Coulomb-coupled quantum-dot thermal transistors

Yanchao Zhang1, Zhimin Yang1, Xin Zhang1, Bihong Lin2, Guoxing Lin1 and Jincan Chen1(a)

1 Department of Physics, Xiamen University - Xiamen 361005, PRC
2 College of Information Science and Engineering, Huaqiao University - Xiamen 361021, PRC

received 13 February 2018; accepted in final form 4 May 2018
published online 25 May 2018

PACS 73.23.Hk – Coulomb blockade; single-electron tunneling
PACS 85.35.Be – Quantum well devices (quantum dots, quantum wires, etc.)
PACS 44.10.+i – Heat conduction

Abstract – A quantum-dot thermal transistor consisting of three Coulomb-coupled quantum dots coupled to the respective electronic reservoirs by tunnel contacts is established. The heat flows through the collector and emitter can be controlled by the temperature of the base. It is found that a small change in the base heat flow can induce a large heat flow change in the collector and emitter. The huge amplification factor can be obtained by optimizing the Coulomb interaction between the collector and the emitter or by decreasing the tunneling rate at the base. The proposed quantum-dot thermal transistor may open up potential applications in low-temperature solid-state thermal circuits at the nanoscale.

Copyright © EPLA, 2018

Introduction. – Controlling heat flow at the nanoscale has attracted significant attention because of its fundamental and potential applications [1–3]. The thermal diode effect and negative differential thermal resistance (NDTR) are the two most important features for building the basic components of functional thermal devices, which are the key tools for the implementation of solid-state thermal circuits [3,4]. The first model of a thermal rectifier/diode was proposed by controlling the heat conduction in one-dimensional nonlinear lattice [5]. Based on different microscopic mechanisms, a very significant rectifying effect was exhibited and the concept of NDTR was also proposed in the subsequent works [6,7]. In recent years, the thermal diode effect and NDTR have been extensively studied in the different systems including quantum-dot systems [8–11], metal-dielectric interfaces [12], metal or superconductor systems [13–16], quantum Hall conductors [17], and spin quantum systems [18]. One of the particularly interesting tasks is to further build and implement a thermal transistor, which is analogous to an electronic transistor and can control the heat flows at the collector and emitter by small changes in the temperature or the heat flow at the base. Since Li et al. put forward the first theoretical proposal for a thermal transistor [7,19], several proposals have been given to design other types of thermal transistors, such as superconductor–normal-metal thermal transistors [20], near-field thermal transistors [21], far-field thermal transistors [22–24], and quantum thermal transistors [25]. Moreover, new concepts for thermal devices such as thermal logical gates [26] and thermal memories [27–29] have also been proposed and demonstrated.

In recent years, the electron and heat transport properties of Coulomb-coupled quantum-dot system have been investigated in detail in the thermoelectric generators [30,31] and refrigerators [32]. Moreover, recent experiments have shown that many new applications for Coulomb-coupled quantum-dot system including rectification [33–35], logical stochastic resonance [36], and thermal gating [37] can be realized by the voltage fluctuation or thermal fluctuation to control and manage the charge current. Ruokola et al. introduced a single-electron thermal diode consisting of two quantum dots in the Coulomb blockade regime. A remarkable feature of the Coulomb-coupled quantum-dot system is that the electron transport through the system is forbidden but the capacitive coupling between the two dots allows electronic fluctuations to transmit heat between the reservoirs [9]. Based on the Coulomb-coupled quantum dots, a thermal management device was recently constructed. A significant advantage of such a device is that it can not only implement thermal diodes separately in two different paths but also perform more thermal management operations, such as heat flow swap, thermal switch, and heat path selector [11].

(a)E-mail: jchene@xmu.edu.cn (corresponding author)
In this paper, we propose a thermal transistor consisting of three Coulomb-coupled quantum dots connected to the respective electronic reservoirs. In our system, the electron transport between the quantum dots is forbidden and the heat transport by phonons is effectively suppressed at sub-kelvin temperature [9]. But the heat transport between the reservoirs is allowed by electrons tunneling into and out of the dots and exchanging energy through Coulomb interaction. Thus, the present model is, in principle, a true thermal transistor.

