The gap parameter for color superconductivity is expected to be a gauge invariant quantity, at least on the appropriate mass shell. Computing the gap to subleading order in the QCD coupling constant, $g$, we show that the prefactor of the exponential in $1/g$ is gauge dependent off the mass shell, and independent of gauge on the mass shell.

1. INTRODUCTION

Quantum chromodynamics (QCD) is the fundamental theory of the strong interactions. In strongly interacting matter at large density or, equivalently, large quark chemical potential $\mu$, asymptotic freedom \[^1\] tells us that the QCD coupling $g$, evaluated at the scale $\mu$, is logarithmically small, $g(\mu) \ll 1$. Then single-gluon exchange, which is attractive in the color-antitriplet channel \[^2\], is the dominant interaction between two quarks at the edge of the Fermi sea. By Cooper’s theorem \[^3\], any attractive interaction destabilizes the Fermi surface and, at sufficiently small temperature $T$, leads to the condensation of Cooper pairs. In QCD, quark-quark Cooper pairs break the color symmetry. In this color superconductor, at $T = 0$ it costs at least $2\phi_0$ to excite a particle-hole pair, where $\phi_0$ is the value of the superconducting gap function at the Fermi surface. This gap $\phi_0$ can be computed using a mean-field approximation to a self-consistent gap equation with single-gluon exchange \[^4\].

Schematically, this gap equation can be written in the form \[^5\]

$$\phi_k^{+} = g^2 \int \frac{dq}{\epsilon_q^{+}} \phi_q^{+} \left[ \zeta \ln \left( \frac{\mu^2}{|\epsilon_k^{+2} - \epsilon_q^{+2}|} \right) + \beta + \beta' \epsilon_q^{+} \ln \left( \frac{\mu}{\epsilon_q^{+}} \right) + \alpha \epsilon_q^{+} \right]. \tag{1}$$

Here, $\phi_k^{+}$ is the gap function for quasiparticles, taken on the quasiparticle mass shell. In general, the gap function $\phi^{+}(K)$ depends on four-momentum $K^\mu \equiv (k_0, \mathbf{k})$. The quasiparticle mass shell is defined by

$$k_0 = \epsilon_k^{+}, \quad \epsilon_k^{+} \equiv \sqrt{(\mu - k)^2 + |\phi_k^{+}|^2}, \tag{2}$$

where $\epsilon_k^{+}$ is the quasiparticle excitation energy. The gap function on the quasiparticle mass shell depends on $k$ only, $\phi_k^{+} \equiv \phi^{+}(\epsilon_k^{+}, \mathbf{k})$. At the Fermi surface, $\phi_{\mu}^{+} \equiv \phi_0$. The
integration variable $q$ in Eq. (1) is the kinetic energy of ultrarelativistic particles. The prefactors $\zeta, \beta, \beta'$, and $\alpha$ are either constants or depend at most logarithmically on $g$. In weak coupling, $g \ll 1$, the solution of Eq. (1) at the Fermi surface is

$$\phi_0 = 2 b \mu \exp \left( -\frac{c}{g} \right) \left[ 1 + O(g) \right].$$

(3)

This solution implies $\ln(\mu/\phi_0) \sim 1/g$, such that the various terms in Eq. (1) differ by powers of $g$. The first term $\sim \zeta$ contains two powers of the logarithm $\ln(\mu/\phi_0)$. One is the well-known BCS logarithm which arises from the integration over $q$,

$$\int \frac{dq}{\epsilon_q^+} \sim \ln \left( \frac{\mu}{\phi_0} \right).$$

(4)

