Adaptive event-triggered control for new type tyre systems based on vehicles

Ruitong Wu\textsuperscript{a} and Kewen Li \textsuperscript{b}

\textsuperscript{a}College of Electrical Engineering, Liaoning University of Technology, Jinzhou, Liao ning, People’s Republic of China; \textsuperscript{b}College of Science, Liaoning University of Technology, Jinzhou, Liao ning, People’s Republic of China

\textbf{ABSTRACT}

In this article, an adaptive event-triggered control scheme is first investigated for new type tyre systems based on vehicle. The intricate systems are transformed into an online arm model to research. The improved event-triggered mechanism (ETM) is proposed to enhance effectively the data transmission capacity of the systems and reduce the communication bandwidth. For the sake of conquering the problem of ‘explosion of complexity’ in traditional adaptive backstepping recursive design process, the dynamic surface control (DSC) technique is introduced in control design process. Based on the Lyapunov stability theory, the developed control scheme guarantees all the variables in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB). Finally, the simulation results verify the rationality and effectiveness of the proposed control scheme.

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\section*{1. Introduction}

At the beginning of 1900, experts such as the United States and France put forward several conjectures that objects could get rid of their own gravity resistance and run efficiently. This is an early model of the magnetic levitation. However, due to the limitations of the technology and the materials at that time, no practical way was proposed to achieve this goal. With a near-one-century development, the magnetic levitation technology has been improved in various fields. Since 2007, China’s tyre industry under the correct guidance of the national macro-economic control policies and driven by the development of related industries, the overall economic operation situation is in a steady upward trend. In recent years, the global auto production and the auto sales are rising steadily (see Group, 2017). The rapid development of Chinese auto industry and the machinery industry have brought about a huge demand for tyres, which has driven the boom of the entire tyre industry. In the coming period of time, the tyre industry will enter a new development period in the world. The core of the tyre and the production technology are safety, the environmental protection, the energy conservation, see (Das et al., 2020). Therefore, it has promoted our research into a new type of tyre.

The tyres are one of the important parts of the vehicle. The traditional tyres have the following functions, supporting the mass of the vehicle, bearing the load of the vehicle, transferring traction and braking torque, which ensure adhesion between tyres and road surface. And the tyres can reduce and absorb the vibration and impact force of the vehicle when the auto is moving, such that prevent the auto parts from severe vibration and breakdown. Based on the Audi model background of Volkswagen Group, see Figure 1. The spherical tyre shows in Figure 2. The studied new type tyre systems are the magnetic levitation ball-type systems, which enables the tyre to be frictionless and noiseless. When the gravity is greater than the gravity of the ball, the ball is pulled up, so as to achieve levitation. Therefore, compared with the traditional tyres, the spherical tyre reduces greatly the noise while driving, and it is increasing the safety, handling the stability, comforting and the energy-saving economic of the vehicles.

The magnetic levitation ball-type systems are the complicated nonlinear systems. Generally speaking, the system consists of the rotor, the sensor, the controller and the actuator, among the actuator includes the electromagnet and the power amplifier. In order to facilitate the following research, the magnetic levitation ball-type systems consists of the electromagnetic coil, the controller, the steel ball and light source in the laboratory, see Yang et al. (2004). In the system, the controller is the core of the whole magnetic levitation ball-type control systems. The system must be stabilized in levitation under various conditions, consequently, it has stronger anti-interference ability than the general systems.
With the rapid development of modern society, the various control methods have been gradually applied to design of the magnetic levitation control systems. For the control of the magnetic levitation systems, analog PID control is more common in (Alimohammadi et al., 2020) and Mughees and Mohsin (2020). Of course, the other control methods of the maglev systems also can achieve different control objectives. As mentioned in Sun et al. (2019), an adaptive neural-fuzzy sliding mode controller is presented for the magnetic suspension system, such that the presented controller distinctly reduces influence of the parameter perturbations and disturbance. As shown in Truong et al. (2020), the proposed the adaptive neural terminal sliding mode control is implemented for tracking control of the magnetic levitation systems with external perturbation. In Adil et al. (2020) has proposed super-twisting sliding mode controllers for the magnetic levitation systems, such that the air gap is maintained at the stable value. It can be seen from Bidikli (2020) that the maglev systems can conduct system controlling without using any knowledge about system parameters. In addition, the magnetic levitation systems has been widely used in the field of various fields. For example, the researches on the basis of symbiotic organism in Sadek et al. (2017).

