B Physics on the Lattice: $\bar{\Lambda}$, $\lambda_1$, $\bar{m}_b(\bar{m}_b)$, $\lambda_2$, $B^0 - \bar{B}^0$ mixing, $f_B$ and all that.

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We present a short review of our most recent high statistics lattice determinations in the HQET of the following important parameters in B physics: the B–meson binding energy, $\bar{\Lambda}$ and the kinetic energy of the b quark in the B meson, $\lambda_1$, which due to the presence of power divergences require a non–perturbative renormalization to be defined; the $\overline{MS}$ running mass of the b quark, $\bar{m}_b(\bar{m}_b)$; the $B^*–B$ mass splitting, whose value in the HQET is determined by the matrix element of the chromo–magnetic operator between B meson states, $\lambda_2$; the B parameter of the $B^0–\bar{B}^0$ mixing, $B_B$, and the decay constant of the B meson, $f_B$. All these quantities have been computed using a sample of 600 gauge field configurations on a $24^3 \times 40$ lattice at $\beta = 6.0$. For $\bar{\Lambda}$ and $\bar{m}_b(\bar{m}_b)$, we obtain our estimates by combining results from three independent lattice simulations at $\beta = 6.0$, $6.2$ and $6.4$ on the same volume.

1. The calculation of $\bar{\Lambda}$ and $\bar{m}_b(\bar{m}_b)$.

The presence of renormalon singularities in the pole mass, $m_b^{pole}$, implies that the intuitive definition $\bar{\Lambda} \equiv m_B - m_b^{pole}$ does not correspond to a physical quantity. Thus, if $\bar{\Lambda}$ is to be used in phenomenological applications, a definition free of renormalon ambiguities has to be given. One can define the B–meson binding energy from the experimentally measured value of some physical quantity, for example the inclusive $B \rightarrow X l \bar{\nu}_l$ lepton spectrum. It is important to notice that such a definition, although correct, depends on the order of the perturbation series that determines the theoretical prediction for the experimental quantity.

Lattice simulations of the HQET provide the opportunity to compute $\bar{\Lambda}$ non–perturbatively by using the definition of $\bar{\Lambda}$ proposed in ref. which is free of renormalon ambiguities. On the lattice, renormalons are absent but matrix elements of lattice operators may contain power ultraviolet divergences. Consider the two–point correlation function $C(t)$ for large values of $t$

$$C(t) = \sum_{\vec{x}} \langle \emptyset | J_B(\vec{x}, t) J_B^\dagger(\vec{0}, 0) | \emptyset \rangle \rightarrow Cte \exp(-\epsilon h)$$

where $J_B$ creates a B–meson from the vacuum. It is tempting to define $\bar{\Lambda} \equiv \epsilon$. However, the ”bare” binding energy $\epsilon$ cannot be a physical quantity because it diverges linearly as $a \rightarrow 0$, $a$ being the lattice spacing $\ell a$. The origin of the linear divergence is the mixing of the operator $\bar{\Lambda}D_4 h$ with the lower dimensional operator $\bar{\Lambda} h$. As pointed out in ref. the linear divergence must be subtracted non–perturbatively in order not to reintroduce renormalon ambiguities. This divergence can be eliminated from any correlation function by defining a renormalized action $\bar{\Lambda}D_4 h$ of the form

$$\bar{\Lambda}D_4 h = \bar{\Lambda}D_4 h - (1 - \exp(-a\delta \bar{m})) \bar{\Lambda} h$$

where $\delta \bar{m}$ is a mass counter–term which can be chosen in many different ways. Our preferred def-

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inition of $\delta \bar{m}$ is

$$- \delta \bar{m} \equiv \lim_{t \to \infty} \frac{1}{a} \ln \left( \frac{S(\vec{x}, t + a)}{S(\vec{x}, t)} \right)$$

(3)

where $S(\vec{x}, t)$ is the heavy–quark propagator in the landau gauge which can be computed in a numerical simulation of the HQET. In words, $\delta \bar{m}$ is the condition that the subtracted heavy–quark propagator has no exponential fall at large times or equivalently that $\langle h(p_4 = 0) | \bar{h}D_h^2 h | h(p_4 = 0) \rangle = 0$. Due to confinement effects, it is not clear that (3) has a finite limit as $t \to \infty$. Our numerical results obtained from three independent high–statistics simulations at $\beta = 6.0, 6.2$ and $6.4$ support our assumption on the infrared behaviour of the heavy–quark propagator: no confining effects are visible up to distances of about 1.5 fm, which is the largest distance which we can reach.

