Affleck-Dine leptogenesis in the radiative neutrino mass model

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Abstract
Radiative neutrino mass models have interesting features, which make it possible to relate neutrino masses to the existence of dark matter. However, the explanation of the baryon number asymmetry in the universe seems to be generally difficult as long as we suppose leptogenesis based on the decay of thermal right-handed neutrinos. Since right-handed neutrinos are assumed to have masses of $O(1)$ TeV in these models, they are too small to generate the sufficient lepton number asymmetry. Here we consider Affleck-Dine leptogenesis in a radiative neutrino mass model by using a famous flat direction $LH_u$ as an alternative possibility. The constraint on the reheating temperature could be weaker than the ordinary models. The model explains all the origin of the neutrino masses, the dark matter, and also the baryon number asymmetry in the universe.

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1 Introduction

Recent observations of the existence of neutrino masses [1] and dark matter (DM) [2] are crucial ingredients to consider physics beyond the standard model (SM). The origin of the baryon number asymmetry in the universe also remains an unsolved problem in the SM [3]. Both the neutrino masses and the baryon number asymmetry are known to be explained in a unified way through the leptogenesis scenario in the framework of the seesaw mechanism [4]. Extensive studies on this subject have been done during recent years [5]. On the other hand, supersymmetry can play a crucial role for the explanation of the DM abundance in the universe [6], although it has been introduced originally to solve the hierarchy problem. Supersymmetric models have a good candidate for the DM as the lightest superparticle (LSP) as long as $R$-parity is conserved. The neutralino LSP has been studied as a DM candidate in both the supersymmetric SM (MSSM) and its singlet extensions [6, 7, 8].

We may consider both the leptogenesis and the explanation of the DM abundance in supersymmetric models, simultaneously. In that case, the out-of-equilibrium decay of thermal heavy neutrinos can generate the sufficient baryon number asymmetry only if the reheating temperature $T_R$ is high enough such as $10^8$ GeV or more [3]. If such a high reheating temperature is required, however, we confront the serious gravitino problem [9, 10]. Various trials to overcome this difficulty have been done by searching scenarios to enhance the $CP$ asymmetry or lower the required reheating temperature [11, 12, 13].

Recently, radiative neutrino mass models gather attentions as candidates for physics at a TeV scale [14, 15, 16]. The right-handed neutrinos and other new fields are assumed to have the masses of $O(1)$ TeV in many of such models. In this framework the small neutrino masses can be generated since the Dirac neutrino masses are assumed to be forbidden at tree-level by some symmetry such as $Z_2$ [14]. Since the lightest one with this odd parity is stable because of this symmetry, it can be one of the DM candidates [15]. This situation is the same as the LSP in the $R$-parity conserved MSSM. Unfortunately, the leptogenesis based on the decay of a thermal right-handed neutrino with the $Z_2$ odd parity to the DM can not generate the sufficient lepton number asymmetry since it is too light. To remedy this fault, a hybrid model using both the radiative seesaw and the ordinary seesaw is proposed in a nonsupersymmetric framework [17]. However, if we try to make this hybrid model supersymmetric, we confront again the gravitino problem as
long as we insist on the decay of the thermal right-handed neutrinos. Thus, it seems to be difficult to reconcile this type of supersymmetric radiative neutrino mass models with the thermal leptogenesis.

In this paper we propose an alternative possibility for the leptogenesis in the supersymmetric radiative neutrino mass model, which can closely relate both the origin of the neutrino masses and the DM abundance. We apply Affleck-Dine (AD) mechanism \cite{18} based on a famous flat direction $LH_u$ to the model. Since the neutrino mass generation is irrelevant to this flat direction in this model, the model shows different features from the ones found in the previous articles \cite{19,20}. We estimate the baryon number asymmetry generated through this leptogenesis based on the AD mechanism and also the constraint on the reheating temperature.

The remaining parts are organized as follows. In section 2 we address the model and the flat direction considered here. In section 3 we estimate the baryon number asymmetry generated through the evolution of this flat direction. We show that the model can give us a simple and consistent picture for the explanations of the neutrino masses, the DM abundance, and the baryon number asymmetry. In section 4 we summarize the paper.

