Energy, momentum and mass outflows and feedback from thick accretion discs around rotating black holes

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ABSTRACT

Using long-duration general relativistic magnetohydrodynamic simulations of radiatively inefficient accretion discs, the energy, momentum and mass outflow rates from such systems are estimated. Outflows occur via two fairly distinct modes: a relativistic jet and a subrelativistic wind. The jet power depends strongly on the black hole spin and on the magnetic flux at the horizon. Unless these are very small, the energy output in the jet dominates over that in the wind. For a rapidly spinning black hole accreting in the magnetically arrested limit, it is confirmed that jet power exceeds the total rate of accretion of rest mass energy. However, because of strong collimation, the jet probably does not have a significant feedback effect on its immediate surroundings. The power in the wind is more modest and shows a weaker dependence on black hole spin and magnetic flux. Nevertheless, because the wind subtends a large solid angle, it is expected to provide efficient feedback on a wide range of scales inside the host galaxy. Empirical formulae are obtained for the energy and momentum outflow rates in the jet and the wind.

Key words: accretion, accretion discs – black hole physics – relativistic processes – methods: numerical – galaxies: jets.

1 INTRODUCTION

1.1 Feedback

Black hole (BH) accretion discs are some of the most energetic objects in the Universe (Frank, King & Raine 2002; Kato, Fukue & Mineshige 2008). Geometrically thin discs (Novikov & Thorne 1973; Shakura & Sunyaev 1973) are radiatively efficient and convert about 10 per cent of the rest mass energy of the accreting gas into radiation. Geometrically thick discs, on the other hand, are advection dominated accretion flows (ADAFs; Narayan & Yi 1994, 1995; Abramowicz et al. 1991; Narayan & McClintock 2008) and produce little radiation relative to their mass accretion rates. Instead, they produce outflows in the form of jets and winds which carry huge amounts of energy, mass and momentum. This is the topic of this paper.

Energy and momentum outflow, and to a lesser extent mass outflow, can affect the BH’s surroundings. This effect is most profound in the case of supermassive black holes (SMBHs) in the centres of galaxies, where a number of observations suggest the existence of strong ‘feedback’ effects from the SMBH accretion disc on the evolution of the entire host galaxy. The best-known evidence for such coupling is the celebrated $M_{\text{BH}} - \sigma_{\text{bulge}}$ relation between the mass, $M_{\text{BH}}$, of the SMBH and the velocity dispersion, $\sigma_{\text{bulge}}$, of its host galaxy bulge (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Gültekin et al. 2009; Kormendy & Ho 2013), and the earlier Magorrian et al. (1998) relation between $M_{\text{BH}}$ and the bulge mass $M_{\text{bulge}}$. Given the very large size ratio $\sim 10^{8}$ and mass ratio $\sim 10^{3}$ between the galaxy bulge and the SMBH, it is reasonable to assume that the coupling occurs via feedback of energy or momentum (e.g. Silk & Rees 1998; King 2003, 2005, 2010; Hopkins, Murray & Thompson 2009).

A second line of evidence is the observed exponential cutoff in the number density of galaxies at the high mass/luminosity end (Schechter 1976), even though there is no cutoff at the equivalent mass scale in the distribution of dark matter haloes. One of the mechanisms invoked to explain the galaxy cutoff is a reduction in the star formation rate in massive systems due to inefficient cooling (White & Rees 1978; White & Frenk 1991). However, cooling effects alone are insufficient (Thoul & Weinberg 1995), and attempts to fit observed luminosity functions need to include additional feedback processes (e.g. Benson et al. 2003; Croton et al. 2006). The most viable process is expulsion of gas from young galaxies in superwinds as a result of feedback from supernovae and/or active galactic nuclei (AGN). Observations of starburst galaxies (e.g. McNamara et al. 2006), and midly relativistic winds in AGN (Tombesi et al. 2010a,b) confirm this picture.

Yet another puzzling aspect of the galaxy population is the fact that the most massive galaxies, typically ellipticals in clusters, are
made of the oldest stars (Bender & Saglia 1999). This ‘downsizing’ is counterintuitive, since it seems to conflict with hierarchical growth of structure in a cold dark matter cosmogony, where massive dark haloes assemble at lower redshift than lower mass haloes (e.g. Lacey & Cole 1993). Again, feedback provides a plausible mechanism; it prevents significant accretion in massive galaxies, thus suppressing star formation at late times (Benson et al. 2003; Springel, Di Matteo & Hernquist 2005).

Finally, the observed X-ray emission in the centres of galaxy clusters implies a cooling time much shorter than the age of the system, suggesting that gas at the centres of these clusters must condense and turn into stars; however, there is no observational evidence for star formation at the required level (Fabian et al. 2001; Peterson et al. 2003; Kaasstra et al. 2004; Croton et al. 2006). While thermal conduction may be part of the explanation (e.g. Narayan & Matteo et al. 2005; Springel et al. 2005), an important clue comes from the observation (Burns, Gregory & Holman 1981) that every cluster with a strong cooling flow also contains an active SMBH in a central radio galaxy. This suggests that energy feedback from the SMBH keeps the cluster gas hot (Ciotti & Ostriker 2001; Brüggen & Kaiser 2002; Ruszkowski & Begelman 2002; Churazov et al. 2004).

The evidence summarized above indicates that feedback from accreting SMBHs plays a crucial role not only on galaxy scales but even on the scale of galaxy clusters. Two kinds of SMBH feedback are discussed in the literature (see Fabian 2012; Kormendy & Ho 2013 for reviews). One occurs in the ‘quasar mode’ when the SMBH accretes at a good fraction (>0.1 to >1) of the Eddington rate and deposits energy in its surroundings either directly through radiation or via radiatively driven winds. The second kind of feedback takes place in the ‘radio mode’ (Croton et al. 2006) or ‘maintenance mode’ (Hopkins 2010). Here, accretion occurs via an ADAF and the radiative luminosity is low. Hence, feedback is almost entirely in the form of mechanical energy and momentum. This ADAF-specific form of feedback is the topic we wish to investigate.

Maintenance mode feedback has been (i) invoked for the ‘cooling flow problem’ in galaxy clusters (Ciotti & Ostriker 2001; Brüggen & Kaiser 2002; Ruszkowski & Begelman 2002; Churazov et al. 2004; Gaspari et al. 2011), (ii) included in semi-empirical models of galaxy formation (Best et al. 2005; Croton et al. 2006; Hopkins et al. 2006; Somerville et al. 2008) and (iii) modelled via simple prescriptions in gas dynamical computer simulations of galaxy/cluster formation in the universe (Di Matteo, Springel & Hernquist 2005; Springel et al. 2005; Cox et al. 2006; Ciotti, Ostriker & Proga 2010; Novak, Ostriker & Ciotti 2011; Scannapieco et al. 2012). However, all these efforts are highly empirical since nobody knows exactly how much mechanical energy or momentum flows out from an accreting SMBH. The general practice is to employ the Bondi model (Springel et al. 2005), or some variant of it (Debuhr et al. 2010), to relate the energy or momentum output from an SMBH to boundary conditions in the surrounding interstellar medium (ISM). However, whether or not the Bondi model is a reasonable description of accretion from an external medium is still very much in debate (see e.g. Igumenshchev & Narayan 2002; Narayan & Fabian 2011). Recently, Gaspari, Ruszkowski & Oh (2013) carried out an in-depth investigation and showed that cold gas condenses out of the hot phase via non-linear thermal instabilities. As a result the cold filaments/clouds collide inelastically and boost the accretion rate, making the Bondi model very unrealistic.

A final important question is the following. Which is more important, energy feedback or momentum feedback? The former is traditionally considered in cosmological simulations (e.g. Di Matteo et al. 2005; Springel et al. 2005), but the latter may also be important (King 2003, 2010; Debuhr et al. 2010; Ostriker et al. 2010). For a given energy flux, the momentum flux is smallest in the case of a relativistic jet and largest for a non-relativistic wind. Thus, the dependence of momentum feedback efficiency on parameters such as the mass accretion rate or the BH spin could be quite different compared to energy feedback efficiency. In this work, we estimate both efficiency factors for ADAFs using numerical simulations.

