New model of kaon regeneration

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Abstract: It is shown that in the standard model of $K_S^0$ regeneration a system of non-coupled equations of motion is used instead of the coupled ones. A model alternative to the standard one is proposed. A calculation performed by means of the diagram technique agrees with that based on exact solution of the equations of motion.

Keywords: equations of motion, regeneration, diagram technique

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1 Introduction

The effect of kaon regeneration has been known since the 1950s. However, in the previous calculations [1–3] a system of non-coupled equations of motion was considered (see Eq. (1)) instead of the coupled equations of motion. This is a fundamental defect because it leads to a qualitative disagreement in the results. This means that the regeneration has not been described at all. The result obtained in Ref. [2] was adduced in Refs. [4, 5] and subsequent papers. In this paper we consider a model based on the exact solution of the coupled equations of motion with the potentials taken in general form. A comparison with the previous model and our calculation performed by means of the diagram technique is given as well.

Let $K_S^0$ fall onto the plate at $t=0$. Our particular interest is in the probability of finding $K_S^0$. Our approach is as follows. Since $K^0\otimes N$- and $K^0\otimes N$-interactions are known, we go to $K^0_L,K^0_S$ representation. The problem is described by coupled equations of motion for $K^0(t)$ and $\bar{K}^0(t)$. We find the corresponding solutions and revert to $K^0_L,K^0_S$ representation.

2 Previous calculations

In the previous calculations the starting equations are (see Eqs. (3) of Ref. [2]):

$$\begin{align}
(\partial_x-ink)\alpha &= 0, \\
(\partial_x-in'k)\alpha' &= 0,
\end{align}$$

(1)

where $n$ and $n'$ are the indexes of refraction for $\alpha$ and $\alpha'$, respectively. In this equation the change of variables $\alpha,\alpha' \rightarrow \alpha_1,\alpha_2$ is performed and the effects of weak interactions are added. As a consequence of the change of variables, Eqs. (5) of Ref. [2] are coupled. The solution of Eqs. (5) in Ref. [2] gives the result in Eq. (6) of Ref. [2] (or Eq. (1) of Ref. [3]). This result is adduced in Eq. (9.32) of Ref. [4], in Eqs. (7.83)–(7.89) of Ref. [5], and subsequent papers.

We consider all the possibilities. If $\alpha$ and $\alpha'$ correspond to $K^0_L$ and $K^0_S$, $\alpha_1$ and $\alpha_2$ describe $K^0$ and $\bar{K}^0$, whereas our interest is in $K^0_S$ and $K^0_L$. The indexes of refraction for $K^0_L$ and $K^0_S$ are unknown.

Let $\alpha = K^0$ and $\alpha' = \bar{K}^0$. Then $\alpha_1 = K^0_S$ and $\alpha_2 = K^0_L$. This variant follows from the initial conditions (see Ref. [2]): $\alpha_1(0) = 1$, $\alpha_2(0) = 0$. There is no off-diagonal mass $\epsilon = (m_L - m_S)/2$. Equations (1) are non-coupled. The non-coupled equations exist only for the stationary states and do not exist for $K^0$ and $\bar{K}^0$.

In any case Eqs. (1) are unrelated to the problem. Our calculation gives the inverse $\Delta \Gamma$- and $\Delta m$-dependences (see Eq. (24) of Ref. [6]). In this paper we consider a model with the potentials taken in general form. A similar question for $\Lambda \Lambda$ oscillations is studied in Refs. [7–10].

3 Our model

Let $K_S^0$ fall onto the plate at $t=0$. We use the model described in the second paragraph of the Introduction. In Ref. [6] the exact wave function $K_S(t)$ of $K_S^0$ has been calculated. The probability of finding $K_S^0$ or, equivalently, the probability of the $K_S^0,K_S^0$ transition, is given by Eq. (12) of Ref. [6]:

$$|K_S(t)|^2 = \frac{1}{4}[V/p|^2 e^{\text{Im}Vt+2\text{Im}Mt}\left[e^{-\text{Im}(pt)}+e^{\text{Im}(pt)}ight]$$

$$-e^{\text{Re}(pt)}-e^{-\text{Re}(pt)}],$$

(2)
This expression is exact; $|K_S(t=0)|^2=0$.

If $\text{Im}V=0$, then $\text{Im}p=0$ as well. In this case

$$|K_S(t)|^2=|V/p|^2e^{-(\mathcal{I} _{K_0}+\mathcal{I} _d)t}\sin^2(\text{Re}(pt)/2). \quad (3)$$

This is a pure oscillation regime. Here regeneration by scattering takes place. The regeneration by absorption is described by $\text{Im}V$.

