The Spectrum of the Nucleons and the Strange Hyperons and Chiral Dynamics

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Abstract

The spectra of the nucleons, $\Delta$ resonances and the strange hyperons are well described by the constituent quark model if in addition to the harmonic confinement potential the quarks are assumed to interact by exchange of the $SU(3)_{F}$ octet of pseudoscalar mesons, which are the Goldstone bosons associated with the hidden approximate chiral symmetry of QCD. In its $SU(3)_{F}$ invariant approximation the pseudoscalar exchange interaction splits the multiplets of $SU(6)_{FS}\times U(6)_{conf}$ in the spectrum to multiplets of $SU(3)_{F}\times SU(2)_{S}\times U(6)_{conf}$. The position of these multiplets differ in the baryon sectors with different strangeness because of the mass splitting of the pseudoscalar octet and the different constituent masses of the u,d and s quarks that breaks $SU(3)_{F}$ flavor symmetry. A description of the whole spectrum, to an accuracy of $\approx 4\%$ or better, is achieved if one matrix element of the boson interaction for each oscillator shell is extracted from the empirical mass splittings. The ordering of the positive and negative parity states moreover agrees with the empirical one in all sectors of the spectrum. A discussion of the conceptual basis of the model and its various phenomenological ramifications is presented.

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1. Introduction

The spectra of the confirmed states of the nucleon and the Λ hyperon separate into a low energy sector of well separated states without nearby parity partners, and a high energy sector with an increasing number of near parity doublets. A natural interpretation of this feature is that the approximate chiral symmetry of QCD is realized in the hidden Nambu-Goldstone mode at low excitation (and temperature) and in the explicit Wigner-Weyl mode at high excitation.

The hidden mode of chiral symmetry is revealed by the existence of the octet of pseudoscalar mesons of low mass, which represent the associated approximate Goldstone bosons. The $\eta'$ (the $SU(3)$-singlet) decouples from the original nonet because of the $U(1)$ anomaly [1, 2]. Another consequence of the spontaneous breaking of the approximate chiral symmetry of QCD is that the valence quarks acquire their dynamical or constituent mass [3, 4, 5, 6, 7] through their interactions with the collective excitations of the QCD vacuum—the quark-antiquark excitations and the instantons. The origin of this dynamical generation of the constituent quark mass is closely related to the origin of the pseudoscalar Goldstone excitations. Thus according to the two-scale picture of Manohar and Georgi [4] the appropriate effective degrees of freedom for the 3-flavor QCD at distances beyond that of spontaneous chiral symmetry breaking (0.2–0.3 fm), but within that of the confinement scale $\Lambda_{QCD}^{-1} \approx 1 \text{fm}$, should be the constituent quarks with internal structure, and the chiral meson fields.

In line with this we have recently suggested [8, 9] that beyond the chiral symmetry spontaneous breaking scale a baryon should be considered as a system of three constituent quarks with an effective quark-quark interaction that is formed of a central confining part, assumed to be harmonic, and a chiral interaction that is mediated by the octet of pseudoscalar mesons between the constituent quarks.

Even in its simplest $SU(3)_F$ invariant form this boson exchange interaction between the constituent quarks leads to a remarkably good description of the whole hitherto measured spectrum of the nucleon, $\Delta$ resonance and $\Lambda$ hyperon [8, 9]. We here develop this model in more detail, with full account of the $SU(3)_F$ breaking caused by the mass splitting of the pseudoscalar octet and
different constituent masses of the u,d and s quarks, and show that it provides
a very satisfactory representation of the known parts of the spectra of the Σ, Ξ
and Ω hyperons as well.

The simplest representation of the most important component of the inter-
action of the constituent quarks that is mediated by the octet of pseudoscalar
bosons in the \( SU(3)_F \) invariant limit is

\[
H_\chi \sim - \sum_{i<j} V(\vec{r}_{ij}) \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j.
\]  

(1.1)

Here the \( \{ \vec{\lambda}_i^F \} \)'s are flavor \( SU(3) \) Gell-Mann matrices and the \( i,j \) sums run
over the constituent quarks. The interaction potential \( V(r) \) will have the usual
Yukawa behavior at long range, but at short range behaves as a smeared ver-
sion of the \( \delta \) function term in the Yukawa interaction for pseudoscalar ex-
change.

If the only interaction between the quarks were the flavor- and spin- indepen-
dent harmonic confining interaction the baryon spectrum would be organized
in multiplets of the symmetry group \( SU(6)_{FS} \times U(6)_{conf} \), as the symmetry of
the 3-quark states in the harmonic oscillator basis is \( U(6)_{conf} \) and the permuta-
tional \( SU(6)_F \) symmetry is uniquely determined by the \( U(6)_{conf} \) symmetry by
the Pauli principle. In this case the baryon masses would be determined solely
by the orbital structure and by the constituent quark masses and the spectrum
would be organized in an alternating sequence of positive and negative parity
states. This multiplet structure of the spectrum is broken by the interaction
(1.1) between the constituent quarks, and in the first order perturbation in the
\( SU(3)_F \) symmetric approximation for the interaction the multiplet structure is
then that of the group \( SU(3)_F \times SU(2)_S \times U(6)_{conf} \). Consequently the baryons
with the same radial structure and the same permutational FS-symmetry but
different flavor or (and) spin symmetries will have different mass.

Because of the flavor dependent factor \( \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \) the chiral boson exchange
interaction (1.1) will lead to orderings of the positive and negative parity states
in the baryon spectra, which agree with the observed ones in all sectors. In
the case of the spectrum of the nucleon the strength of the chiral interaction
between the constituent quarks is sufficient to shift the lowest positive parity
state in the \( N=2 \) band (the \( N(1440) \)) below the negative parity states in the
\( N=1 \) band (\( N(1520), N(1535) \)). In the spectrum of the \( \Lambda \) on the other hand it
is the negative parity flavor singlet states (the \( \Lambda(1405) \) and the \( \Lambda(1520) \)) that
remain the lowest lying resonances, again in agreement with experiment. The
mass splittings between the baryons with different strangeness and between the \( \Lambda \) and the \( \Sigma \) which have identical flavor, spin and flavor-spin symmetries arise from the explicit breaking of the \( SU(3)_F \) symmetry that is caused by the mass splitting of the pseudoscalar meson octet and the different masses of the u,d and the s quarks.

In section 2 below we review the role of chiral symmetry in the quark based models for the baryons and the general justification for considering the baryons to be formed of constituent quarks that interact by exchanging pseudoscalar mesons. This section also contains a comparison between the chiral boson exchange interaction model and the commonly used perturbative gluon exchange interaction model, along with the proof of why the latter leads to incorrect ordering of positive and negative parity states in the spectra. Section 3 contains a description of the chiral boson mediated interaction and section 4 a description of the algebraic structure of the harmonic oscillator basis states. In section 5 the symmetry properties of the interaction are described and a baryon mass formula is derived to first order in the chiral interaction. In section 6 we describe the spectra of the nucleon, the \( \Delta \)-resonance and the \( \Lambda \) hyperon as they are predicted with the \( SU(3)_F \) symmetric chiral boson interaction (1.1). The effect of the \( SU(3)_F \) breaking in the interaction that arises from the the quark and meson mass differences is considered in section 7, where the mass splitting of the baryon octet and decuplet states is considered. This leads to set of new mass formulas for the octet and decuplet states, as well as for the corresponding excited states. In this section we give numerical results for the spectra of all the strange hyperons as well as the nucleon and the \( \Delta \). In section 8 we discuss the role of the tensor force associated to the pseudoscalar-exchange interaction and in section 9 the exceptionally large splitting in the flavor singlet \( \Lambda(1405) \)-\( \Lambda(1520) \) doublet. In section 10 we discuss the role of the exchange current corrections to the baryon magnetic moments that are associated with the pseudoscalar exchange interaction. Section 11 contains a discussion of the framework and implications of the results and section 12 some general comments on the quark model basis for the meson exchange description of the nuclear force, and on the \( q\bar{q} \) interaction and the meson spectrum.

2. Chiral Symmetry and the Quark Model
The importance of the chiral symmetry for strong interactions was realized early on (for an early review and references see [10]). This symmetry, which is almost exact in the light u and d flavor sector is however only approximate in QCD when strangeness is included, because of the large mass of the s-quark. Nevertheless even in 3-flavor QCD the current quark masses may, in a first approximation, be set to zero (the chiral limit), and their deviation from zero treated as a perturbation. The small finite masses of the current quark masses are however very important for the (finite) masses of the mesons. In the chiral limit all members of the pseudoscalar octet ($\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \bar{K}^0, \eta$) would have zero mass, which is most clearly seen in the Gell-Mann-Oakes-Renner [11] relations that relate the pseudoscalar meson masses to the quark condensates:

$$m_{\pi^0}^2 = -\frac{1}{f_\pi^2}(m^0_u <\bar{u}u> + m^0_d <\bar{d}d>) + O(m^0_{u,d}^2),$$

$$m_{\pi^+,\pi^-}^2 = -\frac{1}{f_\pi^2} \frac{m^0_u + m^0_d}{2}(<\bar{u}u> + <\bar{d}d>) + O(m^0_{u,d}^2),$$

$$m_{K^+,\pi^-}^2 = -\frac{1}{f_\pi^2} \frac{m^0_u + m^0_s}{2}(<\bar{u}u> + <\bar{s}s>) + O(m^0_{u,s}^2),$$

$$m_{K^0,\bar{K}^0}^2 = -\frac{1}{f_\pi^2} \frac{m^0_d + m^0_s}{2}(<\bar{d}d> + <\bar{s}s>) + O(m^0_{d,s}^2),$$

$$m_{\eta}^2 = -\frac{1}{3f_\pi^2}(m^0_u <\bar{u}u> + m^0_d <\bar{d}d> + 4m^0_s <\bar{s}s>) + O(m^0_{u,d,s}^2).$$

(2.1)

Here $<\bar{u}u>$, $<\bar{d}d>$ and $<\bar{s}s>$ are the quark condensates of the QCD vacuum which are approximately equal in magnitude [12] ($<\bar{q}q> \approx -(240-250 MeV)^3$). The nonzero values of the quark condensates, which represent the order parameter, is direct consequence of (and evidence for) the spontaneously broken chiral symmetry in the QCD vacuum. Thus all the pseudoscalar mesons above are approximate Goldstone bosons, the nonzero masses of which are determined by the corresponding current quark masses. The masses and structure of the baryons at low energy is quite in contrast mainly determined by the spontaneous breaking of the chiral symmetry and the effective confining interaction, and hence only weakly depend on the current quark masses. The role of the current quark masses in the structure of the baryons is only to break the $SU(3)_F$
symmetry in the baryon spectrum.

The importance of the constraints posed by chiral symmetry for the quark bag models for the baryons [13] was recognized soon after its development. The bag surface term, $\sim \nabla \Psi \delta(R-r)$, breaks the chiral symmetry and requires introduction of a compensating chiral meson field, which couples to the massless quarks on the surface of the bag with [14] or without [13] this field existing inside of a bag. An alternative to this surface coupled version is the volume coupled version [16]. In the early bag models with restored chiral symmetry the massless current quarks within the bag were assumed to interact not only by perturbative gluon exchange but also through meson exchange. Baryon and meson masses and other static properties have been derived in such bag models with pion and gluon exchange interactions e.g. in refs. [17, 18, 19, 20, 21, 22].

If the sharp surface confinement is replaced by a linear scalar confining interaction one obtains a chiral potential model, in which the chiral field is coupled to the massless quarks moving in the confining potential [23]. Robson has shown that good predictions for the splittings in the baryon octet-decuplet as well as some of the excitations in the nucleon spectrum can be achieved with this picture if the quarks are assumed to interact by pseudoscalar meson exchange alone without gluon exchange [24]. In these models the chiral field only has the character of a compensating field rather than a collective low frequency Goldstone quark-antiquark excitation (the possibility of a nonzero quark condensate was not addressed). A general limitation of all bag and bag-like models is of course the lack of translational invariance, which is important for a realistic description of the excited states.

Common to these models is that the breaking of chiral symmetry arises from the confining interaction. This point of view contrasts with that of Manohar and Georgi [4], who pointed out that there should be two different scales in QCD, with 3 flavors. At the first one of these, $\Lambda_{\chi SB} \approx 4\pi f_\pi \approx 1 \text{ GeV}$, the spontaneous breaking of the chiral symmetry occurs, and hence at distances beyond $\frac{1}{\Lambda_{\chi SB}} \approx 0.2 \text{ fm}$ the valence current quarks acquire their dynamical (constituent) mass (called "chiral quarks" in [4]) and the Goldstone bosons (mesons) appear. The other scale, $\Lambda_{QCD} \approx 100 - 300 \text{ MeV}$, is that which characterizes confinement, and the inverse of this scale roughly coincides with the linear size of a baryon. Between these two scales then the effective Lagrangian should be formed out of the gluon fields that provide a confining mechanism as well as of the constituent quark and pseudoscalar meson fields. Manohar and Georgi did not, however,
specify whether the baryons should be described as bound qqq states or as chiral solitons.

The chiral symmetry breaking scale above fits well with that which appears in the instanton liquid picture of the QCD vacuum [3, 4]. In this model the quark condensates (i.e. equilibrium of virtual quark-antiquark pairs in the vacuum state) as well as the gluon condensate are supported by instanton fluctuations of a size $\sim 0.3$ fm. The instanton liquid picture for QCD vacuum is confirmed by recent lattice QCD calculations [25] which show that selective removal of all configurations of the gluon field except for the instantons does not change the vacuum correlation functions of hadronic currents [26] and density-density correlation functions in hadronic bound state. Dyakonov and Petrov [7] suggested that at low momenta (i.e. beyond the chiral symmetry breaking scale) QCD should be approximated by an effective chiral Lagrangian of the sigma-model type that contains valence quarks with dynamical (constituent) masses and meson fields. They considered a nucleon as three constituent quarks moving independently of one another in a self-consistent chiral field of the hedgehog form [27]. In this picture the excited baryon states appear as rotational excitations and no explicit confining interaction is included. A very similar description for the nucleon was suggested within so called ”chiral quark models” [28, 29].

The spontaneous breaking of chiral symmetry and its consequences - the dynamical quark mass generation, the appearance of the quark condensate and pseudoscalar mesons as Goldstone excitations are well illustrated by the Nambu and Jona-Lasinio model [30, 31]. This model lacks a confining interaction, which as argued below is essential for a realistic description of the properties of the baryon physics.

The chiral field interaction (1.1) between the constituent quarks should be contrasted in form with the color-magnetic interaction [32]

$$H_c \sim -\alpha_s \sum_{i<j} \frac{\pi}{6m_im_j} \bar{\lambda}_i^C \cdot \bar{\lambda}_j^C \sigma_i \cdot \sigma_j \delta(\vec{r}_{ij}),$$  \hspace{1cm} (2.2)

where the $\{\bar{\lambda}_i^C\}$s are color SU(3) matrices, and which should be important in the region of explicit chiral symmetry at short distances and high energy. It is in fact this color-magnetic interaction, which has been used in earlier attempts to describe the baryon spectra with the constituent quark model [33, 34, 35]. Although many of the qualitative and some of the quantitative features of the fine structure of the baryon spectra can be described by the interaction (2.2) a
number of outstanding features have proven hard to explain in this approach.

The most obvious one of these is the different ordering of the positive and negative parity resonances in the spectra of the nucleon and delta on the one hand and the Λ hyperon on the other, and in particular the difficulty in describing the low masses of the Λ(1405), N(1440), ∆(1600) and Σ(1660) resonances. A second such feature is the absence of empirical indications for the large spin-orbit interaction that should accompany the color-magnetic interaction (2.2) \cite{32}. Although it has been suggested that the latter problem could be overcome by decreasing of the coupling constant due to smearing of the contact interaction (2.2) and by a partial cancellation against the spin-orbit interaction that is associated with Thomas precession \cite{35}, the first problem is inherent to the color operator structure \( \vec{\lambda}_i \cdot \vec{\lambda}_j \) of the one gluon exchange interaction and to the antisymmetry of the color part of the baryon wave function and cannot be overcome by changing of the radial behavior of the contact interaction (2.2) and of the confining potential. Indeed the interaction (2.2) is attractive in color-spin symmetric quark pair states and repulsive in antisymmetric ones:

\[
< [f_{ij}]_C \times [f_{ij}]_S \vec{\lambda}_i \cdot \vec{\lambda}_j \sigma_i \cdot \sigma_j [f_{ij}]_C \times [f_{ij}]_S > = \begin{cases} 
8 & [11]_C, [11]_S : [2]_{CS} \\
-8 & [11]_C, [2]_S : [11]_{CS} 
\end{cases} .
\]

As a consequence of this and the confining interaction the \( \frac{1}{2}^+ N(1440) \), which belongs to the N=2 band, should have higher mass than the \( \frac{1}{2}^- N(1535) \) (N=1) as both have the same mixed color-spin symmetry. Similarly the \( \frac{3}{2}^+ \Delta(1600) \) (again N=2), the color-spin state of which is totally antisymmetric, should have substantially larger mass than the mixed CS-symmetry N=1 state \( \frac{3}{2}^- \Delta(1700) \). Both of these predictions are in conflict with experiment.

We shall show below that the chiral pseudoscalar interaction (1.1) provides a simpler description of the fine structure of the baryon spectra at the excitation energy up to 1 GeV, that automatically implies the reversal of the ordering of the even and odd parity states between the nucleon and Λ hyperon spectra. Moreover we show that when one matrix element of the effective interaction potential \( V(r) \) in (1.1) for each oscillator shell is extracted from the empirical mass splittings a quite satisfactory description of the fine structure of the whole low lying baryon spectrum is achieved already in lowest order. Finally the overall small spin-orbit splitting in the baryon spectrum is qualitatively explained.
by the absence of any spin-orbit component in the pseudoscalar exchange interaction. This then suggests that it is the chiral field interaction (1.1), which plays the dominant role in ordering the baryon spectrum in the region of hidden chiral symmetry, and that the perturbative gluon exchange interaction becomes important only at length scales smaller than that of the spontaneous chiral symmetry breaking where no constituent quarks.

