Tortoise coordinate and Hawking effect in the Kinnersley spacetime

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Abstract

Hawking effect from the Kinnersley spacetime is investigated using the improved Damour-Ruffini method with a new coordinate transformation. Hawking temperature of the horizons can be obtained point by point. It is found that Hawking temperatures of different points on the horizons are different. Especially, Hawking temperature of Rindler horizon is investigated. The touch between a Kinnersley black hole and its Rindler horizon is considered, and it shows that the phenomenon is related to the third law of thermodynamics.

Keywords: Hawking effect, Kinnersley spacetime, black hole horizon, Rindler horizon, tortoise coordinate

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1 Introduction

It is well known that Hawking effect in a black hole is one of the most striking phenomena\cite{1, 2}. A black hole was found to have thermal property right after the four laws of black hole thermodynamics had been built successfully and Hawking radiation had been discovered. In 1976, Damour and Ruffini proposed a new method with which one can also calculate Hawking radiation\cite{3}. Using this method, Liu et al proved that a Kerr-Newman black hole radiates Dirac particles\cite{4, 5}. In 1990’s, Z Zhao, X. X. Dai and Z. Q. Luo improved Damour-Ruffini method to study Hawking effect from some dynamical black holes. They only investigated several kinds of dynamical spherically symmetric black holes via the improved method\cite{6, 7}.

Some rapid progress has been made on Hawking effect of dynamical black holes in a recent several years\cite{8, 9}. J. L. Huang, et al investigated Hawking effect of a Vaidya black hole using null geodesic method\cite{10}. X. M. Liu, et al studied it via gravitational anomaly method\cite{11}. Using a new tortoise coordinate, Hawking effect of some dynamical spherically symmetric black holes has been investigated in Ref\cite{12}. We got more accurate surface gravity and Hawking temperature.

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It is well known that a stationary black hole has the same temperature on the horizon. A dynamical spherically symmetric black hole has also the same temperature on its horizon at the same time while its temperature varies with time. However, the temperature at different points on the horizon of an axisymmetric dynamical black hole may probably different. Using a new tortoise coordinate, Hawking effect of the Kinnersley spacetime is studied.

The organization of this paper is as follows. In Sec. 2, we will give a brief overview on the Kinnersley spacetime. In Sec. 3, we discuss Hawking effect in the Kinnersley black hole under the new tortoise coordinate transformation. In Sec. 4, the touch between a Kinnersley black hole and its Rindler horizon is studied. The conclusion and discussion are given in the last section.

2 The Kinnersley spacetime

The line element of the Kinnersley spacetime is expressed in Eddington-Finkelstein advanced time coordinate as [13]:

\[
\begin{align*}
\text{ds}^2 &= [1 - 2ar \cos \theta - r^2(f^2 + h^2 \sin^2 \theta) - \frac{2m}{r}]dv^2 - 2dvdr \\
&\quad - 2r^2f dv d\theta - 2r^2h \sin^2 \theta d\vartheta - r^2d\theta^2 - r^2 \sin^2 \theta d\varphi^2,
\end{align*}
\]

where

\[
\begin{align*}
f &= -a(v) \sin \theta + b(v) \sin \varphi + c(v) \cos \varphi, \\
h &= b(v) \cot \theta \cos \varphi - c(v) \cot \theta \sin \varphi, \\
m &= m(v).
\end{align*}
\]

For the case of accelerated rectilinear motion, we have

\[
b(v) = c(v) = 0,
\]

so the line element can be rewritten as

\[
\begin{align*}
\text{ds}^2 &= [1 - 2ar \cos \theta - r^2a^2 \sin^2 \theta - \frac{2m}{r}]dv^2 - 2dvdr \\
&\quad + 2r^2a \sin \theta d\vartheta - r^2d\theta^2 - r^2 \sin^2 \theta d\varphi^2.
\end{align*}
\]

The null hypersurface condition

\[
g^{\mu \nu} \frac{\partial f}{\partial x^\mu} \frac{\partial f}{\partial x^\nu} = 0
\]

can be rewritten as

\[
2\dot{r} - (1 - 2ar \cos \theta - \frac{2m}{r}) + 2ar' \sin \theta - \frac{r'^2}{r^2} = 0,
\]

where \(\dot{r} = \frac{\partial r}{\partial v}, r' = \frac{\partial r}{\partial \theta}\). The Eq. (4) determines the local event horizon of the Kinnersley spacetime.
3 Hawking effect from the black hole horizon

