Intercommutation of Semilocal Strings and Skyrmions

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We study the intercommuting of semilocal strings and Skyrmions, for a wide range of internal parameters, velocities and intersection angles by numerically evolving the equations of motion. We find that the collisions of strings and strings, strings and Skyrmions, and Skyrmions and Skyrmions, all lead to intercommuting for a wide range of parameters. Even the collisions of unstable Skyrmions and strings leads to intercommuting, demonstrating that the phenomenon of intercommuting is very robust, extending to dissimilar field configurations that are not stationary solutions. Even more remarkably, at least for the semilocal U(2) formulation considered here, all intercommutations trigger a reversion to U(1) Nielsen-Olesen strings.

Physical systems can have a large number of spontaneously broken symmetries, some of which may be local (gauged) while others may be global. In cases where the symmetries are only partially gauged, or gauged with unequal strengths, semilocal strings may be present \[1, 2, 3, 4\]. If the gauge couplings are widely disparate, the strings may also be stable. Semilocal string solutions exist in the standard electroweak model, but the SU(2) and U(1) gauge couplings are close enough that the solutions are unstable \[5, 6\]. Recently twisted and current-carrying semilocal string solutions have been discovered that may be stable for other values of parameters \[7, 8\]. Semilocal strings arise in supersymmetric QCD if the number of flavors is larger than the order of the symmetry group \[9\]. In string theory, they can arise as field theoretic realizations of cosmic D-strings \[10\], in which case they may play a role in cosmology. The formation of semilocal strings in a phase transition has been studied in Refs. \[11, 12\], in a cosmological setting in Refs. \[13, 14\], and in superstring cosmology \[15\].

In the case of topological gauged \[16, 17, 18\] and global \[19\] U(1) strings, numerical evolution of the field theory equations show that colliding strings intercommute (Fig. 1) for almost any scattering angle and velocity. Thus the probability of intercommuting is unity for topological strings. Qualitative arguments \[19, 21\] have been given in an effort to understand intercommutation of topological strings. In the context of string theory cosmic strings and QCD strings, intercommuting probabilities have been calculated in Refs. \[22, 23\] within various approximations.

In this paper we study the interactions of semilocal strings, which are topological structures embedded in the solution space of the partially gauged theory, and the related semilocal Skyrmions which are fat and fuzzy objects (still with one “long”, i.e. stringlike dimension) on which strings can terminate, much like strings ending on monopoles. (We shall show below (Figs 1, 2) however, that the termination is dynamic, moving along the string.) The scalar field in a Skyrmion can be arbitrarily close to its vacuum expectation value everywhere, in contrast to a string in which the scalar field magnitude vanishes (which is a nonvacuum value) on some curve. We review below that the embedded topological strings have precisely the dynamics of U(1) Nielsen-Olesen (NO) \[22\] strings. Thus semilocal strings will intercommute on collision with each other. However, the interaction of different objects such as semilocal strings with Skyrmions, or Skyrmions with Skyrmions, is a new consideration. Remarkably we find that intercommuting is extremely robust – strings intercommute with Skyrmions, and Skyrmions intercommute with other Skyrmions. Indeed, even in the parameter regime where the solutions are unstable, we see intercommuting. This establishes, for the first time, that intercommuting is not particular to string solutions, but can happen more generally between any two field configurations. Even more remarkably, at least for the semilocal U(2) formulation considered here, all intercommutations trigger a reversion to U(1) NO strings.

The semilocal model we shall study is the bosonic part of the electroweak model with weak mixing angle \(\theta_w = \pi/2\). The Lagrangian is:

\[
L = |(\partial_\mu - i Y_\mu) \Phi|^2 - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} - \frac{\beta}{2} (\Phi^\dagger \Phi - 1)^2 \tag{1}
\]

where \(\Phi^T = (\phi, \sigma)\) is an SU(2) doublet, \(Y_\mu\) is a U(1) gauge potential and \(Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu\) its field strength. The only parameter in the model is \(\beta\).

