Discriminating hadronic and quark stars through gravitational waves of fluid pulsation modes

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Received 14 March 2014, revised 15 May 2014
Accepted for publication 12 June 2014
Published 14 July 2014

Abstract
One of the key aspects in the comprehension of neutron star (NS) interiors is the identification of observables that may impose constraints on the equation of state (EoS). At present, limits are obtained mainly through the study of the mass–radius relationship, the maximum rotational frequency and the cooling behaviour. However, since gravitational wave (GW) observatories such as Advanced LIGO and Advanced VIRGO will open a new window of the observation of NSs in the very near future, identifying observables that may emerge from the analysis of the GW emission of NSs is crucial. To this end, we investigate non-radial oscillations of hadronic, hybrid and pure self-bound strange quark stars with maximum masses above the mass of the recently observed massive pulsars PSR J1614–2230 and PSR J0348–0432 with $M \approx 2 \, M_\odot$. For the hadronic EoS we employ different parametrizations of a relativistic mean-field model and the EoS of Akmal, Pandharipande and Ravenhall. For quark matter we use the Massachusetts Institute of Technology bag model, including the effect of strong interactions and colour superconductivity. We find that the first pressure mode for strange quark stars has a very different shape than for hadronic and hybrid stars. For strange quarks stars, the frequency of the $p_1$ mode is larger than $\sim 7 \, \text{kHz}$ and diverges at small stellar masses, but for hadronic and hybrid stars it is in the range $\sim 4–7 \, \text{kHz}$. This allows an observational identification of strange stars even if extra information such as the mass, the radius or the gravitational redshift of the object is unavailable or uncertain. Also, we find as in previous works that the frequency of the $g$-mode associated with the quark–hadron discontinuity in a hybrid star is in the range $0.4–1 \, \text{kHz}$ for all masses. Thus, compact objects emitting GWs above $7 \, \text{kHz}$ should be interpreted as strange quark stars, and those emitting a signal within $0.4–1 \, \text{kHz}$ should be interpreted as hybrid stars.
Keywords: neutron stars, quark stars, stellar oscillations, gravitational waves

(Some figures may appear in colour only in the online journal)

1. Introduction

The determination of the mass of the pulsars PSR J1614–2230 with $M = (1.97 \pm 0.04) M_\odot$ [1] and PSR J0348–0432 with $M = (2.01 \pm 0.04) M_\odot$ [2] has brought the necessity to re-examine many aspects of the physics of neutron stars (NSs). In particular, these observations have revived discussions about whether compact stars are purely hadronic, may have quark-matter cores in their interior, or may be pure strange quark stars [3–20].

In fact, several authors have looked over the years for features that may allow us to distinguish unequivocally these stars through e.g. the analysis of the mass–radius relationship and the cooling behaviour since both aspects strongly depend on the microscopic composition. In the case of the mass–radius relationship a discrimination is not easy because the stellar radius is difficult to determine observationally. Moreover, at present, most observed compact stars have masses in a range where many models for hadronic, hybrid and strange stars overlap in the $M – R$ diagram. Cooling studies rely on the fact that the neutrino emission, the specific heat, the thermal conductivity, and other relevant quantities strongly depend on the microscopic composition. However, while different models lead to different thermal evolution, there are many degrees of freedom in the problem and a univocal interpretation of observed data is difficult.

On the other hand, it is now clear that the increase in sensitivity of gravitational wave (GW) detectors such as Advanced LIGO and Advanced VIRGO will bring within the next few years the science of gravitational radiation to a mode of regular astrophysical observation [21]. Since NSs can be conspicuous emitters of gravitational radiation, GWs of NSs will provide in the near future an important piece of information about several aspects of NS physics. In particular, transient phenomena involving the excitation of oscillation modes have long been considered as an important tool for the exploration of NSs interiors because several oscillation modes may emit GWs [22, 23]. A lot of work has been carried out in the last three decades in order to describe the non-radial oscillatory properties of NSs; however, these studies employed the equation of state (EoS) that in most cases renders maximum stellar masses below $2M_\odot$. Recent observations have shifted the maximum stellar mass above $2M_\odot$ and therefore it is worth re-examining the oscillation spectra because the change in the allowed EoS may bring new ways to distinguish hadronic, hybrid and strange stars.

