A General Analytical Model for Hydraulic Computation of Open Channels with Steady State Uniform Flow

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Abstract: In this paper, a mathematical generalised model is developed, which allows for definition in a unit manner an analytical solution of the complex problems of hydraulic computation for the steady state flow open channels. These problems include the design and operational examination for the channels with linear cross-section (trapezoidal, rectangular, and triangular), the channels with curved cross-section (semi-circular, parabolic, and semi-elliptic), as well as the channels with flat sides and a cylindrical bottom. The conditions on hydraulically optimal sections for these channel-types are determined by the authors. The proposed model is programmed easily on microcomputers. Therefore, two computer programs are performed in the FORTRAN programming language for PC-compatible systems, with a view to increase the accuracy and computational efficiency. The advantages to use the proposed programs are explained from two numerical applications for different constructive variants employed in practical engineering.

Keywords: hydraulic structures; open channel; hydraulic computation; hydraulically optimal section; mathematical models.

1. Introduction

Among all the problems that water researchers and hydraulic engineers have faced, water conveyance is considered to be one of not only the inevitable but also the expensive problems. In fact, water conveyance is a means to meet some of the needs of human society, such as irrigation, municipal and flood control. Open channels have been widely used for this purpose because they can be constructed on different topographies and soil conditions and also prevent water from wasting.

Selection of the cross-sectional shape of open channels is one of the primary issues of hydraulic, constructive, economic and operational orders. The comparison performed by Mateescu [1] between different isoperimetric outlines of open channels, each the optimum of its type, shows that the most hydraulically efficient shape is the semi-circle, followed closely by the parabola and the isosceles trapezium with side angles of 60°. For the conveyance of a given water flow rate, the cross-sectional area of a parabolic channel is lower than the hydraulically equivalent trapezoidal channel, as the generic parabola degree is higher.

The comparative study developed by Blidaru et al. [2] between the parabolic and semi-elliptic channels concludes that the semi-elliptic cross-section has better hydraulic characteristics than the parabolic section because the conveyance of the same liquid flow rate has a smaller area.

As a result, it is a known fact that for open channels, modern techniques use both linear cross-sections (trapezoidal, rectangular, and triangular) as well as curved cross-sections (parabolic, semi-elliptic, and semi-circular) [3]. In addition, taking into account the construction deficiencies of trapezoidal channels, the industrialisation necessity of the hydro-amelioration and hydro-urban systems, and the hydraulic advantages [1, 4-6], compound cross-section channels with flat sides and cylindrical bottom were adopted in engineering practice. Design and operational validation of such cross-sections was and still is an active area of research.

Chow [5] and French [7] have published the most hydraulically efficient section relations. Swamee and Bhatia [8] developed optimal design curves for trapezoidal, rounded bottom and rounded corner sections. Loganathan [9] studied optimality conditions for a parabolic channel section. Monadjemi [10] showed that the same optimal section variables can be achieved by minimisation of either flow rate or wetted perimeter. Froehlich [11] recommended simple relations optimum section variables of trapezoidal sections in terms of discharge.

In this paper, a general analytical model is developed that solves in a unitary manner the complex problems of hydraulic computation for open channels with steady state uniform flow and can be easily programmed and implemented on microcomputers. Starting with some theoretical aspects of optimal hydraulic computation of open-channels, the conditions of hydraulically optimal sections are determined for channels with simple curve
sections (circular, parabolic, and semi-elliptic) as well as for compound section channels with flat sides and a cylindrical bottom. To increase the precision and efficiency of the computations, two computer programs are developed in FORTRAN programming language for personal computer (PC)-compatible systems. The advantages to using the proposed programs are highlighted by two examples of comparative design for different channel cross-sections used in practical engineering.

2. Simple channels

2.1. Geometrical elements of the cross-section

- **Trapezoidal isosceles channel**, with the slope 1/m (Fig. 1), has the following geometrical elements:
  - section width at water free-surface: \( B = b + 2m \) \( h \) (1)
  - cross-sectional area: \( A = (2\sqrt{1 + m^2} - m)h^2 \) (2)
  - wetted perimeter: \( P_w = b + 2h\sqrt{1 + m^2} \) (3)
  - hydraulic radius: \( R = \frac{A}{P_w} = \frac{(2\sqrt{1 + m^2} - m)h^2}{b + 2h\sqrt{1 + m^2}} \) (4)

- **Parabolic channel**, in generalised form in Fig. 2, is represented by a parabola of \( \alpha \) degree defined by equation:
  \[ z = px^a \] (5)
where \( p \) is a dimensionless parameter.

