Magnetic field and Rotation effect on thermal stability in an Anisotropic couple-stress fluid saturated Porous Layer under presence of Cross-Diffusion

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Abstract

The present paper is based on the investigation of thermal stability problem under the influence of rotation, solute, heat and magnetic field on electrically conducting fluid saturated rotating horizontal porous media under local thermal non-equilibrium effect. We have considered a rotating horizontal anisotropic porous medium saturated with binary fluid. Process of Heating and salting is made from above to the fluid layer. Governing equations is followed by Darcy Brinkman model for characterizing the convection through porous media under the above said physical configuration. Normal mode technique and Application of Linear stability analysis has been made under the consideration of free-free boundary conditions. Thereafter the characteristic graphs between critical Darcy-Rayleigh number and corresponding wave number for beginning of convection in stationary case has been drawn and the effect of rotation, magnetic and couple stress parameters has been illustrated. At last it has been observed the non-dimensionalise parametres regarding significant components like rotation magnetic field and anisotropic properties of the physical configuration in present study has stabilizing effect.

Keywords: Thermal non-equilibrium, Rotation, Darcy-Rayleigh Number, Finger-ing instability, Cross-diffusion
Nomenclature

Latin Symbols

\( c \)  
Specific heat capacity

\( C \)  
Concentration

\( d \)  
Length of the porous layer

\( C_1 = \frac{\mu_c}{\rho d^2} \)  
coupled stress parameter

\( \Omega \)  
Vertical component of angular velocity

\( T_a = \left( \frac{2\Omega K_a}{\nu} \right)^2 \)  
Length of the porous layer

\( D_m \)  
Solute diffusivity

\( D_f \)  
DuFour coefficient

\( D_r \)  
Soret coefficient

\( D_u \)  
DuFour parameter, \( D_u = \frac{D_f \beta_T}{\kappa_f z \beta_C} \)

\( \ddot{g} \)  
Acceleration due to gravity

\( h_s \)  
Inter-phase heat transfer coefficient

\( H \)  
Non-dimensional inter-phase heat transfer coefficient, \( H = \frac{h_z d^2}{\kappa_f z} \)

\( h_z \)  
Perturbed magnetic field along z-axis

\( \tau_z \)  
Perturbed magnetic field vorticity along z-axis

\( \Gamma_z \)  
Perturbed vorticity along z-axis

\( K \)  
Permeability of porous layer

\( K_x, K_y, K_z \)  
characteristic permeabilities in the x, y and z directions

\( \text{Le} \)  
Lewis number, \( \text{Le} = \frac{\kappa_f z}{D_m} \)

\( p \)  
Reduced pressure

\( Pm \)  
magnetic Prandtl number, \( Pm = \frac{\Lambda}{\kappa_f z} \)

\( Q \)  
Darcy-Chandersekhar number, \( Q = \frac{\mu_m d^2}{\kappa_f z} \frac{\beta_T}{\beta_C} \frac{\Delta T}{K_z} \frac{d}{\kappa_f z} \)

\( R \)  
Darcy-Rayleigh number, \( R = \frac{\rho g \beta_T \Delta T K_z d}{\mu_f z} \frac{\kappa_f z}{\kappa_f z} \)

\( R_c \)  
Critical Rayleigh number, \( R_c \)

\( R_s \)  
Solutal Rayleigh number, \( R_s = \frac{\rho g \beta_C \Delta C K_z d}{\mu_f z} \frac{\kappa_f z}{\kappa_f z} \)

\( a \)  
Horizontal wave number

\( l, m \)  
x-component and y-component of wavenumber

\( \text{Sr} \)  
Soret parameter, \( \text{Sr} = \frac{D_u \beta_c}{D_m \beta_T} \)

