Towards a Model Independent Analysis of Rare $B$ Decays

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We propose to undertake a model-independent analysis of the inclusive decay rates and distributions in the processes $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$ ($B = B^\pm$ or $B_d^0$). We show how measurements of the decay rates and distributions in these processes would allow us to extract the magnitude and sign of the dominant Wilson coefficients of the magnetic moment operator $m_b \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$ and the four-fermion operators $(\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell)$ and $(\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma^5 \ell)$.

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Abstract

We propose to undertake a model-independent analysis of the inclusive decay rates and distributions in the processes $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$ ($B = B^\pm$ or $B_d^0$). We show how measurements of the decay rates and distributions in these processes would allow us to extract the magnitude and sign of the dominant Wilson coefficients of the magnetic moment operator $m_b \bar{s}_L \sigma_{\mu \nu} b_R F^{\mu \nu}$ and the four-fermion operators $(\bar{s}_L \gamma_{\mu} b_L)(\bar{\ell} \gamma_{\nu} \ell)$ and $(\bar{s}_L \gamma_{\mu} b_L)(\bar{\ell} \gamma_{\nu} \bar{\ell})$.

1. The Decay $B \to X_s \gamma$ in SM and Experiment

The measurements of the decay mode $B \to K^* \gamma$, reported last year by the CLEO collaboration [1], having a branching ratio $B(B \to K^* \gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$, and the inclusive decay $B \to X_s \gamma$, reported at this conference [2] with a branching ratio $B(B \to X_s \gamma) = (2.32 \pm 0.51 \pm 0.32 \pm 0.20) \times 10^{-4}$, have put the physics of the electromagnetic penguins on an experimental footing. In the standard model (SM), these transitions are dominated by the short-distance contributions and provide valuable information about the top quark mass and the Cabibbo-Kobayashi-Maskawa (CKM) weak mixing matrix elements $V_{ts} V_{tb}$. The rapport between the SM and experiment may be quantified in terms of the CKM matrix element ratio [3]:

$$0.62 \leq \left| \frac{V_{ts}}{V_{tb}} \right| \leq 1.1,$$

which is consistent with unity, resulting from the unitarity constraints. Alternatively, one can set $V_{ts}/V_{tb} = 1$ to obtain from the CLEO measurement bounds on the Wilson coefficient $C_7(m_b)$ of the effective magnetic moment operator. Using $B(B \to X_s \gamma)$ from [2], one obtains

$$0.22 \leq |C_7(m_b)| \leq 0.30.$$  \hspace{1cm} (2)

Using, however, the 90%-confidence-level range from the CLEO measurement $B(B \to X_s \gamma) = (2.31 \pm 1.1) \times 10^{-4}$ and the theoretical calculation for $B(B \to X_s \gamma)$ from [3], we obtain

$$0.19 \leq |C_7(m_b)| \leq 0.32.$$  \hspace{1cm} (3)

We also remark that the photon energy and hadron mass spectra measured by CLEO are in good agreement with the SM-based calculations in [3]. The bound (3) can be used to constrain the non-SM contribution to the decay rate $B(B \to X_s \gamma)$ as discussed in these proceedings [4, 5].

2. Motivation for a Model Independent Analysis of Rare $B$ Decays

The determination of $|C_7(m_b)|$ from the inclusive branching ratio $B(B \to X_s \gamma)$ is a prototype of the kind of analysis that we would like to propose here to be carried out for the rare $B$ decays in general and for the semileptonic decays $B \to X_s \ell^+ \ell^-$, in particular. First steps towards a model-independent analysis of the FCNC electroweak rare $B$ decays involving these decay modes have recently been proposed in [5], to which we refer for details and references to other related work. Here, we summarize the main assumptions and results.
The main interest in rare $B$ decays is to measure the effective FCNC vertices in order to test the SM and search for new physics. We have argued that with some plausible assumptions these vertices can be parametrized through a limited number of effective parameters, which govern the rates and shapes (differential distributions) in rare $B$ decays $B \to X_s \gamma$, $B \to X_s \ell^+\ell^-$ and $B_s \to \ell^+\ell^-$. The search for physics beyond the SM in these decays can be carried out in terms of three effective parameters, $C_7(\mu)$, $C_9(\mu)$ and $C_{10}(\mu)$, characterizing the strength of the magnetic moment and two four-fermion operators $(\hat{s} L \gamma_\mu \bar{b} L)(\ell \gamma^\mu \ell)$ and $(\hat{s} L \gamma_\mu \bar{b} L)(\ell \gamma^\mu \gamma^5 \ell)$. This can then be interpreted in a large class of models. The presence of non-SM physics may manifest itself by distorting the differential distributions in $B \to X_s \ell^+\ell^-$. Some possible examples of such distortions have been worked out in \cite{8}. Here we present profiles of the Wilson coefficients in the best-motivated extensions of the SM, namely the Minimal Supersymmetric Standard Model (MSSM).

