— Editorial note —

The UK National Conference on Geophysical, Astrophysical and Industrial Magnetohydrodynamics (UK MHD) is an annual meeting of the UK Magnetohydrodynamics research community. The meeting has been organized every year since 1978 with the exception of 1983 and 1987, (http://www.maths.gla.ac.uk/~drf/mhd.htm). The 2013 meeting was held at the School of Mathematics and Statistics, the University of Glasgow on the 23 and 24 May. This document presents an accurate record of the scientific programme of the meeting. The abstracts of presented contributions are listed in chronological order and a full list of participants and their affiliations is included. The Meeting was sponsored by

- The London Mathematical Society, www.lms.ac.uk,
- The Glasgow Journal of Mathematics Trust, www.maths.gla.ac.uk/gmj/gmj-trust,
- The Edinburgh Mathematical Society www.ems.ac.uk,
- The Science and Technology Facilities Council www.stfc.ac.uk.

The conference website (http://www.gla.ac.uk/schools/mathematicsstatistics/events/ukmhd) records some further details of the meeting.

RADO\[\text{\textsc{\textit{S}}}\]

— Programme —

The original version this programme as used at the time of the Meeting can be found online at http://www.gla.ac.uk/schools/mathematicsstatistics/events/ukmhd.

— Thursday, 2013-05-23 —

■ Welcome and Registration

9:00 – 10:25 Arrival, registration, set-up of posters. Refreshments available.
10:25 – 10:30 PROF N.A. HILL

Head of School Welcome message

■ Invited Lecture

Chairperson: David Fearn

10:30 – 11:10 ANDREW M. SOWARD, A.P. Bassom, K. Kuzanyan, D. Sokoloff and S. Tobias

Asymptotic solution of a kinematic \(\alpha\Omega\)-dynamo with meridional circulation

Many stars exhibit magnetic cycles typified by the butterfly diagram characterising our sun's 11 year solar activity cycle. Parker explained the phenomenon by an \(\alpha\Omega\)-dynamo acting in the star's convection zone causing the equatorial propagation of dynamo waves. In contrast, to the many continuing numerical investigations, we adopt a minimalist approach and expand on Parker's original one-dimensional uniform plane layer model. To apply asymptotic methods, we suppose that the dynamo is confined to a thin shell with latitudinal variations of the \(\alpha\Omega\)-sources, whose product the Dynamo number vanishes at the pole and equator. The ensuing linear stability problem is resolved by global stability criteria. Our new results concern the role of meridional circulation. They show that sufficiently large circulation halts the Parker travelling waves leading to non-oscillatory behaviour, a result only predicted previously from numerical integration of the full pde's governing axisymmetric \(\alpha\Omega\)-dynamos.
Session 1: Geophysical and Planetary Applications I

Chairperson: Radostin Simitev

11:10 – 11:25 William Brown, J. Mound, P. Livermore
Jerks abound: Observations of geomagnetic jerks and implications for core dynamics

The geomagnetic field is generated by the constant evolution of the fluid outer core. The secular variation of the geomagnetic field describes the variation of the field at time scales on the order of months to decades and is attributed primarily to flows near the surface of the outer core. The secular variation, at any point on the Earth’s surface, is often characterised as a series of linear trends separated by abrupt turning points known as geomagnetic jerks. These rapid variations in the secular variation are linked to accelerations of flow in the outer core. Various generation mechanisms for these rapid changes have been suggested but none have conclusively explained the phenomena. Our recent study of geomagnetic jerks in observatory data over the period of 1957-2008 indicates that jerks are far more frequent an occurrence than previously suggested and perhaps part of the more rapid end of a spectrum of core dynamics. Whilst jerks are seen to be common, relative peaks in the global number of jerk occurrences are seen in 1968-71, 1973-74, 1977-79, 1983-85, 1989-93, 1995-98 and 2002-03 with the suggestion of further poorly sampled events in the early 1960s and late 2000s. We do not find consistent patterns in the spatial distributions of occurrences suggesting complex origins or the superposition of several discrete individual events. We observe that jerk amplitudes vary through time and their variations are potentially periodic in Europe and North America. This may have implications which help to constrain a source mechanism in the dynamics of the outer core. These signals may be related to the 6yr periods detected independently in the secular variation and length-of-day.

11:25 – 11:40 Robert Teed, C. Jones and S. Tobias
Torsional oscillations in geodynamo simulations

Torsional oscillations are a principal feature of the dynamics of the fluid outer core where the Earth’s magnetic field is generated. These oscillations are Alfvén waves about an equilibrium known as a Taylor state and propagate in the cylindrical radial direction. The change in core angular momentum inferred from geomagnetic observations has a measurable impact on the length of the day, and the small decadal variations in the length-of-day signal confirm the existence of torsional oscillations. In our work we perform three-dimensional spherical dynamo simulations in parameter regimes where Earth-like magnetic fields are produced. Many of our simulations frequently produce the desired torsional oscillations, identified by their movement at the correct Alfvén speed. We find that the frequency, location and direction of propagation of the waves can be influenced by the choice of parameter regime and boundary conditions. Torsional waves are observed within the tangent cylinder region and also have the ability to pass through this theoretical cylinder. Our results indicate that excitation mechanisms for these waves must be available throughout the outer core. We calculate the driving terms in an attempt to better understand these mechanisms.

11:40 – 11:55 Luis Silva, C. Davies, J. Mound
Inner core structure inferred from observations of the geomagnetic field

The short term evolution of the geomagnetic field, as observed at Earth’s surface, contains the information required to infer the velocity field at the top of the core. Although this velocity field cannot be uniquely estimated, several studies have demonstrated that some of its features are robust, not only across assumptions, but also over time. Due to strong rotational effects, the geomagnetic field and the associated core-surface velocities are expected to reflect the dynamics of the bulk of the core, an assumption that has seldom been used. Here we use a very simplified model of Earth’s core and make inferences about its thermal structure starting from observationally constrained core-surface flows. Despite its simplicity, this model is in agreement with much more elaborate data assimilation models making it useful to test assumptions about the force balances inside Earth’s core. Finally, we relate the obtained thermal structure of the outer core to the seismically observed structure of the inner core and propose a possible causal relationship.

