Development of calculation procedure for research and refinement of suspended design natural frequencies by example of vehicle exhaust system

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Abstract. The article presents a finite element model exhaust system developed in the ANSA software package. Experimental research of the dynamic characteristics mounts exhaust system. and developed calculation procedure for improvement of the suspended design natural frequencies based on the experimental definition of boundary conditions by the example of the vehicle exhaust system. Requirements for natural frequencies of the exhaust system are developed; finite element models of the exhaust system are developed; physical and mechanical characteristics of mounts are defined; natural frequencies and oscillation modes of the exhaust system taking into account physical and mechanical characteristics of mounts are calculated.

1. Introduction
In recent years, vehicle NVH requirements have become more stringent. To provide vehicle NVH comfort, the intake and exhaust systems shall meet the main requirements shown in Figure 1.

![Figure 1. NVH requirements for exhaust system.](image)

The minimum natural frequency of the 8-cylinder 4-stroke ICE (internal combustion engine) designs with the 650 rpm idle shall be lower than 43.3 Hz according to formula 1:

\[ f = \frac{n \cdot i}{60 \cdot \tau} = \frac{650 \cdot 8}{60 \cdot 2} = 43.3 \text{ Hz} \]  

where \( n \) – engine crankshaft rpm;  
\( i \) – number of cylinders;
\( \tau = 2 \) for four-stroke engines, \( \tau = 1 \) for two-stroke engines.

Besides, in the mounted (fixed) state ("hard" connection with the ICE and "soft" connection with the vehicle body), the natural frequencies will increase, and therefore it will be necessary to shift frequencies above 43.3 Hz. On the basis of the foregoing, when mounted, they are above the frequency range of 40-75 Hz, since the "cold" engine is idling at 1100 rpm, and the "warmed up" engine is idling at 650 rpm [1-7].

2. Experimental research of boundary conditions

For the calculated definition of the natural frequencies and oscillation modes of the mounted systems, it is necessary to provide the characteristics of the mounts that are non-linear. Without knowledge of both the static and dynamic characteristics of the mounts, it is impossible to model the static and dynamic processes accordingly. Therefore, characteristics of the mounts were investigated, the physical and mechanical properties were obtained.

**Table 1.** Exhaust system mounts for experimental research.

| No. | Denomination               | Reference designation | Appearance |
|-----|----------------------------|-----------------------|------------|
| 1   | Front rubber mount         | FRM                   |            |
| 2   | Rear rubber mount          | RRM                   |            |

The quasistatic and dynamic loadings were performed in the axial directions along the Z and Y axes (see Fig. 2). The sign "+" is tension and the sign "-" is compression. The quasistatic loading is the definition of the force dependence on the deformation (strain) value of the exhaust system lock ring during movement of the bench hydraulic cylinder rod with the speed of no more than 10 mm/min (Table 2).

**Table 2.** Quasistatic loading conditions.

| Axis | Type of the lock ring | Front | Rear |
|------|-----------------------|-------|------|
|      | Samples reference designation | Front loading amplitude | from 0 to +200 N | from +5 to +300 N |
|      | Preliminary cycles    | 3 times up to 200 N | 3 times up to 300 N |
|      | Target values         | F1 = 75 N, S1 = 7.1 mm +/-20% | Smax = 4…6 mm at F = 300 N |
|      |                       | F2 = 150 N, S2 = 14.7 mm +/-20% | |
| Y    | Loading amplitude     | from 0 to +50 N | from +5 to +300 N |
|      | Preliminary Cycles    | 3 times up to 50 N | 3 times up to 300 N |
| Y    | Target values         | F1 = 15 N, S1 = 4.5 mm +/-20% | Smax = 10…16 mm at F = 300 N |
|      |                       | F2 = 30 N, S2 = 10.2 mm +/-20% | |
The dynamic loading is definition of parameters of the amplitude frequency response by means of a program-controlled travel of the hydraulic cylinder rod of the bench according to the harmonic motion law incrementally on each frequency from the required range according to Table 3.

