Pairing and chiral spin density wave instabilities on the honeycomb lattice: a comparative quantum Monte Carlo study

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Using finite-temperature determinantal quantum Monte Carlo calculations, we re-examine the pairing susceptibilities in the Hubbard model on the honeycomb lattice, focusing on doping levels onto and away from the van Hove singularity (VHS) filling. For this purpose, electronic densities of 0.75 (at the hole-doping VHS) and 0.4 (well below the VHS) are considered in detail, where due to a severe sign problem at strong coupling strengths, we focus on the weak interaction region of the Hubbard model Hamiltonian. From analyzing the temperature dependence of pairing susceptibilities in various symmetry channels, we find the singlet $d+id$-wave to be the dominant pairing channel both at and away from the VHS filling. We furthermore investigate the electronic susceptibility to a specific chiral spin density wave (SDW) order, which we find to be similarly relevant at the VHS, while it extenuates upon doping away from the VHS filling.

I. INTRODUCTION

In recent years, graphene\textsuperscript{1–3} has attracted a lot of attentions, due to its unusual electronic properties. At charge neutrality, corresponding to a half-filled lattice in the Hubbard model description of graphene’s π-electron system, a vanishing density of states at the Fermi level (the Dirac points) renders a semi-metallic state stable against instabilities from electron-electron interactions, even in the intermediate coupling regime\textsuperscript{4–7}. In contrast, upon doping well away from the Dirac points through chemical doping\textsuperscript{8} or electrical gating\textsuperscript{9}, correlation effects are expected to no longer be limited to the strong interaction regime. Indeed, various possible phases, such as superconducting instabilities, magnetism or charge/spin density waves have been considered to emerge in doped graphene: Several theoretical studies focused on superconducting states of correlated electrons on the honeycomb lattice of graphene, mainly within a local Hubbard model description\textsuperscript{10}. Based on mean field theory, Black-Schaffer \textit{et al.}\textsuperscript{11}, suggest that graphene may become a $d+id$-wave superconductor over a wide range of doping, while Uchoa \textit{et al.}\textsuperscript{12} suggest extended $s$-wave and $p+ip$-wave pairing states. Functional renormalization group (fRG) theory calculations proposed $f$-wave and $d+id$-wave instabilities\textsuperscript{13}, and variational Monte Carlo\textsuperscript{14,15} and auxiliary-field quantum Monte Carlo study\textsuperscript{16} both support $d+id$-wave pairing, while a variational cluster approximation and a cellular dynamical mean-field theory study\textsuperscript{17} suggests a $p+ip$ pairing symmetry. In general, this problem is thus far from having reached a conclusion. An even more peculiar condition is obtained upon doping the electronic system onto the van Hove singularity (VHS), where the non-interacting extended Fermi surface exhibits perfect nesting. As a consequence, the pairing mechanism may be different from the one at more generic doping levels\textsuperscript{18,19}, and furthermore the electronic system might even host other types of orders, such as a Pomeranchuk instability\textsuperscript{20} or a chiral spin density wave (SDW) order\textsuperscript{21}. Different scenarios have indeed been proposed: A renormalization group study finds $d+id$ pairing at the VHS filling in the weak coupling limit\textsuperscript{22}. Using an fRG approach, Wang \textit{et al.} obtained a chiral SDW in the intermediate interaction region at the VHS filling, while $d+id$ pairing was obtained away from the VHS\textsuperscript{23}. Another fRG study reports possible $d+id$ or SDW instabilities in the intermediate interaction region at the VHS, and $d+id$ or $f$-wave pairing away from the VHS\textsuperscript{24}. More recently a dynamic cluster approximation study suggests that the $d+id$-wave pairing state dominates in the weak-coupling regime, while for stronger interactions, a $p+ip$-wave state strongly competes with the $d+id$-wave state\textsuperscript{25}. However, in this study, SDW instabilities have not been considered. This states of affairs motivates us to examine this problem using finite-temperature determinantal quantum Monte Carlo (FT-DQMC), an essentially unbiased numerical algorithm. The rest of this paper is organized as follows: In Sec. II, we introduce the model that we consider and outline the FT-DQMC approach. Then we analyze in Sec. III various pairing channels of superconducting instabilities, while in Sec. IV, we consider the chiral SDW instability and contrast its behavior to other magnetic ordering channels. Finally, we summarize our results in Sec. V.

