High order quantum decoherence via multi-particle amplitude for boson system

D.L.Zhou, P.Zhang and C. P. Sun\textsuperscript{a,b}
Institute of Theoretical Physics, Academia Sinica,
P.O.Box 2735, Beijing 100080, China

In this paper we depict the high order quantum coherence of a boson system by using the multi-particle wave amplitude, whose norm square is just the high order correlation function. This multi-time amplitude can be shown to be a superposition of several “multi-particle paths”. When the environment or a apparatus entangles with them to form a generalized “which-way” measurement for many particle system, the quantum decoherence happens in the high order case dynamically. An explicit illustration is also given with an intracavity system of two modes interacting with a moving mirror.

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I. INTRODUCTION

A most profound concept in quantum mechanics is coherent superposition of quantum states. This is obviously reflected by the special interference of two or more “paths” in terms of single particle wave function. However, this coherence phenomenon does not sound very marvelous for the same circumstances can also occur in classical case, such as an optical interference. To manifest the intrinsically quantum features of coherence beyond the classical analogue, Glauber’s higher order quantum correlation function (HOQCF) was introduced [1] to accounts for such higher order quantum coherence effect in the Hanbury-Brown-Twiss experiment [2].

The quantum coherence both in the first order version and its higher order one shows the embodying of the wave nature in the world of the microscopic particles, but it is not robust due to the wave function collapse or quantum decoherence caused by a quantum measurement or by the coupling environment. Roughly, in the single particle picture, this decoherence phenomenon losing coherence can be understood according to the quantum entanglement of the considered system with the environment or the measuring apparatus. Obviously, this entanglement implies a “which-path” detection[3]. Precisely speaking, in an initial coherent superposition $|\psi_s\rangle = \sum c_n |n\rangle$, each system state $|n\rangle$ corresponding a “path” is correlated with an environment state $|e_n\rangle$ to form an entangling state $|\psi_T\rangle = \sum c_n |n\rangle \otimes |e_n\rangle$. The existence of the different states $|e_n\rangle$ distinguish among the “paths” of different $|n\rangle$. The interferences terms in $I(x) = Tr((x|\psi_T\rangle\langle\psi_T|x))$ can disappear when the environment state $|e_n\rangle$ are very distinct, i.e., each path is explicitly labelled $\langle e_m|e_n\rangle = \delta_{mn}$.

For the first order decoherence, the above well-known explanation in terms of “wich-path” detection mechanism is more simple, but very profound. However, it is not obvious yet whether this mechanism can account for decoherence in the higher effects: This is because we do not exactly know what is the “paths” and the corresponding “which-path” detection in the high order version. More recently, we have touched the quantum decoherence problem in higher order case [4]. The concrete calculation motivated us to further consider the “which-path” picture for the higher order quantum decoherence. In this paper, for a boson system we introduce the concept of the multi-particle wave amplitude, whose norm square is just the high order correlation function. As an effective wave function, this multi-time amplitude can be shown to be a supposition of several components. When the environment or a apparatus entangles with them, the quantum decoherence occurs in the high order case dynamically. This decoherence process losing the higher order coherence can be explained as a generalized “which-path” measurement for the defined multi-particle paths.

II. MULTI-PARTICLE PATHS FOR PROBABILITY AMPLITUDES

Firstly, we start with considering the meaning of the “path” in the high order quantum correlation. The typical example of the higher order quantum coherence [5] is that the single-component state $|1_k, 1_{k'}\rangle$ of the two independent photons with momenta $k$ and $k'$ shows its quantum coherence in its second order quantum correlation function
\[ G^{(2)} \equiv G^{(2)}(r_1, r_2, t_1, t_2). \] It can just be written as of form of the norm square of the equivalent “two-time wave function”

\[ \psi \equiv \psi(r_1, r_2, t_1, t_2) = \langle 00|E^+(r_2, t_2)E^+(r_1, t_1)|1_k, 1_{k'} \rangle \]

(1)

It was also called the biphoton wave packet for the photon field \( E^+(r, t) \) [6]. Especially, we remark that \( \psi \) is a coherent superposition of several “probability amplitudes”. This result can be promoted to the universal case with quantum systems of identical particles.

