A new method for calculating jet-like QED processes

C. Carimaloa, A. Schillerb, V.G. Serbo∗ †

aLPNHE, IN2P3-CNRS, Université Paris VI, F-75252 Paris, France

bInstitut für Theoretische Physik and NTZ, Universität Leipzig, D-04109 Leipzig, Germany

cNovosibirsk State University, 630090, Novosibirsk, Russia

We consider inelastic QED processes, the cross sections of which do not drop with increasing energy. Such reactions have the form of two-jet processes with the exchange of a virtual photon in the \( t \)-channel. We consider them in the region of small scattering angles \( m/E \lesssim \theta \ll 1 \), which yields the dominant contribution to their cross sections. A new effective method is presented to calculate the corresponding helicity amplitudes. Its basic idea consists in replacing spinor structures for real and weakly virtual intermediate leptons by simple transition vertices for real leptons. The obtained compact amplitudes are particularly suitable for numerical calculations in jet-like kinematics.

1. INTRODUCTION

The subject. Accelerators with high-energy colliding \( e^+e^- \), \( \gamma e \), \( \gamma\gamma \) and \( \mu^+\mu^- \) beams are now widely used or designed to study fundamental interactions. Some processes of quantum electrodynamics (QED) might play an important role at these colliders, especially those inelastic processes the cross sections of which do not drop with increasing energy. For this reason and since, in principle, the planned colliders will be able to work with polarized particles, these QED processes are required to be described in full detail, including the calculation of their amplitudes with definite helicities of all initial and final particles.

These reactions have the form of a two-jet process with the exchange of a virtual photon \( \gamma^* \) in the \( t \)-channel (Fig. 1). The subject of our consideration (for more details see Refs. [1,2] and references therein) is that process at high energies (\( m_i \) is a lepton mass)

\[
s = 2p_1p_2 = 4E_1E_2 \gg m_i^2
\]

for arbitrary helicities of leptons \( \lambda_i = \pm 1/2 \) and \( \lambda_i = \pm 1 \). The emission and scattering angles \( \theta_i \) are small:

\[
m_i/E_i \lesssim \theta_i \ll 1 \, \, , \, \, m_i \lesssim |p_{\perp}| \ll E_i .
\]

The corresponding Feynman diagrams of the discussed process up to \( e^5 \) order are given in [12]. We mention here only such processes as: single and double bremsstrahlung and single pair production in \( e^+e^- \) collisions, single pair production in \( \gamma e \) and double pair production in \( \gamma\gamma \) collisions.

The processes under consideration have large total cross sections. Therefore, they present an essential background and they determine particle losses in the beams and the beam life time. Since all these reactions can be calculated with high
accuracy independently of any model of strong interaction, they can usefully serve for monitoring the luminosity and polarization of colliding beams. Besides, the methods for calculating helicity amplitudes of these QED reactions can be easily translated to several semihard QCD processes such as $\gamma \gamma \rightarrow q\bar{q}Q\bar{Q}$ and $\gamma \gamma \rightarrow M M'$, $\gamma \gamma \rightarrow M q\bar{q}$ (see [33]).

All these properties of the jet-like QED processes justify the growing interest in them from both the experimental and theoretical communities in high-energy physics. Particular problems related to these processes were discussed in a number of original papers and in reviews such as [4]. But only recently (see Ref. [7]) the highly accurate analytical calculation of the helicity amplitudes of all jet-like processes up to $e^4$ was completed. In the above-mentioned original papers different approaches have been used. In our papers [12] we have developed a new simple and effective method to calculate those processes. For simplicity, in this contribution we restrict ourselves to bremsstrahlung processes only.

The form of “the final result”. At high energies the region of scattering angles $\theta_i$ gives the dominant contribution to the cross sections of all QED jet-like processes. In this region we obtain all helicity amplitudes with high accuracy, omitting only terms of the order of

$$\frac{m_i^2}{E_i^2}, \frac{\theta_i^2}{E_i}, \frac{m_i}{E_i}$$

or smaller. The amplitude $M_{fi}$ has a simple factorized form

$$M_{fi} = \frac{s}{q^2} J_1 J_2$$

where the impact factors $J_1$ and $J_2$ do not depend on $s$. We give analytical expressions for $J_1$ and $J_2$. They are not only compact but are also very convenient for numerical calculations, since large compensating terms are already cancelled. It is well known that this problem of large compensating terms is very difficult to manage in all computer packages. The discussed approximation differs considerably from the known approach of the CALCUL group in which terms of the order of $m_i/|p_{i\perp}|$ are neglected.

