CMB anisotropy predictions for a model of double inflation

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We consider a double-inflationary model with two massive scalar fields interacting only gravitationally in the context of a flat cold dark matter (CDM) Universe. The cosmic microwave background (CMB) temperature anisotropies produced in this theory are investigated in great details for a window of parameters where the density fluctuations power spectrum $P(k)$ is in good agreement with observations. The first Doppler (“acoustic”) peak is a crucial test for this model as well as for other models. For values of the cosmological parameters close to those of standard CDM, our model is excluded if the height of the Doppler peak is sensibly higher than about three times the Sachs-Wolfe plateau.

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I. INTRODUCTION

The inflationary paradigm [1] has been an exciting theoretical development in our understanding of some basic open issues in Big Bang cosmology. Indeed, it provides an elegant explanation to the horizon problem as well as to the flatness problem. In addition, it incorporates in a natural way the creation of primordial fluctuations [2] which will eventually grow through gravitational instability into the large scale structures as we observe them today in the Universe on cosmological scales. It is then possible to calculate the fluctuation spectra with great accuracy for all models, using possibly numerical methods [3]. One still has to combine these primordial spectra with some assumptions about the Universe and its matter content. The CDM model [4] has been extensively studied as the simplicity of its basic assumptions is very appealing. However, even before the first COBE DMR data there was observational evidence from the APM survey [5] showing that standard CDM does not agree with observations. When correctly normalized on very large cosmological scales using the COBE DMR data of the CMB anisotropies on large angular scales, standard CDM has too much power on small scales.

Double Inflation [6], [7] is an attempt to reconcile the CDM model with observations in the following way: instead of having a purely scale-invariant (Harrison-Zel’dovich) spectrum of adiabatic fluctuations one has now a spectrum possessing a characteristic scale. As this spectrum rests on a well-defined inflationary model, in other words there is an underlying effective Lagrangian which defines completely the dynamics of the inflationary background and the properties of the various perturbations spectra, it is possible to study in detail, with numerical methods when necessary, the fluctuations spectra generated in this model. This was done already for both density (scalar) fluctuations [8], [9] and tensorial fluctuations or gravitational waves (GW’s) [10]. Another otherwise attractive attempt to reconcile CDM with the observations in the same spirit, is to replace the primordial perturbation spectrum by a tilted one, i.e., one for which \( n < 1 \). Such models were considered in particular after release of the first COBE DMR results which showed that standard CDM would give too much power on small scales, for example \( \sigma_8 \approx 1.3 \). While a tilted spectrum does render CDM more compatible with observations, it is not in accordance with all the observations on large and small scales though a value \( n \approx 0.8 \) comes closest to it [11]. The CDM paradigm is appealing enough so that also tilted CDM models with some change of the cosmological parameters have been considered afterwards. An interesting possibility is a higher baryon fraction in the Universe [12] but it turns out that, in order to have a value for \( \sigma_8 \approx (0.6-0.7) \), one would need \( \Omega_b \sim 15\% \) and, though not excluded, there is no compelling evidence for such a high baryon fraction. Of course, there are still other ways to depart from the standard CDM paradigm, another interesting possibility being a change of the matter content of the Universe, thereby changing the power spectrum as it is seen today (see, for example, [13] and references therein). This can be achieved if one considers a mixture of cold dark matter with a certain amount of hot dark matter. Yet another possibility is to consider universes with open spatial geometries as one would have in any case once there is compelling evidence for \( \Omega < 1 \).

So double inflation is an attempt to cure the problem by a change in the primordial spectrum only, leaving the other parameters otherwise unchanged, or in any case close to their “canonical” values. The purpose of this work is to extend the study of double inflation to the CMB fluctuations up to small angular scales. With the advent of the next generation experiments whose aim is a very high precision measurement of the CMB anisotropies up to \( l \approx 1500 \), like, for example, the satellite mission PLANCK Surveyor, it is clear that the constraints coming from those observations will be crucial regarding the viability of the various existing models.

The outline of this work is as follows. In Sec. II, we review for completeness the model we consider here and some of its peculiarities. In Sec. III, we find the window of allowed parameters after constraining the density power spectrum \( P(k) \) and compare our model with other ones. In Sec. IV, we find the CMB anisotropies for the selected window of parameters. Finally in Sec. V we give a summary and short discussion.

