Boundary states in the Nappi-Witten model

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Abstract

We investigate D-branes in the Nappi-Witten model. Classically symmetric D-branes are classified by the (twisted) conjugacy classes of the Nappi-Witten group, which specify the geometry of the corresponding D-branes. Quantum description of the D-branes is given by boundary states, and we need one point functions of closed strings to construct the boundary states. We compute the one point functions solving conformal bootstrap constraints, and check that the classical limit of the boundary states reproduces the geometry of D-branes.

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1 Introduction

String theory on backgrounds of pp-wave type attracts much attention recently. Geometry of pp-wave type appears in a (pp-wave) limit of Anti-de Sitter (AdS) space and strings on pp-wave background may be solvable. Many works on the string theory have been done after the authors [1] applied this fact to the AdS/CFT correspondence. The spacetime of $AdS_5 \times S^5$ with RR-flux reduces to the maximally supersymmetric pp-wave [2] by the pp-wave limit, and the string theory on the pp-wave background is solvable in the light-cone gauge [3, 4] even with non-trivial RR-flux. In Ref. [1] they compared almost BPS operators in $\mathcal{N} = 4$ super Yang-Mills theory and closed strings on the pp-wave background. D-branes in this background have been also investigated by many authors, for example, in Refs. [5, 6, 7, 8, 9, 10, 11]. In particular, the boundary states are constructed in Refs. [12, 13, 14, 15] using the light-cone gauge.

The Nappi-Witten model [16] is a Wess-Zumino-Witten (WZW) model associated with 4 dimensional Heisenberg group $H_4$, whose target space is 4 dimensional pp-wave with NSNS-flux. The pp-wave limit of $AdS_3 \times S^3$ with NSNS-flux is 6 dimensional generalization of the Nappi-Witten model, and we can also apply this model to the AdS/CFT correspondence [17, 18, 19, 20, 21, 22, 23]. Because the Nappi-Witten model is a WZW model, we can do more than in the case of the pp-waves with RR-flux. The model can be solved without taking the light-cone gauge, and the correlation functions are obtained in Refs. [24, 25, 26] recently. The model itself is also very interesting apart from the application to the AdS/CFT correspondence since it is an example which can be solved and has non-trivial Lorentzian target space-time. In many cases non-trivial Lorentzian theory is defined by analytic continuation of Euclidean theory which may be solvable. In the Nappi-Witten model, however, we can solve the model directly with the Lorentzian signature, and there is no difficulty associated with analytic continuation.

In this paper we investigate D-branes in the Nappi-Witten model. For a non-rational conformal field theory, it is very difficult to solve the theory generally, and in particular, boundary states are constructed only in few examples. Because the Nappi-Witten model is a solvable non-rational conformal field theory, it is worthwhile to construct boundary states also for the respect. We assume that the D-branes preserve the half of the symmetry of the current algebra, then the D-branes are classified by the (twisted) conjugacy classes [27, 28, 29]. After reviewing the geometry of the target space-time and the closed strings of the model in section 2, we examine the classical geometry of possible D-branes in section 3. In section 4 we compute disk one point functions of closed strings and construct

\[1\] For other types of D-branes in the pp-wave with NSNS-flux see Refs. [30, 31, 32, 33].
2 The Nappi-Witten Model

The Nappi-Witten model [16] is a WZW model based on 4 dimensional Heisenberg group $H_4$. The generators of $H_4$ Lie algebra have the following non-trivial commutation relations as

$$[J, P^\pm] = \mp iP^\pm, \quad [P^+, P^-] = -2iF.$$ \hfill (2.1)

A convenient way to parametrize the group element is

$$g(x^+, x^-, y) = e^{\frac{i}{2}x^+J} e^{\frac{i}{2}(y^+P^- + y^P^+) e^{\frac{i}{2}x^+J - 2x^F}},$$ \hfill (2.2)

where the group product is given by

$$g(x^+_1, x^-_1, y_1)g(x^+_2, x^-_2, y_2) = g(x^+_1 + x^+_2, x^-_1 + x^-_2 + \frac{1}{4} \text{Im}(y_1y_2^e^{\frac{i}{2}(x^+_1 + x^+_2)}), y_1 e^{\frac{i}{2}x^+_2} + w_2 e^{-\frac{i}{2}x^+_1}).$$ \hfill (2.3)

The group element leads to the metric of pp-wave type as

$$ds^2 = -2dx^+dx^- - \frac{1}{4}yy^*(dx^+)^2 + dydy^*.$$ \hfill (2.4)

Here we use the definition $ds^2 = \frac{1}{2}(g^{-1}dg, g^{-1}dg)$ with $\langle J, F \rangle = 1, \langle P^+, P^- \rangle = 2$.

The symmetry $g \rightarrow g_R^*gg_{L}$ on the metric is generated by the following differential operators as\(^3\)

$$J_L = \partial_{x^+} - \frac{i}{2}(y\partial_y - y^*\partial_{y^*}), \quad F_L = -\frac{i}{2}\partial_{x^-},$$

$$P_L^+ = -i\sqrt{2}e^{-i\frac{\pi}{4}}(\partial_{y^*} + \frac{1}{2}y\partial_{y^-}), \quad P_L^- = -i\sqrt{2}e^{-i\frac{\pi}{4}}(\partial_y - \frac{1}{4}y^*\partial_{x^-}),$$ \hfill (2.5)

and the right part with replacing $y \leftrightarrow y^*$. These operators are read from the group multiplication law \(^2\).

In the WZW model the symmetry of $H_4$ Lie algebra is enhanced to current algebra, whose generators have the operator product expansions (OPEs)

$$J(z)F(w) \sim \frac{1}{(z - w)^2};$$

$$J(z)P^\pm(w) \sim \mp iP^\pm(w) / (z - w),$$ \hfill (2.6)

$$P^\pm(z)P^-(w) \sim \frac{2}{(z - w)^2} - \frac{2iF(w)}{z - w}.$$
The mode expansions of the currents satisfy
\[ [J_m, F_n] = m \delta_{m+n,0} , \quad [J_m, P^\pm_n] = \mp i P^\pm_{m+n} , \quad [P^+_m, P^-_n] = 2m \delta_{m+n,0} - 2i F_{m+n} , \quad (2.7) \]
where the zero-mode subalgebra is \( H_4 \) Lie algebra. Anti-holomorphic (right-moving) currents are given in the similar way.

