Moiré pattern of spin liquid and Neel magnet in a Kitaev chain

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A moiré pattern occurs when two periodic structures in a system have a slight mismatch period, resulting in the coexistence of distinct phases in different large-scale spatial regions of the same system. Two periodic structures can arise from periodic electric and magnetic fields, respectively. We investigated the moiré pattern via a dimerized Kitaev spin chain with a periodic transverse field, which can be mapped onto the system of dimerized spinless fermions with p-wave superconductivity. The exact solution for staggered field demonstrated that the ground state has two distinct phases: (i) Neel magnetic phase for nonzero field, (ii) Spin liquid phase due to the emergence of isolated flat Bogoliubov–de Gennes band for vanishing field. We computed the staggered magnetization and local density of states (LDOS) for the field with a slight difference period to the chain lattice. Numerical simulation demonstrated that such two phases appear alternatively along the chain with a long beat period. Additionally, we proposed a dynamic scheme to detect the Moiré fringes based on the measurement of Loschmidt echo (LE) in the presence of local perturbation.

I. INTRODUCTION

The moiré patterns emerge due to the superposition of two periodic structures, with either slightly different period or different orientations, and have been realized in materials. Recently there has been a growing interest in the influence of the moiré pattern in physical systems. The moiré pattern as a new way to apply periodic potentials in van der Waals heterostructures to tune electronic properties, has been extensively studied. Many interesting phenomena have been observed in the heterostructure materials with small twist angles and mismatched lattice constants. moiré patterns in condensed matter systems are produced by the difference in lattice constants or orientation of two two-dimensional lattices when they are stacked into a two-layer structure.

While most studies of this field have focused on the quasi-two-dimensional system, the one-dimensional moiré system is by far less well investigated and is expected to be easily realized in an artificial system. Generally speaking, the moiré patterns imply the result of the competition of at least two effects on electrons that are important both for applications and for fundamental physics. These patterns can be the periodic appearance of two different quantum phases, exhibiting unprecedented states of matter. Two periodic structures can arise from periodic electric and magnetic fields, respectively. The periodic potential (or optical lattice) and the strong particle-particle repulsion can compactly array electrons (atoms). The magnetic field with a slight mismatch period affects the spins in two different ways, staggered manner or zero field, depending on the location at the sample. It provides a simple way to demonstrate the moiré patterns in a one-dimensional system which can be seen in Fig. 1.

In this paper, we study the moiré pattern that emerges in a dimerized Kitaev spin chain with a periodic transverse field, which can be mapped onto the system of dimerized spinless fermions with p-wave superconductivity. The exact solution for staggered field demonstrates that there are two distinct phases for the ground states. The strong field results in the Neel order of spin array, while the spin liquid emerges for nonzero field due to the isolated flat Bogoliubov–de Gennes band. When the period of the transverse field slightly mismatches the lattice constant, the local properties, such as the local staggered magnetization and local density of states, vary periodically along the chain with a long beat period, indicating that two phases, Neel and spin liquid, appear alternatively. We also propose a dynamic scheme to detect the Moiré fringes experimentally. The underlying mechanism is based on the relationship between the decay rate of LE and the LDOS when a local perturbation is added. Numerical simulation demonstrates that the decay rate exhibits the same periodic behavior as the other quantities. It provides a method to detect the Moiré fringes in a photonic system.

This paper is organized as follows. In Section II we present a dimerized Kitaev spin chain model with spatially modulated transverse fields. In Section III we introduce the concepts of Magnetization, DOS and string correlation function to characterize the ground state properties. In Section VI we show the Moiré fringes in the model. In Section VII we propose a dynamic scheme to detect the Moiré fringes experimentally. Finally, we give a summary in Section VIII.

II. MODEL HAMILTONIAN

We start our investigation by considering a dimerized Kitaev spin chain with spatially modulated transverse fields

\[
H = \sum_{j=1}^{N} [(1 - \delta)\sigma_{2j-1}^{x}\sigma_{2j}^{x} + (1 + \delta)\sigma_{2j}^{y}\sigma_{2j+1}^{y}] + \sum_{j=1}^{2N} g_{j}\sigma_{j}^{z},
\]

