A comparative study of kalman filtering based observer and sliding mode observer for state of charge estimation

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Abstract. Nowadays, electric mobility is starting to define society and is becoming more and more irreplaceable and essential to daily activities. Safe and durable battery is of a great significance for this type of mobility, hence the increasing interest of research activity oriented to battery studies, in order to assure safe operating mode and to control the battery in case of any abnormal functioning conditions that could damage the battery if not properly managed. Lithium-ion technology is considered the most suitable existing technology for electrical storage, because of their interesting features such as their relatively long cycle life, lighter weight, their high energy density. However, there is a lot of work that is still needed to be done in order to assure safe operating lithium-ion batteries, starting with their internal status monitoring, cell balancing within a battery pack, and thermal management. Tasks that are accomplished by the battery management system (BMS) which uses the state of charge (SOC) as an indicator of the internal charge level of the battery, in order to avoid unpredicted system interruption. Since the state of charge is an inner state of a the battery which cannot be directly measured, a powerful estimation technique is inevitable, in this paper we investigate the performances of tow estimation strategies; kalman filtering based observers and sliding mode observers, both strategies are compared in terms of accuracy, design requirement, and overall performances.

1. Introduction
State of charge is considered one of the crucial information for Battery management system especially in electric vehicles, which have the battery as the only source of energy, however this crucial information is not easily obtained sins it is an inner stat which cannot be directly measured, hence the necessity of developing powerful estimation strategies and approaches, to accurately estimate the state of charge.

In literature the proposed estimation techniques could be divided into three main families. The first category contains the colomb-counting method [1], which is one of the earliest strategies used for state of charge estimation, it’s based on the computation of the amp-hours by using load current integration, however the drawbacks of this approach, such as the unknown initial stat of charge, and its sensitivity to measurement noise made it urgent necessary to search for other strategies, such as the case of the second category which is based on closed loop state observers like kalman filtering [2], sliding mode observer[3], the third category is based on the block box models and the artificial intelligence, such as the artificial neural network [4] and fuzzy logic [5], these strategies are known for the independency of their performances on the battery model, and their capability to be used for almost all battery technologies, however in this paper we will focus on the second category strategies, particularly kalman
filters and sliding mode observers, for their high performances, and their capability of performing well under unconditioned conditions.

The rest of this paper is organized as follows, in section 2 an equivalent circuit model is presented to model the battery. In section 3 the design processes of both unscented kalman filter and sliding mode observer are explained. Section 4 elaborates the simulation results. This manuscript ends with a conclusion.

2. Battery modelling

Both Unscented kalman filter and sliding mode observer are model based observers and requires a prior knowledge of the battery model, hence due to its robustness in modeling the phenomenon inside the battery and its implementation simplicity, the first order thevinin model given in figure 1, is chosen in this paper. The model is composed of a shunt resistor R0, an RC branch to model polarization effect, and a nonlinear voltage source Uoc, which models the nonlinear dependency between the state of charge and the open circuit voltage. In order for the model to mimic the performance of the real battery, the internal parameters (R0, R, C), were identified using Levenberg Marquardt nonlinear least square algorithm, the parameters are given in Table (1).

![Figure 1: Lithium-ion battery model](image)

| Parameter | R0(Ω) | C (F) | R(Ω) |
|-----------|-------|-------|-------|
| Value     | 0.0016| 24000 | 0.0024|

3. State of charge estimation

3.1. Kalman filters

Kalman filter (KF) are online and sometimes considered as the optimal mean to predict and correct time-varying system in a way that minimizes the mean of the squared error. Kalman filter is only valid for linear systems, for nonlinear systems, other variant of KF have been proposed through a linearization procedure, starting from extended kalman filter EKF [2] to sigma point kalman filters SPKF [6, 7]. In this paper the performances of the unscented kalman filter will be presented to illustrate the performances of kalman filters in general. The UKF is based on the unscented transformation as linearization procedure. It is a method for calculating the statistic of a random variable that undergoes a nonlinear transformation, to overcome the nonlinear variation of the OCV with respect to the state of charge. As mentioned above, KF require a dynamic state model of the battery, in order to develop a simulation model for the emulation of battery behavior, in this paper the state space representation used for the UKF is presented by equations (1), (2). UKF algorithm is presented below, for background theory refer to [6].

\[ X(n+1) = f(x_n, i_n) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{T}{RC}} \end{bmatrix} X_n + \begin{bmatrix} -\frac{T}{C} \\ \frac{1}{R(1-e^{-\frac{T}{RC}})} \end{bmatrix} i_b + w_k \]

\[ y(k) = h(x_n, i_n) = U_{oc}(SOC) - X_2(k) - R_0 i_b(k) + v_k \]
**Step 1. Initialization:**

\[
Q(k) = E\{w_k w_k^T\}; \quad R(k) = E\{v_k v_k^T\}; \quad E\{w_k v_k^T\} = 0
\] (3)

\[
P_0 = E\{(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\}
\] (4)

\[
\hat{x}_0 = E\{x_0\}
\] (5)

Where Q(k) and R(k) are diagonal matrices that represent respectively the process noise (wk) covariance and the observation noise (vk) covariance, \(P_0\) is the state covariance matrix, and \(\hat{x}_0\) represents the main.

