First Calculation of Hyperon Axial Couplings from Lattice QCD

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In this work, we report the first lattice calculation of the hyperon axial couplings, using the 2+1-flavor MILC configurations and domain-wall fermion valence quarks. Both the Σ and Ξ axial couplings are for the first time done in lattice QCD, and we find the numbers with greater precision than previous chiral perturbation theory and large-Nc theory estimate: \( g_{\Sigma \Sigma} = 0.450(21)_{\text{stat}}(22)_{\text{syst}} \) and \( g_{\Xi \Xi} = -0.277(15)_{\text{stat}}(16)_{\text{syst}} \). As a side product, we also determine the low-energy chiral parameters \( D \) and \( F \) extracted from these coupling constants: \( D = 0.715(6)_{\text{stat}}(6)_{\text{syst}} \) and \( F = 0.453(5)_{\text{stat}}(5)_{\text{syst}} \).

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I. INTRODUCTION

In recent years lattice QCD calculations have demonstrated remarkable progress in computing hadron properties from first principles. In particular, hadron structure has been a major focus of the lattice QCD community. Recently the nucleon axial coupling \( g_A \) has been computed \([1, 2]\) with good precision and has been shown to be in agreement with the very well known experimental result within the systematic and statistical errors of the calculation. The success of such calculations motivates us to study the axial couplings for all the octet baryons. The experimental knowledge of such couplings is not as good as in the case of \( g_A \) and in certain cases, such as the axial couplings \( g_{\Sigma \Sigma} \) and \( g_{\Xi \Xi} \), their values are not known experimentally, and theoretical estimates are rather imprecise.

The hyperon axial couplings are important parameters entering the low energy effective field theory description of the octet baryons. At the leading order of SU(3) baryon chiral perturbation theory, these coupling constants are linear combinations of the universal coupling constants, \( D \) and \( F \), which enter the chiral expansion of every baryonic quantity, including masses and scattering lengths. Both the non-leptonic decays of hyperons, and the hyperon-nucleon and hyperon-hyperon scattering phase shifts are needed in order to understand the physics of hypernuclei [3]. On the other hand, because of the short lifetimes of hyperons, studying hypernuclei is in certain cases the only way one can obtain experimental information about the hyperon-nucleon and hyperon-hyperon interactions. These interactions are important in understanding the physics of neutron stars where hyperon production may soften the equation of state. At extreme densities, the conversion of neutrons to \( \Sigma^- \)’s is energetically favored by the high neutron Fermi energy. Such hyperon production may result in instability that would cause the neutron star to collapse into a black hole.

Studying the octet baryon axial couplings on the lattice is no more complicated than computing \( g_A \). The lack of experimental information in cases such as \( g_{\Sigma \Sigma} \) and \( g_{\Xi \Xi} \) gives us the opportunity to make predictions using lattice QCD. Previously, there have been two attempts to determine these coupling constants using different theoretical approaches: chiral perturbation theory \([4]\) and the large-\(N_c\) limit \([5, 6]\). M. J. Savage et al.\([4]\) use chiral perturbation theory to work out the one-loop corrections due to SU(3) symmetry breaking, and predict \( 0.35 \leq g_{\Sigma \Sigma} \leq 0.55 \). Similarly, \( 0.18 \leq -g_{\Xi \Xi} \leq 0.36 \), after taking the SU(3) limit, \( g_{\Sigma \Sigma} = F \), which is 0.5 at the leading order. Using the large-\(N_c\) approach, Dai et al.\([5]\) and Flores-Mendieta et al.\([6]\) predict a range of \( 0.30 \leq g_{\Sigma \Sigma} \leq 0.36 \) and \( 0.26 \leq -g_{\Xi \Xi} \leq 0.30 \). Both approaches give very loose bounds on the values of these coupling constants, hence a lattice QCD calculation has the opportunity to make a substantial improvement.

