Modeling topic dependencies in semantically coherent text spans with copulas

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Abstract

The exchangeability assumption in topic models like Latent Dirichlet Allocation (LDA) often results in inferring inconsistent topics for the words of text spans like noun-phrases, which are usually expected to be topically coherent. We propose copulaLDA, that extends LDA by integrating part of the text structure to the model and relaxes the conditional independence assumption between the word-specific latent topics given the per-document topic distributions. To this end, we assume that the words of text spans like noun-phrases are topically bound and we model this dependence with copulas. We demonstrate empirically the effectiveness of copulaLDA on both intrinsic and extrinsic evaluation tasks on several publicly available corpora.

1 Introduction

Probabilistic topic models, such as Latent Dirichlet Allocation (LDA) (Blei et al., 2003), are generative models that describe the content of documents by discovering the latent topics underlying them.

A limitation inherent from the bag-of-words representation in such state-of-the-art models concerns the independence assumption: given their topics, words are assumed to occur independently. While this exchangeability assumption greatly impacts the involved computations and, in particular, the calculations of the conditional probabilities, it is rather naive and unrealistic (Heinrich, 2005). As another limitation caused by the exchangeability assumption, the grouping of words in topically coherent spans, that is contiguous text spans like sentences, is lost.

On the other hand, text structure generally contains useful information that could be leveraged in inference process. Sentences or phrases, for instance, are by definition text spans complete in themselves that convey a concise statement. To better illustrate how text structure could help in topic identification, consider the example of Figure 1. It illustrates the topics inferred by LDA for the words (excluding stop-words) of a sentence drawn from a Wikipedia page. At the sentence level, one could argue that the sentence is generated by the “Cinema” topic since it discusses a film and its authors. LDA, however, fails and assigns several topics to the words of the sentence. Importantly, several of those topics like “Elections” and “Inventions” are unrelated. In finer text granularity, LDA also fails to assign consistent topics in noun-phrases like “film noir classic” and entities like “Brian Donlevy”. A binding mechanism among the topics of the words of a sentence, or a phrase, could have prevented those limitations and taking simple text structure into account would be beneficial.

Motivated by the previous example, we propose to incorporate text structure in the form of sentence or phrase boundaries as an intermediate structure in LDA. We plan to model this binding mechanism with copulas. Copulas have been found to be a flexible tool to model dependencies in the fields of

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Figure 1: Applying LDA on Wikipedia documents.
risk management and finance (Embrechts et al., 2002). They are a family of distribution functions that offer a flexible way to model the joint probability of random variables using only their marginals. This results in decoupling the marginal distributions by the underlying dependency. These properties make them appealing and some preliminary studies have started investigating their integration into different learning tasks (Wilson and Ghahramani, 2010; Tran et al., 2015; Amoualian et al., 2016).

The remainder of the paper is organized as follows: Section 2 presents the related work. The main contribution of this article is presented in Section 3, in which we propose to bind the latent topics that generate the words of a segment using copulas. We show that sampling word topics from copulas offers an elegant way to impose different levels and types of correlation between them. Section 4 then illustrates the behavior of copulaLDA, the copula-based version of LDA introduced in Section 3, while Section 5 concludes the paper.

2 Related Work

Despite the success that vector-space models (Salton et al., 1975) have enjoyed, they come with a number of limitations. We mention, for instance, their inability to model synonymy and polysemy and the sparse, high-dimensional induced representations. Many research studies have researched these problems, and Probabilistic Latent Semantic Analysis (Hofmann, 1999) was among the first attempts to model textual corpora using latent topics. In our work, we build on LDA (Blei et al., 2003), which is often used as a building block for topic models. In its context, the corpus is associated with a set of latent topics, and each document is associated with a random mixture of those topics. The words are assumed exchangeable, that is their joint probability is invariant to their permutation. Previous work proposed a variety of extensions to LDA in order to incorporate additional information such as class labels (Blei and McAuliffe, 2008) and temporal dependencies between stream documents (Wang et al., 2012). Here, our goal is to extend LDA by incorporating simple text structure in its generative and inference processes using copulas.

