Evidence for strange stars from joint observation of harmonic absorption bands and of redshift.

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ABSTRACT

From recent reports on terrestrial heavy ion collision experiments it appears that one may not obtain information about the existence of asymptotic freedom (AF) and chiral symmetry restoration (CSR) for quarks of QCD at high density. This information may still be obtained from compact stars - if they are made up of strange quark matter (SQM). Very high gravitational redshift lines (GRL), seen from some compact stars, seem to suggest high ratios of mass and radius (M/R) for them. This is suggestive of strange stars (SS) and can in fact be fitted very well with SQM equation of state (EOS) deduced with built in AF and CSR. In some other stars broad absorption bands (BAB) appear at about ∼ 0.3 keV and multiples thereof, that may fit in very well with resonance with harmonic compressional breathing mode frequencies of these SS. Emission at these frequencies are also observed in six stars. If these two features of large GRL and BAB were observed together in a single star, it would strengthen the possibility for the existence of SS in nature and would vindicate the current dogma of AF and CSR that we believe in QCD. Recently, in 4U 1700−24, both features appear to be detected, which may well be interpreted as observation of SS - although the group that analyzed the data did not observe this possibility. We predict that if the shifted lines, that has been observed, are from neon with GRL shift \( z = 0.4 \) - then the compact object emitting it is a SS of mass 1.2 \( M_\odot \) and radius 7 km. In addition the fit to the spectrum leaves a residual with broad dips at 0.35 keV and multiples thereof, as in 1E1207−5209 which is again suggestive of SS.

Key words: X-rays: bursts, stars: strange matter- elementary particles

1 INTRODUCTION

QCD, the theory of strong interactions has some peculiar built-in features which should be tested. For example if particles are densely packed, they are believed to be free in the theory due to the property AF. Again the most important building blocks of the theory, namely (u,d,s) quarks, are believed to be nearly massless but acquires a substantial mass when the density is low. The masses are supposed to become small when the density is high (CSR). The third surprise in QCD is that quarks are completely confined in a hadron and one can only question whether in a large system like a compact star there may be partial deconfinement in the sense that quarks are not bound as in a hadron. The study of SS from theory and observations is thus a very interesting challenge - Itoh thought of quarks with parastatistics as constituents of stars as early as 1970, even before QCD. Then Witten (1984) discussed the existence of strange quark matter (SQM) and strange stars (SS) as a result of cosmic separation of phases.

The possible existence of SQM, with AF and CSR, is studied in astrophysics as there are (a) some evidence of compact objects that fit in with SS (Dey et al. 1998, Li et al. 1999a, Li et al. 1999b) and (b) some more evidence from two quasi-periodic peaks in the power spectrum of accreting low mass X-ray binaries (Li et al. 1999b and Mukhopadhyay et al. 2003). Many superbursts are now seen. They are like normal Type I thermonuclear bursts and seen from some compact stars which accrete from their binary partners - but last for hours instead of seconds and are highly energetic. One may need to invoke SS to explain them (Sinha et al. 2002, Sinha et al. 2005, Page & Cumming 2005). A limiting value of the magnetic field can be assigned to strange stars of ∼ 10^{8} G explaining the shortage of radio pulsars with fields less than this value (Mandal et al. 2006).

The situation is worse in heavy ion reactions for testing the features of QCD. The recent reports of the gold on gold collisions from four groups BRAHMS, PHENIX, PHOBOS and STAR (Arsene et al. 2004, Adler et al. 2005, Black et al. 2005, Adams et al. 2003) indicate a phase transition from ordinary matter. But this phase is not the quark gluon plasma with AF and CSR that the protagonists searched for:
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“However, there are a number of features, early on considered as defining the concept of the QGP, that do not appear to be realized in the current reactions, or at least have not (yet?) been identified in experiment. These are associated with the expectations that a QGP would be characterized by a vanishing interaction between quarks and exhibit the features of chiral symmetry restoration and, furthermore, that the system would exhibit a clear phase transition behaviour.” (BRAHMS, Arsene et. al 2004)

The strange star hypothesis may thus be a better laboratory to test the properties of QCD.

