Effects of an external circuit on a MHD slider bearing with couplestress fluid between conducting plates

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Abstract: A MHD Slideer bearing lubricated with conducting couplestress fluid (CCSF) between two electrical conducting plates under the influence of magnetic field in free space is theoretically investigated. A closed form solution for the film pressure and load carrying capacity is obtained analytically in terms of inlet-outlet (IO) film height ratio of slider bearings. The results are presented graphically for different values of operating parameters. The results suggest that the bearings with couplestress fluid as lubricant provide significant load carrying capacity than Newtonian lubricant case. Further, it is observed that the influence of applied magnetic field and induced magnetic field is to increase the load carrying capacity substantially while, the load decreases with increase in IO film ratio. Besides, the conductivity increases the load carrying capacity significantly. The results are compared with the Newtonian Fluid case.

1. Introduction

Slider bearings are designed to support axial loads and are used in hydroelectric generators, steam and gas turbines and similar equipments. The MHD lubrication of finite slider bearings was analysed by Lin [1]. He has shown that the application of the magnetic field signifies an influence in the load carrying capacity, power loss and friction parameter of slider bearing. Kuzma [2] investigated the effect of a non-uniform applied magnetic field on the operation of a parallel plate slider bearing and shown that the optimum magnetic field profile enhances the load-carrying capacity while decreasing the friction factor. Recently several authors have investigated the performance of MHD slider bearing with various film shapes [3-5]. They have shown that the applied magnetic field provide significant improvement in the load carrying capacity, stiffness coefficient and the damping coefficient. Several investigators [6- 8] have studied the lubrication problems of MHD slider bearings.

The study of a couple stress fluids is very useful in understanding various physical problems. Since the classical Newtonian theory cannot accurately describe the rheological behaviour of lubricants blended with various additives, many micro continuum theories are proposed to model the flow rheology. Among these Stokes [9] theory is the simplest theory which accounts for the effects of couple stresses, body couple and asymmetric tensors. Das [10] observed that for slider bearings both the values of maximum load capacity and the corresponding inlet-outlet film ratio depend on couplestress and magnetic parameters. Lin et al.,[11- 13] in their studies have shown that increasing
values of the couple stress parameter increases the load carrying capacity and reduces the required volume flow rate and the friction parameter. Some recent investigations regarding MHD couple stress fluids are mentioned in the studies [14-18]. These studies have shown that the MHD couple stress fluids have better lubricating qualities than the corresponding Non-conducting Newtonian lubricant (NCNL).

Most of the studies assume the bearing surfaces to be electrically non-conducting refs. [1-18] but conductivity influences the performance of bearings. This is been shown by Synder [19] who considered the influence of finite wall conductance on load capacity of the MHD slider bearings. Such an analysis in the case of a hydromagnetic squeeze film has been carried out by Shukla et al., [20] and shown that an increase in load capacity, pressure and time of approach are possible by increasing either the strength of the magnetic field or conductivities or both. In the case of MHD slider bearing, the effect of magnetic field in the free space between two electrically conducting plates was studied by Soundalgekar [21] and it was observed that an increase in the magnetic field in the free space leads to an increase in load carrying capacity of bearing. The effects of external circuit on MHD conducting plates and MHD channel flow with couple stresses on the working of bearing were discussed by Soundalgekar et al.,[22-23].

The objective of the present paper is to study the effects of an External circuit on a MHD slider bearing between electrically conducting surfaces lubricated with conducting couple stress fluids (CCSF), which has not been studied so far.

2. Mathematical Analysis
The configuration of the slider bearing is shown in figure 1. The surfaces of the bearing are assumed to be conducting. The lubricant between the surfaces is an isothermal, incompressible electrically conducting couple stress fluid. The origin is chosen at one end of the lower plate with $x$-axis along the plate and $y$-axis normal to it. A uniform magnetic field is assumed to be applied parallel to the $y$-axis. The plates are assumed to be infinite in extent in the $x$ and $z$ direction.