The Klein-Gordon equation in the Kinnersley spacetime is

\[\begin{align*}
2 \frac{\partial^2 \Phi}{\partial v \partial r} + \frac{2}{r} \frac{\partial \Phi}{\partial v} &+ \left(1 - 2a r \cos \theta - \frac{2m}{r}\right) \frac{\partial^2 \Phi}{\partial r^2} + 2a \sin \theta \frac{\partial^2 \Phi}{\partial \theta \partial r} \\
+ &\frac{2r - 6a r^2 \cos \theta - 2m}{r^2} \frac{\partial \Phi}{\partial r} + \frac{2a \sin \theta}{r} \frac{\partial \Phi}{\partial \Phi} + 2a \cos \theta \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \\
+ &\cot \theta \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{1}{\sin^2 \theta} \frac{\partial^2 \Phi}{\partial \varphi^2} - \mu^2 \Phi = 0, \\
\end{align*}\]

where \(\mu\) is the mass of a Klein-Gordon particle.

It is well known that when Hawking effect from a Schwarzschild black hole is investigated using Damour-Ruffini method, the tortoise coordinate is defined as following [3]

\[r_* = r + 2M \ln \left[\frac{r - 2M}{2M}\right].\]  

(6)

In the Kinnersley case, we suggest a new tortoise coordinate transformation as

\[\begin{align*}
r_* &= r + \frac{1}{2\kappa(v_0, \theta_0)} \ln \left[\frac{r - r_H(v, \theta)}{r_H(v, \theta)}\right], \\
v_* &= v - v_0, \theta_* = \theta - \theta_0, \tag{7}
\end{align*}\]

where both \(v_0\) and \(\theta_0\) are constants under tortoise coordinate transformation. At the same time, \(v_0\) is the moment when the particle escapes from the black hole horizon and depicts evolution of black hole, \(\theta_0\) is the location where the particle escapes from the event horizon of black hole and depicts shape of black hole. According to the new tortoise coordinate transformation, we have

\[\begin{align*}
\frac{\partial}{\partial r} &= \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial}{\partial r_*}, \\
\frac{\partial}{\partial v} &= \frac{\partial}{\partial v_*} - \frac{r^H H}{2\kappa r_H(r - r_H)} \frac{\partial}{\partial r_*}, \\
\frac{\partial}{\partial \theta} &= \frac{\partial}{\partial \theta_*} - \frac{r^H H}{2\kappa r_H(r - r_H)} \frac{\partial}{\partial r_*}, \\
\frac{\partial^2}{\partial r^2} &= \left[1 + \frac{1}{2\kappa(r - r_H)}\right]^2 \frac{\partial^2}{\partial r_*^2} - \frac{1}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*}, \\
\frac{\partial^2}{\partial r \partial v} &= \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial r_* \partial v_*} + \frac{r^H H}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*} - \frac{r^H H}{2\kappa r_H(r - r_H)} \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial r_*^2}, \\
\frac{\partial^2}{\partial r \partial \theta} &= \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial r_* \partial \theta_*} + \frac{r^H H}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*}.
\end{align*}\]
- \frac{rr'H}{2\kappa r_H(r-r_H)}[1 + \frac{1}{2\kappa(r-r_H)}] \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial \theta^2} = \frac{\partial^2}{\partial \theta^2} \left[ - \frac{rr''H}{r_H(r-r_H)} + \frac{rHr_H(r-r_H)}{2\kappa r_H^2 (r-r_H)^2} \right] \frac{\partial}{\partial r_*} - \frac{2rr'}{2\kappa r_H(r-r_H)} \frac{\partial^2}{\partial r_*^2} + \frac{1}{4\kappa r_H^2 (r-r_H)^2} \frac{\partial^2}{\partial r_*^2}.

