The light-cone gauge without prescriptions

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Feynman integrals in the physical light-cone gauge are harder to solve than their covariant counterparts. The difficulty is associated with the presence of unphysical singularities due to the inherent residual gauge freedom in the intermediate boson propagators constrained within this gauge choice. In order to circumvent these non-physical singularities, the headlong approach has always been to call for mathematical devices — prescriptions — some successful ones and others not so much so. A more elegant approach is to consider the propagator from its physical point of view, that is, an object obeying basic principles such as causality. Once this fact is realized and carefully taken into account, the crutch of prescriptions can be avoided altogether. An alternative third approach, which for practical computations could dispense with prescriptions as well as precluding the necessity of careful stepwise watching out of causality would be of great advantage. And this third option is realizable within the context of negative dimensions, or as it has been coined, negative dimensional integration method (NDIM).

\S 1. Introduction

Light-cone gauge in quantum field theory is a fascinating subject. It is a “simple” gauge choice in the sense that the emerging propagator for the boson field has a deceivingly simple structure, when compared to other non-covariant gauge choices. Yet, as it has soon been realized, it hid subtle complications when one actually wanted to work with it.

To make things more concrete, let us analyse it in the framework of vector gauge fields, e.g. the pure Yang-Mills fields, where, after taking the limit of vanishing gauge parameter, the propagator reads,

\[
D_{\mu\nu}^{ab}(k) = -\frac{i\delta^{ab}}{k^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{n_{\mu}k_{\nu} + n_{\nu}k_{\mu}}{k \cdot n} \right] \tag{1.1}
\]

where \(n_{\mu}\) is the arbitrary and constant null four-vector which defines the gauge, \(n \cdot A^a(x) = 0; \quad n^2 = 0\). This generates \(D\)-dimensional Feynman integrals of the following generic form,

\[
I_{lc} = \int \frac{d^Dk_i}{A(k_j, p_l)} \frac{f(k_j \cdot n^*, p_l \cdot n^*)}{h(k_j \cdot n, p_l \cdot n)} \tag{1.2}
\]

where \(p_l\) labels all the external momenta, and \(n^*_\mu\) is a null four-vector, dual to \(n_\mu\).

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We will evaluate, as a pedagogical example, an integral where,
\[
A(k_j, p_l) = (k^2)^{-i} \left[ (k - p)^2 \right]^{-j}, \quad h(k_j \cdot n, p_l \cdot n) = (k \cdot n)^{-l}, \quad f(k_j n^*, p_l n^*) = (k \cdot n^*)^m,
\]
with \(i, j, l\) negative and \(m\) positive or zero.

A conspicuous feature that we need to note first of all, is that the dual vector \(n^*\), when it appears at all, it does so always and only in the numerators of the integrands. And herein comes the first seemingly “misterious” facet of light-cone gauge. How come that from a propagator expression like (1.1), which contains no \(n^*\) factors, can arise integrals of the form (1.2), with prominently seen \(n^*\) factors? Again, this is most easily seen in the framework of definite external vectors \(n\) and \(n^*\). An alternative way of writing the generic form of a light-cone integral is
\[
I_{\mu_1 \cdots \mu_n}^{\mu_1 \cdots \mu_n} = \int \frac{d^D k_j}{A(k_j, p_l)} \frac{g(k_j^{\mu_j}, p_l^{\mu_i})}{h(k_j \cdot n, p_l \cdot n)},
\]
where the numerator \(g(k_j^{\mu_j}, p_l^{\mu_i})\) defines a tensorial structure in the integral. For a vector, we have \(k^{\mu} = (k^+, k^-, k^T)\), where \(k^+ = \lambda(k^0 + k_{D-1})\) and \(k^- = \lambda(k^0 - k_{D-1})\) with \(\lambda\) being a normalization factor. If we choose definite \(n\) and \(n^*\) such that \(n_\mu = (1, 0, \cdots, 1)\), and \(n^*_\mu = (1, 0, \cdots, -1)\), this allows us to write \(k^+ \equiv k \cdot n\) and \(k^- \equiv k \cdot n^*\). We have therefore traced back the origin for the numerator factors containing \(n^*\). By the way, these terms have nothing whatsoever to do with some kind of prescription input as it is in (2.5) below. There, the \(n^*\) factors are added into the structure by hand via the ad hoc prescription.

