Constraining the cosmic radiation density due to lepton number with BBN

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Abstract. We discuss the bounds on the cosmological lepton number from Big Bang Nucleosynthesis (BBN), in light of recent evidences for a non-zero value of the neutrino mixing angle $\theta_{13}$ ($\sin^2 \theta_{13} \gtrsim 0.005$ at $2\sigma$). We compute the largest possible neutrino asymmetries per flavour compatible with $^4$He and $^2$H primordial yields versus the neutrino mass hierarchy and mixing angles, tracing completely the neutrino oscillation dynamics through decoupling and BBN. The interplay between flavour oscillations and scatterings, in general, leads to non-thermal distortions of the neutrino spectra, depending on $\theta_{13}$, increasing the final value for the effective number of neutrinos $N_{\text{eff}}$. Values for $N_{\text{eff}}$ from neutrino asymmetries which are large enough to be detectable with Planck data are found only for the range $\sin^2 \theta_{13} \leq 0.01$. Nowadays flavour neutrino oscillations are well established from the analysis of data from reactor, accelerator, atmospheric and solar neutrino experiments. All neutrino mixing parameters are known with a precision better than 25%, except for the mixing angle $\theta_{13}$. Until very recently only an upper bound on $\sin^2 \theta_{13}$ existed, while in the present year the first indications of non-zero $\theta_{13}$ values appeared from the analysis of global data, further strengthened by recent $\nu_{\mu} \to \nu_e$ searches at the T2K long-baseline experiment [1]. The allowed region for $\sin^2 \theta_{13}$ is approximately from 0.001 to 0.035 – 0.04 at $3\sigma$, depending on the reactor neutrino fluxes and the neutrino mass hierarchy, either normal (NH) or inverted (IH), see e.g. [2].

Neutrino oscillations have implications in many research areas in particle and astroparticle physics. However, their consequences in cosmology are usually not considered important, despite the fact that cosmological neutrinos are almost as abundant as relic photons — if all the neutrino flavours were produced by frequent interactions in the early Universe, with the same momentum spectra, the oscillations, although effective right after neutrino decoupling, do not modify any of the properties of relic neutrinos\(^1\). There exists, however, one situation where flavour oscillations have an impact on cosmological neutrinos, namely when a large flavour neutrino asymmetry was previously created. It is usually assumed that such an asymmetry, parameterized by the number density ratios $\eta_{\alpha} = (n_\alpha - n_{\bar{\alpha}})/n_{\gamma}$ for $\alpha = e, \mu, \tau$, should be of the same order of the cosmological baryon number $\eta_b = (n_b - n_{\bar{b}})/n_{\gamma}$, due to the equilibration by sphalerons of lepton and baryon asymmetries in the very early universe. Thus one does not expect values of $\eta_{\alpha}$ much larger than a few times $10^{-10}$, the value of $\eta_b$ measured by present observations, such as 7-year data from the WMAP satellite and other cosmological measurements [4]. There are,\(^1\) Except for very small effects, at the percent level, from non-instantaneous neutrino decoupling as found in [3].
however, of magnitude larger than $\eta_{b}$ could survive in the neutrino sector (see e.g. [5, 6]) with an influence on fundamental physics in the early universe, such as the QCD transition [7] or a potential relation with large-scale cosmological magnetic fields [8].

Present cosmological observations are not sensitive to a neutrino asymmetry if $|\eta_{\nu}| \lesssim 10^{-2}$, since only larger values lead to a significant enhancement of the contribution of active neutrinos to the radiation energy density $\rho_{r}$ or to changes in the primordial production of light elements at BBN. Cosmological neutrinos influence BBN in two ways. As a background effect, they contribute to $\rho_{r}$, fixing the expansion of the Universe during BBN. This can be parameterized with $N_{\text{eff}}$ as $\rho_{r} = (1 + 7/8(4/11)4^{3/2}N_{\text{eff}})\rho_{\gamma}$ after $e^{+}e^{-}$ annihilations. Non-zero neutrino asymmetries increase $N_{\text{eff}}(\eta_{\nu})$ enhancing the primordial abundance of light elements. On the other hand, any change in the momentum distribution of $\nu_{e}$’s or $\bar{\nu}_{e}$’s influences the neutron-to-proton ratio, crucial for the final $^{4}$He abundance. This is the case of a non-zero $\nu_{e} - \bar{\nu}_{e}$ asymmetry: it shifts the neutron fraction towards larger (smaller) values for negative (positive) values of $\eta_{\nu}$. This sets a stringent bound on $\eta_{\nu}$ which does not apply to the other flavours unless neutrino oscillations are effective before BBN, leaving a total neutrino asymmetry of order unity unconstrained [9].

