Model Prediction of Microcantilever using DOE for Stress and Eigen Frequency Analysis for Force Measurement

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Abstract. In this paper, stress and eigen frequencies are analyzed through prediction of microcantilever model using design of experiment approach. The important factors affecting displacement sensitivity, stress and eigen frequencies were identified such as geometrical dimensions of microcantilever (length, breadth and thickness), modulus of elasticity of materials, applied force. Based on the selection of factors and levels, experiments were designed using L9 orthogonal array. The designed experiments were simulated with finite element method in COMSOL Multiphysics software. Further, optimum values of dimensions of microcantilever and material property are used for stress and eigen frequencies analysis. The evaluated frequencies under eigen frequency analysis revealed that designed and simulated Gallium Arsenide (GaAs) microcantilever is prone to vibrate at these frequencies, whereas stress analysis illustrates the response of structure and material properties towards reliability.

Keyword: - Microcantilever, Force, Design of Experiment, Eigen frequency, stress analysis

1. Introduction

Recently, Micro-electro-mechanical systems (MEMS) technologies have gained a lot of attention due to its extra ordinary features and potential in field of microbotics, automobiles, aerospace, medical sciences, applications [1-4]. MEMS devices are fabricated, by taking advantage of miniaturization, low cost, less power, easy integration, highly resistant of heat, vibration and shock [5]. MEMS technology is mainly categorized into two classes i.e. Sensors and actuators [6]. There are various types of sensing and actuation mechanisms available in the industry. The selection of sensors and actuators depends upon, type application where it is implemented. This paper mainly focused on the sensing part of the MEMS technology. As per current market survey of MEMS sensor market is projected to reach $12.3 billion by 2024, at a CAGR (Compound annual growth rate) of 7% from 2017 to 2024 [7]. Different types of mechanism are available for converting one form of energy into another, for force measurement microcantilever is one of the flexible, versatility, low cost and highly sensitive mechanical structure that can be utilized for vast range of applications including chemical, physical and biological [8-11]. For achieving this various input factors of microcantilever are identified which are affecting displacement, stress sensitivity of microcantilever in correspondence with the applied
force. It is analytically proven from the famous Stoney’s equation that the displacement of the microcantilever depends on dimension of microcantilever and material properties under consideration [12]. Loui et al. reported that microcantilever with length greater than width are optimal for point-loading applications such as microscopy and force measurements [13]. Lim et al.; Nachippan et al. mentioned that by increasing the length reducing the thickness and material with lower modulus of elasticity improves the sensitivity [14-15]. Neethu et al. analyzed from their study that sensitivity of the microcantilever increases by increase in length and reducing thickness [16]. Zhang et al. reported that minimizing the effective mass near the microcantilever’s free end and the clamping width at the fixed end significantly increases the overall sensitivity of microcantilever [17]. From the above mentioned literature different factors affecting the sensitivity of the microcantilever is identified and taken into consideration for investigation in this paper.

Regardless of the above mentioned research work in the field of force measurement with microcantilever, still there exist certain gap towards optimization of microcantilever dimension with appropriate material identification from selected. To conquer this gap, in this paper initially factors affecting displacement and stress of microcantilever are identified, from the literature and analytical proven equation. Further identified factors are utilized for designing an experiment using Taguchi optimization to see the cumulative of various input factors on output. Taguchi approach is utilized as it successfully reduce the design time without compromising on the quality of the products as mentioned by Ghani et al.; Rosa et al. [18-19]. Designed experiments were simulated using COMSOL Multiphysics 5.3a software and the optimized model of microcantilever with respect of its dimension and material is obtained with higher displacement sensitivity. For this investigation materials under consideration are Silicon, Germanium and Gallium Arsenide. At high frequencies Gallium Arsenide (GaAs) microcantilever are less noisy and gives better results than silicon. Due to optical characteristics gallium arsenide material is most widely used in infrared light-emitting diodes, solar cells, laser diodes etc.

2. Design Methodology

2.1. Simulation Consideration

The purpose of this work is to utilize the mechanical properties of microcantilever for the designing of force sensor, which is able to operate in nanoNewton (nN) range. Microcantilever is fixed from one end and free from another end. Force is applied at the moving part of the cantilever and indicates output in form of displacement. As per the investigations conducted by [20-21] rectangular shape of moving part is more suitable for force measurement as compared to other shapes. The specification of the microcantilever is summarized in Table 1, indicates the operating range of sensor along with dimension of microcantilever. Dimension of the microcantilever is to be optimized using Taguchi approach. Mitra et al.; Taguchi et al. reported that Taguchi approach uses a specially design of orthogonal array to study the effect of various parameters with fewer number of experiments [22-23].

