A little theory of everything, with heavy neutral leptons

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Abstract: Recently a new model of “Affleck-Dine inflation” was presented, that produces the baryon asymmetry from a complex inflaton carrying baryon number, while being consistent with constraints from the cosmic microwave background. We adapt this model such that the inflaton carries lepton number, and communicates the lepton asymmetry to the standard model baryons via quasi-Dirac heavy neutral leptons (HNLs) and sphalerons. One of these HNLs, with mass \( \lesssim 4.5 \text{ GeV} \), can be (partially) asymmetric dark matter (DM), whose asymmetry is determined by that of the baryons. Its stability is directly related to the vanishing of the lightest neutrino mass. Neutrino masses are generated by integrating out heavy sterile neutrinos whose mass is above the inflation scale. The model provides an economical origin for all of the major ingredients missing from the standard model: inflation, baryogenesis, neutrino masses, and dark matter. The HNLs can be probed in fixed-target experiments like SHiP, possibly manifesting \( N-\bar{N} \) oscillations. A light singlet scalar, needed for depleting the DM symmetric component, can be discovered in beam dump experiments and searches for rare decays, possibly explaining anomalous events recently observed by the KOTO collaboration. The DM HNL is strongly constrained by direct searches, and could have a cosmologically interesting self-interaction cross section.

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1 Introduction

The standard model (SM) of particle physics is noted for being incomplete in numerous ways. It could be argued that the most urgently missing elements are an inflaton (or other source of primordial density perturbations), a mechanism for baryogenesis, dark matter (DM), and the origin of neutrino masses, since all of these relate to directly observed phenomena as opposed to problems of naturalness. It is tempting to seek relatively simple new physics models that can simultaneously address several of the missing pieces, or perhaps all.

A notable example is the $\nu$MSM [1, 2], in which light sterile neutrinos can accomplish leptogenesis and provide a dark matter candidate while giving neutrino masses. Higgs inflation [3] can be invoked in this framework without needing any additional particles. Another example is the SMASH model [4] that assumes heavy right-handed neutrinos to
explain neutrino mass and thermal leptogenesis, while introducing minimal extra matter content to produce axions as dark matter and a solution to the strong CP problem. The extra scalar field needed for breaking Peccei-Quinn symmetry can combine with the Higgs to give two-field inflation in the early universe.

In the present work we suggest another way of completing the standard model, that does not rely upon leptogenesis as usually defined (through the CP-violating out-of-equilibrium decays of heavy neutrinos). The starting point is a model of inflation in which the Affleck-Dine mechanism [5] for creating a particle asymmetry occurs during inflation [6]. The asymmetry is originally stored in a complex inflaton field, that has the Lagrangian

\[ L = \frac{m_P^2}{2} R (1 + 2\xi|\phi|^2) + |\phi|^2 - m_\phi^2|\phi|^2 - \lambda|\phi|^4 - i\lambda' (\phi^4 - \phi^{*4}) \]

(where \( m_P \) is the reduced Planck scale) including a nonminimal coupling to gravity, needed to flatten the potential at large \( |\phi| \), which makes the inflationary predictions compatible with Planck constraints [7]. In ref. [6] we assumed that \( \phi \) carried baryon number, which was transferred to the SM quarks through colored scalar mediators. Here we consider the case where \( \phi \) carries lepton number, hence giving a new mechanism of leptogenesis. As usual, the resulting lepton asymmetry is transmitted to the baryons through the sphaleron interactions of the SM.

The challenge for such an approach is to find a way of transferring the lepton asymmetry from \( \phi \) to the SM without it being washed out by the lepton-violating effects associated with neutrino mass generation. Indeed, if \( \phi \) decays to heavy right-handed neutrinos that have large Majorana masses, the asymmetry gets washed out immediately and the situation reverts to standard leptogenesis being required. This suggests that \( \phi \) should decay into quasi-Dirac neutrino mediators \( N_i \), that mix with the SM neutrinos to transmit the asymmetry. Among the \( N_i \) mediators, one can be stable and constitute a species of asymmetric dark matter, getting its relic density (partly) from the initial lepton asymmetry. The \( N_i \) are an example of heavy neutral leptons (HNLs), a class of hypothetical particles that is being widely studied both theoretically and by upcoming experiments such as SHiP [8], MATHUSLA [9] and FASER [10].

To deplete the symmetric component of the DM to a viable level, it is necessary to introduce a light mediator, which we take to be a scalar singlet \( s \), so that \( N_i \bar{N}_i \rightarrow ss \) annihilations are sufficiently strong. The DM can be fully or partially asymmetric depending on the coupling strength \( g_s \). We will show that this interaction has interesting implications for direct detection, and for hints of anomalous rare \( K_L \rightarrow \pi^{0}+ \) invisible decays that have recently been reported by the KOTO experimental collaboration [11].

In our proposal, the HNLs do not explain the origin of light neutrino masses, but we hypothesize that their couplings to the SM \( \nu \)'s are related to those of the superheavy Majorana \( \nu \)'s that generate seesaw masses, by a principle similar to minimal flavor violation (MFV) [12]. The setup thereby also addresses the origin of neutrino mass and relates the HNL couplings to it in an essential way. Moreover a direct link is made between the stability of the dark matter candidate and the masslessness of the lightest SM neutrino.
In section 2 we specify the structure of couplings of the HNLs to the inflaton and SM particles, and its relation to neutrino mass generation. In section 3 we discuss constraints on the couplings such that the lepton asymmetry from inflation is transferred to the SM particles without being washed out. It is shown how the resulting baryon asymmetry determines the dark matter asymmetry and its mass. The relations between light $\nu$ properties and the HNL couplings are presented in section 4, and consequent predictions for the phenomenology of the HNLs. In section 5 we compile the experimental limits on the light singlet $s$, and identify a region of parameter space where the KOTO anomaly can be reconciled with DM direct detection limits. The latter are considered in detail in section 6, where we also treat the DM self-interactions. The technical naturalness of our setup is demonstrated in section 7, followed by conclusions in section 8. In appendix A we derive the exact width for HNL decay into different-flavor charged leptons, which was given only in approximate form in previous papers.

