Vibration attenuation control of ocean marine risers with axial-transverse couplings

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ABSTRACT
The target of this paper is designing a boundary controller for vibration suppression of marine risers with coupling mechanisms under environmental loads. Based on energy approach and the equations of axial and transverse motions of the risers are derived. The Lyapunov direct method is employed to formulated the control placed at the riser top-end. Stability analysis of the closed-loop system is also included.

1. INTRODUCTION
Due to its physical structure, a riser basically is modeled as a tensioned beam [1], [2], [3], and [4]. In [5], an active boundary control that produces a vibration-free for an Euler-Bernoulli beam system was designed. Similar use of distributed control can be found in [6]. In [7], the authors used differential evolution optimization to search for the best controller model structure and its parameters for beam control problem. The proposed controller is able to suppress the beam’s vibration without knowledge of the system. However, the searching process is conducted within a set of predefined control structures, no proof of the effectiveness of the control was given.

With efforts to make voltage-source converter (VSC) more efficient in handling distributed parameter systems, sliding-mode control (SMC) was given extra flexibility by adding a neural network and fuzzy control in [8]. The author yield a control law in the form of a mass-damper-spring system at the boundary of a moving string. However, difficulties in selecting proper fuzzy membership functions and a slow convergence speed due to online-tuning might be troublesome when applying the aforementioned controls. After accepting that SMC is non-analytical in the sliding surface, in the first control structure, a boundary layer was defined that enabled fuzzy control by taking a switching function and its derivative as inputs while SMC was activated outside this boundary to achieve fast transient responses. A series of papers with applications of SMC to flexible system can be found in [9], [10], and [11]. A second attempt was made to design a fuzzy neural network control (FNNC) that also employed switching variables as its inputs. The proposed FNNC conducted an online-tuning process to regulate fuzzy reasoning to compromise system uncertainties. Both controls resulted in a variation of axially moving string tension as the control action.

In [12], a beam model representing a tensioned riser is investigated, and a boundary controller consisting of the top-end rise information is designed to achieve exponential stability. Krstic, et al. develop a sys-
2. MATHEMATICAL FORMULATION

The riser kinetic energy is specified by

\[ T = \frac{m_0}{2} \int_0^L \left[ \left( \frac{\partial u(z, t)}{\partial t} \right)^2 + \left( \frac{\partial w(z, t)}{\partial t} \right)^2 \right] dz, \]  

(1)

where \( u(z, t) \) is transverse displacements in the \( X \) direction and \( w(z, t) \) is longitudinal displacement in the \( Z \) direction. \( L \) denote the riser length, \( m_0 = \rho A \) is the riser oscillating mass per unit length, \( A \) is the riser cross-section area, and \( \rho \) represents the mass density of the riser. Assuming that the riser is constrained by constant tension \( P_0 \). The riser potential energy is given as

\[ P = \frac{EI}{2} \int_0^L \left( \frac{\partial^2 u(z, t)}{\partial z^2} \right)^2 dz + \frac{P_0}{2} \int_0^L \left( \frac{\partial u(z, t)}{\partial z} \right)^2 dz + \frac{EA}{2} \int_0^L \left( \frac{\partial w(z, t)}{\partial z} \right)^2 dz + \frac{1}{2} \frac{\partial u(z, t)}{\partial z} + \frac{1}{2} \frac{\partial w(z, t)}{\partial z} \right)^2 dz, \]  

(2)

where \( E \) is the Young’s modulus and \( I \) is the second moment of the riser’s cross section area. The hydrodynamic forces can be given as \([26]\)

\[ f_u(z, t) = f_uD + f_uL, \quad f_w(z, t) = f_wD + f_wL, \]

\[ f_uD = -\Omega_1uD_u(z, t), \quad f_wD = -\Omega_2DW_u(z, t), \]  

(3)

where \( f_uD, f_wD \) and \( f_uL, f_wL \) correspond to the distributed damping and external forces. The work done by the hydrodynamic forces acting on the system is calculated as

\[ W_f = \int_0^L f_u(z, t)u(z, t)dz + \int_0^L f_w(z, t)w(z, t)dz, \]  

