Quantum nonlinear optics in atomic dual arrays

Simon Panyella Pedersen,* Lida Zhang (张理达), and Thomas Pohl†
Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark
(Dated: January 19, 2022)

Atoms in a sub-wavelength lattices have remarkable optical properties that have become of high scientific and technological significance. Here, we show how the coupling of light to more than a single atomic array can expand these perspectives into the domain of quantum nonlinear optics. While a single array transmits and reflects light in a largely linear fashion, the combination of two arrays is found to induce strong photon-photon interactions that can convert an incoming classical beam into highly antibunched light. Such quantum metasurfaces open up new possibilities for coherently generating and manipulating nonclassical light, from optical quantum information processing to exploring quantum many-body phenomena in two-dimensional systems of strongly interacting photons.

The rapidly advancing capabilities to control atomic ensembles at the single-particle level [1] have enabled the development of novel light-matter interfaces. Recent studies of the coupling of individual atoms to nanoscale waveguides and photonic crystals, or the use of atomic arrays [2–20] suggest a promising new type of metamaterial that can be designed and engineered on sub-wavelength scales at the level of individual quantum emitters. In particular, extended two-dimensional lattices of atoms may serve as a virtually ideal quantum optical interface with lossless coupling to freely propagating light fields [14–17], based on the collective interaction of photons with delocalized atomic excitations.

Experiments have demonstrated such strong coherent coupling to subradiant collective excitations in optical lattices [20], which enables coherent manipulations of light and a range of applications from photonic wavefront engineering [18] to highly efficient quantum memories [21]. Yet, the many-body nature of the underlying collective light-matter coupling to a large number of atoms renders such systems intrinsically linear. In fact, the simultaneous photon coupling to many atoms that generates excellent coherence conditions, at the same time, diminishes the otherwise strong optical nonlinearity of individual quantum emitters and, thereby, restricts most applications to the domain of classical optics.

Here, we describe an approach to achieving strong optical nonlinearities in sub-wavelength atomic arrays. Specifically, we will show that combining two atomic arrays, which separately are only weakly nonlinear, can greatly enhance their combined optical nonlinearity to a degree that acts on the level of individual photons. The quantum optical nonlinearity in this system arises from narrow transmission resonances around which photons are strongly confined between the two arrays, which form an effective ultra-high finesse optical resonator. Indeed, we find that this can effectively generate strong anti-bunching, while the statistics of incident photons is left virtually unchanged by individual atomic arrays (cf. Fig. 1b). Owing to their strong emerging photon-photon interaction such dual atomic arrays can serve as powerful and experimentally viable quantum metasurfaces.

![Atomic cavity](image)

**Figure 1.** (a,b) The transmission spectrum $|T|^2$ of a dual array of two-level emitters exhibits characteristic reflection and transmission resonances as a function of the photon frequency detuning, $\Delta$, and the distance, $L$, between the two two-dimensional atomic lattices. (b) For small values of $L$ the system behaves as a single layer of superradiant and subradiant atomic dimers that generate two respective reflection resonances, marked by the two white dashed lines. (a) The large-$L$ limit, on the other hand, leads to a series of narrow transmission resonances of the effective atomic resonator, as indicated by the white dashed line in the upper part of the plot [cf. Eq. (5)]. (c) In the vicinity of such narrow transmission resonances, the optical response becomes highly nonlinear and generates effective photon-photon interactions that can transform an incident coherent beam into highly nonclassical light, as demonstrated by the depicted two-photon correlation function $g^{(2)}(t)$ of the transmitted light (solid line). In contrast, light reflected from a single identical array remains largely uncorrelated (dashed line). The chosen lattice spacing is $a = 0.6\lambda$. The transmission in (a) is obtained for infinitely extended arrays, while the calculations in (c) are performed for finite arrays with $9 \times 9$ atoms driven by a Gaussian laser beam with a waist of $w = 1.5\lambda$, $\Delta = 0.23\gamma$ and $L = 1.55\lambda$. 

[1] Simon Panyella Pedersen, Lida Zhang, and Thomas Pohl, *Quantum nonlinear optics in atomic dual arrays,* arXiv:2201.06544v1 [quant-ph] (January 19, 2022)
that enable the generation and manipulation of highly nonclassical states of light, from the processing of optical quantum information [22] to the realization of correlated quantum many-body phases of light [23, 24].

