Scaling of Black Hole Accretion Discs: From Gamma-Ray Bursts and Black Hole X-Ray Binaries to Active Galactic Nuclei

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Received __________; accepted __________
ABSTRACT

I consider how physical processes scale over eight orders of magnitude in black hole mass, from stellar masses in gamma-ray bursts (GRB) and black-hole X-ray binaries (BHXRB) to supermassive active galactic nuclei (AGN). Accretion rates onto stellar mass black holes range over more than sixteen orders of magnitude, from the lower luminosity BHXRB to GRB. These enormous parameter ranges correspond to qualitative as well as quantitative differences in behavior. The fundamental questions involve the balance between nonequilibrium and thermalized plasmas. When energy fluxes exceed a critical value $\sim 10^{29}$ erg/cm$^2$/s, as in GRB, a black-body equilibrium pair plasma forms. At the lower fluxes found in AGN, BHXRB and microquasars, accretion power electrodynamically accelerates a small number of very energetic particles, explaining their non-thermal spectra and the high energy gamma-ray emission of blazars. Particle acceleration, (particularly of leptons because of the energy dependence of the Klein-Nishina cross-section) is limited by the flux of soft thermal photons, which is necessarily model-dependent. This prohibits the formulation of general scaling laws. Ultra-high energy cosmic rays may be accelerated by massive black holes, otherwise undetectable, with very low thermal luminosities. New-born fast high-field pulsars may be in the black-body equilibrium regime, resembling SGR in permanent outburst. I also consider the question, significant for the acceleration of nonthermal particles in GRB outflows, of whether collisionless plasmas interpenetrate rather than forming hydrodynamic shocks, and propose this as an alternative to internal shock models of GRB. A new appendix attempts to explain why AGN are, proportionally, more efficient accelerators of energetic particles than stellar mass black holes.
1. Introduction

Hundreds of models of active galactic nuclei (AGN) and quasars have been published since the pioneering work of Salpeter (1964), yet none of them is completely satisfactory and generally accepted. This paper\(^1\) investigates the general scaling laws that connect the very different parameter regimes appropriate to AGN and to BHXRB, including microquasars.

GRB, AGN and BHXRB have certain qualitative similarities, despite enormous quantitative differences in their luminosities, masses, and time scales. They all are powered by accretion onto a central black hole. They all produce nonthermal radiation. GRB and AGN and some BHXRB show evidence for relativistic bulk motion. They all fluctuate irregularly in intensity. AGN and some BHXRB (microquasars) are directly observed to produce narrow jets, while energetic arguments and modeling of their afterglows by Frail, et al. (2001) indicate that GRB are also strongly collimated.

These similarities suggest that it may be useful to investigate how a common accretion disc model may manifest itself differently in these objects. The present discussion builds upon the Blandford (1976); Lovelace (1976) model for AGN, although these authors assumed (probably unnecessarily, except to make the problem tractable) that the magnetic dipole moment is aligned with a rotational axis. These models may be scaled to stellar mass BHXRB such as Cyg X-1 and the microquasars.

A single parameter divides all accretion discs into two classes. The fundamental difference between these classes of models is that in GRB and in SGR in outburst (and probably also in young fast high-field pulsars, as yet unobserved) the relativistic wind is thermalized to an equilibrium pair plasma, while at the lower power densities of AGN,

\(^1\)The present paper is the “forthcoming work” cited as Katz (1997b) by Katz (1997).
BHXRB and observed radio pulsars the wind remains transparent and very far from thermodynamic equilibrium with the radiation field, as a comparatively few particles are accelerated but to high energies. The condition for thermalization Katz (1982); Katz, Toole & Unruh (1994); Katz (1996, 1997) is that

\[ \mathcal{I} \equiv \frac{I}{I_{\text{char}}} \gtrsim 1, \]

where \( I \) is the energy flux (power emitted per unit area, whether carried by particles, energetic photons, or low frequency or even DC Poynting flux), so that \( I \) exceeds a characteristic value

\[ I_{\text{char}} \equiv \sigma_{SB} \left[ \frac{m_e c^2}{k_B \ln (6(M/M_\odot) N_{B,\odot} \alpha_G \alpha^2)} \right]^4 \sim \frac{4 \times 10^{29} \text{erg/cm}^2\text{s}}{(1 + 0.05 \ln M/M_\odot)^4}, \]

where \( \sigma_{SB} \) is the Stefan-Boltzmann constant, \( N_{B,\odot} \approx 1.2 \times 10^{57} \) is the number of baryons in a Solar mass, \( \alpha_G \equiv Gm_e^2/hc \approx 1.76 \times 10^{-45} \) is the “gravitational fine structure constant” defined for the electron mass. This corresponds to an equivalent black body temperature exceeding the characteristic value

\[ T_{\text{ch}} \approx \frac{2.9 \times 10^8 \circ K}{1 + 0.05 \ln M/M_\odot} \]

at which a black body equilibrium pair plasma is opaque to Thomson scattering over the characteristic length scale (three Schwarzschild radii) of the source. Because of the steep Boltzmann factor in the equilibrium pair density, the characteristic temperature and intensity are almost universal constants, only weakly dependent on the mass of the source, even over the range from \( 1M_\odot \) to \( 10^8M_\odot \).

The thermalization of energetic particles by scattering in regions of high energy density is auto-catalytic: scattering increases the number density of potential scatterers (by processes such as double Compton scattering, pair production, and curvature radiation). However, these processes only run away to full thermalization if \( \mathcal{I} \gg 1 \).
In these models of AGN and BHXRB the electromagnetic energy is converted to the energy of accelerated particles close to the black hole. This is in contrast to GRB in which electromagnetic energy is converted to a black-body equilibrium pair plasma (Piran, Shemi & Narayan 1993).

In another class of models of AGN and BHXRB the disc radiates vacuum electromagnetic waves instead of energetic particles. At much greater radii these waves accelerate particles, just as in GRB the pair plasma accelerates particles in distant collective interactions. These two classes of models can be comparably efficient particle accelerators. I do not consider the vacuum wave model further because it is less closely analogous to the GRB model (which cannot be a vacuum wave model because the energy density leads to creation of an equilibrium pair plasma), and because external plasma injection or pair breakdown are likely to fill the wave zone with energetic particles.

The remainder of this paper is chiefly concerned with scaling of a variety of physical processes over the very wide range of parameters encountered. It would be difficult to say anything qualitatively new about any of the models of these objects, which have been extensively developed over 45 years, but it may be useful to re-examine their scaling. To this end it is necessary to review the fundamental physical processes involved.

There are also temporal similarities among the various classes of black hole accretion discs. In one BHXRB (Cyg X-1; Weiskopf, et al. 1978) a non-zero time skewness was measured in an X-ray time series. Time skewness of the same sense is found (Nemiroff, et al. 1994) in many GRB. Searches for time skewness in AGN time series have so far been unsuccessful (Press & Rybicki 1997), but do not exclude it. In fact, it was once suggested (Piran & Shaham 1975, 1977) that Cyg X-1 might be a source of GRB. Although such Galactic stellar mass X-ray sources are now known not to be the origin of GRB, Cyg X-1 does show outbursts and flaring behavior in comparable energy
bands (Zdziarski, et al. (2002)).

