THE OPTIMAL PRICING AND SERVICE STRATEGIES OF A DUAL-CHANNEL RETAILER UNDER FREE RIDING

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ABSTRACT. Free riding refers to that in a multi-channel market, consumers enjoy the presale service of a product at one channel but purchase the product at another channel. In this paper, we study the optimal pricing and service strategies for a dual-channel retailer, who sells a product through both a traditional retail channel and an online channel. We assume that the offline channel provides the presale service but the online channel does not. We investigate how the changes of the degree of free riding affect the pricing/service strategies and profits of the two channels under three different scenarios: Stackelberg competition, Bertrand competition and channel integration. Our analysis shows that when the dual-channel retailer operates the two channels separately, no matter under which competitive scenario, free riding has a negative effect on both channels. And it is much more beneficial for the dual-channel retailer to let one channel work as a leader and another channel as a follower than to let the two channels make their decision simultaneous. In contrast, when the dual-channel retailer runs the two channels jointly, i.e., employs the channel-integration scenario, free riding may be beneficial to the retailer. Finally, this paper proposes and analyzes a revenue-sharing contract to coordinate a decentralized dual-channel retailer to achieve beneficial outcomes for both channels.

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1. **Introduction.** The Internet has provided a new avenue to sell products to consumers in the E-commerce market. A number of survey reports show that commerce on the Internet is growing at an attractive rate. For example, According to the survey reported by Forrester Research (Jun 2019), online retail sales in Asia Pacific will almost double over the next 5 years, from $1.3 trillion in 2018 to $2.5 trillion in 2023, with a compound annual growth rate (CAGR) of 14.0%, and will account for 28% of total retail sales.\(^1\) The fact of the increasing development of E-commerce attracts a lot of brick and mortar retailers to devote themselves to online sales, such as Barnes & Noble, GOME and Wal-Mart, etc. Among these dual-channel retailers, some operate the two channels independently. For instance, GOME, one of the largest retailers in China, operates separately with gome.com.cn\(^2\) and Barnes & Noble runs separately with Barnesandnoble.com (Berger et al. [3]). But some such as Gap (Yan [45]) and Office Depot (Gulati et al. [10]) operate the two channels jointly.

In practice, when one retail channel offers the customer services that are helpful to sell a product (such as, fitting service, display of products, the explanation of product features, answering customer’s questions, etc.), another channel may benefit from those services by making the final sale. This is because some consumers may obtain services from one retail channel, but purchase the product from another channel, possibly at a lower price. This phenomenon is called “free riding”, which often occurs in these products, whose presales services can be conducted separately from the actual sale of the products (Shin [31]; Kucuk and Maddux [19]; Pu et al. [28]), such as clothes, shoes, toys and furniture, etc. (Baal and Dach [2]; Heitz-Spaun [13]; Zhou et al. [51]). With the increasing development of Internet technologies, free riding becomes more popular than ever before (Wu et al. [41]). Intuitively, free riding may reduce the offline retail channels’ incentives to promote the pre-sales service. This may, in turn, lead to a decrease in total sales. Some researchers, like Antia et al. [1] and Carlton & Chevalier [5] etc., have also verified that free riding in a retail market has a negative impact on the profit of the offline retailer who provides the pre-sales service.

Nonetheless, many retailers in reality are still providing pre-sales service despite the free-riding problem. For instance, Ikea.com, one of the online home-furnishing retailers in the world, has also opened multiple “Ikea experience centers” except the online channel, where customers can experience the product first, and then decide whether to buy or not, and where to buy— at the traditional channel or free riding to the online channel. These practical cases seemingly conflict with our intuition and observations from many researches, and motivate us to further study how free riding affects the decision of a dual-channel retailer who operates his/her online and offline channel separately or jointly. In our paper, we consider a dual-channel retailer who uses both a traditional (offline) and an online channel in parallel to sell a product. Customers can purchase products through either the online channel or the traditional retail channel. In the traditional channel, the retailer offers customers pre-sales services, such as teaching customers how to use product, free customer experience, etc.. Some customers may enjoy the pre-sales services at the traditional channel, but purchase the products online. We assume the possible reasons of consumer free riding are to reduce product quality uncertainty,

\(^1\)https://go.forrester.com/press-newsroom/retail-sales-in-asia-pacific/
\(^2\)http://www.gome.com.cn/consultation/1000229045.html
or to test taste, color and fit etc., but not necessarily to buy goods at a lower price. With this kind of customer free-riding behavior, we discuss how the two channels make pricing/pre-sales service decisions to maximize their own profits or the total profit of the two channels, depends on whether the dual-channel retailer operates the two channels separately or jointly. We focus on how the degree of free riding influences the pricing/pre-sales service strategies, demands and profits of both channels. First, we analyze two non-cooperative settings—the first one assumes one of the two channels to be a leader, whereas the other one assumes the two channels making decisions simultaneously. Then, we discuss channel integration of the dual-channel retailer. Finally, we propose and analyze a revenue-sharing contract to coordinate a decentralized dual-channel retailer to achieve win-win outcomes for both channels.

2. Literature review. Our study is mainly related to two categories of literature. The first category focuses on multiple channel competition, and the second category is related to free riding.

2.1. Multiple channel competition. In the last few years, many researchers have paid much attention on multiple channel competition. For example, Choi [7] focused on the intra- and inter-channel price competition and its implication on the channel structure and profits. Huang and Swaminathan [17] showed that the multiple channel retailers always set a higher price for the products than do the single channel retailers. Moon et al. [26] argued that when the customers prefer the online channel, the channel conflict become fiercer. Hua et al. [14] examined the optimal decisions of delivery lead time and prices in a centralized and a decentralized dual-channel supply chain. By analyzing various mixed channel structures composed of a monopoly manufacturer and online and offline outlets, Chen et al. [6] found that a manufacturer’s contract with a wholesale price and a price for the direct channel can coordinate the dual-channel supply channel, benefiting the retailer but not the manufacturer. Ma et al. [24] studied a channel bargaining problem where the manufacturer and the retailer negotiate about the wholesale price and order quantity, with equal and unequal bargaining power. Huang et al. [16] showed that the manufacturer always gets better off if he takes timely response to the production cost disruptions, while the retailer gets better off only if the production cost disruption is negative and gets worse off otherwise. Liu et al. [23] developed a multi-period model to analyze the supply chain coordination in logistics service supply chain. Wang et al. [39] investigated performance of two different markup arrangements, namely, percentage and dollar, in a channel system with two competitive manufacturers and one common retailer. Focused on the strategic role played by cooperative advertising in the dual-channel competition of the manufacturer–retailer supply chain, Yan and Pei [46] showed that a cooperative advertising program can be utilized as an effective coordinative mechanism to alleviate dual-channel competition and thus help to improve the channel as a whole and the individual channel members’ performance. Furthermore, Chen [6] also examined the impact of price schemes and cooperative advertising mechanism on the dual-channel supply chain competition, and suggested that a revenue-sharing contract could be applied to eliminate the channel conflict. Soleimani et al. [33] studied the optimal decisions in a dual-channel supply chain under simultaneous demand and production cost disruptions. Li et al. [21] investigated the dual-channel supply chain equilibrium problems regarding retail services and fairness, they showed that channel efficiency
grows with increasing customer loyalty to the retail channel and falls with increases in the retailer’s fairness concerns. Zheng et al. [50] examined the effect of forward channel competition and power structure on dual-channel closed loop supply chains (CLSC), which consists of a manufacturer, a retailer and a collector, and showed that each channel member has an incentive to play the channel leader’s role. Ke and Liu [18] investigated dual-channel supply chain competition with channel preference and sales effort under uncertain environment, and shown that the supplier’s profit first decreases then increases as the expected value of customers’ preference to the direct channel increases, and the retailer’s profit decreases with the increase of the expected value of customers’ preference to the direct channel. Tan and Carrillo [34] studied a retailer owning both digital goods and traditional goods distribution channels in the fixed price model, and compare the profitability and consumer welfare for different pricing schemes. Yang et al. [47] studied the inventory competition in a dual-channel supply chain with delivery lead time, and showed that compared with the centralized scenario, at least one of the supply chain members will overstock in the decentralized scenario and that consumers in the online direct channel enjoy a shorter delivery lead time and hence better service in the decentralized scenario. Zhang and Wang [49] considered a retailer that sells a short-life-cycle product and took the online channel as a supplement to its traditional channel, they introduced a new strategy to address channel conflict, in which two channels are introduced at different periods and the retailer employs different pricing mechanisms for two channels, and showed that the retailer could adjust price difference between two channels to reduce channel conflict. Radhi and Zhang [29] studied optimal pricing policies for a centralized and decentralized dual-channel retailer (DCR) with same- and cross-channel returns, and showed that when a channel with significant customer preference faces a high rate of returns, decentralized channels generate a greater system profit for retailers than coordinated channels that have a unified pricing strategy. However, the above-mentioned literature ignores the impact of the free riding factors on channel competition.

