Determinations of $|V_{ub}|$ and $|V_{cb}|$ from Measurements of $B \rightarrow X_{u,c}\ell\nu$
Differential Decay Rates

Changhao Jin
School of Physics, University of Melbourne
Parkville, Victoria 3052, Australia

Abstract

Methods are described in the framework of light-cone expansion which allow one to determine the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{ub}|$ and $|V_{cb}|$ from measurements of the differential decay rates as a function of the scaling variables in the inclusive semileptonic decays of $B$ mesons. By these model-independent methods the dominant hadronic uncertainties can be avoided and the $B \rightarrow X_u\ell\nu$ decay can be very efficiently differentiated from the $B \rightarrow X_c\ell\nu$ decay, which may lead to precise determinations of $|V_{ub}|$ and $|V_{cb}|$. 
The origins of quark masses, quark flavor mixing and CP violation are among the fundamental issues of particle physics. In the standard model, quark flavor mixing is described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the phase of it is also responsible for the CP violation. The CKM matrix appears to be deeply connected to the origin and the values of quark masses, since it results from the diagonalization of the quark mass matrices. The CKM matrix is unpredictable except for unitarity in the standard model, and precise determinations of the magnitudes and the phases of its elements and a consistency checking in the results are of particular importance.

The magnitudes of two CKM matrix elements \( |V_{ub}| \) and \( |V_{cb}| \) can be determined from inclusive and exclusive semileptonic \( B \) meson decays. Main theoretical uncertainties in these determinations arise from hadronic bound state effects on the underlying weak decays. Precise determinations of \( |V_{ub}| \) and \( |V_{cb}| \) require methods which reduce or avoid hadronic uncertainties. Especially, improved precision in \( |V_{ub}| \) is an even more pressing need.

The light-cone expansion provides a solution to the problem of organizing nonperturbative QCD effects on inclusive semileptonic decays of the \( B \) meson in such a way as to exploit heaviness of the \( B \) meson and allows the selection of observables less affected by hadronization and thus better suited for precise determinations of \( |V_{ub}| \) and \( |V_{cb}| \). In this framework the leading nonperturbative effect is described by the distribution function of the \( b \) quark inside the \( B \) meson. Several important properties of the distribution function are known in QCD. In particular, the distribution function is normalized exactly to unity as a consequence of \( b \) quantum number conservation.

In this paper, we propose to determine \( |V_{ub}| \) and \( |V_{cb}| \) by measuring the differential decay rates as a function of \( \xi_q = (\nu + \sqrt{\nu^2 - q^2 + m_q^2})/M \) in the inclusive semileptonic \( B \) meson decays \( B \to X_q \ell \nu \), where the invariant variable \( \nu \) is defined by \( \nu = q \cdot P/M \), \( q \) stands for the momentum transfer to the lepton pair, \( P \) and \( M \) represent, respectively, the four-momentum and the mass of the \( B \) meson, and \( m_q \) is the final quark mass with \( q = u, c \). We will show that the differential decay rates \( d\Gamma/d\xi_q \) are explicitly proportional to the distribution function. Thus the dominant hadronic uncertainties can be avoided with these observables either by exploiting the known normalization of the distribution function or through the cancellation of the distribution function in the ratio of the differential decay rates. These methods are complementary to other methods. The main advantage of our approach is that the dominant hadronic uncertainties can be avoided, providing model-independent and precise determinations of \( |V_{ub}| \) and \( |V_{cb}| \), and the \( B \to X_u \ell \nu \) decay can be more efficiently differentiated from the \( B \to X_c \ell \nu \) decay for determining \( |V_{ub}| \) than the proposed method to measure \( |V_{ub}| \) by the hadronic invariant mass spectrum.

