Beta Viscose Prescription in Self-Gravitating Disks

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ABSTRACT

Duschl et al. (2000) have shown that the standard model for geometrically thin accretion disks (α-disks) leads to inconsistency if self-gravity play a role. This problem arise from parametrization of viscosity in terms of local sound velocity and vertical disks scale height. The β-viscosity prescription was introduced by Duschl et al. (2000), which has been derived from rotating shear flow experiment (ν = βΩR²). Following the Duschl et al. (2000) suggestion for a β-prescription for viscosity, we apply this model for a thin self-gravitating disk around newborn stars. Our result is quite different with standard alpha disks in the outer part of the disks where the self-gravity becomes important. In the inner part of the disks, our solution converged to the standard α disks. It has been presented that for beta model, Toomre parameter is more than unity everywhere which means that gravitational fragmentation can be occur everywhere. We suggest that the kind of hydrodynamically driven viscosity, β-model, can be used for modeling of accretion disks from proto-stellar disks to AGN and galactic disks. It would be interest to investigate ADAF-type solution for follow any effects by β-viscosity model. An important property of the β-disk is that they are viscously stable.

Key words: ISM: molecules, ISM: structure, instabilities

1 INTRODUCTION

Disk-like or flattened geometries are very common in astrophysics, from large scale of spiral galaxies down to the small scales of Saturn rings. The dynamics underlying the development of such structures is determined by the propagation of density waves and the important role of the disks self-gravity in their development has been clearly recognized. In the standard thin disks model, the effect of self-gravity is neglected, and only pressure supports the vertical structure. In contrast, the theory of self-gravitating accretion disks was less developed. From the observational point of view, there are already some clues that the disk self-gravity could be important both in the context of proto-stellar disks and in the accretion disks around super-massive black holes in the AGN.

The study of self-gravity generally, is difficult and most authors usually study the effects that relate to self-gravity either in the vertical structure of the disk or in the radial direction. Usually self-gravity occurs at large distances from the central objects and mainly in the direction perpendicular to the plan of the disk. But in the accretion disks around young stellar objects or pre-main-sequence stars, self-gravity can be important in all parts of the disk in both vertical and radial directions. Ghanbari & Abbassi (2004) introduced a model that shows self-gravity is an important effect in the equilibrium structure of thick disks around a compact object.

Viscous accretion through disks and the ensuing dissipation is known to be a very efficient process of converting gravitational energy into radiation. In particular accretion into the black holes allows to librate a sizeable fraction of the accreted matter. Duschl et al. (2000) has rest energy. Because detailed modeling of the structure and evolutions of accretion disks depends on the viscosity and its dependence on the physical parameters, choosing the best viscosity model is important. There is a belief that the molecular viscosity is inadequate to describe luminous accretion disks so that some kind of turbulence viscosity is required. Most investigators adopt the so-called α-model introduce by Shakura & Sanyev (1972) and Shakura & Sanyev (1973) that give the viscosity, ν, as the product of the pressure scale height in the disk (h), the velocity of sound, and a parameter α that contains all the unknown physics. One interprets this as some kind of isotropic turbulence viscosity ν = ν₁ = l₁ν₂, where l₁ is an length scale and ν₂ an characteristic velocity of a turbulence. One may then write \( \alpha = \left( \frac{v}{c_s} \right) \cdot \left( \frac{\nu}{\nu_1} \right) \). On general physics grounds neither term in parenthesis can exceed unity so that \( \alpha \leq 1 \). If initially \( \nu_1 > c_s \), shock waves would result in strong damping and hence a return to a subsonic turbulence velocity. The condition \( l_1 > h \) would require anisotropic turbulence since the vertical length scales.
are limited by the disk’s thickness, which is comparable to $h$. The models for the structure and evolution of accretion disks in close binary systems show that $\alpha$-model leads to a result that reproduce the overall observed behavior of the disks quite well. The $\alpha$-prescription is based on turbulence viscosity but there is no physical evidence for this as origin of turbulence.

2 CONDITION FOR SELF-GRAVITY IN ACCRETION DISK

The main properties of accretion disks is that they are often thin. This means that the typical length scale in the vertical direction, the disk thickness $h$, is much smaller than the radial distance from the central object. For example AGN disks are generally very thin, with aspect ratio $h/r \approx 0.001 \pm 0.01$. While proto-stellar disks are comparably a bit thicker, with $h/r \approx 0.1$. This has a significant impact on the conditions under which the disk is self-gravitating, and hence this intrinsic difference between the AGN case and the proto-stellar case will be reflected in a significant different behavior of these two kind of system in the self-gravitating regime.

