Motion-induced inertial effects and topological phase transitions in skyrmion transport

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Abstract
When the skyrmion dynamics beyond the particle-like description is considered, this topological structure can deform due to a self-induced field. In this work, we perform Monte Carlo simulations to characterize the skyrmion deformation during its steady movement. In the low-velocity regime, the deformation in the skyrmion shape is quantified by an effective inertial mass, which is related to the dissipative force. When skyrmions move faster, the large self-induced deformation triggers topological transitions. These transitions are characterized by the proliferation of skyrmions and a different total topological charge, which is obtained as a function of the skyrmion velocity. Our findings provide an alternative way to describe the dynamics of a skyrmion that accounts for the deformations of its structure. Furthermore, such motion-induced topological phase transitions make it possible to control the number of ferromagnetic skyrmions through velocity effects.

Keywords: skyrmions, Monte Carlo, effective mass, topological transition, deformation

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent decades, physical systems with topological protection have been a focus of extensive research [1–4]. Particle-like excitations, characterized by nontrivial topological invariants, cannot be continuously deformed to another state with a different topology unless enough extra energy is injected into the system. Magnetic skyrmions [5, 6] are an example of topologically stable spin structures. These textures, characterized by a topological charge, have been predicted [7–13] in magnetic systems with Dzyaloshinskii–Moriya interactions (DMIs) [14, 15], known as chiral magnets. The observation of magnetic skyrmions in noncentrosymmetric crystals [13, 16–18], cubic helimagnets [19, 20], and ultrathin films [21, 22] has opened the door to potential functionalities such as the ability to serve as information bits in spin-based devices [23–26] or neuromorphic systems [27–29]. Although it is known that applications of skyrmions in spintronics demand specific conditions such as low current densities or temperature gradients [26, 30–41], the difficulties encountered in nucleating skyrmions and displacing them in straight lines along the applied currents [33, 42, 43] have hampered progress in skyrmion-based spintronics.

The dynamics of magnetic skyrmions are usually described by the Thiele’s equation [44], which is derived from the...
Landau–Lifshitz–Gilbert equation \cite{7, 45–48} under the assumption of a rigid structure. However, if the rigidity condition is relaxed, the skyrmions are allowed to deform, and in this case, the magnetic texture becomes dependent on its own velocity. As a result, the effective dynamics are now captured by an inertial Thiele’s equation \cite{36, 49, 61}, i.e., in which the mass appears as the capacity of the system to store kinetic energy during motion \cite{49–52, 50, 51, 53, 54}. The inertial dynamics of skyrmions have been previously theoretically studied \cite{36, 50, 51} and observed in magnetic Co/Pt \cite{53} and FeGe disks \cite{54}. Recently, it has been shown \cite{55} that beyond the small-driving-current regime, skyrmion deformation in the form of an expanding size and a noncircular shape occurs during skyrmion transport. This motion-driven deformation yields interesting nonlinear effects in the relationship between the charge current and skyrmion velocity.

Topological transitions have been predicted for certain types of magnetic textures such as a meron \cite{56, 57}, i.e., a magnetic vortex with a core. It has been shown that under large applied charge currents, the ultrafast switching of the vortex polarity generates a magnetic singularity during vortex annihilation \cite{58–60}. This process, mediated by the appearance of an unstable antivortex, induces a change in the topological charge, \( q = pw \), where \( p \) and \( w \) are the vortex polarity and chirality, respectively. Topological transitions in skyrmion systems have recently been observed in magnetic multilayers \cite{61}, in which the number of skyrmions is thermally controlled. Despite some related efforts \cite{62}, a systematic study of the topological transitions of skyrmions induced by magnetic fields, temperature or charge currents has not been reported.

In this paper, we study the self-induced deformation of moving skyrmions at low temperature. Using Monte Carlo simulations, we characterize the shapes of current-induced skyrmions in terms of the skyrmion velocity. We analyze two main regimes to describe the skyrmion profiles of both Bloch- and Néel-type skyrmions. The obtained results reveal that at low velocities, the predicted effective field generated by a current-induced motion \cite{49} slightly modifies the skyrmion shape. This shape deformation is quantified by a mass term, which is determined as a function of the velocity and found to be approximately constant for very low speeds. For high velocities, we find that Bloch-type skyrmions experience abrupt transitions that increase the total topological charge. This type of topological transition is not observed for Néel-type skyrmions.