The decay rate for the inclusive semileptonic \( B \) meson decay is given by

\[
d\Gamma = \frac{G_F^2 |V_{qb}|^2}{(2\pi)^5 E} L_{\mu\nu} W_{\mu\nu} \frac{d^3k_\ell \, d^3k_\nu}{2E_\ell \, 2E_\nu},
\]

where \( E \) is the energy of the decaying \( B \) meson, \( k_\ell(\nu) \) and \( E_\ell(\nu) \) denote the four-momentum and the energy of the charged lepton (antineutrino), respectively. \( V_{qb} \) is the element of the CKM matrix, which is \( V_{cb} \) for the \( b \to c \) transition induced decay and \( V_{ub} \) for the \( b \to u \) transition induced decay. Our discussion is made in arbitrary frame of reference throughout this work. \( L_{\mu\nu} \) is the leptonic tensor:
\[ L_{\mu\nu} = 2(k^\mu_k^{\nu} + k^\nu_k^{\mu} - g^{\mu\nu} k_\ell \cdot k_\nu + i\varepsilon_{\mu\nu\alpha\beta} k_\ell^{\alpha} k_\nu^{\beta}). \]  

(2)

\( W_{\mu\nu} \) is the hadronic tensor:

\[
W_{\mu\nu} = -\frac{1}{2\pi} \int d^4 y e^{i q \cdot y} \langle B | [j_\mu(y), j_\nu^\dagger(0)] | B \rangle,
\]

(3)

where \( j_\mu(y) = \bar{q}(y)\gamma_\mu(1 - \gamma_5)b(y) \) is the weak current. The \( B \) meson state \( |B\rangle \) satisfies the standard covariant normalization \( \langle B | B \rangle = 2E(2\pi)^3\delta^3(0) \).

The nonperturbative QCD effects on the processes reside in the hadronic tensor. In general, the hadronic tensor can be constructed of scalar structure functions by Lorentz decomposition, and three structure functions \( W_a(\nu, q^2), a = 1, 2, 3 \) contribute to the inclusive semileptonic \( B \) decays under consideration with negligible lepton masses:

\[
W_{\mu\nu} = -g_{\mu\nu}W_1 + \frac{P_\mu P_\nu}{M^2}W_2 - i\varepsilon_{\mu\nu\alpha\beta} \frac{P^\alpha q^\beta}{M^2} W_3.
\]

(4)

The inclusive semileptonic decay of the \( B \) meson involves large momentum transfer over most of the phase space because of heaviness of the decaying \( B \) meson. Therefore, the integral of Eq. (3) is dominated by the light-cone distance in the space-time structure. This leads to a factorization of the matrix element in Eq. (3) into two parts: one characterizing the light-cone space-time singularity and another being the reduced matrix element of the bilocal \( b \) quark operator [4–7]:

\[
\langle B | [j_\mu(y), j_\nu^\dagger(0)] | B \rangle = 2(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\nu\alpha\beta}) [\partial^\alpha \Delta_q(y)] \langle B | \bar{b}(0)\gamma^\beta(1 - \gamma_5)b(y) | B \rangle,
\]

(5)

where \( S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta} \) and \( \Delta_q(y) \) is the Pauli–Jordan function for a free \( q \)-quark of mass \( m_q \). The light-cone expansion of the reduced matrix element provides a systematic way to organize the nonperturbative effects. The leading term of this expansion is given by

\[
\langle B | \bar{b}(0)\gamma^\beta(1 - \gamma_5)b(y) | B \rangle |_{y^2 = 0} = 2P^\beta \int_0^1 d\xi e^{-i\xi y^\mu} f(\xi).
\]

(6)

The function \( f(\xi) \) is known as the distribution function of the \( b \) quark inside the \( B \) meson. A similar distribution function has been introduced by the resummation of the heavy quark expansion [13].