### 2.1 Primary fields

The general states in the WZW model can be constructed by
\[ J_{-n_1}^{A_1} \cdots J_{-n_l}^{A_l} |v\rangle , \quad J_{n}^{A_l} |v\rangle = 0 \quad (\forall \ n > 0) , \quad (2.8) \]
and tensoring the anti-holomorphic part. We use \( n_i > 0 \) and \( A_i = (J, F, \pm) \) with \( J^J = J, J^F = F, J^\pm = P^\pm \). Note that there is no singular vector in general. The state \( |v\rangle \) is a vacuum state labeled by the representation of the zero-mode subalgebra, whose irreducible unitary representations are summarized e.g. in Refs. [35, 36].

The vacuum state can be labeled by two eigenvalues as
\[ J_0 |j, \eta\rangle = ij |j, \eta\rangle , \quad F_0 |j, \eta\rangle = i\eta |j, \eta\rangle , \quad (2.9) \]
and \( P^+_0 \) and \( P^-_0 \) act on the eigenstates as the lowering and rising operators, respectively (see (2.1)). As in Refs. [35, 36], there are the following Hilbert spaces based on three types of unitary representation. One of them includes a lowest weight state \( P^+_0 |j, \eta\rangle = 0 \) and the other vacuum states are given by the action of \( P^-_0 \). The Hilbert space based on this representation is called as \( \mathcal{H}^+_j \) with \( j \in \mathbb{R} \) and \( 0 < \eta < 1 \). We will construct the states with \( \eta > 1 \) by applying the spectral flow in the next subsection. There is a similar Hilbert space \( \mathcal{H}^-_j \) including the highest weight state \( P^-_0 |j, \eta\rangle = 0 \), and the general vacuum states are generated by acting \( P^+_0 \) to the highest weight state. The labels of \( \mathcal{H}^-_j \) range \( j \in \mathbb{R} \) and \( -1 < \eta < 0 \). The conformal weights for these vacuum states are
\[ h^\pm = |\eta|^2 (1 - |\eta|) - j\eta . \quad (2.10) \]
The other Hilbert space \( \mathcal{H}^0_{j,s} \) does not include lowest nor highest weight state. The general vacuum states are generated by acting \( P^\pm_0 \) to a vacuum state with \( -1/2 < j \leq 1/2 \). The other parameter \( s (\geq 0) \) is related to the conformal weight as
\[ h^0 = \frac{s^2}{2} . \quad (2.11) \]
The eigenvalue of \( F_0 \) is zero (\( \eta = 0 \)).

In order to express the primary fields corresponding to the vacuum states, it is convenient to introduce the parameter \( x (\in \mathbb{C}) \) to sum up the representation of zero-mode
subalgebra. In this parametrization we can map the action of the currents to the primary fields into the differential operation as

\[ J^A(z)\Psi(w, x) \sim \frac{D^A_x \Psi(w, x)}{z - w}. \]  \hspace{1cm} (2.12)

Therefore, the constraint of the symmetry may be written in the form of differential equations, and in Ref. [24] the correlation functions are computed heavily using this property. The differential operators are [24]

\[ D^J_x = i(j + x \partial_x), \quad D^F_x = i\eta, \quad D^+_x = \sqrt{2}\eta x, \quad D^-_x = -\sqrt{2}\eta x. \]  \hspace{1cm} (η > 0)  \hspace{1cm} (2.13)

\[ D^J_x = i(j - x \partial_x), \quad D^F_x = i\eta, \quad D^+_x = \sqrt{2}\partial_x, \quad D^-_x = -\sqrt{2}\partial_x. \]  \hspace{1cm} (η < 0)  \hspace{1cm} (2.14)

For η = 0 we use a phase \( x = e^{i\alpha} \) and

\[ D^J_x = i(j - x \partial_x), \quad D^F_x = i\eta, \quad D^+_x = \sqrt{2}\partial_x, \quad D^-_x = -\sqrt{2}\partial_x. \]  \hspace{1cm} (η = 0)  \hspace{1cm} (2.15)

Since the zero-mode of the currents correspond to (2.5), the classical expression of the primary fields are obtained by solving the differential equations (2.12). For \( H^{+}_{j, \eta} \) the solutions are\(^4\)

\[ \Psi^+_{j, \eta} = N^+_{j, \eta} \exp\left(i j x^+ - 2i\eta x^- - \frac{\eta y y^*}{2} + i\eta \left(y x e^{ix^+} + y^* x e^{-ix^+}\right) + \eta x \bar{x} e^{ix^+}\right) \]
\[ = N^+_{j, \eta} e^{i j x^+ - 2i\eta x^- + \frac{\eta y y^*}{2}} \sum_{m,n} \frac{|\eta|^m y^{m-n}}{m!} (i x)^m (-i x)^n e^{i(m+n)x^+} L^m_n(\eta y y^*), \] \hspace{1cm} (2.16)

where \( L^k_n(x) \) is the associated Laguerre polynomial. Similarly for \( H^{-}_{j, \eta} \)

\[ \Psi^-_{j, \eta} = N^-_{j, \eta} \exp\left(i j x^+ - 2i\eta x^- + \frac{\eta y y^*}{2} + i\eta \left(y x e^{-ix^+} + y^* x e^{-ix^+}\right) - \eta x \bar{x} e^{-ix^+}\right) \]
\[ = N^-_{j, \eta} e^{i j x^+ - 2i\eta x^- + \frac{\eta y y^*}{2}} \sum_{m,n} \frac{|\eta|^m y^{m-n}}{m!} (i x)^m (-i x)^n e^{-i(m+n)x^+} L^m_n(|\eta| y y^*), \] \hspace{1cm} (2.17)

The normalization is fixed as

\[ N^\pm_{j, \eta} = \frac{1}{\sqrt{\gamma(|\eta|)}}, \quad \gamma(x) = \frac{\Gamma(x)}{\Gamma(1 - x)}. \] \hspace{1cm} (2.18)

For \( H^{0}_{j, s} \) we have

\[ \Psi^0_{j, s} = \exp\left(i j x^+ + \frac{is}{\sqrt{2}} \left(y e^{i\psi} + y^* e^{-i\psi}\right)\right) \sum_n e^{in(\theta + x^+)} \] \hspace{1cm} (2.19)

\(^4\)We construct the closed string spectrum using the same representation in the holomorphic and antiholomorphic part.
with $2\psi = \alpha - \bar{\alpha}$ and $\theta = \alpha + \bar{\alpha}$.

