Experimental observation of azimuthal shock waves on nonlinear acoustical vortices

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**Abstract.** Thanks to a new focused array of piezoelectric transducers, experimental results are reported here to evidence helical acoustical shock waves resulting from the nonlinear propagation of acoustical vortices (AVs). These shock waves have a three-dimensional spiral shape, from which both the longitudinal and azimuthal components are studied. The inverse filter technique used to synthesize AVs allows various parameters to be varied, especially the topological charge which is the key parameter describing screw dislocations. Firstly, an analysis of the longitudinal modes in the frequency domain reveals a wide cascade of harmonics (up to the 60th order) leading to the formation of the shock waves. Then, an original measurement in the transverse plane exhibits azimuthal behaviour which has never been observed until now for acoustical shock waves. Finally, these new experimental results suggest interesting potential applications of nonlinear effects in terms of acoustics spanners in order to manipulate small objects.
1. Introduction

Since Nye and Berry [1] introduced the concept of phase singularity in wave theory, wave dislocations have been intensively studied in many fields of physics. Phase singularities can be classified into three categories: edge, screw or mixed type dislocations. They are important features of a wave field because they are both generic and structurally stable. Physically, this means, respectively, that they are naturally produced in a wave field, and that a weak perturbation of the field does not eliminate them.

The phase singularity of the screw type has been widely studied in a particular field of optics called singular optics [2]. This screw type dislocation, called optical vortex (OV), confers to the phase a helical shape winding up around a line (Z-axis) where the amplitude vanishes and the phase is not defined. Considering a transverse plane, \((X,Y)\), the phase is defined everywhere except at a single point corresponding to the projection of the beam axis. In this plane, the number of \(2\pi\)-iterations achieved by the phase on a close contour around that point is called the topological charge, denoted \(l\). The sense of the winding gives the sign of the topological charge: it is positive if counterclockwise and negative clockwise [3]. At the axial point, the amplitude of the wave field is null and forms a dark core because of destructive interferences. The fields at two points symmetric in relation to the centre of the beam are dephased by \(\pi\).

By analogy with optics, in acoustics the screw type dislocation is called acoustical vortex (AV). The amplitude and phase of AV of charge \(l = 1\) and \(l = 3\) are illustrated in figure 2. Hefner and Marston [4] demonstrated the possibility of generating single AVs of charge \(l = 1\) and proposed to use them for underwater alignment thanks to the very narrow zero amplitude associated with them. Gspan et al [5] showed that these structures could be produced by optoacoustic generation. Thomas and Marchiano [6] demonstrated that the pseudo-angular momentum and the pseudoenergy of an AV are related to its topological charge. Moreover, this relation was extended to the case of weak nonlinear acoustics, for which the analysis provides a conservation law for the ratio of the topological charge to the harmonic order [6, 7]. Recently, Marchiano et al [8] have developed a temporal analysis of the behaviour and the interaction of AV, and have numerically predicted the existence of an azimuthal shock wave.

In the present work, we report experimental observations of shock waves on AVs. First, we present the nonlinear propagation of AV, from the harmonic generation up to the shock wave formation. Then, we investigate shock waves in the transverse plane \((X,Y)\) in order to characterize the azimuthal shock waves. Finally, we discuss its potential applications as suggested by recent studies about acoustic spanning with AV [9, 10].
2. Experimental setup

As previously done \cite{7}, we use the linear inverse filter technique \cite{11} to synthesize AVs. It is based on knowledge of the medium of propagation between the acoustical sources and the points where the ‘target’ field should be synthesized, namely the control points. Once the propagation operator has been experimentally recorded, an appropriate numerical treatment is applied to compute the signals that the transducers have to emit to synthesize the desired field at the control points. Here, we choose to use the Gauss–Laguerre (GL) beams as the ‘target’ pattern in the control plane to synthesize AV. Indeed, these beams are known to carry screw dislocations \cite{12}. They have a limited spatial extension and, consequently, are of finite energy. In addition, GL beams are exact solutions of the linear paraxial wave equation.

