Conductivity of 2D many-component electron gas, partially-quantized by magnetic field

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Abstract

The 2D semimetal consisting of heavy holes and light electrons is studied. The consideration is based on assumption that electrons are quantized by magnetic field while holes remain classical. We assume also that the interaction between components is weak and the conversion between components is absent. The kinetic equation for holes colliding with quantized electrons is utilized.

It has been stated that the inter-component friction and corresponding correction to the dissipative conductivity $\sigma_{xx}$ do not vanish at zero temperature due to degeneracy of the Landau levels. This correction arises when the Fermi level crosses the Landau level. The limits of kinetic equation applicability were found. We also study the situation of kinetic memory when particles repeatedly return to the points of their meeting.

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Introduction

Since the discovery of quantum Hall effect the problem of 2D electron system in strong magnetic field has attracted big attention. It was generally accepted that the most interesting thing is low-temperature limit when all inelastic processes are frozen out and the system can be treated as the electron-impurity one. Here we concentrate our consideration on the case of semimetal with coexisting electrons and holes. Such systems based on 2D HgTe layers were obtained and have been intensively studied for the recent years [1, 2].

The specificity of semimetal is the presence of electron-hole scattering. Due to large density of the second component this process can be comparable with the impurity scattering. Usually, in the Fermi system at $T = 0$ the interparticle scattering disappears and the friction between components gives temperature additions $\sim T^2$ to the transport coefficients [2, 3]. This is not the case in the system with a degenerate ground state, in particular, caused by the Landau quantization. In such a system the scattering redistributes particles within the degenerate state that needs no energy transfer. As a result, the friction does not disappear even at zero temperature.

Problem formulation

We consider a 2D semimetal with $g_e$ equivalent electron valleys and $g_h$ equivalent hole valleys centered in points $p_{e,i}$ and centered in points $p_{h,i}$, correspondingly. The conduction bands with energy spectra $(p - p_{e,i})^2/2m_e$ overlaps with the valence bands $E_g - \varepsilon_{p-p_{h,i}}$, $\varepsilon_p = p^2/2m_h$ ($E_g > 0$). The hole mass $m_h$ is assumed to be much larger than the electron mass $m_e$. The distances between electron and hole extrema $|p_{h,i} - p_{e,j}|$ are supposed to be large to suppress the electron-hole conversion. At the same time, the scattering between electrons and holes changing the momenta near extrema are permitted. Without the loss of generality, further we shall count the momenta from the band extrema and replace $p - p_{h,j} \rightarrow p$, $p - p_{e,i} \rightarrow p$.

The system is placed in a moderately strong magnetic field, such that the electrons are quantized, while holes stay classical. In other words, the number of the filled hole Landau levels $N_h + 1$ is large, while the analogical electron number $N_e + 1$ has the order of unity. We shall consider the low-temperature limit when the electron transitions occur within the
same Landau level and transitions between different electron Landau levels are forbidden. The energy conservation permits this process only when the Landau level is partially filled, i.e. in a state of compressible Landau liquid.

We shall neglect the rearrangement of the energy spectrum caused by the interaction between electrons and electrons and holes. The Landau levels widening will be also neglected.

The interaction of quantized gas with a classical one is an unusual situation. The kinetics of holes can be described by a classical kinetic equation, while electrons need a quantum description.

The vector-potential of the magnetic field $\mathbf{H} = (0, 0, H)$ is chosen in the form of $\mathbf{A} = (0, Hx, 0)$, electric field $\mathbf{E}$ is directed along $x$ axis. Holes will be described by a momentum $\mathbf{p}$ and distribution function $f_p$. Electrons with Landau level $n$ and momentum $k$ along $y$ axis are described by the distribution function $\varphi_{n,k}$.

Here we should explain some trick we will use below. If one uses the electron states in quantizing magnetic field only, the uniform spatial distributions of electrons gives $\varphi_{n,k}$ not depending on $k$. Thus, there is no non-equilibrium in the electron subsystem until one takes off-diagonal terms of the density matrix into consideration. In fact, the diagonal elements of the density matrix are larger than off-diagonal ones, but they do not determine the current. At the same time, the electron states in the magnetic and electric fields without scattering are localized along the field. So, we can consider them as the first approximation to avoid a loss of electric field in the kinetic equations. Hence, we shall include the electric field in the electron states. Namely, we shall consider $|n, k>$ as the states in the presence of the electric field directed along axis $x$. Bearing in mind the smallness of the electric field we shall expand the results everywhere it can be done.

