Robustly Safe Compilation

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October 24, 2018

Abstract

Secure compilers generate compiled code that withstands many target-level attacks such as alteration of control flow, data leaks or memory corruption. Many existing secure compilers are proven to be fully abstract, meaning that they reflect and preserve observational equivalence. Fully abstract compilation is a strong and useful property that, in certain cases, comes at the cost of requiring expensive runtime constructs in compiled code. These constructs may have no relevance for security, but are needed to accommodate differences between the source language and the target language that fully abstract compilation necessarily regards.

As an alternative to fully abstract compilation, this paper explores a different criterion for secure compilation called robustly safe compilation or RSC. Briefly, this criterion means that the compiled code preserves relevant safety properties of the source program against all adversarial contexts interacting with said program. We show that RSC can be attained easily and results in code that is more efficient than that generated by fully abstract compilers. We also develop three illustrative robustly-safe compilers and, through them, develop two different proof techniques for establishing that a compiler attains RSC. Through these, we also establish that proving RSC is simpler than proving fully abstraction.

To better explain and clarify notions, this paper uses colours. For a better experience, please print or view this paper in colours.

1 Introduction

Control flow manipulation, improper stack modification and unauthorised reading/writing of private memory are just some examples of low-level attacks that programs are subject to. Specific countermeasures such as Control Flow Integrity [3] or Code Pointer Integrity [40] have been devised to address these attacks individually. A secure compiler, in contrast, seeks to defend against entire classes of such attacks. Secure compilers often achieve security by relying on different protection mechanisms, e.g., cryptographic primitives [4, 5, 21, 25], types [9, 10], address space layout randomisation [6, 35], protected module architectures [8, 52, 54] (also known as isolated enclaves [45]), tagged architectures [38],
etc. Once designed, the question researchers face is how to formalise that such a compiler is indeed secure, and how to prove that: we want a criterion that specifies secure compilation. The most widely-used criterion for compiler security is fully abstract compilation (FAC) \cite{1, 33, 51}, which has been shown to preserve many interesting security properties like confidentiality, integrity, invariant definitions, well-bracketed control flow and hiding of local state \cite{8, 35, 52}.

Informally, a compiler is fully abstract if it preserves and reflects observational equivalence from source-level components (i.e., partial programs) to their compiled counterparts. Most existing work instantiates observational equivalence with contextual equivalence: co-divergence of two components in any program context (i.e., in any larger program they interact with). Fully abstract compilation is therefore a very strong property, which preserves source-level reasoning for all source-level abstractions.

Unfortunately, preserving all source-level abstractions also has downsides. In fact, while FAC preserves many relevant security properties, it also preserves a plethora of other non-security ones, and preserving the latter may force inefficient checks in the compiled code. For example, when compiling object-oriented code to assembly, the generated code must have fixed size \cite{8, 52}, lest fully abstract compilation be violated. This requirement seems largely impractical and because of this (and other similar inefficiencies), recent work started questioning FAC as a criterion for secure compilation \cite{31}. Additionally, FAC is not well-suited for source languages with undefined behaviour (e.g., C and LLVM) \cite{38} and if used naively, it can fail to preserve even simple safety properties \cite{55} (though, fortunately, no existing work falls prey to this naivety).

Motivated by this, recent work started investigating alternative secure compilation criteria that overcome these limitations. These security-focused criteria have the form of preservation of hyperproperties or classes of hyperproperties, such as hypersafety properties or safety properties \cite{31}. This paper investigates one of these criteria, namely, Robustly Safe Compilation (RSC) which has clear security guarantees and can be attained efficiently.

Informally, a compiler attains RSC if it is correct and it preserves robust safety of source components in the target components it produces. Robust safety is an important security notion that has been widely adopted to study the security of communication protocols \cite{12, 16, 32} but never in compilation. Before explaining RSC, we explain robust safety as a language property.

Robust Safety as a Language Property Intuitively, a program property is safety if it encodes that “bad” events do not happen when the program executes \cite{11, 57}. A program is robustly safe if it has relevant safety properties despite active attacks from adversaries. As the name suggests, robust safety relies on the notions of safety and robustness.

Safety. As mentioned, safety asserts that “no bad event happens”, so we can characterise a safety property by the set of finite observations where these bad events happen. A whole program has a safety property if its behaviours exclude these bad observations. Many relevant security properties can be encoded as
safety: integrity, weak secrecy and functional correctness.

**Example 1 (Integrity).** Integrity ensures that an attacker does not tamper with the invariants we have set up in our code. For example, consider the code below, which manages electronic payments and, as common nowadays, does not require a PIN for small transactions. We expect this code to have integrity, especially in variable `amount` where the amount to be charged is stored, so that an event `charge_account(100)` only happens if there is a call to `request_pin` beforehand. Bad observations for this code are those where a call to `charge_account(n)` with `n > 10` is not preceded by a call to `request_pin`. Note that this code seems to have this safety property, but it may not have the safety property robustly: If an adversary can transfer control to the middle of the function (to `charge_account(amount)` directly), then this property can be violated.

```plaintext
function pay(amount : Int){
  if amount > 10; {
    request_pin(); }
  charge_account(amount); 
  return;
}
```

**Example 2 (Weak Secrecy).** Weak secrecy asserts that if our program creates a secret that the attacker does not know, the secret never flows explicitly to the attacker. For example, consider the code that manages `network_h`, the handler for a network interface. This code generally does not expose the handler directly to external code but it provides an API to control how the network handler is used. If the handler is directly accessible to outer code, then it can be misused in unintended ways. If the code has weak secrecy wrt `network_h` then we know that the handler is never passed to an attacker. In this case we can define the sets of bad observations to be those where `network_h` is passed to external code (e.g., as a parameter on or the heap).

**Example 3 (Correctness).** Program correctness can also be stated as a safety property. Consider a program that computes the `fibonacci` of some given input. Safety dictates that, for example, returning `13` is only allowed if `7` was passed as input, so the output of `fibonacci` must be correct with respect to its input. Therefore we can define the set of bad observations to be the input/output pairs that are not functionally correct, e.g., `input 4` and `output 5`, `input 3` and `output 6` etc.

This example also indicates that safety really matters at end-to-end points, i.e., from when the code with the safety property starts executing until it returns control to external code. So we choose to model safety as an end-to-end property and not as an internal one. Specifically, we are not interested in how a program internally calculates `fibonacci`, so long as the end result is correct.

Safety properties are not a panacea for security, and there are security properties that are not safety. For example, noninterference, the standard information flow property, is not safety. However, many interesting security properties are safety. In fact, many non-safety properties including noninterference can be
conservatively approximated as safety properties \[19\]. Hence, safety properties are a meaningful goal to pursue for secure compilation.

**Robustness.** In programming languages we often want to reason about a component – which we write – no matter what other programs it interacts with. These other programs are the libraries we link against, or the runtime, and this is modelled as the program context whose hole our component fills. From a security perspective the context represents the attacker in the threat model. When our component links against this context, we have a whole program that can run. A property holds robustly for a component if it holds for any context that our component is linked against.

Let us now discuss robust safety as a compiler property.

**Robust Safety (Preservation) as a Compiler Property** A compiler attains RSC if it maps any source component that has a safety property robustly to a compiled component that has the same safety property robustly. Thus, safety has to hold robustly in the target language, which often does not have the powerful abstractions (e.g., typing) that the source language has. The compiler must insert enough defensive runtime checks into the compiled code to prevent the more powerful target contexts from launching attacks (violations of safety properties) that source contexts could not launch. This is unlike correct compilation, which either does not have a target level context to link a component against (the compiler takes whole programs \[42\]) or only links against contexts that behave like source-level ones \[39, 48, 59\].

As motivated by Example 3, we consider safety to be an end-to-end property. This plays an important role when safety is preserved across compilation, as a compiler can optimise code internally as long as the end result is not affected by the optimisation. When compiling the fibonacci function of Example 3, the compiler can do all sorts of internal optimisation such as storing intermediate results, as long as the end result is correct. However, it is important that these intermediate results cannot be tampered with by the attacker, lest the robust safety be violated.

A RSC-attaining compiler focuses only on preserving security (as captured by robust safety) instead of contextual equivalence (typically captured by full abstraction). So, such a compiler can produce code that is more efficient than code compiled with a fully abstract compiler as it does not have to preserve all source abstractions (we demonstrate this later).

Finally, robust safety scales naturally to thread-based concurrency. Thus RSC also scales naturally to thread-based concurrency (we demonstrate this too). This is unlike FAC, where thread-based concurrency can introduce additional undesired abstractions that also need to be preserved.

RSC is a very recently proposed criterion for secure compilers. Garg *et al.* [31] define RSC abstractly in terms of preservation of program behaviours, but their development is limited to the definition only. Our goal in this paper is to examine how RSC can be realized and established, and to show that in certain cases it leads to compiled code that is more efficient than what FAC
leads to. Specifically, the first contribution of this paper is the presentation of three \(RSC\)-attaining compilers in different settings.

- The first compiler compiles an untyped source language to an untyped target language with support for fine-grained memory protection via so-called capabilities [22, 64]. Here, we guarantee that if a source program is robustly safe, then so is its compilation.

- The second compiler starts from a typed source language where the types already enforce robust safety, and compiles to a target language similar to that of the first compiler. In this instance, both languages also support shared-memory concurrency. Here, we guarantee that all compiled target programs are robustly safe.

- The third compiler starts from the same concurrent typed source language of the second one but it targets an untyped concurrent language with support for memory isolation, which provides a high-level model of what SGX-like isolation provides [45]. Here we also guarantee that all compiled target programs are robustly safe, and in doing so we highlight how the chosen defence mechanism does not play a key role in our theory.

The second contribution of this paper regards the different proof techniques used to prove \(RSC\).

- For the first compiler we prove \(RSC\) using an existing technique for \(FAC\): trace-based backtranslation [8, 54].

- For the second and third compilers, we show that types simplify the proof of \(RSC\), relative to that of the first compiler, so that no backtranslation is needed.

The third contribution of this paper is a comparison between \(RSC\) and \(FAC\).

- For this, we describe changes that would be needed to attain \(FAC\) and argue that these changes make generated code inefficient and also complicate the backtranslation proof significantly.

The rest of the paper starts with the formal definition of \(RSC\) and of robust safety as well as with a discussion of proof techniques for \(RSC\) (Section 2). Then it presents our three \(RSC\)-attaining compilers and their proofs (Section 3, Section 4 and Section 5). Then the paper introduces \(FAC\), discusses its advantages and limitations and shows how to turn the first compiler into a fully abstract one, detailing possible inefficiencies and proofs complications (Section 6). Finally, the paper discusses related work (Section 7) and concludes (Section 8).

**Conventions** We use a blue, sans-serif font for source elements, an orange, bold font for target elements and a black, italic font for elements common to both languages (to avoid repeating similar definitions twice). Thus, \(C\) is a source-level component, \(\overline{C}\) is a target-level component and \(\mathcal{C}\) is generic notation for either a source-level or a target-level component.
We use the term component and program to denote partial programs, when we mean a whole program we always specify the “whole” qualifier.

Necessarily, some definitions are massaged to eliminate tedious technical details and focus only on the relevant bits. Analogously, formalisations are reduced, proofs are completely omitted and, when necessary, only proof sketches that explain the intuition why a proof holds are presented. The interested reader will find all formal details in the appendix.

2 Robustly Safe Compilation

This section first discusses robust safety as a language (not a compiler) property (Section 2.1) and then it presents $RSC$ as a compiler property along with techniques to prove it (Section 2.2).

2.1 Safety and Robust Safety

In order to explain safety we need to present our programming model. Programmers write components and they want to enforce safety properties robustly upon said components. Components are lists of function definitions that must be linked with other function definitions (the context) in order to have a runnable whole program. (So “other” functions are like `extern` functions in C.)

We want safety to specify that the behaviour of a component is good, and this concretely means that relevant parts of the heap remain in good state at function boundaries. So, if our component defines function `fibonacci`, we are interested at heap state between a call to `fibonacci` and its return, or between a call to `fibonacci` and a callback to external code that it makes. This choice is somewhat arbitrary but it serves to illustrate our key ideas; moreover other kinds of properties such as I/O can be modelled e.g., by considering designated buffers in memory for input/output. Concretely, we will rely on a specification of a component behaviour as a `trace`, i.e., a sequence of actions recording function boundary interactions and, especially, the heap at these points. Actions, the items on a trace, have the following grammar:

\[
\begin{align*}
\text{Actions } \alpha & ::= \text{call } f \; v \; H? \mid \text{call } f \; v \; H! \mid \text{ret } H! \mid \text{ret } H?
\end{align*}
\]

These actions respectively capture call and callback to a function $f$ with parameter $v$ when the heap is $H$ as well as return and returnback with a certain heap $H$. We use $?$ and $!$ decorations to indicate whether the control flow of the action goes from the context to the component ($?$) or from the component to the context ($!$). Well-formed traces have alternations of $!$ and $?$ decorated actions, starting with $?$ since execution starts in the context. As the heap is the safety-relevant part of the action, we use function `relevant(·)` to obtain it and disregard other parts of the action that may be accumulated for other purposes (e.g., proofs). Intuitively, `relevant$(\pi)$` returns the list of heaps $\Pi$ mentioned in trace $\pi$. 

6
Some readers may wonder why we do not follow existing work and specify safety as “programmer-written assertions never fail” [29, 32, 44, 61]. Unfortunately, this approach does not yield a meaningful criterion for specifying a compiler, as a compiler could just erase all assertions and the compiled code it generates would be trivially (robustly) safe – no assertion can fail if there are no assertions in the first place!

Once we have a representation of a program behaviour, we need a specification for safety properties in order to check whether a program satisfies said safety property. Generally, properties can be specified just as sets of traces, however safety properties specifically can be specified in a more informative way as security automata, or monitors [57]. We choose this form of specification since monitors are less abstract than sets of traces and they are closer to the enforcement mechanisms used to enforce safety properties, e.g., runtime monitors. Briefly, a monitor is a state machine that transitions states in response to security-relevant events of the program it monitors. By definition, the program is in violation of the safety property coded by the monitor if the monitor is unable to make a transition in response to an event. Schneider [57] argues that all properties codable this way are safety properties and that all enforceable safety properties can be coded this way.

Formally, a monitor $M$ consists of a set of abstract states $\{\sigma \cdots\}$, the transition relation $\leadsto$, an initial state $\sigma_0$, the set of heap locations that matter for the monitor, $\{l \cdots\}$, and the current state $\sigma_c$ (we indicate a set of elements $e$ as $\{e \cdots\}$). The transition relation $\leadsto$ is a set of triples of the form $(\sigma_s, H, \sigma_f)$ consisting of a starting state $\sigma_s$, a final state $\sigma_f$ and a heap $H$. The transition $(\sigma_s, H, \sigma_f)$ is interpreted as “The state $\sigma_s$ transitions to $\sigma_f$ when the heap is $H$”. The heap $H$ occurring in a transition must contain in its domain the locations from $\{l \cdots\}$, so given a larger heap (from a program action), we restrict it to the part that the monitor actually cares about as $H\mid_{\{l \cdots\}}$. We assume that the transition relation is deterministic: for any $\sigma_s$ and $H$, there is at most one $\sigma_f$ such that $(\sigma_s, H, \sigma_f) \in \leadsto$.

Given the behaviour of a program as a trace $\tau$ and a monitor $M$ specifying a safety property, $M \vdash \tau$ denotes that the trace satisfies the safety property. Intuitively, to satisfy a safety property, the sequence of heaps in a trace’s actions must never get the monitor stuck (Rule Valid trace). Every single heap must allow the monitor to step according to its reductions (Rule Monitor Step).

\[
\begin{align*}
&M: \text{relevant}(\tau) \leadsto M' \\
&M \vdash \tau \\
&M = (\{\sigma \cdots\}, \leadsto, \sigma_0, \{l \cdots\}, \sigma_c) \\
&M' = (\{\sigma \cdots\}, \leadsto, \sigma_0, \{l \cdots\}, \sigma_f) \\
&(\sigma_c, H|_{\{l \cdots\}}, \sigma_f) \in \leadsto \\
&M; H \leadsto M'
\end{align*}
\]

With this setup in place, we can formalise safety (Definition 8), attackers (Definition 2) and robust safety (Definition 10).
Definition 1 (Safety).

\[ M \vdash C : \text{safe} \overset{\text{def}}{=} \text{if } \vdash C : \text{whole} \text{ then if } \Omega_0(C) \xrightarrow{\overline{\alpha}} \text{ then } M \vdash \overline{\alpha} \]

A whole program is safe for a monitor if the monitor accepts any trace the program generates from its initial state (indicated as \( \Omega_0(C) \)).

Definition 2 (Attacker).

\[ C \vdash A : \text{atk} \overset{\text{def}}{=} C = \{ l \cdots \}, \overline{F} \text{ and } \{ l \cdots \} \cap \text{fn}(A) = \emptyset \]

An attacker is valid if its free names (captured by \( \text{fn}(\cdot) \)) do not refer to the locations that the component cares about. (This is a basic sanity check: If we allow an attacker to mention heap locations that the component cares about, the attacker will be able to modify those locations, thus causing all but trivial safety properties to not hold robustly.)

Definition 3 (Robust Safety).

\[ M \vdash C : \text{rs} \overset{\text{def}}{=} \forall A. \text{ if } M \Rightarrow C \text{ and } C \vdash A : \text{atk} \text{ then } M \vdash A[C] : \text{safe} \]

A component \( C \) is robustly safe wrt monitor \( M \) if \( C \) composed with any attacker is safe wrt \( M \).

The final condition for this setup to make sense is that the monitor and the component must agree on the locations that are security sensitive, denoted as \( M \Rightarrow C \). If this is not checked, the component could be enforcing the safety property on the wrong set of locations. So, intuitively the components we consider must not only specify code but also a the security-sensitive locations.

2.2 Robustly Safe Compilation

Robustly-safe compilation is concerned that a safety property and its meaning are preserved across compilation. Thus, a first question arises in what it means to preserve meanings across languages. If a source safety property says never write 3 to a location, and we compile to an assembly language by mapping numbers to binary, we would the corresponding target property to say never write 0x11 to an address.

In order to relate properties across languages, we assume a relation \( \approx : v \times v \) between source and target values that is total, so it maps any source value \( v \) to a target value \( v \): \( \forall v. \exists v. v \approx v \). This value relation is used to define a relation between heaps: \( H \approx H \), which intuitively holds when related locations point to related values. This is then used to define a relation between actions: \( \alpha \approx \alpha \), which holds when the two actions are the “same” action (i.e., \( \text{call} \cdots ? \) only relates to \( \text{call} \cdots ? \)) and the arguments of the action (values and heap) are related. Next, we require a relation \( M \approx M \) between source and target monitors, which means that the source monitor \( M \) and the target monitor \( M \) code the same safety property, modulo the relation \( \approx \) on values assumed above. The precise
definition of this relation depends on the source and target languages; specific instances are shown in Sections 3.3.2 and 4.3. Accounting for difference in the representation of safety properties is also something that sets us apart from the work of Garg et al. [31]. They consider the alphabet of trace actions to be shared between source and target languages but in practice properties may need suitable translations across languages [55].

We denote a compiler from language $S$ to language $T$ by $\llbracket \cdot \rrbracket^S_T$. A compiler $\llbracket \cdot \rrbracket^S_T$ attains RSC, if it maps any component $C$ that is robustly safe wrt $M$ to a component $C$ that is robustly safe wrt $M$, provided that $M \approx M$.

**Definition 4 (Robustly Safe Compilation).**

\[
\vdash \llbracket \cdot \rrbracket^S_T : RSC \overset{\text{def}}{=} \forall C, M, M. \text{ if } M \vdash C : rs \text{ and } M \approx M \text{ then } M \vdash [C]^S_T : rs
\]

A consequence of the universal quantification over monitors here is that the compiler cannot be property-sensitive. A robustly-safe compiler preserves all robust safety properties, not just a specific one, e.g., it does not just enforce that fibonacci is correct.

### 2.2.1 RSC Implies Compiler Correctness

As mentioned in Section 1, RSC implies (a form of) compiler correctness. However, this is not apparent from Definition 4, so let us explain why it is the case and what form of compiler correctness is implied by RSC.

Whether concerned with whole programs or partial ones, compiler correctness states that the behaviour of compiled programs refines the behaviour of source ones [17, 34, 39, 43, 48, 59]. So, if $\{\alpha \cdots\}$ and $\{\overline{\alpha} \cdots\}$ are the sets of compiled and source behaviours, then a compiler should force $\{\alpha \cdots\} \sim \{\overline{\alpha} \cdots\}$, where $\sim$ is the composition of $\subset$ and of the action relation $\approx$.

To make the connection with refinement more explicit, Definition 19 presents an alternative, equivalent formulation of RSC. We call this characterisation property-free as it does not mention monitors explicitly (it mentions the relevant($\cdot$) function for reasons we explain below).

**Definition 5 (Property-Free RSC).**

\[
\vdash \llbracket \cdot \rrbracket^S_T : PF-RSC \overset{\text{def}}{=} \forall C, A, \overline{\alpha}, \alpha. \text{ if } [C]^S_T \vdash A : atk \text{ and } \vdash A [\llbracket C \rrbracket^S_T] : \text{whole}
\]

and

\[
\Omega_0 \left( A \left[ \llbracket C \rrbracket^S_T \right] \right) \Rightarrow \_ \text{ then } \exists A, \overline{\alpha}, \alpha. \text{ if } C \vdash A : atk \text{ and } \vdash A [C] : \text{whole}
\]

and

\[
\Omega_0 \left( A [C] \right) \Rightarrow \_ \text{ and relevant} (\overline{\alpha}) \approx \text{relevant}(\alpha)
\]

Specifically, this notion states that the compiled code produces behaviours that refine source level behaviours robustly.

The intuition of why PF-RSC preserves all safety properties is the following. A property is safety if and only if it implies programs not having any trace prefix from a given set of bad prefixes (i.e., finite traces). Hence, not having a safety
property robustly amounts to some context being able to induce a bad prefix. Consequently, preserving all robust safety properties amounts to ensuring that all target prefixes can be generated in the source too.

As they capture the same intuition, we can prove that PF-RSC and RSC coincide (Theorem 7).

**Theorem 1** (PF-RSC and RSC are equivalent).

\[
\forall \llbracket \cdot \rrbracket_T \vdash \llbracket \cdot \rrbracket_T : PF-RSC \iff \vdash \llbracket \cdot \rrbracket_T : RSC
\]

First and foremost, there is an assumption for this implication to be meaningful for correctness: that the relevant(·) function must not erase correctness-relevant actions. For example, if there were additional labels on traces that register read/writes (as often done in compiler correctness work) and the relevant(·) functions blindly erased them, then there would be a problem. For the instances of this paper we consider correctness to be end-to-end relatedness of heaps, and so our relevant(·) function does not prevent correctness.

The reason why PF-RSC covers compiler correctness is hidden in the universal quantification of target contexts (∀A). If we consider a component C that is already a whole program, it can only link against empty contexts, both in the source and in the target. This effectively removes contexts from the definition altogether, and PF-RSC simplifies to standard refinement of traces. So, PF-RSC or, equivalently, RSC implies whole program compiler correctness.

However, PF-RSC (or, equivalently, RSC) does not imply, nor is implied by, any form of compositional compiler correctness (CCC) [39, 48, 59]. CCC requires that the behaviours produced by a compiled component linked against a target context that is related (in behaviour) to a source context can also be produced by the source component linked against the related source context. In contrast, PF-RSC allows picking any source context to simulate the behaviours. Hence, PF-RSC does not imply CCC. On the other hand, PF-RSC universally quantifies over all target contexts, while CCC only quantifies over target contexts related to a source context, so CCC does not imply PF-RSC either. Hence, compositional compiler correctness, if desirable, must be imposed in addition to PF-RSC. Note that this lack of implications is unsurprising: PF-RSC and CCC capture two completely different aspects of compilation—security (against all contexts) and compositional preservation of behaviour (against well-behaved contexts).

### 2.2.2 Proving RSC

Proving that a compiler attains RSC can be done by either adapting existing techniques for establishing fully abstract compilation, or with simpler ones. Specifically, RSC can be established with a backtranslation technique. This proof technique has been often used to prove fully abstract compilation [8, 31, 38, 49, 52, 54] and it aims at building a source context starting from a target one. In fact, our equivalent criterion, PF-RSC, embeds a backtranslation! It can be rewritten (eliding irrelevant details) as:
Essentially, given a target context $A$, a compiled program $\llbracket C \rrbracket_T$ and a target trace $\pi$ that $A$ causes $\llbracket C \rrbracket_T$ to have, we need to construct, or backtranslate to, a source context $A$ that will cause the source program $C$ to simulate $\pi$. Such backtranslation based proofs can be quite difficult. The difficulties depend on the features of the languages and the compiler. However, backtranslation for RSC is not as complex as backtranslation for FAC. We show a backtranslation proof for RSC in Section 3.3.2 and illustrate differences from backtranslation for FAC in Section 6.3.3.

A simpler proof strategy is also viable for RSC when we compile only those source programs that have been verified to be robustly safe (e.g., using a type system). The idea is this: using the verification of the source program, we can establish an invariant which is always maintained by the target code, and which, in turn, implies the robust safety of the target code. For example, if the safety property is that values in the heap always have their expected types, then the invariant can simply be that values in the target heap are always related to the source ones (which have their expected types). This is tantamount to proving type preservation in the target in the presence of an active adversary. This is harder than standard type preservation (because of the active adversary) but is still much easier than backtranslation as there is no need to map target constructs to source contexts syntactically. We illustrate this proof technique in Section 4.

3 RSC: First instance

This section presents our first instance of an RSC-attaining compiler. First, we present our source language $L^U$, a simply-typed, first-order imperative language with hidden local state (Appendix E). Then, we present our target language $L^P$, an untyped imperative target language with a concrete heap, whose addresses are natural numbers. $L^P$ provides hidden local state via a fine-grained capability mechanism on heap accesses (Appendix B). Finally, we present the compiler $\llbracket \cdot \rrbracket_{L^U}$ and prove that it attains RSC (Section 3.3).

To avoid detracting with mundane details, we deliberately use source and target languages that are fairly similar. However, they differ substantially in one key point: the heap model. This affords the target-level adversary attacks like guessing private locations and writing to them that do not obviously exist in the source (and makes our proofs nontrivial). We believe that (with due effort) the ideas here will generalize to source and target languages with larger gaps and more features.

11
Components \( C ::= \ell_\text{root}; F; I \)

Contexts \( A ::= H; F \)

Interfaces \( I ::= f \)

Functions \( F ::= f(x) \rightarrow \text{return} ; \)

Values \( v ::= b \in \{\text{true}, \text{false}\} | n \in \mathbb{N} | (v, v) | \ell \)

Expressions \( e ::= x | v | e \oplus e | e \otimes e | (e, e) | e.1 | e.2 | \ell \)

Statements \( s ::= \text{skip} | s; s | \text{let } x = e \text{ in } s | \text{if } e \text{ then } s \text{ else } s | \text{call } f e | \text{let } x = \text{new } e \text{ in } s | x := e \)

Heaps \( H ::= \emptyset | H; \ell \rightarrow v \)

Monitors \( M ::= (\{\sigma \cdots \}, \leadsto, \sigma_0, \ell_\text{root}, \sigma_c) \)

Labels \( \lambda ::= \epsilon | \alpha \)

Actions \( \alpha ::= \text{call } f v H? | \text{call } f v H! | \text{ret } H! | \text{ret } H? \)

Figure 1: Syntax of \( L^U \) (excerpts). We indicate a list of elements \( e_1, \ldots, e_n \) as \( \overline{e} \).

### 3.1 The Source Language \( L^U \)

\( L^U \) is an untyped imperative while language \([50]\). Its syntax is presented in Figure 1. Components \( C \) are collections of function definitions \( F \), interfaces \( I \) and a special location \( \ell_\text{root} \). An interface is a list of functions that the component relies on the context to provide (similar to \( C \)'s \texttt{extern} functions). The special location \( \ell_\text{root} \) defines the locations that are monitored for safety, as explained below. Attackers \( A \) (program contexts) represent untrusted code that a component interacts with. Function bodies include standard boolean and arithmetic operations, generically written \( \oplus \), heap manipulation statements, (recursive) function calls, etc. Heaps \( H \) are maps from abstract locations \( \ell \) to values \( v \).

\( L^U \) uses monitors that control a dynamically growing set of locations. All in all, monitors \( M \) work as explained in Section 2.1, except that the description of a monitor includes a single location of interest \( \ell_\text{root} \). The use of a single root location is arbitrary and motivated by simplicity, this can be easily extended to a set of locations. However, the heap that is considered in order to make the monitor step is that of locations \textit{reachable} from \( \ell_\text{root} \), this is captured in Rule \( L^U\text{-Monitor Step} \) below. In practice, \( \ell_\text{root} \) may, for instance, be the root of a data structure (e.g., the program’s I/O buffers) that the component being monitored wants to have a safety property on.

\[
\begin{align*}
M &= (\{\sigma \cdots \}, \leadsto, \sigma_0, \ell_\text{root}, \sigma_c) \\
M' &= (\{\sigma \cdots \}, \leadsto, \sigma_0, \ell_\text{root}, \sigma_f) \\
\text{(dom}(H') &= \text{reach}(\ell_\text{root}, H)) \\
M; H \leadsto M'
\end{align*}
\]

We can now precisely define that a monitor and a component agree if they mention the same \( \ell_\text{root} \).

\[
M \models C \overset{\text{def}}{=} (M = (\{\sigma \cdots \}, \leadsto, \sigma_0, \ell_\text{root}, \sigma_c)) \text{ and } (C = (\ell_\text{root}; F; I))
\]

\( L^U \) has a big-step semantics for expressions (\( \models \rightarrow \rightarrow \)) that relies on evaluation contexts, a small-step semantics for statements (\( \models \overset{\lambda}{\longrightarrow} \)) that is labelled and a semantics that accumulates labels in traces (\( \models \overset{\lambda}{\overrightarrow{\longrightarrow}} \)). A program state \( C, H \triangleright (s) \overline{\Omega} \) (denoted with \( \Omega \)) includes the function bodies \( C \), the heap \( H \), a statement \( s \) being
### Rule Set for *L_\text{U}* (Excerpts)

**Definition of Substitution**: Substituting value $v$ for variable $x$ is denoted as $[v / x]$. Dashed lines separate different kinds of rules.

| Rule Type     | Description                                                                 |
|---------------|-----------------------------------------------------------------------------|
| (\text{EL}_\text{U}\text{-val}) | $H \triangleright v \mapsto v$                                               |
| (\text{EL}_\text{U}\text{-op})   | $n \oplus n' = n''$                                                         |
| (\text{EL}_\text{U}\text{-dereference}) | $H \triangleright e \mapsto \ell$ $\ell \mapsto v \in H$                       |
| (\text{EL}_\text{U}\text{-letin})    | $C, H \triangleright ! e \mapsto v \in \text{dom}(H)$                         |
| (\text{EL}_\text{U}\text{-alloc})    | $C, H \triangleright \ell := e \rightarrow C, H' \triangleright \text{skip}$ |
| (\text{EL}_\text{U}\text{-single})  | $\Omega \Rightarrow \Omega'' \quad \Omega'' \xrightarrow{\alpha} \Omega'$          |
| (\text{EL}_\text{U}\text{-silence})  | $\Omega \xrightarrow{\text{silent}} \Omega'$                                 |

Figure 2: Semantics of *L_\text{U}* (excerpts). Substituting value $v$ for variable $x$ is denoted as $[v / x]$. Dashed lines separate different kinds of rules.

executed and a stack of function calls $\overline{T}$ (often omitted in the rules for simplicity). The latter is used to populate judgements of the form $I \vdash f, f' : \text{internal/in/out}$. These determine whether calls and returns are internal (to the attacker or to the
component), directed from the attacker to the component (in) or directed from the component to the attacker (out). This information is used to determine the label of the semantics (see Figure 2).

### 3.2 The Target Language $L^P$

Components $C ::= k_{\text{root}}; F; I$

Values $v ::= n \in \mathbb{N} | \langle v, v \rangle | k$

Expressions $e ::= x | v | e \oplus e | e \otimes e | \langle e, e \rangle | \text{e.1} | \text{e.2} | \text{!e} \text{ with } e$

Statements $s ::= \text{skip} | s; s | \text{let } x = \text{new e in s} | x ::= e \text{ with } e | \text{let } x = e \text{ in s}$

$H ::= \emptyset | H; n \mapsto \vdash v : \eta | H; k$

Monitors $M ::= (\{ \sigma_0 \cdots \}, \sim_0, k_{\text{root}}, \sigma_c)$

Tags $\eta ::= \bot | k$

Actions $\alpha ::= \text{call } f v H | \text{call } f v H! | \text{ret } H! | \text{ret } H$

Figure 3: Syntax of $L^P$ (excerpts).

Figures 4: Semantics of $L^P$ (excerpts).

$L^P$ is an untyped, imperative language that follows the structure of $L^U$ (Figure 3). It has a big-step semantics for expressions ($\mapsto \mapsto$), a labelled small-step semantics ($\xrightarrow{\lambda}$) and a semantics that accumulates traces ($\xrightarrow{\pi}$, Figure 4). However, there are critical differences (that make the compiler interesting!). The main difference is that heap locations in $L^P$ are concrete natural numbers. An adversarial context can guess locations used as private state by a component and clobber them. To support hidden local state, a location can be “hidden” explicitly via the expression $\text{hide } n$. This expression allocates a new capability $k$, an abstract token that grants access to the location $n$ [58]. Subsequently,
all reads and writes to \( n \) must be authenticated with the capability. Unlike locations, capabilities cannot be guessed. To hide a location, a component hides the capability of the location. To bootstrap this hiding process, we assume that a component has one location that can only be accessed by it, a priori in the semantics (in our formalization, we always focus on only one component and we assume that, for this component, this special location is at address 0).

In detail, \( L^P \) heaps \( H \) are maps from natural numbers \( n \) to values \( v \) and a tag \( \eta \). The tag \( \eta \) can be \( \bot \), which means that \( n \) is globally available (not protected) or a capability \( k \), which protects \( n \). A globally available location can be freely read and written (Rule \( EL^P\)-assign-top, Rule \( EL^P\)-deref-top) but one that is protected by a capability requires the capability to be supplied at the time of read/write for the operation to succeed (Rule \( EL^P\)-assign-k, Rule \( EL^P\)-deref-k).

A second difference between \( L^P \) and \( L^U \) is that \( L^P \) has no booleans, while \( L^U \) has them. This makes the compiler and the related proofs interesting, as discussed in the proof of Theorem 9.

In \( L^P \), the locations of interest to a monitor are all those that can be reached from the address 0. 0 itself is protected with a capability \( k_{\text{root}} \) that is assumed to occur only in the code of the component in focus. We can now give a precise definition of component-monitor agreement for \( L^P \) as well as a precise definition of attacker, which must care about the \( k_{\text{root}} \) capability.

\[
\begin{align*}
M \triangleright C & \triangleq (M = (\sigma \cdots , \sim, \sigma_0, k_{\text{root}}, \sigma_c)) \text{ and } (C = (k_{\text{root}}; F; T)) \\
M \vdash A : \text{atk} & \triangleq (M = (\sigma \cdots , \sim, \sigma_0, k_{\text{root}}, \sigma_c), A = F \text{ and } k_{\text{root}} \notin \text{fn}(F))
\end{align*}
\]

### 3.3 Compiler from \( L^U \) to \( L^P \)

We now present \( \llbracket \cdot \rrbracket_{L^U}^{L^P} \), the compiler from \( L^U \) to \( L^P \), detailing how it uses the capabilities of \( L^P \) to achieve RSC (Section 3.3.1). Then, it proves that \( \llbracket \cdot \rrbracket_{L^U}^{L^P} \) attains RSC (Section 3.3.2).

#### 3.3.1 The compiler \( \llbracket \cdot \rrbracket_{L^U}^{L^P} \)

The compiler \( \llbracket \cdot \rrbracket_{L^U}^{L^P} \) takes as input a \( L^U \) component \( C \) and returns a \( L^P \) component. The compiler performs a simple pass on the structure of functions, expressions and statements (Figure 5). Each \( L^U \) location is encoded as a pair of a \( L^P \) location and the capability to access the location; location update and dereference are compiled accordingly. The compiler also codes source booleans: \( \text{true} \) to 0 and \( \text{false} \) to 1.

