Rashba Torque Driven Domain Wall Motion in Magnetic Helices

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Manipulation of the domain wall propagation in magnetic wires is a key practical task for a number of devices including racetrack memory and magnetic logic. Recently, curvilinear effects emerged as an efficient mean to impact substantially the statics and dynamics of magnetic textures. Here, we demonstrate that the curvilinear form of the exchange interaction of a magnetic helix results in an effective anisotropy term and Dzyaloshinskii–Moriya interaction with a complete set of Lifshitz invariants for a one-dimensional system. In contrast to their planar counterparts, the geometrically induced modifications of the static magnetic texture of the domain walls in magnetic helices offer unconventional means to control the wall dynamics relying on spin-orbit Rashba torque. The chiral symmetry breaking due to the Dzyaloshinskii–Moriya interaction leads to the opposite directions of the domain wall motion in left- or right-handed helices. Furthermore, for the magnetic helices, the emergent effective anisotropy term and Dzyaloshinskii–Moriya interaction can be attributed to the clear geometrical parameters like curvature and torsion offering intuitive understanding of the complex curvilinear effects in magnetism.

Assessing spin textures of three-dimensionally curved magnetic thin films1–3, hollow cylinders4–6 or wires7–10 has become a dynamic research field. These 3D-shaped systems possess striking novel fundamental properties originating from the curvature-driven effects, such as magnetochiral effects4,11–13 and topologically induced magnetization patterns13,14,15. To this end, a general fully 3D approach was put forth recently to study dynamical and static properties of arbitrary curved magnetic shells and wires16,17. Due to the curvature and torsion in wires17 (Gaussian and mean curvatures in the case of shells18) two additional interaction terms appear in the exchange energy functional: a geometrically induced anisotropy term which is a bilinear form of the curvature and torsion, and an effective Dzyaloshinskii–Moriya interaction (DMI) term (Lifshitz invariants), which depends linearly on the curvature and torsion. In the framework of this approach, the existence of topologically induced patterns in Möbius rings15 and new magnetochiral effects16,17 were predicted.

In addition to these rich physics, the application potential of 3D-shaped objects is currently being explored as magnetic field sensors for magnetofluidic applications18,19, spin-wave filters20,21, advanced magneto-encephalography devices for diagnosis of epilepsy at early stages22–24 or for energy-efficient racetrack memory devices25,26. The propagation of domain walls in a magnetic wire27 for racetrack memory25,28 or magnetic domain wall logic29,30 applications induced by spin-polarized currents is already widely explored31. In contrast, spin-orbitronics32,33, based on current-induced spin-orbit torques, launches the new concept of low energy spintronic devices.

Caus2ed by the structural inversion symmetry, multilayers consisting of magnetic metal with nonmagnetic metal and oxide on contralateral sides like Pt/Co/AlO can support spin–orbit torques acting on the localized magnetic moments due to the Rashba and spin Hall effects34,35. The Rashba field, produced by a charge current in these structures is considered to be one of the most efficient ways to act on the magnetization patterns34. However, in widely used planar devices, transverse domain walls are not affected by the Rashba effect36. Here, we demonstrate that the impact of the curvilinear effects on the magnetic texture of the domain walls in helical wires allows for their efficient displacement using spin-orbit Rashba torque. The geometrically induced anisotropy and DMI affect both the spatial orientation of the transverse (head-to-head and tail-to-tail) domain walls in helices as well as the magnetization distribution in the domain wall. As a consequence, the chiral symmetry breaking is

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characteristic for the wall structure: the direction of the magnetization rotation in the wall is opposite for the left- and right-handed helices. The domain wall mobility is proportional to the product of curvature and torsion of the wire; it depends on the topological charge of the wall. The direction of the domain wall motion is determined by the sign of the product of the helix chirality and domain wall charge. Furthermore, a remarkable feature of this 3D geometry is that its curvature and torsion are coordinate independent. Therefore, all effects coupled with an interplay between the geometry of the system and the geometry of the magnetic texture may be presented here in a most clear and lucid style. The obtained results are general and valid for any thin wire with nonzero torsion.