\( T_f \)  
Temperature of fluid

\( T_s \)  
Temperature of solid

\( \Delta T \)  
Temperature difference across the porous layer

\( T_l \)  
Temperature of lower surface

\( T_u \)  
Temperature of upper surface

\( t \)  
Time

\( \ddot{q} \)  
Velocity

\( x, y, z \)  
Space Co-ordinates
Greek symbols

κ       Thermal diffusivity
κ₁      Thermal diffusivity for fluid
κ₂      Thermal diffusivity for solid
βₐ      Coefficient of concentration expansion
βₜ      Coefficient of thermal expansion
Λ       Magnetic viscosity
η₁      Fluid thermal conductivity ratio \( (= \frac{κ₁x}{κ₁z} = \frac{κ₁y}{κ₁z}) \)
η₂      Solid thermal conductivity ratio \( (= \frac{κ₂x}{κ₂z} = \frac{κ₂y}{κ₂z}) \)
ε       Porosity
µ       Dynamic viscosity of the fluid
µₚ      Magnetic permeability
ν       Kinematic viscosity
ρ       Density of fluid
ρ₀      Reference density
γ       Porosity modified conductivity ratio, \( \gamma = \frac{εκ₁z}{(1-ε)κ₂z} \)
χ       Diffusivity ratio, \( \chi = \frac{(ρc₁x)κ₁z}{(ρc₁z)κ₂z} \)
ξ       Anisotropy ratio \( (= \frac{K_x}{K_z}, = \frac{K_y}{K_z}) \)
Θ       Amplitude of fluid temperature perturbation
Ψ       Amplitude of solid temperature perturbation
W       Amplitude of fluid concentration

Other symbols

\( D \)       \( d/dz \)
\( \nabla^2 \) \( \frac{∂^2}{∂x^2} + \frac{∂^2}{∂y^2} \)

Subscripts

b       basic state
c       Critical
f       Fluid
s       Solid

Superscripts

'       Perturbed quantity
*       Dimensionless quantity
1 Introduction

Convection through fluid-saturated porous media due to effect of heating and salting simultaneously (called double diffusive convection) has been a very interesting physical phenomena and is of practical importance in many branches of science and technologies viz, Geophysics, applied chemistry and physics and has been discussed in Nield and Bejan [20]. Double diffusive convection through porous media was first studied by Nield [7]. Malashetty et al [16] has studied stability analysis of double diffusive convection when temperature and solute concentration are independent factors of diffusion. But in some cases where these two diffusive components interact each other to give rise cross diffusion the studies has been made by Rudraih et al [22], Rudraih, Malashetty [23], Malashetty Gaikwad [17] under different physical configurations. Another interesting mechanical situation where fluid temperature and pore temperatures differs to interact significantly has been studied by Rees et al [4], Govender and Vadas [12]. Apart from above convection through porous media becomes more interesting in the presence of magnetic field when the fluid considered is taken as electrically conducting the studied regarding this phenomena has been done by many researchers viz, Chandrashekhar [2], Rudraih [22], Bhadauria et al [6],[8]. An unusual double diffusive convection arise due to heating and salting from above giving rise to finger type convective pattern. The realistic situation can be seen when the sun light falls over the surface of ocean. Sarvanan et al [25] have considered the above said mechanical situation in his study of double diffusive convection through porous media under thermal non-equilibrium model. Further Srivastava, Bhadauria [28] extended the work of Sarvanan [25] by taking anisotropic porous media and application of magnetic field. Another interesting force component for convection called Coriolis force thereby arising due to rotating porous layer and that significantly affects the stability criteria has been disused by many authors, Vadas et. al[14], Govender [13], Desaive et al [9],[10], Malasheetty, Raja shekhar Heera [16]. Apart from above said works, some significant finding regarding MHD flow have been done by R.P. Sharma et.al [30],[32] under various Physical configuration. There is very less literature in the study rotation effect on double diffusive convection through porous media with cross diffusion effect under LTNE case. Moreover anisotropic couple stress fluid saturated porous layer is very important physical configuration and very less attention has been done regarding this study. So In the present investigation We have considered the electrically conducting couple stress fluid saturated with rotating porous layer to study the effect of applied magnetic field and
Coriolis force on the starting criteria of double diffusive convection through anisotropic porous media with cross diffusion under LTNE case. So, study in the present physical configuration is more general than as done by A.K. Srivastava[28]. We have structured the paper in the following way. In section 2 equations in view of the said physical configurations has been perturbed and then reduced to non dimensional form. Further Linear stability analysis has been carried out in section 3. In the Results and discussion section and effect of various parameters has been illustrated through graphs.

