The LHC potential for study of the small $x$ gluon physics in ultraperipheral collisions of 3.5 TeV protons.

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Abstract

We argue that already the first year LHC run at $\sqrt{s} = 7\,\text{TeV}$ will provide a quick and effective way to test the dynamics of the color dipole - gluon interactions and the small $x$ behavior of the gluon density in the proton by studying vector meson photoproduction in ultraperipheral proton-proton collisions.

It is expected now that during 2010 the LHC will operate at energies of the colliding proton beams of 3.5 TeV and luminosity $L_{pp} \approx 10^{32}\,\text{cm}^{-2}\,\text{s}^{-1}$. This raises the question whether it will be possible to realize part of the program of small $x$ gluon physics in the ultraperipheriral collisions [1] already during the first year of the operation of the LHC. In this paper we suggest to study large momentum transfer vector meson photoproduction with target dissociation and coherent $J/\psi$ photoproduction in ultraperipheral proton-proton collisions (UPC) at $\sqrt{s} \approx 7\,\text{TeV}$. We show that this would allow one to investigate the behavior of the gluon density in a proton at small $x$ down to $x \approx 10^{-5}$, as well as to determine the energy dependence of the color dipole - gluon elastic scattering amplitude.
Understanding of the small $x \leq 10^{-3}$ dynamics of gluon interactions at moderate virtualities is important for a realistic description of the $pp$ collisions at the LHC energies. The small $x$ processes are a topic of numerous theoretical studies. Currently most of the experimental information about such processes comes from experiments which were performed at the electron-proton HERA collider. It would be natural to extend these studies at the LHC. A possible solution is to use the so-called ultraperipheral collisions (UPC). So far the main focus of the theoretical UPC studies (for a recent review see [1]) was on the heavy ion and proton-ion collisions. Notable exceptions [2], [3] are the theoretical predictions of the exclusive $J/\psi$ production in $pp$ collisions at $\sqrt{s} = 14$ TeV and theoretical and experimental studies in $p\bar{p}$ collisions at energies of Tevatron.

Recently we pointed out that it would be possible to look for nonlinear effects in the interaction of small dipoles with strong gluon fields by studying the large momentum transfer $-t \equiv (p_\gamma - p_{J/\psi})^2$ process: $\gamma + T \rightarrow J/\psi + \text{gap} + X$ in the UPC of heavy ions with proton and nuclear target [4], [5], [6]. Here we want to explore the feasibility of using the same UPC probe for $pp$ scattering at the LHC. In practical terms it has a number of appealing features: (i) the ability to vary $t$ allows one to change the resolution scale by a factor $\sim Q_0^2(1 - t/4m_c^2)$, where $m_c$ is the charm quark mass and $Q_0^2 \sim 2.5 \div 3.5\text{GeV}^2$ is the scale probed in the exclusive photoproduction of $J/\psi$; (ii) the possibility to use triggers involving hadron production. Such measurements will also serve as a benchmark for similar measurements in the heavy ion collisions where this process provides an effective way to probe propagation of the small dipoles through the strong gluon fields.

The cross section of the vector meson photoproduction in the proton-proton UPC is dominated by scattering at large impact parameters $b > 2R_p$ corresponding to the transverse momentum transferred through photon $p_t < 0.1$ GeV. Hence it is reasonable to expect that soft initial and final state inelastic $pp$ interactions are still not too significant. Numerically a reduction factor due to these rescatterings should be of the order of 0.8 as given by the estimates in the literature for exclusive $J/\psi$ production in the $pp$ UPC [2]. We neglect these absorption effects in the current calculation. Then, the cross section for the UPC process $p + p \rightarrow p + \gamma + p \rightarrow p + V + X$ in the first approximation can be written as a convolution of the photon flux from the fast moving proton and the cross section of the large $t$ and rapidity gap
vector meson (V) photoproduction off the proton target

\[ \frac{d\sigma_{pp\rightarrow p+X+V}}{dxdt} \cdot dy = \frac{dN_{\gamma/p}(y)}{dy} \cdot \frac{d\sigma_{\gamma+p\rightarrow X+V}(x,y,t)}{dxdt}. \]  

(1)

The flux of the photons with momentum \( k \) from the fast moving proton was calculated in [7].

