Generation of vortex particles via weak measurements

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While twisted photons with orbital angular momentum can be generated in many ways, their massive counterparts – vortex electrons, neutrons, or atoms – can be obtained so far only via diffraction techniques, not applicable for relativistic energies. Here we show that vortex particles of arbitrary mass, spin, and energy, including hadrons, ions, and nuclei, can be generated during emission in helical undulators, via Cherenkov radiation, in collisions with intense laser beams, in such scattering and annihilation processes as \( e\mu \rightarrow e\mu, e\mu \rightarrow e\mu, e^+ \rightarrow h\hbar, \) scattering on relativistic ions [4], etc. The key observation is that it is largely not the process itself that defines vorticity of a particle, but a post-selection protocol due to quantum entanglement. A final particle acquires a phase vortex if the other final particle’s momentum is not measured with no post-selection of its azimuthal angle. This technique can be adapted for ultrarelativistic beams of modern and future colliders for hadronic and spin studies, and it can also facilitate the development of sources of hard X-ray and \( \gamma \)-range twisted photons at storage rings and free-electron lasers.

Introduction. — Twisted light with orbital angular momentum (OAM) [1] has found numerous applications in quantum optics and information, optomechanics, biology, astrophysics, and so forth [2,8]. Along with the diffraction techniques, such photons can be generated by charged particles in undulators [9,13], via non-linear Thomson or Compton scattering [14,18], during Cherenkov and transition radiation [19], etc. However, despite the potential use in particle and nuclear physics [6], the highest energy of the twisted photons achieved so far does not exceed a few keV [7].

It has been realised that potentially any quantum wave – be it an electron [20], a hadron, an ion, or a spin wave [21] – can be created in a twisted state. The vortex electrons, generated at the 300 keV microscopes [22,24], have attracted much attention outside the microscopy community because of their potential applications in hadronic and spin studies, atomic and high-energy physics, and in accelerator physics [20,23,15]. The possible experiments with vortex muons, hadrons, ions and atoms, cold neutrons, etc. are being discussed [8,29,30,36,39,40], whereas the non-relativistic twisted atoms and molecules have been generated only recently [47]. However, the available diffraction techniques [22,24,18,49] are not applicable for relativistic energies, which severely limits the development of the matter waves physics. To probe the vortex physics at higher energies, there is an urgent need in alternative approaches to generate the twisted states of a wide range of quantum systems.

In this Letter, we put forward a versatile method to generate vortex particles of arbitrary mass, spin, and energy, including \( \gamma \)-rays, relativistic muons, protons, ions and nuclei. Such particles can be obtained during emission of photons, via scattering or annihilation processes, such as \( e\mu \rightarrow e\mu, e\mu \rightarrow e\mu, e^+ \rightarrow h\hbar, \) scattering on relativistic ions [4], etc. The key observation is that it

\[ |e', \gamma \rangle = \hat{S} |\text{in} \rangle = \sum_{\lambda, \lambda'} \int \frac{d^3p'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} S_{fi}|k, \lambda'; p', \lambda \rangle, \]

where \( S_{fi} = \langle k, \lambda'; p', \lambda | \hat{S} | \text{in} \rangle \) is a matrix element with two final plane-wave states with the momenta \( p', k \) and
the helicities $\lambda' = \pm 1/2, \lambda_s = \pm 1$. Without post-selection, the wave function of the evolved state is not factorized into a product of the photon wave function and that of the electron. Indeed, in the momentum representation we have

$$\langle p', k' | e', \gamma \rangle = \sum_{\lambda_s, \lambda'} u'|e S_{fi},$$

where $e \equiv e_{k\lambda_s}$ is a photon polarization vector in the Coulomb gauge, $e \cdot k = 0, e \cdot e^* = 1$, and $u'|e\lambda'\rangle$ is an electron bispinor, $\bar{u}' u' = 2m_e$.

In order to derive the evolved wave function of the photon alone, we need to post-select the electron. If it is done to a state $| e'_d \rangle$, the photon’s wave function is

$$A^{(f)}(k) = \sum_{\lambda_s=\pm 1} e S_{fi},$$

where $S_{fi} = \langle k, \lambda_s' | e_{d} | \bar{S} | \text{in} \rangle$.