II. MODEL AND METHOD

In this paper, we examine the effective pairing susceptibility for various different pairing channels, and identify the dominant pairing channel for doping levels onto and away from the VHS. Moreover, we also consider the chiral SDW instability that was proposed by Li\textsuperscript{21}, and examine, to what extent this chiral SDW instability ef-
fects the behavior at the VHS filling and upon doping away from the VHS point. For this analysis, we consider the Hubbard model on the honeycomb lattice to describe the doped graphene system. This model is given in terms of the Hamiltonian

\[ H = -t \sum_{\langle i,j \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow}, \]

\[ -\mu \sum_i (n_{i\uparrow} + n_{i\downarrow}), \]

where \( t \) is the fermion hopping amplitude between nearest neighbor sites on the honeycomb lattice (here, \( i \) and \( j \) denote lattice vectors), \( U \) denotes an onsite repulsion, and \( \mu \) the chemical potential that allows to tune the electron density, denoted \( \rho \) in the following. We work in units of \( t = 1 \) in the following.

The numerical algorithm used in this paper is the finite-temperature determinantal quantum Monte Carlo (FT-DQMC) method\textsuperscript{26,27}. We consider finite rhombic clusters of the bipartite honeycomb lattice with periodic boundary conditions and with \( N_s = L \times L \times 2 \) lattice sites, mainly for \( L = 6 \) and \( L = 12 \) in order to ensure that both the \( K \) (Dirac) and the \( M \) points of the hexagonal Brillouin zone are included in the discrete lattice momentum space. The simulations were performed at finite temperatures, and we then analyzed the observed tendencies upon lowering the temperature. In particular, upon doping beyond half-filling, FT-DQMC suffers from a severe sign problem, which worsens upon lowering the temperature and increasing the interaction strength. Depending on the doping level, we are limited to the weak to intermediate interaction regime, in order to still access low temperatures that allow us to identify the onset of divergences in the pairing or magnetic susceptibilities. In the following, we mainly focus on the doping level at the VHS, where the electron density is \( \rho = 0.75 \) or \( \rho = 1.25 \). Due to particle-hole symmetry, we considered in our simulations the case of \( \rho = 0.75 \). For comparison, we also performed further simulations at \( \rho = 0.4 \), i.e., a doping level far below the VHS, where the sign problem is less severe\textsuperscript{28}, and where we can extend a bit further towards the stronger interaction regime.

### III. PAIRING CORRELATIONS

In order to probe for superconducting instabilities, we examine the system’s susceptibility towards various previously proposed pairing channels for this model. In particular, we consider the nearest-neighbor (NN) extended \( s \)-wave, \( d + id \)-wave and \( p + ip \)-wave pairing correlations, and consider also next-nearest-neighbor (NNN) \( d + id \)-wave, \( p + ip \)-wave and \( f \)-wave pairings. In real space, these different pairing channels are given in terms of appropriate form factors,

\[ f_{\text{NN},c\alpha}(\bm{\delta}_l) = 1, \]

where the vectors \( \delta_l, l = 1, 2, 3, 4, 5 \) denote the NN (NNN) lattice directions from a given lattice site, and \( \epsilon_s = 0 \) (1) for sites on the A (B) sublattice. Figure 1 shows these various form factors explicitly.

![FIG. 1: (Color online) Phases of the considered pairing channels along the corresponding directions on the honeycomb lattice: (a) NN extended \( s \)-wave, (b) NN \( d + id \)-wave, (c) NN \( p + ip \)-wave, (d) NNN \( d + id \)-wave, (e) NNN \( p + ip \)-wave and (f) NNN \( f \)-wave.](image)

where \( \Delta_{\alpha \delta_l} = \frac{1}{\sqrt{N_s}} \sum_\alpha f_\alpha(\delta_l^{(2)})(c_{i\uparrow}^\dagger c_{i+\delta_l^{(1)}\downarrow} \pm c_{i\downarrow}^\dagger c_{i+\delta_l^{(1)}\uparrow}), \]

where \( + (-) \) for triplet (singlet) pairing, and \( N_\alpha \) are the corresponding normalization factors, with \( N_\alpha = 3 \) \((N_\alpha = 6)\) for the NN (NNN) channels.

