Exploring the deviation of cosmological constant by a generalized pressure parameterization

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Abstract  We bring forward a generalized pressure (GP) parameterization for dark energy to explore the evolution of the universe. This parametric model has covered three common pressure parameterization types and can be reconstructed as quintessence and phantom scalar fields, respectively. We adopt the cosmic chronometer (CC) datasets to constrain the parameters. The results show that the inferred late-universe parameters of the GP parameterization are (within $1 \sigma$): the present value of Hubble constant $H_0 = (72.30^{+1.26}_{-1.37})$ km s$^{-1}$ Mpc$^{-1}$; the matter density parameter $\Omega_m = 0.302^{+0.046}_{-0.047}$, and the bias of the universe towards quintessence. Then we perform a dynamic analysis on the GP parameterization and find that there is an attractor or a saddle point in the system corresponding to the different values of the parameters. Finally, we discuss the ultimate fate of the universe under the phantom scenario in the GP parameterization. It is demonstrated that the three cases of pseudo rip, little rip, and big rip are all possible.

1 Introduction

Over the past two decades, a large number of cosmological observations have confirmed that the expansion of the late universe is speeding up [1–5], which has become one of the greatest challenges of cosmology. In order to explain the accelerated expansion, there are two main approaches: modifying gravity (MG) and adding dark energy (DE). The former means modifying the geometric parts of general relativity (GR), such as scalar-tensor theory [6], $f(R)$ gravity [7,8] and brane cosmology [9,10]. The other method is to add the dark energy, which breaks the strong energy condition and produces a mysterious repulsive force to make the universe accelerate the expansion. The simplest DE model is the $\Lambda$CDM model, where the cosmological constant $\Lambda$ is related to DE, and its equation of state (EoS) is $\omega_{\Lambda} = -1$. The $\Lambda$CDM model provides a fairly good explanation for current cosmic observations. Recently, Planck-2018 reaffirmed the validity of the six-parameter $\Lambda$CDM model in describing the evolution of the universe [11]. Nonetheless, there are two long-term problems with the $\Lambda$CDM model. One is the fine-tuning problem: The observation of dark energy density is 120 orders of magnitude smaller than the theoretical value in quantum field theory [12,13]; the second is the coincidence problem: At the beginning of the universe, the proportion of DE is especially tiny while now the dark energy density and matter density are exactly of the same magnitude. Besides, in recent years, the tension of Hubble constant between the Planck datasets and SHoES has reached 4.4$\sigma$ [14], which are brought about by the $\Lambda$CDM model and the cosmic distance ladder method, respectively. So as to alleviate these problems, many dynamic dark energy models with the time-variation EoS are proposed, including scalar field models (such as quintessence [15–19], phantom [20–22], k-essence [23–25], quintom [26–28] and tachyon [29]), holographic model [30], agegraphic model [31], Chaplygin gas model [32,33].

The model presented in this paper is also a dynamic dark energy model, which parameterizes the total pressure of the universe. Parameterization of the observable is an effective method to explore the characteristics of DE, such as the parameterization of EoS [34–36], the luminosity distance [37,38], dark energy density [39], pressure [40–45] and deceleration factors [46]. Take the pressure parameterization as an example. In general, we can write the pressure parameter equation as $P = \sum_{n=0} P_n x_n(z)$, where $x_n(z)$ expands for the late universe in the following forms: (i) redshift: $x_n = z^n$, (ii) scale factor: $x_n(z) = (1 - a)^n = (z/(1 + z))^n$, (iii) logarithmic form: $x_n(z) = (\ln(1 + z))^n$. The form corresponding to $n = 1$ in (i) and (ii) was proposed by Zhang et al. [42]. Case (iii) for $n = 1$ was given by Wang
and Meng [45]. In order to unify these mainstream parameterization methods, we suggest a three-parameter pressure parameterization model to explore the evolution of the universe.

This paper is organized as follows: Sect. 2 presents a generalized pressure (GP) parameterization of the total pressure and discusses its feature. In Sect. 3, we use CC datasets to impose constraints on the parameters of the GP parameterization. The discussion of fixed points using the GP parameterization is analyzed in Sect. 4. In Sect. 5, we exhibit the end of the universe under the phantom case. Section 6 is for the conclusion.