2 Model

We assume the inflaton carries lepton number 2 (more correctly, $B - L = -2$ since $B - L$ symmetry is not broken by electroweak sphalerons), and couples to $N_N$ flavors of quasi-Dirac HNLs as
\[
g_\phi \phi \bar{N}_{L,i} N^c_{L,i} + g_\phi \phi \bar{N}_{R,i} N^c_{R,i} + \text{H.c.}
\] (2.1)

$N_N$ is a free parameter; hereafter we take $N_N = 3$, which is the minimal number needed to get dark matter and the observed neutrino properties, through consistent assumptions about the flavor structure of the neutrino sector that will be explained presently. The HNLs couple to the SM lepton doublets as
\[
\eta_{\nu,ij} \bar{N}_{R,i} H L_j
\] (2.2)

At energy scales relevant for inflation and below, it is consistent to assume that the only source of lepton number violation is through a small Majorana mass $\epsilon_\nu$ for the standard model neutrinos, which could be generated through the seesaw mechanism, by integrating out very heavy right-handed neutrinos, with mass $M_{\nu_R}$ above the scale of inflation. In the basis $\nu_L, N^c_R, N^c_L$, the neutrino mass matrix is
\[
\begin{pmatrix}
\epsilon_\nu & \eta_\nu^T \bar{\nu} & 0 \\
\eta_\nu \bar{\nu} & 0 & M_N \\
0 & M_N & 0
\end{pmatrix}
\] (2.3)

where $\bar{\nu} \equiv 174$ GeV is the complex Higgs VEV. We assume that $\epsilon_\nu$ has a flavor structure that is aligned with the couplings in (2.2) as
\[
\epsilon_\nu = \bar{\mu}_\nu \eta_\nu^T \eta_\nu
\] (2.4)

where $\bar{\mu}_\nu$ is a scale that we will constrain below. This alignment ensures the stability of dark matter against oscillations with its antiparticle, if $\eta_\nu$ has one vanishing eigenvalue. In
order to justify the ansatz, we will show that it is radiatively stable, due to an approximate SU(3) flavor symmetry for the \( N_i \) leptons, that is broken in a minimal-flavor-violating (MFV) [12] manner, solely by the matrix \( \eta_\nu \). For example, the flavor-diagonal couplings of the inflaton to \( N_i \) could be perturbed by a term proportional to \( \eta_\nu \eta_\nu^T \) without spoiling the viability of the framework.

By solving for the eigenvalues of (2.3), one finds that the light neutrino part \( \epsilon_\nu \) induces a small Majorana mass matrix for the \( N_i \)'s of the form

\[
\delta M = \frac{\bar{\nu}^2}{M_N^2} \epsilon_\nu \eta_\nu^T \epsilon_\nu 
\tag{2.5}
\]

that leads to \( N_i - \bar{N}_i \) oscillations. These are mildly constrained by the need for approximate lepton number conservation during the generation of the lepton asymmetry (apart from electroweak sphalerons), as we consider below.

3 Nonstandard leptogenesis and DM relic density

During inflation \( \phi \) gets an asymmetry determined mostly by the couplings in eq. (1.1) and to a smaller extent by the initial conditions of the inflaton, which provide the source of CP violation in the Affleck-Dine mechanism [5]. The details of asymmetry generation at the level of \( \phi \) are exactly the same as discussed in ref. [6]. The difference here is that the \( \phi \) asymmetry is transferred to the HNLs by the decays \( \phi \to NN \) from the interaction (2.1).

Whether reheating is perturbative or proceeds by parametric resonance is not crucial to the present discussion, where we assume that the created asymmetry results in the observed baryon asymmetry. This can always be achieved by appropriate choice of the \( L \)-violating parameter \( \lambda' \), for example.

3.1 Sharing and preserving the asymmetry

For simplicity, consider the case where \( g_\phi \) is sufficiently small so that perturbative decays are the dominant mechanism for reheating, with reheat temperature of order

\[
T_R \sim g_\phi (m_\phi m_P)^{1/2} \sim 10^{-3} g_\phi m_P \tag{3.1}
\]

using the typical value \( m_\phi \sim 10^{-6} m_P \) identified in ref. [6]. Even for rather small values \( g_\phi \lesssim 0.01 \), this is well above the weak scale. Therefore it is easy for the HNLs to equilibrate with the SM through the interaction (2.2), which transmits the primordial \( B-L \) asymmetry to the SM. The dominant process is \( N_i \) (inverse) decays, whose rate is \( \Gamma_d \cong 10^{-3} \eta_\nu^2 T \) [13] for \( T \gtrsim 100 \text{ GeV} \). Demanding that this comes into equilibrium before sphalerons freeze out, we find the lower bound \( |\eta_\nu| \gtrsim 4 \times 10^{-7} \) on the largest elements of \( \eta_\nu \).

The only \( L \)-violating process operative at scales below that of inflation is \( N-\bar{N} \) oscillations induced by the \( \delta M \) matrix elements (2.5). These would wash out the \( B \) and \( L \) asymmetries if they were in equilibrium before sphaleron freeze out. The rate of \( L \) violation is not simply the same as the oscillation rate \( \sim 1/\delta M \), because flavor-nondiagonal interactions of \( N \) with the plasma can measure the state of the oscillating \( N-\bar{N} \) system before it
has time to oscillate significantly, damping the conversions of $N \to \bar{N}$. The effective rate
of $L$ violation can be parametrized as \cite{14, 15}

$$\Gamma_{\Delta L} \sim \frac{M_N^2 \delta M^2}{M_N^2 \delta M^2 + T^2 \Gamma_m^2} \Gamma_m$$  \hspace{1cm} (3.2)$$

where $\Gamma_m$ is the rate of processes that destroy the coherence of the $N$-$\bar{N}$ system.\(^1\) For $T > T_{EW} \sim 100$ GeV, (inverse) decays are dominant, but these quickly go out of equilibrium as $T$ falls below the mass of the Higgs boson. At temperatures somewhat below $T_{EW}$, the elastic (but flavor-violating) $NL \to NL$ scatterings mediated by Higgs exchange dominate, with $\Gamma_m = \Gamma_{el} \sim (\eta_\nu T^5/m_h^4)$. On the other hand sphalerons are safely out of equilibrium since they are exponentially suppressed by the Boltzmann factor involving the sphaleron energy, which is above the TeV scale. Therefore it is sufficient to show that the rate (3.2) is out of equilibrium in this case, to establish that the washout process is innocuous.

In section 4 we will show that $|\eta_\nu| \lesssim \eta_0 = 7 \times 10^{-5}$ as a consequence of laboratory constraints on the mixing of HNLs with the SM neutrinos. For this value of $\eta_\nu$, the $L$-violating mass is $\delta M \sim 4 \times 10^{-7}$ eV, while $\Gamma_{el} \sim 1 \times 10^{-15}$ GeV. We will also show below that $M_N \lesssim 4.5$ GeV from the relic density requirement for the dark matter. Then $\Gamma_{\Delta L}/H \sim \eta_0^{-1} M_N^2 \delta M^2 m_F / T_{EW}^5 \lesssim 1 \times 10^{-5}$, which is safely out of equilibrium. The lepton-violating effects of $\delta M$ are therefore too small to affect the baryon asymmetry, but they can be observable in collider experiments that we will discuss in section 4.

DM-antiDM oscillations for asymmetric DM have been considered in refs. \cite{16, 17}. They can potentially regenerate the symmetric component of the DM and lead to its dilution through annihilations. We avoid these constraints by the relation (2.5) that causes $\delta M$ to vanish when acting on the $N'$ DM state.

### 3.2 DM asymmetric abundance and maximum mass

The relic density for fully asymmetric DM is determined by its chemical potential, which in our framework is related to the baryon asymmetry in a deterministic way, since the DM initially has the same asymmetry as the remaining two HNLs. The relation between the DM and baryon asymmetries can be found by solving the system of equilibrium constraints, similarly to ref. \cite{18}. We generalize their network to include the extra HNL species, that satisfy the equilibrium condition

$$\mu_N = \mu_h + \mu_L$$  \hspace{1cm} (3.3)$$

from the $\eta_\nu$ interactions. Eq. (3.3) only applies to the unstable HNL species since $N'$ is conserved, and its chemical potential is fixed by the initial lepton asymmetry

$$\mu_{N'} = \frac{1}{6} L_0$$  \hspace{1cm} (3.4)$$

The factor of 6 comes from having three HNL species, each with two chiralities.