The basic equations governing the hydrodynamic flow of the couple-stress lubricant between two conducting plates are:

$$
\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} - J_i B_i = \frac{\partial p}{\partial x}
$$

(1)

$$
\frac{dH}{dy} = -J_z
$$

(2)

where the third term represents the magnetic body force, where $J_z$ is the current density given by Ohm’s law

$$
J_z = \sigma (E_z + uB_i)
$$

(3)

Eliminating $J_z$ between equations (1) and (2) gives

$$
\frac{d^2 u}{dy^2} + \frac{\eta}{\mu} \frac{d^4 u}{dy^4} - \frac{M^2}{h_2^2} u = \frac{1}{\mu} \frac{\partial p}{\partial x} + \frac{\sigma M}{\mu h_2} E_z
$$

(4)
\[
\frac{dH_y}{dy} = -\sigma \left( E_z + uB_y \right)
\]  

(5)

where \( M = B_f h_y \sqrt{\sigma/\mu} \) is the Hartmann number.

The boundary conditions and no-stress conditions for the velocity are:

\[
\begin{align*}
&u = 0 \quad \text{at} \quad y = 0 \quad \text{for channel flow} \\
&u = 0 \quad \text{at} \quad y = h \\
&u = U \quad \text{at} \quad y = 0 \quad \text{for Couette flow} \\
&u = 0 \quad \text{at} \quad y = h
\end{align*}
\]  

(6)

\[
\frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = 0, h
\]  

(7)

and the boundary conditions on the induced magnetic field are

\[
\begin{align*}
&\frac{dH_y}{dy} - \frac{H_z}{\Phi_1 h_2} = -\frac{H_n}{\Phi_1 h_2} \quad \text{at} \quad y = 0 \\
&\frac{dH_y}{dy} + \frac{H_z}{\Phi_2 h_1} = \frac{H_n}{\Phi_2 h_1} \quad \text{at} \quad y = h
\end{align*}
\]  

(9)

**Channel flow**

The solution of equations (4) and (5) subject to conditions (6), (8) and (9) are

\[
U_{ch} = \frac{h_y^2}{M^2} \left( 1 + \frac{\sigma}{\mu} \frac{M}{h_z} E_z \right) f_i (h, l, M, y)
\]  

(10)
\[
\begin{align*}
H_z &= -\sigma E_z (y + \Phi_h z) - \sqrt{\sigma \mu h_z} \left( \frac{1}{\mu} \frac{\partial p}{\partial x} + \frac{\sigma M}{\mu h_z} E_z \right) \left[ f_z (h, l, M, y) - y \right] + H_n, \\
E_c &= \frac{h_z}{\sqrt{\sigma \mu}} \left( \frac{\partial p}{\partial x} \left[ h - f_z (h, l, M) \right] \right) - \frac{\left( H_n - H_\infty \right)}{\sigma} \\
\end{align*}
\]

where \( l = \frac{\eta}{\mu} \) is the couplestress parameter,

\[
A = \sqrt{\frac{1 + \sqrt{1 - 4l^2 M^2}}{2}}, \quad B = \sqrt{\frac{1 - \sqrt{1 - 4l^2 M^2}}{2}}
\]

\[
f_1 (h, l, M, y) = \frac{A^2}{A^2 - B^2} \left( \frac{\cosh \left( \frac{B(2y - h)}{2l} \right)}{\cosh \left( \frac{Bh}{2l} \right)} \right) - \frac{B^2}{A^2 - B^2} \left( \frac{\cosh \left( \frac{A(2y - h)}{2l} \right)}{\cosh \left( \frac{Ah}{2l} \right)} \right) - 1
\]

\[
f_2 (h, l, M, y) = \frac{A^2 l}{B \left( A^2 - B^2 \right)} \left( \frac{\sinh \left( \frac{B(2y - h)}{2l} \right)}{\cosh \left( \frac{Bh}{2l} \right)} + \tanh \left( \frac{Bh}{2l} \right) \right) - \frac{B^2 l}{A \left( A^2 - B^2 \right)} \left( \frac{\sinh \left( \frac{A(2y - h)}{2l} \right)}{\cosh \left( \frac{Ah}{2l} \right)} + \tanh \left( \frac{Ah}{2l} \right) \right)
\]

\[
f_3 (h, l, M) = \frac{2l}{B \tanh \left( \frac{Bh}{2l} \right)} \left( \frac{A^2}{B} \tanh \left( \frac{Bh}{2l} / A \tanh \left( \frac{Ah}{2l} \right) \right) \right)
\]

