Transverse Target Azimuthal Single Spin Asymmetry in Elastic $e - N^\uparrow$ Scattering

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Abstract

Using the Coulomb Eikonal Phase associated with a transversely polarized nucleon, we calculate the azimuthal single spin asymmetry of unpolarized electrons elastically scattered from a transversely polarized nucleon target. The azimuthal asymmetry in this case is attributed to the fact that in a transversely polarized nucleon, the transverse charge density is no longer axially symmetric and is distorted due to the transverse spin of the nucleon. This asymmetry in the nucleon charge density causes a left-right asymmetry in the azimuthal distribution of the scattered electrons. In our approach, we utilize the relativistic Eikonal approximation to calculate the one and two photon exchange amplitudes from which a non zero azimuthal asymmetry appears due to the interference between the two amplitudes.
I. Introduction

Polarized scattering experiments have become a primary tool to extract information about different properties of hadrons in elastic and deep inelastic (DIS) scattering processes where single (and double) spin asymmetry is the main measured quantity in the different scattering processes. Due to time-reversal invariance, this quantity vanishes in the one photon approximation and knowledge of the two photon exchange process is necessary to calculate the asymmetry, beside its importance to achieve the necessary precision needed to understand discrepancies in recent experimental results related to nucleon form factors [1–4]. In inclusive and semi inclusive DIS processes, the information obtained from the observed azimuthal (left right) asymmetry has attracted many current experimental and theoretical attention. Similarly in elastic $e^-p$ scattering, transverse beam or target azimuthal asymmetries have recently become under experimental and theoretical investigation [5–8], where information about nucleon form factors and nucleon structure can be extracted beside accessing the two photon exchange amplitude.

In this work we are interested in the azimuthal transverse (or normal) target SSA in elastic $e^-N^\uparrow$ scattering. A main motivation for this work is the fact that the charge density for a transversely polarized nucleon is no longer axially symmetric in the transverse plane and is disturbed due to the transverse spin of the nucleon [9,10]. This azimuthal asymmetry in the charge density plays a main rule in the observed asymmetry in inclusive and semi inclusive DIS [11–13]. In a similar way, in elastic scattering of electrons from a transversely polarized nucleon, the azimuthal asymmetry in the nucleon charge density is expected to cause an asymmetry in the distribution of the scattered electrons. Moreover, while transverse target SSA in elastic scattering has been extensively studied [14–16], there is no explicit calculation of the azimuthal asymmetry in this case. In our approach, the azimuthal asymmetry of the scattered electrons appears as a consequence of the distortion of the transverse charge density for a transversely polarized target.

In Ref. [11] the electromagnetic potetial associated with a transversely polarized nucleon was calculated, this potential represents the impact parameter Coulomb-Eikonal phase which carries the asymmetry of the transverse charge density of the nucelon and represents the main input for the relativistic Eikonal approximation amplitude [17–19]. By expanding the Eikonal amplitude, one can calculate the one and two photon exchange amplitudes, from which it can be shown that SSA is indeed vanishes at the one photon exchange level, while a non zero asymmetry appears due to the interference of the one and two photon exchange amplitudes.

II. Nucleon Transverse Charge and Magnetization Densities

We start by a introducing the transverse structure of the nucleon [9,20], which is the frame of this work. For a nucleon of mass $M$ transversely polarized (with respect to the plane of the incident beam) in the $x$ direction, the charge density is no longer axially symmetric and the distribution of partons in the transverse plane is distorted which is represented by a dipole term in the parton distribution function for a transversely polarized nucleon. In terms of the relevant generalized parton distributions, GPDs, the parton distribution function is given by [9]

\[
q(x, b_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-ib_\perp \cdot q_\perp} \left[ H^q(x, 0, -q_\perp^2) + \frac{i}{2M} \frac{q_y}{2} E^q(x, 0, -q_\perp^2) \right]
= q(x, b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \mathcal{E}^q(x, b_\perp),
\]

