Study on characterization model of relative permeability based on three parameter Weibull model

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Abstract. Aiming at the problem of large error between relative permeability and water cut in high water cut and low water cut stage. A large number of experiments show that there is a good linear relationship between the double logarithm of water saturation and the logarithm of oil-water relative permeability ratio. On this basis, the three parameter Weibull model is applied to establish a new characterization model of relative permeability. The calculation results of two examples show that the calculated values of the new model are all close to the actual values in low water cut, medium water cut and high water cut stages, and the accuracy is high. This study provides a basis for improving the accuracy of reservoir engineering scheme design and reservoir numerical simulation.

1. Introduction
Relative permeability is one of the most important reference data for parameter design of oil and gas field development scheme, reservoir performance analysis, evaluation of remaining oil distribution, wettability evaluation and reservoir numerical simulation[1]. At present, the commonly used method to measure oil-water relative permeability is unsteady state method. The advantage of this method is that it can better simulate reservoir development performance, and the measurement is simple, the time is short, but the calculation process is complex.

Wang Jianfu[2] used the mercury injection data of tight sandstone core, obtained the typical tight sandstone oil water relative permeability curve through fractal theory, normalization and non standardization calculation. Wang Dongqi[3] selected Willhite empirical formula as the basic fitting formula to study the relationship between oil and water phase index and outlet water saturation, and gradually revised oil and water phase index to improve the fitting accuracy of Willhite empirical formula. Wu Keliu[4] considered the characteristics of oil-water seepage in porous media of low-permeability, ultra-low permeability reservoir, established the calculation model of unsteady oil-water relative permeability of low-permeability, ultra-low permeability reservoir considering the influence of start-up pressure gradient, gravity and capillary force, and calculated the oil-water relative permeability curve
under different influence factors. Guancuo[5] through orthogonal test, the causes of water drive characteristic curve upwarping except for relative permeability curve are deeply analyzed. The relationship between oil-water relative permeability ratio and water saturation is a factor that makes water drive characteristic curve upwarping, but it is not the only factor. Shi Fengxia[6] selected the influencing factors according to the theoretical derivation process of water drive typical curve. Based on orthogonal design and multiple regression analysis, the influence degree of each factor on the warping time of water drive typical curve was discussed, and the warping time of water drive typical curve deviated from straight line was quantitatively characterized.

The above research shows that: many scholars have done a lot of research on relative permeability, but the nonlinear problem of water drive curve in low and high water cut stage is still facing great difficulties[7-12]. To solve this problem, based on the three parameter Weibull prediction model, a new relative permeability partial flow equation is established by linear fitting, and the water cut predicted by the new model is compared with the actual value.

2. The relationship between relative permeability and water saturation

The derivation process of traditional water drive typical curve model assumes oil-water two-phase seepage. Many statistical data of core experiments show that most areas of oil-water two-phase co-permeability area, the oil-water relative permeability ratio has a semi logarithmic linear relationship with water saturation, that is[13]:

\[
\frac{K_{ro}}{K_{rw}} = me^{-as_w}
\]

(1)

However, a large number of field practice and core experiments show that as the oilfield enters the high water cut and ultra-high water cut stage, \(\frac{K_{ro}}{K_{rw}}\) and \(S_w\) deviate from the linear relationship in the semi logarithmic coordinates, and the water drive typical curve deviates from the linear upward warping phenomenon in the high water cut stage(Fig. 1), resulting in the water drive typical curve unable to accurately predict the oilfield development performance.

![Fig. 1 The relative permeability ratio versus water saturation curve](image)

In order to accurately describe the nonlinear relationship between water saturation and relative permeability, it is urgent to establish a new function model to meet the actual production situation of low water cut and high water cut reservoirs.

3. Characterization of relative permeability

Weibull prediction model is a statistical distribution model proposed by Weibull in 1939. The integral of the distribution density function of the model in the random variable \(0<x<+\infty \) interval is 1, and the three parameter Weibull function is[14-15]:

\[
y = \left(1 - e^{-ax^b}\right)^b
\]

(2)
It can be seen from Eq. (2) that when $x$ approaches 0, $y$ approaches 0, and when $x$ approaches $+\infty$, $y$ approaches 1.

\[
y_{x=0} = 0 \quad (3)
\]
\[
y_{x=+\infty} = 1 \quad (4)
\]

Eq. (2) can be rewritten as follows:

\[
\frac{1}{1 - y^{\beta}} = ax^{\beta} \quad (5)
\]

Taking logarithm on both sides of Eq. (5):

\[
\ln \left( \frac{1}{1 - y^{\beta}} \right) = ax^{\beta} \quad (6)
\]

Taking logarithm on both sides of Eq. (6) too.

\[
\ln \left( \ln \left( \frac{1}{1 - y^{\beta}} \right) \right) = \ln a + b \ln x \quad (7)
\]

It can be seen from Eq. (7) that there is a linear relationship between \( \ln \left( \frac{1}{1 - y^{\beta}} \right) \) and \( \ln x \). The slope of the line is $b$ and the intercept is $\ln a$.

