Observation of particle acceleration in laboratory magnetosphere

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The self-organization of magnetospheric plasma is brought about by inward diffusion of magnetized particles. Not only creating a density gradient toward the center of a dipole magnetic field, the inward diffusion also accelerates particles and provides a planetary radiation belt with high energy particles. Here, we report the first experimental observation of a ‘laboratory radiation belt’ created in the Ring Trap 1 (RT-1) device. By spectroscopic measurement, we found an appreciable anisotropy in the ion temperature, proving the betatron acceleration mechanism which heats particles in the perpendicular direction with respect to the magnetic field when particles move inward. The energy balance model including the heating mechanism explains the observed ion temperature profile.

I. INTRODUCTION

Magnetospheres are natural plasma confinement devices ubiquitous in the universe. Yet, their creation mechanism remains to be fully understood. There must be a spontaneous mechanism that transports particles towards the center of a dipole magnetic field [1, 2]. However, such inward diffusion seemingly violates the entropy principle, because it creates a gradient instead of flattening the density distribution. This challenge may be overcome by considering adiabatic invariants: these are the magnetic moment, the bounce action and the longitudinal angular momentum. Typically, the frequencies of the corresponding periodic motions are separated by three orders of magnitude. The third invariant is most fragile, and its violation (i.e. the change of angular momentum) gives rise to the radial transport of particles. When the diffusion of magnetized particles is constrained by the remaining adiabatic invariants, the particle density tends to distribute uniformly on ‘magnetic coordinates’ rather than on Euclidean coordinates. Therefore, the homogeneous density on the former turns out to be inhomogeneous on the latter [3–5].

The same mechanism plays another interesting role, i.e., the acceleration of particles to produce a radiation belt [6–10]. Conserving the remaining adiabatic invariants, the kinetic energy of a particle increases as it moves inwards. An increase in cyclotron frequency (maintaining the magnetic moment constant) results in betatron acceleration of the velocity in the direction perpendicular to the magnetic field, and an increase in bounce frequency (maintaining the bounce action constant) results in Fermi acceleration of the parallel velocity [2] (the electrons are further accelerated to an ultra-relativistic regime by whistler waves [11, 12]). Since the former is stronger than the latter, the inward diffusion results in temperature anisotropy [2]. Numerous satellite observations have provided evidence of anisotropic temperatures in planetary radiation belts. The anisotropic electron temperatures are consistent with the estimates from inward diffusion heating [13–15]; ions also have anisotropic temperatures in the radiation belt of the Earth [16, 17] as well as that of Saturn [18].

In addition to these theoretical and observational studies, the inward diffusion was experimentally verified recently. On the Ring Trap 1 (RT-1) device which is a ‘laboratory magnetosphere’ simulated by a levitated superconducting magnet [19, 20], the peaked electron density profiles were observed and proved to originate from inward diffusion [21]. Similar density profiles were observed in the Levitated Dipole Experiment (LDX), which is another dipole confinement system [22]. On RT-1, distinct proof of inward diffusion was provided by producing a non-neutral (pure electron) plasma [23]; by action-conserving acceleration, electrons diffuse into the central region where the electric potential is higher than their initial energies (such particles absorb energy from fluctuations, which drive radial motion). Here, we report the first experimental observation of the other outcome of the inward diffusion, i.e. the particle acceleration. Specifically, we observed the ion temperature anisotropy and identified its generation mechanism as betatron acceleration. This finding means that we succeeded to demonstrate ‘laboratory radiation belt’.

II. EXPERIMENTAL SET-UP

The experiment was performed on RT-1 device. Figure 1 shows the layout of the device, together with associated plasma images (soft X-ray image and an electron density distribution reconstructed from interferometry). Plasma is produced by electron cyclotron resonance heating (ECH) with an 8.2 GHz microwave (maximum power of 50 kW) and a discharge duration of approximately 1 s. The plasma contains high-temperature (10 ∼ 50 keV) electrons as well as low temperature (≲ 100 eV) electrons; typically, the former occupies about half of the total population [24]. Electrons are confined in a radially elongated and vertically thin region, resembling planetary radiation belts [21, 25]; see Fig. 1(a). From quasi-neutrality, ion density distribution is also considered to be peaked on the equatorial plane (z = 0). In the inward
diffusion process, a concomitant heating mechanism must be in effect to produce high-energy electrons. However, electrons are also heated by ECH (cyclotron resonance occurs just in the vicinity of the levitated ring magnet). It is, therefore, difficult to separate the betatron and Fermi accelerations in the total energy balance of electrons. As a viable alternative, ions may thus be used as an appropriate target for analyzing potential heating mechanisms.

