Absorption of Microwaves by Random-Anisotropy Magnets

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Microscopic model of the interaction of spins with a microwave field in a random-anisotropy magnet has been developed. Numerical results show that microwave absorption occurs in a broad range of frequencies due to the distribution of ferromagnetically correlated regions on sizes and effective anisotropy. That distribution is also responsible for the weak dependence of the absorption on the damping. At a fixed frequency of the ac-field spin oscillations are localized inside isolated correlated regions. Scaling of the peak absorption frequency agrees with the theory based upon Imry-Ma argument. The effect of the dimensionality of the system related to microwave absorption by thin amorphous magnetic wires and foils has been studied.

I. INTRODUCTION

In conventional ferromagnets the ac field can excite spin waves with a finite angular momentum and/or induce the uniform ferromagnetic resonance (FMR) corresponding to the zero angular momentum. In the presence of strong disorder in the local orientation of spins, however, that exists in materials with quenched randomness, such a spin glasses and amorphous ferromagnets, spin waves must be localized while the existence of the FMR becomes non-obvious. On general grounds one should expect that random magnets would exhibit absorption of the ac power in a broad frequency range that would narrow down when spins become aligned on increasing the external magnetic field.

Collective excitation modes have been observed in random magnets in the past\textsuperscript{4,5}. In spin-glasses they were attributed\textsuperscript{4} to the random anisotropy arising from Dzyaloshinskii-Moriya interaction and analyzed\textsuperscript{5} within hydrodynamic theory\textsuperscript{6,7}. Later Suran et al. studied collective modes in amorphous ferromagnets with random local magnetic anisotropy\textsuperscript{8} and reported evidence of their localization\textsuperscript{9}. Longitudinal, transverse and mixed modes have been observed in thin amorphous films. Detailed analysis of these experiments, accompanied by analytical theory of the uniform spin resonance in the random anisotropy (RA) ferromagnet in a nearly saturating magnetic field, has been recently given by Saslow and Sur\textsuperscript{10}. Rigorous approach to this problem requires investigation of the oscillation dynamics of a system of a large number of strongly interacting spins in a random potential landscape. While it was not possible at the time when most of the above-mentioned work was performed, the capabilities of modern computers allow one to address this problem numerically in great detail. Such a study must be worth pursuing because of the absence of the rigorous analytical theory of random magnets and also with an eye on their applications as microwave absorbers.

In this paper we consider dynamics of an amorphous ferromagnet consisting of up to $4 \times 10^5$ spins within the RA model. It assumes (see, e.g., Refs. \textsuperscript{11} and references therein) that spins interact via ferromagnetic exchange but that directions of local magnetic anisotropy axes are randomly distributed from one spin to another. In the past this model was successfully applied to the description of static properties of amorphous magnets, such as the ferromagnetic correlation length, zero-field susceptibility, the approach to saturation, etc\textsuperscript{12,13}.

The essence of the RA model can be explained in the following terms. The ferromagnetic exchange tends to align the spins in one direction but it has no preferred direction. In a crystalline body the latter, in the absence of the magnetic field, is determined by the magnetic anisotropy due to the violation of the rotational symmetry by the crystal lattice. Still, due to the time reversal symmetry, any two states with opposite directions of the magnetization have the same energy. In a macroscopic magnet this leads to the formation of ferromagnetically aligned magnetic domains. Magnetic particles of size below one micron typically consist of one such a domain.

This changes in an amorphous magnet. If no material anisotropy was introduced by a manufacturing process, such a magnet would be lacking global anisotropy axes. Random on-site magnetic anisotropy disturbs the local ferromagnetic order but cannot break it at the atomic scale $a$ because the RA energy per site, $D_R$, is small compared to the exchange energy $J$. The resulting magnetic state can be understood within the framework developed in the seminal papers of Larkin\textsuperscript{14} and Imry and Ma\textsuperscript{15}. Due to random local phases from the RA the magnetization wonders around the magnet at the nanoscale in a random-walk manner (see Fig. \ref{fig:random-walk}), with the ferromagnetic correlation length given by $R_f \propto (J/D_R)^{2/(4-d)}$, where $d$ is the dimensionality of the system.