To see the main physical ideas implied by the above well-known result, without loss of the generality, we define a “measuring” operator of two modes \( V \) and \( H \) [4]

\[ \hat{\phi} = c_V \hat{b}_V e^{-i\omega_V t} + c_H \hat{b}_H e^{-i\omega_H t} \equiv c_V(t) \hat{b}_V + c_H(t) \hat{b}_H \]

(2)

where \( b_H \) and \( b_V \) are the annihilation operators of the boson system. The generalized second order correlation function [5]

\[ \hat{G}^{(2)} = \langle 1_V 1_H|\hat{\phi}^\dagger(t_1)\hat{\phi}^\dagger(t_2)\hat{\phi}(t_2)\hat{\phi}(t_1)|1_V, 1_H \rangle \]

\[ = \langle |0, 0\rangle|\hat{\phi}(t_2)\hat{\phi}(t_1)|1_V, 1_H \rangle|^2 \equiv |\Psi(t_1, t_2)|^2 \]

(3)

The two time wave function \( \Psi(t_1, t_2) \) can be understood in terms of the two “paths” picture from the initial state \( |1_V, 1_H \rangle \) to the final one \( |0, 0 \rangle \):

\[ \begin{array}{c|c|c|c}
|1_V, 1_H \rangle & c_H(t_2) & |1_V, 0_H \rangle & c_v(t_2) \\
\hline
\psi^{c_v}(t_1) & \psi^{c_H}(t_1) & |0, 0 \rangle & \psi^{c_H}(t_2) \\
\hline
|0, 1_H \rangle & & & \\
\end{array} \]

They are just associated with the two amplitudes forming a coherent superposition

\[ \Psi(t_1, t_2) = c_V c_H e^{-i\omega_V t_2 - i\omega_H t_1} + c_H c_V e^{-i\omega_H t_2 - i\omega_V t_1} \]

(4)

Correspondingly, the second order correlation function

\[ G^{(2)} = 2 |c_V c_H|^2 \left[ 1 + \cos[(\omega_V - \omega_H)|t_2 - t_1|] \right] \]

(5)

The above observation for the second order quantum coherence can also be discovered in the higher order case. Our arguments in this paper will be based on two novel observations: a. The equivalent field operator \( \hat{\phi} = \sum c_n \hat{b}_n \) is specified for a quantum measurement to a superposition state \( |\phi \rangle = \sum c_n |n \rangle \). b. For a certain initial single component state \( |s_0 \rangle \) of \( N \) particles system, the \( n \)th order quantum correlation function

\[ G^{(n)}(r_1, r_2, \ldots, r_n, t_1, t_2, \ldots, t_n) = |\psi^{(n)}|^2 \]

(6)

can be written as the norm square of an effective wave function \( \psi^{(n)} \), which is just a superposition of many amplitudes.

For the seek of simplicity, we only consider the third order situation when the initial state is in the state \( |1_H, 2_V \rangle \). Indeed, the generalized third order correlation function

\[ \hat{G}^{[3]} = \langle 2_V 1_H|\hat{\phi}^\dagger(t_1)\hat{\phi}^\dagger(t_2)\hat{\phi}^\dagger(t_3)\hat{\phi}(t_3)\hat{\phi}(t_2)\hat{\phi}(t_1)|2_V, 1_H \rangle \]

\[ = \langle |0, 0\rangle|\hat{\phi}(t_3)\hat{\phi}(t_2)\hat{\phi}(t_1)|2_V, 1_H \rangle|^2 \equiv |\Psi(t_1, t_2, t_3)|^2 \]