(1)

where

\[
q(x, b_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} H^q(x, 0, -q_\perp^2)e^{-ib_\perp \cdot q_\perp},
\]

(2)

\[
\mathcal{E}^q(x, b_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} E^q(x, -q_\perp^2)e^{-ib_\perp \cdot q_\perp},
\]

(3)
using

\[ F_1(t) = \sum_q e_q \int_{-1}^{1} dx H_q(x, \xi, t), \quad F_2(t) = \sum_q e_q \int_{-1}^{1} dx E_q(x, \xi, t), \] (4)

and

\[ \rho_1 (|b_\perp|) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} F_1(q^2_\perp), \quad \rho_2 (|b_\perp|) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} F_2(q^2_\perp), \] (5)

the nucleon transverse charge density in impact parameter space can be written as

\[ \rho (b_\perp) = \rho_1 (|b_\perp|) - \frac{1}{2M} \frac{\partial}{\partial y} \rho_2 (|b_\perp|), \] (6)

the second term in the above equation represents a shift in the transverse charge density in the y direction with respect to the unpolarized charge density, represented by the undistorted first term in the above equation. The charge density in the transverse plane for a transversely polarized nucleon can be written in a more useful form by evaluating the Fourier transform in Eq. (11)

\[ \rho_T(b_\perp) = \rho(b_\perp) - \sin(\phi_\perp - \phi_s) \int_0^\infty \frac{dq_\perp q^2_\perp}{2\pi} \frac{1}{2M} j_1(b_\perp q_\perp) F_2(q^2_\perp). \] (7)

The above form show the explicit azimuthal dependence of the nucleon’s charge density.

III. Coulomb Eikonal Phase and One and Two Photon Exchange Eikonal Amplitudes

For unpolarized beam of electrons elastically scattered from a transversely polarized nucleon target, the scattering amplitude in the relativistic Eikonal approximation is given by [17][19]

\[ f^\uparrow (q_\perp) = -2is \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[ e^{i\chi^\uparrow (b_\perp)} - 1 \right], \] (8)

where \( s \) is the center of mass energy, \( \mathbf{b}_\perp = b_\perp \cos(\phi_\perp) \hat{e}_x + b_\perp \sin(\phi_\perp) \hat{e}_y \), \( q_\perp = q_\perp \cos(\phi_\perp) \hat{e}_x + q_\perp \sin(\phi_\perp) \hat{e}_y \) and \( \chi^\uparrow (b_\perp) \) is the Coulomb/Eikonal phase associated with a nucleon target of spin transverse to the plane of the incident beam with spin vector \( \mathbf{S} = \cos(\phi_s) \hat{e}_x + \sin(\phi_s) \hat{e}_y \). The Coulomb/Eikonal phase is given by

\[ \chi^\uparrow (b_\perp) = -\frac{4\pi e}{2s} \int_{-\infty}^{\infty} dz A^{(0)\uparrow}(b_\perp, z) = -\frac{4\pi e}{2s} A^{(0)\uparrow}(b_\perp), \] (9)

where, \( A^{(0)\uparrow} \) is the electromagnetic transverse potential produced by a transversely polarized nucleon of spin up, and \( e = \frac{1}{137} \) is the electromagnetic coupling constant. From Ref. [11], the transverse potential is given by

\[ A^{(0)\uparrow}(b_\perp) = \frac{-1}{2\pi} \int_0^\infty dq_\perp q_\perp [-J_0(b_\perp q_\perp) + J_0(b_0 q_\perp)] F_1(q^2_\perp) + \frac{\sin(\phi_\perp - \phi_s)}{4\pi M} \int_0^\infty dq_\perp J_1(b_\perp q_\perp) F_2(q^2_\perp), \] (10)

where \( b_0 \) is a reference point for the transverse potential which can be chosen to be the transverse radius of the nucleon since the transverse potential is sufficiently vanishes outside the charge distribution [11]. By expanding the exponential in (8), the Eikonal scattering amplitude takes the form

\[ f^\uparrow (q_\perp) = -2is \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[ e^{i\frac{4\pi e}{2s} A^{(0)\uparrow}(b_\perp)} - 1 \right] = -4\pi \alpha \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} A^{(0)\uparrow}(b_\perp) + \frac{i8\pi \alpha}{s} \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[ A^{(0)\uparrow}(b_\perp) \right]^2 + \cdots. \] (11)