We can see the relation to the basis labeled by eigenstates of $F$ and $J$ using the following expansions as

$$
\Psi^{\pm}_{j,\eta}(z, \bar{z}; x, \bar{x}) = \sum_{n,\bar{n}=0}^{\infty} V^{\pm}_{j,n,\bar{n}}(z, \bar{z}) \left( \frac{x}{\sqrt{n!}} \right)^n \left( \frac{\bar{x}}{\sqrt{\bar{n}!}} \right)^{\bar{n}},
$$

(2.20)

$$
\Psi^{0}_{j,s}(z, \bar{z}; \alpha, \bar{\alpha}) = \sum_{n,\bar{n}=-\infty}^{\infty} V^{0}_{j,n,\bar{n}}(z, \bar{z}) e^{i\alpha n + i\bar{\alpha} \bar{n}},
$$

(2.21)

or inversely

$$
V^{\pm}_{j,n,\bar{n}}(z, \bar{z}) = \frac{|\eta|^2}{\pi^2} \int d^2 x d^2 \bar{x} e^{-|\eta|^2|x|^2} \Psi^{\pm}_{j,\eta}(z, \bar{z}; x, \bar{x}) \left( \frac{x}{\sqrt{n!}} \right)^n \left( \frac{\bar{x}}{\sqrt{\bar{n}!}} \right)^{\bar{n}},
$$

(2.22)

$$
V^{0}_{j,n,\bar{n}}(z, \bar{z}) = \frac{1}{(2\pi)^2} \int_0^{2\pi} d\alpha \int_0^{2\pi} d\bar{\alpha} \Psi^{0}_{j,\eta}(z, \bar{z}; \alpha, \bar{\alpha}) e^{-i\alpha n - i\bar{\alpha} \bar{n}}.
$$

(2.23)

### 2.2 Free field realization and spectral flow

In order to compute physical quantities, it might be useful to use free field realization introduced in Refs. [35, 36]. Using the free fields

$$
X^+(z) X^-(w) \sim \ln(z - w), \quad Y(z) Y^*(w) \sim -\ln(z - w),
$$

(2.24)

we can rewrite the currents as

$$
J = \partial X^-, \quad F = \partial X^+, \quad P^+ = -i \sqrt{2} e^{-iX^+} \partial Y, \quad P^- = -i \sqrt{2} e^{iX^+} \partial Y^*.
$$

(2.25)

We can check that the OPEs (2.6) are reproduced by using (2.24). The primary fields in $H^\pm_{j,\eta}$ can be expressed by

$$
V^{\pm}_{j,n,0} = \exp \left(i j X^+ + i \eta X^-\right) \sigma^{\pm}_{j,\eta},
$$

(2.26)

and the action of $P^{0\pm}_{j,0}$, where we introduce the twist fields

$$
i \partial Y(z) \sigma^+_{\eta}(w) \sim (z - w)^{-\eta \tau^+_{\eta}}(w), \quad i \partial Y^*(z) \sigma^+_{\eta}(w) \sim (z - w)^{\eta - 1 \tau^+_{\eta}}(w),
$$

\begin{align*}
&i \partial Y(z) \sigma^-_{\eta}(w) \sim (z - w)^{-\eta - 1 \tau^-_{\eta}}(w), \quad i \partial Y^*(z) \sigma^-_{\eta}(w) \sim (z - w)^{\eta \tau^-_{\eta}}(w), \quad \text{(2.27)}
\end{align*}

with descendant twist fields $\tau^{\pm}_{\eta}, \tau^{\pm\eta}$. The primary fields in $H^0_{j,\eta}$ are of the form of the plane wave as

$$
\Psi^{0}_{j,s} = \exp \left(i j X^+ + \frac{is}{\sqrt{2}} \left(Y e^{i\psi} + Y^* e^{-i\psi}\right)\right) \sum_{\eta} e^{i\eta(\theta + X^+)}.
$$

(2.28)

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5 Other type of free field realization was proposed in Ref. [25], which is not used in this paper.
therefore the computation involving only the primary fields in $H_{j,\eta}^\pm$ is the same as in the flat space-time case.

In the current algebra (2.7) there is a symmetry to redefine the currents as ($w \in \mathbb{Z}$)

$$
J_n \to J_n , \quad F_n \to F_n - i w \delta_{n,0} , \quad P_n^+ \to P_{n-w}^+ , \quad P_n^- \to P_{n+w}^- , \quad (2.29)
$$

and this operation is called as spectral flow.\(^6\) We should include the representation of the currents generated by the spectral flow because OPEs do not close without including the fields with flowed representation. In the free field realization defined above, we can easily express the primary fields obtained by the spectral flow as ($w = 0, \pm 1, \pm 2$ for $H_{j,\eta}^\pm$)

$$
V_{j,\eta,0}^{\pm,w} = \exp \left( i j X^+ + i (\eta + w) X^- \right) \sigma_\eta^\pm , \quad (2.30)
$$

and the action of $P^-_{-w}$. The corresponding vacuum state satisfies

\begin{align*}
J_0 |j, \eta, w\rangle &= i j |j, \eta, w\rangle , \\
F_0 |j, \eta, w\rangle &= i (\eta + w) |j, \eta, w\rangle , \\
P_n^\pm |j, \eta, w\rangle &= 0 \quad (\forall n \geq -w) , \quad P_n^\mp |j, \eta, w\rangle = 0 \quad (\forall n > w) \quad (2.31)
\end{align*}

We should note that we restricted the range of $w$ even though we could take $w \in \mathbb{Z}$ because there are identities among the Hilbert spaces

$$
H_{j,\eta}^{+,w} \cong H_{j,1-\eta}^{-,w+1} . \quad (2.32)
$$

The primary fields in $H_{j,\eta}^{0,w} (w \in \mathbb{Z})$ are expressed as

$$
\Psi_{j,s}^0 = \exp \left( i j X^+ + i w X^- + \frac{i s}{\sqrt{2}} (Y e^{i \psi} + Y^* e^{-i \psi}) \right) \sum_n e^{i n (\theta + X^+)} , \quad (2.33)
$$

namely, we only include an integer momentum $p^+ = w$.