We find that at certain threshold velocities, the self-induced field provides sufficient magnetic energy to deform the skyrmion texture and increasing its topological charge. Since topological phase transitions are not observed for Néel-type skyrmions, most of the results presented in this work concern Bloch-type skyrmions. It is worth noting that most of the experimental evidence collected from conducting materials describes Néel-type skyrmions \cite{42, 43, 63, 64}. However, Bloch-type skyrmions have recently been observed in Heusler magnets \cite{65} and Fe/Gd multilayers \cite{66}, and their dynamics have been considered in various theoretical works \cite{36, 49, 51, 52}. Montoya et al \cite{66} reported a method of controlling material properties and magnetic fields to obtain dipole-stabilized skyrmions such that a rich variety of domain structures (among them, Bloch-type skyrmions) were obtained in Fe/Gd multilayers. Notably, although we assume that the spin-transfer torque effect induces translational motion, our predictions are not restricted to electrical driving currents. Instead, the analysis can be properly generalized to other mechanisms, such as temperature gradients \cite{41} and magnon-driven skyrmion motion \cite{52, 79}.

This paper is organized as follows. In section 2, we present the theoretical model and details of the Monte Carlo simulations. Section 3 presents our results and related discussions. In section 4, we summarize the conclusions of this study and future research prospects.

2. Model and numerical methods

2.1. Classical spin Hamiltonian

We consider a ferromagnet on a hexagonal lattice, as shown in figure 1. The considered system is described by the Hamiltonian

\[
H = -J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \mu \sum_i \mathbf{B} \cdot \mathbf{S}_i - K \sum_i \mathbf{S}_i^z - \sum_{\langle ij \rangle} \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j),
\]

where \( \langle \cdot \rangle \) denotes summation over nearest neighbor (NN) lattice sites. In equation (1), \( J > 0 \) is the ferromagnetic exchange coupling constant between NN spins, \( K \) is the easy-axis magnetic anisotropy constant, and \( \mathbf{B} \) represents the applied magnetic field along the \( z \)-direction. In addition, DMI are included to stabilize the magnetic skyrmion. The DMI favor a canting orientation between NN spins and are characterized by the vector \( \mathbf{D}_{ij} = D \hat{d}_{ij} \), where \( D \) is the DMI strength. The pattern of the magnetization vector depends on whether \( \hat{d}_{ij} \) is perpendicular (Bloch-type) or parallel (Néel-type) to the vector \( \mathbf{r}_{ij} \) that connects two neighboring magnetic moments. In this work, we consider both Bloch-type (\( r_{ij} \times \hat{d}_{ij} = 0 \)) and Néel-type (\( r_{ij} \times \hat{d}_{ij} = 1 \)) skyrmions. However, most of the presented results concern Bloch-type skyrmions.

2.2. Self-induced field due to skyrmion motion

Let us consider the dynamics of a single skyrmion induced by electrical currents. It is established that in the skyrmion center of mass reference frame, the skyrmionic magnetization profile is affected by an additional magnetic field \cite{49}

\[
\mathbf{B}_{si} = \frac{2 \pi}{\gamma a} \mathbf{S} \times (\mathbf{v} \cdot \nabla) \mathbf{S},
\]

which linearly depends on the skyrmion velocity \( \mathbf{v} \) and its magnetization \( \mathbf{S} \). In the previous expression, \( a \) is the lattice constant, and \( \gamma = g_\mu_B / \hbar \approx 1.76 \times 10^{11} \text{ rad/s/T} \) is the gyromagnetic ratio, where \( g_\mu_B \) is the Landé factor. One can also notice that the strength of the effective field is confined within the perimeter of the skyrmion. Additionally, because the effective torque takes a maximum value in the center of skyrmion, it suffers a distortion of its shape due to the current-induced motion. Thus, as the skyrmion moves, the field \( \mathbf{B}_{si} \) modifies the shape of its spin texture. The magnetization pattern changes
gradually until the system reaches the steady state, resulting in an asymmetrically deformed skyrmion [55].

