The Goldstone theorem protects naturalness, and the absence of Brout-Englert-Higgs fine-tuning, in spontaneously broken SO(2)

Bryan W. Lynn\textsuperscript{1} and Glenn D. Starkman\textsuperscript{2,3}

\textsuperscript{1} University College London, London WC1E 6BT, UK
\textsuperscript{2} ISO/CERCA/Department of Physics, Case Western Reserve University, Cleveland, OH 44106-7079 and
\textsuperscript{3} CERN, CH-1211 Geneva 23, Switzerland

The Gell-Mann-Lévy (GML), Schwinger and Standard Models were previously shown not to suffer from a Brout-Englert-Higgs (BEH) fine-tuning problem due to ultraviolet quadratic divergences, with finite Euclidean cut-off $\Lambda$, because of the symmetries obeyed by all $\mathcal{O}(\Lambda^2)$ contributions. We extend those no-fine-tuning results to finite contributions from certain $M_{\text{Heavy}}^2 \gg m_{\text{BEH}}^2$ particles, in simplified SO(2) versions of GML and Schwinger. We demonstrate explicit 1-loop physical naturalness (i.e. G. 't Hooft's 1979 criteria for no-fine-tuning) for two examples, both SO(2) singlets: a heavy $M_3^2 \gg m_{\text{BEH}}^2$ real scalar $S$ with discrete $S \to -S$ symmetry; a right-handed Type 1 See-saw Majorana neutrino $\nu_R$ with $M_{\nu_R}^2 \gg m_{\text{BEH}}^2$. We prove that for $|q^2| \ll M_{\text{Heavy}}^2$ the heavy degrees of freedom contribute, at worst, marginal operators in spontaneously broken SO(2) Schwinger.

The key Gell-Mann-Lévy lesson, from these two one-loop examples, is that the pseudo Nambu-Goldstone boson (NGB) mass-squared $m_N^2$ must be properly renormalized. A true NGB value, $m_N^2 = 0$, is then protected by the Goldstone theorem. For the SO(2) Schwinger model, two crucial observations emerge: that global Ward-Takahashi identities (WTI) force all relevant operators into Schwinger's symmetric Wigner mode, i.e. into the pseudo-NGB mass-squared $m_N^2 \neq 0$, and WTI enforce the Goldstone theorem by forbidding all relevant operator contributions $- \mathcal{O}(\Lambda^2)$, $\mathcal{O}(M_{\text{Heavy}}^2 \ln \Lambda^2)\mathcal{O}(M_{\text{Heavy}}^2)$ and $\mathcal{O}(M_{\text{Heavy}}^2 \ln M_{\text{Heavy}}^2)$ -- in Schwinger’s spontaneously broken Goldstone mode, i.e. $\pi_{\text{BEH}}$, weak-scale $M_{\text{BEH}}^2, \langle H \rangle^2 \ll M_{\text{Heavy}}^2$, is not-fine-tuned, not just as a stand-alone renormalizable field theory, but also as a low-energy effective theory with certain high-mass-scale extensions. Its "Goldstone Exceptional Naturalness", where all relevant operators vanish identically, is a far more powerful suppression of fine-tuning (i.e. than G 't Hooft's criteria) and is simply another (albeit unfamiliar) consequence of WTI, spontaneous symmetry breaking and the Goldstone theorem.

If Goldstone Exceptional Naturalness can somehow be extended to the Standard Model (SM), there should be no expectation that LHC will discover any Beyond the SM physics unrelated to neutrino mixing, i.e. the only known experimentally necessary modification of the Standard Model plus General Relativity paradigm.

PACS numbers: 11.10.Gh

Naturalness has played a central role in theoretical particle physics over the last several decades. Theories deemed “un-natural” are condemned as fatally Fine-Tuned (FT). It was the widespread belief that the scalar sector of the Standard Model (SM) suffered from the so-called Brout-Englert-Higgs (BEH) FT problem that drove the development of many Beyond the Standard Model (BSM) theories, including Low Energy Supersymmetry and Technicolor. This letter takes a step (i.e. for global theories) toward a non-BSM proposal, based on the Goldstone theorem, that may resolve a perceived crisis due to tension between LHC8 data and simple BSM solutions of that “BEH-FT problem.”

One challenge has been pinning down exactly what it means to be “natural,” what it means to be “fine-tuned,” and how they are related. In this letter we will demonstrate how, when fine-tuned and natural are regarded as mutually exclusive, not only are the Gell-Mann-Lévy Model (GML) and the Schwinger model not FT, with respect to ultraviolet quadratic divergences (UVQD), but neither are a variety of interesting extensions to GML and Schwinger. The key is for any new high-mass-scale physics to respect the same symmetry that protects GML and Schwinger.