Within the QMC simulations, we can directly access the temperature dependence of the pairing susceptibilities for the various channels,

\[ P_\alpha = \frac{1}{N_s} \sum_{i,j} \int_0^\beta d\tau \langle \Delta_{\alpha}^\dagger(\tau) \Delta_{\alpha}(0) \rangle, \]

where \( \Delta_{\alpha}^\dagger(\tau) = e^{\tau H} \Delta_{\alpha}^\dagger(0) e^{-\tau H} \). These pairing susceptibilities are however strongly affected by the enhanced response of the free system at \( U = 0 \). This behavior is illustrated in Fig. 2, which shows the different susceptibilities \( P_\alpha \) as functions of \( T \) on the \( L = 6 \) lattice, for both \( \rho = 0.4 \) and \( \rho = 0.75 \) in the noninteracting limit \( U = 0 \). While there is no superconducting ground states in the noninteracting case, the apparent divergence of the \( P_\alpha \) upon lowering \( T \) provides a background to the susceptibility measurements in the interacting case, in particular...
in the low-coupling regime that we can access in the FT-DQMC simulations. We thus require to examine the various pairing channels based on the effective pairing interaction vertex$^{29}$. In order to extract the corresponding effective pairing susceptibilities, we compute in FT-DQMC also the bare pairing contributions $\tilde{P}_\alpha$, for which two-particle terms $\langle c_{k\uparrow}(\tau) c_{\bar{k}\downarrow}(0) c_{\bar{k}\uparrow}(\tau) c_{k\downarrow}(0) \rangle$ that appear in evaluating the $P_\alpha$ in Eq. 4 are replaced by the decoupled contributions $\langle c_{k\uparrow}(\tau) c_{\bar{k}\downarrow}(0) \rangle \langle c_{\bar{k}\uparrow}(\tau) c_{k\downarrow}(0) \rangle$. The effective pairing susceptibilities are then given as $P_\alpha^{\text{eff}} = P_\alpha - \tilde{P}_\alpha$, and where a positive (negative) value of $P_\alpha^{\text{eff}}$ signals an enhanced (suppressed) tendency towards pairing in the corresponding channel. By definition, for the noninteracting case, the $P_\alpha^{\text{eff}}$ vanishes.

We now turn to examine the interacting system, and begin with the case of an electron density of $\rho = 0.4$, i.e., well below the VHS filling. First, we consider the results obtained for the $L = 6$ lattice with 72 sites. At this density the sign problem is sufficiently moderate, and we can obtain the $P_\alpha^{\text{eff}}$ up to $U/t = 4$, as shown in Fig. 3 (a) to (d) for $U/t = 1$ to $U/t = 4$, respectively. These results for the $L = 6$ lattice exhibit that consistently both the NN and NNN $d + id$-wave pairing susceptibilities are enhanced upon lowering $T$, for all the considered interaction strengths (We also measured the extended $s$-wave channel susceptibility, but it is rather strongly suppressed in all the interacting cases that we considered and we thus do not include it in Fig. 3 or any of the figures below).

To assess the stability of this result with respect to finite size effects, we also performed simulations on the $L = 12$ system with 288 sites, i.e., four times larger than the $L = 6$ lattice. Since in Fig. 3 we find the prevailing pairing channel does not depend on the interaction strengths at $\rho = 0.4$, we concentrate in Fig. 4 to the case of $U/t = 2$ for the $L = 12$ lattice. For $\rho = 0.4$, the results on the $L = 12$ lattice are in accord with the findings on the $L = 6$ lattice, and we conclude that $d + id$-wave pairing forms the dominant pairing channel in this doping regime. This is in good accord with various previous findings, as mentioned in the introduction.

We next perform a similar investigation for the VHS filling, $\rho = 0.75$. Due to the sign problem, we are in this case limited to weaker interactions, and consider explicitly here the cases of $U/t = 1$ and $U/t = 2$. In contrast to the case of $\rho = 0.4$, we observe strong finite-size effects at the VHS filling, even regarding the leading low-temperature effective pairing susceptibility: As shown in Fig. 5(a), for a weak coupling of $U/t = 1$ on the $L = 6$ lat-
momenta within the Brillouin zone, thus better resolving also the effective interactions near the M-point, which – based on the divergence in the density of states (DOS) – is most important at the VHS filling. Another reason for this strong size dependence may be that due to the enhanced DOS at the VHS filling, other electronic instabilities compete with superconductivity. Indeed, based on a recent mean-field theory and RG calculations, a particular interesting chiral SDW state was argued to form the leading magnetic instability of the Hubbard model at the VHS filling. In the following section, we examine this scenario based on FT-DQMC simulations.

