Comment on Radiative Neutrino Mass Matrix
with a Sterile Neutrino

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A mechanism of radiatively generating neutrino masses is implemented in an \(SU(2)_L \times U(1)_Y \times SU(2)'_L \times U(1)'_Y\) model, where the first and second families respect \(SU(2)_L \times U(1)_Y\) while \(SU(2)'_L \times U(1)'_Y\) is specific for the third family. The fourth neutrino, \(\nu_s\), that has a \(U(1)'_Y \times U(1)'_Y\) coupling joins in the model to induce neutrino mixings by additional interactions with e and \(\mu\). The phenomenologically consistent oscillation of \(\nu_s - \nu_e\) requires a dominated coupling of \(\nu_s\) to e.

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I. INTRODUCTION

New era of physics beyond the standard quark-lepton physics has been opened by the SuperKamiokande collaboration who has provided the convincing evidence on atmospheric neutrino oscillations.\textsuperscript{1} There have also been other observed anomalous phenomena suggesting that neutrinos are oscillating, which are often referred to the solar neutrino deficit\textsuperscript{2} and to the LSND signal\textsuperscript{3}. Such neutrino oscillations are only possible if neutrinos are massive\textsuperscript{4}. The well known theoretical frameworks realizing neutrino oscillations are based either on a seesaw mechanism\textsuperscript{5,6} implying a huge mass scale such as the unification scale or on a radiative mechanism\textsuperscript{7} employing extra Higgs bosons at the electroweak scale. These observed oscillations could be explained in general by the simultaneous oscillations among three known neutrinos, \(\nu_e, \nu_\mu, \nu_\tau\)\textsuperscript{8}. However, it seems reasonable to assume that these phenomena really occur as a result of the oscillations involving only two neutrino species. Along this line of thought, it has been suggested that transitions of \(\nu_e \leftrightarrow \nu_s\) (atmospheric), \(\nu_\mu \leftrightarrow \nu_s\) (solar) and \(\nu_\tau \leftrightarrow \nu_s\) (LSND) explain neutrino oscillations indicated by the observed data, where \(\nu_s\) is a sterile neutrino with no interactions in the standard model\textsuperscript{9}.

In the radiative mechanism\textsuperscript{7}, an extra Higgs doublet is required to yield a coupling to a charged Higgs singlet that interacts with a neutrino-charged lepton pair. The smallness of neutrino masses comes from the smallness of charged lepton masses and from feeble couplings associated with interactions of the neutrino-charged lepton pairs. The extensive analyses on neutrino physics arising from the radiative mechanism have been performed in detail\textsuperscript{10} and have shown that the radiative mechanism gives consistent results with the observed data. Therefore, it is of quite importance to construct a gauge model that includes a sterile neutrino. In the recent study done by N. Gaur et al.\textsuperscript{11}, it has been explicitly demonstrated how the sterile neutrino scenario works well in their model with radiatively generated neutrino masses. Since all neutrinos are kept massless at the tree level, a sterile neutrino, which is a gauge singlet in the standard model, should be protected from acquiring a Majorana mass. The simplest mechanism is based on an extra \(U(1)'\) symmetry. If the scenario really shoots the right way to go beyond physics of the standard model, there must exist a physical reason of the need for the \(U(1)'\) symmetry\textsuperscript{12}.

In this report, the mass protection \(U(1)'\) symmetry is identified with another hypercharge in the standard model. The extended gauge symmetry to be studied is \(SU(2)_L \times U(1)_Y \times SU(2)'_L \times U(1)'_Y\), where the first and second families respect \(SU(2)_L \times U(1)_Y\) while \(SU(2)'_L \times U(1)'_Y\) is specific for the third family\textsuperscript{13,14}. Since the third family carries different quantum numbers from those of the first and second families, Through mixing effects due to extra gauge bosons, \(W\) and \(Z\) interactions with the third family will differ from those with the first and second families. Investigation of phenomenological effects due to the non-universality specific to the third family is of great

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mixing angles are defined by $W$ and $V$ for $SU(3)_C$. Its spontaneous breakdown to $SU(2)_L \times U(1)_Y$ is introduced. A possible mechanism will be briefly discussed in the last section. In the present discussions, we concentrate on the study of radiative mechanism by simply assuming that these extra gauge bosons are heavy enough.