Eliminating \( E_c \) between equations (10) and (12), we have the expression for velocity in channel flow as

\[
U_{ch} = \frac{h_z^2}{M^2} \left\{ \frac{1}{\mu} \frac{\partial p}{\partial x} (\Phi h_z + \Phi_z h_z + h) - \frac{M \left( H_n - H_\infty \right)}{h_z \sqrt{\sigma \mu}} \right\} \left[ f_z (h, l, M, y) \right]
\]

**Couette flow**

Equation (4) for plane coquette flow reduces to

\[
\frac{d^2 u}{dy^2} - l_z \frac{d^4 u}{dy^4} - \frac{M^2}{h_z^2} u = \frac{\sigma M}{\sqrt{\mu \ h_z}} E_z
\]

The solution of equations (14) and (5) with respect to the conditions (7), (8) and (9) are

\[
U_{co} = \frac{h_z}{M} \left\{ \frac{\sigma}{\mu} E_z f_z (h, l, M, y) + U_f (h, l, M, y) \right\}
\]

\[
H_z = -\sigma E_z \left[ \Phi h_z + f_z (h, l, M, y) \right] - \frac{M \sqrt{\sigma \mu}}{h_z} U \left[ \Phi h_z + f_z (h, l, M, y) \right] + H_n
\]

\[
E_z = -\frac{M}{h_z} \left\{ \frac{\sqrt{M \ U}}{\sigma} \left[ \frac{f_z (h, l, M)}{2} + \Phi h_z \right] + \frac{\left( H_n - H_\infty \right)}{\sigma} \right\}
\]
where \( f_s(h,l,M,y) = \frac{B^2}{A^2-B^2} \left( \frac{\sinh A(y-h)}{l} \right) - \frac{A^2}{A^2-B^2} \left( \frac{\sinh B(y-h)}{l} \right) \)

\[ f_s(h,l,M,y) = \frac{B^2 l}{A(A^2-B^2)} \left( \frac{\cosh A(y-h)}{l} - \frac{\cosh A h}{l} \right) - \frac{A^2 l}{A^2-B^2} \left( \frac{\cosh B(y-h)}{l} - \frac{\cosh B h}{l} \right) \]

Eliminating \( E_z \) between equations (15) and (17), we have the expression for velocity in couette flow as

\[
U_{co} = \frac{1}{M \sqrt{\sigma \mu}} f_s(h,l,M,y) + U f_s(h,l,M,y)
\]

Rouse [24] has shown in the case of an ordinary slider bearing that the velocity of the lubricant between the bearing plates can be approximated by the superposition of the velocities for channel and plane coquette flow, a valid approximation when the inclination angle between the bearing plates is very small.

Hence, the velocity for the whole region is obtained by superposing equation (13) on equation (18):

\[
u = U_{ch} + U_{co} = f_s(h,l,M, \Phi_1, \Phi_2) f_s(h,l,M,y) + U f_s(h,l,M,y)
\]

where

\[ Q = \int_0^h \nu \, dy \]

Substituting equation (19) in equation (20) and carrying out the integration, we obtain

\[ Q = f_s(h,l,M, \Phi_1, \Phi_2) \left[ f_s(h,l,M) - h \right] + U f_s(h,l,M) \]

But the net current \( I_x \) and the external magnetic field are related by:

\[ I_x = H - H_n \]

where \( H_n \) and \( -H_n \) are the quantities of the net current which return to the channel through an upper and lower path respectively. If the conducting path is solely in the lower region, then \( H_n = 0 \) and \( I_x = H_n \). Hence equation (21) becomes

\[ Q = f_s(h,l,M, \Phi_1, \Phi_2) \left[ f_s(h,l,M) - h \right] + U f_s(h,l,M) \]

where

\[ Q = \int_0^h \nu \, dy = \frac{h^2}{M^2} \frac{1}{\mu} \left( \Phi_1 h_2 + \Phi_2 h_2 + h \right) - \frac{2 h_2}{M \sqrt{\sigma \mu}} \left[ f_s(h,l,M) - h \right] + \frac{\Phi_1 h_2}{M \sqrt{\sigma \mu}} - U f_s(h,l,M) \]

The Volume flow rate is

\[ Q = \int_0^h \nu \, dy \]

Substituting equation (19) in equation (20) and carrying out the integration, we obtain

\[ Q = f_s(h,l,M, \Phi_1, \Phi_2) \left[ f_s(h,l,M) - h \right] + U f_s(h,l,M) \]