2.2. Free riding. With the development of e-commerce, it is rather common that some consumers use the offline channel as a presale information source, but make purchases online or vice versa. This means that free riding phenomenon becomes more and more prevalent. Therefore, many researchers pay their attention on free riding. Telser [35] was probably the first to examine the free-riding problem. He argued that competition might dissuade retailers from offering presale information service. Mittelstaedt [25] showed that free riding leads to a decrease in information service provided to consumers, and therefore to less demand for the product. Singley and Williams [32] found that free riding increases the price disparity between free-riding and service-providing retailers. Through empirical study, Carlton and Chevalier [5] showed that free riding in a retail market hurt the profit of the retailer who provides the service. Kucuk et al.’s [19] empirical study indicated that full-service retailers’ beliefs about online consumers’ choice of purchase outlet are predominantly influenced by online retail price rather than availability of products on the Internet. Heitz-Spahn [13] further studied cross-channel free-riding from a consumer empowerment perspective. Huang et al. [15] investigated the impact of the newly introduced mobile channel on the sales of the incumbent web channel and new consumption from the mobile channel. In these researches, besides empirical research methods are employed, a fundamental viewpoint is that free riding has a negative impact on manufacturers’/retailers’ performance as it reduces the offer of
the service. Unlike the above studies, Wu et al. [41] examined a market where two types of e-retailers sell horizontally differentiated products — Type-1 retailers have ability to provide information service but Type-2 retailers do not offer any information service. They analytically showed that retailers might obtain benefits from providing information service, even in the presence of free riding. Shin [31] studied a market where one retailer who offers sales service and the other who does not sell the same product. He found that the free riding phenomenon might benefit both free-riding and service-providing retailers because it weakens price competition. Bernstein et al. [4] showed that the introduction of a direct channel benefits both the retailer and the manufacturer, even in the presence of free riding. Xing and Liu [43] studied sales effort coordination issues for a supply chain with one manufacturer and two retail channels, where an online retailer offers a lower price and free-rides a brick-and-mortar retailer’s sales effort. They showed that if all with price match, the selective rebate outperforms both the target rebate contract and the wholesale price discount contract in coordinating the brick-and-mortar retailer’s sales effort and improving supply chain efficiency. Dan et al. [9] investigated the impacts of bidirectional free-riding and service competition on members’ decisions, and showed that when a new channel is added, the retailer will always increase his service level to compete with the manufacturer, while whether the manufacturer increases or decreases her service level depends on the relationship with the retailer. He et al. [12] evaluated the impact of consumer free riding on carbon emissions in a product’s life cycle across a dual channel closed loop supply chain, and showed that although manufacturers may gain economic benefits from consumer free riding behavior, total carbon emissions across the supply chain increase too, and a governmental tax on e-commerce can help reduce consumer free riding and total carbon emissions. Setak et al. [30] studied a decentralized supply chain in which a manufacturer sells a common generic product through two traditional and online retailers under free riding market, and showed that through the cooperative advertising program, information sharing between the manufacturer and traditional retailer is always beneficial for all the supply chain members and therefore, the entire supply chain is coordinated except when the traditional retailer is not efficient and the degree of free riding is relatively small. Pu [28] and Zhou et al. [51] investigated the effects of free riding on sales effort in a dual-channel supply chain consisting of one manufacturer and one offline store, and proposed a cost sharing contract to help the manufacturer and the offline store to achieve Pareto improvements. Li et al. [22] investigated the influence of the “showrooming” effect, which means consumers may experience and assess the products in a retail store but free riding to the online channel to buy the products, and showed that the firms can benefit the most from the ex-post service efforts under the free riding. Although all of these studies focused on the impact of free riding, they did not discuss how the degree of free riding affects sellers’ decisions, especially on the sales effort decisions. Neither did they consider retailers who own multiple channels simultaneously or various competitive scenarios. Furthermore, most of the previous research on free-riding behavior considered that manufacturers sale from the dual channels, but didn’t consider that the retailer may sales from his own dual channels. However, in view of the advantages of online and offline sales, many offline retailers have also begun to open up online channels for sales, and some e-retailers have also begun to establish physical stores to sell, and the dual-channel sales model has become increasingly prevalent.
Our paper differs from the previous studies in two sides. First, we focus on the sales effort decisions of the offline store facing free-riding consumers for the dual-channel retailer under the decentralized and centralized setting, and investigate how the changes of the degree of free riding affect the pricing/service strategies and profits of the two channels. Furthermore, we also compare the competitive scenarios with the special channel integration scenario (the traditional channel acts as an experience store). Finally, we proposes and analyzes a revenue-sharing contract to coordinate a decentralized dual-channel retailer to achieve beneficial outcomes for both channels. As such, this yields some different results between their studies and ours, especially the impact of free riding on the dual-channel retailers’ strategies and performance. (1) Our results show that when the dual-channel retailer operates the two channels separately, free riding has always a negative effect on the pre-sales service-providing traditional channel’s profit and the total demand and profit of the two channels, and also even may be detrimental to the free-riding online channel. Furthermore, we show that if the retailer operates the two channels separately, free riding may aggravate price competition between two channels, i.e., “channel conflict”. (2) When the dual-channel retailer runs the two channels jointly, i.e., employs the channel-integration scenario, free riding may be beneficial to both channels, regardless of sales made at the traditional channel or not. (3) Under the presence of free riding, it is much more beneficial for the dual-channel retailer, who operates the two channels separately, to let one channel work as a leader (first announce his decision information) and another channel as a follower than let the two channel make their decision simultaneous. Moreover, the traditional-channel-leader setting is beneficial to both channels if the potential market demand is relatively small, whereas both channels like to act as a last mover if the potential market demand is relatively large. (4) As compared to the competitive scenarios, the special channel-integration scenario, where the traditional channel acts as an “experience store” and sales is made at the online channel, may lead to less total profit of the dual-channel retailer, but the optimal channel-integration scenario can always bring the dual-channel retailer more total profit. (5) Under the optimal channel-integration scenario, the potential market demand is an important factor that influences whether the dual-channel retailer make sales at both channels or not. (6) The decentralized dual-channel retailer can realize a win–win outcome of the traditional and online channel under the revenue-sharing contract.

The remaining of this paper is organized as follows. The model framework is provided in Section 3. In Section 4, we study the impact of free riding on the pricing/pre-sales service strategies, demands and profits of the online and traditional channels under different decentralized competition settings. Section 5 discusses the dual-channel retailer’s decisions under channel integration and illustrates the model with a computational study and presents the sensitivity analysis of parameters. Section 6 proposes a revenue sharing contract to help the decentralized dual-channel retailer to realize a win–win outcome of the traditional and online channel. The conclusions are presented in Section 7.

3. Model framework. Consider a market setting where a dual-channel retailer uses both traditional and online channels in parallel to sell a product. The dual-channel retailer operates the two channels separately, like K-mart and Target, or jointly, like Wal-Mart, Gap and Home Depot. Customers can purchase products
through either the online channel or the traditional retail channel. In the traditional channel, the retailer offers customer services necessary to buy products, such as fitting service, information service about product features, experience, after-sales service etc. Thus, demand in the traditional channel depends not only on two channels’ pricing but also on its service level offered. As known in Section 1, some customers may enjoy the services at the traditional channel, but purchase the final products online. We call these customers free-riding customers. Hence, the customer services offered by the traditional channel also partly stimulate demand in the online channel.

In this setting, the traditional channel chooses its own retail price and service level while the online channel determines its sales price, and then realizes demand. Each channel’s demand is decreasing in its own price but increasing in the sales price of its rival channel. Moreover, an increase of the service level in the traditional channel can increase not only its own demand but also the demand of its rival channel, due to free riding. Like many researchers (e.g., Tsay and Agrawal [37]; Yan [45]; Zhou et al.[51]), we will also discuss a specific linear demand function. Then, the demand functions of the two channels are respectively given by

The demand of the traditional channel: $d_1 = a_1 - p_1 + \theta_1 p_2 + (r-\beta)S$, \hspace{1cm} (1)

The demand of the online channel: $d_2 = a_2 - p_2 + \theta_2 p_1 + \beta S$. \hspace{1cm} (2)

Based on the description above, the profit functions of both the traditional channel and online channel are respectively given by

The profit of the traditional channel: $\pi_1 = (p_1-c_s)[a_1-p_1+\theta_1 p_2 + (r-\beta)S]$, (3)

The profit of the online channel: $\pi_2 = p_2[a_2 - p_2 + \theta_2 p_1 + \beta S]$. (4)

The total profit of the dual-channel retailer will be

$\pi = (p_1 - c_s)[a_1 - p_1 + \theta_1 p_2 + (r-\beta)S] + p_2[a_2 - p_2 + \theta_2 p_1 + \beta S]$. (5)

For convenience, we summarize the notation adopted in this paper in Table 1.

| Notations used in the paper |
|-----------------------------|
| $p_i$ | Retail price of channel $i$ |
| $d_i$ | Demand of the channel $i$ |
| $a_i$ | Market base or potential demand of channel $i$ |
| $\theta_i$ | Cross-price sensitivity for the two channels, where $0<\theta_i<1$ |
| $S$ | The pre-sales service level of the traditional channel |
| $c_s$ | The traditional channel’s service cost incurred by providing service level $S$ |
| $\beta$ | The degree of free riding, where $0\leq \beta \leq r$ |
| $r$ | Service sensitivity parameter |
| $\lambda$ | Revenue sharing rate of the online channel, where $0\leq \lambda \leq 1$ |
| $\pi_i$ | Profit of the channel $i$ |
| $\pi$ | The total profit of the dual-channel retailer |

Subscripts:

$i = 1$ represents the retailer’s traditional channel and $i = 2$ the retailer’s online channel

Where $\theta_i(0<\theta_i<1)$ denotes that own-channel effects are greater than cross-channel effects. $r$ represents the marginal channel demand incurred by the increase of the traditional channel’s service level. $\beta(0\leq \beta \leq r)$ represents the degree of free riding, which measures how many of the demands from customers who enjoy the service at the traditional channel shift to the online channel, just as Pu et al. [28] and Zhou et al. [51]. The traditional channel’s per unit service cost $c_s$
incurred by providing service level $S$ is assumed to be $c_s = \eta S^2 / 2$, where the quadratic form means diminishing returns on such expenditures and $\eta (>0)$ is referred to as “service cost factor”. The assumption for this type of per-unit service cost has been employed in the previous literature by many researchers, such as Tirole [36] and Mukhopadhyay [27]. For the sake of simplicity, we assume that the wholesale price of the product is zero. Also, to maintain analytical tractability, we assume $a_1 = a_2 = a$ and $\theta_1 = \theta_2 = \theta$, which were ever used by many researchers like Xia et al. [42], Mukhopadhyay et al. [27], Yan [45] and Zhou et al. [51]. This assumption can be easily relaxed to the case with $a_1 \neq a_2$ and $\theta_1 \neq \theta_2$. However, such a relaxation cannot bring any significant result.

Since the retailer’s two channels may operate separately or jointly, in what follows, we consider how the two channels make their pricing/service strategy under the two different scenarios when they face free riding. We refer to the two different scenarios as decentralized strategy and integrated strategy.

4. Pricing/Service strategies under different competitive scenarios. In this section, we consider that the retailer operates the traditional channel and online channel separately. Each channel aims to maximize its own channel profit and has separate pricing rights to maximize its profit. Actually, the traditional channel and online channel have different power in the market, and adopt different game strategies, such as Stackelberg and Bertrand competition (Yan [45]). When the dual-channel retailers are offline retailers before opening their online channels, their traditional channels may often have more power, and the traditional channel stackelberg setting will be mostly observed, such as Red Star Macalline. However, there are also some e-retailers who are planning to open or have opened physical stores. For example, Bonobos, a menswear e-retailer in UK, opened a physical store in Soho in 2013, and the online channel stackelberg setting may be observed. Furthermore, with the development of retailers’ traditional channels or online channels, the two channels may have the equal channel power, just as the retailer Watsons, the two channels always can work as a Bertrand competition. We will consider the different competitive scenarios following.