2 Governing Equations

An anisotropic porous layer has been taken with height d from z=0 and z=d, and the whole system is being rotated with constant angular velocity about z axis. The difference of temperature and concentration between lower and upper layers are δT and δS. Applied magnetic field is along positive z direction. According to Darcy Brinkman for couple stress fluid the governing equation under the said conditions with considerations of Boussinesq approximation following the notations (See[25],[28]) are as follows

\[ \nabla \cdot \vec{q} = 0 \]  

(1)

\[
\frac{1}{\varepsilon} \frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2} (\vec{q} \cdot \nabla) \vec{q} + \frac{1}{\rho_0} (\mu - \nabla^2 \mu_c) K \vec{q} + \frac{2}{\varepsilon} \Omega \times \vec{q} = - \frac{1}{\rho_0} \nabla P + \frac{\rho_f}{\rho_0} \vec{q} + \frac{\mu_m}{\rho_0} \left( \vec{H} \cdot \nabla \right) \vec{H} 
\]

(2)

\[
\epsilon (\rho_f \xi_f) \frac{\partial T_f}{\partial t} + \rho_f \xi_f (\vec{q} \cdot \nabla) T_f = \nabla (\epsilon K_f \nabla T_f) + D_f \nabla^2 C + h_s (T_s - T_f) 
\]

(3)

\[
(1 - \epsilon) \left( \rho_c \xi_s \right) \frac{\partial T_s}{\partial t} = \nabla \left( (1 - \epsilon) K_s \nabla T_s \right) - h_s (T_s - T_f) 
\]

(4)

\[
\epsilon \frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = \epsilon D_m \nabla^2 C + \epsilon D_T \nabla^2 T_f 
\]

(5)

\[
\frac{\partial \vec{H}}{\partial t} + \vec{q} \cdot \nabla \vec{H} - \vec{H} \cdot \nabla \vec{q} = \eta \nabla^2 \vec{H} 
\]

(6)

The constants and variables in the above equations have their usual meanings and are given in the nomenclature. The relation between the reference density and temperature is given by

\[
\rho_f = \rho = \rho_0 \left[ 1 - \beta_T (T_u - T_f) + \beta_C (C_u - C_f) \right] 
\]

(7)
where $\beta_T > 0$ and $\beta_C > 0$ are the coefficient of thermal and concentration expansions, respectively. The boundary conditions are

$$T_f = T_s = T_i, \ C = C_i \text{ at } z = 0 \text{ and } T_f = T_s = T_u, \ C = C_u \text{ at } z = d$$  \hspace{1cm} (8)

### 3 Linear Stability analysis

To get the criteria for the onset of convection, we use linear stability analysis. The basic state is perturbed as follows

$$[\vec{q},T_f,T_s,C,\vec{H}] = [\vec{q}_b,T_{fb},T_{sb},C_b,\vec{H}_b] + [\vec{q}',\Theta,\Phi,\Psi,\vec{H}']$$ \hspace{1cm} (9)

where the basic state is assumed to be quisent as given by

$$\vec{q}_b = (0,0,0), T_{fb} = T_{sb} = C_b = z$$ \hspace{1cm} (10)

Using (9) and (10) into the Eqs. (1)-(6) linearizing them, omitting primes, eliminating the pressure term by applying curl once, twice, taking the vertical component, thereafter performing non-dimensionalization of the equations using following scaling parameters, distance by $d$, velocity by $\frac{\epsilon \kappa f z}{(\rho f c f) d}$, pressure by $\kappa f z \mu (\rho f c f) K z$, C by $\Delta C$, time by $\frac{d^2}{(\rho f c f) \kappa f z}$, $T_f$ by $\Delta T$, $T_s$ by $\Delta T$, magnetic field by $H_B$.