In the next section, a gauge model based on $SU(2)_L \times U(1)_Y \times SU(2)'_L \times U(1)'_Y$ is formulated. Neutrino masses are generated by the radiative mechanism based on one-loop diagrams, which is discussed in the section 3. The last section is devoted to summary and discussions.

II. OUTLINE OF THE MODEL

Our extended gauge model with an $SU(2)_L \times U(1)_Y \times SU(2)'_L \times U(1)'_Y$ symmetry is arranged such that the first and second families carry quantum numbers of $SU(2)_L \times U(1)_Y$ while the third family transforms under $SU(2)'_L \times U(1)'_Y$. Its spontaneous breakdown to $SU(2) \times U(1)$ will be induced by Higgs scalars whose quantum numbers are placed as $(SU(2)_L, U(1)_Y, SU(2)'_L, U(1)'_Y)$:

$$\xi_1 : (2, 0, 2, 0), \quad \Phi_S : (1, 1/2, 1, -1/2),$$

for $SU(2)_L \times SU(2)'_L \to SU(2)$ by the vacuum expectation value (VEV) of $\langle 0|\xi_1|0 \rangle \propto I$ and $\langle 0|\Phi_S|0 \rangle$. The gauge bosons, $W^0_\mu$ and $B^0_\mu$, associated with $SU(2) \times U(1)$ are then described by

$$W^0_\mu = \cos \theta_L V_\mu + \sin \theta_L V'_\mu, \quad B^0_\mu = \cos \theta_Y Y_\mu + \sin \theta_Y Y'_\mu,$$

where $V_\mu$, $Y_\mu$, $V'_\mu$ and $Y'_\mu$ are, respectively, gauge bosons of $SU(2)_L$, $U(1)_Y$, $SU(2)'_L$ and $U(1)'_Y$ with the notation of $W^0_\mu = \sum_{a=1}^3 (\tau^{(a)/2}) W^{0(a)}_\mu$ and similarly for $V_\mu$ and $V'_\mu$. Let $g_L$, $g_Y$, $g'_L$ and $g'_Y$ be the gauge couplings, then the mixing angles are defined by

$$\sin \theta_L = g_L/\sqrt{g^2_L + g^2_Y}, \quad \sin \theta_Y = g_Y/\sqrt{g^2_Y + g^2_{Y'}},$$

which give the gauge couplings of $SU(2) \times U(1)$, $g$ and $g'$, as

$$g = \cos \theta_L g_L = \sin \theta_L g'_L, \quad g' = \cos \theta_Y g_Y = \sin \theta_Y g'_Y.$$

In addition to these scalars,

$$\phi_1 : (2, 1/2, 1, 0), \quad \eta_1 : (1, 0, 2, 1/2)$$

are responsible for generating masses of the first and second families and of the third family, respectively.

Denoting extra massive gauge bosons in $SU(2) \times U(1)$ by $W^{0\mu}$ and $B^{0\mu}$,

$$W^{0\mu} = \cos \theta_L V'_\mu - \sin \theta_L V_\mu, \quad B^{0\mu} = \cos \theta_Y Y'_\mu - \sin \theta_Y Y_\mu,$$

we find that

$$g_L V_\mu = g W^0_\mu - G_L W^{0\mu}, \quad g'_L V'_\mu = g W^0_\mu + G'_L W^{0\mu},$$

$$g_Y Y_\mu = g' B^0_\mu - G_Y B^{0\mu}, \quad g'_Y Y'_\mu = g' B^0_\mu + G'_Y B^{0\mu},$$