The general properties of the skyrmions can be captured by considering them as rigid structures, and their dynamics can be described by Thiele’s equation [44], which determines the skyrmion position as a function of an external stimuli (electric current, magnetic field, temperature gradient, etc). Some works analyze the skyrmion dynamics under this framework by considering them as circular-shaped structures (see for instance [48, 53, 72] and references therein). However, micromagnetic simulations [50, 51] and experimental works [43, 73, 74] revealed the existence of inertial effects from skyrmion deformations during its steady motion. Thus, a better description of skyrmion dynamics should consider changes in its shape during its displacement. Therefore, recent theoretical works describing the skyrmion motion consider it as a rigid structure with a non-circular shape [49, 51, 75] that deforms as a function of its velocity. A complete analysis describing the skyrmion motion considering dynamical deformations during its displacement is an open issue. Based on the above, we consider that the magnetization profile is represented by the ansatz $S(r, t) = S_0(r) + \lambda \xi S_0(r) \times (v \cdot \nabla) S_0(r)$, where $S_0(r)$ is the magnetization texture of the static skyrmion. The dimensionless factor $\lambda$ determines the strength with which the skyrmion is deformed, and $\xi = \hbar^2 / J a^2$, where $\ell$ is the skyrmion size. Using this approximation, the elements of the mass tensor $\mathcal{M}$ can be readily obtained from the dissipative term in the Thiele’s equation and the skyrmion charge $Q$. In particular, the diagonal elements describe the effective scalar mass and satisfy $\mathcal{M}_{xx} = \mathcal{M}_{yy} = \lambda \xi D_{xx}$, while the nondiagonal terms are given by $\mathcal{M}_{xy} = -\mathcal{M}_{yx} = 4\pi \lambda \xi \alpha Q$. In addition, since $Q$ is invariant under smooth deformations, the nondiagonal elements of the mass matrix do not change when the skyrmion is moving slowly. However, for high velocities, the shape deformation is sufficiently large that, as we will show later, a topological transition may occur.

2.3. Monte Carlo simulations

To investigate the steady-state profile of a moving skyrmion, we use standard Monte Carlo simulations. Based on the Metropolis algorithm [68], the stable magnetic state for the Hamiltonian given in (1) is determined. The temperature of the system $T$ is assumed to be sufficiently low to obtain an equilibrium magnetization configuration that results in a circular-shaped Neél or Bloch-type skyrmion. The simulation is performed as follows: for a specific site, we propose a random change in the orientation of its magnetic moment. The criterion for accepting a change in the orientation of such a single magnetic moment is given by an acceptance probability of $p = \exp \left[ -\Delta H / k_B T \right]$, where $\Delta H$ is the change in the energy of the system. If $p < r$, where $r$ is a random number in the range $[0, 1]$, then the reorientation is not accepted. One Monte Carlo step (MCS) is defined as $N$ single-site attempts to change the orientations of magnetic spins at different lattice sites. This procedure was performed in eight independent Monte Carlo simulations considering different seeds for the random number sequences. The final result is given by the averaged value of the magnetization obtained from these independent Monte Carlo simulations.

The effects of the skyrmion velocity can be taken into account by considering the presence of the self-induced field $B_\xi$. For simplicity, we assume a constant velocity along the $x$-direction, $v = \nu \xi$. The field $B_\xi$, which is computed from

![Figure 1. Representation of a magnetic skyrmion centered at the hexagonal spin lattice with $N = 631$ sites. The color code indicates the $S_z$ component of the magnetization. The inset indicates the $i$th lattice site and its NNs.](image-url)
Figure 2. (a) and (b) depict the vector field $(S_x, S_y)$ of the magnetization for Bloch and Néel-type skyrmions, respectively. (c) $S_z$ component of the deformed Bloch skyrmion profile and (d) the corresponding $z$-component of the self-induced field $B_{si}$ of a configuration obtained from MC simulations at a low velocity $v' = 0.005$. At panels (e) and (f) we plot, respectively, the same quantities as (c) and (d) for a velocity $v' = 0.3$. Note the asymmetry in the skyrmion shape for the later case.

equation (2), is included in the Hamiltonian (1) in addition to $B$. This new magnetization pattern, in turn, creates a new self-induced field $B_{si}$, thereby generating a new skyrmion texture $S(r, t)$.

This procedure is repeated until the skyrmion profile stabilizes. Once we obtain the stable configuration for the deformed skyrmion, the dissipative matrix $D$ is determined, and consequently, the effective skyrmion mass $M$ is obtained for different velocities. Additionally, we calculate the total topological charge $Q$ of the system to characterize the existing topological states and transitions.

