Prospect for relic neutrino searches

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Abstract

Neutrinos from the Big Bang are theoretically expected to be the most abundant particles in the Universe after the photons of the Cosmic Microwave Background (CMB). Unlike the relic photons, relic neutrinos have not so far been observed. The Cosmic Neutrino Background (CνB) is the oldest relic from the Big Bang, produced a few seconds after the Bang itself. Due to their impact in cosmology, relic neutrinos may be revealed indirectly in the near future through cosmological observations. In this talk we concentrate on other proposals, made in the last 30 years, to try to detect the CνB directly, either in laboratory searches (through tiny accelerations they produce on macroscopic targets) or through astrophysical observations (looking for absorption dips in the flux of Ultra-High Energy (UHE) neutrinos, due to the annihilation of these neutrinos with relic neutrinos at the Z-resonance).

We concentrate mainly on the first possibility. We show that, given present bounds on neutrino masses, lepton number in the Universe and gravitational clustering of neutrinos, all expected laboratory effects of relic neutrinos are far from observability, awaiting future technological advances to reach the necessary sensitivity. The problem for astrophysical searches is that sources of UHE neutrinos at the extreme energies required may not exist. If they do exist, we could reveal the existence, and possibly the mass spectrum, of relic neutrinos, with detectors of UHE neutrinos (such as ANITA, Auger, EUSO, OWL, RICE and SalSA).

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1. Introduction

Neutrinos from the Big Bang are theoretically expected to be the most abundant particles in the Universe after the photons of the Cosmic Microwave Background (CMB). The Cosmic Neutrino Background $\nu B$ can contain the three active neutrinos of the Standard Model of Elementary Particles (SM), which are Dirac or Majorana particles with masses between about 0.01 and 1 eV, one or more sterile neutrinos (present in trivial extensions of the SM) and possibly light bosons coupled to neutrinos. Unlike the CMB, the $\nu B$ has not been yet observed. Its detection would provide insight into early moments of our Universe, from before Big-Bang the Nucleosynthesis (BBN) until now. In fact, the $\nu B$ is the oldest relic from the Big Bang, produced a few seconds after the Bang itself. Thus, it impacts cosmology from the BBN (which finished about 20 minutes later), to the emission the CMB (380 kyr later), to the formation the Large Scale Structure of the Universe (a Gyr later). Due to this impact, relic neutrinos may be revealed indirectly in the near future through cosmological observations [1].

In this talk we concentrate on other proposals, made in the last 30 years, to try to detect the $\nu B$, either in laboratory searches or through astrophysical observations. We will concentrate in the first possibility and mention the second briefly.

In laboratory experiments cosmic neutrinos could be revealed through the tiny accelerations they produce on macroscopic targets, accelerations which are quadratic or linear in the Fermi coupling constant. Forces quadratic in the Fermi constant are for sure present and are largest for Dirac neutrinos. Torques linear in the Fermi coupling constant could be present only if there is a net Lepton number in the background, i.e. a difference between relic neutrinos and antineutrinos. If present, this effect has a comparable magnitude for both Majorana and Dirac neutrinos. We show that, given present bounds on neutrino masses, Lepton number in the Universe and gravitational clustering of neutrinos, all expected laboratory effects of relic neutrinos are far from observability, awaiting future technological advances to reach the necessary sensitivity.

Astrophysical searches would look for absorption dips in the flux of Ultra-High Energy neutrinos, due to the annihilation of these neutrinos with relic neutrinos at the Z-resonance. The problem with this idea is that sources of UHE neutrinos at the extreme energies required ($10^{22}$ eV) may not exist. If they do exist, we could reveal the existence, and possibly the mass spectrum, of relic neutrinos, with detectors of UHE neutrinos, such as ANITA, Auger, EUSO, OWL, Rice and SalSA.
2. The standard relic neutrino background

The standard relic neutrino background is assumed to consist of the three active neutrinos of the SM, with relic abundances dictated purely by the interactions present in the SM, and a negligible Lepton number in the Universe. However we know that there must be neutrino physics beyond the SM, because we know now experimentally that neutrinos have masses, which they do not in the SM. In fact the solar mass-square difference $\Delta m_{12}^2 \approx 8.1 \times 10^{-5}$ eV$^2$ and the atmospheric mass difference $\Delta m_{23}^2 \approx 2.2 \times 10^{-3}$ eV$^2$ [2] imply that there are at least three neutrino mass eigenstates. Since the larger mass entering into a mass-square difference must be larger than or equal to $\sqrt{\Delta m^2}$, we know that two of the three masses must be larger than $0.9 \times 10^{-2}$ eV. Cosmological bounds dictate that all active neutrino masses are smaller than about 1 eV [3].

