Differential equations of the rigid vibratory drum interaction with compacted soil

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Abstract. Differential equations of the rigid vibratory drum dynamic interaction with compacted construction soil are obtained, and relationships of the vibratory drum geometric parameters with soil characteristics and technological parameters of the soil compaction process are determined. The vibration exciter driving radial force has been replaced by the vertical driving medium operating constant force. The vibratory drum interaction with the soil is considered during a vibration exciter eccentric weight half-turn during which the vibratory drum single influence occurs on the compacted soil. The reaction force of the vibratory drum with the soil is written using the linear Hooke’s law, that has allowed to obtain an analytical solution adequate to the real process of the soil compaction by the vibratory drum.

Key-words: differential equation, rigid vibratory drum, transient processes, compaction mode.

1. Introduction
The theory of the rigid vibratory drum interaction is based on the fundamental provisions of the soil mechanics theory [1, 2, 3, 4, 5]. For the first time N.M. Gersevanov (1931) formulated the soil deformation law, establishing a linear dependence of the relative soil deformation ε on the stress σ under the punch. The concept of the general soil deformation modulus $E_0$ [1, 2] has been introduced. The linear deformation principle is valid for the construction soils of categories 1-5, and at present it is the main one in the soil mechanics, and all engineering calculations are based on it.

2. Problem statement
To create the theory of the rigid vibratory drum dynamic interaction with the soil, to determine the vibratory drum transient movement process in the compaction soil in one half-period of the eccentric weight rotation in the soil compaction operating cycle.

3. Theory
In the soil mechanics, the soil linear deformation modulus can be determined by the equation of papers [2, 3, 4]

$$E_0 = \frac{K_\omega \sigma b (1 - \mu^2)}{z},$$

where $E_0$ is the soil linear deformation modulus; $K_\omega$ is the punch area shape factor; $\sigma$ is the average specific pressure under the punch; $b$ is the linear minimum dimension of the punch; $\mu$ is the Poisson’s modulus of the lateral soil deformation; $z$ is the punch settlement.

The soil linear deformation modulus is determined by the indentation of punches of different area and shape into the soil. The parameters of formula (1) allow to obtain expressions for the relative soil deformation $\varepsilon$ under punch [5, 6].
\[ E_0 = \frac{\sigma}{\varepsilon}; \quad \varepsilon = \frac{z}{K_0 b(1 - \mu^2)}; \quad h_0 = \frac{z}{h_0}, \]  

where \( h_0 \) is the thickness of the layer to be compacted, determined by the formula of papers [1, 2, 3]

\[ h_0 = K_0 b(1 - \mu^2). \]

The obtained parameters and characteristics represent the basis for deriving the differential equations of the road roller vibratory drum interaction with the soil.

Let’s consider the example of determining the parameters \( E_0, h_0, \varepsilon \) as per the results of the indentation of a round punch with the diameter of \( d = 0.798 \) m and area of \( A = 0.5 \) m\(^2\) into the soil body. The experimental data are borrowed from paper [3] and have the form (see the following Table).

| Stress under the punch \( \sigma \), kPa | 50  | 100 | 150 | 200 | 250 | 300 |
|----------------------------------------|-----|-----|-----|-----|-----|-----|
| Settlement \( z \), m                  | 0.0014 | 0.0032 | 0.0062 | 0.0089 | 0.0111 | 0.0172 |

According to the Table it is established that the linear zone of the general soil deformation is limited by the pressure \( p = 250 \) kPa.

Figure 1 shows the results of the experimental indentation of the punch aimed at determining the soil characteristics.

Figure 1. Dependence of the relative deformation \( \varepsilon \) and punch settlement \( z \) on the stress \( \sigma \)

Construction soils as opposed to agricultural soils and lands have a sufficiently expressed zone of the linear deformation \( z = f(\sigma) \) and \( \varepsilon = f(\sigma) \). The linear proportionality limit of the relative deformation \( \varepsilon \) versus \( \sigma \) is established from the condition of the deformation deviation from the linear law with the reliability \( R^2 = 0.96 \).

The main soil deformation during the rigid vibratory drum rolling over the deformable soil is the vertical compression, as a result of which the porosity is decreased and density is increased. The study of the soil compaction modes is primarily connected with the contact stress \( \sigma \), settlement \( z \) and relative deformation \( \varepsilon \).

According to the results of the experiments the linear deformation modulus can be determined by formula (2) \( E_0 = \frac{\Delta \sigma}{\Delta \varepsilon} = 12.5 \) MPa.