(4)

The work done by boundary control is

\[ W_m = U_u(L, t)u(L, t) + U_w(L, t)w(L, t), \]  

(5)

where \( U_u(L, t) \) and \( U_w(L, t) \) are the boundary control forces. The total work done on the system is \( W = W_f + W_m \). The extended Hamilton principle is indicated as

\[ \int_{t_1}^{t_2} \delta(T - P + W)dt = 0. \]  

(6)

For the sake of clear presentation, \( (z, t) \) is omitted whenever it is applicable. The kinetic energy variation can be written as

\[ \int_{t_1}^{t_2} \delta T dz = -m_0 \int_{t_1}^{t_2} \int_0^L \left( \frac{\partial^2 u}{\partial t^2} \delta u + \frac{\partial^2 w}{\partial t^2} \delta w \right) dz dt, \]  

(7)
where $\delta u = \delta v = \delta w = 0$ at $t = t_1, t_2$ have been used. For the riser under consideration, ball joints arranged at both ends (Figure 1) implying that bending free. In addition, the lower end stationed at the well-head. The riser dynamics is yielded

$$\begin{align*}
-m_0\ddot{u}_t - E I u_{zzzz} + P_0 u_{zzz} + 3\frac{E A}{2} u_t^2 u_{zz} + E A w_{zz} u_z + E A w_z u_{zz} - \Omega_1 D u_t + f_u &= 0, \\
-m_0\ddot{w}_t - E A w_{zz} + E A u_z u_{zz} - \Omega_2 D w_t + f_w &= 0, \\
-m_0\ddot{w}_t - E A w_{zzz}(L, t) + P_0 u_{zz}(L, t) + \frac{E A}{2} u_z^2(L, t) + E A w_z(L, t) u_z(L, t) = U_u(L, t), \\
E A w_z(L, t) + \frac{E A}{2} u_z^2(L, t) + \frac{E A}{2} v_z^2(L, t) = U_w(L, t), \\
u_{zz}(L, t) = v_{zz}(L, t) = u_{zz}(0, t) = v_{zz}(0, t) = 0, u(0, t) = \dot{v}(0, t) = w(0, t) = 0,
\end{align*}$$

(8)

![Figure 1. Riser coordinates](image)

### 3. CONTROL DESIGN

In order to minimize the riser vibration using measured state and applied forces at the top end, we consider the following Lyapunov candidate function

$$V = \frac{m_0}{2} \int_0^L (u_t^2 + w_t^2)dz + \frac{P_0}{2} \int_0^L u_t^2 dz + \frac{E A}{2} \int_0^L (w_z + \frac{u_t^2}{2})^2dz + \frac{E I}{2} \int_0^L u_{zz}^2dz$$

$$+ \rho_1 \int_0^L uu_t dz + \rho_2 \int_0^L w w_t dz + (k_1 + \frac{k_2 \rho_1}{m_0}) u^2(L, t) + (k_3 + \frac{k_4 \rho_2}{m_0}) w^2(L, t).$$

(9)

Since $\forall t \geq 0$ and $u(0, t) = w(0, t) = 0$, it can be shown that

$$\gamma_1 \rho_1 \int_0^L u_t^2dz \leq 4 L^2 \gamma_1 \rho_1 \int_0^L u_t^2dz, \quad \gamma_2 \rho_2 \int_0^L w_t^2dz \leq 4 L^2 \gamma_2 \rho_2 \int_0^L w_t^2dz.$$

(10)

where $\gamma_1$ and $\gamma_2$ are positive constants, it can be deduced that

$$\begin{align*}
-4L^2 \gamma_1 \rho_1 \int_0^L u_t^2dz - \frac{\rho_1}{\gamma_1} \int_0^L uu_t dz &\leq \rho_1 \int_0^L uu_t dz \leq 4L^2 \gamma_1 \rho_1 \int_0^L u_t^2dz + \frac{\rho_1}{\gamma_1} \int_0^L u_t^2dz, \\
-4L^2 \gamma_2 \rho_2 \int_0^L w_t^2dz - \frac{\rho_2}{\gamma_2} \int_0^L uu_t dz &\leq \rho_2 \int_0^L uu_t dz \leq 4L^2 \gamma_2 \rho_2 \int_0^L w_t^2dz + \frac{\rho_2}{\gamma_2} \int_0^L w_t^2dz.
\end{align*}$$