(1)
where $\sigma_j^x (\alpha = x, y, z)$ are the Pauli operators on site $j$ and $g_j = g \cos[\pi(1 + \Delta)j]$. We take $\sigma_{2N+1}^y \equiv 0$ to impose the open boundary condition. In the zero-field case ($g = 0$), it has been studied in many perspectives. In the staggered-field case ($\Delta = 0$), previous work devoted to the topological feature of the degeneracy lines. For an arbitrary parameter distribution function $g_j$, we have an equivalent dimerized spinless fermion model with the Hamiltonian

$$H = \sum_{j=1}^N [(1 - \delta) (c_{2j-1}^\dagger c_{2j} + c_{2j-1} c_{2j}) + (1 + \delta) (c_{2j+1}^\dagger c_{2j+1}^\dagger + c_{2j+1} c_{2j+1}) + \text{H.c.}] + g_j \sum_{j=1}^{2N} \cos[\pi(1 + \Delta)j](1 - 2c_j^\dagger c_j),$$

which describes the p-wave superconductivity. Here $c_j$ is spinless fermionic operators and this mapping is obtained by the Jordan-Wigner transformation \cite{15}:

$$\sigma_j^x = -i \prod_{l<j} (1 - 2c_l^\dagger c_l)(c_j^\dagger + c_j),$$

$$\sigma_j^y = -i \prod_{l<j} (1 - 2c_l^\dagger c_l)(c_j^\dagger - c_j),$$

$$\sigma_j^z = 1 - 2c_j^\dagger c_j.$$  

The on-site external field $g_j$ is extracted from the continuous field $g(x) = g \cos[\pi(1 + \Delta)x]$ in the continuous coordinate $x$, with the period $2/(1 + \Delta)$. For sufficient small $\Delta$, we have $g_j \approx g_{\text{eff}} \cos(\pi j)$, where $g_{\text{eff}} = g \cos(\pi \Delta f)$ varies slowly. For a small scale, $g_j$ can be regarded as a staggered magnetic field with amplitude $|g_{\text{eff}}|$. For a long scale, $|g_{\text{eff}}|$ is a periodic function of $j$ with beat period $2/\Delta$. To investigated the local properties of the system within a small region, we consider the Hamiltonian as a homogeneous one, i.e., with zero $\Delta$ but varied $g$. The ground state properties with different $g$ reflect the local properties of the original Hamiltonian within different space regions.

We impose periodic boundary condition $\sigma_j^{\alpha} = \sigma_{j+2N}^{\alpha}$ and perform the Fourier transformations for two sublattices, which obeys

$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{ikj} \left\{ \begin{array}{c} \alpha_k, \quad j = 2l - 1 \\ \beta_k, \quad j = 2l \end{array} \right.,$$

where $l = 1, 2, \ldots, N, k = 2m\pi/N, m = 0, 1, 2, \ldots, N - 1$. Thus, spinless fermionic operators in $k$ space $\alpha_k, \beta_k$ can be expressed as

$$\alpha_k = \frac{1}{\sqrt{N}} \sum_j e^{-ikj} c_j, \quad j = 2l - 1$$

$$\beta_k = \frac{1}{\sqrt{N}} \sum_j e^{-ikj} c_j, \quad j = 2l$$

This transformation block diagonalizes the Hamiltonian with translational symmetry, i.e.,

$$H_0 = \sum_k H_k = \sum_k \psi_k^\dagger h_k \psi_k,$$

which based on the basis vector

$$\psi_k = \frac{1}{\sqrt{2}} \begin{pmatrix} -\alpha_k^\dagger + \alpha_k \\ -\beta_k^\dagger + \beta_k \\ -\alpha_k^\dagger - \alpha_k \\ -\beta_k^\dagger - \beta_k \end{pmatrix},$$

where $\gamma_k = (1 - \delta) + (1 + \delta)e^{ik}$. The eigenvector with eigenvalue can be solved as

$$\varepsilon_{\rho\sigma}^k = \frac{\rho}{\sqrt{2}} \sqrt{\Lambda_k + \sigma \sqrt{\Lambda_k^2 - 4g^4}}$$

