Numerical Prediction Methods for Clock Deviation Based on Two-Way Satellite Time and Frequency Transfer Data

GUO Hairong  YANG Yuanxi  HE Haibo

Abstract  Three functional models, polynomial, spectral analysis, and modified AR model, are studied and compared in fitting and predicting clock deviation based on the data sequence derived from two-way satellite time and frequency transfer. A robust equivalent weight is applied, which controls the significant influence of outlying observations. Some conclusions show that the prediction precision of robust estimation is better than that of LS. The prediction precision calculated from smoothed observations is higher than that calculated from sampling observations. As a count of the obvious period variations in the clock deviation sequence, the predicted values of polynomial model are implausible. The prediction precision of spectral analysis model is very low, but the principal periods can be determined. The prediction RMS of 6-hour extrapolation interval is 1 ns or so, when modified AR model is used.

Keywords  time prediction; time transfer; two-way satellite time and frequency transfer

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Introduction

With the development of spread spectrum technology and pseudo-noise code technology, time transfer technology has improved rapidly. Currently, time synchronization precision estimated from two-way satellite time and frequency transfer (TWSTFT) data has come to 0.2~0.3ns \cite{1-3}. Considering that the clock deviation sequence derived from TWSTFT is a random time sequence, the influences of random errors and outlying observations will be changed into a systematic component when these clock deviation data are employed for time prediction. Therefore, numerous observations should be used to reduce the influence of random errors. Robust estimation is applied to control the influences of outlying observations on parameter estimates. Then three functional models: polynomial, spectral analysis, and modified AR model are adopted for parameter fit and time prediction performed over the clock deviation sequence in this paper.

1  Functional models and estimates

1.1  Polynomial model

Assume that the clock deviations are \(x_1, x_2, \cdots, x_n\) at epochs \(t_1, t_2, \cdots, t_n\) respectively. Then the observation equation is expressed as the polynomial model as below:

\[
\Delta x_i = a_0 + a_1 \Delta t + a_2 \Delta t^2 + \cdots + a_m \Delta t^m + e_i \quad (1)
\]
where $\Delta t = t - t_0$, $\Delta x_i = x_i - x_{i_0}$, $a_0, a_1, \ldots, a_m$ are unknown parameters, and $e_i$ is an observation error.

Eq.(1) is also presented in matrix as

$$\Delta X = Ha + e$$

where $\Delta X$ is an observation vector; $H$ is an $n \times m$ design matrix; $a$ is an unknown parameter vector, and $e$ is an observation error vector.

### 1.2 Spectral analysis model

The observation equation expressed using the spectral analysis model is:

$$x_i = a_0 + b_0 t_i + \sum_{k=1}^{p} A_k \sin(2\pi f_k t_i) + \sum_{k=1}^{p} \varphi_k \cos(2\pi f_k t_i) + e_i$$

where $a_0$ and $b_0$ are a constant and a trend term of long-term variation, respectively; $p$ is the number of principal periods; $f_k$ is the frequency of the k-th periodic term; $A_k$ and $\varphi_k$ are the corresponding amplitude and phase; $e_i$ is an observation error. And $p$ and $f_k$ can be determined from the power-frequency chart\(^{[4]}\).

The linear expression of Eq.(3) can be written as follows:

$$x_i = a_0 + b_0 t_i + \sum_{k=1}^{p} (A_k \sin(2\pi f_k t_i) + b_k \cos(2\pi f_k t_i)) + e_i$$

Then Eq.(4) is also presented in matrix as below:

$$X = Ha + e$$

where $X$ is an observation vector made of clock deviation $x_i$ ; $H$ is a design matrix; $a$ is an unknown parameter vector, and $e$ is an observation error vector.

### 1.3 Modified AR model

The observation equation expressed using the AR model is given below:

$$x_i = a_0 x_{i-1} + a_1 x_{i-2} + \cdots + a_m x_{i-m} + e_i$$

where $a_0, a_1, \ldots, a_m$ are unknown parameters, $e_i$ is an observation error. Eq.(6) is also presented in matrix as

$$X = Ha + e$$

where $H = \begin{bmatrix} x_m & x_{m-1} & \cdots & x_i \\ x_{m+1} & x_m & \cdots & x_i \\ \vdots & \vdots & \vdots & \vdots \\ x_{n-1} & x_{n-2} & \cdots & x_{n-m} \end{bmatrix}$ is a coefficient matrix; $X = \begin{bmatrix} x_m \\ x_{m+1} \\ \vdots \\ x_{n-1} \end{bmatrix}$ is an $(n-m)$ observation vector.