This statement, known as the Imry-Ma (IM) argument, works for many systems with quenched randomness, such as disordered antiferromagnets\textsuperscript{16}, flux lattices in superconductor\textsuperscript{17}, charge-density waves\textsuperscript{18}, liquid crystals and polymers\textsuperscript{19,20}, and superfluid $^3$He-A in aerogels\textsuperscript{21}. In the case of the RA ferromagnet it suggests that the RA, no matter how weak, breaks the long range ferromagnetic order for $d = 1, 2, 3$, although the order can persist locally on the scale $R_f$ that can be large compared to the interatomic distance $a$ if $D_R \ll J$. For the same ratio $D_R/J$, the lower is the dimensionality of the system the smaller is the correlated region. (Formally, the long-
range order is restored in higher dimensions, \( d = 4, 5, \ldots \).

Fundamental feature of the RA model is that it can be rescaled in terms of spin blocks of size \( r > a \) with the effective \( D'_{R} \) and \( J' \) different from the original \( D_{R} \) and \( J \) up to the size \( r \) at which \( D'_{R} \sim J' \). This can be useful for numerical work but it also means that the model is intrinsically non-perturbative. It describes a strongly correlated system that cannot be treated perturbatively on small \( D_{R}/J \). The latter is evidenced by the IM result for the ferromagnetic correlation length \( R_{f} \).

One deficiency of the IM model is that it ignores topological defects\(^{23} \) (apparent in Fig. 1) that lead to metastability. It was recently argued that random field (RF) converts a conventional ferromagnet into a topological glass in which ferromagnetically correlated regions (often called IM domains) possess nonzero topological charge\(^{24} \). Although this argument was made for the RF rather than the RA, the two models have much in common due to the fact that the RA creates a local anisotropy field that acts on spins similarly to the RF. The RA model, however, is more nonlinear than the RF model. High metastability, history dependence, and memory effect\(^{25} \) exhibited by ferromagnets with random magnetic anisotropy reveal complex non-ergodic temporal behavior typical of spin glasses\(^{26} \).

Fueled by potential applications, there has been a large body of recent experimental research on the absorption of microwave radiation by nanocomposites comprised of magnetic nanoparticles of various shapes and dimensions, embedded in dielectric matrices\(^{27} \). With the use of more and more exotic shapes and materials, the complexity of such systems has increased dramatically in recent years\(^{28,29} \) but their evaluation as microwave absorbers has been largely empirical and often a matter of luck rather than driven by theory.

Here we investigate the microwave absorption by the RA magnet in a zero external field - situation the least obvious from the theoretical point of view and the least studied in experiments, although it must be the most interesting one for applications. Our goal is to understand the fundamental physics of the absorption of the ac power by the random magnet without focusing on material science. The reference to microwaves throughout the paper is determined by the outcome: The peak absorption happens to be in the microwave range due to the typical strength of the magnetic anisotropy.

We should emphasize that this problem is noticeably different from the microwave absorption by a nanocomposite. Coated magnetic particles or particles dissolved in a dielectric medium are absorbing the ac power more or less independently up to a weak dipole-dipole interaction between them. On the contrary, in the amorphous ferromagnet all spins are coupled by the strong exchange interaction and they respond to the ac field collectively. Metastability and magnetic hysteresis exhibited by the RA magnet\(^{30} \) makes such response highly nontrivial.

Our most interesting observation is that the absorption of the microwave power in the RA magnet is dominated by spin oscillations localized inside well-separated ferromagnetically correlated regions (IM domains) that at a given frequency are in resonance with the microwave field. In that sense there is a similarity with a nanocomposite where certain particles react resonantly to the ac field of a given frequency. However, the pure number of such areas in a random magnet must be greater due to the higher concentration of spins.

The paper is organized as follows. The RA model and the numerical method for computing the absorption of the ac power are introduced in Section II. Results of the computations are given in Section III. Interpretation of the results, supported by snapshots of oscillating spins, is suggested in Section IV. Estimates of the absorbed power and implications of our findings for experiments are discussed in Section V.

