INVITED PAPER

Vortex tubes in turbulence velocity fields at high Reynolds numbers

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Received 17 April 2007, in final form 9 January 2008
Published 6 March 2009
Online at stacks.iop.org/FDR/41/021402

Communicated by Y Tsuji

Abstract

The elementary structures of turbulence, i.e. vortex tubes, are studied using velocity data obtained in laboratory experiments for boundary layers and duct flows at microscale Reynolds numbers \( Re_\lambda = 332–1934 \). Although the past experimental studies focused on intense vortex tubes, the present study focuses on all vortex tubes with various intensities. We obtain the mean velocity profile. The radius scales with the Kolmogorov length. The circulation velocity scales with the Kolmogorov velocity, in contrast to the case of intense vortex tubes alone where the circulation velocity scales with the rms velocity fluctuation. Since these scaling laws are independent of the configuration for turbulence production, they appear to be universal at high Reynolds numbers.

1. Introduction

Turbulence contains vortex tubes as the elementary structures (Frisch 1995, Makihara et al 2002, Sreenivasan and Antonia 1997). Regions of significant vorticity tend to be organized into tubes. They occupy a small fraction of the volume and are embedded in the background fluctuation. Their existence was established at microscale Reynolds numbers \( Re_\lambda \lesssim 200 \), by seeding a turbulent liquid with gas bubbles and thereby visualizing regions of low pressure that are associated with vorticity (Douady et al 1991, La Porta et al 2000).

At low Reynolds numbers, \( Re_\lambda \lesssim 200 \), direct numerical simulations derived basic parameters of vortex tubes. The radii are of the order of the Kolmogorov length \( \eta \). The total lengths are of the order of the correlation length \( L \). The circulation velocities are of the order of the Kolmogorov velocity \( u_K \) or the rms velocity fluctuation \( \langle u^2 \rangle^{1/2} \). Here \( \langle \cdot \rangle \) denotes an average. The lifetimes are of the order of the turnover time for energy-containing eddies \( L/\langle u^2 \rangle^{1/2} \) (Jiménez and Wray 1998, Jiménez et al 1993, Makihara et al 2002, Tanahashi et al 2004, Vincent and Meneguzzi 1991, 1994).

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In particular, the velocity dependence has not been known at high Reynolds numbers. At $Re_c \gtrsim 200$, a direct numerical simulation is not easy for now. The bubble visualization does not have a high enough spatial resolution (La Porta et al 2000), except for the study of the tube length and lifetime.

The more promising approach is velocimetry in laboratory experiments. A probe suspended in the flow is used to obtain a one-dimensional cut of the velocity field. The small-scale velocity variation is enhanced at the positions of vortex tubes. For all vortex tubes with various intensities, the tube behavior has not been known because the parameter values are from Mouri et al (2007).

### Table 1. Turbulence parameters in boundary layers (B1–B6) and duct flows (D1–D5): sampling interval $\delta x$, $\eta$ = $(\nu^3/\langle \epsilon \rangle)^{1/4}$ where $\nu$ is the kinematic viscosity and $\langle \epsilon \rangle = 15v(\partial_k v^2)^2/2$ is the mean energy dissipation rate, Taylor microscale $\lambda = [2(\nu^3/\langle \epsilon \rangle)^{1/4}]^{1/2}$, Kolmogorov velocity $u_K = (\nu^2)^{1/4}$, rms velocity fluctuations $\langle u^2 \rangle^{1/2}$ and $\langle v^2 \rangle^{1/2}$, and microscale Reynolds number $Re_c = \lambda (\nu^2)^{1/2}/\nu$. The parameter values are from Mouri et al (2007).