4.1. The Stackelberg competition. First, we consider the Stackelberg competition, where one channel has more power than another channel and acts as the market leader, and first makes its decision independent of its competing channel, the other channel that acts as the market follower make its decision by taking into account the decision set by the market leader. In this subsection, we consider two settings: 1) the traditional channel acts as the Stackelberg leader, and 2) the online channel acts as the Stackleberg leader.
4.1.1. The traditional channel acts as the Stackelberg leader. In this setting, the traditional channel announces a retail price $p_1$ and sets the service level $S$ to maximize its own profit. In response to $p_1$ and $S$, the online channel posts a retail price $p_2$ to maximize its profit. We refer to this setting as the TC-Stackelberg scenario for brevity. Through the standard backward induction, we can easily derive the equilibrium pricing/service strategies for the two channels in this setting, which are given in theorem 1.

**Theorem 1.** Under the TC-Stackelberg scenario, the optimal equilibrium pricing/service strategies for the two channels are given by

$$\begin{align*}
p_1^{TC} &= A_1/(2-\theta^2)C_1, \quad S^{TC} = 4D_1/C_1, \quad \text{and} \quad p_2^{TC} = B_1/(2-\theta^2)C_1, \quad (6)\\
\text{where} \quad A_1 &= 2\eta a(2-\theta^2)/2+3[2(r-\beta)+\theta\beta], \quad B_1 = 2\eta a(2-\theta^2)[4-\theta^2+2\theta]/2(r-\beta)+\theta\beta/[2\theta(r-\beta)+(8-\theta^2)\beta], \quad C_1 = 4\eta(2-\theta^2), \quad \text{and} \quad D_1 = 2(r-\beta)+\theta\beta.
\end{align*}$$

Under the above equilibrium strategies, the demands and profits of the two channels are respectively given by

$$\begin{align*}
d_1^{TC} &= (A_1-2D_1)/(2C_1), \quad d_2^{TC} = B_1/[2-\theta^2]C_1, \quad \pi_1^{TC} = (A_1-2D_1)^2/[2(2-\theta^2)C_1], \quad (7)\\
\pi_2^{TC} &= B_1^2/[4(2-\theta^2)^2C_1].
\end{align*}$$

In terms of (6) and (7), we can obtain that under the TC-Stackelberg scenario, the equilibrium pricing/service policies and profits for two channels with free riding have the properties as follows.

**Property 1.**

(i) $\partial p_1^{TC}/\partial r > 0$, $\partial p_2^{TC}/\partial r > 0$, $\partial p_1^{TC}/\partial a > 0$, $\partial p_2^{TC}/\partial a > 0$, $\partial p_1^{TC}/\partial \eta < 0$, and $\partial p_2^{TC}/\partial \eta < 0$;

(ii) $\partial d_1^{TC}/\partial r > 0$, $\partial d_2^{TC}/\partial r > 0$, $\partial d_1^{TC}/\partial a > 0$, $\partial d_2^{TC}/\partial a > 0$, $\partial d_1^{TC}/\partial \eta < 0$, and $\partial d_2^{TC}/\partial \eta < 0$;

(iii) $\partial \pi_1^{TC}/\partial r > 0$, $\partial \pi_1^{TC}/\partial a > 0$, $\partial \pi_1^{TC}/\partial \eta < 0$, and $\partial \pi_2^{TC}/\partial \eta < 0$;

(iv) $\partial S^{TC}/\partial r > 0$, $\partial S^{TC}/\partial a = 0$, $\partial S^{TC}/\partial \eta < 0$.

Property 1 indicates that the sales prices, demands and profits of the two channels increase as the market base $a$ and service sensitivity $r$ increase, but decrease as the service cost factor $\eta$ increases. The service level offered by the traditional channel increases as the service sensitivity increases, and decreases as the service cost factor increases. It is interesting to note that the potential market base has no effect on the service level. This is consistent with our intuition. A direct managerial implication from property 1 is that the dual-channel retailer should try best to increase the market base $a$ and the customers’ service sensitivity $r$ or to reduce the service cost factor, such as, increasing their advertisement, offering characteristic service, improving service efficiency, and so on, so that he/she can obtain much more profits.

**Property 2.**

(i) $p_1^{TC}$ and $S^{TC}$ decrease with $\beta$ over $[0, r]$, so do $d_1^{TC}$ and $\pi_1^{TC}$.

(ii) $p_2^{TC}$, $d_2^{TC}$ and $\pi_2^{TC}$ increase with $\beta$ over $[0, \beta^*]$ but decrease with $\beta$ over $[\beta^*, r]$ if $0 < \theta < 3 - \sqrt{5}$, and decrease with $\beta$ over $[0, r]$ if $3 - \sqrt{5} \leq \theta < 1$.

(iii) Define $\Delta p^{TC} = p_1^{TC} - p_2^{TC}$, then $\Delta p^{TC}$ decreases with $\beta$ over $[0, \beta^*]$ but increases with $\beta$ over $[\beta^*, r]$ if $0 < \theta < 0.5985$, and always decreases with $\beta$ over $[0, r]$ if $0.5985 \leq \theta < 1$.

(iv) The total demand of two channels $d^{TC}$ decrease with $\beta$. The total profit of the dual-channel retailer decreases with $\beta$. 

where $\beta^* = r(12\theta - 2\theta^2 - 8)/(20\theta - 4\theta^2 - \theta^3 - 16)$ and $\beta^\wedge = r(2\theta^2 - 24\theta + 32)/(\theta^3 + 10\theta^2 - 44\theta + 40)$.

**Proof.** See Appendix.

Property 2 shows the impact of changes of the degree of free riding on the equilibrium policies, demands and profits of the two channels in the TC-Stackelberg setting. Property 2(i) indicates that the equilibrium retail price, pre-sales service level, demand and profit of the traditional channel will decrease as the degree of free riding increases. It is mainly because when the degree of free riding $\beta$ arises, more customers will move to the online channel, which will force the traditional channel to reduce its retail price and pre-sales service level for keeping its market share. Consequently, the profit of the traditional channel declines. This implies that free riding always has a negative impact on the traditional channel that provides the pre-sales service, even though the traditional channel moves first. It is consistent with our expectation but also supported by empirical results from researchers like Antia et al. [1] and Carlton & Chevalier [5] etc. However, our finding is different from that obtained by Shin [31]. Shin [31] found that both free-riding and pre-sales service-providing retailers could benefit from free riding.

As shown in property 2(ii), in the TC-Stackelberg setting, the impact of changes of the degree of free riding on the equilibrium policies, demands and profits of the online channel depends on the cross-price sensitivity, given by $\theta$. When the cross-price sensitivity of the two channels is relatively small (say, $0 < \theta < 3 - \sqrt{5}$), the sales price, demand and profit of the online channel increases with the degree of free riding $\beta$ if $\beta < \beta^*$ but decreases with the degree of free riding $\beta$ if $\beta > \beta^*$. From this we can obtain two insights. (1) If the price competition between the two channels is not very fierce, allowing a relatively small free riding can have the free rider forgo to compete with the pre-sales service provider on price, which results in that free riding has a positive effect on the online channel. (2) This positive effect of free riding on the online channel will vanish gradually after the degree of free riding exceeds the threshold value $\beta^*$, and ultimately convert to a negative impact when the degree of free riding goes up to $r$. It implies that free riding is not always beneficial to the online channel but sometime even harmful to the online channel, which seems inconsistent with our common expectation. However, if the cross-price sensitivity of the two channels is relatively large (say, $3 - \sqrt{5} \leq \theta < 1$), the sales price, demand and profit of the online channel decrease with the degree of free riding $\beta$. That is to say, when the price competition between the two channels becomes very fierce, free riding has always a negative effect on the online channel and, hence, is not beneficial to any of the two channels, which is utterly out of our intuition.

Property 2(iii) shows that the two channels' retail price difference $\Delta p^TC$ decreases with the degree of free riding if the cross-price sensitivity of the two channels is relatively large, otherwise decreases and then increases with the degree of free riding. We can infer that free riding can increase the price disparity between free-riding and pre-sales service-providing channels only when the cross-price sensitivity is relatively small and the degree of free riding is relatively greater, otherwise will lead to the head-to-head price competition between two channels. Property 2(iv) indicates that under the TC-Stackelberg setting, free riding leads to the decrease of both the total demand and profit of the dual-channel retailer. It means that free riding is always detrimental to the dual-channel retailer, who operates two channels separately, when both channels follow the TC-Stackelberg competition. In next section, we compare
with the benefits gained from the two channels in the TC-Stackelberg setting which be shown in property 3. Their proofs are omitted.

**Property 3.** The profit of the traditional channel is higher than the online channel (i.e., \( \pi^T_C \geq \pi^O_C \)) if and only if \( \beta \leq \beta_X \) and \( a \leq a_X \), otherwise, would always lower than the online channel (i.e., \( \pi^T_C < \pi^O_C \)).

Where \( a_X = (2r - 2\beta + \theta \delta)(2r - 2\beta + \theta \delta)(4 - 2\delta^2)^{1/2} - (6\theta r - 6\theta \beta + 3\theta \beta^2)/(2\eta(2 - \theta)^2)((2 + \theta)(2 - (4 - 2\delta^2)^{1/2}) - \theta^2), \) \( \beta_X = 2r(4 - 2\delta^2)^{1/2} - 3\theta(2 - \theta)^2(4 - 2\delta^2)^{1/2} \).

**Proof.** One can easily prove property 3. We omit its proof here.

Property 3 shows that in the TC-Stackelberg setting, the traditional channel can get more profits than the online channel only when the market base, the cross-price sensitivity and the degree of free riding are all relatively small. It implies that the traditional channel who acts as the leader does not always obtain more profits than does the online channel who is the follower. This finding is different from the corresponding result for the up- and down-stream competition in supply chain.

4.1.2. The online channel acts as the Stackelberg leader. In this setting, the online channel acts as the leader, who first announces its price \( p_2 \) to maximize its profit. In response to \( p_2 \), the traditional channel (the follower) updates its retail price \( p_1 \) and service level \( S \) to maximize its profit. We refer to this setting as the OC-Stackelberg scenario for brevity. Through the backward induction, we can obtain the optimal pricing and service strategies for the two channels, which are given in theorem 2.

**Theorem 2.** In the OC-Stackelberg scenario, the optimal equilibrium pricing/service strategies for the two channels are given by

\[
\pi^O_C = A_2/(2C_2), \quad \pi^T_C = B_2/C_2, \quad \text{and} \quad S^O = (r - \beta)/\eta, \quad \text{where} \quad A_2 = 2\eta a(4 + 2\theta - \theta^2) + 3(4 - \theta^2)(r - \beta)^2 + 4\theta \beta(r - \beta), \quad B_2 = 2\eta a(2 + \theta) + 3\theta(r - \beta)^2 + 4\beta(r - \beta), \quad \text{and} \quad C_2 = C_1 = 4\eta(2 - \theta^2).
\]

**Proof.** The proof of theorem 2 is similar to that of theorem 1, hence is omitted.

Under the above equilibrium strategies, the demands and profits of the two channels are given by

\[
\pi^O_C = A_2/(2C_2), \quad \pi^T_C = B_2/(2\eta), \quad \pi^O_C = A_2/(4C_2), \quad \text{and} \quad \pi^T_C = B_2/(8\eta C_2).
\]

From (8) and (9), we can derive that in the OC-Stackelberg setting, the equilibrium pricing/service policies and profits for the two channels under free riding have similar properties with those in the TC-Stackelberg setting. Their proofs are omitted.