II. THE MODEL AND NUMERICAL METHOD

We consider Heisenberg RA model described by the Hamiltonian

\[
\mathcal{H} = -\frac{J}{2} \sum_{i,j} \mathbf{s}_i \cdot \mathbf{s}_j - \frac{D_R}{2} \sum_i (\mathbf{n}_i \cdot \mathbf{s}_i)^2 - \mathbf{h}(t) \cdot \sum_i \mathbf{s}_i, \tag{1}
\]

where the first sum is over nearest neighbors, \( s \) is a three component spin of a constant length \( s \), \( D_R \) is the strength of the easy axis RA, \( \mathbf{n}_i \) is a three-component unit vector having random direction at each lattice site, and \( \mathbf{h} \)
is the magnetic field in energy units. We assume ferromagnetic exchange, $J > 0$. Factor 1/2 in front of the first term is needed to count the exchange interaction $Js^2$ between each pair of spins once. In our numerical work we consider a chain of equally spaced spins in 1D, a square lattice in 2D, and a cubic lattice in 3D. For the real atomic lattice of square or cubic symmetry the single-ion anisotropy of the form $-(n \cdot s)^2$ would be absent, the first non-vanishing anisotropy terms would be fourth power on spin components. However, in our case the choice of the lattice is merely a computational tool that should not affect our conclusions.

The last term in Eq. (1) describes Zeeman interaction of the spins with the ac magnetic field of amplitude $h_0$ and frequency $\omega$. We assume that the wavelength of the electromagnetic radiation is large compared to the size of the system, so that the time-dependent field acting on the spins is uniform in space. This corresponds to situations of practical interest when the microwave radiation is incident to a thin dielectric layer containing random magnets. As in microscopic studies of static properties of random magnets, we assume that in the absence of net magnetization the dynamics of the spins is dominated by the local exchange and magnetic anisotropy and neglect the long-range dipole-dipole interaction between the spins. Adding it to the problem would have resulted in a considerable slowdown of the numerical procedure.

The effective exchange field acting on each spin from the nearest neighbors in $d$ dimensions is $2dJs$. In our model it competes with the anisotropy field of strength $2sD_R$. The case of a large random anisotropy, $2sD_R \gg 2dJs$, that is, $D_R \gg dJ$, is obvious. It corresponds to a system of weakly interacting randomly oriented spins, each spin aligned with the local anisotropy axis $n$. Due to the two equivalent directions along the easy axis the system possesses high metastability with the magnetic state depending on history.

On the contrary, weak anisotropy, $D_R \ll dJ$, cannot destroy the local ferromagnetic order created by the strong exchange interaction. The direction of the magnetization becomes only slightly disturbed when one goes from one lattice site to the other. As in the random walk problem, the deviation of the direction of the magnetization would grow with the distance. In a $d$-dimensional lattice of spacing $a$ the average statistical fluctuation of the random anisotropy field per spin in a volume of size $R$ scales as $D_{eff} = 2sD_R(a/R)^{d/2}$. Since Heisenberg exchange is equivalent to $J(\nabla s)^2$ in a continuous spin-field model, the ordering effect of the exchange field scales as $2dJs(a/R)^2$. The effective exchange and anisotropy energies become comparable at $R \sim R_f$, where

$$R_f \sim a(dJ/D_R)^{2/(4-d)}$$

(2)
determines the ferromagnetic correlation length. The exact numerical factor in front of $(J/D_R)^{2/(4-d)}$ is unknown but the existing approximations and numerical results suggest that it increases progressively with the dimensionality of the system.

Since magnetic anisotropy has relativistic origin its strength per spin is usually small compared to the exchange per spin. Anisotropy axes in the amorphous ferromagnet are determined by the local arrangement of atoms. When the latter has a short range order the axes are correlated within structurally ordered grains whose size must replace $a$ in the RA model. This results in a greater effective RA, making both limits, $D_R \ll dJ$ and $D_R \gg dJ$, relevant to amorphous ferromagnets.

As to the Zeeman interaction of the ac field with the spins, that is determined by the amplitude of $h$ in Eq. (1), in all situations of practical interest it would be smaller than all other interactions by many orders of magnitude. It is worth noticing, however, that for a sufficiently large system the random energy landscape created by the RA and the ferromagnetic exchange would have all energy scales, including that of $h$. This, in principle, may inject nonlinearity into the problem with a however small amplitude of the ac field.

