Directions for model building from asymptotic safety

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Building on recent advances in the understanding of gauge-Yukawa theories we explore possibilities to UV-complete the Standard Model in an asymptotically safe manner. Minimal extensions are based on a large flavor sector of additional fermions coupled to a scalar singlet matrix field. We find that asymptotic safety requires fermions in higher representations of $SU(3)_C \times SU(2)_L$. Possible signatures at colliders are worked out and include $R$-hadron searches, diboson signatures and the evolution of the strong and weak coupling constants.

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I. INTRODUCTION

Asymptotic freedom plays a central role in the construction of the Standard Model (SM) of particle physics and extensions thereof \[1, 2\]. It predicts that interactions are dynamically switched off at highest energies due to quantum fluctuations. In the language of the renormalisation group, asymptotic freedom corresponds to a free ultraviolet (UV) fixed point. Asymptotic freedom famously requires the presence of non-abelian gauge fields \[3\], together with suitable matter interactions to ensure that Yukawa and scalar couplings reach the free fixed point in the UV alongside the non abelian gauge coupling \[4\]. Identifying viable theories beyond the Standard Model (BSM) with complete asymptotic freedom continues to be an active area of research \[5–7\].

Asymptotic safety states that fundamental quantum fields may very well remain interacting at highest energies \[8, 9\], implying that running couplings reach an interacting (rather than a free) UV fixed point under the renormalisation group evolution. If so, theories remain well-behaved and predictive up to highest energies in close analogy to theories with complete asymptotic freedom. Asymptotic safety has initially been put forward as a scenario for quantum gravity \[9\] where a large amount of evidence has arisen from increasingly sophisticated studies in four dimensions including signatures at colliders (see \[10\] for an overview). More recently, necessary and sufficient conditions for asymptotic safety in general weakly coupled gauge theories (without gravity) have been derived, alongside strict no go theorems \[11\]. Most importantly, it was found that Yukawa interactions together with elementary scalar fields such as the Higgs offer a unique key towards asymptotic safety \[11\]. Moreover, an important proof of existence has been provided in \[12\], and further expanded in \[13\], showing that exact asymptotic safety with a stable ground state can arise in \(SU(N)\) gauge theories under strict perturbative control in the Veneziano limit. The feasibility of asymptotic safety is thus well motivated theoretically and opens intriguing new directions for model building beyond the SM.

In this paper, we make a first step to investigate asymptotically safe extensions of the SM and phenomenological signatures thereof at colliders. Our motivation for doing so is twofold. Firstly, we want to understand whether and how minimal extensions of the SM can be found with weakly interacting UV fixed points. We are particularly interested in the “phase space” of such extensions, and in the concrete conditions under which interacting UV fixed points are connected through well-defined trajectories with the SM at low energies. Secondly, we wish to understand how phenomenological constraints may arise through existing data, and, more generally, the conditions under which asymptotic safety can be tested at colliders. Our investigation is “top-down” in that we begin by requiring conditions under which weakly coupled asymptotic safety can be achieved. Our central new input are BSM fermions and scalars, some of which are charged under the gauge symmetries of the SM. Our approach will be minimal in that we add a single BSM Yukawa coupling whose sole task is to negotiate asymptotically safe UV completions for the SM with \(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\) gauge symmetry.

The paper has the following format. In Sec. \[II\] we discuss the basic perturbative mechanism for asymptotic safety in gauge theories including general conditions for existence. In Sec. \[III\] we investigate minimal extensions of the Standard Model in view of weakly interacting high energy fixed points. In Sec. \[IV\] we explain the conditions under which interacting UV fixed points are connected with the SM at low energies. Phenomenological implications are worked out in Sec. \[V\]. We summarize in Sec. \[VI\]. Appendix \[A\] contains technicalities summarising the perturbative loop coefficients and group theoretical information, and details of UV-IR connecting separatrices.
II. BASICS OF ASYMPTOTIC SAFETY FOR GAUGE THEORIES

In this section, we recall the basic mechanism for asymptotic safety in four-dimensional gauge theories with matter and recall general theorems for asymptotic safety in weakly coupled gauge theories following [11, 12]. We also introduce some notation and conventions.

A. Weakly interacting UV fixed points

We begin with a discussion of asymptotic safety in gauge theories and the renormalisation group running of couplings. In the absence of asymptotic freedom, it is well-known that perturbative couplings would grow towards higher energies thereby limiting predictivity to a highest energy scale $\Lambda$. The main feature of asymptotic safety, however, is that the growth of couplings is tamed, dynamically, through a weakly interacting fixed point. An explicit mechanism which allows quantum fields to avoid the notorious Landau poles of QED-like theories has recently been discovered in [12]. Strict theorems for asymptotic safety in general weakly coupled gauge theories have been derived in [11].

To illustrate the mechanism, and to prepare for our models below, we consider the renormalization group (RG) flow for a simple gauge theory with gauge coupling $\alpha_g = g^2/(4\pi)^2$ interacting with scalars and fermions, with Yukawa coupling $\alpha_y = y^2/(4\pi)^2$. Within perturbation theory, the RG flow in the gauge-Yukawa system to the leading non-trivial order is given by

\[
\beta_g \equiv \frac{d\alpha_g}{d\ln \mu} = (B + C\alpha_g - D\alpha_y)\alpha_g^2,
\]

\[
\beta_y \equiv \frac{d\alpha_y}{d\ln \mu} = (E\alpha_y - F\alpha_g)\alpha_y.
\]

Scalar selfcouplings do not impact on interacting fixed points to leading order at weak coupling and can be neglected. The various loop coefficients $B, C, D, E$ and $F$ depend on the matter content of the theory, which we leave unspecified at this stage. The gauge coupling is asymptotically free (infrared free) provided that the one loop gauge coefficient obeys $B > 0$ ($B < 0$). The two loop gauge coefficient $C$ may take either sign depending on the matter content. Provided that asymptotic freedom is absent, $B < 0$, it has also been shown that $C > 0$ [11]. The other loop coefficients obey $D, E, F > 0$ for any quantum field theory, irrespective of the matter content. Also notice that Yukawa couplings always contribute with a negative sign to the running of the gauge coupling, irrespective of the sign of $B$.

In general, theories with (1) may have various types of fixed points, depending on the matter content. Equating $\beta_i = 0$ for both couplings, three types of fixed points are found. The Gaussian fixed point

\[
(\alpha_g^*, \alpha_y^*) = (0, 0)
\]

always exists, and corresponds to the UV (IR) fixed point provided that $B > 0$ ($B < 0$). An interacting fixed point where Yukawa interactions are switched-off may also exist, with

\[
(\alpha_g^*, \alpha_y^*) = \left(\frac{B}{C}, 0\right).
\]

This is the well-known Caswell–Banks-Zaks fixed point [14, 15] which requires $B \cdot C > 0$ to be
physical and \( B/C \ll 1 \) to be perturbative. It is also known that \( B \cdot C < 0 \) as soon as \( B < 0 \) for any quantum field theory \([III]\). This result has the form of a no go theorem: in four dimensions, weakly coupled gauge theories cannot become asymptotically safe without Yukawa interactions. Hence, Caswell–Banks-Zaks fixed points \([3]\) are invariably IR fixed points.

A fully interacting gauge-Yukawa fixed point may arise provided that the Yukawa coupling is non-vanishing. Requiring \( \beta_y = 0 \), \([1]\) implies that the gauge and Yukawa coupling are proportional to each other, \( \alpha_y = \frac{F}{E} \alpha_g \). This nullcline condition modifies the running of the gauge coupling and turns \([1]\) into

\[
\beta_g = (-B + C' \alpha_g) \alpha_g^2 ,
\]

where the two loop term is effectively shifted \( C \to C' \) owing to Yukawa interactions, with

\[
C' = C - D \frac{F}{E} < C .
\]

This shift term has important implications: Firstly, the fixed point is now fully interacting, with the gauge coupling taking the form \([3]\) with \( C \) shifted as in \([5]\), together with the interacting fixed point for the Yukawa coupling,

\[
(\alpha_g^*, \alpha_y^*) = \left( \frac{B}{C'}, \frac{B}{C'} \frac{F}{E} \right).
\]

Secondly, for theories with asymptotic freedom \( (B > 0) \), and provided that \( C' > 0 \), the gauge-Yukawa fixed point \([6]\) corresponds to an IR fixed point. It can be reached by RG trajectories emanating out of the Gaussian UV fixed point. Finally, for theories without asymptotic freedom \( (B < 0) \) the gauge coupling may now take a viable interacting fixed point \( \alpha_g^* = B/C' > 0 \) as long as \( C' < 0 \). This is the interacting UV fixed point of asymptotic safety (see Tab. \([1]\) for a summary). The result is in stark contrast to theories without Yukawa interactions, where \([3]\) cannot possibly become an UV fixed point. We conclude that the Yukawa interactions are of crucial importance for asymptotic safety \([\Pi]\). Moreover, the necessary condition for asymptotic safety at weak coupling \( B, C' < 0 \), see Tab. \([I]\) c), now translates into a simple condition relating the one and two loop coefficients appearing in \([1]\),

\[
C' < 0 \iff DF - CE > 0 .
\]

In the remaining part of the paper, we evaluate whether the condition \([7]\) can be achieved for extensions of the SM.

### B. Scaling behaviour

In the vicinity of a free (UV or IR) fixed point the running of couplings is logarithmically slow. In the vicinity of interacting (UV or IR) fixed points, instead, the running of couplings is power law like, characterised by universal scaling exponents \( \{\vartheta_i\} \). Linearising the RG flow in the vicinity of a fixed point

\[
\beta_i = \sum_j M_{ij} (\alpha_j - \alpha_j^*) + \text{subleading} ,
\]

\[\text{(8)}\]
Table 1. All weakly interacting fixed points $\alpha_*$ of simple gauge theories with (1) and their dependence on the matter content expressed through the parameters $B, C$ and $C'$, see [11].

| case | parameter | fixed point | info | type          |
|------|-----------|-------------|------|---------------|
| a)   | $B > 0, C > 0$ | IR          | asymptotic freedom | Caswell–Banks-Zaks (BZ) |
| b)   | $B > 0, C' > 0$ | IR          | asymptotic freedom | gauge-Yukawa (GY) |
| c)   | $B < 0, C' < 0$ | UV          | asymptotic safety   | gauge-Yukawa (GY) |

the scaling exponents can be derived as the eigenvalues of the stability matrix $M_{ij} = \partial \beta_i / \partial \alpha_j |_{\ast}$. Eigendirections are termed relevant (irrelevant) provided that $\vartheta < 0$ ($\vartheta > 0$). Marginal couplings have vanishing eigenvalues at linear order [8], reflecting logarithmic running. Whether these are marginally relevant ($\vartheta = 0^-$) such as in QCD, or marginally irrelevant ($\vartheta = 0^+$) such as in QED, is determined beyond leading order. In the vicinity of interacting fixed points couplings scale according to

$$\alpha_i(\mu) = \alpha_i^* + \sum_n c_n V^n_i \left( \frac{\mu}{\mu_0} \right)^{\vartheta_n} + \text{subleading},$$

where $V^n$ are the eigenvectors of the stability matrix with eigenvalue $\vartheta_n$, $\mu$ denotes the RG scale, and $c_n$ are free numbers. The significance of (9) is as follows [16, 17]. In order to achieve a well-defined UV limit, the parameters $c_n$ related to irrelevant eigenvalues must be set identically to zero, or else the UV fixed point cannot possibly be reached from (9) in the limit $\mu \to \infty$. On the other side, the relevant eigendirections are unconstrained and the corresponding numbers $c_n$ are free parameters of the theory. Provided that the number of relevant directions is finite, the theory is predictive with a finite number of free parameters whose values must be determined by experiment.

Returning to the models at hand, three different types of interacting fixed points arise. At a Caswell–Banks-Zaks fixed point [3], scaling exponents are given by

$$\vartheta_1 = -BF/C, \quad \vartheta_2 = B^2/C,$$

(10)

(11)

to leading order in $B/C \ll 1$, with $\vartheta_1 < 0 < \vartheta_2$. Consequently, the fixed point has a relevant direction corresponding to the Yukawa interaction, and an irrelevant one, corresponding to the gauge coupling, see Tab. [1a]. At the gauge-Yukawa fixed point [6], the scaling exponents of the theory are given by

$$\vartheta_1 = B^2/C', \quad \vartheta_2 = BF/C',$$

(12)

(13)

to leading order in $B/C' \ll 1$. For asymptotically free theories, we note that $0 < \vartheta_1 < \vartheta_2$, meaning that both directions are IR attractive, see Tab. [1b]. In the remaining part of the paper we are particularly interested in theories with asymptotic safety where $B < 0$. For these, the eigenvalues are of the form $\vartheta_1 < 0 < \vartheta_2$, see Tab. [1c]. It states that the fixed point has a one

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1 Notice that (12) and (13) do not follow from (10) and (11) by substituting $C \to C'$. 

dimensional UV critical surface characterised by the relevant direction given through $\vartheta_1$ [12].

C. Theorems for asymptotic safety

General theorems for asymptotic safety in weakly coupled gauge theories have recently been derived in [11]. In particular, it has been established that Yukawa interactions offer a unique mechanism towards asymptotic safety. Neither gauge interactions nor scalar self interactions are able to negotiate an interacting UV fixed point at weak coupling. Stated differently, it is impossible to find an asymptotically safe and weakly coupled gauge theory with simple or product gauge groups but without Yukawa interactions. Hence, asymptotic safety in four dimensional gauge theories invariably requires elementary scalars and fermions, besides the gauge fields. Furthermore, fermions must minimally be charged under some or all of the gauge group(s). For general gauge theories with product gauge group $G = G_1 \otimes G_2 \otimes \cdots \otimes G_n$, weakly interacting fixed points arise as solutions to the linear equations [11]

$$B'_i = C_{ij} \alpha_j^*, \text{ subject to } \alpha_j^* \geq 0, \quad (14)$$

where $C_{ij}$ denotes the matrix of two loop gauge contributions, and $B'_i = B_i + 2 Y_{4,i}^*$ the one-loop coefficient shifted by the Yukawa terms $Y_{4,i}^* = \text{Tr}[C_{2i}^{F_2} Y^A (Y^A_*)^\dagger]/d(G_i) \geq 0$ at the interacting fixed point. Here, $C_{2i}^{F_2}$ denotes the Casimir of the fermions, $Y^A$ the matrix of Yukawa couplings, and $d(G_i)$ the dimension of the group $G_i$ following the conventions of [18–21]. It has also been shown in [11] that for any infrared free gauge factor ($B_i < 0$), the necessary condition for asymptotic safety amongst the solutions to (14) is

$$B'_i > 0. \quad (15)$$

It states that Yukawa interactions must effectively change the sign of the one loop coefficient for the infrared free gauge couplings, generalising the necessary condition (7) to general gauge theories. Sufficiency conditions for asymptotic safety, in addition to the mandatory presence of Yukawa couplings, have also been detailed in [11]. These relate to the specifics of the Yukawa sector as well as to the viability of the scalar sector including the stability of the vacuum. It then remains to investigate whether the mandatory and sufficient conditions for asymptotic safety have viable weakly coupled UV fixed points as their solutions. In the remaining part of the paper, we investigate in concrete terms the availability of asymptotically safe solutions of (14), (15) for BSM extensions of the $SU(3)_C \times SU(2)_L$ sector of the SM.

III. ASYMPTOTIC SAFETY BEYOND THE STANDARD MODEL

In this section, we investigate minimal extensions of the SM and conditions under which asymptotic safety becomes available in the deep UV [11]. While designing the structure of the BSM sector, we make use of the properties of the gauge-Yukawa theory where asymptotic safety can be achieved by an interplay between the gauge and Yukawa interactions of vector-like fermions and a scalar matrix field [12].
A. Minimal BSM extensions

Asymptotic safety in BSM extensions minimally require the presence of new matter fields which carry charges under the SM gauge groups and thereby modify the RG running of couplings. Guided by the findings of [11, 12], we consider the existence of $N_F$ flavors of BSM vector-like fermions $\psi$ which minimally couple to the SM gauge bosons. In general, the BSM fermions may carry charges under $SU(3)_C$, $SU(2)_L$, or hypercharge $Y$, meaning

$$\psi_i(R_3, R_2, Y),$$

where $i = 1, \cdots, N_F$ denotes the flavor index. Furthermore, the BSM fermions couple via Yukawa interactions to complex scalar fields $S_{ij}$ which we take to be a singlet under the SM. Since the BSM fermions are taken to be vector-like, anomalies are not an issue. The Yukawa interactions are given by

$$L_{\text{BSM,Yukawa}} = -y \text{Tr} (\bar{\psi}_L S \psi_R + \bar{\psi}_R S^\dagger \psi_L).$$

Here, $y$ denotes the BSM Yukawa coupling, the trace $\text{Tr}$ sums over color and flavor indices, and the decomposition $\psi = \psi_L + \psi_R$ with $\psi_{R/L} = \frac{1}{2} (1 \pm \gamma_5) \psi$ is understood. Yukawa interactions are crucial for asymptotic safety to arise in weakly coupled gauge theories. The BSM sector is invariant under global $U(N_F) \times U(N_F)$ flavor rotations. The full Lagrangean for the BSM extension of the SM is given by

$$L = L_{\text{SM}} + L_{\text{BSM,kin.}} + L_{\text{BSM,pot.}} + L_{\text{BSM,Yukawa}}.$$

Here, $L_{\text{SM}}$ denotes the SM Lagrangean and $L_{\text{BSM,pot.}}$ the interaction Lagrangean of the BSM scalars. The BSM scalars $S$ can mix with the SM Higgs boson through suitable portal coupling contained in $L_{\text{BSM,pot.}}$. The BSM kinetic terms are given by

$$L_{\text{BSM,kin.}} = \text{Tr} (\bar{\psi} i \partial \psi) + \text{Tr} (\partial_{\mu} S^\dagger \partial^{\mu} S).$$

The BSM fermions communicate to the SM through the gauge interactions, provided they are charged accordingly. The scalar fields are taken to be singlets under the SM gauge groups. We assume that the BSM matter fields develop soft scalar $M_S$ and fermion $M_\psi$ masses for the model to be compatible with data.