and

$$|\phi_{\rho\sigma}^k\rangle = \frac{1}{\Omega_{\rho\sigma}} \begin{pmatrix} \varepsilon_{\rho\sigma}^k g\gamma_k \\ \varepsilon_{\rho\sigma}^k (\varepsilon_{\rho\sigma}^k)^2 - g^2 \\ g \varepsilon_{\rho\sigma}^k - g^2 \gamma_k \end{pmatrix},$$
where parameters satisfy $\sigma, \rho = \pm$ and $\Lambda_k = |\gamma_k|^2 + 2g^2$. The normalization factors are $\Omega_{\rho\sigma} = \rho \sqrt{2g (\varepsilon^k_{\rho\sigma})^{-1}} \{[(\varepsilon^k_{\rho\sigma})^2 - g^2] [(\varepsilon^k_{\rho\sigma})^2 - g^2]\}^{1/2}$. There are four Bogoliubov-de Gennes bands from the eigenvalues of $h_k$, indexed by $\rho, \sigma = \pm$. In this study, we consider the case with the band touching points for $\varepsilon^{+}_{\downarrow\downarrow}$ and $\varepsilon^{+}_{\uparrow\uparrow}$, which are the ground state is strongly determined by the magnitude $g$, i.e., analytically studied. In the medium, the Hamiltonian has been systematically studied when $g = 0$. The ground state is spin liquid phase, i.e., $A_x$ or $A_y$ phase for $\delta < 0$ or $\delta > 0$, respectively. In the medium $g$, the crossover ground state can be obtained from exact solution. In this Section, we utilize three parameters, magnetization, DOS and string correlation function (SCF), to characterize the ground state properties.

(i) The staggered magnetization on site $j$ is defined as

$$m_j^z = (-1)^j \langle G| \sigma_j^z |G \rangle$$

for ground state $|G\rangle$. According to the Hellmann-Feynman theorem, we have

$$m_j^z = (-1)^j \frac{1}{2N} \langle G| \frac{\partial h}{\partial j} |G \rangle = (-1)^j \frac{\partial E_g}{\partial g_j},$$

where $E_g$ is the many-particle density of ground state energy. For zero $\Delta$, $m_j^z$ is a uniform function of $(\delta, g)$, i.e.,

$$m_z = m_j^z = \frac{\partial E_g}{\partial g},$$

With exact solution

$$E_g = \frac{1}{4\pi} \int_{-\pi}^{\pi} (\varepsilon^{+}_{\downarrow\downarrow} + \varepsilon^{+}_{\uparrow\uparrow}) \, dk$$

$$= -2 \sqrt{1+g^2} \pi [E(1 - \delta^2)/(1+g^2)],$$

in last Section, $m_z$ can be exactly obtained as

$$m_z = -2g[E(1 - \delta^2)/(1+g^2)]/(\pi \sqrt{1+g^2}).$$

Here, $E(t)$ is the complete elliptic integral. Notably, parameter satisfies $m_z = 0$ when $g = 0$, which demonstrates one characteristic of quantum spin liquid. More phenomena of the parameter $m_z$ are described in Fig. 3.

FIG. 3. Numerical simulations of $m_z$ as a function of on-site potential $g$ for four typical values of $\delta$: $\delta = 0.2$ (blue dashed line), $\delta = 0.4$ (black solid line), $\delta = 0.6$ (orange solid line) and $\delta = 0.8$ (red solid line). The size of the system is $N = 500$ and parameter is $\Delta = 0$.

(ii) On the other hand, the DOS for Bogoliubov-de Gennes band is defined as

$$D(E) = \lambda \frac{dN(E)}{dE},$$

generally, $dN(E)$ indicates that how many energy levels are appeared in interval $[E, E + dE]$ and the normalization factors are $\lambda = \Delta \lambda/4N$. Notably, $D(E) \rightarrow \infty$ accords with the fact that flat bands occur at $g = 0$. Furthermore, Fig. 2 demonstrates it.

(iii) The correlation length of quantum spin model can also demonstrate the character of spin liquid phase. We

III. MAGNETIZATION, DOS AND STRING CORRELATION FUNCTION

For the uniform system $(\Delta = 0)$ what we study here, the ground state is strongly determined by the magnitude $g$. In fact, this viewpoint can be seen from two limits. On the one hand, in strong limit $|g| \gg 1$, the ground state is a Neel magnet with staggered spin alignment. On the other hand, the Hamiltonian has been systematically studied when $g = 0$. The ground state is spin liquid phase, i.e., $A_x$ or $A_y$ phase for $\delta < 0$ or $\delta > 0$, respectively. In the medium $g$, the crossover ground state can be obtained from exact solution. In this Section, we utilize three parameters, magnetization, DOS and string correlation function (SCF), to characterize the ground state properties.

FIG. 2. Plots of the Bogoliubov-de Gennes energy bands for two typical conditions (a) $g = 0.4$, (b) $g = 0.001$, as a function of momentum $k$ (blue line) and $D(E)$ (red line), respectively. Notably, the flat band energy value and the infinite DOS value both occur when $g \rightarrow 0$. The size of the system is $N = 500$, parameters are $\Delta = 0$ and $\delta = 0.2$. 

