Study of tetraquarks in dipole-dipole interaction potential

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In recent years tetraquark and pentaquark states have received much attention due to the significant experimental findings. In this work masses of some heavy tetraquarks are estimated by considering the spin-spin interaction, the dipole-dipole interaction, and the meson-meson interaction. It is found that such interactions give significant mass contributions. The Regge trajectory of X(3872) state is also studied and is found to be non-linear. Masses of some tetraquark states are also proposed.

I. INTRODUCTION

In nature, the most elementary particles (experimentally observed) are quarks, leptons, and gauge particles. The quarks carry color degree of freedom. Because of color confinement one can not observe a free quark in nature. All forms of quark matter in nature are color singlet. All these quark matter around us are made of mesons (quark - antiquark) and baryons (three quarks). In principle any combination of color singlet multiquark systems can exist in nature. Therefore, search for multiquark systems beyond baryons has always been an area of interest for many physicists [1, 2]. First time the existence of these particles was proposed by Gellmann [3]. In early days because of limited experimental as well as computational facilities, many experimental results on tetraquarks became inconclusive. Recently many experimental findings have confirmed the existence of tetraquarks. Tetraquarks are the hadrons containing two quarks and two antiquarks. Since these are very short lifetime particles therefore, their many properties are still unknown. Study of tetraquarks is very important because it helps us to understand the physics of higher order quark matter, nature of color confinement, and strong forces.

In this work we shall consider tetraquarks containing at least one heavy quark. However, in popular compositions, its quark contents are \(qq\bar{q}\bar{Q}\), where \(q\) represents a light (up, down or strange) quark, \(Q\) represents a heavy (charm or bottom) quark, and \(\bar{q}, \bar{Q}\) are the corresponding antiquarks [4]. But the QCD based studies also indicate the presence of tetraquarks with fully heavy flavors [5, 6]. Because of their large mass and very short lifetime, tetraquarks remained undetected till the last century. In 2003 the Belle experiment in Japan discovered a tetraquark state \(X(3872)[7]\), later confirmed by BABAR Collaboration [8], the Collider Detector at Fermilab experimental collaboration (CDF II) [9, 10], D0 experiment[11], LHCB (Large Hadron Collider beauty)[12] and CMS (Compact Muon Solenoid) etc [13]. Following its discovery, many new tetraquark candidates have also been discovered. Later other tetraquark states \(Y(4260)\) at BABAR in 2005 [14], \(Z(3900)\) at BESIII in 2013 [15] were also discovered. Recently the LHCb collaboration observed a new tetraquark state with all heavy quark flavors [16]. So with improved experimental facilities many new tetraquark states are discovered and the quest is still on. Because of the lack of sufficient experimental data, the nature of these states is still unclear and the work is under progress.

Some progress is also reported in lattice QCD approach, which is actually a computational technique on tetraquarks. In a work by Prelovsek and Leskovec it is argued that \(X(3872)\) is a bound state pole of \(DD^*\) scattering [17]. In a separate work HAL QCD collaboration have studied the \(Z_c(3900)\) by the method of coupled-channel scattering in lattice QCD and shown that it is a threshold cusp [18]. In another work Francis et al have studied the possibility of \(qq\bar{b}\bar{b}\) in lattice QCD framework and have shown the existence of strong-interaction-stable \(J^P = 1^+\) tetraquarks [19]. Bicudo, Scheunert, and Wagner have studied spin effects in \(qq\bar{b}\bar{b}\) like systems using the Born-Oppenheimer approximation. In this work they have predicted the existence of \(ub\bar{b}b\) tetraquark system with quantum number \(I(J^P) = 0(1^+)\) [20]. Jumarkar, Mathur, and Padmanath have done a lattice QCD study and estimated mass and energy levels of \(qq\bar{Q}\bar{Q}\) type of tetraquark systems [21]. This way we find that lattice QCD results also support the existence of tetraquarks and are helpful to explain their properties.

