Practical Quantum Simulation of Non-Hermitian Dynamics

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Non-Hermitian quantum systems have recently attracted considerable attentions due to their exotic properties. Though many experimental realizations of non-Hermitian systems have been reported, the non-Hermiticity usually resorts to the hard-to-control environments. An alternative approach is to use quantum simulation with the closed system, whereas how to simulate general non-Hermitian Hamiltonian dynamics remains a great challenge. To tackle this problem, we propose a protocol by combining a dilation method with the variational quantum algorithm (VQA) [30, 31], for practically simulating general non-Hermitian Hamiltonian dynamics. VQA functions by iteratively optimizing a parameterized quantum circuit with limited operations and circuit depth, thus has found plenty of applications in finding ground states and excited states for quantum chemistry [32–35]. Moreover, it has also been used to approximate quantum dynamics of general non-Hermitian Hamiltonians.

In this work, we propose to combine the dilation method [24] with the variational quantum algorithm (VQA) [30, 31], for practically simulating general non-Hermitian Hamiltonian dynamics. VQA functions by iteratively optimizing a parameterized quantum circuit with limited operations and circuit depth, thus has found plenty of applications in finding ground states and excited states for quantum chemistry [32–35]. Moreover, it has also been used to approximate quantum processes and quantum gates [36–39]. Here, we bypass the difficulty of realizing the complex entangled operation in the dilatation method with the use of VQA. This leads to an important advantage that this complex entangled operation can be realized in a much shorter time. To optimize the parameters in VQA, a routine is to apply gradient-based algorithms [40]. However, as the system size and the circuit depth increase, the convergence speed of the conventional gradient-based algorithms usually becomes very slow due to the quickly enlarged parameter space. To mitigate this issue, we utilize the back-propagation strategy, which is widely used in machine learning [41], to accelerate the optimization process.

To verify the effectiveness of our protocol, we apply it to the problem of simulating the evolution of a quantum Ising chain with nonlocal non-Hermitian perturbations [42]. This physical model is important for understanding the quantum phase transition (QPT) of spin systems at nonzero tempera-

I. INTRODUCTION

Recent years have witnessed ongoing interests in exploring non-Hermitian phenomena [1, 2]. Quantum systems driven by non-Hermitian Hamiltonians can lead to unconventional and exclusive features, such as the non-Hermitian topological band [2, 3], the non-Hermitian skin effect [4], the quantum critical phenomenon [5], the chiral population transfer [6, 7], and the anomalous bulk-boundary correspondence [8–10]. However, experimentally investigating non-Hermitian physics is very challenging, because the non-Hermiticity usually originates from particle loss and decoherence assigned by environments, which is hard to manipulate [11–15].

In order to practically explore non-Hermitian physics, there have developed many strategies to simulate non-Hermitian Hamiltonian dynamics using closed quantum systems. The stochastic Schrödinger equation method achieves the non-Hermitian dynamics by averaging the measured results over multiple noise configurations, which needs tremendous experimental resources thus is hard to scale [16–18]. Researchers also suggest dilating a non-Hermitian Hamiltonian into a Hermitian one in a higher-dimensional Hilbert space, but they are only applicable for special types of parity-time-symmetric systems [19–23]. Remarkably, a recent work in Ref. [24] proposes a powerful method that can dilate a general non-Hermitian Hamiltonian into a Hermitian one, by executing a carefully designed quantum circuit on the ancilla-assisted system. Except for single-qubit rotations, this circuit contains a complex entangled unitary operation, which brings a new challenge for experimental realization, especially in the multi-qubit case [24]. Conventional strategies for realizing a general unitary operation include matrix decomposition [25, 26] and quantum simulation using the Suzuki-Trotter formulas [27–29]. However, the former one is actually a highly nontrivial task, and the latter one may not have an available form for specific quantum systems. In addition, both of these two strategies inevitably produce exponentially growing operations, so the accumulated experimental errors will be serious. Therefore, much more efforts should be made on developing feasible methods that enable practical realization of the dynamics of general non-Hermitian Hamiltonians.

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optimization of arbitrary state simulation of applying our protocol to simulate the spin dynamics.

