Mathematical model for production of an industry focusing on worker status

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Abstract. Productivity improvement is posing a great challenge for industry everyday because of the difficulties in keeping track and prioritising the variables that have significant impact on the productivity. The variation in production depends on the linguistic variables such as worker commitment, worker motivation and worker skills. Since the variables are linguistic we try to propose a model which gives an appropriate production of an industry. Fuzzy models aids the relationship between the factors and status. The model will support the industry to focus on the mentality of worker to increase the production.

1. Introduction.
Operational hazards, inability of doing work and mismanagement have led the recent industries to drain pipes upon their priority budget. Employees are the heart of any organization and their smoothness in operation and less concern in interruption comes with employee cooperation which can never be replaced by anything else. Risk managements objective is to assure uncertainty does not deflect the endeavour from business goals[2,5,8,9]. Risks can come from various sources including uncertainty in financial markets, threats from project failures, production, development etc[1]. The three ongoing factors that we have used in out mathematical model are:

1. Worker motivation
2. Worker commitment
3. Worker skills.

The first factors varies upon each individual and also depends upon the environment and the peers with whom the individual is working with. Motivation can be thought upon as the combination of motive and action. It is a process governing choices made by persons or lower organisms among alternative form of voluntary activity [5]. It is a factor, or factors, that cause(s) an employee to pursue work tasks or goals. It's what causes you to act in a certain way.

There are two primary theories of motivation that are often used by employers: extrinsic motivation and intrinsic motivation. Extrinsic motivations are factors external that causes an employee to act toward fulfilment of a work task or goal. Intrinsic motivation, on the other hand, is motivation that comes from within an employee.

The second factor deals with employee commitment which can take in different forms. In case of an organization a commitment can be defined as a psychological state that binds an individual to the organisation. As a result employees are more loyal to an organisation and less likely to leave it, while
in case of a job it can be defined as the probability that someone continues to work in that job and feels psychologically bound to it. This is regardless of whether it is fulfilling or not. These definitions essentially have the same meaning, namely that commitment is a stabilising or binding force (mind-set), which directs behaviour. In order to show how commitment can contribute to improving performance, we use the commitment of certain employee data towards an organisation. In other words, the bond with the organisation.

The third factor deals with worker skill set which the worker has gained from time to time experience. A high-performing employee is motivated to do his job to the best of his ability. He is dedicated to his company and exhibits a number of skills that set him apart from his peers. His employer views him as a valued member of the team who can always be counted on when needed. Skills can be measured according to the various other factors such as communication, team player, confidence, multitasking, leadership, respect, positive attitude, problem solving and many others. In our work we do not deal with these different factors rather confine our study to worker skill as a whole.

2. Fuzzy logic modelling

2.1. Fuzzy logic and fuzzy set theory. Fuzzy logic is an extension of crisp logic. It was Introduced by Dr. Lotfi Zadeh[10] at University of California, Berkeley(USA) in the 1960’s as a means to model the uncertainty of the natural language. The theoretical aspect of fuzzy logic and fuzzy arithmetic have been explained in many ways. Crisp logic is like binary values. That is, variables are yes or no, true or false, black white, Zero or one. Only two values are varying like binary. In case of fuzzy logic, a fuzzy set is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1. It handles the concept of partial truth, where the truth value may range between completely true and completely false. For each input and output variables of a fuzzy inference system (FIS), the fuzzy sets are created by dividing the universe of discourse into a number of sub-regions, namely as linguistic terms (low, medium, high). If fuzzy A set is a pair (U, m) where U is a set and m: U→ [0,1] a membership function. The reference set is called universe of discourse, and for each x ∈ U, the value m(x) is called the grad of membership of x in (U, m). The function m=µ(A) is called membership function of the fuzzy set A=(U, m).

So the concept of fuzzy set theory[4] is a membership function, which represents numerically to what degree an element belongs to a set. To what degree an element belongs to a set is called the membership degree. Knowledge or past experience is represented by IF-THEN rules or IF-ELSE rules in fuzzy rule-based system[7]. Fuzzy rules consist of two parts: an antecedent part stating conditions on the input variables and a consequent part describing the corresponding values of the output variables. The aim of this study is to construct fuzzy knowledge-based models to support the industry to focus on the mentality of worker to increase the production.