We get the linearised format of equations as follows

$$[\left\{\frac{1}{V_a} \frac{\partial \nabla^2}{\partial t} + \left(\nabla^2_i + \frac{1}{\xi} \frac{\partial}{\partial z^2}\right)(1 - C_i \nabla^2)\right\} W + T_i^2 \frac{\partial \Gamma_z}{\partial z} + R_T \nabla_i^2 \Theta - R_s \nabla_i^2 \Phi - Q_m \nabla^2 D h_z] = 0$$ \hspace{1cm} (11)

$$W + \left(\frac{\partial}{\partial t} - \eta_f \nabla^2_i - D^2 + H\right) \Theta - H \Phi - Du \frac{R_s}{R_T} \left(\nabla^2_i + D^2\right) \Psi = 0$$ \hspace{1cm} (12)

$$-\gamma H \Theta + \left(\chi \frac{\partial}{\partial t} - \eta_s \nabla^2_i - D^2 + \gamma H\right) \Phi = 0$$ \hspace{1cm} (13)

and

$$W - \frac{St R_T}{R_S L e} \left(\nabla^2_i + D^2\right) \Theta + \left(\frac{\partial}{\partial t} - \frac{1}{Le} \left(\nabla^2_i + D^2\right)\right) \Psi = 0$$ \hspace{1cm} (14)

$$\left(\frac{1}{V_a} \frac{\partial}{\partial t} + \frac{1}{\xi} (1 - C_i \nabla^2)\right) \Gamma_z - Q_m \frac{\partial \tau_z}{\partial z} = T_i^2 \frac{\partial w}{\partial z}$$ \hspace{1cm} (15)

$$\frac{\partial \tau_z}{\partial t} - P_m \nabla^2 \tau_z = \frac{\partial \Gamma_z}{\partial z}$$ \hspace{1cm} (16)

$$\frac{\partial h_z}{\partial t} - P_m \nabla^2 h_z = \frac{\partial w}{\partial z}$$ \hspace{1cm} (17)
The corresponding boundary conditions reduces to

\[ W = 0 \text{ at } z = 0, 1 \text{ and } \Theta = \Psi = 0 \text{ at } z = 0, 1 \] (18)

where the symbols and non dimensional parameters used are listed in the nomenclature.

To solve the system of Eqs. (11) – (17), we apply normal mode technique, thus express the perturbed variables as given below

\[ \begin{bmatrix} W, \Theta, \Phi, \Psi, \Gamma, \Omega, h \end{bmatrix} = \begin{bmatrix} A_1 f_1, A_2 f_2, A_3 f_3, A_4 f_4, A_5 f_5, A_6 f_6, A_7 f_7 \end{bmatrix} e^{i(lx+my+\sigma t)} \sin \pi z \] (19)

where \( A_1, A_2, A_3, A_4, A_5, A_6, A_7 \) are constants and \( f_1, f_2, f_3, f_4, f_5, f_6, f_7 \) are functions of \( z \) alone which in view of the boundary conditions can be assumed as

\[ f_1(z) = f_2(z) = f_3(z) = f_4(z) = f_5(z) = \sin \pi z, f_6(z) = f_7(z) = \cos \pi z \] (20)

and \( l, m \) are wave numbers in \( x, y \) directions respectively. We Substitute Eq. (19) and (20) into Eqs. (11) – (17) and get the matrix equation \( XA = 0 \) where for a non-trivial solution we have \( \lvert X \rvert \neq 0 \)

\[
\begin{vmatrix}
(a^2 + \frac{1}{2}\sigma^2) (1 + C_1 (a^2 + \sigma^2)) & a^2 R_0 & 0 & -a^2 R_0 & 0 & 0 & Q \rho_m (a^2 + \sigma^2) \\ 1 & n f a^2 + \sigma^2 + \Gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & a^2 R_0 & 0 & 0 & 0 & 0 & 0 \\ 1 & n f a^2 + \sigma^2 + \Gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & R_0 (\sigma^2 + 2\sigma^2) & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\tau (T_a)^{1/2} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\tau & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{vmatrix} = 0.
\]

