The Sum Rules for Structure Functions of Polarized $e(\mu)N$ Scattering: Theory vs. Experiment

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Abstract

The determination of twist-4 corrections to the structure functions of polarized $e(\mu)N$ scattering by QCD sum rules is reviewed and critically analyzed. It is found that in the case of the Bjorken sum rule the twist-4 correction is small at $Q^2 > 5 \text{ GeV}^2$. However, the accuracy of the today experimental data is insufficient to reliably determine $\alpha_s$ from the Bjorken sum rule. For the singlet sum rule $- p + n$ – the QCD sum rule gives only the order of magnitude of twist-4 correction. At low and intermediate $Q^2$ the model is presented which realizes a smooth connection of the Gerasimov-Drell-Hearn sum rules at $Q^2 = 0$ with the sum rules for $\Gamma_{p,n}(Q^2)$ at high $Q^2$. The model is in a good agreement with the experiment.
1. Introduction

In the last few years there is a strong interest to the problem of nucleon spin structure: how nucleon spin is distributed among its constituents - quarks and gluons. New experimental data continuously appear and precision increases (for the recent data see [1], [2]). One of the most important item of the information comes from the measurements of the first moment of the spin-dependent nucleon structure functions $g_1(x)$ which determine the parts of nucleon spin carried by $u$, $d$ and $s$ quarks and gluons. The accuracy of the data is now of a sort that the account of twist-4 terms is of importance when comparing the data with the Bjorken and Ellis-Jaffe sum rules at high $Q^2$. On the other side, at low and intermediate $Q^2$ a smooth connection of the sum rules for the first moments of $g_1(x, Q^2)$ with the Gerasimov-Drell-Hearn (GDH) sum rules [3,4] is theoretically expected. This connection can be realized through nonperturbative $Q^2$-dependence only. In my talk I discuss such nonperturbative $Q^2$-dependence of the sum rules (see also [5]).

Below I will consider only the first moment of the structure function $g_1(x, Q^2)$

$$
\Gamma_{p,n}(Q^2) = \int_0^1 dx \ g_{1;p,n}(x, Q^2) \tag{1}
$$

The presentation of the material is divided into two parts. The first part deals with the case of high $Q^2$. I discuss the determination of twist-4 contributions to $\Gamma_{p,n}$ by QCD sum rules and with the account of twist-4 corrections and of the uncertainties in their values compare the theory with the experiment. In the second part the case of low and intermediate $Q^2 \lesssim 1 GeV^2$ is considered in the framework of the model, which realizes the smooth connection of GDH sum rule at $Q^2 = 0$ with the asymptotic form of $\Gamma_{p,n}(Q^2)$ at high $Q^2$.

2. High $Q^2$

At high $Q^2$ with the account of twist-4 contributions $\Gamma_{p,n}(Q^2)$ have the form

$$
\Gamma_{p,n}(Q^2) = \Gamma_{p,n}^{as}(Q^2) + \Gamma_{p,n}^{tw4}(Q^2) \tag{2}
$$

$$
\Gamma_{p,n}^{as}(Q^2) = \frac{1}{12} \left\{ [1 - a - 3.58a^2 - 20.2a^3 - ca^4][\pm g_A + \frac{1}{3}a_s] + \frac{4}{3}[1 - \frac{1}{3}a - 0.55a^2 - 4.45a^3]\Sigma \right\} - \frac{N_f}{18\pi}\alpha_s(Q^2)\Delta g(Q^2) \tag{3}
$$

$$
\Gamma_{p,n}^{tw4}(Q^2) = \frac{b_{p,n}}{Q^2} \tag{4}
$$

In eq.(3) $a = \alpha_s(Q^2)/\pi, g_A$ is the $\beta$-decay axial coupling constant, $g_A = 1.260 \pm 0.002$ [6]

$$
g_A = \Delta u - \Delta d \quad a_s = \Delta u + \Delta d - 2\Delta s \quad \Sigma = \Delta u + \Delta d + \Delta s. \tag{5}
$$

$\Delta u, \Delta d, \Delta s, \Delta g$ are parts of the nucleon spin projections carried by $u, d, s$ quarks and gluons:
\[ \Delta q = \int_0^1 \left[ q_+(x) - q_-(x) \right] dx \]  