This compiler solely relies on the capability abstraction of the target language as a defence mechanism to attain RSC. Unlike existing secure compilers, \( \llbracket \cdot \rrbracket_{L^U}^{L^P} \) needs neither dynamic checks nor other constructs that introduce runtime overhead to attain RSC [8, 30, 38, 52, 54].
\[
\begin{aligned}
\left[ \ell_{\text{root}}, F; I, M \right]_{L^u}^{L^p} &= k_{\text{root}} : \left[ F \right]_{L^p}^{L^u} : \left[ I \right]_{L^p}^{L^u} : \left[ f(x) \mapsto s \; ; \; \text{return} ; \right]_{L^p}^{L^u} = f(x) \mapsto [s]_{L^p}^{L^u} ; \text{return} ;
\end{aligned}
\]

\[
\begin{aligned}
\left[ \text{true} \right]_{L^p}^{L^u} &= 0 \\
\left[ \text{false} \right]_{L^p}^{L^u} &= 1 \\
\left[ n \right]_{L^p}^{L^u} &= n \\
\left[ \text{let } x = e \text{ in } s \right]_{L^p}^{L^u} &= \text{let } x = [e]_{L^p}^{L^u} \text{ in } [s]_{L^p}^{L^u} \\
\left[ \text{let } x = \text{new } e \text{ in } s \right]_{L^p}^{L^u} &= \text{let } x_{\text{loc}} = \text{new } [e]_{L^p}^{L^u} \text{ in } \text{let } x_{\text{cap}} = \text{hide } x_{\text{loc}} \text{ in } \text{let } x = \langle x_{\text{loc}}, x_{\text{cap}} \rangle \text{ in } [s]_{L^p}^{L^u} \\
\left[ \text{if } e \text{ then } s_1 \text{ else } s_2 \right]_{L^p}^{L^u} &= \text{if } e \left[ e \right]_{L^p}^{L^u} \text{ then } [s_1]_{L^p}^{L^u} \text{ else } [s_2]_{L^p}^{L^u} \\
\left[ x := e' \right]_{L^p}^{L^u} &= \text{let } x_{\text{loc}} = x.1 \text{ in } \text{let } x_{\text{cap}} = x.2 \text{ in } x_{\text{loc}} := e'_{L^p}^{L^u} \text{ with } x_{\text{cap}}
\end{aligned}
\]

Figure 5: The \([\ ]_{L^p}^{L^u}\) compiler from \(L^u\) to \(L^p\) (excerpts).

### 3.3.2 Proof of RSC

The compiler \([\ ]_{L^p}^{L^u}\) attains RSC (Theorem 9). In order to set up this theorem, we need to instantiate the cross-language relation for values, which we write as \(\approx_{\beta}\) here. The relation is parametrised by a partial bijection \(\beta : \ell \times n \times \eta \leftrightarrow \eta\) from source heap locations to target heap locations such that:

- if \((\ell_1, n, \eta) \in \beta\) and \((\ell_2, n, \eta) \in \beta\) then \(\ell_1 = \ell_2\);
- if \((\ell, n_1, \eta_1) \in \beta\) and \((\ell, n_2, \eta_2) \in \beta\) then \(n_1 = n_2\) and \(\eta_1 = \eta_2\).

The bijection determines when a source location and a target location (and its capability) are related.

The partial bijection \(\beta\) grows as we consider successive steps of program execution in our proof. For example, if executing \text{let } x = \text{new } e \text{ in } s\ creates some source location \(\ell\), then executing its compiled counterpart will create some target location \(n\). At this point we add \((\ell, n, \bot)\) to \(\beta\).

More generally, \(\approx_{\beta}\) on values is defined as follows:

- \(\text{true} \approx_{\beta} 0; \text{false} \approx_{\beta} n\) such that \(n \neq 0; n \approx_{\beta} n\)
- \(\ell \approx_{\beta} (n, v)\) if \(\begin{cases} v = k \text{ and } (\ell, n, k) \in \beta \\ v \neq k \text{ and } (\ell, n, \bot) \in \beta \end{cases}\)
- \((v_1, v_2) \approx_{\beta} (v_1, v_2)\) if \(v_1 \approx_{\beta} v_1\) and \(v_2 \approx_{\beta} v_2\).
This relation is used to define the heap, monitor state and actions relations. Heaps are related, written $\mathcal{H} \approx_{\beta} \mathcal{H}$, when related locations in $\beta$ point to related values. States are related, written $\Omega \approx_{\beta} \Omega$, when they have related heaps. The action relation ($\alpha \approx_{\beta} \alpha$) is as defined in Section 2.2.

With this relation we define a backwards simulation lemma (Lemma 1) that is necessary for the RSC proof (and that can also yield whole program compiler correctness).

**Lemma 1** (Backward simulation).

\[
\text{if } C, H \triangleright [s]_{LP}^U \xrightarrow{\alpha!} \Omega' \text{ and } C, H \vdash s \approx_{\beta} C, H \triangleright [s]_{LP}^U \text{ and } \alpha! \approx_{\beta} \alpha! \n\text{then } C, H \vdash s \xrightarrow{\alpha!} \Omega' \text{ and } \Omega' \approx_{\beta} \Omega'.
\]

**Monitor Relation** The monitors in these languages are more concrete than what was described in Section 2.2. Two monitors are related when they can simulate each other on related heaps. Given a monitor-specific relation $\sigma \approx \sigma$ on monitor states, we say that a relation $R$ on source and target monitors is a bisimulation if the following hold whenever $M = (\{\sigma \cdots\}, \leadsto, \sigma_0, \ell_{\text{root}}, \sigma_c)$ and $M = (\{\sigma \cdots\}, \leadsto, \sigma_0, k_{\text{root}}, \sigma_c)$ are related by $R$:

1. $\sigma_0 \approx \sigma_0$, and $\sigma_c \approx \sigma_c$, and
2. For all $\beta$ containing $(\ell_{\text{root}}, 0, k_{\text{root}})$ and all $H, H'$ with $H \approx_{\beta} H'$ the following hold:
   
   (a) $(\sigma_c, H, \_\_\_ \_ \_ \_ \_ \text{)} \in \leadsto$ iff $(\sigma_c, H, \_\_\_ \_ \_ \_ \_ \text{)} \in \leadsto$, and
   
   (b) $(\sigma_c, H, \sigma') \in \leadsto$ and $(\sigma_c, H, \sigma') \in \leadsto$ imply $(\{\sigma \cdots\}, \leadsto, \sigma_0, \ell_{\text{root}}, \sigma_c') \mathcal{R}(\{\sigma \cdots\}, \leadsto, \sigma_0, k_{\text{root}}, \sigma_c')$.

In words, $R$ is a bisimulation only if $M \mathcal{R} M$ implies that $M$ and $M$ simulate each other on heaps related by any $\beta$. In particular, this means that neither $M$ nor $M$ can be sensitive to the specific addresses allocated during the run of the program. However, they can be sensitive to the “shape” of the heap or the values stored in the heap.

Note that the union of any two bisimulations is a bisimulation. Hence, there is a largest bisimulation, which we denote as $\approx$. Intuitively, $M \approx M$ implies that $M$ and $M$ encode the same safety property (up to the relation on values $\approx_{\beta}$ described above). With all the boilerplate for RSC in place, we state our main theorem.

**Theorem 2** ($\llbracket\cdot\rrbracket_{LP}^U$ attains RSC). $\vdash \llbracket\cdot\rrbracket_{LP}^U : \text{RSC}$

We outline our proof of Theorem 9, which relies on a backtranslation $\llbracket\cdot\rrbracket_{LP}^U$. Intuitively, $\llbracket\cdot\rrbracket_{LP}^U$ takes a target trace $\overline{\alpha}$ and builds a set of source contexts such that one of them when linked $C$, produces a related trace $\overline{\alpha}$ in the source (Theorem 10). In prior work, backtranslations return a single context [9, 10, 20,
26, 49, 52, 54]. This is because they all, explicitly or implicitly, assume that \( \approx \) is injective from source to target. Under this assumption, the backtranslation is unique: a target value \( v \) will be related to at most one source value \( v \). We do away with this assumption and thus there can be multiple source values related to any given target value. This results in a set of backtranslated contexts, of which at least one will reproduce the trace as we need it.

The backtranslation requires a lengthy technical setup that, for space constraints, we cannot fully show here. Instead, we only provide an informal description of the backtranslation and why it achieves what it is supposed to, the interested reader will find all details in the appendix.

**Example 4** (Backtranslation of a trace). Below is the trace \( \overrightarrow{\alpha} \) and the the output of \( \langle \langle \alpha \rangle \rangle_{L^P} \).

\[
\begin{align*}
(1) \quad &\text{call } f_0 (1 \rightarrow 4 : \bot, 2 \rightarrow 3 : \bot)? \\
&\text{let } x = \text{new } 4 \text{ in } L :: \langle x, 1 \rangle; \\
&\text{let } x = \text{new } 3 \text{ in } L :: \langle x, 2 \rangle; \quad \text{(1)}
\\
(2) \quad &\text{ret } (1 \rightarrow 4 : \bot, 2 \rightarrow 3 : \bot, 3 \rightarrow 11 : k)! \\
&\text{let } x = \text{new } L(2) \text{ in } L :: \langle x, 3 \rangle; \\
&\text{let } x = \text{new } L(1) \text{ in } x := 55; \quad \text{(2)}
\\
(3) \quad &\text{call } f_2 (1 \rightarrow 55 : \bot, 2 \rightarrow 3 : \bot, 3 \rightarrow 15 : k)? \\
&\text{let } x = \text{new } L(3) \text{ in } x := 15; \\
&\text{call } f_2; \quad \text{(3)}
\end{align*}
\]

\( \langle \langle \alpha \rangle \rangle_{L^P} \) generates empty method bodies for all context methods called by the compiled component. Then it backtranslates each action, generating code blocks that will mimic that action and places that code inside the method bodies. The diagram shows the code blocks generated for each action with the same number. Backtranslated code maintains a support datastructure at runtime, a list of locations denoted with \( L \) where locations are added (\( :: \)) and they are looked up (\( L(n) \)) based on their second field \( n \), which is their target-level address.

In order to backtranslate the first call, we need to setup the heap with the right values and then perform the call. In the diagram, dotted lines describe which source statement generates which part of the heap. The return only generates code that will update the list \( L \) to ensure that the context has access to all the locations it knows in the target too. In order to backtranslate the last call we lookup the locations to be updated in \( L \) so we can ensure that when the \( \text{call } f_2 \) statement is executed, the heap is in the right state.

For the backtranslation to be used in the proof we need to prove its correctness, i.e., that \( \langle \langle \alpha \rangle \rangle_{L^P} \) generates a context \( A \) that, together with \( C \), generates a trace \( \overrightarrow{\alpha} \) related to the input one.

**Theorem 3** (\( \langle \langle \alpha \rangle \rangle_{L^P} \) is correct).

If \( A \mid [C]_{L^P} \Rightarrow \Omega \) then \( \exists A \in \langle \langle \alpha \rangle \rangle_{L^P}. A \mid [C] \Rightarrow \Omega \) and \( \overrightarrow{\alpha} \approx_{\beta} \overrightarrow{\Omega} \) and \( \Omega \approx_{\beta} \Omega \).
This theorem immediately implies that \( \Gamma \vdash \llbracket L^U \rrbracket : PF-RSC \), which, by Theorem 7, implies that \( \Gamma \vdash \llbracket L^P \rrbracket : RSC \).

4 RSC: Second Instance with Types, Concurrency and Simpler Proofs

When the source language has a verification system that enforces robust safety, proving that a compiler attains RSC can be simpler than that of Section 3—it may not require a back translation. To demonstrate this, we consider a specific class of monitors, namely those that enforce type invariants on a specific set of locations. Our source language, \( L^\tau \), is similar to \( L^U \) but it has a type system that ensures that source programs generate traces that the monitor always accepts. Our compiler \( \llbracket L^\tau \rrbracket \) is directed by typing derivations, and its proof of RSC establishes a specific cross-language invariant on program execution, rather than a backtranslation.

A second, independent goal of this section is to show that RSC is compatible with concurrency. Consequently, our source and target languages include constructs for forking threads.

4.1 The Source Language \( L^\tau \)

\( L^\tau \) extends \( L^U \) with concurrency—it has a fork statement \( ([| s ]) \), processes and process soups [18]—and with a more extensive type system (Figure 6). \( L^\tau \) has an unconventional type system that enforces robust type safety [1, 12, 29, 32, 44], which means that no context can cause the static types of sensitive/trusted heap locations to be violated at runtime. Using a special type described below, a program component statically partitions heap locations it deals with into those it cares about (“trusted” locations) and those it does not care about (“untrusted” locations). Call a value shareable if only untrusted locations can be extracted from it using the language’s elimination constructs. The type system then ensures that a program component only ever shares shareable values with the context. This ensures that the context—which may not be well-typed in a canonical sense—cannot violate any invariants (including static types) of the trusted locations, since it can never get direct access to them.

Technically, the type system includes a type called \( UN \), which stands for “untrusted” or “shareable” and contains all values that can be passed to the context. Every type that is not a subtype of \( UN \) is implicitly trusted and cannot be passed to the context. Untrusted locations are explicitly marked \( UN \) at their allocation points in the program. Other types are deemed shareable via subtyping. Intuitively, a type is safe if values in it can only yield locations of type \( UN \) by the language’s elimination constructs. For example, \( UN \times UN \) is a subtype of \( UN \). We write \( \tau \vdash \circ \) to mean that \( \tau \) is a subtype of \( UN \) (see Rule TL\( \tau \)-coercion).
Components \( C ::= \Delta; F; I \)  
Statements \( s ::= \cdots | (\| s) | \text{endorse } x = e \text{ as } \varphi \text{ in } s \)

Heaps \( H ::= \emptyset \mid H; \ell \mapsto v : \tau \)  
Types \( \tau ::= \text{Bool} \mid \text{Nat} \mid \tau \times \tau \mid \text{Ref } \tau \mid \text{UN} \)

Superf. Types \( \varphi ::= \text{Bool} \mid \text{Nat} \mid \text{UN} \times \text{UN} \mid \text{Ref UN} \)

Monitors \( M ::= (\{\sigma \cdots \}, \sim, \sigma_0, \Delta, \sigma_c) \)  
Mon. Trans. \( \rightsquigarrow ::= \emptyset \mid \rightsquigarrow; (\sigma, \sigma) \)

Envs. \( \Gamma ::= \emptyset \mid \Gamma; (x : \tau) \)  
Store Env. \( \Delta ::= \emptyset \mid \Delta; (\ell : \tau) \)

### Typing Judgements

\( \vdash C : \text{UN} \) Component \( C \) is well-typed. \( \Delta, \Gamma \vdash e : \tau \) Expression \( e \) has type \( \tau \) in \( \Gamma \) and \( \Delta \).

\( \tau \vdash \odot \) Type \( \tau \) is insecure. \( C, \Delta, \Gamma \vdash s \) Statement \( s \) is well-typed in \( C, \Gamma \) and \( \Delta \).

### Typing rules

\[ \tau \vdash \odot \]

\[ \begin{align*}
\tau \vdash \odot & \quad (\text{TL}_\text{\text{-pub}}) \\
\text{Bool} \vdash \odot & \quad (\text{TL}_\text{\text{-pub}}) \\
\text{Nat} \vdash \odot & \quad (\text{TL}_\text{\text{-pub}}) \\
\tau \times \tau' \vdash \odot & \quad (\text{TL}_\text{\text{-pub}}) \\
\text{UN} \vdash \odot & \quad (\text{TL}_\text{\text{-pub}})
\end{align*} \]

\[ \begin{align*}
\begin{array}{l}
\Delta, \Gamma \vdash e : \tau \\
\Delta, \Gamma \vdash \text{endorse } x = e \text{ as } \varphi \text{ in } s
\end{array} & \quad (\text{TL}_\text{\text{-coercion}}) \\
\Delta, \Gamma \vdash e : \text{UN} & \quad (\text{TL}_\text{\text{-endorse}}) \\
C, \Delta, \Gamma; (x : \varphi) \vdash s
\end{align*} \]

Figure 6: Syntax, judgements and static semantics of \( L^\tau \) (excerpts).

Further, \( L^\tau \) contains an endorsement statement that dynamically checks the top-level constructor of a value of type \( \text{UN} \) and gives it a more precise superficial type \( \varphi \) \[23\]. This allows a program to safely inspect values coming from the context. It is similar to existing type casts \[47\] but it only inspects one structural layer of the type (this simplifies the compilation).

The operational semantics of \( L^\tau \) \( \lambda \rightarrow \) updates that of \( L^\text{U} \) to deal with concurrency and endorsement. The latter performs a runtime check on the endorsed value \[56\].

The new monitors check at runtime that the set of trusted heap locations have values of their intended static types. Accordingly, the description of the monitor includes a list of trusted locations and their expected types (in the form of an environment \( \Delta \)). The type \( \tau \) of any location in \( \Delta \) must be trusted, so \( \tau \not\vdash \odot \). To facilitate checks of the monitor, every heap location carries a type at runtime (in addition to a value). The monitor transitions should therefore be of the form \( (\sigma, \Delta, \sigma) \), but since \( \Delta \) never changes, we write the transitions as \( (\sigma, \sigma) \).
Agreement between a monitor and a component is defined as follows:

\[ M \triangleleft C \overset{def}{=} (\{\sigma \cdot \cdot \cdot \}, \sim, \sigma_0, \Delta, \sigma_c) \triangleleft (\Delta; \mathbb{P}; \mathbb{I}) \]

As Theorem 11 shows, a well-typed component generates traces that are always accepted.

**Theorem 4** (Typability Implies Robust Safety in \(L^\tau\)).

If \( \vdash C : \text{UN} \) and \( C \triangleleft M \) then \( M \vdash C : rs \)

No other definition needs to change: robust safety, safety and actions are defined as before.

**Richer Source Monitors** In \(L^\tau\), source language monitors only enforce the property of type safety on specific memory locations (robustly). This can be generalized substantially to enforce arbitrary invariants other than types on locations. The only requirement is to find a type system (e.g., based on refinements or Hoare logics) that can enforce robust safety in the source (cfr [61]). Our compilation and proof strategy should work with little modification.

Another simplification that can be lifted with little effort is making the set of locations considered by the monitor grow over time, as for the compiler of Section 3. We did not make these generalizations to retain focus on the compilation without complicating the remaining development.

### 4.2 The Target Language \(L^\pi\)

**Statements**

\[ s ::= \cdots | (\parallel s) | \text{let } x = \text{newhide } e \text{ in } s | \text{destruct } x = e \text{ as } B \text{ in } s \text{ or } s \]

**Patterns**

\[ B ::= \text{nat} | \text{pair} \]

**Monitors**

\[ M ::= (\{\sigma \cdot \cdot \cdot \}, \sim, \sigma_0, H_0, \sigma_c) \]

**Operational semantics**

\[
\begin{align*}
\text{(EL}^\pi\text{-destruct-nat)} \\
C, H \triangleright \text{destruct } x = e \text{ as } \text{nat in } s \text{ or } s' & \rightarrow C, H \triangleright s[n/x] \\
\text{(EL}^\pi\text{-new)} \\
H = H_1; n \mapsto (v, \eta) & \quad H \triangleright e \leftrightarrow v \quad k \notin \text{dom}(H) & \quad s' = s[(n + 1, k) / x] \\
C, H \triangleright \text{let } x = \text{newhide } e \text{ in } s & \rightarrow C, H; n + 1 \mapsto v : k; k \triangleright s'
\end{align*}
\]

Figure 7: \(L^\pi\) additions to syntax and semantics (excerpts).

Our target language, \(L^\pi\), extends the previous target language, \(L^\rho\), with support for concurrency (forking, processes and process soups), atomic co-creation of a protected location and its protecting capability (statement newhide) and for examining the top-level construct of a value according to a pattern, \(B\) (Figure 7). The atomic creation of capabilities is provided to show how modern
4.3 Compiler from $L^r$ to $L^\pi$

The high-level structure of the compiler, $\llbracket \cdot \rrbracket_{L^r}^{L^r}$, is similar to that of our earlier compiler $\llbracket \cdot \rrbracket_{L^\pi}^{L^\pi}$ (Section 3.3). However, $\llbracket \cdot \rrbracket_{L^r}^{L^\pi}$ is defined by induction on the type derivation of the component to be compiled (Figure 8). Additionally, the case for allocation explicitly uses type information to achieve security efficiently, protecting only those locations whose type is not $UN$.

\[
\begin{align*}
\Delta, \Gamma \vdash e : \tau & \quad \Rightarrow \quad \begin{cases}
\text{let } xo = \text{new } \llbracket \Delta, \Gamma \vdash e : \tau \rrbracket_{L^r}^{L^r} & \text{if } \tau = UN \\
\text{in let } x = \langle xo, 0 \rangle & \text{\quad in } \llbracket C, \Delta, \Gamma; x : \text{Ref}\; \tau \vdash s \rrbracket_{L^r}^{L^r} \\
\text{let } x = \text{newhide } \llbracket \Delta, \Gamma \vdash e : \tau \rrbracket_{L^r}^{L^r} & \text{\quad in } \llbracket C, \Delta, \Gamma; x : \text{Ref}\; \tau \vdash s \rrbracket_{L^r}^{L^r} \\
\text{otherwise} & \text{\quad otherwise}
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\Delta, \Gamma \vdash e : UN & \quad \Rightarrow \quad \begin{cases}
\Delta, \Gamma \vdash e : UN & \quad \text{in } \llbracket C, \Delta, \Gamma \vdash \| s \rrbracket_{L^r}^{L^r} \\
\text{let } x = \text{newhide } \llbracket \Delta, \Gamma \vdash e : \tau \rrbracket_{L^r}^{L^r} & \text{\quad in } \llbracket C, \Delta, \Gamma \vdash \| s \rrbracket_{L^r}^{L^r} \\
\text{otherwise} & \text{\quad otherwise}
\end{cases}
\end{align*}
\]

Figure 8: The $\llbracket \cdot \rrbracket_{L^r}^{L^r}$ compiler from $L^r$ to $L^\pi$ (excerpts).

As the monitor definitions have changed, the proof of $RSC$ relies on a different cross-language relation for monitors. We describe that relation below.

**New Monitor Relation** Informally, a source and a target monitor are related if the target monitor can always step whenever the source heap satisfies the types specified in the source monitor (up to renaming by the partial bijection $\beta$).

Consider a typing $\Delta$ for the part of the source heap being monitored (indicated as $\vdash H : \Delta$) and a partial bijection $\beta$ from source to target locations. We say that a target monitor $M = (\{ \sigma_0 \cdots \}, \sim, \sigma_0, H_0, \sigma_e)$ is good, written $\vdash M : \beta, \Delta$, if for all $\sigma \in \{ \sigma_0 \cdots \}$ and all $H \equiv_\beta H'$ such that $\vdash H : \Delta$, there is a $\sigma'$ such that $(\sigma, H, \sigma') \in \sim$. For a fixed partial bijection $\beta_0$ between the domains of $\Delta$ and $H_0$, we say that the source monitor $M$ and the target monitor $M'$ are related, written $M \approx M'$, if $\vdash M : \beta_0, \Delta$ for the $\Delta$ in $M$.

**Theorem 5** (Compiler $\llbracket \cdot \rrbracket_{L^r}^{L^r}$ attains $RSC$). $\vdash \llbracket \cdot \rrbracket_{L^r}^{L^r} : RSC$

To prove that $\llbracket \cdot \rrbracket_{L^r}^{L^r}$ attains $RSC$ we do not rely on a backtranslation. Here, we know statically which locations can be monitor-sensitive: they must all be security architectures such as Cheri [64] (which implement capabilities at the hardware level) would support $RSC$. However, this construct is not necessary and we prove that $RSC$ is attained both by a compiler relying on it and by one that allocates a location and then protects it non-atomically.
trusted, i.e., must have a type \( \tau \) satisfying \( \tau \not\models \circ \). Using this, we set up a simple cross-language relation and show it to be an invariant on runs of source and compiled target components. The relation captures the following:

- Heaps (both source and target) can be partitioned into two parts, a trusted part and an untrusted part;
- The trusted source heap contains only locations whose type is trusted;
- The trusted target heap contains only locations related to source trusted locations and these point to related values; more importantly, every target trusted location is protected by a capability;
- In the target, any capability protecting a trusted location does not occur in attacker code, nor is it stored in an untrusted heap location.

We need to prove that this relation is preserved by reductions both in the compiled code and in the attacker code. The former follows from source robust safety (Theorem 11). The latter is simple since all trusted locations are protected with capabilities, attackers have no access to trusted locations (Figure 8), and capabilities are unforgeable and unguessable (by the semantics of \( \mathcal{L}^\pi \)). At this point, knowing that monitors are related, and that source traces are always accepted by source monitors, we can conclude that target traces are always accepted by target monitors too.

Note that this kind of an argument requires all compilable source programs to be robustly safe and is, therefore, impossible for our first compiler. Avoiding the backtranslation here simplifies the proof significantly.

### 4.3.1 Non-Atomic Allocation of Capabilities

The compiler of Figure 8 uses a new target language construct, `newhide`, that simultaneously allocates a new location and protects it with a capability. This atomic construct simplifies our proof of security in the concurrent setting at hand: if allocation and protection were not atomic, then a concurrent adversary thread could protect a location that had just been allocated and acquire the capability to it before the allocating thread could do so. This would break the cross-language relation we use in our proof. However, note that this is not really an attack since it does not give the adversary any additional power to violate the safety property enforced by the monitor. In fact, the allocating thread just gets stuck when it tries to acquire the capability itself, and it must try to acquire said capability in order to protect the location. Consequently, it is possible to do away with the `newhide` construct (Figure 9).

The price to pay is a slightly more involved cross-language relation, which must relate states where either (i) the heaps are partitioned as before, or (ii) the target execution is stuck on trying to acquire a capability for a location that should be trusted.

We refer the interested reader to the appendix for details of the new relation and the proof that this alternative compiler also attains RSC.
\[
\begin{align*}
\Delta, \Gamma \vdash e : \tau & \quad \Delta, \Gamma ; \text{x : Ref } \tau \vdash s \\
\text{C, } \Delta, \Gamma ; \text{let x = new}_{\tau} e \text{ in } s & \quad \begin{cases}
\text{let x = new } 0 \text{ in} \\
\text{let x}_{k} = \text{hide } x \text{ in}
\end{cases} \\
\text{C, } \Delta, \Gamma ; \text{let x = new } \tau_{e} \text{ in } s & \quad \begin{cases}
\text{let x}_{e} = \llbracket \Delta, \Gamma ; e : \tau \rrbracket_{L^{\pi}} \text{ in } \text{if } \tau \neq \text{UN}
\end{cases} \\
\text{C, } \Delta, \Gamma ; \text{x : Ref } \tau \vdash s & \quad x := x_{c} \text{ with } x_{k}; \\
\llbracket \text{C, } \Delta, \Gamma ; x : \text{Ref } \tau \vdash s \rrbracket_{L^{\pi}} & \quad \end{cases}
\end{align*}
\]

Figure 9: Non-atomic implementation of capability allocation, only the interesting case for \( \tau \neq \text{UN} \) is reported, the other is analogous to the one in Figure 8.

5 \textbf{RSC: Third Instance with Target-level Memory Isolation}

This section presents how changing the target language defence mechanism does not impact achieving and proving \textit{RSC}. To this end, this section defines \( L^{I} \), a new target language that provides memory isolation much like the Intel SGX \cite{45} (Section 5.1). Then it presents \( \llbracket \cdot \rrbracket_{L^{I}} \), the compiler from our previous source language \( L^{\pi} \) to \( L^{I} \) (Section 5.2) and it proves that it attains \textit{RSC}.

5.1 \( L^{I} \), a Target Language with Memory Isolation

Language \( L^{I} \) (Figure 10) is similar to \( L^{\pi} \) except that it replaces capabilities with a simple memory isolation mechanism (which we call \textit{enclave}). This is a very high-level model of an SGX-like enclave, which provides isolated code and memory, abstracting from many tedious low-level details. In an SGX enclave, the isolated code can only be accessed by jumping to the enclave entry point while the isolated data can only be accessed by the code within the enclave. This access control mechanism is exactly what \( L^{I} \) mimics. For simplicity, we use a single enclave, since modelling multiple ones is straightforward \cite{54} and does not bring additional insights.

In order to understand who has the privilege of writing to the isolated memory, programs have an additional parameter \( E \), the list of functions in the enclave. Only functions that are listed in \( E \) can create (Rule \( EL^{I} \)-isolate), read (Rule \( EL^{I} \)-deref-iso) and write (Rule \( EL^{I} \)-assign-iso) locations in the enclave. These functions rely on the crucial judgement \( \text{C} \vdash f : \text{prog} \), which is valid only if the current executing function \( f \) is in \( \text{C}'s \) enclave functions \( E \). Moreover, these functions rely on changing the judgement of the semantics of expressions to carry around information about the current executing program and the enclave functions. The heap of \( L^{I} \) maps locations from integers (and not just natural numbers) to values and the enclave is the portion of the heap whose domain is less than 0. Monitors are unchanged from \( L^{\pi} \).
Components $C ::= H_0; F: I; E$

Enclave functions $E ::= f$

Statements $s ::= \cdots \mid x ::= e \mid \text{let } x = \text{new } e \text{ in } s \mid \text{let } x = \text{newiso } e \text{ in } s$

Expressions $e ::= \cdots \mid !e$

Heaps $H ::= \emptyset \mid H \sto n \mapsto v$

Operational semantics

$\frac{(EL^I\text{-deref})}{(EL^I\text{-deref}-iso)} \quad \begin{align*} &\text{if } n \geq 0 \quad C; H; f \triangleright n \mapsto v \\ &\text{if } n < 0 \quad \vdash f : \text{prog} \quad C; H; f \triangleright n \mapsto v \end{align*}$

$\begin{align*} &\frac{(EL^I\text{-new})}{(EL^I\text{-isolate})} \quad \begin{align*} &\text{if } n \Rightarrow v \in H \quad C; H; f \triangleright n \mapsto v \\ &\text{if } n \not\Rightarrow v \quad \vdash f : \text{prog} \quad C; H; f \triangleright n \mapsto v \\ &\text{let } x = \text{new iso } e \text{ in } s \\ &\text{let } x = \text{new } e \text{ in } s \\ &\text{skip} \\ &\text{new e} \quad C; H; f \triangleright n \mapsto v \end{align*}$

Figure 10: $L^I$ changes to syntax and semantics.

5.2 Compiler from $L^\tau$ to $L^I$

Again the high-level structure of the $[[ \cdot ]]_{L^I}^{L^I}$ compiler is similar to that of its predecessor. The first responsibility of $[[ \cdot ]]_{L^I}^{L^I}$ is ensuring that the right functions populate the target enclave functions $E$ (Rule $([[ \cdot ]]_{L^I}^{L^I}\text{-Component})$). Additionally, when creating the target component the compiler populates the monitor relevant heap $H_0$ based on the information in $\Delta$, as captured by the judgement $\Delta \vdash \phi_0 H_0$. The second responsibility of $[[ \cdot ]]_{L^I}^{L^I}$ is ensuring that only security-relevant locations are stored in the enclave. As before, to achieve this the compiler relies on the typing information from the well-typedness of $C$. Locations whose type are not security relevant ($UN$) are placed outside the enclave while those that are security relevant are placed inside (Rule $([[ \cdot ]]_{L^I}^{L^I}\text{-New})$).

Theorem 6 (Compiler $[[ \cdot ]]_{L^I}^{L^I}$ attains $RSC$). $\vdash [[ \cdot ]]_{L^I}^{L^I} : RSC$

As anticipated, $[[ \cdot ]]_{L^I}^{L^I}$ also attains $RSC$, and the intuition is simple: all monitor-relevant locations are placed in the enclave and no external code can
tamper with them. The proof of Theorem 6 is very similar to that of Theorem 13. In this case we again do not rely on a backtranslation but we carry forward a similar relation to the previous one. The only change is that every location in the target trusted heap is in the enclave. That ensures that locations in the trusted heap cannot be accessed by attacker code and that only compiled code manipulates them according to the source semantics. Since by assumption the source component never violates the safety of locations in the trusted heap, the theorem holds.

With Theorem 6, we can conclude that RSC scales seamlessly to different protection mechanisms and its formulation is not linked to our isolation mechanism of choice.

6 Fully Abstract Compilation

This section defines fully abstract compilation or FAC (Section 6.1). It explains how, in some cases, FAC may result in inefficient compiled code, while RSC does not (Section 6.2). Then, it explains what would be needed to write a fully abstract compiler from $L^U$ to $L^P$ (the languages of our first compiler), and uses this to compare RSC and FAC concretely, both from the perspective of efficiency of compiled code as well as simplicity of proofs (Section 6.3).

It is worth noting that fully abstract compilation is often the desirable property to prove and there are cases in which it has little to no downsides (regarding efficiency and proof complexity). This is especially tangible when the target language is typed [9, 10, 20, 49]. However, many target languages, including those we consider, are not typed, and it is for compilers targeting them that we propose RSC as a more efficient alternative.
6.1 Formalising Fully Abstract Compilation

As stated in Section 1, FAC requires the preservation and reflection of observational equivalence, and most existing work instantiates observational equivalence with contextual equivalence ($\simeq_{\text{ctx}}$). Contextual equivalence and FAC are defined below. Informally, two components $C_1$ and $C_2$ are contextually equivalent if no context $A$ interacting with them can tell them apart, i.e., they are indistinguishable. Contextual equivalence can encode security properties such as confidentiality, integrity, invariant maintenance and non-interference [6, 8, 52, 55].

**Definition 6** (Contextual equivalence $\simeq_{\text{ctx}}$). $C_1 \simeq_{\text{ctx}} C_2 \overset{\text{def}}{=} \forall A. A[C_1] \uparrow \iff A[C_2] \uparrow$, where $\uparrow$ means that the execution diverges.

Informally, a compiler $\llbracket \cdot \rrbracket_S^T$ is fully abstract if it translates (only) contextually-equivalent source components into contextually-equivalent target ones.

**Definition 7** (Fully abstract compilation).

$$\vdash \llbracket \cdot \rrbracket^S_T : \text{FAC} \overset{\text{def}}{=} \forall C_1, C_2. C_1 \simeq_{\text{ctx}} C_2 \iff \llbracket C_1 \rrbracket^S_T \simeq_{\text{ctx}} \llbracket C_2 \rrbracket^S_T$$

The security-relevant part of FAC is the $\Rightarrow$ implication [26]. This part is security-relevant because the proof thesis concerns target contextual equivalence ($\simeq_{\text{ctx}}$). Unfolding the definition of $\simeq_{\text{ctx}}$ on the right of the implication yields a universal quantification over all possible target contexts $A$, which captures malicious attackers. In fact, there may be target contexts $A$ that can interact with compiled code in insecure ways that are impossible in the source language (e.g., jumping into the middle of a function or accessing hidden local state directly). Compilers that attain FAC with untyped target languages often insert checks in compiled code that detect these insecure interactions and respond to them securely [55], often by halting the execution [6, 8, 26, 35, 38, 41, 52]. These checks are often inefficient, but must be performed even when the compiled code is not under attack.

6.2 FAC and Inefficient Compiled Code

We illustrate various ways in which FAC forces inefficiencies in compiled code via a running example. Consider a password manager written in an object-oriented language that is compiled to an assembly-like language. We elide most code details and focus only on the relevant aspects.

```java
private db: Database;

public testPwd( user: Char[8], pwd: BitString): Bool{
    if( db.contains( user )){ return db.get( user ).getPassword() == pwd; }
}

private class Database{ ... }
```

The source program exports the function `testPwd` to check whether a `user`'s stored password matches a given password `pwd`. The stored password is in a
local database, which is represented by a piece of local state in the variable \( db \). The details of \( db \) are not important here, but the database is marked private, so it is not directly accessible to the context of this program in the source language.

**Example 5** (Extensive checks). A fully-abstract compiler for the program above must generate code that checks that the arguments passed to `testPwd` by the context are of the right type [8, 30, 38, 52, 54]. In fact, the code expects an array of characters of length 8, any other parameter (e.g., an array of objects) cannot be passed in the source, so it must also be prevented to be passed in the target. More precisely, a fully abstract compiler will generate code similar to the following for `testPwd` (we assume that arrays are passed as pointers into the heap).

```assembly
label testpwd
  for i = 0; i < 8; i++ // 8 is the length of the user field in the previous snippet
    add r0 i
    load the memory word stored at address r0 into r1
    test that r1 is a valid char encoding
...
```

Basically, this code dynamically checks that the first argument is a character array. Such a check can be very inefficient.

The problem here is that FAC forces these checks on all arguments, even those that have no security relevance. In contrast, RSC does not need these checks. Indeed, neither of our earlier compiler, \( \llbracket L \rrbracket \) nor \( \llbracket L \rrbracket \pi \), insert them. Note that any robustly safe source program will have programmer-inserted checks for all parameters that are relevant to the safety property of interest, and these checks will be compiled to the target. For other parameters, the checks are irrelevant, both in the source and the target, so there is no need to insert them.

