Iterative Joint Beamforming Training with Constant-Amplitude Phased Arrays in Millimeter-Wave Communications

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Abstract—In millimeter-wave communications (MMWC), in order to compensate for high propagation attenuation, phased arrays are favored to achieve array gain by beamforming. beamforming, where transmitting and receiving antenna arrays need to be jointly trained to obtain appropriate antenna weight vectors (AWVs). Since the amplitude of each element of the AWV is usually constraint constant to simplify the design of phased arrays in MMWC, the existing singular vector based beamforming training scheme cannot be used for such devices. Thus, in this letter, a steering vector based iterative beamforming training scheme, which exploits the directional feature of MMWC channels, is proposed for devices with constant-amplitude phased arrays. Performance evaluations show that the proposed scheme achieves a fast convergence rate as well as a near optimal array gain.

Index Terms—Millimeter wave, beamforming, phased array, beamforming training, 60 GHz.

I. INTRODUCTION

WHILE AROUSING increasing attentions in both academia and industry due to abundant frequency spectrum [1], [2], millimeter-wave communications (MMWC) face the challenge of high propagation attenuation caused by the high frequency. To remedy this, phased arrays can be adopted at both the source and destination devices to exploit array gains and compensate for the propagation loss [1], [2].

To achieve a sufficiently high array gain, the antenna weight vectors (AWVs) at both ends need to be appropriately set prior to signal transmissions, which is the joint Tx/Rx beamforming. If the channel state information (CSI) is available at both ends, the optimum AWVs can be directly found under well-known performance criteria, e.g., maximizing receiving signal-to-noise ratio (SNR) [1], [2]. Unfortunately, since the number of antennas is generally large, the channel estimation becomes time-consuming in MMWC. In addition, the computational complexity is high, because the matrix decomposition, e.g., the singular vector decomposition (SVD), is generally required. Owing to this, the beamforming training approach, which has a lower complexity, becomes more attractive to find the AWVs [3]–[5]. Generally, there are two types of joint Tx/Rx beamforming training schemes. One is switched beamforming training based on a fixed codebook [3]. The codebook contains a number of predefined AWVs. During beamforming training, the AWVs at both ends are examined according to a certain order, and the pair that achieves the largest SNR is selected. The other one is adaptive beamforming training [4], [5], which does not need a fixed codebook. The desired AWVs at both ends are found via real-time joint iterative training. It is clear that the switched beamforming training is simpler, while the adaptive one is more flexible.

Most adaptive beamforming training schemes adopt the same state-of-the-art approach, i.e., finding the best singular vector via iterative training without a priori CSI at both ends [4], [5]. This singular vector based training scheme (SGV) requires that both the amplitudes and phases of the AWV are adjustable. On the other hand, in MMWC, phased arrays are usually implemented with the approach that only phases of the antenna branches are adjustable; the amplitudes are set constant to simplify the design and reduce the power consumption of phased arrays [4], [6], [7]. In fact, even in general multiple-input multiple-output (MIMO) systems [8], antenna branches with constant amplitude (CA) are also an optimization objective to reduce implementation complexity [9], [10]. In such a case, SGV becomes infeasible due to the CA phased array. The schemes proposed in [9], [10] cannot be used here either, because these schemes are designed for transmitting beamforming with full or quantized a priori CSI at only the source device, but for MMWC with CA phased arrays, joint beamforming is required without a priori CSI at both the source and destination devices.

In this letter, a steering vector based joint beamforming training scheme (STV), which exploits the directional feature of MMWC channels, is proposed. Performance comparisons show that for line-of-sight (LOS) channels, both STV and SGV have fast convergence rates and achieve the optimal array gain. On the other hand, for non-LOS (NLOS) channels, STV achieves a faster convergence rate at the cost of a slightly lower array gain than SGV, which can also achieve the optimal array gain. In conclusion, STV achieves a fast convergence rate and a near optimal array gain under both LOS and NLOS channels, which highlights its applicability in practice.

