Mechanical behaviour analysis of superconducting magnet in LHD-type reactor FFHR

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Abstract. The force-free helical reactor (FFHR) is a conceptual design of a steady state fusion reactor that has been studied to demonstrate a LHD-type fusion power plant. The helical coil of the FFHR has a major radius of 14 m, a magnetic energy of 120 GJ and a maximum field of 13 T. An aluminium-alloy jacketed Nb$_3$Sn superconductor and indirect cooling using cooling panels within the coil was proposed as a candidate magnet system for the helical coil. Due to the complicated three-dimensional structure of the helical coil winding, it is very important to clarify the mechanical behaviour of the magnet, by considering not only the overall force and deformation but also the detailed stress and strain behaviour in the cross section of the coil. In this study, we evaluated the mechanical behaviour of the helical coil using a 3D axisymmetric finite element method model by considering the non-axisymmetric electromagnetic force.

1. Introduction
The experimental results of the large helical device (LHD) have shown that a LHD-type fusion power reactor has many advantages such as a steady-state nature and a lack of plasma current operation [1]. The force-free helical reactor (FFHR) is a conceptual design of the LHD-type heliotron power reactor developed at the National Institute for Fusion Science under a collaboration program among universities [2, 3]. Figure 1 shows the schematics of the FFHR. The superconducting magnet system in the FFHR consists of one pair of helical and two pairs of poloidal coils. Among several design parameters considered thus far, we focus on just one, FFHR2m1. The helical coil of FFHR2m1 has a major radius of 14 m, a magnetic energy of 120 GJ and a maximum field of 13 T [2]. An aluminium-alloy-jacketed Nb$_3$Sn superconductor and indirect cooling using cooling panels within the coil was proposed as a candidate magnet system for the helical coil [4]. Since the helical coil has a complicated three-dimensional structure, it is difficult to describe the mechanical behaviour of the coil. However, it is important to clarify the mechanical behaviour and rigidity in order to optimize the coil design.

Forced flow cooling is usually used for large-scale fusion experimental devices such as ITER, JT-60SA and Wendelstein 7-X, since a cable-in-conduit conductor (CICC) has the advantage of being mechanically strong. Indirect cooling is an alternative approach, which uses a superconducting magnet that solves the problem of pressure drops in CICC. In some studies of the FFHR, an aluminium-jacketed Nb$_3$Sn superconductor has been proposed for use in the conceptual design of the indirectly cooled superconducting helical coil [4]. In this study, we evaluated the mechanical behaviour of the helical coil in the FFHR with indirect cooling using a three-dimensional axisymmetric model,
considering not only the overall force and deformation but also the detailed stress and strain
distribution in the cross section of the coil.

2. Analytic model

The helical coil of the FFHR has a complex three-dimensional structure. The curvature of the coil
winding changes with the toroidal angle. It is believed that a circular coil that has an average curvature
similar to that of an actual helical coil can sufficiently estimate the mechanical behaviour of the coil.
The average radius of curvature of the helical coil was 5500 mm at the centre of the cross-section of
the coil. The cross-sectional structure of the helical coil introduced in [4] was used for creating the
analytic model. As shown in Figure 2, the structure had a rectangular cross-section of 1.8 m in width
and 0.9 m in height. In addition, there were 432 superconductors made of Nb₃Sn with an aluminium-
alloy jacket. The radius from the central axis to the centre of the coil cross-section was set as 5.5 m.
The current flow in each superconductor was 100 kA. The cooling panels were placed at every two or
four turns of the winding. The insulator used in the superconductors was unknown; therefore, we
assumed that alumina ceramics with resin were used for insulation. A stainless steel coil case with a
thickness of 300 mm at the top and 150 mm at both sides of the coil section was used.

An electromagnetic force was applied as the body force by multiplying the current density and the
magnetic field in the superconducting region, considering the actual magnetic field distribution [3, 5].
The magnetic field can be divided into two directions according to the axial and radial direction of the
cross-section of the coil. As a result of an interaction between the magnetic field and the current flow,
the axial magnetic field produced a hoop force, while the radial one produced an overturning force.
Each magnetic field was applied to every single superconducting region to ensure that the

![Figure 1. Schematics of the FFHR.](image1)

![Figure 2. Geometry of the analytic model.](image2)
electromagnetic force was precisely applied to the coil. ANSYS version 10.0 was used for calculation, and the three-dimensional harmonic axisymmetric solid element was adopted. This element can calculate the sinusoidal displacement or force distribution in the circumferential direction. Using this feature, both the hoop force and the overturning force can be calculated.

2.1. Hoop force
The maximum stress and strain level is observed when the total electromagnetic force reaches the maximum, if the hoop force is distributed in the circumferential direction. Although the magnetic field intensity was different in every cross-section, an averaged magnetic field was applied at every single superconductor position along the circumference. Furthermore, a constant value was added to the averaged magnetic field so that the total hoop force in the cross-section was equal to the maximum overall hoop force. Figure 3 shows the applied magnetic field distribution. The distribution is bilaterally symmetrical about the direction of the coil width. This magnetic field was applied to the superconducting region of the model as the mass density variable in the analysis code, and the current flow was set as the constant gravity variable for the radial direction in the code. The electromagnetic force can be calculated as the body force by multiplying the magnetic field with the current flow in the same way the mass density and gravity are used to calculate the weight as the body force.

2.2. Overturning force
The axisymmetric model does not consider a deformation or a force that changes adjacent to the circumferential direction. However, if the deformation or the force is expressed using a sinusoidal function, it is possible to mathematically calculate stress and displacement using the same axisymmetric model. Here, we used this method via the ANSYS program, which includes a part of this technique.

