Hypersingular integral equation for triple inclined cracks problems in half plane elasticity

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Abstract. Hypersingular integral equation associated with the modified complex potential is formulated to solve the three inclined cracks problems in an elastic half-plane with free traction boundary condition. The modified complex potential possesses two parts: the principal and the complementary parts. The principal part is derived from the original complex potential of the crack problem in an infinite plate. The complementary part eliminates the traction along boundary of half-plane caused by the principal part. The crack opening displacements (COD) is the unknown function and the traction is the right hand terms. The appropriate quadrature formula is adapted to solve the integral equation numerically and the stress intensity factor (SIF) is computed. The behaviour of SIF at crack tips is analysed. Numerical examples show that the SIF increases as the angle of inclined cracks and the distance of cracks from the boundary of half-plane increase. Our results are agreeable with the previous works.

1. Introduction

The study of cracks is one of the important element in fracture mechanics concerning to predict the strength of the materials. It is necessary to analyse the behaviour of stress intensity factor (SIF) as it will indicate the toughness of materials containing cracks. Various methods are proposed by many researchers to formulate the cracks problems. New integral equation with log singular kernels for plane elasticity crack problems was dealt by Chen and Cheung [1]. The boundary element method of a straight and edge cracks was developed by Denda and Dong [2]. The hypersingular integral equation (HSIE) [3] and singular integral equation (SIE) [4] methods were established for the solution of multiple curved cracks problems in plane elasticity. Aridi et al. [5] used the HSIE for the interaction of straight and curved cracks in plane elasticity. Rafar et al. [6] applied the HSIE for the problem of two cracks in circular position in plane elasticity.

A curved crack problem in the upper half-plane was solved in [7] using a weakly singular integral equation (WSIE). Half-plane problems for transversely isotropic piezoelectric materials was investigated using the potential theory method by Huang et al. [8]. Chen et al. [9] solved the curved cracks problems in an elastic half-plane using HSIE. A distributed dipole technique for the analysis of multiple, straight, kinked and branched cracks in half-plane was developed by Hallback and Tofique [10]. Monfared and Ayatollahi [11] analysed the SIF of cracked orthotropic half-plane using the dislocation method involving complex Fourier transform and the Cauchy singularity. The multiple curved cracks problems in an elastic half-plane was formulated into...
SIE using modified complex potentials by Elfakhakhre et al. [12].

In this paper, HSIE together with the modified complex potentials and free traction boundary condition is adapted to formulate the problem of triple inclined cracks in an elastic half-plane. This paper is arranged in the following order; Section 2 deals with mathematical formulation of the problem, Section 3 provides some numerical works, and follow up with the conclusion in Section 4.

2. Mathematical formulation

2.1. Complex variable function method

Complex variable function method is used to formulate the complex potentials. The stresses \((\sigma_x, \sigma_y, \sigma_{xy})\), the resultant force functions \((X, Y)\) and the displacements \((u, v)\) can be described by two complex potentials \(\phi(\eta)\) and \(\psi(\eta)\) as follows [13]:

\[
\begin{align*}
\sigma_x + \sigma_y &= 4 \operatorname{Re}\Phi(\eta) \quad (1) \\
\sigma_y + i\sigma_{xy} &= 2 \operatorname{Re}\Phi(\eta) + \eta\Phi'(\eta) + \Psi(\eta) \quad (2) \\
f &= -Y + iX = \phi(\eta) + i\phi'(\eta) + \psi(\eta) \quad (3) \\
2G(u + iv) &= \kappa\phi(\eta) - \eta\phi'(\eta) - \psi(\eta) \quad (4)
\end{align*}
\]

where \(\Phi(\eta) = \phi'(\eta), \Psi(\eta) = \psi'(\eta)\) are the derivatives of the complex potentials, \(G\) is the shear modulus of elasticity, \(\kappa = (3 - v)/(1 + v)\) is for the plane stress problem, \(\kappa = 3 - 4v\) is for the plane strain problem, and \(v\) is the Poisson’s ratio and a bar over a function denotes the conjugated value. Derivative in a specified direction (DISD) is defined by differentiating equation (3) with respect to \(\eta\) as:

\[
N + iT = \frac{d}{d\eta}(-Y + iX) = \phi'(\eta) + \phi'(\eta) + \frac{d\eta}{d\eta}(\eta\phi''(\eta) + \psi'(\eta)).
\]

where \(N\) and \(T\) are the known normal and tangential tractions.

