Scattering of Several Multiply Charged Extremal $D = 5$ Black Holes

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Abstract

The moduli space metric for an arbitrary number of extremal $D = 5$ black holes with arbitrary relatively supersymmetric charges is found.
I. INTRODUCTION

One of the major advances in string theory over the last year and a half has been the new understanding of the properties of extremal, and some non-extremal, black holes from a microscopic perspective. This began with the microscopic D-brane counting of the entropy of some simple five-dimensional black holes, by Strominger and Vafa [1]. The entropy of innumerable other black holes has since been counted in this way—for review and references see e.g. [2,3]. Hawking radiation, including grey-body factors, has also been successfully derived from the microscopic string theory [4].

In this paper, we will explore the scattering of certain five-dimensional extremal black holes that carry charges of more than one $U(1)$. For most of the paper, we will only discuss the classical, supergravity, scattering; most of the D-brane calculation is left for future work. The standard approach to the supergravity calculation follows the monopole scattering of Manton [5]. The idea is that, for extremal black holes, supersymmetry ensures that there is no force between static black holes; hence any configuration of $N$ black holes has the same energy. So, the positions of the black holes are the $(D - 1)N$ moduli of this sector of the theory (where $D$ is the space-time dimension of the theory), and the slow motion of the black holes is governed by the metric of the moduli space, which is just the kinetic term of the low energy effective action. The first such calculation was done for $D = 4$ extremal Reissner-Nordström black holes in [6]. Shiraishi [7] then generalized this to theories with a dilaton, in arbitrary dimension, and with arbitrary coupling of the dilaton to the $U(1)$ vector field. Khuri and Myers [8] then generalized this result to higher-dimensional branes. In particular, they found that for scattering of branes which admit kappa-symmetric world-volume actions—and so, in particular, for scattering of identical D-branes—there is no interaction; the metric on the moduli space is flat.

More recently, Douglas, Polchinski and Strominger [9] calculated the scattering of D-brane probes off a $D = 5$ extremal black hole that carried three $U(1)$ charges. They considered this from both a supergravity and a D-brane point of view. Their supergravity
calculation was performed by expanding the D-brane (Born-Infeld) action in the background of the black hole. The D-brane calculation involves reducing the black-hole–D-string probe system to a 1+1-dimensional superconformal effective Yang-Mills theory. This calculation correctly reproduced the leading-order supergravity $\frac{q^2 v^2}{r^2}$ interaction, but not the (same order in velocities but weaker) $\frac{q^3 v^2}{r^4}$ interaction. This is still an unresolved problem.

With this motivation, we scatter these black holes off each other, not probes. Hence, this is again a Manton-type calculation. In section II we discuss the string/supergravity theory and the black hole solution. In section III we give the calculation and the result. Our result agrees, in the appropriate limits, with the previous results of [7,8,9]. In section IV we discuss coalescence of two black holes. Section V concludes with some comments about D-brane approaches.

II. SUPERGRAVITY SOLUTION

We will analyze scattering of the black holes of [11]. The black holes arise in compactification of type IIB string theory on $T^5$. All black holes in this theory are U-dual to a black hole with at most three non-zero charges. Black holes with less than three charges are singular at the horizon—the dilaton (or another scalar) blows up there. Even for supersymmetric multi-black hole solutions, one can always perform a U-duality transformation so that only three $U(1)$s are turned on. For most of this paper we will, for simplicity, consider only black holes which carry 10-dimensional NS-NS 5-brane charge and 1-brane (fundamental string) charge, and momentum along the 1-brane. U-duality, and the fact that the cubic invariant of $E_6$ is unique, will then give the moduli space metric for generic (supersymmetric) black

1 After this work was completed, ref. [10] appeared which discusses some further special cases.

2 It is inevitable that someone will worry that the duality group is actually $E_6(\mathbb{Z})$, for which little is known, including uniqueness of the cubic invariant. However, type IIB supergravity is actually invariant under the full $E_6$ duality, and furthermore, the classical black hole solutions can
hole configurations. This will be discussed further below. The ten-dimensional supergravity solution corresponding to these black holes is easily obtained by an S-duality transformation of the corresponding RR-charged black holes of [12,13,9].