where $G_L = g_L \sin \theta_L$, $G'_L = g'_L \cos \theta_L$, $G_Y = g_Y \sin \theta_Y$ and $G'_Y = g'_Y \cos \theta_Y$. The first and second families couple to $V_\mu$ and $Y_\mu$ while the third family, to $V'_\mu$ and $Y'_\mu$. The universality of the $W^0$ and $B^0$ couplings are ensured by $SU(2) \times U(1)$ as expected. The $W'$ and $B'$ masses, $m^{(0)}_{W'}$ and $m^{(0)}_{B'}$ are given by

$$m^{(0)}_{W'} = \sqrt{g^2_L + g^2_Y} v_{\xi_1}/2, \quad m^{(0)}_{B'} = \sqrt{g'^2_L + g'^2_Y} v_s/2,$$

where $v_{\xi_1}$ is a VEV defined by $\langle 0|\xi_1|0 \rangle = (v_{\xi_1}/\sqrt{2}) \text{diag.}(1, 1)$ and $v_s$, by $\langle 0|\Phi_S|0 \rangle = v_s/\sqrt{2}$. After spontaneous breaking due to $\phi$ and $\eta$, $W^0$ and $B^0$ will mix with $W^{0\mu}$ and $B^{0\mu}$ to produce massive $W$ and $Z$ bosons and extra $W'$, $Z'$ and $Z''$ bosons, where $(W^{0(3)}, B^0, W^{0(3)}', B^{0\mu}) \to (Z, \gamma, Z', Z'')$. Our sterile neutrino to be defined in the next section will only couple to $B^{0\mu}$, thus mainly to $Z''$. In case that $v_{\xi_1} >> v_s$, the model is approximated to be described by $SU(2) \times U(1)_Y \times U(1)'_Y$. 

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III. NEUTRINO MASSES

In the original Zee's radiative mechanism [7], an extra Higgs doublet and charged singlet are required. In the present context, interactions with the following extra Higgs bosons:

\[
\begin{align*}
\phi_2 : (2,0,1,1/2), & \quad \eta_2 : (1,1/2,2,0), \\
\xi_2 : (2,1/2,2,1/2), & \quad \chi^+_1 : (1,1,1,0), & \quad \chi^+_2 : (1,1/2,1,1/2), \\
\end{align*}
\]

will generate the mixing of \(\nu_e - \nu_\mu\). New interactions, which are of course invariant under \(SU(2)_L \times U(1)_Y \times SU(2)_R \times U(1)'_Y\), are assumed to be further subject to the conservation of a discrete \(Z_2\) symmetry as a \(\tau\)-parity that is negative for the third family, \(\xi_1\) and \(\xi_2\). All others have the positive parity. The direct coupling of \(\phi_2\) and \(\eta_2\) to quarks and leptons does not respect the gauge invariance; therefore, it is forbidden. Also forbidden is the other possible interaction given by \(\eta_i^T \xi^\dagger_2 \phi_i\) (\(i = 1,2\)), that will give \(\langle \xi_2 \rangle \neq 0\) by a tadpole coupling generated after the spontaneous breaking. By choosing the mass squared for \(\xi_2\) to be positive in the Higgs potential, one can safely set \(\langle 0 | \xi_2 | 0 \rangle = 0\) that would be disturbed if the tadpole interaction were active. It also ensures the absence of the tree-level \(\nu_e^T \nu_\mu\) term arising from \(L(e,\mu) \xi_2 L(\tau)\), where \(L^{(i)}\) denote lepton doublets in three families (\(i = e, \mu, \tau\)).

