Decoherence of Correlation Histories

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Abstract

We use a $\lambda\Phi^4$ scalar quantum field theory to illustrate a new approach to the study of quantum to classical transition. In this approach, the decoherence functional is employed to assign probabilities to consistent histories defined in terms of correlations among the fields at separate points, rather than the field itself. We present expressions for the quantum amplitudes associated with such histories, as well as for the decoherence functional between two of them. The dynamics of an individual consistent history may be described by a Langevin-type equation, which we derive.

Dedicated to Professor Brill on the occasion of his sixtieth birthday, August 1993

1. Introduction

1.1. Interpretations of Quantum Mechanics and Paradigms of Statistical Mechanics

This paper attempts to bring together two basic concepts, one from the foundations of statistical mechanics and the other from the foundations of quantum mechanics, for the purpose of addressing two basic issues in physics:
1) the quantum to classical transition, and
2) the quantum origin of stochastic dynamics.
Both issues draw in the interlaced effects of dissipation, decoherence, noise, and fluctuation. A central concern is the role played by coarse-graining—the naturalness of its choice, the effectiveness of its implementation and the relevance of its consequences.

On the fundations of quantum mechanics, a number of alternative interpretations exists, e.g., the Copenhagen interpretation, the many-world interpretation [1], the consistent history interpretations [2], to name just a few (see [3] for a recent review).

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The one which has attracted much recent attention is the decoherent history approach of Gell-Mann and Hartle [4]. In this formalism, the evolution of a physical system is described in terms of ‘histories’: A given history may be either exhaustive (defining a complete set of observables at each instant of time) or coarse-grained. While in classical physics each history is assigned a given probability, in quantum physics a consistent assignment of probabilities is precluded by the overlap between different histories. The decoherence functional gives a quantitative measure of this overlap; thus the quantum to classical transition can be studied as a process of “diagonalization” of the decoherence functional in the space of histories.

On the foundational aspects of statistical mechanics, two major paradigms are often used to describe non-equilibrium processes (see, e.g., [5, 6, 7, 8]): the Boltzmann theory of molecular kinetics, and the Langevin (Einstein-Smoluchowski) theory of Brownian motions. The difference between the two are of both formal and conceptual character.

To begin with, the setup of the problem is different: In kinetic theory one studies the overall dynamics of a system of gas molecules, treating each molecule in the system on the same footing, while in Brownian motion one (Brownian) particle which defines the system is distinct from the rest, which is relegated as the environment. The terminology of ‘relevant’ versus ‘irrelevant’ variables highlights the discrepancy.

The object of interest in kinetic theory is the (one-particle) distribution function (or the nth-order correlation function), while in Brownian motion it is the reduced density matrix. The emphasis in the former is the correlation amongst the particles, while in the latter is the effect of the environment on the system.

The nature of coarse-graining is also very different: in kinetic theory coarse-graining resides in the adoption of the molecular chaos assumption corresponding formally to a truncation of the BBGKY hierarchy, while in Brownian motion it is in the integration over the environmental variables. The part that is truncated or ‘ignored’ is what constitutes the noise, whose effect on the ‘system’ is to introduce dissipation in its dynamics. Thus the fluctuation-dissipation relation and other features.

Finally the philosophy behind these two paradigms are quite different: In Brownian motion problems, the separation of the system from the environment is prescribed: it is usually determined by some clear disparity between the two systems. These models represent “autocratic systems”, where some degrees of freedom are more relevant than others. In the lack of such clear distinctions, making a separation ‘by hand’ may seem rather ad hoc and unsatisfactory. By contrast, models subscribing to the kinetic theory paradigm represent “democratic systems”: all particles in a gas are equally relevant. Coarse-graining in Boltzmann’s kinetic theory appears less contrived, because information about higher correlation orders usually reflects the degree of precision in a measurement, which is objectively definable.
In the last five years we have explored these two basic paradigms of non-equilibrium statistical mechanics in the framework of interacting quantum field theory with the aim of treating dissipative processes in the early universe \cite{9, 10} and decoherence processes in the quantum to classical transition issue \cite{11}. Here we have begun to explore the issues of decoherence with the kinetic model.

Because of the difference in approach and emphasis between these two paradigms and in view of their fundamental character, it is of interest to build a bridge between them. We have recently carried out such a study with quantum fields \cite{12}. By delineating the conditions under which the Boltzmann theory reduces to the Langevin theory, we sought answers to the following questions:

1) What are the factors conducive to the evolution of a ‘democratic system’ to an ‘autocratic system’ and vise versa? A more natural set of criteria for the separation of the system from the environment may arise from the interaction and dynamics of the initial closed system \cite{13, 14}.

2) The construction of collective variables from the basic variables, the description of the dynamics of the collective variables, and the depiction of the behavior of a coarser level of structure emergent from the microstructures. \cite{13, 14, 16}.

The paradigm of quantum open systems described by quantum Brownian models has been used to analyze the decoherence and dissipation processes, for addressing basic issues like quantum to classical transitions, fluctuation and noise, particle creation and backreaction, which arise in quantum measurement theory \cite{17, 18, 19}, macroscopic quantum systems \cite{20}, quantum cosmology \cite{21} (for earlier work see references in \cite{22}), semiclassical gravity \cite{23, 24, 25} and inflationary cosmology \cite{26}. The reader is referred to these references and references therein for a description of this line of study.

The aim of this paper is to explore the feasibility for addressing the same set of basic issues using the kinetic theory paradigm. We develop a new approach based on the application of the decoherence functional \cite{3, 4} formalism to histories defined in terms of correlations between the fundamental field variables. We shall analyse the decoherence between different histories of an interacting quantum field, a $\lambda\Phi^4$ theory here taken as example, corresponding to different particle spectra and study issues on the physics of quantum to classical transition, the relation of decoherence to dissipation, noise and fluctuation, and the quantum origin of classical stochastic dynamics.

1.2. Quantum to Classical Transition and Coarse-Graining

One basic constraint in the building of quantum theory is that it should reproduce classical mechanics in some limit. (For a schematic discussion of the different criteria of classicality and their relations, see \cite{27}). Classical behavior can be characterized
by the existence of strong correlations between position and momentum variables described by the classical equations of motion [28] and by the absence of interference phenomena (decoherence).

Recent research in quantum gravity and cosmology have focused on the issue of quantum to classical transition. This was highlighted by quantum measurement theory for closed systems (for a general discussion, see, e.g., [29]), the intrinsic incompatibility of quantum physics with general relativity [30], and the quantum origin of classical fluctuations in explaining the large scale structure of the Universe. Indeed, in the inflationary models of the Universe [31], one hopes to trace all cosmic structures to the evolution from quantum perturbations in the inflaton field. More dramatically, in quantum cosmology [32] the whole (classical) Universe where we now live in is regarded as the outcome of a quantum to classical transition on a cosmic scale. In these models, one hopes not only to explain the ‘beginning’ of the universe as a quantum phenomenon, but also to account for the classical features of the present universe as a consequence of quantum fluctuations. This requires not only a theoretical understanding of the quantum to classical transition issue in quantum mechanics, but also a theoretical derivation of the laws of classical stochastic mechanics from quantum mechanics, the determination of the statistical properties of classical noise (e.g., whether it is white or coloured, local or nonlocal) being an essential step in the formulation of a microscopic theory of the structure of the Universe [26].

