Hypersharp Resonant Capture of Anti-Neutrinos

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Recent ideas suggest that the 18.6 keV antineutrino ($\nu_\beta$) line from 2-body decay of $^3$H in crystals is emitted with natural width, motionally narrowed by lattice vibrations as in recoilless emission. It can be resonantly captured in $^3$He with geometrical cross section $\sigma \sim 10^{-17}$ cm$^2$. A key technique solves a basic obstacle for achieving resonance—the chemical difference of H and He in metals. The low $\nu_\beta$ energy, the high $\sigma$ and the hypersharp sensitivity $\Delta E/E \sim 10^{-29}$ make an extraordinary tool for bench scale tests of $\nu_\beta$ $\theta_{13}$ oscillations and predicted Planck length limits on nuclear level widths in models of quantum gravity.

Two-body weak nuclear decays emit monoenergetic lines of antineutrinos ($\nu_\beta$). The two well known modes of such decay are electron capture (EC) and the reverse process of bound-state beta decay (BB)$^1$ in which the $\beta$-electron is captured in an atomic orbital. The question if these lines can also be emitted recoilless was raised immediately after the discovery of the M"ossbauer effect (ME). Visscher considered the EC mode$^2$ in 1959 and, 25 years later, Kells and Schiffer$^3$, the BB mode, particularly that of tritium $^3$H (T). However, these ideas remain yet speculative because of the very stringent, unanswered experimental demands, even for the more favorable case of T. State-of-the-art hydrogen storage technology and materials now suggest a breakthrough in the T case. In a preliminary report$^4$, I proposed a specific approach to observe recoilless resonant capture of the 18.6 keV $\nu_\beta$ emitted in the T-BB in a $^3$He target. The key idea focuses on solving the biggest problem posed by this experiment—the different behaviors of the noble gas He absorber and the chemically bound source T in metals.

With the advantage of recoilless emission, the resonant cross section $\sigma$ for capture is fundamentally determined by the spectral widths of the emitted $\nu_\beta$. The widths are, in turn determined by the broadening induced by various means, but mainly by the spin motions and the fluctuations of local dipolar fields. In ref. 4, a relaxation width measured by NMR in the chosen material was speculated and an effective resonance cross section $\sigma \sim 3 \times 10^{-33}$ cm$^2$ some $10^{10}$ times that for usual $\nu_\beta$ reactions, was derived. While this was very attractive, major experimental challenges remained.

New ideas have recently emerged (see companion paper$^5$), on the origins of the linewidth. The broadening assumed in ref. 4 is appropriate for short lived states (including all ME cases so far) but not for very long lived states for which no data is available yet. Ref. 5 suggests surprisingly, that in these cases, one should actually expect hypersharp $\nu_\beta$ lines of natural line width—not the severe broadening assumed in ref. 4, basically because it ignores the key role of motional averaging via lattice vibrations in full analogy to recoilless emission itself. In this case $\sigma$ rises to the geometrical limit, vastly larger than the previous estimate and dramatically enhances prospects for observing resonant capture of tritium $\nu_\beta$. In this Letter I briefly summarize the theory of ref.5, and on that basis, discuss a radically simplified experimental approach.

Hypersharp $\nu_\beta$ lines offer a basically new tool of unprecedented power, combining the low $\nu_\beta$ energy, the high resonant $\sigma \sim 10^{-17}$ cm$^2$ and the hypersharp energy sensitivity $\Delta E/E \sim 10^{-29}$. It challenges the imagination of new perspectives of the physical universe. One example is tests of $\nu_\beta$ $\theta_{13}$ oscillations with gram- not kiloton scale targets and bench-scale—not km scale baselines. Another is testing the predicted limits on nuclear level widths set by the Planck length.

The $T-H$ system $^3$H(1/2)$^\rightarrow ^3$He(1/2)$^\nu_\beta$ : $[(E(\nu_\beta)$ =18.6 keV; $t (^3$H) $\sim 6 \times 10^8$ s; $\Gamma \sim 10^{24}$ eV$^3$)) is ideal for resonant $\nu_\beta$ capture. It offers a sizable BB branching (~5.4 x10$^{-8}$) to the atomic ground state of $^3$He. The initial T atom has a vacancy in the ls shell for B$^\beta$ decay and the target $^3$He has two 1s electrons one of which can be captured. 1s EC is ideal since $|\Psi(He n -1s)|$ (for BB decay and $\nu_\beta$ capture) is $\propto 1/n^2$ and maximal for n =1. Note that spins of both $^3$H and $^3$He are ½, thus both have zero quadrupole moments. In BB decay$^1$