**Example 6** (Component size in memory). Let us now consider two different ways to implement the `Database` class: as a `List` and as a `RedBlackTree`. As the class is private, its internal behaviour and representation of the database is invisible to the outside. Let \( C_{list} \) be the program with the `List` implementation and \( C_{tree} \) be the program with the `RedBlackTree` implementation; in the source language, these are equivalent. However, a subtlety arises when considering the assembly-level, compiled counterparts of \( C_{list} \) and \( C_{tree} \): the code of a `RedBlackTree` implementation consumes more memory than the code of a `List` implementation. Thus, a target-level context can distinguish \( C_{list} \) from \( C_{tree} \) by just inspecting the sizes of the code segments. So, in order for \( \llbracket S \rrbracket_T \) to be fully abstract, it must produce code of a fixed size [8, 52]. This wastes memory and makes it impossible to compile some components. An alternative would be to spread the components in an overly-large memory at random places i.e., use address-space layout randomization or ASLR, so that detecting different code sizes has a negligible chance of success [6, 35]. However, ASLR is now known to be broken [13, 36].

\( \square \)
Again, we see that FAC introduces an inefficiency in compiled code (pointless code memory consumption) even though this has no security implication here. In contrast, RSC does not require this unless the safety property(ies) of interest care about the size of the code (which is very unlikely in a security context, since security by code obscurity is a strongly discouraged security practice). In particular, the monitors considered in this paper cannot depend on code size.

**Example 7** (Wrappers for heap resources). Assume that the `Database` class is implemented as a `List`. Shown below are two implementations of the `newList` method inside `List` which we call `C_one` and `C_two`. The only difference between `C_one` and `C_two` is that `C_two` allocates two lists internally; one of these (shadow) is used for internal purposes only.

```
public newList(): List{
    ell = new List();
    return ell;
}
```

Again, `C_one` and `C_two` are equivalent in a source language that does not allow pointer comparison. To attain FAC when the target allows pointer comparisons, the pointers returned by `newList` in the two implementations must be the same, but this is very difficult to ensure since the second implementation does more allocations. A simple solution to this problem is to wrap `ell` in a proxy object and return the proxy [8, 46, 52, 54]. Compiled code needs to maintain a lookup table mapping the proxy to the original object. Proxies must have allocation-independent addresses. Proxies work but they are inefficient due to the need to look up the table on every object access.

Another way to attain FAC is to weaken the source language, introducing an operation to distinguish object identities in the source [51]. However, this is a widely discouraged practice, as it changes the source language from what it really is and the implication of such a change may be difficult to fathom for programmers and verifiers.

In this example, FAC forces all privately allocated locations to be wrapped in proxies, however RSC does not require this. Our target languages $L^P$ and $L^\pi$ support address comparison (addresses are natural numbers in their heaps) but $L^P$ and $L^\tau$ just use capabilities to attain security efficiently. On the other hand, for attaining FAC, capabilities alone would be insufficient since they do not hide addresses; proxies would still be required (this point is concretely demonstrated in Section 6.3).

**Example 8** (Strict termination vs divergence). Consider a source language that is strictly terminating while a target language that is not. Below is an extension of the password manager to allow database encryption via an externally-defined function. As the database is not directly accessible from external code, the two implementations below `C_enc` (which does the encryption) and `C_skip` which skips the encryption are equivalent in the source.
public encryptDB (func: Database → Bitstring) : void {
    func(this.db);
    return;
}

If we compile $C_{enc}$ and $C_{skip}$ to an assembly language, the compiled counterparts cannot be equivalent, since the target-level context can detect which function is compiled by passing a `func` that diverges. Calling the compilation of $C_{enc}$ with such a `func` will cause divergence, while calling the compilation of $C_{skip}$ will immediately return.

This case presents a situation where FAC is outright impossible. The only way to get FAC is to make the source language artificially non-terminating, see the work of Devriese et al. [27] for more details of this particular problem. On the other hand, RSC can be easily attained even in such settings since it is completely independent of termination in the languages (note that program termination and nontermination are both different from the monitor getting stuck on an action, which is what RSC cares about). Indeed, if our source languages $L^U$ and $L^\tau$ were restricted to terminating programs only, the same compilers and the same proofs of RSC would still work.

**Remark** It is worth noting that many of the inefficiencies above could be resolved by just replacing contextual equivalence with a different equivalence in the statement of FAC. However, it is not known how to do this generally for arbitrary sources of inefficiency and, further, it is unclear what the security consequences of such instantiations of FAC would be. On the other hand, RSC is uniform and it does address all these inefficiencies.

An issue that can normally not be addressed just by tweaking equivalences is side-channel leaks, as they are, by definition, not expressible in the language. Neither FAC nor RSC deals with side channels, but recent results describe how to account for side channels in secure compilers [14].

### 6.3 Towards a Fully Abstract Compiler from $L^U$ to $L^P$

This section sketches a fully abstract compiler from $L^U$ to $L^P$. Our goal is not to argue that this compiler is actually fully abstract, but to only show some compilation constructs that seem necessary for full abstraction, and to comment on the inefficiency of these constructs and of the ensuing proof of full abstraction (to contrast this with the relatively easier RSC-attaining compiler from Section 3). In fact, in order to define a FAC-attaining compiler the languages need to be adjusted slightly. We motivate each change with an otherwise unrecoverable failure of full abstraction (Section 6.3.1). We informally define the FAC compiler, $\Box L^U$, and comment on its inefficiency in Section 6.3.2. Then, we sketch a proof of FAC for $\Box L^U$, and highlight how it would be more complicated than a RSC proof (Section 6.3.3).
6.3.1 Language Extensions to $L^U$ and $L^P$

This section lists the language extensions required by the compiler. It is not possible to motivate all of them before explaining the details of the compiler, so some of the justification is postponed to Section 6.3.2.

A first concern for full abstraction is that a target context can always determine the memory consumption of two compiled components, analogously to Example 6. To ensure that this does not break full abstraction, we add a source expression $\text{size}$ that returns the amount of locations $\ell$ allocated in the current heap $H$.

In the target language $L^P$, we need to know whether an expression is a pair, whether it is a location, and we need to be able to compare two capabilities. For this, we add the expression constructs $\text{isloc}(e)$, $\text{ispair}(e)$ and $\text{eqcap}(e,e)$, respectively.

Finally, compiled code needs private functions for its runtime checks that must not be visible to the context. $L^P$ does not have this functionality: all functions defined by a component can be called by the context. Now we modify $L^P$ so that all functions $F$ defined in a component are by default private to it. Additionally, each component must explicitly define the list of functions it exports (typically a subset of $F$), so that those are the only ones that can be called by the context and the rest are private to the component.

6.3.2 The $L^U_{LP}$ Compiler

$L^U_{LP}$ is similar to $L^U_{LP}$ but with critical differences. We know that fully abstract compilation preserves all source abstractions in the target language. Here, the only abstraction that distinguishes $L^P$ from $L^U$ is that locations are abstract in $L^P$, but concrete natural numbers in $L^U$. Thus, locations allocated by compiled code must not be passed directly to the context as this would reveal the allocation order (as seen in Example 7). Instead of passing the location $\langle n, k \rangle$ to the context, the compiler arranges for an opaque handle $\langle n', k_{com} \rangle$ (that cannot be used to access any location directly) to be passed. Such an opaque handle is often called a mask or seal in the literature.

To ensure that masking is done properly, $L^U_{LP}$ inserts code at entry points and at exit points to compiled code, wrapping the compiled code in a way that enforces masking. This notion of wrapping is standard in literature on fully abstract compilation [30, 54]. The wrapper keeps a list $\mathcal{L}$ of component-allocated locations that are shared with the context in order to know their masks. When a component-allocated location is shared, it is added to the list $\mathcal{L}$. The mask of a location is its index in this list. If the same location is shared again it is not added again but its previous index is used. So if $\langle n, k \rangle$ is the 4th element of $\mathcal{L}$, its mask is $\langle 4, k_{com} \rangle$. To implement lookup in $\mathcal{L}$ we must compare capabilities too, so we rely on $\text{eqcap}$. To ensure capabilities do not leak to the context, the second field of the pair is a constant capability $k_{com}$ whose location the compiled code does not use otherwise. Technically speaking, this is exactly how...
existing fully abstract compilers operate (e.g., [52]).

As should be clear, this kind of masking is very inefficient at runtime. However, even this masking is not sufficient for full abstraction. Next, we explain additional things the compiler must do.

**Determining when a Location is Passed to the Context.** A component-allocated location can be passed to the context not just as a function argument but on the heap. So before passing control to the context the compiled code needs to scan the whole heap where a location can be passed and mask all found component-allocated locations. Dually, when receiving control the compiled code must scan the heap to unmask it. The problem now is determining what parts of the heap to scan and how. Specifically, the compiled code needs to keep track of all the locations (and related capabilities) that are shared, i.e., (i) passed from the context to the component and (ii) passed from the component to the context. These are the locations on which possible communication of locations can happen. Compiled code keeps track of these shared locations in a list \( S \). Intuitively, on the first function call from the context to the compiled component, assuming the parameter is a location, the compiled code will register that location and all other locations reachable from it in \( S \). On subsequent ? (incoming) actions, the compiled code will register all new locations available as parameters or reachable from \( S \). Then, on any ! (outgoing) action, the compiled code must scan whatever locations (that the compiled code has created) are now reachable from \( S \) and add them to \( S \). We need the new instructions \texttt{isloc} and \texttt{ispair} in \( LP \) to compute these reachable locations. Of course, this kind of scanning of locations reachable from \( S \) at every call/return between components can be extremely costly.

**Enforcing the Masking of Locations** The functions \texttt{mask} and \texttt{unmask} are added by the compiler to the compiled code. The first function takes a location (which intuitively contains a value \( v \)) and replaces (in \( v \)) any pair \( \langle n, k \rangle \) of a location protected with a component-created capability \( k \) with its index in the masking list \( L \). The second function replaces any pair \( \langle n, k_{\text{com}} \rangle \) with the \( n \)th element of the masking list \( L \). These functions should not be directly accessible to the context (else it can \texttt{unmask} any \texttt{mask}’d location and break full abstraction). This is why \( LP \) needs private functions.

**Letting the Context use Masked Locations** Masked locations cannot be used directly by the context to be read and written. Thus, compiled code must provide a \texttt{read} and a \texttt{write} function (both of which are public) that implement reading and writing to masked locations.

As should be clear, code compiled through \( [\square]_{LP}^{L_U} \) has a lot of runtime overhead in calculating the heap reachable from \( S \) and in masking and unmasking locations. Additionally, it also has code memory overhead: the functions \texttt{read}, \texttt{write}, \texttt{mask}, \texttt{unmask} and list manipulation code must be included. Finally,
there is data overhead in maintaining $S$, $L$ and other supporting data structures to implement the runtime checks described above. In contrast, the code compiled through $\llbracket \cdot \rrbracket_{LP}$ (which is just robustly safe and not fully abstract) has none of these overheads.

6.3.3 Proving that $\llbracket \cdot \rrbracket_{LP}$ is a Fully Abstract Compiler

Using $\llbracket \cdot \rrbracket_{LP}$ as a concrete example, we now discuss why proving $\text{FAC}$ is harder than proving $\text{RSC}$. Consider the hard part of $\text{FAC}$, the forward implication, $C_1 \simeq_{\text{ctx}} C_2 \Rightarrow \llbracket C_1 \rrbracket_T \simeq_{\text{ctx}} \llbracket C_2 \rrbracket_T$. The contrapositive of this statement is $\llbracket C_1 \rrbracket_T \not\simeq_{\text{ctx}} \llbracket C_2 \rrbracket_T \Rightarrow C_1 \not\simeq_{\text{ctx}} C_2$. By unfolding the definition of $\not\simeq_{\text{ctx}}$ we see that, given a target context $A$ that distinguishes $\llbracket C_1 \rrbracket_T$ from $\llbracket C_2 \rrbracket_T$, it is necessary to show that there exists a source context $A$ that distinguishes $C_1$ from $C_2$. That source context $A$ must be built (backtranslated) starting from the already given target context $A$ that differentiates $\llbracket C_1 \rrbracket_T$ from $\llbracket C_2 \rrbracket_T$.

A backtranslation directed by the syntax of the target context $A$ is hopeless here since the target expressions $\text{iscap}$ and $\text{isloc}$ cannot be directly backtranslated to valid source expressions. Hence, we resort to another well-known technique [8, 54]. First, we define a fully abstract (labeled) trace semantics for the target language. A trace semantics is fully abstract when its notion of equivalence coincides with contextual equivalence, and thus can be used in place of the latter. Specifically, this means that two components are contextually inequivalent iff their trace semantics differ in at least one trace. We write $\text{TR}(\cdot)$ to denote the traces of the component $C$ in this fully abstract semantics. Given this trace semantics, the statement of the forward implication of full abstraction reduces to:

$$\text{TR}(\llbracket C_1 \rrbracket_{LP}) \neq \text{TR}(\llbracket C_2 \rrbracket_{LP}) \Rightarrow C_1 \not\simeq_{\text{ctx}} C_2.$$  

The advantage of this formulation over the original one is that now we can construct a distinguishing source context for $C_1$ and $C_2$ using the trace on which $\text{TR}(\llbracket C_1 \rrbracket_{LP})$ and $\text{TR}(\llbracket C_2 \rrbracket_{LP})$ disagree. While this proof strategy of constructing a source context from a trace is similar to our proof of $\text{RSC}$ in Section 3.3.2, it is fundamentally much harder and much more involved. There are two reasons for this.

First, fully abstract trace semantics are much more complex than our simple trace semantics of $LP$ from earlier sections. The reason is that our earlier trace semantics include the entire heap in every action, but this breaks full abstraction of the trace semantics: such trace semantics also distinguish contextually equivalent components that differ in their internal private state. In a fully abstract trace semantics, the trace actions must record only those heap locations that are shared between the component and the context. Consequently, the definition of the trace semantics must inductively track what has been shared in the past. In particular, the definition must account for locations reachable...
indirectly from explicitly shared locations. This complicates both the definition of traces and the proofs that build on the definition.

Second, the source context that the backtranslation constructs from a target trace must simulate the shared part of the heap at every context switch. Since locations in the target may be masked, the source context must maintain a map with the source locations corresponding to the target masked ones, which complicates it substantially. We call this map $B$. Now, this affects two patterns of target traces that need to be handled in a special way: call $\text{read} \ v \ H \ ? \cdot \text{ret} \ H \ !$ and call $\text{write} \ v \ H \ ? \cdot \text{ret} \ H \ !$. Normally, these patterns would be translated in source-level calls to the same functions ($\text{read}$ and $\text{write}$), but this is not possible. In fact, the source code has no $\text{read}$ nor $\text{write}$ function, and the target-level calls to those functions need to be backtranslated to the corresponding source constructs ($!$ and $:=$, respectively). The locations used by these constructs must be looked up from $B$ as these are reads and writes to masked locations. Moreover, calls and returns to $\text{read}$ can be simply ignored since the effects of reads are already captured by later actions in traces. Calls and returns to $\text{write}$ cannot be ignored as they set up a component location (albeit masked) in a certain way and that affects the behaviour of the component. We show in Example 9 how to backtranslate calls and returns to $\text{write}$.

Example 9 (Backtranslation of traces). Consider the trace below and its backtranslation.

\begin{align*}
\text{(1)} & \quad \text{call } f \ 0 \ 1 \mapsto 4? \\
\text{(2)} & \quad \text{ret } 1 \mapsto \langle 1, k_{\text{com}} \rangle! \\
\text{(3)} & \quad \text{call write} \ \langle 1, k_{\text{com}} \rangle, 5 \mapsto \langle 1, k_{\text{com}} \rangle? \\
\text{ret } 1 \mapsto \langle 1, k_{\text{com}} \rangle! 
\end{align*}

\begin{align*}
\text{main}(x) \mapsto \begin{array}{c}
\text{let } x = \text{new } 4 \text{ in } L :: \langle x, 1 \rangle \\
\text{call } f 0 \\
\text{let } x =!L(1) \text{ in } B :: \langle x, 1 \rangle \\
!B(1) := 5
\end{array} \\
\text{(1)} & \quad \text{(2)} & \quad \text{(3)}
\end{align*}

The first action, where the context registers the first location in the list $L$, is as in Example 4. Then in the second action the compiled component passes to the context (in location $1$) a masked location with index $1$ and, later, the context writes $5$ to it. The backtranslated code must recognise this pattern and store the location that, in the source, corresponds to the mask $1$ in the list $B$ (action 2). In action 3, when it is time to write $5$ to that location, the code looks up the location to write to from $B$. \(\square\)

It should be clear that this proof of $\text{FAC}$ is substantially harder than our corresponding proof of $\text{RSC}$, which needed neither fully abstract traces, nor tracking any mapping in the backtranslated source contexts.

7 Related Work

In their preliminary work, Garg et al. [31] present new criteria for secure compilation that ensure preservation of subclasses of hyperproperties. Hyperproperties [24] are a formal representation of predicates on programs, i.e., they are predicates on sets of traces. Hyperproperties capture many security-relevant
properties including not just conventional safety and liveness, which are predicates on traces, but also properties like non-interference, which is a predicate on pairs of traces. Modulo technical differences, our definition of RSC coincides with the criterion of safety property preservation by Garg et al. [31]. We show, through concrete instances, that this criterion can be easily realized by compilers, and develop two proof techniques for establishing it. We further show that the criterion leads to more efficient compiled code than does FAC. The criteria in [31] assume that behaviours in the source and target are represented using the same grammar. Hence, the definitions (somewhat unrealistically or ideally) do not require a translation of source properties to target properties. In contrast, we consider differences in the representation of behaviour in the source and in the target. This difference is accounted for in our monitor relation $M \approx M$. A slightly different account of this difference is presented by Patrignani and Garg [55] in the context of reactive black-box programs.

Abate et al. [7] define a variation of robustly-safe compilation called RSCC specifically tailored to the case where (source) components can perform undefined behaviour. RSCC does not target attacks from arbitrary target contexts but from compiled components that can become compromised and behave in arbitrary ways. To demonstrate RSCC, Abate et al. [7] rely on two backends for their compiler: software fault isolation and tag-based monitors. On the other hand, we rely on capability machines and memory isolation. RSCC also preserves (a form of) safety properties and can be achieved by relying on a trace-based backtranslation; it is unclear how whether proofs can be simplified when the source is verified and concurrent, as in our second compiler.

ASLR [6, 35], protected module architectures [8, 41, 52, 54], tagged architectures [38], capability machines [62] and cryptographic primitives [4, 5, 21, 25] have been used as targets for FAC. We believe all of these can also be used as targets of RSC-attaining compilers. In fact, some targets such as capability machines seem to be better suited to RSC than FAC, as we demonstrated.

Ahmed et al. prove full abstraction for several compilers between typed languages [9, 10, 49]. As compiler intermediate languages are often typed, and as these types often serve as the basis for complex static analyses, full abstraction seems like a reasonable goal for (fully typed) intermediate compilation steps. In the last few steps of compilation, where the target languages are unlikely to be typed, one could establish robust safety preservation and combine the two properties (vertically) to get an end-to-end security guarantee.

There are three main other criteria for secure compilation that we would like to mention: securely compartmentalised compilation (SCC) [38], trace-preserving compilation (TPC) [55] and non-interference-preserving compilation (NIPC) [15]. SCC is a re-statement of the “hard” part of full abstraction (the forward implication), but is adapted to languages with undefined behaviour and a strict notion of components. Thus, SCC suffers from much of the same efficiency drawbacks as FAC. TPC is a stronger criterion than FAC, that most existing fully abstract compilers also attain. Again, compilers attaining TPC also suffer from the drawbacks of compilers attaining FAC.

NIPC is a criterion that preserves a single property: non-interference (NI). It
has been described in the setting where NI is achieved by typing in both source and target languages and where the target uses the same security lattice as the source. Since noninterference is not a safety property, it is difficult to compare NIPC to \textit{RSC} directly. However, noninterference can also be approximated as a safety property \cite{19}. So, in principle, \textit{RSC} can be applied to the same end-goals as NIPC.

Swamy \textit{et al.} \cite{60} embed an F* model of a gradually and robustly typed variant of JavaScript into an F* model of JavaScript. Their type-directed compiler is proven to attain memory isolation as well as static and dynamic memory safety. However, they do not consider general safety properties, nor a specific, general criterion for compiler security. Gradual typing supports constructs similar to our endorsement construct in $L^\tau$.

Both our target languages rely on capabilities for restricting access to sensitive locations from the context. These capabilities can be realized in many different ways. First, we could use a capability machine such as Cheri that supports capabilities in hardware \cite{64}. Capability machines have been advocated as a target for efficient secure compilation \cite{28} and preliminary work on compiling C-like languages to them exists, but the criterion applied is \textit{FAC}, not \textit{RSC} \cite{62}. Second, we could use memory isolation (via software-fault isolation \cite{63} or an architecture like Intel’s SGX \cite{45}) to isolate high locations.

8 Conclusion

This paper has examined robustly safe compilation (\textit{RSC}), a soundness criterion for compilers with direct relevance to security. We have shown that the criterion is easily realizable and leads to more efficient code than does the current standard for secure compilers, fully abstract compilation wrt contextual equivalence. We have also presented two different techniques for establishing that a compiler attains \textit{RSC}. One is an adaptation of an existing technique, backtranslation, and the other is based on inductive invariants.
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A The Untyped Source Language: \( \mathbf{L}^U \)

This is a sequential while language with monitors.

A.1 Syntax

**Whole Programs**

\[ P ::= \ell \text{root}; H; F; I \]

**Components**

\[ C ::= \ell \text{root}; F; I \]

**Contexts**

\[ A ::= H; F \]

**Interfaces**

\[ I ::= f \]

**Functions**

\[ F ::= f(x) \mapsto s; \text{return} \]

**Operations**

\[ \oplus ::= + | - \]

**Comparison**

\[ \otimes ::= == | < | > \]

**Values**

\[ v ::= b \in \{\text{true}, \text{false}\} | n \in \mathbb{N} | \langle v, v \rangle | \ell \]

**Expressions**

\[ e ::= x | v | e \oplus e | e \otimes e | \langle e, e \rangle | e.1 | e.2 | !e \]

**Statements**

\[ s ::= \text{skip} | s; s | \text{let } x = e \text{ in } s | \text{if } e \text{ then } s \text{ else } s | \text{call } f e | \text{let } x = \text{new } e \text{ in } s | x := e \]

**Eval. Ctxs.**

\[ E ::= [:] | e \oplus E | E \oplus n | e \otimes E | E \otimes n \]

**Heaps**

\[ H ::= \emptyset | H; \ell \mapsto v \]

**Monitors**

\[ M ::= (\{\sigma \cdots \}, \nvdash, \sigma_0, \ell \text{root}, \sigma_c) \]

**Mon. States**

\[ \sigma \in \mathcal{S} \]

**Mon. Reds.**

\[ \nvdash ::= \emptyset | \nvdash; (s, H, s) \]

**Substitutions**

\[ \rho ::= \emptyset | \rho[v / x] \]

**Prog. States**

\[ \Omega ::= C, H > (s) \]

**Labels**

\[ \lambda ::= e | \alpha \]

**Actions**

\[ \alpha ::= \text{call } f v H? | \text{call } f v H! | \text{ret } H! | \text{ret } H? \]

**Traces**

\[ \pi ::= \emptyset | \pi \cdot \alpha \]

A.2 Dynamic Semantics

Rules \( \mathbf{L}^U \)-Jump-Internal to \( \mathbf{L}^U \)-Jump-OUT dictate the kind of a jump between two functions: if internal to the component/attacker, in(from the attacker to the component) or out(from the component to the attacker). Rule \( \mathbf{L}^U \)-Plug tells how to obtain a whole program from a component and an attacker. Rule \( \mathbf{L}^U \)-Whole tells when a program is whole. Rule \( \mathbf{L}^U \)-Initial State tells the initial state of a whole program. Rule \( \mathbf{L}^U \)-Monitor Step tells when a monitor makes a single step given a heap.
A.2.1 Component Semantics

Expression e reduces to e'.

Statement s reduces to s' and evolves the rest accordingly, emitting label λ.

Program state Ω steps to Ω' emitting trace π.
A.3 Monitor Semantics

Let \( \text{reach}(\ell_0, H) \) return a set of locations \( \{\ell \cdots\} \) in \( H \) such that it is possible to reach any \( \ell \in \{\ell \cdots\} \) from \( \ell_0 \) just by expression evaluation.

\[
\text{reach}(\ell, H) = \{ \ell \mid \exists e. H \triangleright e \iff \ell \land \ell \in \text{dom}(H) \}
\]

To ensure monitor transitions have a meaning, they are assumed to be closed under bijective renaming of locations.

\[
\begin{align*}
M; H \rightsquigarrow M' \\
\text{(LU-Monitor Step)} \\
M = (\{\sigma \cdots\}, \rightsquigarrow, \sigma_0, \ell_\text{root}, \sigma_c) \\
M' = (\{\sigma \cdots\}, \rightsquigarrow, \sigma_0, \ell_\text{root}, \sigma_f) \\
(\sigma_c.H', \sigma_f) \in \rightsquigarrow H' \subseteq H \\
\text{dom}(H') = \text{reach}(\ell_\text{root}, H)
\end{align*}
\]

\[
\begin{align*}
M; H \rightsquigarrow M' \\
\text{(LU-Monitor Step Trace Base)} \\
M; \emptyset \rightsquigarrow M \\
M; \forall H' \rightsquigarrow M' \\
M; H \cdot H \rightsquigarrow M' \\
\text{(LU-valid trace)} \\
M; H \rightsquigarrow M' \\
\text{relevant}(\pi) = \overline{H} \\
M \vdash \overline{\pi}
\end{align*}
\]

Monitor actions are the only part of traces that matter for safety, so we define function \( \text{relevant}(\cdot) \) that takes a general trace and elides all but the heap of actions. This function is used by both languages so we typeset it in black.

\[
\begin{align*}
\text{relevant}(\emptyset) = \emptyset \\
\text{relevant}(\text{call } f v H? \cdot \overline{\pi}) = H \cdot \text{relevant}(\overline{\pi}) \\
\text{relevant}(\text{call } f v H! \cdot \overline{\pi}) = H \cdot \text{relevant}(\overline{\pi}) \\
\text{relevant}(\text{ret } H! \cdot \overline{\pi}) = H \cdot \text{relevant}(\overline{\pi}) \\
\text{relevant}(\text{ret } H? \cdot \overline{\pi}) = H \cdot \text{relevant}(\overline{\pi})
\end{align*}
\]
B  The Target Language: \( L^P \)

B.1  Syntax

Whole Programs \( P \) ::= \( \text{root} : F ; I \)

Components \( C \) ::= \( \text{root} : F ; I \)

Contexts \( A \) ::= \( F [\cdot] \)

Interfaces \( I \) ::= \( f \)

Functions \( F \) ::= \( f(x) \mapsto s \): return;

Operations \( \oplus \) ::= \( + | - \)

Comparison \( \otimes \) ::= \( == | < | > \)

Values \( v \) ::= \( n \in \mathbb{N} | \langle v, v \rangle | k \)

Expressions \( e \) ::= \( x | v | e \oplus e | e \otimes e | \langle e, e \rangle | e.1 | e.2 | !e \) with \( e \)

Statements \( s \) ::= \( \text{skip} | s ; s | \text{let } x = e \text{ in } s | \text{if } z e \text{ then } s \text{ else } s | \text{call } f e \)

| \( x := e \) with \( e | \text{let } x = \text{new } e \text{ in } s | \text{let } x = \text{hide } e \text{ in } s \)

Eval. Ctxs. \( E \) ::= \( [\cdot] | e \oplus E | E \ominus n | e \otimes E | E \otimes n | !E \) with \( v | !e \) with \( E \)

| \( \langle e, E \rangle | (E, v) | E.1 | E.2 \)

Heaps \( H \) ::= \( \emptyset | H; n \mapsto v : \eta | H, k \)

Tag \( \eta \) ::= \( \bot | k \)

Monitors \( M \) ::= \( (\{ \sigma \cdots \}, \Rightarrow, \sigma_0, \text{root}, \sigma_c) \)

Mon. States \( \sigma \in S \)

Mon. Reds. \( \Rightarrow \) ::= \( \emptyset | \Rightarrow (s, H, s) \)

Substitutions \( \rho \) ::= \( \emptyset | \rho[v / x] \)

Prog. States \( \Omega \) ::= \( C, H \triangleright (s_T) \)

Labels \( \lambda \) ::= \( \epsilon | \alpha \)

Actions \( \alpha \) ::= \( \text{call } f \ v H? | \text{call } f \ v H! | \text{ret } H! | \text{ret } H? \)

Traces \( \alpha \) ::= \( \emptyset | \alpha \cdot \alpha \)

B.2  Operational Semantics of \( L^P \)

\[
\begin{array}{c}
\text{(LP-Jump-Internal)} \\
(f' \in T \land f \in T) \lor \\
(f' \notin T \land f \notin T) \\
\hline \\
\text{I} \vdash f, f' : \text{internal} \\
\end{array}
\begin{array}{c}
\text{(LP-Jump-IN)} \\
(f \in T \land f' \notin T) \\
\hline \\
\text{I} \vdash f : \text{in} \\
\end{array}
\begin{array}{c}
\text{(LP-Jump-OUT)} \\
f' \notin T \land f \in T \\
\hline \\
\text{I} \vdash f, f' : \text{out} \\
\end{array}
\]

47
B.2.1 Component Semantics

Expression e reduces to e'.

Statement s reduces to s' and evolves the rest accordingly, emitting label λ.

Program state Ω steps to Ω' emitting trace π.
\[
\text{(ELP-return)}\]
\[
\begin{array}{c}
\text{C.intfs} \vdash f.f' : \text{out} \\
\text{C, } H \triangleright \text{(return)} ; \text{ret } f' \\
\text{C, } H \triangleright \text{(skip)} f'
\end{array}
\]

\[
\begin{array}{c}
\Omega \Longrightarrow \Omega
\end{array}
\]

B.3 Monitor Semantics

Define \( \text{reach}(n_r, k_r, H) \) as the set of locations \( \{ n \cdots \} \) such that it is possible to reach any \( n \in \{ n \cdots \} \) from \( n_r \) using any expression and relying on capability \( k_r \) as well as any capability reachable from \( n_r \). Formally:

\[
\text{reach}(n_r, k_r, H) = \left\{ n \mid H \triangleright e \leftrightarrow \triangleright n \text{ with } v \leftrightarrow v' \right\}
\]

\[
fv(e) = n_r \cup k_r
\]

\[
\begin{array}{c}
\text{(LP-Monitor Step)}
\end{array}
\]

\[
\begin{array}{c}
M = (\{ \sigma \cdots \}, \sim, \sigma_0, k_{\text{root}}, \sigma_c) \\
(\text{sc}, H', s_f) \in \sim H' \subseteq H \\
\text{dom}(H') = \text{reach}(0, k_{\text{root}}, H)
\end{array}
\]

\[
\begin{array}{c}
\text{(LP-Monitor Step Trace Base)}
\end{array}
\]

\[
\begin{array}{c}
M; \emptyset \rightsquigarrow M
\end{array}
\]

\[
\begin{array}{c}
\text{(LP-Monitor Step Trace)}
\end{array}
\]

\[
\begin{array}{c}
M; H \rightsquigarrow M'' \quad M'' ; H \rightsquigarrow M'
\end{array}
\]

\[
\begin{array}{c}
\text{(LP-valid trace)}
\end{array}
\]

\[
\begin{array}{c}
M; H \rightsquigarrow M' \\
\text{relevant}(\pi) = \Pi
\end{array}
\]

\[
\begin{array}{c}
M \vdash \pi
\end{array}
\]
C Language and Compiler Properties

C.1 Safety, Attackers and Robust Safety

These properties hold for both languages are written in black and only once.

Definition 8 (Safety).

\[ M \vdash C : \text{safe} \overset{\text{def}}{=} \]
\[ \quad \text{if } C : \text{whole} \]
\[ \quad \text{then } \Omega_0(C) \xrightarrow{\pi} \_ \]
\[ \quad \text{then } M \vdash \pi \]

A program is safe for a monitor if the monitor accepts any trace the program generates.

Definition 9 ((Informal) Attacker).

\[ C \vdash A : \text{attacker} \overset{\text{def}}{=} \text{no location the component cares about } \in \text{fn}(A) \]

An attacker is valid if it does not refer to the locations the component cares about. We leave the notion of location the component cares about abstract and instantiate it on a per-language basis later on.

Definition 10 (Robust Safety).

\[ M \vdash C : \text{rs} \overset{\text{def}}{=} \forall A. \]
\[ \quad \text{if } M \bowtie C \]
\[ \quad C \vdash A : \text{attacker} \]
\[ \quad \text{then } M \vdash A[C] : \text{safe} \]

A program is robustly safe if it is safe for any attacker it is composed with. The definition of \( M \bowtie C \) is to be specified on a language-specific basis, as the next section does for \( L^U \) and \( L^P \).

C.2 Monitor Agreement and Attacker for \( L^P \) and \( L^U \)

Definition 11 (\( L^U: M \bowtie C \)).

\[ (\{\sigma \cdots\}, \rightsquigarrow, \sigma_0, \ell_{\text{root}}, \sigma_c) \sim (\ell_{\text{root}}; F; I) \]

A monitor and a component agree if they focus on the same initial location \( \ell_{\text{root}} \).

Definition 12 (\( L^P: M \bowtie C \)).

\[ (\{\sigma \cdots\}, \rightsquigarrow, \sigma_0, k_{\text{root}}, \sigma_c) \sim (k_{\text{root}}; \overline{F}; \overline{I}) \]
A monitor and a component agree if they use the same capability \( k_{\text{root}} \) to protect the initial location 0.

**Definition 13** (\( \text{LU} \) attacker).

\[
C \vdash A : \text{attacker} \overset{\text{def}}{=} C = (\ell_{\text{root}}; F; I), A = H; F \quad \ell_{\text{root}} \notin (\text{fn}(F) \cup H)
\]

**Definition 14** (\( \text{LP} \) attacker).

\[
C \vdash A : \text{attacker} \overset{\text{def}}{=} C = (k_{\text{root}}; F; I), A = F \quad k_{\text{root}} \notin \text{fn}(F)
\]

### C.3 Cross-language Relations

Assume a partial bijection \( \beta : \ell \times n \times \eta \) from source to target heap locations such that

- if \((\ell_1, n, \eta) \in \beta \) and \((\ell_2, n, \eta) \) then \( \ell_1 = \ell_2 \);
- if \((\ell, n_1, \eta_1) \in \beta \) and \((\ell, n_2, \eta_2) \) then \( n_1 = n_2 \) and \( \eta_1 = \eta_2 \).

We use this bijection to parametrise the relation so that we can relate meaningful locations.

For compiler correctness we rely on a \( \beta_0 \) which relates initial locations of monitors.

Assume a relation \( \approx_\beta : v \times \beta \times v \) that is total so it maps any source value to a target value \( v \).

- \( \forall v. \exists v. v \approx_\beta v \).

This relation is used for defining compiler correctness. By inspecting the semantics of \( \text{LU} \), Rules EL\( P \)-sequence and EL\( P \)-if-true let us derive that

- \( \text{true} \approx_\beta 0 \);
- \( \text{false} \approx_\beta n \) where \( n \neq 0 \);
- \( \ell \approx_\beta \langle n, v \rangle \) where \( \begin{cases} v = k & \text{if } (\ell, n, k) \in \beta \\ v \neq k & \text{otherwise, so } (\ell, n, \bot) \in \beta \end{cases} \)
- \( \langle v_1, v_2 \rangle \approx_\beta \langle v_1, v_2 \rangle \) iff \( v_1 \approx_\beta v_1 \) and \( v_2 \approx_\beta v_2 \).

We overload the notation and use the same notation to indicate the (assumed) relation between monitor states: \( \sigma \approx_\beta \sigma \).

We lift this relation to sets of states point-wise and indicate it as follows: \( \{ \sigma \ldots \} \approx_\beta \{ \sigma \ldots \} \). In these cases the bijection \( \beta \) is not needed as states do not have locations inside.