Our understanding of the issue of quantum to classical transition has been greatly advanced by the recent development of the decoherent histories approach to quantum mechanics [4]. An essential element of the decoherent histories approach is that the overlap between two exhaustive histories can never vanish. Therefore, the discussion of a quantum to classical transition can only take place in the framework of a coarse grained description of the system, that is, giving up a complete specification of the state of the system at any instant of time.

As a matter of fact, some form of coarse graining underlies most, if not all, successful macroscopic physical theories. This fact has been clearly recognized and exploited at least since the work of Nakajima and Zwanzig [33] on the foundations of nonequilibrium statistical mechanics. Like statistical mechanics, the decoherent histories approach allows a variety of coarse-graining procedures; not all of these, however, are expected to be equally successful in leading to interesting theories. Since the prescription of the coarse graining procedure is an integral part of the implementation of the decoherent histories approach, the development and evaluation of different coarse graining strategies is fundamental to this research program.

When we survey the range of meaningful macroscopic (effective) theories in physics arising from successfully coarse-graining a microscopic (fundamental) theory, one particular class of examples is outstanding; namely, the derivation of the hydrodynamical description of dilute gases from classical mechanics. The crucial step in deriving the
Navier-Stokes equation for a dilute gas consists in rewriting the Liouville equation for the classical distribution function as a BBGKY hierarchy, which is then truncated by invoking a ‘molecular chaos’ assumption. If the truncation is made at the level of the two-particle reduced distribution function, the Boltzmann equation results. In the near-equilibrium limit, this equation leads to the familiar Navier-Stokes theory.

We must stress that in this general class of theories exemplified by Boltzmann’s work, coarse-graining is introduced through the truncation of the hierarchy of distribution functions; i.e., by neglecting correlations of some order and above at some singled-out time \[5, 6\]. This type of coarse graining strategy is qualitatively different from those used in most of the recent work in quantum measurement theory and cosmology, which invoke a system-bath, space-time, or momentum-space separation. In most of these cases, an intrinsically justifiable division of the system from the environment is lacking and one has to rely on case-by-case physical rationales for making such splits. (An example of system-bath split is Zurek’s description of the measurement process in quantum mechanics, where a bath is explicitly included to cause decoherence in the system-apparatus complex \[17\]. Space-time coarse graining has been discussed by Hartle \[34\] and Halliwell et al \[35\]. An example of coarse-graining in momentum space is stochastic inflation \[36\], where inflaton modes with wavelengths shorter than the horizon are treated as an environment for the longer wavelength modes \[37\].)

### 1.3. Coarse-Graining in the Hierarchy of Correlations

In this paper we shall develop a version of the decoherent histories approach where the coarse-graining procedure is patterned after the truncation of the BBGKY hierarchy of distribution functions. For simplicity, we shall refer below to the theory of a single scalar quantum field, with a $\lambda \Phi^4$-type nonlinearity.

The simplest quantum field theoretical analog to the hierarchy of distribution functions in statistical mechanics is the sequence of Green functions (that is, the expectation values of products of $n$ fields) \[9\]. In this approach, the BBGKY hierarchy of kinetic equations is replaced by the chain of Dyson equations, linking each Green function to other functions of higher order.

The analogy between these two hierarchies is rendered most evident if we introduce “distribution functions” in field theory through suitable partial Fourier transformation of the Green functions. Thus, a “Wigner function” \[8\] may be introduced as the Fourier transform of the Hadamard function (the symmetric expectation value of the product of two fields) with respect to the difference between its arguments. It obeys both a mass shell constraint and a kinetic equation, and may be regarded as the physical distribution function for a gas of quasi particles, each built out of a cloud of virtual quanta. Similar constructs may be used to introduce higher “distribution functions” \[8, 9\].
As in statistical mechanics, the part of a given Green function which cannot be reduced to products of lower functions defines the corresponding “correlation function”. Thus the chain of Green functions is also a hierarchy of correlations.

To establish contact between the hierarchy of Green functions and the decoherent histories approach, let us recall the well-known fact that the set of expectation values of all field products contains in itself all the information about the statistical state of the field \[9\]. For a scalar field theory with no symmetry breaking, we can even narrow this set to products of even numbers of fields. This result suggests that a history can be described in terms of the values of suitable composite operators, rather than those of the fundamental field. If products to all orders are specified (binary, quartet, sextet, etc), then the description of the history is exhaustive, and different histories do not decohere. On the other hand, when some products are not specified, or when the information of higher correlations are missing, which is often the case in realistic measurement settings, the description is coarse-grained, which can lead to decoherence.

In this work we shall consider coarse-grained histories where the lower field products (binary, quartic) are specified, and higher products are not. Decoherence will mean that the specified composite operators can be assigned definite values with consistent probabilities. Higher composite operators retain their quantum nature, and therefore cannot be assigned definite values. However, their expectation values can be expressed as functionals of the specified correlations by solving the corresponding Dyson equations with suitable boundary conditions. This situation is exactly analogous to that arising from the truncated BBGKY hierarchy, where the molecular chaos assumption allows the expression of higher distribution functions as functionals of lower ones (e.g., [5, 6]).

For those products of fields which assume definite values with consistent probabilities, these values can be introduced as stochastic variables in the dynamical equations for the other quantities of interest (usually of lower correlation order). This approach would provide a theoretical basis for the derivation of the equations of classical stochastic dynamics from quantum fields. It can offer a justification (or refutation) for a procedure commonly assumed but never proven in some popular theories like stochastic inflation [36, 37]. Moreover, since in general we shall obtain nontrivial ranges of values for the specified products with nonvanishing probabilities, it can be said that our procedure captures both the average values of the field products and the fluctuations around this average. The statistical nature of these fluctuations is a subject of great interest in itself [26].

There is another conceptual issue that our approach may help to clarify. As we have already noted, in the system-bath split approach to coarse-graining, as well as in related procedures, it is crucial to introduce a hierarchical order among the degrees of freedom of the system, in such a way that some of them may be considered relevant,
and others irrelevant. While it is often the case that the application itself suggests which notions of relevance may lead to an interesting theory, in a quantum cosmological model, which purports to be a “first principles” description of our Universe, all these choices are, in greater or lesser degree, arbitrary. Since correlation functions already have a “natural” built-in hierarchical ordering, in this approach the ‘arbitrariness’ is reduced to deciding on which level this hierarchy is truncated, and that in turn is determined by the degree of precision one carries out the measurement. In most cases one still needs to show the robustness of the macroscopic result against the variance of the extent of coarse-graining, and exceptional situations do exist (an example is the long time-tail relaxation behavior in multiple particle scattering of dense gas, arising from a failure of the simple molecular chaos assumption). But in general terms correlational coarse-graining seems to us a less ad hoc procedure compared to the commonly used system-bath splitting and coarse-graining.

