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Transverse Patterns in Nonlinear Optical Resonators

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Abstract

This book is devoted to the formation and dynamics of localized structures (vortices, solitons) and of extended patterns (stripes, hexagons, tilted waves) in nonlinear optical resonators such as lasers, optical parametric oscillators, and photorefractive oscillators. Theoretical analysis is performed by deriving order parameter equations, and also through numerical integration of microscopic models of the systems under investigation. Experimental observations, and possible technological implementations of transverse optical patterns are also discussed. A comparison with patterns found in other nonlinear systems, i.e. chemical, biological, and hydrodynamical systems, is given.

Keywords: Pattern formation, Spatial solitons, Optical vortices, Nonlinear optics

The first chapter of the book (the introduction) and the table of contents are given in this article. The full text of the book is available at:

http://springeronline.com
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1 Introduction

Pattern formation, i.e. the spontaneous emergence of spatial order, is a widespread phenomenon in nature, and also in laboratory experiments. Examples can be given from almost every field of science, some of them very familiar, such as fingerprints, the stripes on the skin of a tiger or zebra, the spots on the skin of a leopard, the dunes in a desert, and some others less evident, such as the convection cells in a fluid layer heated from below, and the ripples formed in a vertically oscillated plate covered with sand [1].

All these patterns have something in common: they arise in spatially extended, dissipative systems which are driven far from equilibrium by some external stress. “Spatially extended” means that the size of the system is, at least in one direction, much larger than the characteristic scale of the pattern, determined by its wavelength. The dissipative nature of the system implies that spatial inhomogeneities disappear when the external stress is weak, and the uniform state of the system is stable. As the stress is increased, the uniform state becomes unstable with respect to spatial perturbations of a given wavelength. In this way, the system overcomes dissipation and the state of the system changes abruptly and qualitatively at a critical value of the stress parameter. The very onset of the instability is, however, a linear process. The role of nonlinearity is to select a concrete pattern from a large number of possible patterns.

These ingredients of pattern-forming systems can be also found in many optical systems (the most paradigmatic example is the laser), and, consequently, formation of patterns of light can also be expected. In optics, the mechanism responsible for pattern formation is the interplay between diffraction, off-resonance excitation and nonlinearity. Diffraction is responsible for spatial coupling, which is necessary for the existence of nonhomogeneous distributions of light.

Some patterns found in systems of very different nature (hydrodynamic, chemical, biological or other) look very similar, while other patterns show features that are specific to particular systems. The following question then naturally arises: which peculiarities of the patterns are typical of optics only, and which peculiarities are generic? At the root of any universal behavior of pattern-forming systems lies a common theoretical description, which is independent of the system considered. This common behavior becomes evident
after the particular microscopic models have been reduced to simpler models, called order parameter equations (OPEs). There is a very limited number of universal equations which describe the behavior of a system in the vicinity of an instability; these allow understanding of the patterns in different systems from a unified point of view.

The subject of this book is transverse light patterns in nonlinear optical resonators, such as broad-aperture lasers, photorefractive oscillators and optical parametric oscillators. This topic has already been reviewed in a number of works [2–10]. We treat the problem here by means of a description of the optical resonators by order parameter equations, reflecting the universal properties of optical pattern formation.

1.1 Historical Survey

The topic of optical pattern formation became a subject of interest in the late 1980s and early 1990s. However, some hints of spontaneous pattern formation in broad-aperture lasers can be dated to two decades before, when the first relations between laser physics and fluids/superfluids were recognized [11]. The laser–fluid connection was established by reducing the laser equations for the class A case (i.e. a laser in which the material variables relax fast compared with the field in the optical resonator) to the complex Ginzburg–Landau (CGL) equation, used to describe superconductors and superfluids. In view of this common theoretical description, it could then be expected that the dynamics of light in lasers and the dynamics of superconductors and superfluids would show identical features.

In spite of this insight, the study of optical patterns in nonlinear resonators was abandoned for a decade, and the interest of the optical community turned to spatial effects in the unidirectional mirrorless propagation of intense light beams in nonlinear materials. In the simplest cases, the spatial evolution of the fields is just a filamentation of the light in a focusing medium; in more complex cases, this evolution leads to the formation of bright spatial solitons [12]. The interest in spatial patterns in lasers was later revived by a series of works. In [13, 14], some nontrivial stationary and dynamic transverse mode formations in laser beams were demonstrated. It was also recognized [15] that the laser Maxwell–Bloch equations admit vortex solutions. The transverse mode formations in [13, 14], and the optical vortices in [15] were related to one another, and the relation was confirmed experimentally (Fig. 1.1) [16, 17]. The optical vortices found in lasers are very similar to the phase defects in speckle fields reported earlier [18, 19].

