Application of the bridged crack model for evaluation of
materials repairing and self-healing

M Perelmuter
Institute for Problems in Mechanics of RAS
prospect Vernadskogo 101-1, Moscow, 119526 Russia

E-mail: perelm@ipmnet.ru

Abstract. The bridged crack model is used for analysis of repairing and self-healing
of cracked structures. Material repairing is treated as insertions of external ligaments
into cracks or placement of the reinforcing patches over cracks. Bonds destruction and
regeneration at the crack bridged zone is evaluated by the thermo-fluctuation kinetic
theory. The healing time is dependent on the chemical reaction rate of the healing
agent, the crack size and the external loads. The decreasing of the stress intensity
factors is used as the measure of the repairing and healing effects. The mathematical
background of the problem solution is based on the methods of the singular integral-
differential equations. The model can be used for the evaluation of composite
materials durability.

1. Introduction

Actions of the external loads, aggressive environments and physical fields lead to degradation of the
physical-mechanical properties of materials and to loss of structures integrity. Maintenance and
recovery of serviceability of materials and structures is the fundamentally scientific and applied
problem. Models of cracks formation and evolution, based on the crack surfaces interaction
accounting (a zone of crack surfaces interaction is named usually as ‘process zone’ or ‘cohesive-
bridged zones’), allows us to investigate processes of destruction in heterogeneous mediums
(materials) at various scale levels. Models of a crack with the developed bridged zone can be used also
in the analysis of the processes connected with repairing of damaged structures and self-healing of
materials.

At presence of the macroscopic cracks-defects (flaws) which can be detected visually or by means of
special sensors, the repairing of serviceability is possible, in particular, with use of special fillers or
repairing cover plates [1]. Accumulation of microscopic defects inside materials and also inside
inaccessible parts of structures can lead to catastrophic destruction. In such cases can be effectively
used self-healed materials [2-3]. Self-healing capability of materials is especially important when the
human intervention is complicated, for example, in space, or in extreme physical or chemical
conditions. This process of self-healing is very important and for nanocomposites which have
excellent functional properties, but it can be degraded under extreme service conditions. Self-healing
of cracks in materials can occur under special external affects (mainly, the heating which is making
active chemical or physical processes in a material [4]).
The other approach is based on the development of new composite materials containing special components (inclusions) which provide self-recovery of cracked material. For example, in the polymeric material made on the basis of epoxy matrix, capsules with "a healing liquid" (dicyclopentadiene), and also microcapsules of catalyzing substance are introducing. In an initial state the catalyst is not in contact with dicyclopentadiene. If there is a crack in a material that grows, it destroys walls of capsules and captures catalyst microcapsules. Dicyclopentadiene outflows and due to capillary effects fills a crack and comes into contact with the catalyst. The polymerisation reaction of dicyclopentadiene leads to liquid solidification and bonds healing. "Healed" material is capable to stand the loads similar to new material (the restoration factor of mechanical properties is about 75%-90% [5]).

An alternative approach to self-healing is based on inserting of hollow fibers system into a material (by analogy to vascular system of living organisms), containing a special reagent [6-7]; level of material restoration in the latter case attain up to 100%. The technique of inserting nanotubes into a polymeric composite material which becomes electroconductive, shows high efficiency. On changing nanotubes conductance system it is possible to determine a place of defect formation and to provide its self-healing due to local heating at an electric current getting through [8]. Nanotubes which are introduced into a polymeric matrix can also contain "healing liquid". In this case they are simultaneously supporting fibers, forming of nanocomposite and provide material healing at crack formation [9].

Numerical and analytical estimations of various factors influence on materials repairing and self-healing processes are necessary for increasing these procedures efficiency. We note, also, that in many cases the experimental investigations can be extremely long and labor-consuming. It is possible to mark out three basic stages of process material repairing and self-healing: 1) formation and growth of cracks in a heterogeneous material under the external loading; 2) insertion into cracks or pores of the repairing media or self-healing agent; 3) strengthening and restoration of ligaments between cracks or pores surfaces, leading to a material or structure restoration.

Experimental results have been obtained on each of these stages, but physical-mechanical models and numerical techniques are only started to develop. Known numerical techniques are based, mostly, on the finite element method [10-12].

The model of a crack with bonds in the bridged zone allows one to determine the crack resistance and the adhesive strength of joints between different materials on the basis of micromechanical properties of the bonds [13-15]. The development of this model with consideration of time-variation in the physical and mechanical properties in the crack bridged zone allows one to estimate the long-term strength and variation over time of materials resistance characteristics [16].

In the present paper the bridged crack model is used for modeling of cracks repairing and materials healing. For cracks repairing we consider quasistatic problem for a bridged crack with increased ligaments stiffness. Healing of material due to bonds restoration is considered on the basis of Zhurkov’s kinetic model of thermal fluctuation fracture [17-18] together with the crack bridged zone model.

