Multifunction acoustic modulation by a multi-mode acoustic metamaterial architecture

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Abstract
Exotic acoustical features, like acoustic transparency, ultrasonic beam focusing, acoustic band gap and super lensing capability using a single metamaterial architecture is unconventional and unprecedented in the literature, demonstrated herein. Conventional metamaterials can focus an ultrasonic beam at specific frequency which results into unwanted distortion of the output wave fields at neighboring sonic frequencies in the host medium. However, ultrasonic wave focusing by virtue of negative refraction and simultaneous transparency of the metamaterial at sonic frequencies are uncommon due to their frequency disparity. To circumvent this problem and to avoid the unwanted distortion of wave at sonic frequencies, metamaterial with an array of butterfly-shaped thin ring resonators are proposed to achieve the beam focusing at ultrasonic frequency (37.3 kHz) and keep the structure transparent to the sonic frequencies (< 20 kHz). The butterfly metamaterial with local ring resonators or butterfly crystals (BC) were previously proposed to create wide band gaps (~7 kHz) at ultrasonic frequencies above 20 kHz. However, in this study a unique sub-wavelength scale wave focusing capability of the butterfly metamaterial utilizing the negative refraction phenomenon is demonstrated, while keeping the metamaterial block transparent to the propagating wave at lower sonic frequencies below the previously reported bandgaps.

Introduction
The last two decades have witnessed unique and careful design of several artificially engineered composite materials called metamaterials. The composite metamaterials are envisioned for various practical applications to manipulate the acoustic waves [1–9] in a unique way. Periodic structures both in photonics and phononics, are capable of significantly alter the wave propagation phenomena in the host media. Thus, it inspired a large group of researchers to realize several mechanisms for wave front modulation [10, 11] using the periodic structures. Some of the practical application includes wave focusing [12], sound isolation, energy harvesting [13, 14], ultrasonography etc. However, most of the research were performed to manipulate the acoustical waves at specific and/or predefined frequencies [15, 16]. The presence of periodic metamaterial structures not only generates unique feature at the designed frequency, it also alters the wave fields inside and outside the metamaterial at off-designed frequencies (outside the band of specified designed frequency), in an unintended/uncontrolled way. Eventually, this limits the application of the metamaterials for wide frequency applications.

To solve this issue, here in this article it is proposed that the periodic structures need to be designed in such a way that the metamaterial system should not affect the propagating wave field until the designed frequency is reached. That means that the metamaterial system should behave as an acoustically transparent media, except at the desired frequency at which it is expected to behave as metamaterial and demonstrate a specific designed phenomenon. The range of frequencies where the incident waves can transmit without any distortion can be called 'Acoustically Transparent Range'.

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On the other hand, the possibilities of focusing the elastic waves through metamaterials, led the researchers to design acoustic phononic [17] lenses embedding intricate geometrical variations in host materials. Similar approaches were also adopted for photonic lenses [18, 19] in electromagnetic applications. Many researchers have successfully demonstrated the acoustic flat lenses using phononic crystals in a periodic fashion. Fascinating wave propagation phenomena such as single and double negative refraction [20, 21], orthogonal wave transportation [7], non-diffracting Bessel beam [22], sub-wavelength scale wave focusing [22] and multiple wave scattering etc, were demonstrated by various periodic structures. Recently, a topologically optimized [23] two-dimensional acoustic lens was successfully implemented in underwater imaging. While the applications of these metamaterials are fully realized, acoustic wave focusing may have larger impact with ultrasonic frequencies in biomedical imaging and surgery [24]. It is established that the resolution of the conventional flat lens is limited by its diffraction limit due to the diminishing evanescent component of the propagating waves [25]. Hence, the possibility of achieving the subwavelength information is a topic of interest, if can be achieved by the metamaterials. So far, one of the promising methods that has been proposed is to utilize the negative refraction property of the meta structures with constituent phononic crystals to construct the Super-lenses [26, 27]. Although the concept of Super-lenses was first coined in photonics, it is now also exploited in acoustical studies.

