Reversible state transfer between superconducting qubits and atomic ensembles

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We examine the possibility of coherent, reversible information transfer between solid-state superconducting qubits and ensembles of ultra-cold atoms. Strong coupling between these systems is mediated by a microwave transmission line resonator that interacts near-resonantly with the atoms via their optically excited Rydberg states. The solid-state qubits can then be used to implement rapid quantum logic gates, while collective metastable states of the atoms can be employed for long-term storage and optical read-out of quantum information.

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Solid state superconducting (SC) qubits are promising candidates for implementing quantum information (QI) processing in a scalable way. Very fast and efficient quantum logic gates can be performed by SC qubits without significant loss of coherence. However, dephasing and decoherence hinders the long-term storage of QI in such qubits. It would therefore be desirable to reversibly transfer the QI from the rather fragile qubits to a longer-lived system for storage and retrieval purposes.

Appealing candidates for such storage are ground electronic (hyperfine) states of ultra-cold (UC) atoms having very long coherence times. QI can then be stored for many seconds as a collective spin excitation of an atomic ensemble. Using stimulated Raman techniques, such as electromagnetically induced transparency, this QI can be optically read out by mapping the collective atomic state onto the generated photon, acting as traveling qubit, whose detection is possible with quantum efficiency approaching unity.

Here we put forward a proposal for reversible transfer of QI between SC qubits and UC atomic ensembles. In our scheme, the coupling between these systems is mediated by a near-resonant microwave transmission line resonator employing optical excitations of atomic Rydberg states. This hybrid scheme, despite considerable challenges, is predicted to be feasible and allow high-fidelity QI processing.

Our solid state qubit is represented by a SC Cooper pair box coupled to a SC electrode via two tunnel junctions at the rate $E_J$ (charge qubit) in the SQUID configuration. At the charge degeneracy point, the energy separation $\hbar \omega_{10} = 2E_J \cos(\pi \Phi/\Phi_0)$ between the qubit states $|0\rangle$ and $|1\rangle$ can be dynamically controlled via an external magnetic field $B_\perp$ that induces flux $\Phi = AB_\perp$ through the SQUID area $A$ ($\Phi_0 = hc/2e$ is the flux quantum). Typical values for $E_J/\hbar$ are the microwave range ($10-20$ GHz). The dipole moment $\gamma_{01}$ for the transition $|0\rangle \leftrightarrow |1\rangle$ is typically very large, $\gamma_{01} \approx 10^4 a_0 e$. Using resonant microwave fields, one can then perform many fast quantum logic gates within the qubit dephasing time $1/\gamma_q \gtrsim 1 \mu s$.

Charge qubits can be embedded in near-resonant SC transmission line resonators, such as a coplanar waveguide (CPW) cavity of Fig. 1(a) [10,11,12], having high quality factor $Q \approx 10^6$. The tight confinement of the cavity field in a small volume (see below) yields very large field per photon $\epsilon_c$ and strong coupling (vacuum Rabi frequency) $\eta_{qc} = (\gamma_{01}/\hbar)\epsilon_c u(r) \sim 2\pi \times 50$ MHz between the cavity field and the qubit located at position $r$ near the field antinode where the cavity mode function $u(r) \lesssim 1$. In the frame rotating with the cavity field frequency $\omega_c$, the Hamiltonian has the form

$$H_{qc} = \hbar \Delta_{qc} \hat{\sigma}^+ \hat{\sigma}^- - \hbar \eta_{qc} (\hat{\sigma}^+ \hat{c} + \hat{c}^\dagger \hat{\sigma}^-),$$

where $\Delta_{qc} = \omega_{10} - \omega_c$, is the externally controlled (via $B_\perp$) detuning, $\hat{\sigma}^- (\hat{\sigma}^+)$ is the qubit lowering (rising) operator, and $\hat{c} (\hat{c}^\dagger)$ is the cavity photon annihilation (creation) operator.

One can incorporate many SC qubits in the same CPW cavity, each qubit located near the cavity field antinode [Fig. 1(a)]. The cavity can then mediate long-range controlled interactions between pairs of resonant qubits, realizing, e.g., the two-qubit $\sqrt{\text{SWAP}}$ gate.