11:55 – 12:10 Wieland Dietrich, T. Gastine and J. Wicht
The nonlinear interaction between geostrophic and ageostrophic zonal flows

Zonal flows on rapidly rotating objects, such as the Gas or the Ice giants, form in a variety of surface patterns and amplitudes. It is thought that equatorial prograde surface zonal flows are created by Reynolds-stress in rotation-dominated convection, whereas the equatorial retrograde jets form in an inertia-dominated regime due to turbulent angular momentum mixing. As far as numerical simulations can reach, simple Boussinesq-models predict only minor variation in the interior zonal flow structure along the axis of rotation according to the Taylor-Proudman-Theorem. Further solar or stellar irradiation might cause a latitudinal variation pattern of the surface temperature. The heat flux is then expectancy
higher at the cooler poles and smaller at the hotter equatorial region. If such a latitudinal temperature/heat flux anomaly at the surface affects also the interior of the fluid, thermal winds will introduce ageostrophic variations of the geostrophic zonal flow structure along the axis of rotation. To investigate the interaction between thermal wind driven ageostrophic and Reynolds-stress driven geostrophic zonal flows, we conduct a series of hydrodynamical simulations using a rotating and convecting spherical fluid shell in the limit of the Boussinesq-approximation. The fluid is heated by a homogeneously distributed interior heat source and cooled by a prescribed mean heat flux at the outer boundary, which additionally varies smoothly with latitude as a axisymmetric spherical harmonic degree two of variable amplitude. Our results suggest, that for weak perturbation amplitudes the flow tends to be the more ageostrophic the stronger the heat flux normally. This results from increasing ageostrophic and decreasing geostrophic zonal flows. Since the introduced thermal wind defines the zonal flow variation along the axis of rotation, the equatorial jet decreases its prograde amplitude. If the anomaly reaches a critical strength, the equatorial jet crosses zero amplitude and can even be reverted into retrograde direction. For perturbation amplitudes stronger than critical, the reverted jet gains in amplitude due to re-established, but reverted Reynolds-stress. Even though the ageostrophic flow is linearly amplified with the anomaly amplitude, the geostrophic flow contributions dominate at maximal heat flux perturbation. We therefore propose a nonlinear backreaction of the thermal wind onto the Reynolds-stress driven zonal flows. Analysis of the main force balance and parameter studies further foster this result.

12:10 – 12:25
MICHAEL PROCTOR
Bounds and scaling for the Archontis dynamo
The Archontis dynamo is characterised by strong magnetic fields closely aligned with the driving velocity field. An interesting question is how far such a balance can be maintained as diffusivity rates change. We present preliminary results giving rigorous bounds on flow quantities and computations showing that the Archontis balance is surprisingly robust.

Lunch Break
12:25 – 14:00
Lunch is provided at “One A The Square”. The restaurant is located at the North-West corner of the Main University Building. Lunch vouchers are enclosed in your conference wallets.

Session 2: Geophysical and Planetary Applications II
Chairperson: Phil Livermore
14:00 – 14:15
GRAEME SARSON, R. Boys, A. Golightly and D. Henderson
Modelling geomagnetic reversals as a Gaussian Cox Process
The mean rate of reversal of the geomagnetic field, as recorded in the Geomagnetic Polarity Time Scale (GPTS), has long been an object of study. The significance of the apparent long-term variations in the mean rate of reversals — including the occurrence of superchrons — has been debated, as has the possible origins of these variations in external control of the geodynamo (e.g. by the time-varying boundary conditions imposed by mantle convection). Here we model the long-term variations in the reversal rate nonparametrically, in terms of an inhomogeneous Poisson process. Specifically, we consider a Gaussian Cox process, a type of doubly-stochastic Poisson process where the mean rate (or intensity) is modelled in terms of a Gaussian process. Such processes are amenable to likelihood-based inference using Bayesian Markov Chain Monte-Carlo (MCMC) methods, which we employ to provide posterior distributions of the model parameters. The specification of a Gaussian process requires a covariance function, relating the intensity at nearby times; crucially, however, the timescale of the covariance function is not prescribed, but appears as a model hyperparameter, whose posterior distribution is an important output of the analysis. For the geodynamo, this hyperparameter should robustly characterise the timescale of long-term variations. Two different types of Gaussian Cox process are considered: a Log Gaussian Cox Process, applied to binned reversal data; and a Sigmoidal Gaussian Cox Process, applied to the discrete reversal data using a technique involving latent variables. Different MCMC algorithms for sampling the posterior distribution of the model parameters are investigated for both types of process, to check (and to optimise) the convergence of the MCMC chains. This analysis is applied to different records of the GPTS, including those of Cande & Kent (1995) and Gradstein & Ogg (1996). The implications of this analysis for the geodynamo, and the possibility of comparable analysis of the output of numerical geodynamo simulations, are discussed.

14:15 – 14:30
JACK WOOD
A layer of fluid with variable conductivity
The electrical conductivity of Jupiter varies over several orders of magnitude from the non conducting outer reaches of the planet to the highly conducting metallic hydrogen region (at approximately 0.8
RJ). In an attempt to identify what effect this variance could have on the dynamics of such a system, I will present a plane layer approximation of a rapidly rotating, inviscid fluid with imposed zonal surface winds and varying conductivity. Limitations of a low \( Rm \) assumption will be discussed along with initial thoughts for a higher \( Rm \) approximation.