Let the initial electron be described as a plane wave propagating along the $z$ axis (say, during Cherenkov emission). If it is post-selected to a plane wave with the momentum $p'$ and the helicity $\lambda'$, the matrix element $S_{fi}^{(pw)}$ and the photon’s evolved wave function are both proportional to the momentum conservation delta-function $[50, 51]$

$$S_{fi}^{(pw)} \propto \delta(p'_{\perp} + k_{\perp}), A^{(f)}(k) \propto \sum_{\lambda_s=\pm 1} e \delta(p'_{\perp} + k_{\perp})\delta(4)$$

When the electron is detected at an azimuthal angle $\phi'$, the photon’s angle is also set to a definite value (see the extended version of this Letter [57] and Fig.1)

$$\phi_k = \phi' \pm \pi.$$ (5)

Thereby, the photon is projected to a plane-wave state without an intrinsic OAM. Thus, the customary post-selection to the plane waves is not suitable for direct comparison with the classical theory from [9,11,16,18]. We comment on this seeming discrepancy in Ref. [57].

The plane-wave post-selection represents a strong (or projective) measurement when all components of the momentum $p' = \{p'_x, p'_y, p'_z\} = \{p'_z, \phi', p'_z\}$ are measured with vanishing errors. After a weak measurement, on the contrary, some of the components are known with a non-vanishing error (see, e.g., [52,55]). Put simply, a weak measurement is a post-selection to a wave packet,

$$| e'_d \rangle^{(w)} = \int \frac{d^3 p'}{(2\pi)^3} \psi(p') | p', \lambda' \rangle.$$ (6)

The function $\psi(p')$ can be written in the Gaussian form, $\psi(p') \propto \prod_{i=x,y,z} \exp\{-(p'_i - \bar{p}'_i)^2/\sigma_i^2\}$. A strong measurement implies that $\sigma_x, \sigma_y, \sigma_z \to 0$. Alternatively, one can use cylindrical coordinates $p'_z, \phi', p'_z$ with the uncertainties $\sigma_z, \sigma_\phi, \sigma_z$. For a weak measurement, at least one of these uncertainties is not vanishing. Let us distinguish the following three scenarios:

(i) We post-select to the plane waves $[51,51]$ and repeat the measurements many times with an ensemble of electrons, each time fixing the detector at a different angle $\phi'$. The emission rate is proportional to

$$\int \frac{2\pi}{2\pi} |S_{fi}^{(pw)}|^2,$$

which represents an incoherent averaging over the angle.

(ii) Another example of a strong measurement is post-selection to a Bessel state $[29]$ with the definite $p'_z, p'_z$, the $z$-projection of the total angular momentum (TAM) $m'$, and the helicity $\lambda'$, but undefined $\phi'$$\text{)} = | p'_z, p'_z, m', \lambda' \rangle = \int \frac{2\pi}{2\pi} i^{-m' - \lambda'} e^{i(m' - \lambda') \phi'} | p', \lambda' \rangle.$ (8)

The corresponding amplitude

$$\int \frac{2\pi}{2\pi} i^{-m' - \lambda'} e^{i(m' - \lambda') \phi'} S_{fi}^{(pw)}$$

represents a coherent averaging over the angle, while the detector is able to measure the TAM with a vanishing error $\sigma_m \to 0$. As the azimuthal angle and the $z$-projection of the angular momentum represent the conjugate variables $[56]$, in this way we also obtain the complete information about the electron, but in the cylindrical basis.

(iii) Consider now an electron emitting a photon in the case we do not measure the electron momentum’s angle $\phi'$. The information about the electron’s state is incomplete, although the energy is well-defined, $\varepsilon' = \sqrt{(p'_z)^2 + (p'_z)^2 + m'^2}$. The corresponding amplitude is also obtained via coherent averaging,

$$\int \frac{2\pi}{2\pi} |S_{fi}^{(pw)}|.$$ (10)

This expression coincides with (9) at $m' - \lambda' = 0$, but its physical meaning is different. In the scheme (ii), we do measure the TAM projection and the helicity, and we can easily obtain $m' - \lambda' = 0$, but during the weak measurement we do not measure the TAM at all, which implies projection to the state

$$| e'_d \rangle^{(w)} = \int \frac{2\pi}{2\pi} | p', \lambda' \rangle.$$ (11)