Spatial distribution of ion temperature was measured by two sets of Doppler spectroscopies; one scanned the plasma on a horizontal plane, and the other on a vertical plane (see Fig. 1). The lines of sight of the former ranged from \( r = 375 \text{ mm} \) to \( 878 \text{ mm} \), and those of the latter ranged from \( r = 375 \text{ mm} \) to \( 808 \text{ mm} \) on the \( z = 0 \) axis (\( r \) and \( z \) are the radial and vertical coordinates, respectively, \( r = 375 \text{ mm} \) is the surface of the superconducting magnet, \( r = 1000 \text{ mm} \) is the vacuum chamber wall, and \( z = 0 \) is the mid plane of the magnetic dipole). He II transition (468.565 nm) was used to measure the Doppler broadening. The horizontal chords measured the perpendicular temperature \( (T_\perp) \) with respect to the ambient magnetic field, while the vertical chords measured the mixture of \( T_\perp \) and the parallel temperature \( (T_\parallel) \). The local temperature profiles of \( T_\perp \) and \( T_\parallel \) were reconstructed from the line integrated spectrum data. (see Appendix for the detailed algorithm).

FIG. 1: (a) The plan (vertical front view) of the device. The superconducting magnet is levitated by the lifting magnet placed on the top of the vacuum chamber. The lines of sight of the vertical spectroscopic measurement cover the greyed vertical plane. On the left-hand side of the figure, the electron density profile (measured by interferometers [21, 25]) is plotted and the superimposed soft X-ray image (the interferometry measurement and the soft X-ray image are from different shots) is shown. (b) The plan (top view) of the device. The lines of sight of the horizontal spectroscopic measurement cover the greyed horizontal plane. The reflectometer (3.9 GHz microwave) measures the density fluctuation near the cutoff density which is \( \sim 1.9 \times 10^{17} \text{ m}^{-3} \) (light blue).

III. OBSERVATION OF ION TEMPERATURE ANISOTROPY

Figure 2(a) and 2(b) shows the two dimensional profiles of \( T_\parallel \) and \( T_\perp \) of the He\(^+\) ion in helium plasma. Because of the mirror effect, \( T_\parallel \) decreases and \( T_\perp \) increases with closer distance to the magnetic poles. The radial temperature profile of the He\(^+\) ion on the equatorial plane is plotted in Fig. 2(c), showing an evident difference between \( T_\perp \) and \( T_\parallel \).

FIG. 2: Two dimensional profiles of (a) parallel temperature \( (T_\parallel) \) and (b) perpendicular temperature \( (T_\perp) \) of He\(^+\) ions. \( T_\perp \) is plotted in a region where the mirror ratio (the local magnetic field normalized by the minimum magnetic field along each field line) is less than 2. (c) The radial temperature profile on the equatorial plane \( (z = 0) \). The translucent regions are the error bands estimated by the covariance of the reconstruction fitting.

Figure 3 shows the dependencies of line averaged ion temperatures and anisotropy on ECH power and neutral helium particle density at two different radial points \( (r \sim 510 \text{ mm} \) and \( r \sim 710 \text{ mm} \)). The neutral helium particle density is estimated as \( n_n = p/k_BT - n_e \), where \( p \) is the filling gas pressure, \( k_B \) is the Boltzmann constant, \( T = 300 \text{ K} \) is the room temperature and \( n_e \) is the electron density measured by the interferometers. From Fig. 3(a) and 3(b), both the temperatures and anisotropy increase as ECH power increases. From Fig. 3(c) and 3(d) both the temperatures and anisotropy decreases as the neutral helium particle density increases.
nisms depends on the driving mechanism of the inward diffusion.

