ROLE OF NONPERTURBATIVE EFFECTS IN DEEP INELASTIC SCATTERING REVISITED

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Abstract

Restrictions imposed on the (electromagnetic or weak) current operator by its commutation relations with the representation operators of the Poincare group are considered in detail. We argue that the present theory of deep inelastic scattering based on perturbative QCD does not take into account the dependence of the current operator on the nonperturbative part of the quark-gluon interaction which cannot be neglected even in leading order in $1/Q$, where $Q$ is the magnitude of the momentum transfer.

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1 Introduction

The present theory of deep inelastic scattering (DIS) has proven rather successful in describing many experimental data. This theory is based on two approaches which are the complement of one another. In the first approach (see e.g. ref. [1] and references therein) one assumes that only Feynman diagrams from a certain class dominate in DIS, and in the second approach
DIS is considered in the framework of the operator product expansion (OPE) \[2\]. Although the assumptions used in the both approaches are natural, the problem of their substantiation remains since we do not know how to work with QCD beyond perturbation theory. In particular, the OPE has been proved only in perturbation theory \[3\] and its validity beyond that theory is problematic (see the discussion in ref. \[4\] and references therein).

At the same time, it is strange that almost all authors investigating DIS do not pay attention to restrictions imposed on the (electromagnetic or weak) current operator by its commutation relations with the representation operators of the Poincare group. In the present paper we investigate these restrictions in detail. The paper is organized as follows. In Secs. \[2\] and \[3\] we discuss the general properties of the current operator in quantum field theory and the properties derived in the framework of canonical formalism. As shown in Sec. \[4\], the latter properties are not reliable since in some cases they are incompatible with Lorentz invariance. In Sec. \[5\] we apply these results to DIS and show that the nonperturbative part of the current operator contributes to deep inelastic scattering even in leading order in \(1/Q\) where \(Q\) is the magnitude of the momentum transfer.

## 2 Relativistic invariance of the current operator

In any relativistic quantum theory the system under consideration is described by some (pseudo)unitary representation of the Poincare group. The current operator \(\hat{J}^\mu(x)\) for this system
(where \( \mu = 0, 1, 2, 3 \) and \( x \) is a point in Minkowski space) should satisfy the following necessary conditions.

Let \( \hat{U}(a) = e^{i\hat{P}_{\mu}a^{\mu}} \) be the representation operator corresponding to the displacement of the origin in spacetime translation of Minkowski space by the four-vector \( a \). Here \( \hat{P} = (\hat{P}^0, \hat{P}) \) is the operator of the four-momentum, \( \hat{P}^0 \) is the Hamiltonian, and \( \hat{P} \) is the operator of ordinary momentum. Let also \( \hat{U}(l) \) be the representation operator corresponding to \( l \in SL(2, C) \). Then

\[
\hat{U}(a)^{-1} \hat{J}^{\mu}(x) \hat{U}(a) = \hat{J}^{\mu}(x - a)
\]

(1)

\[
\hat{U}(l)^{-1} \hat{J}^{\mu}(x) \hat{U}(l) = L(l)^{\mu}_{\nu} \hat{J}^{\nu}(L(l)^{-1}x)
\]

(2)

where \( L(l) \) is the element of the Lorentz group corresponding to \( l \) and a sum over repeated indices \( \mu, \nu = 0, 1, 2, 3 \) is assumed.

Let \( \hat{M}^{\mu\nu} \) be the representation generators of the Lorentz group. Then, as follows from Eq. (2), Lorentz invariance of the current operator implies

\[
[\hat{M}^{\mu\nu}, \hat{J}^{\rho}(x)] = -i \{ (x^\mu \partial^\nu - x^\nu \partial^\mu) \hat{J}^{\rho}(x) + g^{\mu\rho} \hat{J}^{\nu}(x) - g^{\nu\rho} \hat{J}^{\mu}(x) \}
\]

(3)

where \( g^{\mu\nu} \) is the metric tensor in Minkowski space.

The operators \( \hat{P}^{\mu}, \hat{M}^{\mu\nu} \) act in the scattering space of the system under consideration. In QED the electrons, positrons and photons are the fundamental particles, and the scattering space is the space of these almost free particles ("in" or "out" space). Therefore it is sufficient to deal only with \( \hat{P}^{\mu}_{\text{ex}}, \hat{M}^{\mu\nu}_{\text{ex}} \) where "ex" stands either for "in" or "out". However in QCD the scattering space by no means can be considered as a space of almost free fundamental particles — quarks and gluons. For example, even if the scattering space consists of one particle (say the nucleon), this particle is the bound state of quarks and gluons, and the
operators $\hat{P}^\mu, \hat{M}^{\mu\nu}$ considerably differ from the corresponding free operators $P^\mu, M^{\mu\nu}$. It is well-known that perturbation theory does not apply to bound states and therefore $\hat{P}^\mu$ and $\hat{M}^{\mu\nu}$ cannot be determined in the framework of perturbation theory. For these reasons we will be interested in cases when the representation operators in Eqs. (1) and (2) correspond to the full generators $\hat{P}^\mu, \hat{M}^{\mu\nu}$.