We call G. 't Hooft’s widely accepted naturalness/nofine-tuning criteria physical naturalness (PhysNat) because it avoids all reference to unmeasurable quantities: “at any energy scale $\mu$ a [dimensionless] physical parameter or a set of physical parameters $\alpha_\mu$ is allowed to be very small only if the replacement $\alpha_\mu(\mu) = 0$ would increase the symmetry of the system”$^{2,8}$.

We define here a new, far more powerful, suppression of fine-tuning called Goldstone Exceptional Naturalness (GoldExceptNat): “If taking a dimensionless parameter to zero increases the symmetry of the system, and that increased symmetry then forbids all relevant-operator contributions to observables or counter-terms, then a small value for that quantity, and the resultant theory, are GoldExceptNat. GoldExceptNat is simply another (albeit unfamiliar) consequence of Ward-Takahashi identities (WTI), spontaneous symmetry breaking (SSB) and the Goldstone theorem.”

Theories are ordinarily regarded as “Bare Fine Tuned"
if physical observables depend too sensitively on the parameters of the bare Lagrangian \( L^{\text{bare}} \), especially if small dimensionless ratios of physical observables arise through cancellation of large (often divergent, and generally including bare) contributions unrelated by any symmetry. Bare-FT troubles us. \( L^{\text{bare}} \) is only a useful parametric device, a calculation tool without physical reality of its own, expressed in terms of unphysical bare parameters, whose values cannot be determined by any set of experiments. To judge a theory based on how a mathematical fiction depends on unphysical parameters seems unjustified. We argue here, both for the stronger PhysNat definition that avoids reference to unmeasurable quantities, and to explicitly exclude natural theories from being FT. If taking a dimensionless quantity to zero increases the symmetry of a theory, then a small value for that quantity is natural and not FT.

Some time ago, we demonstrated that the SM \( \mathbb{R} \) and its naive zero-gauge coupling limit in the scalar sector, the chiral-symmetric limit of GML \( \mathbb{R} \), are UVQD-PhysNat (and consequently are not FT). This is because in GML all cancellations of UVQD, to obtain weak-scale quantities, are absorbed into the mass-squared \( m_2^2 \) (at \( q^2 = 0 \)) of the three pseudo Nambu-Goldstone bosons (pseudo-NGBs) that share the same SU(2) doublet as the BEH. Other dimensionful contributions to physical quantities are proportional to \( \langle H \rangle^2 \), the square of the vacuum expectation value (VEV) of the BEH field, and to dimensionless couplings that can be fixed by low-energy experiments. Since taking either \( m_2^2 \) or \( \langle H \rangle^2 \) to zero restores chiral SU(2)_L−R symmetry, the smallness of \( m_2^2 \) or \( \langle H \rangle^2 \) compared to a Euclidean-integral UV cutoff-squared \( \Lambda^2 \) is PhysNat, not FT. (Note that UVQD are the only potential source of large dimensionful quantities in the SM \( \mathbb{R} \), since the SM has no intrinsic scale other than \( \langle H \rangle \).) This may seem reminiscent of the argument [10, 11] that the SM, with all masses set to zero, is not FT because it would then acquire classical scale invariance (broken by quantum \( \beta \)-functions); however, our results differ crucially: chiral SU(2)_L−R symmetry is actually a quantum symmetry of the Schwinger Model (that is transmitted to the SM), while scale invariance is not (and probably isn’t for gravity [12]).

To illustrate the absence of FT in GML and how that can be preserved when GML is augmented by high-mass-scale physics, we consider, for pedagogical simplicity, an SO(2) version of GML, with a complex scalar field \( \Phi \), with VEV \( \langle H \rangle / \sqrt{2} \). In linear and unitary representations, the renormalized field \( \Phi \equiv \frac{\langle H + i \sigma_3 \pi_3 \rangle}{\sqrt{2}} \equiv \frac{H}{\sqrt{2}} U \); \( U \equiv \exp \left[ \frac{i \sigma_3 \pi_3}{\langle H \rangle} \right] \) with Pauli matrix \( \sigma_3 \), bare field \( \Phi_0 = Z_0^2 \Phi \), with \( \langle H_0 \rangle = Z_0^2 \langle H \rangle \). The bare Lagrangian

\[
L^{\text{bare}}_{\text{GML}} = \frac{1}{2} \text{Tr} \left[ \partial_\mu \Phi_0 \right]^2 - V^{\text{bare}}_{\text{GML}/\text{Sym}} + L^{\text{bare}}_{\text{PCAC}}
\]

\[
V^{\text{bare}}_{\text{GML}/\text{Sym}} = \frac{\mu_{\phi(0)}^2}{2} \text{Tr} \left[ \Phi_0^\dagger \Phi_0 \right] + \frac{\lambda_{\phi(0)}^2}{4} \left( \text{Tr} \left[ \Phi_0^\dagger \Phi_0 \right] \right)^2.
\]

A Ward-Takahashi Identity (WTI) [13–17] relates bare and renormalized explicit symmetry breaking terms

\[
L^{\text{bare}}_{\text{PCAC}} \equiv c_0 H_0 = \langle H \rangle m_2^2 H = \epsilon H \equiv L^{\text{PCAC}},
\]

with \(-m_2^2 \) the renormalized \( \pi_3 \) inverse propagator at \( q^2 = 0 \). We take the renormalized vacuum, \( \langle H \rangle = \langle H \rangle \), to lie in the \( \langle \pi_3 \rangle = 0 \) direction.