Let us first consider a single two-dimensional square lattice of atoms with a lattice spacing \(a\), and which we assume to infinitely extended in the \(x\)-\(y\) plane. We focus the discussion on two-level systems that are resonantly driven by a coherent cw-field with an electric field amplitude \(E_{\text{in}}\) and spatial mode function \(f(r)\) [25]. At the small lattice spacings considered here, exchange of photons across the array generates strong atomic interactions that can be efficiently described within input-output theory by integrating out the photonic degrees of freedom and using a Born-Markov approximation [26, 27]. This yields a master equation \(\dot{\rho} = -i[H, \rho] + \mathcal{L}[\rho]\) (with \(\hbar = 1\)) for the density matrix, \(\rho\), of the atomic lattice, where the Hamiltonian and Lindblad operator

\[
\hat{H} = -\Delta \sum_n \sigma^+_n \sigma^-_n - \sum_{n} \Omega_n (\sigma^+_n + \Omega^*_n \sigma^-_n)
\]

\[
- \sum_{n,m} J_{nm} \sigma^+_n \sigma^-_m,
\]

\[
\mathcal{L}[\rho] = \sum_{n,m} \Gamma_{nm} (2 \sigma^+_n \rho \sigma^-_m - \{ \sigma^+_m \sigma^+_n, \rho \}),
\]

(1a)

(1b)

describe the exchange of excitations and corresponding collective dipole-dipole processes due to the photon-mediated dipole-dipole interactions between the atoms [28]. Here, \(\sigma_n = |g_n\rangle \langle e_n|\) denotes the transition operator between the ground state, \(|g_n\rangle\), and excited state \(|e_n\rangle\) of an atom at position \(r_n\) in the lattice. The interaction coefficients \(J_{nm}\) and decay rates \(\Gamma_{nm}\) for two atoms at positions \(r_n\) and \(r_m\) are determined by the Green’s function tensor of the free-space electromagnetic field [28, 29]. The atomic transition is driven by the incident light with a frequency detuning \(\Delta\) and a single-atom Rabi frequency \(\Omega_n = dE_{\text{in}} f(r_n)\), which is determined by the driving-field amplitude and the transition dipole moment \(d\) of the two-level emitters. From the solution for \(\hat{\rho}\), one can reconstruct the electromagnetic field generated by the driven atomic dipoles. Choosing \(f(r_n)\) as the detection mode for the transmitted light, one obtains

\[
\hat{E} = E_{\text{in}} + \frac{3\pi i\gamma}{k^2 \eta_d} \sum_n f^*_n (r_n) \hat{\sigma}_n
\]

(2)

for the electric-field amplitude, \(\hat{E}\), of the detected photons, where \(k = 2\pi/\lambda\) is the wave number of the incident light, \(\lambda\) denotes its wavelength, \(\gamma\) is the decay rate of the individual atoms, and \(\eta = \int dr |f(r)|^2\).

For weak plane-wave driving with \(f \sim e^{ikz}\), Eqs. (1a), (1b) and (2) yield simple expressions for the transmission and reflection spectra [29]

\[
t = \langle \hat{E} \rangle / E_{\text{in}}, \quad r = t - 1 = -\frac{i\Gamma}{\Delta - \Delta + i\Gamma},
\]

(3)

that feature a Lorentzian resonance at the collective Lamb shift \(\Delta = -\sum_{n \neq 0} J_{n0}\) with a width \(\Gamma = 3\gamma/\hbar^2\). On resonance, the single atomic layer, thus, reflects incoming photons with unit efficiency and no losses from the incident mode, \(|t|^2 + |r|^2 = 1\).

While this ideal behaviour strictly emerges only in the thermodynamic limit of infinite arrays and plane-wave driving, the atom-light coupling becomes highly coherent already for rather small systems [14]. For example, a 9 \(\times\) 9 atomic array with \(a = 0.6\lambda\) can reflect a focussed Gaussian beam with a waist of \(w = 1.5\lambda\) with a large linear reflection amplitude of \(|r| = 0.998\). The nonlinear response coefficients, on the other hand, are very small as the beam still covers a sizeable number of atoms, and thereby diminishes nonlinear saturation effects. This is seen from the second order correlation function \(g^{(2)}(t)\) of the reflected light, shown by the dashed line in Fig. 1b. One finds only a marginal suppression of simultaneous two-photon reflection, otherwise preserving the classical nature of the incident light (\(g^{(2)} \sim 1\)).