The above determinant equation when expressed gives the functional relation between Raleigh number and wave number for various parameters.

### 3.1 Stationary Case

We now study the steady case, in this case we must have \( \sigma = 0 \) at the neutral stability.

for limited case when \( T_a = 0, C_1 = 0 \) the result is same as obtained by A. K Srivastava et al (2015) for Darcy model. For \( T_a = 0, Q = 0, \xi = 1, C_1 = 0 \), the value of Rayleigh number in terms of wave number obtained by solving the determinant equation 21 coincides with that of Saravanan and Jegajothi (2010).
3.2 Result and discussion

Effect of Couple stress, magnetic filed and rotation parameter has been discussed. Neutral stability curves between Raleigh number and wave number for various parameters has been given in Fig 1, Fig2 and Fig 3.

In Fig 1 to visualize the effect of magnetic on onset of convection three neutral stability curves for $Q = 50, Q = 500, Q = 1000$ has been drawn for different values of rest parameters. It can be easily observed that by increasing Darcy Chandrasekhar number Neutral stability curves shifted upward which implies magnetic field is a stabilizing factor for the onset of convection.

In Fig 2 to visualize the effect of rotation on onset of convection three neutral stability curves for $Ta = 100, Ta = 10000, Ta = 40000$ has been drawn for different values of rest parameters. It can be easily observed that by increasing Taylor number Neutral stability curves shifted upward which implies rotation is a stabilizing factor for the onset of convection.

In Fig 3 to visualize the effect of couple stress on onset of convection three neutral stability curves for $C_1 = 1, C_1 = 2, C_1 = 3$ has been drawn for different values of rest parameters. It can be easily observed that by increasing couple stress parameter Neutral stability curves shifted upward which implies couple stress is a stabilizing factor for the onset of convection.
Figure 1: effect of magnetic field

Figure 2: effect of rotation parameter

Figure 3: effect of couple stress parameter
3.3 Conclusion

Double diffusive convection through Rotating fluid saturated anisotropic porous layer with LTNE case under the presence of magnetic field has been investigated through consideration of darcy Brinkman model for convection in porous media. After observing the graphs following facts has been pointed out.

(i) Effect of rotation is to stabilize the system under consideration.
(ii) Effect of magnetic field is to stabilize the system under consideration.
(iii) Effect of couple stress is to stabilize the system under consideration.

References

[1] Alchaar, S., Vasseur, P. and Bilgen, E., 1995a Effect of a magnetic field on the onset of convection in a porous medium, Heat Mass Trans., 30, pp. 259-267.

[2] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Dover, Inc, New York, 1981.

[3] Alchaar, S., Vasseur, P. and Bilgen, E., 1995b Hydromagnetic natural convection in a tilted rectangular porous enclosure, Numer. Heat Trans., 27(A), pp. 107-127.

[4] Banu, N. and Rees, D.A.S., 2002 Onset of Darcy-Benard convection using a thermal non-equilibrium modal, Int. J. Heat Mass Trans., 45, pp. 2221-2228.

[5] Bhadauria, B.S., 2008 Combined effect of temperature modulation and magnetic field on the onset of convection in an electrically conducting-fluid-saturated porous medium, ASME J. Heat Trans., 130, pp. 1-9(052601).