In addition to the indication against strong gluon exchange interactions at low energy that is provided by the cooled QCD lattice calculations of Chu et al. [25] mentioned above, there is good evidence from the recent lattice QCD calculations by Liu and Dong [36] that the splittings $N - \Delta$ and $\pi - \rho$ are not due to the one-gluon exchange interaction between quarks. To show this Liu and Dong measured these mass splittings with two approximations. The first one is the standard quenched approximation, which neglects sea quark closed loop diagrams generated by gluon lines. This quenched approximation contains however part of the antiquark effects related to the $Z$ graphs mediated of valence quark lines [37]. The second one is so called "valence approximation" where the quarks are limited to propagating only forward in time (i.e. no $Z$-graphs and related quark-antiquark pairs). The gluon exchange and all other possible gluon configurations including instantons are exactly the same within both approximations. As expected finite $N - \Delta$ and $\pi - \rho$ splittings are observed in the quenched approximation, but not in the valence approximation, in which the $\Delta$ and the $N$ and the $\rho$ and the $\pi$ become degenerate within errors [36]. Since the one-gluon exchange is not switched off in the valence approximation this indicates that the hyperfine splitting does not arise from the one-gluon exchange interaction.

3. The Chiral Boson exchange interaction

In an effective chiral symmetric description of baryon structure based on the constituent quark model the coupling of the quarks and the pseudoscalar Goldstone bosons will (in the $SU(3)_F$ symmetric approximation) have the form $ig\bar{\psi}\gamma_5 \vec{X}^F \cdot \vec{\phi}\psi$, where $\psi$ is the fermion constituent quark field operator and $\vec{\phi}$ the octet boson field operator, and $g$ is a coupling constant. A coupling of this form in a nonrelativistic reduction for a constituent quark spinors will -- to lowest order -- give rise to a Yukawa interaction between the constituent quarks, the
spin-spin component of which has the form

\[ V_Y(r_{ij}) = \frac{g^2}{4\pi} \frac{1}{3m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i \cdot \vec{\lambda}_j \left\{ \mu^2 e^{-\mu r_{ij}} - \frac{4\pi \delta(\vec{r}_{ij})}{r_{ij}} \right\}. \]  

(3.1)

Here \( m_i \) and \( m_j \) denote masses of the interacting quarks and \( \mu \) that of the meson. There will also be an associated tensor component, which will be discussed in section 8 below. Because of the short range of the baryon wavefunctions the role of the \( \delta \) function term is of crucial importance, although the latter is expected to be smeared out by the finite size of the constituent quarks and pseudoscalar mesons.

Along with the pseudoscalar bosons chiral symmetry requires an accompanying scalar meson field \( \phi_\sigma \) to complete the chiral multiplet. This also contributes an effective interaction between the constituent quarks. As the main component of this scalar meson exchange interaction is a spin- and flavor-independent attractive interaction it contributes to the effective confining interaction, but not to the fine structure of the spectrum except through the associated weak spin-orbit interaction, which will be discussed in section 9 below.

At short range the simple form (3.1) of the chiral boson exchange interaction cannot be expected to be realistic, and should only be taken to be suggestive. Because of the finite spatial extent of both the constituent quarks and the pseudoscalar mesons that the delta function in (3.1) should be replaced by a finite function, with a range of 0.6-0.7 fm as suggested by the spatial extent of the mesons. In addition the radial behaviour of the Yukawa potential (3.1) is valid only if the boson field satisfies linear Klein-Gordon equation. The chiral symmetry requirements for the effective Lagrangian, which contains constituent quarks as well as boson fields imply that these boson fields cannot be described by linear equations near their source. Therefore it is only at large distances where the amplitude of the boson fields is small that the quark-quark interaction reduces to the simple Yukawa form. At this stage the proper procedure should be to avoid further specific assumptions about the short range behavior of \( V(r) \) in (1.1) and instead to extract the required matrix elements of it from the baryon spectrum and to reconstruct by this an approximate radial form of \( V(r) \). The overall – sign in the effective chiral boson interaction in (1.1) corresponds to that of this short range term in the Yukawa interaction.

The flavor structure of the pseudoscalar octet exchange interaction in (1.1) between two quarks \( i \) and \( j \) should be understood as follows
\[ V(r_{ij}) \vec{\lambda}_i \cdot \vec{\lambda}_j = \sum_{a=1}^{3} V_\pi(r_{ij}) \lambda_i^a \lambda_j^a + \sum_{a=4}^{7} V_K(r_{ij}) \lambda_i^a \lambda_j^a + V_\eta(r_{ij}) \lambda_i^8 \lambda_j^8. \] (3.2)

The first term in (3.2) represents the pion-exchange interaction, which acts only between light quarks. The second term represents the kaon exchange interaction, which takes place in u-s and d-s pair states. The \( \eta \)-exchange, which is represented by the third term, is allowed in all quark pair states. In the \( SU(3)_F \) symmetric limit the constituent quark masses would be equal \( (m_u = m_d = m_s) \), the pseudoscalar octet would be degenerate and the meson-constituent quark coupling constant would be flavor independent. In this limit the form of the pseudoscalar exchange interaction reduces to (1.1), which does not break the \( SU(3)_F \) invariance of the baryon spectrum. Beyond this limit the pion, kaon and \( \eta \) exchange interactions will differ \( (V_\pi \neq V_K \neq V_\eta) \) because of the difference between the strange and u, d quark constituent masses \( (m_{u,d} \neq m_s) \), and because of the mass splitting within the pseudoscalar octet \( (\mu_\pi \neq \mu_K \neq \mu_\eta) \) (and possibly also because of flavor dependence in the meson-quark coupling constant). As pion exchange and kaon exchange takes place only for quark pairs of unique mass (we neglect the possible small mass difference between u and d constituent quarks) the dependence on the quark mass can be absorbed into the corresponding potential functions \( V_\pi \) and \( V_K \). As on the other hand \( \eta \) exchange is possible in all light and strange quark pair combinations with different mass the potential function \( V_\eta \) should be expected to be flavor dependent as it is denoted below by the corresponding subscripts. The source of both the \( SU(3)_F \) symmetry breaking constituent quark mass differences and the \( SU(3)_F \) symmetry breaking mass splitting of the pseudoscalar octet is the explicit chiral symmetry breaking in QCD.

The flavor matrix elements of the interaction are

\[
< [f_{ij}]_F T_{ij} | \vec{\sigma}_i \cdot \vec{\sigma}_j \sum_{a=1}^{8} V^a(r_{ij}) \lambda_i^a \lambda_j^a | [f_{ij}]_F T_{ij} > = \vec{\sigma}_i \cdot \vec{\sigma}_j \times \begin{cases} 
V_\pi + \frac{1}{3} V_{uu}^\pi, & \text{if } [2]_F, \ T_{ij} = 1 \\
2V_K - \frac{2}{3} V_{us}^K, & \text{if } [2]_F, \ T_{ij} = \frac{1}{2} \\
\frac{4}{3} V_{ss}^\eta, & \text{if } [2]_F, \ T_{ij} = 0 \\
-2V_K - \frac{2}{3} V_{us}^\eta, & \text{if } [11]_F, \ T_{ij} = \frac{1}{2} \\
-3V_\pi + \frac{4}{3} V_{uu}^\eta, & \text{if } [11]_F, \ T_{ij} = 0
\end{cases}. \] (3.3)
Here the Young pattern \([f_{ij}]\) denotes the flavor permutational symmetry in the quark pair \(i, j\) (the symbol \([2]\) represents the Young pattern with two boxes in in first row and \([11]\) that with two boxes in one column). The total isospin of the pair state is denoted \(T_{ij}\). The subscripts uu, us, ss on the \(\eta\) exchange potential \(V_\eta\) indicate that the potential acts in pairs of two light, one light and one strange and two strange quarks respectively.

4. Algebraic Structure of the Oscillator Wavefunctions

The confining interaction between two constituent quarks \(i, j\) will be taken to have the harmonic oscillator form

\[
V_{\text{conf}}(\vec{r}_{ij}) = V_0 + \frac{1}{6}m\omega^2(\vec{r}_i - \vec{r}_j)^2, \tag{4.1}
\]

where \(m\) is the mass of the constituent quark and \(\omega\) is the angular frequency of the oscillator interaction. For simplicity we neglect here the mass difference between the light and strange constituent quarks. The Hamiltonian for the unperturbed basis states of the 3 quark system then takes the form

\[
H_0 = \sum_{i=1}^{3} \frac{\vec{p}_i^2}{2m_i} - \frac{\vec{P}_{cm}^2}{6m} + \frac{1}{6} \sum_{i<j} m\omega^2(\vec{r}_i - \vec{r}_j)^2 + 3V_0. \tag{4.2}
\]

Here \(\vec{P}_{cm}\) denotes the total momentum of the baryon. The exact eigenvalues of this Hamiltonian are

\[
E_0 = 3V_0 + (N + 3)\hbar\omega, \tag{4.3}
\]

where \(N\) is the number of excitation quanta in the state.

The overall orbital symmetry of \(A\) particles interacting each other through harmonic forces is \(U(3(A - 1))\) which in the present case reduces to \(U(6)\). The eigenvalue (4.3) is highly degenerate and therefore additional quantum numbers are required to characterize an eigenstate uniquely.

The spatial part of the three body wave function is determined by the following quantum numbers:

\[
|N(\lambda\mu)L[f]X(r)_X >. \tag{4.4}
\]
Here we use the notations of the translationally invariant shell model (TISM) \cite{38}. The wave functions are exact solutions of the three-body Schrödinger equation with the Hamiltonian (4.2) and coincide with the corresponding harmonic oscillator shell model states after removal of the center-of-mass motion from the latter. The Elliott symbol \((\lambda \mu)\) determines the SU(3) harmonic oscillator multiplet with the dimension \(\text{dim}(\lambda \mu) = \frac{1}{2}(\lambda + 1)(\mu + 1)(\lambda + \mu + 2)\) and \(L\) is the total orbital angular momentum. The allowed values of \(L\) that are compatible with a given Elliott symbol \((\lambda \mu)\) are given by the Elliott formula \cite{38, 39}.

The spatial permutational symmetry of the state is indicated by the Young pattern (diagram) \([f]_X\), where \(f\) is a sequence of integers that indicate the number of boxes in the successive rows of the corresponding Young patterns. Thus \([3]\) represents the completely symmetric state, \([111]\) the completely antisymmetric one and \([21]\) states of mixed symmetry. Finally, \((r)_X\) is the Yamanouchi symbol which determines the basis vector of the irreducible representation \([f]_X\) of the permutation group \(S_3\). The Yamanouchi symbol is uniquely connected with the standard Young tableau (i.e. with the Young pattern where the numbers of particles are put in boxes in regular sequence). For the totally symmetrical \([3]\) or totally antisymmetrical \([111]\) representations they are unique - \((111)\) and \((123)\), respectively. For the mixed symmetry state \([21]\) there are two different basis vectors determined by the Yamanouchi symbols \((112)\) (i.e. the first and the second particles are in the first row and the third particle is in the second row of the Young pattern) and \((121)\) (the first and the third particles are in the first row and the second particle is in the second row). All the necessary functions (4.4) are well known and can be found e.g. in \cite{10}.

The Hamiltonian (4.2) does not depend on the spin and flavor degrees of freedom. Thus to provide the full set of zero order wave functions one should construct all possible color-flavor-spin parts that are compatible with a given spatial wave function. The color part of the wave function is totally antisymmetric \(([111])\) and therefore the Pauli principle requires the spatial-flavor-spin part should be symmetric:

\[
[111]_{\text{CXFS}} = [111]_C \times [3]_{\text{XFS}}.
\] (4.5)

The color part of the wave function can be factored out and it will be suppressed in the expressions below as the interaction (1.1) is independent of color. A possible color dependence of the confining interaction of the form \(\vec{\lambda}_i^C \cdot \vec{\lambda}_j^C\) is inessential for baryon states as the corresponding matrix element
\[<[11]_C \bar{\lambda}^c \cdot \bar{\lambda}^c|[11]_C> = -\frac{8}{3} \] (4.6)

is the same for all quark pair states and hence can be absorbed into definition of the effective confining interaction.

The total symmetry of the spatial-flavor-spin wave function implies that the permutational symmetry \([f]_X\) of the orbital part (4.4) and the permutational symmetry \([f]_{FS}\) of the flavor-spin part have to coincide: \([f]_X = [f]_{FS}\). By the general rules the symmetrical spatial-flavor-spin wave functions should be constructed as

\[|N(\lambda\mu)L[f]_X[f]_{FS}[f]_F[f]_S\rangle_Y T >\]

\[= \frac{1}{\sqrt{dim[f]_X}} \sum_{(r)_X=(r)_{FS}} |N(\lambda\mu)L[f]_X(r)_X > |[f]_{FS}[f]_F[f]_S\rangle_Y T(r)_{FS} >, \] (4.7)

where \([f]_F\) and \([f]_S\) denote permutational flavor and spin symmetries, respectively and \(Y\) is hypercharge and \(T\) is isospin of a baryon. Obviously, \(dim[111] = dim[3] = 1, dim[21] = 2\). Note that \([f]_S\) uniquely determines the total spin \(S\) as one half of the difference of the first and second rows in the spin Young pattern above. It is also understood that the orbital momentum \(L\) and spin \(S\) are coupled to the total angular momentum \(J\).

The flavor-spin part of the 3 quark states in (4.7) is easily constructed by the fractional parentage expansion for the separation of one particle

\[|X_{12}T_{12}S_{12}s_{12}S_{3}s_3\rangle_Y T >\]

\[= \sum_{T_{12}t_{12}T_3t_3S_{12}s_{12}S_{3}s_3} \Gamma (T_{12}t_{12}T_3t_3|Tt|)(S_{12}s_{12}S_{3}s_3|Ss). \] (4.8)

Here \([f]_{12}\) denotes Young patterns for the symmetries of the two-particle states. Obviously the flavor-spin symmetry \([f]_{12}\) is uniquely determined by the Yamanouchi symbol \((r)_{FS}\). The quantum numbers for the hypercharge, isospin and spin of the particle pair 12 and the single particle 3 are indicated by subscripts. Finally the fractional parentage coefficient \(\Gamma\) can be presented as a product of two factors \([\Pi]\). The first is the scalar factor of the Clebsch-Gordan coefficient.
for the group $SU(6)_{FS}$ in the reduction $SU(6)_{FS} \supset SU(3)_{F} \times SU(2)_{S}$, which is determined only by invariants of the groups $SU(6)_{FS}$, $SU(3)_{F}$ and $SU(2)_{S}$ and does not depend on isospin, hypercharge nor on the third components of isospin and spin:

$$\left( [f_{12}]_{FS} [f_{12}]_{F} S_{12}; [1]_{FS} [1]_{F} S_{3} = \frac{1}{2} \parallel [f]_{FS} [f]_{F} S \right). \quad (4.9a)$$

The second is the isoscalar factor of the $SU(3)_{F}$ Clebsch-Gordan coefficient:

$$([f_{12}]_{F} Y_{12} T_{12}; [1]_{F} Y_{3} T_{3} \parallel [f]_{F} Y T). \quad (4.9b)$$

The coefficients (4.9a) are listed in Table 1 and the (4.9b) ones in Tables 2a and 2b.

In the following we shall use the quantum numbers above to characterize wave function:

$$\Psi = \mid N(\lambda \mu) L[f]_{X} [f]_{FS} [f]_{F} [f]_{S} YT \rangle \ . \quad (4.10)$$

The explicit indication of the permutational symmetry in the notation for the states is more convenient than the indication of dimension of the corresponding multiplets, which is conventional in baryon and meson spectroscopy. This is because the symmetry properties of the interaction (1.1) (see next Section) together with the permutational symmetries $[f]_{FS}$, $[f]_{F}$ and $[f]_{S}$ makes it very transparent for which states the interaction (1.1) is most attractive, and hence to understand the ordering of the states in the spectrum. All the required dimensions are easily calculated for the given Young patterns according to the general rules [42] and are listed below:

$$SU(2)_{S} : \left\{ \begin{array}{l} 4, \quad [3]_{S} \\
2, \quad [21]_{S} \end{array} \right. \quad (4.11)$$

$$SU(3)_{F} : \left\{ \begin{array}{l} 10, \quad [3]_{F} \\
8, \quad [21]_{F} \\
1, \quad [111]_{F} \end{array} \right. \quad (4.12)$$

$$SU(6)_{FS} : \left\{ \begin{array}{l} 56, \quad [3]_{FS} \\
70, \quad [21]_{FS} \\
20, \quad [111]_{FS} \end{array} \right. \quad (4.13)$$

The symmetry structure of the zero order wave functions is $SU(6)_{FS} \times U(6)_{conf}$. 
Thus, when the only interaction between the quarks is the flavor- and spin-independent harmonic confining interaction and all quarks have equal mass the baryon spectrum would be organized in multiplets of the group above. In this case the baryon masses would be determined solely by the orbital structure and by the constituent quark mass and the spectrum would be organized in an alternating sequence of positive and negative parity states: the ground states (N=0, positive parity), the first excited band (N=1, negative parity), the second excited band (N=2, positive parity) and so on.

If the confining potential is not harmonic, but some other possible monotonically increasing central potential, the symmetry structure of the zero order wave functions reduces to $SU(6)_{FS} \times O(3)$.

5. The Symmetry Structure of the Chiral Boson Exchange Interaction and the Baryon Mass Formula

In the $SU(3)_F$ limit all pseudoscalar octet mesons would be degenerate and also $m_u = m_d = m_s$. In this limit $V_\pi = V_K = V_\eta = V$ in (3.2) and (3.3). The flavor-spin two-quark matrix elements of the the chiral boson exchange interaction are in this case:

\[
< [f_{ij}]_F \times [f_{ij}]_S : [f_{ij}]_{FS} | - V(r_{ij}) \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j | [f_{ij}]_F \times [f_{ij}]_S : [f_{ij}]_{FS} >
\]

\[
= \begin{cases} 
-\frac{4}{3} V(r_{ij}) & [2]_F, [2]_S : [2]_{FS} \\
-8V(r_{ij}) & [11]_F, [11]_S : [2]_{FS} \\
4V(r_{ij}) & [2]_F, [11]_S : [11]_{FS} \\
\frac{8}{3} V(r_{ij}) & [11]_F, [2]_S : [11]_{FS} 
\end{cases} 
\]

(5.1)

From these the following important properties may be inferred:

(i) At short range where $V(r_{ij})$ is positive the chiral interaction (1.1) is attractive in the symmetrical FS pairs and repulsive in the antisymmetrical ones. At large distances the potential function $V(r_{ij})$ becomes negative and the situation is reversed.

