Resource Allocation for a Wireless Powered Integrated Radar and Communication System

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Abstract—The integrated radar and communication system is promising in the next generation wireless communication networks. However, its performance is confined by the limited energy. In order to overcome it, a wireless powered integrated radar and communication system is proposed. An energy minimization problem is formulated subject to constraints on the radar and communication performances. The energy beamforming and radar-communication waveform are jointly optimized to minimize the consumption energy. The challenging non-convex problem is solved by using semidefinite relaxation and auxiliary variable methods. It is proved that the optimal solution can be obtained. Simulation results demonstrate that our proposed optimal design outperforms the benchmark scheme.

Index Terms—Energy beamforming, radar and communication waveform design, wireless power transfer.

I. INTRODUCTION

Integrated radar and communication (IRC) systems are promising in the next generation wireless communication networks since radar and communication functions can be simultaneously performed at a common frequency band in a single platform and the antenna and signal processing hardware can be shared [1]. Recently, it has inspired widely investigations from industry and academia, such as the development of the electronic warfare and intelligent transportation system [2]. In the IRC system, it is of crucial importance to design waveform for improving the radar and communication performance. Up to now, there have been two main categories of waveform design, namely, multiplexing waveform [3] and identical waveform [4]. For the first one, the radar and communication waveforms are multiplex. However, the resource utilization efficiency is low. In contrast, for the identical waveform, the resource efficiency is high since the waveform is shared by radar and communication functions. Thus, we focus on the second one in this paper.

The research was supported by the National Natural Science Foundation of China (61701214, 61561034, and 61701301), the Excellent Youth Foundation of Jiangxi Province (2017BAB212002), the Postdoctoral Science Foundation of Jiangxi Province (2017M610400, 2017KY04, 2017RC17), and Young Elite Scientist Sponsorship Program by CAST.

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Since the identical waveform relies on the traditional communication waveform, and orthogonal frequency division multiplexing (OFDM) waveform has high spectrum efficiency (SE), high modulation flexibility and strong tolerance for inter-symbol interference, there are many literatures devoted to exploiting OFDM to simultaneously perform radar and communication functions for improving SE [5]-[9]. In [5], the authors focused on signal processing in the IRC system. However, the OFDM waveform has not been optimized. The authors in [7] studied the optimization of OFDM waveform and designed flexible waveform. The designed waveform is adaptive to the environment and can improve the performance. Recently, the authors in [8] and [9] studied the resource allocation problem and the robust OFDM radar waveform design problem, respectively. The optimal power allocation strategy and the robust waveform have been designed.

Although the performance of the IRC system can be improved by the designed waveform in [5]-[9], it is still confined by the finite energy of devices. Fortunately, wireless power transfer (WPT) has been proposed to prolong the operation time of devices [10]. Low-power devices can harvest energy from the wireless environment. In [11], it was shown that WPT can improve the energy efficiency (EE) of wireless communication networks. Thus, in order to overcome the finite operational energy problem and improve the performance of IRC systems, a wireless powered IRC system is studied in this paper. The transmitter exploits the collected energy from a wireless power station to simultaneously perform radar and communication functions. Moreover, a protocol called “harvest-then-transmit” is applied in order to facilitate the implementation of the wireless powered IRC system [12].

Different from the works in [8] and [9], it is the first time that a wireless powered IRC system is proposed. The transmission energy of the wireless power station is minimized by jointly optimizing the energy beamforming vector, energy harvesting time and the transmit power. It is proved that the optimal solution can be obtained. Then, by demonstrating that the optimal beamforming matrix after semi-definition relaxation (SDR) is rank-one, we prove that the constraints of the non-convex problem are always achievable. Simulation results show that our proposed optimal design outperforms the benchmark scheme.

