LENSES FOR COMPOSABLE SERVERS

ANDRE VIDELA, MATTEO CAPUCCI

ABSTRACT. We implement the semantics of server operations using parameterised lenses. They allow us to define endpoints and extend them using classical lens composition. The parameterised nature of lenses models state updates while the lens laws mimic properties expected from HTTP.

This first approach to server development is extended to use dependent parameterised lenses. An upgrade necessary to model not only endpoints, but entire servers, unlocking the ability to compose them together.

1. INTRODUCTION

Client-server architectures are the bread and butter of modern Internet applications with HTTP [FGM99] being the underlying technology enabling them. HTTP allows communication between a client and a server, and in that context, servers need to be written, maintained and extended to keep providing services to its clients. However, the implementation and design of servers have been relying on ad-hoc conventions [Fie00] and informal practices to make up for a lack of formal definition of servers. The goal of this paper is to find a data structure that captures the essence of web server operations to construct, extend and compose servers together in an automated manner.

We will see in detail how lenses can be used to fill that role. Thanks to their use as generic data accessors, we can model a typical server as a program that gives access to a resource, and lenses focus on different parts of it. While helpful, that picture is incomplete because servers need to handle state in addition to respond to requests. To represent state, we are going to rely on a variant of lenses called “parametrised lenses”[CHGR21a], they make use of an additional “port” which we use to represent state. From that perspective, servers as
lenses are interaction systems with both an outside client, and a database.

\[ \text{front end} \xleftarrow{\text{HTTP request}} \text{storage} \xrightarrow{\text{HTTP response}} \text{back end} \]  

(1.1)

We rely heavily on graphical intuition to communicate this correspondence between server and lenses. And because those ideas have been implemented, we are also going to rely on code snippets written in Idris [Bra13, Bra21]. Idris was picked for its focus on runnable software, in particular its out of the box support for network primitive while providing the dependent types necessary to implement dependent lenses.

1.1. Contributions. We provide the following contributions:

- A conceptualisation of resources as lenses.
- A definition of servers as parametrised dependent lenses.
- An implementation of a server library using the preceding contributions.

Those contributions will guide the structure of this document. In the first part, Section 2, we introduce plain and parametrised lenses. A reader familiar with them can skip it and jump directly to Section 3 where we introduce resources as parametrised lenses. Resources for web servers are traditionally documents served to a client. But with more dynamical web pages, resources come from a database, and clients ask the server to provide or modify the data living on the database. We are going to view resources as remote data that we can both query and update. In that interpretation, servers act as lenses focusing on different parts of their internal storage. This suggests that we can construct them using the same building blocks used for lenses, unlocking the ability to use the existing ecosystem of lens libraries to build servers.

We notice that lenses are not enough to build servers because they are stateless, whereas servers perform state manipulation, for this we move to parametrised lenses, whose parameter will represent our state. This implies that the server mediates the communication between the client and its state.

While useful, this construction hits a roadblock when we attempt to combine multiple lenses together. In order to build a server we need to step outside of the realm of lenses and write a server engine that perform routing and handling.
To solve this issue we introduce dependent lenses and dependent parametrised lenses in Section 4. They provide us with two new operators that perform the coproduct on parametrised lenses.

We will use this coproduct operation in Section 5 to combine endpoints into larger servers and even combine servers together. Finally, in the last section, Section 6, we look at the benefits of implementing these ideas in a programming language by presenting RECOMBINE a server library written in Idris built upon parametrised dependent lenses and providing an embedded domain specific language to write servers.

2. POLYMORPHIC (PARAMETRISED) LENSES

Lenses in functional programming have the following structure:

- A view function that retrieves the information nested inside a structure.
- An update function that returns a new version of the overall structure after modification using an argument.

Their original intent was to explain view and update in programs such as databases [FGM+07]. In practice, they gained popularity in the context of pure functional programming where mutating state is impossible, yet operations to update nested data are still necessary to implement real-world functionality. Lenses have been implemented into multiple libraries (such as [Kme12, War15]) to allow seamless manipulation of nested data by providing operators to compose lenses and use them to query objects. More recently, lenses have been extensively generalized, to so-called optics, and studied formally using advanced category theory [CEG+20]. Additionally, optics (and therefore lenses) have been found very helpful in modeling interactive systems such as games and machine learning models [CGHR21b, BHZ19] which suggests they are not only suited for data access, but also for dynamical systems.

