The Interference between Virtual Photon and 1−− Charmonium in $e^+e^-$ Experiment

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$e^+e^-$ experiments producing charmonium are reviewed. It is found that the contribution of the continuum amplitude via virtual photon was neglected in almost all the experiments and the channels analyzed. It is shown that the contribution of the continuum part may affect the final results significantly in $\psi(2S)$ and $\psi(3770)$ decays, while the interference between continuum and resonance amplitudes may even affect the $J/\psi$ decays as well as the $\psi(2S)$ and $\psi(3770)$. This should be considered in analyzing the “$\rho\pi$ puzzle” between $J/\psi$ and $\psi(2S)$ decays, and the difference between inclusive hadron and $D\bar{D}$ cross sections in $\psi(3770)$ decays.

1. Introduction

There are three well-known problems in the study of the charmonium decays, namely the relative phase between strong and electromagnetic amplitudes of the 1−− charmonium decays, “$\rho\pi$ puzzle” between $J/\psi$ and $\psi(2S)$ decays, and non-$D\bar{D}$ decays of $\psi(3770)$.

The attempt to understand the strong decays of $J/\psi$ via three-gluon and the electromagnetic decays via one-photon annihilation reveals the relative phase between these two amplitudes is close to 90° [1,2,3,4], while for the radially excited $\psi(2S)$, the phase is 0° or 180° [14]. This indicates there would be no interference between these two amplitudes in $J/\psi$ decays, but strong interference in $\psi(2S)$ decays.

It was found that in $\psi(2S)$ hadronic decays, some decay modes are abnormally suppressed compared with the corresponding $J/\psi$ decays based on perturbative QCD (pQCD) prediction. This suppression was first observed by the Mark-II in vector pseudoscalar (VP) decay modes like $\rho\pi$ and $K^*\bar{K}$ [5], and confirmed by BES [6]. Moreover, BES also observed the suppression in vector tensor (VT) decays of $\psi(2S)$ [7]. This has led to active theoretical efforts in solving the problem [14][8][9]. Unfortunately, most of the models were ruled out by the experiments, while some others need further experimental test.

There is a renewed interest in $\psi(3770)$ studies because of the upcoming high precision measurements by CLEO-c [10] and BES-III [11]. One of the puzzling problems in $\psi(3770)$ decays is that the $D\bar{D}$ cross section may be significantly lower than the inclusive hadronic cross section [12]. This is in contradiction with the commonly accepted picture that $\psi(3770)$ decays predominantly to the OZI allowed $D\bar{D}$ states.

These three topics play important roles in understanding the charmonium decay dynamics. In this paper we examine what the experiments observe and what theories analyze on charmonium produced in $e^+e^-$ experiments. We present a self-consistent analysis by considering the unavoidable background process in $e^+e^-$ experiment, namely, the continuum process. We show that, for exclusive decays of these charmonium states, the contribution of this process could be very important, or even if the direct contribution is relatively small, the interference between this term and other dominant amplitudes may contribute a non-negligible part.

2. Experimentally observed cross section

It is known that $J/\psi$ or $\psi(2S)$ decays into light hadrons via strong and electromagnetic interac-
tions. At the leading order in $\alpha_s(m_c)$ and $1/m_c$, it goes through three-gluon and one-photon annihilation of which the amplitudes are denoted by $a_{3g}$ and $a_\gamma$ respectively \cite{14}. This is also true for $\psi(3770)$ in its OZI suppressed decay into light hadrons. In general, for the resonance $R$ ($R = J/\psi$, $\psi(2S)$ or $\psi(3770)$), the cross section at the Born order is expressed as

$$\sigma_B(s) = \frac{4\pi s \alpha^2}{3} |a_{3g} + a_\gamma|^2,$$

(1)

where $\sqrt{s}$ is the C.M. energy, $\alpha$ is the fine structure constant. If the $J/\psi$, $\psi(2S)$ or $\psi(3770)$ is produced in $e^+e^-$ collision, the process

$$e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}$$

(2)

could produce the same final hadronic states as charmonium decays do \cite{14}. We denote its amplitude by $a_c$, then the cross section becomes

$$\sigma'_B(s) = \frac{4\pi s \alpha^2}{3} |a_{3g} + a_\gamma + a_c|^2.$$ 

(3)

