Baryogenesis in non-extensive Tsallis Cosmology

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\begin{abstract}
Non-extensive Tsallis thermostatistics is a widespread paradigm to describe large-scale gravitational systems. In this work we use Tsallis Cosmology to study thermodynamic gravity and derive modified Friedmann equations. We show that corrections induced by non-extensivity affect the Hubble function evolution during the radiation-dominated epoch. In turn, this leads to non-trivial modifications of the mass density and pressure content of the Universe, which provide a viable mechanism allowing for baryogenesis, even in the presence of the standard interaction between the Ricci scalar and baryon current. By demanding consistency with current observational bounds on baryogenesis, we constrain Tsallis $\delta$ parameter to be $|\delta - 1| \lesssim 10^{-3}$. Based on the recently established connection between Tsallis thermostatistics and the quantum gravitational generalization of the uncertainty principle at Planck scale (GUP), we finally show that this bound is in agreement with the estimation of the GUP parameter predicted by many quantum gravity models.

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\end{abstract}

1. Introduction

There are indications from various scenarios and results, such as black hole mechanics [1], Bekenstein-Hawking (BH) formula [2–4], holographic principle [5,6] and the recent firewall puzzle [7], that the concepts of gravity and entropy are non-trivially connected, possibly intertwined in the would-be theory of quantum gravity. This appears even more evident in the gravity-thermodynamics conjecture by Jacobson [8], who showed that Einstein’s equations describing relativistic gravitation can be naturally derived by assuming that the Bekenstein bound and the laws of thermodynamics hold true. The ensuing research framework is often referred to as thermodynamic gravity.

Thermodynamic gravity has been largely addressed over the years [9,10]. One of its most fascinating implications is the possibility to deduce the cosmological Friedmann equations from the first law of thermodynamics on the apparent horizon of a Friedmann-Robertson-Walker (FRW) Universe (see, for instance, [11–17]). This connection has also been studied in various models of modified gravity, such as Lanczos-Lovelock [18], quadratically extended [19], Gauss-Bonnet and running vacuum [20] models, highlighting the general validity of the gravity-thermodynamics conjecture.

The above analysis has been developed within the framework of standard thermodynamics based on Boltzmann-Gibbs entropy. However, for systems with a divergent partition function, Boltzmann-Gibbs theory cannot be applied, as figured out by Gibbs in 1902. Now, we know that gravitational systems lie within this class. As shown by Tsallis et al. in [21–23], in these cases the usual Boltzmann-Gibbs additive entropy (which is based on the assumption of weak probabilistic correlations and their connections to ergodicity) must be generalized to the non-additive entropy (i.e., the entropy of the whole system is not necessarily the sum of the entropies of its sub-systems), which is given by [24]

$$S_\delta = \gamma \left( \frac{A}{\ell_p^2} \right)^\delta,$$

where $A$ is the area of the system, $\gamma > 0$ a constant factor and $\ell_p$ the Planck length.\footnote{Throughout the manuscript we set $\hbar = c = k_B = 1.$} Deviation from extensivity is quantified by the Tsallis exponent $\delta > 0$, which gives rise to a probability distribution decaying asymptotically as a power law (rather than an exponential). We emphasize that this parameter is not fixed by the theory, the only expectation being $|\delta - 1| \ll 1$. Clearly, the standard BH en-

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entropy is recovered for $\delta \to 1$, provided that $\gamma \to 1/4$ in this limit. Although not contemplated in the original formulation by Tsallis, the possibility of a running $\delta$ has been considered for quantum gravity [25] and field theoretical [26,27] systems, respectively.

Tsallis’s prescription has been providing encouraging results in the description of many complex (strongly correlated) systems [23–33]. Recently, it has aroused a lot of interest also in cosmological contexts and, in particular, in the study of the evolution of the whole Universe, which is a gravitational (and, therefore, nonextensive) system. For instance, in Ref. [34] it has been shown that non-extensivity translates into a modification of the gravitational constant and, accordingly, of the dark content of the Universe. Likewise, by assuming Eq. (1) as entropy law for the Universe apparent horizon, one can derive modified Friedmann equations and a Tsallis driven cosmic evolution [35–37] for any spatial curvature [38]. Interestingly enough, for a particular choice of Tsallis parameter the model of [38] is able to reproduce the cosmic acceleration without invoking dark energy. A quite similar result has been exhibited in Ref. [36], showing that the modified Newton equation on large scales can explain the asymptotic flatness of galaxy rotation curves without the need of dark matter, provided that $\delta \lesssim 1/2$.

