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To cite this article: I V Rudnev et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 456 012087

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Verification and validation of finite-elemental calculation of wooden structures connections on gluedflat steel rods

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Abstract. The article considers the verification and validation processes algorithms of the finite element model calculation results of wooden structures joints with the use of glued steel flat rods. The introduction shows the relevance of the determination processes the accordance between numerical and mathematical models and numerical and physical models accepted for calculation. As an example, methods for calculating multicomponent joints of wooden structures on glued steel flat rods pasted into wood are considered, and structural nodes solutions of wooden trusses on these type connections are given. The second paragraph describes the features of finite element models creation for calculating the stress-strain state of the glued joint of flat rods with wood when they are pulled out with rigid nodes in the conjugation zone of finite elements from various materials and when contact surfaces are created in this zone. A mathematical solution of this problem, reduced to the plane deformation problem for finite canonical domains, is considered. An analytic solution of this three-dimensional problem, reduced to the plane deformation problem for finite canonical domains, is considered. The mathematical solution is based on obtaining functions that are biorthogonal to the Papkovich-Fadl eigenfunctions. The calculation results comparison of the stress-strain state of the finite element model of the steel flat rods’ glued joint with wood, the analytical solution and experiment are given. On the basis of verification and validation it was shown that the proposed approaches to the construction and calculation of the glued joint of steel flat rods with wood can be reasonably applied to determine the stress-strain state and the design of the wooden structures nodes

1. Introduction

Procedure for justifying the correctness of engineering calculations produced by numerical methods proposed particularly by the NAFEMS (National Agency for Finite Element Methods and Standards) organization [1], involves the implementation of verification and validation processes. In particular, such processes allow us to verify the adequacy of finite element models created in CAE-systems and compare the accuracy of calculations with the results obtained on mathematical and physical models with similar parameters [2].

It is especially important to carry out verification and validation in the study of connections and joints of wooden structures with the use of glued steel products as connecting elements, particularly working on pulling out. The numerical models of such connections are quite complex from the point of view of insertion correct initial data, namely: physic mechanical properties of materials in a compound of several components, including the anisotropy of materials, and contact surfaces in
contact areas of different materials. Recently, there was no analytical solution, which could be used to create a mathematical model of pulling a steel rod from a solid wood without assumptions that reduce the calculations accuracy. With the advent of such solution [3], it became possible to conduct calculations verification.

The possibility of using glued steel flat rods in the nodes of wooden structures is justified in a number of works, including works [4]. Constructive solutions of such nodes are developed. Figures 1 and 2 show, for example, sketches of structural solutions of the support unit and the extended joint of the lower belt of trusses, at which the position "1" shows glued into the beam flat steel rods.

![Figure 1. Sketch of the stretched joint of the lower belt of the truss.](image1)

![Figure 2. Sketch of a truss support node.](image2)

The gluing of steel flat rods is carried out on the glue composition based on epoxy ED-20 in the slots previously selected in the wood. Use in the joints of flat rods allows to design a fairly large range of nodes of wooden structures.

2. Verification and validation of the calculation of the glued joint of flat steel rods with wood

The construction of joints using glued steel flat rods caused a necessity of preliminary experimental and numerical study of the stress-strain state of flat rods’ glued joint with wood on specially prepared samples.

For experimental studies of the stress-strain state of the steel flat rods’ glued joint, when they were pulled out from a wooden beam, samples were prepared, which sketch is shown in Figure 3. The sample was tested on a tensile machine, as shown in Figure 4, before destruction, with the fixation of movement $\Delta$ by an electronic indicator with the value of division 0.001 mm.

![Figure 3. Sketch of the sample for testing.](image3)
It was revealed that in the experiment course the destruction of the sample with the parameters shown in Figure 3 occurred at a load of 20 kN. The load is given taking into account statistical processing of tests from 11 samples. The samples broke brittle either from chipping the wood along the fiber in the immediate vicinity of the adhesive composition, or by normal peeling off the surface of the metal. The amount of the plates pulling out from the wood was also insignificant. Its averaged value, determined by four indicators, was 0.124 mm.

Further in the software complex ANSYS Mechanical, solid-state finite element models of the experimental sample with corresponding geometric parameters were created. In the first model, eight node hexahedrons of the "SOLID-185" type are adopted as final elements, which are shown in Figure 5. The conjugation of finite elements from various materials was carried out by rigid nodes.

The second model, shown in Figure 6, was created taking into account the characteristics of the contact surfaces. Target surfaces were assigned to the surfaces of flat steel rods and wood in the contact area with the glue. The target surfaces were covered with volumetric contact elements of the TARGE170 type. The contact surface was the adhesive layer covered with the volumetric contact elements CONTA174. The type of created contact pairs - Bonded - the associated contact, the gap

Figure 4. Sample for testing in grippers of a tensile machine.

Figure 5. Finite element model with rigid nodes.