The light-cone dominance implies that the structure functions are given by [4–6]

\[
W_1 = 2[f(\xi_+) + f(\xi_-)],
\]

(7)

\[
W_2 = \frac{8}{\xi_+ - \xi_-}[\xi_+ f(\xi_+) - \xi_- f(\xi_-)],
\]

(8)

\[
W_3 = -\frac{4}{\xi_+ - \xi_-}[f(\xi_+) - f(\xi_-)],
\]

(9)

where

\[
\xi_\pm = \frac{\nu \pm \sqrt{\nu^2 - q^2 + m_q^2}}{M}.
\]

(10)
Equations (7)–(9) include the leading twist contribution; higher-twist contributions are expected to be suppressed by powers of $q^2$.

The leading nonperturbative QCD effects on the inclusive semileptonic decays is now encoded in the distribution function $f(\xi)$. The detailed form of the distribution function is not known. However, taking $y = 0$ in Eq. (6), $b$ quantum number conservation implies the normalization condition [4,7]:

$$\int_0^1 d\xi f(\xi) = \frac{1}{2M^2} \frac{1}{4\pi^3} \frac{M^5}{E} \left\{ 1 \right\}$$

(11)

The differential decay rate as a function of the scaling variable $\xi_+$ for $B \to X_q\ell\nu$ is calculated in terms of the distribution function as

$$\frac{d\Gamma}{d\xi_+} = \frac{G_F^2 |V_{qb}|^2}{4\pi^3} \frac{M}{E} \left\{ 1 \right\}$$

(12)

with

$$\Phi(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x,$$

(13)

where $r_q = m_q/M$, $\nu$ and $\xi_-$ are related to $\xi_+$ and $q^2$ by

$$\nu = \frac{\xi_+^2 M^2 + q^2 - m_q^2}{2\xi_+ M},$$

(14)

$$\xi_- = \frac{q^2 - m_b^2}{\xi_+ M^2}.$$  

(15)

The kinematic range of $\xi_+$ is $r_q \leq \xi_+ \leq 1$. Note that $\xi_+$ is different kinematic variable for $B \to X_u\ell\nu$ and $B \to X_c\ell\nu$, defined in Eq. (11).

The free quark limit corresponds to $f(\xi) = \delta(\xi - m_b/M)$. Since the heavy $b$ quark inside the $B$ meson is almost on-shell, we expect that the distribution function in reality is very close to the delta function, which is supported by the analysis based on the operator product expansion and the heavy quark effective theory (HQET) [15,16], indicating that the distribution function peaks sharply around $\xi = m_b/M \approx 0.93$ [4,5].

The $f(\xi_-)$ term is a consequence of field theory, corresponding to the creation of quark-antiquark pairs through the $Z$-graph. The contribution of the $f(\xi_-)$ term, present as an integral in Eq. (12), is expected to play less of a role, since (i) in the free quark limit the integral vanishes and (ii) the dominant contribution to the $f(\xi_-)$ integral at a given $\xi_+$ resulting from the large $\xi_-$ region, corresponding to the neighbourhood of the upper integration limit for $q^2$, is suppressed by $\nu^2 - q^2$. This expectation is confirmed by the numerical study discussed below. The spectra $d\Gamma/d\xi_+$ are dominated by the $f(\xi_+)$ term for both $b \to c$ and $b \to u$ transitions and both spectra have a sharp peak at the same peak location as $f(\xi_+)$. In the following we will to good approximation ignore the $f(\xi_-)$ term and so the differential decay rates $d\Gamma/d\xi_+$ given by Eq. (12) are proportional to $f(\xi_+)$.
In the free quark limit, the tree-level and virtual gluon processes $b \to u(c) \ell \nu$ generate a spectrum nonvanishing at only one point, i.e., $\xi_+ = m_b/M$, solely on kinematic grounds. It is gluon bremsstrahlung and hadronic bound state effects that expand the spectrum over the whole range of $\xi_+$ from $m_\ell/M$ to 1. This unique feature implies that measurements of the spectra $d\Gamma/d\xi_+$ would offer the intrinsically most sensitive probe of long-distance strong interactions. Indeed, we have shown on the basis of light-cone expansion that the spectra $d\Gamma/d\xi_+$ are explicitly proportional to the nonperturbative distribution function. Thus their measurements would lead to a direct extraction of the distribution function (see further below). On the other hand, the spectra $d\Gamma/d\xi_+$ also provide the most straightforward and best way to eliminate the dependence on the distribution function since they are proportional to it, so that the dominant hadronic uncertainties can be avoided. Moreover, the $B \to X_\ell \ell \nu$ spectra $d\Gamma/d\xi_+$ are sharply peaked at $\xi_+ = m_b/M$, since the spectra stemming from the tree-level and virtual gluon processes would only concentrate at $\xi_+ = m_b/M$ and gluon bremsstrahlung and hadronic bound state effects smear the spectra about this point, but most of the decay rates remain around $\xi_+ = m_b/M$. This implies that the kinematic cut on the $b \to u$ scaling variable $\xi_+$ will make a very efficient discrimination between $B \to X_u \ell \nu$ and $B \to X_c \ell \nu$ events, even better than the kinematic cut on the hadronic invariant mass of the final state. Consequently, measurements of the decay distribution as a function of the $b \to u$ scaling variable $\xi_+$ would yield, particularly, a high precision $|V_{ub}|$ determination. We will discuss these points in detail below.