(7)

is indeed a norm square of the two time wave function:

\[ \Psi(t_1, t_2, t_3) = \sqrt{2} c_V^2 c_H e^{-i\omega_V (t_3 + t_2) - i\omega_H t_1} + \sqrt{2} c_H c_V e^{-i\omega_H t_2 - i\omega_V (t_3 + t_1)} + \sqrt{2} c_H c_V e^{-i\omega_H t_3 - i\omega_V (t_2 + t_1)} \]

(8)

Each term in the above equivalent wavefunction is contributed by the corresponding one of the four “paths” from \( |2_V, 1_H \rangle \) into \( |0, 0 \rangle \):
field operator $\rho$ describes a specific quantum measurement \[4\] with respect the polarized photon states in a reservoir with different driving forces.

Radiation pressure forces proportional to the photon numbers by $\hat{a}$ as a harmonic oscillator with a small mass. The coupling of fields to the cavity wall (a moving mirror) is just by the field operator.

Two mode cavity field interact with a moving wall of the cavity, which is attached to a spring and can be regarded as a harmonic oscillator with a small mass. The coupling of fields to the cavity wall (a moving mirror) is just by the field operator.

From the above equivalent 3-time-wave function, the third order correlation function is explicitly written down

$$G^{(3)} = 4|c_{V}^2 c_{H}|^2 \left( \frac{3}{2} + \cos[(\omega_V - \omega_H)(t_2 - t_1)] + \cos[(\omega_V - \omega_H)(t_3 - t_1)] + \cos[(\omega_V - \omega_H)(t_2 - t_3)] \right)$$

and shows the quantum interference in the time-domain.

From the above calculations for the second and third order quantum decoherence, the observation can be made that, for a specially-given initial states, a high order correlation function may be explicitly written down as the norm square of the equivalent multi-time-wave function, which is a coherent superposition of several complex components in associated with the generalized many-particle paths. It is pointed out that this kind of many-particle path is not a simple-product of single-particle paths and it can be determined by the concrete measurement.

### III. INTRACAVITY MODEL

With the above introduced concept of “many-particle path”, we can discuss the higher order decoherence problem by considering the many-particle “which-path” measurement. Let us use the following model to sketch this central idea. Our model is consisted of non-dissipative bosons in two modes. The problem is studied via calculating the second order quantum correlation functions in the Heisenberg picture. We take $\hbar = 1$ in this paper.

The model Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}$ is defined by

$$\hat{H}_0 = \omega_V \hat{b}_V^{\dagger} \hat{b}_V,$$

$$\hat{V} = \sum_j \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_j [d_V(\omega_j)\hat{b}_V^{\dagger} \hat{b}_V + d_H(\omega_j)\hat{b}_H^{\dagger} \hat{b}_H](\hat{a}_j^\dagger + \hat{a}_j),$$

where $\hat{H}_0$ is the free Hamiltonian of the system, $\hat{V}$ the free Hamiltonian of the reservoir (or a detector) plus the interaction between the system and the reservoir; and $\hat{b}_V^{\dagger} \hat{b}_V, \hat{b}_H^{\dagger} \hat{b}_H$ the creation (annihilation) operators for two modes with frequencies $\omega_V$ and $\omega_H = 0$. The operators $\hat{a}_j^\dagger (\hat{a}_j)$ are creation (annihilation) operators of the reservoir modes of frequencies $\omega_j$. The frequency-dependent constant $d_H(\omega_j) (d_V(\omega_j))$ measures the coupling constant between $H(\hat{V})$ mode and the $j$ mode of the reservoir. The most important feature of the model is that $[\hat{H}_0, \hat{V}] = 0$, i.e. the system does not dissipate energy to the reservoir, but it can leave imprinting on the reservoir since, for different number states $|n_V, n_H\rangle$, there are different interactions $\sum_j [n_V d_V(\omega_j) + n_H d_H(\omega_j)](\hat{a}_j^\dagger + \hat{a}_j)$ acting on the oscillator reservoir with different driving forces $\sim n_V d_V(\omega_j) + n_H d_H(\omega_j)$.