For the sake of this paper we make a few further simplifying assumptions. Firstly, we limit ourselves to BSM fermions which carry no hypercharge. This assumption can be relaxed without changing the overall picture of results. Secondly, we neglect the role of quartic self interactions of the BSM scalars as well as portal couplings to the Higgs. At weak coupling, neither of these are relevant for the primary existence of the UV fixed point in the gauge-Yukawa sector. Consequently, the free fundamental parameters of the BSM matter sector are given by their group-theoretical representation under $SU(2)_L$ and $SU(3)_C$, and their flavor multiplicity $N_F$,

$$(R_2, R_3, N_F).$$

A key goal of our study will be to identify viable, weakly coupled UV fixed points for the BSM theory $\text{(18)}$ within the parameter space $\text{(20)}$.

$^2$ A detailed analysis of the SM Yukawa and scalar sector will be given elsewhere.
B. Renormalisation group

In order to identify interacting fixed points, we must analyse the RG equations for the theory. Within perturbation theory, weakly interacting fixed points arise for the first time at the two loop level in the gauge sector and at the one loop level in the Yukawa and scalar sectors. Also, interacting UV fixed points necessarily require the presence of a fixed point in the Yukawa interactions. For these reasons, we consider the RG equations for up to second order in both gauge couplings, and up to first order in the BSM Yukawa coupling. This is the lowest order at which a weakly coupled UV fixed point may arise.

To be concrete, we normalise the gauge and Yukawa couplings with the perturbative loop factor and introduce

\[ \alpha_2 = \frac{g_2^2}{(4\pi)^2}, \quad \alpha_3 = \frac{g_3^2}{(4\pi)^2}, \quad \alpha_y = \frac{y^2}{(4\pi)^2}, \tag{21} \]

to denote the weak, strong, and BSM Yukawa coupling, respectively.

A few comments are in order. The one loop gauge coefficients \( B_i \) can take either sign, depending on the BSM matter content. The two loop gauge coefficient \( C_2 \) is positive throughout. The two loop gauge coefficients \( C_3 \) may take either sign if \( B_3 > 0 \), but is strictly positive as soon as \( B_3 \leq 0 \). The two loop gauge mixing terms \( G_i \) as well as the two loop Yukawa contribution \( D_i \) and the one loop Yukawa terms \( E \) and \( F_i \) are always positive in any quantum field theory. The Yukawa couplings always contribute with a negative sign to the running of gauge couplings. This is centrally important for interacting UV fixed points to arise at weak coupling.

Explicit expressions for the various loop coefficients and further details are summarised in the appendix, see (A2) – (A6). In the absence of BSM matter fields, the RG flow reduces to the RG flow of the SM with loop parameters given by (A9). In this limit, the RG flow for the BSM Yukawa coupling becomes obsolete. With (22) at hand, we now turn to a systematic fixed point search within the perturbative regime.

C. UV fixed points at weak coupling

Gauge-Yukawa theories with may display up to four different types of weakly coupled UV fixed points, depending on whether the gauge couplings take free or interacting values in the UV.

\[ \beta_3 \equiv \frac{d\alpha_3}{d\ln \mu} = (-B_3 + C_3 \alpha_3 + G_3 \alpha_2 - D_3 \alpha_y) \alpha_3^2, \]

\[ \beta_2 \equiv \frac{d\alpha_2}{d\ln \mu} = (-B_2 + C_2 \alpha_2 + G_2 \alpha_3 - D_2 \alpha_y) \alpha_2^2, \tag{22} \]

\[ \beta_y \equiv \frac{d\alpha_y}{d\ln \mu} = (E \alpha_y - F_2 \alpha_2 - F_3 \alpha_3) \alpha_y. \]

Our definition for the gauge couplings relates to the more standard definition \( \alpha_s = g_s^2/(4\pi) \) as \( \alpha_s = 4\pi \alpha_3 \), and similarly for \( \alpha_y \).
We refer to the different cases as \( \text{FP}_1 \) – \( \text{FP}_4 \), defined as
\[
\begin{align*}
\text{FP}_1 : & \quad \alpha_3^* = 0, \quad \alpha_2^* = 0, \\
\text{FP}_2 : & \quad \alpha_2^* > 0, \quad \alpha_3^* = 0, \\
\text{FP}_3 : & \quad \alpha_2^* = 0, \quad \alpha_3^* > 0, \\
\text{FP}_4 : & \quad \alpha_2^* > 0, \quad \alpha_3^* > 0,
\end{align*}
\]
see Tab. 2. The Gaussian fixed point \( \text{FP}_1 \), where all couplings vanish, always exists. It qualifies as a candidate for an asymptotically free extension of the SM provided that each gauge sector remains asymptotically free individually. Using the explicit expressions (A2) this condition translates into bounds
\[
\begin{align*}
SU(2)_L : & \quad N_F < 19/(8 S_2(R_2) d(R_3)), \\
SU(3)_C : & \quad N_F < 21/(4 S_2(R_3) d(R_2)).
\end{align*}
\]
In Tabs. 3 and 4 we show the maximal number of BSM vector-like fermions \( \psi(R_3, R_2) \) compatible with asymptotic freedom, \( N_F \leq N_{\text{AF}} \) for \( SU(2)_L \) singlets, doublets and triplets, and for different dimensions of the \( SU(3)_C \) representations. We observe a small window for low-dimensional representations where asymptotic freedom persists. Asymptotic freedom is lost as soon as the BSM fermions transform under higher-dimensional representations of the gauge group. See [5] for a recent analysis of BSM extensions with complete asymptotic freedom.

Theories with (22) may also display weakly interacting fixed points with \( \alpha^* \leq 1 \). These are either partially or fully interacting. Conditions for existence of partially interacting UV fixed points such as \( \text{FP}_2 \) and \( \text{FP}_3 \) then reduce to those given in Sec. II A for simple gauge theories. Analogous conditions of existence arise for the fully interacting fixed point \( \text{FP}_4 \). In either of these cases, for \( \text{FP}_2 \), \( \text{FP}_3 \) or \( \text{FP}_4 \) to qualify as asymptotically safe UV fixed points, the Yukawa coupling must take an interacting fixed point by itself. To the leading non-trivial order in perturbation theory, using

| case   | gauge couplings | Yukawa coupling | type       | info       |
|--------|-----------------|-----------------|------------|------------|
| \( \text{FP}_1 \) | \( \alpha_3^* \) = 0, \( \alpha_2^* = 0 \) | \( \alpha_3^* \) = 0 | G \cdot G | non-interacting |
| \( \text{FP}_2 \) | \( \alpha_2^* > 0 \), \( \alpha_3^* = 0 \) | \( \frac{F_2}{E} \alpha_3^* \) | G \cdot GY | partially interacting |
| \( \text{FP}_3 \) | \( \alpha_2^* = 0 \), \( \alpha_3^* > 0 \) | \( \frac{F_3}{E} \alpha_3^* \) | GY \cdot G | partially interacting |
| \( \text{FP}_4 \) | \( \frac{C'_2 B_3 - B_2 G'_3}{C'_2 C'_3 - G'_2 G'_3} \), \( \frac{C'_3 B_2 - B_3 G'_2}{C'_2 C'_3 - G'_2 G'_3} \), \( \frac{F_3}{E} \alpha_3^* + \frac{F_2}{E} \alpha_2^* \) | | GY \cdot GY | fully interacting |

Table 2. The four different types of UV fixed points \( \text{FP}_1 \) – \( \text{FP}_4 \) in minimal BSM extensions of the SM with (22). The primed and unprimed loop coefficients are defined in App. A. We also indicate how the fixed points can be interpreted as products of the Gaussian (G) and gauge-Yukawa (GY) fixed points when viewed from the individual gauge group factors (see main text).
Table 3. Asymptotic freedom versus asymptotic safety at the partially interacting fixed point FP$_3$: shown are the maximal numbers of BSM fermion flavors compatible with asymptotic freedom, $N_{AF}$, and the smallest number of flavors required for an asymptotically safe fixed point FP$_3$ to exist, $N_{AS}$, both in dependence on the fermion representations $R_2$ and $R_3$ under $SU(2)_L$ and $SU(3)_C$, respectively. $N_{AS}$ values in brackets relate to the absolute lower bound, those without to fixed points with $0 < \alpha_3^*, \alpha_y^* < 1$. Also indicated are the weights $(p, q)$, quadratic Casimir, and Dynkin index under $SU(3)$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$R_3$ & $(p, q)$ & $C_2(R_3)$ & $S_2(R_3)$ & $R_2 = 1$ & $R_2 = 2$ & $R_2 = 3$ \\
\hline
3 & (1,0) & $\frac{4}{3}$ & $\frac{1}{2}$ & 10 & - & 6 & - & 3 & - \\
6 & (2,0) & $\frac{10}{3}$ & $\frac{5}{2}$ & 2 & (29) & 37 & 1 & (60) & 77 & - & (90) & 117 \\
8 & (1,1) & 3 & 3 & 1 & (62) & 96 & - & (127) & 198 & - & (192) & 299 \\
10 & (3,0) & 6 & $\frac{15}{2}$ & - & (16) & 18 & - & (32) & 34 & - & (48) & 51 \\
15 & (2,1) & $\frac{16}{3}$ & 10 & - & (28) & 30 & - & (55) & 60 & - & (82) & 90 \\
15' & (4,0) & $\frac{28}{3}$ & $\frac{35}{2}$ & - & (16) & 18 & - & (32) & 33 & - & (48) & 50 \\
\hline
\end{tabular}
\end{table}

\[(22)\), it follows that the Yukawa coupling at a fixed point is linearly related to the gauge couplings,

\[
\begin{align*}
    \text{FP}_2: & \quad \alpha_y^* = \frac{F_2}{E} \alpha_2^*, \\
    \text{FP}_3: & \quad \alpha_y^* = \frac{F_3}{E} \alpha_3^*, \\
    \text{FP}_4: & \quad \alpha_y^* = \frac{F_2}{E} \alpha_2^* + \frac{F_3}{E} \alpha_3^*,
\end{align*}
\]

\[(25)\)

depending on whether $\alpha_2$, or $\alpha_3$, or both, take interacting fixed points by themselves. Combining \[(25)\] with the vanishing of the gauge beta functions provides explicit expressions for the different fixed points. An overview of fixed points and their properties is given in Tab. 2. Next we analyse minimal conditions that need to be fulfilled in the BSM sector in order to generate partially or fully interacting UV fixed points in the system \[(22)\].

D. Partially interacting fixed points

The partially interacting fixed points FP$_2$ and FP$_3$ are characterised by one of the gauge couplings, say $\alpha_{AS}$, taking an asymptotically safe fixed point in the UV whereby the other gauge coupling, say $\alpha_{AF}$, becomes asymptotically free. The Yukawa couplings must take interacting values, $\alpha_y^* \propto \alpha_{AS}^*$, see \[(25)\]. The beta functions \[(22)\] then take the simplified form

\[
\begin{align*}
    \beta_{AS} &= (-B_{AS} + C_{AS} \alpha_{AS} - D_{AS} \alpha_y) \alpha_{AS}^2, \\
    \beta_y &= (E \alpha_y - F_{AS} \alpha_{AS}) \alpha_y.
\end{align*}
\]

\[(26)\]

These expressions formally agree with \[(1)\] and therefore offer the same type of fixed point solutions. The non-trivial UV fixed point is then of the form \[(5)\], \[(6)\], after substituting the appropriate loop
coefficients. A minimal requirement for partially interacting fixed points to be UV fixed points is the loss of asymptotic freedom in the gauge sector $B_{\text{AS}} < 0$, meaning either

\[
\text{FP}_2: \quad N_F > 19/(8 S_2(R_2) d(R_3)),
\]

or

\[
\text{FP}_3: \quad N_F > 21/(4 S_2(R_3) d(R_2)),
\]

thus reverting the condition (24). Associating suitable charges to the BSM fermions, it is then possible to satisfy either of the conditions in (27). Furthermore, the physicality condition (7) translates into

\[
\text{FP}_2: \quad D_2 F_2 - E C_2 > 0, \\
\text{FP}_3: \quad D_3 F_3 - E C_3 > 0.
\]

It remains to evaluate solutions to the conditions (28) separately for FP$_2$ and FP$_3$, to which we turn next.

**Strong strong and weak weak gauge coupling.** In Fig. 1 we analyse the condition (28) exemplarily for FP$_3$ where the strong coupling remains interacting in the deep UV whereas the weak coupling vanishes asymptotically. We assume that the BSM fermions carry no $SU(2)_L$ charges ($R_2 = 1$), but different $SU(3)_C$ representations $R_3 = 3, 6, 8$ and 10. We observe the following pattern. For fermions in the fundamental, a narrow window of weakly interacting fixed points exists for a low number of flavors $N_F$. These low-$N_F$ solutions come out as IR fixed points in that they relate to settings with asymptotic freedom in both gauge sectors (see the discussion in Sec. IV E). With increasing $N_F$, the fixed point takes negative values and becomes unphysical. Conversely, for fermions in higher-dimensional representations (anything but the fundamental), we find that a fixed point exists for sufficiently large $N_F$. No fixed points exist for intermediate values of $N_F$. Occasionally we find that fixed points can exist for exceptionally low values of $N_F$, in which case the fixed point is IR rather than UV. In Fig. 2 (middle panel), we show the set of parameters $(R_2, R_3, N_F)$ for which FP$_3$ exists as an interacting UV fixed point. In Tab. 3 we

| $R_2$ | $\ell$ | $C_2(R_2)$ | $S_2(R_2)$ | $R_3 = 1$ | $R_3 = 3$ | $R_3 = 6$
|-------|--------|-------------|-------------|------------|------------|------------|
|       |        |             |             | $N_{AF}$  | $N_{AS}$   | $N_{AF}$  |
| 2     | $\frac{3}{2}$ | $\frac{3}{4}$ | $\frac{1}{2}$ | 4          | –          | 1          |
| 3     | 1      | 2           | 2           | 1          | (26) 53    | –          |
| 4     | $\frac{3}{2}$ | $\frac{5}{4}$ | $\frac{5}{4}$ | –          | (7) 9      | –          |
| 5     | 2      | 6           | 10          | –          | (6) 7      | –          |
| 6     | $\frac{5}{2}$ | $\frac{35}{4}$ | $\frac{35}{4}$ | –          | (6) 7      | (16) 17    |

Table 4. Asymptotic freedom versus asymptotic safety at the partially interacting fixed point FP$_2$: shown are the maximal numbers of BSM fermion flavors $N_F < N_{AF}$ compatible with asymptotic freedom, and the smallest number $N_F \geq N_{AS}$ required for a weakly-coupled asymptotically safe fixed point, both in dependence on the fermion representations $R_2$ and $R_3$ under $SU(2)_L$ and $SU(3)_C$, respectively. Values for $N_{AS}$ in brackets relate to the absolute lower bound, those without brackets to settings with $0 < \alpha^2, \alpha^y < 1$. Also indicated are the weight $\ell$, the quadratic Casimir, and the Dynkin index under $SU(2)_L$.
Figure 1. Partially interacting fixed point FP$_3$ with $\alpha^*_3 = 0$, showing the strong coupling $\alpha^*_3$ (blue, solid line) and the BSM Yukawa coupling $\alpha^*_y$ (red, dashed) versus the number $N_F$ of the BSM flavors for $R_2 = 1$ and different $SU(3)_C$ representations $R_3 = 3, 6, 8$ and $10$ (see main text).

summarise the minimum number of BSM fermions $N_{AS}$ which lead to a weakly coupled UV fixed point with $\alpha^* \leq 1$.