The above pioneering works were followed by an increasing number of investigations. Efforts were devoted to deriving an order parameter equation for lasers and other nonlinear resonators; this would be a simple equation capturing, in the lowest order of approximation, the main spatio-temporal properties of the laser radiation. The Ginzburg–Landau equation, as derived
in [11], is just a very simple model equation for lasers with spatial degrees of freedom. Next, attempts were made to derive a more precise order parameter equation for a laser [15, 20], which led to an equation valid for the red detuning limit. Red detuning means that the frequency of the atomic resonance is less than the frequency of the nearest longitudinal mode of the resonator. This equation, however, has a limited validity, since it is not able to predict spontaneous pattern formation: the laser patterns usually appear when the cavity is blue-detuned. Depending on the cavity aperture, higher-order transverse modes [17] or tilted waves [21] can be excited in the blue-detuned resonator.

The problem of the derivation of an order parameter equation for lasers was finally solved in [22, 23], where the complex Swift–Hohenberg (CSH) equation was derived. Compared with the Ginzburg–Landau equation, the CSH equation contains additional nonlocal terms responsible for spatial mode selection, thus inducing a pattern formation instability. Later, the CSH equation for lasers was derived again using a multiscale expansion [24]. The CSH equation describes the spatio-temporal evolution of the field amplitude. Also, an order parameter equation for the laser phase was obtained, in the form of the Kuramoto–Sivashinsky equation [25]. It is noteworthy that both the Swift–Hohenberg and the Kuramoto–Sivashinsky equations appear frequently in the description of hydrodynamic and chemical problems, respectively.

The derivation of an order parameter equation for lasers means a significant advance, since it allows one not only to understand the pattern formation mechanisms in this particular system, but also to consider the broad-aperture laser in the more general context of pattern-forming systems in nature [1].
The success in understanding laser patterns initiated a search for spontaneous pattern formation in other nonlinear resonators. One of the most extensively studied systems has been photorefractive oscillators, where the theoretical background was set out [26], complicated structures experimentally observed [27, 28] and order parameter equations derived [29]. Intensive studies of pattern formation in passive, driven, nonlinear Kerr resonators were also performed [30–33]. Also, the patterns in optical parametric oscillators received a lot of attention. The basic patterns were predicted [34, 35], and order parameter equations were derived in the degenerate [36, 37] and nondegenerate [38] regimes. The connection between the patterns formed in planar- and curved-mirror resonators was treated in [39], where an order parameter equation description of weakly curved (quasi-plane) nonlinear optical resonators was given.

These are just a few examples. In the next section, the general characteristics of nonlinear resonators, and the state of the art are reviewed.

1.2 Patterns in Nonlinear Optical Resonators

The patterns discussed in the main body of the book are those appearing in nonlinear optical resonators only. This particular configuration is characterized by (1) strong feedback and (2) a mode structure, both due to the cavity. The latter also implies temporal coherence of the radiation. Thanks to the feedback, the system does not just perform a nonlinear transformation of the field distribution, where the fields at the output can be expressed as some nonlinear function of the fields at the input and of the boundary conditions. Owing to the feedback, the system can be considered as a nonlinear dynamical system with an ability to evolve, to self-organize, to break spontaneously the spatial translational symmetry, and in general, to show its “own” distributions not present in the initial or boundary conditions.

Nonlinear optical resonators can be classified in different ways: by the resonator geometry (planar or curved), by the damping rates of the fields (class A, B or C lasers), by the field–matter interaction process (active and passive systems) and in other ways. After order parameter equations were derived for various systems, a new type of classification became possible. One can distinguish several large groups of nonlinear resonators, each of which can be described by a common order parameter equation:

1. Laser-like nonlinear resonators, such as lasers of classes A and C, photorefractive oscillators, and nondegenerate optical parametric oscillators. They are described by the complex Swift–Hohenberg equation,

$$\frac{\partial A}{\partial t} = (D_0 - 1) A - A |A|^2 + i (a \nabla^2 - \omega) A - (a \nabla^2 - \omega)^2 A,$$

and show optical vortices as the basic localized structures, and tilted waves and square vortex lattices as the basic extended patterns.
2. Resonators with squeezed phase, such as degenerate optical parametric oscillators and degenerate four-wave mixers. They are described, in the most simplified way, by the real Swift–Hohenberg equation,

\[ \frac{\partial A}{\partial t} = (D_0 - 1) A - A^3 + (a\nabla^2 - \omega)^2 A , \]

and show phase domains and phase solitons as the basic localized structures, and stripes and hexagons as the basic extended patterns.