Strictly speaking, the notion of current is not necessary if the theory is complete. For example, in QED there exist unambiguous prescriptions for calculating the elements of the S-matrix to any desired order of perturbation theory and this is all we need. It is believed that this notion is useful for describing the electromagnetic or weak properties of strongly interacted systems. It is sufficient to know the matrix elements $\langle \beta | \hat{J}^\mu(x) | \alpha \rangle$ of the operator $\hat{J}^\mu(x)$ between the (generalized) eigenstates of the operator $\hat{P}^\mu$ such that $\hat{P}^\mu | \alpha \rangle = P^\mu_\alpha | \alpha \rangle$, $\hat{P}^\mu | \beta \rangle = P^\mu_\beta | \beta \rangle$. It is usually assumed that, as a consequence of Eq. (1)

$$\langle \beta | \hat{J}^\mu(x) | \alpha \rangle = \exp[i(P^\nu_\beta - P^\nu_\alpha)x_\nu] \langle \beta | \hat{J}^\mu | \alpha \rangle$$

where formally $\hat{J}^\mu \equiv \hat{J}^\mu(0)$. Therefore in the absence of a complete theory we can consider the less fundamental problem of investigating the properties of the operator $\hat{J}^\mu$. From the mathematical point of view this implies that we treat $\hat{J}^\mu(x)$ not as a four-dimensional operator distribution, but as a ”nonlocal” operator satisfying the condition

$$\hat{J}^\mu(x) = \exp(i\hat{P}x)\hat{J}^\mu\exp(-i\hat{P}x)$$

The standpoint that the current operator should not be treated on the same footing as the fundamental local fields is advocated
by several authors in their investigations on current algebra (see, for example, ref. [3]). One of the arguments is that, for example, the canonical current operator in QED is given by [3]

\[ \hat{J}^\mu(x) = \mathcal{N}\{\hat{\psi}(x)\gamma^\mu\hat{\psi}(x)\} = \frac{1}{2}[\hat{\psi}(x), \gamma^\mu\hat{\psi}(x)] \] (6)

(where \( \mathcal{N} \) stands for the normal product and \( \hat{\psi}(x) \) is the Heisenberg operator of the Dirac field), but this expression is not a well-definition of a local operator. Indeed, Eq. (6) involves the product of two local field operators at coinciding points, i.e. \( \hat{J}^\mu(x) \) is a composite operator. The problem of the correct definition of the product of two local operators at coinciding points is known as the problem of constructing the composite operators (see e.g. ref. [7]). So far this problem has been solved only in the framework of perturbation theory for special models. When perturbation theory does not apply the usual prescriptions are to separate the arguments of the operators in question and to define the composite operator as a limit of nonlocal operators when the separation goes to zero (see e.g. ref. [8] and references therein). Since we do not know how to work with quantum field theory beyond perturbation theory, we do not know what is the correct prescription.

It is well-known (see, for example, ref. [8]) that it is possible to add to the current operator the term \( \partial_\nu X^{\mu\nu}(x) \) where \( X^{\mu\nu}(x) \) is some operator antisymmetric in \( \mu \) and \( \nu \). However it is usually believed [8] that the electromagnetic and weak current operators of a strongly interacted system are given by the canonical quark currents the form of which is similar to that in Eq. (6).

We will not insist on the interpretation of the current operator according to Eq. (5) and will not use this expression in the
derivation of the formulas, but in some cases the notion of $\hat{J}^\mu$ makes it possible to explain the essence of the situation clearly. A useful heuristic expressions which follows from Eqs. (3) and (5) is

$$[\hat{M}^{\mu \nu}, \hat{J}^\rho] = -i(g^{\mu \rho} \hat{J}^\nu - g^{\nu \rho} \hat{J}^\mu)$$

(7)

3 Canonical quantization and the forms of relativistic dynamics

In the standard formulation of quantum field theory the operators $\hat{P}_\mu, \hat{M}_{\mu \nu}$ are given by

$$\hat{P}_\mu = \int \hat{T}_\mu^\nu(x) d\sigma_\nu(x), \quad \hat{M}_{\mu \nu} = \int \hat{M}_{\mu \nu}^\rho(x) d\sigma_\rho(x)$$

(8)

where $\hat{T}_\mu^\nu(x)$ and $\hat{M}_{\mu \nu}^\rho(x)$ are the energy-momentum and angular momentum tensors and $d\sigma_\mu(x) = \lambda_\mu \delta(\lambda x - \tau)d^4x$ is the volume element of the space-like hypersurface defined by the time-like vector $\lambda$ ($\lambda^2 = 1$) and the evolution parameter $\tau$. In turn, these tensors are fully defined by the classical Lagrangian and the canonical commutation relations on the hypersurface $\sigma_\mu(x)$. In this connection we note that in canonical formalism the quantum fields are supposed to be distributions only relative the three-dimensional variable characterizing the points of $\sigma_\mu(x)$ while the dependence on the variable describing the distance from $\sigma_\mu(x)$ is usual [9].

In spinor QED we define $V(x) = -L_{int}(x) = e\hat{J}^\mu(x)\hat{A}_\mu(x)$, where $L_{int}(x)$ is the quantum interaction Lagrangian, $e$ is the (bare) electron charge and $\hat{A}_\mu(x)$ is the operator of the Maxwell field (let us note that if $\hat{J}^\mu(x)$ is treated as a composite operator
then the product of the operators entering into \( V(x) \) should be correctly defined).