However, in the light of above discussions, the problem of the transmission resources efficiency is not involved. Nowadays, the system research has been developing in the direction of large scale and complexity, which induce higher requirement for data transmission and effective utilization of the resources. Therefore, how to save the transmission of the system resources and reduce the communication bandwidth have become the current research trend. In recent years, the backstepping technique has attracted a lot of attention, due to its flexibility for the systems. At all events, the disadvantage of the backstepping technique is ‘explosion of complexity’ caused by the repeated differences of the virtual inputs (Li et al., 2020). For the sake of avoiding this problem, DSC technique is introduced into each step of the backstepping design process in Li and Li (2020).

In order to keep in step with the rapid development of the auto industry, we study an adaptive event-triggered control problem for the magnetic levitation ball-type systems. Compared with the existing works, the main innovations of this paper can be summarized as follows: (a) The event-triggered control algorithm has been applied in automobile suspension control by Li et al. (2019), Liu and Li (2020) and Ding et al. (2021). In Wang et al. (2021), the investigated fuzzy-based adaptive event-triggered nonlinear systems within fixed-time interval, but it emphasizes on theoretical research. However, our purpose in this paper is to propose an adaptive event-triggered controller for the magnetic levitation ball-type systems (the real systems) firstly, which greatly reduces the communication bandwidth. (b) In contrast with the fix-threshold-based event-triggered mechanism proposed in Peng et al. (2020) and the relative-threshold-based event-triggered mechanism presented in Dolk et al. (2017), the improved relative-threshold-based event-triggered mechanism in this paper, which can increase higher stability for system and reduce the waste of resources.

The rest of the paper is distributed as follows. Section 2 details the problem formulations and preliminaries. Section 3 describes the design procedure for event-triggered controller. Section 4 exhibits stability analysis of model systems. Simulation results are shown in Section 5 to prove the effectiveness of the designed control scheme. Eventually, conclusions are described in Section 6.

2. Problem formulations and preliminaries

2.1. Model of the magnetic levitation system

A one-degree-of-freedom magnetic levitation system is illustrated in Figure 3. According to Yang et al. (2004). The system dynamics is given as

\[ \dot{x} = -\frac{Q_{mag}^2}{2M_{ball}(K_{ball} + x)^2} + g \]
\[ \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \]

where \( \mathbf{x} \) is the state vector, \( \mathbf{A} \) and \( \mathbf{B} \) are the system matrices, and \( \mathbf{u} \) is the control input. Therefore, the model of magnetic levitation system (1) can be rewritten as

\[ \begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + g \\
\dot{x}_3 &= \frac{1}{M_{ball}(K_{ball} + x_1)^3} \\
&\quad \left[ -Q_{ball}\dot{i}_c \left( \frac{Q_{ball}x_2 - R_{ball}(K_{ball} + x_1)^2}{Q_{ball}(K_{ball} + x_1) + L_{ball}(K_{ball} + x_1)^2} \right) \right. \\
&\quad \left. + \frac{K_{ball} + x_1}{Q_{ball} + L_{ball}(K_{ball} + x_1)} u \right] (K_{ball} + x_1) + Q_{ball}\dot{x}_1^2 x_2 \\
\end{align*} \]

(2)

Define

\[ \begin{align*}
\mu(x) &= \frac{1}{M_{ball}(K_{ball} + x_1)^3} \\
&\quad \left[ -Q_{ball}\dot{i}_c \left( \beta(x) + \rho(x)u \right)(K_{ball} + x_1) + Q_{ball}\dot{x}_1^2 x_2 \right] \\
\beta(x) &= \frac{(Q_{ball}x_1^2 - R_{ball}(K_{ball} + x_1)^2)\dot{i}_c}{Q_{ball}(K_{ball} + x_1) + L_{ball}(K_{ball} + x_1)^2} \\
\rho(x) &= \frac{K_{ball} + x_1}{Q_{ball} + L_{ball}(K_{ball} + x_1)} u \end{align*} \]

Therefore, the model of magnetic levitation system (2) can be rewritten as

\[ \begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + g \\
\dot{x}_3 &= \mu(x) \\
y &= x_1 \\
\end{align*} \]

(3)

where \( y \in \mathbb{R} \) is the output variable, i.e., the vertical distance between the steel ball and the coil.

In our daily life, due to the temperature and humidity change of some factors in the environment, the characteristics of the coils and magnetic core cannot be accurately obtained. If parameters are available, they are also not fixed. Consider the parameter uncertainties, the model of the magnetic levitation systems is described as

\[ \begin{align*}
\dot{x}_1 &= x_2 + \Delta_1 \\
\dot{x}_2 &= x_3 + g + \Delta_2 \\
\dot{x}_3 &= \mu(x) + \Delta_3 \\
y &= x_1 \\
\end{align*} \]

(4)

where \( \Delta_i, i = 1, 2, 3 \) are unknown disturbances.

