A Study on the Modeling Consideration Dynamic Properties of Vibration Damper Rubber Parts

Tomoaki Kodama¹, and Yasuhiro Honda²

¹Department of Science and Engineering, School of Science and Engineering, Kokushikan University, 4-28-1 Setagaya Setagaya-ku Tokyo 154-8515 Japan
²Department of Science and Engineering, School of Science and Engineering, Kokushikan University, 4-28-1 Setagaya Setagaya-ku Tokyo 154-8515 Japan

¹kodama@kokushikan.ac.jp, ²honda@kokushikan.ac.jp

Key Words: Damper Pulley; Torsional Vibration; Crankshaft System; Torsional Stiffness; Damping Coefficient; Strain Rate; Loss Factor; Modeling; Rubber; Experiment; Diesel Engine.

Abstract. In this paper, we first describe the dynamic characteristics of damper rubber parts that are necessary for the modeling of torsional vibration. Secondly, we describe an experiment in which two crankshaft pulleys with a torsional rubber damper are fitted to a 6-cylinder, high-speed diesel engine. The torsional waveforms of the damper inertia ring and the pulley are measured by means of phase-shift torsigraph equipment. The measured waveforms are harmonically analyzed and the dynamic characteristics of the stiffness and the damping are investigated from an experimental viewpoint. As a result of comparisons with experimental data, certain dynamic characteristics of pulleys with a torsional rubber damper have been clarified.

1. Introduction

The important requirements in the design stage are that the vibration and noise levels of automobile engines be reduced and that the timbre be improved further [1]-[4]. Torsional vibration dampers of high performance, and crankshaft pulleys with a torsional rubber damper (hereafter called "damper pulleys"), which can reduce both torsional and bending vibrations, and flywheels with a torsional rubber damper have been widely employed in automobile engines [5]-[7]. When designers estimate the amplitude of angular displacement of crankshafts with the abovementioned damper pulleys in the design process, the following issues are of concern:

(1) the estimation of dynamic torsional stiffness and damping of the damper.

These dynamic characteristics are indeterminable [1]-[4]. Therefore, an experimental equation for determining the values has not existed until now and the quantitative values are inaccurate [8]. Unless these values can be accurately estimated to a certain degree, it is difficult to achieve high-precision computation [9]-[13]. We clarify some of the complicated dynamic characteristics of the rubber vibration isolator by measuring the torsional vibrations of the crankshaft system with a damper pulley.

2. Notations

The main symbols used in this paper are as follows:

\( b_{RUB} \): Thickness of the rubber at the representative radius [m], \( C_{RDP} \): Damping coefficient of the rubber part of a rubber damper pulley [Nms/rad], \( I_{RDP} \): Inertia moment of the damper inertia ring [kgm²], \( K_{RDP} \): Dynamic torsional stiffness of the rubber part of a rubber damper pulley [Nm/rad], \( K^*_{RDP} \):
Complex torsional stiffness of the rubber part of a rubber damper pulley [Nm/rad]

\[ K_{\text{RDP}} + j \cdot C_{\text{RDP}} \cdot \omega \cdot j \] (imaginary number),

\[ K_{\text{RDP}} : \text{Absolute torsional stiffness of the rubber part of a rubber damper pulley [Nm/rad]} \]

\[ j, C_{\text{RDP}} \text{ : Imaginary number}, \]

\[ M_{\text{RDP}} : \text{Amplitude ratio} \]

\[ \theta_{\text{RDP},O} \text{ : Outer diameter of the rubber part [m]}, \]

\[ \gamma_{\text{RDP}} : \text{Strain rate [l/s]}, \]

\[ \phi_{\text{RDP}} : \text{Phase angle between the damper inertia ring and pulley [rad]} \]

\[ \rho_{\text{RDP}} : \text{Representative radius of the rubber part of a rubber damper pulley [m]}, \]

\[ \theta_{\text{PUL}} : \text{Torsional angular displacement of the pulley [rad]}, \]

\[ \theta_{\text{RDP}} : \text{Torsional angular displacement of the damper inertia ring [rad]}, \]

\[ \phi_{\text{RDP}} : \text{Relative torsional angular displacement of the rubber part of a rubber damper pulley [rad]} \]

3. Experimental Equipment and Method

This section concerns the main specifications of the test engine and damper pulley, as well as the method of measuring the torsional vibration waveforms. In the experiment, the engines are operated under full load to cause steady forced torsional vibration.