In this work, we study the $f$, $p$ and $g$ modes of hadronic, hybrid and pure self-bound strange quark stars with maximum masses above $2M_\odot$ within the relativistic Cowling approximation. The paper is organized as follows: in section 2 we describe the EoS used for the description of hadronic and quark matter. In section 3 we present the equations that govern non-radial fluid oscillations of compact stars. In section 4 we present our results, and in section 5 a summary and our conclusions.

2. The equation of state

2.1 Hadronic matter

The relativistic mean-field model is widely used to describe hadronic matter in compact stars. In this paper we adopt the following standard Lagrangian for matter composed by nucleons
and electrons, 

\[ \mathcal{L}_H = \sum_B \bar{\psi}_B \left[ i \partial^\mu - g_{\omega B} \omega^\mu - \frac{1}{2} g_{\rho B} \not{\rho} \cdot \vec{\rho} \right] \]

\[ - (m_B - g_{\sigma B} \sigma) \psi_B + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) \]

\[ - \frac{1}{4} \omega^\mu \omega_\mu + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} \vec{\rho} \cdot \vec{\rho} \]

\[ + \frac{1}{2} m_\rho^2 \vec{\rho} \cdot \vec{\rho} - \frac{1}{3} b m_\sigma (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 \]

\[ + \sum_L \bar{\psi}_L \left[ i \gamma^\mu \partial_\mu - m_L \right] \psi_L. \quad (1) \]

Leptons \( L \) are treated as non-interacting and baryons \( B \) are coupled to the scalar meson \( \sigma \), the isoscalar-vector meson \( \omega \), and the isovector-vector meson \( \rho \). For more details about the EoS obtained from the above Lagrangian the reader is referred to e.g. [24] and references therein.

The five constants in the model are fitted to the bulk properties of nuclear matter [25]. In this work we use the parametrizations GM1 [25] and NL3 [26] whose coupling constants are shown in table 1. At low densities we use the Baym, Pethick and Sutherland model [27]. For completeness we will also present results that were obtained with the Akmal, Pandharipande and Ravenhall EoS [28].

2.2. Unpaired quark matter

For non-colour-superconducting quark matter we use the following modified bag model

\[ \Omega_{QM} = \sum_i \Omega_i + \frac{3 \mu_i^4}{4 \pi^2} (1 - a_i) + B, \]

where \( B \) is the bag constant and \( \Omega_i \) is the thermodynamic potential for a free gas of \( u, d, s \) quarks and electrons. The effects of gluon-mediated quantum chromodynamics interactions between the quarks in the Fermi sea are roughly incorporated through the parameter \( a_i \), in the same way as in [29] and [5]. With this EoS we construct hybrid stars and strange quark stars. In the case of hybrid stars we consider that the hadron–quark interphase is a sharp
discontinuity at which the pressure and the Gibbs free energy per baryon are continuous. For strange quark stars, the values of the parameters are chosen in order to fulfill the absolute stability condition \[\text{[30]}\]; i.e., the energy per baryon for three (two) flavour quark matter is smaller (larger) than the energy per baryon of the most stable atomic nucleus.

2.3. Effect of colour superconductivity

We also consider colour flavour locked (CFL) strange stars \[\text{[31, 32]}\] made up of CFL quark matter from the center to the surface of the star. Within the Massachusetts Institute of Technology (MIT) bag model and to order \(\Delta^2\), the thermodynamic potential \(\Omega_{\text{CFL}}\) can be expressed as \[\text{[33]}\]:

\[
\Omega_{\text{CFL}} = \Omega_{\text{free}} - \frac{3}{\pi^2} \Delta^2 \mu^2 + B,
\]

being \(\Omega_{\text{free}}\) the thermodynamic potential of a state of unpaired \(u, d\), and \(s\) quarks in which all of them have a common Fermi momentum \(\nu\), with \(\nu\) chosen to minimize \(\Omega_{\text{free}}\):

\[
\Omega_{\text{free}} = \int_0^\nu \frac{6p^2 dp}{\pi^2} [p - \mu] + \int_0^2 \frac{2p^2 dp}{\pi^2} \left[\sqrt{p^2 + m_s^2} - \mu\right].
\]

The binding energy of the diquark condensate is included in the condensation term proportional to \(\Delta^2 \mu^2\), where the chemical potential \(\mu \equiv (\mu_u + \mu_d + \mu_s)/3\) is related to \(\nu\) through \(\nu = 2\mu - (\mu_u^2 + m_s^2/3)^{1/2}\), being \(m_s\) the mass of the strange quark. We consider \(B, m_s\), and \(\Delta\) as free parameters that fall inside the stability windows presented in figure 2 of \[\text{[33]}\]; i.e., we always obtain self-bound strange stars when integrating the stellar structure equations. Additionally, the parameters satisfy the stability condition \(m_s^2 < 2\mu\Delta\) \[\text{[34]}\].

