Quantum Physics and Fluctuating Topologies: Survey

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Abstract

The spin-statistics connection, quantum gravity and other physical considerations suggest that classical space-time topology is not an immutable attribute and can change in quantum physics. The implementation of topology change using quantum principles has been studied for over two decades by a few of us. There has been a recent revival of interest in some of our work, dating back to as early as 1995. The present paper is meant as a resource article to our major relevant papers. It contains summaries of the contents of the cited papers and the corresponding links wherever available.

1 Introduction

The role of classical topology in quantum physics is not well-understood even after nearly one hundred years since the formulation of the principles of quantum theory. Observables in quantum physics are self-adjoint operators and their spectra, and the mode of emergence of space-time from such data is a central problem of quantum epistemology. A few of us have been collaborating on this topic since late 1970’s and have in fact coauthored two books on this subject \cite{1,2}.

A related issue concerns quantum gravity. It is widely speculated that space-time topology can change in quantum gravity and classical manifolds are just approximate idealizations emergent from quantum states which are localized in neither one nor another topological space. But this idea has remained speculative. The tools tried for its implementation in quantum gravity have been functional integrals. But the latter are ill-defined on smooth Lorentzian manifolds \cite{3} and anyway the Lorentz metric practically excludes topology change because of known theorems on cobordism of manifolds \cite{3,8}. The use of Euclidean functional integrals for gravity are fraught with problems and cannot be regarded as reliable probes of so fundamental an issue.

It is in this context that a few of us began the study of simple quantum mechanical systems and formulate topology change using properties of wave functions. This was in 1995, some 17 years...
 ago [9]. Recent work [10] shows a revival of interest in this paper and also in earlier studies of classical topology in quantum physics by our group of collaborators. With this in mind, we have collected together here some of our relevant publications. They are not exhaustive as regards our work, but do contain adequate citations to follow the trails of our thought.

We have divided this collection into two parts. It is the first which works with systems with finite degrees of freedom and uses basic ideas on differential operators such as the Laplacian on manifolds with boundaries to explore spatial topology and and its transmutation. The second is also involved with spatial topology. In one approach reported here, an algebraic approach extracted from large distance physics is developed for this purpose. It also contains a paper on quantum fields on curved spaces with nontrivial topology and shows how diffeomorphisms can change the domain of the field Hamiltonian and become anomalous.

We now give brief introductions to the papers.

Part 1. Topology Change and Quantum Physics

1.1 Topology Change and Quantum Physics

1. A. P. Balachandran, G. Bimonte, G. Marmo and A. Simoni, Topology change and quantum physics Nucl. Phys., B446 (1995) [9]

In this paper, the role of topology in elementary quantum physics is explored. The attributes of classical spatial topology are shown to emerge from properties of state vectors with suitably smooth time evolution. Equivalently, they emerge from the domain of the quantum Hamiltonian, which is often specified by boundary conditions. Several examples are presented where classical topology is changed by smoothly altering the boundary conditions. When the parameters labeling the latter are treated as quantum variables, quantum states need not give a well-defined classical topology, instead they can give a quantum superposition of such topologies. Arguments of Sorkin based on the spin-statistics connection which indicate the need for topology change in quantum gravity and further arguments presented here suggest that Einstein gravity and its minor variants are effective theories of a deeper description with additional novel degrees of freedom.

1.2 Global Theory of Quantum Boundary Conditions and Topology Change

2. M. Asorey, A. Ibort and G. Marmo, Global Theory of Quantum Boundary Conditions and Topology Change, Int.J.Mod.Phys. A 20, 1001 (2005) [11].

3. M. Asorey, A. Ibort and G. Marmo, The Boundary Grassmannian for Dirac Operators (in preparation).

4. A. Ibort, Three lectures on global boundary conditions and the theory of self-adjoint extensions of the covariant Laplace–Beltrami and Dirac operators on Riemannian manifolds with boundary, AIP Conf. Proc., 1460 (2012) [12].

5. M Asorey, D Garca-Alvarez and J M Munoz-Castaneda, Vacuum energy and renormalization on the edge, J. Phys. A 40, 6767 (2007) [13]

These four are companion papers to the preceding one. In the first, the theory of the self-adjoint Laplacians on a Riemannian manifold with ‘regular’ boundary is systematically
treated. The possible boundary conditions are fully classified and their equivalence with von Neumann’s conditions are established. This analysis and the corresponding analysis of Dirac operators in the second paper are particularly adapted to discuss topology change. A striking result, with implications for topology change, is that for small deformations of the Dirichlet boundary condition, the Laplacian has infinitely many negative energy states localized near the boundary.

The second paper is yet to be published, but its contents as well as those of the first paper are summarized in the third. The fourth paper is particularly important for its treatment of the dynamics of topology change and the boundary renormalization group flow.