In reservoir engineering, the formulas of normalized water saturation ($S_{wn}$) and oil saturation ($S_{on}$) are as follows:

\[
S_{wn} = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \quad (8)
\]
\[
S_{on} = \frac{1 - S_w - S_{or}}{1 - S_{wc} - S_{or}} \quad (9)
\]

The statistical results show that $y$ in Eq. (7) is replaced by normalized water saturation $S_{wn}$, $x$ is replaced by the relative permeability ratio of water to oil $\frac{K_{rw}}{K_{ro}}$. There is a good linear relationship between \( \ln \left( \frac{1}{1 - S_{wn}^{\beta}} \right) \) and \( \ln \left( \frac{K_{ro}}{K_{rw}} \right) \). Therefore, the relationship between relative permeability and normalized water saturation based on three parameter Weibull model can be expressed as follows:

\[
S_{wn} = \left[ 1 - e^{-a \left( \frac{K_{ro}}{K_{rw}} \right)^{\beta}} \right]^{\alpha} \quad (10)
\]

It can be seen from Eq. (10) that when $\frac{K_{rw}}{K_{ro}}$ approaches 0, $S_{wn}$ approximates to 0, and when $\frac{K_{rw}}{K_{ro}}$ approaches $+\infty$, $S_{wn}$ approximates to 1.
Rearranging Eq.(10) and taking double natural logarithms on both sides:
\[
\ln \left( \ln \frac{1}{1 - S_{w}} \right) = b \ln \left( \frac{K_{rw}}{K_{ro}} \right) + \ln a
\]

According to Eq.(13), the formula of oil-water relative permeability ratio is as follows:
\[
\frac{K_{ro}}{K_{rw}} = a^\frac{1}{b} \left( \frac{1}{1 - S_{w}^{\frac{1}{\beta}}} \right)
\]

The formula of water cut expressed by mobility ratio is:
\[
f_{w} = \frac{1}{1 + \frac{\mu_{w} K_{rw}}{\mu_{o} K_{ro}}}\left( \frac{1}{1 - S_{w}^{\frac{1}{\beta}}} \right)\left( \frac{\ln \frac{1}{1 - S_{w}^{\frac{1}{\beta}}} - \ln a}{\mu_{w}} \right)
\]

Eq.(16) is a newly established formula of relative infiltration partial seepage. The values of $\mu_{w}$, $\mu_{o}$, $\beta$, $A$ and $B$ can calculate the water cut $f_{w}$, which improve the prediction accuracy and guide the development of water drive reservoir.

4. Example application

4.1. Case 1
Taking the core from well 30-18 of Ng3 reservoir in Gudao oilfield as an example[16], the viscosity of crude oil used in relative permeability experiment is $\mu_{o}=57.76$ mPa·s, the viscosity of formation water is $\mu_{w}=0.582$ mPa·s. The experimental data of relative permeability of core samples are shown in Table 1.
Table 1. The relative permeability and water cut data of well 30-18 in Ng3 reservoir

| Number | $S_w$ | $K_{rw}$ | $K_{ro}$ | Actual water cut $f_w$ | Predicted water cut $f_w$ |
|--------|-------|--------|--------|----------------|----------------|
| 1      | 0.173 | 0.000  | 1.000  | 0.000           | 0.000           |
| 2      | 0.309 | 0.015  | 0.537  | 0.729           | 0.729           |
| 3      | 0.328 | 0.021  | 0.436  | 0.825           | 0.823           |
| 4      | 0.351 | 0.029  | 0.345  | 0.893           | 0.893           |
| 5      | 0.377 | 0.040  | 0.271  | 0.936           | 0.937           |
| 6      | 0.411 | 0.057  | 0.201  | 0.966           | 0.966           |
| 7      | 0.442 | 0.076  | 0.158  | 0.980           | 0.980           |
| 8      | 0.470 | 0.097  | 0.127  | 0.987           | 0.987           |
| 9      | 0.512 | 0.135  | 0.096  | 0.993           | 0.993           |
| 10     | 0.544 | 0.171  | 0.077  | 0.996           | 0.995           |
| 11     | 0.574 | 0.209  | 0.065  | 0.997           | 0.997           |
| 12     | 0.594 | 0.239  | 0.057  | 0.998           | 0.998           |
| 13     | 0.617 | 0.276  | 0.050  | 0.998           | 0.998           |
| 14     | 0.715 | 0.485  | 0.000  | 1.000           | 1.000           |