14:30 – 14:45  
DALI KONG, K. Zhang and G. Schubert  
Gravitational signature of rotationally distorted Jupiter caused by deep zonal winds  
Both deep zonal winds, if they exist, and the basic rotational distortion of Jupiter contribute to its zonal gravity coefficients \( J_n \), for \( n \) greater than or equal to 2. In order to capture the gravitational signature of Jupiter that is caused solely by its deep zonal winds, one must take into account the full effect of rotational distortion by computing the coefficients \( J_n \) in non-spherical geometry. Based on the model of a polytropic Jupiter with index unity, we compute for the first time Jupiter’s gravity coefficients \( J_2, J_4, J_6, \ldots, J_{12} \) taking into account the full effect of rotational distortion of the gaseous planet. For the model of deep zonal winds on cylinders parallel to the rotation axis, we also compute for the first time the variation of the gravity coefficients \( \Delta J_2, \Delta J_4, \Delta J_6, \ldots, \Delta J_{12} \) caused solely by the effect of the winds on the rotationally distorted Jupiter. It is found that the effect of the zonal winds on lower-order coefficients is weak, \(|\Delta J_n/J_n|\) less than 1%, for \( n = 2, 4, 6 \), but it is substantial for the high-degree coefficients with \( n \) greater than or equal to 8.

Posters and Afternoon Coffee Break  
Chairperson: Andrew Baggaley  
14:45 – 15:20  Posters  
15:20 – 15:50  Coffee break  

Session 3: Solar Applications I  
Chairperson: Ineke De Moortel  
15:50 – 16:10  THOMAS NEUKIRCH, F. Wilson, M. Harrison  
One-dimensional force-free current sheet equilibria: MHD vs Vlasov theory  
One-dimensional current sheet equilibria are often used as initial states for investigations of fundamental plasma processes such as magnetic reconnection. While finding 1D MHD equilibria is very simple, the task of finding the corresponding equilibrium distribution functions in Vlasov theory is non-trivial. In this talk some recent results for constructing exact analytical equilibrium distribution functions for non-linear force-free current sheets will be presented.

16:10 – 16:25  FREDERICK GENT, R. Erdélyi, V. Fedun and S. Mumford  
Flux tube equilibrium  
A single magnetic flux tube emerging from the solar photosphere to the lower corona is modelled in magnetohydrostatic equilibrium with a realistic stratified atmosphere. We solve analytically a 3D structure for the model, with field strength, plasma density, pressure and temperature all consistent with observational and theoretical estimates. Such flux tubes are observed to remain relatively stable for up to an hour or more, and it is our aim to apply the model as the background condition for numerical studies of energy transport mechanisms from the surface to the corona. The model includes a number of free parameters, which makes the solution applicable to a variety of other physical problems and it may therefore be of more general interest.

16:25 – 16:40  EDUARD KONTAR, N. Bian, I. Hannah and N. Jeffrey  
Magnetic fluctuations, acceleration and transport in solar flares  
Plasma turbulence is thought to be associated with various physical processes involved in solar flares, including magnetic reconnection, particle acceleration, and transport. Using X-ray observations, we determine the spatial and spectral distributions of energetic electrons for a flare, which was previously found to be consistent with a reconnection scenario. Energy-dependent transport of tens of keV electrons is observed to occur both along and across the guiding magnetic field of the loop. We show that the cross-field transport is consistent with the presence of magnetic turbulence in the loop, where electrons are accelerated, and estimate the magnitude of the magnetic field fluctuations. The level of magnetic fluctuations peaks when the largest number of electrons is accelerated and is below detectability or absent at the decay phase.

16:40 – 16:55  NIC BIAN, and E. Kontar  
Stochastic acceleration by multi-island contraction during turbulent magnetic reconnection  
The acceleration of charged particles in magnetized plasmas is considered during turbulent multi-island magnetic reconnection. The particle acceleration model is constructed for an ensemble of islands which
produce adiabatic compression of the particles. The model takes into account the statistical fluctuations in the compression rate experienced by the particles during their transport in the acceleration region. The evolution of the particle distribution function is described as a simultaneous first- and second-order Fermi acceleration process. While the efficiency of the first-order process is controlled by the average rate of compression, the second-order process involves the variance in the compression rate. Moreover, the acceleration efficiency associated with the second-order process involves both the Eulerian properties of the compression field and the Lagrangian properties of the particles. The stochastic contribution to the acceleration is nonresonant and can dominate the systematic part in the case of a large variance in the compression rate. The model addresses the role of the second-order process, how the latter can be related to the large-scale turbulent transport of particles, and explains some features of the numerical simulations of particle acceleration by multi-island contraction during magnetic reconnection.

On the relaxation of braided magnetic fields

The braiding of the solar coronal magnetic field by photospheric motions - with subsequent relaxation and plasma heating - is one of the most widely debated ideas of solar physics. We describe here the resistive relaxation of a magnetic field that contains braided magnetic flux, as a model for a coronal loop, subject to line-tied boundary conditions. It is found that the system is unstable, and that following the onset of instability a myriad of thin current layers progressively develops. The current layer structure becomes increasingly complex, and is increasingly long-lived, for higher magnetic Reynolds numbers ($Rm$). For large $Rm$ the evolution resembles a state of decaying turbulence. The final state is found to approximate a non-linear force-free field, implying that the system does not undergo a 'Taylor-like' relaxation. The implications of the results for the heating of the solar corona will be discussed.

On the relaxation of braided magnetic fields

The braiding of the solar coronal magnetic field by photospheric motions - with subsequent relaxation and plasma heating - is one of the most widely debated ideas of solar physics. We describe here the resistive relaxation of a magnetic field that contains braided magnetic flux, as a model for a coronal loop, subject to line-tied boundary conditions. It is found that the system is unstable, and that following the onset of instability a myriad of thin current layers progressively develops. The current layer structure becomes increasingly complex, and is increasingly long-lived, for higher magnetic Reynolds numbers ($Rm$). For large $Rm$ the evolution resembles a state of decaying turbulence. The final state is found to approximate a non-linear force-free field, implying that the system does not undergo a 'Taylor-like' relaxation. The implications of the results for the heating of the solar corona will be discussed.

