Microstates and near-horizon D-brane probes

Joris Raeymaekers
Department of Physics, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan

In [1], Gaiotto, Strominger and Yin proposed a novel way of counting black hole microstates by counting the quantum mechanical ground states of probe branes placed in the near-horizon black hole background. We discuss the generalization of this proposal to the case of two-charge D0-D4 ‘small’ black holes in type IIA. We also describe the construction of BPS D-brane probes in the near-horizon region of the 2-charge D1-D5 system in type IIB. Based on [5],[6].

1 Introduction

String theory has provided a microscopic understanding of large a class of supersymmetric black holes. In [1], Gaiotto, Strominger and Yin (GSY) proposed a novel way of counting black hole microstates by counting the quantum mechanical ground states of probe branes placed in the near-horizon black hole background.

GSY considered a class of black holes in type IIA on CY3 carrying D0 charge q0 and charges pA from D4-branes wrapping a generic 4-cycle in the Calabi-Yau space. The near-horizon region is an AdS2 × S2 × CY3 attractor geometry [2]. They considered the quantum mechanics living on D0-branes placed in this attractor background. The super-isometry group of the background acts as a superconformal symmetry group on the quantum mechanics [3]. It was proposed that the black hole microstates should be identified with the chiral primaries of this superconformal quantum mechanics. This can be seen as a concrete proposal for an AdS2/CFT1 duality.

An important ingredient in implementing this proposal is the property that N D0-branes in the attractor background can clump together to form nonabelian bound state configurations through a form of the Myers effect. For sufficiently large N, these can be described as D2-branes wrapping the S2 and carrying N units of worldvolume magnetic flux [4]. They are static with respect to the global AdS2 time coordinate and the corresponding Hamiltonian has a discrete bound-state spectrum.

Such probe branes experience a magnetic field along the Calabi-Yau directions induced by the Wess-Zumino coupling ∫ C(3) to the D4-branes in the background. The chiral primary states were shown to be in one-to-one correspondence to lowest Landau levels in this magnetic field, and their degeneracy was found to exactly reproduce the leading order entropy formula. However, this counting does not capture the corrections to the entropy formula subleading in the D4-charges pA.

When some of the D4-charges pA are taken to zero, 2-cycles in the attractor geometry shrink to zero size and the above analysis breaks down due to the fact that higher derivative corrections to the supergravity action become important. It is therefore a nontrivial question whether a GSY-inspired approach can still account for the black hole entropy of such black holes. We will consider here two examples of 2-charge ‘small’ black holes, carrying D0-charge and only one type of D4-brane charge. The examples discussed here preserve a large amount of supersymmetry, which is presumably the reason for their tractability. We will show that the GSY proposal correctly accounts for the black hole entropy in these examples [5].

It would also be of great interest to generalize the GSY proposal to account for the microstates of other black holes or D-brane systems. The first step in such a program is the identification of suitable BPS probe branes in the near-horizon geometry. In the second part of this note, we will consider the 2-charge D1-D5

* E-mail: joris@hep-th.phys.s.u-tokyo.ac.jp
system in type IIB, forming a black string in 6 dimensions with near-horizon $AdS_3 \times S^3$ geometry. We give a classification of half-BPS probe branes that are string-like in $AdS_3$ and discuss some of their properties \[6\].

The results discussed here have appeared in \[5, 6\], to which we refer for details and a more complete list of references.

2 Superconformal quantum mechanics of small black holes

The analysis of GSY in \[1\] was performed for ‘large’ black holes, which have a nonvanishing horizon area in the leading supergravity approximation. This is the case if the D4-brane charges $p^A$ are chosen such that $C_{ABC}p^Ap^Bp^C \neq 0$ where $C_{ABC}$ are the triple intersection numbers on CY3. Furthermore, all $p^A$ have to be taken to be nonvanishing and large in order for $\alpha'$ corrections to the background to be suppressed. When $C_{ABC}p^Ap^Bp^C = 0$, the horizon area vanishes in the supergravity approximation and higher derivative corrections cannot be neglected. On general grounds, a horizon is expected to appear once these corrections are taken into account \[7\], hence these objects are called ‘small’ black holes. We will address the GSY-inspired microstate counting in two examples.