II. METHODOLOGY

We consider the task of simulating general non-Hermitian Hamiltonian dynamics with closed quantum systems. The schematic diagram of our protocol is shown in Fig. 1. Briefly speaking, our protocol consists of two parts: (1) Dilate the non-Hermitian Hamiltonian into a Hermitian one using the quantum circuit depicted in Fig. 1(a); (2) Approximate the entangled operation in the above circuit with VQA; see Fig. 1(b). In the following, we introduce our protocol in detail.

A. Procedures of the dilation method

We first describe the principle of dilating a non-Hermitian Hamiltonian into a Hermitian Hamiltonian with one ancillary qubit [24]. For a general non-Hermitian system with Hamiltonian $H_s(t)$, its quantum state $\rho_s(t)$ satisfies the equation

$$i\partial_t\rho_s(t) = H_s(t)\rho_s(t) - \rho_s(t)H_s^\dagger(t).$$

We introduce a dilated state

$$\rho_{s,a}(t) = \rho_s(t)|-\rangle\langle-| + \rho_s(t)\eta^\dagger(t)|+\rangle\langle+| + \eta(t)\rho_s(t)|+\rangle\langle-| + |\eta(t)\rho_s(t)\eta^\dagger(t)||+\rangle\langle+|$$

driven by some dilated Hermitian Hamiltonian $H_{s,a}(t)$, where $|-\rangle = (|0\rangle - i|1\rangle)/\sqrt{2}$ and $|+\rangle = -i(|0\rangle + i|1\rangle)/\sqrt{2}$ are the eigenstates of the Pauli operator $\sigma_z$ with respect to the ancilla, and $\eta(t)$ is an appropriate linear operator. The evolution

$$U_{H_{s,a}}(\theta) = \exp\left(-\frac{i}{\hbar}H_{s,a}(t_0)\theta \right) = \exp\left(-\frac{i}{\hbar}\eta(t_0)\rho_s(t_0)\eta^\dagger(t_0)\theta \right).$$

The joint system then undergoes the evolution driven by $H_{s,a}(t)$, i.e., $U_{H_{s,a}}(t)$. Afterwards, a rotation $R_a(-\pi/2)$ is applied on the ancilla. The non-Hermitian system’s final state $\rho(t)$ can be extracted by measuring the subspace where the ancillary qubit is $|0\rangle|0\rangle$.
of $\rho_s(t)$ can be obtained by measuring the state $\rho_{s,a}(t)$ in the subspace where the ancilla is $|\psi\rangle$. As proved in Ref. [24], the dilated Hermitian Hamiltonian $H_{s,a}(t)$ is not unique and can be chosen as

$$H_{s,a}(t) = \Delta(t) \otimes I + \Gamma(t) \otimes \sigma_z,$$

where $\Delta(t) = \{H_s(t) + [i\frac{d}{dt}\eta(t) + \eta(t)H_s(t)]\eta(t)\}M^{-1}(t)$ and $\Gamma(t) = i[H_s(t)\eta(t) - \eta(t)H_s(t) - i\frac{d}{dt}\eta(t)]M^{-1}(t)$ with $M(t) = \eta(t)\eta(t) + I$. Here, $\sigma_z$ is the Pauli operator and $I$ represents the identity matrix.

Concretely, the above described dilation method can be realized by the quantum circuit shown in Fig. 1(a). The system’s initial state is arbitrary and denoted as $\rho_s(0)$, meanwhile the ancillary qubit is initialized at $|0\rangle$. Two rotations $R_y(\alpha)$ and $R_x(\pi/2)$ are then successively applied on the ancilla, where $R_z(\beta)$ represents a $\beta$-angle rotation along the $z$ axis and $\alpha = 2\arctan(\eta_0)$. Afterwards, the system and the ancilla jointly undergo the evolution driven by $H_{s,a}(t)$. Finally, a rotation $R_x(-\pi/2)$ is applied on the ancillary qubit. With these operations, the total final state

$$\rho_{s,a}(t) = \rho_s(t)|0\rangle\langle 0| + \rho_s(t)\eta^3(t)|0\rangle\langle 1| + \eta(t)\rho(t)|1\rangle\langle 0| + \eta(t)\rho(t)\eta^3(t)|1\rangle\langle 1|,$$

thus $\rho_s(t)$ can be extracted by measuring in the subspace where the ancilla is $|0\rangle$. However, in this quantum circuit, the evolution governed by $H_{s,a}(t)$ is a general multi-qubit unitary operation, which brings a great challenge for experimental realizations. In the following, we discuss how to solve this problem.