### 2.3 Two and three point functions

Two and three point functions of the primary fields are computed in Refs. \cite{24, 25}. They used several methods, such as, free field realizations, the pp-wave limit of $SU(2) \times U(1)$ WZW model and the conformal bootstrap from the four point function obtained by solving Knizhnik-Zamolodchikov equations. Two point functions are only the normalization, and meaningful information is in three point functions.

The non-trivial two point functions are the following. One is the two point function between the primary fields $\Psi_{j,\eta}^+, \Psi_{j,\eta}^-$ as

$$
\langle \Psi_{j_1,\eta_1}^+(z_1, \bar{z}_1; x_1, \bar{x}_1) \Psi_{j_2,\eta_2}^-(z_2, \bar{z}_2; x_2, \bar{x}_2) \rangle = \frac{|e^{-\eta_1 x_1 x_2}|^2 \delta(\eta_1 + \eta_2) \delta(j_1 + j_2)}{|z_1 - \bar{z}_2|^{4h_1^+}} \quad (2.34)
$$

\(^6\)See Refs. \cite{17, 24} for more detail. Also refer to Ref. \cite{37} for $AdS_3$ case.
with the notation $|f(x)|^2 = f(x)f(\bar{x})$. Another is the two point function for $\Psi^0_{j,s}$ as

$$\langle \Psi^0_{j_1,s_1}(z_1, \bar{z}_1; \alpha_1, \bar{\alpha}_1) \Psi^0_{j_2,s_2}(z_2, \bar{z}_2; \alpha_2, \bar{\alpha}_2) \rangle = \frac{(2\pi)^2 e^{-i(j_1+j_2)\theta_2} \delta(\theta_1 - \theta_2) \delta(p_1^2 + p_2^2) \sum_{n=0,1} \delta(j_1 + j_2 - n)}{|z_1 - z_2|^{4h_1}}, \quad (2.35)$$

where we define

$$p^x = s \cos \psi, \quad p^y = s \sin \psi, \quad (2.36)$$

so that the momentum conservation in the $Re\,y$ and $Im\,y$ directions manifest. Note that the non-trivial case is only when $j_2 + j_1 = 0$ or $j_2 = j_1 = \frac{1}{2}$.

The three point functions involving $\Psi^+_j$ or $\Psi^-_j$ are of the form

$$\left\langle \prod_{i=1}^3 \Psi^a_{j_i}(z_i, \bar{z}_i; x_i, \bar{x}_i) \right\rangle = \frac{C^{a_1a_2a_3}(J_1, J_2, J_3)D^{a_1a_2a_3}(x_1, \bar{x}_1, x_2, \bar{x}_2, x_3, \bar{x}_3)}{|z_1 - z_2|^{2(h_1+h_2-h_3)}|z_2 - z_3|^{2(h_2+h_3-h_1)}|z_3 - z_1|^{2(h_3+h_1-h_2)}}, \quad (2.37)$$

where $a_i = \pm$ and $J_i = (j_i, \eta_i)$. The functions $D^{a_1a_2a_3}$ can be fixed only from the symmetry. In other words, they are solutions to the differential equations coming from the relation (2.12) as

$$D^{++-} = |e^{-x_3(\eta_1x_1+\eta_2x_2)}(x_2-x_1)^N|2\delta(\eta_1 + \eta_2 + \eta_3) \sum_{n=0}^{\infty} \delta(N-n), \quad (2.38)$$

$$D^{+-+} = |e^{x_1(\eta_2x_2+\eta_3x_3)}(x_2-x_3)^N|2\delta(\eta_1 + \eta_2 + \eta_3) \sum_{n=0}^{\infty} \delta(N+n), \quad (2.39)$$

where $N = -j_1 - j_2 - j_3$. The coefficients $C^{a_1a_2a_3}(J_1, J_2, J_3)$ cannot be determined from the symmetry and must be computed in other methods. The results are [24, 25]

$$C^{++-}(J_1, J_2, J_3) = \frac{1}{\Gamma(1+N)} \left( \frac{\gamma(-\eta_3)}{\gamma(\eta_1)\gamma(\eta_2)} \right)^{\frac{1}{2} + N}, \quad (2.40)$$

$$C^{+-+}(J_1, J_2, J_3) = \frac{1}{\Gamma(1-N)} \left( \frac{\gamma(\eta_1)}{\gamma(-\eta_2)\gamma(-\eta_3)} \right)^{\frac{1}{2} - N}. \quad (2.41)$$

The OPEs among the primary fields can be read from the above three point functions, though we should care about the non-trivial normalization of the two point functions [23], [23].

\[ \text{The normalization [23] may be read from the three point functions [23], [23]. The limit of } \lim_{j,\eta \to 0} \Psi^\pm_{j,\eta} \text{ in [2.16], [2.17] is } \lim_{j,\eta \to 0} N^\pm_{j,\eta} \cdot 1 \text{ where 1 is the identity. The three point function with one identity must reduce to the two point function, which may lead to the normalization [23]. Another way to fix the normalization may be using the free field realization as in Refs. [24] and [26].} \]
For the three point functions only involving $\Psi^0_{j,s}$, it is not useful to express in the form of (2.37). Since the $\Psi^0_{j,s}$ is of the form of plane wave, we rather write the three point function as

$$\left\langle \prod_{i=1}^{3} \Psi^0_{j,s_i}(z_i, \bar{z}_i; x_i, \bar{x}_i) \right\rangle$$

$$= (2\pi)^3 e^{iN\theta_3} \delta(\theta_1 - \theta_3) \delta(\theta_2 - \theta_3) \delta(p_1^s + p_2^s + p_3^s) \sum_{n=\pm 1, 0} \delta(N - n)$$

$$\frac{1}{|z_1 - z_2|^{2(h_0^3 + h_0^2 - h_0^1)} |z_2 - z_3|^{2(h_0^3 + h_0^2 - h_0^1)} |z_3 - z_1|^{2(h_0^3 + h_0^2 - h_0^1)}}$$

where the momentum conservation is manifest. The normalization is fixed using the fact that the three point function (2.42) should reduce to the two point function (2.35) if one operator is the identity $V^0_{0,0,0}(z, \bar{z})$ (2.23).