\section*{§2. Prescriptions}

Now, the troublesome factors in the denominator, represented by \(h(k_j \cdot n, p_l \cdot n)\), for one-loop, two-point functions are typically products of the form
\[
\frac{1}{h(k_j \cdot n, p_l \cdot n)} \sim \frac{1}{(k \cdot n) (k \cdot p \cdot n)}.
\]

In the standard approach, these are dealt with first by partial fractioning them, a trick sometimes called “decomposition formula” (see, for example,\(^1\)-\(^3\))
\[
\frac{1}{(k \cdot n) (p-k \cdot n)} \rightarrow \frac{1}{p \cdot n} \left[ \frac{1}{k \cdot n} + \frac{1}{(p-k) \cdot n} \right], \quad p \cdot n \neq 0
\]

It is easy to see that such kind of trick becomes very clumsy to deal with when one has higher powers of denominator factors. In fact, the authors of\(^3\) agree that the application of (2.2) complicates considerably the calculation of such Feynman integrals. For example, consider,
\[
\frac{1}{(k \cdot n)^2 (p-k \cdot n)} \rightarrow \frac{1}{(p\cdot n)^2} \left[ \frac{p \cdot n}{(k \cdot n)^2} + \frac{1}{k \cdot n} + \frac{1}{(p-k) \cdot n} \right], \quad p \cdot n \neq 0
\]
and so on and so forth. Yet, the standard approach based upon prescriptions, cannot handle denominator products like these without first decomposing them. As far as we know only Leibbrandt and Nyeo said explicitly before, that “decomposition formulas” like (2.2) and (2.3) are in fact part of the prescription\(^3\). Be it as it is, after the decomposition is made, then and only then, one seeks to handle the isolated denominators calling for prescriptions that would turn them manageable mathematically.

Early efforts in this direction draw heavily from the use of the Cauchy principal value (CPV or PV for short),

\[
\text{PV} \left( \frac{1}{q \cdot n} \right) \rightarrow \frac{1}{2} \lim_{\epsilon \to 0} \left( \frac{1}{q \cdot n + i\epsilon} + \frac{1}{q \cdot n - i\epsilon} \right). \tag{2.4}
\]

Unfortunately, this prescription, although preserves the general structure of the light-cone integral (1.2), and is mathematically consistent, does not provide physically acceptable results in the light-cone gauge\(^4\);\(^5\).

Later on, independently Mandelstam\(^6\) and Leibbrandt\(^7\) proposed new prescriptions to treat the gauge-dependent poles,

\[
\frac{1}{q \cdot n} \rightarrow \frac{1}{q \cdot n + i\epsilon \text{ sgn}(q \cdot n^*)} = \frac{q \cdot n^*}{(q \cdot n)(q \cdot n^*) + i\epsilon}. \tag{2.5}
\]

where the former is the modified Mandelstam prescription and the latter is the Leibbrandt’s one. It can be proven that, in fact, they are completely equivalent to each other, so that sometimes people refer to this as the Mandelstam-Leibbrandt (ML) prescription. A conspicuous feature of these prescriptions is that they introduce the factor \((q \cdot n^*)\) in the denominator of integrals, thus modifying the original structure. This, however, does not seem to affect the end result, as far as physically meaningful results are concerned.

Applying the ML prescription for example, in the one-loop two-point function integrals, double poles still appear but they cancel against each other and one is left with physical poles only. In other words, those singularities that are merely gauge artifacts cancel out when one uses this approach. Experts\(^5\) argue, therefore, that adopting the ML prescription makes the light-cone gauge acceptable, at least perturbatively. Along this line, some two-loop calculations have been performed explicitly and can be found in the pertinent literature\(^8\).