(ii) At short range among the $FS$-symmetrical pairs the flavor antisymmetrical pairs experience a much larger attractive interaction than the flavor-symmetrical ones and among the $FS$-antisymmetrical pairs the strength of the repulsion in flavor-antisymmetrical pairs is considerably weaker than in sym-
metrical ones.

Given these properties we conclude that with the given flavor symmetry the more symmetrical flavor-spin Young pattern for a baryon - the more attractive contribution at short range comes from the interaction (1.1). With two identical flavor-spin Young patterns \([f]_F\) the attractive contribution at short range is larger in the case with the more antisymmetrical flavor Young pattern \([f]_F\).

When the boson exchange interaction is treated in first order perturbation theory the mass of the baryon states takes the form

\[
M = M_0 + N\hbar\omega + \delta M_x, \tag{5.2}
\]

where the chiral interaction contribution is

\[
\delta M_x = \langle \Psi | H_x | \Psi \rangle, \tag{5.3}
\]

and

\[
M_0 = \sum_{i=1}^{3} m_i + 3(V_0 + \hbar\omega). \tag{5.4}
\]

The interaction (1.1) is diagonal in states of definite orbital angular momentum \(L\) and good \([f]_F, [f]_S\) symmetries and thus there is no configuration mixing in zero order perturbation theory of the states that are degenerate in energy at lowest order. The associated tensor interaction, expected to be weak as mentioned above, does however mix states with equal \(J\) and flavor symmetry.

Due to the overall antisymmetry of the wavefunctions \(\Psi\), the chiral contribution (5.3) can be expressed as

\[
\langle \Psi | H_x | \Psi \rangle = 3 \langle \Psi | -V(r_{12})\vec{\lambda}_1 \cdot \vec{\lambda}_2 \cdot \vec{\sigma}_1 \cdot \vec{\sigma}_2 | \Psi \rangle. \tag{5.5}
\]

This is readily evaluated with the explicit expression for the wave function (4.7) and the matrix elements (3.3) for the flavor part. The result can be found in Tables 3-10 below and is a linear combination of the spatial matrix elements of the two-body potential \(V(r_{12})\), defined as

\[
P_{nl}^k = \langle \varphi_{nlm}(\vec{r}_{12})| V_k(r_{12})| \varphi_{nlm}(\vec{r}_{12}) \rangle. \tag{5.6}
\]

Here \(\varphi_{nlm}(\vec{r}_{12})\) represents the oscillator wavefunction with \(n\) excited quanta, and \(k\) the exchanged meson. As we shall only consider the baryon states in the
$N \leq 2$ bands we shall only need the 4 radial matrix elements $P_{00}, P_{11}, P_{20}$ and $P_{22}$ for the numerical construction of the spectrum.
6. The Structure of the Baryon Spectrum

Consider first for the purposes of illustration a schematic model which neglects the radial dependence of the potential function $V(r)$ in (1.1). In this model all the radial integrals $P^k_{nl}$ (5.6) will have the same constant value $C_\chi$.

The 3-quark states in the baryon spectrum have the following flavor-spin symmetries:

\begin{align}
[3]_{FS}[21]_F[21]_S, & \quad [3]_{FS}[3]_F[3]_S, & \quad [21]_{FS}[21]_F[21]_S, \\
[21]_{FS}[3]_F[21]_S, & \quad [21]_{FS}[21]_F[3]_S, & \quad [21]_{FS}[111]_F[21]_S.
\end{align}

(6.1)

For these states the matrix elements (5.5) are $-14C_\chi$, $-4C_\chi$, $-2C_\chi$, $4C_\chi$, $2C_\chi$ and $-8C_\chi$ respectively. The first one of these describes the $S = \frac{1}{2}$ baryons $N, \Lambda, \Sigma$ and $\Xi$ in the ground state band and the second the corresponding $S = \frac{3}{2}$ resonances $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$ and $\Omega$. The third describes the lowest negative parity doublet in all sectors, except for that of the $\Lambda$, in which case the lowest negative parity doublet $\Lambda(1405) - \Lambda(1520)$ is a flavor singlet described by the last one of the flavor-spin states above.

These matrix elements alone suffice to prove that the ordering of the positive and negative parity states in the baryon spectrum will be correctly predicted by the chiral boson exchange interaction (1.1). The constant $C_\chi$ may be determined from the $N - \Delta$ splitting to be 29.3 MeV. When the radial structure of the interaction (1.1) is neglected as above the oscillator parameter $\hbar \omega$ may be determined by the mass differences between the first excited $\frac{1}{2}^+$ states and the ground states of the baryons, which have the same flavor-spin, flavor and spin symmetries (e.g. $N(1440) - N$, $\Lambda(1600) - \Lambda$, $\Sigma(1660) - \Sigma$), to be $\hbar \omega \simeq 250$ MeV. In the $N$ sectors the mass difference between the lowest excited $\frac{1}{2}^+$ ($N(1440)$) and $\frac{1}{2}^-$ states ($N(1535)$) will then be

\begin{equation}
N : \quad m(\frac{1}{2}^+) - m(\frac{1}{2}^-) = 250 \text{ MeV} - C_\chi(14 - 2) = -102 \text{ MeV}, \quad (6.3)
\end{equation}

whereas it for the $\Lambda$ system ($\Lambda(1600), \Lambda(1405)$) should be

\begin{equation}
\Lambda : \quad m(\frac{1}{2}^+) - m(\frac{1}{2}^-) = 250 \text{ MeV} - C_\chi(14 - 8) = 74 \text{ MeV}. \quad (6.4)
\end{equation}
This simple example shows how the chiral interaction (1.1) provides different ordering of the lowest positive and negative excited states in the spectra of the nucleon and the Λ-hyperon. This is a direct consequence of the symmetry properties of the boson-exchange interaction discussed in the previous section. That the $SU(2)_T \times SU(2)_S$ version of the interaction (1.1) may be important for the downshift of the Roper resonance has in fact been noted earlier [43].

Consider now in addition the radial dependence of the potential with the $SU(3)_F$ invariant version (1.1) of the chiral boson exchange interaction (i.e. $V_\pi(r) = V_K(r) = V_\eta(r)$). The contribution to all nucleon, Δ and Λ hyperon states from the boson exchange interaction in terms of the matrix elements $P_{nl}$ (5.6) are listed in Tables 3 and 4. In this approximate version of the chiral boson exchange interaction the Λ − N and the Ξ−Σ mass differences would solely be ascribed the mass difference between the s and u,d quarks since all these baryons have identical orbital structure and permutational symmetries and the states in the Λ-spectrum would be degenerate with the corresponding states in the Σ-spectrum which have equal symmetries.

The oscillator parameter $\hbar\omega$ and the 4 integrals that appear in the two tables are extracted from the mass differences between the nucleon and the Δ(1232), the Δ(1600) and the N(1440), as well as the splittings between the nucleon and the average mass of the two pairs of states $N(1535)$ − $N(1520)$ and $N(1720)$ − $N(1680)$. This procedure yields the parameter values $\hbar\omega = 157.4$ MeV, $P_{00} = 29.3$ MeV, $P_{11} = 45.2$ MeV, $P_{20} = 2.7$ MeV and $P_{22} = -34.7$ MeV. Given these values all other excitation energies (i.e. differences between the masses of given resonances and the corresponding ground states) of the nucleon, Δ- and Λ-hyperon spectra are predicted to within $\sim 15\%$ of the empirical values where known, and well within the uncertainty limits of those values. These matrix elements provide a quantitatively satisfactory description of the Λ-spectrum even though they are extracted from the $N−Δ$ spectrum. The parameter values above should be allowed a considerable uncertainty range in view of the uncertainty in the empirical values for the resonance energies. To illustrate this we note that the description of the resonance energies does not notably deteriorate if instead the following set of parameter values were used: $\hbar\omega = 227.4$ MeV, $P_{11} = 31.2$ MeV, $P_{20} = 22.7$ MeV and $P_{22} = -14.4$ MeV, with the same value for $P_{00}$ as above. These latter values are obtained by taking the Δ(1600) to have an energy of 1700 MeV.

As mentioned above the symmetrical $FS$ pair states experience an attractive
interaction at short range, whereas antisymmetrical ones experience repulsion. This explains why the $[3]_{FS}$ state in the $N(1440)$, $\Delta(1600)$ and $\Sigma(1660)$ positive parity resonances feels a stronger attractive interaction than the mixed symmetry state $[21]_{FS}$ in the $N(1535)$, $\Delta(1700)$ and $\Sigma(1750)$ resonances. Consequently the masses of the positive parity states $N(1440)$, $\Delta(1600)$ and $\Sigma(1660)$ are shifted down relative to the other ones, which explains the reversal of the otherwise expected ”normal ordering”. The situation is different in the case of the $\Lambda(1405)$ and $\Lambda(1600)$, as the flavor state of the $\Lambda(1405)$ is totally antisymmetric. Because of this the $\Lambda(1405)$ gains an attractive energy, which is comparable to that of the $\Lambda(1600)$, and thus the ordering suggested by the confining oscillator interaction is maintained.

The predicted nucleon (and $\Delta$) spectrum, which in Table 3 is listed up to $N = 2$, contains two groups of nonconfirmed and unobserved states. These all belong to the $N = 2$-band. The lowest group is the 4 $\Delta$ states around 1675 MeV, one of which plausibly corresponds to the 1-star $\Delta(1750)$. The predicted $\frac{3}{2}^+$ and $\frac{5}{2}^+$ resonances around 1909 MeV plausibly correspond to the 1– and 2–star resonances $N(1900)$ and $N(2000)$ respectively. The predicted $\frac{4}{2}^+$ state at 1850 MeV corresponds well with the recent evidence in favor of a fourth $P_{11}$ state in the 1750 MeV - 1885 MeV region \cite{45, 44}. Similarly the predicted $\frac{5}{2}^+$ state at 1813 MeV corresponds well with the suggestion of a $P_{13}$ state at 1885 MeV in ref.\cite{45}. The predicted $\Lambda$ spectrum contains one unobserved state in the $N = 1$ band and 8 in the $N = 2$ band. As these are predicted to lie close to observed states with large widths their existence is not ruled out. The structure of the spectra of the $\Sigma$ and $\Xi$ hyperons are predicted to be similar to that of the nucleon and the $\Delta$ resonance in Table 3. However all $\Sigma$ and $\Lambda$ resonances with equal spatial structure and with the same flavor-spin, flavor and spin symmetries are degenerate within the SU(3)-symmetric version of the boson-exchange interaction. This degeneracy is lifted by the $SU(3)_F$ breaking interaction (3.2)-(3.3) that is treated in the following section.

The relative magnitudes and signs of the numerical parameter values can be readily understood. If the potential function $V(\vec{r})$ is assumed to have the form of a Yukawa function with a smeared $\delta$-function term that is positive at short range $r \leq 0.6 - 0.7$ fm, as suggested by the pion size $\sqrt{<r^2_\pi>} = 0.66$ fm, one expects $P_{20}$ to be considerably smaller than $P_{00}$ and $P_{11}$, as the radial wavefunction for the excited S-state has a node, and as it extends further into region of where the potential is negative. The negative value for $P_{22}$ is also natural as the corresponding wavefunction is suppressed at short range and extends well
beyond the expected 0 in the potential function. The relatively small value of
the oscillator parameter (157.4 MeV) leads to the empirical value 0.86 fm for
the nucleon radius $\sqrt{<r^2>} = \sqrt{\hbar/m\omega}$ if the light quark constituent mass is
taken to be 330-340 MeV, as suggested by the magnetic moments of the nucleon.

7. The SU(3)$_F$ Breaking Chiral Boson Interaction

The contributions to the baryon masses in the $N = 0$ band that arise from
the SU(3)$_F$ symmetry breaking version of the chiral boson interaction (3.2) are
listed in Table 5 along with the correction that arises from the difference $\Delta_q$
between the masses of the strange and up, down constituent quarks. In this case
we shall distinguish between the different strengths of the $\eta$-exchange interaction
for pairs of light ($uu$), of one light and one strange ($us$) and two strange ($ss$)
quarks. The matrix elements to be considered are thus (cf. (5.6)) those of the
pion- and kaon exchange interactions $P^\pi_{nl}$ and $P^K_{nl}$ and the $\eta$-exchange interaction
matrix elements $P^u_{nl}, P^u_{nl}$ and $P^{ss}_{nl}$. As indicated by the Yukawa interaction (3.1)
these matrix elements should be inversely proportional to the product of the
quark masses of the pair state (this result is common to all $\sigma$-model based
interactions). Thus

$$P^u_{nl} = \frac{m_u}{m_s} P^{uu}_{nl}, \quad P^{ss}_{nl} = \left(\frac{m_u}{m_s}\right)^2 P^{uu}_{nl}. \quad (7.1)$$

Here the usual assumption of equality between the constituent masses of the up
and down quark ($m_u = m_d$) has been made.

To determine the matrix element $P^K_{00}$ and $P^u_{00}$ we consider the $\Sigma(1385) - \Sigma$
mass difference, which depends only on these two integrals:

$$m_{\Sigma(1385)} - m_\Sigma = 4P^u_{00} + 6P^K_{00} \quad (7.2)$$

With the assumption that $P^u_{00} \simeq P^K_{00}$, which is suggested by the fact that the
quark masses are equal in the states, in which these interactions act, and by the
near equality of the kaon and $\eta$ masses, $\mu_\eta \simeq \mu_K$, we obtain $P^K_{00} = P^u_{00} = 19.6$
MeV. To determine the integral $P^\pi_{00}$ and the quark mass difference $\Delta_q = m_s - m_u$
we consider the $N - \Delta$ and $\Lambda - N$ mass splittings:

$$m_\Delta - m_N = 12P^\pi_{00} - 2P^{uu}_{00}, \quad (7.3a)$$
Elimination of $P_{00}^\pi$ from these two equations yields $\Delta_q = 121$ MeV if the conventional value 340 MeV is given to $m_u$. Solving for $P_{00}^\pi$ then gives the value $P_{00}^\pi = 28.9$ MeV and the quark mass ratio $m_s/m_u = 1.36$. The $\Sigma^- - \Lambda$ and $\Xi^- - \Sigma$ mass differences have the expressions

$$m_{\Sigma} - m_{\Lambda} = 8P_{00}^\pi - 4P_{00}^K - \frac{4}{3}P_{00}^{uu} - \frac{8}{3}P_{00}^{us}. \quad (7.4a)$$

$$m_{\Xi} - m_{\Sigma} = P_{00}^\pi + \frac{1}{3}P_{00}^{uu} - \frac{4}{3}P_{00}^{ss} + \Delta_q. \quad (7.4b)$$

With the matrix element values above these expressions lead to the values 65 MeV and 139 MeV for these two splittings in good agreement with the empirical values 77 MeV and 125 MeV respectively. This explanation of the octet mass splittings is differs from the early suggestion for explaining it in terms of an interaction of the form $\vec{\sigma}_i \cdot \vec{\sigma}_j V(r_{ij})/m_i m_j$, with $V(r_{ij})$ being a flavor independent function [47, 48, 49, 50].

The predictions for the energies of the baryon states in the $N = 0$ band that are obtained with the values for the integrals $P_{00}$ above are listed in Table 5. The predicted values are in remarkably satisfactory agreement with the empirical values. The largest deviation occurs for the $\Omega^-$, the energy of which is underpredicted by 21 MeV (i.e. by 1%). The quality of the fit can be improved by relaxing the requirement that $\Delta_q = m_s - m_u$ be determined to satisfy eqs. (7.3) exactly as above. With the values $\Delta_q = 127$ MeV, $m_u = 340$ MeV (i.e. $m_s/m_u = 1.37$), $P_{00}^\pi = 29.05$ MeV, $P_{00}^K = 20.1$ MeV the deviations between the experimental mass and predicted mass values are within a few MeV except for the $\Sigma$ and $\Xi$ where these deviations are about 10 MeV. The numerical values in Table 5 are very similar to those obtained earlier by Robson [24], who considered a similar pseudoscalar meson-exchange model for the interaction between massless current quarks. In Robson’s model the different strengths of the pion, kaon and $\eta$ exchange interactions at short range were obtained by taking the pseudoscalar meson mediated interactions to be proportional to the inverse square of the appropriate meson decay constants.

When the matrix elements of the boson exchange interaction (3.2) and the quark mass difference $\Delta_q$ are eliminated from the expressions for the $N = 0$ band baryons in Table 5 the following mass relations are obtained:
\[ m_{\Delta} - m_N = m_{\Sigma(1385)} - m_\Sigma + \frac{3}{2}(m_\Sigma - m_\Delta), \quad (7.5a) \]
\[ m_{\Sigma(1385)} - m_\Sigma = m_{\Xi(1530)} - m_\Xi, \quad (7.5b) \]
\[ \frac{1}{3}(m_\Omega - m_\Delta) = m_{\Xi(1530)} - m_{\Sigma(1385)}. \quad (7.5c) \]
All of these are well satisfied: the right and left hand sides of (7.5a) being 293 MeV and 307 MeV respectively, of (7.5b) being 192 MeV and 212 MeV respectively, and of (7.5c) being 147 MeV and 148 MeV respectively. The Gell-Mann-Okubo relation
\[ 3m_\Lambda + m_\Sigma = 2(m_N + m_\Xi), \quad (7.6) \]
and the equal spacing rules
\[ m_\Omega - m_{\Xi(1530)} = m_{\Xi(1530)} - m_{\Sigma(1385)} = m_{\Sigma(1385)} - m_\Delta, \quad (7.7) \]
are recovered in the SU(3)_F symmetric limit of the chiral boson exchange interaction.