In accordance with above-mentioned, we shall use the states in crossed electric and magnetic fields $\psi_{n,k}(\mathbf{r}) = e^{iky} \phi_n((x - X_k))/\sqrt{aL_y}$, where $\phi_n$ are normalized oscillator functions, $L_y$ is the normalizing length of the system in $y$-direction, $X_k = X_k^{(0)} - v_d/\omega_e$, $X_k^{(0)} = -a^2k$ is the coordinate of the center of cyclotron motion, $v_d = c[\mathbf{E}, \mathbf{H}]/H^2$ is the drift velocity, $a = \sqrt{c/eH}$ is the magnetic length, $\omega_e = eH/m_e c$ is the electron cyclotron frequency, $-e$ is the electron charge; we set $\hbar = 1$ and will restore the dimensionality in the final expressions. The corresponding energy is presented by $\varepsilon_{n,k} = \omega_e(n + 1/2) + eEX_k$. 


Dissipative conductivity

The transmission of the momentum between electrons and holes is determined by the scattering processes. The collision term in the kinetic equation for holes in the Born approximation reads

\[
\hat{I}_{he}\{f_p\} = \frac{2\pi}{S^2} \frac{g_e}{2} \sum_{p',q,\gamma,\gamma'} |u_q|^2 \delta_{p',p+q} |J_{\gamma',\gamma}(q)|^2 \delta(\varepsilon_{p'} - \varepsilon_p + \varepsilon_{\gamma'} - \varepsilon_\gamma) \times \\
\left[ f_p (1 - f_{p'}) \varphi_\gamma (1 - \varphi_{\gamma'}) - f_{p'} (1 - f_p) \varphi_{\gamma'} (1 - \varphi_\gamma) \right].
\]

Here \( u_q \) is the Fourier transform of potential of interaction between electron and hole, \( S \) is the system area, \( J_{\gamma',\gamma}(q) = \langle \gamma' | e^{i\mathbf{qr}} | \gamma \rangle \), \( \gamma = (n,k) \), \( \gamma' = (n',k') \), \( \varepsilon_p = p^2/2m_h \) is the hole energy (\( m_h \) is the hole effective mass).

Due to the uniformity of the space the quantity \( \varphi_\gamma \) does not depend on the wave vector and coincides with the equilibrium distribution function. At zero temperature all factors \( \varphi_{n'}(1 - \varphi_n) \equiv 0 \), excluding the contribution with \( n = n' = N_e \), where \( N_e \) is the number of the last partially filled Landau level. The quantities \( N_e \) and \( \varphi_{N_e} \equiv \nu \) can be expressed via the electron density \( n_e \) as \( N_e = [n_e \pi a^2 / g_e] \), \( \nu = \{ n_e \pi a^2 / g_e \} \) (square and figure brackets mean the integer and fractional parts). We shall expand the collision term with respect to weak non-equilibrium, assuming that the electric field and the deviation of distribution function from equilibrium are small.

Expanding Eq. (1) we get

\[
\hat{I}_{he}\{f_p\} = \frac{2\pi}{S^2} \frac{g_e}{2} \sum_{p',n,k} R_n(|\mathbf{p} - \mathbf{p}'|) \left[ \delta(\varepsilon_p - \varepsilon_{p'}) \left[ (\delta f_p - \delta f_{p'}) \varphi_n^{(0)} (1 - \varphi_n^{(0)}) \right] \\
+ \delta'(\varepsilon_p - \varepsilon_{p'}) e \mathbf{a}^2 (p_y' - p_y) \varphi_n^{(0)} (1 - \varphi_n^{(0)}) (f_p^{(0)} - f_{p'}^{(0)}) \right].
\]

Here \( \delta f_p \) is linear in \( E \) correction to the hole distribution function, \( f_p^{(0)} \), \( \varphi_n^{(0)} \) are equilibrium distribution functions of holes and electrons,

\[
R_n(q) = |u_q|^2 (L_n^2 (q^2 a^2 / 2) e^{-q^2 a^2 / 2}),
\]

\( L_n \) are the Laguerre polynomials. The function \( R_n(q) \) has a characteristic size in \( q \)-space \( 1/s \). The parameter \( s \) is determined by the largest of sizes of potential \( L \) and wave functions.
of electrons $a\sqrt{2(n+1)}$. In the coordinate space, $s$ corresponds to the typical impact parameter for scattering.

