Can four neutrinos explain global oscillation data including LSND & cosmology?∗

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Abstract

We present an analysis of the global neutrino oscillation data in terms of four-neutrino mass schemes. We find that the strong preference of oscillations into active neutrinos implied by solar+KamLAND as well as atmospheric neutrino data allows to rule out (2+2) mass schemes, whereas (3+1) schemes are strongly disfavoured by short-baseline experiments. In addition, we perform an analysis using recent data from cosmology, including CMB data from WMAP and data from 2dFGRS large scale structure surveys. These data lead to further restrictions of the allowed regions for the (3+1) mass scheme.

1 Introduction

The neutrino oscillation interpretations of the solar\cite{1, 2} and KamLAND\cite{3} neutrino experiments, atmospheric\cite{4, 5} neutrino data, and the LSND experiment\cite{6} require three neutrino mass-squared differences of different orders of magnitude\cite{7}. Since it is not possible to obtain this within the Standard Model framework of three active neutrinos it has been proposed to introduce a light sterile neutrino\cite{8} to reconcile all the experimental hints for neutrino oscillations. Here we present an analysis of the global neutrino oscillation data in terms of four-neutrino mass schemes, including data from solar, KamLAND and atmospheric neutrino experiments, the LSND experiment, as well as data from short-baseline (SBL) experiments\cite{9, 10, 11} and reactor experiments\cite{12} reporting no evidence for oscillations. We find that for all possible types of four-neutrino schemes different sub-sets of the data are in serious disagreement and hence, four-neutrino oscillations do not provide a satisfactory description of the global oscillation data including LSND. The details of our calculations can be found in Refs.\cite{13, 14, 15}.

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2 Notations and approximations

Four-neutrino mass schemes are usually divided into the two classes (3+1) and (2+2), as illustrated in Fig. 1. We note that (3+1) mass spectra include the three-active neutrino scenario as limiting case. In this case solar and atmospheric neutrino oscillations are explained by active neutrino oscillations, with mass-squared differences $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$, and the fourth neutrino state gets completely decoupled. We will refer to such limiting scenario as (3+0). In contrast, the (2+2) spectrum is intrinsically different, as there must be a significant contribution of the sterile neutrino either in solar or in atmospheric neutrino oscillations or in both.

Neglecting CP violation, in general neutrino oscillations in four-neutrino schemes are described by 9 parameters: 3 mass-squared differences and 6 mixing angles in the unitary lepton mixing matrix. Here we use a parameterisation introduced in Ref. [13], which is based on physically relevant quantities: the 6 parameters $\Delta m^2_{\text{sol}}, \theta_{\text{sol}}, \Delta m^2_{\text{atm}}, \theta_{\text{atm}}, \Delta m^2_{\text{lsnd}}, \theta_{\text{lsnd}}$ are similar to the two-neutrino mass-squared differences and mixing angles and are directly related to the oscillations in solar, atmospheric and the LSND experiments. For the remaining 3 parameters we use $\eta_s, \eta_e$ and $d_\mu$. Here, $\eta_s (\eta_e)$ is the fraction of $\nu_s (\nu_e)$ participating in solar oscillations, and $(1 - d_\mu)$ is the fraction of $\nu_\mu$ participating in oscillations with $\Delta m^2_{\text{atm}}$ (for exact definitions see Ref. [13]). For the analysis we adopt the following approximations: (1) We make use of the hierarchy $\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}} \ll \Delta m^2_{\text{lsnd}}$. This means that for each data set we consider only one mass-squared difference, the other two are set either to zero or to infinity. (2) In the analyses of solar and atmospheric data (but not for SBL data) we set $\eta_e = 1$, which is justified because of strong constraints from reactor experiments [11, 12].

Due to these approximations the parameter structure of the four-neutrino analysis gets rather simple. The parameter dependence of the four data sets solar, atmospheric, LSND and NEV is illustrated in Fig. 2. The NEV data set contains the experiments KARMEN [9], CDHS [10], Bugey [11] and CHOOZ/Palo Verde [12], reporting no evidence for oscillations. We see that only $\eta_s$ links solar and atmospheric data and $d_\mu$ links atmospheric and NEV data. LSND and NEV data are coupled by $\Delta m^2_{\text{lsnd}}$ and $\theta_{\text{lsnd}}$. 