3.1 Main Specifications of Test Engine and Rubber Damper Pulley.

The test engine is a 6-cylinders, in-line, high-speed diesel engine. Figure 1 shows the dimensions of the test damper pulley. This Table also shows the static stiffness and natural frequency obtained from the static and free vibration tests, respectively. Types A and B of the damper pulleys differ in Shore hardness but their rubber parts have the same shape and dimensions. The materials are similar-quality rubber.

3.2 Method of Measuring Torsional Vibration Waveform. An eddy current dynamometer was connected via a universal joint to the flywheel of the engine. The test damper pulley was fitted to the end of the crankshaft. The gears for generating signal pulses were mounted on the damper inertia ring and pulley. The electric frequency signals proportional to engine speed were obtained from the electromagnetic pickup. The measured signals were transmitted to the phase-shift torsiograph equipment via the adapter which numerically calculated the average of angular velocity (the center frequency). The torsional vibration waveforms could be obtained from the torsional angles, which were numerically calculated using the relationship between the measured and center frequencies. The measured torsional waveforms of the damper inertia ring and pulley were harmonically analyzed using the F.F.T. analyzer. The torsional angular displacement was measured under full load from 800 [r/min] to 3200 [r/min]. The indicator diagrams, data from which were necessary for the vibration analysis described later, were measured using the piezotype indicator in the sixth cylinder from the pulley side.

![Fig. 1. Dimensions and Shape of the Test Rubber Damper Pulley](image)

4. Dynamic Characteristics of Rubber Parts of Damper Pulleys
4.1 Measured Torsional Vibration Waveforms. The measured waveform of the pulley is the waveform in which the 6th-order amplitude is amplified due to the measurement at the 6th-order resonant point. On the other hand, the measured waveform of the damper inertia ring via the rubber part shows a slightly different tendency, in which the 6th-order amplitude is not particularly amplified and, in addition, the 3rd- and 4.5th-order amplitudes are large.

4.2 Measured Amplitude Curves of Angular Displacement. Figure 2 shows the measured amplitude curves of angular displacements at the pulley and the damper inertia ring of the crankshaft with the B-type damper pulley. These figures show merely the main-order amplitude curves of angular displacements. The large 6th-order resonant point occurs at 2524 [r/min] and its resonant amplitude is $14.9 \times 10^{-3}$ [rad] in the case of installing no damper pulley. Since the tuning ratio of the B-type damper pulley is not optimum, the 6th-order amplitude is not greatly reduced.

4.3 Numerical Calculation of Dynamic Characteristic Values from Measured Waveform. The values of dynamic torsional stiffness and damping coefficient of damper rubber parts can be obtained from the harmonically analyzed results of waveforms measured at the damper inertia ring and the pulley. The equation of motion at the damper inertia ring is

$$ I_{RDP} \cdot \frac{d^2 \theta_{RDP}}{dt^2} + C_{RDP} \left( \frac{d \theta_{RDP}}{dt} - \frac{d \theta_{FUL}}{dt} \right) + K_{RDP} (\theta_{RDP} - \theta_{FUL}) = 0, $$

where $\theta_{RDP} = \theta_{RDP,0} \cdot e^{i(\omega t - \phi)}$ and $\theta_{FUL} = \theta_{FUL,0} \cdot e^{i\omega t}$. The following Eq. (2) can be obtained by rearranging the expression of Eq. (1) by substituting the above-mentioned relational expression into Eq. (1) and rearranging:

$$ K_{RDP} = \frac{I_{RDP} \cdot \omega_{RDP}^2 \cdot M_{RDP} \left( M_{RDP} - \cos \phi_{RDP} \right)}{M_{RDP}^2 + 1 - 2 \cdot M_{RDP} \cdot \cos \phi_{RDP}}, \quad C_{RDP} = \frac{I_{RDP} \cdot \omega_{RDP}^2 \cdot M_{RDP} \cdot \sin \phi_{RDP}}{M_{RDP}^2 + 1 - 2 \cdot M_{RDP} \cdot \cos \phi_{RDP}}. $$

In Eq. (2), the values of amplitude ratio $M_{RDP}$ and phase angle $\phi_{RDP}$ can be obtained by analyzing harmonically the waveforms measured at the damper inertia ring and the pulley, where the values of $I_{RDP}$, $\omega_{RDP}$ are known. Therefore, the values of dynamic torsional stiffness $K_{RDP}$ and damping coefficient $C_{RDP}$ can be determined from Eq. (2). The dynamic characteristics of the vibration isolator rubber depend generally on (1) temperature, (2) frequency, and (3) average strain and strain amplitude effects on the basis of same shape and material. To reveal the effect of one factor among the above-mentioned factors on the dynamic characteristics, it is necessary to keep the values of the other factors constant in the experiment. In our experiments, the surface temperature of the rubber part was kept at 313 [K], but the other factors could not be controlled due to the vibration characteristics of the crankshaft.
In full consideration of these conditions of the other factors, strain rate $\gamma_{\text{RDP}}$, which is the product of strain amplitude and angular frequency, is defined by the following equation.