The estimation of bonds restoration inside a crack is based on the following assumptions:
- at the initial time instant the polymerization process of healing agent inside of a crack is starting;
- polymerization process rate is defined by kinetic theory;
- restored bonds between crack surfaces are considered as a part of the crack bridged zone;
- variation of bonds density over time in that zone is considered on the basis of the thermal fluctuation mechanism;
- bonds rigidity is proportional to their density at every point of the crack bridged zone.

2. Crack bridged model
The plane elasticity problem on a crack at the interface of two dissimilar joint half-planes is considered. The crack surface interaction exists in the bridged zones, \( \ell - d \leq |x| < \ell \) (Fig.1). As a
simple mathematical model of the crack surfaces interaction we will assume that the linearly elastic bonds act through out the crack bridged zones.

Denote by \( \sigma(x) \) the stresses arising in the bonds

\[
\sigma(x) = \sigma_{yy}(x) - i \sigma_{yx}(x), \quad i^2 = -1, \quad (1)
\]

where \( \sigma_{yy}(x) \) and \( \sigma_{yx}(x) \) are the normal and shear components of bonds stress. The crack opening, \( u(x) \) at \( \ell - d \leq |x| < \ell \) is determined as follows [13]

\[
u(x) = u_{j}(x) - i u_{i}(x) = \frac{H}{E_b} (\phi_1(x)\sigma_{yy}(x) - i \phi_2(x)\sigma_{yx}(x)), \quad (2)
\]

where \( u_{j}(x) \) are the projections of the crack opening on the coordinate axes (Fig.2), \( H \) is a linear scale related to the thickness of the intermediate layer adjacent to the interface, \( E_b \) is the effective elasticity modulus of the bond and \( \phi_{1,2} \) are dimensionless functions of the coordinate \( x \).

By incorporating linearity of the problem one can represent the crack opening \( u(x) \) as follows

\[
u(x) = u_{o}(x) - u_{b}(x), \quad (3)
\]

where \( u_{o}(x) \) and \( u_{b}(x) \) are the crack opening caused by the external loads and bond stresses \( \sigma(x) \), respectively.

By incorporating formulae (2)-(3) one can obtain a system of integral-differential equations relative to bonds stress \( \sigma(x) \).

Introduce the new variable, \( s = x / \ell \), and differentiate relation (3) to obtain

\[
\frac{H}{\ell} \frac{\partial}{\partial s} \left[ \phi_1(s)\sigma_{yy}(s) - \phi_2(s)\sigma_{yx}(s) \right] + u_{b}'(s)E_b = u_{o}'(s)E_b, \quad (4)
\]
where the right side of this relation is the given function of the coordinate.
Taking into account the results given in [13, 19] one can obtain the system of two singular integral-differential equations (SIDE) relative to bonds stresses $\sigma_{xy}(x)$ and $\sigma_{yy}(x)$ in the following form [13]

$$T_y(s, \sigma) \frac{dq_y(s)}{ds} + W_y(s, \sigma)q_y(s) + \int_{-d/\ell}^{d/\ell} G_y(s, t)q_y(t)dt = Z_y(s), \quad i, j = 1, 2$$  \hspace{1cm} (5)

where $q_j(s)$ are unknown function depending on bond stresses as follow

$$\sigma_{yy}(s) - i\sigma_{xy}(s) = (q_y(s) - iq_x(s))\sqrt{1 - s}\left(\frac{1 - s}{1 + s}\right)^{-\beta},$$  \hspace{1cm} (6)

The explicit relations for the equations (5) kernels $T_y, W_y, G_y, Z_y$ are presented in [13], parameters $\varepsilon$ and $\beta$ in (5)-(6) depend on the materials and bonds properties.

For numerical solution of the equations system we use a collocation scheme with piecewise quadratic approximation unknown of the bond stresses. See details in [13].

Having the distribution of the bond stresses $\sigma_{xy}(x)$ and $\sigma_{yy}(x)$ over the crack bridged zone one can calculate the stress intensity factors (SIF) $K_I, K_{II}$ following to [20]

$$K_I + iK_{II} = \lim_{\delta \to 0} \sqrt{2\pi\delta} \left(\sigma_{yy}(\delta) + i\sigma_{xy}(\delta)\right)\delta^{-\beta},$$  \hspace{1cm} (7)

where $\sigma_{yy}(\delta)$ and $\sigma_{xy}(\delta)$ are the stresses ahead the crack tip caused by the external loads and by the bonds stresses, $\delta$ represents the small distance to the crack tip.

