Hot Nuclear Matter in Asymmetry Chiral Sigma Model

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Abstract

In the frame work of SU(2) chiral sigma model, the nuclear matter properties at zero and finite temperature have been investigated. We have analyzed the nuclear matter equation of state by varying different parameters, which agrees well with the one derived from the heavy-ion collision experiment at extreme densities and reliable realistic(DBHF) model at low density region. We have then calculated the temperature dependent asymmetric nuclear matter, also investigated the critical temperature of liquid gas phase transition and compared with the experimental data. We found that the critical temperature in our model is in the range of $14 - 20$ MeV.

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1 Introduction

Recently, nuclear equation of state (EOS) is of great interest in nuclear physics and astrophysics (1 - 3). Specially, in the calculation of nuclear many body problem such as liquid gas phase transition (4, 5) at low density and finite temperature. The EOS is also useful to study the quark gluon plasma (QGP) at extreme densities and temperatures (6 - 8). Very recent experiment has confirmed indirectly more or less the formation of QGP at extreme conditions (9). Also the EOS is a main ingredient to study the evolution of stellar systems, and the global properties of neutron star and supernova (7, 10, 11).

To derive the EOS theoretically, many body approaches have been adopted. These are Hartree-Fock, Thomas-Fermi and mean-field theory type procedures (12, 13). One of them, the relativistic mean-field (RMF) formalism is of great success in the theoretical calculation of finite nuclei and infinite nuclear matter (13, 14, 7, 6). The original Walecka RMF model (15) has been modified to a great extent due to its unrealistic meson nucleon interactions. For example by adding the non-linear self-interaction of scalar mesons in the RMF model (16, 14, 6), one can describe desirable values of saturation properties of nuclear matter such as incompressibility, binding energy, saturation density and effective nucleon mass. Though the non-linear RMF describe well the finite nuclei and the nuclear matter properties at normal density, it deviates from the relativistic Dirac Brueckner Hartree Fock (DBHF) equation of state (17). Because, the DBHF is considered to be the most realistic EOS in the non-relativistic approach at low density realm. Therefore, there are attempts to include vector meson self-coupling to reproduce the EOS, compatible to DBHF. One can not take arbitrarily the vector-scalar and vector-vector interactions for the model to be renormalized. However, these interactions can be included in the RMF model, inspired by the effective field theory (EFT) approach (18).

The chiral sigma (CS) model plays significant role in the high density matter, because chiral symmetry is a good hadron symmetry (19), which is desirable in any theory of dense hadronic matter. The CS model is analogous to RMF, where meson fields are treated in the mean-field approximation. The beauty of this model is that its non-linear terms simulate the three body forces and is essential to reproduce the nuclear matter saturation properties. A decade ago we proposed a SU(2) CS model, where the mass of the isoscalar vector field is generated dynamically (19). The main problem of this theory was unrealistic high incompressibility. To overcome this shortcoming, recently (20) we
made an attempt to introduce the higher order terms of scalar meson field. In this way, we can reproduce the empirical values of incompressibility, effective mass, binding energy and saturation density. Using the same SU(2) CS model we calculate here the EOS for symmetric and asymmetric nuclear matter at zero and finite temperature. In this calculation, we choose the incompressibility 210, 300 and 380 MeV and the effective masses, 0.8, 0.85 and 0.9 of nucleon mass and discuss their applicability to the various heavy-ion collision experiments.

The main interest to study finite temperature nuclear EOS is to observe the liquid gas phase transition\cite{21} near normal nuclear matter density. It is also required to estimate the critical temperature at liquid gas coexistence point. This feature is very much noticeable, because in the medium energy heavy-ion collisions, one of the theoretical studies of dynamics is the liquid gas phase transition. Similar feature has been suggested at very low temperature in the crust of neutron star\cite{11, 22}. In this direction much work has been carried out using both non-relativistic\cite{5} as well as relativistic\cite{23} formalisms. The liquid gas phase transition and the critical temperature were studied extensively based on the the non-relativistic theory\cite{24-26}. The estimated critical temperature is found to be in the range of 15-20 MeV.

