The standard model of star formation applied to massive stars: accretion discs and envelopes in molecular lines

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ABSTRACT

We address the question of whether the formation of high-mass stars is similar to or differs from that of solar mass stars through new molecular line observations and modelling of the accretion flow around the massive protostar IRAS20126+4104. We combine new observations of NH$_3$(1,1) and (2,2) made at the Very Large Array (VLA), new observations of CH$_3$CN(13–12) made at the Submillimeter Array, previous VLA observations of NH$_3$(3,3) and NH$_3$(4,4) and previous Plateau de Bure observations of C$^{34}$S(2–1), C$^{34}$S(5–4) and CH$_3$CN(12–11) to obtain a data set of molecular lines covering 15–419 K in excitation energy. We compare these observations against simulated molecular line spectra predicted from a model for high-mass star formation based on a scaled-up version of the standard disc–envelope paradigm developed for accretion flows around low-mass stars. We find that in accord with the standard paradigm, the observations require both a warm, dense, rapidly rotating disc and a cold, diffuse infalling envelope. This paper suggests that accretion processes around 10 M$_\odot$ stars are similar to those of solar mass stars.

Key words: stars: individual: IRAS 20126+4104 – stars: massive.

1 INTRODUCTION

Does the formation of massive stars differ significantly from that of solar mass stars? As far as we know, stars of all masses form in gravitationally unstable regions of molecular clouds and gain their mass by accretion. A standard model developed for accretion flows around low-mass stars consists of two components, a rotationally supported disc inside a freely falling envelope (Shu, Adams & Lizano 1987; Hartmann 2001). This model has been particularly successful in explaining infrared observations of low-mass star formation. The disc and envelope produce an excess of long-wavelength infrared emission that has been adopted as the identifying signature of accreting protostars in Galactic (Whitney et al. 2003) and extragalactic (Whitney et al. 2008) star-forming regions. Furthermore, because the disc and envelope have different densities and temperatures, the evolutionary state of the protostars can be identified by the shape of the infrared spectral energy distribution: class 0 (envelope dominated) and class I (disc dominated) (Lada 1987; Andre, Ward-Thompson & Barsony 1993).

Does this standard two-component accretion model developed for low-mass stars also describe the accretion flows around massive stars? There are some doubts. The more massive stars are luminous enough to generate radiation pressure and hot enough to ionize their own accretion flows such that the outward radiative and thermal pressures rival the inward pull of the stellar gravity (Larson & Starrfield 1971; Kahn 1974; Keto 2002; Keto & Wood 2006). Do these outward pressures result in accretion flows that are different around more massive stars? In this paper, we compare new and previous molecular line observations of an accretion flow around one massive star against the standard disc–envelope paradigm for accretion flows around low-mass stars.

The previous molecular line observations of the massive protostar IRAS20126+4104,¹ suggest an accretion disc and bipolar outflow around a 7–15 M$_\odot$ protostar embedded in a dense molecular envelope (Cesaroni et al. 1997; Zhang, Hunter & Sridharan 1998; Cesaroni et al. 1999; Kawamura et al. 1999; Zang et al. 1999; Shepherd et al. 2000; Cesaroni et al. 2005; Lebron et al. 2006; Su et al. 2007; Qiu et al. 2008). Previous infrared observations of absorption and scattering also reveal a disc and outflow cavity immediately around the star (Sridharan, Williams & Fuller 2005; deBuizer 2007).

In this paper, we assemble a suite of observations of molecular lines of different excitation temperature in order to compare with molecular spectra predicted from the disc–envelope model. Lines of different excitation temperature are useful because massive stars heat the surrounding molecular gas to observationally significant temperatures (∼100 K) at observationally significant distances from the star ($T_{\text{gas}} \sim T_\cdot (R/R_\odot)^{\alpha}$), and we can exploit the relationship between temperature and radius to distinguish emission from gas at different radii around the star. We expect to identify the emission

¹IRAS20126 is located in the Cygnus-X region at a distance of 1.7 kpc (Wilking, Lada & Young 1989).
from the higher excitation temperature lines with gas in the flow closer to the star and also to separate the emission from the disc and envelope components. In previous observations, Keto, Ho & Haschick (1987) and Cesaroni et al. (1994) used this technique, observing several lines of NH$_3$ to study the accretion flows around very high-mass stars associated with H$_2$ regions.

We present new observations of the NH$_3$ (1,1) and (2,2), inversion transitions made with the National Radio Astronomy Observatory’s Very Large Array (VLA)$^2$ and new observations of CH$_3$CN(13–12) made with the Submillimeter Array (SMA). The new observations of NH$_3$(1,1) and NH$_3$(2,2) have a factor of 3 better sensitivity than the earlier observations of Zhang et al. (1998). Combined with previous observations of NH$_3$(3,3) and NH$_3$(4,4) (Zhang et al. 1999) the four NH$_3$ lines span a range of excitation temperatures from 23 to 200 K. We also have additional lines of lower and higher excitation temperature from previous observations of the C$^{34}$S(2–1) and C$^{34}$S(5–4) lines with energies of 7 and 35 K, respectively, and the ladder of CH$_3$CN ($J = 12$–11) lines with energies ranging from 69 K for ($K = 0$) to 419 K for ($K = 7$). The C$^{34}$S and CH$_3$CN observations were previously presented in Cesaroni et al. (1999) and Cesaroni et al. (2005).