An alternative definition of $\delta \bar{m}$ which does not require any assumption about the large time behaviour of the heavy quark propagator is given in refs. However, with this definition $\bar{X}$ is not a parameter of $O(\Lambda_{QCD})$. Notice also that the divergent part of $\delta \bar{m}$ must be gauge invariant. Moreover, the gauge dependence of possible finite parts is eliminated from physical quantities up to the order of perturbation theory at which the calculation is performed. The renormalized binding energy $\bar{X}$ which is of order $O(\Lambda_{QCD})$, free of renormalon singularities and independent of the renormalization scale is defined as: $\bar{X} \equiv \bar{E} - \Delta \bar{E}$. Our best estimate of $\bar{X}$ is $\bar{X} = 180^{+30}_{-20}$ MeV (see ref. for details).

We can also compute the $\overline{MS}$ running mass of the b–quark, $m_b(\overline{m}_b)$. By comparing the heavy–quark propagator in the continuum with the one on the lattice, we obtain

$$m_b(\overline{m}_b) = (M_B - \overline{X}) \times \overline{\alpha}_s(\delta \bar{m})$$

(4)

where $\overline{\alpha}_s$ is the QCD–HQET matching constant (see ref. for details). Taking $M_B = 5.278$ GeV, we find $m_b(\overline{m}_b) = 4.15 \pm 0.05 \pm 0.20$ GeV, where the first error is statistical and the second is an estimation of the error in the truncation at $O(\overline{\alpha}_s^3)$ of the perturbative factor $\overline{\alpha}_s$, derived from the bubble–resummation approximation. This error can be reduced to about 100 MeV by computing perturbative corrections at two–loop order. This calculation requires the evaluation of the heavy quark propagator in the continuum, already done, and on the lattice at $O(\overline{\alpha}_s^2)$. The Langevin approach of ref. can be used to perform the computation on the lattice.

2. The calculation of $\lambda_1$.

Like $\overline{X}$, $\lambda_1$ has no physical meaning since it depends on the definition that one decides to adopt for the kinetic operator. On the lattice, matrix elements of this operator contain linear and quadratic divergences. The finite kinetic operator, which is free of power divergences, has the form

$$\bar{h}D_h^2 h = \bar{h}D_R^2 h - \frac{c_1}{a} \bar{h}D_R h - \frac{c_2}{a^2} \bar{h}h$$

(5)

where $c_1$ and $c_2$ are subtraction constants which we define by imposing the “physical” non–perturbative renormalization prescription. The condition

$$\langle h(\vec{p} = 0) | \bar{h}D_R^2 h | h(\vec{p} = 0) \rangle = 0$$

(6)

in the landau gauge. This is equivalent to the condition $\rho_{\beta^2}(t) = c_1 + c_2$ where

$$\rho_{\beta^2}(t) \equiv \sum_{\vec{x}, \vec{y}, t=0}^{t} S(\vec{x}, \vec{y}) \frac{\bar{D}_R^2(\vec{t}) S(\vec{y}, 0)}{\sum_{\vec{x}} S(\vec{x}, 0)}$$

(7)

with $x = (\vec{x}, t)$ and $y = (\vec{y}, t')$. To determine $c_1$ and $c_2$, we fit $\rho_{\beta^2}(t)$ for large $t$ to a straight line. Our numerical results show that the infrared limit of the ratio in eq. does indeed exist, in spite of the confinement effects in the heavy–quark propagator. $c_2$ can also be obtained directly by eliminating the sum over $t'$ in eq. and searching for a plateau in $t$. We find that, after subtracting the contribution of contact terms, both methods give indistinguishable results (see ref. for details). Notice also that, for the same reason as for $\delta \bar{m}$, $c_2$ must be gauge independent.