Lattice QCD calculations can now provide much more stringent theoretical estimates of these axial couplings. The various systematic errors that enter such calculations can be controlled, giving us the ability to compute the predictions of QCD very precisely. The systematic errors that we need to control are due to the finite lattice spacing, the finite volume and chiral extrapolations. Lattice calculations are performed on a discrete spacetime in a finite volume, using Monte Carlo integration to directly evaluate the path integral; continuum physics is recovered by taking the lattice spacing to zero \((a \to 0)\) and the volume to infinity \((V \to \infty)\). In addition, using current computer resources, we cannot yet calculate at the physical pion mass. Using chiral perturbation theory and calculations at multiple heavier pion masses which are more affordable, we can extrapolate quantities of interest to the physical limit. Such calculations also help to

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The axial charge for baryon doscalar form factor and $q$ has been successfully employed by LHPC and NPLQCD making them particularly well suited for the purpose of only having to compute the valence quark propagators tor. This way we take advantage of the publicly available fermion action (asqtad) [10–12] for the sea quarks and that can be easily computed. Details of our lattice for-

\[ (B|A_\mu(q)|B) = \pi_B(p') \left[ \gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 q \right] \frac{G_P(q'^2)}{2MB} \times u_B(p)e^{-i\mathbf{q}\cdot\mathbf{x}}, \]

where $u_B$ is the Dirac spinor, $G_P$ is the induced pseudoscalar form factor and $q$ is the momentum transfer. The axial charge for baryon $B$ is defined as $g_{A,BB} = G_A(q^2) = 0.$

On the lattice, we can extract the matrix element $(B|A_\mu|B)_{\text{lat}}$ by calculating a three-point function using zero initial and final momentum for the baryon states. This matrix element needs to be renormalized because we use the local axial current in order to simplify our calculation. The renormalization constant $Z_A$ is a scalar that can be easily computed. Details of our lattice formula-

For the rest of the paper, we will define the renormalized zero momentum transfer matrix elements as following: for the nucleon $g_A = Z_A(N|A_\mu|N)_{\text{lat}}$, the $\Sigma g_{\Sigma\Sigma} = (\Sigma|A_\mu|\Sigma)_{\text{lat}}/2$ (the factor of 2 is coming from a Clebsch-Gordan coefficient so that $g_{\Sigma\Sigma} = F$ in the SU(3) limit) and for the $\Xi g_{\Xi\Xi} = Z_A(\Xi|A_\mu|\Xi)_{\text{lat}}$.

In this calculation, we use (improved) staggered fermion action (asqtad) [10–12] for the sea quarks and domain-wall fermions (DWF) [13–16] for the valence sector. This way we take advantage of the publicly available gauge configurations generated by MILC collaboration, only having to compute the valence quark propagators needed for the matrix elements. In addition, domain-

\[
\begin{array}{cccccc}
\text{m} & \text{m010} & \text{m020} & \text{m030} & \text{m040} & \text{m050} \\
m_e (\text{MeV}) & 354.2(8) & 493.6(6) & 594.2(8) & 685.4(19) & 754.3(16) \\
m_u & 2.316(7) & 3.035(7) & 3.478(8) & 3.822(23) & 4.136(20) \\
m_K / f_\pi & 3.951(14) & 3.969(10) & 4.018(11) & 4.060(26) & 4.107(21) \\
\text{confs} & 612 & 345 & 561 & 320 & 342 \\
g_{A,N} & 1.22(8) & 1.21(5) & 1.195(17) & 1.150(17) & 1.167(11) \\
g_{\Sigma\Sigma} & 0.418(23) & 0.450(15) & 0.451(7) & 0.444(8) & 0.453(5) \\
g_{\Xi\Xi} & -0.262(13) & -0.270(10) & -0.269(7) & -0.257(9) & -0.261(7)
\end{array}
\]

TABLE I: Numbers used in our calculation

One way to probe SU(3) symmetry breaking in the axial couplings is to monitor the quantity $\delta_{\text{SU(3)}}$, defined as

\[
\delta_{\text{SU(3)}} = g_A - 2.0 \times g_{\Sigma\Sigma} + g_{\Xi\Xi} = \sum_n c_n x^n ;
\]

where $x = \frac{m_N^2 - m_{\Xi^0}^2}{4f_\pi^2}$; Figure 1 shows $\delta_{\text{SU(3)}}$ as a function of $x$. Note that the value increases monotonically as we go to lighter pion masses. Our lattice data suggest that $x^2$ dependence is strongly preferred; plotting $\frac{\delta_{\text{SU(3)}}}{x^2}$ shows the consistency. Such an extrapolation to the physical point gives 0.27(5), telling us that SU(3) breaking is roughly 20% at the physical point, where $x = 0.332$ using the PDG [38] for $m^+_\pi$, $m_K$ and $f_\pi$.  