1.2 Blandford–Znajek

It is a remarkable prediction of general relativity that magnetic field lines threading a BH can extract the hole’s rotational energy (Ruffini & Wilson 1975). Rapidly rotating BHs can drive powerful jets. In the standard BH jet model, the jet power scales as

\[ P_{\text{jet}} \sim \Omega^2 \Phi^2_{\text{BH}}, \quad \Omega = \frac{a}{2r_H}, \quad r_H = 1 + \sqrt{1 - a^2}, \]  

(1)

where \( \Omega \) is the angular velocity of the outer BH horizon with radius \( r_H \) and \( \Phi_{\text{BH}} \) is the magnetic flux threading the horizon (Blandford & Znajek 1977; MacDonald & Thorne 1982; Phinney 1982; Thorne, Price & MacDonald 1986). The main predictions of the Blandford–Znajek (BZ) jet model are supported by general relativistic magnetohydrodynamic (GRMHD) simulations (Komissarov 2001; Hirose et al. 2004; De Villiers et al. 2005; McKinney 2005; Hawley & Krolik 2006; Beckwith, Hawley & Krolik 2008; Tchekhovskoy, Narayan & McKinney 2010, 2011; Tchekhovskoy, McKinney & Narayan 2012). The BZ model is a close cousin of the Goldreich & Julian (1969) model for pulsar magnetospheres, a relationship which becomes particularly clear in the membrane formulation of the BZ model (MacDonald & Thorne 1982; Thorne et al. 1986). Recently, it has been observed that the scaling of jet power with BH spin in galactic X-ray binaries is consistent with the BZ model (Narayan & McClintock 2012; Steiner, McClintock & Narayan 2013).

One of the aims of this paper is to check how well jets and winds in simulated ADAFs agree with the scaling shown in equation (1).

1.3 Previous work

Outflows of mass and energy are multidimensional and are best studied with numerical simulations. Multidimensional numerical hydro- and magnetohydrodynamical simulations of hot accretion discs have been performed for more than a decade. Already early works based on pseudo-Newtonian codes with purely hydrodynamic viscosity showed that a significant fraction of the inflowing mass near the equatorial plane can flow out along the poles (Igumenshchev & Abramowicz 1999, 2000; Stone, Pringle & Begelman 1999). Li, Ostriker & Sunyaev (2013, see also earlier work by Proga & Begelman 2003; Janiuk et al. 2009) ran a set of hydrodynamical axisymmetric simulations of low angular momentum gas. For their viscous models they observed conical outflows almost balancing inflow.

Pseudo-Newtonian magneto-hydrodynamic (MHD) simulations have been then performed by a number of authors (Machida, Hayashi & Matsumoto 2000; Machida, Matsumoto & Mineshige 2001; Stone & Pringle 2001; Hawley & Balbus 2002; Igumenshchev, Narayan & Abramowicz 2003). Outflows were observed and it was claimed that the initial configuration of the magnetic field may play an important role in determining the mass outflow rate. On the contrary, in a series of numerical MHD simulations, Pen, Matzner & Wong (2003) and Pang et al. (2011) found
little evidence for either outflows or convection. Even though the entropy gradient was unstable the gas was apparently prevented from becoming convective by the magnetic field. Recently, Yuan, Wu & Bu (2012a) and Yuan, Bu & Wu (2012b) carried out 2D hydrodynamical and MHD simulations of ADAFs which cover a very large range of radius and show fairly strong outflows.

Beginning with the work of De Villiers, Hawley & Krolik (2003), accretion flows have been studied using GRMHD codes. The authors observed two kinds of outflows: bipolar unbound jets and bound coronal flow. The coronal flow supplied gas and magnetic field to the coronal envelope, but apparently did not have sufficient energy to escape to infinity. The jets on the other hand were relativistically escaped easily, though carrying very little mass. Jets have been studied in detail by a number of authors (McKinney & Gammie 2004; De Villiers et al. 2005; McKinney 2006). Tchekhovskoy et al. (2011) simulated a strongly magnetized disc around a rapidly spinning BH, and obtained very powerful jets with energy efficiency \( \eta > 100 \) per cent, i.e. jet power greater than 100 per cent of \( M_{BH} c^2 \), where \( M_{BH} \) is the mass accretion rate on to the BH. Their work showed beyond doubt that at least some part of the jet power had to be extracted from the spin energy of the BH.

Recently, McKinney, Tchekhovskoy & Blandford (2013) have studied a large set of thick accretion disc simulations both for rotating and non-rotating BHs. They found that models initiated with poloidal magnetic field showed mass-loss both in the jet and a magnetized wind. The energy releasing large radii was dominated by the power produced via the magnetic flux penetrating the horizon as in the BZ mechanism.

### 1.4 This work

This paper attempts to obtain via global GRMHD simulations quantitative estimates of the amount of energy, mass and momentum that flow out from an ADAF around a BH. In a previous paper (Narayan et al. 2012), we studied mass outflow from an ADAF around a non-rotating BH, and showed that a wind flows out at relatively large radii, \( r \gtrsim 40 \). Here, we extend our analysis to accretion flows around rotating BHs. In the process, we show that there are two kinds of outflows in such systems: (i) relatively slow winds at larger radii, similar to the winds studied in Narayan et al. (2012) and (ii) relativistic jets which flow out from close to the BH and are uniquely associated with spinning holes. We study in detail the energy, mass and momentum in these two kinds of outflow. In an accompanying paper (Penna, Narayan & Sadowski 2013b), we discuss the physics of relativistic jets and demonstrate that our numerical simulations strongly validate the Blandford & Znajek (1977) mechanism.

The paper is organized as follows. In Section 2, we introduce the numerical scheme we used to simulate the discs. In Section 3, we discuss outflows emerging from them. In particular, in Sections 4.5, 4.6 and 4.7, we present radial profiles of outflowing energy, mass and momentum, respectively, and in Section 4.8 we give approximate formulae for the corresponding fluxes. In Section 5, we compare the efficiencies of generating outflows by thick and thin discs. We conclude with Section 6 discussing implications of our results.

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1 All distances are given in units of \( R_G = G M_{BH} / c^2 \), where \( M_{BH} \) is the BH mass. Thus, the dimensionless radius \( r \) is related to the dimensional radius \( R \) by \( r = R / R_G \). Time is similarly given in units of \( G M_{BH} / c^2 \). We adopt \( G = c = 1 \) and choose the \((-+++)\) signature for the metric.

### Table 1. Disc models.

| Model | BH spin \((a_*)\) | Initial magnetic field | Resolution \((r, \theta, \phi)\) | \(t_{final}\) |
|-------|------------------|-----------------------|-------------------------------|---------------|
| \(a_* = 0.0\) SANE | 0.0 | multiloop | 256 \times 128 \times 64 | 200 000 |
| \(a_* = 0.7\) SANE | 0.7 | multiloop | 256 \times 128 \times 64 | 100 000 |
| \(a_* = 0.9\) SANE | 0.9 | multiloop | 256 \times 128 \times 64 | 50 000 |
| \(a_* = 0.98\) SANE | 0.98 | multiloop | 256 \times 128 \times 64 | 25 000 |
| \(a_* = 0.0\) MAD | 0.0 | single loop | 264 \times 126 \times 60 | 200 000 |
| \(a_* = 0.7\) MAD | 0.7 | single loop | 264 \times 126 \times 60 | 100 000 |
| \(a_* = 0.9\) MAD | 0.9 | single loop | 264 \times 126 \times 60 | 50 000 |

### 2 NUMERICAL SETUP

We performed seven simulations of radiatively inefficient accretion flows around spinning and non-spinning BHs, as listed in Table 1. Following the methods described in Narayan et al. (2012), two distinct initial conditions were used for the seed magnetic field threading the initial gas torus: (i) in one set of simulations the seed field consisted of multiple poloidal loops of magnetic field with changing polarity (multiloop or SANE, which stands for ‘Standard And Normal Evolution’). (ii) In the other set of simulations the initial field was a single poloidal loop threading the entire torus (single loop or MAD, ‘Magnetically Arrested Disc’). SANE runs are designed such that relatively little magnetic flux accumulates around the BH. In contrast, MAD runs quickly saturate at the maximum allowed magnetic flux on the BH for the given mass accretion rate; the back reaction of the saturated field causes the accretion flow to be magnetically arrested and to settle down to the MAD state (Narayan, Igumenshchev & Abramowicz 2003).

All the simulations were performed using the GRMHD code HARM (Gammie, McKinney & Töth 2003). The coordinates, initial torus and magnetic field were set up following Narayan et al. (2012). We used modified Kerr–Schild horizon penetrating coordinates which covered uniformly the full range of azimuthal and polar angles. The radial size of cells increased exponentially with radius, with the innermost radius chosen to fit six cells inside the BH horizon. The outermost radius was set to \( r = 10000 \). The adopted resolutions are given in Table 1. The initial torus of gas (set up following Penna, Kulkarni & Narayan 2013a) was threaded either by multiple and counteroriented, or single poloidal magnetic field loops for SANE and MAD models, respectively.

Two of the simulations discussed here have BH spin \( a_*/M = a_* = 0 \), and are the same as the ones described in Narayan et al. (2012), except that the \( a_* = 0 \) MAD model has been run up to a time \( t_{final} = 200000 \) (to match the corresponding SANE run), which is twice as long as in the previous work. The runs with spinning BHs \( (a_* = 0.7, 0.9, 0.98) \) are all new.