The undamped dynamics of the system is described by the Larmor equation

$$h\dot{s}_i = s_i \times H_{eff,i}, \quad H_{eff,i} \equiv -\frac{\partial H}{\partial s_i}$$

(3)

with $H(t) = h_0 \sin(\omega t)$. Here, one also can include in the usual way a small phenomenological damping $\alpha \ll 1$ by adding the term $-\alpha s_i \times (s_i \times H_{eff,i})$ to the first of Eqs. (3). However, as will be discussed later, a random magnet has a continuous distribution of resonances (normal modes) that makes the power absorption insensitive to the small damping. Using the Larmor equation, one can obtain the relation

$$\dot{H}(t) = -\dot{h}(t) \cdot \sum_i s_i(t),$$

(4)

where the left-hand side is the rate of change of the total energy (power absorption) and the right-hand side is the work of the ac field on the magnetic system per unit of time. For a conservative system this gives two ways of calculating the absorbed power numerically, which is important for checking the self-consistency and accuracy of computations.

When the phenomenological damping is included, the energy of the spin system saturates in a stationary state in which the power absorption is balanced by dissipation. On the contrary, the work done by the ac field continues to increase linearly period after period of the ac field. In this case the work done by the ac field becomes the single measure of the absorbed power. For a large system, a long computing time is needed to reach saturation, especially when the damping is small. Fortunately, in most cases the absorbed power can be already obtained with a good accuracy from a short computation on a conservative system, typically using 5 periods of the ac field.

One other complication arises from the necessity to keep the amplitude of the ac field as small as possible in relation to the exchange and anisotropy to reflect situations of practical interest. In this case, one can expect
normal modes to respond to the ac field independently. However, decreasing the amplitude of the ac field below a certain threshold increases computational errors. Large amplitude of the ac field causes resonant group of the normal modes to oscillate at higher amplitudes, which triggers nonlinear processes of the energy conversion.

We use the following procedure. At the first stage, the magnetic state in zero field is prepared by the energy minimization starting from random orientation of spins. This reflects the process of manufacturing of amorphous ferromagnet by a rapid freezing from the paramagnetic state in the melt. The numerical method\(^{33}\) combines sequential rotations of spins \(s_i\) towards the direction of the local effective field, \(\mathbf{H}_{\text{eff},i}\), with the probability \(\eta\), and the energy-conserving spin flips (overrelaxation), \(s_i \rightarrow 2(s_i \cdot \mathbf{H}_{\text{eff},i})\mathbf{H}_{\text{eff},i}/H_{\text{eff},i}^2 - s_i\), with the probability \(1 - \eta\). We used \(\eta = 0.03\) that ensures the fastest relaxation.

At the end of this stage, a disordered magnetic state with the ferromagnetic correlation length \(R_f\) is obtained (see Fig. 1).

At the second stage, the ac field is turned on and Eq. (3) is solved with the help of the classical fourth-order Runge-Kutta method. We also have tried the 5th order Runge-Kutta method by Butcher\(^{22}\) that makes six function evaluations per time step. This method can be faster as it allows a larger time step for the same accuracy. However, for the RA model it shows instability and has been discarded. The main computation was parallelized for different ac frequencies \(\omega\) in a cycle over 5 periods of the ac field. Long computation was required at low frequencies. Wolfram Mathematica with compilation on a 20-core Dell Precision Workstation was used. In the computations, we set \(s = \hbar = J = 1\). In most cases the integration step was \(\Delta t = 0.1\) or 0.05. We computed the absorbed power in 1D, 2D, and 3D systems with a number of spins \(N\) up to 400,000.