This section will recap the definition of lenses we use and highlight the parts that are relevant for our servers. The code samples will be written in Idris, because the library we present in Section 6 is written in Idris. Idris’ syntax should be familiar to readers accustomed with Haskell, though we are going to depart from some of the traditional notation. In particular, products (pairs) are written using (* at the type level to distinguish them from their constructors at the term-level: (,). Coproducts (the Either type) will be written using (+) at the type level to be symmetric with (*). Idris features records as well as dot postfix notation which mimic syntax from object-oriented programming languages. We will make use of both of them to streamline the use of field projections.

A very small example of lenses is a record for a User which contains a field for an Address, which itself contains fields for its street name, street number and city.
record Address where
    constructor MkAddress
        city : String
        streetName : String
        streetNumber : Int

record User where
    constructor MkUser
        username : String
        address : Address
        birthdate : Date

Trying to update the street number of a user by copying the fields results in unsatisfying code gymnastics:

changeStreetName : String -> User -> User
changeStreetName newName (MkUser un (MkAddress c oldName num) bd) =
    MkUser un (MkAddress newName c num) bd

The entire constructor needs to be destructured and copied, save for the new value. But as this example shows, not only it’s tedious, it’s also error prone since neither the syntax nor the types allow us to catch the fact that we’ve flipped the street name and the city in the example.

Lenses aim to solve this problem of composing nested updates by abstracting the projection and update functions into their own records. A classical implementation for lenses would be:

record Lens (s, t, a, b : Type) where
    constructor MkLens
        view : s -> a
        update : s -> b -> t

The view returns a sub-part of the data. Conversely, given an argument b, which represents a change in the data, the update returns new data. The true power of the lens abstraction, however, is in encapsulating these operation into composable and reusable blocks. Indeed, assuming lenses for each field of our previous example, our change in street name is expressed with the following program:

changeStreetName : User -> Int -> User
changeStreetName = update (userLens |> streetLens)

where (|>) performs lens composition, which we will define in Section 2.1.1.
To better capture the expected behavior of lenses, sometimes one limits themselves to so-called *monomorphic lawful lenses*, which are lenses such that \( s = t \) and \( a = b \) (this is what *monomorphic* means) and that moreover follow a set of laws:

\[
\begin{align*}
\text{-- You get back what you’ve put in} \\
\text{put-get : } \forall x, v, \text{lens. lens.view (lens.update } x \ v) &= v \\
\text{-- Putting things in twice in a row is the same as doing it once} \\
\text{put-put : } \forall x, v, \text{lens. lens.update (lens.update } x \ v) v &= \text{lens.update } x \ v \\
\text{-- if you put in what you see you won’t change anything} \\
\text{get-put : } \forall v, \text{lens. lens.update (lens.view } v) v &= v
\end{align*}
\]

It’s very important that these properties are stable under composition, that is compositions of lawful lenses are still lawful [Ril18].

Throughout this paper we will use three representations for lenses:

2.0.1. *Lenses as morphisms between boundaries.* This notation exposes the nature of lenses as morphisms between boundaries and is written as \((X, S) \rightarrow (Y, R)\). They decompose into functions \(X \rightarrow Y\) which we call the *forward* part, and \(X \rightarrow R \rightarrow S\), which we call the *backward* part.

2.0.2. *Lenses as string diagrams.* We can represent lenses as black boxes with left and right boundaries that compose together. The composition operations will follow shortly. Sometimes it is informative to draw the content of the lens to understand how they are implemented and how the flow of data is directed through them. You see in Figure 1 a typical lens with a view and update function, the top left input \((X)\) is shared between the *view* and *update*. The direction of arrows shows the flow of data through the lens and from and explains why the *view* function is called *forward* and *update* is called *backward* (arrows flow from right to left). In Figure 2 the lens is an *adapter*, adapters do not share the top left input with the backward part.

2.0.3. *Lenses as programs.* Lenses can be implemented as a product of two functions, one for the forward and one for the backward part of the lens. While we could use the implementation given earlier, we choose to represent boundaries explicitly rather than rely on four unrelated
type parameters. We achieve this in Idris by declaring lenses as a record parametrised over two boundaries:

\[ \text{Boundary} : \text{Type} \]
\[ \text{Boundary} = \text{Type} * \text{Type} \]

record Lens (l, r : Boundary) where
    constructor MkLens
    view : l.π₁ -> r.π₁
    update : l.π₁ -> r.π₂ -> 1.π₂

Using boundaries instead of type parameters recalls their notation as morphism between boundaries, though its functions need to be defined in terms of projections of the boundaries where \( \pi₁ : a * b \rightarrow a \) and \( \pi₂ : a * b \rightarrow b \).

