Hard exclusive electroproduction of decuplet baryons in the large $N_c$ limit

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The cross sections and transverse spin asymmetries in the hard exclusive electroproduction of decuplet baryons are calculated in the large $N_c$ limit and found to be comparable to that of octet baryons. Large $N_c$ selection rules for the production amplitudes are derived, leading to new sensitive tests of the spin aspects of the QCD chiral dynamics both in the nonstrange and strange sectors. Importance of such studies for the reliable extraction of the pion form factor from pion electroproduction is explained.

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Recently the QCD factorization theorem was shown to hold for a wide range of two-body exclusive deep inelastic processes with $t = \Delta^2$ and $x_B = Q^2/2p \cdot q$ fixed:

$$\gamma_\ell^T(q) + p \to M(q - \Delta) + B'(p + \Delta).$$

It asserts that at sufficiently large $Q^2$, the amplitude has the form:

$$\sum_{i,j} \int_0^1 dz \int dx_1 f_i/p(x_1, x_1 - x_B; t, \mu) \times H_{ij}(Q^2 x_1/x_B, Q^2 z, \mu) \phi_j(z, \mu) + \text{power-suppressed corrections.}$$

The nonzero momentum transfer $\Delta$ for forward angle scattering introduces a skewedness $\xi$, which is defined through the light cone fraction $\Delta^+ = -(2\xi)P^+$ (with $P = (p_p + p_B')/2$ - the average four-momentum of the initial nucleon and the recoiling hadron $B'$). In Eq. (3), $f$ are the skewed parton distributions (SPD) (see formal definition below), $\phi$ is the minimal Fock component of the light-front wave function of the meson, and $H$ is a hard-scattering coefficient, computable in powers of $\alpha_s(Q)$. By virtue of the factorization theorem, SPD’s are universal and enter in a wide range of hard exclusive two body processes. Model calculations of SPD’s are currently possible within the QCD chiral models for intermediate $x_B$ [4].

One may expect that eventually it would be possible to calculate SPD’s for intermediate $x_B$ using lattice QCD.

Hence reactions (1) provide a new tool to probe the quark-gluon structure of various mesons and baryons as well as the hard QCD dynamics. Many studies focused on the production of vector mesons at HERA energies where this process is primarily sensitive to the gluon dynamics in nucleons at small $x_B$ and to the properties of the vector meson wave functions. Studies at larger $x_B$ - at lower energies - provide a wide range of new opportunities. One of the most promising ones theoretically and feasible experimentally is the production of pseudoscalar mesons. So far the main focus was the reaction $\gamma_\ell^T + N \to \pi(q) + N'$ [5,6] where $N'$ is an octet baryon [7]. The analysis based on the chiral QCD dynamics has demonstrated that the SPD’s which disappear in the case of diagonal parton densities play an important role in these processes, leading in particular to a strong dependence of the differential cross section on the transverse polarization of the target.

One of the intriguing questions of medium-energy QCD dynamics is the differences and similarities in the structure of baryons belonging to the different $SU(3)$-multiplets. In particular, a naive constituent quark model suggests that they are similar, while there are suggestions that due to a strong attraction between the quarks in the spin-isospin zero channel, diquark correlations should be important in the octet baryons but not in the decuplet [8]. At the same time the chiral models suggest that in the large $N_c$ limit, nucleons and $\Delta$ isobars are different rotational excitations of the same soliton [9].

Clearly an ability to compare experimentally the wave functions of the decuplet and octet would be very helpful to discriminate between different ideas. So far only one process could be used for these purposes - electroexcitation of $\Delta$ isobars [10]. Here we want to explore the potential of the process $\gamma_\ell^T + N \to \pi + \Delta$ as well as the DVCS process $\gamma^* + N \to \gamma + \Delta$ for these studies. In the experiments with low resolution in the mass of the recoiling system ($\Delta M \approx 300$ MeV for HERMES in the current set-up), the estimates of $\Delta$ production are necessary to extract the $N \to N$ SPD’s from such data. Note also that the study of these processes would allow to make a more reliable separation of the pion pole contribution in the electroproduction of pions which is mandatory for the measurement of the pion elastic form factor. We use the large $N_c$, to predict a number of striking characteristics of these processes. In particular we predict large absolute cross sections for a number of channels including those with production of octet strange baryons (extension to decuplet strange baryons will be considered in a
future work). We also predict large transverse spin asymmetries for $\Delta$ and $\Lambda$ production, related to the peculiar feature of the chiral QCD. Hence study of these processes can provide unique tests of the soliton type approach to baryon structure. We will mostly focus on the ratios of cross sections for the different channels and on the spin asymmetries which are likely to be less sensitive to higher twist effects and hence could be explored already using the HERMES detector and TJNAF at higher energies. We hope that this letter will encourage upgrades of the detector recoil capabilities which are necessary for the study of some of the discussed effects as well as measurements with transversely polarized targets.

