Diffraction of an Off-axis Vector-Beam by a Tilted Aperture

Ghanasyam Remesh,1 Athira B S.2,† Shyam Gucchait,3,4 Ayan Banerjee,3,2 Nirmalya Ghosh,3,2 and Subhasish Dutta Gupta3,4,5,

1Department of Condensed Matter Physics and Material Science, Tata Institute of Fundamental Research, Mumbai 400005, India
2Center of Excellence in Space Sciences India, Indian Institute of Science Education and Research (IISER) Kolkata. Mohanpur 741246, India.
3Department of Physical Sciences, Indian Institute of Science Education and Research (IISER) Kolkata. Mohanpur 741246, India.
4Tata Centre for Interdisciplinary Sciences, TIFRH, Hyderabad 500107, India
5School of Physics, University of Hyderabad, Hyderabad 500046, India

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Manifestations of orbital angular momentum induced effects in the diffraction of a radially polarized vector beam by an off-axis tilted aperture are studied both experimentally and theoretically. Experiments were carried out to extract the degree of circular polarization, which was shown to be proportional to the on-axis component of the spin angular momentum density. We report a clear separation of the regions of dominance of the right and left circular polarizations associated with positive and negative topological charges respectively, which is reminiscent of the standard vortex-induced transverse shift, albeit in the diffraction scenario. The experimental results are supported by model simulations and the agreement is quite satisfactory. The results are useful to appreciate the orbit-orbit related effects due to unavoidable misalignment problems (especially for vortex beams).

I. INTRODUCTION

It is now well understood that the angular momentum of light can have contributions from two factors. One depends on the polarization of the field, and is related to the spin angular momentum (SAM). The other, drawing its origin from the spatial distribution of the field, being independent of the beam polarization, is related to orbital angular momentum (OAM). SAM is an intrinsic property of the field, i.e., independent of the choice of axis about which the angular momentum is calculated [1]. For a circularly polarized light, the SAM per photon takes the value of ±ℏ. Similarly, Allen et al. [2] showed that for Laguerre Gaussian (LG) beams, each photon has an OAM of ℏl, where l is the topological charge of the vortex. However, in contrast to SAM, the OAM can either be intrinsic or extrinsic in nature, depending on the geometry of the system. The OAM of an LG beam causes a particle to rotate around the beam axis, and a SAM would cause a birefringent material to spin about its own axis, and hence can be used as an ‘optical spanner’ [3]. Thus, it should be noted that the separation of angular momentum into OAM and SAM is not just a theoretical result.

Several interactions happen between this intrinsic and extrinsic angular momentum, thereby giving rise to spin-orbit and orbit-orbit interaction (SOI and OOI, respectively). The interactions can be divided into three categories: between SAM and extrinsic OAM, SAM and intrinsic OAM and intrinsic and extrinsic OAM (related to OOI) [4–6]. This paper deals with the last, where we have a separation in the spatial profile depending on the sign of the topological charge. Owing to both fundamental interests and its potential applications the studies on OOI of light and its manifestations are currently attracting a lot of attention. [4, 7–10].

Several authors study the effects of Goos–Hänchen and Imbert–Fedorov shift in Gaussian and vortex beams [11–16]. A detailed description of these phenomena is covered in [17]. There has been extensive research on intrinsic and extrinsic properties of the angular momentum in the context of LG beams [1–18]. It is well understood that an LG beam falling on an aperture whose center is shifted from the beam center (without any tilt) leads to a change in the distribution of the SAM density inside the aperture. A recent paper by Taira and Zhang [19] has a focus on shifted aperture-beam system without any tilt in order to bring out the ability of such systems to measure the vortex charge of (higher order) LG beams by mixing it with a reference beam. However, there exists no such studies on the general vector beams falling on a tilted aperture. A radial vector beams can be considered to be a superposition of right and left circularly polarized LG beams of equal intensities and opposite OAM densities. Vortex charge dependence of the diffracted pattern projected onto the circular basis manifests itself as OAM induced transverse shift of the LCP and RCP components, which is clearly a manifestation of OOI [6, 14, 20, 22].