The contributions to the baryon resonances in the \( N > 0 \) bands from the chiral boson exchange interaction (3.2) are listed in Tables 6-10. The lowest excited states with \( N > 0 \) in the nucleon and \( \Delta \) spectra are the \( N = 2, L = 0 \) breathing modes. The relevant integrals \( P^k_{20} \) and the oscillator parameter \( \hbar\omega \) can be determined from the \( N(1440) - N, \Delta(1600) - N \) and \( \Lambda(1600) - N \) mass differences. As in the case of the ground state matrix elements we take \( P^\text{us}_{20} = P^\text{K}_{20} \) and thus there are only two independent radial matrix elements for each shell. The other \( \eta \) - exchange matrix elements are determined by the expressions (7.1). For the \( P^\text{us}_{00} \) integrals and constituent masses we use the second set of parameter values above. These splittings lead to the relations
\[ m_{N(1440)} - m_N = \frac{15}{2} P^\pi_{00} - \frac{1}{2} P^\text{uu}_{00} - \frac{15}{2} P^\pi_{20} + \frac{1}{2} P^\text{uu}_{20} + 2\hbar\omega, \quad (7.8a) \]
\[ m_{\Delta(1600)} - m_N = \frac{27}{2} P^\pi_{00} - \frac{3}{2} P^\text{uu}_{00} - \frac{3}{2} P^\pi_{20} - \frac{1}{2} P^\text{uu}_{20} + 2\hbar\omega, \quad (7.8b) \]
\[ m_{\Lambda(1600)} - m_N = \frac{21}{2} P^\pi_{00} - \frac{1}{2} P^{uu}_{00} - 3 P^K_{00} - \frac{9}{2} P^\pi_{20} + \frac{1}{2} P^{uu}_{20} - 3 P^K_{20} + 2 \hbar \omega + \Delta_q, \] (7.8c)

which imply that \( \hbar \omega = 156.7 \) MeV, \( P^\pi_{20} = 2.2 \) MeV, \( P^K_{20} = 0.1 \) MeV. These values may be checked against the \( \Sigma(1660) \), which is the breathing mode of the \( \Sigma \):

\[ m_{\Sigma(1660)} = m_N - \frac{1}{2} P^\pi_{00} - \frac{1}{6} P^{uu}_{00} - \frac{4}{3} P^{us}_{00} - 5 P^K_{00} - \frac{1}{2} P^\pi_{20} - \frac{1}{6} P^{uu}_{20} - \frac{4}{3} P^{us}_{20} - 3 P^K_{20} + 2 \hbar \omega + \Delta_q. \] (7.9)

The result is \( m_{\Sigma(1660)} = 1639 \) MeV in good agreement with the empirical value. Again if the \( P_{20} \) integrals are not determined to satisfy (7.8) exactly (in view of the large uncertainties in the empirical values for the masses of the resonances above considerable freedom should be permitted in this regard) the quality of the fit can be improved: with \( \hbar \omega = 158.5 \) MeV, \( P^\pi_{20} = 3.0 \) MeV, \( P^K_{20} = -2.5 \) MeV we obtain the very satisfactory results \( m_{N(1440)} = 1436 \) MeV, \( m_{\Delta(1600)} = 1604 \) MeV, \( m_{\Lambda(1600)} = 1606 \) MeV, and \( m_{\Sigma(1660)} = 1660 \) MeV. We shall use this set of parameters in the further discussion (see also Tables 6-10). There is no contribution to these as well nor to the \( N = 0 \) states from the tensor forces in first order perturbation theory.

The good quality of the prediction of this breathing mode state suggests that the breathing mode of \( \frac{3}{2}^+ \Sigma \) state should lie around 1748 MeV as seen in Table 8. This predicted state possibly corresponds to the observed two-star \( \Sigma(1690) \) resonance the quantum numbers of which are unknown \[51\]. In the cascade spectrum the breathing mode states for the \( \Xi \) and the \( \Xi(1530) \) are predicted to be at 1798 MeV and 1886 MeV respectively. It is difficult to make definite assignments in the cascade spectrum since the quantum numbers of most of the identified excited states remain unknown and several predicted states have yet to be observed experimentally. We cannot exclude that these predicted breathing states correspond to the observed states \( \Xi(1690) \) and \( \Xi(1950) \), which in that case should be \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \) states respectively. Because of the close similarity between the fine structure corrections to the \( N = 2, L = 0 \) breathing mode excitations and the corresponding states in the ground state band a mass relation of the form (7.5b) also applies to the breathing resonances of the \( \Sigma \)'s and the \( \Xi \)'s:

\[ m_{\Sigma(2(20)0[3]X[3]F_S[3]F_S[3])} - m_{\Sigma(2(20)0[3]X[3]F_S[21]F[21])} = m_{\Sigma(2(20)0[3]X[3]F_S[3]F_S[3])} - m_{\Sigma(2(20)0[3]X[3]F_S[21]F[21])}. \]
\[
= m_{\Xi(2(20)0[3]_L X [3]_F S[3]_F [3]_S)} - m_{\Xi(2(20)0[3]_L X [3]_F S[21]_F [21]_S)}.
\]

If the assignment for \( \Sigma(2(20)0[3]_L X [3]_F S[3]_F [3]_S) \) to be the \( \Sigma(1690) \) is correct, the empirical values for the l.h.s. in (7.10) is only 30 MeV (with a large uncertainty margin), whereas the predicted values in Table 8 gives 88 MeV. Both sets of values suggest that the splitting between the breathing modes of the cascades on the r.h.s should not exceed 100 MeV. Thus if the \( \Xi(1690) \) is the breathing mode of the \( \Xi \), the \( \Xi(1950) \) lies too high to be the breathing mode of the \( \Xi(1530) \) and vice versa, unless the empirical determinations of masses of those two resonances represent an underestimate in the case of the former, and an overestimate in the case of the latter one.

The breathing mode of the \( \Omega^- \) is predicted to be at 2020 MeV. This region of the \( \Omega^- \) spectrum is predicted to contain several states in all quark model based calculations [35]. Empirically the only known excited states of the \( \Omega^- \) are the \( \Omega^-(2250), \Omega^-(2380) \) and the \( \Omega^-(2470) \). As by the present model the spectrum of the \( \Omega^- \) should be similar in structure to that of the \( \Delta \), the excited high lying \( \Omega^- \) states most probably are analogs of the corresponding highly excited \( \Delta \) states above 1900 MeV, and hence none of those represent the breathing mode of the \( \Omega^- \).

The two independent integrals \( P_{11}^\pi \) and \( P_{11}^K \) that are required for the determination of the resonance energies in the \( N = L = 1 \) band can be determined from two empirical energy differences if the assumption \( P_{11}^{\pi} \approx P_{11}^{K} \) is made as above. The large uncertainty limits on empirical energies of the negative parity states implies that the precision of this determination will be low. With the spin-spin component of the pseudoscalar exchange interaction (3.2) (for a discussion of the tensor and spin-orbit interactions we refer to sections 8 and 9 below) one can try to explain qualitatively only the centroids of the spin-flavor multiplets in the \( N = L = 1 \) band. These centroid positions will be shifted by the tensor force, but not much. There is also configuration mixing caused by the tensor interaction and thus the assignments in Tables 3-10 imply only main components for the baryon wave functions. The only multiplets in this band that are unmixed by the tensor and spin-orbit interactions are the \( \Delta(1620) - \Delta(1700) \) doublet and the \( \Lambda(1405) - \Lambda(1520) \) flavor singlet. The excitation energies of these states should therefore in principle be used to determine the P-state matrix elements. A somewhat better overall description of the baryon states in this band is however obtained if the values of the matrix elements are chosen so as to position the \( N(1535) - N(1520) \) doublet correctly: \( P_{11}^\pi = 45.5 \) MeV, \( P_{11}^K = 30.5 \) MeV. The predicted values of other negative parity states in N=1
shell and some of states in the $N = 2, L = 0$ shell which depend on the matrix elements $P_{11}$ (e.g. $N(1710)$, $\Delta(1910)$, $\Lambda(1810)$,...) in Tables 6 – 9 fall within 4% of the corresponding empirical values, and mostly within their uncertainty limits. Thus the two independent matrix elements $P_{11}^\pi$ and $P_{11}^K$ suffice for a satisfactory prediction of the energies of more than 20 confirmed baryon states.

For the final set of integrals $P_{22}$ required to complete the table of baryon states in the $N = L = 2$ band we chose the values $P_{22}^\pi = -35.3$ MeV and $P_{22}^K = -35.7$ MeV, as these values lead to the correct mean energies for the $L=N=2$, $[3]_{FS}[21]_F[21]_S$ nucleon and $\Lambda$ doublets $N(1720) - N(1680)$ and $\Lambda(1890) - \Lambda(1820)$. Again the model is supported by the good predictions for the energies of the confirmed $N = L = 2$ band states $\Delta(1920)$ - $\Delta(1905)$, $\Lambda(2110)$,$\Lambda(2020)$, $\Sigma(1915)$ and $\Sigma(2030)$. The results in Tables 6 - 9 show the predictions for the energies in the $N = L = 2$ band to be satisfactory, with the exception of the so far incompletely determined $S = \frac{3}{2}$ nucleon and $\Delta$ multiplets, the only so far empirically known members of are the $N(1990)$ and the $\Delta(1750)$ states, which are underpredicted by $60 – 130$ MeV. As the energy of these two one star $[51]\Sigma$ resonances remain poorly known and both of them have widths larger than $300$ MeV, their underprediction does not at this stage appear as a problem for the model.

In Table 8 we have suggested assignments for the $\Sigma$ states below $2100$ MeV, for which the spin-parity assignments are known. Some of these assignments could have been made differently however – e.g. $\Sigma(1750),\Sigma(2080)$ – because of the presence of several nearby states. The negative parity states $\Sigma(1940)$, $\Sigma(2000)$ and $\Sigma(2100)$ are not included in Table 9, as they are low lying states in the $N = 3$ band. The one star $\Sigma(1420)$, the quantum numbers of which are unknown, is not included in the table as its existence is uncertain $[51]$. The same applies to the two-star $\Sigma(1560)$ state, which if confirmed probably would be a $\frac{1}{2}^-$ state with the flavor symmetry $[21]_F$. In this latter case the $\Sigma(1620)$ may be the $\frac{1}{2}^-$ member of the multiplet $[21]_{FS}[3]_{F}[21]_S$.

For the cascade states in Table 9 we do not suggest quantum number assignments, with exception for the $\Xi(1820)$, as the quantum numbers of the orbital excitations of the $\Xi$ remain unknown and several predicted states remain to be found empirically. Given only the mass, there are several possible assignments for each resonance.

The predicted excitation spectrum of the $\Omega^-$ hyperon shown in Table 10
begins around 2 GeV. The predicted structure of the $\Omega^{-}$ should as mentioned above correspond to the of the $\Delta$, and also to the $\left|3\right>_F$ parts of the spectra of the $\Sigma$ and the $\Xi$. As at least the first two of these spectra are very satisfactorily predicted, we believe that there is compelling reason for the existence of the predicted $\Omega^{-}$ resonances around and above 2 GeV. The only observed $\Omega^{-}$ resonances, the $\Omega^{-}(2250)$, $\Omega^{-}(2380)$ and the $\Omega^{-}(2470)$ most probably are all positive parity states in the $N = 2$ band, and members of the $L = 0$, $S = \frac{1}{2}$ and $L = 2$, $S = \frac{1}{2}$, $\frac{3}{2}$ multiplets.

8. The Tensor Interaction

The pseudoscalar exchange mechanism that underlies the chiral boson exchange interaction (1.1) has no spin-orbit interaction component associated with it, and can therefore only cause a spin-orbit splitting of the spectrum through the associated tensor interaction. As the empirical splitting of the baryon states in the $N = L = 1$ band is small, and within the present uncertainty limits consistent with 0, with the exception for the anomalously large splitting of the $\Lambda(1405) - \Lambda(1520)$ flavor singlet spin doublet, this tensor component as well as the spin-orbit interaction that should be associated with the scalar harmonic confining interaction (4.1) is a priori expected to be small. The present large uncertainties in the empirical mass values for the baryon resonances in the $N=L=1$ shell makes it difficult to determine the strength of the tensor interaction phenomenologically (even the sign of some of those small spin-orbit splittings within these multiplets are not definitely settled). Therefore we here shall evaluate the effect on these spin-orbit splittings by the tensor interaction by employing the Yukawa interaction model for the pseudoscalar exchange tensor interaction.

The general expression for the tensor component of the pseudoscalar octet mediated interaction is:

$$H_T = \sum_{i<j} \left\{ \sum_{a=1}^{3} V_T^p(r_{ij}) \lambda^a_i \lambda^a_j + \sum_{a=4}^{7} V_T^K(r_{ij}) \lambda^a_i \lambda^a_j + V_T^q(r_{ij}) \lambda^8_i \lambda^8_j \right\} \hat{S}_{ij}. \quad (8.1)$$

Here

$$\hat{S}_{ij} = 3 \vec{\sigma}_i \cdot \hat{r}_{ij} \vec{\sigma}_j \cdot \hat{r}_{ij} - \vec{\sigma}_i \cdot \vec{\sigma}_j \quad (8.2)$$
is the tensor operator and $V_\pi^T, V_K^T$ and $V_\eta^T$ denote the tensor potentials that arise from $\pi$, $K$ and $\eta$ and meson exchange respectively.

The tensor interaction (8.1) will contribute to the energies of the $L = 1$ states for two reasons. The first is its nonvanishing and different matrix elements for the $\frac{1}{2}^-, \frac{3}{2}^-$ and $\frac{5}{2}^-$ states in the multiplets with completely symmetric spin states ($S = \frac{3}{2}$). The second is that its nonvanishing matrix elements between the states with different spin symmetry but equal flavor symmetry causes configuration mixing of the states with equal total angular momentum $J$ and flavor symmetry but different total spin $S$ in the $N = 1$ band. Thus although the diagonal matrix elements with the tensor force for $[21]_S$ states within $N=L=1$ shell vanish, some of these states will experience the tensor force contribution through their mixing with $[3]_S$ states that have the same $J$ and flavor symmetry. For example, the $N(1535) – N(1520)$ doublet the main component of which is $| 1(10)[21]_X[21]_S [21]_F [21]_S >$ will obtain an admixture of the spin-quartet component $| 1(10)[21]_X[21]_F [21]_S [3]_S >$.

For simplicity we adopt for this example the $SU(3)_F$ symmetric version of the spin-spin (1.1) and tensor (8.1) components of the boson-exchange interaction. After diagonalisation of the matrices that cause configuration mixing of the states with $[21]_S$ and $[3]_S$ spin symmetries, the explicit expressions for the chiral interaction contribution to the masses of this doublet are:

\[
\delta M_\chi(N(1535)) = -\frac{9}{2}P_{00} + \frac{9}{2}P_{11} + 4T_{11} - \sqrt{(-\frac{5}{2}P_{00} + \frac{1}{2}P_{11} - 4T_{11})^2 + 64T_{11}^2},
\]

(8.3)

\[
\delta M_\chi(N(1520)) = -\frac{9}{2}P_{00} + \frac{9}{2}P_{11} - \frac{16}{5}T_{11} - \sqrt{(-\frac{5}{2}P_{00} + \frac{1}{2}P_{11} + \frac{16}{5}T_{11})^2 + \frac{32}{5}T_{11}^2}.
\]

(8.4)

Here the matrix element for the tensor interaction is defined as

\[
T_{11} = \langle \varphi_{11m}(\vec{r}_{12})|V_T(r_{12})|\varphi_{11m}(\vec{r}_{12}) \rangle.
\]

The integrals $P_{00}$ and $P_{11}$ are taken from the section 6 to be 29.3 MeV and 45.2 MeV respectively. For this qualitative estimate we assume the pure Yukawa radial form for the tensor interaction:
\begin{equation}
V_Y^T(r_{ij}) = \frac{g^2}{4\pi} \frac{\mu^3}{12 m_i m_j} \left( 1 + \frac{3}{\mu r_{ij}} + \frac{3}{\mu^2 r_{ij}^2} \right) \exp(-\mu r_{ij}) \frac{\mu r_{ij}}{\mu r_{ij}}.
\end{equation}

As the \(\eta\)-exchange contribution to the tensor force matrix element for \(N\) and \(\Delta\) states in the \(N=1\) band is suppressed compared to the \(\pi\)-exchange contribution by the ratio 1:9 it suffices here to use the pure pion mass in the Yukawa potential.

The coupling constant \(g\) can be derived for this estimate from the \(\pi N\) coupling constant. For this we shall use the Goldberger-Treiman relations for both the constituent quark - pion and nucleon - pion couplings:

\begin{equation}
g = g^A \frac{m_u}{f_\pi}, \quad \text{(8.7a)}
\end{equation}

\begin{equation}
g_{\pi N} = g^A \frac{m_N}{f_\pi}.
\end{equation}

(8.7b)

Weinberg has recently shown \cite{46} that the constituent quarks have the bare unit axial coupling constant \((g^A = 1)\) and no anomalous magnetic moment. One thus obtains the relation

\begin{equation}
g = \frac{3}{5} \frac{m_u}{m_N} g_{\pi N}.
\end{equation}

(8.8)

The same expression can also be obtained by assuming that the relation between the pseudovector pion-quark and pion-nucleon coupling constants is \(f = \frac{3}{5} f_{\pi N}\). The factor \(\frac{3}{5}\) here and above comes from the spin-isospin matrix element when we consider the \(\pi N\) interaction as the interaction between the pion and 3 constituent quarks. With \(\frac{g_{\pi N}^2}{4\pi} = 14.2\) one has \(\frac{g^2}{4\pi} = 0.67\).