3. Non-radial fluid oscillations of compact stars

The framework for studying non-radial modes within the theory of general relativity was depicted in the pioneering work of Thorne and Campollataro \[\text{[35]}\], and further extended by other authors (see \[\text{[36]}\] and references therein). The perturbation equations are decomposed into spherical harmonics leading to two classes of oscillations according to the parity of the harmonics. Even (or polar) oscillations produce spheroidal deformations on the fluid, while odd (or axial) oscillations produce toroidal deformations (see \[\text{[23]}\] and references therein). For non-rotating stars composed of a perfect fluid, the fluid axial oscillations lead to a zero frequency trivial solution to the perturbation equations with vanishing pressure and density variations while the space-time axial modes \((w\)-modes or \(w_l\)-modes) are of non-zero frequency. For polar oscillations the linearized field equations inside the star can be cast as a system of three wave equations; two of them corresponding to the perturbations of the space-time and the other one to the density perturbations inside the star \[\text{[23]}\]. If the gravitational field is very weak, the two equations corresponding to the metric perturbation can be neglected while the remaining one describes the oscillations of the fluid. This approach is known as the Cowling approximation and considerably simplifies the analysis. This procedure was first introduced in \[\text{[37]}\] for the study of Newtonian stars and subsequently adapted in \[\text{[38]}\] for the investigation of relativistic stars. A more recent analysis shows that for typical relativistic stellar models the oscillation frequencies obtained by the complete linearized equations of general relativity and by the Cowling approximation differ by less than 20% for \(f\)-modes, around 10% for \(p\)-modes \[\text{[39]}\], and less than a few percent for \(g\)-modes \[\text{[40]}\]. This justifies its wide utilization for studying, for example, slowly and differentially rotating compact stars.
[41], rapidly rotating relativistic stars consisting of a perfect fluid obeying a polytropic EoS [42], elastic modes of oscillation in the crust of an NS [43], and NSs with internal anisotropic pressure [44].

In this work we employ the pulsation equations within the Cowling approximation as derived in [45]. To obtain these equations, fluid perturbations are decomposed into spherical harmonics \( Y_{l\ell} (\theta, \phi) \) and a sinusoidal time dependence \( \exp (i \omega t) \) with frequency \( \omega \). The Lagrangian fluid displacements that represent the infinitesimal oscillatory perturbations of the star are

\[
\xi^i = \left( e^{-\lambda} w_i - V \partial^i - V \sin^2 \theta \partial_i \right) r^{-2} Y_{l\ell}(\ell, \phi),
\]

where \( W \) and \( V \) are functions of \( r \). The pulsation equations read:

\[
\rho' \omega \Phi = \Lambda \Phi' - \Lambda p \rho \omega, \quad (4)
\]

\[
\Phi' = - \Lambda V \omega, \quad (5)
\]

where primes denote the derivatives with respect to \( r \) (for more details see [45]). To close the system we need boundary conditions at the center (\( r = 0 \)) and at the surface (\( r = R \)) of the star. The behaviour of \( W \) and \( V \) near the center of the star can be obtained from the above equations and is given by

\[
W(r) = C_1 r + C_2 r^2, \quad V(r) = - C_3 r + C_4 r^2,
\]

where \( C \) is an arbitrary constant. At the surface of the star the Lagrangian perturbation in the pressure must be zero (\( \Delta P = 0 \)), leading to

\[
a\rho' \omega \Phi + \Phi' = 0, \quad (6)
\]

In the case of hybrid stars, we have to impose additional junction conditions at the density discontinuity between the quark and the hadronic phases. These junction conditions read [45]:

\[
W_i = W_-, \quad (7)
\]

\[
V_i = \frac{\rho_+ + P}{\rho_+ + P} \left[ a^2 \lambda^2 + e^{-\lambda} \Phi' W_- \right] - e^{-\lambda} \Phi' W_-, \quad (8)
\]

where \( R_\gamma \) represents the position of the density discontinuity, and the values of \( W, V, \) and \( \rho \) at both sides of the discontinuity are denoted by: \( W_- \equiv W \left( R_\gamma - 0 \right), V_- \equiv V \left( R_\gamma - 0 \right), \rho_- \equiv \rho \left( R_\gamma - 0 \right), W_+ \equiv W \left( R_\gamma + 0 \right), V_+ \equiv V \left( R_\gamma + 0 \right), \) and \( \rho_+ \equiv \rho \left( R_\gamma + 0 \right) \).