At present we do not have a theory to study tetraquark systems. Therefore, it is studied with some models. The properties of tetraquarks are studied by interpreting their internal structure as cusps, hadron molecules, diquarks-antidiquarks etc, using suitable potential models [22, 23]. Ebert, Faustov, and Galkin have studied the mass spectra
of hidden charm and bottom quark containing tetraquarks in diquark-antidiquark picture. They have shown that the $X(3872)$ can be interpreted as hidden charm tetraquark [24]. In a work Liu et al, have studied the open charm decay modes of state $Y(4630)$ by considering tetraquark with a bound state of a diquark and an anti-diquark [25]. Gutsche, Kesenheimer, and Lyubovitskij1 have studied the $Z_c^+(3900)$ tetraquarks by considering it as a hadronic molecule and interpreted it’s decay widths [26]. Rathaud and Rai have studied some heavy quarks systems as di-hadronic molecules and shown that there should be dipole like interaction between two color neutral states [27]. Mass and decay widths of $bb\bar{b}b$ was estimated by Esposito and Polosa using diquark-antidiquark interaction model [28]. In a separate work the mass spectrum of $c\bar{c}c\bar{c}$ states are also determined using the dynamical diquark model by Giron and Lebed [29]. Recently Cheng et al have estimated the mass spectrum and constraints of double-heavy tetraquark states with heavy diquark-antidiquark symmetry and the chromomagnetic interaction [30]. Hence we see that various models are used to mainly study the mass spectrum and decay properties of tetraquarks for different tetraquark systems. 

In order to study the properties of highly confined quarks, different models are used. The string model of hadrons is one such model which gives the Regge trajectories of hadrons. The study of Regge trajectory is important because it not only gives estimation of hadron masses, also directly correlates mass with the angular momentum of a hadron [1, 2]. It is shown that the Regge trajectories for mesons, baryons, and pentaquarks are non-linear [31–36]. Various theoretical models have been developed to study the Regge trajectories of hadrons. Some of the theoretical models are, string models such as Olsson model [31], Soloviev string quark model [37], non-relativistic quark models such as Inopin model [38, 39], Martin model [40], relativistic and semi-relativistic models like Basdevant model [41], Semay model [42, 43] etc.

In this paper we have estimated masses of different heavy tetraquarks by considering the spin-spin interaction, meson-meson interaction, as well as dipole-dipole interactions. We have found that these interactions cause significant corrections. At the end we have also discussed about the Regge trajectory of $X(3872)$ state.

II. FORMULATION

The masses of different tetraquark states are determined by considering different interactions. Different interactions will have different contribution. In this work the spin-spin, diquark-antidiquark and dipole interactions are considered. Initially the masses are determined using constituent quark model which contains the individual quark masses and spin-spin interactions. A tetraquark system $qq\bar{q}\bar{q}$ ($q = u, d, s, c$ quarks) can have three types of interactions namely, diquark-antidiquark interaction $[qq][\bar{q}\bar{q}]$, dipole-dipole or meson-meson interactions $[qq][\bar{q}\bar{q}]$. Here the dipole interaction can be color as well as electromagnetic. Here the electromagnetic interaction between dipoles is considered and it is assumed that the dipoles formed are sufficiently far from each other so the color interaction can be ignored. Therefore, the total mass of the tetraquark system is

$$M_{th} = \sum_{i=1}^{4} m_i + E_{spin} + \Delta M$$

where, $\Delta M = (E_{diquark} + E_{meson} + E_{dipole})_{avg}$, $E_{spin}$ is the spin-spin interaction energy, $E_{diquark}$ is the diquark-antidiquark interaction energy, $E_{meson}$ is the meson-meson interaction energy and $E_{dipole}$ is the dipole-dipole interaction energy. Let us discuss these contributions separately.

A. Spin-spin interaction

The total mass is due to the constituent quark masses as well as their spin-spin interactions. The standard spin-spin interaction term for quarks can be written as

$$E_{spin} = A' \sum_{j \neq k}^{4} \frac{(S_j \cdot S_k)}{m_j m_k}$$

Let

$$M_{spin} = \sum_{i} m_i + E_{spin}.$$
bottom quark). If $S_{jk} = S_j + S_k$ then

$$S_j \cdot S_k = \frac{S_{jk}^2 - S_j^2 - S_k^2}{2}$$

$j, k$ are the different flavor constituents of the tetraquark.

Here the constituent quark masses are $m_u = 336MeV$, $m_d = 340MeV$, $m_s = 486MeV$, $m_c = 1550MeV$, and $m_b = 4730MeV$ [44]. The masses due to spin-spin interaction is shown in table 2. There will be actually many type of interactions among quarks/antiquarks. Based on their contribution we consider only dominating terms. Here the diquark-antidiquark, meson-meson interactions are considered. Actually in the meson-meson interaction it is found that the color interaction contribution is smaller than the electric dipole-dipole interaction.