We define a new set of SPD's for the axial $N \to \Delta$ (isovector) transition, denoted as the skewed distributions $C_i^{(3)}$ (functions of $x, \xi$, and $\Delta^2$), where we show only the dominant (at large $N_c$) magnetic contributions with transversely polarized targets. In the large $N_c$ limit, the nucleon and $\Delta$ are rotational excitations of the same classical object-soliton. This allows to number a number of relations between $N \to N$ and $N \to \Delta$ SPD's. For $C_1^{(3)}$ and $C_2^{(3)}$, they have the form:

$$C_1^{(3)}(x, \xi, \Delta^2) = \sqrt{3} \bar{H}^{(3)}(x, \xi, \Delta^2),$$

$$C_2^{(3)}(x, \xi, \Delta^2) = \sqrt{3/4} \bar{E}^{(3)}(x, \xi, \Delta^2),$$

where the SPD $\bar{H}^{(3)} = \bar{H}^u - \bar{H}^d$ and analogously for $\bar{E}^{(3)}$. We use notations of Ji [12] for the $N \to N$ SPD's.

For the $N \to \Delta$ DVCS process, besides the axial SPD's of Eq. (3), also vector SPD's enter, which are defined as:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', \lambda | \psi(\lambda n/2) \bar{\sigma} \gamma_5 \psi(\lambda n/2) | N, p \rangle = \frac{\gamma^2(p')}{2} \left[ C_1^{(3)} n_\beta + C_2^{(3)} \frac{\Delta_\beta (n_\beta - \Delta)}{m_N} + \ldots \right] u(p),$$

where we show only the dominant (at large $N_c$) magnetic contribution $H^{(3)}_M$ ($K_{\mu \nu}^M$ is the corresponding Lorentz structure, see [11] for details). At large $N_c$, we find:

$$H^{(3)}_M(x, \xi, \Delta^2) = 2/\sqrt{3} E^{(3)}(x, \xi, \Delta^2),$$

in terms of the isovector unpolarized $N \to N$ SPD $E^{(3)}$, giving comparable $\gamma^* N \to \gamma N$ and $\gamma^* N \to \gamma \Delta$ amplitudes (except that the SPD $H$ [12] is absent for $N \to \Delta$).

Using the large $N_c$ relations of Eqs. (11), one can easily derive the relations between the different cross sections for charged pion production as $\sigma_L^{\gamma^* p \to \pi^+ n}$:

$$\sigma_L^{\gamma^* p \to \pi^+ n} : \sigma_L^{\gamma^* p \to \pi^0} : \sigma_L^{\gamma^* n \to \pi^- p} \approx 1 : 1.25 : 0.8,$$

see also Fig. 1. For the production of neutral pseudoscalars we estimated $\sigma_L^{\gamma^* p \to \eta(\eta')} + \Delta^{(3)}$ and $\sigma_L^{\gamma^* p \to \pi^0}$, and $\sigma_L^{\gamma^* p \to \pi^+ n}$.

**Fig. 1.** Leading order predictions for the $\pi N$, $\pi \Delta$, and $KY$ longitudinal electroproduction cross sections at $t = -0.3 \text{GeV}^2$, as function of $x_B$ (plotted up to the $x_B$ value for which $t = t_{\text{min}}$). Results for pion and charged kaon channels are given using an asymptotic DA. For $K^0 \Sigma^+$, predictions are shown using the CZ DA with antisymmetric part: $\eta_N^* = 0.25$.

Besides the cross section $\sigma_L$, the second observable which involves only longitudinal amplitudes and which is a leading order observable for hard exclusive meson electroproduction, is the single spin asymmetry, $A_{\pi B}$ for a proton target polarized perpendicular to the reaction plane (or the equivalent recoil polarization observable). For the hard electroproduction of $N$ final states, a large value of $A_{\pi N}$ was predicted in [11]. We give here for the first time $A_{\pi \Delta}$ and $A_{KY}$, which also turn out to be large.

