Signature of phase coherence on the electric and magnetic response of 10⁵ non-connected Aharonov-Bohm rings is measured by a resonant method at 350 MHz between 20 mK and 500 mK. The rings are etched in a GaAs-AlGaAs heterojunction. Both quantities exhibit an oscillating behaviour with a periodicity consistent with half a flux quantum Φ₀/2 = h/2e in a ring. We find that electric screening is enhanced when time reversal symmetry is broken by magnetic field, leading to a positive magnetopolarisability, in agreement with theoretical predictions for isolated rings at finite frequency. Temperature and electronic density dependence are investigated. The dissipative part of the electric response, the electric absorption, is also measured and leads to a negative magnetoconductance. The magnetic orbital response of the very same rings is also investigated. It is consistent with diamagnetic persistent currents of 0.25 nA. This magnetic response is an order of magnitude smaller than the electric one, in qualitative agreement with theoretical expectations.

I. INTRODUCTION

At mesoscopic scale and at low temperature, electrons in metallic samples keep their phase coherence on a length Lₚ which is bigger than the sample size. Transport and thermodynamic properties of the system are then sensitive to interference between electronic wave functions and present spectacular signatures of this phase coherence. To study these effects the ring geometry is especially suitable. Indeed in the presence of a magnetic flux Φ through the ring the periodic boundary conditions for electronic wavefunctions acquire a phase factor 2πΦ/Φ₀ with Φ₀ = h/e the flux quantum [1]. As a result, the magnetoconductance of a phase coherent ring exhibits quantum oscillations which periodicity corresponds to one flux quantum through the area of the sample [2]. The phase of the first harmonics of these oscillations is sample specific so that these harmonics do not survive ensemble average. In contrast the second harmonics have a contribution which resists this averaging. This results from the interference between time reversed paths around the ring (weak localisation contribution). These Φ₀/2 periodic oscillations were observed in transport measurements on long cylinders or connected arrays of rings [3]. Their sign corresponds to a positive magnetoconductance in zero field. In the case of singly connected geometries, like full disks, the signature of weak localization consists in a single peak of positive magnetoconductance which width corresponds to Φ₀/2 through the sample [4].

Magneo-transport experiments on connected systems constitute a very sensitive and powerful probe for the investigation of sample specific signatures of quantum transport. However, because of strong coupling between the system and the measuring device, quantum corrections represent a small fraction of the conductance (of the order of 1/g where g is the dimensionless conductance expressed in e²/h units) which is still dominated by the classical Drude response in the diffusive regime g ≫ 1. There exists a number of experiments which can address some of the electronic properties of mesoscopic samples without coupling to macroscopic wires. This is the case of ac conductance experiments where Aharonov Bohm rings are coupled to an electromagnetic field. In contrast with the connected case, the response of an isolated system can be dominated by quantum effects. Moreover, the quasi discrete nature of the energy spectrum and the sensitivity to the statistical ensemble (Canonical or Grand-Canonical) are new features of isolated mesoscopic systems. In particular it has been emphasized that the average absorption of isolated mesoscopic systems is determined by the energy level statistics and its sensitivity to time reversal symmetry breaking by a magnetic field.

The first experiments done in this spirit were performed by coupling an array of disconnected GaAs/GaAlAs rings to a strip-line superconducting resonator [5]. In such a geometry the rings experience both ac magnetic and electric field. The magnetic response of the rings i.e. their orbital magnetism is directly related to persistent currents in the zero frequency limit [6] [7]. On the other hand the electric response of isolated metallic sample is related to the screening of the electric field inside the metal. The induced charge displacement is at the origin of the polarisability α, defined as the ratio between the induced electric dipole d and the applied electric field E (d = αE). The polarisability is known to be essentially determined by the geometry of the sample with correction of the order of λₑ/L, with λₑ the screening length and L the typical size of the system [8]. It has been recently predicted that this quantity is sensitive to phase coherence around the ring [9] [10] and is thus expected to present flux oscillations. The electric contribution can be in the particular case of GaAs rings of the same order of magnitude as the magnetic response [11].

To be able to distinguish between the two types of response we have designed a superconducting LC resonator which capacitive part and inductive part are physically
separated. In this paper we present measurements of both magnetic and electric response of Aharonov-Bohm rings. Note that these experiments are done on the very same array of rings for both types of response, giving us the opportunity to compare them. Preliminary account of measurement of the electric response was given in reference [19].

The paper is organised in the following way. Section II gives a detailed presentation of the sample, an array of Aharonov-Bohm rings, and the resonating technique used to measure the magnetic and electric response. Results on the non-dissipative part of the flux dependent electric response are presented in section III. A comparison with theoretical predictions is given, including frequency dependence. Temperature and electronic density dependence are also investigated. The next section focuses on the dissipative part of the magnetopolarisability of the rings. Theoretical results for this quantity are derived and compared to the experiment. The section IV is devoted to the measurements of the magnetic response of the same rings. Despite the fact that the signal is then smaller, the magnetic response of the rings is detected and compared to predictions on averaged persistent currents. We conclude by a comparison between the magnetic and electric response.

II. EXPERIMENTAL SETUP

A. The sample

1. The rings

We have studied the electric and magnetic susceptibilities of isolated Aharonov-Bohm rings. Our system is an array of $10^5$ 2D rings etched by reactive ion etching in a high mobility AlGaAs/GaAs heterojunction. The characteristics of these rings are given in table I. They are ballistic in the transverse direction and diffusive longitudinally ($l_e < L$ and $l_e > W$). It is important to perform a deep etching of the heterojunction (down to GaAs) in order to minimize high frequency losses, which have been observed to be important in etched AlGaAs. Because of etching the electronic density is strongly depressed. However we are able to recover the nominal density of the heterojunction by illuminating the rings with an infrared diode placed close to the sample in the dilution refrigerator. For each illumination a current of $10\mu A$ is run through the diode during several minutes. Measurements are done at least one hour after the illumination in order to ensure good stability of the sample. An upper value of the estimated illumination power coupled to the sample is $600$ photons/s with a wavelength of $766$ nm. With this setup we are thus able to perform measurements at different electronic density. The control on the density is rather qualitative because of the difficulty to calibrate the illumination procedure. We have checked the effect of illumination on a connected Aharonov-Bohm (AB) ring (figure 4). At zero illumination time the conductance of the ring is zero. On such a sample we can follow the AB oscillations when the resistance decreases by more than an order of magnitude with illumination. As a consequence a clear effect of illuminating the ring is to increase its conductance. The Fourier transform of the resistance of the ring is shown on figure 3. We see for each illumination an oscillation whose periodicity is consistent with a flux quantum $\Phi_0$ in the area of the ring. However the Fourier transform shows that the peak corresponding to this periodicity changes with illumination both in shape and in amplitude. The amplitude increases with illumination due to the increase of AB oscillations. The fact that the shape, and in particular the width, of the peak changes with illumination is an indication that the width of the ring increases with illumination time. To be more precise the width of the rings is multiplied by a factor 2 between the first and last curve of figure 3. Note that the increase of the electronic density has also been shown to induce an increase of the electronic mobility [20].