Let us consider the hole transport. The kinetic equation for $\delta f_p$ reads

$$eE\frac{\partial f_p^{(0)}}{\partial p} + \omega_h[p, \hbar]\frac{\partial \delta f_p}{\partial p} = \hat{I}_{he}\{f_p\}. \quad (4)$$

where $\omega_h = eH/m_hc$, $\hbar = H/H$.

The solution of Eq.(4) can be searched using a usual substitution

$$\delta f_p = pC(\varepsilon_p)\frac{\partial f_p^{(0)}}{\partial \varepsilon_p}. \quad (5)$$

Then Eq.(4) is algebraized

$$C + \omega_h\tau[h, C] = -\frac{e\tau}{m_h}E - \frac{\tau}{\tau_{he}}v_d \equiv -\frac{e\tau}{m_h}E_{ef}. \quad (6)$$

Here we introduced notations: $1/\tau = 1/\tau_i + 1/\tau_{he}$, $\tau_i$ is the impurity transport relaxation time of holes. The hole transport relaxation time, due to hole-electron interaction, is presented by

$$\frac{1}{\tau_{he}} = 2g_e\nu(1-\nu)\sum_{p'}\frac{(p-p')p}{p^2}R_N(|p-p'|)\delta(\varepsilon_p-\varepsilon_{p'}). \quad (7)$$

According to Eq.(7) the relaxation process can be interpreted as energy-conserving collisions of holes with immobile distributed charges with form-factors $R_N(q)$ and density $\nu(1-\nu)/\pi a^2$. The factor $\nu(1-\nu)$ reflects the circumstance that the scattering occurs due to the fluctuations of electron density; fully occupied Landau states produce no fluctuations. One can introduce the quantum cross-section of scattering $\sigma$ by the relation $\tau_{he} = \pi a^2/\sigma\nu(1-\nu)v_{Fh}$ ($v_{Fh}$ is the hole Fermi velocity). The quantity $\sigma$, unlike $s$, characterizes the strength of scattering, rather than the size of the scatterer.

After simplifications,

$$\frac{1}{\tau_{he}} = 2g_e\nu(1-\nu)\frac{m_h}{2\pi^2p^2a^4}\int_0^{2p^2a^2} e^{-t}L_N^2(t) |u_{\sqrt{2t/a}}|^2 \sqrt{t} \frac{dt}{2p^2a^2-t}. \quad (8)$$

The effective electric field consists of the external electric field (the first contribution in the middle expression in Eq.(6)) and the drag force from electrons, drifting across the external field (the second term).

Solving the equation (6) we get

$$C = -\frac{e\tau}{m_h}E_{ef} - \omega_h\tau[h, E_{ef}] \frac{1}{1 + (\omega_h\tau)^2}, \quad (9)$$
The hole current at temperature \(T = 0\) is

\[
j_h = \frac{e^2 n_h \tau}{m_0 h} \left[ \frac{1}{1 + \omega_n^2 \tau^2} \left( (1 - \frac{\tau}{\tau_{he}})E - \frac{1}{\omega_n \tau_{he}} + \omega_n \tau \right) |hE| \right],
\]

where \(n_h\) is the hole concentration, relaxation parameters \(\tau_{he}\) and \(\tau\) are taken at the Fermi energy of holes.

For determination of the dissipative electron current we shall use the Titeika formula

\[
j_{e,x} = 2g_e \frac{e}{S} \sum_{n,k',k} (X_{k'} - X_k) W_{n,k',n,k} \varphi_{n,k'} (1 - \varphi_{n,k}),
\]

where \(W_{\gamma',\gamma}\) is the electron transition probability from \(\gamma'\) to \(\gamma\):

\[
W_{\gamma',\gamma} = 4g_h \frac{\pi}{S^2} \sum_{p,p',q} |v_{q,p'}|^2 \delta(p',p+q) J_{\gamma',\gamma}(q)^2 \delta(\varepsilon_p - \varepsilon_{p'} + \varepsilon_{\gamma'} - \varepsilon_{\gamma}) f_{p'} (1 - f_p).
\]

