Random Double Auction: A Robust Bilateral Trading Mechanism

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I construct a novel random double auction as a robust bilateral trading mechanism for a profit-maximizing intermediary who facilitates trade between a buyer and a seller. It works as follows. The intermediary publicly commits to charging a fixed commission fee and randomly drawing a spread from a uniform distribution. Then the buyer submits a bid price and the seller submits an ask price simultaneously. If the difference between the bid price and the ask price is greater than the realized spread, then the asset is transacted at the midpoint price, and each pays the intermediary half of the fixed commission fee. Otherwise, no trade takes place, and no one pays or receives anything. I show that the random double auction maximizes the worst-case expected profit across all dominant-strategy incentive compatible and ex-post individually rational mechanisms for the symmetric case. I also construct a robust trading mechanism with similar properties for the asymmetric case.

The full paper is available at https://arxiv.org/abs/2105.05427

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At every moment, a huge amount of trades are facilitated by intermediaries charging fees for their intermediary services in matching buyers with sellers. In reality, there are many situations in which the uncertainty of the value of the asset being traded is huge, e.g., a newly public stock, and Tesla’s new model. Intermediaries may then know little of the concerned parties’ willingness to trade and only have an overall estimate about it. Given the huge uncertainty towards the two-sided market, it is natural for the intermediary to seek for a trading mechanism that guarantees a good profit. How should a profit-maximizing intermediary design trading rules in such situations? Would the intermediary still be able to guarantee a positive profit and thus have strict incentives to offer intermediary services?

To answer these questions, I study the design of profit-maximizing bilateral trading mechanisms when the intermediary has limited information about the value distribution of the buyer and the seller. Specifically, I assume that the intermediary knows only the expectations of the private values of the buyer and the seller, but does not know the joint distribution of the private values. I refer to any joint distribution consistent with the known expectations as a feasible value distribution. The intermediary evaluates any trading mechanism by the expected profit in the worst case over all feasible value distributions, referred to as the profit guarantee. The objective of the intermediary is to find a trading mechanism, referred to as a maxmin trading mechanism, that maximizes the profit guarantee across all dominant-strategy incentive compatible (DSIC) and ex-post individually
The main finding is that a random double auction is a maxmin trading mechanism in the symmetric case in which the known expectations of the values of the buyer and the seller sum up to the upper bound of the values, which is normalized to 1. This mechanism essentially works as follows.

**Step 0:** fixed commission fee. The intermediary publicly commits to charging a fixed commission fee \( r > 0 \).

**Step 1:** uniformly random spread. The intermediary publicly commits to randomly drawing a spread \( s \) uniformly on \([r, 1]\). Then a random spread is drawn whose realization is not observed by either the buyer or the seller. The buyer and the seller both know \( r \) and the uniform distribution on \([r, 1]\) from which the random spread is drawn.

**Step 2:** midpoint transaction price. The buyer submits a bid price \( b \), and the seller submits an ask price \( a \), simultaneously. If the difference between the bid price and the ask price is greater than the random spread, or \( b - a > s \), then the seller sells the asset to the buyer at the midpoint price \( \frac{b + a}{2} \), and each pays the intermediary half of the fixed commission fee \( \frac{r}{2} \). Otherwise, no trade takes place, and no one pays or receives anything.

Notably, the features of the random double auction are familiar in the real world. First, there is extensive evidence that the bid-ask spread of a given financial asset is not constant, but varies over time, e.g., the New York Stock Exchange (NYSE) stocks and Chicago Board Options Exchange (CBOE) options. Second, brokerage firms often adopt the fixed-commission practice, e.g., Interactive Brokers offers fixed-commission plans for many financial assets; E*TRADE charges a fixed commission per contract for futures contracts. Third, a double auction is widely used in stock exchanges as well as in dark pools, e.g., the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange (TSE) use a double auction to determine the opening prices; block-trading dark pools such as Liquidnet or POSIT typically match orders at the midpoint of the prevailing bid-ask prices. To my knowledge, the random double auction is a novel trading mechanism that combines the above three commonly observed features.

To show that the random double auction is a maxmin trading mechanism, I reformulate the intermediary’s problem into a zero-sum game between the intermediary and adversarial nature who chooses a feasible value distribution to minimize the expected profit, and then I construct a feasible value distribution, referred to as a worst-case value distribution, such that the random double auction and the worst-case value distribution form a Nash equilibrium of the zero-sum game.

I find that the worst-case value distribution is a symmetric triangular value distribution that can be described as follows. The support is a symmetric triangular subset in the set of joint valuations, which is the same as the trading region of the random double auction. The marginal distribution for the buyer is a combination of a uniform distribution on \((r, 1)\) and an atom on 1, while for the seller is a combination of a uniform distribution on \((0, 1 - r)\) and an atom on 0. The conditional distribution is some truncated generalized Pareto distribution with an atom on 1 (resp, 0) for the buyer (resp, the seller).

I extend my analysis to constructing a maxmin trading mechanism for the asymmetric case in which the known expectations of the values of the buyer and the seller sum up to a number other than 1. I find that the maxmin trading mechanism shares similar properties to the random double auction in that 1) trade is restricted: trade does not take place if the difference between a weighted bid price and a weighted ask price falls below some threshold; 2) trade takes place randomly otherwise, albeit with a different randomization device.