Figure 6. Finite element model with specified surface contact parameters.
between the bodies is automatically closed, the penetration is ignored. The coefficient of friction is adopted according to the technical characteristics of epoxy adhesives based on epoxy resin ED-20 and amounted to 0.15. Stiffness coefficient by the results of the experiment when the plate pulling out of the wooden beam was 0.1.

The load applied in the models to the free ends of the plates corresponded to an experimental breaking load of 20 kN. The following values of physic mechanical characteristics and elastic constants were introduced into the calculation:

- Material of flat rods - low-carbon steel with a module of normal elasticity of the first kind equal to 200,000 MPa, a Poisson's ratio equal to 0.3, a shear modulus equal to 81,000 MPa, a design tensile strength of 235 MPa;
- The material of the beam is the second-grade pine wood with a modulus of elasticity along the fibers equal to 10,000 MPa, an elastic modulus across the fibers equal to 400 MPa, a Poisson's ratio across the fibers (stresses along the fibers) is 0.45, the Poisson's ratio along the fibers (stresses across the fibers) - 0.018, the shear modulus along and across the fibers is 500 MPa, the design tensile strength along the fibers is 14 MPa, the design resistance to local shearing along the fibers in the adhesive joints is 2.1 MPa;
- Glue is an epoxy composition with an elasticity modulus equal to 3000 MPa, a Poisson's ratio 0.17, a shear modulus of 1250 MPa, and a design tensile strength of 20 kN.

In connection with the exact analytical solution absence of the problem connected with the determination of the joint stress-strain state when a flat rod pulling out from a wooden beam in a three-dimensional setting, the problem of plane deformation was solved. Considering the periodic problem of pulling a steel edge out from a wooden plate, assuming that a flat deformation is realized, and taking the plate to be semi-infinite, we obtain a design scheme of the boundary value problem of the elasticity theory for finite canonical domains with corner points of the boundary and an edge of rigidity, shown in Figure 7.

![Figure 7. The task realization of pulling out a flat rod from wood in a three-dimensional setting into the problem of plane deformation.](image)

The boundary conditions for the value problem are obtained from the equilibrium condition for the edge element and from the accepted condition for the problem periodicity. To construct the solution of the boundary value problem; we apply the method of initial functions [5], which makes it possible to obtain expressions for the Papkovich-Fadl functions:

\[
\sigma_1(x, y) = \frac{sx(\lambda_1, y) \cdot e^{\lambda_1 \cdot x}}{M_1} \cdot \sigma_1 \cdot (1 - a_1 \cdot x) - \sum_{k=1}^{n} 2 \cdot \text{Re} \left( \frac{sx(\lambda_k, y) \cdot e^{\lambda_k \cdot x}}{M_k} \cdot \frac{\text{Im}(\lambda_k \cdot e^{\lambda_k \cdot x})}{\text{Im}(\lambda_k)} \right)
\]

\[
\sigma_2(x, y) = \frac{-sx(\lambda_1, y) \cdot e^{\lambda_1 \cdot x}}{M_1 \cdot (\lambda_1)} \cdot \left( \sigma_1 \cdot (1 + a_1 \cdot x) \right) - \sum_{k=1}^{n} \frac{2 \cdot \text{Re} \left( \frac{sy(\lambda_k, y) \cdot (\lambda_k \cdot e^{\lambda_k \cdot x})}{M_k \cdot (\lambda_k)} \cdot \frac{\text{Im}(\lambda_k \cdot e^{\lambda_k \cdot x})}{\text{Im}(\lambda_k)} \right)}{M_k \cdot (\lambda_k)}
\]
\[
\tau_{,x}(x, y) = -\frac{\xi(x, y) \cdot e^{n \cdot x}}{M_1 \cdot \lambda_k} \cdot \sigma_1 \cdot (\lambda_k^2 + 2 \cdot \text{Re} \left( \frac{\xi(x, y) \cdot \lambda_k}{M_k \cdot \lambda_k} \cdot \sigma_1 \cdot Im(\lambda_k) \right))
\]

\[
u(x, y) = \frac{\chi(x, y) \cdot e^{n \cdot y}}{M_1 \cdot \lambda_k} \cdot \sigma_1 \cdot (-1 + a_1 \cdot x) + 2 \cdot \text{Re} \left( \frac{\chi(x, y) \cdot \lambda_k}{M_k \cdot \lambda_k} \cdot \sigma_1 \cdot Im(\lambda_k) \right)
\]

\[
u(x, y) = \frac{\chi(x, y) \cdot e^{n \cdot y}}{M_1 \cdot \lambda_k} \cdot \sigma_1 \cdot (a_1 \cdot (1 + a_1 \cdot x) + \text{Re} \left( \frac{\chi(x, y) \cdot \lambda_k}{M_k \cdot \lambda_k} \cdot \sigma_1 \cdot Im(\lambda_k) \right))
\]

Here \(\xi(\lambda_k, y), \chi(\lambda_k, y)\) etc. are Papkovich-Fadl functions, and the decomposition factor \(M_k\) is determined by the formula:

\[
M_k = \frac{\sqrt{\nu - 3}}{2} \cdot \lambda_k \cdot \sin(\lambda_k) \cdot \left( \frac{\nu - 3}{4} \cdot \left(1 - \frac{3 \cdot \nu}{4} \right) \cdot \sin(2 \cdot \lambda_k) \cdot \frac{\nu - 3}{8} \right) \cdot D + \sin(2 \cdot \lambda_k)
\]

where \(\nu = 0.3\) is the Poisson’s ratio of steel.