For the charmless decay $B \to X_u \ell \nu$, we obtain from Eq. (12)

$$|V_{ub}|^2 f(\xi_u) = \frac{192\pi^3 E}{G_F^2 M^6} \frac{1}{\xi_u^5} \frac{d\Gamma(B \to X_u \ell \nu)}{d\xi_u},$$

(16)

with $\xi_u = (\nu_u + \sqrt{\nu_u^2 - q_{u\ell}^2})/M$. We have ignored the up quark mass. To avoid confusion, quantities for the $b \to u$ ($b \to c$) decay are indicated by a subscript $u$ ($c$); in particular, $\xi_+$ is explicitly denoted as $\xi_{u(c)}$ for the $b \to u$ ($b \to c$) decay.

By integrating Eq. (16) over $\xi_u$ and using the normalization condition (11), one gets rid of the distribution function, obtaining

$$|V_{ub}|^2 = \frac{192\pi^3 E}{G_F^2 M^6} \int_0^1 d\xi_u \frac{1}{\xi_u^5} \frac{d\Gamma(B \to X_u \ell \nu)}{d\xi_u}.$$  

(17)

This implies that the known normalization of the distribution function allows a model independent determination of $|V_{ub}|$ from a measurement of the weighted integral of the decay spectrum $d\Gamma(B \to X_u \ell \nu)/d\xi_u$.

 Likewise, for the charmed decay $B \to X_c \ell \nu$, we obtain from Eq. (12)

$$|V_{cb}|^2 f(\xi_c) = \frac{192\pi^3 E}{G_F^2 M^6} \frac{1}{\xi_c^5} \frac{d\Gamma(B \to X_c \ell \nu)}{d\xi_c},$$

(18)

with $\xi_c = (\nu_c + \sqrt{\nu_c^2 - q_{c\ell}^2 + m^2_{c\ell}})/M$. Integrating Eq. (18) over $\xi_c$ and using the normalization condition (11) yields

$$|V_{cb}|^2 = \frac{192\pi^3 E}{G_F^2 M^6} \int_{r_c}^1 d\xi_c \frac{1}{\xi_c^5} \frac{d\Gamma(B \to X_c \ell \nu)}{d\xi_c},$$

(19)
which is independent of the distribution function. Thus, a measurement of the weighted integral of the charmed differential decay rate as a function of $\xi_c$ would provide a model independent determination of $|V_{ub}|/V_{cb}|$.