The following geometrical elements are obtained:
- section width at water free-surface: \( B = 2x \) (6)
\[ h = z \]  
(7) 
- geometric tangent slope in parabola intersection point with water free-surface plane (side slope) 

\[ \frac{1}{m} = \tan \theta = \frac{dz}{dx} \]  
(8) 
Differentiating Eq. (5) and stating point \( M \) is on the parabola, from Eq. (8), the following results: 

\[ m = \frac{1}{\alpha} h^{\frac{1-\alpha}{\alpha}} = \frac{2^{\alpha-1}}{\alpha p} B^{1-\alpha} \]  
(9) 
- cross-sectional area: 

\[ A = 2 \int_{0}^{b} x(z)dz = 2 \alpha^2 m h^2 \]  
(10) 
- wetted perimeter: 

\[ P_u = 2 \int_{0}^{b} \sqrt{1 + \left(\frac{dx}{dz}\right)^2} dz = 2 \int_{0}^{b} \sqrt{1 + m^2} dz, \]  
(11) 

In the case that the generic parabola is of random form, \( P_u \) can be expressed as: 

\[ P_u = \lambda \sqrt{A} \]  
(12) 
where the parameter \( \lambda = \lambda(\alpha,m) \) can be calculated with a satisfactory approximation using the following relations [3, 12]:

a) for \( m \geq 1 \) and \( \alpha > 1 \):

\[ \lambda = \sqrt{2(1 + \alpha)m} \left[ 1 + \frac{1}{2(2\alpha - 1)m^2} - \frac{1}{8(4\alpha - 3)m^4} \right] \]  
(13) 

b) for \( m < 1 \) and \( \alpha = 2 \):

\[ \lambda = \sqrt{2(1 + \alpha)m} \left[ \frac{8\alpha^2 - 8.25\alpha + 1.625}{(4\alpha - 3)(2\alpha - 1)} + \frac{1}{\alpha} \left( \frac{\alpha}{m^{1-\alpha}} - 1 \right) + \frac{1}{8(3\alpha - 4)} \left( \frac{4-3\alpha}{m^{1-\alpha}} - 1 \right) \right] \]  
(14) 

c) for \( m < 1 \) and \( \alpha < 1 \) (\( \alpha \neq 2 \)):

\[ \lambda = \sqrt{2(1 + \alpha)m} \left[ \frac{8\alpha^2 - 8.25\alpha + 1.625}{(4\alpha - 3)(2\alpha - 1)} + \frac{1}{\alpha} \left( \frac{\alpha}{m^{1-\alpha}} - 1 \right) - \frac{1}{2(\alpha - 2)} \left( \frac{2-\alpha}{m^{1-\alpha}} - 1 \right) + \frac{1}{8(3\alpha - 4)} \left( \frac{4-3\alpha}{m^{1-\alpha}} - 1 \right) \right] \]  
(15) 

- hydraulic radius:

\[ R = \frac{A}{P_u} = \frac{A}{\lambda \sqrt{1 + \alpha}} \]  
(16) 

- Semi-elliptic channel in Fig. 3 is defined by the general ellipse with the following equation:

\[ \frac{(z-a)^2}{a^2} + \frac{x^2}{b^2} - 1 = 0 \]  
(17) 

where: \( a \) is the major semi-axis; \( b \) is the small semi-axis.

