The spin structure of the Λ hyperon in quenched lattice QCD

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It has been suggested to use the production of Λ hyperons for investigating the nucleon spin structure. The viability of this idea depends crucially on the spin structure of the Λ. Using nonperturbatively $O(a)$ improved Wilson fermions in the quenched approximation we have studied matrix elements of two-quark operators in the Λ. We present results for the axial vector current, which give us the contributions of the $u$, $d$, and $s$ quarks to the Λ spin.

1. INTRODUCTION

In the quark model, the Λ hyperon is a $uds$ state, and its spin is completely carried by the $s$ quark. Combining, on the other hand, the experimental results for the $gs$ structure function of the proton with the data on hyperon decay and assuming SU(3) flavour symmetry one finds that only $\approx 60\%$ of the Λ spin is carried by $s$ quarks, while $u$ ($\bar{u}$) and $d$ ($\bar{d}$) quarks contribute $\approx -40\%$. The spin structure of the Λ is of special interest, because its polarisation can easily be measured via the self-analysing weak decay $\Lambda \to p\pi^-$. Indeed, the Λ polarisation has been measured at the $Z^0$ pole in $e^+e^-$ annihilation where it is mainly due to the $s$ quarks, which when produced via $Z^0$ decays have an average polarisation of $-0.91$. Furthermore, Λ polarisation has been studied in deep-inelastic scattering of polarised positrons on unpolarised protons in the current fragmentation region, i.e. selecting Λ’s which most likely originate from the struck quark [3]. While polarised $s$ quarks are rare in the proton and their contribution to electromagnetic processes is suppressed by a factor $Q_s^2 = 1/9$, there are plenty of polarised $u$ quarks contributing with $Q_u^2 = 4/9$. Thus even a small correlation between the spins of the $u$’s and the $Λ$’s could make the $Λ$’s useful as probes of the polarised $u$-quark distribution in the proton [3].

Needless to say, however, that the interpretation of the experiments is complicated by fragmentation effects, Λ’s from the decay of heavier hyperons, etc.

On the theoretical side, for a spin 1/2 baryon the fraction $\Delta q$ of the spin carried by the quarks (and antiquarks) of flavour $q$ is given in terms of the forward matrix element of the axial vector current:

$$\langle p, s|\bar{q}\gamma_\mu\gamma_5 q|p, s\rangle = 2\Delta q \cdot s_\mu,$$  \hspace{1cm} (1)

where $s^2 = -m^2$ and the states are normalised according to $\langle p, s|p', s'\rangle = 2E_p(2\pi)^3\delta(\vec{p} - \vec{p'})\delta_{ss'}$.

In the following we shall present preliminary results of a lattice computation of such matrix elements in the Λ.

2. THE SIMULATION

We have performed quenched simulations with the Wilson gauge action at $\beta = 6.0$ using nonperturbatively $O(a)$ improved fermions (clover fermions) with $c_{SW} = 1.769$. The lattice size was $16^3 \times 32$. We worked with nine combinations of hopping parameters: $\kappa_u = \kappa_d$, $\kappa_s \in \{0.1324, 0.1333, 0.1342\}$ corresponding to bare quark masses of $\approx 166$, 112, 58 MeV, respectively. So we can extrapolate to the chiral limit

\textsuperscript{*}Talk given by M. Göckeler at Lat01, Berlin, Germany.
in $\kappa_u = \kappa_d$ and interpolate in $\kappa_s$ to the physical value $\kappa_s^*$. For the critical hopping parameter $\kappa_c$ we take the value 0.135201 determined from the PCAC quark mass \cite{4}, $\kappa_s^* = 0.134138$ was fixed by requiring the pseudoscalar mass $m_{PS}$ for $\kappa_u = \kappa_d = \kappa_c$ and $\kappa_s = \kappa_s^*$ to be equal to the $K^+$ mass = 494 MeV (with the scale set by the Sommer parameter $r_0 = 0.5$ fm). In the extra- and interpolation we assumed a linear dependence of $m_{PS}$ on the quark mass.

As an interpolating field for the $\Lambda$ we used (employing Euclidean notation from now on)

$$\sum_{x, x_d = t} \epsilon_{ijk} \kappa_i(x) (u_j^T(x) C\gamma_0 d_k(x))$$

with the charge conjugation matrix $C$ ($i, j, k$; colour indices). The quark fields were (Jacobi) smeared to improve the overlap with the $\Lambda$ state.

As usual, the bare matrix elements are determined from ratios of three-point functions over two-point functions. In order to reduce the cutoff effects from $O(a)$ to $O(a^2)$ also in matrix elements, the improvement of the fermionic action has to be accompanied by the improvement of the operator under study. In our case, this is the axial vector current, and for the renormalised improved operator we can take

$$A^R_{\mu} = Z^0_{\Lambda}(1 + b_{A} a m)(A_\mu + a c_A \partial_\mu P)$$

with the bare axial vector current $A_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$, the pseudoscalar density $P(x) = \bar{q}(x)\gamma_5 q(x)$, and the bare quark mass $a m = 1/(2\kappa_s) - 1/(2\kappa_c)$. The improvement term $\partial_\mu P$ vanishes in forward matrix elements, so we do not need the coefficient $c_A$. For $Z^0_{\Lambda}$ and $b_{A}$ we take the values from Ref. \cite{5}: $Z^0_{\Lambda} = 0.807(2)(8)$, $b_{A} = 1.28(3)(4)$.

3. RESULTS

From the $\Lambda$ masses at our nine combinations of $\kappa_d, \kappa_s$ we have computed $\Lambda$ masses at $\kappa_d = \kappa_c$ by linear extrapolation of $m_{\Lambda}^2$ in $1/\kappa_d$. These 12 masses are plotted in Fig.\[1\]. Our value for $\kappa_s^*$ reproduces quite accurately the ratio $m_{\Lambda}/m_p = 1.19$. This fits in nicely with the observation that $m_{\Sigma}/m_p$ and $m_{\Xi}/m_p$ are also rather well reproduced by quenched simulations (see, e.g., \cite{4}).

![Figure 1](image-url)  

The plot shows clearly the breaking of the SU(3) flavour symmetry. In particular, the dependence of the masses on $\kappa_d$ is rather pronounced.

In Fig.\[2\] we plot our (preliminary) results for $\Delta s$ and $\Delta d$ in the $\Lambda$ versus $1/\kappa_s$. In contrast with the case of $m_{\Lambda}/m_p$, the dependence on $\kappa_d$ is rather weak. The values corresponding to $\kappa_d = \kappa_c$ have been obtained by extrapolating the bare matrix elements linearly in $1/\kappa_d$. Interpolating the bare matrix elements for $\kappa_d = \kappa_c$ linearly in $1/\kappa_s$ to $1/\kappa_s^*$ we obtain the desired $\Lambda$ matrix elements given in the last line of the following table:

|          | $\Delta u_{\Lambda}$ | $\Delta d_{\Lambda}$ | $\Delta s_{\Lambda}$ |
|----------|-----------------------|------------------------|------------------------|
| quark model | 0                     | 1                      | 1                      |
| exp. + SU(3)$_F$ | -0.17(3)             | 0.63(3)                |                        |
| MC + SU(3)$_F$  | -0.016(9)            | 0.65(2)                |                        |
| this work | -0.01(4)              | 0.67(3)                |                        |

In the third line we give the $\Lambda$ matrix elements as they follow from our Monte Carlo results for the proton matrix elements by the use of SU(3)$_F$. They agree quite well with the matrix elements computed directly. Since the flavour symmetry breaking effects in the matrix elements are rather small (see Fig.\[2\]), this is hardly surprising. The second line contains the values of the $\Lambda$ matrix...
elements computed from the proton spin structure under the assumption of flavour SU(3)(see, e.g., [6]). All these results differ markedly from the predictions of the (naive) quark model shown in the first line.

Concerning the Monte Carlo results one should keep in mind that they were obtained at a fixed value of the lattice spacing. So a continuum extrapolation is not possible. Furthermore we used the quenched approximation and neglected quark-line disconnected contributions. Therefore it might be more reasonable to compare with estimates of the valence quark contribution, for which, e.g., Ashery and Lipkin [6] find $\Delta u_\Lambda = \Delta d_\Lambda = -0.07(4)$, $\Delta s_\Lambda = 0.73(4)$. Compared with the numbers from the second line of our table, $\Delta d_\Lambda$ gets closer to our Monte Carlo result, while $\Delta s_\Lambda$ becomes somewhat larger.

4. SUMMARY

Our Monte Carlo investigation of the spin structure of the $\Lambda$ hyperon leads to satisfactory agreement with the phenomenological numbers, although the comparison is not straightforward due to the use of the quenched approximation. SU(3) flavour symmetry appears to be much less violated in the matrix elements of the axial vector current than in the baryon masses, in agreement with the empirical observation that the hyperon semileptonic decays can be parametrised rather well assuming SU(3) flavour symmetry. The relevance of our findings for using $\Lambda$'s as a probe of the nucleon spin structure in deep-inelastic lepton-nucleon scattering remains to be studied. But we expect them to shed some light also on the lattice results for the nucleon spin structure.

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