[6] Bhadauria, B.S. and Sherani, A., 2008 Onset of Darcy-convection in a magnetic-fluid saturated porous medium subject to temperature modulation of the boundaries, Trans. in Porous Media, 73, pp. 107-127.

[7] Nield, D A., 1968 Onset of thermohaline convection porous medium water Resour. Res. 4, 553-560, Trans. in Porous Media, 73, pp. 107-127.

[8] Bhadauria, B.S. and Srivastava, A.K., 2010 Magneto-double diffusive convection in an electrically conducting-fluid-saturated porous medium with temperature modulation of the boundaries, Int. J. Heat and Mass Trans., 53, pp. 2530-2538.
[9] Desaive, T., Hennenberg, M. and Lebon, G., 2002 Therma instabilities of a rotating saturated porous medium heated from below and submitted to rotation, The European Physical Journal B, 29, pp. 641-647.

[10] Desaive, T., Hennenberg, M. and Dauby, P.C., 2004 Satbilite thermomagneto-convective d’un ferro-fluide dans une couche poreuse en rotation, Mecanique and Industries, 5, pp. 621-626.

[11] Epherre, J.F., 1975 Critere d’ apparition de la convection naturelle dans une couche poreuse anisotrope, Revue. Gen. Thermique, 168, pp. 949-950.

[12] Govender, S. and Vadasz, P., 2007 The effect of mechanical and thermal anisotropy on the stability of gravity driven convection in rotating porous media in the presence of thermal non-equilibrium, Trans. in Porous Media, 69, pp. 55-66.

[13] Govender, 2003 Coriolis effect on the linear stability of convection in porous layer placed for away from the axis of rotation, Trans. in Porous Media, 51, pp. 315-326.

[14] Vadasz, 1998 Coriolis effect on gravity-driven convection in a rotating porous layer heated from below, J. Fluid Mech., 276 pp. 351–375.

[15] Malashetty, M.S., Shivakumara, I.S. and Sridhar, K., 2005a The onset of Lapwood-Brinkman convection using a thermal non-equilibrium model, Int. J. heat mass Trans., 48, pp. 1155-1163.

[16] Malashetty, M.S., Rajshekhra, Heera., 2008 Effect of rotation on the onset of double diffusive convection in horizontal anisotropic porous layer, Trans. in Porous Media, 74, pp. 105-127.

[17] Gaikwad, S. N, Malashetty, M.S., Prasad, KR 2007 An analytical study of linear and nonlinear double diffusive convection with sorre and dufour effect in couple stress fluid, International Journal of Non-linear Mechanics, 427, pp. 903-913.

[18] Malashetty, M.S., Sridhar, K. and Swamy, M., 2007 Thermal convection in a rotating porous layer using a thermal non-equilibrium model, Phys. Fluids, 19(5), pp. 1-16(054102).

[19] Malashetty, M.S. and Sridhar, K., 2009 The convective instability of Maxwell fluid-saturated porous layer using a thermal non-equilibrium model, J. Non-Newtonian Fluid Mech., 162, pp. 29-37.
[20] Nield, D.A. and Bejan, A., 2006 Convection in Porous Media, New York: Springer-Verlag.

[21] Rees, D.A.S. and Pop, I., 2005 Local thermal non-equilibrium in porous medium convection, Trans. Phenomena in porous media, 3, pp. 147-173.

[22] Rudraiah, N., 1984 Linear and non-linear magnetoconvection in a porous medium, Proc. Indian Acad. Sci.(Math. Sci.), 93, pp. 117-135.

[23] Rudraiah, N. and Malashetty, M.S., 1986 The influence of coupled molecular diffusion on the double diffusive convection in porous medium, ASME J. Heat Trans., 108, pp. 872-876.

[24] Rudraiah, N. and Vortmeyer, D., 1978 Stability of finite-amplitude and overstable convection of a conducting fluid through fixed porous bed, Warme-Stoffubertrag, 11, pp. 241-254.

