Research Article

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Rheological behavior of a bentonite mud

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Abstract: Predicting drilling fluids rheology is crucial to control/optimize the drilling process and the gas extraction from drilling fluids in logging systems. A Couette viscometer measured the apparent viscosity of a bentonite mud at various shear rates and temperatures. The bentonite mud behaved as a yield-pseudoplastic fluid, and a modified Herschel-Bulkley model predicted the shear rate and temperature effects upon the shear stress. A pipe viscometer was built to seek a correlation between the mud flow rate and the pressure drop and thereby determine refined Herschel-Bulkley parameters. Coupling a rheological model to a pipe viscometer enables the continuous acquisition of apparent viscosities of Newtonian or non-Newtonian fluids at a rig-site surface.

Keywords: Drilling mud, Herschel-Bulkley model, Pipe viscometer, Rheology, Yield-pseudoplastic fluid

Symbols

\( A \) Intermediate rheological parameter [K]
\( D \) Diameter of each pipe [cm]
\( k \) Consistency index (rheological parameter) [(dyn/cm\(^2\))·s\(^n\)]
\( k_0 \) Intermediate rheological parameter [(dyn/cm\(^2\))·s\(^n\)]
\( k_1 \) Viscometer torsion constant [386 dyn-cm/degree deflection]
\( k_2 \) Viscometer shear stress constant for the effective bob surface [1.323×10\(^{-2}\) cm\(^{-3}\)]
\( k_3 \) Viscometer shear rate constant [1.7023 s\(^{-1}\) per rpm]
\( L \) Length of each pipe [cm]
\( L_{\text{entrance laminar}} \) Laminar flow entrance length [cm]
\( m \) Generalized Reynolds number parameter

\( n \) Flow index (rheological parameter)
\( n_a \) Intermediate rheological parameter [°C\(^{-1}\)]
\( n_0 \) Intermediate rheological parameter
\( Q_{\text{mud}} \) Mud flow rate [L/min \(\equiv 16.67\) cm\(^3\)/s]
\( Re_{\text{generalized}} \) Generalized Reynolds number
\( T \) Mud temperature [°C]
\( v \) Mud velocity in each pipe [cm/s]

Greek Symbols

\( \Delta P \) Pressure drop [kPa \(\equiv 10^4\) dyn/cm\(^2\)]
\( \dot{\gamma} \) Shear rate \(\equiv\) velocity gradient [s\(^{-1}\)]
\( \eta_a \) Apparent viscosity [cP]
\( \theta \) Viscometer dial reading
\( \rho \) Mud density [g/cm\(^3\)]
\( \tau \) Shear stress [dyn/cm\(^2\)]
\( \tau_w \) Shear stress at the pipe wall [dyn/cm\(^2\)]
\( \tau_0 \) Yield stress (rheological parameter) [dyn/cm\(^2\)]
\( \tau_{0a} \) Intermediate rheological parameter [dyn/(cm\(^2\)·°C)]
\( \tau_{0i} \) Intermediate rheological parameter [dyn/cm\(^2\)]
\( \omega \) Angular velocity [rpm]

1 Introduction

1.1 Drilling fluids

Drilling a well is a complex process that must take into account diverse rock formations and geological structures. In most cases, drilling occurs in very harsh operating conditions, e.g. high temperatures, pressures and vibrations that represent enormous technological challenges. In drilling wells, a specific drilling fluid (known as mud) is pumped down the borehole, in order to carry the drilled cuttings up to the surface, to suspend the cuttings in the mud whenever the circulation stops, to yield hydrostatic pressure preventing the borehole collapse and/or wellbore flooding by formation fluids, and also to clean, cool, and lubricate the drill bit [1]. Furthermore, the analysis of the gas extracted from the drilling mud is essential for the hydrocarbons identification in gas and oil reservoirs. A thumb rule in mud logging states that the drilling mud at the surface carries similar ratios of hydrocarbons in the drilled formations [2]. For in-
stance, the ratios \( \text{CH}_4/\text{C}_2\text{H}_6 \), \( \text{CH}_4/\text{C}_3\text{H}_8 \), and \( \text{CH}_4/\text{C}_4\text{H}_{10} \) imply rock formation variations and reservoirs productivities [3].

There are two main types of drilling fluids, namely water-based mud (commonly referred as bentonite mud), and invert emulsion oil-based drilling fluid. A water-based mud mainly consists of water or substituents, such as seawater, brine, saturated brine, and chemical compounds (e.g. potassium formate, lime or silicate). An oil-based drilling fluid mainly comprises distilled crude oil products, complemented by barite, lime, lignite, and asphaltic or polymeric materials [4].

The knowledge of the mud rheology is essential to control/optimize the drilling process performance. Moreover, the optimization of the gas extraction from the drilling mud in a logging system, such as a gas analyzer system, also urged the prediction of the mud viscosity as a function of the mud temperature and the stirring velocity in the gas trap [5, 6]. The gas analyzer system analyzes the gases extracted from the mud at the rig-site surface to estimate the hydrocarbons contents in the drilled formations. The knowledge of the mud rheology aids promoting an adequate turbulence (by mechanical stirring) in the gas trap of the gas analyzer system. Additionally, the turbulence data enables to get insight on the gas microbubbles dynamics and thereby maximize the gas analyzer system efficiency.