### 3 Geometry of Symmetric D-branes

D-branes are defined by the hypersurface where the boundary of open string sweeps. The worldsheet for open string can be given by a disk or an upper half plane, which can be mapped to each other by conformal transformation. Thus we should assign the boundary condition to the currents in order to preserve the conformal symmetry. Here we only consider the boundary conditions which preserve the half of the current algebra.

As classified in Ref. [28] there are essentially two types of boundary conditions. One type is

$$J = (-1)^{\epsilon} \bar{J} , \quad F = (-1)^{\epsilon} \bar{F} , \quad P^+ = (-1)^{\epsilon} \bar{P}^+ , \quad P^- = (-1)^{\epsilon} \bar{P}^- . \quad (3.1)$$

Since we should perform a conformal mapping to the currents in order to exchange the open and closed string channel (equivalently to exchange the time and space coordinates of worldsheet), there is a phase factor in front of the currents, where $\epsilon = 0$ and $\epsilon = 1$ for the open and closed string channel, respectively. The other type is

$$J = (-1)^{\epsilon} \bar{J} , \quad F = (-1)^{\epsilon} \bar{F} , \quad P^+ = (-1)^{\epsilon} \bar{P}^- , \quad P^- = (-1)^{\epsilon} \bar{P}^+ . \quad (3.2)$$

We do not consider the boundary conditions which reduce to (3.1) or (3.2) by using the inner automorphism.

Let us first examine the boundary condition (3.2). Under the boundary condition, a group element at the boundary can be transformed by the adjoint transformation

$$g^{-1}(x^+, x^-, y)g(x_0^+, x_0^-, y_0)g(x^+, x^-, y)$$

$$= g(x_0^+, x_0^- - \frac{1}{2}|y|^2 \sin x_0^+ + \text{Im}(y_0 g^* e^{\frac{iy}{2}} \cos \frac{x_0^+}{2}), e^{\frac{iy}{2}} (y_0 e^{\frac{iy}{2}} x_0^+ - 2iy \sin \frac{x_0^+}{2})) .$$

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If the parameters \((x^+, y)\) vary, then the right hand side of (3.3) draws a subspace of the Nappi-Witten background. The subspace has the symmetry left after taking the boundary condition, so it must be the geometry of the D-brane corresponding to the boundary condition (3.2). This subspace is classified by the conjugacy class of the group [28].

For \(x_0^+ = 0 \mod 2\pi\), the equation (3.3) leads to

\[
g(0, x_0^- \pm \text{Im}(y_0 y^* e^{\frac{i}{2}x^+}), y_0 e^{ix^+}) .
\]

(3.4)

Therefore we have 0-dimensional instanton for \(y_0 = 0\) and cylindrical 2-dimensional brane for \(y_0 \neq 0\). However we do not consider these branes because their metrics are degenerated.

For \(x_0^+ \neq 0 \mod 2\pi\), we can rewrite (3.3) as

\[
g(x_0^+, x_0^- + \frac{1}{4} \cot \frac{x_0^+}{2} (|y_0|^2 - |a|^2), a)
\]

(3.5)

thus the corresponding branes have 2-dimensional Euclidean worldvolume located at

\[
x^+ = x_0^+ , \quad x^- = x_0^- + \frac{1}{4} \cot \frac{x_0^+}{2} |y|^2 .
\]

(3.6)

If \(x_0^+ = \pi \mod 2\pi\), the worldvolume is flat at \(x^\pm = x_0^\pm\), otherwise the worldvolume is hyperbolic.

Another boundary condition (3.1) implies the invariance of the geometry under the twisted adjoint action \(g \rightarrow (r \cdot g_L)^{-1} gg_R\) where \(r\) generates an outer automorphism

\[
r \cdot g(x^+, x^-, y) \rightarrow g(-x^+, -x^-, -y^*)
\]

(3.7)

as discussed in Ref. [28]. Since we have

\[
g^{-1}(-x^+, -x^-, -\bar{y}) g(x_0^+, x_0^-, y_0) g(x^+, x^-, y) = g(x_0^+ + 2x^+ ,
\]

(3.8)

\[
x_0^- + 2x^- + \frac{1}{2} \text{Im}(\bar{y} e^{\frac{i}{2}(x_0^+ + x^+)} (y_0 + \bar{y}_0 + \bar{y} e^{\frac{i}{2}(x_0^+ + x^+)}, y_0 + 2 \text{Re}(y e^{-\frac{i}{2}(x_0^+ + x^+))},
\]

the corresponding brane is the \((2+1)\) dimensional hypersurface at

\[
\text{Im } y = \text{Im } y_0 \equiv b
\]

(3.9)

with a real parameter \(b\). In summary, we have Lorentzian D2-branes at (3.9) for the boundary condition (3.1) and Euclidean D2-branes at (3.6) for the boundary condition (3.2).
4 One Point Functions and Boundary States

In quantum level the D-branes are expressed by so-called boundary states. In the open string channel, the D-branes are defined by the hypersurface where the ends of the open strings are attached to. On the other hand, in the closed string channel, the D-branes are described by the boundary states which have information how closed strings couple to the D-branes. A general boundary state is given by a linear combination of the Ishibashi states, and the coefficients can be read from the disk (or upper half plane) one point functions of primary fields. In this section, we compute the one point functions with boundary conditions (3.1), (3.2) and construct the boundary states for the Lorentzian D2-branes and Euclidean D2-brane.

4.1 Lorentzian D2-branes

Let us first consider the Lorentzian D2-branes corresponding to the boundary condition (3.1). The boundary condition leads to

\[ (D_{x}^{J,F,±} + \bar{D}_{x}^{J,F,±}) \Psi_{j,s}(z,\bar{z};\alpha,\bar{\alpha}) = 0, \]

(4.1)

because of the relation (2.12). The condition \( D_{x}^{F} + \bar{D}_{x}^{F} = 0 \) means that only the one point functions of the states in \( \mathcal{H}_{j,s}^{0} \) have no-trivial value (even including the spectral flowed states). This is reasonable since D-brane parallel to the light-cone direction couples to the closed strings with vanishing light-cone momentum. The other conditions restrict the form of the one point function to

\[ \langle \Psi_{j,s}^{0}(z,\bar{z};\alpha,\bar{\alpha}) \rangle = U_{j,s}(\alpha,\bar{\alpha}) \theta(z - \bar{z})^{2n}, \quad U_{j,s}(\alpha,\bar{\alpha}) = f_{j,s}(p^{c})\delta(2j - n). \]

(4.2)

This form may be also derived from the free field realization (2.28). In fact, it is the one point function with the Newmann and Dirichlet boundary conditions (in the open string terms) for the Re \( y \) and Im \( y \) directions, respectively.