The liquid gas phase also has been studied by many authors based on the RMF theory\cite{23, 27}. In the original Walecka model, the critical temperature in the symmetric nuclear matter is estimated to be 18.3 MeV\cite{13}. This value was brought down to 14.2 MeV, if the non-linear terms were included in the model. However, recent experiments in heavy-ion collisions for dilute warm nuclear matter report a small liquid gas phase region and low critical temperature~\(13.1 \pm 0.6 \text{ MeV}\)\cite{21}. It has been noticed that the different critical temperature are extracted by various theories. This is because, each theory has its own type of treatments of the nuclear interactions. Therefore, our motivation is to see such properties in the present modified CS sigma model.

The paper is organized as follows: In section II, we present a brief formalism of the CS model. In this section we derive the EOS with zero and finite temperature for symmetric and asymmetric nuclear matter. Results and discussions are displayed in section III. The calculation of the EOS for various parameter sets and its sensitivity have been studied and the liquid gas phase transition has been discussed in this section. We also compare our results with the recent experimental data. The summary and concluding remarks follow in section IV.
2 The Formalism of SU(2) Chiral Sigma Model

The SU(2) chiral sigma Lagrangian can be written as \[19, 20\]

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \pi \cdot \partial^\mu \pi + \partial_\mu \sigma \partial^\mu \sigma) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (x^2 - x_o^2)^2 - \frac{\lambda b}{6m^2} (x^2 - x_o^2)^3 - \frac{\lambda c}{8m^4} (x^2 - x_o^2)^4 - g\sigma \bar{\psi}(\sigma + i\gamma_5 \tau \cdot \pi) \psi + \bar{\psi}(i\gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu) \psi + \frac{1}{2} g_\omega^2 x^2 \omega^\mu \omega_\mu + \frac{1}{24}\xi g_\omega^4 (\omega_\mu \omega^\mu)^2 - D\sigma. \tag{1}
\]

Here \( F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) and \( x^2 = \pi^2 + \sigma^2 \), \( \psi \) is the nucleon isospin doublet, \( \pi \) is the pseudoscalar-isovector pion field, \( \sigma \) is the scalar field, and \( D \) is a constant. We work in natural units with \( \hbar = c = k_B = 1 \).

The Lagrangian includes a dynamically generated mass of the isoscalar vector field, \( \omega_\mu \), that couples to the conserved baryonic current \( j_\mu = \bar{\psi} \gamma_\mu \psi \). The constant parameters \( b \) and \( c \) are included in the higher-order self-interaction of the scalar field to describe the desirable values of nuclear matter properties at saturation point. Henceforth, we define our model as modified non-linear CS model (NCS) in our successive discussions. In the fourth order term of the omega fields, the quantity \( \xi \) is a constant parameter. Throughout our calculations, for simplicity, we set \( \xi \) to zero. In this model the pion mass \( m_\pi \) is zero without symmetry breaking. Thus the last term, \( D\sigma \) in the Lagrangian is zero in our present calculation. The interaction of the scalar and the pseudoscalar mesons with the vector boson generate a mass for the latter through the spontaneous breaking of the chiral symmetry. The masses of the nucleon, scalar and vector meson are respectively given by

\[
m = g_\sigma x_o, \quad m_\sigma = \sqrt{2}\lambda x_o, \quad m_\omega = g_\omega x_o, \tag{2}\]

where \( x_o \) is the vacuum expectation value of the \( \sigma \) field, \( \lambda = (m_\sigma^2 - m_\pi^2)/(2f_\pi^2) \), with \( m_\pi \), the pion mass and \( f_\pi \) the pion decay coupling constant, and \( g_\omega \) and \( g_\sigma \) are the coupling constants for the vector and scalar fields, respectively. In the mean-field treatment we ignore the explicit role of \( \pi \) mesons.