To validate if the magnetorotational instability (MRI) is resolved we calculate the parameters

\[
Q_\psi = \frac{2\pi}{\Delta x^d} \frac{|b^\psi|}{\sqrt{\Delta r \rho}}, \quad Q_\phi = \frac{2\pi}{\Delta x^d} \frac{|b^\phi|}{\sqrt{\Delta r \rho}},
\]

where \( \Delta x^d \) (grid cell size) and \( b^i \) (the magnetic field) are evaluated in the orthonormal fluid frame, \( \Omega \) is the angular velocity and \( \rho \) is the gas density. For SANE runs, the gas inside \( r = 100 \) and within one density scaleheight of the mid-plane have \( Q_\psi \) and \( Q_\phi \) between 10 and 20, for all of the time chunks except the first one (see below for the definition of time chunks). The MAD simulations have \( Q_\psi > 100 \) and \( Q_\phi \sim 50 \) in the same regions. Therefore, MRI is properly resolved in both cases (Hawley, Guan & Krolik 2011).
Fig. 1 shows snapshots of density and Lorentz factor $u'$ for the $a_*=0.7$ SANE and MAD runs. In both models, the disc is geometrically thick (root mean square $h/r \approx 0.4$) and turbulent. However, gas escapes in two fairly distinct structures: a fast collimated laminar flow along the axis, which we refer to as the ‘jet’, and a slower outflow covering a wider range of angles, which we call the ‘wind’. We focus on these two components in the rest of the paper. Compared to the SANE run, the outflowing jet velocity is much higher in the MAD simulation, sometimes exceeding $u' = \frac{v_{jet}}{c} > 0.98c$.

### Table 2. Time chunks.

| Chunk | Time range   | $t_{chunk}$ |
|-------|--------------|-------------|
| T1    | 3000–6000    | 3000        |
| T2    | 6000–12 000  | 6000        |
| T3    | 12 000–25 000| 13 000      |
| T4    | 25 000–50 000| 25 000      |
| T5    | 50 000–100 000 | 50 000    |
| T6    | 100 000–200 000 | 100 000    |

3 ANALYSIS OF SIMULATION OUTPUT

3.1 Averaging

As in our previous work (Narayan et al. 2012), we used time-averaged disc properties to extract radial profiles of quantities of interest. The time averaging was done over logarithmically increasing chunks of time as listed in Table 2, with each successive time chunk being twice as long as the previous one. The two $a_*=0$ simulations were run up to $t_{final} = 200,000$, allowing us to compute time averages for chunks T1–T6. The shorter $a_*=0.7$ runs only went up to chunk T5, the $a_*=0.9$ runs reached chunk T4, while the $a_*=0.98$ run stopped at chunk T3. Apart from averaging over time, we also averaged the data over azimuth and symmetrized it with respect to the equatorial plane.

2 The physical wall time of all the simulations was comparable because of the smaller horizon radius of spinning BHs, which required a correspondingly smaller time step.

3.2 Quantities of interest

We are interested in estimating the amount of mass, energy and momentum flowing out of the accretion disc. The radial flux of rest mass is given by

$$\dot{m} = \rho u' ,$$

where a positive (negative) sign indicates that matter flows away from (towards) the BH. The total energy flux is

$$\dot{E}_{tot} = -T'_r ,$$

where $T'_r$ is the $(r,t)$ component of the magnetohydrodynamical stress energy tensor (e.g. Misner, Thorne & Wheeler 1973)

$$T'_r = (\rho + \Gamma u + b^i u^i - b'b^i)u'_r - b' b^i ,$$

and $b^{i}$ is the magnetic field four-vector (e.g. Gammie et al. 2003). The negative sign in equation (4) is because $u'_r$ is negative; thus, a positive value of $\dot{E}_{tot}$ means that total energy flows outward, and vice versa. Note that $T'_r$ represents the total energy transported by the fluid and the magnetic field, including the rest mass energy of the gas. However, this is not very convenient when considering energy at ‘infinity’ since the rest mass energy plays no role in feedback. Therefore, we consider a different measure of energy flux in which we eliminate the rest mass energy,

$$\dot{e} = \dot{E}_{tot} - \dot{m} = -T'_r - \rho u' ,$$

which we hereafter refer to as ‘the energy flux’. Positive values of $\dot{e}$ correspond to energy lost from the system, i.e. energy flows out into the surrounding medium.

Integrating over $\theta$ and $\phi$ and normalizing by the net mass flow rate at $r=10$ (to avoid numerical issues near the horizon) we obtain the normalized radial profiles of mass and energy flow. The mass accretion rate is thus

$$\dot{M}(r) = \frac{1}{|M_{net}|} \int_\theta \int_\phi \dot{m} \, dA_{\theta\phi} ,$$

and the two energy loss rates are

$$\dot{E}_{tot}(r) = \frac{1}{|M_{net}|} \int_\theta \int_\phi \dot{E}_{tot} \, dA_{\theta\phi} ,$$

$$\dot{E}(r) = \frac{1}{|M_{net}|} \int_\theta \int_\phi \dot{e} \, dA_{\theta\phi} .$$
where \( dA_{\theta \phi} = \sqrt{-g} d\theta d\phi \) is an area element in the \( \theta - \phi \) plane,

\[
M_{\text{net}} = \int_{\theta} \int_{\phi} m(r = 10) dA_{\theta \phi},
\]

(10)

and signs have been chosen so as to make the integrated fluxes positive for outflow. The integrands in the above integrals correspond to time averages over the duration of the time chunk of interest. The \( \phi \) integral is over the range 0 to \( 2\pi \), while the range of the \( \theta \) integral depends on the quantity of interest. When we wish to calculate the net outflow of mass or energy, we integrate over the full range \( \theta = 0 - \pi \). When we are interested in outflow in the jet or the wind, we limit the \( \theta \) range accordingly, as described in the next two subsections. From the integrated rate of outflow of mass and energy, we calculate the integrated momentum in the outflow using the relativistic formula

\[
\dot{P}(r) = \sqrt{(\dot{E}_{\text{int}}(r))^2 - (\dot{M}(r))^2}.
\]

(11)

Finally, we quantify the magnetic field strength at the BH horizon by means of the magnetic flux parameter (Tchekhovskoy, Narayan & McKinney 2011),

\[
\Phi_{\text{BH}}(t) = \frac{1}{2\sqrt{M}} \int_{\theta} \int_{\phi} |\mathbf{B}'(r_{\text{H}}, t)| dA_{\theta \phi},
\]

(12)

where \( \mathbf{B}' \) is the radial component of the magnetic field and \( r_{\text{H}} \) is the radius of the horizon. The integral is over the whole sphere, and the factor of \( 1/2 \) converts the result to one hemisphere. An accretion flow with geometrical thickness \( h/r \approx 0.4 \) transitions to the MAD state once \( \Phi_{\text{BH}} \) reaches \( \sim 50 \) (Tchekhovskoy et al. 2011, 2012). As we show later, the three MAD runs reach this limit quickly and remain there, whereas the four SANE runs are for the most part well below this limit.

3.3 The outflow criterion

Since the simulations extend over only a finite range of radius, we need a criterion to decide whether a particular parcel of fluid can escape to infinity. The quantity we use to determine this is the Bernoulli parameter \( \mu \) (Narayan et al. 2012)

\[
\mu = \frac{T_r \rho u_r \sqrt{g_{\theta \theta}} + T_{\theta \theta} \rho u_\theta \sqrt{g_{\phi \phi}}}{(\rho u_r)^2 g_{rr} + (\rho u_\theta)^2 g_{\theta \theta}} - 1.
\]

(13)

To understand this expression, note that the quantities \( \dot{e} \) and \( \dot{m} \) introduced earlier correspond to the radial components of the respective fluxes. Correspondingly, there are \( \theta \) components of these fluxes, and the two together may be viewed as two-vectors \( \mathbf{e}_r \) and \( \mathbf{m}_r \) in the poloidal \( r\theta \) plane. We see then that \( \mu \) is equal to \( \dot{e}_r \cdot \mathbf{m}_r/\dot{m} \cdot \mathbf{m}_r \), i.e. it is the flux of energy parallel to the flow streamline. This quantity has to be positive for gas to be able to escape to infinity.

Fig. 2 shows profiles of \( \mu \) and \( \dot{m} \) for \( a_* = 0.7 \) SANE (top panel) and MAD (bottom) simulations. The profiles were calculated at \( r = 80 \) and 160, respectively.

![Figure 2](https://example.com/fig2.png)

However, as Fig. 2 shows, the two conditions are practically degenerate, so we could equally well have used just one of them.