The relevant part of the lagrangian yielding \(\nu_e - \nu_\mu\), is given by

\[
\Delta L_{\nu_e - \nu_\mu} = f_{e\mu} L^{(e)T} C i \tau^2 L^{(\mu)} \chi^+_1 + \sum_{i=e,\mu} f_{i\tau} L^{(i)T} C i \tau^2 \xi_2 L^{(\tau)} + \mu (\phi^T_2 i \tau^2) \eta_1 + \eta_2^T i \tau^2) \eta_1) \chi^+_2 + \mu \chi^+_1 \chi^-_1 \Phi_S + H.c.,
\]

where \(i \tau^2 (2) (C)\) denotes the charge conjugation operator in \(SU(2)\) (the Lorentz space). Similarly, an additional interaction given by

\[
\Delta L_{\nu_e - \nu_\ell} = \mu' \text{Tr}(\xi^3_1 \xi^2_1) \chi^+_2 + H.c.
\]

induces \(\nu_e - \nu_\ell (=e,\mu)\). The corresponding one-loop diagrams for \(\nu_e - \nu_\ell \ (\ell, \ell' = e, \mu, \tau)\) with \(\ell \neq \ell'\) are depicted in Fig.1.

To give a desirable neutrino mass matrix as shown in [11], one sterile neutrino, \(\nu_s\), together with other two neutrals, \(N_{1,2}\), joins in the model:

\[
\nu_s : (1,-1/2,1,1/2), \quad N_1 : (1,1/2,1,-1/2), \quad N_2 : (1,0,1,0).
\]

The \(\tau\)-parity is positive for \(\nu_s\) and negative for \(N_{1,2}\). The mass term of \(N_2N_2\) is allowed but \(\nu_sN_1\) is forbidden. The extra neutrals, \(N_{1,2}\), can also get massive by \(N_1 \Phi^\dagger_1 N_2\). The mixing of \(\nu_s\) with \(\nu_{e,\mu}\) is made possible by adding \(\Delta L_{\nu_s - \nu_\ell}\):

\[
\Delta L_{\nu_s - \nu_\ell} = \sum_{i=e,\mu} h_i \ell^{(i)} \nu_s \chi^-_2 + H.c.,
\]

where \(\ell^{(i)} = e \ (i = e)\) and \(\mu \ (i = \mu)\). Shown in Fig.2 are the one-loop diagrams for \(\nu_s - \nu_\ell\). It should be noted that \(\ell^{(e)} \nu_s \chi^-_2\) is forbidden by the \(\tau\)-parity. Our sterile neutrino interacts with \(B_{\mu}^{\mu}\) since it has a \(U(1)_Y \times U(1)'_Y\) coupling. The possible mixing of \(B_{\mu}^{\mu}\) with the \(Z\) boson induces a \(Z - \nu_s\) coupling (also by the \(\nu_s\) mixings with \(\nu_{e,\mu,\tau}\)), which should be greatly suppressed. The suppression can be realized by setting the mass of \(B_{\mu}^{\mu}\) to be much heavier than the \(Z\) mass, say, of the order of 1 TeV. The remaining symmetry, \(SU(2)_L \times SU(2)'_L \times U(1)\), is blind for \(\nu_s\).

These additional interactions are invariant under an accidental global \(U(1)_1\) transformation. The nonvanishing \(U(1)_1\) charges (\(Q_1\)) are given by 1 for leptons, -1 for \(\nu_s\) and -2 for \(\phi_2, \eta_2, \xi_2\) and \(\chi^\dagger_1\), which can be regarded as an extended lepton number. This \(U(1)_1\) will be broken by \(\langle \phi_2 \rangle\) and \(\langle \eta_2 \rangle\). However, since \(U(1)_1\) is broken by VEV's with \(|Q_1| = 2\), there is still the conservation due to a \(Z_2\) symmetry of \(\exp(i\pi Q_1)\) as the \(\ell\)-parity [11] that is negative for leptons (\(Q_\ell = 1\)) and \(\nu_s\) (\(Q_\ell = -1\)). Thus, the mixings of \(N_{1,2}\) with leptons and \(\nu_s\) are totally forbidden. The spontaneous breakdown generates a massless Nambu-Goldstone boson, which becomes massive by an explicit breaking of \(U(1)_1\) served by, for instance, \(\langle \phi_2 \rangle + \eta_2^\dagger \eta_1 \rangle \Phi_S [11].\)