On the other hand, the SIF can be written as follows

$$K_I + iK_{II} = (K_{I}^{ext} + K_{I}^{int}) + i(K_{II}^{ext} + K_{II}^{int}),$$  \hspace{1cm} (8)

where $K_{II}^{ext}$ and $K_{II}^{int}$ are the SIF caused by the external loads and the bond stresses.

By incorporating the formula for the stress distribution ahead the interface crack tip under arbitrary loads [19] and using (7)-(8) we obtain for the external tension load $\sigma_0$ ($K_0$ is SIF modulus)

$$K_I + iK_{II} = \frac{\sqrt{\pi\ell}}{(2\ell)^{\beta}} \left[\sigma_0(1 + 2i\beta) - \frac{2\cosh(\beta)}{\pi} \int_{-\delta/\ell}^{d/\ell} \frac{(q_y(t) + iq_x(t))}{\sqrt{1 - t^2}} dt\right], \quad K_0 = \sqrt{K_I^2 + K_{II}^2}$$  \hspace{1cm} (9)

3. Kinetics of bonds at the crack bridged zone

The restoration of bonds inside of cracks can be considered on the basis of the Zhurkov’s fluctuation model [17, 18]. According to the fluctuation theory of fracture the lifetime of bonds ($\tau_d$) at the point $x$ of the crack bridged zone under the external loading is the exponential function

$$\tau_d = \tau_0 \exp\left(\frac{U_d - A(x)}{kT}\right)$$  \hspace{1cm} (10)
where \( \tau_0 \) is the characteristic time (10\(^{-13}\)-10\(^{-12}\) s), \( k \) is the Boltzmann constant, \( T \) is absolute temperature, \( U_0 \) is the energy of bonds destruction, \( A(x) \) is the work of the deformation per one intermolecular bond at the point \( x \) of the crack bridged zone.

Bonds lifetime at the crack bridged zone is the function of the bond position. It’s assumed that the time increasing of the bonds surface density \( n(x,t) \) is governed by the equation

\[
n(x,t) = N_0 \exp\left(\frac{t}{\tau_b(x)}\right),
\]

where \( \tau_b(x) \) is the lifetime of the bond and \( N_0 \) is a small initial bond density inside of a crack. The increasing of bonds surface density over time can be modelled by the changing of bonds properties. Let’s denote the bond stiffness by \( k_s \). Then the effective stiffness of bonds per unit of an area in the crack bridging zone, \( k \), is determined as follows

\[
k(x,t) = k_n(x,t) = k_0 \exp\left(\frac{t}{\tau_b(x)}\right),
\]

where \( k_0 = k_s N_0 \) is the initial effective stiffness of bonds in the crack bridged zone. Since the effective bonds compliance \( c(x,t) \) is the value reciprocal to its stiffness one can write that

\[
c(x,t) = c_0 \exp\left(-\frac{t}{\tau_b(x)}\right).
\]

This relation enables us to model the kinetics of bonds restoration in the crack bridged zone by means of the bonds compliance variation over time. The system of SIDE for the solving of static crack bridging problems [13] was extended for the time-steps scheme [16]. On each time step the bond compliances depend on the density of bonds according to relation (13) and the system of SIDE is solved numerically. If the initial crack reaches sufficient size in the polymer composite with the healing microcapsules and/or with shape-memory alloy wires then the self-healing process is started. On this stage of bonds restoration the problem for the crack bridged zone is solved. The healing time is dependent on the chemical reaction rate of the healing agent, wires properties, crack size and the external loads. The decreasing of the stress intensity factors is used as the measure of the healing effect. The model can be used for the evaluation of healed composite materials durability. The results of computation modelling of interfacial crack repairing and bonds healing are presented.

4. Computational results

The above proposed approach was used for several problems analysis. A crack on the interface between different materials under the external tension \( \sigma_0 \) was considered. It was assumed that at the initial time instant (when crack surfaces are free of constraints) some healing process is activated inside a crack and bridges between the crack surfaces are built.

The numerical calculations were performed for plane strain conditions and the following elastic constants of the joint materials and bonds (Cu-epoxy polymer): \( E_1 = 25GPa \), \( E_2 = 135GPa \); \( E_b = E_2 \), \( v_1 = v_2 = 0.35 \). The purpose of calculations is the dependence analysis of the repairing and self-healing processes efficiency (the measure of efficiency is the level of SIF at the crack tip) on the bridged zone length (the crack filling with bonds) and on the bonds stiffness.