The experimental setup (figure 1) is made of a new spherically focused array of 127 piezoelectric transducers mounted on a spherical cap with a geometrical focal length of 450 mm (Vermon, Paris, France) immersed in water. The transducers are of hexagonal shape with a 60 mm² surface. They are distributed on a compact hexagonal pattern with a 100 mm period. The amplitude and the phase of each transducer can be driven independently thanks to electronic amplifiers (Lecoeur Electronique, Chuelles, France). The central frequency of the transducers is 1 MHz. In these experiments, they are excited by wave trains centred at \( f_0 = 0.8 \text{ MHz} \) corresponding to a time period \( T_0 = 1.25 \mu\text{s} \) and a wavelength \( \lambda = 1.85 \text{ mm} \).

To measure the instantaneous amplitude of the nonlinear acoustic field, we use a very fast response membrane hydrophone (Precision Acoustics Ltd, UK) with a geometric diameter of 80 mm and an active sensor diameter of 0.2 mm. The recorded signals are sampled at 625 MHz and averaged 32 times to improve the signal-to-noise ratio. At a fixed distance of propagation (\( Z = 500 \text{ mm} \)), step by step motors achieve the displacement of the hydrophone in both transverse directions (\( X, Y \)). In the following, \( P \) will refer to this plane, whose coordinates are (\( X, Y, Z = 500 \text{ mm} \)) as illustrated in figure 1. The field is sampled with 6561 points regularly set on a square grid of 40 \( \times \) 40 mm² with a spatial step of 0.5 mm.
In this work, we focus our attention on single vortices of topological charge $l = 1$ and $l = 3$. The rms amplitude and the phase patterns are given for each case at the fundamental frequency $f_0$ in figure 2. The spatial shape of the rms amplitude recalls that of a ‘doughnut’, a characteristic feature of vortices, either acoustical or optical. At the center of the beam, the pressure is very low whereas it is maximum on the ring. Then, the pressure decreases progressively and tends to vanish. The phase increases linearly with the polar angle $\theta$ and turns around the center of the beam for which the value of the phase is undefined as previously mentioned in the introduction. Note that the radial curvature of equiphase lines is due to a classical diffraction effect as already observed in acoustics [4].

To produce shock waves with ultrasonic waves, high power is required. Indeed, for waves with finite amplitude, propagation is not linear: the speed of propagation depends on the instantaneous value of the pressure [13]. Hence, the parts of the wave with a high amplitude travel faster than those with a low amplitude. As water is a quasi non-dispersive media for acoustic waves, this results in a distortion of the temporal profile and finally to the formation of shock waves. This shock formation process is well known for plane or focused waves, but has never been observed experimentally for AV, even though helical shock fronts have been recently predicted numerically [8]. In terms of frequency spectrum, the counterpart of this nonlinear distortion is the generation of high order harmonics. Even though the nonlinear

**Figure 2.** RMS amplitude (top view) and phase (bottom view) of the pressure field at the fundamental frequency in the plane $P$ for nonlinear AV of charge +1 (left column) and +3 (right column).
Figure 3. (a) Temporal signal of AV with $l = 1$ (figure 2 left). Inset: zoom on a cycle. (b) Corresponding fast Fourier transform exhibiting a wide cascade of harmonics.

distortion of the wave profile is large over long distances of propagation, it comes out only as a cumulative effect of small nonlinear perturbations (because waves are of small amplitude, measured by an acoustical Mach number of the order here of $10^{-3}$). Given the structural stability of screw dislocations, the properties of AVs will remain during the nonlinear propagation and even beyond the shock formation. This will allow us to observe shock waves (an intrinsically nonlinear phenomenon) with a helical shape characteristic and topological charge characteristic for linear AVs. As a consequence, the signals computed at low amplitude by the inverse filter technique (which is a linear technique) to synthesize GL beams, are then sent with a high amplitude on each transducer to observe nonlinear effects such as harmonic generation and shock waves. Then the pressure field is recorded on the observation transverse plane $P$.

3. Experimental results

3.1. Temporal shock waves on AVs

Figure 3(a) shows the temporal profile of the pressure field on a point of the transverse plane $P$ located at $X = 4$ mm, $Y = 12$ mm. The high-sampled temporal recording ($= 625$ MHz) allows us to observe fully developed shock waves.