Pimentel et al. realized that when the physical principle of causality is correctly taken into account — either by careful, stepwise watching out for it to be preserved along the whole computation, or by implementing it directly onto the propagator, via causal considerations the same picture arises, namely, unphysical, gauge-dependent poles cancel out leaving solely physical ones\(^9\);\(^10\). Actually, the procedure corresponds exactly to taking out the zero-frequency mode from the Fourier series expansion for the field operators, ensuring that only positive energy quanta propagate into the future and vice versa. Mathematically this can be accomplished via

\[

\text{PV} \left( \frac{1}{q \cdot n} \right) \rightarrow \frac{1}{2} \lim_{\epsilon \to 0} \left( \frac{1}{q \cdot n + i\epsilon} + \frac{1}{q \cdot n - i\epsilon} \right). \tag{2.4}
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\[
\frac{1}{(q \cdot n)^j} = PV \left( \frac{1}{(q \cdot n)^j} \right) - i\pi \frac{(-1)^{j-1}}{(j-1)!} \delta^{(j-1)}(q \cdot n) \ sgn(q^0), \tag{2.6}
\]

where \(PV\) stands for Cauchy principal value. The second term of this expression is crucial. It is the term that guarantees that the troublesome zero-frequency mode of the field quanta is subtracted out from the energy spectrum. It is exactly tantamount to avoiding the pathological mixing of quanta of opposite energy signs propagating into the future, thus violating causality.

By the very nature in which these prescriptions are introduced, they are completely powerless to deal with products of light-cone poles without ever using the partial fractioning trick. Moreover, in addition to the need to use the “decomposition formula”, the necessity to resort to prescriptions or even clever maneuvers in order to extract out the culprit factor from the original integral is an awkward trail to walk along to reach our objective. Because a prescription is solely a prescription, a mere device to by-pass a problem, prescriptions are usually inconvenient and undesirable. Yet, as awkward and burdensome as it might be, there was no other way we could do things properly in dealing with perturbative light-cone gauge computations.

§3. Negative dimensional approach

Not so now, with the advent of NDIM\(^{11}\). NDIM is a technique wherein the principle of analytic continuation plays a key role. With it we solve a “Feynman-like” polynomial fermionic integral, i.e., a loop integral in negative \(D\)-dimensional space with propagators raised to positive powers in the numerator. Solutions arise as linear combinations of solutions for simple systems of linear equations and these are then analytically continued to allow for negative values of exponents (i.e. propagators now become raised to negative powers, becoming denominator terms in the integrands) and positive dimension\(^{12}\). This procedure, of course, when applied to the light-cone case has to consider the general structure (1.2) above which is characteristic of this gauge. That is, terms like \(f(k \cdot n^*, p \cdot n^*) \sim (q \cdot n)^a [(q-p) \cdot n]^b\) which can appear in the numerator, evidently must remain there; therefore, no exponents of such terms are to be analytically continued to allow for negative values\(^{13}\). Schematically (see for instance second reference in\(^{12}\)),

\[
\int d^Dk_i \ A(k_j, p_l) f(k_j n^*, p_m n^*) h(k_j n, p_m n) \Delta C_i \int \frac{d^Dk_i}{A(k_j, p_l)} \ f(k_j \cdot n^*, p_l \cdot n^*) \ h(k_j \cdot n, p_l \cdot n) \tag{3.1}
\]

where the left-hand side shows the negative dimensional integral to be performed and the right-hand side displays the generic form (1.2) for the light-cone integrals. Note that the factor \(f(k_j \cdot n^*, p_l \cdot n^*)\) remains in the numerator of the integrands in both sides. The left-hand side of (3.1), which is evaluated via NDIM methodology, is defined from projecting out powers of Gaussian type \(D\)-dimensional momentum integrals of propagators\(^{11}\). It is worth mentioning here that the usual Schwinger exponentiation of propagators for the factor \([h(k_j \cdot n, p_l \cdot n)]^{-1}\) in positive dimensional calculation does not work for light-cone integrals\(^{10}\).
Now, one could rightfully ask: how can one get the standard ML results for one and two loop ML dimensionally regulated Feynman-integrals with the procedure of negative dimensions? Substitute (1.3) in (1.2), that is, consider the integral,

\[
B(i, j, l, m) = \int dDq \, (q^2)^i \left((q - p)^2\right)^j \left((q \cdot n)^i \left((q \cdot n^*)^m\right). \tag{3.2}
\]