This situation changes dramatically as we add a second atomic array. Fig. 1a shows the transmission coefficient \(|T|^2\) for this dual-array configuration of two parallel atomic lattices as a function of the detuning \(\Delta\) and the distance \(L\) between the arrays. The calculations reveal a series of narrow transmission resonances that extends towards large values of \(L\) and a pair of sharp reflection resonances at small array distances. Both regimes can be traced back to the photon-mediated interactions between the two arrays.

At small \(L\), atoms in different arrays but at identical lattice sites in each array predominantly interact via their evanescent fields which mediate a strong dipolar excitation exchange with a coupling constant that scales as \(J_L \approx -3\gamma/2(kL)^3\) [29]. We can, thus, define symmetric and antisymmetric superposition states, \(|\pm\rangle_n\), of a single delocalized excitation between two adjacent atoms at a given lattice site \(n\) in each array. Owing to its parity symmetry, the atomic interaction does not couple the two states, \(|\pm\rangle_n\), but shifts their respective energies by \(\pm J_L\). Therefore, the two atomic dimer states become energetically isolated for small \(L\) and separately generate reflection resonances at the collective energies \(\Delta_{\pm} \sim \pm L^{-3}\), as indicated by the dashed lines in Fig. 1a [29]. Their respective widths are given by \(\Gamma_{\pm} = \Gamma [1 \pm \cos(kL)]\), such that one finds an ultra-narrow reflection resonance with \(\Gamma_- \ll \Gamma\), generated by an effective array of subradiant atomic dimer states as \(L\) decreases.

For larger values of \(L\), the evanescent-field coupling vanishes and inter-array interactions are predominantly generated by propagating photons between the two arrays. Analogous to light-induced atomic interactions by optical waveguides [30, 31], the coupling strength \(J_L \approx 3\gamma \cos(kL)/kL\) acquires an oscillating behaviour from the photon propagation phase and leads to an energy difference \(\Delta_+ - \Delta_- \sim \sin(kL)\) of the two collec-
tive dimer states, which varies periodically with the distance between the arrays. As both collective dimer states are excited by the parity-breaking incident field, their interference leads to a series of narrow transmission resonances, akin to electromagnetically induced transparency in three-level systems [29, 32]. One can obtain a simple expression for the dual-array transmission amplitude [29]

$$T = \frac{\ell^2}{1 - r^2 e^{2ikL}}$$

(4)

in terms of the reflection and transmission amplitudes, $r$ and $t$, of the individual arrays. Substituting their explicit expressions (Eq. (3)), we obtain the following condition for perfect transmission ($|T| = 1$)

$$\Delta - \tilde{\Delta} = -\Gamma \tan(kL),$$

(5)

that defines the series of transmission resonances. Along these resonances, transmitted photons can acquire a substantial group delay with a delay time [29]

$$\tau = \frac{2\Gamma}{(\Delta - \tilde{\Delta})^2},$$

(6)

that diverges for resonant detunings $\Delta = \tilde{\Delta}$. The width of the transmission resonances decreases as $\sim 1/\tau$ around these values and, therefore, vanishes at $kL = n\pi$ for an integer $n > 0$. In between the resonances ($kL = (n + 1/2)\pi$), the system features high reflection and behaves largely linear, which can be used to store several delocalized excitations across distant arrays at $L \gg \lambda$ [33].

Note that Eq. (4) coincides with the transmission of a Fabry-Pérot cavity formed by two identical mirrors with reflection and transmission amplitudes $r$ and $t$. The dual-array setting thus acts as an ultra-high finesse resonator. In stark contrast to typical cavities, however, the diverging time $\tau$ for which photons are confined in the resonator can enhance the intrinsic nonlinearity of the effective atomic mirrors and give rise to exceedingly strong photon-photon interactions, as we shall see below.

We study the signatures of such photon-photon interactions via Monte Carlo wave function simulations [34] of the atomic master equation with Eqs. (1a) and (1b) for finite arrays. In the limit of weak driving, we can truncate the underlying Hilbert space at more than 2 excitations, which yields converged results for relevant two-photon observables as $E_{\text{in}} \to 0$. Working with finite arrays and focussed driving beams, generally entails photon losses that tend to broaden the otherwise ultranarrow transmission resonances. These effects can, however, be well mitigated through a proper choice of the atomic lattices [29]. In fact, already rather small lattices of $9 \times 9$ atoms permit to generate narrow transmission resonances with linewidths of $\sim 10^{-2}\gamma$ and high peak transmission of $|T| \sim 0.98$.