2. Homopolar Generators

All of the objects we discuss contain rotating, electrically conducting, magnetized matter, and therefore are homopolar generators with potentials $V_0 \sim \int (v/c) B \, dr$. In general, such generators produce a power $P = V_0^2 / Z$, where the load impedance $Z \equiv E/H = Z_r + iZ_i$ is written as the sum of reactive ($Z_i$) and dissipative ($Z_r$) parts. A fundamental assumption of nearly all astrophysical models of such objects is that $|Z| \sim Z_0 \equiv 4\pi/c = 4.19 \times 10^{-10}$ sec/cm, the impedance of free space in c.g.s. units ($Z_0 \equiv \sqrt{\mu_0/\epsilon_0} = 377$ Ω in m.k.s. units).

The detailed or microscopic justification of $|Z| \sim Z_0$ is model-dependent. A similar result will always be found if the particle energies are large enough that particle multiplication (by processes such as double Compton scattering and collisional and radiative pair production) turn the insulating vacuum into a conducting relativistic plasma. $Z_i \sim Z_0$ is a general result obtainable from Maxwell’s equations by estimating $|\vec{\nabla}| \sim \partial/\partial (ct)$. As long as the system size exceeds $c\Delta t$, where $\Delta t$ is a characteristic (rotation or fluctuation) time scale, the reactive impedance $Z_i \sim Z_0$ will always be in series with $Z_r$.

The dissipative impedance $Z_r$ may be small in a relativistic plasma, but we hypothesize that when the fields are large particle acceleration and multiplication will generally lead to $Z_r \sim Z_0$. This hypothesis, essentially dimensional, is central to the model of this paper and is implicit in most pulsar and AGN models (Goldreich & Julian (1969); Blandford (1976); Lovelace (1976)). For example, application to pulsars gives

$$P \sim \frac{V^2}{Z} \sim \frac{c}{4\pi} (Er)^2 \sim \frac{c}{4\pi} \left(\frac{v}{c} r B\right)^2 \sim \frac{\omega^2 r^4 B^2}{4\pi c} \sim \frac{\omega^2 \mu^2}{4\pi c r^2} \sim \frac{\omega^4 \mu^2}{4\pi c^3},$$

where $\mu$ is the magnetic moment and $B \sim \mu/r^3$ is evaluated at the radius of light cylinder $r = c/\omega$. This is comparable both to the expression for magnetic dipole radiation of a
misaligned rotor in vacuum and to the result of Goldreich & Julian (1969) for an aligned rotor when the vacuum fills with a relativistic plasma. It is also unavoidable on dimensional grounds, provided that the fields are large enough to drive these processes.

In the laboratory \( Z_r \gg Z_0 \) when the vacuum or materials do not break down into a conducting plasma, as when fields and potentials are small and insulators remain insulators, and frequently \( Z_r \ll Z_0 \) when conductors are not relativistic. In relativistic astrophysics \( Z_r \sim Z_0 \) follows from assuming that the power carried to \( r > c\Delta t \) by the Poynting vector is dissipated somewhere, rather than remaining forever as electromagnetic field energy or returning (like an unphysical advanced time solution) to the source.

The first nontrivial part of this problem is to estimate \( B \), which can be done on the basis of dimensional arguments from the total disc power. The second, and most critical (but generally unsolved), part is to determine how the potential drop is distributed in space and how the dissipated power is apportioned among the accelerated particles.

### 3. Nonthermal efficiency

All three classes (AGN, BHXRB and GRB) of objects show a significant amount of nonthermal emission. The nonthermal efficiency, which may be defined as the fraction of the emitted power appearing as non-Planckian radiation or as particle distributions that are far from thermal equilibrium (typically power laws over orders of magnitude in energy) is \( O(1) \). In GRB the observed emission appears to be nonthermal, although it is not the primary radiation emitted by the central engine but rather the consequence of particles accelerated in the relativistic outflow; the high energy density and intensity at the source thermalizes the relativistic wind. Still, the wind is generally believed to be produced by a fundamentally nonthermal process, the coherent radiation of electromagnetic energy by the
fields of the central engine. In many AGN a substantial fraction of the power appears as nonthermal visible synchrotron radiation or high energy gamma-rays \cite{Krolik1999}.

The case for nonthermal X-ray emission in BHXRB is plausible but less compelling. It is a natural explanation of the complex multi-component spectra frequently observed, but it may also be possible to explain such spectra as the sum of thermal spectra emitted by matter distributed over a range of temperatures. For example, \cite{Lynden-Bell1969} gives the classic example of an apparently non-thermal power law spectrum that results from thermal emission from an accretion disc whose temperature is a power law function of radius. Only if the emission extends to very high energy, or is associated with low frequency emission of very high brightness temperature (as in radio pulsars), can the case for nonthermal processes be considered compelling.

As examples of such a compelling case, the superluminal radio components and jets present in some BHXRB (microquasars) certainly require acceleration of relativistic particles. Some, such as Cyg X-3, also show strong outbursts of nonthermal radio emission (there is no direct evidence this particular object contains a black hole, and qualitatively similar effects may be produced by accretion onto a low magnetic field neutron star), but this radio emission is only a very small fraction of their power output.

In GRB the electrodynamic efficiency $\epsilon_e$ (Equation 5 of \cite{Katz1997}) is not directly measured because virtually all the thermal radiation emerges as neutrinos (the thermal luminosity is subject to an Eddington limit that is $\sim 10^{38} M/M_\odot$ erg/s for electromagnetic radiation but many orders of magnitude greater for neutrinos because of the tiny neutrino opacity of matter), and is essentially undetectable. For the lower energy density accretion flows of AGN and BHXRB neutrino emission is negligible, and the thermal radiation produced by viscous heating is directly observable.

\cite{Katz1997} argued that $\epsilon_e \sim 0.1$–0.5 is likely, independent of the magnitude of the
magnetic field (but depending on its unknown orientation and spatial structure). This is consistent with observations of AGN; the likelihood of relativistic beaming precludes quantitative comparisons. This range of $\epsilon_e$ is also consistent with observations of Cyg X-1 and other BHXRB if the harder components of their spectra are either nonthermal or the thermal emission of optically thin matter heated by nonthermal particles.

The measured nonthermal efficiency is also affected by radiation trapping (Katz (1977)). As is well known (and the subject of an extensive literature) if the mass accretion rate exceeds the nominal Eddington rate the excess mass is readily swallowed by the black hole, but the luminosity $L_{th}$ emergent in thermal radiation that diffuses through the accretion flow is limited to slightly less than the Eddington limiting luminosity $L_E$ (cf. Eggum, Coroniti & Katz (1988)). An analogous limit involving the neutrino Eddington limit applies to the unobserved neutrino luminosity. In AGN and BHXRB there can be an apparent nonthermal efficiency $\epsilon_e \rightarrow 1$ and $P_{rw} \gg L_{th}$, because the nonthermal wind luminosity $P_{rw}$ is proportional to the accretion rate and is not subject to the Eddington limit, while the emergent thermal luminosity $L_{th}$ cannot exceed $L_E$ even if the viscous power dissipated $P_{visc} \gg L_E$.

