Intensification of heat transfer in chaotic modes

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Abstract. This paper presents the results of numerical modeling and comparative studies of the effectiveness of diffusion, convective, and turbulent heat transfer modes in a liquid layer. Here the heat transfer physical model was used which is typical for reactors with an internal energy source. Conditions for the continuity of the fluid flow and the equations of heat transfer, analytical and simulation models of these modes are obtained based on the Navier-Stokes equation. Quantitative characteristics have been found using the simulation model, confirming the expected efficiency indicators ratios of these processes in the course of heat transfer. Convective and turbulent heat transfer processes properties are analyzed as control objects basing on the Comsol Multiphysics modeling system. Recommendations have been obtained concerning the multi-alternative algorithms synthesis for automatic control of these processes. In particular, it is advisable to use the heat flux density as a control action, whereas the liquid particle velocity dispersion should be used as an adjustable value. Corresponding control system model demonstrated the efficiency of the proposed algorithms.

1. Introduction
Widespread use of the heat transfer process in modern industrial technologies involves the continuous improvement of its efficiency indicators [1-5]. Most publications describing the activity in this direction are devoted mainly to the constructive changes in the heat transfer devices that give a certain growth of heat transfer coefficient and heat flux density [6-11]. At the same time, the significant and as yet insufficiently studied possibilities for improving these characteristics lie in the implementation of automatic heat transfer control which provides the required process quality indicators under the unsteady operating conditions.

The results of studies accumulated by now in hydromechanics indicate the possibility to increase heat transfer efficiency by means of internal turbulization occurring in the liquid layer under certain conditions. It is established that such a condition is raising the temperature gradient [12,13]. A small temperature gradients heat transfer occurs due to the motion of the liquid particles at the molecular level, i.e., the diffusion. As far as the gradient increases, the convective thermal waves arise involving significant volumes of liquid. With further temperature gradient growth, convective waves collapse causing the disordered, chaotic fluid motion which can be identified as turbulence.

Thus, the heat exchange process is characterized by the possibility of occurrence of qualitatively different, critical internal operating modes and respectively the goal can be set to control these modes using the automation.

This article describes the results of a quantitative study of the critical modes’ emergence pattern basing on the corresponding mathematical and simulation models, as well as a corresponding control system for them.

2. Physical and mathematical models of the heat transfer process
A horizontal liquid layer is offered as a specific physical heat transfer scheme, with layer height \( h = 5 \times 10^{-2} \) m, heated from below by a heat source with heat flux density \( q \). Solid wall with thickness \( \delta_w = 3 \times 10^{-3} \) m and heat transfer coefficient \( \alpha = 500 \) W/(m\(^2\)-K) is used as heat sink to the external environment with a constant temperature \( T_{ex} = 20 \) °C, as shown in figure 1; \( \lambda_w = 100 \) W/(m-K) is the wall thermal conductivity coefficient. This model is surely the best possible variant describing the heat exchangers with an internal energy source (reactors, electric heaters).
For mathematical description of a fluid motion and heat transfer in this physical model, the author used the Navier-Stokes equation [14-17]:

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{g},
\]

the fluid flow continuity equation:

\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0;
\]

and heat transfer equation:

\[
\frac{\partial T}{\partial t} + \nabla (T \mathbf{v}) = k \nabla^2 T,
\]

where: \( \mathbf{v}(x,y,z,t), p(x,y,z,t), \rho(x,y,z,t), T(x,y,z,t) \) - velocity, pressure, density and temperature fields in the liquid layer varying in the space \( xyz \) and in time \( t \); \( g \) - acceleration of gravity in the gravitational field, \( g = 9.8 \text{ m/sec}^2 \); \( k \) - thermal diffusivity of liquid, \( k = 5.8 \times 10^{-8} \text{ m}^2/\text{sec} \); \( \nu \) - kinematic viscosity, \( \nu = 2.6 \times 10^{-6} \text{ m}^2/\text{sec} \).