The renormalized kinetic energy $\lambda_1$ is then given by: $\lambda_1 = Z_{\beta^2} (\lambda_1^{bare} - c_2)$, where $\lambda_1^{bare} = \langle B | \bar{h}D_R^2 h | B \rangle/(2M_B)$ and $Z_{\beta^2}$ is a gauge invariant matching constant known up to one loop order. A convenient way to extract $\lambda_1^{bare}$ is to study the large time behaviour of the ratio of the two–point and the three–point, with an insertion
of the kinetic operator, B–meson correlation functions. The significant reduction of the statistical errors with respect to our previous study allows for a much better identification of the plateau region, which could not be really observed in ref.[3]. We find $\lambda_1(B_d) = 0.09 \pm 0.14$ GeV$^2$ and for the physical parameter $\lambda_1(B_s) - \lambda_1(B_d) = -0.09 \pm 0.04$ GeV$^2$ (see ref.[3] for details). In order to make a meaningful comparison of our results with other theoretical determinations, a perturbative calculation of the relation between the different definitions of $\lambda_1$, and also of $\lambda_2$, is needed at a sufficient degree of accuracy [4]. This calculation is missing to date.

3. The calculation of $\lambda_2$

The chromo–magnetic operator that enters in the calculation of $\lambda_2$ is only logarithmically divergent in the ultraviolet cut–off so that there is no need of a non–perturbative subtraction. Moreover, $\lambda_2$ has a physical meaning because it corresponds directly to the measurable $B^*-B$ mass splitting up to higher power corrections in $1/m_b$,

$$M_{B^*}^2 - M_B^2 = 4\lambda_2 = 4Z_{\sigma \cdot G} \lambda_2^{bare}$$

(8)

where $Z_{\sigma \cdot G}$ is the renormalization constant necessary to remove the logarithmic divergence present in the bare operator. The procedure to extract $\lambda_2^{bare}$ is the same as for $\lambda_1^{bare}$ (see ref.[4] for details). We find $\lambda_2 = 0.07 \pm 0.01$ GeV$^2$ corresponding to a mass splitting which is about one half of the experimental value. In our opinion, there are three possible explanations of the discrepancy, which is common to all lattice results. The first is the fact that, at one loop order, $Z_{\sigma \cdot G}$ is very large indicating that higher–order terms may modify this factor significantly. A second source of systematic error is quenching. The final possibility is that the discrepancy is due to the fact that the mass splitting receives significant contributions from higher terms in the HQET. Further investigation is needed on this subject.

4. The calculation of $B_B$ and $f_B$

We have also computed the parameter $f_B^2 B_B$, which enters the theoretical prediction of the $B^0 - \bar{B}^0$ mixing parameters $x_d$ and $x_s$. We measure $f_B$ and $B_B$ from the ratios of three– and two–point functions with the $\Delta B = 2$ operator fixed at the origin. Due to the high statistics we have accumulated, we can study this ratios at large time distances at which the ground state has been unambiguously isolated. We find a very good signal for all the correlation functions and ratios. However, the main uncertainty in our results comes from the evaluation of the renormalization factors relating the effective operators on the lattice to those in the continuum. For $B_B$, we use the new calculation of the NLO Wilson coefficients of the $O(\alpha_s)$ full theory–HQET matching, which takes into account previously missed contributions [6]. In our opinion, a non–perturbative computation of these factors is crucial to reduce the rather big systematic errors. This work is in progress now [6]. Our best estimates are [6]: $\hat{B}_{B_d} = 1.21 \pm 0.06$ (RGI $B$ parameter), $\hat{B}_{B_s}/B_{B_d} = 1.011 \pm 0.008$ and $f_B^2 B_{B_s}/f_B^2 B_{B_d} = 1.38 \pm 0.07$, where we use the boosted perturbation theory to evaluate the renormalization constants.

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