Results
We describe a helix curve by using its arc-length parametrization in terms of curvature–torsion:

$$\gamma(s) = R \cos \left( \frac{s}{s_0} \right) + \hat{y} R \sin \left( \frac{s}{s_0} \right) + \hat{z} \frac{CPs}{2s_0},$$

where \(s\) is the arc length, \(R\) is the helix radius, \(P\) is the pitch of the helix, \(C = \pm 1\) is the helix chirality and \(s_0 = \sqrt{R^2 + P^2/(2\pi)^2}\). A helix is characterized by the constant curvature \(\kappa = R/s_0^2\) and torsion \(\tau = CP/(2\pi s_0^2)\).

The magnetic properties are described using assumptions of classical ferromagnets with uniaxial anisotropy directed along the wire. The energy of the helix wire reads

$$E = K^{\text{eff}} S \int \hat{e} ds, \quad E = E^{\text{ex}} + E^\text{an}.$$  

Here \(K^{\text{eff}} = K + \pi M_s^2\), where the positive parameter \(K\) is a magnetocrystalline anisotropy constant of easy-tangential type, the term \(\pi M_s^2\) stems from the magnetostatic contribution37–39, \(M_s\) is the saturation magnetization, and \(S\) is the cross-section area. The exchange energy density reads \(E^{\text{ex}} = -\ell \hat{m} \cdot \nabla^2 \hat{m}\), where \(\hat{m}\) is the magnetization unit vector, \(\ell = \sqrt{A/K^{\text{eff}}}\) is the characteristic magnetic length (domain wall width), and \(A\) is an exchange constant. The anisotropy energy density is \(E^\text{an} = -(\hat{m} \cdot \hat{e}_a)\^2\) where \(\hat{e}_a\) is the unit vector along the anisotropy axis, which is assumed to be oriented along the tangential direction. The easy-tangential anisotropy in a curved magnet is spatially dependent. Therefore, it is convenient to represent the energy of the magnet in the curvilinear Frenet–Serret reference frame with \(\hat{e}_t\) being a tangential (T), \(\hat{e}_n\) being a normal (N) and \(\hat{e}_b\) being a binormal (B) vector, respectively (TNB basis).

In the curvilinear frame, the exchange energy has three different contributions37, \(E^{\text{ex}} = E^{\text{ex}}_\alpha + E^{\text{ex}}_\beta + E^{\text{ex}}_\sigma\). The first term \(E^{\text{ex}}_\alpha = \mu_0 F^\text{t}\), describes the isotropic part of the exchange expression, which has the same form as for a straight wire. Here and below the prime denotes the derivative with respect to the dimensionless coordinate \(u = s/\ell\). The second term, \(E^{\text{ex}}_\beta = F_{\alpha\beta} (m_\alpha m_\beta - m_\alpha^* m_\beta^*)\), is a curvature induced effective DMI, where the components of the Frenet–Serret tensor \(F_{\alpha\beta}\) are linear with respect to the reduced curvature and torsion

$$\sigma = \kappa \ell, \quad \tau = \tau \ell,$$

respectively. The last term, \(E^{\text{ex}}_\sigma = F_{\sigma \alpha} m_\alpha m_\beta\), describes a geometrically induced effective anisotropy interaction, where the components of the tensor \(F_{\sigma \alpha}\) are bilinear with respect to the curvature and torsion, see Supplementary Materials for details. Two additional contributions (effective DMI and effective anisotropy) naturally appear in the curvilinear reference frame similar to contributions to the kinetic energy of the mechanical particle in the rotating frame with Coriolis force (linear with respect to velocity) and centrifugal force (bilinear with respect to velocity).

The emergent effective anisotropy leads to the modification of the equilibrium magnetic states37. Here, we consider helices with relatively small curvature possessing quasiisotangential magnetization distribution shown in Fig. 1(a). For further discussion it is instructive to project the magnetization onto the local rectifying surface, which coincides with the supporting surface of the helix [yellow cylinder in Fig. 1(a)]. The top view is plotted for the right-handed helix [\(\sigma > 0\), Fig. 1(c)] and for the left-handed one [\(\sigma < 0\), Fig. 1(d)].

The influence of the curvature and torsion can be treated as an effective magnetic field \(F \propto \sigma \chi \times \chi\) acting along the binormal direction37. This field causes a tilt of the equilibrium magnetization from the tangential direction by an angle37:

$$\psi \approx \sigma \xi, \quad \text{when} \quad |\sigma| \ll 1,$$

see Fig. 1(b) and Supplementary Eq. (S3) for details. The symbols represent the results of the spin-lattice simulations using the package SLaSi without magnetostatics and Nmag simulations of a magnetically soft wire, see Methods for details: the analysis shows that the model is adequate for soft magnets with \(\chi < 0.4\).

