Properties of Odd Gap Superconductors

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A new class of superconductors with the gap function odd under time reversal is considered. Some of the physical properties of these superconductors such as the Meissner effect, composite condensate, gapless spectrum and transition from the odd gap superconductor to the BCS state at lower temperatures are discussed.

Key words: superconductivity, odd frequency gap, composite operator.

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The discovery of high temperature superconductivity has stimulated new approaches to the phenomenon of superconductivity in the strongly correlated systems. Recently a new class of singlet superconductors has been proposed in [1] with a remarkable feature that the gap function $\Delta(k, \omega)$ is odd under inversion of momentum and frequency separately. The original motivation for considering this new class was the fact that this pairing channel is insensitive to the short range Coulomb repulsion and thus might be a relevant pairing channel in strongly correlated systems [2]. This new class of superconductors was called odd frequency gap superconductors, hereafter odd gap superconductors. In the context of the superfluidity of $^3He$ the analogous class of spin triplet odd gap superconductors was considered by Berezinskii some time ago [3].

Here we will discuss some general properties of the odd gap superconductor, such as the Meissner effect, composite operator description of odd gap superconductors and the second phase transition from the odd gap superconductor to the BCS superconductor [4].

We begin with the symmetry equation, from which the existence of the odd class of superconductors follows [1,3]. Consider spin singlet anomalous function $F(k, \omega_n) = \frac{1}{2} \sum_{\alpha, \beta} \int d\mathbf{r} \int_{-\beta}^{\beta} d\tau \ e^{i \omega_n \tau} e^{-i \mathbf{k} \cdot \mathbf{r}} \langle T_\tau c_\alpha(\mathbf{r}, \tau) c_\beta(0, 0) \rangle (i\sigma^y)_{\alpha\beta}$ with $(i\sigma^y)_{\alpha\beta}$ being a spin metric tensor, $\tau$ being the Matsubara time and $\beta = 1/T$. Note that the anomalous Green’s function is explicitly written in a general spin-singlet form. The same set of definitions holds for a gap function $\Delta(k, \omega)$. The only constraint on the possible symmetry of $F(k, \omega)$ and $\Delta(k, \omega)$ comes from the fact that the $c_\alpha(\mathbf{r}, \tau)$ are fermions, and using the definition of the time ordering operator one can show that [1]:

$$F(k, \omega_n) = F(-k, -\omega_n), \quad \Delta(k, \omega_n) = \Delta(-k, -\omega_n) \quad (1)$$

Apart from the standard BCS (even gap) solution of this constraint in terms of the $P$-even (parity) and $T$ even (time reversal), we immediately find that there is a new class of solutions with $P$-odd and $T$-odd behavior of the anomalous function $F$ and gap
\( \Delta \) [1]. In this new class the singlet \( p \) wave pairing state is allowed, inverting relation between spin state and orbital parity. There is no contradiction with the Pauli principle since the equal time wave function vanish in this state.

It is clear that the physical properties of the odd gap superconductors are different from we are familiar with from BCS superconductors. The main difference comes from the fact that the gap function \( \Delta(k, \omega) \), as well as anomalous function \( F(k, \omega) \), are odd functions of frequency (time). We list some of the peculiar properties of odd gap superconductors [1,4]:

1) Broken \( P \) and \( T \) symmetries. By definition singlet odd gap superconductor breaks \( T \) and \( P \).

2) Quasiparticle spectrum. The quasiparticle spectrum of the odd gap superconductor is gapless because the equal time gap vanish \( \Delta(k) = \sum_n \Delta(k, \omega_n) = 0 \). The pole of the Green’s function defines the quasiparticle spectrum on the superconducting state at \( \omega^2 - \epsilon_k^2 - \Delta^2(k, \omega) = 0 \). For linearized \( \Delta(k, \omega) = a_k \omega \) at \( \omega \rightarrow 0 \) we find that the quasiparticle spectrum is given by the renormalized free particle dispersion

\[
\omega_k = \epsilon_k (1 - a_k^2)^{-1/2}
\]

so that quasiparticle spectrum remains gapless in the odd gap superconductor. This also implies that the low temperature specific heat and the penetration depth in this superconductor will be power law functions of temperature.