While being intensively investigated for late-time Cosmology, comparatively less attention has been devoted to the study of Tsallis thermodynamics in the early Universe. One of the most debated problems in this context is the observed baryon (i.e. matter/anti-matter) asymmetry generated in the radiation dominated era. For this asymmetry to occur, it is known that three (Sakharov) conditions are to be satisfied [39], at least in the simplest versions of baryogenesis: (i) baryon number $B$ violation, (ii) C-symmetry and CP-symmetry violation and (iii) out-of-equilibrium interactions. In the standard cosmological model based on Boltzmann-Gibbs entropy, the predicted baryon asymmetry equals zero, due to the last condition not being satisfied. Therefore, despite strong evidences from Cosmic Microwave Background [43] and Big Bang Nucleosynthesis [44] measurements, the origin of this phenomenon is not yet understood, leaving room for disparate explanations [45–49].

Starting from the above premises, in this work we investigate baryogenesis in Tsallis Cosmology. In particular, we adopt the entropy (1) for the horizon degrees of freedom of the FRW Universe and derive modified Friedmann equations. This leads to corrected mass density and pressure content of the Universe, which provide an effective mechanism to break the thermal equilibrium in the radiation dominated era. Along with the conventional coupling between spacetime and baryon current satisfying the first two Sakharov conditions, this scenario ensures that all Sakharov’s criteria are met, thus allowing for baryogenesis. We exploit this result to constrain Tsallis Cosmology by comparison with observational bounds on baryon asymmetry.

It is worth noting that the rationale behind our analysis takes its cue from situations typically occurring in low-energy systems, such as crystalline membranes, where thermal fluctuations are intimately connected with long-range dipole interactions [50], or strong electrolytes, which exhibit long-distance behaviors when driven out of equilibrium by external fields [51]. Here, we are somehow reversing the perspective, exploring the possibility of a deviation from thermality induced by long-range correlations in gravitational systems obeying Tsallis description. In a broader sense, this framework is motivated by the fact that Eq. (1) works well for macroscopic states that exhibit (quasi-)stationarity or metastability and for driven non-equilibrium systems with large-scale temperature fluctuations (superstatistical systems [52–54]).

In passing, we mention that a similar study has been developed in Ref. [49]. In that case the mechanism responsible for baryon asymmetry is identified with quantum gravity corrections embedded in a generalized uncertainty principle (GUP) at Planck scale. By exploiting the recently established connection between Tsallis statistics and GUP [55], we show that our bound on $\delta$ is indeed consistent with the value of the GUP deformation parameter predicted by many quantum gravity models.

The layout of the paper is as follows. In the next Section we derive the modified Friedmann equations in Tsallis Cosmology. Section 3 is devoted to the investigation of baryon asymmetry problem. Consistency between non-extensive Tsallis statistics and Planck-scale deformed uncertainty relations is then discussed in Section 4. Conclusions are summarized in Section 5.

2. Modified Friedmann equations in Tsallis Cosmology

In this Section we derive the modified Friedmann equations in FRW Universe within the framework of Tsallis Cosmology. Suppose the spacetime geometry is given by the $(1+3)$-dimensional metric

$$ds^2 = h_{\nu}dx^\nu dx^\xi + \tilde{r}^2(d\theta^2 + \sin^2\theta d\phi^2), \ b, c = (0, 1),$$

where $h_{\nu} = \text{diag}(-1, a^2/(1 - kr^2))$ is the metric of a $(1+1)$-dimensional subspace, $x^\nu = (t, r), \tilde{r} = a(t)r$, with $a(t)$ being the scale factor, $r$ is the comoving radius and $k$ the (constant) spatial curvature. As shown in [16], the dynamical apparent horizon is fixed by the condition $h_{\nu\xi} \partial_\nu \tilde{r} h_{\xi} = 0$, which gives for the FRW Universe

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \kappa/a^2}},$$

where $H = \dot{a}(t)/a(t)$ is the Hubble parameter, the dot indicating a time derivative.