In order to numerically solve the oscillation equations we proceed as follows. First, we integrate the Tolman–Oppenheimer–Volkoff stellar structure equations for each set of parameters of the EoS in order to obtain the coefficients of the oscillation equations for a given central pressure. Then we solve the oscillation equations by means of the shooting method: we start the numerical integration of equations (4) and (5) for a trial value of \( \omega^2 \) and a given set of values of \( W \) and \( V \) such that the boundary condition at the centre is fulfilled. The equations are integrated outwards trying to match the boundary condition at the star’s surface. After each integration, the trial value of \( \omega^2 \) is corrected through a Newton–Raphson iteration scheme in order to improve the matching of the surface boundary condition until the desired precision is achieved. The discrete values of \( \omega \) for which equation (6) is satisfied are the eigenfrequencies of the star. In order to check our code we have reproduced the results of [47]. In the case of hybrid stars, we employ the shooting to a fitting point method. The numerical integration is started at the centre and the surface of the star towards the density...
discontinuity and the trial value of $\omega^2$ is corrected until the junction conditions in equations (7) and (8) are verified with the desired precision.

4. Results

The polar quasi-normal modes are usually classified according to a scheme in which each family of modes is directly associated with the restoring force that prevails when a fluid element is displaced from its equilibrium position [37]. The most important modes for GW emission are the (pressure) $p$-modes, the (fundamental) $f$-mode, and the (gravity) $g$-modes. The frequencies of $g$-modes are lower than those of $p$-modes, and the two sets are separated by the frequency of the $f$-mode [23]. These are called fluid modes to distinguish them from e.g. purely gravitational modes ($\omega$-modes) for which the fluid motion is barely excited. Since the metric perturbations are set to zero within the Cowling approximation, only $f$, $p$ and $g$ modes can be studied through the equations of the previous section. In chemically homogeneous, zero temperature (and hence isentropic) stars, all the $g$-modes are zero frequency [48], i.e. in the present study they arise only in hybrid stars. In figures 1–5 we show our results for quadrupole oscillations ($l = 2$). For strange quark stars and hybrid stars, the mass of the strange quark has been set to $m_s = 100$ MeV in all calculations, and we spanned the values of the parameters $a_4$, $B$ and $\Delta$ that give stars with a maximum mass above $2M_\odot$.

In figure 1 we show our results for the $f$-mode of hadronic and hybrid stars and in figure 2 of hadronic and strange stars. In figure 1 we see that there is a folding in the curve for hybrid stars at the mass value above which the star has a core of quark matter. Above that mass, the curves for hybrid stars are steeper than the hadronic ones but the models overlap each other. In the upper panel of figure 2 we show $f_\pi$ as a function of the stellar mass for hadronic and strange stars. The shape of the curves is qualitatively different for both types of objects but the results tend to overlap around ~2 kHz in the mass range of interest. However, in some cases it is possible to differentiate strange stars from hadronic/hybrid stars. For example, objects in
Figure 2. Frequencies of the $f$-mode for hadronic stars and strange quark stars as a function of the mass $M$, the gravitational redshift $z$ at the surface of the star, and the square root of the mean density. For strange stars, the labels indicate the values of $B$ in MeV fm$^{-3}$, of $\Delta$ in MeV and of $a_s$. In the lower panel we include the analytic fittings of [22] and [46] for hadronic stars. The results for the APRB2 and BBS1 equations of state for hybrid stars have been extracted from [46].
the mass range $1 \sim 1.5 \, M_\odot$ with $f_\nu$ in the range $2 \sim 3 \, \text{kHz}$ would be strange stars. We also present the behaviour of $f_\nu$ as a function of the gravitational redshift $z$ at the surface of the star (see central panel of figure 2) because $z$ could be inferred through the identification of spectral lines. Finally, in the bottom panel of figure 2 we show $f_\nu$ as a function of the square root of the average density, which is a more natural scaling in the case of hadronic stars [22]. We also show the fitting formulae found in [22] and [46] which are in reasonable agreement with the curves for hadronic stars with maximum masses above $2 \, M_\odot$. The main conclusion that can be obtained from figures 1 and 2 is that there is an overlapping of the results for hadronic, hybrid and strange stars around a frequency of $\sim 2 \, \text{kHz}$, and therefore, in most cases it would be rather difficult to infer the internal composition of a compact object even if its mass or the surface $z$ is determined together with the frequency of the fundamental mode. However, in some cases the identification of strange stars would be possible.