3) Order parameter and composite operator. The odd gap superconductor has a new order parameter. For any odd in time gap function for both spin channels, assuming analyticity of \( \Delta_{\alpha\beta} \) in small relative pair field time \( \tau \), we have \( \Delta_{\alpha\beta}^{\text{odd}} = 2\tau \Delta_{\alpha\beta}^{(1)} + \mathcal{O}(\tau^3) \). Thus the slope \( \Delta_{\alpha\beta}^{(1)} = \langle \dot{c}_{\alpha,k} c_{\beta,-k} - c_{\alpha,k} \dot{c}_{\beta,-k} \rangle \) of the gap function at \( \tau = 0 \) can be taken as the order parameter for odd gap superconductor. To calculate the time derivative of any operator we need to use the Hamiltonian. Let us consider a Kondo lattice model with conduction band in which the superconducting instability takes place and with the set of localized spins in the second band: \( H = \sum_{k,\sigma} \epsilon_k c_\sigma^\dagger(k)c_\sigma(k) + J \sum_{k,p} c_\alpha^\dagger(k -
\[ p c_\alpha(k) \sigma_{\alpha \beta} S_p + H_{\text{spin}}, \] where \( H_{\text{spin}} \) is the Hamiltonian, which describes spins with the propagator \( D(r, \tau) = -\langle T_r S(r, \tau) S(0, 0) \rangle \). Using equation of motion for \( \dot{c}_\alpha(k) \) one finds for the Berezinskii odd gap triplet state with \( K = (\sigma^y \sigma)_{\alpha \beta} \Delta^{(1)}_{\alpha \beta} \).

\[ K = J \sum_{k,p} \langle c_\alpha(k - p) c_\beta(k) \sigma_{\alpha \beta}^y \otimes S_p \rangle \] (3)

The order parameter in Eq.(3) is a \textit{composite operator} describing the pairing in the odd gap superconductor. The spin triplet condensate occurs in the form of a bound state of the \( S = 0 \) Cooper pair (the first part in the product in Eq.(3)) and the \( S = 1 \) spin boson \( S \). Analogous considerations for the spin singlet shows that \textit{composite operator} describes the spin triplet Cooper pair bound with spin \( S = 1 \) boson to yield a total singlet. This form of the \textit{composite operator} was proposed originally by Berezinskii [3] and later considered in [6,7,4,5].

Thus we are led to consider the susceptibility in the \textit{composite operator} channel to reveal the odd gap instability in this Hamiltonian formalism. The simplest minimal model which leads to the mean field instability with the \textit{composite} condensate is quadratic in \( K \) and is analogous to the BCS Hamiltonian for the weak coupling theory:

\[ H = \sum_{k,\sigma} \epsilon_k c^\dagger_\sigma(k) c_\sigma(k) + V \sum_{k,p} c^\dagger_\alpha(k - p) c_\beta(k) \sigma_{\alpha \beta}^y S_p \cdot K + |K|^2 / 2 \] (4)

The order parameter \( K \) describes the new \textit{anomalous vertex} in the superconducting state with two fermions and one boson vanishing in the condensate, similar to the two fermion anomalous Green’s function in the BCS superconductor.

The diagram corresponding to the \( K \) \textit{anomalous vertex} is shown in the Fig.1. The selfconsistency equation for \( K \) involves the \textit{three particle} (two fermions and boson) scattering vertex \( V \) and gives \( K = T^2 \sum_{n,m,p,k} G(\omega_n, p) G(\omega_m, k) D(p + k, \omega_n + \omega_m) \). Taking \( D(p, \omega) \sim \delta(\omega) \delta(p) \) we recover the BCS selfconsistency equation, what corresponds to the case when spins are ferromagnetically ordered and can be factorized in the Eq.(3).

The results we obtain from this phenomenological Hamiltonian are similar to the
predictions made in the Elaishberg theory for odd gap superconductor with spin boson interaction mediating pairing in the odd frequency channel.

4) Critical coupling. We find a critical coupling $g_{\text{crit}} = N_0 V_{\text{crit}} = O(1)$ for the odd frequency superconducting instability. This occurs because the gap function is odd in time (frequency) and the phase space available for pairing interaction is greatly diminished at low frequency. The BCS like infrared instability is directly related to the fact that the gap function $\Delta(k, \omega)$ is nonzero at $\omega \to 0$. Thus for any $\Delta(k, \omega) \to 0$, as $\omega \to 0$ the transition takes place only above some threshold \[^{[1]}\] . The phase transition into the odd gap superconducting phase is of the second order with the order parameter $K$ having the mean field exponent $\nu = 1/2$: $|K| \sim (T_c - T)^{\frac{1}{2}}$, $|K| \sim (g - g_{\text{crit}})^{\frac{1}{2}}$.