The temperature $T = \kappa/2\pi$ associated to such horizon can be derived from the definition of surface gravity [16]

$$\kappa = \frac{1}{2\sqrt{\pi-h}} \partial_\xi \left( \sqrt{h + h_{\nu\xi} \partial_\nu} \right) = -\frac{1}{\tilde{r}_A} \left(1 - \frac{\tilde{r}_A}{2H\tilde{r}_A}\right),$$

which gives

$$T = -\frac{1}{2\pi\tilde{r}_A} \left(1 - \frac{\tilde{r}_A}{2H\tilde{r}_A}\right).$$

The matter/energy content of the Universe is described by a perfect fluid. Denoting by $\rho$ and $p$ its mass density and pressure at equilibrium, respectively, the corresponding energy-momentum tensor is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu},$$

where $u_\mu$ is the four-velocity of the fluid. In turn, the conservation equation $\nabla_\mu T^{\mu\nu} = 0$ for the FRW Universe implies the continuity equation

$$\dot{\rho} = -3H(\rho + p).$$

Now, Friedmann equations in the bulk of the Universe are obtained by considering the first law of thermodynamics on the apparent horizon

$$dE = TdS + WdV.$$
where $E = \rho V$ is the total energy content of the Universe of 3-dimensional spherical volume $V = 4\pi \ell_A^3 / 3$ and horizon surface area $A = 4\pi \ell_A^2$. $W$ is the work done by the volume change of the Universe,

$$W = -\frac{1}{2} T B h_{bc} = \frac{1}{2} (\rho - p).$$ (9)

A comment is now in order: in Tsallis model it has been argued that non-extensivity translates into a non-trivial modification of both core thermodynamic relations and standard definitions of pressure and temperature (see, for instance, [56]). However, following the analysis of [17,36–38,49,57], such corrections can be neglected as a first approximation in cosmological contexts. Having this in mind, we henceforth adopt the usual definitions for $p$ and $T$, as well as the first law of thermodynamics in its traditional form (8).

Plugging Eqs. (5), (7) and (9) into (8), after simple algebra one obtains the first modified Friedmann equation

$$-4\pi G (\rho + p) = \left( \frac{\dot{H}}{G} - \frac{k}{a^2} \right) f'(A),$$ (10)

where $f(A)$ is defined in such a way as

$$S = \frac{f(A)}{4G} \implies \frac{dS}{dA} = f'(A) \frac{A}{4G}.$$ (11)

Notice that $G = l_p^2 = 1/m_p^2$ in our units convention, where $m_p$ is the Planck mass. Similarly, by use of Eq. (7), the second Friedmann equation reads

$$\frac{8\pi G}{3} \rho = -4\pi \int \frac{f'(A)}{A^2} dA.$$ (12)

We remark that at this stage we have not yet used Tsallis definition for the horizon entropy. Thus, Eqs. (10) and (12) hold true for any modified entropy obeying Eq. (11). Clearly, by comparison with Eq. (1), in Tsallis cosmological model we obtain

$$f'(A) \equiv f'_T(A) = 4\gamma \frac{A}{G}.$$ (13)

In the Boltzmann-Gibbs limit of $\delta \to 1$, we have $f'_T(A) \to 1$, consistently with the standard BH area law $S = A/(4G)$. This allows us to straightforwardly recover the usual Friedmann equations.

By use of Eq. (13), the modified Friedmann equations (10) and (12) take the form

$$-4\pi G (\rho + p) = 4\gamma \left( \frac{\dot{H}}{G} - \frac{k}{a^2} \right) \left( \frac{A}{G} \right)^{\delta - 1},$$ (14)

$$\frac{8\pi G}{3} \rho = \frac{16\gamma \delta A^{\delta-2}}{(2-\delta)} G^{\delta-1} + c,$$ (15)

respectively. The integration constant $c$ can be fixed by imposing the boundary condition $\rho \equiv \rho_{vac} \to \Lambda$ for $A \to \infty$ in the vacuum energy dominated era, $\Lambda$ being the cosmological constant. This gives $c = \frac{8\gamma \delta}{3(2-\delta)} \Lambda$, where we have taken into account that $|\delta - 1| \ll 1$ (see below Eq. (1)). We then obtain

$$\frac{8\pi G}{3} (\rho - \Lambda) = \frac{16\gamma \delta A^{\delta-2}}{(2-\delta)} G^{\delta-1}.$$ (16)

