Detection of the vortex-liquid phase in superconducting films by Nernst effect

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Abstract. To clearly detect the vortex-liquid state of a superconducting film, we constructed an experimental system for measuring the Nernst signals in a film sample. We measured Nernst signals $N_{\text{in}}$ in an amorphous Mo$_x$Ge$_{1-x}$ film with a thickness of 300 nm. As the field is increased in the vortex-solid phase, $N_{\text{r}}$ rises from zero at the boundary between the vortex-solid state and the vortex-liquid state, showing a peak in the middle of the liquid phase, and finally decreases. From a linear extrapolation of $N_{\text{r}}$ after the peak to $N_{\text{r}}=0$, we roughly estimated the upper bound of the liquid state. These results imply that the Nernst measurement could be an alternative to a resistivity measurement for detecting the vortex-liquid state. It would be helpful when the resistivity measurement is inconclusive in detecting vortices such as in an insulator phase in the vicinity of the superconductor-insulator transition in two dimensions.

1. Introduction
In the vortex-liquid state, freely mobile vortices give rise to non-zero resistivity in a superconductor. Using the superconducting order parameter with the phase $\theta$ and the amplitude $\Psi$, the liquid state is characterized by large fluctuations of $\theta$ due to the free vortex motion induced by thermal or quantum fluctuations. In the vortex-solid state, by contrast, both $\theta$ and $\Psi$ have global coherence and the resistivity falls to zero. Both states are clearly distinguished from resistivity. On the other hand, the vortex-liquid state shows a crossover to the normal state with increasing temperature $T$ or magnetic field $B$. Both states are thermodynamically equivalent and the crossover field $B_{c2}$ is empirically estimated from resistivity. In the normal state, the resistivity decreases with decreasing temperature because of the superconducting fluctuations of both $\theta$ and $\Psi$, which is described as the Gaussian fluctuations [1, 2]. This behaviour is smoothly connected to the resistivity drop in the vortex-liquid state. Such a gradual change is also seen in the magnetic susceptibility.

A Nernst effect measurement is an alternative tool to detect the vortex-liquid state. When the temperature gradient $\nabla_x T$ is applied to a superconductor, vortices are driven by a thermal force and a voltage $V_{xy}$ is generated transversely to the vortex motion, which is called a vortex Nernst effect [3-10]. The Nernst signal is defined as $N = E_{xy}/\nabla_x T$ using a transverse electric field $E_{xy}$. When the pinning effect is negligible, the contribution of vortices is described by

$$N = E_{xy}/\nabla_x T = B_z v_x/\nabla_x T = \rho_F S_\phi / \Phi_0,$$

where $B_z$ is magnetic field, $v_x$ average velocity of vortices, $\rho_F$ flux-flow resistivity, $S_\phi$ transport
entropy per unit length of a vortex, and $\Phi_0$ quantized magnetic flux. For the last substitution, the balance between the thermal force and the viscous force is taken into account.

Previously, large vortex Nernst signals have been observed in the vortex-liquid state of high-$T_c$ superconductors [5-8], where most of studies have been carried out on bulk samples. For thin films, on the other hand, very limited measurements have been performed so far, especially in the low temperature regime. At low temperature, an applied heat current must be small to prevent a thermal bath from warming up, which makes a resulting $\nabla_x T$ small and Nernst measurement difficult. Pourret et al. have succeeded in Nernst measurements for amorphous NbSi films with thicknesses of 12.5 and 35 nm below 1 K and clearly detected vortex Nernst signals [8-10]. They have roughly estimated $B_{c2}$ from a linear extrapolation of $N$ [10]. However, a boundary between the vortex-solid state and the liquid state, where $N$ falls to zero, was not identified because of the broaden liquid state due to the two dimensionality of the sample.

In this work, we constructed an experimental system for the Nernst effect measurement in a film sample. The measurements were performed on an amorphous Mo$_x$Ge$_{1-x}$ film with a thickness of 300 nm, which is thicker than the amorphous thin films in the previous study [9, 10]. Our sample shows three-dimensional properties and the vortex-liquid state estimated from linear resistivity $\rho$ is narrow [12]. We observed a clear peak of $N$ in the liquid state and estimated $B_{c2}$ in the same manner as the previous studies. Moreover, the solid-liquid boundary was clearly detected from a sharp rise of $N$ as well as from the measurement of $\rho$.

