], \]  

(3.12)

with \((N_{16}, \bar{N}_{16}) = (8, 36)\). According to the mirror construction described in [2], we might have expected this model to be the mirror of our first example where we quotiented by
However, this is not the case, and we are left with the question: where are the models with $(N_{16}, \bar{N}_{16}) = (2, 42)$ and $(N_{16}, \bar{N}_{16}) = (36, 8)$, respectively?

A little trial and error reveals that after dividing our starting model by a $\mathbb{Z}_5$, denoted $H_1$, which acts according to,

$$(\vec{q}_i; \vec{q}_a) = \left( -\frac{4}{5}, 0, 0, 0, \frac{3}{5}, \frac{3}{5}; \frac{4}{5}, 0, 0, -\frac{4}{5}, \frac{1}{5}, \frac{1}{5} \right),$$

we get the following contributions from the twisted sectors to the $\chi_y$ genus.

| $l \setminus a$ | 0 | 1 | 2 | 3 | 4 |
|-----------------|---|---|---|---|---|
| 0               | $1 + 18(y - y^3) - y^4$ | $y^4$ | $-2y^2$ | $2y^2$ | $-1$ |
| 1               | $6y + 6y^2$ | 0 | $y^3$ | $-2y$ | $-3y + 3y^2$ |
| 2               | $-12y^2 - 12y^3$ | $-y^2$ | 0 | 0 | $-2y$ |
| 3               | $12y + 12y^2$ | $2y^3$ | 0 | 0 | $y^2$ |
| 4               | $-6y^2 - 6y^3$ | $-3y^2 + 3y^3$ | $2y^3$ | $-y$ | 0 |

**Table 4:** $\chi_y$ genus for $H_1$.

Again summing the twisted sector contributions gives, $\chi_y = 28(y - y^3)$, and the desired spectrum, $(N_{16}, \bar{N}_{16}) = (36, 8)$, of generations and anti-generations. Acting with a second orbifold action, $H_2$,

$$(\vec{q}_i; \vec{q}_a) = \left( 0, -\frac{4}{5}, 0, 0, \frac{3}{5}, \frac{3}{5}; 0, 0, 0, -\frac{4}{5}, \frac{1}{5}, \frac{1}{5} \right),$$

we obtain a model with spectrum $(N_{16}, \bar{N}_{16}) = (2, 42)$. We can combine the two symmetries $G_{1,2}$ and $H_3$ into a single action $(GH)_{1,2,3}$:

$$(\vec{q}_i; \vec{q}_a) = \left( \frac{1}{5}, -\frac{1}{5}, -\frac{4}{5}, \frac{0}{5}, \frac{3}{5}, \frac{3}{5}; -\frac{1}{5}, \frac{1}{5}, \frac{4}{5}, 0, -\frac{4}{5}, \frac{1}{5}, \frac{1}{5} \right).$$

In this case, the $\chi_y$ genus for each twisted sector is displayed in the following table.

| $l \setminus a$ | 0 | 1 | 2 | 3 | 4 |
|-----------------|---|---|---|---|---|
| 0               | $1 + 15(y - y^3) - y^4$ | $y^4$ | $-2y^2$ | $2y^2$ | $-1$ |
| 1               | 0 | $-3y^2 + 3y^3$ | $y^3$ | $-y$ | $-3y + 3y^2$ |
| 2               | 0 | $-y$ | $-3y + 3y^2$ | $-3y^2 + 3y^3$ | $y^3$ |
| 3               | 0 | $2y^2$ | $-4y$ | $4y^3$ | $-2y^2$ |
| 4               | 0 | $y - 7y^2 - 2y^3$ | $2y^3$ | $-2y$ | $2y + 7y^2 - y^3$ |

**Table 5:** $\chi_y$ genus for $(GH)_{1,2,3}$. 

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This model has \((N_{16}, \bar{N}_{16}) = (18, 14)\). To find a candidate mirror, we must quotient by \(G_{1,2} \times H_3\). Finally, the mirror of our original model is given by the quotient \(G_{1,2} \times G_{2,3} \times G_{3,4}\), which gives a model with massless spectrum: \((N_{16}, \bar{N}_{16}) = (0, 80)\). To summarize, starting from the \((0, 2)\) model described in table 2, we have, by successive orbifolds, obtained the following mirror symmetric family of models:

\[
\begin{array}{|c|c|c|}
\hline
G & N_{16} & \bar{N}_{16} \\
\hline
1 & 80 & 0 \\
G_{1,2} & 42 & 2 \\
H_1 & 36 & 8 \\
(GH)_{1,2,3} & 18 & 14 \\
G_{1,2} \times H_3 & 14 & 18 \\
G_{1,2} \times G_{2,3} & 8 & 36 \\
H_1 \times H_2 & 2 & 42 \\
G_{1,2} \times G_{2,3} \times G_{3,4} & 0 & 80 \\
\hline
\end{array}
\]

Table 6: A family of \((0,2)\) models obtained by orbifolding.

In many ways, this appears to be the \((0, 2)\) analogue of the Greene-Plesser construction for the mirror of a Fermat type \((2, 2)\) theory \[3\]. In the following section, we shall consider models with non-trivial discrete torsion, where we will find a similar pattern.