Modified Friedmann equations can be further manipulated by expressing the horizon surface area as

$$A = 4\pi \ell_A^2 = \frac{4\pi}{H^2 + k/a^2}.$$ (17)

By inserting into Eqs. (14) and (15), we get

$$-4\pi G (\rho + p) = \frac{4\pi^3}{G^{\delta-1}} \frac{1}{\gamma} \left( \frac{\dot{H}}{H^2 + k/a^2} \right)^{1-\delta},$$ (18)

$$\frac{8\pi G}{3} (\rho - \Lambda) = \frac{4\pi^3}{(2-\delta) G^{\delta-1}} \frac{1}{\gamma} \left( H^2 + k/a^2 \right)^{2-\delta}.$$ (19)

If we now apply the above model to the radiation dominated era, we can safely neglect the tiny observed cosmological constant $\Lambda$ and set the spatial curvature constant $k = 0$, consistently with the observed spatially flat Universe. Accordingly, the modified Friedmann equations become

$$-4\pi G (\rho + p) = \frac{4\pi^3}{G^{\delta-1}} \frac{1}{\gamma} \left( \frac{\dot{H}}{H^2} \right)^{1-\delta},$$ (20)

$$\frac{8\pi G}{3} (\rho - \Lambda) = \frac{4\pi^3}{(2-\delta) G^{\delta-1}} \frac{1}{\gamma} H^{2(2-\delta)}.$$ (21)

It is immediate to check that the limit of $\delta \to 1$ gives back the standard cosmological relations, as expected.

As we shall see in the next Section, Eqs. (20) and (21) lie at the heart of the computation of Tsallis-modified mass density and pressure content in the radiation dominated era. The evolution of the Universe in Tsallis Cosmology has been studied in [35], showing that the generalized Friedmann equations contain extra corrections that constitute an effective dark energy sector.

3. Constraints on Tsallis Cosmology from baryon asymmetry

Observational evidences indicate that our Universe is mostly made up of matter, rather than balanced amounts of matter and anti-matter as predicted by Quantum and Relativistic theories [48]. The common view is that this asymmetry is generated dynamically as the Universe expands and cools. Ref. [39] Sakharov listed three necessary conditions for the occurrence of baryogenesis: i) baryon number violation, which is required to produce an excess of baryons over anti-baryons; ii) C and CP-violation; iii) CPT-violation. The first one assures that the interactions which produce more baryons than anti-baryons are not counterbalanced by interactions with reverse effects. CP-violation is similarly needed, since otherwise equal numbers of left-handed baryons and right-handed anti-baryons would be produced, as well as equal numbers of left-handed anti-baryons and right-handed baryons; iii) out-of-thermal-equilibrium interactions, which prevent CP-violation from compensating between processes increasing and decreasing $B$.

To meet the conditions ii) and iii), the conventional approach provides for the introduction of interactions that violate C- and CP-symmetry in vacuum and a period of non-thermal equilibrium for the Universe. Nevertheless, several other possible explanations have been proposed over the years [45–49]. For instance, in Ref. [47] a mechanism has been suggested involving a dynamical breaking of CP in an expanding Universe. The key ingredient is a CP-violating interaction in vacuum between the derivative of the Ricci scalar curvature $\mathcal{R}$ and the baryon number current $j^B$ in the form [47]

$$\frac{1}{M^2_*} \int d^4x \sqrt{-g} j^B \partial_\mu j^\mu \mathcal{R}.$$ (22)

where $M_*$ is the cutoff scale of the effective theory, which is taken to be of order of the reduced Planck mass $M_* = (8\pi G)^{-1/2} \approx 2.4 \times 10^{18}$ GeV.

To generate baryon asymmetry from Eq. (22), it is also required that there be some $B$-violating process. In Ref. [47] this is expected to take place while maintaining thermal equilibrium. In this scheme, it can be shown that the interaction (22) gives opposite
sign energy contributions that differ for particles and antiparticles. Then, a net baryon asymmetry

\[ \frac{1}{M^2} J^\mu \delta \mu \mathcal{R} = \frac{1}{M^2} (n_B - n_{\bar{B}}) \mathcal{R}. \]  

(23)
can be generated, where \( n_B \) and \( n_{\bar{B}} \) denote the baryon and antibaryon number density, respectively.