Theoretical model and principle description. The model of a quantum-dot thermal transistor (QDTT) is illustrated in fig. 1. The QDTT is analogous to an electronic transistor consisting of three terminals: the base (B), the collector (C), and the emitter (E). Each terminal \(a = B, C, E\) consists of an electronic reservoir with temperature \(T_a\) and a quantum dot with the lowest single-particle energy level \(\varepsilon_a\) and the quantum dot is connected to the electronic reservoir through a tunnel barrier. Three quantum dots are capacitively coupled to each other and interact only through the long-range Coulomb force, so that there is no electron transport between the quantum dots. However, the Coulomb interaction \(U_{ab}\) \((a, b = B, C, E, a \neq b)\) between the quantum dots \(a\) and \(b\) allows through electron tunneling into and out of quantum dots \(a\) and \(b\) to transmit the heat between the electronic reservoirs \(a\) and \(b\) with a temperature difference. When a constant temperature bias is applied between the emitter and the collector, the heat flow between the quantum dots is forbidden and the tunneling amplitudes.

The Hamiltonian of the QDTT is described by

\[
H = H_D + H_R + H_T,
\]

where

\[
H_D = \sum_{a=B,C,E} \varepsilon_a d_a^\dagger d_a + \sum_{a,b=\{B,C,E\}, a \neq b} U_{ab} d_a^\dagger d_b d_b^\dagger d_a \tag{2}
\]

is the Hamiltonian of Coulomb-coupled quantum dots, \((d^\dagger, d)\) denote the creation/annihilation operators of quantum dots,

\[
H_R = \sum_k \sum_{a=\{B,C,E\}} \varepsilon_{a k} c_{a k}^\dagger c_{a k} \tag{3}
\]

is the Hamiltonian of reservoirs, \(\varepsilon_{a k}\) is the energy of the noninteracting reservoir electrons with continuous wave number \(k\), \((c^\dagger, c)\) denote the creation/annihilation operators of heat reservoirs,

\[
H_T = \sum_k \sum_{a=\{B,C,E\}} (t_{a k} c_{a k}^\dagger d_a + t_{a k}^* d_a^\dagger c_{a k}) \tag{4}
\]

is the tunneling Hamiltonian between the quantum dots and the reservoirs, and \(t_{a k}\) and its conjugate \(t_{a k}^*\) denote the tunneling amplitudes.

The Coulomb-coupled quantum-dot system is denoted by the charge configurations \((n_B, n_E, n_C)\), where \(n_a\) is the occupation number of quantum dots \(a\). In the Coulomb blockade regime, each of these quantum dots can be occupied only by zero or one electron \((n_a = 0, 1)\). Thus, the dynamics of the quantum-dot system is characterized by eight charge states labeled as \([1] = \{0, 0, 0\}, [2] = \{1, 0, 0\}, [3] = \{0, 1, 0\}, [4] = \{0, 0, 1\}, [5] = \{1, 1, 0\}, [6] = \{1, 0, 1\}, [7] = \{0, 1, 1\}, and [8] = \{1, 1, 1\}\). The occupation probabilities for eight charge states are given by the diagonal elements of the density matrix, \(\rho = (\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7, \rho_8)^T\). In the limit of weak tunneling coupling \((\hbar \gamma \ll k_B T)\), the broadening of energy levels can be neglected and the transmission through tunnel barriers is well described by sequential tunneling rates. The off-diagonal density matrix elements do not contribute to steady-state transport and can be neglected. Thus, the time evolution of occupation probabilities is given by a master equation. The matrix form can be written as \(d\rho/dt = M\rho\), where \(M\) denotes the matrix containing the transition rates and is given by Fermi’s golden rule. The steady-state heat flows from electronic reservoir \(a\) to quantum dot \(a\) are given by the stationary solution of the master equation \(M\rho = 0\). Because the electronic reservoir of the emitter induces the transitions of charge states including \([1] \leftrightarrow [3], [2] \leftrightarrow [5], [4] \leftrightarrow [7]\) and \([6] \leftrightarrow [8]\), the heat flow of the emitter is given by