To overcome the lack of continuous viscosity measurements in the commercial gas analyzer system (TRU-vision), the main aim of this work was the rheological study of a bentonite mud which presented a non-linear behavior with respect to the shear rate (non-Newtonian fluid). Next, a pipe viscometer was built to aid in the mud rheology characterization. Finally, the rheological data collected were correlated by the Herschel-Bulkley model, the parameters of which were determined. This methodology enables a continuous and fairly reliable prediction of the apparent viscosity of a drilling fluid, either in laboratory essays and/or industrial applications.

1.2 Non-Newtonian fluids

Unlike Newtonian fluids, a non-Newtonian fluid at constant pressure and temperature does not display a linear relationship between the shear stress and the shear rate. Instead, it may exhibit various behaviors (Figure 1), e.g. shear-thinning, shear-thickening, and shear-independency, each of them presenting yield stress or not [7–10].

Usually, drilling fluids are designed to behave as shear-thinning yield stress fluids, because shear-thinning limits pressure drops when flowing at high flow rates, whereas the yield stress and gel behavior yields the capability of solids carrying at static or nearly static conditions.

![Figure 1: Rheological behaviors of Newtonian and non-Newtonian fluids.](image)

1.3 Herschel-Bulkley model

A fluid presents a viscoplastic behavior if a shear stress higher than the fluid yield stress must be applied to start deforming it, in other words for the fluid to start flowing [11]. In particular, the Herschel-Bulkley model may describe a yield-pseudoplastic fluid as denoted by Eq. 1 [12–14]:

\[
\begin{align*}
\tau & = \tau_0 + k \dot{\gamma}^n, & |\tau| > |\tau_0| \\
\dot{\gamma} & = 0, & |\tau| < |\tau_0|
\end{align*}
\]

where \( \tau \) is the shear stress, \( \tau_0 \) the characteristic yield stress, \( \dot{\gamma} \) the shear rate, \( n \) the flow behavior index, and \( k \) the consistency coefficient.

As each term of Eq. 1 has the same dimensions, there is a coupling between the units and values of the parameters \( k \) and \( n \). Nelson & Ewoldt [15] preferred to describe the Herschel-Bulkley model by the equation \( \tau = \tau_0 \left[ 1 + \left( \frac{\dot{\gamma}}{\dot{\gamma}_{\text{critical}}} \right)^n \right] \), because all parameters dimensions are equal, regardless of their values. The parameter \( \dot{\gamma}_{\text{critical}} = \left( \frac{\tau_0}{k} \right)^{\frac{1}{n}} \) is physically meaningful as a critical shear rate at which the shear stress is twice the yield stress. Saasen & Ytrehus [16] claimed that the yield stresses of some drilling fluids are too low to be determined by oil well drilling standard procedures. Moreover, the determination of certain shear stresses by typical viscometers is not reliable (measurements at a few shear rates). Besides, the measurements accuracy of typical viscometers at low shear rates is doubtful. Hence, Saasen & Ytrehus [16] extended the dimensionless parameters of Nelson & Ewoldt [15] to be used in drilling fluids. The 1st step was to estimate the yield
stress from the rheogram. The 2nd step was to determine a surplus stress, $\tau_s$, at a relevant shear rate, $\dot{\gamma}$, for most drilling operations (equivalent to that obtained at 100 rpm on most typical viscometers and simultaneously within the shear rate range for drilling fluid flow). Their equation to describe the Herschel-Bulkley model was: $\tau = \tau_0 + \tau_s \left( \frac{\dot{\gamma}}{\dot{\gamma}_s} \right)^n$, where $\tau_s = \tau - \tau_0$ at $\dot{\gamma} = \dot{\gamma}_s$. Nelson & Ewoldt’s model is obtained by setting $\tau_s = \tau_0$ and $\dot{\gamma}_s = \dot{\gamma}_{critical}$.

According to Zamora and Power [17], determination of the yield stress for a given fluid remains a great challenge. Direct measurement is preferable because $\tau_0$ is an intrinsic property of the fluid independent of the rheological model. Besides, $\tau_0$ should be determined simultaneously with rheological data acquisition. However, typical Couette viscometers used in the drilling industry are not appropriate for measuring it. For fluids with a yield stress, the readings at low shear rates are unreliable due to a plug flow region in the viscometer gap. These authors proposed several strategies for reasonably accurate measurements of $\tau_0$:

1. Fann reading at 3 rpm ($R_3$)
2. Fann reading at 6 rpm ($R_6$)
3. Low shear yield point (stress beyond which a material becomes plastic, $2R_3 \cdot R_6$)
4. “Zero” gel strength (no time delay)
5. Initial gel strength (10 sec delay)
6. 10 min gel strength (10 min delay)

The first three are based on stable readings, whilst the last three on gel strength measurements. Low shear yield point is the best from the 1st group and the initial gel strength is the best from the 2nd group.

Skadsem et al. [18] reported that their steady rheological measurements (with a smooth bob and container walls) were overall well represented by the Herschel-Bulkley model, except at low shear rates probably due to apparent slip effects or short measurement time. In particular, the model overestimated shear stresses at shear rates lower than ca. 1 s$^{-1}$.

