Asymptotic flatness in rainbow gravity

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Abstract

Conformal infinity in null and spatial directions is constructed for the rainbow flat spacetime corresponding to doubly special relativity. From this construction a definition of asymptotic DSRness is put forward which is compatible with the correspondence principle of rainbow gravity. Furthermore, a result equating asymptotically flat spacetimes with asymptotically DSR spacetimes is presented.

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1. Introduction

The idea of a fundamental length scale has emerged in multiple approaches to quantum gravity. This length, typically identified as the Planck length $l_p$, is expected to be the scale at which quantum gravitational corrections to our present theories would be required. However, the idea that when probing below the Planck length we will require new physics to describe the resultant phenomena is in direct contradiction with special relativity. How can we reconcile the idea of a fundamental length scale when special relativity allows lengths to contract? This apparent paradox of quantum gravity is what gave the impetus behind the original work in doubly special relativity [1–4]. Doubly special relativity fixes the Planck length and attempts to occupy the position of a flat spacetime limit of quantum gravity.

Recent work on doubly special relativity has been spurred on by current experiments that could provide a fertile testing ground for its results. Experiments such as GLAST and AUGER [5, 6] provide the opportunity to test the GZK cutoff and the constancy of the speed of light, both of which are subjects which DSR is capable of making predictions for. The other reason for the increased excitement in DSR is the possibility of it providing increased insight into quantum gravity. The ingredients that lead to DSR are not dependent upon any particular attempt towards a quantum theory of gravity and in fact are based solely upon attempting to combine the ideas of special relativity and a fundamental length scale. Due to the simplicity of its construction, it is possible that DSR could not only provide hints into the structure of...
spacetime in a complete theory of quantum gravity, but could also place restrictions upon one as well.

The picture of spacetime resulting from DSR is in some ways still an open question. Some approaches to DSR (particularly those that involve Hopf algebras) have resulted in a picture of spacetime which is non-commutative; these approaches though interesting have yet to present any physical predictions, primarily due to the difficulty in construction of large-scale behaviour from non-commutative spacetimes. Fortunately, there is an alternative to non-commutative geometry as the arena in which the spacetime of DSR is understood, by attempting to extend DSR to general relativity, Magueijo and Smolin put forward rainbow gravity. In rainbow gravity, the need for non-commutative geometry is avoided by introducing a metric which ‘runs’ with respect to the energy at which it is probed. Not only does this approach avoid the need for non-commutative geometry to describe the spacetime of DSR, it allows DSR to be extended to a theory of gravity much like general relativity.

The intent of this paper is to explore the implications of rainbow gravity further. Of particular interest is the asymptotic behaviour of the theory as this is an area where other approaches to DSR have not yet produced results. To this end we will investigate the ideas of conformal infinity and asymptotic DSRness (in analogy with asymptotically flat spacetimes in general relativity) in rainbow gravity.

2. Rainbow gravity

Rainbow gravity [7] is an attempt at constructing an extension of DSR into a general relativity framework which has at its foundation the proposal that the geometry of a spacetime ‘runs’ with the energy scale at which the geometry is being probed. The implication of this is that the metric of a spacetime becomes energy dependent. Fortunately, a natural proposal for the manner in which the metric could vary with respect to energy emerged from previous work on DSR [8], which is the form that we shall use.

Rainbow gravity is governed by two principles: the correspondence principle and the modified equivalence principle. These two principles are stated as follows [7].

Correspondence principle. In the limit of low energies relative to the Planck energy, standard general relativity is recovered. That is for any rainbow metric $g_{ab}(E)$ corresponding to a standard metric $g_{ab}$ the following limit holds true:

$$\lim_{E \to 0} g_{ab}(E) = g_{ab}. \quad (1)$$

Modified equivalence principle. Given a region of spacetime with a radius of curvature $R$ such that

$$R \gg E^{-1}_{pl}, \quad (2)$$

then freely falling observers measure particles and fields with energies $E$ observe the laws of physics to be the same as modified special relativity to the first order in $\frac{1}{R}$ so long as

$$\frac{1}{R} \ll E \ll E_{pl}. \quad (3)$$

Thus, they can consider themselves to be inertial observers in a rainbow flat spacetime (to the first order in $\frac{1}{R}$) and use a family of energy-dependent orthonormal frames locally given by

$$e_0 = \frac{1}{f(E_{pl})} \tilde{e}_0. \quad (4)$$
\[ e_i = \frac{1}{g(E)\eta} e_i, \]  