1 Staggered fermions come in four “tastes” which must be removed by taking fractional powers of the fermionic determinant. Although no theoretical proof of the validity of this approach exist, there is significant evidence in the literature that supports the conjecture that the continuum limit of such formulation is QCD. For a recent review see Ref. [36] and references therein.
sible assumptions for the values of some low energy constants the individual axial couplings can be fitted with reasonable $\chi^2$/dof, the combined fit seems to fail to describe the data. The conclusion we derive from this is that SU(3) heavy baryon chiral perturbation theory fails to describe the lattice data. The pion and kaon masses used in our calculation are probably outside the radius of convergence of the perturbative expansion. It is possible that such a unified description of the axial couplings requires the next order in chiral perturbation theory or smaller pion masses. However, since the physical strange quark mass is rather large it seems that SU(3) heavy baryon chiral perturbation theory might be problematic in helping extrapolate the lattice data to the physical point. Here we should not forget that one of the reasons for our failure to fit with continuum heavy baryon SU(3) chiral perturbation theory formulas could be discretization errors. However, such errors should be small and are further suppressed since they enter at NLO in the expansion since in the valence sector we use domain-wall fermions that retain chiral symmetry. Hence we do not expect that finite lattice spacing corrections will change our conclusion.

C. Simple chiral forms

Since the continuum chiral extrapolation fails to describe our data, we take a step back and expand the axial couplings in terms of the SU(3) breaking parameter $x = (m_K^2 - m_\pi^2)/(4\pi^2 f_\pi^2)$ as follows:

$$
g_A = D + F + \sum_n C^{(n)}_N x^n$$

$$
g_{\Sigma \Sigma} = F + \sum_n C^{(n)}_\Sigma x^n$$

$$
g_{\Xi \Xi} = F - D + \sum_n C^{(n)}_\Xi x^n. \tag{3}$$

This form reduces to the known SU(3)-symmetric limit where the axial couplings are simple linear combinations of $F$ and $D$. In addition, not all the $C^{(n)}_B$ (with $B \in \{N, \Sigma, \Xi\}$) are independent parameters. From Sec. III A we found that the constraint $C^{(1)}_N - 2C^{(1)}_\Sigma + C^{(1)}_\Xi = 0$ is preferred by the data. This suggests a 4 parameter fit to $n = 1$ order or a 7 parameter fit to $n = 2$ order if we ignore possible pion mass dependence of $F$ and $D$. In order to have a reasonable fit form with the smallest number of parameters we will focus on the $n = 1$ case. Figure 2 shows our lattice data as a function of $(m_\pi/f_\pi)^2$ with the corresponding chiral extrapolation; the band shows the jackknife uncertainty. The $\chi^2$/dof is 0.83 and the linear fit parameters are very poorly determined as $C^{(1)}_N = 0.02(13)$ and $C^{(1)}_\Sigma = -0.01(6)$. This is not surprising, since a small slope is seen in all three axial charges. At the physical pion point, we find the nucleon axial charge is $g_{A,N} = 1.18(4)$; this is

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B. Continuum chiral extrapolation

Ideally, we should adopt a chiral extrapolation that would correctly describe the discretization errors due to finite lattice spacing as we extrapolate the axial couplings from the pion masses we used in our calculation to the physical point. However, such a calculation does not exist in the literature. Here we consider the next-best available option, which is continuum chiral perturbation theory results.

W. Detmold and C.-J. D. Lin worked out the forward twist-two matrix element extrapolation at one-loop order with finite-volume corrections [39]. From their work, we see that finite-volume corrections contribute at the order of $10^{-4}$ or less for the parameters of our calculation. Therefore, we will only implement the finite-volume chiral perturbation theory and further simplify the formulation by taking $m_{\text{val}} = m_{\text{sea}}$. In these formulae, there are total eight parameters to be determined: three SU(3) coupling constants $(C, D, F)$, and five other low energy constants $(\Delta c_n, \Delta a_n, \Delta \beta_n, \Delta \gamma_n, \Delta \sigma_n)$. We replace three mass splittings between the decuplet and octet with their lattice-measured values and replace $f$ and the chiral perturbation theory scale ($\mu$) with the pseudoscalar decay constants calculated from the same lattice and actions; a similar strategy was adopted by NPLQCD and LHPC [25–32, 40], and they showed that it simplifies the mixed action chiral perturbation theory formulas [18, 20]. We perform a simultaneous fit among multiple axial coupling constants without making further assumptions. We attempted to do simultaneous fitting among all three $g_A$, $g_{\Sigma \Sigma}$ and $g_{\Xi \Xi}$, and found the $\chi^2$/dof for such a fit is of the order of $10^2$. If we limit ourselves to the lightest three pion masses, we can reduce $\chi^2$/dof to the order of 10. Although under certain plausible assumptions for the values of some low energy constants the individual axial couplings can be fitted with reasonable $\chi^2$/dof, the combined fit seems to fail to describe the data. The conclusion we derive from this is that SU(3) heavy baryon chiral perturbation theory fails to describe the lattice data. The pion and kaon masses used in our calculation are probably outside the radius of convergence of the perturbative expansion. It is possible that such a unified description of the axial couplings requires the next order in chiral perturbation theory or smaller pion masses. However, since the physical strange quark mass is rather large it seems that SU(3) heavy baryon chiral perturbation theory might be problematic in helping extrapolate the lattice data to the physical point. Here we should not forget that one of the reasons for our failure to fit with continuum heavy baryon SU(3) chiral perturbation theory formulas could be discretization errors. However, such errors should be small and are further suppressed since they enter at NLO in the expansion since in the valence sector we use domain-wall fermions that retain chiral symmetry. Hence we do not expect that finite lattice spacing corrections will change our conclusion.