At this stage it is not necessary to require that \( \hat{J}^\mu(x) \) is given by Eq. (3), but the key assumption in the canonical formulation of QED is that \( \hat{J}^\mu(x) \) is constructed only from \( \hat{\psi}(x) \) (i.e. there is no dependence on \( \hat{A}_\mu(x) \) and the derivatives of the fields \( \hat{A}_\mu(x) \) and \( \hat{\psi}(x) \)). Then the canonical result derived in several well-known textbooks and monographs (see, for example, ref. [6]) is

\[
\hat{P}^\mu = P^\mu + \lambda^\mu \int V(x)\delta(\lambda x - \tau)d^4x
\]  
(9)

\[
\hat{M}^{\mu\nu} = M^{\mu\nu} + \int V(x)(x^\nu \lambda^\mu - x^\mu \lambda^\nu)\delta(\lambda x - \tau)d^4x
\]  
(10)

It is important to note that if \( A^\mu(x) \), \( J^\mu(x) \) and \( \psi(x) \) are the corresponding free operators then \( \hat{A}^\mu(x) = A^\mu(x) \), \( \hat{J}^\mu(x) = J^\mu(x) \) and \( \hat{\psi}(x) = \psi(x) \) if \( x \in \sigma_\mu(x) \).

As pointed out by Dirac [10], any physical system can be described in different forms of relativistic dynamics. By definition, the description in the point form implies that the operators \( \hat{U}(l) \) are the same as for noninteracting particles, i.e. \( \hat{U}(l) = U(l) \) and \( \hat{M}^{\mu\nu} = M^{\mu\nu} \), and thus interaction terms can be present only in the four-momentum operators \( \hat{P} \) (i.e. in the general case \( \hat{P}^\mu \neq P^\mu \) for all \( \mu \)). The description in the instant form implies that the operators of ordinary momentum and angular momentum do not depend on interactions, i.e. \( \hat{P} = P \), \( \hat{M} = M \) (\( \hat{M} = (\hat{M}^{23}, \hat{M}^{31}, \hat{M}^{12}) \)), and therefore interaction terms may be present only in \( \hat{P}^0 \) and the generators of the Lorentz boosts \( \hat{N} = (\hat{M}^{01}, \hat{M}^{02}, \hat{M}^{03}) \). In the front form with the marked \( z \) axis we introduce the + and - components of the four-vectors as \( x^+ = (x^0 + x^3)/\sqrt{2}, \quad x^- = (x^0 - x^3)/\sqrt{2} \). Then we require that the operators \( \hat{P}^+, \hat{P}^j, \hat{M}_{12}, \hat{M}^{+-}, \hat{M}^{+j} \) \( (j = 1, 2) \) are the same.
as the corresponding free operators, and therefore interaction terms may be present only in the operators $\hat{M}^{-j}$ and $\hat{P}^{-}$.

In quantum field theory the form of dynamics depends on the choice of the hypersurface $\sigma_\mu(x)$. The representation generators of the subgroup which leaves this hypersurface invariant are free since the transformations from this subgroup do not involve dynamics. Therefore it is reasonable to expect that Eqs. (9) and (10) give the most general form of the Poincare group representation generators in quantum field theory if the fields are quantized on the hypersurface $\sigma_\mu(x)$, but in the general case the relation between $V(x)$ and $L_{\text{int}}(x)$ is not so simple as in QED. The fact that the operators $V(x)$ in Eqs. (9) and (10) are the same follows from the commutation relations between $\hat{P}_\mu$ and $\hat{M}^{\mu\nu}$.

The most often considered case is $\tau = 0, \lambda = (1, 0, 0, 0)$. Then $\delta(\lambda x - \tau)d^4x = d^3x$ and the integration in Eqs. (9) and (10) is taken over the hyperplane $x^0 = 0$. Therefore, as follows from these expressions, $\hat{P} = \hat{P}$ and $\hat{M} = \hat{M}$. Hence such a choice of $\sigma_\mu(x)$ leads to the instant form [10].

The front form can be formally obtained from Eqs. (9) and (10) as follows. Consider the vector $\lambda$ with the components
\[
\begin{align*}
\lambda^0 &= \frac{1}{(1 - v^2)^{1/2}}, \\
\lambda^j &= 0, \\
\lambda^3 &= -\frac{v}{(1 - v^2)^{1/2}} (j = 1, 2)
\end{align*}
\] (11)

Then taking the limit $v \to 1$ in Eqs. (9) and (10) we get
\[
\begin{align*}
\hat{P}_\mu &= P_\mu + \omega_\mu \int V(x)\delta(x^+)d^4x, \\
\hat{M}^{\mu\nu} &= M^{\mu\nu} + \int V(x)(x^\mu \omega^\nu - x^\nu \omega^\mu)\delta(x^+)d^4x
\end{align*}
\] (12)

where the vector $\omega$ has the components $\omega^- = 1, \omega^+ = \omega^j = 0$. 

It is obvious that the generators (12) are given in the front form and that’s why Dirac [10] related this form to the choice of the light cone \( x^+ = 0 \).

In ref. [10] the point form was related to the hypersurface \( t^2 - x^2 > 0, t > 0 \), but as argued by Sokolov [11], the point form should be related to the hyperplane orthogonal to the four-velocity of the system under consideration. We shall not discuss this question in the present paper.

4 Incompatibility of canonical formalism with Lorentz invariance for spinor fields

In canonical formalism the key property of the current operator for the spinor field is that \( \hat{J}^\mu(x) = J^\mu(x) \) if \( x \in \sigma_\mu(x) \). The purpose of this section is to show that this property is not correct since it is incompatible with Lorentz invariance.