A decade ago, it was shown that flavour neutrino conversions in the early Universe are suppressed by effects at large temperatures, and it is only at $T \lesssim 10$ MeV that oscillations could lead to strong flavour conversions before BBN [10, 11, 12]. An example of the evolution of $\eta_{\nu_{\alpha}}$ for $\mathcal{O}$(MeV) temperatures is given in Fig. 1, found solving numerically the evolution of the momentum distributions of flavour neutrinos including both oscillations and interactions with the rest of the plasma ($\nu$’s and $e^{\pm}$). See refs. [10, 13, 14] for details about the corresponding kinetic equations and the approximations made. Note that $\nu_{\tau} - \nu_{\mu}$ mixing, driven by the mixing parameters $\Delta m_{23}^{2}$ and $\theta_{23}$ is effective at $T \simeq 15$ MeV, when weak interactions are fully effective [10]. Thus, our numerical calculations start at $T = 10$ MeV with initial asymmetries $\eta^{\text{in}}_{\nu_{e}} = \eta^{\text{in}}_{\nu_{\tau}}$ and $\eta^{\text{in}}_{\nu_{\mu}}$. The total neutrino asymmetry $\eta_{\nu} = \sum_{\alpha} \eta_{\nu_{\alpha}}$ is unchanged by oscillations or collisions.

The onset of flavour oscillations involving $\nu_{e}$’s is initially fixed by $\Delta m_{31}^{2}$ and $\theta_{13}$. If $\Delta m_{31}^{2} > 0$ (NH) neutrino oscillations follow an MSW conversion when the vacuum term overcomes the matter potential and the degree of conversion depends in this case on the value of $\theta_{13}$ [10, 11, 12], being very efficient compared with $\theta_{13} = 0$ for values close to the upper bound. This can be seen in Fig. 1 for one particular case with $\eta_{b} = 0$ and varying $\theta_{13}$, while the conversion for non-zero $\theta_{13}$ is more evident if $\Delta m_{31}^{2} < 0$ (IH), due to the resonant character of the MSW transition. Indeed, for IH we found that if $\sin^{2}2\theta_{13} \gtrsim 5 \times 10^{-3}$ equipartition of the total lepton asymmetry among the three neutrino flavours is quickly achieved. Finally, for negligible $\theta_{13}$ flavour oscillations driven by $\Delta m_{23}^{2}$ and $\theta_{12}$ are not effective until $T \lesssim 3$ MeV.

The moment when flavour oscillations become effective is important not only to establish $\eta_{\nu_{e}}$ at the onset of BBN, but also to determine whether weak interactions can still keep neutrinos and $e^{+}e^{-}$ in good thermal contact. Oscillations redistribute the asymmetries among the flavours, but only if they occur early enough would interactions preserve Fermi-Dirac spectra for neutrinos, so that a chemical potential $\mu_{\nu_{\alpha}}$ is well defined for each $\eta_{\nu_{\alpha}}$. For instance, for initial flavour asymmetries with opposite signs, neutrino conversions will tend to reduce the asymmetries, decreasing in turn $N_{\text{eff}}$. But if flavour oscillations begin at temperatures close to neutrino decoupling, this would not hold and an extra contribution of neutrinos to radiation is expected [13].

Prompted by recent indications of non-zero $\theta_{13}$ and the hints of possible extra radiation from cosmological data [4], we have recently found the BBN bounds on the cosmological lepton number for a range of initial $\eta_{\nu_{\alpha}}$ [14, 15]. Both the allowed region for the total neutrino asymmetry and its maximum contribution to the radiation content of the Universe were calculated for the $\theta_{13}$ values favoured by oscillation data, as well as considering both neutrino mass hierarchies.