FIG. 1: (a) CPW cavity with strip-line length $L$ and electrode distance $w$. SC qubits are placed at the antinodes of the standing wave field and ensembles of UC atoms are trapped near the CPW surface. (b) SC qubit (left) and ensemble qubit (right) can couple to a common mode of the CPW cavity.
which together with the single qubit rotations form the universal set of logic gates in such a quantum computer. However, due to rapid dephasing and relaxation, neither SC qubits nor the cavity mode can carry out reliable long term storage of QI. In what follows, we show that this task can be accomplished by reversibly transferring the CPW cavity and SC qubits \cite{16}.\footnote{We envision a small trapping volume containing \( N \approx 10^6 \) atoms of \(^{87}\text{Rb} \) with the ground state hyperfine splitting \( \omega_{\text{gg}}/2\pi = 6.83 \text{GHz} \) between \( |F = 1\rangle \equiv |g\rangle \) and \( |F = 2\rangle \equiv |s\rangle \). Let us choose the frequency of the CPW cavity to be near-resonant with that of the atomic transition \( |g\rangle \leftrightarrow |s\rangle \) \cite{17}. When the atomic ensemble is near the field antinode, with the spacial dimension of the cloud being small compared to the mode wavelength, all the atoms couple symmetrically to the cavity field. The corresponding interaction Hamiltonian, \( H_{\text{ac}} \), can be expressed as

\[
H_{\text{ac}} = \hbar \Delta_{\text{ac}} \hat{s}^\dagger \hat{s} + \hbar \eta_{\text{ac}} \hat{s}^\dagger \hat{g} \hat{c} + \hat{c}^\dagger \hat{g}^\dagger \hat{s},
\]

where \( \Delta_{\text{ac}} = \omega_{\text{sg}} - \omega_c \) is the detuning and \( \eta_{\text{ac}} = \frac{\hbar}{\omega_c} \epsilon_c u(\mathbf{r}) \) is the coupling rate between the cavity field and a (single) atom at position \( \mathbf{r} \). Since \( |F = 1\rangle \leftrightarrow |F = 2\rangle \) is a magnetic dipole transition, the corresponding matrix element is small, \( \langle \mathbf{r}| \hat{\sigma}| \mathbf{r}' \rangle \sim \frac{\hbar}{\omega_c} \epsilon_c u(\mathbf{r}) \) with \( \alpha = 1/137 \), which yields \( \eta_{\text{ac}} \sim 2\pi \times 20 \text{ Hz} \) \cite{18}. The operators \( \hat{s} \) \((\hat{g}^\dagger)\) and \( \hat{c} \) \((\hat{c}^\dagger)\) annihilate (create) an atom in the corresponding state \( |g\rangle \) and \( |s\rangle \); these essentially bosonic operators live in a space of completely symmetrized states \( |n_g,n_s\rangle \) with \( n_g \) atoms in state \( |g\rangle \) and \( n_s \) atoms in state \( |s\rangle \), while \( n_g + n_s = N \).

Apparently \cite{2}, the most direct approach to state transfer from the SC qubit to the atomic ensemble would be to prepare all the atoms in state \( |g\rangle \) in the cavity field in vacuum \( |0\rangle_c \), and choose the frequencies of the cavity mode and the atomic hyperfine transition to be the same, \( \Delta_{\text{ac}} = 0 \). Then, by tuning the SC qubit frequency to resonance with the cavity, \( \Delta_{\text{sc}} = 0 \), during time \( \tau_{\text{qc}} \) such that \( 2\eta_{\text{sc}} \tau_{\text{qc}} = \pi \), an arbitrary quantum state \( |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \) will be transferred from the SC qubit to the CPW cavity field \cite{19}. Next, it follows from Eq. \( (2) \) that the collective coupling rate of the cavity field and the atomic ensemble via the transition \( |n_g,N,n_s = 0; 1\rangle \rightarrow |n_g,N-1,n_s = 1; 0\rangle \) is given by \( \sqrt{N} \eta_{\text{ac}} \sim 2\pi \times 20 \text{ KHz} \). Thus, during time \( \tau_{\text{sc}} = \frac{\pi}{2\sqrt{N} \eta_{\text{ac}}} \sim 12 \mu s \) the cavity photon will be absorbed by the atoms and we will have achieved our goal. The time \( \tau_{\text{sc}} \) is, however, comparable to the photon lifetime in the CPW cavity, \( \kappa^{-1} = Q/\omega_c \sim 20 \mu s \). Thus the photon will be lost with high probability \( P_{\text{LOSS}} \approx \kappa \tau_{\text{sc}} \sim 0.5 \) before being coherently absorbed by the atoms. It is therefore necessary to improve the CPW cavity by increasing its quality factor \( Q \) and thereby decreasing the photon decay rate \( \kappa \).