4. Particle acceleration

In this section I collect and summarize results (many of which are spread throughout the very extensive literature) concerning nonthermal acceleration processes that may occur in black hole accretion discs. The purpose is to develop criteria to determine if the condition $I \gg 1$ necessarily leads to black-body equilibration, which is energetically allowed, but not required by any fundamental physical law. Unfortunately, the rates of many processes depend sensitively on the angular, spatial and (in the Klein-Nishina case) spectral distributions of any soft photon background flux, such as may be emitted by clouds
of comparatively cool thermal matter. These fluxes depend on detailed modeling that cannot be developed from first principles, and for which we do not generally have sufficient information to enable even phenomenological models.

Pulsars and GRB (in the present model) have large electric fields which lead to pair production. A supermassive black hole in a galactic nucleus, or a stellar mass black hole in a mass transfer binary, is surrounded by a complex accretional gas flow. Significant sources of mass include the companion star (in the BHXRB), the galactic interstellar medium and possibly disrupted stars (in the AGN) and the surface of the outer parts of the accretion disc. Although the flow is not understood in detail, it is plausible that some of this gas has sufficiently little angular momentum (or loses its angular momentum at large enough radii) to permit accretion on the axis of rotation, and can fill all directions around the black hole, as was found in the calculations of Eggum, Coroniti & Katz (1988). A pulsar-like vacuum is not likely. The space charge density required to neutralize the corotational electric field (Goldreich & Julian (1969)) is small, and may readily be supplied by this plasma. Pair production is therefore not required for the extraction of energy from the rotating disc in AGN and BHXRB, although it may occur.

The homopolar generator of a rotating magnetized fluid (Goldreich & Julian (1969); Blandford (1976); Lovelace (1976)) implies a large electric field with a nonzero curl in an inertial (observer’s) frame. This cannot be canceled or screened everywhere by space charge density. In some regions the electric field must be nonzero (and large); these regions are conventionally called “gaps” because the field purges them of plasma. Charged particles entering or created (Sturrock (1971); Ruderman & Sutherland (1975)) in these gaps are accelerated. The fundamental unsolved problems of compact magnetized rotating astronomical objects are the distribution of the homopolar potential drop and the acceleration of particles in gaps.
If the entire circuit were composed of dense thermal plasma then $Z_r$ would be the plasma resistivity, which would be $\ll Z_0$. $Z_i$ would depend on the circuit dimensions. If these were much less than the characteristic light travel dimension for time variations (as often the case in the laboratory, where circuits of cm or m dimensions may oscillate at 60 Hz, or even be nearly DC) then $Z_i$ may be tiny ($\gg 1$ $\Omega$). The power $P = V_0^2 Z_r/|Z|^2 \approx V_0^2/Z_r$ could be extremely high. This may describe some GRB (perhaps short GRB or subpulses in longer GRB, though it is hard to be sure because their magnetic fields are unknown), in which vacuum gaps may fill with equilibrium pair plasma, or electromagnetically driven supernovae. However, if the circuits close at the light travel distance or are open beyond it, as in classical pulsar models [Goldreich & Julian (1969)], then $Z_i \sim Z_0$ and $P \lesssim V_0^2/Z_0$ (as in Eq. 4). This is probably the case for AGN and BHXRS, in which $V_0$ is largely determined by fields near the light travel distance.

Near a luminous object accelerated electrons and positrons are slowed by Compton scattering on the thermal radiation field ([Jones (1965); Katz & Salpeter (1974)]). As a result, they are not accelerated to the limiting energy $E_0 \equiv eV_0 \sim e \int (v/c)B dr$. As an extreme bound we assume that this electric field is parallel to $\vec{B}$. In the presence of a significant ([Goldreich & Julian (1969)]) density of charged particles the parallel component of $\vec{E}$ will be much less. The following results will remain as upper bounds and may be useful as such, even though they are likely to be overestimates of the actual energies achieved.

### 4.1. Electron Retardation by Compton Scattering; Thomson Limit

If the Thomson cross-section is applicable the energy loss length $\ell_C$ of an electron with Lorentz factor $\gamma$ in an isotropic radiation field of intensity $L_{th}/(4\pi r^2)$ near a mass $M$ is

$$\ell_C = \frac{m_e}{m_p} \frac{1}{\gamma} \frac{L_{th}}{GM} \frac{r^2}{c^2}. \quad (5)$$
The first two factors are each $\ll 1$, and the next two are not much greater than unity in the inner disc of a luminous object, so that $\ell_C \ll r$. This result is also approximately valid for anisotropic radiation fields, except in the extreme case of a particle moving accurately in the direction of a narrowly collimated beam of radiation.

Given a value for the magnetic field $B$, it is possible to calculate the maximum energy an electron achieves, and to estimate its radiation. To estimate $B$ equate the magnetic stress to that required to supply a bolometric luminosity $L_b$ (including relativistic wind, thermal radiation and radiation advected into the black hole) yields

$$\frac{B^2}{8\pi} \sim \frac{1}{2} \frac{L_b}{L_E} \left(\frac{GM}{rc^2}\right)^{3/2} \frac{r}{\hbar GM\kappa} \sim 3 \times 10^7 \frac{L_b}{L_E} \left(\frac{10GM}{rc^2}\right)^{3/2} \frac{r}{10\hbar} \frac{10^8 M_\odot}{M} \text{ erg cm}^{-3}; \number{6}$$

where $\kappa$ is the Thomson scattering opacity and $h$ the disc thickness. We assume only that the viscosity is magnetic and that $B^2 \sim \langle B_r B_\phi \rangle$. The magnetic field does not depend on an assumption of equipartition or on the value of $\alpha$, and is derived from the accretion rate implied by $L_b$ alone. This estimate of $B$ is supported by polarimetric observation of the BXHS Cyg X-1 (Gnedin, et al. (2003)).

Although the estimate of $B$ (and especially of $\langle B_r B_\phi \rangle$) is robust, the process by which accretional energy is dissipated remains mysterious. It is plausible, or even likely (in analogy with the magnetic heating of the Solar convective zone or the tidal dissipation in the terrestrial oceans), that dissipation occurs in a disc corona rather than in its deep interior. In a low density corona the power dissipated may be coupled to a few particles accelerated to high energy while power dissipated in the deep interior heats matter in thermal equilibrium. Depending on the (unknown) spatial distribution of impedance and magnetic field, these loads combine in a complex and unpredictable manner. For example, dense thermal plasma of low impedance may be in series with or in parallel to vacuum gaps; in the former case most of the dissipation occurs in the gaps, while in the latter case most of it occurs in the thermal plasma.
The tendency of large electric fields to purge space of plasma, either by collisional resistive heating and subsequent expansion or by acceleration of collisionless particles, combined with the pinch effect that concentrates the current into narrow filaments, suggests that there will be regions of near-vacuum in which a few particles are accelerated to high energy. Hence we must consider the limits on particle acceleration in those regions.