Function names are related when they are the same: \( f \approx_\beta f \).
Variables names are related when they are the same: \( x \approx_\beta x \).
Substitutions are related when they replace related values for related variables: \([v / x] \approx_\beta [v / x]\) iff \( v \approx_\beta v \) and \( x \approx_\beta x \).

\[
\begin{array}{c}
\alpha \approx_\beta \beta
\end{array}
\]

\( (\text{Call relation}) \)
\[
\begin{array}{c}
f \approx_f \quad v \approx_\beta v \quad H \approx_\beta H
\end{array}
\]
\( \text{call } f v H? \approx_\beta \text{call } f v H? \)

\( (\text{Callback relation}) \)
\[
\begin{array}{c}
f \approx_f \quad v \approx_\beta v \quad H \approx_\beta H
\end{array}
\]
\( \text{call } f v H! \approx_\beta \text{call } f v H! \)

\( (\text{Return relation}) \)
\[
\begin{array}{c}
h \approx_\beta h
\end{array}
\]
\( \text{ret } h! \approx_\beta \text{ret } h! \)

\( (\text{Returnback relation}) \)
\[
\begin{array}{c}
h \approx_\beta h
\end{array}
\]
\( \text{ret } h! \approx_\beta \text{ret } h! \)

\( (\text{Epsilon relation}) \)
\[
\begin{array}{c}
\epsilon \approx_\beta \epsilon
\end{array}
\]

**Definition 15** \((\mathcal{M}, \mathcal{R}, \mathcal{M})\). Given a monitor-specific relation \( \sigma \approx_\sigma \) on monitor states, we say that a relation \( \mathcal{R} \) on source and target monitors is a bisimulation if the following hold whenever \( M = (\{\sigma \ldots\}, \sim, \sigma_0, \ell_{\text{root}}, \sigma_c) \) and \( M = (\{\sigma \ldots\}, \sim, \sigma_0, k_{\text{root}}, \sigma_c) \) are related by \( \mathcal{R} \):

1. \( \sigma_0 \approx_\sigma \sigma_0 \), and
2. \( \sigma_c \approx_\sigma \sigma_c \), and
3. For all \( \beta \) containing \((\ell_{\text{root}}, 0, k_{\text{root}})\) and all \( H, H \) with \( H \approx_\beta H \) the following hold:
   
   (a) \( (\sigma_c, H, \_ ) \in \sim \) iff \( (\sigma_c, H, \_ ) \in \sim \), and
   
   (b) \( (\sigma_c, H, \sigma') \in \sim \) and \( (\sigma_c, H, \sigma') \in \sim \) imply \((\sigma \ldots\), \( \sim, \sigma_0, \ell_{\text{root}}, \sigma' \)\) \( \mathcal{R} \) \((\sigma \ldots\), \( \sim, \sigma_0, k_{\text{root}}, \sigma' \).

**Definition 16** \((\mathcal{M} \approx \mathcal{M})\). \( \mathcal{M} \approx \mathcal{M} \) is the union of all bisimulations \( \mathcal{M}, \mathcal{R}, \mathcal{M} \), which is also a bisimulation.

\[
\begin{array}{c}
H \approx_\beta H
\end{array}
\]

\( (\text{Heap relation}) \)
\[
\begin{array}{c}
H \approx_\beta H_1; H_2 \\
\ell \approx_\beta (n, \_ ) \\
v \approx_\beta v
\end{array}
\]
\( H = H_1; n \mapsto v : \eta ; H_2 \)
\( H; \ell \mapsto v \approx_\beta H \)

\( (\text{Empty relation}) \)
\[
\begin{array}{c}
\emptyset \approx_\beta \emptyset
\end{array}
\]

The heap relation is crucial. A source heap \( H \) is related to a target heap \( H \) if for any location pointing to a value in the former, a related location points to a related value in the target (Rule Heap relation). The base case (Rule Empty relation) considers that in the target heap we may have keys, which are not related to source elements.

As additional notation for states, we define when a state is stuck as follows

\[
\begin{array}{c}
\Omega = M; \mathcal{T}; \mathcal{T} > s \\
s \neq \text{skip} \\
\mathcal{\Omega}', \lambda, \Omega \xrightarrow{\lambda} \mathcal{\Omega}'
\end{array}
\]

\( \Omega^* \)
A state that terminated is defined as follows; this definition is given for a concurrent version of the language too (this is relevant for languages defined later):

\[ \Omega = M; F; T; H \triangleright \text{skip} \]

(\text{Terminated state})

\[ \Omega \Downarrow \]

(\text{Terminated soup})

\[ \Omega = M; F; T; H \triangleright H \quad \forall \pi \in H. M; F; T; H \triangleright \pi \]

To define compiler correctness, we rely on a cross-language relation for program states. Two states are related if their monitors are related and if their whole heap is related (Rule Related states – Whole).

(\text{Related states – Whole})

\[ \Omega = M; F; T; H \triangleright s \]

\[ \Omega = M; F; F'; T; H \triangleright s \]

\[ M \approx_{\beta} M \]

\[ H \approx_{\beta} H \]

\[ \Omega \approx_{\beta} \Omega \]

C.4 Correct and Robustly-safe Compilation

Consider a compiler to be a function of this form: \([\_]^{S} : C \rightarrow C\), taking a source component and producing a target component.

Definition 17 (Correct Compilation).

\[ \vdash [\_]^{S} : CC \overset{\text{def}}{=} \forall \beta. \exists \beta_{0}. \]

if \[ \Omega_{0} ([\_]^{S}_{T}) \overset{\beta}{\Rightarrow} \Omega \]

\[ \Omega \Downarrow \]

\[ \Omega_{0} (C) \approx_{\beta_{0}} \Omega_{0} ([\_]^{S}_{T}) \]

then \[ \Omega_{0} (C) \overset{\beta}{\Rightarrow} \Omega \]

\[ \beta_{0} \subseteq \beta \]

\[ \Omega \approx_{\beta} \Omega \]

\[ \bar{\pi} \approx_{\beta} \bar{\pi} \]

\[ \Omega \Downarrow \]

Technically, any sequence \(\bar{\pi}\) above is empty, as \(\bar{I}\) is empty (the program is whole).
Definition 18 (Robust Safety Preserving Compilation).
\[ \vdash [\cdot]_T^S : RSC \overset{def}{=} \forall C, M, M. \]
if \( M \vdash C : rs \)
\( M \approx M \)
then \( M \vdash [C]_T^S : rs \)

C.4.1 Alternative definition for RSC
Definition 19 (Property-Free RSC).
\[ \vdash [\cdot]_T^S : PF-RSC \overset{def}{=} \forall C. \]
if \( \forall A, \pi. \)
\[ [C]_T^S \vdash A : attacker \]
\[ \vdash A [C]_T^S : whole \]
\[ \Omega_0 \left( [C]_T^S \right) \overset{\pi}{\Rightarrow} _ \]
then \( \exists A, \pi. \)
\( C \vdash A : attacker \)
\( \vdash A [C] : whole \)
\[ \Omega_0 (C) \overset{\pi}{\Rightarrow} _ \]
relevant(\( \pi \)) \( \approx_\beta \) relevant(\( \pi \))

The property-free characterisation of RSC is equivalent to its original characterisation.

Theorem 7 (PF-RSC and RSC are equivalent).
\[ \forall [\cdot]_T^S, \vdash [\cdot]_T^S : PF-RSC \iff \vdash [\cdot]_T^S : RSC \]

C.4.2 Compiling Monitors
We can change the definition of compiler to also compile the monitor so we are not given a target monitor related to the source one, but the compiler gives us that monitor. Consider this compiler to have this type and this notation:
\[ [\cdot]_T^S : C \rightarrow C. \]

Definition 20 (Robustly-safe Compilation with Monitors).
\[ \vdash [\cdot]_T^S : rs-pres(M) \overset{def}{=} \forall C, M. \]
if \( M \vdash C : rs \)
then \( [M]_T^S \vdash [C]_T^S : rs \)
Definition 21 (Compiler $L^U$ to $L^P$). $[[\text{C}]]_{L^P} : C \rightarrow C$

$[[\ell_{\text{root}}; F; I; M]]_{L^P} = k_{\text{root}}; [[F]]_{L^P}; [[I]]_{L^P}$ if $\ell_{\text{root}} \approx \beta \langle 0, k_{\text{root}} \rangle$

$[[f(x) \rightarrow s; \text{return}]]_{L^P} = f(x) \rightarrow [[s]]_{L^P}; \text{return}$

$[[f]]_{L^P} = f$

**Expressions**

$[[\text{true}]]_{L^P} = 0$ if $\text{true} \approx_{\beta} 0$  

$[[\text{false}]]_{L^P} = 1$ if $\text{false} \approx_{\beta} 1$  

$[[n]]_{L^P} = n$ if $n \approx_{\beta} n$  

$[[x]]_{L^P} = x$  

$[[\ell]]_{L^P} = \langle n, v \rangle$ if $\ell \approx_{\beta} \langle n, v \rangle$  

$[[\langle e_1, e_2 \rangle]]_{L^P} = \langle [[e_1]]_{L^P}, [[e_2]]_{L^P} \rangle$  

$[[e.1]]_{L^P} = [[e]]_{L^P}.1$  

$[[e.2]]_{L^P} = [[e]]_{L^P}.2$  

$[[e \oplus e']]_{L^P} = [[e]]_{L^P}.1$ with $[[e]]_{L^P}.2$  

$[[e \& e']]_{L^P} = [[e]]_{L^P} \& [[e']]_{L^P}$  

$[[e \& e']]_{L^P} = [[e]]_{L^P} \& [[e']]_{L^P}$  

$[[\text{skip}]]_{L^P} = \text{skip}$  

$[[s; s]]_{L^P} = [[s]]_{L^P}; [[s]]_{L^P}$  

$[[\text{let } x = e \text{ in } s]]_{L^P} = \text{let } x = [[e]]_{L^P} \text{ in } [[s]]_{L^P}$  

$[[\text{if } e \text{ then } s_1 \text{ else } s_2]]_{L^P} = \text{if } [[e]]_{L^P} \text{ then } [[s_1]]_{L^P} \text{ else } [[s_2]]_{L^P}$  

$[[\text{let } x = \text{ new } e \text{ in } s]]_{L^P} = \text{let } x_{\text{loc}} = \text{ new } [[e]]_{L^P} \text{ in }$  

$\text{let } x_{\text{cap}} = \text{ hide } x_{\text{loc}} \text{ in }$  

$\text{let } x = \langle x_{\text{loc}}, x_{\text{cap}} \rangle \text{ in } [[s]]_{L^P}$
\[
[x := e']^L_U = \begin{align*}
\text{let } x1 &= x.1 \\
\text{let } x2 &= x.2 \\
x1 &:= [e]^L_U \text{ with } x2
\end{align*}
\text{(}[\_\_\_]^L_U-Assign)}
\]
\[
\text{call f } e^L_U = \text{call f } [e]^L_U \text{ (}[\_\_\_]^L_U-call)}
\]

Note that the case for Rule ([\_\_\_]^L_U-New) only works because we are in a sequential setting. In a concurrent setting an adversary could access \text{xloc} before it is hidden, so the definition would change. See Rule ([\_\_\_]^L_T-New) for a concurrent correct implementation.

\[
\begin{align*}
[[v / x]]^L_U &= \left[ [v]^L_U \right] / x
\end{align*}
\]

Optimisation

We could optimise Rule ([\_\_\_]^L_U-Deref) as follows:

- rename the current expressions except dereferencing to \text{b};
- reform expressions both in \(L^r\) and \(L^P\) as \(e ::= b | \text{let } x = b \text{ in } e | !b\). In the case of \(L^P\) it would be \(\cdots | !b \text{ with } b\).
  
  This allows expressions to compute e.g., pairs and projections.
- rewrite the Rule ([\_\_\_]^L_P-Deref) case for compiling \(!b\) into:
  
  \[
  \text{let } x = [b]^L_P \text{ in } \text{let } x1 = x.1 \text{ in } \text{let } x2 = x.2 \text{ in } !x1 \text{ with } x2.
  \]
- as expressions execute atomically, this would also scale to the compiler for concurrent languages defined in later sections.

We do not use this approach to avoid nonstandard constructs.

D.1 Properties of the \([\_\_\_]^L_U\) Compiler

Theorem 8 (Compiler \([\_\_\_]^L_U\) is \(CC\)). \(\vdash [\_\_\_]^L_U : CC\)

Theorem 9 (Compiler \([\_\_\_]^L_U\) is \(RSC\)). \(\vdash [\_\_\_]^L_U : RSC\)

D.2 Back-translation from \(L^P\) to \(L^U\)

D.2.1 Values Backtranslation

Here is how values are back translated.

\[
\langle \_\_\_ \rangle^L_U : v \rightarrow v
\]

\[
\langle 0 \rangle^L_U = \text{true}
\]
The backtranslation is nondeterministic, as \( \approx \) is not injective. In this case we cannot make it injective (in the next compiler we can index it by types and make it so but here we do not have them). This is the reason why the back-translation algorithm returns a set of contexts, as backtranslating an action that performs call \( f \) \( v \) \( H \) could result in either call \( f \) true \( H \) or call \( f \) 0 \( H \). Now depending on \( f \)'s body, which is the component to be compiled, supplying true or 0 may have different outcomes. Let us assume that the compilation of \( f \), when receiving call \( f \) \( v \) \( H \) does not get stuck. If \( f \) contains if \( x \) then \( s \) else \( s' \), supplying 0 will make it stuck. However, because we generate all possible contexts, we know that we generate also the context that will not cause \( f \) to be stuck. This is captured in Lemma 2 below.

**Lemma 2** (Compiled code steps imply existence of source steps).

\[
\forall \Omega'' \approx_{\beta} \Omega''
\]

\[
\Omega'' \xrightarrow{\alpha?} C, H \triangleright [s]_{L^P}^{L_U} : s' \rho \\
C, H \triangleright [s]_{L^P}^{L_U} : s' \rho \xrightarrow{\alpha!} \Omega'
\]

\[
\{ \alpha? \cdots \} = \{ \alpha? | \alpha? \approx_{\beta} \alpha? \} \\
\{ \rho \cdots \} = \{ \rho | \rho \approx_{\beta} \rho \}
\]

then \( \exists \alpha_j \in \{ \alpha? \cdots \}, \rho_y \in \{ \rho \cdots \}, C_j, H_j, s_j; s'_j \rho'_j \).

\[
\Omega'' \xrightarrow{\alpha?} C_j, H_j \triangleright s_j; s'_j \rho_y \approx_{\beta} C, H \triangleright [s]_{L^P}^{L_U} : s' \rho \\
C_j, H_j \triangleright s_j; s'_j \rho_y \xrightarrow{\alpha!} \Omega'
\]

\[
\alpha! \approx_{\beta} \alpha! \\
\Omega' \approx_{\beta} \Omega'
\]

**D.2.2 Skeleton**

\[
\llbracket \cdot \rrbracket_{L^P}^{L_U} : I \rightarrow F
\]

\[
\llbracket f \rrbracket_{L^U}^{L_P} = f(x) \mapsto \text{incrementCounter(); return}; \\
\llbracket f \rrbracket_{L^U}^{L_P} \text{-fun}
\]

58
Functions call `incrementCounter()` before returning to ensure that when a
returnback is modelled, the counter is incremented right before returning and
not beforehand, as doing so would cause the possible execution of other bac-
translated code blocks. Its implementation is described below.

\[
\langle \langle \cdot \rangle \rangle_{LPU} : I \rightarrow A
\]

\[
\langle \langle T \rangle \rangle_{LPU} = \begin{array}{l}
\ell_i \mapsto 1; \\
\ell_{glob} \mapsto 0 \\
\text{main}(x) \mapsto \text{incrementCounter}(); \text{return}; \\
\text{incrementCounter}() \mapsto \text{see below} \\
\text{register}(x) \mapsto \text{see below} \\
\text{update}(x) \mapsto \text{see below}
\end{array}
\]

\[
\langle \langle f \rangle \rangle_{LPU} \quad \forall f \in I
\]

We assume compiled code does not implement functions `incrementCounter`,
`register` and `update`, they could be renamed to not generate conflicts if they were.
The skeleton sets up the infrastructure. It allocates global locations \(\ell_i\), which
is used as a counter to count steps in actions, and \(\ell_{glob}\), which is used to keep
track of attacker knowledge, as described below. Then it creates a dummy for all
functions expected in the interfaces \(I\) as well as a dummy for the `main`. Dummy
functions return their parameter variable and they increment the global counter
before that for reasons explained later.

**D.2.3 Single Action Translation**

We use the shortcut `ak` to indicate a list of pairs of locations and tag to access
them \(\langle n, \eta \rangle\) that is what the context has access to. We use functions `.loc` to
access obtain all locations of such a list and `.cap` to obtain all the capabilities
(or 0 when \(\eta = \bot\)) of the list.

We use function `incrementCounter` to increment the contents of \(\ell_i\) by one.

\[
\text{incrementCounter}(\ ) \rightarrow \\
\text{let } c = !\ell_i \text{ in let } l = \ell_i \text{ in } l := c + 1
\]

Starting from location \(\ell_g\) we keep a list whose elements are pairs locations-
numbers, we indicate this list as \(L_{glob}\).

We use function `register(\langle \ell, n \rangle)` which adds the pair \(\langle \ell, n \rangle\) to the list \(L_{glob}\).

Any time we use this we are sure we are adding a pair for which no other
existing pair in \(L_{glob}\) has a second projection equal to \(n\). This function can be
defined as follows:

\[
\text{register}(x) \mapsto \\
\text{let } x_1 = x.1 \\
in \text{let } x_n = x.2 \\
in L_{\text{glob}} :: (x_1, x_n)
\]

\(L_{\text{glob}}\) is a list of pair elements, so it is implemented as a pair whose first projection is an element (a pair) and its second projection is another list; the empty list being \(0\). Where :: is a recursive function that starts from \(\ell_{\text{glob}}\) and looks for its last element (i.e., it performs second projections until it hits a \(0\)), then replaces that second projection with \(\langle \langle x_1, x_n \rangle, 0 \rangle\).

**Lemma 3** (*register(\(\ell, n\)) does not add duplicates for \(n\)). For \(n\) supplied as parameter by \(\langle \langle \cdot \rangle \rangle_{L_U} C; H \triangleright \text{register}(\ell, n) \overset{\ell}{\rightarrow} C; H' \triangleright \text{skip}\) and \(\langle _, n \rangle \notin L_{\text{glob}}\).

**Proof.** Simple analysis of Rules \(\langle \langle \cdot \rangle \rangle_{L_U} -\text{call}\) to \(\langle \langle \cdot \rangle \rangle_{L_U} -\text{ret-loc}\). \(\square\)

We use function \(\text{update}(n, v)\) which accesses the elements in the \(L_{\text{glob}}\) list, then takes the second projection of the element: if it is \(n\) it updates the first projection to \(v\), otherwise it continues its recursive call. If it does not find an element for \(n\), it gets stuck.

**Lemma 4** (*update(n, v) never gets stuck*). \(C; H \triangleright \text{update}(n, v) \overset{\ell}{\rightarrow} C; H' \triangleright \text{skip}\) for \(n\) and \(v\) supplied as parameters by \(\langle \langle \cdot \rangle \rangle_{L_U}\) and \(H' = H[\ell \mapsto v / \ell \mapsto _{\_}]\) for \(\ell \approx_{\beta} (n, _)\).

**Proof.** Simple analysis of Rules \(\langle \langle \cdot \rangle \rangle_{L_U} -\text{call}\) and \(\langle \langle \cdot \rangle \rangle_{L_U} -\text{retback}\). \(\square\)

We use the meta-level function \(\text{reachable}(H, v, ak)\) that returns a set of pairs \(\langle n \mapsto v : n, e \rangle\) such that all locations in are reachable from \(H\) starting from any location in \(ak \cup v\) and that are not already in \(ak\) and such that \(e\) is a sequence of source-level instructions that evaluate to \(\ell\) such that \(\ell \approx_{\beta} (n, _)\).

**Definition 22** (*Reachable*).

\[
\text{reachable}(H, v, ak) = \begin{cases} 
\langle n \mapsto v : k, e \rangle & \text{if } n \in \text{reach}(n_{st}, k_{st}, H) \\
\text{where } n_{st} \in v \cup ak.loc \\
\text{and } k_{st} \in k_{\text{root}} \cup ak.cap \\
\text{and } n \mapsto v : k \in H \\
\text{and } H \triangleright e \iff \text{skip with } k \\
\text{and } \forall H. H \approx_{\beta} H \\
H \triangleright \langle e \rangle_{L_U} \overset{\ell}{\rightarrow} \ell \\
\text{and } (\ell, n, k) \in \beta
\end{cases}
\]
Intuitively, \texttt{reachable}(\cdot) finds out which new locations have been allocated by the compiled component and that are now reachable by the attacker (the first projection of the pair, \( n \mapsto v : \eta \)). Additionally, it tells how to reach those locations in the source so that we can \texttt{register}(\cdot) them for the source attacker (the backtranslated context) to access.

In this case we know by definition that \( e \) can only contain one \( \cdot.1 \) or \( \cdot.2 \). The base case for values is as before.

\[
\llangle x \rrangle_{L^p} : e \to e
\]

\[
\llangle e \rrangle_{L^p} = e \\
\llangle e.1 \rrangle_{L^p} = \llangle e \rrangle_{L^p}.1 \\
\llangle e.2 \rrangle_{L^p} = \llangle e \rrangle_{L^p}.2
\]

The next function takes the following inputs: an action, its index, the previous attacker knowledge and the stack of functions called so far. It returns a set of: code, the new attacker knowledge, its heap, the stack of functions called and the function where the code must be put. In the returned parameters, the attacker knowledge, the heap and the stack of called functions serve as input to the next call.

\[
\llangle \cdot \rrangle_{L^p} : \alpha \times n \in \mathbb{N} \times H \times n \times \eta \times f \to \{ s \times n \times \eta \times H \times f \times f \ldots \}
\]

\[
\llangle \text{call f v H?}, n, H_{\text{pre}}, ak, f' \rrangle_{L^p} = \begin{cases} 
\text{if } \ell_i \text{ == n then} \\
\text{incrementCounter()} \\
\text{let } x_1 \text{ = new } v_1 \text{ in register}(\llangle x_1, n_1 \rrangle) \\
\hspace{1cm} \ldots \\
\text{let } x_j \text{ = new } v_j \text{ in register}(\llangle x_j, n_j \rrangle) \\
\text{update}(m_1, u_1) \\
\hspace{1cm} \ldots \\
\text{update}(m_i, u_i) \\
\text{call f v} \\
\text{else skip} \\
\end{cases}
\]

where \( H \setminus H_{\text{pre}} = H_n \)
\[
H_n = n_1 \mapsto v_1 : \eta_1, \ldots, n_j \mapsto v_j : \eta_j
\]

and \( H \cap H_{\text{pre}} = H_c \)
\[
H_c = m_1 \mapsto u_1 : \eta'_1, \ldots, m_1 \mapsto u_1 : \eta'_1
\]

and \( ak' = ak, (n_1, \eta_1), \ldots, (n_j, \eta_j) \)
and $\overline{t} = f^{\overline{f}}$

\[
\langle \langle \text{call } f \text{ v } H!, n, H_{\text{pre}}, ak, \overline{t} \rangle \rangle_{L^P}^{L_U} = \begin{cases} 
\text{if } !\ell_1 == n \text{ then} \\
\quad \text{incrementCounter()}
\end{cases}
\]

\[
\begin{aligned}
&\text{let } l_1 = e_1 \text{ in register } ((l_1, n_1)) \\
&\quad \ldots \\
&\text{let } l_j = e_j \text{ in register } ((l_j, n_j)) \\
&\quad \text{else skip} \\
&\end{aligned}
\]

\[
(\langle \langle \cdot \rangle \rangle_{L^P}^{L_U} - \text{callback-loc})
\]

if $\text{reachable}(H, v, ak) = (n_1 \mapsto v_1 : \eta_1, e_1), \ldots, (n_j \mapsto v_j : \eta_j, e_j)$

and $ak' = ak, (n_1, \eta_1), \ldots, (n_j, \eta_j)$

\[
\langle \langle \text{ret } H!, n, H_{\text{pre}}, ak, f, \overline{t} \rangle \rangle_{L^P}^{L_U} = \begin{cases} 
\text{if } !\ell_1 == n \text{ then} \\
\quad /\!/ \text{no incrementCounter() as explained}
\end{cases}
\]

\[
\begin{aligned}
&\text{let } x_1 = \text{new } v_1 \text{ in register } ((x_1, n_1)) \\
&\quad \ldots \\
&\text{let } x_j = \text{new } v_j \text{ in register } ((x_j, n_j)) \\
&\quad \text{update}(m_1, u_1) \\
&\quad \ldots \\
&\text{update}(m_1, u_1) \\
&\quad \text{else skip} \\
&\end{aligned}
\]

\[
, ak', H, \overline{t}, f
\]

\[
(\langle \langle \cdot \rangle \rangle_{L^P}^{L_U} - \text{retback})
\]

where

$H \setminus H_{\text{pre}} = H_a$

$H_a = n_1 \mapsto v_1 : \eta_1, \ldots, n_j \mapsto v_j : \eta_j$

and $H \cap H_{\text{pre}} = H_c$

$H_c = m_1 \mapsto u_1 : \eta'_1, \ldots, m_1 \mapsto u_1 : \eta'_1$

and $ak' = ak, (n_1, \eta_1), \ldots, (n_j, \eta_j)$

\[
\langle \langle \text{ret } H!, n, H_{\text{pre}}, ak, f, \overline{t} \rangle \rangle_{L^P}^{L_U} = \begin{cases} 
\text{if } !\ell_1 == n \text{ then} \\
\quad \text{incrementCounter()}
\end{cases}
\]

\[
\begin{aligned}
&\text{let } l_1 = e_1 \text{ in register } ((l_1, n_1)) \\
&\quad \ldots \\
&\text{let } l_j = e_j \text{ in register } ((l_j, n_j)) \\
&\quad \text{else skip} \\
&\end{aligned}
\]

\[
(\langle \langle \cdot \rangle \rangle_{L^P}^{L_U} - \text{ret-loc})
\]
if \text{reachable}(H, 0, ak) = \langle n_1 \mapsto v_1 : \eta_1, e_1 \rangle, \ldots, \langle n_j \mapsto v_j : \eta_j, e_j \rangle

and \(ak' = ak, \langle n_1, \eta_1 \rangle, \ldots, \langle n_j, \eta_j \rangle\)

and \(f = f'f'\)

This is the back-translation of functions. Each action is wrapped in an if statement checking that the action to be mimicked is that one (the same function may behave differently if called twice and we need to ensure this). After the if, the counter checking for the action index \(ℓ_i\) is incremented. This is not done in case of a return immediately, but only just before the return itself, so the increment is added in the skeleton already. (there could be a callback to the same function after the return and then we wouldn’t return but execute the callback code instead)

When back-translating a ?-decorated, we need to set up the heap correctly before the call itself. That means calculating the new locations that this action allocated \((H_n)\), allocating them and registering them in the \(L_{glob}\) list via the \(\text{register(·)}\) function. These locations are also added to the attacker knowledge \(ak'\). Then we need to update the heap locations we already know of. These locations are \(H_c\) and as we know them already, we use the \(\text{update(·)}\) function.

When back-translating a !-decorated action we need to calculate what part of the heap we can reach from there, and so we rely on the \(\text{reachable(·)}\) function to return a list of pairs of locations \(n\) and expressions \(e\). We use \(n\) to expand the attacker knowledge \(ak'\) as these locations are now reachable. We use \(e\) to reach these locations in the source heap so that we can \(\text{register}\) them and ensure they are accessible through \(L_{glob}\).

Finally, we use parameter \(\overline{f}\) to keep track of the call stack, so making a call to \(f\) pushes \(f\) on the stack \((f; \overline{f})\) and making a return pops a stack \(f; \overline{f}\) to \(\overline{f}\). That stack carries the information to instantiate the \(f\) in the return parameters, which is the location where the code needs to be allocated.

\[
\langle \langle \cdot \rangle \rangle_{L^P}^{L_P} : \alpha \times n \in \mathbb{N} \times H \times n \times \eta \times f \rightarrow \{s, \overline{f}, \ldots\}
\]

\[
\langle \langle \cdot \rangle \rangle_{L^P}^{L_P} \left( \langle \langle \cdot \rangle \rangle_{L^P}^{L_P} \right) = \emptyset
\]

\[
\langle \langle \alpha, n, H_{pre}, ak, \overline{f} \rangle \rangle_{L^P}^{L_P} = \begin{cases} s, f; s, \overline{f} & | s, ak', H', \overline{f}, f = \langle \langle \alpha, n, H_{pre}, ak, \overline{f} \rangle \rangle_{L^P}^{L_P} \\ s, \overline{f} \in \langle \langle \alpha, n + 1, H', ak', \overline{f} \rangle \rangle_{L^P}^{L_P} \end{cases}
\]

This recursive call ensures the parameters are passed around correctly. Note that each element in a set returned by the single-action back-translation has the same \(ak'\), \(H\) and \(f'\), the only elements that change are in the code \(s\) due to the backtranslation of values. Thus the recursive call can pass those parameters taken from any element of the set.

63
D.2.4 The Back-translation Algorithm $\langle \langle \cdot \rangle \rangle_{LU}^{LP}$

$$\langle \langle \cdot \rangle \rangle_{LU}^{LP} : \mathcal{T} \times \mathcal{T} \rightarrow \{ A \cdots \}$$

$$\langle \langle \mathbf{T}, \mathbf{\pi} \rangle \rangle_{LU}^{LP} = \left\{ \begin{array}{l} A = A_{\text{skel}} \ni \mathbf{s}, \mathbf{t} \\
\text{for all } \mathbf{s}, \mathbf{t} \in \{ \mathbf{s}, \mathbf{t}, \cdots \} \\
\text{where } \{ \mathbf{s}, \mathbf{t}, \cdots \} = \langle \langle \mathbf{\pi}, 1, H_0, \emptyset, \text{main} \rangle \rangle_{LU}^{LP} \\
H_0 = 0 \mapsto 0 : k_{\text{root}} \\
A_{\text{skel}} = \langle \langle \mathbf{T} \rangle \rangle_{LU}^{LP} \end{array} \right\}$$

This is the real back-translation algorithm: it calls the skeleton and joins it with each element of the set returned by the trace back-translation.

$$\ni: A \times \mathbf{s}, \mathbf{t} \rightarrow A$$

$$A \ni \emptyset = A \quad (\langle \langle \cdot \rangle \rangle_{LU}^{LP} - \text{join})$$

$$H; F_1; \cdots; F; \cdots; F_n \ni \mathbf{s}, \mathbf{t}; f = H; F_1; \cdots; F; \cdots; F_n \ni \mathbf{s}, \mathbf{t}$$

$$\text{where } F = f(x) \mapsto s'; \text{return};$$

$$F' = f(x) \mapsto s; s': \text{return};$$

When joining we add from the last element of the list so that the functions we create have the concatenation of if statements (those guarded by the counter on $\ell_i$) that are sorted (guards with a test for $\ell_i = 4$ are before those with a test $\ell_i = 5$).

D.2.5 Correctness of the Back-translation

**Theorem 10** ($\langle \langle \cdot \rangle \rangle_{LU}^{LP}$ is correct).

$$\forall \Omega_0 \left( A \left[ [C]_{LU}^{LP} \right] \xrightarrow{\mathbf{\pi}} \Omega \right)$$

$$\Omega \xrightarrow{\ell} \Omega', \quad \mathbf{T} = \text{names}(A)$$

$$\mathbf{\alpha} \equiv \overline{\mathbf{\alpha}}', \overline{\mathbf{\alpha}}?$$

$$\ell_i; \ell_{\text{glob}} \notin \beta$$

then $\exists A \in \langle \langle \mathbf{I}, \mathbf{\pi} \rangle \rangle_{LU}^{LP}$ such that

$$\Omega_0 (A \mid C) = \mathbf{\pi} \xrightarrow{\mathbf{\pi}} \Omega$$
The back-translation is correct if it takes a target attacker that will reduce to a state together with a compiled component and it produces a set of source attackers such that one of them, that together with the source component will reduce to a related state performing related actions. Also it needs to ensure the step is incremented correctly.

**D.2.6 Remark on the Backtranslation**

Some readers may wonder whether the hassle of setting up a source-level representation of the whole target heap is necessary. Indeed for those locations that are allocated by the context, this is not. If we changed the source semantics to have an oracle that predicts what a `let x = new e in s` statement will return as the new location, we could simplify this. In fact, currently the backtranslation stores target locations in the list $L_{glob}$ and looks them up based on their target name, as it does not know what source name will be given to them. The oracle would obviate this problem, so we could hard code the name of these locations, knowing exactly the identifier that will be returned by the allocator. For the functions to be correct in terms of syntax, we would need to pre-emptively allocate all the locations with that identifier so that their names are in scope and they can be referred to.

However, the problem still persists for locations created by the component, as their names cannot be hard coded, as they are not in scope. Thus we would still require `reach` to reach these locations, `register` to add them to the list and `update` to update their values in case the attacker does so.

Thus we simplify the scenario and stick to a more standard, oracle-less semantics and to a generalised approach to location management in the back-translation.
E  The Source Language: \( L^\tau \)

This is an imperative, concurrent while language with monitors.

Whole Programs \( P ::= \Delta; H; \pi \)

Components \( C ::= \Delta; f; I \)

Contexts \( A ::= H; F; I \)

Interfaces \( I ::= f \)

Functions \( F ::= f(x : \tau) \to s; \text{return}; \)

Operations \( @ ::= + | - \)

Comparison \( \otimes ::= == | < | > \)

Values \( v ::= b \in \{\text{true}, \text{false}\} | n \in \mathbb{N} | \langle v, v \rangle | \ell \)

Expressions \( e ::= x | v | e \oplus e | e \otimes e | !e | \langle e, e \rangle | e.1 | e.2 \)

Statements \( s ::= \text{skip} | s; s | \text{let } x : \tau = e \text{ in } s | \text{if } e \text{ then } s \text{ else } s | x ::= e | \text{let } x = \text{new}_\tau e \text{ in } s | \text{call } f e \)

Types \( \tau ::= \text{Bool} | \text{Nat} | \tau \times \tau | \text{Ref} \tau | \text{UN} \)

Superficial Types \( \phi ::= \text{Bool} | \text{Nat} | \text{UN} | \text{UN} \times \text{UN} | \text{Ref} \text{UN} \)

Eval.Ctxs. \( E ::= [] | e \oplus E | E \otimes n | e \otimes E | E \otimes n | \langle e, E \rangle | \langle E, v \rangle | E.1 | E.2 \)

Heaps \( H ::= \emptyset | H, \ell \mapsto v : \tau \)

Monitors \( M ::= (\{\sigma \cdots\}, \rightsquigarrow, \sigma_0, \Delta, \sigma_c) \)

Mon. States \( \sigma \in \mathcal{S} \)

Mon. Reds. \( \rightsquigarrow ::= \emptyset | \rightsquigarrow; (s, s) \)

Environments \( \Gamma, \Delta ::= \emptyset | \Gamma; (x : \tau) \)

Store Env. \( \Delta ::= \emptyset | \Delta; (\ell : \tau) \)

Substitutions \( \rho ::= \emptyset | \rho[v/x] \)

Processes \( \pi ::= (s)_{\tau} \)

Soups \( \Pi ::= \emptyset | \Pi \parallel \pi \)

Prog. States \( \Omega ::= C, H \triangleright \Pi \)

Labels \( \lambda ::= \epsilon | \alpha \)

Actions \( \alpha ::= \text{call } f v ? | \text{call } f v! | \text{ret } ! | \text{ret } ? \)

Traces \( \overline{\alpha} ::= \emptyset | \overline{\alpha} \cdot \alpha \)

We highlight elements that have changed from \( L^U \).
E.1 Static Semantics of $\text{L}^T$

The static semantics follows these typing judgements.

- $\vdash C : \text{UN}$ Component $C$ is well-typed.
- $C \vdash F : \tau$ Function $F$ takes arguments of type $\tau$ under component $C$.
- $\Delta, \Gamma \vdash \varnothing$ Environments $\Gamma$ and $\Delta$ are well-formed.
- $\Delta \vdash \text{ok}$ Environment $\Delta$ is safe.
- $\tau \vdash \circ$ Type $\tau$ is insecure.
- $\Delta, \Gamma \vdash e : \tau$ Expression $e$ has type $\tau$ in $\Gamma$.
- $C, \Delta, \Gamma \vdash s$ Statement $s$ is well-typed in $C$ and $\Gamma$.
- $C, \Delta, \Gamma \vdash \pi$ Single process $\pi$ is well-typed in $C$ and $\Gamma$.
- $C, \Delta, \Gamma \vdash \Pi$ Soup $\Pi$ is well-typed in $C$ and $\Gamma$.
- $\vdash H : \Delta$ Heap $H$ respects the typing of $\Delta$.
- $\vdash M$ Monitor $M$ is valid.

E.1.1 Auxiliary Functions

We rely on these standard auxiliary functions: $\text{names}(\cdot)$ extracts the defined names (e.g., function and interface names). $\text{fv}(\cdot)$ returns free variables while $\text{fn}(\cdot)$ returns free names (i.e., a call to a defined function). $\text{dom}(\cdot)$ returns the domain of a particular element (e.g., all the allocated locations in a heap). We denote access to the parts of $C$ and $P$ via functions $\text{funs}$, $\text{intfs}$ and $\text{mon}$.

