On p- Open Sets with Respect to an Ideal

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Abstract
In this paper we introduce and investigate, the notion of \( \alpha - p \)-open, semi-\( lp \)-open and pre-\( lp \)-open sets via idealization by using \( p \)-local function and studied their some properties.

Keywords: Semi open, Pre open, Alpha open, P–Local function.

Introduction
Ideal in topological space have been considered since 1930 by Kuratowski [1] and Vaidyanathaswamy [2]. After that ideal topology generalized in general topology by Jankovi and Hamleet [3]. In 2005 Hatir and Noir introduced the \( \alpha - l \)-open set, semi-\( l \)-open set, pre-\( l \)-open set [4]. Finally in 2014 \( \alpha - l \)-open, semi-\( l \)-open, pre-\( l \)-open sets were introduced by R.Shanthi and M.Rameshkumar [5]. In this paper we introduced the notion of \( \alpha - lp \)-open, semi-\( lp \)-open, pre-\( lp \)-open set and studied some properties of their.

Preliminaries
Let (\( X, \tau \)) be topological space with no separation properties assumed. For a subset of topologicalspace (\( X, \tau \)), Cl (A) and Int (A) denote the closure and interior of A in (\( X, \tau \)) resp. An ideal I of topological space is collection of non-empty subset of X together with the following.

(i) \( x \in \text{Cl}(A) \) implies \( x \in \text{Cl}(B) \) for every \( B \subseteq A \).
(ii) \( A \subseteq X \) implies \( B \subseteq X \) for every \( B \subseteq A \).
(iii) \( A \subseteq X \) implies \( A \subseteq X \) for every \( B \subseteq A \).

The triplet forms (\( X, \tau, I \)) is called the ideal topological space where \( \tau \) is topological space of X with an ideal I. Given a topological space (\( X, \tau \)) with an ideal I on X If P(x) is the set of all subset of X, a set operator (\( x \subseteq P(x) \) called a local function [5] of A with respect to \( \tau \) and I is defined as follows: for \( A \subseteq X \), \( A_{\tau} = \{ x \in X \mid x \in A \} \) for every \( U \subseteq \tau \) wherer \( (x) = \{ u \in x \mid x \in U \} \). Additionally, \( \text{cl}^*(A) = A \cup (A \subseteq \tau \) defines kuratowski closure operator for a topology \( \tau \) (I. \( \tau \)), called the \( ^* \)-topology and finer than \( \tau \).
Definition 2.1
Let \((X, \tau)\) be a topological space. A subset \(A\) of \(X\) is said to be a \(p\)-open set if there exists an open set \(U\) in \(X\) such that \(U \subseteq A \subseteq \text{int}(\text{Cl}(A))\). The complement of a \(p\)-open set is \(p\)-closed. The collection of all \(p\)-open sets in \(X\) is denoted by \(pO(X)\) and is called the \(p\)-local function. The semi closure of \(A\) in \((X, \tau)\) is denoted by the intersection of all \(p\)-closed sets containing \(A\) and is denoted by \(pc\).

Definition 2.2
For \(A \subseteq X\), the set \(\{x \in X / U \cap A \neq \emptyset\}\), for every \(U \in pO(X)\) where \(pO(X) = \{U \cap pO(X) / x \in U\}\), we write \(A\) instead of \(A \cap (U \cap A)\). The closure operator \(Cl^p\) for a topology \(T\) is defined as follows \(Cl^p(A) = AUA\). For a topology \(T \subseteq (I) \subseteq r^p(I)\) and \(Int^p(A)\) denotes the interior of the set \(A\) in \((X, r^p, I)\).

Definition 2.3
A subset of topological space \(X\) is said to be,
- \(pre - open\) if \(A \subseteq \text{int}(\text{Cl}(A))\)
- \(semi-open\) if \(A \subseteq \text{Cl}(\text{int}(A))\)
- \(alpha - open\) if \(A \subseteq \text{int}(\text{Cl}(\text{int}(A)))\)

Definition 2.4
A subset of topological space \(X\) is said to be,
- \(alpha-1\)-open if \(A \subseteq \text{int}(\text{Cl}(\text{int}(A)))\)
- \(pre-1\)-open if \(A \subseteq \text{int}(\text{Cl}(A))\)
- \(semi-1\)-open if \(A \subseteq \text{Cl}(\text{int}(A))\)

Lemma: For a subset of topological space, the following properties hold.
- \(pc(A) = AUA\text{int}(\text{Cl}(A))\)
- \(pc(A) = \text{int}(\text{Cl}(A))\), if \(A\) is open

Lemma: Let \(A\) be a topological space and \(A, B\) be subsets of \(X\), then the following properties hold:
- if \(A \subseteq B\) then \(A \subseteq B\).
- if \(U \in r\) then \(U \cap A \subseteq (U \cap A)\).
- \(A \subseteq \text{Cl}(A)\) the \(A\) is \(p\)-closed in \(X\).

\(A \subseteq A\).
\(AUB = AUB\).
if \(I = \{\emptyset\}\), then \(A = \text{Cl}(A)\)

\(a-1\_p\)-open, \(semi-1\_p\)-open, \(pre-1\_p\)-open
In this we define the \(a-1\_p\)-open sets, \(pre-1\_p\)-open, \(semi-1\_p\)-open and studied some properties of their.

Definition 3.1
A subset of topological space \(X\) is said to be.
- \(alpha-1\_p\)-open if \(A \subseteq \text{int}(Cl^p(\text{int}(A)))\)
- \(pre-1\_p\)-open if \(A \subseteq \text{int}(Cl^p(A))\)
- \(semi-1\_p\)-open if \(A \subseteq Cl^p(\text{int}(A))\)
Proposition 3.2
For a subset of an ideal topological space the following hold:
*Every \( \alpha - I_p \)-open set is \( \alpha \)-open.