1
II. THE DOUBLE-INFLATIONARY BACKGROUND AND THE FLUCTUATIONS

We give here a short description of our model of double inflation, starting with the homogeneous background. We consider the following Lagrangian density describing matter and gravity

\[ L = -\frac{R}{16\pi G} + \frac{1}{2} (\phi_{h,\mu} \phi_{h}^\mu - m_{h}^2 \phi_{h}^2) + \frac{1}{2} (\phi_{l,\mu} \phi_{l}^\mu - m_{l}^2 \phi_{l}^2), \]  

where \( \mu = 0, ..., 3, \) and \( c = \hbar = 1. \) The background space-time metric has the form

\[ ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j, \quad i, j = 1, 2, 3. \]

The first period of inflation is driven by the heavy scalar field and we will see that the interesting part of the spectrum corresponds to the end of this first inflation. The homogeneous background is treated classically, it is determined by the scale factor \( a(t) \) and the two scalar fields \( \phi_h, \phi_l. \)

A crucial ingredient of any inflationary model is the generation of inhomogeneous perturbations, of quantum mechanical origin, superimposed on the homogeneous background. For their description we consider a perturbed FRW background whose metric, in the longitudinal gauge, reads

\[ ds^2 = (1 + 2\Phi) dt^2 - a^2(t)(1 - 2\Psi) \delta_{ij} dx^i dx^j, \]

(in Bardeen’s notation \( \Phi = \Phi_A, \Psi = -\Phi_H \)). We are interested in the spectrum of growing adiabatic perturbations which arise from the vacuum fluctuations of the scalar fields \( \phi_h \) and \( \phi_l. \) They are Gaussian and the power spectrum \( \Phi^2(k) \) of the gravitational potential, defined through

\[ \langle \Phi_k \Phi_{k'}^* \rangle = \Phi^2(k) \delta(k - k'), \]

characterizes them completely. Some of the features of the spectrum when the intermediate matter-dominated stage is pronounced are relevant also to cases where it is not. For scales crossing the Hubble radius when both scalar fields are in the slow rolling regime, the spectrum of growing adiabatic perturbations, when those scales are outside the Hubble radius during the matter-dominated stage (assuming \( a(t) \propto t^2 \) at the present time), is given by

\[ k^3 \Phi(k) \simeq \frac{\sqrt{24\pi G m_h^2}}{5} s \ln \frac{k_f}{k}, \quad k \ll k_f. \]

where the rhs has to be taken at \( t = t_k, \) the first Hubble radius crossing time and the quantity \( s(t) \) is the total number of \( e \)-folds from time \( t \) till the end of the second inflation. The wave number \( k_f \) corresponds to the scale crossing the Hubble radius near the end of the first inflation. An important point is seen from Eq. (5), namely that the upper part of the spectrum, corresponding to small \( k' \)s or very large scales, is not flat but has a logarithmic dependence \( \propto \ln \frac{k_f}{k}. \) This is why the naive picture of two plateaus fails and one has to calculate the spectrum with accuracy if the model is to be confronted with observations. This remains true when the intermediate matter-dominated stage is absent and has crucial observational consequences.

We introduce the parameter \( p \equiv \frac{m_h}{m_l}, \) higher \( p \) values correspond to bigger “steps” (see figure 1). The height of the “step” \( \Delta k \) between a scale on the upper “plateau” (still inside our visible universe today) and a scale at the beginning of the lower plateau, is given by

\[ \Delta k \simeq 0.13p \ln \frac{k_f}{k}, \quad k \ll k_f. \]

For \( p < 25, \) the observationally interesting values, the evolution of the background during the transition between the two main inflationary stages is still inflationary, in the sense that \( \dot{a} > 0 \). In order to make accurate comparison with the observations for models without a pronounced intermediate matter-dominated stage, physical scales of our spectra are defined with the help of the
quantity \( k_0 \), the scale where the extrapolated upper part intersects the lower plateau. If \( \phi_1 \simeq 3M_P \) towards the end of the first inflation, the second inflationary stage has the right amount of expansion and puts the excess of power on the right scales. A tiny change in this initial value is enough to shift the spectrum in \( k \) space while leaving the form of the spectrum practically unaltered.