So what truly contribute to the experimentally measured cross section are three classes of diagrams, i.e. the three-gluon decays, the one-photon decays, and the one-photon continuum process, as illustrated in Fig. 1, where the charm loops stand for the charmonium state, and the photons and gluons are highly off-shell and can be treated perturbatively. To analyze the experimental results, we must take into account three amplitudes and two relative phases.

For an exclusive mode, $a_c$ can be expressed by

$$a_c(s) = \frac{\mathcal{F}(s)}{s} e^{i\phi'},$$

(4)

where $\phi'$ is the phase relative to $a_{3g}$; $\mathcal{F}(s)$ depends on the individual mode, and for simplicity, the phase space factor is incorporated into $|\mathcal{F}(s)|^2$. The one-photon annihilation amplitude can be written as

$$a_\gamma(s) = \frac{3\Gamma_{ee} \mathcal{F}(s)/(\alpha\sqrt{s})}{s - m_R^2 + im_R \Gamma_l} e^{i\phi},$$

(5)

where $m_R$ and $\Gamma_l$ are the mass and the total width of $R$, $\Gamma_{ee}$ is the partial width to $e^+e^-$, $\phi$ is the phase relative to $a_{3g}$. The strong decay amplitude $a_{3g}$ is defined by $C \equiv |a_{3g}/a_\gamma|$, which is the relative strength to $a_\gamma$, so

$$a_{3g}(s) = C \frac{3\Gamma_{ee} \mathcal{F}(s)/(\alpha\sqrt{s})}{s - m_R^2 + im_R \Gamma_l}.$$ 

(6)

For resonances, $C$ can be taken as a constant.

In principle, $a_{3g}$, $a_\gamma$ and $a_c$ depend on individual exclusive mode both in absolute values and in relative strengths. In this paper, for illustrative purpose, following assumptions are used for an exclusive hadronic mode: $\mathcal{F}(s)$ is replaced by $\sqrt{R(s)}$, where $R(s)$ is the ratio of the inclusive hadronic cross section to the $\mu^+\mu^-$ cross section measured at nearby energy \cite{15}; in Eq. 6,

$$C = \sqrt{\frac{B(R \rightarrow ggg \rightarrow \text{hadrons})}{B(R \rightarrow \gamma \rightarrow \text{hadrons})}},$$

(7)

Here $B(R \rightarrow \gamma \rightarrow \text{hadrons}) = B_{\mu^+\mu^-} R(s)$, where $B_{\mu^+\mu^-}$ is the $\mu^+\mu^-$ branching ratio; while $B(R \rightarrow ggg \rightarrow \text{hadrons})$ is calculated as following: we first estimate the branching ratio of $B(R \rightarrow \gamma gg) + B(R \rightarrow ggg)$ by subtracting the lepton pairs, $\gamma \rightarrow \text{hadrons}$, and the modes with charmonium production from the total branching ratio (100%). Then using pQCD result \cite{10}.
Table 1
Estimated amplitudes at \( J/\psi, \psi(2S) \) and \( \psi(3770) \) peaks.

| \( \sqrt{s} \) | \( m_{J/\psi} \) | \( m_{\psi(2S)} \) | \( m_{\psi(3770)} \) |
|---|---|---|---|
| \( |a_3 (m_{R}^2)|^2 \) | 70% \( \sigma_{J/\psi} \) | 19% \( \sigma_{\psi(2S)} \) | \( \sim 1\% \sigma_{\psi(3770)} \) |
| \( |a_1 (m_{R}^2)|^2 \) | 13% \( \sigma_{J/\psi} \) | 1.6% \( \sigma_{\psi(2S)} \) | \( 2.5 \times 10^{-5}\sigma_{\psi(3770)} \) |
| \( |a_0 (m_{R}^2)|^2 \) | 20 nb | 14 nb | 14 nb |