\[
J_E = (\varepsilon_E - \mu_E) (\Gamma_{31}\tilde{\rho}_1 - \Gamma_{13}\tilde{\rho}_3) + (\varepsilon_E + U_{EB} - \mu_E) (\Gamma_{52}\tilde{\rho}_2 - \Gamma_{25}\tilde{\rho}_5) + (\varepsilon_E + U_{CE} - \mu_E) (\Gamma_{74}\tilde{\rho}_4 - \Gamma_{47}\tilde{\rho}_7) + (\varepsilon_E + U_{EE} + U_{CE} - \mu_E) (\Gamma_{86}\tilde{\rho}_6 - \Gamma_{68}\tilde{\rho}_8). \tag{5}
\]

The electronic reservoir of the collector triggers the transitions of charge states including \([1] \leftrightarrow [4], [2] \leftrightarrow [6]\),
is written as \( T \) and served. This means that the thermal transistor is able to divide by the variation of the heat flow applied at the collector. This thermal transistor effect is characterized by an amplification factor \( \alpha \) with the energy conservation.

\[
J_C = (\varepsilon_C - \mu_C)(\Gamma_{41} \rho_1 - \Gamma_{14} \rho_4) + (\varepsilon_C + U_{CB} - \mu_C)(\Gamma_{62} \rho_3 - \Gamma_{26} \rho_6) + (\varepsilon_C + U_{CB} - \mu_C)(\Gamma_{37} \rho_5 - \Gamma_{73} \rho_7) + (\varepsilon_C + U_{CB} + U_{CE} - \mu_C)(\Gamma_{85} \rho_5 - \Gamma_{58} \rho_8). \tag{6}
\]

The electronic reservoir of the base drives the transitions of charge states including \(|1\rangle \leftrightarrow |2\rangle, |3\rangle \leftrightarrow |5\rangle, |4\rangle \leftrightarrow |6\rangle \) and \(|7\rangle \leftrightarrow |8\rangle \), and the heat flow of the base is expressed as

\[
J_B = (\varepsilon_B - \mu_B)(\Gamma_{21} \rho_1 - \Gamma_{12} \rho_2) + (\varepsilon_B + U_{CB} - \mu_B)(\Gamma_{53} \rho_3 - \Gamma_{35} \rho_5) + (\varepsilon_B + U_{EB} - \mu_B)(\Gamma_{64} \rho_4 - \Gamma_{46} \rho_6) + (\varepsilon_B + U_{CB} + U_{EB} - \mu_B)(\Gamma_{87} \rho_7 - \Gamma_{78} \rho_8). \tag{7}
\]

In eqs. (5)–(7), \( \Gamma_{ji} = \gamma_0 f_a(E_{ji}) \) \((i, j = 1, 2, \ldots, 8 \) and \( i < j \)\) is the transition rate from the charge state \(|i\rangle\) to \(|j\rangle\) with an electron from the electronic reservoir \(a\) into the quantum dot \(a\), and \( \Gamma_{ij} = \gamma_a(1 - f_a(E_{ji})) \) when an electron leaves the quantum dot \(a\) into the electronic reservoir \(a\), where \( f_a(x) = \text{exp}[x - \mu_a)/(k_BT_a)] + 1 \)^{-1} \(\) is the Fermi-Dirac distribution function with the chemical potential \(\mu_a\) and temperature \(T_a\), \( E_{ji} = E_j - E_i \), \( E_i \) is the energy of the charge state \(i\), \( k_B \) is the Boltzmann constant. \(\gamma_0\) is the tunneling rate that is effectively the parameter for all tunneling transitions in each terminal \(a\). In the present model, the electron transport between the quantum dots is forbidden. Through the etched trenches of about 150 nm, the particle exchange between the quantum-dot subsystems can be effectively prevented in the experiment \cite{33,36}. Hence, there is no Joule heating. Moreover, the heat dissipation by phonons is effectively suppressed at sub-kelvin temperature \cite{9}. Thus, the heat flows fulfill \( J_C + J_E + J_B = 0 \), which complies with the energy conservation.