The resulting neutrino mass matrix turns out to be in the form of

\[
\begin{bmatrix}
0 & a & b & d \\
-a & 0 & c & e \\
b & -c & 0 & f \\
d & -e & -f & 0
\end{bmatrix},
\]
where \(a, b\) and \(c\) come from Fig.1 and \(d, e\) and \(f\) from Fig.2. The masses are computed to be

\[
a = f_{e\mu} \mu' \left( m_\mu^2 - m_e^2 \right) \frac{v_{\phi_1}}{v_{\phi_1}} G(m_{\phi_1^-}, m_{\phi_1^+}) (0|\Phi_S|0),
\]

\[
b = f_{e\tau} \mu'' \left[ m_\mu^2 \frac{v_{\phi_1}}{v_{\phi_1}} G(m_{\phi_1^-}, m_{\phi_1^+}) - m_\tau^2 \frac{v_{\phi_2}}{v_{\phi_1}} G(m_{\phi_2^-}, m_{\phi_2^+}) \right] (0|\xi_1^0|0),
\]

\[
c = f_{\mu\tau} \mu'' \left[ m_\mu^2 \frac{v_{\phi_2}}{v_{\phi_1}} G(m_{\phi_2^-}, m_{\phi_2^+}) - m_\mu^2 \frac{v_{\phi_2}}{v_{\phi_1}} G(m_{\phi_2^-}, m_{\phi_2^+}) \right] (0|\xi_1^0|0),
\]

where \(m_{e,\mu,\tau}\) represent the masses of charged leptons and

\[
G(x, y) = \frac{1}{16\pi^2} \frac{1}{x-y} \left[ \log x - \log y - \frac{1}{x} \right].
\]

The explicit form of \(G\) is subject to the assumption that \(m_{\chi_1} \sim m_{\chi_2} \sim m_{\xi_1} \sim m_{\xi_2}\). The remaining entries are given by

\[
d = f_{e\mu} \mu' h_\mu F(m_{\chi_2^-}, m_{\chi_2^+}) (0|\Phi_S|0),
\]

\[
e = -f_{e\mu} \mu' h_\mu F(m_{\chi_2^-}, m_{\chi_2^+}) (0|\Phi_S|0),
\]

\[
f = -\sum_{i=e,\mu} f_{i\tau} \mu'' h_i F(m_{\chi_i^-}, m_{\chi_i^+}) (0|\xi_i^0|0),
\]

where

\[
F(x, y) = \frac{1}{16\pi^2} \frac{\log x - \log y}{x-y}.\]

It should be noted that the mixings of \(\nu_i\) are controlled by either \(m_e\) or \(m_\mu\) but not by \(m_\tau\) because of the absence of \(\tau \nu_s \chi_2\) ensured by the \(\tau\)-parity. It provides more suppression than what would be expected \([11]\) owing to \(m_{e,\mu} \ll m_\tau\).

An example of getting phenomenologically acceptable neutrino masses and mixings is given by adjusting various parameters such that \(|e| \gg |b, c| \gg |a|, |c| \gg |d, f|, |f| \gg |b|,\) and \(|ef| \gg |cd| \gg |ab|\) are satisfied \([17]\). The resulting neutrino mass spectrum consists of almost degenerate two massive neutrinos and two extremely light ones: \((-2ab/c, c + (ab + ef)/c, -c + (ab + ef)/c, -2ef/c)\). The parameter setting that reproduces current neutrino-oscillation data is supplied by

\[
|a| \sim 10^{-5} \text{eV}, \quad |b| \sim 10^{-2} \text{eV}, \quad |c| \sim 1 \text{eV},
\]

\[
|d| \sim 10^{-4} \text{eV}, \quad |e| \sim 10^{-2} \text{eV}, \quad |f| \sim 10^{-1} \text{eV},
\]

which give

\[
|f_{e\mu}|/16\pi^2 \sim 10^{-8}, \quad |f_{e\tau}|/16\pi^2 \sim 10^{-7}, \quad |f_{\mu\tau}|/16\pi^2 \sim 10^{-5},
\]