\(^1\)We will introduce an additional elastic scattering channel mediated by a singlet scalar $s$ below. These flavor-conserving interactions are not relevant for decohering the $N$-$\bar{N}$ oscillations \cite{16}. 

Repeating the analysis of [18] we find the following equilibrium relations (setting the $W$ boson potential $\mu_W = 0$ since $T > T_{EW}$):

\[
L = \frac{13}{3} \mu_\nu + \mu_h + 2\mu_{N'} = \frac{95}{27} \mu_\nu + 2\mu_{N'},
\]

\[
B = -\frac{4}{3} \mu_\nu,
\]

\[
\mu_{u_L} = -\frac{1}{9} \mu_\nu, \quad \mu_h = \frac{4}{27} \mu_\nu
\]

\[
\mu_N = \frac{11}{27} \mu_\nu
\]

(3.5)

where $L, B$ are the respective total potentials for lepton and baryon number, $\mu_\nu$ is the sum of light neutrino chemical potentials, and $\mu_h$ is that of the Higgs. Since $B - L$ is conserved by sphalerons, we can relate these to the initial lepton asymmetry $L_0 = 6\mu_{N'} = (L - B)$: $\mu_\nu = \frac{84}{125} \mu_{N'}, B = -\frac{112}{125} \mu_{N'}$. This allows us to determine the maximum mass of $N'$ that gives the observed relic density:

\[
m_{N'} = M_N \leq \left| \frac{B}{\mu_{N'}} \right| \Omega_c \Omega_b m_n = 4.5 \text{ GeV}
\]

(3.6)

using the values $\Omega_c = 0.265$ and $\Omega_b = 0.0493$ from ref. [19] and the nucleon mass $m_n$.

The inequality (3.6) is only saturated if the symmetric DM component is suppressed to a negligible level. Otherwise a smaller value of $m_{N'}$ is needed to compensate the presence of the symmetric component. We turn to the general case next.

### 3.3 Dark matter annihilation and relic density

In order to reduce the symmetric component of the DM to avoid overclosure of the universe, an additional annihilation channel is needed. The Higgs-mediated annihilations $N' \bar{N}' \rightarrow LL$ are not strong enough, in light of the bound $|\eta_\nu| \lesssim 10^{-4}$ mentioned above. We need an additional particle with sufficiently strong couplings to the DM.

The simplest possibility is to introduce a singlet scalar $s$, with interactions

\[
g_s s \bar{N}_i N_i + \frac{1}{4} \lambda_s (s^2 - v_s^2)^2 + \lambda_h s h^2 s^2
\]

(3.7)

that at tree level are diagonal in the $N_i$ flavors, and lead to mixing of $s$ with the Higgs $h$. We assume that $m_s < m_{N'}$ so that $N' \bar{N}' \rightarrow ss$ is allowed; otherwise the annihilation cross section is too small, unless $m_s \approx 2m_{N'}$ to resonantly enhance the $s$-channel process. Away from this point, the $s$-channel amplitude for $N' \bar{N}' \rightarrow c \bar{c}$ (where $c$ is the charm quark, the most strongly coupled kinematically accessible final state) is of the same order of magnitude as that for $N'$-nucleon scattering, which is strongly constrained by direct detection (section 6), making this contribution too small to be sufficient for annihilation.

The cross section for $N' \bar{N}' \rightarrow ss$ is $p$-wave suppressed. Parameterizing the Mandelstam variable as $s = 4m_{N'}^2(1 + \epsilon)$ we find in the limit $m_s \ll m_{N'}$ and $\lambda_s \ll g_s$

\[
\sigma \approx \frac{3g_s^4}{64\pi m_{N'}^2} \frac{\epsilon^{1/2}}{(1 + \epsilon)^2}
\]

(3.8)
(this is an analytic approximation to the exact result, which is more complicated). Carrying out the thermal average \[20\] with \( x = m_{N'}/T \) gives

\[
\langle \sigma v \rangle \approx \frac{3g_4^4}{16\pi^2 m_{N'}^2} F(x) \tag{3.9}
\]

\[
F(x) = \frac{x}{K_2(x)^2} \int_0^\infty \frac{e^{3/2}}{(1+e)^5/2} K_1(2x\sqrt{1+e}) \tag{3.10}
\]

\[
\approx 0.058 - 0.002x + 3.25 \times 10^{-5}x^2 - 1.87 \times 10^{-7}x^3
\]

which is a good numerical approximation in the region \( 15 < x < 70 \). For values \( x \sim 20 \) typical for freezeout, \( F \approx 0.03 \).

To find the relic abundance including both symmetric and asymmetric components, one can solve the Boltzmann equation for their ratio \( r \) given in ref. \[21\], which depends upon \( \langle \sigma v \rangle \). Then as shown there, the fractional contribution of \( N' \) to the energy density of the universe is

\[
\Omega_{N'} = \epsilon \frac{\eta_B m_{N'}}{\rho_{\text{crit}}} \frac{s}{1 + r} \frac{1 + r - r}{1 - r} \tag{3.11}
\]

where \( \eta_B = 8.8 \times 10^{-11} \) is the observed baryon-to-entropy ratio, \( s \) is the entropy density, and \( \epsilon = \eta_{N'}/\eta_B = 123/112 \) in our model (see below eq. (3.5)). Using ref. \[22\], we checked whether the DM annihilation cross section might be Sommerfeld-enhanced since \( m_s < m_{N'} \), but this was a negligible effect in the relevant parts of parameter space that we will specify below. In Figure 1 we plot contours of \( \Omega_{N'} \), the fractional contribution of the DM to the energy density of the universe, in the plane of \( m_{N'} \) versus \( g_s \). For \( g_s \gtrsim 0.14 \) the maximum value in eq. (3.6) is achieved, while for lower \( g_s \), the symmetric component abundance is increased (while the asymmetric abundance remains fixed), corresponding to lower DM masses.