Now we can rotate the reference frame in a local rectifying surface by the angle \(\psi\) (see Supplementary for details). The magnetization in the rotated \(\psi\)-frame \([\hat{e}_t, \hat{e}_n, \hat{e}_b]\) reads

$$m = (m_1, m_2, m_3) = (\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi),$$

where magnetization angular variables \(\theta\) and \(\phi\) depend on the spatial and temporal coordinates. Using this reference frame we can diagonalize the effective anisotropy energy density of the helix wire (Supplementary Eqs. (S3)–(S5) for details):
\[ \mathcal{E} = \frac{m^T m}{2} - \mathcal{K}_1 m_1^2 - \mathcal{K}_2 m_2^2 + \mathcal{D}_1 (m_2 m_3 - m_3 m_2) + \mathcal{D}_2 (m_1 m_3 - m_3 m_1). \]  

Figure 1. Magnetization distribution in a helix. (a) Schematics of magnetic helix with the easy-tangential anisotropy (magnetic moments are shown with red arrows, TNB-basis is shown with green arrows. (b) The rotation angle \( \psi \) for different torsions \( \sigma \) and curvatures \( \kappa \). The onion state is energetically preferable in the grey region. Solid lines correspond to analytics, filled circles and open squares correspond to SLAn and Nmag simulations, respectively; see Methods for details. (c,d) Discrete magnetic moments at equilibrium for right- and left-handed helices, respectively. The effective anisotropy axis is shown with thin black arrow \( e_1 \).}

The coefficient \( \mathcal{K}_1 \) characterizes the strength of the effective easy-axis anisotropy while \( \mathcal{K}_2 \) gives the strength of the effective easy-surface anisotropy. The parameters \( \mathcal{D}_1 \) and \( \mathcal{D}_2 \) are the effective DMI constants. We note that the energy (2) has the general form of the energy density for 1D biaxial magnets with an intrinsic DMI and contains the complete set of the Lifshitz invariants. Hence, effective DMI constants \( \mathcal{D}_1 \) and \( \mathcal{D}_2 \) can include other contributions, e.g. the intrinsic DMI or DMI due to the structural inversion asymmetry.

In the case of small curvature and torsion the geometrically induced anisotropy and DMI constants can be attributed to the geometrical parameters of the object:

\[ \mathcal{K}_1 \approx 1 + \sigma^2 - \kappa^2, \quad \mathcal{K}_2 \approx \kappa^2, \quad \mathcal{D}_1 \approx 2\sigma, \quad \mathcal{D}_2 \approx 2\kappa. \]

The possible static magnetization structures can be found by variation of the total energy functional with density (2). The homogeneous equilibrium state (quasitangential state) is described by \( \theta^h = 0 \) and \( \theta^h = \pi \), which corresponds to the two possible directions of the helix magnetization.

**Static domain wall.** One of the simplest inhomogeneous magnetization distribution in a nanowire is a transverse domain wall, which connects two possible equilibrium states. We start our analysis with general remarks about the domain wall described by the energy functional with the density (2), which can be applied for a wide class of 1D magnets also with the intrinsic DMI.
The structure of the domain wall can be described analytically for $D_2 = K_2 = 0$. This case corresponds to the uniaxial ferromagnet with an additional $D_1$ DMI term. For such a system there is an exact analytical solution of static equations of the domain wall type:

$$\cos \theta^{dw}(u) = -p \tanh \frac{u}{\delta}, \quad \phi^{dw}(u) = \Phi + \gamma u.$$  

(3)

Here $p = \pm 1$ is a domain wall topological charge: $p = 1$ corresponds to kink (head-to-head domain wall) and $p = -1$ corresponds to antikink (tail-to-tail domain wall). The domain wall width $\delta$ and the slope $\gamma$ are as follows:

$$\delta = \frac{1}{\sqrt{K_1 - D_1^2/4}}, \quad \gamma = D_1/2.$$  

(4)

In the uniaxial magnet with the anisotropy parameter $K_1$, the typical domain wall width without DMI reads $\delta = 1/\sqrt{K_1}$. One can see that the presence of DMI causes broadening of the wall. Furthermore, the domain wall is not perpendicular to the wire length and is tilted by an angle determined by $D_1$ constant. The slope of the azimuthal angle $\phi' = -D_1/2$. This behaviour is similar to the known domain wall inclination in magnetic stripes caused by the intrinsic DMI.