2 Theoretical model

Pressure parameterization describes our universe in the following ways: First, hypothesize a relationship between the pressure $P$ and the redshift $z$. Then the expression of the density $\rho$ can be derived from the conservation equation $\dot{\rho} + 3(\dot{a}/a)(\rho + P) = 0$. Finally, by utilizing the Friedmann equations $H^2 = 3/(8\pi G) \sum \rho_i$ and the EoS $\omega = P/\rho$, we can get the form of the Hubble parameter $H$ and $\omega$, respectively. Here we take the speed of light $c = 1$. At this point, a closed system of cosmic evolution has been established which is described by the Friedmann equations, the conservation equation, and the EoS form.

It is worth noting that there are still some deviations between the $\Lambda$CDM and actual data (e.g., $H_0$ tension), but the physical mechanism behind it is not clear. Taking advantage of this kind of ad hoc model, we can probe the possible deviations further between the dynamic case and the cosmological constant case without a specific premise. In this work, we propose a generalized pressure parameterization of the total energy components in a spatially flat Friedmann–Robertson–Walker (FRW) universe:

$$P(z) = P_1 - P_2 \left[ \frac{(1 + z)^{-\beta - 1}}{\beta} \right], \quad \beta \neq 0,$$

where $P_1$, $P_2$ and $\beta$ are free parameters. Notice that $P_1$ is the current value of the total pressure in the universe, and $P_2$ represents the deviation of $P(z) - z$. The model degenerates into the $\Lambda$CDM model as $P_2 = 0$. We point out that this parametric form of $P(z)$, i.e., Eq. (1), is inspired by a generalized equation of state for dark energy [36]. When specific limits are given to $\beta$, this model returns to the three models mentioned in Sect. 1, i.e.,

$$P(z) = \begin{cases} 
P_a + P_b z, & \text{for } \beta = -1, \\
P_a + P_b \ln(1 + z), & \text{for } \beta \rightarrow 0 \\
P_a + P_b \left( \frac{z}{1 + z} \right), & \text{for } \beta = +1.
\end{cases}$$

By using Eq. (1), the relationship of the scale factor $a (a = 1/(1 + z))$ and the conservation equation $(\dot{\rho} + 3(\dot{a}/a)(\rho + P) = 0)$, we can get the density:

$$\rho(a) = \frac{3a^2 P_2}{(3 + \beta)\beta} - \frac{P_2}{\beta} - P_1 + Ca^{-3},$$

where $C$ is the integral constant. We assume $\rho_0$ is the current total density, i.e. $\rho(a = 1) = \rho_0$. Finally, the total density and total pressure can, respectively, be written in the following form:

$$\rho(a) = \rho_0 (P_a a^\beta + \Omega_{m0} a^{-3} + 1 - P_a - \Omega_{m0}),$$

$$P(a) = \rho_0 \left[ -1 - P_a - \Omega_{m0} \right] - \frac{3 + \beta}{3} P_a a^\beta.$$  

The parameters $P_1$ and $P_2$ have been replaced here by the new parameters $P_a$ and $\Omega_{m0}$, where $P_a \equiv 3P_2/(3 + \beta)\beta\rho_0$, $\Omega_{m0} \equiv (\beta \rho_0 + P_a + \beta P_1 - 3P_2/(3 + \beta))/\beta\rho_0$. In the density expression (4), the item $\Omega_{m0} a^{-3}$ corresponds to the matter density $\rho_m$. So $\Omega_{m0} a^{-3}$ signifies the physical meaning of the parameter $\Omega_{m0}$; it is the present-day matter density parameter. The term $\rho_0 (P_a a^\beta + 1 - P_a - \Omega_{m0})$ accords with the dark energy density $\rho_{de}$, and the constant part $1 - P_a - \Omega_{m0}$ looks similar to the $\Lambda$CDM case. The term $P_a a^\beta$ makes $\rho_{de}$ change with time: The larger $|P_a|$ is, the higher the deviation from the $\Lambda$CDM model will be; the larger $\beta$ is, the faster the dark energy density will change. Accordingly, this cosmological model only includes matter and dark energy components, and the pressure of the dark energy $P_{de}$ is the total pressure $P$. Note that when $\beta = -3$, the density of the part of the dark energy is expressed as the matter density, and the total density $\rho(a)$ of our GP parameterization has the same form as the $\Lambda$CDM model.

