Limits of Quasi-Static Approximation in Modified-Gravity Cosmologies

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We investigate the limits of applicability of the quasi-static approximation in cosmologies featuring general models of dark energy or modified gravity. We show that the quasi-static approximation always breaks down outside of the sound horizon of the dark-energy, rather than the cosmological horizon as is frequently assumed. When the sound speed of dark energy is significantly below that of light, the quasi-static limit is only valid in a limited range of observable scales and this must be taken into account when computing effects on observations in such models. In particular, in the analysis of data from today’s weak-lensing and peculiar-velocity surveys, dark energy can be modelled as quasi-static only if the sound speed is larger than order 1% of that of light. In upcoming surveys, such as Euclid, it should only be used when the sound speed exceeds around 10% of the speed of light.

I. INTRODUCTION

Dynamical dark energy (DE) and modified gravity (MG), modelling the observed acceleration of late-time cosmological expansion, usually require a new degree of freedom beyond those present in standard Λ-cold dark matter (ΛCDM) cosmology. The full dynamics of this new degree of freedom in general models of DE/MG are usually quite complicated. However, since data are still mostly available only on scales small compared to the cosmological horizon, the quasi-static (QS) approximation is frequently used to approximate the full DE/MG dynamics and interpret observations. Roughly, this amounts to neglecting terms involving time derivatives in the Einstein equations for perturbations and only keeping spatial derivatives. Such a procedure should be valid on sufficiently small scales, typically assumed to be those well inside the cosmological horizon. It is well known from numerical studies that in the case of quintessence [4, 5], $f(R)$ gravity [6] or the covariant galileon models [7, 8] this approximation is good enough on observable linear scales (see e.g. [9–11] and in particular ref. [12]). On the other hand, general perfect-fluid DE can show some discrepancy between the full and QS solutions at late times [13].

On non-linear scales, N-body simulations have been used to study symmetron-screened [14] non-minimally coupled quintessence [15, 16] and $f(R)$ gravity [17] with largely the same conclusion: the effects of the time-derivative terms are small. But how general are these results?

The solutions in the dust-dark-energy dynamical system which models the late universe can indeed always be described using two functions of space and time [18, 22], without reference to the extra degree of freedom. Taking the QS limit allows one to remove the dependence on the initial conditions for the DE/MG and to reinterpret what is a parameterisation of a particular solution as a universal description of a model. In the extreme QS limit, all the scale-dependence is also neglected and the DE model is described using just two functions of time: the effective Newton’s constant $\mu$ and the gravitational slip parameter. Allowing for scale dependence in principle gives a closer description on a wider set of scales [24–28], but the question arises to what extent all of these approximations are valid.

Recently, effective-field-theory (EFT) methods [29–34] have been developed to efficiently encode the full dynamics of DE/MG models. Linear structure formation in a very general class of DE/MG models can be fully described in this way using a small set of functions of time only. We can thus use this formulation to compare QS and full solutions in a general way in order to understand the approximations properly. In what follows, we will use the term “dark energy” to refer to any one of the models described by the EFT, i.e. to also most models of modified gravity.

In this paper, we show that the domain of validity of the QS approximation is determined by the sound horizon of dark energy: the QS approximation can only be trusted inside the sound horizon. The scale of this sound horizon can be calculated from an algebraic relationship between the parameters of the EFT or, alternatively, is a functional of the action for the DE model. We derive a condition determining when the QS approximation can be used safely to analyse a particular dataset and apply this result to parameters of current and future surveys.