In the following we proceed with the investigation of the finite curvature effects on the magnetization distribution in domain walls in helices. We will apply a variational approach by using (3) as a domain wall Ansatz with the static equations of the domain wall type:

$$\frac{E^{dw}}{K^{eff} s \ell} = \frac{2}{\delta} + \frac{2\delta \gamma^2}{4} + \frac{2\delta K_1 + \delta K_2 (1 + C_1 \cos 2\Phi) - 2\delta D_1 \gamma + p C_2 D_2 \cos \Phi}{\sinh(\pi \gamma \ell/2)}.$$  

(5)

The variational parameters (5) coincide with parameters (4) of the exact solution obtained in the case $D_2 = K_2 = 0$. Thus, the approximation of vanishing curvatures describe the domain wall statics for small enough $\kappa$ and $\sigma$.

The comparison of these predictions with the 3D spin-lattice simulations using package SLaSi and micromagnetic simulations using Nmag confirms our theory, see Fig. 2 (the details of simulations are described in Methods). Figure 2(a) represents the untwisted view of the domain wall. The magnetization direction corresponds to the ground state along $e_i$ inside two domains. Inside the head-to-head domain wall the magnetization is directed outward the helix (opposite to $e_i$). Qualitatively this is explained by the fact that such a configuration minimizes the magnetization gradient and, therefore, the exchange energy. For the tail-to-tail domain wall the direction of the magnetization tilt is opposite to the head-to-head one. The dependence of the phase slope $\gamma$ on the torsion $\sigma$ is in good agreement with the simulation data, solid line in Fig. 2(b). Symbols correspond to the results of the simulations carried out for $\kappa = 0.1$. We performed the spin-lattice simulations without magnetostatics (green circles) and with magnetostatics for a magnetically soft sample (blue filled triangles, the quality factor $Q = K/2\pi M_s^2$ equals to zero) as well as magnetically hard sample with $Q = 4$. The micromagnetic simulations of a thin Permalloy wire (diamonds) are also in good agreement with the spin-lattice simulations and theory. The static head-to-head and tail-to-tail domain walls are well described by the Ansatz (3) with optimal parameters determined by (5), see solid lines in Fig. 2(c,d), for $\kappa = 0.1, \sigma = 0.5$.

**Domain wall dynamics driven by the Rashba spin-orbit torque.** Here, we describe the domain wall dynamics in the Rashba spin-orbit system, where the magnetic wire is adjacent to a nonmagnetic conductive layer with a strong spin-orbit interaction. Spin-orbit interaction is well known to be a source of two possible symmetries of torques, acting on magnetization and field-like torque. Depending on the microscopic nature of the spin-orbit interaction, the antidamping or Slonczewski torque can be caused by the spin Hall effect or indirect Rashba effect. In contrast, the field-like torque is due to the spin Hall effect or Rashba effect. The relative importance of these interactions depends on the geometry and types of interfacing materials. For example, in thin magnetic films with spin-independent electron scattering, the antidamping spin transfer torque vanishes. Accordingly to ref. 36 (see Table 1 of ref. 36), the antidamping torque relying on the spin Hall and indirect Rashba effects does not lead to the motion of head-to-head domain walls independent of the injection geometry of charge current (parallel or perpendicular to the wire). Furthermore, head-to-head domain walls can be moved by the field-like torque relying on the Rashba effect only in the case of perpendicular injection. In stark contrast to the planar systems, here, we demonstrate that the head-to-head domain walls can be efficiently moved in helix wires relying on the Rashba effect even in the case of the parallel charge current injection.
The Rashba effect typically appears in systems with inversion symmetry broken spin-orbit interaction. We consider the parallel geometry, in which the ferromagnetic wire is parallel to the spin-orbit layer on the whole length of the wire. The sketch of the system is shown in Fig. 3(a). The magnetic wire is wound around the conductive layer forming a helix. Electrical charge current flows along the magnetic wire in the tangential direction. Under the action of the field-like torque caused by the Rashba effect, the magnetic subsystem is affected by the effective Rashba field

\[ \mathbf{H}_R^\alpha = \alpha \beta \mu_0 \mathbf{M}_s \times [\mathbf{j} \times \mathbf{n}] \]  

with \( \alpha \) being the Rashba parameter, \( \beta \) being the polarization of the carriers in the ferromagnetic layer, \( \mu_0 \) being the Bohr magneton and \( \mathbf{n} \) being the unit vector perpendicular to the spin-orbit layer. Note that the Rashba parameter, see Eq. (6), depends on the material properties of the interface and does not depend on the thickness of the conductive layer.