### B. VQA for approximating the evolution governed by $H_{s,a}(t)$

To realize a general unitary operation, a direct way is to decompose it into basic operations, usually in terms of single-qubit gates and CNOT gates. However, it has been proved that synthesizing any $N$-qubit $U(2^N)$ gate needs an order of $O(4^N)$ basic operations [25, 43]. This matrix decomposition task is highly complicated, and the exponentially increasing basic gates will introduce serious accumulated errors. An alternative way is to use quantum simulation with the Trotter-Suzuki formulas [27–29]. Assume a general Hamiltonian that consists of many local-interaction terms, i.e., $H = \sum_i H_i$. The first step is to break up the total evolution time $t$ into a sufficiently large number of equal slices, and the duration of each slice is denoted as $\Delta t$. Thus the evolution governed by $H$ can be calculated by $U = (e^{-iH\Delta t})^{t/\Delta t}$. As in most cases $[H_i, H_i^\dagger] \neq 0$, we then need to approximate $e^{-iH\Delta t}$ by the Trotter-Suzuki formulas. For example, the first-order Trotter-Suzuki formula leads to $U \approx (\prod_i e^{-iH_i\Delta t})^{t/\Delta t}$. It is worth mentioning that the local term $H_i$ may not be available in specific quantum systems, thus more operations are needed to make a transformation. Therefore, there will be a huge number of separated operations, especially when the higher-order Trotter-Suzuki formulas are applied for better precision, which can lead to very large errors.

As the above mentioned two approaches face great challenges in efficiently realizing a general unitary gate, we thus need more practical methods. The VQA is routinely used to find the eigenstates of molecules, but it can also be applied to many other problems, such as simulating dynamics of quantum systems and solving linear equations [31]. Here, we utilize the VQA to approximate the evolution governed by $H_{s,a}(t)$, namely $U_{H_{s,a}(t)}$. Usually, the VQA parametrizes a quantum circuit $U(\theta)$ with $L$ sequentially applied layers (called ansatz), with each layer involving a limited number of single-qubit rotations and nonlocal gates. The specific structure of an ansatz is generally relevant to the problems at hand, but some ansätze can work even when no relevant information is readily known. Here, we use such an ansatz called hardware-efficient ansatz, as shown in Fig. 2(a). The hardware-efficient ansatz determines the form of the nonlocal gates by the connectivity and interactions specific to a quantum hardware, which can reduce the circuit depth, thus has witnessed many practical applications [32, 44]. Concretely,
the parameterized quantum circuit can be expressed as

\[ U(\theta) = \prod_{l=1}^{L} U_l, U_l = U_{\text{ent}}[\otimes_{n=1}^{N} R^{n}(\theta^{l}_{n})], \]

where \( R^{n}(\theta^{l}_{n}) = R^{n}_{x}(\theta^{l}_{n,3}) R^{n}_{y}(\theta^{l}_{n,2}) R^{n}_{z}(\theta^{l}_{n,1}) \) with \( R^{n}_{\gamma}(\theta^{l}_{n,k}) \) representing \( \theta^{l}_{n,k} \) rotation along the \( \gamma \) axis applied on the \( n \)th qubit of the \( l \)th layer \((\gamma = x, y, z)\), and \( U_{\text{ent}} \) represents the hardware-specific nonlocal gate.

On the other hand, as \( H_{s,a}(t) \) is time-dependent, we calculate the target evolution operator \( U_{H_{s,a}(t)} \) by dividing the evolution time \( t \) into sufficiently large \( M \) equal segments, with the duration of each segment denoting as \( \Delta t \). Thus we can get

\[ U_{H_{s,a}(t)} = T e^{-i \int_0^t dt H_{s,a}(t)} = \prod_{m=1}^{M} e^{-i \Delta t H_{s,a}(m \Delta t)}. \]

It is worth noting that we simply treat the evolution \( U_{H_{s,a}(t)} \) as a whole, so that the approximated operator \( U(\theta) \) is very likely to be realized in a much shorter time. This is a great advantage for exploring the long-time behavior of the non-Hermitian Hamiltonian dynamics.