16:55 – 17:10  DAVID PONTIN, A.L. WILMOT-SMITH and G. HORNIG

On the relaxation of braided magnetic fields

The braiding of the solar coronal magnetic field by photospheric motions - with subsequent relaxation and plasma heating - is one of the most widely debated ideas of solar physics. We describe here the resistive relaxation of a magnetic field that contains braided magnetic flux, as a model for a coronal loop, subject to line-tied boundary conditions. It is found that the system is unstable, and that following the onset of instability a myriad of thin current layers progressively develops. The current layer structure becomes increasingly complex, and is increasingly long-lived, for higher magnetic Reynolds numbers ($Rm$). For large $Rm$ the evolution resembles a state of decaying turbulence. The final state is found to approximate a non-linear force-free field, implying that the system does not undergo a 'Taylor-like' relaxation. The implications of the results for the heating of the solar corona will be discussed.

17:10 – 17:15  KEKE ZHANG

Brief announcement

Conference Dinner at The Bothy

18:30 – 19:20  Drinks at the Maths & Stats Common Room
19:30 –  Dinner at The Bothy

— Friday, 2014-05-23 —

Invited Lecture

Chairperson: David Hughes

9:00 – 9:40  CHRIS A. JONES

Dynamo models of Jupiter’s magnetic field

Numerical dynamo models have had some success in reproducing important features of the Earth’s magnetic field. Here we report on simulations of Jupiter’s magnetic field using the anelastic approximation, which takes into account the large density variation across the dynamo region. The reference state used in these models is a Jupiter model taken from ab initio calculations of the physical properties of Jupiter’s magnetic field (French et al. 2012), which makes the reasonable assumption that the interior is close to adiabatic. The French et al. work also gives an electrical conductivity profile which is adopted here. Dynamo simulations depend on the dimensionless input parameters, particularly the Ekman number, Rayleigh number, the Prandtl number and magnetic Prandtl number. Many different types of field have been found, some of which will be described. The most relevant models are those which produce a Jupiter-like strong dipole dominated field. These are found at low Ekman number, Rayleigh numbers large enough for the convective heat flux to dominate the radiative flux, low Prandtl number and moderate magnetic Prandtl number. Another important issue is the driving heat flux source. Here we assume that Jupiter evolves through a sequence of adiabats, leading to a distributed entropy source throughout the planet, rather than basal heating from the small rocky core. The interaction between the magnetic field, the zonal flow and the convection appears to be crucial in determining the type of magnetic field found.

Session 4: Solar Applications II

Chairperson: David Tsklauri

9:40 – 9:55  ANVAR SHUKUROV, P. BUSHBY, L. COLE and Y. JI

Asymptotics for the mean-field dynamo in a slab
Boundary-layer asymptotics for the kinematic mean-field dynamo equations in a slab are revisited, earlier results revised, and some misconceptions identified to achieve a consistent understanding of the dynamo action in a broad range of dynamo numbers.

9:55 – 10:10
**ANDREW GASCOYNE and R. Jain**

*p*-Mode driven sausage waves

We investigate the generation and propagation of sausage tube waves within the solar convection zone and chromosphere by the buffeting of *p* modes. The tube waves propagate along the many magnetic fibrils which are embedded in the convection zone and expand into the chromosphere due to the fall in density with height of the surrounding plasma. The magnetic fibrils form a waveguide for these waves to freely propagate up and down the tube, those waves propagating upward pass through the photosphere into the chromosphere and enter the upper atmosphere, where they can be measured as loop oscillations and other forms of propagating coronal waves. We treat the magnetic fibrils as vertically aligned, thin flux tubes embedded in a two layer polytropic-isothermal atmosphere to investigate the coupling of *p*-mode driven sausage waves; which are excited in the convection zone and propagate into the overlying chromosphere. The excited tube waves carry energy away from the *p*-mode cavity resulting in a deficit of *p*-mode energy which we quantify by computing the associated damping rate of the driving *p* modes. We calculate the damping rates and compare them with observations and previous theoretical studies of this nature.

10:10 – 10:25
**PETER WYPER and R. Jain**

The effects of current sheet asymmetry on 3D null reconnection

Asymmetric current sheets are prevalent in both astrophysical and laboratory plasmas with complex three dimensional (3D) magnetic topologies. In 3D, reconnection rate is measured by $\left| \int E \cdot dl \right|_{\text{max}}$ along all field lines threading the non-ideal region. Therefore, current sheet dimensional asymmetry likely plays an important role in the manner and rate of 3D reconnection. This work studies the importance of current sheet asymmetry for the fan and spine modes of 3D null point reconnection. We find that asymmetric fan reconnection is characterised by an asymmetric reconnection of flux past each spine line and a bulk flow of plasma through the null point. In contrast, asymmetric spine reconnection is inherently equal and opposite in how flux is reconnected across the fan plane. The rich nature of the spine solutions necessitates in some circumstances a dual definition for the reconnection rate: a local rate quantifying the “total” rate that flux crosses the fan plane and a global rate quantifying the rate an observer “sees” flux transferred far from the null. Both models suggest that current sheet asymmetry has a profound effect on 3D null reconnection.

10:25 – 10:40
**CHRISTINA DAVIES**

Short wavelength magnetic buoyancy instability

We consider the magnetic buoyancy instability in the short-wavelength limit of Gilman (1970). In this limit the perturbation equations (a system of coupled ODEs) can be reduced to a single algebraic dispersion relation, with coefficients depending on height. Put otherwise it seems that, in this limit, a problem that would have been treated as an eigenvalue problem requiring a set of boundary conditions can be reduced to a single equation for which the boundary conditions are unimportant. Here I present asymptotics and numerical calculations to illustrate the link between the two systems, which can be viewed as being analogous to the more familiar problem of the quantum harmonic oscillator.