Following the dimensional reduction procedure of [14,15], the relevant bosonic terms in the 5-dimensional Einstein action are found to be

\[
S = \frac{1}{16\pi G} \int d^5x \sqrt{-g} \left\{ R - \frac{4}{3} (\partial_\mu \varphi)^2 + \frac{1}{8} \text{Tr}(\partial_\mu M L \partial^\mu M L) \\
- \frac{1}{4} e^{\frac{2}{3}\varphi} \tilde{F}_\mu \tilde{F}^{\mu} - \frac{1}{4} e^{-\frac{4}{3}\varphi} \mathcal{F}^i (LML)_{ij} \mathcal{F}^{j\mu\nu} \right\} .
\]

(2.1)

Here, \( \tilde{F} = d\tilde{A} \) is the \( U(1) \) field strength that is dual (in 5-dimensions) to the NS-NS antisymmetric tensor field; an NS-NS 5-brane which wraps the \( T^5 \) appears to the 5-dimensional observer to be carrying electric \( \tilde{A} \) charge. The \( U(1) \) field strengths \( \mathcal{F}^i = dA^i, i = \{1, \ldots, 10\} \), are Kaluza-Klein fields. The first five come from \( \hat{g}_{\mu I} \), and the second five come from \( \hat{B}_{\mu I} \) where \( \hat{g} \) and \( \hat{B} \) are respectively the 10-dimensional metric and NS-NS antisymmetric tensor, \( \mu, \nu, \ldots = 0, \ldots, 4 \) are \( D = 5 \) space-time indices and \( I, J, \ldots = 5, \ldots, 9 \) label directions on the internal torus. In particular, the 1-branes are chosen to wind around the 5-direction, and hence carry electric \( A^6 \) charge, and the momentum on the string turns into electric \( A^1 \) charge. The 5-dimensional string coupling constant is \( g = e^\varphi \) where \( \varphi \) is the 5-dimensional dilaton. The matrix of scalars \( M_{ij} \) is given by

\[
M = \begin{pmatrix}
\mathcal{G}^{-1} & -\mathcal{G}^{-1}\mathcal{B} \\
-\mathcal{B}^T\mathcal{G}^{-1} & \mathcal{G} + \mathcal{B}^T\mathcal{G}^{-1}\mathcal{B}
\end{pmatrix},
\]

(2.2)

where \( \mathcal{G}_{IJ} \) and \( \mathcal{B}_{IJ} \) are the internal components of the \( D = 10 \) metric and antisymmetric tensor, respectively, and

\[
L = \begin{pmatrix}
0 & \mathbb{I} \\
\mathbb{I} & 0
\end{pmatrix},
\]

(2.3)

be continuously deformed to solutions with non-quantized charges. Continuity then requires that the moduli space metric be invariant under the full \( E_6 \).
Finally, $G$ is the 5-dimensional Newton constant.

The static multi-black hole solution with the desired charges is given by (we set $g = \alpha' = R = V = 1$, where $R$ is the radius of the circle in the 5-direction and $V$ is the volume of the torus in the 6–9-directions)

$$ds^2 = -\psi_1^{-\frac{5}{4}}\psi_5^{-\frac{5}{4}}\psi_R^{-\frac{5}{4}}dt^2 + \psi_1^{\frac{1}{4}}\psi_5^{\frac{1}{4}}\psi_R^{\frac{1}{4}}d\vec{x} \cdot d\vec{x},$$

$$\tilde{A} = \psi_5^{-1}dt,$$

$$A^1 = \psi_R^{-1}dt,$$

$$A^6 = \psi_1^{-1}dt,$$

$$e^\varphi = \psi_5^{\frac{1}{2}}\psi_1^{-\frac{1}{2}}\psi_R^{-\frac{1}{2}},$$

$$\mathcal{G}_{55} = \psi_1^{-1}\psi_R.$$}

All other fields are zero. The $\psi_\alpha$'s, ($\alpha = 1, 5, R$) are harmonic functions; for a solution with $N$ black holes,