## B. Diquark-antidiquark interaction

A diquark is a two quark composition forming a bound state. The formation of bound state is mainly due to one-gluon exchange potential [46]. According to SU(3), diquarks can form color triplet or color sextet with color factor $-\frac{2}{3}$ (triplet) and $\frac{1}{3}$ (sextet). Diquark-antidiquark systems are considered as two-body systems and the potential chosen in this case is Cornell-like potential. The sextet color configuration is considered because only then the potential will show confining nature.

$$V_{Cornell} = ar - ks \frac{b}{r}$$

This potential is solved by Kuchin et al, [45] using the Schroedinger’s equation, and the expression for energy is,

$$E_{diquark} = \frac{3a}{\delta} - \frac{(ksb + 3a)}{2\mu} \frac{2\mu}{(2n + 1) \pm \sqrt{1 + 4l(l + 1) + \frac{8\mu a}{3\delta}})^2}$$

Here $a, b,$ and $\delta$ are constants, $a = 0.2GeV^2$, $b = 1.244$ and $\delta = 0.231GeV$ (for charm quark) and $b = 1.569$ and $\delta = 0.378GeV$ (for bottom quark) [45] and $ks = \frac{1}{3}$. In calculation we have chosen the negative terms only because the positive term causes unacceptable errors.

## C. Meson-meson interaction (using Cornell potential)

Just like diquarks, quark, and antiquarks also interact via one-gluon exchange potential. The color factor depends on the color state of the quarks. It can form a color singlet or octet. The color factor can take values $-\frac{4}{3}$ (singlet) or $\frac{1}{6}$ (octet). The octet-octet interactions are considered to include the color interaction. The energy due to mesonic interactions are determined using the diquark energy expression.

## D. Meson-meson interaction (using Yukawa potential)

In this section the Yukawa potential of the following form is considered,

$$V(r) = -V_0 \frac{e^{-ar}}{r}$$

where, $V_0 = 0.5$. Hamzavi et al had obtained the solution of the Yukawa potential as [47],

$$E_{nl} = -\frac{a^2}{2m} \frac{(mV_0a - (n + 1)^2 - l(2n + l + 2))^2}{(n + l + 1)^2}$$

In this work $n = 0, l = 0$ and $a = 0.686fm^{-1}$. 
E. Dipole-dipole interaction

Here the tetraquark is considered as the two dipoles interacting. Let us consider a tetraquark as two electromagnetically interacting dipoles. It is assumed that the separation between dipoles (inside a tetraquark) is large so the color interaction can be ignored. The dipoles may interact with all orientations due to thermal effect. Hence it must should be thermally averaged. The thermally averaged dipole-dipole interaction potential is given by,

\[ V(r) = -\frac{C}{r^6} \]

where,

\[ C = \frac{2 \mu_A^2 \mu_B^2}{3 (4\pi\epsilon_0)^2} \frac{1}{k_B T} \]

\( \mu_A \) and \( \mu_B \) are the dipole moment of two dipoles and \( r \) is the distance between the center of two dipoles.

According to Gao [48] on solving the radial Schroedinger’s equation for \( -\frac{C}{r^6} \) potential, the expression for energy can be determined [48].

\[ E = \Delta \times 16 \times \frac{\hbar^2}{2\mu} \frac{1}{\beta^2} \]

Here,

\[ \beta = \left( \frac{2\mu C}{\hbar^2} \right)^{\frac{1}{2}} \]

\( \Delta \) is the critical scaled energy and \( \mu \) is the reduced mass of the dipoles. The value of \( \Delta \) for different angular momentum states are shown in Table I. The simplified expression for energy in (MeV) can be expressed as,

\[ E = 3.7 \times 10^7 \sqrt{\frac{k_B T}{\mu^3}} \frac{\Delta}{f_A f_B d_A d_B} \]  \hspace{1cm} (1)

Here, \( T \) and \( \mu \) are expressed in MeV and \( d_A \) and \( d_B \) are the dipole lengths expressed in fm. The critical scaled energy for different angular momenta \( l \) is given in the following table. Here the temperature is at \( T = 0.1MeV \). We have chosen the low temperature because we are interested in ground state energy or lowest mass.