In order to model and predict clock deviations with AR model, the clock deviation sequence should be converted into a random time sequence. Considering that there are obvious long-term trends, daily and semi-daily periods in the clock deviation sequence, the differential operation should be as follows\(^{[5,6]}\):

$$\nabla \nabla_t X = (X_i - X_{i-1}) - 2(X_{i-1} - X_{i-2}) + (X_{i-2} - X_{i-3})$$

where $\tau$ is one of the longest periods in the time sequence.

Obviously, the linear and quadratic terms are eliminated from the time sequence by the above operation. And the principal periods are also removed. Then a random time sequence is obtained and the AR model can be applied for time prediction.

### 1.4 Parameter estimation

The least squares estimator of the unknown parameter vector in Eqs.(2), (5) and (7) is

$$\hat{a} = (H^T PH)^{-1} H^T PX$$

$$\hat{\sigma}_0^2 = \frac{V^T PV}{n-m}$$

where $P$ is the weight matrix of observation vector; $V$ is the residual vector; $n$ is the number of observations; $m$ is the number of unknown parameters.

If a robust equivalent weight is applied based on the residuals of the observations, then the significant influences of outlying observations can be controlled partially. Then the robust estimator for unknown parameters is:

$$\hat{a}_r = (H^T \overline{P} H)^{-1} H^T \overline{P} X$$

$$\sigma_r^2 = \frac{V^T \overline{P} V}{n-m-s}$$

where $s$ is the number of $\bar{p}_j = 0$; $\overline{P}$ is equivalent weight matrix and its element is expressed as\(^{[7]}\):

$$\bar{p}_i = \begin{cases} p_i, & |v_i/\sigma_v| \leq k_0 \\ p_i k_0 - k_i |v_i/\sigma_v|, & k_0 < |v_i/\sigma_v| \leq k_i \\ 0, & |v_i/\sigma_v| > k_i \end{cases}$$

where $k_0 = 1.0 - 2.0$, $k_i = 2.5 - 4.5$. 


2 Determination of the orders of functional models

The linear or quadratic model was applied to GPS clock deviation prediction. Considering that the characteristics of the adopted clocks are inferior to those of GPS clocks, the linear, quadratic and third order polynomial models have been used for analyzing clock deviation prediction and precision analysis.

The energy spectral analysis can be used for determining the long-term trend and principal periods in the clock deviation sequence. The primary calculations show that there are 6.8-hour, 3.8-hour, 2.84-hour and 2.27-hour periods in the 1.5 second sampling or smoothed clock deviation sequence. And there are 6.8-hour, 4.27-hour, 2.84-hour and 2.27-hour periods in the 30 second and 1 minute sampling or smoothed clock deviation sequence. Therefore, principal periods are slightly changed for different sampling interval data. High-frequency sampling data can be used for discovering short period variations. At the same time, these data are unfavorable in determining long period variations. However, low-frequency sampling data is favorable for determining long period variations, at the same time, short period variations will also be covered. Then the 15 second, 30 second and 1 minute sampling or smoothed clock deviation sequences are used for time prediction analysis with the modified AR model.

It is found from experience that the order of AR model should be less than \( \sqrt{n} \) or \( n/10 \), where \( n \) is the number of observations\[^8\]. Then the 20 order AR model is adopted in the following calculation.

3 Prediction model and precision estimation

3.1 Polynomial prediction

After the unknown parameter vector has been estimated, the predicted clock deviation \( \hat{x}_{t+1} \) can be calculated with the polynomial model as follows:

\[
\hat{x}_{t+1} = x_{t0} + H^T_{t+1} \hat{a}
\]  

(14)

where \( H^T_{t+1} = [\Delta t_{t1}, \Delta t_{t2}, \ldots, \Delta t_{tn}] \), \( \Delta t_{t+1} = t_{t+1} - t_0 \).

The root mean square of prediction residuals is:

\[
\text{RMS} = \sqrt{\frac{\sum_{t=1}^{l} (x_{t+1} - \hat{x}_{t+1})^2}{l}}
\]

(15)

3.2 Spectral analysis prediction

The predicted clock deviation is calculated with the spectral analysis model as follows:

\[
d\hat{X}_{t+1} = H^{T}_{t+1} \hat{a}
\]

(16)

where \( \hat{a} = [\hat{a}_0, \hat{b}_0, \hat{a}_1, \hat{b}_1, \ldots, \hat{a}_p, \hat{b}_p] \),

\[
H^T_{t+1} = [1 \ t_{t+1} \ \sin(2\pi f_{t+1}) \ \cos(2\pi f_{t+1}) \ \ldots \ \sin(2\pi f_{t+1}^p) \ \cos(2\pi f_{t+1}^p)]
\]

And the corresponding root mean square of prediction residuals can be calculated using Eq.(15).