A possible objection against the derivation of Eqs. (9) and (10) is that the product of local operators at one and the same value of \( x \) is not a well-defined object. For example, if \( x^0 = 0 \) then following Schwinger [12], instead of Eq. (9), one can define \( J^\mu(x) \) as the limit of the operator

\[
J^\mu(x) = \frac{1}{2} \left[ \bar{\psi}(x + \frac{1}{2}) \gamma^\mu \exp(ie \int_{x-\frac{1}{2}}^{x+\frac{1}{2}} A(x')dx') \psi(x - \frac{1}{2}) \right] \quad (13)
\]

when \( l \to 0 \), the limit should be taken only at the final stage of calculations and in the general case the time components of the arguments of \( \hat{\psi} \) and \( \hat{\psi} \) also differ each other (the contour integral in this expression is needed to conserve gauge invariance). Therefore there is a ”hidden” dependence of \( \hat{J}^\mu(x) \) on \( \hat{A}^\mu(x) \) and hence Eqs. (9) and (10) are incorrect.
However, any attempt to separate the arguments of the \( \hat{\psi} \) operators in \( \hat{J}^\mu(x) \) immediately results in breaking of locality. In particular, at any \( l \neq 0 \) in Eq. (13) the Lagrangian is nonlocal. We do not think that locality is a primary physical condition, but once the Lagrangian is nonlocal, the whole edifice of local quantum field theory (including canonical formalism) becomes useless. Meanwhile the only known way of constructing the generators \( \hat{P}^\mu, \hat{M}^{\mu\nu} \) in local quantum field theory is canonical formalism. For these reason we first consider the results which formally follow from canonical formalism and show that these results are inconsistent.

In addition to the properties discussed above, the current operator should also satisfy the continuity equation \( \partial \hat{J}^\mu(x)/\partial x^\mu = 0 \). As follows from this equation and Eq. (1), \([\hat{J}^\mu(x), \hat{P}_\mu] = 0\). The canonical formalism in the instant form implies that if \( x^0 = 0 \) then \( \hat{J}^\mu(x) = J^\mu(x) \). Since \( J^\mu(x) \) satisfies the condition \([J^\mu(x), P_\mu] = 0\), it follows from Eq. (1) that if \( \hat{P}^\mu = P^\mu + V^\mu \) then the continuity equation is satisfied only if

\[
[V^0, J^0(x)] = 0
\]  

(14)

where

\[
V^0 = \int V(x)d^3x, \quad V(x) = -eA(x)J(x)
\]  

(15)

We take into account the fact that the canonical quantization on the hypersurface \( x^0 = 0 \) implies that \( A^0(x) = 0 \).

As follows from Eqs. (11) and (3), the commutation relation between the operators \( \hat{M}^{0i}(i = 1, 2, 3) \) and \( J^0(x) \) should have the form

\[
[\hat{M}^{0i}, J^0(x)] = -x^i[\hat{P}^0, J^0(x)] - \imath J^i(x)
\]  

(16)
Since
\[ [M^{0i}, J^0(x)] = -x^i[P^0, J^0(x)] - x^i J^i(x) \]  \hspace{1cm} (17)

it follows from Eqs. (10), (14) and (15) that Eq. (16) is satisfied if
\[ \int y^i A(y) [J(y), J^0(x)] d^3 y = 0 \]  \hspace{1cm} (18)

It is well-known that if the standard equal-time commutation relations are used naively then the commutator in Eq. (18) vanishes and therefore this equation is satisfied. However when \(x \to y\) this commutator involves the product of four Dirac fields at \(x = y\). The famous Schwinger result \([12]\) is that if the current operators in question are given by Eq. (13) then
\[ [J^i(y), J^0(x)] = C \frac{\partial}{\partial x^i} \delta(x - y) \]  \hspace{1cm} (19)

where \(C\) is some (infinite) constant. Therefore Eq. (18) is not satisfied and the current operator \(\hat{J}^\mu(x)\) constructed in the framework of canonical formalism does not satisfy Lorentz invariance.

At the same time, Eq. (19) is compatible with Eqs. (14) and (15) since \(\text{div}(A(x)) = 0\). One can also expect that the commutator \([\hat{M}^{0i}, J^k(x)]\) is compatible with Eq. (3). This follows from the fact \([13]\) that if Eq. (19) is satisfied then the commutator \([J^i(x), J^k(y)]\) does not contain derivatives of the delta function.

While the arguments given in Ref. [12] prove that the commutator in Eq. (19) cannot vanish, one might doubt whether the singularity of the commutator is indeed given by the right hand side of this expression. Of course, at present any method of calculating such a commutator is model dependent, but the result that canonical formalism is incompatible with Lorentz
invariance (see Eq. (16)) follows in fact only from algebraic considerations. Indeed, Eqs. (14), (16) and (17) imply that if 
\[ \hat{M}^{\mu\nu} = M^{\mu\nu} + V^{\mu\nu} \]
then
\[ [V^{0i}, J^0(x)] = 0 \quad (20) \]

Since \( V^{0i} \) in the instant form is a nontrivial interaction dependent operator, there is no reason to expect that it commutes with the free operator \( J^0(x) \). Moreover for the analogous reason Eq. (14) will not be satisfied in the general case.

To better understand the situation in spinor QED it is useful to consider scalar QED [14]. The formulation of this theory can be found, for example, in ref. [15]. In contrast with spinor QED, the Schwinger term in scalar QED emerges canonically [12, 8].