II. SYSTEM AND CHANNEL MODELS

Without loss of generality, we consider a MMWC system with half-wave spaced uniform linear arrays (ULAs) of $M$ and $N$ elements at the source and destination devices, respectively. The ULAs are phased arrays where only the phase can be controlled. A single RF chain is tied to the ULA at each of the

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source and destination devices. According to the measurement results of channels for MMWC in [1], [11], only reflection contributes to generating multipath components (MPCs), while scattering and diffraction effects are negligible due to the extremely small wave length of MMWC. Thus, the MPCs in MMWC have a directional feature, i.e., different MPCs have different physical transmitting steering angles $\phi_{\ell,t}$ and receiving steering angles $\phi_{\ell,r}$. Consequently, the channel model is expressed as

$$\mathbf{H} = \sqrt{NM} \sum_{\ell=0}^{L-1} \lambda_{\ell} \mathbf{g}_{\ell} \mathbf{h}_{\ell}^H,$$

(1)

where $L$ is the number of multipath components, $(\cdot)^H$ is the conjugate transpose operation, $\lambda_{\ell}$ are the channel coefficients, and $\mathbf{g}_{\ell}$ and $\mathbf{h}_{\ell}$ are the receiving and transmitting steering vectors [12], [13] that are given by $\mathbf{g}_{\ell} = \left\{ e^{j \pi (n-1) \Omega_{\ell,t} } / \sqrt{N} \right\}_{n=1,2,\ldots,N}$ and $\mathbf{h}_{\ell} = \left\{ e^{j \pi (m-1) \Omega_{\ell,r} } / \sqrt{M} \right\}_{m=1,2,\ldots,M}$, respectively. Note that $\Omega_{\ell,t}$ and $\Omega_{\ell,r}$ represent the transmitting and receiving cosine angles of the $\ell$th MPC, respectively [13], i.e., $\Omega_{\ell,t} = \cos(\phi_{\ell,t})$ and $\Omega_{\ell,r} = \cos(\phi_{\ell,r})$.

Given the transmitting AWV $\mathbf{t}$ and the receiving AWV $\mathbf{r}$, where $\| \mathbf{t} \| = \| \mathbf{r} \| = 1$, the received signal $y$ is given by $y = \mathbf{r}^H \mathbf{H} \mathbf{s} + \mathbf{r}^H \mathbf{n}$, where $s$ is the transmitted symbol, $\mathbf{n}$ is the noise vector. The target of beamforming training is to find appropriate transmitting and receiving AWVs to obtain a high receiving SNR, which is given by $\gamma = |\mathbf{r}^H \mathbf{H} \mathbf{s}|^2 / \sigma^2$, where $\sigma^2$ is the noise power.

III. SINGULAR-VECTOR BASED SCHEME

Let us introduce the SGV scheme first. It is known that the optimal AWVs to maximize $\gamma$ is the principal singular vectors of the channel matrix $\mathbf{H}$, [4], [5]. Denote the SVD of $\mathbf{H}$ as

$$\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H = \sum_{k=1}^{K} \rho_k \mathbf{u}_k \mathbf{v}_k^H,$$

(2)

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices with column vectors (singular vectors) $\mathbf{u}_k$ and $\mathbf{v}_k$, respectively, $\Sigma$ is an $N \times M$ rectangular diagonal matrix with nonnegative real values $\rho_k$ on the diagonal, i.e., $\rho_1 \geq \rho_2 \geq \ldots \geq \rho_K \geq 0$ and $K = \min(\{N,M\})$. The optimal AWVs are $\mathbf{t} = \mathbf{v}_1$ and $\mathbf{r} = \mathbf{u}_1$.