Morning Coffee Break

10:40 – 11:10

Session 5: Solar Applications III

Chairperson: Paul Bushby

11:10 – 11:25
**INEKE DE MOORTEL, D.J. Pascoe, A.W. Hood, A.N. Wright, M.S. Ruderman and J. Terradas**

Wave damping due to mode coupling in Solar coronal loops

Recent observations of the loops in the Sun’s atmosphere reveal ubiquitous transverse velocity perturbations that undergo strong damping as they propagate. Using numerical (and analytical) modelling, we demonstrate that these can be understood in terms of coupling of different wave modes in the inhomogeneous boundaries of the loops: we perform 3D numerical MHD simulations of footpoint-driven transverse waves propagating in a coronal plasma with a cylindrical density structure. Mode coupling in the inhomogeneous boundary layers of the loops leads to the coupling of the transversal (kink) mode to the azimuthal (Alfven) mode, observed as the decay of the transverse kink oscillations. Both the numerical and analytical results show that the initial damping is Gaussian in nature, before tending to linear...
exponential damping at large heights. To use the observed waves as a seismological tool, we propose a general spatial damping profile (based on the analytical results) that accounts for the initial Gaussian stage of damped kink waves as well as the asymptotic exponential stage. The applicability of this profile for coronal seismology purposes is demonstrated by a full numerical parametric study of the relevant physical parameters.

11:25 – 11:40

DAVID TSIKLAURI

3D, linearly polarised, Alfven wave dissipation in Arnold-Beltrami-Childress magnetic field

Previous studies (e.g. Malara et al ApJ, 533, 523 (2000)) considered small-amplitude Alfvenic wave (AW) packets in WKB approximation. They draw a distinction between 2D AW dissipation via phase mixing and 3D AW dissipation via exponentially divergent magnetic field lines. In the former case AW dissipation time scales as $S^{1/3}$ and in the latter as $\log(S)$, where $S$ is the Lundquist number. In this work, using MHD numerical simulation and Arnold-Beltrami-Childress magnetic field, we verify the $\log(S)$ scaling via direct MHD numerical simulation for the first time. Implications for the MHD wave heating of solar corona are also discussed.

11:40 – 11:55

SAMPLUEL HUNTER, D. Hughes and S. Griffiths

Waves in shallow water MHD

Models of the solar dynamo predict the presence of a strong magnetic field permeating this structure. Effects of fluid motion in the tachocline occur on all time-scales: from the short, where perturbations from convective overshooting displace fluid; through the medium, in which the sun's 11-year cycle takes place; to the long, where solar structure is determined. Here we adopt the shallow water MHD equations [Gilman, 2000], which are an extension of the well-studied hydrodynamic shallow water system to incorporate the extra effects of magnetised fluid. Waves are a fundamental property of fluid systems, and such shallow astrophysical layers almost certainly exhibit wave-like behaviour. Two and three layer models have been used extensively to analyse wave motion in the ocean. Here we augment this system to describe waves in the presence of a magnetic field; magnetically modified internal and surface modes result, that are combinations of Alfven and gravity waves.

11:55 – 12:10

MICHAEL BAREFORD and A. Hood

Shock heating in the Solar corona

We explore the process by which a coronal loop can become heated in response to an ideal magnetic field instability. A three-dimensional magnetohydrodynamic Lagrangian-remap code is used to simulate the evolution of a specific line-tied field configuration, which is based on a zero-net-current cylindrical loop model. The initial loop state is known to be linearly kink unstable. In addition, the field surrounding the loop is potential (the external field is parallel to the initial loop axis). The kink instability rapidly leads to the formation of slow mode shocks within the loop interior, where the pressure and sound speed are low compared to the loop boundary. We investigate how these slow mode shocks influence the heating process. In general, slow mode shocks act to release magnetic energy in the form of currents, which then, via the Lorentz force, concentrate plasma flows, giving rise to steep velocity gradients at the loop boundary. It is this last feature that causes shock heating, which is represented in the code as an artificial viscosity. Our model also incorporates thermal conduction, radiation and gravity; thus, we can forward model our results to show the appearance of the loop within the 171Å passband used by the AIA instrument onboard the Solar Dynamics Observatory.

12:10 – 12:25

DAVID MACTAGGART

Finite deformation in ideal MHD: analytical twisted current layers

Numerical studies of magnetic relaxation take, as their initial conditions, stressed magnetic fields out of equilibrium. To produce the required initial condition, flows are often imposed on the boundaries of the computational domain to deform a simple magnetic field into the required shape. In this talk we shall look at performing such nonlinear deformations analytically. We shall examine how to deform the magnetic field and the current density and demonstrate this with the example of deforming a straight magnetic field into one that produces twisted current layers. The technique can be applied to other magnetic fields, e.g. null points, and will prove useful to modellers in setting up initial conditions for relaxation problems.

Lunch Break

12:25 – 14:00

Lunch is provided at “One A The Square”. The restaurant is located at the North-West corner of the Main University Building. Lunch vouchers are enclosed in your conference wallets.
Session 6: Models and Methods

Chairperson: Graeme Sarson

14:00 – 14:15  
**STUART MUMFORD**  
3D Simulations of MHD waves in low solar atmospheric flux tubes driven by photospheric motions  
Recent ground- and space-based observational results reveal the presence of small-scale motion between convection cells in the solar photosphere. In these regions small-scale magnetic flux tubes are generated due to the interaction of granulation motion and background magnetic field. In this talk we present numerical simulations of magnetic flux tubes in these regions driven by photospheric motion. A new method for analysing MHD wave modes in 3D simulations is presented which decomposes the velocity perturbations into three components defined as parallel to the magnetic field, perpendicular to the magnetic field and flux surface and an azimuthal component perpendicular to the field inside the surface. Allowing the accurate identification of MHD modes in complex 3D geometries. This method is implemented and applied to the numerical simulations and then used to identify excited wave modes and the energy flux of each is calculated. A comparison of the excited wave modes and the energy flux contributed by each for a uniform torsional driver, a Archimedian spiral-type and a logarithmic spiral-type drivers with each other and with vertical and horizontal drivers is presented here.