For the thickness determination of the layer to be compacted \( h_0 \) we may use the known formula of papers [5]
\[ h_0 = \frac{zE_0(1 - \mu^2)}{\sigma} = 0.505 \text{ m.} \] (4)

Soil with such parameters according to the construction strength classification can be attributed to loamy soils of category 1 of normal humidity [5, 6].

The conducted analysis indicates a sufficient set of characteristics of deformable soils used for studying the vibratory drum interaction with the soil [7].

The soil compaction principle by a static vibratory drum consists in a single vertical impact of the vibratory drum on the soil during the movement over the compacted soil, when the porosity is decreased, settlement and compaction of the soil occur [8–13].

The peculiarity of the vibration compaction of the soil by the vibratory drum is in the additional multiple dynamic influence of the vibratory drum vibration exciter on the compacted material during a single pass of the vibratory drum over the compacted surface.

Figure 2 shows the design diagram of the vibratory drum interaction with the soil. On shaft 2 inside vibratory drum 1 inertial exciters 3, 4 of rotational movement are located, which create the radial dynamic force \( P_d \). During the time of one half-turn of the eccentric weight the vertical force \( P_{dz} = P_d \sin \pi t \) is formed, directed downward, which compacts the material during the time \( 0 \leq t < 0.5T \) of a half-period of one revolution of the vibration exciter rotation.

\[ \begin{align*}
R_z &= \text{soil reaction on the vibratory drum; } h_0 \text{ is the thickness of the compacted soil layer; } z \text{ is the soil settlement; } b \text{ is the width of the vibratory drum contact with the soil; } D \times L \text{ is the vibratory drum diameter and length, respectively.}
\end{align*} \]

Normal vertical stresses on the side of the soil on the vibratory drum can be specified in the form of the linear Hooke’s law

\[ \sigma = \sigma_0 + \frac{z}{h_0}E_0, \] (5)

where \( \sigma_0 \) is the initial normal stress of the soil, characterizing the initial strength before compaction.

According to formula (5), during compaction the soil acquires additional strength. For the differential equation derivation, we use the basic equation of the Newton’s dynamics in projection onto the vertical \( z \)-axis.
\[ m\ddot{z} = \sum F_{kz}, \]  

(6)

where \( m\ddot{z} \) is the product of the vibratory drum mass and acceleration; \( \sum F_{kz} \) is the sum of projections of forces on the \( z \)-axis.

On the right-hand side of equation (6), the use of the harmonic function \( P_{dz} = P_d \sin pt \) and other constant components leads to obtaining a differential equation that has no analytical solution and is solved by numerical methods.

In the general theory of vibrations, forced vibrations of a mechanical system are considered taking into account only harmonic effects in equation (6). Taking into consideration the constant components in equation (6) leads to differential equations that don’t have an analytical solution.

In this paper for obtaining the analytical solution of the differential equation, the harmonic driving force of the vibration exciter \( P_{dz} \) is replaced by the average operating equivalent constant force as per

\[ P_{dz} = P_d \cdot 0.637, \]

where \( P_d \) is the radial driving force.

The resultant equivalent force \( P_{dz} \) shown in Figure 2 while compacting the soil during the compaction time \( t = 0.5T \) is the constant value \( P_{dz} = P_d \cdot 0.637 = \text{const} \). The constant gravity force \( mg \) and viscous resistance force of the soil \( \mu \dot{z} \) act on the vibratory drum.

For the vibratory drum the basic dynamics equation (6) according to Figure 2 has the form

\[ mgP_bLzh \dot{z} + \sigma - \mu \dot{z} = 637, 0 \cdot 2 ) ( \]

(7)

After transformation and introduction of designations, equation (7) takes the following form

\[ \ddot{z} + 2n\dot{z} + \omega^2 z = h + g - \frac{\sigma_0 bL}{m}, \]

(8)

where \( \omega \) is the circular frequency of vertical vibrations of the vibratory drum on the soil

\[ \omega = \sqrt{\frac{E_0 bL}{h_0 m}}, \]

(9)

where \( m \) is the vibratory drum mass.

The reduced amplitude of the forced acceleration \( h \) in equation (8) is determined by the formula

\[ h = \frac{2P_d \cdot 0.637}{m}. \]

(10)

Equation (8) is a nonhomogeneous second-order differential equation with the constant right-hand side.

