Pairing in nuclei

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Abstract
Simple generic aspects of nuclear pairing in homogeneous media as well as in finite nuclei are discussed. It is argued that low-energy nuclear structure is not sensitive enough to resolve fine details of nuclear nucleon–nucleon (NN) interaction in general and pairing NN interaction in particular which allows for regularization of the ultraviolet (high-momentum) divergences and a consistent formulation of effective superfluid local theory. Some aspects of (dis)entanglement of pairing with various other effects as well as forefront ideas concerning isoscalar pairing are also briefly discussed.

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1. Introduction

The pioneering works of Bohr et al [1], Belyaev [2] and in particular, the ultimate success of early ‘large-scale’ BCS calculations by Nilsson and Prior [3] explaining simultaneously odd–even mass staggering and moments of inertia neatly settled pairing just at the heart of nuclear physics. The success of the Nilsson-plus-BCS approach using a simple constant matrix element pairing interaction confirms the absolute dominance of the s-wave (monopole) part of the in-medium effective particle–particle (p-p) NN interaction at the Fermi energy. There is, however, yet another very general argument pointing toward the simplicity of nuclear pairing. Indeed, since pairing modifies the nucleonic motion essentially only in the closest vicinity of the Fermi energy, $E_F - \Delta \leq \langle p_F \pm \delta p \rangle^2/2m \leq E_F + \Delta$ it gives rise to uncertainty in momentum space, $\delta p \sim \Delta/\sqrt{\nu_F}$, which translates to uncertainty in coordinate space of the order of $\xi \sim (\hbar c)^2 E_F/(mc^2) \Delta \sim 50$ fm. The quantity $\xi$, which is known as the coherence length, defines the spatial extension of the nucleonic Cooper pair. Note that (in fact, due to $E_F \gg \Delta$) the value of $\xi$ exceeds by far the typical interaction range $\xi \gg r_0 \sim 1/k_F$ which is known as the weak coupling limit. The above argumentation holds also for finite nuclei where $\xi \sim 2R \gg r_0$ ($R$ denotes nuclear radius), i.e. nucleonic Cooper pairs are spatially very extended objects. They are therefore not bosons, they are overlapping pairs which, for example, cannot form Bose–Einstein condensates. Since nuclear pairing is characterized by a small parameter $\epsilon \equiv r_0/\xi \ll 1$ it should posses an intrinsic simplicity. In particular, it should be fairly insensitive to fine details of the NN interaction or, alternatively, should be well described by a local theory. In fact, since $r_0 \ll 2R$, arguments speaking in favour of a local approximation can be extended over to the particle–hole (p-h) channel as well, see [4] and references quoted therein.

The paper is organized as follows. In section 2, we discuss the independence of $^1S_0$ pairing with respect to details of the NN interaction in homogeneous medium. Section 3 presents possible regularization schemes leading to a cutoff parameter independent superfluid local density approximation (SLDA). In section 4, we briefly discuss the perplexing problem of (dis)entanglement between pairing and various other effects. Finally, section 5 is a brief overview of current ideas regarding isoscalar pairing.

2. The $^1S_0$ pairing gap in an infinite homogeneous medium

Several bare NN potentials are available nowadays that fit the two-body scattering data with very high precision. In spite of substantial differences among them they all predict essentially the same values for the $^1S_0$ pair-gap over a rather wide range of Fermi momenta $k_F$, see examples in figure 1. It appears that the detailed NN interaction is not needed at all to determine the $^1S_0$ gap, as nicely demonstrated in [5]. The decisive point is that the $^1S_0$ NN scattering is characterized by a large negative scattering length indicating the presence of a nearly bound resonant state at zero scattering energy. Around this low-energy pole, the NN interaction can be well approximated by a separable interaction $v(k, k') \approx \gamma(v(k) v(k'))$ for which the so-called inverse scattering problem can be solved. It means that both $v(k)$ as well as $\Delta(k_F)$ are in fact fully determined.
by means of the phase shifts \( \delta(k) \). Although, at first glance, this approximation seems to be valid (i) only at low energies and requires (ii) the knowledge of \( \delta(k) \) at, in principle, all energies it appears to work surprisingly well up to \( k_F \sim 1.4 \text{ fm}^{-1} \). Note however, that the effective range approximation to the phase shifts works well only at low-densities up to \( k_F \sim 0.6 \text{ fm}^{-1} \), see figure 1.