4. Examples with Non-trivial Discrete Torsion

In order to include discrete torsion, we need to choose a finite group \(G\) such that \(H^2(G, U(1))\) is non-trivial. Perhaps the simplest choice is to take \(G\) to be the product of two simple abelian groups. For example, since \(H^2(\mathbb{Z}_5 \times \mathbb{Z}_5, U(1)) = \mathbb{Z}_5\), the models obtained by orbifolding by \(G_{1,2} \times G_{2,3}, H_1 \times H_2\), and \(G_{1,2} \times H_3\) admit the introduction of a non-trivial discrete torsion.

\[\epsilon(\vec{\alpha}, \vec{\beta}) = e^{\pi i(\vec{\alpha}Q\vec{\beta})}\]

\[Q = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & \frac{2m}{5} \\ 0 & -\frac{2m}{5} & 0 \end{pmatrix}, \quad m \in \{0, \ldots, 4\}\]

---

1 We thank M. Flohr for helping us with a C-code program to calculate all 625 twisted sectors of this model.
Including the discrete torsion has the effect of changing the projections in the various twisted sectors. After a tedious calculation, we arrive at the following spectra for these models:

| $m$ | 0   | 1    | 2    | 3    | 4   |
|-----|-----|------|------|------|-----|
| $G_{1,2} \times G_{2,3}$ | (8, 36) | (23, 3) | (23, 3) | (23, 3) | (23, 3) |
| $H_1 \times H_2$ | (2, 42) | (17, 9) | (19, 11) | (19, 11) | (17, 9) |
| $G_{1,2} \times H_3$ | (14, 18) | (16, 8) | (14, 6) | (14, 6) | (16, 8) |

**Table 7:** \((N_{16}, \bar{N}_{16})\) with the inclusion of discrete torsion.

For \(G_{1,2} \times G_{2,3} \times G_{3,4}\), we shall consider the following choices for discrete torsion, which are captured in the following Q matrix,

\[
\epsilon(\vec{\alpha}, \vec{\beta}) = e^{\pi i \langle \vec{\alpha} Q \vec{\beta} \rangle}
\]

\[
Q = \begin{pmatrix}
4 & 0 & 0 & 0 \\
0 & 0 & \frac{2m_1}{5} & -\frac{2m_2}{5} \\
0 & -\frac{2m_1}{5} & 0 & \frac{2m_3}{5} \\
0 & \frac{2m_2}{5} & -\frac{2m_3}{5} & 0
\end{pmatrix}
\]

\(m_i \in \{0, \ldots, 4\}\).

Because of the permutation symmetry, there are redundant choices for the three parameters \(m_i\). Taking permutations into account leaves nineteen independent triples. However, only six of the resulting models have different spectra, and these cases are displayed below.

| \((m_1, m_2, m_3)\) | (0, 0, 0) | (1, 1, 1) | (2, 2, 2) | (1, 2, 2) | (1, 3, 3) | (1, 2, 3) |
|---------------------|----------|----------|----------|----------|----------|----------|
| \((N_{16}, \bar{N}_{16})\) | (0, 80) | (9, 17) | (11, 19) | (8, 16) | (6, 14) | (3, 23) |

**Table 8:** The spectra of the \(G_{1,2} \times G_{2,3} \times G_{3,4}\) orbifold with discrete torsion.

The spectra for these models again form a mirror symmetric set, even with the inclusion of discrete torsion.

If \((2, 2)\) mirror symmetry is any indicator, we have only uncovered a small part of what is likely to be a very rich area to study. There are many avenues to explore: perhaps, the most natural question is whether a reasonable combinatoric conjecture for \((0, 2)\) mirror pairs can be formulated, analogous to Batyrev’s conjecture for Calabi-Yau spaces constructed from toric varieties \([21]\). In a similar vein, could a T-duality be responsible for \((0, 2)\) mirror symmetry \([22]\)? There seem to be many interesting subtleties in even trying to define, in the Calabi-Yau phase, the operation which corresponds to the orbifold action in the Landau-Ginzburg phase. This is a topic that deserves detailed study. In the cases
where we have corresponding geometric models, there are numerous more mathematical questions to answer: for instance, desingularizing the corresponding geometric models, and checking that their spectra agree with the Landau-Ginzburg computations. A topic barely explored in (2,2) mirror symmetry is the question of non-abelian quotients. It would be interesting to extend this analysis to incorporate non-abelian orbifolds.

Constructing more examples of the kind we have presented here would also be interesting. Perhaps, a more systematic study of the properties of the elliptic genus for this class of models would provide insight into the general structure of (0,2) mirror symmetry, as in [23]. Further, it does not seem unlikely that some deformation theoretic couplings are immune to instanton corrections. Can we use (0,2) mirror symmetry to check this possibility?

As a final comment, we suspect that (0,2) mirror pairs will be very helpful in unraveling part of the structure of N=1 dualities in four dimensions. For such applications, it is interesting to consider (0,2) models on base spaces which are elliptically-fibered. Imposing the condition that the base admit an elliptic-fibration undoubtedly provides interesting constraints on the allowed bundle structure for these theories.

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