By adopting mean-field approximation, the equation of motion of fields are obtained. This approach has been used extensively to evaluate the EOS in any theoretical
models for high density matter. Using the ansatz of the mean-field, the equation of motion for the scalar field ($\sigma_0$) in terms of $m^*/m \equiv x/x_o$ is

$$
(1 - Y^2) - \frac{b}{m^2 c_\omega}(1 - Y^2)^2 + \frac{c}{m^4 c_\omega^2}(1 - Y^2)^3 + \frac{2c c_\sigma n_B^2}{m^2 Y^4} - \frac{2c_\sigma n_S}{m Y} = 0,
$$

where $m^* \equiv Ym$ is the effective mass of the nucleon and $c_\sigma \equiv g_\sigma^2/m_\sigma^2$ and $c_\omega \equiv g_\omega^2/m_\omega^2$ are scalar and vector coupling constants respectively. $n_S$ is the scalar density defined in equation (7).

The equation of motion for the isoscalar vector field is

$$
\omega_0 = \frac{n_B}{g_\omega x^2},
$$

where in the mean-field limit $\omega = \omega_0$. The quantity $k_F$ is the Fermi momentum and $\gamma$ is the nucleon spin degeneracy factor and $n_B$ is the baryon density defined in the next section.

### 2.1 Equation of state at zero temperature

The EOS is calculated from the diagonal components of the conserved total stress tensor corresponding to the Lagrangian together with the mean-field equation of motion for the Fermion field and a mean-field approximation for the meson fields. The total energy density, $\varepsilon$, and pressure, $P$, of the many-nucleon system are the following:

$$
\varepsilon = \frac{m^2(1 - Y^2)^2}{8c_\sigma} - \frac{b}{12c_\omega c_\sigma}(1 - Y^2)^3 + \frac{c}{16m^2 c_\omega^2 c_\sigma}(1 - Y^2)^4
+ \frac{c_\omega n_B^2}{2Y^2} + \frac{\gamma}{2\pi^2} \int_0^{k_F} k^2 dk \sqrt{k^2 + m^*^2},
$$

$$
P = -\frac{m^2(1 - Y^2)^2}{8c_\sigma} + \frac{b}{12c_\omega c_\sigma}(1 - Y^2)^3 - \frac{c}{16m^2 c_\omega^2 c_\sigma}(1 - Y^2)^4
+ \frac{c_\omega n_B^2}{2Y^2} + \frac{\gamma}{6\pi^2} \int_0^{k_F} \frac{k^4 dk}{\sqrt{k^2 + m^*^2}},
$$

The energy per nucleon is $E/A = \varepsilon/n_B$, where $\gamma = 4$ for symmetric nuclear matter and $\gamma = 2$ for neutron matter.
The baryon density $n_B$ and scalar density $n_S$ are defined as

$$n_B = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k,$$

and

$$n_S = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} \frac{m^* d^3k}{\sqrt{k^2 + m^*}},$$

respectively, which are used in eq.(3).

### 2.2 Equation of state at finite temperature

The EOS for finite temperature can be defined in the same manner as zero temperature, these are as follows:

$$\varepsilon(T) = \frac{m^2(1 - Y^2)^2}{8c_\sigma} - \frac{b}{12c_\omega c_\sigma}(1 - Y^2)^3 + \frac{c}{16m^2c^2_\omega c_\sigma}(1 - Y^2)^4$$

$$+ \frac{c_\omega n_B^2}{2Y^2} + \frac{\gamma}{2\pi^2} \int_0^\infty k^2 dk \sqrt{k^2 + m^*}(f(T) + \bar{f}(T)),$$

$$P(T) = -\frac{m^2(1 - Y^2)^2}{8c_\sigma} - \frac{b}{12c_\omega c_\sigma}(1 - Y^2)^3 - \frac{c}{16m^2c^2_\omega c_\sigma}(1 - Y^2)^4$$

$$+ \frac{c_\omega n_B^2}{2Y^2} + \frac{\gamma}{6\pi^2} \int_0^\infty \frac{k^4 dk}{\sqrt{k^2 + m^*}}(f(T) + \bar{f}(T)).$$