The thermal history of neutrinos starts at temperatures $T \gg \text{MeV}$, at which the three active neutrinos $\nu_\alpha$ of the SM ($\alpha$ stands for e, $\mu$ or $\tau$) were in equilibrium, its reaction rate being larger than the expansion rate of the Universe, $\Gamma_\nu \gg H$. Thus neutrinos had an equilibrium distribution

$$ f_{\nu_\alpha}(p) = \left[ \exp \left( \frac{E - \mu_\alpha}{T} \right) + 1 \right]^{-1} \quad (1) $$

where $E \approx p$, since $m_\alpha \ll \text{MeV}$ and $\mu_\alpha$’s are the chemical potentials. The standard assumption is that $\mu_\alpha = 0$. At $T \approx \text{MeV}$, the neutrino interaction rate, $\Gamma_\nu = \langle \sigma_\nu n_\nu \rangle$, falls below the expansion rate, $H = \sqrt{8\pi \rho/3M_P^2}$, and neutrinos decouple at a temperature between 2 and 3 MeV, depending on the flavour. But even after neutrinos are decoupled, while they are relativistic they maintain the equilibrium distributions $f_{\nu_\alpha}(p)$. Just after neutrinos decouple, at $T \approx m_e = 0.5 \text{ MeV}$, $e^\pm$ pairs annihilate and transfer their entropy into photons, increasing their temperature $T$ with respect to the temperature of neutrinos, which becomes $T_\nu = (4/11)^{1/3} T$. Therefore now $T_\nu = 1.9^oK = 1.7 \times 10^{-4}\text{eV}$, which means that at least two of the active neutrinos in the CMB (with masses above $10^{-2}\text{eV}$) are non-relativistic. The Dirac or Majorana nature of neutrinos becomes important for non-relativistic neutrinos.

From this history we obtain the usual expressions for the number density

$$ n_\nu = n_{\nu e} = \frac{3}{22} n_\gamma = \frac{3\zeta(3)}{11\pi^2} T^3 = 56/\text{cm}^3 \quad (2) $$

the relativistic energy density (for $T_\nu > m_\nu$)

$$ \rho_\nu + \rho_\gamma = \frac{7}{8} \left( \frac{T_\nu}{T} \right)^4 \rho_\gamma = \frac{7\pi^2}{120} \left( \frac{4}{11} \right)^{4/3} T^4 \quad (3) $$
and the non-relativistic energy density (for $T_{\nu} < m_{\nu}$) $\rho_{\nu} = m_{\nu} n_{\nu}$ (using $\Omega$, the density in units of the critical density, and $h$, the reduced Hubble constant), i.e.

$$\Omega_{\nu} h^2 = \sum_{\alpha} m_{\alpha} / 94\text{eV},$$

which, with only the interactions present in the SM, are the same for Majorana or Dirac neutrinos. The reason is that although Dirac neutrinos have four possible states and Majorana only two, the two additional states of Dirac neutrinos are not populated in the early Universe, because of how small neutrino masses are.

The number of relativistic neutrinos species, $N_{\nu}$, is used to parametrize $(\rho_{\text{relativistic}} - \rho_{\gamma})$ in terms of the present density of one relativistic standard neutrino species (computed in the limit of instantaneous decoupling), i.e.

$$\rho_{\text{relativistic}} = \rho_{\gamma} + \rho_{\nu} + \rho_x = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\nu}\right] \rho_{\gamma}.$$ \hspace{1cm} (5)

Lower bounds on $N_{\nu}$ larger than zero have been obtained, both during BBN as well as from CMB measurements. The BBN two-$\sigma$ bound $1.6 \leq N_{\nu} \leq 3.2$ [4] constitutes, in fact, a detection of relic neutrinos, since during BBN at least $\nu_e, \bar{\nu}_e$ are needed (for the weak interactions of p, n). However, the two-$\sigma$ bound obtained by WMAP from measurements of the CMB anisotropy, $0.9 \leq N_{\nu} \leq 8.3$ [4], measures only the relativistic energy density in the Universe at the time of CMB emission (380 kyr after the Bang), which may not necessarily consist of relic neutrinos.

3. Non-Standard neutrino backgrounds

A non-standard relic neutrino background can have less or more neutrinos that the standard background. In inflationary models, the beginning of the radiation dominated era of the Universe results from the decay of coherent oscillations of a scalar field, and the subsequent thermalization of the decay products into a thermal bath with the so called “reheating temperature”, $T_R$. This temperature may have been as low as 0.7 MeV [5] (a very recent analysis strengthens this bound to $\sim 4$ MeV [6]). It is well known that a low reheating temperature inhibits the production of particles which would have become non-relativistic or decoupled at $T$ above or close to $T_R$ [7, 8]. The final number density of active neutrinos starts departing from the standard number for $T_R \simeq 8$ MeV, stays within 10% of it for $T_R > 5$ MeV, and for $T_R = 1$ MeV the number of $\nu_{\mu,\tau}$ is about 2.7% of the standard number. We may even have no relic neutrinos left in extreme models in which neutrinos annihilate into light boson at late times [9].
A non-standard relic neutrino background with a neutrino asymmetry would have more neutrinos than a standard background (in which the neutrino chemical potentials are assumed to be zero). While charge neutrality requires the asymmetry in charged leptons to be the same as that in protons, for which \((n_B - n_{\bar{B}})/n_\gamma \simeq O(10^{-10})\), no such requirement limits the asymmetry in neutrinos. With a relic neutrino asymmetry, the number of neutrinos and antineutrinos of the same flavour is different, \(n_\nu \neq n_{\bar{\nu}}\). The relic neutrino energy density always increases with a neutrino asymmetry,

