Polarization-dependent orbital angular momentum flipping in fibers with acousto-optic interaction

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Abstract. In this paper we consider the evolution of linearly polarized optical vortices in circular optical fibers with a propagating fundamental flexural acoustic wave. A new polarization-dependent mode conversion is found out, in which the sign of the topological charge (and orbital angular momentum) of the outgoing vortex beam is governed by the direction of the incident linear polarization. This effect can be used for implementing polarization-controlled orbital angular momentum flipping. This paves the way to implementation of the all-fibre stable controlled-NOT gate, in which the linear polarization carries the control qubit and the topological charge carries the target. Such a gate is able to produce optical beams with entanglement between polarization and orbital degrees of freedom in regime of linear optics. Yet, such orbital angular momentum controlling should be useful in micromechanics, classical and quantum information encoding, and classical simulation of quantum algorithms.

1. Introduction

Nowadays, the orbital angular momentum (OAM)-bearing beams [1], in particular optical vortices (OVs) [2], are commonly recognized as highly perspective carries of information encoded in the orbital degrees of freedom of light [3] in both free space [4, 5, 6, 7, 8, 9] and optical fibres [10, 11, 12, 13, 14, 15, 16]. It is connected with the fact that OAM of an OV, defined in the simplest case as $\ell \hbar$, where topological charge $\ell = 0, \pm1, \pm2, \ldots$, has theoretically unlimited range of values. Naturally, a successful application of OVs in information technologies requires the means for controlling OAM, which is necessary for information encoding as well as for performing the basic set of logic operations. Besides, since the orthogonal states of OVs with different OAM values form a multidimensional space, this question turns out to be closely related to quantum computations. Indeed, the possibility of modeling quantum calculations by means of classical optical fields has been proven in lots of works [17, 18, 19, 20], including simulation of local [20] and nonlocal entanglement [21].

Variety of practically relevant operations with OAM can be implemented on the basis of specially designed optical fibres. A very promising type of such fiber systems is presented by optical fibers endowed with the acousto-optic interaction (AOI), which is induced by the so-called flexural acoustic wave (FAW). It is connected with the fact that the fibers guide both optical and acoustic waves. Besides, the FAW is primarily used in fiber acousto-optics because it produces an axially-asymmetric perturbation, which enables the well-known coupling of the input fundamental mode LP₀ to the higher-order LP₁ modes [22, 23]. As a relevant example, an efficient generation
of an optical vortex beam of topological charge $+1$ or $-1$ directly from the input fundamental mode was theoretically predicted in [24], in which both the spin and orbital angular momentum of the produced beam are defined by the handedness of the incident polarization. The experimental confirmation of this polarization-dependent mode transformation was given in [25]. The same researches showed [26] that a circular fiber with the AOI is able to efficiently convert the incident fundamental mode to the cylindrical vector beams.

Surprisingly, the propagation of the incident optical vortices through the well-studied circular fibers with the fundamental FAW has not been considered yet. Here we aim at demonstrating that a conventional circular fiber endowed with the lowest-order FAW can be used for polarization-controlled wavelength-tunable flipping of OAM of an incident OV with a unity topological charge. As a relevant example, this effect can be used for implementing the all-fibre fundamental controlled-NOT gate, in which the direction of linear polarization carries the control qubit, while the topological charge - the target one. Note that such a CNOT gate is able to produce optical beams with local entanglement between their polarization and OAM in regime of linear optics.

2. Fiber model and resonance modes
The proposed only recently [27] proper expression for the permittivity of a circular fiber with a propagating fundamental linearly polarized FAW [28] (see Fig. 1) is given by:

$$\hat{\varepsilon}(r,\varphi,z,t) = \varepsilon_0(r) + 2\Delta \varepsilon_{co}u_0 f_r \cos \varphi \cos(Kz - \Omega t) + \varepsilon_{co}^2 u_0 K \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \sin(Kz - \Omega t).$$

(1)

