Optimal Power Allocation for A Massive MIMO Relay Aided Secure Communication

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Abstract—In this paper, we address the problem of optimal power allocation at the relay in two-hop secure communications under practical conditions. To guarantee secure communication during the long-distance transmission, the massive MIMO (M-MIMO) relaying techniques are explored to significantly enhance wireless security. The focus of this paper is on the analysis and design of optimal power assignment for a decode-and-forward (DF) M-MIMO relay, so as to maximize the secrecy outage capacity and minimize the interception probability, respectively. Our study reveals the condition for a nonnegative the secrecy outage capacity, obtains closed-form expressions for optimal power, and presents the asymptotic characteristics of secrecy performance. Finally, simulation results validate the effectiveness of the proposed schemes.

I. INTRODUCTION

The open nature of the wireless channel facilitates a multi-user transmission, but also incurs the security problem. Recently, as a complement of traditional upper-layer encryption techniques, physical layer security (PHY-security) has been proposed to realize secure communications by making use of the characteristics of wireless channels, i.e., noise, fading and interference [1].

From an information-theoretic viewpoint, the performance of PHY-security is determined by the rate difference between the legitimate channel and the eavesdropper channel [2] [3]. Therefore, to enhance wireless security, it makes sense to simultaneously increase the legitimate channel rate and decrease the eavesdropper channel rate. Inspired by this, various physical layer techniques have been introduced to improve the secrecy performance. Wherein, MIMO relaying technique gains considerable attention due to the following two reasons. First, the relay can provide a diversity gain and shorten the accessing distance, and thus improve the secrecy performance [4]. Second, MIMO techniques, such as spatial beamforming, can reduce the information leakage to the eavesdropper [5]. The beamforming schemes at the MIMO relay based on amplify-and-forward (AF) and decode-and-forward (DF) relaying protocols were presented in [6] and [7], respectively. It is worth pointing out that the optimal beam design at the relay requires global channel state information (CSI) [8]. Yet, the CSI, especially eavesdropper CSI is difficult to obtain, since the eavesdropper is usually passive and keeps silent. Therefore, it is impossible to realize absolutely secure communications over fading channels. In this context, statistical secrecy performance metrics, e.g., secrecy outage capacity and interception probability, are adopted to evaluate wireless security [9].

Recently, an advanced MIMO relaying technology, namely massive MIMO (M-MIMO) relaying, is introduced into secure communications to further improve the secrecy performance [10]. Even without eavesdropper CSI, M-MIMO relaying can generate a very high-resolution spatial beam, and thus the information leakage to the eavesdropper is quite small. More importantly, the secrecy performance can be enhanced by simply adding more antennas. Hence, the challenging issue of short-distance interception in secure communications can be well solved. Note that in two-hop secure communications, the transmit power at the relay has a great impact on the secrecy performance, since it affects the signal quality at both the destination and the eavesdropper. An optimal power allocation scheme for a multi-carrier two-hop single-antenna relaying network was given in [11] by maximizing the sum secrecy rate. However, the power allocation for a multi-antenna relay, especially an M-MIMO relay, is still an open issue. In this paper, we focus on power allocation for DF M-MIMO secure relaying systems under very practical assumptions, i.e., no eavesdropper CSI and imperfect legitimate CSI. The contributions of this paper are three-fold:

1) We reveal the relation between the secrecy outage capacity and a defined relative distance-dependent path loss, and then derive the condition for a nonnegative secrecy outage capacity and the constraint on the minimum number of antennas.
2) We derive closed-form expressions for the optimal power at the relay in the sense of maximizing the secrecy outage capacity and minimizing the interception probability, respectively.
3) We present clear insights into the secrecy performance through asymptotic analysis. We show that the maximum secrecy outage capacity is an increasing function of the source power while the minimum interception probability keeps constant.