5) Meissner effect. There is the Meissner effect for the odd gap superconductors. We have at the moment two distinct approaches to the calculation of the Meissner effect in these superconductors. The straightforward use of the normal and anomalous Green's functions in the current-current correlator yields a negative Meissner effect for the spatially homogeneous ground state in the vicinity of $T_c$ (see however \[^{[2]}\] ). On the other hand using the composite operator description of the odd gap superconductor, as in Eq.(4) it is easy to show the existence of the positive Meissner effect in the homogeneous phase with momentum independent $K$ \[^{[3]}\] . This difference reflects the choice of the spatial structure of the ground state in composite operator .

6) Transition from the odd gap superconductor to the BCS superconductor. The effective theory of odd gap superconductor with composite order parameter has a second phase transition into the BCS superconductor at lower temperatures. The anomalous vertex $K$ is a charge $2e$ operator. Thus any other condensate with charge $2e$ is allowed in this phase and the transition could take place at any temperature below the first transition into the odd gap phase. To see this consider the even frequency spin singlet anomalous Green’s function $F_{\text{BCS}}(r,t) = -\langle T_r c_\alpha(r,\tau) c_\beta(0,0) \rangle \sigma^y_{\alpha\beta}$. The second order expansion in $K$ in the trace inside the brackets in $F_{\text{BCS}}$ yields the term which is a
source for the spin singlet BCS anomalous function (see Fig.2) with:

\[ F_{BCS}(k, n) = -TK^2G(k, n)G(-k, -n) \sum_{m, p} F^*_{BCS}(p, m)D(p - k, m - n) \]  (5)

It follows that the spin triplet odd gap superconductor might be unstable at lower temperatures towards the even frequency spin singlet superconductor. How relevant this new pairing channel is will be investigated. On the other hand the direct exchange between particles due to \( D(k, \omega_n) \) can produce only spin triplet even gap and one has to compare the free energies of different states. Because the symmetry of the BCS order parameter is different from the symmetry of the odd gap, we generally expect the second transition to occur at temperatures away from the \( T_c \) for the odd gap superconductor. The effective coupling in the BCS channel is proportional to \( K^2 \) and is small near \( T_c \) for the odd gap superconductor thus making second transition temperature \( T_{c}^{BCS} < T_c \).

The crucial difference of Eq.(5) from the BCS selfconsistency equation is that the vertex is anomalous and as a result we have \( F_{BCS} \) on the l.h.s. related to \( F^*_{BCS} \) on the r.h.s. of Eq.(5), whereas in the BCS theory anomalous function is expressed through itself in the selfconsistency equation. This also means that the phase of \( K \) is related to the the phase of \( F_{BCS} \).

It is possible that this hierarchy of transitions is an inherent property of any odd gap superconductor. It might be that this phenomenon is related to the second phase transition observed in some of the heavy fermions far inside the superconducting phase.

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REFERENCES

[1] A.V. Balatsky and E. Abrahams, Phys. Rev. B 45, 13125 (1992).

[2] E. Abrahams, A.V. Balatsky, J.R. Schrieffer and P. B. Allen, Phys. Rev. B 47, 513, (1993).

[3] V.L. Berezinskii, JETP Lett. 20, 287 (1974); [ZhETP Pisma 20, 628 (1974)].

[4] E. Abrahams, A.V. Balatsky, D.J. Scalapino and J.R. Schrieffer, to be published.

[5] P. Coleman, E. Miranda and A. Tsvelik, Phys. Rev. Lett. 70, 2960, (1993).

[6] V.J. Emery and S. Kivelson, Phys. Rev. B 46, 10812, (1992).

[7] A.V. Balatsky and J. Bonca, Phys. Rev. B 47, September 1, (1993).
FIGURES

FIG. 1. The anomalous vertex corresponding to the composite operator which describes the binding of the Cooper pair with boson in the odd gap superconductor condensate.

FIG. 2. The second order in K diagram which leads to the nontrivial solution for BCS spin singlet gap and to Eq.(5) for weakly momentum dependent K. The vertex in this diagram is anomalous and does not conserve the number of particles, which makes this diagram different from the gap equation in the standard Eliashberg theory.