But the net current \( I_x \) and the external magnetic field are related by:

\[ I_x = H - H_n \]

Where \( H_n \) and \( -H_n \) are the quantities of the net current which return to the channel through an upper and lower path respectively. If the conducting path is solely in the lower region, then \( H_n = 0 \) and \( I_x = H_n \). Hence equation (21) becomes

\[ Q = f_s(h,l,M, \Phi_1, \Phi_2) \left[ f_s(h,l,M) - h \right] + U f_s(h,l,M) \]

where

\[ Q = \int_0^h \nu \, dy = \frac{h^2}{M^2} \frac{1}{\mu} \left( \Phi_1 h_2 + \Phi_2 h_2 + h \right) - \frac{2 h_2}{M \sqrt{\sigma \mu}} \left[ f_s(h,l,M) - h \right] + \frac{\Phi_1 h_2}{M \sqrt{\sigma \mu}} - U f_s(h,l,M) \]
Equation (22) can be solved for $\frac{\partial p}{\partial x}$. But the resulting expression cannot be integrated in closed form.

Hence, attention is restricted to the case of high Hartmann number. For a large $M$, the expression $\frac{\partial p}{\partial x}$ in non-dimensional form is given by

$$\frac{\partial p^*}{\partial x^*} = M^2 \left( 2I + \frac{G(I', M)}{2} + \Phi_1 \right) M^2 \left( Q' - \frac{G(I', M)}{2} \right) \frac{(\Phi_1 + \Phi_2 + G(I', M))}{(\Phi_1 + \Phi_2 + h^*)}$$

(23)

where $G(I', M) = \frac{I'}{(A^2 - B^2)} \left( \frac{A^2 - B^2}{B' - A'} - \frac{B^2}{A^2} \right)$

$$A^* = \sqrt{1 + \frac{1 - I'^2}{M^2}}, \quad B^* = \sqrt{1 - \frac{1 - I'^2}{M^2}}$$

From figure 1, the relation between $h^*$ and $x^*$ is

$$h^* = a - (a-1)x^*$$

(24)

where $a = \frac{h_1}{h_2}$

The relevant boundary conditions on pressure $p^*$ are:

$$p^* = p_e \quad \text{at} \quad h^* = a$$

$$p^* = p_e \quad \text{at} \quad h^* = 1$$

(25)

Substituting equation (24) in equation (23) and carrying out the integration using the boundary conditions (25), we get

$$Q' = \frac{2I + \frac{G(I', M)}{2} + \Phi_1}{\log \left( \frac{G(I', M) - a}{G(I', M) - 1} \right)} \frac{\Phi_1 + \Phi_2 + a}{(\Phi_1 + \Phi_2 + h^*)}$$

(26)

and

$$p^* - p_e = M^2 \left( 2I + \frac{G(I', M)}{2} + \Phi_1 \right) A_1 \quad M^2 \left( Q' - \frac{G(I', M)}{2} \right) A_2$$

(27)

where $A_1 = \log \left( \frac{\Phi_1 + \Phi_2 + a}{\Phi_1 + \Phi_2 + h^*} \right)$

$A_2 = \log \left( \frac{G(I', M) - a}{G(I', M) - h^*} \right) \frac{(\Phi_1 + \Phi_2 + a)}{(\Phi_1 + \Phi_2 + h^*)}$
The load carrying capacity of the bearing is

$$ W = \int_{0}^{L} (p - p_0) \, dx $$

The non-dimensional form of load carrying capacity is

$$ W' = \int_{0}^{1} \left( p' - p_0' \right) \, dx' $$

Substituting for \( p' - p_0' \) from equation (27) and for \( h' \) from equation (24), and carrying out the integration, we get

$$ W' = \frac{M^2 \left( 2I + \frac{G(I', M)}{2} + \Phi_1 \right) (A_s + (a-1)) + M^2 \left( Q' - \frac{G(I', M)}{2} \right) (A_s + A_s + 2(a-1))}{(a-1)^2} $$

Where

$$ A_s = \left( \Phi_1 + \Phi_2 + 1 \right) \log \left( \Phi_1 + \Phi_2 + a \right) $$

$$ A_s = \left( G(I', M) + 1 \right) \log \left( \frac{G(I', M) - 1}{G(I', M) - a} \right) $$

3. Results and discussion

The results are presented graphically for different values of operating parameters namely viz. Hartmann number \( M \), couplestress parameter \( I' \), conductivity parameters \( \Phi_1, \Phi_2 \), IO film height ratio \( a \), and external current \( I \). Various graphs have been drawn to understand load bearing behaviour.