In such parallel geometry the Rashba field is always directed perpendicular to the wire. For a straight wire the direction of the Rashba field is transversal to the domain magnetization, hence the field can not push the wall. However, for the helix geometry the equilibrium magnetization direction deviates from the wire direction. The energy density of the interaction with the effective Rashba field is

\[ \mathcal{E} = - h \sin \psi \]  

with \( h = H R^\alpha / H_A \) being the reduced field normalized by the anisotropy field. There are two components of the magnetic field: \( h_1 = h \sin \psi \) is parallel along the domain, hence it pushes the wall. Another one, \( h_2 = h \cos \psi \) is directed along the wire. In general, magnetic fields with the transversal component result in the deformation of the domain wall profile and other changes of the characteristic parameters like Walker field and maximal domain wall velocities. In the case of weak fields, we can limit our consideration to the parallel field \( h_1 \) only and neglect the dynamical changes of the wall width. Furthermore, we will not take into account the influence of Ørsted fields generated by the charge current.

Far below the Walker limit, we can use the generalized \( q - \Phi \) model, cf. (3):

\[ \cos \varphi_{sw} (u, \bar{\iota}) = - p \tanh \frac{u - q(\bar{\iota})}{\delta}, \quad \varphi_{sw} (u, \bar{\iota}) = \Phi (\bar{\iota}) - \bar{T} [u - q(\bar{\iota})], \]  

where \( \bar{T} = \omega_0 t \) and \( \omega_0 = \gamma c K^{eff} / M_s \), \( \gamma c \) being the gyromagnetic ratio.
Using \((q, \Phi)\) as a pair of time dependent collective coordinates, we obtain the stationary motion of the domain wall (see Methods for details)

\[
\nu = \frac{dq}{dt} \quad (t \to \infty) = \frac{2ph\delta}{\eta} \cdot \frac{\sin \psi}{1 + \delta^2 \tau^2}.
\]  

(8)

We checked the theoretically predicted velocities for the domain wall motion (8) by SLaSi and Nmag simulations in the range of effective fields, \(|h| \leq 0.02\), see Fig. 3(b–d) and Methods for details. Symbols correspond to SLaSi and Nmag simulations, solid lines correspond to the theoretical predictions, obtained accordingly to Eq. (8), see also Supplementary Eq. (S3). The domain wall velocity is almost linear with the field, see Fig. 3(b) with a fixed damping constant \(\eta = 0.1\). The inverse linear dependence \(\eta \propto 1/v^{1/2}\) is well pronounced in Fig. 3(c).

The maximal velocity \(v = 0.1\) shown in Fig. 3(b) for \(h = 0.02\) corresponds to 35 m/s for Permalloy.

The most intriguing effect in the domain wall dynamics is the torsion dependence of the wall motion. The mobility of the domain wall \(\mu = \nu/h\) as a function of the helix torsion is plotted in Fig. 3(d,e) for different helix curvatures. In the case of small curvature and torsion \(\sigma = (c, 1)\), the wall mobility, accordingly to (8), has the following asymptotic:

\[
\mu \approx \frac{2p\delta}{\eta} \cdot \sigma. \tag{9}
\]

Therefore, the domain wall can move only under the joint action of the curvature and torsion. The direction of the domain wall motion depends on the helix chirality \(C\), see Fig. 4(a,b), where the head-to-head domain wall position is shown at different time moments and Fig. 4(c,d), where the domain wall position is shown as a function of time for different torsions and values of \(p\). The initial domain wall displacement occurs in the positive direction, while the steady-state motion is described by Eq. (8). That is why the close positions of the domain walls in Fig. 4(a,b) occur at different time of 9 ns and 14 ns.

In some respect, the effect of chirality sensitive domain wall mobility is similar to the recently found chiral-induced spin selectivity effect in helical molecules due to the Rashba interaction.