$$\psi_\alpha = 1 + \sum_{a=1}^{N} \frac{q_{\alpha a}}{r_a^2},$$

where $q_{\alpha a}$ is the $\alpha$th charge of the $a$th black hole, and $\vec{r}_a$ is its position, or more precisely, the location of the horizon. This coordinate patch does not cover the interior of the black holes. This solution is BPS saturated and generically preserves $\frac{1}{8}$th of the original $N = 8$, 5-dimensional supersymmetries. If no black hole carries one or two of the three types of charges, then the preserved supersymmetry increases to $\frac{1}{4}$ or $\frac{1}{2}$. Saturation of the Bogomol’nyi bound gives the mass of the black holes as

$$m_a = q_{1a} + q_{5a} + q_{Ra}.$$}

The equations of motion for this solution require that the Laplacian of each $\psi_\alpha$ vanish. This is true everywhere but at the horizon, $\vec{x} = \vec{r}_a$. There is no contradiction, however, as the coordinate system is singular at the horizon; the equations of motion hold everywhere the coordinates are good. The equations of motion will be satisfied in a coordinate system which is non-singular at the horizon as well.
The reason we make this observation is that the black hole scattering calculation requires the regularization of self-interaction infinities. This is achieved by smoothing out the black holes by adding charged dust. To find the correct couplings and normalization, we add in sources at the horizons of the black holes in the equations of motion. Integrating the equations of motion gives

$$S_{\text{source}} = -\frac{4\pi^2}{16\pi G} \sum_{a=1}^{N} \int ds \left\{ -q_a R A^1_\mu \frac{dx^\mu_a}{ds} - q_{a1} A^6_\mu \frac{dx^\mu_a}{ds} - q_{a5} \bar{A}_\mu \frac{dx^\mu_a}{ds} ight.$$

$$+ e^{-\frac{4}{3} \phi} q_{a5} + e^{\frac{2}{3} \phi} G^{-\frac{1}{2}}_{55} q_{aR} + e^{\frac{2}{3} \phi} G^{-\frac{1}{2}}_{55} q_{a1} \right\}. \quad (2.7)$$

It must be emphasized that equation (2.7) is non-physical and we include it only to smooth it out for regularization. Smoothing out equation (2.7), gives the (bosonic part) of the dust action

$$S_{\text{dust}} = \frac{4\pi^2}{16\pi G} \int d^5 x \sqrt{-g} \left\{ \rho_R A^1_\mu u^\mu + \rho_1 A^6 u^\mu + \rho_5 \bar{A}_\mu u^\mu ight.$$

$$- e^{-\frac{4}{3} \phi} \rho_5 - e^{\frac{2}{3} \phi} G^{-\frac{1}{2}}_{55} \rho_R + e^{\frac{2}{3} \phi} G^{-\frac{1}{2}}_{55} \rho_1 \right\}. \quad (2.8)$$

Here, $u^\mu = \frac{dx^\mu}{dt}$ is the 5-velocity of the dust. Also, we have suppressed Lagrange multipliers and factors of $\sqrt{-u^\mu u_\mu} = 1$ that are required to obtain the correct equations of motion from equation (2.8).

**III. EFFECTIVE ACTION AND SCATTERING**

The scattering calculation is very similar to that of [6,7], but we will describe it in detail for completeness. When the velocities are small, the solution describing scattering black holes will be a perturbation around the solution (2.4) and (2.5). Furthermore, as there are no linear terms in the action, solving equations of motion to $O(\vec{v} = \frac{dx}{dt})$ is sufficient, when substituting back into the action, to get the effective action to $O(\vec{v}^2)$. Then, by Galilean invariance, the only perturbations of the fields are of the form

$$\delta ds^2 = 2\psi_1 - \frac{2}{3} \psi_5 - \frac{7}{3} \psi_R - \frac{2}{3} \tilde{Q} \cdot d\bar{x} dt, \quad (3.1a)$$

$$\delta \bar{A}^1 = \tilde{P}_R - \psi_R^{-1} \tilde{Q}, \quad (3.1b)$$
\[ \delta \vec{A}^6 = \vec{P}_1 \psi_1^{-1} \vec{Q}, \quad (3.1c) \]
\[ \delta \vec{A} = \vec{P}_5 \psi_5^{-1} \vec{Q}, \quad (3.1d) \]

where \( \vec{Q} \) and \( \vec{P}_\alpha \) are first-order quantities to be determined.