3. Lasers with a slow population inversion \( D \) (class B lasers). They cannot be described by a single order parameter equation, but can be described by two coupled equations,

\[ \frac{\partial A}{\partial t} = (D - 1) A + i (a\nabla^2 - \omega) A - (a\nabla^2 - \omega)^2 A , \]

\[ \frac{\partial D}{\partial t} = -\gamma \left( D - D_0 + |A|^2 \right) , \]

and their basic feature is self-sustained dynamics, in particular the “restless vortex”.

4. Subcritical nonlinear resonators, such as lasers with intracavity saturable absorbers or optical parametric oscillators with a detuned pump. The effects responsible for the subcriticality give rise to additional terms in the order parameter equation, which in general has the form of a modified Swift–Hohenberg equation,

\[ \frac{\partial A}{\partial t} = F \left( D_0, A, |A|^n, \nabla^2 \right) + i (a\nabla^2 - \omega) A - (a\nabla^2 - \omega)^2 A , \]

where \( F \) represents a nonlinear, nonlocal function of the fields. Its solutions can show bistability and, as consequence, such systems can support bistable bright spatial solitons.

This classification is used throughout this book as the starting point for studies of pattern formation in nonlinear optical resonators. The main advantage of this choice is that one can investigate dynamical phenomena not necessarily for a particular nonlinear resonator, but for a given class of systems characterized by a common order parameter equation, and consequently by a common manifold of phenomena.

In this sense, the patterns in nonlinear optics can be considered as related to other patterns observed in nature and technology, such as in Rayleigh–Bénard convection [40], Taylor–Couette flows [41], and in chemical [42] and biological [43] systems. The study of patterns in nonlinear resonators has been strongly influenced and profited from the general ideas of Haken’s synergetics [44] and Prigogine’s dissipative structures [45, 46]. On the other hand, the knowledge achieved about patterns in nonlinear resonators provides feedback to the general understanding of pattern formation and evolution in nature.

Next we review the basic transverse patterns observable in a large variety of optical resonators. It is convenient to distinguish between two kinds of
patterns: localized structures, and extended patterns in the form of spatially periodic structures.

### 1.2.1 Localized Structures: Vortices and Solitons

A transverse structure which enjoys great popularity and on which numerous studies have been performed, is the optical vortex, a localized structure with topological character, which is a zero of the field amplitude and a singularity of the field phase.

Although optical vortices have been mainly studied in systems where free propagation occurs in a nonlinear material (see Sect. 1.3), some works have treated the problem of vortex formation in resonators. As mentioned above, the early studies of these fascinating objects [15–19] strongly stimulated interest in studies of pattern formation in general. The existence of vortices indicates indirectly the analogy between optics and hydrodynamics [22, 47–49]. It has been shown that the presence of vortices may initiate or stimulate the onset of (defect-mediated) turbulence [27, 50–53]. Vortices may exist as stationary isolated structures [54, 55] or be arranged in regular vortex lattices [17, 23, 28]. Also, nonstationary dynamics of vortices have been reported, both of single vortices [56, 57] and of vortex lattice structures [58]. Recently, optical vortex lattices have been experimentally observed in microchip lasers [59].

Another type of localized structure is spatial solitons, which are non-topological structures. Although such structures do not appear exclusively in optical systems [60–62], they are now receiving tremendous interest in the field of optics owing to possible technological applications. A spatial soliton in a dissipative system, being bistable, can carry a bit of information, and thus such solitons are very promising for applications in parallel storage and parallel information processing.

Spatial solitons excited in optical resonators are usually known as cavity solitons. Cavity solitons can be classified into two main categories: amplitude (bright and dark) solitons, and phase (dark-ring) solitons. Investigations of the formation of bright localized structures began with early work on bistable lasers containing a saturable absorber [63, 64] and on passive nonlinear resonators [65].

