Bath induced phase transition in a Luttinger liquid

Saptarshi Majumdar
LPTMS, Université Paris-Saclay
Work done with Alberto Rosso (LPTMS), Laura Foini (IPhT) and Thierry Giamarchi (University of Geneva)

Link: https://doi.org/10.48550/arXiv.2210.01590
Motivation: Localisation and dissipation

- Bray and Moore (PRL, 1982), Caldeira and Leggett (Physica A, 1983): Dissipative two-state systems.

  System: single spin-1/2 particle in constant magnetic field along x and z direction

  Bath: collection of simple harmonic oscillators

Spectral function of the bath:

\[ J(\Omega) \approx \pi \alpha \Omega^s \] (for small \( \Omega \))

- \( s = 1 \): ohmic bath
- \( 0 < s < 1 \): subohmic bath \( \Rightarrow \) Localisation of system!

Question: What happens in one dimension?
Description of the microscopic system

Hamiltonian:

\[ H = H_S + H_B + H_{SB} \]

\[ H_S = \sum_{j=1}^{N} J_z S_j^z S_{j+1}^z + J_{xy} \left( S_j^x S_{j+1}^x + S_j^y S_{j+1}^y \right) \]

\[ H_B = \sum_{jk} \frac{P_{jk}^2}{2m_k} + \frac{m_k \Omega_k^2}{2} X_{jk}^2 \]

\[ H_{SB} = \sum_{j=1}^{N} S_j^z \sum_{k} \lambda_k X_{jk} \]

Schematic diagram:

Dissipative baths at T=0

XXZ spin chain

Strictly local, ohmic baths (s=1).
Last step : Path integral

Spin chain \[ \xrightarrow{\text{Bosonisation}} \xrightarrow{\text{Path integral}} \xrightarrow{\text{2D statistical field theory}} \xrightarrow{\text{Integrate out bath modes}} \text{Effective field theory} \]

\[ S_{\text{eff}} = S_{\text{LL}} + S_{\text{diss}} \]

\[ S_{\text{LL}}[\phi] = \frac{1}{K} \int \left[ \frac{1}{u} \left( \partial_{\tau} \phi(x, \tau) \right)^2 + u \left( \partial_x \phi(x, \tau) \right)^2 \right] dx d\tau \]

Properties:
- Luttinger liquid.
- Gapless spectrum.
- Delocalised, perfectly conducting (metallic) phase.
Last step: Path integral

\[ S_{\text{eff}} = S_{\text{LL}} + S_{\text{diss}} \]

\[ S_{\text{diss}} = -\alpha \int \cos \left( 2 \left( \phi(x, \tau) - \phi(x, \tau') \right) \right) \frac{dxd\tau d\tau'}{|\tau - \tau'|^2} \]

\[ J(\Omega) \approx \pi \alpha \Omega^s \]

Properties:

- Long-range in nature.
- Only along \( \tau \) (imaginary time direction) – bath is local in nature!
- For a generic ohmicity s, the exponent is 1+s.
- Action valid for finite magnetic field sector.
Cazalilla et al [PRL, 2006]:
• Perturbative RG study on the action.
• $K_c = 0.5$, BKT transition ($z=1$).

Schollwock et al [PRL, 2014]:
• Numerical study on the quantum model.
• $z=2$ type transition at $K=1$. 
Approaching the problem: Numerics

Langevin dynamics:

\[
\frac{d\phi_{ij}(t)}{dt} = -\delta S[\phi_{ij}(t)] + \eta_{ij}(t) \quad (i,j = \text{discretised space and imaginary time}, t = \text{langevin time})
\]

\[
= \frac{u}{K\pi} [\phi_{i+1,j} + \phi_{i-1,j} - 2\phi_{i,j}] + \frac{1}{uK\pi} [\phi_{i,j+1} + \phi_{i,j-1} - 2\phi_{i,j}]
+ \frac{\alpha}{\pi^2} \sum_{j'} D(|j - j'|) \sin [2(\phi_{ij'} - \phi_{ij})] + \eta_{ij}(t) \quad (D(|j-j'|) = 1/\tau^2 \text{discretised kernel with PBC})
\]

\[
\langle \eta_{ij}(t) \rangle = 0 \quad \langle \eta_{ij}(t)\eta_{i'j'}(t') \rangle = 2\delta_{i,i'}\delta_{j,j'}\delta(t-t')
\]