The process \( \gamma + p \rightarrow V + \text{"gap"} + X \) belongs to a class of reactions where the selection of large \( -t \gg 1/r_N^2 \) ensures that the dipole quark-antiquark component of the photon wave function which transforms into vector meson has the small size \( r_V \approx 1/\sqrt{-t} \). Besides, the important consequence of such a selection is that the transverse momenta remain large in all rungs of the gluon ladder and therefore two gluons are attached to one parton in the target. An attachments of the ladder to two different partons of the proton is suppressed by a power of \( t \). As a result the cross section can be written in a factorized form as a product of the gluon density in the proton \( g_p(x) \) (with a small correction for scattering off the quarks which we will not write down explicitly) and the elastic cross section of scattering of a small dipole off a gluon [8]:

\[ \frac{d\sigma_{\gamma p\rightarrow VX}}{dtdx} = \frac{d\sigma_{\gamma g\rightarrow Vg}}{dtdg} g_p(x,t). \]  

(2)

Here fraction \( x \) of the target proton momentum which carries the parton interacting with the dipole is

\[ x = -t/(M_X^2 - m_N^2 - t), \]  

and \( M_X \) is the invariant mass of the produced hadron system.

Experimentally it is rather difficult to measure \( M_X \) directly but at fixed \( t \) and \( M_X^2 >> -t >> m_N^2 \) it can be expressed with a good accuracy through the rapidity interval \( \Delta y \), occupied by the system \( X \)

\[ \Delta y = \ln \left( \frac{M_X^2}{m_N\sqrt{-t}} \right) = \ln \left( \frac{\sqrt{-t}}{xm_N} \right). \]  

(4)

Hence, accurate measurement of \( \Delta y \) ensures determination of \( M_X \) and the value of \( x \) of the interacting gluon.

Theoretical analysis of the elastic dipole-gluon scattering through the exchange by the gluon ladder leads in the leading and next-to-leading log
approximations to the amplitude which increases with energy at fixed \( t \) as a power of \( s, f(s,t) \propto (s_{\gamma g}/|t|)^{\alpha(t)-1} \), where \( s_{\gamma g} = xs_{\gamma N} \). The parameter \( \alpha(t) = \alpha(0) + \alpha' t \) can be treated as a trajectory of the hard effective pomeron responsible for the elastic dipole-gluon scattering. Calculations of intercept \( \alpha(0) \) in the BFKL approach give different estimates ranging from value \( \alpha(0) \approx 1.4 \) in leading order BFKL to \( \alpha(0) \approx 1.1 \) in next-to-leading order and intermediate value \( \alpha(0) \approx 1.2 \) in the resummed next-to-leading order BFKL; for a review see [9]. In our analysis of the HERA data for the large \( t \) and rapidity gap process \( \gamma + p \rightarrow J/\psi + X \) we were trying to extract the value of \( \alpha(0) \) from fitting the data with the following parametrization [5]

\[
\frac{d\sigma_{\gamma+g \rightarrow J/\psi+g}}{dt} = \frac{C}{(t_0 - t)(M^2_{J/\psi} - t)^3} \cdot \left[ \frac{xW^2_{\gamma p}}{\sqrt{(t_0 - t)(M^2_{J/\psi} - t)}} \right]^{2(\alpha(t)-1)}. \tag{5}
\]

We fixed the scale \( t_0 = 1 \text{ GeV}^2 \) and the quantities \( C, \alpha(0) \) and \( \alpha' \) were used as free parameters. A rather narrow energy interval \( 80 \text{ GeV} < W_{\gamma p} < 200 \text{ GeV} \) studied in the HERA experiments and the experimental cuts imposed due to specific acceptances of the detectors do not allow one to perform stringent tests of the factorization approximation and result in a very modest sensitivity to the energy dependence of the dipole - gluon elastic cross section. In particular, we found [5] that the data restrict the value of the slope parameters \( \alpha' \approx 0.005 \div 0.01 \) but allow change of the intercept \( \alpha(0) \) in the range from 1.0 to 1.2. Similar results have been obtained in [10] where we analyzed with the same model \( \rho \) meson photoproduction (for other recent data analyses and references see Refs. [11], [12]).