The first term in the above expansion represents the one photon exchange amplitude while the second corresponds to the two photon exchange amplitude. In the following section we will explicitly calculate these two amplitudes from which one can evaluate the single spin asymmetry.
IV. Evaluation of the One and Two Photon Exchange Eikonal Amplitudes

The potential in Eq. (10) can be written as a sum of unpolarized and polarized terms

\[ A^{(0)}_u(b_\perp) = A_u^{(0)\uparrow}(b_\perp) + A_p^{(0)\uparrow}(b_\perp), \]

where

\[ A_u^{(0)\uparrow}(b_\perp) = \frac{-1}{2\pi} \int_0^\infty dq_\perp \frac{d}{q_\perp} [-J_0(b_\perp q_\perp) + J_0(b_0 q_\perp)] F_1(q_\perp^2), \]

\[ A_p^{(0)\uparrow}(b_\perp) = \frac{\sin(\phi_{b_\perp} - \phi_s)}{4\pi M} \int_0^\infty dq_\perp J_1(b_\perp q_\perp) F_2(q_\perp^2). \]  

(12)

Thus, the \( 1\gamma \) amplitude (the first term in eq. (11)) becomes,

\[ f_{(1\gamma)}^\dagger = f_{(1\gamma)u}^\dagger + f_{(1\gamma)p}^\dagger, \]

where

\[ f_{(1\gamma)u}(q_\perp) = -4\pi\alpha \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} A_u^{(0)\uparrow}(b_\perp), \]

\[ f_{(1\gamma)p}(q_\perp) = -4\pi\alpha \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} A_p^{(0)\uparrow}(b_\perp), \]

(14)

the integration over \( \phi_{b_\perp} \) in the unpolarized term gives \( 2\pi J_0(b_\perp q_\perp) \), thus we have

\[ f_{(1\gamma)u}(q_\perp) = -8\pi^2\alpha \int db_\perp b_\perp J_0(b_\perp q_\perp) A_u^{(0)\uparrow}(b_\perp), \]

\[ = -8\pi^2\alpha I_{u}^{(1\gamma)}, \]

(15)

where \( I_{u}^{(1\gamma)} \) is the integral in Eq. (15). In the polarized term, the integration over \( \phi_{b_\perp} \) gives \( -2\pi iJ_1(b_\perp q_\perp) \sin(\phi_{q_\perp} - \phi_s) \) and the corresponding amplitude becomes

\[ f_{(1\gamma)p}(q_\perp) = i8\pi^2\alpha \sin(\phi_{q_\perp} - \phi_s) \int db_\perp b_\perp J_1(b_\perp q_\perp) \frac{A_p^{(0)\uparrow}(b_\perp)}{\sin(\phi_{b_\perp} - \phi_s)} \]

\[ = i8\pi^2\alpha \sin(\phi_{q_\perp} - \phi_s) I_p^{(1\gamma)}, \]

(16)

where \( I_p^{(1\gamma)} \) is the integral in Eq. (16). Thus, the \( 1\gamma \) amplitude becomes

\[ f_{(1\gamma)}(q_\perp) = 8\pi^2\alpha \left[ -I_{u}^{(1\gamma)} + i\sin(\phi_{q_\perp} - \phi_s) I_p^{(1\gamma)} \right]. \]

(17)

Next we consider the \( 2\gamma \) exchange amplitude which, from Eq. (11), is given by

\[ f_{(2\gamma)}(q_\perp) = \frac{i8\pi^2\alpha^2}{s} \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[ A^{(0)\uparrow}(b_\perp) \right]^2. \]