In order to study the disorder average we have measured conductance of a single ring and a mesh, representing a two-dimensional square array, etched in the same type of heterojunction than the rings. The magnetoresistance is shown on figure 3. As expected the AB effect disappears under ensemble averaging. The $\Phi_0/2$ oscillations on the other hand remains on the mesh. In this case the triangular shape of the magnetoresistance is attributed to weak localisation in the wire of the mesh.
TABLE I. Characteristics of the rings after illumination.

| Characteristic                          | Value                      |
|----------------------------------------|----------------------------|
| Nominal density \( n \)                | \( 3 \times 10^{11} \) cm\(^{-2} \) |
| Thomas Fermi screening length \( \lambda_s \) | \( 16 \) nm                  |
| Etched width                           | \( 0.5 \) \( \mu \)m          |
| Effective width \( W \)                | \( 6.5 \) \( \mu \)m          |
| Phase coherence length \( L_\Phi \)    | \( 80 \) mK or \( 1.66 \) GHz |
| Mean free path \( l_e \)               | \( 3 \) \( \mu \)m            |
| Diffusion coefficient \( D \)          | \( 3 \) \( \mu \)m per \( \mu \)m \( \times 100 \) |
| Mean level spacing \( \Delta \)        | 5.6                         |
| Thouless energy \( E_c \)              |                             |
| Effective width \( W \)                |                             |
| Phase coherence length \( L_\Phi \)    |                             |

2. Superconducting micro-resonator

To measure the electric or magnetic response of the rings we couple them to a superconducting micro-resonator and detect the changes in its properties. This resonator is made by optical lithography. It consists of a niobium strip-line deposited on a sapphire substrate. This substrate has been preferred to silicon or GaAs because it induces the weakest temperature dependence of the resonance frequency and gives the best quality factor due to the quality of the niobium layer on sapphire. A schematic drawing is given in figure 4. The width of the wire constituting the resonator is \( 2 \) \( \mu \)m, the thickness \( 1 \) \( \mu \)m and the spacing between two adjacent wires is \( 4 \) \( \mu \)m. The total length of the capacitance or the inductance is 10 or 20 cm. In this kind of resonator the inductance is physically separated from the capacitance by a distance of 300 \( \mu \)m, allowing to submit the sample only to an electric field (or to a magnetic field) to measure its electric (or magnetic) response. This separation between magnetic and electric response has been checked by deposition of a paramagnetic system (DPPH) alternatively on the capacitive and inductive part of the resonator. A magnetic spin resonance signal was only observed when DPPH was on the inductive part. The resonance frequency of the bare resonator varies between 200 MHz and 400 MHz depending on the geometry. Its quality factor is 10000 at 4.2 K and 200000 at 20 mK. The resonator can be modelled by an LC circuit of resistance \( r \), inductance \( L \), capacitance \( C \), whose resonance frequency is \( f_0 = 1/2\pi \sqrt{LC} \) and quality factor \( Q = \omega_0/\Delta \). From the value of the higher resonance frequencies of the resonator we have estimated that the residual capacitance of the meander line is at least 10 times smaller than \( C \). Due to the Meissner effect, the dc field just above the resonator is multiplied by 5 or 100. FIG. 2. Fourier transform of the magnetoconductance of an Aharonov Bohm ring at different illumination. The shape and amplitude of the \( \Phi_0 \) peak is strongly dependent on illumination. The curves are shifted for clarity.

2. Superconducting micro-resonator

To measure the electric or magnetic response of the rings we couple them to a superconducting micro-resonator and detect the changes in its properties. This resonator is made by optical lithography. It consists of a niobium strip-line deposited on a sapphire substrate. This substrate has been preferred to silicon or GaAs because it induces the weakest temperature dependence of the resonance frequency and gives the best quality factor due to the quality of the niobium layer on sapphire. A schematic drawing is given in figure 4. The width of the wire constituting the resonator is \( 2 \) \( \mu \)m, the thickness \( 1 \) \( \mu \)m and the spacing between two adjacent wires is \( 4 \) \( \mu \)m. The total length of the capacitance or the inductance is 10 or 20 cm. In this kind of resonator the inductance is physically separated from the capacitance by a distance of 300 \( \mu \)m, allowing to submit the sample only to an electric field (or to a magnetic field) to measure its electric (or magnetic) response. This separation between magnetic and electric response has been checked by deposition of a paramagnetic system (DPPH) alternatively on the capacitive and inductive part of the resonator. A magnetic spin resonance signal was only observed when DPPH was on the inductive part. The resonance frequency of the bare resonator varies between 200 MHz and 400 MHz depending on the geometry. Its quality factor is 10000 at 4.2 K and 200000 at 20 mK. The resonator can be modelled by an LC circuit of resistance \( r \), inductance \( L \), capacitance \( C \), whose resonance frequency is \( f_0 = 1/2\pi \sqrt{LC} \) and quality factor \( Q = \omega_0/\Delta \). From the value of the higher resonance frequencies of the resonator we have estimated that the residual capacitance of the meander line is at least 10 times smaller than \( C \). Due to the Meissner effect, the dc field just above the resonator is multiplied by 5 or 100. FIG. 2. Fourier transform of the magnetoconductance of an Aharonov Bohm ring at different illumination. The shape and amplitude of the \( \Phi_0 \) peak is strongly dependent on illumination. The curves are shifted for clarity.

2. Superconducting micro-resonator

To measure the electric or magnetic response of the rings we couple them to a superconducting micro-resonator and detect the changes in its properties. This resonator is made by optical lithography. It consists of a niobium strip-line deposited on a sapphire substrate. This substrate has been preferred to silicon or GaAs because it induces the weakest temperature dependence of the resonance frequency and gives the best quality factor due to the quality of the niobium layer on sapphire. A schematic drawing is given in figure 4. The width of the wire constituting the resonator is \( 2 \) \( \mu \)m, the thickness \( 1 \) \( \mu \)m and the spacing between two adjacent wires is \( 4 \) \( \mu \)m. The total length of the capacitance or the inductance is 10 or 20 cm. In this kind of resonator the inductance is physically separated from the capacitance by a distance of 300 \( \mu \)m, allowing to submit the sample only to an electric field (or to a magnetic field) to measure its electric (or magnetic) response. This separation between magnetic and electric response has been checked by deposition of a paramagnetic system (DPPH) alternatively on the capacitive and inductive part of the resonator. A magnetic spin resonance signal was only observed when DPPH was on the inductive part. The resonance frequency of the bare resonator varies between 200 MHz and 400 MHz depending on the geometry. Its quality factor is 10000 at 4.2 K and 200000 at 20 mK. The resonator can be modelled by an LC circuit of resistance \( r \), inductance \( L \), capacitance \( C \), whose resonance frequency is \( f_0 = 1/2\pi \sqrt{LC} \) and quality factor \( Q = \omega_0/\Delta \). From the value of the higher resonance frequencies of the resonator we have estimated that the residual capacitance of the meander line is at least 10 times smaller than \( C \). Due to the Meissner effect, the dc field just above the resonator is multiplied by 5 or 100. FIG. 2. Fourier transform of the magnetoconductance of an Aharonov Bohm ring at different illumination. The shape and amplitude of the \( \Phi_0 \) peak is strongly dependent on illumination. The curves are shifted for clarity.

2. Superconducting micro-resonator

To measure the electric or magnetic response of the rings we couple them to a superconducting micro-resonator and detect the changes in its properties. This resonator is made by optical lithography. It consists of a niobium strip-line deposited on a sapphire substrate. This substrate has been preferred to silicon or GaAs because it induces the weakest temperature dependence of the resonance frequency and gives the best quality factor due to the quality of the niobium layer on sapphire. A schematic drawing is given in figure 4. The width of the wire constituting the resonator is \( 2 \) \( \mu \)m, the thickness \( 1 \) \( \mu \)m and the spacing between two adjacent wires is \( 4 \) \( \mu \)m. The total length of the capacitance or the inductance is 10 or 20 cm. In this kind of resonator the inductance is physically separated from the capacitance by a distance of 300 \( \mu \)m, allowing to submit the sample only to an electric field (or to a magnetic field) to measure its electric (or magnetic) response. This separation between magnetic and electric response has been checked by deposition of a paramagnetic system (DPPH) alternatively on the capacitive and inductive part of the resonator. A magnetic spin resonance signal was only observed when DPPH was on the inductive part. The resonance frequency of the bare resonator varies between 200 MHz and 400 MHz depending on the geometry. Its quality factor is 10000 at 4.2 K and 200000 at 20 mK. The resonator can be modelled by an LC circuit of resistance \( r \), inductance \( L \), capacitance \( C \), whose resonance frequency is \( f_0 = 1/2\pi \sqrt{LC} \) and quality factor \( Q = \omega_0/\Delta \). From the value of the higher resonance frequencies of the resonator we have estimated that the residual capacitance of the meander line is at least 10 times smaller than \( C \). Due to the Meissner effect, the dc field just above the resonator is multiplied by 5 or 100. FIG. 2. Fourier transform of the magnetoconductance of an Aharonov Bohm ring at different illumination. The shape and amplitude of the \( \Phi_0 \) peak is strongly dependent on illumination. The curves are shifted for clarity.
The quality factor is determined by:

\[ \frac{1}{Q} = r + \frac{Nk_e\alpha''(\omega_0)}{LC\omega_0} \]

so that, with \( LC\omega_0^2 = 1 \) at resonance:

\[ \delta\frac{1}{Q} = Nk_e\alpha''(\omega_0) - \frac{1}{Q^2}k_eN\alpha'(\omega_0) \approx Nk_e\alpha''(\omega_0) \]

provided that \( Q \gg 1 \).