We use \( \varphi(x) \) to denote the operator of the scalar complex field at \( x^0 = 0 \). The canonical calculation yields
\[
\hat{J}^0(x) = J^0(x) = i[\varphi^*(x)\pi^*(x) - \pi(x)\varphi(x)], \\
\hat{J}^i(x) = J^i(x) - 2eA^i(x)\varphi^*(x)\varphi(x), \\
J^i(x) = i[\varphi^*(x) \cdot \partial^i \varphi(x) - \partial^i \varphi^*(x) \cdot \varphi(x)] \quad (21)
\]
where \( \pi(x) \) and \( \pi^*(x) \) are the operators canonically conjugated with \( \varphi(x) \) and \( \varphi^*(x) \) respectively. In contrast with Eq. (13), the operator \( V(x) \) in scalar QED is given by
\[
V(x) = -eA(x)J(x) + e^2A(x)^2\varphi^*(x)\varphi(x) \quad (22)
\]
However the last term in this expression does not contribute to the commutator (16). It is easy to demonstrate that as pointed out in Ref. [14], the commutation relations (3) in scalar QED are satisfied in the framework of the canonical formalism. Therefore the naive treatment of the product of local operators at coinciding points in this theory is not in conflict with the canonical
commutation relations. The key difference between spinor QED and scalar QED is that in contrast with spinor QED, the spatial component of the canonical current operator is not free if \( x^0 = 0 \) (see Eq. (21)). Just for this reason the commutator \([\hat{M}^{0i}, J^0(x)]\) in scalar QED agrees with Eq. (23) since the Schwinger term in this commutator gives the interaction term in \( \hat{J}^i(x) \).

Now let us return to spinor QED. As noted above, the canonical formalism cannot be used if the current operator is considered as a limit of the expression similar to that in Eq. (13). In addition, the problem exists what is the correct definition of \( V(x) \) as a composite operator. One might expect that the correct definition of \( J^\mu(x) \) and \( V(x) \) will result in appearance of some additional terms in \( V(x) \) (and hence in \( V^0 \) and \( V^{0i} \)). However it is unlikely that this is the main reason of the violation of Lorentz invariance. Indeed, as noted above, for only algebraic reasons it is unlikely that both conditions (14) and (20) can be simultaneously satisfied. Therefore, taking into account the situation in scalar QED, it is natural to conclude that the main reason of the failure of canonical formalism is that either the limit of \( \hat{J}^\mu(x^0, x) \) when \( x^0 \to 0 \) does not exist or this limit is not equal to \( J^\mu(x) \) (i.e. the relation \( \hat{J}^\mu(x) = J^\mu(x) \) is incorrect).

The fact that the relation \( \hat{J}^\mu(x) = J^\mu(x) \) cannot be correct follows from simpler considerations. Indeed, assume first that this relation is valid. Then we can use canonical formalism in the framework of which the generator of the gauge transformations is \( \text{div} \mathbf{E}(y) - J^0(y) \), and if \( \mathbf{J}(x) \) is gauge invariant then \( [\text{div} \mathbf{E}(y) - J^0(y), \mathbf{J}(x)] = 0 \). The commutator \([J^0(y), \mathbf{J}(x)]\) cannot be equal to zero [12] and therefore \( \mathbf{J}(x) \) does not commute with \( \text{div} \mathbf{E}(y) \) while the free operator \( \mathbf{J}(x) \) commutes with \( \text{div} \mathbf{E}(y) \). The relation \( \hat{J}^\mu(x) = J^\mu(x) \) also does not take place in
explicitly solvable two-dimensional models [9]. In addition, once we assume that the field operators on the hypersurface $\sigma(\mu)$ are free we immediately are in conflict with the Haag theorem [10, 9]. However for our analysis of the current operator in DIS in Sec. 5 it is important that $\hat{J}^{\mu}(x) \neq J^{\mu}(x)$ as a consequence of Lorentz invariance.

By analogy with ref. [12] it is easy to show that if $x^+ = 0$ then the canonical current operator in the front form $J^+(x^-, \mathbf{x}_\perp)$ (we use the subscript $\perp$ to denote the projection of the three-dimensional vector unto the plane 12) cannot commute with all the operators $J^i(x^-, \mathbf{x}_\perp)$ ($i = -, 1, 2$). As easily follows from the continuity equation and Lorentz invariance (3), the canonical operator $J^+(x^-, \mathbf{x}_\perp)$ should satisfy the relations

$$[V^-, J^+(x^-, \mathbf{x}_\perp)] = [V^{-j}, J^+(x^-, \mathbf{x}_\perp)] = 0 \quad (j = 1, 2) \quad (23)$$

By analogy with the above consideration we conclude that these relations cannot be simultaneously satisfied and therefore either the limit of $\hat{J}^{\mu}(x^+, x^-, \mathbf{x}_\perp)$ when $x^+ \to 0$ does not exist or this limit is not equal to $J^{\mu}(x^-, \mathbf{x}_\perp)$. Therefore the canonical light cone quantization does not render a Lorentz invariant current operator for spinor fields.

Let us also note that if the theory should be invariant under the space reflection or time reversal, the corresponding representation operators in the front form $\hat{U}_P$ and $\hat{U}_T$ are necessarily interaction dependent (this is clear, for example, from the relations $\hat{U}_P P^+ \hat{U}_P^{-1} = \hat{U}_T P^+ \hat{U}_T^{-1} = \hat{P}^-$). In terms of the operator $\hat{J}^{\mu}$ one can say that this operator should satisfy the conditions

$$\hat{U}_P(\hat{j}^0, \hat{J})\hat{U}_P^{-1} = \hat{U}_T(\hat{j}^0, \hat{J})\hat{U}_T^{-1} = (\hat{j}^0, -\hat{J}) \quad (24)$$

Therefore it is not clear whether these conditions are compat-
ible with the relation $\hat{J}^\mu = J^\mu$. However this difficulty is a consequence of the difficulty with Eq. (2) since, as noted by Coester [17], the interaction dependence of the operators $\hat{U}_P$ and $\hat{U}_T$ in the front form does not mean that there are discrete dynamical symmetries in addition to the rotations about transverse axes. Indeed, the discrete transformation $P_2$ such that $P_2 x := \{x^0, x_1, -x_2, x_3\}$ leaves the light front $x^+ = 0$ invariant. The full space reflection $P$ is the product of $P_2$ and a rotation about the 2-axis by $\pi$. Thus it is not an independent dynamical transformation in addition to the rotations about transverse axes. Similarly the transformation $TP$ leaves $x^+ = 0$ invariant and $T = (TP)P_2R_2(\pi)$.