Fuzzy modelling consist three parts:

a. Fuzzification
b. Fuzzy inference rule
c. Defuzzification

2.2. Fuzzification. Fuzzification is the first step in fuzzy inference process. In conventional systems or real-world problems, the inputs will be a numeric value. This numeric value is know as crisp values, so fuzzification converts crisp values into a membership grade in one or more fuzzy sets.
2.3. Fuzzy Inference rule. Fuzzy inference rule[3] is the process, which maps a given input to an output using fuzzy logic. The mapping then provides basis from which decisions can be made. The process of fuzzy inference involves all membership functions, fuzzy logic operators and if -then rules.

For example, in case of single fuzzy if-then rule assumes the form:If x is A then y is B, where A and B are linguistic values defined by fuzzy sets and x and y are input variables. The if-part of the rule “x is A” is called the antecedent or premise, while the then-part of the rule “y is B” is called the consequent or conclusion.

2.4. Defuzzification. Defuzzification is the process that converts the fuzzy set into a crisp value. In general, the membership functions are sampled to find the membership grade used in the fuzzy logic equations to define an outcome region, by deducing the crisp output. The crisp output is the result of the implication and aggregation steps of the fuzzy output, which is the union of all the outputs of individual rules.

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3. Proposed Model

As shown in the below figure ,there are three input variables namely Worker Motivation(WM), Worker Skill(WS), Worker Commitment(WC) are considered for the prediction of the output of the industry to focus on the mentality of the worker to increase the production. The fuzzy logic toolbox of MATLAB was used. The linguistic variables used for fuzzification of WM are Low Extent(LE), Medium Extent(ME) and High Extent(HE), for WS are Beginner(B), Leaner(L), Programmer(P), Trainer(T) and for WC are No Commitment(NC), Partially Commitment(PC), Commitment with Question(CQ) and Fully Commitment(FC). These linguistic variables were used for input. The linguistic variables used for fuzzification of output are Low(L), Medium(M) and High(H). Several Defuzzification techniques have been suggested over the years like Max- membership (height method), Center of area(COA) or Centroid method, weighted average method and last of maxima. For the study, we used centroid method Because in centroid method ,the crisp value u is taken to be the geometrical centre of the output fuzzy value µout(u). It is based on finding a balance point of the

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Figure 1. Fuzzy theory
property that can be the total geometric figure which represents the area of each fuzzy set. It is given by

\[ u^* = \frac{\sum_{i=1}^{N} (u_i \mu_{OUT}(u_i))}{\sum_{i=1}^{N} (\mu_{OUT}(u_i))} \]

Where, \( \mu_{OUT} \) = fuzzy set of the final output of one linguistic or fuzzy variable, \( u \) = location of each singleton on the universe of discourse.

For the three inputs and one output, a fuzzy rule is developed. The main advantage of fuzzy Logic is that it can be tuned and adapted if necessary, thus enhancing the degree of freedom of the system Rajagopalan et al., 2003 [6].

![Fuzzy modelling](image)

**Figure 2. Fuzzy modelling**

### 3.1 Membership functions.

A membership function (MF) is a curve that defines how each point in the input space (universe of discourse) is mapped to a membership value or degree of membership between 0 and 1. There are several types of MFs, namely triangular, trapezoidal function, gaussian function, quadratic function and sigmoid curve. But we used trapezoidal function. MFs selected for WM, WS, WC and Production are represented by trapezoidal distribution function in MATLAB(FIS editor).
Figure 3. MATLAB representation of MF for WM

Figure 4. MATLAB representation of MF for WS
Figure 5. MATLAB representation of MF for WC
Figure 6. MATLAB representation of MF for production

If we consider WM input variable there are three fuzzy sets namely LE, ME, HE similarly for WS input variable there are four fuzzy sets namely B, L, P, T. Similarly for WC input variable, there are four fuzzy sets namely NC, PC, CQ, FC. So there are three input variable and first input variable consists of three fuzzy sets and second and three input variables consists of four fuzzy sets. Hence there will be $3 \times 4 \times 4$ propositions. Thus there are 48 propositions. For example

- IF WM is LE AND WS is B AND WC is NC THEN Production is L.
- IF WM is ME AND WS is P AND WC is CQ THEN Production is M.

If we are taking value of input WM as 95% and WS as 86% and WC as 43%. Then the numerical value of input is converted into fuzzy membership. The fuzzified values of 95% WS is 1 HE. The fuzzified value of 86% WS are 0.8 P and 0.2 T. The fuzzified value of 43% WC are 0.4 PC and 0.6 CQ.