The electric coupling coefficient is estimated in appendix \( \[ \] \). Knowing this value and the number of rings coupled to the resonator, it is possible to evaluate quantitatively the polarisability of the rings by measuring the resonance frequency shift (equation \( \[ \] \)) and the variation of the quality factor (equation \( \[ \] \)).

4. Magnetic coupling with the resonator

When the rings are placed on top of the inductance \( L \) of the resonator, this inductance is shifted because of their magnetic response \( \chi(\omega) = \chi'(\omega) - i\chi''(\omega) \) according to:

\[ \frac{\delta L}{L} = Nk_m\chi \]

with \( N \) the number of rings coupled to the resonator, \( k_m \) the magnetic coupling coefficient between one ring and the inductance, which has the dimension of the inverse of a volume. Note that, properly defined, the coupling coefficient \( k_m \) is of the same order of magnitude than \( k_e \). More precisely the estimation of \( k_e \) and \( k_m \) done in appendix leads to \( k_m \approx \epsilon_0\epsilon_r k_e \), as expected from reference \( \[ \] \). Following the same reasoning than for the electric coupling, the properties of the resonator are modified according to:

\[ \delta f = -\frac{1}{2}Nk_m\chi' \]

\[ \delta\left(\frac{1}{Q}\right) = Nk_m\chi'' \]

From previous equations it is in principle possible to measure the absolute value of \( \alpha \) or \( \chi \). However when a GaAs sample is on the inductive or capacitive part of the resonator the modification of the resonance is dominated by the influence of the substrate. As a consequence it is very difficult to have an accurate absolute measurement. Nevertheless relative measurements are possible so that the variation of the electric or magnetic response with magnetic field can be detected in a reliable way.
FIG. 5. Rf circuit for measuring the reflected signal from the resonator

FIG. 6. Experimental setup used to lock the frequency of the RF generator to the resonance frequency.

B. Measurement of the resonance frequency and the quality factor

The reflected signal of the resonator is measured with the setup of figure 6 and used in a feedback loop to lock the frequency of a RF generator to the resonance frequency. The setup is summarized in figure 6. The resonator is coupled capacitively to the external circuit using on-chip capacitances. In order to preserve the quality factor of the resonator we work in a configuration where the resonator is undercoupled. The RF power injected is sufficiently low (≈ 10 pW) so as not to heat the sample.

1. Detection of the resonance frequency

The frequency of the RF generator is modulated at Ω and the signal from the resonator is detected by a lock-in detector at the frequency of the modulation. The lock-in signal is to first approximation the derivative of the resonance peak: it gives an error signal i.e. this signal is zero at resonance, and changes sign when the frequency of the generator is higher or lower than the resonance frequency. Using this signal in a feedback loop the frequency of the RF generator is locked to the resonance frequency. This way, by measuring the feedback signal, one has direct access to the shift of the resonance frequency. To enhance the accuracy we modulate the magnetic field by a 1G AC field oscillating at 30 Hz, produced by a small superconducting coil close to the sample, and detect the modulated resonance frequency with a lock-in detector.

2. Detection of the quality factor

At this point we consider that the frequency of the generator is locked to the resonance frequency by the previous setup. The signal measured is the signal reflected from the resonator. As a consequence it is related to the reflexion coefficient \((Z(\omega) - Z_0)/(Z(\omega) + Z_0)\), with \(Z(\omega)\) the impedance of the resonator and the coupling capacitance and \(Z_0 = 50 \, \Omega\) the impedance matched by the external circuit. We assume that near the resonance frequency the impedance \(Z(\omega)\) reads:

\[
Z(\omega_0 + \delta \omega) = \frac{RQ^2}{1 + 2iQ \frac{\delta \omega}{\omega_0}} \tag{8}
\]

with \(\omega_0\) the resonance frequency. In the limit \(Z(\omega) \ll Z_0\), which correspond to a very undercoupled resonator, the reflected signal is a linear function of \(Z(\omega)\). As a consequence if the RF signal is frequency modulated at Ω around the resonance frequency \(\omega_0\) the reflected signal at 2Ω is related to the second derivative of the real part \(Z(\omega)\), which is proportional to \(Q^2\). This way by measuring the signal at 2Ω we have access to the quality factor. However when the frequency modulation is not small compared to the width of the resonance peak or the resonator is not very undercoupled to the external circuit, the relation between the signal at 2Ω and the quality factor is not straightforward and needs calibration.

III. FLUX DEPENDENT POLARISABILITY

In this part we present measurements of the flux dependent polarisability of the rings, which are placed on the capacitive part of the resonator as described in section II. In this configuration the resonance frequency is decreased.
expected signature of phase coherence, a numerical high pass filter with a cut-off frequency of 0.05 G\(^{-1}\) (corresponding to the arrow on figure (c)) is applied and the signal is then numerically integrated in order to have the frequency shift due to the rings (figure (d)). This shift is proportional to the variation of polarisability versus magnetic field according to formula 2. We will return to the aperiodic signal in the section devoted to illumination effect.

A. Magneto-polarisability

The frequency shift due to the rings is periodic with a period of approximately 12.5 G. From the Fourier transform (figure (c)) the period of the oscillation is deduced to be consistent with half a flux quantum \(\Phi_0/2\) in a ring with no signature of \(\Phi_0\) periodicity, as expected for an Aharonov-Bohm effect averaged over many rings \(\Phi\). Note the extra broadening (by more than a factor 2) of this \(\Phi_0/2\) peak compared to the measurements on a single connected ring. We interpret this as resulting from the dispersion in circumferences in the different rings. The sign of frequency shift is negative at low magnetic field which means according to formula \(2\) that the magneto-polarisability is positive, i.e. \(\alpha’(H) - \alpha (0) > 0\) at low magnetic field. The screening is thus better when time reversal symmetry is broken by magnetic field. The scale of the signal is given by the amplitude of the first oscillation. From figure (d) we deduced \(\delta \Phi f/f = (f(6.3\text{ G}) - f(3 G))/f = -2.5 \times 10^{-7}\). Note that this value means detecting a frequency shift of 100 Hz on a frequency of 350 MHz. With the coupling coefficient estimated in appendix 3 it leads to the value of the magneto-polarisability \(\delta \Phi \alpha /\alpha_{1D} = 5 \times 10^{-4} \pm 2 \times 10^{-4}\), where \(\alpha_{1D} = \epsilon_0 \pi^2 R^3 / \ln(R/W)\) is the calculated polarisability of a quasi one dimensional (quasi-1D) circular ring of radius \(R\).

B. Theoretical predictions

Our experiment shows that there is a flux correction to the polarisability of the rings, which is positive at low field. Let’s now compare this result to recent theoretical predictions. Since we are using a ring geometry we are going alternatively from a situation where the system presents time reversal symmetry (at flux values of \(\Phi = n \Phi_0/2\), with \(n \in \mathbb{Z}\)) to the case where time reversal symmetry is broken by magnetic field. In the Random Matrix Theory (RMT) the first case corresponds to Gaussian Orthogonal Ensemble (GOE) whereas the second is related to the Gaussian Unitary Ensemble (GUE). So the quantity to be compared with theoretical predictions, which evaluate the variation of a physical variable \(A\) between GOE and GUE, is \(\delta \Phi A\) defined as \(A(\Phi_0/4) - A(0)\).
Note that since the rings are semi-ballistic, the transition with magnetic field may not be exactly from GOE to GUE.

The polarisability of small metallic grains was studied using RMT first by Gor'kov and Eliashberg [24]. The sensitivity of the electrostatic properties of mesoscopic systems to quantum coherence has been emphasized by Büttiker for connected geometries [25]. The phase coherent correction to the polarisability of isolated systems was recently theoretically investigated. Efetov found that it is possible to relate self consistently this correction to the flux dependence of the screened potential [15]. Two recent works have calculated this effect in the diffusive regime using linear response formalism (Noat [17,27]) or supersymmetry techniques (Blanter and Miram [16,26]).

