Characteristic Analysis of Monobit Signal

Pengfei Ji	extsuperscript{1*}, Xiao Yang	extsuperscript{2}, Chenglong Xia	extsuperscript{3}, Huan Lv	extsuperscript{1}

	extsuperscript{1}State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System Technology, National University of Defense Technology, Changsha, Hunan, 410073, China.

	extsuperscript{2}School of Electronic Science, National University of Defense Technology, Changsha, Hunan, 410073, China.

	extsuperscript{3}Military Representative Office of Rocket Force in Changsha, Changsha, Hunan, 4100205, China.

*Corresponding author’s e-mail: pfji@foxmail.com

**Abstract.** This paper analyzes the characters of monobit signals and gives the reason why the spurious free dynamic ranges (SFDR) of monobit signals is much lower than those of multi-bit signals. According to our analysis and simulation, if the third harmonic of a monobit signal can be filtered, the SFDR of it can be improved well. Therefore, we give the way to obtain the desired band, which do not contain any third harmonics, and the simulation results are in agreement with our design. Such way can be used to improve the performance of monobit digital receivers in engineering.

1. Introduction
Monobit receivers are one of these receivers because of their large instantaneous bandwidth, low power consumption, good real-time, and very simple structure. However, the main defect of monobit receiver is their poor instantaneous dynamic ranges (IDR) caused by the nonlinear quantization, which can generate a series of higher harmonics [1]-[5]. Many experts focus on suppressing or eliminating these higher harmonics to improve the IDR, and some good results have been obtained [6-12]. This paper aims at analyzing the characters of monobit signals and explains the reason why the IDRs of the monobit receivers are so poor from a mathematical viewpoint. At last, some advices are given for reference.

2. Analysis of Monobit Signals
Firstly, the procedure of 1-bit sampling is given here. A continuous-time sinusoidal signal is selected as the measured signal. Then, a comparator is used to quantize the measured signal, compared with the ground (zero) and the output is as (1), where $T$ is the fundamental period of the signal and $N$ is an integer.

$$x(t) = \begin{cases} 
1, & |t| \leq T/4 + NT \\
0, & T/4 + NT < |t| < 3T/4 + NT 
\end{cases} \quad (1)$$

The Fourier series expansion of (1) can be denoted as (2), where $a_k$ is the Fourier series coefficients, which determine the amplitudes of all the fundamental and harmonic components.
The Fourier series coefficient can be furtherly worked out as (3).

\[
a_k = \begin{cases} 
  \frac{1}{2}, & k = 0 \\
  \sin(\pi k/2), & k \neq 0 \\
  \frac{k\pi}{\sin(\pi k)}, & k \neq 0
\end{cases}
\]

where \(a_0\) is the average value of \(x(t)\) over one period or can be called DC component; \(a_k\) equals to 0 if \(k\) is even; \(a_{\pm 1}\) are the coefficients of the fundamental components; \(a_{\pm 3}\) are the coefficients of the third harmonic components; \(a_{\pm 5}\) are the coefficients of the fifth harmonic components, etc.

Equation (3) gives the amplitudes of all the harmonic components when continuous periodic square wave signal. It is obvious that the absolute amplitude of the fundamental components is largest except DC component, which means the 1-bit signal can be distinguished and detected correctly, while the absolute amplitudes of the third harmonic components are just 4.77 dB lower than those of the fundamental components, which results in the very poor SFDRs of 1-bit signals. Other higher order harmonics also have an impact on SFDRs, but the third harmonic components are the most serious ones. Therefore, the main work in this section is to analyze the effect of the third harmonic component after 1-bit sampling on SFDRs.

Here, we define that the sampling rate is \(f_s\) and the frequency of input signal is \(f\). Mathematically, the 1-bit sampling process in time domain is convolving the continuous signal with a periodic impulse signal in frequency domain, which means the spectrum of the 1-bit sampled signal should be periodic and the period is \(f_s\). The whole frequency spectrum of the original signal before sampling is replicated in each period, such that the third harmonic components, which may belong to the fundamental period or the other period, will inevitably appear in the fundamental frequency band. Especially, the fundamental frequency band here should range from 0 to \(f_s/2\) because the sampled data are real. Case I, Case II and Case III are given as follows to clarify this furtherly, according to the frequency of the third harmonic component associated with the fundamental component in the fundamental frequency band, which is also the target frequency band we concerned.