Amplitude solitons can be excited in subcritical systems under bistability conditions, and can be considered as homoclinic connections between the lower (unexcited) and upper (excited) states. They have been reported for a great variety of passive nonlinear optical resonators, such as degenerate [66–68] and nondegenerate [69, 70] optical parametric oscillators, and for second-harmonic generation [71–73] (Fig. 1.2), where the bistability was related to the existence of a nonlinear resonance [37]. In some systems, the interaction of solitons and their dynamical behavior have been studied [73–75]. Resonators containing Kerr media also support amplitude solitons, as a result of either Kerr [76] or polarization (vectorial) [77] instabilities.
In active systems, bright solitons have been demonstrated in photorefractive oscillators [78–80] and in lasers containing saturable absorbers [81, 82] or an intracavity Kerr lens [83]. A promising system for practical applications is the vertical cavity surface emission laser (VCSEL), which forms a microresonator with a semiconductor as a nonlinear material. The theoretically predicted patterns for this system [84–89] were recently experimentally confirmed in [90].

The required subcriticality condition is usually achieved by introducing an intracavity absorbing element. However, recently, stable solitons in the absence of an additional medium have been reported in cascade lasers [91].

Besides the amplitude solitons in subcritical nonlinear resonators, a different type of bistable soliton exists in supercritical resonators. Such systems are characterized by a broken phase symmetry of the order parameter, and solutions with only two possible phase values are allowed. In this case the solitons connect two homogeneous solutions of the same amplitude but of opposite phase. Such phase solitons, which are round, stable phase domains of minimum size, appear as a dark ring on a bright background. This novel type of optical soliton is now receiving a lot of interest, since these solitons are seemingly much easier to realize experimentally than their bright counterparts in subcritical systems.

One of the systems most investigated has been the degenerate optical parametric oscillator (DOPO), either in the one-dimensional case [92, 93] or in the more realistic case of two transverse dimensions [94–97]. Also, the soliton formation process [98–100] and its dynamical behavior [101, 102] have been analyzed. Optical bistability in a passive cavity driven by a coherent external field is another example of a system supporting such phase solitons [103–107]. Both the DOPO and systems showing optical bistability are systems described by a common order parameter equation, the real Swift–Hohenberg

Fig. 1.2. Interaction of two moving amplitude solitons in vectorial intracavity second-harmonic generation: (a) central collision, (b) noncentral collision. From [73], ©1998 American Physical Society
equation [108]. Systems with a higher order of nonlinearity, such as vectorial Kerr resonators, have also been shown to support phase solitons [109–111].

Phase solitons can form bound states, resulting in soliton aggregates or clusters [94, 112]. Phase solitons in a cavity are seemingly much easier to excite than their counterparts in subcritical systems. In fact, such phase solitons have already been experimentally demonstrated in degenerate four-wave mixers [113–115] (Figs. 1.3 and 1.4).

1.2.2 Extended Patterns

Besides the localized patterns, vortices and solitons, to which the book is mainly devoted, extended patterns in optical resonators have been also extensively studied. In optical resonators, two main categories of patterns can be distinguished. One class of patterns appears in low-aperture systems, characterized by a small Fresnel number, such as a laser with curved mirrors. Since this is the most typical configuration of an optical cavity, this phenomenon
was observed in the very first experimental realizations, although a systematic study was postponed to a later time [16]. The patterns of this kind are induced by the boundary conditions, and can be interpreted as a weakly nonlinear superposition of a small number of cavity modes of Gauss–Hermite or Gauss–Laguerre type.

Theoretical predictions based on modal expansions of the field [14, 116, 117] have been confirmed by a large number of experiments, some of them reported in [118–122]. Owing to the particular geometry of the cavity, this kind of pattern is almost exclusively optical. If the aperture is increased, the number of cavity modes excited can grow, and so the spatial complexity of the pattern grows [123].

The other class of extended optical patterns is typical of large-aperture resonators, formed by plane mirrors in a ring or a Fabry–Pérot configuration. The transverse boundary conditions have a weak influence on the system dynamics, in contrast to what happens in small-aperture systems. Consequently, the patterns found in these systems are essentially nonlinear, and the system dynamics can be reduced to the evolution of a single field, called the order parameter.