3.4 Three regions: disc, wind and jet

We identify the region of the solution where the gas flows in (\( \mu < 0 \)) as the ‘disc’. In our radiatively inefficient ADAF-like simulations, the disc region is geometrically thick and extends over a range of \( \theta \sim \pm 0.6 \) rad around the mid-plane (see Fig. 2). Outside the disc zone we have outflowing gas, which we further subdivide into two components, a slowly moving ‘wind’ and a rapidly moving ‘jet’. The distinction between wind and jet is motivated by the shapes of the \( \mu \) and \( \dot{m} \) profiles in Fig. 2. In both the SANE and MAD simulations, we see that for \( \theta \) values within about 0.4 rad of the poles, \( \mu \) is large and \( \dot{m} \) is small. The outflowing material here is clearly relativistic and has a large energy per unit mass; we call it
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\[ \dot{e} = |T_\parallel + \rho u' | \]

Figure 3. Contours of \( \mu \) in the poloidal plane for the \( a_*=0.7 \) MAD model plotted on top of the magnitude of the radial flux of energy \( \dot{e} \). Five contours are plotted corresponding to \( \mu = 0.3 \) (thinnest blue line), 0.1, 0.05, 0.02 (thickest blue line) and 0.0 (green line).

a ‘jet’. The rest of the outflowing region, which lies in between the jet and the disc, has a low value of \( \mu \) and larger \( \dot{m} \). This is a slowly moving outflow, which we call a ‘wind’.

How do we determine the boundary between the jet and the wind? Unfortunately, there is no unique way of separating these regions since the transition is smooth and the regions of high mass flux (wind) and high energy flux (jet) overlap each other. One possibility is to follow streamlines of the poloidal energy flux \( T_\parallel \), and to identify all outflowing streamlines anchored on the horizon as the jet and the remaining outflowing streamlines that originate in the disc as the wind (cf. the bottommost right-hand panel of Fig. 5 and see Penna et al. 2013b). This is similar to the approach used by Tchekhovskoy et al. (2011) and is a convenient way of distinguishing the part of the outflow that is powered by the BH from that which is powered by the disc. However, magnetic fields extract rotational energy from the BH at all latitudes (Penna et al. 2013b), even in the disc region where the inflowing rest mass flux overwhelms this effect and causes the flux of total energy to be negative.

Another approach, which we follow in this paper, is to choose a critical value of \( \mu \), or equivalently a critical value of the ‘velocity at infinity’, and to identify all outflowing gas with \( \mu \) larger than this critical value as the jet and the rest as the wind. We choose

\[ \mu_{\text{crit}} = 0.05, \]  

(14)

which at infinity corresponds to \( \beta_{\text{crit}} = v_{\text{crit}}/c \approx 0.3 \), a reasonable demarcation point between jet and wind. In Fig. 3, we plot five contours of \( \mu \) on top of the outflowing energy fluxes for the \( a_*=0.7 \) MAD simulation. Our particular choice of \( \mu_{\text{crit}} = 0.05 \) reasonably separates the region of high energy flux, which corresponds to the jet, and the region of low energy flux, which corresponds to the wind.

3.5 Radius of inflow equilibrium

Since each of our simulations has been run for only a finite duration, the solutions reach inflow equilibrium only within a limited volume. Gas outside this volume has not had enough time to be influenced by the accretion flow structure near the horizon. To quantify the range of radii over which the disc solution is reliable, we adopt the ‘loose’ equilibrium criterion from Narayan et al. (2012), i.e. we search for an inflow equilibrium radius \( r_{\text{eq}} \) which satisfies

\[ r_{\text{eq}} = |v'(r_{\text{eq}})| t_{\text{chunk}}. \]  

(15)

where \( v'(r_{\text{eq}}) \) is the density-weighted average velocity at a given radius and \( t_{\text{chunk}} \) is the duration of the last time chunk for the particular simulation. We carry out this calculation separately for the disc, the wind and the jet. In the case of the wind and jet \( v'(r_{\text{eq}}) \) is outward, so technically \( r_{\text{eq}} \) is the outflow equilibrium radius, but the principle remains the same.

Table 3 gives values of the limiting inflow/outflow equilibrium radii \( r_{\text{eq}} \) for each of the runs for each of the three regions. These radii decrease with increasing BH spin, since the durations of the simulations become shorter. In all cases, the wind region reaches equilibrium to larger radii than the disc since the radial velocity of the gas is larger. Similarly, since the jet has a relativistic velocity, this region reaches equilibrium to very large radii (essentially the entire domain of the simulation). Another systematic effect is that the MAD simulations, because of their larger radial velocities (see Narayan et al. 2012), are in inflow equilibrium over a significantly larger volume compared to the SANE simulations.

4 RESULTS

4.1 Accretion rate and magnetic flux

All the simulations were initialized with an equilibrium gas torus threaded by a weak poloidal magnetic field. Once the MRI develops, gas accretes towards the BH and the inner regions of the torus are depleted of matter. The accretion rate on the BH thus decreases with time, the variation being determined by the density profile assumed in the initial torus (Narayan et al. 2012). The upper panel of Fig. 4 shows the mass accretion rate \( \dot{M} \) on the BH versus time for all the radiatively inefficient models studied here. The solid and dotted lines correspond to MAD and SANE models, respectively. For a given BH spin, the accretion rate evolution is roughly the same for SANE and MAD models. However, at any given time, the higher the spin, the lower is the accretion rate. This appears to be because of the increasing mass-loss rate (discussed in Section 4.6).

By monitoring the magnetic flux \( \Phi_{\text{BH}} \) threading the BH horizon (equation 12) we can evaluate whether a particular simulation is in the SANE or MAD state. The bottom panel of Fig. 4 shows the evolution of \( \Phi_{\text{BH}} \) for each of the models. The magnetic flux at the horizon for the MAD runs (solid lines) remains always near \( \Phi_{\text{BH}} \approx 50 \), showing that the flux has saturated at the maximum allowed value, as appropriate for the MAD state (Tchekhovskoy et al. 2011). In contrast, the SANE models are characterized by lower values of \( \Phi_{\text{BH}} \). However, once in a while even these simulations show \( \Phi_{\text{BH}} \) approaching the MAD limit, though the flux subsequently falls

| Model | Disc | Wind | Jet |
|-------|------|------|-----|
| \( a_* = 0.0 \) SANE | 110 | 210 | – |
| \( a_* = 0.7 \) SANE | 70 | 130 | 21 000 |
| \( a_* = 0.9 \) SANE | 50 | 110 | 16 000 |
| \( a_* = 0.98 \) SANE | 50 | 80 | 8000 |
| \( a_* = 0.0 \) MAD | 340 | 720 | 65 000 |
| \( a_* = 0.7 \) MAD | 160 | 320 | 29 000 |
| \( a_* = 0.9 \) MAD | 140 | 260 | 13 000 |
as an oppositely polarized magnetic loop reaches the BH. For all the three SANE models with non-zero BH spin, $\Phi_{BH}$ approaches the saturation value appropriate to the MAD state near the end of the simulation. Thus, despite our efforts to avoid the MAD state in our SANE simulations, the accretion flow apparently has a tendency to be pushed towards the MAD limit.

4.2 Structure of the outflow regions

In this section, we discuss the general properties of the outflow regions. Fig. 5 shows the magnitude of mass and energy fluxes ($\dot{m}$, $\dot{e}$ and $\dot{e}_{\text{crit}}$) for $a_*=0.7$ SANE and $a_*=0.7$ MAD models together with corresponding streamlines in the poloidal plane. The blue contours denote the border between wind and jet regions (equation 14) while the green contours separate the outflow and inflow regions.

The left-hand column shows the rest mass flux. At large radii mass flows mostly inside the bulk of the disc. However, the streamlines clearly show that the inflowing accretion rate is not constant in all models – some rest mass is lost from the inflow region and forms the wind. Narayan et al. (2012) have shown that for $a_*=0$ models such a magnetically driven wind from the accretion disc itself does not extend all the way towards the BH but stops around $r=40$.

The mass outflows are enhanced for rotating BHs. For the $a_*=0.7$ SANE model the extra energy flux along the polar axis (middle panel) suppresses accretion in the polar region. However, the accretion flow wants to deposit the same amount of gas as in the non-rotating case (locally the disc does not feel the impact of spin at large radii). The surplus of gas has to find its way out of the system at large radii. The locations of both jets are visualized in Fig. 6 showing the density distribution in the left-hand plot and outflows of rest mass and energy in the right-hand part of the plot with magenta and blue, respectively. It is clear that the strong energy flux region is surrounded by the region where the mass-loss is most efficient.

The right-hand column in Fig. 5 shows the magnitude and streamlines of the total energy flux $\dot{e}_{\text{out}}$, i.e. the sum of the first two columns. Inside the disc the inflowing stream of rest mass dominates and the net energy flux points inward. In contrast, in the wind and jet the total energy flux is positive.

The solid angles of the jet regions in the $a_*=0.7$ simulations are $\Omega_{\text{jet}} \lesssim 1.0$ sr for the SANE and MAD models, respectively. The corresponding solid angles covered by the wind are $\Omega_{\text{wind}} \gtrsim 5.0$ sr. The wind thus subtends a significantly larger solid angle.