IV. MAGNETIC CORRELATIONS

The chiral SDW state considered in Refs. 21,23 is characterized by the three independent nesting vectors \( \mathbf{Q}_i \), \( i = 1, 2, 3 \) of the free-system’s Fermi surface at the VHS filling, which (folded back to the first Brillouin zone) correspond to the three independent \( M \) points at the centers of the Brillouin zone edges. In terms of the reciprocal lattice vectors \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \), these are \( \mathbf{Q}_1 = \frac{1}{2} \mathbf{b}_1 \), \( \mathbf{Q}_2 = \frac{1}{2} \mathbf{b}_2 \), \( \mathbf{Q}_3 = \frac{1}{2} (\mathbf{b}_1 + \mathbf{b}_2) \). For lattice sites on the \( A \) and \( B \) sublattices within a unit cell centered at position \( \mathbf{R} \), the mean-field expectation values of the local spin operator in the chiral SDW state are proportional (up to a global rotation in spin space) to the local direction vectors

\[
\langle S_{R,i} \rangle_{\text{SDW}} = \frac{1}{\sqrt{3}} (\hat{x} e^{\text{i} Q_1 \cdot \mathbf{R}} + \hat{y} e^{\text{i} Q_2 \cdot \mathbf{R}} + \hat{z} e^{\text{i} Q_3 \cdot \mathbf{R}}),
\]

\[
\langle S_{R,i} \rangle_{\text{SDW}} = \frac{1}{\sqrt{3}} (\hat{x} e^{\text{i} Q_1 \cdot \mathbf{R}} - \hat{y} e^{\text{i} Q_2 \cdot \mathbf{R}} - \hat{z} e^{\text{i} Q_3 \cdot \mathbf{R}}),
\]

where the \( \hat{x} \), \( \hat{y} \) and \( \hat{z} \) are the three mutually orthogonal unit vectors in spin space. This state exhibits four different spin directions \( \hat{x} + \hat{y} + \hat{z} \), \( -\hat{x} - \hat{y} + \hat{z} \), \( \hat{x} - \hat{y} - \hat{z} \), and \( -\hat{x} + \hat{y} - \hat{z} \), the magnetic unit cell thus contains eight lattice sites, and we require \( L \) to be even in order to accommodate this spin structure within the finite rhombic clusters. In order to probe for this chiral SDW within the FT-DQMC simulations, we monitor a corresponding structure factor

\[
S_{\text{SDW}} = \frac{1}{N_s} (\sum_{\mathbf{R}} M_{\mathbf{R,A}}) (\sum_{\mathbf{R}} M_{\mathbf{R,B}})
\]

in terms of the projections \( M_{\mathbf{R}} = M_{\mathbf{R,A}} + M_{\mathbf{R,B}} \), with \( M_{\mathbf{R,A}}(B) = (S_{\mathbf{R,A}}(B))_{\text{SDW}} \) of the local spin operators onto the chiral SDW texture. Here, \( S_{\mathbf{R,A}}(B) \) denote the local spin operator on the \( A \) (\( B \)) sublattice unit within the cell at position \( \mathbf{R} \); for a lattice site at position \( i \) this is given as \( S_{i} = \frac{1}{2} \sum_{\alpha, \beta} \sigma_{\alpha, \beta} g_{i, \alpha} g_{i, \beta} \) in terms of fermionic operators and the vector \( \mathbf{g} \) of Pauli matrices. In the following, we consider for comparison also the corresponding antiferromagnetic structure factor \( S_{\text{AF}} \) for the antiferromagnetic Néel state, which is defined similarly to \( S_{\text{SDW}} \), but with (up to a global spin rotation) \( (S_{\mathbf{R,A}})_{\text{AF}} = \hat{z} \), and \( (S_{\mathbf{R,B}})_{\text{AF}} = -\hat{z} \), respectively.

FIG. 5: (Color online) Temperature dependence of the effective pairing susceptibilities at the VHS on the \( L = 6 \) lattice for (a) \( U/t = 1 \) and (b) \( U/t = 2 \).