Figure 1: The six four-neutrino mass spectra, divided into the classes (3+1) and (2+2).
Figure 2: Parameter dependence of the different data sets in our parameterisation.

3 (2+2): ruled out by solar and atmospheric data

The strong preference of oscillations into active neutrinos in solar and atmospheric oscillations leads to a direct conflict in (2+2) oscillation schemes. We will now show that thanks to the latest solar neutrino data (especially from SNO) in combination with the KamLAND experiment, and the improved SK statistic on atmospheric neutrinos the tension in the data has become so strong that (2+2) oscillation schemes are essentially ruled out.\footnote{Details of our analyses of the solar, KamLAND and atmospheric neutrino data can be found in Refs. [15, 16]. For an earlier four-neutrino analysis of solar and atmospheric data see Ref. [17].}

The thin lines in the left panel of Fig. 3 show the $\Delta \chi^2$ of latest solar data as a function of $\eta_s$, the parameter describing the fraction of the sterile neutrino participating in solar neutrino oscillations. The 99% CL bounds $\eta_s \leq 0.44$, when the SSM constraint on the $^8$B-flux is included, and $\eta_s \leq 0.61$ for free $^8$B-flux reflect the strong preference for active neutrino oscillations of solar data. Recently, the outstanding results of the KamLAND reactor experiment confirmed the LMA solution of the solar neutrino problem\cite{16, 18}. Apart from this very important result the sensitivity of KamLAND to an admixture of a sterile neutrino is rather limited (see thin lines in Fig. 3). The combined analysis leads to the 99% CL bound $\eta_s \leq 0.5$ for free $^8$B flux, whereas in the SSM constrained case the bound is unchanged.

In contrast, in (2+2) schemes atmospheric data imply $\eta_s \geq 0.65$ at 99% CL, in clear disagreement with the bound from solar data. In the right panel of Fig. 3 we show the $\chi^2$ for solar data and for atmospheric combined with SBL data as a function of $\eta_s$. Furthermore, we show the $\chi^2$ of the global data defined by

$$\chi^2(\eta_s) \equiv \Delta \chi^2_{\text{SOL}}(\eta_s) + \Delta \chi^2_{\text{ATM+SBL}}(\eta_s).$$

From the figure we find that only if we take both data sets at the 99.95% CL a value of $\eta_s$ exists, which is contained in the allowed regions of both sets. This follows from the $\chi^2$-value $\chi^2_{\text{PC}} = 12.2$ shown in the figure. In Refs. [14, 19] we have proposed a statistical method to evaluate the disagreement of different data sets in global analyses. The parameter goodness of fit (PG) makes use of the $\bar{\chi}^2$ defined in Eq. (1). This criterion evaluates the GOF of the combination of data sets, without being diluted by a large number of data points, as it happens for the usual GOF criterion (for details see Ref. [19]). We find $\chi^2_{\text{PG}} \equiv \chi^2_{\text{min}} = 23.5$, leading to the marginal PG of $1.3 \times 10^{-6}$. We conclude that
Figure 3: Left: $\Delta \chi^2$ from solar and KamLAND data as a function of $\eta_s$. Right: $\Delta \chi^2_{\text{sol}}$, $\Delta \chi^2_{\text{atm+sbl}}$ and $\chi^2_{\text{global}}$ as a function of $\eta_s$ in (2+2) oscillation schemes.

(2+2) oscillation schemes are highly disfavoured by the disagreement between the latest solar and atmospheric neutrino data. This is a very robust result, independent of whether LSND is confirmed or disproved.²

4 (3+1): strongly disfavoured by SBL data

It is known for a long time[21, 22] that (3+1) mass schemes are disfavoured by the comparison of SBL disappearance data[10, 11] with the LSND result. In Ref. [23] we have calculated an upper bound on the LSND oscillation amplitude $\sin^2 2\theta_{\text{LSND}}$ resulting from NEV and atmospheric neutrino data. From Fig. 4 we see that this bound is incompatible with the signal observed in LSND at the 95% CL. Only marginal overlap regions exist between the bound and global LSND data if both are taken at 99% CL. An analysis in terms of the parameter goodness of fit[14] shows that for most values of $\Delta m^2_{\text{LSND}}$ NEV and atmospheric data are compatible with LSND only at more than 3$\sigma$, with one exception around $\Delta m^2_{\text{LSND}} \sim 6$ eV², where the PG reaches 1%. These results show that (3+1) schemes are strongly disfavoured by SBL disappearance data.