2 A flat direction in the model

2.1 The radiative neutrino mass model

We consider an extension of the MSSM with three singlet chiral superfields $N_i$, two extra doublet chiral superfields $\eta_u$ and $\eta_d$, and also an additional singlet chiral superfield $\phi$. \cite{21,22} Lepton number $L$ is assigned to these superfields as $L(N_i) = L(\phi) = 0$ and $L(\eta_u) = -L(\eta_d) = -1$. All their scalar components are assumed to have no vacuum expectation values. We introduce a $Z_2$ symmetry in addition to the ordinary $R$-parity. The charge assignment for these discrete symmetries are summarized in Table 1. As a result of these discrete symmetries, we have two DM components, that is, the lightest ordinary neutralino $\chi$ and the lightest $Z_2$ odd neutral field. In the following discussion, we assume that the lightest $N_i$ is lighter than $\eta_{u,d}$ and $\phi$ among the $Z_2$ odd fields. We require that the gauge invariant superpotential constructed by these chiral superfields

\footnote{A similar supersymmetric model considered in a different context can be found in \cite{23}.}
should also be invariant under the $R$-parity and also the $Z_2$ symmetry.

The invariant renormalizable superpotential is expressed as

\[
W = h^U_{ij}Q_i\bar{U}_jH_u + h^D_{ij}Q_i\bar{D}_jH_d + h^E_{ij}Q_i\bar{E}_jH_d + \mu_H H_uH_d, \\
+ h^N_{ij}L_i\bar{N}_j + \lambda_u\eta_u H_d\phi + \lambda_d\eta_d H_u\phi + \mu_\eta\eta_\eta + \frac{1}{2}M_iN_i^2 + \frac{1}{2}\mu_\phi\phi^2,
\]

(1)

where all couplings and mass parameters are supposed to be real, for simplicity. The MSSM superpotential is contained in the first line. The second line includes the additional terms to the MSSM. They are relevant to the radiative neutrino mass generation. Following the lepton number assignment to the fields shown in Table 1, the lepton number is violated only through the Yukawa couplings $\lambda_u\eta_u H_d\phi$ and $\lambda_d\eta_d H_u\phi$. Since detailed discussion on the neutrino mass generation and the DM abundance in this model can be found in [22], we do not repeat it. Here we review only some important features of the model for the following discussion on the leptogenesis based on the Affleck-Dine mechanism.

The neutrino masses are generated through the one-loop diagrams generated by the terms in the second line of eq. (1). An interesting point in this mass generation is that the tri-bimaximal MNS matrix is automatically realized if the simple flavor structure is assumed for the neutrino Yukawa couplings such as

\[
h^N_{\alpha i} = 0, \quad h^N_{\mu i} = h^N_{\tau i} (i = 1, 2), \quad h^N_{\alpha 3} = h^N_{\mu 3} = -h^N_{\tau 3}.
\]

(2)

Moreover, if we consider that chiral superfield $\phi$ is much heavier than the chiral superfields $N_i$ and $\eta_{u,d}$, the dominant contribution to the neutrino masses takes a very simple form

\[
\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} (h^2_{\tau 1}\Lambda_1 + h^2_{\tau 2}\Lambda_2) + \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} h^2_{\tau 3}\Lambda_3.
\]

(3)
The scales for the neutrino masses are determined by $\Lambda_i$ which is defined as

$$\Lambda_i = \frac{\bar{\lambda} v^2 M_i}{16\pi^2} \left( g(M_i, m_{\eta^+}) - g(M_i, m_{\eta^-}) \right), \quad \bar{\lambda} = \lambda_u \lambda_d \tan \beta,$$



$$g(m_a, m_b) = \frac{m_b^2 - m_a^2 + m_a^2 \ln(m_a^2/m_b^2)}{(m_b^2 - m_a^2)^2},$$



(4)

where $\langle H_u^0 \rangle = v \sin \beta$ and $\langle H_d^0 \rangle = v \cos \beta$. $m_{\eta^\pm}$ are the mass eigenvalues of the neutral scalar components of $\eta_u, \eta_d$, which are defined as $m_{\eta^\pm}^2 = \mu_{\eta}^2 + m_0^2 \pm B \mu_{\eta}$ by using supersymmetry breaking parameters $m_0^2$ and $B$. As long as $\mu_{\eta}$ and $M_i$ are assumed to be $O(1)$ TeV and $\bar{\lambda}$ is sufficiently suppressed as $\bar{\lambda} = O(10^{-8})$, this neutrino mass matrix can explain the neutrino oscillation data consistently with both constraints from the lepton flavor violating processes and the DM abundance [22]. In that case, since the Yukawa couplings $\lambda_u$ and $\lambda_d$ take very small values of $O(10^{-4})$ for the $O(1)$ neutrino Yukawa couplings $h_{ij}^N$, the lepton number violation in eq. (1) is found to be largely suppressed.