(6)

where \( q_+(x), q_-(x) \) are quark distributions with spin projection parallel (antiparallel) to nucleon spin and a similar definition takes place for \( \Delta g \). The coefficients of perturbative series were calculated in [7-10], the numerical values in (3) correspond to the number of flavours \( N_f = 3 \), the coefficient \( c \) was estimated in [11], \( c \approx 130 \). In the \( \overline{\text{MS}} \) renormalization scheme chosen in [7-10] \( a_8 \) and \( \Sigma \) are \( Q^2 \)-independent. In the assumption of the exact \( SU(3) \) flavour symmetry of the octet axial current matrix elements over baryon octet states \( a_8 = 3F - D = 0.59 \pm 0.02 \) [12].

Strictly speaking, in (3) the separation of terms proportional to \( \Sigma \) and \( \Delta g \) is arbitrary, since the operator product expansion (OPE) has only one singlet in flavour twist-2 operator for the first moment of the polarized structure function – the operator of singlet axial current \( j^{(0)}_{\mu5}(x) = \sum \bar{q}(x)\gamma_{\mu}\gamma_5 q \), \( q = u, d, s \). The separation of terms proportional to \( \Sigma \) and \( \Delta g \) is outside the framework of OPE and depends on the infrared cut-off. The expression used in (3) is based on the physical assumption that the virtualities \( p^2 \) of gluons in the nucleon are much larger than light quark mass squares, \( |p^2| \gg m_q^2 \) [13] and that the infrared cut-off is chosen in a way providing the standard form of axial anomaly [14].

Twist-4 corrections to \( \Gamma_{p,n} \) were calculated by Balitsky, Braun and Koleshichenko (BBK) [15] using the QCD sum rule method.

BBK calculations were critically analyzed in [5], where it was shown that there are few possible uncertainties in these calculations: 1) the main contribution to QCD sum rules comes from the last accounted term in OPE – the operator of dimension 8; 2) there is a large background term and a much stronger influence of the continuum threshold comparing with usual QCD sum rules; 3) in the singlet case, when determining the induced by external field vacuum condensates, the corresponding sum rule was saturated by \( \eta \)-meson, what is wrong. The next order term – the contribution of the dimension 10 operator in the BBK sum rules was estimated by Oganesian [16]. The account of the dimension-10 contribution to the BBK sum rules and estimation of other uncertainties results in (see [5]):

\[ b_{p-n} = -0.006 \pm 0.012 \text{ GeV}^2 \]  

(7)

\[ b_{p+n} = -0.035(\pm 100\%) \text{ GeV}^2 \]  

(8)

As is seen from (7), in the nonsinglet case the twist-4 correction is small (\( \lesssim 2\% \) at \( Q^2 \gtrsim 5\text{GeV}^2 \)) even with the account of the error. In the singlet case the situation is much worse: the estimate (8) may be considered only as correct by the order of magnitude.

I turn now to comparison of the theory with the recent experimental data. In Table 1 the recent data obtained by SMC [1] and E 154(SLAC) [2] groups are presented.
In the second line of Table 1 the results of the performed by SMC [1] combined analysis of SMC [1], SLAC-E80/130 [17], EMC [18] and SLAC-E143 [19] data are given. The data presented in Table 1 refer to $Q^2 = 5 GeV^2$. In all measurements each range of $x$ corresponds to each own mean $Q^2$. Therefore, in order to obtain $g_1(x, Q^2)$ at fixed $Q^2$ refs. [1,2] use the following procedure. At some reference scale $Q_0 = 1 GeV^2$ in [1] and $Q_0 = 0.34 GeV^2$ in [2]) quark and gluon distribution were parametrized as functions of $x$. (The number of the parameters was 12 in [1] and 8 in [2]). Then NLO evolution equations were solved and the values of the parameters were determined from the best fit at all data points. The numerical values presented in Table 1 correspond to $\overline{MS}$ regularization scheme, statistical, systematical, as well as theoretical errors arising from uncertainty of $\alpha_s$ in the evolution equations, are added in quadratures. In the last line of Table 1 the Ellis-Jaffe (EJ) and Bjorken (Bj) sum rules prediction for $\Gamma_p$, $\Gamma_n$ and $\Gamma_p - \Gamma_n$, correspondingly are given. The EJ sum rule prediction was calculated according to (3), where $\Delta s = 0$, i.e., $\Sigma = a_8 = 0.59$ was put and the last-gluonic term in (3) was omitted. The twist-4 contribution was accounted in the Bj sum rule and included into the error in the EJ sum rule. The $\alpha_s$ value in the EJ and Bj sum rules calculation was chosen as $\alpha_s(5 GeV^2) = 0.276$, corresponding to $\alpha_s(M_Z) = 0.117$ and $\Lambda^{(3)}_{\overline{MS}} = 360 MeV$ (in two loops). As is clear from Table 1, the data, especially for $\Gamma_n$, contradict the EJ sum rule. In the last column, the values of $\alpha_s$ determined from the Bj sum rule are given with the account of twist-4 corrections.