5 Comparing (3+1), (2+2) and (3+0) hypotheses

With the methods developed in Ref. [13] we are able to perform a global fit to the oscillation data in the four-neutrino framework. This approach allows to statistically compare the different hypotheses. Let us first evaluate the GOF of (3+1) and (2+2) spectra with the help of the PG method described in Ref. [19]. We divide the global oscillation data

²Sub-leading effects beyond the approximations adopted here should not affect this result significantly. Allowing for additional parameters to vary might change the ratio of some observables; however, we expect that the absolute number of events relevant for the fit will not change substantially.
Figure 4: Upper bound on $\sin^2 2\theta_{\text{LSND}}$ from NEV and atmospheric neutrino data in (3+1) schemes compared to the allowed region from global LSND data and decay-at-rest (DAR) LSND data.

Table 1: Parameter GOF and the contributions of different data sets to $\chi^2_{\text{PG}}$ in (3+1) and (2+2) neutrino mass schemes.

| Scheme | SOL | ATM | LSND | NEV | $\chi^2_{\text{PG}}$ | PG          |
|--------|-----|-----|------|-----|-----------------|-------------|
| (3+1)  | 0.0 | 0.4 | 7.2  | 7.0 | 14.6            | $5.6 \times 10^{-3}$ |
| (2+2)  | 14.8| 6.7 | 2.2  | 9.7 | 32.4            | $1.6 \times 10^{-6}$ |

The parameter goodness of fit is now obtained by evaluating $\chi^2_{\text{PG}}$ for 4 DOF. This number of degrees of freedom corresponds to the 4 parameters $\eta_s, d_\mu, \theta_{\text{LSND}}, \Delta m^2_{\text{LSND}}$ describing the coupling of the different data sets (see Eq. (2) and Fig. 2). The best GOF is obtained in the (3+1) case. However, even in this best case the PG is only 0.56%.
PG of $1.6 \times 10^{-6}$ for (2+2) schemes shows that these mass schemes are essentially ruled out by the disagreement between the individual data sets.

Although we have seen that none of the four-neutrino mass schemes can provide a reasonable good fit to the global oscillation data including LSND, it might be interesting to consider the relative status of the three hypotheses (3+1), (2+2) and the three-active neutrino scenario (3+0). This can be done by comparing the $\chi^2$ values of the best fit point (which is in the (3+1) scheme) with the one corresponding to (2+2) and (3+0). First we observe that (2+2) schemes are strongly disfavoured with respect to (3+1) with a $\Delta \chi^2 = 17.8$, implying a CL of 99.87% for 4 DOF. Further we find that (2+2) is only slightly better than (3+0), which is disfavoured with a $\Delta \chi^2 = 20.0$ with respect to (3+1).

6 Adding information from cosmology

Recently a precision measurements of the cosmic microwave background (CMB) radiation has been published by the WMAP collaboration[25]. Under certain assumptions about the cosmological model and in combination with other cosmological data a stringent upper bound on the sum of the neutrino masses $\sum m_\nu < 0.7$ eV is obtained[25, 26]. Since this bound is of the same order as $\sqrt{\Delta m^2_{\text{LSND}}}$ it will lead to further restrictions in four-neutrino mass schemes[27]. To combine this information with data from oscillation experiments we use the results of an analysis performed by S. Hannestad[28]. There, WMAP[25] results have been combined with other CMB data[29], data from the large scale structure survey 2dFGRS[30], the HST Hubble key project[31], and Type Ia supernova observations[32]. It has been found that a non-zero $\sum m_\nu$ can be compensated partially by an increase in the effective number of neutrino species. Therefore, the original bound obtained in Ref. [25], which was calculated assuming three neutrinos, cannot be applied in a straightforward way. In the case of four neutrinos a somewhat weaker bound of $\sum m_\nu < 1.38$ eV (95% CL) is obtained[28]. To combine this result with data from oscillation experiments we use the $\chi^2$ from cosmology for four neutrinos, shown in Fig. 6 of Ref. 28.