The inclusion of in-medium polarization and screening corrections appears to be extremely difficult. So far no consensus has been reached on how to consistently compute these corrections, see [6, 7] and references therein. However, instead of deriving an in-medium gap equation \( \Delta(k_F) \) from the bare NN interaction one can attack the problem starting directly from an effective interaction like Gogny or local density-dependent delta interaction (DDDD). Indeed, as shown by Garrido et al [8] the pair-gap calculated using the Gogny force with parameter set D1S fitted directly to finite nuclei [9] follows rather closely the pair-gap calculated using the bare Paris-force with Brueckner–Hartree–Fock spectrum. The difference between the gaps increases with increasing density reflecting, most likely, an enhancement due to the averaging over the low-density skin region which is implicitly taken into account through the parameters of the effective Gogny interaction. In [8] it was demonstrated that DDDD is also flexible enough to follow the Paris-force (and Gogny) results, although only after careful adjustment of the cutoff parameter. A need for cutoff within the local pairing theory is neither satisfactory nor unambiguous and spoils to a large extent its simplicity. It is interesting to observe, however, that local interactions can be safely used in p-h channel (Skyrme force) without any artificial cutoff and that in fact in this channel it is equivalent, at least according to effective theory principles, to the finite-range Gogny interaction.

3. Toward local superfluid effective theory

A guiding principle underlying any effective theory aiming to describe the low-energy limit of a deeper, more fundamental theory can be formulated in the following way: the low-energy (infrared) phenomena are not sensitive enough to resolve high-energy (ultraviolet) dynamics. It means that short-range (high-momentum) dynamics can be removed from the theory and be replaced (renormalized) by a few local corrections. In momentum space one can therefore expand a short-range (SR) interaction as:

\[
v_{SR}(q^2) \approx g + g_2 q^2 + g_4 q^4 + \ldots,
\]

expressing it formally by means of a few constants \( g, g_2, g_4, \ldots \) which need to be carefully readjusted to a selected set of low-energy data. In coordinate representation interaction equation (1) can be rewritten as:

\[
v_{SR}(r) \approx c d_0^2 \delta_d(r) + d_1 a^4 \nabla^2 \delta_d(r) + d_2 a^4 \nabla \delta_d(r) \times \nabla + \ldots + h_1 a^4 r^2 \nabla^6 \delta_d(r) + \ldots,
\]

where \( \delta_d(r) \) denotes an arbitrary model of the Dirac delta function while \( c, d_1, d_2, h_1, \ldots \) denote empirically adjustable constants. In nuclear structure, due to the non-singular and very short range nature of the effective interaction, one can in fact use the strict limit \( \lim_{r \to 0} \delta_d(r) = \delta(r) \) which corresponds to the well-known Skyrme interaction (first three terms in equation (2)). On the other hand by retaining in equation (2) only the first term modelled by a sum of attractive and repulsive Gaussians of different ranges one obtains the well-known finite-range Gogny force. Of course both forces must be augmented by the density dependent and spin–orbit terms as well as by space-, spin-, and isospin-exchange terms. The Skyrme and Gogny forces are therefore two realizations (among infinitely many) of effective nuclear interactions which are used in practical nuclear structure calculations.