We specify the two-component accretion model in terms of six parameters: the scalefactors for the density and temperature of the envelope and of the disc, the angular momentum of the envelope and the stellar mass. We use our molecular line emission code MOLLIE to predict molecular line spectra from the parametrized model, and we use a least-squares fitting procedure to adjust the parameters to fit the observations.

We find that the two-component, disc–envelope model can successfully describe the observations, but a single-component model cannot. A warm, dense rotationally supported disc is required to obtain sufficient brightness and width in the high-excitation lines, and a cold, large-scale envelope is required to match the emission from the lower excitation lines. We find no evidence that the accretion flow around IRAS20126 is profoundly altered by the outward force of radiation pressure or by ionization. At 10 M$_\odot$, the star might simply not be luminous enough or hot enough for its radiation pressure or ionization to significantly affect the accretion process. Based on this example, the accretion flows around 10 M$_\odot$ stars are quite similar to flows around lower mass stars.

2 THE DATA

2.1 Ammonia inversion lines

We observed the IRAS20126+4104 region, right ascension (RA) (J2000) = 20:14:26.06, declination (Dec.) (J2000) = 41:13:31.50, in the inversion transitions of NH$_3$ ($J, K = 1, 1$), (2,2), (3,3) and (4,4) on 1999 March 27 and 1999 May 29 with the VLA in its D configuration. We used the 2IF correlator mode to sample both the right- and left-hand polarizations, a spectral bandwidth of 3.13 MHz and a channel width of 24 kHz or 0.3 km s$^{-1}$. The primary beam of the VLA at the NH$_3$ line frequencies is about 2 arcmin. Quasars 3C48 and 3C286 were used for flux calibration, 3C273 and 3C84 for bandpass calibration and quasar 2013+370 for phase calibrations. The NH$_3$(3,3) and NH$_3$(4,4) observations were previously discussed in Zhang et al. (1999).

The VLA data were processed using the NRAO Astronomical Image Processing System (AIPS) package. Data from the two days were combined to achieve an rms of 3 mJy per 0.3 km s$^{-1}$ channel for the NH$_3$(1,1), (2,2) lines and 6 and 4 mJy per 0.6 km s$^{-1}$ channel for the NH$_3$(3,3) and (4,4) lines, respectively. Fig. 1 shows two images of IRAS20126 in the integrated intensity of NH$_3$(1,1) and (2,2). The protostar is located within the bright core of the larger scale cloud. Also shown in Fig. 1 is the image of the bipolar outflow in SiO(2–1) from Cesaroni et al. (1999). The NH$_3$ spectra used in the analysis are taken at positions offset from the phase centre by $-0.23, 1.00$ (centre panel), $-3.10, 3.25$ (left-hand panel) and $2.50, -0.94$ (right-hand panel) in arcseconds of RA and Dec. The corresponding absolute positions are RA (J2000) = 20:14:26.02, Dec. (J2000) = 41:13:32.8 (centre panel), RA (J2000) = 20:14:25.79, Dec. (J2000) = 41:13:30.8 (right-hand panel) and RA (J2000) = 20:14:26.28, Dec. (J2000) = 41:13:35.0 (left-hand panel). The locations of the spectra are shown as red stars in Fig. 1.

2.2 Methyl Cyanide and Carbon Sulfide lines

The CH$_3$CN(13–12) observations were made with the Submillimeter Array (SMA; the Submillimeter Array is a joint project between the Smithsonian Astrophysical Observatory and the Academia
The density is related to the mass accretion rate (equation 11 of Keto 2007):

\[ M = \rho_0(R_d)4\pi R_d^2 v_k, \]

where \( v_k \) is the Keplerian velocity at \( R_d \).

The model for the disc assumes that a rotationally supported disc forms at the radius where the centrifugal force in the rotating envelope equals the gravitational force of the point mass,

\[ \Gamma^2 = \frac{GM}{R_d^3}, \]

and \( \Gamma = v_k R \). The disc is truncated at \( R_d \).