2.2. Modified complex potentials for crack problem in an elastic half-plane

Modified complex potentials (MCP) is applied for the elastic half-plane with free traction boundary condition. MCP consists of two parts which are principal and complementary parts, expresses as:

\[
\begin{align*}
\phi(\eta) &= \phi_p(\eta) + \phi_c(\eta) \quad (6) \\
\phi'(\eta) &= \phi'_p(\eta) + \phi'_c(\eta) \quad (7) \\
\psi(\eta) &= \psi_p(\eta) + \psi_c(\eta) \quad (8) \\
\psi'(\eta) &= \psi'_p(\eta) + \psi'_c(\eta) \quad (9)
\end{align*}
\]

where \(\phi_p(\eta), \phi'_p(\eta), \psi_p(\eta), \psi'_p(\eta)\) are the principal parts and \(\phi_c(\eta), \phi'_c(\eta), \psi_c(\eta), \psi'_c(\eta)\) are the complementary parts. Principal part is derived from the crack opening displacements (COD)
distribution along the crack $L$ in an original problem of infinite plate. Let the complex potentials of the principal part be given as follows \[9\] :

\[
\phi_p(\eta) = \frac{1}{2\pi} \int_L \frac{g(t)dt}{t-\eta}, \quad (10)
\]

\[
\phi_p'(\eta) = \frac{1}{2\pi} \int_L \frac{g(t)dt}{(t-\eta)^2}, \quad (11)
\]

\[
\psi_p(\eta) = \frac{1}{2\pi} \int_L \frac{g(t)dt}{t-\eta} + \frac{1}{2\pi} \int_L g(t) \left( \frac{dl}{t-\eta} - \frac{idt}{(t-\eta)^2} \right), \quad (12)
\]

\[
\psi_p'(\eta) = \frac{1}{2\pi} \int_L \frac{g(t)dt}{(t-\eta)^2} + \frac{1}{2\pi} \int_L g(t) \left( \frac{dl}{(t-\eta)^2} - \frac{2idt}{(t-\eta)^3} \right), \quad (13)
\]

where the unknown function, $g(t)$ is the crack opening displacement (COD) defined by :

\[
g(t) = \frac{2G}{i(\kappa+1)} \left[(u(t) + iv(t))^+ - (u(t) + iv(t))^-\right], \quad (t \in L) \quad (14)
\]

$(u(t) + iv(t))^+$ and $(u(t) + iv(t))^-$ denote the displacements at a point $t$ of the upper and lower parts of crack faces. The complementary part eliminates the traction along the boundary of half-plane caused by the principal part. From equation (3), the traction free condition along the boundary of half-plane ($L_0$) can be described as :

\[
\phi(\eta) + \eta \phi' (\eta) + \psi(\eta) = 0, \quad \eta \in L_0 \quad (15)
\]

Then, using equations (6), (7), and (8), condition (15) can be rewritten as :

\[
\left[\phi_p(\eta) + \phi_c(\eta)\right] + \eta \left[\phi_p'(\eta) + \phi_c'(\eta)\right] + \left[\psi_p(\eta) + \psi_c(\eta)\right] = 0, \quad \eta \in L_0 \quad (16)
\]

Substituting equations (10), (11), (12) and (13) into equation (16), after some manipulation, gives :

\[
\phi_c(\eta) = -\psi_p(\eta) - \eta \phi_p'(\eta), \quad (17)
\]

\[
\phi_c'(\eta) = -\psi_p'(\eta) - \psi_p''(\eta) - \eta \phi_p''(\eta), \quad (18)
\]

\[
\psi_c(\eta) = -\phi_p(\eta) + \eta \phi_p'(\eta) + \eta \psi_p(\eta) + \eta^2 \phi_p''(\eta), \quad (19)
\]

where $\phi_p'(\eta)$ is an analytic function denoted by $\phi_p'(\eta) = \phi'(\eta)$. From the known complex potentials $\phi_p(\eta)$ and $\psi_p(\eta)$, one gets $\phi_c(\eta)$ and $\psi_c(\eta)$. Hence $\phi(\eta)$ and $\psi(\eta)$ can be established from (6) and (8). This process is called modified complex potential (MCP).