**Property 4.**

(i) \( p_1^O, S^O, d_1^O \) and \( \pi_1^O \) always decrease with \( \beta \).

(ii) \( p_2^O, d_2^O \) and \( \pi_2^O \) increase with \( \beta \) on \((0, \beta^#)\) but decrease with \( \beta \) on \((\beta^#, r)\) if \( 0 < \theta < 2/3 \); \( p_2^T, d_2^T \) and \( \pi_2^T \) always decrease with \( \beta \) on \((0, r)\) if \( 2/3 \leq \theta < 1 \).

(iii) Define \( \Delta p^O = p_1^O - p_2^O \), then \( \Delta p^O \) decreases with \( \beta \) over \([0, \beta^#] \) but increases with \( \beta \) over \([\beta^#, r) \).

(iv) The total demand of the two channels decreases with \( \beta \) on \((0, r) \). The total profit of the dual-channel retailer decreases with \( \beta \).

Where \( \beta^# = r(3\theta - 2)/(3\theta - 4) \) and \( \beta^# = r(16 - 8\theta - 3\theta^2)/(20 - 10\theta - 3\theta^2) \).

From Property 4, we can see that when the online channel acts as the leader, the impact of the model’s parameters on the pricing/service policies, demands and profits of the two channels is similar with that in the TC-Stackelberg setting. In
The total demand of the two channels decreases with the traditional channel (i.e., \( \pi_2^{OC} \geq \pi_1^{OC} \)) if and only if \( a \leq a_Y, \beta \geq \beta_Y \), otherwise, would always lower than the traditional channel (i.e., \( \pi_2^{OC} < \pi_1^{OC} \)).

Where \( a_Y = (r - \beta)[3 \theta(3r - \beta) + 4 \beta(4r - 2\theta^2)]^{1/2}/[2 \eta(2 + \theta)[2 - (4 - 2\theta^2)] - \theta^2} \}, \beta_Y = 3r[4 - \theta^2 - \theta(4 - 2\theta^2)^{1/2}]/{12 - 3\theta^2 - 4\theta + (4 - \theta)(4 - 2\theta)^{1/2}] \).

Comparing the profits between the two channels, we can obtain property 5 easily. Similar with property 3, property 5 shows that in the \( OC \)-Stackelberg setting the leader does not always get more benefits than the follower, too.

### 4.2. The Bertrand competition

Under the Bertrand competition, both the online and traditional channels have equal power in the market, and the two channels make their individual decisions simultaneously. The traditional channel determines its retail price \( p_1 \) and service level \( S \) to maximize its profits, viewing the online channel’s retail price as a given parameter. At the same time, the online channel determines its sales price \( p_2 \) to maximize its profits, viewing the traditional channel’s retail price and service level as given parameters. Through some straightforward derivation, we obtain the Nash equilibrium pricing and service strategies for the two channels, which are given in theorem 3.

**Theorem 3.** Under the Bertrand competition, the equilibrium pricing and service strategies for the two channels under free riding are given by

\[
p_1^B = A_3/C_3, \quad p_2^B = B_3/(2C_3), \quad S^B = (r - \beta)/\eta. \quad (10)
\]

Where \( A_3 = \eta \beta(2 + \theta) + \beta(3r - \beta)^2 + 3\theta(r - \beta), \quad B_3 = B_2 = 2\eta \beta(2 + \theta) + 3\theta(r - \beta)^2 + 4\beta(r - \beta), \) and \( C_3 = \eta(4 - \theta^2) \).

Under the above equilibrium strategies, the demands and profits of the two channels are given by

\[
d_1^B = D_3/(2C_3), \quad d_2^B = B_3/(2C_3), \quad \pi_1^B = D_3^2/(4C_3^2) \) and \( \pi_2^B = B_3^2/(4C_3^2) \). \quad (11)
\]

Where \( D_3 = 2\eta \beta(2 + \theta) + (2 + \theta^2)(r - \beta)^2 + 2\beta \theta(r - \beta) \).

Similarly, we can derive from (10) and (11) that in the Bertrand setting the equilibrium pricing/service policies and profits of the two channels have the following properties.

**Property 6.**

(i) \( p_1^B, S^B, d_1^B \) and \( \pi_1^B \) always decrease with \( \beta \).

(ii) \( p_2^B, d_2^B \) and \( \pi_2^B \) increase with \( \beta \) on \( (0, \beta^#) \) but decrease with \( \beta \) on \( (\beta^#, r) \) if \( 0 < \theta < 2/3; \) \( p_1^B, d_1^B \) and \( \pi_1^B \) always decrease with \( \beta \) on \( (0, r) \) if \( 2/3 \leq \theta < 1 \).

(iii) Define \( \Delta p^B = p_1^B - p_2^B \), then \( \Delta p^B \) decreases with \( \beta \) over \( [0, \beta^\circ] \) but increases with \( \beta \) over \( [\beta^\circ, r] \).

(iv) The total demand of the two channels decreases with \( \beta \) on \( (0, r) \). The total profit of the dual-channel retailer decreases with \( \beta \).

Where \( \beta^# = r(3\theta - 2)/(3\theta - 4) \) and \( \beta^\circ = 4r/5 \).

Property 6 shows that when the two channels follow the Bertrand competition, the impact of the model’s parameters on the pricing/service policies, demands and profits of the two channels is also similar with those in the two Stackelberg settings.

In next section, we compare with the benefits gained from the two channels in the Bertrand competition setting which be showed in property 7. Their proofs are omitted.
Property 7. The profit of the traditional channel is higher than the online channel (i.e., $\pi_1^T > \pi_2^O$) if $\beta \leq \beta_2$, otherwise, lower than the online channel (i.e., $\pi_1^T < \pi_2^O$) if $\beta > \beta_2$, where $\beta_2 = r(1-\theta)/(3-\theta)$.

Property 7 shows that in the Bertrand competition setting, the degree of free riding determines the profit splitting between the two channels. Moreover, a smaller $\beta$ is beneficial to the traditional channel whereas a greater $\beta$ is profitable to the online channel.

4.3. Comparison of the pricing/service strategies and profits under different settings. In the two subsections above, we have discussed the pricing/service strategies and corresponding profits of the two channels under the three settings. In this subsection, we make a comparison of the pricing/service strategies and profits for the two channels under different settings. The results are shown in Theorem 4.

Theorem 4.

(i) Among the three settings, the traditional channel sets the highest pre-sales service level in the TC-Stackelberg setting, and sets the same pre-sales service level under the other two settings, i.e., $S^T > S^O = S_B$.

(ii) Among the three settings, the traditional channel sets the highest retail price in the TC-Stackelberg setting, and the lowest retail price in the Bertrand competition setting, i.e., $p_1^T > p_1^O > p_2^B$. The traditional channel’s profit is always higher in the TC/OC-Stackelberg setting than in the Bertrand competition setting, i.e., $\pi_1^T > \pi_1^O$ whereas its profit in the TC-Stackelberg setting is higher than in the OC-Stackelberg setting (i.e., $\pi_1^T > \pi_1^O$ if $a \leq a_x$, but lower than in the OC-Stackelberg setting (i.e., $\pi_1^T < \pi_1^O$) if $a > a_x$.

(iii) The online channel’s sales price is higher in the TC/OC-Stackelberg setting than in the Bertrand competition setting, i.e., $p_2^T > p_2^O$ and $p_2^O > p_2^B$, whereas its sales price in the TC-Stackelberg setting is higher than in the OC-Stackelberg setting (i.e., $p_2^T > p_2^O$ if $a \leq a_y$ but lower than in the OC-Stackelberg setting (i.e., $p_2^T < p_2^O$) if $a > a_y$. Among the three settings, the profit of the online channel is the highest in the TC-Stackelberg setting but the lowest in the Bertrand competition setting, i.e., $\pi_2^T > \pi_2^O > \pi_2^B$.

Where $a_x = \{2[2\theta + 2(r - \beta)]^2 - (r - \beta)(4 - 2\theta^2)^{1/2} + 3(4 - \theta^2)(r - \beta)][2(4 + 2\theta - \theta^2)(1 + \theta^2 + \theta^3)]/[2\theta(4 + 2\theta - \theta^2)(1 + \theta^2 + \theta^3)]$, $a_y = \{6\theta(r - \beta) + \beta^2(2 - \theta^2)]/[2\theta(2 - \theta^2)]$.

Theorem 4(i) indicates that the traditional channel sets a higher pre-sales service level in the TC-Stackelberg setting than in the other two settings. This finding means that under free riding the traditional channel has an incentive to offer a higher pre-sales service level to attract more customers when it acts as the leader. From Theorem 4 (ii) and (iii), we can see that the pricing of the traditional channel is higher when it acts as the leader than when it acts as the follower. However, the pricing of the online channel is not always like this. The sales price of the online channel will be lower when it acts as the leader than when it acts as the follower, if the potential market demand is relatively small. It also shows that the pricing of the two channels is always higher in the TC/OC-Stackelberg setting than in the Bertrand competition setting. It implies that the price competition or channel conflict between the two channels is fiercer in the Bertrand competition setting than in the TC/OC-Stackelberg setting. Consequently, the profits of the two channels are both higher in the TC/OC-Stackelberg setting than in the Bertrand competition setting. The management insight obtained from this result is that when there is free riding, it is beneficial for the dual-channel retailer, who operates the two channels
separately, to let one channel act as a leader and another as a follower. Therefore, the dual-channel retailer, who operates the two channels separately, should choose the TC/OC-Stackelberg competition setting rather than the Bertrand competition setting under free riding.