2 THE MODEL

’t Hooft (1974) suggested the use of inverse of the number of colours 1/NC as an expansion parameter since there are no free parameters in QCD. A baryonic system like stars can be explored in relativistic tree level calculation with a potential which may be obtained from the meson sector phenomenology in (Witten 1979) using 1/NC theory. We have done this with Richardson potential - modified to have different scales for AF and confinement - deconfinement mechanism (CDM) with parameters are fitted from baryon magnetic moments (Bagchi et al. 2005). So there are no free potential parameters left in SS calculations. In this model, a density dependent mass is chosen and from the two body potential a mean field EOS is derived for beta stable chargeless (u-d-s) matter. The beta stability and charge neutrality demands a self consistent mean field EOS is derived for beta stable chargeless (u-d-s) matter.

Our interaction potential is due to Richardson (1979) and is given by,

\[ V_{ij} = \frac{12\pi}{27} \left( \frac{1}{\ln(1 + \frac{q_i^2}{\Lambda^2})} - \frac{1}{q_i^2} \right) \]

(1)

\[ q_i \equiv k_i - k_j \text{ being the momentum transfer between } i^{th} \text{ and } j^{th} \text{ quarks.} \]

The scale parameter \( \Lambda \) is \( \simeq 400 \text{ MeV} \) for hadron phenomenology (Crater & van Albite 1984, Dey et. al 1986). But in SS calculation \( \Lambda \) is taken \( \sim 100 \text{ MeV} \) (Dey et. al 1998). This discrepancy ultimately leads us to modify the potential and the modified form of the potential is,

\[ V_{ij} = \frac{12\pi}{27} \left[ \frac{1}{q_i^2\ln(1 + \frac{q_i^2}{\Lambda^2})} - \frac{\Lambda^2}{q_i^2} + \frac{\Lambda'^2}{q_i^2} \right] \]

\( \Lambda \) is the scale parameter representing the AF as the first two terms are asymptotically zero for large \( q \). \( \Lambda' \) is the scale parameter corresponding to confinement as the third term reduces to a linear confinement for small \( q \). From Bagchi et al., (2004), we get \( \Lambda = 100 \text{ MeV} \) and \( \Lambda' = 350 \text{ MeV} \). The modified potential is used in SS calculation (Bagchi et. al, 2005b). The potential is screen due to gluon propagation in a medium and \( q_i^2 \) is replaced by \( [q_i^2 + D^{-2}] \). Inverse screening length \( D^{-1} \) (i.e. gluon mass \( m_g \)) is:

\[ (D^{-1})^2 = \frac{2\alpha_0}{\pi} \sum_{i=u,d,s} k_i^f \sqrt{(k_i^f)^2 + M_i^2} \]

(2)

Fermi momenta are related to number density:

\[ k_i^f = (n_i \pi^2)^{1/3} \]  

(3)

\[ k_e^f = (n_e 3\pi^2)^{1/3} \]  

(4)

The subscript \( e \) stands for the electron. Perturbative quark gluon coupling, \( \alpha_0 \) is 0.2 in the unmodified potential and 0.55-0.65 for the modified potential. The quark mass, \( M_i \) is taken to be density dependent to restore chiral symmetry at high density:

\[ M_i = m_i + M_Q \text{ sech} \left( \frac{n_B}{N_0} \right), \quad i = u, d, s. \]

(5)

where the constituent quark mass \( M_Q \) lies between 300-350 MeV for each quark, \( n_B = (n_u + n_d + n_s)/3 \) is the baryon number density, \( n_0 \) is the normal nuclear matter density and \( N \) is a parameter. The current quark masses, \( m_i \) are taken as 4, 7 and 150 MeV for u, d, s quarks respectively. From the charge neutrality and beta equilibrium conditions, \( k_i^f \) and \( \mu_i \) (chemical potentials) of quarks and electron are obtained as a function of \( n_B \) and performing a relativistic Hartree-Fock calculation, the energy density \( e \) of the SQM is obtained. The first law of thermodynamics gives the EOS of SQM at zero temperature as:

\[ P = \sum_i (\mu_i n_i - e_i) + (\mu_e n_e - e_e) \]