In the following, it is convenient to define

\[ \hat{\Psi}_{j,s}^{0}(z,\bar{z};\psi) \equiv \frac{1}{2\pi} \int_{0}^{2\pi} d\theta e^{ij\theta} \Psi_{j,s}^{0}(z,\bar{z};\alpha,\bar{\alpha}), \]

(4.3)

with \( \hat{j} = 0, 1/2 \), and compute its one point function

\[ \langle \hat{\Psi}_{j,s}^{0}(z,\bar{z};\psi) \rangle = \frac{f_{j,s}(p^{c})}{|z - \bar{z}|^{2n}}. \]

(4.4)

We also fix the normalization of the one point function such as

\[ \langle V_{0,0,0,0}^{0}(z,\bar{z}) \rangle = 1, \]

(4.5)
namely, the one point function of the identity is set to be one.

In order to compute the coefficient $f_{j,s}$, we utilize the conformal bootstrap method \cite{38, 39, 40}. We consider the following two point function on the upper half plain

$$
\langle \hat{\Psi}_0^{j_1,s_1}(z_1, \bar{z}_1; \psi_1) \hat{\Psi}_0^{j_2,s_2}(z_2, \bar{z}_2; \psi_2) \rangle.
$$

(4.6)

This function can be computed in two ways. One way is using the OPE of the inserted closed strings

$$
\hat{\Psi}_0^{j_1,s_1}(z, \bar{z}; \psi_1) \hat{\Psi}_0^{j_2,s_2}(w, \bar{w}; \psi_2) \sim \sum_{j_3-j_1-j_2=0, \pm 1} \int dp_3^c dp_3^s \hat{C}_{123}^{000}(w, \bar{w}; \psi_3) \frac{1}{|z-w|^{2(h_1+h_2-h_3)}}
$$

(4.7)

up to the contribution of descendants, where the three point coefficient is given by

$$
\hat{C}_{123}^{000} = \delta(p_1^c + p_2^c - p_3^c)\delta(p_1^s + p_2^s - p_3^s).
$$

(4.8)

Another way is to use the OPEs between the bulk operators and boundary operators. If the boundary operator is the identity, then the coefficient of the OPE is the same as the one point function (4.4) under the normalization (4.5). Comparing the two ways of computing the two point function (4.6), we have the equality

$$
\int dp_3^c dp_3^s \hat{C}_{123}^{000} f_{j_3,s_3}(w, \bar{w}; \psi_3) \mathcal{F}_{21}^{12}(z) = f_{j_1,s_1} \delta(p_1^c) f_{j_2,s_2} \delta(p_2^c) \mathcal{F}_{22}^{11}(1-z)
$$

(4.9)

as long as the propagating boundary operator is the identity.\footnote{We changed the normalization (4.3) such that the identity appears as the propagating boundary operator with the correct normalization.}

We denote $\mathcal{F}(z)$ as the four point conformal blocks

$$
\mathcal{F}_{21}^{12}(z) = \mathcal{F}_{22}^{11}(1-z) = z^{p_1^c p_2^s + p_1^s p_2^c} (1-z)^{-(p_1^c)^2 + (p_1^s)^2},
$$

(4.10)

which correspond to the choice of $z_1 = z, z_2 = 0, \bar{z}_2 = \infty, \bar{z}_1 = 1$. Solution to the equation (4.9) is

$$
f_{j,s} = \exp(ip^s b)
$$

(4.11)

with a parameter $b$. We should notice that the normalization is consistent with (4.5). In summary we have the non-trivial one-point function of the closed strings for $\mathcal{H}_{j,s}^0$ as

$$
\langle \Psi_0^{j,s}(z, \bar{z}; \alpha, \bar{\alpha}) \rangle_b = \frac{e^{ip^s b - i j \theta} \delta(p^c) \sum_{n=0,1} \delta(2j - n)}{|z-\bar{z}|^{2d_0}}
$$

(4.12)

for the Lorentzian D2-branes corresponding to the boundary condition (3.1).\footnote{Note that sin $\psi$ only set the sign in front of $s$ because of the delta function $\delta(s \cos \psi)$ in (4.4).}
The boundary state for the Lorentzian D2-brane is defined to reproduce the one point function \( (4.12) \), namely
\[
\langle \langle b | j, s, \alpha, \bar{\alpha} \rangle = |z - \bar{z}|^{2 h_0} \langle \Psi^0_{j,s}(z, \bar{z}; \alpha, \bar{\alpha}) \rangle_b ,
\]
where \(|j, s, \alpha, \bar{\alpha}\rangle\) is closed string state corresponding to the primary fields \( \Psi^0_{j,s}(\alpha, \bar{\alpha}) \). Noticing that classically \( \langle j, s, \alpha, \bar{\alpha} | x^+, x^-, y \rangle = \Psi^0_{j,s} \) in \((2.19)\), we have
\[
\langle \langle b | x^+, x^-, y \rangle \sim \frac{1}{(2\pi)^2} \int_{1/2}^{1} \frac{d j}{j} \int_{0}^{2\pi} s d s \int_{0}^{2\pi} d \alpha \int_{0}^{2\pi} d \bar{\alpha} \langle \Psi^0_{j,s}(\alpha, \bar{\alpha}) \rangle \Psi^0_{j,s}(\alpha, \bar{\alpha}) \]
\[
\sim \delta(\text{Im } y - b) .
\]
Therefore, we can conclude that the boundary state \((4.13)\) reproduces the classical geometry of the D2-branes \((3.9)\), where we identify the parameter \( b \) in \((4.11)\) as the position of the D2-brane.

### 4.2 Euclidean D2-branes

The boundary condition \((3.2)\) also leads to the condition
\[
(D^{J,F}_x - D^{J,F}_{\bar{x}}) \langle \Psi^0_{j}(z, \bar{z}; \alpha, \bar{\alpha}) \rangle = 0 ,
\]
which is similar to \((4.1)\). In this case the condition \( D^F_x - D^F_{\bar{x}} = 0 \) gives no constraint on \( \eta \), which implies that all the closed strings couple to the brane contrary to the previous case. The other conditions restrict the form of one point functions as before.