The physical meaning of the formula (11) is obvious: the scattering of an electron changes its coordinate of cyclotron center \(X_k\). After expansion we find

\[
W_{n,k',n,k} (1 - \varphi_{n,k}) \approx \frac{4 \pi g_h}{S^2} \sum_{p,p'} \delta(k'-k, p_y - p_y) R_n (|p - p'|) (F_1 + F_2),
\]

\[
F_1 = \delta(\varepsilon_p - \varepsilon_{p'}) [\delta f_p (1 - f_p^{(0)}) - \delta f_{p'} f_p^{(0)}] \varphi_p^{(0)} (1 - \varphi_{n}^{(0)}),
\]

\[
F_2 = -e E a^2 (p_y' - p_y) \delta(\varepsilon_p - \varepsilon_{p'}) f_{p'}^{(0)} (1 - f_p^{(0)}) \varphi_{p'}^{(0)} (1 - \varphi_p^{(0)}).
\]

The contribution \(F_1\) is connected with the absence of equilibrium in the hole subsystem. It could be calculated with using the electron states at \(E = 0\). The electric field enters this contribution indirectly, via \(\delta f_p\). The term \(F_2\) is determined by the direct perturbation of electronic states by the electric field. These contributions are independent and appear or do not appear in different kinetic problems.

Generally speaking, the first contribution to the current is not bound to the direction of the electric field. The specific choice of gauge \(A = (0, Hx, 0)\) gives possibility to calculate only \(x\) component of this current. The \(y\) component of this current can be found owing to the fact that the resulting current is independent on the gauge, by means of the choice \(A = (-Hy, 0, 0)\). Collecting \(j_x\) and \(j_y\) components we find the electron current density caused by the nonequilibrium of holes

\[
j_{e}^{(1)} = 2g_e gh \frac{e}{S^2} \nu (1 - \nu) \sum_{p,p'} [(p' - p) \ h] \delta(\varepsilon_{p'} - \varepsilon_p) R_N (|p' - p|) (\delta f_{p'} - \delta f_p)
\]
Substituting (5) and (9) into Eq. (13) we get to

\[ j_e^{(1)} = \frac{1}{\omega h \tau_{he}} [j_h h]. \]  

For the dissipative current caused by the direct action of electric field we find:

\[ j_e^{(2)} = 2 g_e g_h \frac{e^2 a^2}{2 S^2} \nu(1 - \nu) E \sum_{\mathbf{p}, \mathbf{p}'} \delta(\varepsilon_{\mathbf{p}' - \varepsilon_{\mathbf{p}})} \delta_2(\varepsilon_{\mathbf{p}' - \varepsilon_{F,h}}) (\mathbf{p}' - \mathbf{p})^2 R_{N_e}(|\mathbf{p}' - \mathbf{p}|). \]  

Here \( \varepsilon_{F,h} \) is the Fermi energy of holes.

Taking into account Eq. (7) one can reduce Eq. (15) to the form

\[ j_e^{(2)} = \frac{n_h e^2}{m_h \omega_h^2 \tau_{he}} E. \]  

Using the preceding formulas we arrive at the expressions for the dissipative conductivity of the system with interacting electrons and holes

\[ \sigma_{xx} = n_h \pi a^2 \frac{e^2}{\pi \hbar} \frac{1 - \tau / \tau_{he}}{\tau_{he} \omega_h} \frac{1 + \tau \tau_{he} \omega_h^2}{1 + \tau^2 \omega_h^2}. \]  

Here \( n_h \pi a^2 / g_h \approx N_h + 1 \) is a large number. The dissipative conductivity Eq. (17) vanishes if \( 1/\tau_i = 0 \). Indeed, in the absence of impurities the Lorentz invariance permits to exclude electric field by the transition to the frame of reference drifting across the electric field with the velocity \( v_d \). The evident absence of the energy losses in this frame proves the absence of \( \sigma_{xx} \).