(6)

To obtain a realistic EOS, the model parameters \( \alpha_0, M_Q \) and \( N \) are chosen in such a way that the minimum value of energy per baryon \( (E/A \equiv e/n_B) \) for uds quark matter is less than that of the most stable element Fe\textsuperscript{56} i.e. 930 MeV. Thus uds quark matter can construct stable stars. The minimum value of \( E/A \) is obtained at the star surface where the pressure is zero. The presence of zero pressure indicates the existence of a sharp surface of the strange star in contrast to fuzzy surfaces of the neutron stars. The surface is sharp since strong interaction dictates the deconfinement point.

On the contrary, for the same parameters, the minimum value of \( E/A \) for ud quark matter is greater than that of Fe\textsuperscript{56} so that Fe\textsuperscript{56} remains the most stable element in the non-strange world. Using this EOS for strange quark matter we obtain the Mass-Radius relation of the strange star by solving the hydrostatic equilibrium equation (TOV equation) with appropriate boundary conditions.

Among different EOSs obtained from our SS model, varying the parameters, only one (EOS A) is used in the present work and the parameters for this EOS are listed in table I. The variation for other parameter sets are unimportant.

3 HARMONIC BANDS FOR ABSORPTION.

In our SS model, the energy per baryon has a minimum at the star surface and this surface can vibrate (Sinha et al., 2003 and Ray et al., 2004). A Taylor expansion of the energy per baryon about \( r = R \) can be written as

\[ E(r)/A = E(R)/A + \frac{1}{2} k(R) (r - R)^2 \]

(7)

which gives \( \left( \frac{dE}{dr} \right)_{r=R} = 0 \) and \( k(R) = \left( \frac{d^2E}{dr^2} \right)_{r=R} \). Here \( r \) is the radial coordinate, \( R \) is the radius of the star and \( E(R)/A \) is the energy per baryon at \( r \). The fundamental frequency of the vibration is given by:

\[ \omega = \sqrt{\frac{E(R)}{m_{\text{skin}}}} \]

(8)

and the energy of the vibrational mode as \( \hbar \omega \). This fundamental mode and its harmonics then should be observable in the spectrum of the star.

To calculate \( \omega \), pressure (p) is written as:
Table 1. EOS A using the modified potential. $\Lambda$ is 100 MeV. $\epsilon_c/c^2$ is the central density.

| EOS Label | $N'$ | $M_q$ | $N$ | $\alpha_q$ | $(E/A)_{min}^{uds}$ | $(E/A)_{min}^{ud}$ | $M_{max}$ | $R$ for $M_{max}$ | $\epsilon_c/c^2$ for $M_{max}$ |
|-----------|------|------|----|-----------|-----------------|-----------------|---------|-----------------|----------------|
| A         | 350  | 325  | 3.0| .55       | 874             | 942             | 1.53    | 7.41            | 42.98          |

\[ p = -\frac{dE}{dV} = -\frac{1}{4\pi r^2} \frac{dE}{dr} \]  

(9)

using the first law of thermodynamics for zero temperature system. Then $k(R)$ can be written as

\[ k(R) = -4\pi R^2 \left( \frac{dp}{dr} \right)_{r=R} \]  

(10)

\( \frac{dp}{dr} \) is obtained by solving the TOV equation as mentioned in the previous section. The mass of the skin $m_{skin}$ is calculated from the properties of electron cloud around the star (see Sinha et al. 2003 for details).

The existence of absorption bands at 0.35 keV with harmonics at 0.7, 1.05, 1.4 etc, in 1E 1207$-$5209 were interpreted in Sinha et al. (2003) as evidence of surface vibration from strange stars. This is an isolated star and presumably there is no accretion disk associated with this star.

It was then observed that six stars accreting stars show enhanced harmonic band emission at energies corresponding to the same vibrational modes (Ray et al. 2004). More evidences of such kind of surface vibration will help to establish strange star model. These stars are accreting and thus there is a disk above it. In SS, the electrons tend to flow out from the strongly held quarks since they do not feel the QCD forces. Only the net resulting balancing positive charge of the star holds the electrons close to the star - they form an envelope of about few hundred Fermi above the star as first shown by Alcock, Farhi and Olinto (1986).