FIG. 3. Numerical simulations of $m_z$ as a function of on-site potential $g$ for four typical values of $\delta$: $\delta = 0.2$ (blue dashed line), $\delta = 0.4$ (black solid line), $\delta = 0.6$ (orange solid line) and $\delta = 0.8$ (red solid line). The size of the system is $N = 500$ and parameter is $\Delta = 0$. 
introduce the SCF for the ground state, which is defined as
\[ O_x^{(2r+1)}(2j - 1, 2(j + r)) = \langle G \sigma_x^{2j-1} \prod_{n=2j}^{2(j+r)-1} \sigma_x^n \sigma_x^{2(j+r)} | G \rangle. \quad (20) \]

For estimating the correlation length of SCF, we consider a perturbated Hamiltonian
\[ H^{(2r+1)} = H + c \sum_{2j-1} \sigma_x^{2j-1} \prod_{n=2j}^{2(j+r)-1} \sigma_x^n \sigma_x^{2(j+r)}, \quad (21) \]
where the extra term stands for long range spin-spin coupling with strength c. The corresponding Schrodinger Equation for ground state is
\[ H^{(2r+1)} \begin{pmatrix} G^{(2r+1)} \\ \end{pmatrix} = E^{(2r+1)}_{g,j} \begin{pmatrix} G^{(2r+1)} \\ \end{pmatrix}, \quad (22) \]
where the many-particle density of ground state energy \( E^{(2r+1)}_{g,j} \) can be exactly obtained as
\[ E^{(2r+1)}_{g,j} = -\frac{1}{4\pi} \int_{-\pi}^{\pi} \left\{ (1 - \delta)^2 + (1 + \delta)^2 + c^2 + 2(1 - \delta)(1 + \delta) \cos k + 2c(1 - \delta) \right. \cos(\rho k) + 2c(1 + \delta) \cos(\rho + 1) k + 4g^2 \right\}^{1/2} \partial c \partial g \partial k. \quad (23) \]

by the translational symmetry of \( H^{(2r+1)} \). Meanwhile, the translational symmetry of \( G^{(2r+1)} \) results in
\[ O_x^{(2r+1)}(2j - 1, 2(j + r)) = O_x^{(2r+1)}(2(j + l) - 1, 2(j + r + l)) = O_x^{(2r+1)}. \quad (24) \]

According to the Hellmann-Feynman theorem, SCF can be expressed as
\[ O_x^{(2r+1)} = \left[ \frac{\partial E^{(2r+1)}_g}{\partial c} \right]_{c=0} = -\int_{-\pi}^{\pi} \frac{1}{4\pi} \frac{1}{\sqrt{2}} \left\{ (1 - \delta) \cos(\rho k) + (1 + \delta) \cos(\rho + 1) k + 2g^2 \right\}^{1/2} \partial c \partial g \partial k. \quad (25) \]
The \( O_x^{(2r+1)} \) satisfies the relation
\[ O_x^{(2r+1)} \propto e^{-\frac{\delta}{\xi}}. \quad (26) \]
which can be transformed as
\[ \xi = 2[\ln O_x^{(2r+1)} - \ln O_x^{(2r+3)}]^{-1}. \quad (27) \]

Fig. 4 numerically demonstrates it. Actually, \( \xi \) is also determined by \( g \) and \( \xi = 0 \) occurs at a large limit of \( g \), we also numerically demonstrate it [5].

In a conclusion, this system has a large number of degeneracy ground states with zero magnetization, which are the characterization of spin liquid phase, as claimed in previous work[2]. On the other hand, \( D(E) = 0 \) vanishes, \( \xi \to 0 \) and \( m_z \to 1 \), when \( g \to \infty \), which all indicate the Neel phase.