Here the unperturbed fiber permittivity looks like

$$\varepsilon_0(r) = \varepsilon_{co}[1 - 2\Delta f(r)],$$

(2)

where $\Delta = (\varepsilon_{co} - \varepsilon_{cl})/2\varepsilon_{co}$ is the normalized index difference, $\varepsilon_{co}$ and $\varepsilon_{cl}$ are the core and cladding values of the permittivity, respectively, and $f(r)$ is the fiber’s profile function. In the second term of Eq. (1) $f_r = df/dr$ and $u_0$, $K$, and $\Omega$ are the amplitude, wavevector and frequency of the FAW, respectively. In the last term for silica at the wavelength $\lambda = 0.63 \mu m$ the constant of photoelasticity is $p = -0.075$. The cylindrical coordinates $(r, \varphi, z)$ are implied.
On basis of the previously developed resonance perturbation theory for a non-stationary fiber permittivity [29], the following analytical expressions of the eigenmodes of the fiber model in the problem have been obtained:

\[
\begin{align*}
|\Psi_{1}^{(\sigma)}\rangle &= \left[\sin\theta|\text{LP}_{0}^{\sigma}\rangle + \cos\theta|\text{LP}_{1}^{\sigma,\sigma}\rangle e^{i(\Omega t - Kz)}\right] e^{i(\beta z - \omega t)}, \\
|\Psi_{2}^{(\sigma)}\rangle &= \left[\cos\theta|\text{LP}_{0}^{\sigma}\rangle - \sin\theta|\text{LP}_{1}^{\sigma,\sigma}\rangle e^{i(\Omega t - Kz)}\right] e^{i(\beta z - \omega t)}, \\
|\Psi_{3}^{(\sigma)}\rangle &= \left[\sin\theta|\text{LP}_{1}^{\sigma,\sigma}\rangle - \cos\theta|\text{LP}_{1}^{\sigma,\sigma}\rangle e^{i(Kz - \Omega t)}\right] e^{i(\beta z - \omega t)}, \\
|\Psi_{4}^{(\sigma)}\rangle &= \left[\cos\theta|\text{LP}_{1}^{\sigma,\sigma}\rangle + \sin\theta|\text{LP}_{1}^{\sigma,\sigma}\rangle e^{i(Kz - \Omega t)}\right] e^{i(\beta z - \omega t)}, \\
|\Psi_{5}^{(\sigma)}\rangle &= |\text{LP}_{1}^{\text{od},\sigma}\rangle e^{i(\beta z - \omega t)}. 
\end{align*}
\]

Here in the basis of linear polarizations \(|\Psi_{k}^{(\sigma)}\rangle = (E_{x}, E_{y})^{T}\) the standard even and odd LP modes read as

\[
\begin{align*}
|\text{LP}_{\ell}^{\text{ev},x}\rangle &= \sqrt{2} F_{\ell}(r)(\cos \varphi, 0)^{T}, \\
|\text{LP}_{\ell}^{\text{ev},y}\rangle &= \sqrt{2} F_{\ell}(r)(0, \cos \varphi)^{T}, \\
|\text{LP}_{\ell}^{\text{od},x}\rangle &= \sqrt{2} F_{\ell}(r)(\sin \varphi, 0)^{T}, \\
|\text{LP}_{\ell}^{\text{od},y}\rangle &= \sqrt{2} F_{\ell}(r)(0, \cos \varphi)^{T},
\end{align*}
\]

where \(\sigma = x, y\) specifies the direction of linear polarization, \(F_{\ell}(r)\) is the well-known radial function [30], the radial number is omitted and \(T\) stands for the transposition. The energy distribution within the hybrid modes in Eq. (3) is governed by the parameter \(0 < \theta \leq \pi/4\) defined as

\[
\cos 2\theta = (\epsilon / \sqrt{\epsilon^2 + Q^2}),
\]

where the detuning from the resonance \(\epsilon = K - \bar{K}\) and the resonance value of the acoustic wave vector

\[
\bar{K} = \bar{\beta}_{0} - \bar{\beta}_{1},
\]

defined through the well-known scalar propagation constants \(\bar{\beta}_{\ell}\) [30] of the modes (4). The parameter \(Q\), which characterizes the coupling strength between the unperturbed fiber modes, is:

\[
Q = Q_{g} + Q_{p}, Q_{g} = \sqrt{\frac{\epsilon_{0}}{2r_{0}N_{0}N_{1}}} k \Delta u_{0}, Q_{p} = -\sqrt{\frac{\epsilon_{0}^{2}}{8r_{0}N_{0}N_{1}}} pKu_{0} \int_{0}^{\infty} R \frac{dF_{0}}{dR} F_{1} dR,
\]

where \(k = 2\pi/\lambda, r_{0}\) is the fiber core’s radius and the mode normalization \(N_{\ell} = \int_{0}^{\infty} RF_{\ell}^{2} dR\). Here we implied the conventional step-index fibers with a profile function \(f(r) = \Theta(r/r_{0} - 1)\), where \(\Theta\) is the unit step function. As follows form Eq. (3), the resonance fiber modes are represented by the superposition of the identically linearly polarized fundamental modes \(|\text{LP}_{0}\rangle\) and the frequency shifted higher-order states \(|\text{LP}_{1}\rangle\). Naturally, then strongest coupling between the zero-order modes 4 within the hybrid modes takes place at resonance condition \(\epsilon = 0, 5\), when \(\theta = \pi/4\).

The propagation constants of modes (3) are found to be:

\[
\begin{align*}
\beta_{1}^{x,y} &= \bar{\beta}_{0} + (1/2)(\epsilon + \sqrt{\epsilon^2 + Q^2}) \pm (Q_{p}/2) \sin 2\theta, \\
\beta_{2}^{x,y} &= \bar{\beta}_{0} + (1/2)(\epsilon - \sqrt{\epsilon^2 + Q^2}) \mp (Q_{p}/2) \sin 2\theta, \\
\beta_{3}^{x,y} &= \bar{\beta}_{1} + (1/2)(-\epsilon - \sqrt{\epsilon^2 + Q^2}) \mp (Q_{p}/2) \sin 2\theta, \\
\beta_{4}^{x,y} &= \bar{\beta}_{1} + (1/2)(-\epsilon + \sqrt{\epsilon^2 + Q^2}) \pm (Q_{p}/2) \sin 2\theta.
\end{align*}
\]
Here the upper sign at the last terms corresponds to the $x$-polarized modes in Eq. (3). It should be noted that the propagation constants of the orthogonally polarized resonance modes appear to be split by a coupling constant $Q_p$, which comes from the last tensor correction in fiber’s permittivity (1).

3. Polarization-dependent OAM flipping and CNOT gate

Now, let the OV linearly polarized along $x$ or $y$-axis

$$|LV^\sigma_\ell\rangle = |LP^{ev,\sigma}_1\rangle + i|LP^{od,\sigma}_1\rangle,$$

at frequency $\omega$ be propagating in the fiber with the above described AOI:

$$|\Psi_{in}\rangle = |LV^\sigma_\ell\rangle e^{-i\omega t}.$$

It will excite in the fiber the superposition of eigenstates (3) with the same polarization $\sigma$:

$$|\Psi(z)\rangle = \sum_k a_k |\Psi_\sigma^k(z)\rangle.$$

Invoking the simplified boundary conditions

$$|\Psi_{in}\rangle = \sum_k a_k |\Psi_\sigma^k(z = 0)\rangle,$$

one can determine the decomposition coefficients $a_k$ and the field in the AOI region, which within a phase factor, can be brought to the following form:

$$|\Psi(z)\rangle = \left[ c_0^\sigma(z) |LP^{\sigma}_0\rangle e^{i(Kz - \Omega t)} + c_\ell^\sigma(z) |LV^\sigma_\ell\rangle + c_{-\ell}^\sigma(z) |LV^{-\sigma}_{-\ell}\rangle \right] e^{-i\omega t},$$

where the coefficients $c_i(z)$ are given by:

$$c_0^\sigma(z) = (i/2) \sin 2\theta \sin(\eta_\sigma z),$$

$$c_\ell^\sigma(z) = (1/2) \{ [\cos(\eta_\sigma z) + i \cos 2\theta \sin(\eta_\sigma z)] e^{-0.5i\epsilon z} + 1 \},$$

$$c_{-\ell}^\sigma(z) = (1/2) \{ [\cos(\eta_\sigma z) + i \cos 2\theta \sin(\eta_\sigma z)] e^{-0.5i\epsilon z} - 1 \},$$