Although the three-parameter Herschel-Bulkley model yields an adequate relationship between the shear stress and the shear rate of drilling fluids [19], more complex four-parameter rheological models have been proposed [19, 20]. Even though these models predict more accurately drilling fluids rheological behavior than the widely accepted Herschel-Bulkley model, they have not been broadly applied due to the complexity in finding analytical solutions for the differential equations of motion, deriving velocity profiles and pressure drops in circular cylinders or cylindrical annulus, and seeking an appropriate Reynolds number and a criterion for the transition from laminar to turbulent flow [21].

While the molecular theory has been useful for calculations in gases at low density, it does not apply for liquids, suspensions, pastes, and other fluids. When experimental data is absent, it is necessary to use empiricisms of varying reliability. For liquids, a long-used empirical formula states that the viscosity of a liquid diminishes with the absolute temperature increase, $\eta = A \exp(B/T)$, because a nearly exponential dependence of the viscosity on the inverse absolute temperature is often observed. Nevertheless, errors up to 30% are frequent, and this empiricism should not be applied for long slender molecules [22].

### 1.3.1 Consistency coefficient, $k$

The temperature effect upon the consistency coefficient is commonly described by an Arrhenius type equation [23, 24]:

$$k = k_0 \exp \left( \frac{A}{\tau_0 T} \right)$$

where $k_0$ and $A$ are parameters and $T$ is the temperature (°C).

### 1.3.2 Yield stress, $\tau_0$

The literature reports that the yield stress exhibits a linear decreasing trend with the temperature increase [23, 24], i.e.:

$$\tau_0 = \tau_{01} - \tau_{02} T$$

where $\tau_{01}$ and $\tau_{02}$ are parameters.

### 1.3.3 Flow behavior index, $n$

The flow behavior index revealed a linear rising trend with the temperature increase in previous studies [23, 24]:

$$n = n_a T + n_0$$

where $n_a$ and $n_0$ are parameters.

The volumetric flow rate of a Herschel-Bulkley fluid in steady laminar flow is given by Eq. 5, where $D$ is the tube diameter, $Q_{mud}$ is the mud flow rate, and $\tau_w$ the shear stress at the tube wall. This equation was derived by averaging the fluid velocity over a tube cross section area, and assuming
null shear rate if the shear stress is inferior to the yield stress [25, 26].

\[
Q_{\text{mud}} = \left[ \pi \left( \frac{D}{2} \right)^3 \right] / 256 \left( \frac{4n}{3n+1} \right) \times \left( \tau_w / k \right)^{1/n} \left( 1 - \tau_0 / \tau_w \right)^{(1/n)} \times \left( 1 - \frac{\tau_0}{\tau_w} \left[ 1 + \frac{2n}{n+1} \left( \tau_0 / \tau_w \right) \left( 1 + n \tau_0 / \tau_w \right) \right] \right)
\]

(5)

On the other hand, at steady state the pressure drop, \( \Delta P \), and the shear stress at the tube wall, \( \tau_w \), are correlated by Eq. 6, where \( L \) is the tube length.

\[
\tau_w = \frac{\Delta P D^3}{4L}
\]

(6)

Generalized Reynolds numbers were determined inside the pipes of the pipe viscometer [27], using the velocities given by:

\[
v = \frac{Q_{\text{mud}}}{\pi(D/2)^2}
\]

(7)

where \( v \) is the mud velocity in each pipe. It is worth remarking that this work aimed to determine mud viscosity from pipe flow measurements.

The shear stress distribution in pipes and narrow concentric annuli are well defined, regardless of the fluid rheology. Only for Newtonian fluids and power law fluids flowing within pipes, or narrow concentric annuli, the shear rate at the wall may be derived from the average velocity and the flow path dimensions. It corresponds to a lower limit for the shear rate at the wall of non-Newtonian fluids, provided they are shear-thinning (most drilling fluids). The Newtonian shear rate at the wall is extremely sensitive to the pipe diameter or annular size, and to the fluid flow rates, thus it may substantially vary (\textit{vd}. Table 4-1 in [28]). Typically, shear rates in drilling pipe flow are in the order of 500-1000 s\(^{-1}\), whereas in well annuli flow they are typically in the order of 6-250 s\(^{-1}\).

1.4 Pipe viscometers

It is essential to characterize the rheological behavior of drilling fluids pumped down the wells to control the hydraulic pressure there-in. In oilfields, rheological measurements are conducted by off-line viscometers dating from more than 50 years ago. Most off-line viscometers/rheometers fail to characterize drilling fluids due to clogging, abrasion, non-homogeneity, and slip. On the other hand, on-line measurements ease the hydraulic pressure prediction almost instantly and early troubleshooting.

Despite these benefits, studies are still scarce concerning on-line flow measurements of drilling fluids, mainly because they usually present harsh issues, \textit{e.g.} most of them are non-Newtonian and thixotropic, are opaque and/or dense due to insoluble matter in suspension, and can be water- or oil-based. Therefore, only a few devices have been designed to monitor drilling fluids [29].

Measurements are usually executed at atmospheric pressure and temperature, thus they are not representative of actual downhole pressure and temperature. High-Pressure High-Temperature (HPHT) viscometers may be used at drilling fluid design to determine its rheological properties at downhole conditions. However, these properties can vary during drilling due to variation in mud maintenance and addition of cuttings and contaminations. Besides, installation of a HPHT viscometer at the rig-site is impractical due to its cost, maintenance, calibration, and lack of skilled personnel [30].