The rest of this paper is organized as follows. We first give an overview of the DF M-MIMO secure relaying system in Section II, and then derive two optimal power allocation schemes for the relay in Section III. In Section IV, we present simulation results to validate the effectiveness of the proposed schemes. Finally, we conclude the paper in Section V.
II. SYSTEM MODEL

We consider a time division duplex (TDD) two-hop massive MIMO (M-MIMO) relaying system, where a single antenna source transmits message to a single antenna destination with the aid of a relay with \(N_r\) antennas, while a passive single antenna eavesdropper intends to intercept the information. Note that the number of antennas at the relay in such an M-MIMO relaying system is quite large, i.e., \(N_r = 100\) or even bigger. It is assumed that there is no direct transmission between the source and the destination due to a long propagation distance. The relay system works in a half-duplex mode, which means any successful transmission requires two time slots. Specifically, the source sends the signal to the relay in the first time slot, and then the relay forwards the post-processing signal to the destination in the second time slot. In this paper, we assume the eavesdropper is far from the source and close to the relay, since it was concluded that the signal comes from the relay directly. Therefore, it is reasonable to assume that the eavesdropper only monitors the transmission from the relay to the destination.

We use \(\sqrt{\alpha_{S,R}} \mathbf{h}_{S,R}, \sqrt{\alpha_{R,D}} \mathbf{h}_{R,D}\) and \(\sqrt{\alpha_{R,E}} \mathbf{h}_{R,E}\) to represent the channels from the source to the relay, the relay to the destination, and the relay to the eavesdropper, respectively, where \(\alpha_{S,R}, \alpha_{R,D}\) and \(\alpha_{R,E}\) are the distance-dependent path losses and \(\mathbf{h}_{S,R}, \mathbf{h}_{R,D}\) and \(\mathbf{h}_{R,E}\) denote the channel fading coefficient vectors with independent and identically distributed (i.i.d.) zero mean and unit variance complex Gaussian entries. It is assumed that the channels remain constant during a time slot and fade independently over slots. Thus, the received signal at the relay in the first time slot can be expressed as

\[
y_R = \sqrt{P_S} \alpha_{S,R} \mathbf{h}_{S,R} s + \mathbf{n}_R, \tag{1}
\]

where \(s\) is the normalized Gaussian distributed transmit symbol, \(P_S\) is the transmit power at the source, and \(\mathbf{n}_R\) stands for the additive white Gaussian noise with unit variance at the relay. We assume that the relay has perfect CSI about \(\mathbf{h}_{S,R}\) by channel estimation. Then, maximum ratio combination (MRC) decoding is performed to recover the information. Specifically, the received signal is multiplied by a vector \(\mathbf{w}_R = \mathbf{h}_{S,R}/\|\mathbf{h}_{S,R}\|\).

During the second time slot, the relay forwards the decoded signal \(s\) through maximum ratio transmission (MRT). We assume that the relay has partial CSI about \(\mathbf{h}_{R,D}\) due to channel reciprocity impairment in TDD systems. The relation between the estimated CSI \(\hat{\mathbf{h}}_{R,D}\) and the real CSI \(\mathbf{h}_{R,D}\) is given by

\[
\hat{\mathbf{h}}_{R,D} = \sqrt{\hat{\mathbf{h}}_{R,D}} + \sqrt{1-\rho} \mathbf{e}, \tag{2}
\]

where \(\mathbf{e}\) is the error vector with i.i.d. zero mean and unit variance complex Gaussian entries, and is independent of \(\hat{\mathbf{h}}_{R,D}, \rho\), scaling from 0 to 1, is the correlation coefficient between \(\mathbf{h}_{R,D}\) and \(\hat{\mathbf{h}}_{R,D}\). Thus, the received signals at the destination and the eavesdropper are given by

\[
y_D = \sqrt{P_R \alpha_{R,D}} \mathbf{h}_{R,D}^H \mathbf{r} + n_D, \tag{3}
\]

\[
y_E = \sqrt{P_R \alpha_{R,E}} \mathbf{h}_{R,E}^H \mathbf{r} + n_E, \tag{4}
\]

respectively, where \(P_R\) is the transmit power of the relay, \(\mathbf{r} = \mathbf{v}_R \hat{s}\) is the forwarded signal with \(\mathbf{v}_R = \hat{\mathbf{h}}_{R,D}/\|\hat{\mathbf{h}}_{R,D}\|\) being a MRT beamforming vector. \(n_D\) and \(n_E\) are additive white Gaussian noise (AWGN) samples with unit variance at the destination and the eavesdropper, respectively.