with

$$\eta_{x,y} = 0.5(\sqrt{\epsilon^2 + Q^2} \pm Q_p \sin 2\theta).$$

It is important noting that

$$|c_0^\sigma|^2 + |c_1^\sigma|^2 + |c_2^\sigma|^2 = 1,$$

which expression is the conservation energy law once one takes into account that the energy stored in the partial beams in Eq. (13) is determined as:

$$W_\sigma^\delta = |c_n^\sigma|^2,$$

where $n = 0, \pm \ell$. Eq. (13) shows that the field in the fiber is composed of the incident OV mode $|LV^\sigma_\ell\rangle$ and the generated OV $|LV^\sigma_{-\ell}\rangle$ with the topological charge of an opposite sign as well as the frequency upshifted fundamental mode $|LP^\delta_0\rangle$.

When (i) the resonance regime $\epsilon = 0$ is implemented and (ii) the fiber has the optimal length $z = L_m^\sigma$ (see Fig. 2 and Fig. 3), where

$$L_m^x = \frac{2(2m + 1)\pi}{Q_g + 2Q_p}, \quad L_m^y = \frac{2(2m + 1)\pi}{Q_g},$$

4
Figure 2. Transmission spectra for the generated OVs $W^\sigma_{\ell}$ with (a) $x$ and (b) $y$ direction of linear polarization. The fiber has a fixed length that is the optimal one for a certain acoustic power $P_0$, optical wavelength $\lambda_0$, at which $\varepsilon(\lambda_0) = 0$, and for the given polarization according to Eq. 18. The fiber’s parameters: $V = 4.16$, $\Delta = 0.001$, $r_0 = 6.3$ $\mu$m, the acoustic power $P_0 = 50$ mW, $L_x^0 = 4.26$ cm and $L_y^0 = 3.56$ cm.

Figure 3. The dependence of the energy $W_0^\sigma$ stored in the state $|LV_\ell\rangle$ on the fiber’s length. The fiber’s parameters: $V = 4.16$, $\Delta = 0.001$, $r_0 = 6.3$ $\mu$m, the acoustic power $P = 50$ mW and $L_0 = 21$ cm.

\[
W_0^\sigma (0, L_0^m) = W_0^\sigma (0, L_m^0) = 0 , W_{-\ell}^\sigma (0, L_m^0) = 1 ,
\]

all the incident energy becomes stored in the generated OV mode with the topological charge opposite to the incident one:

\[
|LV_\ell\rangle e^{-i\omega t} \rightarrow |LV_{-\ell}\rangle e^{-i\omega t} .
\]

It is important to note that the corresponding conversion length is polarization-dependent,
as is seen from Eq. (18). Since $Q_g/Q_p \sim 2k\Delta/n_{co}pK$ and $K \sim \sqrt{2\Delta/r_0}$ [30], one gets:

$$Q_g/Q_p \sim 100\sqrt{\Delta}(r_0/\lambda) \gg 1,$$

(21)

for all reasonable parameters of weakly-guiding fibers, so that the relative difference in conversion lengths is small (see Fig. 3) For example, at the waveguide parameter $V = 4.16$, the normalized index difference $\Delta = 0.001$, the core radius $r_0 = 6.3 \text{ \mu m}$ and the acoustic power $P = 50 \text{ mW}$, one has: $L_0^x = 4.26 \text{ cm}$, $L_0^y = 3.56 \text{ cm}$, hence $L_0^x - L_0^y = 7 \text{ mm}$.

Nevertheless, such a subtle difference between conversion lengths may be relevant at the corresponding fiber lengths. Indeed, as can be easily shown, at the fiber’s length $z = L_k$ given by:

$$L_k = \frac{4\pi}{Q_g} \left[ (k + 1/2) \frac{Q_g}{2Q_p} \right],$$

(22)

where $[x]$ is the ceil function and $k = 0, 1, \ldots$, one gets (see Fig. 3 and Fig. 4):

$$W^x_\ell(0, L_k) \approx W^y_\ell(0, L_k) \approx 1, W^x_\ell(0, L_k) \approx W^y_\ell(0, L_k) \approx W^y_\ell(0, L_k) \approx 0,$$