Here it is useful to give a remark on the parameters in eq. (1). In the above review of the neutrino masses, one might consider that a lot of ad hoc assumptions have made for the coupling constants and the mass parameters. However, as discussed in [22], they could be justified if we suppose to embed the $Z_2$ symmetry in an anomalous U(1) symmetry. In that case the superpotential $W$ is considered as an effective one induced from nonrenormalizable interaction terms as a result of the spontaneous breaking of the anomalous U(1) symmetry due to vacuum expectation values (VEVs) of singlet fields $\Sigma^+_\pm$ and $\Sigma^-_\pm$ at high energy regions. The coupling constants and the mass parameters in the superpotential $W$ are expressed by using $\langle \Sigma^+_\pm \rangle$ or $\langle \Sigma^-_\pm \rangle$ as

$$h_{ijk} = y_{ijk} \left( \frac{\langle \Sigma^\pm \rangle}{M_{pl}} \right)^{n_{ijk}}, \quad n_{ijk} = -\frac{X_i + X_j + X_k}{X_{\Sigma^\pm}}$$

for $h_{ijk} \Psi_i \Psi_j \Psi_k,$

$$\mu_{ij} = y_{ij} M_{pl} \left( \frac{\langle \Sigma^+ \rangle}{M_{pl}} \right)^{n_{ij}}, \quad n_{ij} = -\frac{X_i + X_j}{X_{\Sigma^+}}$$

for $\mu_{ij} \Psi_i \Psi_j,$

(5)

where $X_i$ stands for the anomalous U(1) charge of the chiral superfield $\Psi_i$. The coupling constants $y_{ijk}$ and $y_{ij}$ in the nonrenormalizable interaction terms in the original superpotential at high energy regions are supposed to be $O(1)$ naturally. If the anomalous U(1) charge is assigned appropriately and the singlet scalars $\Sigma^\pm_\pm$ obtain favorable VEVs, these VEVs cause the hierarchical structure in the Yukawa couplings of quarks and leptons, which realizes the favorable mass eigenvalues and mixing angles. Moreover, several parameters including $\lambda_u, \lambda_d$ in the superpotential $W$ are properly suppressed through eq. (5).
Such examples can be found in [22]. We adopt this picture and use the example given there. This example gives the following values for the parameters relevant to the present discussion:

\[ h_{ij}^N = O(1), \quad M_i, \, \mu_\eta = O(1) \text{ TeV}, \quad \lambda_{u,d} = O(10^{-4}), \quad \mu_\phi = O(10^8) \text{ TeV}. \]  

(6)

2.2 A flat direction

We now consider a flat direction of this model which is defined by a single complex field \( \phi \) as

\[ L_i = \begin{pmatrix} \varphi \\ 0 \end{pmatrix}, \quad H_u = \begin{pmatrix} 0 \\ \varphi \end{pmatrix}, \]  

(7)

where the scalar components of other chiral superfields are fixed to be zero. The AD mechanism based on this flat direction and others has been studied in the MSSM and its extensions [19, 20, 26]. In such studies the flat direction is closely related to the neutrino masses. In particular, the lightest neutrino mass is constrained by the relation to the reheating temperature. This aspect can be changed in this model since the relevant operate \( L_i H_u \) has nothing to do with the neutrino mass generation as discussed above.

The flat direction is lifted by a nonrenormalizable interaction and also both supersymmetry breaking terms due to hidden sector dynamics and finite energy density of an inflaton field. As a result, the initial value of \( \phi \) is fixed and \( \phi \) evolves following the potential minimum determined by the evolution of the inflaton. As such a nonrenormalizable superpotential consistent with the imposed symmetry discussed above, we find

\[ W_{nr} = \frac{\xi}{M} (L_i H_u)^2, \]  

(8)

where \( M \) and \( \xi \) can be determined by the symmetry imposed on the model. In fact, if the model is considered to be invariant under the anomalous U(1) symmetry as discussed at the end of the previous part, \( M = M_{pl} \) and \( |\xi| = O(10^{-6}) \) may be expected. We use these in the following discussion. It should be noted that the small value of \( \xi \) is naturally realized in this picture.

The scalar potential for \( \phi \) is induced by \( W_{nr} \) and also by the above mentioned supersymmetry breaking effects. The latter one is induced by the hidden sector dynamics which is characterized by the gravitino mass \( m_{3/2} \) of \( O(1) \) TeV and also the inflaton finite
energy density which is characterized by the Hubble parameter $H$ [19]. The feature of the AD mechanism is determined by this scalar potential at the inflation era characterized by $H = H_I$ and also at the period after the inflation, that is, from the time when the inflation ends to the time $H \simeq m_{3/2}$ when $\varphi$ is expected to start moving toward a true minimum of the potential. This scalar potential is expressed as

$$V(\varphi, \varphi^*) = (m_{\varphi}^2 - cH^2)|\varphi|^2 + \left( \frac{Am_{3/2} + aH}{M_{\text{pl}}} \xi \varphi^4 + \text{h.c.} \right) + \frac{\xi^2}{M_{\text{pl}}^2} |\varphi|^6,$$

(9)

where $m_{\varphi}^2 = |\mu_H|^2 + m_0^2$ which is considered to be $O(1)$ TeV. All the dimensionless constants $A$, $a$ and $c$ except for $\xi$ are considered to have values of $O(1)$. We define the phases of $A$ and $a$ as $A\xi \equiv \tilde{A}e^{i\theta_A}$ and $a\xi \equiv \tilde{a}e^{i\theta_a}$. In this scalar potential, the lepton number is violated by $\Delta L = 2$ through the terms in the parenthesis. Moreover, the same terms also violate the CP invariance through the phases $\theta_A$ and $\theta_a$.