| \( l \) | \( \Delta \) |
|-------|-------|
| \( s \) | 9.654418\times10^{-2} |
| \( p \) | 1.473792\times10^{-1} |
| \( d \) | 4.306921\times10^{-1} |
| \( f \) | 1.580826 |
| \( g \) | 2.073296 |
The energy corrections due to diquark-antiquark, meson-meson (using Cornell potential) and dipole-dipole interactions are determined and are given in Table II. The energy corrections due to diquark-antiquark, meson-meson (using Yukawa potential) and dipole-dipole interactions are determined and are given in Table III.

| State | $M_{\text{spin}}$ (MeV) | $E_{\text{diquark}}$ (MeV) | $E_{\text{meson}}$ (MeV) | $E_{\text{dipole}}$ (MeV) | $\Delta M$ (MeV) | $M_{\text{th}}$ (MeV) | % error |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------|
| $\bar{c}ud\bar{s}$ $X_0(2900)$ | 2558               | $ud - \bar{c}s$ -81 | $\bar{c}u - \bar{d}s$ 51 50 | $\bar{c}u - d\bar{s}$ 367 392 | 260                | 2818               | 2.8     |
| $c\bar{c}u\bar{u}$ $X(3872)$ | 3672               | $cu - \bar{c}u$ 88   | $c\bar{c} - u\bar{u}$ 177 39 | $c\bar{c} - u\bar{u}$ 194 87 | 195                | 3867               | 0.13    |
| $uc\bar{d}\bar{c}$ $Z_c^+(3900)$ | 3677               | $uc - \bar{d}\bar{c}$ 88 | $uc - \bar{d}\bar{c}$ 41 177 | $uc - \bar{d}\bar{c}$ 153 | 3830               | 1.8     |
| $cs\bar{c}s$ $Y(4140)$ | 4013               | $cs - \bar{c}s$ 105 120 | $cs - s\bar{s}$ 193 120 | $cs - s\bar{s}$ 250 138 | 269                | 4282               | 3.4     |
| $dc - \bar{u}c$ $Z(4430)$ | 3677               | $dc - \bar{u}c$ 88   | $dc - \bar{u}c$ 41 177 | $dc - \bar{u}c$ 153 | 3830               | 13.5    |
| $cs\bar{c}s$ $X(4700)$ | 4013               | $cs - \bar{c}s$ 105 120 | $cs - s\bar{s}$ 193 120 | $cs - s\bar{s}$ 250 138 | 269                | 4282               | 8.8     |
| $cc\bar{c}\bar{c}$ $(6900)$ | 6185               | $cc - \bar{c}\bar{c}$ 186 | $cc - \bar{c}\bar{c}$ 271 271 | $cc - \bar{c}\bar{c}$ 41 41 | 270                | 6455               | 6.4     |
Table III. Energy corrections due to spin-spin, diquark, meson (using Yukawa potential) and dipole interactions

| State        | $E_{\text{spin}}$ | $E_{\text{diquark}}$ | $E_{\text{meson}}$ | $E_{\text{dipole}}$ | $\Delta M$ | $M_{\text{th}}$ | $\%$ |
|--------------|-------------------|-----------------------|---------------------|----------------------|------------|-----------------|------|
| $\bar{c}u\bar{u}d\bar{s}$ | X(2900)            | 2558                  | -81                 | $\bar{c}u - d\bar{s}$ | $\bar{c}\bar{d} - u\bar{s}$ | 367   | 392             | 223  |
| $\bar{c}\bar{u}\bar{u}\bar{d}$ | X(3872)            | 3672                  | 88                  | $\bar{c}\bar{d} - u\bar{u}$ | $\bar{c}\bar{c} - \bar{u}\bar{d}$ | 194   | 87              | 97   |
| $\bar{u}\bar{c}\bar{d}\bar{c}$ | Z$^+$ (3900)       | 3677                  | 88                  | $\bar{u}\bar{d} - \bar{c}\bar{c}$ | $\bar{u}\bar{c} - \bar{c}\bar{d}$ | 4     | 3681            | 5.6  |
| $\bar{c}\bar{s}\bar{c}\bar{s}$ | Y(4140)            | 4013                  | 105                 | $\bar{c}\bar{s} - \bar{s}\bar{s}$ | $\bar{c}\bar{s} - \bar{s}\bar{s}$ | 249   | 138             | 129  |
| $\bar{d}\bar{u}\bar{c}\bar{c}$ | Z(4430)            | 3677                  | 88                  | $\bar{d}\bar{c} - \bar{c}\bar{c}$ | $\bar{d}\bar{c} - \bar{c}\bar{c}$ | 4     | 3681            | 17   |
| $\bar{c}\bar{s}\bar{c}\bar{s}$ | X(4700)            | 4013                  | 105                 | $\bar{c}\bar{s} - \bar{s}\bar{s}$ | $\bar{c}\bar{s} - \bar{s}\bar{s}$ | 249   | 138             | 129  |
| $\bar{c}\bar{c}\bar{c}\bar{c}$ | (6900)             | 6185                  | 185                 | $\bar{c}\bar{c} - \bar{c}\bar{c}$ | $\bar{c}\bar{c} - \bar{c}\bar{c}$ | 41    | 41              | 1    |