The other one is the logarithm multiplying $\zeta$, when the momentum $q$ is near the Fermi surface and $\epsilon_q^+ \sim \epsilon_k^+ \sim \phi_0$. This logarithm is special to theories with long-range interactions, like the exchange of almost static magnetic gluons in QCD. Its origin is a collinear singularity when integrating over the angle between incoming and outgoing quark momenta in the gap equation. Since $\ln^2(\mu/\phi_0) \sim 1/g^2$, the term proportional to $\zeta$ is the leading term on the right-hand side of the gap equation (1); together with the prefactor $g^2$ it is of order $O(\phi_0)$, and thus matches the order of magnitude of the left-hand-side of the gap equation. The value of the coefficient $\zeta$ determines the constant $c$ in Eq. (3). This constant was first computed by Son,

$$c = \frac{\pi}{2 \bar{g}} \ , \ \bar{g} \equiv \frac{g}{3\sqrt{2}}.$$  

(5)

The second and third terms in Eq. (1) contain subleading contributions to the gap equation. These are proportional to a single power of the logarithm $\ln(\mu/\phi_0) \sim 1/g$. For the term proportional to $\beta$, this logarithm is the BCS logarithm from Eq. (3). In the other term, proportional to $\beta'$, this logarithm is explicitly present, but the BCS logarithm is absent, because the additional factor $\epsilon_q^+$ cancels the one in the denominator of the integration measure, $dq/\epsilon_q^+$. Together with the prefactor $g^2$, these terms constitute a contribution of order $O(g\phi_0)$ to the right-hand-side of the gap equation, i.e., an $O(g)$ correction to the leading contribution.

The contribution proportional to $\beta$ arises from the exchange of non-static magnetic and static electric gluons. Both types of interactions are short-range: they are screened on a distance scale $m_g^{-1}$, where $m_g$ is the gluon mass; $m_g^2 = N_f g^2 \mu^2/(6\pi^2)$, $N_f$ is the number of quark flavors. Consequently, the collinear logarithm characteristic for long-range interactions is absent, and one is left with the BCS logarithm. The contribution proportional to $\beta'$ arises from the quark self-energy. The coefficients $\beta$ and $\beta'$ in Eq. (1) determine the constant $b$ in Eq. (3). Son was the first to give an estimate for the constant $b$,

$$b = \frac{b_0}{g^5},$$

(6)

with a constant $b_0$ of order $O(1)$, which could not be determined in the approach of Ref. [4]. In [4,5] the constant $b_0$ was computed by solving the QCD gap equation including...
non-static magnetic and static electric gluon exchange, but without taking into account
the quark self-energy. In other words, all terms $\sim \beta$ in Eq. (1) were collected, but the
term $\sim \beta'$ was neglected. The result is

$$b_0 = 256 \pi^4 \left(\frac{2}{N_f}\right)^{5/2} b'_0,$$

with an undetermined constant $b'_0$ of order $O(1)$. In \[3,5\], i.e., without effects from the
quark self-energy, $b'_0 = 1$. In \[8,9\], it was shown that the quark self-energy gives rise to
a term $\sim \beta'$ in Eq. (1). As this is parametrically also of subleading order in the gap
equation, it modifies the constant $b'_0$,\n
$$b'_0 = \exp\left(-\frac{\pi^2 + 4}{8}\right) \simeq 0.177.$$  

The fourth term in Eq. (1) summarizes sub-subleading contributions. These are of order
$O(g^2 \phi_0)$. It was argued in \[4,5,10\] that at this order gauge-dependent terms enter the
mean-field gap equation for the color-superconducting gap parameter. However, the gap
parameter is in principle an observable quantity, and thus gauge independent. Therefore,
it was concluded that one has to go beyond the mean-field approach to compute gauge-
dependent sub-subleading contributions to the gap parameter. It was also shown \[11\]
that effects from the finite lifetime of quasiparticles in the Fermi sea influence the value of
$\phi_0$ at this order. In weak coupling, the terms $\sim \alpha$ in Eq. (1) are suppressed by one power
of $g$ compared to the subleading terms and therefore constitute an order $O(g)$ correction
to the prefactor $b$, as indicated in Eq. (3).