$$\theta_{\text{RDP}} = \theta_{\text{RDP}1} \cdot \omega_{\text{RDP}} \cdot \rho_{\text{RDP}} / b_{\text{RDP}}$$

$$\rho_{\text{RDP}} = \frac{3}{4} \left( \frac{1}{\rho_{\text{RDP}2}} - \frac{1}{\rho_{\text{RDP}3}} \right)$$

Equation (4) is obtained for the uniform, hollow, circular cross section under the condition that the torsional moment (in such a case that the shearing stress in the representative radius $\rho_{\text{RDP}}$ is uniformly distributed over the entire cross section) is equal to that (in the case that shearing stress distribution $\tau_{\text{RDP}} = G_{\text{RDP}} \cdot \rho_{\text{RDP}} \cdot C_{\text{RDP} \cdot \omega_{\text{RDP}}}$ caused by the simple shearing deformation in the circumferential direction.

Figure 3 shows the relationship between absolute torsional stiffness and strain rate. Figure 4 illustrates the relationship between loss factor and strain rate. Both Figs. 3 and 4 are aimed at expressing the absolute torsional stiffness and the loss factor as a function of the strain rate on the condition that the nonlinear dynamic characteristic values of the damper rubber part are assumed to be approximately expressed by linear equations. The values of the dynamic torsional stiffness $K_{\text{RDP}}$ and the damping coefficient $C_{\text{RDP}}$ (or $C_{\text{RDP}} \cdot \omega_{\text{RDP}}$), which are necessary as input data for the simulation analysis in section 4, are arranged in the same way as shown in Figs. 3 and 5, respectively. The value of absolute torsional stiffness can be determined by $|K'_{\text{RDP}}| = \sqrt{(K_{\text{RDP}})^2 + (C_{\text{RDP}} \cdot \omega_{\text{RDP}})^2}$, but the following expression applies under the condition that the value of $C_{\text{RDP}} \cdot \omega_{\text{RDP}}$ is fairly small in comparison with the value of $K_{\text{RDP}}$: $|K'_{\text{RDP}}| = K_{\text{RDP}}$. Therefore, the values of $|K'_{\text{RDP}}|$ and $K_{\text{RDP}}$ approximately agree in the figure. The curves obtained by the curve-fitting method are illustrated in Figs. 3, 4 and 5. The values of the correlation coefficients against the curves of Dampers A and B are in the regions of 0.82-0.89 and 0.86-0.94, respectively.
5. Analytical Investigation of Torsional Vibration Characteristics of Engine Crankshaft with Damper Pulley

5.1 Input Data Necessary for Numerical Calculation of Torsional Vibration Waveforms. Figure 6 shows the equivalent torsional vibration system of a crankshaft with a rubber damper pulley replaced according to the analytical method described in section 4.1. The rubber part of the damper pulley is replaced with a Voight model. Since the simulation program can yield the torsional vibration waveform for a given engine speed, the value of the strain rate is numerical calculated using the measured dominant-order amplitude of angular displacement at a given engine speed. Then, the values of the dynamic torsional stiffness $K_{RDP}$ and damping coefficient $C_{RDP}$ of the rubber part can be obtained by referring to Figs. 3 and 5, respectively. These obtained values of $K_{RDP}$ and $C_{RDP}$ are used as the input data for the numerical calculation. Since the curve-fitted curves in Figs. 3 and 5 are representative of all the experimental data, the obtained values of $K_{RDP}$ and $C_{RDP}$ can be regarded as the mean values.

Fig. 6. Equivalent Torsional Vibration System of an Engine Crankshaft System with a Rubber Damper Pulley

5.2 Numerical Calculated Results and Considerations. Figure 7 shows the main-order amplitude curves of the angular displacements at the damper inertia ring and the pulley obtained by analyzing harmonically the numerical calculated torsional vibration waveforms of the crankshaft with the rubber damper pulley. The numerical calculated amplitude curves in this figure correspond to the measured amplitude curves of the angular displacements in Fig. 2. As compared with the experimental results, these calculated results contain slight error, but this degree of error is allowable in practice.