Note that in the HERA experiments the gap \( \delta y \) between produced mass and \( J/\psi \) was only 2-3 rapidity units that is obviously insufficient for developing the BFKL dynamics (emitting of one additional gluon in the ladder requires at least two units of rapidity). Simple estimates show that at different LHC detectors it is possible to study this process with \( \delta y \approx 2 \div 6 \). Hence, exploring the ultraperipheral pp or ion-ion collisions at LHC significantly extends the available kinematical region and can provide the capability to obtain more detailed and precise information about the physics of the hard effective pomeron.

We suggest to explore in the LHC experiments two different options for studying the large momentum transfer and rapidity gap vector meson photoproduction in the proton-proton UPC.

First, one can fix \( \Delta y \) that corresponds to fixed \( M_X \) and, hence, fixed \( x \) of
the target gluon participating in the process and study the cross section as a function of the rapidity of the vector meson. This kinematics allows one to investigate the energy dependence of the dipole-gluon elastic scattering through the exchange by the gluon ladder provided that the rapidity gap between the vector meson and produced mass $M_X$ is large enough. If a detector has sufficiently good acceptance it is possible to increase statistics by summing events with the produced mass $M_X \leq M_X^{\text{max}}$. In theoretical estimates this procedure corresponds to the integration of the cross section over $x$ in the range $x_{\text{min}} < x < 1$ where $x_{\text{min}}$ can be found from Eq. (3) using the value of $M_X^{\text{max}}$. Since the integral

$$I(x_{\text{min}}) = \int_{x_{\text{min}}}^{1} x^{2\alpha(t)-2} g_p(x) dx$$

grows with an increase of $M_X^{\text{max}}$ (decrease of $x_{\text{min}}$) due to the growth of the gluon density at small $x$, the choice of larger $M_X^{\text{max}}$ will also increase the counting rate. However, because of the increasing uncertainties in the gluon density distributions with decrease of $x$, larger $M_X^{\text{max}}$ leads to larger uncertainties in the analysis of the data. In our calculations we used $M_X^{\text{max}} = 25$ GeV that corresponds to $x_{\text{min}} \approx 0.005$ at $-t = 3$ GeV$^2$. In the interval $0.005 \leq x \leq 1$ all modern sets of the gluon distributions give close results, so, we calculated the cross sections in the approximation described above with gluon density distribution given by CTEQ6L [13]. The results are presented in Fig.1 for the production of $J/\psi$ at $-t = 3$ GeV$^2$ and of the $\rho$ meson at $-t = 6$ GeV$^2$. In the case of $\rho$ meson we choose larger $t$ in order to have large enough scale justifying the perturbative QCD description. The standard method of $J/\psi$ detection is through the dilepton decays; hence, to estimate the counting rate for $J/\psi$ one has to account for the branching factor $6 \cdot 10^{-2}$. We estimate that it will be possible to collect tens of thousands of $J/\psi \rightarrow \mu^+\mu^-$ events in 1 year of running. A more accurate estimate requires accounting for acceptance and other specifics of the detectors. In spite of the decrease of cross section due to increase of $t$ (asymptotically the cross section is scaled as $t^{-4}$) the number of events in measurements of $\rho$ photoproduction can be significantly larger if the detector is capable of registering pions from $\rho$ decay in the forward direction.