One may identify two lines of research to address the limitations due to the exchangeability assumption in LDA: extensions to account for the boundaries of text spans like sentences and extensions to account for the word order. With respect to the first line, (Wang et al., 2009) combine a unigram language model with topic models over sentences so that the latent topics are represented by sentences instead of terms. In (Griffiths et al., 2004), the authors investigate a combination of a topic model with a Hidden Markov Model (HMM). They assume that the HMM generates the words that handle the long-range dependencies (semantic dependencies) and the topic model the words that handle the short range dependencies (syntactic dependencies). Also, (Boyd-Graber and Blei, 2009) proposed the Syntactic Topic Model whose goal is to integrate the text semantics and the syntax in a non-parametric topic model. In another effort, (Zhu et al., 2006) propose TagLDA, where they replace the unigram word distributions by a factored representation that is conditioned on the topic and the part-of-speech tag of a term. Recently, (Balikas et al., 2016) introduced senLDA, that assumes that the terms occurring within a sentence are generated by the same topic. In our work here, we integrate part of the text structure in LDA by relying only on the boundaries of contiguous text spans like sentences, which can be obtained without deep linguistic analysis like the one required in the Syntactic Topic Model. Also, differently from senLDA, we do not restrict the words of the spans to be generated by the same topic. Instead, using copulas we pose correlations between those topics, which is more flexible.

The second line of research investigates how topic models can be extended to incorporate word order. In (Shafiei and Milios, 2006), the authors propose a four-level hierarchical structure where the latent topics of paragraphs are decided after performing a nested word-based LDA operation. In a similar context, (Wang et al., 2007) study how the word order in the form of n-grams can be leveraged to better capture a document’s topical content. Their topical n-gram model extends LDA by determining unigram words and phrases based on context and assigning mixture of topics to both individual words and n-gram phrases.

Another interesting line of research studied the task of discovering and partitioning text in topically coherent spans. In (Du et al., 2010; Du et al., 2013) the authors rely on hierarchical Bayesian models to accomplish it. In this work, contrary to identifying such spans, we assume them to be topically coherent a priori, and we investigate how to leverage and incorporate this information to LDA.
Lately, there is an increasing interest over the integration of copulas in machine learning applications (Elidan, 2013) such as classification (Elidan, 2012) or structure learning (Liu et al., 2009). Interestingly, (Wilson and Ghahramani, 2010) have shown how to incorporate copulas in Gaussian processes in order to model the dependency between random variables with arbitrary marginals with a practical application on predicting the standard deviation of variables in the financial sector (volatility estimation). In another generic framework, (Tran et al., 2015) have shown the benefits of using copulas to model complex dependencies between latent variables in the general variational inference setting. The idea of using copulas with topic models was recently investigated in (Amoualian et al., 2016). In the context of document streams they proposed a topic model where the dependencies between the topic distributions of two consecutive documents are captured by copulas.

3 Integrating text structure to LDA using copulas

In this section we develop copulaLDA (hereafter copLDA), that extends LDA by integrating simple text structure in the model using copulas. We assume that the topics that generate the terms of coherent text spans are bound. A strong binding signifies high probability for the terms to have been generated by the same topic. Therefore, as we show, the conditional independence of topics given the per-document topic distributions does not hold. Before presenting the generative and inference processes of copLDA, we shortly discuss the idea of coherent text spans.

Each sentence is a coherent, meaningful segment of text and we consider them as coherent text spans in this study. However, each sentence can be further decomposed into smaller segments through syntactic analysis. Figure 2 illustrates the output of a shallow parsing step of the example sentence of Figure 1, generated using the Stanford Parser.1 Among these different segments, noun phrases play a particular role as they are, for instance, at the basis of terminology extraction that aims at capturing concepts from a document. Noun phrases usually constitute a semantic unit, pertaining to a given concept related to few, related topics. For this reason, we also consider noun phrases as coherent text spans in this study. Another advantage of the two types of coherent text spans we consider (whole sentences and noun phrases) is that they can be easily extracted using shallow parsing techniques, and one needs not resort to complex syntactic analysis in practice.