It is useful to subtract from (17) the trivial contribution of holes in the absence of electron-hole scattering \( \sigma_{xx}|_{\tau_{he} \to \infty} \) which corresponds to \( \nu = 0 \) and can be easily found experimentally in the incompressible phase. The result is

\[ \delta \sigma_{xx} = n_h \pi a^2 \frac{e^2}{\pi \hbar} \frac{\tau_{he} + \tau_i}{\omega_h(1 + \tau_i^2 \omega_h^2)(\tau_{he} + \tau_i)^2 + \tau_{he} \tau_i^2 \omega_h^2)}. \]  

Though the searched effect is caused by the friction between components it vanishes if \( \tau_{he} \to \infty \) or \( \tau_i \to \infty \). The applicability of Eq. (17) is limited by the large magnetic field because \( H \) should quantize electrons.

The figure shows the correction to the dissipative conductivity \( \delta \sigma_{xx} \) conductivity in units of conductance quantum \( e^2 / \pi \hbar \) versus the inverse magnetic field in the neutrality point \( n_e = n_h \). We utilized the screened Coulomb potential \( u_q = 2 \pi e^2 / (\chi(q + \varkappa)) \), where \( \chi \) is the dielectric constant. For calculations we approximate the screening parameter \( \varkappa \) by
the value $2/a_{B,h}$ ($a_{B,h}$ is the Bohr radius for holes). The parameters of HgTe from $\chi = 20$, $m_e = 0.025m_0$, $m_h = 0.15m_0$, $g_e = 1$, $g_h = 2$ have been taken. For the energy gap we have chosen the value $E_g = 0.003$ eV. The quantity $\delta \sigma_{xx}$ is finite when the Fermi level gets to the Landau level.

![FIG. 1: Correction to the dissipative conductivity $\delta \sigma_{xx}$ due to e-h scattering versus the magnetic field for $\tau_i = 10^{-12}s$ (thick line) and for $\tau_i = 3 \cdot 10^{-12}s$ (thin line).](image)

**Finite Landau level width**

The crucial point for previous consideration, in particular, for temperature-independent contribution of e-h scattering to the dissipative conductivity is the presence of the Landau levels degeneracy. The potential fluctuations lift this degeneracy and the the Landau levels get width $\gamma_n$. For long-range potential this width is proportional to the amplitude of potential. For developed fluctuations the self consistent Born approximation gives the width of the Landau levels proportional to $\sqrt{n_i}$ ($n_i$ is the impurity concentration) and the potential of individual impurity. The exception is the short-range impurities with $\delta$-like potential for which the part of the Landau level states $1/\pi a^2 - n_i$ remains degenerate if $1/\pi a^2 > n_i$, while $n_i$ states form a band of localized with a finite width.

In the case of the Landau level with a finite width $\gamma_N$ the interparticle scattering depends on the ratio of the temperature $T$ to the width. If $T \ll \gamma_n$ the e-h scattering is suppressed and if $T \gg \gamma_n$ the scattering does not notice the width. Thus, all previous consideration of e-h scattering remains valid for intermediate temperature $\hbar \omega_e > T > \gamma_n$. 
The scattering processes work in the dissipative conductivity of quantized electrons in a parallel manner. One can sum up the contributions to electron $\sigma_{xx}$ conductivity caused by impurity scattering and electron-hole processes. According [12], the contribution to electron dissipative current caused by short-range impurity scattering is

$$(\sigma_{xx})_{ei} = \frac{g_e e^2}{\pi^2 \hbar} (N_e + 1/2)(1 - \mu^2). \quad (19)$$

The reduced distance between the Fermi level and the Landau level with number $N_e$, $\mu = (\varepsilon_{F,e} - (N_e + 1/2)\omega_e)/\gamma N$, is connected with the quantity $\nu$ by the equation

$$\nu = \frac{1}{2\pi} \left( \pi + 2\mu \sqrt{1 - \mu^2} + 2 \arcsin \mu \right).$$

The result Eq. (19) should be added to (16). The ratio of the contribution to conductivity (16) to (19) in their maxima is

$$\frac{\pi g_h (N_h + 1)}{g_e (N_e + 0.5)} \frac{1}{\omega_h \tau_{he}}. \quad (20)$$

The first factor is large by assumptions, the second one can be as large, so small. The total dissipative conductivity is the sum of (17) and (19).

In the case of short-range impurities with small concentration $n_i \ll 1/\pi a^2$ the absence of widening leads to the validity of the results obtained in the previous section up to zero temperature. In more wide range $n_i < 1/\pi a^2$ the scattering rate $1/\tau_{he}$ should be corrected by the factor $1 - n_i \pi a^2$ reflecting the fraction of degenerate states.