If both $H > m_{\varphi}$ and $c > 0$ are satisfied, the scalar potential (9) has a nontrivial minimum at $\varphi_0(H) = |\varphi_0(H)|e^{i\theta_{\varphi_0}(H)}$ where $|\varphi_0(H)|$ is determined as a function of $H$ as

$$|\varphi_0(H)| = \frac{M_{\text{pl}}}{\sqrt{3|\xi|}} \left[ \frac{2\tilde{a}H}{M_{\text{pl}}} + \left\{ \left( \frac{2\tilde{a}H}{M_{\text{pl}}} \right)^2 + \frac{3c|\xi|^2 H^2}{M_{\text{pl}}^2} \right\}^{1/2} \right]^{1/2} \simeq \left( \frac{M_{\text{pl}}H}{|\xi|} \right)^{1/2},$$

(10)

and the initial value of $\theta_{\varphi_0}$ can be expressed as

$$\theta_{\varphi_0}(H_I) = \frac{(2n + 1)\pi}{4} - \frac{\theta_a}{4},$$

(11)

where $n$ is an integer. The energy density of the universe is considered to be dominated by the inflaton during the evolution of $\varphi$. Thus, following the inflaton motion, the value of the Hubble parameter $H$ changes to induce the shift of the potential minimum $\varphi_0(H)$. The field $\varphi$ follows this minimum. Once the universe reaches the time at which $H < m_{\varphi}$ is satisfied, a true minimum of the potential $V(\varphi)$ appears at $\varphi = 0$ and $\varphi$ starts the motion toward this minimum with or without rotating around $\varphi = 0$.

The $\theta_\varphi$ dependence of the potential $V(\varphi)$ changes from $\cos(\theta_a + 4\theta_\varphi)$ to $\cos(\theta_A + 4\theta_\varphi)$ when the universe changes over from the period $H > m_{3/2}$ to the period $H < m_{3/2}$. Thus, during this transient time torque is generated for the motion of $\varphi$ as long as $\theta_A$ is not equal.
to $\theta_a$. If $|\theta_A - \theta_a|$ takes a larger value, the larger torque can be caused and $\varphi$ could start rotating around $\varphi = 0$. Since the lepton number density $n_L$ stored in the $\varphi$ configuration is expressed as $n_L = -2\dot{\varphi}|\varphi|^2$, the substantial lepton number is expected to be generated through this evolution of $\theta_A$. In the next section, we estimate this induced lepton number by studying the evolution of the flat direction $\varphi$.

3 Leptogenesis based on the AD mechanism

We follow the negative squared mass scenario given in [19] and then $c > 0$ is assumed as in the above discussion here. In this case, the potential minimum during the inflation exists at $\varphi_0$ given in eqs. (10) and (11). Here we use $H_I \simeq 10^{14}$ GeV as the Hubble parameter during the inflation, which is required by the density perturbation found in the CMB anisotropy observation. After this inflation period, the evolution of $\varphi$ is described by the equation of motion which can be expressed as

$$
\frac{d^2 \varphi}{dt^2} + 3H \frac{d\varphi}{dt} + \frac{dV(\varphi, \varphi^*)}{d\varphi^*} = 0,
$$

(12)

where the potential $V(\varphi, \varphi^*)$ is given in eq. (9). We are interested in the evolution of $\varphi$ during the period relevant to the lepton number generation. This period is characterized by $H \sim m_{3/2}$. We suppose that the Hubble parameter $H$ is larger than the inflaton decay width $\Gamma_I$ in this period here. This is the case if we confine our study to the case with the reheating temperature $T_R < 10^{10}$ GeV. In this case the universe is dominated by the matter due to the coherent oscillation of the inflaton. Thus, we can use $H = \frac{2}{3t}$ in eq. (12).