$$A(Z-1) \rightarrow \nu_\beta + A(Z)+ e^\prime (in \, bound\, state) \qquad (1),$$

the $\beta^-$ electron of $A(Z-1)$ in is inserted in a vacant orbit in $A(Z)$. A $\nu_\beta$ line is emitted with the unique energy:

$$E_{\nu_\beta} = Q + B_Z - E_R \qquad (2)$$

where $B_z$ is the shell binding energy is gained in inserting an electron in $A(Z)$. $E_R$ is a deficit due to nuclear recoil. Mikaelyan et al$^6$ first noted the reverse reaction

$$\nu_\beta + A(Z)+ e (bound\, orbital) \rightarrow A(Z-1) \qquad (3),$$

and its resonant character at the $\nu_\beta$ energy:
\[ E(\tilde{\nu}_{\text{reco}}) = Q + B_Z + E_K \]  \hspace{1cm} (4).

\( Q \) is the maximum \( \tilde{\nu} \) energy (= \( M_{Z-1} - M_Z \)) in the \( \beta \)-decay \( \Lambda(Z-1) \rightarrow \Lambda(Z) \). The \( \tilde{\nu} \) from (1) has exactly the excess energy \( B_Z \) in (2) in order to remove the same electron in the capture (3). As is well known, the double deficit \( 2E_K \) remains. The basic idea of ref. 4 (and 3) is that the \( \tilde{\nu} \) line from T - B decay is resonant if the \( \tilde{\nu} \) emission and absorption are recoiless.

The resonance cross section \( \sigma \) is determined directly by the spectral density of the incident beam at the resonance energy i.e., \( \nu_{\text{e}} \), the line width of the \( \tilde{\nu} \) from the BB-decay (3) -- broader it is, less the \( \sigma \). A width \( \sim 10^{12} \) times the natural width was assumed in ref. 4. Based on ref. 5 a \( \nu_{\text{e}} \) of natural width \( \Gamma \) should be emitted in long lived states such as T (see below). Then the spectral density at resonance is maximal, 1/\( \Gamma \). Thus, \( \sigma \) is maximal-- the geometrical value \( \sigma_0 = \frac{2\pi^2k^2}{2} \) as in \( \gamma \)-ray resonance. The cross section \( \sigma_0 \sim 2 \times 10^{-17} \text{ cm}^2 \) saturates absorption in g/cm² thick \( ^3\text{He} \) targets. Such a “black” \( \nu_{\text{e}} \) absorber is a novel experience in \( \tilde{\nu} \) research.

Time-dependent fields such as dipolar interactions (\( \propto 1/r^3 \) the interatomic distance) create a fluctuating field from nuclear motions because of lattice vibrations. The side bands that create line broadening require T - B decay is resonant if the \( \tilde{\nu} \) emission and absorption are recoiless.

If the energy shifts \( E_T \) and \( E_{\text{He}} \) are unique, static, and identical in source/absorber, a deficit \( (E_T - E_{\text{He}}) \) in the BB decay T \( \rightarrow \) He at \( \tilde{\nu} \) emission is self-compensated exactly by \( (E_{\text{He}} - E_T) \) in the reverse He \( \rightarrow \) T e-capture in \( \tilde{\nu} \) absorption. An example of this effect is the role of the shell electron energy B in eq. 1 and 3.

Recoiless and hypersharp \( \tilde{\nu} \) emission in TBB \( \rightarrow 3\text{He} \) requires T and \( ^3\text{He} \) (normally gases) to be embedded in solids. Metal tritides offer a practical approach. Hydrogen (T) reacts with metals to form hydrides (tritides) and creates a uniform population of T in the bulk of the metal. As the tritide ages, the \( ^3\text{He} \) daughter grows and populates the lattice (the “tritium-trick” TT). The He site in the source is its birth site – that of its parent T. The absorber is made in an identical manner. However, the absorber site of He, an insoluble mobile inert atom, is typically different and indeed, non-unique. It rapidly diffuses away and forms clusters/micro-bubbles, sites very different from regular lattice sites in T, thus basically unsuitable for hypersharp \( \tilde{\nu} \) resonance. In bcc metals (Ti, Nb, V), the T sits only in tetragonal interstitial sites (TIS) whereas in fcc metals (Pd, Ni), it finds octahedral interstitial sites (OIS).