Three basic ideas: (i) a convenient decomposition of all 4-momenta into large and small components (using the so-called Sudakov or light-cone variables); (ii) gauge invariance of the amplitudes is used in order to combine large terms into finite expressions; (iii) the calculations are considerably simplified in replacing the numerators of lepton propagators by vertices involving real leptons.

All these ideas are not new. In particular, the last one is the basis of the equivalent-electron approximation [5] and has been used to calculate some QCD amplitudes with massless quarks. However, the combination of these ideas leads to a very efficient way in calculating the amplitudes of interest just in the jet kinematics.

2. METHOD OF CALCULATION

Sudakov or light-cone variables. We use light-like 4-vectors $P_1$ and $P_2$:

$$P_1 = p_1 - \frac{m_1^2}{s} p_2, \quad P_2 = p_2 - \frac{m_2^2}{s} p_1,$$

$$P_1^2 = P_2^2 = 0,$$  \hspace{1cm} (5)

and decompose any 4-vector $A$ as

$$A = x_A P_1 + y_A P_2 + A_\perp, \quad A^2 = s x_A y_A + A^2_\perp,$$  \hspace{1cm} (6)

where $x_A$ and $y_A$ are the so-called Sudakov variables. The 4-vectors $p_i$ of particles from the first jet have large components along $P_1$ and small ones along $P_2$, i.e.

$$x_i = \frac{2 p_i P_2}{s} = \frac{E_i}{E_1}, \quad y_i = \frac{2 p_i P_1}{s} = \frac{m_i^2 + p_{i\perp}^2}{s x_i}.$$ \hspace{1cm} (7)

Therefore, in the limit $s \to \infty$ the parameters $x_i$ are finite, whereas $y_i$ are small. The Sudakov variable $x_i$ is the fraction of energy of the first incoming particle carried by the $i$-th final particle. The Sudakov parameters $x_q$ and $y_q$ for the virtual photon are small.

Let $e \equiv e^{(A)}(k)$ be the polarization 4-vector of the final photon in the first jet. Using gauge invariance, this vector can be presented in the form

$$e = y_e P_2 + e_\perp, \quad y_e = \frac{-2k_\perp e_\perp}{s x_k}, \quad x_k = \frac{2k_\parallel P_2}{s},$$ \hspace{1cm} (8)
where
e_\perp \equiv e^{(\Lambda)}_\perp = -\frac{\Lambda}{\sqrt{2}} \left(0, 1, i\Lambda, 0\right) = -e^{(-\Lambda)^*}_\perp. \quad (9)

Therefore, \(e_\perp\) does not depend on the 4-momentum of the photon \(k\) contrary to the polarization vector \(e\) itself.

**Factorization of amplitudes.** The amplitude of Fig. 1 can be written as

\[ M_{f1} = M_1^\mu \frac{g_{\mu\nu}}{q^2} M_2^\nu, \quad (10) \]

where \(M_1^\mu\) and \(M_2^\nu\) are the amplitudes of the upper and lower block in Fig. 1. It is not difficult to show that this amplitude can be presented in the simple factorized form (10) with

\[ J_1 = \frac{\sqrt{2}}{s} M_1^\mu P_{2\mu}, \quad J_2 = \frac{\sqrt{2}}{s} M_2^\nu P_{1\nu}. \quad (11) \]

At high energies, the impact factor \(J_1\) depends on \(x_1, p_{i\perp}\) with \(i \in \text{jet}_1\) and on the helicities of the first particle and of the particles in the first jet. We use exactly this form for the concrete calculations.

Due to gauge invariance, we have \(M_1^\mu P_{2\mu} = -M_1^\mu q_{\perp\mu}/y_q\), and, therefore, we obtain another form of the impact factors:

\[ J_1 = -\frac{\sqrt{2}}{sy_q} M_1^\mu q_{\perp\mu}, \quad J_2 = -\frac{\sqrt{2}}{sx_q} M_2^\nu q_{\perp\nu}. \quad (12) \]

This representation is important, since it shows that at small transverse momentum of the exchanged photon the \(J_{1,2}\) behave as

\[ J_{1,2} \propto |q_{\perp}| \text{ at } q_{\perp} \rightarrow 0. \quad (13) \]

In our further analysis we will combine various contributions of the impact factor into expressions which clearly exhibit such a behaviour.