Optimal well operations require frequent and accurate viscosity and density measurements of drilling fluids. Currently, the standard methods by American Petroleum Institute (API) are used for drilling fluids properties measurements. However, these measurements are sporadic, labor-intensive and the data quality strongly depends on the operator, thus automated, continuous and practical methods of measuring and monitoring drilling fluid properties should be implemented [31].

Rogers \textit{et al.} [32] conducted flow tests of crosslinked fracturing fluids (“gels” with complex flow behavior) on a coiled-pipe viscometer designed for the purpose in 1985. The large use of crosslinked fracturing gels in the 1970s urged for accurate assessment of their rheology, since conventional rotational viscometers developed for simpler non-Newtonian fluids, were unsuitable for such a purpose. Gel responses to various temperatures and shear rates were determined by the pipe viscometer, which revealed several attributes: gel preparation within the viscometer, thus each gel element was subjected to identical shear during mixing; gel developed in-line as it moved to the viscometer test section without stagnant periods; and heat transfer occurred from the conduit walls to the flowing fluid, as in a fracture. Their results showed the likelihood of stress-induced slip flow in a fracture and the coiled-pipe viscometer proved to simulate fluid preparation and flow in fracturing.

Saasen \textit{et al.} [33] built a large-scale flow loop to measure several drilling fluid properties including the viscosity. The authors used a Couette viscometer as well. Although many properties were studied, no comparisons were made between on-line data and standard bench off-line data.

Broussard \textit{et al.} [34] developed a density and viscosity meter device for drilling fluids, and compared on-line data
and off-line data from standard bench devices (Couette method). The deviations between on-line and off-line data were attributed to the geometry and drifting forces in the on-line device.

Rondon et al. [35] built a prototype to be inserted into a drilling column to measure pressure drops, and the rheological behavior of drilling fluids in real time at downhole conditions. Their on-line data were compared to standard benchmark devices, only for polymeric solutions not for drilling fluids.

Carlsen et al. [36] installed pressure sensors along a rig-site, measuring the relative pressure and the pressure drop at several spots during drilling. They predicted the apparent viscosity from those pressure readings by hydraulic modeling. The on-line data were compared to standard benchmark devices and they showed similar behavior to Broussard’s work [34].

Vajargah and co-authors [30, 37, 38] proposed a method to determine mud rheological parameters in real-time and at actual downhole pressure and temperature by using downhole sensor data. The well was envisaged as a large annulus pipe viscometer with pressure drop and temperature measurements along its length by multiple sensors at strategic spots in the drillstring. Pressure drops were recorded at several flow rates and the rheological parameters of Herschel-Bulkley model (which most accurately describes the drilling fluids majority) were derived. The results were compared to off-line data collected by an off-line HPHT rheometer. Furthermore, time-dependent fluid characteristics, such as the gel strength, were also determined. This approach was a huge step for full automation of drilling fluid property monitoring, not requiring human interaction nor surface measurement equipment and describing accurately the downhole pressure and temperature. The advantages were obvious given the relevance of accurate rheology characterization in annular pressure management using Managed Pressure Drilling (MPD) and Dual Gradient Drilling (DGD) techniques.

Magalhães Filho et al. [29] developed two on-line viscometers to assess the rheological behavior of drilling fluids in real-time, simulating well drilling operations. The performances of a modified Couette viscometer and a standard pipe viscometer were compared with FANN 35A, an off-line Couette viscometer commonly used as benchmark in oilfields. For Newtonian fluids, there was agreement between the viscosity data taken by all devices. For power law fluids, there were divergences in the parameters yielded by each instrument both for a drilling fluid (with suspended solids) and a polymeric solution. These divergences were attributed to non-homogeneity, slip, and interactions in the fluid/gap interfaces but they were negligible for pressure drop calculations. The modified Couette viscometer optimized for drilling fluids, highly automated, without flow rate limits, and working up to 200 psi and 145°C, was suitable to continuously measure on-line the rheological behavior of drilling fluids on the surface and predict pressure losses in real-time.

Gul et al. [31] presented a vertical helical-pipe viscometer for automated mud measurements. An automated flow loop was built to perform flow tests of Newtonian and non-Newtonian fluids, either in two straight-pipe and two helical-pipe test sections. Pressure drop data was determined at each section simultaneously. The toroidal geometry led to secondary flow due to centrifugal forces, thus high pressure drops (for Re > 200) and delayed transition to turbulent flow occurred in the helical-pipe viscometer, and in turn the accuracy of low shear rheological parameter estimations augmented. A machine-learning regression model was trained using the experimental pressure loss data to predict friction factor. Using the trained machine-learning model, an algorithm was conceived to prove its applicability for automated rheological property determination. Excellent accuracy was noticed when comparing the helical-pipe viscometer and standard Couette viscometer measurements. Summing up, helical-pipe viscometers can be used in oil and gas field for automated, nearly real-time characterization of Newtonian and non-Newtonian fluids rheology. They are also appropriate for the hydraulic design of coiled-pipe and slim-hole operations. Helical-pipe viscometers display advantages over standard pipe viscometers, e.g. compact size and more versatile pressure loss profiles.

2 Experimental

The bentonite mud tested consisted of water (70.5% m/m), barite (26.4% m/m), bentonite (1.8% m/m), low viscosity polyanionic cellulose (PAC LV, 1.1% m/m, viscosifier), and Xanthan Gum polymer (0.2% m/m, viscosifier). The mud displayed a density of 1.25 g/cm³, as determined by the pycnometer method.