2. Experimental
An amorphous Mo$_x$Ge$_{1-x}$ film with a thickness of 300 nm was sputtered onto a glass substrate mounted on a rotating stage kept at 10°C with water cooling. Since the pinning strength is weak and the pinning sites are randomly distributed in our amorphous sample, we can detect intrinsic vortex motion [13, 14]. Nernst signals from quasiparticles are also negligible because
Figure 3. Field dependences of (a) the resistivity at 10 µA and (b) the Nernst signal at ∆T = 0.10 K. Both were obtained at 5.00 K. The gray plots in (a) contain both positive and negative values, indicating that they are below the sensitivity of our experiment. The Nernst signal rises at the boundary between the vortex-solid and the liquid states marked by the dashed line, which is determined by the sharp rise of the resistivity from zero. The dotted line shows an extrapolation of the linear part of the Nernst signal after showing the peak.

a mean free path of the quasiparticles is the order of the interatomic distance. The transition temperature where ρ falls to zero is 6.75 K at zero field.

A measurement system for the Nernst effect in a film sample is schematically illustrated in figure 1. Using this apparatus, the Nernst effect measurement and a standard four-terminal measurement of ρ were performed. One side of the substrate was attached to a Cu thermal bath and the other side was connected to a heater to supply heat current. We used a glass substrate of low thermal conductivity with a thickness of 0.15 mm. A temperature difference, ∆T = T_{high} − T_{low}, was measured by two RuO2 thermometers spaced by 4.4 mm (= L), which were mounted on the substrate in the vicinity of the high-temperature side and the low-temperature side of the sample film. An additional thermometer and heater were also glued on the Cu bath to control the bath temperature. V_{xy} was measured between Ag electrodes (+H and -H) spaced by 1.7 mm (= l) using a nanovoltmeter. The magnetic field was applied perpendicular to the film plane. Nernst signals were obtained from $N = \frac{E_{xy}}{3} = \frac{V_{xy}/l}{\Delta T/L}$.

3. Results and discussions

First, we performed an experiment to test the linear relationship between $V_{xy}$ and ∆T, which is necessary to obtain N from the measured $V_{xy}$ and ∆T. Figure 2 shows a typical response of $V_{xy}$ as a function of ∆T at 5.00 K and 3.5 T, where the large N is observed. A linear behaviour between the $V_{xy}$ and ∆T is clearly visible. At each measurement, the background thermoelectric voltage generated at ∆T = 0 is subtracted.

Next, we show ρ measured as a function of field at 5.00 K and 10 µA in figure 3(a). ρ rises from the background level at 2.9 T as marked by the dashed line, which corresponds to the boundary between the vortex-solid state and the vortex-liquid state. With further increasing field, ρ steeply increases and above 3.5 T gradually reaches the normal-state resistivity $\rho_n = 160 \mu\Omega$ cm.
Figure 3(b) shows a field dependence of $N$ at $T = 5.00$ K and $\Delta T = 0.10$ K ($T_{\text{high}} = 5.05$ K and $T_{\text{low}} = 4.95$ K). To subtract the thermoelectric voltage caused by misalignment of Hall electrodes, we extracted antisymmetric contributions with respect to magnetic field reversal. With increasing field, $N$ also rises from zero at 2.9 T and then decreases after showing a peak around 3.6 T. The behaviour is consistent with what is expected from the vortex Nernst effect in the vortex-liquid state [3-10]. The rapid growth of $N$ from the solid-liquid boundary results from the reduced pinning effects due to thermal fluctuations. The decrease of $N$ is attributed to a decrease of $S_\phi$.

According to the theory, $S_\phi$ is proportional to $(B_{c2}^2 - B)$ near $B_{c2}$ and vanishes at $B_{c2}$ [15, 16]. To roughly estimate a value of $B_{c2}$ from $N$, we extrapolated the linear part of the data after showing the peak as shown with the dotted line [7, 10]. The deviation of $N$ from the dotted line becomes pronounced above 4.3 T, which probably originates from the Nernst signals due to the Gaussian fluctuations [8-11]. We can estimate $B_{c2}$ to be around 4.7 T from the extrapolation to $N = 0$. This value corresponds to the field at which $\rho = 0.96 \rho_n$. From these results, we consider that we detected the vortex-liquid state in the thick amorphous Mo$_x$Ge$_{1-x}$ film using the Nernst measurements. This technique can be applied to thinner (2D) films, for example, undergoing the superconductor-insulator transition (SIT). It is of interest to explore the vortices on an insulating side of the SIT, the so-called Bose-insulator phase, which is a long-standing problem [17].

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