Without \( L_{\text{PCAC}} \), \( L^{\text{bare}}_{\text{GML}} \) has an SO(2) chiral symmetry, under which \( \Phi \to \exp \left[ \frac{i \sigma_3 \theta}{2} \right] \Phi \exp \left[ \frac{i \sigma_3 \theta}{2} \right] \), easily understood in the unitary representation as a “shift” symmetry \( \pi_3 \to \pi_3 + \langle H \rangle \theta \). \( L_{\text{PCAC}} \) explicitly breaks this chiral SO(2) symmetry, sourcing only Partial Conservation of the Axial-vector Current (PCAC) \( \partial_\mu J_\mu = \langle H \rangle m_2^2 \pi_3 \).

In renormalizing \( L^{\text{bare}}_{\text{GML}} \) we compute quantum loop corrections to its operators and replace the 3 bare parameters \( (\mu_{\phi(0)}^2, \lambda_{\phi(0)}^2, c_0) \) by 3 experimentally measurable quantities \( E_i \) \((i \leq 3)\) e.g. the quartic coupling \( \lambda_\phi^2 \) and the renormalized masses-squared \( m_2^2 \), \( m_3^2 \) of the two physical degrees of freedom \( h \equiv H - \langle H \rangle \) and \( \pi_3 \). Any other measurable quantity \( E_j \) \((j \geq 4)\) is a function exclusively of \( E_i \). We call a theory “Physically Fine-Tuned” if, for at least one of \( E_i \), \( \partial_\theta H/\partial \ln E_i \gg 1 \). (Certain non-perturbative observables, such as the sphaleron-driven rate of baryon-number violation in the SM, are excused because of their exponential dependence on perturbative observables.) We make a crucial exception: an observable \( E_k \) is not (even bare!) fine-tuned if it is PhysNat, i.e. if setting \( E_k = 0 \) increases the symmetry of the theory.

In [4], we examined the renormalization with finite Euclidean integral-cutoff \( \Lambda \) of the UVQD that appear in the full GML model \( \mathbb{R} \) (i.e. with a complex scalar doublet, \( SU(2)_L \times SU(2)_R \) broken explicitly to \( SU(2)_{L+R} \) by \( L_{\Lambda S B} \)). Renormalizability is transparent only in the linear representation, to which we therefore adhere. We observed that the UVQD \( \sim \Lambda^2 \) of the theory were all absorbed into \( m_2^2 \), the SU(2) equivalent of \( m_3^2 \). Since setting either \( m_2^2 \) or \( \langle H \rangle \) to zero restores the chiral \( SU(2)_{L+R} \) symmetry of the theory, GML is PhysNat and neither physically nor bare FT. We extended these results to all perturbative loop-orders. We observed that including SM fermions (whose Yukawa couplings break the symmetry to \( SU(2)_L \)) did not alter these conclusions. (A 4th SM generation fermion with \( m_2^2 \gg \langle H \rangle^2 \) might be argued FT, but at the expense of in-calculability, with non-perturbative Yukawa coupling \( y_f = \sqrt{2} m_f / \langle H \rangle \gg 1 \), so we ignore that case) GML and Schwinger with SM fermions have no BEH-FT problem from UVQDs \( \mathbb{R} \). Neither does the SM \( \mathbb{R} \).

This conclusion is at odds with the usual viewpoint, which prefers to cast FT in terms of bare parameters (e.g. \( \mu_{\phi(0)}^2 \)) instead of physical ones (e.g. \( m_2^2 \)); which prefers the unitary representation of the BEH doublet (with which nobody knows how to renormalize) to the linear one (in which renormalization is straightforward and GML/Schwinger-Wagner mode \( \langle H \rangle \to 0 \) makes sense); and which prefers to ignore \( m_2^2 \) rather than treat it as a
physical parameter. That view misses the crucial observation: that UVQD are all absorbed into $m_f^2$, and that a zero value for $m_f^2$ is protected by the Goldstone theorem.