2.1 Case I
In first case, the third harmonic component is limited in the target frequency band, such that the frequency of it ranges from 0 to \(f_s/2\), which means the frequency of sampled signal should be 0\(\leq f \leq f_s/6\). Fig.1 gives an example, where the frequency of the harmonic in the target frequency band is \(3f\) and 0\(\leq 3f \leq f_s/2\). According to this analysis, the frequency under the case can be divided as (4), where \(m\) is a positive integer. So that each frequency band conforming to (4) can be clean and the effect of the third harmonic on SFDRs can be eliminated in these band.

\[
\frac{f_s}{2 \times 3^{m+1}} \leq f \leq \frac{f_s}{2 \times 3^{m}}.
\]

2.2 Case II
Considering that the sampling data is real, the frequency spectrum should be of bilateral symmetry over one period, as shown in Fig. 1. It is to say that there are two pairs of fundamental components and the third harmonic components over one period and they are symmetrical about the center of the spectrum. So, the second case is that the third harmonic in the target frequency band is caused by the right half part of the frequency spectrum in a period, such as Fig.2. That is, the frequency of the third harmonic caused by the
fundamental component1 in Fig. 2 should range from \( f_s/2 \) to \( f_s \). In other word, the third harmonic2 appearing in the fundamental frequency band should be \( f_s - 3f \) and the frequency of the fundamental1 should range from \( f_s/6 \) to \( f_s/3 \). Then, it is clear that the frequency bands without third harmonics can be concluded as (5) and (6). If the receiving band of a receiver can be designed as (5) or (6), it is unnecessary to worry about the effect of third harmonics and the SFDR of measured 1-bit signals can be raised.

\[ \frac{f_s}{6} \leq f \leq \frac{f_s}{4}. \]  \hspace{1cm} (5)

\[ \frac{f_s}{4} \leq f \leq \frac{f_s}{3}. \]  \hspace{1cm} (6)

Figure 1. Fig. 1 Relationship between the fundamental component and the third harmonic component in Case I.

Figure 2 Relationship between the fundamental component and the third harmonic component in Case II.

Figure 3 Relationship between the fundamental component and the third harmonic component in Case III.

2.3 Case III

The third case is illuminated by Fig. 3, where the frequency of the third harmonic1 associated with fundamental component1 is \( 3f \) and \( f_s \leq 3f \leq 3f_s/2 \). Thereby, the third harmonic3 in the target frequency band belongs to the fundamental component3 over the previous adjacent period and the frequency is \( 3f - f_s \). So that, this case can be denoted by (7), where \( m \) is a positive integer, to obtain the frequency band without third harmonics.

\[ \frac{(3^m - 1)f_s}{2 	imes 3^m} \leq f \frac{(3^{m+1} - 1)f_s}{2 	imes 3^{m+1}}. \]  \hspace{1cm} (7)
3. Simulation

In this section, the simulation about Case I, Case II and Case III is conducted. Especially, because Case I and Case II contain an infinite number of small divided frequency bands, we cannot simulate them all. Therefore, we choose the two largest bands from them for simulation. Thereby, all the bands we used for simulation include $[f_s/18, f_s/6]$, $[f_s/6, f_s/4]$, $[f_s/4, f_s/3]$, $[f_s/3, 4f_s/9]$ and the sampling rate ($f_s$) is set as 12853MHz. Although many small bands have been discarded, the four remaining bands, which is 5 GHz at all, still cover 77.8% of the Nyquist sampling bands. 1024-point DFT with a 1024-point Hanning window is used to obtain the spectrum. The results are shown in Fig.4, Fig.5, Fig.6 and Fig.7, where different color curves denote the normalized amplitude-frequency curves of different signals (only frequency is different) and four black lines are used to calibrate the theoretical amplitudes of fundamental components (line1), third harmonic components (line2), fifth harmonic components (line3) and seventh harmonic components (line4). Besides, each figure contains 31 different signals and the simulated frequencies are evenly spaced in each figure.