More generally, in the weak-measurement scheme we post-select to a wave packet (w) with the finite uncertainties $\sigma_z, \sigma_\phi, \sigma_z$, whereas in Eq. (11) we have taken a limiting case $\sigma_\phi \to 2\pi$ (as explained in Ref. [57]).
photons become twisted. In this case, the photon is projected onto a twisted state. The conservation law of the TAM also takes place 

\[ A^\text{(f)}(k) = (-1)^{\lambda - \lambda'}ieN(2\pi)^3\delta(\varepsilon - \varepsilon' - \omega)\delta(p|p' - k|) + 2\lambda 2\lambda'\varepsilon - m_e\varepsilon' - m_e + [F - n(nF)] \] 

where \( \chi_0, \chi_{\pm 1} \) are eigenvectors of the spin operator \( \hat{s}_z \) with the eigenvalues 0, \pm 1, given in Eq. (46) of [57]. The terms with \( \chi_0 e^{i(\lambda - \lambda')\phi_k} \) and \( \chi_0 e^{-i(\lambda + \lambda')\phi_k} \) are eigenvectors of \( \hat{s}_z \) with the eigenvalues 0 and 2\lambda, and eigenfunctions of the OAM projection \( \hat{L}_z = -i\partial/\partial\phi_k \) with the eigenvalues \( \lambda - \lambda' \) and \( -\lambda - \lambda' \), respectively. Therefore, the vector \( F \) is an eigenvector of the TAM operator \( \gamma_j^\text{(f)} = \hat{s}_z + \hat{L}_z \) with the eigenvalue \( \lambda - \lambda' \). Similarly, one can prove that \( n(nF) \) is an eigenvector of \( \gamma_j^\text{(f)} \).

Hence

\[ \gamma_j^\text{(f)} A^\text{(f)} = (\lambda - \lambda') A^\text{(f)}. \]

The transverse momentum of this Bessel beam with the spin-orbit interaction (SOI) is defined by the electron scattering angle, \( k_\perp = |\mathbf{p}'|\sin\theta' \). As this angle is very small, \( \theta' \ll 1 \), it is technically challenging to precisely measure the angle \( \phi' \), and in this case the Cherenkov photons become twisted.

The conservation law of the TAM also takes place

\[ p = \{\varepsilon, 0, 0, |\mathbf{p}|\}, \quad p' = \{\varepsilon', p_\perp \cos \phi', p_\perp \sin \phi', p_z\}, \]

\[ p'_\perp = |\mathbf{p}'|\sin\theta', \quad k = \{\omega, k_\perp \cos \phi_k, k_\perp \sin \phi_k, k_z\}, \]

where \( N \) is a normalization constant and

\[ |\mathbf{k}| = \sqrt{k_\perp^2 + k_z^2} = \omega n(\omega), \]

while \( n(\omega) \) is a refractive index.

The azimuthal angles of the final momenta are correlated in the strong-measurement scheme and, thus, we know which detector in Fig. 1 registers the photon. If the electron is detected in the above weak-measurement scheme, we do not know the photon’s azimuthal angle, and its evolved state is (see Eq. (3))

\[ A^\text{(f)}(k) = \int 0^{2\pi} \frac{d\theta'}{2\pi} \sum_{\lambda, \pm 1} \chi^{\text{(pw)}} \hat{\gamma} u \mu \epsilon, \]

where we employ the Coulomb gauge with \( \epsilon^\mu = (0, \varepsilon) \),

\[ \sum_{\lambda, \pm 1} \varepsilon_i \varepsilon_j^* = \delta_{ij} - n_i n_j, \quad \mathbf{n} = \mathbf{k}/|\mathbf{k}|. \]

Substituting Eq. (45) from [57] to Eq. (14), we arrive at

\[ A^\text{(f)}(k) = \int 0^{2\pi} \frac{d\theta'}{2\pi} \sum_{\lambda, \pm 1} \chi^{\text{(pw)}} \hat{\gamma} u \mu \epsilon, \]

for the mixed states with \( \langle \hat{j}_z \rangle \in [-1/2, 1/2], \langle \hat{j}_z \rangle \in [-1/2, 1/2] \). Then, instead of Eq. (16) we have

\[ \langle \hat{j}_z \rangle^\text{(f)} = \langle \hat{j}_z \rangle - \langle \hat{j}_z \rangle, \]

where the photon’s TAM lies in the interval \( \langle \hat{j}_z \rangle^\text{(f)} \in [-1, 1] \). In particular, when the initial electron is unpolarized, \( \langle \hat{j}_z \rangle = 0 \), and the final electron’s TAM is not measured, the photon’s TAM is vanishing, although it is still twisted.