Our analysis is based on an effective Hamiltonian of the form

$$H_{eff}(b \to sX) = -\frac{4G_F}{\sqrt{2}} \lambda_i \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \cdot (4)$$

where $X$ stands for $q\bar{q}$, $\gamma$, gluon and $\ell^+ \ell^-$ and $\lambda_i = V_{tb}^* V_{tb}$. The operator basis $O_{1 \cdots 10}$ is given in \cite{3} and is the same as in the SM, thereby restricting our analysis to cases, in which the effective Hamiltonian may be written as \cite{3}.

3. Analysis of the Decays $B \to X_s \gamma$ and $B \to X_s \ell^+\ell^-$

The experimental quantities we consider in this paper are the following: (i) Inclusive radiative rare decay branching ratio $B(B \to X_s \gamma)$; (ii) Invariant dilepton mass distributions in $B \to X_s \ell^+\ell^-$; (iii) Forward-backward (FB) charge asymmetry $A(\hat{s})$ in $B \to X_s \ell^+\ell^-$. The FB asymmetry $A(\hat{s})$ is defined with respect to the angular variable $z \equiv \cos \theta$, where $\theta$ is the angle of the $\ell^+$ with respect to the $b$-quark direction in the centre-of-mass system of the dilepton pair. It is obtained by integrating the doubly differential distribution $d^2B/(dz d\hat{s})(B \to X_s \ell^+\ell^-)$ \cite{3}:

$$A(\hat{s}) \equiv \int_0^1 dz \int_{-1}^0 d\hat{s} \frac{d^2B}{dz d\hat{s}} - \int_0^1 dz \int_{-1}^0 d\hat{s} \frac{d^2B}{dz d\hat{s}} \cdot (5)$$

where $\hat{s} = (p_1 + p_2)^2/m_b^2$ and $p_1$ and $p_2$ denote, respectively, the momenta of the $\ell^+$ and $\ell^-$. We remark that the decay rate $B(B \to X_s \gamma)$ puts a bound on the absolute value of the coefficient $C_7(\mu)$. However, the radiative $B$ decay rate by itself is not able to distinguish between the solutions $C_7(\mu) > 0$ (holding in the SM) and the solutions $C_7(\mu) < 0$, which, for example, are also allowed in the MSSM as one scans over the allowed parameter space. We recall that the invariant dilepton mass distribution and the forward-backward asymmetry in $B \to X_s \ell^+\ell^-$ are sensitive to the sign and magnitude of $C_7(\mu)$ \cite{3}. Using $H_{eff}$ given in \cite{3}, one obtains for the dilepton invariant mass distribution

$$\frac{d\mathcal{B}}{ds} = \mathcal{B}_{sl} \frac{3\alpha^2}{8\pi^2} \lambda_i \frac{\hat{\nu}(\hat{s})^2}{f(m_c/m_b)} \times \left[(|C_9 + Y(\hat{s})|^2 + C_{10}^2) \alpha_1(\hat{s}, \hat{m}_s) + \frac{4}{3} C_7^2 \alpha_2(\hat{s}, \hat{m}_s) + 12 \alpha_3(\hat{s}, \hat{m}_s) C_7(C_9 + \text{Re } Y(\hat{s})) \right],$$

where the auxiliary functions $\alpha_i$ depend only on the kinematic variables and $Y(\hat{s})$ depends on the coefficients $C_1, \cdots, C_6$ of the four quark operators (see \cite{3}).