In this note, we first present the gap equation for the quasiparticle and quasi-antiparticle
gap, including the gauge-dependent terms, and review previous arguments \[3\] on why the
gauge dependence enters at sub-subleading order in the gap equation. These arguments
were actually incorrect, in that they neglected additional powers of the gluon momentum
in the gap equation. Naively correcting for these powers, one obtains that the gauge depen-
dence enters already at subleading order, giving rise to an extra prefactor $\sim \exp(3 \xi_C/2)$
to the gap parameter \(\phi_0\), where $\xi_C$ is the gauge parameter for the gluon propagator in
a general Coulomb gauge. This result is similar to other claims made in the literature
\[7\]. Finally, we demonstrate that a careful calculation of the gauge-dependent term on
the correct quasiparticle mass shell shows that the gauge dependence indeed enters only
beyond subleading order in the gap equation. Consequently, the gauge dependence does
not affect the $O(1)$ result for the prefactor of the gap, as was originally claimed.

Our units are $\hbar = c = k_B = 1$, the metric tensor is $g^{\mu\nu} = \text{diag}(+, -, -, -)$, and we
work in a general Coulomb gauge, with gauge parameter $\xi_C$. We denote 4-vectors with
capital letters, $K^\mu \equiv (k_0, \mathbf{k})$, $k \equiv |\mathbf{k}|$, $\hat{k} \equiv \mathbf{k}/|\mathbf{k}|$.

2. GAUGE-DEPENDENT TERMS IN THE QCD GAP EQUATION

For the sake of simplicity, let us focus on the condensation of quarks with two massless
flavors, forming Cooper pairs with total spin zero. For the discussion of the gauge de-
pendence, terms in the gap equation arising from the quark self-energy \[3\] can be safely
The traces can be readily evaluated, thus, the QCD gap equation reads, cf. Eq. (29) of \[5\],
\[
\phi^e_h(K) = \frac{2}{3} g^2 T \sum_{q_0} \int \frac{d^3q}{(2\pi)^3} \Delta_{\mu\nu}(K - Q) \left\{ \frac{\phi^e_h(Q)}{q_0^2 - (\epsilon^e_q)^2} \text{Tr} \left[ P^e_h(k) \gamma^\mu P^{-e}_h(q) \gamma^\nu \right] \\
+ \frac{\phi^{-e}_h(Q)}{q_0^2 - (\epsilon^e_q)^2} \text{Tr} \left[ P^e_h(k) \gamma^\mu P^{-e}_h(q) \gamma^\nu \right] \right\} .
\] (9)

Here, $\phi^e_h(K)$ is the gap function for condensation of quarks with chirality $h$ and energy $e$, $\epsilon^e_q \equiv \sqrt{(\mu - e q)^2 + |\phi^e_h(Q)|^2}$, and
\[
P^e_h(k) \equiv \frac{1 + h \gamma_5}{2} \left( 1 + \frac{e \gamma_0 \gamma \cdot \hat{k}}{2} \right)
\] (10)
are projectors \[13\] onto states with chirality $h = \pm = r, \ell$ and energy $e = \pm$. The Matsubara sum $\sum_{q_0}$ runs over fermionic Matsubara frequencies $q_0 \equiv -i(2n + 1)\pi T$. We first perform the Matsubara sum and then consider the limit $T \to 0$. The gluon propagator in a general Coulomb gauge is, cf. Eq. (30) of \[4\],
\[
\Delta_{00}(P) = \Delta_{\ell}(P) + \xi_{C} \frac{p_0^2}{p^4},
\]
\[
\Delta_{0\ell}(P) = \xi_{C} \frac{p_0 p_\ell}{p^4},
\]
\[
\Delta_{\ell\ell}(P) = (\delta_{\ell\ell} - \hat{p}_\ell \hat{p}_\ell) \Delta_{\ell}(P) + \xi_{C} \frac{p_\ell p_\ell}{p^4}.
\] (11)