Our calculations indicate that the ultraperipheral collisions of 3.5 TeV protons at the LHC will allow one to explore the region of $W_{\gamma p}$ from 100 GeV up to $\approx 1000 \div 2000$ GeV. Note, at HERA such measurements were
Figure 1: Rapidity distributions for the large $t$ $J/\psi$ and $\rho$ meson photoproduction in the ultraperipheral proton-proton collisions at LHC at $\sqrt{s} = 7$ TeV. Solid line: calculations with $\alpha(0) = 1.0$; short dashed line: $\alpha(0) = 1.1$; long dashed line: $\alpha(0) = 1.2$. At rapidity of vector meson $y = 0$ the rapidity gap between vector meson and produced system $X$ is $\delta y = 3$. 
restricted to $80 \, \text{GeV} < W_{\gamma p} < 200 \, \text{GeV}$. Hence it appears that the discussed measurements will allow one to determine the value of $\alpha(0)$ with a good precision.

The second possible strategy is to fix the rapidity of the vector meson and momentum transfer $t$ and increase the mass $M_X$ of the system $X$ produced by target dissociation. In this case the energy of the dipole-gluon collision remains practically constant, while the rapidity interval occupied by the produced system $X$ grows with an increase of $M_X$ - a decrease of $x$ at fixed $t$, $x \approx \frac{2t}{M_X^2}$. Hence such measurements allow to study the behavior of the nucleon gluon density at small $x$ at the scale given by the value of $t$.

We performed calculations with the value of intercept $\alpha(0) = 1.1$ and considered photoproduction of $J/\psi$ with fixed rapidity $y = 4$ at the momentum transfer $-t = 3 \, \text{GeV}^2$. The number of $J/\psi \rightarrow \mu^+ \mu^-$ events as a function of $\Delta y$ for a 1 year running period at luminosity $10^{32} \, \text{cm}^{-2} \text{s}^{-1}$ is shown in Fig. 2 for gluon density distributions CTEQ6L and CTEQ6M. The interval of change of $M_X$ from 25 GeV to 350 GeV corresponds to scanning of the gluon density in the proton in the range $5 \cdot 10^{-3} > x > 3 \cdot 10^{-5}$. Obviously, accuracy of such scanning of the gluon density essentially depends on the capability of the detector to measure with good precision the value of $t$, say with $\Delta t \leq 1 \, \text{GeV}^2$, and the interval of rapidities $\Delta y$ occupied by the diffractive produced system $X$ which determines $x$ of the gluon. Also, to reach the region of very small $x < 0.0001$ still keeping the reasonable value of rapidity gap ($\delta y \geq 2$) one needs to detect the forward $J/\psi$ with the very high energy. In principle this is possible with the forward muon spectrometer of the ALICE detector but the acceptance will probably be small.

It seems that more preferable will be the study of the small $x$ behavior of the gluon density in the proton from the measurement of the coherent $J/\psi$ photoproduction in the proton-proton UPC. Feasibility of such measurements has recently been demonstrated by CDF Collaboration at energies of Tevatron [3]. The cross sections for this process have been also predicted for the energy $\sqrt{s} = 14 \, \text{TeV}$ in [2]. Here we demonstrate that the reasonable statistics can be accumulated already during the first year running of the LHC at the energy $\sqrt{s} = 7 \, \text{TeV}$.

The specific feature of the coherent photoproduction in the symmetric UPC at the collider is that there are two contributions since both colliding proton can be sources of photons and targets. Hence, $J/\psi$ at fixed rapidity can be produced in the interaction of the low energy photon with the large $x$
Figure 2: The large rapidity gap $J/\psi$ photoproduction in the ultraperipheral proton-proton collisions at $\sqrt{s} = 7$ TeV - number of $J/\psi \rightarrow \mu^+\mu^-$ events which could be accumulated for the 1 year running period at luminosity $10^{32} cm^{-2}s^{-1}$. 
Figure 3: Rapidity distributions for the coherent $J/\psi$ photoproduction in the ultraperipheral proton-proton collisions at LHC at $\sqrt{s} = 7$ TeV. The dashed curve shows one-side contribution of $J/\psi$ photoproduction off the proton target.
gluon and in the interaction of the high energy photon with the low \( x \) gluon when source of photons and target are interchanged. Then the cross section can be written in the form