**Vertices instead of spinor lines.** Let us consider a virtual electron in the amplitude \(M_1\) with \(p = (E, p), E > 0\) and virtuality \(p^2 - m^2\). Due to jet kinematics, \(|p^2 - m^2| \ll E^2\). We introduce an artificial energy \(E_p = \sqrt{m^2 + p^2}\) and the bispinors \(\bar{u}^{(\Lambda)}_p\) and \(v^{(\Lambda)}_p\) corresponding to a real electron and a real positron with 3-momentum \(p\) and energy \(E_p\). In the high-energy limit \(E = E_p = (p^2 - m^2)/(2E)\) and

\[ \hat{p} + m \approx u^{(\Lambda)}_p \bar{u}_p^{(\Lambda)} + \frac{p^2 - m^2}{4E^2} v^{(\Lambda)}_p \bar{v}_p^{(\Lambda)}. \quad (14) \]

Using this equation for all virtual electrons, we are able to substitute the numerators of all spinor propagators by vertices involving real electrons and real positrons. These generalized vertices are finite in the limit \(s \rightarrow \infty\). On the contrary, a numerator like \(\hat{p} + m\) is a sum of a finite term \(\hat{p}_\perp + m\) and an unpleasant combination \(E(\gamma^0 - p_2\gamma_2)\) of large terms that requires special care. Therefore, those replacements significantly simplify all calculations.

More detailed considerations show that only three types of vertices are needed to calculate the impact factors involving the emission of real photons. The numerator of the spinor propagator \(\hat{p}_\perp + m\) in (14) consists of two terms. The first term corresponds to the simple replacements

\[ \hat{p} + m \rightarrow u^{(\Lambda)}_p \bar{u}_p^{(\Lambda)} \]

and leads to the vertex for the transition \(e(p) + \gamma^*(q) \rightarrow e(p')\) (where \(\gamma^*(q)\) is a virtual photon with energy fraction \(x_q = 0\) and an “effective polarization vector” \(e_\perp = \sqrt{2}p_2/s\))

\[ V(p) \equiv V_{\lambda\nu}(p) = \bar{u}_p^{(\lambda)} \bar{e}_q u_p^{(\lambda)} = \sqrt{2} \frac{E_p'}{E_1} \delta_{\lambda\nu} \Phi \quad (16) \]

and to the vertex for the transition \(e(p) \rightarrow e(p') + \gamma^*(k)\) (where \(\gamma^*(k)\) is a real photon with helicity \(\Lambda\))

\[ V(p, k) \equiv V_{\lambda\nu}(p) \approx \bar{u}_p^{(\lambda)} e^{(\lambda)} p \bar{e}_q u_p^{(\lambda)} = \\
= \left[ \delta_{\lambda\nu} 2 \left(e^{(\lambda)} \ast p\right) + (1 - x_\Lambda - 2\lambda) + \right. \]

\[ + \left. \delta_{\lambda\Lambda} 2 \delta_{\lambda,2\lambda} \sqrt{2} m x \right] \Phi \quad (17) \]

with \(x = \omega/E\) and

\[ \Phi = \sqrt{\frac{E}{E'}} e^{(\lambda') \ast (\lambda')}} e p = e_\perp \left(\hat{p}_\perp - \frac{k_\perp}{x}\right) \quad (18) \]

The second term in \(\hat{p}_\perp + m\) of (14) corresponds to the more complicated replacement

\[ \hat{p} + m \rightarrow \frac{p^2 - m^2}{4E^2} v^{(\Lambda)}_p \bar{v}_p^{(\Lambda)} \approx \frac{p^2 - m^2}{4E^2} \hat{P}_2. \quad (19) \]

Since this expression contains a factor proportional to the denominator of the spinor propagator, that denominator is cancelled and a new
vertex with four external lines (incoming and outgoing leptons and two emitted photons) can be introduced:

\[ V(p, k_1, k_2) = \frac{1}{4(E - \omega_1)E_2} \delta^{(\lambda)} \epsilon^{(\lambda_2)}(k_2) \hat{P}_2 \hat{e}^{(\lambda_1)*}(k_1) 1^{(\lambda)}_p \]

\[ = - \frac{E'}{E - \omega_1} \delta_{\lambda_1, 2\lambda} \delta_{\lambda_1, 2\lambda} \Phi \]

with \( \Phi \) defined in (18). This vertex is similar to a vertex with four external particles in scalar QED.