When there is only one mode in external system we can physically realized this model as an intracavity model\[4,8\]: Two mode cavity field interact with a moving wall of the cavity, which is attached to a spring and can be regarded as a harmonic oscillator with a small mass. The coupling of fields to the cavity wall (a moving mirror) is just by the radiation pressure forces proportional to the photon numbers by $\hat{b}_H^{\dagger} \hat{b}_H$ and $\hat{b}_V^{\dagger} \hat{b}_V$.

The second order coherence is directly determined by the second order correlation.

$$G[t, t', \hat{\rho}(0)] = Tr(\hat{\rho}(0)\hat{B}^{\dagger}(t)\hat{B}^{\dagger}(t')\hat{B}(t')\hat{B}(t)),$$

which is defined as a functional of the density operator $\hat{\rho}_S(0)$ of the whole system at a given time 0. Here, the bosonic field operator

$$\hat{B}(t) = \exp(i\hat{H}t)[c_1 \hat{b}_H + c_2 \hat{b}_V] \exp(-i\hat{H}t) = \exp(i\hat{V}t)[c_1 \hat{b}_H + c_2 \hat{b}_V] \exp(-i\omega_V t) \exp(-i\hat{V}t)$$

describes a specific quantum measurement\[4\] with respect the polarized photon states $|+\rangle = c_1 |H\rangle + c_2 |V\rangle$ and $|-\rangle = c_2 |H\rangle - c_1 |V\rangle$ where $c_1$ and $c_2$ satisfy the normalization relation $|c_1|^2 + |c_2|^2 = 1$. Without loss of the generality, we take $c_1 = c_2 = 1/\sqrt{2}$ standing for a given measurement as follows.
To examine whether the macroscopic feature of the reservoir causes the second order decoherence or not, we consider the whole system in an initial state

$$|\psi(0)\rangle = |1_H, 1_V\rangle \otimes |\{0_j\}\rangle,$$

where $|\{0_j\}\rangle$ is the vacuum state of the reservoir. Here, we have denoted the general Fock states of the many mode field by $|\{n_j\}\rangle \equiv |n_1, n_2, \ldots\rangle$.

For the total system, instead of defining the equivalent “two-time wave function” in the above section, we define an effective two-time state vector

$$|\psi_B(t, t')\rangle = \hat{B}(t')\hat{B}(t)|\psi(0)\rangle,$$

(15)

to re-write the second order correlation function as

$$G[t, t', \hat{\rho}(0)] = \langle \psi_B(t, t')|\psi_B(t, t')\rangle$$

(16)

It is interested that the effective state vector can be evaluated as the superposition

$$|\psi_B(t, t')\rangle = \frac{1}{2} e^{i\hat{V}(0,0)t'} \left[ \exp(-i\omega V t') e^{-i\hat{V}(1,0)t'} e^{i\hat{V}(1,0)t} + \exp(-i\omega V t) e^{i\hat{V}(0,0)t'} e^{-i\hat{V}(1,0)t'} e^{-i\hat{V}(1,1)t'} |\{0_j\}\rangle \otimes |0_H, 0_V\rangle \right],$$

(17)

and

$$\hat{V}(m, n) = \sum_j \hat{V}_j(m, n) = \sum_j \omega_j \hat{a}_j \hat{a}_j + \sum_j (d_V(\omega_j)m + d_H(\omega_j)n)(\hat{a}_j^\dagger + \hat{a}_j),$$