14:15 – 14:30  
**WAYNE ARTER**  
Potential vorticity formulation of compressible Magnetohydrodynamics  
Compressible ideal magnetohydrodynamics (MHD) is formulated in terms of the time evolution of potential vorticity and magnetic flux per unit mass using a compact Lie bracket notation. It is demonstrated that this simplifies analytic solution in at least one very important situation relevant to magnetic fusion experiments. Potentially important implications for analytic and numerical modelling of both laboratory and astrophysical plasmas are also discussed.

14:30 – 14:45  
**ASHLEY WILLIS**  
Optimising the dynamo  
What velocity field is 'best' for generating a magnetic field? What is the lowest possible magnetic Reynolds number for a magnetic dynamo? To answer such fundamental questions requires optimisation over the full space of permitted velocity fields. Theoretical studies have considered many parametrized velocity fields in several geometries, including Ponomorenko, Roberts, Arnold-Beltrami-Childress (ABC), and Dudley and James flows. In this work, optimisation is shown to be possible without need for the specification of a parametrized set of acceptable flows [1]. This enables a lower bound on the magnetic Reynolds number to be identified for a dynamo, here for the case of a periodic box. [1] A. P. Willis (2012), PRL, 109, 251101.

14:45 – 15:00  
**MICHAEL GRIFFITHS, V. Fedun and R. Erdélyi**  
A fast MHD code for gravitationally stratified media using multiple graphical processing units: SMAUG  
Parallelisation techniques have been exploited most successfully by the gaming/graphics industry with the adoption of graphical processing units (GPUs) possessing hundreds of processor cores. The opportunity has been recognised by the computational sciences and engineering communities who have recently harnessed extremely well the numerical performance of GPUs. For example, parallel magnetohydrodynamic (MHD) algorithms are important for numerical modelling of highly inhomogeneous solar, geophysical and astrophysical plasmas. Here we describe the implementation of the new GPU-based MHD code, the Sheffield Magnetohydrodynamics Algorithm Using GPUs (SMAUG). SMAUG is a 3D nonlinear MHD code capable of modelling magnetised and gravitationally strongly stratified plasmas. The objective of this presentation is to introduce the numerical methods used and the techniques for porting the code to this novel and highly parallel compute architecture. We show the recent development enabling the MHD code to utilise multiple GPUs and we describe the techniques implemented to enable communication between the GPUs. The methods applied are justified by the performance benchmarks and validation results demonstrating that the code successfully simulates the physics for a range of test scenarios including a full 3D realistic model of MHD wave propagation in the strongly stratified solar atmosphere.

15:00 – 15:15  
**KACPER KORNET and A. Potherat**  
A new spectral method for direct numerical simulations of magnetohydrodynamic channel flows  
Simulations of liquid metal flows in channel and duct configurations under a strong magnetic field pose a difficult problem for existing numerical methods. The main obstacle is the linear increase in number
of modes required to resolve thin Hartmann boundary layers with the intensity of the magnetic field \( B \). To overcome this problem we developed a new approach to the numerical calculations describing these flows. The solution of the flow is expressed in a base of eigenfunctions of the linear part of the governing equations and its adjoint. We show that in this approach the computational cost does not depend on the thickness of boundary layer and therefore it allows for performing calculations for high magnetic fields.

### End of Meeting Coffee Break

15:15 – 16:00

### School of Mathematics and Statistics Colloquium

16:00 – 17:00  
**Prof Herbert Huppert, FRS**  
Fluid modelling of carbon dioxide sequestration  
*Note: This is a School event. UK MHD Participants are most welcome to attend. The Colloquium takes place in the lecture theatre next door - Maths 516.*

Current global anthropogenic emissions of carbon dioxide are approximately 32 Gigatonnes annually. The influence of this green-house gas on climate has raised concern. A means of reducing environmental damage is to store carbon dioxide somewhere until well past the end of the fossil fuel era. Storage by injection of liquid, or supercritical, carbon dioxide into porous reservoir rocks, such as depleted oil and gas fields and regional saline aquifers, is being considered. The presentation will discuss the rate and form of propagation to be expected and quantify some of the risks involved. The talk builds on theoretical and experimental investigations of input of liquid of one viscosity and density from a point source above an impermeable boundary, either horizontal or slanted, into a heterogeneous porous medium saturated with liquid of different viscosity and density. In the Sleipner natural gas field, carbon dioxide has been injected at a rate of 1 Mt/yr since 1996. We will briefly show how to apply our results to interpret these field observations. One of the best controlled field experiments, the Otway Project, commenced on 2 April 2008 in Victoria, Australia. Approximately sixty thousand tonnes of carbon dioxide was injected into a slanted sill over a period of just over a year. We will show how accurately some of our theoretical models predict the field data obtained so far. The talk will be illustrated by colour movie sequences of laboratory experiments and some simple desk-top demonstrations of aspects of flow of multi-phase fluids into a porous ambient.

--- Posters ---

Posters will remain on display in the Maths and Stats Common Room on both days.

**Poster**  
**Homayon Aryan, M.A. Balikhin, A. Taktakishvili and T.L. Zhang**  
CME associated shocks  
The interaction of CMEs with the solar wind can lead to the formation of interplanetary shocks. Ions accelerated at these shocks contribute to the solar energetic protons observed in the vicinity of the Earth. Recently a joint analysis of Venus Express (VEX) and STEREO data by C. T. Russell and co-authors have shown that the formation of strong shocks associated with Co-rotating Interaction Regions (CIRs) takes place between the orbits of Venus and the Earth as a result of coalescence of weaker shocks formed earlier. The present study uses VEX and Advanced Composition Explorer (ACE) data in order to analyse shocks associated with CMEs that occurred on 29th and 30th of July 2007 during the solar wind conjunction period between Venus and the Earth. For these particular cases it is shown that the above scenario of shock formation proposed for CIRs also takes place for CMEs. Contradiction with shock formation resulting from MHD modelling is explained by inability of classical MHD to account for the role of wave dispersion in the formation of the shock.