The Klein-Gordon Eq(5) can be rewritten as

\[ r_H^2 + \left[ 1 + 2\kappa(r-r_H) \right] \left\{ -2rr_H(\hat{r}_H + ar'_H \sin \theta) + r_H^2 \left[ 1 + 2\kappa(r-r_H) \right] (1 - 2ar \cos \theta - \frac{2m}{r}) \right\} \frac{\partial^2 \Phi}{\partial r^2} + 2 \frac{\partial^2 \Phi}{\partial v_* \partial \theta_*} + \frac{1}{r_H^2 [1 + 2\kappa(r-r_H)] (r-r_H)} \left[ 2rr_Hr_H r(r-r_H) - r_H^2 (1 - 2ar \cos \theta - \frac{2m}{r}) + 2ar_H^2 r_H \sin \theta \right] \frac{\partial \Phi}{\partial r_*} + \frac{1}{r_H^2 (1 + 2\kappa(r-r_H)) \sin \theta - 2r_H' \frac{\partial^2 \Phi}{\partial r^2} + \frac{2\kappa(r-r_H)}{1 + 2\kappa(r-r_H)} \frac{\partial \Phi}{\partial v_*} + \frac{2a \sin \theta \frac{\partial \Phi}{\partial \theta_*}}{r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta_*^2} + \frac{\cot \theta \frac{\partial \Phi}{\partial \phi_*}}{r^2 \sin^2 \theta} + 2 \frac{\partial^2 \Phi}{\partial \phi_*^2} - \mu^2 \Phi = 0. \] (8)

From the null hypersurface condition Eq(1) of Kinnersley spacetime, the numerator of the coefficient on the term \( \frac{\partial^2 \Phi}{\partial \phi_*^2} \) approaches to zero at the horizon \( r_H \). Therefore we can calculate the limit of the coefficient using L’Hospital law. Assuming the limit is equal to an undetermined constant \( K \), so when \( \kappa \) is selected as

\[ \kappa = \frac{1}{2r_H} \frac{m}{r_H} - a \cos \theta - \frac{r_H^2}{2r_H} + \frac{1}{2r_H} \frac{m}{r_H} + a \cos \theta + \frac{r_H^2}{2r_H}, \] (9)

we have \( K = 1 \).

When \( r \) approaches to \( r_H \), the Klein-Gordon Eq(8) can be transformed into

\[ \frac{\partial^2 \Phi}{\partial r_*^2} + 2 \frac{\partial^2 \Phi}{\partial v_* \partial r_*} + B \frac{\partial^2 \Phi}{\partial r_* \partial \theta_*} - G \frac{\partial \Phi}{\partial r_*} = 0, \] (10)

where

\[ B = 2(a \sin \theta_0 - \frac{r_H'}{r_H^2}), \]
\[ G = -\frac{1}{r_H} \frac{r_H'}{r_H^2} + \frac{2m}{r_H} + \frac{r_H \cot \theta_0}{r_H^2} + \frac{r_H''}{r_H^2} - \frac{r_H^2}{r_H^3} + \frac{r_H^2}{r_H^2}. \]

Separate variables as following

\[ \Phi = R(r_*) \Theta(\theta_*) e^{i\nu_r - i\omega \nu_*}. \] (11)
where $\omega$ is the energy of the Klein-Gordon particle, $l$ is the projection of angular momentum on $\varphi$-axis, we can get

$$
\Theta' = \lambda \Theta, \\
R'' + (\lambda B - G - 2i\omega)R' = 0,
$$

(12)

where the constant $\lambda$ is introduced by the separation of variables.

Assuming $\lambda = \lambda_1 + i\lambda_2$, $\lambda_1, \lambda_2 \in R$, the Eqs[12] can be written as

$$
\Theta' = (\lambda_1 + i\lambda_2)\Theta, \\
R'' + [(\lambda_1 + i\lambda_2)B - G - 2i\omega]R' = 0,
$$

(13)

and its solution is

$$
\Theta = c_1 e^{(\lambda_1 + i\lambda_2)\theta_*}, \\
R = c_2 e^{-[(\lambda_1 + i\lambda_2)B - G - 2i\omega]r_*} + c_3,
$$