4.3 Angle-integrated mass and energy fluxes

Fig. 7 shows radial profiles of various mass flow rates for the $a_*=0.7$ SANE and MAD simulations. The red lines show the absolute value of the normalized net mass accretion rate (equation 7 with $\theta$ integrated over the full range $0$ to $\pi$) while the black lines show the magnitude of the total inflowing mass flux (same integral but over inflowing gas only). The curves are terminated at the limiting inflow equilibrium radius of the disc. As expected, the net $\dot{M}$ is equal to unity (because of the normalization) and is independent of radius (indicative of steady state), though constancy is violated near the outer edge, where complete equilibrium has not been achieved.

The blue curves show the mass outflow rate in the jet; in this case the integral in equation (7) is limited to those values of $\theta$ where the Bernoulli parameter $\mu$ exceeds the critical value $\mu_{\text{crit}}$ and $\dot{n} > 0$. Since the jet originates close to the BH, we see that the blue lines asymptote to a constant value at large radii. Thus, the simulations provide a reliable estimate of the mass-loss rate in the jet. The rate is about 10 per cent of the accretion rate for the SANE model and roughly equal to the accretion rate for the MAD model.

In contrast to the jet, the mass outflow rate in the wind (orange lines) increases steadily with radius and there is no sign of convergence at large radii. This confirms that mass-loss in the wind is dominated by large radii. Since the simulations reach inflow equilibrium at best out to a few hundred $R_G$, whereas real flows in nature extend to outer radii $R_{\text{out}} \sim 10^{-6} R_G$ or even larger, this means that estimates of mass-loss rates require considerable extrapolation. This is discussed below.

Fig. 8 shows corresponding results for the energy flow rate. Here, the red lines show the efficiency of the accretion flow, i.e. the energy that flows out to infinity normalized by the net mass accretion rate. We see that the $a_*=0.7$ SANE run has an efficiency of about 5 per cent whereas the MAD run has a much larger efficiency of about 70 per cent. The stronger magnetic flux around the BH in the latter enables much more efficient tapping of the BH spin energy.

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Figure 4. Accretion rate $\dot{M}$ into the BH (top) and magnetic flux $\Phi_{BH}$ threading the BH horizon (bottom) versus time for SANE (dotted lines) and MAD (solid lines) models with different BH spins.
The energy outflow rates in the jet (blue curves) are well converged, just like the mass. Even the energy outflow rates in the wind (orange curves) seem reasonably well converged. This is consistent with the simple analysis we present next.

### 4.4 Radial scalings of outflows

Let us define the mass inflow rate $\dot{M}_{\text{in}}(r)$ at a given radius as the sum of the net mass accretion rate, which we call $\dot{M}_{\text{BH}}$, and the
mass outflow rate in the jet and wind $\dot{M}_{\text{wind}}$ (e.g. Stone et al. 1999; Yang et al. 2013). Thus, $\dot{M}_{\text{in}}$ is the mass accretion rate we would calculate if we restricted the $\theta$ integral in equation (7) to the regions with $\alpha < 0$. It is reasonable to assume that $\dot{M}_{\text{wind}}$ scales with radius as a power law (Blandford & Begelman 1999). Hence, let us assume that

$$\dot{M}_{\text{wind}}(r) = \dot{M}_{\text{BH}} \left( \frac{r}{r_{\text{in}}} \right)^s,$$

$$\dot{M}_{\text{in}}(r) = \dot{M}_{\text{BH}} \left[ 1 + \left( \frac{r}{r_{\text{in}}} \right)^s \right],$$

where we expect the index $s$ to lie in the range $0 - 1$, and $r_{\text{in}}$ is some characteristic radius, typically of order tens of $R_G$. The black curves in Fig. 7 show the variation of $\dot{M}_{\text{in}}$ versus $r$ for the two simulations. Note the approximate power-law behaviour, with $s \gtrsim 0.5$.

From equation (16) it is evident that the mass outflow rate is dominated by large radii. Therefore, unless we have a reliable estimate of the value of $s$, we cannot hope to obtain an accurate estimate of the mass-loss rate in an ADAF. The situation is better in the case of energy outflow. The differential mass-loss at a given radius is given by

$$\text{d}M_{\text{wind}} = \dot{M}_{\text{wind}} \left( \frac{r}{r_{\text{in}}} \right)^s \text{d}r,$$

We expect that any mass that flows out at radius $r$ will carry with it an energy equal to some fraction $\xi$ of the local potential energy. Thus, we estimate the local energy loss rate in the wind to be

$$\text{d}E_{\text{wind}} = \text{d}M_{\text{wind}} \frac{\xi c^2}{r} = \xi s \dot{M}_{\text{BH}} c^2 \frac{r^{s-2}}{r_{\text{in}}} \text{d}r,$$

which gives a cumulative energy loss rate of

$$E_{\text{wind}}(r) = \frac{\xi s}{1-s} \dot{M}_{\text{BH}} c^2 \left[ 1 - \left( \frac{r_{\text{in}}}{r} \right)^{1-s} \right] \approx \frac{\xi s}{1-s} \dot{M}_{\text{BH}} c^2 r_{\text{in}}.$$ (20)

We see that the energy loss in the wind is dominated by the innermost regions (as confirmed in Fig. 8), hence the simulations ought to provide reliable estimates of the energy feedback rate into the surroundings. The momentum outflow rate has a scaling intermediate between those of mass and energy outflow.

GRMHD simulations of BH accretion discs are limited by computational power. Even in the best cases (e.g. the simulations discussed here), the inflow equilibrium region of a simulation extends only to radii of order a few hundred $R_G$ (Table 3). Because of this limitation, we can obtain meaningful estimates of the total mass-loss rate in the wind only if the mass outflow behaves in a self-similar fashion and the range of inflow equilibrium of the simulation is far enough from the BH that we can obtain a reliable estimate of the index $s$. Having this caveat in mind, we discuss first the more reliable energy outflow.

### 4.5 Energy outflow

Fig. 9 shows radial profiles of energy outflow in the jet and wind regions in the final three time chunks of our simulations: chunks T4,
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Figure 7. Radial profiles of the mass flux $\dot{M}$ for $a_*= 0.7$ SANE (top) and MAD (bottom panel) at time chunk T5. The red, orange and blue lines show the net mass accretion rate, mass outflow in the wind and mass outflow in the jet, respectively. All fluxes are normalized by $\dot{M}$ at $r=10$, and each line is terminated at the corresponding radius of inflow or outflow equilibrium (Table 3). The black lines show the total inflow flux, i.e. the integral over sphere of the inflowing rest mass (equation 7).

Figure 8. Similar to Fig. 7 but for the energy flux $\dot{E}$.

T5, T6 for the $a_*= 0$ SANE and MAD runs, T3, T4, T5 for the two $a_*= 0.7$ runs, and T2, T3, T4 for the two $a_*= 0.9$ runs. We see that the wind energy profiles (orange curves) are poorly converged, in agreement with Narayan et al. (2012) who found that outflows from discs around non-rotating BHs showed poor convergence with time. On average, the longer the duration of a given simulation, the further from BH the winds originate. There is non-monotonic behaviour in the $a_*= 0.7$ SANE and $a_*= 0.9$ MAD models, but we believe this is simply because the magnetic flux around the BH increased in the last time chunk for these two simulations (cf. Fig. 4). Note that the energy lost in the wind for the SANE models, and to a lesser extent for the MAD models, is independent of radius, i.e. the energy budget is dominated by the innermost region of the wind, in agreement with the scalings discussed in Section 4.4.

The energy profiles $E(r)$ of the simulated jets show very good spatial convergence, asymptoting to a constant value at large radii. This indicates that the jet criterion we adopted ($\mu > \mu_{\text{crit}}$) closely follows the energy flux streamlines at large radii. Variation among different time chunks is due to fluctuations in the magnetic flux at the horizon, as discussed further below.

Because of the lack of convergence with time of the wind regions in the simulations, it is not obvious how one should compare different simulations to study the effect of BH spin. In the following, we choose to compare simulations at the same physical time, viz., time chunk T4. Fig. 10 shows the results. The power of both the jet and the wind increases with BH spin for both SANE and MAD simulations. The variation in the case of the wind is modest, whereas jet power shows a very strong dependence on BH spin. In fact, the jet power in the $a_*= 0.9$ MAD simulation exceeds $\dot{M}c^2$, showing that the jet is powered by more than accretion energy. At least some part of the power must come directly from the BH (Tchekhovskoy et al. 2011).

Theoretical jet models indicate that the power extracted from a spinning accreting BH scales as (Blandford & Znajek 1977; Tchekhovskoy & McKinney 2012; Penna et al. 2013b)

$$\eta_{\text{jet}} = \frac{P_{\text{jet}}}{\dot{M}c^2} \sim \frac{0.05}{4\pi c} \Phi_{\text{BH}}^2 \Omega_{\text{H}}^2,$$

(21)

where $\Phi_{\text{BH}}$ is the magnetic flux threading the horizon (equation 12) and $\Omega_{\text{H}}$ is the angular velocity of the outer BH horizon (equation 1). The jet powers in our simulations are generally consistent with the spin dependence in this formula; the $a_*= 0.9$ has the strongest jet and $a_*= 0$ has essentially no jet. Note that the non-zero jet power we find for the $a_*= 0.0$ MAD model is an artefact caused by the activation of numerical floors at the polar axis and the corresponding injection of mass and energy.