In the common case that $\mathbf{H}$ is unavailable, iterative beamforming training can be adopted to find the optimal AWVs. According to [4], [5], $\mathbf{H}^{2m} \approx (\mathbf{H}^H \mathbf{H})^{m} = \sum_{k=1}^{K} \rho_k^{2m} \mathbf{u}_k \mathbf{v}_k^H$, which can be obtained by $m$ iterative trainings utilizing the reciprocal feature of the channel. When $m$ is large, $\mathbf{H}^{2m} \approx \rho_1^{2m} \mathbf{v}_1 \mathbf{v}_1^H$. Thus, the optimal transmitting and receiving AWVs can be obtained by normalizing $\mathbf{H}^{2m} \mathbf{t}$ and $\mathbf{H} \times \mathbf{H}^{2m} \mathbf{r}$, respectively.

The SGV scheme is described in Algorithm1. The iteration number $\varepsilon$ depends on practical channel response, which will be shown in Section V. It is clear that although the SGV scheme is effective, it is required that both the amplitudes and phases of AWV are adjustable, which cannot be satisfied when CA phased arrays are used, where only phases are adjustable.

IV. STEERING-VECTOR BASED SCHEME

In fact, the SGV scheme is a general one suitable for an arbitrary channel $\mathbf{H}$. It does not use the specific feature of MMWC channels. In MMWC, the channel has a directional feature, i.e., $\mathbf{H}$ can be naturally expressed as in (1), which is similar to the expression in (2). The difference is that in (1), the vectors $\{ \mathbf{g}_i \}$ and $\{ \mathbf{h}_i \}$ are CA steering vectors, not orthogonal bases, but in (2), $\{ \mathbf{u}_k \}$ and $\{ \mathbf{v}_k \}$ are strict non-CA orthogonal bases. Nevertheless, according to [13], $|\mathbf{g}_m^H \mathbf{g}_n|$ and $|\mathbf{h}_m^H \mathbf{h}_n|$ are approximately equal to zero given that $|\Omega_{\ell,m} - \Omega_{\ell,n}| \geq 1/N$ and $|\Omega_{\ell,m} - \Omega_{\ell,n}| \geq 1/M$, respectively, i.e., the receiving and transmitting angles can be resolved by the arrays, which is the common case in MMWC. Consequently, as a suboptimal approach, the steering vectors of the strongest MPC can be adopted as the transmitting and receiving AWVs at the source and destination devices, which leads to the proposed STV scheme. The advantage of STV is that the elements of the steering vector have a constant envelope, which is suitable for the devices with CA phased arrays. Moreover, although the transmitting and receiving angles are required to be resolved by the arrays in the following analysis, the STV scheme can work even when there exists angles that cannot be resolved, because two or more MPCs associated with sufficiently close angles that cannot be resolved actually build a single equivalent MPC.

Assuming that $\mathbf{H}$ is available in advance, the background of STV is presented as follows. Using the directional feature of MMWC channels, we have $\mathbf{H}^{2m} \approx \sum_{k=1}^{L} \sqrt{MN} \lambda_k^{2m} |\mathbf{h}_k^H \mathbf{h}_k|^2$, for a positive integer $m$. Suppose the $k$th MPC is the strongest one. For $\ell \neq k$, $|\lambda_{\ell}^{2m} / \lambda_k|^{2m}$ exponentially decreases. This means that the contribution to the matrix product $\mathbf{H}^{2m}$ from the the other $L - 1$ MPCs exponentially decreases, compared

\(^1\)The SVD on $\mathbf{H}$ gives a set of orthogonal transmitting and receiving AWV pairs, as well as the energies projected to these AWV pairs.
with the strongest one. Therefore, we have \( \lim_{m \to \infty} H^{2m} = |\sqrt{MN} \lambda_k|^2 m h_k h_k^H \). Thus, for a sufficiently large \( m \) and an arbitrary initial transmitting AWV \( t \), we have

\[
H^{2m} t = |\sqrt{MN} \lambda_k|^2 m h_k h_k^H t = \left( |\sqrt{MN} \lambda_k|^2 m h_k^H t \right) h_k,
\]

which is \( h_k \) multiplied by a complex coefficient. It is noted that \( h_k \) is a constant-envelope steering vector. Hence, the desired transmitting AWV can be obtained by the signature estimation\(^3\) where \( e_t = exp(j \angle (H^{2m} t)) / \sqrt{M} \) is to be estimated. Here, \( \angle(x) \) represents the angle vector of \( x \) in radian. In fact, the signature estimation can be carried out by the entry-wise normalization on \( H^{2m} t \).