This paper is organized as follows: In Sec. 2 we discuss the implementation of our procedure for the simple case of a $\lambda \Phi^4$ theory in flat space-time. We then derive the formulae for the quantum amplitude associated with a set of correlation histories and the decoherence functional between two such histories. In Sec. 3 we discuss the decoherence of correlation histories between binary histories and derive the classical stochastic source describing the effect of higher-order correlations on the lower-order ones, arriving at a Langevin equation for classical stochastic dynamics. In Sec. 4 we summarize our findings.

2. Quantum Amplitudes for Correlation Histories and Effective Action

2.1. Quantum Mechanical Amplitudes for Correlation Histories

In this section, we shall consider the quantum mechanical amplitudes associated with different histories for a $\lambda \Phi^4$ quantum field theory, defined in terms of the values of time-ordered products of even numbers of fields at various space-time points. Let us begin by motivating our ansatz for the amplitudes of these correlation histories.

In the conceptual framework of decoherent histories [4], the “natural” exhaustive specification of a history would be to define the value of the field $\Phi(x)$ at every space-time point. These field values are complex numbers. The quantum mechanical amplitude for a given history is $\Psi[\Phi] \sim e^{iS[\Phi]}$, where $S$ is the classical action. The decoherence functional between two different specifications is given by $D[\Phi, \Phi'] \sim \Psi[\Phi]\Psi[\Phi']^*$. Since $|D[\Phi, \Phi']| \equiv 1$, there is never decoherence between these histories.

A coarse-grained history would be defined in general through a “filter function” $\alpha$, which is basically a Dirac $\delta$ function concentrated on the set of exhaustive histo-
ries matching the specifications of the coarse-grained history. For example, we may have a system with two degrees of freedom $x$ and $y$, and define a coarse-grained history by specifying the values $x_0(t)$ of $x$ at all times. Then the filter function is $\alpha[x, y] = \prod_{t \in R} \delta(x(t) - x_0(t))$. The quantum mechanical amplitude for the coarse-grained history is defined as

$$\Psi[\alpha] = \int D\Phi \ e^{iS[\Phi]}$$

(1)

where the information on the quantum state of the field is assumed to have been included in the measure and/or the boundary conditions for the functional integral. The decoherence functional for two coarse-grained histories is

$$D[\alpha, \alpha'] = \int D\Phi D\Phi' e^{i(S(\Phi) - S(\Phi'))} \alpha[\Phi] \alpha'[\Phi']$$

(2)

In this path integral expression, the two histories $\Phi$ and $\Phi'$ are not independent; they assume identical values on a $t = T = $ constant surface in the far future. Thus, they may be thought of as a single, continuous history defined on a two-branched “closed time-path” \[39, 40, 41, 42\], the first branch going from $t = -\infty$ to $T$, the second from $T$ back to $-\infty$. Alternatively, we can think of $\Phi = \Phi^1$ and $\Phi' = \Phi^2$ as the two components of a field doublet defined on ordinary space time \[8\], whose classical action is $S[\Phi^a] = S[\Phi^1] - S[\Phi^2]$. This notation shall be useful later on.

Let us try to generalize this formalism to correlation histories. We begin with the simplest case, where only binary products are specified. In this case a history is defined by identifying a symmetric kernel $G(x, x')$, which purports to be the value of the product $\Phi(x)\Phi(x')$ in the given history, both $x$ and $x'$ defined in Minkowsky space - time. By analogy with the formulation above, one would write the quantum mechanical amplitude for this correlation history as

$$\Psi[G] = \int D\Phi \ e^{iS} \prod_{x > x'} \delta(\Phi(x)\Phi(x') - G(x, x'))$$

(3)

(In this equation, we have introduced a formal ordering of points in Minkowsky space - time, simply to avoid counting the same pair twice.)

But this straightforward generalization for the correlation history amplitude is unsatisfactory on at least two counts. First, it assumes that the given kernel $G$ can actually be decomposed (maybe not uniquely) as a product of c number real fields at different locations; however, we wish to define amplitudes for kernels (such as the Feynman propagator) which do not have this property. Second, (which is related to the first point,) it is ambiguous, since we do not have a unique way to express higher even products of fields in terms of binary products, and thus of applying the $\delta$ function constraint.
To give an example of this, observe that, should we expand the exponential of the action in powers of the coupling constant $\lambda$, the second order term $\int dx \, dx' \Phi(x)^4 \Phi(x')^4$ could become, after integration over the delta function, either
\[
\int dx \, dx' \, G(x, x)^2 G(x', x')^2, \\
\int dx \, dx' \, G(x, x)^4,
\]
or any other combination; of course, if $G$ could be decomposed as a product of fields, this would be unimportant.

Let us improve on these shortcomings. The general idea is to accept Eq. (3) as the definition of the amplitude in the restricted set of kernels where it can be applied, and to define the amplitude for more general kernels through some process of analytical continuation. To this end, we must rewrite the quantum mechanical amplitude in a more transparent form, which we achieve by using an integral representation of the $\delta$ function. Concretely, we redefine
\[
\Psi[G] = \int DK \int D\Phi \, e^{iS + \frac{i}{2} \int dx dx' \, K(x, x') (\Phi(x) \Phi(x') - G(x, x'))}
\]
where the filter function in the Gell-Mann Hartle scheme is replaced by an integration over “all” symmetric non-local sources $K$. Eq. (3) is not yet a complete definition, since one must still specify both the path and the measure to be used in the $K$ integration. Performing the integration over fields, we obtain
\[
\Psi[G] = \int DK \, e^{i(W[K] - (1/2)KG)}
\]
where $W[K]$ is the generating functional for connected vacuum graphs with $\lambda \Phi^4$ interaction, and $(\Delta^{-1} - K)^{-1}$ for propagator (see below). Here $\Delta^{-1} = -\nabla^2 + m^2$ is the free propagator for our scalar field theory (our sign convention for the flat space-time metric is $- + + +$).

The path integral over kernels can be computed through functional techniques. For example, for a free field, $\lambda = 0$,
\[
W[K] = -i \ln \text{Det}[(\Delta^{-1} - K)^{-1/2}] + \text{constant}
\]
Through the change of variables
\[
(\Delta^{-1} - K) = \kappa G^{-1}
\]
we obtain
\[ \Psi[G] = \text{constant} \left[ \det G \right]^{-1/2} e^{(-i/2)\Delta^{-1}G} \]  

When the self coupling \( \lambda \) is not zero, the evaluation of \( \Psi[G] \) is more involved; however, if we are interested in the leading behavior of the amplitude only, we can simply evaluate the functional integral over \( K \) by saddle point methods. The saddle lies at the solution to

\[ \frac{\partial W[K]}{\partial K} = \frac{1}{2} G \]  

We recognize immediately that the exponent, evaluated at the saddle point, is simply the 2 Particle Irreducible (2PI) effective action \( \Gamma \), with \( G \) as propagator (see below). Including also the integration on gaussian fluctuations around the saddle, we find

\[ \Psi[G] \sim \left[ \det \left\{ \frac{\partial^2 \Gamma}{\partial G^2} \right\} \right]^{1/2} e^{i\Gamma[G]} \]  

This is our main result.