The pattern of results is easily understood once $N_F$ is sufficiently large. The necessary condition for existence $^{(28)}$ of FP$_3$ turns into a quadratic polynomial in $N_F$ after inserting the explicit expressions for the loop coefficients,

$$X N_F^2 + Y N_F - Z < 0,$$

with coefficients

$$X = Z C_2(R_3) \left[5 - 2C_2(R_3)\right]/52,$$

$$Y = Z^2 \left[C_2(R_3) + 5\right]/52 - C_2(R_3),$$

$$Z = C_2(R_3)d(R_3)d(R_2),$$

and with $C_2(R)$ and $d(R)$ defined in $^{(A8)}$. For sufficiently large $N_F$, the sign of the coefficient $X$ dictates whether the condition $^{(29)}$ provides an upper or a lower bound on $N_F$. If $X > 0$, the condition $^{(29)}$ provides an upper bound on the number of the BSM fermions. However, we observe that $X > 0$ if and only if the BSM fermions transform under the fundamental representation of $SU(3)_C$ (see Tab. $^{3}$ for explicit values of the Casimir invariant for several $R_3$ of the lowest dimension). In this case it is readily confirmed that a solution to $^{(29)}$ is incompatible with the lower bound from $^{(24)}$ for any choice of $R_3$, meaning that such a fixed point is necessarily an IR fixed point. We conclude that asymptotic safety via a partially interacting fixed point cannot
Figure 2. Availability of weakly interacting UV fixed points FP$_2$ (left panel), FP$_3$ (middle panel), and FP$_4$ (right panel) in dependence on the representation $(R_2, R_3)$ and the flavor multiplicities $N_F$ of BSM fermions. The pattern of results continues to higher $(R_2, R_3)$. Partially interacting fixed points FP$_2$ are absent for any $N_F$ as soon as $R_2 = 1$ or 2; FP$_3$ is absent whenever $R_3 = 1$ or 3; fully interacting UV fixed points FP$_4$ are absent for $R_3 = 1$ or $(R_2, R_3) = (1, 8), (2, 3),$ and $(1, 3)$.

be achieved within the fundamental representation of $SU(3)_C$. On the other hand, for higher-dimensional representations the coefficient $X$ becomes negative. Consequently, (29) provides a lower bound on the number of BSM fermions required to achieve asymptotic safety, $N_F \geq N_{AS}$. The case $X = 0$ has no physical solutions. Exemplary values for the lower bound, $N_F \geq N_{AS}$ for different representations $R_3$ and $R_2$ are given in Tab. 3 where we additionally require weak coupling $\alpha_i^* < 1$ at the fixed point.

Strong weak and weak strong gauge coupling. Next we turn to FP$_2$ where the weak sector remains interacting in the deep UV whereas the strong coupling becomes asymptotically weak. Qualitatively, our findings for FP$_2$ are very similar to those discussed previously for FP$_3$. The absence of asymptotic freedom in the $SU(2)_L$ gauge sector, (27), requires a minimal number of BSM fermion flavors $N_F \geq N_{AF}$. In Fig. 2 (left panel), we show the set of parameters $(R_2, R_3, N_F)$ for which FP$_2$ exists as an interacting UV fixed point. In Tab. 4 we provide $N_{AF}$ for $SU(3)_C$ singlets, triplets and sextets, and for different dimensions of the $SU(2)_L$ representations. Since the SM contribution to the one-loop gauge coefficient is larger for $SU(2)_L$ than for $SU(3)_C$, lower values for $N_F$ and lower dimensions of representations are required to lose asymptotic freedom for $SU(2)_L$. Similarly, from (28) we find that asymptotic safety cannot be achieved with fermions in the fundamental representations of $SU(2)_L$. The minimal number of BSM fermion flavors required for a weakly-coupled asymptotically safe fixed point, $N_{AS}$, are given in Tab. 4 for various choices of $R_2$ and $R_3$. 
Next we discuss the fate of the gauge coupling which vanishes at partially interacting fixed points FP₂ or FP₃, and which we denote for notational simplicity as $\alpha_{AF}$. The coupling $\alpha_{AF}$ must be asymptotically free for a partially interacting fixed point to be viable, or else the UV fixed point cannot be reached by any finite RG trajectory along the $\alpha_{AF}$ direction. In general, we find that $B_{AF}$ becomes negative as soon as BSM fermions carry charges of both gauge groups. However, the sign of $B_{AF}$ plays no role, as it no longer dictates whether this sector remains asymptotically free or not. Rather, to leading order in the asymptotically free gauge coupling, we have

$$\beta_{AF} = \left(-B_{AF} + G_{AF} \alpha_{AS}^* - D_{AF} \alpha_y^*\right) \alpha_{AF}^2 + O(\alpha_{AF}^3),$$

(31)

showing that the one loop coefficient $B_{AF}$ is replaced by $B'_{AF} = B_{AF} - G_{AF} \alpha_{AS}^* + D_{AF} \alpha_y^*$. We stress that this shift is a consequence of partially interacting fixed points. It arises from residual interactions at the UV fixed point due to asymptotic safety of the gauge coupling $\alpha_{AS}$ and the BSM Yukawa coupling. Their residual interactions modify the running of the asymptotically free coupling owing to fermions which carry charges under both gauge groups. Provided that the shifted one loop coefficients $B'$ take positive values,

$$B'_{AF} > 0,$$

(32)

the non-interacting gauge sector becomes asymptotically free in the deep UV. We also stress that the BSM Yukawa interactions play a central role: only Yukawa couplings add negatively to the beta function (31). Without them, (32) cannot be achieved starting from $B_{AF} < 0$. Using (22),

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Asymptotic freedom at partially interacting fixed points FP₃. Shown is the condition for asymptotic freedom (32) for the effective coefficient $B'_{2}$ in units of the two loop coefficient $C₂ > 0$ at FP₃ for, exemplarily, $R₂ = 1$ (left panel) and $R₂ = 3$ (right panel), and as a function of $R₃$ (color coding given in the legend). We observe that asymptotic freedom in the weak sector is regained as soon as $R₂ > 1$ and $R₃ > 3$, for any $N_F$. For $R₂ = 1$, a lower bound on $N_F$ is found (left panel). Qualitatively and quantitatively similar results are obtained at FP₂ (not displayed).}
\end{figure}

E. Regaining asymptotic freedom
we have the following expressions for the shifted one loop coefficients

\[
\begin{align*}
\text{FP}_2: \quad & B_3 \rightarrow B_3' = B_3 - G_3 \alpha_2^* + D_3 \alpha_y^*, \\
\text{FP}_3: \quad & B_2 \rightarrow B_2' = B_2 - G_2 \alpha_3^* + D_2 \alpha_y^*.
\end{align*}
\] (33)

We conclude that (32), (33) are necessary conditions for the corresponding partially interacting fixed point to qualify as UV completions of the SM.

In Fig. 3 the condition for asymptotic freedom (33) at FP_3 is shown for models with \( R_2 = 1 \) (left panel) and \( R_2 = 3 \) (right panel) and various \( R_3 > 3 \) (recall that there are no viable UV fixed points FP_3 for \( R_3 \leq 3 \), Fig. 2). If \( R_2 = 1 \), we observe that \( B_2' \) is positive for sufficiently large \( N_F \), and negative for sufficiently low \( N_F \), thus leading to a lower bound. Conversely, if \( R_2 = 3 \) (or larger), the sign of \( B_2' \) is always positive. In this case asymptotic freedom is guaranteed without any further constraints as soon as \( \alpha_3 \) is asymptotically safe. The same pattern of results holds true for FP_2. We conclude that as soon as the BSM fermions carry a non-trivial charge under the asymptotically free coupling \( R_{AF} \neq 1 \), for any \( R_{AS} \neq 1 \), the condition (32) follows from the condition for asymptotic safety for \( \alpha_{AS} \) (28). For BSM fermions with \( R_{AF} = 1 \), (32) entails an additional lower bound on \( N_F \).

F. Fully interacting fixed points

Finally we consider the case FP_4. In the case where both gauge couplings and the BSM Yukawa coupling remain weakly interacting at the fixed point in the asymptotic UV the overall behaviour of the system depends on the interplay between one- and two-loop coefficients. Using the results of Tab. 2, the necessary condition for a fixed point can be stated as

\[
\begin{align*}
\alpha_2^* &= \frac{C'_3 B_2 - B_3 G'_2}{C'_2 C'_3 - G'_2 G'_3} > 0, \quad \alpha_3^* = \frac{C'_2 B_3 - B_2 G'_3}{C'_2 C'_3 - G'_2 G'_3} > 0,
\end{align*}
\] (34)

with primed two-loop coefficient given in (A11). Unlike the condition for partially interacting fixed points (28), those for fully interacting ones involve ratios of differences of Yukawa-shifted loop coefficients. In particular, fully interacting fixed points may exist even if only one of the conditions (28) is satisfied. For the purpose of this work, we have investigated the conditions (34) numerically. In Fig. 2 (right panel), we show the set of parameters \( (R_2, R_3, N_F) \) for which FP_4 exists as an interacting UV fixed point. Our results for the lowest number of flavor multiplicities \( N_F \geq N_{AS} \) are summarised in Tab. 5.

G. Large-\( N_F \) approximation

Some analytical insights about interacting UV fixed points can be obtained in the limit of many flavors of fermions \( N_F \gg 1 \), which we discuss separately for either type of fixed point.

**Partially interacting fixed points.** For a partially interacting fixed point, and using the explicit solution for FP_3 as given in Tab. 2 the large-\( N_F \) approximation leads to

\[
(\alpha_3^*, \alpha_2^*, \alpha_y^*) \bigg|_{N_F \gg 1} = \frac{1}{X_3} \left( \frac{1}{3}, 0, \frac{2C_2(R_3)}{N_F} \right) + \text{subleading},
\] (35)
Table 5. The minimal number of the BSM fermions flavors \( N_F \geq N_{AS} \) required for the fully interacting fixed point \( FP_4 \) to exist, in dependence on the fermion representations \( R_2 \) and \( R_3 \) under \( SU(2)_L \) and \( SU(3)_C \). The values for \( N_{AS} \) in brackets relate to the absolute lower bound, and those without to settings with \( 0 < \alpha_3^*, \alpha_2^*, \alpha_y^* < 1 \).

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
FP_4 & R_3 = 1 & R_3 = 3 & R_3 = 6 & R_3 = 8 & R_3 = 10 \\
\hline
R_2 & N_{AS} & N_{AS} & N_{AS} & N_{AS} & N_{AS} \\
\hline
1 & – & – & (130) 130 & – & (21) 21 \\
\hline
2 & – & – & (29) 35 & (45) 56 & (27) 39 \\
\hline
3 & – & (23) 28 & (27) 30 & (38) 43 & (33) 35 \\
\hline
4 & – & (17) 18 & (26) 28 & (36) 39 & (37) 39 \\
\hline
5 & – & (15) 16 & (27) 28 & (36) 38 & (40) 42 \\
\hline
\end{array}
\]

where \( X_3(R_3) = 2C_2(R_3) - 5 \). The subleading terms are at least one power in \( N_F \) smaller than the leading order terms. Several observations can now be made. First of all, positivity of the fixed point couplings requires \( X_3(R_3) > 0 \) or \( C_2(R_3) > \frac{5}{2} \). Hence, our result confirms that asymptotic safety cannot be achieved within the fundamental representations of \( SU(3)_C \) even at large-\( N_F \), owing to \( X_3(\text{fund.}) < 0 \). Secondly, we observe that the Yukawa coupling scales like \( 1/N_F \) and can always be made arbitrarily small. Conversely, the size of the gauge coupling is solely determined by the quadratic Casimir \( C_2(R_3) \), and independent of \( N_F \) in the large-\( N_F \) limit. We stress that (35) is parametrically close to the Gaussian fixed point, provided that \( X_3 \) becomes parametrically large.

In addition, the necessary condition for asymptotic freedom (32) for the weak coupling simplifies to leading order at large-\( N_F \) and reads \( C_2(R_3) > \frac{5}{2} \). Interestingly, the condition for asymptotic freedom exactly coincides with the condition for asymptotic safety of (35) at large-\( N_F \),

\[
C_2(R_3) > \frac{5}{2}, \quad \text{or} \quad R_3 \geq 6 . \tag{36}
\]

We conclude that higher dimensional representations under \( SU(3)_C \) with (35) are favoured for the theory to display a perturbative UV fixed point in the \( SU(3)_C \) coupling, and for \( SU(2)_L \) sector to regain asymptotic freedom at the partially interacting UV fixed point \( FP_3 \), see Fig. 1. Analogous results are established for \( FP_2 \), where the large-\( N_F \) expansion starts off with

\[
\left(\alpha_3^*, \alpha_2^*, \alpha_y^*\right)_{N_F \gg 1} = \frac{1}{X_2} \left(0, \frac{1}{2}, \frac{3C_2(R_2)}{N_F}\right) + \text{subleading} , \tag{37}
\]

and \( X_2(R_2) = 3C_2(R_2) - 5 \). Again, subleading terms are suppressed by at least one additional power in \( N_F \) over the leading terms. A necessary condition for asymptotic safety is \( X_2(R_2) > 0 \), thus excluding the fundamental representation owing to \( X_2(\text{fund.}) < 0 \). We also conclude that \( FP_2 \) is parametrically close to the Gaussian fixed point in the limit of high-dimensional representations \( X_2 \). Furthermore, to leading order at large-\( N_F \) the condition for asymptotic freedom (32) for the strong coupling becomes

\[
C_2(R_2) > \frac{5}{3}, \quad \text{or} \quad R_2 \geq 3 . \tag{38}
\]
Once more, this secondary condition coincides with the condition for asymptotic safety of 37).

**Fully interacting fixed points.** For the fully interacting fixed point, using the explicit solution for FP\textsubscript{4} as given in Tab. 2 and performing a large-$N\textsubscript{F}$ limit, we find

$$
\left(\alpha_{3}^{\ast}, \alpha_{2}^{\ast}, \alpha_{y}^{\ast}\right)\bigg|_{N\textsubscript{F} \gg 1} = \frac{1}{X_{32}} \left(\frac{1}{3}, \frac{1}{2}, \frac{2C_{2}(R_{3}) + 3C_{2}(R_{2})}{N\textsubscript{F}}\right) + \text{subleading},
$$

with $X_{32} = 2C_{2}(R_{3}) + 3C_{2}(R_{2}) - 5$. The above expression holds true provided that $R_{3} \neq 1$ and $R_{2} \neq 1$. The requirement of asymptotic safety results in an inequality $X_{32} > 0$. Furthermore, the result also shows that the fully interacting fixed point is parametrically close to the Gaussian provided that $X_{32}$ is large. The explicit result explains why a fully interacting fixed point with asymptotic safety can be achieved even with BSM fermions in the fundamental representation of $SU(3)_{C}$, as long as they transform under $SU(2)_{L}$ in a representation of a dimension higher than the fundamental. Analogously, FP\textsubscript{4} exists for BSM fermions in the fundamental representation of $SU(2)_{L}$ provided that $R_{3} > 3$. Note that these large-$N\textsubscript{F}$ estimates are in very good agreement with the numerical findings in Tab. 5. In the special case where $R_{3} > 1$ and $R_{2} = 1$, and instead of (39), one obtains

$$
\left(\alpha_{3}^{\ast}, \alpha_{2}^{\ast}, \alpha_{y}^{\ast}\right)\bigg|_{N\textsubscript{F} \gg 1} = \left(\frac{1}{3X_{32}}, \frac{19}{35} - \frac{24}{35X_{32}}, \frac{2C_{2}(R_{3})}{N\textsubscript{F}X_{32}}\right) + \text{subleading},
$$

**Figure 4.** Summary of weakly interacting UV fixed points of (22) in dependence on the fermion representation and flavor multiplicities ($R_{3}, R_{2}, N\textsubscript{F}$). The different symbols relate to FP\textsubscript{2} (gray circle), FP\textsubscript{3} (blue square) and FP\textsubscript{4} (red diamond). Overlapping symbols indicate that either type of fixed point can exist, with lower-lying symbols relating to fixed points which arise at a higher number of fermion flavors $N\textsubscript{F}$, see also Fig. 2.
with \( X_{32} = 2C_2(R_3) + 3C_2(R_2) - 5 \) as before. Notice that this fixed point is parametrically close to the “would-be” Banks-Zaks fixed point in the \( SU(2)_L \) sector of the SM. The condition for existence is now given by \( X_{32} > \frac{24}{73} \) which translates into \( C_2(R_3) > \frac{3}{8} \). Solutions are given by the \( R_3 = 6 \) and \( R_3 \geq 10 \) representations under \( SU(3)_C \). Curiously, the adjoint representation \( R_3 = 8 \) with \( R_2 = 1 \) is not a solution of (40) owing to the “would-be” Banks-Zaks IR fixed point. Finally, for \( R_3 = 1 \) and \( R_2 \geq 1 \) one readily confirms that \( \alpha_3^c \) and \( \alpha_5^c \) cannot simultaneously take positive values meaning that an asymptotically safe fixed point does not arise at large \( N_F \).

### H. Synopsis of UV fixed points

We are now in a position to summarise the main results for weakly interacting UV fixed points in extensions of the SM of the form (18). We have observed that interacting UV fixed points can arise as partially or fully interacting ones. In either of these cases, necessary conditions for their existence have been found, providing us with constraints on the remaining BSM parameters \((R_3, R_2, N_F)\). We have also observed that for fixed \((R_3, R_2)\), UV fixed points typically exists for all \( N_F \) down to limiting values specified in Tab. 3, 4 and 5. Fig. 4 shows a summary of our findings, in dependence on the fermion representation \((R_3, R_2)\) under \( SU(3)_C \otimes SU(2)_L \) with different symbols relating to the different fixed points \( FP_2, FP_3, \) and \( FP_4 \). Broadly speaking, results show the existence of fixed points both with increasing dimensionality of the fermion representation, and with increasing flavor multiplicities. Similarly, fixed points come out less strongly coupled the larger their dimensionality \( R_2, R_3 \) and the flavor multiplicity \( N_F \). We also observe that different types of fixed points might coexist for BSM fermions with the same set of representations \((R_2, R_3)\), starting from a lowest value for \( N_F \) where the fixed point arises for the first time. In Fig. 4 the possibility of coexistence is indicated by overlapping symbols: the lower lying symbol relates to a fixed point which arises for larger \( N_F \).