The baryon and scalar density at finite temperature are respectively modified as

$$n_B(T) = \frac{\gamma}{(2\pi)^3} \int_0^\infty d^3k(f(T) - \bar{f}(T)),$$

and

$$n_S(T) = \frac{\gamma}{(2\pi)^3} \int_0^\infty \frac{m^* d^3k}{\sqrt{k^2 + m^*}}(f(T) + \bar{f}(T)).$$

The nucleon and anti-nucleon distribution functions $f(T)$ and $\bar{f}(T)$, are respectively, expressed as

$$f(T) = \frac{1}{\exp[(E^* + \nu)/T] + 1}$$

and

$$\bar{f}(T) = \frac{1}{\exp[(E^* - \nu)/T] + 1}.$$
where $E^* = \sqrt{k^2 + m^2}$, $T$ is temperature and the chemical potential $\nu = \mu - g_w w_o$. These distribution functions are used in eqs. (8, 10).

### 2.3 Asymmetric nuclear matter

For asymmetric matter, the extra contribution to Lagrangian eq.(1) due to the interaction of the isospin triplet $\rho$–meson is given as

$$ -\frac{1}{4} G_{\mu\nu} \cdot G^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \rho_{\mu} \cdot \rho^\mu - \frac{1}{2} g_{\rho} \bar{\psi}(\rho_{\mu} \cdot \tau^\mu)\psi .$$

(13)

where $G_{\mu\nu} \equiv \partial_{\mu}\rho_{\nu} - \partial_{\nu}\rho_{\mu}$. This term accounts for asymmetric nuclear matter with mixture of protons and neutrons only.

Similarly, the equation of motion for $\rho$–meson is obtained from eq.(13) as:

$$ \rho_3 = \frac{g_{\rho}}{2m_{\rho}^2}(n_p - n_n),$$

(14)

where $n_p$ and $n_n$ are number density of proton and neutron, respectively. The inclusion of the $\rho$–meson in the Lagrangian will contribute the term

$$ + m_{\rho}^2(\rho_3^2)^2/2$$

(15)

to the energy density and pressure as given above eqs.(5, 8) for both zero and finite temperature. From the semi-empirical nuclear mass formula, the symmetric energy coefficient is

$$ a_{\text{sym}} = \frac{c_{\rho} k_F^3}{12\pi^2} + \frac{k_F^2}{6\sqrt{(k_F^2 + M^*2)}},$$

(16)

where $c_{\rho} \equiv g_{\rho}^2/m_{\rho}^2$ and $k_F = (6\pi^2 n_B/\gamma)^{1/3}(n_B = n_p + n_n)$. We fix the coupling constant $c_{\rho}$ by requiring that $a_{\text{sym}}$ correspond to the empirical value, $32 \pm 6 \text{ MeV}$ [28]. This gives $c_{\rho} = 4.66 \text{ fm}^2$ for $a_{\text{sym}}=32 \text{ MeV}$. The chemical potential is redefined as $\nu = \mu - g_w w_o + \tau^3 g_{\rho} \rho_3^3$, due to presence of asymmetric nuclear matter, where $\tau^3$ is $+1/2$ for neutron and $-1/2$ for proton.

Also we introduce the asymmetric parameter, $\alpha$ to describe the asymmetric nuclear matter. This is defined as

$$ \alpha = \frac{n_n - n_p}{n_n + n_p},$$

(17)
where $\alpha=0$ for symmetric matter nuclear matter ($\gamma = 4$) and $\alpha=1$ for the pure neutron matter ($\gamma = 2$).