The matrix element (8.5) with the potential (8.6) is then

\[
< \varphi_{11m}(\vec{r}_{12}) | V_Y^T(r_{12}) | \varphi_{11m}(\vec{r}_{12}) > = \frac{g^2}{4\pi} \frac{\mu^3}{12 m_0^2} \sqrt{\frac{2}{\pi}} \left[ -\frac{1}{3b\mu} + \frac{b\mu}{3} + \frac{1}{b^2\mu^2} \sqrt{\frac{\pi}{2}} \exp\left(\frac{b^2\mu^2}{2}\right) \text{erfc}\left(\frac{b\mu}{\sqrt{2}}\right) \right],
\]

(8.9)

where \(b\) is nucleon mean-square radius for which we take the value \(b = 0.86\) fm (see section 6). This yields the value \(T_{11} \simeq 4.2\) MeV, which is much smaller than the corresponding radial matrix elements of the spin-spin interaction. The contribution from the tensor force will become even smaller by the natural regularization effect of the the finite size of the constituent quarks and the
pseudoscalar meson. Any vector-octet-like exchange interaction component between the constituent quarks, would also reduce the net tensor interaction at short range as the contributions to the tensor interaction from pseudoscalar and vector exchange mechanisms tend to cancel, whereas they add in the case of the spin-spin component. These modifications of the the tensor interaction at short range may even lead to a sign change of the matrix element (8.9), but in any case to a smaller absolute value than above.

The contribution from the tensor forces to the baryons with spatial structure $|0(00)0[3]_X >$ and $|2(20)0[3]_X >$ is identically zero in first order perturbation theory. The tensor force will however cause a small admixture of an $L = 2$ component in the ground state wave functions as well as in the breathing mode states when the calculations are performed beyond the first order perturbation theory. Such D-wave admixtures will bring along a small quadrupole moments for the spin quartet states ([3]_S) and are responsible for the observed $E2 N \rightarrow \Delta$ transition.

This estimate for the matrix element $T_{11}$ implies a small splitting in the $N(1535) - N(1520)$ doublet, $m_{N(1535)} - m_{N(1520)} = -6.4$ MeV, and a downshift of its centroid by 4.7 MeV. The admixture of the [3]_S state in the $N(1535)$ and $N(1520)$ wave functions is 5.2% and 1.9% respectively. At the same time the centroid of the $N(1650) - N(1700) - N(1675)$ triplet is shifted up by 7.6 MeV to 1640 MeV, which lies within the uncertainty limits of the empirical value. The same result also applies to the $\Lambda$ spectrum and as seen from the Table 4 the tensor force shifts the $\Lambda(1670) - \Lambda(1690)$ doublet and the $\Lambda(1800) - \Lambda(?) - \Lambda(1830)$ triplet in the right directions.

Thus taking into account the tensor interaction component of the pseudoscalar exchange interaction actually leads to a small improvement of the predicted baryon spectrum. The tensor forces do not contribute to the states $|1(10)1[21]_X [21]_FS[3]_F[21]_S >$. This implies that the splitting of the $\Delta(1620) - \Delta(1700)$ doublet should vanish in first order perturbation theory, a prediction which is consistent with observation because of the large uncertainty in the corresponding empirical mass values.

9. The $\Lambda(1405) - \Lambda(1520)$ Splitting
Although the pseudoscalar exchange interaction considered above provides an explanation of the relatively low energy of the centroid of the $\Lambda(1405) - \Lambda(1520)$ flavor singlet the tensor interaction that is expected to be associated with it cannot explain its spin-orbit splitting in first order perturbation theory as its matrix element for that state vanishes. The exceptionally large spin-orbit splitting of the $\Lambda(1405) - \Lambda(1520)$ flavor singlet suggests a dynamical origin that is specific to that state. That is to be expected a priori, as the $\Lambda(1405)$, which lies slightly below the $KN$ threshold, may be described as a $KN$ bound state \cite{53, 54, 55, 56, 57}. That implies that it has an appreciable 3 quark + octet meson component in addition to the basic 3 quark component and that therefore the chiral meson field cannot be completely integrated out in the case of $\Lambda(1405)$ as in the other baryons.

In this section we investigate the other possibility that the large spin-orbit splitting in this case might be ascribed to the effective vector-meson-exchange like interactions, that are naturally expected to arise in the second iteration of the pseudoscalar exchange interaction.

The fact that the lowest $\frac{1}{2}^-$ $\Lambda$ state lies below the lowest $\frac{3}{2}^-$ $\Lambda$ state indicates that the spin-orbit splitting cannot be due to the spin-orbit interaction that is associated with the scalar confining interaction alone, as that would lead to the opposite ordering as in the case of the corresponding spin-orbit interaction component of the nucleon-nucleon interaction (see also \cite{52}). To obtain a spin-orbit splitting that gives a lower energy for the $\frac{1}{2}^-$ than for the $\frac{3}{2}^-$ states, and thus the right sign for the $\Lambda(1405) - \Lambda(1520)$ splitting one therefore has to invoke the spin-orbit component that is associated by exchange of the vector octet between the constituent quarks. Inclusion of that vector meson-like spin-orbit interaction leads to a total spin-orbit interaction of the form

$$H_{LS} = -\sum_{i<j} \frac{1}{2} \left( \vec{\sigma}_i + \vec{\sigma}_j \right) \cdot \vec{L}_{ij}$$

$$\times \left\{ V_{LS}^S(r_{ij}) + \sum_{a=1}^3 V_{LS}^\rho(r_{ij}) \lambda_i^a \cdot \lambda_j^a + \sum_{a=4}^7 V_{LS}^{K^*}(r_{ij}) \lambda_i^a \cdot \lambda_j^a + V_{LS}^{\omega}(r_{ij}) \lambda_8^i \lambda_8^j \right\}. \quad (9.1)$$

Here $\vec{L}_{ij}$ is the orbital momentum of the relative motion of the quark pair $ij$. The potential $V_{LS}^S$ denotes the spin-orbit interaction that arises from the scalar confining interaction and the potentials $V_{LS}^\rho$, $V_{LS}^{K^*}$ and $V_{LS}^{\omega}$ denote those that arise from exchanges of systems with the quantum numbers of the $\rho$, $K^*$ and $\omega$.
mesons respectively.

The large splitting in the $\Lambda(1405) - \Lambda(1520)$ doublet implies that this spin-orbit force should be strong. A vector-octet-like spin-orbit interaction that were sufficiently strong to explain the large splitting of the flavor doublet above would however also lead to large – and empirically contraindicated – spin-orbit splittings for the other multiplets in the $N = L = 1$ band. The question is therefore whether or not the effect of such a large spin-orbit interaction can be compensated by the tensor force in the case of other multiplets.

The spin-orbit splittings of the multiplets in the $N = L = 1$ band of the spectrum can be expressed in terms of the following integrals of the spin-orbit and tensor potentials defined in (9.1) and (8.1):

$$V_{k11}^L = <\varphi_{11m}|V_{LS}^k(r_{12})|\varphi_{11m}>,$$

$$T_{k11}^L = <\varphi_{11m}|V_{T}^k(r_{12})|\varphi_{11m}>.$$

(9.2a) (9.2b)

The explicit expressions for the matrix elements of the tensor and spin-orbit potential for the the baryon states in the $N = L = 1$ band that arise from the spin-spin interaction (3.3), the spin-orbit interaction (9.1) and the tensor force (8.1) are listed in Table 11. The asterisk (*) on the matrix elements in the table indicate that they are the net matrix elements for the $N, \Lambda, \Sigma$ and $\Xi$ sectors that are defined in Table 12.

After diagonalization of the matrices in Table 11 that cause configuration mixing of states in the different multiplets with $[21]_S$ and $[3]_S$ spin symmetry the explicit expressions for the spin-orbit splitting of the $1^+ - \frac{3}{2}^-$ doublet states are

$$\delta_{[111]}(\frac{1}{2} - \frac{3}{2}) = -3V^*_{11},$$

$$\delta_{[21]}(\frac{1}{2} - \frac{3}{2}) = -\frac{3}{2}V^*_{11} + \frac{36}{5}T^*_{11}$$

$$+ \frac{1}{2} \sqrt{\left(\Delta - \frac{3}{2}V^*_{11} - \frac{32}{5}T^*_{11}\right)^2 + 40 \left(\frac{4}{5}T^*_{11} + \frac{1}{4}V^*_{11}\right)^2}$$

$$- \frac{1}{2} \sqrt{\left(\Delta - \frac{3}{2}V^*_{11} + 8T^*_{11}\right)^2 + 4 \left(-8T^*_{11} + \frac{1}{2}V^*_{11}\right)^2}.$$  

(9.3a) (9.3b)
Here the flavor-symmetry is indicated by the subscripts on the splittings and $\Delta$ is

$$\Delta = \delta M_\chi([21]_F[3]_S) - \delta M_\chi([21]_F[21]_S), \quad (9.4)$$

where the corresponding corrections $\delta M_\chi$ to the energies of those states from the spin-spin interaction (3.2) are listed in Tables 7-10 for the different baryon sectors. The first splitting (9.3a) is that for the $\Lambda(1405) - \Lambda(1520)$ doublet, which remains unmixed. The latter (9.3b) is that for the $N(1535) - N(1520)$, $\Lambda(1670) - \Lambda(1690)$ and corresponding $\Sigma$ and $\Xi$ doublets, which will obtain $S = \frac{3}{2}$ components by configuration mixing.

The corresponding spin-orbit splittings between the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ and the $\frac{1}{2}^-$ and $\frac{5}{2}^-$ states in the triplets are

$$\delta_{[21]}(\frac{1}{2} - \frac{3}{2}) = -\frac{3}{2} V_{11}^\ast + \frac{36}{5} T_{11}^*$$

$$-\frac{1}{2} \sqrt{\left( \Delta - \frac{3}{2} V_{11}^\ast - \frac{32}{5} T_{11}^* \right)^2 + 40 \left( \frac{4}{5} T_{11}^* + \frac{1}{4} V_{11}^\ast \right)^2}$$

$$+ \frac{1}{2} \sqrt{\left( \Delta - \frac{3}{2} V_{11}^\ast + 8 T_{11}^* \right)^2 + 4 \left( -8 T_{11}^* + \frac{1}{2} V_{11}^\ast \right)^2}, \quad (9.5a)$$

$$\delta_{[21]}(\frac{1}{2} - \frac{5}{2}) = -\frac{1}{2} \Delta - \frac{13}{4} V_{11}^\ast + \frac{12}{5} T_{11}^*$$

$$+ \frac{1}{2} \sqrt{\left( \Delta - \frac{3}{2} V_{11}^\ast + 8 T_{11}^* \right)^2 + 4 \left( -8 T_{11}^* + \frac{1}{2} V_{11}^\ast \right)^2}. \quad (9.5b)$$

Below for simplicity we again assume the $SU(3)_F$ limit and take $V_{LS}^\rho \simeq V_{LS}^{\omega} \simeq V_{LS}^{K^*}$.

To get the large spin-orbit splitting 115 MeV for the doublet $\Lambda(1405) - \Lambda(1520)$ would require that the effective matrix element $V_{11}^\ast$ in the $\Lambda$ sector be as large as 38 MeV. At the same time the centroid of this doublet is shifted down by $0.5 V_{11}^\ast$, which is favourable. To maintain the small spin-orbit splittings of the mixed flavor symmetry $[21]_F$ doublet and triplet states would then require that it be balanced by a correspondingly large tensor interaction matrix element $T_{11}^*$. Sufficiently small net splittings of those states can in principle be obtained if $T_{11}^*$ is taken to be 13 MeV, but only at the price of shifts of the order 27 MeV and 9 MeV down of the centroids of the $[21]_S$ and $[3]_S$ multiplets respectively. This downshift can however be compensated by increase of the
$P_{11}$ matrix element by a few MeV. Thus the only criterium here can be the empirical separation between the centroids of the $S = \frac{1}{2}$ and $S = \frac{3}{2}$ multiplets. With such large values for $V_{11}^*$ and $T_{11}^*$ this separation is about 120 MeV which should be compared with the corresponding empirical separation of 148 MeV in the nucleon sector and of 135 MeV in the Λ sector.

If on the other hand the strength of the spin-orbit potential is taken to be smaller, $V_{11}^* \simeq 25$ MeV, a value that explains most (75 MeV) of the Λ(1405) – Λ(1520) splitting, a much weaker tensor interaction is required to compensate the unfavorable splittings in the [21]F multiplets. For example, with $\hbar \omega = 180$ MeV, $P_{00} = 27.4$ MeV and $P_{11} = 50$ MeV and taking $T_{11}^*$ to be 7 MeV we get the centroids of the $N(1535) - N(1520)$ and $\Lambda(1670) - \Lambda(1690)$ doublets at 1518 MeV and 1694 MeV respectively, the centroids of the $N(1650) - N(1700) - N(1675)$ and $\Lambda(1800) - \Lambda(?) - \Lambda(1830)$ triplets around 1660 MeV and 1835 MeV respectively, and the following splittings within these multiplets: $m_{N(1535)} - m_{N(1520)} = m_{\Lambda(1670)} - m_{\Lambda(1690)} = -18$ MeV, $m_{N(1650)} - m_{N(1675)} = m_{\Lambda(1800)} - m_{\Lambda(1830)} = -40$ MeV, $m_{N(1650)} - m_{N(1700)} = m_{\Lambda(1800)} - m_{\Lambda(?)} = 44$ MeV.

Thus an at least qualitative explanation of the existing spin-orbit splittings can in principle be achieved with the assumption of a sizeable vector meson octet like interaction between the constituent quarks. Attempting a quantitative explanation of the the spin-orbit splittings in this way may of course require going beyond first order perturbation theory.

As noted above this qualitative explanation above for the larger part the spin-orbit splitting of the Λ(1405)–Λ(1520) doublet is of course only suggestive. Several other possible mechanisms may generate large spin-orbit splittings of the flavor-singlet baryons and at the same time small (or vanishing) ones for baryons with mixed or complete flavor symmetry. One such mechanism is the short range instanton-induced three - quark 't Hooft interaction [2], which involves all three flavors simultaneously in the totally antisymmetric state and which does not contribute to states with mixed or complete flavor symmetry. Finally, there remains the much discussed possibility that the Λ(1405) and the Λ(1520) states contain appreciable 5 quark components, as implied e.g. by the bound state soliton model [58], which automatically leads to a large (100 – 200 MeV) spin-orbit splitting for that doublet [59, 60].

10. Exchange Current Corrections to the Magnetic Moments
A flavor dependent interaction of the form (1.1) will imply the presence of an irreducible two-body exchange current operator, as seen e.g. directly from the continuity equation, by which the commutator of the interaction and the single particle charge operator equals the divergence of the exchange current density \[61\]. Because this commutator vanishes with interparticle separation \[62\] this exchange current is however a priori expected to be of less importance for baryons, than for nuclei, in which the longer range of the wave functions can lead to large matrix elements of the pion exchange current operator. This is one contributing reason for why the naive constituent quark model provides such a successful description of the magnetic moments. There has nevertheless been considerable discussion of the pion exchange current operator for quark pair states in the literature \[63, 64, 69, 75\].

The general form of the octet vector exchange current operator that is associated with the complete octet mediated interaction (3.2) will have the form

\[
\vec{\mu}_{\text{ex}} = \mu_N \{ \vec{V}_\pi(r_{ij})(\vec{\tau}_i \times \vec{\tau}_j)_3 \\
+ \vec{V}_K(r_{ij})(\lambda_i^4 \lambda_j^5 - \lambda_i^5 \lambda_j^4)\} (\vec{\sigma}_i \times \vec{\sigma}_j).
\]

\[10.1\]

Here \(\vec{V}_\pi(r)\) and \(\vec{V}_K(r)\) are dimensionless functions that describe \(\pi\) and \(K\) exchange respectively. At long range where the interaction between quarks can be described by a pure Yukawa potential the function \(\vec{V}_\pi(r)\) approaches the pion exchange form

\[
\vec{V}_\pi(r_{ij}) \rightarrow \frac{g^2}{4\pi} \frac{\mu m_N}{3 m_i m_j} \frac{1}{2\mu r_{ij} - 1} e^{-\mu r_{ij}},
\]

\[10.2\]

which includes both the pionic current and the pair current term.

The exchange current operator (10.1) will give rise to the following corrections to the magnetic moments of the ground state baryon octet:

\[
\mu_{\text{ex}}(p) = -\mu_{\text{ex}}(n) = -4 < \varphi_{000}(\vec{r}_{12})|\vec{V}_\pi(r_{12})|\varphi_{000}(\vec{r}_{12}) > \mu_N,
\]

\[10.3a\]

\[
\mu_{\text{ex}}(\Lambda) = -\mu_{\text{ex}}(\Sigma^0) = 2 < \varphi_{000}(\vec{r}_{12})|\vec{V}_K(r_{12})|\varphi_{000}(\vec{r}_{12}) > \mu_N,
\]

\[10.3b\]

\[
\mu_{\text{ex}}(\Sigma^+) = -\mu_{\text{ex}}(\Xi^0) = -4 < \varphi_{000}(\vec{r}_{12})|\vec{V}_K(r_{12})|\varphi_{000}(\vec{r}_{12}) > \mu_N,
\]

\[10.3c\]
\[ \mu^{ex}(\Sigma^-) = \mu^{ex}(\Xi^-) = 0, \]  

(10.3d)

\[ \mu^{ex}(\Sigma^0 \to \Lambda) = -\frac{4}{\sqrt{3}} |\varphi_{000}(\vec{r}_{12})|\bar{V}_\pi(r_{12})|\varphi_{000}(\vec{r}_{12})| > \mu_N \]

\[ -\frac{2}{\sqrt{3}} |\varphi_{000}(\vec{r}_{12})|\bar{V}_K(r_{12})|\varphi_{000}(\vec{r}_{12})| > \mu_N. \]  

(10.3e)

Here the notation for the matrix elements is the same as in eq. (5.6). The exchange current operator (10.1) cannot contribute to the magnetic moments of the ground state decuplet baryons, which have completely symmetric flavor and spin states. The absence of an exchange current correction to the magnetic moments of the \( \Sigma^- \) and \( \Xi^- \) is an immediate consequence of the fact that they are formed only of \( d \) and \( s \) quarks, which have equal charge.