(14)

where $c_1, c_2$ and $c_3$ are integral constants, $\theta_*$ is polar angle. Its radial ingoing and outgoing components are respectively

$$
\psi_{in} = e^{-i\omega v_*}, \\
\psi_{out} = e^{-i\omega v_*}e^{2i(\omega - \omega_0)r_*}e^{(G - \lambda_1 B)r_*},
$$

(15)

where

$$
\omega_0 = \frac{1}{2}\lambda_2 B = \lambda_2 (a \sin \theta_0 - \frac{r'_H}{r_H}).
$$

(16)

The outgoing wave is rewritten as

$$
\psi_{out} = e^{-i\omega v_*}e^{2i(\omega - \omega_0)r_*}e^{\frac{\bar{A} r}{r_H}}e^{\frac{(G - \lambda_1 B)r_*}{r_H}}e^{rac{\pi}{2\kappa}} e^{-\frac{\pi \lambda_2 B}{2\kappa}} e^{\frac{\pi (\omega - \omega_0)}{\kappa}},
$$

(17)

where $\bar{A} = G - \lambda_1 B$. It is obvious that the outgoing wave is not analytical at the horizon $r_H$. Extending the outgoing wave from outside to inside of the horizon analytically through the negative half complex plane, we get

$$
\tilde{\psi}_{out} = e^{-i\omega v_*}e^{2i(\omega - \omega_0)r_*}e^{\frac{\bar{A} r}{r_H}}e^{i\frac{\pi \lambda_2 B}{2\kappa}} e^{\frac{\pi (\omega - \omega_0)}{\kappa}}.
$$

(18)

The scattering probability of outgoing wave at the horizon is

$$
\left| \frac{\psi_{out}}{\tilde{\psi}_{out}} \right|^2 = e^{-\frac{2\pi (\omega - \omega_0)}{\kappa}}.
$$

(19)

According to explanation of Sannan[14], the outgoing wave has black body spectrum

$$
N_\omega = \frac{1}{e^{\frac{\omega - \omega_0}{\kappa}T} \pm 1},
$$

(20)

$$
T = \frac{\kappa}{2\pi k_B}.
$$

(21)
4 Touch between a Kinnersley black hole and its Rindler horizon

If \( m = 0 \) in Eq.(11), we will get Rindler horizon equation as

\[
2\dot{r} - (1 - 2a\cos \theta) + 2ar' \sin \theta - \frac{r'^2}{r^2} = 0. \tag{22}
\]

So the Klein-Gordon Eq.(23) can be rewritten as

\[
\begin{align*}
  2\frac{\partial^2 \Phi}{\partial v \partial r} + & \frac{2}{r} \frac{\partial \Phi}{\partial v} + (1 - 2a\cos \theta) \frac{\partial^2 \Phi}{\partial v^2} + 2a \sin \theta \frac{\partial^2 \Phi}{\partial \theta \partial r} \\
  + & \frac{2r - 6a^2 \cos \theta}{r} \frac{\partial \Phi}{\partial r} + \frac{2a \sin \theta}{r} \frac{\partial \Phi}{\partial \theta} + 2a \cos \theta \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \\
  + & \frac{\cot \theta}{r^2} \frac{\partial \Phi}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta \partial \varphi^2} - \mu^2 \Phi = 0. \tag{23}
\end{align*}
\]

To study the equation outside the Rindler horizon \((r < r_H)\), the tortoise coordinate should be written as

\[
r_* = r + \frac{1}{2\kappa(v_0, \theta_0)} \ln \left[ \frac{r_H(v, \theta)}{r_H(v_0, \theta)} \right],
\]

\[
v_* = v - v_0, \theta_* = \theta - \theta_0. \tag{24}
\]