As argued in Ref. [47], dynamical CPT-violation modifies thermal equilibrium in a similar fashion as a chemical potential \( \mu_B = -\mu_{\bar{B}} = -\frac{\mathcal{R}}{M^2} \). Once the temperature drops below the decoupling temperature \( T_D \) at which \( B \)-violation goes out of equilibrium, the imbalance

\[ n_B - n_{\bar{B}} = \frac{g_B}{6} \mu_B T^2 \]

(24)
gets frozen, where \( g_B \sim O(1) \) is the number of intrinsic degrees of freedom of baryons.\(^3\) Baryon asymmetry can be then quantified as [42]

\[ \frac{\eta}{T} = \frac{n_B - n_{\bar{B}}}{s} = \frac{15 g_B}{4 \pi^2 g_* T^2} \left( \frac{\mathcal{R}}{T} \right) \text{ at } T = T_D, \]

(25)
where

\[ s = \frac{2 \pi^2 g_*}{45} T^3, \]

(26)
is the standard entropy density and \( g_* \) the number of degrees of freedom of particles contributing to the entropy of the Universe in the radiation dominated era. As noted in Ref. [42], \( g_* \) is nearly equal to the total number \( g_s \) of degrees of freedom of relativistic Standard Model particles, i.e. \( g_* \approx g_s \approx 106 \).

Now, from Eq. (25) it is clear that baryogenesis occurs, provided that \( \mathcal{R} \neq 0 \). In the standard cosmological model based on thermodynamic Boltzmann-Gibbs entropy, \( \mathcal{R} = 0 \) in the radiation dominated era, due to thermal equilibrium still being satisfied. On the other hand, by using Tsallis entropy for long-range correlated gravitational systems, departure from thermality could be naturally allowed, as it appears from next calculation.

Following [49], we parameterize departure from thermal equilibrium by mass density and pressure variations, which are indicated by \( \delta \rho \) and \( \delta p \), respectively. The total mass density \( \rho \) and pressure \( p \) then read

\[ \rho = \rho_0 + \delta \rho, \quad p = p_0 + \delta p, \]

(27)
where \( \rho_0 \) and \( p_0 \) denote the corresponding quantity at equilibrium, i.e.

\[ \rho_0 = \frac{3 H^2}{8 \pi G}, \quad p_0 = w \rho_0. \]

(28)
The latter relation defines the well-known equation of state parameter at equilibrium. At the radiation dominated era, we have \( w = 1/3 \).

Based on the previous considerations, we expect that both these mass density and pressure fluctuations depend on the parameter \( \delta \) in such a way that \( \delta \rho, \delta p \to 0 \) as \( \delta \to 1 \). The explicit expressions of such corrections can be derived by plugging Eqs. (27) into the modified Friedmann equations (20) and (21). Specifically, from Eq. (21) we obtain

\[ \delta \rho = \frac{1}{3} \rho_0 \left( -3 + \frac{2^{3-\delta} \times 3^{\delta} \gamma \delta}{(2 - \delta) G^{2(\delta-1)}} \rho_0^{1-\delta} \right). \]

(29)
Requiring \( \delta \rho \to 0 \) for \( \delta \to 1 \) fixes definitively the constant \( \gamma = 1/4 \), giving

\[ \delta \rho = \frac{1}{3} \rho_0 \left( -3 + \frac{2^{1-\delta} \times 3^{\delta}}{(2 - \delta) G^{2(\delta-1)}} \rho_0^{1-\delta} \right). \]

(30)
Similarly, we get for the pressure fluctuations

\[ \delta p = \rho_0 \left( -w + \frac{2}{3} \frac{1}{(2 - \delta) G^{2(\delta-1)}} \rho_0^{1-\delta} \right), \]

(31)
which becomes for the radiation dominated era \( (w = 1/3) \)

\[ \delta p = \frac{1}{9} \rho_0 \left[ -3 + \frac{2^{1-\delta} \times 3^{\delta} (5 - 4\delta)}{(2 - \delta) G^{2(\delta-1)-1}} \rho_0^{1-\delta} \right]. \]

(32)
Notice that in the above treatment we have included all the \( \delta \)-dependent terms in \( \delta \rho \) and \( \delta p \). We stressed that both these quantities vanish as \( \delta \to 1 \), consistently with the recovery of thermal equilibrium in the radiation dominated era within standard Cosmology.