The experimental data on $\Gamma_p$ presented in Table 1 are not in a good agreement. Particularly, the value of $\Gamma_p$ given by E154 Collaboration seems to be low: it does not agree with the old data presented by SMC [20] ($\Gamma_p = 0.136 \pm 0.015$) and E143 [19] ($\Gamma_p = 0.127 \pm 0.011$). Even more strong discrepancy is seen in the values of $\alpha_s$, determined from the Bj sum rules. The value which follows from the combined analysis is unacceptably low: the central point corresponds to $\alpha_s(M_Z) = 0.126 \pm 0.009$. Therefore, I come to a conclusion that at the present level of experimental accuracy $\alpha_s$ cannot be reliably determined from the Bj sum rule in polarized scattering.

Table 2 shows the values of $\Sigma$ – the total nucleon spin projection carried by $u, d$ and $s$-quarks found from $\Gamma_p$ and $\Gamma_n$ presented in Table 1 using eq.(3). (It was put $g_A = 1.260, a_8 = 0.59$, the term, proportional to $\Delta g$ is included into $\Sigma.$).
Table 2: The values of $\Sigma$

|             | From $\Gamma_p$ | From $\Gamma_n$ | From $\Gamma_p$ | From $\Gamma_n$ |
|-------------|----------------|----------------|----------------|----------------|
|             | $\alpha_s(5GeV^2) = 0.276$ | $\alpha_s(5GeV^2) = 0.276$ | $\alpha_s(5GeV^2) = 0.264$ | $\alpha_s(5GeV^2) = 0.266$ |
| SMC         | 0.256          | 0.254          | 0.264          | 0.266          |
| Comb.       | 0.350          | 0.250          | 0.145          | 0.225          |
| E154        | 0.070(0.14; 0.25) | 0.133(0.20; 0.30) | 0.19(0.25; 0.14) | 0.144 (0.205; 0.10) |

In their fitting procedure [2] E154 Collaboration used the values $a_8 = 0.30$ and $g_A = 1.09$. The values of $\Sigma$ obtained from $\Gamma_p$ and $\Gamma_n$ given by E154 at $a_8 = 0.30, g_A = 1.26$ and $a_8 = 0.30, g_A = 1.09$ are presented in parenthesis. The value $a_8 = 0.30$ corresponds to a strong violation of SU(3) flavour symmetry and is unplausible; $g_A = 1.09$ means a bad violation of isospin and is unacceptable. As seen from Table 1, $\Sigma$ is seriously affected by these assumptions. The values of $\Sigma$ found from $\Gamma_p$ and $\Gamma_n$ using SMC and combined analysis data agree with each other only, if one takes for $\alpha_s(5GeV^2)$ the values given in Table 1 ($\alpha_s = 0.116$ for combined data), what is unacceptable. The twist-4 corrections were not accounted in $\Sigma$ in Table 2: their account, using eqs.(7), (8), results in increasing of $\Sigma$ by 0.04 if determined from $\Gamma_p$ and by 0.03 if determined from $\Gamma_n$.

To conclude, one may say, that the most probable value of $\Sigma$ is $\Sigma \approx 0.3$ with an uncertain error. The contribution of gluons may be estimated as $\Delta g(1GeV^2) \approx 0.3$ (see [5], [21], [22]). Then $\Delta g(5GeV^2) \approx 0.6$ and the account of gluonic term in eq.(3) results in increasing of $\Sigma$ by 0.06. At $\Sigma = 0.3$ we have $\Delta u = 0.83, \Delta d = -0.43, \Delta s = -0.1$.