The semilocal model has \([SU(2) \times U(1)_Y]/Z_2\) symmetry and only the \(U(1)_Y\) is gauged. If \(\Phi^T\) gets a vacuum
then tells us whether the strings intercommute or pass through.

Initial data and boundary conditions are based on suitable superpositions of single, boosted, static string solutions. Static solutions $\Phi^T = (g, f e^{i\theta})$ and $Y_0 = (v/r) \hat{e}_\theta$ for semilocal strings with $\beta = 1$ can be found from:

$$f' + \frac{v - 1}{r} f = 0$$

$$v' + r \left[ f^2 \left( 1 + \frac{g^2}{r^2} + g^2 - 1 \right) \right] = 0.$$  

where $f$, $g$ and $v$ are functions of $r$ only and $g = q_0 f/r$. To set up initial data and boundary conditions one could numerically solve Eqs. (4) and (5) and construct a table to be used for the superposition of string solutions. We choose instead parametrized approximate solutions to the equations. That is,

$$f = \left[ 1 - a_1 \left( \frac{r}{1 + r^2} \right)^2 \right] f_{NO}$$

where $f_{NO} = |\tan (r R_f/2)|^{E_f}$, $R_f = 1 + (a_2 - 1) e^{-r/r_o}$ and $E_f = a_3 + (1 - a_4) e^{-r/r_o}$ with similar expressions for $v$. The parameters in these expressions are found by minimizing the total energy for a single string.

Note that the fields for semilocal strings with non-zero $q_0$ approach their vacuum values only as power laws ($r^{-2}$ for $\Phi$, and $r^{-4}$ for the magnetic field strength). This is to be contrasted with the exponential approach for NO strings (which is the $q_0 = 0$ case). Nonetheless these polynomial fall-offs are still fast enough that we anticipate being able to construct initial data by superposing fields for widely separated strings, with minimal change in the total energy. The scheme is remarkably straightforward. (An early reference to a similar scheme for NO strings is Abrikosov 29. See also Matzner 16.)

Next we consider how to superpose the scalar fields of two well-separated strings, labeled by $i = 1, 2$. Let the two string scalar field configurations be $\Phi_1^r = (\phi_1, \sigma_1)$ and $\Phi_2^r = (\phi_2, \sigma_2)$, respectively. Since the $\phi_i$ vanish as a power law at infinity, and $\sigma_i$ go to unity, we superpose the scalar field string configurations using the scheme $\Phi^T = (\phi_1 + \phi_2, \sigma_1 \sigma_2)$. Because the axial gauge field $Y_\mu$ satisfies an equation which is linear, the superposition of $Y_\mu$ is taken to be $Y_\mu = Y_\mu^{(1)} + Y_\mu^{(2)}$.

The actual data are set as follows. With the energy minimization procedure described above, we start by constructing two single string solutions with parameters $q_0^{(1)}$ and $q_0^{(2)}$ in coordinates $(x, y, z)$. The strings are parallel to the $z$ axis and pass through the origin. We henceforth assume that $Y_\mu$ is written in the Lorentz gauge: $\partial_\mu Y^\mu = 0$. This gauge is linear and Lorentz invariant.

The string $q_0^{(2)}$ is then rotated around the $x$-axis by an angle $\Theta$, with $\Theta = 0$ (parallel strings) and $\Theta = \pi$ (antiparallel strings). Next, both strings are off-set along the $x$-axis a distance $x^{(1)} = -D/2$ and $x^{(2)} = D/2$, respectively. Finally, the strings are Lorentz boosted toward
FIG. 1: Field $\sigma$ for a collision of $q_0 = 1$ (“top”) and $q_0 = 3$ semilocal strings, at $\Theta = 90^\circ$, with collision velocity $V = 0.9$. The contours are $1 - |\sigma|^2$ at values 0.1, 0.2, 0.5. Note that $|\sigma|^2 + |\phi|^2 = 1$ is the vacuum. The strings reconnect.