\( B(R \to \gamma gg)/B(R \to gg) \approx 6\% \) we obtain \( B(R \to gg \to \text{hadrons}) \). Table 1 lists all the estimations used as inputs in the calculations, where \( \sigma_R^\psi \) is the total resonance cross section of Born order at \( s = m_R^2 \) obtained from

\[
\sigma_0^R(s) = \frac{12\pi e^4 \Gamma_t}{(s - m_R^2)^2 + m_R^4 \Gamma_t^2} . \tag{8}
\]

The cross section by \( e^+e^- \) collision incorporating radiative correction on the Born order is expressed by \( 17 \)

\[
\sigma_{\text{r.c.}}(s) = \int_0^{x_m} dx F(x, s) \sigma_0(s(1 - x)) \left[ 1 - \Pi(s(1 - x)) \right]^2 , \tag{9}
\]

where \( \sigma_0 \) is \( \sigma_B \) or \( \sigma'_B \) by Eq. \( 11 \) or \( 5 \), \( F(x, s) \) has been calculated in Ref. \( 17 \) and \( \Pi(s) \) is the vacuum polarization factor \( 18 \); the upper limit of the integration \( x_m = 1 - s_m/s \) where \( \sqrt{s_m} \) is the experimentally required minimum invariant mass of the final state \( f \) after losing energy to multi-photon emission. In this paper, we assume that \( \sqrt{s_m} \) equals to 90% of the resonance mass, i.e. \( x_m = 0.2 \).

For narrow resonances like \( J/\psi \) and \( \psi(2S) \), one should consider the energy spread function of \( e^+e^- \) colliders:

\[
G(\sqrt{s}, \sqrt{s'}) = \frac{1}{\sqrt{2\pi} \Delta} e^{-\frac{(\sqrt{s} - \sqrt{s'})^2}{2\Delta^2}} , \tag{10}
\]

where \( \Delta \) describes the C.M. energy spread of the accelerator, \( \sqrt{s} \) and \( \sqrt{s'} \) are the nominal and actual C.M. energy respectively. Then the experimentally measured cross section

\[
\sigma_{\text{exp}}(s) = \int_0^\infty d\sqrt{s'} G(\sqrt{s}, \sqrt{s'}) d\sqrt{s'} . \tag{11}
\]

The radiative correction reduces the maximum cross sections of \( J/\psi, \psi(2S) \) and \( \psi(3770) \) by 52%, 49% and 29% respectively. The energy spread further reduces the cross sections of \( J/\psi \) and \( \psi(2S) \) by an order of magnitude. The radiative correction and energy spread also shift the maximum height of the resonance peak to above the resonance mass. Take \( \psi(2S) \) as an example, from Eq. \( 5 \), \( \sigma_{\psi(2S)} = 7887 \) nb at \( \psi(2S) \) mass; substitute \( \sigma_0(s) \) in Eq. \( 3 \) by \( \sigma_0^R(s) \) in Eq. \( 5 \), \( \sigma_{\text{r.c.}} \) reaches the maximum of 4046 nb at \( \sqrt{s} = m_{\psi(2S)} + 9 \) keV; with the energy spread \( \Delta = 1.3 \) MeV at BES/BEPC, combining Eqs. \( 3 \) \( 11 \), \( \sigma_{\text{exp}} \) reaches the maximum of 640 nb at \( \sqrt{s} = m_{\psi(2S)} + 0.14 \) MeV. Similarly, at \( J/\psi \), with BES/BEPC energy spread \( \Delta = 1.0 \) MeV, the maximum of \( \sigma_{\text{exp}} \) is 2988 nb. At DORIS, the maximum of \( \sigma_{\text{exp}} \) at \( J/\psi \) is 2190 nb (\( \Delta = 1.4 \) MeV), and at \( \psi(2S) \), it is 442 nb (\( \Delta = 2.0 \) MeV). In this paper, we calculate \( \sigma_{\text{exp}} \) at the energies which yield the maximum inclusive hadronic cross sections.