In the following analysis we consider (3+1) schemes of the hierarchical type (first two spectra in Fig. 11) and neglect the masses of the three lower mass states, such that $\sum m_\nu \approx \sqrt{\Delta m^2_{\text{LSND}}}$. The $\Delta \chi^2_{\text{cosmology}}$ from Ref. 28 under this assumption is shown in the right panel of Fig. 5 as a function of $\Delta m^2_{\text{LSND}}$. From the left panel of this figure one finds that cosmology excludes the region $\Delta m^2_{\text{LSND}} > 3$ eV$^2$. However, the islands at $\Delta m^2_{\text{LSND}} \sim 0.91$ eV$^2$ and 1.7 eV$^2$ are practically unaffected by cosmological data. Performing a GOF analysis shows that adding cosmological data gives $\chi^2_{\text{PG}} = 17.1$ and 19.6 for the two islands. This should be evaluated for 5 DOF[19], leading to a PG of 0.43% and 0.15%, respectively, which is comparable to the value given for (3+1) in Tab. 1. We note, however, that a modest improvement of the bound on $\sum m_\nu$ by future cosmological data will drastically worsen the fit of the last remaining islands in the four-neutrino parameter space.

A further cosmological constraint on four-neutrino schemes comes from Big Bang nucleosynthesis (BBN)[22, 33]. In the absence of any non-standard mechanism, like a large lepton number asymmetry, (3+1) as well as (2+2) schemes will lead to an effective number of neutrino species $N_\nu = 4.34$. This is in conflict with the upper bound on $N_\nu$

\[3\text{Note that all other four neutrino schemes will be much more disfavoured, since more mass states will contribute at the LSND mass scale.}\]
Figure 5: Left: Allowed regions at 90% and 99% CL for (3+1) schemes without (solid and dashed lines) and including data from cosmology (coloured regions). The grey region is the 99% CL region of LSND[6]. Right: $\Delta \chi^2$ from cosmological data[28] as a function of $\Delta m^2_{LSND}$.

derived in recent analyses[28,35,36] including CMB and BBN data. Taking into account the uncertainty related to the primordial abundances of He and D, upper bounds in the range $N_\nu < 3.1 - 3.3 \ (2\sigma)$[36] are obtained.

7 Conclusions

Performing a global analysis of current neutrino oscillation data we find that four-neutrino schemes do not provide a satisfactory fit to the data:

- The strong rejection of non-active oscillation in the solar+KamLAND and atmospheric neutrino data rules out (2+2) schemes, independent of whether LSND is confirmed or not. Using an improved goodness of fit method especially sensitive to the combination of data sets we obtain a GOF of only $1.6 \times 10^{-6}$ for (2+2) schemes.

- (3+1) spectra are disfavoured by the disagreement of LSND with short-baseline disappearance data, leading to a marginal GOF of $5.6 \times 10^{-3}$. If LSND should be confirmed we need more data on $\nu_e$ and/or $\nu_\mu$ SBL disappearance to decide about the status of (3+1).

- Recent cosmological data lead to further restrictions of the allowed parameter space. Only two islands around $\Delta m^2_{LSND} \sim 0.91 \ eV^2$ and $1.7 \ eV^2$ in hierarchical (3+1) schemes survive.

Our analysis brings the LSND hint to a more puzzling status, and the situation will become even more puzzling if LSND should be confirmed by the up-coming MiniBooNE experiment[37]. Recently an explanation in terms of five neutrinos has been proposed[38]. In such a (3+2) scheme solar and atmospheric data are explained dominantly by active oscillations, similar to the (3+1) case. The fit of SBL data is improved with respect to (3+1) by involving two large mass splittings $\Delta m^2_{41} \sim 0.9 \ eV^2$ and $\Delta m^2_{51} \sim 20 \ eV^2$. Alternative explanations of LSND by anomalous muon decay[39] seem to be in disagreement with the negative result of KARMEN, or require a very radical modification of standard
physics, like CPT violation\cite{40}.

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