The gas density in the disc is (equation 3.14 of Pringle 1981)

\[ \rho_{\text{disc}}(\rho, R) = \rho_0(R) \exp(-\rho^2/2H^2), \]

where \( \rho_0(R) \) is the density in the mid-plane, \( \theta = \pi/2 \), at any radius \( R \). We use \( R = \sqrt{x^2 + y^2} \) to denote the cylindrical radius, and \( r = \sqrt{x^2 + y^2 + z^2} \) for the polar radius. In the thin-disc theory, the scaleheight, \( H \), is

\[ H^2 = \frac{c_s^2}{G} \frac{R^3}{M_{\text{env}}}, \]

where \( c_s \) is the sound speed. We follow Whitney et al. (2003) and use a modified version of this equation,

\[ H = H_0(R/R_*)^{3.25} \]

with \( H_0 = 0.01 R_\ast \). The density in the mid-plane of the accretion disc, \( \rho_0(R) \), is

\[ \rho_0 = \rho_0(R_d) R^{2.25} \]

where \( \rho_0(R) \) is the density in the mid-plane at \( R_d \). Again following Whitney et al. (2003), we use an exponent of 2.25 in equation (8) rather than 2 which is derived in the thin-disc theory. As explained in Whitney et al. (2003) the modifications to the equations for the scaleheight and mid-plane densities are based on the fits to numerical models of disc structure.

The densities in the disc and envelope at radius \( R_d \) are related by a factor \( A_\rho \),

\[ \rho_{\text{env}} = A_\rho \rho_0. \]

For example, in the steady-state flow, the disc would have a higher density if, as should be the case, the inward velocities in the rotationally supported disc were lower than in the freely falling envelope. Because the inward velocities in the disc are not described by the thin-disc theory, we leave \( A_\rho \) as an adjustable parameter. The total gas density at any point in the model is the sum of the densities in the disc and envelope,

\[ \rho_{\text{gas}} = \rho_{\text{disc}} + \rho_{\text{env}}. \]

### 3.2 The temperature

We assume that the envelope is heated by the star. The envelope temperature is (equation 7.36 of Lamers & Cassinelli 1999)

\[ T_{\text{env}} = T_s(R_s/2r)^{-(3+p)}, \]

where \( p \) in our model is an adjustable parameter related to the dust opacity and the geometry of the flow. If the density structure were spherically symmetric, then \( p \) would be the exponent in the frequency dependence, \( v^p \), of the Planck mean opacity of the dust. In a flattened flow, \( p \) can be negative if the geometrical dilution of the radiation in the accretion flow is greater than \( r^{-2} \). This can happen if the dust at each radius absorbs and isotropically re-emits the
outward flowing radiation. Radiation that is emitted perpendicular to the disc escapes from the flow. Only the radiation that is emitted in the direction along the flattened flow continues to heat the dust at larger radii. The result is a decrease in the radiation in the disc faster than \( r^{-2} \).

In the thin-disc theory, the disc is heated by dissipation related to the accretion rate. The disc temperature is (equation 3.23 of Pringle 1981)

\[
T_{\text{disc}} = B_T \left[ \left( \frac{3GM \dot{M}}{4\pi R^3 \sigma} \right) \left( 1 - \sqrt{\frac{R_*}{R}} \right) \right]^{1/4}.
\]

(12)

We include an adjustable factor, \( B_T \), to allow for additional, or possibly less, disc heating. For example, the observations constrain the gas density through the observed optical depth of \( \text{NH}_3 \). This means that the density and therefore the accretion rate, \( \dot{M} \), are dependent on the assumed \( \text{NH}_3 \) abundance which is not well known. The adjustable factor, \( B_T \), decouples the disc temperature from the assumed molecular abundance. Also the disc temperature may be raised by stellar radiation (passive heating). The factor, \( B_T \), allows for these effects in an approximate way.

The gas temperature in the model is the density-weighted average of the envelope and disc temperatures,

\[
T_{\text{gas}} = \frac{T_{\text{disc}} \rho_{\text{disc}} + T_{\text{env}} \rho_{\text{env}}}{\rho_{\text{disc}} + \rho_{\text{env}}}.
\]

(13)

3.3 The velocity

The three components of the gas velocity in the envelope are given in spherical coordinates by equations (4)–(6) of Keto (2007), but there are errors in earlier papers. In particular, equations (5) and (6) of Keto (2007) and equation (8) of Ulrich (1976) are incorrect. Equation (8) of Ulrich (1976) (same as equation 5 of Keto 2007) does not result in energy conservation, \( m v^2/2 = GM/r \), when combined with the other velocity components. The error is small, 1 part in 10\(^4\). The following equations, same as in Mendoza et al. (2004), are exact:

\[
v_r(r, \theta) = \left( \frac{G M_\ast}{r} \right)^{1/2} \left[ 1 + \frac{\cos \theta}{\cos \theta_0} \right]^{1/2},
\]

(14)

\[
v_\theta(r, \theta) = \left( \frac{G M_\ast}{r} \right)^{1/2} \frac{\cos \theta - \cos \theta}{\sin \theta} \left[ 1 + \frac{\cos \theta}{\cos \theta_0} \right]^{1/2},
\]

(15)

\[
v_\phi(r, \theta) = \left( \frac{G M_\ast}{r} \right)^{1/2} \frac{\sin \theta}{\sin \theta_0} \left[ 1 - \frac{\cos \theta}{\cos \theta_0} \right]^{1/2}.
\]

(16)

The velocity in the disc is simply the Keplerian velocity,

\[
v_{\text{disc}}(R) = \sqrt{\frac{G M_\ast}{R}},
\]

(17)

where the velocity in the disc is purely azimuthal. We assume that the radial velocity in the rotationally supported disc is comparatively small. The gas velocity in the model is the density-weighted average of the envelope and disc velocities,

\[
v_{\text{gas}} = \frac{v_{\text{disc}} \rho_{\text{disc}} + v_{\text{env}} \rho_{\text{env}}}{\rho_{\text{disc}} + \rho_{\text{env}}},
\]

(18)

where \( v_{\text{env}} \) is given by equations (14)–(16) and \( v_{\text{disc}} \) is given by (17).