3.1 Copulas and random variables

Copulas are interesting because they separate the dependency structure of random variables from their marginals. Formally (Nelsen, 2007; Trivedi and Zimmer, 2007), a \( p \)-dimensional copula \( C \) is a \( p \)-variate distribution function with \( C : [0, 1]^p \rightarrow [0, 1] \) whose univariate marginals are uniformly distributed on \( I \) and \( C(u_1, \ldots, u_p) = P(U_1 \leq u_1, \ldots, U_p \leq u_p) \). Copulas allow one to explicitly relate joint and marginal distributions, through Sklar’s theorem (Sklar, 1959):

**Theorem 3.1** Let \( F \) be a \( p \)-dimensional distribution function with univariate margins \( F_1, \ldots, F_p \). Let \( A_j \) denote the range of \( F_j \). Then there exists a copula \( C \) such that for all \( (x_1, \ldots, x_p) \in \mathbb{R}^p \)

\[
F(x_1, \ldots, x_p) = C(F_1(x_1), \ldots, F_p(x_p))
\]

(1)

Furthermore, when \( F_1, \ldots, F_p \) are all continuous, then \( C \) is unique.

As a result any multivariate distribution \( F \) can be decomposed into its marginals \( F_i, \ i \in \{1, \ldots, p\} \) and a copula, allowing to study the multivariate distribution independently of the marginals. Sklar’s theorem also provides a way of sampling multivariate distributions with a large number of random variables using copulas: \( F(x_1, \ldots, x_p) = F(F_1^{-1}(u_1), \ldots, F_p^{-1}(u_p)) = P[U_1 \leq u_1, \ldots, U_p \leq u_p] = C(u_1, \ldots, u_p) \). Hence, to sample \( F \) it suffices to sample the dependence structure modeled by copulas and then transform

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1http://nlp.stanford.edu/software/lex-parser.shtml
Figure 3: The transformation of a random vari-
te to multinomial (or arbitrary) marginals. The
arrows illustrate the generalized inverse; the his-
tograms in y (resp. x) axis depict the distribu-
tions of the initial (resp. transformed) samples.

Figure 4: The positive correlation imposed to
two random variates when sampling from a
Frank copula with $\lambda = 25$. The histograms
in x (resp. y) axis show the distributions of
each of the variates that generate the scatterplot.

the obtained sample in the marginals of interest using the probabilistic integral transform. We illustrate this transformation for one variable in Figure 3. Sampling the copula returns, for each variate, a sample as the one indicated in the histogram of the y axis. One can then transform the sample using the quantile ($F^{-1}$) of an arbitrary marginal.

Before proceeding further, we visit some extreme conditions of dependence illustrating the respective copulas that model them: (1) Independence, which is a frequently assumed simplification in topic models and is obtained with $\prod_{i=1}^{p} u_i$, and (2) Co-monotonicity, which is the complete, positive correlation between the random variables $u_p$, obtained with $\min(u_1, \ldots, u_p)$.

In the rest of our development we will be using a particular family of copulas, the Archimedean copulas. Archimedean copulas are widely used copulas and are defined with respect to a generator function $\psi$. They take the form: $C(u_1, \ldots, u_d) = \psi^{-1}(\psi(u_1) + \cdots + \psi(u_d))$. A special case of Archimedean copulas corresponds to Frank copulas, which are obtained by setting: $\psi_\lambda(u) = \frac{1}{\lambda} \log(1 - (1 - e^{-\lambda})e^{-u})$.