Inspired by the acoustic wave manipulation phenomenon, the authors realized the need for a multifunctional and multi-mode acoustic metamaterial architecture. In this article, a butterfly shaped engineered metamaterial consists of an array of stainless steel split ring resonators of different sizes embedded in epoxy matrix [28] is envisioned for the wave modulations discussed above. Many researchers designed metamaterial architectures that demonstrated a single acoustic feature, whereas just by using one meta structure orientation, Butterfly metamaterial is proposed to simultaneously demonstrate multiple acoustic features such as acoustic transparency, wave focusing and superlensing at respective different frequencies. To achieve the acoustic features and create the dependencies at multiple frequencies, individual full and split ring resonators that create the local anisotropy at multiple scales is introduced in a butterfly shape. To achieve the transparency at lower frequencies it is necessary to have near isotropic behavior of the meta structure in global scale, whereas to achieve wave focusing and superlensing behavior an anisotropy needs to be created at the local scale. Hence, the shape and the orientation of the proposed model are purposefully designed for acoustic wave bifurcation and focusing of the wave field, when the geometrical configurations (e.g. ring thickness) and the number of repeating unit cells are constructed for low frequency acoustic transparency. Furthermore, the presence of wave focusing capability dictates the negative refraction property of the structure which resulted superlensing phenomenon.

This article is divided into four sections. Initially, eigenfrequency analysis has been reported to identify the dispersion behavior of the proposed butterfly structure within the first Brillouin zone. In section two, a frequency domain study has been performed to validate the simulated configuration. As no wave should pass through the bandgaps, a material block constructed with butterfly structure shows that indeed the wave was totally blocked at the band gap frequencies. Also, acoustic transparency behavior of the proposed structure and a frequency domain study below ~18 kHz are demonstrated in this section. In section three, the modal behaviors around ~37 kHz is identified and a possible wave bifurcation and wave focusing capabilities have been predicted in the structure. A frequency domain study was performed to confirm the wave focusing phenomenon, and subsequently the range of focusing frequencies were identified. The last section of this article identifies the negative refraction phenomenon and the superlensing capability of the proposed butterfly structure.

**Dispersion curve of the butterfly structure**

Since the mode shapes and the respective group velocities are crucial parameters to understand the wave propagation through a structure, a dispersion relation for the proposed unit cell (figure 1 (a)) is obtained by performing an eigenfrequency analysis of the structure proposed. The geometric dimensions and the material specifications reported in reference [28] are used in this article and are summarized in table 1. One of the objectives of selecting this unique butterfly structure is the presence of local geometric anisotropy at the material level but possess the near isotropic behavior at the global scale. While performing the eigenfrequency study, a rectangular periodicity of the unit cell is assumed. Using Comsol Multiphysics V4.3 software, the dispersion relation is obtained by performing eigenfrequency analysis for different wave vector directions. The dispersion band structure shown in figure 1 (b) was computed for the MGX boundary of the first Brillouin zone [29] (figure 1 (c)) using the Bloch-Floquet periodic boundary condition [30].

While investigating the figure 1 (b), a large band gap from ~26 kHz to ~32 kHz is evident which was also reported previously in reference [28]. A further investigation of the figure 1 (b) reveals the near linear dispersion (frequency versus normalized wave vector) relation below ~18 kHz, irrespective of ΓX (dashed window) or XM directions. This demonstrates the near isotropic behavior which is found in classical bulk isotropic materials [31]. Applying similar Bloch-Floquet periodic boundary condition on the base Epoxy material (i.e. without the
butterfly constituents), dispersion relation was computed at the M–XM boundary of the first Brillouin zone (figure 1(d)). Upon comparing the dispersion relation in ΓX (dashed window) or XM direction for both the butterfly structure and the base Epoxy material, it is evident that both geometries have similar dispersion behavior. Hence an infinitely repeated butterfly unit cell placed in a 2D epoxy medium can act as a single isotropic material, and the presence of these unit cells will almost be unrealized below the ∼18 KHz. Since the linear dispersion behavior is an indicative of spherical wave fronts in isotropic materials, it can be assumed that the proposed butterfly structure will disperse the transmitted waves linearly and will result nearly undisturbed wave fronts below ∼18 KHz. As a result, the proposed structure will be acoustically transparent within this frequency range.