As a first example, we consider type IIA compactified on $T^2 \times K_3$. The 4-dimensional effective theory is an $N = 4$ supergravity. The 2-charge system of interest consists of $q_0$ D0-branes and $p^1$ D4-branes, the latter wrapped on $K_3$. Such configurations are half-BPS and have a heterotic dual microscopic description as BPS excitations in the fundamental string spectrum, the Dabholkar-Harvey states \[8\].

In the supergravity approximation, the corresponding solution has vanishing horizon area, but a horizon is generated when one includes the leading 1-loop correction to the prepotential \[9\]. The near-horizon geometry is determined in terms of the charges by generalized attractor equations \[10\]. The resulting ten-dimensional IIA background is $AdS_2 \times S^2 \times T^2 \times K_3$ with nonzero 2-form and 4-form RR flux:

$$ds^2 = R^2 \left( -r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sin^2 \theta d\phi^2 \right) + 2dzd\bar{z} + 2r g_{ab} dz^a d\bar{z}^b$$

$$F^{(4)} = \frac{p^1}{4\pi} \sin \theta d\theta d\phi \wedge \omega_1; \quad F^{(2)} = \frac{R}{g_s} dr \wedge dt$$

Here, we have chosen coordinates $(z, \bar{z})$ on $T^2$ and $(z^a, \bar{z}^\bar{a})_{a, \bar{a}=1,2}$ on $K_3$, $g_{ab}$ is proportional to the asymptotic Ricci-flat metric on $K_3$, and $\omega_1$ is the normalized volume form on $T^2$. In units in which $2\pi\sqrt{\alpha'} = 1$, the radius $R$ of $AdS_2 \times S^2$ is given by $R = \frac{g_s}{2\pi} \sqrt{\frac{E}{q_0}}$. It’s important to note that the volume of $K_3$ is not fixed to a finite value at the horizon but varies like $r^2$. This is a consequence of the fact there are no D4-branes wrapped on the 2-cycles dual to the 2-cycles in $K_3$, hence the size of these cycles is not fixed by the attractor mechanism. This constitutes an important difference with the large black holes studied in \[1\], where all internal four-cycles have a large number of D4-branes wound on them.

As in \[1\], we will consider the quantum mechanics of a nonabelian configuration of $N$ D0-brane probes in the background \[1\] forming a fuzzy sphere of radius $R$. For $R^2/N \ll 1$, this system has an equivalent description in terms of a D2-brane wrapping the $S^2$ with $N$ units of flux turned on on its worldvolume. The terms contributing to the bosonic worldvolume action are

$$S = T_2 \int d^3 \sigma e^{-\phi} \sqrt{-\det(G + F)} + T_2 \int d^2 z C^{(3)} + T_2 \int d^2 z F \wedge C^{(1)}$$

The bosonic Hamiltonian, to quadratic order in derivatives and in the limit $R^2/N \ll 1$ is

$$H = \frac{1}{8RT} p_\xi^2 + \frac{R}{T^2 \xi^2} (P_z - A_z)(P_{\bar{z}} - A_{\bar{z}}) + \frac{32\pi^4 R^5}{g_s^2 N \xi^2} + \frac{R}{T} p_a \alpha^{a\bar{b}} P_{\bar{b}}$$

where $\xi = 1/\sqrt{7}$, $T = \frac{2\pi}{g_s} \sqrt{(4\pi R^2)^2 + N^2}$ and we have introduced a $U(1)$ gauge potential $A$ on $T^2$ obeying $dA = 2\pi p^1 \omega_1$. We denoted the canonical momenta conjugate to $z, z^\bar{a}$ by $P_z, P_{\bar{a}}$ respectively.
One notes that the Hamiltonian (2) is a sum of two decoupled parts: the first three terms describing the dynamics on $\mathbb{R} \times T^2$ (the $\mathbb{R}$ representing the the radial $AdS_2$ direction), and the last term describing the motion on $K_3$. This decoupling is a direct consequence of the radial dependence of the $K_3$ volume modulus in (1). The full quantum mechanics also contains sixteen fermions, which we have not displayed here.