3. Low and Intermediate $Q^2$

The problem of a smooth connection of the Gerasimov-Drell-Hearn (GDH) sum rules [3,4] which holds at $Q^2 = 0$, and the sum rules at high $Q^2$ attracts attention in the last years [5, 23-25].

In order to connect the GDH sum rule with $\Gamma_{p,n}(Q^2)$ consider the integrals [26]

$$I_{p,n}(Q^2) = \int_{Q^2/2}^{\infty} \frac{d\nu}{\nu} G_{1,p,n}(\nu, Q^2)$$

Changing the integration variable $\nu$ to $x$, (9) can be also identically written as

$$I_{p,n}(Q^2) = \frac{2m^2}{Q^2} \int_0^1 dx g_{1,p,n}(x, Q^2) = \frac{2m^2}{Q^2} \Gamma_{p,n}(Q^2)$$

At $Q^2 = 0$ the GDH sum rule takes place

$$I_p(0) = -\frac{1}{4} \kappa_p^2 = -0.8035; \quad I_n(0) = -\frac{1}{4} \kappa_n^2 = 0.9149; \quad I_p(0) - I_n(0) = 0.1144$$

where $\kappa_p$ and $\kappa_n$ are proton and neutron anomalous magnetic moments.

The schematic $Q^2$ dependence of $I_p(Q^2), I_n(Q^2)$ and $I_p(Q^2) - I_n(Q^2)$ is plotted in Fig.1. The case of $I_p(Q^2)$ is especially interesting: $I_p(Q^2)$ is positive, small and decreasing at $Q^2 \gtrsim 3GeV^2$ and negative and relatively large in absolute value at $Q^2 = 0$. With
The situation is similar. All this indicates large nonperturbative effects in $I(Q^2)$ at $Q^2 \lesssim 1 \text{GeV}^2$.

In [23] the model was suggested, which describes $I(Q^2)$ (and $\Gamma(Q^2)$) at low and intermediate $Q^2$, where GDH sum rules and the behaviour of $I(Q^2)$ at large $Q^2$ where fullfilled. The model had been improved in [24]. (Another model with the same goal was suggested by Soffer and Teryaev [25]).

Since it is known, that at small $Q^2$ the contribution of resonances to $I(Q^2)$ is of importance, it is convenient to represent $I(Q^2)$ as a sum of two terms

$$I(Q^2) = I^{\text{res}}(Q^2) + I'(Q^2),$$

where $I^{\text{res}}(Q^2)$ is the contribution of baryonic resonances. $I^{\text{res}}(Q^2)$ can be calculated from the data on electroproduction of resonances. Such calculation was done with the account of resonances up to the mass $W = 1.8 \text{GeV}$ [27].

In order to construct the model for nonresonant part $I'(Q^2)$ consider the analytical properties of $I(Q^2)$ in $q^2$. As is clear from (9),(10), $I(Q^2)$ is the moment of the structure function, i.e. it is a vertex function with two legs, corresponding to ingoing and outgoing photons and one leg with zero momentum. The most convenient way to study of analytical properties of $I(q^2)$ is to consider a more general vertex function $I(q_1^2, q_2^2; p^2)$, where the momenta of the photons are different, and go to the limit $p \to 0$, $q_1^2 \to q_2^2 = q^2$. $I(q_1^2, q_2^2; p^2)$ can be represented by the double dispersion relation:

$$I(q^2) = \lim_{q_1^2 \to q_2^2 = q^2, p^2 \to 0} I(q_1^2, q_2^2; p^2) = \left\{ \int ds_2 \int ds_1 \frac{\rho(s_1, s_2; p^2)}{(s_1 - q_1^2)(s_2 - q_2^2)} + P(q_1^2) \int \frac{\varphi(s, p^2)}{s - q_2^2} ds + P(q_2^2) \int \frac{\varphi(s, p^2)}{s - q_1^2} ds \right\}_{q_1^2 = q_2^2 = q^2, p \to 0}$$