III. OPTIMAL POWER ALLOCATION

In this section, we aim to optimize the secrecy performance through allocating the relay power \(P_R\), since it affects the signal quality at both the destination and the eavesdropper. In what follows, we analyze and design the power allocation schemes in the sense of maximizing the secrecy outage capacity and minimizing the interception probability, respectively.

A. Secrecy Outage Capacity Maximization Power Allocation

Based on the received signal in (1), when performing MRC decoding at the relay, the channel capacity from the source to the relay can be expressed as

\[
C_{S,R} = W \log_2 (1 + \gamma_R), \tag{5}
\]

where \(W\) is the spectral bandwidth and \(\gamma_R = P_S \alpha_{S,R} \|\mathbf{h}_{S,R}\|^2\) is the signal-to-noise ratio (SNR). Similarly, according to (3) and (4), channel capacities from the relay to the destination and from the relay to the eavesdropper are given by

\[
C_{R,D} = W \log_2 (1 + \gamma_D), \tag{6}
\]

\[
C_{R,E} = W \log_2 (1 + \gamma_E), \tag{7}
\]

respectively, where \(\gamma_D = P_R \alpha_{R,D} \|\mathbf{h}_{R,D}^H \mathbf{h}_{R,D}/\|\hat{\mathbf{h}}_{R,D}\|\|^2\) and \(\gamma_E = P_R \alpha_{R,E} \|\mathbf{h}_{R,E}^H \mathbf{h}_{R,D}/\|\hat{\mathbf{h}}_{R,D}\|\|^2\).

Then, for the secrecy outage capacity in a DF M-MIMO secrecy relaying system, we have the following theorem:

Theorem 1: Given an outage probability bound by \(\varepsilon\), the secrecy outage capacity is approximated as

\[
C_{soc} = W \log_2 (1 + \min(P_S \alpha_{S,R} N_R, P_R \alpha_{R,D} \rho N_R)) - W \log_2 (1 + P_R \alpha_{R,E} \ln \varepsilon), \tag{8}
\]

where \(N_R\) is the number of relay antennas.

Proof: Please refer to Appendix I.

Note that the secrecy outage capacity may be negative from a pure mathematical view. Hence, it makes sense to find the condition that the secrecy outage capacity is nonnegative.

For notational simplicity, we let \(\rho \alpha_{R,D} N_R = A, -\alpha_{R,E} \ln \varepsilon = A \cdot r_1, P_S \alpha_{S,R} N_R = B\), where \(r_1 = -\alpha_{R,E} \ln \varepsilon / \rho \alpha_{R,D} N_R\) is defined as the relative distance-dependent path loss. Then, the secrecy outage capacity can be rewritten as

\[
C_{soc} = \begin{cases} W \log_2(1 + B) - W \log_2(1 + P_R A r_1), & B < P_R A \\ W \log_2(1 + P_R A) - W \log_2(1 + P_R A r_1), & B \geq P_R A \end{cases}
\]

Observing the above secrecy outage capacity, we get the following theorem:
\textbf{Theorem 2:} If and only if \(0 < r_1 < 1\), the secrecy outage capacity in such a DF M-MIMO secure relaying system in presence of imperfect CSI is nonnegative.

\textit{Proof:} Please refer to Appendix II.

Notice that \(0 < r_1 < 1\) is the precondition for power allocation in such a secure relaying system. Given channel conditions and outage probability requirements, in order to fulfill \(0 < r_1 < 1\), the number of antennas \(N_R\) must be bigger than \(-\frac{\log R}{\rho_{R,D}}\). For an M-MIMO relaying system, it is always possible to meet the condition of \(0 < r_1 < 1\) by adding more antennas, which is one of its main advantages. In what follows, we only consider the case of \(0 < r_1 < 1\).