As a check, it is interesting to compare the saddle method expression with our exact result for free fields. For a free field \( \Gamma[G] = (-i/2) \ln \det G - (1/2) \Delta^{-1} G \), and therefore \( \Gamma,G = (-i/2)(G^{-1} - i\Delta^{-1}) \), \( \Gamma,G,G = (i/2)G^{-2} \), so

\[ \left[ \det \left\{ \frac{\partial^2 \Gamma}{\partial G^2} \right\} \right]^{1/2} e^{i\Gamma[G]} = \left[ \det G \right]^{-1/2} \left[ \det G \right]^{1/2} e^{(-i/2)\Delta^{-1}G} \]  

which is exactly the earlier result, Eq. (9).

### 2.2. Quantum Amplitudes and Effective Actions

Eq. (11) is the natural generalization to correlation histories of the quantum mechanical amplitude \( e^{iS} \) associated to a field configuration. Let us consider its physical meaning.

The effective action is usually introduced in Field Theory books [43] as a compact device to generate the Feynman graphs of a given theory. Indeed, all Feynman graphs appear in the expansion of the generating functional

\[ Z[J] = \int D\Phi e^{i(S + J\Phi)} \]  

in powers of the external source \( J \) [here, \( J\Phi = \int d^4 x \ J(x)\Phi(x) \)]. \( Z \) has the physical meaning of a vacuum persistence amplitude: it is the amplitude for the in vacuum (that is, the vacuum in the distant past) to evolve into the out vacuum (the vacuum...
in the far future) under the effect of the source $J$. Thus, after proper normalization, $|Z|$ will be unity when the source is unable to create pairs out of the vacuum, and less than unity otherwise.

A more compact representation of the Feynman graphs is provided by the functional $W[J] = -i \ln Z[J]$; the Taylor expansion of $W$ contains only connected Feynman graphs. Thus $W$ developing a (positive) imaginary part signals the instability of the vacuum under the external source $J$.

The external source will generally drive the quantum field $\Phi$ so that its matrix element

$$
\phi(x) = \frac{\langle 0_{\text{out}} | \Phi(x) | 0_{\text{in}} \rangle}{\langle 0_{\text{out}} | 0_{\text{in}} \rangle}
$$

between the in and out vacuum states will not be zero. Indeed, it is easy to see that

$$
\phi = \frac{\partial W}{\partial J}
$$

The transformation from $J$ to $\phi$ is generally one to one, and thus it is possible to consider the matrix element, and not the source, as the independent variable. This is achieved by submitting $W$ to a Legendre transformation, yielding the effective action $\Gamma[\phi] = W[J] - J\phi$ [$J$ and $\phi$ being related through Eq. (15)]. This equation can be inverted to yield the dynamic law for $\phi$

$$
\frac{\partial \Gamma}{\partial \phi} = -J
$$

Eq. (16) shows that $\Gamma$ may be thought of as a generalization of the classical action, now including quantum effects. In the absence of external sources, the in and out vacua agree, so $\phi$ becomes a true expectation value; its particular value is found by extremizing the effective action. Indeed, in this case it can be shown that $\Gamma$ is the energy of the vacuum.

$\Gamma[\phi]$ can be defined independently of the external source through the formula

$$
\Gamma[\phi] = S[\phi] + (i/2) \ln \text{Det} \left( \frac{\partial^2 S}{\partial \phi^2} \right) + \Gamma_1[\phi]
$$

where $\Gamma_1$ represents the sum of all one particle irreducible (1PI) vacuum graphs of an auxiliary theory whose classical action is obtained from expanding the classical action $S[\phi + \varphi]$ in powers of $\varphi$, and deleting the constant and linear terms. Eq. (17) shows that $\Gamma$ is related to the vacuum persistence amplitude of quantum fluctuations around the matrix element $\phi$. Therefore, an imaginary part in $\Gamma$ also signals a vacuum instability. This situation closely resembles the usual approach to tunneling
and phase transitions, where an imaginary part in the free energy signals the onset of instability \[45\].

Observe that each of the transformations from \(Z\) to \(W\) to \(\Gamma\) entails a drastic simplification of the corresponding Feynman graphs expansions, from all graphs in \(Z\) to connected ones in \(W\) and to 1PI ones in \(\Gamma\). Roughly speaking, it is unnecessary to include non 1PI graphs in the effective action, because the sum of all one-particle insertions is already prescribed to add up to \(\phi\). Now the process can be continued: if we could fix in advance the sum of all self energy parts, then we could write down a perturbative expansion where only 2PI Feynman graphs need be considered. This is achieved by the 2PI effective action \[46\].

Let us return to Eq. (13), and add to the external source a space-time dependent mass term

\[
Z[J,K] = \int D\Phi e^{i(S+J\Phi+(1/2)\Phi^2)}
\]

where \(\Phi K\Phi = \int dx \, dx' \, \Phi(x)K(x,x')\Phi(x')\). Also define \(W[J,K] = -i \ln Z[J,K]\). Then the variation of \(W\) with respect to \(J\) defines the in-out matrix element of the field, as before, but now we also have

\[
\frac{\partial W}{\partial K}(x,x') = \frac{1}{2}[\phi(x)\phi(x') + GF(x,x')]
\]

where \(GF\) represents the Feynman propagator of the quantum fluctuations \(\varphi\) around the matrix element \(\phi\). As before, it is possible to adopt \(G\) as the independent variable, instead of \(K\). To do this, we define the 2PI effective action (in schematic notation)

\[
\Gamma[\phi,GF] = W[J,K] - J\phi - (1/2)K[\phi^2 + GF].
\]

Variation of this new \(\Gamma\) yields the equations of motion \(\Gamma,\phi = -J - K\phi\), \(\Gamma,GF = -(1/2)K\).

We can see that the 2PI effective action generates the dynamics of the Feynman propagator, and in this sense it plays for it the role that the classical action plays for the field. In this sense we can say that Eq. (14) generalizes the usual definition of quantum mechanical amplitudes.

The perturbative expansion of the 2PI effective action reads \[46\]

\[
\Gamma[\phi,GF] = S[\phi] + (i/2) \ln \text{Det}G_{F}^{-1} + \left(\frac{1}{2}\right) \text{Tr} \frac{\partial^2 S}{\partial \phi^2} G_{F} + \Gamma_2[\phi,GF] + \text{constant}
\]

where \(\Gamma_2\) is the sum of all 2PI vacuum graphs of the auxiliary theory already considered, but with \(GF\) as propagator in the internal lines. As we anticipated, to replace \(GF\) for the perturbative propagator amounts to adding all self energy insertions, and therefore no 2PI graph needs be explicitly included.
Like its 1PI predecessor, the 2PI effective action has the physical meaning of a vacuum persistence amplitude for quantum fluctuations $\varphi$, constrained to have vanishing expectation value and a given Feynman propagator. Therefore, an imaginary part in the 2PI effective action also signals vacuum instability.

The description of the dynamics of a quantum field through both $\phi$ and $G_F$ simultaneously, rather than $\phi$ alone, is appealing not only because it allows one to perform with little effort the resummation of an infinite set of Feynman graphs, but also because for certain quantum states, it is possible to convey statistical information about the field through the nonlocal source $K$. This information is subsequently transferred to the propagator. For this reason, the 2PI effective action formalism is, in our opinion, a most suitable tool to study statistical effects in field theory, particularly for out-of-equilibrium fields [9, 47]. In our earlier studies the object of interest is the on-shell effective action, that is, the effective action for propagators satisfying the equations of motion. Here, in Eq. (11), we find a relationship between the quantum mechanical amplitude for a correlation history and the 2PI effective action which does not assume any restriction on the propagator concerned.