3.1. Fluid film pressure

Figure 2 shows the effect of wall conductance \( \Phi_1, \Phi_2 \) and couplestress parameter \( I' \) on non-dimensional film pressure when \( a, M \) and \( I \) are constant. It is noticed that increasing values of \( I' \) increases pressure for \( \Phi_1 \leq \Phi_2 \). The variation of dimensionless pressure with \( x' \) for various values of Hartmann number \( M \) and couplestress parameter \( I' \) are displayed in figure 3. It indicates that combined effect of \( M \) and \( I' \) increases the pressure significantly.

The variation of dimensionless pressure with \( x' \) for different values of couplestress parameter \( I' \) and external current \( I \) is shown in figure 4. It is observed that the external current \( I \) plays a prominent role in enhancing the pressure in the presence of couplestress parameter \( I' \) and it shows that the pressure is more in the presence of external circuit \( I \neq 0 \) than in its absence \( I = 0 \). Figure 5 depicts the variation of pressure with respect to \( x' \) for different values of \( I' \) and \( a \). It shows an increase in pressure as \( I' \) and \( a \) increases.
Figure 2. Variation of dimensionless pressure $p^* - p_e$ with $x^*$ for different values of $l^*$, $\Phi_1$ and $\Phi_2$ at $a=1.5$, $M=10$ and $l=2$.

Figure 3. Variation of dimensionless pressure $p^* - p_e$ with $x^*$ for different values of $l^*$ and $M$ at $a=1.5$, $I=2$, $\Phi_1 = 0.6$ and $\Phi_2 = 0.6$. 
Figure 4. Variation of dimensionless pressure $p^* - p_*$ with $x^*$ for different values of $I^*$ and $I$ at $a = 1.5$, $M = 10$, $\Phi_1 = 0.6$ and $\Phi_2 = 6$.

Figure 5. Variation of dimensionless pressure $p^* - p_*$ with $x^*$ for different values of $I^*$ and $a$ at $l = 2$, $M = 10$, $\Phi_1 = 0.6$ and $\Phi_2 = 6$. 
3.2. **Load carrying capacity**

Variation of load carrying capacity with current \( I \) for increasing values of \( l' \) for \( \Phi_1 \leq \Phi_2 \) is shown in figure 6. We observe from this figure that the load increases with increasing values of \( l' \) and the effects are more prominent for \( \Phi_1 \geq \Phi_2 \) than for \( \Phi_1 < \Phi_2 \). From the results presented in figure 7 for the variation of load with current \( I \) for various values of \( l' \) and \( M \), it can be concluded that \( M \) and \( l' \) enhances the load carrying capacity of the bearing. Also with the increasing values of \( I \), \( W' \) increases significantly.

Figure 8 gives the load profile with respect to \( a \) for different values of wall conductance \( \Phi_1, \Phi_2 \) and couplestress parameter \( l' \). It is clear from this figure that the bearing suffers on account of IO film ratio \( a \), as a result the load carrying capacity decreases considerably. But it is also clear that load increases with increasing \( l' \) and it is more prominent for \( \Phi_1 \geq \Phi_2 \) than \( \Phi_1 < \Phi_2 \). Figure 9 presents the values of \( W' \) at various IO film ratios \( a \) of slider bearings for different values of \( l' \) and \( M \). It is observed that load decreases with increasing IO film ratios which can be compensated by increasing the magnetic field \( M \) and the couplestress \( l' \) of the fluid.

![Figure 6. Variation of dimensionless load \( W' \) with \( I \) for different values of \( l' \), \( \Phi_1 \) and \( \Phi_2 \) at \( a=1.5 \) and \( M=10 \).](image-url)
Figure 7. Variation of dimensionless load $W^*$ with $I$ for different values of $l^*$ and $M$ at $a=1.5$, $\Phi_1=0.6$ and $\Phi_2=6$.