Based on Theorem 1, the secrecy outage capacity maximization power allocation is equivalent to the following optimization problem:

\[
J_1 : \max_{P_R} \quad C_{soc}
\]

\[\text{s.t. } P_R \leq P_{max}, \quad (8)\]

where \(P_{max}\) is the transmit power constraint at the relay.

For the optimal solution of the above optimization problem, we have the following theorem:

\textbf{Theorem 3:} The optimal power at the relay is \(P_R^* = \min \left(\frac{P_S\alpha_{S,R}}{\rho_{R,D}}, P_{max}\right)\), and the corresponding maximum secrecy outage capacity is

\[
C_{soc}^\max = W \log_2 \left(1 + \min \left(\frac{P_S\alpha_{S,R}}{\rho_{R,D}}, P_{max}\right) A R_l\right).
\]

\textit{Proof:} Please refer to Appendix III.

\textbf{Remark:} It is found that when eavesdropper CSI is unavailable, it is optimal for the DF secure relaying system to let the two hops have the same channel capacity, resulting in \(P_R^* = \min \left(\frac{P_S\alpha_{S,R}}{\rho_{R,D}}, P_{max}\right)\).

\section{B. Interception Probability Minimization Power Allocation}

In this subsection, we analyze the optimal power allocation from the perspective of minimizing the interception probability. In general, interception probability is defined as the probability of information leakage, namely the probability of \(C_D < C_E\). Then, interception probability is equivalent to the secrecy outage probability when \(C_{soc} = 0\) in (18). Thus, it can be computed as

\[
P_0 = \exp \left(-\frac{2C_D/W - 1}{P_R\rho_{R,E}}\right), \quad (9)
\]

where \(C_D = W \log_2 \left(1 + \min \left(P_S\alpha_{S,R}N_R, P_R\alpha_{R,D}pN_R\right)\right)\) is the legitimate channel capacity.

Then, interception probability minimization power allocation can be described as the following optimization problem:

\[
J_2 : \min_{P_R} \quad P_0
\]

\[\text{s.t. } P_R \leq P_{max}, \quad (10)\]

Since \(\exp(x)\) is a monotonously increasing function of \(x\) and \(\min_x(-f(x))\) is equivalent to \(\max_x(f(x))\), \(J_2\) can be transformed to the following problem:

\[
J_3 : \max_{P_R} \quad \min \left(P_S\alpha_{S,R}N_R, P_R\alpha_{R,D}pN_R\right) \quad \frac{P_R\rho_{R,E}}{P_{max}}
\]

\[\text{s.t. } P_R \leq P_{max} \quad (11)\]

By solving the above optimization problem, we have the following theorem:

\textbf{Theorem 4:} From the perspective of minimizing interception probability, the optimal transmit power at the relay \(P_R^*\) belongs to a region \(\left[0, \min \left(\frac{P_S\alpha_{S,R}}{\rho_{R,D}}, P_{max}\right)\right]\), and the corresponding minimum interception probability is \(P_{0}^\min = \exp \left(-\frac{\rho_{R,D}}{\alpha_{R,E}}\right)\).

\textit{Proof:} Please refer to Appendix IV.

\textbf{Remarks:} It is found that when eavesdropper CSI is unavailable, the optimal power minimizing the interception probability is not unique. However, from the perspective of maximizing the secrecy outage capacity, \(P_R = \min \left(\frac{P_S\alpha_{S,R}}{\rho_{R,D}}, P_{max}\right)\), namely the upper bound, is optimal. Thus, it is better to let \(P_R^* = \min \left(\frac{P_S\alpha_{S,R}}{\rho_{R,D}}, P_{max}\right)\) in the sense of jointly optimizing interception probability and secrecy outage capacity.

\section{C. Asymptotic Characteristic}

In the above, we prove that the optimal relay power \(P_R^*\) in the sense of maximizing the secrecy outage capacity and minimizing the interception probability is a function of the source power \(P_S\). Thus, the source power has a great impact on the secrecy performance, as described in Theorem 3 and 4.

In order to get some clear insights, we carry out an asymptotic performance analysis with respect to the source power.