In Fig. 11, we test the scaling of jet power with $\Phi_{\text{BH}}^2$. The horizontal axes show the magnetic flux $\Phi_{\text{BH}}$, and the vertical axes show the corresponding injection of mass and energy.
Fluxes of energy ($\dot{E}$) for SANE (top) and MAD (bottom) simulations with given value of BH spin. The blue and orange lines correspond to the jet and wind regions, respectively. On each subpanel three sets of lines are plotted corresponding to three most recent chunks of time for each simulation.

Radial profiles of the energy flux ($\dot{E}$) for SANE (top) and MAD (bottom) models at time chunk $T_4$. Profiles for three values of BH spins are presented. All fluxes are normalized to $\dot{M}$ at $r = 10$ and the lines are terminated at the radius of outflow equilibrium of the corresponding $a_*=0.9$ model.

Simulation (only two points in the case of the $a_*=0.98$ model because of the very short duration of this run). The horizontal error bars reflect the standard deviation of the magnetic flux within the particular time chunk. Colours denote BH spin. The triangles and crosses correspond to SANE and MAD simulations, respectively.

The top panel in Fig. 11 shows the energy loss rate in the jet as a function of magnetic flux at the horizon. Within each model and for a given spin, the jet power follows the $\Phi_{\mu\nu}^2$ scaling quite well, validating this prediction of theory. The bottom panel of the same figure shows the energy carried away by the wind measured at a common radius of $r=80$. This quantity again increases with the magnetic flux $\Phi_{\mu\nu}$, but the dependence is much weaker than for the jet. This shows that the wind receives only some of its energy from the BH, the rest coming directly from the gravitational energy released by the accreting gas.

### 4.6 Mass outflow

Fig. 12 shows radial profiles of mass outflow for SANE and MAD models with BH spin $a_* = 0, 0.7$ and $0.9$. For each model, the last three time chunks are shown. Lack of convergence of the mass outflow rate in the wind as a function of time is clearly visible for most of the models – the orange lines move steadily outward with time. The mass outflow rate in the jet is well converged in space, i.e. it quickly saturates at a constant value. However, it varies with time (most profound for $a_* = 0.9$ SANE model) as a result of changing magnetic flux threading the horizon (Fig. 4).

Fig. 13 shows profiles of mass outflow in the jet and wind for time chunk $T_4$ and three BH spins, $a_* = 0, 0.7$ and $0.9$. In the SANE simulations (top panels), where the jet is weak, mass-loss is dominated by the wind at all radii. Mass-loss is strongest and starts closest to the BH for the highest value of BH spin. In the MAD solutions (bottom panels), mass outflow in the jet (cocoon) dominates at smaller radii (except for $a_* = 0$), and the wind takes over only at larger radii. Outflow rates initially overlap each other and then diverge showing the same dependence on BH spin.
for a given BH spin, there is essentially no correlation between the mass-loss in the wind and the magnetic flux. This is reasonable. Winds flow out from relatively large radii in the disc, where the gas is not sensitive to the magnetic flux near the BH. Note, however, that there is a correlation between the wind mass-loss rate and BH spin – on average higher spin leads to stronger mass outflow.

4.7 Outflows of momentum

The relativistic momentum flux given by equation (11) reduces for particles to

$$\dot{P} = \frac{M_v}{\sqrt{1 - v^2}},$$

(22)

and in the non-relativistic limit gives $\dot{P} = M_v$. Therefore, one could expect that the profiles of outflowing momentum are similar to the profiles of lost rest mass but normalized by a coefficient related to the total gas velocity $v$.

For the mildly relativistic wind region the dominant component of gas velocity is the azimuthal component ($v \approx r^{-1/2}$). Fig. 15 shows the radial profiles of the integrated momentum flux $P$. Indeed, the momentum lost in the wind (orange lines) resembles profiles of the rest mass lost in that region (Fig. 12). Its magnitude is noticeably smaller and follows the rescaling with characteristic gas velocity, e.g., at $r = 100$ is a factor of $v = 0.1$ lower. As a result of the radial dependence of the azimuthal velocity the profiles are less steep than for the rest mass flux.

The gas in the jet region is highly relativistic (Fig. 1) and its velocity is dominated by the radial component. In a similar way, the radial profiles of momentum lost in the jet region should resemble the rest mass-loss rates with the scaling factor depending on the gas velocity. For the jet region with the characteristic Lorentz factor $\gamma' \approx 2$ this factor is close to unity. Fig. 15 shows that indeed the momentum lost in the jet region (blue lines) is quantitatively similar to the profiles of rest mass lost in this region. The fact that both are constant in radius for $r \geq 50$ proves that the characteristic velocity in the jet region does not change.

4.8 Approximate model of outflows

Outflows in a typical strongly magnetized ADAF around a rotating BH can be divided into a slow wind at large radii and a relativistic jet along the poles. Such a structure is schematically shown in Fig. 16. Most of the energy is lost via the jet; for large enough BH spins, the power of the jet may even exceed the total energy budget $M_c^2$ of the accretion flow. Mass is lost through both the wind and the ‘cocoon’ surrounding the jet. At large radii, mass-loss is dominated by the wind.

In the case of the jet, as discussed in previous sections, the energy and mass outflow are sensitive both to the BH spin as well as the magnetic flux at the horizon. Outflows in the wind also depend on these parameters, but less sensitively.

According to the BZ model (Section 1.2) the power extracted from the BH (equation 21), $P_{\text{BH}}$, is proportional to the square of the magnetic flux threading the horizon $\Phi_{\text{BH}}$ and the square of the angular velocity of the BH horizon $\Omega_\text{BH}$. Using this scaling as a guide and keeping in mind the discussion in Sections 4.5–4.7, we construct here simple empirical fits which reasonably reproduce the simulation results. Except for $\dot{P}_{\text{wind}}$, all the other outflow rates are good to within a factor of order unity (Fig. 17). This is true even for $P_{\text{wind}}$ so long as $s$ is not very different from $1/2$. 

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**Figure 11.** Outflow rates of the energy flux ($\dot{E}$) in the jet (top) and the wind (bottom panel) as a function of the magnetic flux at BH horizon averaged over the duration of given time chunk. For each model, values corresponding to the three most recent time chunks are presented. The triangles and crosses are for SANE and MAD models, respectively. Colours denote BH spin. The outflow rate in the wind was measured at $r = 80$ while the jet power is averaged over $r = 100 \div 1000$ and the vertical error bars show the minimum and maximum values in this range.

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jet mass-loss rate in particular shows noticeable dependence on the BH spin.

The top panel of Fig. 14 shows the relation between the magnetic flux at the horizon $\Phi_{\text{BH}}$ and the mass-loss rate in the jet. There is a strong correlation, with a dependence approximately $\propto \Phi_{\text{BH}}^{5/2}$, which is similar to the scaling of the jet energy loss rate ($\Phi_{\text{BH}}^{5/2}$) but a little shallower. The slightly different slope is because the terminal Lorentz factor of the jet ($\gamma'$) scales with the magnetic flux roughly as $\Phi_{\text{BH}}^{0.5}$, i.e. jets in MAD solutions are typically more relativistic than those in SANE solutions.

Thus, the mass in the jet cocoon is driven essentially by energy outflow from the BH horizon. There is also a clear dependence on BH spin, with larger spins giving stronger mass-loss in the jet. The mass-loss in the jet for the $s = 0$ MAD model is an artefact, and represents leakage of mass from the disc wind into the jet region.

The bottom panel shows the mass-loss rate in the wind as measured at $r = 80$ (which lies inside the wind outflow equilibrium region for all the simulations) as a function of $\Phi_{\text{BH}}$. It is clear that,
All the estimates given in this subsection are normalized by the mass accretion rate on the BH, $\dot{M}_{BH}$. Thus, energy outflow rates are given in units of $\dot{M}_{BH}c^2$, momentum outflow rates in units of $\dot{M}_{BH}c$ and mass outflow rates in units of $\dot{M}_{BH}$.
Outflows from accreting BHs

Figure 15. Similar to Figs 9 and 12 but for fluxes of momentum ($\dot{P}$) for SANE (top) and MAD (bottom) simulations.

Figure 16. Schematical picture of the disc structure near a rotating BH.