Figure 4 shows the frequency spectrum of the density fluctuations detected by a reflectometer. A flat spectrum extending up to around 10 kHz was observed. The bounce frequency of a He\(^+\) ion is typically around 15 kHz (assuming \(T\| \sim 10\) eV). On the other hand, the ion cyclotron frequency is typically 360 kHz (for \(T\| = 20\) eV and \(B = 150\) G); thus, the betatron acceleration is deemed to be ideally in effect. Hence, it was concluded that in this instance, the Fermi acceleration is at most very weak in comparison with the betatron acceleration.

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{fig4}
\caption{Time evolution of the frequency spectrum measured by the reflectometer for the total lifetime of the plasma (ECH power is injected from 0.5 s through 1.5 s). The density cutoff resides at \(r \sim 640\) mm. The inset is the clip between \(t = 1.001584\) s and \(t = 1.063193\) s. The fluctuation has a flat spectrum up to around 30 kHz (which ranges beyond the bounce frequency). However, the fluctuations in the range of the ion cyclotron frequency are very weak.}
\end{figure}

Other competitive processes are (1) the thermal equilibration of He\(^+\) ions and the cold component of electrons, (2) the thermal equilibration of He\(^+\) ions and He\(^+\) ions immediately after ionization from neutral helium particles which have isotropic temperature \(T_0\), (3) the loss of He\(^+\) ions by charge exchange with neutral particles and (4) the isotropization of \(T\|\) and \(T\perp\). The effect of hot electrons may be neglected because the thermal equilibration time is on the order of 10\(^3\) s, which is considerably longer than the lifetime of the plasma. In this instance, no assumption is made of any ‘anomalous transport’; thus, the only energy loss mechanism at play is charge exchange loss.

Taking into account the aforementioned processes, a one-dimensional model of energy transport on the radial
coordinate $r$ may be considered:

$$\frac{dT_{\perp}}{dt} = \frac{T_0 - T_{\perp}}{\tau_{\text{ei}}} + \frac{T_0 - T_{\parallel}}{\tau_{\text{ii}}} - \frac{T_{\perp} - T_{\parallel}}{\tau_{\text{iso}}} - \frac{T_{\parallel}}{\tau_{\text{cx}}} + P_{\text{betatron}},$$

$$\frac{1}{2} \left( \frac{d}{dt} \left( \frac{T_{\parallel}}{2} \right) \right) = \frac{T_0 - T_{\parallel}}{2\tau_{\text{ei}}} + \frac{T_0 - T_{\parallel}}{2\tau_{\text{ii}}} + \frac{T_{\perp} - T_{\parallel}}{\tau_{\text{iso}}} - \frac{T_{\parallel}}{2\tau_{\text{cx}}},$$

where $d/dt$ is the temporal derivative to be evaluated on each volume element co-moving with the plasma on a Lagrangian coordinate; for a stationary state, we may evaluate $d/dt = V_r d/dr$ with an inward diffusion speed $V_r$. All coefficients of equations (2) and (3) are evaluated by experimentally measured local plasma parameters.

The electron-ion thermal equilibration time ($\tau_{\text{ei}}$) is a function of the ion temperature ($\sim 10$ eV), the cold electron density ($n_e \sim 10^{17}$ m$^{-3}$ measured by interferometers) and the cold electron temperature $T_e$ [26]. Figure 5 shows the spatial profile of $T_e$ measured by line integrated He I line ratios (728.1/706.5 and 667.8/728.1 nm) [27] for different plasma condition from that of Fig. 2. Unlike the ion temperature profile, $T_e$ is higher near to the dipole field magnet. We only have the single spatial point measurement $T_e = 30.3$ eV at $r = 795$ mm for the same condition as Fig. 2. Therefore, we infer that the cold electron temperature is distributed as $30 \lesssim T_e \lesssim 40$ eV in the plasma for the present study. Finally, $\tau_{\text{ei}}$ was determined to be $\sim 0.1$ s. The thermal equilibration time among He$^+$ ions ($\tau_{\text{ii}}$) is estimated as $\sim 1$ ms by the ion temperature and density ($\approx n_e$). The ion isotropization time ($\tau_{\text{iso}}$) is on the order of 1 ms, as estimated by the ion temperatures ($T_{\perp} \sim 20$ eV and $T_{\parallel} \sim 10$ eV) [26].