First, for the secrecy outage capacity in a DF M-MIMO secure relaying system, there are the following asymptotic characteristics:

\textbf{Proposition 1:} In the low \(P_S\) regime, the optimal relay power \(P_R^*\) and maximum secrecy outage capacity \(C_{soc}^\max\) asymptotically approach zero. In the high \(P_S\) regime, the maximum secrecy outage capacity will be saturated and is independent of \(P_S\). Furthermore, \(C_{soc}^\max\) is an increasing function of \(P_S\).

\textit{Proof:} In the low \(P_S\) regime, the optimal relay power and the corresponding secrecy outage capacity are reduced as \(P_R^* = \frac{P_S\alpha_{S,R}}{\rho_{R,D}}\) and \(C_{soc}^\max = W \log_2 \left(\frac{1 + P_S\alpha_{S,R}N_R}{1 + P_S\alpha_{S,R}N_R}\right)\), respectively. Both of them asymptotically approach zero as \(P_S\) tends to zero. Otherwise, in the high \(P_S\) regime, the optimal relay power is limited by \(P_{max}\), then \(C_{soc}^\max\) is constant. In other words, the secrecy outage capacity becomes saturated. In addition, because of \(0 < r_1 < 1\), \(C_{soc}^\max\) is an increasing function of \(P_S\).

Second, for the interception probability, we have the following asymptotic characteristics:

\textbf{Proposition 2:} The minimum interception probability is independent of \(P_S\).

\textit{Proof:} As proved in Theorem 4, although the optimal relay power is a function of the source power, the final interception probability is a constant independent of \(P_S\) and \(P_R\).
IV. Simulation Results

To examine the effectiveness of the proposed power allocation schemes for DF M-MIMO secure relaying systems, we present several simulation results for the following scenarios. We set $N_R = 100$, $W = 10$ kH, $\rho = 0.9$ and $\epsilon = 0.01$ without extra statements. For convenience, we normalize the path loss from the source to the relay as $\alpha_{S,R} = 1$ and use $\alpha_{R,D}$ and $\alpha_{S,E}$ to denote the relative path loss. Specifically, $\alpha_{R,E} > \alpha_{R,D}$ means that the eavesdropper is closer to the relay than the destination. In addition, we use $SNR_S = 10 \log_{10} P_S$, $SNR_R = 10 \log_{10} P_R$ and $SNR_{\text{max}} = 10 \log_{10} P_{\text{max}}$ to represent the SNR in dB at the source, the relay and the constraint at the relay, respectively.

First, we verify the accuracy of the theoretical expression in Theorem 1 with $SNR_S = SNR_R = 10$ dB and $\alpha_{R,D} = 1$. As seen in Fig. 1, the theoretical results are well consistent with the simulations in the whole $\alpha_{R,E}$ region with different outage probability requirements, which proves the high accuracy of the derived results. Given an outage probability bound by $\epsilon$, as $\alpha_{R,E}$ increases, the secrecy outage capacity decreases gradually. This is because the interception ability of the eavesdropper enhances due to the short interception distance. In addition, for a given $\alpha_{R,E}$, the secrecy outage capacity improves with the increase of $\epsilon$, since the outage probability is an increasing function of the secrecy outage capacity.

Next, we show the performance gain of the proposed optimal power allocation schemes compared to a fixed power allocation with $SNR_S = 10$ dB, $SNR_{\text{max}} = 15$ dB and $\alpha_{R,E} = 5$. It is worth pointing out that the fixed scheme uses a fixed power $P_R = 15$ dB regardless of channel conditions and system parameters. As seen in Fig. 2, the secrecy outage capacity maximization power allocation scheme performs better than the fixed power allocation scheme. Especially in the high $\alpha_{R,D}$ regime, the performance of the proposed scheme improves sharply, while that of the fixed allocation scheme nearly remains unchanged. This is because the legitimate channel capacity is limited by the source-relay channel capacity under this condition, but the fixed scheme is regardless of $\alpha_{S,R}$ and $P_S$. In the low $\alpha_{R,D}$ regime, the secrecy outage capacities of both schemes approach zero due to $\eta_1 > 1$, which verifies Theorem 2 again. In terms of interception probability, as shown in Fig. 3, the proposed scheme also outperforms the fixed power allocation scheme. Consistent with the theoretical claims, the interception probability approaches zero when $\alpha_{R,D}$ is large enough.