![Figure 1: The scalar and vector potentials for various parameter sets of NCS model with baryon density. The DBHF result is from the Bonn A potential and NL3 parameter set is from the relativistic mean-field theory.](image)

### 3 Results and discussions

In the EOS eqs.\[5\] for both zero and finite temperature, the four parameters are: the nucleon coupling to the scalar and the vector fields, $c_\sigma$ and $c_\omega$, and the coefficients in the scalar potential terms, $b$ and $c$. These are obtained by fitting at the saturation point: the binding energy/nucleon ($B/A = -16.3$ MeV), baryon density ($n_0 = 0.153$ fm$^{-3}$), incompressibility ($K = 300$ MeV) and effective (Landau) mass ($m^* = 0.85M$)\[28\]. In our calculation we have chosen the effective mass from $0.8 - 0.9M$, to observe the
Table 1: Different parameter sets for the NCS model.

| set | $c_\sigma$ ($fm^2$) | $c_\omega$ ($fm^2$) | $b/m^2$ ($fm^2$) | $c/m^4$ ($fm^4$) | $K$ ($MeV$) | $m^*/m$ |
|-----|-------------------|-------------------|-------------------|-------------------|-------------|--------|
| I   | 8.86              | 1.99              | -12.24            | -31.59            | 210         | 0.85   |
| II  | 6.79              | 1.99              | -4.32             | 0.165             | 300         | 0.85   |
| III | 5.36              | 1.99              | 1.13              | 22.01             | 380         | 0.85   |
| IV  | 8.5               | 2.71              | -9.26             | -40.73            | 300         | 0.8    |
| V   | 2.33              | 1.04              | 9.59              | 46.99             | 300         | 0.9    |

sensitivity of EOS at high density region. Another interesting point we note that by changing the effective mass, the EOS can be compared well with the recent one which has been extracted from the heavy-ion collisions data\cite{29}. We will discuss this below. The nuclear incompressibility is somewhat uncertain at saturation and therefore we take in the range of $210 - 380$ MeV. The desirable values of effective mass and nuclear matter incompressibility are chosen in accordance with recent heavy-ion collision data\cite{30 29}. These parameters are listed in Table I.

3.1 At zero temperature limit

The scalar $U_s (= g_\sigma \sigma_0)$ and vector $U_v (= g_\omega \omega_0)$ potentials versus baryon density are displayed in Fig. 1 for five parameter sets as listed in Table I. We compare these potentials with the more realistic Dirac–Brueckner–Hartree–Fock (DBHF) (Bonn-A parametrization)\cite{31} and the standard $\sigma – \omega$ non-linear relativistic mean-field (NL3 parameter set)\cite{32} potential, that are available in literature. It is shown in Refs.(\cite{33 - 35}) that the standard $\sigma – \omega$ model with scalar self-couplings describes the saturation point and the data for finite nuclei successfully, do not follow the trends of the DBHF properly. In the RMF model, the vector potential increases linearly with density and gets stronger as it does not depend on the non-linear terms of the vector meson. However, in DBHF it bends down (see Fig. 1), because it has density dependent potentials. The scalar potential overestimates the DBHF result at high density in order to compensate for the strong repulsion in the vector channel. The $U_s$ and $U_v$ results obtained by NCS model are quite
low at near and below the nuclear matter saturation density. At high density, the result obtained by parameter set IV is comparable with the DBHF calculation, whereas other sets underestimate the results of both NL3 and DBHF models. In the scalar case, all the parameter sets of NCS model give a low value due to the strong scalar coupling. The smaller value of $U_v$ is counter balanced by the higher $U_s$ and gives a similar magnitude of the total potential, compared to DBHF. For example, at $\sim 3n_0$ the total potential is $\sim -58$ MeV for DBHF, whereas this is $\sim -73$ MeV in set II of NCS model. Similarly, for all other sets this varies from $-17$ to $-200$ MeV for NCS model and for NL3, it is $-327$ MeV. This feature reflects in the EOS (discussed in next figure).

