Effects of chemical degradation on longitudinal fracture of non-homogeneous beams: an analytical study

V Rizov

1Department of Technical Mechanics, University of Architecture, Civil Engineering and Geodesy, 1046 - Sofia, Bulgaria

E-mail: v_rizov_fhe@uacg.bg

Abstract. An analytical study of the chemical degradation of the material and its effects on longitudinal fracture behaviour of continuously non-homogeneous cantilever beam is carried out. The chemical degradation is described by decreasing of the elasticity modulus of the non-homogeneous material with time. The fracture is studied by analyzing of the strain energy release rate. For this purpose, the strain energy release rate is derived as a function of time with taking into account the chemical degradation and the non-homogeneity of the material. The time-dependent fracture is analyzed also by applying the J-integral with considering the chemical degradation for verification. The variation of the strain energy release rate with the time due to the chemical degradation is studied and discussed in detail.

1. Introduction
Continuously non-homogeneous materials are used very often in various load-bearing structural applications in modern engineering. These materials exhibit continuous variation of their properties in one or more coordinates in the structural member. Hence, the material properties (the modulus of elasticity in this paper) are smooth functions of the coordinates. The scientific literature shows that among the various kinds of non-homogeneous materials, functionally graded materials have attracted significant attention in the last decades [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. In fact, the interest in continuously non-homogeneous materials is due in a high degree to the extensive utilization of functionally graded materials in advanced engineering technologies. The functionally graded materials are continuously non-homogeneous composites made usually of two or more phases. Gradual change of microstructure and material properties of functionally graded materials is formed by gradual variation of volume fractions of the constituent materials during manufacturing.

The analysis of fracture behaviour of continuously non-homogeneous (functionally graded) materials plays an important role in safety design of engineering structures. The objective of this paper is to analyze the effect of chemical degradation of material on longitudinal fracture in continuously non-homogeneous beam structures since this problem has not been studied in the previous publications [12, 13, 14]. One of the motives for carrying-out this analysis is that during their lifetime, continuously non-homogeneous beams are often exposed to the influence of aggressive chemical agents such as acid rain attack, carbonation, sulphate attack.

2. Non-homogeneous beam with longitudinal crack under chemical degradation
The continuously non-homogeneous cantilever beam subjected to chemical degradation is under consideration (figure 1). The cross-section of the beam is a rectangle with dimensions, b x h. The beam has length, l. The static scheme of the beam is cantilever with clamping in section, D.
longitudinal vertical crack of length , \( a \), is located arbitrary in the beam as displayed in figure 1. The cross-sections of the right-hand and left-hand crack arms are rectangles with different widths designated by \( b_1 \) and \( b_2 \), respectively. Besides the chemical degradation, the beam structures is loaded also by external loading that includes two bending moments, \( M_1 \) and \( M_2 \), applied at the free ends of the two crack arms, as displayed in figure 1.

![Figure 1. Geometry and loading of the non-homogeneous cantilever beam.](image)

The beam is made of material that is continuously non-homogeneous in both thickness and length directions. The distribution of the elasticity modulus, \( E \), along the beam thickness is written as

\[
E = E_U + \frac{E_L - E_U}{h^n} \left( \frac{h}{2} + z_4 \right)^n
\]  

(1)

where \(-h/2 \leq z_4 \leq h/2\), \( E_U \) and \( E_L \) are, respectively, the values of the modulus of elasticity at the upper and lower surfaces of the beam, \( n \) is a material property that controls the variation of the modulus of elasticity along the thickness. The distributions of \( E_U \) and \( E_L \) along the length of the beam are expressed as
\[ E_U = E_{UF} + \frac{E_{UC} - E_{UF}}{l^m} x_4^m \]  
\[ E_L = E_{LF} + \frac{E_{LC} - E_{LF}}{l^f} x_4^f \]  

where \( 0 \leq x_4 \leq l \), \( E_{UF} \) and \( E_{UC} \) are, respectively, the values of \( E_U \) at the free and clamped ends of the beam, \( E_{LF} \) and \( E_{LC} \) are, respectively, the values of \( E_L \) at the free and clamped ends of the beam, \( m \) and \( f \) control the variations of \( E_U \) and \( E_L \) along the beam length, respectively.

The upper and lower surfaces of the structure are exposed to influence of aggressive chemical agent. The chemical degradation of the material leads to decrease of the elasticity modulus with time. The decrease of \( E_{UF} \), \( E_{UC} \), \( E_{LF} \) and \( E_{LC} \) are expressed as \[ E_{UF} = E_{UF0} e^{-\beta_{UF} t + \psi_{UF} t} \]  
\[ E_{LF} = E_{LF0} e^{-\beta_{LF} t + \psi_{LF} t} \]  
\[ E_{UC} = E_{UC0} e^{-\beta_{UC} t + \psi_{UC} t} \]  
\[ E_{LC} = E_{LC0} e^{-\beta_{LC} t + \psi_{LC} t} \]  

where \( t \) is time, \( \beta_{UF} \), \( \psi_{UF} \), \( \beta_{LF} \), \( \psi_{LF} \), \( \beta_{UC} \), \( \psi_{UC} \), \( \beta_{LC} \) and \( \psi_{LC} \) are material properties which control the change of the moduli of elasticity with time, \( E_{UF0} \), \( E_{UC0} \), \( E_{LF0} \) and \( E_{LC0} \) are, respectively, the values of \( E_{UF} \), \( E_{UC} \), \( E_{LF} \) and \( E_{LC} \) at \( t = 0 \).