The (super-)isometries of the background (1) act as symmetries on the quantum mechanics, giving the symmetry algebra a superconformal structure. Due to the decoupling of the $\mathbb{R} \times T^2$ and $K_3$ parts of the Hamiltonian, the symmetry group naturally splits into a group acting on the $\mathbb{R} \times T^2$ and $K_3$ parts of the wavefunction respectively. It turns out that the symmetry algebra of the $\mathbb{R} \times T^2$ part is the $N = 4$ superconformal algebra $su(1,1|2)_Z$, where $Z$ indicates the presence of a central charge. It contains the conformal algebra $sl(2,R)$, an $su(2)$ R-symmetry and and 8 fermionic generators. The motion on $K_3$ has the structure of an $N = 4$ supersymmetric quantum mechanics (SQM). The GSY proposal made in (1) states that the chiral primaries of the near-horizon D0-brane quantum mechanics are to be identified with the black hole microstates. In the case at hand, we should tensor the chiral primaries of $su(1,1|2)_Z$ with the supersymmetric ground states of the $N = 4$ SQM.

As in (1), the chiral primaries of $su(1,1|2)_Z$ can be shown to be in one-to-one correspondence with lowest Landau level wavefunctions for a particle moving on $T^2$ in the presence of the magnetic field $A$. The number of independent lowest Landau level wavefunctions is given by an index theorem and is equal to the first Chern number $\frac{1}{2π} \int_{T^2} dA = p^1$. These states should be tensored with supersymmetric ground states of the $N = 4$ SQM on $K_3$. A standard construction maps the supersymmetric ground states of $N = 4$ SQM on any Kähler manifold to Dolbeault cohomology classes, the even and odd forms corresponding to bosons and fermions respectively. Hence on $K_3$ we have an $N = 4$ SQM with 24 bosonic supersymmetric ground states.

Tensoring those together we find a total of $24p^1$ bosonic chiral primaries. Since the number of ground states doesn’t depend on the background D0-charge $q_0$, one can take $q_0 \to 0$ so that all of the D0 charge comes from the probes and is equal to $N$. There is a large degeneracy of states coming from the many ways the total number of D0-branes charge $N$ can be partitioned into smaller clusters, each cluster corresponding to a wrapped D2-brane that can reside in any of the $24p^1$ chiral primaries. The ground state degeneracies $d_N$ can be summarized in a generating function

$$Z = \sum_N d_N q^N = \prod_n (1 - q^n)^{-24p^1}.$$  

This gives the asymptotic degeneracy at large $N$

$$\ln d_N \approx 4π\sqrt{Np^1}$$

which indeed equals the known asymptotic degeneracy obtained from microscopic counting [8] or from the supergravity description incorporating higher derivative corrections [9]. We note that the known subleading corrections to the entropy are not captured by the above partition function, and their incorporation in this framework remains an open problem.

The above analysis can be repeated for $K_3$ replaced by a four-torus $T^4$, under some additional assumptions. The corresponding small black hole is 1/4 BPS in the effective 4-dimensional $N = 8$ supergravity. Since all corrections to the prepotential vanish in this case, the corrections that generate the horizon are expected to come from non-holomorphic corrections to the supergravity equations, and it is not known how to incorporate these systematically at present. We shall be cavalier and simply assume that the near-horizon limit of the corrected background is still of the form (1), with the $K_3$ metric now replaced by the flat metric on $T^4$ and possibly with a different value of the constant $R$. The above analysis can then be repeated, the only difference coming from the counting of ground states of the $N = 4$ SQM, now corresponding to the Dolbeault cohomology of $T^4$. This gives 8 bosonic and 8 fermionic ground states. The partition function is now

$$Z = \prod_n \left( \frac{1 + q^n}{1 - q^n} \right)^{8p^1}.$$
This gives the asymptotic degeneracy

\[
\ln d_N \approx 2\sqrt{2}\pi \sqrt{Np^1}
\]

which is in agreement with the known degeneracy from microscopic counting \([8]\).