(5)  

with a metric  

\[ g(E) = \eta^{ab} e_a \otimes e_b. \]  

(6)  

We shall assume the existence of these two functions \( f(E) \) and \( g(E) \), which are strictly greater than zero for small values of \( E \); the small range of restriction is to allow for the possibility that at significantly greater energies the geometry of spacetime could take on a significantly different character, and the restrictions on this assumption will be explored further in the context of asymptotic flatness. These functions will also be subject to an additional restriction of invertibility, discussed in section 4. It should also be noted that our assumption that these functions depend only upon the energy, and not momenta, is rooted in the idea that as our family of metrics (instead of just a single spacetime) is to be the dual to the momentum space, the form of it should not depend on the momenta.

The result of these principles is that the rainbow metric for any govern spacetime is actually a family of metrics given by energy-dependent orthonormal frame fields—as presented above—which must satisfy a ‘rainbow Einstein equation’  

\[ G_{\mu\nu}(E) = 8\pi G(E)T_{\mu\nu}(E) + g_{\mu\nu}\Lambda(E), \]  

(7)  

where Newton’s constant and the cosmological constant are now allowed to vary with the energy so long as they obey the correspondence principle.

This is the form of rainbow gravity which will be used to study the idea of asymptotic DSRness in the following sections. It should be noted that conformal mappings of rainbow gravity spacetimes pointwise with respect to the energy (at specific energies instead of treating the energy as a dimension) are possible due to the similarity between the rainbow Einstein equations and the original Einstein equations. All rainbow metrics are actually solutions to Einstein’s equations in a mathematical sense (treating the functions solely as mathematical concepts, instead of allowing them to correspond to physical quantities) with the caveat that for energies where \( G(E) \) varies from Newton’s constant, the equations are slightly modified, but not in a manner which would impact the behaviour of solutions under conformal mappings, nor their compactifications.

### 3. Conformal infinity in rainbow Minkowski spacetimes

In order to understand the asymptotic behaviour of spacetimes in rainbow gravity it is useful to have a consistent manner in which to evaluate quantities ‘infinitely far away’ in null and spatial directions. Additionally, in order to be able to ascribe the title of ‘asymptotically’ DSR to a spacetime we need to be able to identify the behaviour of the flat DSR spacetime at these asymptotic locations. To do this we shall extend the ideas of \( \mathcal{I} \) and \( \mathcal{I}_0 \) from asymptotic flatness to DSR, proceeding in a similar manner to previous demonstrations of these completions of Minkowski spacetime [9].

We begin by considering the components of the metric of the flat DSR spacetime (we shall call this the deformed Minkowski spacetime or rainbow spacetime) \( g_{ab} \),  

\[ ds^2 = \frac{-dt^2}{f^2(E)} + \frac{dr^2}{g^2(E)} + \frac{r^2}{g^2(E)} d\Omega^2, \]  

(8)
where $d\Omega^2$ is the angular component of the spatial directions. By setting this equal to zero (and likewise setting the angular component to zero) we are able to identify the speed of light as a function of the energy by

$$c = \frac{dr}{dt} = \frac{g(E)}{f(E)}.$$  \hspace{1cm} (9)

This allows us to construct (energy-dependent) null coordinates given by

$$v = t + \frac{f(E)}{g(E)} r,$$

$$u = t - \frac{f(E)}{g(E)} r,$$  \hspace{1cm} (10)

and change the metric accordingly to

$$ds^2 = \frac{1}{f^2(E)} \left( -dv \, du + \frac{1}{4} (v - u)^2 \, d\Omega^2 \right).$$  \hspace{1cm} (12)