In the grand canonical ensemble (GCE) the chemical potential in each ring is supposed to be constant. It describes a situation where the rings are connected to a reservoir of particles. A priori this is not the case in the experiment where the rings are isolated but as the theory is simpler in GCE we recall first the result in this statistical ensemble. No flux dependence for the polarisability is simple in GCE we recall first the result in this statistical regime using linear response formalism (Noat [17,27]). However when \( \omega \gg \gamma \) the magnetopolarisability is related to the flux dependence of the diagonal matrix element of the screened potential:

\[
\delta \alpha_{\text{GCE}} = -\frac{2e^2}{E^2\Delta} \delta \phi \left( < |F_{\alpha\alpha}|^2 >_\mu \right) \tag{9}
\]

\(< |F_{\alpha\alpha}|^2 >_\mu \) is the disorder averaged square of the diagonal matrix element of the screened potential \( F \) at energy \( \mu \), the mean chemical potential of the rings. \( E \) is the applied electric field. We note \( \psi_\alpha \) the eigenstates of the unperturbed system. This matrix element is then given by:

\[
< |F_{\alpha\alpha}|^2 >_\mu = \int d\mathbf{r}_1 \int d\mathbf{r}_2 F(\mathbf{r}_1)F(\mathbf{r}_2)
\]

\[
< |\psi_\alpha(\mathbf{r}_1)|\psi_\alpha(\mathbf{r}_2)|^2 >_\mu \tag{10}
\]

From this expression it appears that the magnetopolarisability is related to the difference of correlation function of the eigenstates with and without time reversal symmetry. This correlation function has been computed in the diffusive regime within a supersymmetric \( \sigma \)-model approach [28,29]:

\[
V^2 < |\psi_\alpha(\mathbf{r}_1)|\psi_\alpha(\mathbf{r}_2)|^2 >_\mu = \left[ 1 + 2k_d(\mathbf{r}_1) \right] \left[ 1 + 2\Pi_D(\mathbf{r}_1, \mathbf{r}_2) \right] \tag{GOE} \tag{11}
\]

\[
V^2 < |\psi_\alpha(\mathbf{r}_1)|\psi_\alpha(\mathbf{r}_2)|^2 >_\mu = \left[ 1 + k_d(\mathbf{r}_1) \right] \left[ 1 + \Pi_D(\mathbf{r}_1, \mathbf{r}_2) \right] \tag{GUE} \tag{12}
\]

with \( V \) the volume of the sample, \( k_d(\mathbf{r}) \) a short range function which decays on the length scale of the mean free path and \( \Pi_D(\mathbf{r}_1, \mathbf{r}_2) \) the diffusion propagator. The correction due to the short range term \( k_d(\mathbf{r}) \) has been shown to be negligible [15]. By considering only the diffusion term the magnetopolarisability is given by:

\[
\delta \alpha_{\text{GCE}}' = \frac{2e^2}{E^2\Delta V^2} \int d\mathbf{r}_1 \int d\mathbf{r}_2 F(\mathbf{r}_1)F(\mathbf{r}_2)\Pi_D(\mathbf{r}_1, \mathbf{r}_2) \tag{13}
\]

Note that this derivation of the magnetopolarisability is equivalent to the one used by Noat et al. [10] based on the following RMT argument:

\[
\delta \phi \left( < |F_{\alpha\alpha}|^2 > \right) \approx -\frac{1}{2} < |F_{\alpha\alpha}|^2 >_{\text{GOE}} \tag{14}
\]

This relation can also be obtained from [1] and [2] using the fact that:

\[
\int d\mathbf{r} F(\mathbf{r}) = 0 \tag{15}
\]

due to symmetry properties of the screened potential. The calculation of the magnetopolarisability using formula [3] for the case of a quasi-1D ring (appendix [3]) leads to:

\[
\frac{\delta \alpha_{\text{GCE}}'}{\alpha_{1D}} = \epsilon_r f \left( \frac{L}{W} \right) \frac{\lambda}{W} \frac{\Delta}{E_\text{c}} \tag{16}
\]

\( f(x) \) is a function related to the geometry and the dimension of the sample. Using this expression and the value of table [3] we have \( \delta \alpha_{\text{GCE}}'/\alpha_{1D} = 1.2 \times 10^{-3} \).

In our experiment the rings are isolated, the number of electrons in each ring is supposed to be constant, so that the result of the canonical ensemble (CE) should apply. At T=0 and zero frequency the flux dependent correction

![Graph](image.png)

**FIG. 8.** Calculated evaluation of \( \delta \phi \alpha_{\text{GCE}}(\omega)/\delta \phi \alpha_{\text{GCE}} \) at different value of the parameter \( \gamma/\Delta \). Note that the value of magnetopolarisability is zero at zero whatever the level broadening.
to polarisability is found to be zero. However at $\omega \gg \Delta$ the GCE result is recovered. The complete frequency dependence of the magnetopolarisability in the CE has been recently derived by Blanter and Mirlin [23]. Following their reasoning but taking into account the level broadening $\gamma$ we can write:

$$\delta \Phi \alpha_{CE}(\omega) = \delta \Phi \alpha_{GCE} F(\omega)$$ (17)

with $F(\omega)$ a function which depends only on the statistic of energy levels:

$$F(\omega) = 1 + \int_0^\infty d\epsilon \frac{1}{\epsilon} \left( \frac{\epsilon(\epsilon + \omega) + \gamma^2}{(\epsilon + \omega)^2 + \gamma^2} + \frac{\epsilon(\epsilon - \omega) + \gamma^2}{(\epsilon - \omega)^2 + \gamma^2} \right) \left[ \delta \Phi R_2(\epsilon) + \int_0^\epsilon d\epsilon_1 \delta \Phi R_3(\epsilon, \epsilon_1) \right]$$ (18)

$R_2(\epsilon)$ and $R_3(\epsilon, \epsilon_1)$ are respectively the two and three levels correlation function, known from RMT [30,31]. By evaluating this expression versus frequency at different level broadening we get the results shown on figure 8.

The behaviour at low value of the level broadening is in qualitative agreement with result of reference [23]. In particular the magnetopolarisability is found to be zero at zero frequency (in our calculation the value of $\delta \Phi \alpha_{CE}(\omega = 0)$ is at least 25 time smaller than $\delta \Phi \alpha_{GCE}$). The present experiment was performed at $\omega/\Delta = 0.2$, the CE magnetopolarisability is equal at most to 50% of the GCE value (in the limit of small level broadening). As a consequence the expected value for $\delta \Phi \alpha_{CE}/\alpha_{1D}$ is then $6 \times 10^{-4}$ which is of the same order of magnitude as the experimental value. Note that the measurement is not sufficiently accurate to give an estimate of the level broadening by comparing the experimental result with the curves of figure 8. A very interesting extension of the experiment would be to study the magnetopolarisability at different frequencies in order to test the theoretical predictions. This could be done by working with resonators with smaller inductances.

C. Effect of temperature

The temperature dependence of the signal is also investigated. The magnetopolarisability decreases with temperature (inset of figure 8). Theoretically the effect of temperature on magnetopolarisability has not been studied yet. We will base our analysis of the temperature dependence on the hypothesis that the amplitude of the signal is related to the phase coherence length $L_\Phi$, in the same way as weak-localization. In this case the amplitude of the $\Phi_0/2$ component of the signal is proportional to $\exp(-2L/L_\Phi(T))$ [15]. In figure 9 the temperature dependence of this component is shown. We have tried to fit it using two laws for $L_\Phi(T)$. First using the behaviour deduced from the measurements on connected wires [12] we have tried the experimental value $L_\Phi^{exp}(T)$ which exhibits a $T^{-1/3}$ dependence, as expected for 1D system [23]. It leads to a poor agreement with experimental points. Using for the phase coherence time the result of electron-electron interaction in quantum dots (OD system) [14] $\tau_\Phi(T) \propto T^{-2}$, leading in the diffusive regime to $L_\Phi(T) \propto 1/T$, gives a better agreement. In this case the temperature scale is found to be 90 mK. We deduced from this value $\gamma = 1/\tau_\Phi = D/L_\Phi^2 \approx 0.8$ mK at 18 mK, i.e. much smaller than the level spacing. The phase coherence length deduced from this analysis is 10 times higher than the length measured on connected sample [12]. We relate this difference between the non-connected and connected case to the fact that whereas the connected samples are one dimensional with a continuous energy spectrum due to the strong coupling with the reservoirs, the spectrum of the non-connected rings is discrete. As a concluding remark on this temperature dependance, we want to emphasize the need of a deeper theoretical analysis of the behaviour of the magnetopolarisability versus temperature.