3.4 The density singularities in the accretion model

The density in the Ulrich model is singular in the mid-plane of the disc at the centrifugal radius and also at the origin. These singularities are caused by the convergence of the streamlines in the simple mathematical description of the flow. This convergence is not expected on physical grounds because gas pressure, neglected in the Ulrich model, would prevent it. We handle the singularities in two ways. We define the computational grid to have an even number of cells so that the centres of the middle cells are above and below the mid-plane and around the origin. Secondly, we smooth the density in the radial direction with a Gaussian with a width of \( R_d/2 \).

3.5 The model parameters

Based on the analyses of the previous observations cited in the introduction, we assume that the disc–envelope is viewed edge-on. The model contains six adjustable parameters:

1. \( \rho_0 \) sets the density of the envelope (equation 1) and the mass accretion rate (equation 3);
2. \( p \) sets the exponent of the power-law decrease of the temperature in the envelope (equation 11);
3. \( \Gamma = v_f \) is the specific angular momentum (equation 4) of the envelope flow;
4. \( M_\ast \) is the stellar mass;
5. \( A_p \) sets the ratio (equation 9) of the disc density to the envelope density at \( R_d \);
6. \( B_T \) is factor multiplying the disc temperature (equation 12).

4 FITTING THE MODEL TO THE \( \text{NH}_3 \) DATA

The disc enters into the model additively, and we can test for the presence of a disc by fitting models to the data with and without the disc. In the first case, with the disc, we adjust all six model parameters for both the envelope and the disc, and in the second case, without the disc, we adjust only the first four parameters for the envelope. The fitting is done independently in each case, and the four envelope parameters are therefore different in the two cases.

The procedure for fitting the data is the same as described in Keto et al. (2004). We use a fast simulated annealing algorithm to adjust the model parameters to minimize the summed squared difference (\( \chi^2 \)) between the data and the model spectra.

The model spectra for each particular set of model parameters are generated by our radiative transfer code MOLLIE (Keto 1990; Keto et al. 2004). We assume local thermodynamic equilibrium (LTE) conditions for the \( \text{NH}_3 \) and \( \text{CH}_3\text{CN} \) lines. The LTE approximation is appropriate for \( \text{NH}_3 \) because in the absence of strong infrared radiation the level populations are expected to be mostly in the ‘metastable’ states which are the lowest \( J \) state of each \( K \) ladder. Since radiative transitions between \( K \) ladders are forbidden, the coupling between the metastable states is purely collisional and the population approximates a Boltzmann distribution as in LTE (Ho & Townes 1983). \( \text{CH}_3\text{CN} \) is also a symmetric top and transitions across the \( K \) ladders are similarly forbidden; however, unlike \( \text{NH}_3 \), the upper states are easily populated in warm gas (\( > 100 \) K). While the justification for the LTE approximation is not as strong for \( \text{CH}_3\text{CN} \) as for \( \text{NH}_3 \), we find that \( \text{CH}_3\text{CN} \) always traces hot, very dense gas where collisional transitions should be important. For \(^{13}\text{C} \text{S} \) we use the accelerated lambda iteration algorithm (ALI) of Rybicki & Hummer (1991) to solve for the non-LTE level populations.

In comparing the model to the data, we take into account the spatial averaging of the brightness by the width of the observing...
Data (red) and model (blue) spectra of NH$_3$ (1,1) (top row) NH$_3$ (2,2) (second row) NH$_3$ (3,3) (third row) and NH$_3$ (4,4) (bottom row) at three positions across the mid-plane of IRAS20126 for the accretion flow including both an infalling envelope and a flared disc. The spectra in the middle column are towards the centre of the flow, and the columns to the left- and right-hand side show spectra on either side of the centre, 6200 au to the left-hand side, and 5700 au to the right-hand side. The three locations are shown in Fig. 1.

We compute spectra over a grid of positions and smooth the result by convolution with a Gaussian with the full width at half-maximum (FWHM) equivalent to the observing beam of each observation, Section 2.

We simultaneously fit 12 NH$_3$ spectra, four transitions at three locations. The three locations are marked in Fig. 1. We ran 20,000 trial models for each of the two cases with and without the disc. The spectra of the two best-fitting models (with and without the disc) are shown in Figs 2 and 3. Parameters for both cases are listed in Table 1. Fig. 4 shows the temperature and density in the mid-plane of our best fitting models. Fig. 5 shows the density and velocity of the disc–envelope model on planes parallel and perpendicular to the rotation.