Figure 8. Variation of dimensionless load $W^*$ with $a$ for different values of $l^*$, $\Phi_1$ and $\Phi_2$ at $I=2$ and $M=10$. 
3.3. Study of the variation of conductivities of bearing surfaces

Numerical values of conductivities of plates $\Phi_1 \leq \Phi_2$ have been calculated and entered in table 1-table 3.

Table 1 and table 2 shows the variation of dimensionless pressure and load carrying capacity, when the conductivities of both surfaces are different ($\Phi_1 \neq \Phi_2$). It is observed that, the dimensionless pressure and load carrying capacity effects are more prominent for $I = 0.3$ than $I = 0.0$ for both $\Phi_1 \geq \Phi_2$. Further, for fixed $l'$ and $\Phi_1$ [ $\Phi_2$, an increase in $\Phi_2$] $\Phi_1$ leads to decrease [ increase in pressure for both $\Phi_1 \geq \Phi_2$. Also, it is observed that there is significant increase in dimensionless pressure and load carrying capacity for increasing values of $l'$ in either the cases.

Table 3 shows the variation of dimensionless pressure and load carrying capacity, when both surfaces have same conductivities i.e, $\Phi_1 = \Phi_2 = \Phi$. It is interesting to note that pressure and load carrying capacity increases with increasing values of fixed $l'$ and $\Phi$ and the effects are more prominent in the presence of external current $I$.

Table 4 shows the variation of non dimensional pressure and load carrying capacity with increasing values of $M$ and $l'$. Combined effect of $M$ and $l'$ is to enhance the non-dimensional pressure and load carrying capacity significantly as compared to non magnetic case and Newtonian case.

Table 5 shows the prominent effects of couplestresses $l'$ and the results are compared with Soundalgekar case [21]. As $l' \rightarrow 0$ the present analysis reduces to Newtonian case as studies by Soundalgekar [21].
Table 1. Variation of dimensionless pressure $p^* - p_o$ for unequal conductivities at $a = 1.5$, $x = 0.2$ and $M = 10$.

| $l^*$ | $\phi_1$ | $\phi_2$ | $I = 0$ | $I = 0.3$ |
|---|---|---|---|---|
| 0.2 | 0.159984 | 0.129368 | 0.105593 | 0.47995 | 0.388102 | 0.316778 |
| 0.4 | 0.261657 | 0.211029 | 0.172718 | 0.575645 | 0.464263 | 0.37998 |
| 0.6 | 0.359042 | 0.289224 | 0.23738 | 0.666792 | 0.537131 | 0.440849 |
| 0.2 | 0.176485 | 0.141569 | 0.115164 | 0.176485 | 0.282619 | 0.384049 |
| 0.4 | 0.282619 | 0.2263 | 0.184636 | 0.611584 | 0.489709 | 0.399551 |
| 0.6 | 0.384049 | 0.307366 | 0.251529 | 0.706116 | 0.565126 | 0.462463 |
| 0.2 | 0.198275 | 0.157527 | 0.127629 | 0.554282 | 0.440372 | 0.356791 |
| 0.4 | 0.310145 | 0.24621 | 0.200123 | 0.658515 | 0.522765 | 0.424912 |
| 0.6 | 0.416771 | 0.33097 | 0.269887 | 0.757379 | 0.601457 | 0.490455 |

Table 2. Variation of dimensionless load capacity $W^*$ for unequal conductivities at $a = 1.5$ and $M = 10$.

| $l^*$ | $\phi_1$ | $\phi_2$ | $I = 0$ | $I = 0.3$ |
|---|---|---|---|---|
| 0.2 | 1.11988 | 1.09896 | 1.07713 | 1.43985 | 1.41295 | 1.38488 |
| 0.4 | 1.76802 | 1.73043 | 1.69403 | 2.02675 | 1.98367 | 1.94193 |
| 0.6 | 2.14705 | 2.10716 | 2.0686 | 2.35823 | 2.31442 | 2.27206 |
| 0.2 | 1.18347 | 1.15986 | 1.13554 | 1.51913 | 1.48882 | 1.45761 |
| 0.4 | 1.84684 | 1.80676 | 1.76801 | 2.11609 | 2.07017 | 2.02577 |
| 0.6 | 2.23249 | 2.1905 | 2.14994 | 2.45152 | 2.40542 | 2.36087 |
| 0.2 | 1.2663 | 1.23913 | 1.21152 | 1.6223 | 1.5875 | 1.55213 |
| 0.4 | 1.94887 | 1.90554 | 1.86373 | 2.23172 | 2.1821 | 2.13422 |
| 0.6 | 2.34286 | 2.29815 | 2.25499 | 2.57202 | 2.52294 | 2.47556 |
Table 3. Variation of dimensionless pressure $p^* - p_*$ and dimensionless load capacity $W^*$ with equal conductivities i.e., $\Phi_1 = \Phi_2 = \Phi$ at $a = 1.5$, $x = 0.2$ and $M = 10$.