\[
N_\nu = 3 + \frac{15}{7} \sum_\alpha \left[ 2 \left( \frac{\xi_\alpha}{\pi} \right)^2 + \left( \frac{\xi_\alpha}{\pi} \right)^4 \right] = 3 + \sum_\alpha 0.22 \left( 2 \xi_\alpha^2 + 0.10 \xi_\alpha^4 \right). \tag{6}
\]

We see here that for any non-zero value of the dimensionless chemical potential \(\xi_\alpha \equiv \mu_\nu_\alpha/T\) (chosen as parameter because it is constant while the expansion of the Universe is adiabatic), \(N_\nu\) is larger that for a zero value, even if for any value of \(\xi\) smaller than 1 the increase is very small. For example, \(\Delta N_\nu = 4 \times 10^{-3}\) for \(|\xi| = 0.1\) and \(\Delta N_\nu \simeq 1\) for \(|\xi| = 1.5\).

The net lepton number,

\[
L_\alpha \equiv \frac{n_\nu_\alpha - n_{\bar{\nu}_\alpha}}{n_\gamma} = \frac{\pi^2}{12\zeta(3)} \left( \xi_\alpha + \frac{\xi_\alpha^3}{\pi^2} \right) \left( \frac{T_\nu}{T_\gamma} \right)^3 = 0.25 \left( \xi_\alpha + 0.10 \xi_\alpha^3 \right) \tag{7}
\]

can be sizeable even with values of \(\xi\) somewhat smaller than 1. For example \((n_\nu_\alpha - n_{\bar{\nu}_\alpha}) \simeq 10/\text{cm}^3\) for \(|\xi| = 0.1\) and \((n_\nu_\alpha - n_{\bar{\nu}_\alpha}) \simeq 190/\text{cm}^3\) for \(|\xi| = 1.5\).

So \(|\xi| \geq 0.1\) produce small \(\nu\)-density increases but significant \(\nu\)-asymmetries. This is important for what follows, because the most conservative upper bound on \(\xi\) is about 0.1.

In the absence of significant extra contributions to the radiation density of the Universe during BBN (except for that implied by the neutrino asymmetries), bounds from BBN alone allow for larger chemical potentials for \(\nu_\mu\) and \(\nu_\tau\) than for \(\nu_e\): \(|\xi_e| < 0.2\), \(|\xi_{\mu,\tau}| < 2.6\). The reason is that the effects of \(\xi_e\) and of \(\xi_{\mu,\tau}\) compensate each other, but while the neutron to proton ratio increases with \(\sqrt{\rho_{\text{rad}}}\) (which increases with \(\xi_{\mu,\tau}\) of any sign), it is proportional to \(\exp(-\xi_e)\), thus it decreases faster with positive \(\xi_e\) [35]. However, due to the large mixings between neutrinos, fast neutrino oscillations equilibrate all \(\xi\), so the bound on all neutrino asymmetries must be equal to the smallest upper bound imposed by BBN, \(|\xi_{e,\mu,\tau}| < 0.1\) [11]. These oscillations could be suppressed [12] if neutrinos are coupled to light bosons (Majorons [13]), whose interactions produce a large effective potential for neutrinos in the early Universe, and, if so, the previously mentioned bounds would apply. Even without the suppression of
oscillations, the smallest upper bound imposed by BBN can be larger than 0.1, if an
independent source of radiation is present during BBN, so that $\Delta N_\nu = N_\nu - 3$ and the
neutrino chemical potentials are independent parameters. Then, BBN combined with
CMB data (provided by WMAP) require $-0.1 \leq \xi_e \leq 0.3$ for $-2 \leq \Delta N_\nu \leq 5$ [14].

Thus, in the following we will take as conservative upper bounds $\xi_{e,\mu,\tau} \simeq 0.1$ which
implies $(n_\nu - n_\bar{\nu}) \simeq 10/cm^3$ and $(n_\nu + n_\bar{\nu}) \simeq 112/cm^3$, or, as an extreme upper bound
$\xi_{\mu,\tau} \simeq 3$, i.e. $L \simeq 2.5$, $(n_\nu - n_\bar{\nu}) \simeq n_\nu \simeq 1,050/cm^3$.