**Assumption 2.1 (Lin et al., 2010):** The model disturbances \( \Delta_i, (i = 1, 2, 3) \) are bounded. There exists a constant satisfies \( |\Delta_i| < T_i, (i = 1, 2, 3) \), and \( T_i, (i = 1, 2, 3) \) are unknown positive design parameters.
Remark 2.1: Assumption 2.1 is commonly used in the existing results about the adaptive control for nonlinear systems with unknown disturbances, for example in Lin et al. (2010).

In order to overcome the influence caused by parameter uncertainty, a parameter estimator need to be designed to estimate the bound of unknown parameter. The estimation error is \( \hat{T}_i = T_i - \hat{T}_i \) \((i = 1, 2, 3)\), \( \hat{T}_i \) is the estimation of \( T_i \).

3. Control design

3.1. Adaptive dynamic surface control design

In this subsection, for the sake of obtaining the desired control objective, the backstepping design technique is adopted to design the valid adaptive event-triggered controller. Generally, define the following coordinate transformation as

\[
\begin{align*}
    z_1 &= x_1 \\
    z_i &= x_i - \xi_i \\
    s_i &= \xi_i - \alpha_{i-1}, \quad i = 2, 3
\end{align*}
\]

where \( s_i \) is the first-order filter output error, \( z_i \) is the virtual surface error, \( \xi_i \) is new state variable, and \( \alpha_{i-1} \) is the virtual control law. The design process is described as follows.

**Step 1:** In retrospect (4) and (5), \( \dot{z}_1 \) is given as

\[
\begin{align*}
    \dot{z}_1 &= x_1 \\
    &= z_2 + \xi_2 + \Delta_1 \\
    &= z_2 + s_2 + \alpha_1 + \Delta_1
\end{align*}
\]

Choose the Lyapunov function \( V_1 \) candidate as

\[
V_1 = \frac{1}{2} z_1^2 + \frac{1}{2\eta_1} \hat{T}_1^2
\]

where \( \eta_1 \) is a positive design parameter.

It can be derived from (6) and (7) that

\[
\dot{V}_1 = z_1 \dot{z}_1 + \frac{1}{\eta_1} \hat{T}_1 \dot{\hat{T}}_1
\]

\[
= z_1 (z_2 + s_2 + \alpha_1 + \Delta_1) + \frac{1}{\eta_1} \hat{T}_1 \dot{\hat{T}}_1 \tag{8}
\]

In consideration of Young’s inequality, it follows that

\[
z_1 (z_2 + s_2) \leq z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} s_2^2 \tag{9}
\]

According to Assumption 2.1, we gain

\[
z_1 \Delta_1 \leq z_1 \text{sign}(z_1) (\hat{T}_1 + \hat{T}_1) \tag{10}
\]

Substituting (9) and (10) into (8), we can obtain

\[
\dot{V}_1 \leq z_1 (z_1 + \alpha_1 + \text{sign}(z_1) \hat{T}_1) + \frac{1}{2} z_2^2 + \frac{1}{2} z_2^2 + \frac{1}{2} s_2^2 \tag{11}
\]

Consider the Step 1, we design the virtual control signal \( \alpha_1 \) and the parameter adaptive law of \( \hat{T}_1 \) as

\[
\alpha_1 = -(c_1 + 1) z_1 - \text{sign}(z_1) \hat{T}_1 \\
\hat{T}_1 = \text{sign}(z_1) \eta_1 z_1 - \gamma_1 \hat{T}_1 \tag{12, 13}
\]

where \( c_1, \gamma_1 > 0 \) are design parameters.

Substituting (12) and (13) into (11), one has

\[
\dot{V}_1 \leq -c_1 z_1^2 + \frac{\gamma_1}{\eta_1} \hat{T}_1 \hat{T}_1 + \frac{1}{2} z_2^2 + \frac{1}{2} \hat{T}_1^2 \tag{14}
\]

In order to address repeatedly differentiating of \( \alpha_1 \), consider the first-order filter as follows

\[
\tau_2 \dot{\xi}_2 + \xi_2 = \alpha_1, \quad \xi_2(0) = \alpha_1(0) \tag{15}
\]

where \( \tau_2 \) is a given constant.