In an alternative setup, the SC qubit and the atoms are tuned to be resonant with each other, \( \Delta_{\text{qc,ac}} \sim \Delta \), but detuned from the cavity mode frequency. For large detuning \( \Delta \gg \eta_{\text{qc}} \), the adiabatic elimination of the cavity mode yields an effective photon decay rate \( \kappa_{\text{eff}} = \kappa\eta_{\text{qc}}^2/\Delta^2 \), while the corresponding second-order interaction Hamiltonian, \( V_{\text{qc,ac}}^{(2)} \), describes an effective swap of an excitation between the SC qubit and atomic ensemble with the rate \( \sqrt{N} \eta_{\text{eff}} \), mediated by virtual photon exchange in the cavity \cite{3}. Thus the effective coupling is reduced by a factor of \( \Delta/\eta_{\text{qc}} \), while the decoherence rate is reduced by a factor of \( (\Delta/\eta_{\text{qc}})^2 \). For \( \Delta = 10\eta_{\text{qc}} \) we then have \( \sqrt{N} \eta_{\text{eff}} \sim 2\pi \times 2 \text{ KHz} \) while \( \kappa_{\text{eff}} \sim 2\pi \times 100 \text{ Hz} \), which yields a low probability of photon decay during the excitation swap, \( P_{\text{LOSS}} \sim \kappa_{\text{eff}} \pi/(2\sqrt{N} \eta_{\text{eff}}) = 0.08 \).

Recall, however, that the SC qubit dephasing \( \gamma_g \ll 1 \text{ MHz} \) is much larger than \( \sqrt{N} \eta_{\text{eff}} \), hampering the foregoing scheme. In turn, \( \gamma_g \) is much smaller than the coupling \( \eta_{\text{qc}} \), hence this decoherence does not pose a problem during the resonant excitation exchange between the SC qubit and CPW cavity. This rapid stage may therefore be accomplished with high fidelity, as opposed to the much slower stage of excitation transfer from the cavity to the atoms with weak magnetic dipole transition.

Very strong atom–cavity field coupling can be achieved at microwave frequencies for electric-dipole transitions between highly-excited Rydberg states \cite{18,19}. Let us therefore select a pair of Rydberg states \( |i\rangle \) and \( |r\rangle \) such that the frequency \( \omega_{\text{ir}} \) of transition \( |i\rangle \leftrightarrow |r\rangle \) is close to the cavity mode frequency \( \omega_c \), the corresponding detuning being \( \Delta_{\text{ir}} = \omega_{\text{ir}} - \omega_c \). We envision a level scheme sketched in Fig. \( \text{2}(a) \), where the transition \( |g\rangle \leftrightarrow |i\rangle \) is driven by an external optical field with Rabi frequency \( \Omega_{\text{gi}} \) and detunings \( \Delta_{\text{ig}} \). The Hamiltonian reads

\[
H_{\text{ac}} = \hbar \Delta_{\text{ig}} \hat{i}^\dagger \hat{i} + \hbar (\Delta_{\text{ir}} + \Delta_{\text{ig}}) \hat{i}^\dagger \hat{r} - \hbar \Omega_{\text{gi}} (\hat{i}^\dagger \hat{r} + \hat{r}^\dagger \hat{i} + \text{H.c.}),
\]

where \( \eta_{\text{ac}} = \frac{\hbar}{\omega_c} \epsilon_c u(\mathbf{r}) \) is the atom-cavity field coupling rate, \( \psi_{\text{ir}} \) being the corresponding dipole matrix element, while operators \( \hat{i} \) \((\hat{i}^\dagger)\) and \( \hat{r} \) \((\hat{r}^\dagger)\) annihilate (create) an atom in state \( |i\rangle \) and \( |r\rangle \), respectively.