- System size: scale $\beta$ as $L$
- Spatially homogenous: Use $u = 1$.
- Algorithm: Stochastic 2nd order Runge-Kutta for white noise [Honeycutt, PRA 1992]
- Generate equilibrated configurations to calculate correlation functions.
Approaching the problem: calculations

Variational ansatz: Obtain an effective quadratic action by minimising the free energy!

\[ G^{-1}_{\text{LL}}(q, \omega_n) = \frac{1}{2\pi K_r} \left( u_r q^2 + \frac{\omega_n^2}{u_r} \right) \]

\[ G^{-1}_{\text{ord}}(q, \omega_n) = \frac{u_r q^2}{2\pi K_r} + \frac{\alpha_r}{\pi^2} |\omega_n| + a_1 |\omega_n|^{3/2} + a_2 \omega_n^2 \]

| Observable | Luttinger liquid (\(\alpha = 0\)) | Dissipative phase (\(\alpha \neq 0\)) |
|------------|-----------------------------------|-------------------------------------|
| \(\chi = \lim_{q \to 0} \lim_{\omega_n \to 0} \frac{q^2}{\pi^2} \langle \phi(q, \omega_n)\phi(-q, -\omega_n) \rangle\) | \(K_r / (u_r \pi)\) | \(K_r / (u_r \pi)\) |
| \(C(\omega_n) = (1/\pi L) \sum_q \langle |\phi(q, \omega_n)|^2 \rangle\) | \(K_r / 2\omega_n\) | \(\sqrt{K_r / 8\pi u_r (\alpha_r \omega_n / \pi^2 + a_1 \omega_n^{3/2} + a_2 \omega_n^2)^{-1/2}}\) |
| \(\langle \cos(\phi) \rangle\) | \(L - K_r / 4\) | \(c_1 + c_2 / \sqrt{L} + c_3 / L\) |
Results

$K = 0.75, \alpha = 1$ : (LL Phase)

$K = 0.75, \alpha = 8$ : (Ordered Phase)
Results

\[ K_r / u_r, K_r, \langle \cos(\phi) \rangle, C(\omega_n) \]

\[ \alpha = 0.5, K = 0.75, \alpha = 3, \alpha = 4 \]

Luttinger Liquid

Critical region

Dissipative Phase

Ordered phase

Tomonaga–Luttinger liquid

Disordered phase
Transport properties

Conductivity:

\[ \sigma(\omega) = \frac{e^2}{\pi^2 \hbar} [\omega_n G(q = 0, \omega_n)]_{i\omega_n \rightarrow \omega + i\epsilon} \]

| Phase            | DC conductivity (Re(\(\sigma(\omega \rightarrow 0)\)) | Possibility of Localisation! |
|------------------|------------------------------------------------------|-----------------------------|
| LL               | \((e^2 u K / \hbar) \delta(\omega)\)               |                             |
| Ohmic bath       | \(e^2 / \hbar \alpha_r\)                            |                             |
| Subohmic bath    | 0                                                    |                             |
Conclusions

• In one dimensional interacting systems, local baths have an effect on the existing Luttinger liquid phase.
• For an ohmic bath, there exists a KT phase transition for $K_c=0.5$.
• For $K > K_c$, above a critical $\alpha$ the system enters into an ordered phase with unaltered susceptibility (compressibility) and reduced conductivity.
• For subohmic bath, there exists a possibility of the conductivity reducing to zero, which is signature of localization in the system.
Future directions

• Checking the subohmic finite magnetic field spin chain case.
• Checking the zero magnetic field spin chain case.
• Understanding the possibility of existence of a third phase (“disordered phase” of Cazalilla).
THANK YOU !
Appendix: Jordan-Wigner transformation