We also have CH$_3$CN and C$^{34}$S data from the observations of Cesaroni et al. (1999). The way the radiative transfer simulation program operates, we cannot use our automated search algorithm on more than one molecule at a time. The program is recompiled for each molecule. We also have not implemented the automated search for CH$_3$CN. We could fit the model to the C$^{34}$S data, but this line does not have the hyperfine structure of NH$_3$ that is so useful in constraining the optical depth and temperature. Therefore we opted for a different strategy. We do not use the CH$_3$CN or the C$^{34}$S data in the fitting. We use the data on these other lines as a check on the model derived from the NH$_3$ data. We simulate the CH$_3$CN and C$^{34}$S emission using the same two models (with and without the disc) previously derived from the NH$_3$ observations for comparison with the predicted spectra against the data. The models derived from the NH$_3$ fitting fix the temperature, densities and velocities, but we still need to assume abundances for CH$_3$CN and C$^{34}$S. For CH$_3$CN we assume an abundance of $6 \times 10^{-8}$. The assumed abundance of C$^{34}$S is $9.0 \times 10^{-11}$, chosen to match the brightness of the (2–1) line.

### 4.1 Comparison of the model with the observed spectra

The comparison of Figs 2 and 3 shows that the higher temperature and density disc and the lower temperature and density envelope are both required to fit both the high- and low-excitation NH$_3$ spectra. Without the disc, there is not enough warm, dense gas to reproduce the (4,4) line brightness. Without the large-scale envelope, there is not enough cold gas to fit the observed ratios of the NH$_3$ (1,1) hyperfine lines.

Further evidence for the disc component comes from the CH$_3$CN spectrum (Fig. 6). The results again show that the warm, high-density, disc component is required to get enough brightness in lower frequency (higher velocity) $K$ transitions. Based on the observed linewidth, the CH$_3$CN (12–11) emission comes from gas very close to the star. The observed CH$_3$CN (12–11) lines, which are not blended with other molecular lines, have linewidths of 10.0 km s$^{-1}$. The two lines that appear much broader in the data contain contaminating emission from transitions of CH$_{13}$CN (12–11) and HNCO (Cesaroni et al. 1999). Rotational velocities >10 km s$^{-1}$ are only found at radii closer than 600 au ($<GM/v^2$) around a 10 M$_{\odot}$ star. Fig. 4 shows that the gas in the disc has a temperature of greater
Table 1. Model parameters.

| Parameter                                | Symbol | Disc       | No disc    |
|------------------------------------------|--------|------------|------------|
| Envelope density at $R_d$ (cm$^{-3}$)    | $\rho_{e0}$ | $7.9 \times 10^4$ | $7.9 \times 10^5$ |
| Temperature power law exponent           | $p$    | $< -1$     | 0.4        |
| Angular momentum (au km s$^{-1}$)        | $\Gamma$ | 8100       | 3500       |
| Stellar mass ($M_\odot$)                 | $M_*$  | 10.7       | 7.3        |
| Disc density ratio                       | $A_{\rho}$ | 5.1        | –          |
| Disc temperature factor                  | $B_{T}$ | 15.0       | –          |
| Centrifugal radius (au)                  | $R_d$  | 6900       | 1900       |
| Velocity at $R_d$ (km s$^{-1}$)          | $v_k$  | 1.2        | 1.8        |
| Total mass$^a$ within 0.128 pc ($M_\odot$) |        | 12.6       | 10.1       |
| Disc mass ($M_\odot$)                    |        | 2.5        | –          |
| Envelope accretion rate ($M_\odot$ yr$^{-1}$) |        | $7.6 \times 10^{-5}$ | $1.0 \times 10^{-4}$ |

$^a$The mass estimates include the envelope and the disc for the case with a disc, and the envelope only for no disc. Both cases assume an NH$_3$ abundance of $1.0 \times 10^{-7}$.

Figure 3. Spectra in the same format as Fig. 2 for the accretion flow with an envelope only without a flared disc.

The CH$_3$CN(13–12) spectra (Fig. 7) is too noisy to help discriminate between the two cases. The peak signal-to-noise ratio is about 6 after smoothing by every other channel. None the less, the models are at least consistent with these observations.

The brightness ratio of the C$^{14}$S(5–4) and (2–1) lines also suggests the presence of a warm, dense disc (Fig. 8). The model with the disc reproduces the observed brightness ratio, 6:1 for the two lines, although the linewidth is less than observed. The model without the disc is not able to generate sufficient brightness in the higher excitation line, and the shapes of the line profiles do not match the data. In particular, the strong splitting that is seen in the model profiles without the disc is not seen in the data. This difference is also seen in the NH$_3$ and CH$_3$CN spectra of the two models, although not as prominently. We usually associate split line profiles with spatially unresolved observations of Keplerian discs (Beckwith & Sargent 1993), whereas here the disc produces a triangular profile. What happens here is that the disc component is spatially resolved and has a very high density. Thus within the beam through the centre of the model there is a lot of high-density gas from the outer part of the disc with very low velocity projected along the line of sight. This gas disc creates the peak in the spectrum around zero
velocity. In the envelope-only model without the high-density disc, the model is optically thin in C$^{34}$S. In this case, we get the usual split line profiles from the rotation and infall in the envelope.