We set the detunings as \( \Delta_{\text{ir}} \approx -\Delta_{\text{ig}} \approx \Delta \). Then, given a photon in the cavity, the external field \( \Omega_{\text{gi}} \) and the cavity field induce a two-photon transition from the ground state \( |g\rangle \) to the Rydberg state \( |r\rangle \) via non-resonant intermediate Rydberg state \( |i\rangle \). If \( \Delta \gg \eta_{\text{qc}}, \sqrt{N} \Omega_{\text{gi}}, \) state \( |i\rangle \) is never populated, and we obtain an effective interaction Hamiltonian \( V_{\text{ac}}^{(2)} = \hbar \eta_{\text{ac}} \hat{i}^\dagger \hat{g} \hat{c} + \hat{c}^\dagger \hat{g}^\dagger \hat{i} \), with \( \eta_{\text{ac}} = \Omega_{\text{gi}} \eta_{\text{ac}} / \Delta \). Thus, starting from the initial state of the system \( |n_g = N,n_{r,s} = 0; 1\rangle_c \), by pulsed \( \Omega_{\text{gi}} \) for time \( \tau_{\text{gi}} = \pi/(2\sqrt{N} \eta_{\text{eff}}) \), the cavity photon will be coherently absorbed and a single atom from the ensemble will be excited to the Rydberg state \( |r\rangle \). Next, another (bichromatic) external field with Rabi frequency
Ω_{rs} pulsing for a time \( \tau_{rs} = \pi/(2\Omega_{rs}) \) can resonantly transfer the single collective Rydberg excitation of the atomic ensemble to the storage state \( |s⟩ \): this process is described by \( H_{rs} = -\hbar \Omega_{rs} \hat{r} \hat{s} + \text{H.c.} \). At a later time, when required, the reverse process can add a single photon in the cavity while all the atoms will up in state \( |g⟩ \). This single photon excitation can then be quickly transferred to the SC qubit, as described above.

Before proceeding, we survey the relevant experimental parameters. An elongated trapping volume \( V_0 \sim d \times d \times l \), with \( d \approx 5 \mu m \) and \( l \approx 1 \text{ mm} \), contains \( N \approx 10^6 \) atoms at density \( \rho_0 \approx 4 \times 10^{13} \text{ cm}^{-3} \). The atomic lower states \( |g⟩ \) and \( |s⟩ \) correspond to the \( |F = 1, M_F = -1⟩ \) and \( |F = 2, M_F = 1⟩ \) sublevels of the ground electronic state \( 5s_{1/2} \) of \(^{87}\text{Rb} \). We choose the Rydberg states \( |i⟩ \equiv |np_{1/2}, F = 2, M_F = -1⟩ \) and \( |r⟩ \equiv |(n+1)s_{1/2}, F = 1, M_F = 0⟩ \) with \( n = 68 \) the principal quantum number. The quantum defect for the \( s_{1/2} \) and \( p_{1/2} \) Rydberg states of \(^{87}\text{Rb} \) are \( \delta_s = 3.131 \) and \( \delta_p = 2.6545 \) \(^{18}\), with which the corresponding transition frequency is \( \omega_{ir} = 2\pi \times 12.2 \text{ GHz} \). Calculation of the relevant transition dipole matrix element involving the radial and angular parts gives \( \omega_{ir} \approx 1520 \text{ a.u.} \).

With the strip-line length \( L \approx 1 \text{ cm} \) and effective dielectric constant \( \varepsilon_r \approx 6 \), the frequency of the \( n \text{th} \) standing-wave mode of the cavity is \( \omega_c = \pi n c/L \sqrt{\tau_r} \) [Fig. (a)]. The grounded SC electrodes at distance \( w \approx 10 \mu m \) confine the cavity field within the effective volume \( V_c = \int d^3r |u(r)|^2 \approx \frac{2}{\varepsilon_0} w^2 L \) yielding \( \varepsilon_c = \sqrt{\hbar \omega_c / 2e_0 V_c} \approx 0.5 \text{ V/m} \). Taking the full-wavelength \( (m = 2) \) linearly polarized cavity mode with \( \omega_{ir}/2\pi \approx 12.16 \text{ GHz} \), we estimate \( \omega_{ir} \) that at the position of atomic cloud about \( 10 \mu m \) above the CPW surface the mode function \( u(r) \approx e^{-r} \) which yields the vacuum Rabi frequency \( \eta_{ac} = (\sqrt{\pi} \hbar / 2) e_{uv}(r) \approx 2\pi \times 3.85 \text{ MHz} \) and appropriately large detuning \( \Delta_{ri} \approx 10\eta_{ac} \).