At this point we see that the only difference between this metric and the Minkowski spacetime metric in null coordinates is a factor of $\frac{1}{f^2(E)}$. This means that we can perform a conformal mapping of this spacetime into a restriction of the Einstein static universe by using a conformal factor of

$$\Theta^2 = \frac{4}{f^2(E)(1 + v(E)^2)(1 + u(E)^2)},$$  \hspace{1cm} (13)

such that the new metric $\hat{g}_{ab}$ is related to the old by

$$\hat{g}_{ab} = \Theta^2 g_{ab}.$$  \hspace{1cm} (14)

The only restrictions arising from this being that

$$\frac{f(E)}{g(E)} \neq 0,$$

so that our null coordinates are well defined for all $E$ and that

$$f(E) \neq 0,$$  \hspace{1cm} (15)

and that both $\frac{f(E)}{g(E)}$ and $f(E)$ are finite, so that our conformal factor $\Theta^2$ is well defined for all $E$. This mapping is made clear by choosing new coordinates of

$$T(E) = \tan^{-1}(v(E)) + \tan^{-1}(u(E))$$

$$R(E) = \tan^{-1}(v(E)) - \tan^{-1}(u(E)).$$  \hspace{1cm} (17)

The ranges of the new coordinates are

$$-\pi < T(E) + R(E) < \pi$$

$$-\pi < T(E) - R(E) < \pi$$

$$0 \leq R,$$  \hspace{1cm} (19)

and the components of the new metric are

$$d\hat{s}^2 = -dT^2 + dR^2 + \sin^2(R) \, d\Omega^2.$$  \hspace{1cm} (22)

From here we are able to extend the original spacetime to the boundary of the larger spacetime to yield an identification of the ‘infinity’ of the deformed Minkowski spacetime as follows:
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Future null infinity ($I^+$) is identified with $T(E) = \pi - R(E)$ for $0 < R < \pi$.

Past null infinity ($I^-$) is identified with $T(E) = -\pi + R(E)$ for $0 < R < \pi$.

Spatial infinity ($i^0$) is identified with $R(E) = \pi, T(E) = 0$.

We therefore now have an identification of conformal infinity of the deformed Minkowski spacetime with two reasonably physical restrictions given by equations (15) and (16). This allows us to examine asymptotic properties, and additionally examine the concept of asymptotic DSRness in curved spacetimes.

4. Asymptotically DSR spacetimes

We now wish to build on the identification of $I^+, I^-$ and $i^0$ for deformed Minkowski spacetime by using them to define an ‘asymptotically DSR spacetime’. To do this we shall rely on the definitions of asymptotically flat spacetimes [9–11] and expand them to incorporate the running metric of rainbow gravity.

There is a concern however that as rainbow gravity requires two as of yet unknown functions—$f(E)$ and $g(E)$—that we cannot assume that our definitions of $I^+, I^-$, and $i^0$ hold for all values of $E$ in the deformed Minkowski space.

We therefore shall define an interval $\chi$ by

$$\frac{E}{E_{pl}} = \varphi, \quad f(E) \text{ and } g(E) \text{ fail to satisfy the restrictions given in section 3.}$$

We will only address the concept of asymptotic DSRness where $\frac{E}{E_{pl}} \in \chi$ as our concepts of conformal infinity are ill defined outside of this interval. This corresponds to the deterioration of the ability of classical general relativity to describe the universe in high energy situations corresponding to short-distance phenomena. Such a restriction on the domain in which we can define asymptotic structures is natural given the intent of rainbow gravity. It is of course natural to consider possible values of $\varphi$ in the definition of the interval $\chi$, and it would be logical to suggest that values of order $E_{pl}$ are strong candidates for such an upper bound.

The interval $\chi$ also allows us to adequately describe our restriction of invertibility on $f(E)$ and $g(E)$. For these two functions to produce a mapping $U$ from momentum space to itself satisfying the restrictions outlined in [12], $(Ef(E), pg(E))$ must be invertible on the interval $\chi$, which therefore adds the additional restriction that

$$g(E) \neq 0, \quad \text{ (23)}$$

for all $E > 0$.