3. IMPACT FACTOR FOR THE SINGLE BREMSSTRAHLUNG

The impact factor for the single bremsstrahlung along the direction of the first electron corresponds to the virtual Compton scattering \( e(p_1) + \gamma^*(q) \rightarrow e(p_3) + \gamma(k) \) and has the form

\[ J_1(\epsilon_{\lambda_1} + \gamma^* \rightarrow \epsilon_{\lambda_2} + \gamma_{\lambda_3}) = 4\pi \alpha \times \]

\[ \hat{u}_3 \left( \frac{\hat{e}_3(p_1 - \hat{k} + m)\hat{e}^*_3}{2p_1k} - \frac{\hat{e}^*_3(p_3 + \hat{k} + m)\hat{e}_3}{2p_3k} \right) u_1. \]

Here for \( \hat{p}_1 - \hat{k} + m \) and \( \hat{p}_3 + \hat{k} + m \) we can use the simple substitution (16) that allows us to eliminate the numerators of the two spinor propagators and to introduce the vertices \( V(p) \) and \( V'(p, k) \):

\[ J_1 = 4\pi \alpha \times \]

\[ \left[ \frac{V(p_1, k)V(p_1 - k)}{2p_1k} - \frac{V(p_1, k)V(p_3 + k, k)}{2p_3k} \right]. \]

This impact factor depends on the energy fractions \( x = \omega/E_1, X_3 = E_3/E_1 \), and the transverse momenta \( k_\perp, p_{3\perp} \) of the final particles in the first jet with the relations: \( x + X_3 = 1 \) and \( k_\perp + p_{3\perp} = q_\perp \). In particular, the denominators in (22) are:

\[ 2p_1k = (m^2x^2 + k_\perp^2)/x, \]

\[ 2p_3k = [m^2x^2 + (k_\perp - xq)^2]/[x(1 - x)]. \]

We further rearrange \( J_1 \) into an expression which clearly exhibits the behaviour (18). Using the simple relation

\[ V(p_3 + k, k) = V(p_1 + q, k) = \]

\[ V(p_1, k) + 2 \left( q_\perp \epsilon_\perp^{(\lambda_2)*} \right) (1 - x \delta_{\lambda_1, -2\lambda_1}) \delta_{\lambda_1, \lambda_3}, \]

we immediately obtain the final result

\[ J_1 = \sqrt{2} 4\pi \alpha \left\{ \delta_{\lambda_1, \lambda_2} 2 \left( \epsilon^{(\lambda_1)*}p_1 \right) \times \right. \]

\[ \left. (1 - x \delta_{\lambda_1, -2\lambda_1}) + \delta_{\lambda_1, -\lambda_3} \delta_{\lambda_1, \lambda_3} \sqrt{2} m \right\} A_1 \]

\[ + q_\perp B_1 \right\} \Phi_{13}, \]

where

\[ A_1 = \frac{1 - x}{2p_1k} - \frac{1}{2p_3k} \quad \Phi_{13} = \frac{1}{\sqrt{X_3}} \epsilon^{(\lambda_3\varphi_3 - \lambda_1\varphi_1)} \]

\[ B_1 = - \frac{\epsilon_\perp^{(\lambda_1)*}}{p_{3\perp}} (1 - x \delta_{\lambda_1, -2\lambda_1}) \delta_{\lambda_1, \lambda_3}. \]

The impact factor (20) is a simple and compact expression for all 8 helicity states written in a form that all individual large (compared to \( q_\perp \)) contributions are cancelled. Indeed, the last term in \( J_1 \) is directly proportional to \( q_\perp \) and \( A_1 \propto q_\perp \) due to (23).

4. IMPACT FACTOR FOR THE DOUBLE BREMSSTRAHLUNG

The impact factor \( J_1 \) for the double bremsstrahlung along the direction of the first electron \( e(p_1) + \gamma^*(q) \rightarrow e(p_3) + \gamma(k_1) + \gamma(k_2) \) corresponds to six diagrams, three of them are shown in Fig. 2.