To simplify the notation in the following, let us write the QCD gap equation \[4\] in the form
\[
\phi^e_h(K) = R^e_h(K) + \xi_{C} X^e_h(K),
\] (12)
where $R^e_h(K)$ is the right-hand side of \[4\] for $\xi_{C} = 0$, and introducing $P \equiv K - Q$,
\[
X^e_h(K) \equiv \frac{2}{3} g^2 T \sum_{q_0} \int \frac{d^3q}{(2\pi)^3} \frac{P_\mu P_\nu}{p^4} \left\{ \frac{\phi^e_h(Q)}{q_0^2 - (\epsilon^e_q)^2} \text{Tr} \left[ P^e_h(k) \gamma^\mu P^{-e}_h(q) \gamma^\nu \right] \\
+ \frac{\phi^{-e}_h(Q)}{q_0^2 - (\epsilon^e_q)^2} \text{Tr} \left[ P^e_h(k) \gamma^\mu P^{-e}_h(q) \gamma^\nu \right] \right\} .
\] (13)

The traces can be readily evaluated,
\[
P_\mu P_\nu \text{Tr} \left[ P^e_h(k) \gamma^\mu P^{-e}_h(q) \gamma^\nu \right] = \frac{1 + \frac{\hat{k} \cdot \hat{q}}{2}}{2} \left[ p_0^2 - (k - q)^2 \right],
\]
\[
P_\mu P_\nu \text{Tr} \left[ P^e_h(k) \gamma^\mu P^{-e}_h(q) \gamma^\nu \right] = \frac{1 - \frac{\hat{k} \cdot \hat{q}}{2}}{2} \left[ p_0^2 - (k + q)^2 \right].
\] (14)

Since the gap equations for right- and left-handed gaps decouple, we shall drop the index $h$ in the following. We furthermore focus on the gap equation for the quasiparticle gap function $\phi^+(K)$, i.e., $e = +$ in Eqs. (12) and (13). In the next section, we argue that quasiparticle contributions to the gap equation for the quasiparticle gap are negligible. We then proceed to estimate the magnitude of the gauge-dependent terms in Section 4.
3. NEGLECTING THE QUASI-ANTIPARTICLE MODES

It is permissible to neglect the contribution from the quasi-antiparticle excitations \( \sim \phi^-(Q) \) in the gap equation (11), as has been done in Eq. (1). The reason is that the quasi-antiparticle gap \( \phi^-(K) \) is suppressed with respect to the quasiparticle gap \( \phi^+(K) \) by at least one power of \( g \). This can be checked by simply power-counting the contributions to the gap equation for \( \phi^-(K) \) using the arguments presented in the introduction. Taking the gap function on the quasi-antiparticle mass shell, \( k_0 = \epsilon_k^- \), i.e., \( \phi^-(K) \equiv \phi^- (\epsilon_k^-, k) \equiv \phi_k^- \), and performing the Matsubara sum over \( q_0 \) (which puts the internal gap functions on the mass shell, \( \phi^\pm (Q) \equiv \phi^\pm (\epsilon_q^\pm, q) \equiv \phi_q^\pm \)) and the integral over the angle between external quark three-momentum \( k \) and internal quark three-momentum \( q \), this gap equation has a similar structure as Eq. (1), with the obvious replacements \( \phi^+_k \rightarrow \phi^-_k \), \( \epsilon^+_k \rightarrow \epsilon^-_k \). (The contribution from quasi-antiparticles to this gap equation, which is still present in Eq. (9), can be discarded, as it does not even have a BCS logarithm.) Since \( \epsilon_k^- \sim \mu + k \), the collinear logarithm from almost static, magnetic gluon exchange in Eq. (1) is only of order \( O(1) \). This leaves the BCS logarithm. At the Fermi surface

\[
\phi^-_\mu \sim g^2 \ln \left( \frac{\mu}{\phi_0} \right) \phi_0 .
\] (15)

With Eq. (3) one obtains \( \phi^-_\mu \sim g \phi_0 \), which proves our assertion.