Equating the energy $eE\ell_C$ gained in a length $\ell_C$ to the energy $\gamma m_e c^2$ lost by Compton scattering, using Eq. 5, yields the maximum electron Lorentz factor

$$\gamma_C \sim \left( \frac{eE}{m_pc^2GM} \frac{r}{L_E} \right)^{1/2}. \quad (7)$$

Using $E \sim vB/c \sim B(GM/rc^2)^{1/2}$ and Eq. 6 yields

$$\gamma_C \sim \left( \frac{r^2}{GM} \right)^{1/8} \left( \frac{L_{th}}{L_E} \right)^{-1/2} \left( \frac{r}{\hbar} \right)^{1/4} \left( \frac{Gnbm^2_c}{e^2} \right)^{1/4} \left( \frac{L_{tb}}{L_E} \right)^{1/4} \sim 1.0 \times 10^4 \left( \frac{M}{M_\odot} \right)^{1/4} \left( \frac{rc^2}{10GM} \right)^{1/8} \left( \frac{L_{th}}{L_E} \right)^{-1/2} \left( \frac{r}{10\hbar} \right)^{1/4} \left( \frac{L_{tb}}{L_E} \right)^{1/4}, \quad (8)$$

where $N_B \approx 1.2 \times 10^{57} M/M_\odot$ is the ratio of the black hole mass to the proton mass. It is possible to express $\gamma_C$ in terms of fundamental constants, dropping factors of order unity which depend on the properties of the individual object, and noting that if $M$ equals the Chandrasekhar mass $M_{Ch}$, then $N_B \approx (\hbar c/Gm_p^2)^{3/2}$:

$$\gamma_C \sim \left( \frac{m_e}{m_p} \right)^{1/2} \left( \frac{e^2}{\hbar c} \right)^{-1/4} \left( \frac{Gm_p^2}{\hbar c} \right)^{-1/8} \left( \frac{M}{M_{Ch}} \right)^{1/4}. \quad (9)$$

**4.1.1. Compton gamma-rays**

Eq. 8 directly gives an upper bound on the Compton scattered photon energy $\gamma_C m_e c^2$. This limit is $\sim 10^{12}$ eV for typical AGN masses and $\sim 10^{10}$ eV for typical BHXRB masses if the factors in parentheses other than $M/M_\odot$ are of order unity. In AGN typically $L_{th} \ll L_E$ so that significantly more energetic electrons and Compton scattered gamma-rays may be
produced. This explains, at least qualitatively, the production of TeV gamma-rays in AGN such as Mrk 421 (Gaidos, et al. (1996)) and Mrk 501 (Quinn, et al. (1996)).

The actual spectral cutoff depends on the spectrum of thermal photons; if their energy $\hbar \omega_{th} \sim m_e c^2 / \gamma_C$ the cutoff will be $\sim \gamma_C m_e c^2$; if $\hbar \omega_{th} < m_e c^2 / \gamma_C$ the cutoff will be $\sim \gamma_C^2 \hbar \omega_{th}$; if $\hbar \omega_{th} > m_e c^2 / \gamma_C$ the cutoff is $\sim \gamma_C m_e c^2$ but Eq. 8 then underestimates $\gamma_C$ because of the reduction in Klein-Nishina cross-section and the discreteness of the energy loss. The observed spectra of AGN and BHXRB are so complicated (it is also unclear which components are emitted at the small radii at which electron acceleration is assumed to occur) that it is difficult to be quantitative. Visible radiation from AGN and soft X-rays from BHXRB place Compton scattering marginally in the Klein-Nishina range (note that both the black body $\hbar \omega_{th}$ and $m_e c^2 / \gamma_C$ scale $\propto M^{-1/4}$), but quantitative estimates depend on the uncertain factors in parentheses in Eq 8.

Because the Compton scattering power of an electron and the frequency of the scattered photon each are proportional to $\gamma^2$, while the rate of energy gain in an electric field is independent of $\gamma$, there is expected to be a broad peak in $\nu F_\nu$ around the cutoff frequency, with $F_\nu \propto \nu^{1/2}$ at lower frequencies (the same slope as for synchrotron radiation discussed in 4.1.2 for the same reasons). In principle, the weak dependence of $\gamma_C$ on $M$, $L_{th}$ and $L_b$ in Eq. 8 could be tested by comparing AGN and BHXRB if $M$ could be estimated independently of the luminosity.

The majority of the power which goes into lepton acceleration may appear as Compton scattered gamma-rays of energy $\sim \gamma_C m_e c^2$. This is half $P_{rw}$ in a proton-electron wind and all of $P_{rw}$ if pairs are accelerated. As discussed in 8 this can far exceed $L_{th}$ if the disc is undergoing highly supercritical accretion. This may explain the dominance of the emitted power by energetic gamma-rays in some AGN. Supercritical accretion by black holes of comparatively low mass also permits more rapid variability than accretion at the Eddington
limit by a more massive black hole, so that, in principle, masses could be estimated from the variability time scale.

4.1.2. Synchrotron radiation

Electrons with Lorentz factors up to that given by Eq. 8 radiate synchrotron radiation in the magnetic field Eq. 6. Assuming an isotropic distribution of pitch angles, the characteristic synchrotron frequencies extend up to

\[ \nu_{\text{synch}} \sim \left( \frac{3}{8\pi^2} \right)^{1/2} \left( \frac{GM}{rc^2} \right)^{1/2} \frac{L_b}{L_{th}} \frac{r}{m_e c^3} \sim 7 \times 10^{22} \frac{L_b}{L_{th}} \left( \frac{10GM}{rc^2} \right)^{1/2} \frac{r}{10h} \text{ Hz} \]  

(10)

The usual relation between the particle distribution function and the synchrotron spectral index for \( \nu < \nu_{\text{synch}} \) would predict a spectral index of \(-1/2\), because a uniform accelerating electric field produces a particle distribution function with energy exponent zero. However, the low frequency component of the synchrotron radiation function increases the spectral index to \(-1/3\), in analogy to the predicted (Katz (1994b)) and observed (Galama, et al. (1998); Frail, Waxman & Kulkarni (2000); Barraud, et al. (2003)) low frequency spectra of gamma-ray bursts.

The ratio of synchrotron to Compton scattering powers, assuming an isotropic electron distribution, is the ratio of the magnetic to the thermal energy densities \( U_B/U_{th} \):

\[ \frac{P_{\text{synch}}}{P_{\text{Compt}}} = \frac{U_B}{U_{th}} \sim \frac{1}{2} \frac{L_b}{L_{th}} \frac{r}{h} \left( \frac{GM}{rc^2} \right)^{-1/2} \]  

(11)

This is generally \( \geq 1 \).

Unlike the Compton cutoff, the synchrotron cutoff Eq. (10) is independent of the mass of the black hole, and is in that sense “universal”, although it depends on other parameters, most notably the ratio \( L_b/L_{th} \). Eqs. (10) and (11) suggest the possibility of synchrotron radiation with significant power up to \( \sim \text{GeV energies} \). This is probably a great
overestimate, both of the power and of the radiation frequency, because the assumption of an isotropic distribution of electron momenta is unlikely to be valid. Electrons may be effectively accelerated only parallel to the magnetic field—a component of $E$ perpendicular to $B$ does not effectively accelerate charged particles unless it varies at their cyclotron frequency, unlike the nearly steady corotational electric field—and readily lose their transverse momentum by synchrotron radiation.

Internal dissipation in the relativistic wind may be as essential to radiation in AGN as in GRB, for in them plasma turbulence may partially isotropize the electron distribution, making effective synchrotron radiation possible. If the pitch angles remain small the frequency of the synchrotron radiation is reduced and it is emitted nearly parallel to the direction of the electrons’ motion, the magnetic field, and the Compton scattered gamma-rays. This can be described as relativistic bulk motion of the electrons and their associated radiation field, and may be necessary to avoid absorption of the photons by gamma-gamma pair production.