### 2.3. Hypersingular integral equation for triple inclined cracks in half-plane elasticity

The formulation of cracks problems using HSIE is defined by two parts which are $[N(t_0) + iT(t_0)]_p$ and $[N(t_0) + iT(t_0)]_c$. For $[N(t_0) + iT(t_0)]_p$, substituting equations (10)-(13) into equation (5) and for $[N(t_0) + iT(t_0)]_c$ substituting equations (17)-(19) into equation (5), then letting $\eta$ approach $t_0$ and changing $d\eta/d\eta$ by $dt/dt$. The tractions are obtained by taken the observation point $t_0$ on the crack $L$. Summing the principal and complementary parts yields the following equations for a single crack problem :

\[
[N(t_0) + iT(t_0)] = \frac{1}{\pi} \int_L \frac{g(t)dt}{(t - t_0)^2} + \frac{1}{2\pi} \int_L \mu_1(t, t_0) g(t) dt + \frac{1}{2\pi} \int_L \mu_2(t, t_0) g(t) dt, \quad t_0 \in L \quad (20)
\]
where the kernels are defined as

\[
\mu_1(t, t_0) = -\frac{1}{(t-t_0)^2} - \frac{1}{(t-t_0)^2} - \frac{2(t_0 - t)}{(t-t_0)^3} - \frac{1}{(t-t_0)^2} \frac{d\bar{t}}{dt} - \frac{1}{(t-t_0)^2} \frac{d\bar{t}}{dt}
\]

\[
+ \frac{dt_0}{dt_0} \left( \frac{1}{(t-t_0)^2} \frac{d\bar{t}}{dt} + \frac{1}{(t-t_0)^2} \frac{d\bar{t}}{dt} + \frac{2(3\bar{t}_0 - 2t_0 - t)}{(t-t_0)^3} + \frac{2(t_0 - t)}{(t-t_0)^3} \right)
\]

\[
\mu_2(t, t_0) = -\frac{1}{(t-t_0)^2} - \frac{1}{(t-t_0)^2} - \frac{1}{(t-t_0)^2} \frac{d\bar{t}}{dt} + \frac{1}{(t-t_0)^2} \frac{d\bar{t}}{dt} + \frac{2(t_0 - t)}{(t-t_0)^3} + \frac{2(t_0 - t)}{(t-t_0)^3} \frac{d\bar{t}}{dt}
\]

\[
+ \frac{dt_0}{dt_0} \left( \frac{1}{(t-t_0)^2} + \frac{1}{(t-t_0)^2} + \frac{2(t_0 - t)}{(t-t_0)^3} \right)
\]

For triple cracks problems, let \(N_j(t_{j0}) + iT_j(t_{j0})\) represent the tractions applied at the point \(t_{j0}\) of the crack-\(j\) for \(j = 1, 2, 3\). By superposition of the COD distribution \(g_j(t_j)\) along the crack-\(j\), the system of HSIEs for triple cracks problems is obtained as follows:

\[
\frac{1}{\pi} \int_{L_j} g_j(t_j) \frac{dt_j}{(t_j - t_{j0})^2} + \frac{1}{2\pi} \int_{L_j} \mu_1(t_j, t_{j0}) g_j(t_j) \frac{dt_j}{(t_j - t_{j0})^2} + \frac{1}{2\pi} \int_{L_j} \mu_2(t_j, t_{j0}) g_j(t_j) \frac{dt_j}{(t_j - t_{j0})^2}
\]

\[
+ \sum_{k=1}^{3} \left\{ \frac{1}{\pi} \int_{L_k} g_k(t_k) \frac{dt_k}{(t_k - t_{j0})^2} + \frac{1}{2\pi} \int_{L_k} \mu_1(t_k, t_{j0}) g_k(t_k) \frac{dt_k}{(t_k - t_{j0})^2} + \frac{1}{2\pi} \int_{L_k} \mu_2(t_k, t_{j0}) g_k(t_k) \frac{dt_k}{(t_k - t_{j0})^2} \right\}
\]

\[
+ \sum_{m=1}^{3} \left\{ \frac{1}{\pi} \int_{L_m} g_m(t_m) \frac{dt_m}{(t_m - t_{j0})^2} + \frac{1}{2\pi} \int_{L_m} \mu_1(t_m, t_{j0}) g_m(t_m) \frac{dt_m}{(t_m - t_{j0})^2} + \frac{1}{2\pi} \int_{L_m} \mu_2(t_m, t_{j0}) g_m(t_m) \frac{dt_m}{(t_m - t_{j0})^2} \right\}
\]

\[
= N_j(t_{j0}) + iT_j(t_{j0}), \quad t_{j0} \in L_j
\]