(11)

(12)
The (9) can be lower and upper bounded by

\[
V \geq \left( \frac{m_0}{2} - \frac{\rho_1}{\gamma_1} \right) \int_0^L u_t^2 \, dz + \left( \frac{m_0}{2} - \frac{\rho_2}{\gamma_2} \right) \int_0^L w_t^2 \, dz + \left( \frac{P_0}{2} - 4L^2 \gamma_1 \rho_1 \right) \int_0^L u_z^2 \, dz + \left( \frac{EA}{2} - 4L^2 \gamma_2 \rho_2 \right) \int_0^L w_z^2 \, dz + \frac{EA}{8} \int_0^L u_z^4 \, dz + \frac{EA}{4} \int_0^L w_z^2 u_z^2 \, dz + \frac{EI}{2} \int_0^L u_z^2 \, dz + \frac{1}{2} \left( k_1 + \frac{k_2 \rho_1}{m_0} \right) u^2(L, t) + \left( k_3 + \frac{k_4 \rho_2}{m_0} \right) w^2(L, t),
\]

(13)

and

\[
V \leq \left( \frac{m_0}{2} + \frac{\rho_1}{\gamma_1} \right) \int_0^L u_t^2 \, dz + \left( \frac{m_0}{2} + \frac{\rho_2}{\gamma_2} \right) \int_0^L w_t^2 \, dz + \left( \frac{P_0}{2} + 4L^2 \gamma_1 \rho_1 \right) \int_0^L u_z^2 \, dz + \left( \frac{EA}{2} + 4L^2 \gamma_2 \rho_2 \right) \int_0^L w_z^2 \, dz + \frac{EA}{8} \int_0^L u_z^4 \, dz + \frac{EA}{4} \int_0^L w_z^2 u_z^2 \, dz + \frac{EI}{2} \int_0^L u_z^2 \, dz + \frac{1}{2} \left( k_1 + \frac{k_2 \rho_1}{m_0} \right) u^2(L, t) + \left( k_3 + \frac{k_4 \rho_2}{m_0} \right) w^2(L, t).
\]

(14)

If we select \( \rho_1, \rho_2, \gamma_1, \) and \( \gamma_2 \) such that:

\[
\frac{m_0}{2} - \frac{\rho_1}{\gamma_1} = c_1, \quad \frac{m_0}{2} - \frac{\rho_2}{\gamma_2} = c_2, \quad \frac{P_0}{2} - 4L^2 \gamma_1 \rho_1 = c_3, \quad \frac{P_0}{2} - 4L^2 \gamma_2 \rho_2 = c_4,
\]

(15)

where \( c_i \), for \( i = 1 \ldots 4 \), are strictly positive constants. Differentiating (9) and taking (8) into account yields

\[
\dot{V} = \left( u_t(L, t) + \frac{\rho_1}{m_0} u(L, t) \right) \left( -EI u_{zzz}(L, t) + P_0 u_z(L, t) + \frac{EA}{2} u_z^3(L, t) \right) + EAw_z(L, t) u_z(L, t) + \frac{EA}{8} \int_0^L w_t^4 \, dz - \left( \Omega_1 \gamma_1 - \rho_1 \right) \int_0^L u_z^2 \, dz - \left( \Omega_2 \gamma_1 \rho_1 - \frac{\rho_2}{m_0} \right) \int_0^L u_z^2 \, dz - \rho_1 E \frac{P_1}{m_0} \int_0^L u_z^2 \, dz - \rho_1 E \frac{P_1}{m_0} \int_0^L u_z^2 \, dz
\]