By zooming on one cycle (inset), the rise time is measured about 20 ns. Although its theoretical value is incalculable in these conditions\(^4\), it is interesting to notice that the measured rise time is somewhat longer than that calculated for plane waves in pure water (about 3–4 ns according to Rudenko and Soluyan [13]). This is due indeed to the resolution limit in time of the membrane hydrophone whose bandwidth is about 50 MHz. Nevertheless, it is clearly visible on figure 3(b) that, starting from a sine wave at the source, a high number of harmonics generated by the nonlinear effects are observable.

Figure 4 shows the phase pattern measured on each point of the transverse plane $P$ for the third, sixth, ninth and twelfth harmonics for nonlinear AV of charge $l = 1$ (left row) and charge $l = 3$ (right row).

\(^4\) AV are not plane waves and the focused nature of the beam makes the amplitude estimation difficult.
Figure 4. Phase for the (a) third, (b) sixth, (c) ninth, and (d) twelfth harmonic of nonlinear AV \( (l = 1, \text{left column}) \) and \( (l = 3, \text{right column}) \) in the transverse plane \( \mathcal{P} \).
These figures allow us to access to the value of the topological charge for each harmonic frequency (respectively \(l = 3, 6, 9, 12\) for an AV of charge +1, and \(l = 9, 18, 27, 36\) for an AV of charge +3). Theoretically, the ratio between the total topological charge and the frequency has to be constant for propagation in an inviscid and isotropic medium \([6]\). Hence, the \(p\)th harmonic of the fundamental will display a topological charge \(pq\), if the charge of the fundamental is \(q\). This law has already been checked in experiments for weak nonlinear effects (before the shock formation distance \([6, 7]\)) and extended to the case of parametric interaction between two AV \([14]\). Figure 4 shows that this law is here recovered, at least until the twelfth harmonics and so even for an AV of charge \(l = 3\): its phase is made of \(3 \times 12 = 36\) blades (figure 4(d)). However, the limited spatial sampling of these measurements does not allow us to observe higher harmonics. Indeed, the information contained by the phase becomes inaccessible as illustrated by the blurred zone which grows with harmonic order.

These observations show that the law of conservation of the ratio between the frequency and the topological law remains valid beyond the shock formation distance. This result is far from obvious as it is in apparent contradiction with the theoretical demonstration \([6]\) valid only for an isotropic and inviscid propagation medium. Here, the assumption of a non-dissipative medium is violated by the jump of entropy and the dissipation processes occurring through any shock. That result shows that the shocks do not destroy the stable structure of the AV and reinforces the concept of structural stability of AV with respect to nonlinear perturbations.

3.2. Azimuthal shock waves on AVs

The main result of our paper is about the experimental highlighting of shock waves in a transverse plane, called azimuthal shock waves. Indeed, our measurements of the instantaneous pressure in the transverse plane \(P\) reveal sharp transitions in space, e.g. spatial shock waves, whose number corresponds to the topological charge \(l\) (figure 5). In addition, the latter parameter sets their angular velocity as illustrated in figure 5 showing the instantaneous pressure at four different times: \(t, t + T_0/3, t + 2T_0/3, t + T_0\). On one hand, for \(l = 1\), the single shock undergoes one revolution (\(\delta \theta = 2\pi\)) in one period \(T_0\) exactly. On the other hand, for \(l = 3\), each of the three shocks propagates three times slower as illustrated by the black arrow in figure 5 (\(\delta \theta = 2\pi/3\)). Videos of the evolution in time of these azimuthal shock waves (see attached movie file) show their rotation in the transverse plane \(P\).

To get a picture of these azimuthal shock waves without animation, we show here a three-dimensional figure (figure 6). With an appropriate algorithm, we are able to extract the position of the shock front in the transverse plane \(P\) for an AV of charge \(l = 1\) during one period \(T_0\). Each time of measurement corresponds to a spatial position along the beam axis allowing to unroll the AV structure in space.\(^5\) Thus, shock waves allow us to visualize the helical wavefront, whose step is equal to one wavelength (\(\lambda = 1.85\) mm). It is important to notice that this structure seems stable during the propagation: once the shock waves are formed, the shock front remains an helix. Indeed, it is well known that shock wave may mitigate linear diffraction and for instance prevent or delay focusing \([15]\). This nonlinear effect is weaker for acoustical shock waves whose Mach number is here around \(10^{-4}–10^{-3}\). Nevertheless, in some case it may lead to special features in the wave fields \([16]\).