(21)

where \(j \neq k \neq m\).

The first integrals is hypersingular, and the remaining are regular. The first three integrals represent the effect on crack-\(j\) caused by crack-\(j\) itself, while the second three integrals show the effect of the crack-\(k\) on crack-\(j\) and the last three integrals describe the effect of the crack-\(m\) on crack-\(j\) for \(j = 1, 2, 3\) where \(j \neq k \neq m\). All the integrals will be mapped on the real axis \(s\) with the interval \(2a\) by using the curve length coordinate method defined as [4] :

\[
g_j(t_j)_{|t_j=t_{j0}(s_j)} = \sqrt{a^2 - s_j^2} H_j(s_j) \quad \text{where} \quad H_j(s_j) = H_{j1}(s_j) + iH_{j2}(s_j) \quad \text{for} \quad j = 1, 2, 3
\]

(22)
3. Numerical examples

The stress intensity factor (SIF) at \( D_{jk} \) of crack-\( j \) for \( j = 1, 2, 3 \) and crack tips-\( k \) for \( k = 1, 2 \) be given as follows:

\[
K_{D_{jk}} = (K_1 - iK_2)_{D_{jk}} = -\sqrt{2\pi} \lim_{t\to t_{D_j}} \sqrt{|t-t_{D_j}|} g_j^*(t_j) = \sqrt{\pi a_j} F_{D_{jk}}
\]

(23)

where \( F_{D_{jk}} = (F_{1D_{jk}} + iF_{2D_{jk}}) \), \( F_{1D_{jk}} \) and \( F_{2D_{jk}} \) are Mode I and Mode II of nondimensional SIF of tips \( k \) of crack \( D_j \).

![Diagram](image)

**Figure 1.** Three inclined cracks in the upper half-plane.

3.1. Example 1

Figure 1(a) illustrates three inclined cracks in series in the upper half-plane with the remote stress \( \sigma_x^\infty = p \) and the traction is free at the boundary. The length of each crack is \( 2R \), \( d \) is the distance between each crack and \( h/R \) is the distance between the tips of cracks to the boundary. \( A_1, A_2, B_1, B_2, C_1 \) and \( C_2 \) are the cracks tips and \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are the angles of inclined cracks.

Table 1 shows Mode I of nondimensional SIF \( F_1 \) and Mode II of nondimensional SIF \( F_2 \) for three inclined cracks in series when \( \alpha_1 = \alpha_2 = \alpha_3 = 90^\circ \) and \( h/R \) varies. From Table 1, it is found that \( F_1 \) at crack tip \( A_1 \) is equal to \( F_1 \) at crack tip \( C_1 \) and \( F_1 \) at crack tip \( A_2 \) is equal to \( F_1 \) at crack tip \( C_2 \). Whereas \( F_2 \) is equal to zero at all crack tips. As the distance of cracks is far from the boundary of half-plane, the SIF approaches to the SIF for cracks in an infinite plane. This observation is in good agreement with those of Denda and Dong [2].

Figures 2(a) and 2(b) represent the behaviour of nondimensional SIF when \( \alpha \) varies and \( h/R \) is 0.5 as defined in Figure 1(a). It can be observed that the nondimensional SIF for \( F_1 \) at crack tip \( A_1 \) is equal to \( F_1 \) at crack tip \( C_1 \), \( F_1 \) at crack tip \( A_2 \) is equal to \( F_1 \) at crack tip \( C_2 \) and \( F_2 \) at crack tip \( B_1 \) is equal to \( F_2 \) at crack tip \( B_2 \) is equal to zero for \( \alpha_1 = \alpha_2 = \alpha_3 = 90^\circ \). Figure 2(b), shows that the nondimensional SIF for \( F_1 \) at all cracks tips increases as the inclined angle increases. Meanwhile the nondimensional SIF for \( F_2 \) at all cracks tips decreases when \( \alpha_2 = \alpha_3 < 50^\circ \) and increases for \( \alpha_2 = \alpha_3 > 50^\circ \).