In the opposite regime \( \lambda_s \gg g_s \), the annihilation could in principle be dominated by the \( s \)-channel diagram, giving the cross section

\[
\langle \sigma v \rangle \approx \frac{1}{\pi} \left( \frac{3\lambda_v v_s g_s}{8 m_{N'}^2} \right)^2 \bar{F}(x), \tag{3.12}
\]

in the case where \( m_s \ll m_{N'} \), with

\[
\bar{F}(x) = \frac{x}{K_2(x)^2} \int_0^\infty \frac{e^{3/2}}{(1+e)^5/2} K_1(2x\sqrt{1+e}) \tag{3.13}
\]

For \( x \sim 20 \), \( F \approx 0.01 \) leading to the requirement that \( g_s \) must be significantly larger than in the previous case to suppress the symmetric DM component. Such values are excluded by direct DM search constraints to be discussed in section 6 below. Hence there is no practical enlargement of the allowed parameter space from including the \( s \)-channel contribution.
4 Neutrino properties and HNL constraints

Below the scales of electroweak symmetry breaking and the HNL mass $M_N$, the light neutrino mass matrix gets generated,

\[ m_\nu \cong \epsilon_\nu - \delta M' \]
\[ \delta M' \equiv \frac{\bar{v}^2}{M_N} \eta_\nu^T \epsilon_\nu \eta_\nu \]  

(4.1)

However $|\eta_\nu|\bar{v}/M_N \ll 1$ is the magnitude of the mixing between the light neutrinos and the HNLs, as we will discuss below, so that the correction $\delta M' \ll \epsilon_\nu$ can be ignored. We reiterate that $\epsilon_\nu$ is generated by the usual seesaw mechanism, integrating out sterile neutrinos whose mass is above all the other relevant scales in our model.

Recall that the stability of the dark matter $N'$ requires $\eta_\nu$ to be a matrix with one vanishing eigenvalue, which implies that the lightest neutrino is massless. This is an exact statement, not relying upon the neglect of $\delta M'$, since $\epsilon_\nu$ and $\delta M'$ are simultaneously diagonalizable by construction. This is a consequence of our MFV-like assumption that $\eta_\nu$ is the only source of flavor-breaking in the HNL/neutrino sector.

4.1 Explicit $\eta_\nu$ and HNL mixings

Using eq. (2.4) we can solve for $\eta_\nu$ in terms of the neutrino masses and mixings,

\[ \eta_\nu = O \left( \frac{D_\nu}{\mu_\nu} \right)^{1/2} U_{\nu \text{PMNS}}^{-1} \]  

(4.2)
where $D_\nu$ is the diagonal matrix of light $\nu$ mass eigenvalues, and $U_{PMNS}$ is the $3 \times 3$ PMNS matrix. The orthogonal matrix $O$ is undetermined since the $N_i$ are practically degenerate; for simplicity we set it to 1 in the following. Since we have assumed that one eigenvalue is vanishing, the other two are known,

$$D_{\nu 11} = 0, \quad D_{\nu 22} = \sqrt{\Delta m_{21}^2}, \quad D_{\nu 33} = \sqrt{\Delta m_{31}^2}, \quad \text{NH}$$

$$D_{\nu 33} = 0, \quad D_{\nu 22} = \sqrt{\Delta m_{32}^2}, \quad D_{\nu 11} \approx \sqrt{\Delta m_{32}^2 - \Delta m_{21}^2}, \quad \text{IH} \quad (4.3)$$

for the normal and inverted hierarchies, respectively.

The light neutrinos mix with $N_i$, with mixing matrix elements given by

$$U_{\ell i} \approx \eta_{\nu, \ell i} \bar{v}$$

(4.4)

where $\ell = e, \mu, \tau$ and $i = 1, 2, 3$. Constraints on $U_{\ell i}$ arise from a variety of beam dump experiments and rare decay searches, summarized in refs. [8, 23]. For masses $M_N > 2$ GeV, the most stringent limit comes from searches for $Z \to N\nu$ boson decays by the DELPHI Collaboration [24]. Defining $\bar{U}_\ell = (\sum_i |U_{\ell i}|^2)^{1/2}$, at our largest allowed mass $M_N = 4.5$ GeV, the bound is

$$\bar{U} \equiv \left( \sum_\ell \bar{U}_\ell^2 \right)^{1/2} < 0.0039, \quad (4.5)$$

since DELPHI was sensitive to the total rate of $N_i$ production from $Z \to N_i\nu_\ell$ decays, times the total (semi)leptonic rate of $N_i$ decays.

Using eqs. (4.2, 4.4), the bound (4.5) can be approximately saturated if $\mu_\nu = 5.7 \ (9.8)$ MeV for the normal (inverted) hierarchy. Taking the PDG central values of the neutrino masses

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.pdf}
\caption{Left: minimum allowed mass scale $\bar{\mu}_\nu(M_N)$, predicted by our model for the normal mass hierarchy case, compatible with current constraints on the HNL mixings to light neutrinos [8]. The shaded gray region is excluded. Right: the ratio $r$ showing how the maximum allowed mixings (4.8) at $M_N = 4.5$ GeV are rescaled at lower $M_N$.}
\end{figure}
and mixings [25], we find

\[ \eta^T_\nu \approx 10^{-5} \begin{pmatrix} 0 & 2.1 & -1.0 - 0.9 i \\ 0 & 2.3 - 0.2 i & 6.9 \\ 0 & -2.2 - 0.1 i & 6.1 \end{pmatrix} \]

\[ \bar{U}_e \approx 0.00099, \quad \bar{U}_\mu \approx 0.0028, \quad \bar{U}_\tau \approx 0.0025 \]  

(4.6)

at \( M_N = 4.5 \) GeV for the normal hierarchy, and

\[ \eta^T_\nu \approx 10^{-5} \begin{pmatrix} 5.8 & 3.9 & 0 \\ -2.8 - 0.6 i & 3.8 - 0.4 i & 0 \\ 2.8 - 0.6 i & -4.6 - 0.4 i & 0 \end{pmatrix} \]

\[ \bar{U}_e \approx 0.0027, \quad \bar{U}_\mu \approx 0.0019, \quad \bar{U}_\tau \approx 0.0021 \]  

(4.7)

for the inverted hierarchy. In each case the column of zeros corresponds to the absence of coupling to the DM state \( N' \); hence we identify \( N' = N_1 \) for the normal hierarchy and \( N' = N_3 \) for the inverted hierarchy.

For the lighter mass range \( M_N \sim (0.4 - 2) \) GeV, beam dump experiments such as CHARM [26] and NuTEV [27] give the strongest limits for electron and muon flavors, roughly \( U_{ei}, U_{\mu i} \lesssim 6 \times 10^{-4}(M_N/\text{GeV})^{-1.14} \). The largest allowed magnitudes of the HNL mixings \( U_{\ell i} \) can be expressed as a function of \( M_N \),

\[ |U_{\ell i}| \approx r(M_N) \begin{pmatrix} 0 & 0.00083 & 0.00054 \\ 0 & 0.00090 & 0.0027 \\ 0 & 0.00087 & 0.0024 \end{pmatrix} \]  

(4.8)

focusing on the normal hierarchy case. We determined the minimum allowed value of \( \bar{\mu}_\nu \) for lower \( M_N \), and the consequent scaling factor \( r(M_N) = (5.7 \) MeV/min(\( \bar{\mu}_\nu \))\(^{1/2} \), from the limits summarized in figs. 4.10-4.12 of ref. [8]. These limits were rescaled and combined to account for the fact that our model has two HNLs, each of which mixes with all of the light flavors rather than just one \( N_i \) that can mix with only one flavor at a time. The functions \( \min(\bar{\mu}_\nu) \) and \( r(M_N) \) are plotted in Figure 2. The various constraints on the HNL mixing with electron neutrinos in the mass range relevant for our model are shown for two choices of \( \bar{\mu}_\nu \) in Figure 3, including future constraints from FCC-ee [28], DUNE [29] and SHiP [30].