The following expressions for cross-section elements are obtained:

- section width at water free-surface:

\[ B = 2b \]  
(18) 

- water depth:

\[ h = a \]  
(19) 

- cross-sectional area:

\[ A = \frac{\pi}{2} ab = \frac{\pi}{4} B h \]  
(20)
The following dimensionless parameters are defined:

- for trapezoidal section:
  \[ \beta = \frac{b}{h} \]  
  \[ \varphi = \beta + 2m \]  
  \[ \psi = \beta + m \]  
  \[ f = \beta + 2\sqrt{1 + m^2} \]  

- for parabolic section:
  \[ \varphi = 2\alpha m \]  
  \[ \psi = \frac{2\alpha^2 m}{1 + \alpha} \]  
  \[ f = \sqrt{2\lambda\alpha} \sqrt{\frac{m}{1 + \alpha}} \]  

- for semi-elliptic section:
  \[ \varphi = \frac{B}{h} \]  
  \[ \psi = \frac{\pi}{4}\varphi \]  
  \[ f = \frac{\pi}{4} \left[ 1.5(\varphi + 2) - \sqrt{2\varphi} \right] \]

Each dimensional geometrical element can be expressed depending on one sole dimensional element and two other dimensionless parameters, according to Table 1.

For \( m=0 \) and \( \beta=0 \), in Eqs. (23)-(26), the particular case of a rectangular channel and a triangular channel, respectively, is obtained. Substituting \( \varphi=2 \) in Eqs. (30)-(32) correspond to the case of a semi-circular channel.

### 2.2 Hydraulically optimal section

The hydraulically optimal cross-section [13] of an open channel is a section that for the same area \( A \), the same bottom slope \( i \) and the same roughness coefficient \( n \), conveys the maximum discharge \( Q \). Maximum discharge is obtained when the hydraulic radius is maximal and when the wetted perimeter is minimal.

For the trapezoidal channel, writing the minimum condition:

\[ \frac{dP_u}{dh} = 0, \]

is obtained:
Table 1 Geometrical element expression of simple cross-sections

| Elements | A function of: |
|----------|----------------|
|          | $h$ | $B$ | $A$ | $P_u$ | $R$ |
| 0        | 1   | 2   | 3   | 4     | 5   |
| $h$      | $h$ | $\frac{1}{\varphi}B$ | $\frac{1}{\varphi^2}\sqrt{A}$ | $\frac{1}{f}P_u$ | $\frac{f}{\psi}R$ |
| $B$      | $\varphi h$ | $B$ | $\frac{\varphi}{\varphi^2}A$ | $\frac{\varphi}{f^2}P_u$ | $\frac{\varphi}{\psi}R$ |
| $A$      | $\frac{\varphi}{h^2}$ | $\frac{\varphi}{\varphi^2}B^2$ | $A$ | $\frac{\varphi}{f^2}P_u^2$ | $\frac{f^2}{\psi}R^2$ |
| $P_u$    | $f h$ | $\frac{1}{\varphi}B$ | $\frac{1}{\varphi^2}\sqrt{A}$ | $P_u$ | $\frac{f^2}{\psi}R$ |
| $R$      | $\frac{\varphi h}{f}$ | $\frac{\varphi}{f}B$ | $\frac{\varphi^2}{f^2}A$ | $\frac{\varphi}{f^2}P_u$ | $R$ |

$$b = 2h \tan \frac{\theta}{2}$$  \hspace{1cm} (34)

Eq. (34) shows that the hydraulically optimal cross-section, for a given slope, corresponds to:

$$\beta_o = \frac{b}{h} = 2 \tan \frac{\theta}{2}$$  \hspace{1cm} (35)

If the minimal perimeter condition is imposed, then $\theta = 60^\circ$.

For the parabolic channel, the hydraulically optimal cross-section condition is obtained by determining the minimum of function (12), considering $A = \text{const.}$, which is reduced to calculate the minimum of function $\lambda(\alpha, m)$. For this purpose, the values $\lambda(\alpha, m)$ were calculated using Eqs. (13)-(15) for 80 values of $\alpha$ and 340 values of $m$ using a computer program.

The minimum value $\lambda(\alpha, m)$ for a given degree $\alpha$ of the generic parabola corresponds to the reverse optimal side slope $m_o$, of which the values are presented in Table 2 and represent the condition of hydraulically optimal parabolic cross-section.

![Table 2](attachment:image2)

The minimal value $\lambda(\alpha, m)$ for a given slope $1/m$ also corresponds to $\alpha_o$, being the optimal degree of a generic parabola. The optimal value $\alpha_o$ is listed in Table 3 and represent the hydraulically optimal parabolic cross-section condition for a given side slope.