3.3 Modified AR prediction

The predicted clock deviation is calculated with the modified AR model as below:

\[
\hat{X}_{t+1} = X_{t+1} + 2(X_{t+1} - X_{t+1-1})
\]

\[-(X_{t+1-1} - X_{t+1-2}) + \sum_{j=1}^{m} a_j \nabla \nabla x_{t+1}
\]

(17)

And the corresponding root mean square of prediction residuals can also be calculated using Eq.(15).

4 Computation and analysis

4.1 Data preprocessing

In order to understand the characteristics of clock deviation sequence and the approximate distribution of the corresponding observation errors, a preliminary analysis should be performed on the observed clock deviation sequence. A one-day clock deviation sequence derived from TWSTFT is used for analyzing the prediction characteristic in this paper. The data gaps are interpolated using linear function. And the Sagnac effect induced by satellite motion and the zero value of equipments has to be corrected in advance\[^9\].

Then the variations of the clock deviations, clock velocities, clock accelerations and Sagnac effect in one day are depicted in the following figures.

From Figs.1~3, we find that there are obvious long-term trend and principal periods in the clock deviation sequence. Additionally, large outlying observa-
tions are also obvious. Fig. 4 shows that the Sagnac effect has a periodic characteristic and the corresponding amplitude is as small as 0.1 ns. That means that the Sagnac effect contributes little to the periodic variations of clock deviation sequence and other influencing factors should be researched further in the future.

4.2 Result analysis

The preprocessed clock deviation sequences in different sampling intervals and different spans were adopted for clock deviation prediction with the polynomial, spectral analysis and the modified AR model. The unknown model parameters were estimated by the least squares method and robust estimation respectively. Firstly, the clock deviation sequences, where the linear and quadratic terms have been removed were given in the following:

From Figs. 5–6, we can see that there are obvious periodic variations with the amplitude of 0–70 ns in the preprocessed clock deviation sequence. The calculation and analysis show that the polynomial model cannot be used for time prediction owing to the obvious periodic variations in the clock deviation sequence. It is known that the linear or quadratic model is well applied to GPS satellite clock prediction since GPS satellite clocks have better time stability and are insensitive to environment changes. In order to predict the clock deviation using the polynomial model, the periodic variations should first be removed from the time sequence of the clock adopted in this paper.

The spectral analysis model can be used for determining the principal variations in the clock deviation sequence, but the model is inappropriate for time prediction on account of other period variations also presented in the clock deviation sequence. The calculations also show that the RMS of predicted clock
deviation is in the order of 60 ns when the 6 hour clock deviation data are used for extrapolation during the 6 hour prediction period.

The modified AR model is proper for time prediction on account of the long-term trend and the principal periods have been removed from the clock deviation sequence by the differential operation. The prediction precision of robust estimation is much better than that of LS estimation since there are outlying observations in clock deviation sequence. And the corresponding results of robust estimation are given in the following table:

Table 1  Prediction precision of different sampling, different span using the modified AR Model/\(\text{ns}\)

| Sampling scheme | 1 h-1 h RMS | 3 h-3 h RMS | 6 h-6 h RMS | Sampling scheme | 1 h-1 h RMS | 3 h-3 h RMS | 6 h-6 h RMS |
|-----------------|------------|------------|------------|-----------------|------------|------------|------------|
| 0.2s            | 0.46       | 0.34       | 0.45       | 15 s Sampling   | 1.6        | 1.4        | 1.2        |
| 1s Sampling     | 1.0        | 1.0        | 0.94       | 30 s Sampling   | 1.9        | 1.43       | 1.34       |
| 1s Smoothness   | 0.65       | 0.8        | 0.7        | 30 s Smoothness | 0.6        | 0.76       | 0.56       |
| 5s Sampling     | 1.0        | 1.24       | 1.1        | 5s Smoothness   | 0.3        | 0.9        | 0.65       |

From Table 1, the prediction RMS of different sampling and different span is 1 ns or so when the modified AR model is used for clock deviation prediction. The RMS calculated from sampling observations is 1 ns or so. The RMS calculated from smoothed observations is less than 1 ns. Therefore, the prediction precision calculated from smoothed observations is much higher than that calculated from sampling observations, but the former needs to store more observations than the latter.

In this paper, only half-of-the-day clock deviation data has been used for parameter estimation and time prediction with the modified AR model when the semi-daily period has been removed from one-day clock deviation sequence by the differential operation. Thus, in order to improve the prediction precision, the clock deviation sequence should be at least twice as long as the longest principal period so that the principal periods can be removed.

5 Conclusions

By calculation and analysis using one day clock deviation data, some preliminary conclusions are drawn. It is important that the prediction method is presented for the adopted clock in this paper. It is hoped that the more reliable predicted results will be given if longer clock deviation data are analyzed. Additionally, the determination of the AR model order still needs further research.

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