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FIG. 1: (Top) The SU(3) symmetry breaking measure $\delta_{SU(3)}$. The circles are the measured values at each pion mass, the square is the extrapolated value at the physical point, and the shaded region is the quadratic extrapolation and its error band. (Bottom) $\delta_{SU(3)}/x^2$ plot. Symbols as above, but the band is a constant fit.
is consistent with what LHPC obtained, 1.23(8), from SU(2) chiral perturbation extrapolation [40]. The extrapolated coupling constants $g_{\Sigma}$ and $g_{\Xi}$ are 0.450(21) and $-0.277(15)$ respectively. These numbers are consistent with the existing predictions from chiral perturbation theory [4] and large-$N_c$ calculations but have much smaller errors. The low-energy chiral parameters are $D = 0.715(6)$ and $F = 0.453(5)$, which are not consistent with the recent determination using semileptonic decay data: $D = 0.804(8)$ and $F = 0.463(8)$ with SU(3) symmetry [41]; however, the two calculations do agree within the range of the SU(3) breaking effect of 20%, which we observed in the previous section.

Since we know that the SU(3) breaking in Eq. 2 is quadratic in $x$, we expect that one needs to go at least to $n = 2$ in order to capture this effect. Hence, when taking the $n = 2$ expansion, it is not surprising to see the $\chi^2$/dof improves to 0.57; however, the $C_B(n)$ remain poorly determined (most of them are consistent with zero within the errorbar). Note that $C_N^{(2)} - 2C_{\Sigma}^{(2)} + C_{\Xi}^{(2)} = 1.9(6)$ which is consistent with what we found in the curvature of $\delta SU(3)$ in Sec. IIIA. The final results, $D = 0.711(7)$ and $F = 0.452(5)$, are consistent with the $n = 1$ case but are better determined. The axial couplings in this case are $g_A = 1.28(6)$, $g_{\Sigma} = 0.39(6)$ and $g_{\Xi} = -0.24(4)$. The discrepancy between the results of $n = 1$ and $n = 2$, which is at one standard deviation, might be taken as an indication of the systematic error of such extrapolations. However, it is hard to make an honest determination of such systematic error without further study at lighter pion masses and higher statistics.

Finally, one can consider that $D$ and $F$ have $m_{\pi}^2$ dependence as

$$D = D_0 + c_D m_{\pi}^2/(4\pi^2 f_{\pi}^2), \quad F = F_0 + c_F m_{\pi}^2/(4\pi^2 f_{\pi}^2),$$

with $D_0$ and $F_0$ to be the chiral limit numbers. Such fit forms have more parameters that we can possibly determine with our data. To better understand how strong this dependence is, let us assume $C_B^{(n)} = 0$ in Eq. 3. This gives us $c_D = -0.03(7)$ and $c_F = 0.01(5)$, consistent with zero. In both scenarios, the axial coupling constants are consistent with the $n = 1$ extrapolation. The discrepancies in the extrapolated results for all fitting forms we used is always at the level of one standard deviation. Therefore, we will assign an upper limit for the systematic error the same amount as the statistical error due to the extrapolation and keep the $n = 1$ results as our central values.