We denote access to parts of $M$ with a dot notation, so $M.\Delta$ means $\Delta$ where $M = (\{\sigma \cdots \}, \rightsquigarrow, \sigma_0, \Delta, \sigma_c)$.

E.1.2 Typing Rules

\[
\begin{align*}
\vdash C & \quad \text{Component $C$ is well-typed.} \\
C \equiv \Delta; \overline{F}; \overline{I} & \quad C \vdash \overline{F} : \text{UN} & \quad \text{names}(\overline{F}) \cap \text{names}(\overline{I}) = \varnothing & \quad \Delta \vdash \text{ok} \quad \vdash C : \text{UN} \\
\end{align*}
\]

\[
\begin{align*}
C \vdash F : \text{UN} & \quad \text{(TL}$^T$-function) \\
F \equiv f(x : \text{UN}) \rightarrow s; \text{return;} & \quad C, \Delta; x : \text{UN} \vdash s \\
C \equiv \Delta; \overline{F}; \overline{I} & \quad \forall f \in \text{fn}(s), f \in \text{dom}(C.funs) \lor f \in \text{dom}(C.intfs) \\
\vdash C : \text{UN} & \quad \Delta, \Gamma \vdash \varnothing
\end{align*}
\]
Notes  Monitor typing just ensures that the monitor is coherent and that it can’t get stuck for no good reason.
E.1.3 UN Typing

Attackers cannot have new, t terms where r is different from UN.

\[
\Delta, \Gamma \vdash \text{UN e : UN}
\]

\[
\begin{align*}
A &= H, F [] \quad & \Delta \vdash \text{UN A} \\
\text{dom}(H) \cap \text{dom}(\Delta) &= \emptyset & \text{dom}(\Delta) \cap (\text{fv}(F) \cup \text{fv}(H)) &= \emptyset
\end{align*}
\]

\[
\begin{align*}
(\text{TUL}^\top\text{-base}) & \quad & \\
(\text{TUL}^\top\text{-true}) & \quad & \\
(\text{TUL}^\top\text{-false}) & \quad & \\
(\text{TUL}^\top\text{-nat}) & \quad & \\
(\text{TUL}^\top\text{-var}) & \quad & \\
(\text{TUL}^\top\text{-loc}) & \quad & \\
(\text{TUL}^\top\text{-op}) & \quad & \\
(\text{TUL}^\top\text{-cmp}) & \quad & \\
(\text{TUL}^\top\text{-sequence}) & \quad & \\
(\text{TUL}^\top\text{-dereference}) & \quad & \\
(\text{TUL}^\top\text{-assign}) & \quad & \\
(\text{TUL}^\top\text{-letin}) & \quad & \\
(\text{TUL}^\top\text{-function-call}) & \quad & \\
(\text{TUL}^\top\text{-new}) & \quad & \\
(\text{TUL}^\top\text{-fork}) & \quad &
\end{align*}
\]

\[
\begin{align*}
C, \Delta, \Gamma \vdash \text{UN s} & \\
(\text{TUL}^\top\text{-skip}) & \quad & \\
\text{(f} \in \text{dom}(C.funs)) \lor (f \in \text{dom}(C.intfs)) & \quad & C, \Delta, \Gamma \vdash \text{UN e : UN}
\end{align*}
\]

\[
\begin{align*}
C, \Delta, \Gamma \vdash \text{UN e : Bool} & \\
(\text{TUL}^\top\text{-if}) & \quad & C, \Delta, \Gamma \vdash \text{UN s}_t \quad C, \Delta, \Gamma \vdash \text{UN s}_e \quad C, \Delta, \Gamma \vdash \text{UN s}
\end{align*}
\]
E.2 Dynamic Semantics of $L^\tau$

Function $\text{mon-care}(\cdot)$ returns the part of a heap the monitor cares for (Rule $L^\tau$-Monitor-related heap). Rules $L^\tau$-Jump-Internal to $L^\tau$-Jump-OUT dictate the kind of a jump between two functions: if internal to the component/attacker, in (from the attacker to the component) or out (from the component to the attacker). Rule $L^\tau$-Plug tells how to obtain a whole program from a component and an attacker. Rule $L^\tau$-Initial State tells the initial state of a whole program. Rule $L^\tau$-Initial-heap produces a heap that satisfies a $\Delta$, initialised with base values. Rule $L^\tau$-Monitor Step tells when a monitor makes a single step given a heap.

\[
\text{mon-care}(\cdot) \quad \text{(L$^\tau$-Monitor-related heap)}
\]
\[
H' = \{ \ell \mapsto v : \tau \mid \ell \mapsto v : \tau \in H \}
\]
\[
\vdash \text{mon-care}(H, \Delta) = H'
\]

\[
\text{Helpers} \quad \text{(L$^\tau$-Jump-Internal)}
\]
\[
\vdash f, f' : \text{internal}
\]
\[
A \equiv H; F[\cdot] \quad C \equiv \Delta; F'; I
\]
\[
\vdash C, F : \text{whole} \quad \Delta \vdash H_0 \quad \text{main}(x : \text{UN}) \rightarrow s; \text{return}; \in F
\]
\[
A[C] = \Delta; H \cup H_0; F; F'; I
\]

\[
\text{Helpers} \quad \text{(L$^\tau$-Jump-IN)}
\]
\[
(f' \in I \land f \in I) \lor (f' \notin I \land f \notin I)
\]
\[
\vdash f, f' : \text{in}
\]
\[
\text{Helpers} \quad \text{(L$^\tau$-Jump-OUT)}
\]
\[
(f' \notin I \land f \in I)
\]
\[
\vdash f, f' : \text{out}
\]

\[
\text{Helpers} \quad \text{(L$^\tau$-Plug)}
\]
\[
\Delta \vdash H_0
\]
\[
\Delta, \ell : \tau \vdash H; \ell \mapsto v : \tau
\]

\[
\text{Helpers} \quad \text{(L$^\tau$-Whole)}
\]
\[
\text{Name} = \emptyset
\]
\[
\text{Name} \subseteq \text{Name}(F) \cup \text{Name}(F')
\]
\[
\vdash C, F : \text{whole}
\]

\[
\text{Helpers} \quad \text{(L$^\tau$-Initial State)}
\]
\[
\text{Name} = \emptyset
\]
\[
\Omega_0(P) = C, H \triangleright \text{call main } 0
\]

\[
\text{Helpers} \quad \text{(L$^\tau$-Initial-heap)}
\]
\[
\Delta \vdash H
\]
\[
\emptyset \vdash v : \tau
\]
\[
\Delta, \ell : \tau \vdash H; \ell \mapsto v : \tau
\]

\[
M; H \rightsquigarrow M'
\]

\[
\text{Helpers} \quad \text{(L$^\tau$-Monitor Step)}
\]
\[
M = (\{\sigma \cdots\}, \sim, \sigma_0, \Delta, \sigma_c) \quad M' = (\{\sigma \cdots\}, \sim, \sigma_0, \Delta, \sigma_f)
\]
\[
(\sigma_c, \sigma_f) \in \sim
\]
\[
\vdash H : \Delta
\]

\[
M; H \rightsquigarrow M'
\]

71
E.2.1 Component Semantics

H \triangleright e \mapsto e' \quad \text{Expression } e \text{ reduces to } e'.

C, H \triangleright \pi \mapsto C', H' \triangleright \pi' \quad \text{Process } \pi \text{ reduces to } \pi' \text{ and evolves the rest accordingly.}

C, H \triangleright \Pi \mapsto C', H' \triangleright \Pi' \quad \text{Soup } \Pi \text{ reduce to } \Pi' \text{ and evolve the rest accordingly.}

\Omega \triangleright \Omega' \quad \text{Program state } \Omega \text{ steps to } \Omega' \text{ emitting trace } \pi.

C, H \triangleright e \mapsto e'
F: \( L^\pi \): Extending \( L^P \) with Concurrency and Informed Monitors

F.1 Syntax

This extends the syntax of Appendix B.1 with concurrency and a memory allocation instruction that atomically hides the new location.

Whole Programs \( P ::= H_0; F; I \)

Components \( C ::= H_0; F; I \)

Statements \( s ::= \cdots | (|| s) | \text{destruct } x = e \text{ as } B \text{ in } s \text{ or } s\)

| let \( x = \text{newhide } e \text{ in } s \)

Patterns \( B ::= \text{nat} | \text{pair} \)

Monitors \( M ::= (\{ \sigma \cdots \}, \rightsquigarrow, \sigma_0, H_0, \sigma_c) \)

Single Process \( \pi ::= (s) \)

Processes \( \Pi ::= \emptyset | \Pi || \pi \)

Prog. States \( \Omega ::= C, H \triangleright \Pi \)

F.2 Dynamic Semantics

Following is the definition of the \( \text{mon-care}(\cdot) \) function for \( L^\pi \).

\[
\text{mon-care}(\cdot)
\]

\[
H' = \{ n \mapsto v : \eta \mid n \in \text{dom}(H_0) \text{ and } n \mapsto v : \eta \in H \}
\]

\[
\text{mon-care}(H, H_0) = H'
\]

Helpers

\[
A \equiv F [\cdot]
\]

\[
C \equiv H_0; F; I
\]

\[
\vdash C, F : \text{whole}
\]

\[
\text{main}(x) \mapsto \text{return}; s \in F
\]

\[
\forall n \mapsto v : k \in H_0, k \in H_0
\]

\[
A [C] = H_0; F; F; I
\]

\[
\text{(L\textsuperscript{\pi}-Initial State)}
\]

\[
\Omega_0(P) = P, H_0 \triangleright \text{call main 0}
\]

\[
M; H \rightsquigarrow M'
\]
F.2.1 Component Semantics

\( C, H \triangleright \Pi \xrightarrow{\sim} C', H' \triangleright \Pi' \) Processes \( \Pi \) reduce to \( \Pi' \) and evolve the rest accordingly.
G Extended Language Properties and Necessities

G.1 Monitor Agreement for \( L^\tau \) and \( L^\pi \)

Definition 23 \((\L^\tau: M \bowtie C)\).

\[
\{(\sigma \cdots), \rightsquigarrow, \sigma_0, \Delta, \sigma_c\} \sim (\Delta; \overline{F}; \overline{I})
\]

A monitor and a component agree if they focus on the same set of locations \( \Delta \).

Definition 24 \((\L^\pi: M \bowtie C)\).

\[
\{(\sigma \cdots), \rightsquigarrow, \sigma_0, \sigma_0, \Delta_0, \sigma_c\} \sim (\Delta_0; \overline{F}; \overline{I})
\]

A monitor and a component agree if they focus on the same set of locations, protected with the same capabilities \( \Delta_0 \).

G.2 Properties of \( L^\tau \)

Definition 25 \((L^\tau \text{ Semantics Attacker})\).

\[
C \vdash_{\text{attacker}} A \overset{\text{def}}{=} \begin{cases} 
\forall \ell \in \text{dom}(C, \Delta), \ell \notin \text{fn}(A) \\
\text{no let } x = \text{new}, e \text{ in } s \text{ in } A \text{ such that } \tau \neq \text{UN}
\end{cases}
\]

This semantic definition of an attacker is captured by typing below, which allows for simpler reasoning.

Definition 26 \((L^\tau \text{ Attacker})\).

\[
C \vdash_{\text{att}} A \overset{\text{def}}{=} C = \Delta; \overline{F}; \overline{I}, \Delta \vdash_{\text{UN}} A
\]

\[
C \vdash_{\text{att}} \pi \overset{\text{def}}{=} \pi = (s)_{s}^{f} \text{ and } f \in C.\text{its}
\]

\[
C \vdash_{\text{att}} \Pi \rightarrow \Pi' \overset{\text{def}}{=} \Pi = \Pi_1 \parallel \pi \parallel \Pi_2 \text{ and } \Pi' = \Pi_1 \parallel \pi' \parallel \Pi_2
\]

The two notions of attackers coincide.

Lemma 5 (Semantics and typed attackers coincide).

\[
C \vdash_{\text{attacker}} A \iff (C \vdash_{\text{att}} A)
\]

Theorem 11 (Typability Implies Robust Safety in \( L^\tau \)).

\[
\forall C, M \quad \text{if } \vdash C : \text{UN} \quad C \bowtie M \quad \text{then } M \vdash C : \text{rs}
\]
G.3 Properties of $L_π$

Definition 27 ($L_π$ Attacker).

\[
C \vdash \text{att } A \overset{\text{def}}{=} C \equiv H_0; \overline{F}; \overline{I}, \forall k \in H_0. k \notin \text{fv}(A)
\]

\[
C \vdash \text{att } \pi \overset{\text{def}}{=} \pi = (s)_{\text{itfs}} \quad \text{and} \quad f \in C. \text{itfs}
\]

\[
C \vdash \text{att } \Pi \rightarrow \Pi' \overset{\text{def}}{=} \Pi = \Pi_1 \parallel \pi \parallel \Pi_2 \quad \text{and} \quad \Pi' = \Pi_1 \parallel \pi' \parallel \Pi_2
\]

\[\text{and} \quad C \vdash \text{att } \pi \quad \text{and} \quad C \vdash \text{att } \pi'\]
H Compiler from $L^\tau$ to $L^\pi$

H.1 Assumed Relation between $L^\tau$ and $L^\pi$ Elements

We can scale the $\approx_\beta$ relation to monitors, heaps, actions and processes as follows.

\[
\begin{align*}
M &\approx M \\
(\text{Ok Mon}) &
M = (\{\sigma \ldots \}, \rightsquigarrow, \sigma_0, H_0, \sigma_c) \\
&\forall \sigma \in \{\sigma \ldots \}, \forall \text{mon-care}(H; \Delta) \approx_\beta \text{mon-care}(H, H_0). \\
&\text{if } \vdash H : \Delta \text{ then } \exists \sigma', (\sigma, \text{mon-care}(H, H_0), \sigma') \in \rightsquigarrow \\
&\beta, \Delta \vdash M \\
(\text{Monitor relation}) &
M = (\{\sigma \ldots \}, \rightsquigarrow, \sigma_0, \Delta, \sigma_c) \\
&\beta_0, \Delta \vdash M \\
&\beta_0 = (\text{dom}(\Delta), \text{dom}(H_0), H_0, \eta) \\
M &\approx M
\end{align*}
\]

\[
\begin{align*}
\Delta &\vdash_\beta H_0 \\
\Delta, H &\vdash v : \tau \\
(\text{Initial-heap}) &
\Delta \vdash H \\
&\Delta, H \vdash_\beta v : \tau \\
&\ell \approx_\beta (n, k) \\
&\Delta, \ell : \tau \vdash_\beta H; n \mapsto v : k \\
\end{align*}
\]

\[
\begin{align*}
(\tau \equiv \text{Bool} \land v \equiv 0) &\lor \\
(\tau \equiv \text{Nat} \land v \equiv 0) &\lor \\
(\tau \equiv \text{Ref} \land v \equiv n' \land n' \mapsto v' : k' \in H \land \ell' \approx_\beta (n', k') \land \ell : \tau \in \Delta, \Delta, H \vdash v' : \tau) &\lor \\
(\tau \equiv \tau_1 \times \tau_2 \land v \equiv (v_1, v_2) \land H \vdash v_1 : \tau_1 \land H \vdash v_2 : \tau_2) &\lor \\
\end{align*}
\]

\[
\begin{align*}
\Pi &\approx_\beta \Pi \\
(\text{Single process relation}) &
\tilde{f} \approx \tilde{f} \\
(\text{Process relation}) &
\Pi \approx_\beta \Pi \\
\Pi &\vdash_\beta \Pi \\
&\Pi \vdash_\beta \Pi \\
\Pi &\parallel_\beta \Pi \\
\end{align*}
\]

H.2 Compiler Definition

Definition 28 (Compiler $L^\tau$ to $L^\pi$). \[ ]^{\text{L}^\tau}_{\text{L}^\pi} : \mathbb{C} \rightarrow \mathbb{C} \]
Given that $C = \Delta; F; I$ if $\vdash C : \text{UN}$ then $\llbracket C \rrbracket_{L^T}$ is defined as follows:

\[
\begin{align*}
\text{(\text{TL}^T\text{-component})} \\
C & \equiv \Delta; F; I \\
\Delta & \vdash F : \text{UN} \\
\text{names}(F) \cap \text{names}(I) & = \emptyset \\
\vdash C : \text{UN} \\
\end{align*}
\]

\[
\llbracket C \rrbracket_{L^T} = \text{H}_0 : \llbracket F \rrbracket_{L^T} : \llbracket I \rrbracket_{L^T} \quad \text{if } \Delta \vdash \beta_0 \text{ H}_0
\]

\[
\begin{align*}
\text{(\text{TL}^T\text{-function})} \\
F & \equiv f(x : \text{UN}) \rightarrow s; \text{return;} \\
C, \Delta; x : \text{UN} & \vdash s \\
\forall f \in \text{fn}(s), f \in \text{dom}(C.\text{funs}) & \quad \forall f \in \text{dom}(C.\text{intfs}) \\
\vdash C : F : \text{UN} \rightarrow \text{UN} \\
\end{align*}
\]

\[
\llbracket F \rrbracket_{L^T} = f(x) \mapsto [C; \Delta; x : \text{UN} \vdash s]_{L^T}; \text{return;} \\
\llbracket f \rrbracket_{L^T} = f 
\]

\[
\text{Expressions}
\]

\[
\begin{align*}
\text{(\text{TL}^T\text{-true})} \\
\Delta, \Gamma & \vdash \text{true} : \text{bool} \\
\llbracket \text{true} \rrbracket_{L^T} & = 0 \quad \text{if } \text{true} \approx_\beta 0
\end{align*}
\]

\[
\text{(\text{TL}^T\text{-false})} \\
\Delta, \Gamma & \vdash \text{false} : \text{bool} \\
\llbracket \text{false} \rrbracket_{L^T} & = 1 \quad \text{if } \text{false} \approx_\beta 1
\]

\[
\text{(\text{TL}^T\text{-nat})} \\
\Delta, \Gamma & \vdash n : \text{Nat} \\
\llbracket n \rrbracket_{L^T} & = n \quad \text{if } n \approx_\beta n
\]

\[
\text{(\text{TL}^T\text{-var})} \\
\Delta, \Gamma & \vdash x : \tau \\
\llbracket x \rrbracket_{L^T} & = x
\]

\[
\text{(\text{TL}^T\text{-loc})} \\
\Delta, \Gamma & \vdash \ell : \tau \\
\llbracket \ell \rrbracket_{L^T} & = \langle n, v \rangle \quad \text{if } \ell \approx_\beta \langle n, v \rangle
\]

\[
\text{(\text{TL}^T\text{-pair})} \\
\Delta, \Gamma & \vdash e_1 : \tau \\
\Delta, \Gamma & \vdash e_2 : \tau' \\
\llbracket \langle e_1, e_2 \rangle \rrbracket_{L^T} & = \llbracket \Delta, \Gamma \vdash e_1 : \tau \rrbracket_{L^T} : \llbracket \Delta, \Gamma \vdash e_2 : \tau' \rrbracket_{L^T}
\]

80
\[
\begin{align*}
\Delta, \Gamma \vdash e : \tau \times \tau' & \quad \frac{}{\Delta, \Gamma \vdash e \cdot 1 : \tau} \quad (\text{TL}^-\text{-proj-1}) \\
\Delta, \Gamma \vdash e : \tau \times \tau' & \quad \frac{}{\Delta, \Gamma \vdash e \cdot 2 : \tau'} \quad (\text{TL}^-\text{-proj-2}) \\
\Delta, \Gamma \vdash e : \tau & \quad \frac{}{\Delta, \Gamma \vdash \text{Ref } \tau} \quad (\text{TL}^-\text{-dereference}) \\
\Delta, \Gamma \vdash e : \tau & \quad \frac{\Delta, \Gamma \vdash e : \tau} {\Delta, \Gamma \vdash \text{Proj } \tau e} \quad (\text{TL}^-\text{-proj-1}) \\
\Delta, \Gamma \vdash e : \text{Nat} & \quad \frac{}{\Delta, \Gamma \vdash e : \text{Nat}} \quad (\text{TL}^-\text{-op}) \\
\Delta, \Gamma \vdash e : \text{Nat} & \quad \frac{\Delta, \Gamma \vdash e : \text{Nat}} {\Delta, \Gamma \vdash e : \text{Nat} \oplus \Delta, \Gamma \vdash e : \text{Nat}} \quad (\text{TL}^-\text{-cmp}) \\
\Delta, \Gamma \vdash e : \tau & \quad \frac{\Delta, \Gamma \vdash \tau \vdash \circ \quad \text{UN}} {\Delta, \Gamma \vdash e : \text{UN}} \quad (\text{TL}^-\text{-coercion}) \\
\Delta, \Gamma \vdash e : \text{Nat} & \quad \frac{}{\Delta, \Gamma \vdash e : \text{Nat}} \quad (\text{TL}^-\text{-skip}) \\
\Delta, \Gamma \vdash e : \tau & \quad \frac{}{\Delta, \Gamma \vdash e : \tau} \quad (\text{TL}^-\text{-new}) \\
\end{align*}
\]

**Statements**

\[
\begin{align*}
\Delta, \Gamma \vdash \text{skip} & \quad \frac{}{\Delta, \Gamma \vdash \text{skip}} \quad (\text{TL}^-\text{-Skip}) \\
\Delta, \Gamma \vdash e : \tau & \quad \frac{\Delta, \delta, \Gamma, x : \text{Ref } \tau \vdash s} {\Delta, \Gamma \vdash \text{let } x = \text{new} \tau \text{ in } s} \quad (\text{TL}^-\text{-New}) \\
\end{align*}
\]
\[
\begin{align*}
\lL^\text{-function-call} & \quad \Delta, \Gamma \vdash \text{call } f \ [\Delta, \Gamma \vdash e : \text{UN}]_{L^\tau}^L \\
\lL^\text{-if} & \quad \Delta, \Gamma \vdash e : \text{Bool} \\
& \quad \begin{cases} 
\Delta, \Gamma \vdash s_1 & \text{if } e \text{ then } s_1 \text{ else } s_2 \\
\Delta, \Gamma \vdash s_2 & \text{else} 
\end{cases} \\
\lL^\text{-sequence} & \quad \begin{cases} 
\Delta, \Gamma \vdash s_1 \quad & \text{let } x_1 \vdash \tau_1 \\
\Delta, \Gamma \vdash s_2 \quad & \text{let } x_2 \vdash \tau_2 \\
\Delta, \Gamma \vdash s_{12} & \text{inlet } x_1 := [\Delta, \Gamma \vdash e : \tau_1]_{L^\tau}^L \\
& \quad \text{with } x_2 \\
\lL^\text{-letin} & \quad \begin{cases} 
\Delta, \Gamma \vdash x : \tau \\
\Delta, \Gamma \vdash e : \tau \\
\Delta, \Gamma \vdash \text{letin } x := e \quad & \text{let } x_1 = x.1 \\
\Delta, \Gamma \vdash \text{let } x_2 := x.2 \\
\end{cases} \\
\lL^\text{-assign} & \quad \Delta, \Gamma \vdash x : \text{Ref } \tau \\
& \quad \begin{cases} 
\Delta, \Gamma \vdash e : \tau \\
\Delta, \Gamma \vdash x := e \\
\end{cases} \\
\lL^\text{-fork} & \quad \begin{cases} 
\Delta, \Gamma \vdash s \quad & \text{C, } \Delta, \Gamma \vdash s \\
\end{cases} \\
\lL^\text{-proc} & \quad \begin{cases} 
\Delta, \Gamma \vdash s \quad & \text{C, } \Delta, \Gamma \vdash (s)_{\tau} \\
\end{cases} \\
\lL^\text{-soup} & \quad \begin{cases} 
\Delta, \Gamma \vdash \pi \quad & \text{C, } \Delta, \Gamma \vdash \pi \\
\end{cases} \\
\lL^\text{-unify} & \quad \Delta, \Gamma \vdash \pi_1 \parallel \pi_2 \\
\end{align*}
\]
We write \texttt{wrong} as a shortcut for a failign expression like \texttt{3 + true}.

The remark about optimisation for \llbracket L\rrbracket in Appendix D is also valid for the Rule (\llbracket L\rrbracket-Deref) case above. As expressions are executed atomically, we are sure that albeit inefficient, dereferencing will correctly succeed.

We can add reference to superficial types and check this dynamically in the source, as we have the heap there. But how do we check this in the target? We only assume that reference must be passed as a pair: location- key from the attacker. Thus the last case of Rule (\llbracket L\rrbracket-Endorse), where we check that we can access the location, otherwise we’d get stuck.
NonAtomic Implementation of New-Hide  We can also implement Rule (\llbracket \tau; \pi \rrbracket_{L^\tau} - New) using non-atomic instructions are defined in Rule (\llbracket \tau; \pi \rrbracket_{L^\tau} - New-nonat) below.

\[
\begin{cases}
\text{(TL^\tau-new)} & \Delta, \Gamma \vdash e : \tau \\
\text{C, \Delta, \Gamma; x : Ref \, \tau \vdash s} & \text{if } \tau = \text{UN} \\
\end{cases}
\]

H.3 Properties of the L^\tau-L^\pi Compiler

**Theorem 12** (Compiler \llbracket \cdot \rrbracket_{L^\tau} is CC). \vdash \llbracket \cdot \rrbracket_{L^\tau} : CC

**Theorem 13** (Compiler \llbracket \cdot \rrbracket_{L^\tau} is RSC). \vdash \llbracket \cdot \rrbracket_{L^\tau} : RSC

H.4 Cross-language Relation \approx_\beta

We define a more lenient relation on states \approx_\beta analogous to \approx_\beta (Rule Related states – Whole) but that ensures that all target locations that are related to secure source ones only vary accordingly: i.e., the attacker cannot change them.

\[
\Omega \approx_\beta \Omega
\]

\[
\begin{array}
\hline
\text{(L^\tau-Secure heap)} & H' = \{\ell \mapsto v : \tau \mid \ell \mapsto v : \tau \in H \text{ and } \tau \not\in \mathcal{C}\} \\
\hline
\text{(L^\tau-Low Location)} & \vdash \text{secure}(H) = H' \\
\hline
\#\ell \in \text{secure}(H) & \ell \approx_\beta \langle n, \_ \rangle \quad n \in \text{dom}(H) \\
\hline
\hline
\text{(L^\tau-High Location)} & \ell \in \text{secure}(H) \\
\hline
\ell \approx_\beta \langle n, k \rangle & n \mapsto _2 : k \in H \\
\hline
\hline
\text{(L^\tau-High Capability)} & \ell \in \text{secure}(H), \ell \approx_\beta \langle n, k \rangle \\
\hline
\ell \approx_\beta \langle n, k \rangle & n \mapsto _2 : k \in H \\
\hline
\end{array}
\]
There is no secure(·) function for the target because they would be all locations that are related to a source location that itself is secure in the source. An alternative is to define secure(·) as all locations protected by a key k but the point of secure(·) is to setup the invariant to ensure the proof hold, so this alternative would be misleading.

Rule $L^\pi$-Low Location tells when a target location is not secure. That is, when there is no secure source location that is related to it. This can be because the source location is not secure or because the relation does not exist, as in order for it to exist the triple must be added to β and we only add the triple for secure locations.

The intuition behind Rule Related states – Secure is that two states are related if the set of locations they monitor is related and then: for any target location $n$ that is high (i.e., it has a related source counterpart $\ell$ whose type is secure and that is protected with a capability $k$ that we call a high capability), then we have: (1) the capability $k$ used to lock it is not in any attacker code; (2) for any target level location $n'$: (2a) either it is locked with a high capability $k$ (i.e., a capability used to hide a high location) thus $n'$ is also high, in which case it is related to a source location $\ell$ and the values $v, v$ they point to are related; or (2b) it is not locked with a high capability, so we can derive that $n'$ is a low location and its content $v$ is not any high capability $k'$.

Lemma 6 (A target location is either high or low).

$$\forall$$

if $H \simeq \beta H$

$$n \mapsto v : \eta \in H$$

then either $H, H \vdash \text{low-loc}(n)$

or $\exists \ell \in \text{dom}(H)$.

$H, H \vdash \text{high-loc}(n) = \ell, \eta$

Proof. Trivial, as Rule $L^\pi$-Low Location and Rule $L^\pi$-High Location are duals. \qed
I The Second Target Language: \( L^I \)

For clarity, we use a pink, italics font for \( L^I \).

I.1 Syntax

Whole Programs \( P ::= H_0; F; I; E \)

Components \( C ::= H_0; F; I; E \)

Contexts \( A ::= F[\cdot] \)

Interfaces \( I ::= f \)

Enclave functions \( E ::= f \)

Functions \( F ::= f(x) \mapsto s; \text{return} \)

Operations \( \odot ::= + | - \)

Comparison \( \otimes ::= == | < | > \)

Values \( v ::= n \in \mathbb{Z} | \langle v, v \rangle | k \)

Expressions \( e ::= x | v | e \odot e | e \otimes e | \langle e, e \rangle | e.1 | e.2 | !e \)

Statements \( s ::= \text{skip} | s; s | \text{let } x = e \text{ in } s | \text{ifz } e \text{ then } s \text{ else } s | \text{call } f \ e \)

\quad | | (\parallel s) | \text{destruct } x = e \text{ as } B \text{ in } s \text{ or } s

\quad | | x := e | \text{let } x = \text{new } e \text{ in } s | \text{let } x = \text{newiso } e \text{ in } s

Patterns \( B ::= \text{nat} | \text{pair} \)

Eval.Ctxs. \( E ::= [\cdot] | e \odot E | E \otimes n | e \otimes E | E \otimes n | e E \)

\quad | | \langle e, E \rangle | \langle E, v \rangle | E.1 | E.2

Heaps \( H ::= \emptyset | H; n \mapsto v \)

Monitors \( M ::= (\{ \sigma \cdot \cdot \cdot \}, \leadsto, \sigma_0, H_0, \sigma_c) \)

Mon. States \( \sigma \in S \)

Mon. Reds. \( \leadsto ::= \emptyset | \leadsto; (s, H, s) \)

Substitutions \( \rho ::= \emptyset | \rho[v/x] \)

Single Process \( \pi ::= (s) \pi \)

Processes \( \Pi ::= \emptyset | \Pi \parallel \pi \)

Prog. States \( \Omega ::= C, H \triangleright \Pi \)

Labels \( \lambda ::= \epsilon | \alpha \)

Actions \( \alpha ::= \text{call } f \ v \ H? | \text{call } f \ v \ H! | \text{ret } H! | \text{ret } H? \)

Traces \( \overline{\alpha} ::= \emptyset | \overline{\alpha} \cdot \alpha \)

I.2 Operational Semantics of \( L^I \)
I.2.1 Component Semantics

Expression $e$ reduces to $e'$.

Processes $\Pi$ reduce to $\Pi'$ and evolve the rest accordingly.

Program state $\Omega$ steps to $\Omega'$ emitting trace $\pi$.

---

C; H; f \triangleright e \rightarrow e'

\((EL^1_{\text{val}})\)

C; H; f \triangleright v \rightarrow v

\((EL^1_{\text{p2}})\)

C; H; f \triangleright \langle v, v' \rangle .1 \rightarrow v'

\((EL^1_{\text{comp}})\)

if $n \otimes n' = \text{true}$ then $n'' = 0$ else $n'' = 1$

C; H; f \triangleright n \otimes n' \rightarrow n''

\((EL^1_{\text{deref-iso}})\)

n \rightarrow v \in H \quad n < 0 \quad C \vdash f : \text{prog}

C; H; f \triangleright \! n \rightarrow v

\((EL^1_{\text{ctx}})\)

C; H; f \triangleright E[e] \rightarrow E[e']
We elide the suffix with the stack of functions when obvious.

\[
\begin{array}{ll}
\text{(EL}\,^1\text{-sequence)} & C, H \triangleright s \xrightarrow{\:\varepsilon\:} C, H \triangleright s \\
\text{(EL}\,^1\text{-step)} & C, H \triangleright s \xrightarrow{\:\lambda\:} C, H \triangleright s' \\
\text{(EL}\,^1\text{-if-true)} & C, H \triangleright \text{if } e \text{ then } s \text{ else } s' \xrightarrow{\:\varepsilon\:} C, H \triangleright s \\
\text{(EL}\,^1\text{-if-false)} & C, H \triangleright \text{if } e \text{ then } s \text{ else } s' \xrightarrow{\:\varepsilon\:} C, H \triangleright s' \\
\text{(EL}\,^1\text{-letin)} & C, H \triangleright \text{let } x = e \text{ in } s \xrightarrow{\:\varepsilon\:} C, H \triangleright s[v/x] \\
\text{(EL}\,^1\text{-new)} & H = H_1; n \rightarrow _\varepsilon C, H \triangleright e \xrightarrow{\:v\:} v \\
\text{(EL}\,^1\text{-isolate)} & C, H \triangleright \text{let } x = n \text{ in } s \xrightarrow{\:\varepsilon\:} C, H[n + 1] \rightarrow v \triangleright s[n + 1/x] \\
\text{(EL}\,^1\text{-assign)} & C, H \triangleright (\text{let } x = \text{newiso } e \text{ in } s) \xrightarrow{\:\varepsilon\:} C, H \triangleright (n - 1) \rightarrow v \triangleright (s[n - 1/x])_\eta \\
\text{(EL}\,^1\text{-assign-iso)} & C, H \triangleright (n := e) \xrightarrow{\:\varepsilon\:} C, H' \triangleright \text{skip} \\
\text{(EL}\,^1\text{-call-internal)} & C, H \triangleright (\text{call } f \ e) \xrightarrow{\:\varepsilon\:} C, H \triangleright (s; \text{return } \in \text{C.funs})_\eta \\
\text{(EL}\,^1\text{-callback)} & C, H \triangleright (\text{call } f \ e) \xrightarrow{\:\varepsilon\:} C, H \triangleright (\text{return } \in \text{F}) \\
\end{array}
\]

88
I.3 Monitor Semantics
\[ H' = \{ n \mapsto v : \eta \mid n \in \text{dom}(H_0) \text{ and } n \mapsto v : \eta \in H \} \]

\[ \text{mon-care}(H, H_0) = H' \]

\[ M; H \leadsto M' \]

\[ \text{mon-step}(H, H_0, M) = (\{ \sigma \cdot \ldots \}, \leadsto, \sigma_0, H_0, \sigma_c) \]

\[ M = (\{ \sigma \cdot \ldots \}, \leadsto, \sigma_0, H_0, \sigma_c) \]

\[ M' = (\{ \sigma \cdot \ldots \}, \leadsto, \sigma_0, H_0, \sigma_f) \]

\[ (s_c, \text{mon-care}(H, H_0), s_f) \in \leadsto \]

\[ M; H \leadsto M' \]

\[ M; \emptyset \leadsto M \]

\[ M; H \cdot H \leadsto M' \]

\[ M; H' \leadsto M'' \]

\[ M; H' \leadsto M'' \]

\[ M; H \leadsto M' \]

\[ \text{valid trace} \]

\[ M; H \leadsto M' \]

\[ M; H \cdot H \leadsto M' \]

\[ M; H \leadsto M' \]

\[ M \vdash \text{mon } H \]

**I.4  Monitor Agreement for \( L^I \)**

**Definition 29 (\( L^I \): M \sim C).**

\[ (\{ \sigma \cdot \ldots \}, \leadsto, \sigma_0, H_0, \sigma_c) \sim (H_0; F; I; E) \]

A monitor and a component agree if they focus on the same set of locations \( H_0 \).

**I.5  Properties of \( L^I \)**

**Definition 30 (\( L^I \) Attacker).**

\[ C \vdash \text{att } A \overset{\text{def}}{=} C = H_0; F; I; E, A = F', \text{names}(F) \cap \text{names}(F') = \emptyset \]

\[ C \vdash \text{att } \pi \overset{\text{def}}{=} \pi = (s)_{f,s} \text{ and } f \in C.\text{itfs} \]

\[ C \vdash \text{att } \Pi \rightarrow \Pi' \overset{\text{def}}{=} \Pi = \Pi_1 \parallel \pi \parallel \Pi_2 \text{ and } \Pi' = \Pi_1 \parallel \pi' \parallel \Pi_2 \]

\[ \text{and } C \vdash \text{att } \pi \text{ and } C \vdash \text{att } \pi' \]

**J  Second Compiler from \( L^\tau \) to \( L^I \)**

For this compiler we need a different partial bijection, which we indicate with \( \varphi \) and its type is \( \ell \times n \). It has the same properties of \( \beta \) listed in Appendix C.3.