\[
|h_\mu| \sim 1, \quad |h_\mu| \sim 10^{-4}, \quad \mu \sim M_0/10,
\]

where, for simplicity, \(v_{\phi_1} (v_{\phi_2}) = v_{\phi_2} (v_{\phi_2})\) is taken and the mass parameters, \(\mu', \mu''\), charged scalar masses appearing in \(G(x, y)\) and \(F(x, y)\) and VEV’s of \(\xi_1\) and \(\Phi_S\), are all set equal to \(M_0\) presumably of the order of 1 TeV. These numerical values are so chosen that

\[
m_{\nu_e} \sim 2 \times 10^{-7} \text{eV}, \quad m_{\nu_e} \sim 2 \times 10^{-3} \text{eV}, \quad m_{\nu_\mu} \sim m_{\nu_\tau} \sim 1 \text{eV},
\]

are realized. An appropriate mixing of \(\nu_s\) and \(\nu_e\) is found to be controlled by the dominated coupling of \(\nu_s\) to \(e\), i.e. \(|h_e| \sim 1\). For \(\nu_{\tau} \leftrightarrow \nu_\tau\), they are maximally mixed with each other by its squared mass difference \(\sim 4(ab + ef) \sim 4 \times 10^{-3} \text{eV}^2\). The remaining mixing angles to be denoted by \(\theta\) for \(\nu_e \leftrightarrow \nu_s\) and \(\nu_e \leftrightarrow \nu_\mu\), respectively, are given by, \(|\theta| \sim |\theta - (bc + af)|/|ef|\) and \(|\theta| \sim |b/e|\), which can be cast into the right order of the magnitudes roughly controlled by \(s^2 \theta \sim 10^{-3} - 10^{-2}\), as have been discussed in Ref. \([11]\) for \(\nu_e \leftrightarrow \nu_s\).
IV. SUMMARY AND DISCUSSIONS

Summarizing our discussions, we have formulated a gauge model based on $SU(2)_L \times U(1)_Y \times SU(2)_L' \times U(1)_{Y}'$ in order to equip with the radiative mass generation mechanism for neutrino oscillations. To make this mechanism effective, one needs extra Higgs scalars, especially, $SU(2) -$ singlet charged ones that connect neutrinos with the charged leptons. The interactions are requested to be subject to the conservation of the $\tau$ - parity and of the global $U(1)_\chi$ symmetry. As a result, $\nu_e$ mixes with $\nu_{e,\mu,\tau}$ via $e$ and $\mu$ only. The spontaneously broken $U(1)_\chi$ symmetry is replaced by the remnant $Z_2$ symmetry, which forbids the mixings of $\nu_{e,\mu,\tau}$ with the massive $N_{1,2}$. Our scenario implies a rather large coupling of $\nu_\tau$ to $e$, $h_\tau^2 \sim 3g^2 \sim 1$, that favors a consistent $\nu_\tau - \nu_e$ mixing with an expected solution of the solar neutrino problem. Therefore, among the $\nu_\tau - e$ transitions supposedly described by the exchanges of bosons whose masses are $\mathcal{O}(1 \text{ TeV})$, one expects that the $\chi_2^2$ - exchange gives dominant contributions, which alter an effective number of neutrino species, $N_{\nu}$, in the big bang nucleosynthesis. The strength is roughly given by $h_\tau^2/m_\chi^2$ corresponding to the decoupling temperature of $\mathcal{O}(100) \text{ MeV}$ for $m_\nu \sim 1 \text{ TeV}$ and $|h_\nu| \sim 1$. It will add $\mathcal{O}(0.1)$ to $N_{\nu}$, which is allowed to be larger than 3 by the recent refined analyses. 