**Simulation setup to prove the ‘acoustic transparency’**

To concrete the possibility of introducing ‘Acoustic Transparency’ using the proposed butterfly metamaterial model, a frequency domain simulation was performed. However, to verify the simulation model, initially a
frequency domain study was performed choosing an arbitrary frequency from the bandgap region. Since incident plane waves from any direction do not propagate through the material made of butterfly shaped crystals (BC) at the band gap frequencies, a frequency domain study should reveal this phenomenon and would simultaneously verify the accuracy of the model. To achieve this objective, a 112 mm × 100 mm epoxy plate was modeled using COMSOL Multiphysics V4.3. A butterfly crystal (BC) region, designed by an array of unitary cells identical to the one considered in figure 1(a), is placed in the base plate. In particular, the BC arrangement consists of 22 rows and 6 columns of butterfly unit cells. A plane wave front was generated by the periodic displacement of a rectangular source with a dimension of 127 × 1.27 mm². A perfectly matched layer boundary condition was considered at all the boundaries of the base plate domain to approximate negligible wave reflection from all the edges. Figure 2(a) shows the geometric configuration of the setup without an actuation of the rectangular exciter. Figure 2(b) shows the simulation outcome performed between the frequency ranges ~26 kHz to ~32 kHz (only one frequency at 30 kHz in shown in the figure 2(b)). It is clearly evident that no displacement is observed neither at the BC nor at the Epoxy and the host material on the right. This confirms that no wave is transmitted through the BC within the band gap region.

Next, to demonstrate the acoustic transparency, plane crested wave was generated at the audible frequency range in Γ–X direction (along x-direction). Additionally, the unit cells made of base Epoxy material having the butterfly structure was considered. Simulation was performed between 0 and ~20 kHz frequency within the audible frequency range. Results of these two simulations are presented in figure 3 to have a visible comparison between the wave fields generated in the base material with and without the presence of the BC. Normalized total displacement amplitudes are plotted in figure 3. Figure 3 shows the wave fields (total normalized displacement) at four acoustic frequencies with and without the BC arrangements.

Comparing figures 3(a-1) and (a-2), at frequency ~5 kHz, it is evident that very similar circular wave fronts are generated in the base material and in the material with the BCs. This indicates that the presence of BC region does not affect the wave propagation at ~5 kHz. Similarly, at ~15 kHz and ~18 kHz, transmission of the circular wave fronts is also visible. Despite the presence of the BCs, patterns in the wave fields are unaffected which are shown in figures 3(b) and (c), when the microstructures are comparable to the respective wave lengths at respective frequencies. While the presence of the butterfly metamaterial region is unrealized by the incident waves at or below ~18 kHz, circular wave front starts to alter due to the presence BCs at or beyond the ~19 kHz, i.e. towards the end of the audible frequency range. At ~20 kHz, the effect (figures 3(d-1) and (d-2)) of BC is strongly realized. Therefore, the proposed structure acts as an acoustically transparent media throughout the 90% of the audible frequency range but demonstrate meta structure features beyond ~20 KHz. It is designed such a way that the individual resonant frequencies of the steel-balls, split rings and the closed elliptical steel rings are achieved at frequencies higher than ~20 kHz, which resulted in acoustic transparency feature below the ultrasonic range. In ultrasonic frequency range (>20 KHz), the propagation of the wave fronts and the wave fields are significantly affected by the BC region and, as an example, a large band gap region is shown in figure 1(b). The acoustic transparency feature is particularly important in such cases where the effect of BC is desirable only in the ultrasonic frequency ranges when the material block is undetectable at the audible ranges.

Figure 2. (a) Geometric configuration without excitation, (b) simulation of wave field at a frequency from the band gap at a frequency of 30 kHz.
Analysis of mode shapes for wave bifurcation and wave focusing

In this section higher order dispersion curves, i.e. frequencies beyond the complete band gap, are studied along the ΓX and MΓ direction inside the first Brillouin zone. During this analysis, the concept of acoustic energy of a propagating elastic wave which directly depends on the group velocity was utilized, qualitatively. It is well known that higher group velocity of a propagating wave results higher acoustical energy \[32\]. To understand the qualitative measure of the acoustic energy, the mode shapes at different points on the dispersion curves at the frequency ranges between \(\sim 37.085\) kHz and \(\sim 37.43\) kHz were analyzed. In figure 4, a portion of the dispersion curve in ΓX direction along with the mode shapes at \(\sim 37.3\) kHz is presented. In the top section of figure 4, mode shapes at points 'a' through 'f' are shown next to the frequency versus normalized wave vector plot in ΓX direction. Here, three distinct types of mode shapes are identified. Mode shapes at points 'a', 'b' and 'c', located on mode 18, have the same type of particle displacement at the unit cell level. Similarly, points 'd' and 'e' are marked on mode 17 and have the similar displacement pattern. The last point 'f' which is on the mode 16 has different mode shape than that of all the other points. While points 'a', 'c' and 'e' are located outside the intended frequency range, identifying the mode shapes at these points is required to understand the influence of the dispersion behavior found in mode 16, 17 and 18. It can be noted that points 'b', 'e' and 'f' are located on three different modes around at equal frequency level. The assumption that the mode shape patterns of these points are the determinant of the wave propagation direction, can be deduced from the respective acoustic energy requirement. Since the measure of the group velocity directly depends on the slope of the dispersion curve, it can be observed that