Table 1. Design parameter consideration of Piezoresistive MEMS Force sensor.

| Parameter Considered         | Value                                                                 |
|------------------------------|----------------------------------------------------------------------|
| Force range                  | 0-100 nanoNewton (nN)                                                |
| Dimension of Microcantilever | L μm × B μm × H μm (To be optimized by considering different values) |
| Simulated Platform           | COMSOL Multiphysics 5.3a                                             |

The procedure of Microcantilever optimization using Taguchi approach is illustrated in the Figure 1.
2.1.1. Selection of factors for Microcantilever optimization: Applied force at the tip of the microcantilever is responded in form of displacement. The deflection or displacement of the microcantilever is given by the relation shown below in equation (1) [24]

\[ \Delta = \frac{4FL^3}{EIW^3} \]  

Where is the deflection or displacement of the microcantilever, F is the applied force, E is young’s modulus of elasticity, L, W and H is the length, width and height of the cantilever respectively. This equation indicates that Force (F) and deflection (\( \Delta \)) are input and output quantities respectively. Further L, W, H is associated with microcantilever dimension optimization whereas E is material dependent parameter. So, the selected factors for optimization are L, W, H and E of the microcantilever. For optimizing the microcantilever length, breadth and thickness parametric sweep is applied in (50-300) \( \mu m \), (5-30) \( \mu m \) and (1-5) \( \mu m \) ranges respectively in COMSOL Multiphysics 5.3a. The levels are selected on the bases of observed major variations in simulated results as illustrated in Table 2.

Table 2. Factors and levels selected for Microcantilever Optimization.

| Factors         | Level 1   | Level 2   | Level 3   |
|-----------------|-----------|-----------|-----------|
| Length (\( \mu m \)) [A] | 100 \( \mu m \) | 150 \( \mu m \) | 200 \( \mu m \) |
| Breadth (\( \mu m \)) [B] | 10 \( \mu m \) | 15 \( \mu m \) | 20 \( \mu m \) |
| Thickness (\( \mu m \)) [C] | 1 \( \mu m \) | 2 \( \mu m \) | 3 \( \mu m \) |
| Material [D]    | GaAs (Gallium Arsenide) | Ge (Germanium) | Si (Silicon) |

Material properties are also one of the important factors that need to be considered for simulation for designing of microcantilever. For this investigation the in-built properties of Silicon, Germanium, Gallium Arsenide in COMSOL Multiphysics software is tabulated in Table 3, indicates the young’s modulus of elasticity, density and poisson’s ratio.
Table 3. Materials and its Properties for Simulation in COMSOL Multiphysics 5.3a.

| Material Properties | Material Name   | Silicon(Si) | Germanium(Ge) | Gallium Arsenide(GaAs) |
|---------------------|----------------|-------------|---------------|-----------------------|
| Young’s Modulus of elasticity (GPa) | 170 | 103 | 85.5 |
| Density [kg/m³] | 2329 | 5323 | 5320 |
| Poisson’s Ratio | 0.28 | 0.275 | 0.31 |

2.2. Numerical modeling

For stress and eigen frequency analysis type of study selected in COMSOL Multiphysics 5.3a software is stationary and eigen frequency respectively. The device is modeled using solid mechanics module, as it is based on solving problem using Navier’s equation and compute results in form of stress, displacement and strain. For boundary conditions, all boundaries are participating for the designed model. Boundary load is applied to boundary 4 whereas fixed constrain is boundary 1. As force is a vector quantity the direction of applied force is also taken into consideration. Force is applied in x-direction on the boundary 4. Free tetrahedral fine meshing shown in Figure 2, is selected with minimum and maximum element size of 0.04 and 4 required for finite element analysis.

Figure 2. Free tetrahedral meshing of designed and simulated microcantilever

3. Design of Experiments Using Taguchi Approach

This work has adopted Taguchi approach for the design of experiment. It utilizes a special design of orthogonal array to study various input factors with fewer number of experiments. Three different types of signal to noise ratio methods are used to study the effect of various input factors on output i.e. Larger-the-Better, Smaller-the-Better, Nominal-the-better. In this paper larger-the-better approach has been used, as in accordance with the applied force displacement required to be highest. Depending upon the number of factors and levels orthogonal array using Taguchi approach is selected. For this investigation with four different factors at three different levels are there and Taguchi approach reduces the number of experiments from $3^4=81$ to 9 experiments. Taguchi design of experiment methodology reduces the number of experiment from 81 to 9
without compromising the quality of output [21-22]. By using different combination of length, breadth, thickness and material the designed L9 orthogonal is shown in Table 4.