In three dimensions, the ferromagnetic correlation length at small \(D_R\) is very large and can easily become longer than the system size. In this case, the magnetization per spin \(m = |\mathbf{m}|\), where \(\mathbf{m} \equiv (1/N) \sum_i s_i\) is the average spin polarization, may be far from zero at the energy minimum. For the sake of uniformity, the ac field was always applied in the direction perpendicular to \(\mathbf{m}\) for which the power absorption is stronger than in the direction parallel to \(\mathbf{m}\). The absorbed power \(P_{\text{abs}}\) was obtained by numerically integrating the left or right side of Eq. (4) during \(N_T\) periods of the ac field and dividing the result by \(N_T T\) with \(T = 1/f = 2\pi/\omega\). In most cases we used \(N_T = 5\). It was numerically confirmed that \(P_{\text{abs}} \propto h_0^2\), so, in the plots we show \(P_{\text{abs}}/h_0^2\) per spin.

### III. NUMERICAL RESULTS

To test our short-time method of computing the absorbed power, we performed longer computations and plotted the absorbed energy vs time for the integer number of periods of the ac field, \(t = nT, n = 0, 1, 2, \ldots\) For the undamped model, \(\alpha = 0\), both methods of computing the absorbed energy discussed in the previous section give the same result, which proves sufficient computational accuracy. For the damped model, the energy of the system saturates at long times while the magnetic work, obtained by integrating the right-hand side of Eq. (4) continues to increase linearly.

An example of these tests is shown in Fig. 2 for a 2D system of 300 \(x\) 300 spins with \(D_R/J = 1\), \(\hbar \omega/J = 0.2\), the ac-field amplitude \(h_0/J = 0.01\), and different values of the damping constant \(\alpha\). For the unrealistically high damping \(\alpha = 0.1\) the absorbed energy line goes higher than the other dependences. For \(\alpha = 10^{-2}, 10^{-3},\) and \(10^{-4}\) the magnetic work is practically the same. This confirms our conjecture about a continuous distribution of resonances for which the absorption does not depend of the resonance linewidths, see the next section.

The plot in Fig. 2 at \(\alpha = 10^{-4}\) shows the increase of the energy of the system, \(\Delta E\), for the damping that saturates after about 100 periods of the ac field. Such a long computing time would be impractical for the computation of the dependence of the absorbed power on frequency and RA strength. Fortunately, the short-time behavior of the absorbed energy is the same in all cases, except for the extremely high damping at \(\alpha = 0.1\). This allowed us to compute the frequency dependence of the absorbed power by using the undamped model and the short computing time \(t = N_T T\) with \(N_T = 5\) or 10. Still, the computation for a large system and low frequency was rather long.

The choice of the amplitude of the ac-field is important for numerical work. Large \(h_0\) reduces numerical noise while leading to the flattening of the absorption maxima due to a partial saturation. Small \(h_0\) increases computational errors. We have chosen the values \(h_0/J = 0.0001\) in 1D (where numerical errors are the smallest), \(h_0/J = 0.0003\) in 2D, and \(h_0/J = 0.003\) in 3D.

The frequency dependence of the absorbed power for in a 1D RA ferromagnet is shown in Fig. 3. This is the easiest case computationally. One can use a long chain of spins (here \(N = 30000\)) that is much longer than the magnetic correlation radius \(R_f\) in 1D. Thus, after the energy minimization the system remains well disordered, \(m \ll 1\). Fig. 3 shows broad absorption maxima shifting to lower frequencies with decreasing \(D_R\). The heights of the maxima are approximately the same. At large frequencies, there is a cut-off at the highest spin-wave frequency, \(\omega_{\text{max}} = 4dJ \Rightarrow 4J\) in 1D.

Frequency dependence of the absorbed power for the 2D model is shown in Fig. 4. Qualitatively the results are the same as in 1D, only we used a larger system of \(N = 340 \times 300 = 102000\) spins. Here, we were able to go down to the RA only as small as \(D_R/J = 0.2\) as compared to 0.1 in 1D because the absorption maximum
is shifting to very low frequencies on decreasing RA.

Frequency dependence of the absorbed power for the 3D model is shown in Fig. 4. Again, the absorption curves are similar to 1D and 2D. However, the 3D model is the hardest to crack numerically because the ferromagnetic correlation length $R_f^2$, given by Eq. 2 with $d = 3$, becomes very large at small $D_R$. We had to use 3D systems of a much greater number of spins, $68 \times 74 \times 80 = 402560$ and $75 \times 80 \times 84 = 504000$, but of smaller lateral dimensions than 1D and 2D systems that we have studied. The lowest RA for which we could observe the absorption maximum in 3D was $D_R/J = 2$. For lower $D_R$ the absorption maxima shift to very low frequencies for which computation becomes impractically long and inhibited by the accumulation of numerical errors.