In this paper, we study the effects of both tilt and shift on orbit-orbit coupling due to diffraction of a vector beam by an off-centered tilted aperture. In particular, we explore the tilt- and shift- dependence of the topological charge-induced transverse shifts of the circular polarization components. In the experiment, we perform Stokes...
FIG. 1. Schematics of the setup used for the numerical calculations. The orientation of the different axes as defined in the text are shown. The primed coordinate system is attached to the beam, where as the un-primed coordinate system describes the aperture plane. The rotation between the coordinate systems is assumed to be in the $x-z$ plane. Hence, both $\hat{x}$ and $\hat{x}'$ would lie on the plane of this paper, while $\hat{y} = \hat{y}'$ projects out of the this plane. The center of the aperture is shifted from the origin by $r_0$ as shown in the figure.

imaging to extract the degree of circular polarization for a radially polarized vector beam diffracted by a tilted circular aperture. Further, we do numerical calculations of the same. For a comparison of our experimental results with the theoretically computed SAM, we establish a link between the experimentally measured Stokes parameter and the spin-angular momentum density. We also discuss in detail the procedure with due attention to the complications arising from the tilt and shift of the aperture. Since it is almost impossible to center a vortex beam exactly around a point, we assume in our numerical calculations that the beam is off-centered by a small distance. The high degree of resemblance between the experimentally observed and simulated results suggest that we can indeed trace the origins of the observed behavior to the tilt of the aperture and the shift of the beam. It should be noted that the finite aperture thickness has not been taken into account in any of the numerical calculations.

II. FORMULATION OF THE PROBLEM

For a general EM field, the linear and angular momentum of light can be broken up into two parts, spin and orbital angular momentum, as mentioned in the introduction. Recall that the angular momentum density of light is given by $\text{r} \times \text{p}$, where $\text{p}$ is the linear momentum density of light given by $\text{p} = \frac{\epsilon_0}{2} \text{Re} \, [\text{E}^* \times \text{B}]$. Using Maxwell’s equations and vector identities, we can break up $\text{p}$ into $\text{p}_{\text{orb}}$ and $\text{p}_{\text{sp}}$. The structures for the spin and orbital parts of the linear momentum densities are,

$$\text{p} = \text{p}_{\text{orb}} + \text{p}_{\text{sp}},$$

where

$$\text{p}_{\text{orb}} = \frac{\epsilon_0}{2\omega} \text{Im} \, [\text{E}^* \cdot \nabla \text{E}],$$

$$\text{p}_{\text{sp}} = \frac{\epsilon_0}{2\omega} \text{Im} \, \left[ \frac{1}{2} \nabla \times (\text{E}^* \times \text{E}) \right].$$

In eq. 2 we have used the notation, $[\text{A} \cdot \nabla \text{B}]_j = A_i \frac{\partial B_j}{\partial x_i}$. Accordingly, the angular momentum density can also be broken up into two parts.

$$\text{j} = \text{j}_{\text{OAM}} + \text{j}_{\text{SAM}} = \text{r} \times \text{p}_{\text{orb}} + \text{r} \times \text{p}_{\text{sp}}.$$

Here, the first term $\text{j}_{\text{OAM}}$ leads to the intrinsic or extrinsic spin depending on the geometry and is independent of the polarization of the field. The second term represents the spin angular momentum density of light $\text{j}_{\text{SAM}}$, which is always intrinsic in nature. Our focus is mainly on SAM and we show that it can be related to one of the Stokes parameters at the end of this section. Assuming the electromagnetic field to be transverse, we can rewrite the expression for spin angular momentum density by integrating over space to find the total spin angular momentum using the boundary condition that the fields go to zero at the edge of the aperture and beyond. Finally, we have 23

$$\text{j}_{\text{SAM}} = \frac{\epsilon_0}{2\omega} \text{Im} \, [\text{E}^* \times \text{E}] = -\frac{\epsilon_0}{\omega} \text{Im} \, [\text{E}^*_y \text{E}_x].$$

For a field that satisfies the paraxial equation, such as for the LG beam, it can be seen that the ratio of SAM density to photon density depends on the polarization of the field, and the corresponding value for $\text{j}_{\text{OAM}}$ depends on $l$. Hence, the physical origins of the two spins are clearly different.