Discussion

First, we discuss the consequence of the interplay between the curved geometry of the helical wire with the magnetic texture of the transverse domain walls:

(i). The geometrically induced effective anisotropy causes the tilt of the equilibrium magnetization by the angle \(\psi\) with respect to the tangential direction. This rotation angle depends on the product of the curvature and the torsion. Due to the nonzero value of \(\psi\) there appears a Rashba field component \(h_{\parallel} = h \sin \psi\) along the magnetization of one of the domains. The field \(h_{\parallel}\) pushes the domain wall and thus, the geometrically induced
The effective anisotropy is the origin of the Rashba field induced domain wall motion in a magnetic helix. There appears curvature induced easy-surface anisotropy. For the helix geometry the anisotropy tends to orient the magnetization within the rectifying surface, i.e. tangentially to the cylinder surface. Additionally, the geometry caused easy-axis anisotropy, favours the orientation of the magnetization along the $e_1$ direction.

The more intriguing features of the geometry are connected to the curvatures induced Dzyaloshinskii–Moriya interaction. Two effective DMI terms in the energy (2) correspond to all possible Lifshitz invariants in the 1D case. In this respect our analysis is valid also for 1D systems with an intrinsic DMI as well as for the DMI induced due to the structural inversion asymmetry. Using SI units, one estimates that
\[
\pi \approx + D A P R P^2 / (R^2 + P^2)^2
\]
Using typical values $A = 10 \text{ pJ/m}$, we obtain that $D = 0.28 \text{ mJ/m}^2$ for a helix with the radius $R = 50 \text{ nm}$ and the pitch $P = 300 \text{ nm}$; $D = 0.14 \text{ mJ/m}^2$ for $R = 100 \text{ nm}$, $P = 600 \text{ nm}$. These values are comparable to those estimated from the $ab\ initio$ calculations for multilayer systems.

It is instructive to compare the geometrically induced DMI in helices with the intrinsic DMI for the untwisted objects. In this work we restricted ourselves by considering the quasitangential ground state of the helix, which is realized for the relatively weak curvatures and torsions (weak effective DMI). In case of strong DMI, the helix favours the onion ground state, where the magnetization is almost homogeneous (in the physical space) due to the strong exchange interaction. At the same time, the magnetization rotates in the curvilinear reference frame. Such a state is an analogue of the spiral state in straight magnets with intrinsic DMI.

The geometrically induced DMI drastically changes the internal structure of the transverse domain wall: the azimuthal magnetization angle $\phi$ rotates inside the wall, see Supplementary Fig. S1. While the domain wall orientation in its centre is determined by the domain wall topological charge $p$, the direction of the magnetization rotation (i.e. magnetochirality $\mathcal{C} = - sgn \mathcal{T} = - sgn \sigma$) mainly depends on the helix torsion $\sigma$. One can interpret the sign of $\sigma$ as the helix chirality $\mathcal{C}$ (different for right-handed helix when $\sigma > 0$ and left-handed one when $\sigma < 0$). Therefore, the magnetochirality of the domain wall is always opposite to the helix chirality, $\mathcal{C} = - \mathcal{C}$.

In order to elucidate the role of the geometrically induced DMI we compare the domain wall structure in a helix with the domain wall in a straight wire of a biaxial magnet without DMI. Figure S5 shows the comparison of the magnetization distribution for these two geometries obtained by the SLaSi simulations. The panel (b) represents the data for a straight wire with the energy (2), where the anisotropy coefficients $K_1, K_2$ correspond to the effective anisotropies in the helix, and DMI constants $D_1 = D_2 = 0$. The panel (c) represents the data for a helix with $\alpha = 0.1$, $\sigma = 0.5$. While for the straight wire the magnetization always lies in the plane, $m_n = 0$, the competition between the easy-plane anisotropy and DMI results in the essential coordinate dependence of both normal and binormal magnetization components.

The chiral symmetry breaking strongly impacts the domain wall dynamics and allows the motion of domain walls under the action of the Rashba spin-orbit torque: the direction of motion if determined by the product
of the helix chirality and the wall charge ($v \propto \sigma p$). Thus, domain walls can be moved only under the combined action of the Rashba effect and geometrical effects, caused by finite curvature and torsion. The wall does not move in the limit of a planar wire, see Fig. 3. The head-to-head and tail-to-tail domain walls move in opposite directions, see Supplementary Video. Our theory describes the domain wall motion both in magnetically hard and soft helices, see comparison in Fig. 2(b) for the phase slope and 3(b,c) for the domain wall mobility, and also Supplementary Fig. S2. The results obtained for this test system are valid well beyond the considered here specific case of helical wires. The Rashba torque driven domain wall motion will be characteristic for any transverse wall present in a curvilinear system with non-zero torsion.