3. Results

We consider a system with $N = 631$ Heisenberg spins on a two-dimensional triangular lattice with a hexagonal boundary shape (see figure 1) and helical boundary conditions [69, 70]. This system yields a configuration with 15 particles at the side of the lattice. The magnetic parameters used are defined in terms of $J$, i.e., an external out-of-plane magnetic field $\mu B/J = 0.1$, DMI and anisotropy constants given by $D/J = 0.32$ and $K/J = 0.07$, respectively. Stable states are obtained for a thermal energy of $k_B T/J = 0.001$, which corresponds to an absolute temperature of $\sim 80$ mK. As discussed in reference [69], these parameters are adequate to generate a stable skyrmion in a hexagonal lattice. For each set of parameters, we performed $5 \times 10^5$ MCS. To perform our simulations, it is convenient to introduce a dimensionless field $B_{si}$ from multiplying both sides of equation (2) by $\mu/J$. In this case, the right side of equation (2) yields a dimensionless velocity, $v' = (2\pi \mu / (J\gamma a))v$. In the simulations, we considered parameter values similar to those experimentally estimated [22, 69] for Pd/Fe/Ir(111), given by $\mu = 3\mu_B$, $J = 7$ meV, $a = 2.7 \times 10^{-10}$ m. We first validated our numerical approach by performing simulations for a skyrmion at rest, i.e., $|B_{si}| = 0$. As expected, the magnetic configuration of the system consists of a circularly symmetric Bloch-type skyrmion, as shown in figures 2(a) and (b).

3.1. Skyrmion deformation and inertial mass

Now, we focus on the effects originating from the self-induced field $B_{si}$ in the skyrmion profile. From the performed simulations, we find that despite the different orientations of the
magnetization vector fields of Néel- and Bloch-type skyrmions, the results for the shape deformation and inertial mass are the same (at least for velocities $v' < 0.5$). Therefore, in this section, we present only our results for Bloch-type skyrmions. The initial configuration for the simulations with $|B_s| = 0$ consists of a skyrmion pattern obtained for $v = 0$. For a low velocity, $v' = 0.005 \hat{x}$ ($v = 1.5 \text{ m/s}$), the $z$ component of the resulting skyrmion profile is displayed in figure 2(c). It can be seen that the deformation in the skyrmion shape is small, and thus, its profile is almost axially symmetric. This observation is consistent with the low values obtained for the field $|B_s| \approx 0.001 J$ (see figure 2(d)), which are notoriously smaller than other relevant magnitudes in the system; therefore, the skyrmion mass is expected to keep almost constant up to a certain value of the velocity. In this case, due to the small deformations, the skyrmion can move in the so-called linear regime [55]. Indeed, experimental observations have predicted that in the limit of small velocities, the skyrmion magnetization profile presents a circular-shaped structure [73]. This effect will be evident subsequently when the dissipative force term is calculated. However, larger values of the skyrmion velocity lead to significant changes in its shape. This effect will be evident subsequently when the dissipative force term is calculated. Indeed, as shown in figure 2(e), the magnetization profile is clearly deformed for $v' = 0.3 \hat{x}$. The shape deformation is also confirmed by calculating the field $B_s$ in figure 2(f). From the experimental point of view, the increase in the skyrmion velocity yields large deformations to its shape [43, 73].

Since an increasing skyrmion velocity leads to progressively larger changes in its shape, we now study the motion-induced effects on the effective mass. The skyrmion effective mass is endowed by the topological confinement of the skyrmion and the energy associated with its size change [53]. As discussed in section 2.2 (and in reference [35]), the skyrmion mass can be directly related to the diagonal elements of the matrix $D$. The results depicted in figure (3) indicate that the normalized dissipative parameters $D_{xx}/D_r$ and $D_{yy}/D_r$ remain almost constant ($D_r = 4.78 \pi$) up to $v' \approx 0.06$. In this case, a small split between $D_{xx}$ and $D_{yy}$ is observed. Nevertheless, a clear split between $D_{xx}$ and $D_{yy}$ occur for $v' \geq 0.09$. This result indicates that higher anisotropic resistance to the skyrmion motion occurs as $v'$ increases. Thus, the effective mass will be larger along the direction perpendicular to the skyrmion velocity. This behavior of the skyrmion mass is directly linked to the increase in the driving forces and skyrmion mobility under high driving currents [55]. This effect can be related to a velocity-dependent Hall angle [43] in the skyrmion displacement. It is important to mention that our results do not include temperature effects that might induce changes in the material parameters [71] and, thus, in the skyrmion velocity [73].

