Comment on "Superconducting decay length in a ferromagnetic metal"

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In the paper "Superconducting decay length in a ferromagnetic metal" by Gusakova, Kupriyanov and Golubov [Pis’ma v ZhETF 83, 487 (2006); cond-mat/0605137], the authors claim that they solved the linearized Eilenberger equation in the ferromagnetic region of an S/F heterostructure at arbitrary mean free path. In this comment we show that the solution suggested by the authors is not correct and explain details of the exact solution found by us in an earlier work several years ago (Ref.[2]).

I. INTRODUCTION

In a recent paper [1] Gusakova, Kupriyanov and Golubov studied theoretically the decay length over which the condensate function $f$ decreases in the ferromagnetic region of a S/F structure (S and F denote a superconductor and ferromagnet). In order to determine the decaying length they have solve the linearized Eilenberger equation representing the solution in the form

$$f(x) = C(\theta) \exp(-x/\xi),$$

where $\theta$ is an angle between the direction of the momentum and the $x$-axis (the $x$-axis is perpendicular to the S/F interface). In this solution the function $C(\theta)$ depends only on $\theta$ and the decay length $\xi$ is assumed to be independent of this angle. Substituting this Ansatz into the linearized Eilenberger equation the authors obtain expressions for $C(\theta)$ and the decay length $\xi$. They analyze the dependence of the decay length on the exchange field $h$ and the mean free path for an arbitrary magnitude of the product $h \tau$ (we set the Planck constant equal to 1).

In this Comment we would like to point out that:

1. The solution of the form (1) for the linearized Eilenberger equation is not correct and may serve only for rough estimates for the decay length $\xi$ in a particular limit.

2. The linearized Eilenberger equation can be solved exactly for an arbitrary value of product $h \tau$. The exact solution has been obtained in Ref. [2] for arbitrary values of $h \tau$ (contrary to the statement of the authors of Ref.[1] that the solution was obtained in [2] in the clean limit only) and used later in subsequent theoretical [3, 4] and experimental studies [5, 6].

In Ref. [2] the solution of the linearized Eilenberger equation for the Josephson S/F/S junction is presented explicitly. In order to avoid a literal repetition of that result we use the method of Ref. [2], to obtain the exact solution for a simpler S/F structure considered in [1], thus directly comparing the result with "the solution" of Eq. (1) from that work. From a general formula we obtain simple expressions for $f(x)$ in the limiting cases of small and large values of the product $h \tau$ (the diffusive and quasi-ballistic cases correspondingly). To make the analysis more transparent, we neglect the spin-orbit interaction.

The solution of the linearized Eilenberger equation $\hat{f}(x)$ can be represented as a sum of a symmetric $\hat{s}(x)$ and antisymmetric $\hat{a}(x)$ parts: $\hat{f}(x) = \hat{s}(x) + \hat{a}(x)$. The functions $\hat{f}(x)$, $\hat{s}(x)$ and $\hat{a}(x)$ are matrices in the particle-hole space. The antisymmetric part $\hat{a}(x)$ is related to the symmetric one via the formula

$$\hat{a} = -\text{sgn}\omega (\mu l/\kappa\omega) \hat{\tau}_3 \partial_x \hat{s}$$

and the symmetric part $\hat{s}(x)$ in the ferromagnetic region ($x > 0$) obeys the equation [2]
\[
\mu^2 l^2 \partial^2_{xx} \hat{s} - \kappa^2 \omega \hat{s} = -\kappa \omega \langle \hat{s} \rangle \tag{3}
\]

where \( l = v_F \tau \) is the mean free path, \( \mu = \cos \theta \), \( \kappa = (1 + 2|\omega_m|\tau) - \text{sgn}\omega 2i\hbar \tau \), \( \omega_m = \pi T (2m + 1) \) is the Matsubara frequency. The angle brackets mean the angle averaging

\[
\langle \hat{s} \rangle = (1/2) \int_{-1}^{1} d\mu \hat{s}(\mu) \tag{4}
\]