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1 Although modifications of general-relativistic (GR) constraints also exist, e.g. refs [13].
II. THE QUASI-STATIC SOLUTION

We model the late universe as a mixture of pressureless dust with density fraction \( \Omega_m \), representing both dark matter and baryons between which we do not differentiate, and the dark energy with density fraction \( \Omega_{DE} = 1 - \Omega_m \), which we assume comprises a single degree of freedom. The background expansion history is constrained by observations (e.g. ref. [33]) to be that corresponding to a source with approximately constant pressure and no spatial curvature. In the linear perturbed Einstein equations describing the evolution of structure on largest scales, an extra scalar degree of freedom can be integrated out by solving the constraints. This procedure, described in detail in ref. [33], reduces the exact perturbation equations into a second-order differential equation for the gravitational potential \( \Phi \) coupled to the pressureless matter. The full form for such an equation for general DE models is given in [33, 36]. However, for the purpose of this work, it is enough to write as a proxy an approximation valid at smaller scales, where it can be written as

\[
E[\Phi] = \Phi'' + \left( \frac{\gamma + H'}{H} \right) \Phi' + \left( \frac{M^2}{H^2} + c_s^2 k_H^2 \right) \Phi = (1)
\]

where we use primes to denote derivatives w.r.t. e-folding time, \( \delta \) and \( v \) are the matter density contrast and velocity perturbation, \( a \) the scale factor, \( k_H \equiv k/aH \) is the comoving mode number in units of the cosmological horizon, while \( \gamma \) is a model-dependent friction coefficient. The sound speed \( c_s > 0 \) is the speed at which high-frequency longitudinal (scalar) perturbations in the DE propagate, while \( M \) is some effective mass of the extra degree of freedom related to parameters of the model and the background expansion history (see e.g. [33]). The potential is coupled to matter through an effective Newton’s constant, \( \mu_S \), and an equivalent coupling to the matter velocity potential, \( \mu_v \). We take both of these to be constant, but time evolution does not add any qualitative changes. The system is completed by the standard EMT conservation equations for dust

\[
\delta' = k_H^2 H v + 3 \Phi', \quad H v' = -\Phi, \quad (2)
\]

where we have assumed that there is no gravitational slip, \( \Phi = \Phi^{QS} \). The two gravitational potentials are always related through a constraint rather than a dynamical equation, and therefore generalising this result to models with non-vanishing slip does not change qualitatively the results presented below.

The coupling to the velocity \( \mu_v \) always appears when the Einstein (00) and (0i) equations are used to eliminate the DE scalar perturbation and it does not in general disappear in comoving gauge. Secondly, we stress that since the modification of gravity contains an additional scalar on top of the standard scalar density perturbations in matter, the time derivatives of the potentials \( \Phi \) depend on two time-scales: (i) that of the pressureless collapse of the dust, which is just \( H \) and does not produce any oscillations since there is no pressure (ii) of the modified-gravity scalar degree of freedom, which is related to its mass and the sound horizon and thus to the pressure support it provides. This is somewhat different than in ref. [33], where the time-scale intrinsic to the scalar was not considered to determine the oscillations of the gravitational potentials.

In principle, to solve the system (1-2), we should find the eigenmodes and their initial conditions. This is in general impossible. We instead can define the QS solution by splitting the Newtonian potential into an oscillating part \( \varphi \) and the quasi-static part \( \Phi^{QS} \)

\[
\Phi = \varphi + \Phi^{QS}, \quad (3)
\]

choosing \( \Phi^{QS} \) such that the oscillating part \( \varphi \) has no source in its equation of motion, \( E[\varphi] = 0 \), with \( E \) denoting the homogeneous part of eq. (1). This gives us one decoupled equation for \( \varphi \) and the dependent eqs. (1, 2). Since the equation for \( \varphi \) is source free, provided that its coefficients are appropriate, the oscillations \( \varphi \) can be expected to decay away over time, leaving us with only what we have called the QS solution. This is close but not exactly the same definition as is used when time derivatives are just neglected. We include extra information which allows us to make a definite statement about the region of validity, inside of which the two solutions are the same.

As we will see, there are some corrections to \( E[\varphi] \) which are suppressed by \( (c_s k_H)^{-2} \). These corrections will therefore place an ultimate limit to the scales on which the QS limit can be considered to be valid.