The last two terms in (13) are the subtraction terms in the double dispersion relation, $P(q^2)$ is the polynomial. According to (10), $I(q^2)$ decreases at $|q^2| \to \infty$, $P(q^2) = \text{Const}$ and the constant subtraction term in (13) is absent. We are interesting in $I(Q^2)$ dependence in the domain $Q^2 \lesssim 1 \text{GeV}^2$. Since after performed subtraction, the integrals in (13) are well converging, one may assume, that at $Q^2 \lesssim 2 - 3 \text{GeV}^2$ the main contribution comes from vector meson intermediate states, so the general form of $I'(Q^2)$ is

$$I'(Q^2) = \frac{A}{(Q^2 + \mu^2)^2} + \frac{B}{Q^2 + \mu^2},$$

where $A$ and $B$ are constants, $\mu$ is $\rho$ (or $\omega$) mass. The constants $A$ and $B$ are determined from GDH sum rules at $Q^2 = 0$ and from the requirement that at high $Q^2 \gg \mu^2$ takes place the relation

$$I(Q^2) \approx I'(Q^2) \approx \frac{2m^2}{Q^2} \Gamma^{\text{as}}(Q^2),$$

where $\Gamma^{\text{as}}(Q^2)$ is given by (3). ($I^{\text{res}}(Q^2)$ fastly decreases with $Q^2$ and is very small above $Q^2 = 3 \text{GeV}^2$). These conditions are sufficient to determine in unique way the constant $A$ and $B$ in (14). For $I'(Q^2)$ it follows:
\[ I'(Q^2) = 2m^2 \Gamma^{as}(Q^2_m) \left[ \frac{1}{Q^2 + \mu^2} - \frac{c\mu^2}{(Q^2 + \mu^2)^2} \right], \]  
\[ c = 1 + \frac{\mu^2}{2m^2} \frac{1}{\Gamma^{as}(Q^2_m)} \left[ \frac{1}{4\kappa^2 + I^{res}(0)} \right], \]

where \( I^{res}(0) = -1.03, I^{res}_n(0) = -0.83 \) [24].

The model and eq.16 cannot be used at high \( Q^2 \gg 5 GeV^2 \): one cannot believe, that at such \( Q^2 \) the saturation of the dispersion relation (13) by the lowest vector meson is a good approximation. For this reason there is no matching of (16) with QCD sum rule calculations of twist-4 terms. (Formally, from (16) it would follow \( b_{p-n} \approx -0.15, b_{p+n} \approx -0.07 \). It is not certain, what value of the matching point \( Q^2_m \) should be chosen in (16). This results in 10% uncertainty in the theoretical predictions. Fig.'s 2,3 shows the predictions of the model in comparison with recent SLAC data [28], obtained at low \( Q^2 = 0.5 \) and 1.2\( GeV^2 \) as well as SMC and SLAC data at higher \( Q^2 \). The chosen parameters are \( \Gamma_p(Q^2_m) = 0.142, \Gamma_n(Q^2_m) = -0.061 \), corresponding to \( c_p = 0.458, c_n = 0.527 \) in (16),(17). The agreement with the data, particularly at low \( Q^2 \), is very good. The change of the parameters only weakly influences \( \Gamma_{p,n}(Q^2) \) at low \( Q^2 \).

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Figure Captions

Fig. 1  The $Q^2$-dependence of integrals $I_p(Q^2), I_n(Q^2), I_p(Q^2) - I_n(Q^2)$. The vertical axis is broken at negative values.

Fig. 2  The $Q^2$-dependence of $\Gamma_p = \Gamma'_p + \Gamma^{res}_p$ (solid line), described by eqs.(12,16,17). $\Gamma^{res}_p$ (dotted) and $\Gamma'_p$ (dashed) are the resonance and nonresonance parts. The experimental points are: the dots from E143 [28], the square - from E143 (SLAC) [19], the cross - SMC-SLAC combined data [1], the triangle from SMC [1].

Fig. 3  The same as in Fig.2, but for neutron. The experimental points are: the dots from E143 (SLAC) measurements on deuteron [28], the square at $Q^2 = 2GeV^2$ is the E142(SLAC) [29] data from measurements on polarized $^3He$, the square at $Q^2 = 3GeV^2$ is E143(SLAC) [30] deuteron data, the cross is SMC-SLAC combined data [1], the triangle is SMC deuteron data [1].
Figure 1:
Figure 2:
Figure 3: