FOUR LECTURES ON M-THEORY

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Synopsis: (i) how superstring theories are unified by M-theory; (ii) how superstring and supermembrane properties follow from the D=10 and D=11 supersymmetry algebras; (iii) how D=10 and D=11 supergravity theories determine the strong coupling limit of superstring theories; (iv) how properties of Type II p-branes follow from those of M-branes.

1 M-theory unification

Over the decade from 1984-94 superstring theory came to be regarded as the most promising approach to a unification of all the fundamental forces. The most compelling argument in its favour is that it naturally describes an ultraviolet finite and unitary perturbative quantum gravity. That this is true only in a spacetime of ten dimensions (D=10) is not so much a problem as an invitation to find a realistic model of particle physics by compactification to D=4. However, D=10 'superstring theory' is not a single theory but actually a collection of five of them. They are

(i) Type IIA,
(ii) Type IIB,
(iii) $E_8 \times E_8$ heterotic,
(iv) $SO(32)$ heterotic,
(v) Type I.

While the $E_8 \times E_8$ heterotic string is the one favoured in attempts to make contact with particle physics, the other four are equally acceptable as perturbative theories of quantum gravity (whereas the bosonic string is not because of the tachyon in its spectrum).

In the infinite tension limit each of these five superstring theories is approximated by its effective field theory, which is an anomaly-free D=10 supergravity theory. There are also five of these but one of them, N=1 supergravity/Yang-Mills (YM) theory with gauge group $U(1)^{496}$, is not the

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effective field theory of any superstring theory. The other four are as follows, labelled according to the superstring theory with which each is associated:

(i) Non-chiral N=2 supergravity (IIA)
(ii) Chiral N=2 supergravity (IIB),
(iii) N=1 supergravity/YM with $E_8 \times E_8$ gauge group,
(iv) & (v) N=1 supergravity/YM with $SO(32)$ gauge group.

Observe that two of the five superstring theories have equivalent effective field theories. This ‘coincidence’ led to some early speculation that perhaps these two superstring theories are really the same theory. If so, the equivalence must be non-perturbative because the perturbative spectra are quite different, the heterotic string being a closed oriented string and the Type I string an unoriented one that may be open or closed.

Unfortunately, or perhaps inevitably, superstring theories are defined only as divergent asymptotic power series in the string coupling constant $g_s$, which is related to the expectation value of the massless dilaton field $\phi$ appearing in the effective supergravity theory, specifically $g_s = e^{\langle \phi \rangle}$. Thus, superstring theories fail to qualify as truly unified theories on two counts: (i) there is more than one of them, and (ii) they are defined only as asymptotic expansions. The essence of recent progress on the unification front is the realization that by solving problem (ii) we also solve problem (i), i.e. all five superstring theories are asymptotic expansions around different vacua of a single non-perturbative theory. This is not to say that problem (ii) has been solved, but a convincing picture of how the different asymptotic expansions are unified by a single theory has emerged. The surprising feature is that this unified theory is actually 11-dimensional at almost all points in its moduli space of vacua! The well-known fact that D=10 is the critical dimension of superstring theory means only that the dimension of spacetime is at least ten, because there may be dimensions that are invisible in perturbation theory. Considerations of supersymmetry imply that the spacetime dimension cannot exceed eleven, so we are left with either D=10 or D=11 as the actual dimension of spacetime. In this first lecture I shall attempt to provide an overview of this D=11 superunification and, since the idea of superunification in D=11 is quite an old one, I shall begin with a brief sketch of its history.

The idea first arose from the 1978 construction of D=11 supergravity and was closely tied to the revival of the Kaluza-Klein (KK) program. This program faced insuperable difficulties, e.g. the non-renormalizability of D=11 supergravity and its failure to admit chiral compactifications. Both problems were resolved by superstring theory but the price was a retreat from D=11
to D=10. Only the fact that IIA supergravity is the dimensional reduction of D=11 supergravity offered hope of a possible role for D=11. But whereas the 2-form potential of D=10 supergravity theories is naturally associated with a string, the 3-form potential of D=11 supergravity is naturally associated with a membrane. While it was known how to incorporate spacetime-supersymmetry into string theory, via the Green-Schwarz (GS) worldsheet action, it was unclear how to generalize this to higher-dimensional objects. Progress came from consideration of effective actions for extended objects in supersymmetric field theories; it was known that the D=4 GS action could be re-interpreted as the effective action for Nielsen Olesen vortices in an N=2 supersymmetric abelian Higgs model, but this model is the dimensional reduction to D=4 of a D=6 field theory for which the ‘vortices’ are what we would now call 3-branes. The effective action for this D=6 3-brane is necessarily a higher-dimensional generalization of the GS action, and its construction showed how to overcome the ‘string barrier’. This led to the construction of the D=11 supermembrane action and the interpretation of D=11 supergravity as the effective field theory of a hypothetical supermembrane theory. The subsequent demonstration that the GS action for the IIA superstring is the ‘double-dimensional reduction’ of the D=11 supermembrane action suggested an interpretation of the IIA superstring as a membrane wrapped around the circular 11th dimension. The case for this interpretation was strengthened by the construction of the extreme membrane solution of D=11 supergravity, together with the demonstration that it reduces in D=10 to the extreme string solution of IIA supergravity, which had earlier been identified as the field theory realization of the fundamental string. A further important development on this front was the construction of a fivebrane solution of D=11 supergravity, which was subsequently shown to be geodesically complete. The fivebrane is the ‘magnetic’ dual of the ‘electric’ membrane in D=11, in agreement with the general formula that the dual of a p-brane is a \( \tilde{p} \)-brane with \( \tilde{p} = D - p - 4 \).

These connections between D=10 and D=11 physics were mostly classical; it still seemed impossible that the quantum IIA superstring theory, with D=10 as its critical dimension, could be 11-dimensional. In addition, the non-renormalizability problem of D=11 supergravity appeared to have been replaced by the difficulty of a continuous spectrum for the first quantized supermembrane. An indication that the D=11 supermembrane might after all be relevant to quantum superstring theory arose from consideration of the soliton spectrum of compactified D=11 supergravity. It was noted that the inclusion of wrapping modes of the membrane and fivebrane led to a spectrum of solitons identical to that of the IIA superstring if, as could be argued on
other grounds, the latter includes the wrapping modes of the D=10 p-branes carrying Ramond-Ramond charges. But if this is taken to mean that IIA superstring theory really is 11-dimensional then its non-perturbative spectrum in D=10 must include the Kaluza-Klein excitations from D=11. These would have long range ten-dimensional fields and so would have to appear as (BPS-saturated) ‘0-brane’ solutions of IIA supergravity. Such solutions, and their 6-brane duals, were already known to exist and it was therefore natural to interpret them as the field realization of the KK modes and the KK 6-branes needed for the D=11 interpretation of IIA superstring theory.

Because of the connection between the string coupling constant and the dilaton it was clear that an improved understanding of the role of the dilaton would be crucial to any advance in non-perturbative string theory. The fact that IIA supergravity is the dimensional reduction of D=11 supergravity leads to a KK interpretation of the dilaton as a measure of the radius $R_{11}$ of the 11th dimension, and hence to a relation between the string coupling constant $g_s$ and $R_{11}$:

$$R_{11} = g_s^{\frac{2}{3}}.$$  \hspace{1cm} (1)

This shows clearly that a power series in $g_s$ is an expansion about $R_{11} = 0$, so that the 11th dimension is indeed invisible in string perturbation theory. In retrospect it is clear that that the connection between $R_{11}$ and $g_s$ should have been exploited much earlier by supermembrane enthusiasts. Ironically, the obstacle was the membrane itself; the problem is that the area of a wrapped membrane, and hence its energy, is proportional to $R_{11}$, leading one to expect the tension in D=10 to be proportional to $R_{11}$ too. But if this were the case the tension of the wrapped membrane would vanish in the $R_{11} \rightarrow 0$ limit. Thus it seemed necessary to fix $R_{11}$ at some non-zero value, thereby precluding any connection with perturbative string theory. What this overlooks is that the energy as measured in D=10 superstring theory differs from that measured in D=11 by a power of $R_{11}$ which is precisely such as to ensure that the D=10 string tension is independent of $R_{11}$, and hence non-zero in the $R_{11} \rightarrow 0$ limit. This rescaling also ensures that the 0-brane mass is proportional to $1/R_{11}$, as required for its KK interpretation.

In the strong coupling limit, in which $R_{11} \rightarrow \infty$, the vacuum is 11-dimensional Minkowski and the effective field theory is D=11 supergravity. Some authors refer to this special point in the moduli space of vacua, or a neighbourhood of it, as ‘M-theory’, in which case superstring theories and M-theory are on a somewhat similar footing as different approximations to the underlying unified theory. This can be quite convenient but it leaves us without a name for the ‘underlying unified theory’. One might continue to call it ‘superstring theory’ with the understanding that it is now non-perturbative,
but this terminology is inappropriate because strings do not play a privileged role in the new theory even in vacua that are effectively 10-dimensional, and it
is membranes rather than strings that are important in the D=11 Minkowski vacuum. Consequently, I shall adopt the other usage, in which M-theory is the ‘underlying unified theory’, i.e. M-theory is the quantum theory that unifies the five superstring theories and D=11 supergravity. So ‘defined’, M-theory is still 11-dimensional in the sense that almost all its vacua are 11-dimensional, although some of these dimensions may be compact. The theory with the D=11 Minkowski vacuum will be called ‘uncompactified M-theory’. Superstring theories can then be viewed as ‘compactifications of M-theory’.

One might wonder how a chiral theory like the IIB superstring theory can be obtained by compactification of M-theory. This is a special case of a more general problem of how chiral theories arise upon compactification from D=11, given the no-go theorem for Kaluza-Klein (KK) compactification of D=11 supergravity. It seems that there are two ways in which this no-go theorem is circumvented by some M-theory compactifications, and both involve the membrane or the fivebrane. One way stems from the fact that one can consider compactifications of M-theory on orbifolds, whereas KK theory was traditionally restricted to manifolds. The other way, and the one most directly relevant to the IIB superstring, is that chiral theories can emerge as limits of non-chiral ones as a consequence of massive modes not present in the KK spectrum. For example, D=11 supergravity compactified on $T^2$ consists of D=9 N=2 supergravity coupled to a KK tower of massive spin 2 multiplets. In the limit in which the area of the torus goes to zero, at fixed shape, one obtains the (non-chiral) D=9 supergravity theory. In contrast, M-theory compactified on $T^2$ also includes massive spin 2 multiplets coming from membrane ‘wrapping’ modes on $T^2$. These additional massive modes become massless in the above limit, in such a way that the effective theory of the resulting massless fields is the ten-dimensional and chiral IIB supergravity! Since this is a chiral theory there are two equivalent versions of it, either left-handed or right-handed; which one we get depends on the choice of sign of the Chern-Simons term in the D=11 supergravity Lagrangian or, equivalently, the choice of sign of a related ‘Wess-Zumino’ term in the supermembrane action. Thus, M-theory incorporates an intrinsically ‘membrany’ mechanism that allows the emergence of chirality upon compactification.

To examine this in more detail let us denote by $R_{10}$ and $R_{11}$ the radii of the torus in the M-theory compactification. The limit in which $R_{10} \to \infty$, at fixed $R_{11}$, leads to the IIA theory with coupling constant given by (1), i.e.

$$g_s^{(A)} = R_{11}^{3/2}$$ (2)
where we now call the coupling constant $g_{s(A)}$ to distinguish it from the IIB coupling constant to be given below. For finite $R_{10}$ it is a result of perturbative superstring theory\[4,5\] that the IIA theory is equivalent to the the IIB theory compactified on a circle of radius $1/R_{10}$ (in units in which $\alpha' = 1$); i.e. the IIB theory is the T-dual of the IIA theory. It follows that the $S^1$-compactified IIB theory can also be understood as $T^2$-compactified M-theory; the limit in which $R_{11} \to 0$ and $R_{10} \to 0$, at a fixed ratio, then leads to the uncompactified IIB theory with string coupling constant

$$g_{s(B)} = \frac{R_{11}}{R_{10}}. \quad (3)$$

We may assume that $g_{s(B)} \leq 1$ since the interchange of $R_{10}$ and $R_{11}$ is simply a reparametrisation of the torus. In other words, the IIB theory at coupling $g_{s(B)}$ is equivalent to the IIB theory at coupling $1/g_{s(B)}$. More generally, the discrete $SL(2; Z)$ group of global reparametrizations of the torus implies an $SL(2; Z)$ symmetry of the IIB theory, originally conjectured\[22\] on the basis of the $SL(2; R)$ symmetry of IIB supergravity\[36\]. The way in which the IIA and the IIB theories are found as $T^2$ compactifications of M-theory is shown in the following diagram:\[32\]

A generic point on this diagram corresponds to an 11-dimensional vacuum. The exceptions are those points with $R_{11} = 0$ but non-vanishing $R_{10}$, corresponding to free string theories which may be ignored, and $(R_{10}, R_{11}) = (0, 0)$, corresponding to the uncompactified IIB superstring theory, for which the vacuum is 10-dimensional Minkowski. Actually, $(R_{10}, R_{11}) = (0, 0)$ is not really a
‘point’ in the moduli space because the IIB coupling constant depends on how the $(R_{10}, R_{11}) \to (0, 0)$ limit is taken. Thus while the IIA theory has a more or less straightforward interpretation as an M-theory compactification, as does the $S^1$-compactified IIB theory, the uncompactified IIB theory is a singular limit of an M-theory compactification.

Of course, it is equally true that the uncompactified M-theory is a limit of a IIB ‘compactification’ since, as the above figure illustrates, the $S^1$-compactified IIB theory belongs to the same moduli space as the $T^2$-compactified M-theory. In this sense, the two limiting theories are on equal footing: neither is more fundamental than the other. In taking the IIB theory as the starting point for an exploration of the moduli space of M-theory one might now wonder how the D=11 membrane could emerge from the IIB theory. The answer lies in the fact that the IIB theory is not just a theory of strings: as we shall see in some detail later it also contains other objects, one of which is a 3-brane.

Upon $S^1$ compactification, the 3-brane can wrap around the circle to produce a membrane. But this resolution of the puzzle raises another one; if the 3-brane does not wrap around the circle then it remains a 3-brane and we now have to find this object in the $T^2$-compactified M-theory. This is resolved by the fact that the uncompactified M-theory is a theory not just of membranes but also, as mentioned above, of fivebranes. A fivebrane wrapped around $T^2$ is a 3-brane. Of course, this fivebrane need not wrap around the $T^2$, so that we now have yet more branes to account for in the IIB theory. The end result of this type of analysis is that all branes appearing in one compactification also appear in the other as required by the equivalence of the two compactifications. Conversely, it is precisely the existence of the various branes that makes this equivalence possible.

It is clear from the description of IIB superstring theory as a limit of $T^2$-compactified M-theory that the complex structure of the torus, viewed as a Riemann surface, will survive the limit to become a parameter determining the choice of IIB vacuum. In fact this parameter is the vacuum value of the complex IIB supergravity field

$$\tau = \ell + i e^{-\phi},$$

where $\phi$ is the scalar dilaton and $\ell$ is a pseudoscalar ‘axion’ field. If $\tau$ were assumed to be single-valued in the upper half plane then it would have to be constant over the compact KK space in any compactification of the IIB theory. But, as its M-theory origin makes clear, $\tau$ actually takes values in the fundamental domain of the modular group of the torus, so it need not be single valued in the upper half plane. A class of compactifications that exploits this possibility has been called ‘F-theory’. Since F-theory has been associated
with a hypothetical 12-dimensional theory, which would appear to place it in an entirely different category, it is worthwhile to make a detour to consider how F-theory fits into M-theory. An example will suffice. D=11 supergravity, and hence (presumably) M-theory, can be compactified on a Ricci-flat four dimensional $K_3$ manifold\cite{[2]}. For some Ricci-flat metrics, $K_3$ can be viewed as an elliptic fibration of $\mathbb{CP}^1$, i.e. as a fibre bundle where the fibre is a torus whose complex structure $\tau$ varies over a Riemann sphere. Generically, there will be 24 singular points on the Riemann sphere at which the torus degenerates but these are merely coordinate singularities as long as no two singular points are coincident. Thus, there exist M-theory compactifications on manifolds that are locally isomorphic to $T^2 \times S^2$. If the 2-torus is again shrunk to zero area we arrive at an $S^2$ compactification of the IIB theory in which the scalar field $\tau$ varies over $S^2$. More generally, given a Ricci-flat manifold $E$ that is an elliptic fibration of a compact manifold $B$, one can define ‘F-theory on $E$’ as IIB theory on $B$ with $\tau$ varying over $B$ in the way prescribed by its identification as the complex structure of the torus in the description of $E$ as an elliptic fibration. Formally, this would appear to define ‘F-theory’ as a 12-dimensional theory, but this is indeed purely formal.

Having seen how the Type II superstring theories are unified by M-theory, it remains for us to see how the superstring theories with only N=1 D=10 supersymmetry fit into this scheme. Firstly, we can ask how they are related to each other. It is known\cite{[5]},\cite{[6]} (at least to all orders in perturbation theory) that the two heterotic string theories are related by T-duality. The compactification of either theory on a circle allows a non-vanishing ‘Wilson line’ $\int A$ around the circle, where $A$ is the Lie-algebra-valued gauge field of the effective supergravity/YM theory; this amounts to choosing a non-zero expectation value for the component of $A$ in the compact direction. This expectation value must lie in the Cartan subalgebra of either $E_8 \times E_8$ or $SO(32)$. Generically, this will break the gauge group to $U(1)^{16}$ but special choices result in non-abelian groups, e.g. $SO(16) \times SO(16)$. An $SO(16) \times SO(16)$ heterotic theory obtained in this way by compactification of the $SO(32)$ heterotic string theory on a circle of radius $R$ can be similarly obtained by compactification of the $E_8 \times E_8$ theory on a circle of radius $1/R$. Thus, the uncompactified $SO(32)$ and $E_8 \times E_8$ heterotic string theories are theories with vacua that are limiting points in a single connected space of vacua. We have already mentioned that the $SO(32)$ heterotic and Type I theories are potentially equivalent non-perturbatively. We shall later see some of the evidence for this. Anticipating this result, we see that there are really only two distinct uncompactified D=10 superstring theories with N=1 supersymmetry, one with $SO(32)$ gauge group and one with $E_8 \times E_8$ gauge group. We shall call these the $SO(32)$ and $E_8 \times E_8$ superstring theories.
Another clue from perturbative string theory is the fact that the Type I theory is an ‘orientifold’ of the Type IIB theory. The Type IIB string action is invariant under a worldsheet parity operation, Ω, which exchanges the left and right movers. We can therefore find a new string theory by gauging this symmetry\(^4\). This projects out the worldsheet parity odd states of the Type IIB superstring theory, leaving the states of the closed string sector of the Type I theory. This sector is anomalous by itself, but one can now add an open string sector, which can be viewed as an analogue of the twisted sector in the more conventional orbifold construction. An anomaly free theory is found by the inclusion of SO(32) Chan-Paton factors at the ends of open strings\[^5\]. This is the Type I string theory. By construction it is a theory of unoriented closed and open strings. From its origin in the IIB theory it is clear that the \(S^1\)-compactified Type I string must be 11-dimensional too. But since the IIA theory has the more direct connection to D=11, and since this is the T-dual of the IIB theory, we might expect to understand the 11-dimensional nature of the Type I theory more readily by considering its T-dual, which is called the Type IA (or Type I') theory\[^3\].

The Type IA theory has some rather peculiar features. To understand them it is convenient to start with the Type I theory in which the \(SO(32)\) gauge group is broken to \(SO(16) \times SO(16)\) by the introduction of Wilson lines. Let \(Y(t, \sigma)\) be the map from the string worldsheet to the circle. T-duality exchanges \(Y\) for its worldsheet dual \(\tilde{Y}\). Since \(Y\) was a worldsheet scalar, \(\tilde{Y}\) is a pseudoscalar, i.e.

\[
\Omega[\tilde{Y}](t, \sigma) = -\tilde{Y}(t, -\sigma).
\]  

(5)

Let \(\tilde{y}\) be the constant in the mode expansion of \(Y(\sigma)\); then \(\Omega[\tilde{y}] = -\tilde{y}\). The gauging of worldsheet parity now implies that a point on the circle with coordinate \(\tilde{y}\) is identified with the point with coordinate \(-\tilde{y}\), so the circle becomes the orbifold \(S^1/\mathbb{Z}_2\) in which the \(\mathbb{Z}_2\) action has two fixed points at \(\tilde{y} = 0, \pi\). In fact, since \(S^1/\mathbb{Z}_2\) is just the closed interval \(I = [0, \pi]\), the fixed ‘points’ are actually 8-plane boundaries of the 9-dimensional space, called ‘orientifold’ planes because the \(\mathbb{Z}_2\) action on \(S^1\) is coupled with a change of orientation on the worldsheet.

Thus, the Type IA theory is effectively the Type IIA theory compactified on the orbifold \(S^1/\mathbb{Z}_2\); closed strings that wind around the circle become open strings stretched between two 8-plane boundaries, each of which is associated with an \(SO(16)\) gauge group. Actually, for reasons explained below, the open strings in the Type IA theory do not end on the orientifold 8-plane boundaries as such but rather on 8-branes which happen to coincide with them. Leaving this point aside for the moment, we are now in a position to connect the N=1 superstring theories to M-theory. Since a IIA superstring is an \(S^1\)-wrapped
The supermembrane in a D=11 KK spacetime, the open strings of the 1A theory must be wrapped D=11 supermembranes stretched between two S\(^1\)-wrapped 9-plane boundaries of the D=11 KK spacetime. Let \(L\) be the distance between these boundaries and let \(R\) be the radius of the circular dimension, as measured in the D=11 metric. Then we can identify the Type 1A theory as the \(R \to 0\) limit of M-theory compactified on a cylinder of radius \(R\) and length \(L\). The stretched membrane described above is effectively wrapped on the cylinder and has a closed string boundary on each of the two \((S^1\)-wrapped\) 9-plane boundaries. Clearly, each string boundary must carry an \(SO(16)\) current algebra in order that an \(SO(16)\) gauge theory emerge in the \(R \to 0\) limit.

Suppose that we now increase \(R\) at fixed \(L\). The cylindrical D=11 supermembrane will eventually be transformed from a long tube of length \(L\) and small radius \(R\) to a long strip of length \(R\) and width \(L\). For small \(L\) the two string boundaries of the supermembrane will appear as a single closed string in a D=10 spacetime carrying an \(SO(16) \times SO(16)\) current algebra. This is the M-theory description of the heterotic string with \(E_8 \times E_8\) broken to \(SO(16) \times SO(16)\); taking \(R \to \infty\) we recover the uncompactified D=10 heterotic string with unbroken \(E_8 \times E_8\) gauge group. Its M-theory description is as a supermembrane stretched between two 9-plane boundaries of the 10-dimensional space, separated by a distance \(L\), with an \(E_8\) current algebra on each of its two string boundaries\(^\text{27}\). The string coupling constant turns out to be \(g_s = L^7\), so that the 11th dimension, now taken to be the interval of length \(L\), is indeed invisible in perturbation theory. Of course, the limiting process just described leads to an infinite heterotic string; a finite closed string has the interpretation as a cylindrical D=11 supermembrane that is stretched between the 9-plane boundaries but is not otherwise wrapped.

Let us now return to the \(E_8 \times E_8\) heterotic string theory compactified on a circle of (large) radius \(R\) with \(E_8 \times E_8\) broken to \(SO(16) \times SO(16)\). As we have seen, this has a description as M-theory compactified on a cylinder of radius \(R\) and length \(L = g_s^{2/3}\). If \(R\) is now continued from large to small values, at fixed small \(L\), we may switch to the T-dual description as an \(SO(32)\) heterotic string compactified on a circle of radius \(1/R\), with \(SO(32)\) similarly broken to \(SO(16) \times SO(16)\). The coupling constant of this theory turns out to be \(g_s^{\text{het}} = L/R\), which is still small as long as \(R \gg L\). If we continue to reduce \(R\) we eventually move into the region for which \(R \ll L\). The heterotic string coupling constant is now large but we can switch to the dual Type I description for which the string coupling constant is

\[
g_s^I = 1/g_s^{\text{het}} = \frac{R}{L}. \tag{6}
\]
In the limit in which both $R$ and $L$ go to zero at fixed small $g_s^I$, we recover the uncompactified Type I theory from which we started. Thus, the moduli space of M-theory compactified on a cylinder includes all superstring theories with $N=1$ supersymmetry, as illustrated by the following figure:

The generic vacuum in this moduli space is 11-dimensional but a 10-dimensional theory with gauge group $SO(32)$ is obtained in the limit in which the cylinder shrinks to zero area at fixed shape.

We now return to the issue of open strings in the Type IA theory. T-duality exchanges the Neumann boundary conditions on $Y$ at the ends of an open string to Dirichlet boundary conditions on $\tilde{Y}$, i.e.

$$\partial_t \tilde{Y}(t, 0) = 0 \quad \partial_t \tilde{Y}(t, \pi) = 0.$$  \hfill (7)

It follows that open strings must now start at some fixed value of $\tilde{Y}$ and end at some other, or the same, fixed value, i.e. open strings have their ends tethered to some number of parallel 8-planes. Unlike Neumann boundary conditions, Dirichlet boundary conditions do not prevent the flow of energy and momentum off the ends of the string, so that the 8-planes on which the strings end must be dynamical objects. They are called D-branes or D-p-branes when we wish to specify the spatial dimension of the object. In this case, the open strings end on D-8-branes. The $N=2$ supersymmetry of the IIB theory is broken to $N=1$ in the Type I theory because of the restriction on the IIB fermion fields.
at the ends of open strings. The N=2 supersymmetry of the IIA theory is similarly broken to N=1 in the Type IA theory, but with the crucial difference that since the ends of open strings lie in the D-8-branes it is only on these branes that the N=2 supersymmetry is broken; elsewhere, we have the unbroken N=2 supersymmetry of the IIA theory. Thus, the Type IA theory is effectively equivalent to the IIA theory on an interval with some number of D-branes. In fact, this number is 16, but to get an idea why we shall need to understand some properties of D-branes.

It is typical of soliton solutions of supersymmetric field theories that they carry conserved central charges of the supersymmetry algebra. The supersymmetry algebra then implies a bound on the mass, for fixed charge, that is typically saturated by the soliton solution; the soliton is then said to be ‘BPS-saturated’. BPS-saturated solitons preserve some fraction, often half, of the supersymmetry of the vacuum. A further important, and related, feature is that the force between static BPS-saturated solitons vanishes so that there also exist static, and BPS-saturated, multi-soliton solutions. There is a generalization of all this to Type II supergravity theories in which a ‘multi-soliton’ is replaced by a solution representing a number of parallel infinite planar p-branes. The central charge in the supersymmetry algebra becomes a p-form charge. Some of these p-branes, which all preserve half the D=10 N=2 supersymmetry, carry the charges associated with (p+1)-form gauge potentials coming from the Ramond-Ramond (R ⊗ R) sector of the Type II superstring theory; they are the R ⊗ R branes. There are R ⊗ R p-branes of IIA supergravity for p = 0, 2, 4, 6, 8 and R ⊗ R p-branes of IIB supergravity for p = 1, 3, 5, 7. As for p-branes in general, the long wavelength dynamics is governed by an effective (p+1)-dimensional field theory, but a feature peculiar to R ⊗ R branes is that this worldvolume field theory includes a U(1) gauge potential. This has a simple string theoretic explanation: the R ⊗ R branes of Type II supergravity theories are the field theory realization of the Type II superstring D-branes, and the (electric) U(1) charges on the brane are the ends of open Type II strings. This allows a string theory computation of the bosonic sector of the effective worldvolume field theory, and the full action is then determined by supersymmetry and ‘kappa-symmetry’. The result (upon partial gauge fixing) is a non-linear supersymmetric U(1) gauge theory of Born-Infeld type, except that the fields now depend only on the (p+1) worldvolume coordinates of the brane.

In the case of parallel multi D-branes there can be open strings with one end on one brane and the other end on another brane. Classically, such a string has a minimum energy proportional to the distance between the branes. Supersymmetry ensures that this remains true quantum-mechanically, so addi-
tional massless states can appear only when two or more D-branes coincide. In fact, they do appear, and in just such a way that the $U(1)^n$ gauge group associated with $n$ coincident D-branes is enhanced to $U(n)$. Also, if this $n$ D-brane system approaches an orientifold plane then further massless states (associated with strings stretched between the D-branes and their mirror images) appear in just such a way that $U(n)$ is enhanced to $SO(2n)$. The relevance of these results to the Type IIA theory is due to the fact noted above that this string theory is just the Type IIA theory compactified on an interval of length $L$ with some number of parallel D-8-branes. In the uncompactified theory we could allow any number, $n$, of parallel D-8-branes. The number $n$ can be interpreted as the total $R \otimes R$ charge; in general it will equal the number of branes minus the number of anti-branes (although a configuration with both branes and antibranes could not be static). On a compact ‘transverse’ space, which is one-dimensional in this instance, the total $R \otimes R$ charge must vanish, so $n = 0$ unless there are singular points of the compact space carrying non-zero $R \otimes R$ charge. In compactification on $S^1/Z_2$ it turns out that the orientifold planes each carry $R \otimes R$ charge $-8$, so that precisely 16 D-8-branes are needed to achieve a vanishing total charge. To cancel the charge locally we must put 8 branes on one orientifold plane and 8 on the other. This leads to an $SO(16)$ gauge group associated with each orientifold plane; this is the Type IA theory discussed above, i.e. the T-dual of the Type I theory with $SO(32)$ broken to $SO(16) \times SO(16)$.

It would be possible to arrange for the total 8-brane charge to vanish without it vanishing locally by simply moving the 8-branes apart and/or away from the orientifold planes. The generic configuration of this type would be one in which the $SO(16) \times SO(16)$ symmetry is broken to $U(1)^{16}$; this is the T-dual of the generic $S^1$ compactified Type I theory in which $SO(32)$ is broken to its maximal abelian subgroup by an adjoint Higgs field. One could also move all 16 8-branes to one end in which case the gauge group would be enhanced to $SO(32)$; this is the T-dual of the $S^1$ compactified Type I theory with unbroken gauge group. In view of these possibilities, an obvious question is why, in our earlier discussion, we needed to select the particular Type IA theory with $SO(16) \times SO(16)$ gauge group. One might suppose that some other configuration would be related to a version of M-theory in which 9-branes on the D=10 boundaries of $S^1/Z_2$ compactified M-theory are moved away from the boundary. However, a special feature of sources of 8-brane charge, i.e. 8-branes or orientifold 8-planes, is that the dilaton grows with distance from the source in such a way that the effective string coupling diverges at finite distance $L$. At fixed relative positions of the 8-branes the absolute distances between them will grow with the distance $L$ between the orientifold planes so that the
8-brane configuration is effectively constrained, as $L \to \infty$, to approach the one in which the 8-brane charge is canceled locally.

We have now seen how all superstring theories are unified by a single theory, M-theory, whose vacua are generically 11-dimensional. The moduli spaces of vacua of the superstring theories with $N=1$ and $N=2$ supersymmetry are connected by the special $D=11$ Lorentz invariant vacuum of uncompactified M-theory. The connections between this $D=11$ uncompactified M-theory and the $D=10$ superstring theories are illustrated by the following figure (in which the two $N=1$ string theories with $SO(32)$ gauge group are considered as a single non-perturbative $SO(32)$ string theory):

This completes our overview of how superstring theories and $D=11$ supergravity are unified by M-theory, and why ‘branes’ are crucial to this unification. In the next lecture we backtrack to explain how considerations of $D=10$ and $D=11$ supersymmetry algebras both provide an explanation of why there are five $D=10$ superstring theories and suggest a role for a $D=11$ supermembrane. In the third lecture we shall see how the new scenario for superunification via M-theory is supported by supergravity considerations. In the final lecture we shall see how various features of D-branes, and other superstring p-branes, are consequences of properties of the ‘M-branes’ of M-theory.

2 Superstrings and the supermembrane

It will be helpful to begin by reviewing how the various $D=10$ superstring theories arise. Our starting point will be the $N=1$ and $N=2$ superspaces, which can be identified with the supertranslation groups. The supertranslation algebras
are spanned by the 10-momentum $P_\mu$ and one or more Lorentz spinor charges. The minimal spinor in $D=10$ is both Majorana and chiral. A Majorana spinor $Q$ is one for which $\bar{Q} = Q^T C$, where the bar indicates the Dirac conjugate and $C$ is the antisymmetric real charge conjugation matrix. There exists a representation of the Dirac algebra, the Majorana representation, in which the Dirac matrices are real; in this representation $C = \Gamma^0$ and a Majorana spinor is a real 32-component spinor. A chiral spinor $Q_\pm$ is one for which

$$\Gamma_{11} Q_\pm = \pm Q_\pm$$

(8)

where $\Gamma_{11}$ is the product of all ten Dirac matrices. It satisfies $(\Gamma_{11})^2 = 1$ and is clearly real in the Majorana representation, so chirality is compatible with reality in $D=10$. A chiral Majorana spinor has 16 independent real components.

The $N=1$ supertranslation algebra is

$$\{Q^+_\alpha, Q^+_\beta\} = (CT^\mu P^+)_{\alpha\beta} P_\mu$$

(9)

where $Q^+$ is a chiral Majorana spinor and $P^+$ projects onto the positive chirality subspace; the choice of positive or negative chirality is of course purely convention. An element of the supertranslation group is obtained by exponentiation of the algebra element

$$X^\mu P_\mu + \theta_+ Q^+$$

(10)

where $X^\mu$ are the $D=10$ spacetime coordinates and $\theta_+$ is an anti-chiral and anticommuting Majorana spinor coordinate. There are two $N=2$ supertranslation algebras, according to whether the two supersymmetry charges have the same or opposite chirality. If they have opposite chirality we can assemble them into a single non-chiral Majorana charge $Q$. This leads to the IIA algebra

$$\{Q_\alpha, Q_\beta\} = (CT^\mu P^+)_{\alpha\beta} P_\mu .$$

(11)

If the two supersymmetry charges have the same chirality we can assemble them into the $SO(2)$ doublet $Q^+_I$, $(I = 1, 2)$ to arrive at the IIB algebra

$$\{Q^+_I, Q^+_J\} = \delta^{IJ} (CT^\mu P^+)_{\alpha\beta} P_\mu .$$

(12)

With these supertranslation algebras in hand we can now turn to the construction of superstring worldsheet actions in the Lorentz-covariant Green-Schwarz (GS) formulation in which the fields are maps from the worldsheet to
superspace. We first introduce supertranslation invariant superspace 1-forms on the three possible superspaces:

\[
\Pi^\mu = \begin{cases} 
  dX_\mu - i \bar{\theta} \Gamma_\mu d\theta & \text{(heterotic)} \\
  dX_\mu - i \bar{\theta} \Gamma_\mu d\theta & \text{(IIA)} \\
  dX_\mu - i \delta_{\mu j} \bar{\theta}_j \Gamma_\mu d\theta^j & \text{(IIB)}
\end{cases}
\]  

(13)

As indicated, the N=1 superspace case is relevant to the heterotic strings, the Type I superstring being derived from the IIB superstring. Let \( \xi^i = (t, \sigma) \) be the worldsheet coordinates and let \( \Pi_\mu^i \) denote the 10-vector components of the induced worldsheet 1-forms. For example, in the heterotic case we have

\[
\Pi_\mu^i = \partial_i X^\mu - i \bar{\theta}_+ \Gamma_\mu \partial_i \theta_+
\]

(14)

where \( \{ X^\mu(\xi), \theta_+^\alpha(\xi) \} \) are the worldsheet fields. Setting the string tension to unity, for convenience, we can now write down the supersymmetrized Nambu-Goto part of the superstring action,

\[
S_{NG} = - \int d^2 \xi \sqrt{- \det(\Pi_i \cdot \Pi_j)} .
\]

(15)

For reasons reviewed elsewhere, this is not the complete action; it must be supplemented by a ‘Wess-Zumino term’. To construct it we must search for super-Poincaré invariant closed forms on superspace. This search reveals the following possibilities. Firstly, we have some 3-forms

\[
h_{(3)} = \begin{cases} 
  \Pi^\mu \bar{\theta}_+ \Gamma_\mu d\theta_+ & \text{(heterotic)} \\
  \Pi^\mu d\bar{\theta}_+ \Gamma_\mu d\theta_+ & \text{(IIA)} \\
  \tilde{S}_{IJ} \Pi^\mu d\bar{\theta}_+ \Gamma_\mu d\theta^J_+ & \text{(IIB)}
\end{cases}
\]

(16)

where \( \Gamma_{11} \) is the product of the ten Dirac matrices \( \Gamma^\mu \), and \( \tilde{S}_{IJ} \) are the entries of the 2 \times 2 matrix

\[
\tilde{S} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .
\]

(17)

Secondly, we have the heterotic 7-form

\[
h_{(7)} = \Pi^{\mu_1} \cdots \Pi^{\mu_p} d\bar{\theta}_+ \Gamma_{\mu_1 \cdots \mu_p} d\theta_+ .
\]

(18)

The crucial fact about these forms is that they are closed (by virtue of Dirac matrix identities valid in D=10) so locally we can write \( h = db \). In fact, these forms are exact, but this is a special feature of the chosen background. Given a \((p + 2)\)-form \( h_{(p+2)} \) we can find a super-Poincaré invariant Wess-Zumino (WZ)
type action for a p-dimensional object, i.e. a 'p-brane', by integrating the 
\((p+1)\)-form \(b_{(p+1)}\) over the \((p+1)\)-dimensional worldvolume. Thus \(h_{(7)} = db_{(6)}\) could only be relevant to a 5-brane. In fact, there is a heterotic 5-brane but
our present concern is with the 3-forms \((16)\) relevant to 1-branes, i.e. strings.

Let \(b_{ij}\) denote the components of the worldsheet 2-form induced from \(b\). We can now write down the Wess-Zumino part of the GS superstring action:

\[
S_{\text{WZ}} = \frac{1}{2} \int d^2 \xi \varepsilon^{ij} b_{ij}.
\] (19)

The combined action

\[
S = S_{\text{NG}} + S_{\text{WZ}}
\] (20)

has a fermionic gauge invariance, usually called '\(\kappa\)-symmetry', which allows half the components of \(\theta\) to be gauged away. On choosing a physical gauge one finds that half of the original spacetime symmetries are linearly realized worldsheet supersymmetries; without the \(\kappa\)-symmetry, they would all be non-linearly realized. Thus, \(\kappa\)-symmetry is essential for equivalence with the worldsheet supersymmetric NSR formulation of superstring theory. This is one reason why the WZ term is essential to the construction of a physically acceptable GS superstring theory; and it is this that restricts the construction to \(N=1\) and \(N=2\) superspaces. While it is possible to introduce supertranslation algebras for \(N > 2\), a super-Poincaré invariant closed 3-form \(h\) exists only for \(N=1\) and \(N=2\).

The Type II superstrings are closed strings whose covariant GS action is just (20), and for which the worldsheet fields are all periodic (recall that the fermions are actually worldsheet scalars in this formulation; they become worldsheet spinors only after gauge fixing the \(\kappa\)-symmetry). The heterotic strings are closed strings based on the action (20) for \(N=1\) superspace, but conformal invariance of the first quantized string requires the addition of a 'heterotic' action \(S_{\text{het}}\) involving 32 worldsheet chiral fermions \(\zeta^A\), \((A = 1 \ldots 32)\). If these are chosen to transform as half-densities then

\[
S_{\text{het}} = \frac{1}{2} \int d^2 \xi \zeta^A \partial_+ \zeta^B \delta_{AB}
\] (21)

where \(\partial_+\) is a chiral worldsheet derivative. Thus

\[
S = S_{\text{NG}} + S_{\text{WZ}} + S_{\text{het}}
\] (22)

is the GS action for the heterotic strings. The worldsheet fermions \(\zeta^A\) may be periodic or anti-periodic so there are, a priori, many possible sectors in the full Hilbert space of the first-quantized string. However, quantum consistency
requires a truncation to sectors with common boundary conditions on groups of eight fermions, and further considerations along these lines leads to the $SO(32)$ and $E_8 \times E_8$ heterotic strings as the only ones with a spacetime supersymmetric spectrum. This accounts for the Type II and heterotic string theories.

We have been considering these superstrings in a particular (Minkowski) background. More generally, any solution of the associated effective supergravity theory provides a possible background, at least to leading order in an expansion in powers of the inverse string tension $2\pi\alpha'$. We shall discuss these supergravity theories in the following lecture. For the present it will be sufficient to note that they all have in common the fields of $N=1$ supergravity, for which the bosonic fields are the metric $g_{\mu\nu}$, an antisymmetric tensor gauge field $B_{\mu\nu}$ and a scalar ‘dilaton’ field $\phi$. We can regard the background considered so far as one for which $g$ is the Minkowski metric, $B$ vanishes and $\phi$ is constant. Omitting worldsheet fermions, the worldsheet action for a general background involving these fields is

$$S = -\frac{1}{4\pi\alpha'} \int d^2\xi \left\{ \sqrt{-\gamma} \gamma^{ij} g_{ij} + \varepsilon^{ij} B_{ij} + \alpha' \phi \sqrt{-\gamma} R^{(2)} \right\},$$

where we use here the ‘sigma model’ formulation of the action in which $\gamma_{ij}$ is an independent (but auxiliary) worldsheet metric with scalar curvature $R^{(2)}$. The sigma model ‘coupling constants’ $g_{ij}$, $B_{ij}$ and $\phi$ are the pullbacks to the worldsheet of the spacetime fields. In the full action including worldsheet fermions the 2-form $B$ combines with the superspace 2-form $b$; both are part of the complete WZ term.

For the heterotic string, the general bosonic background will also include a background $SO(32)$ or $E_8 \times E_8$ gauge potential. In the $SO(32)$ case this background is easily accommodated by modifying $S_{het}$ in (22) such that $\partial_+$ becomes a covariant derivative constructed from the pullback of the spacetime gauge potential. In the $E_8 \times E_8$ case only an $SO(16) \times SO(16)$ subgroup can be dealt with this way. For the Type II strings there is a more serious omission, owing to the fact that spacetime bosons in the string spectrum arise from two distinct sectors, the Neveu-Schwarz/Neveu-Schwarz ($NS \otimes NS$) sector and the Ramond/Ramond ($R \otimes R$) sector. For present purposes we may define these sectors according to whether the spacetime boson couples to a boson bilinear ($NS \otimes NS$) or to a fermion bilinear ($R \otimes R$). The $R \otimes R$ fields are abelian $(p+1)$-form potentials for various values of $p$ (as explained in the next lecture) which couple to the worldsheet only through their $(p+2)$-form field strengths (this is the only possibility compatible with gauge invariance). This has the important consequence that $R \otimes R$ charges are not carried by the Type II strings themselves. In contrast, since the 2-form potential $B$ couples ‘minimally’ to
the heterotic and Type II strings, they carry the charge

$$Q_1 = \int_{S^7} \star H ,$$

(24)

where (locally) $H = dB$ and $\star$ is the Hodge dual of spacetime. The integral is over a 7-sphere surrounding the string, as shown schematically below:

Because the heterotic and Type II strings are charged, in the sense that $Q_1 \neq 0$, they cannot break; if this were to happen the 7-sphere could be slid off the string and contracted to a point, which would imply $Q_1 = 0$. Actually, this argument needs some qualification for Type II strings since these can have endpoints on the D-branes which we encountered earlier.

We now turn to the remaining Type I superstring theory. Note that $\tilde{S}$ of (17) is not an $SO(2)$ invariant tensor, so the $SO(2)$ invariance of the IIB supertranslation algebra is broken by the IIB superstring action. On the other hand, the minus sign in the definition of $\tilde{S}$ means that the IIB action is invariant under a worldsheet parity operation $\Omega$, induced by $\sigma \rightarrow -\sigma$, where the fields $X^\mu$ are assigned positive parity (i.e. they are true scalars) and $(\theta_1^1 \pm \theta_2^1)$ is assigned parity $\pm 1$. That is, suppressing Lorentz spinor and vector indices,

$$\Omega[X](t, \sigma) = X(t, -\sigma)$$
$$\Omega[(\theta_1^1 \pm \theta_2^1)](t, \sigma) = \pm(\theta_1^1 \pm \theta_2^1)(t, -\sigma).$$

(25)

As mentioned earlier, the existence of this $Z_2$ symmetry means that we can find another superstring theory, the Type I theory, as an orientifold of the IIB theory. The states of the closed string sector of the Type I theory are
found by projection onto the even worldsheet parity subspace of the Type IIB Fock space. The $NS \otimes NS$ two-form $B$ is projected out since the worldsheet boson bilinear to which it couples, $\varepsilon^{ij}\partial_iX^\nu\partial_jX^\nu$, has odd parity. There are two parity-even fermion bilinears, both involving the antisymmetric product of three Dirac matrices, but (as explained in the next lecture) only one of them $(\bar{\theta}^1_+\Gamma^{\mu\nu}\theta^2_+)$ couples to a 3-form field strength from the $R \otimes R$ sector of the string theory. This is the only $R \otimes R$ field to survive the projection. Thus, the massless fields of the closed Type I superstring are exactly those of $N=1$ supergravity, but with the 2-form gauge potential coming from the $R \otimes R$ sector.