It can be seen from (5), we get

\[
\dot{\xi}_2 = \frac{\alpha_1 - \dot{\xi}_2}{\tau_2} = -\frac{\xi_2}{\tau_2} \quad \text{and} \quad \xi_2 = \xi_2 - \dot{\xi}_2 = -\frac{\dot{\xi}_2}{\tau_2} + Y_2(z_1, s_2, \hat{T}_1) = -\frac{\dot{\xi}_2}{\tau_2} + Y_2(\cdot), \quad \text{where} \quad Y_2(\cdot) = -\dot{\alpha}_1 = c_1 \dot{z}_1 + z_1 + \dot{\hat{T}}_1 \text{ is a continuous function.}
\]

**Step 2:** According to (5), we can obtain \( z_2 \) as follows

\[
\begin{align*}
    \dot{z}_2 &= \dot{x}_2 - \dot{\xi}_2 \\
    &= z_2 + \xi_2 + \Delta_2 - \dot{\xi}_2 \\
    &= z_2 + s_2 + \alpha_2 + \Delta_2 - \dot{\xi}_2 \tag{16}
\end{align*}
\]

Choose the Lyapunov function candidate as follows

\[
V_2 = V_1 + z_2^2 + \frac{1}{2} s_2^2 + \frac{1}{2\eta_2} \hat{T}_2^2 \tag{17}
\]

where \( \eta_2 \) is a positive design parameter.

According to (5), (16) and (17), the time derivative of \( V_2 \) satisfies

\[
\dot{V}_2 = \dot{V}_1 + \dot{z}_2 \dot{z}_2 + s_2 \dot{s}_2 + \frac{1}{\eta_2} \hat{T}_2 \dot{\hat{T}}_2
\]

\[
= \dot{V}_1 + \dot{z}_2 (z_2 + s_2 + \alpha_2 + \Delta_2 - \dot{\xi}_2)
\]

\[
+ s_2 \dot{s}_2 + \frac{1}{\eta_2} \hat{T}_2 \dot{\hat{T}}_2 \tag{18}
\]

By utilizing Young’s inequality, we get

\[
z_2 (z_2 + s_2) \leq z_2^2 + \frac{1}{2} s_2^2 + \frac{1}{2} \hat{T}_2^2 \tag{19}
\]

According to Assumption 1, it follows that

\[
z_2 \Delta_2 \leq z_2 \text{sign}(z_2) (\hat{T}_2 + \hat{T}_2) \tag{20}
\]
Substituting (19) and (20) into (18) results in
\[
V_2 \leq -c_1 z_1^2 + \frac{\gamma_1}{\eta_1} \hat{T}_1 \hat{T}_1 + \frac{1}{2} s_1^2 + \frac{1}{2} \hat{z}_2^2
+ \frac{1}{2} s_2^2 + z_2 \left( \frac{3}{2} z_2 + \alpha_2 + g + \text{sign}(z_2) \hat{T}_2 - \hat{z}_2 \right)
+ s_2 \hat{s}_2 + \frac{1}{\eta_2} \hat{T}_2 (\text{sign}(z_2) \eta_2 z_2 - \hat{z}_2)
\]
(21)

Design the virtual control signal \( \alpha_2 \) and the parameter adaptive law of \( \hat{T}_2 \) as
\[
\alpha_2 = - (c_2 + \frac{3}{2}) z_2 - g - \text{sign}(z_2) \hat{T}_2 + \hat{z}_2
\]
(22)
\[
\hat{T}_2 = \text{sign}(z_2) \eta_2 z_2 - \gamma_2 \hat{T}_2
\]
(23)

where \( c_2 > 0 \) and \( \gamma_2 > 0 \) are design parameters.

Similar to (11)–(14), substituting (22) and (23) into (21), we can get
\[
V_2 \leq - \sum_{j=1}^{2} c_j z_j^2 + \sum_{j=1}^{2} \frac{\gamma_j}{\eta_j} \hat{T}_j \hat{T}_j + \frac{1}{2} \sum_{j=1}^{2} s_j^2 + \frac{1}{2} \hat{z}_2^2 + s_2 \hat{s}_2
\]
(24)

Similar to (15), define the first-order filter as follows
\[
r_3 \dot{\xi}_3 + \xi_3 = \alpha_2, \quad \xi_3(0) = \alpha_2(0)
\]
(25)

where \( r_3 \) is a given constant.

From (5), one can obtain \( \dot{s}_3 = \dot{\xi}_3 - \dot{\alpha}_2 = - \frac{s_3}{r_3} + Y_3(\cdot) \) and \( \ddot{\xi}_3 = - \frac{s_3}{r_3} \), where \( Y_3(\cdot) = Y_3(z_1, z_2, z_2, s_3, \hat{T}_1, \hat{T}_2) \) is a continuous function.