FIG. 1. The primordial power spectrum \( k^{3/2}\Phi(k) \) is plotted for different values of \( p \): from top to bottom on the right, \( p = 8, 10, 12, 15, 20, 25 \). All spectra have the characteristic scale \( k_0 = 1 \, h \text{Mpc}^{-1} \), and are normalized to COBE. At small scales, the power spectra decrease quickly with growing \( p \). At large scales the effect of \( p \) is opposite, but almost negligible.

We turn now our attention to the power spectrum \( P(k) \) defined through \( \langle \delta_k \delta_k^* \rangle = P(k) \delta(\mathbf{k} - \mathbf{k}') \). Perturbations in the linear regime grow at different rates depending on the relation between their wavelengths, the Jeans length and the Hubble radius. The resulting amplitude for different scales is encoded in the so-called transfer function \( T(k, t_0) \)

\[
P(k, t_0) = \frac{4}{9} \frac{k^4}{H_0^4} \Phi^2(k) T^2(k),
\]

where, by definition, \( T(k \to 0) = 1 \). \( T(k) \) is computed numerically once assumptions are made about the matter content of the Universe and other cosmological parameters like \( \Omega_0 \) and \( h \). We do not use here the transfer function for the standard CDM model given by \( T(q) = \ln(1 + 2.34q)^{-\frac{1}{2}} \), where \( q \equiv \frac{k}{\Omega_0 h^2 \text{Mpc}^{-1}} \).

\[
T(q) = \frac{\ln(1 + 2.34q)}{2.34q} [1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.72q)^4]^{-\frac{1}{2}},
\]

where \( q \equiv \frac{k}{\Omega_0 h^2 \text{Mpc}^{-1}} \), \( \Omega_0 = 1 \), \( h = 0.5 \), nor even for different shape parameters \( \Gamma \). We use rather the more accurate transfer function that is computed numerically with the help of the code cmbfast (Seljak and Zaldarriaga [16]). So we repeat the study of the observational constraints on \( P(k) \), in order to find the allowed region in the \( (p, k_0) \) plane, with an accuracy that matches the precision of the CMB anisotropies computation by Seljak’s and Zaldarriaga’s code cmbfast. We assume tacitly everywhere \( \Omega_0 = 1 \), in accordance with the standard picture of inflation.
Another point that deserves special attention is the production of a primordial gravitational waves (GW’s), or tensorial perturbations, background in this model \[10\]. The tensorial perturbations $h_{ij}$ are given by

$$h_{ij} = \sqrt{32\pi G} \varphi_{e_{ij}},$$  \tag{9}$$

where $\varphi$ is a massless scalar field while the polarization tensor satisfies $e_{ij}e^{ij} = 1$. Later, we will need their power spectrum $h(k)$ defined by

$$\langle h_{k,\lambda} h^{*}_{k',\lambda'} \rangle \equiv h^2(k) \delta^{(3)}(k-k') \delta_{\lambda \lambda'},$$  \tag{10}$$

where $\lambda = 1, 2$ denotes the two polarization states. Usually the importance of this background lies mainly in the fact that when its contribution to the temperature anisotropy on large angular scales is not too small, this balance between scalar and tensorial perturbations allows a smaller power spectrum $P(k)$ for given normalization. Though in our model the contribution of the GW background to the temperature anisotropy is definitely subdominant compared to that of the scalar adiabatic perturbations, it is interesting to note that the relations for small multipoles $l$ which hold for single-field slow-roll inflation, do not apply in this case. In our models we have in particular

$$\frac{\langle |a_{2m}|^2 \rangle_{GW}}{\langle |a_{2m}|^2 \rangle_{AP}} = \frac{C_T}{C_S} \ll \frac{1}{|n_T|} \simeq 7(1-n) \approx 1,$$  \tag{11}$$

as can be seen in figure 4, $n_T$, resp. $n$ being the spectral indices of the GW, resp. the scalar perturbations ($n \equiv 1 + \frac{d \ln k^3 \varphi^2(k)}{d \ln k}$, $n_T \equiv \frac{d \ln(k^3 h^2(k))}{d \ln k}$). Relation (11) would become a powerful discriminative test between single field inflation and our double inflationary model provided the GW contribution to the CMB anisotropy can be separated from the adiabatic scalar perturbations contribution.