The corresponding differential asymmetry as defined in \cite{3} is

$$A(\hat{s}) = -\mathcal{B}_{sl} \frac{3\alpha^2}{8\pi^2} \frac{1}{f(m_c/m_b)} \hat{\nu}(\hat{s})^2 C_{10} \times \left[\hat{s}(C_9 + \text{Re } Y(\hat{s})) + 4C_7(1 + \hat{m}_s^2) \right].$$

We first present the partial branching ratio $B(\Delta s)$ and partial FB asymmetry $A(\Delta s)$, where $\Delta s$ defines an interval in the dilepton invariant mass. In order to minimize long-distance effects we shall consider the kinematic regime for $s$ below the $J/\psi$ mass (low invariant mass) and for $s$ above the mass of the $\psi'$ (high invariant mass). Integrating \cite{3} over these regions for the invariant mass one finds

$$B(\Delta s) = A(\Delta s) \left(C_9^2 + C_{10}^2\right) + B(\Delta s) C_9 + C(\Delta s),$$

where $A$, $B$ and $C$ are fixed in terms of the Wilson coefficients $C_1 \cdots C_6$ and $C_7$. For the numerical analysis we use $m_b = 4.7$ GeV, $m_c = 1.5$ GeV, $m_s = 0.5$ GeV. The resulting coefficients $A$, $B$, and $C$ are listed in Table \cite{3} for the decays $B \to X_s e^+e^-$ and $B \to X_s \mu^+\mu^-$. For a measured branching fraction $B(\Delta s)$, one can solve the above equation for $B(\Delta s)$, obtaining a circle in the $C_9$-$C_{10}$ plane, with centre lying at $C_9^* = B(\Delta s)/(2A(\Delta s))$ and $C_{10}^* = 0$. The radius of this circle is proportional to $\sqrt{B(\Delta s) - B_{min}(\Delta s)}$, where the minimum branching fraction

$$B_{min}(\Delta s) = C(\Delta s) - \frac{B^2(\Delta s)}{4A(\Delta s)}$$

is determined mainly by the present data on $B \to X_s \gamma$, i.e. by $|C_7|$. 


| $\Delta s$ | $C_7$ | $\alpha(\Delta s)$ | $\beta(\Delta s)$ |
|----------|-------|-----------------|-----------------|
| $4m^2_J < s < m^2_{J/\psi}$ | $+0.3$ | $-0.12$ | $24.0$ |
| $4m^2_J < s < m^2_{J/\psi}$ | $-0.3$ | $-0.12$ | $-64.0$ |
| $m^2_{J/\psi} < s < (1 - m^2_Z)$ | $+0.3$ | $-0.09$ | $0.276$ |
| $m^2_{J/\psi} < s < (1 - m^2_Z)$ | $-0.3$ | $-0.09$ | $1.37$ |

Table 2. Values for the coefficients $\alpha(\Delta s)$ and $\beta(\Delta s)$ (in units of $10^{-8}$).

To further pin down the Wilson coefficients, one could perform a measurement of the forward-backward asymmetry $A$. Integrating (8) over a range $(\Delta s)$ yields

$$A(\Delta s) = C_{10} (\alpha(\Delta s)C_9 + \beta(\Delta s)).$$

For a fixed value of $A(\Delta s)$, one obtains hyperbolic curves in the $C_9$-$C_{10}$ plane; like the coefficients $A$, $B$ and $C$, the parameters $\alpha$ and $\beta$ are given in terms of the Wilson coefficients $C_1 \ldots C_6$, $C_7$ and $\Delta s$; their values are presented in Table 2.

Given the two experimental inputs, the branching fraction $B(\Delta s)$ and the corresponding asymmetry $A(\Delta s)$, one obtains a fourth-order equation for the Wilson coefficients $C_9$ and $C_{10}$, which admits in general four solutions, which can be plotted as contours for a fixed value for the branching fraction $B(\Delta s)$ and the FB asymmetry $A(\Delta s)$. The possible solutions for $C_9$ and $C_{10}$ are given by the intersections of the circle corresponding to the measured branching fraction and the hyperbola, corresponding to the measured asymmetry. Details can be seen in [3].

We stress that the spectrum itself is very sensitive to the values of the Wilson coefficients and to the sign of $C_7$. In fig. 4 we plot the various contributions to the spectrum, for positive and for negative $C_7$.

In a similar way, it may become possible to measure also the differential asymmetry $\alpha$, the various contributions to $A(s)$ are shown in fig. 3.

4. Model Predictions for the Wilson Coefficients

As an illustrative example we shall consider here the MSSM; we shall show, how the predictions for the Wilson coefficients $C_7$, $C_9$, and $C_{10}$ are altered in this model.