If the long-range potential fluctuation case is realized the degeneracy disappears. In the absence of e-h scattering the model of adiabatic transport is valid when electron cyclotron centers are drifting along the lines of constant potential. Without the external field only one infinite fractal level line of the fluctuating potential corresponding to the percolation threshold exists. In the presence of the finite electric field this level line decays to independent infinite entangled lines going across the external electric field. The drift does not depend on the charge of particles: the velocities and trajectories of the cyclotron centers of quantized electrons and classical holes are the same. The dissipative conductivity of electrons vanishes, while the Hall conductivity changes stepwise between quantized $Ne^2/h$ values. In the lack of degeneracy the temperature-independent e-h scattering also disappears.

The dissipative conductivity could result from the jumps between different energy levels with the transfer of abundant energy between electrons and holes. However, electrons and holes exactly repeat trajectories of each other and corresponding contributions cancel.
Memory effects

Here we discuss the memory effects caused by the scattering between components. In the previous sections the validity of the kinetic equation for holes was assumed. The presence of strong enough classical magnetic field violates the applicability of the kinetic equation. The inapplicability is reasoned by the lack of randomness of motion of a charged particle due to recurrent returns to the starting point. Let the scatterer has a finite size. The particle either never meets a scatterer or returns to the scatterer if the particle has met the scatterer once.

Starting from the paper [6], these "memory" effects were subject of investigation in relation to the impurity scattering (see also more recent papers [7–11]). In a strong magnetic field the picture of classical transport of holes interacting with quantized electrons is similar to the 2D hole system with short-range impurities. The inelasticity of scattering is absent due to the degeneracy of the Landau level and electrons play a role of immobile scatterers for holes. Their effective density is \( g_e \nu (1-\nu)/\pi a^2 \). The parameter \( x = g_e \nu (1-\nu) \) is the main factor governing the number of these scatterers. The quantity \( s \) serves as a spatial size of scatterers. If \( s \) is less than the cyclotron radius, the electrons can be treated as short-range scatterers for holes. The total density of scatterers \( \tilde{n} \) is the sum of the impurity \( n_i \) and active electron densities \( \tilde{n} = n_i + x/\pi a^2 \).

The kinetic equation is applicable if holes experience multiple collisions on their cyclotron path \( 2\pi r_h = 2\pi \sqrt{2(N_h+1)a} \), where \( N_h \gg 1 \) is the number of the last-filled hole Landau level. In other words, the quantity \( l = 1/(2s\tilde{n}) \) should be less than \( 2\pi r_h \). Assuming that \( s \sim a \sqrt{2(N_e+1)} \) we get the condition

\[
8(\pi n_x a^2 + x)\sqrt{(N_e+1)(N_h+1)} \gg 1. \tag{21}
\]

If \( 8\pi n_x a^2 \sqrt{(N_e+1)(N_h+1)} \gg 1 \) the kinetic equation is applicable for any \( \nu \). Otherwise, the kinetic equation is valid in the domain \( 8x \sqrt{(N_e+1)(N_h+1)} \gg 1 \) that covers (since \( N_h \gg 1 \)) the majority of the range of variation of \( \nu \), including the center of the Landau level \( \nu = 0.5 \). The applicability of the kinetic equation is violated on the wings of the Landau level, where \( \nu \ll 1 \) or \( 1 - \nu \ll 1 \).

If the density of scatterers is small, \( 8(\pi n_x a^2 + x)\sqrt{(N_e+1)(N_h+1)} \ll 1 \), the most part of holes, namely \( \exp(-2\pi r_h/l) \), will not collide the scatterers at all. The remainder
1 − \exp(−2\pi r_h/l) \text{ collides the scatterers. The criterium } \text{[21]} \text{ differs from the criterium of a strong field from the kinetic equation point of view } \omega_h \tau \gg 1. \text{ Even if to suppose the equality of cross-section } \sigma \text{ and } 2s, \text{ the inequality } \omega_h \tau \gg 1 \text{ gives } r_h \ll l, \text{ that differs from } 2\pi r_h \ll l \text{ by the numerical factor } 2\pi.