III. MAXIMUM DOMAIN OF VALIDITY

In general, we cannot find an exact solution for the particular integral of (1). We can, however, obtain the Padé approximant to this solution, valid at least on some scales. Therefore, for the QS solution, we will be seeking an approximate solution valid inside some set of small scales to some accuracy.

We start off by considering the typical evolution of cosmological-perturbation variables inside the cosmological horizon, \( k_H \gg 1 \). In the usual case, we have \( Hv \sim O(k_H^{-2} \delta) \) by the virtue of the continuity equation (2). This gives a natural hierarchy for these variables
which will allow us to carry out an expansion. In what follows, we will therefore consider
\[
\delta \sim O(\epsilon^0), \quad H v \sim O(\epsilon), \tag{4}
\]
identifying the order parameter $\epsilon$ roughly with $k_{H}^{-2}$. This also gives us $\Phi \sim O(\epsilon)$. By definition, our approximation fails at most at the cosmological horizon.

Given this hierarchy, we can choose an ansatz for the QS solution, which includes the contributions lowest-order in $\epsilon$ and is compatible with eq. (1),
\[
k_{H}^{2} \Phi_{\text{QS}} \equiv -3 \left( A_{1} + \frac{\epsilon A_{2}}{k_{H}^{2}} \right) \Omega_{m} \delta - \frac{3 \epsilon B_{1}}{2} \Omega_{m} H v. \tag{5}
\]
The time-dependent coefficients $A_{1}, A_{2}, B_{1}$ are chosen in such a way that eq. (1) can be written as $E[\varphi] = 0$ up to the relevant precision defined by $\epsilon$ when $\Phi$ is replaced according to eq. (5).

In the extreme quasi-static limit, $k_{H} \to \infty$, one typically neglects the $A_{2}$ and $B_{1}$ terms, since they are suppressed by $\epsilon$. However, if we wish to include, for example, the corrections resulting from the mass $M^{2}$, one needs to include terms such as $A_{2}$ and therefore also $B_{1}$ since both of these corrections appear at the same order. Moreover, as we will see, including $B_{1}$ allows us to estimate the domain of validity for the QS solution.

One now substitutes the QS ansatz (3) into eqs (1), (2) and (3), keeping all terms involving $\delta$ and $v$ using the conservation equations. The evolution equation (1) can then be rewritten as
\[
\varphi'' + \left( \gamma + \frac{H'}{H} \right) \varphi' + \left( \frac{M^{2}}{H^{2}} + \frac{3}{2} A_{1}(1 + 3 \epsilon_{A}^{2}) \Omega_{m} + \frac{c_{s}^{2} k_{H}^{2}}{\epsilon} \right) \varphi = \left( \frac{F_{1}}{\epsilon} + \frac{F_{2}}{k_{H}^{2}} \right) \Omega_{m} \delta + \frac{3}{2} F_{3} \Omega_{m} H v + O(\epsilon),
\]
with the $F_{i}$ functions of time. According to how we have defined the QS solution, we need to pick such coefficients $A_{1}, A_{2}$, and $B_{1}$ that all the $F_{i} = 0$. The particular choices we require are:
\[
A_{1} = \mu s, \quad A_{2} = \frac{3}{2} \Omega_{m} \mu s^{2} - \mu s \left( 1 - \frac{H'}{H} + \frac{M^{2}}{H^{2}} - \gamma \right), \quad B_{1} = c_{s}^{2} \mu v + \mu s (4 - \gamma). \tag{7}
\]
This choice fixes all the freedom in ansatz (3) and removes the external source in the evolution equation for $\varphi$, eq. (8), decoupling it from the matter perturbations. This choice also matches the standard choice for the leading-order terms of the QS solution. In this new description, $\varphi$ evolves with no source and, given certain additional conditions, $\varphi$ decays away and the QS solution is eventually reached. We discuss these additional conditions in section IV.