Let us estimate neutral helium gas density profile to evaluate the time constant $\tau_{\text{cx}}$ which scales the time of charge transfer between the He$^+$ ion and the neutral helium gas. The mean free path of the neutral particle is $\lambda = \frac{n_e}{n_{\text{e}} (\sigma v)_{\text{ion}}}$, where $v_n$ is the thermal velocity of the neutral gas and $(\sigma v)_{\text{ion}}$ is the ionization rate coefficient. The neutral helium temperature was found to be almost spatially homogeneous with 0.7 eV by the Doppler broadening of the He I (471.315 nm) line as shown in Fig. 6. Using $n_e$ of the shot in Fig 2 and $30 \leq T_e \leq 40$ eV, $\lambda$ is evaluated as $\sim 1$ m near the dipole field magnet and $\sim 10$ m at the chamber wall, implying that the neutral gas density is spatially flat. In addition, the absolute value of the neutral helium density must be evaluated. We solved the rate equation of ionization, recombination and charge exchange processes of helium, varying neutral helium gas density. The stationary solutions are shown in Fig. 7. Since adequate light emission was observed from both the He I and He II lines, the neutral gas density was determined to be roughly in the range of $0.1 n_e \lesssim n_\lambda \lesssim 10 n_e$. If $n_\lambda$ is too small, He$^{2+}$ dominates and He I line would be scarce. Or if $n_\lambda$ is too large, He II line would disappear (since He$^{2+}$ does not emit light, we have yet to determine how much of He$^{2+}$ is present). Therefore, the neutral helium gas density is homogeneously $n_\lambda \sim 10^{17}$ m$^{-3}$. $\tau_{\text{cx}}$ was determined to be $\sim 1$ ms.

![FIG. 5: The spatial profile of $T_e$ measured by line integrated He I line ratios (728.1/706.5 and 667.8/728.1 nm). The plasma condition is different from that of Fig. 2.](image)

![FIG. 6: Radial profile of the line averaged temperature of the neutral helium particles evaluated from the Doppler broadening of the He I line spectrum (471.315 nm) on the horizontal line of sight.](image)

The inward diffusion speed $V_r$ is required to evaluate $P_{\text{betatron}}$. This was measured in the initial formation phase. At the beginning of the discharge, the density has a broad distribution, but subsequently it begins to concentrate into the central region, thereby creating a clump of particles. Figure 8(a) shows the time evolution of the line integrated electron density measured by the three chord interferometers around the onset of the plasma discharge. Two dimensional density profile of $n_e$ is reconstructed at the two different time points $t = 0.98300$ s and $t = 0.98348$ s. The electron density evolves in accordance with the continuity equation with the inward diffusion velocity $V$:

$$\frac{\partial n_e}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n_e V_r) + \frac{\partial}{\partial z} (n_e V_z) = 0. \quad (4)$$
We assume \( V \) as
\[
V = V_{\psi 0} \left( \frac{\psi}{\psi_1} \right)^\alpha \frac{\nabla \psi}{|\nabla \psi|}.
\]

The parameters \( V_{\psi 0} \) and \( \alpha \) are optimized so that the solution of (4) with the initial condition as \( n_e \) at \( t = 0.98300 \) s in Fig. 8 becomes closer to \( n_e \) at \( t = 0.98348 \) s. Choosing appropriate parameters, the density evolution is replicated well as shown in Fig. 8(a), thus the speed \( V_r \) that transports the particles is deduced. In the stationary phase, we do not have a direct measurement of \( V_r \), but we assume that the same estimate applies. Figure 8(b) shows optimized \( V_r \) on the equatorial plane which will be used to evaluate \( P_{\text{betatron}} \).

\[ \tau_{iso} \] is smaller than the inward diffusion time scale in this region, the isotropization also decreases \( T_\| \), and at the same time, increases \( T_\perp \). Here the exact value of the neutral gas density (which has a homogeneous distribution in the plasma) is tuned to \( 1 \times 10^{17} \) m\(^{-3} \) to match the peak value of \( T_\perp \).

FIG. 7: Ionization ratio of helium as a function of neutral particle density, obtained by solving the rate equation of ionization, recombination and charge exchange processes. The electron temperature was set to 30 eV from the measurement.