The distribution function cancels in the ratio of the differential decay rates at $\xi_u = \xi_c$ and we obtain from Eqs. (16) and (18)

$$\frac{|V_{ub}|}{V_{cb}} = \Phi \left( \frac{r_c}{\xi_c} \right) \left[ \frac{d\Gamma(B \rightarrow X_u\ell\nu)}{d\xi_u} \right]_{\xi_u=\xi_c} \left[ \frac{d\Gamma(B \rightarrow X_c\ell\nu)}{d\xi_c} \right].$$

This provides a model independent method for determining the ratio $|V_{ub}|/V_{cb}|$.

Equation (18) shows that a measurement of the decay spectrum $d\Gamma(B \rightarrow X_c\ell\nu)/d\xi_c$ can also be used to extract the distribution function directly. We note that the distribution function can also be extracted from a measurement of the charmless decay distribution with respect to $\xi_u$, as shown by Eq. (16), though the decay rate is much smaller than the $b \rightarrow c$ decay rate.

The quantity $\xi_q(q = u, c)$ is a combination of two kinematic variables $\nu$ and $q^2$, which can be measured through neutrino reconstruction. Except the lower end point $\xi_q = r_q$ corresponding to $\nu = 0$ and $q^2 = 0$, different values of $\nu$ and $q^2$ correspond to the same value of $\xi_q$, as shown in Fig. 1, thereby giving the same physical value for the differential decay rate $d\Gamma(B \rightarrow X_q\ell\nu)/d\xi_q$. This implies that the spectrum can be well described by the light-cone dynamics as long as it can originate from processes with sufficiently large momentum transfer. From Fig. 1, we see that this is the case for the dominant spectrum over not too small values of $\xi_q$. Thus departures from the light cone are expected to bring little theoretical uncertainty in the methods previously described. Experimentally, one can

FIG. 1. Contours for $\xi_{u,c}$ in the ($\nu, q^2$) phase space. The dashed (dotted) lines from right to left refer to $\xi_{u(c)} = 1, 0.9, 0.8, 0.65, 0.5$, taking $m_c = 1.6$ GeV.
FIG. 2. The spectra $d\Gamma(B \to X_u \ell\nu)/d\xi_u$ (left) and $d\Gamma(B \to X_c \ell\nu)/d\xi_c$ (right) calculated using the parametrization of [5] for the distribution function for $\alpha = \beta = 1$, $a = 0.953$, $b = 0.00560$, and $m_c = 1.6$ GeV. The radiative QCD corrections are not included. The absolute scale is arbitrary, but the relative scale between the $b \to u$ and $b \to c$ spectra is fixed to $|V_{ub}/V_{cb}| = 0.08$.

To verify that the contributions of the $f(\xi)$ term in Eq. (12) are negligible, we have made a numerical study by using the HQET-constrained parametrization of [5] for the distribution function. As expected, the contributions of the $f(\xi)$ term to the integrated decay rates in Eqs. (17) and (19) are both at the level below 1%, yielding negligible theoretical uncertainties in $|V_{ub}|$ and $|V_{cb}|$ obtained by these two methods.

Figure 2 shows the spectra $d\Gamma(B \to X_u \ell\nu)/d\xi_u$ and $d\Gamma(B \to X_c \ell\nu)/d\xi_c$. We see that both spectra peak sharply at the same value for $\xi_u$ and $\xi_c$.

This feature does not depend on the detailed form of the distribution function. However, the shapes of both spectra are sensitive to the form of the distribution function. Numerical studies show that including the $f(\xi)$ term makes almost no difference in the spectra, and the tiny contribution from the $f(\xi)$ term is meaningful (although still invisible in Fig. 2) only for the spectra very close to their two end points, where the spectra become vanishingly small. Therefore, measurements...
of both $\xi_u$ and $\xi_c$ distributions in the neighborhood of maximum, where the $f(\xi_-)$ term is negligible, would be appropriate for a reliable determination of $|V_{ub}/V_{cb}|$ by the method of Eq. (20). The smallness of the $f(\xi_-)$ term has already been found in the parton model [8].