The \( g_{ti} \) and \( A_i \) equations of motion determine the exterior derivatives of \( \vec{Q} \) and \( \vec{P}_\alpha \) to be

\[ d\vec{Q} = -\psi_1 \psi_5 \psi_R \sum_\alpha \psi_\alpha^{-1} dK_\alpha, \quad (3.2a) \]
\[ d\vec{P}_\alpha = -\psi_1 \psi_5 \psi_\alpha^{-1} \sum_{\beta \neq \alpha} \psi_\beta^{-1} dK_\beta, \quad (3.2b) \]

where

\[ \vec{K}_\alpha \equiv \psi_\alpha \vec{v} = -4\pi^2 \vec{\nabla}^{-2} (\psi_1^{\frac{2}{3}} \psi_5^{\frac{2}{3}} \psi_R^{\frac{2}{3}} \rho_\alpha \vec{v}). \quad (3.2c) \]

Here we have used the equations of motion of the smoothed out action \( S + S_{\text{dust}} \),

\[ \vec{\nabla}^2 \psi_\alpha = -4\pi^2 \psi_1^{\frac{2}{3}} \psi_5^{\frac{2}{3}} \psi_R^{\frac{2}{3}} \rho_\alpha. \quad (3.3) \]

In principle, there should be functions of integration on the right hand sides of equations (3.2a) and (3.2b); however, the exterior derivative of the equations results in homogenous differential equations for the functions of integration, and they can therefore be consistently set to zero.

Substituting equations (3.2) and (3.1) into the action \( S + S_{\text{dust}} \) of equations (2.1) and (2.8) gives, after some tedious algebra and integration by parts

\[ S = \frac{1}{16\pi G} \int d^5x \left\{ \psi_1 \psi_5 \psi_R \left[ -\sum_{\alpha<\beta} \psi_\alpha^{-1} \psi_\beta^{-1} \partial_t \psi_\alpha \partial_t \psi_\beta + 2\pi^2 \psi_1^{\frac{2}{3}} \psi_5^{\frac{2}{3}} \psi_R^{\frac{2}{3}} \sum_\alpha \psi_\alpha^{-1} \rho_\alpha \vec{v}^2 \right] \right. \\
+ \left[ \sum_\alpha \partial_t \vec{P}_\alpha \cdot \vec{\nabla} \psi_\alpha + 4\pi^2 \sum_\alpha \psi_1^{\frac{2}{3}} \psi_5^{\frac{2}{3}} \psi_R^{\frac{2}{3}} \rho_\alpha \vec{P}_\alpha \cdot \vec{v} \right] \\
+ \psi_1^{-1} \psi_5^{-1} \psi_R^{-1} \left[ -\frac{1}{2} |d\vec{Q}|^2 - \frac{1}{4} \sum_\alpha \psi_\alpha^{-2} |d\vec{P}_\alpha|^2 + \frac{1}{2} \sum_\alpha \psi_\alpha d\vec{P}_\alpha \cdot d\vec{Q} \right] \left\} , \quad (3.4) \right. \]

up to total derivatives. We would like to write this entirely in terms of \( \psi_\alpha, \rho_\alpha, \vec{v}^2 \) and \( \vec{K}_\alpha \).

This is done by using equations (3.2), and by noting that time derivatives can be eliminated using \( \partial_t = -\vec{v} \cdot \nabla \).
Having done this, we take the black hole limit, which is equation \((2.3)\) and

\[
\psi_1 \frac{2}{3} \psi_2 \frac{3}{2} \psi_3 \frac{3}{2} \psi_5 \frac{3}{2} \rho_a \hat{v}_a^2 \to \sum_a q_{aa} \delta(4)(\vec{x} - \vec{r}_a)^2, \tag{3.5a}
\]

\[
\vec{K}_a \to \sum_a \frac{q_{aa}}{r_a^2} \vec{v}_a^2, \tag{3.5b}
\]
as clearly follows from equations \((3.3)\) and \((3.2c)\). Integrating over space gives the final result:

\[
S_{\text{eff}} = \frac{\pi}{4G} \int dt \left\{ -\sum_a m_a + \frac{1}{2} \sum_a m_a \vec{v}_a^2 + \frac{1}{2} \sum_{a,b} (q_{1a}q_{5b} + q_{1a}q_{RB} + q_{5a}q_{RB}) \frac{|\vec{v}_a - \vec{v}_b|^2}{|\vec{r}_a - \vec{r}_b|^2} \right.
\]
\[
+ \frac{1}{4} \sum_{a,b,c} (q_{1a}q_{5b}q_{RC} + q_{1a}q_{RB}q_{5c} + q_{5a}q_{RB}q_{1c}) |\vec{v}_a - \vec{v}_b|^2
\]
\[
\times \left[ \frac{1}{|\vec{r}_a - \vec{r}_b|^2|\vec{r}_a - \vec{r}_c|^2} + \frac{1}{|\vec{r}_a - \vec{r}_b|^2|\vec{r}_b - \vec{r}_c|^2} - \frac{1}{|\vec{r}_a - \vec{r}_c|^2|\vec{r}_b - \vec{r}_c|^2} \right] \right\}. \tag{3.6}
\]

Clearly, \(a = b\) does not contribute in either sum. Also, in the triple sum, when \(c = a\) or \(b\), the divergent terms cancel leaving a finite two-body interaction. We also note that if two of the \(N\) black holes are coincident with identical velocities, then equation \((3.6)\) gives the result for the corresponding configuration with \(N - 1\) black holes.

When \(q_{5a} = q_{1a} = q_{Ra}\) for each \(a\), the multi-black hole reduces to Shiraishi’s \(a = 0\) black hole \([7]\). In this case, equation \((3.4)\) agrees with the corresponding result of \([7]\). If two of the charges are zero—for example for scattering of 5-branes—then the scattering is trivial, as given in \([8]\). If one of the charges is zero, then equation \((3.6)\) reproduces the scattering of \((4||0)\)-brane bound states of \([10]\), when the latter is reduced from six to five dimensions. Also, if we only consider scattering of two black holes, and let the charges (and therefore the mass) of one of the black holes be much less than those of the other, the configuration reduces to that of \([9]\) and equation \((3.6)\) agrees with the results of that paper.

Equation \((3.6)\) is easily made U-duality invariant by comparison to \([11]\). After inserting factors of \(\alpha', g = e^{\varphi_\infty}\), where the subscript denotes evaluation at spatial infinity, and

\[
l_p = \left( \frac{g^2 \alpha'^4}{VR} \right)^{1/3} \tag{3.7}
\]

where \(R\) is the radius of the circle in the 5-direction, and \(V\) is the volume of the \(T^4\) in the
6–9 directions, and in particular, rescaling the charges appropriately (as was done carefully in [9]) the result is

$$S_{\text{eff}} = \int dt \left\{ -\sum_a m_a + \frac{1}{2} \sum_a m_a \vec{r}_a^2 + \frac{1}{2} \sum_{a<b} (m_a m_b \nu^3 - q_{\Lambda a} (\mathcal{M}_\infty^{-1})^{\Lambda \Sigma} q_{\Sigma b}) \frac{\vec{v}_a - \vec{v}_b}{|\vec{r}_a - \vec{r}_b|^2} \\
+ \frac{1}{4} \sum_{a<b} \sum_c d^{\Lambda \Sigma \Gamma} q_{\Lambda a} q_{\Sigma b} q_{\Gamma c} \vec{r}_p^3 |\vec{v}_a - \vec{v}_b|^2 \\
\times \left[ \frac{1}{|\vec{r}_a - \vec{r}_b|^2 |\vec{r}_a - \vec{r}_c|^2} + \frac{1}{|\vec{r}_a - \vec{r}_b|^2 |\vec{r}_b - \vec{r}_c|^2} - \frac{1}{|\vec{r}_a - \vec{r}_c|^2 |\vec{r}_b - \vec{r}_c|^2} \right] \right\}, \quad (3.8)$$

where $d^{\Lambda \Sigma \Gamma}$ is proportional to the cubic $E_6$ invariant, and $\mathcal{M}$ is the kinetic matrix for the vector fields in the full, manifestly U-dual, 5-dimensional Einstein bosonic supergravity action.