The scale \( \bar{\mu}_\nu \) determines how the couplings \( y_\nu = k \eta_\nu \) of the light neutrinos to the heavy Majorana neutrinos (as restricted by our MFV-like assumption) are enhanced relative to \( \eta_\nu \) by a proportionality factor, \( k = (M_{\nu_R} \bar{\mu}_\nu / v^2)^{1/2} \). Perturbativity of \( y_\nu \) limits \( k \lesssim 10^5 \), hence the scale of the heavy neutrinos is bounded by \( M_{\nu_R} \lesssim 10^{11} \) GeV for the largest value of \( \bar{\mu}_\nu \) in Figure 2. This is not restrictive, and can be made consistent with our assumption that the heavy neutrinos do not play a role during inflation or reheating, if the reheat temperature is sufficiently low.

### 4.2 \( N-\bar{N} \) oscillations

As mentioned in sect. 3.1, the \( L \)-violating mass term \( \delta M \) causes \( N-\bar{N} \) oscillations, at a rate that is too small to destroy the lepton asymmetry in the early universe, but fast enough to
Figure 3. Summary of constraints on HNL mixing with electron neutrinos, over mass range of interest for our model (left: normal hierarchy, right: inverted hierarchy). Solid and dot-dashed curves show the model’s predictions for $U_{e2}$ ($U_{e1}$) (solid curves) and $U_{e3}$ ($U_{e2}$) (dot-dashed) in the normal (inverted) mass hierarchy, for two choices of the parameter $\bar{\mu}_\nu$ that determines the mixing through eqs. (4.2, 4.4). $U_{e1} \equiv 0$ ($U_{e3} \equiv 0$) for the normal (inverted) hierarchy since $N_1 = N'$ ($N_3 = N'$) denotes the dark matter HNL. Sensitivity regions of future experiments FCC-ee [28], DUNE [29] and SHiP [30] are bounded by dashed curves.

possibly be detected in laboratory searches. For the values of $\bar{\mu}_\nu$ and $\eta_\nu$ in eq. (4.6), the largest eigenvalue of $\delta M$ is given by \footnote{The eigenvalue of $\delta M$ computed in eq. (4.9) is the maximum value allowed by current experimental constraints because $\delta M \propto \bar{\mu}_\nu^{-1}$ from eqs. (2.5, 4.2) and the minimum of $\bar{\mu}_\nu$ is reached at $M_N = 4.5$ GeV as shown in Figure 2.}

$$\delta M = 3.1 \times 10^{-6} \text{eV} \left( \frac{2 \text{GeV}}{M_N} \right)^2,$$  

(4.9)

It was recently shown by ref. [31] that this is an interesting value for inducing observable $N$-$\bar{N}$ oscillations at the SHiP experiment. These would be seen by production of $N\ell^+$ in a hadronic collision, followed by semileptonic decays $N \rightarrow \bar{N} \rightarrow \ell^+\pi$ (where $\pi$ represents a generic hadron). The smoking gun is the presence of like-sign leptons in the decay chain, that can only occur if $N$ oscillates to $\bar{N}$ within the detector.

4.3 HNL decays

One can estimate the lifetime of the unstable $N_i$ leptons, which undergo 2- or 3-body decays $N_i \rightarrow \ell^-q\bar{q}$ (with $q\bar{q}$ hadronizing into a meson) and $N_i \rightarrow \nu\ell^+\ell^-$ by $W$ and $Z$ exchange, due to mixing of $N_i$ with the active neutrinos with mixing angles $U_{\ell i}^T \approx -U_{\ell i}$. Then the decay rate is of order

$$\Gamma_{N_i} \approx \sum_\ell \frac{|U_{\ell i}|^2 G_F^2 M_N^5}{192 \pi^3}.$$  

(4.10)

This gives a lifetime of $\sim 10^{-3} - 10^{-4}$ s for $M_N \sim 1$ GeV, making such $N_i$ decays harmless for BBN or CMB.
Figure 4. Top: Minimum lifetime (left) and decay length (right) of the HNLs $N_2$ and $N_3$ for the case of normal neutrino mass hierarchy. Decay length assumes energy $E = 25$ GeV, appropriate for SHiP experiment. (The wiggles in the mass range $0.2 < M_N < 0.4$ GeV come from the E949 bound [36] present in Figure 4.11 of ref. [8], which also appear in fig. 2.) Bottom: branching ratios for $N_2$ (left) and $N_3$ (right) into various final states involving photon, hadrons, light neutrinos or charged leptons.

More quantitatively, we have evaluated the partial widths for $N_i \rightarrow \nu\gamma$, $N_i \rightarrow h^0\nu$, $N_i \rightarrow h^+\ell^-$, $N_i \rightarrow 3\nu$, $N_i \rightarrow \nu\ell^+\ell^-$, including the hadronic final states with $h^0 = \pi^0, \eta, \eta', \rho^0$, $h^+ = \pi^+, K^+, \rho^+, D^+$ as computed in ref. [32] and [33]. 3 Focusing on the normal hierarchy case, we use the mixing matrix given by eq. (4.4) with $\bar{\mu}_\nu$ shown in Figure 2, that leads to different lifetimes for the two unstable HNLs $N_2$ and $N_3$. The lifetimes are plotted in Figure 4, along with decay lengths in the case of HNLs with energy $E = 25$ GeV that would be relevant for the SHiP experiment. For $M_N \lesssim 0.3$ GeV, the lifetimes start to exceed 1 s, which for generic models of HNLs would come into conflict with nucleosynthesis. In our model, this need not be the case since the HNL abundance is suppressed by $N_i\bar{N}_i \rightarrow ss$ annihilations. Then it is the singlet that should decay before BBN, which generally occurs as long as $m_s > 2m_e$. However even if $m_s < 2m_e$, which typ

\[m_\ell_1 \text{ is negligible compared to } m_\ell_2. \] This is not as good an approximation for the case $\ell_1 = \mu, \ell_2 = \tau$ as for $\ell_1 = e, \ell_2 = \mu$. We provide the exact formula formula in Appendix A.
Figure 5. Constraints on a light singlet mediator, in the $m_s$-$\theta_s$ plane. The four plots consider different values of the DM mass $m_{N^{'}} = 1.5, 2.5, 3.5, 4.5$ GeV, for which the direct detection constraints (black dotted line) differ; all other constraints are the same. The dark blue regions are favored at 1 $\sigma$ and 2 $\sigma$ for the KOTO anomaly. The red, cyan, green and brown regions are excluded by CHARM [37], E949 [48], LHCb [38] and BaBar experiments [39], respectively. The violet and light-green regions are excluded by BBN [40] and supernova data [41]. Sensitivity projection for the SHiP experiment is indicated by the dashed blue-gray boundary. The experimental bounds, along with the projected sensitivity, are taken from ref. [41].

ically leads to singlet lifetimes greater than 1 s, ref. [34] shows that $s$ is so weakly coupled to the SM in these cases that it never thermalizes, and can therefore have a suppressed abundance.