**NDTR and thermal transistor effect.** - Let us now turn to the discussion of a thermal transistor, where the small change in the base heat flow or the base temperature can control the heat flows through the emitter and collector. This thermal transistor effect is characterized by an amplification factor \( \alpha_{E/C} \), which is defined as the change of the emitter heat flow or the collector heat flow divided by the variation of the heat flow applied at the base. In the present QD model shown in fig. 1, the temperatures of the collector and emitter are fixed at \( T_C \) and \( T_E \) (\( T_C < T_E \)), respectively. The base at temperature \( T_B \) (\( T_C < T_B < T_E \)) can be used to control the heat flows \( J_E \) and \( J_C \) with the help of the base heat flow \( J_B \). Thus, the amplification factor \( \alpha_{E/C} \) is defined as

\[
\alpha_{E/C} = \frac{\partial J_{E/C}}{\partial J_B}. \tag{8}
\]

When \( |\alpha_{E/C}| > 1 \), a thermal transistor effect will be observed. This means that the thermal transistor is able to amplify a small thermal signal. In other words, a small change of the heat flow through the base can yield a large change in the heat flows through the emitter and collector.

We defined the heat flows from the emitter into the quantum dot and from the quantum dot into the collector as positive, and the differential thermal resistances are positive along the direction of the increase of the temperature difference. The differential thermal resistances (DTRs) of the emitter and collector are defined as

\[
R_E = \left( \frac{\partial J_E}{\partial T_B} \right)^{-1}_{T_E} \tag{9}
\]

and

\[
R_C = \left( \frac{\partial (-J_C)}{\partial T_B} \right)^{-1}_{T_C}. \tag{10}
\]

By using eqs. (9) and (10), the amplification factor in eq. (8) can be rewritten as

\[
\alpha_{E/C} = \frac{-R_{E}}{R_{C} + R_{E}}. \tag{11}
\]

The thermal transistor effect, \( i.e., \ |\alpha_{E/C}| > 1 \), implies that there exists a NDTR. The DTRs of the emitter and collector as a function of the base temperature \( T_B \) are plotted in fig. 2(a).

It is shown that a NDTR is observed at the emitter. This then makes it possible that, over a wide regime of parameters, not only the emitter heat flow \( J_E \) but also...
the collector heat flow $J_C$ may increase when the base temperature $T_B$ increases, as shown in figs. 2(b) and (c). This is because in the configuration of the present system, the available thermal energy from the collector reservoir at the low temperature $T_C$ is much smaller than the required energy $\mu C - \varepsilon C$. But the probability of the electron tunneling into the collector reservoir is effectively controlled by the base temperature $T_B$ because the Coulomb interaction energy $U_{CB}$ can compensate for the energy required by the collector electron. When the base temperature $T_B$ is increased, the probability of the base electron tunneling increases. This can effectively energize the electron tunneling into the collector reservoir and thus efficiently transfer energy from emitter to collector through the Coulomb interaction energy $U_{CE}$. This asymmetric Coulomb blockade configuration is the origin of the NDTR [9,11].