Fig. 7. Numerical Calculation Amplitude Curves of Angular Displacement [with B-Type Damper Pulley]

6. Summary
Accurate estimation of the dynamic characteristics of the rubber part of the damper pulley are problems in the numerical calculation of torsional vibration of a crankshaft with a rubber damper pulley. The results of investigation of these characteristics from experimental and analytical viewpoints are as follows:

1) We expressed the absolute torsional stiffness and damping of the rubber part of the damper pulley as a function of the strain rate under the conditions of the same kind rubber material, constant temperature and invariable shape factor.

7. References

[1] Yasuhiro Honda, Tomoaki Kodama, Katsuhiko Wakabayashi: Relationships between Rubber Shapes and Dynamic Characteristics of some Torsional Vibration Rubber Dampers for Diesel Engines, *The 15th Pacific Automotive Engineering Conference – APAC15*, No.0320, (2009), p. 1-8.

[2] Yasuhiro Honda, Katsuhiko Wakabayashi, Tomoaki Kodama, Hiroshi Okamura: An Experimental Study of Dynamic Properties of Rubber Specimens for a Crankshaft Torsional Vibration of Automobiles, *ASME 1999 International Design Engineering Technical Conference and The Computers and Information in Engineering Conference, 17th Biennial Conference on Mechanical Vibration and Noise Symposium on Dynamics and Vibration of Machine Systems, Session VIB-00037: Dynamics and Vibration of Machine System 02*, DETC99/VIB-08125, (1999), p. 1-14.

[3] Yasuhiro Honda, Katsuhiko Wakabayashi, Tomoaki Kodama, Shinkichi Hama, Shoichi Iwamoto: An Experimental Study on Dynamic Properties of Rubber Test Specimens for Design of Shear-type Torsional Vibration Dampers, *The Ninth International Pacific Conference on Automotive Engineering*, Volume 2, No. 971400 (Abstract Code: 00088), (1997), p. 217-222.

[4] Tomoaki Kodama, Yasuhiro Honda, Katsuhiko Wakabayashi, Shoichi Iwamoto: A Calculation Method for Torsional Vibration of a Crankshafting System with a Conventional Rubber Damper by Considering Rubber Form, *SAE 1996 International Congress and Exposition, SAE Technical Paper Series No. 960060*, (1996), p. 103-121.

[5] George Nerubenko: Tuned Torsional Vibration Damper, *SAE 2013 Noise and Vibration Conference and Exhibition*, SAE Technical Paper 2013-01-1897, DOI: https://doi.org/10.4271/2013-01-1897, (2013), p. 1-8.

[6] Mohammad Hijawi: Dual Torsional Vibration Damper, *SAE 2012 World Congress and Exhibition*, 2012-01-0065, DOI: https://doi.org/10.4271/2012-01-0065, (2012), p. 1-9.

[7] George Nerubenko: Engine Noise Reduction Using Self-Tuning Torsional Vibration Damper, *SAE 2016 World Congress and Exhibition*, SAE Technical Paper 2016-01-1063, DOI: https://doi.org/10.4271/2016-01-1063, (2016), p. 1-8.

[8] Jaroslaw Pankiewicz and Bogumil Chilinski: Model Aided Design of Tuned Rubber TVD, *Journal of KONES Powertrain and Transport*, Vol. 23, No. 4, DOI: 5604/12314005.1217250, (2016), p. 367-374.

[9] Hanmant. S. Tamkhade and G. S. Kondhalkar: Theoretical and Experimental Validation of Viscous Torsional Damper on Turbocharged Inline Six Cylinder Engine, *International Journal of Innovations in Engineering Research and Technology*, Vol. 4, Issue 2, (2017), p. 59-72.
[10] John Licari, Carlos E. Ugalde, Janaka B. Ekanayake and Nick Jenkins: Comparison of the Performance and Stability of Two Torsional Vibration Dampers for Variable-Speed Wind Turbines, *Window Energy*, John Wiley & Sons, No. 18, (2015), p. 1545-1559.

[11] Jing Sheng, Meng-da He and Mei-hua Yao: Simulation Study on the Performance of a Rubber-type Torsional Vibration Damper, *2017 2nd International Conference on Applied Mechatronics Engineering*, (2017), p. 79-84.

[12] Tieshn Zhang and Mingsong Li: A Design Study on the Torsional Vibration of the Vehicle Powertrain, *5th International Conference on Advanced Engineering Materials and Technology*, Atlantis Press, (2015), p. 429-433.

[13] Lionel Manin, Regis Dufour and Sebastien Schultz: Pulley Torsional Vibration Damper Characterization, *Mechanics & Industry*, AFM, EDP Sciences, Vol. 14, DOI: 10.1051/meca/2013057, (2013), p. 151-155.