In terms of the operator $\hat{J}^\mu$ the results of this section are obvious. Indeed, since at $x = 0$ the Heisenberg and Schrodinger pictures coincide then in view of Eq. (3) one might think that the operator $\hat{J}^\mu$ is free, i.e. $\hat{J}^\mu = J^\mu$. However there is no reason for the interaction terms in $M^{\mu\nu}$ to commute with the free operator $J^\mu$ (see Eq. (4)). Therefore the results of this section show that the algebraic reasons based on Eq. (2) are more solid than the reasons based on formal manipulations with local operators, and in the instant and front forms $\hat{J}^\mu \neq J^\mu$. Note also that although the model considered in this section is spinor QED, the above results are not very important for QED itself since, as pointed out in Sec. 2, in QED it is sufficient to consider only commutators involving $\hat{P}_c^{\mu}$ and $\hat{M}_c^{\mu\nu}$. However it will be clear in the next section that the above considerations are important for investigating the properties of the current operator for strongly interacting particles.
5 Current operator in DIS

If \( q \) is the momentum transfer in DIS then the DIS cross-section is fully defined by the hadronic tensor

\[
W^{\mu\nu} = \frac{1}{4\pi} \int e^{igx} \langle P' | [\hat{J}^\mu (\frac{x}{2}), \hat{J}^\nu (\frac{-x}{2})] | P' \rangle d^4x
\]

(25)

where the initial nucleon state \( | P' \rangle \) is the eigenstate of the operator \( \hat{P} \) with the eigenvalue \( P' \) and the eigenstate of the spin operators \( \hat{S}^2 \) and \( \hat{S}^z \) which are constructed from \( \hat{M}^{\mu\nu} \). In particular, \( \hat{P}^2 | P' \rangle = m^2 | P' \rangle \) where \( m \) is the nucleon mass. Therefore the four-momentum operator indeed necessarily depends on the nonperturbative part of the interaction which is responsible for binding of quarks and gluons in the nucleon.

Suppose that the Hamiltonian \( \hat{P}^0 \) contains the nonperturbative part of the quark-gluon interaction and consider the well-known relation \([\hat{M}^{0i}, \hat{P}^k] = -i\delta_{ik}\hat{P}^0 \ (i, k = 1, 2, 3)\). Then it is obvious that if \( \hat{P}^k = P^k \) then all the operators \( \hat{M}^{0i} \) inevitably contain the nonperturbative part and vice versa, if \( \hat{M}^{0i} = M^{0i} \) then all the operators \( \hat{P}^k \) inevitably contain this part. Therefore in the instant form all the operators \( \hat{M}^{0i} \) inevitably depend on the nonperturbative part of the quark-gluon interaction and in the point form all the operators \( \hat{P}^k \) inevitably depend on this part. In the front form the fact that all the operators \( \hat{M}^{-j} \) inevitably depend on the nonperturbative part follows from the relation \([\hat{M}^{-j}, \hat{P}^l] = -i\delta_{jl}\hat{P}^+ \ (j, l = 1, 2)\). Of course, the physical results should not depend on the choice of the form of dynamics and in the general case all ten generators can depend on the nonperturbative part of the quark-gluon interaction.

In turn, as follows from Eq. (3) and the results of Sec. 4, the operators \( \hat{J}^\mu (x) \) in the instant form and \( \hat{J}^\mu (x^-, x_\perp) \) in the
front one inevitably depend on the nonperturbative part of the quark-gluon interaction. If it is possible to define $\hat{J}^\mu$ in the point form then as follows from Eq. (7), the relation $\hat{J}^\mu = J^\mu$ does not contradict Lorentz invariance but, as follows from Eq. (5), the operator $\hat{J}^\mu(x)$ in that form inevitably depend on the non-perturbative part. The fact that the same operators ($\hat{P}^\mu, \hat{M}^{\mu\nu}$) describe the transformations of both the operator $\hat{J}^\mu(x)$ and the state $|P\rangle$ guaranties that $W^{\mu\nu}$ has the correct transformation properties.

We see that the relation between the current operator and the state of the initial nucleon is highly nontrivial. Meanwhile in the present theory they are considered separately. In the framework of the approach based on Feynman diagrams the possibility of the separate consideration follows from the factorization theorem [18] which asserts in particular that the amplitude of the lepton-parton interaction entering into diagrams dominating in DIS depend only on the hard part of this interaction. Moreover, in leading order in $1/Q$, where $Q = |q|^2^{1/2}$, one obtains the parton model up to anomalous dimensions and perturbative QCD corrections which depend on $\alpha_s(Q^2)$ where $\alpha_s$ is the QCD running coupling constant.

It is well-known that the parton model is equivalent to impulse approximation (IA) in the infinite momentum frame (IMF). This fact is in agreement with our experience in conventional nuclear and atomic physics according to which in processes with high momentum transfer the effect of binding is not important and the current operator can be taken in IA. However this experience is based on the nonrelativistic quantum mechanics where only the Hamiltonian is interaction dependent and the other nine generators of the Galilei group are free. Note also that in
the nonrelativistic case the kinetic energies and the interaction operators in question are much smaller than the masses of the constituents.