| Data | $\delta x$ (cm) | $\eta$ (cm) | $\lambda$ (cm) | $u_K$ (m s$^{-1}$) | $\langle u^2 \rangle^{1/2}$ (m s$^{-1}$) | $\langle v^2 \rangle^{1/2}$ (m s$^{-1}$) | $Re_c$ |
|------|----------------|-------------|----------------|------------------|-----------------|-----------------|--------|
| B1   | 0.0378         | 0.0539      | 1.93           | 0.0262           | 0.283           | 0.242           | 332    |
| B2   | 0.0312         | 0.0335      | 1.46           | 0.0423           | 0.582           | 0.475           | 488    |
| B3   | 0.0265         | 0.0198      | 1.04           | 0.0716           | 1.18            | 0.973           | 716    |
| B4   | 0.0230         | 0.0152      | 0.919          | 0.0934           | 1.80            | 1.46            | 945    |
| B5   | 0.0180         | 0.0120      | 0.776          | 0.118            | 2.46            | 1.98            | 1080   |
| B6   | 0.0164         | 0.0104      | 0.742          | 0.137            | 3.14            | 2.51            | 1304   |
| D1   | 0.0355         | 0.0288      | 1.52           | 0.0489           | 0.694           | 0.666           | 719    |
| D2   | 0.0255         | 0.0177      | 1.15           | 0.0798           | 1.38            | 1.34            | 1098   |
| D3   | 0.0217         | 0.0133      | 0.986          | 0.107            | 2.11            | 2.04            | 1416   |
| D4   | 0.0216         | 0.0111      | 0.895          | 0.128            | 2.84            | 2.69            | 1693   |
| D5   | 0.0212         | 0.00955     | 0.826          | 0.149            | 3.46            | 3.32            | 1934   |

However, for most of the tube parameters, universality has not been established because the behavior has not been known at high Reynolds numbers. Since the data were long, $(1–4) \times 10^4$ points, their statistics are expected to be significant. Table 1 lists turbulence parameters that are to be used here.

The experiments were done in a wind tunnel of the Meteorological Research Institute. We use coordinates $x$, $y$, and $z$ in the streamwise, spanwise, and floor-normal directions.

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3 Other vortical structures, i.e. vortex sheets, are not important at least statistically. This is already known for the case of intense velocity variation (Mouri et al 2007, Noullez et al 1997) and is to be demonstrated here for the case of velocity variation with arbitrary intensity (section 4).
The origin \( x = y = z = 0 \, \text{m} \) is on the tunnel floor at the entrance to the test section. Its size was \( \delta x = 18 \, \text{m}, \delta y = 3 \, \text{m} \) and \( \delta z = 2 \, \text{m} \). We simultaneously measured the velocity fluctuations \( u \) and \( v \) in the streamwise and spanwise directions, by using a hot-wire anemometer with a crossed-wire probe. The wires were 5 \( \mu \text{m} \) in diameter, 1.25 mm in sensing length, 1.4 mm in separation and oriented at \( \pm 45^\circ \) to the streamwise direction. Taylor’s frozen-flow hypothesis was used to convert temporal variations into spatial variations. The sampling interval \( \delta x \) was set to be as small as possible, on the condition that high-wave-number noise was not significant in the power spectrum.

2.1. Boundary layers (data B1–B6)

Over the entire floor of the tunnel test section, we placed blocks as roughness elements. Their size was \( \delta x = 0.06 \, \text{m}, \delta y = 0.21 \, \text{m} \) and \( \delta z = 0.11 \, \text{m} \). Their spacing was \( \delta x = \delta y = 0.5 \, \text{m} \). The measurement position was at \( x = 12.5 \, \text{m} \), where the boundary layer had been well developed, and \( z = 0.25–0.35 \, \text{m} \) in the log-law sublayer.\(^4\) We obtained the data B1–B6 at \( Re_c = 332–1304 \) by changing the incoming-flow velocity from 2 to 20 m s\(^{-1}\).

2.2. Duct flows (data D1–D5)

At \( x = -2 \, \text{m} \), we placed a rectangular duct with width \( \delta y = 1.3 \, \text{m} \) and \( \delta z = 1.4 \, \text{m} \). The duct center was on the tunnel axis. The measurement position was at \( x = 15.5 \, \text{m} \) and \( z = 0.6 \, \text{m} \), where the flow had become turbulent. We obtained the data D1–D5 at \( Re_c = 719–1934 \) by changing the duct-exit flow velocity from 11 to 55 m s\(^{-1}\).

3. Model for vortex tubes

Using the Burgers vortex, an idealized model for vortex tubes, we discuss what information is available from a one-dimensional cut of the velocity field (for a similar discussion based on a direct numerical simulation, see Mouri et al (1999)). The Burgers vortex is an axisymmetric steady circulation in a strain field. In cylindrical coordinates, the circulation \( u_\Theta \) and strain field \((u_R, u_Z)\) are

\[
 u_\Theta \propto \frac{v}{a_0 R} \left[ 1 - \exp \left( -\frac{a_0 R^2}{4v} \right) \right] \quad \text{and} \quad (u_R, u_Z) = \left( -\frac{a_0 R}{2}, a_0 Z \right).
\]

Here \( v \) is the kinematic viscosity and \( a_0(>0) \) is a constant. The circulation is maximal at \( R = R_0 = 2.24(v/a_0)^{1/2} \). Thus, \( R_0 \) is regarded as the tube radius. We do not use other models for vortex tubes, e.g. spirals of Lundgren (1982), because detailed information about individual vortex tubes is anyway not available from one-dimensional velocity data.