4. Gravitational clustering of relic neutrinos

Gravitational clustering of neutrinos in our galaxy or galaxy cluster may enhance the
relic neutrino density making it easier to detect the C$\nu$B on Earth. Already in 1979,
Tremaine and Gunn [15] produced a kinematical constraint, which shows that neutrinos
as light as we now know they are, would not significantly cluster. Light neutrinos with
masses $m_\nu < eV$ can be gravitationally bound only to the largest structures, large
clusters of galaxies. We can see this using simple velocity arguments. Only cosmic
neutrinos with velocities smaller than the escape velocity of a given structure can be
bound to it. The escape velocity from a large galaxy like ours is about 600 km/s and
from a large cluster of galaxies is about 2,000 km/s. Considering that the average
velocity modulus of non-relativistic neutrinos of mass $m$ and temperature $T_\nu$ is (using
Maxwell-Boltzman distribution) $\langle |\vec{v}_\nu| \rangle = \sqrt{8kT_\nu/\pi m} = \sqrt{4.3 \times 10^{-4} eV/m}$ (namely $\langle |\vec{v}_\nu| \rangle = 6,200$ km/s for $m = 1eV$, and $\langle |\vec{v}_\nu| \rangle = 19,600$ km/s for $m = 0.1eV$), it is obvious
that only about a third of 1eV mass neutrinos, and a very small fraction of lighter
neutrinos, could be gravitationally bound to large clusters at present. Fermi degenerate
neutrinos (those with $\xi > 1$) may have even larger average velocities depending on
their chemical potential, but the conclusions remain the same. For $\xi >> 1$, we have
$\langle |\vec{v}_\nu| \rangle = \sqrt{6kT_\nu/5m} \simeq \sqrt{1.68 \times 10^{-4} eV/m}$ (namely $\langle |\vec{v}_\nu| \rangle = \sqrt{\xi} 12,300$ km/s for $m = 0.1eV$), and both expressions coincide for $\xi = 2.5$. In all cases the amount of neutrinos
in the tail of the velocity distribution with velocities smaller that 600km/s, which would
be gravitationally bound to galaxies, is much smaller.

More recently, in 2002, Singh and Ma [16] studied the clustering of neutrinos in cold
dark matter halos. They found that neutrino overdensities decrease with cluster halo
mass and distance to the center (and we are in the periphery of the Virgo supercluster),
so overdensities close to Earth could be at most of $O(1)$ for neutrino masses close to 1
eV, and for $m_\nu \leq 0.1$ eV neutrino clustering is insignificant.
5. Prospects for laboratory searches: effects of $O(G_F^2)$

Given the characteristic relic neutrino energy $\langle E_{\nu_\alpha} \rangle \simeq T_\nu \simeq 10^{-4} \text{ eV}$, the relic $\nu$-nucleon cross sections are very small. For Dirac neutrinos,

$$\sigma_{\nu-N} \approx \begin{cases} 
G_F^2 m_\nu^2 / \pi \simeq 10^{-56} (m_\nu / \text{eV})^2 \text{cm}^2 & \text{for (NR- Dirac)} \\
G_F^2 E_\nu^2 / \pi \simeq 5 \times 10^{-63} \text{cm}^2 & \text{for (R)} 
\end{cases} \quad (8),$$

where R stands for relativistic, NR for non relativistic and $G_F$ is the Fermi coupling constant. Majorana and Dirac neutrinos are indistinguishable while relativistic. For NR Majorana neutrinos the cross sections are even smaller: since only the $\gamma \mu \gamma_5$ weak interaction coupling remains, a factor $\beta^2_\nu$ appears, where $\beta$ is the neutrino velocity.

With $(n_\nu + n_\bar{\nu}) \simeq 100 \text{ cm}^{-3}$, incoherent scattering off nucleons leads to rates smaller than $10^{-6} \text{ yr}^{-1}$ per kiloton, for the most favourable case NR Dirac neutrinos with eV mass.

Nuclear coherence enhancement factors, of order $A^2 \simeq 10^4$ (with $A$ the atomic number), do not help much. But coherence over the relic $\nu$ wavelength, $\lambda_\nu = 2\pi h / 4T_\nu \approx 2.4 \text{ mm}$ (or 1.2 mm$\times$eV/m$_\nu$ for clustered neutrinos), makes an enormous difference since a volume $\lambda_\nu^3$ contains more than $10^{20}$ nuclei. Since destructive interference occurs if target size is larger than $\lambda_\nu$ the largest enhancement is obtained with a material less than half filled with grains of size $\lambda_\nu = [17, 18]$.

Even with this sizeable cross section, the net momentum imparted by relic neutrinos on a target on Earth would be zero on average, if the C$\nu$B would be on average at rest with respect to the Earth. But this is clearly not so. A reasonable guess is that the C$\nu$B is at rest with the CMB, and the Sun’s motion with respect to the CMB (derived from COBE-DMR dipole anisotropy) is $v_{\text{sun}} = 369.0 \pm 2.5 \text{ km/sec}$, i.e. the speed of the Earth with respect to the C$\nu$B is $\beta_{\text{earth}} = 1.231 \times 10^{-3}$.