Moreover, the spatial extension of these azimuthal shock waves is measured. To do that, we scan the vortex ring by varying the polar angle \(\theta\) at a fixed radius \(R\) from the center of the beam:

\(^5\) We do not show the ‘3D’ figure for an AV of charge \(l = 3\) because of the poor signal/noise ratio.

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Figure 5. Instantaneous pressure of nonlinear AV ($l = 1$ left column), ($l = 3$ right column) in the plane $P$ at four different times: $t$ (a), $t + T_0/3$ (b), $t + 2T_0/3$ (c) and $t + T_0$ (d), see the movie available from stacks.iop.org/NJP/11/013002/mmedia. The black arrows indicate the position of one of the three shocks.
Figure 6. Shock front reconstruction from pressure measurements done in the transverse plane $P$ during one period $T_0$ for an AV of charge $l = 1$.

Figure 7. Pressure versus polar angle $\theta$ at fixed radius $R = 8.5$ mm (a) and at $R = 16$ mm (b) in the transverse plane $P$ for nonlinear AV ($l = 1$ and $l = 3$, respectively).

$R = 8.5$ mm for $l = 1$ and $R = 16$ mm for $l = 3$. To ensure the best spatial resolution, we scan both AV with a fine spatial step about $R\delta \theta \sim 0.1$ mm. As expected, figure 7 presents clearly $l$ sharp transition(s) in the pressure along these circles: one shock for $l = 1$ and three shocks for $l = 3$. The shock extends in a very small angle range as shown in the inset: $\delta \theta \sim 0.05$ rad corresponding to $R\delta \theta \sim 0.4$ mm. In this case, the precision of the measurement of the shock wave extension is limited by the size of the active element of the membrane hydrophone ($\sim 0.2$ mm in diameter).
4. Discussion

Recently Volke-Sepúlveda et al [9] demonstrated that AV ($l = 1$ and $l = 2$) can transfer angular momentum to matter under free field conditions. Their results indicate that, for small objects, the torque exerted by the first-order vortex is larger than that exerted by the second-order one, but the opposite occurs when the object is larger than approximately $0.15\,\lambda$. Skeldon et al [10] demonstrated angular momentum transfer between an acoustic beam with helical phase ($l = \pm 1$) and a suspended tile.

The pseudo angular momentum of AV is directly proportional to the transverse velocity, which itself is proportional to the spatial derivative of the pressure field. From our data (see figure 5), we plot the transverse velocity measured in the plane $\mathcal{P}$ for both linear and nonlinear AV (figure 8). One can clearly observe that the transverse velocity is much higher in the case of the shock wave regime. Although the magnitude is limited by the resolution of our scan (step = 0.5 mm), which tends to smooth the azimuthal shock wave and, consequently, tends to reduce the associated pressure gradient, this experimental result provides a qualitative picture in terms of acoustical spanning by means of nonlinear fields.

5. Conclusion

The highlight of this paper is the experimental observation of azimuthal shock waves on nonlinear AV of different topological charge ($l = 1$ and $l = 3$). After recovering the conservation law between the topological charge and the harmonic order on fully developed shock waves, we investigated the helical structure of the azimuthal shock waves that proves stable during propagation. Moreover, our measurements on transverse velocities illustrate the promising nature of azimuthal shock waves as efficient acoustical tweezers. Lastly, all the present experimental results were obtained in a quasi-stationary regime (long wave train about 10 cycles). So, it could also be interesting to investigate the structure of these nonlinear AV in a transient regime when the burst is reduced to one or two cycles.

**Figure 8.** Transverse velocity (black arrows) of AV of charge $l = 1$ in the plane $\mathcal{P}$ for both linear (fundamental component) (a) and nonlinear (azimuthal shock) (b) regimes.
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