In the VQA, the parameterized circuit \( U(\theta) \) needs to be optimized with some appropriate optimization algorithm by setting a suitable cost function. Here, we set the quantum gate fidelity as a cost function, namely

\[ F(\theta) = |\text{Tr}(U_{H_{s,a}(t)}^{\dagger}(U(\theta)))^2|/4^N. \]

Generally speaking, the optimization algorithms that have been used in the VQA can be classified into three categories, i.e., gradient-based algorithms \([40, 45, 46]\), gradient-free algorithms \([47]\) and machine learning \([48, 49]\). In our protocol, we utilize the common gradient-based algorithm to accomplish the optimization. However, as the system size becomes larger and the number of to-be-optimized parameters increases, the convergence speed of the gradient-based algorithm can be significantly reduced.

To mitigate this issue, we combine the backpropagation strategy \([50]\), which is widely used in machine learning, to accelerate the convergence process. The backpropagation computes the gradients of the cost function by the chain rule, and then iterates backwardly from the last term to avoid redundant calculations of the intermediate terms in the chain rule \([51]\).

Specifically, the gradients of the cost function in Eq. (7) with respect to the parameters \( \theta^{l}_{n,k} \) can be calculated by the chain rule

\[ \frac{\partial F}{\partial \theta^{l}_{n,k}} = \frac{\partial F}{\partial F} \frac{\partial U}{\partial U} \frac{\partial U_{\text{ent}}}{\partial \theta^{l}_{n,k}} \frac{\partial R^{n}}{\partial \theta^{l}_{n,k}}, \]

as shown in Fig. 2(b). During the optimization, there exist many gradient nodes that are repeatedly calculated. For example, in each iteration, the calculations of \( \partial F/\partial \theta^{l}_{n,k} \) with the same \( l \) and \( n \) actually share the same gradient nodes \( \partial F/\partial U \), \( \partial U/\partial U_{\text{ent}} \), and \( \partial U_{\text{ent}}/\partial R^{n} \). The backpropagation strategy suggests computing the above shared gradient nodes only once and recording them for further use. This strategy can significantly save the computing time, and thus accelerates the convergence.

III. NUMERICAL SIMULATIONS

To demonstrate the effectiveness of our protocol, we apply it to simulate the evolution dynamics of an Ising chain with non-Hermitian perturbations \([42]\). This physical model indicates that the QPT can be completely preserved at finite temperatures, and it presents an alternative approach for understanding the QPT of quantum spin systems at nonzero temperatures \([42]\). However, it is a great challenge to realize such non-Hermitian Hamiltonians, especially for the multi-qubit case. In the following, we detailly describe that how our protocol can accomplish this task.

A. Model and methods

We consider the \( N_s \)-qubit Ising model with non-Hermitian perturbations under open boundary condition, which can be described by the following Hamiltonian

\[ H_s = H_0 + \kappa D_n \]

with

\[ H_0 = -J \sum_{n=1}^{N_s-1} \sigma_x^n \sigma_x^{n+1} + g \sum_{n=1}^{N_s} \sigma_z^n \]

and

\[ D_n = \prod_{i<n} (-\sigma_z^i) \sigma_z^n - i \prod_{i<n-s} (-\sigma_z^i) \sigma_y^n \sigma_z^{n-s+1}, \]

where \( \sigma_\gamma^n (\gamma = x, y, z) \) are the Pauli operators applied on the \( n \)th site, \( J \) represents the coupling strength, \( g \) \((g \geq 0)\) denotes the field strength, and \( \kappa \) is a real constant that satisfies \( \kappa \ll g \).

To simulate the evolution dynamics of the above non-Hermitian Hamiltonian using our protocol, we first use the method introduced in Sec. II.A to dilate it into an \( N \)-qubit Hermitian Hamiltonian with one ancillary qubit. Next, in order to approximate \( U_{H_{s,a}(t)} \) with the VQA, we need to focus on a specific quantum platform. Taking the nuclear magnetic resonance (NMR) system as an example, the nonlocal gate in this system can be conveniently expressed as

\[ U_{\text{ent}} = e^{-i H_{\text{ent}} t_s}, H_{\text{int}} = \sum_{i<j} \pi J_{i,j} \sigma_z^i \sigma_z^j / 2, \]

where \( J_{i,j} \) is the coupling between the \( i \)th spin and the \( j \)th spin, and \( t_s \) is the evolution time. As the NMR samples have fixed molecular structures, the couplings between the spins remain unchanged. Thus, the above nonlocal gate is simply realized by letting the system undergo free evolution for a time period \( t_s \), which is very favorable for the use of the hardware-efficient ansatz; see Fig. 2(a). For an \( N \)-qubit parameterized quantum circuit with \( L \) layers, we thus need \( 3NL \) single-qubit rotations and \( N \) nonlocal gates. To optimize these rotation parameters, we apply the gradient-based algorithm with the backpropagation strategy \([52]\).
For exploring the QPT in the above physical model, we routinely introduce the Loschmidt echo (LE) [53], i.e.,

$$L(t) = \left[ \text{Tr} \sqrt{\rho_s(0) \rho_s(t) \sqrt{\rho_s(0)}} \right]^2.$$  \hspace{1cm} (13)