FIG. 6: (Color online) Temperature dependence of the effective pairing susceptibilities at the VHS on the \( L = 12 \) lattice for (a) \( U/t = 1 \) and (b) \( U/t = 2 \).

Given lattice, upon lowering the temperature, the effective pairing susceptibility in the NNN \( f \)-wave channel gets strongly enhanced, while all other channels get suppressed, which suggests \( f \)-wave pairing to dominate at the VHS in the weak coupling region. If the interaction strength is increased to \( U/t = 2 \) in Fig. 5(b), the dominant pairing on the \( L = 6 \) system still appears in the \( f \)-wave channel, however, the error bars are larger, due to a more severe sign problem. Considering however the larger \( L = 12 \) system at the VHS filling, as shown in Fig. 6, we find that the dominant pairing channel switches from the \( f \)-wave observed on the \( L = 6 \) system to the NN and NNN \( d + id \)-wave pairings when the lattice size is increased. The reason for this behavior may be the fact, that on the larger lattice size, we resolve a more narrow grid of
antiferromagnetic Néel state is well known to emerge in the half-filled system for sufficiently strong interactions. However, here we first focus on the behavior of the chiral SDW structure factor $S_{cSDW}$, considering the two specific electronic densities $\rho = 0.75$ and $\rho = 0.4$ as above.

In Fig. 7, we show the FT-DQMC results for $S_{cSDW}$ as functions of $T$ for the two densities $\rho = 0.75$ and $\rho = 0.4$ at both $U/t = 1$ and $U/t = 2$ on the $L = 6$ lattice. For $U/t = 2$ we also performed simulations on the $L = 12$ lattice in order to examine finite-size effect in $S_{cSDW}$. We find that upon lowering the temperature, $S_{cSDW}$ increases at $\rho = 0.75$, whereas it does not significantly increase, but is even weakly suppressed at $\rho = 0.4$. Since the corresponding magnetic instabilities can occur only at $T/t = 0$ (due to the SU(2) symmetry of the Hamiltonian $H$), this results suggest that the chiral SDW order, while possibly stable at $\rho = 0.75$, is not favored at $\rho = 0.4$. A similar picture also emerges from analyzing the corresponding chiral SDW susceptibility

$$\chi_{cSDW} = \frac{1}{N_s} \int_0^\beta d\tau \langle (\sum_R M_R(\tau))^\dagger (\sum_R M_R(0)) \rangle,$$

(7)

where $M_R^\dagger(\tau) = e^{\tau H} M_R(0) e^{-\tau H}$. Here, we need to again account for the enhanced response of the free system at $U = 0$. This is shown in Fig. 8: for both densities, $\chi_{cSDW}$ at $U = 0$ exhibits an apparent divergence upon lowering the temperature. Similarly to the case of the pairing susceptibilities, we thus examine the corresponding effective chiral SDW susceptibility, which is obtained as $\chi_{cSDW}^{eff} = \chi_{cSDW} - \chi_{cSDW}^{bare}$, where $\chi_{cSDW}^{bare}$ denotes the bare chiral SDW susceptibility. This procedure is similar to the antiferromagnetic case considered in Ref. 29.

As shown in Fig. 9 for $U/t = 1$ and $U/t = 2$, the effective susceptibility $\chi_{cSDW}^{eff}$ at $\rho = 0.75$ strongly increases in the low-$T$ region for both system sizes, while $\chi_{cSDW}^{eff}$ at $\rho = 0.4$ does not show a similarly strong enhancement, whereas for $U/t = 2$ it is even weakly suppressed at low $T$ for the larger system. Unfortunately, the sign-problem does not allow us to perform simulations on larger system sizes in order to perform a throughout finite-size scaling analysis of, e.g., $S_{cSDW}$ at low temperatures, which would be required in order to assess, if a chiral SDW ground state exists in the thermodynamic limit. Note that this case is different from the case of pairing instabilities, which may in principle set in at a finite (but still low) temperature regime. Nevertheless, our findings provide indication that at the VHS filling the system may exhibit an instability to the chiral SDW order, whereas away from the VHS filling, this instability is eventually suppressed.