For $\pi \Delta$ final states we find:

$$A_{\pi \Delta} = -2 \int \frac{\sigma_{\Delta L}}{\pi} \frac{Im(A B^*) 2x m_{\pi}^2}{D_{\pi \Delta}},$$

$$D_{\pi \Delta} = |A|^2 m_{\pi}^4 (1 - \xi)^2 + |B|^2 \xi^2 \left[ 2x - 2t (m_{\pi}^2 + m_N^2) + (m_{\pi}^2 - m_N^2) \right] + \Re (A B^*) 2x m_{\pi}^2 \left( t - \xi (3m_{\pi}^2 + m_N^2) - t - m_{\pi}^2 + m_N^2 \right),$$

For $\pi^0 \Delta^0$ electroproduction , $A$ and $B$ are given by:

$$A_{\pi^0 \Delta^0} = \int_{-1}^1 dx C_{1}^{(3)} \left\{ \frac{e_a}{x - \xi + i\epsilon} + \frac{e_d}{x + \xi - i\epsilon} \right\},$$

$$B_{\pi^0 \Delta^0} = \int_{-1}^1 dx C_{2}^{(3)} \left\{ \frac{e_a}{x - \xi + i\epsilon} + \frac{e_d}{x + \xi - i\epsilon} \right\},$$

where the functions $C_{1}^{(3)}$, $C_{2}^{(3)}$ are as defined in Eqs. (13). Eq. (14) implies that the pion pole contribution to $B_{\pi^0 \Delta^0}$
is given by $B_{\pi^0}^{pole} = \sqrt{3}/4 B_{\pi^n}^{pole}$. We use an asymptotic pion distribution amplitude (DA), for which $B_{\pi^n}^{pole}$ is given by:

$$B_{\pi^n}^{pole} = -3/(2\xi) g_A (2m_N)^2 / (-t + m_N^2),$$  \hspace{1cm} (11)$$

with $g_A \approx 1.267$. For electroproduction of $\pi^- \Delta^{++}$ one has - up to a global isospin factor - analogous expressions as Eqs. (4,10), by making the replacement $e_u \leftrightarrow e_d$.

In Fig. 1 we plot $A_{\pi^n}$ and $A_{\pi^+ \Delta^0}$ at several $t$, as a function of $x_B$. We use the large $N_c$ relations Eqs. (4,10) for the $N \to \Delta$ SPD’s. For the $N \to N$ SPD’s, we use the phenomenological $\xi$-dependent ansatz used in [13]. Fig. 2 shows that the predicted $A_{\pi^+ \Delta^0}$ is large and comparable with the results of [9] in which the SPD’s computed in the chiral quark soliton model [8] were used. Fig. 2 furthermore shows that $A_{\pi^- \Delta^0}$ has the opposite sign compared with $A_{\pi^n}$. Although the magnitude of $A_{\pi^+ \Delta^0}$ is smaller than $A_{\pi^n}$, as anticipated in [9], it is still sizeable. Similarly, we find $A_{\pi^- \Delta^{++}} \approx 0.5 A_{\pi^+ \Delta^0}$.

![FIG. 2. Transverse spin asymmetry for the longitudinal electroproduction of $\pi^0 n$ and $\pi^0 \Delta^0$, at different values of $t$ (indicated on the curves in (GeV/c)^2).](image)

Measurement of $A_{\pi N}$ can provide an important help in the extraction of the pion form factor. For $x_B \leq 0.15$ where one reaches values $t_{min} \sim 2m_N^2$, we find that the pion pole constitutes about 70% to the longitudinal cross section. Measurement of the asymmetry, which is an interference between the pseudoscalar (PS) and pseudovector (PV) contributions, would help to constrain the non-pole term, and in this way help to get a more reliable extraction of the pion form factor.

We next turn to strangeness hard electroproduction reactions. In leading order, the charged kaon electroproduction channels involve the amplitudes:

$$A_{K^{+Y}} = \int_{-1}^{1} dx \frac{e_u}{x - \xi + i\epsilon} + \frac{e_s}{x + \xi - i\epsilon},$$  \hspace{1cm} (12)$$

where $e_s$ is the $s$-quark charge. To estimate the SPD $H^{+Y}$, we use the $SU(3)$ relations of [4]. The SPD $E^{+Y}$ contains a charged kaon pole contribution, given by:

$$B_{K^{+Y}}^{pole} = -3/(2\xi) \eta_{K} f_K g_{KNY} (2m_N)/(-t + m_K^2),$$  \hspace{1cm} (14)$$

with $f_K \approx 150$ MeV, and where $g_{KXY}$ are the KNY coupling constants. In line with our use of $SU(3)$ relations to estimate $\hat{H}$, we use also $SU(3)$ predictions for the coupling constants: $g_{KXY}/\sqrt{4\pi} \approx -3.75$ and $g_{KXY}/\sqrt{4\pi} \approx 1.09$, which are compatible with those obtained from a Regge fit to high energy kaon photoproduction data [13]. In Eqs. (14), $\eta_{K}^{*}$ is defined as:

$$\{ \eta_{K}^{*} \} = \frac{1}{3} \int_{-1}^{1} d\xi \left\{ \frac{\Phi_{K}^{*}(\xi)}{\Phi_{K}(\xi)} \right\} \frac{1}{1 - \xi^2},$$  \hspace{1cm} (15)$$

where $\Phi_{K}^{*}(\xi)$ is the symmetric (in $\xi$) part of the kaon DA. In Eq. (15), we have also defined - for further use - $\eta_{K}^{*}$ for the antisymmetric part $\Phi_{K}(\xi)$ of the kaon DA. The latter is due to $SU(3)$ symmetry breaking effects. One has $\eta_{K}^{*} = 1$ for an asymptotic DA $[\Phi_{K}^{*}(e) = 3/4(1 - \xi^2)]$ and $\eta_{K}^{*} = 7/5$ for the CZ kaon DA [14].

For the $K^{0}\Sigma^{+}$ electroproduction, we give the expressions for $A$ and $B$, allowing for both a symmetric and antisymmetric component in the kaon DA, which yields:

$$\left\{ \begin{array}{l} A_{K^{0}\Sigma^{+}} \\ B_{K^{0}\Sigma^{+}} \end{array} \right\} = \int_{-1}^{1} dx \left\{ \frac{H^{+\Sigma^{+}}}{E^{+\Sigma^{+}}} \right\} \left[ \frac{1 - \eta_{K}^{*}/\eta_{K}^{*}}{x - \xi + i\epsilon} + \frac{1 + \eta_{K}^{*}/\eta_{K}^{*}}{x + \xi - i\epsilon} \right].$$  \hspace{1cm} (16)$$

In contrast to $\pi^{0}$ electroproduction, $K^{0}$ electroproduction can contain a pole contribution, which is given by:

$$B_{K^{0}\Sigma^{+}}^{pole} = \frac{4}{3} \eta_{K}^{*} \left\{ \frac{3}{2\xi} \right\} \frac{f_K g_{KNY} (2m_N)}{-t + m_K^2},$$  \hspace{1cm} (17)$$

and which vanishes when the kaon DA is symmetric (i.e. when $\eta_{K}^{*} = 0$). Therefore, the $K^{0}$ pole contribution to $B_{K^{0}\Sigma^{+}}$, provides a direct measure of the antisymmetric component of the kaon DA.

In Fig. 1, we compare the leading order predictions for pion and kaon hard electroproduction reactions at $Q^2 = 10$ GeV$^2$. The charged pion and kaon channels obtain a large contribution in the range $x_B \approx 0.1$ from the pion (kaon) pole. This largely determines the ratio between these channels at larger $x_B$. For values of $-t$ in the range $0.1 \to 0.5$ GeV$^2$, this yields $\pi^0 n: K^{+}\Lambda \approx 7 : 1 \to 1.8 : 1$, using an asymptotic DA for both $\pi$ and $K$. The kaon DA is not well known however, and the results with a $CZ$ kaon DA yield $K^{+}$ cross sections larger by a factor $(7/5)^{4} \approx 3.8$, for the pole contribution. The ratio $K^{+} \Lambda : K^{+}\Sigma^{0}$ at large $x_B$ is determined from the ratio of the couplings:
\[ \frac{\sigma_{KnA}}{\sigma_{KnN}} \approx 12. \] For the \( K^0\Sigma^+ \) channel, the pole contribution is absent if \( \eta_K^p = 0 \) (as for \( \pi^0 \)). In this case, the ratio \( \pi^0 p : K^0\Sigma^+ \) is determined by the PV contribution and is very sensitive to the input valence quark distribution into \( \tilde{H} \). For \( \Delta u_V \approx -\Delta d_V \) expected in the large \( N_c \) limit, \( \pi^0 : K^0 \approx 1 : 3 \), while for \( \Delta u_V \approx -2\Delta d_V \) preferred by the global fit to DIS of Ref. [16], \( \pi^0 : K^0 \approx 3 : 1 \). The sensitivity of this ratio to the polarized quark distributions might be interesting to provide cross-checks on such global fits from DIS. In Fig. 1, we show the results for \( K^0\Sigma^+ \) by using the polarized distributions of [16] as input for \( \tilde{H} \) (as in [13]). Besides the PV contribution, \( K^0\Sigma^+ \) electroproduction has also a pole contribution, if \( \eta_K^p \neq 0 \). We include the pole contribution of Eq. (17), and use the CZ kaon DA with \( \eta_K^p = 0.25 \). The resulting \( K^0 \) pole contribution provides a sizeable enhancement of the \( K^0\Sigma^+ \) cross section (it is roughly half the value of the PV contribution at the largest \( x_B \).)