\[
\frac{dw}{dk} \Big|_b > \frac{dw}{dk} \Big|_f > \frac{dw}{dk} \Big|_e
\]

i.e.,

\[C^b_g > C^f_g > C^e_g\]

in other words,

\[E_b > E_f > E_e\]

Therefore, it is indicative that the point 'e' possesses lower resonant energy compared to other two points. While the influence of three different mode shapes exists at this frequency point, to propagate the wave with mode shape other than point 'e' requires higher acoustic energy. Considering this argument, mode shape of point 'e' can be taken into consideration for wave to be propagated in ΓX direction. A closer observation of the lowest energy mode shape at point 'e' reveals that the two non-oscillating region exists alternatively among three higher-order oscillating regions; and all these oscillations are aligned along a line parallel to the ΓX direction. At point 'e', the strength of the oscillating region found at the middle of the unit cell is lower than that of the...
oscillations found at the top and bottom section of the unit cell. The displacement patterns of this mode shape indicate that the transmitting wave needs to be propagated in orthogonal direction to \( \Gamma X \). Therefore, the mode shape of point ‘e’ directs the propagating wave to transmit in both \( +y \) and \( -y \) direction locally. However, the transmitting wave from the first unit cell to its adjacent second unit is again dominated by the mode shape of point ‘e’. Therefore, the second unit cell locally bends the transmitting wave orthogonally while maintaining the global wave propagation in \( \Gamma X \) direction. The lower part of figure 4 (boxed part) explains this feature where ‘E’ and ‘P’ indicates the ‘Excitation’ and ‘Propagation’ direction respectively. Since the butterfly unit cell can shift the transmitting wave orthogonally, incident plane wave is bifurcated into upward (\( +y \) direction) and downward (\( -y \) direction) directions locally while preserving the global wave propagation direction (i.e. the wave vector) in \( x \)-direction. Therefore, there exists three principle propagation direction of the transmitted waves; one along \( x \)-direction and the other two along the \( +y \) and \( -y \) directions. Resultant of the two principle directions, i.e., \( x \) and \( +y \) directions, enables the wave to follow in \( +45^\circ \) direction. Similarly, resultant of other two driving directions, i.e., \( x \) and \( -y \) directions, enables the wave to follow in \( -45^\circ \) direction. Hence, in a global case scenario, it is necessary that the transmitted wave should bifurcates inside the BC structure.

To verify the above claim, a frequency domain study was performed for a frequency range of \( \sim 37.085 \) kHz to \( \sim 37.43 \) kHz. The dimension and the rectangular arrangements of the unit cells were kept similar as described in earlier section. At \( \sim 37.3 \) kHz, a remarkable wave focusing phenomenon was observed which is shown in figure 5. This makes the proposed butterfly structure to act as a flat lens so as it can focus the incident plane wave in a single focal point. In figure 5(a), it can be observed that the excited plane wave initially bifurcates into two wave directions inside the BC, and afterwards these two waves converge into a single point at the outside of the BC.

**Inside BC region**
At this frequency, when a plane wave excited at zero-degree angle impinges on Epoxy-BC interface, the first lobe of the transmitted wave was strongly dominated by the mode shape of point ‘e’ (figure 4).

![Figure 4. Analysis of mode shape and possible wave propagation direction.](image)

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**Figure 4.** Analysis of mode shape and possible wave propagation direction.
onward lobes of the transmitted waves show the displacement mode shape pattern as described earlier. However, these lobes consist of both rotated and non-rotated mode shapes of point ‘e’ which prevent them from nested bifurcation of the transmitted wave. In figure 5(a), the zoomed-in image of the first lobe of the transmitted wave clearly identifies the dominated mode shape. The particle mode shapes at the first and second columns of the BC region are exactly similar to the mode shape found at point ‘e’ which is consistent with the hypothesis of lowest acoustic energy requirement. When these bifurcated waves come out of the interface formed by the BC and the epoxy, the transmitted waves (dashed red arrow) cross each other to make a prominent focus point as shown in figure 5(a).