Table 3. Dynamic loading conditions.

| Axis | Type of the lock ring | Front | Rear |
|------|-----------------------|-------|------|
|      | Samples reference designation | Front rubber mount | Rear rubber mount |
| Z    | Amplitude | ±0.1 mm |  |
|      | Preloading | Sequentially 50 N, 100 N, 150 N | 100 N |
|      | Frequency | from 1 to 500 Hz with the step of 2 Hz |  |
|      | Target values | Dynamic stiffness (C dyn) no more than 32 N/mm up to 40 Hz | - |

The static creeping is a definition of the sample elongation at higher temperatures when exposed to constant load during the specified period according to Table 4.
Table 4. Static loading conditions.

| Axis | Type of the lock ring | Front | Rear |
|------|-----------------------|-------|------|
| Z    | Samples designation   | reference | Front mount | Rear mount |
|      | Preloading            | 30 N  | 100 N |
|      | Temperature           | 60 °С |
|      | Exposure time         | 48 hours |
| Target values | Constant elongation L no more than 5 mm, $\Delta L = L(48 \text{ h}) - L(20 \text{ s})$ | Constant elongation L no more than 1,5 mm, $\Delta L = L(48 \text{ h}) - L(10 \text{ s})$ |

The determination of the parameters of the elastic and damping, static and dynamic characteristics of the exhaust system mounts was performed on the hydraulic testing machine MTS (hereinafter – the MTS elastomer bench) /Automated 831.50 Elastomer Test System/ (type 333).

The elastic characteristics of the exhaust system mounts at quasistatic loading along the Z axis are presented as the dependence diagrams (Fig. 3) of the deformation (strain) value on the arising force therewith and in Table 5.

Table 5. Dependence of deformation value on the arising force.

| Sample | Estimated stiffness | Inlet energy | Restored energy | Energy loss |
|--------|---------------------|--------------|----------------|-------------|
|        | N/mm                             | N-mm         | N-mm           | N-mm        |
| Front  | 11.43                           | 1773.68      | 1439.35        | 334.32      |
| Rear   | 65.08                           | 719.32       | 596.68         | 122.64      |

Figure 3. Characteristics in the course of quasistatic loadings along the Z-axis of the (a) front and (b) rear mount of the exhaust system.

The elastic characteristics of the exhaust system mount samples in the course of quasistatic loading along the Y axis are presented as the dependence diagrams (Fig. 4) of the deformation (strain) value on the arising force therewith and in Table 6.
Table 6. Dependence of the deformation (strain) value on the arising force.

| Sample  | Estimated stiffness | Inlet energy | Restored energy | Energy loss |
|---------|---------------------|--------------|-----------------|-------------|
|         | N/mm                | N-mm         | N-mm            | N-mm        |
| Front   | 2.68                | 427.89       | 330.93          | 96.96       |
| Rear    | 21.57               | 2049.35      | 1543.57         | 505.78      |

Figure 4. Characteristics in the course of quasistatic loading along the Y axis of the (a) front and (b) rear exhaust system mounts.

The elastic characteristics of the exhaust system mount samples in the course of dynamic loading are presented as the dependence diagrams (Fig. 5) of the dynamic stiffness ($K^*$, N/mm) and phase (Phase, deg.) values on the disturbing frequency at the constant displacement amplitude of ±0.1 mm.

Figure 5. Characteristics in the course of dynamic loading, preload 100 N, of the (a) front and (b) rear exhaust system mounts.

Based on the conducted experimental research, it can be concluded that:

- in the course of quasistatic loading along the Z axis, the estimated stiffness of the exhaust system mount samples amounted to: 11.4 N/mm for the front one and 65.08 N/mm for the rear one. The energy loss (consumed energy) amounted to: 334.32 for the front one and 122.64 N-mm;
- in the course of quasistatic loading along the Y axis, the estimated stiffness of the exhaust system mounts samples amounted to: 2.68 for the front one and 21.57 N/mm for the rear one. The energy loss (consumed energy) amounted to: 96.96 for the front one and 505.78 N-mm;
in the course of dynamic loading, the exhaust system mounts samples have the following stiffness:
front – up to 370 N/mm (with preload of 50 N), up to 380 N/mm (with preload of 100 and 150 N), the
maximum values are marked at the frequencies of 330-340 Hz, rear – up to 255 N/mm (with preload
of 100 N), and upon that the dependence of the stiffness on the frequency of the rear mount has a
linear character, and the maximum stiffness values are noted at the maximum examined frequency of
500 Hz;

Having defined the physical and mechanical characteristics of the mounts, the natural frequencies
and oscillation modes of the exhaust system design were calculated using these characteristics.