Now we compare the EOS of our calculations with the NL3 and DBHF models in Fig. 2. Here we find that sets IV and V match with NL3 and DBHF, respectively up to three times the nuclear matter density. It is to be noted that the EOS obtained by DBHF is trusted upto two times of nuclear matter density[36, 37]. The difference between the two sets IV (stiff) and V (soft) is only due to the different effective mass for the same

\[ -\frac{E}{n_B} - M (\text{MeV}) \]

Figure 2: Same as figure 1 but for energy per particle
incompressibility ($K = 300$ MeV). The all other three sets namely, I ($K=210$ MeV), II ($K=300$ MeV) and III ($K=380$ MeV) are having same effective mass with different incompressibilities. From this graph, we note that the stiffness or softness of EOS is insignificant with incompressibilities in comparison to NL3 and DBHF.

![Graph](image)

**Figure 3:** The pressure versus baryon density for different sets of NCS model. The shaded block represents the EOS consistent with experimental data [29].

In Fig. 3 we compare our EOS with the predicted experimental values obtained from the heavy-ion collisions data [29]. The overall EOS are good fit to experimental data. If we consider more vividly, then we notice that the EOS having incompressibility, $K=300$ MeV and $m^*/m=0.9$ (set V) fits well. However, $K =210$ (set I) and 300 (set II) MeV with $m^*/m=0.85$ also agree, but slightly deviate from the data at low density. In addition, $K=300$ MeV and $m^*/m=0.8$ (set IV) shows more stiffer EOS. In general, the set II ($K=300$ MeV and $m^*/m=0.85$) explains EOS fairly well and hence could be the ideal parameterization (set II). Note that the value for EOS predicted by experiment may change due to the momentum dependent potential as given in Ref. [30]. We recall
here that all the sets considered in the present calculations (set I–V) compare well with the DBHF prediction at low density (see Fig. 2). Whereas this figure represents a clear picture of the EOS at high densities.

![Graph](image)

Figure 4: For different five parameter sets of NCS model, the effective mass is shown with baryon density.

For the sake of completeness, the effective masses for various sets as a function of baryon density are compared in Fig. 4. The features are similar to those as given in Fig. 2. That means, sets IV and V give drastically different curves owing to different effective masses at saturation density for same incompressibility, as expected. However, we get marginal changes for other three sets I–III beyond nuclear matter density, which represent the different incompressibilities. It is interesting to note that $m^*$ for set IV again increases with density for $n_B \sim 2n_0$ due to strong repulsive force. The effective mass of set V decreases monotonically (see Fig. 4), which has a strong attractive force.
### 3.2 At finite temperature limit

Figure 5: The left panel of figure represents pressure versus baryon density and the right panel shows the energy per particle as a function of baryon density for the set I for different temperature.

Here we analyze the effect of temperature on the EOS, explicitly near the nuclear matter density, such as liquid gas phase transition. The pressure versus baryon density is plotted to show the liquid gas phase transition for various temperature ranging from 0 to 20 MeV in left panel of Fig. 5(a). In this figure we have taken the parameter set I for symmetric matter. At zero temperature, there is a nice pocket (isotherm), which means that the liquid and gas phase are well separated with each other by an unphysical region, where the pressure is negative. One can make a smooth transition from liquid to gas state by making a Maxwell construction. This pocket gradually decreases with increasing temperature. At a particular temperature, the pocket vanishes and is marked as the pure gas state. At this point the pressure gradient with respect to density (inflection point) is zero ($\partial P/\partial n_B|_{T_c} = \partial^2 P/\partial n_B^2|_{T_c} = 0$) and is noted as the critical point for liquid gas phase transition. In other words, the point where the two phases can not be distinguished from each other for a particular temperature is called the critical point. In this figure, the critical temperature $T_c = 14.2$ MeV which corresponds to pressure $P_c = 0.14$ MeV/fm$^3$ and density $n_c = 0.035$ fm$^{-3}$.