![Table 3](attachment:image3)

For the semi-elliptic channel, the hydraulically optimal section condition is obtained by considering $A = \text{const.}$ and determining the minimum of function (21) rewritten under general form:

$$P_u = \frac{\sqrt{\pi}}{2} \left( 1.5 \varphi + \frac{2}{\sqrt{\varphi}} - \sqrt{2} \right) \sqrt{A}$$  \hspace{1cm} (36)

The following is obtained:

$$\varphi_o = 2$$  \hspace{1cm} (37)

Condition (37) leads to the limit case when the semi-ellipse tends to a semi-circle.
2.3 Mathematical model

Expressing water velocity \( V \) using Chézy’s formula and adopting the Pavlovski’s formula for the hydraulic resistance coefficient \( C \) yields the well-known flow equation:

\[
Q = \frac{1}{n} AR^{y+0.5} i^{0.5} \tag{38}
\]

where: \( n \) is the Manning’s roughness coefficient; \( i \) is the channel bottom slope; \( y \) is an exponent in Pavlovski’s formula, calculated with the following equation:

\[
y = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R\sqrt{n} - 0.10} \tag{39}
\]

Substituting the \( A \) and \( R \) expressions depending on \( h \) (Table 1) in Eq. (38), the general equation for hydraulic computation of simple cross-section channels with steady state uniform flow is obtained:

\[
Q = \frac{1}{n} f^{y+1.5} h^{y+2.5} i^{0.5} \tag{40}
\]

If the water velocity \( V \) obtained after design is not enclosed between admissible limits \( V_m \) and \( V_M \), indicated in the literature, the channel bottom slope \( i \) is increased or decreased according to the following equation:

\[
i = \frac{n^2 V_m^2}{R^{2y+1}}, \tag{41}
\]

and the water depth \( h \) is determined with the following equation:

\[
h = \frac{1}{n^y} \frac{Q}{V_m^{0.5}} \tag{42}
\]

The design of a channel with a given shape requires determination of the variables \( h \), \( B \) and \( V \) when the elements \( Q \), \( i \), \( n \) and parameters \( \alpha \) and \( p \) or \( m \) (parabolic section), \( \varphi \) (semi-elliptic or semi-circular section), \( m \) (trapezoidal section), \( m \) (triangular section), or \( \beta \) (rectangular section) are known. This can be easily performed applying the iteration method. In the case of the operation checking problem, the geometrical elements \( B \), \( h \), \( n \), \( m \) and \( \alpha \) are given and the hydraulic elements \( Q \) and \( V \) are computed.

Taking into account the calculation formulas presented, an algorithm to solve the outstanding problems of the hydraulic computation of simple cross-section (linear and curve) open channels with steady state uniform flow was developed. On the basis of this algorithm the computer program CANDES1 [12] was elaborated in FORTRAN programming language for PC microsystems.

3. Compound channels

3.1. Geometrical elements of the channels with flat sides and a cylindrical bottom

Taking into account the notations in Fig. 4 and the tangency condition of trapezium sides to the circle arch, the cross-section elements can be deduced:

\[
m = \cot \theta \tag{43}
\]

\[
b = 2r \sin \theta = f_1(m)r \tag{44}
\]

\[
h = 2r \sin \frac{\theta}{2} = f_2(m)r \tag{45}
\]

\[
H - h = \alpha r \tag{46}
\]

Fig. 4 Channel with flat sides and a cylindrical bottom
\[ H = \frac{\alpha + f_2(m)}{f_2(m)} h = \left[ \alpha + f_2(m) \right] r \quad (47) \]

\[ B = b + 2m(H - h) = \left[ 2m\alpha + f_1(m) \right] r \quad (48) \]
in which:

\[ f_1(m) = \frac{4}{m'} \quad (49) \]

\[ m' = 2\sqrt{1 + m^2} \quad (50) \]

\[ f_2(m) = 1 - \frac{2m}{m'} \quad (51) \]

\[ \alpha = \frac{H - f_2(m)}{r} = f_2(m) \left( \frac{H}{h} - 1 \right) \quad (52) \]

where: \( \theta \) is the side angle; \( m \) is the reciprocal of the side slope; \( r \) is the circle arch radius; \( H \) is the water depth; \( B \) is the section width at the water free-surface.