**Poster**  
**Jonathan Hodgson**  
A formalism for computing symmetric MHD equilibria with anisotropic pressure  
We present a general formalism that allows the calculation of MHD equilibria with translational or rotational symmetry and anisotropic pressure. The pressure tensor is assumed to be of CGL form, and external forces such as gravity or centrifugal forces are included. If field-aligned currents are assumed to vanish, the fundamental equation is of Grad-Shafranov type. The inclusion of field-aligned currents results in a more complex mathematical problem with no obvious analogy in the theory of MHD equilibrium with isotropic pressure. Magnetospheres of giant, fast rotating planets could be a possible application of this theoretical formalism.
**Poster**

**Agnieszka Hudoba, J. Priede, S. Aleksandrova and S. Molokov**

Instabilities in the buoyant convective flows subject to high magnetic fields

The project encapsulates the study of instabilities in the buoyant convective flows in the presence of high magnetic fields. The study is of particular importance as the phenomenon plays a fundamental role in such industrial applications as fusion reactor blankets, semiconductor crystal growth or electromagnetic processing of materials. Buoyant magnetohydroconvection has been studied numerically involving spectral methods. The linear stability results show that the instability critically depends on the electrical and thermal boundary conditions, and on the Prandtl and Hartmann numbers. The problem is first solved in the inductionless approximation, where the flow cannot disturb the imposed magnetic field. Then the higher magnetic Reynolds number is considered in order to study the magnetic field evolution and possible Alfvénic oscillations.

**Poster**

**David Skinner**

Double-diffusive magnetic buoyancy instability in a quasi-two-dimensional Cartesian geometry

Magnetic buoyancy, believed to occur in the solar tachocline, is both an important part of large-scale solar dynamo models and the picture of how sunspots are formed. Given that in the tachocline region the ratio of magnetic diffusivity to thermal diffusivity is small it is important, for both the dynamo and sunspot formation pictures, to understand magnetic buoyancy in this regime. Furthermore, the tachocline is a region of strong shear and such investigations must involve structures that become buoyant in the double-diffusive regime which are generated entirely from a shear flow. Silvers et al. (2009) have illustrated that shear-generated double-diffusive magnetic buoyancy instability is possible in the tachocline. However, this study was severely limited due to the computational requirements of running three dimensional magnetohydrodynamics simulations over diffusive timescales. A more comprehensive investigation is required to fully understand the double-diffusive magnetic buoyancy instability and its dependency on a number of key parameters; such an investigation requires the consideration of a reduced model. Here we consider a quasi-two dimensional model where all gradients in the direction are set to zero. We show how the instability is sensitive to changes in the thermal diffusivity and also show how different initial configurations of the forced shear flow affect the behaviour of the instability. Finally, we conclude that if the tachocline is thinner than currently stated then the double-diffusive magnetic buoyancy instability can more easily occur.

**Poster**

**Christopher Davies, L. Silva and J. Mound**

Modelling the influence of a translating inner core on outer core convection

It has recently been proposed that the hemispheric seismic structure of the inner core can be explained by a self-sustained rigid-body translation of the inner core material, resulting in melting of the solid at the leading face and a compensating crystallisation at the trailing face. This process induces a hemispherical variation in the release of light elements and latent heat at the inner-core boundary, the two main sources of thermochemical buoyancy thought to drive convection in the outer core, suggesting a possible influence of inner core translation on outer core dynamics. The outer core is also likely subjected to lateral variations in heat-flux imposed from above by the convecting mantle. We use numerical models of nonmagnetic thermal convection in a rotating spherical shell to investigate possible long-term effects of inner core translation and mantle convection on the outer core.

**Poster**

**Jonathan Hagan**

Two-dimensional nonlinear travelling waves in MHD channel flow

The present study is concerned with the stability of a flow of viscous conducting liquid driven by a pressure gradient between two parallel walls in the presence of a transverse magnetic field. This magnetohydrodynamic counterpart of the classic plane Poiseuille flow is generally known as the Hartmann flow. Although the magnetic field has a strong stabilizing effect, the turbulence is known to set in this flow similarly to its hydrodynamic counterpart well below the threshold predicted by the linear stability theory. Such a nonlinear transition to turbulence is thought to be mediated by unstable equilibrium flow states which may exist in addition to the base flow. In the plane Poiseuille flow, such states are known to bifurcate from the base flow as 2D travelling waves that appear due to the subcritical instability at the critical Reynolds number $Re_c = 5772$ and exist down to $Re_2 = 2939$. The aim of the present study is to investigate how these subcritical states are affected by the magnetic field. We find that in the Hartmann flow, 2D nonlinear travelling waves exist starting from the linear stability threshold $Re_c \approx 4.83 \times 10^4 Ha$ down to $Re_2 \approx 6.5 \times 10^3 Ha$, where $Ha$ is the Hartmann number. Note that $Re_2$ is still by an order of magnitude greater than the experimentally observed value for the onset of turbulence in this flow. Three-dimensional travelling waves bifurcating either from 2D ones or infinity are expected to extend beyond $Re_2$ and give a more adequate prediction for the onset of turbulence.
Withdrawn Contributions

A small number of contributions were withdrawn for valid reasons. Their details are included below.