Our results for the $f$-mode of hybrid/hadronic stars are in agreement with previous calculations [45, 49] which give $f_\nu \approx 1.5 \sim 3.5 \, \text{kHz}$; however, notice that most of that models have maximum masses well below $2 \, M_\odot$. Additionally, our calculations were performed for many values of the stellar mass and therefore our curves show clearly the behaviour near the maximum mass and in the case of hybrid stars the folding at the mass value above which the star has a core of quark matter. For strange stars, [49] found results in agreement with ours; in particular, they show that strange stars can be differentiated from hadronic/hybrid stars in some cases. However, their models have maximum masses that never exceed $1.8 \, M_\odot$. Notice that we also explored the effect of colour superconductivity that was not addressed previously.

In figures 3–4 we show our results for the first pressure mode. The shape of the curves for hadronic and hybrid stars is different, as can be seen in figure 3. Above $M \sim 1.5 \, M_\odot$, the branches corresponding to hybrid stars emerge over the curves corresponding to the hadronic models, i.e. for a given hadronic EoS the frequencies for hybrid stars are larger than for hadronic stars. However, as seen in figure 3, the curves overlap if we consider several hadronic and hybrid models, and again, it would be rather difficult to infer the existence of a
quark matter core inside a given compact star even if $M$ or $z$ are measured together with $f_{p1}$. The situation is different if we compare strange quark stars with hybrid or hadronic stars. For hadronic and hybrid stars, the frequencies are $\sim 7$ kHz near the maximum mass and decrease to $\sim 4$ kHz for small masses. For strange stars the frequencies are $\sim 7$ kHz near the maximum mass but are considerably larger for smaller masses. NSs observed up to date have masses in the range 1.0–2.0 $M_\odot$. Therefore, the observation of a $p_1$-mode with a frequency significantly larger than $\sim 7$ kHz would be clear evidence in favor of a strange quark star, even if the mass, the radius or the gravitational redshift of the object are unknown.

Notice that our results for the $p_1$-mode are consistent with previous studies. In [46] purely hadronic stars and hybrid stars were described using some hadronic EoS [25, 28, 50] and the MIT bag model for quark matter, giving $f_{p1} \sim 5$–6 kHz. They also considered few strange star models with masses below 1.5 $M_\odot$ and obtained $f_{p1} \sim 8$–11 kHz, in agreement with our results for low mass objects. Sotani et al. [45] also present calculations for hadronic and hybrid models with very low maximum masses that are consistent with our results.
In Figure 5 we present the results for the $g$-mode of hybrid stars. The frequencies cover the range 0.4–1 kHz, in agreement with previous calculations for polytropic EoS [51] and for hybrid stars with low maximum masses [45]. The frequency interval of the $g$-modes for different parametrizations of the EoS is clearly separated from the $f$-mode frequencies. Additionally, other $g$-modes such as those associated with a non-homogeneous composition in the outer layers of the star, or those associated with a thermal profile, have lower frequencies than the here-studied quark–hadron discontinuity $g$-modes [51]. Thus, the observation of oscillations with frequency in the range 0.4–1 kHz would be an evidence of a hybrid star.

5. Summary and conclusions

In this paper we have investigated non-radial fluid oscillations of hadronic, hybrid and strange quark stars with maximum masses above the mass of the recently observed pulsars PSR J1614–2230 and PSR J0348–0432 with $M \approx 2 M_\odot$. For the hadronic EoS we employed two different parametrizations of a relativistic mean-field model with nucleons and electrons. For quark matter we have included the effect of strong interactions and colour superconductivity within the MIT bag model. The equations of non-radial oscillations were integrated within the Cowling approximation in order to determine the frequency of the $f$, $p_1$ and $g$-modes.