3. Calculated analysis

The eigenvalues and eigenvectors were found by means of the Lanczos method. The first 6 natural
frequencies and oscillation modes of the models were subject to finding; the results are given in Table 7.

| Mode | Frequency, Hz | Mode | Frequency, Hz | Mode | Frequency, Hz |
|------|---------------|------|---------------|------|---------------|
| 1    | 9.5           | 3    | 11.6          | 5    | 13.1          |
| 2    | 10.1          | 4    | 12.9          | 6    | 14.2          |

Figure 6 shows an example of the first two oscillation modes of the exhaust system design.

As can be seen in Table 8, the first natural frequency is lower than the target value. Increasing of
the natural frequencies and oscillation modes of the mounted exhaust system is possible by means of
decreasing of the oscillatory or vibrating mass in the places with the maximum vibratory displacement
amplitudes, by increasing of the stiffness in the areas with the maximum strain energy values, or by
means of these methods combining.

The analysis of the natural frequencies and oscillation modes in the mounted state shows that the
maximum vibration displacements in the frequency range are located on the rear mufflers. Decreasing
of the rear mufflers weight is not possible due to technological and acoustic reasons. Therefore, we
will use the method of stiffness increase in the areas with the maximum strain energy values. As the
elements with high strain energy values indicate the places of high elastic deformations (strain
energies). These elements have the highest direct effect on deformation (strain) at this frequency.
Therefore, increasing of the stiffness of the elements with the maximum strain energy will increase the
exhaust system vibration (oscillation) frequency.

It is known that the strain energy is substantially the energy accumulated in the design elements in
the course of elastic deformation. If considering the quasistatic spring compression with the F force
when the loaded element moves over δ, then the energy can be recorded as follows

\[ U = 0.5 \cdot F \cdot \delta \]

As \( F = k \cdot \delta \) then
\[ U = 0.5k\delta^2 \] (3)

It can be easily shown that modifying the element with the maximum strain energy results in the maximum effect, for example, in terms of the maximum design deflections, etc.

Figures 7-9 show deformation (strain) localization areas and deformation distribution patterns in the exhaust system load-bearing parts.

**Figure 7.** Strain energy localization on the natural frequency of (a) 9.5 Hz and (b) 10.1 Hz.

Figure 7 shows that the main elements defining the stiffness at design vibration (oscillation) at the frequency of 9.5 (a) and 10.1 Hz (b) are the pipes before the mufflers and rear muffler-to-body mounting brackets while the maximum strain energy amplitude is located on the pipe bends.

**Figure 8.** Strain energy localization on the natural frequency of (a) 11.6 Hz and (b) 12.9 Hz.

Figure 8 shows the maximum strain energy amplitudes located on the pipe bends before the mufflers and on the rear muffler-to-body mounting brackets.

**Figure 9.** Deformation (strain) localization on the natural frequency (a) 13.1 Hz and (b) 14.2 Hz.

Figure 9 shows the maximum strain energy amplitudes located also on the pipe bends before the mufflers and on the front muffler partitions (baffles).
Based on the analysis of the strain energies of the first six modes, the exhaust system design elements were identified: the pipes before the mufflers and muffler-to-body mounting brackets. In order to increase the natural frequencies, the stiffnesses of these elements were increased by means of increasing of the pipe thickness, adding stiffening ribs and increasing the thicknesses of the exhaust system-to-body mounting bracket plates shown in Figures 10-12.

4. Conclusion
The developed calculation method allows achieving target natural frequencies and vibration modes of design of exhaust systems of modern vehicles based on experimental investigations of physical and mechanical characteristics of mounts. This method can be used in research and academic institutions, as well as in manufacturing enterprises specializing in development and production of vehicle intake and exhaust systems, as well as by automobile manufacturers.

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