\[
\frac{d\sigma_{pp \rightarrow ppJ/\psi}}{dt dy} = \frac{dN_{\gamma/p}(y)}{dy} \cdot \frac{d\sigma_{\gamma p \rightarrow ppJ/\psi}(y,t)}{dy} + \frac{dN_{\gamma/p}(-y)}{dy} \cdot \frac{d\sigma_{\gamma p \rightarrow ppJ/\psi}(-y,t)}{dt},
\]

where in the leading order the coherent \( J/\psi \) photoproduction off the proton target can be described by the perturbative QCD formula \[14\]

\[
\frac{d\sigma_{\gamma p \rightarrow pJ/\psi}}{dt} = \frac{\Gamma_{ee}M_{J/\psi}^3\pi^3}{48\alpha_{em}} \cdot \frac{\alpha_s^2(Q^2)}{Q^8} \left[ xg_N(x, \bar{Q}^2) \right]^2 \exp[B_{J/\psi}(s)t].
\]

Here the slope \( B_{J/\psi} \) is parametrized by the expression

\[
B_{J/\psi} = 3.1 + 0.25 \log_{10}(s/s_0),
\]

with \( s_0 = 100 \text{ GeV}^2 \) reasonably describing the data.

To give the prediction for the expected counting rates we calculated the coherent cross section using the QCD motivated formula \[15\] :

\[
\frac{d\sigma_{\gamma N \rightarrow J/\psi N}(s,t)}{dt} = 280 \cdot \left[ 1 - \frac{(m_{J/\psi} + m_N)^2}{s} \right]^{1.5} \cdot \left( \frac{s}{10000 \text{ GeV}^2} \right)^{0.415} \cdot \left[ \Theta(s_0 - s) \right]^{1 - \frac{t}{t_0}} + \Theta(s - s_0) \exp(B_{J/\psi}(s)t).
\]

with free parameters fitted to the existing data \[16\]. The scale parameter \( t_0 \) was fixed, \( t_0 = 1 \text{ GeV}^2 \), and the slope parameter for \( J/\psi N \) scattering was taken to be \( B_{J/\psi}(s) = 4.5 \text{ GeV}^{-2} \). Based on (Eq\[8\]) prediction \[15\] of the cross section of \( J/\psi \) photoproduction in ultraperipheral \( \text{AuAu} \) collisions at energies of RHIC was recently confirmed by experimental data obtained by PHENIX at rapidity \( y = 0 \) \[17\]. Also at \( \sqrt{s} = 1.96 \text{ TeV} \) this parametrization gives the cross section 2.53 nb at midrapidity which is about 30% lower than the CDF result \( \frac{d\sigma(y = 0)}{dy} = 3.92 \pm 0.25(\text{stat}) \pm 0.52(\text{syst}) \text{ nb} \) \[3\].