In addition, we have

\[
H \times H^{2m} t = \left( \lambda_k \sqrt{MN} |\lambda_k| \sqrt{MN} |^{2m} h_k^H t \right) g_k. \tag{3}
\]

Thus, the desired receiving AWV can be obtained by the signature estimation of \( e_r = exp(j \angle (H \times H^{2m} t)) / \sqrt{N} \).

It is clear that given full CSI, the AWVs steering along the strongest MPC in both ends can be obtained. In practical MMWC, however, \( H \) is basically unavailable in both ends; thus we propose the joint iterative beamforming training process of STV, which is shown in Fig.1 and the corresponding algorithm is described in Algorithm 2.\(^2\) The iteration number \( \varepsilon \) depends on practical channel response. According to the simulation results in Section V, \( \varepsilon = 2 \) or \( 3 \) can basically guarantee convergence.

It is noted that STV is tailored for MMWC devices with CA phased arrays based on SGV. Thus, STV and SGV have common features, e.g., both schemes need iteration. However, their mathematical fundamentals are different. SGV is to find the principal singular vectors of the channel matrix \( H \), which is optimal and applicable for arbitrary channels, while STV is to find the CA steering vectors of the strongest MPC by exploiting the directional feature, which is sub-optimal and only feasible under MMWC channels. Thus, in each iteration, STV requires the signature estimation, which is to estimate the CA steering vector of the strongest MPC. Meanwhile, in order to make STV feasible for CA phased arrays, it adopts the DFT matrices in transmitting and receiving training sequences, because the entries of them have a constant envelope.

V. PERFORMANCE EVALUATION

In this section we evaluate the performances of STV, including array gain and convergence rate, and compare them with those of SGV via simulations. In all the simulations, the channel is normalized as \( E(\sum_{i=1}^{\ell} |\lambda_i|^2) = 1 \). The transmitting SNR is thus \( \gamma_t = 1/\sigma_t^2 \), and the array gain becomes the ratio of receiving SNR to transmitting SNR, i.e., \( \eta = \gamma / \gamma_t = |r|^2/|Ht|^2 \). The initial transmitting AWVs in the two schemes are selected under the principle that its power evenly projects on the \( M \) basis vectors of the receiving matrices at the source, i.e., \( I_M \) and \( F_M \), respectively. Thus, the initial transmitting AWV for SGV is \( I_M / \sqrt{M} \), while that for STV is a normalized constant-amplitude-zero-autocorrelation (CAZAC) sequence with length \( M \).

The array gain is empirically found using the ratio of the average receiving SNR to the average transmitting SNR over \( 10^3 \) realizations of channels. Furthermore, the SVD upper bound is obtained by averaging the squares of the principal singular values of these channel realizations. Channel realizations are generated under the Rician and Rayleigh fading models for the LOS and NLOS channels, respectively. For the LOS channel, the power of the LOS MPC is \( |\lambda_1|^2 = 0.7692 \), and the average powers of the NLOS MPCs are \( E(\{ |\lambda_i|^2 \}_{i=2,3,4}) = [0.0769 0.0769 0.0769] \). For the NLOS channel, \( E(\{ |\lambda_i|^2 \}_{i=1,2,3,4}) = [0.25 0.25 0.25 0.25] \). The transmitting and receiving steering angles are randomly generated within \([0, 2\pi)\) in each realization.

The left and middle figures in Fig.2 show the achieved

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\(^2\)There are other approaches to obtain the desired AWV here. The presented one is a simple one in implementation.