\[ \gamma h_p \rightarrow K^+ \Lambda, K^0 \Sigma^+ \]

FIG. 3. Transverse spin asymmetry, for \( K^+ \Lambda \) and \( K^0 \Sigma^+ \) longitudinal electroproduction for different values of \( t \) (indicated on the curves in \((\text{GeV/c})^2\)). For \( K^+ \Lambda \), thick (thin) lines are the predictions with asymptotic (CZ) kaon DA. For \( K^0 \Sigma^+ \), thick (thin) lines are the predictions with CZ type kaon DA, with antisymmetric part: \( \eta_K^p = 0.25 \) (0.1).

For the strangeness channels, \( A_{KY} \) is given by:

\[ A_{KY} = \frac{2|\Delta_1|}{\pi} \frac{Im(AB^\ast)4\xi_m N}{D_{KY}}, \quad (18) \]

\[ D_{KY} = |A|^2 m_N^2 (1 - \xi^2) + |B|^2 \xi^2 \left[-t + (m_N - m_N)^2\right] - Re(AB^\ast) 4\xi m_N \left[\xi(m_N + m_N) + m_Y - m_N\right], \]

where \( A \) and \( B \) are as defined before. Interestingly, that in the case of hyperon production, the same interference azimuthal spin asymmetry can be measured on an unpolarized target by measuring the polarization of the recoiling hyperon through its decay angular distribution. \( A_{K^+A}, A_{K^0\Sigma^+} \) are shown in Fig. 3. They are as large as for \( \pi^+ n \). We also find \( A_{K^+\Sigma^0} \sim A_{K^+\Lambda} \). For \( K^0 \) production, the sensitivity to the \( SU(3) \) symmetry breaking effects in the kaon DA is illustrated (lower panel of Fig. 3), by plotting \( A_{K^0\Sigma^+} \) for two values of \( \eta_K^p \). Because \( A_{K^0\Sigma^+} \) is directly proportional to \( \eta_K^p \), it provides a very sensitive observable to extract the \( K^0 \) form factor.

To summarize, we have shown that yields for hard exclusive production of decuplet and octet baryons are similar. Strange and nonstrange channels can be comparable and in some channels strange can even dominate (depending on DA and polarized parton distributions), in contrast to low-energy strangeness production. Large transverse spin asymmetries are predicted for many of these reactions. Several tests of validity of the large \( N_c \) approximation in QCD would be possible.

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[1] J.C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D 56, 2982 (1997).
[2] V.Y. Petrov, P. Pobylitsa, M.V. Polyakov, I. Börnig, K. Goeke, and C. Weiss, Phys. Rev. D 57, 4325 (1998).
[3] M. Penttinen, M.V. Polyakov and K. Goeke, hep-ph/9909489.
[4] L.L. Frankfurt, M.V. Polyakov, M. Strikman, hep-ph/9808449.
[5] L. Mankiewicz, G. Piller and A. Radyushkin, Eur. Phys. J. C 10, 307 (1999).
[6] L.L. Frankfurt, P.V. Pobylitsa, M.V. Polyakov and M. Strikman, Phys. Rev. D 60, 014010 (1999).
[7] M. Eides, L. Frankfurt, M. Strikman, Phys. Rev. D 59, 114025 (1999).
[8] T. Schafer and E.V. Shuryak, Rev. Mod. Phys. 70, 323 (1998).
[9] G. Adkins, C. Nappi and E. Witten, Nucl. Phys. B228, 552 (1983); D.I. Diakonov, V.Y. Petrov and P.V. Pobylitsa, Nucl. Phys. B306, 809 (1988).
[10] V.V. Frolov et al., Phys. Rev. Lett. 82, 45 (1999).
[11] M.V. Polyakov and M. Vanderhaeghen, in preparation.
[12] X. Ji, J. Phys. G24, 1181 (1998).
[13] M. Vanderhaeghen, P.A.M. Guichon and M. Guidal, Phys. Rev. D 60, 094017 (1999).
[14] M. Guidal, J.-M. Laget and M. Vanderhaeghen, Nucl. Phys. A 627, 645 (1997).
[15] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173 (1984).
[16] E. Leader, A.V. Sidorov and D.B. Stamenov, Phys. Rev. D 58, 114028 (1998).