Measurement of Positronium Hyperfine Splitting with Quantum Oscillation

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Abstract. It is well known that a coherent system can bring a quantum interference between different energy eigenstates into an observable oscillation. Since an energy difference and a frequency are related with each other by the equation $\Delta E = \hbar \omega$, we can make use of the oscillation to measure the energy difference of quantum states. In this experiment, the hyperfine splitting of positronium is measured using the quantum oscillation. The result is $203.324 \pm 0.044$ (stat.) $\pm 0.028$ (sys.) GHz. This value is consistent with the theoretical calculations.

1. Introduction

Positronium (Ps) is a bound state of the electron and the positron. It has triplet states and singlet state. The triplet state is called “Ortho-Positronium (o-Ps)” and the singlet state is called “Para-Positronium (p-Ps)”. O-Ps has three degenerated eigenstates $|s = 1, m_z = \pm 1\rangle, |s = 1, m_z = 0\rangle$. Because of the charge conjugation conservation in the electromagnetic interactions, o-Ps mainly decays into three gamma rays, which leads to the relatively longer lifetime of 142 ns in vacuum. On the other hand, p-Ps has only one eigenstate $|s = 0, m_z = 0\rangle$ and shorter lifetime of 125 ps, mainly decaying into two 511 keV-gamma rays.

The energy difference between o-Ps and p-Ps is called “Positronium Hyperfine Splitting (Ps HFS)”, which is about 0.84 meV (203.4 GHz). It has been measured in several experiments, mostly using Zeeman effect as follows; Figure 1 shows the energy level of Ps as function of a static magnetic field. In a static magnetic field, $|s = 1, m_z = 0\rangle$ and $|s = 0, m_z = 0\rangle$ get mixed into two new eigenstates $|+\rangle, |\rangle$, while $|s = 1, m_z = \pm 1\rangle$ states stay in the same energy. The energy difference between $|+\rangle$ and $|s = 1, m_z = \pm 1\rangle$ is, \[
\Delta_{\text{mix}} = \frac{\Delta_{\text{HFS}}}{2}(\sqrt{1+\chi^2} - 1),
\] (1)

where $\chi = \frac{g'\mu_B H}{\Delta_{\text{HFS}}}$, $H$ is the magnetic field strength, $\Delta_{\text{HFS}}$ is Ps HFS, $\mu_B$ is the Bore magneton and $g'$ is the electron spin g-factor with correction for bound systems. Since $\chi$ is small in a weak magnetic field, we can assume that $\Delta_{\text{mix}}$ is approximately proportional to the HFS. $\Delta_{\text{mix}}$ can be measured with a few GHz-RF wave. This Zeeman effect was used in the previous measurements.
performed by Mills [1] and Ritter [2] and \( \Delta_{\text{HFS}} = 203.3865(67) \) was obtained. On the other hand, V.G.Baryshevsky [4] proposed to make use of the quantum oscillation between these states instead of observing the transition, and later, he observed the oscillation experimentally [5]. In 1996, S.Fan [6] measured Ps HFS with this method, obtaining \( 202.5 \pm 3.5 \text{GHz} \).

Positrons emitted from \( \beta^+ \) source are polarized to their momentum-direction because of Parity-violation in weak interaction. Higher polarization can be obtained with \( \text{Ge-Ga} \) source whose endpoint energy is 1.9 MeV (polarization ratio \( P = \beta \)). When the positron stops in a material, it forms polarized positronium which is in a superposed state of \( |+\rangle \) and \( |-\rangle \). Such a superposition oscillates with the frequency corresponding to the energy difference. In this experiment, we produce a coherent superposition of \(|+\rangle\) and \(|-\rangle\) efficiently using polarized \( \beta^+ \) perpendicularly to the magnetic field.

Oscillation will be observed as follow.

Let us take the momentum direction of the positron as \( x \)-axis and that of the magnetic field as \( z \)-axis. We define \( \theta = \arccos(\frac{z}{\sqrt{x^2+y^2}}) \) and \( \phi = \arctan(\frac{y}{x}) \). The quantum oscillation modulates the angular distribution of three gamma rays emitted from o-Ps decay (Figure 2). As a result, the decay curve of o-Ps beats with the oscillation. The oscillating amplitude is the biggest at \( \theta = 0 \) and \( \phi = \frac{\pi}{2} \) (the adjacent pairs have the opposite phases). Unlike the muon precession, in which the decay amplitude of \( \mu \rightarrow e \) rotates, this oscillation changes its angular distribution as a “vibration” in the \( yz \)-plane. This property is unique to Spin-1 systems.