The impulse approximation expressions for the magnetic moments of the ground state octet baryons and their experimental values are listed in Table 13. If these expressions are used to determine the mass ratios \( m_N/m_u \) and \( m_N/m_s \) so as to reproduce the experimental values of the magnetic moments of the proton and the \( \Lambda \) (i.e. \( m_N/m_u = 2.79 \), \( m_N/m_s = 1.83 \)), the quark model predictions for the \( \Sigma^- \) and the cascade hyperons as well as for those decuplet states, the magnetic moments of which are experimentally known (\( \Omega \) and \( \Delta^{++} \)) will differ from the experimental values by 15-30% [65]. These values for the mass ratios moreover imply that the quark mass difference \( \Delta_q \) should be 183 MeV, which is much larger than the values \( \Delta_q \simeq 130 \text{ MeV} \) required by the spectrum (Table 5).

A more natural approach is to determine the mass ratios \( m_N/m_u \) and \( m_N/m_s \) to fit the experimental values of the magnetic moments of the \( \Sigma^- \) and \( \Xi^- \) octet and the \( \Omega \) and \( \Delta^{++} \) (\( \mu_\Omega = -2.019 \pm 0.054 \mu_N \) [66], \( \mu_{\Delta^{++}} = 4.52 \pm 0.50 \mu_N \) [67]) decuplet baryons, which are unaffected by the exchange current operator (10.1). While with only two independent variables it is not possible to fit all four experimental magnetic moments exactly, the best overall fit

\[ \mu_{\Sigma^-} = -1.00 \mu_N, \]

\[ \mu_{\Xi^-} = -0.59 \mu_N, \]

\[ \mu_{\Omega^-} = -2.01 \mu_N, \]

\[ \mu_{\Delta^{++}} = 5.52 \mu_N \]
happens to be obtained with precisely the ratios $m_N/m_u = 2.76$ and $m_N/m_s = 2.01$, which used for constituent quark masses to fit baryon spectrum in section 7 ($m_u = 340$ MeV and $m_s = 467$ MeV).

With the given value for the strange quark mass the impulse value for the magnetic moment of the $\Lambda$ hyperon is $\mu^I_{\Lambda} = -0.67 \mu_N$. As this exceeds the experimental value $-0.61 \mu_N$ by only 10% in magnitude the implication is that the K-exchange radial matrix element in (10.3b) should be no larger than $<\varphi_{000}(\vec{r}_{12})|\tilde{V}_K(r_{12})|\varphi_{000}(\vec{r}_{12})> = 0.03$. The meson (kaon) - exchange contribution to the magnetic moment of $\Lambda$ thus does not exceed 10%.

With the light quark mass value above the differences between the experimental proton and neutron magnetic moments and the corresponding impulse approximation predictions are also very small: $\mu^e_p - \mu^I_p = 0.03 \mu_N$ and $\mu^e_n - \mu^I_n = -0.07 \mu_N$. This implies that the pion-exchange current contribution should be very small, and that the corresponding radial integral in (10.3a) should be $<\varphi_{000}(\vec{r}_{12})|\tilde{V}_\pi(r_{12})|\varphi_{000}(\vec{r}_{12})> = -(0.008 - 0.018)$.

With the negative value for $<\varphi_{000}(\vec{r}_{12})|\tilde{V}_\pi(r_{12})|\varphi_{000}(\vec{r}_{12})>$ the meson-exchange current contribution improves the theoretical value for the $N \rightarrow \Delta$ transition,

$$\mu^e(N \rightarrow \Delta) = -4\sqrt{2} \frac{a}{m_N} = -0.045 - 0.102 \mu_N, \quad (10.4)$$

somewhat. The sign of this MEC correction is opposite to that found with a pure Yukawa model for a pion-exchange interaction in ref. [39). This again confirms the crucial importance of the smearing of a $\delta$-function term in the Yukawa potential. The value above is however not large enough to explain the whole difference between the experimental transition magnetic moment, $\mu^e(N \rightarrow \Delta) = 3.1 - 3.2 \mu_N$, and the impulse (one body) contribution, $\mu^I(N \rightarrow \Delta) = 2\sqrt{2} \mu_N = 2.6 \mu_N$.

With the values for the $\pi$- and $K$-exchange contributions extracted above the results in Table 13 show that the predictions for the magnetic moments of the other octet baryon also are improved as compared to the impulse approximation results. The present phenomenological analysis suggests that the meson exchange current contribution to the octet magnetic moments does not exceed...
10%, which agrees with the expectation above. As the discussion here has been purely phenomenological it has however left open the task of constructing a model potential functions \( V_\pi \) and \( V_K \) in (3.3), which have the matrix elements \( P_{nl}^k \) as required by the spectrum, and which should lead to associated exchange current operators with radial behavior \( \tilde{V}_\pi(r) \) and \( \tilde{V}_K(r) \), with \( S \)-state matrix elements in the oscillator basis of the required magnitude. The construction of the radial part of the exchange current operator from the interaction potential can in principle be carried out using the methods of ref. [68].

11. Discussion

The agreement between the empirical baryon spectra and those predicted above treating the chiral boson exchange interaction (3.2) in first order perturbation theory is quite remarkable. While it should suffice to prove that the structure of the interaction mediated by the pseudoscalar octet of Goldstone bosons is essential for the understanding of the fine structure of the spectrum it also suggests that the baryon spectrum can be understood in the following way.

If the approximate chiral symmetry of the underlying QCD were realized in the explicit (Wigner-Weyl) mode, all hadron states should appear with nearby parity partners. The fact that the low lying part of both the meson and the baryon spectra lack this feature thus implies that the approximate chiral symmetry is spontaneously broken and realized in the hidden (Nambu-Goldstone) mode. This implies the generation of the dynamical mass of the valence quarks and the presence of octet of pseudoscalar Goldstone bosons, which are coupled directly to the constituent quarks. In this low energy region the gross structure of the spectrum is caused by the confining interaction, and (most of) the fine structure by the interaction (1.1) (or (3.3)) that is mediated by the octet of pseudoscalar Goldstone bosons, which are associated with the hidden mode of chiral symmetry.

Without the chiral interaction, the harmonic confining interaction would organize the baryon spectrum into equidistant shells of alternating parity. The chiral boson exchange interaction between the constituent quarks shifts some of the negative parity states in the \( N=1 \) shell and positive parity states in
the N=2 shell towards each other, which leads to approximate parity doublets. Among these are the $\frac{1}{2}^-$ near parity doublet $\Lambda(1800) - \Lambda(1810)$, the $\frac{3}{2}^-$ parity doublet $N(1700) - N(1720)$ and the $\frac{5}{2}^-$ parity doublets $N(1675) - N(1680)$ and $\Lambda(1830) - \Lambda(1820)$. This demonstrates the role of the pseudoscalar interaction for partial restoration of chiral symmetry.

The role of the pseudoscalar mesons for the partial restoration of chiral symmetry was recognized early on. Thus the continuum states formed of the baryons and odd numbers of these pseudoscalar mesons form the approximate parity partners of the low lying baryon states. The divergence of the axial current does not vanish as in the explicit mode of chiral symmetry but it is proportional to the pseudoscalar meson field (PCAC) and does vanish in the limit $m_\pi \to 0$. The smallness of the breaking of the underlying chiral symmetry is revealed by the remarkable accuracy of the Goldberger-Treiman relation for the pion-nucleon coupling. The present model achieves the partial restoration of chiral symmetry at a more microscopic level, which leads to the explanation of the appearance of the near parity doublets in the spectrum.

The (still poorly mapped) high energy part of the baryon spectrum on the other hand, which is formed of a gradually increasing number of near degenerate parity doublets (or more generally multiplets), should reveal the explicit Wigner-Weyl mode of chiral symmetry, which is due to the indistinguishability between left- and right-handed massless quarks in QCD. The remaining small splitting of the degeneracy between the parity partners is then due to the small mass of the current quarks and the gradually vanishing hidden mode of chiral symmetry.

The baryon spectrum suggests that the phase transition between the Nambu-Goldstone and Wigner-Weyl mode of chiral symmetry is gradual, as there already in the hidden mode appears a partial restoration of chiral symmetry and as the mass difference between the nearest neighbors with opposite parity falls to zero only gradually with increasing resonance energy. The clearest signal for this is that while the splitting within the $\Lambda(1600) - \Lambda(1670)$ parity doublet is still 70 MeV, the splittings within the $J^P = \frac{1}{2}^\pm$ and $J^P = \frac{5}{2}^\pm$ Λ-resonance parity doublets around 1800 MeV are only 10 MeV. This is an indication of the amorfic (disordered) structure of the QCD vacuum and its quark condensate. The implication would then be that there is a gradual chiral restoration phase transition. The disordered quark condensate structure of the QCD vacuum appears in the instanton liquid model of the QCD vacuum \[3, 4\]. Because
of the gradual character of this phase transition no definite transition energy can be defined. If the onset of the parity doubling in the resonance spectrum is taken to be at about 500 MeV above the ground state, as suggested by the mass difference between the Λ and the lowest parity doublet formed by the Λ(1600) and the Λ(1670), or by the mass difference between the nucleon and the N(1440) − N(1535) pair, the approximate transition energy would then by of the order 500 MeV. The absence of structure in the baryon spectrum above 2 GeV excitation energy suggests that the Nambu-Goldstone mode has totally disappeared in that energy range.

The present results indicate that the role of the one gluon exchange interaction, which should be important in the Wigner-Weyl mode and for current quarks, for the ordering of the baryon spectrum is small. If it is included in the model as a phenomenological term the value of the effective coupling strength α_s should be much smaller than the values ∼ 1 that have been typically employed [33, 34, 35].

Quark-quark interactions that involves the flavor degrees of freedom have been found to arise in the instanton induced interaction between quarks [2]. This interaction taken between constituent quarks has recently been applied directly to baryon structure [71, 72, 73, 74]. It differs in a crucial aspect from the pseudoscalar octet mediated interaction (1.1) in that it vanishes in flavor symmetric pair states. As a consequence it fails to account for the fine structure in the Δ-spectrum, as exemplified e.g. in the prediction of the wrong ordering of the Δ(1600) and the negative parity pair Δ(1620) − Δ(1700) [71].

It proves instructive to consider the symmetry structure of the harmonic confining + chiral octet mediated interaction (1.1) model presented here in view of the highly satisfactory predictions obtained for the baryon spectra. The symmetry group for the orbital part of a harmonically bound system of A quarks is U(3(A − 1)), which in the present case reduces to U(6). In the absence of the fine-structure interaction (1.1), and with equal u,d and s- quark masses, the baryon states would form unsplit multiplets of the full symmetry group SU(6)_FS × U(6)_{conf}. The SU(3)_F symmetrical version of the chiral interaction (1.1) reduces this degeneracy within the multiplets to those that corresponding to SU(3)_F × SU(2)_S × U(6)_{conf} and is in fact strong enough to shift some of the N=2 states below the N=1 states and to mix positions of different multiplets. Thus the N=2 resonance N(1440) is shifted down below the N=1 resonance N(1535) etc. As noted above when this shifting moves states from adjacent
N-levels close to each other near degenerate parity doublets appear. The model thus suggests an explicit explanation of the observed near parity doubling of the spectrum already in the Nambu-Goldstone mode. Within the constituent quark model the most natural suggestion for the appearance of the near parity doublets is that the Hamiltonian that is formed of the confining harmonic interaction and the chiral field interaction (1.1) contains an additional symmetry of higher rank than $SU(3)_F \times SU(2)_S \times U(6)_{\text{conf}}$, which combines the spatial and flavor-spin degrees of freedom. This conjecture is supported by the relative insensitivity of the predicted spectra to the parameter values used. The most natural suggestion is that this "unification" is related to the $SU(3)_F^L \times SU(3)_F^R \times U(1)^F$ symmetry of the underlying QCD in the Wigner-Weyl mode.

The mass splittings between the different members of the same $SU(3)_F \times SU(2)_S \times U(6)_{\text{conf}}$ multiplet arise due to both the constituent quark mass difference in (5.4) and the different strength of the meson-exchange interaction $V_\pi \neq V_K \neq V_\eta$ beyond the $SU(3)_F$ limit. Thus even those states in the $\Lambda$ and $\Sigma$ spectrum which have identical quark content and equal spatial, flavor, spin and flavor-spin symmetries, get different contributions from the interaction (3.2)-(3.3) and consequently different masses.

There is no fundamental reason for why the effective confining interaction between the constituent quarks should have to be harmonic. The low-lying part of the baryon spectrum is not very sensitive to the form of the confining interaction, but the very satisfactory numerical predictions obtained here for the baryon spectra up to about 1 GeV excitation energy suggest that any anharmonic corrections should be small. Quantitative study of the detailed form of the confining interaction would require a simultaneous specification of the detailed short range part of the chiral interaction (1.1), and would presumably also need increased accuracy for the empirical resonance energies. If the harmonic confining interaction is replaced by a nonharmonic form, the $U(6)$ spatial symmetry of the confining form is reduced to $O(3)$.

The low lying part of the baryon spectrum depends to a much higher degree on the chiral boson exchange interaction than on the confining interaction. This can be illustrated by the fact that only about a quarter of the mass difference between the nucleon and the lowest $\frac{1}{2}^-$ state $N(1535)$ is due to the confining central interaction, whereas the remaining 3 quarters are due to the spin-spin interaction (1.1). This relative "weakness" of the confining interaction is the reason for why the oscillator parameter in the present model is much smaller ($\simeq$
160 MeV) than in the models that are based on perturbative gluon exchange between the quarks. As in the latter the color magnetic interaction (2.2) contributes very little to the $N(1535) - N$ mass splitting the oscillator parameter in such models is much larger (≈ 500–600 MeV). This difference in the oscillator parameter value is the reason for why the present chiral interaction based model leads to the correct nucleon radius (0.86 fm), whereas the gluon exchange based model leads to underestimates (≈ 0.5 fm).

Be it as it may, the present organization of fine structure of the baryon spectrum based on the quark-quark interaction that is mediated by the octet of pseudoscalar mesons, which represent the Goldstone bosons associated with the hidden mode of the approximate chiral symmetry of QCD is both simple and phenomenologically successful. The predicted energies of the states in the nucleon and strange hyperon spectra agree with the empirical values, where known, to within a few percent. The accuracy of the predictions can readily be improved both by readjustment of the required matrix elements of the fine structure interaction and by carrying the calculation to second order.

12. Outlook

The very satisfactory predictions obtained here for the baryon spectrum suggest a solution to the long standing problem of finding a quark model basis for the phenomenologically successful meson exchange description of the nucleon-nucleon interaction. This problem can be approached by describing the nucleon-nucleon system as a six-quark system with the quark-cluster ansatz for the six-quark wave function (resonating group method (RGM) or related generator coordinate method) [76], in which the effective interaction is formed of direct interquark interactions and quark interchanges between clusters. Previous work based on this approach has since the early work of Oka and Yazaki [76] attempted to describe the effective repulsive short-range part of the NN system as a combination of the color-magnetic interaction (2.2) and the quark interchanges. From the present perspective it is interesting to note that with this approach a quantitatively satisfactory description of the nucleon-nucleon interaction and nucleon-nucleon scattering observables requires the presence of a pion or chiral field interaction between the quarks in addition to the confining and possible gluon exchange terms [43, 77, 78, 79, 80]. The present results indi-
cate that the effective nucleon-nucleon interaction can be described in terms of the chiral meson field mediated interaction between the constituent quarks and the short range quark interchanges alone, without any need for a perturbative gluon exchange component.

In this context it is worth emphasizing that if the confining quark-quark interaction is harmonic, it does not contribute to the effective interaction between the three quark clusters that form the nucleons. This is because no color Vander-Waals forces appear in in the one-channel RGM approach and because the nonlocal RGM kernel that obtains with a harmonic interquark potential is proportional to the normalizing kernel, and therefore cancels out in the RGM wave equation [76]. It then follows that in this approximation the meson exchange interaction between the quarks gives rise to a pure meson exchange interaction between the nucleons with the addition of quark interchanges at short range. The latter appear as a consequence of the Pauli principle at the quark level and are essential for the short range repulsion in the two-nucleon system. Experiments for the testing of the quark-interchange contributions directly have been proposed in refs. [81, 82, 83, 84].

A natural final question that arises is that of the effective interaction between quarks and antiquarks. The general relation between meson-exchange models for the quark-quark and quark-antiquark interactions is that their components that describe exchange of systems with even \( G \)-parity have the same and those with odd \( G \)-parity have the opposite sign. The discussion of the spin-orbit interaction in section 9 above suggests that the effective meson exchange interaction (1.1) could be formed not only of pseudoscalar exchange but also of a vector-meson-like exchange component with the same sign. In the \( q\bar{q} \) channel the pseudoscalar exchange term would have opposite sign because of the odd \( G \)-parity of the pseudoscalar octet, and therefore the corresponding spin-spin interaction should be much weaker because of the partial cancellation between its pseudoscalar and vector-meson-like exchange components. On the other hand the flavor-dependent tensor interaction (8.1), which is weak because of the partially cancelling pseudoscalar and vector exchange components should be much stronger in the \( q\bar{q} \)-channel, in which the two components have the same sign. As a consequence of this the effective flavor dependent meson exchange interaction between quarks and antiquarks should be very different from that between quarks. Hence no immediate conclusion concerning the spectrum of the vector - and heavier meson can be drawn from the present results. No simple two-particle interaction model should in any case be expected to apply to
the lowest pseudoscalar mesons, which are approximate Goldstone bosons and thus collective $q\bar{q}$ excitations but not simple two-body $q\bar{q}$-systems.