When the water saturation in table (1) is substituted into Eq.(8), the normalized water saturation can be obtained, and then the values $\frac{K_{rw}}{K_{ro}}$ can be calculated. The calculated normalized water saturation $S_{wn}$ and $\frac{K_{rw}}{K_{ro}}$ are substituted into Eq.(13) in turn. When the linear correlation is best, the coefficients of Eq.(13) can be obtained: $a=0.17238$, $b=0.8147$, $\beta=0.2941$. The relative permeability partial flow rate equation of well 30-18 in Ng3 reservoir of Gudao oilfield is as follows:

$$f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o} \left( \frac{1}{\beta} \ln \left( \frac{1}{1 - S_{wn}} \right) \right)^{1/\beta} + 0.001164 \times \left( \ln \frac{1}{1 - S_{wn}} \right)^{-1.227446}}$$ \quad \text{(17)}

Where: $S_{wn} = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{ar}} = 1.845S_w - 0.3192$ \quad \text{(18)}

Comparing the calculated value of Eq.(17) with the actual, it can be seen that the theoretical calculated value of water cut is close to the actual and has high accuracy in the low, medium and high water cut stages(Fig. 2).

![Fig.2 The comparison of actual and calculated water cut of well 30-18](image)
4.2. Case 2
Taking X reservoir in reference [17] as an example, the reservoir is developed by water injection with a five spot well pattern, the original water saturation is 0.1, the average permeability is 10mD, and the crude oil viscosity is \( \mu_o = 1 \text{mPa·s} \), formation water viscosity is \( \mu_w = 0.5 \text{mPa·s} \). The relationship between oil-water relative permeability, water cut and water saturation is shown in Table 2.

**Table 2. The relative permeability and water cut data of X reservoir**

| Number | \( S_w \) | \( K_{rw} \) | \( K_{ro} \) | Actual water cut \( f_w \) | Predicted water cut \( f_w \) |
|--------|----------|-----------|-----------|----------------|----------------|
| 1      | 0.10     | 1.000     | 0         | 0              | 0.0000         |
| 2      | 0.30     | 0.373     | 0.070     | 0.2729         | 0.2760         |
| 3      | 0.40     | 0.210     | 0.169     | 0.6168         | 0.6107         |
| 4      | 0.45     | 0.148     | 0.226     | 0.7533         | 0.7531         |
| 5      | 0.50     | 0.100     | 0.300     | 0.8571         | 0.8575         |
| 6      | 0.55     | 0.061     | 0.376     | 0.9250         | 0.9260         |
| 7      | 0.60     | 0.033     | 0.476     | 0.9665         | 0.9672         |
| 8      | 0.65     | 0.012     | 0.600     | 0.9901         | 0.9896         |
| 9      | 0.70     | 0.000     | 0.740     | 1.0000         | 1.0000         |

Using the similar method in example 1, when the linear correlation is the best, the coefficients of Eq.(13) can be obtained in X reservoir: \( a = 1.91115, b = 0.17998, \beta = 3.9621 \). Then the relative permeability partial flow equation of X reservoir is as follows:

\[
 f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o} \left( \ln \frac{1}{1 - S_{wn}} \right)^{\frac{1}{\beta}}} = \frac{1}{1 + 18.2765 \times \left( \ln \frac{1}{1 - S_{wn}^{0.25239}} \right)}^{0.5562} \quad (19)
\]

Where:

\[
 S_{wn} = \frac{S_w - S_{we}}{1 - S_{we} - S_w} = 1.667S_w - 0.1667 \quad (20)
\]

Similarly, the theoretical calculation value of water cut in X reservoir is close to the actual and has high accuracy (Fig. 3).

**Fig. 3** The comparison of actual and calculated water cut in X reservoir
5. Conclusion

(1) In the low and high water cut stage of water flooding reservoir, the relationship between actual oil-water relative permeability ratio and water saturation is nonlinear, which leads to the traditional fitting calculation formula is no longer applicable. The results show that: there is a good linear relationship between $\ln \left( \frac{1}{1 - S_{\text{sw}}^\beta} \right)$ and $\ln \left( \frac{K_{\text{ro}}}{K_{\text{rw}}} \right)$, and the relationship conforms to the three parameter Weibull function.

(2) Based on the three parameters Weibull model, a new calculation model of relative permeability partial seepage is established. The calculation results of case 1 and case 2 show that the theoretical water cut values calculated by the new model are close to the actual data at low, medium and high water cut stages, and the prediction accuracy is high, which improves the accuracy of reservoir production performance prediction.

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