**Notations.** \( J_1 \) depends only on the energy fractions \( x_{1,2} = \omega_{1,2}/E_1, X_3 = E_3/E_1 \) with \( x_1 + x_2 + X_3 = 1 \) and on the transverse momenta of the final particles in the combinations:

\[ q_\perp = k_{\perp 1} + k_{\perp 2} + p_{3\perp}, \quad r_j = X_3k_{\perp j} - x_jp_{3\perp}. \]

The denominators of the spinor propagators are:

\[ a_j \equiv -(p_1 - k_j)^2 + m^2 = \frac{1}{x_j} (m^2x_1^2 + k_{\perp j}^2), \]

\[ b_j \equiv (p_3 + k_j)^2 - m^2 = \frac{1}{x_j} (m^2x_3^2 + r_{\perp j}^2), \]

\[ a_{12} = a_{21} \equiv -(p_1 - k_1 - k_2)^2 + m^2 = \]

\[ = a_1 + a_2 - \frac{1}{x_1x_2} (x_1k_{2\perp} - x_2k_{1\perp})^2, \]

\[ b_{12} = b_{21} \equiv (p_3 + k_1 + k_2)^2 - m^2 = \]

\[ = b_1 + b_2 + \frac{1}{x_1x_2} (x_1k_{2\perp} - x_2k_{1\perp})^2. \]
Figure 2. Feynman diagrams for the impact factor related to the double bremsstrahlung diagrams with \( k_1 \leftrightarrow k_2 \) photon exchange have to be added.

**General formula.** To calculate the impact factor \( J_1 \), we follow along the electron line from left to right in the diagrams of Fig. 2 and write down the corresponding vertices:

\[
\frac{J_1}{(4\pi\alpha)^{3/2}} = \frac{V(p_1, k_1)V(p_1 - k_1, k_2)V(p_3 - q)}{a_1a_{12}} - \frac{V(p_1, k_1)V(p_1 - k_1)V(p_1 - k_1 + q, k_2)}{a_1b_2} + \frac{V(p_1)V(p_1 + q, k_1)V(p_1 - k_1 + q, k_2)}{b_1b_{12}} - \frac{V(p_1, k_1, k_2)V(p_3 - q)}{a_{12}} + \frac{V(p_1)V(p_1 + q, k_1, k_2)}{b_{12}} + (k_1 \leftrightarrow k_2). \tag{29}
\]

Then we present \( J_1 \) in the form

\[
J_1 = \sqrt{2} (4\pi\alpha)^{3/2} X_3 (1 + P_{12}) \times M_{\Lambda_1 \Lambda_2}^{\Lambda_1 \Lambda_2} (x_1, x_2, k_{1\bot}, k_{2\bot}, p_{3\bot}) \Phi_{13}, \tag{30}
\]

where we introduce the permutation operator

\[
P_{12}f(k_1, e_1; k_2, e_2) = f(k_2, e_2; k_1, e_1) \tag{31}
\]

and the factor \( \Phi_{13} \) from (20) which includes the common phase. This allows us to omit below all factors \( \Phi \) from the vertices \( V(p), V(p, k) \) and \( V(p, k_1, k_2) \).

Using relations similar to (24), we transform \( M \) into an expression which clearly exhibits the behaviour (13):

\[
X_3 M_{\Lambda_1 \Lambda_2}^{\Lambda_1 \Lambda_2} = + A_2 V_{\Lambda_1 \Lambda_2}^{\Lambda_1 \Lambda_2} (p_1, k_1) V_{\Lambda_2 \Lambda_3}^{\Lambda_2 \Lambda_3} (p_1 - k_1 + q, k_2) +
+ q_1 B_{2\Lambda_1 \Lambda_2}^{\Lambda_1 \Lambda_2} + \tilde{A}_2 V_{\Lambda_1 \Lambda_2}^{\Lambda_1 \Lambda_2} (p_1, k_1, k_2), \tag{32}
\]

where

\[
A_2 = \frac{X_3}{a_{12}} \frac{1 - x_1}{1 - b_2} + \frac{1}{b_1b_2}, \tag{33}
\]

and the transverse 4–vector \( B_2 \) is

\[
B_2^{2\Lambda_1 \Lambda_2} = - X_3 \frac{2e^{(\Lambda_2^* \Lambda_2)}}{a_{12}} V_{\Lambda_1 \Lambda_3}^{\Lambda_1 \Lambda_3} (p_1, k_1) \times \left( 1 - \frac{x_2}{1 - x_1} \delta_{\Lambda_2 - 2\Lambda_3} \right) + \frac{2e^{(\Lambda_1^* \Lambda_1)}}{b_1b_{12}} V_{\Lambda_1 \Lambda_3}^{\Lambda_1 \Lambda_3} (p_1 - k_1 + q, k_2) \times \left( 1 - x_1 \delta_{\Lambda_2 - 2\Lambda_3} \right). \tag{34}
\]

Now it is not difficult to check that \( A_2 \propto q_\perp \), \( \tilde{A}_2 \propto q_\perp \) and, therefore, \( J_1 \propto q_\perp \) as well.