Finally, we check the asymptotic characteristics with $\alpha_{R,D} = 1$. As shown in Fig. 4 as $P_S$ tends to zero, the maximum secrecy capacities with different $\alpha_{R,E}$ approach zero. In the large $P_S$ regime, the maximum secrecy outage capacity will be saturated, which is in agreement with Proposition 1 again. From Fig. 5 it is seen that the minimum interception probability is independent of $P_S$. Additionally, the interception probability floor becomes higher with the increase of $\alpha_{R,E}$.

V. Conclusion

In this paper, we have first presented a secrecy performance analysis for a DF M-MIMO secure relaying system with imperfect CSI. We proved that in order to guarantee a nonnegative secrecy outage capacity, there is a constraint
The channel capacity can be computed as (5) and from the relay to the destination in (6), the legitimate maximization of the secrecy outage capacity and minimizing the on the minimum number of antennas at the relay. Then, by maximizing the secrecy outage capacity and minimizing the interception probability, we proposed two optimal relay power allocation schemes. At last, we revealed the asymptotic characteristics of maximum secrecy outage capacity and minimum interception probability with respect to the source power.

**APPENDIX A**

**PROOF OF THEOREM 1**

Based on channel capacities from the source to the relay in (5) and from the relay to the destination in (6), the legitimate channel capacity can be computed as

\[
C_D = \min(C_{S,R}, C_{R,D}), \quad (12)
\]

\[
= \min\left(W \log_2(1 + P_S \alpha_{S,R} ||h_{S,R}||^2), W \log_2\left(1 + P_{R \alpha_{R,D}} \left(\sqrt{\rho_h^H h_{R,D}^H + \sqrt{1 - \rho_e^H} \frac{h_{R,D}}{||h_{R,D}||^2} \right)^2\right)\right) (13)
\]

\[
= \min\left(W \log_2(1 + P_S \alpha_{S,R} ||h_{S,R}||^2), W \log_2\left(1 + P_{R \alpha_{R,D}} (\rho ||h_{R,D}||^2 + 2 \sqrt{(1 - \rho) \rho R(e^H h_{R,D})} + (1 - \rho) ||e^H h_{R,D}||^2 / ||h_{R,D}||^2)\right)\right)
\]

\[
\approx \min\left(W \log_2(1 + P_S \alpha_{S,R} ||h_{S,R}||^2), W \log_2\left(1 + P_{R \alpha_{R,D}} ||h_{R,D}||^2\right)\right) (14)
\]

\[
\approx W \log_2\left(1 + \min(P_S \alpha_{S,R} N_R, P_{R \alpha_{R,D}} \rho N_R)\right). \quad (15)
\]

where \(\mathcal{R}(x)\) denotes the real part of \(x\). \(h_{R,D}\) has been replaced with \(\sqrt{\rho_h^H h_{R,D}^H + \sqrt{1 - \rho_e^H} \frac{h_{R,D}}{||h_{R,D}||^2} \right)^2\) scales with the order \(O(\rho N_R)\) as \(N_R \to \infty\) while \(2 \sqrt{(1 - \rho) \rho R(e^H h_{R,D})} + (1 - \rho) ||e^H h_{R,D}||^2 / ||h_{R,D}||^2\) scales as the order \(O(1)\), which is negligible. Eq. (15) holds true because of \(\lim_{N_R \to \infty} \frac{||h_{R,D}||^2}{N_R} = 1\) and \(\lim_{N_R \to \infty} \frac{||h_{R,D}||^2}{N_R} = 1\), namely channel hardening (12).