Let us now discuss the following question. Since the current operator depends on the nonperturbative part of the quark-gluon interaction then this operator depends on the integrals from the quark and gluon field operators over the region of large distances where the QCD running coupling constant $\alpha_s$ is large. Is this property compatible with locality? In the framework of canonical formalism compatibility is obvious but, as shown in the preceding section, the results based on canonical formalism are not reliable. Therefore it is not clear whether in QCD it is possible to construct local electromagnetic and weak current operators beyond perturbation theory. However the usual motivation of the parton model is that, as a consequence of asymptotic freedom (i.e. the fact that $\alpha_s(Q^2) \to 0$ when $Q^2 \to \infty$), the partons in the IMF are almost free and therefore, at least in leading order in $1/Q$, the nonperturbative part of $\hat{J}^\mu(x)$ is not important. We will now consider whether this property can be substantiated in the framework of the OPE.

In this framework the commutator of the currents entering into Eq. (25) can be written symbolically as

$$[\hat{J}(\frac{x}{2}), \hat{J}(-\frac{x}{2})] = \sum_i C_i(x^2) x_{\mu_1} \cdots x_{\mu_n} \hat{O}^{\mu_1 \cdots \mu_n}_i$$

(26)

where $C_i(x^2)$ are the $c$-number Wilson coefficients while the operators $\hat{O}^{\mu_1 \cdots \mu_n}_i$ depend only on field operators and their covariant derivatives at the origin of Minkowski space and have the same form as in perturbation theory. The basis for twist two
operators contains in particular

\[ \hat{O}_V^\mu = \mathcal{N}\{\hat{\psi}(0)\gamma^\mu\hat{\psi}(0)\} \quad \hat{O}_A^\mu = \mathcal{N}\{\hat{\psi}(0)\gamma^\mu\gamma^5\hat{\psi}(0)\} \quad (27) \]

where for simplicity we do not write flavor operators and color and flavor indices.

As noted above, the operator \( \hat{J}^\mu(x) \) necessarily depends on the nonperturbative part of the quark-gluon interaction while Eq. (26) has been proved only in the framework of perturbation theory. Therefore if we use Eq. (26) in DIS we have to assume that either nonperturbative effects are not important to some orders in \( 1/Q \) and then we can use Eq. (26) only to these orders (see e.g. ref. [19]) or it is possible to use Eq. (26) beyond perturbation theory. The question also arises whether Eq. (26) is valid in all the forms of dynamics (as it should be if it is an exact operator equality) or only in some forms.

In the point form all the components of \( \hat{P} \) depend on the nonperturbative part of the quark-gluon interaction and therefore, in view of Eqs. (4) or (5), it is not clear why there is no nonperturbative part in the \( x \) dependence of the right hand side of Eq. (26), or if (for some reasons) it is possible to include the nonperturbative part only into the operators \( \hat{O}_i \) then why they have the same form as in perturbation theory.

One might think that in the front form the \( C_i(x^2) \) will be the same as in perturbation theory due to the following reasons. The value of \( q^- \) in DIS is very large and therefore only a small vicinity of the light cone \( x^+ = 0 \) contributes to the integral (25). The only dynamical component of \( \hat{P} \) is \( \hat{P}^- \) which enters into Eq. (26) only in the combination \( \hat{P}^- x^+ \). Therefore the dependence of \( \hat{P}^- \) on the nonperturbative part of the quark-gluon interaction is of no importance. These considerations are
not convincing since the integrand is a singular function and the operator $\tilde{J}^\mu(x^-, x_\perp)$ in the front form depends on the nonperturbative part, but nevertheless we assume that Eq. (26) in the front form is valid.

If we assume as usual that there is no problem with the convergence of the OPE series then experiment makes it possible to measure each matrix element $\langle P' | \hat{O}_V^{\mu_1\cdots\mu_n} | P' \rangle$. Let us consider, for example, the matrix element $\langle P' | \hat{O}_V^\mu | P' \rangle$. It transforms as a four-vector if the Lorentz transformations of $\hat{O}_V^\mu$ are described by the operators $\hat{M}^\mu\nu$ describing the transformations of $| P' \rangle$, or in other words, by analogy with Eq. (7)

$$[\hat{M}^\mu\nu, \hat{O}_V^\rho] = -i(g^{\mu\rho} \hat{O}_V^\nu - g^{\nu\rho} \hat{O}_V^\mu)$$

(28)

It is also clear that Eq. (28) follows from Eqs. (1), (3) and (25). Since the $\hat{M}^{-j}$ in the front form depend on the nonperturbative part of the quark-gluon interaction, the results of Sec. 4 make it possible to conclude that at least some components $\hat{O}_V^\mu$, and analogously some components $\hat{O}_V^{\mu_1\cdots\mu_n}$, also depend on the nonperturbative part. Since Eq. (28) does not contain any $x$ or $q$ dependence, this conclusion has nothing to do with asymptotic freedom and is valid even in leading order in $1/Q$ (in contrast with the statement of the factorization theorem [18]). Since the struck quark is not free but interacts nonperturbatively with the rest of the target then, in terminology of ref. [1], not only "handbag" diagrams dominate in DIS but some of "cat ears" diagrams or their sums are also important (in other words, even the notion of struck quark is questionable).