The Gogny force can be unambiguously used also in the p-p channel. Indeed, the finite-range \( r_{co} \sim 1 \text{ fm} \) automatically discriminates states above \( E_c \sim p_c^2/2m \sim h^2/mr_{co} \sim 40 \text{ MeV} \) \( (m_r = m/2 \text{ is reduced mass}) \) since \( r_{co} p_c \sim h \). The Gogny interaction was proved to be very successful in numerous practical applications. In particular, the average Hartree–Fock–Bogolyubov (HFB) pair-gaps calculated using D1S Gogny follows very accurately empirical three-point OES \( \Delta(N) \equiv (-1)^N [B(N-1) + B(N+1) - 2B(N)]/2 \) calculated for odd-\( N \), see [10] and figure 2.

Local delta-type or DDDD type pairing interactions are successfully used particularly in conjunction with Skyrme interaction. These applications explicitly invoke a cutoff parameter to avoid divergences. It appears, however, that the
divergence can be rather easily identified and subsequently regularized and a cutoff-free local superfluid theory can be constructed.

To illustrate a possible regularization schemes let us consider first the effective theory approach of Papenbrock and Bertsch [11]. In the contact-force approximation which takes into account only the first term in equation (1) the BCS gap equation takes the following form:

\[ 1 = -\frac{gV}{2(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{\sqrt{(E_k - \lambda)^2 + \Delta^2}}. \]  

(3)

where \( V \) and \( \lambda \) denote volume and chemical potential, respectively. The integral in equation (3) is ultraviolet (high-momentum) divergent. It appears, however, that the equation for scattering length, \( a \), is also divergent within the contact approximation:

\[ -\frac{mgV}{4\pi a} + 1 = -\frac{gV}{2(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{E_k}. \]  

(4)

and that the divergences are of the same type. Hence relation (4) can be used as a counter-term to regularize the gap equation:

\[ m = -\frac{1}{2(2\pi)^{\frac{3}{2}}} \int d^3k \left\{ \frac{1}{\sqrt{(E_k - \lambda)^2 + \Delta^2}} - \frac{1}{E_k} \right\}. \]  

(5)

This is a very elegant example of regularization connecting the pair-gap (and contact-force strength \( g \)) directly to the free two-particle scattering length. However, the formalism applies only to dilute homogeneous media. Finite range corrections (i.e. higher order expansion terms in equation (1)) are difficult to handle but can be, at least in principle, systematically implemented and regularized order by order. Judging from figure 1 the lowest order finite range corrections are expected to extend the validity of this scheme till \( k_F \sim 0.6 \text{ fm}^{-1} \).

The problem of ultraviolet type divergence in anomalous density matrix persist also in finite nuclei:

\[ v(r_1, r_2) = \sum_i v_i^*(r_1)u_i(r_2) \sim \frac{1}{|r_1 - r_2|}. \]  

(6)

Here, the situation seems to be even more complex since (i) realistic single-particle (sp) spectra must be used right from the beginning and (ii) it is not at all obvious what physical quantities need to be used in order to regularize divergent terms. The appropriate regularization scheme was proposed recently by Bulgac and Yu [12, 13]. Their scheme is built upon the local density approximation (LDA), i.e. takes automatically into account the dominant \( p \)-\( h \) channel. The idea is to introduce cutoff (\( E_c \equiv (\hbar k_F)^2/2m \)) dependent counter-terms leading to standard local HFB formalism with cutoff parameters but with a gap equation dependent on the effective running coupling constant

\[ v_c(r) = \sum_{E_i \geq 0} E_i v_i^*(r)u_i(r), \]  

(7)

\[ \Delta(r) = -g_{\text{eff}}(r)v_c(r), \]  

(8)

\[ \frac{1}{g_{\text{eff}}(r)} = \frac{1}{g[\rho(r)]} - \frac{m(r)k_c(r)}{2\pi^2\hbar^2} \times \left\{ 1 - \frac{k_S(r)}{k_c(r)} \right\} \ln \left\{ \frac{k_c(r) + k_S(r)}{2k_c(r) - k_S(r)} \right\}. \]  

(9)

Introducing a running coupling constant implies that the cutoff dependence is only formal and disappears for sufficiently large \( E_c \) [12, 13]. This cutoff free SLDA approach is now in phase of extensive tests [13, 14].