Let us evaluate it in great detail following the steps thoroughly described in our previous papers\textsuperscript{12). Let our starting point be,

\[
G = \int dDq \exp \left[ -\alpha q^2 - \beta (q - p)^2 - \gamma q \cdot n - \theta q \cdot n^* \right] = \left(\frac{\pi}{\lambda}\right)^{D/2} \exp \left[ -\frac{1}{\lambda} \left( \alpha \beta p^2 + \beta \gamma p \cdot n + \beta \theta p \cdot n^* - \frac{1}{2} \gamma \theta n \cdot n^* \right) \right], \tag{3.3}
\]

where \(\lambda = \alpha + \beta\). Taylor expanding the exponentials above and solving for the integral \(B(i, j, l, m)\) we obtain,

\[
B(i, j, l, m) = (\pi)^{D/2} i! j! l! m! \sum_{\{X,Y\}=0}^{\infty} \frac{(p^2)^{X_1} (p^+)^{X_2} (p^-)^{X_3}}{X_1! X_2! X_3! X_4! Y_1! Y_2!} \left(\frac{-nn^*}{2}\right)^{X_4} \times \delta_{a,i} \delta_{b,j} \delta_{c,l} \delta_{d,m} \delta_{e,\sigma}, \tag{3.4}
\]

where \(\sigma = i + j + l + m + D/2, a = X_1 + Y_1, b = X_1 + X_2 + X_3 + Y_2, c = X_2 + X_4, d = X_3 + X_4, e = X_1 + X_2 + X_3 + X_4\). This system of linear algebraic equations has six possible solutions – since the number of unknowns is bigger than the number of equations. In all six cases eliminating the deltas leaves us with one remaining sum, a hypergeometric series \(3F_2\). Three out of these six series have as its variable,

\[
z = \left(\frac{p^2 nn^*}{2p^+ p^-}\right),
\]

while the other three have as variable

\[
w = z^{-1} = \left(\frac{2p^+ p^-}{p^2 nn^*}\right).
\]

In our previous work\textsuperscript{13)}, we have considered the simplest one-loop integral in the light-cone gauge, evaluating it via NDIM method, with one- and two-degree violation of Lorentz covariance (more specifically, with \(n_\mu\) only and with both \(n_\mu\) and its dual \(n^*_\mu\)). For the first case we obtained the usual PV prescription result, whereas in the second case we obtained the Mandelstam-Leibbrandt prescription result. However, that toy integral is easily evaluated in the NDIM scheme, since there are no left over summation indices, that is, the number of linear equations in the system equals the number of unknowns and the result can be expressed just in terms of product of gamma functions. The integrals we consider here are more complicated; they generate several linearly independent and dependent solutions which need to be sorted out carefully. That is, the space of solutions spanned by base functions are not all linearly independent ones. This is characteristic of these particular cases of

\[
\text{The light-cone gauge without prescriptions} \quad 5
\]
integrals we are considering here, and this is the reason we treat them at length and in depth.

From our previous works\textsuperscript{12} we know that the Feynman integral will be represented by a linear combination of linearly independent series. Moreover, these representations are related through analytic continuation. Let us consider the representation of $B(i, j, l, m)$ in terms of $z^{-1}$,

$$B(i, j, l, m) = B_1 + B_2,$$

where

$$B_1 = (-\pi)^{D/2} \frac{\Gamma(1 + j)\Gamma(1 + l)\Gamma(1 + m)(p^2)^i(p^+)^{i+l+D/2}(p^-)^{j+m+D/2}}{\Gamma(1 + j + l + D/2)\Gamma(1 + j + m + D/2)\Gamma(1 - j - D/2)} \times \left( -\frac{nn^*}{2} \right)^{-j-D/2} 3F_2(a_1, b_1, c_1; e_1, f_1 | w),$$

and

$$B_2 = (-\pi)^{D/2} \frac{\Gamma(1 + i)\Gamma(1 + j)\Gamma(1 + m)(1 - \sigma - D/2)(p^2)^{\sigma-m}(p^-)^{-l+m}}{\Gamma(1 - j - l - D/2)\Gamma(1 - i - m - D/2)\Gamma(1 - l + m)\Gamma(1 + \sigma - m)} \times \left( -\frac{nn^*}{2} \right)^{l} 3F_2(a_2, b_2, c_2; e_2, f_2 | w),$$