The complete contribution from quasi-antiparticle excitations to the equation for the quasiparticle gap is suppressed relative to the quasiparticle contribution even more than just by a single power of \( g \). Neglecting a possible angular dependence entering through the traces in Eq. (3) and performing the Matsubara sum which picks up the poles at \( q_0 = \pm \epsilon_q^- \sim \pm (\mu + q) \), the quasi-antiparticle contribution is proportional to

\[
\frac{\phi^-_q}{\epsilon_q^-} \sim \frac{g \phi^+_q}{\epsilon_q^+} \sim \frac{g \epsilon_q^+ \phi^-_q}{\mu + q \epsilon_q^+} ,
\] (16)

i.e., at the Fermi surface it is suppressed by a factor \( \sim g \phi_0 / (2\mu) \) relative to the quasiparticle contribution (which is proportional to the last factor in Eq. (16)).

4. THE GAP IS GAUGE DEPENDENT OFF THE MASS SHELL ...

The argument of (3), as to why the gauge-dependent terms do not enter into the gap equation at leading and subleading order, starts with neglecting the terms proportional to \( p^2_0 \) in Eq. (14). This is because on the mass shell such terms are of order \( p^2_0 \equiv (k_0 - q_0)^2 \sim (\epsilon_k^\pm \pm \epsilon_q^\pm)^2 \sim \phi_0^2 \). One is then left with the spatially longitudinal terms, cf. Eq. (134) of (3).

\[
\text{Tr} \left[ P^+_h (k) \gamma \cdot \hat{p} P^-_h (q) \gamma \cdot \hat{p} \right] \sim - \frac{1 + \hat{k} \cdot \hat{q} (k - q)^2}{2 p^2} .
\] (17)

As \( k \) and \( q \) approach the Fermi surface, this term vanishes, completing the argument that gauge-dependent terms do not enter at leading and subleading order in the gap equation.

This is, however, a very general argument. So let us explicitly compute \( X^+(K) \) from Eq. (13) with (17), just to make sure. We had made the seemingly innocuous assumption
of neglecting terms \( \sim p_0^2 \). In doing so, implicitly we will be working off the mass shell, and so find that the gap is gauge dependent. In the next section, we show that this goes away on the mass shell.

Under the present approximations, there is no term depending on \( q_0 \) except for the denominator in (13). Thus, one can immediately perform the Matsubara sum over \( q_0 \) using

\[
T \sum_{q_0} \frac{\phi^+(Q)}{q_0^2 - (\epsilon_q^+)^2} \equiv -\frac{\phi^+_q}{2\epsilon_q^+} \left[ 1 - 2n_F \left( \frac{\epsilon_q^+}{T} \right) \right],
\]

where \( n_F(x) \equiv (e^x + 1) \) is the Fermi distribution. For \( T = 0 \), \( n_F(\epsilon_q^+/T) \) vanishes, as \( \epsilon_q^+ \geq \phi_q^+ > 0 \). Accounting for an additional factor \( 1/p^2 \) from Eq. (13), one then integrates over \( d^3q \). The integrand does not depend on the polar angle; therefore \( d^3q \equiv 2\pi dq q^2 d\cos \theta \). Here, \( \cos \theta \equiv \hat{k} \cdot \hat{q} \). Substituting \( p \equiv |k - q| \) for \( \cos \theta \), the following integral appears in Eq. (13).

\[
\int_{|k - q|}^{k+q} dp \frac{(k + q)^2 - p^2}{p^4} = \frac{2kq}{(k - q)^2} - \ln \frac{k + q}{|k - q|}.
\]

As this term is multiplied by \((k - q)^2\) on account of (17), the singularity at \( k = q \) is rendered harmless. Inserting this into Eq. (13), one obtains

\[
X^+(k) = \frac{g^2}{24 \pi^2} \int dq \frac{\phi_q^+}{\epsilon_q^+} \left[ \frac{q}{k} - \frac{(k - q)^2}{2k^2} \ln \frac{k + q}{|k - q|} \right].
\]