**IV. GEODESIC MOTION AND BLACK HOLE CAPTURE**

If we consider the case of two black holes scattering off each other, the effective action reduces to

$$S_{2b} = \int dt \left\{ -M + \frac{1}{2} M \dot{V}^2 + \frac{1}{2} f(r) \vec{v}^2 \right\}, \quad (4.1)$$

where $M = m_1 + m_2$, $V = (m_1 v_1 + m_2 v_2)/M$, $r = |\vec{r}_1 - \vec{r}_2|$, $\vec{v} = \vec{v}_1 - \vec{v}_2$,

$$f(r) = \left[ \mu + \frac{\Gamma_{II}}{r^2} + \frac{\Gamma_{III}}{r^4} \right], \quad (4.2a)$$

$$\Gamma_{II} = (q_{1,1} q_{5,2} + q_{5,1} q_{R,2} + q_{R,1} q_{1,2}) + (1 \leftrightarrow 2), \quad (4.2b)$$

$$\Gamma_{III} = (q_{1,1} q_{5,2} q_{R,2} + q_{R,1} q_{1,2} q_{5,2} + q_{5,1} q_{R,2} q_{1,2}) + (1 \leftrightarrow 2), \quad (4.2c)$$

and $\mu$ is the reduced mass. This is the appropriate generalization of the results of [9]. Therefore, the metric on the two-body moduli space is (neglecting center of mass motion)

$$ds_{2b}^2 = f(r)(dr^2 + r^2 d\Omega^2) \quad (4.3)$$

As noted in [9], although this metric is euclidean, and therefore has no horizons, generically there is a second asymptotic region around $r = 0$. This can be seen by transforming to a new radial coordinate $\rho = 1/r$, in which the metric takes the form
\[ ds_{2b}^2 = \left( \Gamma_{III} + \frac{\Gamma_{II}}{\rho^2} + \frac{\mu}{\rho^4} \right) (d\rho^2 + \rho^2 d\Omega^2). \] (4.4)

The analysis of geodesic motion in this moduli space proceeds in exactly the same manner as in \cite{9}. Given two black holes approaching each other with asymptotic velocity \( v_\infty \) and impact parameter \( b \), there is a critical impact parameter \( b_c \) (independent of \( v_\infty \)) below which the black holes must coalesce. This is given by

\[ \frac{b_c^2}{\mu} = \Gamma_{II} + 2\sqrt{\mu \Gamma_{III}}. \] (4.5)

Black holes which scatter at this critical impact parameter will orbit each other at radius

\[ r_c^2 = \sqrt{\frac{\Gamma_{III}}{\mu}}. \] (4.6)

V. DISCUSSION

We have derived the U-duality invariant metric on the multi-black hole moduli space from the supergravity point of view. It would be interesting to derive this from a D-brane perspective. We will conclude this paper by discussing a couple of possible approaches.

One approach is a simple generalization of the D-brane probe calculations of \cite{9}. The main observation is that the calculation of \cite{9} involves a \( U(q_5) \), 1 + 1-dimensional gauge theory, that is essentially a dimensionally reduced 5-brane worldvolume theory, with extra fields due to the 1-branes and their excitations. There were also extra fields representing the D-string probe and its interactions. Considering instead multiple black holes essentially only requires consideration of a \( U(\prod_a q_{5a}) \) gauge theory, that is Higgsed to \( U(q_{51}) \times \ldots \times U(q_{5N}) \). However, this calculation is (by construction!) so similar to the one performed in \cite{9} that it will most likely similarly fail to reproduce the cubic interaction.

An alternative approach is motivated by the observation that the cubic interaction involves factors of \( \frac{1}{\rho^2} \) that are suggestive of strings stretched between D-branes. Therefore, it is reasonable to hypothesize that a string analysis of three spatially separated black holes,
each with a single non-trivial charge turned on, may be able to reproduce the cubic inter-
action. This analysis has not been done, and is qualitatively different from that of [4], and 
therefore is an interesting direction for future research.

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