(19)

can label the different paths and thus lead to the higher order quantum decoherence. The above result clearly demonstrates that, in presence of the reservoir, the difference probability amplitudes ($\sim \exp(-i\omega V t')$ and $\exp(-i\omega V t)$) from $|1_H, 1_V\rangle$ to $|0_H, 0_V\rangle$ entangle with the different states ($\frac{1}{2} e^{i\hat{V}(0,0)t'} e^{-i\hat{V}(1,0)t'} e^{i\hat{V}(1,0)t} e^{-i\hat{V}(1,1)t'} |\{0_j\}\rangle$ and $\frac{1}{2} e^{i\hat{V}(0,0)t'} e^{-i\hat{V}(1,0)t'} e^{i\hat{V}(1,0)t} e^{-i\hat{V}(1,1)t'} |\{0_j\}\rangle$) of the reservoir. This is just physical source of the higher order quantum decoherence. In the following section an explicit calculation of the second order correlation function will be given to show this crucial observation.

**IV. DYNAMIC DECOHERENCE IN HIGHER ORDER CASE**

In our calculation, the second order correlation function

$$G[t, t', \hat{\rho}(0)] = \frac{1}{2} + \frac{1}{4} e^{i\omega V (t-t')} \prod_j F_j + \frac{1}{4} e^{-i\omega V (t-t')} \prod_j F_j^*,$$

(20)

is firstly expressed as a factorization form[9] where each factor

$$F_j = \langle 0_j | \hat{u}_j^0(t_6) | 0_j \rangle \equiv \langle 0_j | e^{i\hat{V}_j(1,1)t} e^{-i\hat{V}_j(0,0)t} e^{i\hat{V}_j(0,1)t} e^{-i\hat{V}_j(1,0)t} e^{i\hat{V}_j(1,1)t} | 0_j \rangle,$$

(21)

is a two-time transition amplitude of the $j'th$ mode of the reservoir. Obviously, the term $\prod_j F_j$ determines the extent of coherence and decoherence in the second order case, which is called “the decoherence factor” and plays the role just as the same as that in the first order decoherence[9].

In the following, to given the factor $F_j$ explicitly, we adopt the Wei-Norman method [10,11] to calculate the equivalent time evolution defined by $\hat{u}_j^0(t_6)$. It can be image as an evolution governed by a discrete time-dependent Hamiltonian $H(t)$ dominated by $\hat{V}_j(1,1), -\hat{V}_j(1,0), \hat{V}_j(0,1), -\hat{V}_j(0,0), \hat{V}_j(0,0)$ and $-\hat{V}_j(1,1)$ in six time-intervals $[t_0 = 0, t_1 = t], [t_1, t_2 = 2t], [t_2, t_3 = 2t + t'], [t_3, t_4 = 3t + 2t'], [t_4, t_5 = 3t + 2t'], [t_5, t_6 = 4t + 2t']$ respectively. In the $k$-th step of calculation, we take the final state of $(k-1)$-th step as its initial state. Therefore, we obtain $\hat{u}_j^0(t_6)$ as the sixth step evolution

$$\hat{u}_j^0(t_6) = e^{g_{2j}^6(t_6)} \hat{a}_j \gamma_{j}^{6,\dagger}(t_6) e_{2j}^6(t_6) \hat{a}_j \gamma_{j}^{6}(t_6) e_{2j}^6(t_6)$$

(22)
Here, $g_{kj}^{6}(t_6)(k = 1, 2, 3, 4)$ are the coefficients that can be explicitly obtained, but for the calculation of the $j$-th component $F_j = \exp[g_{kj}^{6}(t_6)]$ of the decoherence factor, we only need to know $g_{kj}^{6}$. The detailed discussion in the appendix gives

$$g_{kj}^{6}(t_6) = -2\left[\frac{d_H(j) - d_V(j)}{\omega_j^2}\right] \sin^2\left[\frac{\omega_j(t' - t)}{2}\right] + \frac{d_V^2(j) - d_H^2(j)}{\omega_j^2} \left[\omega_j(t' - t) + 2(1 - \cos(\omega_j |t' - t|)) \sin(\omega_j t) + (1 - 2 \cos \omega_j t) \sin(\omega_j |t' - t|)\right].$$

Since the real part of $g_{kj}^{6}$ is no more than zero, the norm of $F_j$ is no more than one. From the same argument as in the first order decoherence [9], the universal factorization structure of the decoherence factor implies the second order decoherence in the macroscopic limit.