The momentum dependence of the gap function restricts the \( q \)-integration to a narrow interval around the Fermi surface, \( \mu - \delta \leq q \leq \mu + \delta, \delta \ll \mu \). At the Fermi surface, \( k = \mu \), and introducing \( \xi \equiv q - \mu \) one obtains

\[
X^+(\mu) = \frac{g^2}{12 \pi^2} \int_0^\delta d\xi \frac{\phi_\xi^+}{\epsilon_\xi} \left( 1 - \frac{\xi^2}{4\mu^2} \ln \frac{4\mu^2}{\xi^2} \right),
\]

where we have neglected some terms of order \( \xi/\mu \leq \delta/\mu \ll 1 \), and where \( \epsilon_\xi \equiv \sqrt{\xi^2 + |\phi_\xi^+|^2} \).

As one approaches the Fermi surface, \( \xi \to 0 \), the second term in parentheses vanishes. Consequently, the first term is the dominant one, and it gives rise to a BCS logarithm,

\[
X^+(\mu) = \frac{g^2}{12 \pi^2} \int_0^\delta d\xi \frac{\phi_\xi^+}{\epsilon_\xi} \sim g^2 \phi_0 \ln \left( \frac{\mu}{\phi_0} \right).
\]

Such a term is of the same order as terms \( \sim \beta \) in Eq. (4), i.e., of subleading order in the gap equation! It thus affects the prefactor \( b \) in Eq. (4). A careful analysis shows that \( b \) is multiplied by a factor \( \exp(3\xi_C/2) \). In pure Coulomb gauge, \( \xi_C = 0 \), this factor is unity, and one obtains the previous result for \( b \), Eqs. (4) and (7). A similar dependence of \( b \) on the gauge parameter was also reported for covariant gauges (4). The appearance of a gauge dependence in those calculations is, however, not surprising, as the gap parameter is not calculated on the quasiparticle mass shell, but for Euclidean (imaginary) energies. Away from the quasiparticle mass shell, the gap is not an observable quantity, and therefore may depend on the choice of gauge.

The above argument is, however, still incorrect, as terms proportional to \( p_0^2 \) were neglected. Therefore, as mentioned previously, we are also computing off the mass shell. Restoring these terms and putting \( k_0 \) on the quasiparticle mass shell, we shall see that the gauge dependence enters neither at leading nor subleading order in the gap equation.
5. ... AND INDEPENDENT OF GAUGE ON THE MASS SHELL

We now compute the gap on the correct mass shell. To this end, the seemingly innocuous terms \( \sim p_0^2 \) in Eq. (14) must not be neglected, since \( p_0 = k_0 - q_0 \). To perform the Matsubara sum over these terms, one uses the following trick:

\[
T \sum_{q_0} \frac{p_0^2}{q_0^2 - (\epsilon_q^+)^2} = \lim_{\tau \to 0} (k_0 + \partial_\tau)^2 T \sum_{q_0} e^{-q_0 \tau} \frac{\phi^+(Q)}{q_0^2 - (\epsilon_q^+)^2} . \tag{23}
\]

The Matsubara sum can be computed in the standard way. Taking \( T \to 0 \), the final result is

\[
\lim_{T \to 0} T \sum_{q_0} \frac{p_0^2}{q_0^2 - (\epsilon_q^+)^2} = -\frac{\phi_q^+}{2\epsilon_q^+} (k_0 - \epsilon_q^+)^2 . \tag{24}
\]

Putting everything together and taking the external energy \( k_0 \) on the mass shell, \( k_0 = \epsilon_k^+ \), one obtains for \( X_k^+ \equiv X^+(\epsilon_k^+, k) \) the following equation:

\[
X_k^+ = \frac{g^2}{48 \pi^2 k^2} \int dq \frac{\phi_q^+}{\epsilon_q^+} \left[ \frac{2kq}{(k - q)^2} - \ln \frac{k + q}{|k - q|} \right] \left[ (k - q)^2 - (\epsilon_k^+ - \epsilon_q^+)^2 \right] . \tag{25}
\]