In what follows, we show that $\text{j}_{\text{SAM}}$ can directly be extracted from the final Stokes parameter, $V$. It is well known that for an an electric field vector given by $\text{E} = E_x \hat{x} + E_y \hat{y}$, the Stokes parameter, $I$ and $V$ are given by,

$$I = |E_x|^2 + |E_y|^2,$$

$$V = 2 \text{Im} \, [E_x^* E_y].$$

Thus, both SAM and $V$ have the same structure, and thus, by means of Stokes mapping we can reveal the character of spin angular momentum of the diffracted beam. The numerical computation results can then be compared with the measured $V$.

In what follows, we give a brief sketch of our theoretical calculations and describe the corresponding experimental setup. Our experiment directly measures the values of $V$ and $I$, as will be detailed in section II B.

A. Theoretical approach

In contrast to the existing studies 11 19, this paper incorporates the tilt of the aperture with respect to the
beam axis. We study the diffraction of a radial vector beam propagating in the $\hat{z}'$ direction and falling on a tilted aperture, as shown in Fig. 1. The image is captured in a plane that is taken to be perpendicular to the direction of propagation of the beam. For ease of calculations, we assume the plane of the aperture to pass through the center of the beam waist. As mentioned before, the center of the aperture is assumed to be shifted by $r_0 = x_0 \hat{x} + y_0 \hat{y}$ from $(0,0)$, i.e., the center of the beam waist. Here $\hat{x}$ and $\hat{y}$ refer to unit vectors that lie on the aperture plane, and $\hat{z}$ is the normal to this plane, i.e., the unprimed coordinate system is attached to the aperture. Similarly, $\hat{x}'$ and $\hat{y}'$ lie on the image plane, and $\hat{z}'$ is the normal to this plane, i.e., the primed coordinate system is assigned to the beam. We assume the origins of both coordinate systems to be coincident. The angle between $\hat{z}$ and $\hat{z}'$ is taken to be $\theta$.

Vector beams have been studied extensively and the analytical expression for the electric field can be written as [24],

$$E(x', y', z') = E(r', z') [\cos(\theta + \phi) \hat{x}' + \sin(\theta + \phi) \exp(i\delta) \hat{y'}],$$  \hspace{1cm} (8)

where,

$$E(r', z') = \frac{2\sqrt{2}}{\omega_0} \frac{\exp(-\frac{r'^2}{2\omega_0^2(z')})}{\sqrt{2\pi}z'} \exp{\{-i\psi(z')\}} \exp\{ikz'\},$$  \hspace{1cm} (9)

and $r'^2 = x'^2 + y'^2$ and $\phi = \tan^{-1}\frac{y'}{x'}$. We introduce nominal deviation from the radial vector beam to account for the imperfections in the experimental scenario. Thus, $\theta$ and $\delta$ in eq. (8) account for ellipticity of the beam. Note that for $\theta = \delta = 0$, the beam polarization is in the radial direction.

Let a vector beam as described by eq. (8) fall on the tilted aperture as shown in Fig. 1. Our goal is to calculate the diffraction pattern formed by this aperture at a screen located at a distance of $z'$. The method of using angular spectrum decomposition to propagate the field at an angle to the aperture plane has been discussed in detail in [25]. In what follows, we lay out the procedure and the mathematics involved in calculating the diffraction pattern. We also briefly discuss the complications that the tilted aperture problem presents, as opposed to the case without the tilt.