When $\lambda \to 0$, the Frank copula approaches the independency copula; when $\lambda \to \infty$ it approaches the co-monotonicity copula. Hence, the Frank copula allows one to model all dependencies between complete independence to perfect dependence while varying $\lambda$ from 0 to $\infty$. Therefore, $\lambda$ can be seen as an additional hyper-parameter to be tuned or learned from the data. Figure 4 illustrates the positive dependence between two random variables sampled from a Frank copula with $\lambda = 25$. To sample from the Archimedean copulas, we rely on the algorithm proposed by (Marshall and Olkin, 1988), which was further improved in (McNeil, 2008; Hofert, 2011) and implemented in the R language (Hofert et al., 2011).

3.2 Extending LDA with copulas

As mentioned above, copulas provide a nice way to bind random variables. We are making use of them here to bind word-specific topics (the $z$ variables in LDA) within coherent text spans, the rationale being that coherent text spans cannot be generated by many different, uncorrelated topics. This leads us to the following generative model:

- For each topic $k \in [1, K]$, choose a per-word distribution: $\phi_k \sim \text{Dir}(\beta)$, with $\phi_k, \beta \in \mathbb{R}^{|V|}$

- For each document $d_i, i \in \{1, \ldots, D\}$:
  - Choose a per-document topic distribution: $\theta_i \sim \text{Dir}(\alpha)$, with $\theta_i, \alpha \in \mathbb{R}^{|K|}$
  - Sample number of segments in $d_i$: $S_i \sim \text{Poisson}(\xi)$
  - For each segment $s_{i,j}, j \in \{1, \ldots, S_i\}$:
* Sample number of words: \( N_{i,j} \sim \text{Poisson}(\xi_d) \);
* Sample topics \( Z_{i,j} = (z_{i,j,1}, \ldots, z_{i,j,N_{i,j}}) \) from a distribution admitting \( \text{Mult}(1, \theta_i) \) as margins and \( C \) as copula;
* Sample words \( W_{i,j} = (w_{i,j,1}, \ldots, w_{i,j,N_{i,j}}) \): \( w_{i,j,n} \sim \text{Mult}(1, \phi_{z_{i,j,n}}) \), \( 1 \leq n \leq N_{i,j} \).

There are two main differences between \( \text{copLDA} \) and LDA. Firstly, the former assumes a hierarchical structure in the documents: the topics that generate the words in the coherent segments exhibit topical correlation, hence the conditional independence assumption between the terms of a segment given the document per-topic distribution \( (\theta_i) \) no longer holds. Secondly, this topical correlation is modeled using copulas. Figure 5 provides the graphical model for \( \text{copLDA} \). For clarity, we draw each word in a coherent segment \( S \) \( (w_1, \ldots, w_N) \) to make the dependencies explicit. Notice how the topics of those words depend on both the copula parameter \( \lambda \) and the per-document topic distribution \( \theta \).

The hyper-parameters \( \alpha \) and \( \beta \) correspond to priors of the model. Following \cite{Blei2003}, we assume them here to be symmetric and we fix them to \( \frac{1}{K} \), with \( K \) the number of topics retained. The hyper-parameter \( \lambda \) is chosen after exploration of a grid of possible values, and is the same for the whole corpus. We choose the value that minimizes perplexity.

### 3.3 Inference with Gibbs sampling

The parameters of the above model, that are \( \phi, \theta \) and the topics of each segment \( Z_{i,j} = (z_{i,j,1}, \ldots, z_{i,j,N_{i,j}}) \), can be directly estimated through Gibbs sampling. Denoting \( \Omega \) and \( \Psi \) the count matrices such that \( \Omega = (\Omega_{i,k}) \) (resp. \( \Psi = (\Psi_{k,v}) \)) represents the count of word belonging to topic \( k \) assigned to document \( d_i \) (resp. the count of word \( v \) being assigned to topic \( k \)), the Gibbs updates for \( \theta \) and \( \phi \) are the same as the ones for the standard LDA model \cite{Blei2003}:

\[
\theta_i \sim \text{Dir}(\alpha + \Omega_i) \quad \text{and} \quad \phi_k \sim \text{Dir}(\beta + \Psi_k)
\]  