Let us now turn to the evolution equation for $\delta$, or equivalently, the equation for the growth factor, which is usually the equation of interest when the QS approximation is used. Using the QS ansatz given by (5) with the coefficient values given in eqs (7), we eliminate the matter velocity potential $v$ from the matter conservation equations (2), obtaining
\[
\delta'' + \left( 2 + \frac{H'}{H} + \epsilon F_{3} \right) \delta' + \frac{3}{2} \mu s \Omega_{m} (1 + \epsilon F_{3}) \delta = S[\varphi] \tag{8}
\]
where the source is determined by the deviation away from the QS solution,
\[
S[\varphi] \equiv - \left( k_{H}^{2} - \frac{9}{2} \epsilon \mu s \Omega_{m} \right) \varphi + 3 \epsilon \left( 2 + \frac{H'}{H} \right) \varphi' + 3 \epsilon \varphi'',
\]
and
\[
c_{s}^{2} k_{H}^{2} F_{3} \equiv \Omega_{m} \left( \frac{3}{2} \mu s (\gamma - 4) \right) + O(c_{s}^{2}),
\]
\[
c_{s}^{2} k_{H}^{2} F_{5} \equiv \frac{M^{2}}{H^{2}} + \frac{3}{2} \mu s \Omega_{m} + 1 - \frac{H'}{H} - \gamma + O(c_{s}^{2}).
\]
In the small scale limit of eq. (5), $k_{H} \to \infty$, we recover the standard equation for the growth factor with sources which all involve the oscillating part of the potential, $\varphi$. Thus if indeed we can show that $\varphi$ decays away at these scales, the QS solution for the growth factor will be valid once that has occurred (see section IV).

As we move away from the extremely small scales, corrections to the evolution appear. In particular, if $M^{2} \gg H^{2}$, then the corrections due to this mass are as expected from the form of the coefficient of $\Phi$ in eq. (1) and as is currently taken in $f(\mathcal{R})$ gravity models. There are also some additional corrections of order $\mu s H^{2}$ to the mass which can only be relevant when the QS approximation breaks down by construction, close to the cosmological horizon. Thus the naive QS estimate of the effective Newton’s constant is essentially valid at cosmological subhorizon scales.

However, there are also corrections to the friction term, $\delta'$. The corrections are of order $\mu s$ and are only suppressed by the sound horizon for the dark energy, $c_{s} k_{H}$ rather than the cosmological horizon $k_{H}$. Thus, at scales near the sound horizon, the response of the growth function to some particular value of the effective Newton’s constant is different than what would be expected from the naive guess, even if the quasi-static solution for $\Phi$ were valid. The typical sign of this correction is negative, thus the growth rate increases as the sound horizon is approached. This suppression by only the sound horizon is driven by the fact that the solution for the coefficient $B_{1}$ contains terms of order...
\[ c_s^{-1} \] and thus such corrections are missed when the dust velocity potential \( \nu \) is ignored.

We should also add that similar corrections appear to the evolution equation for \( \varphi \), eq. (9) at order \( \epsilon \). The coefficient of the friction term is also modified by a term \( \mathcal{F}_1/c_s^{-2}k_H^2 \), thus leading to a similar decrease in the friction and therefore a relative enhancement of the amplitude of the oscillation \( \varphi \). Thus the sound horizon also provides a limiting scale for the quasi-static approximation to hold for the potential \( \Phi \).

**IV. EVOLUTION OF OSCILLATIONS**

In the previous section, we demonstrated that, inside the dark-energy sound horizon, the quasi-static solution can in principle model the evolution of the full dark-energy-dust system. Choosing a scale well inside the sound horizon, \( c_s k_H \gg 1 \), is a necessary condition for the existence of this QS solution. It is not sufficient, however. In addition, the deviation from the QS solution \( \varphi \) must decay faster than the QS solution, such that the QS solution is asymptotically reached after sound-horizon crossing. In this section, we will demonstrate the conditions required for this decay to occur.