When the initial electron is twisted with the TAM \( m = \pm 1/2, \pm 3/2, \ldots \) [20], its bispinor \( u \equiv u_{p\lambda} \) with \( p = \{p_\perp \cos \phi, p_\perp \sin \phi, p_z\} \) transforms as (see Eq. (8))

\[ u_{p\lambda} \rightarrow u_{p_\perp, m_\lambda} = i^{-(m - \lambda)} \int 0^{2\pi} \frac{d\phi}{2\pi} e^{i(m - \lambda)\phi} u_{p\lambda}. \]

As a result, the TAM transforms as \( \lambda - \lambda' \rightarrow m - \lambda' \). Clearly, the same protocol can also be realized for
other emission processes, including transition radiation, diffraction radiation, undulator radiation, and so on.

Non-linear Compton scattering. — In a circularly polarized laser wave with the potential [50, 58]
\[
A^\mu = a^\mu_0 \cos(kx) + a^\mu_1 \sin(kx), \quad kx = \omega t - \mathbf{k} \cdot \mathbf{r},
\]
\[
A^2 = a^2_1 = a^2_2 = -a^2 < 0, \quad (a_1 a_2) = 0,
\] (19)
an electron is described with a Volkov state [50, 58, 60]
\[
\psi_{p \lambda}(\mathbf{r}, t) = N_e \left( 1 + \frac{e}{2\hbar}\frac{1}{(pk)}(\gamma k)(\gamma A) \right) \times \exp\left\{ -ipx - \frac{ie}{(pk)} \int dp \left( (pA) - \frac{e}{2} A^2 \right) \right\}. \tag{20}
\]
The final photon’s wave function is \( \mathcal{A}^\mu = N_e e^{i\mu} e^{-i\omega' t + ik' \cdot \mathbf{r}} \). The matrix element is
\[
S_{fi}^{(pw)} = -ie \int d^4x \bar{\psi}_{p' \lambda'} \gamma'^\mu \psi_{p \lambda}(\mathcal{A}^\mu)^* \tag{21}
\]
where the final electron is also in the Volkov state.

The results of this exactly solvable problem can be applied for an approximate description of similar problems where an electron also moves along a helical path, the simplest example being emission in a helical undulator.

The radiation characteristics of both the processes are quantitatively similar for ultrarelativistic electrons with \( \varepsilon / m \gg 1 \) and the small recoil \( \omega' / \varepsilon \ll 1 \) [60]. Note that although the electron is usually post-selected to the Volkov state [50, 58, 59], this implies a strong measurement of its quasi-momentum \( q' \). This may not necessarily happen in the quasi-classical regime if we do not have complete information about the final electron’s state. When the recoil is small, the electron stays in the field, its energy is implicitly measured, but its azimuthal angle \( \phi' \) may not be known. In this regime, the photon is post-selected to the twisted state.

Following the standard procedure [50, 58, 59], we expand the matrix element into a series over the harmonic numbers \( s \), reorganize the terms with the same indices of the Bessel functions \( J_s \) and \( J_{s \pm 1} \), and get
\[
S_{fi}^{(pw)} = \sum_{s=1}^{\infty} S_{fi}^{(s)} = -ie N e^{i\pi} \sum_{s=1}^{\infty} 4 \delta(4)(q + sk - q' - k') \times \sum_{\sigma = 0, \pm 1} (e_{\mu})^* u^\prime \Gamma^\sigma_\mu u J_{s + \sigma}(\Sigma) e^{i(s + \sigma) \xi}, \tag{22}
\]
where \( N = N_e N_c N_G \), the vertex \( \Gamma^\sigma_\mu \) is given in Eq.(76) of [57], and
\[
q' = p' + e^2 k'^2 a^2 / (2(pk)) \quad (q'^\mu) = (p'^\mu) + e^2 k'^2 a^2 / (2(pk)),
\]
\[
\Sigma^2 = e^2 \left( \frac{(pa_1)}{(pk)} - \frac{(p'a_1)}{(pk)} \right)^2 + e^2 \left( \frac{(pa_2)}{(pk)} - \frac{(p'a_2)}{(pk)} \right)^2,
\]
\[
\xi = \arctan\left( \frac{(pa_2)}{(pk)} - \frac{(p'a_2)}{(pk)} / \frac{(pa_1)}{(pk)} - \frac{(p'a_1)}{(pk)} \right). \tag{23}
\]
Eq. (22) is a standard expression [50, 58, 59], just written differently. Here, we only study the head-on collision with \( \xi = \phi' = \phi_{k'} \pm \pi \) where \( \phi_{k'} \) is the angle of the final photon’s momentum \( \mathbf{k}' = \omega' \mathbf{n}' \).