Hence, four distinct cases arise: (i) For high dimensional representations, starting from \( R_2 = 3 \) and \( R_3 = 6 \) onward, all three types of fixed points are realised starting from some lowest value for \( N_F \). For fixed \((R_2, R_3)\) but with increasing \( N_F \) results show that the fully interacting fixed point \( FP_4 \) is achieved first, followed by the partially interacting \( FP_3 \) for the strong coupling, and ultimately followed by the partially interacting fixed point \( FP_2 \) in the weak coupling. (ii) If the fermions are in the fundamental representation of one of the two gauge groups, we find that the corresponding partially interacting fixed point is absent throughout. However, the other two fixed points still exist and the order in which they appear, with increasing \( N_F \), is exactly the same as the order observed for the higher dimensional representations. (iii) If the BSM fermions are uncharged under \( SU(3)_C \), only the partially interacting fixed point \( FP_2 \) can arise, starting from \( R_2 = 3 \) onwards. (iv) If the BSM fermions are uncharged under \( SU(2)_L \), we find that \( FP_3 \) arises first, followed by \( FP_4 \), while \( FP_2 \) is absent throughout. This holds true for all \( R_3 = 6 \) or higher, except for \( R_3 = 8 \) where only \( FP_3 \) appears.

Finally, we discuss the status of interacting fixed points for low numbers of flavors \( N_F \). The low-\( N_F \) partially interacting fixed points \( FP_3 \) at \( R_2 = 1 \) with \( R_3 = 3, 6 \) and \( 8 \) all have \( B_2, B_3 > 0 \), [24]. Hence the theory remains asymptotically free in both gauge group factors, and the interacting fixed point is formally an IR fixed point of the type discussed in Tab. 1b). Similarly, for \( FP_2 \) and \( FP_4 \) we find a handful of low-\( N_F \) fixed points all of which occur where asymptotic freedom persists in both gauge groups [24]. Conversely, we also have low-\( N_F \) fixed points \( FP_3 \) with \( R_2 = 2, \cdots, 10 \) which have asymptotic freedom only in the strong gauge coupling, while the weak sector has become infrared free. Such fixed points are not of phenomenological interest because they cannot
be linked with any finite $\alpha_2 \neq 0$ in the IR and shall be dropped.

This completes our investigation of weakly coupled UV fixed points of (18) to the leading non-trivial order in perturbation theory, (22). In the next section, we explain whether and how these fixed points are connected with the SM at low energies under the RG evolution of couplings.

IV. MATCHING ONTO THE STANDARD MODEL

In this section, we evaluate the conditions under which BSM trajectories emanating out of interacting UV fixed points are connected with the SM at low energies.

A. Matching conditions

Any RG trajectory emanating from free or interacting UV fixed points qualifies as a UV complete quantum field theory. The UV critical surface then determine the set of UV-safe trajectories. The relevant or marginally relevant couplings in the UV determines the dimensionality of the UV critical surface $\mathfrak{d}$. Conversely, the irrelevant couplings are uniquely fixed by the relevant couplings in the UV. Consequently, the number of fundamentally free parameters which characterise the UV-safe trajectories is given by the dimensionality of the UV critical surface. At low energies, physically viable BSM trajectories must connect with those of the SM, see Fig. 5 as soon as the BSM matter fields have decoupled.

It remains to check whether the UV fixed points discovered in the previous section are connected
through well-defined RG trajectories to the SM at low energies. Away from the fixed point, BSM matter fields will develop scalar and fermion masses $M_S$ and $M_\psi$ which are independent parameters of the theory. Phenomenological constraints for $M_S$ and $M_\psi$ are worked out in Sec. $V$ below. For RG scales much larger than the masses, the BSM matter fields are effectively massless, and the RG flow is given by (22). Conversely, for RG scales much lower than the masses, the BSM fields are taken to be infinitely heavy and decouple. The RG flow (22) reduces to the one of the SM, also restoring confinement of QCD at low energies. Hence, the BSM contributions to the running gauge couplings decouple as soon as $\mu$ is of the order of the BSM fermion mass. Furthermore, threshold effects are subleading to the overall picture and will be neglected. Consequently, the RG flow of the SM is matched onto the RG flow of the BSM extension at the matching scale $M$, 

$$\mu = M \approx M_\psi$$  \hspace{1cm} (41) 

below which the BSM fermions decouple. This leads to matching conditions between the RG flow of the SM at scales below (41), and BSM flows (22) above the mass scale (41),

$$\alpha_i(\mu = M) \bigg|_{\text{SM}} = \alpha_i(\mu = M) \bigg|_{\text{BSM}},$$  \hspace{1cm} (42) 

for $i = 2, 3$ (or $i=\text{AF, AS}$ in settings with partially interacting fixed points). There will be no matching condition for the BSM Yukawa coupling since it is not part of the SM. Rather, after the decoupling of the BSM fields, the Yukawa coupling will “freeze out” at its value at decoupling. For the quantitative studies below, we use PDG SM reference values at the scale of the $Z$ pole mass $\mu = m_Z$,

$$\alpha_2(\mu = m_Z) = 2.7 \times 10^{-3},$$  

$$\alpha_3(\mu = m_Z) = 9.5 \times 10^{-3},$$  \hspace{1cm} (43) 

together with the two loop perturbative running of gauge couplings in the SM, using (22) with $N_F = 0$. Fig. 5 illustrates the SM running between the mass of the $Z$ boson ($m_Z = 91.19$ GeV) and Planckian energies. Note that equality of gauge couplings

$$\alpha_2(\mu) = \alpha_3(\mu)$$  \hspace{1cm} (44) 

arises in the SM at the GUT scale $\mu_{GUT} \approx 4 \times 10^{15}$ GeV. We emphasize that the matching of the BSM extension (22) onto the SM (42) takes place at perturbatively small couplings.

**B. Partially interacting fixed points**

As has been detailed in Sec. $III_B$ at partially interacting fixed points FP$_2$ and FP$_3$, one of the two gauge couplings becomes asymptotically free, while the other one becomes asymptotically safe. Moreover, the asymptotically (free) safe coupling is (marginally) relevant and, hence, the UV critical surface is invariably two-dimensional. On the other hand, the BSM Yukawa coupling $\alpha_y$ is irrelevant and fully specified by the asymptotically safe coupling in the UV.

In this light, a convenient choice for the two fundamentally free dimensionless parameters which characterise UV-safe trajectories running out of the fixed point are the deviations of the gauge
Table 6. UV fixed points and matching characteristics for various benchmark scenarios. An asterisk indicates that a matching is permitted at any scale including low (TeV) energy scales.

| model | parameter $(R_3, R_2, N_F)$ | UV fixed points $\alpha_3^\ast$, $\alpha_2^\ast$, $\alpha_y^\ast$ | type (Fig. 4) | info |
|-------|-----------------------------|-------------------------------------------------|-----------------|------|
| A     | $(1, 4, 12)$                | 0.2407, 0.1158, 0.0995                           | $\bullet$ FP$_2$| low scale$^\ast$ |
| B     | $(10, 1, 30)$               | 0.3317, 0.0752, 0.1500                           | $\bullet$ FP$_3$| low scale$^\ast$ |
| C     | $(10, 4, 80)$               | 0.0503, 0.0752, 0.0292                           | $\bullet$ FP$_4$| high scale |
| D     | $(3, 4, 290)$               | 0.0416, 0.0615, 0.0056                           | $\bullet$ FP$_2$| low scale$^\ast$ |
| E     | $(3, 3, 72)$                | 0.1499, 0.2181, 0.0471                           | $\bullet$ FP$_4$| low scale |

Table 6. UV fixed points and matching characteristics for various benchmark scenarios. An asterisk indicates that a matching is permitted at any scale including low (TeV) energy scales.

couplings from their UV fixed point values at some high scale $\mu = \Lambda$,

$$\delta \alpha_i(\Lambda) = \alpha_i^\ast - \alpha_i(\Lambda), \quad (45)$$

with $i = 2, 3$ (or $i=\text{AF,AS}$). We take the practical view that the high scale is essentially given by the Planck scale. Quantum gravity effects should be retained at scales close to and above $\Lambda$. The BSM Yukawa coupling is an irrelevant coupling and entirely dictated by the UV hypercritical surface relating it with $\alpha_{\text{AS}}$ and $\alpha_{\text{AF}}$,

$$\alpha_y = F_y(\alpha_{\text{AS}}, \alpha_{\text{AF}}). \quad (46)$$

The parameters (45) will be used to match trajectories onto the SM. Specifically, the parameter $\delta \alpha_{\text{AS}}$ controls at which energy scale the asymptotically safe coupling is crossing over from the UV fixed point towards the Gaussian IR fixed point of (22). For $0 < \delta \alpha_{\text{AS}} \ll 1$, $\alpha_{\text{AS}}$ will start out of the UV fixed point along the separatrix which connects the UV fixed point with the Gaussian in the IR. In the immediate vicinity of the UV fixed point the RG flow is of the power-law type and thus fast, controlled by the relevant scaling exponent. Further away from the fixed point, as soon as $\alpha_{\text{AS}} \approx \frac{2}{3} \alpha_{\text{AS}}^\ast$ and below [13], we observe a cross-over whereby the running becomes logarithmically slow instead, dominated by the “would-be” Gaussian IR fixed point of (22). Hence, the parameter $\delta \alpha_{\text{AS}}$ allows us to chose at which scale $\alpha_{\text{AS}}(M)$ has reached the desired SM value. Notice that this discussion is largely independent of $\alpha_{\text{AF}}$ provided the latter remains small.

The running of $\alpha_{\text{AF}}$ out of the UV fixed point is controlled by the RG flow (31), which in turn is largely determined by the parameter $\delta \alpha_{\text{AF}}$, together with the coefficients $B_{\text{AF}}$ and $B_{\text{AF}}'$. Integrating (31) close to the UV fixed point gives

$$\frac{1}{\alpha_{\text{AF}}(\mu)} = \frac{1}{\delta \alpha_{\text{AF}}(\Lambda)} + B_{\text{AF}}' \ln(\mu/\Lambda). \quad (47)$$
One might expect that the two free parameters \(^{45}\) are sufficient to match the RG flow in the UV to two preset values at low energies. We stress, however, that a matching may fail if the “would-be” asymptotically free coupling \(\alpha_{AF}\) runs into Landau poles at intermediate energies. Thus, we must explain how Landau poles are avoided. In the deep UV, we have that \(B'_{AF} > 0\), \(^{32}\). However, the coefficient \(B'_{AF}\) depends on the asymptotically safe gauge coupling \(\alpha_{AS}\) and on the Yukawa coupling. Both of these run out of the UV fixed point and induce an effective running of \(B'_{AF} \to B'_{AF}(\mu)\) owing to \(^{51}\). For sufficiently small \(\alpha_{AS}\) such as close to the matching scale \(\mu = M\), the coefficient \(B'_{AF}\) falls back onto the BSM one loop coefficient \(B'_{AF} \to B_{AF}\). Close to the matching scale the one loop approximation is viable and we have

\[
\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + B_i \ln(\mu/M) \tag{48}
\]

for both \(i = AF, AS\). If \(B_{AF} > 0\), meaning that the gauge sector remains asymptotically free, we have that \(B'_{AF}(\mu) > 0\) at all intermediate scales and matching will always be possible. If \(B_i < 0\), the one loop running \(^{48}\) for the gauge couplings reach a “would-be” Landau pole at

\[
\frac{\mu_i}{M} = \exp \left( -\frac{1}{B_i \alpha_i(M)} \right). \tag{49}
\]

For \(\alpha_{AS}\) the Landau pole is avoided automatically owing to the two loop Yukawa terms: with growing energy, once the scale \(\mu_{AS}\) is reached, the two loop terms kick in and \(\alpha_{AS}\) settles into its UV fixed point, see Sec. \(\Pi\). For \(\alpha_{AF}\) it is not guaranteed that \(B'_{AF}(\mu)\) changes sign in time for \(\alpha_{AF}\) to avoid the Landau pole. We find that \(\alpha_{AF}\) avoids a Landau pole provided that

\[
B_{AF} \cdot \alpha_{AF}(M) < B_{AS} \cdot \alpha_{AS}(M). \tag{50}
\]

The condition \(^{50}\) ensures that the “would-be” one loop Landau pole for \(\alpha_{AF}\) arises at higher scales \(\mu_{AF} > \mu_{AS} > M\) than the one for \(\alpha_{AS}\). The crucial point about scales \(\mu \approx \mu_{AS}\) is that the two loop terms have become active. Two loop terms also contribute to the running of \(\alpha_{AF}\) and thereby ensure that the sign of \(B'_{AF}\) has become positive. We conclude that \(^{50}\) is sufficient to provide an upper bound for viable matchings to the SM, ensuring that neither of the couplings escapes a successful matching through a Landau pole at intermediate energies. Within the confines of \(^{50}\), this enables us to match SM and BSM running onto each other essentially at any scale between TeV and Planckian energies.

### C. Fully interacting fixed points

All fully interacting UV fixed points are characterized by a stability matrix \(^{8}\) with a single relevant eigenvalue. This important result states that a linear combination of the gauge groups’ kinetic terms together with the BSM Yukawa interaction term in the fundamental Lagrangean \(^{18}\) correspond to the sole UV relevant operator in the theory. This result has important implications. Unlike in asymptotically free theories (or in asymptotically safe theories at partially interacting fixed points FP\(_2\) or FP\(_3\)) where every gauge coupling corresponds to a UV relevant direction, here, instead, the UV critical surface is of a lower dimensionality. This new effect is a consequence of competing gauge interactions in the UV. Most notably, it entails that the number of fundamentally independent parameters is reduced, leading to an enhanced level of predictivity.

In our models, the UV critical surface at fully interacting UV fixed points becomes one-
dimensional, parametrised by a single free parameter. Consequently, only one out of the three couplings \((\alpha_3, \alpha_2, \alpha_y)\) may be considered as an independent variable. For \(\text{FP}_4\), and without loss of generality we chose this to be \(\alpha_3\). The UV critical surface then uniquely determines the weak and the Yukawa coupling as functions of the strong coupling,

\[ \alpha_i = F_i(\alpha_3) \quad \text{for} \quad i = 2, y. \]  

Most importantly, the UV critical surface imposes a relation between the two gauge couplings which arises as a strict consequence of asymptotic safety at a fully interacting fixed point \(\text{FP}_4\). We may then use the dimensionless parameter

\[ \delta \alpha_3(\Lambda) = \alpha_3^* - \alpha_3(\Lambda) \]  

at the high scale \(\Lambda\) to parametrise all UV safe trajectories running out of the fully interacting UV fixed point \(\text{FP}_4\). The UV-IR connecting separatrix

\[ (\alpha_3, \alpha_2, \alpha_y)(\mu) \equiv (\alpha_3, F_2(\alpha_3), F_y(\alpha_3))(\mu) \]  

uniquely determines the relation between the strong and the weak gauge coupling for all scales above the matching scale. The role of the free parameter \(\delta \alpha_3(\Lambda)\) is to determine at which scale the curves \([53]\) display a cross-over from UV dominated running towards IR dominated running. The task to identify trajectories which can be matched onto the SM at some matching scale \(\mu = M\) reduces to analysing the separatrix \([53]\). Given that the set of determining equations is over-constrained, a successful matching cannot be guaranteed from the outset, meaning that the viability needs to be checked for each \(\text{FP}_4\). On the other hand, if trajectories emanating out of \(\text{FP}_4\) can be matched, the BSM extension implies a fundamental relation between both gauge couplings, which would not exist otherwise. In settings where \(\text{FP}_4\) exists alongside \(\text{FP}_2\), or \(\text{FP}_3\), or both, either of the partially interacting fixed points is the more relevant UV fixed point. Their UV critical dimensions are larger, UV-IR connecting trajectories can be found which link \(\text{FP}_2\) or \(\text{FP}_3\) with \(\text{FP}_4\). An example for this is discussed below in Fig. III.