Figure 2. (a) The characteristic time dependence of the two-photon correlation function, $g^{(2)}(t)$, can be understood by the dynamics of the short-lived ($|+\rangle$) and long-lived ($|−\rangle$) single atomic excitation that is symmetrically ($|+\rangle$) and anti-symmetrically ($|−\rangle$) delocalized between the two arrays. (b) Following the detection of a photon, the subsequent population dynamics, $|c_+|^2$ (blue) and $|c_-|^2$ (red), of these two states agrees with the characteristic time dependence of $g^{(2)}(t)$ shown in panel (c) (see text for more details). The parameters are the same as in Fig. 1c.

Fig. 1b shows the calculated second order correlation function

$$g^{(2)}(t) = \frac{\langle \hat{E}^\dagger(t')\hat{E}^\dagger(t' + t)\hat{E}(t' + t)\hat{E}(t') \rangle}{\langle \hat{E}^\dagger(t')\hat{E}(t') \rangle^2}$$

(7)

of the transmitted light for a Gaussian cw-beam [29], whose waist is centred right in between the two arrays. Its dependence on the time delay, $t$, between consecutively detected photons indicates the generation of highly nonclassical light. Interestingly, one finds a sharp initial drop to small values $g^{(2)} \sim 0$ leading to strong antibunching over a broad range of delay times between two transmitted photons. Within the Monte Carlo wave function treatment, such characteristic temporal correlations can be understood as follows. Let us denote the steady state of the two arrays as $|\psi\rangle$. Detection of a transmitted photon in the steady state after a long time $t'$ then projects this state onto $|\tilde{\psi}\rangle = \hat{E}|\psi\rangle/\sqrt{\langle \psi | \hat{E}^\dagger \hat{E} | \psi \rangle}$. The correla-
from the time evolved state $|−t⟩$ of a photon at time $t$ indicated by the color coding. Panel (d) shows the minimum different indicated points along this transmission resonance, as indicated by the color coding. Panel (d) shows the minimum value $g^2_{\text{min}} = \min_{t} g^2(t)$. The color bars in panel (e) show the long timescale on which the correlation functions eventually approach unity, which matches the photon delay time, or photon confinement time, $τ$. Its asymptotic value for infinite arrays in given by Eq. (6), while the dashed line in panel (e) has been obtained numerically for the 9×9 arrays.

The two-photon correlation function can thus be obtained as

$$g^2(t) = \frac{⟨ψ(t′ + t)|\hat{E}^† \hat{E} |ψ(t′ + t)⟩}{⟨ψ(t′)|\hat{E}^† \hat{E} |ψ(t′)⟩}$$

from the time evolved state $|ψ(t′ + t)⟩$ following detection of a photon at time $t′$. For weak driving this state is predominantly determined by the collective ground state, $|0⟩$, and the single-excitation manifold. It can hence be expressed as a superposition, $|ψ⟩ = c_0 |0⟩ + c_+ |+⟩ + c_{−} |−⟩$, of the collective superradiant ($|+⟩$) and subradiant ($|−⟩$) states of a single atomic excitation that is shared (anti)symmetrically across the two arrays, as discussed above. Indeed, the time evolution of the populations $|c_{±}|^2$ resembles the temporal photon correlations, see Fig. 2. The initial drop of $g^2(t)$ thus reflects the fast decay of the superradiant excitation on a short timescale $τ_+$. Its subsequent slow rise, on the other hand, can be traced back to the repopulation of the long-lived subradiant cavity state, $|−⟩$, on a long timescale $τ_−$ given by the inverse width of the transmission resonance, which corresponds to the photon delay time, discussed above (cf. Fig. 3e and see [29]).

Consequently, we expect stronger effects of photon-photon interactions around more narrow transmission lines. This is demonstrated in Fig. 3, where we show the two-photon correlation function $g^2(t)$ while scanning the frequency detuning, $Δ$, and the array distance, $L$, along the transmission maximum of one of the resonances, as illustrated in Fig. 3b. Indeed, we find stronger antibunching as the resonance becomes more narrow and the delay time increases. Apart from the rapid superradiant short-time dynamics, the limit of perfect antibunching, thus, resembles a giant atom with a single excited state $|−⟩$ that is strongly coupled to the incident photon mode. While the dual array naturally supports more than a single atomic excitation, it nevertheless behaves close to a single saturable quantum emitter due to the vast timescale separation of $τ_−$.