(18)

Similar to the \( 1\gamma \) exchange case, this amplitude can be decomposed into three parts as follows

\[ f_{(2\gamma)u_1}(q_\perp) = \frac{i8\pi^2\alpha^2}{s} \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[ A_u^{(0)\uparrow}(b_\perp) \right]^2, \]

\[ f_{(2\gamma)u_2}(q_\perp) = \frac{i8\pi^2\alpha^2}{s} \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[ A_p^{(0)\uparrow}(b_\perp) \right]^2, \]

\[ f_{(2\gamma)p}(q_\perp) = \frac{i16\pi^2\alpha^2}{s} \int d^2b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} A_p^{(0)\uparrow}(b_\perp) A_u^{(0)\uparrow}(b_\perp), \]

(19)
integrating over $\phi_{b\perp}$ in each of the above integrals, one obtains

$$f_{(2\gamma)u1}(q_{\perp}) = \frac{i 16\pi^3\alpha^2}{s} \int db_{\perp} b_{\perp} J_0(b_{\perp} q_{\perp}) \left[A_u^{(0)\dagger}(b_{\perp})\right]^2$$

$$= \frac{i 16\pi^3\alpha^2}{s} f_{u1}^{(2\gamma)}$$

$$f_{(2\gamma)u2}(q_{\perp}) = \frac{i 8\pi^3\alpha^2}{s} \int db_{\perp} b_{\perp} \left[J_2(b_{\perp} q_{\perp}) \cos(2\phi_{q_{\perp}} - 2\phi_s) + J_0(b_{\perp} q_{\perp})\right] \frac{[A_p^{(0)\dagger}(b_{\perp})]^2}{\sin^2(\phi_{b_{\perp}} - \phi_s)}$$

$$= \frac{i 8\pi^3\alpha^2}{s} \left[I_{a2a}^{(2\gamma)} \cos(2\phi_{q_{\perp}} - 2\phi_s) + I_{u2b}^{(2\gamma)}\right],$$

$$f_{(2\gamma)p}(q_{\perp}) = \frac{32\pi^3\alpha^2}{s} \sin(\phi_{q_{\perp}} - \phi_s) \int db_{\perp} b_{\perp} J_1(b_{\perp} q_{\perp}) \frac{A_p^{(0)\dagger}(b_{\perp}) A_u^{(0)\dagger}(b_{\perp})}{\sin(\phi_{b_{\perp}} - \phi_s)} A_u^{(0)\dagger}(b_{\perp})$$

$$= \frac{32\pi^3\alpha^2}{s} \sin(\phi_{q_{\perp}} - \phi_s) f_{p}^{(2\gamma)},$$

where in the second integral we used

$$\int_0^{2\pi} \sin^2(\phi_{b_{\perp}} - \phi_s)e^{-iq_{\perp} b_{\perp} \cos(\phi_{b_{\perp}} - \phi_{q_{\perp}})} = \pi \left[J_2(b_{\perp} q_{\perp}) \cos(2\phi_{q_{\perp}} - 2\phi_s) + J_0(b_{\perp} q_{\perp})\right].$$

Therefore, the $2\gamma$ exchange amplitude becomes

$$f_{(2\gamma)p}(q_{\perp}) = \frac{16\pi^3\alpha^2}{s} \left[i \left(I_{u1}^{(2\gamma)} + \frac{I_{a2a}^{(2\gamma)}}{2} \cos(2\phi_{q_{\perp}} - 2\phi_s) + \frac{1}{2} I_{u2b}^{(2\gamma)}\right) + 2 \frac{f_{u2b}^{(2\gamma)}}{s} \sin(\phi_{q_{\perp}} - \phi_s)\right]$$

(21)

From Eqs. (17, 21) it is clear that the one and two photon amplitudes are azimuthally asymmetric, however the corresponding cross sections are azimuthally symmetric while the azimuthal asymmetry appears in the interference term between the $1\gamma$ and $2\gamma$ amplitudes as we shall see in the next section.