Within the interval $\chi$, we shall require that for a spacetime to be considered asymptotically DSR at spatial infinity it must satisfy the following requirements (analogous to those of asymptotic flatness conditions [10, 11])

**Definition 1.** A ‘rainbow spacetime’ $(M, g_{ab}(E))$ is considered asymptotically DSR at spatial infinity if there exists a set of spacetimes defined by the parameter $E$ $(\hat{M}(E), \hat{g}_{ab}(E))$ where each spacetime is smooth everywhere except at a point $i^0(E)$ where $\hat{M}$ is $C^1$ and $\hat{g}_{ab}$ is $C^0$, and that there exists an imbedding of $M(E)$ into its respective $\hat{M}$ satisfying the following for all values of $\frac{E}{E_{pl}} \in \chi$.

**Requirement 1.** The union of the closures of the causal future and causal past of $i^0(E)$ is equal to the complement of $M(E)$ in $\hat{M}(E)$, i.e. that

$$\hat{J}^+(i^0(E)) \cup \hat{J}^-(i^0(E)) = \hat{M} - M.$$
Requirement 2. There exists a function $\Theta$ on $\hat{M}$ that is $C^2$ at $i^0(E)$ and smooth everywhere else satisfying

\begin{align*}
\hat{g}_{ab} &= \Theta^2 g_{ab} \\
\Theta_0 &= 0 \\
\hat{\nabla}_a \Theta_0 &= 0 \\
\hat{\nabla}_a \hat{\nabla}_b \Theta_0 &= 2\hat{g}_{ab}(\Theta_0).
\end{align*}

The motivation behind this definition is the desire for rainbow gravity to be consistent at each value of $E$. Given any single value of $E$ within $\chi$ all standard rules of general relativity should apply and therefore the requirement for a spacetime to be asymptotically DSR should be that it be able to be mapped to the deformed Minkowski space in a manner corresponding to that in which asymptotically flat spacetimes are mapped to Minkowski space through the Einstein static universe. For a rainbow spacetime to be asymptotically DSR however, it must satisfy this requirement at all energies within the interval $\chi$ as we desire a definition which is dependent upon the spacetime, not upon the specific energy at which it is being probed. It should be noted that this requires that only spacetimes which are asymptotically flat in the low energy limit can be asymptotically DSR.

Expanding this approach, we are able to expand the standard definition of an asymptotically flat and empty spacetime as follows.

Definition 2. A ‘rainbow spacetime’ $(M, g_{ab}(E))$ is considered asymptotically empty and DSR at null and spatial infinity if it is asymptotically DSR at spatial infinity (as defined above) and satisfies the following for all values of $E$ within $\chi$.

Requirement 1. On the union of boundaries of the causal future and causal past of $i^0(E)$, $\Theta = 0$ and excepting $i^0(E)$, $\hat{\nabla}_a \Theta \neq 0$ on the same.

Requirement 2. There exists a neighbourhood $N(E)$ of the union of boundaries of the causal future and causal past of $i^0(E)$ in $\hat{M}(E)$ such that $(N(E), \hat{g}_{ab}(E))$ is strongly causal and time orientable, and in the intersection of $N(E)$ and the image of $M(E)$ in $\hat{M}$, $R_{ab}(E) = 0$ (where $R_{ab}(E)$ corresponds to the original physical metric $g_{ab}(E)$).

Requirement 3. The map of null directions at $i^0(E)$ into the space of integral curves of $n^a = \hat{g}^{ab} \hat{\nabla}_b \Theta$ on $I^+(E)$ and $I^-(E)$ is a diffeomorphism.

Requirement 4. For a smooth function, $\omega$, on the complement of $i^0(E)$ in $\hat{M}(E)$ with $\omega > 0$ on the union of the image of $M(E)$ in $\hat{M}(E)$ with $I^+(E)$ and $I^-(E)$ which satisfies $\hat{\nabla}_a (\omega^4 n^a) = 0$ on the union of $I^+(E)$ and $I^-(E)$ the vector field $\omega^{-4} n^a$ is complete on the union of $I^+(E)$ and $I^-(E)$.

These two definitions provide a means of identifying whether those concepts that are defined through asymptotic behaviour (such as the energy of an isolated system) carry over to rainbow gravity in a consistent manner.