The Klein-Gordon Eq.(23) can be rewritten as

\[
\begin{align*}
  r_H^2 + & \left[ 1 + 2\kappa(r - r_H) \right] \left[ -2rr_H(r_H + ar_H' \sin \theta) + r_H^2 \left[ 1 + 2\kappa(r - r_H) \right] (1 - 2a \cos \theta) \right] \\
  & \times \frac{\partial^2 \Phi}{\partial r_*^2} + 2 \frac{\partial \Phi}{\partial v_* \partial r_*} + \left\{ \frac{1}{r_H^2 \left[ 1 + 2\kappa(r - r_H) \right]} \right\} \left[ 2rr_H^2(r - r_H) \right. \\
  & \left. - 2r_Hr_H^2 r(r - r_H) \right. \\
  & - 2a^2 r_H v_H r(r - r_H) \sin \theta \\
  & - 2r_H^2 r_H^2(r - r_H) \cos \theta \\
  & + 2a^2 [r_H'(r - r_H) + r_H'(r - r_H)] \\
  & + \frac{2}{r} - 4a \cos \theta \right\} \frac{\partial \Phi}{\partial r_*} \\
  & + 2rr_H \left[ 1 + 2\kappa(r - r_H) \right] a \sin \theta - 2r_H' \frac{\partial^2 \Phi}{\partial r_* \partial \theta_*} \\
  & + \frac{2\kappa(r - r_H)}{1 + 2\kappa(r - r_H)} \left\{ \frac{2}{r} \frac{\partial \Phi}{\partial v_*} + \frac{2a \sin \theta}{r} \frac{\partial \Phi}{\partial \theta_*} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta_*^2} \\
  & + \frac{\cot \theta}{r^2} \frac{\partial \Phi}{\partial \theta_*} + \frac{1}{r^2 \sin^2 \theta \partial \varphi^2} - \mu^2 \Phi \right\} = 0. \tag{25}
\end{align*}
\]

Due to the Rindler horizon Eq.(22), the numerator of the coefficient on the first term of above equation approaches to zero at the Rindler horizon \( r_H \). Therefore we can calculate the limit of the coefficient using L'Hospital law. Following above procedure, we obtain

\[
\kappa = \frac{1}{2r_H} \frac{-a \cos \theta - \frac{v_H^2}{2r_H}}{a \cos \theta + \frac{v_H^2}{2r_H}} + \frac{1}{2r_H} \frac{a \cos \theta + \frac{v_H^2}{2r_H}}{a \cos \theta + \frac{v_H^2}{2r_H}}. \tag{26}
\]

Then the equation can be transformed into

\[
\frac{\partial^2 \Phi}{\partial r_*^2} + 2 \frac{\partial^2 \Phi}{\partial v_* \partial r_*} + B \frac{\partial^2 \Phi}{\partial r_* \partial \theta_*} - C \frac{\partial \Phi}{\partial r_*} = 0, \tag{27}
\]
where
\[ B = 2(a \sin \theta_0 - \frac{r'_H}{r''_H}), \]
\[ G = -\frac{1}{r_H} + \frac{r'_H \cot \theta_0}{r''_H} + \frac{r''_H}{r_H} - \frac{r'^2_H}{r''_H} + \frac{r'^2_H}{r''_H}. \]

Separating variables as
\[ \Phi = R(r_*) \Theta(\theta_*) e^{i\varphi - i\omega v_*}, \]
we have radial solution of the equation
\[ \Psi_{in} = e^{-i\omega v_*}, \]
\[ \Psi_{out} = e^{-i\omega v_*} e^{2i\omega r_*} e^{Gr_*}. \]

The outgoing wave solution \( \Psi_{out} \) can be rewritten as
\[ \tilde{\Psi}_{out} = e^{-i\omega v_*} e^{2i\omega r_*} e^{Gr_*} e^{i\pi G/2} e^{-\pi \omega / \kappa}, \]
which is not analytical at the horizon. Extending the outgoing wave from outside \((r < r_H)\) to inside \((r > r_H)\) of the horizon analytically by turning \(+\pi\) angle through the positive half complex plane, we get
\[ \tilde{\Psi}_{out} = e^{-i\omega v_*} e^{2i\omega r_*} e^{Gr_*} e^{i\pi G/2} e^{-\pi \omega / \kappa}. \]

The relative scattering probability of outgoing wave at the horizon is
\[ \left| \frac{\Psi_{out}}{\Psi_{out}} \right|^2 = e^{2\pi \omega / \kappa}, \]
so Hawking radiation spectrum is
\[ N_\omega = \frac{1}{e^{\pi \omega / \kappa} + 1}, \]
\[ T = -\frac{\kappa}{2\pi k_B} \]
\[ = \frac{1}{2\pi k_B} \left( \frac{1}{2r_H} \frac{a \cos \theta + \frac{r'^2_H}{r_H}}{a \cos \theta + \frac{r'^2_H}{2r_H}} + \frac{1}{2r_H} \frac{-a \cos \theta - \frac{r'^2_H}{2r_H} + \frac{1}{2r_H}}{a \cos \theta + \frac{r'^2_H}{2r_H}} \right). \]