Another interesting finding from Theorem 4 is that the traditional channel has the first-mover advantage as the potential market demand is relatively small and the last-mover advantage as the potential market demand is relatively large, whereas the online channel has always the last-mover advantage. This implies that when a dual-channel retailer operates the two channels separately, the TC-Stackelberg setting is beneficial to both channels if the potential market demand is relatively small and both channels like to act as a last mover if the potential market demand is relatively large. The interpretation of this result is as following. When the market is scarce (for example, a brand new product first launches on the market.), it is very necessary for the dual-channel retailer to foster the market. In such case, if the traditional channel moves first, more total demand can be generated by providing high level experience pre-sales service (known from Theorem 4(i)), which weakens price competition between two channels but also enhances the profits of both channels. Conversely, if the online channel moves first, it can attract more customers only by reducing the sales price. This results in price competition between two channels and lower level pre-sales service as well as total demand, which makes both channels worse off. When the potential market size is sufficient, it is not necessary to stimulate demand. At that moment, the information advantage of the follower can bring more benefits from the market to him/her.

5. Channel integration of the dual-channel retailer. In the case of channel integration, the two channels are integrated and operated jointly by the dual-channel retailer. Therefore, the dual-channel retailer will determine the optimal pricing/service strategy for both the online and traditional channels so that the total profit of the two channels can be maximized. In practice, there are many cases of channel integration. For instance, Suning, one of the largest retailers in China, its traditional channel is combined with its online channel Suning.com, and aimed to maximized the dual channel’s total profit.

From (5), one can easily derive that for any given service level the total profit function of the two channels is jointly concave with respect to \( p_1 \) and \( p_2 \). Hence, for any given service level, the first-order optimality conditions will give the optimal pricing policies of the two channels as

\[
p_1^I = \frac{\eta S^2}{4} + \left[a(1+\theta) + (r - \beta)S + \beta S\theta\right]/\left[2(1-\theta^2)\right], \quad p_2^I = \left[a(1+\theta) + (r - \beta)\theta + \beta S\right]/\left[2(1-\theta^2)\right].
\] (12)

One easily derives from (12) \( dp_1^I/dS>0 \) and \( dp_2^I/dS>0 \). This indicates that as the service level provided by the traditional channel arises, the retail prices of both the traditional channel and online channel will increase. It seemingly makes sense, because the increased service level leads to the increase of demand, which will induce the raise of the retail price. Under the retailer’s optimal pricing strategy for the two channels, the demands for both the traditional channel and the online channel are respectively given by

\[
d_1 = \frac{a + \beta \theta S}{2} \quad \text{and} \quad d_2 = \frac{a + \beta S + \theta \eta S^2/2}{2}. \] (13)

Property 8. If \( S \geq S_o \), then \( d_1 = 0 \), where \( S_o = \left( (r - \beta) + \left[(r - \beta)^2 + 2a\eta\right]^{1/2}\right)/\eta \).

Property 8 reports an interesting finding: if the decision maker of two channels set a relative high service level for traditional retailer (i.e., \( S \geq S_o \)), the actual
demand of the offline store is zero and all the customers who visit the offline store will take freeriding, which means that the offline store actually become an experience store. For instance, Dell is one of the examples that multi-channel sellers open experience stores (Dell opened the first flagship experience store in Shanghai of China in 2009). To avoid the trivial case, it is necessary to request that the traditional channel’s demand is non-negative, i.e., \( d_1 \geq 0 \). We can then get that 
\[ 0 < S_2 \leq S_o, \]
where \( S_o = \{(r - \beta) + [(r - \beta)^2 + 2a\eta]^1/2\}/\eta \) is referred to as the highest service level the traditional channel could provide.

5.1. Special case: Make sales at online channel only. Consider a special integrated system, where the traditional channel acts as an experience store and the dual-channel retailer makes sales only at the online channel. In this special setting, the optimal pricing/service policies of the two channels are given by
\[
p_1^{SI} = \eta S_o^2/4 + [a(1 + \theta) + (r - \beta)S_o + \beta S_o \theta]/[2(1 - \theta^2)], \quad p_2^{SI} = [a(1 + \theta) + (r - \beta)S_o \theta + \beta S_o]/[2(1 - \theta^2)], \quad \text{and} \quad S^{SI} = S_o. \tag{14}
\]

The dual-channel retailer’s total profit will be
\[
\pi^{SI} = [(a - \eta S_o^2)/2 + (r - \beta)S_o^2 + (a + \theta \eta S_o^2/2 + \beta S_o)(a(1 + \theta) + [\beta + 2\theta(r - \beta)]S_o - \theta \eta S_o^2/2)]/4(1 - \theta^2). \tag{15}
\]

From (13), (14) and (15), we can derive that in the special case the optimal pricing/service policies and profits of the two channels have the following properties.

Property 9.
(i) \( p_2^{SI}, \quad d_2^{SI} \) and \( \pi^{SI} \) increase with \( \beta \) if \( a \geq a_z = r^2/[2\eta(1 - \theta)^2] \), and increase with \( \beta \) as \( \beta < \beta_o \), but decrease with \( \beta \) as \( \beta \geq \beta_o \) if \( a < a_z = r^2/[2\eta(1 - \theta)^2] \), where
\[
\beta_o = a\eta(1 - \theta)/r + (1 - 2\theta)r/[2(1 - \theta)].
\]
(ii) \( S_0 \) always increases with \( a \) and \( r \) but decreases with \( \beta \) and \( \eta \).
(iii) \( p_2^{SI}, \quad d_2^{SI} \) and \( \pi^{SI} \) always increase with \( a \) and \( r \) but decreases with \( \beta \) and \( \eta \).

Property 9 shows that in the special setting both the potential market demand and pre-sales service sensitivity have a positive effect on the pre-sales service level at the offline store, the sales price and demand at the online channel, and the total profit of the dual-channel retailer, but the pre-sales service cost factor has a negative impact on them. This is consistent with that in the previous competition settings. Additionally, property 9 indicates that in many cases free riding has a positive effect on the sales price and demand at the online channel and the total profit of the dual-channel retailer. Hence, in most cases, free riding can benefit the dual-channel retailer if he/she makes sales only at the online channel. However, when the potential market demand is relatively small (i.e., not exceed the threshold value \( a_z \)), a relatively large degree (i.e., greater than the threshold value \( \beta_o \)) of free riding on the offline store may be detrimental to the dual-channel retailer (i.e., online channel), which is inconsistent with our expectation.

In order to investigate whether it is always better for the dual-channel retailer to employ this special scenario than to use the previous competitive scenarios, we need to compare the dual-channel retailer’s profit under this special scenario with that under each of the previous scenarios. Property 10 shows the compared results.

Property 10.
(i) The profit of the dual-channel retailer in the special case of the offline store working as the experience store is higher than or equals to that in the TC-Stackelberg setting if \( \chi \leq 4\eta^2 a^2 \varepsilon^2 + 4a\eta(\omega - \zeta) + \xi \), i.e., \( \pi^{SI} \geq \pi^{TC}_1 + \pi^{TC}_2 \), but lower than that in the TC-Stackelberg setting if \( \chi > 4\eta^2 a^2 \varepsilon^2 + 4a\eta(\omega - \zeta) + \xi \), i.e., \( \pi^{SI} < \pi^{TC}_1 + \pi^{TC}_2 \).
(ii) The profit of the dual-channel retailer in the special case of the offline store working as the experience store is higher than or equals to that in the OC-Stackelberg setting if $\chi_2 \leq 4\eta^2 a^2/\pi^2 + 4\eta a(\omega_2 - \xi_2)$, i.e., $\pi^S \geq \pi^O_{\text{OC}}$, but lower than that in the OC-Stackelberg setting if $\chi_2 > 4\eta^2 a^2/\pi^2 + 4\eta a(\omega_2 - \xi_2)$, i.e., $\pi^S < \pi^O_{\text{OC}}$.

(iii) The profit of the dual-channel retailer in the special case of the offline store working as the experience store is higher than or equals to that in the Bertrand setting if $\chi_2 \leq 4\eta^2 a^2/\pi^2 + 4\eta a(\omega_2 - \xi_2)$, i.e., $\pi^S \geq \pi^B$, but lower than that in the Bertrand setting if $\chi_2 > 4\eta^2 a^2/\pi^2 + 4\eta a(\omega_2 - \xi_2)$, i.e., $\pi^S < \pi^B$.

Where $\varepsilon = \theta^2(2-\theta^2)^2(1+\theta)(8+4\theta-3\theta^2-\theta^3)$, $\beta = 5\theta^4 - 2\theta^6 - 6\theta^3 - 7\theta^2 + 14\theta + 12$, $\omega = 4(4-\theta^2)^2/\{4(2-\theta^2)^2\}$, $\chi = (2r - 2r\beta + \theta\beta)^2(1-\theta^2)[4(r - \beta)^2(4 + 7\theta^2) + 4\beta(\theta - (28 - 5\theta^2) + \beta^2(64 - 12\theta^2 - 8\theta^3))$, $\lambda_1 = (1-\theta^2)(r - \beta)^2[9(r - \beta)^2(16 - 4\theta^2 - \theta^4) + 24\beta(\theta - (8 - 3\theta^2) + 16\beta^2(4 - \theta^2))$, $\lambda_2 = (r - \beta)^2[(r - \beta)^2(4 + 13\theta^2 + \theta^4) + 4\beta(\theta - (8 + \theta^2) + 4\beta^2(4 + \theta^2))]$, $\xi = 4(2-\theta^2)^2\{K^2\{[1-\theta^2][K-4(1-\beta^2) + 4(r - \beta^2) + 2\beta(\theta - (r - \beta^2)] + 4\beta^2(32 + 24\theta - 8\theta^2 - 6\theta^3 - \theta^4)]\}$, $\zeta = 4(2+\theta)^2[(r - \beta)^2(1+\theta)(8 + 16\theta + 2\theta^2 - 5\theta^3) + 4\beta(\theta - (16 + 16\theta + 4\theta^2 - 29\theta^3 + 39\theta^4) + \beta^2(32 + 24\theta - 8\theta^2 - 6\theta^3 - \theta^4))$, $\eta = (2\theta^2)(1-\theta^2)[4(r - \beta)^2(8 + 16\theta + 2\theta^2 - 5\theta^3) + 4\beta(\theta - (16 + 16\theta + 4\theta^2 - 29\theta^3 + 39\theta^4) + \beta^2(32 + 24\theta - 8\theta^2 - 6\theta^3 - \theta^4))]$, $\eta_2 = 4(2+\theta)^2(1+\theta)[\eta_2 \geq \eta]$.

Property 10 indicates that as compared to the previous competitive scenarios, the special channel integration scenario cannot always bring the dual-channel retailer more benefits. Conversely, under certain specified conditions, the competitive scenarios, whatever the Stackelberg competitive scenario or the Bertrand competitive scenario, may be better for the dual-channel retailer than the special channel integration scenario. This can explain to some extent why some retailers run their dual channels separately in practice.