Now we introduce a dimensionless Hubble parameter $x = \frac{H}{H_I}$ to rewrite eq. (12) as its differential equations for the dimensionless fields $\phi_{R,I}(x)$ which are defined as $\varphi(x) \equiv \frac{|\varphi_0(H_I)|}{\sqrt{2}}(\phi_R(x) + i\phi_I(x))$. Thus, eq. (12) can be expressed as

$$
\frac{d^2 \phi_R}{dx^2} + \frac{4}{9x^4} \frac{\partial V}{\partial \phi_R} = 0, \quad \frac{d^2 \phi_I}{dx^2} + \frac{4}{9x^4} \frac{\partial V}{\partial \phi_I} = 0,
$$

(13)

where $\partial V/\partial \phi_R$ and $\partial V/\partial \phi_I$ are given by

$$
\frac{\partial V}{\partial \phi_R} = \left(\frac{m_{3/2}^2}{H_I^2} - cx^2\right)\phi_R + \frac{2m_{3/2}|\Phi_0|^2}{M_{pl}H_I^2}(\tilde{A}\cos \theta_A + \frac{\tilde{a}H_Ix}{m_{3/2}}\cos \theta_a)(\phi_R^2 - 3\phi_I^2)\phi_R
- \frac{2m_{3/2}|\Phi_0|^2}{M_{pl}H_I^2}(\tilde{A}\sin \theta_A + \frac{\tilde{a}H_Ix}{m_{3/2}}\sin \theta_a)(3\phi_R^2 - \phi_I^2)\phi_I + \frac{3|\xi|^2|\Phi_0|^4}{4M_{pl}^2H_I^2}(\phi_R^2 + \phi_I^2)^2\phi_R,
$$
\[
\frac{\partial V}{\partial \phi_I} = \left( m_{3/2}^2 \frac{H_I^2}{H_I^2} - c x^2 \right) \phi_I - \frac{2m_{3/2} |\Phi_0|^2}{M_{\text{pl}} H_I^2} \left( \tilde{A} \cos \theta_A + \frac{\tilde{a} H_I x}{m_{3/2}} \cos \theta_a \right) \left( 3 \phi_R^2 - \phi_I^2 \right) \phi_I \\
- \frac{2m_{3/2} |\Phi_0|^2}{M_{\text{pl}} H_I^2} \left( \tilde{A} \sin \theta_A + \frac{\tilde{a} H_I x}{m_{3/2}} \sin \theta_a \right) \left( \phi_R^2 - 3 \phi_I^2 \right) \phi_R + \frac{3|\xi|^2|\Phi_0|^4}{4M_{\text{pl}}^2 H_I^2} \left( \phi_R^2 + \phi_I^2 \right)^2 \phi_I,
\]

where we use the definition \( \Phi_0 = |\varphi_0(H_I)| \).

The behavior of \( \phi_{R,I} \) is found by solving these equations numerically. Unfixed free parameters in these equations are \( \theta_A \) and \( \theta_a \) only. The initial value of \( \theta_\varphi \) at \( x = 1 \) is determined by \( \theta_a \) as found from eq. (11). Although only \( \theta_A \) is relevant to the potential minimum at \( x \lesssim m_{3/2} H_I (\equiv x_i) \), \( \theta_\varphi \) could have any values there since the global potential minimum should be realized at \( \varphi = 0 \). On the other hand, since \( \tilde{A} = \tilde{a} = |\xi| \) is supposed here, both \( \theta_A \) and \( \theta_a \) play an equal role in the potential at the transient period \( x \sim x_i \). This could make a local potential minimum appear at \( \varphi \neq 0 \) depending on the value of \( \theta_A \). Since such a local minimum appears at a separated place for a larger \( |\theta_A - \theta_a| \), the larger torque could be induced to make \( \dot{\theta}_\varphi \) larger as mentioned before. In that case, sufficient lepton number could be generated in the flat direction and \( \varphi \) is expected to rotate around \( \varphi = 0 \) at \( x \ll x_i \). On the other hand, if sufficient torque for the motion of \( \varphi \) is not induced because of a small \( |\theta_A - \theta_a| \) value, the lepton number might not be sufficiently generated in the \( \varphi \) and also the \( \varphi \) might not show the rotating motion around \( \varphi = 0 \) at \( x < x_i \). In such a case, the generated lepton number might not be released into the plasma at the appropriate time for leptogenesis. This could occur since the large mass of fields induced by \( \varphi_0 \) prohibits the evaporation of the flat direction into such fields. In order to generate the lepton number in the plasma, \( \varphi \) has to store the sufficient lepton number when \( \varphi \) starts the oscillation around \( \varphi = 0 \). We need to examine these points through the numerical study.