A momentum of the order of the neutrino momentum, $\Delta \vec{p} \approx \vec{p}_\nu$, is imparted in each neutrino collision. Due to the bulk velocity of the “neutrino wind” on Earth, $-\vec{\beta}_{\text{earth}}$, there is a preferred direction for $\Delta \vec{p}$, thus $\langle \Delta \vec{p} \rangle \approx -\vec{\beta}_{\text{earth}} (3T_\nu / c)$ (or $\approx -\vec{\beta}_{\text{earth}} c m_\nu$ for clustered neutrinos).

The resulting accelerations for relativistic (Dirac or Majorana) neutrinos are [19]

$$a_R \approx 2 \times 10^{-33} \text{ cm/sec}^2 f (\rho / 10). \quad (9)$$

For non-clustered non-relativistic Dirac neutrinos (i.e for most relic neutrinos), the cross-sections are [19]

$$a_{\text{NC-NR}} \approx 3 \times 10^{-27} \text{ cm/sec}^2 f (m_\nu / \text{eV})^2, (\rho / 10), \quad (10)$$
and for clustered non-relativistic Dirac neutrinos, the cross sections are [19]

\[ a^D_{C-NR} \simeq 10^{-26}\text{cm/sec}^2 f(\rho/10) . \]  

(11)

In these equations the factor \( f \) accounts for the possible enhancement due to clustering (as well as, possibly, a large lepton asymmetry), \( 1 \leq f \equiv (n_\nu + n_{\nu c})/100\text{cm}^{-3} < 10. \)

As mentioned above, non-relativistic Majorana neutrinos have only a \( \gamma_\mu\gamma_5 \) coupling. The coherent interactions due to the static limit of this coupling are, however, suppressed by \( \beta_\nu \), the ratio of the “small” and “large” components of the spinor. Recall that for non-relativistic spinors the lower components are “smaller” than the upper components by a factor of \( \beta \). Thus the analog of Eqs. (10) and (11) for non-relativistic Majorana neutrinos is suppressed by an extra factor of \( \beta_\nu^2 \approx 10^{-6} \).

6. Prospects for laboratory searches: effects linear in \( G_F \)

Coherent interactions of a low energy neutral particle, with a medium in which the interatomic spacing is much smaller than the deBroglie particle wavelength (recall \( \lambda_\nu \approx 2.4\text{mm} \)), change the particle momentum from \( p \) to \( p' \). Then, one can define an index of refraction \( n = p'/p \), and \( n - 1 \sim G_F \). However, early proposals to use “neutrino optics”, either total reflection [20] or refraction (or refraction in a superconducting surface which would induce a current) [21] were incorrect. Cabibbo and Maiani [22] and Langacker, Leveille and Sheiman [23] in 1982, proved that the force due to linear momentum or energy exchange on a target immersed in a uniform neutrino field cancel to order \( G_F \), in fact [22]

\[ \vec{F} = -\frac{\Delta \vec{p}_\nu}{\Delta t} \simeq G_F \int d^3x \rho_A(x) \vec{\nabla} n_\nu(x) , \]  

(12)

where \( \rho_A \) is the atomic number density of the target, and \( \vec{\nabla} n_\nu(x) \) is the gradient of the local neutrino number density. This gradient is zero (since \( n_\nu \) due to gravitational effects is uniform at the scale of possible detectors), except for scattered waves due to weak interactions, which are of order \( G_F \), and thus lead to forces \( F \sim G_F^2 \). In fact, Smith and Lewin [18] proposed in 1983 to generate large artificial neutrino density distortions to induce a neutrino density gradient and thus a force, but there is no known way to do this.

There is only one possible mechanical effect of order \( G_F \), proposed by L. Stodolsky [24] in 1974: a torque of order \( G_F \) can arise if both the target (e.g. consisting of
magnetized iron) and the $\nu$-background have a polarization. The $\nu$-background must have a non-zero net flux of weak-interactions-charge (i.e. of neutrinos minus antineutrinos) to produce a net torque on polarized electrons. Since the Earth is moving with respect to the $C\nu B$, there is a net flux of particles reaching us, with $\langle \vec{\beta}_\nu \rangle = -\vec{\beta}_{earth}$. Thus we only need a lepton asymmetry in those particles to have a net flux of weak-charge $\sim -\vec{\beta}_{earth}(n_\nu - n_\bar{\nu})$.

The Stodolsky effect consists of an energy split of the two spin states of non-relativistic electrons in the $C\nu B$. This energy split is proportional to the difference between the densities of neutrinos and antineutrinos in the neutrino background for Dirac neutrinos and relativistic Majorana, and proportional to the net helicity of the background for non-relativistic Majorana neutrinos, as we will now see. The distinction of Dirac or Majorana neutrinos is important only for non-relativistic neutrinos.