In the remainder of this letter, we extend the SO(2) simplification of our GML results \[8\] to include certain physics at a new finite scale $M_{Heavy}^2 \gg m_f^2$, and construct the effective low-energy Lagrangian $L^{Eff} = L^{bare} + L^{1-loop}$, which emerges after integrating out the heavy degrees of freedom. We keep all quadratic $O(\Lambda^2)$ and logarithmic $O(\ln \Lambda^2)$ divergences, never taking the limit $\Lambda^2 \to \infty$. We ignore $5$ classes of finite operators $O^{Ignores} = O^{Light} + O^{Heavy}_{marginal} + O^{Heavy}_{constant} + O^{Heavy}_{irrelevant} + p^M$; $O^{Light}$ arise entirely from the light degrees of freedom. Although important for computing physical observables (e.g. the successful 1-loop high precision SM predictions for the top-quark and BEH masses from Z-pole physics \[18\] in 1984 and the $W^\pm$ mass \[19\] in 1980) they are not the point of this letter; $O^{Heavy}_{marginal}$ are marginal operators $\sim \ln (M_{Heavy}^2)$, e.g. $1$st differentials $\Pi_{hh}(q^2), \Pi^\prime_{33}(q^2)$ of 2-point scalar self-energies evaluated at low $\langle q^2 \rangle \lesssim m_f^2$; $O^{Heavy}_{constant} \sim [M_{Heavy}^2]_0$ are analogous with the SM gauge-sector $S$ and $U$ oblique parameters \[20\] and might reveal heavy particles via the scalar sector; $O^{Heavy}_{irrelevant}$ are irrelevant operators that vanish as $m_f^2/M_{Heavy}^2 \to 0$, e.g. $\Pi_{hh}(q^2), \Pi^\prime_{33}(q^2)$, $2$nd and higher differentials with respect to $q^2$ evaluated at low $\langle q^2 \rangle$; $p^M$ are operators that approach a constant as $q^2$ approaches the physical pole of a 2-point Green’s function, and therefore do not contribute to physical observables \[22\]. None of these can spoil PhysNat not-FIT in GML or the SM.

As shown previously for UVQD \[8\], our 1-loop examples will demonstrate explicitly that the WTI \[1\] forces all relevant operator terms - $O(\Lambda^2)$, $O(M_{Heavy}^2)$, $O(\ln \Lambda^2)$, $O(M_{Heavy}^2 \ln \Lambda^2)$ and $O(M_{Heavy}^2 \ln m_f^2)$ - into the renormalized pseudo-NGB mass-squared $m_3^2$, which appears with renormalized $\langle H \rangle$ and $\lambda_3^2$ in the renormalized effective potential $V_{GML}$:

$$L^{Eff}_{GML} = \frac{1}{2} Tr |\partial_\mu \Phi|^2 - V^{Eff}_{GML} + O^{Ignores}$$

$$V^{Eff}_{GML} = \frac{\lambda_3^2}{4} \left[ H^2 + \pi_3^2 - \left( \langle H \rangle^2 - \frac{m_f^2}{\lambda_3^2} \right) \right]^2 - \langle H \rangle m_3^2 H$$

$$m_3^2 = m_3^2(0) + 2\lambda_3^2 \langle H \rangle^2$$

(4)

at $q^2 = 0$ in \[4\]. The WTI \[1\] ensures the vanishing of the “tadpole” term (the term linear in $h$) in \[3\], automatically \[4\] enforcing a vacuum stability condition: the BEH cannot simply disappear into the vacuum. We see clearly that SO(2) symmetry is restored in \[3\] by taking either $m_3^2 \to 0$ or $\langle H \rangle \to 0$. Consequently, values of $m_3^2$ or $\langle H \rangle^2$ and thus $m_3^2 \ll (\Lambda^2, M_{Heavy})$ are (at least) PhysNat, and neither physically nor bare FT. A crucial observation: GML/Schwinger Goldstone mode ($m_3^2 \to 0, \langle H \rangle^2 \neq 0, m_f^2 \to 2\lambda_3^2 \langle H \rangle^2$) is also GoldExactNat, with far more powerful suppression of FT.

A so-called FT Problem arises when the term $\langle H \rangle m_3^2 H$ is mistakenly ignored while minimizing $V^{Eff}_{GML}$ in \[4\]. The incorrect result, $\langle H \rangle^2_{PT} = \left( \frac{1}{2} \langle H \rangle^2 - \frac{m_f^2}{\lambda_3^2} \right)$, violates GML and Schwinger WTI, violates stationarity \[13\] of the true minimum at $\langle H \rangle$, and destroys the theory’s renormalizability and unitarity, which require \[13\] \[24\] wavefunction renormalization $\langle H \rangle_{bare}$.

We now examine the consequences of extending SO(2) GML to include a wide class of high-mass-scale $M_{Heavy}^2 \gg m_f^2$ physics. We display explicit 1-loop results for examples of a heavy scalar and a heavy fermion.