The definition of the QS solution we have proposed in section III was constructed in such a way that the variable \( \varphi \) — describing the deviation of the gravitational potential from the QS solution — has no source in its evolution equation up to some precision. This decouples the evolution of \( \varphi \) from the dust part of the system and therefore its behaviour can be considered in isolation. Taking the small-scale limit, \( k_H \to \infty \), where the QS solution should be valid, if it is valid anywhere at all, the variable \( \varphi \) obeys the homogeneous part of eq. (6),

\[ \ddot{\varphi} + (\gamma - 1)H \dot{\varphi} + c_s^2 k^2 \varphi = 0, \tag{9} \]

where we have re-expressed eq. (6) in conformal time \( \eta \), with the overdot signifying a derivative w.r.t. it, while \( H \equiv a \dot{H} \) is the conformal Hubble parameter.

For any particular model with its appropriate background, eq. (9) can be solved numerically. Here, we will only perform an estimate in a situation where closed-form solutions are available: assuming power-law evolution where necessary. The conformal Hubble parameter evolves as \( H = 2/(1 + 3w_{\text{tot}}) \eta \) where \( w_{\text{tot}} \neq -1/3 \) is the total equation of state for the universe. We are also going to assume that the dark-energy sound speed evolves as a power law with redshift, \( c_s^2 = c_{s0}^2(1 + z)^{-\nu} = c_{s0}^2 \eta^{2\nu/(1+3w_{\text{tot}})} \). We note here that for \( \nu < -1 \), the comoving sound horizon would actually be shrinking during matter domination and therefore the modes would be leaving the sound horizon. For any particular mode, the super-sound-horizon corrections to eq. (9) would eventually take over and the QS limit would be violated completely by the results of section III. We shall only consider below such cases where the comoving sound horizon grows.

The friction term \( \gamma \) for general scalar-tensor models is determined by the action and given in full in ref. [33]. In MG models, where the kineticity is small compared to the braiding, \( \alpha_K \ll \alpha_B \) (e.g. \( f(R) \)), \( \gamma \approx 3 \); on the other hand, in perfect-fluid DE models, such as k-essence/quintessence \( \gamma \approx 4 \). This value is model-dependent, so we will only assume it is a constant during the era being considered. We can then rewrite eq. (9) in (one of the) standard forms of the Bessel equation, the solutions of which are oscillatory with a decaying envelope. Expressing the result as a function of redshift, the envelope evolves as \( \varphi_{\text{env}} \propto (1 + z)^\nu \) with decay exponent

\[ \nu = \frac{p}{4} + \frac{\gamma - 1}{2}, \] (10)

when inside the sound horizon. Note that this is independent of \( w_{\text{tot}} \) and is also a good approximation for the \( \Lambda \)CDM background. So if we take \( \gamma = 3 \), then \( p > -4 \) for a decaying \( \varphi \). Thus the sound speed must decrease rapidly for \( \varphi \) to grow. Beware that the comoving cosmological horizon shrinks during acceleration \( (z < 0.65) \) in \( \Lambda \)CDM with \( \Omega_m = 0.31 \) and the modes begin to exit the sound horizon, potentially invalidating the QS limit.

We discuss the impact of the above current results and upcoming surveys in section V.

**V. APPLICATION TO SURVEYS**

In the preceding, we have built up a picture where, outside the dark-energy sound horizon, the configuration of the dark energy degree of freedom is significantly different from the QS solution. Provided that the comoving sound horizon is growing, the mode will cross it. The deviation from the QS solution will then decay away with the exponent given by eq. (10), eventually becoming negligible given some required theoretical precision.