In Fig. 3 for different values of the relative bonds stiffness \( \kappa = \ell/H \) the dependencies of the SIF module (it can be obtained from eq. (9)) versus the relative bridged zone length are shown. For bonds with relative stiffness more than 10 the repairing efficiency reaches the saturation if the crack has filled with bonds more than for the half of it’s length.
The evolution of the healing process as the dependence of the SIF module versus the relative bond stiffness (in logarithmic scale) is shown in Fig. 4, where the scale factor $\kappa_0 = 1$ corresponds to $\ell = H$. Saturation of the repairing effect is observed for bonds with rather big stiffness.

The next result shows the healing process for the whole crack over time. Computation of the SIF dependence on the healing time were performed for the following problem parameters: the activation energy of the healing process - $U_H = 100 \text{kJ/mole}$, $\tau_0 = 10^{-12} \text{s}$ (see relation (10)), the initial bonds density is $n_0 = 10^{17} \text{m}^{-2}$, the size of the healed crack is $2\ell = 10^{-5} \text{m}$. The SIF dependence on the healing time (in logarithmic scale) is shown in Fig. 5. During the healing time the initial bonds stiffness increases by 20 times. Therefore, the SIF module decreases due to bonds restoration.

The results of computations on the basis of the crack bridged model allow us to estimate the SIF module variation due to the repairing of cracks and bonds restoration over time. The results based on the kinetic approach to bonds restoration might be helpful for durability analysis of adhesion joints. Since the computational parameters of the kinetic model strongly depend on the initial data (which is caused by the exponential dependence of the durability in formula (10)), the comparative analysis of different healing agents and processes under appropriate loading conditions is of greatest practical interest in this method.
Acknowledgements

Financial support for this research was partly provided by RFBR, Project No. 17-08-01312.

References

[1] Wang C H 2000 Fatigue crack closure analysis of bridged cracks representing composite repairs, Fract. Engng. Mater. Struct 23, 477-488
[2] White S R, Sottos N R, Geubelle P H, Moore J S, Kessler M R, Sriram S R, Brown E N and Viswanathan S, 2001 Autonomic healing of polymer composites, Nature 409, 794–797
[3] Syrett J A, Beecer C R and Haddleton D M 2010 Self-healing and self-mendable polymers, Polymer Chemistry 1, 978-987
[4] Volynsky A L and Bakeev N F 2009 Healing of an interphase surface in polymeric systems, High-molecular compounds, Series A 51, 1783-1816
[5] Blaiszik B J, Sottos N R, White S R 2008 Nanocapsules for self-healing materials, Composites Science and Technology 68, 978-986
[6] Toohey, K S, Sottos N R, Lewis J A, Moore J S and White S R 2007 Self-healing materials with microvascular networks, Nature, 6, 581-585
[7] Trask R S, Williams G J and Bond I P 2007 Bioinspired Self-Healing of Advanced Composite Structures Using Hollow Glass Fibres, J. Royal Society, Interface 4, 363-371
[8] Zhang W, Sakalkar V. and Koratkara N. 2007 In situ health monitoring and repair in composites using carbon nanotube additives, Applied physics letters 91, 133100-133102
[9] Lanzara G, Yoon Y, Liu H, Peng S and Lee W-I 2009 Carbon nanotube reservoirs for self-healing materials, Nanotechnology 20, 335704-7
[10] Maiti S and Geubelle P. H. 2006 Cohesive modelling of fatigue crack retardation in polymers - Crack closure effect, Eng. Fract. Mech. 73, 22-41
[11] Ural A, Krishnan V R and Papoulia K D 2009 A cohesive zone model for fatigue crack growth allowing for crack retardation Int. J. of Solids and Structures 46, 2453-2462
[12] Ozaki S, Osada T and Nakao W 2016 Finite element analysis of the damage and healing behavior of self-healing ceramic materials, Int. J. of Solids and Structures 100–101, 307-318
[13] Goldstein R V and Perelmuter M N 1999 Modeling of bonding at the interface crack, Int. J. Fracture 99 53-79
[14] Perelmuter M N 2007 A criterion for the growth of cracks with bonds in the end zone J. Appl. Math. Mech. 71 (1) 137-153
[15] Goldstein R V and Perelmuter M N 2009 Modeling of crack resistance of composite materials (in Russian), Computational continuum mechanics 2, 22-39
[16] Goldstein R V and Perelmuter M N 2012 Kinetics of crack formation and growth on the material interface, Mech. Solids 47, 400-414
[17] Zhurkov S N 1965 Kinetic concept of the strength of solids Int. J. Fracture Mech. 1, 311-323
[18] Regel V G, Slutsker A I and Tomashevskii E E 1974 Kinetic Nature of Strength of Solids (Moscow: Nauka), in Russian
[19] Slepijn L I 1981 Mechanics of Cracks (Leningrad: Sydostroenie), in Russian
[20] Rice J R 1988 Elastic fracture mechanics concepts for interfacial cracks Trans. ASME J. Appl. Mech. 55 98-103