4.1.3. Curvature radiation

The electrons also radiate curvature radiation (on the magnetic field lines of radii of curvature $\sim r$) at frequencies up to

$$\nu_{\text{curve}} \sim \frac{1}{2\pi} \left( \frac{GM}{rc^2} \right)^{5/8} \left( \frac{L_{\text{th}}}{L_E} \right)^{-3/2} \left( \frac{L_b}{L_E} \right)^{3/4} \left( \frac{r}{\hbar} \right)^{3/4} m_e^{3/2} c^3 m_p^{3/4} c^3 (GM)^{1/4}$$

$$\sim 3 \times 10^{15} \left( \frac{10GM}{rc^2} \right)^{5/8} \left( \frac{L_{\text{th}}}{L_E} \right)^{-3/2} \left( \frac{L_b}{L_E} \right)^{3/4} \left( \frac{r}{10\hbar} \right)^{3/4} \left( \frac{M_\odot}{M} \right)^{1/4} \text{ Hz.} \quad (12)$$

Curvature radiation is insignificant, and its power is small, when the electron energy is limited by Compton scattering in the Thomson limit. It cannot cause pair production, in contrast to the case of pulsars (which have much larger fields than do accretion discs).
4.1.4. Pair production

The most energetic gamma-rays (produced when the electrons Compton scatter thermal photons) of energy $E_\gamma \sim \gamma C m_e c^2$ may produce electron-positron pairs by interacting with the thermal photons. The condition for this to occur (assuming an isotropic thermal radiation field)

$$E_\gamma \hbar \omega_{th} \sim \gamma C m_e c^2 \hbar \omega_{th} > (m_e c^2)^2, \quad (13)$$

is equivalent to the condition for the breakdown of the Thomson approximation to Compton scattering. It appears to be met in AGN and BHXRB, taking the observed thermal spectra and assuming all factors in parentheses in Eq. 8, except that involving the mass, are $O(1)$. If the thermal radiation field is assumed to be a black body then Eq. 13 can be rewritten

$$\left( \frac{GM}{rc^2} \right)^{3/8} \left( \frac{r}{\hbar} \right)^{1/4} \left( \frac{L_b}{L_{th}} \right)^{1/4} \left( \frac{e^2}{\hbar c} \right)^{-3/4} > 1. \quad (14)$$

It is clear that this condition is generally met; if (as is likely) the thermal spectrum is harder than that of a black body at the effective temperature the inequality holds even more strongly. Pair production by interaction between gamma-rays produced by Compton scattering of the accelerated electrons and thermal photons takes the place of pair production by curvature radiation which occurs in pulsars.

4.2. Electron Retardation by Compton Scattering; Klein-Nishina Case

As pointed out in 4.1.1, Compton scattering of the accelerated electrons by thermal photons may be at sufficiently high energy that the Thomson cross-section is inapplicable, and the full Klein-Nishina cross-section must be used instead. In this case $\ell_C$ depends on the frequency distribution of the thermal radiation as well as on its luminosity. Eq. 8 is replaced by

$$\ell_{KN} = \frac{\gamma}{3 \ln (\gamma \hbar \nu_{th} / m_e c^2)} \frac{\hbar \nu_{th}}{m_e c^2} \frac{L_E}{L_{th}} \frac{\hbar \nu_{th} r}{GM m_p r}. \quad (15)$$
The dependence on the thermal photon frequency $\nu_{th}$ is nearly quadratic, one power coming from the reciprocal relation between the photon number density and their frequency (at fixed $L_{th}$) and the other power coming from the energy dependence of the Klein-Nishina cross-section. We have assumed a thermal radiation intensity $L_{th}/4\pi r^2$, as would be produced by energy dissipated at radii comparable to that of particle acceleration. Thermal radiation produced at greater radii (for example, in the broad line regions of AGN) is diluted, giving a correspondingly lower effective value of $L_{th}$.

Because of this sensitivity to $\nu_{th}$, information about the spectrum is required to evaluate Eq. 15. For example, for near-Eddington limited accretion onto a $10^8 M_\odot$ black hole $\ell_{KN} \gg r$ if $h\nu_{th} = 10$ KeV, but $\ell_{KN} \ll r$ if $h\nu_{th} = 100$ eV. A small amount of soft radiation has a large effect. For electrons of Lorentz factor $\sim 10^6$, suggested in Eq. 15, even scattering by visible light (below the black body spectral peak for luminous accretion onto a supermassive black hole) or the “blue bump” observed in some AGN spectra is in the Klein-Nishina regime, and the efficacy of Compton scattering in limiting electron acceleration is sensitive to the actual spectral distribution of the radiation.

The rate of electron energy loss is essentially proportional to the scattering rate, because an electron loses most of its energy in a single scattering. The Compton scattering rate is given by $\int \sigma_{KN}(F_{\nu}/\nu) \, d\nu$, where $\sigma_{KN} \propto \ln (h\nu/m_e c^2)/\nu$. If the thermal spectrum is self-absorbed below a frequency $\nu_{abs}$, follows a thin bremsstrahlung or synchrotron spectrum at higher frequencies, and $\nu_{abs} > \nu_{KN}$ where the characteristic Klein-Nishina frequency $\nu_{KN} \equiv m_e c^2/\gamma h$, then most of the Compton energy loss is attributable to photons of frequency $\nu \sim \nu_{abs}$ and $L_{th}$ in Eqs. 17 and 18 should be replaced by the luminosity at frequencies of this order. If $\nu_{abs} < \nu_{KN}$ then (provided the spectral index in the optically thin regime does not exceed 1) most of the Compton energy loss is attributable to photons of frequency $\nu \sim \nu_{KN}$, and $L_{th}$ should be replaced by the luminosity at frequencies of this
order. These are reasonable rough approximations to the full integrals of the Compton energy transfer function (Blumenthal & Gould (1970)) over the radiation spectrum.

4.2.1. Ultimate (Goldreich-Julian) limit

The reduction in the Compton energy losses when the bulk of the thermal luminosity is emitted at frequencies above $\nu_{KN}$ implies that the maximum Lorentz factor is larger than indicated in Eqs. 7, 8 and 9. The available potential drop is given by Goldreich & Julian (1969) and accelerated particles of mass $m$ reach a limiting Lorentz factor

$$\gamma_{GJ} \sim \frac{e}{2mc^2} \left(\frac{\Omega r}{c}\right)^2 rB,$$

in place of Eq. 7. Using the naïve estimate Eq. 6 for $B$ leads to a Lorentz factor of accelerated electrons, in place of Eq. 8,

$$\gamma_{GJe} \sim \left(\frac{rc^2}{GM}\right)^{-3/4} \left(\frac{r}{\hbar}\right)^{1/2} \left(\frac{Gm_pm_e}{e^2}\right)^{1/2} \left(\frac{L_b}{L_E}\right)^{1/2} \sim 2 \times 10^{10} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{rc^2}{10GM}\right)^{-3/4} \left(\frac{r}{10\hbar}\right)^{1/2} \left(\frac{L_b}{L_E}\right)^{1/2}.$$ (17)