Spin chain \( \overset{\text{Jordan-Wigner transf.}}{\longrightarrow} \) Interacting Fermionic system

\[
S_j^+ \equiv S_j^x + iS_j^y \rightarrow c_j^+ e^{i\pi \sum_{i<j} n_i} \quad S_j^z \rightarrow n_j - \frac{1}{2}
\]

\[
H_S = \frac{J_{xy}}{2} \sum_i c_{i+1}^+ c_i + \text{h.c} + J_z \sum_i \left( n_i - \frac{1}{2} \right) \left( n_{i+1} - \frac{1}{2} \right)
\]

\[
H_{SB} = \sum_i \left( n_i - \frac{1}{2} \right) \sum_k \lambda_k X_{ik}
\]
Apendix: Bosonisation

1D Fermionic system \( \leftrightarrow \) Bosonisation field theory

- Technique specifically designed for 1D systems. [Giamarchi, Quantum Physics in one dimension]
- Captures low energy excitation physics of the system.

Simple observation: \( \rho^\dagger(q) = \sum_k c_k^\dagger q c_k \) is bosonic in nature.
Appendix: Bosonisation (recipe)

1. \( c_j = \frac{1}{\sqrt{N}} \left[ \sum_{k \approx k_F} e^{ikx} c_k + \sum_{k \approx -k_F} e^{ikx} c_k \right] = [\psi_+(x) + \psi_-(x)] \)

2. \( \psi_r(x) = \frac{1}{\sqrt{2\pi}} e^{i(r(k_Fx - \phi(x)) + \theta(x))} \)

Bosonic field operators
Appendix: Variational ansatz

Variational ansatz: Obtain an effective quadratic action by optimising the free energy [Feynmann, statistical mechanics: a set of lectures, 1998]!

Recipe:

1. Assume: \[ S_{\text{var}} = \frac{1}{2\beta L} \sum_{q, \omega_n} \phi^*(q, \omega_n) G_{\text{var}}^{-1} \phi(q, \omega_n) \]

2. Calculate: \[ F_{\text{var}} = \frac{1}{\beta} \left( - \sum_{q, \omega_n} \log G_{\text{var}} + \langle S - S_{\text{var}} \rangle_{S_{\text{var}}} \right) \]

3. Minimise: \[ \frac{\partial F_{\text{var}}}{\partial G_{\text{var}}} = 0 \]
**Appendix : Results for K=0.55**

K=0.55, \(\alpha = 0.05\) :

K=0.55, \(\alpha = 10\) :

---

**FIG. 1.** Calculation of different quantities for \(K = 0.55\) that characterizes LL (\(\alpha = 0.05\), top row) and dissipative phase (\(\alpha = 10\), bottom row). Blue and red points correspond to \(L = \beta = 384\) and \(L = \beta = 128\), respectively. **Left**: Due to symmetry, \(\pi \frac{\chi}{u_r} = \frac{K_r}{u_r}\) is equal to \(K/u = 0.55\) for all values of \(\alpha\) and all length scales. **Middle**: For \(\alpha = 0.05\), \(\omega_\pi C(\omega_\pi)\) saturates to \(K_r/2 = 0.273\) as \(\omega_\pi \to 0\); whereas for \(\alpha = 10\), \(\sqrt{\omega_\pi C(\omega_\pi)}\) saturates to \(\left(\frac{K_r\pi}{8\omega_\pi u_r}\right)^{1/2} = 0.156\). The other fitting constants are \(a_1 = 16.61\) and \(a_2 = 571.4\). **Right**: For \(\alpha = 0.05\), \(\langle \cos(\phi) \rangle\) decays as a power law, which allows us to extract \(K_r = 0.546\), consistent with the fit of \(\omega_\pi C(\omega_\pi)\). For \(\alpha = 10\) it saturates to a constant, as predicted by the variational ansatz (the fit gives \(c_1 = 0.788, c_2 = 0.215\) and \(c_3 = 0.012\)).
Appendix : Results for $K=0.55$

$Luttinger$ $liquid$ $\quad Critical$ $\quad Dissipative$ $phase$

$K_c = 0.5$

$K_r/u_r$

$K_r \cos(\phi)$

$K_r C(\omega_n)$

$\alpha_r$