Notations: Boldface capital letters represent matrices; boldface lower case letters represent vectors. I and 0 denote the identity matrix and the matrix with zero entries (their size is determined from context), respectively. \( \mathbf{x}^\dagger \) represents the conjugate transpose of a vector \( \mathbf{x} \). \( \mathbb{E}\{\cdot\} \) represents the expectation operator. \( \text{Tr}\{\cdot\} \) is the trace of the square matrix.
where the center frequency with its unit being Hz; \(N_c\) is the number of subcarrier; \(a_m\) is the amplitude of the \(m\)th subcarrier that needs to be designed; \(c_{m,n}\) is the phase encoding of the \(m\)th subcarrier at the \(n\)th OFDM signal; \(\Delta f\) is the interval between two adjacent OFDM subcarriers in the frequency domain; \(T_s\) is the duration of each completed OFDM symbol, and \(\text{rect}[t]\) is rectangle function, which is equal to one for \(0 \leq t \leq 1\); otherwise, it is zero.

For radar target detection and valuation, the conditional mutual information (MI) is an appropriate metric to evaluate the estimation accuracy of the target impulse response. Let \(g(t)\) denote the impulse response of an extended target. The signal received by the radar receiver is given as

\[
y(t) = \int_{-\infty}^{+\infty} g(\tau)z(t - \tau) d\tau + n(t),
\]

where \(n(t)\) is complex AWGN with zero mean and power spectral density \(N(f)\). In order to improve the estimation accuracy of the target impulse response, the conditional MI between the received signal and the impulse response should be as large as possible [6]. The conditional MI can be expressed as

\[
I_1(g(t); y(t)|z(t)) = \frac{1}{2} \Delta f T_p \sum_{m=0}^{N_c-1} \log_2(1 + p_m \varpi_m),
\]

where \(p_m = |a_m|^2\) denotes the \(m\)th subcarrier power with its unit being W; \(T_p = N_c T_s\) is the total duration of the transmitted OFDM signal, namely, \(T_p = \tau_2\); \(\varpi_m = N_c T_s^2 |G(f_m)|^2/(N(f_m) T_p)\) can be regarded as the signal to noise ratio (SNR); \(f_m = f_c + m \Delta f\) is the frequency of the \(m\)th subcarrier, and \(G(f)\) is the Fourier transform of \(g(t)\).

For the communication process, the data information rate (DIR) is an important performance metric, and the frequency selective fading channel is considered. Thus, the total DIR is given as

\[
C_t = \Delta f T_\tau \sum_{m=0}^{N_c-1} \log_2(1 + p_m \varpi_m),
\]

where \(\varpi = |h_m|^2/\sigma^2\) can be interpreted as the SNR; \(h_m\) is the channel gain of the \(m\)th subchannel, and \(\sigma^2\) is the noise power at the user receiver. In order to guarantee the radar and communication performance, the minimum MI and DIR are required to be considered. They can be expressed as

\[
C1 : \frac{1}{2} \Delta f T_\tau \sum_{m=0}^{N_c-1} \log_2(1 + p_m \varpi_m) \geq R_r,
\]

\[
C2 : \Delta f T_\tau \sum_{m=0}^{N_c-1} \log_2(1 + p_m \varpi_m) \geq R_c,
\]

where \(R_r\) and \(R_c\) are the minimum performance requirements for the radar and communication process, respectively. Moreover, due to the energy harvesting causal constraint, the consumption energy cannot be larger than the harvesting energy, namely,

\[
C3 : \tau_2 \sum_{m=0}^{N_c-1} p_m \leq \eta \tau_1 |h^H w_e|^2.
\]
III. ENERGY BEAMFORMING AND WAVEFORM DESIGN

In this section, an energy beamforming and waveform design problem is formulated. In order to minimize the energy of the wireless power station, the optimization problem is formulated as $P_0$, given as

$$P_0: \begin{array}{ll}
\min & \tau_1 \text{Tr}(w_e w_e^H) \\
\text{s.t.} & C1 - C3,
\end{array}$$

where $\tau = [\tau_1, \tau_2]$. $C4$ is the peak power constraint due to the nonlinearity of power amplifiers in practice. It is given to protect the transmitter [16]. $C5$ is the total time constraint.