(2)

The update for the variables \( z \) is obtained as follows:

\[
\begin{align*}
    p(Z_{i,j} | Z_{-i,j}, \Theta, \Phi, \alpha, \beta, \lambda) &= p(Z_{i,j}, Z_{-i,j}, W_{i,j} | \Theta, \Phi, \alpha, \beta, \lambda) \\
    p(Z_{i,j}, W_{i,j} | Z_{-i,j}, \Theta, \Phi, \lambda)p(Z_{-i,j}, W_{-i,j} | \Theta, \Phi, \lambda) &= p(Z_{-i,j}, W_{-i,j} | \Theta, \Phi, \lambda) \\
    p(Z_{i,j}, W_{i,j} | Z_{-i,j}, \Theta, \Phi, \lambda) &= \sum_{Z_{i,j}} p(Z_{i,j}, W_{i,j} | \Theta, \Phi, \lambda) \\
    p(W_{i,j} | Z_{i,j}, \Theta, \Phi)p(Z_{i,j} | \Theta, \lambda) &= \sum_{Z_{i,j}} p(W_{i,j} | Z_{i,j}, \Theta, \Phi)p(Z_{i,j} | \Theta, \lambda) \\
    p(W_{i,j} | Z_{i,j}, \Phi)p(Z_{i,j} | \Theta, \lambda) &= \sum_{Z_{i,j}} p(W_{i,j} | Z_{i,j}, \Phi)p(Z_{i,j} | \Theta, \lambda)
\end{align*}
\]  

(3)

where \( W, \Theta \) and \( \Phi \) stand for the whole parameter set of \( w, \theta \) and \( \phi \) and the probability outside the product in the last step admits a copula \( C_{\lambda} \) and \( \text{Mult}(1, \theta_i) \) as margins. As is standard in topic models, the notation \( \neg i,j \) means excluding the information for \( i,j \). Note that in case where \( \lambda \to 0 \), the words of a segment become conditionally independent given the per-document distribution and one recovers the non collapsed Gibbs sampling updates of LDA.

From the expression of Eq. (3), a simple acceptance/rejection algorithm can be formulated: (1) Sample a random variable of pdf \( p(Z_{i,j} | \Theta, \lambda) \) using copula, and (2) Accept the sample with probability \( p(W_{i,j} | Z_{i,j}, \Phi) = \prod_{n=1}^{N_{i,j}} \phi_{w_{i,j,n}, z_{i,j,n}} \). Algorithm 1 summarizes the inference process.
3.4 Computational Considerations

As the values of $\phi_{w_{i,j},n,z_{i,j}} \times \cdots \times \phi_{w_{i,j},n,z_{i,j}}$ tend to be very low, the acceptance/rejection sampling step described above is very slow in practice (see below). We propose here to speed it up by considering, for each word $w_{i,j,n}$ in a given segment, not the exact probability of $z_{i,j,n}$, but its mean (noted $M$) over all the other words in the segment:

$$M(z_{i,j,n}\mid Z_{-i,j}, W, \Theta, \Phi, \alpha, \beta, \lambda) = \sum_{w_{i,j,l} \neq n} \sum_{z_{i,j,l} \neq n} P(Z_{i,j}\mid Z_{-i,j}, W, \Theta, \Phi, \alpha, \beta, \lambda) \propto \phi_{w_{i,j,n}} \theta_{d,z_{i,j,n}}$$

as $\sum_{w_{i,j,l}} \phi_{w_{i,j,l}} = 1$. Note that the above form is a marginalization of $P(Z_{i,j}\mid Z_{-i,j}, W, \Theta, \Phi, \alpha, \beta, \lambda)$ and thus defines a valid probability and a valid Gibbs sampler, even though on a joint distribution that slightly differs from the original one.