The transition \( |g⟩ \rightarrow |i⟩ \) is driven by linearly \( \pi \)-polarized UV field with wavelength \( \lambda_{uv} \approx 298 \text{ nm} \) and detuning \( \Delta_{ig} = -\Delta_{ri} \). To optimize the transition rate, its Rabi frequency is chosen as \( \sqrt{\hbar \Omega_{gi}} \approx \eta_{ac} \), with which the transfer time is \( \tau_{gr} \approx 0.65 \mu s \). The required UV field intensity at the atomic cloud is \( I_{gi} = 0.46 \text{ W cm}^{-2} \). Next, \( |r⟩ \rightarrow |s⟩ \) is a two-photon transition via non-resonant intermediate state \( |np_{1/2}, F = 2, M_F = 0⟩ = |e⟩ \).

The wavelengths are \( \lambda_{re} \approx 476 \text{ nm} \) (linearly \( \pi \)-polarized field) and \( \lambda_{es} \approx 795 \text{ nm} \) (circularly \( \sigma- \)-polarized field). With the corresponding intensities \( I_{re} = 440 \text{ W cm}^{-2} \) and \( I_{es} = 2.25 \text{ mW cm}^{-2} \) and intermediate detuning \( \Delta_{es} = 2\pi \times 25 \text{ MHz} \), the two-photon Rabi frequency is \( \Omega_{es} = 2\pi \times 250 \text{ KHz} \) leading to the transfer time of \( \tau_{rs} \approx 1 \mu s \). Note that \( \tau_{gr} \) and \( \tau_{rs} \) are short compared to the lifetimes of cavity photon \( 1/\kappa \approx 10 \mu s \) and Rydberg states \( 1/\Gamma_R \approx 100 \mu s \) \(^{18}\).

Employing a master equation approach \(^{4}\), we have simulated the dynamics of transfer process in the system with above parameters and temperature-dependent initial population \( \langle \hat{n}_c(0) \rangle \) of the CPW cavity photon field. Figure 3 shows the results of numerical integration of the equations for density operator \( \hat{\rho}(t) \) whose evolution is governed by Hamiltonians \( H_{ac}, H_{re} \) of \(^{19}\) and \( H_{rs} \).

As seen, in the case of \( \langle \hat{n}_c(0) \rangle = 0 \), the state transfer is nearly ideal, with the small final error probability \( P_{err} \lesssim 0.04 \) due to relaxation of the qubit, the cavity field and the atoms. However, in the case of finite temperature, \( \langle \hat{n}_c(0) \rangle = 0.5 \), during the transfer, as expected, the cavity field and the collective atomic state occupation numbers \( \langle \hat{n}_c \rangle \) and \( \langle \hat{n}_{rs} \rangle \), exceed unity, and the resulting error probability \( P_{err} \approx 0.3 \) is large.

We can characterize the transfer process for a given initial state \( |\psi\rangle_q \) of the SC qubit by the conditional fidelity

![Figure 2](image1.png)

**FIG. 2:** (a) Atomic Rydberg states \(|i⟩\) and \(|r⟩\) and relevant couplings for the excitation transfer from the CPW cavity to the atomic storage state \(|s⟩\). (b) Propagation geometry of the corresponding optical fields and the beam-stop and metallic mirrors on top of the SC electrodes of the CPW cavity.