The determination of $|V_{ub}|$ by the method proposed in this paper may still suffer from large theoretical systematic errors. The light-cone expansion parameter is $\Lambda_{QCD}^2/q^2$. For $B \rightarrow X_c\ell\nu$, the maximum value of $q^2$ is $(M - M_D)^2$ with $M_D$ being the $D$ meson mass. It means that $q^2$ is not large enough to neglect the higher order corrections. The $\Lambda_{QCD}^2/q^2$ correction may be necessary to achieve 5% accuracy for $|V_{ub}|$. Actually the semileptonic $b \rightarrow c$ decay rate is dominated by a few exclusive decay modes ($D, D^*$ and $D^{(*)}\pi$), which suggests that the light-cone picture cannot be valid point by point. The theoretical prediction obtained in the light-cone expansion is not quite accurate and refers only to the smeared spectrum, not point by point. A related problem is the uncertainty in the charm quark mass. The charm quark mass is an important source of uncertainty when one measures $\xi_c$, although, in principle, this uncertainty can be eliminated experimentally by using the multivariate freedom in $\xi_c$, as discussed previously. Since the spectrum $d\Gamma(B \rightarrow X_c\ell\nu)/d\xi_c$ has a sharp dependence on $\xi_c$, the measurement precision required for the elimination of the charm quark mass uncertainty will be challenging experimentally.

The method proposed in this paper by measurements of the differential decay rate as a function of $\xi_u$ is, on the other hand, theoretically very reliable for a high precision $|V_{ub}|$ determination. The light-cone expansion works much better for the $B \rightarrow X_u\ell\nu$ decay because a much larger momentum transfer with the maximum value of $q^2$ being $M^2$ can occur in the $B \rightarrow X_u\ell\nu$ decay than in the $B \rightarrow X_c\ell\nu$ decay. Many final hadronic states contribute to the $\xi_u$ spectrum above the charm threshold, without any preferential weighting towards the low-lying resonance states. Thus, we feel more confident in the method of Eq. (17) for determining $|V_{ub}|$. Applying the kinematic cut $\nu > M - M_D$, which leads to $\xi_u > 1 - M_D/M = 0.65$ (see Fig. 1), one can discriminate between the inclusive $b \rightarrow u$ signal and $b \rightarrow c$ background. We find that about 99% of the $b \rightarrow u$ events pass $\xi_u > 1 - M_D/M$, since the $\xi_u$ distribution peaks sharply at $\xi_u$ close to 1, as demonstrated in Fig. 2. This discrimination between $b \rightarrow u$ and $b \rightarrow c$ events is even better than the method of the hadronic invariant mass spectrum where about 90% of the $b \rightarrow u$ events survive the cut on the hadronic invariant mass $M_X < M_D$ [8–13]. Thus, $b \rightarrow c$ background can be very efficiently suppressed and improved statistics can be achieved in the measurement of the spectrum $d\Gamma(B \rightarrow X_u\ell\nu)/d\xi_u$, permitting a precise determination of $|V_{ub}|$. We also anticipate that a reliable value for the inclusive charmless semileptonic branching fraction of the $B$ meson may be obtained through this measurement.

The residual hadronic uncertainty in $|V_{ub}|$ due to higher-order, power-suppressed corrections is expected to be at the level of 1%. The precision of this determination of $|V_{ub}|$ will mainly depend on its experimental feasibility. The accuracy in $\nu$ and $q^2$, which determine $\xi_u$, in the experiments seems essential for this method to work. The experimental technique of neutrino reconstruction could well make this way of extracting $|V_{ub}|$ experimentally feasible. If the neutrino can be reconstructed kinematically by inferring its four-momentum from the missing energy and missing momentum in each event, then it is possible to measure $\nu$ and $q^2$ and thus the scaling variable $\xi_u$. To reach 10% precision in $|V_{ub}|$ requires, for instance, a measurement of $\nu$ and $\sqrt{q^2}$ with an error around 4%.