An immediate consequence of this difference is that the Type I string does not carry the charge $Q_1$ defined above and hence can break. In fact, the closed string sector is anomalous by itself, but we can find an anomaly free theory by the addition of an open string sector with $SO(32)$ Chan-Paton factors. The states of the open string sector of the Type I theory are found by quantization of the IIB worldsheet fields $Z^M(t, \sigma)$ subject to the constraint $Z = \Omega[Z]$. From (25) we see that this constraint implies that

$$X'(t, \sigma) = -X'(t, -\sigma)$$

$$[\theta^1_+-\theta^2_+](t, \sigma) = -[\theta^1_+ - \theta^2_+](t, -\sigma)$$

where the prime indicates differentiation with respect to $\sigma$. This implies, in turn, that $X' = 0$ and $\theta^1_+ = \theta^2_+$ at $\sigma = 0, \pi$, which are the standard boundary conditions at the ends of an open superstring.

It is now time to address the discrepancy between the symmetries of the IIB superstring action and those of the IIB supertranslation algebra (12). Recall that the latter has an $SO(2)$ symmetry not shared by the former. In fact, the discrepancy is illusory because the algebra (12) is that relevant to the Minkowski vacuum. The algebra of supersymmetry charges deduced as Noether charges of the IIB superstring action contains an additional term arising from the fact that the WZ Lagrangian is not invariant but changes by a total derivative. The algebra found this way is

$$\{Q^I_\alpha, Q^J_\beta\} = \delta^{IJ}(CT^\mu P^+)_{\alpha\beta}P_\mu + S_{IJ}(CT^\mu P^+)_{\alpha\beta}Z^\mu$$

where $Z^\mu$ is the 1-form charge

$$Z^\mu = \oint dX^\mu,$$

with the integral being taken over the image of the closed string in spacetime. This charge is non-zero for strings that wind around a homology 1-cycle in space. Effectively, this means that the charge $Z$ is relevant only for the $S^1$-compactified IIB superstring, but one can then take the limit of infinite radius.
to deduce that $Z$ is a 1-form charge carried by an infinite string in $D=10$ Minkowski spacetime. Thus, the supersymmetry algebra in the presence of an infinite IIB superstring has the same symmetries as the IIB superstring itself.

Clearly a similar 1-form charge must appear in all the superstring theories for which the worldsheet action contains a WZ term, i.e. all but the Type I superstring. For example, for the IIA superstring we find that the algebra is modified to

$$\{Q_\alpha, Q_\beta\} = (CT^\mu)_{\alpha\beta} P_\mu + (CT^{\mu\Gamma_{11}})_{\alpha\beta} Z_\mu .$$

(29)

It will prove instructive to rewrite the Type II algebras in terms of the spinor charge $Q^+$ of the $N=1$ algebra and a second spinor charge $S^\pm$, where $S^+$ is the second charge of the IIB algebra and $S^-$ of the IIA algebra. In either case the supertranslation algebra is then

$$\{Q^\alpha_+, Q^\beta_+\} = (CT^\mu P^+)_{\alpha\beta} (P + Z)_\mu$$

$$\{S^\alpha_+, S^\beta_+\} = (CT^\mu P^\pm)_{\alpha\beta} (P - Z)_\mu$$

(30)

where $P^\pm$ are the projection operators onto the spinor subspaces of positive or negative chirality. Note that the $N=1$ subalgebra (of $Q^+$) is invariant under the interchange

$$P \leftrightarrow Z .$$

(31)

Of course, this symmetry is a classical one; in the quantum theory the spectrum of $P$ and $Z$ as operators will generally be different, in which case (31) would make no sense. For example the momentum in an uncompactified direction can take any value while the corresponding winding number has only one allowed value, zero. Suppose, however, that the $X^9$ direction is a circle of radius $R$. Then the spectrum of $P_9$ is isomorphic to that of the winding number operator $Z^9$; the isomorphism involves the exchange of $R$ with $1/R$ since the eigenvalues of $P_9$ are multiples of a unit proportional to $1/R$ while those of $Z^9$ are multiples of a unit proportional to $R$. In fact, it is known that a heterotic string theory on a circle of radius $R$ is equivalent to the same theory on a circle of radius $\alpha' / R$. This $Z_2$ symmetry of the heterotic string is called T-duality; it is actually a subgroup of a much larger $SO(1,17;Z)$ discrete symmetry group of the generic $S^1$-compactified heterotic string theory which is also called the T-duality group. The invariance of the supersymmetry algebra under the interchange $P \leftrightarrow Z$ is clearly necessary for this to be possible.

If we had taken the $N=1$ supersymmetry algebra of the $S^\pm$ charges the conclusion would have been the same, with the exchange symmetry being $P \leftrightarrow -Z$, but the combined $N=2$ algebra has no analogous symmetry. It follows that neither the IIA nor the IIB superstring, compactified on a circle of radius $R$, is mapped to itself under the T-duality transformation $R \rightarrow 1/R$. However, if
we replace $S^\pm$ by $\Gamma^9 S^\mp$ at the same time that we make the exchange $P \leftrightarrow Z$, then we recover the IIA algebra if we started with the IIB one, and vice-versa (note that multiplication by $\Gamma^9$ maps a spinor of one D=10 chirality to one of the other chirality). In other words, the combined transformation

$$P^a \leftrightarrow Z^a \quad S^\pm \leftrightarrow \Gamma^9 S^\mp$$

(32)

maps the IIA algebra into the IIB algebra, and vice versa. As before this transformation makes sense in the quantum theory only if $X^9$ is the coordinate of a circle, and then it must be accompanied by $R \rightarrow 1/R$. In fact, it is known from perturbative string theory that the IIA and IIB theories are interchanged by the T-duality transformation $R \rightarrow 1/R$.

Finally, we turn to the D=11 supermembrane. A convenient starting point is again the supertranslation algebra. In D=11 the minimal algebra is spanned by the 11-momentum $P_M$ and a 32-component Majorana spinor of the D=11 Lorentz group $Q_{\alpha}$ obeying the anticommutation relation

$$\{Q_\alpha, Q_\beta\} = (\mathcal{C} \Gamma^M)_{\alpha\beta} P_M .$$

(33)

As before, we can introduce the supertranslation invariant 11-vector-valued 1-form on superspace

$$\Pi^M = dX^M - i \bar{\theta} \Gamma^M d\theta .$$

(34)

We now search for super-Poincaré invariant closed forms on superspace. The only possibility is the 4-form

$$h^{(4)} = \Pi^M \Pi^N d\bar{\theta} \Gamma_{MN} d\theta ,$$

(35)

which leads us to expect a membrane rather than a string. The word ‘membrane’ has been used in the past to refer both to a generic p-brane and to a domain wall in D spacetime dimensions, i.e. a $(D-2)$-brane. Here we use the word ‘membrane’ to mean exclusively a 2-brane.

The Nambu-Goto string action has an obvious p-brane generalization. The $p=2$, i.e. membrane, case was first considered by Dirac, so this type of action is sometimes called the Dirac action; its supersymmetric version (for unit surface tension) is

$$S_D = - \int d^3 \xi \sqrt{- \det(\Pi_i \cdot \Pi_j) .}$$

(36)

It turns out that there is a $\kappa$-invariant supermembrane action of the form

$$S = S_D + S_{WZ}$$

(37)
where $S_{WZ}$ is constructed from the 3-form $b_{(3)}$ for which $h_{(4)} = db_{(3)}$. As for the heterotic and Type II superstrings, the presence of the WZ term implies a modification of the supersymmetry algebra. This time we find that

$$\{Q_\alpha, Q_\beta\} = (CT^M)_{\alpha\beta} P_M + (CT_{MN})_{\alpha\beta} Z^{MN}_{(2)} \quad (38)$$

where $Z_{(2)}$ is a 2-form charge. This D=11 supertranslation algebra can be rewritten as a D=10 algebra by the simple expedient of splitting all charges into their representations under the D=10 subgroup of the D=11 Lorentz group. The D=11 supersymmetry charge becomes a D=10 Majorana spinor charge while

$$P_M = (P_\mu, P_{11})$$
$$Z^{MN}_{(2)} = (Z_{(2)}^\mu Z_{(2)}^{\nu 11}) = Z^\nu \quad (39)$$

In this new notation the algebra (38) reads

$$\{Q_\alpha, Q_\beta\} = (CT^\nu)_{\alpha\beta} P_\mu + (CT^{\mu} \Gamma_{11})_{\alpha\beta} Z_\mu + (CT^{11})_{\alpha\beta} P_{11} + (CT_{\mu\nu})_{\alpha\beta} Z_{(2)}^{\mu\nu} \quad (40)$$

Note the similarity to (38), but in addition to the 1-form charge $Z$ associated to the IIA string we also find a 0-form charge $P_{11}$ and a 2-form charge $Z_{(2)}$. This suggests not only that the IIA superstring is really a D=11 supermembrane but also that the non-perturbative D=10 theory is a theory not just of strings but also of 0-branes and 2-branes. This line of inquiry will be followed up in the last lecture.

We have seen in this lecture that the construction of string and membrane actions with manifest spacetime supersymmetry requires the existence of a closed 3-form or 4-form on the relevant superspace, and that this requirement severely restricts the possibilities. In fact, one can determine, for any spacetime dimension D and each N, the values of $p$ for which there exists a closed superspace $(p+2)$-form (of the required dimension). The resulting table of possibilities (the ‘old branescan’) includes those D=10 and D=11 cases discussed above, but it is now time to admit that the full story is rather more complicated. It turns out that a closed $(p+2)$-form on superspace is necessary only if (after gauge fixing the $\kappa$-symmetry) the worldvolume fields consist exclusively of scalars and spinors. While this is the case for p-brane solutions of flat space field theories, and for some p-brane solutions of supergravity theories, it is not true in general, as was originally discovered by an analysis of the small fluctuations about 5-brane solutions of Type II supergravity theories. Examples in D=10 are provided by D-branes, whose worldvolume field content is
that of the D=10 vector multiplet dimensionally reduced to (p+1) dimensions. Another example is the fivebrane solution of D=11 supergravity, for which the field content is that of a 6-dimensional antisymmetric tensor multiplet. These examples can be viewed as having a common origin since the D=11 fivebrane is a type of M-theory D-brane in the sense that it is an object on which a membrane can have a boundary.

These additional possibilities for p-branes in D=10 and D=11 supergravity theories are also associated with p-form extensions of the supersymmetry algebra. For example, the D=11 superfivebrane is associated with a 5-form extension of the D=11 supersymmetry algebra. Thus, the full D=11 supertranslation algebra is

\[ \{ Q_\alpha, Q_\beta \} = (CT^M)_{\alpha\beta} P_M + (CT_{MN})_{\alpha\beta} Z^{MN} + (CT_{MNPQR})_{\alpha\beta} Z^{MNPQR}. \]

Note that the total number of algebraically independent charges that could appear on the right hand side is 528. The number actually appearing is

\[ 11 + 55 + 462 = 528 \]

so the algebra is ‘maximally extended’. The three types of charge appearing on the right hand side are those associated with the supergraviton, the supermembrane and the superfivebrane, which are the three basic ingredients of M-theory. It is therefore natural to regard as the ‘M-theory superalgebra’.

### 3 Effective supergravities and strong coupling limits

In this lecture we shall see how consideration of the D=10 and D=11 supersymmetry algebras, and the associated supergravity theories, essentially determines the strong coupling limits of all uncompactified superstring theories. Our starting point will be N=1 or N=2 D=10 superfields, which are superfunctions of definite Grassman parity on the corresponding superspaces. The gauge-invariant fields of D=10 supergravity theories are components of a single real scalar superfield, subject to certain constraints. This description becomes quite involved for the full non-linear theories but is simple at the linearized level and a linearized analysis is sufficient to reveal the field content. The restriction to N=1 and N=2 arises in this context because the superfield expansion would otherwise contain high spin gauge fields with field equations that are consistent only in flat space.

Consider first a real scalar superfield \( \phi(X, \theta^+) \) on N=1 superspace. This has the \( \theta \)-expansion

\[ \phi(X, \theta^+) = \phi + i\tilde{\theta}^+ \lambda^+ + i(\tilde{\theta}^+ \Gamma^{\mu\rho} \theta^+) H_{\mu\rho} + \ldots \]
The first component is a scalar, the ‘dilaton’ $\phi$, followed by the dilatino $\lambda^+$. Since $\theta_+$ has 16 components, the total number of components at the $\theta^2$ level is 120, which is precisely the number of components of the 3-form field $H$. The constraints on the superfield $\phi$ therefore occur at the $\theta^3$ level, where we find the gravitino field strength. The Riemann tensor of the D=10 metric appears at the $\theta^4$ level, and thereafter all higher dimension components are just derivatives of the lower dimension ones. In addition, the constraints imply the Bianchi identity $dH = 0$, allowing us to write $H = dB$, where $B$ is a 2-form gauge potential. In fact, the constraints also imply the field equations of $B$ and the other fields in the graviton supermultiplet, of which the bosonic fields are $(\phi, g_{\mu\nu}, B_{\mu\nu})$.

For $N=1$ we also have the possibility of a YM supermultiplet. The YM field strength 2-form $F$ is contained in a Lie-algebra valued anti-chiral spinor superfield $\chi_+(X, \theta_+)$ with the $\theta$-expansion

\[
\chi_+(X, \theta_+) = \chi_+ + \Gamma^{\mu\nu} \theta_+ F_{\mu\nu} + \ldots.
\]

The constraints on this superfield, which occur at the $\theta^1$ level, imply that there are no further independent components and that $F$ satisfies both the YM Bianchi identity and field equation. When the YM multiplet is coupled to the graviton supermultiplet the superfield constraints on both are modified in such a way that, inter alia, the Bianchi identity $dH = 0$ is replaced by an ‘anomalous’ one, equivalent to a modification of $H$ to include a YM Chern-Simons (CS) term $^{66}$.

Classically, $N=1$ supergravity can be coupled to a YM supermultiplet for any choice of the gauge group $G$, but in the quantum theory cancellation of gravitational anomalies requires $G$ to have dimension 496. If the group is non-abelian then there are additional gauge and mixed anomalies that can be cancelled by the GS mechanism only for $G = SO(32)$ or $G = E_8 \times E_8$, and then only by the inclusion of additional Lorentz CS terms. Supersymmetry then requires the inclusion of an infinite number of further higher-order interactions, and the full supersymmetric anomaly-free theory is not known. This is a complicating feature of $N=1$ that is fortunately absent for $N=2$.