The cross-language relation \( \approx \) is unchanged but for the relation of locations, as they are no longer compiled as pairs:

- \( \ell \approx \varphi n \) if \( (\ell, n) \in \varphi \)
Actions relation is unchanged from Rule Call relation etc.
Heaps relation is unchanged (modulo the elision of capabilities) from Rule Heap relation.
Process relation is unchanged from Rule Single process relation etc.
State relation is unchanged from Rule Related states – Whole.
The monitor relation $M \approx M$ is defined as in Rule Monitor relation.
Some auxiliary functions are changed:

\[
\begin{align*}
\Delta \vdash H & \quad \Delta, H \vdash v : \tau \\
(\text{Initial-heap}) & \\
\Delta \vdash H & \quad \Delta, H \vdash v : \tau \\
\ell \approx \varphi & \\
\Delta, \ell : \tau \vdash H; n \mapsto v \\
(\text{Initial-value}) & \\
(\tau \equiv \text{Bool} \land v \equiv 0) \quad \lor \\
(\tau \equiv \text{Nat} \land v \equiv 0) \quad \lor \\
(\tau \equiv \text{Ref} \tau \land v \equiv n' \land n' \mapsto v' \in H \land \ell' \approx \varphi & \\
\Delta, \ell : \tau \vdash H, n \mapsto v' \\
(\tau \equiv \tau_1 \times \tau_2 \land v \equiv (v_1, v_2) \land \Delta, H \vdash v_1 : \tau_1 \land \Delta, H \vdash v_2 : \tau_2) \\
\Delta, H \vdash v : \tau
\end{align*}
\]

\[\Delta \vdash v : \tau \]

\[\]
\[
\begin{align*}
\text{\textbf{(TL'}^-\text{-false})} & \quad \Delta, \Gamma \vdash \text{false} : \text{Boo} \\
& \quad L^T \quad = 1 \quad \text{if } \text{false} \approx_\varphi 1 \quad (\text{\textbf{[]}_{L^T}^-\text{-False}}) \\
\text{\textbf{(TL'}^-\text{-nat})} & \quad \Delta, \Gamma \vdash n : \text{Nat} \\
& \quad L^T \quad = n \quad \text{if } n \approx_\varphi n \quad (\text{\textbf{[]}_{L^T}^-\text{-Nat}}) \\
\text{\textbf{(TL'}^-\text{-var})} & \quad \Delta, \Gamma \vdash x : \tau, x \in \Gamma \\
& \quad L^T \quad = x \quad (\text{\textbf{[]}_{L^T}^-\text{-Var}}) \\
\text{\textbf{(TL'}^-\text{-loc})} & \quad \Delta, \Gamma \vdash \ell : \tau, \ell \in \Delta \\
& \quad L^T \quad = n \quad \text{if } \ell \approx_\varphi n \quad (\text{\textbf{[]}_{L^T}^-\text{-Loc}}) \\
\text{\textbf{(TL'}^-\text{-pair})} & \quad \Delta, \Gamma \vdash e_1 : \tau, \Delta, \Gamma \vdash e_2 : \tau' \\
& \quad L^T \quad = \langle [\Delta, \Gamma \vdash e_1 : \tau]_{L^I}^T, [\Delta, \Gamma \vdash e_2 : \tau']_{L^I}^T \rangle \quad (\text{\textbf{[]}_{L^T}^-\text{-Pair}}) \\
\text{\textbf{(TL'}^-\text{-proj-1})} & \quad \Delta, \Gamma \vdash e : \tau \times \tau', \Delta, \Gamma \vdash e.1 : \tau \\
& \quad L^T \quad = [\Delta, \Gamma \vdash e : \tau \times \tau']_{L^I}^T \cdot 1 \quad (\text{\textbf{[]}_{L^T}^-\text{-P1}}) \\
\text{\textbf{(TL'}^-\text{-proj-2})} & \quad \Delta, \Gamma \vdash e : \tau \times \tau', \Delta, \Gamma \vdash e.2 : \tau' \\
& \quad L^T \quad = [\Delta, \Gamma \vdash e : \tau \times \tau']_{L^I}^T \cdot 2 \quad (\text{\textbf{[]}_{L^T}^-\text{-P2}}) \\
\text{\textbf{(TL'}^-\text{-dereference})} & \quad \Delta, \Gamma \vdash e : \text{Ref } \tau, \Delta, \Gamma \vdash e.1 : \tau \\
& \quad L^T \quad = ![\Delta, \Gamma \vdash e : \text{Ref } \tau]_{L^I}^T \cdot 1 \quad (\text{\textbf{[]}_{L^T}^-\text{-Deref}}) \\
\text{\textbf{(TL'}^-\text{-op})} & \quad \Delta, \Gamma \vdash e : \text{Nat}, \Delta, \Gamma \vdash e' : \text{Nat} \\
& \quad \Delta, \Gamma \vdash e \oplus e' : \text{Nat} \\
& \quad L^T \quad = [\Delta, \Gamma \vdash e : \text{Nat}]_{L^I}^T \oplus [\Delta, \Gamma \vdash e' : \text{Nat}]_{L^I}^T \quad (\text{\textbf{[]}_{L^T}^-\text{-op}}) \\
\text{\textbf{(TL'}^-\text{-cmp})} & \quad \Delta, \Gamma \vdash e : \text{Nat}, \Delta, \Gamma \vdash e' : \text{Nat} \\
& \quad \Delta, \Gamma \vdash e \otimes e' : \text{Bool} \\
& \quad L^T \quad = [\Delta, \Gamma \vdash e : \text{Nat}]_{L^I}^T \otimes [\Delta, \Gamma \vdash e' : \text{Nat}]_{L^I}^T \quad (\text{\textbf{[]}_{L^T}^-\text{-cmp}}) \\
\text{\textbf{(TL'}^-\text{-coercion})} & \quad \Delta, \Gamma \vdash e : \tau, \tau \vdash o \\
& \quad \Delta, \Gamma \vdash e : \text{UN} \\
& \quad L^T \quad = [\Delta, \Gamma \vdash e : \tau]_{L^I}^T \quad (\text{\textbf{[]}_{L^T}^-\text{-Coerce}})
\end{align*}
\]
\[
\begin{align*}
\text{[\text{TL}^*-\text{skip}]} & \quad \vdash_{L^T} \text{skip} & \quad (\text{[\text{TL}^*-\text{Skip}]}) \\
\text{[\text{TL}^*-\text{new}]} & \quad \begin{cases}
\text{let } x = \text{new } \begin{cases}
\Delta, \Gamma \vdash e : \tau \\
\text{in } \begin{cases}
\Delta, \Gamma; x : \text{Ref } \tau \vdash s
\end{cases}
\end{cases}
\text{if } \tau = \text{UN}
\end{cases} & \quad (\text{[\text{TL}^*-\text{New}]}) \\
\text{[\text{TL}^*-\text{function-call}]} & \quad \begin{cases}
\text{call } f \begin{cases}
\Delta, \Gamma \vdash e : \text{UN}
\end{cases}
\text{if } \tau = \text{UN}
\end{cases} & \quad (\text{[\text{TL}^*-\text{call}]}) \\
\text{[\text{TL}^*-\text{if}]} & \quad \begin{cases}
\text{ifz } \begin{cases}
\Delta, \Gamma \vdash e : \text{Bool}
\end{cases}
\text{then } \begin{cases}
\Delta, \Gamma \vdash s_1
\end{cases}
\text{else } \begin{cases}
\Delta, \Gamma \vdash s_2
\end{cases}
\end{cases} & \quad (\text{[\text{TL}^*-\text{If}]}) \\
\text{[\text{TL}^*-\text{sequence}]} & \quad \begin{cases}
\Delta, \Gamma \vdash s_u
\end{cases} & \quad (\text{[\text{TL}^*-\text{Seq}]}) \\
\text{[\text{TL}^*-\text{letin}]} & \quad \begin{cases}
\text{let } x = \begin{cases}
\Delta, \Gamma \vdash e : \tau
\end{cases}
\text{in } \begin{cases}
\Delta, \Gamma; x : \tau \vdash s
\end{cases}
\end{cases} & \quad (\text{[\text{TL}^*-\text{Letin}]}) \\
\text{[\text{TL}^*-\text{assign}]} & \quad \begin{cases}
\Delta, \Gamma \vdash x : \text{Ref } \tau
\end{cases} & \quad (\text{[\text{TL}^*-\text{Assign}]})
\end{align*}
\]
We use \textit{wrong} as before for \textit{wrong}.

\section*{J.1 Properties of the $L^T$-$L^I$ Compiler}

\textbf{Theorem 14} (Compiler $\llbracket \cdot \rrbracket_{L^I}$ is $CC$), $\vdash \llbracket \cdot \rrbracket_{L^I} : CC$
Theorem 15 (Compiler $\llbracket L^I \rrbracket$ is RSC). $\vdash \llbracket L^I \rrbracket : RSC$

J.2 Cross-language Relation $\cong_\varphi$

As before, we define a more lenient relation on states $\cong_\varphi$

\[
\Omega \cong_\varphi \Omega
\]

\[\begin{array}{c}
(\text{L}^I\text{-Low Location}) \\
\not\exists \ell \in \text{secure}(H) \quad \ell \cong_\varphi n \quad n \geq 0
\end{array}\]

\[\vdash H, H \vdash \text{low-loc}(n)\]

\[\begin{array}{c}
(\text{L}^I\text{-High Location})
\ell \in \text{secure}(H) \quad \ell \cong_\varphi n \quad n < 0
\end{array}\]

\[\vdash H, H \vdash \text{high-loc}(n) = \ell\]

(Related states – Secure)
\[\Omega = \Delta; F; F'; I; H \triangleright \Pi \quad \Omega = H_0; F; \llbracket F' \rrbracket_{L^I}; I; E; H \triangleright \Pi \quad \Delta \vdash_\varphi H_0\]
\[\forall n, \ell, \text{ if } H, H \vdash \text{high-loc}(n) = \ell \text{ then } n \mapsto v \in H \text{ and } \ell \mapsto v : \tau \in H \text{ and } v \cong_\varphi v\]

\[\Omega \cong_\varphi \Omega\]

We change the definition of a “high location” to be one that is in the enclave, i.e., whose address is less than 0.

The intuition behind Rule Related states – Secure is that high locations only need to be in sync, nothing is enforced on low locations. Compared to Rule Related states – Secure, we have less conditions because we don’t have to track fine-grained capabilities but just if an address is part of the enclave or not.

♠

Lemma 7 (A L$^I$ target location is either high or low).

\[\forall\]
\[\text{if } H \cong_\varphi H\]
\[n \mapsto v \in H\]

then either $H, H \vdash \text{low-loc}(n)$

or $\exists \ell \in \text{dom}(H)$.

$H, H \vdash \text{high-loc}(n) = \ell$

\[\text{Proof. Trivial, as Rule L}^I\text{-Low Location and Rule L}^I\text{-High Location are duals. }\square\]
K Proofs

K.1 Proof of Theorem 7 (PF-RSC and RSC are equivalent)

Proof. $\Rightarrow$ HP

if $\forall A, \pi. [C]_T^S \vdash A :\text{attacker}$

$\vdash A [C]_T^S :\text{whole} \Omega_0 ([C]_T^S) \xrightarrow{\pi} \Omega$

then $\exists A, \pi C \vdash A :\text{attacker}$

$\vdash A [C] :\text{whole} \Omega_0 (A [C]) \xrightarrow{\pi} \Omega$

relevant($\pi$) $\approx_\beta$ relevant($\pi$)

TH

if $M \approx_\beta M$

$\forall A, \pi. \vdash A [C] :\text{whole}$ if $\Omega_0 (C) \xrightarrow{\pi} \Omega$

then $M \vdash \pi$

then $\forall A, \pi. \vdash A [C]_T^S :\text{whole}$ if $\Omega_0 ([C, M]_T^S) \xrightarrow{\pi} \Omega$

then $M \vdash \pi$

We proceed by contradiction and assume that $M \not\approx_\beta \pi$ while $C \vdash \pi$.

By the relatedness of the traces, by Rules Call relation to Returnback relation we have $H \approx_\beta H$ for all heaps in the traces.

But if the heaps are related and the source steps (by unfolding $M \vdash \pi$), then by point 3.b in Definition 15 ($M \approx M$) we have that the target monitor also steps, so $M \vdash \pi$.

We have reached a contradiction, so this case holds.

$\Leftarrow$ Switch HP and TH from the point above.

Analogously, we proceed by contradiction:

- $\forall A, \pi. \vdash A [C] :\text{whole}$ and $\Omega_0 (A [C]) \xrightarrow{\pi} \Omega$ and relevant($\pi$) $\not\approx_\beta$ relevant($\pi$)

By the same reasoning as above, with the HP we have we obtain $M \vdash \pi$ and $M \vdash \pi$.

Again by 3.b in Definition 15 ($M \approx M$) we know that the heaps of all actions in the traces are related.

Therefore, relevant($\pi$) $\approx_\beta$ relevant($\pi$), which gives us a contradiction. $\square$
K.2 Proof of Theorem 8 (Compiler $L^{u}_L$ is CC)

Proof. The proof proceeds for $\beta_0 = (\ell, 0, k_{\text{root}})$ and then, given that the languages are deterministic, by Lemma 9 (Generalised compiler correctness for $L^{u}_L$) as initial states are related by definition. \hfill \Box

Lemma 8 (Expressions compiled with $L^{u}_L$ are related).

$$\forall$$

if $H \approx_{\beta} H$\hfill

$H \triangleright e\rho \iff \nu$\hfill

then $H \triangleright [e]^{u}_{L^{u}_L} [\rho]^{u}_{L^{u}_L} \iff [\nu]^{u}_{L^{u}_L}$

Proof. This proof proceeds by structural induction on $e$.

Base case: Values

true By Rule ($[.]^{u}_{L^{u}_L}$-True), $[true]^{u}_{L^{u}_L} = 0$.

As $true \approx_{\beta} 0$, this case holds.

false Analogous to the first case by Rule ($[.]^{u}_{L^{u}_L}$-False).

$n \in \mathbb{N}$ Analogous to the first case by Rule ($[.]^{u}_{L^{u}_L}$-nat).

$x$ Analogous to the first case, by Rule ($[.]^{u}_{L^{u}_L}$-Var) and by the relatedness of the substitutions.

$\ell$ Analogous to the first case by Rule ($[.]^{u}_{L^{u}_L}$-Loc).

$\langle \nu, \nu \rangle$ By induction on $\nu$ by Rule ($[.]^{u}_{L^{u}_L}$-Pair) and then it is analogous to the first case.

Inductive case: Expressions

e $\oplus$ $e'$ By Rule ($[.]^{u}_{L^{u}_L}$-op) we have that

$[e \oplus e']^{u}_{L^{u}_L} = [e]^{u}_{L^{u}_L} \oplus [e']^{u}_{L^{u}_L}$

By HP we have that $H \triangleright e\rho \iff n$ and $H \triangleright e'\rho \iff n'$.

By Rule $EL^{u}_L$-op we have that $H \triangleright n \oplus n' \iff n''$.

By IH we have that $H \triangleright [e]^{u}_{L^{u}_L} [\rho]^{u}_{L^{u}_L} \iff [n]^{u}_{L^{u}_L}$ and $H \triangleright [e']^{u}_{L^{u}_L} [\rho]^{u}_{L^{u}_L} \iff [n']^{u}_{L^{u}_L}$.

By Rule $EL^{u}_P$-op we have that $H \triangleright [n]^{u}_{L^{u}_L} \oplus [n']^{u}_{L^{u}_L} \iff [n'']^{u}_{L^{u}_L}$.

So this case holds.

e $\otimes$ $e'$ Analogous to the case above by IH, Rule ($[.]^{u}_{L^{u}_L}$-cmp), Rule $EL^{u}_L$-comp and Rules $EL^{P}_L$-op and $EL^{P}_L$-if-true.
Analogous to the case above by IH twice, Rule $[\llbracket \cdot \rrbracket_{LP}^{L} \text{-Deref})$, Rule $EL^{P} \text{-dereference}$ and Rules $EL^{P} \text{-p1}$, $EL^{P} \text{-p2}$ and $EL^{P} \text{-letin}$ and a case analysis by Rules $EL^{P} \text{-deref-top}$ and $EL^{P} \text{-deref-k}$.

$\langle e, e \rangle$ Analogous to the case above by IH and Rule $[\llbracket \cdot \rrbracket_{LP}^{L} \text{-Pair})$.

e.1 By Rule $([\llbracket \cdot \rrbracket_{LP}^{L} \text{-P1})$ $[\llbracket e \rrbracket_{LP}^{L}] = [\llbracket e \rrbracket_{LP}^{L}.1$.

By HP $H \triangleright e.1 \rho \iff \langle v_1, v_2 \rangle \iff v_1$.

By IH we have that $H \triangleright [\llbracket e \rrbracket_{LP}^{L}]_1 \rho \rightarrow [\llbracket (v_1, v_2) \rrbracket_{LP}^{L}.1$.

By Rule $([\llbracket \cdot \rrbracket_{LP}^{L} \text{-P2})$ we have that $[\llbracket (v_1, v_2) \rrbracket_{LP}^{L}.1 = [\llbracket (v_1) \rrbracket_{LP}^{L}, [v_2]_{LP}^{L}.1$.

Now $H \triangleright \llbracket (v_1) \rrbracket_{LP}^{L}, [v_2]_{LP}^{L}.1 \rightarrow \llbracket (v_1) \rrbracket_{LP}^{L}$.

So this case holds.

e.2 Analogous to the case above by Rule $([\llbracket \cdot \rrbracket_{LP}^{L} \text{-P2})$, Rule $EL^{L} \text{-p2}$ and Rule $EL^{P} \text{-p2}$.

\[\square\]

Lemma 9 (Generalised compiler correctness for $[\llbracket \cdot \rrbracket_{LP}^{L})$).

Proof.

$$\forall \ldots \exists \beta'$$

if $\vdash C : \text{whole}$

$$C = \Delta; \mathcal{F}; 
\llbracket C \rrbracket_{LP}^{L} = k_{\text{root}}: \mathcal{F}; \mathcal{I} = C$$

$$C, H \triangleright s \approx_{\beta} C, H \triangleright [s]_{LP}^{L}$$

$$C, H \triangleright \rho \rightarrow C', H' \triangleright s' \rho'$$

then $C, H \triangleright [s]_{LP}^{L}, [\rho]_{LP}^{L} \rightarrow C', H' \triangleright [s']_{LP}^{L}, [\rho']_{LP}^{L}$

$$C' = k_{\text{root}}: \mathcal{F}; \mathcal{I}$$

$$C, H \triangleright s' \rho' \approx_{\beta'} C, H \triangleright [s']_{LP}^{L}, [\rho']_{LP}^{L}$$

$$\beta \subseteq \beta'$$

The proof proceeds by induction on $C$ and the on the reduction steps.

Base case

skip By Rule $([\llbracket \cdot \rrbracket_{LP}^{L} \text{-Skip}) this case follows trivially.

Inductive
let $x = \text{new } e \text{ in } s$

By Rule $([\ell]_L^P,\text{New})$ $[\text{let } x = \text{new } e \text{ in } s]_L^P =$

let $x_{\text{loc}} = \text{new } [e]_L^P$ in

let $x_{\text{cap}} = \text{hide } x_{\text{loc}}$ in

let $x = \langle x_{\text{loc}}, x_{\text{cap}} \rangle$ in $[s]_L^P$

By HP $H \vdash e \rho \leftrightarrow \nu$

So by Lemma 8 we have HPE: $H \vdash [\beta]_L^P [\rho]_L^P \vdash [\nu]_L^P$ and HPV $\nu \approx_{\beta'} [\nu]_L^P$.

By Rule EL$^P$-alloc: $C; H \vdash \text{let } x = \text{new } e \text{ in } s \overset{\ell}{\rightarrow} C ; H ; \ell \vdash \nu \triangleright s[\ell / x]$.

So by HPE:

$C; H \vdash \text{let } x_{\text{loc}} = \text{new } [e]_L^P$ in

let $x_{\text{cap}} = \text{hide } x_{\text{loc}}$ in

let $x = \langle x_{\text{loc}}, x_{\text{cap}} \rangle$ in $[s]_L^P, \rho$

Rule EL$^P$-new

$\overset{\ell}{\rightarrow} C; H ; n \vdash [\nu]_L^P : \bot \triangleright \text{let } x_{\text{cap}} = \text{hide } x_{\text{loc}}$ in

let $x = \langle x_{\text{loc}}, x_{\text{cap}} \rangle$ in $[s]_L^P, [\rho]_L^P [n / x_{\text{loc}}]$

$\equiv C; H ; n \vdash [\nu]_L^P : \bot \triangleright \text{let } x_{\text{cap}} = \text{hide } n$ in

let $x = \langle n, x_{\text{cap}} \rangle$ in $[s]_L^P, [\rho]_L^P$

Rule EL$^P$-hide

$\overset{\ell}{\rightarrow} C; H ; n \vdash [\nu]_L^P : k \triangleright \text{let } x = \langle n, x_{\text{cap}} \rangle$ in $[s]_L^P, [\rho]_L^P [k / x_{\text{cap}}]$

$\equiv C; H ; n \vdash [\nu]_L^P : k \triangleright \text{let } x = \langle n, k \rangle$ in $[s]_L^P, \rho$

Rule EL$^P$-letin

$\overset{\ell}{\rightarrow} C; H ; n \vdash [\nu]_L^P : k \triangleright [s]_L^P, [\rho]_L^P [\langle n, k \rangle / x]$

Let $\beta' = \beta \cup (\ell, n, k)$.

By definition of $\approx_{\beta'}$ and by $\beta'$ we get HPL $\ell \approx_{\beta'} \langle n, k \rangle$.

By a simple weakening lemma for $\beta$ for substitutions and values applied to HP and HPV we can get HPVB $\nu \approx_{\beta'} [\nu]_L^P$.

As $H \approx_\beta H$ by HP, by a simple weakening lemma get that $H \approx_{\beta'} H$ too and by Rule Heap relation with HPL and HPVB we get $H' \approx_{\beta'} H'$.

We have that $\rho' = \rho[\ell / x]$ and $\rho' = [\rho]_L^P [\langle n, k \rangle / x]$.

So by HPL we get that $\rho' \approx_{\beta'} \rho'$. 

99
let x = e in s
Analogous to the case above by IH, Rule \( [\llbracket L \rrbracket U L P - \text{Seq}] \) and a case analysis on what \( s \) reduces to, either with Rule \( E L U - \text{sequence} \) and Rule \( E L P - \text{sequence} \) or with Rule \( E L U - \text{step} \) and Rule \( E L P - \text{step} \).

\[ x := e' \]
Analogous to the case above by Rule \( [\llbracket L \rrbracket U L P - \text{Assign}] \) and Rule \( E L U - \text{letin} \) and Rule \( E L P - \text{letin} \).

if e then s else s'
Analogous to the case above by IH, Rule \( [\llbracket L \rrbracket U L P - \text{If}] \) and then either Rule \( E L U - \text{if-true} \) and Rule \( E L P - \text{if-true} \) or Rule \( E L U - \text{if-false} \) and Rule \( E L P - \text{if-false} \).

call f e
By Rule \( [\llbracket L \rrbracket U L P - \text{call}] \) \( \llbracket \text{call f e} \rrbracket_{L U} = \text{call f} \llbracket e \rrbracket_{L U} \).

By HP \( H \triangleright e \rho \mapsto v \) and HPR \( v \approx_{\beta} \llbracket v \rrbracket_{L U} \).

We instantiate \( \rho' \) with \( \rho[\llbracket v / x \rrbracket_{L U}] \). We can use IH1 to conclude
\[ \llbracket s; \text{return}; \rho[\llbracket v / x \rrbracket_{L U}] \rrbracket_{f} \approx_{\beta} C, H \triangleright (s; \text{return}; \rho[\llbracket v / x \rrbracket_{L U}])_{T_{\beta}f} \]

As \( \beta' = \beta \), this case holds.
K.3 Proof of Theorem 9 (Compiler $\llbracket \cdot \rrbracket_{LP}$ is RSC)

Proof. HPM: $M \approx_\beta M$

HP1: $M \vdash C : rs$

TH1: $M \vdash \llbracket C \rrbracket_{LP} : rs$

We can state it in contrapositive form as:

HP2: $M \nvdash \llbracket C \rrbracket_{LP} : rs$

TH2: $M \nvdash C : rs$

By expanding the definition of $rs$ in HP2 and TH2, we get

HP21 $\exists A, \alpha. M \vdash A :\text{attacker}$ and either $\nvdash A \llbracket C \rrbracket_{LP} :\text{whole}$ or

HPR T1 ($\Omega_0 (A \llbracket C \rrbracket_{LP}) \triangleright\triangleright_\beta \Omega$ and HPRMT1 $M \nvdash \pi$)

TH21 $\exists A, \alpha. M \vdash A :\text{attacker}$ and either $\nvdash A [C] :\text{whole}$ or TH2 ($\Omega_0 (A [C]) \triangleright\triangleright_\beta _\pi$)

and TH4 $M \nvdash \pi$)

We consider the case of a whole $A$, the other is trivial.

We can apply Theorem 10 ($\llbracket \cdot \rrbracket_{LP}$ is correct) with HPR T1 and instantiate $A$ with a $A$ from $\llbracket \cdot \rrbracket$ and we get the following unfolded HPs

HPRS $\Omega_0 (A [C]) \triangleright\triangleright_\beta \Omega$

HPRel $\pi \approx_\beta \pi$.

So TH3 holds by HPRS.

We need to show TH4

Assume by contradiction HPBOT: the monitor in the source does not fail:

$M \vdash \pi)$

By Rule $\text{L}^u$-valid trace we know that forall $\alpha \in \pi$ such that $\text{relevant}(\alpha) = H$, this holds: HPHR $M; H \rightsquigarrow M'$.

We can expand HPHR by Rule $\text{L}^u$-Monitor Step and get:

HPMR: $(\sigma_c, H', \sigma_f) \in \rightsquigarrow$

for a heap $H' \subseteq H$

By HPM $M \approx_\beta M$ for initial states.

By Definition 16 ($M \approx M$) and the second clause of Definition 15 ($M \underline{\text{RM}}$) with HPMR we know that $M \approx_\beta M$ for the current states.

By the first clause of Definition 15 ($M \underline{\text{RM}}$) we know that

HPM R BI: $(\sigma_c, H, \_ \_ \_) \in \rightsquigarrow \iff (\sigma_c, H, \_ \_ \_) \in \rightsquigarrow$

By HPM R BI with HPMR we know that

HPMRTC: $(\sigma_c, H', \sigma_f) \in \rightsquigarrow$

However, by HPRMT1 and Rule $\text{L}^p$-valid trace we know that

HPNR: $M; H \not\rightsquigarrow$

so we get

HPCON: $\not\exists (\sigma_c, H', \sigma_f) \in \rightsquigarrow$

By HPCON and HPMRTC we get the contradiction, so the proof holds. \qed
K.4 Proof of Lemma 2 (Compiled code steps imply existence of source steps)

Proof. The proof proceeds by induction on \( \alpha! \)

Base case: \( \alpha! \)

By Rule ELP-single we need to prove the silent steps and the \( \alpha! \) action.

\( \epsilon \)

The proof proceeds by analysis of the target reductions.

**Rule ELP-sequence** In this case we do not need to pick and the thesis holds by Rule ELU-sequence.

**Rule ELP-step** In this case we do not need to pick and the thesis holds by Rule ELU-step.

**Rule ELP-if-true** We have: \( H \triangleright [e]_{LP} \rho \triangleleft 0 \)

We apply Lemma 10 (Compiled code expression steps implies existence of source expression steps) and obtain a \( v \approx_\beta 0 \)

By definition we have \( 0 \approx_\beta 0 \) and \( \text{true} \approx_\beta 0 \), we pick the second.

So we have \( H \triangleright e \rho \triangleleft \text{true} \)

We can now apply Rule ELU-if-true and this case follows.

**Rule ELP-if-false** This is analogous to the case above.

**Rule ELP-assign-top** Analogous to the case above.

**Rule ELP-assign-k** This is analogous to the case above but for \( v = \ell \approx_\beta (n, k) \).

**Rule ELP-letin** This follows by Lemma 10 and by Rule ELU-letin.

**Rule ELP-new** This follows by Lemma 10 and by Rule ELU-alloc.

**Rule ELP-hide** By analysis of compiled code we know this only happens after a \textbf{new} is executed.

In this case we do not need to perform a step in the source and the thesis holds.

**Rule ELP-call-internal** This follows by Lemma 10 and by Rule ELU-call-internal.

**Rule ELP-ret-internal** In this case we do not need to pick and the thesis holds by Rule ELU-ret-internal.

\( \alpha! \)

The proof proceeds by case analysis on \( \alpha! \)

**call f v H!** This follows by Lemma 10 (Compiled code expression steps implies existence of source steps) and by Rule ELU-callback.

**ret H!** In this case we do not need to pick and the thesis holds by Rule ELU-return.

Inductive case: This follows from IH and the same reasoning as for the single action above.
Lemma 10 (Compiled code expression steps implies existence of source expression steps).

\[ \forall \]

if \( H \triangleright [e]_{L^P}^{U \rho} \leftrightarrow v \)

and if \( \{ \rho \cdots \} = \{ \rho \mid \rho \approx_\beta \rho \} \)

\( v \approx_\beta v \)

\( H \approx_\beta H \)

then \( \exists \rho_j \in \{ \rho \cdots \} . H \triangleright e \rho_j \leftrightarrow v \)

Proof. This proceeds by structural induction on \( e \).

Base case: true  This follows from Rule (\( [[\_]_{L^P}^{U}\text{-True}] \)).

false  This follows from Rule (\( [[\_]_{L^P}^{U}\text{-False}] \)).

\( n \in \mathbb{N} \)  This follows from Rule (\( [[\_]_{L^P}^{U}\text{-nat}] \)).

\( x \)  This follows from the relation of the substitutions and the totality of \( \approx_\beta \) and Rule (\( [[\_]_{L^P}^{U}\text{-Var}] \)).

\( \langle v, v' \rangle \)  This follows from induction on \( v \) and \( v' \).

Inductive case: \( e \oplus e' \) By definition of \( \approx_\beta \) we know that \( v \) and \( v' \) could be either natural numbers or booleans.

We apply the IH with:

IHV1 \( n \approx_\beta n \)

IHV2 \( n' \approx_\beta n' \)

By IH we get

IHTE1 \( H \triangleright [e]_{L^P}^{U \rho} \leftrightarrow n \)

IHTE2 \( H \triangleright [e']_{L^P}^{U \rho} \leftrightarrow n' \)

IHSE1 \( H \triangleright e \rho_j \leftrightarrow n \)

IHSE2 \( H \triangleright e' \rho_j \leftrightarrow n' \)

By Rule (\( [[\_]_{L^P}^{U}\text{-op}] \)) we have that \( [e \oplus e']_{L^P}^{U} = [e]_{L^P}^{U} \oplus [e']_{L^P}^{U} \).

By Rule \( E_{L^P}^{U}\text{-op} \) with IHTE1 and IHTE2 we have that \( H \triangleright [e]_{L^P}^{U} \oplus [e']_{L^P}^{U} \leftrightarrow n'' \) where IHVT \( n'' = n \oplus n' \) if \( n'' = n \oplus n' \)

This follows from IHVT and IHV1 and IHV2.
e ⊗ e' As above, this follows from IH and Rule ([ ]^u_{LP}-cmp) and Rule El^u_{LP}-comp.

⟨e, e'⟩ As above, this follows from IH and Rule ([ ]^u_{LP}-Pair).

e.1 As above, this follows from IH and Rule ([ ]^u_{LP}-P1) and Rule El^u_{LP}-p1.

e.2 Analogous to the case above.
!
e As above, this follows from IH and Rule ([ ]^u_{LP}-Deref) and Rule El^u_{LP}-dereference but with the hypothesis that e evaluates to a v related to a ⟨n, v⟩.

\[ \]
if ℓᵢ == n then
  incrementCounter()
  let x₁ = new v₁ in register((x₁, n₁))
  ...  
  let xᵣ = new vᵣ in register((xᵣ, nᵣ))
call f v
else skip

As $H_{pre}$ is $\emptyset$, no updates are added.

Given that $ℓᵢ$ is initialised to 1 in Rule ($\langle \cdot \rangle_{L^P_{skel}}$), this code is executed and it generates action $\text{call } f v H$ where $H = ℓ₁ \mapsto v₁; \cdots; ℓᵣ \mapsto vᵣ$ for all $n_i \in \text{dom}(H)$ such that $ℓᵢ \approx_{β} \langle nᵢ, _₁ \rangle$ and:

HPHR $H \approx_{β} H$

By HPHR, Lemma 12 (Backtranslated values are related) and Lemma 2 (Compiled code steps imply existence of source steps) with HPF we get THA, THE and TH1.

By Rule Related states – Secure, THS holds too.

Execution of $\text{incrementCounter()}$ satisfies THC.

**Inductive case:**

We know that (eliding conditions HP that are trivially satisfied):

IHP₁ $Ω₀ \left( A \left[ [C]_{L^P_{U}} \right] \right) \overset{π}{\longrightarrow} Ω' \overset{α₁}{\longrightarrow} Ω'' \overset{α₂}{\longrightarrow} Ω$

And we need to prove:

ITH₁ $Ω₀ \left( \langle I, π₀ \rangle_{L^P_{U}} \left[ C \right] \right) \overset{π}{\longrightarrow} Ω' \overset{α₁}{\longrightarrow} Ω'' \overset{α₂'}{\longrightarrow} Ω$

ITHA $\overline{α₀}α₀? \approx_{β} \overline{α₀}α₀$

ITHS $Ω \approx_{β} Ω$

And the inductive HP is (for $\emptyset \subseteq β'$):

IH-HP1 $Ω₀ \left( A \left[ [C]_{L^P_{U}} \right] \right) \overset{π}{\longrightarrow} Ω'$

IH-TH1 $Ω₀ \left( \langle I, π₀ \rangle_{L^P_{U}} \left[ C \right] \right) \overset{π}{\longrightarrow} Ω'$

IH-THA $\overline{α₀} π \approx_{β'} \overline{α₀} π$

IH-THS $Ω' \approx_{β'} Ω'$

By IH-HP1 and HPF we can apply Lemma 2 (Compiled code steps imply existence of source steps) and so we can apply the IH to get IH-TH1, IH-THA and IH-THS.
We perform a case analysis on $\alpha!$, and show that the back-translated code performs $\alpha!$.

By IH we have that the existing code is generated by Rule (\text{\langle \langle \cdot \rangle \rangle}_{L^U}\text{-listact-i}): \text{\langle \langle \pi, n, H_{pre}, ak, \overline{t} \rangle \rangle}_{L^U}$.

The next action $\alpha!$ produces code according to:

HPF $\text{\langle \langle \alpha!, n, H_{pre}, ak, \overline{t} \rangle \rangle}_{L^U}$.

By Rule (\text{\langle \langle \cdot \rangle \rangle}_{L^U}\text{-join}), code of this action is the first if statement executed.

call f v H! By Rule (\text{\langle \langle \cdot \rangle \rangle}_{L^U}\text{-callback-loc}) this code is placed at function $f$ so it is executed when compiled code jumps there

if $!\ell_i == n$ then
  incrementCounter()
  let $l1 = e_1$ in $\text{register}(\langle l1, n_1 \rangle)$
  ... 
  let $lj = e_j$ in $\text{register}(\langle lj, n_j \rangle)$
  else skip

By IH we have that $\ell_i \mapsto n$, so we get

IHL $\ell_i \mapsto n + 1$

By Definition 22 (Reachable) we have for $i \in 1..j$ that a reachable location $n_i \in \text{dom}(H)$ has a related counterpart in $\ell_i \in \text{dom}(H)$ such that $H \triangleright e_i \implies \ell_i$.

By Lemma 11 ($L^T$ attacker always has access to all capabilities) we know all capabilities to access any $n_i$ are in $ak$.

We use $ak$ to get the right increment of the reach.

ret H! In this case from IHF we know that $\overline{t} = f'\overline{f}$.

This code is placed at $f'$, so we identify the last called function and the code is placed there. Source code returns to $f'$ so this code is executed Rule (\text{\langle \langle \cdot \rangle \rangle}_{L^U}\text{-ret-loc})

if $!\ell_i == n$ then
  incrementCounter()
  let $l1 = e_1$ in $\text{register}(\langle l1, n_1 \rangle)$
  ... 
  let $lj = e_j$ in $\text{register}(\langle lj, n_j \rangle)$
  else skip

This case now follows the same reasoning as the one above.

So we get (for $\beta' \subseteq \beta''$):

HP-AC! $\alpha! \approx_{\beta''} \alpha!$
By IH-THS and Rule Related states – Whole and HP-AC! we get HP-OM2:

HP-OM2: \( \Omega'' \approx_{\beta''} \Omega'' \)

The next action \( \alpha? \) produces code according to:

IHF1 \( \langle \langle \alpha?, n + 1, H_{\text{pre}}', ak', \pi \rangle \rangle_{L^P}^{L^U} \).