The simplest patterns in these systems consist of a single tilted or traveling wave (TW), which is the basic transverse solution in a laser [21], although more complex solutions formed by several TWs have been found [124, 125]. The predicted laser TW patterns have been observed in experiments with large-Fresnel-number cavities [126–128]. The TW solutions are also found in passive resonators described by the same order parameter equation, such as nondegenerate optical parametric oscillators (OPOs) [35, 129]. The effect of an externally injected signal in a laser has been also studied [130, 131], showing the formation of more complex patterns, such as rolls or hexagons.

Roll, or stripe, patterns are commonplace for a large variety of nonlinear passive cavities, such as degenerate OPOs [34], four-wave mixers [37], systems showing optical bistability [31, 132] and cavities containing Kerr media [133]. Patterns with hexagonal symmetry are also frequently found in such resonators [134, 135]. Both types of pattern are familiar in hydrodynamic systems, such as systems showing Rayleigh–Bénard convection.

Another kind of traveling solution existing in optical resonators corresponds to spiral patterns, such as those found in lasers [136, 137] and in OPOs [138, 139], which are typical structures in chemical reaction–diffusion systems.

When more complex models, including additional effects are considered, a larger variety of patterns, sometimes of exotic appearance, is found. Some such models generalize the above cited models by considering the existence of competition between different parametric processes [140, 141] or between scalar and vectorial instabilities [142], the walk-off effect due to birefringence in the medium [143–145], or external temporal variation of the cavity parameters [146].
Some systems allow the simultaneous excitation of patterns with different wavenumbers. These systems form patterns with different periodicities that have been called quasicrystals [147, 148] and daisy patterns [149] (Fig. 1.5).

The experimental conditions for large-aspect-ratio resonators are not easy to achieve. Most of the experiments performed have studied multimode regimes involving high-order transverse modes. The formation of the patterns described above was reported in lasers [126–128] and OPOs [151, 152]. The observed patterns correspond well to the numerical solutions of large-aspect-ratio models. Conditions for boundary-free, essentially nonlinear patterns were obtained in [78,153] with the use of self-imaging resonators, which allowed the experimenters to obtain Fresnel numbers of arbitrarily high value.

All the patterns reviewed above are two-dimensional, the light being distributed in the transverse space perpendicular to the resonator axis, and evolving in time. Recently, the possibility of three-dimensional patterns was demonstrated for OPOs [154], nonlinear resonators with Kerr media [155, 156], optical bistability [157] and second-harmonic generation [158].

Finally, the problem of the effect of noise on the pattern formation properties of a nonlinear resonator has also been treated. One can expect that noise, which is present in every system, will bring about new features in the spatio-temporal dynamics of the system. First, noise can modify (shift) the threshold of pattern formation [159]. Second, owing to noise, the precursors of patterns can be seen below the pattern formation threshold [160–162]. While a noiseless pattern-forming system below the pattern formation thresh-

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**Fig. 1.5.** Experimentally observed hexagonal patterns with sixfold and twelvefold symmetry (quasipatterns), in a nonlinear optical system with continuous rotational symmetry. From [150], ©1999 American Physical Society.
1.3 Optical Patterns in Other Configurations

In parallel with the studies on nonlinear resonators, pattern formation problems have been considered in other optical configurations. These configurations can be divided into the following categories, according to their geometry and complexity.

1.3.1 Mirrorless Configuration

When an intense light beam propagates in a nonlinear medium, it can experience filamentation effects, leading to periodic spatial distributions [168], or develop into self-trapped states of light, or solitons. The self-focusing action of the nonlinearity compensated by diffraction results in self-sustained bright spatial solitons [12], which can exist as isolated states or form complex ensembles, sometimes interacting in a particle-like fashion [169–175]. Also, dark solitons [176–184] and optical vortices [185–196] have been described and experimentally observed. In such a mirrorless configuration feedback is absent, and one obtains not a spontaneous pattern formation, but just a nonlinear transformation of the input distribution. This nonlinear transformation can be very complicated, and can be described by complicated integro-differential equations. However, every transformation remains a transformation, and without feedback it does not lead to spontaneous pattern formation. Some other mirrorless schemes, where optical pattern formation has been predicted, are based on the interaction of two counterpropagating pumping waves in a nonlinear medium. It has been shown that the waves that appear through nonlinear mixing processes have their lowest threshold at certain angles with respect to the pumping waves, and may result in a wide variety of patterns, either extended, such as rolls or hexagons [197–204](Fig. 1.6), or localized [205]. Experimental confirmation has been obtained using various nonlinear media, such as atomic vapors and photorefractive crystals.