Owing to the finite size of the array, the linear transmission decreases as we approach this regime by varying $L$ (Fig. 3a). The associated losses tend to broaden the transmission lines and, as shown in Fig. 3e, lead to a maximum delay time of $τ \sim 1000/γ$, instead of the diverging behaviour (cf. Eq. (6)) discussed above for infinite arrays and plane wave driving. Larger arrays yield longer photon confinement times, $τ$, for a given transmission maximum, which enhances both the temporal extend and the strength of photon-photon correlations. Remarkably, however, one can reach strong antibunching under conditions of high photon transmission already for moderate system sizes, which are achieved in ongoing optical lattice experiments [1, 20].

Sub-wavelength atomic lattices have outstanding optical properties, and in this work we have shown how these extend into the domain of quantum nonlinear optics by confining light between two such arrays. While nonlinearities may be generated in atomic ensembles via strong additional atomic interactions [35–37] or by using tightly focussed beams [38], the present mechanism offers a robust approach that does not cause additional photon losses. This is made possible by ultra-narrow transmission resonances that emerge from interference between collective superradiant and subradiant states of the dual array setting, bearing analogies to electromagnetically induced transparency [32] and the physics of Fano resonators [39]. The resulting photon-photon interactions offer a number of perspectives. We have identified a regime in which the system behaves approximately like a giant saturable quantum emitter that supports only a single subradiant excitation with strong single-mode coupling to freely propagating photons and highly suppressed losses. Such a capability offers immediate applications as nonlinear quantum optical elements for quantum communication protocols [40] as well as photonic quantum information processing [22]. Given this strong single-mode coupling and the versatility of optical atom traps, such nonlinear elements may serve as suitable building blocks for larger quantum networks and realizations of waveguide QED to explore exotic propagation effects and emerging nonclassical states of light [41–45] at the view-photon level. While we have focussed here on the single-mode dynamics of two-photon processes, the multi-mode limit...
of extended arrays also yields an ideal platform for exploring the many-body physics of multiple photons in the two-dimensional plane of the effective resonator described above. For example, this motivates future work on the formation and nonlinear dynamics of cavity polaritons [23] in the dual-array setting. Hereby, the present results suggest possibilities to form strongly interacting photons in two dimensions, while offering additional possibilities to engineer topological bands [46–50] for atomic excitations, photons, or both combined.

We thank Kristian Knakkergaard and Aurélien Dantan for valuable discussions. This work was supported by the Carlsberg Foundation through the ‘Semper Ardens’ Grant agreement no.: 113601 (2017).

C. Gross and I. Bloch, Quantum simulations with ultracold atoms in optical lattices, Science 357, 995 (2017).

H. Zheng and H. U. Barranger, Persistent Quantum Beats and Long-Distance Entanglement from Waveguide-Mediated Interactions, Physical Review Letters 110, 113601 (2013).

J. D. Thompson, T. G. Tiecke, N. P. de Leon, J. Feist, A. V. Akimov, M. Gullans, A. S. Zibrov, V. Vuletić, and M. D. Lukin, Coupling a Single Trapped Atom to a Nanoscale Optical Cavity, Science 340, 1202 (2013).

J. Petersen, J. Volz, and A. Rauschenbeutel, Chiral nanophotonic waveguide interface based on spin-orbit interaction of light, Science 346, 67 (2014).

T. G. Tiecke, J. D. Thompson, N. P. de Leon, L. R. Liu, V. Vuletić, and M. D. Lukin, Nanophotonic quantum phase switch with a single atom, Nature 508, 241 (2014).

A. Goban, C.-L. Hung, J. D. Hood, S.-P. Yu, J. A. Muniz, O. Painter, and H. J. Kimble, Superradiance for atoms trapped along a photonic crystal waveguide, Phys. Rev. Lett. 115, 063601 (2015).

J. S. Douglas, H. Habibian, C.-L. Hung, A. V. Gorshkov, H. J. Kimble, and D. E. Chang, Quantum many-body models with cold atoms coupled to photonic crystals, Nature Photonics 9, 326 (2015), arXiv:1312.2435.

R. J. Coles, D. M. Price, J. E. Dixon, B. Royall, E. Clarke, P. Kok, M. S. Skolnick, A. M. Fox, and M. N. Makhonin, Chirality of nanophotonic waveguide with embedded quantum emitter for unidirectional spin transfer, Nature Communications 7, 11185 (2016).