### 2.3. Quantum Amplitudes for More General Correlation Histories: 2PI CTP Effective Action

One of the peculiarities of the ansatz Eq. (3) for the amplitude of a correlation history is that the kernel $G$ must be interpreted as a time-ordered binary product of fields. This results from the known feature of the path integral, which automatically time orders any monomials occurring within it. Before we proceed to introduce the decoherence functional for correlation histories, it is convenient to discuss how this restriction could be lifted, as well as the restriction to binary products.

The time ordering feature of the path integral is also responsible for the fact that the c-number field $\phi$ in Sec. 2.2 is a matrix element, rather than a true expectation value. As a matter of fact, the Feynman propagator $G_F$ discussed in the previous section is also a matrix element

$$G_F(x, x') = \frac{\langle 0_{out}|T[\varphi(x)\varphi(x')]|0_{in}\rangle}{\langle 0_{out}|0_{in}\rangle} \tag{21}$$

Because $\phi$ and $G_F$ satisfy mixed boundary conditions, the dynamic equations resulting from the 2PI effective action are generally not causal. This drawback has placed limitations in their physical applications.

Schwinger [39] has introduced an extended effective action, whose arguments are true expectation values with respect to some in quantum state. Because the dynamics of these expectation values may be formulated as an initial value problem, the
equations of motion resulting from the Schwinger-Keldysh effective action are causal. Schwinger’s idea is also the key to solving the restrictions in our definition of quantum amplitudes for correlation histories.

Schwinger’s insight was to apply the functional formalism we reviewed in Sec. 2.2 to fields defined on a “closed time-path”, composed of a “direct” branch $-T \leq t \leq T$, and a “return” branch $T \geq t \geq -T$ (with $T \to \infty$) [38, 40]. Actually, we have already encountered this kind of path in the discussion of the decoherence functional for coarse-grained histories. Since the path doubles back on itself, the in vacuum is the physical vacuum at both ends; the formalism may be generalized to include more general initial states, but we shall not discuss this possibility [9].

The closed time-path integral time-orders products of fields on the direct branch, anti-time-orders fields on the return branch, and places fields on the return branch always to the left of fields in the direct branch. To define the closed time-path generating functional, we must introduce two local sources $J_a$, and four nonlocal ones $K_{ab}$ (as in Sec. 2.1 an index $a,b = 1$ denotes a point on the first branch, while an index 2 denotes a point on the return part of the path). These sources are conjugated to c number fields $\phi^a$ and propagators $G_{ab}$, which stand for $\langle 0\text{in}|\Phi(x)\Phi(x')|0\text{in}\rangle$ and $\langle 0\text{in}|\phi^a(x)\phi^b(x')|0\text{in}\rangle$. Explicitly, decoding the indices, the propagators are defined as (here and from now on, we assume that the background fields $\phi^a$ vanish):

\begin{align*}
G^{11}(x,x') &= \langle 0\text{in}|T[\Phi(x)\Phi(x')]|0\text{in}\rangle \\
G^{12}(x,x') &= \langle 0\text{in}|\Phi(x')\Phi(x)|0\text{in}\rangle \\
G^{21}(x,x') &= \langle 0\text{in}|\Phi(x)\Phi(x')|0\text{in}\rangle \\
G^{22}(x,x') &= \langle 0\text{in}|(T[\Phi(x)\Phi(x')])^{\dagger}|0\text{in}\rangle
\end{align*}

They are, respectively, the Feynman, negative- and positive-frequency Wightman, and Dyson propagators. The definition of the closed time-path (CTP) or in-in 2PI effective action follows the same steps as the ordinary effective action discussed in the previous section, except that now, besides space-time integrations, one must sum over the discrete indexes $a,b$. These indexes can be raised and lowered with the “metric” $h_{ab} = \text{diag}(1, -1)$. Similarly, the “propagator” to be used in Feynman graph expansions is the full matrix $G^{ab}$, and the interaction terms should be read out of the CTP classical action $S[\Phi^1] - S[\Phi^2]$, discussed in Sec. 2.1.

In the case of vacuum initial conditions, these can be included into the path integral by tilting the branches of the CTP in the complex $t$ plane (the direct branch should
acquire an infinitesimal positive slope, and the return branch, a negative one.

The CTP boundary condition, that the histories at either branch should fit continu-
ously at the surface \( t = T \), may also be explicitly incorporated into the path integral
as follows. We first include under the integration sign a term
\[
\prod_{x \in \mathbb{R}^3} \delta(\Phi^1(x,T) - \Phi^2(x,T))
\]
which enforces this boundary condition; then we rewrite Eq. (26) as
\[
\exp\left\{\frac{(-1/\alpha^2) \int d^3x (\Phi^1(x,T) - \Phi^2(x,T))^2}{\alpha^2}\right\}
\]
where \( \alpha \to 0 \). This term has the form
\[
\exp\{i \int d^4x \, d^4x' K_{ab}(x,x') \Phi^a(x) \Phi^b(x')\},
\]
where
\[
K_{ab}(x,x') = (i/\alpha^2) \delta(x - x') \delta(t - T) [2\delta_{ab} - 1].
\]
In this way, we have traded the boundary condition by an explicit coupling to a non
local external source.

As before, variation of the CTP 2PI effective action yields the equations of motion
for background fields and propagators. The big difference is that now these equations
are real and causal.

We can now see how the CTP technique solves the ordering problem in the definition
of quantum amplitudes for correlation histories. One simply considers the specified
kernels as products of fields defined on a closed time-path. In this way, we may define
up to four different kernels \( G^{ab} \) independently, to be identified with the four different
possible orderings of the fields (for simplicity, we assume the background fields are
kept equal to zero). If the kernels \( G^{ab} \) can actually be decomposed as products of
c-number fields on the CTP, then we associate to them the quantum amplitude
\[
\Psi[G^{ab}] = \int D\Phi^a e^{iS} \prod_{x \neq x', ab} \delta(\Phi^a(x) \Phi^b(x') - G^{ab}(x, x'))
\]
(\( S \) stands for the CTP classical action) The path integral can be manipulated
as in Sec. 2.1 to yield
\[
\Psi[G^{ab}] \sim \left[\text{Det}\left(\frac{\partial^2 \Gamma}{\partial G^{ab} \partial G^{cd}}\right)\right]^{(1/2)} e^{i\Gamma[G^{ab}]}
\]
where $\Gamma$ stands now for the CTP $2\Pi$ effective action. This last expression can be analytically extended to more general propagator quartets, and, indeed, even to kernels which do not satisfy the relationships $G^{11}(x, x') = G^{21}(x, x') = G^{12*}(x, x') = G^{22*}(x, x')$ for $t \geq t'$, which follow from their interpretation as field products.