This may be rewritten in terms of fundamental constants, in analogy to Eq. 9

$$\gamma_{GJe} \sim \left(\frac{e^2}{\hbar c}\right)^{-1/2} \left(\frac{Gm_p^2}{\hbar c}\right)^{-1/4} \left(\frac{M}{M_{Ch}}\right)^{1/2}.$$ (18)

For a black hole of mass $M \sim 10^8M_\odot$, as expected for AGN, electrons may be accelerated to energies $\sim 10^{20}$ eV. For black holes of stellar mass the limiting electron energy is $\sim 10^{16}$ eV. If this energy is achieved in microquasars, they may produce (by Compton scattering) gamma-rays approaching this energy.
4.2.2. Curvature radiation

The extreme energies of Eqs. 16, 17 and 18 are possible only in the limit in which Compton drag is negligible, either because $L_{th}$ is small or because $\nu_{th}$ is large (Eq. reflkn). If such extreme electron energies are achieved then even the curvature radiation may be powerful and energetic. Equating the accelerating power $eEc \sim e\nu B$ to the curvature radiation loss $2e^2c^4/(3r^2)$ and using Eq. 6 for $B$ yields a limiting Lorentz factor

$$\gamma_{\text{curv}} \sim \left( \frac{r c^2}{GM} \right)^{1/16} \left( \frac{r}{\hbar} \right)^{1/8} \left( \frac{L_b}{L_E} \right)^{1/8} \left( \frac{r}{r_e} \right)^{3/8} \left( \frac{m_p}{m_e} \right)^{1/8}$$

$$\sim 4 \times 10^7 \left( \frac{r c^2}{10GM} \right)^{7/16} \left( \frac{r}{10h} \right)^{1/8} \left( \frac{L_b}{L_E} \right)^{1/8} \left( \frac{M}{M_{\odot}} \right)^{3/8},$$

where $r_e \equiv e^2/m_e c^2 = 2.82 \times 10^{-13}$ cm is the classical electron radius. Just as for pulsars, if Compton drag is negligible the electron energy will be limited by curvature radiation losses, and most of the power of electron acceleration will appear as curvature radiation.

The characteristic frequency of curvature radiation is then

$$\nu_{\text{curv}} \sim \frac{1}{2\pi} \left( \frac{10GM}{rc^2} \right)^{1/8} \left( \frac{r}{10h} \right)^{1/4} \left( \frac{L_b}{L_E} \right)^{1/4} \left( \frac{c^3}{10GM_{\odot}} \right)^{3/4} \left( \frac{m_p}{m_e} \right)^{1/4}$$

$$\sim 6 \times 10^{18} \left( \frac{10GM}{rc^2} \right)^{1/8} \left( \frac{M_{\odot}}{M} \right)^{1/4} \left( \frac{r}{10h} \right)^{1/4} \left( \frac{L_b}{L_E} \right)^{1/4} \text{Hz.}$$

This frequency, typically hard X-rays for stellar-mass objects and very soft X-rays for AGN, is not a prominent feature of their spectra. This implies that curvature radiation is not usually the dominant electron energy loss mechanism in these objects, and supports the customary assumption that Compton loss is dominant.

4.2.3. Pair production

Very energetic electrons lose more energy on a background radiation field by pair production ($\gamma e^- \rightarrow e^- e^-$) than by Compton scattering (Mastichiadis, Marscher & Brecher).
These authors find that this is the case only far into the Klein-Nishina regime, when $\gamma_e > 550 m_e c^2 / h \nu_{th}$. Using their results, we obtain a limiting Lorentz factor

$$\gamma_{e,pair} \sim 6 \times 10^{11} \left( \frac{r c^2}{GM} \right)^{0.98} \left( \frac{r}{h} \right)^{0.65} \left( \frac{L_b}{L_E} \right)^{0.65} \left( \frac{L_E}{L_{th}} \right)^{1.30} \left( \frac{h \nu_{th}}{m_e c^2} \right)^{1.60} \left( \frac{M}{M_\odot} \right)^{0.65}. \tag{21}$$

Of course, if this $\gamma_{e,pair}$ exceeds the Goldreich-Julian limit Eq. 16 then radiation drag is negligible and the actual limit is $\gamma_e < \gamma_{GJ e}$.

Because of the sensitivity of $\gamma_{e,pair}$ to $\nu_{th}$ even a small amount of soft radiation has a large effect on the limiting electron and positron energy, and hence on the photon energy of Compton gamma rays and of curvature radiation produced by these light leptons. This is the same difficulty encountered when Compton drag in the Klein-Nishina limit is dominant. Without a detailed geometrical and spectral model it is not possible to estimate $\gamma_{e,pair}$ even to order of magnitude.

### 4.3. Proton acceleration

If protons are accelerated their energy may be radiated as high energy gamma-rays following collisional pion production, either directly from $\pi^0$ decay or by Compton scattering of $e^\pm$ produced by decay of $\pi^\pm$. In the latter case neutrinos are also produced, with power and energy comparable to that of the gamma-rays. Alternatively, the proton kinetic energy may be degraded and coupled to electrons in a collisionless shock, as is generally assumed to occur in gamma-ray bursts, and then radiated by Compton scattering or by synchrotron radiation. The proton kinetic energy may much exceed the values given by Eqs. 7, 8 and 9 and may approach the limiting energies given in Eq. 21. Gamma-rays (and neutrinos) following from pion production will have energies approaching a tenth of the proton energies. On the other hand, proton synchrotron radiation will occur at much lower energies than electron synchrotron radiation, and is generally negligible.
If there were no pair production, the accelerated plasma would consist of protons (and nuclei) and electrons. Even in the presence of pair production, protons may be accelerated along with the positrons. This is important in intense sources of thermal radiation, such as BHXRB and AGN, because proton-photon scattering is negligible below the pion production threshold. Even above threshold, the effective energy loss cross-section (Greisen (1966)) is a fraction \( f \sim 10^{-4} \) of the Thomson cross-section. This permits accretion discs around black holes in AGN and BHXRB to be efficient proton accelerators (as has previously been discussed by Lynden-Bell (1969); Kazanas & Ellison (1986); Katz (1991) in other models). Very high energy gamma-rays may then result from photoproduction of \( \pi^0 \) by thermal radiation. Collisions of protons with nucleons make pions, leading to high energy radiation directly from \( \pi^0 \) decay and indirectly by Compton scattering of the very energetic \( e^\pm \) produced by \( \pi^\pm \to \mu^\pm \to e^\pm \) (Katz (1991); Dar & Laor (1997)). Similarly, very high energy neutrinos are produced in the pion and muon decays.