The electron cloud outside and the positive central charge leads to a electrostatic field which may be as strong as $5 \times 10^{17} \text{V cm}^{-1}$ decreasing to $10^{11} \text{V cm}^{-1}$ as one goes out radially by $10^{-8} \text{ cm}$ (Xu, Zhang and Qiao 2001). The accretion disk on top of the electron layer gathered by the star from their binary partner lies typically within the range $10^{-11} M_\odot \text{ yr}^{-1} < \dot{M} < 10^{-8} M_\odot \text{ yr}^{-1}$ (Cumming and Bildsten, 2000).

There is a coupling of the surface vibration of the SS and the accretion disk that leads to the enhancement in select harmonic bands of the emission spectrum. We hope more data will become available in near future.

### 3.1 4U 1700+24

In Fig. we display the curve given in Tiengo et al. (2005) for the spectral analysis of 4U 1700+24 for which possibly a line with redshift $z \sim 0.4$ was observed (will be discussed in the next section). Note the residual at the bottom where there are the absorption bands visible up to 2 keV. We hope in future, when much better data will become available, one will be able to test our conjecture that these bands are also due to surface vibration as the emission bands in six stars and the absorption bands in 1E1207$-$5209 discussed above.

![Figure 1](image1.png)

Figure 1. The PN count rate spectrum of 4U 1700+24 and residuals in units of $\sigma$ with respect to the best fit model. The individual model components are also shown. The presence of absorption lines is clear from the residuals. This graph is taken from Tiengo et al. (2005).

Table 2. Coefficients of the Gaussian used to fit the data given by Mereghetti.

| $r$'s | 0.36 | 0.72 | 1.08 | 1.44 | 1.80 | 2.16 | 2.52 | 2.88 |
|------|------|------|------|------|------|------|------|------|
| $a$'s | 2.8  | 10.0 | 1.5  | 6.0  | 1.0  | 2.0  | 3.0  | 1.0  |

$\sigma$'s | 800 | 35 | 110 | 55 | 700 | 90 | 10 | 70 |

![Figure 2](image2.png)

Figure 2. The fit of the residual provided by Mereghetti with a Gaussian.
3.2 1E1207−5209 Revisited.

As already mentioned, the absorption bands at 0.35 keV with harmonics at 0.7, 1.05, 1.4 etc, in 1E1207−5209 were interpreted in Sinha et al. (2003) as evidence of surface vibration from strange stars.

Now, the residuals for 1E1207−5209, as supplied by S. Mereghetti, is plotted in Fig. 3 and are fitted with an empirical form for harmonic oscillations with an arbitrary envelope of the form

\[ \chi(E) = \sum_{j=1,8} a_j \exp[-(E - \sigma_j)^2] \]

where \(a_j\) and \(\sigma_j\) are the amplitudes and central frequencies with widths \(\sqrt{2}/\sigma_j\). \(a_j\) and \(\sigma_j\) are given in the table. It is clear that the fundamental and even harmonics are suppressed while the odd harmonics are dominant. This may be due to the preferential detection of the signal receiver, in which case the observed form will change when a different satellite is used. On the other hand if it is due to preferential absorption in the interstellar medium, the form could change even with the same detector.

To us the second alternative seems preferred at the moment, since at different times with change of phase, shown in different colours in Bignami et al. (2004). The intensity in these phases vary. But a clear understanding of the physical phenomena behind it is yet needed.