In conclusion, we have proposed new methods for determining $|V_{ub}|$ and $|V_{cb}|$ by mea-
suring the differential decay rates as a function of $\xi_{u,c}$ in the inclusive semileptonic decays $B \to X_{u,c} \ell \nu$. These methods are based on the light-cone expansion, which are, in principle, model independent as it is in deep inelastic scattering. We have shown that the differential decay rates as a function of the scaling variables $\xi_{u,c}$ for the inclusive semileptonic $B$ meson decays $B \to X_{u,c} \ell \nu$ are proportional to the distribution function, which describes the leading nonperturbative contribution, because of the light-cone dominance. This unique feature makes these observables ideally suitable for eliminating the dependence on the distribution function and thus avoiding the dominant hadronic uncertainties. The integrated decay rates are, to a large extent, free of hadronic uncertainties by using the known normalization of the distribution function from current conservation, permitting precise determinations of $|V_{ub}|$ and $|V_{cb}|$, respectively. Moreover, the distribution function cancels in the ratio of the differential decay rates at specific kinematic points, permitting a precise determination of $|V_{ub}/V_{cb}|$. The whole methods are then free from model parameters. Since theoretical uncertainties associated with nonperturbative strong interactions are, to a large extent, avoided and perturbative corrections are calculable, these methods could yield very accurate values of $|V_{ub}|$ and $|V_{cb}|$. There will be smaller residual theoretical uncertainties from higher-twist terms and perturbative corrections.

We wish to emphasize that the method by measurements of the spectrum $d\Gamma(B \to X_u \ell \nu)/d\xi_u$ is especially promising for a high precision $|V_{ub}|$ determination. Theoretically, this method is very reliable since the theoretical description based on the light-cone expansion works much better for $b \to u$ decays than $b \to c$ decays. With the dominant hadronic uncertainty being avoided, there is a residual hadronic uncertainty in $|V_{ub}|$ only at the level of 1%. Experimentally, it gives access to almost all $b \to u$ events, even more than the selection against charm background based on the hadronic invariant mass spectrum. Therefore, at least potentially, this theoretically sound, clean and experimentally efficient method allows for a model-independent determination of $|V_{ub}|$ with a minimum overall (experimental and theoretical) error.

The decay spectra $d\Gamma(B \to X_u \ell \nu)/d\xi_u$ and $d\Gamma(B \to X_c \ell \nu)/d\xi_c$ offer, on the other hand, the intrinsically most sensitive probe of long-distance strong interactions. Measurements of the differential decay rates as a function of $\xi_{u,c}$ can be also used to extract the distribution function directly. This procedure is crucial for improving the theoretical accuracies on the determinations of $|V_{ub}|$ and $|V_{cb}|$ from the charged lepton energy spectra and the hadronic invariant mass spectrum in inclusive semileptonic $B$ meson decays. A reliable value for the inclusive charmless semileptonic branching fraction of the $B$ meson could also be obtained by measuring the spectrum $d\Gamma(B \to X_u \ell \nu)/d\xi_u$.

ACKNOWLEDGMENTS

I would like to thank Emmanuel Paschos for discussions. This work was supported in part by the Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Bonn, FRG under grant 057DO93P(7), and by the Australian Research Council.
REFERENCES

[1] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[2] See, for example, H. Fritzsch, Nucl. Phys. B 155 (1979) 189; B. Stech, Phys. Lett. 130B (1983) 189; M. Shin, Phys. Lett. B 145 (1984) 285; M. Gronau, R. Johnson, and J. Schechter, Phys. Rev. Lett. 54 (1985) 2176; C.H. Jin, in Proceedings of the 25th International Conference on High Energy Physics, Singapore, 1990, edited by K.K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1991), p. 795.