The beam under chemical degradation exhibits non-linear elastic behaviour that is described by the following non-linear stress-strain relationship [15]:

\[ \sigma = \frac{E \varepsilon}{\sqrt{1 + \varepsilon^2}} \]  

where \( \sigma \) is the stress, \( \varepsilon \) is the strain. In fact, (8) is a stress-strain-time relationship since the elasticity modulus varies with time as a result of the chemical degradation of the material.

The longitudinal fracture in the beam under chemical degradation is time-dependent and is studied by analysing of the strain energy release rate. In this relation, a time-dependent analysis of the strain energy release rate, \( G \), that accounts for the chemical degradation of the material is developed by considering of the balance of the energy

\[ M_1 \delta \phi_1 + M_2 \delta \phi_2 = \frac{\partial U}{\partial a} \delta a + G h \delta a \]  

where \( \phi_1 \) and \( \phi_2 \) are the rotation angles of the free ends of the two crack arms, \( \delta a \) is a small increase of the crack length, \( U \) is the strain energy in the beam. From (9), one obtains

\[ G = \frac{M_1}{h} \frac{\partial \phi_1}{\partial a} + \frac{M_2}{h} \frac{\partial \phi_2}{\partial a} - \frac{1}{h} \frac{\partial U}{\partial a} \]  

The strain energy is found as
\[ U = \iiint_{V_1} u_{01} dV + \iiint_{V_2} u_{02} dV + \iiint_{V_3} u_{03} dV \]  

where \( u_{01}, u_{02} \) and \( u_{03} \) are, respectively, the strain energy densities in the two crack arms and in the un-cracked beam portion, \( V_1, V_2 \) and \( V_3 \) are the volumes of the right-hand and left-hand crack arms and in the un-cracked beam portion, respectively. The density of the strain energy in the right-hand crack arm under chemical degradation is obtained as [14]

\[ u_{01} = E \left( \sqrt{1 + \varepsilon^2} - 1 \right) \]  

It should be specified that (12) is time-dependent because the elasticity modulus changes continuously with time due to the chemical degradation according to (4), (5), (6) and (7). The Bernoulli’s hypothesis for plane sections is applied for describing the distribution of the strains since beams of high length to thickness ratio is considered in the present paper. Thus, the distribution of strains in direction of the thickness of the right-hand crack arm is written as

\[ \varepsilon = \kappa_1 (z_1 - z_{in}) \]  

where the curvature, \( \kappa_1 \), and the coordinate of the neutral axis, \( z_{in} \), are determined by using the following equations of equilibrium of elementary forces in the cross-section of the right-hand crack arm:

\[ N_1 = \iiint_{A_1} \sigma da, \quad M_1 = \iiint_{A_1} \sigma z_1 da \]  

where \( A_1 \) is the area of the cross-section, \( N_1 \) is the axial force, \( z_1 \) is the vertical centroidal axis of the cross-section. Apparently, \( N_1 = 0 \) (figure 1).

\[ \frac{G}{E_{\text{p,0}}} b \times 10^4 \]

\[ 0.0 \quad 0.06 \quad 0.12 \quad 0.18 \quad 0.24 \quad 0.30 \quad 0.36 \]

\[ 0.0 \quad 7 \quad 14 \quad 21 \quad 28 \]

\[ \beta_{UF} t \]

**Figure 2.** The non-dimensional strain energy release rate displayed as a function of non-dimensional time (curve 1 – at \( a/l = 0.3 \), curve 2 – at \( a/l = 0.5 \) and curve 3 – at \( a/l = 0.7 \).
After substituting of $\sigma$ in (14), the two equations are solved with respect to $\kappa_1$ and $z_{in}$ at various values of time by using the MatLab computer program. The curvatures, the neutral axis coordinates and the strain energy densities in the left-hand crack arm and in the un-cracked beam portion are found in analogical way.

The angles of rotation which are involved in (10) are obtained by using the integrals of Maxwell-Mohr

$$\varphi_1 = \int_0^a k_1 dx_3 + \int_a^l k_3 dx_3, \quad \varphi_2 = \int_0^a k_2 dx_3 + \int_a^l k_3 dx_3$$

(15)

where $\kappa_2$ and $\kappa_3$ are the curvatures of the left-hand crack arm and the un-cracked beam portion, respectively.

Equation (10) represents a solution of the strain energy release rate that depends on time since the elasticity modulus is function of time through (4), (5), (6) and (7) when the beam is under chemical degradation. Thus, (10) can be used to calculate the strain energy release rate at various values of time.