FIG. 8: (a) Time evolution of the line integrated electron density measured by the interferometers at each chords around the onset of the plasma discharge. The bullets at \( t = 0.98348 \) s are the solution of (4) with the parameters \( (V_{\psi 0}, \alpha) = (3000, -7) \) and the initial condition as the bullets at \( t = 0.98300 \) s. (b) Radial distribution of the estimated inward diffusion speed \( V_r \) on the equatorial plane.

V. NUMERICAL SIMULATION OF THE ENERGY TRANSPORT MODEL

Equations (2) and (3) are both ultimately solved with the aforementioned parameters. An initial condition at \( r = 1000 \) mm (edge of the plasma) is assumed; using the measurement, \( T_\perp = T_\| = 8 \) eV. This initial temperature may be equivalent to the temperature of ions immediately after ionization from neutral helium particles. Therefore we set \( T_0 = 8 \) eV. The solution is shown in Fig. 9(a) and it turns out that the model explains the experimental profiles presented in Fig. 2(c). In the first half of the inward transport \( (1000 > r > 700 \) mm), the betatron acceleration is stronger than the charge exchange loss because of the sufficiently fast \( V_r \). This results in the increase in \( T_\perp \). In the last half \( (700 > r > 375 \) mm) where \( V_r \) is rather smaller, on the other hand, the charge exchange loss overcomes the betatron acceleration. Thus, \( T_\perp \) decreases. Since \( \tau_{iso} \) is smaller than the inward diffusion time scale in this region, the isotropization also decreases \( T_\perp \), and at the same time, increases \( T_\| \). Here the exact value of the neutral gas density (which has a homogeneous distribution in the plasma) is tuned to \( 1 \times 10^{17} \) m\(^{-3} \) to match the peak value of \( T_\perp \).

Figure 9(b)-9(e) illustrate the dependencies of temperatures and anisotropy calculated from our model on the initial temperature at \( r = 1000 \) mm and the neutral helium particle density. The results are consistent with the experimental scaling in Fig. 3. The increase in the ECH power implies an increase in the initial ion temperature through the relaxation between cold electrons; then \( P_{\text{betatron}} \) becomes larger, resulting in larger \( T_\perp \) and anisotropy. The increase in \( T_\| \) is given by the isotropization. The decrease in the ion temperature with the increase in neutral helium gas density is consistent with the charge exchange loss term in (2) and (3). Since \( P_{\text{betatron}} \).
is independent of the neutral gas density, an increase in the charge exchange loss results in small anisotropy. Therefore all the dependencies were found to be consistent with the assessed model underpinning this study.

In conclusion, the anisotropic ion temperature observed in the laboratory radiation belt is experimental proof of the betatron acceleration concomitant with inward radial diffusion of particles.

![Graphs showing radial profiles and dependencies](image)

**FIG. 9:** (a) The radial profiles of $T_\perp$ and $T_\parallel$ calculated by the energy transport model (2)–(3). The result is compared with the experimental observation shown in Fig. 2(c). Dependence of (b) the ion temperature and (c) the anisotropy on the initial temperature at $r = 1000$ mm, and the dependence of (d) the ion temperature and (e) the anisotropy on the neutral helium gas density. The temperatures and anisotropy are evaluated at $r = 700$ mm. The results are compared with the experimental scaling shown in Fig. 3.

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Appendix A: Algorithm for the reconstruction of the local profile

To reconstruct the local temperature profile from the line averaged measured data, the following procedure was used. The local profile on the equatorial plane \((z = 0)\) is reconstructed from the spectrum data obtained via the horizontal chord. From the axisymmetric property, the spectrum on the equatorial plane can be written as \(f_{\text{eq}}(\lambda, \psi(r, 0))\) where \(\lambda\) is the wavelength, \(\psi(r, z)\) is the inverse of magnetic flux and \(r\) and \(z\) are the radial and vertical coordinates, respectively. The innermost flux surface contacting the dipole field magnet is denoted as \(\psi_0\), and the outermost flux contacting the vacuum chamber at \(r = 1000\) mm is denoted as \(\psi_1\). Hereafter, the subscript \(\text{eq}\) means the value on the equatorial plane.