Note: The corrections are given in MeV, and the errors are in parentheses.
Table IV. Predicted mass spectra of some tetraquark states

| State   | $M_{\text{spin}}$ | $E_{\text{diquark}}$ | $E_{\text{meson}}$ | $E_{\text{dipole}}$ | $\Delta M$ | $M_{\text{th}}$ |
|---------|-------------------|------------------------|---------------------|----------------------|------------|-----------------|
| (Exp)   | (MeV)             | (MeV)                  | (MeV)               | (MeV)                | (MeV)      | (MeV)           |
| $ss\bar{s}$ | 2960              | $ss - \bar{s}s$       | $-678$              | $-678$               | 941        | 941             | -142            | 2818           |
| $uu\bar{d}$ | 6872              | $uu - \bar{d}b$       | $-568$              | $-568$               |            | -984            | 5888            |
| $dd\bar{u}$ | 6867              | $dd - \bar{u}b$       | $-568$              | $-568$               | -981       | 5886            |
| $\bar{b}u\bar{s}$ | 6797         | $ud - \bar{u}s$       | $-492$              | -488                 |            | 903             | 5894            |
| $us\bar{s}$ | 6758              | $us - \bar{s}b$       | $-486$              | -428                 | -826       | 5932            |
| $uu\bar{s}$ | 6800              | $uu - \bar{s}b$       | $-488$              | -488                 | -602       | 6198            |
| $ss\bar{s}$ | 6748              | $ss - \bar{s}s$       | $-426$              | -426                 | 430        | 430             | -220            | 6528           |
| $bb\bar{b}$ | 18931             | $bb - \bar{b}b$       | $-5$                | -5                   | 30         | 30              | -49             | 18882          |
Table V. Comparison between Cornell and Yukawa potential corrections

| State  | Expt (MeV) | $M_{th}$ (Cornell) (MeV) | $M_{th}$ (Yukawa) (MeV) |
|--------|------------|--------------------------|--------------------------|
| $\bar{c}ud$ | $X_0 (2900)$ | 2818 | 2781 |
| $cc\bar{u}u$ | $X (3872)$ | 3867 | 3769 |
| $ucd\bar{c}$ | $Z_c^+ (3900)$ | 3830 | 3681 |
| $cs\bar{c}s$ | $Y (4140)$ | 4282 | 4142 |
| $d\bar{u}cc$ | $Z (4430)$ | 3830 | 3681 |
| $cs\bar{c}s$ | $X (4700)$ | 4282 | 4142 |
| $cc\bar{c}$ | (6900) | 6455 | 6186 |

Table VI. Comparison of masses of tetraquarks, in units of MeV, calculated in this work and values from other different works.

| State  | present work | other work |
|--------|---------------|-------------|
| $ss\bar{s}\bar{s}$ | 2818 | 2200-2900 [50] |
| $uu\bar{d}\bar{b}$ | 5888 | 5977-6503 [51] |
| $dd\bar{u}\bar{b}$ | 5886 | 5977-6503 [51] |
| $\bar{b}ud\bar{s}$ | 5894 | 5584±137 [52] |
| $uss\bar{b}$ | 5932 | 6388-6728 [51] |
| $uu\bar{s}\bar{b}$ | 6198 | 6194-6602 [51] |
| $ss\bar{b}\bar{s}$ | 6528 | 6682-6826 [51] |
| $bb\bar{b}\bar{b}$ | 18882 | 18690 [53] |