The fuzzy rule (propositions) for the above inputs values are as follows:

1. IF WM is HE AND WS is P AND WC is PC THEN Production is M.
2. IF WM is HE AND WS P AND WC is CQ THEN Production is H.
3. IF WM is HE AND WS is T AND WC is PC THEN Production is M.
4. IF WM is HE AND WS is T AND WC is CQ THEN Production is H.

These are the rule number 39,40,43,44 respectively.

3.1.1 For rule 1. When rule 39 is evaluated, the truth of the predicate is taken as minimum of the membership grades of the three input fuzzy sets (1 HE and 0.8 P and 0.4 PC). That is $\min\{1,0.8,0.4\}=0.4$. Thus the corresponding fuzzy set M is selected with the truth of 0.4. For rule 1 the main output is” production is medium(M)”.
The membership function for output by rule 1 is denoted as $\mu_1$, is given by $\mu_1(\text{production})= \min\{0.4, MF(\text{production})\}$. This MF defines the implied fuzzy set for rule 1.

3.1.2 For rule 2. When rule 2 is evaluated, the truth of the predicate is taken as minimum of the membership grades of the three input fuzzy sets (1 HE and 0.8 P and 0.6 CQ). That is $\min\{1,0.8,0.6\}=0.6$. Thus the corresponding fuzzy set M is selected with the truth of 0.6. For rule 2 the main output is” production is high(H)”.
The membership function for output by rule 2 is denoted as $\mu_2$, is given by $\mu_2(\text{production})= \min\{0.4, MF(\text{production})\}$. This MF defines the implied fuzzy set for rule 2.

3.1.3 For rule 3. When rule 3 is evaluated, the truth of the predicate is taken as minimum of the membership grades of the three input fuzzy sets (1 HE and 0.2 T and 0.4 PC). That is $\min\{1,0.2,0.4\}=0.2$. Thus the corresponding fuzzy set M is selected with the truth of 0.2. For rule 3 the main output is” production is medium(M)”.
The membership function for output by rule 3 is denoted as $\mu_3$, is given by $\mu_3(\text{production})= \min\{0.2, MF(\text{production})\}$. This MF defines the implied fuzzy set for rule 3.

3.1.4 For rule 4. When rule 4 is evaluated, the truth of the predicate is taken as minimum of the membership grades of the three input fuzzy sets (1 HE and 0.2T and 0.6 CQ). That is $\min\{1,0.2,0.6\}=0.2$. Thus the corresponding fuzzy set M is selected with the truth of 0.2. For rule 4 the main output
is” production is high(H)”. The membership function for output by rule 4 is denoted as \( \mu_4 \), is given by 
\[ \mu_4(production) = \min\{0.2, MF(production)\} \]. This MF defines the implied fuzzy set for rule 4.

It is noticed that rules (1), (2), (3) and (4) can apply to the current situation. We used Mamdani min for output fuzzy set. The output fuzzy region will be the aggregated fuzzy region that is obtained by combining the round down fuzzy sets resulted due to execution of all valid proposition. Then by centre of area (COA) defuzzification we can obtained numerical value of output from output fuzzy region.

Formula of COA is given as follows:
\[ u^* = \frac{\sum_{i=1}^{N}(u_i \mu_{OUT}(u_i))}{\sum_{i=1}^{N}(\mu_{OUT}(u_i))} \]

In our example, the Defuzzification output value of the round down fuzzy set is 36.34.

The diagram of the whole computational fuzzy modelling is presented in the below Figure.

![Figure 7. Fuzzy rule viewer](image)

When all the variables of WM, WS and WC get executed by the fuzzy inference engine (FIE), a decision surface can generated by the value of the output variable production.

The surface viewer is the dynamic combination of WS and WM affecting production as shown in the below figure. This is a 3-dimensional curve representing the mapping from WM and WS as inputs. WM on X-axis and WS on Y axis and production as an output on Z-axis. In varying condition of WM and WS inputs variables, the maximum value of the production will be change according to the inference engine mechanism. Production of an industry increased as increased in workers mentality.
4. Conclusion
In this paper an effort has been made to develop a fuzzy logic approach for the prediction of appropriate Production considering the WM, WS and WC as input parameters which is necessary for the production. In comparison to other predictive modeling techniques, fuzzy models have the advantage of being simple due to the Fuzzy Inference System (FIS) capability to deal with problems that are based on user knowledge and experience. This system can be developed further by increasing the knowledge rules and by addition of Genetic-Fuzzy and Neuro-Fuzzy to the system.

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