We perform a case analysis on \( \alpha? \) and show that the back-translated code performs \( \alpha? \):

\[ \text{ret } H? \]
By Rule \( \langle \langle \cdot \rangle \rangle_{L^U}^{L^P} \)-retback, after \( n \) actions, we have from IHF1 that \( \pi = f'\pi' \) and inside function \( f' \) there is this code:

\[
\begin{align*}
\text{if } &! \ell_i == n \text{ then } \\
&\text{let } x_1 = \text{new } v_1 \text{ in register}(\langle x_1, n_1 \rangle) \\
&\ldots \\
&\text{let } x_j = \text{new } v_j \text{ in register}(\langle x_j, n_j \rangle) \\
&\text{update}(m_1, u_1) \\
&\ldots \\
&\text{update}(m_l, u_l) \\
\text{else skip} \\
\end{align*}
\]

By IHL, \( \ell_i \rightarrow n + 1 \), so the if gets executed.

By definition, forall \( n \in \text{dom}(H) \) we have that \( n \in H_n \) or \( n \in H_c \) (from the case definition).

By Lemma 11 (\( L^? \) attacker always has access to all capabilities) we know all capabilities to access any \( n \) are in \( ak \).

We induce on the size of \( H \); the base case is trivial and the inductive case follows from IH and the following:

\( H_n \): and \( n \) is newly allocated.
In this case when we execute \( C; H' \triangleright \text{let } x_1 = \text{new } v_1 \text{ in register}(\langle x_1, n_1 \rangle) \rightarrow^L C; H'; \ell'' \rightarrow \langle \langle v_1 \rangle \rangle_{L^P}^{L^U} \triangleright \text{register}(\langle \ell'', n_1 \rangle) \) and we create \( \beta'' \) by adding \( \ell'', n, \eta'' \) to \( \beta' \).

By Lemma 3 (register(\( L, n \) does not add duplicates for \( n \)) we have that:

\( C; H'; \ell'' \rightarrow \langle \langle v_1 \rangle \rangle_{L^P}^{L^U} \triangleright \text{register}(\langle \ell'', n_1 \rangle) \rightarrow^L C; H'; \ell'' \rightarrow \langle \langle v_1 \rangle \rangle_{L^P}^{L^U} \triangleright \text{skip} \) and we can lookup \( \ell'' \) via \( n \).

\( H_c \): and \( n \) is already allocated.
In this case \( C; H' \triangleright \text{update}(m_1, u_1) \rightarrow^L C; H' \triangleright \text{skip} \)

By Lemma 4 (update(\( n, v \)) never gets stuck) we know that \( H'' = H'[\ell'' \rightarrow_\_ / \ell'' \rightarrow u_1] \) and \( \ell'' \) such that \( (\ell'', m_1, \eta'') \in \beta' \).

By Lemma 12 (Backtranslated values are related) on the values stored on the heap, let the heap after these reduction steps be \( H \), we can conclude
HPRH $H \approx_{\beta''} H$.

As no other if inside $f$ is executed, eventually we hit its return statement, which by Rule $(\llbracket \cdot \rrbracket_{L^P} -$join and Rule $(\llbracket \cdot \rrbracket_{L^P} -$fun) is `incrementCounter(); return;`. Execution of `incrementCounter()` satisfies THC.

So we have $\Omega'' \xrightarrow{\text{ret}} \Omega$ (by Lemma 12) and with HPRH.

call $f \triangleright H$? Similar to the base case, only with update bits, which follow the same reasoning above.

So we get (for $\beta'' \subseteq \beta$):

HP-AC? $\alpha ? \approx _{\beta} \alpha$?

By IH-THA and HP-AC! and HP-AC? we get ITHA.

Now by Rule Related states – Whole again and HP-AC? we get ITHS and ITH1, so the theorem holds.

\[\square\]

\[\star\]

\textbf{Lemma 11 (L}^T\text{ attacker always has access to all capabilities).}

\[\forall\]

\[\text{if } \llbracket (\alpha, n, H, ak, p) \rrbracket_{L^U}^{L^P} = \{s, ak', H', \overline{p}, f \ldots\}\]

\[k. n \rightarrow v : k \in H\]

then $n \in \text{reach}(ak'.\text{loc}, ak'.\text{cap}, H)$

\textit{Proof.} Trivial case analysis on Rules $(\llbracket \cdot \rrbracket_{L^U} -$ret-loc) to $(\llbracket \cdot \rrbracket_{L^U} -$callback-loc). 

\[\square\]

\[\star\]

\textbf{Lemma 12 (Backtranslated values are related).}

\[\forall v, \beta.\]

\[\llbracket v \rrbracket_{L^U}^{L^P} \approx_{\beta} v\]

\textit{Proof.} Trivial analysis of Appendix D.2.1.

\[\square\]

\[\star\]
K.6 Proof of Theorem 11 (Typability Implies Robust Safety in $L^\tau$)

Proof. $HP1 \vdash C : UN$

$HP2 \ C \vdash M$

$TH M \vdash C : rs$

We expand TH: $\forall A, \pi, M \vdash A$ : attacker and $\vdash A [C] : whole$ if $HPR \Omega_0 (A [C]) \xrightarrow{\pi} _{-}$ then $THM M \vdash \pi$

By definition of $relevant(\pi)$ and by Rule $L^\tau$-valid trace we get a $H$ to induce on.

The base case holds by Rule $L^\tau$-Monitor Step Trace Base.

In the inductive case we are considering $\vdash H \cdot H$ and the IH covers the first part of the trace.

By Lemma 13 ($L^\tau$-$\alpha$ reductions respect heap typing), given that the state generating the action is $C, H \triangleright \Pi$ we know that, $HPH \vdash mon\text{-}care(H, \Delta) : \Delta$

By Rule $L^\tau$-valid trace and by Rule $L^\tau$-Monitor Step we need to show that $\vdash H : \Delta$.

This follows by HPH.

We thus need to prove that the initial steps are related heaps are secure.

By Rule $L^\tau$-Plug we need to show that the heaps constituting the initial heap - both $H$ and $H_0$ - are well typed.

The latter, $\vdash H_0 : \Delta$, holds by Rule $L^\tau$-Plug.

The former holds by definition of the attacker: Rules $TUL^\tau$-base and $TUL^\tau$-loc.

\[\Box\]

K.6.1 Proof of Lemma 5 (Semantics and typed attackers coincide)

Proof. This is proven by trivial induction on the syntax of $A$.

By the rules of Appendix E.1.3, points 1 and 3 follow, point 2 follows from the HP Rule $L^\tau$-Plug.

\[\Box\]

Lemma 13 ($L^\tau$-$\alpha$ reductions respect heap typing).

if $C \equiv \Delta; \cdots$

$\vdash mon\text{-}care(H, \Delta) : \Delta$

$C, H \triangleright \Pi \rho \xrightarrow{\pi} C', H' \triangleright \Pi' \rho'$

then $\vdash mon\text{-}care(H', \Delta) : \Delta$

Proof. The proof proceeds by induction on $\pi$.
**Base case** This trivially holds by HP.

**Inductive case** This holds by IH plus a case analysis on the last action:

- \text{call } f \; \nu? \text{ This holds by Lemma 18 (L}\tau^-? \text{ actions respect heap typing).}
- \text{call } f \; \nu! \text{ This holds by Lemma 19 (L}\tau^-! \text{ actions respect heap typing)}
- \text{ret } ! \text{ This holds by Lemma 19 (L}\tau^-! \text{ actions respect heap typing)}
- \text{ret } ? \text{ This holds by Lemma 18 (L}\tau^-? \text{ actions respect heap typing)}

---

**Lemma 14** (L\tau \text{ An attacker only reaches UN locations}).

\[
\forall \\
\text{if } \ell \mapsto \nu : \text{UN} \in H \\
\text{then } \nexists e \\
H \triangleright e \iff \ell' \\
\ell' \mapsto \nu : \tau \in H \\
\tau \neq \text{UN}
\]

**Proof.** This proof proceeds by contradiction.

Suppose \( e \) exists, there are two cases for \( \ell' \):

- \( \ell' \) was allocated by the attacker:
  
  This contradicts the judgements of Appendix E.1.3.

- \( \ell' \) was allocated by the compiled code:
  
  The only way this was possible was an assignment of \( \ell' \) to \( \ell \), but Rule TL\tau-assign prevents it.

---

**Lemma 15** (L\tau \text{ attacker reduction respects heap typing}).

\[
\text{if } C \equiv \Delta; \cdots \\
C \Gamma_{\text{att}} \Pi \rightarrow \Pi' \\
C, H \triangleright \Pi \rho \xrightarrow{\lambda} C, H' \triangleright \Pi \rho' \\
\text{then } \text{mon-care}(H, \Delta) = \text{mon-care}(H', \Delta)
\]
Proof. Trivial induction on the derivation of \( \Pi \), which is typed with \( \vdash_{\text{UN}} \) and by Lemma 14 (\( L^\tau \) An attacker only reaches \text{UN} locations) has no access to locations in \( \Delta \) or with a type \( \tau \vdash \alpha \).

Lemma 16 (\( L^\tau \) typed reduction respects heap typing).

if \( C \equiv \Delta ; \cdots \)
\( C, \Gamma \vdash s \)
\( C, \Gamma \vdash s' \)
\( \vdash \text{mon-care}(H, \Delta) : \Delta \)
\( C, H \triangleright s \xrightarrow{\Delta} C', H' \triangleright s' \rho' \)
then \( \vdash \text{mon-care}(H', \Delta) : \Delta \)

Proof. This is done by induction on the derivation of the reducing statement.

There, the only non-trivial cases are:

Rule \( TL^\tau\)-new By IH we have that
\( H \triangleright e \rho \xrightarrow{} v \)
So
\( C; H \triangleright \text{let } x = \text{new}_\tau e \text{ in } s \xrightarrow{\ell} C; H\ell \triangleright v : \tau \triangleright s[\ell / x] \)
By IH we need to prove that \( \vdash \text{mon-care}(\ell \triangleright v : \tau, \Delta) : \Delta \)
As \( \ell \notin \text{dom}(\Delta) \), by Rule \( L^\tau\)-Heap-ok-i this case holds.

Rule \( TL^\tau\)-assign By IH we have (HPH) \( H \triangleright e \xrightarrow{} v \)
such that \( \ell : \text{Ref } \tau \) and \( v : \tau \).
So
\( C; H \triangleright x := e \rho \xrightarrow{\ell} C; H' \triangleright \text{skip} \)
where \( [x / \ell] \in \rho \) and
\( H = H_1; \ell \mapsto v' : \tau; H_2 \)
\( H' = H_1; \ell \mapsto v : \tau; H_2 \)
There are two cases

\( \ell \in \text{dom}(\Delta) \) By Rule \( L^\tau\)-Heap-ok-i we need to prove that \( \ell : \text{Ref } \tau \in \Delta \).
This holds by HPH and Rule \( L^\tau\)-Initial State, as the initial state ensures that location  \( \ell \) in the heap has the same type as in \( \Delta \).

\( \ell \notin \text{dom}(\Delta) \) This case is trivial as for allocation.
**Rule TL\(\tau\)-coercion** We have that \(C, \Gamma \vdash e : \tau\) and HPT \(\tau \vdash \circ\).

By IH \(H \triangleright e \leftrightarrow v\) such that \(\vdash \text{mon-care}(H', \Delta) : \Delta\).

By HPT we get that \(\text{mon-care}(H) = \text{mon-care}(H')\) as by Rule \(L\tau\)-Secure heap function \(\text{mon-care}(\cdot)\) only considers locations whose type is \(\tau \not\vdash \circ\), so none affected by \(e\).

So this case by IH.

**Rule TL\(\tau\)-endorse** By Rule EL\(\tau\)-endorse we have that \(H \triangleright e \leftrightarrow v\) and that \(C, H \triangleright \text{endorse } x = e \text{ as } \varphi \text{ in } s \leftrightarrow C, H \triangleright s[v/x]\).

So this holds by IH.

\[\square\]

\[\blacklozenge\]

**Lemma 17** (\(L\tau\) any non-cross reduction respects heap typing).

If \(C \equiv \Delta; \cdots\)

\(\vdash \text{mon-care}(H, \Delta) : \Delta\)

\(C, H \triangleright \Pi_\rho \xrightarrow{\lambda} C', H' \triangleright \Pi'_\rho'\)

Then \(\vdash \text{mon-care}(H', \Delta) : \Delta\)

**Proof.** By induction on the reductions and by application of Rule EL\(L\tau\)-par. The base case follows by the assumptions directly. In the inductive case we have the following:

\(C, H \triangleright \Pi_\rho \xrightarrow{\lambda} C'', H'' \triangleright \Pi''_\rho'' \xrightarrow{\lambda} C', H' \triangleright \Pi'_\rho'\)

This has 2 sub-cases, if the reduction is in an attacker function or not.

\(C \vdash_{\text{att}} \Pi'' \rightarrow \Pi\): this follows by induction on \(\Pi''\) and from IH and Lemma 15 (\(L\tau\) attacker reduction respects heap typing).

\(C \not\vdash_{\text{att}} \Pi'' \rightarrow \Pi\): In this case we induce on \(\Pi''\).

The base case is trivial.

The inductive case is \((s)\tau \parallel \Pi\), which follows from IH and Lemma 16 (\(L\tau\) typed reduction respects heap type)

\[\square\]

\[\blacklozenge\]
Lemma 18 (L^? actions respect heap typing).

if \( C \equiv \Delta; \cdots \)
\[
C, H \triangleright \Pi \rho \xrightarrow{\alpha?} C, H' \triangleright v'
\]
then \( \text{mon-care}(H, \Delta) = \text{mon-care}(H', \Delta) \)

Proof. By Lemma 17 (\( L^? \) any non-cross reduction respects heap typing), and a simple case analysis on \( \alpha? \) (which does not modify the heap). □

Lemma 19 (L^! actions respect heap typing).

if \( C \equiv \Delta; \cdots \)
\[
C, H \triangleright \Pi \rho \xrightarrow{\alpha!} C', H' \triangleright v'
\]
then \( \vdash \text{mon-care}(H, \Delta) : \Delta \)

Proof. By Lemma 17 (\( L^? \) any non-cross reduction respects heap typing) and a simple case analysis on \( \alpha! \) (which does not modify the heap). □

K.7 Proof of Theorem 12 (Compiler \([\cdot]_{L^?} \) is CC)

Proof. By definition initial states have related components, related heaps and well-typed, related starting processes, for \( \beta_0 = (\text{dom}(\Delta), \text{dom}(H_0), H_0, \eta) \) so we have:

\[
\text{HRS } \Omega_0 (C) \approx_{\beta_0} \Omega_0 \left( [C]_{L^?} \right).
\]

As the languages have no notion of internal nondeterminism we can apply Lemma 21 (Generalised compiler correctness for \([\cdot]_{L^?} \)) with HRS to conclude. □

Lemma 20 (Expressions compiled with \([\cdot]_{L^?} \) are related).

\[
\forall \quad \text{if } H \approx_{\beta} H \\
H \triangleright e \rho \rightarrow v \\
\text{then } H \triangleright [e]_{L^?} \xrightarrow{[\rho]_{L^?}} [v]_{L^?}
\]
Proof. The proof is analogous to that of Lemma 8 (Expressions compiled with $llbracket L_p \rrbracket$ are related) as the compilers perform the same steps and expression reductions are atomic. 

Lemma 21 (Generalised compiler correctness for $llbracket L_p \rrbracket$).

\[ \forall \ldots \exists b' \]

if \hspace{1em} $C; \Gamma \vdash \Pi$, \hspace{1em} $\vdash C : \text{whole}$

$C = \Delta; \overline{F}; \overline{I}$

$\beta \subseteq \beta'$

then

\[ \vdash C, H \triangleright [C; \Gamma \vdash \Pi]^{L_p}_{L_p} \Rightarrow C, H' \triangleright [C; \Gamma \vdash \Pi']^{L_p}_{L_p} \]

Proof. This proof proceeds by induction on the typing of $\Pi$ and then of $\pi$.

Base Case $\text{skip}$ Trivial by Rule ($llbracket L_p \rrbracket$-Skip).

Inductive Case

In this case we proceed by induction on the typing of $s$

Inductive Cases Rule $T L_p$-new There are 2 cases, they are analogous.

\[ \tau = \text{UN} \] By HP

$\Gamma \vdash e : \tau$

$H \triangleright e \leftrightarrow v$

$C, H \triangleright \text{let } x = \text{new}_\tau e \text{ in } s \rho \rightarrow C, H, \ell \triangleright v : \tau \triangleright s[\ell / x] \rho$

By Lemma 20 we have:

IHR1 $H \triangleright [\Gamma \vdash e : \tau]^{L_p}_{L_p} \leftrightarrow [\Gamma \vdash v : \tau]^{L_p}_{L_p}$

By Rule ($llbracket L_p \rrbracket$-New) we get

let xo = new $[\Gamma \vdash e : \tau]^{L_p}_{L_p}$

in let x = xo

in $[C, \Gamma; x : \text{Ref } \tau \vdash s]^{L_p}_{L_p}$

So:

$C, H \triangleright \text{let xo } = \text{new} [\Gamma \vdash e : \tau]^{L_p}_{L_p}$

in let x = xo

in $[C, \Gamma; x : \text{Ref } \tau \vdash s]^{L_p}_{L_p}$
$\xrightarrow{} C, H; n \mapsto [[\Gamma \vdash \tau]_{L^\tau} : \perp \triangleright \text{let } \mathbf{x} = \langle n, 0 \rangle$

in $[C, \Gamma; x : \text{Ref } \vdash s]_{L^\tau}$

$\xrightarrow{} C, H; n \mapsto [[\Gamma \vdash \tau]_{L^\tau} : \perp \triangleright [C, \Gamma; x : \text{Ref } \vdash s]_{L^\tau} \langle \langle n, 0 \rangle / \mathbf{x} \rangle$

For $\beta' = \beta \cup (\ell, n, \perp)$, this case holds.

else The other case holds follows the same reasoning but for $\beta' = \beta \cup (\ell, n, k)$ and for $H' = H; n \mapsto [C, \Gamma \vdash \tau]_{L^\tau} : k; k$.

Rule $TL^\tau$-sequence By HP

$\Gamma \vdash s; \Gamma \vdash s'$

$C, H \triangleright s \rho \Rightarrow C', H' \triangleright s'' \rho''$

There are two cases

$s'' = \text{skip}$ Rule $EL^u$-sequence

$C', H' \triangleright \text{skip } \rho''; s' \rho \xrightarrow{} C', H' \triangleright s' \rho$

By IH

$C, H \triangleright [[\Gamma \vdash s]_{L^\tau} [\rho]_{L^\tau} \Rightarrow C', H' \triangleright [[\Gamma \vdash \text{skip}]_{L^\tau} [\rho'']_{L^\tau}}$

By Rule ($[\ell]_{L^\tau}$-Seq)

$[C, \Gamma \vdash s]_{L^\tau} : \langle C, \Gamma \vdash s' \rangle_{L^\tau}$

So

$C, H \triangleright [C, \Gamma \vdash s]_{L^\tau} [\rho]_{L^\tau} : [C, \Gamma \vdash s']_{L^\tau} [\rho']_{L^\tau} \Rightarrow [C', H' \triangleright [[\Gamma \vdash \text{skip}]_{L^\tau} [\rho'']_{L^\tau} : [C, \Gamma \vdash s']_{L^\tau} [\rho']_{L^\tau} \xrightarrow{} C', H' \triangleright [C, \Gamma \vdash s']_{L^\tau} [\rho']_{L^\tau}$

At this stage we apply IH and the case holds.

else By Rule $EL^u$-step we have

$C, H \triangleright s; s' \rho \Rightarrow C', H' \triangleright s''; s'$

This case follows by IH and HPs.

Rule $TL^\tau$-function-call Analogous to the cases above.

Rule $TL^\tau$-letin Analogous to the cases above.

Rule $TL^\tau$-assign Analogous to the cases above.

Rule $TL^\tau$-if Analogous to the cases above.

Rule $TL^\tau$-fork Analogous to the cases above.

Rule $TL^\tau$-coercion By Rule ($[\ell]_{L^\tau}$-Coerce), this follows from IH directly.

Rule $TL^\tau$-endorse This has a number of trivial cases based on Rule ($[\ell]_{L^\tau}$-Endorse) that are analogous to the ones above.
K.8 Proof of Theorem 13 (Compiler $\llbracket \cdot \rrbracket_{\mathit{L}_\tau}$ is $\mathit{RSC}$)

Proof. Given:

- HP1: $M \vdash C : rs$
- HPM: $M \equiv_\beta M$

We need to prove:

- TP1: $M \vdash \llbracket C \rrbracket_{\mathit{L}_\tau} : rs$

We unfold the definitions of $rs$ and obtain:

$$\forall A. M \vdash A : \text{attacker}, \vdash A \llbracket C \rrbracket_{\mathit{L}_\tau} : \text{whole}$$

HPE1: if $\Omega_0 (A \llbracket C \rrbracket_{\mathit{L}_\tau}) = _\approx \_ \Rightarrow \_ \text{ then } M \vdash \text{relevant}(\pi)$

$$\forall A. M \vdash A : \text{attacker}, \vdash A \llbracket C \rrbracket_{\mathit{L}_\tau} : \text{whole}$$

THE1: if $\text{HPTR} \Omega_0 (A \llbracket C \rrbracket_{\mathit{L}_\tau}) = _\approx \_ \Rightarrow \_ \text{ then } \text{THE1 } M \vdash \text{relevant}(\pi)$

By definition of the compiler we have that

HPISR: $\Omega_0 (A \llbracket C \rrbracket_{\mathit{L}_\tau}) \approx_\beta \Omega_0 (A \llbracket C \rrbracket_{\mathit{L}_\tau})$

for $\beta = \text{dom}(\Delta), H_0$ such that $M = (\{\sigma \cdots\} , \sim, \sigma_0, \Delta, \sigma_c)$ and $M = (\{\sigma \cdots\} , \sim, \sigma_0, H_0, \sigma_c)$

By $\text{relevant}(\pi)$ and Rule $\mathit{L}_\tau$-valid trace we get a $\pi$ to induce on.

Base case: this holds by Rule $\mathit{L}_\tau$-Monitor Step Trace Base.

Inductive case: By Rule $\mathit{L}_\tau$-Monitor Step Trace, $M; \pi \leadsto M'$ holds by IH, we need to prove $M''; H \leadsto M'$.

By Rule $\mathit{L}_\tau$-Monitor Step e need to prove that THMR: $\exists \sigma'. (\sigma, \text{mon-care}(H, H_0), \sigma') \in \leadsto$.

By HPISR and with applications of Lemmas 23 and 24 we know that states are always related with $\approx_\beta$ during reduction.

So by Lemma 25 ($\approx_\pi$ implies relatedness of the high heaps) we know that HPHH $\text{mon-care}(H, \Delta) \approx_\beta \text{mon-care}(H, H_0)$, for $H, H$ being the last heaps in the reduction.

By HPM and Rule Monitor relation we have $\beta_0, \Delta \vdash M$.

By this and Rule Ok Mon we have that HPHR $\forall \text{mon-care}(H, \Delta) \approx_\beta \text{mon-care}(H, H_0)$.

if $\vdash H : \Delta$ then $\exists \sigma'. (\sigma, \text{mon-care}(H, H_0), \sigma') \in \leadsto$ so by HPHH we can instantiate this with $H$ and $H$.

By Theorem 11 (Typability Implies Robust Safety in $\mathit{L}_\tau$) applied to HPE1, as $\llbracket \cdot \rrbracket_{\mathit{L}_\tau}$ operates on well-typed components, we know that HPMR: $M \vdash \text{relevant}(\pi)$ for all $\pi$.

So by Rule $\mathit{L}_\tau$-Monitor Step with HPMT we get HPHD $\vdash H : \Delta$ for the $H$ above.

By HPHD with HPHR we get THMR $\exists \sigma'. (\sigma, \text{mon-care}(H, H_0), \sigma') \in \leadsto$, so this case holds.

□
Lemma 22 ($\equiv_\beta$ implies relatedness of the high heaps).

\[
\text{if } \Omega = \Delta; \Gamma, \Gamma', \gamma; H \triangleright \Pi \\
\Omega = H_0; \Gamma, \llbracket \Gamma \rrbracket_{L^\tau}; \gamma; H \triangleright \Pi \\
\Omega \approx_\beta \Omega
\]

then \text{mon-care}(H, \Delta) \approx_\beta \text{mon-care}(H, H_0)

Proof. By point 2a in Rule Related states – Secure.

Lemma 23 ($L^\tau$-compiled actions preserve $\approx_\beta$).

\[
\forall...
\text{if } C, H \triangleright \Pi \triangleright \Pi' \rho'
\]

\[
C, H \triangleright \llbracket C; \Gamma \vdash \Pi \rrbracket_{L^\tau} \llbracket \rho \rrbracket_{L^\tau} \quad \triangleleft \quad C, H' \triangleright \llbracket C; \Gamma \vdash \Pi' \rrbracket_{L^\tau} \llbracket \rho' \rrbracket_{L^\tau}
\]

\[
C, H \triangleright \Pi \rho \parallel C, H \triangleright \llbracket C; \Gamma \vdash \Pi \rrbracket_{L^\tau} \llbracket \rho \rrbracket_{L^\tau}
\]

\[
C, \Gamma \vdash \Pi
\]

then \[
C, H' \triangleright \Pi' \rho' \parallel_\beta C, H' \triangleright \llbracket C; \Gamma \vdash \Pi' \rrbracket_{L^\tau} \llbracket \rho' \rrbracket_{L^\tau}
\]

Proof. Trivial induction on the derivation of $\llbracket \cdot \rrbracket$, analogous to Lemma 21 (Generalised compiler correctness for $\llbracket \cdot \rrbracket$).

Rule $T_{L^\tau}$-New There are 2 cases, they are analogous.

$\tau = \text{UN}$ By HP

\[
\Gamma \vdash e : \tau \\
H \triangleright e \mapsto v
\]

$C, H \triangleright \text{let } x = \text{new}_\tau e \text{ in } s \rho \mapsto C, H; \ell \mapsto v : \tau \triangleright s[\ell / x] \rho$

By Lemma 20 (Expressions compiled with $\llbracket \cdot \rrbracket_{L^\tau}$ are related) we have:

IHR1 $H \triangleright \llbracket \Gamma \vdash e : \tau \rrbracket_{L^\tau} \llbracket \rho \rrbracket_{L^\tau} \quad \leftrightarrow \quad \llbracket \Gamma \vdash v : \tau \rrbracket_{L^\tau}$

By Rule ($\llbracket \cdot \rrbracket_{L^\tau}$-New) we get

let xo = new $\llbracket \Gamma \vdash e : \tau \rrbracket_{L^\tau}$

in let x = (xo, 0)

in $\llbracket C, \Gamma; x : \text{Ref } \tau \vdash s \rrbracket_{L^\tau}$

So:

\[
C, H \triangleright \text{let } xo = \text{new } \llbracket \Gamma \vdash e : \tau \rrbracket_{L^\tau}
\]

in let x = (xo, 0)

in $\llbracket C, \Gamma; x : \text{Ref } \tau \vdash s \rrbracket_{L^\tau}$
$\xrightarrow{\sim} C, H; n \mapsto \llbracket \Gamma \vdash v : \tau \rrbracket_{L^\tau} \downarrow \triangleright \text{let } x = \langle n, 0 \rangle$

in $\llbracket C, \Gamma; x : \text{Ref } \tau \rrbracket_{L^\tau} = \llbracket C, \Gamma; \vdash s \rrbracket_{L^\tau}[(n, 0) / x]$

For $\beta' = \beta$, this case holds.

else The other case holds follows the same reasoning but for $\beta' = \beta \cup (\ell, n, k)$ and for $H' = H; n \mapsto \llbracket C, \Gamma \vdash v : \tau \rrbracket_{L^\tau} : k; k$.

We need to show that this preserves Rule Related states – Secure, specifically it preserves point $(2a)$: $\ell \approx_{\beta} \langle n, k \rangle$ and $\ell \mapsto v : \tau \in H$ and $v \approx_{\beta} v$

These follow all from the observation above and by Lemma 20 (Expressions compiled with $\llbracket . \rrbracket_{L^\tau}$ are related).

\[\square\]

\textbf{Lemma 24} ($L^P$ Attacker actions preserve $\approx_{\beta}$).

\begin{align*}
\forall \ldots \\
&\text{if } C, H \triangleright \Pi^\rho \xrightarrow{\lambda} C, H' \triangleright \Pi'^{\rho'} \\
&\quad \quad C, H \triangleright \Pi^\rho \xrightarrow{\lambda} C, H' \triangleright \Pi'^{\rho'} \\
&\quad \quad C, H \triangleright \Pi^\rho \approx_{\beta} C, H \triangleright \Pi^\rho \\
&\quad \quad C \vdash \Pi^\rho \xrightarrow{\lambda} \Pi'^{\rho'} \\
&\quad \quad C \vdash \Pi^\rho \xrightarrow{\lambda} \Pi'^{\rho'} \\
&\text{then } C, H' \triangleright \Pi'^{\rho'} \approx_{\beta} C, H' \triangleright \Pi'^{\rho'}
\end{align*}

\textit{Proof.} For the source reductions we can use Lemma 17 ($L^\tau$ any non-cross reduction respects heap typing) to know that $\text{mon-care}(H) = \text{mon-care}(H')$, so they don’t change the interested bits of the $\approx_{\beta}$.

Suppose this does not hold by contradiction, there can be three clauses that do not hold based on Rule Related states – Secure:

- violation of $(1)$: $\exists \pi \in \Pi. C \vdash \pi : \text{attacker}$ and $k \in \text{fv}(\pi)$.

  By HP5 this is a contradiction.

- violation of $(2a)$: $n \mapsto v : k \in H$ and $\ell \approx_{\beta} \langle n, k \rangle$ and $\ell \mapsto v : \tau \in H$ and $\neg(v \approx_{\beta} v)$

  To change this value the attacker needs $k$ which contradicts points $(1)$ and $(2b)$.

- violation of $(2b)$: either of these:
Since Rule Lπ-High Location does not hold, by Lemma 6 this is a contradiction.

- \( v = k' \) for \( H, H \vdash \text{high-cap}(k') \)
  This can follow from another two cases
  * forgery of \( k \): an inspection of the semantics rules contradicts this
  * update of a location to \( k' \): however \( k' \) is not in the code (contradicts point (1)) and by induction on the heap \( H \) we have that \( k' \) is stored in no other location, so this is also a contradiction.

\[\square\]

\[\spadesuit\]

### K.9 Proofs for the Non-Atomic Variant of Lτ (Appendix H.2)

The only proof that needs changing is that for Lemma 23: there is this new case.

For this we weaken \( \cong_\beta \) and define \( \sim_\beta \) as follows:

\[
\begin{align*}
\Omega \sim_\beta \Omega
\end{align*}
\]

Two states are now related if:

- either they are related by \( \cong_\beta \)

- or the red process is stuck on a hide \( n \) where \( n \mapsto v \mapsto k \) but \( \ell \sim \langle n, k \rangle \) does not hold for a \( \ell \) that is secure, and we have that \( \ell \sim \langle n, 0 \rangle \) (as this was after the new). And the hide on which the process is stuck is not in attacker code.

Having this in proofs would not cause problems because now all proofs have an initial case analysis whether the state is stuck or not, but because it steps it’s not stuck.

This relation only changes the second case of the proof of Lemma 23 for Rule (\([\llbracket \cdot \rrbracket]_{L^\tau} - \text{New-nonat}\)) as follows:
Proof. `new` is implemented as defined in Rule ([LL]_τ^{Lπ}-New-nonat).

τ ≠ UN By HP

Γ ⊢ e : τ

H ⊢ e →→ v

C, H ⊢ let x = new_τ e in sρ →→ C, H ; ℓ ↦→ v : τ ↦→ s[ℓ / x]ρ

By Lemma 20 we have:

IHR1 H ⊢ [Γ ⊢ e : τ]^{Lτ}_{Lπ}; [ρ]^{Lτ}_{Lπ} →→ [Γ ⊢ v : τ]^{Lτ}_{Lπ}

By Rule ([L]^{Lτ}_{Lπ}-New-nonat) we get

let x = new 0 in
let xk = hide x in
let xc = [Δ, Γ ⊢ e : τ]^{Lτ}_{Lπ} in
x := xc with xk;

[C, Δ, Γ ⊢ s]^{Lτ}_{Lπ}

So:

C, H ⊢ let x = new 0 in
let xk = hide x in
let xc = [Δ, Γ ⊢ e : τ]^{Lτ}_{Lπ} in
x := xc with xk;

[C, Δ, Γ ⊢ s]^{Lτ}_{Lπ}

And β' = β ∪ (ℓ, n, 0).

Now there are two cases:

- A concurrent attacker reduction performs hide n, so the state changes.

C, H, n ↦→ 0 : k; k ⊢ let xk = hide n in
let xc = [C, Γ ⊢ e : τ]^{Lτ}_{Lπ} in
x := xc with xk;

[C, Δ, Γ ⊢ s]^{Lτ}_{Lπ}

At this stage the state is stuck: Rule EL^{P}-hide does not apply.
Also, we have that this holds by the new $\beta'$: $(\ell \sim_{\beta'} (n, 0))$
And so this does not hold: $(\ell \sim_{\beta'} (n, k))$
As the stuck statement is not in attacker code, we can use Rule Non Atomic State Relation -stuck to conclude.

\begin{itemize}
  \item The attacker does not. In this case the proof continues as in Lemma 23.
\end{itemize}

\section{Proof of Theorem 14 (Compiler $[\cdot]_{LI}^{LT}$ is $CC'$\textsuperscript{1})}

\begin{proof}
Analogous to that of Appendix K.7.
\end{proof}

\section{Proof of Theorem 15 (Compiler $[\cdot]_{LI}^{LT}$ is $RSC$\textsuperscript{1})}

\begin{proof}
Given:
\begin{align*}
  \text{HP1: } & M \vdash C : rs \\
  \text{HPM: } & M \approx_{\varphi} M
\end{align*}
We need to prove:
\begin{align*}
  \text{TPI: } & M \vdash [C]_{LI}^{LT} : rs \\
  \text{We unfold the definitions of } & rs \text{ and obtain: } \\
  \forall A. M \vdash A : attacker, \vdash A [C] : whole
\end{align*}
HPE1: if $\Omega_{0} (A | C) \xrightarrow{\pi} _{\varphi}$ then $M \vdash \text{relevant}(\pi)$
\begin{align*}
  \forall A. M \vdash A : attacker, \vdash A [C]_{LI}^{LT} : whole
\end{align*}
THE1: if $\text{HPRT } \Omega_{0} \left( A \left[ C \right]_{LI}^{LT} \right) \xrightarrow{\pi} _{\varphi}$ then $\text{THE1 } M \vdash \text{relevant}(\pi)$

By definition of the compiler we have that
\begin{align*}
  \text{HPISR: } & \Omega_{0} (A | C) \approx_{\varphi} \Omega_{0} \left( A \left[ C \right]_{LI}^{LT} \right)
\end{align*}
for $\varphi = \text{dom} (\Delta)$, $H_{0}$ such that $M = (\{\sigma \cdots \}, \leadsto, \sigma_{0}, \Delta, \sigma_{c})$ and $M = (\{\sigma \cdots \}, \leadsto, \sigma_{0}, H_{0}, \sigma_{c})$
By $\text{relevant}(\pi)$ and Rule $L^{I}$-valid trace we get a $\mathcal{H}$ to induce on.

\textbf{Base case:} this holds by Rule $L^{I}$-Monitor Step Trace Base.

\textbf{Inductive case:} By Rule $L^{I}$-Monitor Step Trace, $M; H \leadsto M''$ holds by IH, we need to prove $M''; H \leadsto M'$.

By Rule $L^{I}$-Monitor Step we need to prove that $\text{THMR } \exists \sigma'. (\sigma, \text{mon-care}(H, H_{0}), \sigma') \in \leadsto$.

By HPISR and with applications of Lemmas 26 and 27 we know that states are always related with $\approx_{\varphi}$ during reduction.

So by Lemma 25 ($\approx_{\varphi}$ implies relatedness of the high heaps) we know that $\text{HPHH mon-care}(H, \Delta) \approx_{\varphi} \text{mon-care}(H, H_{0})$, for $H, H$ being the last heaps in the reduction.