D. Effect of illumination

Using the procedure described in [14] we are able to study the influence of electronic density on magnetopolarisability. On figure 11 the Fourier transform of the derivative of frequency shift, when the base line due to the resonator is removed, is shown at different illumination time. As expected the Fourier transform exhibits a $\Phi_0/2$ peak. Note also the low frequency component which corresponds to the aperiodic signal seen on fig-
FIG. 10. Fourier transform of the derivative of the resonance frequency versus magnetic field at different illumination. The curves are shifted for clarity.

FIG. 11. Amplitude of the frequency shift due to the rings at different illumination time.

The \( \Phi_0/2 \) peak depends on electronic density. Its amplitude shows first an increase and then decreases at high illumination. Moreover the width of the \( \Phi_0/2 \) peak increases showing that the rings widen with illumination. Following the procedure described in section III we measured the amplitude of magnetopolarisability versus illumination time. It yields figure 11 which shows the change of the amplitude of the magnetopolarisability. At first the signal increases, then for illumination time above 1400 s the amplitude of oscillation decreases. We interpret this behaviour in the following way. Before illumination the electronic density in the rings obtained after deep etching of the 2DEG is strongly depressed compared to the nominal value. As a consequence an important fraction of the rings are likely to be localized and do not contribute to the magnetopolarisability. In this regime the signal is small. After illumination the number of rings contributing to the signal increases so that the frequency shift due to the rings increases. At high enough electronic density when the rings are sufficiently populated so that they contain delocalized electrons the theoretical results obtained in the diffusive regime are expected to be valid leading to a \( 1/g \) dependence (formula 16), with \( g = E_c/\Delta \) the dimensionless conductance. This is a possible explanation for the decrease of the magnetopolarisability observed at high illumination level. Note that we cannot exclude also a reduction of the screening length \( \lambda_s \) due to illumination. From formula 16 we deduce then a decrease of the signal. However we believe that the screening length changes very weakly with illumination because this length is essentially determined by the density of states at the Fermi energy, which is independent of energy for a 2D system (see table I).

We have analysed so far the \( \Phi_0/2 \) periodic component of the magnetopolarisability signal obtained after filtering low frequency (see figure 7). On the other hand the whole integrated signal is depicted in figure 12. One can clearly see a triangular shape dependence of the signal with magnetic field superimposed to the oscillations, very similar to the weak-localisation conductance of the connected mesh shown in figure 3. Note that this behaviour is only present at low temperature, it completely disappears for temperature higher than 300 mK. The amplitude of this extra signal due to the finite width of the rings strongly increases and sharpens with illumination. We think that it is reasonable to attribute this evolution to the increase of the width of the rings. Note that a similar evolution has been previously observed in the AC magnetoconductance of ballistic GaAs squares [35].

IV. ELECTRIC ABSORPTION

By measuring the quality factor of the resonator versus magnetic field, we have access to the flux dependent
electric absorption (formula [3]), which is related to the conductance, in the case of an electric dipole, through:

\[ G_e = \frac{\omega \alpha''}{a^2} \]  

(19)

The contribution due to the rings (figure 13) exhibits the same periodicity as the frequency shift, which corresponds to half a quantum flux in a ring. The low field signal decreases. It corresponds to a negative magnetoconductance, i.e. opposite to weak-localization. This surprising sign was pointed out in the context of the magnetoconductance of rings submitted to an oscillating magnetic flux in the discrete spectrum limit [36–38]. To explain this result one has to take into account the level spacing distribution in a disordered system [24,39]. The sign and amplitude of the typical variation of electric absorption are understandable using the fact that level repulsion in a disordered system is higher in GUE than in GOE. Following reference [18] the flux dependent electric absorption in a system described by eigenvalues \( \epsilon_\alpha \) and the corresponding eigenfunctions \( \psi_\alpha \) could be written in a linear response regime:

\[ \delta \phi \alpha'' = -\frac{2e^2}{E^2} \delta \phi \left( \sum_{\alpha \neq \beta} \frac{f_\alpha - f_\beta}{\epsilon_{\alpha \beta}} \frac{\gamma \omega}{(\epsilon_{\alpha \beta} + \omega)^2 + \gamma^2} |F_{\alpha \beta}|^2 \right. 
\]

\[ + \left. \frac{\gamma \omega}{\gamma^2 + \omega^2} \sum_\alpha \frac{\partial f_\alpha}{\partial \epsilon_\alpha} |F_{\alpha \alpha}|^2 \right) \]  

(20)

with \( \epsilon_{\alpha \beta} = \epsilon_\alpha - \epsilon_\beta \). We will first consider this expression in the GCE where an average over the chemical potential is computed. With this procedure \( \langle (f_\alpha - f_\beta)/(\epsilon_{\alpha \beta}) \rangle = -1/\Delta \mu \) and \( \langle \partial f_\alpha / \partial \epsilon_\alpha \rangle = -1/\Delta \mu \), where \( \Delta \mu \) is the range over which the average over the chemical potential is done. The first term in the right hand side of equation (21) then reads:

\[ \frac{2e^2}{E^2 \Delta \mu} \delta \phi \left( \sum_{\epsilon_\alpha \neq \epsilon_\beta} \frac{\gamma \omega}{(\epsilon_{\alpha \beta} + \omega)^2 + \gamma^2} |F_{\alpha \beta}|^2 \right) \]  

(21)

Note that in this sum the energies \( \epsilon_\alpha \) and \( \epsilon_\beta \) have to belong to the range \( [\mu - \Delta \mu/2, \mu + \Delta \mu/2] \). Using this constraint we replace the sum by an integral:

\[ \sum_{\epsilon_\alpha < \epsilon_\beta} = \frac{\Delta \mu}{\Delta} \int_0^{\infty} \frac{de}{\Delta} R_2(e) \]  

(22)

with \( R_2(e) \) the two energy level correlation function. In this expression we neglect the flux dependence of the matrix element and note its average value \( \langle |F_{\alpha \beta}|^2 \rangle > \mu, \omega \). With this approximation equation (21) reads:

\[ \frac{2e^2}{E^2 \Delta} \langle |F_{\alpha \beta}|^2 \rangle > \mu, \omega \int_0^{\infty} \frac{de}{\Delta} \left( \frac{\gamma \omega}{(\epsilon + \omega)^2 + \gamma^2} \right. 
\]

\[ + \left. \frac{\gamma \omega}{(\epsilon - \omega)^2 + \gamma^2} \right) \delta \phi R_2(e) \]  

(23)
The Debye term of equation 24 is equal in the GCE to:

$$\frac{2e^2}{\hbar^2} \frac{\gamma \omega}{\omega^2 + \gamma^2} \delta \Phi (|F_{\alpha\alpha}|^2)$$

(24)

In the GCE in the dynamical regime the flux correction to the polarisability is given by formula 3 at $T = 0$. Using the following correlation function 24:

$$V^2 < \psi_\alpha^* (r_1) \psi_\beta (r_1) \psi_\alpha (r_2) \psi_\beta^* (r_2) >_{\mu,\omega} = k_d (r) + [1 + k_d (r)] \Pi_D (r_1, r_2) \quad (\text{GOE})$$

(25)

we have $\delta \Phi (|F_{\alpha\alpha}|^2 >_{\mu,\omega} \approx - < |F_{\alpha\beta}|^2 >_{\mu,\omega}^{\text{GOE}}$. Hence:

$$\frac{\delta \Phi \alpha''_{\text{GCE}} (\omega)}{\delta \Phi \alpha_{\text{GCE}}} = - \frac{\gamma \omega}{\omega^2 + \gamma^2} + \int_0^\infty \frac{d \epsilon}{\Delta} \left[ \frac{\omega \gamma}{(\epsilon + \omega)^2 + \gamma^2} \right] \delta \Phi R_2 (\epsilon)$$

(26)

which can be evaluated numerically (figure 3).