5. Correspondence between asymptotic flatness and DSR

It is a natural question to inquire into whether an asymptotically flat spacetime will—when extended to rainbow gravity—correspond to an asymptotically DSR spacetime. Fortunately, we are able to show that for any metric derived from the conditions outlined in section 2, this is the case.
**Theorem 1.** Given any rainbow gravity metric derived from orthonormal frame fields

\[ e_0 = \frac{1}{f(E_{pl})} \tilde{e}_0 \] (29)

\[ e_i = \frac{1}{g(E_{pl})} \tilde{e}_i, \] (30)

with a metric

\[ g(E) = \eta^{ab} e_a \otimes e_b. \] (31)

If the metric is asymptotically flat in the limit as \( E_{pl} \) goes to zero with a conformal factor of \( \Omega^{-2} \), the rainbow gravity metric is asymptotically flat within the interval \( \chi \).

**Proof.** We may freely label the frame fields in such a way that our metric becomes

\[ ds^2 = -A^2(t, x, y, z) \, dt^2 + \frac{B^2(t, x, y, z)}{g^2(E_{pl})} \, dx^2 + \frac{C^2(t, x, y, z)}{g^2(E_{pl})} \, dy^2 + \frac{D^2(t, x, y, z)}{g^2(E_{pl})} \, dz^2. \]

We then change coordinates into a radial form defined by

\[ x = r \cos(\theta) \sin(\phi) \]
\[ y = r \sin(\theta) \sin(\phi) \]
\[ z = r \cos(\phi). \]

Our metric in this form (with the dependences of functions \( A, B, C, D, f, g \) omitted) becomes

\[ ds^2 = -A^2 \frac{1}{f^2(E)} \, dr^2 + \frac{1}{g^2(E)} \left[ B^2 \cos^2(\theta) \sin^2(\theta) + C^2 \sin^2(\theta) \sin^2(\phi) + D^2 \sin^2(\phi) \right] \, d\theta^2 \]
\[ + \frac{r^2}{g^2} \left[ B^2 \sin^2(\theta) \sin^2(\phi) + C^2 \cos^2(\theta) \sin^2(\phi) \right] \, d\phi^2 \]
\[ + \frac{r^2}{g^2} \left[ -2B^2 \cos(\theta) \sin(\theta) \sin^2(\phi) + 2C^2 \sin^2(\theta) \sin(\theta) \cos(\theta) \right] \, dr \, d\theta \]
\[ + \frac{r^2}{g^2} \left[ 2B^2 \cos^2(\theta) \sin(\phi) \cos(\phi) + 2C^2 \sin^2(\theta) \cos(\phi) \sin(\phi) - 2D^2 \sin(\phi) \cos(\phi) \right] \, dr \, d\phi \]
\[ + \frac{r^2}{g^2} \left[ -2B^2 \cos(\theta) \sin(\theta) \cos(\phi) \sin(\phi) + 2C^2 \cos(\theta) \sin(\theta) \sin(\phi) \sin(\phi) \right] \, d\theta \, d\phi. \]

We now find the speed of light by setting the distance to zero,

\[ \frac{dr}{dt} = \frac{gA}{fF}, \]

where the function \( F \) is given by

\[ F^2 = \frac{B^2 \cos^2(\theta) \sin^2(\theta) + C^2 \sin^2(\theta) \sin^2(\phi) + D^2 \sin^2(\phi)}{g^2}, \]

and is by that nature energy independent. Similarly, we can define the functions \( H, K, L, M \), and \( N \) to be