Since the operators $\hat{O}_V^{\mu_1\cdots\mu_n}$ depend on the nonperturbative part of the quark-gluon interaction, then by analogy with the above considerations we conclude that the operators in Eq. (27)
are ill-defined and the correct expressions for them involve integrals from the field operators over large distances where the QCD coupling constant is large. Therefore it is not clear whether the operators $\hat{O}_{i}^{\mu_{1}...\mu_{n}}$ are local and whether the Taylor expansion at $x = 0$ is correct, but even it is, the expressions for $\hat{O}_{i}^{\mu_{1}...\mu_{n}}$ will depend on higher twist operators which contribute even in leading order in $1/Q$.

6 Discussion

As follows from the results of Sec. 4, the current operator non-trivially depends on the nonperturbative part of the interaction responsible for binding of quarks and gluons in the nucleon. Then the problem arises whether it is possible to construct a local current operator $\hat{J}^{\mu}(x)$ beyond perturbation theory and whether the nonperturbative part of the interaction entering into $\hat{J}^{\mu}(x)$ contributes to DIS. Our consideration shows that the dependence of $\hat{J}^{\mu}(x)$ on the nonperturbative part of the interaction makes the OPE problematic. Nevertheless we assume that Eq. (26) is valid beyond perturbation theory but no form of the operators $\hat{O}_{i}^{\mu_{1}...\mu_{n}}$ is prescribed. Then we come to conclusion that the nonperturbative part contributes to DIS even in leading order in $1/Q$.

To understand whether the OPE is valid beyond perturbation theory several authors (see e.g. ref. [4] and references therein) investigated some two-dimensional models and came to different conclusions. We will not discuss the arguments of these authors but note that the Lie algebra of the Poincare group for 1+1 space-time is much simpler than for 3+1 one. In particular, the
Lorentz group is one-dimensional and in the front form the operator \( M^{+-} \) is free. Therefore Eqs. (7) and (28) in the ”1+1 front form” do not make it possible to conclude that the operators \( \hat{J}^{\mu} \) and \( \hat{O}^{\mu}_{V} \) necessarily depend on the nonperturbative part of the quark-gluon interaction. At the same time the full space reflection \( P \) in the 1+1 front form is an independent dynamical transformation, in contrast with the situation in the 3+1 front form (see Sec. 4).

Note also that in solvable models considered in the literature the operators \( \hat{P}^{\mu}, \hat{M}^{\mu\nu} \) were not explicitly constructed; in particular, it is not clear whether the current operators in these models satisfy the conditions (1,2).

Since the operators \( \hat{O}^{\mu_{1}...\mu_{n}}_{i} \) in Eq. (27) should depend on the nonperturbative part of the quark-gluon interaction then, as noted above, there is no reason to think that these operators are local but even if they are then twist (dimension minus spin) no longer determines in which order in \( 1/Q \) the corresponding operator contributes to DIS. This is clear from the fact that the dependence on the nonperturbative part implies that we have an additional parameter \( \Lambda \) with the dimension of momentum where \( \Lambda \) is the characteristic momentum at which \( \alpha_{s}(\Lambda^{2}) \) is large.

Nevertheless if we assume that (for some reasons) Eq. (26) is still valid and consider only the \( q^{2} \) evolution of the structure functions then all the standard results remain. Indeed the only information about the operators \( \hat{O}^{\mu_{1}...\mu_{n}}_{i} \) we need is their tensor structure since we should correctly parametrize the matrix elements \( \langle P' | \hat{O}^{\mu_{1}...\mu_{n}}_{i} | P \rangle \). However the derivation of sum rules in DIS requires additional assumptions.

Let us consider sum rules in DIS in more details. It is well-known that they are derived with different extent of rigor. For
example, the Gottfried and Ellis-Jaffe sum rules [20] are essentially based on model assumptions, the sum rule [21] was originally derived in the framework of current algebra for the time component of the current operator while the sum rules [22] also involve the space components. As shown in Sec. 4, the operator \( \hat{J}(x) \) is necessarily interaction dependent and there exist models in which \( \hat{J}^0(x) \) is free. Therefore in the framework of current algebra the sum rule [21] is substantiated in greater extent than the sum rules [22] (for a detailed discussion see refs. [23, 8]). Now the sum rules [21, 22] are usually considered in the framework of the OPE and they have the status of fundamental relations which in fact unambiguously follow from QCD. However the important assumption in deriving the sum rules is that the expression for \( \hat{O}_V^\mu \) coincides with \( \hat{J}^\mu \), the expression for \( \hat{O}_A^\mu \) coincides with the axial current operator \( \hat{J}_A^\mu \) etc. (see Eqs. (6) and (27)). Our results show that this assumption has no physical ground. Therefore although (for some reasons) there may exist sum rules which are satisfied with a good accuracy, the statement that the sum rules [21, 22] unambiguously follow from QCD is not substantiated.

For comparing the theoretical predictions for the sum rules with experimental data it is also very important to calculate effects in next-to-leading order in \( 1/Q \). As shown in ref. [24] there exist serious difficulties in calculating such effects in the framework of the OPE, and the authors of ref. [24] are very pessimistic about the possibility to overcome these difficulties (while in our approach problems exist even in the leading order).

The current operator satisfying Eqs. (1) and (3) can be explicitly constructed for systems with a fixed number of interacting relativistic particles [23]. In such models it is clear when the
corresponding results and the results in IA are similar and when they considerably differ [20].

We conclude that the present theory of DIS based on perturbative QCD does not take into account the dependence of the current operator on the nonperturbative part of the quark-gluon interaction which cannot be neglected even in leading order in $1/Q$. On the other hand, as already noted, the present theory has proven rather successful in describing many experimental data. It is very important to understand why this situation takes place.

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