1.3.2 Single-Feedback-Mirror Configuration

The presence of a mirror introduces feedback into the system. Unlike the case in the previous schemes, here nonlinearity and diffraction act at different spatial locations. The most typical configuration is formed by a thin slice
of a Kerr medium and a mirror at some distance. Theoretical studies have predicted structures mainly with hexagonal symmetry [206–209] (Fig. 1.7), although more complex solutions have been found [210,214]. From the experimental side, various nonlinear media, such as atomic vapors [211,212], and Kerr [213] and photorefractive [214] media have been used successfully. Also, this configuration led to the first realization of localized structures in nonlinear optics [215]. The dynamics and interaction of these localized structures have been extensively investigated [216–219] (Fig. 1.8).
1.3 Optical Patterns in Other Configurations

Fig. 1.8. Dissipative solitons observed experimentally in sodium vapor with a single feedback mirror. From [219], ©2000 American Physical Society

Fig. 1.9. Experimental patterns in an optical system with two-dimensional feedback. (a) Hexagonal array, (b)–(d) “black-eye” patterns, (e) island of bright localized structures, (f) optical squirms. From [224], ©1998 American Physical Society

1.3.3 Optical Feedback Loops

Another configuration, somewhat between the single feedback mirror and the nonlinear resonator, is the feedback loop. In such a configuration, one has the possibility of acting on the field distribution on every round trip through the loop, continuously transforming the pattern distribution. Some typical two-dimensional transformations are the rotation, translation, scaling and filtering of the pattern. The first work obtained pattern formation by controlling the spatial scale and the topology of the transverse interaction of the light field in a medium with cubic nonlinearity [220–222], by controlling the phase of the field with a spatial Fourier filter [223, 224] (Figs. 1.9 and 1.10), and by introducing a medium with a binary-type refractive nonlinear response [225].
A very versatile system is a feedback loop with a liquid-crystal light valve acting as a phase modulator with a Kerr-type nonlinearity. The conversion from a phase to an intensity distribution can be performed by two means: by free propagation (diffractive feedback) or by interference with reflected waves (interferential feedback), as shown in Fig. 1.11. In both cases, a great variety of kaleidoscope-like patterns have been obtained theoretically and experimentally. The patterns can also be controlled by means of nonlocal interactions, via rotation or translation of the signal in the feedback loop, giving rise to more exotic solutions such as quasicrystals and drifting patterns. The existence of spatial solitons and the formation of bound states of solitons have also been reported experimentally in the liquid-crystal light valve system, as shown in Fig. 1.13.
1.4 The Contents of this Book

In Chaps. 2 and 3, the order parameter equations for broad-aperture lasers and for other nonlinear resonators are obtained. These chapters are relatively mathematical; however, the OPEs derived here pave the way for the subsequent chapters of the book. The derivation of the OPEs for class A and class C lasers is given in Chap. 2. For completeness, two techniques of derivation are given: one based on the adiabatic elimination of the fast variables, and one based on multiscale expansion techniques. Both procedures lead to the complex Swift–Hohenberg equation as the OPE for lasers.
The CSH equation describes the spatio-temporal dynamics of the complex-valued order parameter, which is proportional to the envelope of the optical field. In Chap. 3, the OPEs for optical parametric oscillators and photorefractive oscillators (PROs) are derived. In the degenerate case, the resulting equation is shown to be the real Swift–Hohenberg equation, first obtained in a hydrodynamic context. For large pump detuning values, a generalized model including nonlinear resonance effects is obtained. In the case of PROs, the adiabatic elimination technique is used to derive the CSH equation. The order parameter equations derived in Chaps. 2 and 3 divide nonlinear optical resonators into distinct classes, and thus allow one to study pattern formation phenomena without necessarily considering every nonlinear optical system separately; instead, one can consider classes of the systems.