We start with the field distribution in the opening of the aperture using eq. (8) and the transformation equations between the two coordinate systems. For any point $(x, y, z = 0)$ which lies on the aperture plane, a simple coordinate transformation from the unprimed to the primed systems yields the relations,

$$x' = x \cos \theta_i, \quad y' = y, \quad z' = x \sin \theta_i.$$  \hspace{1cm} (10)

The field at any point inside the aperture can be evaluated by using eq. (10) and eq. (8),

$$E(x, y, z = 0) = E(r', x \sin \theta_i) [\cos(\theta + \phi) \hat{x}' + \sin(\theta + \phi) \exp(i\delta) \hat{y'}].$$  \hspace{1cm} (11)

FIG. 2. A schematic of the setup for observing orbit-orbit mixing of vector beam by a tilted aperture. P1 and P2, rotatable linear polarizers.

where $r'^2 = (x \cos \theta_i)^2 + y^2$ and $\phi = \tan^{-1}\frac{y}{x \cos \theta_i}$. The aperture in numerically implemented such that every point on the aperture plane that lies inside a circle of radius $a$ defined by $(x - x_0)^2 + (y - y_0)^2 < a^2$ retains its field value as given by eq. (11) whereas any point on or outside the boundary of the circle is assumed to have a zero field. It should be noted that this is exactly the same boundary condition as in Fresnel–Kirchhoff diffraction theory. The Fourier spectrum of the field in the aperture plane can now be obtained using standard 2D Fast Fourier Transform (FFT) algorithms.

$$\tilde{E}(k_x, k_y) = \mathcal{F}\mathcal{T}\{E(x, y, z = 0)\}. \hspace{1cm} (12)$$

Since our goal is to get the image in the plane perpendicular to the path of the beam (in the $x'y'$ plane), we perform a rotation from un-primed to primed coordinate system in the $k$-space to get $\tilde{E}(k_x', k_y')$. The transformation relations we use are given by,

$$k_{x'} = k_x \cos \theta_i - k_z \sin \theta_i, \quad k_{y'} = k_y,$$

$$k_z = k_z \cos \theta_i + k_x \sin \theta_i.$$  \hspace{1cm} (13)

Finally, we propagate the fields by a distance of $z'$ and calculate the field in the image plane, $E_I$, by taking an inverse Fourier transform as follows,

$$E_I(r', z') = \mathcal{F}^{-1}\{J(k_x', k_y') E(k_x', k_y')e^{ik_z'z'}\}. \hspace{1cm} (14)$$

The Jacobian in eq. (14) is related to the rotation in $k$-space such that $dk_x dk_y = J(k_x', k_y') dk_x' dk_y'$ given by

$$J(k_x', k_y') = \left|\cos \theta_i - \frac{k_z'}{k_z} \sin \theta_i\right|.$$  \hspace{1cm} (15)

Note that while evaluating the forward transform in eq. (12) we have taken evenly spaced $k_x$ values. However, from the nature of eq. (14) it is clear that $k_{x'}$ has a non-linear dependence on $k_x$. Hence, a uniformly sampled $k_x$ values yields unevenly distributed $k_{x'}$ values. Hence, in order to employ standard fast Fourier algorithms to calculate the inverse Fourier transform, we first interpolate the data from $E(k_{x'}, k_{y'})$ to calculate the field at uniformly spaced values of $k_{x'}$. Furthermore, the transformation causes the $k_{x'}$ values to fold on itself at the point where $k_{x'}'/k_z$ goes to zero. In other words, we find that two distinct values of $k_x$ transform to the same value of $k_{x'}$, each with opposite signs for $k_{x'}$. We ignore contributions from all points where $k_{x'}'/k_z < 0$, since they cannot form an image at the destination plane.
B. Experimental realization

As mentioned in the introduction, the goal of this paper is to study orbit-orbit mixing both theoretically and experimentally. The above theoretical formulation was implemented in order to explain the observed results of the experiment we performed to study orbit-orbit coupling. A schematic of the experimental arrangement is shown in Fig. 3. Fundamental Gaussian mode of a 632.8 nm line of a He-Ne laser (HNL225R – He-Ne Laser, 632.8 nm, 22.5 mW, Random) is passed through a tilted aperture (P300D, Thorlabs, USA) of diameter 300 µm. The radially polarized beam was generated from the fundamental Gaussian mode by using a combination of a linear polarizer, P1 (GTH10M, Thorlabs, USA) and S-waveplate (RPC-515-06-46, Altechna). The radially polarized beam is then passed through a tilted aperture (P300D, Thorlabs, USA) of diameter 300 µm. The diffraction pattern is then passed through a combination of quarter waveplate (WPQ10M-633, Thorlabs, USA) and linear polarizer, P2. The spatial variation of Stokes parameters $[I \ Q \ U \ V]^T$ of the diffraction pattern are determined by the quarter waveplate and the linear polarizer combination. The measurements were performed for a range of tilt angle of the aperture ($\theta_i = 0^\circ, 7^\circ, 14^\circ, 40^\circ$). The polarization resolved diffraction pattern are imaged into a CCD camera (1024 × 768 square pixels, pixel dimension 4.65 µm, Thorlabs, USA) at a distance of $z' = 15$ cm. The image collected by the CCD is compared with the theoretical simulations.