The energy loss length of a proton is, in analogy to Eq 5,

\[
\ell_p \approx \frac{1}{\gamma f} \frac{L_E}{L_{\text{th}}} \frac{rc^2}{GM^2 r}.
\]  

(22)

Unlike Thomson scattering, this process has an energy threshold determined by the \( \pi^0 \) rest mass of 135 MeV. If the thermal spectrum consists of visible light, the acceleration of protons is not restrained until \( \gamma_p \sim 10^8 \). The result analogous to Eq. 8 for the limiting Lorentz factor of protons slowed by photopion production in a thermal radiation field is

\[
\gamma_p \sim \left( \frac{rc^2}{GM} \right)^{1/8} \left( \frac{L_{\text{th}}}{L_E} \right)^{-1/2} \left( \frac{r}{\hbar} \right)^{1/4} \left( \frac{GNm_e^2}{e^2} \right)^{1/4} \left( \frac{L_b}{L_E} \right)^{1/4} f^{-1/2} 
\]

\[
\sim 1.0 \times 10^6 \left( \frac{M}{M_\odot} \right)^{1/4} \left( \frac{rc^2}{10GM} \right)^{1/8} \left( \frac{L_{\text{th}}}{L_E} \right)^{-1/2} \left( \frac{r}{10\hbar} \right)^{1/4} \left( \frac{L_b}{L_E} \right)^{1/4}.
\]

(23)

For an AGN with \( M \sim 10^8 M_\odot \) this yields a limiting \( \gamma_p \sim 10^8 \), approximately the threshold at which \( \pi^0 \) photoproduction begins. Just as for pair production, the product of \( \gamma_p \) and the black-body thermal photon energy is independent of \( M \), so that Eq. 20 may be (barely)
applicable at all $M$.

4.3.1. Proton Radiation

These possible $\sim 10^{17}$ eV protons in AGN could produce $\sim 10^{16}$ eV photons from photoproduced $\pi^0$, but these are not observable at the greatest distances because of pair production on the microwave background radiation. However, at closer distances these gamma-rays may be observable. In BHXRB, perhaps including microquasars, the corresponding proton energies are $\sim 10^{15}$ eV, which may produce photons of $\sim 10^{14}$ eV.

4.3.2. Underluminous black holes as sources of UHE cosmic rays?

If most of the thermal radiation is ineffective at slowing the protons because it is below the photopion threshold (in the protons’ frame) or if $L_{th}$ is small, then, just as for electrons in the Klein-Nishina case, it may be possible for protons to approach the limiting Lorentz factor Eq. 16. The low luminosity black hole powering Sgr A* in our Galactic Center demonstrates that black holes with $L_{th}/L_E \ll 1$ may exist even in the presence of sufficient mass to support high accretion rates, and they would generally be undetectable at extragalactic distances. Ratios $L_{th}/L_E$ even lower than that of Sgr A* may occur. For parameters appropriate to supermassive black holes the proton energies of $\sim 10^{20}$ eV, and energies of heavier nuclei greater by a factor of $Z$, may be sufficient to explain the highest energy cosmic rays.
4.3.3. Stellar-mass black holes as TeV sources?

Just as underluminous supermassive black holes may accelerate protons and nuclei to ultra-high energies, underluminous stellar-mass black holes may accelerate them to energies sufficient to be sources of TeV radiation (if the hadrons collide with other hadrons to make $\pi^0$ or $\pi^\pm$ whose decay electrons undergo Compton scattering). Such objects, resembling blazars (rather than isolated neutron stars, the more conventional explanation) might be the origin of the unidentified Galactic TeV sources (Ubertini (2005)).

5. Are there shocks in GRB?

In most models of relativistic astronomical flows it is assumed that the fluids behave as if they were collisional, even though the single-particle collision lengths are generally orders of magnitude greater than the spatial extent of the fluids involved. Collective collisionless processes (beam-driven electromagnetic or longitudinal two-stream instabilities) are assumed to lead to momentum transfer on much shorter scales, so that the flows can be described by the hydrodynamic equations. Kinematic constraints require some such interaction in order to convert the kinetic energy of relativistic motion to radiation. In particular, in a hydrodynamic supersonic flow the interaction takes the form of shocks. Such shocks are assumed in nearly all models of GRB.

GRB are the only identified objects near whose central engines $I \gg 1$. The observed radiation is produced at distances several orders of magnitude greater (typical estimates are $\mathcal{O}10^{15}$ cm), at which $I \ll 1$. Under these conditions nonthermal particle acceleration is possible, and in fact is required by their observed nonthermal spectra. Further development of models requires investigation of the conditions under which thermal flows lead to the acceleration of nonthermal particles. Shock processes have been
extensively investigated, but we should also ask if shocks (hydrodynamic discontinuities between two fluids each with thermodynamic equilibrium distribution particle functions, though generally transparent and with photon energy densities many orders of magnitude less than Planckian at the matter temperature) form at all. The kinematic constraints cannot be avoided, but it may be that the hydrodynamic assumption is invalid. It has been demonstrated (Fenimore, Madras & Nayakshin (1996); Sari & Piran (1997)) that the complex multipeaked temporal structure of GRB can only be explained, given the hydrodynamic assumption, by internal shocks. External shocks may be associated with, or even defined as, the origin of afterglows (Katz (1998)). Yet elementary kinematics (Katz (1997); Kobayashi & Piran (1997); Fan & Piran (2006)) shows that, unless the ratios of Lorentz factors are very large, the energetic efficiency of internal shocks is low, generally no more than 10–20%. More efficient radiation would require the emission of a shell of low Lorentz factor followed by one of much higher Lorentz factor. It is implausible that this occurs regularly, but any deviation from this sequence (for example, a random distribution of Lorentz factors and of proper masses) would imply low efficiency. This would increase the energetic requirements of GRB and (perhaps worse) would raise the question of why the remaining energy does not appear as afterglows many times more energetic than GRB themselves.

One possible escape from this “Efficiency Crisis” (Ioka, et al. (2006)) is to reject the assumption of hydrodynamic flow. Recall the reason why an external shock (on external matter initially at rest or moving non-relativistically) cannot explain complex substructure: After the external matter is hit by a shell of relativistic debris, it must be accelerated in a shock to relativistic speeds (Lorentz factor > 100) to explain the observed properties of GRB emission (spectrum and avoidance of gamma-gamma pair production). Then the external matter is no longer available as a static or non-relativistically moving target for the impact of a second shell, and the duration of emission of the first subpulse (from
impact of the first shell) will overlap that of a second subpulse, even if a second shell is emitted cleanly separated from the first shell \cite{FenimoreMadrasNavakshin1996,SariPiran1997}.

This conclusion may be avoided if the first relativistic shell interacts only with a fraction of the matter at each point surrounding it, thus satisfying the kinematic constraints while violating the hydrodynamic assumption that the target behaves as a single fluid with a single (relativistic) equilibrium particle distribution function. It is possible that the collective plasma processes that mediate the interaction couple, at any time, only to a small fraction of the matter (for example, to a narrow subrange of the velocity distribution) in the external medium. This is, at least qualitatively, consistent with the intermittent and fluctuating nature of most observed plasma-physical instabilities, both in the laboratory and in Nature.

This hypothesis suggests that as the density of target matter is gradually eroded by the passage through it of successive shells of relativistic debris the parameters of the accelerated particles and the properties of their radiation may also change. Although the details of these changes are unknown (because the collective interactions between debris and target are not understood from first principles), this does suggest a progressive evolution of the characteristics of GRB emission through the pulse, consistent with the usual (but not universal) observations that the emission gradually softens.