We have now seen that the bosonic fields of the combined $N=1$ supergravity/YM theory are

$$(\phi, g_{\mu\nu}, b_{\mu\nu}; A_\mu),$$

where $A$ is a YM 1-form taking values in the Lie algebra of $SO(32)$ or $E_8 \times E_8$. Omitting fermions (and neglecting higher-derivative terms in the $\alpha'$ expansion) the action, for a particular choice of units, is

\[
S_{het} = \int d^{10}x \sqrt{-g} e^{-2\phi} \left[ R + 4|d\phi|^2 - \frac{1}{3}|H|^2 - \alpha' tr |F|^2 \right].
\]
Notice that the constant vacuum value of $\phi$ is not fixed by the field equations. In fact, the action is invariant under the dilation

$$
\phi \to \phi + 4\lambda \\
g_{\mu\nu} \to e^{2\lambda} g_{\mu\nu} \\
b_{\mu\nu} \to e^{2\lambda} b_{\mu\nu} \\
A_\mu \to e^{\lambda} A_\mu ,
$$

and this invariance extends to the full action. Since the vacuum, in which $\phi$ takes a particular value, is not invariant, this symmetry is a spontaneously broken one for which the dilaton is the Nambu-Goldstone boson, hence its name.

We have chosen to write the action (46) in a way that is appropriate to the heterotic strings. Observe that the scalar curvature and all other kinetic terms, appear multiplied by the factor $e^{-2\phi}$. It will be important for what follows to understand why this factor is there. Observe that for $\phi$ equal to its vacuum value $\langle \phi \rangle$ the worldsheet action (23) includes the term $67 - \langle \phi \rangle \chi$, where

$$
\chi = \frac{1}{4\pi} \int d^2 \xi \sqrt{-\gamma} R^{(2)}
$$

is the worldsheet Euler number. For a closed Riemann surface of genus $g$ we have $\chi = 2 - 2g$, so the Euclidean path-integrand $e^{-S}$ acquires a factor of $g_s^{(2g-2)}$, where we have set

$$
g_s = e^{\langle \phi \rangle} .
$$

Since the genus $g$ orders the perturbation series of closed string theories we can identify $g_s$ as the closed string coupling constant. In particular, classical closed string theory is associated with the Riemann sphere for which $g = 0$. This leads to a factor of $g_s^{-2}$ in the closed string effective action, which is consistent with the $\phi$-dependence of the spacetime action (46).

Next, we turn to IIA supergravity. The gauge-invariant fields are again contained in a single real scalar IIA superfield $\phi(X, \theta)$. Its $\theta$-expansion is

$$
\phi(X, \theta) = \phi + i \bar{\theta} \lambda + i \bar{\theta} \theta M + i \bar{\theta} \Gamma^{\mu\nu} \Gamma_{11} \theta K_{\mu\nu} + i \bar{\theta} \Gamma^{\mu\nu\rho} \Gamma_{11} \theta H_{\mu\nu\rho}
$$

$$
+ i \bar{\theta} \Gamma^{\mu\nu\rho\sigma} \theta G_{\mu\nu\rho\sigma} + \ldots
$$

Note that there are a total of 496 possible components at the $\theta^2$ level. In fact, only 376 appear, so there is a constraint that sets to zero a 3-form field at the $\theta^2$ level in the $\theta$-expansion. Apart from the gravitino field-strength and the Riemann tensor there are again no further independent components. The constraints also imply Bianchi identities for the field-strengths at the $\theta^2$ level.
In particular, $dM = 0$, so the scalar $M$ is just a constant. This is actually the cosmological constant of the ‘massive’ IIA supergravity. We shall set $M = 0$ in these lectures. The other Bianchi identities imply that $K = dC$ for 1-form potential $C$ (of KK origin in D=11), $H = dB$ for 2-form potential $B$ and (at the linearized level) $G = dA$ for 3-form potential $A$. Thus, the bosonic field content of IIA supergravity consists of the fields of $N=1$ supergravity,

$$(\phi, g_{\mu\nu}, B_{\mu\nu}) ,$$

which are also those of the $NS \otimes NS$ sector of the IIA string theory, together with the gauge potentials

$$(C_\mu, A_{\mu\nu\rho}) ,$$

which are the fields from the $R \otimes R$ sector of the IIA string theory. As in the $N=1$ case, the superfield constraints actually imply the full field equations, but the bosonic field equations can also be derived from the component action,

$$S_{IIA} = \int d^{10}x \left\{ \sqrt{-g} e^{-2\phi} \left[ R + 4|d\phi|^2 - \frac{1}{3}|H|^2 \right] - \sqrt{-g} \left[ |K|^2 + \frac{1}{12}|G|^2 \right] \right\} + \frac{1}{144} \int G \wedge G \wedge B ,$$

where $G = dA + 12B \wedge K$ is the non-linear version of the 4-form field strength.

Observe that the terms in (53) involving the $R \otimes R$ fields are not multiplied by a factor of $e^{-2\phi}$. Since this property of $R \otimes R$ fields has important consequences it deserves comment. In the worldsheet supersymmetric NSR formulation of Type II string theories the $R \otimes R$ fields do not couple to the string through local worldsheet interactions but rather through bilinears of spin fields. These create cuts on the Riemann surface which invalidate the conclusion we arrived at previously that tree level closed string interactions are proportional to $g_s^{-2}$ (in the GS formulation the RR fields do couple through local interactions but the $\kappa$-symmetry makes the quantization of the GS superstring problematic). Supersymmetry can be used to show that the $R \otimes R$ fields must appear in the action as above, but it is also possible to show this directly from string theory.

We turn now to IIB supergravity. The field-strength superfield of linearized IIB supergravity is again a real constrained scalar superfield $\phi(X, \theta^I_\perp)$ with the $\theta$-expansion

$$\phi(X, \theta^I_\perp) = \phi + i \bar{\theta}^l_+ \lambda^l_+ + \epsilon^{IJ} i \bar{\theta}^l_+ \Gamma_{\mu}^I \theta^J_+ L_\mu + i \bar{\theta}^l_+ \Gamma_{\mu \nu \rho \sigma} \theta^J_+ \bar{H}^{IJ}_{\mu \nu \rho \sigma} + \epsilon^{IJ} i \bar{\theta}^l_+ \Gamma_{\mu \nu \rho \sigma \lambda} \theta^J_+ M^I_{\mu \nu \rho \sigma} + \ldots$$

(54)
The 5-form field $M^+$ is self-dual; this is not a constraint on the superfield but rather an automatic consequence of the chirality of the two $\theta$ coordinates. The tilde on $\tilde{H}$ indicates that this $SO(2)$ tensor is tracefree, i.e. $\delta_{IJ}\tilde{H}^{IJ} = 0$, or

$$
\tilde{H}^{IJ} = \begin{pmatrix} H & H' \\ H' & -H \end{pmatrix}.
$$

(55)

This is a constraint because, as in the IIA case, it means that of the possible 496 components that could appear at the $\theta^2$ level only 376 actually do appear, viz. $L, H, H', M^+$. The superfield constraints imply various Bianchi identities for these fields. In particular, $dL = 0$, which implies that $L = d\ell$ for pseudoscalar $\ell$, and (at the linearized level) $dM^+ = 0$ which, because of the self-duality, implies not only that $M^+ = dC^+$ but also the linearized field equation for the 4-form $C^+$. The other Bianchi identities imply that $H = dB$ and $H' = dB'$ for two 2-form potentials $B$ and $B'$. Thus, the $NS \otimes NS$ fields of the IIB theory are the same as those of the IIA superstring while the $R \otimes R$ fields are

$$
(\ell, B'_{\mu\nu}, C^+_{\mu\nu\rho\sigma}),
$$

(56)

where the superfix on $C^+$ is to remind us that its 5-form field strength is self-dual. Note that we are regarding the pseudoscalar $\ell$ as a gauge field here because it appears only through its field strength $L$.

The self-duality of $M^+$ complicates the construction of an action for IIB supergravity. There are some ways around this problem but they are rather unwieldy so we shall adopt the simpler procedure in which the self-duality condition is temporarily dropped, thus allowing us to use the standard Lagrangian for $C^+$. The self-duality condition is then simply added to the field equations that follow from the variation of this action. With this understanding, and omitting fermions, the IIB supergravity action is

$$
S_{IIB} = \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ R + 4|d\phi|^2 - \frac{1}{3}|H|^2 \right] - 2|d\ell|^2 - \frac{1}{3}|H'|^2 - \ell H|^2 - \frac{1}{60}|M^+|^2 \right\} - \frac{1}{48} \int C^+ \wedge H \wedge H',
$$

(57)

where the full non-linear Bianchi identity satisfied by $M^+$ is now $dM^+ = H \wedge H'$. By combining this ‘modified’ Bianchi identity with the self-duality condition on $M^+$ we deduce that $d \ast M^+ = H \wedge H'$, which is just the $C^+$ field equation. Thus, the modification of the Bianchi identity is needed for consistency with the self-duality condition. Notice that the $R \otimes R$ fields again appear in the action without the factor of $e^{-2\phi}$. 

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We are now in a position to discuss the effective field theory of the Type I superstring theory. The field content is necessarily the same as that of the effective field theory of the $SO(32)$ heterotic string, but the 2-form gauge potential is the field $B'$ from the $R \otimes R$ sector of the IIB theory, so that its kinetic term must appear without the factor of $e^{-2\phi}$. Moreover, since the YM fields couple to the string endpoints, their tree-level amplitudes are associated with the disc (rather than the Riemann sphere) which has Euler number equal to 1, and this leads to a factor of $e^{-\phi}$ (rather than $e^{-2\phi}$) multiplying the YM terms in the effective action. Thus, the bosonic sector of the Type I effective action is (to leading order in an $\alpha'$ expansion)

$$S_I = \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left( R + 4|d\phi|^2 \right) - e^{-\phi} \text{tr} |\mathcal{F}|^2 - \frac{1}{3} |H'|^2 \right\}. \tag{58}$$

We have now found the bosonic sectors of the effective supergravity theories of all five $D=10$ superstring theories. As we shall see, this provides a powerful tool in the analysis of the possible strong coupling limits of these theories. All that we need assume of a given superstring theory is that it provides an asymptotic expansion in $g_s$ to some theory, or theories, defined for all $g_s$. Given the existence of a non-perturbative theory, one can continue $g_s$ from small to large values. Since $1/g_s$ is now small it is reasonable to expect that the theory can now be approximated by another asymptotic expansion in $g_s$ to some theory, or theories, defined for all $g_s$. Given the existence of a non-perturbative theory, one can continue $g_s$ from small to large values. Since $1/g_s$ is now small it is reasonable to expect that the theory can now be approximated by another asymptotic expansion in $g_s$. What is this new perturbation theory? One can first ask what its massless sector will be. For sufficiently small $g_s$ the massless quanta are those of the initial superstring theory. One might imagine that some of them could acquire masses as $g_s$ is increased, but massless quanta can become massive only if their number, charges, and spins are such that they can combine to form massive multiplets, which are all larger than the irreducible massless ones. This condition is not met by the quanta associated to the massless fields of any $D=10$ supergravity theory which must, therefore, remain massless for all $g_s$, in particular for large $g_s$. Thus, the only issue to be addressed is whether any other massless quanta appear at some non-zero value of $g_s$ (or as $g_s \to \infty$).

Let us consider this question first for the Type IIB theory. All supermultiplets of massive one-particle states of the IIB supersymmetry algebra contain states of at least spin 4. There are some indications that higher-spin massless field theories might be consistent if all spins are present but then only in the presence of a cosmological constant. This makes it rather unlikely that additional massless states could appear as the IIB string coupling constant is increased, so we conclude that the massless states at strong coupling are almost certainly the same as those at weak coupling. If so, the effective field theory at strong coupling must again be IIB supergravity, since this is the only
possibility permitted by supersymmetry. We can conclude that there must exist a symmetry of IIB supergravity which maps large negative $\phi$ to to large positive $\phi$, i.e. small $g_s$ to large $g_s$.

In fact, there is such a symmetry. It is most easily discussed in terms of the ‘Einstein-frame’ metric

$$g^{(E)}_{\mu\nu} = e^{-\frac{1}{2}\phi} g_{\mu\nu},$$

for which the action reads

$$S^{(E)}_{IIB} = \int d^{10}x \sqrt{-g} \left\{ R - 2[(d\phi)^2 + e^{2\phi}|dl|^2] - \frac{1}{60}|M^+|^2 - \frac{1}{3}e^{-\phi}|H|^2 \\
- \frac{1}{3}e^{\phi}|H' - \ell H|^2 \right\} - \frac{1}{48} \int C^+ \wedge H \wedge H'.$$

The action for $\phi$ and $\ell$ may now be recognised as that of a sigma-model with target space $Sl(2; R)/U(1)$. The $Sl(2; R)$ group acts on $\phi$ and $\ell$ by fractional linear transformations on the complex scalar $\tau = \ell + ie^{-\phi}$, i.e.

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d},$$

where

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sl(2; R).$$

The full action is also $Sl(2; R)$ invariant provided that the 2-form-valued row vector $(B, -B')$ transforms as an $Sl(2; R)$ doublet:

$$\begin{pmatrix} B' \\ B \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B' \\ B \end{pmatrix}.$$

This $Sl(2; R)$ symmetry can be extended to the complete IIB supergravity action including fermions. By choosing the special $Sl(2; R)$ matrix for which $a = d = 0$ and $b = -c = 1$, we see that there is a symmetry of IIB supergravity that takes $\phi \rightarrow -\phi$ for $\ell = 0$ and is still such that large negative $\phi$ is mapped to large positive $\phi$ for $\ell \neq 0$, as predicted.

We have just seen that the symmetries of IIB supergravity are consistent with the earlier deduction concerning the strong coupling limit of the IIB superstring theory. We have not yet made any assumption about the microscopic theory in this limit, but a now obvious guess is that the strongly coupled IIB superstring theory is another IIB superstring theory. Ultimately, consistency of this guess implies that only a discrete $Sl(2; Z)$ subgroup of $Sl(2; R)$ can be
realized as a symmetry of the non-perturbative IIB superstring theory. This discrete symmetry is itself non-perturbative and therefore a surprise from the point of view of conventional superstring theory. The embedding of $Sl(2; Z)$ in $Sl(2; R)$ depends on the vacuum expectation value of $\ell$. When $\langle \ell \rangle = 0$ the $Sl(2; Z)$ group is the one for which the entries of the matrix (62) are integers; otherwise it is a similarity transformation of an integer $Sl(2; R)$ matrix with the similarity transformation depending on $\langle \ell \rangle$. Although the full $SL(2; Z)$ symmetry cannot be checked directly, a $Z_2$ subgroup mapping weak coupling to strong coupling must be a symmetry of the full non-perturbative theory if this theory exists because, as we have seen, this is required by supersymmetry. This $Z_2$ ‘duality’ group is precisely the one that takes $\phi \to -\phi$ when $\ell = 0$. It is instructive to note that this $Z_2$ transformation also takes $B$ to $B'$, and hence maps the string charge $Q_1$ of (24) into a similar charge $Q'_1$ defined with $B'$ replacing $B$. Thus the weak to strong coupling duality of IIB superstring theory requires the existence of a new type of string carrying $R \otimes R$ charge; this is just the D-string, to be discussed in the following lecture. Given the existence of the D-string, T-duality implies the existence of p-branes carrying all other $R \otimes R$ charges of either the IIB or the IIA superstring theory, so the existence of the Type II D-branes is a direct consequence of IIB superstring duality which is virtually a direct consequence of the structure of the IIB supersymmetry algebra!