| $l'$ | $\Phi_1$ | $\Phi_2$ | $p^* - p_*$ | $W^*$ |
|------|----------|----------|--------------|-------|
|      | $l = 0$  | $l = 0.3$| $l = 0$      | $l = 0.3$|
| 0.0  | 2        | 38.8765  | 37.2401      | 35.7355 |
|      | 4        | 52.7162  | 51.1525      | 49.6789 |
|      | 6        | 60.0654  | 58.6907      | 57.3775 |
| 0.2  | 2        | 45.3946  | 43.466       | 41.6943 |
|      | 4        | 61.1578  | 59.3325      | 57.6129 |
|      | 6        | 69.4952  | 67.8973      | 66.3713 |
| 0.4  | 2        | 53.5197  | 51.2219      | 49.113  |
|      | 4        | 71.5589  | 69.4081      | 67.3827 |
|      | 6        | 81.0558  | 79.1821      | 77.3931 |

Table 4. Variation of dimensionless pressure $p^* - p_*$ and dimensionless load capacity $W^*$ with increasing values of $M$ and $l'$.

| $l = 2$, $x = 0.2$, $a = 1.5$ | $p^* - p_*$ | $W^*$ |
|-------------------------------|--------------|-------|
|                               | $l' = 0.0$   | $l' = 0.3$| $l' = 0.6$| $l' = 0.0$| $l' = 0.3$| $l' = 0.6$|
| $\Phi_1 = 0.4$                | $M = 0$      | 0      | 0         | -0.004   | -0.004   | -0.004   |
| $\Phi_2 = 0.6$                | $M = 4$      | 0.380815 | 0.406239 | 0.448517 | 18.2418  | 19.8523  | 22.2517  |
| $(\Phi_1 < \Phi_2)$           | $M = 6$      | 0.668056 | 0.712202 | 0.774099 | 26.4535  | 30.2914  | 35.2278  |
| $(\Phi_1 = \Phi_2)$           | $M = 8$      | 1.06118  | 1.13179  | 1.22078  | 34.8129  | 41.8313  | 49.9733  |
| $\Phi_1 = 2$                  | $M = 0$      | 0      | 0         | -0.004   | -0.004   | -0.004   |
| $\Phi_2 = 2$                  | $M = 4$      | 0.678979 | 0.722703 | 0.807565 | 34.3429  | 37.4121  | 41.997   |
| $(\Phi_1 < \Phi_2)$           | $M = 6$      | 1.17115  | 1.25429  | 1.37171  | 49.603   | 56.8606  | 66.2165  |
| $(\Phi_1 = \Phi_2)$           | $M = 8$      | 1.84457  | 1.97703  | 2.1444   | 65.1431  | 78.3715  | 93.749   |
| $\Phi_1 = 4$                  | $M = 0$      | 0      | 0         | -0.004   | -0.004   | -0.004   |
| $\Phi_2 = 0.9$                | $M = 4$      | 0.799443 | 0.853535 | 0.943547 | 39.4106  | 42.8594  | 47.9917  |
| $(\Phi_1 < \Phi_2)$           | $M = 6$      | 1.39759  | 1.49133  | 1.62349  | 57.3358  | 65.5956  | 76.2013  |
\[
\begin{array}{ccccccc}
(\phi_1 > \phi_2) & M = 8 & 2.21628 & 2.36604 & 2.55493 & 75.5773 & 90.7288 & 108.271 \\
\end{array}
\]

Table 5. Comparison of the present analysis with Newtonian case (NC) by Soundalgekar [21] with \( I = 2 \), \( x = 0.2 \), \( M = 10 \), \( \phi_1 = 0.6 \), \( \phi_2 = 6 \).