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**Algorithm 2 The STV Scheme**

1) **Initialize:**

Pick an initial transmitting AWV \( t \) at the source device. This AWV may be chosen either randomly or with the approach specified in Section V.

2) **Iteration:** Iterate the following process \( \varepsilon \) times, and then stop.

Keep sending with the same transmitting AWV \( t \) at the source device over \( N \) slots. Meanwhile, use discrete Fourier Transform (DFT) matrix \( F_M \) as the receiving AWVs at the destination device, i.e., the \( n \)-th column of \( F_M \) as the receiving AWV at the \( n \)-th slot. Note that \( I_M \) cannot be used for the receiving AWVs here, due to its non-constant-envelope entries, but other unitary matrices with constant-envelope entries are feasible. Consequently, we receiving a vector \( r = F_M^H t + F_M^H n_t \), where \( n_t \) is the noise vector. Estimate the signature \( e_r = exp(j \angle (F_M r)) / \sqrt{N} \) and assign \( e_t \) to \( r \).

Keep sending with the same transmitting AWV \( r \) at the destination device over \( M \) slots. Meanwhile, use DFT matrix \( F_M \) as the receiving AWVs at the source device. Consequently, we receiving a new vector \( t = F_M^H H^H r + F_M^H n_r \), where \( n_r \) is the noise vector. Estimate the signature \( e_t \) as \( e_t = exp(j \angle (F_M t)) / \sqrt{M} \) and assign \( e_t \) to \( t \).

3) **Result:**

\( t \) is the AWV at the source device, and \( r \) is the AWV at the destination device.
array gains of SGV and STV under the LOS and NLOS channels, respectively, with different numbers of iterations, where \( M = N = 16 \). The right figure in Fig. 2 shows the comparison of convergence rate between SGV and STV under the LOS and NLOS channels with a high transmitting SNR, i.e., 25 dB, in the cases of \( M = N = 16 \) and \( M = N = 32 \), respectively. From the left and right figures, it is found that under the LOS channel, both schemes achieve fast convergence rates and approach the optimal array gain, i.e., the SVD upper bound. From the middle and right figures, it is observed that under the NLOS channel, both the two schemes have slower convergence rates, and STV achieves a faster convergence rate at the cost of a slightly lower array gain than SGV that also approaches the SVD upper bound. It is noted that, although not shown in these figures, similar results are observed with a smaller or larger number of antennas.

Explanations for these observations are as follows. Under the LOS channel, there is one and only one strong MPC, and the steering vectors of this MPC are almost the optimal AWVs. Thus, STV can achieve the optimal array gain. But under the NLOS channel, there are several MPCS with different steering angles (or steering vectors) and the STV scheme obtains one of them as an AWV, which is not optimal. Hence, STV cannot achieve the optimal array gain in such a case. On the other hand, since the SGV is based on the principal singular vector, it can surely achieve the SVD upper bound once convergence has been achieved. Besides, the fact that STV achieves a faster convergence rate in NLOS channel indicates that the signature estimation in each iteration of STV is more robust against noise, while the AWV estimation of SGV is more sensitive to noise.

In brief, although STV is tailored for MMWC devices with CA phased arrays, where SGV is infeasible, it has comparable performances to SGV in terms of the convergence rate and array gain, under both the LOS and NLOS channels. On the other hand, it is noted that a single iteration consumes \( M + N \) training slots, which may significantly degrade the system efficiency, especially when the number of antennas is large. Hence, even if there is no CA constraint, i.e., both phase and amplitude are adjustable and thus SGV is feasible, STV may still be favored in the case that the iteration number is constrained to be 1 or 2 to save training time, because it achieves a higher array gain according to the right figure of Fig. 2.