Quantum amplitudes for correlation histories including higher order products are defined following a similar procedure. For example, four particle correlations are specified by introducing 16 kernels $[9]$

$$G_{abcd} \sim \Phi^a \Phi^b \Phi^c \Phi^d - G^{ab} G^{cd} - G^{ac} G^{bd} - G^{ad} G^{bc}$$ (32)

If the new kernels are simply products of the binary ones, then the amplitude is given by

$$\Psi[G^{ab}, G^{abcd}] = \int D\Phi^a e^{iS} \prod_{ab} \delta(\Phi^a \Phi^b - G^{ab}) \prod_{abcd} \delta(\Phi^a \Phi^b \Phi^c \Phi^d - G^{ab} G^{cd} - G^{ac} G^{bd} - G^{ad} G^{bc} - G^{abcd})$$ (33)

(In the last two equations, we have included the space-time index $x$ and the branch index $a$ into a single multi-index). Here, each pair appears only once in the product, as well as each quartet $abcd$. Exponentiating the $\delta$ functions we obtain

$$\Psi[G^{ab}, G^{abcd}] = \int DK_{abcd} \int DK_{ab} \int D\Phi \exp\{i[S + \frac{1}{2} K_{ab}(\Phi^a \Phi^b - G^{ab})
+ \frac{1}{24} K_{abcd}(\Phi^a \Phi^b \Phi^c \Phi^d - G^{ab} G^{cd} - G^{ac} G^{bd} - G^{ad} G^{bc} - G^{abcd})]\}$$ (34)

Now the integral over fields yields the CTP generating functional for connected graphs, for a theory with a non local interaction term. Thus

$$\Psi[G^{ab}, G^{abcd}] = \int DK_{abcd} \int DK_{ab} \exp\{i[W[K_{ab}, K_{abcd}] - \frac{1}{2} K_{ab} G^{ab}
- \frac{1}{24} K_{abcd}(G^{ab} G^{cd} + G^{ac} G^{bd} + G^{ad} G^{bc} + G^{abcd})]\}$$ (35)

The integral may be evaluated by saddle point methods, the saddle being the solution to $W_{,K_{ab}} = (1/2)G^{ab}$, $W_{,K_{abcd}} = \frac{1}{24}(G^{ab} G^{cd} + G^{ac} G^{bd} + G^{ad} G^{bc} + G^{abcd})$. To evaluate the exponential at the saddle is the same as to perform a Legendre transform on $W$—it yields the higher order CTP effective action $\Gamma[G^{ab}, G^{abcd}]$. Variation of $\Gamma$ yields the equation of motion for its arguments, which are also the inversion of the saddle
Thus up to quartic correlations, the quantum mechanical amplitude is given by

$$\Psi[G^{ab}, G^{abcd}] \sim e^{i\Gamma[G^{ab}, G^{abcd}]}$$

(37)

This expression can likewise be extended to more general kernels.

As a check on the plausibility of this result, let us note the following point. Since quantum mechanical amplitudes are additive, it should be possible to recover our earlier ansatz Eq. (11) for binary correlation histories from the more general result Eq. (37), by integration over the fourth order kernels. Within the saddle point approximation, integration amounts to substituting these kernels by the solution to the second Eq. (36) for the given $G^{ab}$, with $K^{abcd} = 0$, and with null initial conditions. (Indeed, since initial conditions can always be included as delta function-like singularities in the external sources, the third condition is already included in the second.) This procedure effectively reduces the fourth order effective action to the 2PI CTP one [9], as we expected.

A basic point which emerges here relevant to our study of decoherence is that, while quantum field theory is unitary and thus time reversal invariant, the evolution of the propagators derived from the 2PI CTP effective action is manifestly irreversible [9, 49]. The key to this apparent paradox is that, while the evolution equations are indeed time reversal invariant, when higher order kernels are retained as independent variables, their reduction to those generated by the 2PI effective action involves the imposition of trivial boundary conditions in the past. Thus the origin of irreversibility in the two point functions is the same as in the BBGKY formulation in statistical mechanics [3]. The lesson for us in the present context is that there is an intrinsic connection between dissipation and decoherence [26, 23]. Knowledge that the evolution of the propagators generated by the 2PI effective action is generally dissipative leads us to expect that histories defined through binary correlations will usually decohere. We proceed now to a detailed study of this point.
3. Decoherence of Correlation Histories

3.1. Decoherence Functional for Correlation Histories

Having found an acceptable ansatz for the quantum mechanical amplitude associated with a correlation history, we are in a position to study the decoherence functional between two such histories. As was discussed in the Introduction, if the decoherence functional is diagonal, then correlation histories support a consistent probability assignment, and may thus be viewed as classical (stochastic) histories.

For concreteness, we shall consider the simplest case of decoherence among histories defined through (time-ordered) binary products. Let us start by considering two histories, associated with kernels \( G(x, x') \) and \( G'(x, x') \), which can in turn be written as products of fields. Taking notice of the similarity between the quantum amplitudes Eqs. (1) and (3), we can by analogy to Eq. (2) define the decoherence functional for second correlation order as

\[
D[G, G'] = \int d\Phi d\Phi' e^{i(S[\Phi] - S[\Phi'])} \prod_{x \gg x'} \delta((\Phi(x)\Phi(x') - G(x, x'))\delta((\Phi'(x)\Phi'(x') - G'^{*}(x, x')))
\]

(38)

Recalling the expression Eq. (30) for the quantum amplitude associated with the most general binary correlation history, we can rewrite Eq. (38) as

\[
D[G, G'] = \int DG^{12} DG^{21} \Psi[G^{11} = G, G^{22} = G'^{*}, G^{12}, G^{21}] (39)
\]

This expression for the decoherence functional can be extended to arbitrary kernels.

In the spirit of our earlier remarks, we use the ansatz Eq. (31) for the CTP quantum amplitude and perform the integration by saddle point methods to obtain

\[
D[G, G'] \sim e^{i\Gamma[G^{11} = G, G^{22} = G'^{*}, G^{12}, G^{21}]} (40)
\]

where the Wightman functions are chosen such that

\[
\frac{\partial \Gamma}{\partial G^{12}} = \frac{\partial \Gamma}{\partial G^{21}} = 0 (41)
\]

for the given values of the Feynman and Dyson functions. These last two equations are the sought-for expression for the decoherence functional.

As an application, let us study the decoherence functional for Gaussian fluctuations around the vacuum expectation value (VEV) of the propagators for a \( \lambda \Phi^4 \) theory,
carrying the calculations to two-loop accuracy. Gaussian fluctuations means that we only need the closed time-path 2PI effective action to second order in the fluctuations $\delta G^{ab} = G^{ab} - \Delta_0^{ab}$, where $\Delta_0^{ab}$ stands for the VEVs. Since the effective action is stationary at the VEV, there is no linear term. Formally

$$\Gamma[\delta G^{ab}] = (1/2)\{\Gamma_{(aa),(bb)}\delta G^{aa}\delta G^{bb} + 2\Gamma_{(a\neq b),(cc)}\delta G^{a\neq b}\delta G^{cc} + \Gamma_{(a\neq b),(c\neq d)}\delta G^{a\neq b}\delta G^{c\neq d}\}$$

so the saddle point equations (41) become

$$\{\Gamma_{(a\neq b),(c\neq d)}\} \delta G^{c\neq d} = -\Gamma_{(a\neq b),(ee)}\delta G^{ee}$$

The formal Feynman graph expansion of the 2PI effective action is given in Eq. (20). To two-loop accuracy, we find

$$\Gamma_2[G^{ab}] = -\frac{\lambda}{8} h_{abcd} \int d^4x \ G^{ab}(x,x)G^{cd}(x,x)$$

$$+ \frac{i\lambda^2}{48} h_{abcd} h_{efgh} \int d^4x \ d^4x' G^{ae}(x,x')G^{af}(x,x')G^{cg}(x,x')G^{dh}(x,x')$$

where $h_{ab}, h_{abcd} = 1$ if $a = b = c = d = 1$, $-1$ if $a = b = c = d = 2$, and vanish otherwise.