The LE characterizes the degree of distinguishability between $\rho(0)$ and $\rho(t)$, which allows us to quantify the sensitivity of quantum evolution to perturbations. To study nonzero temperature QPT, we set the initial state as the thermal state $\rho_s(0) = e^{-\beta H_0}/\text{Tr} e^{-\beta H_0}$ at temperature $\beta$ for the pre-quench Hamiltonian $H_0$. As the system state $\rho_s(t)$ obeys the equation $i\partial \rho_s(t)/\partial t = H_s \rho_s(t)$ and $H_s$ is time-independent, the system’s final state with normalization can be directly calculated by $\rho_s(t) = e^{-iH_s t} \rho_s(0) e^{iH_s t}/\text{Tr}[e^{-iH_s t} \rho_s(0) e^{iH_s t}]$. In this way, we can get the theoretical LE, marked as $L_{\text{th}}(t)$. On the other hand, when executing our protocol introduced in Sec. II, the system’s final state obtained from the dilated circuit also needs to be normalized. This final state then leads to the simulated LE, marked as $L_{\text{sim}}(t)$.

### B. Simulation results

We first consider the simulations for $H_s = H_0 + \kappa D_1$, where $D_1$ is the dominant term of $D_n, n = 1, 2, ..., N_s$. With $N_s = 5$ qubits, we apply the above introduced method to simulate its dynamical evolution. As for realizing $U_{\text{ent}}$ in the VQA with the NMR system, we choose the parameters of the seven-qubit sample $^{13}$C-labeled crotonic acid and set $t_s = 0.012$ for the simulations [54–56]. With sufficiently large number of layers $L = 400$ and the initial guess $\theta_{0,s,k} = 0$, we obtain the cost function $F \geq 0.9995$. As described above, we use the LE to characterize the QPT in this physical model. The theoretical LEs and the simulated LEs are shown in Fig. 3(a). It can be seen that the LEs tend to different values for different $g$. Specifically, when $g > 1$, the system is in the paramagnetic phase; the non-Hermitian perturbation does not substantially affect the dynamics, hence the LEs satisfy $L(t) \approx L(0) = 1.0$. On the contrary, the system is in the ferromagnetic phase when $g < 1$, and the dominant non-Hermitian term $D_1$ leads to the exceptional point dynamics. The exceptional point makes the thermal state approach to its half component in the ferromagnetic phase [42], thus the LEs converge to 0.5. Generally speaking, the simulated LEs (markers) and the theoretical LEs (solid lines) are well matched, revealing that our protocol can almost exactly simulate the evolution dynamics of this non-Hermitian physical model. In addition, the total operation time of the quantum circuit using our protocol is only approximately $400t_s = 4.8$, which is much shorter than that using the dilated circuit but without resorting to the VQA. This advantage makes our protocol more favorable for simulating long-time behavior of the non-Hermitian Hamiltonian dynamics.

Next, we consider the Hamiltonian $H_s = H_0 + \kappa D_n$ with $n = 1, 2, 3, 4, 5$. We set $N_s = 5$ and use the same simulation parameters as above, i.e., $t_s = 0.012, L = 400, \theta_{0,s,k} = 0$. The cost functions for all the cases satisfy $F \geq 0.9995$. The theoretical LEs and the simulated LEs are shown in Figs. 3(b)-