Suppose the dark energy is a scalar field $\phi$ that changes with time. The corresponding pressure and density are equivalent to $\rho_{de} = (n/2)\frac{\dot{\phi}^2}{2} + V(\phi)$ and $P_{de} = (n/2)\frac{\dot{\phi}^2}{2} - V(\phi)$, separately, where $n = 1$ or $-1$ corresponds to the quintessence and phantom scalar field, respectively. The calculation shows that $\dot{\phi}^2 = -(n\beta/3)\rho_0 P_a a^\beta$, so $\beta P_a < 0$ fits quintessence, and $\beta P_a > 0$ matches the phantom model.

Additionally, for the GP parameterization, the EoS of the dark energy $\omega_{de}$, the dimensionless Hubble parameter $E$, the deceleration parameter $q$ and the jerk parameter $j$, respectively, take the form

$$\omega_{de} = -1 - \frac{\frac{1}{3} P_a a^\beta}{1 - \frac{1}{3} P_a a^\beta - \Omega_{m0} + P_a a^\beta},$$

$$E = \left( \Omega_{m0} a^{-3} + P_a a^\beta + 1 - P_a - \Omega_{m0} \right)^{1/2},$$

$$q \equiv -\frac{\ddot{a}}{a H^2} = -1 + \frac{3}{2} \Omega_m - \frac{\beta P_a}{2E^2} a^\beta,$$

$$j \equiv \frac{\dddot{a}}{a H^3} = \frac{1 + 2(q + 1)(2q - 1) - a^2 P_a a^\beta((\beta + 3)}{2E^2}.$$
Table 1  Cosmic chronometers data used in this paper

| z   | $H(z)$  | $\sigma_H(z)$ | References | z   | $H(z)$  | $\sigma_H(z)$ | References |
|-----|---------|---------------|------------|-----|---------|---------------|------------|
| 0.07 | 69      | 19.68         | [47]       | 0.36 | 81.2    | 5.9           | [48]       |
| 0.09 | 69      | 12            | [49]       | 0.382 | 83    | 13.5         | [50]       |
| 0.1  | 69      | 12            | [51]       | 0.4  | 95     | 17           | [52]       |
| 0.12 | 68.6    | 26.2          | [47]       | 0.4004 | 77    | 10.2         | [50]       |
| 0.17 | 83      | 8             | [52]       | 0.4247 | 87.1 | 11.2         | [50]       |
| 0.1791 | 75      | 4             | [48]       | 0.4497 | 92.8 | 12.9         | [50]       |
| 0.1993 | 75      | 5             | [48]       | 0.47 | 89     | 50           | [53]       |
| 0.2  | 72.9    | 29.6          | [47]       | 0.4783 | 80.9 | 9           | [50]       |
| 0.27 | 77      | 14            | [52]       | 0.48  | 97     | 62           | [51]       |
| 0.28 | 88.8    | 36.6          | [47]       | 0.5929 | 104   | 13           | [48]       |
| 0.3519 | 83.0   | 14            | [48]       | 0.6769 | 92    | 8           | [48]       |

The data. The differences of $H_0$ between our results and SHoES [14] and the Planck base-$\Lambda$CDM [11] are 0.9 $\sigma$ and 3.5 $\sigma$, respectively. This suggests that the GP parameterization is more supportive of the results of SHoES. The $\Lambda$CDM model is not the best form in this general framework and shows some difference from the best form.

The evolution of the DE EoS parameter $\omega_{\text{de}}$, the DE density parameter $\Omega_{\text{de}}$, the deceleration parameter $q$ and the jerk parameter $j$ with $1\sigma$ error propagation from data fitting (Table 3) are shown in Fig. 2. From Fig. 2 we find that these results are acceptable, except for the DE density parameter $\Omega_{\text{de}}$, which is too high at the beginning of the universe and contradicts the facts we now know. This parameterization is an approximation of the late universe, and the result of the DE density parameter suggests that it does not apply to the early universe. In addition, from $\{(\rho_{\text{de}}/\rho_{m})_{\text{d}} \rightarrow 0, \beta > -3 \rightarrow 0$ we can see that, for the case of $\beta > -3$, the DE accounts for a small percentage in the early universe and the previous problem is avoided.