Suppose that velocity data are obtained on a one-dimensional cut of a flow field that consists of vortex tubes and the background random fluctuation, as illustrated in figure 1(a). The one-dimensional cut is along the \( x \)-axis, the tube position is \((x_0, y_0)\) and the tube inclination is \((\theta_0, \phi_0)\). The circulation flows \( u_\Theta \) of the vortex tubes induce small-scale variations in the \( v \) signal.

If we consider intense velocity variations above a high threshold (Mouri et al 2007), their scale and amplitude correspond to the radius and circulation velocity of intense vortex

\(^4\) The measurement position might appear to have been too close to the floor of the tunnel, but this is not serious at all. Firstly, this is not in contradiction to our purpose, which is to find features of vortex tubes that are independent of the large-scale flow. Secondly, not invalid is the basic assumption of our study that the spatial distribution of vortex tubes was similar among experiments (section 3). Since the measured ratio \( (u^2)/\langle v^2 \rangle \) is not far from unity (table 1), the tube distribution at the measurement position should have been almost isotropic in all the experiments.
4. Mean velocity profile

The spanwise-velocity increment

\[ v(x + \delta x_0/2) - v(x - \delta x_0/2) \]

over a small scale \( \delta x_0 \) varies at the position \( x = x_0 \) of a vortex tube, regardless of its intensity. We try to use all of such variations and study vortex tubes as a whole. The increment is smoothed with the Gaussian window function \( \exp(-x^2/2\delta x_0^2) \). We determine the tube positions \( x_0 \) as local maxima and minima of the smoothed increment and then analyze the unsmoothed velocity data. The smoothing is to make sure that each of the local maxima and minima corresponds to each vortex tube. Also, the smoothing reduces the background fluctuation. We set \( \delta x_0 \) to be a multiple of the sampling interval \( \delta x_s \) that is close to the mean radius \( 6\eta \) estimated for
intense vortex tubes in Mouri et al (2007). This estimate is consistent with those in Jiménez et al (1993), Belin et al (1996), Jiménez and Wray (1998) and Tanahashi et al (2004), if we consider the difference in the definition of tube radius.

Figure 2 compares the probability density of the absolute velocity increment $|v(x + \delta x) - v(x)|$ over the sampling interval $\delta x$ for the entire data with that for the subdata from $x = x_0 - \delta x_0$ to $x_0 + \delta x_0 - \delta x$ around all tube positions $x_0$. The latter distribution accounts for the tail of the former distribution where vortex tubes should be dominant (Mouri et al 2007), and does not account for the center of the former distribution where the background fluctuation should be dominant. It is thereby expected that we have surely captured vortex tubes.

Figure 3 shows the mean velocity profiles obtained by averaging signals centered at individual tube positions $x_0$. The $v$ profile is similar to that in figure 1b. Hence, the contribution from vortex tubes is surely dominant. The contribution from vortex sheets is not dominant. If it were dominant, the $v$ profile in figure 3 should exhibit some kind of step (Mouri et al 2007, Noullez et al 1997). Exceptionally, to the extended tails of the $v$ profile, the contribution from vortex sheets might be dominant.

By fitting the $v$ profile in figure 3 around its peaks by the $v$ profile of the Burgers vortex for $x_0 = y_0 = \theta_0 = 0$ (dotted line), we estimate the radius $R_0$ and maximum circulation.
velocity \( V_0 \). The measured velocity is considered to have been smoothed over the probe size in the streamwise direction, 1 mm. Table 2 lists the \( R_0 \) and \( V_0 \) values. Although the \( R_0 \) value is greater than the true mean radius, the \( V_0 \) value is less than the true mean circulation velocity, because of the contribution from vortex tubes with \( |y_0| \gg R_0 \) or \( \theta_0 \gg 0 \). The \( R_0 \) and \( V_0 \) values are still proportional to the true mean values (section 3).