In Figure 4 the branching ratios for $N_2$ and $N_3$ to decay into the various final states (summing over flavors within each category) is also shown. Leptonic final states dominate for $M_{N^{'}} > 2$ GeV, while hadronic ones dominate for lower $M_{N^{'}}$.

5 Constraints on singlet

In recent years there have been intensive efforts to constrain the possible existence of light mediators connecting the SM to a hidden sector, the scalar singlet with Higgs portal being
a prime example. The parameter space of $m_s$ and $\theta_s$ (the singlet-Higgs mixing angle) is constrained by a variety of beam-dump, collider and rare decay experiments, and by cosmology (big bang nucleosynthesis), astrophysics (supernova cooling) and dark matter direct searches. A large region of parameter space with $\theta_s \lesssim 10^{-3}$ and $m_s \lesssim 10\text{ GeV}$ remains open, and parts of this will be targeted by the upcoming SHiP experiment [8]. In Figure 5 we show some of the existing constraints, reproduced from ref. [41].

The KOTO experiment has searched for the rare decay $K_L \to \pi^0 \nu \nu$ and set a new stringent upper limit of $3 \times 10^{-9}$ on its branching ratio [42, 43]. Recently four candidate events above expected backgrounds were reported [11], far in excess of the standard model prediction ($\text{BR} = 3 \times 10^{-11}$ [25]). These could potentially be explained by the exotic decay mode $K_L \to \pi^0 s$, if $s$ is sufficiently long-lived to escape detection, or if it decays invisibly. Such an interpretation has been previously considered in refs. [44–46]. In Figure 5, the 1σ and 2σ regions estimated in ref. [44] for explaining the KOTO excess are shown in blue. Parts of these regions are excluded by other experiments, notably NA62 [47] and E949 [48], but a range around $m_s \sim (120 - 160)\text{ MeV}$ and $\theta_s \sim (2 - 9) \times 10^{-3}$ remains viable.

The four plots in Figure 5 pertain to different choices of the DM mass $m_{N'}$, for the purpose of showing constraints from direct detection, that we describe in the following section. It can be seen that the region favored by the KOTO excess events is excluded by DM direct searches except for light DM, with $m_{N'} \lesssim 2.5\text{ GeV}$.

6 DM Direct detection and self-interaction

In general, the interactions of DM with nucleons versus with other DM particles are independent processes, whose cross sections need not be related. In our model however, both are mediated by exchange of the singlet $s$, so it is natural to consider them together.

6.1 DM-nucleon scattering

The mixing of $s$ with the Higgs boson leads to spin-independent DM interactions with nucleons. In particular, the cross section for scattering on nucleons is

$$
\sigma_{\text{SI}}^p = \frac{g_s^2 m_N^2 m_n^4 \sin^2 2\theta_s f_n^2}{4\pi (m_{N'} + m_n)^2 \bar{v}^2} \left( \frac{1}{m_h^2} - \frac{1}{m_s^2} \right)^2,
$$

where $m_n = 938\text{ MeV}$ is the nucleon mass, $f_n = 0.30$ is the relative coupling of the Higgs to nucleons [49, 50], $m_h = 125\text{ GeV}$ is the SM-like Higgs mass, $m_s$ is the singlet mass and $\theta_s$ is the $s$-$h$ mixing angle. (Recall that $\bar{v} \approx 174\text{ GeV}$ is the complex Higgs field VEV.)

The strongest constraints from direct detection, in the mass range $m_{N'} < 4.5\text{ GeV}$ predicted in our model, come from the experiments CRESST II [51], CDMSlite II [52] and LUX [53]. In the future these limits will be improved by SuperCDMS [54]. In all cases the sensitivity rapidly drops with lower DM mass because of the threshold for energy deposition. The DarkSide experiment [55] claims limits below those mentioned above, but their validity has been questioned in ref. [56], and we omit them from our analysis.

Recently ref. [57] cast doubts on the robustness of direct constraints on light dark matter in light of astrophysical uncertainties, especially that of the local escape velocity,
that has been revisited using Gaia data [58]. It is claimed that the 2017 cross section bound from XENON1T [59] at DM mass 4 GeV is uncertain by six orders of magnitude. We checked their results using DDCalc [60], finding only two orders of magnitude uncertainty. More importantly, the astrophysical uncertainty on the more relevant newer constraints is only a few percent (due to the much lower thresholds of those experiments), hence irrelevant.

For a given value of DM mass $m_{N'}$, we can use the relic density constraint shown in Figure 1 to determine the coupling $g_s$. Then the predicted direct detection cross section (6.1) leads to a constraint in the $m_s$-$\theta_s$ plane, that we plot as a dashed curve in Figure 5. As mentioned above, for larger values of $m_{N'}$ the direct detection constraint is strongest, and the region favored by the KOTO anomaly is excluded.

### 6.2 DM self-interactions

Dark matter can also interact with itself by exchange of the $s$, which is of interest for addressing small-scale structure problems of collisionless cold dark matter (see ref. [61] for a review). Ref. [62] showed that the self-interaction cross section can be at an interesting level for solving these problems, while obtaining the right DM relic density, if both $m_{N'}$ and $m_s$, are light,

$$m_s \approx 1 \text{ MeV} \times \left\{ \begin{array}{ll}
\left( \frac{m_{N'}}{0.55 \text{ GeV}} \right)^{3/4}, & \sigma/m_{N'} = 1 \text{ cm}^2/\text{g} \\
\left( \frac{m_{N'}}{0.25 \text{ GeV}} \right)^{3/4}, & \sigma/m_{N'} = 0.1 \text{ cm}^2/\text{g}
\end{array} \right. \quad (6.2)$$

These relations, valid for approximately symmetric DM, correspond to self-interaction cross section per mass in the range $\sigma/m_{N'} = 0.1 - 1 \text{ cm}^2/\text{g}$, that are relevant for suppressing cusps in density profiles of dwarf spheroidal to Milky Way-sized galaxies.

Such light singlets in the MeV mass range are strongly constrained by direct detection. The prediction (6.1) is modified by the fact that the momentum transfer $q$ is no longer negligible compared to $m_s$, hence $m_s^2 \rightarrow m_s^2 + q^2$ in eq. (6.1). We take $q = m_{N'} v_{N'}$ with DM velocity $v_{N'} \sim 300 \text{ km/s}$ to account for this. Figure 5 shows that for low $m_s$ there is an allowed window for $\sin \theta_s \sim 2 \times (10^{-5} - 10^{-4})$ between the BBN and E949 constraints (which persists to smaller values of $m_s \gtrsim 1 \text{ MeV}$ before being nominally excluded by BBN as $m_s$ falls below the threshold for $s \rightarrow e^+ e^-$ decay).