Figure 6 compares the perplexity scores achieved in 200 documents from the Wikipedia dataset “Wiki46” of Table 1 by the copLDA model, when considering noun-phrases as coherent spans, with and without rejection sampling. We repeat the experiment 10 times and also plot the standard deviation. We first note that approximating Algorithm 1 by ignoring the rejection sampling step results in slightly worse performance. On the other hand, without the rejection sampling, copLDA converges faster in terms of iterations. Furthermore, the cost in terms of running time of a single iteration is significantly smaller: for instance, for 30 iterations with rejection sampling, Algorithm 1 by ignoring the rejection sampling, copLDA converges faster in terms of running time of a single iteration.

**Algorithm 1: A Gibbs Sampling iteration for copLDA**

| Input: documents’ words grouped in segments, $\alpha, \beta, K$, Copula family and its parameter $\lambda$ |
| //Initialize counters $\Psi, \Omega$ |
| **for** document $d, i \in [1, D]$ **do** |
| **for** segment $s_{i,j} : j \in \{1, \ldots, S_i\}$ **do** |
| Draw a random vector $U = (U_1, \ldots, U_{N_{i,j}})$ that admits a copula $C_\lambda$ |
| **do** /* If the mean approximation is used, the loop is done once, ignoring the acceptance condition */ |
| **for** words $w_{i,j,k}, k \in [1, W_{N_{i,j}}]$ in $s_{i,j}$ **do** |
| Decrease counter variables $\Psi, \Omega$ |
| Get $z_{i,j,k}$ by transforming $U_k$ to Mult. marginals with the generalized inverse |
| Assign topic $z_{i,j,k}$ to $w_{i,j,k}$ |
| Increase counters $\Psi, \Omega$ |
| **end** |
| **while** Accept the new segment topic assignments with probability $\phi_{w_{i,j,n},z_{i,j}} \times \cdots \times \phi_{w_{i,j,n},z_{i,j,n}}$ |
| **end** |

4 Experimental study

**Models** In our experiments, we compare the following topic models: (1) copLDA$_{sen}$ that considers sentences as coherent segments, (2) copLDA$_{np}$ that considers noun-phrases as coherent segments, (3) LDA as proposed in (Blei et al., 2003) using the collapsed Gibbs sampling inference of (Griffiths and Steyvers, 2004), and (4) senLDA described in (Balikas et al., 2016) using its public implementation. For copLDA$_{a}$ models, we use the Frank copula which was reported to obtain the best performance in similar tasks (Amoualian et al., 2016) and was also found to achieve the best performance in our local validation settings compared to Gumbel and Clayton copulas. We have implemented the models using Python; for sampling the Frank copulas we used the R copula package (Hofert et al., 2011) and rPY. As mentioned in Section 3.2, $\lambda$ is set to 2 for copLDA$_{sen}$ and to 5 for copLDA$_{np}$ (values which we found to perform well in every dataset we tried). Furthermore, the hyper-parameters $\alpha$ and $\beta$ where set to $1/K$, where $K$ is the number of topics, which was selected from $\{50, 100, 200, 300, 400\}$ for each dataset. For the shallow parsing step, required for copLDA$_{np}$, we used the Stanford Parser (Klein and Manning, 2003). The text pre-processing steps performed are: lower-casing, stemming using the Snowball Stemmer and removal of numeric strings.