121
By HPM and Rule Monitor relation (adjusted for $L^I$) we have $\phi_0, \Delta \vdash M$.

By this and Rule Ok Mon (adjusted for $L^I$) we have that

$$\forall \text{mon-care}(H, \Delta) \approx_{\phi} \text{mon-care}(H, H_0).$$

if $\vdash H : \Delta$ then

$$\exists \sigma', (\sigma, \text{mon-care}(H, H_0), \sigma') \in \sim$$

so by HPHH we can instantiate this with $H$ and $H$.

By Theorem 11 (Typability Implies Robust Safety in $L^\tau$) applied to HPE1, as $\llbracket \cdot \rrbracket_{L^\pi}$ operates on well-typed components, we know that HPMR: $M \vdash \text{relevant}(\alpha)$ for all $\alpha$.

So by Rule $L^\tau$-Monitor Step with HPMT we get $\exists \sigma'. (\sigma, \text{mon-care}(H, H_0), \sigma') \in \sim$, so this case holds.

\[\square\]

\[\blacktriangledown\]

\textbf{Lemma 25} ($\equiv\phi$ implies relatedness of the high heaps).

$$\begin{align*}
\text{if } \Omega &= \Delta; F; F'; \Gamma; H \triangleright \Pi \\
\Omega &= H_0; F; \llbracket F' \rrbracket_{L^I}; \Gamma; E; H \triangleright \Pi \\
\Omega &\equiv_{\phi} \Omega \\
\text{then } \text{mon-care}(H, \Delta) &\approx_{\phi} \text{mon-care}(H, H_0)
\end{align*}$$

\textit{Proof.} By Rule Related states – Secure. \[\square\]

\[\blacktriangledown\]

\textbf{Lemma 26} ($L^\tau$-compiled actions preserve $\equiv_{\phi}$).

$$\forall \ldots$$

if

$$\begin{align*}
C, H \triangleright \Pi \rho &\xrightarrow{\lambda} C, H' \triangleright \Pi' \rho' \\
C, H \triangleright \llbracket C; \Gamma \vdash \Pi \rrbracket_{L^I} \llbracket \rho \rrbracket_{L^I} &\xrightarrow{\lambda} C, H' \triangleright \llbracket C; \Gamma \vdash \Pi' \rrbracket_{L^I} \llbracket \rho' \rrbracket_{L^I} \\
C, H \triangleright \Pi \rho &\equiv_{\phi} C, H \triangleright \llbracket C; \Gamma \vdash \Pi \rrbracket_{L^I} \llbracket \rho \rrbracket_{L^I} \\
C; \Gamma \vdash \Pi &\equiv_{\phi} C; \Gamma \vdash \Pi \rho
\end{align*}$$

then

$$\begin{align*}
C, H' \triangleright \Pi' \rho' &\equiv_{\phi} C, H' \triangleright \llbracket C; \Gamma \vdash \Pi' \rrbracket_{L^I} \llbracket \rho' \rrbracket_{L^I}
\end{align*}$$
Proof. Trivial induction on the derivation of $\Pi$, analogous to Lemma 21 (Generalised compiler correctness for \llbracket \cdot \rrbracket$ and Lemma 23 ($L^\tau$-compiled actions preserve $\approx_\beta$).

\[ \blacksquare \]

Lemma 27 ($L^P$ Attacker actions preserve $\approx$).

\[ \forall \ldots \]
\[ \text{if } C, H \gg \Pi \rho \xrightarrow{\lambda} C, H' \gg \Pi' \rho' \]
\[ C, H \gg \Pi \rho \xrightarrow{\lambda} C, H' \gg \Pi' \rho' \]
\[ C, H \gg \Pi \rho \approx_\varphi C, H \gg \Pi \rho \]
\[ C \vdash_{\text{att}} \Pi \rho \xrightarrow{\lambda} \Pi' \rho' \]
\[ C \vdash_{\text{att}} \Pi \rho \xrightarrow{\lambda} \Pi' \rho' \]
\[ \text{then } C, H' \gg \Pi' \rho' \approx_\varphi C, H' \gg \Pi' \rho' \]

Proof. For the source reductions we can use Lemma 17 ($L^\tau$ any non-cross reduction respects heap typing) to know that $\text{mon-care}(H) = \text{mon-care}(H')$, so they don’t change the interested bits of the $\approx_\varphi$.

Suppose this does not hold by contradiction, there can be one clause that does not hold based on Rule Related states – Secure:

- two related high-locations $\ell$ and $n$ point to unrelated values.

Two cases arise: creation and update of a location to an unrelated value. Both cases are impossible because Rule EL$^1$-assign-iso and Rule EL$^1$-isolate check $C \vdash f : \text{prog}$ and Rule L$^1$-Whole ensures that the attacker defines different names from the program, so the attacker can never execute them.

\[ \blacksquare \]
A Fully Abstract Compiler from $L^U$ to $L^P$

We make these changes to both $L^U$ and $L^P$ which only simplify the technical development for this section:

- we make functions defined in components to be private except for those exported via the exports list ($E$);
- we take that all parameters of functions are pointers.

### L.1 The Source Language $L^U$

In $L^U$ we need to add a functionality to get the size of a heap, as that is an observable that exists in the target. In fact, in the target if one allocates something, that reveals how much it’s been allocated entirely.

\[
\begin{align*}
\text{Components } & C ::= F; I; E \\
\text{Exports } & E ::= f \\
\text{Expressions } & e ::= \cdots \mid \text{size} \\
\end{align*}
\]

\[
\begin{align*}
\{L^U\text{-Size}\} \\
\|H\| = n \\
H \triangleright \text{size} \leftrightarrow n
\end{align*}
\]

#### Helpers

\[
\begin{align*}
& (L^U\text{-Jump-Internal}) \\
& ( (f' \in I \land f \in I) \lor (f' \in E \land f \in E)) \quad \text{I;} E \vdash f, f': \text{internal} \\
& (L^U\text{-Jump-IN}) \\
& f \in I \land f' \notin E \quad \text{I;} E \vdash f, f': \text{in} \\
& (L^U\text{-Jump-OUT}) \\
& f \in E \land f' \in I \\
& \text{I;} E \vdash f, f': \text{out} \\
& (L^U\text{-Plug}) \\
& A \equiv \text{H;} F[\cdot] \quad C \equiv \text{F}; I; E \quad \vdash C, F : \text{whole} \\
& \forall f \in E, f \notin \text{fn}(F) \\
& \text{A}[C] = \text{H;} F; F'; I; E \\
& (L^U\text{-Whole}) \\
& C \equiv \text{F}; I; E \\
& \text{names}(F) \cap \text{names}(F') = \emptyset \\
& \text{names}(F) \subseteq \text{names}(F) \cup \text{names}(F') \\
& \text{fv}(F) \cup \text{fv}(F') = \emptyset \\
& \vdash C, F : \text{whole} \\
& (L^U\text{-Initial State}) \\
& P \equiv \text{H;} F; I; E \\
& C \equiv \text{F}; I; E \\
& \Omega_0(P) = C; H \triangleright \text{call main 0}
\end{align*}
\]

The semantics is unchanged, it only relies on the new helper functions above.
L.2 The Target Language $L^p$

L.2.1 Syntax Changes

Components $C ::= F; I; E; k_{\text{root}}, k_{\text{com}}$

Exports $E ::= f$

Expressions $e ::= \cdots | \text{isloc}(e) | \text{ispair}(e) | \text{eqcap}(e, e)$

Trace states $\Theta ::= (C; H; \pi \triangleright (t)_{\triangleright})$

Trace bodies $t ::= s | \text{unk}$

Trace labels $\delta ::= \varepsilon | \beta$

Trace actions $\beta ::= \text{call } f v H? | \text{call } f v H! | \text{ret } H! | \text{ret } H? | \downarrow | \uparrow | \text{write}(v, n)$

Traces $\mathcal{B} ::= \emptyset | \mathcal{B} \cdot \mathcal{B}$

We assume programs are given two capabilities they own: $k_{\text{root}}$ and $k_{\text{com}}$ and that the attacker does not have. The former is used to create a part of the heap for component-managed datastructures. The latter does not even hide a location, we need it as a placeholder.

Traces in this case have the same syntactic structure as before, but they do not carry the whole heap. So we use a different symbol ($\mathcal{B}$), to visually distinguish between the two traces and the kind of information carried by them.

We need a write label $\text{write}(v, n)$ that tells that masked location $n$ has been updated to value $v$. This captures the usage of compiler-inserted functions to read and write masked locations (concepts that will be clear once the compiler is defined). The read label is not needed because its effect are captured anyway by call/return.

Trace states are either operational semantics states or an unknown state, mimicking the execution in a context. The former has an additional element $\bar{n}$, the list of locations shared with the context. The latter carries the information about the component and the heap comprising the one private to the component and the one shared with the context. It also carries the stack of function calls, where we add symbol $\text{unk}$ to indicate when the called function was in the context.

Helper functions are as above.

L.2.2 Semantics Changes

In $L^p$ we need functionality to tell if a pair is a location or not and to traverse values in order to extract such locations.

$$(H \triangleright e \leftrightarrow (n, v) \quad n \rightarrow _{\cdot} \eta \in H \quad \eta = v \text{ or } \eta = \bot) \Rightarrow b = \text{true}$$

otherwise $b = \text{false}$

$H \triangleright \text{isloc}(e) \leftrightarrow b$
These are used to traverse the value stored at a location and extract all sublocations stored in there. There may be pairs containing pairs etc, and thus when we need to know if something is a pair before projecting out. Also, we need to know if a pair is a location or not, in order to know whether or not we can dereference it.

Additionally, we need a functionality to tell if two capabilities are the same. Now, this could be problematic because it could reveal capability allocation order and thus introduce observations that we do not want. However, the compiler will ensure that the context only receives $k_{\text{com}}$ as a capability and never a newly-allocated capability. So the context will not be able to test equality of capabilities generated by the compiled component as it will effectively see only one.

\[
\begin{align*}
(L^P\text{-ispair}) \\
H \triangleright e & \leftrightarrow \langle v, v \rangle \Rightarrow b = \text{true} \\
\text{otherwise } b & = \text{false} \\
H \triangleright \text{ispair}(e) & \leftrightarrow b
\end{align*}
\]

L.2.3 A Fully Abstract Trace Semantics for $L^P$

\[
\begin{align*}
\Theta \xrightarrow{\beta} \Theta' & \quad \text{State } \Theta \text{ emits visible action } \beta \text{ becoming } \Theta'. \\
\Theta \xrightarrow{\beta} \Theta' & \quad \text{State } \Theta \text{ emits trace } \beta \text{ becoming } \Theta'.
\end{align*}
\]

\[
\begin{align*}
(H \triangleright e & \leftrightarrow k) \quad H \triangleright e' \leftrightarrow k & \\
L^P\text{-eqcap-true} & \\
H \triangleright \text{eqcap}(e, e') & \leftrightarrow \text{true} \\
H \triangleright e & \leftrightarrow k \quad H \triangleright e' \leftrightarrow k' \quad k \neq k' & \\
L^P\text{-eqcap-false} &
\end{align*}
\]

\[
\begin{align*}
(\text{Reachable}) & \\
H \vdash \text{reachable}(n, H') & \quad n \in \text{reach}(n_{st}, k_{st}, H) \quad n_{st} \mapsto \_ : \_ \in H' \\
k_{st} \in k_{\text{root}} \cup H' & \\
H & = H_{\text{priv}} \cup H_{\text{sha}} \quad H' = H_{\text{priv}} \cup H'_{\text{sha}} \cup H_{\text{new}} \\
H'' = H'_{\text{sha}} \cup H_{\text{new}} & \\
\text{dom}(H) = \pi & \\
\text{dom}(H'') = \pi' & \\
\forall k \in H_{\text{sha}} \iff k \in H'_{\text{sha}} & \\
\forall k' \in H_{\text{new}}, k' \notin H_{\text{priv}} \cup H_{\text{sha}} & \\
\forall n \mapsto v; \eta \in H_{\text{sha}}, n \mapsto v''; \eta \in H'_{\text{sha}} \land \vdash \text{valid}(v'', H) & \\
\forall n' \mapsto v'; \eta' \in H_{\text{new}}, \vdash \text{valid}(v', H_{\text{priv}} \cup H'_{\text{sha}}) & \\
H'' \vdash \text{reachable}(v', H_{\text{priv}} \cup H'_{\text{sha}}) & \\
\vdash \text{validHeap}(H, H', H'', \pi, \pi')
\end{align*}
\]
\( \varnothing \xrightarrow{\tau} \Theta' \)

\[
\begin{array}{c}
\text{(L^P-Traces-Silent)} \\
(C; H; \overline{\pi} \triangleright (s) \xrightarrow{\cdot} (C; H'; \overline{\pi} \triangleright (s'))_{\overline{\tau}}) \\
\text{(L^P-Traces-Call)} \\
C = F; I; E \quad f \in E \quad f(x) \mapsto s; \text{return}; \in F \\
\overline{f} = \overline{f}, f \\
\vdash \text{validHeap}(H, H', H', \overline{\pi}, \overline{\tau}) \\
\end{array}
\]

\[
\begin{array}{c}
(C; H; \overline{\pi} \triangleright (s; \overline{\tau}) \xrightarrow{\text{call } f \ v \ H''} (C; H'; \overline{\pi} \triangleright (s; \overline{\tau}')) \\
\text{(L^P-Traces-Returnback)} \\
\overline{f} = \overline{f}, f \\
\vdash \text{validHeap}(H, H'', H', \overline{\pi}, \overline{\tau}') \\
\end{array}
\]

\[
\begin{array}{c}
(C; H; \overline{\pi} \triangleright (s; \overline{\tau}) \xrightarrow{\text{ret } H'} (C; H'; \overline{\pi} \triangleright (s; \overline{\tau}')) \\
\text{(L^P-Traces-Callback)} \\
s = \text{call } f \ e \\
C = F; I; E \\
\overline{f} = \overline{f}, f \\
\pi \subseteq \overline{\pi}' = \{ n \mid H \vdash \text{reachable}(n, H) \} \\
H' = H_{\overline{\tau'}} \\
\end{array}
\]

\[
\begin{array}{c}
(C; H; \overline{\pi} \triangleright (s; \overline{\tau}) \xrightarrow{\text{ret } H} (C; H; \overline{\pi} \triangleright (s; \overline{\tau}')) \\
\text{(L^P-Traces-Terminate)} \\
(C; H; \overline{\pi} \triangleright (s; \overline{\tau}) \xrightarrow{\cdot} )_2 \\
\end{array}
\]

\[
\begin{array}{c}
\forall n. \quad (C; H; \overline{\pi} \triangleright (s; \overline{\tau}) \xrightarrow{\cdot} n (C; H'; \overline{\pi} \triangleright (s'; \overline{\tau}')) \\
\text{(L^P-Traces-Diverge)} \\
\end{array}
\]

\[
\begin{array}{c}
(C; H; \overline{\pi} \triangleright (s; \overline{\tau}) \xrightarrow{\cdot} (C; H; \overline{\pi} \triangleright (s; \overline{\tau}')) \\
\text{(L^P-Traces-Write)} \\
C = F; I; E \\
\text{write} \in E \\
\text{write}(x) \mapsto s; \text{return}; \in F \\
\end{array}
\]

\[
\begin{array}{c}
C = H; s n / x ; \text{return}; \rightarrow \star C; H' \triangleright \text{return}; \\
\end{array}
\]

\[
\begin{array}{c}
(C; H; \overline{\pi} \triangleright (s; \overline{\tau}) \xrightarrow{\text{write}(v, n)} (C; H'; \overline{\pi} \triangleright (s; \overline{\tau}')) \\
\text{(L^P-Traces-Read)} \\
\end{array}
\]

\[
\begin{array}{c}
C = F; I; E \\
\text{read} \in E \\
\text{read}(x) \mapsto s; \text{return}; \in F \\
\end{array}
\]

\[
\begin{array}{c}
\end{array}
\]
L.2.4 Results about the Trace Semantics

The following results hold for $C_1 = \langle C_1 \rangle_{L_p}$ and $C_2 = \langle C_2 \rangle_{L_p}$.

**Property 1** (Heap locations). As mentioned, the trace semantics carries the whole shared heap: locations created by the compiled component and then passed to the context and locations created by the context and passed to the compiled component. We can really partition the heap as follows then:

| location \ creator | \ $\theta_0 (C)$ \ $\Rightarrow$ \ $\Rightarrow$ _ | A |
|---------------------|---------------------------------|-----|
| private             | (1) to $\langle C \rangle_{L_p}$ | (2) to A |
| shared              | (3) with A                       | (4) with $\langle C \rangle_{L_p}$ |

Now, for compiled components there never are locations of kind 3. That is because those locations are masked and never passed, never made accessible to the context. So really, the trace semantics only collects locations of kind 4 on the traces.

**Lemma 28** (Correctness).

if $C_1 \simeq_{cctx} C_2$
then $\text{TR}(C_1) = \text{TR}(C_2)$

*Proof Sketch.* By contraposition:

if $\text{TR}(C_1) \neq \text{TR}(C_2)$
then \( \exists A. \ A[C_1]\downarrow \land A[C_2]\uparrow \) (wlog)

We are thus given \( \beta_1' = \beta \cdot \beta_1 \) and \( \beta_2' = \beta \cdot \beta_2 \) and \( \beta_1 \neq \beta_2 \).

We can construct a context \( A \) that replicates the behaviour of \( \beta \) and then performs the differentiation.

This is a tedious procedure that is analogous to existing results \([37, 53]\) and analogous to the backtranslation of Appendix D.2.

The actions only share the heap that is reachable from both sides, the heap that is private to the component is never touched, so reconstructing the heap is possible. The reachability conditions on the heap also ensure this.

The differentiation is based on differences on the actions which are visible and reachable, so that is also possible.

\[\Box\]

**Lemma 29** (Completeness).

\[
\text{if } \text{TR}(C_1) = \text{TR}(C_2) \quad \text{then } C_1 \simeq_{ctx} C_2
\]

**Proof Sketch.** By contradiction let us assume that

\[\exists A. \ A[C_1]\downarrow \land A[C_2]\uparrow \) (wlog)

Contexts are deterministic, so they cannot behave differently based on the values of locations that are never shared with \( C_1 \) or \( C_2 \).

The semantics forbids guessing, so a context will never have access to the locations that \( C_1 \) or \( C_2 \) do not share.

Thus a context can exhibit a difference in behaviour by relying on something that \( C_1 \) modified unlike \( C_2 \) and that can be:

- a parameter passed in a call.
  
  This contradicts the hypothesis that the trace semantics is the same as that parameter is captured in the call \( f \nu H! \) label.

- the value of a shared location.
  
  This contradicts the hypothesis that the trace semantics is the same as all locations that are reachable both by the context and by \( C_1 \) and \( C_2 \) are captured on the labels

Having reached a contradiction, this case holds.

\[\Box\]

**Lemma 30** (Full abstraction of the trace semantics for compiled components).

\[
\text{TR}(C_1)_{LP}^U = \text{TR}(C_2)_{LP}^U \iff C_1_{LP}^U \simeq_{ctx} C_2_{LP}^U
\]

**Proof.** By Lemmas 28 and 29.

\[\Box\]
L.3 The Compiler

\[
\begin{align*}
\text{let } x = \text{new } e \text{ in } s & \quad \Rightarrow \quad \text{let } x_{\text{loc}} = \text{new } [e]_{LP} \text{ in } \\
\text{let } x_{\text{cap}} = \text{hide } x_{\text{loc}} \text{ in } \\
\text{register}(x_{\text{loc}}, x_{\text{cap}}); \\
\text{let } x = \langle x_{\text{loc}}, x_{\text{cap}} \rangle \text{ in } [s]_{LP} \\
\text{call } f \ e & \quad \Rightarrow \quad \text{let } x = [e]_{LP} \text{ in } s_{\text{add}}(x); s_{\text{post}}; \text{call } f \ x; s_{\text{pre}}
\end{align*}
\]

So the compiler is mostly unchanged.
The compiled code will maintain the following invariant:

- no locations (even though protected by capabilities) are ever made accessible “in clear” to the context;
- “made accessible” means either passed as a parameter or through a shared location;
• instead, before passing control to the context, all component-created locations that are shared with the context are masked, i.e., their representation \( (n, k) \) is replaced with \( (n', k_{\text{com}}) \), where \( n' \) is their index in the list of shared locations that the compiled component keeps.

• when receiving control from the context, the compiled component ensures that all component-created locations that are shared are unmasked, i.e., upon regaining control the component replaces all values \( (n', k_{\text{com}}) \) that are sub-values of reachable locations with \( (n, k) \), which is the \( n' \)th pair in the list of component-allocated locations;

• what is a “component-shared” location? A shared location is a pair \( (n, k) \) where (i) \( k \) is a capability created by the compiled component and (ii) the pair is stored in the heap at a location that the context can dereference (perhaps not directly).

• In order to define what is a shared location, the compiled component keeps a list of all the locations that have been passed to it and that the context created. These locations can only be in \( (n, _) \) form, where _ is either a capability or not depending whether the context hid the location. These locations can only be pairs since we know that a compiled component will only use pairs as locations, mimicking the source semantics. We normally do not know what locations will be accessed, but given a parameter that is a location, we can scan its contents to understand what new locations are passed.

• The compiled component thus can keep a list of “shared” locations: those whose contents are accessible both by the context and by itself. These locations created by the context are acquired as parameters or as locations reachable by a parameter. These locations created by the component are tracked as those hidden with a component-created capability and reachable from a shared location.

• The only concern that can arise is if we create location \( n \) and then add it to the list of shared locations at index \( n' \). That location \( (n, k) \) would be masked as \( (n', k_{\text{com}}) \), which grants the context direct access to it. This is where we need to use \( k_{\text{com}} \) as leaking different capabilities would lead to differentiation between components. Fortunately, the context starts execution and, in order to call the compiled component, it must allocate at least one location, so this problem cannot arise.

L.3.1 Syntactic Sugar

The languages we have do not let us return directly a value. In the following however, for readability, we write

\[
\text{let } x = \text{func } v \text{ in } s
\]

131
to intend: call function \texttt{func} with parameter \texttt{v} and store its returned value in \texttt{x} for use in \texttt{s}.

We indicate how that statement can be expressed in our language with the following desugaring:

\begin{verbatim}
let y = new 0 in let z = \langle v, y \rangle in call func z; let x =!z.2 with 0 in s
\end{verbatim}

\subsection{L.3.2 Support Data Structures}

The compiler relies on a number of data structures it keeps starting from location 0, which is accessible via \texttt{k\textsubscript{root}}.

These data structures are:

- a list of capabilities, which we denote with \(\mathbb{K}\). These capabilities are those that the compiled component has allocated.
- a list of component-allocated locations, which we denote with \(\mathbb{L}\). These are locations \(\langle n, k \rangle\) that are created by the compiled component and whose \(k\) are elements of \(\mathbb{K}\).
- a list of shared locations, which we denote with \(\mathbb{S}\). These are either (i) locations that are created by the context and passed to the compiled component or (ii) locations that are created by the compiled component and passed to the context.

Given a list \(L\) of elements \(e\), we use these helper functions:

- \texttt{indexof} \((L, e)\) returns \(n\), the index of \(e\) in \(L\), or 0 if \(e\) is not in \(L\);
- \(L(n)\) returns the \(n\)th element \(e\) of \(L\) or 0 if the list length is shorter than \(n\);
- \(L :: e\) if \(e\) is not in \(L\), it adds element \(e\) to the list, increasing its length by 1;
- \texttt{rem} \((L, e)\) removes element \(e\) from \(L\);
- \(e \in L\) returns true or false depending on whether \(e\) is in \(L\) or not.

We keep this abstract syntax for handling lists and do not write the necessary recursive functions as they would only be tedious and hardly readable. Realistically, we would also need a temporary list for accumulating results etc, again, this is omitted for simplicity and readability.
L.3.3 Support Functions

Read

\[
\begin{align*}
\text{s}_{\text{read}} &= \text{let } x_n &= x.1.1 \text{ in} \\
&\quad \text{let } x_k &= x.1.2 \text{ in} \\
&\quad \text{let } x_{\text{real}} &= \overline{L}(x_n) \text{ in} \\
&\quad \text{let } x_{\text{dest}} &= x.2.1 \text{ in} \\
&\quad \text{let } x_{\text{dcap}} &= x.2.2 \text{ in} \\
&\quad \text{let } x_{\text{val}} &= !x_{\text{real}} \text{ with } x_k \text{ in} \\
&\quad x_{\text{dest}} := x_{\text{val}} \text{ with } x_{\text{dcap}}
\end{align*}
\]

In order to read a location \( \langle n, k \rangle \), we receive that as the first projection of parameter \( x \). Because we do not explicitly return values, we need the second projection of \( x \) to contain the destination where to target receives the result of the read.

We split the pair in the masking index \( x_n \) and in the capability to access the location \( x_k \). Then we lookup the location in the list of component-defined locations and return its value. We do not need to mask its contents as we know that they have already been masked when this location was shared with the context (line 5 of the postamble). We do not need to add its contents to the list of shared locations as that is already done in lines 2 and 3 of the postamble.

Write

\[
\begin{align*}
\text{s}_{\text{write}} &= \text{let } x_n &= x.1.1 \text{ in} \\
&\quad \text{let } x_k &= x.1.2 \text{ in} \\
&\quad \text{let } x_{\text{real}} &= \overline{L}(x_n) \text{ in} \\
&\quad x_{\text{real}} := x.2 \text{ with } x_k;
\end{align*}
\]

In order to write value \( v \) a location \( \langle n, k \rangle \) we receive a parameter structured as follows: \( x \equiv \langle n, k \rangle, v \). Then we unfold the elements of the parameter and lookup element \( n \) in the list of component-defined locations. We use this looked-up element to write the value \( v \) there.

We do not need to mask \( v \) because it cannot point to locations that are created by the compiled component.

At this stage, \( v \) may contain new locations created by the context and that are now shared. We do not add them now to the list of shared locations because we know that upon giving control again to the compiled component, the preamble will do this.

Mask

\[
\begin{align*}
\text{s}_{\text{mask}} &= \forall \langle n, k \rangle \in x. \text{isloc}(\langle n, k \rangle) \\
&\quad \text{if } k \in \overline{K} \\
&\quad \text{replace } \langle n, k \rangle \text{ with } \langle \text{indexof}(\overline{L}, n), k_{\text{com}} \rangle
\end{align*}
\]
We use the abstract construct `replace...` to indicate the following. We want to keep the value passed as parameter `x` unchanged but replace its subvalues that are pairs and, more specifically, component-created locations, with a pair with its location masked to be the index in the list of component-allocated locations.

This can be implemented by checking the sub-values of a value via the `ispair` and `isloc` expressions, we omit its details for simplicity. To ensure \( \in K \) is implementable, we use the `eqcap` expression.

Masked locations cannot mention their capability or they would leak this information and generate different traces for equivalent compiled programs.

**Unmask**

\[
s_{\text{unmask}} = \forall (n, k) \in x \\
\quad \text{if } k = k_{\text{com}} \\
\quad \text{replace } \langle n, k \rangle \text{ with } L(n)
\]

In the case of unmasking, we receive a value through parameter `x` and we know that there may be subvalues of it of the form `\langle n, k \rangle` where `n` is an index in the component-created shared locations. So we lookup the element from that list and replace it in `x`.

### L.3.4 Inlined Additional Statements (Preamble, Postamble, etc)

**Adding**

\[
s_{\text{add}}(x) = \text{if } \text{isloc}(x) \text{ then} \\
\quad S :: x; \\
\quad \text{if } x.2 \in K \text{ then } L :: x \text{ else skip}
\]

This common part ensures that the parameter `x` is added to the list of shared locations (line 1) and then, if the capability is locally-created, it is also added to the list of locally-shared locations (line 2).

The second line is for when this code is called before a \( \text{call } f \).

**Registration**

\[
s_{\text{register}}(x_{\text{loc}}, x_{\text{cap}}) = K :: x_{\text{cap}};
\]

This statement registers capability `x_{\text{cap}}` in the list of component-created capabilities.

**Preamble** The preamble is responsible of adding all context-created locations to the list of shared locations and to ensure that all contents of shared locations
are unmasked, as the compiled code will operate on them.

\[
s_{\text{pre}} = \forall (n, k) \in \text{reach}(\overline{S}). \text{isloc}(\langle n, k \rangle) \\
\quad \text{if } \langle n, k \rangle \notin \overline{S} \text{ then } \overline{S} :: \langle n, k \rangle; \text{ else skip} \\
\forall (n, k) \in \overline{S}. \text{isloc}(\langle n, k \rangle) \\
\quad \text{let } x = \text{unmask}(!n \text{ with } k) \text{ in } n := x \text{ with } k
\]

First any location that is reachable from the shared locations (line 1) and that is not a shared location already is added to the list of shared locations (line 2). By where this code is placed we know that these new locations can only be context-created.

Then, for all shared locations (line 3), we unmask their contents using the \texttt{unmask} function (line 4).

\textbf{Postamble} The postamble is responsible of adding all component-created locations to the list of shared locations and of component-created shared locations and to ensure that all shared locations are masked as the context will operate on them.

\[
s_{\text{post}}(x) = \forall (n, k) \in \text{reach}(\overline{S}). \text{isloc}(\langle n, k \rangle) \\
\quad \text{if } \langle n, k \rangle \notin \overline{S} \text{ then } \overline{S} :: \langle n, k \rangle; \overline{L} :: \langle n, k \rangle; \text{ else skip} \\
\forall (n, k) \in \overline{S}. \text{isloc}(\langle n, k \rangle) \\
\quad \text{let } x = \text{mask}(!n \text{ with } k) \text{ in } n := x \text{ with } k
\]

Then for all locations that are reachable from a shared location (line 1), and that are not already there (line 2), we add those locations to the list of shared locations and to the list of component-created shared locations (line 2). Then for all shared locations (line 3), we mask their contents using the \texttt{mask} function (line 4).

\textbf{L.4 The Trace-based Backtranslation: } \langle \langle \text{value} \rangle \rangle_{L^P_U}

Value backtranslation is the same, so \langle \langle \text{value} \rangle \rangle_{L^P_U} = \langle \langle \text{value} \rangle \rangle_{L^P_U}.

\textbf{L.4.1 The Skeleton}

The skeleton is almost as before (Appendix D.2.2), with the only addition of another list \texttt{B} explained below.

The only additions are two functions \texttt{terminate} and \texttt{diverge}, which do what their name suggests:

\[
\texttt{terminate}(x) \mapsto \text{fail} \\
\texttt{diverge}(x) \mapsto \text{call diverge 0}
\]
L.4.2 The Common Prefix

call f v H? As in Rule $\langle \langle \cdot \rangle \rangle_{L^P}$-call, we keep a list of the context-allocated locations and we update them. Also, we extend that list.

call f v H! This is analogous to Rule $\langle \langle \cdot \rangle \rangle_{L^P}$-callback-loc) but with a major complication.

Now this is complex because in the target we don’t receive locations $\langle n, k \rangle$ from the compiled component, but masked indices $\langle i, k_{\text{com}} \rangle$. (using $i$ as a metavariable for natural numbers outputted by the masking function) We need to extract them based on where they are located in memory, knowing that the same syntactic structure is maintained in the source. So what before was relying on the relation on values $\ell \approx_\beta \langle n, k \rangle$ now is no longer true because we have $\ell \approx_\beta \langle i, k_{\text{com}} \rangle$ which cannot hold. We need to keep a this relation as a runtime argument in the backtranslating and base it solely on the syntactic occurrences of $\langle i, k_{\text{com}} \rangle$. So this runtime relation maps target masking indices to source locations.

So this relation is really a list $B$ where each entry has the form $\langle \langle i \rangle \rangle_{L^P}, \ell \rangle$.

Intuitively, consider heap $H$ from the action. For all of its content $n \mapsto v : \eta$, we do a structural analysis of $v$. This happens at the meta-level, in the backtranslation algorithm. $v$ may contain subvalues of the form $\langle i, k_{\text{com}} \rangle$, and accessing this subvalue we know is a matter of $\cdot$ etc. So we produce an expression $e$ with the same instructions ($\cdot$ etc) in the source in order to scan at runtime the heap $H$ we receive after the callback is done. (so after the action here is executed and where backtranslation code executes) Given that expression $e$ evaluate to location $\ell$, we now need to add to $B$ the pair $\langle i, \ell \rangle$ (also given that $i=\langle \langle v \rangle \rangle_{L^P}$).

ret !H As above.

write(v, i) In this case we need to make use of the runtime-kept relation $B$. We need to know what source location $\ell$ corresponds to $i$ so we can produce the correct code: $\ell := \langle \langle v \rangle \rangle_{L^P}$.

$\ell$ is looked up as $B(\langle \langle i \rangle \rangle_{L^P})$.

L.4.3 The Differentiator

The differentiator needs to put the right code at the right place. The backtranslation already carries all necessary information to know what the right place is, this is as in previous work: the index of the action $i$ (at the meta level) stored in location $\ell_i$ (at runtime) and the call stack $\tilde{f}$.
We now go over the various cases of trace difference and see that the differentiation code exists. We consider $\alpha_1$ to be the last action in the trace of $[C_1]_T$, while $\alpha_2$ is the last one of $[C_2]_T$, both made after a common prefix.

$\alpha_1 = \text{call } f \nu H$ and $\alpha_2 = \text{call } g \nu H$ Code if $!\ell_i == i$ then call terminate 0 else skip is placed in the body of $f$ while the code if $!\ell_i == i$ then call diverge 0 else skip is placed in the body of $g$.

$\alpha_1 = \text{call } f \nu H$ and $\alpha_2 = \text{call } f \nu H'$ Code

$$ \begin{align*}
&\text{if } !\ell == i \text{ then } \\
&\quad \text{if } x == \langle \langle v \rangle \rangle_{L^P} \text{ then call terminate 0 else call diverge 0 else skip}
\end{align*}$$

is placed in $f$.

$\alpha_1 = \text{call } f \nu H$ and $\alpha_2 = \text{call } f \nu H'$ Here few cases can arise, consider $H = H_1, n \mapsto v : \eta, H_2$ and $H' = H_1, n' \mapsto v' : \eta', H'_2$:

$v \neq v'$ We use shortcut $\text{L}_\text{glob}(n)$ to indicate the location bound to name $n$ in the list of shared locations (same as in Appendix D.2.3).

Code

$$ \begin{align*}
&\text{if } !\ell_i == i \text{ then } \\
&\quad \text{let } x = \text{L}_\text{glob}(\langle \langle n \rangle \rangle_{L^P}) \text{ in } \\
&\quad \quad \text{if } x == \langle \langle v \rangle \rangle_{L^P} \text{ then call terminate 0 else call diverge 0 else skip}
\end{align*}$$

is placed in the body of $f$.

$n \neq n'$ In this case one of the two addresses must be bigger than the other.

Wlog, let’s consider $n = n' + 1$.

So $H_1 = H'_1, n' \mapsto v', \eta'$ and $H_2 = \emptyset$ (otherwise we’d have a binding for $n$ there).

The code in this case must access the location related to $n$, it will get stuck in one case and succeed in the other:

$$ \begin{align*}
&\text{if } !\ell_i == i \text{ then let } x = \text{update}(\langle \langle n \rangle \rangle_{L^P}, 0) \text{ in call diverge 0 else skip}
\end{align*}$$

$\eta \neq \eta'$ Two cases arise:

- the location is context-created: in this case the tag must be the same, so we have a contradiction;
- the location is component-created, but in this case we know that no such location is ever passed to the context (see Property 1), so we have a contradiction.

$\alpha_1 = \text{ret } H$ and $\alpha_2 = \text{ret } H$ As above.
\( \alpha_1 = \text{call } f \rhd v \ \text{H! and } \alpha_2 = \text{ret } \) \!
Code if \(!l_i \equiv i\) then call terminate 0 else skip
is placed at \( f \) while if \(!l_i \equiv i\) then call diverge 0 else skip
is placed at the top of \( \overline{f} \).

\( \alpha_1 = \text{call } f \rhd v \ \text{H! and } \alpha_2 = \downarrow \) \!
Code if \(!l_i \equiv i\) then call diverge 0 else skip
is placed at \( f \).

\( \alpha_1 = \text{call } f \rhd v \ \text{H! and } \alpha_2 = \uparrow \) \!
Code if \(!l_i \equiv i\) then call terminate 0 else skip
is placed at \( f \).

\( \alpha_1 = \text{ret } \text{H! and } \alpha_2 = \downarrow \) \!
Code if \(!l_i \equiv i\) then call diverge 0 else skip
is placed at the top of \( \overline{f} \).

\( \alpha_1 = \text{ret } \text{H! and } \alpha_2 = \uparrow \) \!
Code if \(!l_i \equiv i\) then call terminate 0 else skip
is placed at the top of \( \overline{f} \).

\( \alpha_1 = \downarrow \text{ and } \alpha_2 = \uparrow \) \!
Nothing to do, the compiled component performs the differentiation on its own.