[3] For recent reviews, see P.S. Drell, to appear in Proceedings of the XVIII International Symposium on Lepton-Photon Interactions, Hamburg, Germany, 1997, hep-ex/9711020; R. Poling, to appear in Proceedings of the 2nd International Conference on B Physics and CP Violation, Honolulu, Hawaii, USA, 1997; L.K. Gibbons, in Proceedings of the 28th International Conference on High Energy Physics, Warsaw, Poland, 1996, edited by Z. Ajduk and A.K. Wroblewski (World Scientific, Singapore, 1997), p. 183; J.R. Patterson, in Proceedings of the 28th International Conference on High Energy Physics, Warsaw, Poland, 1996, edited by Z. Ajduk and A.K. Wroblewski (World Scientific, Singapore, 1997), p. 871; J.D. Richman and P.R. Burchat, Rev. Mod. Phys. 67 (1995) 893; T.E. Browder and K. Honscheid, Prog. Part. Nucl. Phys. 35 (1995) 81.

[4] C.H. Jin and E.A. Paschos, in Proceedings of the International Symposium on Heavy Flavor and Electroweak Theory, Beijing, China, 1995, edited by C.H. Chang and C.S. Huang (World Scientific, Singapore, 1996), p. 132; hep-ph/9504375.

[5] C.H. Jin, Phys. Rev. D 56 (1997) 2928.

[6] C.H. Jin and E.A. Paschos, Eur. Phys. J. C 1 (1998) 523.

[7] C.H. Jin, Phys. Rev. D 56 (1997) 7267.

[8] A. Bareiss and E.A. Paschos, Nucl. Phys. B 327 (1989) 353; C.H. Jin, W.F. Palmer, and E.A. Paschos, Phys. Lett. B 329 (1994) 364.

[9] V. Barger, C.S. Kim, and R.J.N. Phillips, Phys. Lett. B 251 (1990) 629.

[10] A.F. Falk, Z. Ligeti, and M.B. Wise, Phys. Lett. B 406 (1997) 225.

[11] C.S. Kim, Phys. Lett. B 414 (1997) 347.

[12] R.D. Dikeman and N. Uraltsev, Nucl. Phys. B 509 (1998) 378; I. Bigi, R.D. Dikeman, and N. Uraltsev, Eur. Phys. J. C 4 (1998) 453.

[13] C.H. Jin, Phys. Rev. D 57 (1998) 6851.

[14] M. Neubert, Phys. Rev. D 49 (1994) 3392; 49 (1994) 4623; T. Mannel and M. Neubert, Phys. Rev. D 50 (1994) 2037; I.I. Bigi, M.A. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Int. J. Mod. Phys. A 9 (1994) 2467.

[15] N. Isgur and M.B. Wise, Phys. Lett. B 232 (1989) 113; B 287 (1990) 1527; E. Eichten and B. Hill, Phys. Lett. B 234 (1990) 511; B 243 (1990) 427; H. Georgi, Phys. Lett. B 240 (1990) 447; A.F. Falk, B. Grinstein, and M.E. Luke, Nucl. Phys. B 357 (1991) 185; T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. B 368 (1992) 204.

[16] The heavy quark effective theory in inclusive semileptonic decays of heavy hadrons is discussed, e.g., in J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B 247 (1990) 399; I.I. Bigi, N.G. Uraltsev, and A.I. Vainshtein, Phys. Lett. B 293 (1992) 430; 297 (1993) 477E; I.I. Bigi, M.A. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Phys. Rev. Lett. 71 (1993) 496; A.V. Manohar and M.B. Wise, Phys. Rev. D 49 (1994) 1310; B. Blok, L.
Koyrakh, M.A. Shifman, and A.I. Vainshtein, Phys. Rev. D 49 (1994) 3356; 50 (1994) 3572(E); T. Mannel, Nucl. Phys. B 413 (1994) 396.