For the heavy scalar we consider an SO(2) singlet real scalar $S$, with $(S \to -S)$ Z$_2$ symmetry, wavefunction renormalization $Z_S$, $M^2_S \gg m_f^2$, and either $\langle S \rangle = 0$ or $\langle S \rangle \neq 0$. We add to the renormalized theory $L_S = \frac{1}{2} (\partial_\mu S)^2 - V_{S\bar{S}}$, with $V_{S\bar{S}} = \frac{1}{2} m_S^2 S^2 + \frac{1}{4} \lambda_S^2 S^4 + \frac{i}{\sqrt{2}} \lambda_{S3} S^2 \left[ Tr(\Phi^\dagger \Phi) - \langle H \rangle^2 \right]$.

For the heavy fermion we consider an SO(2) singlet right-handed Majorana neutrino $\nu_R$, with $M^2_{\nu_R} \gg m_f^2$, involved in a Type I See-Saw with a left-handed neutrino $\nu_L$, with Yukawa coupling $y_L$ and resulting Dirac mass $m_D = y_L \langle H \rangle / \sqrt{2}$. We add $L_V = L^{free} + L^{ukawa}$ to the renormalized theory, with $L^{ukawa} = y_L \nu_L \nu_R^\dagger \nu_L^\dagger \nu_R / 2$. $L^{free}$ is a physical observable, and $L^{ukawa}$ is also.

The relevant operator contributions $O(\Lambda^2)$, $O(M_{Heavy}^2 \ln \Lambda^2)$, $O(M_{Heavy}^2 \ln m_f^2)$ and $O(M_{Heavy}^2)$ to the effective GML Lagrangian of low-mass scalar fields $h$ and $\pi_3$ are examined. We show that, for low-energy $|\langle q^2 \rangle| \ll M_{Heavy}^2$ physics, heavy degrees of freedom contribute at worst marginal operators $O(\ln M_{Heavy}^2)$.

Tadpole renormalization, as enforced by the WTI \[1\], relates the renormalized finite $m_3^2$, the bare mass-squared $m_3^2(0)$, and the $\pi_3$ self-energy at $q^2 = 0$:

$$m_3^2 = m_3^2(0) - \Pi_{33}(0).$$

(5)

Neither $m_3^2(0)$ nor $\Pi_{33}(0)$ is a physical observable, and their precise functional forms may differ from model to model. Suppressing the contributions of other fermions necessary to make the theory anomaly-free, we assemble the contributions of $h, \pi_3, \nu_L$ and SO(2) singlets $S, \nu_R$. The crucial calculations appear in the literature \[26\] \[30\]: we adapted those for SO(2) with neutrinos and $(S) \neq 0$.

$$m_3^2(0) = \mu^2_{\phi(0)} Z_\phi + \lambda^2_{\phi(0)} Z_\phi^2 \langle H \rangle^2 + \frac{1}{2} \lambda^2_S S(0) Z_\phi Z_S S \langle S \rangle^2$$

$$16 \pi^2 \Pi_{33}(0) = -\lambda_3^2 \left[ 3A(m_h) + A(m_t) \right] - \frac{1}{2} \lambda_{S3} A(M_S)$$

$$+ y_L^2 \left[ A(M_{\nu_R}) + A(0) - M^2_{\nu_R} (\ln \frac{\lambda^2_S}{M^2_{\nu_R}} + 1) - 2m_f^2 \right]$$

$$\frac{\lambda^2_S}{M^2_{\nu_R}}$$
Because $\Pi_{33}(q^2)$ is UVQD, Passarino-Veltman’s 25 dimension-2 function $A(m) = \Lambda^2 - m^2 \left( \ln \frac{\Lambda^2}{m^2} + 1 \right)$, for finite Euclidean cutoff $\Lambda$, appears. Eqn. 3 then includes all relevant operators: $O(\Lambda^2)$, $O(M_{\text{Heavy}}^2 \ln \Lambda^2)$, $O(M_{\text{Heavy}}^2 \ln M_{\text{Heavy}}^2)$ and $O(M_{\text{Heavy}}^2)$. It has been falsely claimed that dimensional regularization (DR) eliminates such UVQD. We call this misconception the “dim-reg herring”, because it sounds good, but is simply untrue. As shown by MJG Veltman 26, DR associates UVQD with poles at $\mathcal{O}(\ln \Lambda^2)$. The smallness of the $M_{\text{Heavy}}^2$ physical value for that, although each of the self-energies $\Pi_{\lambda}^q$ is the light field (identified with the physical BEH) that reflects the possibility that the physical BEH is a linear in terms of the physical fields (h). The smallness of $m_{\text{Heavy}}^2$/2 is to be “small,” i.e. $\ll M_{\text{Heavy}}^2$, a cancellation must be arranged between $m_{\text{Heavy}}^2(0)$ and $\Pi_{33}(0)$ in 4. The smallness of $m_{\text{Heavy}}^2/M_{\text{Heavy}}^2$ would normally be called Bare-FT, but setting $m_{\text{Heavy}}^2$ = 0 in 4 restores chiral SO(2) shift symmetry. Therefore $m_{\text{Heavy}}^3/M_{\text{Heavy}}^2 \ll 1$ is PhysNat, and neither physically nor bare FT. If $f_{\text{bare}}$ GML is extended in a way that respects that SO(2) symmetry, $m_{\text{Heavy}}^3 \rightarrow 0$ remains a symmetry restoration, so is PhysNat not FT. The importance of maintaining SO(2) symmetry when extending the model, to preserve PhysNat and thus the absence of FT, is a primary lesson of this letter.