We turn now to the IIA theory. In the absence of additional massless fields appearing for large $g_s$, the effective field theory at strong coupling would have to be IIA supergravity again. But unlike IIB supergravity, there is no symmetry that maps large positive $\phi$ to large negative $\phi$, so this possibility is ruled out. It must be the case that additional massless fields appear as $g_s \to \infty$. The main difference between IIA and IIB in the analysis of this question is that there is the possibility of a central charge in the IIA algebra; as we saw from our earlier discussion of the IIA superalgebra it has an interpretation as a KK charge. Centrally charged multiplets can have maximum spin two and there is a (unique) consistent coupling of IIA supergravity to massive centrally charged spin two supermultiplets: it is the coupling determined by the compactification of D=11 supergravity to D=10. We conclude that the effective action at strong coupling must be D=11 supergravity. An immediate corollary is that, in contrast to the IIB case, the strong coupling limit of the Type IIA superstring theory cannot be another superstring theory.

The consistency of these conclusions can be checked by considering the dimensional reduction of D=11 supergravity. Omitting fermions, the D=11
supergravity action is

\[ S = \frac{1}{\kappa^2} \int d^{11}x \left\{ \sqrt{-g} \left[ R - \frac{1}{12} |F|^2 \right] + \frac{2}{(72)^2} e^{M_1\ldots M_{11}} F_{M_1\ldots M_4} F_{M_5\ldots M_8} A_{M_9 M_{10} M_{11}} \right\}, \quad (64) \]

where \( \kappa \) is the D=11 gravitational coupling constant. In general, dimensional reduction to D=10 is possible once we assume that the D=11 background has a \( U(1) \) isometry with Killing vector field \( k \), such that the 4-form \( F \) is also invariant, i.e \( \mathcal{L}_k F = 0 \), where \( \mathcal{L}_k \) is the Lie derivative with respect to \( k \). In coordinates \( x^M = (x^\mu, y) \) for which \( k = \partial/\partial y \), we can write the D=11 bosonic fields as

\[
\begin{align*}
&ds^2 = e^{-\frac{2}{3}\phi(x)} dx^\mu dx^\nu g_{\mu\nu}(x) + e^{\frac{4}{3}\phi(x)} (dy - dx^\mu C_{\mu}(x))^2 \\
&A = \frac{1}{6} dx^\mu \wedge dx^\nu \wedge dx^\rho A_{\mu\nu\rho}(x) + \frac{1}{2} dx^\mu \wedge dx^\nu \wedge dy B_{\mu\nu}(x),
\end{align*}
\]

(65)

from which we can identify the D=10 bosonic fields. Note that they coincide with the \( NS \otimes NS \) fields \((\phi, g_{\mu\nu}, B_{\mu\nu})\) and the \( R \otimes R \) fields \((C_{\mu}, A_{\mu\nu})\) of IIA supergravity. Substituting the Kaluza-Klein (KK) ansatz (65) into the D=11 action (64) leads precisely to the IIA action (53) (in units for which \( R_{11} = \kappa^2 \)).

Since we have supposed that \( k = \partial/\partial y \) is the Killing vector field of a \( U(1) \) isometry, the coordinate \( y \) is periodically identified and we may choose some standard identification without loss of generality, e.g. \( y \sim y + 2\pi \). It then follows from (65) that the radius of the 11th dimension is \( e^{\phi(x)} \). This is generally \( x \)-dependent but in a KK vacuum we may set \( \phi = \langle \phi \rangle \). In view of the relation (49) between the dilaton and the string coupling constant we deduce that

\[ R_{11} = (g_s^{(A)})^{\frac{2}{3}}, \quad (66) \]

which is precisely the relation of (3). This confirms that the effective action of the IIA superstring theory in its strong coupling limit is uncompactified D=11 supergravity, but it provides no clue to the nature of the D=11 quantum theory for which this is the effective field theory. One possibility is a supermembrane theory because, as we shall explore further in the next lecture, the IIA superstring transmutes at strong coupling into a D=11 supermembrane. But one should distinguish between a superstring or supermembrane theory and the superstring or supermembrane itself. The absence of a dilaton in D=11 means that there is no small parameter in terms of which one might define a perturbation theory, so it is not obvious that the presence of a membrane in D=11 implies the existence of a supermembrane theory. We shall return

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briefly to this point in the epilog to these lectures but it is important to appre-
ciate that, however things turn out, the major premise of M-theory, for which
the circumstantial evidence is now overwhelming, is that there exists some con-
sistent supersymmetric quantum theory in D=11 containing membranes and
fivebranes, with D=11 supergravity as its effective field theory.

As mentioned earlier, it is known that the IIA and IIB superstring theories
are equivalent, order by order in perturbation theory, after compactification
on a circle. Since this equivalence involves an interchange of KK modes and
winding modes it does not extend to the respective $S^1$-compactified supergrav-ity
theories, but the massless modes in D=9 are unaffected by this exchange
so the D=9 N=2 supergravity obtained by dimensional reduction of IIA super-
gravity must be equivalent to that obtained from IIB supergravity. This also
follows from supersymmetry because D=9 N=2 supergravity is unique up to
field redefinitions, but to find the map from IIA fields to IIB fields and vice-
versa one must compare the two dimensionally reduced supergravity theories.
If we denote by $R_A$ and $R_B$ the radii of the circles in the $S^1$ compactified IIA
and IIB supergravity theories, respectively, and by $\phi_A$ and $\phi_B$ the respective
dilatons, then one finds that

$$e^{-\phi_A} R_A = e^{-\phi_B} R_A = 1/R_B .$$

Given that the IIA theory is $S^1$-compactified M-theory it follows that the IIB
theory can be found by a $T^2$ compactification, as discussed in the first lecture.
To determine the relation between the radius $R_{10}$ appearing in that discussion
and the radius $R_A$ of the IIA compactification we write the 10-metric of IIA
supergravity as $ds^2_{10} = ds^2_9 + R_A^2 (dx)^2$ where $x$ is the coordinate of the circle
(such that $x \sim x + 2\pi$). Since $ds^2_{10}$ appears in the KK ansatz (64) with a factor
of $e^{-2\phi/3}$ we deduce that the radius of the circle from the D=11 perspective is

$$R_{10} = e^{-\frac{2}{3}\phi_A} R_A .$$

Combining this with (67) we have

$$e^{\phi_B} = e^{\frac{2}{3}\phi_A} / R_{10} ,$$

and hence a formula for $g_s^{(B)}$ in terms of $g_s^{(A)}$ and $R_{10}$. Using (69) to eliminate
$g_s^{(A)}$ from this formula we find that

$$g_s^{(B)} = R_{11} / R_{10} ,$$

which is precisely the formula (3).
We now turn to the issue of the strong coupling dynamics of superstring theories with N=1 supersymmetry. Let us consider first the Type I theory. At weak coupling the effective field theory is (omitting fermions) given by (58). We must again address the question of whether additional massless fields can appear at strong coupling. Because there is no possible central charge there is also no possibility of a shortened massive supermultiplet, but because we now have only N=1 supersymmetry the maximum spin of a massive supermultiplet could be as low as two. As just explained, additional spin two fields becoming massless signals the decompactification of an extra dimension. This is now an unlikely possibility because the only higher dimensional supersymmetric field theory is D=11 supergravity, which has double the required number of supersymmetries and no gauge fields. Thus, the most likely possibility is that the effective field theory at strong coupling is equivalent to the one at weak coupling. This would mean that it must be obtainable by some field redefinition that involves $\phi \rightarrow -\phi$. In contrast to the case of IIB supergravity, there is no field redefinition of this type which takes the effective field theory of the Type I superstring theory into itself, i.e. there is no strong-to-weak coupling symmetry. However, there is a field redefinition of this type that transforms the Type I effective field theory into the $SO(32)$ heterotic effective field theory, (46). It is

$$
\begin{align*}
g_{\mu\nu} & \rightarrow e^{-\phi}g_{\mu\nu} \\
\phi & \rightarrow -\phi \\
B' & \rightarrow B \\
A & \rightarrow \alpha' A. 
\end{align*}
$$

The equivalence of the two effective field theories is not in itself surprising because the N=1 supergravity/YM theory is unique up to field redefinitions once the gauge group is specified. However, the fact that the required field redefinition involves a change of sign of the dilaton is significant. It shows that the strong coupling limit of the Type I string theory is a theory with the same effective field theory as the SO(32) heterotic string theory. It is a now obvious guess that the strongly coupled Type I string theory is the SO(32) heterotic string theory, and vice-versa. This is certainly the only possibility if the strong coupling limit of one string theory is another string theory. Thus, subject to this assumption (for which there is now plenty of additional evidence), the Type I and SO(32) heterotic string theories are just the weak and strong coupling expansions of a single non-perturbative ‘SO(32) superstring theory’.

It remains for us to determine the strong coupling limit of the D=10 $E_8 \times E_8$ heterotic string theory. In this case there is neither a weak-to-strong coupling symmetry of its effective field theory nor a transformation that maps the latter
into the effective field theory of any other string theory. Thus, the strongly
coupled \( E_8 \times E_8 \) superstring theory cannot be another string theory. If the
effective field theory at strong coupling were a KK theory it would have to be
a compactification of D=11 supergravity, but the only conventional compactifi-
cation is on \( S^1 \) and this leads, as we have seen, to the IIA theory. The strong
coupling limit of the \( E_8 \times E_8 \) superstring theory is therefore the most puzzling
of the five. As explained briefly in the earlier overview of M-theory unif-
ication, this puzzle is resolved by the interpretation\(^2\) of the \( E_8 \times E_8 \) superstring theory
as a compactification of M-theory on \( S^1/Z_2 \).

4 Branes from M-theory

We have now seen how the picture sketched earlier in which all five superstring
theories are asymptotic expansions of a single 11-dimensional theory, M-theory,
is supported, and suggested, by the effective supergravity theories. In fact, we
have only just begun to mine the information contained in these effective field
theories. For example, much more information is contained in the solutions
admitted by them\(^7\). For each \((p+1)\)-form in the Lagrangian there is an
associated electric-type \( p \)-brane solution and a magnetic-type \((6-p)\)-brane
solution, carrying charges \( Q_p \) and \( Q_{(6-p)} \) respectively. Actually, there are
families of such solutions in which the \( p \)-volume tension can be varied at will
subject only to a BPS-type bound. For reasons mentioned briefly in our M-
theory overview, the solutions of most interest are the ‘BPS-saturated’ \( p \)-branes
for which, as the name suggests, the bound is saturated. The values of \( p \) for
which such solutions of a given theory exist are given in the ‘M-theory brane-
scan’ of Table 1. Only those \( p \)-branes with \( p \leq 6 \) appear in electric/magnetic
pairs, and these will be the only ones to be discussed here (the IIB 3-brane is an exception because it is self-dual, as indicated by the ‘+’ superscript).
The IIB 7-brane and IIA 8-brane are included in the table only for the sake
of completeness: the IIB 7-brane is important for F-theory, the IIA 8-brane is
associated with the massive IIA theory.

The D=10 \( p \)-branes have been labelled in Table 1 with the subscript \( F \), \( D \),
or \( S \), according to whether they are ‘Fundamental’, ‘Dirichlet’ or ‘Solitonic’.
These adjectives are indicative of the string theory interpretation of the various
supergravity solutions. The Fundamental strings and Solitonic 5-branes carry
the electric or magnetic charges of the \( NS \otimes NS \) 2-form potential \( B \), and are
therefore present for all but the Type I theory. In particular, we expect \( N=1 \)
supergravity/YM solutions to represent the long range fields of the heterotic
string and its 5-brane dual, although their identification is not straightforward
in this case because of the Lorentz Chern-Simons terms required for anomaly
Table 1: The M-Theory Branscan

| D=11 | 2 | 5 |
|------|---|---|
| IIA  | 0_D | 1_F | 2_D | 4_D | 5_S | 6_D | 8_D |
| IIB  | 1_F, 1_D | 3_D^+ | 5_S, 5_D | 7_D |
| Type I | 1_D | 5_D |
| Het  | 1_F | 5_S |

Cancellation, and the consequent infinite series of higher derivative terms then required by supersymmetry. Thus, for the heterotic string theories one should rather seek massless field configurations that define conformal field theories \[ \mathcal{C} \]; these will be approximated by solutions of the effective field theory. In contrast, the Type II supergravity p-brane solutions define sigma-models with (4,4) worldsheet supersymmetry, which are automatically conformally invariant. The Dirichlet branes are those carrying the \( R \otimes R \) charges, which appear in all but the heterotic string theories. Their string theory interpretation is in terms of open strings with mixed Dirichlet/Neumann boundary conditions, as discussed in other contributions to the school proceedings. Since the supergravity solutions will also be covered in other contributions we shall not enter into details of them either. For our purposes it will suffice to observe that the dependence of the p-volume tension \( T \) on the string coupling constant \( g_s \), for the string-frame metric, can be essentially read off from the supergravity Lagrangians given previously. The result is

\[
T \sim \begin{cases} 
1 & \text{for a Fundamental string} \\
1/g_s & \text{for a Dirichlet p-brane} \\
1/g_s^2 & \text{for a Solitonic 5-brane}
\end{cases} \tag{72}
\]

Note that all but the ‘Fundamental’ string are non-perturbative in \( g_s \), as required for consistency since there is no sign of any other extended object in
string perturbation theory.

The chief purpose of these lectures is to show how M-theory unifies, and encompasses, superstring theories. Although we made a start on this in the previous lectures it should now be clear that part of our goal must be to provide an M-theory explanation for all the superstring p-branes. We have already seen some reasons for believing that the IIA superstring is a D=11 supermembrane wrapped around the 11th dimension, but Table 1 suggests that we should also expect to be able to interpret the IIA D-4-brane as a wrapped D=11 fivebrane. Furthermore, properties of these IIA branes, e.g. their dependence on the string coupling constant, should follow from properties of the D=11 branes, which we shall refer to collectively as ‘M-branes’. Our knowledge of M-branes is rather limited at present but the effective worldvolume action for the supermembrane is known and some features of the fivebrane action are also known.