To measure an exclusive mode in \( e^+e^- \) experiment, the contribution of the continuum part should be subtracted from the experimentally measured \( \sigma_{\text{exp}} \) to get the physical quantity \( \sigma_{\text{exp}} \), where \( \sigma_{\text{exp}} \) and \( \sigma_{\text{exp}}' \) indicate the experimental cross sections calculated from Eqs. \( 6 \) \( 11 \) with the substitution of \( \sigma_B \) and \( \sigma'_B \) from Eqs. \( 11 \) and \( 8 \) respectively for \( \sigma_0 \) in Eq. \( 9 \). Up to now, most of the measurements did not include this contribution and \( \sigma_{\text{exp}}' = \sigma_{\text{exp}} \) is assumed at least at \( J/\psi \) and \( \psi(2S) \). As a consequence, the theoretical analyses are based on \( \sigma_{\text{exp}} \), while the experiments actually measure \( \sigma_{\text{exp}}' \).

We display the effect from the continuum amplitude and corresponding phase for \( J/\psi, \psi(2S) \) and \( \psi(3770) \) respectively. To do this, we calculate
Figure 2. (a) $k$ as a function of $\phi$ and $\phi'$ for $\psi(2S)$, with input from Table 1 and (b) $k$ as a function of $\phi$ ($\phi = \phi'$) for different ratios of $|a_{3g}|$ to $|a_\gamma|$: line 1 to 5 for $a_{3g} = 0$, $|a_{3g}| = |a_\gamma|$, $|a_{3g}| = 3.4|a_\gamma|$, $|a_{3g}| = 5|a_\gamma|$ and $|a_{3g}| = 10|a_\gamma|$, respectively.

3. Continuum contribution for charmonium decay

We now discuss separately the effect of continuum amplitude for $\psi(3770)$, $\psi(2S)$ and $J/\psi$.

At $\psi(3770)$, the maximum resonance cross section of inclusive hadrons is 8 nb which predominantly decays into $D\bar{D}$, while the continuum cross section is 14 nb which mainly goes to light hadrons. Assuming 1% of $\psi(3770)$ decays to non-$D\bar{D}$ interferes with the continuum amplitude, it could bring an effect of maximum 1.9 nb in the observed cross section. Such large constructive interferences could be responsible for the larger cross section of inclusive hadrons by direct measurement of $e^+e^- \rightarrow \psi(3770) \rightarrow$ hadrons than the $D\bar{D}$ cross section [12]. As to the exclusive decays, it could make some of the decay modes with small branching ratios more observable at the resonance. For example, if $B(\psi(3770) \rightarrow \rho\pi) \approx 4 \times 10^{-4}$ (or equivalently, $\sigma(\psi(3770) \rightarrow \rho\pi) \approx 0.003$ nb) as suggested in Ref. [14], and $\sigma(e^+e^- \rightarrow \rho\pi) \approx 0.014$ nb at Born order by the model of Ref. [14], then the maximum interference could be 0.011 nb, much larger than the pure contribution from $\psi(3770)$ decays.