After the summation over helicities, the evolved state of the photon at the 2nd harmonic within the weak-measurement scheme becomes (see details in [57])
\[
A_{(f,s)}^{(w)}(\mathbf{k}') = \int \frac{d\phi'}{2\pi} A_{(f,s)}^{(w)}(\mathbf{k}') = -ie(1)^{\lambda + \lambda'} N(2\pi)^3 \delta(q') \delta(q - q') \delta(q' - q') \delta(q + sk - q' - k') \times \frac{1}{k_{\perp}} (p'_{\perp} - k'_{\perp}) n' \times \left[ n' \times \sum_{\sigma = 0, \pm 1} J_{s + \sigma}(p'_{\perp} k'_{\perp}) e^{i(s + \sigma) \phi_{k'}} \left( d^{(1/2)}_{\lambda'} G^{(\uparrow \uparrow)} e^{i\lambda \phi_{k'}} + d^{(1/2)}_{\lambda'} G^{(\uparrow \downarrow)} e^{-i\lambda \phi_{k'}} \right) \right], \tag{24}
\]
where \( p'_{\perp} = e\alpha / (p'k) \) is a radius of the classical helical trajectory of the final electron in the wave [61]. We have also used the representation \( u_{\sigma \lambda}^{(\sigma \prime)} \Gamma_{\sigma \prime} u = G^{(\uparrow \downarrow)} \delta_{\lambda, \sigma'} + G^{(\uparrow \uparrow)} \delta_{\lambda, -\sigma'} \) with the functions \( G^{(\uparrow \downarrow)}, G^{(\uparrow \uparrow)} \), given in Eq.(91) of [57]. Eq.(24) describes a Bessel beam with the SOI and the following TAM projection:
\[
\mathcal{J}^{(\gamma)} A_{(f,s)}^{(w)}(\mathbf{k}') = (s + \lambda - \lambda') A_{(f,s)}^{(w)}. \tag{25}
\]
When the incoming electron is twisted with the TAM \( m \) (being in the Bessel-Volkov state [28]), we obtain \( s + \lambda - \lambda' \rightarrow s + m - \lambda' \), whereas the most general formula for the mixed states is
\[
\langle \mathcal{J}_z \rangle^{(\gamma)} = s + \langle \mathcal{J}_z \rangle - \langle \mathcal{J}_z \rangle. \tag{26}
\]
For the “unpolarized” electrons with \( \langle \mathcal{J}_z \rangle = 0 \) and the photon’s TAM is simply \( s \). This result is in agreement with the classical theory [9, 11, 16, 18].

Heavy particles. — We take the process
\[
e^-(p_1) + \mu^-(p_2) \rightarrow e^-(p_3) + \mu^-(p_4) \tag{27}
\]
as an example, where the electron and the muon have the momenta \( p_1 = \{ \varepsilon_1, 0, 0, |p_1| \} \) and \( p_2 = \{ \varepsilon_2, 0, 0, -|p_2| \} \), respectively. When the former is detected in the scheme (iii), the evolved wave function of the muon is
\[
\psi_{\mu}^{(f)} = \sum_{\lambda = \pm 1 / 2} \int \frac{d\phi}{2\pi} u_{\lambda} S_{fi}^{(pw)}, \tag{28}
\]
where $u_4 \equiv u_{p_4,\lambda_4}$ and the matrix element reads
\[
S_{fi}^{(pw)} = i (2\pi)^4 N \delta^{(4)}(p_1 + p_2 - p_3 - p_4)
\times \frac{4\pi e^2}{q^2} \langle \bar{u}_3 \gamma^\alpha u_1 \rangle \langle \bar{u}_4\gamma_\alpha u_2 \rangle, \quad q = p_1 - p_3, \tag{29}
\]
where the propagator is taken in the Feynman gauge.