| \( a \) | NC [21] | Present Analysis |
|---|---|---|
| \( \rho^* - \rho_c \) | \( l' = 0.0 \) | \( l' = 0.3 \) | \( l' = 0.6 \) | \( l' = 0.9 \) |
| 1.5 | 1.59383 | 1.59384 | 1.69909 | 1.82112 | 1.93841 |
| 2.0 | 2.27418 | 2.27419 | 2.40186 | 2.54735 | 2.68474 |
| 2.5 | 2.59222 | 2.59222 | 2.72257 | 2.86969 | 3.00729 |
| 3.0 | 2.736 | 2.736 | 2.86295 | 3.00537 | 3.13777 |
| 3.5 | 2.78688 | 2.78689 | 2.90846 | 3.04431 | 3.17012 |
| \( W^* \) | 44.2083 | 44.2089 | 55.1589 | 67.645 | 78.4136 |
| 2.0 | 23.0164 | 23.0167 | 28.5174 | 34.4237 | 39.6776 |
| 2.5 | 16.3005 | 16.3007 | 19.8933 | 23.7563 | 27.1979 |
| 3.0 | 13.0629 | 13.0631 | 15.7132 | 18.5663 | 21.1116 |
| 3.5 | 11.161 | 11.161 | 13.2516 | 15.5046 | 17.5168 |

Conclusions

An analysis of the combined effects of external circuit and couplestresses on MHD slider bearing between conducting plates is presented in this paper. From the theoretical results presented in this article, we can conclude that

- The effect of couplestresses \( l' \), applied magnetic field \( M \) and magnetic field in the free space \( I \) increases the non-dimensional fluid film pressure and load carrying capacity considerably. Further, as \( l' \to 0 \) the present analysis reduce to Newtonian case [21].
- The IO film ratio increases the non-dimensional pressure whereas the reverse trend is seen in non-dimensional load carrying capacity.
- For fixed conductivity of upper plate, an increase in conductivity of lower plate leads to increase in pressure and load capacity while an opposite effect is seen when the conductivity of upper plate is increased by fixing that for lower plate. i.e the non-dimensional pressure and load carrying capacity is more prominent in case of \( \phi_1 \geq \phi_2 \) than \( \phi_1 < \phi_2 \).
- The presence of \( I \) increases the dimensionless pressure and load carrying capacity significantly in comparison with \( I = 0 \).

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Nomenclature
\(a\) ratio of heights \(h_1/h_2\)
\(B_z\) applied magnetic field
\(E_z\) induced electric field in the \(Z\)-direction
\(h\) film thickness
\(h_1\) film thickness at the inlet
\(h_2\) film thickness at the outlet
\(h'\) non-dimensional film thickness \(h/h_2\)
\(H_x\) induced magnetic field in the \(X\)-direction
\(H_{x_0}\) magnetic field below the lower plate in free space
\(H_{x_1}\) magnetic field above the upper plate in free space
\(H^*\) non-dimensional magnetic field \(\left(\frac{H_x}{\sigma UB_h}\right)\)
\(I_x\) external current \(\left(H_{x_0} - H_{x_1}\right)\)
\(I\) non-dimensional external current \(\left(I_x/\sigma UB_h\right)\)
\(I^*\) couplestress parameter \(\left(\frac{\eta}{\mu}\right)^\frac{1}{2}\)
\(l^*\) non-dimensional couplestress parameter \(\left(2l/h_2\right)\)
\(L\) length of the bearing
\(L_1\) thickness of the lower plate
\(L_2\) thickness of the upper plate
\(M\) Hartmann number \(\left(R_h h_2 \frac{\sqrt{\sigma}}{\mu}\right)\)
\(p\) pressure in the film region
\(p^*\) non-dimensional pressure \(\left(wh_2^2/\mu UL\right)\)
\(Q\) rate of flow per unit length
\(Q^*\) non-dimensional rate of flow \(\left(Q/wh_2\right)\)
\(U\) velocity of the lower plate
\(\mu\) viscosity of lubricant
\(\eta\) material constant characterizing couple stress
\(\sigma\) electrical conductivity of the fluid
\(\sigma_1\) electrical conductivity of the lower plate
\(\sigma_2\) electrical conductivity of the upper plate
\(\Phi_1\) electrical conductance ratio of the lower plate
\(\Phi_2\) electrical conductance ratio of the upper plate