3.2. Motion-induced topological phase transitions

Litzius et al [73] showed that the maximum skyrmion velocity in ferromagnetic devices is limited by a mechanism based on skyrmion surface tension and deformation. In this case, the skyrmion shape is deformed into a stripe. Therefore, in this section, we investigate deformations in the skyrmion texture with a focus on the high-velocity regime. Our simulations reveal that at velocities up to $v' = 0.5$, the shape of a Bloch-type skyrmion is strongly distorted without changing the topological charge ($Q = 1$) of the system. However, for velocities in the range $0.5 \leq v' \leq 0.6$, the number of skyrmions is altered. This effect is observed in figure 4(a) for $v' = 0.6$, where we display a pair of skyrmions with a total charge of $Q = 2$. On the other hand, in the case of Néel-type skyrmions, no change in the number of skyrmions is observed. Indeed, an increase in the velocity up to $v' = 1.2$ yields larger deformations, but no topological transition is observed for Néel-type skyrmions. Therefore, from here on, we will discuss only results for Bloch-type skyrmions. In this case, the increase in the number of skyrmions originates from the large self-induced field, which results in an abrupt deformation of the Bloch-type skyrmion texture. Since a change in $Q$ characterizes a transition, the appearance of a new skyrmion is considered a topological transition. Note that the pair of skyrmions in figure 4(a) does not return to the original state, characterized by $Q = 1$, even when the velocity is again reduced to zero. Indeed, although the $Q = 1$ state has lower energy than states with $Q = 2$ for $v' = 0$, there is an energy cost to be overcome (topological protection) for the system to return to its original state. Therefore, in the absence of any extra energy source, the system remains in the new state, and no new topological transition is observed. In the limit of higher velocities, e.g., $v' = 2$, the skyrmion texture is destroyed, in agreement with the results of reference [55]. The increase in the number of skyrmions at high current densities and impurities has previously been observed in reference [76], where the authors showed that at a certain current density threshold, the skyrmion suffers a substantial distortion, and consequently, skyrmion multiplication is observed. Since skyrmion motion can be induced, for example, by electrical currents [77], we estimate the current density necessary to induce the transition (shown in figure 4) as $j \approx 1.64 \times 10^{12} \text{ A/m}^2$. This value is obtained by considering $\Delta \rho_v^\infty \approx 3 \times 10^{-11} \text{ cm}^2$ and $\Delta \rho_y^\infty \approx 4 \times 10^{-11} \text{ cm}^2$ for Pd/Fe/Ir(111).
Figure 4. Topological transition of Bloch-type skyrmions. (a) $S_z$ component of the magnetization profile of a system with 15 sites per side, at $v' = 0.6$. (b) $S_z$ component of the magnetization profile of a system with 30 sites per side, at $v' = 0.8$. (c) Topological charge of the system as a function of the skyrmion velocity. At the inset we display the topological charge as a function of MCS for various velocities.

We now study the role of the system size and skyrmion velocity in determining the number of skyrmions nucleated due to such a topological transition. Because the skyrmion size is typically comparable to the length of the spiral determined by the competing influences of the DMI and the exchange, anisotropy and Zeeman interactions [7, 32], one might expect that the number of skyrmions should be limited by the lattice size. Indeed, for a system with 15 sites per side, a pair of skyrmions emerges in the stable state, as shown in figure 4(a). However, for a system with 30 sites per side and a velocity of $v' = 0.8$, simulations reveal that $Q = 9$, as shown in figures 4(b) and (c), after a total of $N_{\text{MC}} = 9 \times 10^7$ MCS starting from the same initial conditions as in the case presented in figure 4(a). Nevertheless, in addition to the lattice size, the skyrmion velocity also contributes to determining the total number of skyrmions. The relation between the velocity and the number of skyrmions can be determined from the total topological charge when the system reaches the steady state. Figure 4(c) depicts the value of $Q$ as a function of the skyrmion velocity for $N_{\text{MC}} = 9 \times 10^7$ MCS. The inset depicts the evolution of the topological charge with increasing MCS for various velocities. Note that $Q$ saturates at $v' \geq 1.1$. This upper bound ($Q_u$) is due to the lattice size, and when 30 sites per side are considered, we find that $Q_u = 13$, which corresponds to the maximum number of skyrmions that can appear in the system. The maximum number of skyrmions can be determined based on the skyrmion radius, $\rho \approx 2J/D = 6.25$ atomic sites [32]. Thus, the area of each skyrmion is approximated as $A_s = \pi \rho^2 \approx 123$ sites. The total area of a lattice with 30 sites per side contains $N = 3018$ sites. Accordingly, under the assumption that the spacing between two neighboring skyrmions occupies approximately the same number of sites as one skyrmion, we find that the maximum number of skyrmions in a lattice with 30 sites per side is $N_s \approx 3018 / 125 \approx 12.3$ skyrmions, close to the value obtained from our simulations.