Again equations (31) and (32) represent a very simple and compact expression for all 16 helicity states, where all individual large (compared to \( q_\perp \)) contributions have been rearranged into finite expressions.

**Explicit expressions.** To find the amplitudes with given initial and final helicities, it is sufficient to substitute in the above equations the expressions for the vertices. As a result, we find:

\[
M_{++} = 2 \left\{ A_2 \frac{K_1 R_2^*}{x_1 x_2 X_3} + \frac{K_1 Q^*}{x_1 a_{12}} - \frac{Q R_2^*}{x_2 X_3 b_1 b_{2}} \right\}, \tag{33}
\]

\[
M_{+-} = X_3 (M_{++}^*)^*, \tag{34}
\]

\[
M_{+} = -2(1 - x_1) \times \left( A_2 \frac{K_1 R_2^*}{x_1 x_2 X_3} + \frac{K_1 Q^*}{x_1 a_{12}} - \frac{Q R_2^*}{x_2 X_3 b_1 b_{2}} \right), \tag{35}
\]

\[
M_{+-} = \frac{X_3}{(1 - x_1)^2} (M_{++}^*)^* + \frac{X_3}{(1 - x_1)^2} (M_{++}^*)^* \tag{36}
\]
\[ + \frac{2}{1 - x_1} \left( m^2 A_2 x_1 x_2 \frac{x_3}{X_3} - \tilde{A}_2 \right), \]

\[ M_{++}^{++} = 2 m x_1 \left( A_2 \frac{R_2}{x_2 X_3} + \frac{Q}{a_1 a_{12}} \right), \]

\[ M_{-+}^{--} = 2 m \frac{x_2}{X_3} \left( A_2 \frac{K_1}{x_1} - \frac{Q}{b_{12} b_2} \right), \]

\[ M_{++}^{++} = 0, \]

\[ M_{-+}^{--} = -\frac{1}{1 - x_1} \left( X_3 M_{-+}^{+-} + M_{-+}^{++} \right)^*. \]

Other amplitudes can be found due to the parity conservation relation

\[ M_{-\lambda_1 -\lambda_2}^{\Lambda_1 -\Lambda_2} = -(-1)^{\lambda_1 + \lambda_2} \left( M_{-\lambda_1 \lambda_2}^{\Lambda_1 \Lambda_2} \right)^*. \] (36)

The generalization of the results obtained for the single and double bremsstrahlung to the bremsstrahlung of \( n \) photons can be done straightforwardly. To demonstrate this, we have considered in detail the case \( n = 3 \) in [1].

5. SUMMARY

1. We have formulated a new effective method to calculate all helicity amplitudes for bremsstrahlung jet-like QED processes at tree level. The main advantage of our method consists in using simple universal “building blocks” — transition vertices with real leptons. Those vertices replace efficiently the spinor structure involving leptons of small virtuality in the impact factors, making the calculations short and transparent for any final helicity state.

2. In the considered case of bremsstrahlung we have found that only three nonzero transition vertices are required. The properties of these vertices determine all nontrivial general properties of the helicity amplitudes (for details see [12]).

3. We have outlined in this contribution how to calculate the impact factors for single and double bremsstrahlung. The case of triple bremsstrahlung can be found in [1]. It allows us to give a complete analytic and compact description of all helicity amplitudes in \( e^- e^\pm \) scattering with the emission of up to three photons along the directions of each initial leptons (in that case \( 2^5 \times 2^5 = 1024 \) different helicity amplitudes are involved). The corresponding calculations for the case of lepton pair production can be found in [2].

4. Since by construction individual large (compared to \( q_\perp \) ) contributions have been rearranged into finite expressions, the formulae obtained for the amplitudes are very convenient for numerical calculations of various cross sections.

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