In order to demonstrate the above arguments quantitatively, we give the numerical results for the second order decoherence for different numbers $N$ of the quantum oscillators. As $N$ increase, these results are illustrated in FIG.1. In the numerical calculation, the coupling constants $\{d_V(j)\}$ take random values in the domain $[0.8, 1.0]$, the coupling constants $\{d_H(j)\}$ in $[0.2, 0.4]$, and the frequencies $\{\omega_j\}$ in $[0.5, 1.5]$. The other parameters is given in the caption of the figure.

![FIG. 1. The horizontal axe denotes time period $t' - t$, the vertical axe denotes the second order correlation function $G(t, t', \hat{\rho}(0))$, parameters $\omega_V = 1.0$, (a) $N = 5, t = 0$, (b) $N = 5, t = 5$, (c) $N = 10, t = 0$, (d) $N = 10, t = 5$. From the above illustration, we see clearly that the second order correlation is not only a function of the time period $t' - t$, but also a function of time $t$. The amplitude of the second order correlation function is mainly determined by the number of the quantum oscillators, the second order coherence vanishes faster and faster, and the quantum revival amplitude becomes smaller. With extrapolation, it can be expected that, when the number of the quantum oscillators limit to infinity, i.e., in the macroscopic limit of the reservoir, the second order coherence will decoherence in a short time and no quantum revival can be observed.](image-url)
APPENDIX:

In this appendix, the Wei-Norman method \cite{10,11} is adopted to calculate the second order decoherence factor $F_j$. The calculation is completed in six steps. During the time period $[t_{k-1}, t_k]$, the evolution for $W_j^k(t, t')$ is dominated by the Hamiltonian

$$
\dot{h}_j^k = \alpha_j^k \hat{a}_j^\dagger \hat{a}_j + \beta_j^k \hat{a}_j^\dagger + \gamma_j^k \hat{a}_j, \quad \{k = 1, 2, \cdots, 6\}. \quad (A1)
$$

The coefficients $\alpha_j^k, \beta_j^k, \gamma_j^k$ and the time intervals $T_k = t_k - t_{k-1}$ take different values in different steps:

$$
\begin{align*}
\alpha_j^1 &= \omega_j, \quad \beta_j^1 = \gamma_j^1 = d_v(\omega_j), \quad T_1 = t, \\
\alpha_j^2 &= -\omega_j, \quad \beta_j^2 = \gamma_j^2 = d_v(\omega_j), \quad T_2 = t, \\
\alpha_j^3 &= \omega_j, \quad \beta_j^3 = \gamma_j^3 = d_v(\omega_j), \quad T_3 = t', \\
\alpha_j^4 &= -\omega_j, \quad \beta_j^4 = \gamma_j^4 = -d_v(\omega_j), \quad T_4 = t', \\
\alpha_j^5 &= \omega_j, \quad \beta_j^5 = \gamma_j^5 = d_v(\omega_j), \quad T_5 = t, \\
\alpha_j^6 &= \omega_j, \quad \beta_j^6 = \gamma_j^6 = -d_v(\omega_j) - d_v(\omega_j), \quad T_6 = t.
\end{align*}
\quad (A2)
$$

Due to the fact that $\hat{a}_j^\dagger \hat{a}_j, \hat{a}_j^\dagger, \hat{a}_j, 1$ form a closed algebra - the Heisenberg-Weyl algebra, the unitary time evolution operator at every step takes the following form

$$
\hat{u}_j^k(T) = e^{g_{i_1}^k(T) \hat{a}_j^\dagger e^{g_{i_2}^k(T) \hat{a}_j e^{g_{i_3}^k(T) \hat{a}_j^\dagger}}}.
\quad (A3)
$$

for $T \in [t_{k-1}, t_k]$ in a special sequence. The benefit of the above form is that only the coefficient $g_{i_3}^k(T)$ is needed to be known in the calculation of the average value of the vacuum state.