The Hamiltonian density of the neutrino-electron interaction is

$$\mathcal{H}(x) = \frac{G_F}{\sqrt{2}} (\bar{\nu}\gamma^\mu(g_V - g_A\gamma_5)e) \left[ \bar{\nu}\gamma_\mu(1 - \gamma_5)\nu \right]. \quad (13)$$

In the non-relativistic limit the electron current $(\bar{e}\gamma^\mu(\gamma_5)e)$ yields a factor $\vec{\sigma}_e \cdot \vec{\beta}_e$, $\vec{\sigma}_e$ (from the time and space components of the current). For Dirac neutrinos (for which $\nu \neq \bar{\nu}$), the $\gamma_\mu$ term dominates, and $[\bar{\nu}\gamma_\mu \nu]$ yields $\bar{\nu}1\nu, ... \sim (n_\nu - n_\bar{\nu})$, which is non zero if there is a net Lepton number in the $C\nu B$. The effect, originally derived by Stodolski [24] only for Dirac neutrinos, is proportional to the product $\vec{\sigma}_e \cdot \vec{\beta}_e(n_\nu - n_\bar{\nu})$. For Majorana neutrinos (for which $\nu = \bar{\nu}$), only the $\gamma_\mu\gamma_5$ term remains, and $[\bar{\nu}\gamma_\mu\gamma_5\nu]$ yield in the non-relativistic limit $\bar{\nu}(\vec{\sigma}_e \cdot \vec{\beta}_e, \vec{\sigma}_e)\nu \sim (n_{\nu_L} - n_{\nu_R})$, which is non-zero only if there is a net helicity in the $C\nu B$ [19].

Here we call left (right) chirality eigenstates $\nu_L(\nu_R)$, and left (right) helicity eigenstates $\nu_e(\nu_\bar{e})$. The general expression for the energy of one electron in the $C\nu B$, in the electron-rest frame, to first order in $\beta_{earth}$, both for Dirac and Majorana neutrinos were first derived in Ref.[19]. Let us see the most relevant particular cases, starting from early times, before the decoupling of neutrinos.

Since neutrinos are lighter than about 1 eV, they were relativistic at decoupling ($T_{dec} \geq O(\text{MeV})$). Relativistic neutrinos are only in left-handed chirality states (and anti- neutrinos only in right-handed chirality states). These are the only states produced by weak interactions. For relativistic neutrinos chirality and helicity coincide (up to mixing terms of order $m_\nu/E_\nu \simeq m_\nu/T_\nu$). Thus, at decoupling, neutrinos $\nu_L$ were only in left-handed helicity states, and antineutrinos $\nu_{\bar{L}}$ (or $\nu_R$ in the case of
Majorana neutrinos) in right-handed ones. In this case, the term in the Hamiltonian of one electron in the CνB linear in the spin of the electron \( \vec{s}_e \), is

\[
H^D_R = H^M_R = -\sqrt{2}G_F g_A 2\vec{s}_e \cdot \vec{\beta}_{\text{earth}}(n_{\nu_L} - n_{\nu_R}).
\]

(14)

Helicity is an eigenstate of propagation and, therefore, it does not change while neutrinos propagate freely, even if they become non-relativistic. Recall that two of the neutrinos mass eigenstates are non-relativistic at present. For Majorana neutrinos chirality acts as lepton number, so we are calling “neutrinos” those particles produced at \( T > T_{\text{dec}} \) as \( \nu_L \), and “anti-neutrinos” those produced as \( \nu_R \). Thus, neglecting intervening interactions, non-relativistic background neutrinos are in left-handed helicity eigenstates (which consist of equal admixtures of left- and right-handed chiralities) and anti-neutrinos are in right-handed helicity eigenstates (which also consist of equal admixtures of left- and right-handed chiralities). If the non-relativistic neutrinos are Dirac particles, only the left-handed chirality states (right for anti-neutrinos) interact, since the other chirality state is sterile, while if the neutrinos are Majorana, both chirality states interact (the right-handed “neutrino” state is the right-handed anti-neutrino). In the most favourable case of a large \( \xi = \mu/T \), for \( m_\nu \leq 0.1eV \), the term in the Hamiltonian linear in the electron spin, for non-clustered non-relativistic relic neutrinos (which are most of them), in the electron rest frame is

\[
H^D_{NC-NR} \simeq \frac{1}{2} H^M_{NC-NR} \simeq 0.85 \langle |\vec{\beta}_\nu|\rangle^{-1} H^D_R \leq \frac{7}{\sqrt{\xi}} H^D_R,
\]

(15)

where \( \langle |\vec{\beta}_\nu|\rangle = \sqrt{6\xi T_\nu/5m} \simeq \sqrt{\xi 1.7 \times 10^{-4}eV/m} \) is the characteristic velocity (the average of the velocity modulus) of relic neutrinos in the CνB rest frame. The dominant term shown here, comes from the space part of the neutrino current (\( \vec{\sigma}_\nu \) in the helicity base is \( h_i\vec{\beta}_\nu \sim \langle |\vec{\beta}_\nu|\rangle^{-1} \), where \( \vec{\beta}_\nu \) is the unit vector of the neutrino velocity in the relic neutrino rest frame). Notice that there is no \( \beta \) factor penalty for Majorana neutrinos.