**Methods**

**Spin-lattice and micromagnetic simulations.** Numerically we study the magnetization textures in a helix and its dynamics using the in-house developed spin-lattice simulator SLaSi\textsuperscript{44} for anisotropic samples and Nmag\textsuperscript{45} for magnetically soft samples.

When using SLaSi we consider a classical chain of magnetic moments $m_i$ with $i = 1, N$, situated on a helix (1). We use the anisotropic Heisenberg Hamiltonian taking into account the exchange interaction, easy-tangential anisotropy and Rashba field. The dynamics of this system is described by a set of $N$ vector Landau-Lifshitz ordinary differential equations, see ref. 59 for the general description of the SLaSi simulator and ref. 37 for details of the helix simulations. To study the static magnetization distribution spin chains of $N = 2000$ sites are considered. The domain wall is placed in the centre of the chain. To simulate the magnetization dynamics spin chains of 4000 sites are considered. The domain wall is placed at the 300-th site from one end of the helix and is pushed by the field-like torque to another end. The velocity is measured at the steady state of the domain wall motion before it is driven out off the helix. In all simulations the magnetic length $\ell = 15a$ with $a$ being the lattice constant and damping $\eta = 0.01$ is used except the case when studying the velocity dependence on damping, where $\eta = 0.01 \ldots 0.1$. For all simulations with magnetostatics the exchange length $\ell_{ex}$ is used to obtain the effective magnetic length $\ell = \sqrt{A/K_{eff}} = 2\ell_{ex}/\sqrt{1 + 2Q} = 15a$.

The simulations using the Nmag are performed with the following parameters: exchange constant $A = 13$ pJ/m, saturation magnetization $M_s = 860$ kA/m and damping $\eta = 0.01$ which correspond to Permalloy (Ni$_{81}$Fe$_{19}$). These parameters result in the effective anisotropy field of $H_{an}^N = 0.54$ T and exchange length $\ell_{ex} \approx 3.7$ nm. Samples of radius 5 nm and length 1 $\mu$m are studied. Thermal effects and anisotropy are neglected. The typical Rashba field $h = 0.02$ (using SI units $H_s \approx 10.8$ mT) corresponds to the electrical charge current density $j = 10.8$ mA/µm$^2$ for the polarization of carriers $P = 0.5$ and Rashba parameter $\alpha = 100$ peV $\mu$m$^4$. The static and dynamical properties of the domain walls on a helix are studied in the same way as for the classical chain described above.

The simulations are performed using the computer clusters of the Bayreuth University\textsuperscript{60}, Taras Shevchenko National University of Kyiv\textsuperscript{61}, Bogolyubov Institute for Theoretical Physics of the National Academy of Sciences of Ukraine\textsuperscript{62}.

**Domain wall dynamics.** We use the generalized collective coordinate $q = \Phi$ approach\textsuperscript{43} based on the effective Lagrangian formalism. Inserting the Ansatz (7) into the “microscopic” Lagrangian with the density $L = -\cos \theta \dot{\phi} - E$ and the dissipative function $F = \frac{\rho}{2} [\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2]$, after integration over the wire, we obtain the effective Lagrangian and the effective dissipative function, normalized by $K_{eff}\ell$, as follows:

![Diagram](https://www.nature.com/scientificreports/)
Here and below overdot means the derivative over \( \dot{t} \). The effective equations of motion are then obtained as the Euler–Lagrange–Rayleigh equations

\[
\frac{\partial \mathcal{L}_{\text{eff}}}{\partial X_i} - \frac{d}{dt} \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \dot{X}_i} = \frac{\partial \mathcal{E}_{\text{eff}}}{\partial X_i}, \quad X_i = (q, \Phi).
\]

These equation describe the steady motion of the domain wall \( q = \dot{q}_0 + \nu \dot{t} \) with the constant velocity (8). The corresponding phase \( \Phi = \text{const} \) is determined by the equation

\[
2C^2K^2 \sin[2\Phi + \nu \dot{t}] - 2C^2D^2 \sin \Phi = -\nu (p - \delta \dot{\eta} \dot{t}).
\]

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Author Contributions
O.P. and D.S. formulated the theoretical problem and performed the analytical calculations. O.P. performed spin-lattice simulations. K.Y. performed micromagnetic simulations. O.P., D.S., V.K., K.Y., D.M. and Y.G. contributed to the discussion and writing of the manuscript text.

Additional Information
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