**Step 2. Compute sigma points**

A set of weighted sigma points are deterministically computed, as shown in the following equation:

\[
X_{k-1} = [\bar{x}_{k-1}, \bar{x}_{k-1} + \sqrt{(n + \lambda)P_{x,k-1}}, \bar{x}_{k-1} - \sqrt{(n + \lambda)P_{x,k-1}}]
\] (6)

Where \(\lambda\) is scaling parameter that represents how far the sigma points are separated from the mean, and \(n\) is the system dimension, in our case \(n=2\), for more background theory refer to [8].

**Step 3. Prediction phase.**

1) Propagating sigma points through the system/state function \(f(x, i_{bk})\):

\[
X_{k|k-1}^{(i)} = f(\bar{X}_{k|k-1}^{(i)}, i_{bk})
\] (7)

2) Calculating the propagated mean and covariance:

\[
\bar{x}_{k|k-1} = \sum_{i=0}^{2n} W_m X_{k|k-1}^{(i)}
\] (8)

\[
P_{x,k|k-1} = \sum_{i=0}^{2n} W_m [X_{k|k-1}^{(i)} - \bar{x}_{k|k-1}] [X_{k|k-1}^{(i)} - \bar{x}_{k|k-1}]^T + Q_{k-1}
\] (9)

Where \(W_m\) and \(W_c\) represents the weights of sigma points.

3) Computing the new sigma points matrix using \(\bar{x}_{k|k-1}^{(i)}\) and \(P_{x,k|k-1}^{(i)}\).

\[
X_{k|k-1} = [\bar{x}_{k|k-1}, \bar{x}_{k|k-1} + \sqrt{(n + \lambda)P_{x,k|k-1}^{(i)}}, \bar{x}_{k|k-1} - \sqrt{(n + \lambda)P_{x,k|k-1}^{(i)}}]
\] (10)

4) Propagating the computed sigma points through the nonlinear function \(H(x, i_{bk})\)

\[
y_{k|k-1}^{(i)} = H(\bar{x}_{k|k-1}^{(i)}, i_{bk})
\] (11)

5) Calculating the mean of the output variable.

\[
y_{k|k-1} = \sum_{i=0}^{2n} W_m^o y_{k|k-1}^{(i)} \quad \text{For } i=1...2n
\] (12)

**Step 4. Measurement update**

1) Calculating the covariance of the measurement vector and the cross covariance respectively.

\[
P_{y,k} = \sum_{i=0}^{2n} W_c[y_{k|k-1}^{(i)} - \bar{y}_{k|k-1}] [y_{k|k-1}^{(i)} - \bar{y}_{k|k-1}]^T
\] (13)

\[
P_{xy,k} = \sum_{i=0}^{2n} W_c [x_{k|k-1}^{(i)} - \bar{x}_{k|k-1}] [y_{k|k-1}^{(i)} - \bar{y}_{k|k-1}]^T
\] (14)

2) Calculating the Kalman gain, in order to correct the estimations

\[
K_k = P_{xy,k} P_{y,k}^{-1}
\] (15)

3) Update the estimated Stats with kalman gain.

\[
\hat{x}_k = \bar{x}_{k|k-1} + K_k(y_k - \bar{y}_{k|k-1})
\] (16)

4) Update the propagated state covariance using kalman gain.

\[
\hat{P}_k = P_{x,k|k-1} - K_k P_{y,k} K_k^T
\] (17)

### 3.2 Sliding mode observer

Sliding mode observer SMO [9] is a robust nonlinear state estimator that has the same principal as the sliding mode control, it is widely known for the capability of compensating the effects of nonlinearity and uncertainty in battery models. This observer relies on the determination of the appropriate feedback.
switching gain associated with system uncertainty bound, which balances between the chattering levels, and the error convergence speed, to achieve better SOC estimation accuracy.

Based on figure 1 the dynamic state model used in this section is given by equations (18, 19, 20).

\[
\begin{align*}
\dot{V}_t &= -b_1V_t + b_1U_{oc}(SOC) - a_1.l_b + \Delta f_1 \\
SOC &= b_2V_t - b_2U_{oc}(SOC) + b_2V_{RC} + \Delta f_2 \\
\dot{V}_{RC} &= -b_1V_{RC} + a_2.l_b + \Delta f_3
\end{align*}
\]

Where \( b_1 = 1/(R.C) \), \( b_2 = 1/(R_o.C_b) \), \( a_1 = \frac{R}{R.C} + \frac{K}{C_b} + \frac{1}{c} \), \( a_2 = \frac{1}{c} \), and \( \Delta f_1, \Delta f_2, \Delta f_3 \) are modelling uncertainties.