The thermal transistor effect is shown in fig. 3. In fig. 3(a), the base heat flow $J_B$ vs. the base temperature $T_B$ is plotted. It is shown that the base heat flow $J_B$ is significantly smaller than the emitter heat flow $J_E$ and the collector heat flow $J_C$, and a small change of the heat flow $J_B$ corresponds to a large change of the heat flows $J_E$ and $J_C$. This leads to a noticeable amplification effect for the base heat flow $J_B$. The amplification ability of the thermal transistor is described by the amplification factors $\alpha_E$ and $\alpha_C$, as shown in fig. 3(b). We first emphasize that this effect is the result of negative differential thermal resistance. This effect can be explained qualitatively as follows: The base heat flow $J_B$ comes from two different contributions. One part is the negative (opposite to the positive direction) heat flow from the emitter to the base through the Coulomb interaction $U_{EB}$ when there is a temperature gradient $T_E - T_B$, and the other part is the positive heat flow from the base to the collector through the Coulomb interaction $U_{CB}$ when there is a temperature gradient $T_B - T_C$. The base temperature is the same as that of the collector at the beginning, and there is no heat flow between the base and the collector. However, the emitter has heat flow into the base by the Coulomb interaction $U_{EB}$, so the base heat flow $J_B$ is negative at this time. As the base temperature increases, although the temperature gradient $T_E - T_B$ is reduced, the heat flow from the emitter to the base by the Coulomb interaction $U_{EB}$ continues to increase due to negative differential thermal resistance. Consequently, the base heat flow $J_B$ continues to decrease (absolute value increase) until the positive heat flow from the base to the collector by the Coulomb interaction $U_{CB}$ increases to a greater degree. At this point the base heat flow $J_B$ reaches a minimum value. The base heat flow $J_B$ then increases with the increase of the base temperature. Specific analysis shows that the amplification factors $\alpha_E$ and $\alpha_C$ diverge and lead to an infinite amplification factor at $T_B^m \approx 285 \text{ mK}$, which is due to the fact that the base heat flow $J_B$ has a minimum at this point. In other words, the change rate of $J_B$ at this point is zero, as shown in fig. 3(a). In the range of $T_B > T_B^m$, the amplification factors $\alpha_E$ and $\alpha_C$ gradually decrease with the increase of the base temperature $T_B$. In particular, the base heat flow $J_B = 0$ at $T_B \approx 333 \text{ mK}$. This means that there is no heat flow through the base, but the amplification effect will still occur at this point since $\alpha_E \approx 31$ and $\alpha_C \approx -32$. So within the range of these parameters, the system shows a good magnification effect.

**Optimization of amplification factors.** – In order to obtain good thermal transistor effect, two feasible methods can be implemented: i) by reducing the base tunneling rate $\gamma_B$ or ii) by adjusting the Coulomb interaction energy $U_{CE}$.

i) Reducing the base tunneling rate $\gamma_B$ can effectively suppress energy transport at the base. This results in a decrease in the base heat flow, which can increase the amplification factor. Figure 4 shows the amplification factors $\alpha_E$ and $\alpha_C$ as functions of the base tunneling rate $\gamma_B/\gamma$. It is found that the decrease of the base tunneling rate $\gamma_B$ can significantly increase the amplification factors $\alpha_E$ and $\alpha_C$. The regulation tunneling rate is the result of the system structure or parameter control. A detailed description was given in ref. [31]. The experimental demonstration of the direct control of the tunneling rate is well suited to this purpose [31].

ii) Adjusting the Coulomb interaction $U_{CE}$ between the collector and the emitter can directly control the quantized packets of energy $U_{CE}$ to transfer from the emitter into the collector. Adjusting the Coulomb interaction $U_{CE}$ can be done experimentally by electrostatic bridging of two subsystems [38,39]. The DTRs of the emitter $R_E$ and the collector $R_C$ as functions of $U_{CE}$ are shown in fig. 5(a).
Coulomb-coupled quantum-dot thermal transistors

Fig. 4: (Color online) The amplification factors $\alpha_E$ and $\alpha_C$ as functions of the base tunneling rate $\gamma_B/\gamma$ for $T_B = 280$ mK. The values of other parameters are the same as those used in fig. 2.

Fig. 5: (Color online) (a) The differential thermal resistances of the emitter $R_E$ and the collector $R_C$ as functions of $U_{CE}$ for $T_B = 280$ mK. (b) The amplification factors $\alpha_E$ and $\alpha_C$ as functions of $U_{CE}$ for $T_B = 280$ mK. The values of other parameters are the same as those used in fig. 2.