where $a_1 = -i, b_1 = D/2 + j, c_1 = \sigma + D/2, e_1 = 1 + j + l + D/2, f_1 = 1 + j + m + D/2, a_2 = -l, b_2 = -i - j - l - D/2, c_2 = i + m + D/2, e_2 = 1 - l + m, f_2 = 1 - j - l - D/2$ and $3F_2$ is the generalized hypergeometric function\textsuperscript{14}. This is the result in the negative dimension region and $i, j, l, m$ positive. Now, in order to be physically meaningful it must be analytically continued to positive dimension and $i, j, l$ negative, while the exponent $m$ must be kept untouched. To carry out this analytic continuation one must rewrite the gamma factors as Pochhammer symbols and use one of its properties

$$(a|j) \equiv (a)_j = \frac{\Gamma(a + j)}{\Gamma(a)}, \quad (a| - j) = \frac{(-1)^j}{(1 - a|j)}.$$

Performing these simple operations one gets,

$$B^{AC}(i, j, l, m) = \pi^{D/2} \frac{(-j| - l - D/2)(-l|j + l + D/2)}{(1 + m|j + D/2)} (p^2)^i(p^+)^{i+l+D/2}$$

$$\times (p^-)^{j+m+D/2} \left( \frac{nn^*}{2} \right)^{-j-D/2} 3F_2(a_1, b_1, c_1; e_1, f_1 | w)$$

$$+ \pi^{D/2} \frac{(-j|i + j + m + D/2)(-i|i + j + l + D/2)}{(1 + m| - l)}$$

$$\times (\sigma + D/2) - 2\sigma - D/2 + m)(p^2)^{\sigma-m}(p^-)^{-l+m} \left( \frac{nn^*}{2} \right)^{l}$$

$$\times 3F_2(a_2, b_2, c_2; e_2, f_2 | w).$$

In fact, from the theory of hypergeometric functions\textsuperscript{14}, we know that the hypergeometric differential equation for $pF_q$ has up to $p$ linearly independent solutions,
so, in writing down $B(i, j, l, m)$ we would expect it to contain up to three terms. Indeed, among the three series of functions $3F_2(-;|w)$ there is a third term, namely,

$$B_3 = (-\pi)^{D/2} \frac{\Gamma(1 + i)\Gamma(1 + j)\Gamma(1 + l)\Gamma(1 - \sigma - D/2)(p^2)^i + j + m + D/2}{\Gamma(1 + i + j + m + D/2)\Gamma(1 + l - m)\Gamma(1 - j - m - D/2)} \times \frac{(p^+)^{l-m}(\frac{-nn^*}{2})^m}{\Gamma(1 - i - l - D/2)} \left[ 3F_2(a_3, b_3, c_3; e_3, f_3|w) \right], \quad (3.10)$$

where $a_3 = -i - j - m - D/2$, $b_3 = -m$, $c_3 = i + l + D/2$, $e_3 = 1 + l - m$, $f_3 = 1 - j - m - D/2$. However, this solution is not a linearly independent one. Indeed, from the fact that $m$ is always positive or zero we can rewrite $B_3$ using eq.(6), section 3.2 of ref.\textsuperscript{14} — if we choose $a_1 = c_3$, $a_2 = a_3$, $n = -e_3$, $p_1 = f_3$ — as a linear combination of $B_1$ and $B_2$.

The integral we are considering here was studied by Lee and Milgram\textsuperscript{15}. Our result can also be written in terms of the so-called $G$–function or Meijer’s function and agrees with their calculation. Taking the special case where $i = j = l = -1$ and $m = 0$ one gets,

$$B(-1, -1, -1, 0) = -(-\pi)^{D/2} \frac{\Gamma(2 - D/2)\Gamma(D/2 - 1) (p^+)^{D/2-2} (p^-)^{D/2-1}}{\Gamma(D/2)} \times (\frac{-2}{nn^*})^{D/2-1} 2F_1(1, D - 3; D/2|w) - \pi^{D/2}\Gamma(D/2 - 1) \times \frac{\Gamma(D/2 - 2)\Gamma(3 - D/2)}{\Gamma(D - 3)} (p^2)^{D/2-3} p^- (\frac{-2}{nn^*}) \times 2F_1(1, D/2 - 1; 2|w). \quad (3.11)$$