**Step 3:** Similar to the previous two steps, we obtain
\[
\ddot{z}_3 = \dot{x}_3 - \ddot{\xi}_3
= \mu + \Delta_3 - \ddot{\xi}_3
\]
(26)

Choose the following Lyapunov function candidate
\[
V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2} s_3^2 + \frac{1}{2} \hat{T}_3^2
\]
(27)

where \( \eta_3 \) is a positive design parameter.

In the light of (5), (26) and (27), the time derivative of \( V_3 \) satisfies
\[
\dot{V}_3 = \dot{V}_2 + z_3 \ddot{z}_3 + s_3 \ddot{s}_3 + \frac{1}{\eta_3} \hat{T}_3 \ddot{T}_3
= \dot{V}_2 + z_3 (\mu + \Delta_3 - \ddot{\xi}_3) + s_3 \ddot{s}_3 + \frac{1}{\eta_3} \hat{T}_3 \ddot{T}_3
\]
(28)

Similar to (10), the following inequality holds
\[
z_3 \Delta_3 \leq z_3 \text{sign}(z_3) (\hat{T}_3 + \ddot{T}_3)
\]
(29)

Substituting (24) and (29) into (28), we can get
\[
\dot{V}_3 \leq - \sum_{j=1}^{2} c_j z_j^2 + \sum_{j=1}^{2} \frac{\gamma_j}{\eta_j} \hat{T}_j \hat{T}_j + \frac{1}{2} \sum_{j=1}^{2} s_j^2 + s_3 \hat{s}_3
+ z_3 \left( \mu + \frac{1}{2} \Delta_3 + \text{sign}(z_3) \hat{T}_3 - \ddot{\xi}_3 \right) + s_3 \ddot{s}_3
+ \frac{1}{\eta_3} \hat{T}_3 (\text{sign}(z_3) \eta_3 z_3 - \ddot{T}_3)
\]
(30)

To assist us obtain the actual event-triggered controller, we initially design the virtual control signal \( \varphi \) and the parameter adaptive law of \( \hat{T}_3 \) as follows
\[
\varphi = - \left( c_3 z_3 + \frac{1}{2} z_3 + \text{sign}(z_3) \hat{T}_3 - \ddot{\xi}_3 \right)
= \left( \frac{Q_{ball} c_2}{M_{ball} (K_{ball} + x_1)^2} + \frac{Q_{ball} c_2^2 k_2}{M_{ball} (K_{ball} + x_1)^3} \right)
\]
(31)

\[
\hat{T}_3 = \text{sign}(z_3) \eta_3 z_3 - \gamma_3 \hat{T}_3
\]
(32)

where \( c_3 > 0 \) and \( \gamma_3 > 0 \) are design parameters.

### 3.2. Event-triggered controller design

Based on the past experience, by considering the ETC scheme, the event-triggered controller is given as
\[
u(t) = \omega(t_{kq}), \quad \forall t \in [t_{kq}, t_{kq+1})
\]
(33)

Moreover, the defined measurement error is \( e(t) = \omega(t) - u(t) \). We design the event-triggered mechanism as
\[
t_{kq+1} = \inf \left\{ t \in R, t > t_{kq} | e(t) \geq \lambda |u(t)| \right\}
\]
\[
+ \frac{M_{ball} (K_{ball} + x_1)^2}{Q_{ball} c_2} \sigma \}
\]
\[
t_1 = 0
\]
(34)

where \( t_{kq}, kq \in Z^+ \), and \( \lambda \) meets \( 0 < \lambda < 1 \), it is the transition control law. When (36) triggered, \( u(t) \) will be updated to \( \omega(t_{kq+1}) \).

Be convenient for the discussion later, we will think that the actuator normally operates, in short, it satisfies \( |e(t)| \leq \lambda |u(t)| + \frac{M_{ball} (K_{ball} + x_1)^2}{Q_{ball} c_2} \sigma \).

When \( u(t) > 0 \), we have
\[
- \lambda |u(t)| - \frac{M_{ball} (K_{ball} + x_1)^2}{Q_{ball} c_2} \sigma \leq \omega(t) - u(t)
\]
\[
\leq \lambda |u(t)| + \frac{M_{ball} (K_{ball} + x_1)^2}{Q_{ball} c_2} \sigma
\]
(35)

\[
\omega(t) - u(t) = m(t) \left( \lambda u(t) + \frac{M_{ball} (K_{ball} + x_1)^2}{Q_{ball} c_2} \sigma \right)
\]
(36)

where \( m(t) \in [-1, 1] \).
When \( u(t) < 0 \), the following inequalities hold
\[
\lambda |u(t)| - \frac{M_{\text{ball}}(K_{\text{ball}} + x_1)^2}{Q_{\text{ball}} c \rho} \leq \omega(t) - u(t)
\]
\[
\leq -\lambda |u(t)| + \frac{M_{\text{ball}}(K_{\text{ball}} + x_1)^2}{Q_{\text{ball}} c \rho}
\]
where
\[
\omega(t) - u(t) = m(t) \left( \lambda u(t) - \frac{M_{\text{ball}}(K_{\text{ball}} + x_1)^2}{Q_{\text{ball}} c \rho} \right)
\]
(37)