We first consider the primary fields in \( \mathcal{H}_{j,s}^0 \), whose one point function is of the form\(^{10}\)
\[
\langle \Psi^0_{j,s}(z, \bar{z}; \alpha, \bar{\alpha}) \rangle = f_j(\theta) \frac{\delta(s)}{s} .
\]
Just like the previous case it is useful to define
\[
\hat{\Psi}^0_{j,n}(z, \bar{z}) = \frac{1}{(2\pi)^2} \int_{0}^{2\pi} d \psi \int_{0}^{2\pi} d \theta e^{-i n \theta} \Psi^0_{j,0}(z, \bar{z}; \alpha, \bar{\alpha}) ,
\]
and compute its one point function
\[
\langle \langle \hat{\Psi}^0_{j,n}(z, \bar{z}) \rangle = \hat{f}_{j,n} .
\]
We set the normalization as \((4.3)\) (even though the normalization of the one point function is different from the previous one in general). Because the OPE involving \((4.17)\) is given by
\[
\hat{\Psi}^0_{j,n}(z, \bar{z}) \sim \hat{\Psi}^0_{j_2,n_2}(w, \bar{w})\hat{\Psi}^0_{j_1+j_2+m,n_1+n_2-m}(w, \bar{w}) + \cdots
\]
\(^{10}\)The conditions \((4.13)\) allow the form of \( f_{j,s} \delta(\theta) \) other than \((4.16)\), however we do not use it because it is not consistent with the classical picture \((3.6)\).
(m is chosen so that \( -1/2 < j_1 + j_2 + m \leq 1/2 \)), we have the conformal bootstrap constraint

\[
\hat{f}_{j_1+j_2+m,n_1+n_2-m} = \hat{f}_{j_1,n_1} \hat{f}_{j_2,n_2}
\]

as in (4.19). Solution to the constraint is given by

\[
\hat{f}_{j,n} = \exp(i(j + n)x^+_0). \tag{4.21}
\]

Since the wave function corresponding to (4.17) is

\[
\hat{\Psi}^0_{j,n}(z,\bar{z};x_0) = \exp(i(j + n)x^+_0).
\]

(4.22)

the one point function implies that the brane is at \( x^+ = x^+_0 \), which is consistent with the classical analysis (3.6).

Let us move to the closed strings in \( H^\pm_{j,\eta} \). From the conditions (4.15), the form of the one point function is restricted as

\[
\langle \Psi^\pm_{j_1,n_1}(z,\bar{z};x_0) \rangle = U^\pm_{j_1,n_1}e^{|\eta|^2|z-\bar{z}|^2h^\pm}. \tag{4.23}
\]

As before we compute the two point function

\[
\langle \Psi^+_{j_1,n_1}(z_1,\bar{z}_1;x_1) \Psi^-_{j_2,n_2}(z_2,\bar{z}_2;x_2) \rangle \tag{4.24}
\]

in order to obtain constraints for \( U^\pm_{j_1,n_1} \) Here we assume \( \eta_1 + \eta_2 > 0 \).\(^{11}\) Computing two ways and comparing the both, we obtain

\[
\sum_{n=0}^\infty C^{+-+-} + (j_1,\eta_1;j_2,\eta_2;-j_1-j_2+n,-\eta_1-\eta_2) U^\pm_{j_1+j_2-n,\eta_1+\eta_2} F_{12}^{ij}(n,z,x) \]

\[
= \int_0^\infty \int_0^\infty \int_0^\infty dsdx F_{12}^{ij}(s,1-z,x), \tag{4.25}
\]

where \( C^{+-+-}_B \) represents the bulk-boundary two point function with \( C^{+-+-}_B(j,\eta;0,0) = U^\pm_{j_1,n_1} \). We used the conformal blocks \( F_{12}^{ij}(n,z,x) \) and \( F_{11}^{ij}(s,1-z,x) \) defined in Ref. 24 for \( \langle ++-\rangle \) amplitudes, whose intermediate states belong to \( H^\pm_{j,\eta} \) and \( H^0_{j,s} \), respectively.\(^{12}\)

The transformation of the conformal blocks can be computed as \( (N = -j_1-j_2-j_3-j_4) \)

\[
F_{34}^{ij}(n,z,x) = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty ds \left(C_n^N(s)\right)^{-1} F_{32}^{ij}(s,1-z,x), \tag{4.26}
\]

\(^{11}\)The other cases \( \eta_1 + \eta_2 < 0 \) and \( \eta_1 + \eta_2 = 0 \) give other constraints. The former case leads to the similar constraint, and the latter case gives the relation between (4.16) and (4.23). It is very important to check also in these cases although we will not do it in this paper. Another important constraint may come from the two point function with \( \Psi^0_{j,s} \) and \( \Psi^\pm_{j,\eta} \).

\(^{12}\)The definition of the identity state in Ref. 24 is not the same as the one in this paper, and hence the normalization of the one point functions (4.16) and (4.23) may be different.
where
\[
(C_n^N(s))^{-1} = \frac{(-1)^{N/2} n! \Xi^{N+1}}{A^{n+N+1} B^n} \left( \frac{s^2}{2} \right)^{\frac{N}{2}} e^{\frac{s^2}{2}(\psi(\eta_2)+\psi(1-\eta_1)-2\psi(1))} I_n^N \left( \frac{s^2}{2} \Xi \right). \tag{4.27}
\]

Here we have used \( \psi(x) = \frac{d}{dx} \ln \Gamma(x) \) and defined
\[
\Xi = \pi \sin \pi (\eta_1 + \eta_2) \sin \pi \eta_1 \sin \pi \eta_2, \quad A = \frac{\Gamma(1 + \eta_2) \Gamma(\eta_1)}{\Gamma(\eta_1 + \eta_2)}, \quad B = \frac{\Gamma(1 - \eta_1 - \eta_2)}{\Gamma(1 - \eta_1) \Gamma(-\eta_2)}. \tag{4.28}
\]

With the help of the transformation matrix \( \text{(4.26)} \) with \( N = 0 \) and \( s = 0 \), the relation \( \text{(4.25)} \) leads to
\[
\sum_{n=0}^{\infty} \sqrt{\Xi} U_{j_1, j_2 - n, \eta_1 + \eta_2}^+ = U_{j_1, \eta_1}^+ U_{j_2, \eta_2}^-. \tag{4.29}
\]

Solutions to the above equation are given by
\[
U_{j, \eta}^\pm = \sqrt{\frac{\pi}{\sin \pi \eta}} \sum_{n=0}^{\infty} e^{i(j+n)x_0^0 - 2in\eta x_0^-} = \sqrt{\frac{\pi}{\sin \pi \eta}} \frac{1}{1 - e^{\pm i\eta}} e^{i j x_0^+ - 2n\eta x_0^-} \tag{4.30}
\]
with parameters \( x_0^- \) and \( x_0^+ \neq 0 \) mod \( 2\pi \).