Computing the necessary derivatives, we find

$$\frac{\partial^2 \Gamma}{\partial G^{ab}(x,x') \partial G^{cd}(x'',x''')} =$$

$$\left(\frac{-1}{2}\right)[i(G^{-1})_{ac}(x,x'')(G^{-1})_{db}(x'',x')]$$

$$+ (1/2)\lambda h_{abcd} \delta(x' - x) \delta(x'' - x) \delta(x''' - x)$$

$$- (i/2)\lambda^2 h_{ac} h_{bdf} \delta(x'' - x) \delta(x''' - x') \delta_e(x'') \delta_f(x'') \delta_0^{eg}(x,x') \delta_0^{df}(x,x')$$

These derivatives are evaluated at $G^{ab} = \Delta_0^{ab}$, where

$$(\Delta_0^{-1})_{ab}(x,x') = i[h_{ab}(-\nabla^2 + m^2 - i h_{ab} \epsilon) \delta(x' - x)$$

$$+ (\lambda/2) h_{abcd} \delta(x' - x) \Delta_0^{cd}(x,x)$$

$$- (i/6)\lambda^2 h_{ac} h_{bdf} \Delta_0^{ef}(x,x') \Delta_0^{cg}(x,x') \Delta_0^{dh}(x,x')]$$

$$+ \frac{1}{2}\alpha^2 \delta(x' - x) \delta(t - T)[2\delta_{ab} - 1]$$

where it is understood that the limits $\epsilon, \alpha \to 0, T \to \infty$ are taken. The first infinitesimal is included to enforce appropriate Feynman/Dyson orderings, the second to carry the CTP boundary conditions in the far future.
In computing the Feynman graphs in these expressions, the usual divergences crop up. They may be regularized and renormalized by standard methods, which we will not discuss here. The “tadpole” graph \( \Delta_{0}^{cd}(x,x) \) can be made to vanish by a suitable choice of the renormalization point, which we shall assume.

Let us narrow our scope to a physically meaningful set of histories, namely, those describing ensembles of real particles distributed with a position-independent spectrum \( f(k) \), \( k \) being the four momentum vector. Such ensembles are described by propagators [9]

\[
\delta G(x, x') = 2\pi \int \left( \frac{d^4k}{(2\pi)^4} \right) e^{ik(x-x')}\delta(k^2 + m^2)f(k)
\] (47)

The distribution functions \( f \) are real, positive, and even in \( k \). We wish to analyze under what conditions it is possible to assign consistent probabilities to different spectra \( f(k) \). To this end we must compute the decoherence functional between the propagator in Eq. (47) and another, say, associated with a function \( f' \).

Let us begin by investigating Eqs (43) for the missing propagators \( G^{12} \) and \( G^{21} \). We shall first disregard the boundary condition enforcing terms in these equations, introducing them at a later stage. When this is done, the right hand side of Eqs. (43) vanishes, since \((-\nabla^2 + m^2)G^{aa}(x, x') \equiv 0 \) in the present case.

On the other hand, we only need the left hand side to zeroth order in \( \lambda \), since any other term would be of too high an order to contribute to the decoherence functional at the desired accuracy. With this in mind, Eq. (43) reduces to the requirement that the unknown propagators should be homogeneous solutions to the Klein-Gordon equation on both of their arguments.

To determine the proper boundary conditions for these propagators, we may consider the boundary terms in Eq. (42), or else appeal to their physical interpretation. We shall choose the second approach.

To this end, we observe that the physical meaning of the propagators as (non standard) products of fields, Eqs. (22) to (25), entails the identity \( G^{12} + G^{21} = G^{11} + G^{22} \), which is consistent in this case, since both sides solve the Klein-Gordon equation. Actually, this identity is satisfied by the VEV propagators, so it can be imposed directly on their variations.

Physically, a change in the propagators reflects a corresponding change in the statistical state of the field. To zeroth order in the coupling constant, however, the commutator of two fields is a c-number, and does not depend on the state. Therefore, to this accuracy, \( G^{12} - G^{21} \) should not change; that is, \( \delta G^{12} \) should be equal to \( \delta G^{21} \). We thus conclude that the correct solution to Eq. (43) is
\[ \delta G^{12} = \delta G^{21} = \left( \frac{1}{2} \right) \{ \delta G^{11} + \delta G^{22} \} \]  

(48)

Consideration of the CTP boundary conditions would have led to the same result.

We may now evaluate the second variation of the 2PI CTP effective action, Eq. (42). We should stress that the Klein-Gordon operator annihilates all propagators involved, and that the \( O(\lambda) \) term in \( \Delta_0^{-1} \) vanishes because of our choice of renormalization point. Therefore the second (mixed) term in Eq. (42) is of higher than second order and may be disregarded. The same holds for terms of the form \( (\Delta_0)^{-1}_a \delta G^{ad} (\Delta_0)^{-1}_b \delta G^{ab} \), disregarding boundary terms.

The remaining terms can be read out of Eq. (45), with the input of the “fish” graph \[43, 9\]

\[ \Sigma(x, x') = (\Delta_0^{11})^2(x, x') = \frac{i\mu'}{(4\pi)^2} \int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')} \left[ \frac{\sigma^2}{\epsilon} + \ln \frac{m^2}{4\pi \mu^2} - \psi(1) \right] \]

\[ -k^2 \int_{4m^2}^{\infty} \frac{d\sigma^2}{\sigma^2(\sigma^2 + k^2 + \epsilon)} \sqrt{1 - \frac{4m^2}{\sigma^2}} \]

(49)

where \( \epsilon = d - 4 \) and \( \mu \) is the renormalization scale. Clearly, the local terms in \( \Sigma \) can be absorbed into a coupling-constant renormalization.

The important thing for us to realize is that the \( O(\lambda) \) terms in Eq. (15), as well as the imaginary part of \( \Sigma \), contribute only to the phase of the decoherence functional, and thus are totally unrelated to decoherence. The only contribution to a decoherence effect comes from the real part of \( \Sigma \). Reading it out of Eq. (49), we obtain the sought for result

\[ |D[f, f']| \sim \exp \left\{ \frac{-\pi \lambda^2}{8} \int \frac{d^4p}{(2\pi)^8} \frac{d^4q}{(2\pi)^8} \delta(p^2 + m^2) \delta(q^2 + m^2) \left[ (f(p) - f'(p))(f(q) - f'(q)) \theta[-((p + q)^2 + 4m^2)] \right] \sqrt{1 + \frac{4m^2}{(p + q)^2}} \right\} \]

(50)

where \( \theta \) is the usual step function. As expected, we do find decoherence between different correlation histories. Moreover, decoherence is related to dissipative processes, which in this case arise from pair production \([49]\). Indeed, the real part of the kernel \( \Sigma \) is essentially the probability of a real pair being produced out of quanta with momenta \( p \) and \( q \), with \( p + q = k \) \([43]\).
Let us mention two obvious consequences of our result for the decoherence functional. The first point is that decoherence is associated with instability of the vacuum: the distribution functions whose overlap is suppressed represent ensembles which are unstable against non trivial scattering of the constituent particles. This scattering produces correlations between particles. Therefore, truncation of the correlation hierarchy leads to an explicitly dissipative evolution. This would not be the case if there were no scattering.