The muon current can be presented as
\[
J \equiv \bar{u}_4 e^{-i(\lambda_2+\lambda_3)\psi_4}, \quad \text{where } J_\alpha^{(\mu)} \text{ is given in Eq.(49) of \cite{57}}.
\]
After the integration, the electron current becomes
\[
\int_0^{2\pi} d\phi_3 \delta(\phi_3 - (\phi_4 \pm \pi)) \bar{u}_3 \gamma_\alpha u_1 = J_\alpha^{(e)}(\phi_3 = \phi_4 \pm \pi) e^{i(\lambda_1 - \lambda_3)(\phi_4 \pm \pi)}. \tag{30}
\]
Taking into account that $\chi_{-2\lambda_2} \cdot \chi_{2\lambda_1} = -\delta_{\lambda_2\lambda_1}$, it is easy to check that $\mathcal{J} \equiv J_\alpha^{(e)}(\phi_3 = \phi_4 \pm \pi)(J^{(\mu)})^\alpha$ does not depend on $\phi_4$. Therefore,
\[
\psi^{(f)}_\mu \propto \sum_{\lambda_4 = \pm 1/2} \bar{u}_4 e^{i(\lambda_1 - \lambda_2 - \lambda_3)\phi_4}, \tag{31}
\]
where $\bar{u}_4 = u_4 e^{-i\lambda_4\phi_4}$ has a vanishing TAM (see Eq.(40) of \cite{57}). As a result, the muon is twisted,
\[
\mathcal{J}_{4,\pm}^{(f)} = (\lambda_1 - \lambda_2 - \lambda_3)\psi^{(f)}_\mu. \tag{32}
\]
Similarly, for the twisted incoming electron with the TAM $m$, the TAM of the final muon becomes $m - \lambda_2 - \lambda_3$. Dealing with the unpolarized initial muon and the final electron, the incoming electron’s TAM is simply transferred to the muon, $\mathcal{J}_\varphi^{(\mu)} = m$.

Finally, we examine scattering off a proton (see Fig.2),
\[
e(p_1) + p(p_2) \rightarrow e(p_3) + p(p_4), \tag{33}
\]
in the same head-on geometry. Generalization to other hadrons, nuclei, or to inelastic processes is straightforward. The matrix element is
\[
S_{fi}^{(pw)} = i (2\pi)^4 N \delta^{(4)}(p_1 + p_2 - p_3 - p_4)
\times \frac{4\pi e^2}{q^2} \langle \bar{u}_3 \gamma^\mu u_1 \rangle \langle \bar{u}_4 \Gamma_\mu u_2 \rangle, \tag{34}
\]
where $\Gamma_\mu = F_1 \gamma_\mu + F_2 \sigma_\mu \sigma_\nu q^\nu$ is a hadronic vertex, which can be represented via the form-factors $F_1 = F_1(q^2, P^2), F_2 = F_2(q^2, P^2)$ \cite{50,51}, where $q^2 = (p_1 - p_3)^2, P^2 = (p_4 + p_2)^2 / 4$ do not depend on the angle $\phi_4$ of the final proton. One can prove that
\[
\bar{u}_3(\phi_3 = \phi_4 \pm \pi) \gamma^\mu u_1 \langle \bar{u}_4 \Gamma_\mu u_2 \rangle \propto e^{i(\lambda_1 - \lambda_2 - \lambda_3)\phi_4} \tag{35}
\]
and within the same weak-measurement scheme the proton is twisted,
\[
\mathcal{J}_{4,\pm}^{(f)} = (\lambda_1 - \lambda_2 - \lambda_3)\psi^{(f)}_\mu, \tag{36}
\]
or $\lambda_1 - \lambda_2 - \lambda_3 \rightarrow m - \lambda_2 - \lambda_3$ when the initial electron is so. Analogously, for the unpolarized particles the electron’s TAM can be transferred to the hadron, $\mathcal{J}_\varphi^{(\mu)} = m$. This transfer can be realized for neutrons, pions, ions, nuclei, and so forth, and the transverse momentum of the vortex particle always equals that of the particle the angle of which is weakly measured.

Conclusion. — We have shown that twisted photons, vortex leptons and hadrons of arbitrary mass and energy can be generated in a number of processes, simply by employing the post-selection protocol with a weak measurement of the momentum’s azimuthal angle. It is this scheme that adequately describes the generation of twisted photons in the quasi-classical regime with small recoil. One can envisage the implementation of this technique at the SASE3 undulator beamline of the European XFEL, at such powerful laser facilities as the Extreme Light Infrastructure, at synchrotron radiation facilities with the helical undulators, and at the existing and future lepton and hadron colliders.

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