The \( u \) profile in figure 3 is separated for \( \partial_x u > 0 \) \( (u^+) \) and \( \partial_x u \leq 0 \) \( (u^-) \) at \( x = 0 \). These \( u^\pm \) profiles have larger amplitudes than the \( u^\pm \) profiles in figure 1b. This is a signature of vortex tubes with \( |y_0| \gg R_0 \) or \( \theta_0 \gg 0 \), especially of tubes passing the probe with some incidence angles relative to the mean flow direction, \( \tan^{-1}[v/(U+u)] \) (Belin et al 1996). The radial inflow \( u_R \) of the strain field is not discernible, except that the \( u^- \) profile has a larger amplitude than the \( u^+ \) profile. Unlike the Burgers vortex, a real vortex tube is not always

5 We have decomposed the \( u^\pm \) profiles into symmetric and antisymmetric components and show only the antisymmetric components (Mouri et al 2007). This is because, although the \( u^\pm \) profiles for vortex tubes should be antisymmetric, a symmetric positive excursion is induced by the contamination with the \( w \) velocity that is perpendicular to the \( u \) and \( v \) velocities (Sassa and Makita 2005). The two wires of the hot-wire anemometer individually respond to all the \( u \), \( v \) and \( w \) velocities. Since the measured \( u \) velocity corresponds to the sum of the responses of the two wires, it is contaminated with the \( w \) velocity. Since the measured \( v \) velocity corresponds to the difference of the responses, it is free from the \( w \) velocity.

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oriented to the stretching direction (Jiménez and Wray 1998, Jiménez et al. 1993, Tanahashi et al. 2004, Vincent and Meneguzzi 1991, 1994).

5. Scaling laws of tube parameters

The dependence of tube parameters on the microscale Reynolds number $Re$ and on the configuration for turbulence production, i.e. boundary layer or duct flow, is studied in figure 4. Each quantity was normalized by its value in the duct flow at $Re = 1394$ (D5). That is, we avoid the prefactors that are not equal to the true values (section 4). We instead focus on scaling laws.

The radius $R_0$ scales with the Kolmogorov length $\eta$ as $R_0 \propto \eta$ (figure 4(a)). This is partly because vortex tubes were identified on a scale $\delta x_0 \simeq 6\eta$ (section 4), but the scatter of $R_0/\eta$ is less than the scatter of $\delta x_0/\eta$ as in table 2.

The maximum circulation velocity $V_0$ scales with the Kolmogorov velocity $u_K$ as $V_0 \propto u_K$ (figure 4(b)) rather than with the rms velocity fluctuation $\langle v^2 \rangle^{1/2}$ as $V_0 \propto \langle v^2 \rangle^{1/2}$ (figure 4(c)), if we consider the entire $Re_x$ range. This result is reasonable because $u_K$ is a characteristic of small-scale motions. At $Re_x \gtrsim 1000$, the scaling $V_0 \propto \langle v^2 \rangle^{1/2}$ is also significant. Since $V_0 \propto \langle v^2 \rangle^{1/2}$ is observed for intense vortex tubes (section 6), $V_0 \propto \langle v^2 \rangle^{1/2}$ is also observed for all vortex tubes if the mean $v$ profile is biased toward intense vortex tubes. This is more likely at higher $Re_x$ because $Re_x \propto \langle v^2 \rangle/u_K^2$. However, since intense vortex tubes are rare (section 6), $V_0 \propto \langle v^2 \rangle^{1/2}$ does not represent vortex tubes as a whole. We do not consider the scaling $V_0 \propto \langle v^2 \rangle^{1/2}$.

Direct numerical simulations of homogeneous isotropic turbulence and turbulent channel flows at $Re_x \lesssim 200$ derived the scalings $R_0 \propto \eta$ and $V_0 \propto u_K$ for all vortex tubes (Makihara et al. 2002, Tanahashi et al. 2004). We have found that, regardless of the configuration for turbulence production, those scalings extend at least up to $Re_x \simeq 2000$.