![Fig. 4. Numerical simulations of a quantity \( \ln O_x \) about correlation length from Eq. \( 20 \) on uniform system, for four typical classes of parameters \( \delta = 0.2 \) (blue dashed line), \( \delta = 0.4 \) (black solid line), \( \delta = 0.6 \) (orange solid line) and \( \delta = 0.8 \) (red solid line), as a function of the long range length \( r \). The size of the system is \( N = 500 \) and the parameter is \( g = 0.4 \). Note that the \( \ln O_x \) decreases linearly with \( r \) in finite \( r \), which are associated with the conclusion as given in Eq. \( 26 \).](image1)

![Fig. 5. Numerical simulations of correlation length \( \xi \) as a function of on-site potential \( g \). The correlation length \( \xi \) is finite when \( g \to 0 \), while \( \xi \) decays rapidly to zero as \( |g| \) increases. The size of system is \( N = 500 \), parameter is \( \Delta = 0 \) and \( \delta = 0.2 \).](image2)

### IV. MOIRÉ FRINGES

The aim of this work is to present the Moiré fringes of the system with nonzero \( \Delta \), properties of system for zero \( \Delta \) but different \( g \), discussed above, should appear in the different locations along the chain. Consider magnetization \( m_z^g \) is a function of the coordinate, then we introduce LDOS to replace DOS. For calculating such parameters, the solution of \( H \) is necessary. We utilize Majorana fermion operators
\[ a_j = c_j^+ + c_j, b_j = -i(c_j^+ - c_j), \quad (28) \]
which satisfy the relations
\[ \{a_j, a_{j'}\} = 2\delta_{j,j'}, \{b_j, b_{j'}\} = 2\delta_{j,j'} \]  
(29)
and
\[ \{a_j, b_{j'}\} = 0, a_j^2 = b_j^2 = 1. \]  
(30)
The inverse transformation obeys
\[ c_j^\dagger = \frac{1}{2}(a_j + ib_j), c_j = \frac{1}{2}(a_j - ib_j). \]  
(31)
Then the Majorana representation of Hamiltonian is
\[ H = \psi^T h \psi, \]  
(32)
where \( \psi^T = (ia_1, b_1, ia_2, b_2, ia_3, b_3, \ldots, ia_{2N}, b_{2N}) \). Here, \( h \)
represents a \( 4N \times 4N \) matrix, which can be explicitly wrote as
\[
h = \frac{g}{2} \sum_{j=1}^{2N} (-1)^j \cos(j\pi \Delta) |a, j\rangle \langle b, j|
+ \frac{1}{2} \sum_{j=1}^{N} [(1 + \delta) |a, 2j\rangle \langle b, 2j + 1|
+ (1 - \delta) |a, 2j\rangle \langle b, 2j - 1|] + \text{H.c.},
\]  
(33)
where \( |\lambda, j\rangle \) is an orthonormal complete set, which satisfies
\( \langle \lambda, j | \lambda', j'\rangle = \delta_{\lambda\lambda'} \delta_{j,j'}, j \in [1, 2N], \lambda = a, b \) and \( \langle b, 2N + 1\rangle \equiv 0 \). Obviously, \( h \) describes a single-particle tight-binding SSH chain with side-couplings and cosine-modulated couplings. The schematic of the Majorana lattice system is sketched in Fig. 6.

Then ground state energy \( E_g \), \( m^2_g \) and \( D_j \) (LDOS) can also be obtained by diagonalizing the matrix of the lattice system. For the ground state, Majorana operators \( a_{2j-1} \) and \( b_{2j} \) are free in \( g_j = 0 \) region, leaving large numbers of degenerate ground states. We depict the magnetization as a function of \( j \) (Fig. 7 (a)) and compute \( D_j \) by using the method(Fig. 7 (b)). Inspired by the characters of string correlation function, we consider an artificial model with non-neighbour interactions, and the Hamiltonian obeys
\[ h_j^{(2r+1)} = h + \frac{c}{2} \langle a, 2(j + r) | \langle b, 2j - 1| + \text{H.c.} \].
(34)
Density of ground state energy \( E_g^{(2r+1)} \) for many-particle system equals the sum of all negative eigenvalues of from \( h_j^{(2r+1)} \) with the coefficient \( 1/(2N) \), which can be numerically obtained. The corresponding SCF and \( \xi \) can be computed from
\[ O_{x,j}^{(2r+1)} = \lim_{c \to 0} \frac{\partial E_{g,j}^{(2r+1)}}{\partial c}, \]  
(35)
thus, we get the equation
\[ \xi_j = 2[\ln O_{x,j}^{(2r+1)} - \ln O_{x,j}^{(2r+3)}]^{-1}, \]  
(36)
and depict \( \xi_j \) as a function of \( j \) (Fig. 7 (c)).