Let us start with the supermembrane; the bosonic sector of its worldvolume action is

$$S = -\frac{1}{2\pi} \int d^3\xi \sqrt{-\det g_{ij}^{(11)}} - \frac{1}{6} \varepsilon^{ijk} A_{ijk}^{(11)}$$

(73)

where $g_{ij}^{(11)}$ and $A_{ijk}^{(11)}$ are pullbacks to the worldvolume of the spacetime metric and 3-form of D=11 supergravity. The overall factor has been chosen for later convenience. We shall take the spacetime fields to be of the form given by the KK ansatz (65), so that

$$g_{ij}^{(11)} = e^{-\frac{4}{3} \phi} \partial_i X^\mu \partial_j X^\nu g_{\mu\nu} + e^{\frac{4}{3} \phi} (\partial_i y - \partial_i X^\mu C_\mu) (\partial_j y - \partial_j X^\mu C_\mu)$$

$$A_{ijk}^{(11)} = \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho A_{\mu\nu\rho} + 3 \partial_i X^\mu \partial_j X^\nu \partial_k y B_{\mu\nu}$$

(74)

where the square brackets indicate total antisymmetrization (with ‘strength one’). To obtain the action for a string in the D=10 background provided by (53) we must dimensionally-reduce the (2+1)-dimensional supermembrane action to (1+1) dimensions. The standard dimensional reduction ansatz would take all the worldvolume fields to be independent of one of the worldvolume space coordinates, say $\rho$. This results in a $\rho$-independent two-dimensional Lagrangian, but not one that can be identified with the usual superstring Lagrangian. However, this ansatz is not appropriate for a membrane wound around the 11th dimension. There is another way to achieve a $\rho$-independent Lagrangian that makes use of the fact that a U(1) isometry of the D=11 background implies an invariance of the membrane action under the transformation generated by the U(1) Killing vector field $k$. Instead of requiring the worldvolume fields to be $\rho$-independent we can set

$$\partial_\rho X^M = k^M(X).$$

(75)
If we choose spacetime coordinates such that \( k = \partial/\partial y \), where \( y \) is the 11th coordinate, as in (74), then (75) reduces to the condition that all worldvolume fields are \( \rho \)-independent except \( y(\xi) \), which is linear in \( \rho \); we can then choose it to be proportional to \( \rho \) by a partial gauge choice. We can also choose the period of identification of \( \rho \) to be the same as that of \( y \) without loss of generality. Thus, (75) becomes

\[
\partial_\rho X^\mu = 0 \quad y = \nu \rho
\]  

for some integer \( \nu \), which is the winding number of the membrane around the \( S^1 \) factor of the D=11 spacetime. The choice \( \nu = 0 \) corresponds to standard dimensional reduction while \( \nu \neq 0 \) corresponds to a Scherk-Schwarz dimensional reduction, which is called ‘double-dimensional reduction’ in the context of worldvolume actions\(^1\).

Now let \( \xi^i = (\sigma^\alpha, \rho) \). Using (76) we then find that the induced \( 3 \times 3 \) metric \( g^{(11)}_{ij} \) is

\[
g^{(11)}_{ij} = \left( \begin{array}{ccc}
e^{-\frac{2}{3}\phi}(g_{\alpha\beta} + e^{2\phi}C_\alpha C_\beta) & \nu e^{\frac{4}{3}\phi}C_\alpha \\
\nu e^{\frac{4}{3}\phi}C_\beta & \nu^2 e^{\frac{4}{3}\phi} \end{array} \right),
\]  

from which we compute

\[
\sqrt{-\text{det} g^{(11)}_{ij}} = \nu \sqrt{-\text{det} g_{\alpha\beta}}.
\]  

Similarly, (76) implies that

\[
\frac{1}{6} \varepsilon^{ijk} A^{(11)}_{ijk} = \frac{1}{2} \nu \varepsilon^{\alpha\beta} B_{\alpha\beta}.
\]  

The double-dimensionally reduced membrane action is therefore

\[
S = -\nu \int d^2 \sigma \left\{ \sqrt{-\text{det} g_{\alpha\beta}} - \frac{1}{2} \varepsilon^{\alpha\beta} B_{\alpha\beta} \right\},
\]  

but this is just \( \nu \) times the string action (to leading order in \( \alpha' \) and with \( 2\pi\alpha' = 1 \)) in the background provided by the NS-NS fields of IIA supergravity. Applied to the full supermembrane action, the same procedure yields the complete GS action for the IIA superstring in a general D=10 IIA supergravity background.

Note that the string tension is proportional to \( \nu \), which was to be expected for a membrane wound \( \nu \) times around a circle. Note also that, since all \( \phi \)-dependence has cancelled from the action, the tension is \( g_s \)-independent, as required for a ‘Fundamental’ string. This is clearly a special feature of the \( 3 \times 3 \) matrix (77). If we were to double-dimensionally reduce the D=11 fivebrane
action we would have a similar calculation to perform but with a $6 \times 6$ matrix. In this case we would find that
\[
\sqrt{-\det g^{(11)}_{ij}} = \nu e^{-\phi} \sqrt{-\det g_{\alpha\beta}}.
\] (81)

This is sufficient to show that the double-dimensionally reduced fourbrane action will have a tension proportional to $1/g_s$, as required for its interpretation as a IIA D-brane.

We now have evidence that the IIA superstring and the IIA D-4-brane are just the wrapped membrane and fivebrane of M-theory, but if we are to take this seriously, we should consider all the implications. For example, we cannot arbitrarily restrict the double dimensional reduction of the supermembrane to a single choice of the winding number $\nu$. Clearly, the $\nu = 1$ string is the one that should be identified as the IIA superstring, and it might seem that there is no place for the strings with higher winding numbers. However, one should recall that a single charged field can create any number of charged particles, which can appear as a single particle of higher charge if they happen to be coincident. Similarly, a single string field can create any number of coincident strings. It is a feature of supersymmetry that the force between these strings is zero, so a superposition of $\nu$ unit tension strings would appear to be a single string of tension $\nu$. In principle, we should also allow $\nu = 0$. The action (80) vanishes when $\nu = 0$ but a proper treatment of the $\nu = 0$ case leads to the action of a tensionless string. This a potential difficulty because there is no place in IIA superstring theory for a tensionless string. However, the $\nu = 0$ string is not a membrane wound around the compact 11th direction; it is rather a toroidal membrane that has collapsed to a string. This is equally possible, in principle, for a membrane in an uncompactified D=11 spacetime, so if there is indeed a tensionless string it must already be present in D=11. But we are almost certainly stepping outside the domain of validity of the supermembrane action when we consider configurations of membranes collapsed to strings.

The same caveat applies to a membrane collapsed to a point, but let us nevertheless consider this possibility. The action for such collapsed configurations, as deduced from the supermembrane action itself, is that of the massless D=11 superparticle, for which the bosonic part of the action, in a bosonic D=11 supergravity background can be written in the (hamiltonian) form
\[
S = \int dt [\dot{X}^M P_M - \frac{1}{2} \tilde{v} g^{MN} P_M P_N]
\] (82)
where $\tilde{v}$ is an independent worldline density and $P_M$ is the momentum conjugate to $X^M$. In the supersymmetric case one finds that the states of the
quantum theory correspond to the massless fields of D=11 supergravity, and this was one reason for thinking that D=11 supergravity might be the effective field theory of a quantum supermembrane theory. As mentioned above, the starting point of this approach is suspect because the supermembrane action is likely to be only an effective one, but what we currently know about M-theory requires us to postulate that its effective action is indeed D=11 supergravity.

Now, massive KK modes in D=10 can be interpreted as massless quanta of D=11 with non-zero momentum in the compact direction, so the action [82] can be used to determine some features of the KK spectrum of $S^1$ compactified D=11 supergravity. The KK masses will be integral multiples of a unit proportional to $1/R_{11}$, where $R_{11}$ is the radius of the circle, but this is the mass unit as measured in the D=11 metric. To determine the mass in terms of the D=10 string-frame metric we choose the D=11 KK background of (65) and set $P_M = (P_{\mu}, P_y)$. Since $y$ is periodically identified with period $2\pi$, the eigenvalues of its conjugate variable $P_y$ are integers. We therefore set $P_y = n$ for integer $n$; the term $\dot{y}P_y$ is then a total derivative which we may discard. Defining $v = e^{2\phi/3}\tilde{v}$, we thereby arrive at the action

$$S = \int dt \left\{ \dddot{X}^\mu P_{\mu} - \frac{1}{2} v \left[ (P - nC)^2 - (n e^{-\phi})^2 \right] \right\},$$

which is that of a charged massive particle in a 10-dimensional spacetime. Setting $\phi$ equal to its vacuum value $\langle \phi \rangle$ we see that the mass $M$ and charge $Q$ of this particle are given by

$$M = \frac{n}{g_s} \quad Q = n.$$

As expected from its origin, the particle is charged with respect to the KK vector field, which is the $R \otimes R$ vector field of IIA superstring theory. For $n = 1$, its mass is precisely that required for identification as a D-0-brane. Consideration of the complete D=11 massless superparticle action [83] leads to an extension of (83) that includes a supersymmetry WZ term; this term implies an extension of the supersymmetry algebra to include the charge $Q$ as a central charge. Standard arguments can then be used to derive a BPS-type bound on the mass in terms of the charge; this bound is saturated by (84), which was to be expected from the fact that KK modes are BPS-saturated. Actually, the D-0-branes provide only the $n = 1$ KK states whereas M-theory requires the existence of KK states for each $n \geq 1$. The $n \geq 2$ states must appear as bound states in the $n$ D-0-brane system. The absence of forces between static D-0-branes implies that these bound states must be at threshold. The issue of whether there exist bound states at threshold is a delicate one and this prediction of M-theory still awaits verification.

40
We are not yet finished with extracting the consequences of having replaced the IIA superstring by a D=11 supermembrane. We have still to confront the most obvious consequence of this idea. A membrane can as easily move in ten dimensions as eleven so there must also exist a D=10 membrane in the non-perturbative IIA superstring theory. We can determine its effective action from that of the D=11 supermembrane. Here we shall consider only the bosonic action. It will be convenient to rewrite this action in the equivalent form

\[ S = \frac{1}{4\pi} \int d^3 \xi \left\{ v^{-1} \det g_{ij}^{(11)} - v + \frac{1}{3} \varepsilon^{ijk} A_{ijk}^{(11)} \right\} \]  

where \( v \) is an independent worldvolume density. The (classical) equivalence of this action to (73) follows by elimination of \( v \) by means of its Euler-Lagrange equation. As before we take the D=11 supergravity fields to be given by the KK ansatz. This implies that the induced fields are those of (74), which we rewrite as

\[ g_{ij}^{(11)} = e^{-\frac{2}{3} \phi} g_{ij} + e^{\frac{2}{3} \phi} Y_i Y_j \]
\[ A_{ijk}^{(11)} = A_{ijk} + 3B_{[ij} Y_{k]} - 3B_{[ij} C_{k]} , \]  

where \( g_{ij}, A_{ijk} \) and \( B_{ij} \) are the worldvolume fields induced by the D=10 space-time fields, and we have defined

\[ Y \equiv dy + C . \]  

It follows, since \( g_{ij} \) is \( 3 \times 3 \), that

\[ \det g_{ij}^{(11)} = e^{-2\phi} \det \left[ g_{ij} + e^{2\phi} Y_i Y_j \right] . \]  

Using properties of \( 3 \times 3 \) matrices we can rewrite this as

\[ \det g_{ij}^{(11)} = (\det g_{ij}) \left[ e^{-2\phi} + |Y|^2 \right] , \]  

where \( |Y|^2 = Y_i Y_j g^{ij} \). The action (85) is then found to be

\[ S = \frac{1}{4\pi} \int d^3 \xi \left\{ v^{-1} e^{-2\phi} \det g_{ij} - v + \frac{1}{3} \varepsilon^{ijk} [A_{ijk} - 3B_{ij} C_k] \right\} \]  

Note that the one-form \( Y \) in the above action is just shorthand for the expression in (87). As such, it satisfies the identity

\[ d(Y - C) \equiv 0 . \]
We can elevate $Y$ to the status of an independent field if we impose this identity by a Lagrange multiplier. We can do this by adding to the action the term

$$-\frac{1}{2\pi} \int F \wedge (Y - C)$$

for closed two-form $F$. If $F$ were an independent field, it would be a Lagrange multiplier for the constraint $(Y - C) = 0$, whereas what we need is the weaker constraint $d(Y - C) = 0$. This constraint could be imposed by taking $F$ to be an exact 2-form, i.e.

$$F = dV,$$

for some 1-form $V$ but this is slightly too strong a condition on $F$. If we instead write $F = dV + 2\pi \omega^{(2)}$, where the closed 2-form $\omega^{(2)}$ belongs to an integral cohomology class of the membrane’s worldvolume, then (92) acquires the extra term

$$\Delta S = \int \omega^{(2)} \wedge (Y - C).$$

But the periodic identification of $y$ means that $dy/2\pi$, and hence $(Y - C)/2\pi$, also belongs to an integral cohomology class (of the worldvolume after the pullback of forms from spacetime). Thus, $\Delta S/2\pi$ is an integer. This implies that $\exp(i\Delta S) = 1$ and hence that the addition to $F$ of $2\pi \omega^{(2)}$ has no effect on the path-integral. We can take this freedom in the definition of $F$ into account by allowing the 1-form gauge potential $V$ to be defined only locally, such that the flux of $F/2\pi$ over any 2-cycle is an integer. This is equivalent to the statement that $iV/2\pi$ is a $U(1)$ gauge potential (as against merely an abelian one).

Now that we have settled the question of the nature of the gauge potential $V$ introduced by the Lagrange multiplier term (92) we add this term to (90) to obtain the equivalent action

$$S = \frac{1}{8\pi^2} \int d^3\xi \left\{ v^{-1} e^{-2\phi} \det g_{ij} - v + \frac{1}{3} \varepsilon^{ijk} [A_{ijk} + 3F_{ij}C_k] 
+ v^{-1} (\det g_{ij}) |Y^2| - \varepsilon^{ijk} F_{ij}Y_k \right\},$$

where we have defined the ‘modified’ field strength

$$F_{ij} = F_{ij} - B_{ij}.$$

Here, in accordance with our condensed notation, $B$ should be understood to be the pullback to the worldvolume of the spacetime 2-form potential $B$. 
Since $Y$ is an independent field in the new action, it can be eliminated by its algebraic, and linear, Euler-Lagrange equation

$$Y^i = \frac{v}{2 \det g} \varepsilon^{ijk} F_{jk}. \quad (97)$$

The resulting action can then be simplified by use of the $3 \times 3$ matrix identity

$$\det[g_{ij} \pm J_{ij}] \equiv (\det g_{ij}) \left[1 + \frac{1}{2} |J|^2 \right] \quad (98)$$

where $J_{ij}$ is any antisymmetric matrix and $|J|^2 = g^{ij} g^{kl} J_{ik} J_{jl}$. The result of these manipulations is

$$S = \frac{1}{4 \pi^2} \int d^3 \xi \left\{ - \dot{v} e^{-2\phi} + \dot{v}^{-1} \det(g_{ij} + F_{ij}) + \frac{1}{3} \varepsilon^{ijk} \left[A_{ijk} + 3F_{ij}C_k \right] \right\}. \quad (99)$$

where $\dot{v} = -\det(g_{ij})/v$. Finally, elimination of $\dot{v}$ yields

$$S = \frac{1}{4 \pi^2} \int d^3 \xi e^{-\phi} \sqrt{-\det(g_{ij} + F_{ij})} + \frac{1}{4 \pi^2} \int_w (A + F \wedge C) \quad (100)$$

where the final ‘Wess-Zumino’ term has now been written as an integral of a 3-form over the worldvolume $w$. The dependence on $F$ in the first term is reminiscent of the Born-Infeld action for ‘non-linear electrodynamics’, so the full action is called the Dirac-Born-Infeld (DBI) action. Setting $\phi$ to its vacuum value we see that the 2-brane tension is proportional to $1/g_s$, as required for its interpretation as the IIA D-2-brane.