2.1 FANN viscometer measurements

A rotational Couette viscometer (FANN Model 35 with a R1-B1-F1 rotor-bob-torsion spring combination; errors of 0.25 cP at 600 rpm and 50 cP at 3 rpm [39]) was used to study the rheological behavior of the bentonite mud. FANN 35 viscometer is well known as the industry rig standard
for measuring drilling fluids viscosities [40]. Fernandes et al. [41] obtained reliable measurements in general by using FANN 35 for steady state measurements. Nevertheless, they observed that the rheological data at 3 rpm might depend on the drilling fluid rheology, thus a great error might be expected at this shear rate.

The mud flow rates (0.50-3.31 L/min) were measured by a device consisted of three pipes of distinct diameters ($D$, $D_1$, $D_2$). Similarly to Vajargah and co-authors’ work [11], a Coriolis flowmeter, as recommended for non-Newtonian fluids [39], was connected to differential pressure indicators (Freescale MPX2100DP Case 344C-01). The bentonite mud was allowed to rest for 1 hour between consecutive measurements. The shear rate ($\dot{\gamma}$), shear stress ($\tau$), and apparent viscosity ($\eta_a$) were estimated by the following equations:

$$\tau = k_1 k_2 \dot{\gamma}$$

$$\dot{\gamma} = k_3 \omega$$

$$\eta_a = \frac{\tau}{\dot{\gamma}} \times 100$$

where $k_1$ corresponds to the torsion constant (386 dyn·cm/degree deflection), $k_2$ to the shear stress constant for the effective bob surface (1.323×10^{-2} cm^{-2}) and $k_3$ to the shear rate constant (1.7023 s^{-1} per rpm) according to the manufacturer [39].

By comparing the experimental data of shear stress and apparent viscosity against the shear rate with typical rheograms, the bentonite mud was classified as a yield-pseudoplastic fluid in the temperature range of 10-70°C (usual temperature range of drilling fluids returned from the borehole to the surface). Thereafter, the Herschel-Bulkley model (Eq. 1) was applied in that temperature range, with increments of 10°C, to estimate the rheological parameters, $\tau_0$, $k$ and $n$, based on the viscometer data.

### 2.2 Pipe viscometer measurements

The mud flow rates (0.50-3.31 L/min) were measured by a Coriolis flowmeter, as recommended for non-Newtonian fluids [43]. To correlate the pressure loss with the mud flow rate, a pipe viscometer was easily assembled (Figure 2), similarly to Vajargah and co-authors’ work [30, 37, 38]. This device consisted of three pipes of distinct diameters ($D$) and lengths ($L$). The diameters were 18 mm (pipe 1), 9 mm (pipe 2), and 14.5 mm (pipe 3), whereas their lengths were 1.54 m (pipe 1), 0.325 m (pipe 2), and 0.62 m (pipe 3). Couples of...
3 Results and Discussion

3.1 FANN viscometer measurements

The rheological characterization of a bentonite mud was accomplished by inserting the viscometer dial readings, at various angular velocities (3-600 rpm) and common drilling mud temperatures at rig-site surface (10-70°C), into Eqs. 8-9 to estimate the corresponding shear rates and shear stresses. The mud rheograms depicted (Figure 4) were compared with typical rheograms (Figure 1) definitely pointing out a yield-pseudoplastic behavior. In Figure 4, the shear stress increased with the shear rate increase and/or the temperature decrease. Although most drilling fluids exhibit thixotropy and viscoelasticity [42, 49, 50], the tests executed did not take into account these phenomena.

The apparent viscosity of the bentonite mud was estimated for each shear rate (Eq. 10) and plotted in Figure 5, corroborating the yield-pseudoplastic behavior, i.e. the mud revealed viscoplasticity and shear-thinning. Hence, the mud may be designated as a yield-pseudoplastic or a Herschel-Bulkley fluid. In Figure 5, the mud viscosity decreased with the shear rate and/or the temperature increases. According to Busch et al. [42], PAC solutions displayed Newtonian viscosity plateaus at both low and high shear rates due to its polymeric nature, similar to the observations at high shear rates in the present study.

The Herschel-Bulkley model appeared to be a good approach to describe the viscoplasticity and shear-thinning of the bentonite mud. Thus, the rheological parameters of the Herschel-Bulkley model, \( \tau_0, k \) and \( n \), were estimated for each temperature by curve fitting of viscometer readings using a non-linear generalized reduced gradient Solver tool (from Microsoft Excel 2013 software) and were listed thereafter in Table 1.

As the parameters \( \tau_0, k \) and \( n \) exhibited a strong scattering with respect to the temperature, they were refined by minimizing the sum of the quadratic deviations be-
between the experimental shear stress (FANN viscometer data) and the data predicted by the empirical Eqs. 2-4. Thereby, the intermediate rheological parameters were estimated: $k_0$ ($3.50 \times 10^{-2}$ dyn/(cm$^2$·s$^{n}$)), $A$ ($1.38 \times 10^{3}$ K), $n_a$ ($4.12 \times 10^{-4}$ °C$^{-1}$), $n_0$ ($6.47 \times 10^{-1}$), $\tau_{0i}$ ($1.12 \times 10^2$ dyn/cm$^2$), and $\tau_{0a}$ ($8.00 \times 10^{-2}$ dyn/(cm$^2$·°C)).