The scalings of the radius $R_0$ and circulation velocity $V_0$ lead to a scaling of the Reynolds number $Re_0 = R_0 V_0/\nu$ that characterizes stability of vortex tubes (Jiménez and Wray 1998, Jiménez et al. 1993):

$$Re_0 = \text{constant} \quad \text{if} \quad R_0 \propto \eta \quad \text{and} \quad V_0 \propto u_K. \quad (2a)$$

$$Re_0 \propto Re_x^{1/2} \quad \text{if} \quad R_0 \propto \eta \quad \text{and} \quad V_0 \propto \langle v^2 \rangle^{1/2}. \quad (2b)$$

\begin{table}[h]
\centering
\caption{Parameters of vortex tubes in boundary layers (B1–B6) and duct flows (D1–D5): identification scale $\delta x_0$, radius $R_0$, maximum circulation velocity $V_0$ and Reynolds number $Re_0 = R_0 V_0/\nu$. For parameters of intense vortex tubes alone, see table 2 of Mouri et al. (2007).}
\begin{tabular}{ccccccc}
Data & $\delta x_0/\eta$ & $R_0/\eta$ & $V_0/u_K$ & $V_0/\langle v^2 \rangle^{1/2}$ & $Re_0$ & $Re_0/Re_{K}^{1/2}$ \\
B1 & 6.30 & 8.45 & 2.43 & 0.263 & 20.5 & 1.13 \\
B2 & 5.59 & 8.47 & 2.39 & 0.213 & 20.2 & 0.914 \\
B3 & 5.36 & 8.59 & 2.47 & 0.182 & 21.2 & 0.792 \\
B4 & 6.06 & 9.21 & 2.73 & 0.175 & 25.1 & 0.817 \\
B5 & 6.00 & 8.93 & 2.83 & 0.169 & 25.3 & 0.770 \\
B6 & 5.30 & 8.28 & 2.88 & 0.157 & 23.8 & 0.659 \\
D1 & 6.16 & 8.56 & 2.43 & 0.179 & 20.8 & 0.776 \\
D2 & 5.76 & 8.91 & 2.56 & 0.152 & 22.8 & 0.688 \\
D3 & 6.52 & 9.12 & 2.79 & 0.146 & 25.4 & 0.675 \\
D4 & 5.84 & 8.72 & 2.91 & 0.138 & 25.4 & 0.617 \\
D5 & 6.66 & 8.81 & 3.18 & 0.143 & 28.0 & 0.637 \\
\end{tabular}
\end{table}
If we consider the entire $Re_\lambda$ range, our result favors the former scaling (figure 4(d)) rather than the latter (figure 4(e)). This is in accordance with the observed scalings of $R_0$ and $V_0$ (figures 4(a) and 4(b)). The constancy of $Re_0$ implies that vortex tubes as a whole are stable against an increase of the Reynolds number $Re_\lambda$. At $Re_\lambda \gtrsim 1000$, the scaling $Re_0 \propto Re_\lambda^{1/2}$ is also significant. This scaling is not important to us because it is related with $V_0 \propto \langle v^2 \rangle^{1/2}$ discussed before (figure 4(c)).
6. Comparison with intense vortex tubes

The present result for all vortex tubes with various intensities is compared with a past result for intense vortex tubes alone. We use the result of Mouri et al. (2007), which was based on the same experimental data. For a similar experimental result for intense vortex tubes, see Belin et al. (2004).

To identify intense vortex tubes, Mouri et al. (2007) imposed a threshold on the absolute velocity increment $|v(x + \delta x) - v(x)|$. The threshold was such that 0.1 or 1% of the increments were used for the identification. These increments are included in the increments used for all vortex tubes as shown in figure 2. Thus, intense vortex tubes studied by Mouri et al. (2007) are included in all vortex tubes studied here.

Mouri et al. (2007) estimated the radius $R_0$ and maximum circulation velocity $V_0$ from mean velocity profiles. They were obtained by averaging signals centered at individual positions where $|v(x + \delta x) - v(x)|$ was above the threshold. Since the mean velocity profiles were dominated by vortex tubes with $|v_0| \lesssim R_0$ and $\theta_0 \simeq 0$, the $R_0$ and $V_0$ values are close to the true mean radius and true mean circulation velocity (section 3). The dependence of $R_0$ and $V_0$ on the threshold was discussed in Mouri et al. (2007).

The radius for intense vortex tubes, $R_0/\eta \simeq 5–7$ (Mouri et al. 2007), is less than that for all vortex tubes, $R_0/\eta \simeq 8–9$ (table 2). While the former is close to the true mean radius, the latter is greater than the true mean radius (section 4). The true mean radius appears not to be significantly different between intense and all vortex tubes. Both of them obey the scaling $R_0 \propto \eta$. The circulation flows of vortex tubes are always of the smallest scales of turbulence.