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Table 1

The scalar factors of the Clebsch-Gordan coefficients for the group $SU(6)_{FS}$ in the reduction $SU(6)_{FS} \supset SU(3)_F \times SU(2)_S$ defined in eq. (4.9a).

| $[f]_{FS}$ | $[f]_{FS}$ | $[f_{12}]_{FS} = [2]$ | $[f_{12}]_{FS} = [11]$ |
|------------|------------|---------------------|---------------------|
| $[3]$      | $[3]_{\frac{3}{2}}$ | 1                   |                     |
| $[21]_{\frac{1}{2}}$ | $\sqrt{\frac{1}{2}}$ | $\sqrt{\frac{1}{2}}$ |                     |
| $[21]_{\frac{3}{2}}$ | 1                   |                      |                     |
| $[111]_{\frac{1}{2}}$ | 1                   | 1                   |                     |
| $[111]_{\frac{3}{2}}$ | $\sqrt{\frac{1}{2}}$ | $-\sqrt{\frac{1}{2}}$ | $\sqrt{\frac{1}{2}}$ |
| $[111]$    | $[21]_{\frac{1}{2}}$ |                      | $\sqrt{\frac{1}{2}}$ |
| $[111]_{\frac{3}{2}}$ |                      | $-\sqrt{\frac{1}{2}}$ | 1                   |
Table 2a

Isoscalar factors of the Clebsch-Gordan coefficients for the group $SU(3)_F$ in the canonical reduction defined in eq. (4.9b).

| $[f_{12}]_F = [11]$ | $Y_{12}$ $T_{12}$ | $Y_3$ $T_3$ |
|---------------------|-------------------|--------------|
| $[f]_F$             | $YT$              | $-\frac{1}{3} \frac{2}{3}$ $-\frac{2}{3} 0$ | $\frac{2}{3} 0$ $-\frac{2}{3} 0$ | $-\frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3}$ | $\frac{2}{3} 0 \frac{1}{3} \frac{1}{3}$ |
| [21]                | $-1\frac{1}{2}$   | 1            |               |               |               |
| [21]                | 00                | $\sqrt{\frac{2}{3}}$ | $-\sqrt{\frac{1}{3}}$ |               |               |
| [111]               | 00                | $\sqrt{\frac{1}{3}}$ | $\sqrt{\frac{2}{3}}$ |               |               |
| [21]                | 01                |               | $\sqrt{\frac{2}{3}}$ | 1             |               |
| [21]                | $1\frac{1}{2}$   |               |               |               | 1             |
## Table 2b
Isoscalar factors of the Clebsch-Gordan coefficients for the group $SU(3)_F$ in the canonical reduction defined in eq. (4.9b).

| $[f_{12}]_F = [2]$ | $Y_{12}$ | $T_{12}$ | $Y_3$ | $T_3$ |
|------------------|---------|---------|------|------|
| $[f]_F$ | $YT$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $-\frac{4}{3}$ | $-\frac{2}{3}$ |
| [3] | $-20$ | | | | | | | |
| [3] | $-\frac{1}{2}$ | $\sqrt{\frac{1}{3}}$ | | | | | | 1 |
| [21] | $-\frac{1}{2}$ | $\sqrt{\frac{2}{3}}$ | | | | | | |
| [21] | $00$ | | | | | | 1 | |
| [3] | $01$ | | $\sqrt{\frac{1}{3}}$ | | $\sqrt{\frac{2}{3}}$ | | | |
| [21] | $01$ | | $-\sqrt{\frac{2}{3}}$ | | $\sqrt{\frac{1}{3}}$ | | | |
| [21] | $\frac{1}{2}$ | | | | | | 1 | |
| [3] | $\frac{3}{2}$ | | | | | | 1 | |

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Table 3

The structure of the nucleon and ∆ resonance states up to $N = 2$ as predicted with the $SU(3)_F$ invariant version of the chiral boson interaction. The 11 predicted unobserved or nonconfirmed states are indicated by question marks. The predicted energy values (in MeV) are given in the brackets under the empirical ones.
| $N(\lambda \mu) L[f]_X [f]_{FS}[f]_F [f]_S$ | LS multiplet | average energy | $\delta M_x$  |
|------------------|-------------|----------------|------------|
| 0(00)0[3]_X [3]_{FS}[21]_F [21]_S | $\frac{1}{2}^+, N$ | 939 | $-14P_{00}$ |
| 0(00)0[3]_X [3]_{FS}[3]_F [3]_S | $\frac{3}{2}^+, \Delta$ | 1232 | $-4P_{00}$ |
| 2(20)0[3]_X [3]_{FS}[21]_F [21]_S | $\frac{1}{2}^+, N(1440)$ | 1440 | $-7P_{00} - 7P_{20}$ |
| 1(10)1[21]_X [21]_{FS}[21]_F [21]_S | $\frac{1}{2}^-, N(1535); \frac{3}{2}^-, N(1520)$ | 1527 | $-7P_{00} + 5P_{11}$ |
| 2(20)0[3]_X [3]_{FS}[3]_F [3]_S | $\frac{3}{2}^+, \Delta(1600)$ | 1600 | $-2P_{00} - 2P_{20}$ |
| 1(10)1[21]_X [21]_{FS}[3]_F [21]_S | $\frac{1}{2}^-, \Delta(1620); \frac{3}{2}^-, \Delta(1700)$ | 1660 | $-2P_{00} + 6P_{11}$ |
| 1(10)1[21]_X [21]_{FS}[21]_F [3]_S | $\frac{1}{2}^-, N(1650); \frac{3}{2}^-, N(1700)$ | 1675 | $-2P_{00} + 4P_{11}$ |
| 2(20)2[3]_X [3]_{FS}[3]_F [3]_S | $\frac{1}{2}^+, \Delta(1750?); \frac{3}{2}^+, \Delta(?)$ | 1750? | $-2P_{00} - 2P_{22}$ |
| 2(20)2[3]_X [3]_{FS}[3]_F [3]_S | $\frac{1}{2}^+, \Delta(1720); \frac{3}{2}^+, N(1680)$ | 1700 | $-7P_{00} - 7P_{22}$ |
| 2(20)0[21]_X [21]_{FS}[21]_F [21]_S | $\frac{1}{2}^+, N(1710)$ | 1710 | $-\frac{7}{2}P_{00} - \frac{7}{2}P_{20} + 5P_{11}$ |
| 2(20)0[21]_X [21]_{FS}[21]_F [3]_S | $\frac{3}{2}^+, N(1900?); \frac{5}{2}^+, N(2000)?$ | 1950? | $-\frac{7}{2}P_{00} - \frac{7}{2}P_{22} + 5P_{11}$ |
| 2(20)2[21]_X [21]_{FS}[21]_F [21]_S | $\frac{1}{2}^+, N(?); \frac{3}{2}^+, N(?)$ | 1990? | $-P_{00} - P_{22} + 4P_{11}$ |
| 2(20)2[21]_X [21]_{FS}[21]_F [3]_S | $\frac{1}{2}^+, N(?); \frac{3}{2}^+, N(1900?)$ | 1910 | $-P_{00} - P_{20} + 6P_{11}$ |
| 2(20)0[21]_X [21]_{FS}[3]_F [21]_S | $\frac{1}{2}^+, \Delta(1910)$ | 1910 | $-P_{00} - P_{20} + 6P_{11}$ |
| 2(20)2[21]_X [21]_{FS}[3]_F [21]_S | $\frac{3}{2}^+, \Delta(1920); \frac{5}{2}^+, \Delta(1905)$ | 1912 | $-P_{00} - P_{22} + 6P_{11}$ |
The structure of the Λ-hyperon states up to $N = 2$ predicted with the $SU(3)_F$ invariant version of the chiral boson exchange interaction. The 10 predicted unobserved or nonconfirmed states are indicated by question marks. The predicted energies (in MeV) are given in the brackets under the empirical values.

| $N(\lambda\mu)L[f]_X[f]_{FS}[f]_F[f]_S$ | LS multiplet | average energy | $\delta M_X$ |
|----------------------------------------|--------------|----------------|-----------|
| 0(00)0[3]_X[3]_FS[21]_F[21]_S         | $\frac{1}{2}^+, \Lambda$ | 1115           | $-14P_{00}$ |
| 1(10)1[21]_X[21]_FS[111]_F[21]_S      | $\frac{1}{2}^-, \Lambda(1405); \frac{3}{2}^-, \Lambda(1520)$ | 1462 (1512)   | $-12P_{00} + 4P_{11}$ |
| 2(20)0[3]_X[3]_FS[21]_F[21]_S         | $\frac{1}{2}^+, \Lambda(1600)$ | 1600 (1616)   | $-7P_{00} - 7P_{20}$ |
| 1(10)1[21]_X[21]_FS[21]_F[21]_S       | $\frac{1}{2}^-, \Lambda(1670); \frac{3}{2}^-, \Lambda(1690)$ | 1680 (1703)   | $-7P_{00} + 5P_{11}$ |
| 1(10)1[21]_X[21]_FS[21]_F[3]_S        | $\frac{1}{2}^-, \Lambda(1800); \frac{3}{2}^-, \Lambda(?); \frac{5}{2}^-, \Lambda(1830)$ | 1815 (1805)   | $-2P_{00} + 4P_{11}$ |
| 2(20)0[21]_X[21]_FS[111]_F[21]_S      | $\frac{1}{2}^+, \Lambda(1810)$ | 1810 (1829)   | $-6P_{00} - 6P_{20} + 4P_{11}$ |
| 2(20)2[3]_X[3]_FS[21]_F[21]_S         | $\frac{3}{2}^+, \Lambda(1890); \frac{5}{2}^+, \Lambda(1820)$ | 1855 (1878)   | $-7P_{00} - 7P_{22}$ |
| 2(20)0[21]_X[21]_FS[21]_F[21]_S       | $\frac{1}{2}^+, \Lambda(?)$ | ?              | $-\frac{7}{2}P_{00} - \frac{7}{2}P_{20} + 5P_{11}$ |
| 2(20)0[21]_X[21]_FS[21]_F[3]_S        | $\frac{3}{2}^+, \Lambda(?)$ | ?              | $-P_{00} - P_{20} + 4P_{11}$ |
| 2(20)2[21]_X[21]_FS[21]_F[3]_S        | $\frac{1}{2}^+, \Lambda(?)$ | ?              | $-P_{00} - P_{20} + 4P_{11}$ |
| 2(20)2[21]_X[21]_FS[21]_F[3]_S        | $\frac{3}{2}^+, \Lambda(?)$ | ?              | $-P_{00} - P_{22} + 4P_{11}$ |
| 2(20)2[21]_X[21]_FS[111]_F[21]_S      | $\frac{3}{2}^+, \Lambda(?)$ | 2020 (2026)   | $-6P_{00} - 6P_{22} + 4P_{11}$ |
| 2(20)2[21]_X[21]_FS[21]_F[21]_S       | $\frac{3}{2}^+, \Lambda(?)$ | 2110 (2085)   | $-\frac{7}{2}P_{00} - \frac{7}{2}P_{22} + 5P_{11}$ |

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The contributions to the masses of the baryon states in the $N = 0$ band given by the $SU(3)_F$ symmetry breaking chiral interaction (3.2). The mass difference between the s and u,d quarks is denoted $\Delta_q$. The superscripts uu,us and ss on the $\eta$-exchange matrix elements indicate that it applies to pair states of two light, one light and on strange and two strange quarks respectively. The predicted mass values (in MeV) for the parameter set (I) $\Delta_q = 121$ MeV, $m_u = 340$MeV, $P^{\pi}_{00} = 28.9$ MeV, $P^K_{00} = 19.6$ MeV and that for the set (II) $\Delta_q = 127$ MeV, $m_u = 340$MeV, $P^{\pi}_{00} = 29.05$ MeV, $P^K_{00} = 20.1$ MeV are given in the corresponding columns.

| $N(\lambda\mu)L[f]_X[f]_F S[f]_F S[f]_S$ (mass) | State | Predicted mass I | Predicted mass II | $\delta M_X$ |
|-----------------------------------------------|-------|------------------|------------------|-------------|
| 0(00)0[3]$_X$[3]$_F$S[21]$_F$[21]$_S$ (939) | N     | input            | input            | $-15P^{\pi}_{00} + P^{uu}_{00}$ |
| 0(00)0[3]$_X$[3]$_F$S[3]$_F$[3]$_S$ (1232) | $\Delta$ | 1232             | 1232             | $-3P^{\pi}_{00} - P^{uu}_{00}$ |
| 0(00)0[3]$_X$[3]$_F$S[21]$_F$[21]$_S$ (1116) | $\Lambda$ | 1116             | 1120             | $-9P^{\pi}_{00} - 6P^{K}_{00} + P^{uu}_{00} + \Delta_q$ |
| 0(00)0[3]$_X$[3]$_F$S[21]$_F$[21]$_S$ (1193) | $\Sigma$ | 1181             | 1181             | $-P^{\pi}_{00} - 10P^{K}_{00} - \frac{1}{3}P^{uu}_{00} - \frac{8}{3}P^{us}_{00} + \Delta_q$ |
| 0(00)0[3]$_X$[3]$_F$S[3]$_F$[3]$_S$ (1385) | $\Sigma(1385)$ | 1377         | 1382             | $-P^{\pi}_{00} - 4P^{K}_{00} - \frac{1}{3}P^{uu}_{00} + \frac{4}{3}P^{us}_{00} + \Delta_q$ |
| 0(00)0[3]$_X$[3]$_F$S[21]$_F$[21]$_S$ (1318) | $\Xi$ | 1320             | 1327             | $-10P^{K}_{00} - \frac{8}{3}P^{ss}_{00} - \frac{4}{3}P^{us}_{00} + 2\Delta_q$ |
| 0(00)0[3]$_X$[3]$_F$S[3]$_F$[3]$_S$ (1530) | $\Xi(1530)$ | 1516         | 1528             | $-4P^{K}_{00} + \frac{4}{3}P^{ss}_{00} - \frac{4}{3}P^{us}_{00} + 2\Delta_q$ |
| 0(00)0[3]$_X$[3]$_F$S[3]$_F$[3]$_S$ (1672) | $\Omega^-$ | 1651       | 1670             | $-4P^{ss}_{00} + 3\Delta_q$ |
The structure of the nucleon and Δ resonance states in the $N = 1, 2$ bands as predicted with the $SU(3)_F$ breaking version of the chiral boson interaction (2.3). The $\eta$-exchange matrix elements are have the superscript uu to indicate that they apply to pair states of light constituent quarks. The predicted energy values (in MeV) are given in the brackets under the empirical ones. The parameter Set is the following: set (II) from the Table 5 plus $P^\pi_{11} = 45.5$ MeV, $P^K_{11} = 30.5$ MeV, $P^\pi_{20} = 3.0$ MeV, $P^K_{20} = -2.5$ MeV, $P^\pi_{22} = -35.3$ MeV, $P^K_{22} = -35.7$ MeV
| $N(\lambda\mu) L[f]_{FS} [f]_{F} [f]_{S}$ | LS multiplet | average energy | $\delta M_X$ |
|----------------------------------|---------------|----------------|----------------|
| 2(20)0[3]_{FS} [21]_{F} [21]_{S} | $\frac{9}{2}^+$, $N(1440)$ | 1440 (1436) | $-\frac{1}{15} P^\pi_{00} + \frac{1}{3} P^\mu_{00}$ |
| 1(10)1[21]_{FS} [21]_{F} [21]_{S} | $\frac{5}{2}^-$, $N(1535); \frac{3}{2}^-$, $N(1520)$ | 1527 (1527) | $-\frac{1}{3} P^\pi_{00} + \frac{1}{2} P^\mu_{00}$ |
| 2(20)0[3]_{FS} [3]_{F} [3]_{S}   | $\frac{9}{2}^+$, $\Delta(1600)$ | 1600 (1604) | $-\frac{1}{3} P^\pi_{00} - \frac{1}{2} P^\mu_{00}$ |
| 1(10)1[21]_{FS} [3]_{F} [21]_{S} | $\frac{5}{2}^-$, $\Delta(1620); \frac{3}{2}^-$, $\Delta(1700)$ | 1660 (1716) | $-\frac{1}{7} P^\pi_{00} + \frac{1}{2} P^\mu_{00}$ |
| 1(10)1[21]_{FS} [3]_{F} [3]_{S} | $\frac{5}{2}^-$, $N(1650); \frac{3}{2}^-$, $N(1700)$ | 1675 (1632) | $-\frac{1}{7} P^\pi_{00} - \frac{1}{2} P^\mu_{00}$ |
| 2(20)2[3]_{FS} [3]_{F} [3]_{S}   | $\frac{9}{2}^+$, $\Delta(1750)\?; \frac{5}{2}^+$, $\Delta(?)$ | 1750 (1684) | $-\frac{1}{7} P^\pi_{00} - \frac{1}{2} P^\mu_{00}$ |
| 2(20)2[3]_{FS} [21]_{F} [21]_{S} | $\frac{5}{2}^+$, $N(1720); \frac{3}{2}^+$, $N(1680)$ | 1700 (1700) | $-\frac{1}{7} P^\pi_{00} + \frac{1}{2} P^\mu_{00}$ |
| 2(20)0[21]_{FS} [21]_{F} [21]_{S} | $\frac{3}{2}^+$, $N(1710)$ | 1710 (1776) | $-\frac{1}{7} P^\pi_{00} + \frac{1}{2} P^\mu_{00}$ |
| 2(20)0[21]_{FS} [3]_{F} [3]_{S}   | $\frac{3}{2}^+$, $N(?)$ | ? (1818) | $-\frac{3}{4} P^\pi_{00} - \frac{1}{2} P^\mu_{00}$ |
| 2(20)2[21]_{FS} [21]_{F} [21]_{S} | $\frac{5}{2}^+$, $N(1900)\?; \frac{3}{2}^+$, $N(2000)\?$ | 1950 (1908) | $-\frac{3}{7} P^\pi_{00} + \frac{1}{2} P^\mu_{00}$ |
| 2(20)2[21]_{FS} [21]_{F} [3]_{S}   | $\frac{3}{2}^+$, $N(?)\?; \frac{5}{2}^+$, $N(990)\?$ | 1990 (1858) | $-\frac{3}{7} P^\pi_{00} - \frac{1}{2} P^\mu_{00}$ |
| 2(20)0[21]_{FS} [3]_{F} [21]_{S}   | $\frac{1}{2}^+$, $\Delta(1910)$ | 1910 (1902) | $-\frac{3}{7} P^\pi_{00} - \frac{1}{2} P^\mu_{00}$ |
| 2(20)2[21]_{FS} [3]_{F} [21]_{S}   | $\frac{3}{2}^+$, $\Delta(1920); \frac{5}{2}^+$, $\Delta(1905)$ | 1912 (1942) | $-\frac{3}{7} P^\pi_{00} - \frac{1}{2} P^\mu_{00}$ |
Table 7