Through the discussions of the above two cases, it holds that
\[
\omega(t) - u(t) = m_1(t) \lambda u(t) + m_2 \frac{M_{\text{ball}}(K_{\text{ball}} + x_1)^2}{Q_{\text{ball}} c \rho}
\]
where \( m_1 = m_2 = m \), \( u(t) > 0 \). \( m_1 = m_2 = -m \), \( u(t) < 0 \).

Therefore, \( u(t) \) and \( \omega(t) \) can be given as
\[
u(t) = \frac{\omega(t)}{1 + m_1 \lambda} - \frac{m_2 \sigma}{1 + m_1 \lambda}
\]
(40)
\[
\omega(t) = \frac{M_{\text{ball}}(K_{\text{ball}} + x_1)^2}{Q_{\text{ball}} c \rho} (1 + \lambda)
\]
\[
[\varphi \tanh \left( \frac{z_3 \sigma}{\epsilon} \right) + \tilde{\sigma} \tanh \left( \frac{z_3 \tilde{\sigma}}{\epsilon} \right)]
\]
(41)

4. Stability analysis

Via 3-step backstepping design, the properties of the developed adaptive event-triggered control scheme can be summarized as following Theorem.

**Theorem 4.1:** For the systems (1) and (4), consider Assumptions 2.1 and 2, the proposed virtual controllers (12), (22) and (31), the parameter adaptive laws (13), (23) and (32), the adaptive event-triggered controller (33) with the designed ETM (34), can ensure that all signals in the model system are bounded. Moreover, by selecting the appropriate parameters, the state variable converge to a small neighbourhood of zero. Besides, the Zeno behaviour can be avoided.

**Proof:** Substituting (31) and (32) into (30), we have
\[
\dot{V}_3 \leq -3 \sum_{j=1}^3 c j \dot{x}_j^2 + 3 \sum_{j=1}^3 \frac{\gamma_j^2}{\eta_j} \dot{t}_j \dot{x}_j + \frac{1}{2} \sum_{j=1}^2 s_j \dot{s}_{j+1} + \sum_{j=2}^3 \dot{s}_j
\]
\[
+ z_3 \left( \mu - \varphi + \frac{Q_{\text{ball}} c \beta}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} \right)
\]
\[
- \frac{Q_{\text{ball}} c x_2}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3}
\]
(42)

From (2)–(40) and (41), it follows that
\[
z_3 \mu
\]
\[
= z_3 \frac{1}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3} \left[ -Q_{\text{ball}} c \beta \right]
\]
\[
\left( \frac{Q_{\text{ball}} c x_2 - R_{\text{ball}}(K_{\text{ball}} + x_1)^2}{Q_{\text{ball}}(K_{\text{ball}} + x_1) + L_{\text{ball}}(K_{\text{ball}} + x_1)^3} \right)
\]
\[
+ \frac{K_{\text{ball}} + x_1}{Q_{\text{ball}} + L_{\text{ball}}(K_{\text{ball}} + x_1)} u \left( K_{\text{ball}} + x_1 + Q_{\text{ball}} x_2 \right)
\]
\[
= z_3 \left( -\frac{Q_{\text{ball}} c \beta}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} + \frac{Q_{\text{ball}} c x_2}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3} \right)
\]
\[
- z_3 \frac{Q_{\text{ball}} c \rho}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} u
\]
\[
= z_3 \left( -\frac{Q_{\text{ball}} c \beta}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} + \frac{Q_{\text{ball}} c x_2}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3} \right)
\]
\[
+ z_3 \left( -\frac{Q_{\text{ball}} c \beta}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} + \frac{Q_{\text{ball}} c x_2}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3} \right)
\]
\[
+ z_3 \left[ -\frac{Q_{\text{ball}} c \rho}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} \left( \frac{1 + \lambda}{1 + m_1 \lambda} \right)
\]
\[
+ \tilde{\sigma} \tanh \left( \frac{z_3 \tilde{\sigma}}{\epsilon} \right) \right] + \frac{m_2 \sigma}{1 + m_1 \lambda}
\]
(43)