The key design problem is thus the search for a metal system where He sites are lattice sites identical to the T. A search was made using measured parameters
sites can be randomly filled with equal probability. The(evaporation) and EST(OIS) for He (Table 2). Thus, both the He would normally reside only in OIS (bcc or fcc) as e.g. in TiT. However, c2 (pair clusters) and c3 (bubbles). These equations were solved numerically for a variety of tritides, focusing on NbT and TiT. Fig. 1 shows the results for NbT. We see the growth of He for 200 days at which time the T is switched off by desorption. Thereafter, the He in the T-free sample has different ratios of (interstitial sites IS/(bubbles) = c1/(c2+c3) at different temperatures, exemplified by a flat c1 (as at 200K) or a decaying c1 indicating loss to growing bubbles c2+c3. The latter is shown explicitly in the lower curves for T>235K. The 200K results in Fig. 1 are not very sensitive to the exact parameter values: x100 larger D and/or a smaller E1 = 0.8 eV do not change the results. Thus, in NbT the He reside only at unique IS sites if T <200K, grows indefinitely and remains without bubble formation after the T is removed. This behavior in NbT is exceptional. In most other tritides e.g. in the well known PdT, bubble formation dominates already at >20K, normally leaving no He at regular lattice sites.

Fig 1 ³He is generated in NbT for 200 days, when the He generating T is desorbed. The figure shows the ³He concentration in interstitial sites (c1)() and that in clusters/bubbles (c2+c3) () for different ambient temperatures and time t. The He in the T-free absorber below 220K is almost all interstitial and above 235K, almost all in clusters/bubbles.

In the bcc NbT source, the T and thus, also the just-born daughter He reside in the TIS and OIS (see Table 2 for the self-trapping energy EST at TIS and OIS). In the target the He would normally reside only in OIS (bcc or fcc) as e.g. in TiT. However, Nb is unique with degenerate EST(TIS) and EST(OIS) for He (Table 2). Thus, both sites can be randomly filled with equal probability. The site identifications have been verified by ion channeling. There are 6 TIS and 3 OIS in the bcc unit cell. The ZPE at the two sites (Table 2) differ by ~0.01eV so that the 33% He in OIS are off resonant, a small cost. Thus, NbT aged below 200K offers unique, TIS He sites identical to that of the emitter T-He, and (most of) the absorber He. In contrast, in TiT the emitter T is in a TIS but the absorber He sits only in the off-resonant OIS. The NbT system thus uniquely meets the stringent demands of a viable T-He matrix.

**Table 1 He transport parameters in NbT at 200K**

| M= Nb | E1 eV | E2 eV | E3 eV | D/cm² s |
|-------|-------|-------|-------|---------|
| 0.9⁰ | 0.13⁰ | 0.43⁰ | 1.1E-26⁰ |

Ref 7, ⁰Ref 8, ¹Assumes tritium pre-exponential D₀ (ref. 7)

**Table 2. Theoretical lattice energy data for T and ³He in Nb interstitial sites (IS) (Ref. 10)**

| Site   | EST(eV) | ZPE(eV) |
|--------|---------|---------|
| T      | He      |
| TIS    | -0.133  | -0.906  | 0.071  | 0.093  |
| OIS    | -0.113  | -0.903  | 0.063  | 0.082  |

The hypersharp fraction \( \gamma \approx f \), basically because in the best present judgement, no significant source of non-harmonic fluctuations (that do not lead to motional narrowing) is evident. The fearsome inhomogeneous broadening from multipole static fields is absent in the T-He case. However, motion need not be exclusively harmonic. Stochastic motions such as sudden jumps in diffusion are known in tritides at high temperatures and high concentrations. They are less applicable in the low concentrations here. These motions are not narrowed, they create kHz broadening. Other types of relaxation unrelated to the nuclear coordinate \( r \), thus, to lattice vibrations, may also exist. The time scales of harmonic (THz) and stochastic (kHz) and other types of motions are so different that they do not interfere. The harmonic motional \( \nu_e \) is emitted with hypersharp widths and cross sections. The \( \nu_e \) from non-harmonic relaxation modes are broadened orders of magnitude wider with vastly smaller \( \sigma \). Thus the effect is only equivalent to a \( \nu_e \) flux loss which can be coped with by stronger sources/better geometry etc.