Frequency dependence of the power on the right side of the absorption maximum allows scaling shown in Fig. 6. We have found that $P(\omega)$ in this region follows the power law:

$$P_{abs} \propto \frac{D_R^2}{J^2} \left( \frac{J}{\hbar \omega} \right)^{(4-d)/2}.$$  (5)

up to the high-frequency cutoff determined by the strength of the exchange interaction. Away from the
maximum the absorption in this high-frequency region is lower in higher dimensions. However the heights of the absorption maxima are comparable in all dimensions, see figures 3, 4, and 5. The maximum absorption has weak dependence on the strength of the RA and the strength of the exchange interaction. By order of magnitude it is given by \( P_{\text{max}} \sim h_0^2 N / \hbar \).

**IV. INTERPRETATION OF THE RESULTS**

**A. Independence of the phenomenological damping**

One remarkable feature of the ac power absorption by the RA magnet is its independence of the damping within a broad range of the damping constant \( \alpha \ll 1 \), see Fig. 2. While we do not have a rigorous theory that explains this phenomenon, it can be understood along the lines of the qualitative argument presented below.

The absorption power by a conventional ferromagnet near the FMR frequency, \( \omega \approx \omega_0 \), has a general form

\[
P(\omega) \propto h_0^2 \omega_0^2 \left( \frac{\alpha \omega_0^2 G}{(\omega^2 - \omega_0^2)^2 + (\alpha \omega_0^2 G)^2} \right)
\]

with \( G \) being a geometrical factor depending on the polarization of the ac-field and the structure of the magnetic anisotropy. At \( \alpha \to 0 \) it becomes

\[
P(\omega) \propto h_0^2 \omega_0^2 \delta(\omega^2 - \omega_0^2) \propto h_0^2 \omega_0 \delta(\omega - \omega_0).
\]

In an amorphous ferromagnet one should expect many resonances characterized by some distribution function \( f(\omega_0) \) satisfying

\[
\int d\omega_0 f(\omega_0) = 1.
\]

For the power absorption at a frequency \( \omega \) one obtains

\[
P(\omega) \propto \int d\omega_0 f(\omega_0) h_0^2 \omega_0 \delta(\omega - \omega_0) = h_0^2 \omega f(\omega)
\]

which is independent of \( \alpha \). The function \( f(\omega_0) \) for the RA magnet is unknown. It is related to a more general poorly understood problem of excitation spectrum of systems characterized by a random potential landscape that we are not attempting to solve here. Based upon our numerical results, an argument can be made, however, that sheds light on the physics of the absorption by the RA magnet, see below.

**B. Estimation of the maximum-absorption frequency**

The spin field in the RA ferromagnet resembles to a some degree a domain structure or magnetization of a sintered magnet comprised of densely packed single-domain magnetic particles, see Fig. 1. The essential difference is the absence of boundaries between IM domains. They are more of a reflection of the disordering on the scale \( R_f \) than the actual domains. If one nevertheless thinks of the IM domains as independent ferromagnetically ordered regions of size \( R_f \), their FMR frequencies, in the absence of the external field, would be dominated by the effective magnetic anisotropy, \( D_{\text{eff}} \), due to statistical fluctuations in the distribution of the RA axes. In this case the most probable resonance frequency that determines the maximum of \( P(\omega) \) must be given by

\[
\omega_{\text{max}} \sim D_{\text{eff}} \sim D_R (a/R_f)^k/2.
\]
Substituting here $R_f \sim k(d)(J/D_R)^{2/(4-d)}$, with the factor $k(d)$ increasing progressively with $d$, we obtain

$$\frac{\omega_{\text{max}}}{J} = \frac{1}{kd^{d/2}} \left( \frac{D_R}{J} \right)^{4/(4-d)}.$$  \hspace{1cm} (11)

It suggests that $\omega_{\text{max}}/J$ must scale as $(J/D_R)^{4/3}$ in one dimension, as $(J/D_R)^2$ in two dimensions, and as $(J/D_R)^4$ in three dimensions.