We find that the fundamental mode is sensitive to the internal composition, but due to the uncertainties in the EoS, there is an overlapping of the curves corresponding to hadronic, hybrid and strange quark stars for stellar masses larger that $\sim 1 M_\odot$. As a consequence it would be difficult to distinguish hybrid and hadronic stars through the $f$-mode frequency, even if the mass or the surface $z$ of the object is determined concomitantly with $f_f$. However, there are features that in some cases may allow a differentiation between strange stars and hadronic/hybrid stars. For example, strange stars cannot emit GWs with frequency below $\sim 1.7$ kHz for any value of the mass. Also, sources with a mass in the range $1–1.5 M_\odot$ emitting a signal in the range 2–3 kHz would be strange stars. The frequency of the $p_1$ mode is much more affected by the internal composition of the star. For hadronic and hybrid stars, we find that $f_{p_1}$
is in the range 4–7 kHz for objects with masses in the range 1–2 $M_\odot$, but for strange quark stars it can be significantly larger than $\sim 7$ kHz. Thus, a compact object emitting a signal above $\sim 7$ kHz could be identified as a strange star even if its mass or gravitational redshift are unknown. High frequency $g$-modes are only present in hybrid stars and fall in the range 0.4–1 kHz. Thus, they are clearly distinguishable from the fundamental mode, and of low-frequency $g$-modes associated with chemical inhomogeneities in the outer layers or thermal profiles. Our results are summarized in table 2 and show that based on the simultaneous analysis of the frequency of the $f$, $p_1$ and $g$-modes, it would be possible to discriminate between hadronic, hybrid and strange quark stars.

Concerning the observability of the GWs, a plausible detection scenario has already been depicted in [22]; however, a few words are in order regarding more recent results. Detectable amounts of gravitational radiation can be expected because pulsation modes in rotating compact stars can be driven unstable under the emission of GWs by the so-called Chandrasekhar–Friedman–Schutz mechanism. Recently, Gaertig et al [52] presented the first calculation of the growth rate of the $f$-mode instability in relativistic stars and analyzed the stellar parameter space (instability window) where the mode growth due to GWs overcomes dissipative effects such as shear and bulk viscosity. For the dominant $l = m = 4$ mode the instability is present for $\Omega/\Omega_K \approx 0.92–1$, where $\Omega_K$ is the Kepler frequency, and for temperatures $\sim (10^9 - 2 \times 10^{10})$ K. Also, Passamonti et al [53] addressed the influence of the magnetic field on the $f$-mode instability and the presence of an unstable $r$-mode. They considered more massive models and studied the $l = m = 2$ $r$-mode and the $l = m = 4$ $f$-mode, finding that GWs produced during the instability could be detected by the Einstein telescope for sources located in the Virgo Cluster, where around 30–60 supernovas are expected per year. Moreover, for some models, a detection of the $l = m = 3$ $f$-modes would be possible even with the advanced LIGO/VIRGO [53].

Finally, it should be noticed that the spectrum of a pulsating compact star is very rich and therefore the possibility of having other kinds of modes in the same frequency range of the here-studied fluid modes should be addressed in more detail in the light of modern EoS leading to stellar models compatible with the recent observations of PSR J1614–2230 and PSR J0348–0432. If other modes are present in the same range, the criterion presented in the present work may be less effective. Additionally, rotation is known to change the frequency range of the modes, but these modifications are not expected to qualitatively alter the scenario presented in this paper (see e.g. [54] and references therein). In particular, the conclusion that compact objects emitting a signal above 7 kHz should be interpreted as strange quark stars and those emitting a signal in the range $\sim 0.4–1$ kHz should be interpreted as hybrid stars looks quite robust.

| Class. Quantum Grav. 31 (2014) 155002 | C Vásquez Flores and G Lugones |

| Table 2. Discrimination between hadronic, hybrid and strange quark stars based on the observation of the $f$, $p_1$ and $g$ modes. |
|---|---|---|
| Strange stars | $f_f \sim 2$ kHz | $f_{p_1} \geq 7$ kHz | not present |
| Hybrid stars | $f_f \sim 2$ kHz | $f_{p_1} \sim 4$–7 kHz | $f_g \sim 0.4$–1 kHz |
| Hadronic stars | $f_f \sim 2$ kHz | $f_{p_1} \sim 4$–7 kHz | not present |
Acknowledgments

CVF and GL acknowledge the financial support received from FAPESP-Brazil.

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