When \( a \) is constant, we will get Rindler horizon equation of an observer which has uniformly accelerated rectilinear motion
\[ -(1 - 2ar \cos \theta) + 2ar' \sin \theta - \frac{r'^2}{r^2} = 0. \]

Its solution is rotating parabolic surface
\[ r = \frac{1}{a(1 + \cos \theta)}. \]
From Eq(34), the Hawking temperature of the Rindler horizon is constant

\[ T = \frac{-\kappa}{2\pi k_B} = \frac{a}{2\pi k_B}. \] (37)

Now we consider the touch between a Kinnersley black hole and its Rindler horizon. While \( r_H \) reaches extreme value, i.e. \( r'_H = 0 \), Eq(41) can be rewritten as

\[ (2a \cos \theta_1)r_H^2 - (1 - 2\dot{r}_H)r_H + 2m = 0, \] (38)

where \( \theta_1 \) is angle when \( r_H \) reaches extreme value. The solution is

\[ r_H = \frac{(1 - 2\dot{r}_H) \pm \sqrt{(1 - 2\dot{r}_H)^2 - 16ma \cos \theta_1}}{4a \cos \theta_1}. \] (39)

When \( \dot{r}_H = 0 \) and \( ma \ll 1 \), the solution can be rewritten as

\[ r_{H1} \approx \frac{1}{2a \cos \theta_1}, r_{H2} = 2m, \] (40)

obviously \( r_{H2} \) belongs to black hole horizon. Comparing Eq(36) with Eq(10), we find \( r_{H1} \) belongs to Rindler horizon and \( \theta_1 = 0 \). The extreme value of \( r_H \) is

\[ r_{H1} = \frac{(1 - 2\dot{r}_{H1}) + \sqrt{(1 - 2\dot{r}_{H1})^2 - 16ma}}{4a}, \]
\[ r_{H2} = \frac{(1 - 2\dot{r}_{H2}) - \sqrt{(1 - 2\dot{r}_{H2})^2 - 16ma}}{4a}. \] (41)

As black hole horizon touches Rindler horizon, i.e. \( r_{H1} = r_{H2} \), we will have

\[ (1 - 2\dot{r}_H)^2 = 16ma, r_{H1} = r_{H2} = \frac{1 - 2\dot{r}_H}{4a}, \] (42)
\[ a = \frac{m}{r_{H1}^2} = \frac{m}{r_{H2}^2}. \] (43)

Eq(9) can be rewritten as

\[ T_2 = \frac{\kappa_2}{2\pi k_B} \]
\[ = \frac{1}{2\pi k_B} \frac{1}{2r_{H2}} \left( \frac{m}{r_{H2}^2} - a - \frac{m}{r_{H2}^2} - a - \frac{1}{2r_{H2}} \right), \] (44)

and Eq(34) can be rewritten as

\[ T_1 = \frac{-\kappa_1}{2\pi k_B} \]
\[ = \frac{1}{2\pi k_B} \frac{1}{2r_{H1}} \left( \frac{m}{r_{H1}^2} - a + \frac{m}{r_{H1}^2} + a - \frac{1}{2r_{H1}} \right). \] (45)

So the temperature of touch point is

\[ T = \frac{1}{2\pi k_B} \frac{1}{2a - \frac{1}{2r_H}} = -\frac{\dot{r}_H}{2\pi k_B(1 - 2\dot{r}_H)r_H}. \] (46)
5 Conclusion and discussion

Following Zhao’s method, Hawking temperature of the Kinnersley spacetime is investigated. It is found that the temperature relies on both time and angle. The phenomenon of touch between black hole horizon and Rindler horizon is similar to collision between two black holes. When $\dot{r}_H = 0$, the temperature of the touch point in Eq(46) is equal to zero. It will violate the third law of thermodynamics, so maybe the touch is impossible.

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