5.2. Optimal strategy under channel integration. As shown in section 5.1, the channel-integration scenario where the sales is made at the online channel only is sometimes not a good option for the dual-channel retailer, as compared to the competitive scenarios. Then, a natural question is whether or under what conditions it is a better option to make sales to both channels when the dual-channel retailer integrates two channels. In what follows, we will show the answers. Substituting (12) into (5) will yield the retailer’s total profit as

\[ \pi^T = \{[a - \eta S^2/2 + (r - \beta)S]^2 + (a + \theta\eta S^2/2 + \beta S)\{a(1+2\theta) + [\beta + 2\theta(r - \beta)]S - \theta\eta S^2/2\}] / [4(1-\theta^2)]. \]

which is a four degree polynomial of $S$. This polynomial function has the following property.

Theorem 5.

(i) When $2(2+\theta)(r - \beta)^2 > (1-\theta^2)(1+\theta)$, the retailer’s total profit function (shown in (13)) is increasing convex with respect to $S$ over the interval $[0, S_o]$. In that case, the retailer’s optimal service level is $S_o$.

(ii) When $2(2+\theta)(r - \beta)^2 < (1-\theta^2)(1+\theta)$, the retailer’s total profit function (shown in (13)) is an increasing convex function of $S$ over the interval $[0, S_o]$ if $\Psi \leq 0$, and a horizontal S-shaped function of $S$ over the interval $[0, S_o]$ otherwise. Thus, the retailer’s optimal service level that maximizes $\pi(S)$ over the interval $[0, S_o]$ is $S_o$ if $\Psi \leq 0$ and $S^* = \arg\max\{\pi(S_o), \pi(S_o)\}$ otherwise.

Where $S_2 = [(r - \beta) + \{\Delta/3(\theta^2 - 1)\}^{1/2}] / \eta$, $S_3 = [(r - \beta) - [\cos(k/3) - 3^{1/3}\sin(k/3)] \{\Delta/3(\theta^2 - 1)\}^{1/2}] / \eta$, $\Psi = 9(1-\theta^2)[\beta + \theta(r - \beta)]\{r + (r - \beta)(3\theta + \theta(r - \beta)) + \eta(1+\theta) + \eta \}^{1/2}$. 

and $\eta$

We assume the initial values of parameters four scenarios and his/her optimal decision under the channel-integration scenario. The optimal channel-integration scenario always increases with the degree of free riding, except when the potential market size is relatively small. This is opposite to the corresponding results under the previous competitive scenarios. When the potential market size is relatively small, the dual-channel retailer’s profit and the retail price at the online channel under the optimal channel-integration scenario firstly

$\Delta[3(\theta^2-1)\Delta]^{1/2}$, $\Delta = 2[\beta + \theta(r - \beta)]^2 - (1-\theta^2)[(r-\beta)^2 + 2a\eta]$ and $k = \arccos(\theta^2 - 1)[\beta + \theta(r - \beta)][(r - \beta)(\beta + \theta(r - \beta)] + a\eta(\theta+1)]/(\Delta - 3(\theta^2-1)\Delta)^{1/2}$.

Proof. See Appendix.

After noting that $2[\beta + \theta(r - \beta)]^2 \geq (1-\theta^2)[(r - \beta)^2 + 2a\eta]$ is equivalent to $a \leq \Omega \equiv (2[\beta + \theta(r - \beta)]^2 - (1-\theta^2)(r - \beta)^2)/(2(1-\theta^2)\eta)$. Theorem 5 shows that if the dual-channel retailer operates the traditional and online channels jointly, the traditional channel should act only as an “experience store” when the potential demand is relatively small. In that time, the traditional channel provides the highest service level and charges the highest price. All customers enjoy the service at the traditional channel but move to the online channel to purchase. The dual-channel retailer makes sales only at the online channel. Another point obtained from Theorem 5 is that it may be more profitable for the dual-channel retailer to make sales at both channels only when the potential market demand is relatively large. Theorem 5 also shows the specified condition under which the dual-channel retailer should make sales at both channels. For instance, Wal-Mart is a typical example of successfully making sales at both online and offline channels.

It is worth to mention that unlike the special channel integration scenario, the optimal channel integration scenario discussed in this subsection will be always superior to the competitive scenarios. Although we cannot show its analytical proof due to its complexity, we will present the numerical illustrations in the following section. Given $r = 7$, $\beta = 2$, $\theta = 0.4$ and $\eta = 0.1$. Assume the potential market demand takes the values of 40, 70, 500, 3500 and 5000, respectively.

Based on the presented model, we obtain the optimal pricing/service policies and profits for two channels and the dual-channel retailer’s maximum profit under the various scenarios discussed in the paper, which are shown in Table 2. Table 2 can further illustrate or verify the analytical results (eg., Theorem 5) obtained in the previous sections. Except these, from Table 2 we can observe the following.

(i) The optimal channel-integration strategy is always more beneficial to the dual-channel retailer than any discussed competitive strategy, which is not affected by the potential market demand. (ii) The optimal pricing and service level for two channels are higher under the optimal channel-integration strategy than under the three competitive strategies.

In what follows, we will further use the numerical experiments to illustrate how the degree of free riding $\beta$, the pre-sales service cost factor $\eta$ and the pre-sales service sensitivity parameter $\kappa$ impact the dual-channel retailer’s profit under the presented four scenarios and his/her optimal decision under the channel-integration scenario. We assume the initial values of parameters $r$, $\beta$, $\theta$ and $\eta$ are $r = 7$, $\beta = 2$, $\theta = 0.4$ and $\eta = 0.1$, respectively. We then change $\beta$ over $(0, r)$ with the step of 0.5, $\eta$ in $(0, 1.5)$ with the step of 0.2, and $r$ in $(2, 7)$ with the step of 0.5, respectively. Our numerical experiments are implemented under various potential market sizes. The results are shown in Figures 2-4.

From Figures 2-4, we can observe the following. (i) The dual-channel retailer’s profit is always greater in the optimal channel-integration scenario than in any competitive scenario discussed. (ii) The dual-channel retailer’s profit in the optimal channel-integration scenario always increases with the degree of free riding, except when the potential market size is relatively small. This is opposite to the corresponding results under the previous competitive scenarios. When the potential market size is relatively small, the dual-channel retailer’s profit and the retail price at the online channel under the optimal channel-integration scenario firstly
| $\alpha$ | $\Omega$ | Channel integration | TO-Stackelberg | OC-Stackelberg | Bertrand |
|---|---|---|---|---|---|
| | | | $p_1^T$ | $S^T_1$ | $p_2^T$ | $S^T_2$ | $\pi_1^{TC} (\times 10^4)$ | $\pi_2^{TC} (\times 10^4)$ | $\pi^{TC} (\times 10^4)$ | $p_1^O$ | $S^O_1$ | $p_2^O$ | $S^O_2$ | $\pi_1^{OC} (\times 10^4)$ | $\pi_2^{OC} (\times 10^4)$ | $\pi^{OC} (\times 10^4)$ | $p_1^B$ | $S^B_1$ | $p_2^B$ | $S^B_2$ | $\pi_1^{B} (\times 10^4)$ | $\pi_2^{B} (\times 10^4)$ | $\pi^{B} (\times 10^4)$ |
| 40 | 65.5 | | 101 | 289 | 1.0235 | | 284 | 58.7 | 1.7305 | 1.1390 | 1.3515 | 2.4907 | 231 | 50 | 116 | 1.1179 | 1.3490 | 2.3095 |
| 70 | 65.5 | | 112.7 | 326 | 8.9311 | | 304 | 58.7 | 1.7305 | 1.1390 | 1.3515 | 2.4907 | 231 | 50 | 116 | 1.1179 | 1.3490 | 2.3095 |
| 500 | 65.5 | | 161.8 | 802 | 54.017 | | 584 | 58.7 | 1.7305 | 1.1390 | 1.3515 | 2.4907 | 231 | 50 | 116 | 1.1179 | 1.3490 | 2.3095 |
| 3500 | 65.5 | | 387.5 | 3248 | 1146.6 | | 254 | 58.7 | 1.7305 | 1.1390 | 1.3515 | 2.4907 | 231 | 50 | 116 | 1.1179 | 1.3490 | 2.3095 |
| 5000 | 65.5 | | 518.8 | 140.4 | 2258.0 | | 351 | 58.7 | 1.7305 | 1.1390 | 1.3515 | 2.4907 | 231 | 50 | 116 | 1.1179 | 1.3490 | 2.3095 |
Figure 2a-2b The impact of $\beta$ on the dual-channel retailer’s profit under the four scenarios and his/her optimal decision under the channel-integration scenario.

Channel structures of dual-channel retailers

Figure 3a-3b The impact of $\eta$ on the dual-channel retailer’s profit under the four scenarios and his/her optimal decision under the channel-integration scenario.
increase and then decrease with the degree of free riding. This is mainly because the dual-channel retailer’s optimal channel-integration strategy is just the special channel-integration scenario when the potential market size is relatively small. Consequently, Property 9(i) can explain this observation. However, under the optimal channel-integration strategy, the pre-sales service level at the offline channel will increase if the dual-channel retailer makes sales at both channels and otherwise always decrease with the degree of free riding. (iii) When the dual-channel retailer implements the optimal channel-integration strategy, the pre-sales service cost factor has a negative effect on the pricing/pre-sales service level of two channels and the dual-channel retailer’s profit, which is not out of our intuition. (iv) Under the channel-integration scenario, the dual-channel retailer will prefer to make sales at both channels when the potential market size is relatively large but the degree of free riding, the cross-price sensitivity or the pre-sales service sensitivity is relatively small or when the potential market size and the pre-sales service cost factor are both relatively large. (v) Under the optimal channel-integration strategy, the pre-sales service sensitivity always has a positive effect on the pricing, pre-sales service level and the dual-channel retailer’s profit which is similar as in the previous three competitive scenarios.

