Cluster duality and mirror symmetry for Grassmannians

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We consider the Grassmannian $X = \text{Gr}(n-k,n)$ and its mirror dual Landau—Ginzburg model $(X', W)$ as constructed by Marsh—Rietsch. In this Landau—Ginzburg model, $X'$ is the complement of a particular anti-canonical divisor in a Langlands dual Grassmannian, and the superpotential $W$ is expressed very naturally in terms of Plucker coordinates. We describe a natural and explicit duality correspondence between the positive chart $\Phi_G$ from $(\mathbb{C}^*)^N 4 \to X$, and the cluster chart $\Phi'_G$ from $(\mathbb{C}^*)^N \to X'$, where $N = k(n-k)$ and $G$ is a plabic graph. For any projective embedding of $X$ and positive chart Delta there is an associated Newton—Okounkov polytope which is determined by giving its integer lattice points. Therefore to each chart $\Phi_G$ as above and any positive integer $r$, we obtain a Newton—Okounkov polytope $\text{NO}_G^r$. On the other hand, using the cluster chart $\Phi'_G$ for the mirror $X'$, and the same positive integer $r$, we can obtain a polytope $Q'_G$—described in terms of inequalities—by “tropicalizing” the superpotential $W$. Our main result is that the polytopes $\text{NO}_G^r$ and $Q'_G$ coincide.

This is joint work with Konstanze Rietsch.