![Figure 3](image2.png)

**FIG. 3:** Numerical simulations of the three-step excitation transfer from the SC qubit to the UC atomic ensemble. Top panel illustrates the sequence of \((\pi-)\) pulses: first the SQ qubit is brought to resonance with the CPW cavity, by pulsing \( \Delta_{qc}(t) = 0 \) for time \( \tau_{qc} \); next, the \( \Omega_{qi}(t) \) field is pulsed for time \( \tau_{gr} \); finally, the \( \Omega_{rs}(t) \) field is pulsed for time \( \tau_{rs} \). Central and lower panels show the dynamics of occupation numbers for the qubit exited state \( |\sigma^{+} \sigma^{-}\rangle \), the cavity field \( \langle \hat{n}_c \rangle \equiv \langle \hat{c}^\dagger \hat{c} \rangle \) and the collective atomic states \( \langle \hat{n}_{rs} \rangle \equiv \langle \hat{r}^\dagger \hat{r} \rangle \) and \( \langle \hat{n}_{es} \rangle \equiv \langle \hat{s}^\dagger \hat{s} \rangle \). Initially, the mean thermal photon number is \( \langle \hat{n}_c(0) \rangle = 0 \) in the central panel, and \( \langle \hat{n}_c(0) \rangle = 0.5 \) in the lower panel.
the photon lifetime still remains long compared to the
cavity.

However, even after such significant reduction of inten-
sity, the residual absorption would remain too large. To
covered by a few μm thick layer of dielectric fol-
lar waveguide cavity via optically excited Rydberg tran-

To conclude, we have proposed a promising approach
for hybridizing solid-state and atomic quantum devices,
thereby implementing efficient quantum state transfer
between superconducting charge qubits and mesoscopic
ultracold atomic ensembles coupled to a microwave copla-
nar waveguide cavity via optically excited Rydberg transi-
tions. Our scheme is scalable to many superconducting
qubits, whose merit is very high flipping rates, coupled
to atomic ensembles serving as reliable storage qubits.

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FIG. 4: Conditional $F_\psi$ and mean $F$ transfer fidelities vs temperature $k_B T$ (lower horizontal axis) or mean thermal photon number $\langle \hat{n}_c \rangle$ (upper horizontal axis). In $F_\psi$, $\psi = 0, 1, \pm$ correspond to states $|0\rangle$, $|1\rangle$ and $\pm c [0\pm (1)|1\rangle] = \langle 0\rangle$. The dependence of $F_\psi$ and $F$ on the CPW cavity temperature $T_c$, or the mean thermal photon number $\langle \hat{n}_c \rangle = (e^{\hbar \omega_c / k_B T} - 1)^{-1}$, is shown in Fig. 4. Below $k_B T / \hbar \omega_c \simeq 0.2$, corresponding to $\langle \hat{n}_c \rangle \lesssim 0.01$, the transfer fidelity is fairly high, $F > 98\%$, but then it quickly degrades due to the detrimental effect of even a small number of thermal photons. For the above parameters, this critical temperature is $T_c \sim 0.1 \text{K}$ necessitating cryogenic conditions. We note that the temperature of atomic cloud is much smaller, $T_a \lesssim 1 \text{μK}$, resulting in the lifetime of atomic hyperfine coherence in excess of $1 \text{s}$ [2].

The practical realization of our scheme can be hindered by absorption of optical fields, driving the atomic transitions, by the SC electrodes of the CPW cavity. If not eliminated, the photon absorption would break up many Cooper pairs and produce quasi-particles resulting in drastic reduction of the cavity $Q$ factor. Consider a geometry where the strong $\pi$-polarized fields propagate in the direction parallel to the CPW surface and perpendicular to the electrodes, in front of which an opaque barrier of height $\sim 10 \text{μm}$ serves as a beam-stop [Fig. 2(b)]. Due to the Fresnel diffraction on the barrier edge, some light will still reach the shade area behind the barrier, but we estimate that the light intensity at the SC electrodes, each about $10 \text{μm}$ wide, will be reduced by a factor of 300. However, even after such significant reduction of intensity, the residual absorption would remain too large. To completely eliminate the absorption, the SC electrodes can be covered by a few μm thick layer of dielectric followed by a thin metallic mirror. A moderate reduction of the cavity $Q$ factor up to 10 times is tolerable, since the photon lifetime still remains long compared to the transfer times $\tau_{qc} + \tau_{qr} \sim 0.7 \mu s$, but the resulting fidelity will decrease to $F \gtrsim 70\%$. 

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