4.2 Goodness of fit

How well do the data constrain the model parameters? Some appreciation can be gained by plotting $\chi^2$ obtained from all the trial models versus each model parameter. Each panel of Fig. 9 shows the fits obtained (ordinate) for each value of a single parameter (abscissa) as all the other parameters are varied. The lowest value of $\chi^2$ (ordinate) at each parameter value (abscissa) is the best fit that can be obtained for that parameter value, for any combination of all the other parameters. The formal error of a model parameter is proportional to the second derivative of $\chi^2$ with respect to the model parameter. Thus the curvature or width of the lower boundary of the collection of points in each figure is a qualitative measure of the sensitivity of the model to the parameter.

5 DISCUSSION

5.1 The density

The gas density is constrained by the optical depths of the NH$_3$ lines, which can be determined from the brightness ratios of the hyperfine lines. The density, $\rho_0$ at $R_d$ is $7.9 \times 10^9$ (Table 1) assuming an NH$_3$ abundance of $10^{-7}$. With this abundance, the mass of the disc is 2.5 M$_\odot$, the mass of the envelope is 12.6 M$_\odot$, and the accretion rate (equation 3), assumed to be the same in the envelope and the disc, is $M = 7.6 \times 10^{-3}$ M$_\odot$ yr$^{-1}$. The total mass in the accretion flow is mass of the disc and envelope together which is 15.1 M$_\odot$. This mass estimate is the total mass within the model boundary of 26,000 au. Cesaroni et al. (1999) estimates that there is between 0.6 and 8.0 M$_\odot$ within a radius of 5000 au.

The mass estimate derived from molecular line observations is subject to a large uncertainty. The radiative transfer modelling determines the column density of NH$_3$ rather than H$_2$. Therefore, the gas density, the masses of the disc and envelope and the accretion rate depend inversely on the assumed abundance of NH$_3$ which is not well known. Estimates from models and observations of similar clouds range from $10^{-9}$ to $10^{-6}$ (Herbst & Klemperer 1973; Keto 1990; Estalella et al. 1993; Caproni, Abraham & Vilas-Boas 2000; Galvan-Madrid et al. 2009). Thus the abundance of NH$_3$ may be uncertain by more than an order of magnitude, yet a factor of 2 in the masses of the disc and envelope is significant in an interpretation of the accretion dynamics. Furthermore, the abundance of NH$_3$ could be different in the disc and envelope. Estimates of the NH$_3$ abundance are generally the lowest in colder clouds and the highest in warm gas around massive stars. Some NH$_3$ may be frozen on to dust grains in colder gas and sublimated into the gas phase as
Figure 6. Data (red) and model (blue) spectra of \( \text{CH}_3\text{CN}(12–11) \). The left-hand panel shows the spectrum for the model with both an accretion envelope and flared disc. The model is the same as shown in Figs 2, 4 and 5. The right-hand panel is for the model with only an accretion envelope and without a disc (Figs 3 and 4). In the observed spectrum, the line that is anomalously wide \((K = 6,\) second from right-hand side) is contaminated by HNCO emission. Additionally, contamination from weak \( \text{CH}_3\text{CN} \) emission shows up to the red (right-hand panel) of four of the lines. The velocity of the \( K = 6 \) hyperfine line has been set to zero. The observed spectra have been shifted in velocity to match.

Figure 7. Spectra of \( \text{CH}_3\text{CN}(13–12) \) in the same format as Fig. 6 except that we have only the four lowest \( K \) transitions. The \( K = 0 \) and 1 transitions are blended together in the bluest peak. The left-hand panel shows the spectrum for the model with both an accretion envelope and flared disc. The right-hand panel is for the model with only an accretion envelope and without a disc (Figs 3 and 4). The signal-to-noise ratio of the observed spectra, about 6 at the peak, is not high enough to discriminate between the two models. At least the models are consistent with the data. The data are from T. K. Sridharan (private communication).

Figure 8. \( \text{C}^{34}\text{S}(2–1) \) (blue) and \( \text{C}^{34}\text{S}(5–4) \) (red) spectra of the data (solid lines) and the model (dashed lines). Left-hand panel: model with both the accretion envelope and flared disc. Right-hand panel: model with only an accretion envelope and without a flared disc. The brightness of the \( \text{C}^{34}\text{S}(5–4) \) spectra, both observed and modelled, have been divided by 6. The comparison shows that the model with the disc produces the correct brightness ratio. Without the disc, the \( \text{C}^{34}\text{S}(5–4) \) line is not bright enough and both lines have the wrong profile shape. The zero velocity refers to the model. The observed spectra have been shifted to match.