4 Dynamic analysis

In this section, we will construct a self-consistent dynamical system to analyze the cosmological evolution of the GP parameterization. Let us select $\Omega_{\text{de}}$ and $E$ as independent variables. By Friedmann equation, we can get the self-consistent dynamic system

$$\dot{\Omega}_{\text{de}} = \left(1 - \Omega_{\text{de}}\right) \left(\beta \Omega_{\text{de}} - \frac{P_{\beta} \beta}{E^2} + 3 \Omega_{\text{de}}\right),$$

$$E' = \frac{E}{2} \left[\beta \Omega_{\text{de}} - \frac{P_{\beta} \beta}{E^2} - 3(1 - \Omega_{\text{de}})\right],$$

where $P_{\beta} \equiv 1 - P_\alpha - \Omega_{\text{m0}}$, $\dot{\ln(a)}$ represents the derivative of $\ln(a)$. The following are common methods for finding fixed points and its stability of a system. Let $\dot{\Omega}_{\text{de}} = E' = 0$, then,
Fig. 1 The one-dimensional and two-dimensional marginalized probability distributions of the GP parameterization by using CC observations. The dark blue and light blue areas represent 1σ and 2σ errors, respectively.

Table 3 Best-fit parameters $H_0$, $\Omega_{m0}$, $P_a$ and $\beta$ from CC datasets. Here we also list $\chi^2_{\text{min}}$.

| Parameter | Best-fit value with 1σ error |
|-----------|-----------------------------|
| $H_0$     | 72.30$^{+1.26}_{-1.37}$    |
| $\Omega_{m0}$ | 0.30$^{+0.046}_{-0.047}$   |
| $P_a$     | 0.249$^{+0.160}_{-0.180}$  |
| $\beta$  | $-3.87^{+1.77}_{-1.66}$    |
| $\chi^2_{\text{min}}$ | 14.4053                   |

for $P_b \geq 0$, there is a fixed point ($\Omega_{\text{de}} = 1$, $E = P_b^{1/2}$) in the dark energy dominant period. Perform the perturbation expansion of the system near the fixed point and we get the Jacobian matrix

\[
M \equiv \begin{pmatrix}
\frac{\partial^2 \Omega_{\text{de}}}{\partial E^2} & \frac{\partial \Omega_{\text{de}}}{\partial E} \\
\frac{\partial \Omega_{\text{de}}}{\partial E} & \frac{\partial E}{\partial E}
\end{pmatrix} = \begin{pmatrix}
-3 & 0 \\
\frac{\beta + 3 P_b^{1/2}}{E^2} & \beta
\end{pmatrix}. \tag{13}
\]

Its eigenvalues are $\lambda_1 = -3$ and $\lambda_2 = \beta$. The stability of the system depends on the sign of the eigenvalues $\lambda_1$ and $\lambda_2$. For the GP parameterization, $\beta$ is a real number and not equal to zero, so there are two situations: When $\beta < 0$, the point $\{1, P_b^{1/2}\}$ is an attractor, and when $\beta > 0$, the point $\{1, P_b^{1/2}\}$ is a saddle point. Meanwhile, for $P_b < 0$, we can determine the properties of the fixed point by observing its figure. On the other side, for the case of ΛCDM, the parameter $P_a = 0$, so $\beta \Omega_{\text{de}} - P_b^{1/2} = \beta P_a^{1/2} E^2 = 0$, the dynamic system goes to

\[
\Omega_{\text{de}} = 3 \Omega_{\text{de}} (1 - \Omega_{\text{de}}), \tag{14}
\]

\[
E' = -\frac{3}{2} E (1 - \Omega_{\text{de}}). \tag{15}
\]
Figure 3a–f illustrate the evolution of the dynamics system for \( \beta = -1, 1 \) and \( P_b = -1, 0, 1 \); Fig. 3g shows the situation of \( \beta = -3.87 \) and \( P_b = 0.45 \), the red line displays the evolution under the best-fit values listed in Table 3 and the red dot is the current location; Fig. 3h exhibits the evolutional trajectories of the \( \Lambda \)CDM model as a comparison. From Fig. 3a, d, it can be found that there is a saddle line and red dashed line correspond to the GP parameterization and the \( \Lambda \)CDM model, respectively, with the best-fit values listed in Table 3.

In this section, we are interested in the rip case of our GP parameterization. For the GP parameterization, the Hubble constant is written as
\[
H = \frac{\dot{a}}{a} = H_0(\Omega_{m0}a^{-3} + Pa a^\beta + 1 - Pa - \Omega_{m0})^{1/2}. \tag{16}
\]

In Sect. 2, we have pointed out that \( \beta P_a > 0 \) for phantom case of the GP parameterization. Next we discuss each situation separately.