The dependence of $\omega_{\text{max}}/J$ on $J/D_R$ for $d = 1, 2, 3$ derived from figures 3, 4, and 5 is shown in Fig. 7. In 2D and 3D there is a full agreement with the above argument. In 1D the best fit seems to be the $3/2$ power of $J/D_R$ instead of the expected $4/3$ power. Given the qualitative nature of the argument presented above and good fit for $d = 2, 3$ the agreement is nevertheless quite good. The small factor in front of the power of $J/D_R$, that becomes progressively smaller as one goes from $d = 1$ to $d = 3$, correlates with the established fact that $k(d)$ increases with $d$.

It is suggested that spin deviations occur inside resonant IM domains. Noticeable deviations occur at discrete locations. Their amplitude is apparently determined by how well the frequency of the ac field matches the resonant frequency of the IM domain at that location. The oscillating domains appear to be well separated in space.

One of the oscillating regions of Fig. 8 is zoomed at in Fig. 9. It shows that the oscillations quickly go to zero away from the region. The width of the region correlates with the expected value of $R_f$.

**C. Visualization of the local dynamical modes**

Further evidence of the validity of our picture that the power absorption occurs inside resonant IM domains comes from the analysis of the spatial dependence of local spin deviations from the initial state $s_i^{(0)}$, defined as $\Delta s_i^2 = (s_i - s_i^{(0)})^2$. They are related to local spin oscillations and are illustrated in Fig. 8 for a 1D RA ferromagnet. Noticeable deviations occur at discrete locations. Their amplitude is apparently determined by how well the frequency of the ac field matches the resonant frequency of the IM domain at that location. The oscillating domains appear to be well separated in space.

One of the oscillating regions of Fig. 8 is zoomed at in Fig. 9. It shows that the oscillations quickly go to zero away from the region. The width of the region correlates with the expected value of $R_f$.

**Figure 8:** Spatial dependence of spin deviations in a 1D RA ferromagnet.

**Figure 9:** Spatial dependence of spin deviations in one area of resonant absorption.

**Figure 10:** Spatial dependence of spin deviations in a 2D RA ferromagnet.

**Figure 10** shows oscillating regions in a 2D RA ferromagnet. Here again the spin regions that absorb the ac power are well separated in space. This is in line with our picture that they correspond to the resonant IM domains in which the effective magnetic anisotropy due to statistical fluctuations of easy axis directions matches the frequency of the ac field.

Note that the peaks in figures 8, 9, and 10 are stationary, they go up and down in time, but their locations in space are fixed. The snapshots shown in these figures have been taken at particular moments in time.

At present we do not have the theory of the full dependence of the power on frequency. Apparently, it is related
to the size distribution of ferromagnetically correlated regions (Imry-Ma domains), which remains a challenging unsolved problem of statistical mechanics. Our numerical findings, however, may shed some partial light onto this problem.

Indeed, we have found that at large frequencies the power absorption follows Eq. (5): \( P \propto \omega^{(d/2) - 2} \). If it is related to the precession of IM domains of size \( R \), then according to Eq. (10) the frequency of this precession scale as \( \omega \sim R^{-d/2} \). Since this is a high-frequency regime, it must correspond to small \( R \). According to Eq. (9) \( P \propto \omega f(\omega) \). Combined with Eq. (5) it gives \( f(\omega) \propto \omega^{(d/2) - 3} \) at large \( \omega \). If distribution of IM domains is given by \( F(R) \) satisfying \( \int dRF(R) = \int d\omega f(\omega) = 1 \), then, using the above formulas we obtain

\[
F(R) = f(\omega) \frac{d\omega}{dR} \propto R^{-\left(\frac{d}{2} - 1\right)^2}.
\]  

This suggests \( R^{-1/4} \) distribution of small-size IM domains in 1D and 3D, and independence of \( R \) (up to a log factor) in 2D.