To investigate further how the chiral SDW order be-
has at and beyond the VHS filling, we next fix an accessible, low temperature $T/t = 1/12$ and monitor how $S_{\text{cSDW}}$ and $\chi_{\text{eff}}^\alpha$ vary with the electronic density $\rho$. For this purpose, Fig. 10 shows $S_{\text{cSDW}}$ as a function of $\rho$ for different values of $U/t$. These results indicate that upon increasing $U/t$, a peak in $S_{\text{cSDW}}$ gradually builds up near the VHS filling, such that the chiral SDW is indeed most pronounced at the VHS filling. This observation complies to the fact that the three characteristic momentum vectors $Q_i$, $i = 1, 2, 3$ of the chiral SDW state form the nesting vectors of the Fermi surface at the noninteracting system at the VHS filling. For comparison, we also show in this figure the antiferromagnetic structure factor $S_{\text{AF}}$, which in contrast to $S_{\text{cSDW}}$ displays a monotonic increase with increasing electron density. At half-filling, $\rho = 1$, the Hubbard model on the honeycomb lattice is well known to harbor a quantum phase transition to an insulating antiferromagnetic phase for $U/t > 3.76^{30}$. While in Fig. 10, we remain below this critical value of $U$, the antiferromagnetic correlations already display a clear tendency to grow with increasing $U$. Furthermore, at $U/t = 2$, the antiferromagnetic structure factor exceeds the chiral SDW structure factor at (and close to) half-filling, while upon doping further below half-filling, towards the VHS filling, the chiral SDW correlations become more dominant.

We observe a similar enhancement in the chiral SDW response near the VHS filling also for the effective susceptibility $\chi_{\text{cSDW}}^\alpha$, cf. Fig. 11, strengthening the above interpretation of the structure factor data. Note that in Fig. 11, the $L = 6$ data exhibits two kinks around $\rho \approx 0.5$ and $\rho \approx 0.95$. These appear to be due to finite-size effects – compare to the data for the $L = 12$ lattice, where both kinks are absent. Such peculiar finite-size effects can in fact also be observed in a plot of the electronic density as a function of the chemical potential $\mu$ in Fig. 12: on the $L = 6$ lattice, the density as a function of $\mu$ shows two plateaus near $\rho = 0.5$ and $\rho = 0.95$, whereas on the larger lattice, those plateaus have disappeared. We consider these finite-size plateaus to be the reason also for the two kinks seen in Fig. 11 for the $L = 6$ lattice. For the $L = 12$ lattice the density plateaus are absent, and $\chi_{\text{cSDW}}^\alpha$ decreases steadily upon doping away from the VHS filling, again suggesting that the chiral SDW instability is important when the filling is at (and maybe also close to) the VHS value. For comparison, the effective antiferromagnetic susceptibility $\chi_{\text{AF}}^\alpha$ is also shown in Fig. 11 (where $\chi_{\text{AF}}^\alpha$ is defined similarly as the effective susceptibility for the chiral SDW case). While on the $L = 6$
system, this quantity shows similar finite-size anomalies as the effective chiral SDW susceptibility \( \chi_{\text{eff}}^{\text{SDW}} \), on the \( L = 12 \) system it instead shows a monotonic decrease when doping away from half-filling, as anticipated from the behavior of the antiferromagnetic structure factor.

V. SUMMARY

To conclude, we used finite-temperature determinantal quantum Monte Carlo simulations to examine the electronic pairing channels and magnetic instabilities of doped graphene within the Hubbard model description. Due to the sign problem, we restricted to the weak coupling regime at the VHS filling, while at lower fillings beyond the VHS, we also accessed the weak to intermediate coupling regime. In both cases, we find NN and NNN \( d + id \)-wave pairing as the dominant pairing channels on the larger system sizes. However, at the VHS filling, we observed strong finite-size effects in the dominant pairing symmetry. This may be taken as indication, that at this filling, due to the logarithmically diverging density of state and a nested Fermi surface also other electronic instabilities may be relevant. In fact, we observe from measuring appropriate structure factors and magnetic susceptibilities that a previously proposed chiral spin density wave state shows a robust enhancement near the VHS filling, but weakens quickly upon doping away from the VHS point. This is in accord with the result in Ref. 31, which suggests on the mean-field level that upon doping away from a DOS peak, instabilities within the particle-particle channel (superconducting orders) survive decisively further than those in the particle-hole channel (magnetic or charge orders). For the future, it will be interesting to extend also dynamical cluster approximation studies to consider the competition among the superconducting and magnetic instabilities of the doped honeycomb lattice Hubbard model.

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