The line integrated spectrum \(g_{\text{eq}}(\lambda, y)\) is thus calculated from the local spectrum as

\[
g_{\text{eq}}(\lambda, y) = 2 \int_0^1 f_{\text{eq}}(\lambda, \psi_{\text{eq}}(\sqrt{x^2 + y^2})) dx,
\]

where \(x\) denotes the auxiliary coordinate along the line of sight. We assume

\[
f_{\text{eq}}(\lambda, \psi_{\text{eq}}(r)) = \begin{cases} A_{\text{eq}}(\psi_{\text{eq}}(r)) \exp \left[- \frac{(\lambda - \lambda_0 - B_{\text{eq}}(\psi_{\text{eq}}(r)))^2}{C_{\text{eq}}(\psi_{\text{eq}}(r))} \right] & (\psi_0 \leq \psi_{\text{eq}}(r) \leq \psi_1) \\ 0 & (\psi_1 \leq \psi_{\text{eq}}(r)) \end{cases},
\]

where \(\lambda_0\) is the central wavelength. The intensity \(A_{\text{eq}}(\psi_{\text{eq}}(r))\) and the shift \(B_{\text{eq}}(\psi_{\text{eq}}(r))\) are assumed to be

\[
A_{\text{eq}}(\psi_{\text{eq}}(r)) = a_1 \frac{(\psi_{\text{eq}}/\psi_0 - 1)^{a_2(1 - \psi_{\text{eq}}/\psi_1)^a_3}}{(\psi^*/\psi_0 - 1)^{a_2(1 - \psi^*/\psi_1)^a_3}} \left( \psi^* = \frac{a_3 \psi_0 + a_2 \psi_1}{a_2 + a_3} \right)
\]

\[
B_{\text{eq}}(\psi_{\text{eq}}(r)) = b_1 + b_2 \psi_{\text{eq}} + b_3 \psi_{\text{eq}}^2,
\]

with the perpendicular temperature on the equatorial plane assumed to be

\[
T_{\text{eq}}(\psi_{\text{eq}}) = (t_1 - 0.5 t_4) \left[ \psi_{\text{eq}}/\psi_0 - 1 \right]^{t_2(1 - \psi_{\text{eq}}/\psi_1)^{t_3}} \left[ \psi^*/\psi_0 - 1 \right]^{t_2(1 - \psi^*/\psi_1)^{t_3}} + 0.5 t_4 \left[ 1 + \text{erf} \left( \frac{\psi - \psi^*}{0.2(\psi_1 - \psi_0)} \right) \right] \left( \psi^* = \frac{t_3 \psi_0 + t_2 \psi_1}{t_2 + t_3} \right).
\]

The Doppler broadening \(C_{\text{eq}}(\psi_{\text{eq}})\) is determined by \(T_{\text{eq}}(\psi_{\text{eq}})\). The functions \(A_{\text{eq}}, B_{\text{eq}}\) and \(T_{\text{eq}}\) indicate that the spectral emission is zero at both boundaries, the temperature is zero at \(\psi = \psi_0\) and \(t_1\) at \(\psi = \psi_1\), and the toroidal flow speed may be finite at both boundaries. The observed spectrum data set for the horizontal chord is written as \((\lambda_i, y_j, \psi_{\text{eq}ij})\). The parameters \((a_1, a_2, a_3, b_1, b_2, b_3, t_1, t_2, t_3, t_4)\) are optimized so as to minimize \(|g_{\text{eq}ij} - g_{\text{eq}}(\lambda_i, y_j)|\) for each \(\lambda_i\) and \(y_j\).

Next we build a vertical reconstruction algorithm. Using \(A_{\text{eq}}\) and \(T_{\text{eq}}\) obtained by the horizontal reconstruction, a two-dimensional profile is reconstructed. The position of the collimator is denoted as \((r_c, z_c)\) (Fig. 1(a)).