The experimental shear stress and the data predicted by the Herschel-Bulkley model (including the intermediate rheological parameters) is plotted in Figure 6, confirming that this model yielded a fairly good fitting of the experimental data with a coefficient of determination of 0.9762 in the temperature range of 10-70°C [51].

The correlations of the rheological parameters $\tau_{0i}$, $k$, and $n$ (determined by inserting the intermediate rheological parameters in Eqs. 2-4) with the temperature are displayed in Figure 7, being similar to the trends referred in the literature [23, 24].

### 3.2 Pipe viscometer measurements

On the other hand, the pipe viscometer yielded experimental data of mud flow rates and the corresponding pressure drops. The generalized Reynolds numbers in the three pipes were calculated (Eqs. 12-13) as well. Their minimum values were 3, 10, and 2, and their maximum values were 14,
137, and 9, in the pipes 1, 2 and 3, respectively, confirming steady laminar flow in each pipe throughout all essays \((\text{Re}_{\text{generalized}} < 2100)\).

The experimental and predicted pressure drops (Eqs. 5-6) were plotted against the mud flow rate in Figure 8. Both pressure drops naturally augmented with the mud flow rate rise, although the match between the experimental and predicted data was slightly better for the pipes 1 and 2 especially at low flow rates.

3.3 Unified model algorithm

Finally, the sum of the quadratic deviations between the experimental and predicted pressure losses was minimized and the intermediate parameters were refined using a non-linear generalized reduced gradient Solver tool:

\[ k_0 \left( 34.51 \times 10^{-1} \right) \text{ dyn/(cm}^2 \cdot \text{s}^n \right), A \left( 9.57 \times 10^2 \right), n_0 \left( 5.97 \times 10^{-3} \right) \text{ °C}^{-1}, \ n_0 \left( 2.51 \times 10^{-1} \right), \tau_0 \left( 1.70 \times 10^2 \right) \text{ dyn/cm}^2, \text{ and } \tau_{0a} \left( 1.53 \right) \text{ dyn/(cm}^2 \cdot \text{°C}) \). In summary, the modified Herschel-Bulkley model for the bentonite mud (most common drilling fluid) in the temperature range of 10-70°C (typical temperature range of drilling fluids returned from the borehole to the surface) is given by:

\[
\tau \left( \text{dyn/cm}^2 \right) = 170 - 1.53 \ T \left( \text{°C} \right) + 0.3451 \ e^{0.00597 T(°C)+0.251}
\]

Using the refined intermediate rheological parameters, comparison between the predicted and experimental shear stresses is depicted in Figure 9 (against the shear rate and temperature) and Figure 10.

The conclusion to be drawn from Figures 9 and 10 is that the modified Herschel-Bulkley model is in fair agreement with the experimental data with a coefficient of determination of 0.9365. On average, it predicted lower shear stresses \((\text{ca.} 2.2\%)\) than the experimental data in the temperature range of drilling fluids returned from the borehole to the surface is given by:

\[
y = 0.9781x
\]

\[ R^2 = 0.988 \]

**Table 1: Rheological parameters (estimated with FANN viscometer readings) in the temperature range of 10-70°C.**

| \( T \) (°C) | \( \tau_0 \) (dyn/cm\(^2\)) | \( k \) (dyn/(cm\(^2\)·s\(^n\))) | \( n \) |
|---------|-----------------|-----------------|---|
| 10      | 91.28           | 11.15           | 0.52 |
| 15      | 92.7            | 10.52           | 0.51 |
| 20      | 82.4            | 8.41            | 0.54 |
| 25      | 88.3            | 3.77            | 0.66 |
| 30      | 87.58           | 4.66            | 0.62 |
| 35      | 89.46           | 4.73            | 0.60 |
| 40      | 99.58           | 2.45            | 0.69 |
| 50      | 87.68           | 7.98            | 0.49 |
| 55      | 89.38           | 8.63            | 0.47 |
| 60      | 111.52          | 4.44            | 0.56 |
| 65      | 127.62          | 9.89            | 0.41 |
| 70      | 137.94          | 6.95            | 0.46 |

**Figure 8:** Comparison between the experimental and predicted pressure drops in the pipes 1, 2, and 3 vs. mud flow rate.

**Figure 9:** Experimental shear stress and data predicted by the modified Herschel-Bulkley model vs. the shear rate in the temperature range of 10-70°C.

**Figure 10:** Comparison between the experimental shear stress and the data predicted by the modified Herschel-Bulkley model. Dashed line: graph diagonal; solid line: linear trend of \( \tau_{\text{predicted}} \) vs. \( \tau_{\text{exp}} \).
perature range of 10-70°C. Below 40°C, the predicted shear stresses were higher than the experimental data, whilst above 40°C, the predicted shear stresses were lower than the experimental data.

The major outcome of this work was the development of an affordable pipe viscometer that enables the continuous and fairly reliable measurement of the apparent viscosity of Newtonian or non-Newtonian fluids. A patent application was submitted on this subject [52]. Furthermore, the rheological parameters determined allowed the estimation of the Reynolds number in the gas trap of the gas analyzer system, a clue of utmost importance to assess the turbulence therein [6].