We have performed an analysis of the effects of flavour neutrino asymmetries on the BBN outcome, solving the evolution of neutrino spectra in a wide range of values for the total (lepton) asymmetry $\eta_{\nu}$ and the initial $\nu_{e}$ asymmetry $\eta^{\text{in}}_{\nu_{e}}$. The obtained time-dependent neutrino
The allowed region for $\eta$ in the NH values of order within the region favoured by oscillation data, due to the resonant character of the conversions. NH (IH). Note, however, that in the IH this result approximately holds for any value of $\sin^2 \theta_{13}$ region for $\sin^2 \theta_{13}$ of the allowed region from a quasi-horizontal one for zero mixing to an almost vertical rotation

Figure 1. Left: Evolution of the flavour neutrino asymmetries when $\eta_{\nu_e}^{in} = -0.82$ and $\eta_{\nu} = 0$. The outer solid curves correspond to $\theta_{13} = 0$ (black), while the inner ones (red) were calculated in the NH for two values of $\sin^2 \theta_{13}$: 0.04 (left) and 0.02 (right). The same two values of $\sin^2 \theta_{13}$ are shown in the IH (blue dotted). Right: 95% C.L. contours from our BBN analysis in the $\eta_{\nu} - \eta_{\nu_e}^{in}$ plane for $\sin^2 \theta_{13} = 0$ (black solid line), 0.04 and NH (red solid), 0.04 and IH (blue dotted). The case of no flavour oscillations is shown for comparison (black dashed contour).

The allowed region for $\eta_{\nu_e}$ is severely constrained by $^3$He data, arising from a narrow region for the electron neutrino degeneracy, $-0.018 \leq \xi_e \leq 0.008$ at 68% C.L. Instead, the asymmetry for other neutrino flavours could be much larger, since the absolute value of the total asymmetry is only restricted to $|\eta_{\nu}| \lesssim 2.6$ [17, 18]. Flavour oscillations modify this picture and an initially large $\eta_{\nu_e}^{in}$ can be compensated by an asymmetry in the other flavours with opposite sign. The restrictive BBN bound on the $\eta_{\nu}$ value, which must be very close to zero, applies then to $\eta_{\nu}$. This can be seen graphically as a rotation of the allowed region from a quasi-horizontal one for zero mixing to an almost vertical region for $\sin^2 \theta_{13} = 0.04$, in particular for the IH. In all cases depicted in Fig. 1 the allowed regions are mainly fixed by $^3$He that cuts a narrow degenerate band, while data on primordial $^2$H is crucial for closing the regions.

For values of $\theta_{13}$ close to the experimental upper limits, the combined effect of oscillations and collisions leads to an efficient mixing of all neutrino flavours before BBN. Therefore, the individual neutrino asymmetries have similar values, $\eta_{\nu_e} \approx \eta_{\nu}/3$. The BBN bound on $\eta_{\nu}$ applies to all flavours, and in turn to $\eta_{\nu}$ as considered in [9, 10, 19, 20]. We find that for $\sin^2 \theta_{13} = 0.04$ the allowed region at 95% C.L. is $-0.17(-0.1) \leq \eta_{\nu} \leq 0.1(0.05)$ for neutrino masses following a NH (IH). Note, however, that in the IH this result approximately holds for any value of $\sin^2 \theta_{13}$ within the region favoured by oscillation data, due to the resonant character of the conversions. Instead, in the NH values of order $|\eta_{\nu}| \approx 0.6$ are still compatible with BBN if $\theta_{13}$ is very small [14]. The allowed region for $\eta_{\nu}$ and the largest values of $N_{eff}$ from neutrino asymmetries compatible
with BBN, as a function of the neutrino mass hierarchy and the mixing angle $\theta_{13}$ are reported in Fig. 2. If the true value of $\theta_{13}$ lies in the upper part of the region favoured by oscillation experiments (in particular T2K) or $\Delta m^2_{31} < 0$, the presence of primordial asymmetries can not lead to a contribution to the radiation energy density $N_{\text{eff}}> 3.1$. Instead, for the NH and very small values of $\theta_{13}$, larger values of $N_{\text{eff}}$ are still compatible with BBN, up to 3.43 at 95% C.L.

In conclusion, we have found the BBN constraints on the cosmological lepton number and the associated contribution to the radiation energy density, taking into account the effect of flavour neutrino oscillations. In particular, we have shown that pinpointing the value of $\theta_{13}$ is crucial to establish the degree of equilibration of flavour neutrino asymmetries in the early Universe.

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