In the numerical study we fix the free parameters as \( \theta_A = \pi, \frac{\pi}{4} \) and \( \theta_a = \frac{\pi}{8} \) as typical examples for a while. The evolution of \( \varphi \) in these cases is shown in Fig. 1. In the left figures the trajectory of \( \varphi \) is plotted in the \((\phi_R, \phi_I)\) plane. We also plot \( |\varphi|, \frac{\Phi_0}{\sqrt{2}} |\phi_R| \) and \( \frac{\Phi_0}{\sqrt{2}} |\phi_I| \) as functions of the dimensionless Hubble parameter \( x(\equiv H_I^2) \) in the right figures. In case of \( \theta_A = \pi \), \( |\varphi| \) starts the oscillation around the origin \( \varphi = 0 \) and \( \dot{\theta}_\varphi \) can have large values at \( x \lesssim x_i \) as expected. Thus, the sufficient lepton number can be generated in the \( \varphi \) condensate. On the other hand, in case of \( \theta_A = \frac{\pi}{4} \), \( |\varphi| \) does not oscillate around \( \varphi = 0 \).
As a result, few lepton number is generated and it tends to decrease at the $x < x_i$ region. These suggest that a large amount of lepton number is not be expected to be generated for the case with small values of $|\theta_A - \theta_a|$.

The averaged value of the generated lepton number $n_L$ at $x$ may be estimated by using these solutions as

$$\left\langle \frac{n_L}{n_{\varphi}} \right\rangle_x = \frac{1}{\Delta_i + \Delta_f} \int_{x-\Delta_i}^{x+\Delta_f} dx \left( \frac{-3H_I x^2}{2m_{\varphi}} \right) \left( \frac{\dot{\varphi}_R \dot{\varphi}_I - \dot{\varphi}_R \dot{\varphi}_I}{\dot{\varphi}_R^2 + \dot{\varphi}_I^2} \right),$$  

(15)

where $\Delta_i$ and $\Delta_f$ is fixed so as to include a few oscillation cycles if $|\varphi|$ oscillates. In Fig. 2 this averaged value of $\frac{n_L}{n_{\varphi}}$ is plotted as a function of $x$ for two typical values of $\theta_A$. The figure shows that $\left\langle \frac{n_L}{n_{\varphi}} \right\rangle_x$ takes an almost constant value at the region $x \lesssim x_i$ for $\theta_A = \frac{\pi}{4}$. On the other hand, the generated lepton number suddenly decreases in case of $\theta_A = \frac{\pi}{2}$. The generated lepton number can take largely different values depending on the relative value of $\theta_A$ and $\theta_a$ as mentioned above.
The averaged lepton number $\langle n_L/n_\varphi \rangle_x$ generated in the plat direction $\varphi$ as a function of $x$. Each line corresponds to the case $\theta_A = \pi$ and $\pi/4$, respectively. $\theta_a$ is fixed to $\pi/8$.

In the left figure of Fig. 3, we show how the averaged value $\langle n_L/n_\varphi \rangle_{x=0.1x_i}$ depends on $\theta_a$ which determines the initial value of $\theta_{\varphi_0}$. The figure shows that $\langle n_L/n_\varphi \rangle$ can have the values of $O(1)$ as long as the pair of $\theta_A$ and $\theta_a$ takes values in the suitable regions. Such regions are not so narrow as found from this figure.

If we write the energy densities of the AD field and the inflaton as $\rho_\varphi$ and $\rho_I$ respectively, they can be expressed as $\rho_I = s T_R$ and $\rho_\varphi = m_\varphi n_\varphi$ by using the entropy density $s$ and the reheating temperature $T_R$. Taking account of these relations and the fact that the inflaton dominates the energy of the universe, we obtain the ratio of the lepton number density to the entropy density at $H \sim m_{3/2}$ as

$$\frac{n_L}{s} \bigg|_{H \sim m_{3/2}} \sim \frac{n_L T_R}{n_\varphi m_\varphi} \frac{\rho_\varphi}{\rho_I} \sim \frac{n_L T_R \varphi^2(x_i)}{n_\varphi m_{3/2} M_{\text{pl}}^2}.$$  

(16)

where we use $\rho_I \simeq (m_{3/2} M_{\text{pl}})^2$ and $\rho_\varphi \simeq m_{3/2}^2 \varphi^2$ at $H \simeq m_{3/2}$. Since the sphaleron transition can be in the thermal equilibrium after the reheating ($H \leq \Gamma_I$), it causes the reprocessing from the $B - L$ asymmetry to the $B$ asymmetry through the relation $B = \frac{1}{4}(B - L)$ in the present model.** Thus, the generated baryon number asymmetry

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**We can check that this quantity has the same behavior for the pair of $\theta_A$ and $\theta_a$ which satisfies $\theta_A - \theta_a = \pi$. If we use this feature and Fig. 3, we can know the value of $\langle n_L/n_\varphi \rangle_x$ for any set of $\theta_A$ and $\theta_a$.