The critical temperature obtained by the density dependent relativistic mean-field
Figure 6: Same as figure 5 but for set II.

theory \cite{27} \((T_c = 12.66 \text{ MeV})\) and the experimental value \((T_c = 13.1 \pm 0.6 \text{ MeV})\) \cite{21} are comparable to the \(T_c\) obtained in our calculation with set I. As we emphasized in section I, if one consider the original Walecka model \cite{13} (no non-linear terms), the critical temperature is \(T_c \approx 18.3\). This can be reduced to \(T_c \approx 14.2 \text{ MeV}\) \cite{39}, when one introduces the non-linear terms in the scalar field. The derivative scalar coupling model \cite{40} gives a low critical temperature ranging from \(T_c = 13.6 - 16.5 \text{ MeV}\) depending on the parameter sets. The critical temperature extracted by DBHF approach is 15.0 MeV \cite{17}. Also, it is reported by Baldo et al \cite{41} that a very low \(T_c\) of about 8 – 9 MeV is obtained in relativistic Dirac-Brueckner calculation. Therefore, it can be concluded from the above models that the critical temperature varies from 8 – 19 MeV depending on the formalisms and the parameters used. In our present investigation, we also find a large range of \(T_c\) from 14 – 20 MeV depending on the parameter sets, which will be discussed below. Thus our model is compatible with the other relativistic and non-relativistic models.

The right panel of Fig. 5(b) displays the energy per particle versus baryon density for the symmetric matter with set I. With increasing temperature, the system becomes less bound in comparison to zero temperature. At \(T = T_c\), the energy per particle is \(-2.04 \text{ MeV}\).

In Figs. 6(a) (set II) and 7(a) (set III), we show the similar plot like Fig. 5(a) for
symmetric matter. The $T_c$ obtained are 16.8 MeV and 20.4 MeV. The corresponding $P_c$ are 0.22 and 0.36 MeV/fm$^3$ and $n_c$ are 0.044 and 0.051 fm$^{-3}$, respectively for set II and III. From these values we observe that the critical temperature $T_c$ increases with incompressibility. The critical point shifts toward lower density and pressure for softer EOS. These are comparable to other relativistic\[38\] and non-relativistic models\[24\]. Similarly, the energy per particle are presented in Figs. 6(b) and 7(b) for the sets II and III, at $T = T_c$, the energy per particle are 4.8 and 13.3 MeV, respectively. From the above figures (Figs. 5(b)-7(b)), one may notice that the system becomes less bound with increasing incompressibility.

As we mentioned earlier, sets I and II give reasonably good fit to the DBHF and experimental data (see Figs. 2 and 3), we consider hereafter set II for the rest of our discussions. The reason to choose set II is that it is compatible with the description of heavy-ion collision data \[30\]. In our further discussions, we study the asymmetric, $\alpha$ dependence of the system for a fixed temperature, say for example $T = 10$ MeV. Moreover in this section, we analyzed the effect of $\rho$–meson on the nuclear system. Also, the behaviour of effective mass with temperature and the critical temperature $T_c$ as a function of asymmetric parameter $\alpha$ are studied.

In Fig. 8(a), the pressure versus baryon density for different $\alpha$ at a fixed temperature $T=10$ MeV using parameter set II is displayed. In this case, we have not included the
Figure 8: The pressure as a function of the baryon density for fixed $T=10$ MeV with various asymmetric parameter $\alpha$. The left panel is without $\rho$–meson and the right panel is with $\rho$–meson inclusion for set II.

The liquid gas phase transition disappears at $\alpha \sim 0.6$, below which the pressure shows a minimum with respect to density, that means there is a phase boundary between two phases as shown in Fig. 8(a). Also for different $\alpha$, we plot pressure versus baryon density for fixed $T=10$ MeV with inclusion of $\rho$–mesons in the nuclear matter for parameter set II. The graph (Fig. 8(b)) looks very similar to Fig. 8(a), but the pressure rapidly increases as the extra repulsive force generated from the $\rho$–mesons and hence, the liquid gas phase transition vanishes at $\alpha \sim 0.2$.