In general, the decay curve of o-Ps which is observed by a detector with finite acceptance can be written as,

\[
 f(t) = C_1 e^{-\gamma_1 t} + C_2 e^{-\gamma_2 t} + C_3 e^{-\Omega t} \sin(\Omega t + \theta_0) + C_4, 
\]

where \( \gamma_1 \) is the decay rate of \(|+\rangle\), \( \gamma_2 \) is that of \(|-\rangle\), \( \Omega \) is the angular frequency of the oscillation, \( C_3 \) is the oscillation depth, which is proportional to the initial polarization ratio of positrons. Since \(|+\rangle\) state can decay into two gamma rays, \( \gamma_1 \) is larger than \( \gamma_2 \).

We can calculate Ps HFS from \( \Omega \) and the magnetic field strength.

2. Experimental Setup

Figure 3 shows a schematic diagram of the setup with a coordinate axis and size, a photograph of vacuum chamber is shown in Figure 4. We use \( \text{Ge-Ga} \) positron source (30 kBq) which is sealed with \( t=13 \mu \text{m} \) Titanium. A positron emitted to the \( x \)-direction is tagged by the plastic scintillator (\( t=500 \mu \text{m}, \text{NE102} \)), producing a light pulse which is directed to two photomultipliers through the light guide. The positron stops in the silica aerogel (0.11 g/cm\(^3\), \( \phi=10 \text{mm}, d=10 \text{mm} \)) and forms a positronium. Ps-formation assembly is put in a vacuum chamber in order to reduce the pick-off effect.

Gamma rays emitted from o-Ps decay are detected by six \( \text{LaBr}_3(\text{Ce}) \) scintillators (\( \phi=1.5 \text{ inch}, d=2 \text{ inch} \)) as shown in Figure 5, 6. Four of them are placed at \( (\theta, \phi) = \left( \frac{\pi}{2}, \frac{\pi}{2} \right), (\frac{3\pi}{4}, \frac{\pi}{4}), (\frac{3\pi}{4}, -\frac{\pi}{4}), (\frac{\pi}{4}, -\frac{\pi}{4}) \) to observe the oscillating curves. The rest of the detectors are placed at \( (\theta, \phi) = \left( \frac{\pi}{2}, \frac{\pi}{2} \right), (\frac{3\pi}{4}, \frac{\pi}{4}), (\frac{3\pi}{4}, -\frac{\pi}{4}) \) to observe the decay curve without oscillation. The average energy resolution of the \( \text{LaBr}_3 \) detectors is 4\% (FWHM) at 511 keV in the magnetic field strength of 100 mT. The timing resolution of 511 keV-511 keV coincidence between two \( \text{LaBr}_3 \) detectors is 200 ps (FWHM). However, because timing resolution of plastic scintillator is limited in our setup, \( \beta-\gamma \) timing resolution is 3.8 ns (FWHM) at 511 keV in the magnetic field strength of 100 mT.

The magnetic field is provided with a superconducting magnet in KEK, which has an excellent uniformity of 20 ppm over the volume of the silica aerogel. The magnetic field strength is
Figure 1. Energy level of positronium in its ground state as function of the static magnetic field strength. $\Delta_{\text{HFS}}$ represents the HFS and $\Delta_{\text{mix}}$ shows the Zeeman shift between $|+\rangle$ and $|s = 1, m_s = \pm 1\rangle$.

Figure 2. Angular distribution of the gamma rays emitted from $\alpha$-Ps decay. The radius stands for the amplitude of emitting direction. It changes with time as is shown in the solid and dotted lines. The magnetic direction goes from left to right, the polarizing direction is perpendicular to the sheet.

Figure 3. Schematic diagram of the vacuum chamber. $\beta^+$ emitted from $^{68}$Ge-Ga passes through the plastic scintillator, producing a start signal. $\beta^+$ stops in the aerogel and forms a positronium.

Figure 4. Photograph of the vacuum chamber.

measured with a NMR magnetometer. Uncertainty of the absolute calibration of the magnetic field strength is 70 ppm.