The general solution of nonhomogeneous equation (8) has the form

\[ z = z^* + z_1, \]

(11)

where \( z^* \) is a particular solution of homogeneous equation (8) without the right-hand side

\[ \ddot{z} + 2n\dot{z} + \omega^2 z = 0. \]

(12)

The general solution of equation (8) without the right-hand side can be found in the form

\[ z^* = C_1 \cos \omega t + C_2 \sin \omega t. \]

(13)

A particular solution of equation (8) with the constant right-hand side can be found in the form

\[ z_1 = A. \]

(14)

First we determine the coefficient \( A \), for this purpose we insert solution (14) into equation (8)
\[ \omega^2 A = h + g - \frac{\sigma_0 b L}{m}. \]  

(15)

From which

\[ A = \frac{h}{\omega^2} + \frac{g}{\omega^2} - \frac{\sigma_0 b L}{m \omega^2}. \]  

(16)

Let’s write the general solution of equation (8) as follows

\[ z = C_1 \cos \omega t + C_2 \sin \omega t + \frac{h}{\omega^2} + \frac{g}{\omega^2} - \frac{\sigma_0 b L}{m \omega^2}. \]  

(17)

For the determination of the integration constants \( C_1, C_2 \) it is necessary to write the differential equation of the vibratory drum speed

\[ \dot{z} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t. \]  

(18)

Initial conditions for the vibratory drum: at \( t=0 \) \( z=z_0=0 \); \( \dot{z} = \ddot{z}_0 = 0 \), the eccentric weights in Figure 2 are in a horizontal position and directed to the left. By inserting the initial conditions into equations (17), (18), we determine

\[ C_1 = \frac{\sigma_0 b L}{m \omega^2} - \frac{h}{\omega^2} - \frac{g}{\omega^2}; \quad C_2 = 0. \]  

(19)

The final solution of the differential equations of the vibratory drum interactions with the compacted soil has the following form

\[ z = \frac{\sigma_0 b L}{m \omega^2} - \frac{h}{\omega^2} - \frac{g}{\omega^2} \cdot (1 - \cos \omega t). \]  

(20)

\[ \dot{z} = \left( \frac{\sigma_0 b L}{m \omega^2} - \frac{h}{\omega^2} - \frac{g}{\omega^2} \right) \cdot \omega \sin \omega t. \]  

(21)

The peculiarity of obtained equations (20), (21) is that they are valid for the system of forces (see Figure 2) when the vibratory drum moves downward. This means that the transient processes relative to equations (20), (21) are considered on the half-period of vibrations during the time

\[ 0 \leq t < 0.5T; \quad 0.5T = \pi/p, \]  

(22)

where \( p \) is the rotation frequency of the eccentric weights, for example, \( p = 201.6 \) 1/s (\( f = 32 \) Hz).

During its operation the vibration exciter provides multiple successive vertical impacts of the driving force \( P_{dv} \) on the contact area of the vibratory drum with the soil during one pass over the compacted soil.

The impact time of the vibratory drum on the compacted material during one pass over the compacted surface is determined by the formula

\[ t_c = \frac{b}{V_m}, \]  

(23)

where \( b \) is the width of the vibratory drum contact area with the soil; \( V_m \) is the speed of the vibratory drum movement.

In Figure 2 when the vibratory drum enters the contact area, which length is equal to the contact width \( b \), the first vertical impact on the compacted material takes place and first settlement of the soil occurs which is equal to \( z_c \), calculated by formula (20).

The number of vertical impacts of the vibratory drum on the compacted material for the time \( t_c \) is determined by the formula

\[ n_c = t_c / T, \]  

(24)
where \( T \) is the time of one revolution of the eccentric weight.

The vibratory drum settlement \( z \) is connected with the vibratory drum contact width \( b \) with the soil and is determined by the geometry formula

\[
b = D \sqrt{1 - \left(\frac{0.5D - z}{0.5D}\right)^2}.
\] (25)

Obtained differential equations (20), (21) allow to consider in detail the operating process of the vibratory drum downward movement during the soil compaction. Analytical expressions (20), (21) allow to obtain information on the vibratory drum downward movement in the form of the transient process \( z = f(t) \) and information on the vibratory drum downward movement speed \( \dot{z} = f(t) \) during the soil compaction operation performance by the vibratory drum.

4. Results discussion

Figure 3, 4, 5 show different operating modes of the vibratory drum of a heavy duty road roller with the vibratory drum dimensions: \( D \times L = 1.55 \times 2.14 \) m during the compaction of construction soils of strength categories 1 - 4.

Figure 3 shows a resonant operating mode of the vibratory drum, when the vibration frequency of the vibratory drum and rotation frequency of the vibration exciter coincide \( \omega = \nu \).

Figure 4 shows a pre-resonant operating mode when \( \omega < \nu \), and Figure 5 shows a post-resonant operating mode for \( \omega > \nu \).