$P_0$ is a non-convex problem due to the existence of couples among different optimization variables, such as $w_e$ and $\tau_1$. In order to solve it, several auxiliary variables are used, namely, $Q_e = w_e w_e^H$, $\bar{Q}_e = \tau_1 Q_e$, $\gamma_m = \tau_2 p_m$, $m = 0, 1, \ldots, N_c - 1$. Thus, $P_0$ can be equivalently as $P_1$, given as

$$P_1: \begin{array}{ll}
\min & \text{Tr}(\bar{Q}_e) \\
\text{s.t.} & C1 - C3,
\end{array}$$

where $C7$ and $C8$ are given to guarantee that $\bar{Q}_e$ is a semi-definite and rank-one matrix since the matrix of the form $w_e w_e^H$ must be semi-definite and rank-one [14]. The problem $P_1$ is still non-convex due to the constraint $C8$. We use the SDR technique to relax the non-convex optimization problem [15]. $P_1$ can be expressed as

$$P_2: \begin{array}{ll}
\min & \text{Tr}(\bar{Q}_e) \\
\text{s.t.} & C1 - C7.
\end{array}$$

Note the inequality constraints in $C1$ and $C2$ are convex, since they are the perspective function of $\log_2(1 + x)$. Moreover, other constraints are linear. It is easy to prove that $P_2$ is convex and can be efficiently solved by using the interior-point method [16]. Let $(\bar{Q}_e^{\text{opt}}, \tau_e^{\text{opt}}, \gamma_m^{\text{opt}})$ denote the optimal solution of $P_2$. Based on solving $P_2$, Theorem 1 can be obtained.

**Theorem 1:** Provided that $P_2$ is feasible, the optimal solution $\bar{Q}_e^{\text{opt}}$ always exists and is rank-one.

**Proof:** In order to prove that the optimal solution $\bar{Q}_e^{\text{opt}}$ is rank-one, the following problem is firstly considered.

$$\begin{array}{ll}
P_3: \min & \text{Tr}(\bar{Q}_e) \\
\text{s.t.} & \sum_{m=0}^{N_c-1} \gamma_m^{\text{opt}} \leq \eta \text{Tr}(hh^H \bar{Q}_e),
\end{array}$$

Obviously, $P_3$ is convex and can achieve the optimal solution $\bar{Q}_e^{\text{opt}}$. It is seen from eq. (11) and eq. (12) that $\bar{Q}_e^{\text{opt}}$ is also a feasible solution of $P_3$. It indicates that $\text{Tr}(\bar{Q}_e) \leq \text{Tr}(\bar{Q}_e^{\text{opt}})$. Combining the constraints of $P_2$, we can obtain $\text{Tr}(\bar{Q}_e) \leq \tau_1^{\text{opt}} P$. Thus, $(\bar{Q}_e^{\text{opt}}, \tau_e^{\text{opt}}, \gamma_m^{\text{opt}})$ is also a feasible solution of $P_2$ and we can obtain $\text{Tr}(\bar{Q}_e^{\text{opt}}) \geq \text{Tr}(\bar{Q}_e)$. Hence, we have $\text{Tr}(\bar{Q}_e^{\text{opt}}) = \text{Tr}(\bar{Q}_e)$; in other words, $(\bar{Q}_e^{\text{opt}}, \tau_e^{\text{opt}}, \gamma_m^{\text{opt}})$ is also the optimal solution of $P_2$. Next, we need to prove that $\bar{Q}_e^{\text{opt}}$ satisfies the rank-one property.

For $P_3$, the Lagrangian function is given as

$$L(\bar{Q}_e, Y, \mu) = \text{Tr}(\bar{Q}_e) - \text{Tr}(\bar{Q}_e Y) + \mu (\eta \text{Tr}(hh^H \bar{Q}_e) - \sum_{m=0}^{N_c-1} \gamma_m^{\text{opt}}),$$

where $\mu \in \mathbb{R}_+$. and $Y \in H_+^3$ are dual variables associated with (12b) and (12c), and we define $\rho = \frac{1}{\eta} \sum_{m=0}^{N_c-1} \gamma_m^{\text{opt}}$. The corresponding KKT conditions are

$$Y = I - \mu (hh^H),$$

$$Y \bar{Q}_e = 0, \quad \bar{Q}_e \geq 0. \quad (14b)$$

Since the rank of the $hh^H$ is one, $Y$ has at least $N_t - 1$ positive eigenvalues, and one has

