Phase transitions, memory and frustration in a Sznajd-like model with synchronous updating

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We introduce a consensus model inspired by the Sznajd Model. The updating is synchronous and memory plays a decisive role in making possible the reaching of total consensus. We study the transition between the state with no-consensus to the state with total consensus.

Key words: computer simulation, opinion dynamics, phase transitions

Basic features of Sznajd Model

It is well understood, within human societies, that it is generally easier to change someone’s opinion by acting within groups than by acting alone. For example, a single person stopping on the street and staring at the sky is usually ignored (or perhaps considered eccentric). However, if several people stare into the sky, they readily induce others to do the same.

Equally it is not unusual to come across door-to-door sale agents working in couples rather than on an individual basis. In democratic electoral campaigns, small groups (typically couples) of political party activists may visit, door-to-door, potential electors and seek to gain their votes. Trade union movements often try to coordinate their actions in order to strengthen their position against management than if everybody try to negotiate it alone.

The underlying principle, in all of the above-mentioned examples may be captured by the famous Abraham Lincoln’s injunction: “United we stand, divided we fall”. This principle was developed into a computational model by Katarzyna Sznajd and her father$[1]$.

The simplest (non-trivial) version of their model can be implemented on a two-dimension lattice of spins. Each site carries a spin, $S$, that is either up or down. This represents one of the two possible opinions on any question. Two neighbouring parallel spins, i.e. two neighbouring people sharing the
same opinion, convince their neighbours of this opinion. If they do not have the same opinion, then they do not influence their neighbours.

The system evolves from one time step to another through a random sequential updating mechanism.

The system always reaches an overall consensus, within a sufficiently long time and ends up with all spins up (or down) if the initial fraction of up opinions is larger (smaller) than $\frac{1}{2}$ [2]. Furthermore, the smaller the size of the lattice, the smoother is the transition.

If the random sequential updating is replaced by a synchronous updating mechanism the possibility of reaching a consensus is reduced quite dramatically.

The updating is performed by going systematically through the lattice to find the first member of the pair, then choosing randomly the second member of the pair within the neighborhood of the first. Having in this way completed the assembly of couples, each agent then orients her/himself according to her/his neighbours at time step $t$. Like-minded couples will induce their neighbours to turn to the same state (opinion). However a single agent may often belong, simultaneously, to the neighbourhood of more than one couple (of likeminded agents). In this case, if the couples have different opinions, she/he doesn’t know what to do (frustration) and ends up doing nothing, i.e. sticks with her/his previous opinion. Frustration may prevent the system from reaching total consensus.

**Memory**

A feature of the agents in the models discussed above is the complete absence of memory. The past plays no role.

In this note we assume that agents are endowed with memory. For the sake of simplicity, they are all thought to have the same memory span $T$ and updating mechanism is synchronous. Agents are keen to change opinion when in the neighbourhood of a like-minded couple, as in the models above. An agent resorts to her/his individual history when frustration occurs. In that case, the new state turns out to be the most frequent of her/his own $T + 1$ most recent ($T$ accounts for the past, one for the present). Of course, this rule proves to be more efficient when $T$ is an even number.
Phase transitions

In a two-dimensional Sznajd model with asynchronous updating, the system always ends up at a fixed point: either all spins point up or they all point down. When synchronous updating is chosen things are different. The system converges to a fixed point (all spins up or all spins down), only if the asymmetry $\Delta p$ (absolute difference) in the initial distribution of opinions is above a critical value $\Delta p_c$ that depends not only on the size of the lattice [3] but also on the memory length $T$ [fig.1].

Fig.2 shows $1-\Delta p_c$ as a function of $L$ (the lattice side size). The slope decreases as $T$ increases. For small values of $L$ the three curves displayed seem to converge to the same point. This is easily understood because for small lattices one expects the impact of frustration on total consensus to be small. As a consequence, the role of the memory $T$ is not as important as it would be for large values of $L$.

For large $L$ the curves monotonically decay to zero, this suggests that asymptotically (for infinitely large lattice) no consensus is possible.

Fig.3 shows $\Delta p_c$ as a function of $T$. The slope of the two curves appears to be independent of $L$ - an aspect that requires further study.

Conclusions

We have studied a model based on the Sznajd consensus model with synchronous updating that includes memory. This feature plays a very important role and helps overcome frustration and achieve total consensus.

The memory length, $T$, affects the phase transition from the no-consensus state to the total consensus state. The bigger $T$ the closer to zero is the critical point. For a given side size $L$ and $T$ enough large an always-consensus situation, as with the random sequentially updated Sznajd model, may be attained. Anyway, for infinitely large (but with finite memory) lattices, no consensus is possible, just as in the case without memory.

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Fig. 1.1

The phase transition from a no-consensus state to a total consensus state is driven by the parameter $\Delta p$ for $L=17$ and $T=0, 2, 8$. The transition point is shifted towards zero as the agent memory length $T$ increases.

Fig. 1.2

The phase transition from a no-consensus state to a total consensus state is driven by the parameter $\Delta p$ for $L=101$ and $T=0, 2, 8$. The transition point is shifted towards zero as the agent memory length $T$ increases.

Fig. 1.3

The phase transition from a no-consensus state to a total consensus state is driven by the parameter $\Delta p$ for $L=301$ and $T=0, 2, 8$. The transition point is shifted towards zero as the agent memory length $T$ increases.

Fig. 1.4

The phase transition from a no-consensus state to a total consensus state is driven by the parameter $\Delta p$ for $L=1001$ and $T=0, 2, 8$. The transition point is shifted towards zero as the agent memory length $T$ increases.

Fig. 1.5

The phase transition from a no-consensus state to a total consensus state is driven by the parameter $\Delta p$ for $L=50$ and $T=0, 2, 8, 100, 500, 1000$. The transition point is shifted towards zero as the agent memory length $T$ increases.

Fig. 2

Variation with $L$ of $1-\Delta p$ (one minus the absolute value of the difference in the initial probabilities for +1 and -1) for which in half of the cases a consensus was reached. That may be seen as the phase transition point from the state without consensus to the state with consensus. The estimated slope is $-0.39$ for $T=0$, $-0.21$ for $T=2$, $-0.11$ for $T=8$.

Fig. 3

Variation with $T$ of the difference in the initial probabilities for which in half of the cases a consensus was reached. That may be seen as the phase transition point from the state without consensus to the state with consensus. The estimated slope is 0.46 for $L=17$ and 0.47 for $L=50$. 
L=17

% of samples reaching total consensus vs. Δp with different memory lengths:
- T=0
- T=2
- T=8

Memory length

`Memory length`

- T=0
- T=2
- T=8
Memory length

- $T=0$
- $T=2$
- $T=8$
The graph illustrates the relationship between $1 - \text{critical } \Delta p$ value and memory length ($L$) for different time indices ($T$). The graph shows three distinct lines corresponding to $T=0$, $T=2$, and $T=8$.

- **T=0**: Represented by diamond-shaped markers (●). The line is the steepest among the three.
- **T=2**: Represented by square-shaped markers (■). The line is less steep compared to T=0.
- **T=8**: Represented by triangle-shaped markers (▲). The line is the least steep among the three.

The y-axis represents the $1 - \text{critical } \Delta p$ value, ranging from 0.001 to 1.0. The x-axis represents the memory length ($L$), ranging from 1 to 10,000.
L=301

% of samples reaching total consensus

Δp

Memory length

- T=0
- T=2
- T=8