To compute Tsallis-corrected derivative of the Ricci scalar \( \mathcal{R} \), the trace of Einstein equation is derived by

\[ \mathcal{R} = -8 \pi G T_g, \]

(33)
where

\[ T_g = \rho - 3p \]

(34)
is the trace of the energy-momentum stress tensor. Using Eqs. (30) and (31), it follows that

\[ \mathcal{R} = \frac{2^{6-\delta} \times 3^{\delta-1} \pi \delta (1 - \delta)}{(2 - \delta) G^{2(\delta-1)}} \rho_0^{2-\delta}, \]

(35)
where we have omitted for simplicity time-dependence. Therefore, the time derivative of the Ricci scalar is

\[ \frac{\dot{\mathcal{R}}}{\mathcal{R}} = \frac{2^{19/2-\delta} \times 3^{\delta-3/2} \pi^{3/2} \delta (\delta - 1)}{(2 - \delta) G^{2(\delta-7/2)}} \rho_0^{5/2-\delta}. \]

(38)
Following [49], here we have used the equilibrium form of the second Friedmann equation and of the continuity equation, since corrections to the latter would contribute to orders higher than that considered in the present analysis (see below).

Equation (38) indicates that non-extensive Tsallis prescription can induce a non-trivial deviation from thermality and generate baryon asymmetry, in compliance with the last Sakharov condition [39].

Let us now substitute the time derivative of the Ricci scalar (38) into the baryon asymmetry formula (25). A direct calculation leads to

\[ \eta = 35 \times 2^{15/2-\delta} \times 3^{\delta-1/2} \rho_0 (\delta - 1) \left( \frac{g_B}{g_* M^2 T} \right) \rho_0^{5/2-\delta} \]

(39)
We express the equilibrium mass density \( \rho_0 \) in terms of the temperature as [42]

\[ \rho_0 (T) = \frac{\pi^2 g_* T^4}{30}. \]

(40)

\(^3\) See Ref. [47] for more details on how baryon asymmetry occurs.
to obtain

$$\eta = 1792 \times 3^{23/3} \times 5^{\delta - 3/2} \times \pi^{11/2 - 2\delta} \delta |\delta - 1| \times \frac{g_b}{g_*^{3/2}} \left( \frac{T_D}{m_p} \right)^{9-4\delta}.$$  \hspace{1cm} (41)

Here we have used \( g_* \approx g_b \), as discussed above.

Since departure of \( \delta \) from unity is expected to be small (see constraints in Table 1), we can reasonably expand \( \eta \) for \( |\delta - 1| \ll 1 \), yielding to the leading order

$$\eta \approx 1792 |\delta - 1| \times \frac{g_b}{3^{51/2} \times \pi^{11/2} \times 5^{\delta - 3/2}} \left( \frac{M_1}{m_p} \right)^5.$$  \hspace{1cm} (42)

In order to estimate Tsallis parameter, we evaluate this expression at the decoupling temperature \( T_D = M_1 \approx 3.3 \times 10^{16} \text{ GeV} \), which is the upper bound on tensor mode fluctuations at inflation\(^4\) [47]. By plugging numerical values, we get

$$\eta \approx 2.38 \times 10^{-8} |\delta - 1|.$$  \hspace{1cm} (43)

We can now constrain Tsallis exponent by comparing the value of \( \eta \) predicted by our model with the measured baryon asymmetry [49,59–61]

$$5.7 \times 10^{-11} \lesssim \eta \lesssim 9.9 \times 10^{-11}.$$  \hspace{1cm} (44)

From Eq. (43), we obtain up to \( O(10^{-3}) \) (see Fig. 1)

$$0.002 \lesssim |\delta - 1| \lesssim 0.004.$$  \hspace{1cm} (45)

One might argue that this interval excludes the Bekenstein-Hawking limit \( \delta \to 1 \). The reason for that is because in our model we are assuming that baryogenesis is only due to corrections induced by Tsallis entropy, while keeping the coupling between the Ricci scalar and baryon current in its standard form. In this setting, the condition \( \delta = 1 \) implies \( \eta = 0 \) (see Eq. (42)), which is however in contrast with the experimental constraint (44). On the other hand, the lower bound on \( \delta - 1 \) disappears if we consider more elaborated cosmological models, in which baryon asymmetry can also be produced by other mechanisms presented in the literature [40–42], not associated to Tsallis holographic entropy itself.