If \( \tilde{n} \pi r_h^2 \ll 1 \) the particle being scattered once, returns the scatterer repeatedly. The trajectory of a hole consists of circular segments composing, in a general case, an open aperiodic rosette. During a long time the trajectory covers a circle of radius approximately equal to \( 2r_h \). Such trajectories are localized and \( \sigma_{xx} = 0 \).

This picture is valid if there are no other scatterers inside this circle. Otherwise, the rosettes around two or more scatterers join together. When the scatterers whose distance from each other is less than \( 2r_h \) compose a infinite chain, the rosettes join an infinite cluster, the hole gets possibility to travel on the infinite distance and the finite conductivity \( \sigma_{xx} \) appears.

An accurate criterium of localization is given by the theory of percolation. According to [13–15], the percolation threshold reads \( \tilde{n} \pi r_h^2 = B_c \), where \( B_c \) is some number. If to supposed that all impurities and electrons are randomly distributed the corresponding mathematical problem is the percolation threshold for the uniform distribution of disks of radii \( r_h \), where \( B_c = 1.128 \). The critical value of \( x \) is \( x_c = B_c / 2(N_h+1) - \pi n_i a^2 \), assuming that \( 2\pi n_i a^2(N_h+1) < B_c \). Since \( N_h \gg 1 \), the critical values of \( \nu \) or \( 1-\nu \) are small: the holes become localized when the Fermi level gets to the wings of electron Landau levels.

Above the threshold (if \( 2\pi n_i a^2(N_h+1) > B_c \) or \( x > x_c \)) the rosettes overlap and the diffusion chain is formed: a particle executes a motion around one scatterer, then it jumps to a rosette around the other, etc. If \( 2\pi n_i a^2(N_h+1) < B_c \) and \( x_c < x \ll (8\pi \sqrt{(N_e+1)(N_h+1)})^{-1} \) the non-colliding trajectories of holes are most frequent, but the colliding trajectories form the infinite cluster.

Near the threshold the conductivity in the conductive phase can be estimated. The parameter \( x - x_c \ll x_c \) characterizes the proximity with the threshold. Only a small fraction of holes \( 2\pi r_h/l \) can collide the scatterers. This fraction can form the rosettes, while others play a passive role. The holes colliding with electrons have a mean free path of \( 2\pi r_h \).

Let all scatterers be situated in a lattice with a period less than \( 2r_h \). In this case the rosettes around scatterers unite and the conductivity can be estimated as a conductance of
the elementary cell
\[ \sigma_0 = \frac{e^2 \tau_{he}}{m_h n_h} \frac{2\pi r_h}{l} . \]

If the scatterers are randomly distributed the percolational cluster near the threshold of its formation is rarefied. Hence, the conductivity of the system is less than \( \sigma_0 \). Over the distance larger than \( 2r_h \) the medium can be treated as a continuum mixture of conducting cells with conductivity \( \sigma_0 \) and insulating cells. The effective conductivity can be found using the critical index for 2D conductivity \( t \): \( \sigma_{xx} = \sigma_0 (x - x_c)^t \), \( t = 1.2 \).

Summarizing, the hole conductivity has the symmetric dependence on the quantity \( 0 < \nu < 1 \) via the factor \( x = \nu(1 - \nu) \) and the maximum at \( \nu = 0.5 \). The conductivity critically vanishes on the far wings \( x < x_c \ll 1 \). Near the threshold \( (x - x_c \ll x_c) \) the conductivity has a power-like growth. Farther from the edges the domains of the coexistence of localized circular motion and diffusion on combined infinite rosettes are situated. The central region corresponds to the usual transport described by the kinetic equation.

**Conclusions**

We have studied the influence of electron-hole interaction on transport in the system where electrons are quantized and holes are not. In these conditions, the second type of carriers plays its role as an additional (or exceptional) channel of scattering. Weak electron-hole interaction can be considered in the Born approximation, despite the degeneracy of the Landau levels, in contrast to the impurity mechanism which is not perturbative in the quantizing magnetic field, even for a weak potential. The scattering of holes on quantized non-interacting electrons occurs if, and only if, the Landau level is partially filled. The chaotization results from the random distribution of electrons in the momentum space and corresponding entropy and is not frozen out at zero temperature and remains finite. The scattering of holes can be considered by means of kinetic equation approximation when the Fermi level is near the center of the Landau levels; the kinetic approximation loses applicability apart from the center and on the far wings the holes become localized.

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