III. CONSTRAINING THE POWER SPECTRUM $P(k)$

A. Power spectrum normalization

A quite accurate normalization of the power spectrum is now possible using past years measurements of the CMB anisotropies on angular scales of a few degrees, in particular COBE DMR. We normalize our spectra to the value of $C_{10}$ extrapolated from the experimental bounds on $Q_{rms-ps|n=1}$, the quadrupole predicted for a Harrison-Zel’dovich spectrum, since this multipole is minimally dependent on the spectral indices $n$ and $n_T$. A joint analysis of Tenerife Dec=+40° and two-year COBE data gives \[15\] $Q_{rms-ps|n=1} = 21 \pm 1.6 \mu K$, which means $10(10+1)C_{10} = (9.5 \pm 1.9) \times 10^{-10}$ (the error bars take into account both sample and cosmic variances). The four-year COBE DMR result \[19\] is smaller: $Q_{rms-ps|n=1} = 18 \pm 1.6 \mu K$, i.e., $10(10+1)C_{10} = (6.6 \pm 1.2) \times 10^{-10}$. In the following, we will take $10(10+1)C_{10} = 6.6 \times 10^{-10}$, keeping in mind the possibility of a higher value.

B. The window in parameter space

Once normalized to COBE DMR, a robust constraint on the matter power spectrum $P(k)$ comes from the value of $\sigma_8$ (the variance of the total mass fluctuation in a sphere of radius $R = 8h^{-1}$Mpc), for bright galaxies this quantity is close to unity \[20\]. This is what is usually called the “optical” $\sigma_8$. White, Efstathiou, and Frenk \[21\] find

$$\sigma_8 = 0.57 \pm 0.06.$$  \tag{12}$$
These bounds single out a region for our two free parameters $p$ and $k_b$ as can be seen in figure 3 for $h = 0.5, \Omega_B h^2 = 0.015$ and $C_{10} = 6.6 \times 10^{-10}$ (small variations of $h = 0.5$ and $\Omega_B$ will be considered in Sec. [13]). We get a lower bound for $p$: $p > 8$ is required to obtain small enough power on this scale.

An upper bound can be found using some constraints at even smaller scales (but still using the power spectrum obtained in the framework of linear theory and extrapolated to these scales), deduced from observations of galaxies and quasars at high redshifts. In order to explain the formation of these objects, one must put a lower limit on the linear power spectrum at the corresponding scale. For instance, estimates of the mass fraction in host galaxies of quasars at $z = 4$ [22] require the following lower bound [23]:

$$\sigma(10^{11} M_\odot) \geq 2.2 \pm 0.5,$$

where $\sigma(M)$ stands for the variance of the total mass fluctuations in a sphere of mass $M$ today, assuming linear evolution. Similarly, since large galaxies seem to have formed as early as $z = 1$, we have the lower bound [22]:

$$\sigma(10^{12} M_\odot) \geq 2.0 \pm 0.4. \quad (14)$$

The limit $\sigma(10^{12} M_\odot) = 1.6$, which turns out to be even more constraining for our model, is plotted in figure 3 for our standard set of parameters and we see that in order to have enough power at these small scales, we must exclude any $p > 20$.

Redshift surveys provide us with a huge amount of data giving indications about the power spectrum at scales ranging from 15 to 300 $h^{-1}$ Mpc. We compare our model with the count-in-cell analysis of large scale clustering of the Stromolo-APM redshift survey. After normalization of the spectra to $\sigma_8 = 1$, in order to get the correlation function of optical galaxies in redshift space, we compute the variance $\sigma_l^2$ in cells of size $l h^{-1}$ Mpc, and compare it with the Stromlo-APM data [24], consisting of nine points, assumed to be independant, with error bars treated as 2$\sigma$ ones. A $\chi^2$ analysis selects the best-fitting parameters $p$ and $k_b$. Since we have seven degrees of freedom, $\chi^2 \leq 7$ means that we are in very good agreement with the data. Inside our previously allowed window, this requires $p > 10$ and $k_b \leq 2.5 h$ Mpc$^{-1}$. If we apply the same test, for instance, to tilted models, it turns out that $\chi^2 \leq 7$ implies a too low spectral index, namely $n \leq 0.6$.