Linear density/temperature dependence is adopted in this model: \( \rho = \rho_0 - \rho_0 \gamma (T - T_0) \), where: \( \gamma \) - the temperature coefficient of volume expansion, \( \gamma = 7.3 \times 10^{-4} \text{ 1/K} \); \( T_0 \) - the temperature for which the reference value of density \( \rho_0 \) was determined: for \( T_0 = 100 \text{ °C} \), \( \rho_0 = 832 \text{ kg/m}^3 \) was chosen.

Boundary conditions correspond to a horizontal liquid layer enclosed between the side walls. The walls are either thermally insulated for modeling the diffusion mode or isothermal when heat transfer simulation in a moving fluid is made. All velocity components on the wetted surfaces of the side walls are equal to zero.

Here the two-dimensional model is adopted: horizontal coordinate \( x = [0; 0.3] \text{ m} \), vertical \( y = [0; 0.05] \text{ m} \); any changes in velocity, pressure, density, and temperature along the \( z \) coordinate are absent.

### 3. Critical modes study

The resulting mathematical description of the heat transfer process was used to build its simulation model in the Comsol Multiphysics package. Simulation results allowed us to establish that there are three possible modes of heat transfer in the considered system, viz., the diffusion, convective and turbulent modes.

Figure 2 shows the distribution of the temperature field in the diffusion mode.

![Figure 2. The temperature field in diffusion mode.](image1)

This mode occurs at very small temperature gradients and its practical value is small, however, it is considered here with the purpose to compare heat transfer quality indicators in all three critical modes.

In the convective mode the occurrence of alternating ascending and descending fluid flows is observed along with the diffusion heat transfer process. These flows generate stable convective waves (figure 3) which provide, in comparison with the diffusion mode, a significant increase in the intensity of mass and heat transfer.
The presence in the convection mode of zones with an alternating direction of fluid circulation leads to a corresponding periodicity of the temperature field with a repeated increase and decrease of temperature relative to its average value along two coordinate axes. In figure 4 the areas with higher temperatures correspond to shaded areas with closely spaced isotherms.

Figure 4. Temperature field in convection mode.

Figure 5 shows the velocity changes during the transient process (about 4000 sec) and after its completion at a point on the \( x0y \) plane with the coordinates \( x = 0.04 \) m and \( y = 0.005 \) m. Steady state convection is characterized by the stationary value of fluid particle velocity at each point.

As soon as the heat flux density \( q \) increases, the lower wall temperature increases, the unevenness of the temperature field increases, and convective waves gradually collapse. Fluid flow becomes turbulent, as shown in figure 7.

Temperature field unevenness becomes chaotic in the turbulent mode [18-22] which leads, in comparison with the convection mode, to the spatial alignment of time-average temperature values (figure 8), and helps to gain heat exchangers operational reliability.
The turbulent flow is characterized by chaotic pulsations of the particle velocity which provide the rapid transfer of the momentum and heat, as in figure 6.

An average fluid particles velocity in the process of turbulent heat transfer is several times higher than the convective transfer velocity; whereas the amplitude of the velocity pulsations exceeds significantly the range of molecular motions (see figures 5 and 6). Thus, one should expect an increase in the efficiency of turbulent heat transfer in comparison with the convective mode.

Due to the continuity of the heat flux, we can write:

\[ q = r(T_1 - T_2) \]  for fluid layer;
\[ q = \frac{\lambda_w}{\delta_w}(T_2 - T_3) \]  for upper wall;
\[ q = \alpha(T_3 - T_{ex}) \]  for the boundary with the environment.

In these expressions, \( T_1, T_2 \) and \( T_3 \) are the \( x \)-coordinates of the temperature of the surface heated from below and the lower and upper surfaces of the wall, respectively, in accordance with figure 1; where \( r \) is the heat transfer coefficient of the liquid layer.

Using these conditions for all simulated heat transfer modes, the expression was obtained:

\[ r = \frac{T_2 - T_{ex}}{T_1 - T_2} \left( \frac{\delta_w + 1}{\lambda_w} \right) \]

and the heat transfer coefficient values shown in table 1 were calculated.

| Heat transfer mode | \( T_1 \) (°C) | \( T_2 \) (°C) | \( q \) (W/m²) | \( r \) (W/(m²·K)) |
|--------------------|----------------|----------------|--------------|----------------|
| Diffusion          | 116            | 20.4           | 200          | 2.06           |
| Convection         | 35             | 20.4           | 200          | 13.49          |
| Turbulence         | 150            | 24             | 2000         | 15.73          |

Simulation results reflect well the physical features of the considered modes and confirm the expected gain in the heat transfer efficiency in a chaotic mode.