Using Eqs. (47) and (48), the following is obtained:

\[ \beta = \frac{B}{H} = \frac{2m\alpha + f_1(m)}{\alpha + f_2(m)} \quad (53) \]

\[ \alpha = \frac{f_1(m) - f_2(m)\beta}{\beta - 2m} \quad (54) \]

According to Eq. (52), if \( h=H \) (circular channel) and \( h=0 \) (trapezoidal channel), then \( \alpha \) has the value 0 and \( \infty \), respectively.

Introducing restrictions \( \alpha=0 \) and \( \alpha=\infty \) in Eq. (53), the problem’s compatibility condition is obtained:

\[ 2m < \beta < \frac{4}{m' - 2m} \quad (55) \]

Eq. (55) expresses the domain in which the link between the geometric and hydraulic characteristics can be analysed.

For the wetted perimeter \( P_w \), area \( A \) and hydraulic radius \( R \), the following expressions are derived:

\[ P_w = 2\sqrt{1 + m^2} (H - h) + 20r = \left[ m'\alpha + 2f_2(m) \right] r \quad (56) \]

\[ A = \left[ b + m(H - h) \right] (H - h) + \frac{2\theta - \sin 2\theta}{2} r^2 = \left[ m\alpha^2 + f_1(m)\alpha + f_4(m) \right] r^2 \quad (57) \]

\[ R = \frac{A}{P_w} = \frac{m\alpha^2 + f_1(m)\alpha + f_4(m)}{m'\alpha + 2f_3(m)} r \quad (58) \]
in which:

\[ f_3(m) = \arctg \frac{1}{m} \quad (59) \]

\[ f_4(m) = f_3(m) - \frac{2m}{m'} \quad (60) \]

Using the following notations:

\[ \varphi = \alpha + f_2(m) \quad (61) \]

\[ \tau = 2m\alpha + f_1(m) \quad (62) \]

\[ \psi = \sqrt{m\alpha^2 + f_1(m)\alpha + f_4(m)} \quad (63) \]

\[ f = m'\alpha + 2f_3(m) \quad (64) \]
every dimensional geometric element is a function of only one dimensional element and two other dimensionless parameters \( m \) and \( \alpha \), according to Table 4.

### 3.2 Mathematical model

Substituting the expression for geometrical elements \( A \) and \( R \) depending on \( H \) (Table 4) in the discharge equation (38), the general formula is obtained for the hydraulic computation of channels with flat sides and a cylindrical bottom in steady state uniform flow:

\[ Q = \frac{1}{n} \varphi^{2r+3} \psi^{2r+2.5} f^{r+0.5} H^{y+2.5} t^{0.5} \quad (65) \]

The expression of radius \( r \) obtained in Eq. (57) is substituted in Eq. (56) to yield the following notation:

\[ \lambda = \frac{f}{\psi} = \frac{m'\alpha + 2f_3(m)}{\sqrt{m\alpha^2 + f_1(m)\alpha + f_4(m)}} \quad (66) \]
Table 4 Geometrical element expression of the cross-section

| Elements | A function of: |
|----------|----------------|
| r        | 1/\( \varphi \) |
| H        | 1/\( \tau \) |
| B        | \( \varphi / \psi \) |
| A        | \( \varphi / f \) |
| Pu       | \( \varphi / f \) |
| R        | \( \varphi / f \) |

Thus, expression (56) of the wetted perimeter becomes:

\[ P_u = \lambda \sqrt{A} \] (67)

In the case of a hydraulically optimal section that corresponds to the minimal wetted perimeter for the same \( A, n \) and \( i \) values, the dimensionless parameter \( \lambda \) becomes minimal and \( \alpha = \alpha_o \) is obtained by determination of the (66) function minimum from condition \( \partial \lambda / \partial \alpha = 0 \):

\[ \alpha_o = \frac{f_3(m)f_5(m) - m'f_4(m)}{1/2m'} = 2m' = \frac{m}{\sqrt{1 + m^2}} \] (68)

Substituting Eq. (68) in Eq. (53) results in the condition of the hydraulically optimal section for a given side slope:

\[ \beta_o = m' = 2\sqrt{1 + m^2} \] (69)

The values of optimal relative width \( \beta_o \) determined in Eq. (69) for different values of \( m \) are presented in Table 5.