The cumulative energy, mass and momentum outflow at radius $r$ may be written as a sum of contributions from the jet and the wind,

\[ \dot{E} = \dot{E}_{\text{jet}} + \dot{E}_{\text{wind}}, \]
\[ \dot{M} = \dot{M}_{\text{jet}} + \dot{M}_{\text{wind}}, \]
\[ \dot{P} = \dot{P}_{\text{jet}} + \dot{P}_{\text{wind}}. \]

As discussed in Section 4.4, our simulations provide reliable estimates of the two energy fluxes. The energy loss from the system is dominated by the outflow in the jet (Fig. 9) which is driven by energy extracted from the BH through the BZ process. We approximate the jet energy as

\[ \dot{E}_{\text{jet}} \approx 0.5 \Phi_{\text{BH}}^2 \Omega_{\text{H1}}^2, \]

where

\[ \Phi_{\text{BH}} = \Phi_{\text{BH}}/50 \]

\[ \dot{E}_{\text{wind}} \approx 0.005(1 + 3 \Phi_{\text{BH}}^2 \Omega_{\text{H1}}^2), \]

\[ \dot{M}_{\text{jet}} \approx 0.5 \Phi_{\text{BH}}^2 \Omega_{\text{H1}}^2, \]

\[ \dot{M}_{\text{wind}} \approx 0.005(1 + 3 \Phi_{\text{BH}}^2 \Omega_{\text{H1}}^2), \]

\[ \dot{P}_{\text{jet}} \approx 0.5 \Phi_{\text{BH}}^2 \Omega_{\text{H1}}^2, \]

\[ \dot{P}_{\text{wind}} \approx 0.005(1 + 3 \Phi_{\text{BH}}^2 \Omega_{\text{H1}}^2). \]

is the magnetic flux threading the horizon, normalized by the characteristic value for a MAD disc, and

\[ \dot{E}_{\text{wind}} = \dot{E}_{\text{wind}}/0.2 \]

is the horizon angular velocity (equation 1), normalized by the angular velocity for a spin $a_s = 0.7$ BH $\Omega_{\text{H1}}(a_s = 0.7) \approx 0.2$. As the top panel of Fig. 17 shows, the coefficient 0.5 in equation (26) provides a good fit to the results presented in Section 4.5.3

The energy loss in the wind is much weaker (Fig. 9) and we estimate it as

\[ \dot{E}_{\text{wind}} \approx 0.005(1 + 3 \Phi_{\text{BH}}^2 \Omega_{\text{H1}}^2). \]

3 In Section 4.3, we discussed energy flux in jet for $a_s = 0.7$ MAD disc having efficiency $\eta = 70$ per cent. This is consistent with equation (26) because the magnetic flux for this simulation $\Phi_{\text{BH}} \approx 60$ and the $\Phi_{\text{BH}}^2$ factor is not unity.
where the empirical factor \((1 + 3 \Phi_1^2 \Omega_1^2)\) accounts for the increase of the wind power with BH spin and magnetic flux (see the bottom panel of Fig. 11). The middle panel of Fig. 17 validates the choice of the coefficient.

The mass-loss rates we measure are robust only in the jet region. Following the discussion in Section 4.5, we approximate it as

\[
\dot{M}_{\text{jet}} \approx 0.7 \Phi_1^{1.5} \Omega_1,
\]

where the coefficient 0.7 is our best guess from the simulation results (see bottom panel of Fig. 17).

The radial profiles of the mass-loss rate in the wind in the various simulations are not converged in time and are therefore less reliable. Even estimating the power-law index \(s\) and the characteristic radius \(r_\text{in}\) (equation 16) is difficult because the region where the profiles show power-law behaviour is close to the radius \(r_\text{conv}\) defined in equation (15). Roughly, it appears that \(0.5 < s < 0.7\) (in agreement with Yuan et al. 2012b). Let us write

\[
M_{\text{wind}} \approx \left( \frac{L}{L_{\text{Edd}}} \right)^s,
\]

where \(r_\text{in}\) is the radius where the mass flux in the wind equals the net accretion rate on the BH: \(M_{\text{wind}}(r_\text{in}) = 1\). From Fig. 12 it appears that \(r_\text{in} \gtrsim 100\) and that it tends to be smaller (outflows stronger) for rotating BHs. Better estimation of \(r_\text{in}\) is not possible due to the limitations of our simulations.

Finally, the fluxes of momentum in the jet and wind are approximately related to the corresponding mass-loss rates by

\[
\dot{P}_{\text{jet}} \approx \dot{E}_{\text{jet}},
\]

\[
\dot{P}_{\text{wind}} \approx M_{\text{wind}} r^{-0.5},
\]

where we have assumed \(u' \approx 2\) in the jet, that the characteristic velocity of the wind originating at radius \(r\) is \(v \approx v_\phi = r^{-0.5}\) and that \(s > 0.5\).

## 5 COMPARISON WITH THIN DISCS

So far, we have shown that outflows of mass and energy are common in thick accretion discs. This class of discs corresponds to radiatively inefficient flows forming at very low \((L \lesssim 0.01L_{\text{Edd}}\) Narayan & McClintock 2008) or very high \((L \gtrsim L_{\text{Edd}})\) accretion rates; the latter regime is known as the slim disc (Abramowicz et al. 1988). However, many BHs in microquasars and galactic nuclei are known to accrete with moderate accretion rates which correspond to a radiatively efficient, geometrically thin disc. Do such discs produce outflows similar to those found in the case of thick discs?

Jets in Galactic BHs are quenched around the time they change state from the low-hard, presumably geometrically thick, to the high-soft state corresponding to a geometrically thin disc (e.g. Remillard & McClintock 2006). This fact may suggest that relativistic outflows are not characteristic for such class of discs.

Numerical studies of geometrically thin discs have been limited so far because of the requirement of including radiative transfer. All GRMHD simulations of thin discs performed so far (e.g. Shafee et al. 2008; Penna et al. 2010; Noble et al. 2011; Zhu et al. 2012) are based on an artificial cooling function which drives discs to an arbitrarily chosen entropy corresponding to a required given disc thickness. More sophisticated treatment of radiation in the context of thin accretion flows is being developed (e.g. Fragile et al. 2012; Sadowski et al. 2013).

To compare the power of outflowing mass and energy between thick and thin discs we have performed two additional simulations which use the cooling function as described in Zhu et al. (2012) to drive the disc towards \(h/r \approx 0.1\) which corresponds to \(L \approx 0.3L_{\text{Edd}}\). We tested two values of BH spin \(a_*= 0.0\) and 0.9. The initial magnetic field was set in a similar way to the SANE simulations described earlier, i.e. its poloidal component formed a set of counterorientated loops. Details of the simulations are given in Table 4.

### Table 4. Thin disc models.

| Model | BH spin \((a_*)\) | Initial magnetic field | Resolution | \(t_\text{final}\) |
|-------|-----------------|------------------------|------------|-----------------|
| \(a_* = 0.0\) thin | 0.0 | multiloop | \(264 \times 64 \times 64\) | 50 000 |
| \(a_* = 0.9\) thin | 0.9 | multiloop | \(264 \times 64 \times 64\) | 50 000 |
that geometrically thin discs are less efficient in generating relativistic outflows of rest mass and energy than equivalent thick discs. Because of the very limited region of inflow equilibrium in Fig. 19, we cannot address the efficiency of generating wind-like outflows except in the innermost regions (where there is no wind).

6 DISCUSSION AND SUMMARY

In this paper, we presented a number of GRMHD simulations of magnetized non-radiative geometrically thick accretion discs around BHs, and used them to investigate the outflow of energy, momentum and mass from these systems. The simulations covered a range of BH spin parameters $a_* = a/M$ or, equivalently, BH horizon angular frequencies $\Omega_H$, and thus probed the effect of this important parameter on outflows. We also tested two initial configurations of the magnetic field, which led, respectively, to configurations with a weak magnetic field around the BH (SANE configuration) and a saturated magnetic field (MAD configuration). This enabled us to study the dependence of outflow properties on the magnetic flux $\Phi_{BH}$ around the BH.

Each simulation was run for a long time in order to reach steady state over as wide a range of radius as possible (see Table 3 for estimates of the size of the inflow and outflow equilibrium region). In each simulation, we divided the flow into an inflowing ‘disc’, an outflowing ‘wind’ and ‘jet’. The latter two zones were distinguished on the basis of their velocity at infinity. The jet consists of all matter that flows out with $v \gtrsim 0.3$ at infinity and the wind is the rest of the outflowing matter with asymptotic $v \lesssim 0.3$. More precisely, the boundary is defined by a critical Bernoulli parameter, $\mu_{crit} = 0.05$ (equation 13). Fig. 16 shows the three zones schematically.

Our results are summarized in Section 4.8 in the form of approximate fitting functions. In the case of the jet, we are able to provide fairly reliable estimates of its outflow properties. Specifically, we give fitting functions for the jet energy outflow $E_{jet}$, momentum outflow $P_{jet}$ and mass outflow $M_{jet}$. The energy and momentum outflow rates vary proportional to $\Phi_{BH}^{-1} \Omega_H^{-2}$, as predicted by the BZ model.