G. Calajó, F. Ciccarello, D. Chang, and P. Rabl, Atom-field dressed states in slow-light waveguide qed, Phys. Rev. A 93, 033833 (2016).

H. Zoubi and K. Hammerer, Quantum nonlinear optics in optomechanical nanoscale waveguides, Phys. Rev. Lett. 119, 123602 (2017).

A. R. Hamann, C. Müller, M. Jerger, M. Zanner, J. Combes, M. Pletyukhov, M. Weides, T. M. Stace, and A. Fedorov, Nonreciprocity realized with quantum non-linearity, Physical Review Letters 121, 123601 (2018).

D.-P. Yu, J. A. Muniz, C.-L. Hung, and H. J. Kimble, Two-dimensional photonic crystals for engineering atom–light interactions, Proceedings of the National Academy of Sciences 116, 12743 (2019).

R. Jones, G. Buonaiuto, B. Lang, I. Lesanovsky, and B. Olmos, Collectively enhanced chiral photon emission from an atomic array near a nanofiber, Phys. Rev. Lett. 124, 093601 (2020).

R. J. Bettles, S. A. Gardiner, and C. S. Adams, Enhanced optical cross section via collective coupling of atomic dipoles in a 2D array, Physical Review Letters 116, 103602 (2016).

G. Facchinetti, S. D. Jenkins, and J. Ruostekoski, Storng light with subradial correlations in arrays of atoms, Phys. Rev. Lett. 117, 243601 (2016).

E. Shahmoon, D. S. Wild, M. D. Lukin, and S. F. Yelin, Cooperative Resonances in Light Scattering from Two-Dimensional Atomic Arrays, Physical Review Letters 118, 113601 (2017).

A. Asenjo-Garcia, M. Moreno-Cardoner, A. Albrecht, H. J. Kimble, and D. E. Chang, Exponential improvement in photon storage fidelities using subradiance and "selective radiance" in atomic arrays, Physical Review X 7, 031024 (2017).

K. E. Ballantyne and J. Ruostekoski, Optical Magnetism and Huygens’ Surfaces in Arrays of Atoms Induced by Cooperative Responses, Physical Review Letters 125, 143604 (2020).

T. L. Patti, D. S. Wild, E. Shahmoon, M. D. Lukin, and S. F. Yelin, Controlling interactions between quantum emitters using atom arrays, Physical Review Letters 126, 223602 (2021).

J. Rui, D. Wei, A. Rubio-Abadal, S. Hollerith, J. Zeiher, D. M. Stamper-Kurn, C. Gross, and I. Bloch, A subradiant optical mirror formed by a single structured atomic layer, Nature 583, 369 (2020).

M. T. Manzoni, M. Moreno-Cardoner, A. Asenjo-Garcia, J. V. Porto, A. V. Gorshkov, and D. E. Chang, Optimization of photon storage fidelity in ordered atomic arrays, New Journal of Physics 20, 083048 (2018).

T. C. Ralph, I. Söllner, S. Mahmoudian, A. G. White, and P. Lodahl, Photon sorting, efficient bell measurements, and a deterministic controlled-\(x\) gate using a passive two-level nonlinearity, Phys. Rev. Lett. 114, 173603 (2015).

I. Carusotto and C. Ciuti, Quantum fluids of light, Reviews of Modern Physics 85, 299 (2013).

C. Noh and D. G. Angelakis, Quantum simulations and many-body physics with light, Reports on Progress in Physics 80, 016401 (2017).

This can be realized [20] by applying a sufficiently strong magnetic field to ensure that only one atomic transition is near-resonant with the incident driving field of a given polarization and excitation exchange on other dipole transitions is energetically suppressed.

R. H. Lefemberg, Radiation from an N -Atom System. I. General Formalism, Physical Review A 2, 883 (1970).

M. Gross and S. Haroche, Superradiance: An essay on the theory of collective spontaneous emission, Physics Reports 93, 301 (1982).

A. Asenjo-Garcia, J. D. Hood, D. E. Chang, and H. J. Kimble, Atom–light interactions in quasi-one-dimensional nanostructures: A Green’s-function perspective, Physical Review A 95, 033818 (2017).
See supplementary information for details on the linear response of single and dual arrays, the effects of interactions between two arrays, group delay of transmitted photons, finite size effects, and the numerical analysis of subradiant and superradiant states in finite dual arrays.