The line of sight passing the equatorial point \((y, 0)\) is given as

\[
r = \frac{r_c - y}{z_c} (z - z_c) + r_c.
\]

The line integrated spectrum \(g(\lambda, y, z)\) is calculated from the local spectrum \(f(\lambda, r, z)\) as

\[
g(\lambda, y, z) = \int_{-z_c}^{z_c} f(\lambda, r(z), z) \sqrt{1 + \left( \frac{dr}{dz} \right)^2} dz.
\]

The local spectrum is assumed to be

\[
f(\lambda, r, z) = \begin{cases} A(r, z) \exp \left[- \frac{(\lambda - \lambda_0)^2}{C(r, z)} \right] & (\psi_0 \leq \psi \leq \psi_1) \\ 0 & (\psi_1 \leq \psi) \end{cases},
\]

In the vertical chord, the Doppler shift can be ignored because of the absence of the mean poloidal flow in the dipole magnetic field configuration. We use the magnetic field coordinate to treat the vertical direction, and assume \(A(r, z)\) to be the power function of the magnetic field strength \(B\).

\[
A(r, B(r, z)) = A_{\text{eq}}(\psi_{\text{eq}}) \left( \frac{B(r, z)}{B_{\text{eq}}} \right)^{-\tilde{a}}
\]

where \(B_{\text{eq}}\) is the magnetic field strength of the same field line on the equatorial plane. In the \(z \neq 0\) region, the effective temperature \(T_{\text{eff}}\) causing the Doppler broadening is a combination of \(T_\perp\) and \(T_\parallel\). Defining the angle
θ between the line of sight and the line normal to the magnetic field, we may write

$$(T_{\text{eff}})^2 = (T_\perp \cos \theta)^2 + (T_\parallel \sin \theta)^2. \quad (A6)$$

In the same manner as (A5), we assume the perpendicular temperature profile as

$$T_\perp (r, B(r, z)) = T_{\perp \text{eq}} \left( \frac{B}{B_{\text{eq}}} \right)^{\tilde{c}_1}. \quad (A7)$$

While $T_{\perp \text{eq}}$ is already determined by the horizontal reconstruction, we need the profile of $T_\parallel$ on the equatorial plane. We assume $T_{\parallel \text{eq}}$ in a similar manner as $T_{\perp \text{eq}}$:

$$T_{\parallel \text{eq}}(\psi_{\text{eq}}(r)) = (\tilde{t}_1 - 0.5\tilde{t}_4) \left( \psi_{\text{eq}}/\psi_0 - 1 \right)^{\tilde{t}_2} \left( 1 - \psi_{\text{eq}}/\psi_1 \right)^{\tilde{t}_3} \left( \psi_{\text{eq}} / \psi_0 - 1 \right)^{\tilde{t}_2} \left( 1 - \psi_{\text{eq}} / \psi_1 \right)^{\tilde{t}_3} + 0.5\tilde{t}_4 \left[ 1 + \text{erf} \left( \frac{\psi - \psi^*}{0.2(\psi_1 - \psi_0)} \right) \right] \frac{\psi = \tilde{t}_3 \psi_0 + \tilde{t}_2 \psi_1}{\tilde{t}_2 + \tilde{t}_3}. \quad (A8)$$

In the same manner as (A5) and (A7), we assume

$$T_{\parallel}(r, B(r, z)) = T_{\parallel \text{eq}} \left( \frac{B}{B_{\text{eq}}} \right)^{-\tilde{c}_2}. \quad (A9)$$

We write the observed spectrum data set for the vertical chord as $(\lambda_i, y_j, g_{ij})$. The parameters $(\tilde{a}, \tilde{c}_1, \tilde{c}_2, \tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \tilde{t}_4)$ are optimized to minimize $|g_{ij} - g(\lambda_i, y_j)|$ for each $\lambda_i$ and $y_j$. A two-dimensional profile of $A(r, z)$, $T_\perp(r, z)$ and $T_\parallel(r, z)$ is then ultimately obtained.

Figure 10 shows the fitting precision of Fig. 2(c). The solid lines are the line averaged temperatures estimated by $g_{\text{eq}}(\lambda, y)$ and $g(\lambda, y)$, respectively, with optimized parameters, while the bullets are the line averaged temperatures estimated by the measured data $g_{\text{eq,ij}}$ and $g_{ij}$, respectively. The translucent regions are the error bands determined by multiplying a constant by the covariance of the reconstruction fitting. The corresponding error bands for the local profile are illustrated in Fig. 2(c).
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