4  THE REDSHIFT FOR VARIOUS STAR MASS AND RADIUS.

Another check for the existence of SS is the mass-radius \((M/R)\) ratio which can be inferred if the gravitational redshift is measured for some spectral lines. The gravitational redshift is given by:

\[ z = \left(1 - \frac{2GM}{c^2R}\right)^{-1/2} - 1 \]  

(12)

From the spectral analysis of X-ray bursts from EXO 0748−676 Cottam, Paerels and Mendez (2002) found a redshift of \(z = 0.35\). Later, Tiengo et. al (2005) performed a spectral analysis of an XMM-Newton observation of the X-ray binary 4U 1700+24 during an outburst in August 2002. At low energies they detected a prominent soft excess, which they modeled with a broad Gaussian centered at \(\sim 0.5\) keV. In the high resolution RGS spectrum they detected an emission line, centered at 19.19\(^{\pm}0.05\) Å . The authors gave two possible interpretations for this line: O VIII at redshift \(z \sim 0.012\) or Ne IX at redshift \(z \sim 0.4\). But the authors argued O VIII to be a better candidate than Ne for the observed emission line. On the contrary, while modeling the spectrum of three LMXBs 4U 0614+091, 2S 0918−549, and 4U 1543−624 Juett et al. (2003) proposed that there is excess neon local to each of these sources as in the 4U 1626−67. So there is no reason to exclude the possibility of 19.2 Å line from 4U 1700+24 to be a Ne one giving \(z \sim 0.4\) and the star to be a strange star. In Fig. 4, we plotted two lines one for \(z = 0.35\) and the other for \(z = 0.40\) in the Mass(M)-Radius(R) parameter space which cuts the M-R relation obtained from our EOS giving realistic values of mass and radius. For \(z = 0.35\) we find a star of mass slightly larger than 1.1 \(M_\odot\) (fulfilling the condition M \(\geq 1.1\) for EXO 0748−676 as demanded by Cottam, Paerels and Mendez (2002)) and radius 7.0 km. Again, for 0.4 we get a star with mass 1.2 \(M_\odot\) and radius 7.5 km. So the possibility of EXO 0748−676 and 4U 1700+24 to be strange stars are quite open.

On the other hand, Sanwal et. al (2002) discussed that the absorption lines from 1E1207−5209 have \(z = 0.12 − 0.23\) which corresponds to \(M \sim 0.36 − 0.76 \ M_\odot\) and \(R \sim 5.19 − 6.56 \ km\). These values may appear to be unrealistic, but the fact is that the authors explained the absorption lines as a result of atomic transitions while in SS model the origin of absorption lines are completely different (as discussed in the previous section).

Therefore, contradicting current quotes in the literature (Tiengo et al. 2005, Cottam et al. 2002 and Miller 2002), our SS model can explain redshifts observed from some compact stars.

In the next figure we plot with mass and radius of the strange star and see that for \(z = 0.6\) one can get a star of mass 1.55 \(M_\odot\) and the radius is somewhat smaller - about 7.4 km. We hope that a look at these curves will dispel the mistaken notion (Cottam et al. 2002 & Miller, 2002) that strange stars cannot fit EXO 748−676.
5 DISCUSSION

We now discuss the possibility of anti strange stars discussed by Boehm et al. (2003) and Oaknin & Zhitnitsky (2005). If such stars exist then they will have an atmosphere of positrons rather than electrons spilling outside the star radius by a few hundred $\text{fm}$. These positrons will then interact with electrons of the normal matter accreting onto the strange star from its binary partner and positron-electron annihilation will produce the $511 \text{ keV} \gamma$-pair. The only snag in this scenario was that for these anti-quark stars to be stable the surface tension has to be large (Alcock & Olinto, 1989). Recently we have shown that in our ReSS model the surface tension is large unlike the bag model (Bagchi et al. 2005b) and such anti-strange stars are indeed likely to be stable. This partially solves the problem of baryonic dark matter and baryon asymmetry of the universe.

One must affirm that it is quite possible that there are two types of compact stars, namely neutron stars (NS) and strange stars. Indeed, a clear possibility of existence of these two kinds of stars is discussed very recently by Bombaci, Parenti and Vidana (2004) and these authors showed a phase transition from NS to SS can account for delayed gamma ray bursts.

6 SUMMARY AND CONCLUSION

We have shown that the observed gravitational redshift $z$ can be explained by our SS model with built in properties of QCD. Also harmonic bands in the spectrum may convincingly prove the existence of strange stars.

We hope that with the profuse data that is flowing in, one will find more stars like 4U 1700+24 so that the strange star hypothesis can be put through a stringent test. Further data on residuals for 1E1207–5209 will also be very interesting.

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