It is clearly shown that the DTRs of the emitter $R_E$ and the collector $R_C$ flip each other at the point of $U_{CE}^0 = 8.1k_BT$. At this point, the amplification factors $\alpha_E$ and $\alpha_C$ are zero at the same time, as shown in fig. 5(b), which shows the curves of amplification factors $\alpha_E$ and $\alpha_C$ vs. the Coulomb interaction $U_{CE}$. It is found that $U_{CE}$ has an optimized interval, which can give a significantly greater amplification factor. But, the amplification factors $\alpha_E$ and $\alpha_C$ gradually decrease with the increase of $U_{CE}$ and eventually lead to the loss of the amplification effect. When the Coulomb interaction $U_{CE} = 0$, there will be no direct heat transfer from the emitter to the collector. Although both of them exchange energy with the base, the base only controls the heat transport and the system loses the transistor effect.

Fig. 6: (Color online) The amplification factors $\alpha_E$ and $\alpha_C$ as functions of the base temperature $T_B$ for $U_{CE} = 10k_BT$. The values of other parameters are the same as those used in fig. 2.

The amplification factors $\alpha_E$ and $\alpha_C$ as functions of the base temperature $T_B$ for $U_{CE} = 10k_BT$ are illustrated in fig. 6. It is found that there is a significant increase for the amplification factors throughout the entire interval of the base temperature by optimizing $U_{CE}$. Compared with the non-optimized case, as shown in fig. 3(b), the amplification factors of the optimized case are increased by almost ten times. It is also found that by optimizing $U_{CE}$, the amplification factors $\alpha_E/C$ are larger than those in the models of the near-field thermal transistor [21] and quantum thermal transistor [25]. Even though the proposed model is based on the same general idea in ref. [25], the underlying microscopic mechanism in the present model is different from that in ref. [25], in which the thermal transistor is composed of three two-level systems coupled with each other, each of them being connected to a Bose reservoir. Every two-level system is always occupied and exchanges heat through spin interactions. In our model, since quantum dots can exchange an electron with an electron reservoir, the system has completely different quantum states from those in ref. [25]. Three quantum dots are capacitively coupled to each other and exchange energy through the Coulomb interaction. Recently, the Coulomb-coupled quantum-dots system has been experimentally realized and extensively studied [31,33,36,37]. Thus, our work will be sufficiently large so as to encourage the experimental work in the near future.

Conclusions. – The performance of a thermal transistor consisting of three Coulomb-coupled quantum dots has been theoretically analyzed. It is found that the phenomenon of the NDTR constitutes the main ingredient for the operation of a thermal transistor. A small change in the heat flow through the base can control a large heat flow change in the collector and emitter. Such a thermal transistor is able to amplify a small heat signal that...
is injected into the base. The thermal transistor effect can be significantly improved by reducing the base tunneling rate or optimizing the Coulomb interaction between the collector and the emitter. Extremely high amplification factors can be obtained in a wide range of the base temperature. These results will stimulate interest for the quantum-dot thermal transistor and open up potential applications in low-temperature solid-state thermal circuits at the nanoscale.

***

This work was supported by the National Natural Science Foundation (No. 11675132), People’s Republic of China.