To understand the evolution of a Bloch-type skyrmion through the topological transition, we obtained the magnetization profile as a function of MCS and the corresponding magnetic energy obtained from equation (1), i.e., $E = H$ with $B$ including the contribution of the self-induced field $B_{si}$. Figure 5(a) presents the obtained results for a specific simulation near the moment of topological transition. In this case, we find that for a lattice with 15 sites per side, the topological transition occurs between $1.1 \times 10^7$ MCS and $1.15 \times 10^7$ MCS. An analysis of figure 5(a) I, II, and III reveals that the topological transition is mediated by a continuous deformation of the skyrmion, which takes a so-called worm-like shape [73, 76, 78]. The deformation evolves until skyrmion break-up occurs (see figure 5(a) IV, V, and VI). As expected, the topological transition is associated with an abrupt change in energy, as illustrated in figure 5(c).

To complete our analysis, we have studied the underlying mechanism for the absence of topological transition in Neel-type skyrmions. We have then performed simulations in which
the initial state of the Néel skyrmion is the worm-like state, as that one shown in figure 5(a)-I, for the Bloch skyrmion. Because the simulations regarding Néel-type skyrmion do not naturally reach the worm-like configuration, we have artificially started the magnetization pattern of a Néel skyrmion from this state. To construct this initial profile, we change the phase of all magnetic moments presented in figure 5(a)-I by an angle of $\pi/2$, obtaining the profile depicted in figure 5(b)-I. Monte Carlo simulations revealed that Néel skyrmions do not reach this state because it increases the magnetic energy with respect to the state shown in figure 5(b)-VI. The reduction in the Néel skyrmion’s energy when it recovers the elliptical shape is shown in figure 5(d), evidencing that the topological transitions do not occur due to the energetic cost to reach the worm-like pattern, which is the natural way to yield a skyrmion break-up. A complete understanding of Néel and Bloch skyrmions regarding the topological transition is an open issue.

Finally, from the above described results, one can notice that the used approach is able to depict effects of current-driven motion of the skyrmions. Indeed, our results show that the self-induced magnetic field appearing during the skyrmion motion does not change only the shape of skyrmions (deviation from the prefect circular shape) (figure 2(e)), but also the internal structure of the domain wall, the surface (enclosed area) of the skyrmion (figures 2(e) and 5(a)), and the internal structure of the domain wall (figure 5(a)).

4. Conclusions

Using Monte Carlo simulations, we studied the influence of the motion of a skyrmion on its shape in the ranges of low and high velocities. Our results show that, depending on the velocity, a non-negligible self-induced effective field ($B_{si}$) is generated, which induces changes in the skyrmion shape.

In the low-velocity regime, we determined this effective field and used it to obtain the skyrmion mass. For velocities above $v' = 0.3$ (corresponding to $v = 27.4$ m s$^{-1}$), the field $B_{si}$ induces deformations in the skyrmion shape. These deformations break the circular symmetry and lead to an anisotropic mass. The effective mass $M_{xx}$ decreases as a function of the velocity, while $M_{yy}$ increases. If the mechanism that enables skyrmion motion is turned off in the low-velocity regime, the skyrmion returns to its original circular symmetric shape. For very low velocities (i.e., lower than $v = 27.4$ m s$^{-1}$), $|B_{si}|$ is small compared to the other competing interactions, and the mass of the skyrmion remains almost constant. These results are valid for both Bloch- and Néel-type skyrmions.

We also showed that above a certain threshold velocity, the self-induced field is large enough to modify the winding number of Bloch-type skyrmions. This leads to a topological transition in which the total number of new skyrmions is limited by the lattice size. Just before the topological transition, the skyrmion shape is deformed, showing the so-called worm-like structure. This topological transition is not observed in...
Néel-type skyrmions. We also showed that the skyrmion velocity plays a crucial role in determining the final topological charge of the system, since the number of skyrmions increases with the velocity. Nevertheless, there is an upper bound on the topological charge due to the lattice size, e.g., $Q_{\phi} = 13$ for a system with 30 sites per side. We also confirmed that when the skyrmion velocity drops back to zero, the system does not return to its original state, i.e., a single skyrmion. Instead, the number of skyrmions remains constant. The mechanism underlying the transition, e.g., the emergence of singularities during skyrmion division [58–60], and the dynamics of the topological change are open issues for future investigation.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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