Observe that when $D = 4 - 2\epsilon$ there are simple poles in these two terms but they cancel since,

$$\lim_{\epsilon \to 0} \Gamma(\epsilon) + \Gamma(-\epsilon) = O(1).$$

To extract the finite part one can proceed in the same way as we did in the second paper of ref.\textsuperscript{12}. The other representation for the integral $B(i, j, l, m)$ also coincide with that presented in\textsuperscript{15}. In fact, they are related through analytic continuation (see ref.\textsuperscript{14} for such formulas) — as all representations provided by NDIM for Feynman integrals in covariant\textsuperscript{12} and non-covariant gauges\textsuperscript{13}.

Consider now an integral containing two factors of the form $(k \cdot n)$,

$$T_2(i, j, l, m) = \int d^D q (q - p)^{2k}(q \cdot n)^j [(q - p) \cdot n]^{l} (q \cdot n^*)^m, \quad (3.12)$$

where the exponent $m \geq 0$. The standard approach makes use of decomposition formulas (2.2) and (2.3). On the other hand, NDIM can solve all integrals of such kind at the same time, giving in positive dimension\textsuperscript{17},

$$T_2(i, j, l, m) = \pi^{D/2} \left( \frac{2p^+ p^-}{n \cdot n^*} \right)^{D/2+i} \frac{\Gamma(i + l + D/2)\Gamma(1 + m)}{(p^+)^{-j-l}(p^-)^{-m}\Gamma(-i)\Gamma(-j)}.$$
\[ \times \frac{\Gamma(-i - j - l - D/2)}{\Gamma(1 + i + m + D/2)}. \] (3.13)

observe that the Pochhammer symbol which contain \((1 + m)\) was not analytically continued since \(m\) must be either positive or zero.

### §4. Conclusion

Finally, in concluding this work we would like to emphasize some important points concerning the application of NDIM technique to light-cone integrals. First of all, why one needs to define the original Gaussian integral (3.3) with two-degree violation of Lorentz covariance in order to get “causality” preserving results? The answer is related to the very definition of light-cone gauge condition where one has the external four-vector \(n_\mu\) such that it is chosen to be light-like, i.e., \(n^2 = 0\). However, the “light-likeness” condition \(n^2 = 0\) does not uniquely define the needed external vector \(n_\mu\) to implement the gauge condition. The reason for this is more easily seen considering a particular case where \(n_\mu = (n_0, n_3, 0, 0)\), in which case the condition \(n^2 = 0\) gives as solutions either \(n_0 = +n_3\), or \(n_0 = -n_3\) with \(n_0 > 0\). Therefore, the components of the “light-like” vector are not linearly independent; hence the two possibilities: either \(n_\mu \equiv (n_0, +n_3, 0, 0)\) or \(n^*_\mu \equiv (n_0, -n_3, 0, 0)\).

These peculiar light-cone properties have been shown by G. Leibbrandt\(^{16}\) to be connected to the Newman-Penrose tetrad formalism in the context of gravitation and cosmology, where a four-dimensional basis is spanned entirely by null vectors. In his work, Leibbrandt demonstrated that the two-dimensional vector sub-space \((n_0, n_3)\) cannot be spanned solely by the vector \(n_\mu\); it is not sufficient for this purpose because this vector possesses linearly dependent components. A thorough discussion, including not only the sub-space \((n_0, n_3)\), but the entire four-dimensional space, is found in the reference above given.

This is the reason why our NDIM calculation done in\(^{13}\) with only one degree violation of Lorentz covariance failed to be “causal”, reproducing the well-known “causality” breaking PV result. The cure for this pathological result could only be achieved by introducing the needed additional \(n^*_\mu\) vector, so that the basis for spanning the entire four-dimensional vector space be unique and well-defined. Therefore our calculations here always included this needed dual vector \(n^*_\mu\), so that our results are well-defined and unambiguous.