Once the gluino contributions (as well as the analogous ones from neutralino exchange) are neglected, the flavour violation in the supersymmetric models is completely specified by the familiar CKM matrix. The one-loop supersymmetric corrections to the Wilson coefficients $C_7$, $C_9$, and $C_{10}$ are given by two classes of diagrams: charged-Higgs exchange and chargino exchange [4].

The charged-Higgs contribution is specified by two input parameters: the charged-Higgs mass ($m_{H^+}$) and the ratio of Higgs vacuum expectation values ($v_2/v_1 \equiv \tan \beta$). This contribution alone corresponds to the two-Higgs doublet model which has also been considered in [5].

In addition to the diagrams with charged-Higgs
exchange, the MSSM leads also to chargino-mediated diagrams. The chargino contribution is specified by six parameters. Three of them enter the $2 \times 2$ chargino mass matrix:

$$m_{\chi^+} = \left( \begin{array}{cc} M & m_W \sqrt{2} \sin \beta \\ m_W \sqrt{2} \cos \beta & \mu \end{array} \right).$$

(11)

Following standard notations, we call $\tan \beta$ the ratio of vacuum expectation values, the same that appears also in the charged-Higgs sector, and $M$, $\mu$ the gaugino and higgsino mass parameters, subject to the constraint that the lightest chargino mass satisfies the LEP bound, $m_{\chi^+} > 45 \text{ GeV}$. The squark masses

$$m_{\tilde{q}_i}^2 = \tilde{m}^2 + m_q^2 \pm A\tilde{m}_q$$

(12)

contain two additional free parameters besides the known mass of the corresponding quark $m_q$: a common supersymmetry-breaking mass $\tilde{m}$ and the coefficient $A$. The last parameter included in our analysis is a common mass $m_\tilde{\ell}$ for sleptons, all taken to be degenerate in mass, with the constraint $m_\tilde{\ell} > 45 \text{ GeV}$. Therefore the version of the MSSM we are considering is defined in terms of seven free parameters.

We have computed the Wilson coefficients in the MSSM and then varied the seven above-defined parameters in the experimentally allowed region. The results of our analysis are presented in fig. 3 which shows the regions of the $C_9 - C_{10}$ plane allowed by possible choices of the MSSM parameters. The upper plot of fig. 3 corresponds to parameters which give rise to positive (same sign as in the SM) values of $C_7$, consistent with experimental results on $b \to s\gamma$ ($0.19 < C_7 < 0.32$), while the lower plot corresponds to values of $C_7$ with opposite sign ($-0.32 < C_7 < -0.19$). We also show how our results are affected by an improvement in the experimental limits on supersymmetric particle masses, as can be expected from the Tevatron and LEP 200. Fig. 3 also shows the $C_9 - C_{10}$ regions allowed by the MSSM if the further constraints $m_{H^+} > 150 \text{ GeV}$, $m_\tilde{t}$, $m_{\chi^+}$, $m_\tilde{\ell} > 100 \text{ GeV}$ are imposed.

The regions shown in fig. 3 illustrate the typical trend of the supersymmetric corrections. If supersymmetric particles exist at low energies, we can expect larger values of $C_{10}$ and smaller (negative) values of $C_9$ than those predicted by the SM. This is the general feature, although the exact boundaries of the allowed regions depend on the particular model-dependent assumptions one prefers to use. However, the most interesting feature of supersymmetry is that solutions with negative values of $C_7$ are possible and are still consistent with present data. Moreover, values of the other two coefficients $C_9$ and $C_{10}$ sufficiently different from the SM are allowed, leading to measurable differences in the decay rates and distributions of $B \to X_s \ell^+\ell^-$ and $B_s \to \ell^+\ell^-$.  

**Figure 3.** The region in the $C_9-C_{10}$ plane obtained by varying the MSSM parameters. The upper (lower) plot corresponds to solutions that satisfy the $b \to s\gamma$ experimental constraint with positive (negative) $C_7$ given in eq.(2) and the present bounds $(m_{H^+} > 80 \text{ GeV}, m_\tilde{t}, m_{\chi^+}, m_\tilde{\ell} > 45 \text{ GeV})$. The smaller areas limited by the short-dashed line correspond to the region of the MSSM parameter space that will survive an unsuccessful search for supersymmetry at the Tevatron and LEP 200 ($m_{H^+} > 150 \text{ GeV}, m_\tilde{t}, m_{\chi^+}, m_\tilde{\ell} > 100 \text{ GeV}$).

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