\(^{4}\) The upper bound on tensor mode fluctuations constrains the inflationary scale to be \( M_1 \lesssim 3.3 \times 10^{16} \text{ GeV} \) [47]. In this setting, one has \( T_0 \lesssim T_{RD} \lesssim M_1 \), where \( T_{RD} \) is the temperature of the radiation dominated era. Baryon asymmetry can be large enough even for \( M_1 \approx m_0 \) if \( T_D \approx M_1 \) (which is the case we are considering here). Such a scenario predicts that tensor mode fluctuations could be soon observed [47].

### Table 1

| Bound | Physical framework | Ref. |
|---|---|---|
| \( \delta \lesssim 0.5 \) | Galactic rotation curves | [36] |
| \( \delta < 0.5 \) | Late-time accelerated expansion | [36] |
| Running | Boson mixing | [26] |
| \( 0.993 \lesssim \delta \lesssim 0.994 \) | \(^3\)Li Abundance | [57] |
| \( \delta = 1.222 \) | Cosmic ray observations | [62] |
| \( 0.607 \leq \delta \leq 4.2 \) | Entanglement measurements | [63] |
| \( |\delta - 1| \lesssim \times 10^{-13} \) | Non-commutative geometry | [64] |
| \( \delta \geq 1.203 \) | Quark coalescence | [65] |
| \( 1 \leq \delta < 1.333 \) | High-energy collisions | [58] |
| \( \delta \geq 1.218 \) | Black holes | [66] |
| Running | Fermion mixing | [27] |
| \( \delta \lesssim 1.222 \) | Unruh effect | [67] |
| \( \delta \approx 1.01 \) | Big Bang Nucleosynthesis | [57] |
| \( 0.966 \lesssim \delta \lesssim 1.001 \) | \(^4\)He, \(^7\)H Abundance | [57] |
| \( \delta \gtrsim 0.999 \) | Dark Matter relic density | [57] |

In an equal fashion, Eq. (45) reads as

$$|\delta - 1| \simeq 10^{-3}.$$  \hspace{1cm} (46)

A posteriori, this corroborates the expansion in Eq. (42). The obtained result is in agreement with other bounds recently obtained in cosmological scenarios and, in particular, in the study of primordial abundances of \(^4\)He, \(^2\)H and \(^3\)Li (see Table 1). On the other hand, it does not overlap with the constraint \( \delta \lesssim 0.5 \) (\( \delta < 0.5 \)) needed to explain the asymptotic flatness of galactic rotation curves (accelerated expansion of the present Universe) without resorting to dark matter (dark energy) [36]. Such an apparent inconsistency could be explained by invoking some mechanism occurred during the evolution of the Universe that may have reduced Tsallis parameter. This finds support in the recent result of [25], where a modified Tsallis Cosmology with a running exponent has been proposed to trace the thermal history of the Universe.

### 4. Connecting Tsallis thermostatistics and generalized uncertainty relations at Planck scale

Several models of quantum gravity, such as String Theory, Loop Quantum Gravity, Quantum Geometry, Doubly Special Relativity and black hole physics [68–73] agree in predicting a deformation of Heisenberg principle due to gravity effects at Planck energy. The ensuing relation entails a minimal length and/or a maximum momentum, consistently with the would-be discrete structure of spacetime arising at Planck scale.

The most common form adopted for GUP reads

$$\Delta x \Delta p \geq \frac{1}{2} \left[ 1 + \beta \left( \frac{\Delta p}{m_p} \right)^2 \right].$$  \hspace{1cm} (47)

The (dimensionless) deformation parameter \( \beta \) is not fixed by the theory, although investigation along this line is active both at theoretical [74–79] and experimental level [80–83]. Quite debated is also the issue of the sign of \( \beta \), with arguments in favor of either positive [68,70–72,76,77] or negative [73,78,84,85] \( \beta \).

The emergence of a minimal length predicted by the GUP (47) should somehow affect the phase-space structure by modifying the elementary cell volume occupied by each quantum state. In turn, this has implications on the statistical (microscopic) properties of quantum systems. Particularly, in [86] it has been observed that generalized statistics with a quadratic correction over Gaussian profile naturally arise as a consequence of Eq. (47), under the condition that the total phase-space volume is kept invariant. Connections between departure from Gaussianity and GUP have
been later explored within the specific framework of Tsallis statistics [67], considering the Unruh effect as a playground. In that case, it has been shown that modifications to Unruh temperature induced by GUP [87] can be rephrased in terms of a generalized vacuum distribution obeying Tsallis statistics.