The frequency of vertical vibrations of the vibratory drum is determined by formula (9), and depends on the soil linear deformation modulus \( E_0 \), contact width \( b \), vibratory drum length \( L \), thickness of the compacted layer \( h_0 \), and the vibratory drum mass \( m \).

Let’s consider the technological process of the soil compaction in the resonant mode (Figure 3). The compaction process starts when the eccentric weights are in a horizontal position in Figure 2, when the vertical driving force of the eccentric weights \( P_{dc} \) appear, directed downward \( P_{dc} = P_d \sin \nu t \). The vibratory drum rests on the soil, initial conditions: vibratory drum initial movement \( z = z_0 = 0 \); vibratory drum initial speed \( \dot{z} = \dot{z}_0 = 0 \).
In Figure 3,a the process is considered for the initial soil strength $\sigma_0 = 5000$ kPa; in Figure 3b – for $\sigma_0 = 10000$ kPa.

During one half-turn of the eccentric weight rotation, i.e. the time $t = 0.5T$, the vibratory drum monotonously moves downward, acquiring the settlement $z$, during this period time in Figure 3,a the vibratory drum speed was increased to a certain maximum value and then was decreased to zero. The process of the soil compaction was completed, as the vibratory drum moved downward and in the lower position the vibratory drum speed was $\dot{z} = 0$.

The vibration exciters are in a horizontal position and start to form the vertical driving force $Pdz$ of the vibratory drum idle operation mode. The vibratory drum during the idle mode can remain stationary on the compacted soil, as the driving lifting force $Pdz$, directed upward is insufficient to lift the vibratory drum off the soil. The vibratory drum idle operation ends upon completion of one turn of the eccentric weights, when the time $t=T$, where $T=2\pi/p$.

After the end of the idle operation period, the eccentric weights return to their initial horizontal position and vibration exciters are in the initial position for the second compaction cycle. In the considered case, the harmonic process of the sequential performance of operations is not violated. The stability of the compaction process depends on the soil stability and other factors.

5. Consideration of the results

Figure 4 shows two operating modes of the vibratory drum at low frequencies of the vibratory drum $\omega = 72.2$ 1/s (Figure 4,a) and $\omega = 83.69$ 1/s (Figure 4,b) for soils with the initial strength $C_0 = 8000$ kPa and $C_0 = 5000$ kPa.
From the graphs in Figure 4 it can be determined that the vibratory drum makes the vertical movement \( z \) downward during the half-period of the vibration exciter rotation \( t = 0.5T \). However, during the downward stroke time \( t = 0.5T \) the vibratory drum speed is increased monotonously and acquires the certain final value \( \dot{z} = 0.58 \) m/s (Figure 4,a); \( 1.865 \) m/s (Figure 4,b). Thus, the pre-resonant mode is characterized by the fact that in the lower position at the maximum settlement \( z = z_{\text{max}} \) the vibratory drum acquires the speed \( \dot{z} \), directed downward.

This article doesn’t consider the compaction process efficiency, however, it should be noted that the compaction process takes place with the appropriate parameters of movement and speed of the vibratory drum.

Figure 5 shows the post-resonant mode of operation at the high vibration frequency of the vibratory drum \( \omega = 239.27 \) 1/s (Figure 5,a) and \( \omega = 273.75 \) 1/s (Figure 5,b).

It can be seen from the graphs that the downward working travel \( z_{\text{max}} \) of the vibratory drum was changed in nature, and at the end of the working cycle at the time \( t = 0.5T \) it was slightly decreased in comparison with an intermediate value.

The vibratory drum speed \( \dot{z} \) for the same time \( t = 0.5T \) has a harmonic nature of change and at the end of the compaction process it changes the movement direction.

The high-frequency operation mode is characterized by a sharp change of the characteristics of the travel \( z \) and vibratory drum speed \( \dot{z} \) in comparison with the resonant operation mode. The obtained analytical solutions of the differential equations of the vibratory drum are an important tool for further improving the design and operating modes.

6. Conclusion

1. The process of the soil compaction by the vibratory drum is characterized by the following physical and mathematical parameters: general soil deformation modulus \( E_0 \), soil settlement under the punch \( z \), relative soil deformation \( \varepsilon \), thickness of the compacted layer \( h_0 \), etc.

2. The use of the Hooke’s law for the description of the soil linear deformations allowed to obtain differential equations and perform analytical solutions.

3. The resonant operating mode of the vibratory drum is determined by the coincidence of the vibratory drum vertical vibration frequency with the vibration exciter rotation frequency.

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