Withdrawn talk  
Aditi Sood and E. J. Kim  
Dynamic model of dynamo (magnetic activity) and rotation

A dynamic model of dynamo and rotation is investigated to understand the observational data of the dependence of the magnetic activities and the differential rotation $\Delta \Omega$ on the rotation rate $\Omega$. Specifically, we propose a minimal seventh order non-linear dynamical system for magnetic fields and differential rotation $\Delta \Omega$ by parameterizing the generation and destruction of magnetic fields by $\alpha - \Omega$ effect and magnetic flux loss from stars and by including quenching of $\alpha$ effect and differential rotation $\Delta \Omega$ due to Lorentz force. By examining different forms of alpha quenching and flux loss, we study how the strength and frequency $\omega$ of magnetic fields, and the differential rotation $\Delta \Omega$ change with the rotation rate $\Omega$ through dynamo number. In particular, among the three cases with (i) $\alpha$-quenching and no flux loss (ii) flux loss and no $\alpha$-quenching; (iii) $\alpha$-quenching and flux loss, our results show that the best agreement with observations is obtained in the case (iii) with equal amount of alpha quenching and poloidal and toroidal magnetic flux losses with quadratic nonlinear dependence on $|B|$. Specifically, in this case, the frequency spectrum of magnetic field has a well-localized frequency of the maximum intensity which scales as $\omega \propto \Omega^{0.80}$, in agreement with observation of Noyes et al. (1984), while magnetic field and mean differential rotation exhibit the tendency of saturation for high rotation. Implication of our results in mean of necessary dynamic balance is discussed. Detailed investigations leads us to conclusions that dynamic balance between $\alpha$ source term and destruction of magnetic field as well as various transport coefficients is necessary for a stellar dynamo to operate near marginal stability which could be result of self organizing process.

Withdrawn talk  
Grace Cox P. Livermore and J. Mound  
Forward models of torsional waves: dispersion and geometric effects

Alfvén waves are a set of transverse waves that propagate in an electrically conducting fluid in the presence of an ambient magnetic field. Studies of such waves in the Earth’s interior are important because inverse methods allow us to make inferences about the structure and physical properties of the core that would otherwise remain inaccessible. We produce 1D forward models of cylindrical torsional Alfvén waves in the Earth’s core, also known as torsional oscillations, and study their evolution in a full sphere and an equatorially symmetric spherical shell. Here we find that travelling torsional waves undergo significant geometric dispersion that increases with successive reflections from the boundaries such that an initial wave pulse becomes unidentifiable within three transits of the core. We investigate the relationship between geometric dispersion and wavelength, concluding that long wavelength features are more dispersive than short wavelength features. This result is particularly important because torsional waves that have been inferred in the Earth’s core from secular variation are relatively long wavelength, and are therefore likely to undergo significant dispersion within the core. When stress free boundary conditions on angular velocity are applied, waves are reflected at the equator of the core-mantle boundary ($s = r_e$) with the same sign as the incident wave. Waves that pass through the rotation axis undergo a pseudo-reflection and display more complicated behaviour due to a phase shift. In an equatorially symmetric shell, we identify a weak reflection at the tangent cylinder due to geometric effects.

List of Participants

1. Michael Proctor, University of Cambridge  
2. Andrew M. Soward, University of Exeter  
3. Radostin Simitev, University of Glasgow  
4. Andrew Baggaley, University of Glasgow  
5. David Fearn, University of Glasgow  
6. J B Taylor, Radwin, Wallingford  
7. Lintao Zhang, Coventry University  
8. Kelig Aujogue, Coventry University  
9. Kacper Kornet, Coventry University  
10. Ivan Pakhotin, University of Sheffield  
11. Homayon Aryan, University of Sheffield  
12. Peter Wyper, University of Sheffield  
13. Dali Kong, University of Exeter  
14. Keke Zhang, University of Exeter  
15. Andrew Fletcher, Newcastle University  
16. Mitchell Berger, University of Exeter  
17. Wayne Arter, CCFE, Abingdon  
18. Samuel Hunter, Univesity of Leeds  
19. Michael Griffiths, University of Sheffield  
20. Michael Bareford, University of St Andrews  
21. Phil Livermore, University of Leeds  
22. David MacTaggart, University of Abertay Dundee  
23. David Tsiklauri, Queen Mary University of London  
24. Stuart Mumford, University of Sheffield  
25. Lara Silvers, City University London  
26. William Brown, University of Leeds  
27. David Hughes, University of Leeds  
28. Christina Davies, University of Leeds
29. Frederick Gent, University of Sheffield
30. Robert Teed, University of Leeds
31. Paul Bushby, Newcastle University
32. Moritz Linkmann, University of Edinburg
33. Lee Braiden, Coventry University
34. Grace Cox, University of Leeds
35. Laura Burgess, University of Leeds
36. Jonathan Hodgson, University of St Andrews
37. Wieland Dietrich, University of Leeds
38. Chris Jones, University of Leeds
39. David Skinner, City University London
40. Thomas Neukirch, University of St Andrews
41. Agnieszka Hudoba, Coventry University
42. Jack Wood, University of Leeds
43. Andrew Gascoyne, University of Sheffield
44. Ineke De Moortel, University of St Andrews
45. Sam Durston, University of Exeter
46. Luis Silva, University of Glasgow
47. Benjamin Favier, University of Cambridge
48. Christopher Davies, University of Leeds
49. Graeme Sarson, Newcastle University
50. Anvar Shukurov, Newcastle University
51. David Pontin, University of Dundee
52. Jonathan Hagan, Coventry University
53. Eduard Kontar, University of Glasgow
54. Nic Bian, University of Glasgow
55. Natasha Jeffrey, University of Glasgow
56. Philip McGavin, University of Dundee
57. Iain Hannah, University of Glasgow
58. Hugh Hudson, University of Glasgow
59. Ashley Willis, University of Sheffield
60. Scott Richardson, University of Dundee
61. Alasdair Wilson, University of Glasgow
62. Heather Ratcliffe, University of Glasgow