Finally, we compare our spectra with the results of Kolatt and Dekel [25] derived from peculiar velocities of galaxies. Using the Mark III catalog and the reconstruction of the density field from POTENT, a direct estimate of the power spectrum is given for three values of $k$, independently of the bias. For all values of $p$ and $k_b$ inside the previously found window, the double-inflationary power spectrum is too low in order to agree at the 1$\sigma$ level with all three points. However, for the highest $k_b$’s, the disagreement is rather small. Indeed, for $k_b > 2 h$ Mpc$^{-1}$, the spectrum agrees with all three points at the 1.5$\sigma$ level.
Finally, our preferred region is then $2 \, h^{-1} \, \text{Mpc} \leq k_b \leq 2.5 \, h^{-1} \, \text{Mpc}$ and $10 < p < 20$, as can be seen in figure 2. In other words, the position of the step in real space should be

$$ 2.4 \, h^{-1} \, \text{Mpc} < \frac{2\pi}{k_b} < 3 \, h^{-1} \, \text{Mpc}, $$

and the mass of the heavy inflaton

$$ m_h = (3.6 \pm 0.2) \times 10^{-6} \, \text{M}_\text{P}. $$

C. Comparison with other models

In table I, we compare the results of the previous tests for several models: our model (we choose parameters at the center of the previously found window, viz. $p = 12$, $k_b = 2.3 \, h^{-1} \, \text{Mpc}$), “standard CDM” with a flat spectrum, and two tilted models (with and without tensor contribution); all models have the same transfer function with $h = 0.5$, $\Omega_B h^2 = 0.015$. In order to make the relevant comparison, we select the values of the spectral indices that give $\sigma_8 = 0.60$. We find $n = 0.70$ (without tensors) or $n = 0.845$ (with tensors, assuming $n_T = n - 1$ and $C_2^T / C_2^S = 7(1 - n)$).
TABLE I. The results of a few tests are given together with the observational bounds for several models: standard CDM, double-inflation with $p = 12$, $k_b = 2.3 \, h \, \text{Mpc}^{-1}$, and tilted models which are found to be consistent with $\sigma_8 = 0.60$, i.e., $n = 0.70$ (without tensors) and $n = 0.845$ (with tensors). All these results are based on $h = 0.5$, $\Omega_B h^2 = 0.015$ and four-year COBE normalization. The wavenumber $k$ is expressed in $h \, \text{Mpc}^{-1}$ and $P(k)$ in $h^{-3} \, \text{Mpc}^3$.

|                   | sCDM | D.I.  | tilted (S) | tilted (S+T) | Obs.  |
|-------------------|------|-------|------------|-------------|-------|
| $\sigma_8$       | 1.13 | 0.60  | 0.60       | 0.60        | 0.57±0.06 |
| $\sigma(10^{11} \, M_\odot)$ | 7.84 | 2.24  | 2.94       | 3.51        | $\geq 1.7$ |
| $\sigma(10^{12} \, M_\odot)$ | 5.72 | 1.84  | 2.31       | 2.66        | $\geq 1.6$ |
| $\chi^2$         | 84   | 6.5   | 13         | 26          | $\leq 7$ |
| $P(k = 0.061)$    | 9360 | 4200  | 3650       | 3200        | 8157±3127 |
| $P(k = 0.102)$    | 7590 | 2850  | 2540       | 2390        | 4620±1240 |
| $P(k = 0.172)$    | 4800 | 1450  | 1370       | 1400        | 1968±495  |

It is clear from the table that our double-inflationary model gives better results than the chosen tilted models, namely a lower $\chi^2$ and higher peculiar velocities. Of course, most authors favor higher values of the spectral index [26], but then the constraint (1 2) on $\sigma_8$ is violated. Therefore, in the framework of a flat CDM universe with its standard cosmological parameters ($h \simeq 0.5$, $\Omega_B h^2 \simeq 0.015$), no inflationary scenario with less than two free parameters (in addition to the overall normalisation) is compatible with observations while, with respect to constraints considered so far, double-inflation does.