AT BC and Epoxy interface
At this interface, incident waves were impinged at MI direction (in both +45° and −45° direction to x-axis). Therefore, the mode shapes were largely influenced by both x and y components of the wave vectors. To identify the dominant mode shapes, the portion of the dispersion curve related to MI direction was analyzed. At 37.3 kHz, the mode shapes of mode 16, 17 and 18 are presented in figure 5(b). Slope at point ‘a’ on mode 16 has lowest value which indicates the lowest group velocity compared to the points ‘b’ and ‘c’. Therefore, the mode shape at the point ‘a’ possess lowest acoustic energy. Thus, propagating wave requires lowest energy to propagate with the mode shape similar to the point ‘a’. Hence, mode shape at point ‘a’ is the dominant mode shape in MI direction. The complex oscillation pattern of the dominating mode shape at point ‘a’ can be utilized to explain the wave propagation direction. Considering the displacement pattern of the mode shape at the point ‘a’, it can be observed that the displacement of the particles along the two diagonals have different values. While the displacement along one diagonal is zero, the displacement of another diagonal is positive. The diagonalized displacement pattern creates 45° local wave propagation direction indicated by negative 45° black arrows in figure 5(b). Therefore, any wave incident at positive 45° at BC-Epoxy interface, transmits at negative 45° direction. By symmetry, waves incident at negative 45° at the BC-Epoxy interface, transmits at positive 45° direction. As the waves come out of the BC region and transmit through the isotropic epoxy media, the wave propagation direction remains same and eventually cross each other to form a circular focal region.
Determination of frequency at maximum focal point intensity

At this stage, the intensity of the focal points was calculated in terms of particle displacement at the intended range of focusing frequencies. The center of the focal region was determined at a location which was 125 mm away from the right interface of the BC and 533 mm above the bottom line of the BC. The particle displacements were numerically calculated at this point and were plotted against the increasing frequency as shown in figure 5(c). A maximum oscillating displacement of 478 μm was found at ∼37.28 kHz which is where the mode shapes were analyzed.

Negative refraction at BC and Epoxy interphase

It can be observed that the wave impinged about 45° on the BC and Epoxy interphase had been refracted negatively. In figure 6, the negative refraction capability of the Butterfly structure has been illustrated. Figure 6(a) shows the incident and negative refraction angle of the transmitted wave at 37.285 kHz. Figures 6(b-1) and (b-2) demonstrate the wave transmission coefficient at BC and Epoxy interphase and at the focal distance, respectively. Clearly, in figures 6(b-1), transmission coefficient increases at the regions where the incident wave impinges. Similarly, at the focal distance the transmission coefficient reaches to the peak at the focal region only. Thus figures 6(a) and (b) confirm the negative refraction phenomenon. To strengthen this claim, equifrequency surfaces have been plotted for a frequency band of 35 kHz to 39 kHz in figure 6(c). From these surfaces, equifrequency contours have been constructed in figure 6(d) for a frequency range of 37 kHz to 37.5 kHz. It can be noted that the direction of the energy flux is outward from the center of the Brillouin zone (Γ point) since the size of the equifrequency contours increases as the frequency increases. Therefore, any line passing through Γ and M points would intersect the 37.28 kHz equifrequency contour resulting in outward energy flux and the normal to this intersection point determines the direction of negatively refracted waves.

Figure 6. Demonstration of negative refraction phenomena. (a) Shows the incident and refracted wave directions, (b-1) wave transmission at the BC and Epoxy interphase, (b-2) wave transmission at the focal distance, (c) three dimensional equifrequency surfaces, (d) Equifrequency contours for a frequency range of 37 kHz to 37.5 kHz.
Superlensing: beyond the diffraction limit

The formation of acoustic focal points indicates the existence of negative refraction property of the proposed butterfly structure. When an acoustic image forms, the smallest feature that can be identified by a conventional acoustic flat lens is limited by the spatial frequency. In 1873, Ernst Abbe proposed a fundamental ‘diffraction limit’ for optics which is the half of a wavelength of the propagating energy. To overcome this diffraction limit, a structure having negative refraction property can be utilized as a super lens. To investigate the possibility of superlensing capability of the BCs, two different simulation configurations was designed. First, only one rectangular exciter of 12.7 mm × 12.7 mm was excited in the host media with a frequency of 37.285 KHz. Next, two exciters (figure 6) of size 3.81 mm × 3.81 mm placed in the base Epoxy medium were excited at the same frequency of 37.285 kHz. The distance, $d$, between two extreme points of two exciters was kept $0.4\lambda$ which was smaller than the wavelength of the base Epoxy material. To calculate the p-wave velocity in Epoxy, p-wave elastic modulus was used in the following equation:

$$V_p = \sqrt{\frac{E(1-\vartheta)}{(1+\vartheta)(1-2\vartheta)}}$$

where,

$$M = \frac{E(1-\vartheta)}{(1+\vartheta)(1-2\vartheta)}$$

Using this equation for the material properties of the base Epoxy, the p-wave velocity was found as $1990.78 \text{ m s}^{-1}$. As, wavelength, $\lambda = V_p/f$, at $f = 37.285 \text{ kHz}$, the calculated wave length was found as $53.39 \text{ mm}$. In the numerical simulation (figure 7), the distance $d$ was kept as $19.75 \text{ mm}$ which is about $0.37 \lambda$. To demonstrate superlensing capability, schematic diagrams of the simulation results have been plotted in figure 8. Figures 8(a-1) and (b-1) show the schematic of wave propagation direction originated from one and two source excitation system respectively. The blue rectangular boxes in these two Figures have been further enlarged in respective 8(a-2) and 8(b-2) images with the displacement amplitudes at the circular points. While plotting the displacements, a binary format is used. Any displacement amplitude measured at the focal point(s) greater than $125 \mu\text{m}$ was marked as 1 or else 0 in the figures 8(a-2) and (b-2). In case of one source configuration, a prominent single focal point was found with a displacement amplitude of 228.6 $\mu\text{m}$, and thus a single object with a peak value of 1 is shown in figures 8(a-2). This indicates that the butterfly structure can identify an object with a diffraction limit of 0.5 $\lambda$. However, for the two-source configuration, clearly four focal points are generated as shown in the schematic diagram of figures 8(b-2). This in simulation result, the displacement amplitudes of these four points were calculated numerically which are $190.5 \mu\text{m}$, $177.8 \mu\text{m}$, $167.1 \mu\text{m}$ and $127.0 \mu\text{m}$. Since the displacement amplitudes of these focal points are greater than $125 \mu\text{m}$, they are marked as 1 and rest of the points were valued zero as shown in figures 8(b-2). Hence, 4 focal points with peak value 1 were plotted. Therefore, it is clear that the BC can create focal points of acoustic sources that are separated by $0.37\lambda$ of the base Epoxy material which is an evidence its super lensing capability beyond the diffraction limit.
Conclusion

In summary, a butterfly shaped engineered metamaterial consisting of an array of steel resonators at multiple-length scales embedded in Epoxy matrix was analyzed which was previously proposed for creating a wide band gap. The numerical analysis showed that the structure remained acoustically transparent to the propagating wave below $\sim 18$ kHz. A comparison of wave propagation simulation along with the dispersion curves confirmed this feature with or without the repeating structure. To verify the simulation configuration, wave propagation behavior was presented in the band gap region where no displacements were detected for the particles located in and out of the butterfly crystals. In a frequency range of $\sim 36.9$ kHz to $\sim 37.43$ kHz, mode shapes were analyzed to understand the wave propagation behavior by establishing the relationship between the acoustic energy and the group velocity. Based on this analysis, the dominant mode shape in the resultant displacement field was identified which predicted an apparent wave bifurcation phenomenon inside the BC. In accordance with this concept, negative refraction feature of the unit cell arrangement was observed which attributed to the formation of acoustic focal point. The numerical results were shown to illustrate the predicted wave bifurcation and wave focusing phenomena. Thus, it had been established that the proposed structure was capable of focusing plane wave front incident at the normal direction to the BC interface. In this focusing frequency range, the maximum displacement of the focusing points was found to be 478 $\mu$m at $\sim 37.28$ kHz. By measuring the displacement amplitude at the focal points created by these two sources, the superlensing capability of the BC structure has been demonstrated. In brief, the capability of creating multiple acoustic features such as acoustic transparency, wide acoustic band gap, acoustic flat lens and superlensing phenomenon can make this butterfly structure a suitable candidate for biomedical ultrasonic imaging and tailored wave guiding with super lensing capability. Hence, a multifunctional acoustic modulation of multimode metamaterial may unveils a future research exploration.

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Figure 8. Demonstration of superlensing capability of butterfly structure, (a) design configuration with one exciter, (b) design configuration with two sources.
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