First, the averaged magnetic field distribution is obtained in the same way as the hoop force (shown in Figure 4) as the axisymmetric force to determine the internal stress distribution without outward force. Next, the non-axisymmetric overturning deformation was calculated individually and added together. Since the non-axisymmetric force must be a sinusoidal function, the Fourier series expansion was used. The magnetic field expanded by the Fourier series is expressed as

\[
B(\theta) = \frac{a_0}{2} + \sum_{m=1}^{n} [a_m \cos(m\theta) + b_m \sin(m\theta)]
\]

\[
a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} B(\theta) \cos(m\theta) d\theta
\]

\[
b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} B(\theta) \sin(m\theta) d\theta
\]
where $B(\theta)$ is the magnetic field that generates an overturning force, and $m$ is the order of the harmonic waves. Figure 5 shows the expanded result. The horizontal line represents the circumferential angle of the model coil. Here, we assumed that $B(\theta)$ had only a $b_m$ term (i.e. $a_m = 0$) and the approximation was performed to the eighth order. The total value expresses the calculation in [3] very well. The magnitude of each harmonic wave was transformed to the magnitude of the radial magnetic field, and each load case was calculated. Finally, the obtained stress, strain and displacement were added together to determine the total mechanical behaviour.

3. Material property

The material properties of the superconducting region were selected according to the rule of mixture. The other components were treated as isotropic materials. As shown in Figure 2, the superconductor had a 50 mm square shape and a 32 mm square Nb$_3$Sn superconducting region. The superconductor included an 18-mm-thick aluminium alloy and 1-mm-thick insulation. Using the material properties of the components at a cryogenic temperature (4 K) [6, 7], the properties shown in tables 1 and 2 were used in the analytic model.

| Table 1. Material properties of components. |
|--------------------------------------------|
| **Young’s modulus (GPa)** | **Poisson’s ratio** | **Notes** |
| Nb$_3$Sn | - | - | See Table 2 |
| Aluminium | 77 | 0.327 | 6061-T6 |
| Insulator | 80 | 0.3 | Alumina w/epoxy |
| Stainless steel | 208 | 0.284 | SS316 |

| Table 2. Elastic modulus of Nb$_3$Sn wire region.* |
|------------------------------------------|
| **Young’s modulus (GPa)** | **Poisson’s ratio** | **Shear modulus (GPa)** |
| $E_x$, $E_y$, $E_z$ | $\nu_{xy}$, $\nu_{yx}$, $\nu_{zx}$ | $G_{yz}$, $G_{zx}$, $G_{xy}$ |
| Nb$_3$Sn | 40.5 | 82.4 | 0.32 | 0.33 | 31.0 | 15.4 |

*assumptions: cross-sectional composition; Nb$_3$Sn: 0.8, indium: 0.2.

4. Result

Figures 6–9 show the results of the hoop force analysis with respect to the radial displacement, the stress distribution in the radial direction, the hoop stress distribution and the hoop strain, respectively. The maximum radial displacement was approximately 8.7 mm at the bottom. The superconductor was subjected to a transverse compression of approximately 100 MPa, as shown in Figure 7. A maximum hoop stress of 336 MPa appeared in the stainless steel cooling panel in the innermost area. The strain by the hoop force was 0.173% at the bottom centre of the superconductor.
Figure 6. Radial displacement by the radial electromagnetic force.

Figure 7. Radial stress by the radial electromagnetic force.

Figure 8. Hoop stress by the radial electromagnetic force.

Figure 9. Circumferential strain by the radial electromagnetic force.

Figure 10. Axial displacement by the axial electromagnetic force.

Figure 11. Radial stress by the axial electromagnetic force.

Figure 12. Axial stress by the axial electromagnetic force.

Figure 13. Hoop stress by the axial electromagnetic force.
Maximum stress is observed when the overturning force reaches the peak value through the circumferential angle. Figures 10–13 show the results at the circumferential angle of 28 degrees in the overturning force analysis with respect to the axial displacement, the stress distribution in the radial direction, the hoop stress distribution and the hoop stress, respectively. With the outward overturning force, the coil deformed in the axial direction, and the maximum displacement was around 2.75 at the bottom corner. A compressive stress of 229 MPa was applied to the cooling panel section, as shown in Figure 12. The stress in the radial or circumferential direction was not as high as the compressive stress, except that at the side wall.

5. Discussion
The stress and strain values in the coil were sufficiently low compared with the allowable level for each material. The hoop force was more effective than the overturning force for the generation of stress and strain. The stress and strain level in this analysis was slightly lower than that of the calculation result in [4], which was performed using an infinite solenoid model. This difference is believed to be mainly caused by the effect of the axial magnetic field distribution. In the solenoid coil model, the magnetic field in the axial direction was kept constant, while that in the proposed model was varied. This result means that, owing to the ‘force free’ design of the FFHR, the internal stress distribution was also reduced. This is greatly beneficial in magnet design.

In the overturning force analysis, the maximum stress in the superconductor appeared at the corner. This was an effect of the assumption that the cross-section was maintained in the plane after deformation even though there was a possibility that the cross-section deforms in an out-of-plane direction in the actual helical coil. However, the stress level was negligible, and therefore this effect can be overlooked.

6. Conclusions
Stress and strain distributions in the helical coil of the FFHR were investigated using a three-dimensional axisymmetric FEM model. Considering sinusoidal deformation by expanding the magnetic field along the circumferential angle to the Fourier series, it was possible to consider not only the electromagnetic hoop force but also the overturning force. As a result of stress analysis, owing to the force-free geometry, the stress and strain levels for each component were all within the allowable values. The thickness of the structural material of the coil case can be reduced further than that assumed in the FEM model. Indirect cooling with the aluminium-jacketed Nb₃Sn superconductor can be used for the helical coil in the FFHR.

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