\[ H^2 = B^2 \sin^2(\theta) \sin^2(\phi) + C^2 \cos^2(\theta) \sin^2(\phi) \]
\[ K^2 = B^2 \cos^2(\theta) \cos^2(\phi) + C^2 \sin^2(\theta) \cos^2(\phi) + D^2 \sin^2(\phi) \]
\[ L^2 = -2B^2 \cos(\theta) \sin(\theta) \sin^2(\phi) + 2C^2 \sin^2(\theta) \cos(\theta) \sin(\phi) \]
\[ M^2 = 2B^2 \cos^2(\theta) \sin(\phi) \cos(\phi) + 2C^2 \sin^2(\theta) \cos(\phi) \sin(\phi) - 2D^2 \sin(\phi) \cos(\phi) \]
\[ N^2 = -2B^2 \cos(\theta) \sin(\theta) \cos(\phi) \sin(\phi) + 2C^2 \cos(\theta) \sin(\theta) \cos(\phi) \sin(\phi). \]
where each of these functions is also energy independent. This allows us to write the metric more succinctly as
\[
ds^2 = -\frac{A^2}{f^2(E)} \, dt^2 + \frac{F^2}{g^2} \, dr^2 + \frac{r^2 H^2}{g^2} \, d\theta^2 \\
+ \frac{r^2 K^2}{g^2} \, d\phi^2 + \frac{r L^2}{g^2} \, dr \, d\theta + \frac{r M^2}{g^2} \, dr \, d\phi + \frac{r^2 N^2}{g^2} \, d\theta \, d\phi.
\]
Thus this allows us to much more easily convert to null coordinates (again the energy dependence of these will be omitted in formulae)
\[
u = t - \frac{f F}{g A} \, r \\
v = t + \frac{f F}{g A} \, r,
\]
and making our metric in the \((u, v, \theta, \phi, )\) coordinates:
\[
ds^2 = -\frac{A^2}{4f^2(E)} \, (du + dv)^2 + \frac{A^2}{4f^2(E)} \, (dv - du)^2 + \frac{A^2 H^2}{4F^2 f^2} (v - u)^2 \, d\theta^2 \\
+ \frac{A^2 K^2}{4F^2 f^2} (v - u)^2 \, d\phi^2 + \frac{A^2 L^2}{4F^2 f^2} (v - u)(dv - du) \, d\theta \\
+ \frac{A^2 M^2}{4F^2 f^2} (v - u)(dv - du) \, d\phi + \frac{A^2 N^2}{4F^2 f^2} (v - u)^2 \, d\theta \, d\phi.
\]
However, in this form the energy dependence is entirely within the coordinates and a factor of \(\frac{1}{f^2(E)}\) common to all terms. This means that, within the interval \(\chi\), the metric differs by a conformal factor from the metric of the space in the low energy limit. We can therefore find a conformal factor for our energy-dependent space (which we will call \(\Omega^2(E)\)) by making a conformal mapping of our rainbow spacetime to an analogue of the original asymptotically flat spacetime (but with energy-dependent coordinates) and then composing this with a mapping by the conformal factor of \(\Omega^2\). The resultant conformal factor obeys all of the requirements for the space to be asymptotically DSR (as the energy dependence of the coordinates has no impact on the pointwise definition of asymptotic DSRness), and the rainbow spacetime is therefore asymptotically DSR with conformal factor
\[
\Omega^2(E) = \frac{1}{f^2(E)} \, \Omega^2.
\]
We therefore have a multitude of examples of asymptotically DSR spacetimes. It furthermore means that due to the conditions of the definition of asymptotic DSRness given above, the set of asymptotically DSR spacetimes is in a 1–1 correspondence with the set of asymptotically flat spacetimes.

\section{6. Conclusion}

The process of conformally mapping DSR into the Einstein static universe allowed for a natural recreation of the definitions of the conformal infinities and allowed the definitions for asymptotic DSR behaviour to follow from natural requirements. The definitions introduced here, though highly stringent in their pointwise nature, are put forward as minimal, given the current knowledge of the functions \(f(E)\) and \(g(E)\). Should rainbow gravity undergo further refinement it could be possible that these restrictions could be relaxed based upon the running of the metric being ‘well behaved’. These restrictions correspond directly with the concept of an energy bound, relating to the origins of rainbow gravity from doubly special relativity and therefore play closely in the defining of asymptotically flat spacetimes.
The direct correspondence between asymptotically flat and asymptotically DSR spacetimes is an interesting and powerful result allowing work in general relativity to be applied to rainbow gravity with little work.

Further research into asymptotic structure in rainbow gravity, particularly concerning any modification of the asymptotic symmetries of the spacetime, would be a logical next step from this work, along with investigations of the asymptotic definitions of energy and angular momentum.

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