For isolated rings we have to apply the result of CE. It is then possible to estimate the correction to electric absorption by using the same treatment as for the real part of polarisability. It leads to:

$$\frac{\delta \Phi \alpha''_{\text{GCE}} (\omega)}{\delta \Phi \alpha_{\text{GCE}}} = \int_0^\infty \frac{d \epsilon}{\Delta} \frac{1}{\epsilon} \left[ \frac{\omega \gamma}{(\epsilon + \omega)^2 + \gamma^2} \right] \delta \Phi R_2 (\epsilon) + \int_0^\infty \frac{d \epsilon}{\Delta} \delta \Phi R_3 (\epsilon, \epsilon_1)$$

(27)

Numerical estimation of this formula at different value of the level broadening leads to the behaviour shown on figure 13. The electric absorption is always negative at low frequency and may change sign at low value of $\gamma/\Delta$.

In order to compare these calculations with our experimental result we will compute the ratio $\delta \Phi \alpha'' / \delta \Phi \alpha$. It is worth noting that in our experimental this quantity is given according to equations 23 by $\delta \Phi (1/Q)/(2 \delta \Phi f/f)$ and is independent of the number of rings coupled to the resonator or the electric coupling coefficient $k_e$. Experimentally we find $\delta \Phi \alpha'' / \delta \Phi \alpha = -0.2$ at illumination time zero and -0.23 after 420 s of illumination. Theoretically, at $\omega/\Delta = 0.2$, $\delta \Phi \alpha''_{\text{GCE}} (\omega) = 0.5 \delta \Phi \alpha_{\text{GCE}}$ and $\delta \Phi \alpha'' (\omega)/\delta \Phi \alpha_{\text{GCE}} = -0.5$ so that the expected value of $\delta \Phi \alpha'' (\omega)/\delta \Phi \alpha_{\text{GCE}} (\omega)$ is around 1. It corresponds to small $\gamma/\Delta$. For higher value of this parameter the ratio is of the same amplitude or higher. As a consequence the predicted behaviour is consistent with the experimental value for the sign, but the theoretical amplitude is too high by more than a factor 2. This conclusion is different from our previous statement where the frequency dependence of the real part of magnetopolarisability was not taken into account 19.

V. MAGNETIC RESPONSE

Due to the design of the resonator we have also the opportunity to investigate the magnetic response of the same Aharonov-Bohm rings used for the measurements of the electric response. In this case the rings are placed on top of the inductive part of the resonator. Note that to do so we have to warm-up, cool down and re-illuminate the rings. As a consequence, strictly speaking, the rings are not the same than for the measurement of the electric response because the electronic density and the disorder realisation in each ring is not exactly the same from one run to the other. Nevertheless due to the fact that we are measuring an ensemble average quantity the change in disorder realisation of each ring does not modify the result of the experiment. Moreover we have checked (on the electric response measurement) that, for the same illumination procedure, the result varies within a 15 % range.
A. Flux dependent orbital magnetism

The signal measured at zero illumination, after subtracting the base line due to the resonator, is shown in the lower part of figure 16. inspired by our previous analysis we decompose the measured field dependent part of the signal into an aperiodic and a periodic part which corresponds to $\Phi_0/2$ in a ring (figure 14). We interpret the $\Phi_0/2$ component as the contribution of electronic trajectories enclosing the whole ring. On the other hand the triangular shape signal could be due to the contribution of trajectories confined in the finite width of the ring. The amplitude of the $\Phi_0/2$ periodic component of the signal is $\delta f/f = -1.5 \times 10^{-8}$. We deduce from formula (4) and evaluation of the magnetic coupling done in appendix A that the flux dependent magnetic response of the ring is $\delta \Phi = 5.4 \times 10^{-24} \pm 2.1 \times 10^{-24}$ m$^{-3}$. In the following we first assume that the main contribution to this signal is due to the flux derivative of the persistent currents [4] and then discuss finite frequency effects. If the flux dependence of persistent currents is $I(\Phi) = I_0 \sin (4\pi \Phi/\Phi_0)$, we deduce:

$$I_0 = -\frac{\delta \Phi \chi \Phi_0}{2\mu_0 4\pi S^2}$$

with $S$ the surface of the ring. We find then a diamagnetic average persistent current, the amplitude of which is $|I_0| = 0.25 \pm 0.1$ nA. The aperiodic component of the signal corresponds on the other hand to low field paramagnetism.

B. Persistent currents

Let’s now compare our result for the average persistent currents to other experimental results and to theoretical predictions. A $\Phi_0/2$ periodic diamagnetic persistent current has been also observed in arrays of metallic rings [1,2]. The expected amplitude of the averaged current due to repulsive electron-electron interactions from first order Hartree-Fock calculation [4] is $E_c/\Phi_0 = 1.5$ nA, this value is expected to be decreased by higher order terms. Considering on the other hand theoretical predictions for non interacting electrons [2] the expected value is between $\sqrt{\Delta E_c}/\Phi_0 = 0.6$ nA and $\Delta/\Phi_0 = 0.3$ nA. In both cases the currents are predicted to be paramagnetic. The rather small difference between interacting and non interacting electrons is very specific to the GaAs rings where the number of electrons is small. The measured signal is consistent for the amplitude but not for the sign (unless assuming attractive interactions) with theoretical predictions.

It may also be important to take into account an effect of frequency for the flux dependent orbital magnetism. In fact by applying a formalism very similar to the one used for magnetopolarisability the variation of the real part (non-dissipative) of the susceptibility of a ring submitted
to an oscillating magnetic flux in CE without interactions is given by [43]:

$$\delta \phi \chi'(\omega) = \delta \phi \left( \sum_{\alpha \neq \beta} \frac{f_{\alpha} - f_{\beta} \epsilon_{\alpha\beta}(\epsilon_{\alpha\beta} + \omega) + \gamma^2}{\epsilon_{\alpha\beta} (\epsilon_{\alpha\beta} + \omega)^2 + \gamma^2} |J_{\alpha\beta}|^2 \right)$$ (29)

with \(J_{\alpha\beta}\) the matrix element of the current operator. It is then possible to apply the same reasoning as for the real part of polarisability, and to use the fact that \(\delta \phi(\langle |J_{\alpha\alpha}|^2 \rangle) = \langle |J_{\alpha\beta}|^2 \rangle\), so that:

$$\frac{\delta \phi \chi'(\omega = 0)}{\delta \phi \chi(\omega = 0)} = \frac{1}{2} \left( 1 - \int_0^\infty \frac{d\epsilon}{\epsilon} \left( \frac{e(\epsilon + \omega) + \gamma^2}{(\epsilon + \omega)^2 + \gamma^2} - \frac{e(\epsilon - \omega) + \gamma^2}{(\epsilon - \omega)^2 + \gamma^2} \right) \left[ \delta \phi R_2(\epsilon) + \int_0^\epsilon d\epsilon_1 \delta \phi R_3(\epsilon, \epsilon_1) \right] \right)$$ (30)

The evaluation of this expression is easily deduced from the evaluation of magnetopolarisability and leads to figure [17]. Frequency induces a strong decrease of the magnetic signal for frequencies of the order of \(\Delta\) but does not seem to induce a sign change of \(\delta \phi \chi'.\) Note that in contrast with the electric response the magnetic susceptibility is maximum at zero frequency, which corresponds to persistent currents. It would be important to investigate the effect of finite frequency on the contribution due to electron-electron interactions on persistent currents.

Recently it has been suggested that the measured currents may be due to a rectifying behaviour of the rings: a high frequency noise leads then to a DC current [43]. Noise also induces dephasing. A recent paper by Kravtsov and Altshuler [44] predicts that those two quantities, average persistent current and dephasing measured by the saturation of the phase coherence time, are related in a simple way \(I = Ce/\tau_\Phi(T = 0)\). \(C\) is a constant giving the sign of the persistent current and \(\tau_\Phi(T = 0)\) the dephasing time at zero temperature. Using the value of \(\tau_\Phi = 1.5\) ns at 20 mK, deduced from measurements on connected sample, and considering the orthogonal case (absences of spin orbit, then \(C = -4/\pi\)) we deduce an expected value for persistent currents of 0.14 nA. The predicted persistent current is then diamagnetic. The sign and amplitude are then consistent with our experimental findings. On the other hand if we take the value deduced from the temperature dependence of the magnetopolarisability of non-connected rings, which is not the case considered theoretically, we deduce \(I_0 = -0.02\) nA, smaller by an order of magnitude than the experimental value.