6. **An extension: Coordinate the retailer’s dual-channel with revenue sharing contract.** Because the online channel free rides the offline channel’s service level, and increases the online channel demand. So that the online channel has sufficient motivation to further enhance the level of offline channel’s service efforts by sharing parts of his channel revenue with the physical channel. In this section, we would try to use the revenue sharing contract to coordinate the dual channel
Under the RS-Stackelberg scenario, the optimal equilibrium pricing/service strategies for the two channels are given by

\[ p^{RS}_1 = \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right], \quad p^{RS}_2 = \left[ \frac{2 \eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right], \]

\[ S^{RS} = \left[ \frac{2 \eta (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right]. \]

Theorem 6. Under the RS-Stackelberg scenario, the optimal equilibrium pricing/service strategies for the two channels are given by

\[ p^{RS}_1 = \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right], \quad p^{RS}_2 = \left[ \frac{2 \eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right], \]

\[ S^{RS} = \left[ \frac{2 \eta (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right]. \]

Where \( A_4 = 4(1-\theta)(\beta - \theta^3)(\beta - \theta^2)(\beta + \theta (2-\theta^2)) = 4 \eta a^2 (2-\theta^2)(\beta - \theta^2)(\beta ^3 + \theta (2-\theta^2)), \)

\[ \gamma_1 = \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right], \quad \gamma_2 = \left[ \frac{2 \eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right] - \left[ \frac{\eta a (2-\theta^2)}{2 \eta (1-\theta) \beta \lambda (2-\theta^2)} \right], \]

\[ C_4 = \left[ 2 \eta a (2-\theta^2)(\beta - \theta^2)(\beta + \theta (2-\theta^2)) \right] - \left( \eta a (2-\theta^2)(\beta - \theta^2)(\beta + \theta (2-\theta^2)) \right), \]

\[ \Phi = \left[ 2 \eta a (2-\theta^2)(\beta - \theta^2)(\beta + \theta (2-\theta^2)) \right] - \left( \eta a (2-\theta^2)(\beta - \theta^2)(\beta + \theta (2-\theta^2)) \right). \]

The retailer can realize a win–win outcome of the traditional and online channel when \( \lambda \) is within \([\lambda^*, \lambda^*]\).

Property 11. The retailer can realize a win–win outcome of the traditional and online channel when \( \lambda \) is within \([\lambda^*, \lambda^*]\).

Proof. See Appendix.
Property 11 shows that the retailer’s traditional and online channel can both get more profit under the RS-Stackelberg scenario than under the TC-Stackelberg scenario, which also effectively improves the retailer’s overall revenue. Furthermore, we can also get the similar conclusion when the online channel sharing parts of his channel revenue with the physical channel under the OC-Stackelberg setting and the Bertrand competition setting. Due to space limitations, we would not repeat them.

Furthermore, the following numerical examples demonstrate the effect of revenue sharing rate $\lambda$ on the strategy and profit of two channels, where $a = 15, r = 2, \beta = 0.8, \theta = 0.1$ and $\eta = 0.4$, respectively.

Fig. 5 illustrates the effect of revenue sharing rate $\lambda$ on the retailer’s strategy and profit in a coordinated setting, showing that the online channel’s profit always increases by $\lambda$, and the traditional channel’s profit always decreases by $\lambda$, which is not out of intuition. Furthermore, we can get that when revenue sharing rate $\lambda$ is within a certain threshold ($\lambda^* \leq \lambda \leq \lambda^\wedge$), the retailer’s traditional and online channel can both get more profit under the RS-Stackelberg scenario than under the TC-Stackelberg scenario, which also effectively improves the retailer’s overall revenue, the results are consistent with the conclusion of property 11.

Fig. 5 also indicates that the traditional channel sets a higher pre-sales service level in the RS-Stackelberg scenario than in the TC-Stackelberg scenario. This finding means that the traditional channel has an incentive to offer a higher pre-sales service level to attract more customers when the online channel share certain revenues to the traditional channel under free riding. Then the pricing of the traditional and online channel are higher in the RS-Stackelberg scenario than in the TC-Stackelberg scenario, thereby making up for the loss of increased service costs caused by the improvement of service levels.

7. Conclusions.

7.1. Summary. With the presence of E-commerce, more and more traditional retailers, including large retailers like Wal-Mart, K-Mart and Suning etc., have opened online channels. These dual-channel retailers run the two channels separately or jointly in practice. Except price competition between two channels, these dual-channel retailers face the free-riding problem while the customer service activities
necessary to sell a product are conducted only in one channel. In this paper, we study how the degree of both free riding and price competition affects the pricing/service strategy and performance of a dual-channel retailer, who sells products through a service-providing traditional channel and a free-riding online channel. We discuss the channel-decentralized scenario as well as the channel-integration scenario. Under the channel-decentralized scenario, we consider the TC-Stackelberg, OC-Stackelberg, and Bertrand settings, respectively. We build a model that explicitly accounts for the effects of free riding under different settings and investigate its implications for the dual-channel retailer’s scenario choice. We also explore what settings are beneficial to the dual-channel retailer under both decentralization and integration scenarios. Finally, we propose a revenue sharing contract to coordinate the two channels of the dual-channel retailer.

Our analysis shows the following results. If the two channels are operated separately, free riding will aggravate price competition between two channels unless the cross-price sensitivity is relatively small and the degree of free riding is relatively high. Under such case, free riding has a negative effect on the service-providing channel’s profit and the total profit of the two channels, even on the free-riding channel’s profit under certain conditions. If the two channels are run jointly, free riding may be beneficial to both channels, regardless of sales made at the traditional channel or not. We also find that when the two channels are operated separately, it is much more beneficial for the dual-channel retailer to follow the Stackelberg competition scenario than follow the Bertrand competition scenario. Moreover, the TC-Stackelberg setting is beneficial to both channels if the potential market demand is relatively small; otherwise, both channels like to act as a last mover. These findings are not affected by free riding. As compared to the competitive scenarios, the special channel-integration scenario may lead to less total profit of the retailer, but the optimal channel-integration scenario can always bring the retailer more total profit. Under the optimal channel-integration scenario, whether the retailer makes sales at both channels or not depends on the model’s parameters, especially the potential market demand. We show the specified conditions under which the dual-channel retailer should make sales at both channels. The decentralized dual-channel retailer can realize a win–win outcome of the traditional and online channel under the revenue-sharing contract.

7.2. Managerial implications. The results of this study have managerial implications for practice, and can help decision-makers in different circumstances. The first practical implication of this study’s results is that when the dual channel is operate decentralized, offline store managers should understand that they will be vulnerable to free-riding on their sales efforts by their online competitors. Thus, managers can take some actions to address free riding, such as spending more on services such as spare parts and free maintenance for consumers who purchased products in their offline store. Second, when the two channels of the retailer are operated separately, it is much more beneficial for the channel managers to follow the Stackelberg competition scenario than follow the Bertrand competition scenario. Third, under the decentralized or centralized structure, the free riding may negative or positive to the dual-channel retailer’s total profit, and the managers need to dialectically see the impact of free-riding behavior on itself, and adopt different strategies to deal with the free-riding phenomenon under the different channel structures. Finally, the online channel managers should recognize that compared to online stores, offline stores usually have higher operational costs from labor and capital resources, and
their sales efforts are a critical factor in expanding the market. Thus, they need to design an incentive contract, such as revenue sharing contrite, to encourage offline stores to invest more in sales efforts and realize Pareto improvements.

7.3. Limitations and directions for future research. Our study has limitations and can be extended in several directions. First, our model considers a single dual-channel retailer only. In real business world, there often exist multiple competitive dual-channel retailers, such as Wal-Mart, Carrefour, Gome and Suning in China, etc. Therefore, a possible extension is to consider the situation with multiple dual-channel retailers. Second, the cashback affiliate is commonly found in practice and has an important effect on the retailer’s strategies and profits. Thus, incorporating this affiliate into our model would be another interesting direction. Third, this paper only considered the situation that the online channel free rides the traditional channel’s presales service. However, in the real life, it is also very common that consumers switch to the traditional channel to buy the product after searching the product information at the online channel. Therefore, a possible extension of this paper is to consider the bidirectional free riding behavior in the dual-channel supply chain.

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Appendix

Proof of Property 2.

Proof. (ii) From (6) we have
\[ \frac{\partial p_{1}^{TC}}{\partial \beta} = (2r-2)(8\beta-\beta^2+6\theta r-6\theta \beta) + (2r-2\beta+\beta \theta)(8\theta^2-6\theta) /[8\eta (2-\theta^2)^2] \] (A.1)
Solving \[ \frac{\partial p_{1}^{TC}}{\partial \beta} = 0 \] yields \[ \beta^* = (r(12\theta-2\theta^2-8))/(20\theta^2-3\theta^2-16). \] Since \( \theta \in (0,1), 20\theta-4\theta^2-\theta^3-16<0. \)

If \( 0<\theta<3-\sqrt{5}, \) one easily derive \( 12\theta-2\theta^2-8<0 \) and \( 0<\beta^*<r. \) Hence, from (A.1) we have that \( p_{1}^{TC} \) is increasing on \( (0, \beta^*) \) and decreasing on \( (\beta^*, r). \) If \( 3-\sqrt{5}\leq\theta<1, \) then \( 12\theta-2\theta^2-8>0 \) and \( \beta^*<0<r, \) which means that \( p_{1}^{TC} \) is always decreasing in \( \beta \) over \( (0, r). \)

For \( \partial \pi_{1}^{TC}/\partial \beta = \partial p_{1}^{TC}/\partial \beta \) and \( \partial \pi_{2}^{TC}/\partial \beta = \{B_1/[(2-\theta^2)C_1]\}\partial p_{2}^{TC}/\partial \beta, \) we can obtain the remaining part of property 2(ii).

(iii) Based on (6), it is easy to have
\[ \frac{\partial (p_{1}^{TC} - p_{2}^{TC})}{\partial \beta} = [(r-2)(12r-12\theta+12\theta^2+6r\theta+6\beta r+2r\theta^2-8\beta)+(12\theta-12+\theta^2-8)(2r-3\theta+\beta)/(8\eta (2-\theta^2)^2)]. \] (A.2)
Solving \( \partial (p_{1}^{TC} - p_{2}^{TC})/\partial \beta = 0 \) will give \( \beta^* = \frac{2r(\theta^2-12\theta^2+12\theta^2+6r^2-12\theta)-20)}{[(\theta-2)(\theta^2+12\theta^2-20)]. \)

One easily derives \( 0.8<\beta^*<r \) if \( 0<\theta<0.5985 \) and \( r \leq \beta^* \) if \( 0.5985<\theta<1. \) Thus, from (A.2) we can get property 2(iv).