Chapters 4 and 5 are devoted to the patterns of the first class of systems, that described by the CSH equation, i.e. lasers, photorefractive oscillators and nondegenerate OPOs. The localized patterns in this class of systems are optical vortices: these are zeros of the amplitude of the optical field, and are simultaneously singularities of the field phase. Optical vortices dominate the dynamics of the system in near-resonant cases (when the detuning is close to zero). The CGL equation in this near-resonant limit can be rewritten in a hydrodynamic form. Owing to this analogy between laser and hydrodynamics, the dynamics of the transverse distribution of the laser radiation are very similar to the dynamics of a superfluid. It is shown that optical vortices of the same topological charge rotate around one another; a pair of vortices of the same charge translate in parallel through the aperture of the laser or annihilate, depending on the parameters.

In Chap. 5, the limit of large or moderate detuning is considered. The CSH equation cannot be rewritten in a hydrodynamic form, but the dynamics of the fields can still be well interpreted by hydrodynamic means. For large detuning, tilted waves are excited. In hydrodynamic terms, flows with a velocity of fixed magnitude but arbitrary direction are favored. This results, in particular, in counterpropagating flows separated by vortex sheets. This also leads to optical vortices advected by the mean flow, and similar phenomena. Such phenomena are analyzed theoretically and demonstrated numerically. A pattern of square symmetry, called a square vortex lattice, consisting of four counterpropagating flows in the form of a cross, is also described and discussed.

In Chap. 6, the effects of the curvature of the mirrors of the resonator are analyzed. The majority of theoretical investigations of pattern formation in nonlinear optics, including those in the largest part of this book, have been performed by assuming a plane-mirror cavity model. However, in experiments resonators with curved mirrors are often used. Therefore a model of a laser with curved mirrors is introduced. The presence of curved mirrors results in an additional term in the order parameter equation, proportional to the total curvature of the mirrors in the resonator. This term produces a coordinate-
dependent (parabolic) phase shift of the order parameter during propagation in the resonator. The presence of the parabolic potential allows one to expand the field of the resonator in terms of the eigenfunctions (transverse modes) of the potential. Although this mode expansion is strictly valid for linear resonators only, the nonlinearity in the resonator results in a weakly nonlinear coupling of the complex amplitudes of the modes. As a result, an infinite set of coupled ordinary differential equations for complex-valued mode amplitudes is derived. This gives an alternative way of investigating the transverse dynamics of a laser, by solving the equations for the mode amplitudes instead of solving the partial differential equations. The technique of mode expansion is shown to be extremely useful when one is dealing with a small number of transverse modes. In particular, the transverse dynamics of class A lasers and photorefractive oscillators are considered; the phenomena of transverse mode pulling and locking are observed. Chapter 6 also deals with degenerate resonators, such as self-imaging and confocal resonators. In such resonators, the longitudinal mode separation is an integer multiple of the transverse mode separation. It is shown, by analysis of the corresponding ABCD matrices, that self-imaging resonators are equivalent to planar resonators of zero length. This insight opened up new possibilities for experimenting with transverse patterns in nonlinear optical systems, and allowed the first experimental realization of a number of phenomena predicted theoretically for nonlinear resonators.

Chapter 7 deals with patterns in class B lasers. Class B lasers are not describable by the CSH equation. Owing to the slowness of the population inversion, the order parameter equation in this case is not a single equation belonging to one of the classes defined above, but a system of two coupled equations, resembling those derived for excitatory or oscillatory chemical systems, where the (slow) population inversion plays the role of the recovery variable, and the fast optical field plays the role of the excitable variable. An analysis of such self-sustained spatio-temporal dynamics in a class B laser is performed. The vortices, which are stationary in a class A laser, perform self-sustained meandering in a class B laser, a phenomenon known as the “restless vortex”. Also, vortex lattices experience self-sustained oscillatory dynamics. Either the vortices in the lattice oscillate in such a way that neighboring vortices rotate in antiphase, thus resulting in an “optical” mode of vortex lattice oscillation, or the vortex lattice drifts spontaneously with a well-defined velocity, thus resulting in an “acoustic” oscillation mode.