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2The models used in this paper are available for research purposes at [https://github.com/balikasg/topicModelling](https://github.com/balikasg/topicModelling)

3[https://pypi.python.org/pypi/rpy2](https://pypi.python.org/pypi/rpy2)
Datasets We have used the following publicly available data collections to test the performance of the topic models: (1) 20NG (20 news groups), which is a standard text dataset for such tasks as provided by (Bird et al., 2009), (2) Reuters (Reuters-21578, the “ModApte” version), also discussed in (Bird et al., 2009), (3) TED, that is transcriptions of TED talks released in the framework of the International Workshop on Spoken Language Translation 2013 evaluation campaign⁴ (we have merged the train, development and test parts and we selected the transcriptions with at least one associated label among the 15 most common in the data), (4) Wikiₓ, with x ∈ {15, 37, 46} and PubMed, both excerpts⁵ from the Wikipedia dataset of (Partalas et al., 2015) and the PubMed dataset of (Tsatsaronis et al., 2015) used in (Balikas et al., 2016), and (5) “Austen”, where we concatenated three books written by Jane Austen, available from the Gutenberg project (each paragraph is considered as a document). Table 1 presents some basic statistics for these datasets.

Manual inspection of the topics We begin by comparing LDA and copLDAₓ. For presentation purposes, we train the two topic models using the Wiki₁₇ dataset with 10 topics and we illustrate the top-10 words learned for each topic by the two models in Table 2. As one can note, since the two models have been trained on the same data with the same training parameters, the identified topics are very similar. This said, copLDAₓ manages to produce arguably better topics. This is for example the case for the topic “Birth”; although both models assign high probability to words like “born” and “american” due to the content of the dataset, copLDAₓ manages to identify several words corresponding to months which makes the topic more thematically consistent and easier to interpret compared to its LDA counterpart. In the same line, Table 3 visualizes the inferred topics for parts of the Wiki₁₇ dataset. Notice here that given the topic interpretations of Table 2, both models manage to identify intuitive topics. Note however how in most of the cases the text structure information used by copLDAₓ helps to obtain consistent topics to generate noun-phrases like “crime thriller film” and “raspy voice”, a consistency that LDA is lacking.

Intrinsic evaluation: perplexity We present in Table 1 the perplexity scores achieved by the 4 models in

### Table 1: The basic statistics, the perplexity and the classification scores of the datasets used.

| Datasets  | Basic Statistics | Perplexity Scores | Classification (MiF₁) scores |
|-----------|------------------|-------------------|-----------------------------|
|           | Docs. | [N] | [V] | Classes | senLDA | copLDAₓ | LDA | copLDAₓ | senLDA | copLDAₓ | LDA | copLDAₓ |
| 20NG      | 19,056 | 1.7M | 54.4K | 20 | 2636 | 2083 | 2200 | 1483 | 0.5622 | 0.6328 | 0.6246 | 0.6490 |
| TED       | 1,096  | 1.16M | 30.4K | 15 | 2099 | 1812 | 1805 | 1758 | 0.4612 | 0.4768 | 0.4633 | 0.4764 |
| PubMed    | 5,498  | 1.09M | 28.7K | 50 | 1601 | 1385 | 1384 | 1058 | 0.6666 | 0.7425 | 0.7146 | 0.7431 |
| Reuters   | 10,788 | 875K  | 21.4K | 90 | 579  | 512  | 501  | 499  | 0.7504 | 0.7692 | 0.7893 | 0.7851 |
| Wiki15    | 1,198  | 162K  | 13.4K | 15 | 2988 | 2766 | 2640 | 2397 | 0.6920 | 0.7230 | 0.74 | 0.7403 |
| Wiki37    | 2,459  | 371K  | 19.7K | 37 | 3103 | 2871 | 2711 | 2395 | 0.5717 | 0.6483 | 0.6447 | 0.6520 |
| Wiki46    | 3,657  | 478K  | 23.4K | 46 | 2220 | 2280 | 2135 | 1978 | 0.5352 | 0.6170 | 0.6599 | 0.6326 |
| Austen    | 5,262  | 170K  | 6.3K  | - | 1110 | 898  | 798  | 805  | - | - | - | - |