This NDIM technique is therefore the most powerful and beautiful machinery ever to come to front until now to handle the light-cone Feynman integrals. All the desirable features are embedded in it and we can list them as follows: \(i)\) No “decomposition formula” like (2.2) is needed to separate gauge dependent poles before the integral is evaluated; \(ii)\) It preserves the general structure of the light-cone integral, namely, no factors containing \(n^*\) are introduced in the denominators; \(iii)\) No prescription is needed to handle \((q \cdot n)^{-\alpha} = 0\) type singularities to solve the integral; \(iv)\) There are no parametric integrals to perform; \(v)\) There is no need to split the dimensionality of space-time to work out integrations over component sub-spaces like in\(^2\); \(vi)\) Results are obtained for arbitrary negative exponents of propagators and
The light-cone gauge without prescriptions

gauge-dependent poles, so that special cases – which agree \(^1\), \(^4\), \(^13\), \(^17\) with the ones calculated using ML prescription – are contained in them; \(^vii\) Results are always within the context of dimensional regularization, i.e., preserving gauge symmetry.

The power of NDIM is therefore readily apparent. If one remembers how integrals over Grassmann variables are introduced, he/she can recall that the underlying property employed to define them is translation invariance. It is an outstanding thing that this sole property in the fermionic integration is able to guarantee a prescription-less light-cone and more important than that, a causality preserving method of direct computation of Feynman integrals.

Of course, like in any gauge, two-loop integrals are more demanding when compared to the one-loop case, and specially in the light-cone gauge. Preliminary results we have show that NDIM can also handle them with more easiness than in the usual standard approaches.

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References

1) G.Leibbrandt, Rev.Mod.Phys. 59 (1987) 1067. G.Leibbrandt, Non-covariant gauges: Quantization of Yang-Mills and Chern-Simons theory in axial type gauges, World Scientific (1994).
2) G.Leibbrandt, S-L.Nyeo, J.Math.Phys. 27 (1986) 627.
3) G.Leibbrandt, S-L.Nyeo, Z.Phys.C30 (1986) 501.
4) A.Bassetto, G.Nardelli, R.Soldati, Yang-Mills theories in algebraic non-covarinat gauges, World Scientific (1991)
5) A.Bassetto, in Lecture notes in Physics, 61, P.Gaigg, W.Kummer, M.Schweda (Eds.), Springer-Verlag (1989).
6) S.Mandelstam, Nucl.Phys.B 213 (1983) 149.
7) G.Leibbrandt, Phys.Rev.D 29 (1984) 1699.
8) D.M.Capper, D.R.T.Jones, and A.T.Suzuki, Z.Phys. C29 (1985) 585. G.Heinrich, and Z.Kunszt, Nucl.Phys. B519 (1998) 405. A.Basseto, G.Heinrich, Z.Kunszt, and W.Vogelsang, Phys.Rev. D58 (1998) 094020.
9) B.M.Pimentel, A.T.Suzuki, Phys.Rev. D42 (1990) 2115. C.G.Bollini, J.J.Giambiagi, A.González Dominguez, J.Math.Phys. 6 (1965) 165. B.M.Pimentel, A.T.Suzuki, Mod. Phys. Lett. A6 (1991) 2649.
10) A.T.Suzuki, Mod.Phys.Lett. A8 (1993) 2365.
11) I.G.Halliday, R.M.Ricotta, Phys.Lett. B193 (1987) 241.
12) A.T.Suzuki, A.G.M.Schmidt, Eur.Phys.J.C5 (1998) 175. A.T.Suzuki, A.G.M.Schmidt, J.Phys.A31 (1998) 8023. A.T.Suzuki, A.G.M.Schmidt, JHEP 09 (1997) 002. A.T.Suzuki, A.G.M.Schmidt, Phys.Rev.D58 (1998) 047701.
13) A.T.Suzuki, A.G.M.Schmidt, R.Bentín, Nucl.Phys. B537 (1999) 549.
14) Y.L.Luke, The special functions and their approximations, Vol.I, (Academic Press, 1969).
15) H.C.Lee, M.S.Milgram, J.Comp.Phys. 71 (1987) 316.
16) G.Leibbrandt, Phys.Rev.D30. (1984) 2167; Can.J.Phys. 64 (1986) 606; E.T.Newman, R.Penrose, J.Math.Phys.3 (1962) 566; J.Math.Phys.4 (1963) 998.
17) A.T.Suzuki, A.G.M.Schmidt, Eur.Phys.J.C12 (2000) 361.