As in Ref. [6] we put $p \approx V+2e^2/V$ ($\epsilon=\Delta m/2$).

$$\Delta m=m_L-m_S,$$ where $m_L$ and $m_S$ are the masses of stationary states; $\mathcal{I} _{K_0}=\mathcal{I} _{K_S}=\mathcal{I} _d$; $m_{K_0}=m_{K_S}=m$ ($m$ and $\mathcal{I} _d$ are the mass and decay width, respectively, of $K^0$). Let us denote $\mathcal{I} _{K_0}$ and $\mathcal{I} _{K_S}$ as widths of absorption (not decay) of $K^0$ and $K_S$, respectively. Then

$$2\text{Im}M=-(\mathcal{I} _{K_0}+\mathcal{I} _d), \quad (4)$$

$$\text{Re}V=\text{Re}U_{K_0}-\text{Re}U_{K_S}, \quad (5)$$

$$\text{Im}V=-\frac{\Delta \Gamma}{2}, \quad (6)$$

where

$$\Delta \Gamma=\mathcal{I} _{K_0}-\mathcal{I} _{K_S} \quad (7)$$

Equation (2) gives:

$$|K_S(t)|^2=Re\omega, \quad (8)$$

$$R=\frac{1}{4}|V/p|^2e^{-(\mathcal{I} _0+\mathcal{I} _d)t}, \quad (9)$$

$$\omega=e^{-\Gamma(K_L-K_S)t}+e^{(\Gamma(K_L-K_S)t-\Delta \Gamma)t/2-2e^{-\Delta \Gamma/2}\cos(\text{Re}(pt))}, \quad (10)$$

where

$$\Gamma(K_L-K_S)=\frac{e^2}{|V|^2}\Delta \Gamma, \quad (11)$$

$$\text{Re}p=\text{Re}V+2e^2\text{Re}V \quad (12)$$

$\Gamma(K_L-K_S)$ is the width of $K_LK_S$ transition (regeneration). The value $\Delta m$ is involved in $\Gamma(K_L-K_S)$ and $\cos(\text{Re}(pt))$.

Let

$$\Delta \Gamma t \gg 1. \quad (13)$$

In this case

$$\omega=e^{-\Gamma(K_L-K_S)t}. \quad (14)$$

The $t$-dependence is given by an exponential decay law. It is significant that $\text{Re}V \neq 0$, in contrast to Ref. [6]. (Note that Eq. (14) is valid if $\Delta \Gamma >2\epsilon$.)

### 4 Connection between models based on the diagram technique and the exact solution

The calculation presented above is cumbersome and formal, so verification is required. In Ref. [11] an approach based on perturbation theory was proposed. Regeneration followed by the decay $K_L^0 \rightarrow K_S^0 \rightarrow \pi\pi$ was considered. A similar approach is used for the $n\bar{n}$ transition in a medium followed by annihilation (see Refs. [12-15]). The process amplitude $M(K_L^0 \rightarrow K_S^0 \rightarrow \pi\pi)$ is

$$M(K_L^0 \rightarrow K_S^0 \rightarrow \pi\pi)=\frac{e}{V}M_d(K_S^0 \rightarrow \pi\pi). \quad (15)$$

(See the second term of Eq. (23) of Ref. [11].) Here $M_d(K_S^0 \rightarrow \pi\pi)$ is the in-medium amplitude of the decay $K^0_S \rightarrow \pi\pi$. The corresponding process width is

$$\Gamma(K_L^0 \rightarrow K_S^0 \rightarrow \pi\pi)=\frac{e^2}{|V|^2}M_d(K_S^0 \rightarrow \pi\pi), \quad (16)$$

where $M_d(K_S^0 \rightarrow \pi\pi)$ is the width of the decay $K^0_S \rightarrow \pi\pi$.

Consider now the connection between the models based on the diagram technique and the exact solution. In this case we write Eq. (16) in the form

$$\Gamma(K_L^0 \rightarrow K_S^0 \rightarrow \pi\pi)=\frac{e^2}{|V|^2}M_d(K_S^0 \rightarrow \pi\pi)\Delta \Gamma=\Gamma(K_L \rightarrow K_S)W, \quad (17)$$

$$W=\frac{\Gamma_d(K_S^0 \rightarrow \pi\pi)}{\Delta \Gamma}, \quad (18)$$

where $W$ is the probability of the $K^0_S$ decay in the channel $K^0_S \rightarrow \pi\pi$. The physical sense of Eq. (17) is obvious: the multistep process $K_L^0 \rightarrow K_S^0 \rightarrow \pi\pi$ involves the subprocess of $K_LK_S$ transition (regeneration). Equation (17) is verification of the models considered above.