The structure of the Λ-hyperon states in the $N = 1, 2$ bands predicted with the $SU(3)_F$ breaking version of the chiral boson exchange interaction. The superscripts uu and us on the $\eta$ exchange matrix elements indicate that they apply to pair states of two light and one light and one strange quark respectively. The predicted energies (in MeV) are given in the brackets under the empirical values. For the parameter set see Table 6.
| $N(\lambda \mu)L[f]_{FS}[f]_{F}F_{S}$ | LS multiplet | average energy | $\delta M_x$ |
|-----------------------------------|--------------|----------------|------------|
| 1(10)1[21]_{FS}[111]_{F}[21]_{S} | $1/2^-$, $\Lambda(1405)$; $3/2^-$, $\Lambda(1520)$ | 1462 (1498) | $-9/2 P_{00} - 9/2 P_{02} + 9/2 P_{10} - 6P_{20} + 3 P_{K22}$ |
| 2(20)0[3]_{FS}[21]_{F}[21]_{S} | $1/2^+$, $\Lambda(1600)$ | 1600 (1605) | $-9/2 P_{00} + 9/2 P_{02} - 3P_{10}$ |
| 1(10)1[21]_{FS}[21]_{F}[3]_{S} | $1/2^-$, $\Lambda(1670)$; $3/2^-$, $\Lambda(1690)$ | 1680 (1629) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
| 1(10)1[21]_{FS}[21]_{F}[3]_{S} | $1/2^-$, $\Lambda(1800)$; $3/2^-$, $\Lambda(1830)$ | 1815 (1756) | $+3 P_{00} - 3 P_{02}$ |
| 2(20)0[111]_{FS}[21]_{F}[21]_{S} | $1/2^+$, $\Lambda(1810)$ | 1810 (1797) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
| 2(20)2[3]_{FS}[21]_{F}[21]_{S} | $3/2^+, \Lambda(1890)$; $5/2^+, \Lambda(1820)$ | 1855 (1855) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
| 2(20)0[21]_{FS}[21]_{F}[21]_{S} | $1/2^+$, $\Lambda(?)$ | 1870 (1872) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
| 2(20)0[21]_{FS}[21]_{F}[3]_{S} | $3/2^+, \Lambda(?)$ | 1937 (1937) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
| 2(20)2[111]_{FS}[21]_{F}[3]_{S} | $1/2^+, \Lambda(?)$; $3/2^+, \Lambda(?)$; $7/2^+, \Lambda(2020)$ | 2020 (1970) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
| 2(20)2[111]_{FS}[21]_{F}[3]_{S} | $3/2^+, \Lambda(?)$; $5/2^+, \Lambda(2110)$ | 2110 (2005) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
| 2(20)2[21]_{FS}[21]_{F}[21]_{S} | $3/2^+, \Lambda(?)$; $5/2^+, \Lambda(2110)$ | 2110 (1996) | $+3 P_{11} - 3 P_{10} + 3 P_{K11}$ |
Table 8

The structure of the Σ-hyperon states in the $N = 1, 2$ bands predicted with the $SU(3)_F$ breaking version of the chiral boson exchange interaction. The superscripts uu and us on the $\eta$ exchange matrix elements indicate that they apply to pair states of two light and one light and one strange quark respectively. The predicted energies (in MeV) are given in the brackets under the empirical values. For the parameter set see Table 6.
| $N(\lambda\mu)L[f]_{FS}[f]_{F}f_{S}$ | LS multiplet | average energy | $\delta M_\chi$ |
|---------------------------------|--------------|----------------|-----------------|
| 2(20)0[3]_{FS}[21]_{F}[21]_{S} | $\frac{1}{2}^+$, $\Sigma(1660)$ | 1660 (1660) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} - \frac{4}{3}P_{00u} - 5P_{00K}$ |
| 1(10)1[21]_{FS}[21]_{F}[21]_{S} | $\frac{1}{2}^-$, $\Sigma(1620)$; $\frac{3}{2}^-$, $\Sigma(1580)$ | 1600 (1667) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} - \frac{4}{3}P_{00u} - 5P_{00K}$ |
| 2(20)0[3]_{FS}[3]_{F}[3]_{S} | $\frac{3}{2}^+$, $\Sigma(1750)$ | (1748) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 1(10)1[21]_{FS}[3]_{F}[21]_{S} | $\frac{1}{2}^-$, $\Sigma(1750)$; $\frac{3}{2}^-$, $\Sigma(1775)$ | 1750 (1798) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 1(10)1[21]_{FS}[21]_{F}[3]_{S} | $\frac{1}{2}^-$, $\Sigma(1770)$; $\frac{3}{2}^-$, $\Sigma(1840)$; $\frac{5}{2}^+$, $\Sigma(1750)$ | 1732 (1798) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 2(20)2[3]_{FS}[3]_{F}[3]_{S} | $\frac{3}{2}^+$, $\Sigma(1805)$ | (1819) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 2(20)2[3]_{FS}[21]_{F}[21]_{S} | $\frac{3}{2}^+$, $\Sigma(1915)$ | 1915 (1997) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 2(20)0[21]_{FS}[21]_{F}[21]_{S} | $\frac{1}{2}^+$, $\Sigma(1880)$ | 1880 (1906) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 2(20)0[21]_{FS}[21]_{F}[3]_{S} | $\frac{3}{2}^+$, $\Sigma(1985)$ | (1887) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 2(20)2[21]_{FS}[21]_{F}[21]_{S} | $\frac{3}{2}^+$, $\Sigma(2065)$; $\frac{5}{2}^+$, $\Sigma(2080)$; $\frac{7}{2}^+$, $\Sigma(2030)$ | 2060 (2020) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |
| 2(20)2[21]_{FS}[3]_{F}[21]_{S} | $\frac{3}{2}^+$, $\Sigma(2080)$; $\frac{5}{2}^+$, $\Sigma(2070)$ | 2075 (2017) | $-\frac{1}{2}P_{00} - \frac{1}{6}P_{00u} + \frac{2}{3}P_{00u} - 2P_{00K}$ |

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Table 9

The structure of the Ξ-hyperon states in the $N = 1, 2$ bands predicted with the $SU(3)_F$ breaking version of the chiral boson exchange interaction. The superscripts $u$s and $s$s on the $\eta$-exchange matrix elements indicate that the interaction acts in pair states of one light and one strange and two strange quarks respectively. The predicted energies (in MeV) are given in the brackets under the empirical values. For the parameter set see Table 6.
| \(N(\lambda\mu)L[f]_{FS}[f][f]_{S}\) | LS multiplet | average energy | \(\delta M_X\) |
|--------------------------------|--------------|---------------|----------------|
| 2(20)0[3]_{FS}[21]_{F}[21]_{S}\ | \(\frac{1}{2}^+, \Xi(?)\) | \(?\) | \(-\frac{1}{3}P_{00} + \frac{2}{3}P^{ss}_{00} - \frac{8}{3}P^K_{00}\) |
| | \(\frac{3}{2}^-, \Xi(?)\) | \(?\) | \(-\frac{1}{3}P_{00} + \frac{2}{3}P^{ss}_{00} - \frac{8}{3}P^K_{00}\) |
| 1(10)1[21]_{FS}[21]_{F}[21]_{S}\ | \(\frac{1}{2}^+, \Xi(?)\); \(\frac{3}{2}^-, \Xi(?)\) | \(?\) | \(+2P^{ss}_{11} + \frac{3}{2}P^K_{11}\) |
| 2(20)0[3]_{FS}[3]_{F}[3]_{S}\ | \(\frac{3}{2}^+, \Xi(?)\) | \(?\) | \(+\frac{2}{3}P_{00} - \frac{2}{3}P^{ss}_{00} - \frac{1}{3}P^K_{00}\) |
| | \(\frac{1}{2}^+, \Xi(?)\) | \(?\) | \(-\frac{1}{3}P_{00} + \frac{2}{3}P^{ss}_{00} - \frac{1}{3}P^K_{00}\) |
| 2(20)2[3]_{FS}[3]_{F}[3]_{S}\ | \(\frac{3}{2}^+, \Xi(?)\); \(\frac{5}{2}^+, \Xi(?)\) | \(?\) | \(+\frac{2}{3}P_{00} - \frac{2}{3}P^{ss}_{00} - \frac{2}{3}P^K_{00}\) |
| 2(20)2[3]_{FS}[21]_{F}[21]_{S}\ | \(\frac{3}{2}^+, \Xi(?)\); \(\frac{5}{2}^+, \Xi(?)\) | \(?\) | \(+\frac{2}{3}P_{00} - \frac{2}{3}P^{ss}_{00} - \frac{2}{3}P^K_{00}\) |
| 2(20)0[1]_{FS}[21]_{F}[21]_{S}\ | \(\frac{1}{2}^+, \Xi(?)\) | \(?\) | \(-\frac{1}{3}P_{00} + \frac{2}{3}P^{ss}_{00} - \frac{1}{3}P^K_{00}\) |
| | \(\frac{3}{2}^+, \Xi(?)\) | \(?\) | \(-\frac{1}{3}P_{00} + \frac{2}{3}P^{ss}_{00} - \frac{1}{3}P^K_{00}\) |
| 2(20)0[1]_{FS}[3]_{F}[3]_{S}\ | \(\frac{3}{2}^+, \Xi(?)\) | \(?\) | \(+\frac{2}{3}P_{00} - \frac{2}{3}P^{ss}_{00} - \frac{3}{2}P^K_{00}\) |
| | \(\frac{3}{2}^+, \Xi(?)\); \(\frac{1}{2}^+, \Xi(?)\) | \(?\) | \(+\frac{2}{3}P_{00} - \frac{2}{3}P^{ss}_{00} - \frac{3}{2}P^K_{00}\) |
| 2(20)2[1]_{FS}[21]_{F}[21]_{S}\ | \(\frac{3}{2}^+, \Xi(?)\); \(\frac{5}{2}^+, \Xi(?)\) | \(?\) | \(+\frac{2}{3}P_{00} - \frac{2}{3}P^{ss}_{00} - \frac{3}{2}P^K_{00}\) |
| 2(20)2[1]_{FS}[3]_{F}[3]_{S}\ | \(\frac{3}{2}^+, \Xi(?)\); \(\frac{5}{2}^+, \Xi(?)\) | \(?\) | \(+\frac{2}{3}P_{00} - \frac{2}{3}P^{ss}_{00} - \frac{3}{2}P^K_{00}\) |

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Table 10

The structure of the $\Omega^-$ hyperon states in the $N = 1, 2$ bands predicted with the $SU(3)_F$ breaking version of the chiral boson exchange interaction. The predicted energies (in MeV) are given in the brackets under the empirical values. For the parameter set see Table 6.

| $N(\lambda\mu)L[f]_F[f]_F[f]_S$ | LS multiplet | average $\delta M_x$ energy |
|----------------------------------|--------------|-----------------------------|
| 2(20)0[3]_F[3]_F[3]_S           | $\frac{3}{2}^+$, $\Omega^-$ (?) | ? (2020) | $-2P_{00}^{ss} - 2P_{20}^{ss}$ |
| 1(10)1[21]_F[3]_F[21]_S         | $\frac{1}{2}^-$, $\Omega^-$ (?); $\frac{3}{2}^-$, $\Omega^-$ (?) | ? (1991) | $-2P_{00}^{ss} + 6P_{11}^{ss}$ |
| 2(20)2[3]_F[3]_F[3]_S           | $\frac{1}{2}^+$, $\Omega^-$ (?); $\frac{3}{2}^+$, $\Omega^-$ (?); $\frac{5}{2}^+$, $\Omega^-$ (?); $\frac{7}{2}^+$, $\Omega^-$ (?) | ? (2068) | $-2P_{00}^{ss} - 2P_{22}^{ss}$ |
| 2(20)0[21]_F[3]_F[21]_S         | $\frac{1}{2}^+$, $\Omega^-$ (?) | ? (2166) | $-P_{00}^{ss} - P_{20}^{ss} + 6P_{11}^{ss}$ |
| 2(20)2[21]_F[3]_F[21]_S         | $\frac{3}{2}^-$, $\Omega^-$ (?); $\frac{5}{2}^-$, $\Omega^-$ (?) | ? (2190) | $-P_{00}^{ss} - P_{22}^{ss} + 6P_{11}^{ss}$ |
Table 11

The contributions to the baryon energies from the interactions (3.3), (8.1) and (9.1), with inclusion of the nondiagonal matrix elements of the spin-orbit and tensor interactions. The terms $\delta M_\chi$ are the contributions to the corresponding states from the spin-spin interaction (3.3), which are listed in Tables 6-9. The net spin-orbit ($V_{11}^*$) and tensor interaction ($T_{11}^*$) matrix elements for the different sectors of the baryon spectrum are listed in Table 12. The $[3]_F$ states are absent here as both tensor and spin-orbit forces do not contribute in this case.

| $[f]_F [s]_F [f]_S$ | Potential matrix |
|---------------------|------------------|
| $[21]_F [21]_F [21]_S : \frac{1}{2}^-$ | $\delta M_\chi([21]_F [21]_S) - V_{11}^* - 8T_{11}^* + \frac{1}{2}V_{11}^*$ |
| $[21]_F [21]_F [3]_S : \frac{1}{2}^-$ | $-8T_{11}^* + \frac{1}{2}V_{11}^*$ |
| $[21]_F [21]_F [3]_S : \frac{3}{2}^-$ | $\delta M_\chi([21]_F [3]_S) + 8T_{11}^* - \frac{5}{2}V_{11}^*$ |
| $[21]_F [111]_F [21]_S : \frac{1}{2}^-$ | $\delta M_\chi([111]_F [21]_S) - 2V_{11}^*$ |
| $[21]_F [111]_F [21]_S : \frac{3}{2}^-$ | $\delta M_\chi([111]_F [21]_S) + V_{11}^*$ |
| $[21]_F [21]_F [3]_S : \frac{5}{2}^-$ | $\delta M_\chi([21]_F [31]_S) + \frac{8}{5}T_{11}^* + \frac{3}{2}V_{11}^*$ |
Table 12

The contributions to the net spin-orbit ($V_{11}^*$) and tensor ($T_{11}^*$) interaction matrix elements from the different exchange interactions in eqs. (8.1) and (9.1) for the $N = L = 1$ baryon states. The superscripts uu and us on the matrix elements of the $\omega$-exchange-like spin-orbit interaction and of the $\eta$-exchange tensor interaction indicate that they apply to pair states of two light and one strange quark respectively.

|       | $V_{11}^*$                        | $T_{11}^*$                        |
|-------|-----------------------------------|-----------------------------------|
| $N$   | $-V_{11}^S + 3V_{11}^\rho - \frac{1}{3}V_{11}^{uu}$ | $-3T_{11}^\pi + \frac{1}{3}T_{11}^{uu}$ |
| $\Lambda([111]_F)$ | $-V_{11}^S + V_{11}^\rho + \frac{4}{3}V_{11}^{K*} - \frac{1}{9}V_{11}^{uu} + \frac{4}{9}V_{11}^{us}$ | $-T_{11}^\pi - \frac{2}{3}T_{11}^{K} + \frac{1}{9}T_{11}^{uu} - \frac{4}{9}T_{11}^{us}$ |
| $\Lambda([21]_F)$ | $-V_{11}^S + 2V_{11}^\rho + \frac{2}{3}V_{11}^{K*} - \frac{2}{9}V_{11}^{uu} + \frac{2}{9}V_{11}^{us}$ | $-2T_{11}^\pi - \frac{2}{3}T_{11}^{K} + \frac{2}{9}T_{11}^{uu} - \frac{2}{9}T_{11}^{us}$ |
| $\Sigma$ | $-V_{11}^S + 2V_{11}^{K*} + \frac{2}{3}V_{11}^{uu}$ | $-2T_{11}^K - \frac{2}{3}T_{11}^{us}$ |
| $\Xi$ | $-V_{11}^S + 2V_{11}^{K*} + \frac{2}{3}V_{11}^{us}$ | $-2T_{11}^K - \frac{2}{3}T_{11}^{us}$ |
Table 13

Magnetic moments of the baryon octet (in nuclear magnetons). Column IA contains the quark model impulse approximation expressions, column ”exp” the experimental values, column I the impulse approximation predictions, column II the exchange current contribution with $<\varphi_{000}(\vec{r}_{12})|\hat{V}_\pi(r_{12})|\varphi_{000}(\vec{r}_{12})>= -0.018$ and $<\varphi_{000}(\vec{r}_{12})|\hat{V}_K(r_{12})|\varphi_{000}(\vec{r}_{12})>= 0.03$ and column III the net predictions. All magnetic moments are given in nuclear magnetons.

|     | IA          | exp | I     | II    | III   |
|-----|-------------|-----|-------|-------|-------|
| $p$ | $\frac{m_N}{m_u}$ | +2.79 | +2.76 | +0.07 | +2.83 |
| $n$ | $-\frac{2}{3}\frac{m_N}{m_u}$ | -1.91 | -1.84 | -0.07 | -1.91 |
| $\Lambda$ | $-\frac{1}{3}\frac{m_N}{m_s}$ | -0.61 | -0.67 | +0.06 | -0.61 |
| $\Sigma^+$ | $\frac{8}{9}\frac{m_N}{m_u} + \frac{1}{9}\frac{m_N}{m_s}$ | +2.42 | +2.68 | -0.12 | +2.56 |
| $\Sigma^0$ | $\frac{2}{9}\frac{m_N}{m_u} + \frac{1}{18}\frac{m_N}{m_s}$ | ?     | +0.72 | -0.06 | +0.66 |
| $\Sigma^0 \rightarrow \Lambda$ | $-\frac{1}{\sqrt{3}}\frac{m_N}{m_u}$ | 1.61  | -1.59 | +0.01 | -1.58 |
| $\Sigma^-$ | $\frac{-4}{9}\frac{m_N}{m_u} + \frac{1}{9}\frac{m_N}{m_s}$ | -1.16 | -1.00 | 0     | -1.00 |
| $\Xi^0$ | $\frac{-2}{9}\frac{m_N}{m_u} - \frac{4}{9}\frac{m_N}{m_s}$ | -1.25 | -1.51 | +0.12 | -1.39 |
| $\Xi^-$ | $\frac{1}{9}\frac{m_N}{m_u} - \frac{4}{9}\frac{m_N}{m_s}$ | -0.65 | -0.59 | 0     | -0.59 |