**In this derivation we suppose that the lepton number violating interactions $\lambda_u \eta_u H_d \phi$ and $\lambda_d \eta_d H_u \phi$ are out-of-equilibrium. This point is discussed below.
Fig. 3 The $\theta_a$ dependence of the generated lepton number $\langle \frac{n_L}{n_R} \rangle_x$ and the value of $|\varphi(x)|$ at $x = 0.1 x_i$. In the right figure each line represents the same one as in the right figure of Fig. 1. $\theta_A$ is fixed to $\pi$ here.

can be estimated as\footnote{The sign is not crucial here since we can find that the other pair of $\theta_A$ and $\theta_a$ can generate the same values of $|\varphi(x)|$ and also $|\langle \frac{n_L}{n_R} \rangle_x|$ with opposite sign from Fig. 3.}

$$\frac{n_B}{s} \bigg|_x = \frac{1}{4} \langle \frac{n_L}{n_T} \rangle_x \frac{T_R \varphi^2(x)}{m_{3/2} M_{pl}^2} \approx 4.3 \times 10^{-10} \left( \frac{n_L / n_T}{1} \right) \left( \frac{\varphi(x)}{10^{13} \text{ GeV}} \right)^2 \left( \frac{T_R}{10^5 \text{ GeV}} \right).$$

This formula and both figures in Fig. 3 suggest that the favorable $\frac{n_B}{s}$ can be generated for suitable values of $\theta_a$, which include the previously discussed example $\theta_A = \pi$ and $\theta_a = \frac{\pi}{8}$.

We find that rather low reheating temperature $T_R \approx 10^{5-6} \text{ GeV}$ is acceptable as long as $\varphi$ is evaporate to the plasma during $0.1 x_i \lesssim x \lesssim x_i$. Such a reheating temperature $T_R$ is high enough for the sphaleron process to be in the thermal equilibrium. It is also sufficiently low to escape the gravitino problem. Although $\langle \frac{n_L}{n_T} \rangle$ seems to have much smaller values than $O(1)$ at $0.1 x_i \lesssim x \lesssim x_i$ for $|\theta_a| \gtrsim 1$ which includes the equivalent case to the previous example $\theta_A = \frac{\pi}{4}$ and $\theta_a = \frac{\pi}{8}$, $|\varphi(x)|$ takes larger values than $10^{13} \text{ GeV}$ and then it might seem to make $\frac{n_B}{s}$ be of $O(10^{-10})$ even for $T_R \approx 10^{4-6} \text{ GeV}$. However, in that case, the generated lepton number could not be successfully released into the plasma as discussed below.

It is useful to present a remark on this reheating temperature $T_R$ here. Since $|\xi|$ is strongly suppressed in this scenario, the large value of $\varphi(x_i)$ can be realised. It makes the value of $T_R$ required for the generation of the appropriate baryon number asymmetry lower in comparison with the ordinary scenario based on the $LH_u$ direction\footnote{Ref. 19}. It should also be noted that there is no constraint on the neutrino masses since the $LH_u$ direction
is irrelevant to the neutrino masses in this model.

Next, we need to examine the conditions which are required to identify the value of $n_\nu/s$ given by eq. (17) with the observed baryon number asymmetry in the universe. Eq. (17) is estimated under the assumption that all the lepton number generated at the time $H \sim m_{3/2}$ is transformed to the baryon number asymmetry. This estimation gives the correct answer only if the following conditions are satisfied:

(i) The evaporation of the flat direction due to both the decay and the scattering with the plasma yielded from the inflaton decay should be forbidden before the time $H \sim m_{3/2}$. Otherwise, the sufficient $n_\nu/s$ cannot be generated.

(ii) The lepton number violating interactions with the flat direction $\varphi$ should decouple during the period from $H \sim m_{3/2}$ to the sphaleron decoupling time.

We examine the condition (i) at first. If the relevant processes occur, $\Gamma > H$ and also the kinematical condition $\sum_i E_i > \sum_f m_f$ should be satisfied, where $\Gamma$ is the reaction rate of the relevant processes. $E_i$ and $m_f$ represent the energy of the fields included in the initial state and the mass of fields included in the final state, respectively. The energy of the thermal plasma before the reheating is estimated by the temperature $T_r \simeq k_r (M_{\text{pl}} H T_r^2)^{1/4}$ where $k_r = \left(\frac{72}{5\pi^2 g_*}\right)^{1/8}$ and $T_R$ is the reheating temperature [25]. Since the scattering rate of $\varphi$ with this plasma is roughly estimated as $\Gamma \simeq \alpha_y^2 T_r$ where $\alpha_y = \frac{y^2}{4\pi}$ and $y$ is a relevant coupling constant, $\Gamma \gtrsim H$ is satisfied for