The calculated cross section of coherent \( J/\psi \) photoproduction in ultraperipheral proton-proton collisions at \( \sqrt{s} = 7 \text{ TeV} \) is shown in Fig\[3\]. Contrary to the \( J/\psi \) photoproduction in heavy ion UPC in the proton-proton case there is dominance of contribution to the cross section from production of \( J/\psi \) by the high energy photon and small \( x \) gluon (the dashed curve in Fig\[3\]). This is because of the presence of the high energy photons in the
photon flux of the very fast moving proton due to the rather slow drop of the proton form factor comparing to that of heavy ions. As a result contributions from the small $x$ gluons from different protons is reasonably well separated in the rapidity distribution. If the LHC luminosity at $\sqrt{s} = 7$ TeV will be $\approx 10^{32}$ cm$^{-2}$ s$^{-1}$ about $10^4$ events of coherent photoproduction of $J/\psi$ decaying into the dimuon channel can be accumulated for the 1 year running period of the ALICE detector in the interval of the $J/\psi$ rapidities $2 < y < 4$ at estimated acceptance $\approx 0.05$. Hence, basing on the perturbative QCD analysis (Eq.7) and measurement of the cross section with $J/\psi$ rapidities in the range $2 < y < 4$ one will be able to determine with reasonable accuracy behavior of the gluon density in the proton at $x$ ranging down to $x \approx 10^{-5}$, i.e. in the region which was not experimentally available so far. This conclusion relies on the assumption that contribution of the coherent photoproduction given by UPC mechanism dominates the discussed cross section. A generally accepted method to suppress / estimate the background processes (in the situation when the forward proton is not detected by the Roman pot detectors) is to use Zero Degree Calorimeters (ZDC). In the background processes the probability of proton breakup is high and a large fraction of these processes ($\geq 50\%$) leads to detecting of particles in ZDC and other forward detectors.

An example of the background production mechanism discussed in the literature is the odderon-Pomeron interaction resulting in the exclusive $J/\psi$ production in pp collisions. It was recently considered for the Tevatron and the LHC energies [18]. We note that, in spite of numerous efforts, no experimental evidence for existence of the odderon was found so far. The odderon-Pomeron interaction in pp collisions is a strong interaction process characterized by much smaller impact parameters than those in the UPC photoproduction. Within the model used in [18] the suppression factor, the odderon/photon ratio of $J/\psi$ production cross sections integrated over the transverse momenta $p_t$ of the outgoing protons, is proportional to $\alpha_s^3 \cdot S^2$. Under the assumption that the strong interaction coupling constant $\alpha_s = 0.75$ and the gap survival probability $S^2(LHC) = 0.03$ the authors of [18] estimate this suppression factor to be $\sim 0.1$. Comparing these values of $\alpha_s$ and $S^2$ to $\alpha_s \sim 0.2 \div 0.3$ used in the charmonium phenomenology and the currently accepted value of $S^2(LHC) \leq 1.5\%$ one could reasonably expect that the contribution of the odderon-Pomeron mechanism will constitute at most a few percent correction in the kinematics we consider in the paper. As estimated in [18] with an increase of $p_t$, say to $p_t \approx 0.4 \div 0.5$ GeV/c, the odderon exchange $J/\psi$ production cross section drops by 1 order comparing to the
value at small $p_t$ but the photon induced mechanism at large $p_t$ is suppressed more significantly. Hence, the selection of the large transverse momentum of the outgoing protons could give one the chance to look for revealing of the odderon in the considered processes of exclusive $J/\psi$ production, but this definitely will not be possible in the 2010 LHC run. The odderon exchange mechanism is also enhanced in a case when the proton in the proton-odderon vertex dissociates into hadrons since the photon exchange inelastic transitions $p \rightarrow M_X$ comparing to the elastic one are suppressed by a factor $p_t^2$ due to the gauge invariance. This contribution can be experimentally rejected by a veto from ZDC and other forward multiplicity detectors installed in ATLAS, CMS and ALICE. Of course, in studies of the large $t$ and rapidity gap $J/\psi$ photoproduction with dissociation of a target proton one has to apply such a veto to the emission in one direction only. Note also that odderon - Pomeron interactions can lead only to production of isoscalar ($I = 0$) states. Hence it does not contribute to the production of $\rho$-mesons and the comparison of the $\rho/J/\psi$ ratio in the large $t$ kinematics at the LHC and at HERA could help to probe contribution of the odderon.

In conclusion, we would like to emphasize that experimental measurement of the coherent and the large momentum transfer and rapidity gap photoproduction of vector mesons in ultraperipheral proton-proton collisions at $\sqrt{s} = 7$ TeV during the first year of the LHC operation opens new opportunities for the study of small $x$ physics.

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