We turn next to the properties of the BEH. In a wide range of $M_{\text{Heavy}}^2$ extensions to GML, $L_{\text{eff}}^{\text{quad}}$ can readily be shown (using coupled oblique 2-point Dyson’s dispersion equations) to be that of free particles, when written in terms of the physical fields (h, $m_{\text{Heavy}}$) and their renormalized masses-squared ($m_{\text{Heavy}}^2, m_{\text{Heavy}}^3$). The underscore on $h$ reflects the possibility that the physical BEH is a linear combination of $h$ and one or more other CP + 1 scalars. In GML (and SM), $h$ = $h$. So too for a singlet $\nu_R$ with $M_{\text{Heavy}}^2 \gg m_{\text{Heavy}}^2$, and for an SO(2) singlet scalar $S$ with $M_S^2 \gg m_S^2$ and $\langle S \rangle = 0$. In other cases the $h$ mixes with a $M_{\text{Heavy}}^2$ field, e.g. the SO(2) singlet S when $\langle S \rangle \neq 0$. $h$ is the light field (identified with the physical BEH) that results from the diagonalization of that mixing, and $m_{\text{Heavy}}^2$ is its pole-mass squared. $\pi_{33}$ might also mix with other CP = -1 fields, but here we confine our attention to theories with no such mixing.

The pole of the $h$ propagator is given by the relations

$$m_{\text{h}}^2 = m_{\text{Heavy}}^2 + 2\lambda_{\text{phi}}(0) Z_{\text{phi}}^2 \langle H \rangle - \Pi_{\lambda}(m_{\text{h}}^2) + O_{\text{Ignore}}$$

$$m_{\text{h}}^2 = m_{\text{h}}^2 - k_{\text{Sh}} \langle H \rangle^2$$  (7)

where $\Pi_{\lambda} \equiv \Pi_{\lambda h} - \Pi_{\lambda g}$. SO(2) invariance guarantees that, although each of the self-energies $\Pi_{\lambda h}(m_{\text{h}}^2)$ and $\Pi_{\lambda g}(m_{\text{h}}^2) \sim \Lambda^2$, the combination $\Pi_{\lambda} \sim \ln \Lambda^2$, which renormalization group (RG) log divergences are absorbed by the bare Lagrangian term in 4. This matches exactly the absorption, into $\lambda_{\text{phi}}^2$, of the same RG divergences $\sim \ln \Lambda^2$ in 4-scalar and 3-scalar scattering.

The term $-k_{\text{Sh}} \langle H \rangle^2$ in 7 is the expected downward shift in $m_{\text{h}}^2$ when $h$ mixes with the higher mass $S$ to give the physical $h$. In the specific case of our real SO(2) singlet $S$, with $S \rightarrow -S$ symmetry and $\langle S \rangle \neq 0$, $k_{\text{Sh}} = \lambda_{\text{phi}}^2/(2\lambda_S^2)$. The coupled Dyson dispersion relations show that $\lambda_{\text{phi}}^2(q^2)$ is to be evaluated at $q^2 = m_{\text{h}}^2$ in determining $k_{\text{Sh}}$. Most important is that $-k_{\text{Sh}} \langle H \rangle^2$ is $O_{\text{Heavy}} \sim [M_{\text{Heavy}}^2] \sim O_{\text{Ignore}}$. Eqn. 7 can then be written as in 3.

What we have found, by explicit 1-loop calculation, for these examples is generic. Extend GML with SO(2) representations having $M_{\text{Heavy}}^2 \gg m_{\text{h}}^2$ that respect the chiral SO(2) shift symmetry $\pi_{33} \rightarrow \pi_{33} + \langle H \rangle$. Then, although each of $\Pi_{h h}(m_{\text{h}}^2), \Pi_{33}(m_{\text{h}}^2)$ is a relevant operator $O(\Lambda^2)$ or $O(M_{\text{Heavy}}^2)$, $\Pi_{\lambda}(m_{\text{h}}^2)$ in 7 is at worst $O_{\text{Heavy}}$. The result can then be written, up to $O_{\text{Ignore}}$, as 3.