The qualitative argument leading to Eq. (12) is based upon the picture of independently oscillating IM domains. In reality there are no boundaries between ferromagnetically correlated regions. Our derivation suggests a large fraction of compact correlated regions of size that is small compared to the ferromagnetic correlation length \( R_f \). It is supported by Fig. 1 but is different from the prediction of the exponentially small number of such regions in the random-field xy model made within variational approach\(^{33}\).

V. DISCUSSION

We have studied the power absorption by the random-anisotropy ferromagnet in a microwave field in one, two, and three dimensions. The one-dimensional problem describes a thin wire of diameter smaller than the 1D ferromagnetic correlation length \( R_f \) and of length greater than \( R_f \). The two-dimensional problem corresponds to a film of thickness that is small compared to the 2D ferromagnetic correlation length and of lateral dimension large compared to \( R_f \). The three-dimensional problem corresponds to a particle of amorphous ferromagnet of size large compared to the 3D ferromagnetic correlation length.

Our main finding agrees with the statements made by experimentalists\(^{34}\). It elucidates the physics of the microwave absorption by a RA ferromagnet. The absorption is localized inside well separated regions. Scaling of the peak absorption frequency with the strength of the RA points towards the mechanism of the absorption in which oscillations of spins are dominated by isolated ferromagnetically correlated regions (Imry-Ma domains) that are in resonance with the microwave field.

Broad distribution of sizes of ferromagnetically correlated regions results in the broad distribution of resonance frequencies. It makes the absorption broadband, with the frequency at half maximum covering two orders of magnitude. Another consequence of such a broad distribution is independence of the absorption on the damping of spin oscillations within a few orders of magnitude of the damping constant.

A remarkable observation is that the maximum of the absorbed power in a random magnet has a weak dependence on basically all parameters of the system, such as dimensionality, damping, the strength of the RA, and the strength of the exchange interaction. By the order of magnitude it is determined solely by the total number of spins absorbing the microwave energy and the amplitude of the microwave field. This again is a consequence of the broad distribution of sizes of ferromagnetically correlated regions, causing broad distribution of the effective magnetic anisotropy and effective exchange interaction.

A practical question is whether the RA (amorphous) magnets have a good prospect as microwave absorbers. The power absorption by the random magnet occurs in the broad frequency range. Frequencies that provide the maximum of the absorption depend on the strength of the RA. The latter can be varied by at least two orders of magnitude by choosing soft or hard magnetic materials in the process of manufacturing an amorphous magnet. It must allow the peak absorption in the range from a few GHz to hundreds of GHz.

As we have already mentioned, the power absorption at frequencies near the absorption maximum depends weakly on the parameters of the system. By order of magnitude it equals

\[
P_{\text{max}} \sim h_0^2 N/\hbar = 4\mu_B^2 S^2 B_0^2 n_0 Ad/\hbar.
\]  

Here we have written \( h_0 = 2\mu_B S B_0 \) in terms of the length of the dimensionless spin \( S \) and the dimensional amplitude, \( B_0 \), of the microwave field, with \( \mu_B \) being the Bohr magneton. The number of spins in the absorbing layer, \( N = n_0 Ad \), has been expressed in terms of the concentration of spins \( n_0 \), the area \( A \) and the thickness \( d \) of the layer.

The incoming microwave delivers to the layer the power \( P_m = (eB_0^2/2\mu_0)A \), where \( \mu_0 \) is the permeability of free space\(^{35}\). This gives for the ratio of the absorbed and incoming powers

\[
\frac{P_{\text{max}}}{P_m} \sim \frac{8S^2 \mu_0 \mu_B^2 n_0 d}{\hbar c}.
\]  

For a rough estimate, at \( S = 2 \) and \( n_0 \sim 2 \times 10^{27} \text{m}^{-3} \), this ratio becomes of order unity at \( d \sim 5 \text{mm} \). It suggests that a layer of such thickness composed of magnetically dense amorphous wires, foils, and particles, thinly coated to prevent reflectivity due to the electric conductance, may be a strong absorber of the microwave radiation. Parameter-dependent numerical factor of order unity that we omitted in Eq. (13) may slightly favor low-dimensional systems in that respect.
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