4 Conclusions

A FANN viscometer measured the apparent viscosity of a bentonite mud (most common drilling fluid) at various shear rates (3-600 rpm) and typical temperatures at rig-site surface (10-70°C). The mud behaved as a yield-pseudoplastic fluid, thus the Herschel-Bulkley model was applied to predict the effects of the shear rate and temperature upon the shear stress.

As the rheological parameters $k$, $n$, and $\tau_0$ revealed a temperature dependency, a pipe viscometer was assembled to obtain a correlation between the pressure drop and the mud flow rate, in order to refine the parameters of a modified Herschel-Bulkley model including the temperature effect. The model was in fair agreement with the experimental data predicting lower shear stresses (ca. 2.2%). The predicted shear stress was superior to the experimental data under 40°C, whereas it was inferior over 40°C.

Summing up, an effective and affordable method was proposed for the determination of rheological parameters and continuous and reliable apparent viscosities of Newtonian or non-Newtonian fluids. The continuous data acquisition also granted the instantaneous determination of the Reynolds number and assessment of the turbulence in the gas trap of the gas analyzer system.

For the pipe viscometer future development, it is mandatory checking the validity of the modified Herschel-Bulkley model for drilling muds with distinct compositions and rheological behaviors.

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References

[1] Baker Hughes INTEQ. Drilling Engineering Workbook. Houston, Texas, USA; 1995. p. 212-251.
[2] Pixler BO. Baroid Mud Analysis Logging. The Log Analyst 1967;7(05).
[3] Hammerschmidt SB, Wiersberg T, Heuer VB, Wendt J, Erzinger J, Kopf A. Real-time drilling mud gas monitoring for qualitative evaluation of hydrocarbon gas composition during deep sea drilling in the Nankai Trough Kumano Basin. Geochemical Transactions 2016;15(15).
[4] ASME Shale Shaker Committee. Drilling fluids processing handbook. Elsevier: Gulf Professional Publishing; 2011.
[5] Marum DM, Afonso MD, Ochoa BB. Enhancement of the Gas Extraction for Reservoir Identification in a New Mud Logging System. GeoConvention 2019; 2019 May 13-15; Calgary, Alberta, Canada. Available from: https://geoconvention.com/wp-content/uploads/abstracts/2019/GC2019_021_Enhancement_of_the_Gas_Extraction_for_Reservoir_Identification_in_New_Mud_Locking_System.pdf.
[6] Marum DM, Afonso MD, Ochoa BB. Optimization of the Gas Extraction Process in a New Mud-Logging System. Society of Petroleum Engineers, SPE-198909-PA, SPE Drilling & Completion 2020;35(1):1-13. doi:10.2118/198909-PA.
[7] Torres C, André C. Power Consumption of a Rushton Turbine Mixing Viscous Newtonian and Shear-thinning Fluids: Comparison between Experimental and Numerical Results. Chemical Engineering & Technology: Industrial Chemistry-Plant Equipment-Process Engineering-Biotechnology 1998;21(7):599-604.
[8] Sochi T. Non-Newtonian flow in porous media. The International Journal for the Science and Technology of Polymers 2010;51(22):5007-23. doi: 10.1016/j.polymer.2010.07.047.
[9] Partal P, Franco JM. Rheology - Encyclopedia of life support systems. Wolss Publishers Co. Ltd., Oxford, UK; 2010.
[10] Hammad KI. The Flow Behavior of a Biofluid in a Separated and Reattached Flow Region. Journal of Fluids Engineering 2015;137(6):1-8. doi: 10.1115/1.4029727.
[11] Pakzad L, Ein-Mozaffari F, Uprei SR, Lohi A. Agitation of Herschel-Bulkley fluids with the Scaba-anchor coaxial mixers. Chemical Engineering Research and Design 2013;91(5):761-77. doi: 10.1016/j.cherd.2012.09.008.
[12] Chhabra R. Non Newtonian Fluids: An Introduction. In: “Rheology of Complex Fluids”. Springer, New York, USA; 2010. p. 3-34. doi: 10.1155/2013/583809.
[13] Garakani AK, Mostoufi N, Sadeghi F, Hosseinizadeh M, Fatourechi H, Sarrafzadeh MH, Mehrnia MR. Comparison between different models for rheological characterization of activated sludge (Vol. 8). Springer, Tehran, Iran; 2011.
[14] Rao, A. Rheology of Fluid, Semisolid, and Solid Foods Principles and Applications. Springer, Oxford, UK; 2014.
Nelson AZ, Ewoldt RH. Design of yield-stress fluids: a rheology-to-structure inverse problem. Soft Matter 2017;13: 7578-94. doi: 10.1039/c7sm00758b.

Saasen A, Ytrehus JD. Rheological Properties of Drilling Fluids: Use of Dimensionless, Shear Rates in Herschel-Bulkley and Power-law Models. Appl. Rheol. 2018;28:54515. doi: 10.3933/ApplRheol-28-54515.

Zamora M, Power D. Making a Case for AADE Hydraulics. 2002 Technology Conference “Drilling & Completion Fluids and Waste Management”;

Vajargah AK, van Oort E. Determination of drilling fluid rheology under downhole conditions by using real-time distributed pressure data. Journal of Natural Gas Science and Engineering 2015;24:400-11. doi: 10.1016/j.jngse.2015.04.004.