Data acquisition is started when at least one signal of the LaBr$_3$ scintillator and that of the plastic scintillator coincide within $-50$ ns to 1650 ns. The timing information of the plastic and LaBr$_3$ scintillators are measured by direct clock TDCs, which have good integral and differential linearities. Start signal is produced with $\beta$-timing and stop signal is with $\gamma$-timing. These TDCs can work on as fast as 8 GHz clock, but most of data-acquisition are performed on 5 GHz clock to obtain good performance.

The experiments are conducted in the magnetic field strength of 0 mT, 100 mT, 118 mT, 135 mT, 138 mT with both $+z$ polarization and $-z$ polarization by changing the emitting...
direction of $\beta^+$. The period of each run is about 3 days and total data acquiring period is about 22 days with 740 Hz of DAQ rate. The trigger rate is about 3.0 kHz.

3. Analysis

Detectors at $(\theta, \phi) = \left( \frac{\pi}{4}, \frac{\pi}{2} \right), \left( \frac{3\pi}{4}, \frac{\pi}{2} \right), \left( \frac{3\pi}{4}, -\frac{\pi}{2} \right), \left( \frac{\pi}{4}, -\frac{\pi}{2} \right), \left( \frac{\pi}{2}, \frac{\pi}{2} \right), \left( \frac{\pi}{2}, -\frac{\pi}{2} \right)$ are indexed 1 $\sim$ 6 respectively. The pairs of 1st and 3rd detectors, 2nd and 4th detectors observe the oscillation with the same phases. 5th and 6th detectors observe the exponential decay curve without oscillation. $\chi^2$ function is defined to fit the histograms of all the 6 detectors at once. The fitting function for n-th detector is,

$$
\begin{align*}
    f(t, n) &= A_n e^{-\gamma_1 t} + B_n e^{-\gamma_2 t} + C_n e^{-\frac{\gamma_1 + \gamma_2}{2} t} \times \sin(\Omega t + \theta_n) + D_n \\
    &\quad \text{for } n = 1, 2, 3, 4, \\
    f(t, n) &= A_n e^{-\gamma_1 t} + B_n e^{-\gamma_2 t} + D_n \\
    &\quad \text{for } n = 5, 6.
\end{align*}
$$

The two decay rates $\gamma_1, \gamma_2$ and the angular frequency of the oscillation $\Omega$ are defined as common variables (see equation (2)), and the others are defined independently. The fitting range is 30 ns to 1430 ns. Figure 7 shows an example of the timing histograms with the best fit functions.

Figure 8 shows the oscillating period($\frac{2\pi}{\Omega}$) as function of magnetic field strength. The black line is the best fit result. The combined result is

$$
\text{HFS} = 203.324 \pm 0.044\text{(stat.)}(215\text{ ppm}) \\
\pm 0.028\text{(sys.)}(140\text{ ppm}) \text{ GHz}
$$

Main source of the systematic error is absolute calibration of the NMR magnetometer (140 ppm). Accuracy of this measurement is improved by the factor of 70 than the previous measurement conducted by S.Fan [6].

4. Conclusion

Ps HFS is measured with its quantum oscillation. We have used the uniform magnetic field, $\beta^+$ source which has high endpoint energy and experimental setup which has good timing resolution.
Figure 7. Timing histogram obtained in the run of 100 mT, +z polarization. Red line shows the decay curve with normal phase and blue line shows the one with the opposite phase. The black lines are the best fit results.

Figure 8. Measured oscillating periods in the experiment. Red circles show the data points. Errorbars are smaller than the circles. The black line is the best fit result.

to obtain big oscillating amplitude. The result is $HFS = 203.324 \pm 0.044$ (stat.) $(215$ ppm) $\pm 0.028$ (sys.) $(140$ ppm) GHz, which improves the result of the previous experiment conducted with the same method by the order of two. This result agrees with the theoretical calculations.

There have reported several applications using quantum oscillation of positronium as a probe in material science. See [7], for example.

References
[1] Mills A P Jr and Bearman G H 1975 Phys. Rev. Lett. 34 246, Mills A P Jr 1983 Phys. Rev. A 27 262.
[2] Ritter W M 1984 Phys. Rev. A 30 1331.
[3] Melnikov K 2001 Phys. Rev. Lett. 86 1498.
[4] Baryshevsky G V 1989 J. Phys. B 22 2835.
[5] Baryshevsky G V 1989 J. Phys. A 136 428.
[6] Fan S 1996 Phys. Rev. A 216 129.
[7] Ivanov E 2008 Appl. Surf. Sci. 225 179.