Given the above, we can estimate the minimum dark-energy sound speed which is necessary for the QS approximation to be appropriate in the analysis of observations given particular survey parameters. We will estimate this requirement by assuming that

- the cosmological background is exactly \( \Lambda \)CDM with parameters approximating the Planck 2015 results, \( \Omega_m = 0.31 \) and \( z_{\text{eq}} = 3371 \) [35];
- the sound speed of dark energy \( c_s \) is constant, \( p = 0 \); we take a model with \( \gamma = 3 \), i.e. with the exponent of the decay of oscillations of eq. (10) constant, \( \nu = 1 \);
• the value of the effective Newton's constant in the QS solution is in principle redshift dependent and
given by \( \mu_\delta(z) \)

• the survey uses some longest mode \( k_{\text{min}} \equiv 2\pi/r_{\text{max}} \)
for their DE-related analysis using a sample of
galaxies within a redshift bin centred on \( z_{\text{surv}} \).

We then require that, for the longest mode \( k_{\text{min}} \), the
oscillating part of the potential constitutes no more than
fraction \( \epsilon \) of the QS value, i.e. the DE sound speed needs
to be large enough to satisfy

\[
\varphi(z_*) / \Phi_{\text{QS}}(z_{\text{surv}}) \left[ 1 + z_{\text{surv}} \right] / \left[ 1 + z_* \right] \gtrsim \epsilon
\]

(11)

where \( z_* \) is the redshift at which the mode \( k_{\text{min}} \) crossed
the DE sound horizon, obtained by solving \( c_s k_{\text{min}} = H(z_*) \). The deviation from the QS solution at sound-
horizon crossing can be roughly estimated by assuming that
above its sound horizon, the dark energy will behave
roughly as dust (since there is no pressure support), so
one would expect \( f_H^2 \Phi \sim -\frac{3}{2} (\Omega_m + \Omega_{\text{DE}}) \delta \). On the other hand, the QS result is given by
\( f_H^2 \Phi_{\text{QS}} = -\frac{3}{2} \mu_0 \Omega_m \delta \). Thus

\[
\varphi(z_*) = \Phi - \Phi_{\text{QS}} \sim -\mu_\delta^{-1} \left( \mu_\delta - 1 + \frac{\Omega_{\text{DE}}}{\Omega_m} \right) \Phi_{\text{QS}}(z_*).
\]

(12)

Let us now parameterise the effective Newton's constant
as

\[
\mu_\delta = 1 + \mu_0 (1 + z)^{-s}
\]

(13)

and assume that \( \mu_0 \) is not too different from one, so that the
potential is approximately constant during matter
domination. Also, we assume that the dark energy is not
of tracking type and therefore \( \Omega_{\text{DE}}(z_*) \ll \Omega_m(z_*) \). In
order to obtain an analytic solution, we will also approxi-
mate the Hubble parameter as \( H \approx H_0 \sqrt{\Omega_m(1 + z)} \),
which is a reasonable approximation during matter
domination. Putting the above together, we obtain from con-
dition (11)

\[
c_s \gtrsim \sqrt{\Omega_m} \frac{H_0}{k_{\text{min}}} \left( \frac{\mu_0}{\epsilon} \right)^{1/(s + \nu)} (1 + z_{\text{surv}})^{\nu/(s + \nu)}.
\]

(14)

This should be thought of as an order-of-magnitude con-
servative approximation, which should be compared with
numerical results in \[4\]. We can easily see that if the QS
solution is to be good enough, we need higher sound
speeds for high-redshift surveys, large maximum length-
scales surveyed, larger deviations from \( \mu_0 = 0 \) and more
slowly evolving effective Newton’s constants, just as one

would naively expect. The minimum sound speed is
weakly dependent on the survey redshift, but strongly on
the lowest mode \( k_{\text{min}} \) being used in the analysis. Eq. (13)
is the main result of this paper, allowing for a simple es-
timate as to whether the QS approximation can be used
for a given dataset and DE model.