\[
- \frac{\rho_1 E A}{2m_0} \int_0^L u_z^2 \, dz - \frac{\Omega_2 \gamma_1 \rho_1}{m_0} \int_0^L u_z^2 \, dz - \frac{\rho_2 E A}{m_0} \int_0^L w_z^2 \, dz - \rho_2 \frac{\Omega_2 \gamma_1 \rho_1}{m_0} \int_0^L w_z^2 \, dz - \frac{\Omega_2 \gamma_2 \rho_2}{m_0} \int_0^L w_z^2 \, dz + \frac{\Omega_2 \gamma_2 \rho_2}{m_0} \int_0^L w_z^2 \, dz + \frac{\Omega_2 \gamma_2 \rho_2}{m_0} \int_0^L w_z^2 \, dz
\]

\[
+ \frac{k_1 + \frac{k_2 \rho_1}{m_0}}{m_0} \int_0^L w(L, t) w_z(L, t) + \left( k_3 + \frac{k_4 \rho_2}{m_0} \right) u(L, t) u_z(L, t).
\]

(16)

Since

\[
\int_0^L \left( \frac{\Omega_1 \gamma_1 \rho_1}{m_0} \int_0^L u_z \, dz \right) u_z \, dz \leq 4L^2 \Omega_1 \gamma_1 \rho_1 \gamma_3 \int_0^L u_z^2 \, dz + \Omega_1 \gamma_1 \rho_1 \gamma_3 \int_0^L u_z^2 \, dz,
\]

(17)

\[
\int_0^L \left( \frac{\Omega_2 \gamma_2 \rho_2}{m_0} \int_0^L u_z \, dz \right) u_z \, dz \leq 4L^2 \Omega_2 \gamma_2 \rho_2 \gamma_4 \int_0^L u_z^2 \, dz + \Omega_2 \gamma_2 \rho_2 \gamma_4 \int_0^L u_z^2 \, dz,
\]

(18)

and noted that \(-EI u_{zzz}(L, t) + P_0 u_z(L, t) + \frac{EA}{2} u_z^3(L, t) + EAw_z(L, t) u_z(L, t) = u(L, t) \) and \( EAw_z(L, t) + \frac{EA}{2} u_z^3(L, t) = w(L, t) \), the boundary controls are designed as follows,

\[
U_u = -k_1 u(L, t) - k_2 u_z(L, t), \quad U_w = -k_3 w(L, t) - k_4 w_z(L, t),
\]

(19)
From (20), the designed parameters are selected such that
\[
V \leq \frac{k_1p_1}{m_0} u^2(L, t) - k_2 u_1^2(L, t) - \frac{k_4 p_2}{m_0} w^2(L, t) - k_4 w_1^2(L, t) - (\Omega_{1D} - \frac{p_1}{\gamma_3 m_0}) \int_0^L u_1^2 dz
\]
\[
- (\Omega_{2D} - \frac{p_2}{\gamma_4 m_0}) \int_0^L u_2^2 dz - \frac{p_1 E I}{m_0} \int_0^L u_1^2 dz - \left(\frac{p_1 P_0}{m_0} - \frac{4 L^2 \Omega_{1D} p_1 c_1}{m_0}\right) \int_0^L u_2^2 dz
\]
\[
- \left(\frac{p_1 E A}{m_0} - \frac{4 L^2 \Omega_{2D} p_2 c_1}{m_0}\right) \int_0^L u_1^2 dz - \frac{p_1 E A}{2m_0} \int_0^L u_2^2 dz - \frac{E A}{m_0} \left(\frac{p_1 + p_2}{2}\right) \int_0^L u_2^2 dz
\]
\[
+ \frac{p_2}{m_0} \int_0^L u_1^2 w_1^2 dz.
\]