Slow enough non-relativistic neutrinos eventually fall into gravitational potential wells, become bound and, after a characteristic orbital time, their helicities become well mixed up, since momenta are reversed and spins are not. Thus, gravitationally bound relic neutrinos have well mixed helicities, no net helicity remains. For these clustered non-relativistic neutrinos, the Stodolski effect cancels for Majorana neutrinos, but only changes by a factor for Dirac neutrinos.

\[
H^D_{C-NR} = \frac{1}{2} H^D_R, \quad H^M_{C-NR} = 0.
\]

(16)
As we argued above, most relic neutrinos however are not gravitationally bound at present, because they are too light.

The Stodolski effect requires a Lepton asymmetry \( n_\nu - n_\nu^c = f \times 100 \text{ cm}^{-3} \) (where the maximum possible overdensity factor is \( f(\xi)_{\text{max}} = 0.1 - 10 \)) to obtain an energy difference, \( \Delta E \), between the two helicity states of an electron in the direction of the bulk velocity of the neutrino background, \( < \beta_\nu^c > = -\bar{\beta}_\text{earth} \). In the case of Dirac or relativistic Majorana neutrinos of density \( n_\nu \), with a very large lepton asymmetry favouring, say, neutrinos \( \nu_L \) (so that \( n_\nu_L = n_\nu^c = n_\nu \)) we have

\[
\Delta E \approx f^2 \sqrt{2} G_F g_A |\bar{\beta}_\text{earth}| n_\nu .
\]  

This is the equation we will use to estimate the maximum possible accelerations due to this effect. We should recall however that \( \Delta E_{\text{NC-NR}}^M \leq 8\Delta E \) and that \( \Delta E_{\text{C-NR}}^M = 0 \).

The energy difference \( \Delta E \) implies a torque of magnitude \( \Delta E / \pi \) applied on the spin of the electron. Since the spin is “frozen” in a magnetized macroscopic piece of material with \( N \) polarized electrons, the total torque applied to the piece has a magnitude \( \tau = N\Delta E / \pi \). Given a linear dimension \( R \) and mass \( M \) of the macroscopic object, its moment of inertia is parametrized as \( I = MR^2 / \gamma \), where \( \gamma \) is a geometrical factor. In the typical case of one polarized electron per atom in a material of atomic number \( A \), the number \( N \) above is \( N = (M / gr)N_{AV} / A \) (using cgs units), where \( N_{AV} \) is Avogadro’s number. Thus, the effect we are considering would produce an angular acceleration of order \( \alpha = \tau / I \) and a linear acceleration of order \( a_{GF} = R\alpha \) in the magnet given by

\[
a_{GF} \approx 10^{-27} \text{cm/ sec}^2 f \cdot \left( \frac{\gamma}{10} \right) \left( \frac{100}{A} \right) \left( \frac{\text{cm}}{R} \right) \left( \frac{\beta_\text{earth}}{10^{-3}} \right).
\]  

The local density enhancement factor due to a net lepton number or clustering is \( 0 \leq f \equiv (n_\nu - n_\nu^c) / 100\text{cm}^{-3} < 100 \).

We should compare the accelerations mentioned in Eqs. (10), (11) and (18) above with the smallest measurable acceleration at present, with “Cavendish” type torsion balances, which is about \( 10^{-12} \text{ cm/sec}^2 \)[25]. This is about 15 orders of magnitude larger.

As an example of an attempt to produce a relic neutrino detector, we can mention a particular design of a torsion oscillator proposed by C. Hagmann [26] in 1999. In this proposed detector, the uncertainty principle gives a minimum measurable acceleration (QL stands for quantum limit),

\[
a_{QL} = 5 \times 10^{-24} \text{cm/s}^2 (10\text{kg/m})^{1/2} (1\text{day}/\tau_0)^{1/2} (10^6 \text{s}/\tau)
\]  

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where $\tau_0$ is the oscillation period, and $\tau$ measurement time. This acceleration is larger by a factor $(10^3$-$10^4)f^{-1}$ than the largest possible accelerations mentioned above. We would need a detector with a lower quantum limit, and that can operate at that limit. To beat the quantum limit P. Smith proposed to go to tons of mass target [18, 27] or to the opposite, to sub-micron granules whose single displacements can be measured (which involves nanotechnology not yet in place) [28].

Still there are other difficulties. Seismic and gravitational variations are a problem. For example, the gravity gradient due to the Moon produces a varying torque about $10^{10}$ larger than the relic neutrino force. A possible solution would be a concentric balance [28]. Solar neutrinos would produce equal or larger accelerations (and possibly dark matter particles could too) [19]. So directionality would be needed to separate a signal from relic neutrinos (Smith and Lewin [27] proposed using laminated materials).