We again introduce the variable \( \xi \equiv q - \mu \) and neglect terms \( \sim \xi/\mu \ll 1 \). At the Fermi surface, \( k = \mu \), we are then left with

\[
X_\mu^+ = \frac{g^2}{12 \pi^2} \int_0^\delta d\xi \frac{\phi_\xi^+}{\epsilon_\xi} \left( 1 - \frac{\xi^2}{4\mu^2} \ln \frac{4\mu^2}{\xi^2} \right) \left[ 1 - \frac{\left( \phi_0 - \epsilon_\xi \right)^2}{\xi^2} \right] . \tag{26}
\]

As in Eq. (21), the logarithm in parentheses is completely innocuous because of the prefactor \( \xi^2 \), and can be neglected in the following.

The difference to Eq. (21) is the second term in brackets, which arises from the \( p_0^2 \) term in Eq. (14). Expanding this term around the Fermi surface (\( \xi = 0 \)), one realizes that it is of order \( \xi^2 \), and therefore negligible compared to the first term in brackets. Naively, one would now conclude that there is a large BCS logarithm \( \int d\xi/\epsilon_\xi \sim \ln(\mu/\phi_0) \), just like in Eq. (22), and the gauge-dependent term contributes to subleading order in the gap equation. This is, however, incorrect. The BCS logarithm “builds up” as \( \xi \) approaches the Fermi surface at \( \xi = 0 \). This build-up requires a coefficient of order \( O(1) \) over the whole range of integration.

In contrast to Eq. (21), this is not the case here, because the factor 1 in Eq. (21) is replaced by \( 1 - (\phi_0 - \epsilon_\xi)^2/\xi^2 \) in Eq. (26). To see the effect on the magnitude of the gauge-dependent terms in the gap equation, split the integral over \( \xi \) into two parts, one from 0 to \( \kappa \phi_0 \), with \( \kappa \gg 1 \), and one from \( \kappa \phi_0 \) to \( \delta \). In the first integral

\[
1 - \frac{(\phi_0 - \epsilon_\xi)^2}{\xi^2} \sim O(1) . \tag{27}
\]

In the second integral,

\[
1 - \frac{(\phi_0 - \epsilon_\xi)^2}{\xi^2} \sim \frac{2 \phi_0 \epsilon_\xi}{\xi^2} . \tag{28}
\]
Inserting this into (26), one obtains the order-of-magnitude estimate

\[ X_\mu^+ \sim g^2 \phi_0 \left[ \int_0^{\kappa \phi_0} \frac{d\xi}{\epsilon_\xi} + \phi_0 \int_0^{\delta} \frac{d\xi}{\xi^2} \right] \simeq g^2 \phi_0 \left[ \ln(2\kappa) - \frac{\phi_0}{\delta} + \frac{1}{\kappa} \right] \sim g^2 \phi_0. \tag{29} \]

In the first integral, the integration measure would in principle give rise to a BCS logarithm, if the upper limit of integration was large, \( \sim \delta \), and not small, \( \sim \phi_0 \). In the second integral, the factor \( \epsilon_\xi \) in the numerator of (28) cancels with the one in the integration measure \( d\xi/\epsilon_\xi \) and thus prevents the “build-up” of the BCS logarithm.

Comparing the final result (29) with the discussion in the introduction, we conclude that the gauge-dependent term is obviously of sub-subleading order in the gap equation (1). In other words, the mean-field gap equation for the color-superconducting gap in QCD is gauge independent to leading and subleading order. Consequently, the gauge-dependent terms influence the prefactor of the gap in Eq. (3) only at order \( O(g) \). The present note thus confirms by analytical means the numerical results of Ref. [10].

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