[25] Saravanan, S. and Jegajothi, R., 2010 Stationary fingering instability in a non-equilibrium porous medium with coupled molecular diffusion, Trans. in porous media, 84(3), pp. 755-771.

[26] Saravanan, S. and Yamaguchi, H., 2005 Onset of centrifugal convection in a magnetic-fluid-saturated porous medium, Phys. Fluids, 17, pp. 1-9(084105).

[27] Gaikwad, S.N., Malashetty, M.S. and Prasad, K. R., 2011 An analytical Study of linear and non-linear double diffusive convection with sore and dufour effects in couple stress fluid, Int. J. Non-linear Mechanics, 42, pp. 903-913.

[28] Srivastava, A. K., Bhadaura, B.S. , 2016 Influence of magnetic field on fingering instability in a a porous medium with cross diffusion effect: Athermal non-equilibrium approach, Journal of Applied Fluid Mechanics Mechanics, 9, pp. 2845-2853.

[29] Khader, M. M., Sharma, R. P. (2021). Evaluating the unsteady MHD micropolar fluid flow past stretching/shirking sheet with heat source and thermal radiation: Implementing fourth order predictor–corrector FDM. Mathematics and Computers in Simulation, 181, 333-350.

[30] Sharma, R. P., Raju, M. C., Ghosh, S. K., Mishra, S. R., Tinker, S. (2020). Time-dependent oscillatory MHD flow over a porous vertical sheet with heat source and
chemical reaction effects. Indian Journal of Pure Applied Physics (IJPAP), 58(12), 877-884.

[31] Sharma, R. P., Raju, M. C., Ghosh, S. K., Mishra, S. R., Tinker, S. (2020). Time-dependent oscillatory MHD flow over a porous vertical sheet with heat source and chemical reaction effects. Indian Journal of Pure Applied Physics (IJPAP), 58(12), 877-884.

[32] Rao, P. S., Prakash, O., Mishra, S. R., Sharma, R. P. (2020). The transient free convection magnetohydrodynamic motion of a nanofluid over a vertical surface under the influence of radiation and heat generation.

[33] Kundu, S., Ghoshal, K. (2012). An analytical model for velocity distribution and dip-phenomenon in uniform open channel flows. International Journal of Fluid Mechanics Research, 39(5).

[34] Sharma, R. P., Paul, A. (2019). Transient natural convection magnetohydrodynamic motion over an exponentially accelerated vertical porous plate with heat source. Indian Journal of Pure Applied Physics (IJPAP), 57(3), 205-211.

[35] Krishna, P. M., Sandeep, N., Sharma, R. P., Makinde, O. D. (2017). Thermal radiation effect on 3D slip motion of AlCu-Water and Cu-Water nanofluids over a variable thickness stretched surface. In Defect and Diffusion Forum (Vol. 377, pp. 141-154). Trans Tech Publications Ltd.

[36] Sharma, R. P., Makinde, O. D., Animasaun, I. L. (2018). Buoyancy effects on MHD unsteady convection of a radiating chemically reacting fluid past a moving porous vertical plate in a binary mixture. In Defect and Diffusion Forum (Vol. 387, pp. 308-318). Trans Tech Publications Ltd.

[37] Das, K., Sharma, R. P., Duari, P. R. (2017). Hydromagnetic rarefied fluid flow over a wedge in the presence of surface slip and thermal radiation. International Journal of Applied Mechanics and Engineering, 22(4), 827-837.

[38] Krishna, P. M., Sharma, R. P., Sandeep, N. (2017). Boundary layer analysis of persistent moving horizontal needle in Blasius and Sakiadis magnetohydrodynamic radiative nanofluid flows. Nuclear Engineering and Technology, 49(8), 1654-1659.

[39] Das, K., Sharma, R. P., Sarkar, A. (2016). Heat and mass transfer of a second grade magnetohydrodynamic fluid over a convectively heated stretching sheet. Journal of Computational Design and Engineering, 3(4), 330-336.