The power spectra corresponding to these four models are plotted in figure 3. In order to visualize the expected shape of the spectrum, we also plot the redshift surveys data compilation of Peacock and Dodds [27], rescaled to $\sigma_8 = 0.60$. This plot clearly shows that the success of our double-inflationary model in this respect is linked to the decrease of the effective index towards growing $k$, when $k < 2 \, h \, \text{Mpc}^{-1}$ (see eq.(5)).
IV. COSMIC MICROWAVE BACKGROUND ANISOTROPIES

We compute the scalar and tensor CMB anisotropies using cmbfast, the fast Boltzmann code by Seljak and Zaldarriaga [16], extended to non-power-law primordial spectra.

The temperature anisotropies multipoles in Fourier space are calculated at the present time, for scalar and tensor components: \( \Delta^{(S,T)}_{Tl}(k) \). Taking into account the correct normalization factors, and Fourier conventions, the scalar and tensor modes multipoles then read in our notation:

\[
C_{l}^{S} = \frac{2}{\pi} \int \frac{dk}{k^2} \phi^2(k) |\Delta^{(S)}_{Tl}(k)|^2, \quad C_{l}^{T} = \frac{1}{2\pi} \int \frac{dk}{k^2} h^2(k) |\Delta^{(T)}_{Tl}(k)|^2. \tag{17}
\]

We first plot in figure 4 the ratio of tensor and scalar multipoles for parameters inside the allowed window.

We see that \( C_{10}^{T}/C_{10}^{S} = 0.077 \), in excellent agreement with the analytic result of [10]. As already claimed in [10], the relation \( C_{2}^{T}/C_{2}^{S} \approx 7(1 - n) \) is strongly violated in our case for which \( C_{2}^{T}/C_{2}^{S} = 0.1 \) while \( n_{\text{eff}} \approx 0.85 \) for large scales.

![Graph showing the ratio of tensor to scalar multipoles](image)

**FIG. 4.** Ratio of tensor and scalar multipoles \( C_{l}^{T}/C_{l}^{S} \) for double inflation. We have, in particular, \( C_{2}^{T}/C_{2}^{S} = 0.1 \ll 7 |n_{\text{eff}}| \approx 7 (1 - n) \approx 1 \) in contrast with single-field slow-roll inflation.

We then compute the total anisotropies for various parameters inside the allowed window. The anisotropies only depend on \( p \) and \( k_b \) through the primordial power spectrum, so they increase with \( k_b \) and hardly depend on \( p \) at relevant scales. In fact they increase very slightly with \( p \) since \( 2k_b/a_0H_0 \gg 1500 \) and hence all the \( l \)'s containing the three peaks correspond to scales, much smaller than \( k_b \), where the primordial spectra do not differ much as can be seen in figure 1. Since on the other hand there is a precise constraint on \( k_b \), we obtain sharp predictions for the multipoles. For instance, the position and amplitude of the first two peaks are within the ranges:

1. for the first peak, located at \( l = 207 \pm 1 \),

\[
\frac{l(l+1)C_{l}}{100C_{10}} = 2.6 \pm 0.1; \tag{18}
\]
2. for the second peak, located at $l = 505$,

$$\frac{l(l+1)C_l}{110C_{10}} = 1.2 \pm 0.1 .$$

(19)

The full $(l(l+1)C_l/2\pi)^{1/2}$ curve is given on figure 5 for $p = 12$ and $k_b = 2.3 \, h \, Mpc^{-1}$, together with standard CDM and tilted models.

We include a few measurements of the anisotropies: COBE [28], Tenerife [29], South Pole [30], Saskatoon (with the recent recalibration) [31,32], MAX [33], MSAM’s third flight [34], and new preliminary points from CAT [35] and OVRO [36]. A complete analysis of the presently available data set can be found in [37].

As expected from the previous section, the amplitude of the peaks is higher in the double-inflationary case than in the tilted one. However, it is clear from the figure that the improvement is not sufficient to agree with observations at little scales. The Doppler peak is approximately at $4.5\sigma$ under Saskatoon and $2\sigma$ under MSAM measurements. The secondary acoustic peaks, or Sakharov oscillations, are around $2\sigma$ under CAT and Ovro.