It is easy to see that $m_{\text{h}}^2$ is PhysNat and not FT: 7 can be re-written $m_{\text{h}}^2 = m_{\text{h}}^2 + 2\lambda_{\text{phi}}^2 \langle H \rangle^2$ where $\lambda_{\text{phi}}^2$ is the renormalized four-point coupling. Both of these terms are naturally small, as either $m_{\text{h}}^2 \rightarrow 0$ or $\langle H \rangle \rightarrow 0$ restores SO(2) symmetry.

The $SU(2)_L \times SU(2)_R$ Schwinger model 3 is the GML model with $L_{\text{NSB}} \equiv 0$ (i.e. $e_{\text{NSB}} = 0$). Over 4 decades, B.W.Lee 13, K.Symanzik 14, 17, and C.Itzykson and J.B. Zuber 17 have emphasized that proper renormalization of GML and Schwinger requires the PCAC term to avoid the BEH Non-Analyticity Problem, namely that

$$\frac{\partial \langle H \rangle^2}{\partial \mu^2} = -\infty \text{ and } \frac{\partial m_{\text{h}}^2}{\partial \mu^2} = +\infty$$  (8)

at the classical scale-invariant point ($m_{\text{h}}^2 = m_{\text{h}}^2$ = $\langle H \rangle^2$ = 0, so that $\epsilon = m_{\text{h}}^2 = \langle H \rangle = 0$). (Note the return to the notation $m_{\text{h}}^2$ for the pseudo-NGB mass-squared in the $SU(2)_L \times SU(2)_R$ case.) **Renormalized Schwinger must be understood as the chiral-symmetric limits, $m_{\text{h}}^2 \rightarrow 0$ or $\langle H \rangle \rightarrow 0$, of renormalized GML. This shows the importance of including the PCAC term, even in the chiral-symmetric limit.**

According to our two examples, the properly renormalized spontaneously broken SO(2) GML/Schwinger low-energy effective Lagrangian is 3 with $m_{\text{h}}^2 = 0$.

$$L_{\text{Eff; Goldstone Schwinger}}^{\text{Goldstone Schwinger}} = \frac{1}{2} Tr \partial_{\mu} \Phi^{\dagger} \partial_{\mu} \Phi - V_{\text{Eff; Goldstone Schwinger}} + O_{\text{Ignore}}$$

$$V_{\text{Eff; Goldstone Schwinger}}^{\text{Goldstone Schwinger}} = \lambda_{\text{phi}}^2 \left[ \frac{h^2 + n_{\text{phi}}^2}{2} + \langle H \rangle \right]^2$$  (9)

includes all $O(\Lambda^2), O(\ln \Lambda^2)$ UV divergences and all finite relevant operators $O_{\text{Heavy}} \sim M_{\text{Heavy}}^2 \sim 2\lambda_{\text{phi}}^2 \langle H \rangle^2$, but they have all vanished! GML/Schwinger Goldstone mode therefore suppresses FT far more powerfully than PhysNat, and is instead GoldExceptNat, even when extended to include our two heavy-particle examples. Explicit 1-loop calculation shows that marginal
operators $O^{\text{Heavy}}_{\text{marginal}} \sim \ln M_{\text{Heavy}}^2$ are absorbed by $\lambda_0^2$ after renormalization. The heavy fields $S$ and $\nu_R$ completely decouple \cite{13} as $M_S^2$ and $M_{\nu_R}^2 \to \infty$. Management of $O(\Lambda^2)$, $O(M_{\text{Heavy}}^2, \ln \Lambda^2)$ and all finite relevant operators $O^{\text{relevant}}_{\text{Heavy}} \sim M_{\text{Heavy}}^2$ by the WTI \cite{2} has been crucial. New (i.e. largely unfamiliar to modern audiences) UVQD \cite{6} and $O(M_{\text{Heavy}}^2, \ln \Lambda^2)$ operators, $O^{\text{relevant}}_{\text{Heavy}}$, arise in the divergence of the SO(2) GML ($\epsilon_0 \neq 0$) axial-vector current $\partial \mu J^5_\mu$, which do not arise in Schwinger ($\epsilon_0 = 0$) axial-vector current conservation. However, WTI \cite{2} ensures that these new $O(\Lambda^2)$, $O(M_{\text{Heavy}}^2, \ln \Lambda^2)$ and $O^{\text{relevant}}_{\text{Heavy}}$ terms, together with those in $m_3^2$ in \cite{27}, obey the PCAC relation $\partial \mu J^5_\mu = \langle H \rangle m_3^2 \eta_3$, and collectively vanish as $m_3^2 \to 0$. Eq. \cite{2} is the first of a tower of recursive WTI governing connected amputated one-particle-irreducible Greens functions \cite{13, 27}. These guarantee that SO(2) symmetry is restored exactly in the limits $m_3^2 \to 0$ and/or $\langle H \rangle \to 0$. Vast experimental and observational evidence shows that the SM must be extended to include at least classical General Relativistic (GR) gravity. The appearance of quantum gravity’s affect on these results. BWL thanks Jon Butterworth and the Physics/Astronomy Dept. at University College London for support. GDS is partially supported by CWRU grant [doc:sc0009946] and thanks the CERN theory group for hospitality during 2012-2013. Acknowledgments: We thank A. Matas and R. Stora for illuminating conversations, and L. Alvarez-Gaume and J. Donoghue for conversations regarding quantum gravity’s affect on these results. BWL thanks Jon Butterworth and the Physics/Astronomy Dept. at University College London for support. GDS is partially supported by CWRU grant [doc:sc0009946] and thanks the CERN theory group for hospitality during 2012-2013.