**Figure 3.** Comparison of the LEs with different Hamiltonian parameters. The solid lines and the markers represent the theoretical LEs ($L_{\text{th}}(t)$) and the simulated LEs ($L_{\text{sim}}(t)$), respectively. (a) Results for $H_s = H_0 + \kappa D_1$ with different $g$. The corresponding parameters are $N_s = 5, J = 1, \kappa = 0.1$ and $\beta = 10$. The profiles of the LEs in different phases are distinct, converging to 1.0 for the paramagnetic phase and 0.5 for the ferromagnetic phase, respectively. (b–e) Results for $H_s = H_0 + \kappa D_n$ ($n = 1, 2, 3, 4, 5$) with different $g$. The corresponding parameters are $N_s = 5, J = 1, \kappa = 0.1$ and $\beta = 1$. The profiles of the LEs all converge to 1.0 for the paramagnetic phase when (d) $g = 1.1$ and (e) $g = 1.5$. However, for the case of ferromagnetic phase (b) $g = 0.1$ and (c) $g = 0.5$, the LEs for small $n$ remain near 1.0, while the LEs for large $n$ gradually tend to 0.5.
This is because small values enhance the edge effect under the open boundary condition, which is a clear demonstration of the bulk-boundary correspondence at nonzero temperatures. Here we set $\kappa = 0.1$, $J = 1$, $\beta = 10$ and $\tau = 500, T = 500$. It reveals that the average LEs tend to the prediction in the thermodynamic limit as $N_s$ increases.

As the VQA is a key part of our protocol, we here analyze its computational cost. In the above simulations, we set a sufficiently large number of layers $L = 400$ to make sure that the approximated $U(\theta)$ is as close to the target gate $U_{s,a}^{H_s}U$ as possible, resulting in $F \geq 0.9995$ for all the cases. However, the actual applications may do not need such high cost functions, we thus can reduce the number of layers needed. This reduction is beneficial to the experimental applications, as real quantum platforms have finite coherence times, which limits the number of quantum operations that can be faithfully realized. In addition, the VQA parameters can also be significantly reduced, which can accelerate the optimization process. Here, we explore the number of circuit layers required to achieve distinct values of the cost function $F$ for different number of qubits $N$. The simulation results are shown in the
It can be seen that the number of layers can be reduced about 9%-30% or 33%-55%, if decreasing $F = 0.9999$ to 0.999 or 0.99, respectively. Therefore, we need to make a balance between the accuracy of the cost function and the circuit layers a quantum platform can afford for specific applications.

We also explore the convergence speeds of the gradient-based algorithm with the backpropagation strategy for finding the optimal VQA parameters. Here we set a sufficiently large number of circuit layers $L$ for each number of qubits $N$ to guarantee that the optimization algorithm can converge in an acceptable time; see the results in Fig. 5(b). It can be seen that the cost function $F$ for different number of qubits $N$ quickly converge within 120 iterations, reaching $F \geq 0.995$ for all the cases. This reveals that the backpropagation-enhanced optimization algorithm has a very fast convergence speed, and the increasing number of the system dimension and the VQA parameters do not influence the convergence speeds very much. This is an important feature of our protocol for efficiently simulating multi-qubit non-Hermitian dynamics.

**IV. DISCUSSIONS AND OUTLOOK**

In summary, we propose a practical protocol for efficiently simulating the general non-Hermitian Hamiltonian dynamics. The basic idea is to first dilate the non-Hermitian Hamiltonian into a Hermitian one with a carefully designed quantum circuit, then use the VQA with the hardware-efficient ansatz to approximate the complex entangled operation in this circuit. To optimize the VQA parameters, we apply the gradient-based optimization algorithm enhanced by the backpropagation strategy. To show the effectiveness of our protocol, we use it to simulate the dynamics of an Ising chain with nonlocal non-Hermitian perturbations. With simulations up to five qubits, we demonstrate that the simulated results using our protocol are well matched with the theoretical predictions. We hope the proposed protocol can soon find other applications and experimental verifications on various quantum platforms.

There are several aspects worth to make further efforts. First, for specific problems at hand, problem-inspired ansätze [31] instead of the hardware-efficient ansatz may be more efficient to approximate the target unitary gate, which can significantly decrease the optimization complexity. Therefore, suitable ansätze need to be chosen according to the problems and the features of the available physical platform. Second, there may exist many sources of noise in realizing the optimized VQA, so it is helpful to explore the behavior of our protocol when including typical noises in optimizing the VQA parameters [57, 58]. Third, as we are now in the noisy intermediate-scale quantum era [59], considering the problem of simulating the non-Hermitian Hamiltonian dynamics up to tens of qubits will be highly interesting and demanded. This suggests us to combine more efficient time-evolution simulation strategies, more effective VQA ansätze and more powerful optimization algorithms to accomplish the task.

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