**Remark:** It is noted that the authors of [26] use the assumption the riser is always stretched in order to conclude that \( \int_0^L u_2^2 w_2 dt \) is positive. This is not the case in practice since the riser can be bulked or stretched according to external disturbance. Considering the following term
\[
\Delta = -\Delta_1 \int_0^L u_1^2 dt + \Delta_2 \int_0^L u_2^2 dt + \Delta_3 \int_0^L u_2^2 w_2 dt
\]

where
\[
\Delta_1 = \left(\frac{p_1 E A}{m_0} - \frac{4 L^2 \Omega_{2D} p_2 c_1}{m_0}\right), \quad \Delta_2 = \frac{p_1 E A}{2m_0}, \quad \Delta_3 = \frac{E A}{m_0} \left(\frac{p_1 + p_2}{2}\right)
\]
\[
\Delta \text{ can be written as}
\]
\[
\Delta = \Delta_1 \int_0^L u_1^2 dt - (\Delta_2 - \frac{1}{4} \Delta_3) \int_0^L u_2^2 dt + \Delta_3 \int_0^L (w - \frac{1}{2} u_2^2) dz
\]

To remove the requirement of positive tension, we use the following property [27] that
\[
w_1^2 + \frac{1}{4} \geq 0
\]

From (20), the designed parameters are selected such that
\[
\Omega_{1D} - \frac{p_1}{\gamma_3 m_0} = c_5, \quad \Omega_{2D} - \frac{p_2}{\gamma_4 m_0} = c_6,
\]
\[
\frac{p_1 P_0}{m_0} - \frac{4 L^2 \Omega_{1D} p_1 \gamma_4}{m_0} = c_7, \quad \frac{p_3 P_0}{m_0} - \frac{4 L^2 \Omega_{2D} \gamma_3}{m_0} = c_8, \quad \Delta_2 - \frac{1}{4} \Delta_3 = c_9
\]
where
\[
\beta_1 = \left\{ \frac{EA}{m_0} \left( \frac{\rho_1 + \rho_2}{2} \right), k_1 \frac{\rho_1}{m_0}, k_3 \frac{\rho_2}{m_0} \right\}, \quad \beta_2 = \left\{ \frac{1}{2} \left( k_1 + \frac{k_2 \rho_1}{m_0} \right), \frac{1}{2} \left( k_3 + \frac{k_4 \rho_2}{m_0} \right) \right\}.
\] (28)

**Remark:** Different from [16], the control design process is carried out in this chapter without any assumptions on boundedness of time and spatial derivatives of the riser system.

Equation (26) can be written as
\[
\dot{V} \leq -k_1 \frac{\rho_1}{m_0} u^2(L, t) - k_2 u_1^2(L, t) - k_3 \frac{\rho_2}{m_0} w^2(L, t) - k_4 u_1^2(L, t) - cV + \Delta_c,
\] (29)
where
\[
\Delta_c = \frac{\rho_1}{m_0} \int_0^L u f_u dz + \frac{\rho_2}{m_0} \int_0^L w f_w dz + \int_0^L u_t f_u dz + \int_0^L w_t f_w dz.
\] (30)

An upper bound of \(\Delta_c\) can be written as
\[
\Delta_c \leq \frac{1}{\gamma_5} \int_0^L u_1^2 dz + \frac{1}{\gamma_6} \int_0^L f_u^2 dz + \frac{4L^2 \rho_1}{m_0 \gamma_5} \int_0^L u_2^2 dz + \frac{6 \rho_1}{m_0} \int_0^L f_u^2 dz + \frac{1}{\gamma_7} \int_0^L w_2^2 dz + \frac{4L^2 \rho_2}{m_0 \gamma_7} \int_0^L w_2^2 dz + \frac{\gamma_8 \rho_2}{m_0} \int_0^L f_b^2 dz.
\] (31)

There exists a strictly positive constant \(\xi\) such that the following inequality holds
\[
\Delta_c \leq \xi \left( \int_0^L u_1^2 dz + \int_0^L u_2^2 dz + \int_0^L w_2^2 dz + \int_0^L w_2^2 dz \right) + \frac{1}{\xi} \left( \gamma_6 \frac{\rho_1}{m_0} \right) \int_0^L f_u^2 dz + \frac{1}{\xi} \left( \gamma_7 \frac{\rho_2}{m_0} \right) \int_0^L f_b^2 dz.
\] (32)