The following chapters, Chaps. 8 to 11, are devoted to amplitude and phase domains, as well as amplitude and phase solitons in bistable nonlinear optical systems. The general theory of subcritical spatially extended systems is developed in Chap. 8, where two mechanisms of creation of subcriticality in optical resonators are described: one due to the presence of a saturable absorber, and one due to the presence of a nonlinear resonance. A discussion in terms of order parameter equations is given.
In Chap. 9, a theoretical description and experimental evidence of domain dynamics and spatial solitons in lasers containing a saturable absorber are presented. Two different resonator configurations are used: a self-imaging resonator where both nonlinearities (due to the gain and to saturable absorption) are placed at the same location on the optical axis of the resonator, and a self-imaging resonator where the two nonlinearities are placed at Fourier-conjugated locations. For spatially coincident nonlinearities, the evolution of domains is demonstrated numerically and experimentally, with the eventual appearance of spatial solitons. For nonlinearities placed in conjugate locations in the resonator, the competition, mutual interaction and drift of solitons are investigated, also both theoretically and experimentally.

In Chap. 10, a subcriticality mechanism different from saturable absorption is studied, in this case related to the existence of a nonlinear resonance due to nonresonant pumping. As an example, the order parameter equation obtained in Chap. 3 for a degenerate OPO with a detuned pump is considered. The nonlinear resonance implies that the pattern wavenumber depends on the intensity of the radiation. With appropriate values of the detuning, the nonlinear resonance can lead to bistability, and thus allow the excitation of amplitude domains and spatial solitons. Numerical results from the DOPO mean-field model are given for comparison.

In Chap. 11, the dynamics of phase domains in supercritical real-valued order parameter systems, such as the degenerate OPO, are analyzed. These systems should properly be described by the real Swift–Hohenberg equation. It is demonstrated that the domain boundaries, the lines of zero intensity separating domains of opposite phase, may contract or expand depending on the value of the resonator detuning. In this way, the domain boundaries behave as elastic ribbons, with the elasticity coefficient depending on the detuning. Contracting domains, observed for small values of the detuning, eventually disappear. Expanding domains are found for large values of the detuning, and their evolution results in labyrinthine structures. For intermediate values of the detuning, the contracting domain boundaries stop contracting at a particular radius. The latter scenario results in stable rings of domain boundaries, which are phase solitons. The experimental confirmation of the predicted phenomena is described.

In Chap. 12, the Turing pattern formation mechanism, typical of chemical reaction–diffusion systems, is shown to exist also in nonlinear optics. The pattern formation mechanism described in most of the chapters of the book is based on an off-resonance excitation. The Turing mechanism, however, is based on the interplay between the diffusion and/or diffraction of interacting components. In particular, the emergence of Turing-like patterns is predicted to occur in active and passive systems, concrete examples being lasers with a strongly diffusing population inversion, and degenerate OPOs with a strongly diffracting pump wave. In both cases, one field plays the role of activator, and the other the role of inhibitor. It is also shown that the
effect of diffusion and/or diffraction contributes to the stabilization of spatial solitons and allows the existence of complex states resembling molecules of light.

In Chap. 13, we describe the three-dimensional structures of light predicted to occur in resonators described by the three-dimensional Swift–Hohenberg equation. This order parameter equation describes a class of non-linear optical resonators including the synchronously pumped OPO. Various structures embedded in the envelopes of spatio-temporal light pulses are discussed, in the form of extended patterns (lamellae and tetrahedral patterns), light bubbles (the analogue of the phase solitons in two dimensions) and vortex rings. These structures exist when the OPO resonator length is matched to the length of the pump (mode-locked) laser, which emits a continuous or finite train of picosecond pulses. A three-dimensional modulation can develop on the subharmonic pulses generated, depending on several parameters such as the detuning from the resonance of the OPO cavity, and the mismatch of the resonator lengths for the pump and OPO lasers.

The final chapter, Chap. 14, deals with the influence of noise on spatial structures in nonlinear optics. Noise, which is not considered in the rest of the book, is always present in a real experiment, in the form of vacuum noise (always inevitable) or noise due to technological limitations. It is shown that the noise affects the pattern formation in several ways. Above the modulation instability threshold, where extended patterns are expected, the noise destroys the long-range order in the pattern. Rolls and other extended structures still exist in the presence of noise, but they may display defects (such as dislocations and disclinations) with a density proportional to the intensity of the noise. Also, below the modulation instability threshold, where no patterns are expected in the ideal (noiseless) case, the noise is amplified and can result in (noisy) patterns. The symmetry of a pattern may show itself even below the pattern formation threshold, thanks to the presence of noise. This can be compared with a single-transverse-mode laser, where the coherence in the radiation develops continuously, and where the spectrum of the luminescence narrows continuously when the generation threshold is approached from below.

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