4. Heat exchange process control system

The analysis of critical modes of the heat transfer process makes it possible to draw the following conclusions regarding the possible structure of its control system:
the goal of control contains two practically significant alternatives, namely: maintaining the either convective or turbulent mode of heat transfer in the system, i.e. one of two critical modes;
it is advisable to choose the heat flux density \( q \) as a control action;
the distinctive feature by which one can recognize the convective or turbulent mode of heat transfer is the amplitude of pulsations of the fluid particles velocity. Therefore, variance \( \sigma^2(t) \) of this velocity can be taken as a controlled quantity;
the most significant disturbing influences are the ambient temperature \( T_{ex} \) and the heat transfer coefficient \( \alpha \) of the apparatus walls both of which inevitably change during its operation.
Thus, for the implementation of multi alternative control, a control system with a variable structure should be synthesized containing two closed loops for the deviation of the controlled variable and providing for their switching to select the heat transfer mode [23-27].

Figure 9 shows a variant of the proposed control system structure. In this version, a heat flux with some calculated nominal density \(q_n\) is supplied to the system input and the instantly calculated dispersion \(\sigma^2(t)\) of the velocity \(w(t)\) is supplied to the of nonlinear controllers inputs (any component \(v(x,y,z,t)\) or its module can be selected as \(w(t)\)).

Static characteristic of controller \(R_t\) (the upper one in figure 9) provides a growing heat flux density at small \(\sigma^2(t)\) values, i.e., this regulator is designed to maintain turbulent heat transfer. Controller \(R_c\) decreases the nominal value of density \(q_n\) for large \(\sigma^2(t)\), i.e. providing the maintenance of the convective heat transfer mode.

Figure 9. Heat exchange control system structural diagram.

The linear part of the regulators is common for both modes being an integrating link which provides the property of astatism in the system with respect to the disturbances \(T_{ex}(t)\) and \(\alpha(t)\).

The ability of the system to fend off disturbances and control heat transfer modes is illustrated in figure 10 which shows the change in the velocity of the fluid particles at the selected sensor installation point \((x = 0.06\ m, y = 0.005\ m)\) in the time intervals with different modes listed in table 2.

**Table 2.** Situations in which the system performance was checked.

| Time interval (sec) | \(T_{ex}\) (°C) | \(\alpha\) (W/m²-K) | \(q\) (W/m²) | System condition | Heat transfer mode |
|---------------------|----------------|-------------------|-------------|-----------------|------------------|
| 0-4000              | 20             | 500               | 1000        | Open            | Turbulent        |
| 4000-17000          | 50             | 250               | 1000        | Open            | Convective       |
| 17000-20000         | 50             | 250               | 2000        | Closed          | Turbulent        |

Attention should be paid to the expectedly noticeable inertia of the system which is about 1000 sec for this object.

5. Conclusion
Heat transfer intensification task which is traditionally solved by means of structural changes in technological equipment can be solved successfully on the basis of the liquid coolant critical states process control. Such conditions include in particular the internal turbulization which occurs in the liquid layer under certain conditions.
The results of a comparative analysis of convective and turbulent heat transfer modes allowed to establish that:
the quantitative criterion which makes it possible to distinguish between the indicated modes is, the absence or presence, respectively, of chaotic pulsations of fluid particles velocity. In this case, the amplitude of the velocity pulsations significantly exceeds the range of the molecular motions of liquid, i.e. the specified criterion has high selective properties;
it is advisable to use the density of the heat flux supplied to the heat exchanger from the heat source as a control action, providing stabilization of the selected condition of the coolant.

Controlled maintenance of the required heat transfer mode can be successfully performed on the basis of multi alternative control. For the task discussed above such control is implemented in the form of a system having a variable structure.

6. References
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