The second point is that $|D|$ remains unity on the diagonal. Thus, at least for Gaussian fluctuations, and to two-loop accuracy, all histories are equally likely. What this means physically is that the two-point functions to be perceived by an observer after the quantum to classical transition need not be close to their VEV in any stringent sense. Indeed, what is observed will not even be “vacuum fluctuations” in the proper sense of the word; they are real physical particles whose momenta are on shell, and may propagate to the asymptotic region, if they manage not to collide with other particles.

### 3.2. Beyond Coarse Graining

For the observer confined to a single consistent history, as is the case for the quantum cosmologist, questioning the probability distribution of histories is somewhat academic. What would be relevant is one’s ability to predict the future behavior of one’s particular history. This ability is impaired by the lack of knowledge about the coarse-grained elements of the theory, which, in our case, are the higher correlations of the field.

As we have already seen, variation of the 2PI CTP effective action, i.e., of the phase of the decoherence functional, yields the evolution equations for the VEVs of the two-point functions. These equations should be regarded as the Hartree-Fock approximation to the actual evolution, since in them the effect of higher correlations is represented only in the average. Deviations of the actual evolution from this ideal average may be represented by adding a source term to the Hartree-Fock equation. As the detailed state of the higher correlations is unknown, this right hand side should take the form of a stochastic binary external source.

The non-diagonal terms of the decoherence functional represented in Eq. (50), while not contributing to the Hartree-Fock equations, contain the necessary information to build a phenomenological model of the back reaction of the higher correlations on the relevant sector. To build this model, we compare the actual form of the decoherence functional against that resulting from the coupling of the propagators to an actual gaussian random external source [51].

The result of this comparison is that higher correlations react on the propagators as
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if these obey a Langevin-type equation

$$\frac{\partial \Gamma[\delta G^{11} = \delta G, \delta G^{22} = \delta G^\star, \delta G^{12}, \delta G^{21}]}{\partial (\delta G(x, x'))} = \frac{-1}{2v} F(x - x') J(x - x')$$  \hspace{1cm} (51)

where, after the variational derivative is taken, we must take the limit \( \delta G' \to \delta G \). In Eq. (51) \( v \) is (formally) “the space-time volume”, the gaussian stochastic source \( J \) has autocorrelation \( \langle J(u) J(u') \rangle = \delta(u - u') \), and

$$F^2(u) = \lambda^2 \int_0^\infty \frac{ds}{(4 \pi s)^2} \int_{4m^2}^{\infty} d\sigma^2 \sin(\sigma^2 - \frac{u^2}{4s}) \sqrt{1 - \frac{4m^2}{\sigma^2}}$$  \hspace{1cm} (52)

Because the limit \( \delta G' \to \delta G \) is taken, the imaginary terms of the CTP effective action reproduced in Eq. (50) do not contribute to the left hand side of Eq. (51); as far as the “Hartree-Fock” equations are concerned, they could as well be deleted from the effective action.

However, the stochastic source in the right hand side of Eq. (51) modifies the quantum amplitude associated with the correlation history by a factor

$$\exp\{\frac{i}{2} \int d^4u \, F(u) J(u) \delta G(u)\},$$  \hspace{1cm} (53)

where \( G(u) = (1/v) \int d^4X \, G(X + (u/2), X - (u/2)) \). Correspondingly, the decoherence functional gains a factor

$$\exp\{\frac{i}{2} \int d^4u \, F(u) J(u) (\delta G(u) - \delta G'(u))\}.$$  \hspace{1cm} (54)

Upon averaging over all possible external sources, each having a probability

$$\exp\{-\frac{1}{2} \int d^4u \, J^2(u)\},$$  \hspace{1cm} (55)

the new factor in the decoherence functional becomes

$$\exp\{-\frac{1}{8} \int d^4u \, F^2(u) (\delta G(u) - \delta G'(u))^2\},$$  \hspace{1cm} (56)

which exactly reproduces Eq. (50). Observe that the assumed form for the right hand side of Eq. (51), and the requirement of recovering Eq. (50) upon averaging, uniquely determines the function \( F \).

In this way, Eq. (51) yields the correct, if only a phenomenological, description of the dynamics of classical fluctuations in the aftermath of the quantum to classical
transition. It should be obvious that nonlinearity is essential to the generation of these fluctuations.

4. Discussion

This paper presents three main results. The first is the ansatz Eq. (11) for the quantum amplitude associated with a correlation history. The second is the ansatz Eq. (40) for the decoherence functional between two such histories. On the basis of this ansatz, we have shown in Eq. (50) that the quantum interference between histories corresponding to different particle spectra is suppressed whenever these spectra differ by particles whose added momenta go above the two particle threshold $4m^2$, $m^2$ being the one-loop radiative-corrected physical mass. The third result is the phenomenological description in Eq. (51) of the dynamics of an individual consistent correlation history.

What we have presented in the above, despite its embryonic form, is a framework for bringing together the correlational-hierarchy idea in non-equilibrium statistical mechanics and the consistent-history interpretation of quantum mechanics. This framework puts decoherence and dissipation due to fluctuations and noise (manifested here through particle creation) on the same footing. It suggests a natural (intrinsic) measure of coarse-graining which is commensurate with ordinary accounts of dissipative phenomena, and with it addresses the issue of quantum to classical transition. It also provides a theoretical basis for the derivation of classical stochastic equations from quantum fluctuations, and identifies the nature of noise in these equations.

It should be noticed that a formal identity exists between the present results and those previously obtained from the influence functional formalism [51, 20, 11]. Indeed, our decoherence functional has the same structure as the influence functional, with the non diagonal terms in Eq. (50) playing the role of the “noise kernel”. This is more than an analogy, as it should be clear from the discussions above and elsewhere.

While for reasons of clarity and economy of space, we have focused on a simple application from quantum field theory to develop our arguments, the implications on quantum mechanics and statistical mechanics go beyond what this example can show. The theoretical issues raised here in the context of quantum mechanics and statistical mechanics, as well as the consequences of problems raised in the context of quantum and semiclassical (especially the inflationary universe) cosmology, which motivated us to make these inquiries in the first place, will be explored in greater detail elsewhere.

This work is part of an on-going program which draws on many year’s worth of pondering on the role of statistical mechanics ideas in quantum cosmology, using quantum field theoretical methods while placing the issues in the larger context of general physics. The project began in 1985, when one of us (EC) was invited by
Dieter Brill to join the General Relativity Group at Maryland. It is therefore an honour and a pleasure for us to dedicate this paper to him on this happy occasion.

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