⁴ [http://workshop2013.iwslt.org/59.php](http://workshop2013.iwslt.org/59.php)
⁵ Technology, Culture, Science, Global Issues, Design, Business, Entertainment, Arts, Politics, Education, Art, Creativity, Health, Biology and Music.
⁶ [https://github.com/balikasg/topicModelling/tree/master/data](https://github.com/balikasg/topicModelling/tree/master/data)
⁷ We used the books: Emma, Persuasion, Sense. We considered each paragraph as a document.
Table 1: the classification results for the datasets used. The reported scores are the averages of 10-fold cross-validation. We use the per-document topic distributions as classification features fed to Support Vectors Machines (SVMs). We have used the implementation of (Pedregosa et al., 2011) with $C = 1$ for the SVM regularization parameter. For the multi-label datasets (TED and PubMed) we employed one-versus-rest: the SVMs return every category with a positive distance from the separating hyper-planes. As one can note, copLDA can and LDA achieve the highest MiF scores in most of the datasets, without a clear advantage to one vs the other. Binding the topics of sentence words with copulas improves over the

| Profession | Science | Books | Art | Cinema | Places | Music | Birth | Elections | Inventions |
|------------|---------|-------|-----|--------|--------|-------|-------|-----------|------------|
| professor  | univers | book  | art | film   | state  | record | born   | elect     | california |
| world      | research| new   | new | new    | new    | new    | new    | new       | new        |
| football   | scient  | work  | work| work   | role   | role   | role   | role      | role       |
| wrestl     | professor| publish| paint| appear | township| album | best | member | invent |
| play       | author  | york  | american | actor | us | school | song | good | flower |
| born       | also    | also  | also | also   | best | also | best | also     | also       |
| american   | also    | also  | also | also   | also | also | also | also     | also       |
| championship | | | | | | | | | |

Table 2: The top-10 words of copLDA (upper half) and LDA (lower half) in the Wiki46 dataset.

Kiss of Death is a 1995 crime thriller film starring David Caruso Samuel L. Jackson and Nicolas Cage. The film is a very loosely based remake of the 1947 film noir classic of the same name that starred Victor Mature, Brian Donle... [full text]

Table 3: The discovered topics underlying the words of example documents for LDA (left) and copLDA (right). The parts of the documents in italics indicate the noun-phrases obtained by the Stanford Parser. The text colours refer to the topics described in Table 2.

| Known | Wrestl | Born | Profession | American | American | Best | Championship |
|-------|--------|------|------------|----------|----------|------|--------------|
| known | wrestl | born | profession | american | american | best | championship |
| univers | book | new | work | new | new | study | writer |
| research | work | role | work | role | work | magazine | american |
| book | art | film | township | record | play | football | elect |
| art | film | township | record | play | football | elect | work |
| film | township | record | play | football | elect | work | first |
| township | record | play | football | elect | work | first | year |
| record | play | football | elect | work | first | year | photographs |

Extrinsic evaluation: text classification To further highlight the merits of copLDA, we also present in Table 1 the classification results for the datasets used. The reported scores are the averages of 10-fold cross-validation. We use the per-document topic distributions as classification features fed to Support Vectors Machines (SVMs). We have used the implementation of (Pedregosa et al., 2011) with $C = 1$ for the SVM regularization parameter. For the multi-label datasets (TED and PubMed) we employed one-versus-rest: the SVMs return every category with a positive distance from the separating hyper-planes. As one can note, copLDA and LDA achieve the highest MiF scores in most of the datasets, without a clear advantage to one vs the other. Binding the topics of sentence words with copulas improves over the

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results of senLDA: copLDA_{sen} performs only slightly worse than LDA and copLDA_{np} on most datasets and outperforms them, only slightly again, on one dataset.

5 Conclusions

We proposed copLDA that extends LDA to incorporate the topical dependencies within sentences and noun-phrases using copulas. We have shown empirically the advantages of considering text structure and incorporating it in LDA with copulas. In our future work we plan to integrate procedures to learn the $\lambda$ parameter of Frank copulas and to investigate ways to model not only dependencies within text segments like noun-phrases, but also dependencies between such segments with nested copulas.

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