Tsallis-GUP correspondence has been rigorously formalized in [55] through the study of coherent states for generalized uncertainty relations. As a result, it has been argued that the probability distribution associated with the coherent states for the quadratic GUP (47) is a Tsallis-like distribution, the non-extensivity parameter being monotonically related to $\beta$ by

$$\delta = \frac{\beta \gamma_{\Delta P}}{m_p^2 + \beta^2 \gamma_{\Delta P}} + 1. \quad (48)$$

Here, the scale-dependent parameter $\gamma_{\Delta P}$ is defined as

$$\gamma_{\Delta P} = \frac{2(\Delta p)^2}{1 + \beta (\Delta p)^2 / m_p^2}, \quad (49)$$

where $(\Delta p)$ is the momentum-fluctuation scale of the system being in the coherent state $\psi$. From the relativistic equipartition theorem, one has $(\Delta p)^2 \approx 12 T^2$ [55].

We can now insert Eq. (49) into (48) and solve for $\beta$

$$\beta = \frac{(\delta - 1)}{12(5 - 3\delta)} \left( \frac{m_p^2}{T} \right)^2. \quad (50)$$

By setting the temperature $T = T_D \simeq M_1$ as in the previous computation and using the estimate (46) for Tsallis parameter, this yields $|\beta| \sim O(10)$. We notice that this value of the GUP parameter is consistent with the result of [76], $\beta = 82\pi /3$, derived from quantum corrections to the Newtonian potential. Likewise, it fits with predictions from Caianiello’s theory of maximal acceleration [77], $\beta = 8\pi^2 /9$, and non-commutative geometry in Schwarzschild spacetime [88], $\beta = 7\pi^2 /2$. Remarkably, our constraint is more stringent than those obtained in other cosmological/astrophysical contexts (see [99,100] for a detailed review). In this sense, it is much closer to predictions of string theory, $\beta \sim O(1)$, which set quantum gravity corrections at Planck scale (see Eq. (47)).

5. Discussion and conclusions

The baryon asymmetry produced in the radiation dominated era of the Universe can be explained under the assumption of a mechanism satisfying the three Sakharov conditions. In this work we have speculated on a possible Tsallis-induced asymmetry. In our picture, $CP$ is violated by the coupling between the derivative of the Ricci scalar and the baryon current. Taking into account interactions which also break the baryon number $N$, departure from thermal equilibrium is generated by non-extensive Tsallis entropy. Corrections are quantified by computing mass density and pressure fluctuations via the modified Friedmann equations (20) and (21). These variations enter the expression of the baryon asymmetry parameter $\eta$ through its dependence on the time derivative of the Ricci scalar, which is found to be $\delta$-dependent and non-vanishing (as opposed to the standard Boltzmann-like description). Requiring consistency with observed baryon asymmetry, we have constrained Tsallis parameter to be $|\delta - 1| \approx 10^{-3}$. This is in line with other bounds recently appeared in literature in Cosmology. Based on the connection between non-extensive Tsallis statistics and gravitational generalizations of Heisenberg relation, we have then transferred this result to the GUP parameter, yielding $\beta \sim O(10)$. Although this estimate is one order higher than predictions of string theory, it turns out to be more stringent if compared to other bounds from gravitational experiments and cosmological studies (see for instance [90]). Therefore, we conclude that our result provides one of the best estimates on $\beta$ of cosmological origin.

Apart from its own interest, the above formalism can find non-trivial applications in other cosmological problems. For instance, a demanding perspective is to look for signatures of tensor perturbations originated at inflation and propagated during Tsallis cosmological era in future experiments on primordial gravitational waves.

There exist other extended statistic besides Tsallis proposal. For example, a largely used statistics is that introduced by Kaniadakis, which comes from relativistic corrections to the Boltzmann theory [91]. It would be interesting to explore how modified Friedmann equations affect the mass density/pressure content of the Universe in that context [92]. Along this line, preliminary considerations could be derived by making use of the relation between Tsallis and Kaniadakis entropic measures [93]. Work along these directions is in progress.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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