Proof
Let \( A \) be a \( \alpha - I_p \)-open set. Then, we have \( A \subseteq \text{int}(\text{Cl}^\alpha(\text{int}(A))) = \text{int}((\text{int}(A) \cup \text{int}(A))) \subseteq \text{int}(\text{Cl}(\text{int}(A)) \cup \text{int}(A)) \subseteq \text{int}(\text{Cl}(\text{int}(A))). \) Thus, \( A \) is an \( \alpha \)-open set.

Remark 3.3
Converse of the above proposition need not be true as seen from the following example.

Example 3.4
Let \( X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}, X\} \) and \( I = \{\emptyset, \{b\}, \{c\}, \{bc\}\}. \) Then the set \( A = \{b, c\}, B = \{a, b, c, d\}, C = \{a\}. \) \( A \) is \( \text{Semi} - I_p \)-open, but not \( \text{Semi} - I_p \)-open, \( B \) is \( \text{Pre} - I_p \)-open, but not \( \alpha - I_p \)-open.

Proposition 3.5
Every open set of an ideal topological space is an \( \alpha - I_p \)-open set.

Proof:
Let \( A \) be a \( \text{Semi} - I_p \)-open set. Then, we have \( A \subseteq \text{int}(\text{Cl}^\alpha(A)) = \text{int}((\text{int}(A) \cup \text{int}(A))) \subseteq \text{int}(\text{Cl}(\text{int}(A)) \cup \text{int}(A)) \subseteq \text{int}(\text{Cl}(\text{int}(A))). \) Then \( A \) is an \( \alpha - I_p \)-open set.

Remark 3.4
Converse of the above proposition 3.3 need not be true as seen from the following example.

Example 3.6
Let \( X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, X\} \) and \( I = \{\emptyset, \{b\}, \{c\}, \{bc\}\}. \) Set \( A = \{a, c\}, \) is \( \alpha - I_a \)-open, but \( A \notin r. \)

Proposition 3.7
Every \( \alpha - I_p \)-open set is both \( \text{Semi} - I_p \)-open set and \( \text{Pre} - I_p \)-open set.

Proof
The proof is obvious.

Remark 3.8
Converse of the above proposition 3.7 need not be true as seen from the following example.
Example 3.9
Let \( X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}, X\} \) and \( I = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}. \) then the set \( A = \{a\} \) is a pre-\( I_p \)-open, but not \( a-I_p \)-open and \( A \) is Semi-open, but not \( a-I_p \)-open.

Proposition 3.10
For a subset of an ideal topological space the following hold:
Every \( a-I_p \)-open set is \( a-I \)-open.
Every Semi-\( I_p \)-open set is Semi-\( I \)-open.
Every Pre-\( I_p \)-open set is Pre-\( I \)-open.

Proof:
The proof is obvious.

Remark 3.11
Converse of the proposition 3.10 need not be true. DFFD

Proposition 3.12
Let \((X, \tau, I)\) be an ideal topological space and \( A \) an open subset of \( X. \) Then the following hold, if \( I = \{\emptyset\}, \) then
1. \( A \) is \( \epsilon - \gamma - \) open set if and only if \( A \) is a \( a - open. \)

Proof
If \( I = \{\emptyset\}, \) \( A = p Cl(A) \) for any subset \( A \) of \( X \) and hence \( Cl^{\tau}(A) = A \cup A = p Cl(A). \) By proposition 3.2. Every \( a-I_p \)-open set is an \( a- \) open set. Conversely if \( A \) is a \( a- \) open set. Then \( A \subseteq \text{int} (Cl(\text{int}(A))) = \text{acl}\text{(int}(A)) = (Cl^{\tau}(\text{int}(A))). \) Hence \( A = \text{int}(A) \subseteq \text{int} (Cl^{\tau}(\text{int}(A))). \) Therefore, \( A \) is \( a-I_p \)-open. Thus, \( A \) is \( a-I_p \)-open set if and only if \( A \) is \( a-open. \)

2. \( A \) is Semi-\( I_p \)-open set if and only if \( A \) is a Semi-open.

Proof
If \( I = \{\emptyset\}, \) \( A = p Cl(A) \) for any subset \( A \) of \( X \) and hence \( Cl^{\tau}(A) = A \cup A = p Cl(A). \) By proposition 3.2. Every Semi-\( I_p \)-open set is an Semi-\( \) open set. Conversely is \( fA \) is Semi-\( \) open set. Then \( A \subseteq Cl(\text{int}(A)). \) Hence \( A = \text{int}(A) \subseteq \text{int} (Cl(\text{int}(A))) = \text{acl}(\text{int}(A)) = (Cl^\tau(\text{int}(A))). \) Therefore, \( A \) is \( a-I_p \)-open. Thus, \( A \) is Semi-\( I_p \)-open set if and only if Semi-\( open. \)

3. \( A \) is Pre-\( I_p \)-open set if and only if \( A \) is a Pre-open.

Proof
If \( I = \{\emptyset\}, \) \( A = p Cl(A) \) for any subset \( A \) of \( X \) and hence \( Cl^{\tau}(A) = A \cup A = p Cl(A). \) By proposition 3.2. Every Pre-\( I_p \)-open set is an Pre-open set. Conversely if \( A \) is Pre-open set. Then \( A \subseteq \text{int}(Cl(A)) = aCl(A) = Cl^{\tau}(A). \) Hence \( A = \text{int}(A) \subseteq \text{int}(Cl^\tau(A)). \) Therefore, \( A \) is Pre-\( I_p \)-open. Thus, \( A \) is Pre-\( I_p \)-open set if and only if Pre-open.

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