VI. CONCLUSIONS

Since the existing SGV scheme cannot be used in MMWC with CA phased arrays, the STV scheme has been proposed in this study, which effectively exploits the directional feature of MMWC channels. Performance comparisons showed that under LOS channel, both the schemes achieve fast convergence rates and achieve the optimal array gain; under NLOS channel, STV achieves a faster convergence rate at the cost of a slightly lower array gain than SGV that can still approach the optimal array gain. In summary, while the proposed STV scheme is well-suited to MMWC with CA phased arrays, it has comparable performances to SGV in terms of the convergence rate and array gain under both the LOS and NLOS channels.

REFERENCES

[1] S. K. Yong, P. Xia, and A. Valdes-Garcia, 60GHz Technology for Gbps WLAN and WPAN: from Theory to Practice. Wiley, 2011.
[2] Z. Xiao, “Suboptimal spatial diversity scheme for 60 GHz millimeter-wave WLAN,” IEEE Communications Letters, vol. 17, no. 9, pp. 1790–1793, 2013.
[3] J. Wang, Z. Lan, C. Pyo, T. Baykas, C. Sum, M. Rahman, J. Gao, R. Fundada, F. Kojima, and H. Harada, “Beam codebook based beamforming protocol for multi-gbps millimeter-wave WPAN systems,” IEEE Journal on Selected Areas in Communications, vol. 27, no. 8, pp. 1390–1399, 2009.
[4] P. Xia, S. Yong, J. Oh, and C. Ngo, “A practical SDMA protocol for 60 GHz millimeter wave communications,” in Asilomar Conference on Signals, Systems and Computers. IEEE, 2008, pp. 2019–2023.
[5] Y. Tang, B. Vucetic, and Y. Li, “An iterative singular vectors estimation scheme for beamforming transmission and detection in MIMO systems,” IEEE Communications Letters, vol. 9, no. 6, pp. 505–507, 2005.
[6] A. Valdes-Garcia, S. T. Nicolson, J.-W. Lai, A. Natarajan, P.-Y. Chen, S. K. Reynolds, J.-H. C. Zhan, D. G. Kanti, D. Liu, and B. Floyd, “A fully integrated 16-element phased-array transmitter in SiGe BICMOS for 60-GHz communications,” IEEE Journal of Solid-State Circuits, vol. 45, no. 12, pp. 2757–2773, 2010.
[7] E. Cohen, C. Jakobson, S. Ridavid, and D. Ritter, “A thirty two element phased-array transceiver at 60GHz with RT-IF conversion block in 90nm flip chip CMOS process,” in IEEE Radio Frequency Integrated Circuits Symposium (RFIC). IEEE, 2010, pp. 457–460.
[8] L. Bai and J. Choi, “Lattice reduction-based MIMO iterative receiver using randomized sampling,” IEEE Transactions on Wireless Communications, vol. 12, no. 5, pp. 2160–2170, 2013.
[9] X. Zheng, Y. Xie, J. Li, and P. Stoica, “MIMO transmit beamforming under uniform elemental power constraint,” IEEE Transactions on Signal Processing, vol. 55, no. 11, pp. 5395–5406, 2007.
[10] J. Lee, R. U. Nabar, J. P. Choi, and H.-I. Lu, “Generalized co-phasing for multiple transmit and receive antennas,” IEEE Transactions on Wireless Communications, vol. 8, no. 4, pp. 1649–1654, 2009.
[11] A. Maltsev, R. Maslennikov, A. Sevastyanov, A. Lomayev, A. Khoryaev, A. Davydov, and V. Ssorin, “Characteristics of indoor millimeter-wave channel at 60 GHz in application to perspective WLAN system,” in European Conference on Antennas and Propagation (EuCAP). IEEE, 2010, pp. 1–5.

[12] M. Park and H. Pan, “A spatial diversity technique for IEEE 802.11 ad WLAN in 60 GHz band,” IEEE Communications Letters, vol. 16, no. 8, pp. 1260–1262, 2012.

[13] D. Tse and P. Viswanath, Fundamentals of wireless communication. Cambridge university press, 2005.