REFERENCES

[1] Giazotto F., Heikilä T. T., Luukanen A., Savin A. M. and Pekola J. P., Rev. Mod. Phys., 78 (2006) 217.
[2] Wang L. and Li B., Phys. World, 21 (2008) 27.
[3] Li N., Ren J., Wang L., Zhang G., Hänggi P. and Li B., Rev. Mod. Phys., 84 (2012) 1045.
[4] Chang C. W., Okawa D., Majumdar A. and Zettl A., Science, 314 (2006) 1121.
[5] Terraneo M., Peyrard M. and Casati G., Phys. Rev. Lett., 88 (2002) 094302.
[6] Li B., Wang L. and Casati G., Phys. Rev. Lett., 93 (2004) 184301.
[7] Li B., Wang L. and Casati G., Appl. Phys. Lett., 88 (2006) 143501.
[8] Scheirner R., König M., Reuter D., Wieck A. D., Gould C., Buhmann H. and Molenkamp L. W., New J. Phys., 10 (2008) 083016.
[9] Ruokola T. and Ojanen T., Phys. Rev. B, 83 (2011) 241404(R).
[10] Jiang J. H., Kulikarni M., Segal D. and Imry Y., Phys. Rev. B, 92 (2015) 045309.
[11] Zhang Y., Zhang X., Ye Z., Lin G. and Chen J., Appl. Phys. Lett., 110 (2017) 153501.
[12] Ren J. and Zhu J., Phys. Rev. B, 87 (2013) 241412(R).
[13] Giazotto F. and Bergeret F. S., Appl. Phys. Lett., 103 (2013) 242602.
[14] Fornieri A., Martínez-Pérez M. J. and Giazotto F., Appl. Phys. Lett., 104 (2014) 183108.
[15] Martínez-Pérez M. J., Fornieri A. and Giazotto F., Nat. Nanotechnol., 10 (2015) 303.
[16] Fornieri A., Timossi G., Bosio R., Solinas P. and Giazotto F., Phys. Rev. B, 93 (2016) 134508.
[17] Sánchez R., Söthmann B. and Jordan A. N., New J. Phys., 17 (2015) 075006.
[18] Ordonez-Miranda J., Ezzahri Y. and Joulain K., Phys. Rev. E, 95 (2017) 022128.
[19] Chung Lo W., Wang L. and Li B., J. Phys. Soc. Jpn., 77 (2008) 054402.
[20] Saira O. P., Meschke M., Giazotto F., Savin A. M., Möttönen M. and Pekola J. P., Phys. Rev. Lett., 99 (2007) 027203.
[21] Ben-Abdallah P. and Biehs S. A., Phys. Rev. Lett., 112 (2014) 044301.
[22] Joulain K., Ezzahri Y., Drevillon J. and Ben-Abdallah P., Appl. Phys. Lett., 106 (2015) 133505.
[23] Ordonez-Miranda J., Ezzahri Y., Drevillon J. and Joulain K., Phys. Rev. Appl., 6 (2016) 054003.
[24] Prod’homme H., Ordonez-Miranda J., Ezzahri Y., Drevillon J. and Joulain K., J. Appl. Phys., 119 (2016) 194502.
[25] Joulain K., Drevillon J., Ezzahri Y. and Ordonez-Miranda J., Phys. Rev. Lett., 116 (2016) 200601.
[26] Wang L. and Li B., Phys. Rev. Lett., 99 (2007) 177208.
[27] Wang L. and Li B., Phys. Rev. Lett., 101 (2008) 267203.
[28] Kubytskyi V., Biehs S. A. and Ben-Abdallah P., Phys. Rev. Lett., 113 (2014) 074301.
[29] Dyakov S. A., Dai J., Yan M. and Qi M., J. Phys. D: Appl. Phys., 48 (2015) 305104.
[30] Sánchez R. and Büttiker M., Phys. Rev. B, 83 (2011) 085428.
[31] Thierschmann H., Sánchez R., Söthmann B., Arnold F., Heyn C., Hansen W., Buhmann H. and Molenkamp L. W., Nat. Nanotechnol., 10 (2015) 854.
[32] Zhang Y., Lin G. and Chen J., Phys. Rev. E, 91 (2015) 052118.
[33] Hartmann F., Pfeffer P., Höfling S., Kamp M. and Worschech L., Phys. Rev. Lett., 114 (2015) 146805.
[34] Sánchez R., Thierschmann H. and Molenkamp L., Phys. Rev. B, 95 (2017) 241401(R).
[35] Sánchez R., Thierschmann H. and Molenkamp L., New J. Phys., 19 (2017) 113040.
[36] Pfeffer P., Hartmann F., Höfling S., Kamp M. and Worschech L., Phys. Rev. Appl., 4 (2015) 041011.
[37] Thierschmann H., Arnold F., Mitterüller M., Maier L., Heyn C., Hansen W., Buhmann H. and Molenkamp L. W., New J. Phys., 17 (2015) 113003.
[38] Chan I. H., Westervelt R. M., Maranowski K. D. and Gossard A. C., Appl. Phys. Lett., 80 (2002) 1818.
[39] hübel A., Weis J., Dietsche W. and Klitzing K. V., Appl. Phys. Lett., 91 (2007) 102101.