V. Azimuthal Transverse Target Single Spin Asymmetry in Elastic $e - p^\uparrow$ Scattering

Single spin asymmetry is defined as

$$A_n = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}.$$

(22)

The spin up and spin down cross section are given by

$$\sigma^+ = |f^\dagger_{1\gamma} + f^\dagger_{2\gamma}|^2 = |f^\dagger_{1\gamma}|^2 + |f^\dagger_{2\gamma}|^2 + f^\dagger_{1\gamma} f^\dagger_{2\gamma} + f^{\dagger\dagger}_{1\gamma} f^{\dagger\dagger}_{2\gamma},$$

$$\sigma^- = |f^\dagger_{1\gamma} + f^\dagger_{2\gamma}|^2 = |f^\dagger_{1\gamma}|^2 + |f^\dagger_{2\gamma}|^2 + f^\dagger_{1\gamma} f^\dagger_{2\gamma} + f^{\dagger\dagger}_{1\gamma} f^{\dagger\dagger}_{2\gamma},$$

(23)

The spin down amplitude is obtained by rotating the spin vector by $\pi$. Thus, from Eqs. (17, 21), one obtains

$$f^\dagger_{1\gamma} = f^\dagger_{1\gamma} \Rightarrow f^\dagger_{1\gamma} = f^\dagger_{1\gamma},$$

$$f^\dagger_{2\gamma} = -f^\dagger_{2\gamma} \Rightarrow f^\dagger_{2\gamma} = -f^\dagger_{2\gamma}.$$  

(24)

Therefore, we get

$$\sigma^+ - \sigma^- = 2f^\dagger_{1\gamma} f^\dagger_{2\gamma} + 2f^\dagger_{1\gamma} f^\dagger_{2\gamma},$$

$$\sigma^+ + \sigma^- = 2|f^\dagger_{1\gamma}|^2 + 2|f^\dagger_{2\gamma}|^2,$$

and the asymmetry takes the form

$$A_n = \frac{2Re[f^\dagger_{1\gamma} f^\dagger_{2\gamma}]}{|f^\dagger_{1\gamma}|^2 + |f^\dagger_{2\gamma}|^2}.$$  

(26)
The figures below show the proton transverse target azimuthal SSA for various center of mass energies and momentum transfer. From the figures one notes that the asymmetry is positive and it increases with $q_{\perp}$ but decreases with the center of mass energy, as expected. On the other hand, the magnitudes of the amplitudes in the figures are consistent with the expected values for the transverse target SSA with unpolarized electrons beam [2].

![Graphs showing proton transverse target azimuthal SSA](image)

**Figure 1**: Proton transverse target azimuthal SSA for various center of mass energies and momentum transfers using GPD parametrization for the proton form factors

**VI. Conclusion**

We study single spin azimuthal asymmetry in elastic electron-nucleon scattering for the case of transversely polarized nucleon with unpolarized electrons beam. The asymmetry in this case appears due to the interference between the $1\gamma$ and $2\gamma$ exchange amplitudes. To calculate the (1) and $(2\gamma)$ Eikonal scattering amplitudes, we used the Coulomb/Eikonal phase associated with a transversely polarized nucleon, this phase is asymmetric in impact parameter space; consistent with the fact that the nucleon charge density (or the impact parameter dependent parton distribution function) is transversely distorted due to the transverse spin of the nucleon, which explains the existence of SSA in this case. Figure 1 shows the azimuthal target single spin asymmetry for different values of the center of mass energy and momentum transfer, as expected, the asymmetry increases with $q_{\perp}$ and decreases with the center of mass energy. The amplitudes in the figures are consistent with the expected values of the transverse target SSA with unpolarized electrons beam. In the calculations, the transverse radius of the proton was taken from Ref. [20] and the GPD parametrization of the proton form factors from Ref. [21] while the integrals were evaluated numerically using the CUBA library for multidimensional numerical integration [22].
Appendix A  Parametrization of Form Factors Using Generalized Parton Distributions