1. \( \beta < 0, Pa < 0 \):

   With the growth of cosmic time, \( a \to \infty \) and
   \[
   H \to H_0(1 - Pa - \Omega_{m0})^{1/2}, \tag{17}
   \]
   i.e. \( H(t) \) tends to be a constant, and the universe will approach the de Sitter universe infinitely. So this case is a pseudo-rip.

2. \( \beta > 0, Pa > 0 \):

   When \( a \to \infty \),
   \[
   H \to H_0(P_a a^\beta)^{1/2}. \tag{18}
   \]

   By solving the above differential equation, we can get the relation between the scale factor \( a \) and time \( t \), which can be written as
   \[
a = \left(1 - \frac{\beta P_a^{1/2} H_0}{2}(t - t_0)^{2\beta} \right)^{-2/\beta}, \tag{19}
   \]
   where \( t_0 \) is the present value of time. Substituting Eq. (19) for Eq. (18), one obtains
   \[
   H = \frac{H_0 P_a^{1/2}}{1 - \frac{\beta P_a^{1/2} H_0}{2}(t - t_0)}. \tag{20}
   \]

   So when \( t \to \frac{2}{\beta P_a^{1/2} H_0} + t_0 \), \( H(t) \to \infty \) which means the universe will have a big rip after \( t - t_0 = \frac{2}{\beta P_a^{1/2} H_0} \). Thus, the lifetime of the universe is determined by two parameters, \( \beta \) and \( P_a \), regardless of the matter density \( \Omega_{m0} \). Let us make a rough estimate. Take \( \beta = 1, P_a = 0.02, H_0 = 70 \text{ km s}^{-1} \).
Fig. 3 The dynamic system evolution of the GP parameterization. a–f
The evolution of the dynamics system for $\beta = -1, 1$ and $P_b = -1, 0, 1$. g The situation of $\beta = -3.87$ and $P_b = 0.45$, the red line displays the evolution under the best-fit values listed in Table 3 and the red dot is the current location. For comparison, we also exhibit the scenario of the $\Lambda$CDM model in h. The arrows stand for the direction of time. The black dots represent the fixed point.

Mpc$^{-1}$, then the universe will be torn apart after 198 Gyr. If 13.8 Gyr is the current age of the universe, then the universe has spent only 6.5% of its life.

3. $\beta \to 0$:

In this case, the GP parameterization degenerates into the model in [45]. According to the discussion in [45], the ultimate fate of the universe is the little rip.

To sum up, there are three possible fates under the phantom of the GP parameterization:

1. Big rip for $\beta > 0, P_a > 0$.
2. Little rip for $\beta \to 0$.
3. Pseudo-rip for $\beta < 0, P_a < 0$. 

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6 Conclusions

In principle, it is interesting to insert models or theories into a more general framework to test their validity. Not only does this reveal a new set of solutions, but it may also enable more accurate consistency checks for the original model. This paper has made this attempt by expanding the ΛCDM model to a generalized pressure (GP) parameterization. The GP parameterization has three independent parameters: The present value of the matter density parameter Ωm0, the parameter Pα, which represents the deviation from the ΛCDM model, and the parameter β. Picking different values of the parameter β can produce three common pressure parametric models. By using the cosmic chronometer (CC) datasets to constrain the parameters, it shows that the Hubble constant is H0 = (72.30^{−1.37}_{+1.27}) km s^{-1} Mpc^{-1}. The differences of H0 between our results and SHOES [14] and Planck base-ΛCDM [11] are 0.9 σ and 3.5 σ, respectively. In addition, for the GP parameterization, the matter density parameter is Ωm0 = 0.302^{+0.046}_{−0.047}, and the universe shows bias towards quintessence. Then we explore the fixed point of this model and find that there is an attractor or a saddle point corresponding to the different values of the parameters. Next, we analyze the rip of the universe in the phantom case and draw the conclusion that there are three possible endings of the universe: Pseudo-rip for β < 0, Pα < 0, big rip for β > 0, Pα > 0 and little rip for β → 0. Finally, we estimate that, for the big rip case, the universe has a life span of 198 Gyr.

Dark energy has been proposed for 20 years, but its nature remains unknown. With this parametric model, we can probe the possible deviation further between the dynamic case and the cosmological constant condition through existing data.

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