We can now sketch the bare experiment. We envision NbT sources and absorbers (gram scale to avoid crystal recoil to hypersharp precision!) made in an identical manner by the TT method of growing He by the decay of T. This implies a large T content in the absorber which will be a background (Tβ) against the signal of \( \nu_e \) induced T activity (Rβ). A chief design goal is to achieve a large Rβ/Tβ. The source and absorber are set on a suitable baseline (~1cm in the initial phase) in
the same cryogenic bath at temperatures $\ll 200\text{K}$. The signal is derived from the activation 18.6 keV betas (R$\beta$) from the absorber and the deviation caused by its growth from the known decay profile of T-betas (T$\beta$).

Using the ZPE (Z) data in Table 2 for He and T in Nb(TIS) the recoil free fraction $f = f(\text{He})f(T) = \text{exp} - \{27E_G/16\} [1/Z(T)+1/Z(\text{He})] \approx 0.076$. Table 3 shows signal rates in a primitive longitudinal geometry of source-absorber. The 1cm baseline (top line) can test most of the design optimization. The 1kCi source is state of the art. The He absorber is made with the TT method and uses only a 1Ci T source to grow 1pg of He in 100 days. The signal rate grows linearly (a signature of $\nu_e$ activation) and after a delay of 100 days, the signal R$\beta$ is 100 Hz vs. the background T$\beta$ of 37 kHz. The S/B is $\sim 1/400$, sufficient to confirm the signal growth vs. the T-decay.

For longer baselines, e.g. 10m needed to observe $\theta_{13}$ oscillations, the signal rate must be upped by $10^6$ mostly via the target mass. In the TT method this entails GHz T$\beta$ rates in the absorber that present serious counting problems. Desorption of the T activity by $\sim 10^6$ via H exchange is considered practical. An alternate He loading approach, low energy low temperature ion implantation of He in NbT$^+$, could be practical because of the very small doses of He involved. Both T desorption and implantation eliminate the deadweight of T activity in the absorber altogether. Applied to short baselines (line 1 in Table 3) this can similarly drastically reduce the source strength needed.

The estimates of Table 3 show good prospects that hypersharp resonant $\nu_e$ capture could be observed in the not too distant future, revolutionizing $\nu_e$ experiments. An important objective is oscillations via $\theta_{13}$ mixing, in the foreground now with large reactor based experiments on km-scale baselines (e.g., Dayabay, Double-Chooz). The low energy of the tritium $\nu_e$ needs only 9.3m for equivalent sensitivity. The energy specificity of the signal makes it a text book test of $\nu_e$ oscillations. Active-sterile mixing fits well in the program since tests for a sterile state with $\Delta m^2 \sim 1\text{eV}^2$ needs a baseline of only $\sim 6\text{ cm}$. The search for sterile states can thus be extended to a wide area of the $\Delta m^2$ parameter space.

Mead suggested that a fundamental length (Planck length) $\ell$ in nature would limit the ultimate widths of nuclear states i.e. $\Delta E/E \approx \ell/c$ (with the definition $\ell = (G\ h/c^3)^{1/2} \sim 10^{-33}\text{ cm}$, Mead predicts $\Delta E/E(\ell) = \ell/\ell(R) \beta \sim 10^{-20}$ (for $\beta = 1$) to $\Delta E/E(\ell) \sim 10^{-40}$ (for $\beta = \ell/\ell(R)$) ($R$ is the nuclear radius). The form of $\beta$ (in general ($\ell/R)^2$), depends on the quantum gravity model. The hypersharp tritium $\nu_e$ resonance offers the ideal probe of this prediction. Indeed, a width $\Delta E/E\sim 10^{-20}$ implies a Planck broadening by $10^9$ of the tritium $\nu_e$ resonance. A dilution of $\sigma$ by this large a factor can be easily detected. Observation of the T $\nu_e$ resonance would already preclude $\beta = 1$. Further results from scans of the resonance can influence quantum gravity models.

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**Table 3.** T-$^3$He hyper sharp capture rates. $\Delta t$ is the counting delay after start of activation. TT is the "tritium trick".

| He abs. | Base line | T | $^3$He | R$\beta$/d | T$\beta$/d |
|---------|-----------|---|--------|-----------|-----------|
| TT      | 1 cm      | 1kCi | 1 pg   | 10$^4$ ($\Delta t=100$ d) | 3.7x10$^{-10}$ |
| He Implant | 1cm     | 1mCi | 1 pg   | 10 ($\Delta t=0$ d) | $\sim 0$ |
| He Implant | 10m     | 1kCi | 1 pg   | 10 ($\Delta t=0$ d) | $\sim 0$ |