Table 5 Values of optimal relative width \( \beta_o \)

| \( m \) | 0   | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 2.00 | 2.50 | 3.00 | 4.00 |
|--------|-----|------|------|------|------|------|------|------|------|------|------|
| \( \beta_o \) | 2.00 | 2.00 | 2.24 | 2.50 | 2.83 | 3.20 | 3.61 | 4.47 | 5.39 | 6.32 | 8.25 |

If the resulting water velocity \( V \) after design process is not within admissible limits \( V_m \) and \( V_M \), then the channel bottom slope \( i \) is increased or decreased according to Eq. (41), and the water depth \( H \) is determined by the equation:

\[ H = \frac{\varphi}{\psi} \left( \frac{Q}{V_{m(M)}} \right) \] (70)

The mathematical model described was implemented in the computer program CANDES2 [12] written in FORTRAN programming language for PC microsystems.

4. Numerical applications
a) To enhance the advantage of the CANDES1 program, the design of a channel was performed considering different cross-sections (trapezoid al, rectangular, triangular, parabolic, semi-elliptic, and semi-circular) and using the following data: \( Q = 5 \text{ m}^3/\text{s} \), \( i = 0.0006 \), \( n = 0.014 \), \( V_m = 0.7 \text{ m/s} \) and \( V_M = 4 \text{ m/s} \). The numerical results of the computations are summarised in Table 6.

**Table 6 Characteristic elements provided by the CANDES1 program**

| No. | Computed elements | Cross-section shapes |
|-----|-------------------|----------------------|
|     |                   | Trapezoidal | Rectangular | Triangular | Parabolic | Semi-elliptic | Semi-circular |
| 1   | \( h \) [m]       | 1.405       | 1.359       | 1.922      | 1.320     | 1.668        | 1.488        |
| 2   | \( B \) [m]       | 3.976       | 2.718       | 3.844      | 3.181     | 2.668        | 2.976        |
| 3   | \( A \) [m²]      | 3.612       | 3.694       | 3.694      | 3.500     | 3.493        | 3.477        |
| 4   | \( P_e \) [m]     | 5.140       | 5.436       | 5.436      | 4.750     | 4.728        | 4.673        |
| 5   | \( l \) [m/s]     | 1.389       | 1.359       | 1.359      | 1.433     | 1.436        | 1.442        |
| 6   | \( i \) [%]       | 0.060       | 0.060       | 0.060      | 0.060     | 0.060        | 0.060        |
| 7   | \( b \) [m]       | 1.166       | 2.718       | –          | –         | –            | –            |
| 8   | \( p \) [m⁻¹]     | –           | –           | –          | 0.130     | –            | –            |

b) The design of a hydraulically optimal section of a channel with flat sides and a cylindrical bottom was obtained using CANDES2 program for \( Q = 10.0 \text{ m}^3/\text{s} \), \( i = 0.0006 \), \( n = 0.0225 \), \( m = 1 \), \( V_m = 0.50 \text{ m/s} \), \( V_M = 1.50 \text{ m/s} \) and \( \varepsilon = 0.001 \text{ m} \). The following numerical results are obtained: \( \beta_o = 2.828 \), \( r = 2.373 \text{ m} \), \( h = 0.695 \text{ m} \), \( b = 3.355 \text{ m} \), \( H = 2.373 \text{ m} \), \( B = 6.711 \text{ m} \), \( V = 1.126 \text{ m/s} \) and \( i = 0.0006 \).

5. Conclusions

The proposed mathematical models have a higher degree of generality, which allows for performing a more precise unitary hydraulic computation for the channels with different cross-section shapes and to establish their hydraulically optimal profile. The optimal hydraulic computation of these channels can be typically applied for the precast conveyor troughs with different shapes within hydro-amelioration systems.

The general hydraulic calculation of channels with flat sides and a cylindrical bottom is based on Eq. (65). This calculation is performed in a manner analogous to the hydraulically optimal cross-section calculation developed in this study. The difference is that the most adequate relations between the geometrical elements \( m, \alpha, H, B, r, A \) and \( R \) are selected.

Some aspects concerning the hydraulic computation for random degree parabolic channels can be applied in the case of symmetrical river beds [14].

The developed computer programs allow for performing an efficient computation in a manner that is more precise than the traditional methods, when different constructive variants are compared.

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