The complex roots of the characteristic equation are denoted by \(\lambda_k\):

\[
\sin^2 \lambda_k - D \frac{\lambda_k}{4} \left( (\nu - 3) \cdot \frac{\sin 2\lambda_k}{2} + (1 + \nu) \lambda_k \right) = 0,
\]

where \(\lambda\) is the real root of the characteristic equation. A stroke above a symbol indicates complex conjugation, the constant \(D\) is determined by the formula:

\[
D = \frac{E_f}{2Gt}.
\]

Below in Table 1 the values of the first five non-zero roots of equation (2) are given, calculated for \(D=0.8\). This value of \(D\) is obtained if we consider the modulus of elasticity of the first kind of a steel flat rod \(E_1 = 200000\) MPa, the modulus of shear along the fibers \(G = 500\) MPa, the thickness of the bar on each side of the plate \(t = 0.04\) m, the cross-sectional area of the bonded plate is \(f = 0.004 \times 0.04\) m.

| № root \(\lambda_k\) | Re \(\lambda_k\) | Im \(\lambda_k\) |
|----------------------|----------------|---------------|
| 1                    | 0.955663736694| 0.000000      |
| 2                    | 3.7919858546  | 1.02289780844|
| 3                    | 6.97447268966 | 1.3240208737 |
| 4                    | 10.1363916285 | 1.5086258873 |
| 5                    | 13.2904346598 | 1.6431247512 |

The final solution, with the exception of the elementary part, is represented in the form of expansions in the Papkovich-Fadl functions.

\[
\xi(\lambda_k, y) = \left( \frac{1 + \nu}{4} \cos \lambda_k + \frac{\nu - 3}{4} \sin \lambda_k \right) \cdot \cos \lambda_k y + \frac{1 + \nu}{4} \cdot \sin \lambda_k \sin \lambda_k y,
\]

\[
\chi(\lambda_k, y) = \left( \frac{1 + \nu}{4} \right) \cdot (-\cos \lambda_k \sin \lambda_k y + \sin \lambda_k \cos \lambda_k y),
\]

\[
s_1(\lambda_k, y) = \left( \frac{1 + \nu}{2} \lambda_k \cos \lambda_k \frac{3 + \nu}{2} \sin \lambda_k \right) \cdot \cos \lambda_k y + \frac{1 + \nu}{2} \cdot \lambda_k \sin \lambda_k \sin \lambda_k y,
\]
The technique of constructing the solution can be found, for example, in the article [3]. The values of the main controlled parameters based on the experiment results, analytical and finite-element calculations are summarized in Table 2.

### Table 2. Results comparison of the options for calculating the basic parameters of the stress-strain state when pulling out steel flat rods from wood.

|                      | Results of the experiment | Analytical calculation | Finite element model with rigid nodes in contact areas | Finite element model with contact surfaces |
|----------------------|---------------------------|------------------------|-------------------------------------------------------|-------------------------------------------|
| Stress-strain state  |                           |                        |                                                      |                                           |
| parameters           |                           |                        |                                                      |                                           |
| Breaking load, \(F_f\), kN | 20                         | 20                     | 20                                                   | 20                                        |
| Shearing destructive stresses, \(\tau_{xy}\), MPa | -                          | 2.2                    | 2.35                                                 | 2.1                                       |
| Normal destructive stresses, \(s_y\), MPa   | -                          | 5.6                    | 5.3                                                  | 5.7                                       |
| Plate displacement, \(\Delta\), mm    | 0.124                      | 0.116                  | 0.115                                                | 0.128                                      |

Verification showed that the convergence of the calculation destructive stresses results by analytical and numerical methods is in the range from 0 to 7%. In this case, the values of destructive tangential and normal stresses are more significant in the model with rigid nodes. The displacement of the flat rod cross-section at the end of the beam relative to the end itself according to the results of the experiment and the calculation of the finite element model with contact surfaces are correlated with a coefficient equal to 3.2%, with rigid nodes - 7.2%. Thus, it can be noted that the discrepancies between the controlled parameters values, the model with rigid components and the contact problem are no more than 4%. It should be noted that the analytical solution is rather cumbersome and practically not applicable in engineering practice. The process of creating contact pairs with changing the periodic finite element mesh as a whole also complicates the process and time for creating the model. Verification and validation of the glued joint showed that the calculation and design of wooden structures on glued steel flat rods can be reasonably performed on solid-state finite element models with rigid nodes of interface of finite elements from various materials.

This approach allows to significantly reducing the calculation time and, consequently, the design of complex nodes of wooden structures in the process of sufficient correctness and calculations accuracy ensuring.
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