(23)

which entails:

$$|LV^x_\ell e^{-i\omega t} \rightarrow |LV^x_\ell e^{-i\omega t},
|LV^y_\ell e^{-i\omega t} \rightarrow |LP^y_\ell e^{-i\omega t}.$$

(24)

This transformation describes a novel type of the optical polarization-dependent mode

![Figure 4](image_url)

**Figure 4.** Transmission spectra for the generated OVs $W^\sigma_\ell$ for a fiber of the optimal length $L_0$. The fiber has a fixed length that is the optimal one for a certain acoustic power $P_0$, optical wavelength $\lambda_0$, at which $\varepsilon(\lambda_0) = 0$, and for the given polarization according to Eq. 18. The fiber’s parameters: $V = 4.16$, $\Delta = 0.001$, $r_0 = 6.3 \text{ \mu m}$, the acoustic power $P_0 = 50 \text{ mW}$, $L_0^x = 4.26 \text{ cm}$ and $L_0^y = 3.56 \text{ cm}$.

conversion in fiber acousto-optics based on the particular structure of the modes (3) and non-degenerate propagation constants (8). The key feature of such a conversion is that the topological
charge $\ell$ or $-\ell$ of the generated field $|LV^\sigma_{\ell}\rangle$ is determined by the direction of linear polarization $\sigma$ of the incident vortex beam.

Since the OAM per photon of the paraxial OVs is defined through $L_z = \hbar \ell$, Eq. (24) unveils the effect of polarization-dependent OAM flipping for an input OV: $\hbar \ell \rightarrow -\hbar \sigma \ell$. It reveals the possibility of all-fiber dynamic switching of the OAM of the generated vortex beam through changing the direction of the input polarization. This type of all-fiber wavelength-tunable stable optical vortex generation and controlling should be especially useful in such applications as micromechanics, classical and quantum information encoding, and classical simulation of quantum algorithms. As a striking example, the polarization-dependent OV transformation

$|\sigma\rangle \rightarrow -|\sigma\rangle$

Figure 5. The scheme of the logic CNOT gate: the state of linear polarization $|x, y\rangle = |\pm 1\rangle$ plays the role of the control qubit, while the topological charge of an OV $|\ell\rangle = |\pm |\ell|\rangle$ carries the target qubit.

in Eq. (24) paves the way to implementation of the all-fibre stable wavelength tunable logic element CNOT (controlled-NOT) gate, in which the direction of linear polarization carries the control qubit (the states $|\sigma\rangle = |x, y\rangle$ may correspond to the first qubit logic states $\{|0\rangle, |1\rangle\}$) and the topological charge carries the target (the states $|\ell\rangle = |\pm |\ell|\rangle$ match the second qubit basis states $\{|0\rangle, |1\rangle\}$) (see Fig. 5). Such a gate is able to produce optical beams with local entanglement between their polarization and OAM in regime of linear optics. Indeed, let the following superposition of OVs with the completely separable polarization and orbital degrees of freedom

$|\varphi_{\text{in}}\rangle = |LV^x_\ell\rangle + |LV^y_\ell\rangle = (|x\rangle + |y\rangle)|\ell\rangle$. \hspace{1cm} (25)$

be incident on the fiber with the AOI performing the CNOT operation. Then, using Eq. (24), the transmitted beam,

$|\varphi_{\text{out}}\rangle = |LV^x_{-\ell}\rangle + |LV^y_{-\ell}\rangle \neq |p\rangle|\ell'\rangle$, \hspace{1cm} (26)$

where $|p\rangle$ and $|\ell'\rangle$ are some polarization and pure OAM states, respectively, happens to be in the maximally [31] entangled state.