The temperature rises. Therefore, it is possible that the \( \text{NH}_3 \) abundance is higher in the warm disc than the cold envelope. If so, the mass of the envelope could be higher than 12.6 \( M_\odot \), assuming a lower abundance, without necessarily implying a higher mass for the disc.

5.2 The disc density factor

In the model with the flared disc, the density in the mid-plane of the disc at the radius of the disc boundary \( R_d \) is five times more dense than the smoothed density of the envelope at the same
Figure 9. The value of $\chi^2$ as a function of a single parameter. The plots show $\chi^2$ for the better fitting disc–envelope models as a function of a single parameter. The width of the collection of points in each figure is a qualitative measure of the sensitivity of the models to each parameter. Values of the parameters outside the range shown by the lower $\chi^2$ values produce very poor fits. The units of the density are the log of cm$^{-3}$, the units of stellar mass are M$_\odot$ and the units of angular momentum are au km s$^{-1}$ divided by 1000. The parameters are listed in Table 1.

5.3 The stellar mass

The stellar mass, 10.7 M$_\odot$, is constrained primarily by the disc velocities (equation 17) required to match the observed linewidths (about 10 km s$^{-1}$) and the gas temperature that determines the brightness ratios of the low- and high-excitation lines. The gas velocities and therefore linewidths due to unresolved motions within the beam depend on the stellar mass because the velocities are proportional to the square root of the mass. The gas temperature depends on the mass of the star because the stellar temperature is a strong function of the stellar mass and because the accretion rate depends on the stellar mass. In the disc, heating by both the star and by accretion (dissipation) are important (equations 11–13). In the best-fitting model, the disc temperature would be about 1/3 lower without the passive heating from the star. The temperature of the infalling envelope is determined entirely by the stellar temperature (equation 11).

5.4 The angular momentum of the envelope

The angular momentum of the envelope is determined from the widths of the spectral lines and from the velocity with respect to the local standard of rest ($V_{\text{LSR}}$) of the NH$_3$ spectra at the locations left- and right-hand side of the centre. If there were too much rotation then both of the off-centre spectra would have an incorrect $V_{\text{LSR}}$, too little, and all the NH$_3$ linewidths would be too narrow. The disc radius, $R_d = 6900$, is set by the angular momentum in the envelope, $\Gamma$, and the stellar mass (equation 4). The velocity at $R_d$, $v_k = 1.2$ km s$^{-1}$, increases inward as $r^{-1/2}$. We assume a microturbulent broadening of 1 km s$^{-1}$, added in quadrature to the thermal broadening. Since the observed linewidths are about 10 km s$^{-1}$ most of the width of the spectral lines comes from rotation and infall in the model.

The model radius of 6900 au seems large, and we would regard this as an upper limit. First, the Ulrich flow conserves angular momentum whereas real accretion flows probably involve some braking of the spin-up. If the spin-up were slower, then the disc would be found at a smaller radius. Secondly, the high density of the disc is helpful in providing optical depth to strengthen the NH$_3$ outer hyperfine lines. The Ulrich flow has a density singularity in the mid-plane which we have handled as described in Section 3.4 by a combination of gridding and smoothing. May be our mid-plane density in the envelope is too low, and to compensate, the thin disc is bigger.

5.5 The exponent of the temperature power law

The modelling suggests that the temperature of the envelope falls off very quickly away from the star, as $r^{-1}$. This implies that the dilution of the radiation is faster than spherical, $r^{-2}$. The rapid dilution is consistent with the rotationally flattened geometry. At
each radius, radiation is absorbed and re-emitted by dust in the flow. In a flattened flow, much of this reprocessed radiation escapes vertically out of the flow. In contrast, in a spherical flow, all the reprocessed radiation continues to interact with the flow at larger radii. If the gas temperature decreases outward fast enough that most of the envelope is at the minimum gas temperature, then a larger value of the exponent (more negative value of the parameter $p$ in equation 11) would have no further effect. We assume a minimum temperature of $10 \, \text{K}$, a typical temperature of molecular gas. We derive an upper limit $p < -1$.

### 5.6 The disc temperature multiplier

The disc temperature multiplier determines the relative importance of active disc heating, owing to accretion and dissipation (equation 12), and passive heating from the protostar (equation 11). With active disc heating, the temperature in the disc is about 50 per cent above that from passive heating alone.

### 6 CONCLUSIONS

This investigation shows that a standard model of a freely falling, rotationally flattened accretion envelope around a rotationally supported disc is able to reproduce the NH$_3$, CH$_3$CN and C$^{34}$S spectral line observations of IRAS20126. Both the disc and envelope components are required to fit the observed brightness of both the low- and high-excitation lines, and to fit the observed linewidths.

This disc–envelope model was developed for low-mass stars and is quite successful in explaining many of their observable characteristics. The success of this model in explaining the molecular line observations of the massive star IRAS20126 suggests that at least up to $10 \, \text{M}_\odot$, the accretion processes of massive stars are similar to those of solar mass stars.