To understand more explicitly how a matching of \(\text{FP}_4\) onto the SM depends on the fermion representations and flavor multiplicities, we evaluate the link between the gauge couplings as dictated by the UV critical surface, \([9]\). Since in the general case the separatrix cannot be resolved analytically, we use the critical surface approximation of \([9]\), see App. A for the technicalities, keeping in mind that far from the UV fixed point the critical surface may deviate from the one given by the separatrix. The relation \([51]\) then takes the simple linear form

\[ \alpha_2(M) = -X + Y \alpha_3(M), \]

\[ Y = -V_2/V_3, \]

\[ X = -\alpha_2^* + Y \alpha_3^*. \]  

The parameters \(V_i\) are related to the UV relevant eigendirection \([9]\) which characterises the UV critical surface. Using \([22]\), we find the explicit expressions

\[ X = \frac{B_3 D_2 - B_2 D_3}{C_2 D_3 - G_3 D_2}, \quad Y = \frac{C_3 D_2 - G_2 D_3}{C_2 D_3 - G_3 D_2}, \]  

in terms of the perturbative loop parameters. Whether a matching of trajectories \((\alpha_2, \alpha_3)(\mu)\) with
Figure 6. Low scale matching of the partially interacting fixed point FP2 onto the SM for the benchmark scenario A with $(R_3, R_2, N_F) = (1, 4, 12)$ and $M = 2$ TeV. BSM (SM) running is shown by full (dashed) lines. Around the cross-over scale the BSM running of $\alpha_2$ and $\alpha_y$ slows down from power-law to logarithmic. Notice that the running of the strong coupling is not modified by BSM fermions.

Equation (54) onto the SM is possible or not depends on the signs and magnitude of $X$ and $Y$. In the large-$N_F$ limit, we find

\[
X \bigg|_{N_F \gg 1} = \frac{21}{5} \frac{C_2(R_2) - \frac{19}{172} C_2(R_3)}{C_2(R_3) C_2(R_2) d(R_3) d(R_2)} \frac{1}{N_F} + \text{subleading},
\]

\[
Y \bigg|_{N_F \gg 1} = \frac{3}{2} + \text{subleading}. \tag{56}
\]

With increasing $N_F$ we observe that $Y > 0$ rapidly approaches the large $N_F$ limit $Y = \frac{3}{2}$. Conversely, $X$ may have either sign depending on the representation $(R_3, R_2)$, though not on $N_F$. Also, $X$ becomes parametrically small for high dimensional representations and for large flavor multiplicities $N_F$. If $X < 0$ we have $\alpha_2(M) > \alpha_3(M)$ indicating that a matching below GUT-type scales is impossible. Furthermore, (54) also implies that $\alpha_2(M) > -X$, stating that a matching becomes impossible at any scale if $-X$ becomes too large. On the other hand, if $X > 0$ matchings can be found to SM values, in particular at low scales where $\alpha_2(M) < \alpha_3(M)$. For intermediate and large values of $N_F$ the condition $X > 0$ becomes

\[
C_2(R_2) - \frac{19}{172} C_2(R_3) > 0. \tag{57}
\]

A few comments are in order:

(i) If $R_3 < 10$, the condition (57) is satisfied for any $R_2$. Non-trivial constraints arise from (57) once $R_3 = 10$ or higher (for example, $R_3 = 10$ necessitates $R_2 \geq 3$, and similarly for higher dimensional representations $R_3 > 10$).

\[\text{Note that this ratio is a direct consequence of the } SU(3)_C \text{ and } SU(2)_L \text{ gauge groups of the SM.}\]
(ii) An increasing flavor multiplicity $N_F$ is required for models with a low dimensional $SU(3)_C$ representation $R_3$ or high dimensional $SU(2)_L$ representation $R_2$ to ensure that the magnitude of $X$ stays within the limits compatible for a matching onto SM values. Also, increasing the dimension of the matter representations lowers $\alpha_3(M)$. For these cases it then follows that the matching can only take place at a high scale.

(iii) The linear approximation for the separatrix \([54]\) becomes exact once $R_2 = 1$. This can be seen as follows. Using \([55]\) we find $X = -\frac{19}{35}$ at $FP_4$ for any viable $(R_3, N_F)$. The exact same result is found if \([54]\) is evaluated at the Banks-Zaks IR fixed point of the weak gauge coupling in the SM $(\alpha_2^*, \alpha_3^*) = (\frac{19}{35}, 0)$, see \([A10]\). This result indicates that the UV critical surface \([54]\) at $FP_4$ coincides with the critical surface at the Banks-Zaks IR fixed point. The latter therefore controls the running of the weak coupling away from the fully interacting UV fixed point by directing all UV safe trajectories straight into the Banks-Zaks fixed point. As is shown more explicitly below, it is for this reason that a matching of $FP_4$ with $R_2 = 1$ onto the SM is impossible.

(iv) For all scenarios considered in this work, we find that the condition \([57]\) is a good estimator for the availability of a matching with the SM.

This completes the general discussion of matching conditions for UV safe trajectories onto the SM.

D. Benchmark scenarios

Let us now illustrate how the matching works in practice for a selection of benchmark scenarios, summarised in Tab. 6 covering low scale and high scale matchings.

**Benchmark scenario A.** For this setting we assume that the BSM fermions do not carry $SU(3)_C$ charges. Following on from our earlier discussion, $FP_2$ is the sole UV fixed point which may arise and neither $FP_3$ nor $FP_4$ are available, see Fig. 4. We consider the parameters

\[
(R_3, R_2, N_F) = (1, 4, 12)
\]

with RG trajectories displayed in Fig. 6. The matching scale $M$ may take any value between TeV and Planckian energies. In Fig. 6 for illustration, we have set it to the low value $M = 2$ TeV (vertical dashed line). Evidently, the running of the strong coupling is not modified by BSM matter and remains SM-like throughout. Once the matching scale is fixed, the model predicts the value of the (otherwise unconstrained) Yukawa coupling. For $M = 2$ TeV one obtains $\alpha_y(M) = 0.022$.

We have also indicated the cross-over scale where the running of the asymptotically safe couplings changes from power-law behavior in the deep UV to logarithmic behavior towards the IR (dashed vertical line). The cross-over scale is found to be around $\mu_{cr} \sim 3 \times 10^5$ GeV and much larger than the matching scale.

**Benchmark scenario B.** For this case we assume that the BSM fermions do not carry $SU(2)_L$ charges. From Fig. 4 it follows that solely $FP_3$, possibly in conjunction with $FP_4$ can arise. Informed by the results of Tabs. 3 and 5 we chose the parameters

\[
(R_3, R_2, N_F) = (10, 1, 30)
\]

to ensure that the model has both types of UV fixed points, $FP_3$ and $FP_4$. The partially interacting fixed point $FP_3$ can always be matched onto the SM at any scale. In Fig. 7 this is illustrated for a low matching scale $M = 2$ TeV. In this model, the running of the weak gauge coupling is
Figure 7. Low scale matching from the partially interacting fixed point FP$_3$ onto the SM for the benchmark scenario $B$ with $(R_3, R_2, N_F) = (10, 1, 30)$ and $M = 2$ TeV. BSM (SM) running is shown by full (dashed) lines. Around the cross-over scale the BSM running of $\alpha_3$ and $\alpha_y$ slows down from power-law to logarithmic. Notice that the running of the weak coupling is not modified by the BSM fermions.

Figure 8. Shown is the running of the gauge and BSM Yukawa couplings along the UV-IR connecting separatrix emanating from FP$_4$ for the benchmark model $B$, (59). The scale $\mu_0$ may take any value determined by the free parameter $\delta\alpha_3(\Lambda)$. The weak gauge coupling is attracted towards its “would-be” Banks-Zaks fixed point (weak BZ), see (A10), indicated by the dashed line. Consequently, the interacting UV fixed point cannot be matched onto the SM.
\( R_3 = 10, R_2 = 4, N_F = 80 \)

**Figure 9.** Low scale matching from the partially interacting fixed point FP\(_3\) onto the SM for the benchmark scenario \( C \) with \((R_3, R_2, N_F) = (10, 4, 80)\) and \( M = 2 \) TeV. BSM (SM) running is shown by full (dashed) lines. Around the cross-over scale the BSM running of \( \alpha_3 \) and \( \alpha_y \) slows down from power-law to logarithmic. Notice that the approach towards asymptotic freedom of the weak coupling is enhanced by BSM fermions.

not modified by BSM matter to the leading orders in perturbation theory. The crossover scale \( \mu_{\text{cr}} \sim 1.6 \times 10^4 \) GeV is an order of magnitude larger than the matching scale. Elsewise the same reasoning as in Fig. 6 applies. In contrast, the impossibility for a matching at \( \text{FP}_4 \) is illustrated in Fig. 8. The scale \( \mu_0 \) is arbitrary and can take any value upon tuning the UV parameter \( \delta \alpha_3(\Lambda) \). However, the UV safe trajectory emanating out of \( \text{FP}_4 \) is attracted towards the strongly coupled domain, owing to the Banks-Zaks IR fixed point at \((\alpha_2^*, \alpha_3^*) = (19, 0)\) in the weak sector, \( (A10) \). Consequently, the weak coupling cannot become weak in the IR and a matching to SM values is impossible at any intermediate scale due to the dominance of the Banks-Zaks fixed point. We stress that this pattern is a direct consequence of the BSM fermions being uncharged under \( SU(2)_L \). The non-availability of \( \text{FP}_4 \) persists for all models with \( R_2 = 1 \), in line with our discussion in Sec. IV C.

We now turn to benchmark scenarios where the BSM fermions carry both \( SU(2)_L \) and \( SU(3)_C \) charges. In these cases we find realisations for either of the partially interacting fixed points \( \text{FP}_2 \) and \( \text{FP}_3 \), as well as for the fully interacting fixed point \( \text{FP}_4 \).

**Benchmark scenario** \( C \). As soon as \( R_2 \geq 2 \) and \( R_3 \geq 3 \), and for sufficiently large \( N_F \), all three types of UV fixed points arise, see Fig. 4. To illustrate such settings, we consider the benchmark scenario \( C \) with parameters

\[
(R_3, R_2, N_F) = (10, 4, 80) \tag{60}
\]

which displays \( \text{FP}_2 \), \( \text{FP}_3 \) and \( \text{FP}_4 \) within the perturbative domain, see Tab. 6. In Fig. 9, we begin with \( \text{FP}_3 \) where \( \alpha_2^* = 0 \) in the deep UV. Once more we observe that the matching condition (50) on the one loop BSM parameters for the strong and the weak coupling can be satisfied at any scale between a few TeV and Planckian energies including the low matching scale \( M = 2 \) TeV chosen in
Fig. 10. High scale matching of a fully interacting fixed point FP\(_4\) onto the SM for the benchmark scenario \(C\) with \(M = 2 \times 10^{11}\) GeV. Once BSM matter fields are active, the weak coupling approaches asymptotic safety more rapidly than the strong coupling. See also Fig. 9 and Fig. 11.

Fig. 9. Furthermore, the weak coupling \(\alpha_2\) continues to decrease even directly below the matching scale, for any matching scale. For the weak gauge coupling the approach towards asymptotic freedom is accelerated over the SM rate owing to the two loop BSM Yukawa contributions which are winning over the contributions by the strong gauge coupling along the entire UV-IR connecting separatrix into the fixed point. This pattern is consistent with the matching condition (50), which is fulfilled for any intermediate scale. For the example shown in Fig. 9, the cross-over scale and the matching scale are separated by an order of magnitude.

In Fig. 10 we turn to the matching of the fully interacting fixed point FP\(_4\) corresponding to the same parameter set \(\{60\}\). Couplings run out of the UV fixed point FP\(_4\) along a unique separatrix (53) connecting FP\(_4\) with the Gaussian fixed point. The separatrix thereby imposes a link between \(\alpha_2\) and \(\alpha_3\). On the separatrix, and close to the fully interacting fixed point, the weak coupling is genuinely stronger than the strong coupling. At crossover, it becomes rapidly weaker than the strong coupling. The gauge couplings also weaken more rapidly than the BSM Yukawa coupling. For the parameters \(\{60\}\), the separatrix dictates that the unique matching scale onto SM values comes out comparatively high, with \(M \approx 2 \times 10^{11}\) GeV.

In Fig. 11 we consider the matching with FP\(_2\) where \(\alpha_3^* = 0\) in the UV. In this model, we find that a matching at FP\(_2\) is more strongly constrained compared to a generic partially interacting fixed point. The reason for this is the influence of FP\(_4\) on UV-IR connecting trajectories and the necessity to avoid an early Landau pole in the strong sector. Specifically, starting at some high scale \(\Lambda\) we observe that the weak and the Yukawa couplings decrease with energy, while the strong coupling increases towards the IR. For too small \(\delta\alpha_3(\Lambda)\) (45) the strong coupling does not grow fast enough. For too large \(\delta\alpha_3\) the strong coupling runs into a Landau pole at intermediate scales. Within a narrow window for \(\delta\alpha_3\), however, the growth of \(\alpha_3\) is tamed due to FP\(_4\). Then, trajectories are close to the separatrix connecting FP\(_2\) with FP\(_4\), with a cross-over scale \(\mu \approx 5 \times 10^{11}\) GeV, see Fig. 11. Below the cross-over scale, couplings are attracted towards FP\(_4\) (see Tab. \(\{6\}\) for the fixed
point values) which, however, is not reached exactly. Instead, at scales about \( \mu \approx 1.2 \times 10^{11} \) GeV the couplings are driven away from FP\( _4 \), now following the separatrix which connects FP\( _4 \) with the Gaussian. In consequence, we find that couplings can be matched onto the SM at a high matching scale of about \( M = 5 \times 10^{10} \) GeV, close to the matching scale found for FP\( _4 \).

This result is consistent with the “Landau pole avoidance condition” \( \text{(50)} \) derived in Sec. \( \text{IV B} \) which for the parameters \( \text{(60)} \) has no solutions for low matching scales. For example, at \( M = 2 \) TeV the SM predicts \( \alpha_3^\text{SM}(M) \approx 0.0067 \), yet the upper boundary \( \text{(50)} \) reads \( \alpha_3(M) \approx 0.004 \). Only for sufficiently high matching scales such as \( M \approx 5 \times 10^{10} \) GeV, the condition \( \text{(50)} \) eases up and allows a consistent matching without the strong coupling prematurely running into a perturbative Landau pole at intermediate scales.

We conclude that all three fixed points qualify as UV completions for the SM, although the specifics of the UV completion differ due to finer details of the fixed point structure.

**Benchmark scenario D.** For models with \( R_3 = 3 \) and \( R_2 \geq 3 \), and for sufficiently large \( N_F \), we have observed that the UV fixed points FP\( _2 \) and FP\( _4 \) coexist, see Fig. \( 4 \). To illustrate the matching procedure for these settings, we set the parameters as

\[
(R_3, R_2, N_F) = (3, 4, 290). \tag{61}
\]

The partially interacting fixed point FP\( _2 \) can be matched onto the SM, particularly at low matching scale \( M \) (not displayed). Results for FP\( _4 \) are displayed in Fig. \( 12 \). Couplings run out of the UV fixed point along the separatrix \( \text{(53)} \). Unlike the previous example of benchmark C \( \text{(60)} \), in this case couplings display a cross over at much lower energies. In particular, a matching to SM values is possible at a low scale of about \( M \approx 2.4 \) TeV. As explained after \( \text{(55)} \), the price to pay

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Figure 11. High-scale matching of a partially interacting fixed point FP\( _2 \) onto the SM for the benchmark scenario C with \( (R_3, R_2, N_F) = (10, 4, 80) \). Trajectories emanate out of the UV fixed point FP\( _2 \) and initially cross over into the vicinity of FP\( _4 \) (indicated by arrows) at around \( 1.5 \times 10^{11} \) GeV. Subsequently, trajectories display a second cross over to match with the SM at about \( M = 5 \times 10^{10} \) GeV.
Figure 12. Low scale matching from the fully interacting fixed point FP$_4$ onto the SM for the benchmark scenario $D$ with $(R_3, R_2, N_F) = (3, 4, 290)$ and matching scale $M = 2.4$ TeV. Once BSM matter fields are active, the weak coupling approaches asymptotic safety more rapidly than the strong coupling.

Figure 13. The matching procedure at FP$_4$ for the benchmark model $E$ with parameters (62). The thick lines show the BSM running along the UV-IR connecting separatrix as a function of the scale $\mu_0$, which may take any value determined by the free parameter $\delta \alpha_3(\Lambda)$. The dashed lines show the SM running of couplings from Fig. 5 provided that $\mu_0 = 1$ GeV. It is observed that SM and BSM values both coincide at the matching scale $M \approx 1.6$ TeV, see (42), indicated by the short-dashed vertical line.
Figure 14. Summary of matching conditions at fully interacting UV fixed points FP\(_4\) of the RG system \(^{(22)}\) in dependence on the BSM parameters \((R_3, R_2, N_F)\). Blue diamonds indicate low-scale matchings in the multi-TeV regime. Red dots stand for a high matching scale beyond the reach of present day colliders. Black triangles indicate scenarios where a matching onto SM values is not available despite of both gauge couplings approaching the Gaussian. Gray triangles indicate the unavailability of a matching due to strong coupling phenomena in the weak gauge sector \((R_2 = 1)\). Arrows additionally illustrate how the number of BSM fermion flavors \(N_F\) (blue arrow) and the matching scale \(\mu\) (red arrow) vary with the representation to ensure a successful matching.

is the necessity for a large multiplicity of flavors, significantly larger than \(N_{AS} = 18\) as minimally required for weakly coupled asymptotic safety to arise (see Tab. \([5]\)). It is worth contrasting the successful matching at FP\(_4\) with the failure for the benchmark scenario \(B\) \(^{(59)}\): unlike the weak sector of the SM, the strong sector does not display a “would-be” Banks Zaks IR fixed point, see \(^{(A9)}\). Consequently, trajectories emanating out of FP\(_4\) are attracted towards the Gaussian fixed point rather than being diverted by an interacting fixed point as in \(^{(59)}\). We conclude that the (non)-availability of a matching with FP\(_4\) in the benchmark scenario \(D\ (B)\) is dictated by features of the SM rather than the specifics of the BSM extension.