C. Effect of illumination

The influence of electronic density on the magnetic response of the rings has been investigated by illuminating them. Different illumination times are shown in figure [18]. We observed that the triangular envelope of the signal changes sign and increases with illumination. For each illumination the Fourier transform of the signal exhibits a component which is consistent with half a flux quantum periodicity (figure [19]). One sees however that with illumination the shape of the \(\Phi_0/2\) peak in the Fourier transform is modified. The peak broadens with electronic density indicating that the width of the rings increases. Note that this width is always consistent with the one deduced from etching and depletion effects. The power of the Fourier transform integrated in the \(\Phi_0/2\) zone is constant within 10 %.

VI. CONCLUSION

We have presented measurements of electric and magnetic responses of Aharonov-Bohm rings etched in a 2DEG. They present a flux dependent correction to screening. This correction is positive in low field which means that screening is enhanced when time reversal symmetry is broken by a magnetic field. The sign of the effect is consistent with theory for isolated rings at finite frequency. The value of magnetopolarisability is \(\delta \phi \alpha/\alpha_{1D} = 5 \times 10^{-4} \pm 2.3 \times 10^{-4}\), with \(\alpha_{1D} = \epsilon_0 \pi^2 R^3/\ln(R/W)\) the calculated polarisability of a quasi-1D circular ring of radius R. The temperature dependence of magnetopolarisability is consistent with \(L_\Phi \propto 1/T\). The behaviour versus electronic density is compatible with a 1/g dependence of magnetopolarisability.

The magnetic response has been measured on the very same array of rings. The rings exhibits a signal consistent with diamagnetic average persistent currents of amplitude \(|I_0| = 0.25 \pm 0.1\) nA.

Because the measurements are done on the same rings it is possible to compare the electric and the magnetic signal. The experimental ratio between the frequency shift due to the electric or magnetic response is around 10, a value consistent with theoretical expectations taking into account the electric and magnetic coupling coefficient (appendix A and B) and the ratio between the typical matrix element of the screened potential and the current operator which leads to [18]:

$$\frac{\delta \phi \chi}{\delta \phi \alpha/\epsilon_0} \approx (Z_0 G_D)^2 \approx \alpha^2 g^2$$ (31)
with $Z_0 = \sqrt{\mu_0/\epsilon_0} = 377\Omega$ the vacuum impedance, $G_D = ge^2/h$ the Drude conductance and $\alpha \approx 1/137$ the fine structure constant. We have thus shown that the mesoscopic electromagnetic response of GaAs rings is dominated by the flux dependent polarisability instead of orbital magnetism. This would not be the case in metallic rings, where, due to the very short screening length, the mesoscopic electric response is negligible. The low field diamagnetic sign of the orbital magnetism needs further investigation both on the experimental and theoretical sides.

VII. ACKNOWLEDGMENTS

We thank B. Etienne for the fabrication of the heterojunction. We acknowledge fruitful discussions with L.P. Lévy, Ya.M. Blanter, S. Guérin and G. Montambaux and technical help of M. Nardone and P. Demianozuck.

APPENDIX A: EVALUATION OF THE MAGNETIC COUPLING

In this appendix we evaluate the magnetic coupling of one square ring with the resonator in the configuration of the experiment. The inductance $\mathcal{L}$ is modelled by two cylindrical wires separated by a distance of $2d$ (see figure 20 (a)). A ring is submitted to the magnetic field of these wires. Let’s first evaluate the mutual inductance $\mathcal{M}$ between the ring and the resonator. Using Ampere theorem the magnetic field generated by a current $I$ in the the inductance is easily calculated. The flux of this magnetic field through a ring of size $a$ located at the $(0,0)$ point is then:

$$ \Phi = \mathcal{M}I = \frac{\mu_0 a}{\pi} \ln \left( \frac{2d + a}{2d - a} \right) I \quad (A1) $$

If the ring is located at a point $(x,y)$ $\mathcal{M}$ reads:

$$ \mathcal{M} = \frac{\mu_0 a}{4\pi} \ln \left( \frac{(x + d + \frac{a}{2})^2 + y^2}{(x - d - \frac{a}{2})^2 + y^2} \right) $$

$$ \left( \frac{(x + d - \frac{a}{2})^2 + y^2}{(x - d + \frac{a}{2})^2 + y^2} \right) \quad (A2) $$

The ring submitted to a magnetic field $B$ acts as a magnetic dipole $m = \chi B/\mu_0$. This dipole is equivalent to the ring with a current $m/\alpha^2$, so that the flux in the inductance is now $\Phi = (\mathcal{L} + \mathcal{M}^2 \chi/\alpha^4)I$. We deduce from these results that $k_m = \mathcal{M}^2/(\mu_0 a^4 \mathcal{L})$ with $\mathcal{L}$ the inductance of the meander-line. From the resonance frequency and the calculation of the capacitance we deduce $\mathcal{L} = 0.05\mu H$. The rings are not perfectly well coupled to the inductance so that they are not all located at $x = 0$. Moreover because of the mylar sheet inserted between the rings and the resonator the rings are not in the plane of the resonator. To take this into account the inductance is averaged over the x position of the rings and $1.5\mu m < y < 2.5\mu m$. Within this approximations: $k_m = 1.3 \times 10^{-11} \pm 0.5 \times 10^{-11} \text{ m}^{-3}$.

APPENDIX B: EVALUATION OF THE ELECTRIC COUPLING

In this appendix we evaluate the electric coupling coefficient $k$ of one ring with the capacitance $\mathcal{C}$ of the resonator. The capacitance is modelled by two cylindrical wires of radius $r$ and separated by a distance $2d$, one wire with a linear charge of $\lambda$, the other one $-\lambda$. The electric field outside the wires is the one generated by two lines of linear charge with a linear charge $\lambda$ and $-\lambda$ separated by a distance $2d_1$ determined by $d_1 = \sqrt{d^2 - \lambda^2}$. In our case $\mathcal{C} = 10 \text{ pF}$. Using Gauss theorem we can easily calculate the electric field in the plane of the rings in every point $(x,y)$ outside the wires:

$$ E(x,y) = \frac{\lambda}{2\pi \epsilon_0} \left( \frac{x + d_1}{(x + d_1)^2 + y^2} - \frac{x - d_1}{(x - d_1)^2 + y^2} \right) \quad (B1) $$

A ring submitted to this field generates an electric dipole $P = \alpha E$ with $\alpha$ the polarisability of one ring, so that the rings submitted to electric field act as an ensemble of dipole. We model them by two line of linear charge $\sigma$ and $-\sigma$ separated by a distance $a$, and such that $\sigma b = \alpha E/a$ (see figure 20). Note that to do so the electric field has to be constant on the scale of the rings: this is roughly the case. Evaluating the potential $\delta V$ created by these two wires between each side of the capacitance, and using the relation $\delta V = -V \delta C/\mathcal{C}$ we have for rings located at
(x,y) :

$$\frac{\delta C}{C} = \frac{\sigma}{2\lambda} \ln \frac{((d-r-x)^2+y^2)((d-r+x-y)^2+y^2)}{((d-r+x-y)^2+y^2)((d-r+y^2)^2+y^2)}$$

(B2)

To have the capacitance shift induced by one ring we have to divide the previous result by the number of rings $N = 1/b$ with $b$ the length of the capacitance. Moreover as the ring are imbedded in GaAs-AlGaAs we have to divide our result by the dielectric constant of the substrate $\varepsilon_r = 12.85$. We can now evaluate the electric coupling coefficient $k_c$ defined by $\delta C/C = Nk_c \alpha$ by averaging over the x-position of the rings and considering that the rings are located between 1.5 $\mu$m and 2.5 $\mu$m in the y-direction from the resonator. Within these approximations $\varepsilon_0 \varepsilon_r k_c = 8 \times 10^{10} \pm 3.4 \times 10^{10}$ $m^{-3}$. Note that the previous result is very close to the value of the magnetic coupling coefficient $k_m$.