4. Conclusion

To conclude, a novel polarization-dependent optical mode conversion in circular optical fibers with the acousto-optic interaction is unveiled. It is shown that the orbital angular momentum of the outgoing optical vortex ($\pm \hbar$ per photon) is determined by the direction of the incident linear polarization. This effect can be used for implementing polarization-controlled orbital angular momentum flipping. As a relevant application of this effect, the all-fibre stable controlled-NOT gate, in which the linear polarization carries the control qubit and the topological charge carries the target, is proposed.
5. References

[1] Yao A and Padgett M 2011 Adv. Opt. Photon. 3 161-204
[2] Soskin M and Vasnetsov M 2001 Progress in Optics 42(4) 219-276
[3] Willner A E, Huang H, Yan Y, Ren Y, Ahmed N, Xie G, Bao C, Li L, Cao Y, Zhao Z, Wang J, Lavery M P J, Tur M, Ramachandran S, Molisch A F, Ashrafni N and Ashrafi S 2015 Adv. Opt. Photon. 7 66-106
[4] Molina-Terriza G, Torres J P and Torner L 2002 Phys. Rev. Lett. 88 013601
[5] Gibson G, Courtial J, Padgett M, Vasnetsov M, Pas'ko V, Barnett S and Franke-Arnold S 2004 Opt. Express 12 5448-5456
[6] Bouchal Z and Celechovsky R 2004 New J. Phys. 6 131
[7] Wang J, Yang J Y, Fazal I M, Ahmed N, Yan Y, Huang H, Ren Y, Yue Y, Dolinar S, Tur M and Willner A E 2012 Nature Photonics 6 488-496
[8] Tamburini F, Mari E, Sponselli A, Thid B, Bianchini A and Romanato F 2012 New Journal of Physics 14 033001
[9] Yan Y, Xie G, Lavery M P J, Huang H, Ahmed N, Bao C, Ren Y, Cao Y, Li L, Zhao Z, Molisch A F, Tur M, Padgett M J and Willner A E 2014 Nature Communications 5 4876
[10] Bouchal Z, Haderka O and Celechovsky R 2005 New Journal of Physics 7 125
[11] Ramachandran S, Kristensen P and Yan M F 2009 Opt. Lett. 34 2525
[12] Bozinovic N, Golowich S, Kristensen P and Ramachandran S 2012 Opt. Lett. 37 2451
[13] Alexeyev C N 2012 J. Opt. 14 085702
[14] Bozinovic N, Yue Y, Ren Y, Tur M, Kristensen P, Huang H, Willner A E and Ramachandran S 2013 Science 340 1545
[15] Ung B, Vaity P, Wang L, Messaddeq Y, Rusch L A and LaRochelle S 2014 Opt. Express 22 18044
[16] Barshak E V, Alexeyev C N, Lapin B P and Yavorsky M A 2015 Phys. Rev. A 91 033833
[17] Cerf N J, Adami C and Kwiat P G 1998 Phys. Rev. A 57 R1477-R1480
[18] Spreeuw R J C 1998 Foundations of Physics 28 361-374
[19] Kwiat P G, Mitchell J R, Schwindt P D D and White A G 2000 Journal of Modern Optics 47 257-266
[20] Spreeuw R J C 2001 Phys. Rev. A 63(6) 062302
[21] Lee K F and Thomas J E 2002 Phys. Rev. Lett. 88 097902
[22] Kim B Y, Blake J N, Engan H E and Shaw H J 1986 Opt. Lett. 11 389-391
[23] Zhao J and Liu X 2006 Opt. Lett. 31 1609-1611
[24] Yavorsky M A 2013 Opt. Lett. 38 3151-3153
[25] Zhang W, Huang L, Wei K, Li P, Jiang B, Mao D, Gao F, Mei T, Zhang G and Zhao J 2016 Opt. Lett. 41 5082-5085
[26] Wei K, Zhang W, Huang L, Mao D, Gao F, Mei T and Zhao J 2017 Opt. Express 25 2733-2741
[27] Yavorsky M A, Vikulin D V, Barshak E V, Lapin B P and Alexeyev C N 2019 Opt. Lett. 44 598-601
[28] Birks T A, Russell P S J and Culverhouse D O 1996 J. Lightwave Technol. 14 2519-2529
[29] Alexeyev C N, Barshak E V, Volyar A V and Yavorsky M A 2010 J. Opt. 12 115708
[30] Snyder A W and Love J D 1985 Optical waveguide theory (Chapman and Hall, London)
[31] Bashkirov E K and Mastyugin M S 2013 Entanglement of two superconducting qubits interacting with two-mode thermal field Computer Optics 37(3) 278-285

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