Due to a strong absorption of $K^0$ and zero momentum transfer in the $K^0K^0$ transition vertex, the description of competition between scattering and absorption is of particular importance. In this regard the diagram technique has some advantages over the model based on the equations of motion (see Refs. [14,15]).

### 5 Limiting case and numerical results

Let us consider the limiting case $t \rightarrow 0$. Expanding Eq. (10) to the terms $\sim t^2$ we have

$$\omega=1-\cos(\text{Re}(pt)))(2-\Delta \Gamma t)+\omega_1 t^2/2, \quad (19)$$

$$\omega=1-\cos(\text{Re}(pt))\Delta \Gamma^2/2-\cos(\text{Re}(pt)). \quad (20)$$

Let $\text{Re}V \neq 0$ and $\Delta \Gamma=0$. Then $\Gamma(K_L \rightarrow K_S)=\omega_1=0$ and $|K_S(t)|^2$ coincides with Eq. (3). The opposite case when $\text{Re}V=0$ is more interesting. Then

$$\omega=\omega_1 t^2/2 \quad (21)$$

and

$$|K_S(t)|^2=\frac{1}{8}|V/p|^2e^{-(\mathcal{I} _{K_0}+\mathcal{I} _d)t^2}. \quad (22)$$

Here regeneration by absorption takes place. Comparing Eq. (19) and Eq. (21) we see that $\text{Re}V \neq 0$ violates $t^2$-dependence.
where \( N \) and \( \mathcal{N} \).

**Theorem:**

by standard expressions which follow from the optical theorem:

\[
\mathcal{N}_{K^0} = N_n v \sigma(K^0n) + N_p v \sigma(K^0p)
\]

and

\[
\mathcal{N}_{\bar{K}^0} = N_n v \sigma(\bar{K}^0n) + N_p v \sigma(\bar{K}^0p)
\]

where \( N_n \) and \( N_p \) are the number of neutrons and protons in a unit of volume, respectively; \( \sigma(K^0n) \) and \( \sigma(\bar{K}^0n) \) are the total cross sections of \( K^0n \)- and \( \bar{K}^0n \)-interactions, and \( v \) is the velocity of the \( K^0 \) meson. By way of illustration we take \( \sigma(K^0n) = \sigma(K^0p) = 15 \text{ mb} \) [16]. As in Ref. [2], we use \( \sigma(K^0n) = \frac{1}{2} \sigma(K^0n) \).

Instead of cross sections one can use the forward scattering amplitudes of kaons by the molecules of the medium. In this case

\[
V = U_{K^0} - U_{\bar{K}^0} = \frac{2 \pi}{m} N_m f_{21},
\]

\( f_{21} = f - \bar{f} \). Here \( N_m \) is the number of molecules in a unit of volume, and \( f \) and \( \bar{f} \) are the forward scattering amplitudes of \( K^0 \) and \( \bar{K}^0 \), respectively.

For a copper absorber the probability of finding \( K^0 \) is shown in Figs. 1 and 2. Im\( V \) is determined by Eqs. (6), (7), (23) and (24), and Re\( V \) is the parameter. The solid and dot-dashed curves correspond to \( |\text{Re}V| = \Delta \mathcal{N}/2 \) and \( \text{Re}V = 0 \), respectively. The dashed curve corresponds to the copper plate and \( V \) defined from Eq. (25). The amplitudes \( f \) and \( \bar{f} \) are taken from Ref. [17]. In the case \( \text{Re}V = 0 \) only regeneration by absorption takes place. It is seen that \( \text{Re}V \) leads to the suppression of regeneration. \( |K_S(t)|^2 \) is about 10 times smaller than in Ref. [2] (although comparison with Ref. [2] is meaningless for the reasons given above.)

### 6 Conclusion

The main results of this paper are given in the abstract. The most distinctive feature of the model presented above is the inverse \( \Delta \mathcal{N} \) and \( \Delta m \)-dependences of the amplitude of regenerated \( K^0 \), or parameter regeneration \( r \) (see Eqs. (19)–(24) of Ref. [6]). The main uncertainty in the numerical results is conditioned by the uncertainty in the cross sections \( \sigma(K^0n) \) and \( \sigma(\bar{K}^0n) \). The same is also true for the previous results [1–5] since they have been obtained by means of the above-mentioned cross sections as well. In this connection we would like to recall that \( \Delta m \) is extracted from free-space oscillations without recourse to the potentials of \( K^0 \) and \( \bar{K}^0 \). Nevertheless, in any case the regeneration should be described correctly.

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