The specific feature of our model is that the third family is placed in $SU(2)_L' \times U(1)_{Y}'$ while the first and second families are in $SU(2)_L \times U(1)_Y$. Because of this feature, the $b$-mixing is not accommodated. The $b$-mixing can be generated, for example, by introducing a vectorlike down quark, $b'$, as well as extra Higgs scalars, $\phi_b$, $\eta_b$ and $\Phi_b$, whose quantum numbers are taken as $b'_{L,R}: (1, -1/6, 1, -1/6)$, $\phi_b: (2, 1/3, 1, 1/6)$, $\eta_b: (1, 1/6, 1, 1/6)$ and $\Phi_b: (1, 1/6, 1, -1/6)$. The interactions are given by

$$\sum_{i=e,\mu,\tau} \left( f_{b'hivable} \phi_b^{(i)} Q_L^{(i)} + f_{b'hivable} \phi_b^{(i)} \right) + f_{b'b'Q_L^{(i)}} Q_L^{(i)} + f_{b'b'Q_L^{(i)}} Q_L^{(i)} + M_{b'b'} Q_L^{(i)}, \tag{27}$$

where $Q_L^{(i)}$ (q_{R}^{(i)}) denote quark doublets (singlets) in three families $(i = e, \mu, \tau)$, $f_{b'b'}$ are couplings and $M_{b'b'}$ is a mass of $b'$. The mixing of $b$ with $d$ and $s$ is induced via $b'$ by the seesaw-like mechanism for a dominated $M_{b'}$ in mass scales of down quarks.

Phenomenologically interested is to estimate effects from extra particles other than those in the standard model. Some of them cause dangerous flavor - changing interactions that must be greatly suppressed. The suppression of $Z - \nu_\tau$ will impose the severe constraint on the mass of $Z'$, which mainly arises from the gauge bosons associated with $U(1)_Y \times U(1)_{Y}'$. The phenomenology of the extra weak bosons to be characterized by “anomalies” in $t$-, $b$- and $\tau$-interactions will be discussed elsewhere.

\[1\] SuperKamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81 (1998), 1562; Phys. Lett. B 433 (1998), 9 and 436 (1998), 33.
\[2\] See for example, J.N. Bahcall, P.I. Krastev and A.Yu. Smirnov, Phys. Rev. D 58 (1998), 096016-1; J.N. Bahcall, astro-ph/9808162 (Aug., 1998).
\[3\] C. Athanassopoulos, Phys. Rev. Lett. 75 (1995), 2650; 77, 3082 (1996); 81 (1998), 1774.
\[4\] B. Pontecorvo, JETP (USSR) 34 (1958), 247; Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962), 870; B. Pontecorvo, Zh. Eksp. Teor. Piz. 53 (1967), 1717; V. Gribov and B. Pontecorvo, Phys. Lett. 28B (1969), 493.
\[5\] T. Yanagida, in Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe edited by A. Sawada and A. Sugamoto (KEK Report No.79-18, Tsukuba, 1979), p.95; Prog. Theor. Phys. 64 (1980), 1103; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity edited by P. van Nieuwenhuizen and D.Z. Freedmann (North-Holland, Amsterdam 1979), p.315; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980), 912.
\[6\] For recent works on neutrinos in higher dimensions, K.R. Dienes, E. Dudas and T. Gherghetta, hep-ph/9811402 (Nov., 1998); N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russel, hep-ph/9811406 (Nov., 1998).
\[7\] A. Zee, Phys. Rev. D 35 (1983), 389; 161B (1985), 141; L. Wolfenstein, Nucl. Phys. B175 (1980), 92; S.P. Petcov, Phys. Lett. 115B (1982), 401; K.S. Babu and V.S. Mathur, Phys. Rev. Lett. B 196 (1987), 218; K.S. Babu, Phys. Lett. B 203 (1988), 132; J. Liu, Phys. Lett. B 216 (1989), 367; W. Grimus and H. Neufeld, Phys. Lett. B 237 (1990), 521; B.K. Pal, Phys. Rev. D 44 (1991), 2261; W. Grimus and G. Nardulli, Phys. Lett. B 271 (1991), 161; A.Yu. Smirnov and Z. Tao, Nucl. Phys. B426 (1994), 415.
\[8\] See for example, T. Teshima and T. Sakai, Prog. Theor. Phys. 101 (1999), 147; R.P. Thun and S. McKee, Phys. Lett. B 439 (1998), 123; V. Barger and K. Whisnant, hep-ph/9811227 (Dec., 1998) and references therein.
[9] See for example, D.O. Caldwell and R.N. Mohapatra, Phys. Rev D 48 (1993), 3259; 50 (1994), 3477; J.T. Peltoniemi and J.W. Valle, Nucl. Phys. B406 (1993), 409; E. Ma and P. Roy, ibid. 52 (1995), R4780; E. Chun, A.S. Joshipura and A. Yu. Smirnov, Phys. Lett. B 357 (1995), 608; Z. Berezhiani and R.N. Mohapatra, Phys. Rev. D 52 (1995), 607; R. Foot and R. Volkas, ibid. 52 (1995), 6595.

