The Use of Distance Between Soil Layers in Predicting the Hydraulic Conductivity of Granular Soils

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Abstract: This study presents a new empirical approach that can be used to determine the hydraulic conductivity of saturated granular soils. This empirical approach can be obtained through the relationship between the uniformity coefficient of the soil (Cu) and the distance between soil layers (S). For the granular filter materials that have the same particle size distribution, there can be different distances between layers of the soil, depending upon the soil’s density. The distance between soil layers can be obtained by dividing the length of the imaginary line by the number of intersected particles within the length. By using the distance between soil layers, the whole grain size distribution and the soil porosity are taken into consideration, which reflects the degree of compaction. Therefore, this empirical approach is a good representation of hydraulic conductivity.

1. Introduction

Hydraulic conductivity is the property that describes the case with which flow takes place through a porous medium[1]. It is one of the most important physical properties of soil used in geotechnical engineering. Measuring the hydraulic conductivity of soils in laboratory and field is costly and time-consuming. However, before some years ago, the scholars started to estimate the hydraulic conductivity based on the grain size distribution of soils. Many techniques are conducted for determination of hydraulic conductivity in laboratory or field condition, accurate estimation of hydraulic conductivity in the field environment is limited by the lack of precise knowledge of aquifer geometry and hydraulic boundaries[2, 3]. Methods of estimating hydraulic conductivity from an empirical formula based on grain size distribution characteristics have been developed and used to overcome these problems[4]. The best estimation of hydraulic conductivity is indicated by using several empirical formulas based on grain size analysis to determine the hydraulic conductivity of aquifer materials in the Jimeta-yola area[5].

Many empirical formulas have been established to determine hydraulic conductivity; (Vukovic and Soro 1992) presented a general formula [7].
\[ k = \frac{g}{v} \times C \times f(n) \times d_e^2 \quad (1) \]

Where \( k \) = hydraulic conductivity; \( g \) = acceleration due to gravity; \( v \) = kinematic viscosity; \( C \) = sorting coefficient; \( f(n) \) = porosity function, and \( d_e \) = effect grain diameter.

Based on sand filter experiments, (Hazen 1892) developed a formula for the determination of hydraulic conductivity of uniformly graded sand[8].

\[ k = \frac{g}{v} \times 6 \times 10^{-4} \left( 1 + 10(n - 0.26) \right) d_{10}^2 \quad (2) \]

Where \( n \) is porosity, \( d_{10} \) is a particle size for which 10% of the soil is finer.

Kozeny-Carman equation is one of the most widely accepted equations for determination of the hydraulic conductivity. This equation was proposed by Kozeny first and then modified by Carman (1937, 1956) to become the Kozeny-Carman equation[9, 10].

\[ k = \frac{g}{v} \times 8.3 \times 10^{-3} \left( \frac{n^3}{(1-n)^2} \right) d_{10}^2 \quad (3) \]

Terzaghi (1964) proposed an empirical formula. However, related to Cheng et al. [11], this formula is most applicable for large-grain sand[12].

\[ k = \frac{g}{v} \times C_t \times \left( \frac{n + 0.13}{(1-n)^3} \right) d_{10}^2 \quad (4) \]

Where \( C_t \) is sorting coefficient and \( 61 \times 10^{-3} < C_t < 107 \times 10^{-3} \).

Idraratna et al. (2012) presented a new semi-empirical approach for predicting the hydraulic conductivity of noncohesive soil through a constriction size-based technique[6].

\[ k = 3.691 \left( D_{cm}^m \right)^{1.644} \quad (5) \]

Where \( D_{cm}^m \) is mean constriction size that determined from the constriction size distribution curves of the soils.

2. Distance between soil layers

The distance between soil layers is normally assumed to be equal to the soil particle diameter corresponding to the 50% mass finer \( D_{50} \) [13, 14], or it may be assumed equal to \( D_{50} \) based on the surface area of the particle [15], or \( D_{50} \) based on number of particles [16]. None of these methods have a solid physical base, and none of them take the effect of density. Moraci et al. (2012) took the effect of density in consideration and can be obtained from Eq.(6) and Eq.(7) [17], as shown in Figure 1.

\[ S = 0.82D_{50} \quad \text{for the relative density} \ 100\% \quad (6) \]

\[ S = D_{50} \quad \text{for the relative density} \ 0\% \quad (7) \]

Wu et al. (2012) suggested another method to determine the distance between soil layers[18].

\[ S = 0.5D_{50} \quad \text{for the loosest state (relative density} \ 0\%) \quad (8) \]

\[ S = 0.85S_{\text{loosest state}} \quad \text{for the densest state (relative density} \ 100\%) \quad (9) \]

The distance between soil layers can be determined experimentally, and it only requires compacting the soil in a transparent cylinder to the desired density, then simply count the number of particles that intersected by a straight line of a certain length as shown in Figure 2.