4. Entanglement

Pairing gaps are not directly accessible in experiment. Hence, various indirect methods must be applied to extract information about them. The major difficulty is that all these indirect methods entangle pairing with various effects including shape and shape-polarization effects, \( sp \) splitting, time-odd fields, or beyond mean-field residual interaction effects making life rather perplexing and, in fact, introducing in a natural way uncertainties into our knowledge of nuclear pairing. It is therefore desirable to hunt for simple physical situations or phenomena where at least some of these contaminants are either decoupled or can be relatively well controlled. Superdeformation (SD) is one of the most prominent examples of such a phenomenon. Indeed, stability of nuclear shape along the SD band allows one to study pairing correlations from the static to a dynamic regime. Let us recall that such concepts and techniques like double-stretched quadrupole pairing [15], the surface-active DDDI [16], the Lipkin–Nogami (LN) number-projection [17] were applied for the first time in a systematic way in SD bands in \( \text{Hg-Pb} \) nuclei [15, 18–20]. Afterwards these methods became standard in large-scale calculations in high-spin physics.

High-spin isomers (HSI) open yet another and so far unexplored venue to study pair correlations and blocking phenomena [21, 22]. Thanks to their structural simplicity both configuration and shape can be kept rather well under control. In turn, shape and pairing polarization due to blocking can be studied in detail and traditional average gap method [23] used to determine pairing strength, \( G_{\text{LN}} \), \( G_{\text{MN}} \). In particular, it was found that inclusion of these polarization effects requires \( \sim 10\% \) larger \( G \) as compared to \( G_{\text{MN}} \) [24] to reproduce experimental OES.

Recently the HSI have been systematically observed in \( N = 83 \) nuclei with \( 60 \leq Z \leq 67 \) [25]. This unique data set enables us to study for the first time OES both at the ground states (GS) as well as at high-spins. The most striking feature of this data set is the almost constant excitation energy of the HSI which implies that \( \Delta_{\text{GS}(Z)} \sim \Delta_{\text{HSI}(Z)} \), see figure 3. Conventional interpretation of this result in terms of pairing-gap suggests the absence of blocking phenomenon. The pair-gaps \( \Delta_{\text{LN}} - \lambda_2 \) calculated using a diabatic Strutinsky type method involving self-consistent blocking and LN particle-number projection [22, 24] shown in the lower part of figure 3 clearly show that this conventional interpretation is oversimplified. Contributions to OES from pairing (blocking), \( sp \)-proton energy splitting and residual proton-neutron (pn) interaction must all be taken into account to reproduce experimental data in a satisfactory way, see the upper part of
5. Hunting for fingerprints of isoscalar pn pairing collectivity

The existence of isoscalar \( t = 0 \) pairing phases in atomic nuclei is still a fascinating open issue. Various phenomena are discussed in this context including an onset of \( t = 0 \) pairing driven by nuclear rotation. However, neither the suggested shifts in ground-band S-band crossing frequency nor substantial changes in moments of inertia were convincingly confirmed by experiment. In contrast, standard calculations seem to work reasonably well, in particular in the most promising area of heavy deformed \( A \sim 80, N \sim Z \) nuclei [27, 28]. There are, however, new data indicating rather unusual band crossing phenomena in e.g. \(^{73}\text{Kr} [29].\) Indeed, standard mean-field calculations [29] can reproduce the \(^{73}\text{Kr}\) data assuming two different structures below (one-quasi-particle (QP) band built upon a negative parity Nilsson-level originating from \( \nu (f_{5/2}p_{3/2}) \) subshell) and above backbending where the odd-neutron occupies a positive-parity Nilsson state originating from the \( \nu g_{9/2} \) subshell while protons form 2QP structure involving \( \pi (f_{5/2}p_{3/2}) \otimes \pi g_{9/2} \). Strong E2 transitions connecting these structures (i) cannot be explained within standard mean-field calculations and (ii) indicate unusually strong configuration mixing. Whether or not this configuration mixing can be accounted for within the mean-field approximation invoking \( t = 0 \) pn-pairing, i.e. novel spontaneous symmetry breaking mechanism resulting in pn-mixing, is under study [30].