Unfortunately, this is not an unambiguous test of the hypothesis because the parameters and properties of the central engine are also changing as its mass is gradually accreted on to a central black hole, and this can be an alternative explanation of any progressive changes through GRB pulses. The statistics of sub-pulses may be different in the internal shock and non-shock models, but in both cases depend on the unknown statistics of the activity of the central engine.
6. Discussion

It is apparent from comparing the results of 4.1 to those of 4.2 that the actual particle energies achieved depend sensitively on the flux and spectral distribution of any soft radiation in the acceleration region. The power and spectral distribution of radiation produced by interaction (Compton scattering, pion photoproduction) with this soft radiation also depend on the properties of the soft radiation, partly directly, and partly because it affects the distribution of energy of the energetic particles. A small amount of dense nonrelativistic plasma, producing a black body or thin bremsstrahlung flux may have a large effect on the more energetic radiation of an accreting black hole, even though the thermal radiation is comparatively insignificant in power. The kinematic reason for this is similar to that encountered in the study of GRB external shocks, in which a proper mass $E/\Gamma^2c^2$ at rest is sufficient to dissipate inelastically a kinetic energy $E$ in matter moving with Lorentz factor $\Gamma$.

In the present problem $\Gamma$ may be as large as $\sim 10^{11}$ for at least a few of the energetic particles, and generally $\Gamma \gg 1$ by orders of magnitude, so that a tiny amount of isotropic thermal radiation (substituting for mass at rest) may have a large effect on the energetic particles if the interaction cross-sections are sufficient. This extraordinary sensitivity implies that predictive quantitative models are difficult to build. For example, for supermassive black holes possible objects range from classical quasars (with $L_{th} \sim L_E \sim 10^{46}$ erg/s) to sources of the most energetic cosmic rays with almost no electromagnetic luminosity at all.

Emergent spectra depend on such poorly understood and essentially unpredictable variables as the flux and angular distribution of thermal radiation (and hence on the location and parameters of thermal gas) and on temporally fluctuating collective interactions between essentially collisionless interpenetrating streams of matter with relativistic relative velocities. Qualitatively, this sensitivity is consistent with the complex time structure
observed in AGN, BHXRB and GRB, but makes it difficult to predict the emergent spectra. However, linear polarization is a general property of most nonthermal radiation processes (most familiarly, synchrotron radiation), and is predicted if the X-ray emission of BHXRB is nonthermal in some of their states.

AGN may show QPO, just as suggested for GRB (Katz (1997)), but with typical periods of order hours to days, depending on the black hole masses and angular momenta (Shapiro & Teukolsky (1983)). No such QPO have been found in the extensive body of visible light data on AGN. This may be explained if the visible light is produced far from the central object or by thermal radiation from a nearly axisymmetric gas disc. It may be more fruitful (though more difficult observationally) to search for QPO in the energetic gamma-rays produced as particles are accelerated along magnetic field lines closer to the rotating disc. It is the magnetic field that would be expected to show the greatest deviation from axisymmetry, as in pulsars.

The considerations of this paper have led to the prediction of two novel kinds of objects:
1. Young, high field, rapidly rotating pulsars (with \( \mu^2 \omega^6 \gg 12\pi c^5 I_{\text{char}} \approx 10^{73} \text{ erg cm}^3/\text{s}^6 \)), that produce a black-body equilibrium pair gas wind rather than the few but very energetic particles produced by radio pulsars. Pulsars satisfying this condition on \( \mu^2 \omega^6 \) may resemble a SGR in permanent outburst for a lifetime \( \sim \mathcal{I} \omega^4/(8\pi c^2 I_{\text{char}}) \approx 3 \times 10^{-7} \omega^4 \text{ s}^5 \), where \( \mathcal{I} \) is the moment of inertia. The minimum luminosity of such an object is \( 4\pi r^2 I_{\text{char}} \sim 10^{42} \text{ erg/s} \), so they would be detectable (following supernovae or quiet stellar collapse) as rapid (\( \sim 1 \) KHz) periodic X-ray sources at distances approaching that of the Virgo cluster (\( \approx 20 \) Mpc).
2. Supermassive black holes with most of their luminosity in ultra high energy particles, and perhaps not recognizable as AGN. If these are the source of UHE cosmic rays, their arrival directions will be clumped (subject to propagation effects) and perhaps correlated with point sources of UHE neutrinos and gamma-rays.
Acknowledgements

I thank T. Piran, M. A. Ruderman and R. Sari for discussions, Washington University for the grant of sabbatical leave, and the Hebrew University for hospitality and a Forchheimer Fellowship.

A. Conditions for Particle Acceleration

The central mystery of AGN is why so much of their accretional power appears as particle acceleration. In order to accelerate energetic particles, necessary both for incoherent emission (as in AGN and radio sources) and for coherent emission (as in pulsars and FRB, if these particles drive a plasma instability), it is necessary that they gain energy from an electric field faster than they lose it to interaction with ambient plasma (by “Coulomb drag”) (Atzeri, Schiavi & Davies 2009). For a relativistic electron the ratio of these quantities defines an acceleration parameter

$$A \approx \frac{E_m c^2}{4\pi e^3 n \ln \Lambda}, \quad (A1)$$

where $E$ is the electric field, $n$ the plasma particle density and $\Lambda$ is $2m_e c^2/I$ with $I$ the ionization potential in a neutral medium or $m_e c^2/\hbar \omega_p$ in a plasma. $\ln \Lambda \approx 20$ in most astronomical environments, and is insensitive to their parameters. $A > 1$ is a necessary condition for particle acceleration.

For the midplane of an accretion disc, adopting the simple scaling model presented here and taking

$$E \sim \frac{v_{orb}}{c} B \sim \sqrt{\frac{4\pi L}{L_E}} \left(\frac{GM}{rc^2}\right)^{5/4} \left(\frac{r}{h}\right)^{1/2} \left(\frac{c^4}{GM\kappa}\right)^{1/2}, \quad (A2)$$

where $L$ is the accretional luminosity, $L_E$ the Eddington luminosity, $r$ the distance from the central mass $M$, $h$ the local disc thickness and $\kappa$ the opacity (from the definition of $L_E$),
and using the same scaling model for the density
\[ n \sim \frac{L}{L_E} \sqrt{\frac{rc^2}{GM} \frac{1}{\alpha} \frac{3}{4\pi hr_e^2}}, \tag{A3} \]
where \( \alpha \) is the dimensionless viscosity parameter of the disc and \( r_e \) the classical electron radius.

Combining these expressions
\[ A \sim \frac{\sqrt{4\pi m_e c^2}}{e^3 \ln \Lambda} \left( \frac{L}{L_E} \right)^{-1/2} \left( \frac{GM}{rc^2} \right)^{7/4} \left( \frac{r}{h} \right)^{-1/2} \left( \frac{c^4}{GMk} \right)^{1/2} \frac{rr_e^2 \alpha}{3}. \tag{A4} \]
This is generally \( \gg 1 \), but is not realistic because in the generally applicable magnetohydrodynamic regime \( E \ll (v/c)B \).

The low density corona of an accretion disc, where magnetohydrodynamics may not be applicable, offers a more favorable environment for the acceleration of energetic particles. The scaling \( A \propto M^{1/2} \) in Eq. A4 with \( r \propto M \) and the other factors either dimensionless and of order unity or physical constants, may explain why AGN are proportionally more efficient accelerators of energetic particles (their relativistic jets and double radio lobes representing much of their total accretional power) than stellar mass black holes (most of whose accretional power is emitted as thermal X-rays).
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