In table \[8\] we have summarised a selection of surveys
and the constraints on the sound speed given the require-
ment of quasi-staticity. It shows that today’s surveys are
well inside the region where the QS limit is valid for mod-
els with sound speed \( c_s > 0.02 \). There is an interesting
effect whereby, for the surveys centred on lower redshifts,
such as BOSS, the QS approximation is better than ex-
pected since the comoving horizon shrinks for \( z < 0.65 \)
and therefore the modes \( k_{\text{min}} \) entered the sound hori-
zon at higher redshift and are already leaving it at the
time of the survey. Once Euclid data are available, the
quasi-static approximation will only be appropriate for
dark-energy models with sound speeds close to that of
light.

On the other hand, the integrated Sachs-Wolf (ISW)
effect has its kernel peaked at approximately \( k = 0.004 \text{ Mpc}^{-1} \) for \( \ell = 20 \) and \( k = 0.0005 \text{ Mpc}^{-1} \) for \( \ell = 2 \)
(see e.g. \[40\], fig. 4). Thus one should not use the QS
solution at all if the sound speed of the DE is subluminal.
On the other hand, the impact on parameter estimation
is limited because of cosmic variance at such scales.

VI. DISCUSSION AND CONCLUSIONS

Adopting the QS approximation essentially means that
the DE degree of freedom with a finite propagation speed
is replaced by a modification of the constraints of gen-
eral relativity, i.e. with instantaneous propagation. The
advantage is that the dynamics are much simpler and
expressible in terms of simple parameters. However,
this approximation must fail whenever causality places
a limit, i.e. beyond the dark energy sound horizon. We
have shown that for current surveys, the QS limit is suf-
cient in the analysis whenever the DE model has sound
speed \( c_s \gtrsim 0.02 \). As we enter the era of much wider
surveys, such as Euclid, this approximation could be-
come inappropriate even for models with sound speeds
close to those of light and could result in misleading con-
straints, both because the sound horizon is too close to
the scales involved in the analysis and because the close-
ness of the cosmological horizon invalidates the QS ap-
proximation. Moreover, the ISW effect probes the univer-
sal at the largest scales already today and the DE QS
limit is inappropriate for models with sound speed below
that of light. We readily admit, however, that, at the
largest scales, cosmic variance is large and the impact of
a mismodelling of these scales on parameter estimation
might always be small.

On the other hand, most of the well-studied models

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3 Allowing for redshift dependence of \( \mu_\delta \) does not change \( A_1 \) in
eq. (8), which is all that is necessary for the discussion here.
of dark energy, e.g. quintessence and $f(R)$, have sound speed equal to that of light. For these, using the QS approximation to analyse surveys is valid, as shown in the literature previously. Thus the effects such as a scale-dependent Newton’s constant in $f(R)$ gravity when $M^2 \gg k^2/a^2$ do exist and might possibly be observed [9].

We have provided condition [13] as a simple test allowing us to determine whether the use of the QS approximation is valid when we use observational data to test models of gravity beyond the simplest ones such as $f(R)$ and quintessence — for example, those defined by their EFT parameters. As we reach larger scales in observations in the near future with Euclid, LSST and the SKA, an EFT-like formulation including all the time-domain behaviour will increasingly become necessary to connect consistently initial conditions at large scales (e.g. the microwave background) with late-time observables.

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Table I. Minimal dark energy sound speed $c_s$ which allows for the use of the quasi-static approximation for given example survey parameters. Current surveys can be safely analysed using the QS solution for $c_s \geq 0.02$ with accuracy better than $\epsilon/\mu_0 = 1\%$. Future surveys will probe much closer to the cosmological horizon and this sort of accuracy can only be achieved for $c_s \geq 0.1$, depending on the largest modes included in the analysis. When modes comparable to the horizon are included, the QS limit should not be used at all.

| Survey          | BOSS CFHTLenS | DES Euclid |
|-----------------|---------------|------------|
| $z_{\text{surv}}$ | 0.57          | 0.75       |
| $k_{\min}/h\text{Mpc}^{-1}$ | 0.04          | 0.2       |
| $k_{\min}/H(z_{\text{surv}})$ | 140          | 700       |

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