One can estimate the importance of self-gravity by comparing the respective contributions to the local gravitational accelerations in the vertical and radial directions. The vertical gravitational acceleration at the disk surface is $2\pi G \Sigma \approx \frac{2\pi \Sigma}{M_*}$, respectively, for the self-gravitating and the purely Keplerian case. Self-gravitation is thus dominated in the vertical direction when

$$\frac{M_d}{M_*} \approx \frac{\pi^2 \Sigma}{M_*} > \frac{1}{2} \frac{h}{2r}$$

For typical proto-stellar and AGN disks with $h/r \sim 0.001 \sim 0.1$ this conditions translates into a condition for occurrence of vertical self-gravitation of $M_d/M_* > 0.0005 \sim 0.05$. Similar consideration leads us a similar condition for occurrence of self-gravity in radial direction. So when the mass of the disk is comparable with the mass of central star self-gravity in all kind of thin disks, from AGN to proto-stellar disks play an important role.

3 VISCOSE ENIGMA IN ACCREATION DISKS

The evolution of the disk and its time scale are governed by the value of the viscosity parameter $\nu$. The viscose timescale $\tau_{\nu\text{isc}}$ is given by:

$$\tau_{\nu\text{isc}} = \frac{R^2}{\nu}$$

It is not disputed that molecular viscosity is too small by many orders of magnitude and leads in almost all relevant situations to timescales surpassing the Hubble time. Some people believed that some kind of turbulence viscosity can solve this problem. This impasse was solved originally by Shakura & Sanyev (1973). The $\alpha$-prescription is based on the insight the molecularly viscous accretion disks are prone to exceedingly large Reynolds number, indicative of the onset of turbulence. It is worth noting that widely used $\alpha$-prescription for turbulence viscosity dose note invoke any particular mechanism at the origin of turbulence, it is a simple parametrization which is tailored to yield turbulence velocities that remain subsonic for $\alpha < 1$.

Despite a number of successful applications of the $\alpha$ prescription (for instance in explaining the dwarf nova phenomenon), this parametrization suffers from a number of inconveniences.

Duschl et al (2000) have shown that the standard model of thin accretion disk based on $\alpha$ model lead to inconsistencies if self-gravity play an important role. This problem arises from the parametrization of viscosity in terms of local sound velocity and vertical disk scale height.

For these multiple reasons it makes sense to investigate the basic properties of accretion disks built with the alternate $\beta$-prescription, which is observed in laboratory experiment shear flows. Recently some laboratory experiments show that in a self-gravitating flow Reynolds number is extremely high, it was thought that hydrodynamically turbulence viscosity probably plays an important role in the distribution of angular momentum in the accretion disks (Richard & Zahn 1999, Hure et al. 2001). Duschl et al. (2000) have presented a generalized accretion disks viscosity prescription based on the hydrodynamically-driven turbulence at the critical effective Reynolds number, the $\beta$-model, which is applied to both self-gravitating and non-self-gravitating disks and they have shown that it leads to standard $\alpha$-model.

4 COMPARING THE $\alpha$ AND $\beta$- VISCOSITIES

The laboratory experiment have been set up to study of hydrodynamical instabilities in differential rotating flows: a fluid is sheared between two cylinders rotating at different speeds. Only a few experiments have been run in the case where the angular momentum increase inwards, as in a Keplerian accretion disks, but it appears that the turbulence viscosity scales as:

$$\nu_t \propto R^3 \left| \frac{d\Omega}{dR} \right|$$

where $R$ is the distance from the rotation axis and $\Omega$ is angular velocity (Richard & Zahn 1999). In the Keplerian disks this formula is equivalent to:

$$\nu_\beta = \beta \Omega R^2 = \beta R v_\phi$$

which we shall call $\beta$-prescription.

One important property of the $\beta$-viscosity is that it depends only on the radius in a Keplerian disk, and does not involve local physical conditions while the standard $\alpha$-prescription depends on the local values of pressure scale height $H$ and sound velocity $c_s$:

$$\nu_\alpha = \alpha c_s H$$

Duschl et al. 2000 makes the sensible choice of equating the parameter $\beta$ with the inverse the critical Reynolds number $Re$, which they assume to be the same as in plane parallel shear flow ($Re \approx 10^5$ and therefore $\beta \approx 10^5$). However Hure et al. (2001) have shown that more smaller value for $\beta$ can be obtained which is derived from Couette-Taylor experiment. The only experimental data available in the case where angular momentum increases outward (such a Keplerian disk), are those obtained by Wendt (1933) and Taylor...
(1936), in which the inner cylinder is at rest; for this one derive $\beta = 10^{-5}$.