3 Supersymmetric D-branes in the D1-D5 background

In order to put the GSY proposal on a sounder footing and address some of its difficulties, it would be of great interest to extend it to other black holes or D-brane systems. Here, we will consider the well-studied 2-charge D1-D5 system in type IIB compactified on \(M\) (where \(M\) can be \(T^4\) or \(K3\)), forming a black string in 6 dimensions. The first step is the identification of suitable BPS probe branes in the near-horizon geometry which is \(AdS_3 \times S^3 \times M\). Even though the D1-D5 system is T-dual to the small D0-D4 black holes considered above, T-duality does not commute with taking the near-horizon limit, making a direct mapping between near-horizon microstates difficult. We will instead look directly for supersymmetric D-branes in the near horizon region with properties similar to the near-horizon microstates of the D0-D4 small black holes.

The near-horizon geometry reads, in Poincaré coordinates:

\[
ds^2 = r_1 r_5 [u^2 (-dt^2 + dx^2) + \frac{du^2}{u^2} + d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\phi^2)] + \frac{r_1}{r_5} ds_M^2
\]

\[
F^{(3)} = \frac{2r_3^2}{g} \left[ u dt \wedge dx \wedge du + \sin^2 \psi \sin \theta d\psi \wedge d\theta \wedge d\phi \right]; \quad e^{-\phi} = \frac{1}{g} \frac{r_5}{r_1}
\]

We restrict attention to branes that are string-like in \(AdS_3\) and span an \(AdS_2\) subspace. These can be parameterized as

\[
u = \frac{C}{x}
\]

for some constant \(C\). Such branes are static with respect to the global time coordinate, and furthermore lead to good open string boundary conditions \([13]\). We allow the branes to extend in the compact directions and carry arbitrary worldvolume fluxes (and hence also induced lower-dimensional D-brane charges).

The near-horizon geometry preserves 16 supersymmetries, which split into 8 ‘Poincaré’ supersymmetries that extend to the full asymptotically flat solution and 8 ‘conformal’ supersymmetries that exist only in the near-horizon limit. The condition for a D-brane probe to preserve some supersymmetry can be written as \([11]\)

\[(1 - \Gamma) \epsilon = 0\]

where \(\Gamma\) (satisfying \(\text{tr} \Gamma = 0, \ \Gamma^2 = 1\)) is the operator entering in the \(\kappa\)-symmetry transformation rule on the D-brane and depends on the brane embedding as well as the worldvolume gauge fields; \(\epsilon\) are the Killing spinors of the background pulled back to the world-volume.

Referring to \([6]\) for calculational details, one finds in this manner a large variety of D-branes preserving half of the 16 near-horizon supersymmetries. All of them turn out to preserve half of 8 the ‘Poincaré’ supersymmetries as well. They are summarized in the following table which lists the submanifold spanned by the brane as well as possible restrictions on the embedding and/or worldvolume gauge fields.

| brane | \(AdS_3\) | \(S^3\) | \(M\) | restrictions |
|-------|----------|--------|-------|-------------|
| D1    | \(AdS_2\) | \cdot   | \cdot  |             |
| D3    | \(AdS_2\) | \cdot   | 2-cycle \(\Sigma\) | \(\Sigma\) holomorphic |
| D5    | \(AdS_2\) | \cdot   | \(M\)   |             |
| D7    | \(AdS_2\) | \(S^2\) | \(M\)   | \(F_M\) antiselfdual |
The solutions come in two types: branes of the first type are pointlike on the $S^3$ while branes of the second type wrap an $S^2$ within $S^3$. The latter are dipolar as the $S^2$ is contractible and is stabilized by worldvolume flux \([12]\).

In the second category, let’s discuss the D3-branes spanning an $AdS_2 \times S^2$ in a little more detail (see also \([13, 14]\)). The electric and magnetic worldvolume fields induce fundamental string charge $q$ and D-string charge $p$. They can be seen as $(p, q)$ strings puffed up to form a dipolar D3-brane through a form of the Myers effect. The size of the $S^2$ is related to the fundamental string charge $q$ and its maximum value leads to an “exclusion bound” $q \leq Q_5$. This bound is similar to the upper bound on the angular momentum for lowest Landau levels in the D0-D4 system discussed above, which is set by the background D4-charge. However, it is not clear whether the $AdS_2 \times S^2$ branes are related to microstates of the D1-D5 system. Since they intersect the boundary of $AdS_3$, they are not states in the dual CFT, but rather conformal defects as in \([15]\). It would be interesting to have a better understanding of these and the other branes in the above table from the point of view of the dual CFT.

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