[1] L. Susskind, Phys. Rev. D20 (1979) 2619.
[2] L. Susskind acknowledgement of private communication by K. Wilson in \cite{1}.
[3] J. Lykken and M. Spiropulu, Supersymmetry and the Crisis in Physics, Scientific American, May 2014, pg. 22.
[4] G. Bell-Mann and M. Levy, Nuoc. Cim. 16, 705 (1960).
[5] J. Schwinger, Annals of Physics 2, 332-347 (1957).
[6] B.W. Lynn, G.D. Starkman, K. Freese & D.I. Podolsky, arXiv: [hep-ph] 1112.2150, submitted to Phys. Rev. D.
[7] G’t Hooft Proc. of 1979 Cargese Institute on Recent Developments in Gauge Theories, page 135, New York (1980) Plenum Press as quoted, for example in \cite{6}.
[8] G. Alvarez-Guaine and M.A. Vazquez-Mozo, An Invitation to Quantum Field Theory, Springer-Verlag, Berlin-Heidelberg, 2012.
[9] B. W. Lynn, arXiv: 1106.6354; submitted to Phys. Rev. D, 02 July 2011.
[10] W.A. Bardeen, FNAL Conf. 391-T C95-08-27 (1995).
[11] J. Lykken, MITP Workshop, March 18-22, 2013.
[12] S. Weinberg & E. Witten, Phys. Lett. B96 (1980) 59.
[13] B.W. Lee, July 1970 Cargese Summer Inst. Chiral Dynamics, Gordon and Breach, NY, London, Paris, 1972.
[14] K. Symanzik, Comm. Math. Phys. 16 (1970) 48.
[15] K. Symanzik, July 1970 Cargese Summer Institute. .
[16] A. Vassiliev, July 1970 Cargese Summer Institute.
[17] C. Itzykson and J.-B. Zuber, Quantum Field Theory, McGraw Hill, New York, 1980.
[18] B.W. Lynn and R.G. Stuart, Nucl. Phys. B253 (1985)216; ICTP preprint 84-46 (1984).
[19] A. Sirlin, Phys.Rev.D22 (1980)971
[20] D.C. Kennedy and B.W. Lynn, SLAC-PUB-4039, Jan 1988, Nucl. Phys. B322 (1989) 1.
[21] ME Peskin and T. Takuchi, Phys.Rev.Lett. 65(1990)964
[22] P. Ramond, Journeys Beyond the Standard Model, Westview/Perseus Press, Cambridge MA, 2004.
[23] A. Messiah, Quantum Mechanics Volume II, Chapter XVI, John Wiley, NY, North Holland (1958).
[24] J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill, NY 1965.
[25] G. Passarino, MJG Veltman, Nucl. Phys. B160(1979)151.
[26] MJG Veltman, Acta Phys. Pol. B (1985)216; MJG Veltman, ICTP preprint 84-46 (1984).
[27] B.W. Lynn, G.D. Starkman and R. Stora, in preparation.
[28] B.W. Lynn, G.D. Starkman and R. Stora, in preparation.
[29] B.W. Lynn and R.G. Stuart, Nucl. Phys. B253 (1985)216; ICTP preprint 84-46 (1984).
[30] A. Sirlin, Phys.Rev.D22 (1980)971
[31] D.C. Kennedy and B.W. Lynn, SLAC-PUB-4039, Jan 1988, Nucl. Phys. B322 (1989) 1.
[32] ME Peskin and T. Takuchi, Phys.Rev.Lett. 65(1990)964
[33] P. Ramond, Journeys Beyond the Standard Model, Westview/Perseus Press, Cambridge MA, 2004.
[34] A. Messiah, Quantum Mechanics Volume II, Chapter XVI, John Wiley, NY, North Holland (1958).
[35] J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill, NY 1965.