We can construct the boundary state like \( \text{(4.13)} \) and compute the coupling with localized closed strings as
\[
\langle \langle x_0^+, x_0^- | x^+, x^-, y \rangle \rangle \sim \int_{-\infty}^{\infty} dj \int_{-1}^{1} d\eta \frac{\eta^2}{\pi^2} \int d^2 x d^2 \bar{x} \langle \Psi_{-j, -\eta}^+ (- \bar{x}^*, -x^*) \rangle \Psi_{j, \eta}^>(x, \bar{x}) = \int_{-1}^{1} d\eta \frac{\pi \delta(x^+ - x_0^0) \Gamma(1 - |\eta|)}{2 \sin^2 \frac{\eta^+}{2}} e^{-2i\eta(x^- - x_0^- + \frac{1}{4} \cot \frac{x_0^+}{2} |y|^2)}. \tag{4.31}
\]
To see its classical behavior, it is convenient to reintroduce \( \alpha' = 2 \) and rewrite such as \( \sqrt{\frac{\alpha'}{2}} \eta, \sqrt{\frac{\alpha'}{2}} x^- \). Then, in the classical limit \( \alpha' \to \infty \), the small \( \eta \) region is expanded, and the integration gives delta function \( \delta(x^- - x_0^- + \frac{1}{4} \cot \frac{x_0^+}{2} |y|^2) \). Identifying the two free parameters \( x_0^0 \) as those of \( \text{(3.6)} \), we successfully reproduce the classical geometry of the Euclidean D2-brane from the boundary state.

For the couplings with the flowed states, it is useful to use the free field realization as mentioned in section 2.2. Then, we can see from the above results that \( X^- \) field satisfies the Dirichlet boundary condition (in the open string terms) and the one point function is proportional to \( \exp(-2ix_0 (\eta + w)) \). Therefore, the one point functions are obtained as
\[
\langle \Psi_{j, \eta}^{0, w}(z, \bar{z}; \alpha, \bar{\alpha}) \rangle = \sum_n e^{in\theta + i(j+n)x_0^0 - 2iwx_0^-} \delta(s) \tag{4.32}
\]
for \( \mathcal{H}_{j, \eta}^{0, w} \) with \( w \in \mathbb{Z} \) and
\[
\langle \Psi_{j, \eta}^{\pm, w}(z, \bar{z}; x, \bar{x}) \rangle = \sqrt{\frac{\pi}{\sin \pi \eta}} \sum_{n=0}^{\infty} e^{i(j\pm n)x_0^0 - 2i(\eta + w)x_0^-} e^{\eta|x|^2} \frac{e^{\eta|x|^2}}{|z - \bar{z}|^{2\eta}} \tag{4.33}
\]
for \( \mathcal{H}_{j, \eta}^{\pm, w} \) with \( w = 0, \pm 1, \pm 2, \cdots \).
5 Conclusion

We investigated D-branes in the Nappi-Witten model [16], which is a WZW model associated with 4 dimensional Heisenberg group $H_4$. Only the symmetric D-branes have been considered, whose boundary conditions preserve the half of the current algebra (3.1); (3.2). These branes are classified by the (twisted) conjugacy classes [28], and we have Lorentzian D2-brane at (3.9) and Euclidean D2 brane at (3.6). In this paper we computed the one point functions of closed strings on the upper half plane. It is rather difficult to calculate the one point function directly, so we computed them by utilizing conformal bootstrap constraints [38, 39, 40]. Two point function on the upper half plane can be mapped to four point function on the full plane, whose conformal block is obtained by solving Knizhnik-Zamolodchikov equations [24]. Computing the two point functions in two ways and comparing the both, we have constraints to the one point functions. Solving the constraints, we obtained the one point functions (4.12) for the Lorentzian D2-branes and (4.32), (4.33) for the Euclidean D2-branes. We constructed the boundary states based on the one point functions and checked that the classical limits reproduce the geometry of the corresponding D-branes.

The methods we used to obtain the boundary states in the Nappi-Witten model might be useful to investigate some time-dependent D-branes such as D-branes with rolling tachyon [41, 42]. This is because the Nappi-Witten model is solvable and its target spacetime is Lorentzian, and hence we do not need to perform analytic continuation of Euclidean results, which may give rise to difficulty. An interesting case may be the D-branes in $AdS_3$, where the boundary states in the Euclidean $AdS_3$ is given in Refs. [43, 44]. Since the pp-wave limit of $AdS_3 \times S^3$ is very similar to our model, our results may give insights in the construction of boundary states in the Lorentzian $AdS_3$ (see Ref. [29] for a classical argument). The most difficult task with Lorentzian signature is to perform the modular transformation because the worldsheet should be also Lorentzian. Even in our case, it is difficult to read the open string spectrum by using the modular transformation of one-loop amplitude, even though the cylinder amplitude in the closed string channel is easy to compute using our boundary states. It would be interesting to try to manage the difficulty.

$^{13}$D-branes in Lorentzian $SL(2, \mathbb{R})/U(1)$ WZW model (which is a coset of the Lorentzian $AdS_3$ model) are also worth to study because of the non-trivial time-dependence of D-branes [45]. In order to construct the boundary states, we have to perform analytic continuation to their Euclidean counterparts, which are analyzed in Ref. [46].

$^{14}$It might be possible to directly analyze the open strings in the Nappi-Witten model. In particular, we may obtain two and three point functions by utilizing conformal bootstrap as in Ref. [44].

$^{15}$We may be able to perform the modular transformation only in the light-cone gauge as in Ref. [13].
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