Gul S, Erge O, van Oort E. Helical Pipe Viscometer System for Automated Mud Rheology Measurements. Society of Petroleum Engineers, SPE-199572-MS, IADC/SPE International Drilling Conference and Exhibition; 2020 March 3-5; Galveston, Texas, USA. doi: 10.2118/199572-MS.

Rogers RE, Veatch JR RW, Nolte KG. Pipe Viscometer Study of Fracturing Fluid Rheology. Society of Petroleum Engineers, SPE-10258-PA, 1984;24(5). doi: 10.2118/10258-PA.

Saasen A, Omland TH, Ekrene S, Breivie J, Villiard E, Kaagezon-Loe N, Tehrani A, Cameron J, Freeman M, Growcof P, Patrick A, Stock T, Swaco MI, Jørgensen T, Reinholdt F, Scholz N, Amundsen HEF, Steel A, Meeten G. Automatic Measurement of Drilling Fluid and Drill Cuttings Properties. IADC/SPE Drilling Conference (2009). SPE Drilling & Completion, 2009;24(4):611-25.

Broussard S, Gonzalez P, Murphy R, Marvel C. Making Real-Time Fluid Decision with Real-Time Fluid Data at the Rig Site. Society of Petroleum Engineering (SPE), SPE Drilling Conference and Exhibition; 2010; Abu Dhabi, UAE.SPE 137999.

Rondon J, Barrufet MA, Falcone G. A novel downhole sensor to determine fluid viscosity. Flow Measurement and Instrumentation 2012;23:9-18.

Carlsen LA, Nygaard G. Utilizing Instrumented Stand Pipe for Monitoring Drilling Fluid Dynamics for Improving Automated Drilling Operations. Proceedings of the 2012 IFAC Workshop on Automatic Control in Offshore Oil and Gas Production; 2012; Norwegian University of Science and Technology, Trondheim, Norway.

Vajargah AK, van Oort E. Automated Drilling Fluid Rheology Characterization with Downhole Pressure Sensor Data. SPE/IADC Drilling Conference and Exhibition; 2015; Society of Petroleum Engineers, London, United Kingdom. SPE-173085-MS.

Vajargah AK, Sullivan G, van Oort E. Automated Fluid Rheology and ECD Management. SPE Deepwater Drilling & Completions Conference; 2016; Society of Petroleum Engineers, Galveston, Texas, USA. SPE-180331-MS.

Fann Instrument Company. Model 35 Viscometer. Instruction Manual. Houston, Texas, USA; 2016.

Gjerstad K, Time R. Simplified Explicit Flow Equations for Herschel-Bulkley Fluids in Couette-Poiseuille Flow - For Real-Time Surge and Swab Modeling in Drilling. Society of Petroleum Engineers Journal 2015;20(3):610-27.

Fernandes RR, Turezo G, Andrade DEV, Franco AT, Negrão COR. Are the rheological properties of water-based and synthetic drilling fluids obtained by the Fann 35A viscometer reliable? Journal of Petroleum Science and Engineering 2019;177:872-9. doi: 10.1016/j.petrol.2019.05.039.

Busch A, Myrseth V, Khatibi M, Skjetne P, Hovda S, Johansen T, Carlsen LA, Nygaard G, Khatibi M, Skjetne P, Hovda S, Johansen T. Are the rheological properties of water-based and synthetic drilling fluids obtained by the Fann 35A viscometer reliable? Journal of Petroleum Science and Engineering 2019;177:872-9. doi: 10.1016/j.petrol.2019.02.063.

Fyrripi I, Owen I, Escudier M. Flowmetering of non-Newtonian Liquids. Flow Measurement and Instrumentation 2004;3(15):131-8.

Çengel Y, Cimbala J. Fluid Mechanics - Fundamentals and Applications (1st Ed. in SI units). McGraw-Hill, New York, USA; 2006.

Madlener K, Frey B, Ciezki HK. Generalized Reynolds number for non-Newtonian fluids. Progress in Propulsion Physics 2009;1:237-50. doi: 10.1051/eucass/200901237.
[46] Rudman M, Graham L, Blackburn H, Pullum L. Non-Newtonian Turbulent and Transitional Pipe Flow. 15th International Symposium on Hydrotransport incorporating the 11th International Symposium of Freight Pipelines. Banff, Canada; 2002.

[47] Cabral R, Gut J, Telis VT-R. Non-Newtonian flow and pressure drop of pineapple juice in a plate heat exchanger. Brazilian Journal of Chemical Engineering 2010;24(4).

[48] Freescale. Freescale Semiconductor, Inc. 100 kPa On-Chip Temperature Compensated and Calibrated Silicon Pressure Sensors. München, Germany; 2008.

[49] Tehrani, A. Thixotropy in Water-Based Drilling Fluids. Annual Transactions of the Nordic Rheology Society 2008;16.

[50] Werner B. The Influence of Drilling-Fluid Rheology on Cuttings-Bed Behavior [PhD dissertation]. Norwegian University of Science and Technology, Norway; 2018. doi: 10.13140/RG.2.2.19105.30566.

[51] Srivastava M, Sen A. Regression Analysis - Theory, Methods and Applications. Springer (ST Statistics, Ed.), New York, USA; 1990.

[52] Marum DM, Ochoa B. Methods and Systems for Monitoring Drilling Fluid Rheological Characteristics. Patent no. US 62/597503, Germany; 2017 September 18.