Take into consideration \( 0 < 1 + m_1 \lambda < 1 + \lambda, \tilde{\sigma} > \left| \frac{\sigma}{1 - \lambda} \right| \) and \( \frac{\mu_{\text{max}}}{m_{\text{max}}} \leq \left| \frac{\sigma}{1 - \lambda} \right| \), since \( 0 \leq |\lambda| - \epsilon \tanh(\tilde{\sigma}) \leq 0.2785 \epsilon \) (Sui & Tong, 2020), (43) can be expressed as
\[
z_3 \mu \leq z_3 \left( -\frac{Q_{\text{ball}} c \beta}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} + \frac{Q_{\text{ball}} c x_2}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3} \right)
\]
\[
- z_3 \varphi \tanh \left( \frac{z_3 \sigma}{\epsilon} \right) - z_3 \tilde{\sigma} \tanh \left( \frac{z_3 \tilde{\sigma}}{\epsilon} \right) + |z_3 \tilde{\sigma}|
\]
\[
\leq 0.557 \epsilon - |z_3 \varphi| + z_3
\]
\[
\left( -\frac{Q_{\text{ball}} c \beta}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} + \frac{Q_{\text{ball}} c x_2}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3} \right)
\]
(44)

Substituting (44) into (42), we can get
\[
\dot{V}_3 \leq -3 \sum_{j=1}^3 c j \dot{x}_j^2 + 3 \sum_{j=1}^3 \frac{\gamma_j^2}{\eta_j} \dot{t}_j \dot{x}_j + \frac{1}{2} \sum_{j=1}^2 s_j \dot{s}_{j+1} + \sum_{j=2}^3 \dot{s}_j
\]
\[
+ z_3 \left( -\frac{Q_{\text{ball}} c \beta}{M_{\text{ball}}(K_{\text{ball}} + x_1)^2} + \frac{Q_{\text{ball}} c x_2}{M_{\text{ball}}(K_{\text{ball}} + x_1)^3} \right)
\]
\[
+ \frac{1}{2} \sum_{j=1}^3 \dot{s}_j \dot{s}_j + 0.557 \epsilon
\]
(45)

By utilizing Young’s inequality, it follows that
\begin{equation}
\bar{T}_j \dot{\bar{T}}_j \leq -\frac{1}{2} \frac{T_j^2}{T_j^2} + \frac{1}{2} \frac{T_j^2}{T_j^2}
\end{equation}

\begin{equation}
\dot{s}_j \dot{s}_j = \dot{s}_j \left( -\frac{s_j}{T_j} + Y_j(\cdot) \right) \leq -\frac{s_j^2}{T_j} + \frac{1}{2a} \frac{s_j^2 y_j^2}{\eta_j} + \frac{1}{2} a
\end{equation}

where \( Y_j(\cdot) \) satisfies \(|Y_j(\cdot)| \leq H_j\) with constant \( H_j > 0\). \( a \) denotes a positive constant.

Substituting (46) and (47) into (45), one has

\[
\dot{V}_3 \leq -\sum_{j=1}^{3} c_j \dot{s}_j^2 - \frac{1}{2} \sum_{j=1}^{2} \frac{\eta_j}{s_j} \dot{T}_j^2 + \frac{1}{2} \sum_{j=1}^{3} \frac{\eta_j}{s_j} \dot{T}_j^2
\]

\[
+ \frac{1}{2} \sum_{j=1}^{2} \frac{s_j^2}{T_j^2} + \frac{3}{2} \sum_{j=1}^{3} \frac{\eta_j}{s_j} \dot{T}_j^2
\]

\[
+ 0.557 \varepsilon
\]

\[
\leq -\sum_{j=1}^{3} c_j \dot{s}_j^2 - \frac{1}{2} \sum_{j=1}^{2} \frac{\eta_j}{s_j} \dot{T}_j^2
\]

\[
- \sum_{j=2}^{3} \left( \frac{1}{T_j} - \frac{1}{2a} H_j^2 - \frac{1}{2} \right) \dot{s}_j^2
\]

\[
+ \frac{1}{2} \sum_{j=1}^{3} \frac{\eta_j}{s_j} \dot{T}_j^2 + 0.557 \varepsilon + a
\]

**Table 1.** Parameters of the magnetic levitation system.

| \( M_{\text{ball}} \) | \( g \) | \( K_{\text{ball}} \) | \( Q_{\text{mag}} \) | \( I_{\text{mag}} \) | \( R_{\text{mag}} \) |
|---|---|---|---|---|---|
| 0.54 | 9.8 | 0.008114 | 0.001624 | 0.7987 | 11.88 |
| kg | m/s² | m | km | H | Ω |