There are some differences. Although the mass of the disc is uncertain owing to its dependence on the NH$_3$ abundance, the disc mass is a significant fraction of the mass of the star. Furthermore, the extent of the disc is quite large, ~6000 au. This suggests that self-gravity in the disc is dynamically important, and therefore, the disc may be unstable to local fragmentation and the formation of companion stars.

The physical model in combination with the molecular line radiative transfer presented in this paper has a further application to other massive star-forming regions. The spatial resolution and sensitivity of the present generation of interferometers cannot spatially resolve accretion discs in massive protostellar objects at multi-kpc distances. However, spectral lines sampling a wide range of densities and temperatures can still provide constraints to the physical structure of the core and disc. As shown in IRAS20126, the spectral lines formed in the higher density and temperature regions confirm the presence of an accretion disc. While future observatories such as the Atacama Large Millimeter Array (ALMA) will be able to spatially resolve the flow on the disc scale, before the science commissioning of ALMA, this method is a promising technique to probe the spatially unresolved regions of the flow and disc using data from the current interferometers.

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### REFERENCES

Andre P., Ward-Thompson D., Barsony M., 1993, ApJ, 406, 122
Beckwith S. V. W., Sargent A. I., 1993, ApJ, 402, 280
Caproni A., Abraham Z., Vilas-Boas J. W. S., 2000, A&A, 361, 685
Cesaroni R., Churchwell E., Hofner P., Walmsley C. M., Kurtz S., 1994, A&A, 288, 903
Cesaroni R., Felli M., Walmsley C. M., Olmi L., 1997, A&A, 325, 725
Cesaroni R., Felli M., Jenness T., Neri R., Olmi L., Robberto M., Testi L., Walmsley C. M., 1999, A&A, 345, 949
Cesaroni R., Neri R., Olmi L., Testi L., Walmsley C. M., Hofner P., 2005, A&A, 434, 1039
deBuizer J. M., 2007, ApJ, 654, L147
Estalella R., Mauersberger R., Torrelles J. M., Anglada G., Gomez J. F., Lopez R., Murders D., 1993, ApJ, 419, 698
Galvan-Madrid R., Kato E., Zhang Q., Kurtz S., Rodriguez L. F., Ho P. T. P., 2009, ApJ, 706, 1036
Hartmann L., 2001, Accretion Processes in Star Formation. Cambridge Univ. Press, Cambridge
Herbst E., Klemperer W., 1973, ApJ, 185, 505
Ho P. T. P., Townes C. H., 1983, ARA&A, 21, 239
Kahn F. D., 1974, A&A, 37, 149
Kawamura J. H., Hunter T. R., Tong C.-Y. E., Blundell R., Zhang Q., Katz C., Papa D. C., Sridharan T. K., 1999, PASP, 111, 1088
Keto E., 1990, ApJ, 355, 190
Keto E., 2002, ApJ, 580, 980
Keto E., 2007, ApJ, 666, 976
Keto E., Wood K., 2006, ApJ, 637, 850
Keto E., Ho P., Haschick A., 1987, ApJ, 318, 712
Keto E., Rybicki G. B., Bergin E. A., Plume R., 2004, ApJ, 613, 355
Lada C. J., 1987, in Peimbert M., Jugaku J., eds, IAU Symp. 115, Star Forming Regions. Kluwer, Dordrecht, p. 1
Lamers H. J. G. L. M., Cassinelli J. P., 1999, Introduction to Stellar Winds. Cambridge Univ. Press, Cambridge
Larson R. B., Starrfield S., 1971, A&A, 13, 190
Lebron M., Beuther H., Schilke P., Stanke T., 2006, A&A, 448, 1037
Mendoza S., Canto J., Raga A. C., 2004, Revista Mexicana Astron. Astrofisica, 40, 147
Pingle J., 1981, ARA&A, 19, 137
Qiu K., Zhang Q., Megech S. T., Gutermuth R. A., Beuther H., Shepherd D. S., Testi L., De Pree C. G., 2008, ApJ, 685, 1005
Rybicki G. B., Hummer D. G., 1991, A&A, 245, 171
Shepherd D. S., Yu K. C., Bally J., Testi L., 2000, ApJ, 535, 833
Shu F. H., Adams F. C., Lizano S., 1987, ARA&A, 25, 23
Sridharan T. K., Williams S. J., Fuller G. A., 2005, ApJ, L73
Su Y.-N., Liu S.-Y., Chen H.-R., Zhang Q., Cesaroni R., 2007, ApJ, 671, 571
Ulrich R., 1976, ApJ, 210, 377
Whitney B. A., Wood K., Bjorkman J. E., Wolff M. J., 2003, ApJ, 591, 1049
Whitney B. A. et al., 2008, AJ, 136, 18
Wilking B. A., Lada C. J., Young E. T., 1989, ApJ, 340, 823
Zhang Q., Hunter T. R., Sridharan T. K., 1998, ApJ, 505, L151
Zhang Q., Hunter T. R., Sridharan T. K., Cesaroni R., 1999, ApJ, 527, L117