In Figure 6 we show the predicted spin-independent cross section versus $m_{N'}$ for several choices of $\theta_s$ in the allowed range, fixing $g_s$ as in Figure 1 to give the right relic density, and $m_s$ as a function of $m_{N'}$ using (6.2). We see that cosmologically interesting self-interactions can be compatible with all other constraints for $m_{N'} \lesssim 0.3 \text{ GeV}$ and $m_s \lesssim 0.7 \text{ MeV}$. The effect of considering lower self-interaction cross sections ($\sigma/m_{N'} = 0.1 \text{ cm}^2/\text{g}$) is to increase the allowed region for $m_{N'}$ and especially $m_s$ to somewhat larger values, as fig. 6 (right) shows. Although the higher cross section $\sigma/m_{N'} = 1 \text{ cm}^2/\text{g}$ is potentially in tension with BBN constraints on the light singlet, since $m_s < 2m_e$ and its lifetime exceeds 1 s, we noted above that this need not be ruled out because of the suppressed nonthermal abundance of the singlet [34]. The smaller cross section case can be more robustly safe in this respect.

The previous determination holds in the region $m_{N'} \lesssim 3 \text{ GeV}$ where the DM is to a good approximation symmetric, corresponding to the linearly increasing branch of the
relic density contour in Figure 1. For nearly asymmetric DM, the horizontal branch with $m_{N'} \equiv 4.5$ GeV applies. Instead of eq. (6.2), the desired self-interaction cross section requires a roughly linear relation $g_s \cong 0.75 + 4.43 m_s / \text{GeV}$ (valid for $m_s \sim 0.2 - 0.3$ GeV), that we determine by applying a Sommerfeld enhancement factor [64] to the tree-level, phase-space averaged transport scattering cross section given in ref. [62], and requiring that the resulting cross section is $\sigma/m_{N'} = 1 \text{ b/GeV}$ for a mean DM velocity of 10 km/s, corresponding to dwarf spheroidal galaxies. To satisfy the CDMS-Lite constraint $\sigma_{SI} < 1 \times 10^{-41} \text{ cm}^2$ at $m_{N'} = 4.5$ GeV [65], it is necessary to take small mixing $\theta_s \lesssim 6 \times 10^{-6}$, since $g_s \sim 2$ for $m_s \sim 0.2 - 0.3$ GeV, from imposing the desired value of $\sigma/m_{N'}$.

Hence we find two allowed regions for strong self-interactions, one marginal since $m_s \gtrsim 1$ MeV close to BBN limits, with $\sigma/m_{N'} \sim 0.1 \text{ cm}^2/\text{g}$ at the low end of the range desired for small scale structure, and $m_{N'} \sim 0.2$ GeV. The other allows for a larger $\sigma/m_{N'} \gtrsim 0.6 \text{ cm}^2/\text{g}$, with singlet parameters close to the SN1987A exclusion curve and $m_{N'} \cong 4.5$ GeV.

7 Naturalness

In our proposal, the flavor structure of neutrinos is controlled by the same matrix $\eta_{\nu,ij}$ that governs the HNL couplings, up to a proportionality constant, in the spirit of MFV. In order for DM to be stable, $\eta_{\nu,ij}$ must have rank two. The HNL mass matrix is proportional to the identity, up to corrections going as $\eta^2$. We do not provide any fundamental explanation of the origin of this structure; instead we content ourselves with the feature that it is technically natural in the sense of ’t Hooft: all radiative corrections are consistent with our assumptions.
The stability of DM is most easily seen in the basis (4.6, 4.7), where $N'$ obviously decouples from the SM leptons. We assume this coincides with the mass eigenbasis, which is consistent since there are no interactions that can induce mass-mixing between $N'$ and the remaining $N_i$'s. Self-energy corrections involving $s$ exchange are flavor-diagonal. Those involving Higgs and leptons in the loop leave $m_{N'}$ unchanged, while renormalizing the $N_i$ mass matrix by

$$M_N \delta_{ij} \rightarrow M_N \delta_{ij} + O(1) \times \eta_{\nu,ik} m_{\ell_i} \frac{m_{\ell_i}}{16\pi^2} \eta_{\nu,kj}$$

(7.1)

where $m_{\ell_i}$ are the charged lepton masses. Given the smallness of $\eta_\nu \lesssim 10^{-4}$, these corrections are unimportant. Similarly the one-loop corrections to $\eta_\nu$ are negligible,

$$\eta_\nu \rightarrow \eta_\nu + \frac{O(1)}{16\pi^2} \eta_\nu \eta_\nu$$

(7.2)

and cannot induce couplings to $N'$. The only particles to which $N'$ couples are the singlet and the inflaton, eqs. (2.1, 3.7), and these interactions are assumed to be flavor-conserving at tree level. Flavor-changing corrections to $g_s$ and $g_\phi$ of $O(\eta_\nu^2/(16\pi^2)) \times g_{s,\phi}$ arise at the one-loop level and are negligible for our purposes.

There remains the infamous naturalness problem of the Higgs mass (weak scale hierarchy). This problem was addressed in the context of the seesaw mechanism in ref. [66], where the weak scale was linked to that of the heavy Majorana neutrinos by radiative generation of the Higgs potential. A low scale for their masses is needed, $M_{UR} \lesssim 10^7$ GeV [67], which would require a low reheat temperature in our scenario, and consequently small coupling $g_\phi \lesssim 10^{-8}$. Although peculiarly small, this value would still be compatible with the requirements of technical naturalness since it can only be multiplicatively renormalized.

The very light singlet could pose an analogous problem of fine-tuning. The first threshold encountered when running the renormalization scale up from low values is that of $N_i$, which contributes of order $\delta m_s \sim g_s M_N/(4\pi)$ to $m_s$. This can easily be compatible with the tree-level values of $m_s$ desired for large parts of the allowed parameter space (see figs. 1 and 5).

Next one encounters the Higgs threshold, which further shifts $m_s$ through the coupling $\lambda_{hs}$. The correction is of order $\delta m_s \sim \sqrt{\lambda_{hs}} m_h/4\pi$ which is related to the mixing angle by $\theta_s \sim \lambda_{hs} v_s / m_h^2$, where $v$ and $v_s$ are the respective VEVs of the Higgs and the singlet. In turn, $v_s$ depends upon the $s$ self-coupling through $m_s^2 \sim \lambda_s v_s^2$. Using these and demanding that $\delta m_s \lesssim m_s$ gives the constraint $\sqrt{\lambda_s} \lesssim 16\pi^2 m_s^3 v / (\theta_s m_h^4)$. This can always be satisfied by choosing small enough $\lambda_s$, but the latter has a minimum natural value given by its one-loop correction $\delta \lambda_s \sim g_s^2/(16\pi^2)$.

Putting all of these together, we get a naturalness bound on the singlet mixing angle

$$\theta_s \lesssim \left( \frac{4\pi m_s}{m_h} \right)^3 \left( \frac{1}{\sqrt{\lambda_h g_s^2}} \right) \sim 0.008$$

(7.3)

(taking $m_s \sim 0.3$ GeV and $g_s \sim 0.1$) which is compatible with the regions of interest for future discovery, including the anomalous KOTO events. Thus, somewhat surprisingly,
the light scalar does not introduce a new hierarchy problem analogous to that of the Higgs mass, due to its relatively weak couplings.

We do not address the smallness of $\theta_{QCD}$ in our “theory of everything,” which was a motivation for ref. [4] to choose the QCD axion as their dark matter candidate. This neglect is consistent with our philosophy of focusing on technical naturalness rather than aesthetic values of couplings, since $\theta_{QCD}$ is known to be highly stable against radiative corrections [68].