The $\alpha$-prescription has been designed such that the Mach number is smaller than unity for $\alpha \leq 1$. In contrast, subsonic turbulence is not guaranteed with the $\beta$-prescription. This restriction on the subsonic turbulence velocities is dictated by the fact that supersonic turbulence would be highly dissipative and therefore difficult to sustain. Moreover, the turbulence viscosity presented here, $\beta$-prescription, has been measured in a liquid, and its applications to compressible fluid can only be justified in subsonic range.

5 DISCUSSION AND REMARKS

The $\beta$-prescriptions is based on the assumption that the effective Reynolds Number of the turbulence does not fall bellow the critical Reynolds number. We think that this prescription yields a consistent model for fully self-gravitating disks where Duschl et al. have been shown the standard $\alpha$ model leads a inconsistency. Such $\beta$-disks model may be relevant for proto-planetary disks and Galactic disks. In the case of proto-planetary disk Duschl et al. have yield spectra that are considerably flatter than those due to non-self-gravitating disks, in better agreement with the observed spectra of these objects.

Self-gravitation which modifies the hydrodynamical equilibrium of accretion disks and affected all dynamical behavior and structure. In standard disk model self-gravitation was ignored. But to reach a more realistic picture of accretion disk, it should be take into account. In the proto-planetary disks self-gravity should be important in outer part of the disks where the disks mass is comparable with central accretor mass. Disk in AGN are thought to be relatively light in the sense that $\alpha$ central accretor mass. In AGN disks self-gravity paly a role but at large distance from central object (Shlosman & Begelman 1987).

It may seems that using other form of viscosity is not an important issue, because one should just change the mathematical form of the equations. But this changes the governing equations of the system that can effect the dynamical behaviors of the disks. This led us to explore the self-gravitating disks using other viscose prescriptions. However all viscose prescription have phenomenological backgrounds rather than physical confirmed backgrounds. $\beta$-viscose model have a good theoretical and experimental background.

Following the Duschl et al. (2000) suggestion for a $\beta$-prescription for viscosity, Abbassi et al have applied this model for a thin self-gravitating disk around newborn stars. Their results is quite different with standard alpha disks in the outer part of the disks where the self-gravity becomes important. In the inner part of the disks their solution converged to a standard $\alpha$ disks. They have shown that for $\beta$-disk Toomre parameter, $Q$, is less than one in all radial distances, while is more than one $\alpha$ model. As we know, when $Q < 1$ the gravitational fragmentation can be accrue. So gravitational fragmentation can be accrue in all radial distances and it can be used for planet formation in proto-planetary disks. They also show that in a $\beta$ model accretion rate and radial flow is more than the standard $\alpha$ model which means that this new prescription is more effective than the old version. It can reproduce accretion system in a less timescales.

Although the $\beta$-viscosity and the $\alpha$-viscosity may have comparable magnitude, they have very different effects both on the structure and on the stability steady Keplerian disks. It would be interest to investigate ADAF-type solution for follow any effects by $\beta$-viscosity model. An important property of $\beta$-disk is that they are viscously stable. Also unlike the $\alpha$-disks they are thermally stable for ideal cooling processes, such as Thompson scattering and free-free absorption (Hure et al. 2001).

In order to study to model and derive a realistic picture of a thin self-gravitating disk, one must investigate the energy exchange of the disks with environment. This require a mechanism to transfer the thermal energy from the disk to the outside.

Both $\alpha$ and $\beta$ models are phenomenological prescriptions for disk viscosity. In an actual model of viscosity, it is possible to combine these two model and establish and exact description for different regions of the disks. Also it can be prove by SPH simulation of a self-gravitating flow.

ACKNOWLEDGMENT

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are limited by the disk’s thickness, which is comparable to \( h \). The models for the structure and evolution of accretion disks in close binary systems show that \( \alpha \)-model leads to a results that reproduce the overall observed behavior of the disks quite well. The \( \alpha \)-prescription is based on turbulence viscosity but there is no physical evidence for this as origin of turbulence.

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\frac{M_d}{M_*} \sim \frac{\pi^2 \Sigma}{M_*} > \frac{1}{2} \frac{h}{r}
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For typical proto-stellar and AGN disks with \( h/r \approx 0.001 – 0.1 \) this conditions translates into a condition for occurrence of vertical self-gravitation of \( M_d/M_* > 0.0005 – 0.05 \). Similar consideration leads us a similar condition for occurrence of self-gravity in radial direction. So when the mass of the disks is comparable of the mass of central star self-gravity in all kind of thin disks, from AGN to proto-stellar disks plays an important role.

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