D. E. Chang, L. Jiang, A. V. Gorshkov, and H. J. Kimble, Cavity QED with atomic mirrors, New Journal of Physics 14, 063003 (2012).

E. Shahmoon and G. Kurizki, Nonlinear theory of laser-induced dipolar interactions in arbitrary geometry, Physical Review A 89, 043419 (2014).

M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Electromagnetically induced transparency: Optics in coherent media, Rev. Mod. Phys. 77, 633 (2005).

P.-O. Guimond, A. Grankin, D. V. Vasilyev, B. Vermersch, and P. Zoller, Subradiant Bell states in distant atomic arrays, Physical Review Letters 122, 093601 (2019).

K. Mølmer, Y. Castin, and J. Dalibard, Monte Carlo wave-function method in quantum optics, Journal of the Optical Society of America B 10, 524 (1993).

M. Moreno-Cardoner, D. Goncalves, and D. E. Chang, Quantum nonlinear optics based on two-dimensional Rydberg atom arrays (2021), arXiv:2101.01936 [quant-ph].

L. Zhang, V. Walther, K. Mølmer, and T. Pohl, Photon-photon interactions in Rydberg-atom arrays, arXiv:2101.11375 [quant-ph] (2021), arXiv:2101.11375 [quant-ph].

Y. Solomons and E. Shahmoon, Multi-channel waveguide qed with atomic arrays in free space (2021), arXiv:2111.11515 [quant-ph].

L. A. Williamson, M. O. Borgh, and J. Ruostekoski, Superatom picture of collective nonclassical light emission and dipole blockade in atom arrays, Phys. Rev. Lett. 125, 073602 (2020).

M. F. Limonov, M. V. Rybin, A. N. Poddubny, and Y. S. Kivshar, Fano resonances in photonics, Nature Photonics 11, 543 (2017).

D. Witthaut, M. D. Lukin, and A. S. Sørensen, Photon sorters and QND detectors using single photon emitters, EPL (Europhysics Letters) 97, 50007 (2012).

S. Mahmoodian, M. Čepulkovskis, S. Das, P. Lodahl, K. Hammerer, and A. S. Sørensen, Strongly Correlated Photon Transport in Waveguide Quantum Electrodynamics with Weakly Coupled Emitters, Phys. Rev. Lett. 121, 143601 (2018).

A. S. Prasad, J. Hinney, S. Mahmoodian, K. Hammerer, S. Rind, P. Schneeweiss, A. S. Sørensen, J. Volz, and A. Rauschenbeutel, Correlating photons using the collective nonlinear response of atoms weakly coupled to an optical mode, Nature Photonics 14, 719 (2020).

S. Mahmoodian, G. Calajó, D. E. Chang, K. Hammerer, and A. S. Sørensen, Dynamics of Many-Body Photon Bound States in Chiral Waveguide QED, Phys. Rev. X 10, 031011 (2020).

O. A. Iversen and T. Pohl, Strongly Correlated States of Light and Repulsive Photons in Chiral Chains of Three-Level Quantum Emitters, Phys. Rev. Lett. 126, 083605 (2021).

O. A. Iversen and T. Pohl, Self-ordering of individual photons in waveguide qed and rydberg atom arrays (2021), arXiv:2110.12961 [quant-ph].

T. Karzig, C.-E. Bardyn, N. H. Lindner, and G. Refael, Topological Polaritons, Physical Review X 5, 031001 (2015).

J. Perczel, J. Borregaard, D. E. Chang, H. Pichler, S. F. Yelin, P. Zoller, and M. D. Lukin, Topological Quantum Optics in Two-Dimensional Atomic Arrays, Physical Review Letters 119, 023603 (2017).

R. J. Bettles, J. c. v. Minár, C. S. Adams, I. Lesanovsky, and B. Omlos, Topological properties of a dense atomic lattice gas, Phys. Rev. A 96, 041603 (2017).

J. Perczel, J. Borregaard, D. E. Chang, S. F. Yelin, and M. D. Lukin, Topological Quantum Optics Using Atomlike Emitter Arrays Coupled to Photonic Crystals, Physical Review Letters 124, 083603 (2020).

C.-R. Mann and E. Mariani, Topological transitions induced by cavity-mediated interactions in photonic valley-Hall metasurfaces, arXiv:2010.01636 (2020).