As we can see from these four figures, the power of fundamental component is always the largest one and stable around 50 dB while a lot of harmonics are distributed over the whole spectrum according to a certain rule, so that the frequency of the single-tone signal can always be measured correctly. According to (3), if the amplitude of fundamental component is determined, the amplitudes of harmonics can be determined too. It is our observation that the results in the four figures are in agreement with the theoretical results worked out by (3). Especially, all the third harmonic components in each figure are outside of the simulated frequency band, which is the desired results and correspondent with the analysis in Section II. Then, the largest harmonics over the simulated frequency band in Fig.4 and Fig.7 are the fifth harmonic components and those in Fig.5 and Fig.6 are the seventh harmonic components. However, the probability
of these harmonics occurring in the simulated band is smaller than that of the fundamental components, so that the fifth and seventh harmonic components can be suppressed by other methods, such as the compensation matrix way in [9], [12].

Anyway, the SFDRs over the four bands are higher than the whole Nyquist band. Therefore, it is our advice that a channelized monobit receiver can be designed and the channels can be divided as Case I, Case II and Case III in Section II to obtain higher SFDRs for 1-bit signals.

4. Conclusion
This paper analyzes the characteristic of monobit signals by using Fourier Series and designs the frequency band without third harmonics to obtain higher SFDRs. The results in Fig.4, Fig.5, Fig.6 and Fig.7 demonstrate the distribution of third harmonics is with a certain regularity and can be always out of the designed band, which is useful for the design of a channelized monobit receiver with a high instantaneous dynamic range.

References
[1] M. Young, The Technical Writer's Handbook. Mill Valley, CA: University Science, 1989. J. B. Y. Tsui: Digital Techniques for Wideband Receiver, 2nd, Norwood, MA: Artech House, 2001, pp. 397–419.
[2] J. Grajal, R. Blazquez, G. Lopez-Risueno, J.M. Sanz, M. Burgos, and A. Asensio, "Analysis and Characterization of a Monobit Receiver for Electronic Warfare.", IEEE Trans. On Aerospace and Electronic Systems, vol. 39, No. 1, pp. 244–258, Jan. 2003.
[3] A. Host-Madsen and P. Handel, "Effects of sampling and quantization on single-tone frequency estimation," in IEEE Transactions on Signal Processing, vol. 48, no. 3, pp. 650-662, March 2000, doi: 10.1109/78.824661.
[4] S. Hoyos, B. M. Sadler and G. R. Arce, "Monobit digital receivers for ultrawideband communications," in IEEE Transactions on Wireless Communications, vol. 4, no. 4, pp. 1337-1344, July 2005, doi: 10.1109/TWC.2005.850270.
[5] J. Reneau and R. R. Adhami, "Differential Phase Measurement Accuracy of a Monobit Receiver," in IEEE Access, vol. 6, pp. 69672-69681, 2018.
[6] J. B. Y. Tsui, David H. Kaneshiro, and John J. Schamus, “Monobit receiver," U. S. Patent US005963164A, Oct. 5, 1999.
[7] D. Pok, C.-I. H. Chen, J. Schamus, C. Montgomery and J. B. Y. Tsui, “Chip Design for Monobit Receiver,” IEEE Trans. Microwave Theory Tech, vol. 45, no. 12, pp. 2283–2295, Dec. 1997.
[8] J. B. Y. Tsui, “Two signal monobit electronic warfare receiver,” U.S. Patent US5793323.
[9] C.-I. H. Chen, K. George, W. McCormick, J. B. Y. Tsui, S. L. Hary, and K. M. Graves, "Design and Performance Evaluation of a 2.5-GSPS Digital Receiver.", IEEE Trans. on Instrument. and Measurement, Vol. 54, No. 3, pp. 1089–1099, June 2005.
[10] D. Pritsker and C. Cheung, "Monobit Wideband Receiver with Integrated Dithering in FPGA," in 2019 IEEE 27th Annual International Symposium on Field-Programmable Custom Computing Machines (FCCM), San Diego, CA, USA, 2019, pp. 332-332.
[11] S. Sharma, V. Bhatia, K. Deka and A. Gupta, "Sparsity-Based Monobit UWB Receiver Under Impulse Noise Environments," in IEEE Wireless Communications Letters, vol. 8, no. 3, pp. 849-852, June 2019.
[12] M. Yu and S. Dong, "An Improved Harmonic Suppression Method Based on Adaptive Compensation Algorithm for Monobit Receiver," 2019 IEEE International Conference on Signal, Information and Data Processing (ICSIDP), Chongqing, China, 2019, pp. 1-6, doi: 10.1109/ICSIDP47821.2019.9172939