We have now shown that the D=11 supermembrane action requires the D-2-brane of IIA superstring theory to have an effective worldvolume action of the form

$$S = S_{DBI} + S_{WZ} \quad (101)$$

where $S_{DBI}$ is the DBI action with tension of order $1/g_s$, and $S_{WZ}$ is a ‘Wess-Zumino’ term. This prediction of M-theory is verifiable by a string theory calculation, which also shows that it is a general feature. The WZ term provides the coupling of the D-brane to the $R \otimes R$ fields. The ‘leading’ term in the WZ term is always of the form $\int C^{(p+1)}$, i.e. a minimal coupling of the D-brane to the $R \otimes R$ potential $C^{(p+1)}$, implying that the D-brane is a charged source for $C^{(p+1)}$. We have now seen two examples of this: the D-0-brane, which is a source for the 1-form potential $C^{(1)} = C$, and the D-2-brane, which is a source
for the 3-form potential \( C^{(3)} = A \). In the latter case, a magnetic source of \( F \) on the D-2-brane is also a source of \( C \); this has some interesting implications but we shall have to pass over them here.

Just as the existence of a membrane in D=11 implies the existence of one in D=10, the existence of a fivebrane in D=11 implies the existence of a D=10 5-brane. Returning to (86) but interpreting the induced metric as one on the 6-dimensional worldvolume of a fivebrane we have

\[
\det g_{ij}^{(11)} = e^{-4\phi} \det [g_{ij} + e^{2\phi} Y_i Y_j] \tag{102}
\]

in place of (88). Determination of the full D=10 5-brane action from M-theory is complicated by the fact that the D=11 fivebrane action is not yet fully known; its worldvolume fields include a 2-form potential with self-dual 3-form field strength \( K \). Nevertheless, its Lagrangian will include a term of the standard Dirac form and this, together with (102), is sufficient to show that the tension of the D=10 5-brane is \( 1/g_s^2 \), as expected from its ‘Solitonic’ interpretation in string theory. We have still to consider the D-6-brane and the D-8-brane. The D-6-brane does not have an M-brane interpretation, although it does have a simple M-theory interpretation as a generalized KK monopole. The M-theoretic interpretation of the D-8-brane is currently problematic since its long range fields solve the equations of the ‘massive’ IIA supergravity which, as far as we can see, cannot be obtained from D=11 supergravity. Hopefully, this mystery will be cleared up in the near future. In any case, the M-theory predictions agree with results obtainable from IIA superstring theory in so far as it is currently possible to check.

We turn now to the IIB branes. Their worldvolume actions can be determined indirectly from M-theory by virtue of the fact that the IIB theory and the IIA theory are T-dual. For example, if the D-2-brane action given above is compactified on \( S^1 \) and the IIA background is replaced by its T-dual IIB background then we obtain the action for the D-1-brane, or D-string. If this D-string action is compactified on \( S^1 \) and the IIB background is replaced by the original IIA background then we recover the D-0-brane action. This last step provides the simplest illustration of the procedure, so we shall consider some of the details. To do this we must depart slightly from the logic in which the D-brane actions are derived from M-theory by first postulating the (bosonic sector of the) D-string action and then showing that it leads to the same D-0-brane action as we previously derived from M-theory. Actually, in order to illustrate an additional point we shall start with the action for a D-string with an integer \( n \) times the tension of a single D-string. This is

\[
S = -\frac{n}{2\pi} \int d^2\sigma \left\{ e^{-\phi_B} \sqrt{-\det(g_{ij} + F_{ij})} + \frac{1}{2} \varepsilon^{ij} (B'_{ij} + \ell F_{ij}) \right\} \tag{103}
\]
where $\mathcal{F}$ is the ‘modified’ 2-form field strength introduced in (96). We shall proceed by first converting this action to Hamiltonian form. The procedure for doing this is standard so we go straight to the final result, which is

$$S = -\frac{1}{2\pi} \int dt \int_0^{2\pi} d\sigma \left\{ \dot{X} \cdot \vec{P} + \dot{V}_\sigma E + V_t E' + s X' \cdot P - \frac{1}{2} v \left[ (P - nB')^2 + (X')^2 \left[ (E - n\ell)^2 + n^2 e^{-2\phi A} \right] \right] \right\}$$  

where

$$B_\mu = (X')_\nu B_{\mu\nu},$$

$$F'_\mu = (X')_\nu B'_{\mu\nu},$$

and the Lagrange multipliers $v$ and $s$ are the analogues of the lapse and shift functions of General Relativity. The variables $P_\mu$ and $E$ are the conjugate momenta to $X^\mu$ and $V_\sigma$, respectively. Thus $E$ is effectively the BI electric field and the constraint imposed by $V_t$ is the 1+1 dimensional version of the usual Gauss’ law constraint of electrodynamics. Note that a prime is used to denote differentiation with respect to $\sigma$ except in $B'$ where it distinguishes the $R \otimes R$ 2-form potential from the $NS \otimes NS$ one.

With a view to double dimensional reduction we now suppose that the IIB background is of KK type, i.e. admits a $U(1)$ Killing vector field $k = \partial/\partial u$. If $u$ is identified with period $2\pi$ then the radius of the compact direction is $R_B = \sqrt{k^2}$. We then take all worldsheet fields to be $\sigma$-independent with the exception of $u$, which we set equal to $\sigma$. The constraint imposed by the ‘shift’ function $s$ now reduces to $k \cdot P = 0$, so the use of this constraint removes the conjugate pair $(u, k \cdot P)$ from the action. Moreover, since the Lagrangian is now $\sigma$-independent, the $\sigma$ integration can be trivially done, leading to a factor of $2\pi$. At this point, we have a particle action in a background provided by the fields of IIB supergravity, but we may now use the ‘T-duality rules’ to express the background in terms of IIA fields. We have already come across a subset of these rules in (67). These can be extended to the full set of IIA and IIB supergravity fields [31]. We shall not go into the details here except to say that $(B - \ell B')$ becomes the IIA KK 1-form $C$; the net result of using the T-duality rules in the double-dimensionally reduced D-string action (104) is

$$S = -\int dt \left\{ \dot{X} \cdot \vec{P} - \frac{1}{2} v \left[ (P - nC)^2 + n^2 e^{-2\phi A} \right] \right\}$$  

where $\dot{X}^\mu = (X^\mu, V_\sigma)$ with $\bar{\mu} = 0, 1, \ldots, 8$ and $\dot{P}_\mu = (P_\mu, E)$. The IIA metric is also of KK form with Killing vector field $k = \partial/\partial V_\sigma$, and $k^2 = R^2_{A}$. This is
precisely the D-0-brane action \(^{(83)}\) in a D=10 KK background provided that \(R_A\) can be identified as the radius of the compact 10th dimension, which it can be if \(V_{\sigma}\) is an angular variable with period \(2\pi\).

We saw earlier that the M-theory origin of the D-2-brane requires \(iV/2\pi\) to be a \(U(1)\) gauge potential. It is then a consequence of T-duality that \(iV/2\pi\) is equally a \(U(1)\) gauge potential for any D-brane, in particular the D-string. Because of this, the 1-form \(V\) of the D-string action is defined only up to the \(U(1)\) gauge transformation

\[
\frac{iV}{2\pi} \rightarrow \frac{iV}{2\pi} + g^{-1}dg, \quad (g(t, \sigma) \in U(1)).
\]  

(107)

We may choose \(g = e^{i\sigma}\), in which case the gauge transformation becomes

\[
V_{\sigma} \rightarrow V_{\sigma} + 2\pi.
\]  

(108)

Since this is a gauge transformation, we must identify \(V_{\sigma}\) with its gauge transform \(V_{\sigma} + 2\pi\). Thus \(V_{\sigma}\) is the coordinate of a compact direction with the standard identification, so \(2\pi R_A\) is the length of the closed orbit of \(k = \partial/\partial V_{\sigma}\), i.e. \(R_A\) is the radius of the compact dimension, as required.

We have now established the relation of the IIB D-string to the IIA D-0-brane. It is similarly related to the D-2-brane. Let us now investigate its relation to the IIB Fundamental string, or ‘F-string’. Since \(V_{\sigma}\) in \(^{(104)}\) is identified with period \(2\pi\), the eigenvalues of its conjugate variable \(E\) are integers. Let us choose

\[
E = m;
\]  

(109)

the \(V_{t}E'\) term is then zero and the \(\dot{V}_{\sigma}E\) term becomes a total derivative which may be neglected. The Lagrangian density of the action \(^{(104)}\) is thereby reduced to

\[
\mathcal{L} = \dot{X} \cdot P + s X' \cdot P - \frac{1}{2} v \mathcal{H}
\]  

(110)

where

\[
\mathcal{H} = (P - nB' + mB)^2 + (X')^2[(m - n\ell)^2 + n^2 e^{-2\phi_B}]\]  

(111)

is the ‘Hamiltonian’ constrained to vanish by the Lagrange multiplier \(v\). Setting the background scalar fields to their vacuum values, we see that this is the action for a string with tension

\[
T = \frac{1}{2\pi} \sqrt{n^2/g_s^2 + (m - n(\ell))^2}
\]  

(112)

and charge \((m,n)\) with respect to \((B,-B')\), exactly as required by the \(Sl(2;Z)\) symmetry of the IIB theory. In particular, the string with charge \((1,0)\) is just the Fundamental IIB string.
It would make no sense to set \( n = 0 \) in the original D-string action (103), but this is a defect of that action rather than a physical limitation. One of the virtues of the Hamiltonian form of the action is that it makes this fact manifest. Indeed, setting \( n = 0 \) in (111) and eliminating all auxiliary variables from the action one recovers, provided that \( m \neq 0 \), the fundamental string action (80) with \( \nu = m \), i.e. with tension \( T = m/2\pi \); that action was derived from M-theory as the action of the IIA string but since we are omitting fermions it is equally the bosonic sector of the action for the IIB string. Thus, the fundamental string tension can be identified as the lowest non-zero eigenvalue of \( E \). The semi-classical equivalent of an eigenvalue of \( E \) is the circulation of the classical variable \( E(\sigma) \) around the string, \( \frac{1}{2\pi} \int d\sigma E(\sigma) \). This equals the flux of the BI 2-form through the string worldsheet. Thus, up to a normalization factor, the fundamental string tension is the quantized flux of the BI 2-form through the string worldsheet.

No further details of IIB p-branes will be given here, but some mention must be made of the IIB self-dual D-3-brane. In many respects, this plays as crucial a role in the IIB theory as the D-2-brane does in the IIA theory. The effective action for the D-3-brane is of the form (101). An important feature of its equations of motion (the ‘branewave’ equations) is that they exhibit an \( SL(2; Z) \) ‘duality’ in the sense that an \( SL(2; Z) \) transformation of the worldvolume fields, which acts by a generalization of electromagnetic duality on the BI 2-form field strength and its Hodge dual, effects a \( SL(2; Z) \) transformation of the IIB supergravity background. Thus, the \( SL(2; Z) \) invariance of IIB supergravity (actually \( SL(2; R) \) but this is broken to \( SL(2; Z) \) in the quantum superstring theory) extends to the combined supergravity plus branewave equations. Given that M-theory predicts both the \( SL(2; Z) \) symmetry (as the modular group of a 2-torus) and the 3-brane (as a \( T^2 \)-wrapped 5-brane), this result is clearly a consequence of M-theory. From this perspective it is also clear that M-theory equally predicts an \( SL(2; Z) \) duality ‘on the brane’ for \( n \) coincident 3-branes, for which the BI \( U(1) \) group is enhanced to \( U(n) \). After gauge-fixing the \( \kappa \)-symmetry and ignoring all but the leading order terms in an \( \alpha' \) expansion, the worldvolume field theory of this multi-3-brane is just an \( N=4 \) D=4 super-YM theory, for which we can interpret the predicted \( SL(2; Z) \) duality as the conjectured S-duality of this theory.

Let us also call the \( SL(2; Z) \) duality of IIB superstring theory ‘S-duality’, since in both the D=10 and D=4 contexts there is a \( Z_2 \) subgroup that interchanges weak and strong coupling. As just explained, S-duality of IIB superstring theory can be ‘derived’ from the electromagnetic S-duality of the 3-brane in essentially the same way that spacetime T-duality is derived from duality on the worldsheet of the fundamental string. In the former case duality ‘on the
brane’ exchanges a D=4 vector potential with its electromagnetic dual vector potential whereas in the latter case it exchanges a scalar for its dual scalar. In both cases, a duality transformation ‘on the brane’ results in a duality transformation of the background spacetime fields. We have also seen in this lecture how a vector to scalar duality on the worldvolume of the D-2-brane results in a transformation from the background fields of D=10 IIA supergravity to those of D=11 supergravity. Let us call the latter transformation ‘M-duality’. Then, as illustrated in Table 2, all the Type II dualities can be seen to have a common origin in dualities ‘on the brane’. It has been argued that there is only one other M-theory or superstring duality that is ‘independent’ of these Type II dualities, and that it can be taken to be the Type I to $SO(32)$ heterotic string duality. Using these four dualities one can get to any brane on the M-theory brane scan from any other one. In other words, M-theory as we now know it is a ‘p-brane democracy’.

5 Epilog

The main aim of these lectures has been to explain how the five D=10 superstring theories are unified by 11-dimensional M-theory. Pedagogical expediency has dictated the omission of many other interesting topics, in particular connections between superstring and M-theory compactifications in lower dimensions. Perhaps the gravest omission is a definition of M-theory. One excuse for this is that whereas definitions may come first in mathematics they usually come last in physics. It therefore seems appropriate to end these lectures with a brief mention of recent progress on this front. The obvious starting point for a definition of M-theory is the D=11 supermembrane. In the past, there were two major objections to a fundamental supermembrane theory. These were (i) that the $(2+1)$-dimensional worldvolume action is non-renormalizable and (ii) that the spectrum of the first quantized supermembrane is continuous. Both these problems now have answers.

The non-renormalizability problem has been overcome by an interpretation of both the $(1+1)$-dimensional D=10 superstring actions, and the $(2+1)$-
dimensional D=11 supermembrane action as effective actions of the so-called ‘n=2 heterotic strings’. In this approach, the classical supermembrane equations emerge as the conditions required for conformal invariance of the n=2 heterotic string sigma-model action, so the first quantized supermembrane is interpreted as a second-quantized string theory. The second quantized supermembrane would presumably then emerge from a ‘third-quantized n=2 heterotic string theory’. Since we have little idea what this might be, the non-renormalizability problem might appear to have been solved only at the cost of introducing a new problem. On the other hand, there are some indications from an alternative approach that first quantization of the supermembrane (and hence second-quantization of the n=2 heterotic string) might be sufficient.

This alternative approach makes use of the observation that the large N matrix model approximation to the supermembrane Hamiltonian can be re-interpreted as the Hamiltonian for N coincident D-0-branes; the continuity of the spectrum is then seen to be a consequence of the no-force condition between D-0-branes. This suggests a re-interpretation of the Hilbert space of the first quantized supermembrane as the Hilbert space of an interacting multi-particle (and multi-membrane) theory that one would normally expect to arise only on second quantization. Remarkably, it seems possible to extract sensible results for the scattering of D=11 gravitons, and to recover both the membrane and the fivebrane as collective excitations in this approach. Is this the long sought theory of quantum gravity? If past experience is anything to go by, the future holds plenty of surprises in store for us.

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