The systematic errors due to finite-volume effects are expected to be small. LHPC calculated the nucleon axial coupling constant using a chiral extrapolation with finite-volume correction [8]; less than a 1% effect is observed. Furthermore, finite-volume effects (including the $\Sigma$ and $\Xi$ baryons) are also estimated in Ref. [39], using heavy baryon chiral perturbation theory. The finite-volume effects come in at a magnitude no larger than $10^{-4}$. Thus, we take 1% to be an upper bound for the finite-volume systematic error in our calculation. Such effect is negligible given our statistical errors and systematics due to chiral extrapolation.

The final source of systematic errors is the continuum extrapolation. Precise estimate of such errors requires computations at several lattice spacings. However, we can estimate the discretization errors which are $O(a^2 A_{QCD}^2)$. Taking $A_{QCD} \approx 300$ MeV and the value of our lattice spacing which is $a^{-1} \approx 1588$ MeV [37], one expects to have such a systematic error of the order of 4%.

IV. CONCLUSION

In this work, we calculate the axial coupling constants for $\Sigma$ and $\Xi$ strange baryons using lattice QCD for the first time. We do the calculation using 2+1-flavor dynamical configurations with pion mass as light as 350 MeV. We discussed various potential chiral extrapolations and various sources of systematic errors; we conclude that $g_A = 1.18(4)_{\text{stat}}(4)_{\text{syst}}, g_{\Sigma} = 0.450(21)_{\text{stat}}(22)_{\text{syst}}$ and $g_{\Xi} = -0.277(15)_{\text{stat}}(16)_{\text{syst}}$; and $D = 0.715(6)_{\text{stat}}(6)_{\text{syst}}$ and $F = 0.453(5)_{\text{stat}}(5)_{\text{syst}}$. The axial charge coupling of $\Sigma$ and $\Xi$ baryons predicted with significantly smaller errorbars than estimated in the past.

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[1] D. Pleiter et al., PoS LAT2007, 129 (2007).
[2] R. G. Edwards and B. Joo (SciDAC), Nucl. Phys. Proc. Suppl. 140, 832 (2005), hep-lat/0409003.
[3] S. R. Beane, P. F. Bedaque, A. Parreno, and M. J. Savage, Nucl. Phys. A747, 55 (2005), nucl-th/0311027.
[4] M. J. Savage and J. Walden, Phys. Rev. D55, 5376 (1997), hep-ph/9611210.
[5] J. Dai, R. F. Dashen, E. E. Jenkins, and A. V. Manohar, Phys. Rev. D53, 273 (1996), hep-ph/9506273.
[6] R. Flores-Mendieta, E. E. Jenkins, and A. V. Manohar, Phys. Rev. D58, 094028 (1998), hep-ph/9805416.
[7] H.-W. Lin (2007), arXiv:0707.3844 [hep-lat].
[8] R. G. Edwards et al. (LHPC), Nucl. Phys. Proc. Suppl. 140, 832 (2005), hep-lat/0409003.
[9] S. R. Beane, P. F. Bedaque, K. Orginos, and M. J. Savage (NPLQCD), Phys. Rev. D73, 054503 (2006), hep-lat/0506013.
[10] S. R. Beane et al., Phys. Rev. D74, 114503 (2006), hep-lat/0607036.
[11] S. R. Beane, P. F. Bedaque, K. Orginos, and M. J. Savage, Phys. Rev. D75, 094501 (2007), hep-lat/0606023.
[12] J.-W. Chen, D. O'Connell, and A. Walker-Loud (2007), arXiv:0706.3026 [hep-lat].
[13] D. B. Kaplan, Phys. Lett. B288, 342 (1992), hep-lat/9206013.
[14] D. B. Kaplan, Nucl. Phys. Proc. Suppl. 30, 597 (1993).
[15] Y. Shamir, Nucl. Phys. B406, 90 (1993), hep-lat/9303005.
[16] V. Furman and Y. Shamir, Nucl. Phys. B439, 54 (1995), hep-lat/9405004.
[17] O. Bar, C. Bernard, G. Rupak, and N. Shoresh, Phys. Rev. D72, 054502 (2005), hep-lat/0503009.
[18] J.-W. Chen, D. O'Connell, R. S. Van de Water, and A. Walker-Loud, Phys. Rev. D73, 074510 (2006), hep-lat/0510024.
[19] C. Aubin, J. Laiho, and R. S. Van de Water, Phys. Rev. D75, 034502 (2007), hep-lat/0609009.
[20] J.-W. Chen, D. O'Connell, and A. Walker-Loud (2007), arXiv:0706.0335 [hep-lat].
[21] D. B. Renner et al. (LHP), Nucl. Phys. Proc. Suppl. 140, 255 (2005), hep-lat/0409130.