Generalized parton distribution (GPDs) can be considered as generalization of ordinary parton distributions. The formal definition of GPDs for transversely polarized nucleon but unpolarized quarks is given by [23]

\[
\left\langle p', S' \left| \hat{O}_q(x, b_\perp) \right| p, S \right\rangle = \frac{1}{2P^+} \bar{u}(p', S') \left( \gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M} E_q(x, \xi, t) \right) u(p, S) \tag{27}
\]

Where \( \hat{P}^\mu = \frac{1}{2}(P^\mu + P'^\mu) \) represents the average momentum of the target, \( q^\mu = P'^\mu - P^\mu \) is the four momentum transfer, \( t = q^2 \) is the invariant momentum transfer and \( \xi = -\frac{q^+}{\hat{P}^+} \) is the change in the longitudinal component of the target momentum and is called the skewness. The nucleons form factors can be decomposed as follows

\[
F_i^p = e_u F_i^u + e_d F_i^d + e_s F_i^s, \quad F_i^n = e_u F_i^u + e_d F_i^d + e_s F_i^s
\tag{28}
\]

Where \( i = 1, 2 \) and \( e_u = \frac{2}{3}, \quad e_d = e_s = \frac{1}{3} \). The Dirac and Pauli flavor form factors at zero skewness are give by the following sum rules

\[
F_1^p(t) = \int_{-1}^{1} dx \ H^q(x, \xi, t), \quad F_2^p(t) = \int_{-1}^{1} dx \ E^q(x, \xi, t) \tag{29}
\]

The result of integration is independent of \( \xi \). Also the integration region can be reduced to \( 0 < x < 1 \) by introducing the non-forward parton densities

\[
H^q(x, t) = H^q(x, 0, t) + H^q(-x, 0, t), \quad E^q(x, t) = E^q(x, 0, t) + E^q(-x, 0, t) \tag{30}
\]

Where \( q = u, d \) and \( H^q(x, t) \) reduces to the usual valence quark densities for \( t \to 0 \) for the up and down quarks. Thus, the form factors become

\[
F_1^p(t) = \int_{0}^{1} dx \ H^q(x, t), \quad F_2^p(t) = \int_{0}^{1} dx \ E^q(x, t) \tag{31}
\]

The magnetic densities satisfies the following normalization conditions

\[
\kappa_q = \int_{0}^{1} dx \ E^q(x), \quad \kappa_u = 2\kappa_p + \kappa_n = +1.673, \quad \kappa_d = \kappa_p + 2\kappa_n = -2.033
\tag{32}
\]

\[
F_2^p(t = 0) = 1.793, \quad F_2^n(t = 0) = -1.913
\]

Following [21], the anzats for the GPDs are The ansatz for the GPDs is [21]

\[
H^q(x, t) = q_v(x)x^{-\alpha'(1-x)t}, \quad \kappa_q \frac{1}{N_q} (1 - x)^{\eta_q} q_v x^{-\alpha'(1-x)t}, \tag{33}
\]

The normalization constants \( N_q \) satisfies

\[
N_q = \int_{0}^{1} dx (1 - x)^{\eta_q} q_v(x), \tag{34}
\]

and the unpolarized parton distributions are parametrized as

\[
u_v(x) = 0.262x^{-0.69}(1 - x)^{3.50}(1 + 3.83x^{0.5} + 37.65x) \tag{35}
\]

\[
u_d(x) = 0.061x^{-0.65}(1 - x)^{4.03}(1 + 49.05x^{0.5} + 8.65x)
\]

The parameters used in this fit are \( \alpha' = 1.105, \eta_u = 1.713, \eta_d = 0.566 \). The following is a plot of \( F_1 \) and \( F_2 \) as a function of the momentum transfer.
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