As well, the distance between layers can be determined in the numerical model constructed by PFC3D, as shown in Figure 3, by counting the number of the soil balls that intersected by a line pass through the soil sample.
Figure 1. The distance between soil layers.

Figure 2. The experimental approach to determine the distance between soil layers.

Figure 3. The numerical sample constructed by PFC3D (a) top view, (b) side view, (c) the particle intersected by the straight line.

Tomkeiff (1945) gave a mathematical approach to determining the average length $g$ of each particle diameter $x$ taking in consideration different positions and directions through the soil, and AB is a straight line pass through the granular soil then it will intersect soil particles as well as it will pass through the voids among the particles as shown in Figure [19].

\[ g = 4ax \]  

(10)

Figure 4. AB line through a soil sample.

Where $a$ is a constant $= 0.125$ for natural soil, while it equals to $1/6$ for spherical particles [20]. So the number of intersected particles through the soil by the line AB of length L can be computed as Aberg [20].

\[ N_g = \frac{(1-n)l}{4a} \int_0^1 \frac{dy}{x(y)} \]  

(11)

Where $n$ is the soil porosity; $x(y)$ is the particle diameter, and $dy$ is the difference in finer percent between the particles with diameters $x$ and $(x + dx)$, as shown in Figure. The distance between soil layers can be determined by using the next equation.

\[ S = l/N_g = 1/(\frac{(1-n)}{4a} \int_0^1 \frac{dy}{x(y)}) \]  

(12)
This equation considers the whole grain size distribution curve; it also takes porosity, which reflects the degree of compaction.

3. The empirical approach of hydraulic conductivity

In this section, the author established a new equation to determine the hydraulic conductivity of soil $k$ depending on the distance between soil layers $S$ and the uniformity coefficient of the soil $Cu$, based on the results of 34 tests conducted on granular soils by the former researches (Sherard et al. 1984; Indraratna et al. (1996). The GSD curves of soils for the 34 tests are shown in Figure 6.6 and Figure 6.7. The gradations of tested soils for 34 tests are numbered and listed in Table 1. Uniformity coefficient, $Cu$, porosity, $n$, $D_{10}$, and measured permeability are also listed in Table 1.

| ID | Grad | $Cu$ | $D_{10}$ | $n$ | $S$ | $k$ (cm/s) measured | $S/Cu^{0.5}$ | K-C model$^{b}$ | Hazen model | Current model |
|----|------|------|----------|-----|-----|---------------------|--------------|--------------|-------------|--------------|
| 1  | 1    | 1.34 | 0.11     | 0.41| 0.12| 0.012               | 0.100        | 0.019        | 0.012       | 0.011        |
| 2  | 2    | 1.39 | 0.22     | 0.41| 0.24| 0.030               | 0.201        | 0.080        | 0.050       | 0.037        |
| 3  | 3    | 1.4  | 0.65     | 0.41| 0.70| 0.310               | 0.595        | 0.676        | 0.423       | 0.238        |
| 4  | 4    | 1.45 | 0.1     | 0.405| 0.11| 0.008               | 0.091        | 0.015        | 0.010       | 0.010        |
| 5  | 7    | 2.31 | 0.09     | 0.445| 0.14| 0.009               | 0.092        | 0.018        | 0.009       | 0.010        |
| 6  | 8    | 2.3  | 0.2     | 0.44| 0.31| 0.032               | 0.203        | 0.082        | 0.040       | 0.038        |
| 7  | 9    | 2.3  | 0.45    | 0.43| 0.68| 0.200               | 0.449        | 0.409        | 0.202       | 0.147        |
| 8  | 10   | 1.87 | 0.1     | 0.42| 0.13| 0.010               | 0.097        | 0.019        | 0.010       | 0.011        |
| 9  | 15   | 2.22 | 0.1     | 0.43| 0.18| 0.010               | 0.121        | 0.020        | 0.010       | 0.016        |
| 10 | 16   | 2.66 | 0.1     | 0.44| 0.17| 0.011               | 0.102        | 0.021        | 0.010       | 0.012        |
| 11 | 17   | 3.23 | 0.1     | 0.44| 0.15| 0.012               | 0.084        | 0.022        | 0.010       | 0.008        |
| 12 | 18   | 4.01 | 0.1     | 0.44| 0.19| 0.022               | 0.097        | 0.022        | 0.010       | 0.011        |
| 13 | 19   | 1.85 | 0.1     | 0.421| 0.13| 0.009               | 0.097        | 0.018        | 0.010       | 0.011        |
| 14 | 22   | 3.8  | 0.11    | 0.435| 0.21| 0.022               | 0.107        | 0.025        | 0.011       | 0.013        |
| 15 | 23   | 3.25 | 0.1     | 0.449| 0.18| 0.009               | 0.102        | 0.024        | 0.011       | 0.012        |
| 16 | 33   | 3.4  | 0.22    | 0.44| 0.41| 0.091               | 0.220        | 0.108        | 0.049       | 0.043        |