From the lower bound of \(V\), it is shown that
\[
\xi \left( \int_0^L u_1^2 dz + \int_0^L u_2^2 dz + \int_0^L w_2^2 dz + \int_0^L w_2^2 dz \right) \leq \frac{V}{\zeta},
\] (33)
where
\[
\zeta = \min \left\{ c_1, c_2, c_3, c_4, \frac{EA}{8} \frac{EI}{L^2}, \frac{1}{2} \left( k_1 + \frac{k_2 \rho_1}{m_0} \right), \frac{1}{2} \left( k_3 + \frac{k_4 \rho_2}{m_0} \right) \right\}.
\] (34)

Substituting (32) and (33) into (29) gives
\[
\dot{V} \leq -k_1 \frac{\rho_1}{m_0} u^2(L, t) - k_2 u_1^2(L, t) - k_3 \frac{\rho_2}{m_0} w^2(L, t) - k_4 u_1^2(L, t) - (c - \frac{\xi}{\zeta}) V + \frac{1}{\xi} Q,
\] (35)
where
\[
Q = \left( \gamma_5 + \frac{\gamma_6 \rho_1}{m_0} \right) Q_1 + \left( \gamma_7 + \frac{\gamma_8 \rho_2}{m_0} \right) Q_2,
\] (36)
and
\[
Q_1 = \max_{t \geq 0} \int_0^L f_u^2 dz, \quad Q_2 = \max_{t \geq 0} \int_0^L f_b^2 dz.
\] (37)

If \(\xi\) is picked such that \(\bar{c} = c - \frac{\xi}{\zeta}\) is strictly positive, then:
\[
\dot{V} \leq -\bar{c} V + \frac{1}{\xi} Q.
\] (38)

Inequality (38) implies that \(V(t)\) exponentially converges to nonnegative constant \(\frac{1}{\xi} Q\). Using Inequality A.2 [26], it can be conclude that all terms \(|u(z, t)|\) and \(|w(z, t)|\) are bounded and exponentially converge to a non-negative constant defined be the value of external disturbances.
4. NUMERICAL SIMULATIONS

At this stage, we illustrate the advantages of the proposed control through a set of simulations. The marine riser system parameters are given as in Table 1. The linear current velocity vector in a form of $V = \left[ \frac{1}{2} s, \frac{1}{2} s, 0 \right]^T$ is employed in numerical simulations. The hydrodynamic forces can be given as [26]. Simulations are carried out without the proposed control and with the control by set $k_1 = k_2 = 500$. The riser displacements in the X and Z directions for uncontrolled and controlled cases are plotted in Figure 2 and Figure 3, respectively. It can be observed that when the control is activated, displacement magnitudes in all directions (X and Z) are reduced. The reduction in displacement magnitudes illustrates the effectiveness of the proposed control in driving the riser to the vicinity of its equilibrium position. It also can be observed in Figure 4 that the control forces required to drive the risers are reasonable for the riser under consideration.

| Nomenclature | Description | Value       |
|--------------|-------------|-------------|
| $L$          | Length      | 1000m       |
| $D_0$        | Diameter    | 0.61m       |
| $D_i$        | Diameter    | 0.575m      |
| $D_H$        | Diameter    | 0.87m       |
| $\rho_w$     | Density     | 1025 kg/m$^3$ |
| $\rho_m$     | Density     | 1205 kg/m$^3$ |
| $E$          | Young’s modulus | $2 \times 10^5$ kg/m$^2$ |
| $P_0$        | Tension     | $2.15 \times 10^6$ N |

Figure 2. The riser’s motions without control: (a) $u(z, t)$ and (b) $w(z, t)$

Figure 3. The riser’s motions with control: (a) $u(z, t)$ and (b) $w(z, t)$
5. CONCLUSIONS

The paper copes with minimizing vibration of the marine riser. After deriving the set of equations specifying the riser dynamics, the boundary controller applied at the riser top end is designed thank to Lyapunov’s direct method without the assumption of positive tension applied to the riser. The ability in stabilizing the riser at its equilibrium position of the boundary control is validated analytically and illustrated numerically.

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