Figure 3(a) shows the nondimensional SIF when \( \alpha_1 = \alpha_2 = \alpha_3 = 45^\circ \) and \( h/R \) varies for the problem presented in Figure 1(a). The nondimensional SIF for \( F_1 \) at all cracks tips decreases as \( h/R \) increases. It is observed that at crack tip \( A_1 \) and \( A_2 \), \( F_2 \) increases and decreases respectively.
$F_2$ increases when $h/R$ increases at cracks tips $B_1$, $C_1$ and $C_2$. The nondimensional SIF for different values of $h/R$ and $\alpha_1 = \alpha_2 = \alpha_3 = 60^\circ$ is represented by Figure 3(b). As $h/R$ increases, $F_1$ decreases while $F_2$ increases at cracks tips $A_1$, $B_1$ and $C_1$. The nondimensional SIF for $F_1$ decreases at cracks tips $A_2$, $B_2$ and $C_2$. Meanwhile the nondimensional SIF for $F_2$ at crack tip $A_2$ decreases, at crack tip $B_2$ constant when $h/R > 0.4$ and at crack tip $C_2$ increases.

Table 1. SIF for three inclined cracks in series when $h/R$ varies (Figure 1(a)).

| $h/R$ | SIF   | 2R/d  |
|-------|-------|-------|
|       |       | 0.1   | 0.2   | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   |
| 0.8   | $F_{1A}^*$ | 0.88643 | 0.87068 | 0.85424 | 0.84081 | 0.82986 | 0.81978 | 0.80958 |
|       | $F_{1A}^*$ | -0.85937 | -0.84186 | -0.81976 | -0.79783 | -0.77840 | -0.76226 | -0.74941 |
|       | $F_{1B}^*$ | 0.88239 | 0.85824 | 0.83533 | 0.81947 | 0.80812 | 0.79691 | 0.78326 |
|       | $F_{1B}^*$ | -0.85528 | -0.82766 | -0.79412 | -0.76339 | -0.73935 | -0.72234 | -0.71106 |
| 12.0  | $F_{1A}^*$ | 0.88643 | 0.87068 | 0.85424 | 0.84081 | 0.82986 | 0.81978 | 0.80958 |
|       | $F_{1A}^*$ | -0.85937 | -0.84186 | -0.81976 | -0.79783 | -0.77840 | -0.76226 | -0.74941 |
|       | $F_{1B}^*$ | 0.99785 | 0.98576 | 0.96670 | 0.94375 | 0.91943 | 0.89565 | 0.87359 |
|       | $F_{1B}^*$ | 0.99800 | 0.98601 | 0.96698 | 0.94404 | 0.91973 | 0.89595 | 0.87389 |
| 16.0  | $F_{1A}^*$ | 0.99785 | 0.98576 | 0.96670 | 0.94375 | 0.91943 | 0.89565 | 0.87359 |
|       | $F_{1A}^*$ | 0.99698 | 0.98444 | 0.96530 | 0.94233 | 0.91801 | 0.89424 | 0.87222 |
|       | $F_{1A}^*$ | 0.99709 | 0.98458 | 0.96545 | 0.94249 | 0.91817 | 0.89441 | 0.87239 |
|       | $F_{1B}^*$ | 0.99448 | 0.97403 | 0.94304 | 0.90558 | 0.86562 | 0.82628 | 0.78958 |
|       | $F_{1B}^*$ | 0.99462 | 0.97419 | 0.94322 | 0.90577 | 0.86583 | 0.82649 | 0.78980 |
|       | $F_{1C}^*$ | 0.99698 | 0.98444 | 0.96530 | 0.94233 | 0.91801 | 0.89424 | 0.87222 |
|       | $F_{1C}^*$ | 0.99709 | 0.98458 | 0.96545 | 0.94249 | 0.91817 | 0.89441 | 0.87239 |
| 20.0  | $F_{1A}^*$ | 0.99650 | 0.98379 | 0.96461 | 0.94162 | 0.91730 | 0.89356 | 0.87156 |
|       | $F_{1A}^*$ | 0.99655 | 0.98385 | 0.96468 | 0.94169 | 0.91738 | 0.89363 | 0.87163 |
|       | $F_{1B}^*$ | 0.99386 | 0.97334 | 0.94228 | 0.90481 | 0.86489 | 0.82560 | 0.78895 |
|       | $F_{1B}^*$ | 0.99392 | 0.97341 | 0.94236 | 0.90490 | 0.86499 | 0.82570 | 0.78905 |
|       | $F_{1C}^*$ | 0.99650 | 0.98379 | 0.96461 | 0.94162 | 0.91730 | 0.89356 | 0.87156 |
|       | $F_{1C}^*$ | 0.99655 | 0.98385 | 0.96468 | 0.94169 | 0.91738 | 0.89363 | 0.87163 |
| $\infty$ | $F_{1A}^{**}$ | 0.99687 | 0.98379 | 0.96430 | 0.94100 | 0.91650 | 0.89254 | 0.87041 |
|       | $F_{1B}^{**}$ | 0.99410 | 0.97306 | 0.94156 | 0.90361 | 0.86789 | 0.82333 | 0.78603 |