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The curves of \( x_1 \) (black) and \( x_2 \) (red).}
\end{figure}
According to (52), we can get

$$|e(t)| = \int_{t_{kq}}^{t_{kq+1}} |\dot{e}(t)| \, dt$$

$$\leq \int_{t_{kq}}^{t_{kq+1}} \dot{\varphi} \, dt$$

$$\leq \dot{\varphi}^* (t_{kq+1} - t_{kq})$$

(53)

In consideration of (34), the upper bound of the measures error is

$$\lim_{t \to \infty} \frac{|e(t)|}{\lambda |u(t)| + \frac{M_{ball}(K_{ball} + x_1)^2}{Q_{ball} k_p}} \sigma.$$  

Then, we obtain

$$t_{kq+1} - t_{kq} \geq \frac{\sigma}{\dot{\varphi}^*} > 0$$

(54)

Consequently, it can be deduced from (54) that the lower bound of the interval of events exists.

To sum up, the Zeno behaviour can be avoided in this paper.

5. Simulation results

Due to the limitations of the laboratory, we can refer to the data in Yang et al. (2004), i.e. the parameters of the device in Figure 3, see Table 1. It should be noted that the selected data in this paper will be closer to the real value of the vehicle tyre. It is used to verify that the effectiveness of the proposed adaptive ETC scheme for new type tyre systems.

According to the designed control scheme, select the following appropriate reference data $M_{ball} = 0.9$ kg, $g = 10 \text{ m/s}^2$, $K_{ball} = 0.3$ m, $Q_{mag} = 0.04$ Hm, $L_{mag} = 0.5$ H,

Figure 6. The curve of $x_3$.

Figure 7. The time intervals $t_{kq+1} - t_{kq}$ of triggering events.
Choose the design parameters in controller (35), virtual controllers (12), (22) and (33), adaptive laws (13), (23) and (34) as $c_1 = 0.5$, $c_2 = 25$, $c_3 = 25$, $\eta_1 = 2$, $\eta_2 = 5$, $\eta_3 = 3$, $\gamma_1 = 6.1$, $\gamma_2 = 17$, $\gamma_3 = 139$, $\varepsilon = 5$, $\lambda = 0.5$, $\tilde{\sigma} = 60$, $t_2 = 4$, $t_3 = 0.1$, $i_c = \sin(t)$, $\Delta_1 = 0.01 \sin(t)$, $\Delta_2 = 0.01 \cos(t)$, $\Delta_3 = 0.01 \sin(t)$. The initial conditions are $x_1(0) = 0.5$, $x_2(0) = 0.1$, $x_3(0) = 1$, $\xi_2(0) = 0$, $\xi_3(0) = 0.2$, $\tilde{T}_1(0) = 1$, $\tilde{T}_2(0) = 1$, $\tilde{T}_3(0) = 1$.

From the simulation results of Figures 5–9, we can be distinct to see that the proposed adaptive event-triggered control scheme can achieve the expected design effect of the systems, where Figure 5 exhibits the trajectories of $x_1$ and $x_2$. As shown in Figure 6, the trajectory of $x_3$ is more stable. The triggering time intervals $t_{k+1} - t_k$ is presented in Figure 7, and it is clear to achieve that the value of the minimum time interval is 0.005 s, which proves that there is no Zeno behaviour. Figure 8 plots the control signal $u$. Figure 9 depicts the coil current $i_c$.

Compared with the existing methods (Yang et al., 2004), the proposed adaptive ETC method in this paper further saves the communication bandwidth. The application of the scheme to the tyre systems can further maintain the stability of the vehicle in running. It can be seen that the simulation results can demonstrate the validity of the proposed control scheme.
6. Conclusion

In this article, we have investigated the problem of adaptive event-triggered control for new type tyre systems based on vehicle. The DSC technique has been introduced to overcome the ‘explosion of complexity’ problem in the traditional backstepping. Utilizing the improved event triggering mechanism, the designed control scheme cannot only reduce the communication burden for the new type tyres, but also ensure all the signals of the model system are bounded. The stability of the closed-loop system has been proved through Lyapunov stability analysis. In addition, the Zeno behaviour has also been precluded as usual.

The proposed method in this paper may interfere with neighbouring information in the multi-agent systems, this control method could not be applied to multi-agent systems due to the differences in event-triggered control channel. Therefore, the research on multi-agent will be carried out in the future. Then, we can also further research for the proposed model with prescribed performance in the future, it may produce better control effects. According to the current research status, the future research directions of the ETC are that the application will extend to other aspects of the vehicles, see Peng et al. (2018), Min et al. (2020) and Zhang and Jing (2020). Moreover, applying the event-triggered control scheme to vehicle cooperative formation control also will be one of the directions of our future research.

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ORCID

Kewen Li http://orcid.org/0000-0003-0925-3314

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