III. RESULTS AND DISCUSSION

In tetraquark mass calculation constituent quark masses, spin-spin, diquark-antidiquark, meson-meson, and dipole-dipole interactions are considered. From results it is clear that the main contribution is due to constituent quark masses and spin-spin interaction. The spin-spin interaction takes care of color as well as electromagnetic interactions simultaneously. Now for the diquark-antidiquark interactions the Cornell potential with appropriate color factors as mentioned earlier is chosen. In Table II the $E_{diquark}$ is calculated between diquark and antidiquark using the Cornell potential. In Table III the same is done with Yukawa potential. From tables II and III it is clear that the diquark-antidiquark interaction energy is very small compared to the meson-meson interaction where the tetraquark
is considered as two interacting mesons. The decay modes of tetraquark also support this because most of the decay mode show that decay of tetraquark leads to at least two mesons. On the other hand for diquark-antidiquark picture there should also be possibility of two baryons after tetraquark decay. In Table V it is also interesting to see that the mass corrections due to the Cornell potential is smaller than the Yukawa potential corrections. However, the results with the Cornell potential are much closer to the experimental values. So the color interaction picture appears to be more realistic. For tetraquarks two types of mesonic interactions are possible. Hence both the possibilities are considered with equal probability. Actually the probability of a particular configuration can be taken from the branching ratio of decay modes obtained from experiments. In meson-meson case dipole interactions are also possible. It is interesting to see that such interactions also cause significant mass corrections.

From the tables it is clear that for low mass tetraquarks the major contributions are coming due to the diquark-antidiquark, meson-meson, dipole-dipole, and spin-spin interactions. The difference from experimental results are very small. On the other hand if we go to larger mass states the difference gradually increases which tells that other effects will also be dominating. For \( Z(4430) \) and \( Z_c^+(3900) \) states the dipole interaction contribution is very small (< MeV) so it is ignored.

In Table IV the masses of some tetraquark states are proposed and also compared with other theoretical results in Table VI. In this case only the Cornell potential for meson and diquark interactions is considered. Our results are in good agreement with other theoretical works.

\( X \rightarrow \bar{D}^0 D^0 \) is one of the observed decay modes of \( X(3872) \) state [49]. These \( \bar{D}^0 \) and \( D^0 \) mesons are considered as two dipoles. Using the energy equation, the interaction energy for \( X^{(3872)} \) tetraquark state can be determined.

It is clear from the energy equation that the interaction energy depends on temperature \( T \) and the reduced mass \( \mu \).

For \( X(3872) \) tetraquark, the energy correction for the \( s \) state with temperature 1 eV to 200 MeV ranges from 0.3 MeV to 4000 MeV. Figure 2 shows that the mass correction increases with temperature and it also increases as we go to higher angular momentum state. If the heavy tetraquarks are considered, the energy correction is less and for light tetraquarks, the energy correction is more. In this framework for \( X(3872) \) tetraquark state the energy corrections and mass in different higher angular momentum states are determined and is mentioned in Table VII.

| \( l \) | Mass correction (MeV) | Mass of the state (MeV) |
|-------|----------------------|------------------------|
| \( s \) | 140                  | 3868                   |
| \( p \) | 213                  | 3941                   |
| \( d \) | 623                  | 4351                   |
| \( f \) | 2288                 | 6016                   |
| \( g \) | 3000                 | 6728                   |

Figure 2 shows the mass correction for different angular momentum states at different temperatures (Eqn(1)). As expected for higher angular momentum states and temperature the mass corrections are large. This tells that the mass of a tetraquark also depends upon temperature of medium. Therefore, mass of tetraquarks produced in deconfined medium will be larger than the mass observed after thermal freeze out. Figure 3 shows the energy correction versus reduced mass of dipoles for different angular momentum states (Eqn(1)). The mass correction decreases with increase in reduced mass. It indicates that the tetraquarks containing light quarks have significant mass contribution due to the dipole-dipole interaction. The Regge trajectory for the \( X(3872) \) system is shown in figure 4. It is found to be highly non-linear. The same idea can be extended for Regge trajectories for different tetraquark systems.
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