**Benchmark scenario E.** For all settings with \(R_2 \geq 2\) and \(R_3 \geq 3\), the theory can display all three types of interacting UV fixed points. In any of these cases, FP\(_4\) arises at the lowest possible value for \(N_F\), see Fig. \([4]\). It is then interesting to evaluate scenarios where FP\(_4\) is the sole UV fixed point. Using our results from Tab. \([5]\) we consider exemplarily the case

\[
(R_3, R_2, N_F) = (3, 3, 72) .
\]  

(62)

The UV-IR connecting separatrix is displayed in Fig. \([13]\). We observe that both gauge couplings decrease towards the IR. The scale \(\mu_0\) is a free parameter and solely fixed by the free parameter \(\delta\alpha_3(\Lambda)\) in the deep UV. We confirm once more that the hierarchy \(\alpha_2 > \alpha_3\) in the deep UV invariably transforms into \(\alpha_2 < \alpha_3\) once the RG flow falls below the cross-over scale. Also shown is the SM
running of gauge couplings (dashed lines) taken from Fig. 5 with data points starting from $\mu = 1$ TeV (corresponding to the choice $\mu_0 = 1$ GeV on the lower axis). Tuning the value of $\delta \alpha_3(\Lambda)$ (or $\mu_0$) along the separatrix amounts to shifting the separatrix in its entirety parallel to the lower axis. In Fig. 13 values have been chosen to exemplify that the separatrix can match SM values, at the matching scale $M \approx 1.6$ TeV. We stress once more that the shape of the separatrix, and hence the fixed point behavior of the theory in the deep UV, uniquely dictates the scale at which a matching to the SM can be made.

**E. Synopsis of matching conditions**

To summarise, we have established that partially interacting fixed points $FP_2$ and $FP_3$ can comfortably be connected with the SM at low energies provided there are no nearby competing fixed points in the phase diagram of the theory. Moreover, in these cases the matching scale and thus the masses of BSM matter fields remain freely adjustable parameters. The underlying reason for this is that both gauge couplings remain relevant couplings in the deep UV. Typical examples for this are shown in Fig. 6, 7 and 9 for $FP_2$ of benchmark $A$, and $FP_3$ of benchmark $B$ and $C$, respectively. On the other hand, a matching to the SM becomes more contrived, or even impossible, if nearby competing fixed points influence the running of couplings. An example for this is shown in Fig. 11 for $FP_2$ of benchmark $C$, where the nearby fully interacting fixed point $FP_4$ impacts on the UV-safe trajectories emanating out of the partially interacting fixed point $FP_2$, thereby enforcing a high matching scale.

The matching of fully interacting fixed points $FP_4$ to the SM is qualitatively different. The reason for this is that only one of the gauge couplings remains a relevant coupling, which reduces
the number of freely adjustable parameters in the UV by one. Unlike partially interacting ones, fully interacting fixed points predict a relation between the gauge couplings. The availability of a matching to the SM is then encoded in the UV-IR connecting separatrix and must be checked on a case by case basis, see Fig. 12. In Fig. 14 we summarise our results for the matching conditions at FP$_4$ in dependence on the BSM parameters ($R_3, R_2, N_F$). Low-scale matchings in the multi-TeV regime are indicated by blue diamonds. Examples for this relate to benchmark $D$ and $E$ displayed in Fig. 12 and 13 respectively. This is contrasted with matchings at a high scale, beyond the reach of present day colliders (red dots). An example for the latter is furnished by benchmark $C$ as shown in Fig. 10. On the other hand, matchings fail if gauge couplings along the separatrix never hit SM values, Fig. 5 despite both gauge couplings approaching the Gaussian. In Fig. 14 such scenarios are indicated by black triangles. Finally, nearby competing fixed points may distort the UV-safe separatrix and disallow a matching to the SM. In Fig. 14 the unavailability of matching due to strong coupling in the weak gauge sector ($R_2 = 1$) is indicated by gray triangles. An example for this is benchmark $B$ in Fig. 8 where the competing fixed point is the “would-be” Banks-Zaks fixed point of the SM weak sector. Arrows have been added in Fig. 14 to illustrate how the number of BSM fermion flavors $N_F$ (blue arrow) and the matching scale $\mu$ (red arrow) vary with the BSM matter representation to ensure a successful matching.

We briefly come back to the perturbativity of interactions in the fixed point regime. We have found that the gauge couplings for all benchmarks take small values $\alpha \approx 0.04 - 0.8$, see Tab. 6. Moreover, in the large-$N_F$ and large representation limit, fixed point couplings are parametrically small, $\alpha \approx \frac{1}{N_F}$). In models which permit an asymptotic large-$N$ Veneziano limit, it has also been shown that perturbativity in $N_F \cdot \alpha \ll 1$ is guaranteed \[12\]. Here, at finite $N_F$, the products $N_F \cdot \alpha$ come out of order $O(1 - 10)$ for all benchmarks, hinting towards the onset of strong coupling. Ultimately, this pattern of result reflects the unavailability of a Veneziano limit because fixed points necessitate representations higher than the fundamental, Fig. 4. Future studies should therefore include loop corrections beyond the leading orders, and non-perturbative effects.

Finally, we comment on the role of the $U(1)_Y$ hypercharge. The SM predicts a Landau pole for the hypercharge many orders of magnitude beyond the Planck scale $M_{Pl} \sim 10^{19}$ GeV. In our setup the BSM fields do not carry hypercharge. The interesting case where BSM fields carry hypercharge will be detailed elsewhere. Nevertheless, the running of the hypercharge nevertheless differs from SM running above the matching scale because the strong or the weak or both gauge coupling(s) will grow and eventually settle at interacting fixed points. Interacting fixed points accelerate the running of the hypercharge due to contributions at two loop. To exclude that a Landau pole may arise below Planckian energies, we assume a “worst case” scenario in which $(\alpha_2, \alpha_3)$ take fixed point values already at a very low scale of 1.5 TeV (bounds are softened if fixed point values are reached at higher scales). This leads to a conservative exclusion plot shown in Fig. 15 where the shaded area indicates the forbidden region of values for the UV fixed point. We observe that gauge sectors must become strongly coupled already at low energies to inflict a Landau pole below Planckian energies for the hypercharge. For comparison, we also indicate the location of UV fixed points for the benchmark models $A, B, C, D$ and $E$ as given in Tab. 6. Quantitatively, none of the benchmark models reach a Landau pole for the hypercharge below $10^{26}$ GeV. We conclude that all benchmark models are deeply in the UV-safe region of parameter space.
V. PHENOMENOLOGY

In this section we discuss experimental signatures of asymptotically safe SM extensions. We assume that the BSM sector can at least partially be accessed at the LHC, which implies a low matching scale and masses of the BSM matter fields in the multi-TeV range. An order of magnitude heavier states can be considered at future colliders [23]. Because of the flavor symmetry the BSM fermions are stable in the model (18). Allowing the flavor symmetry to be broken, the lightest BSM fermion is still stable as long as Yukawa interactions with SM fermions are absent. The latter holds except for a few low-dimensional representations with tuned hypercharge of the BSM fermions. As we assume that the BSM fields do not carry hypercharge, these exceptional cases cannot be realized. Without mixing with SM fermions, flavor physics constraints are not relevant to our models. We further assume \( R_3 \neq 1 \). If the new fermions would be colorless, their production at hadron colliders would be of higher order and suppressed. Scenarios with \( R_3 = 1 \) are certainly suitable for study at an \( e^+e^- \)-machine operating at high energies [23–25].

An obvious search strategy is to look for asymptotically safe BSM physics by probing the strong running coupling evolution and the weak interaction. We discuss various constraints and opportunities from the running gauge couplings, from the weak sector, from direct searches for long-lived QCD-bound states composed out of BSM fermions and SM partons, and from LHC diboson searches, offering further constraints on BSM matter including the \((M_\psi, M_S)\) parameter space.

A. Strong coupling constant evolution

The presence of a large number of fermions charged under \( SU(3)_C \times SU(2)_L \) changes the running of the corresponding gauge couplings drastically, as illustrated in Figs. 6–13. The deviation from the SM, shown in Fig. 5, kicks in rather quickly with an order one increase in slope of the asymptotically safe coupling and provides a smoking gun signature of BSM physics considered in this work. Threshold corrections are not expected to change this picture qualitatively, although the onset of BSM effects may be somewhat smoother.

The CMS collaboration has extracted the value of the QCD running coupling up to the scale 2 TeV [26] using the measurement of the inclusive jet cross section for proton-proton collisions at a centre-of-mass energy of 7 TeV with data corresponding to an integrated luminosity of 5.0 fb\(^{-1}\) [27]. This determination is consistent with the SM. Also other measurements, including the one of the inclusive 3-jet production differential cross section [28], have not observed deviations from the two-loop running predicted in the SM up to 1.5 TeV. Therefore, modulo threshold corrections, the lower limit on the mass of new colored matter reads

\[
M_\psi \gtrsim 1.5 \text{ TeV}.
\]

In the following subsections we work out further experimental constraints on \( M_\psi \), which in turn imply limits on the matching scale. We recall that scenarios with partially interacting fixed points FP\(_2\) and FP\(_3\) can generically be matched at any scale (except in specific circumstances, see Fig. 11), whereas the matching scale is uniquely fixed for fully interacting fixed points FP\(_4\). The latter scenarios are therefore subject to stronger experimental constraints.

In Fig. 16 we show the running of the strong coupling (black dashed line) and its uncertainty (green band) as determined by CMS [26]. In addition, we show the running of \( \alpha_3 \) in the asymp-
Figure 16. SM running of the QCD coupling constant (black dashed line) and its uncertainty (green band) as determined by CMS [26]. Colored solid lines indicate the running of $\alpha_3$ in asymptotically safe benchmark scenarios $B$, $C$ and $E$ summarized in Tab. 6 with a low matching scale around 1.5 to 2 TeV.

totically safe benchmarks $B$, $C$ and $E$ introduced in Tab. 6 for low matching scales around 1.5 to 2 TeV. Note that benchmark $E$ (blue curve) relates to a fully interacting UV fixed point whose matching scale is fixed at 1.6 TeV. As can be seen, benchmark $E$ is already being probed experimentally. Threshold corrections may allow to evade the CMS limit, as the data near 2 TeV are also losing statistics.

B. The weak sector

The experiments at the LEP collider have probed the SM’s electroweak sector with scrutiny and found no significant deviation up to $\sim 209$ GeV [29]. The LHC has extended related SM tests into the several $\mathcal{O}(100)$ GeV regime [30], still allowing for weakly-interacting uncolored vector-like fermions below the TeV-scale. Within asymptotically safe models, this can happen, for instance, in benchmark $A$ by noting that since the fixed point is partial only, the matching scale can be different than the one shown in Fig. 6. Electroweak vector boson scattering at the LHC and at future lepton colliders [25] is sensitive to such BSM effects.

For $R_2 \neq 1$ contributions to the $\rho$-parameter arise if the BSM fermions encounter $SU(2)_L$ breaking due to mass splitting $\delta M \ll M_\psi$ in the fermion multiplet. This implies [22]

$$N_F d(R_3) S(R_2) \delta M^2 \lesssim (40 \text{ GeV})^2,$$

a splitting below percent level for TeV-ish fermion masses and higher.

In Fig. 17 we show the running of the weak coupling (black dashed line) and, schematically, the region with agreement with the SM’s weak theory denoted by the hatched green band. The solid colored lines correspond to the asymptotically safe benchmarks $A$, $D$ and $E$ summarized in Tab. 6 for a low matching scale around 1.5 to 2 TeV.
Figure 17. SM running of the weak coupling constant (black dashed line) and schematically indicated by the green hatched band the region where the weak sector of the SM has passed experimental tests, see text for details. Colored solid lines indicate the running of $\alpha_2$ in asymptotically safe benchmark scenarios introduced in Tab. 6 that allow for a low matching scale around 1.5 to 2 TeV.

Constraints from rare decays can be evaded as long as the BSM fermions do not couple directly to SM Higgs, quarks, or leptons – as is the case in our setup. For BSM fermions with $R_2 > 1$ a contribution to the anomalous magnetic moment of the muon arises at 2-loop in the electroweak interactions. We estimate this as $\Delta a_\mu \sim d(R_3) S(R_2) N_F [\alpha/(4\pi)(m_\mu/M_\psi)^2]$. Comparison with data $\Delta a_\mu^{\exp} \sim (2 - 3) \cdot 10^{-9}$ [22] yields the constraint

$$d(R_3) S(R_2) N_F \left(\frac{\text{TeV}}{M_\psi}\right)^2 \lesssim 10^4,$$

which is satisfied for all our benchmarks in Tab. 6 and for $M_\psi$ above a TeV.

Below the BSM mass threshold the effects of the BSM fermions can be studied indirectly through electroweak precision tests. Charged and neutral current Drell-Yan (DY) processes offer a promising way to test such corrections [31], as they are both experimentally clean and very well understood theoretically. The oblique parameter $W$ [32, 33] is of particular interests since its impact increases with energy allowing high precision studies at present and future colliders [34]. $W$ is directly related to the BSM contribution that modifies the electroweak beta function,

$$W = -\frac{\alpha_2}{10} \frac{M_W^2}{M_\psi^2} B_2^{\text{BSM}},$$

where $B_2^{\text{BSM}}$ denotes the BSM contribution to the 1-loop coefficient $B_2$ of $\beta_2$, see A2.

Fig. 18 displays $W$ versus the matching scale $M_\psi$ for the benchmark scenarios defined in Tab. 6. In the left panel the experimental 95% C.L. upper limits [31] from LEP (red dashed line) and LHC 8 TeV (blue dashed line) are shown. Constraints on negative $W$ exist but are not relevant here. For the fully-interacting fixed points FP4 of benchmark D and E (full dot), a low matching scale
at around 2 TeV dictates a high multiplicity of BSM fermions, and a large contribution to \( W \). These scenarios turn out to be excluded. For scenarios with partially interacting fixed points (FP\(_2\) or FP\(_3\)) the matching scale is a free parameter. For benchmark models \( C \) (magenta line) and \( D \) (green line) the matching scale must be larger than about 8 TeV to satisfy LEP constraints.

Benchmark \( A \) (red line) is not yet constrained by the data owing to the low number of BSM fermion species \( N_F \), while benchmark \( B \) (black line) is not probed at this order because its BSM fermions are \( SU(2)_L \) singlets.

The right panel of Fig. 18 shows the projected sensitivities of the LHC at 13 TeV with 3 ab\(^{-1}\) integrated luminosity (dashed blue line), the ILC 500 with 3 ab\(^{-1}\) (dashed red line), and a 100 TeV collider with 10 ab\(^{-1}\) (dashed gray line). The precision of the \( W \) determination is expected to increase by two orders of magnitude, requiring the matching scale in all allowed benchmark scenarios to be above around 10 TeV.

While this analysis demonstrates the importance of DY measurements for the type of BSM scenarios outlined here, constraints based on \( W \) must be taken with a grain of salt. The reason for this is that the corrections to \( W \) are only known to one loop order. In our framework, competing two loop corrections in the gauge beta functions play an important role as they are responsible for the fixed point. To estimate two loop effects, we replace \( B_{BSM}^2 \) in \( W \) by the effective coefficient \( B'_2 \) evaluated on the RG trajectory near the matching scale. For benchmark models \( C \) where \( \alpha_3 \) becomes asymptotically safe, we find that the bound on the matching scale softens, from about 8 TeV to about 5 TeV. Also, benchmark \( B \) now contributes negatively to \( W \) and can be probed in the future. Similar two loop effects are expected for the other benchmark models. A complete two loop analysis of the oblique parameter \( W \), although desirable, is beyond the scope of this work.

![Figure 18](image-url)
C. \textit{R}-hadrons

We assume that at least some of the BSM fermions can be pair-produced,
\[ 2M_{\psi} < \sqrt{s}, \]  
where $\sqrt{s}$ denotes the accessible center of mass energy at the collider. At least the lightest of the fermions has a long life, longer than a typical hadronization time scale, and forms colorless QCD bound states with ordinary partons (quarks and gluons), the so-called $R$-hadrons.

Both the ATLAS and CMS collaborations searched for heavy long-lived charged $R$-hadrons using a data sample corresponding to 3.2 fb\(^{-1}\) of proton-proton collisions at $\sqrt{s} = 13$ TeV. No significant deviations from the expected background have been observed which allowed to put a model-independent 95\% confidence level (C.L.) upper limits on the production cross section of long-lived $R$-hadrons. In the framework of supersymmetry, those results have been translated into a lower bound on the mass of the fermionic partner of the gluon (gluino), which read 1.5 TeV for CMS \cite{35} and 1.6 TeV for ATLAS \cite{36}. Recently CMS has updated their analysis to 12.9 fb\(^{-1}\) of data \cite{37}, with the corresponding limit on the gluino mass increased to 1.7 TeV.