![FIG. 6. Lattice geometries for the Majorana models, which is described in Eq. (33). Solid (empty) circle indicates (anti) Majorana modes. Panel (a) indicate a SSH chain with side-couplings \((1 \pm \delta)/2\) and \(\pm g/2\), while panel (b) with \(g = 0\).](image)

V. Dynamic Detection

Motivated by the relationship between the decay rate of LE and the LDOS when the local Hamiltonian is perturbed slightly, we want to demonstrate the veiled information inside the model of this work. This investigation may provide the scheme to experimentally detect the Moiré fringes. It is a challenge to realize a quantum spin chain with the Hamiltonian \( H \). However, the Majorana lattice \( h \) appears as a relative simpler structure, which can be arranged in photonic system. it is based on the analogy between light propagating through a photonic crystal and the tight binding Hamiltonian. For instance, topological effects in some electronic systems can be observed in their photonic counterparts. In a photonic platform, Pauli exclusion is not obeyed, a single-particle state can be amplified by the large population of photons. It allows for a high degree of control over the system parameters.

In this Section, we consider the Hamiltonian \( h_\eta \) with a slight perturbation, which has the form
\[ h_\eta = h + \eta \sum_{j=1}^{2N} \langle b, j | \langle a, j| + \text{H.c.}, \]  
(37)
where the perturbation is a shift field with strength \( \eta \). We employ the LE to investigate the dynamical signature of the moiré pattern. The LE for an initial state \( |\psi_0\rangle \) is defined as
\[ M(t) = |\langle \psi_0 | \exp(ih_\eta t) \exp(-iht)|\psi_0\rangle|^2. \]  
(38)
We take the site-state \( |\psi_0\rangle = |b, 2j\rangle \) as the initial state, thus, \( M(t) \) is a function of position, which labeled by \( M_j(t) \). Numerical simulations demonstrate the LE decays in the form
\[ M_j(t) = e^{-\gamma_j t^2}, \]  
(39)
FIG. 7. Numerical simulations of four typical parameters: (a) magnetization ($m_z$), (b) local density of state (LDOS), (c) correlation length ($\xi$) and (d) decay of the LE ($\gamma$), as a function of site $j$ respectively. Notably, these four panel are the main results of this work. Panel (a), on-site potential amplitude parameters are $g = -2.5$ (blue dashed line), $g = -1$ (black solid line) and $g = -0.5$ (orange solid line). The top (or valley) of the magnetization depend on the value of $g$ in panel (a). All panels exhibit the unitary moiré patterns for infinite site chain. Parameters in panel (d) is $\eta = 0.005$. The size of system is $N = 500$, the common parameters are $\Delta = 0.002$, $g = 0.4$ and $\delta = 0.2$.

The parameter $\Gamma_j(t)$ is introduced to present the decay behaviour, as a function of evolution time $t$ (Fig. 8). Furthermore, we depict the decay of LE ($\gamma_j$) as a function of site $j$ (Fig. 7 (d)). For clarity, we demonstrate the relations of calculated LE for a serious $j$ and $t$ values (Fig. 9). Notably, the performances of $\Gamma$, $\gamma$ and $M$, which are described in above three figures can not be affected by the varied parameters $\delta$ or $g$. It is worth mentioning briefly that such a moiré pattern can be implemented through two-dimensional array of evanescently coupled optical waveguides. Additionally, the LE of photons can be observed in a binary waveguide, by exchanging the two sublattices after some propagation distance.

FIG. 8. along the site $j$ from 1 to 125. The system parameters are $N = 500$, $\Delta = 0.002$, $\eta = 0.005$, $g = 0.4$ and $\delta = 0.2$.

FIG. 9. Numerical simulation of LE $M(t)$ as a function of site $j$ and evolution time $t$ in Majorana model. The periodic behavior is obvious, exhibiting the moiré pattern as expected. The size of system is $N = 500$, the parameters are $\Delta = 0.002$, $\delta = 0.2$, $g = 0.4$, and $\eta = 0.005$.

thus, we have the equation

$$\Gamma_j(t) = \sqrt{-\ln M_j(t)} = \sqrt{\gamma_j} t.$$  (40)

VI. SUMMARY

In summary, we have demonstrate a super periodicity in the coordinate space along a quantum spin chain is imposed on it if the period of the external sinusoidal magnetic field has a slight difference with the lattice constants. There are two quantum phases in each period, one is Neel phase, another is spin liquid phase. Additionally, we have proposed a dynamic scheme to detect the Moiré fringes based on the measurement of LE with the respect to local perturbation. It provides a method to detect the Moiré fringes in a photonic system, which is different from the real quantum condensed-matter system, but can reproduce almost all condensed-matter experiments.

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