APPENDIX C: MAGNETOPOLARISABILITY FOR A QUASI-1D RING

In this part we evaluate the magnetopolarisability given by formula (3) for a quasi-1D ring. The diffusion propagator at frequency $\omega$ is given by:

$$\Pi_D(r,r',\omega) = \frac{\Delta S}{\pi} \sum_n \frac{\psi_n^*(r)\psi_n(r')}{-i\omega + E_n}$$

(S and $\Delta$ are respectively the surface of the ring and the mean level spacing. $E_n$ and $\psi_n$ are the eigenvalues and eigenvectors of the diffusion equation:

$$-hD \Delta \psi_n(r) = E_n \psi_n(r)$$

We consider a 2D ring of perimeter $L$, radius $R$ and width $W$, with $W \ll L$. In this case the solutions of the diffusion equation are:

$$\psi_{m,n}(x,y) = \sqrt{\frac{2}{LW}} \cos \frac{pn}{W} y \exp i2\pi n \frac{x}{L}$$

(C3)

with $m \in \mathbb{N}^*$ and $n \in \mathbb{Z}$. The modes corresponding to $m=0$ are given by:

$$\psi_{m=0,n}(x,y) = \sqrt{\frac{1}{LW}} \exp i2\pi n \frac{x}{L}$$

(C4)

$x$ is the coordinate along the ring, and $y$ the radial coordinate. In our description the ring corresponds to $y \in [0,W]$. We consider reflecting border in the y direction. The corresponding eigenvalue is:

$$E_{m,n} = E_c \left[ 2\pi n^2 + m^2 \frac{\pi}{2} \left( \frac{L}{W} \right)^2 \right]$$

(C5)

$E_c = hD/L^2$ is the Thouless energy. The mean charge density (average over the width of the ring) in the ring submitted to an electric field $E$ is:

$$\rho(x = R \cos \theta, y) = \frac{\varepsilon_0 \pi R E}{W \ln (R/W)} \cos \theta$$

(C6)

Note that using this density we recover the classical result for a quasi-1D ring $\alpha_{1D} = \varepsilon_0 \pi^2 R^3/\ln(R/W)$. In the Thomas-Fermi approximation we deduce the mean screened potential:

$$F(x = R \cos \theta, y) = \frac{R \lambda_x E}{2W \ln (R/W)} \cos \theta$$

(C7)

Using this relation and the formula for the diffusion one can do the calculation analytically. Because of the form of $F$ only the mode $(m=0,n=1)$ remains and leads to:

$$\frac{\delta \alpha'}{\alpha_{1D}} = \varepsilon_r f \left( \frac{L}{W} \frac{\lambda_x}{W} \frac{\Delta}{E_c} \right)$$

(C8)

with $f(x) = 1/(4\pi^2 \ln x/2\pi)$.

[1] See, e.g., Mesoscopic Phenomena in Solids edited by B.L. Altschuler, P.A. Lee and R.A. Webb (Elsevier, Amsterdam, 1991); Y. Imry, Introduction to Mesoscopic Physics Oxford UP, New York, 1997.
[2] S. Washburn and R.A. Webb, Adv. Phys. 35(4), 375-422 (1986).
[3] D.Yu. Sharvin and Yu.V. Sharvin, JETP Lett. 34, 272 (1981).
[4] A.G. Aronov and Yu.V. Sharvin, Rev. Mod. Phys. 59(4), 755 (1987).
[5] A.M. Chang, H.U. Baranger, L.N. Pfeiffer and K.W. West, Phys. Rev. Lett. 73(15), 2111 (1994).
[6] Y. Lee, G. Faini and D. Mailly, Phys. Rev. B 56(15), 9805 (1997).
[7] B. Reulet, M. Ramin, H. Bouchiat and D. Mailly, Phys. Rev. Lett. 75, 124 (1995).
[8] M. Büttiker, Y. Imry and R. Landauer, Phys. Rev. 96, 365 (1983).
[9] For a review see , e.g., U. Eckern and P. Schwab, Adv. Phys. 44(5), 387 (1995).
[10] V. Chandrasekhar et al., Phys. Rev. Lett. 67, 3578 (1991).
[11] E.M.Q. Jariwala, P. Mohanty, M.B. Ketchen and R.A. Webb, Phys. Rev. Lett. 86(8), 1594 (2001).
[12] D. Mailly, C. Chapelier and A. Benoit, Phys. Rev. Lett. 70, 2020 (1993).
[13] L.P. Lévy, G. Dolan, J. Dunsmuir and H. Bouchiat, Phys. Rev. Lett. 64, 2074 (1990).
[14] S. Strässler, M.J. Rice and P. Wyder, Phys. Rev. B 6, 2575 (1972); M.J. Rice, W. R. Schneider and S. Strässler, Phys. Rev. B 8, 474 (1973).
[15] K.B. Efetov, Phys. Rev. Lett. 76, 1908 (1996).
[16] Y. Noat, B. Reulet and H. Bouchiat, Europhys. Lett. 36(9), pp. 701-706 (1996).
[17] Ya.M. Blanter and A.D. Mirlin, Phys. Rev. B 57, 4566 (1998).
[18] Y. Noat, B. Reulet, H. Bouchiat and D. Mailly, Superlattices and Microstructures 23, 621 (1998).
[19] R. Deblock, Y. Noat, H. Bouchiat, B. Reulet and D. Mailly, Phys. Rev. Lett. 84(23), 5379 (2000).
[20] T. Sajoto, Y.W. Suen, L.W. Engel, M.B. Santos and M. Shaye, Phys. Rev. B 41(12), 8449 (1990).
[21] Ando, Fowler and Stern, Rev. Mod. Phys. 54(2), 437-672 (1982).

[22] These values are deduced from weak localization measurements on connected wires etched in the same heterojunction than the rings.

[23] J.P. Carini, K.A. Muttalib and S.R. Nagel, Phys. Rev. Lett. 53, 102 (1984); A. Kamenev and Y. Gefen, Phys. Rev. B 49, 14474 (1994).

[24] L.P. Gor’kov and G.M. Eliashberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 1407-1418 (1965) [Sov. Phys.-JETP 21, 940 (1965)].

[25] M. Büttiker, Phys. Scr. 54, 104 (1994). M. Büttiker and C.A. Stafford, Phys. Rev. Lett. 76, 495 (1996).

[26] Y. Noat, R. Deblock, B. Reulet and H. Bouchiat, to be published [cond-mat/0107408].

[27] Ya.M. Blanter and A.D. Mirlin, Phys. Rev. B 63, 113315 (2001).

[28] Ya.M. Blanter and A.D. Mirlin, Phys. Rev. B 53, 12601 (1996).

[29] A.D. Mirlin, Physics-Reports. 326(5-6), 259-382 (2000).

[30] M.L. Mehta, Random Matrices and the Statistical Theory of Energy Levels (New York, Academic Press, 1967).

[31] Ya.M. Blanter, Phys. Rev. B 54, 12807 (1996).

[32] B. Reulet, H. Bouchiat and D. Mailly, Europhys. Lett. 31(5-6), pp. 305-310 (1995).

[33] B.L. Altshuler and A.G. Aronov, in Electron-Electron interactions in Disordered Systems (North Holland, Amsterdam, 1985).

[34] U. Sivan, Y. Imry and A.G. Aronov, Europhys. Lett. 28, 115 (1994).

[35] Y. Noat, H. Bouchiat, B. Reulet and D. Mailly, Phys. Rev. Lett. 80 (22), 4955 (1998).

[36] N. Trivedi and D.A. Browne, Phys. Rev. B 38(14), 9581 (1988).

[37] A. Kamenev and Y. Gefen, Phys. Rev. Lett. 70, 1976 (1993).

[38] A. Kamenev, B. Reulet, H. Bouchiat and Y. Gefen, Europhys. Lett. 28 (6), 391 (1994).

[39] U. Sivan and Y. Imry, Phys. Rev. B 35, 6074 (1987).

[40] B. Reulet and H. Bouchiat, Phys. Rev. B 50(4), 2259 (1994).

[41] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. 65, 381 (1990).

[42] B.L. Altshuler, Y. Gefen and Y. Imry, Phys. Rev. Lett. 66, 88 (1991).

[43] V.E. Kravtsov and V.I. Yudson, Phys. Rev. Lett. 70, 210 (1993).

[44] V.E. Kravtsov and B.L. Altshuler, Phys. Rev. Lett. 84, 3394 (2000).

[45] Landau and Lifchitz, Electrodynamics of Continuous Media (MIR edition, Moscou, 1969).