$$H \lesssim \left(k_r^4 \alpha_y^8 M_{\text{pl}} T_R^2\right)^{1/3} \sim m_{3/2} \left(\frac{y}{0.25}\right)^{16/3} \left(\frac{T_R}{10^5 \text{ GeV}}\right)^{2/3}. \quad (18)$$

where we use $g_* \simeq 100$ as the relativistic degrees of freedom. This means that $\varphi$ could evaporate through the exchange of top quark which has $y \simeq 1$ before the time $H \sim m_{3/2}$ if $T_R \sim 10^5$ GeV is assumed. However, we need to note that the masses of the fields in the final state are induced as $y|\varphi_0(H)|$ through the interaction with the flat direction, where $y$ is a relevant coupling constant with the flat direction $\varphi$. The measure for this kinematical condition is given by [19]

$$\left.\frac{y|\varphi_0(H)|}{T_r}\right|_{H \sim m_{3/2}} \simeq \left(\frac{m_{3/2} M_{\text{pl}}}{|\xi| T_R}\right)^{1/2} \sim \frac{y}{3 \times 10^{-6}} \left(\frac{10^5 \text{ GeV}}{T_R}\right)^{1/2}. \quad (19)$$

Even if we assume $y \sim 10^{-5}$ which corresponds to the electron case and gives the severest condition, $y|\varphi_0| > T_r$ is satisfied even for $T_R \sim 10^5$ GeV. From this discussion, we find
that $\frac{n_\nu}{n_\nu}$ can reach a suitable value before the evaporation of the flat direction keeping the consistency with the reheating temperature which should satisfy $T_R \geq 10^2$ GeV.

We also note that the flat direction can evaporate to the thermal plasma before the sphaleron decoupling at $T \sim 10^2$ GeV. Since the $\varphi$ starts the oscillation around the potential minimum $\varphi_0 = 0$ for suitable values of $\theta_A$ and $\theta_a$ once $H < m_{3/2}$ is fulfilled, any contribution to the masses of the final states is not induced through the interaction with $\varphi$. Fig. 1 gives such a concrete example. As long as such a situation is realized, we know from the discussion on eq. (18) that the lepton number asymmetry stored in $\varphi$ is released into the plasma immediately through the lepton number conserving scattering processes. Such processes contain $\varphi \tau \rightarrow \tilde{\tau} \nu_\tau$ and $\varphi b \rightarrow \nu_\tau \tilde{b}$ through a Higgsino $\tilde{H}_d$ exchange.

The lepton number violating interactions are given by the $\lambda_{u,d}$ terms in eq. (1). The decay of the flat direction through the interaction $\lambda_d q_d H_u \phi$ is kinematically forbidden since the mass of the singlet field $\phi$ is large enough. The rate of the $\varphi$ scattering through the $\phi$ exchange is proportional to $|\lambda_{u,d}|^4$. Since $|\lambda_{u,d}|$ is considered to be $O(10^{-4})$ as discussed in the previous part, $\Gamma < H$ is satisfied throughout the relevant period. Thus, the dangerous lepton number violating processes are irrelevant to the present scenario and the condition (ii) is also fulfilled. All the lepton number asymmetry induced in the flat direction at $H \sim m_{3/2}$ is distributed in the plasma and then it is converted to the baryon number asymmetry through the sphaleron process as discussed above. Favorable features found in the AD mechanism for the $LH_u$ flat direction in the MSSM are kept even for the rather low reheating temperature such as $T_R \sim 10^5$ GeV in the present framework.

4 Summary

The supersymmetric radiative neutrino mass model is an interesting extension of the SM. It can give a consistent explanation for both the origin of the small neutrino masses and the DM abundance by relating them closely. In this model, however, it seems to be difficult to generate the required baryon number asymmetry in the universe through the ordinary thermal leptogenesis. The right-handed neutrinos with the masses of $O(1)$ TeV are too light to generate the sufficient lepton number asymmetry through their out-of-thermal equilibrium decay.

‡‡Nonperturbative effects might play an important role in the decay of the flat direction [26].
In this paper we have considered the leptogenesis based on the AD mechanism as an alternative possibility. We have applied the AD mechanism to the famous flat direction $LH_u$. Since this flat direction is irrelevant to the neutrino mass generation in this model, any constraint on the neutrino masses does not appear in the relation to the AD mechanism. This is largely different from the previous work. Our analysis shows that the model can produce the sufficient baryon number asymmetry through the AD mechanism based on this flat direction for the rather low reheating temperature such as $10^5$ GeV. It is interesting that the crucial parameters for this mechanism can be related to other parameters in the model by introducing the anomalous U(1) symmetry. We would like to stress that this symmetry is embedded in the present model so as to give the consistent explanation for the problems which remain as unsolved ones in the SM, that is, the neutrino masses, the dark matter abundance and also the baryon number asymmetry in the universe.

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