For $\psi(2S)$, as can be seen in Fig. 2 the ratio $\sigma'_{exp}/\sigma_{exp}$ could deviate from 1 substantially. In general, $a_{3g}$, $a_\gamma$ and $a_c$ are different for different exclusive mode, so $k$ could be different. This must be taken into account in the fitting of $a_\gamma$, $a_{3g}$ and the phase in between. It is noticeable that the observed cross sections of some electromagnetic processes, such as $\psi(2S) \rightarrow \pi^+\pi^-$, $\omega\pi^0$, and the famous puzzling $\psi(2S) \rightarrow \rho\pi$, are three to four orders of magnitude smaller than the total hadronic cross section of the continuum process, which is about 14 nb. Form factor estimation [20] gives these cross sections at continuum comparable to the ones measured at the resonance [21].

It implies that a substantial part of the experimentally measured cross section could come from the continuum amplitude $a_c$ instead of the $\psi(2S)$ decays, and interference between these two amplitudes may even affect the measured quantity further. Therefore it is essential to measure the
production rate of $\pi^+\pi^-$, $\omega\pi^0$ and $\rho\pi$ at the continuum in order to get the correct branching ratios of the $\psi(2S)$ decays. The same holds for VT decays of $\psi(2S)$.

As for $J/\psi$, the interference between the amplitude $a_c$ and the resonance is at the order of a few percent. It is smaller than the statistical and systematic uncertainties of current measurements. Nevertheless, for future high precision experiments such as CLEO-c [10] and BES-III [11], when the accuracy reaches a few per mille or even smaller level, it should be taken into account.

4. Dependence on experimental conditions

Here we emphasize the dependence of the observed cross section in $e^+e^-$ collision on the experimental conditions. The most crucial ones are the accelerator energy spread and the beam energy setting for the narrow resonances like $J/\psi$ and $\psi(2S)$.

Fig. 3 depicts the observed cross sections of inclusive hadrons and $\mu^+\mu^-$ pairs at $\psi(2S)$ in actual experiments. Two arrows in the figure denote the different positions of the maximum heights of the cross sections. The height is reduced and the position of the peak is shifted due to the radiative correction and the energy spread of the collider. However, the energy smear hardly affects the continuum part of the cross section. The $\mu^+\mu^-$ channel is further affected by the interference between resonance and continuum amplitudes. As a consequence, the relative contribution of the resonance and the continuum varies as the energy changes. In actual experiments, data are naturally taken at the energy which yields the maximum inclusive hadronic cross section. This energy does not coincide with the maximum cross section of each exclusive mode. So it is important to know the beam spread and beam energy precisely, which are needed in the delicate task to subtract the contribution from $a_c$.

It is worth noting that in principle if $a_c$ is not considered correctly, different experiments will give different results for the same quantity, like the exclusive branching ratio of the resonance, due to the dependence on beam energy spread and beam energy setting. The results will also be different for different kinds of experiments, such as production of $J/\psi$ and $\psi(2S)$ in $p\bar{p}$ annihilation, or in $B$ meson decays. This is especially important since the beam spreads of different accelerators are much different [2] and charmonium results are expected from $B$-factories.

5. Summary and perspective

In summary, the continuum amplitude $a_c$, by itself or through interference with the resonance, could contribute significantly to the observed cross sections in $e^+e^-$ experiments on charmonium physics. Its treatment depends sensitively on the experimental details, which has not been fully addressed in both $e^+e^-$ experiments and theoretical analyses. So far, most of the mea-
measurements have large statistical and systematic uncertainties, so this problem has been outside the purview of concern. Now with large \( J/\psi \) and \( \psi(2S) \) samples from BES-II [22] and forthcoming high precision experiments CLEO-c [10] and BES-III [11], the effect of \( a_c \) needs to be treated properly. To study it, the most promising way is to do energy scan for every exclusive mode in the vicinity of the resonance, so that both the amplitudes and the relative phases could be fit simultaneously. In case this is not practicable, data sample off the resonance with comparable integrated luminosity as on the resonance should be collected to measure \( |a_c| \), which could give an estimation of its contribution to the decay modes studied. The theoretical analyses based on current available \( e^+e^- \) data, particularly on \( \psi(2S) \) may need to be revised correspondingly.

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