The above equations are obtained by considering that there exists a piecewise linear relationship between the OCV and the SOC in a certain range of the SOC. Therefore, the OCV is expressed as a function of the SOC using piecewise linearization method.

\[
U_{oc}(SOC) = K.SOC + v
\]

Where the values of \( K \) and \( v \) are constants in every 10% of SOC, which allow us to represent the time derivative of \( U_{oc} \) in each 10% of SOC.

\[
Uoc'(SOC) = -K \frac{b}{C_b}
\]

In order to accurately estimate the state of charge, the SMO equations are proposed as follows.

\[
\begin{align*}
\dot{V}_t &= -b_1V_t + b_1U_{oc}(SOC) - a_1.l_b + \hat{f}_1sgn(e_{vt}) \\
\dot{SOC} &= b_2V_t - b_2U_{oc}(SOC) + b_2V_{RC} + \hat{f}_2sgn(e_{Uoc}) \\
\dot{V}_{RC} &= -b_1V_{RC} + a_2.l_b + \hat{f}_3sgn(e_{VRC})
\end{align*}
\]

Where

\[
sgn(e) = \begin{cases} 
+1, & e > 0 \\
-1, & e < 0
\end{cases}
\]

And stats error is

\[
\begin{align*}
e_{vt} &= V_t - \hat{V}_t \\
e_{Uoc} &= U_{oc}(SOC) - U_{oc}(\hat{SOC}) = Ke_{SOC} \\
e_{RC} &= V_{RC} - \hat{V}_{RC}
\end{align*}
\]

The dynamics of the error \( \dot{e}_{vt} \), \( \dot{e}_{Uoc} \), \( \dot{e}_{VRC} \) are obtained by subtracting equations. (18, 19, 20) from equations. (23, 24, 25).

\[
\begin{align*}
\dot{e}_{vt} &= -b_1e_{vt} + b_1e_{Uoc} + \Delta f_1 - \hat{f}_1sgn(e_{vt}) \\
\dot{e}_{SOC} &= b_2e_{vt} - b_2Ke_{SOC} + b_2e_{VRC} + \Delta f_2 - \hat{f}_2sgn(e_{SOC}) \\
\dot{e}_{VRC} &= -b_1e_{VRC} + \Delta f_3 - \hat{f}_3sgn(e_{VRC})
\end{align*}
\]

Based on the Lyapunov stability theory, the asymptotic convergence of the terminal voltage error can be proved by choosing the following candidate of Lyapunov function.

\[
V_{vt} = \frac{1}{2}e_{vt}^2
\]

To guarantee the existence of sliding regime, the time derivative of the candidate of Lyapunov function must be negative, and after developing the expressions of \( \dot{V}_{vt} \), the value of \( \Gamma_1 \) that guarantees \( \dot{V}_{vt} < 0 \) is \( \Gamma_1 \gg \Delta f_1 \). Once the sliding surface is reached then \( \dot{e}_{vt} = e_{vt} = 0 \), and the unmeasurable SOC error can be derived by replacing \( \dot{e}_{vt} \) and \( e_{vt} \) with zeros in equation (30).

\[
e_{SOC} = \left( \frac{\Gamma_1}{Kb_1} \right) sgn(e_{vt})_{eq}
\]

Similarly \( e_{VRC} \) can be derived from equation. (31) by replacing \( \dot{e}_{Uoc} \) and \( e_{Uoc} \) with zero.
\[ e_{vr} = \left\{ \left[ \frac{\Gamma_2}{b^2} \right] \text{sgn}\left( \left[ \frac{\Gamma_1}{Kb_1} \right] \text{sgn}(e_{vt}) \right) \right\}_{eq} \]  

Finally the SMO observer is obtained by substituting equations (34, 35) into equations (24, 25).

4. Simulation and results discussion

Following the design processes, both sliding mode observer and unscented kalman filter were tested and simulated under matlab Simulink environment for a li-ion battery of 2Ah capacity. Simulation results are presented in figure 2, 3, 4 and 5.

![Figure 2: State of charge estimation using SMO](image1)

![Figure 3: State of charge estimation error using SMO](image2)

![Figure 4: SOC estimation using UKF](image3)

![Figure 5: SOC estimation error using UKF](image4)

From the above simulation results, it is clear that both strategies perform well in SOC estimation, although the sliding mode observer (SMO) is slightly more accurate with maximal error value less than 7%, at the beginning of the estimation, and mean error less than 1.7%, compared to maximal error value less than 6% and mean error less than 2% in the case of UKF, which is due to the robustness of the SMO against modeling errors and internal parameters uncertainties. On the other
hand it is clear that SMO suffers from high frequency chattering around the true SOC, which blurs the SOC estimation and affects the stability of the observer, and unlike UKF, SMO has a relatively slow convergence time.

5. Conclusion
The accurate estimation of state of charge, is a key factor of managing batteries with high efficiency. In this paper, a comparative study of kalman filters and the sliding mode observer was carried out, from design challenges to estimation accuracy. From the obtained results, it can be concluded that both strategies are competitive in terms of precision, the SMO is more robust against model uncertainties if the feedback gain is correctly chosen. From implantation point of view and based on work presented in [10, 11], the UKF requires a powerful platform, on the other hand the SMO is easy to implement since it only requires the computation of the feedback gain.

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