$$\text{Rank}(Y) = \text{Rank}(I - \mu (hh^H)) \geq N_t - 1. \quad (15)$$

Based on (14), one has that $\text{Rank}(Y)$ is either $N_t$ or $N_t - 1$. For $\text{Rank}(Y) = N_t$, $\bar{Q}_e$ must be $0$. It is contradictory with (12b) in $P_3$. For $\text{Rank}(Y) = N_t - 1$, $\bar{Q}_e$ lies in the null space of $Y$, whose dimension is one. This means that the optimal solution $\bar{Q}_e^{\text{opt}}$ must be rank-one.

Since $\bar{Q}_e^{\text{opt}}$ is rank-one, the optimal solution of $P_2$ is also the optimal solution of $P_1$. Moreover, this solution is the globally optimal since $P_2$ is convex. Furthermore, the optimal solution of $P_0$, $w_e w_e^H$, can be obtained via singular value decomposition.

IV. SIMULATION RESULTS

In this section, simulation results are presented to demonstrate the effectiveness of the proposed design. The simulation parameters are based on the work in [21], given as: $N_c = 128$, $\Delta f = 2.5 \times 10^8$ Hz, $T_s = 5 \times 10^{-8}$ s, $T = 1 \times 10^{-4}$ s, $N_t = 5$, $P = 50$ W and $\eta = 0.5$. Both the frequency response of radar target and the frequency response of the communication channels are subject to the standard normal distribution, and the channel coefficients between each antenna of the wireless power station and IRC transmitter follow Rayleigh fading with the distribution $CN(0, 0.2)$. For a comparison, the benchmark scheme that the transmit power is equally allocated in each sub-channel is applied.

Fig. 2(a) compares the minimum transmission energy of the wireless power station required by using our proposed...
scheme with that achieved by using the benchmark scheme. Our proposed scheme is denoted by “OP” while the benchmark scheme is represented by “EQ”. The minimum communication performance requirement $R_c$ is set as 150 bits and the communication SNR is set at 10 dB. It is seen that the transmission energy increases with the minimum required MI. The reason is that the harvesting energy needs to be increased in order to satisfy the MI requirement. It is also seen that the EQ scheme requires a larger transmission energy than our proposed scheme. The reason is that our proposed scheme jointly optimizes all the variables related to the wireless resource. When the minimum required MI is lower than 220 bits and the radar SNR is 15dB, it is noted that the transmission energy required for the EQ scheme is almost constant. The reason is that the minimum MI achieved by the EQ scheme is 216 bits when the radar SNR is 15dB.

Fig. 2(b) shows the WPT time slot $\tau_1$ and IRC time slot $\tau_2$ versus the minimum required MI under different radar SNR and power allocation schemes. It is seen that, with the increase of the minimum required MI, the WPT time slot $\tau_1$ increases but the IRC time slot $\tau_2$ decreases. When the SNR is the same, the equal scheme requires a larger energy harvesting time $\tau_1$ than the optimal scheme and $\tau_2$ determines the upper bound of the system’s performance. Therefore, given the total time and transmission power, our proposed optimal scheme can achieve a greater MI than the equal power allocation scheme.

Fig. 2(c) shows the minimum transmission energy of the wireless power station versus the minimum required DIR. The minimum communication performance requirement $R_c$ is set as 150 bits and the radar SNR is set as 10 dB. It is seen that the transmission energy increases with the minimum required DIR, irrespective of the power allocation schemes. When the required DIR is lower than 160 bits and the communication SNR is 12dB, the transmission energy required for the OP scheme is almost constant. The reason is that the minimum DIR achieved by the OP scheme is 121 bits since $R_c$ is set as 150 bits. It is also seen that our proposed scheme outperforms the benchmark scheme in terms of energy consumption.

V. CONCLUSION

A wireless powered OFDM IRC system was studied. The optimal energy beamforming and waveform scheme was designed for minimizing the consumed energy. It was proved that the rank-one solutions can be obtained. Simulation results have verified the efficiency of our proposed scheme.

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