Similarly, the eavesdropper channel capacity is given by

\[
C_E = W \log_2\left(1 + \min(P_S \alpha_{S,R} N_R, P_{R \alpha_{R,D}} ||h_{R,E}^H h_{R,D}||^2 / ||h_{R,D}||^2)\right). \quad (16)
\]

Then, the secrecy outage capacity \(\varepsilon\) with respect to a secrecy outage capacity \(C_{SOC}\) can be computed as (18) at the top of next page, where (17) follows from the fact that \(h_{R,E}^H h_{R,D} / ||h_{R,D}||^2\) is \(\chi^2\) distributed with 2 degrees of freedom, and (18) holds true since exp \(\left(-\frac{P_{S \alpha_{S,R} N_R}}{P_{R \alpha_{R,D}}}\right)\) approaches zero when \(N_R\) is sufficiently large. Based on (18), it is easy to get the Theorem 1.

**APPENDIX B**

**PROOF OF THEOREM 2**

Based on the secrecy outage capacity in Theorem 1, when \(P_R \geq \frac{P_{S \alpha_{S,R}}}{P_{R \alpha_{R,D}}}\), we have \(C_{SOC} = W \log_2\left(1 + P_S \alpha_{S,R} N_R - W \log_2\left(1 + P_R \rho \alpha_{R,D} N_R r_l\right)\right).\) To guarantee \(C_{SOC} \geq 0\), the following condition \(P_{SOC} N_R \geq P_{R \alpha_{R,D} N_R r_l}\) must be satisfied, which is equivalent to \(0 < r_l < 1\) in the case of \(P_R > \frac{P_{S \alpha_{S,R}}}{P_{R \alpha_{R,D}}}\).

Otherwise, when \(P_R \leq \frac{P_{S \alpha_{S,R}}}{P_{R \alpha_{R,D}}}\), the secrecy outage capacity is changed as \(C_{SOC} = W \log_2\left(1 + P_R \rho \alpha_{R,D} N_R\right) - W \log_2\left(1 + P_R \rho \alpha_{R,D} N_R r_l\right)\). Only when \(0 < r_l < 1\), \(C_{SOC}\) is nonnegative.

Above all, \(0 < r_l < 1\) or \(N_R > \frac{\alpha_{R,E} \ln \varepsilon}{\rho \alpha_{R,D}}\) is the precondition that the nonnegative secrecy outage capacity exists. Therefore, we get the Theorem 2.
Thus, min is independent of Interestingly, it is found that the objective function equivalent to secrecy outage capacity be an arbitrary value belonging to at the relay, the optimal power at the relay is . Considering the constraint on the transmit power , the optimal power at the relay is . Furthermore, by substituting into the expression of secrecy outage capacity, we can obtain the maximum secrecy outage capacity as shown in Theorem 3.

**APPENDIX C**

**PROOF OF THEOREM 3**

According to the Theorem 1, when , the secrecy outage capacity is maximized when , since is a monotonously decreasing function of .

When , the secrecy outage capacity is an increasing function of under the condition . Thus, is the optimal power at the relay.

Considering the constraint on the transmit power at the relay, the optimal power at the relay is . Furthermore, by substituting into the expression of secrecy outage capacity, we can obtain the maximum secrecy outage capacity as shown in Theorem 3.

**APPENDIX D**

**PROOF OF THEOREM 4**

First, when , the optimization problem is equivalent to

\[
G_1 : \max_{P_R} \frac{\alpha_{R,D} \rho N_R}{\alpha_{R,E}} \quad \text{s.t.} \quad P_R \leq \min \left( \frac{P_{\alpha_{R,D} N_R}}{\rho_{\alpha_{R,D}} P_{\max}}, P_{\max} \right).
\]

Interestingly, it is found that the objective function is independent of . Hence, the optimal solution of can be an arbitrary value belonging to . Second, when , the optimization problem is reduced as

\[
G_2 : \max_{P_R} \frac{P_{\alpha_{R,D} N_R}}{P_{\alpha_{R,E}}} \quad \text{s.t.} \quad P_R \leq P_{\max}.
\]

The optimal solution of is , since the objective function is a decreasing function of .

Thus, the optimal transmit power at the relay is . Hence, we get the Theorem 4.

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