8 Conclusions

It is interesting to construct scenarios that link the different particle physics ingredients known to be missing from the standard model, since it can lead to distinctive predictions. Here we have constructed a minimal scenario that explains inflation, baryogenesis, dark matter and neutrino masses, is highly predictive, and can be tested in numerous experimental searches for heavy neutral leptons, light dark matter, and light scalar mediators. At low energies, the only new particles are three quasi-Dirac HNLs, one of which is DM (and exactly Dirac), and a light singlet scalar.

One prediction of the model is that no new source of CP-violation is required for baryogenesis, which occurs through a novel form of leptogenesis here. In contrast to ordinary leptogenesis, the asymmetry is formed during inflation, and the right-handed neutrinos that generate light neutrino masses are too heavy to be produced during reheating. CP is spontaneously broken by the inflaton VEV during inflation, and the light HNLs transmit the lepton asymmetry from the inflaton to the SM. In ref. [6] it was shown that observable isocurvature perturbations can arise, depending on the inflaton potential and initial conditions. In the present model, these would appear as correlated dark matter isocurvature and adiabatic perturbations.

Another is that the two unstable HNLs $N_i$ should be degenerate to very high precision with the dark matter $N'$, split only by the correction (7.1) of order $10^{-2}$ eV. Similarly, the $N_i$ are Dirac particles to a very good approximation, with a lepton-violating Majorana mass of order $10^{-6}$ eV. This is too small to be detectable in neutrinoless double beta decay, but large enough to allow for a distinctive signature of lepton violation through $N$-$\bar{N}$ oscillations. The two $N_i$ HNLs can mix strongly enough with SM neutrinos to be discoverable at upcoming experiments like SHiP. The stability of $N'$ is directly linked to the masslessness of the lightest neutrino.

In our framework, the dark matter $N'$ is partially asymmetric, and has a mass bounded by $m_{N'} \lesssim 4.5$ GeV. The bound is saturated when $N'$ is purely asymmetric, and its mass is determined by the observed value of the baryon asymmetry. Light DM can be accommodated by taking small values of the coupling $g_s$ between $N'$ and the singlet $s$, which controls $N'\bar{N}' \rightarrow ss$ annihilation; see Figure 1. In the mass range $(1 - 4.5)$ GeV, significant constraints are already placed by direct DM searches.

The light scalar singlet, whose mass must be less than $m_{N'}$ for efficient $N'\bar{N}' \rightarrow ss$ annihilation, can lead to striking signatures. For example the decay $K_L \rightarrow \pi s$ can explain anomalous excess events recently observed by the KOTO experiment, but only if $m_{N'} \lesssim$
2.5 GeV; otherwise direct detection constraints rule out this mode at the level suggested by the KOTO events, where $m_s \sim (100 - 200)$ MeV and $s$ mixes with the Higgs at the level $\theta_s \sim 5 \times 10^{-4}$. (The preferred parameter region for the KOTO anomaly is only a small part of the full allowed space of our model.) In a different part of parameter space with $m_s \sim (0.2 - 0.3)$ GeV, $m_{N'} \sim 4.5$ GeV and $\theta_s \lesssim 6 \times 10^{-6}$, the singlet mediates DM self-interactions with a cosmologically interesting cross section, $\sigma/m_{N'} \sim 0.6 \text{ cm}^2/\text{g}$.

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A Decay rate for $N_i \to \nu \ell^+ \ell^-$

The matrix element for the process $N_i \to \nu_\beta \ell_\beta^+ \ell_\alpha^-$, where $\alpha, \beta = e, \mu, \tau$, is

$$M = \frac{g_\nu^2}{8 M_W^2} \left[ \bar{u}(p_{\nu_\beta}) \gamma^\mu (1 - \gamma^5) u(p_{N_i}) U_{i\alpha}^* \right] \left[ \bar{u}(p_{\ell_\beta}) \gamma_\mu (1 - \gamma^5) u(p_{\ell_\alpha}) \right]$$

(A.1)

whose square reduces to

$$\langle |M|^2 \rangle = \frac{G_F^2}{16} |U_{i\alpha}|^2 M_i E_{\beta} \left[ \frac{M_i^2 + m_{\beta}^2 - m_{\alpha}^2}{2} - M_i E_{\beta} \right]$$

(A.2)

after averaging over the initial spin, summing over final spins and setting $m_{\nu_\beta} = 0$. Here, $G_F$ is the Fermi constant, $E_{\beta}$ is the energy of $\ell_\beta^+$ and we have defined for simplicity $M_i \equiv M_{N_i}$, $m_\alpha \equiv m_{\ell_\alpha^-}$ and $m_\beta \equiv m_{\ell_\beta^+}$. The decay rate $\Gamma$ can be obtained by plugging eq. (A.2) in the standard decay formula (see ref. [25]) and computing the three-body phase space integral. The common assumption made in the literature is to consider $m_\beta = 0$, which is well motivated for $\alpha = e, \mu$ and $\beta = \mu, e$. In these cases, the decay rate is [32, 35]

$$\Gamma = \frac{G_F^2 M_i^5}{192 \pi^3} |U_{i\alpha}|^2 \left( 1 - 8 x_\alpha^2 + 8 x_\alpha^6 - x_\alpha^8 - 12 x_\alpha^4 \log(x_\alpha^2) \right)$$

(A.3)

where $x_\alpha = m_\alpha/M_i$. Such a simplified formula does not hold for $\alpha = \mu, \tau$ and $\beta = \tau, \mu$, where the muon mass is not negligible compared to the tau mass. The general expression
reads
\[
\Gamma = \frac{G_F^2 M_i^5}{192 \pi^3} |U_{i\alpha}|^2 \left\{ 12 |x_\beta^2 - x_\alpha^2| (x_\beta^2 + x_\alpha^2) \log \left[ \frac{x_\beta^2 + x_\alpha^2 - (x_\beta^2 - x_\alpha^2)^2}{2 x_\beta x_\alpha} \right] - 12 \left[ x_\beta^4 + x_\alpha^4 - 2 x_\beta^4 x_\alpha^4 \right] \log \left[ \frac{1 - x_\beta^2 - x_\alpha^2 - \sqrt{1 - (x_\beta - x_\alpha)^2} \sqrt{1 - (x_\beta + x_\alpha)^2}}{2 x_\beta x_\alpha} \right] + \sqrt{(1 - (x_\beta - x_\alpha)^2)(1 - (x_\beta + x_\alpha)^2)} \right] \left[ 1 - 7 \left( x_\beta^2 + x_\alpha^2 \right) \left( 1 + x_\beta^2 x_\alpha^2 \right) - 7 \left( x_\beta^4 + x_\alpha^4 \right) + 12 x_\beta^2 x_\alpha^2 + x_\beta^6 + x_\alpha^6 \right] \right\} \]
\]
(A.4)

where \( x_\alpha \equiv m_\alpha/M_i \) and \( x_\beta \equiv m_\beta/M_i \). It is easy to check that this formula reduces to eq. (A.3) in the limit \( m_\beta \to 0 \).

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