At the LHC any colored and hypercharge-neutral BSM fermion would be produced in the same way. Therefore one can easily recast the experimental limits for gluino searches in the framework of asymptotically safe scenarios considered in this study. In the leading order the $\psi \bar{\psi}$ pairs can be produced by gluon fusion or by quark-anti-quark annihilation, where the former mechanism is a dominant one. We can additionally assume that the main contribution to $pp \to \psi \bar{\psi}$ comes from a $t$-channel exchange of a BSM fermion. In this case the production cross section $\sigma_{\psi \bar{\psi}}$ depends on $(R_3, R_2, N_F)$ and scales proportionally to the factor $C_3$,
\[ \sigma_{\psi \bar{\psi}} \sim N_F C_3 \text{ with } C_3 = |C_2(R_3)|^2 d(R_3) d(R_2). \]  

We can then put lower limits on the BSM fermion mass using the experimental limits for gluinos provided in \cite{35} and \cite{36}, rescaling the gluino production cross section by $C_3$. Notice also that it is possible for real representations, such as those with $(p, p)$ for $R_3$, that $\psi$ is a Majorana fermion; in all other cases we ignore the differences with respect to the gluinos in our estimates.

In Tab. 7 we show the lower bounds on $M_\psi$ in dependence of $R_2$ and $R_3$ together with $C_3$ for $N_F = 1$. The lower bound increases with increasing $d(R_3)$. For $R_3 = 15'$ and $d(R_2) > 1$ it reads 2.3 TeV. We conclude that colored BSM fermions must be heavier than at least 1.5 TeV, consistent with (63). For larger $N_F$, the bounds get stronger. For example, for benchmarks $B$, $D$ and $E$ defined in Tab. 6 we find $M_{\psi}^{\text{min}} = 2.6, 2.4$ and 2.1 TeV, respectively. The limit for benchmark $C$ is beyond 2.8 TeV. For benchmarks $C$ and $D$ the constraints from DY processes obtained in Sec. V B are stronger than the $R$-hadron ones.

D. Diboson spectra and resonances

Consider the situation where the BSM scalars are lighter than twice the mass of the fermions and can be resonantly produced,
\[ M_S < 2M_\psi, \quad \text{and} \quad M_S < \sqrt{s}. \]
\[ \psi(R_3, R_2) \]

| \( R_3 \) | \( C_3 \) | \( M^{\text{min}}_\psi \) (TeV) | \( C_3 \) | \( M^{\text{min}}_\psi \) (TeV) | \( C_3 \) | \( M^{\text{min}}_\psi \) (TeV) |
|---|---|---|---|---|---|---|
| 3  | 5\( \frac{1}{3} \)  | (1.3) | 10\( \frac{2}{3} \)  | (1.4) | 16 | 1.5 |
| 6  | 66\( \frac{2}{3} \)  | 1.7 | 133\( \frac{1}{3} \)  | 1.8 | 200 | 1.9 |
| 8  | 72  | 1.7 | 144  | 1.8 | 216 | 1.9 |
| 10 | 360  | 2.0 | 720  | 2.1 | 1080 | 2.2 |
| 15 | 426\( \frac{2}{3} \)  | 2.0 | 853\( \frac{1}{3} \)  | 2.1 | 1280 | 2.2 |
| 15’| 1306\( \frac{2}{3} \)  | 2.2 | 2313\( \frac{1}{3} \)  | 2.3 | 3920 | 2.4 |

**Table 7.** Lower limits on the mass of the lightest BSM fermion, \( M^{\text{min}}_\psi \), as derived from the searches for long-lived charged particle by CMS [35, 37] and ATLAS [36] for \( N_F = 1 \). We make explicit a dependence on the fermion representations \( R_2 \) and \( R_3 \) under SU\( (2)_L \) and SU\( (3)_C \), respectively. The dominant contribution to the production cross section is proportional to \( C_3 \), (68), which is also given. Values in parentheses correspond to scenarios \( (R_3, R_2) \) with no weakly interacting UV fixed points, see Fig. 4.

In such a case the scalars cannot decay on-shell to any of the BSM fermions and because its mixing with the SM Higgs boson is negligible, the only possible decay channels are loop-mediated decays into pairs of gauge bosons

\[ GG = gg, \gamma\gamma, ZZ, Z\gamma, \text{ or } WW. \]  

(70)

The cross sections for these, as well as their relative strengths, depend directly on transformation properties of the BSM fermions under SU\( (3)_C \) and SU\( (2)_L \). Since \( S \) does not couple directly to the SM fermions its dominant production mechanism is gluon fusion which proceeds through the loops containing \( \psi_i \). This process is schematically depicted in Fig. 19. Due to the particular flavor structure of the asymptotically safe BSM sector, one needs to consider a simultaneous production of \( N_F^2 \) scalars \( S_{ij} \), each of them coupled to exactly one fermion pair \( \bar{\psi}_i \psi_j \). However, since flavor is conserved in the fermion-gauge boson interactions, only diagonal couplings are allowed in this process and the number of simultaneously produced scalars is reduced to \( N_F \). Due to interference effects between the \( N_F \) diagrams, it is useful to investigate separately the limiting cases of maximal and no interference.

**Maximum interference.** Provided that all scalars \( S_{ii} \) have the same mass \( M_S \) and total decay width \( \Gamma_S \), the interference between them is maximal and the cross section for a diboson signal \( GG \) (70) is given by [38, 39]

\[ \sigma(pp \rightarrow S_1, ..., N_F \rightarrow GG) = N_F^2 \sigma(pp \rightarrow S_1)BR(S_1 \rightarrow GG) = \frac{N_F^2 \pi^2}{8 M_S^3 \Gamma_S} I_{pdf} \Gamma^1_{GG} \Gamma^1_{gg}. \]  

(71)

Notice that (71) scales as \( N_F^2 \) times the cross section for one individual flavor. In the above, \( \Gamma^1_{GG} \) denotes the partial decay width into two gauge bosons with only one generation of BSM fermions in the loop. Similarly, \( \Gamma^1_{gg} \) stands for the corresponding partial width into two gluons, and \( I_{pdf} \) is the integral of parton (gluon) distribution function in proton, evaluated at the energy scale \( \mu = M_S \) with the center of mass energy \( \sqrt{s} \).
No interference. Interference effects are absent provided that the masses of the scalars $S_{ii}$ are narrowly spaced with mass differences $\Delta M$ below the detector mass resolution $\Gamma_{\text{det}}$, and provided that individual widths do not overlap. Consequently, the production cross section becomes

$$
\sigma(pp \to S_1, \ldots, S_{N_F} \to GG) = N_F \sigma(pp \to S_1) \text{BR}(S_1 \to GG) = \frac{N_F \pi^2}{8M_S^3 \Gamma_{\text{det}}} I_{\text{pdf}} \Gamma_{GG}^1 \Gamma_{gg}^1.
$$

Notice that the total cross section scales as $N_F$ times the cross section for one individual flavor. As such, the absence of interference effects reduces the total cross section parametrically by a factor of $N_F$ over (71). We expect that settings with partial interference effects are well covered within the limits (71) and (72).

The relevant energy scale in the process is $\mu \sim M_S$, that is, the diboson invariant mass. The matching scale $M$ is of the order $M_\psi$. For $M_S$ below $M$, roughly $M_S \ll M_\psi$ (“low $M_S$”), the gauge couplings assume SM evolution. For $M_S$ above $M$ (“high $M_S$”), the gauge couplings follow the BSM fixed-point trajectory. The kinematical range for the high $M_S$ scenario is

$$
M_\psi \ll M_S \leq 2M_\psi.
$$

Since characteristic diboson signatures can arise in many BSM scenarios, they have been intensively searched for at the LHC.

Recently both ATLAS and CMS updated their 95\% C.L. limits on the fiducial cross section times branching ratio ($\sigma \times BR \times$ acceptance $A$ for a dijet analysis) for a general scalar resonance decaying into $gg$ [40, 41] (updating [42–44]), $Z\gamma$ [45, 46], $ZZ$ [47, 48], $WW$ [49, 50] and $\gamma\gamma$ [51, 52]. The exact limit in each case depends on the mass of the resonance, as well as on its total width. In the following we choose $M_S = 1.5$ TeV unless otherwise stated, and $\Gamma_S \leq \Gamma_{\text{det}}$. First we consider limits on $\sigma(pp \to GG)$ provided by dijet searches. The partial width of $S$ into gluons [53] reads

$$
\Gamma_{gg} = \frac{\alpha_s^2 M_S^3}{32\pi^3} \left| \frac{yS_2(R_3)d(R_2)}{M_\psi} A_{1/2}(x) \right|^2,
$$

where $\alpha_s = 4\pi\alpha_3$ and the loop function is defined as $A_{1/2}(x) = \frac{2}{x^2}[x + (x - 1)\arcsin(\sqrt{x})^2]$ and $x = M_S^2/(4M_\psi^2)$. In Fig. 20 we show $\sigma(pp \to S \to gg)$ versus the mass of the BSM fermions $M_\psi$ for $M_S = 1.5$ TeV. Solid curves correspond to the limit of maximal interference, while dashed ones...
Figure 20. Dijet cross section as a function of the BSM fermion mass $M_\psi$ for benchmark $B$ (thick blue curves), and benchmark $D$ (thin green curves) for $M_S = 1.5$ TeV. Solid curves correspond to the maximal interference between $N_F$ scalars, while dashed ones to no interference. The upper and lower horizontal dashed line denotes the ATLAS 95% C.L. limit [40] on the dijet cross section assuming 50% and 100% acceptance, respectively.

Figure 21. Exclusion limits in the $M_S - M_\psi$ plane from $R$-hadron searches (green horizontal stripe) and the dijet cross section limit from ATLAS 95% C.L. — exemplarily for the benchmark $B$ ($R_3 = 10, R_2 = 1, N_F = 30$) —, also comparing settings with maximal interference (red solid line) and no interference (red dashed line) and $A = 100\%$. Moreover, solid and dashed black lines represent the borders of (73) where $M_S = 2M_\psi$ and $M_S = M_\psi$, respectively. The excluded areas are below the solid black lines and to the left and below the red lines. The dark and light blue areas indicate, respectively, the searchable parameter space (69) at the LHC (with...
Figure 21. Excluded regions in the $M_\psi - M_S$ plane combining $R$-hadron searches (green horizontal stripe) with dijet cross section limits from ATLAS 95% C.L. [10] for the benchmark $B$ ($R_3 = 10$, $R_2 = 1$, $N_F = 30$). Excluded regions are also given for the limiting cases of maximal interference (below red solid curve) and no interference (left of red dashed curve), with $A = 100\%$. The dark (light) blue area indicates the searchable parameter space [69] at the LHC (future colliders). The strip [73] with $M_\psi \lesssim M_S < 2M_\psi$ where an enhancement of $\alpha_s$ due to asymptotically safe running takes place corresponds to the region between the full and dashed black lines.

$\sqrt{s} = 14$ TeV) and at future colliders [23]. For masses within the range [73] — corresponding to the band between the full and dashed black lines —, the strong coupling $\alpha_s$ is already of non-SM type and enhanced, $\alpha_s(M_S) \gtrsim \alpha_s(M_\psi)$. This regime allows to probe higher values of $M_S$.

If the BSM fermions transform non-trivially under $SU(2)_L$, two additional effects arise. Firstly, the lower dijet bound on the fermion mass increases since the cross section $\sigma(pp \to S \to gg)$ scales with $d(R_2)$. Secondly, decays into electroweak gauge bosons $VV = \gamma\gamma, WW, ZZ, Z\gamma$ become possible. In order to discuss decays into weak gauge bosons in more detail, it is convenient to introduce the reduced decay widths

$$\bar{\Gamma}_{VV} = \frac{1}{F} \frac{\Gamma_{VV}}{\Gamma_{gg}}, \quad \text{with} \quad F = \left(\frac{4 C_2(R_2)}{3 C_2(R_3)}\right)^2,$$

which expresses the widths $\Gamma_{VV}$ in units of $\Gamma_{gg}$ together with a group theoretical factor $F$ which takes into account the quadratic Casimirs of the BSM fermions. In terms of (75), we find

$$\bar{\Gamma}_{WW} = \frac{\alpha_1^2}{\alpha_3^2}, \quad \bar{\Gamma}_{ZZ} = \frac{\alpha_2^4}{2(\alpha_1 + \alpha_2)^2 \alpha_3^2}, \quad \bar{\Gamma}_{Z\gamma} = \frac{\alpha_1 \alpha_2^3}{(\alpha_1 + \alpha_2)^2 \alpha_3}, \quad \bar{\Gamma}_{\gamma\gamma} = \frac{\alpha_1^2 \alpha_2^2}{2(\alpha_1 + \alpha_2)^2 \alpha_3}, \quad (76)$$

where $\alpha_1 = g_Y^2/(4\pi)^2$ is the hypercharge coupling. The reduced decay widths depend, in general, on the three gauge couplings. We note that $\bar{\Gamma}_{WW}$ stands out in that it is independent of $\alpha_1$, and only sensitive to the ratio of the other two gauge couplings.

For low $M_S$ below the matching scale the ratios $\bar{\Gamma}_{VV}$ are solely determined by the SM gauge
couplings. In this case $F$ can be determined from any of the $VV$ modes, providing information about $R_2$ and $R_3$. Measuring more than one mode serves as a consistency check. In Fig. 22 we show the ratios $\Gamma_{WW}/\Gamma_{gg}$ (blue), $\Gamma_{ZZ}/\Gamma_{gg}$ (green), $\Gamma_{Z\gamma}/\Gamma_{gg}$ (red), and $\bar{\Gamma}_{\gamma\gamma}/\Gamma_{gg}$ (orange), depending on $d(R_2)$ for $R_3 = 10$ (left panel) and $R_3 = 3$ (right panel) for low $M_S$. The hierarchies among the different modes are fixed in this regime. Deviations can arise from switching on hypercharges of the fermions, or from running above the matching scale, at high $M_S$, where the gauge couplings experience BSM running. This modifies the running of $\alpha_2$ and $\alpha_3$ whereas the running of $\alpha_1$ remains SM-like. We may neglect its slow logarithmic running and can take $\alpha_1$ constant for the following considerations.

To discuss further BSM effects, we simplify (76) by exploiting that $(\alpha_1/\alpha_2)^2 \lesssim 0.08$ at TeV energies below the matching scale. We find

$$\Gamma_{WW} = \frac{\alpha_2^2}{\alpha_3^2}, \quad \Gamma_{ZZ} \approx \frac{1}{2} \frac{\alpha_2^2}{\alpha_3^2}, \quad \Gamma_{Z\gamma} \approx \frac{\alpha_1 \alpha_2}{\alpha_3 \alpha_3}, \quad \bar{\Gamma}_{\gamma\gamma} \approx \frac{1}{2} \frac{\alpha_2^2}{\alpha_3^2},$$

where “≈” means equality up to relative corrections of order $\mathcal{O}(\alpha_1^2/\alpha_2^2)$. We observe that $\bar{\Gamma}_{\gamma\gamma}$ is no longer sensitive to $\alpha_2$ as it has reduced to a ratio of the other two gauge couplings, similarly to $\Gamma_{WW}$. It follows that $\bar{\Gamma}_{ZZ} \propto \Gamma_{WW}$ and that $\bar{\Gamma}_{Z\gamma} \propto (\bar{\Gamma}_{ZZ} \cdot \bar{\Gamma}_{\gamma\gamma})^{1/2}$.

For models with fully interacting fixed points FP$_4$ we recall that the weak and strong gauge couplings start growing with RG scale above the matching scale, dictated by the underlying separatrix into the UV fixed point, see Fig. 12. Their ratio increases from $\alpha_2/\alpha_3 < 1$ below the matching scale to $\alpha_2/\alpha_3 \rightarrow 3/2$ sufficiently above the matching scale, invariably inverting the SM hierarchy. In particular, we have that $\alpha_2(\mu)/\alpha_3(\mu) > \alpha_2(M)/\alpha_3(M)$ for $\mu > M$ which implies that both $\bar{\Gamma}_{WW}$ and $\bar{\Gamma}_{ZZ}$ increase accordingly with increasing $\mu > M$. On the other hand $\bar{\Gamma}_{\gamma\gamma}$ becomes suppressed. For $\Gamma_{Z\gamma}$, the situation is ambiguous: the growth of $\bar{\Gamma}_{ZZ}$ competes with the suppression of $\bar{\Gamma}_{\gamma\gamma}$ and the outcome in the cross-over region will be model-dependent. Quantitatively, for the fully interacting fixed points of benchmark $D$ (benchmark $E$) we find that both $\bar{\Gamma}_{WW}$ and $\bar{\Gamma}_{ZZ}$ grow from $\mu = M$ to $\mu = 2M$ by factors of about 12 (3), and that $\bar{\Gamma}_{\gamma\gamma}$ is suppressed by factors of about 13 (2). $\bar{\Gamma}_{Z\gamma}$ is very mildly suppressed only.