Eqs \((2 - 3)\) are complemented by the boundary condition \([7]\)

\[
\hat{a} \mid_{x=0} = \gamma(\mu) \text{sgn} \omega \langle \hat{\tau}_3 \hat{f}_s \rangle , \tag{5}
\]

where \( \gamma(\mu) = T(\mu)/4 \) and \( T(\mu) \) is the transmission coefficient. The matrix \( \hat{f}_s = i\hat{\tau}_2 \Delta / \sqrt{\omega_m^2 + \Delta^2} \equiv i\hat{\tau}_2 f_s \) is the matrix condensate function of the superconductor (the BCS-function). The boundary condition \([5\) \] is satisfied automatically due to the last term on the right-hand side. For the Fourier transform \( s_k = \int dx s(x) \exp(-ikx) \) we get from Eq.\((6)\)

\[
s_k(\mu) = \frac{\kappa_\omega}{M(\mu)} \{ \langle s_k(\mu) \rangle + 2\gamma(\mu)\mu f_s \delta(x) \} \tag{7}
\]

where \( M(\mu) = (kl\mu)^2 + \kappa^2 \).

Averaging \( s_k(\mu) \) over \( \mu \) and substituting the result into Eq.\((7)\), we obtain finally

\[
s_k(\mu) = \frac{2\kappa_\omega \mu f_s}{M(\mu)} \left( \frac{\kappa_\omega}{1 - \kappa_\omega(1/M)} \right) \left( \frac{\gamma(\mu)\mu}{M} \right) + \gamma(\mu)\mu \right\} \tag{8}
\]

The inverse Fourier transformation

\[
s(x, \mu) = \int \frac{dk}{2\pi} s_k(\mu) \exp(ikx) \tag{9}
\]

determines the exact solution for Eq.\((6)\) at arbitrary mean free path. It is obvious that the function \( s(x, \mu) \) cannot be represented as a product of two functions, such that one of them would depend only on \( \mu \) and the other one only on \( x \).

The average \( \langle 1/M \rangle \) can easily be calculated

\[
\langle 1/M \rangle = \frac{1}{kl \kappa_\omega} \tan^{-1} \frac{kl}{\kappa_\omega} \tag{10}
\]

Let us consider limiting cases, where one can obtain analytical expressions for the condensate function \( s(x, \mu) \).

a) diffusive limit: \( h\tau << 1 \).

In this case, characteristic wave vectors \( k \) are much smaller than \( 1/l \). Expanding the function on the right-hand side in Eq.\((3)\) in powers of \( kl \), we obtain \( 1 - \kappa_\omega (1/M) \approx k^2 (\kappa^2 + \kappa_\omega^2)/3 \). The behavior of \( s(x, \mu) \) at large \( x \) (larger than the mean free path \( l \)) is determined by the pole in the first term in Eq.\((3)\). Calculating the residue at this pole, we get
\[ s(x, \mu) \approx \frac{3f_s}{k_h l} \langle \gamma(\mu) \mu \rangle \exp(-k_h x) \] (11)

where \( k_h^2 = 2|\omega_m| - \text{sgn}\omega_m i\hbar|/D \), \( D = lv/3 \) is the diffusion coefficient. This result can be easily obtained from the Usadel equation. One can see that \( s(x, \mu) \) is small and the linearization of the Eilenberger equation is possible provided the condition \( 3\sqrt{3}\langle \gamma(\mu) \mu \rangle /\sqrt{h \tau} \ll 1 \) is fulfilled. Only in this case the characteristic length of the condensate decay (at large \( x \)) does not depend on the angle \( \theta \). At \( x \) of the order of the mean free path the solution of the Eq. (6) depends on \( \theta \). Thus, a solution of the form Eq. (11) is only valid in the diffusive limit and for large \( x \). However, in the case of strong ferromagnets or not too short mean free path the solution depends on angles at any \( x \), as we will show.

b) quasi-ballistic case: \( h \tau > > 1 \).

In this case the parameter \( |\kappa_\mu| \approx h \tau \) is large and therefore the first term in the curly brackets in Eq. (8) is small. Calculating the integral in Eq. (9), we get

\[ s(x, \mu) \approx \gamma(\mu)f_s \exp(-k_\omega x/l\mu), \quad \mu > 0 \] (12)

We see that in this case the decay length \( \xi_F \) which is equal to \( \xi_F = \min\{v\mu/T, \mu l\} \) is angle dependent. The condensate function oscillates in space with a small (compared to \( \xi_F \)) period \( \xi_{osc} = l\mu/(2\pi h \tau) \). It is clear that the condensate function cannot be presented in the form (11). In a general case of arbitrary product \( h \tau (h \tau \approx 1) \) the spatial dependence of the Cooper pair wave function \( f \) can be found by numerical integration of Eqs. (7-8).

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