Figure 3. Experimental OES (dots, solid lines) versus calculated (triangles) OES (upper part) and calculated value of mean proton LN gap \( \Delta_{LN} - \lambda_2 \) (lower part). Filled (open) symbols denote GS (HSI) values, respectively. Grey triangles indicate theoretical GS values of OES corrected by residual pn interaction.

Figure 4. Calculated energy differences \( \Delta E_{exp} - \Delta E_{th} \) between terminating states, \( \Delta E = E(d_{7/2}f_{7/2}^{+1}) - E(f_{7/2}^{-1}) \) in \( A \sim 50 \) mass region relative to experimental data. Open symbols denote state of the art SM results (shifted by 475 keV) while filled symbols denote the SHF calculations using the SkO parameterization with spin-orbit term reduced by 5%. Grey dots denote the SHF results in \( N = Z \) nuclei corrected phenomenologically for isospin breaking effect, see insert and [26] for more details.

Also calculated energy differences \( E = E(d_{7/2}f_{7/2}^{+1}) - E(f_{7/2}^{-1}) \) between band-terminating states in \( A \sim 50 \) seem to provide new evidence for \( t = 0 \) pairing [26]. As shown in figure 4 the values of \( \Delta E_{exp} - \Delta E_{th} \) calculated using a state of the art shell-model (SM) and Skyrme–Hartree–Fock (SHF) method follow, up to a constant offset of \( \sim 475 \) keV, each other very closely. The quantitative difference between the accuracy of theoretical predictions in \( N \neq Z \) as compared to \( N = Z \) nuclei seen both in the SM as well as in the SHF calculations suggests, most likely, an enhanced \( t = 0 \) pair scattering from sd to fp shell which is beyond the present SM space. A similar effect was suggested in [31].

One of the most promising signals of \( t = 0 \) pairing comes from binding energies in \( N = Z \) nuclei, where the problem of the Wigner energy (WE) (extra binding energy) is known to plague mean-field masses, see [32] and references therein. It is relatively well established from nuclear SM studies that the WE is predominantly due to \( \delta \nu = \delta Z \) isoscalar pn-pairs. Strong

\[
\Delta (Z) = E(d_{7/2}f_{7/2}^{+1}) - E(f_{7/2}^{-1})
\]

isoscalar \( \delta \nu \) pairing in terms of symmetry conserving pairing interaction consisting only of \( L = 0, S = 1, T = 0 \) [35] or by \( J = 1, T = 0 \) [34] isoscalar pn-pairs. Within the mean-field model, which is entirely based upon the concept of spontaneous symmetry breaking, the definition of \( t = 0 \) pairing in terms of symmetry conserving pairing interaction consisting only of \( L = 0, S = 1, T = 0 \) (or \( J = 1, T = 0 \)) pairs is not at all justified. Although the form of effective \( t = 0 \) pn-pairing interaction appropriate for mean-field calculations is an open issue it is rather well established qualitatively that the mean-field model augmented by \( t = 0 \) pairing is capable to heal the mass-defect problem around the \( N \sim Z \) line [36–38]. A quantitative estimate of \( t = 0 \) pairing within mean-field requires, however, a reliable evaluation of corrections due to isospin-symmetry restoration which go beyond mean-field. Within the random-phase-approximation these corrections roughly restore the linear term in the nuclear
symmetry energy (NSE) giving rise to $\sim T(T+1)$ dependence of the NSE [39]. It appears however that at least part of the linear term ($\sim T$) is incorporated already at the level of the (self-consistent) mean-field. Hence, further progress in the field is impossible without thorough understanding of the NSE, currently under intense study [40, 41–43].

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