For models with partially interacting fixed points FP$_2$ we generically observe $\alpha_2(\mu > M) > \alpha_2(M)$ and $\alpha_3(\mu > M) < \alpha_3(M)$, e.g. Fig. 6. This implies that all reduced decay widths increase with increasing $\mu > M$, albeit with different factors, see (77). Conversely, for models with FP$_3$ we have $\alpha_3(\mu > M) > \alpha_3(M)$ and $\alpha_2(\mu > M) < \alpha_2(M)$. As can be deduced from the explicit expressions in (76) and (77), all four reduced decay widths decrease relative to Fig. 22.

We conclude that diboson searches involving pairs of electroweak gauge bosons can provide stronger limits than the dijet ones if $d(R_2)$ is sufficiently large. Due to the a priori unknown hierarchy between $M_S$ and $M_\psi$, correlations of $VV$ with dijet limits cannot be interpreted unambiguously. On the other hand, an observation of a $GG$-resonance determines $M_S$, while a breakdown of SM-running of $\alpha_3$, perhaps together with a similar effect in the weak coupling, determines $M_\psi$. In these cases, extracting $F$ is feasible at low $M_S$.

Resonance-induced diboson signatures can arise as well from decays of $(\psi \bar{\psi})$-bound states, which are expected to form somewhat below center-of mass energies of $2M_\psi$ for $R_3 \neq 1$ [55]. In our model such $\psi$-onia can start at about $2M_\psi \gtrsim 3$TeV, which is within LHC limits [56]. Relative decay widths are as in the case of the decays of the scalar $S$ resonance (76). Further analysis is beyond the scope of this work.
VI. SUMMARY

The concept of an interacting UV fixed point in quantum field theory is of high interest per se; for particle physics it opens up “theory space” for model building. Here, we have investigated asymptotically safe extensions of the Standard Model by adding new fermions and scalar singlet fields. The new matter fields also interact, minimally, via a single Yukawa coupling to help generate interacting UV fixed points. A large variety of stable high energy fixed points emerges where either the strong, or the weak, or both couplings assume finite values, see Figs. 2, 4. Those where one of the gauge couplings remains asymptotically free can flow into the Standard Model at any scale above $\mathcal{O}(1 - 2)$ TeV, modulo nearby competing fixed points. Many of the fully interacting fixed points can also be matched onto the Standard Model including at TeV scales, Fig. 14. Specifically, with fermions charged under $SU(3)_C \times SU(2)_L$, we found that they must carry representations higher than the fundamental in at least one of the gauge sectors, Fig. 4. Also, fully interacting fixed points cannot arise if the fermions are charged under $SU(2)_L$ only. An intriguing feature of models with fully interacting UV fixed points is a relation between gauge couplings, dictated by asymptotic safety. The number of fundamentally free parameters is thereby reduced offering an enhanced degree of predictivity compared to the Standard Model, quite similar to the idea of unification. Our results have been obtained at two loop accuracy where couplings remain small for all scales, though not parametrically small such as in the Veneziano limit [12]. Of course, further study is needed to explore the full potential of this new direction.

There are several opportunities to look for asymptotically safe BSM physics at colliders. The presence of a large number of new fermionic degrees of freedom from higher representations of $SU(3)_C \times SU(2)_L$ with large multiplicities implies striking new physics at the corresponding mass despite being weakly coupled, e.g. Figs. 6, 13. Irrespective of the choice of benchmark models, the qualitative features from the model ansatz laid out in Sec. 111 are rather generic. For low scale matching BSM physics can be just around the corner, as close as $\mathcal{O}(1 - 2)$ TeV: $R$-hadron signals arise and the strong coupling evolution itself is altered and further collider tests should be pursued, see Fig. 16. For $SU(2)_L$-charged fermions the weak interaction is modified, schematically shown in Fig. 17. Corresponding shifts in electroweak observables, including $WW$-production

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**Figure 22.** The ratios $\Gamma_{WW}/\Gamma_{gg}$ (blue), $\Gamma_{ZZ}/\Gamma_{gg}$ (green), $\Gamma_{Z\gamma}/\Gamma_{gg}$ (red), and $\Gamma_{\gamma\gamma}/\Gamma_{gg}$ (orange) versus $d(R_2)$ for $R_3 = 10$ (left) and $R_3 = 3$ (right) for low $M_S$ (see main text).
appear above threshold. Loop-induced diboson spectra involving the scalar resonance Fig. 19 are sensitive to about an order of magnitude higher scales Fig. 21. While the actual limits are rather model-dependent, this demonstrates that the phenomenology of asymptotically safe BSM can be probed at the LHC at Run 2 and beyond. Tests of the weak interaction are also encouraged at high energy $e^+e^-$ colliders 23–25.

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Appendix A: Technicalities

The appendix summarises group theoretical formulæ and loop coefficients, together with a discussion of UV-IR connecting separatrices.

Loop coefficients and group theoretical factors

We summarise formulæ for perturbative loop coefficients. We have exploited general expressions as given in [18–21]. We consider the SM matter fields, together with $N_F$ vector BSM fermions in the $R_3$ and $R_2$ representation under $SU(3)_C$ and $SU(2)_L$, respectively. The beta functions are stated in [22]. We reproduce them here for completeness,

$$\beta_3 = (-B_3 + C_3 \alpha_3 + G_3 \alpha_2 - D_3 \alpha_y) \alpha_3^2,$$
$$\beta_2 = (-B_2 + C_2 \alpha_2 + G_2 \alpha_3 - D_2 \alpha_y) \alpha_2^2,$$
$$\beta_y = (E \alpha_y - F_2 \alpha_2 - F_3 \alpha_3) \alpha_y.$$  \hfill (A1)

The gauge one loop coefficients read

$$B_3 = 14 - \frac{8}{3} N_F S_2(R_3) d(R_2),$$
$$B_2 = \frac{19}{3} - \frac{8}{3} N_F S_2(R_2) d(R_3).$$ \hfill (A2)

At two loop level for the gauge couplings we have the “diagonal” gauge contributions

$$C_3 = -52 + 4N_F S_2(R_3) d(R_2) (2C_2(R_3) + 10),$$
$$C_2 = \frac{35}{3} + 4N_F S_2(R_2) d(R_3) \left(2C_2(R_2) + \frac{20}{3}\right).$$ \hfill (A3)
together with the “mixing” gauge contributions
\[ G_3 = 9 + 8N_F S_2(R_3) C_2(R_2) d(R_2), \]
\[ G_2 = 24 + 8N_F S_2(R_2) C_2(R_3) d(R_3). \] (A4)

Furthermore, the BSM Yukawa couplings also contribute at two loop level to the running of the
gauge couplings with coefficients
\[ D_3 = 4N_F^2 S_2(R_3) d(R_2), \]
\[ D_2 = 4N_F^2 S_2(R_2) d(R_3). \] (A5)

The running of the BSM Yukawa coupling receives one loop contributions from itself as well as
from the gauge couplings with coefficients
\[ E = 2[N_F + d(R_2) d(R_3)], \]
\[ F_3 = 12C_2(R_3), \]
\[ F_2 = 12C_2(R_2). \] (A6)

In the above expressions, \( C_2(R) \), \( S_2(R) \) and \( d(R) \) denote the quadratic Casimir invariant, the
Dynkin index and dimension of the representation \( R \), respectively. They are related by
\[ S_2(R) = d(R) C_2(R)/d(\text{Adj}), \] (A7)
to the dimension of the adjoint representation \( d(\text{Adj}) \). It is also convenient to parametrize the
loop coefficients through the weights \((p,q)\) for irreducible \( SU(3) \) representations \( R_3 \), and, similarly,
through the highest weight \( \ell \) for \( SU(2) \) representations \( R_2 \),
\[ d(R_3) = \frac{1}{2}(p+1)(q+1)(p+q+2), \]
\[ C_2(R_3) = p+q + \frac{1}{2}(p^2 + q^2 + pq), \quad \text{with} \quad p, q = 0, 1, \ldots, \] (A8)
\[ d(R_2) = 2\ell + 1, \]
\[ C_2(R_2) = \ell(\ell + 1), \quad \text{with} \quad \ell = 0, \frac{1}{2}, 1, \ldots. \]

Evidently, in the absence of BSM matter fields \((N_F = 0)\), the perturbative loop coefficients reduce
to their SM values
\[ B_3^{\text{SM}} = 14, \quad B_2^{\text{SM}} = 19/3, \]
\[ C_3^{\text{SM}} = -52, \quad C_2^{\text{SM}} = 35/3, \] (A9)
\[ G_3^{\text{SM}} = 9, \quad G_2^{\text{SM}} = 24, \]
together with \( E^{\text{SM}} = F_2^{\text{SM}} = F_3^{\text{SM}} = 0 \). In this limit and at two loop accuracy, we observe that
the \( SU(2)_L \) sector displays a “would-be” Banks-Zaks type IR fixed point at
\[ (\alpha_2^*, \alpha_3^*) = \left( \frac{19}{35}, 0 \right). \] (A10)
The $SU(3)_C$ sector does not display signs of a Banks-Zaks type fixed point owing to $B_3/C_3 < 0$. Fingerprints of \((A10)\) become visible for scenarios with BSM matter uncharged under $SU(2)_L$.

At places, primed two loop coefficients arise. They relate to their unprimed counterparts as

\[
\begin{align*}
C'_3 &= C_3 - D_3 F_3/E, \\
C'_2 &= C_2 - D_2 F_2/E, \\
G'_3 &= G_3 - D_3 F_2/E, \\
G'_2 &= G_2 - D_2 F_3/E. \\
\end{align*}
\]

\((A11)\)

In the gauge beta functions, the primed terms arise as shifts of the unprimed coefficients induced by the BSM Yukawa coupling which takes a fixed point proportional to the gauge couplings.

**UV-IR connecting separatrices**

Next, we summarise formulae and results related to the running of couplings along UV safe trajectories emanating out of an interacting UV fixed point. We are particularly interested in the running of the relevant gauge coupling along the UV-IR connecting hypercritical surface down to energy scales $\mu$ close to the mass $M$ of the new matter fields, $\mu \approx M$, where the model connects with the SM. We approximate the beta functions \((A1)\) as

\[
\begin{align*}
\partial_t \alpha &= \alpha^2 (-B + C \alpha - D \alpha_y), \\
\partial_t \alpha_y &= \alpha_y (E \alpha_y - F \alpha). \\
\end{align*}
\]

\((A12)\)

These equations are applicable for partially interacting fixed points where one of the gauge couplings becomes asymptotically safe and the other one asymptotically free. The contribution of the latter is neglected. Quantitatively, corrections from the asymptotically free coupling are sub-leading in the UV and numerically small for the models discussed here. The coefficients $B, C, D$ and $F$ then take the values corresponding to the asymptotically safe gauge coupling. To find the exact UV-IR connecting separatrix, the system \((A12)\) must be solved numerically. However, for most cases of interest, approximate estimates can be obtained as well. We discuss two strategies.

**UV critical surface approximation.** Firstly, we may approximate $\alpha_y$ along the separatrix through its values along the UV critical surface. The virtue of this approximation is that it becomes sufficiently exact close to the UV fixed point. Quantitatively, on the UV hypercritical surface, the BSM Yukawa coupling is determined via the gauge coupling as

\[
\alpha_y = C_y (\alpha - \alpha^*) + \alpha^*_y. \\
\]

\((A13)\)

In this expression, $C_y$ is defined via the relevant eigendirection at the UV fixed point, with $(1,C_y)^T$ denoting the eigenvector with negative eigenvalue of the stability matrix $M$ at the fixed point in the basis $(\alpha, \alpha_y)^T$. In terms of the perturbative loop coefficients, it reads

\[
C_y = 2 \frac{F}{E} \left( 1 + \frac{1 - \frac{2BF(C^2E^2 - 3CDEF + 2D^2F^2) - B^2C^2E^2}{F^2(DF - CE)^2} + \frac{BCE}{F(DF - CE)}}{1 - \frac{2BF(C^2E^2 - 3CDEF + 2D^2F^2) - B^2C^2E^2}{F^2(DF - CE)^2}} \right)^{-1}. \\
\]

\((A14)\)
In order to map out the UV-IR connecting separatrix we insert (A13) into (A12) to find
\[ \partial_t \alpha = \alpha^2 (-\tilde{B} + \tilde{C} \alpha), \] (A15)
with
\[ \tilde{B} = B + D (\alpha^*_y - C_y \alpha^*), \]
\[ \tilde{C} = C - D C_y. \] (A16)
Solving the RG flow (A15) analytically, we find \( \alpha(\mu) \) at any RG scale \( \mu \) in terms of \( \alpha(M) \) determined at some reference mass scale \( M \),
\[ \left( \frac{\mu}{M} \right)^{-\vartheta} = \frac{\alpha^* - \alpha(M)}{\alpha^* - \alpha(\mu)} \alpha(\mu) \exp \left( \frac{\alpha^*}{\alpha(M)} - \frac{\alpha^*}{\alpha(\mu)} \right). \] (A17)
Furthermore, the UV relevant scaling exponent \( \vartheta \) is given by
\[ \vartheta = \frac{\tilde{B}^2}{\tilde{C}} < 0 \] (A18)
in terms of (A16). The result (A17) can be resolved for \( \alpha(\mu) \) with the help of the Lambert function,
\[ \alpha(\mu) = \frac{\alpha^*}{W(\mu, M, \vartheta)}, \] (A19)
where \( W(\mu, M, \vartheta) \equiv W_L[z(\mu, M, \vartheta)] \) with \( W_L[z] \) denoting the Lambert function, defined implicitly through \( z = W_L \exp W_L \). The variable \( z(\mu, M, \vartheta) \) is given explicitly by
\[ z(\mu, M, \vartheta) = \left( \frac{\mu}{M} \right)^\vartheta \left( \frac{\alpha^*}{\alpha_M} - 1 \right) \exp \left( \frac{\alpha^*}{\alpha_M} - 1 \right), \] (A20)
with the relevant scaling exponent \( \vartheta \) given in (A18). For any value of \( \alpha_M \equiv \alpha(\mu = M) > 0 \) there is a unique branch of the Lambert function connecting \( \alpha(\mu) \) inbetween \( \alpha_M \) and \( \alpha^* \). The expression (A19) can now be used to approximately determine the mass scale \( M \) by matching it to values of the SM.

**Yukawa nullcline approximation.** Alternatively, we may use the Yukawa nullcline to estimate \( \alpha_y \) along the UV-IR connecting separatrix. In the system (A12), the Yukawa nullcline is given by
\[ \alpha_y = \frac{F}{E} \alpha. \] (A21)
The virtue of using (A21) to approximate the separatrix is twofold. Firstly, close to the Gaussian fixed point, the UV-IR connecting separatrix and the nullcline coincide, meaning that (A21) is a very good approximation if the gauge coupling is matched to the SM at scales where \( \alpha \ll \alpha^* \). Secondly, rewriting (A21) as
\[ \alpha_y = \frac{F}{E} (\alpha - \alpha^*) + \alpha_y^*, \] (A22)
Figure 23. UV-IR connecting trajectories at the example of a partially interacting fixed point FP$_3$ and parameters $R_2 = 1$, $R_3 = 10$, $N_F = 30$, showing the exact separatrix (red line) in comparison with the UV hypercritical surface approximation (magenta), (A13), and the Yukawa nullcline approximation (blue), (A22). The Yukawa nullcline offers a good global approximation for the exact UV-IR connecting separatrix.

we conclude that the nullcline also coincides with the hypercritical surface at the UV fixed point. Hence, (A21) can be viewed as a “global linear approximation” for the UV-IR connecting separatrix. Comparing this approximation with the UV hypercritical surface (A13) in the limit $B/C' \ll 1$ (with $C' = C - DF/E$), we observe that (A14) becomes $C_y = F/E + O(B/C')$, establishing that the hypercritical surface (A22) exactly coincides with the Yukawa nullcline. For finite $B/C' < 1$, however, the slopes of the hypercritical surface and the nullcline differ. Inserting (A21) into the running of the gauge coupling (A12) we find

$$\partial_t \alpha = \alpha^2 (-B + C' \alpha).$$  \hfill (A23)

The analytical solution to (A23) with initial condition $\alpha(\mu = M) = \alpha_M$ is given by (A19) with (A20), the sole difference being the value for the parameter $\vartheta$ which now reads

$$\vartheta = B^2/C' < 0$$  \hfill (A24)

instead of (A18).

Quantitatively, our results are illustrated in Fig. 23 at the example of a partially interacting fixed point FP$_3$ with $R_2 = 1$, $R_3 = 10$, $N_F = 30$. We compare the exact numerical solution for $\alpha_3(\mu)$ (full red line) with the hypercritical surface approximation (magenta) and with the Yukawa nullcline approximation (blue). We observe that the UV region (IR region) is well-approximated by the UV critical surface (Yukawa nullcline), respectively. We also observe that the exact separatrix is globally well approximated by the Yukawa nullcline, corresponding to (A19) together with (A20) and (A24). This approximation offers good quantitative estimates for the matching scale $M$. 

- [ ] UV-IR connecting separatrix
- [ ] UV hypercritical surface
- [ ] Yukawa nullcline

\begin{align*}
\alpha_3 & \quad \alpha_3^*
\end{align*}

\begin{align*}
\ln (\mu/M)
\end{align*}
Throughout the main body, we have used the exact numerical separatrix for our results. We have also confirmed that the critical surface and the nullcline approximations offer very good accuracy in their respective domains of applicability.

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