*Current study
**[2]

3.2. Example 2
Figure 1(b) represents three parallel inclined cracks in the upper half-plane subjected to remote stress $\sigma_x^\infty = p$ in the upper half-plane with free traction boundary condition. Each crack has length $2R$, and the distance between each crack is $d$. The distance from the center of lower crack to the boundary of half-plane is $h/R$. The cracks tips are defined as $A_1$, $A_2$, $B_1$, $B_2$, $C_1$ and
$C_2$, $\alpha_1$, $\alpha_2$ and $\alpha_3$ are denoted as the angles of inclined cracks.

Figures 4(a) and 4(b) portray the behaviour of nondimensional SIF when $\alpha$ varies from $0^\circ$ to $135^\circ$ and $h/R = 0.5$ for the problem defined in Figure 1(b). As the inclined angle increases, the nondimensional SIF does not show any significant differences in Figure 4(a). In Figure 4(b), the nondimensional SIF for $F_1$ at all cracks tips increases for $\alpha_1 = \alpha_2 = \alpha_3 < 80^\circ$ and decreases when $\alpha_1 = \alpha_2 = \alpha_3 > 100^\circ$. The nondimensional SIF for $F_2$ increases at all cracks tips when $\alpha_1 = \alpha_2 = \alpha_3 > 45^\circ$. It is found that when $\alpha_1 = \alpha_2 = \alpha_3 = 90^\circ$ the nondimensional SIF for $F_2$ at all crack tips is equal to zero.

Figures 5(a) and 5(b) illustrate the behaviour of nondimensional SIF when the inclined angle, $\alpha_1$, $\alpha_2$ and $\alpha_3$ are fixed and $h/R$ varies as defined in Figure 1(b). As $h/R$ increases, the nondimensional SIF increases. Figure 5(a) shows the crack tip $A_1$ has the highest value of nondimensional SIF for $F_1$, whereas crack tip $C_2$ has the lowest value of nondimensional SIF for $F_1$ when $h/R = 0.9$. In Figure 5(b), $F_2$ increases at all cracks tips but decreases at crack tip $C_2$ when $h/R > 0.8$.

![Figure 2](image1.png)  
**Figure 2.** Nondimensional SIF for $h/R = 0.5$ and angle, $\alpha$ varies (Figure 1(a)).

![Figure 3](image2.png)  
**Figure 3.** Nondimensional SIF for fixed angle, $\alpha$ and $h/R$ varies (Figure 1(a)).
4. Conclusion
In this paper, three inclined cracks subjected to remote stress in the upper half-plane is considered. By applying the modified complex potential method, and making use the free stress at the boundary, the HSIE is formulated. These equations are solved numerically using the appropriate quadrature formulas. As a conclusion, the nondimensional SIF at the cracks tips is influenced by the inclined angle of cracks and the distance between cracks and the boundary of upper half-plane. SIF increases as the cracks approaching the boundary of half-plane and the inclined angle of cracks increases.

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