(iv) From (7), one has
\[ \frac{\partial d_2^{TC}}{\partial \beta} = \theta(3\theta^2-6\theta^3+3\theta^4-\theta^5)/[4 \eta (2-\theta^2)^2] \]
\[ \frac{\partial \pi_2^{TC}/\partial \beta}{\partial \beta} = \theta(3\theta^2-6\theta^3+3\theta^4-\theta^5)/[4 \eta (2-\theta^2)^2] \]
\[ \{4\eta (2-\theta^2)(\theta^2+3\theta^4+6\theta^2-12\theta)+\beta(-32+16\theta^2-20\theta^3+6\theta^4-\theta^5)\} + (r-\beta)^3 \]
\[ \{4\eta (2-\theta^2)(\theta^2+3\theta^4+6\theta^2-12\theta)+\beta(-32+16\theta^2-20\theta^3+6\theta^4-\theta^5)\} + (r-\beta)^3 \]
\[ \{4\eta (2-\theta^2)(\theta^2+3\theta^4+6\theta^2-12\theta)+\beta(-32+16\theta^2-20\theta^3+6\theta^4-\theta^5)\} + (r-\beta)^3 \]
\[ \{4\eta (2-\theta^2)(\theta^2+3\theta^4+6\theta^2-12\theta)+\beta(-32+16\theta^2-20\theta^3+6\theta^4-\theta^5)\} + (r-\beta)^3 \]
\[ \{4\eta (2-\theta^2)(\theta^2+3\theta^4+6\theta^2-12\theta)+\beta(-32+16\theta^2-20\theta^3+6\theta^4-\theta^5)\} + (r-\beta)^3 \]
\[ \{4\eta (2-\theta^2)(\theta^2+3\theta^4+6\theta^2-12\theta)+\beta(-32+16\theta^2-20\theta^3+6\theta^4-\theta^5)\} + (r-\beta)^3 \]

Due to \( 0<\theta<1, \) it is easily verified that each of the last four terms in the brace of (A.3) is less than zero. The sign of the first term in the brace of (A.3) is the same with the sign of \( f(\beta) = r \Theta_1(\theta) + \beta \Omega_1(\theta), \) where \( \Theta_1(\theta) = -32\theta+16\theta^3-6\theta^4 \)
and $\Omega_1(\theta) = -32 + 32\theta + 16\theta^2 - 20\theta^3 + 6\theta^4 - \theta^5$. Since $0 < \theta \leq 1$, we can easily derive that $\Theta_1(\theta)$ is decreasing with respect to $\theta$ and less than zero, and that $\Omega_1(\theta)$ is increasing with respect to $\theta$, less than zero if $0 < \theta < 0.9579$ but greater than or equal to zero if $0.9579 \leq \theta < 1$. Hence, if $0 < \theta < 0.9579$, we have $f(\beta) < 0$, i.e., $\partial \pi^{TC}/\partial \beta < 0$. When $0.9579 \leq \theta < 1$, $\Omega_1(\theta) - \Omega_1(1) = 1$, and $\Theta_1(\theta) \leq \Theta_1(0.9579) = 21.6414$. Therefore, we have $f(\beta) \leq 21.6414 r + \beta < 0$ (because of $\beta < r$), i.e., $\partial \pi^{TC}/\partial \beta < 0$.

**Proof of Theorem 5.**

Proof. Let $\pi_t(S), \pi_m(S)$ and $\pi m(S)$ be the first, second and third order derivatives of the total profit function $\pi$ (shown in (16)) with respect to $S$, respectively. From (16), one can easily derive the following.

\begin{align*}
\pi_t(S) &= \{\eta^2(1-\theta^2)S^3 - 3\eta (r-\beta)(1-\theta^2)S^2 + [-2\eta \eta (1-\theta^2) + 2(r-\beta)^2 + 2\beta^2 + 4\beta \theta (r-\beta)]S + 2\eta (1+\theta)\}/[4(1-\theta^2)] \quad \text{(A.4)} \\
\pi_m(S) &= \{3\eta^2(1-\theta^2)S^2 - 6\eta (1-\theta^2)(r-\beta)S - 2\eta \eta (1-\theta^2) + 2(r-\beta)^2 + 2\beta^2 + 4\beta \theta (r-\beta)] \} / [4(1-\theta^2)] \quad \text{(A.5)} \\
\pi m(S) &= \{3\eta^2(1-\theta^2)S^2 - 6\eta (1-\theta^2)(r-\beta)S - 2\eta \eta (1-\theta^2) + 2(r-\beta)^2 + 2\beta^2 + 4\beta \theta (r-\beta)] \} / 2 \quad \text{(A.6)}
\end{align*}

It is clear from (A.6) to have $\pi m(S) > 0$ for $0 < S < (r-\beta)/\eta$ and $\pi m(S) > 0$ for $S \in ((r-\beta)/\eta, S_o)$.

This means that $\pi m(S)$ decreases with $S$ as $S \in (0, (r-\beta)/\eta)$ and increases with $S$ as $S \in ((r-\beta)/\eta, S_o)$. Hence, $\pi m(S)$ reaches at $S = (r-\beta)/\eta$ its minimum, $\Delta = 2[\beta + \theta (r-\beta)]^2 - (r-\beta)^2 + 2\beta \theta (r-\beta)]$, in the interval $(0, S_o)$.

(1) If $2[\beta + \theta (r-\beta)]^2 \geq (r-\beta)^2 + 2\beta \theta (r-\beta)]$, i.e., $\Delta \geq 0$, then $\pi_t(S) \geq 0$ in the interval $(0, S_o)$, which means that $\pi(S)$ is convex in the interval $(0, S_o)$ and that $\pi_t(S)$ increases with $S$ in the interval $(0, S_o)$. This will yield $\pi(S) > 0$ for each $S \in (0, S_o)$ due to $\pi_t(0) > 0$. Hence, the retailer’s total profit function $\pi$ is an increasing function of $S$ in the interval $(0, S_o)$. It means that the retailer’s optimal pre-sales service level is $S_o$.

(2) If $2[\beta + \theta (r-\beta)]^2 < (r-\beta)^2 + 2\beta \theta (r-\beta)]$, i.e., $\Delta < 0$, then there exist two roots (say, $S_1$ and $S_2$) to equation $\pi_t(S) = 0$. From (A.5) we have

\begin{align*}
S_1 &= \{(r-\beta) - \{\Delta /[3(1-\theta^2)]\}^{1/2}\}/\eta, \quad S_2 = \{(r-\beta) + \{\Delta /[3(1-\theta^2)]\}^{1/2}\}/\eta. \quad \text{(A.7)}
\end{align*}

From (A.7) it is easy to have $S_1 < S_2 < S_o$. Since $\Delta < 0$, we have $1-\theta^2 \leq \{\Delta /[3(1-\theta^2)]\}^{1/2}$ and $S_1 < S_2$.

Thus, (i) when $2[\beta + \theta (r-\beta)]^2 \leq (r-\beta)^2 + 2\beta \theta (r-\beta)]$, we have $\pi m(S) < 0$ as $S \in (0, S_2)$ and $\pi m(S) > 0$ as $S \in (S_2, S_o)$. That is, $\pi_t(S)$ decreases with $S$ over $(0, S_2)$ but increases with $S$ over $(S_2, S_o)$. Hence, $\pi_t(S)$ reaches at $S = S_2$ its minimum, $\pi m(S_2)$, in the interval $[0, S_o]$.

If $\Psi \leq 0$, it is easy to derive $\pi_t(S_2) \geq 0$. Then, $\pi_t(S)$ is an increasing function of Sover $(0, S_o)$, which gives that the retailer’s optimal pre-sales service level is $S_o$.

If $\Psi > 0$, then $\pi_t(S_2) \geq 0$. Then, there exist two roots (say, $S_3$ and $S_4$) to equation $\pi_t(S) = 0$ in the interval $(0, S_o)$ due to $\pi_t(0) > 0$ and $\pi_t(S_o) > 0$. From (A.4) solving $\pi_t(S) = 0$ will give $S_3 = \{(r-\beta) - \{\cos(k/3) - 3^{1/3}\sin(k/3)\} \{\Delta /[3(\theta^2 - 1)]\}^{1/2}\}/\eta$, $S_4 = \{(r-\beta) - \{\cos(k/3) + 3^{1/3}\sin(k/3)\} \{\Delta /[3(\theta^2 - 1)]\}^{1/2}\}/\eta$. Hence, $\pi_t(S) > 0$ for $S \in (0, S_3) \cup (S_4, S_o)$ and $\pi_t(S) < 0$ for $S \in (S_3, S_4)$. It indicates that $\pi(S)$ is increasing in the intervals $(0, S_3)$ and $(S_4, S_o)$ and decreasing in the interval $(S_3, S_4)$. That is, $S_3$ and $S_4$ are the maximum and minimum points of $\pi(S)$ in the interval $[0, S_o]$, respectively. Therefore, the retailer’s optimal pre-sales service level is $S = \arg \max \{\pi(S_3), \pi(S_4)\}$.  

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(ii) When \(2(\beta + \theta(r-\beta))^2>(1-\theta^2)[2\alpha \eta - 2(r-\beta)^2]\), we have \(0<S_1<S_2<S_o\). This will give \(\pi_T(S)<0\) as \(S \in (S_1, S_2)\) and \(\pi_T(S)>0\) as \(S \in (0, S_1) \cup (S_2, S_o)\). That is, \(\pi_T(S)\) decreases with \(S\) over \((S_1, S_2)\) but increases with \(S\) over \((0, S_1) \cup (S_2, S_o)\). Hence, \(\pi_T(S)\) reaches at \(S=S_2\) its minimum, \(\pi_T(S_2)\), in the interval \([0, S_0]\) due to \(\pi_T(0)>0\). Through similar discussions with those in case (i), we can obtain the retailer’s optimal pre-sales service level is \(S_o\) if \(\pi_T(S_2)\geq 0\) and \(S = \arg\max\{\pi(S_3), \pi(S_o)\}\) otherwise.

**Proof of Property 11.**

**Proof.** Rational constraints of traditional channel: \(\pi_T^{RS} \geq \pi_T^{TC}\). Based on (7) and (20), it is easy to have \(\lambda \leq \lambda^\wedge\), where \(\lambda^\wedge = \left[ \frac{1}{16\eta^2 (2-\theta^2)^2 (\theta^2 \lambda - 3 \theta^2 + 4)^2 (A_1 - 2 D_2)^2 - 2 C_1^2 \Phi_1 (2-\theta^2)] \right] / \left[ 2 C_1^2 (2-\theta^2)^2 (\Phi_1 + 2 \theta \Phi_2) \right] \).

Rational constraints of online channel: \(\pi_2^{RS} \geq \pi_2^{TC}\). Based on (7) and (20), it is easy to have \(\lambda \geq \lambda^\wedge, \lambda^* = \left[ \frac{1}{16\eta^2 B_1^2 (2-\theta^2)^4 (\theta^2 \lambda - 3 \theta^2 + 4)^2 (4 C_1^2 (2-\theta^2)^2 (\Phi_4 + 2 \theta \Phi_2) \right] / \left[ 2 \eta \theta (2-\theta^2)^2 (\theta^2 \lambda - 3 \theta^2 + 4) + \Phi_2) \right] \).

So it is easy to get that: When \(\lambda^* \leq \lambda \leq \lambda^\wedge\), the retailer’s traditional and online channel can both get more profit under the RS-Stackelberg scenario than under the TC-Stackelberg scenario, which also effectively improves the retailer’s overall revenue.

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