According to the Schrödinger equation $i \frac{d}{dT} \hat{u}_j^k = \hat{h}_j^k \hat{u}_j^k$, the coefficients satisfy the equations:

$$
\begin{align*}
\frac{d}{dT} g_{i_2}^k &= -i\alpha_j^k, \\
\frac{d}{dT} g_{i_1}^k - g_{i_2}^k \frac{d}{dT} g_{i_2}^k &= -i\beta_j^k, \\
\quad e^{-g_{i_2}^k} \frac{d}{dT} g_{i_3}^k &= -i\gamma_j^k \\
\frac{d}{dT} g_{i_3}^k - g_{i_2}^k e^{-g_{i_2}^k} \frac{d}{dT} g_{i_3}^k &= 0
\end{align*}
\quad (A4)
$$

Using the results

$$
\begin{align*}
\frac{d}{dT} g_{i_1}^k &= -i\alpha_j^k g_{i_1}^k - i\beta_j^k, \\
\frac{d}{dT} g_{i_2}^k &= -i\gamma_j^k g_{i_2}^k
\end{align*}
\quad (A5)
$$

obtained by simplifying the system of equations, we get the solution

$$
\begin{align*}
g_{i_1}^k(T) &= (g_{i_1}^k(t_{k-1}) + \frac{\beta_j^k}{\alpha_j^k} e^{-i\alpha_j^k(T-t_{k-1})} - \frac{\beta_j^k}{\alpha_j^k}) e^{-i\alpha_j^k(T-t_{k-1}) - 1} + i\frac{\beta_j^k \gamma_j^k(T-t_{k-1})}{\alpha_j^k}, \\
g_{i_2}^k(T) &= g_{i_2}^k(t_{k-1}) + \frac{\gamma_j^k}{\alpha_j^k} (g_{i_1}^k(t_{k-1}) + \frac{\beta_j^k}{\alpha_j^k})(e^{-i\alpha_j^k(T-t_{k-1}) - 1} + i\frac{\beta_j^k \gamma_j^k(T-t_{k-1})}{\alpha_j^k})
\end{align*}
\quad (A6)
$$

Notice that we have used the step-initial conditions

$$
\begin{align*}
g_{i_1}^k(t_{k-1}) &= g_{i_1}^{k-1}(t_{k-1}), \\
g_{i_2}^k(t_{k-1}) &= g_{i_2}^{k-1}(t_{k-1})
\end{align*}
\quad (A7)
$$

and the initial conditions $g_{i_3}^0(t_0) = g_{i_3}^0(t_0) = 0$. Then we obtain a set of iteration equations
\begin{align*}
g_{ij}^k(t_k) &= (g_{ij}^{k-1}(t_{k-1}) + \frac{\beta_j^k}{\alpha_j^k})e^{-\alpha_j^k T_k} - \frac{\beta_j^k}{\alpha_j^k}, \\
g_{ij}^k(t_k) &= g_{ij}^{k-1}(t_k - 1) + \frac{\gamma_j^k}{\alpha_j^k}(g_{ij}^{k-1}(t_{k-1}) + \frac{\beta_j^k}{\alpha_j^k}(e^{-\alpha_j^k T_k} - 1) + i\frac{\beta_j^k \gamma_j^k T_k}{\alpha_j^k}).
\end{align*}

(A9)

Iterating six times with different initial conditions and coefficients, the final result of $g_{ij}^6(t_6)$ is obtained as the equation (23).

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\begin{itemize}
  \item[a] Electronic address: suncp@itp.ac.cn
  \item[b] Internet www site: http://www.itp.ac.cn/~suncp
\end{itemize}

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