![Figure 3](image-url)

**Figure 3.** The non-dimensional strain energy release rate displayed as a function of $\frac{E_{UC0}}{E_{UF0}}$ ratio (curve 1 – at $E_{LC0}/E_{LF0} = 0.5$, curve 2 – at $E_{LC0}/E_{LF0} = 1.0$ and curve 3 – at $E_{LC0}/E_{LF0} = 2.0$).

The $J$-integral approach with considering the chemical degradation of the material is applied to verify the strain energy release rate determined by using equation (10). In this relation, the $J$-integral is solved along the integration contour, $\Gamma$, shown by a dashed line in figure 1. It should be noted that the $J$-integral value is exact match of the strain energy release rate calculated by applying (10). This fact proves the correctness of the time-dependent analysis of the strain energy release rate that accounts for the chemical degradation of the material.
3. Parametric study
The aim of the parametric study is to evaluate the effects of the chemical degradation of the material on the longitudinal fracture behaviour of the continuously non-homogeneous beam shown in figure 1. For this purpose, calculations of the strain energy release rate are carried-out by applying the time-dependent solution (10). The strain energy release rate that takes into account the chemical degradation is presented in non-dimensional form by using the formula $G_N = G/(E_{UF0}b)$. The variation of the strain energy release rate with time induced by the chemical degradation is analysed. The influences of the location and length of the crack and the material non-homogeneity in the thickness and length directions on strain energy release rate under chemical degradation are analysed too. The following data are used: $b = 0.010$ m, $h = 0.012$ m, $l = 0.150$ m, $n = 0.7$, $m = 0.7$, $f = 0.7$, $M_1 = 6$ Nm and $M_2 = 4$ Nm.

The effect of the chemical degradation of the material on the longitudinal fracture in the beam is illustrated in figure 2 where the non-dimensional strain energy release rate is plotted against the non-dimensional time for three $a/l$ ratios (the $a/l$ ratio characterizes the relative length of the crack). It is evident from figure 2 that the chemical degradation leads to increase of the strain energy release rate with time. One can observe also in figure 2 that when $a/l$ ratio increases, the strain energy release rate decreases.

![Figure 4](image_url)

**Figure 4.** The non-dimensional strain energy release rate displayed as a function of $b_l/b$ ratio (curve 1 – at $E_{LF0}/E_{UF0} = 0.5$, curve 2 – at $E_{LF0}/E_{UF0} = 1.0$ and curve 3 – at $E_{LF0}/E_{UF0} = 2.0$).

In order to evaluate how the material non-homogeneity along the length of the beam influences the longitudinal fracture behaviour under chemical degradation, the non-dimensional strain the energy release rate is displayed as a function of $E_{UC0}/E_{UF0}$ ratio in figure 3 at three $E_{LC0}/E_{LF0}$ ratios. The curves in figure 3 indicate that when $E_{UC0}/E_{UF0}$ ratio increases, the strain energy release rate
decreases. It can also be observed in figure 3 that the increase of $E_{UC0}/E_{UF0}$ ratio leads to decrease of the strain energy release rate when the beam is subjected to chemical degradation.

The influences of the crack location in the width direction and the material non-homogeneity in the thickness directions on the longitudinal fracture under chemical degradation are also evaluated. In this relation, the non-dimensional strain energy release rate is plotted against $b_l/b$ ratio in figure 4 for three $E_{LF0}/E_{UF0}$ ratios. The curves in figure 4 show that the strain energy release rate (when the beam is exposed to chemical degradation) is minimal at $b_l/b = 0.44$. It can be seen in figure 4 that when $E_{LF0}/E_{UF0}$ ratio increases, the strain energy release rate under chemical degradation decreases gradually.

4. Conclusions

The chemical degradation of the material and its effects on the strain energy release rate in a continuously non-homogeneous beam are analysed. The chemical degradation is described by decreasing the elasticity modulus with time. A time-dependent analysis of the strain energy release rate that accounts for the chemical degradation is developed. The $J$-integral approach with considering of the chemical degradation is used to verify the solution. The variation of the strain energy release rate with time due to the chemical degradation of the material is studied. The calculations performed show that the chemical degradation leads to increase of the strain energy release rate with time. The analysis reveals that when $a/l$, $E_{UC0}/E_{UF0}$, $E_{LC0}/E_{LF0}$ and $E_{LF0}/E_{UF0}$ ratios increase, the strain energy release rate in the beam under chemical degradation decreases. In general, the results obtained indicate that the strain energy release rate increases as a result of the chemical degradation. This finding is in a good agreement with prior studies [15]. The analysis presented in this paper is applicable mainly for functionally graded concrete beams. The degradation of concrete due to chemical causes such as sulphate reactions, alkali-reaction and carbonation affects the mechanical properties of concrete. Fracture behaviour of concrete is also deteriorated [16]. In this context, the paper helps for understanding of the influence of chemical degradation on longitudinal fracture behaviour of the functionally graded concrete beams.

5. References

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