The situation is less clear in the case of the wind. We believe our estimates of the wind energy outflow $E_{wind}$ and, to a lesser extent, the wind momentum outflow, $P_{wind}$, are fairly reliable. However, the mass outflow in the wind is highly uncertain. This is because mass outflow is dominated by large radii and our simulations do not go out to a large enough radius to enable us to extrapolate reliably to the putative outer edge of the accretion flow at (say) the Bondi radius. Specifically, in equation (31), we cannot determine the value of $s$ with any certainty. The situation is exacerbated by the fact that, as the simulation progresses in time, the wind launch radius appears to track the limiting radius of inflow equilibrium in the disc. That is, the wind seems always to begin within a factor of about 2 of that radius, but this is where the results are least reliable. Modulo this serious uncertainty, it appears that the energy, momentum and mass outflow rates in the wind have some dependence on the BH angular velocity $\Omega_H$ and the BH magnetic flux $\Phi_{BH}$.

SMBH ‘feedback’ in galaxy clusters is believed to play an important role in keeping the cluster gas hot and preventing catastrophic star formation (Ciotti & Ostriker 2001; Brüggen & Kaiser 2002; Ruszkowski & Begelman 2002; Churazov et al. 2004). Observational evidence suggests that the feedback occurs via relativistic jets. The scaling relations given in this paper for $E_{jet}$ and $P_{jet}$ are likely to be useful in this context.

SMBH feedback also appears to occur inside AGN host galaxies (Silk & Rees 1998; King 2003; Hopkins et al. 2009), but in this case it is less obvious that the jet is important. Jets are much too

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Fig. 18. Accretion rate (top) and vertical magnetic flux threading the horizon (bottom) history for thin disc simulations with $a_* = 0.0$ and 0.9.

Fig. 19 shows the magnitude of the local rest mass flux ($\dot{m}$, equation 3), the energy flux ($\dot{e}$, equation 6), and the total energy flux ($\dot{e}_{tot}$, equation 4) in the poloidal plane, averaged over $t = 25000 \div 50000$, for thin disc simulations with $a_* = 0.0$ and 0.9, respectively. Because of the much lower radial velocity in thin discs, the range of inflow equilibrium in the disc region is limited to $r \lesssim 15$. Even with this limitation, we believe the simulations should be sufficient to provide useful estimates of the outflow in a jet. It is thus significant that we see no sign of a jet in either of the two simulations, suggesting that thin discs are not conducive to producing relativistic jets. We also do not see any outflowing mass or energy in a wind. However, this result is less significant since, even in the geometrically thick ADAF runs described earlier, the wind usually begins only at relatively large radii. Our thin disc simulations have not reached inflow equilibrium at such radii.

The thin disc simulations described here were set up with multiple poloidal magnetic field loops which, as explained earlier, results in unsaturated magnetic field around the BH and thus are destined to produce weaker jets. Initializing the magnetic field with a single loop, as in the MAD ADAF runs, should result in stronger magnetic flux at the horizon. This would be the thin disc equivalent of the MAD state. (Note that, historically, the original conceptual paper on the MAD solution by Narayan et al. (2003) considered thin discs, whereas the numerical simulations by Igumenshchev et al. (2003) which motivated this paper dealt with ADAFs.) Such an experiment is worthwhile and is left for future work. However, comparing the flow structure of the two thin disc simulations discussed here (Fig. 19) with the SANE ADAF simulations discussed earlier (e.g. third row of Fig. 5) already suggests that mean outflows evolve in unsaturated magnetic field around the BH and thus are destined
The result to have much of an effect, e.g. our jets subtend less than 10 per cent of the solid angle around the BH at a radius $r \sim 10^2$, and the solid angle at larger radii is much less because of continued collimation. On the other hand, the wind in our simulations covers nearly 50 per cent of the solid angle around the BH (the other 50 per cent being covered by the thick disc). Thus, the wind is likely to have a strong effect on the host galaxy, especially since, unlike radiation which can escape through optically thin regions of the galaxy, a wind is certain to deposit all its energy and momentum in the ISM of the galaxy. The scaling relations given in this paper for $\dot{E}_{\text{wind}}$ and $\dot{P}_{\text{wind}}$ (of which the energy scaling is more reliable) are thus relevant.

We also presented simulations of artificially cooled thin accretion discs ($h/r \approx 0.1$) for two BH spins: $a_*=0, 0.9$. Neither simulation showed any evidence for either a jet or a wind. We note that both simulations were in the SANE regime and the radius of inflow equilibrium was only $r \approx 15$. Nevertheless, our preliminary conclusion from these simulations is that thin discs are significantly less efficient in producing relativistic jets. We cannot say anything about winds from thin discs.

The flowchart in Fig. 20 summarizes our qualitative conclusions from this study of outflows from accreting BHs. Geometrically thin discs appear not to produce relativistic outflows, nor do they have magnetically driven winds from small radii. Mass-loss through magnetically or line-driven winds at larger radii is certainly possible, but is beyond the scope of this work. Geometrically thick ADAFs exhibit two kinds of outflows: wind and jet. The wind originates from relatively large radii in the flow. It is responsible for most of the mass outflow, and carries a modest amount of energy and momentum. The properties of the wind depend relatively weakly on the BH spin and the magnetic flux at the horizon. In favourable cases – rapid BH spin and large magnetic flux (MAD limit) – the flux of energy and momentum in the jet are very much larger than the corresponding fluxes in the wind. However, because of the strong collimation, jet feedback is probably important only on the largest scales, e.g. galaxy clusters.

Apart from the mass accretion rate $\dot{M}_{\text{BH}}$ and the BH angular velocity $\Omega_1$, the present study confirms the importance of a third parameter, the dimensionless magnetic flux $\Phi_{\text{BH}}$ at the BH horizon. In principle, different systems might have different values of $\Phi_{\text{BH}}$, making it much more difficult to come up with useful predictions for individual systems. One mitigating factor is that the numerical simulations described here as well as other recent work (Tchekhovskoy et al. 2011, 2012; Narayan et al. 2012; McKinney et al. 2013) suggest that ADAFs easily reach the MAD state in which the magnetic flux saturates at its limiting value $\Phi_{\text{BH}} \sim 50–60$. Thus, it is conceivable that the majority of observed systems would be in the MAD limit and have a similar value of $\Phi_{\text{BH}}$, thus eliminating this uncertainty. Note that plenty of magnetic flux is available in the external medium, more than enough to saturate the field at the horizon (Narayan et al. 2003), and theoretical arguments suggest that a geometrically thick ADAF is likely to transport the magnetic field inward efficiently (Guilet & Ogilvie 2012, 2013).

We conclude with two caveats. First, our formulae for the energy and momentum outflow in the jet and wind are expressed in terms of the net mass accretion rate on the BH, $\dot{M}_{\text{BH}}$. In practice, to use these relations, one needs to be able to estimate $\dot{M}_{\text{BH}}$ given the mass supply rate at the feeding zone of the BH, $\dot{M}_{\text{in}}$. Unfortunately, there is considerable uncertainty on this issue. In the language of equation (16), if the Bondi accretion rate at $R = R_B$ is $\dot{M}_B$, then very roughly we expect

$$\dot{M}_B = \dot{M}_{\text{in}}(r_B) = \dot{M}_{\text{BH}} \left[ 1 + \left( \frac{R_B}{r_{\text{in}}} \right)^s \right].$$

(34)
Outflows from accreting BHs

Depending on the values of $s$ and $r_{in}$, the ratio $\dot{M}_{BH}/\dot{M}_{g}$ could vary over a wide range. What can be done about this uncertainty? There is hope that $r_{in}$ could be determined via simulations. However, it might be much harder to obtain a reliable estimate of $s$, at least through GRMHD simulations. Some authors have obtained fairly good estimates of $s$ via large dynamic range hydrosimulations (e.g. Yuan et al. 2012b, obtain $s \sim 0.5$), but it is uncertain if their results will carry over to MHD. This problem needs to be resolved before accretion disc simulation results can be used for galaxy feedback studies.

Secondly, the simulations presented here correspond to non-radiative flows, whereas ADAFs in nature do produce at least some radiation. Dibi et al. (2012) included radiative cooling in their simulations and found that it introduces significant differences in the structure of the flow near the BH for accretion rates above $10^{-7}$ Eddington. Their result, however, depends on assumptions regarding the electron temperature in the two-temperature plasma. More conservatively, one might expect noticeable effects from radiation for accretion rates $\gtrsim 10^{-4}$ Eddington. Radiation can also have a strong effect at and beyond the Bondi radius. Specifically, X-rays produced by inverse Compton scattering near the BH can heat gas near the Bondi radius and significantly modify its thermal properties (Sazonov, Ostriker & Sunyaev 2004; Yuan, Xie & Ostriker 2009). In turn, this can significantly modify the mass supply at the Bondi radius, thereby forming a feedback loop.

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Figure 20. Qualitative description of outflows in BH accretion discs.