1.3 Quantum Boundary Conditions and Spectral Action Principle

6. L. C. de Albuquerque, P. Teotonio-Sobrinho and S. Vaidya, Quantum topology change and large N gauge theories, JHEP, 0410 (2004) [14].

This paper addresses the question of recovering usual topologies from dynamics on the space of boundary conditions $Q_{TOP}$. The explicit model at hand is a one-dimensional model consisting of a collection of $N$ unit segments, generalizing the original example of Balachandran et al [9]. Among all possible boundary conditions, there are special ones that glue the line segments into circles of various sizes. These points of $Q_{TOP}$ correspond to a classical topology. The formalism of spectral triples in noncommutative geometry then provides the motivation for dynamics on $Q_{TOP}$, thus making the system closely analogous to Euclidean quantum gravity for fluctuating topologies.

The spectral action governing the dynamics on $Q_{TOP}$ corresponds to a $U(N)$ lattice gauge theory with a single plaquette where the holonomy $g$ characterizes the boundary condition. The partition function has a single free parameter $\beta$ as usual. We show that topology becomes classical and localized in the limit $\beta \to \infty$ for all values of $N$. For large $N$ the system has a third order phase transition at $\beta_c = 1$. Although topology is not exactly localized for finite $\beta$, one of the phases imposes constraints on classical topology. It turns out that for $\beta > \beta_c$, the only classical circles that can occur have finite sizes. In particular, the probability distribution for large $\beta$ is dominated by classical circles of unit length.

Part 2: Quantum Geons: Spin-Statistics, Topology Change, Anomalies

1. A. Balachandran, E. Batista, I. Costa e Silva and P. Teotonio-Sobrinho, Quantum topology change in (2+1)-dimensions, Int. J. Mod. Phys., 2000, A15, 1629 (2000) [18].

2. A. P. Balachandran, E. Batista, I. Costa e Silva and P. Teotonio-Sobrinho, The Spin-Statistics Connection in Quantum Gravity, Nucl. Phys., B566, 441 (2000) [19].

3. A. P. Balachandran, E. Batista, I. Costa e Silva and P. Teotonio-Sobrinho, A Novel Spin-Statistics Theorem in (2 + 1)d Chern-Simons Gravity, Mod. Phys. Lett., A16, 1335 (2001) [20].

In these papers, instead of adopting dynamical boundary conditions for standard operators like the Laplacian, an algebra of operators capturing topological degrees of freedom
is introduced as follows. Starting from a Yang-Mills-Higgs theory with gauge group $G$ on
an oriented, non-compact Riemannian surface $\Sigma$ of genus $g$ and one asymptotic region,
one assumes further that the symmetry is broken down to a finite subgroup $H \subset G$, and
considers the low energy limit of the theory. The configuration space then splits into dis-
joint sectors. They are characterized by vortices with fluxes valued in $H$. Introducing also
large diffeomorphisms which can change these fluxes, one gets a topological algebra suitable
for large distance physics. It is quantized, giving a description of single vortex irreducible
representations (IRR’s). Each such IRR describes a single irreducible vortex. Its representa-
tion on the two-vortex vector states is then given by a ‘coproduct’ (the algebras being
Hopf [21]). This representation can be decomposed in turn into IRR’s and in general con-
tains many IRR’s even when the starting single vortex transforms irreducibly. Each such
IRR is interpreted as describing a composite irreducible vortex. In this way, we have an
effective description of two (or many) irreducible vortices fusing and changing into single
irreducible vortices, and that is topology change.

Each genus $g$ surface supports a solitonic excitation called a geon, whose quantum attributes
were first studied by Friedman and Sorkin [22,23]. In the absence of topology change, geons
can violate the usual spin-statistics connection. In later work by us, the above ideas on
topology change were applied to these geon manifolds as well, thereby achieving topology
change. A new spin-statistics theorem for these two-dimensional geons, which differs from
the standard one, is also proved.

4. A. P. Balachandran and A. R. de Queiroz. Quantum Gravity: Mixed States from Diffeo-
morphism Anomalies, JHEP, 11 (2011) [24].

As alluded to above, geons are solitonic excitations on spatial manifolds (of any dimension)
with non-trivial topology with remarkable properties. They were first investigated in the
context of quantum gravity by Friedman and Sorkin.

Now in a manifold with geons, a ‘large’ diffeomorphism (an element of the mapping class
group) may change the domain of a self-adjoint operator such as the Laplacian. If it does so,
it is anomalous just as axial ‘symmetry’ transformations in QCD are anomalous. This paper
studies this question by examining the action of the mapping class group on the fundamental
group of manifolds with geons. In this manner, it is established that the mapping class group
can in fact become anomalous. When it does so, conventional quantum gravity based on
pure states loses diffeomorphism invariance. The paper also discusses how the use of suitable
mixed states [25] may enable the construction of an anomaly-free quantum gravity.

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