The circulation velocity for intense vortex tubes, $V_0/(v^2)^{1/2} \simeq 0.4–0.8$ (Mouri et al. 2007), is greater than that for all vortex tubes, $V_0/(v^2)^{1/2} \simeq 0.1–0.3$ (table 2). This is mainly due to the difference in the true mean circulation velocity. In addition, the $V_0$ value for all vortex tubes is less than the true mean circulation velocity (section 4). While all vortex tubes obey the scaling $V_0 \propto u_k$, intense vortex tubes obey the scaling $V_0 \propto (v^2)^{1/2}$ in the same $Re_\lambda$ range.

The Reynolds number for intense vortex tubes scales as $Re_0 \propto Re_\lambda^{1/2}$, which is explained through equation (2b) by the scalings $R_0 \propto \eta$ and $V_0 \propto (v^2)^{1/2}$. With an increase of $Re_\lambda$, intense vortex tubes progressively have higher $Re_0$ and are more unstable. The situation is different in the case of all vortex tubes, for which $Re_0$ is constant.

Direct numerical simulations at $Re_\lambda \lesssim 200$ derived $R_0 \propto \eta$, $V_0 \propto (v^2)^{1/2}$, and hence $Re_0 \propto Re_\lambda^{1/2}$ for intense vortex tubes (Jiménez and Wray 1998, Jiménez et al. 1993), whereas $R_0 \propto \eta$, $V_0 \propto u_k$, and hence $Re_0 \propto \eta$ for all vortex tubes (Makihara et al. 2002, Tanahashi et al. 2004). This difference in the scalings of $V_0$ and $Re_0$ is the same as that obtained here up to $Re_\lambda \simeq 2000$.

The difference in the scalings of $V_0$ and $Re_0$ between intense and all vortex tubes implies that their roles are different. Since $V_0 \propto (v^2)^{1/2}$, intense vortex tubes are responsible for small-scale intermittency. This is especially the case at high $Re_\lambda$ because $Re_\lambda \propto (v^2)/u_k^2$. In fact, at high $Re_\lambda$, small-scale intermittency is significant. However, since $Re_0 \propto Re_\lambda^{1/2}$, intense vortex tubes have short lifetimes and thus rare at high $Re_\lambda$ (Jiménez and Wray 1998, Jiménez et al. 1993, Mouri et al. 2007). On the other hand, vortex tubes as a whole obey the scalings $V_0 \propto u_k$ and $Re_0 \propto \eta$. They are always ubiquitous and responsible for an important fraction of energy at the smallest scales.
7. Conclusion

Using velocity data obtained in boundary layers at \( Re_\lambda = 332–1304 \) and duct flows at \( Re_\lambda = 719–1934 \) (Mouri et al. 2007), we have studied vortex tubes, i.e. the elementary structures of turbulence. Although the past experimental studies focused on intense vortex tubes, the present study focuses on all vortex tubes with various intensities. We have obtained the mean velocity profile, estimated the radius \( R_0 \) and maximum circulation velocity \( V_0 \), and then obtained the scalings \( R_0 \propto \eta \), \( V_0 \propto u_K \) and \( Re_0 = R_0 V_0 / \nu = \text{constant} \). They are in contrast to the scalings for intense vortex tubes alone, i.e., \( R_0 \propto \eta \), \( V_0 \propto \langle v^2 \rangle^{1/2} \) and \( Re_0 \propto Re_\lambda^{1/2} \). Since those scalings for all vortex tubes are independent of the configuration for turbulence production, they appear to be universal at high Reynolds numbers \( Re_\lambda \). The implication of the scalings is that vortex tubes as a whole are always ubiquitous and responsible for an important fraction of energy at the smallest scales.

The present study has some ambiguities because only one-dimensional data of the velocity field are available. To proceed further, two- or three-dimensional velocity data are necessary but are not available for now. The advent of an array of hot-wire probes (Sassa and Makita 2005) or a particle image velocimeter (Tanahashi et al. 2002) that is applicable to vortex tubes at high Reynolds numbers is desirable. Such a technique would enlarge our knowledge of vortex tubes. For example, the study of the probability density distributions of the tube radius and maximum circulation velocity is of interest.

Acknowledgments

We are grateful to the anonymous referees for useful comments.

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