In the case of a higher multipole normalization, already considered in Sec. III B, we obtain a slight increase of the first and third peaks, however not a very significant one, since the global increase is damped by the shift of allowed $k_b$’s to lower values. It is clear then that our model is not viable in the context of CDM with standard values of the cosmological parameters.

Let us see whether this remains true when small variations of $h$ and $\Omega_b$ are considered. Keeping $h$ fixed, an increase in $\Omega_b$ will enhance the radiation transfer function on the one hand and damp the matter transfer function on the other hand, therefore requiring higher $k_b$’s in order to keep enough
power and satisfy large scale structure tests. Both effects contribute to increase CMB anisotropies. If $\Omega_B h^2 = 0.025$, within the corresponding allowed window, the highest peak is given by $p = 12$, $k_b = 4 \, h \, \text{Mpc}^{-1}$:

$$\frac{l(l+1)C_l}{110C_{10}} = 3.5, \quad l = 213$$  \hspace{1cm} (20)

This improvement is too small to agree with the observations: the peak is still $3\sigma$ under Saskatoon and more than $1\sigma$ under MSAM.

Going to higher $h$, with $\Omega_B h^2$ fixed, just yields the opposite effect: the radiation transfer function and the shift in the allowed parameters both lower the anisotropies. When $h = 0.6$ and $\Omega_B h^2 = 0.015$ (resp. 0.025), the peak is as low as $l(l+1)C_l/110C_{10} = 1.9$ (resp. 2.5) in the best case.

V. CONCLUSION

We have studied here a model of double inflation using constraints from both large scale surveys and CMB anisotropies. The primordial spectrum in such a model has a characteristic length with more power towards large scales. The primary purpose of this model is to reconcile CDM with observations in the following sense: we keep the canonical values of standard CDM, possibly allowing a small departure from these, and we take the primordial spectrum of scalar perturbations generated during the inflationary stage in our model instead of a nearly scale invariant (Harrison Zel’dovich) spectrum. So, it should be stressed that once the underlying theory is given, there are two more free parameters as compared to standard CDM, hence all the spectra, including that one of the gravitational waves, are computed from first principles, and no ad hoc assumptions are made.

The observations of the CMB anisotropies on the one hand and of the density fluctuation power spectrum $P(k)$ on the other hand, give independent tests of the primordial fluctuations spectra. It turns out that the power spectrum $P(k)$ in our model meets well the large scales structures observations, as summarized in figure 3, though as found earlier the peculiar velocities are low. It does certainly better than tilted models if one insists on the value $\sigma_8 \approx 0.6$. We have also studied the CMB anisotropies produced in our model and we find that the observations of CMB anisotropies on intermediate and small angular scales tremendously constrain our model. The Saskatoon and MSAM data on angular scales corresponding to the Doppler peak seem to imply a much higher peak than obtained in our model. If these observations are to be confirmed, then it is clear that in the framework of a flat CDM universe with values of the cosmological parameters close to those of standard CDM, our model is to be excluded. Note that the computed CMB anisotropies are obtained using linear perturbation theory so that the results are very reliable and put severe constraints on any model if the CMB anisotropies on those angular scales are measured with great accuracy even after cosmic variance is taken into account. This is the goal of the future satellite experiments PLANCK Surveyor and MAP and their observations will tremendously constrain all models proposed. As stressed above, the observations of CMB anisotropies on the one hand and of the density fluctuation power spectrum, on the other hand, give independent constraints of the primordial fluctuations spectra.

One could then ask whether this implies that all inflationary models yielding a primordial spectrum with a characteristic scale should be rejected, again if we keep the same CDM (flat) universes. Actually, this turns out not to be the case. Indeed, the problems in our model arise due to the form of the spectrum on large scales. Therefore while all models with a spectrum analogous to ours will yield similar CMB anisotropies, and are therefore to be excluded, on the other hand, a spectrum closer to a steplike spectrum with a flat or slightly increasing ($n > 1$) upper plateau is expected to give better agreement with observations. A possibility, based on the spatial distribution of rich Abell clusters, is a spike in the initial spectrum followed by a stepdown as proposed in [39].
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