[10] A. Yu. Smirnov and M. Tanimoto, Phys. Rev. D 55 (1997), 1665. For analyses based on $\nu_{e,\mu,\tau}$, C. Jarlskog, M. Matsuda, S. Skadhauge and M. Tanimoto, hep-ph/9812282 (Dec., 1998).

[11] N. Gaur, A. Ghosal, E. Ma and P. Roy, Phys. Rev. D 58 (1998), 071301; P. Roy, hep-ph/9810448 (Oct., 1998).

[12] E. Ma, Mod. Phys. Lett. A 11 (1996), 1893; Phys. Lett. B 433 (1998), 74; E. Ma and D.P. Roy, Phys. Rev. D 58 (1998), 095005.

[13] For an extra $U(1)$ symmetry specific to the third family, see B. Holdom, Phys. Lett. B 339 (1994), 114; P.H. Frampton, M.B. Wise and B.D. Wright, Phys. Rev. D 54 (1996), 5820; E. Mallawi, T. Tait and C.-P. Yuan, Phys. Lett. B 385 (1996), 304; A. A. Andrianov, P. Osland, A.A. Pankov, N.V. Romanenko and J. Sirkka, Phys. Rev. D 58 (1998), 075001. For a recent status of $b$ in the $Z$ decay, see P.B. Renton, hep-ph/9811417 (Nov., 1998)

[14] For an extra color $SU(3)$ symmetry specific to the third family, see C.T. Hill, Phys. Lett. B 345 (1995), 483; B.A. Dobrescu and C.T. Hill, Phys. Rev. Lett. 81 (1998), 2634; R.S. Chivukula, B.A. Dobrescu, H. Georgi and C.T. Hill, hep-ph/9809470 (Sep., 1998).

[15] A. A. Andrianov et al. in [3]. See also M. Bisset, O.C.W. Keng, C. Macesanu and L.H. Orr, hep-ph/9811498 and 9811499 (Nov., 1998).

[16] Y. Okamoto and M. Yasu`e, in preparation.

[17] For three families with $b = 0$, see H. Terazawa, KEK Preprint 98-157 (Sep., 1998).

[18] S. Sarkar, Rep. Prog. Phys. 59 (1996), 1493; P. Kernan and S. Sarkar, Phys. Rev. D 54 (1996), 3681; K.A. Olive and D. Thomas, hep-ph/9811444 (Nov., 1998).

[19] A. Donini, F. Feruglio, J. Matias and F. Zwirner, Nucl. Phys. B507 (1997), 51 and other references therein.
Figure 1: One loop radiative diagrams for $v_\ell - v'_\ell$ ($\ell, \ell' = e, \mu$) with $\ell \neq \ell'$ and for $v_\tau - v_\ell$, where $\xi_2^+ = (\xi_2)_{11}$ and $\xi_2^- = (\xi_2)_{22}$.

Figure 2: One loop radiative diagrams for $v_\tau - v_{\ell, \tau}$. 