**Table 4.** Taguchi L9 (34) orthogonal array for selected factors and levels

| Experiment No. | [A] (µm) | [B] (µm) | [C] (µm) | [D] (Material) |
|----------------|----------|----------|----------|----------------|
| 1              | 100      | 10       | 1        | GaAs           |
| 2              | 100      | 15       | 2        | Ge             |
| 3              | 100      | 20       | 3        | Si             |
| 4              | 150      | 10       | 2        | Si             |
| 5              | 150      | 15       | 3        | GaAs           |
| 6              | 150      | 20       | 1        | Ge             |
| 7              | 200      | 10       | 3        | Ge             |
| 8              | 200      | 15       | 1        | Si             |
| 9              | 200      | 20       | 2        | GaAs           |

The above tabulated nine experiments are designed and simulated using COMSOL Multiphysics 5.3a software using structural mechanism module followed by assigning boundary conditions and fine tetrahedral meshing. COMSOL software indicates the output in form of displacement corresponding with each designed experiment. The mean SN ratio of simulated outputs is calculated using Minitab 17 software by considering larger-the-better approach. The simulation output is used to evaluate sensitivity of force sensor design.

### 4. Results and Discussion

From the results it can be revealed that whenever there is need of conversion of one form of physical quantity (force) in another form (displacement) microcantilver is one of the simplest and cost effective mechanisms to do so. From this investigation, dimension of microcantilever is optimized using larger the better mean SN (Signal to noise) ratio. Combination of 200µm length, 10µm breadth, 1µm thickness and Gallium arsenide (GaAs) material develops with maximum displacement as compare to other selected levels and factors in force range from 1nN-100 nN as indicated in Figure 3.

![Main Effects Plot for SN ratios](image)

**Figure 3.** The Larger –the-better SN ratio for Microcantilever Dimension Optimization
Above mentioned dimensions and material both helpful in designing an optimized model of microcantilever for force measurement. From this optimized model two different types of analysis observed are indicated as follows:

4.1. Eigen Frequency analysis

Eigen frequencies are analyzed to identified the exact value of frequencies at which microcantilever is prone to vibrate[22]. When an object vibrates at a frequency equal to its Natural Frequency, its vibration amplitude increases appreciably this could lead to irreversible damage. The expression of the eigen frequency is given by the relation shown in equation (2)

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{E}{\rho l^2}} \]  

Where \( f_n \) is natural frequency, E is young’s modulus of elasticity, \( \rho \) is mass density, t is thickness and is the length of microcantilever. By simulating optimized Gallium arsenide microcantilever having dimensions 200µm length, 10µm breath, 1µm thickness first six eigen frequencies all in (KiloHertz) are identified and Tabulated in Table 5, indicates both the theoretical and the simulating frequencies. Theoretical frequency values are calculated using equation (2).

| Natural Frequency | Theoretical Frequency(KHz) | Simulated Frequency(KHz) |
|-------------------|-----------------------------|--------------------------|
| 1.                | 15.60                       | 16.293                   |
| 2.                | 101.26                      | 102.09                   |
| 3.                | 161.15                      | 162.17                   |
| 4.                | 284.92                      | 285.87                   |
| 5.                | 559.11                      | 560.28                   |
| 6.                | 606.10                      | 607.57                   |

The simulated six natural frequencies all in Killohertz(KHz) is shown below in Figure 4, indicating the affect different frequencies on microcantilever.
4.2. Stress Analysis

Stress causes deformation in the structure, can be observed in form of displacement shown in Figure 5, indicates that maximum stress is present at the joint of fixed and moving edges and as we move away from the joint stress reduces can easily be seen from the stress diagram with the variation in colour. Stress analysis is helpful in identifying the cause of the structure failure which is very important before specifying for an application where it can be implemented.

Figure 5. Optimized GaAs Microcantilever Von Mises stress (N/\(\mu m\))^2
The range of force for which optimized model is operated is from 1nN-100nN. Figure 6(a) and (b) shows the linear fit curve in increasing order of force and their corresponding residual v/s independent plot. Similarly Figure 6(c) and (d) shows the plots for above mentioned parameters in decreasing order of force and their corresponding residual v/s independent plot. The relation between force (operating range) and displacement for the optimized gallium arsenide microcantilever model shows the linear relationship. As far as convergence study is concern the designed optimized models takes 11 iterations to solve the simulated model with a time period of 15 minues and 0.0016% of linear error, the linear error lies in the acceptable range.

5. Conclusion

From this investigation it can be concluded that, Gallium Arsenide microcantilever with optimized dimensions of 200µm length, 10µm breadth, 1µm thickness responds to nanoNewton range forces with high sensitivity. The displacement sensitivity of 3.47E+04µm/nN is achieved with the optimized model. It is one of the cost effective method of converting force into...
displacement. Eigen frequency analysis provided a range of frequency from 15-608 KHz at which optimized model is not operated. Above and below this frequency range the optimized model can be operated. Stress analysis illustrates the response of structure and material properties towards reliability. Due to optical properties of the Gallium Arsenide material microcantilever is well suited in optical applications i.e. X-rays detection, solar cells, solar cars, optical fibers.

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