The asymmetric parameter, $\alpha$ versus $T_c$ is displayed in Fig. 9 for the set II without considering $\rho$–meson. The value of critical temperature $T_c$ reduces from nuclear matter $\alpha=0$ ($T_c=16.8$ MeV) to neutron matter $\alpha=1$ ($T_c=11.2$ MeV). It shows that the liquid gas phase transition is more probable in neutron matter than the pure symmetric nuclear matter. The similar behaviour has been reported in Ref. [23] within the effective nuclear model based on the mean-field approximation. That means the pressure generated from repulsive saturation force plays vital role to undergo early phase transition.

The effect of high temperature on the effective mass, which play a dominant role in the EOS (eq.8) is shown in Fig. 10. Also EOS at high temperature is useful to study the supernova simulation, such as the mechanism and whole phenomena of supernova
In the left panel of Fig. 10 (10(a)), we show the variation of $m^*$ with $n_B$ at $T = 25, 50$ and $100$ MeV in set II. The trend of the curves up to temperature $T = 100$ remains similar. It is clear from the figure that $m^*$ increases gradually with $T$. This effect is attributed to the pair formation due to anti-particle production. For example, the change of effective mass from zero to three times nuclear matter density, is around 25% of nucleon mass upto $T = 100$ MeV. In the right panel of Fig. 10 (10(b)), the pressure and energy are plotted for different temperatures, which are function of baryon density. From this figure, we observe that the EOS gets stiffer with increasing temperature. The reason of stiffness is that the extra thermal energy and pressure contribution comes from the anti-baryons.

![Figure 9: The $T_c$ as a function of asymmetric parameter $\alpha$ for the set II](image)

Figure 9: The $T_c$ as a function of asymmetric parameter $\alpha$ for the set II
Figure 10: The left panel shows effective mass versus baryon density at $T = 25, 50$ and 100 MeV and the pressure as a function of energy is displayed in the right panel for the same temperature.

4 Summary and conclusions

We presented a microscopic calculation of EOS in a relativistic framework based on the modified SU(2) chiral sigma model with different parameter sets. In this model, we adopted an approach in which the mass of the isoscalar vector field is generated dynamically. To ensure the empirical value of incompressibility at saturation, we added higher-order terms of scalar meson field. Thus the nucleon effective mass acquires a self-consistent density dependence on the scalar and vector meson fields. Based on this model, we studied the effect of incompressibility ($K = 210 - 380$ MeV) and effective mass ($m^*/m = 0.8 - 9$) on EOS near three times the nuclear matter density. We compared our results with the realistic EOS, prediction at low density region and also with the recently extracted EOS from the heavy-ion collisions[29] at high density. The EOS obtained by NCS models overall agreed well with the experimental data and realistic potential DBHF model. Among these NCS models, we found that the sets I and II are in agreement with the predicted EOS. In our discussions, we considered set II EOS for the analysis of warm asymmetric nuclear system. The reason is that it is compatible with recent heavy-ion collisions data[30]. The change of effective mass is discussed as a function of the baryon density and temperature. We found that it decreased with
density around two times nuclear matter density and the variation was slow thereafter.

The liquid gas phase transition is studied within NCS model (set I, II and III). We found that the critical temperatures are 14.2, 16.8 and 20.4 MeV. The corresponding $P_c$ and $n_c$ are 0.14, 0.22 and 0.36 MeV fm$^{-3}$ and 0.035, 0.044 and 0.051 fm$^{-3}$, respectively. These are in the range of recent experimental observation, $13.1 \pm 0.6$ [21]. Precisely, set I is close to this value. The binding energy per particle is also discussed with various temperature up to 25 MeV. We observed that the system becomes less bound with increasing temperature.

The EOS is also shown with variation of asymmetric parameter, $\alpha$ for a fixed temperature, with and without $\rho$—meson contribution. The critical point decreased due to increase of $\alpha$. With inclusion of $\rho$—meson along with $\alpha$, it is reduced further. This decrease in critical temperature in presence of $\rho$—meson is because of the strong repulsive force. This model worked well at low density region such as liquid gas phase transition. The success of this model could be revisited for the study of finite nuclei and at extreme densities and temperature. Works [43] are in progress to verify the validity of this model.

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