Sherard et al. (1984)

| ID | Grad | $Cu$ | $D_{10}$ | $n$ | $S$ | $k$ (cm/s) measured | $S/Cu^{0.5}$ | K-C model$^{b}$ | Hazen model | Current model |
|----|------|------|----------|-----|-----|---------------------|--------------|--------------|-------------|--------------|
| 17 | 5$b$ | 3.51 | 0.15     | 0.49| 0.55| 0.049               | 0.294        | 0.238        | 0.058       | 0.071        |
| 18 | 5$b$ | 3.51 | 0.15     | 0.40| 0.47| 0.022               | 0.250        | 0.094        | 0.058       | 0.054        |
| 19 | 7$b$ | 5.43 | 0.24     | 0.50| 0.72| 0.051               | 0.309        | 0.334        | 0.076       | 0.077        |
In the paper published by Sherard et al. (1984), permeability tests were done on different granular soils. For each soil, tests were done on two different specimens, with about 70% relative density and zero relative density, as shown in Table 1, using Subscript 0 and 70 to distinguish them[21].

The author believes that the hydraulic conductivity, \( k \), is closely related to the distance between soil layers, so Eq.(12) is used to establish the empirical equation of \( k \). Table 1 shows the computed values of the distance between soil layers for the 34 tested soils.

Potting the data for both hydraulic conductivity and the distance between soil layers over the square root of the uniformity coefficient of the soil (S/Cu\(^{0.5}\)) on logarithmic scales, it is found that the data points satisfy a polynomial function as shown in Figure 6, the generalized polynomial function defined by

\[
k = \alpha (S/Cu^{0.5})^3 + \beta (S/Cu^{0.5})^2 + y \left( \frac{S}{Cu^{0.5}} \right) + \delta
\]

(13)

where \( \alpha, \beta, y, \) and \( \delta \) are constants, and the unit of \( S \) is mm in general.
The best-fit values of the parameters $\alpha$, $\beta$, $\gamma$, and $\delta$ can get from the regression analysis. For the samples given in Table 1, the best-fitted parameters $\alpha$, $\beta$, $\gamma$, and $\delta$ are -0.0357, 0.4817, 0.2328, and -0.0384, respectively. The hydraulic conductivity can be written as:

$$k = -0.0375(S/Cu^{0.5})^3 + 0.4817(S/Cu^{0.5})^2 + 0.2328 \left( \frac{S}{Cu^{0.5}} \right) - 0.0384$$  \hspace{1cm} (14)

Using Eq.(14), hydraulic conductivities for the 34 soils of Table 1 can be calculated, and are also listed in Table 1. It is seen that the predictions from Eq.(14) is very close to the test data.

To verify Eq.(14) further, the relation for 34 data of the tested and predicted by Eq.(14) against $(S/Cu^{0.5})$ are plotted and shown in Figure 7. The predictions agree well with the test data.

To further reveal the superiority of Eq.(13), the author uses the Kozeny-Carman model and Hazen model to calculate the hydraulic conductivities of the 34 soils, and compare them with the test data and $k$ predicted from Eq.(13). Table 1 shows the calculation of hydraulic conductivity from the Kozeny-Carman model, Hazen model, and the current model (Eq.(13)). It is seen that the predictions from the current model for most data are much closer to the test data than those from the two other models.

To compare the differences more clearly between test data and predictions by the Kozeny-Carman model, Hazen model, and current model, Figure 8 illustrates the relation between test data and predictions from three models. The Figure also shows that the predictions from the current model are much closer to the test data than predictions from the two other models.
Figure 8. Comparison between measured and predicted hydraulic conductivity

Eq.(13) is based on the 34 tested soils tabulated in Table 1, whose $1.34 \leq Cu \leq 5.43$ and $0.09 \text{mm} \leq D_{10} \leq 9.6 \text{ mm}$. It is important to note that the distance between soil layers $S$ is very useful to determine the hydraulic conductivity $k$ because it takes the whole grain size distribution and considers porosity.

4. Conclusion
The hydraulic conductivity of granular soils is often determined based on the empirical formulations that can be obtained from laboratory tests. Most of the empirical formulations consider in their formula one or two diameters of the soil particles; however, in our model, we considered the distance between soil layers that takes diameters of whole-grain size distribution and porosity of the soil.

The proposed empirical model is more realistic, as it considers the entire particle gradation together with the corresponding relative density, thereby giving a better alternative for determining the hydraulic conductivity representing a wider array of particle sizes and shapes.

Based on the analysis of soil samples data, this study has demonstrated that the proposed model can give very good accurate values of hydraulic conductivity, and it is more accurate than the traditional Hazen or Kozeny–Carmen theoretical equations.

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