III. EXPERIMENTAL AND NUMERICAL RESULTS AND DISCUSSIONS

A. Comparison between experimentally measured and simulation results

We use a nearly radially polarized vector beam to model the experimental beam we use. By examining the Stokes mapping of the bare vector beam used in the experiment (not shown), a small eccentricity has been included in our numerical calculations to account for the experimentally unavoidable errors. In particular, in order to have proper correspondence between theory and experiment we first studied the bare experimental beam and chose $\delta$ and $\theta$ to yield the theoretical intensity distribution that best matches the experimental pattern.
FIG. 5. (a) Experimentally measured diffraction pattern from an aperture tilted at an angle $\theta_i = 14^\circ$. (b) Theoretical diffraction pattern from an aperture tilted at an angle $\theta_i = 14^\circ$ ($x_0 \sim 6\lambda$ and $y_0 \sim 19.1\lambda$). (c) Experimentally measured V/I, where V is the difference between left and right polarized light intensities. (d) Theoretical V/I. The CG of the intensity pattern for LCP and RCP components are superposed using white and black dots. These values are used for all the Stokes images. The same approach is used to arrive at the value of the shift. This non-zero eccentricity for the polarization ellipse formed by $E_x'$ and $E_y'$ is modeled using $\theta = 2^\circ$ and $\delta = 5^\circ$ in eq. 8.

In order to have a clear idea about the individual effects of shift and tilt on the displacement of the beam we calculated the center of gravity (CG) of the beams (first moments) only theoretically since experimental realization of a given set of values for tilt and shift is practically impossible. Moreover, so as to have a parallel with Stokes images for degree of circular polarization, we projected the electric field $\vec{E}$ onto the circular basis, $\vec{E} = E_{LCP} \hat{e}_{LCP} + E_{RCP} \hat{e}_{RCP}$, where $E_{LCP,RCP} = (E_x \mp iE_y)/\sqrt{2}$. The first moment of the intensity distribution in the circular basis can then be calculated using

$$<x_{LCP,RCP}> = \frac{\int x|E_{LCP,RCP}|^2 dx dy}{\int |E_{LCP,RCP}|^2 dx dy}$$

$$<y_{LCP,RCP}> = \frac{\int y|E_{LCP,RCP}|^2 dx dy}{\int |E_{LCP,RCP}|^2 dx dy}$$

Figs. 3a, 4a, 5a and 6a show the experimentally measured intensity plots for a vector beam diffracted from a tilted aperture with a diameter of 300 $\mu$m for increasing angles. The corresponding simulated intensity plots are shown in Figs. 3b, 4b, 5b and 6b. Figs. 3c, 4c, 5c and 6c show the experimentally measured Stokes parameter plots for the corresponding intensity. However, due to the null intensity at the center of the vector beam (resulting in the inaccuracy of pinning down the center), there could be a small error (of the order of a maximum of $40\lambda$ or $25 \mu$m) in assessing the distance between the centers of the vector beam and the aperture. We thus include a small shift between the two centers in our numerical calculations, shown in Figs. 3d, 4d, 5d and 6d. The arrow in the theoretical plots show the direction of the shifts. Surprisingly, this small shift between the two centers causes a sufficiently large splittings between the left and right circularly polarized lights in the Stokes plot, as can be seen in these figures. We see that with increasing split between the centers, the left and right circularly polarized lights separates out in the direction perpendicular to the shift. The remarkable agreement between the experimental and numerical calculations leads us to conclude that the measured splitting is a result of the shift between the two centers. It should be noted that though the intensity plots for the angles $\theta_i = 0^\circ$, $7^\circ$ and $14^\circ$
seems to be comparable with the numerical results, the one for \( \theta_i = 40^\circ \) seems to be different. This could be due to the slight asymmetry in the bare beam used in the experimental setup.

B. Numerical results studying the effects of tilt and shift

In the previous section, we saw that a shift between the centers of the beam waist and the aperture with or without tilt results in splitting of the centers of left and right circularly polarized light associated with opposite topological charges, which is a 'fingerprint' of OOI of light. We now present the results on the effect of the tilt of the aperture on the OOI for a fixed shift.

We first look at a scenario where the two centers are perfectly aligned, with different tilts for the aperture. Note that the pattern shown in Fig. 7a is affected by the small amount of eccentricity of the bare beam, thus deviating from a perfectly radial vector beam. The variation of the Stokes parameter (or the SAM) in the direction \((x' - y')/\sqrt{2}\) for different angles of tilt is shown in Fig. 7b. As can be seen, the tilting increases the amount of SAM, though by a very small amount till about 29\(^\circ\). Further tilting reduces the amount of SAM, as depicted by the \( \theta_i = 40^\circ \) (Black dotted) curve. We have superposed the CG of the two helicity components in Figs. 7a and 7c. It is clear from Fig. 7a that a tilt of the aperture in the absence of any shift from the axis clearly does not create a significant change in position of the CG of the intensity for each helicities. When we include a small shift between the two centers, the separation between the CG’s is more prominent at \( \theta_i = 0^\circ \). In contrast to the previous case, tilting the slit decreases the amount of SAM, as can be seen in Fig. 7d. We also calculated the corresponding degree of circular polarization which confirms the same results (see Fig. 8).

The intensity patterns with superposed CG (white dot) along with the Stokes image for the degree of circular
polarization for typical parameters are shown in Fig. 9. The top row shows the results for a perfectly aligned system with null shift and tilt, while the left, middle and right columns show, respectively, the LCP, RCP and the degree of circular polarization. The middle row gives the same for null shift with \( \theta = 20^\circ \), while the bottom row depicts the same for null tilt but with shift \( x_0 = 20 \lambda \). It is clear from a comparison of Figs. 9a and 9b, along with panels [3] and [4], for null shift that helicity does not play any role in the behavior of the CG even for tilted systems, while the OO coupling for finite shift leads to opposite movement of the CG, which is also confirmed by the Stokes image (see Fig. 9c).

A more detailed picture is presented in Fig. 10, where the top (bottom) row shows the variation of position of the CG as functions of the tilt angle \( \theta_i \) for \( y_0 = x_0 = 0 \lambda \) (blue line) and for \( y_0 = 0 \lambda, x_0 = 20 \lambda \) (Red line). (c), (d) \( < x > \) and \( < y > \) as a function of the displacement from the center \( x_0 \), for \( y_0 = 0 \lambda \). The tilt angle has been taken to be \( \theta_i = 0^\circ \) (blue line) and \( \theta_i = 20^\circ \) (Red line). Circles (Crosses) represent the CGs of intensity field after projecting onto the LCP (RCP) basis.

Recall that the helicity dependence of the CG of the diffraction pattern of a vector beam is actually an effect of the different \( l \) values associated with each helicity.
IV. CONCLUSIONS

In this article, we have studied the diffraction of an off-axis vector beam from a tilted aperture. The broken symmetry is shown to result in mixing of the intrinsic and extrinsic orbital angular momentum of light, leading to a splitting between the LCP and RCP components of the incident beam. Note that these polarization components are associated with opposite topological charges. We report the increase in this splitting with increasing separation between the beam and aperture centers. Our numerical results have been validated using both experimental techniques, as well as using brute force integration. We also shed light on the connection between the spin angular momentum density and the Stokes parameter V. We have presented a thorough analysis of the individual and combined effects of shift and tilt on the diffraction pattern, on the corresponding Stokes images and also on the center of gravity of the polarization components.

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