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each other with velocity $V$, so that the string intersection will occur in the middle of the computational cube. Initial time derivatives are computed to second order in the timestep by repeating the data construction process at time $\Delta t/2$ into the future and $\Delta t/2$ into the past and taking $1/\Delta t$ times the difference of the two field configurations.

The boundary data are naturally set as part of the initial setup process. In subsequent time steps, the boundary values are computed as if this procedure were repeated, with the strings advanced at the boundary as if in free motion with the initial velocity. This means that even after the collision in the center of the cube, the boundaries continue to act as if the strings had passed through one another, regardless whether they did in fact pass through one another, or reconnected. However our computational domains are large enough that the region where the strings collide is causally disconnected from the boundaries of the computational domain.

Our numerical code solves the field equations of motion

for $\Phi^T = (\phi, \sigma)$ and $Y_\mu$ as a coupled set of first-order differential equations in time:

$$\partial_t \eta_A = \nabla^2 \phi_A - Y^2 \phi_A - \partial_{\phi_A} U - 2\epsilon_{AB}(Y^t \eta_B + Y^t \partial_t \phi_B)$$

$$\partial_t \kappa_A = \nabla^2 \sigma_A - Y^2 \sigma_A - \partial_{\sigma_A} U - 2\epsilon_{AB}(Y^t \kappa_B + Y^t \partial_t \sigma_B)$$

$$\partial_t W_\mu = \nabla^2 Y_\mu - Y_\mu (\phi^2 + \sigma^2) - \epsilon_{AB}(\partial_\mu \phi_B + \sigma_A \partial_\mu \sigma_B),$$

where $\partial_t \phi_A = \eta_A$, $\partial_t \sigma_A = \kappa_A$ and $\partial_t Y_\mu = W_\mu$. The subscripts $A, B = 1, 2$ denote respectively real and imaginary parts, $\epsilon_{AB}$ is the completely antisymmetric tensor in two indices with $\epsilon_{12} = 1$ and $U = \beta(\phi^2 + \sigma^2 - 1)^2/2$. We use a second order accurate, both in space and time,
discretization. The temporal updating is done via a standard leap-frog method.

Our first set of numerical experiments were done with strings of the same internal parameter $q_0$ ( $q_0(2) = q_0(1)$ ). We first verified that $q_0 = 0$ strings (NO strings) reconnect, as had been found previously. We also considered collisions between $q_0 = 1$ strings. Just like NO strings, these $q_0 = 1$ semilocal strings always intercommuted for the set of velocities $V \in \{0,1,0.5,0.9\} \equiv \{V\}$ and all intersection angles tested: $\{30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ\} \equiv \{\Theta\}$. Next, we consider the case $q_0(2) \neq q_0(1)$. Our simulations now depend on four parameters: $V, \Theta, q_0(1)$ and $q_0(2) - q_0(1)$. To reduce the parameter space, we only considered $q_0(1) = 1$, and $q_0(2) \in \{0,0.5,0.9,1.0,1,1.1,2,3,0\}$. In these cases also we find that for all velocities $V \in \{V\}$ and all angles $\Theta \in \{\Theta\}$, reconnection always occurs. The dynamics involved in the reconnection are interesting. In particular, the value of $q_0$ for both strings appears to relax (by radiation) toward $q_0 = 0$. The intercommutation converts these to NO strings! Figures 1 and 2 show frames from simulations of a right angle collision at $V = 0.9$, between $q_0 = 1$ and $q_0 = 3$ semilocal strings showing this behavior. We also considered collisions between unstable ($\beta > 1$) Skyrmions. Again, intercommutation always occurred, and the rever-son to NO always occurred. The full range of the collision simulations is available online at http://gravity.psu.edu/numrel/strings/

We have numerically studied collisions of strings and Skyrmions in the semilocal model. Our results demonstrate that collisions of any two objects in this model, whether they are identical or not, whether they are stable or unstable configurations, leads to intercommuting for any set of collisional parameters. Additionally, the simulations all show a triggered collapse of the interacting objects toward a NO configuration. During the collision we also observe a large amount of energy lost in radiation. This may provide an additional signature for cosmological semilocal strings.

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