The evident conclusion is that laboratory effects of the C$\nu$B are still far from observability, therefore awaiting future technology.

6. Prospects for Astrophysical Searches

Only at the Z-resonance the cross section of astrophysical neutrinos with the C$\nu$B is large enough to reveal its existence [29]. A simple argument is that the cross section at the Z-resonance, $\sigma_{\text{annih}}(E_{\text{res}}) \simeq 4 \times 10^{-32}$ cm$^2$, yields a mean free path for Ultra-High Energy (UHE) neutrinos larger but not by much than the Hubble distance, $\ell_{\text{Hubble}}$, the size of the visible Universe, $\ell_{\text{m.f.p.}} \simeq 35 \times \ell_{\text{Hubble}}$. Thus the probability of interaction is non-negligible, about 0.03. Otherwise, for interactions outside the Z-resonance, the Universe is transparent to UHE$\nu$.

If an intense enough flux of UHE$\nu$ would exist the resonant annihilation with the C$\nu$B would leave an absorption dip at

$$E_{\nu_i}^{\text{res}} = \frac{m_Z^2}{2m_{\nu_i}} = 4.2 \times 10^{22} \text{ eV} \left( \frac{0.1 \text{ eV}}{m_{\nu_i}} \right).$$

A recent examination [30] of the possibility of relic neutrino absorption spectroscopy through this mechanism shows that, if intense enough sources of UHE$\nu$ at energies $E_{\nu} \geq 10^{22}$ eV and above would exist, the different masses of relic neutrinos could produce separated absorption dips. Experiment such as ANITA, Auger, EUSO, OWL, RICE, and SalSA able to detect such UHE neutrinos are expected to be available in the near future [31]. The problem with this idea, is that the only sources proposed to
produce such UHEν fluxes are topological defects. Even Active Galactic Nuclei are not thought to be able to produce neutrinos with energies above $10^{21}$ eV.

Another signature of the annihilation of UHEν with relic neutrinos at the Z-resonance, is the emission of UHE p, n, γ and ν, in what T. Weiler called “Z-bursts”. In fact, Z-bursts were proposed [32] as the possible origin of UHE Cosmic Rays above the “Greisen-Zatsepin Kuzmin cutoff” at $E_{GZK} \approx 5 \times 10^{19}$ eV observed, for example, by AGASA [33]. This mechanism is now considered unlikely to be correct (and if it is correct, it would imply a relic neutrino mass $m_\nu \approx 0.3$ eV [34]). If they do not produce the UHECR, Z-bursts are subdominant, and thus not a good signal to detect the CνB.

7. Conclusions

In the past, neutrinos were thought to be sufficiently massive ($m_\nu \geq 20$ eV) to cluster in our galaxy and make up the local dark matter halo. If so, they could have had large local overdensities, even $f \approx 10^7$. Also chemical potentials had a much larger upper bound until a few years ago, $\xi \leq 6.9$ [35]. All this made for much more optimistic predictions of the laboratory effects of relic neutrinos.

Now, we know that relic neutrinos have sub-eV masses, the enhancement factors $f$ due to clustering could only be at most of $O(1)$, and the maximum allowed chemical potential is likely $\xi \leq 0.1$. We know also, that at least two of the three active neutrino mass eigenstates have masses $m_{\nu_{2,3}} > 10^{-2}$ eV, thus most relic neutrinos are non-relativistic and non-clustered, and have an average velocity with respect to the Earth similar to that of the CMB, $\beta_\nu \approx 10^{-3}$. These values lead to estimations of the the effect of the “cosmic neutrino wind” on macroscopic targets that are smaller than those made even a few years ago.

There are for sure forces on macroscopic targets of order $G_F^2$, and possibly torques of order $G_F$. $O(G_F^2)$ forces are largest when due to coherent elastic scattering out of target grains of the size of the neutrino deBroglie length $\lambda_\nu \approx 0.2$ cm. The largest effect is for Dirac neutrinos with mass as large as possible (i.e. close to eV), as can be seen in Eqs. (10) and (11). The only $O(G_F)$ effect is a torque on a polarized target, if there is a net flux of weak charge due to the CνB. It requires a lepton asymmetry in the relic neutrinos. The effect is largest for non-relativistic non-clustered Majorana or Dirac neutrinos, for which the acceleration could be up to one order of magnitude larger than in Eq. (18). These accelerations are tiny, and there are difficult backgrounds. New experimental ideas are needed to reach the required sensitivity.
With respect to the prospects for astrophysical searches, Z-pole absorption dips in UHEν flux could reveal relic neutrinos and their masses. The problem with this idea is the existence of sources of UHEν with \( E_\nu \geq 10^{22} \text{ eV} \). If these sources exist, this would be the most promising search mechanism in the foreseeable future, using detectors such as ANITA, Auger, EUSO, OWL, RICE, and SalSA.

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