2.1. Operations on lenses. As anticipated, the strength of lenses lies in their compositional properties. The two operations that we will use on lenses are sequential composition and parallel composition. The first one stitches two lenses together one after the other, assuming their connecting boundaries are the same. For data accessors, this is used to focus on deeper parts of a record. Instead, parallel composition is used to focus on two different parts of a record simultaneously.

2.1.1. Composition. Sequential composition chains two lenses together when their boundaries agree. The first lens must have a right boundary that is compatible with the left boundary of the second lens. This means composition is a map like this

\[ \triangleright : ((X, T) \rightarrow (Y, S)) \times ((Y, S) \rightarrow (Z, R)) \rightarrow (X, T) \rightarrow (Z, R). \]

\((\triangleright) : (a : \text{Lens } l \ x) \rightarrow (b : \text{Lens } x \ r) \rightarrow \text{Lens } l \ r\)
\((\triangleright) \ (\text{MkLens } v₁ \ u₁) \ (\text{MkLens } v₂ \ u₂) = \text{MkLens} \)
\((v₂ . v₁)\)
\((\\text{\lambda st, val} \Rightarrow u₁ \ st \ (u₂ \ (v₁ \ st) \ \text{val}))\)
2.1.2. Parallel composition. The parallel composition of two lenses produces a lens that focuses on two distinct parts of a product. In order to properly combine the boundaries, we perform the product on boundaries that we define as the point-wise product of its components.

\[
(x, y) \times (a, b) = (x \times a, y \times b)
\]

\[
\text{parallel} : \text{Lens} \, l_1 \, r_1 \rightarrow \text{Lens} \, l_2 \, r_2 \rightarrow \text{Lens} \, (l_1 \times l_2) \, (r_1 \times r_2)
\]

\[
\text{parallel} \, (\text{MkLens} \, v_1 \, u_1) \, (\text{MkLens} \, v_2 \, u_2) = \text{MkLens}
\]

\[
\text{bimap} \, v_1 \, v_2
\]

\[
\langle s_1, s_2, b_1, b_2 \rangle \mapsto \langle u_1 \, s_1 \, b_1, u_2 \, s_2 \, b_2 \rangle
\]

2.2. Parameterised lenses. The final piece of technology we need to introduce is parametrised lenses. They are lenses augmented with an additional boundary on the top. This version of lenses arises by applying the Para construction on lenses, as detailed in [CGHR21a]. For our purposes, the parametr exposes the state of the server. Adding a parameter to our lenses changes the nature of the lens algebra allowing the composition of lenses along an extra
dimension. Effectively, the parameter boundary is attached to the left boundary using the product of boundaries outlined above:

\[(P, Q) \times (X, S) \rightarrow (Y, R)\]

\[
\text{ParaLens : Boundary } \rightarrow \text{Boundary } \rightarrow \text{Boundary } \rightarrow \text{Type}
\]

\[
\text{ParaLens left para right } = \text{Lens} \left(\text{left } * \text{ para}\right) \text{ right}
\]

\[
\begin{array}{c}
\text{Q} \\
\text{P} \\
\text{X} \\
\text{S}
\end{array}
\quad
\begin{array}{c}
\rightarrow \text{Y} \\
\rightarrow \text{R}
\end{array}
\]

From now on, we refer non-parametrised lenses as \textit{plain lenses}. In what follows we explain operators on parametrised lenses.

2.2.1. \textit{Embedding plain lenses}. We observe that plain lenses can be interpreted as parametrised lenses with a unit top boundary: 

\[
\text{toPara } ((X, S) \rightarrow (Y, R)) = ((\,\,\,), () \times (X, S) \rightarrow (Y, R))
\]

\[
\text{toPara : Lens l r } \rightarrow \text{Para l ((), () r}
\]

\[
\begin{array}{c}
\text{toPara lens } = \\
\text{MkLens (lens.view . } \pi_1) (\lambda x \rightarrow (\,\,\,),(\,) . \text{ lens.update x.}\pi_1)
\end{array}
\]

2.2.2. \textit{Composition of parametrised lenses}. The addition of a parameter seems innocuous but it changes composition. Composed parametrised lenses combine their parameter with a product, that means that the resulting lens has access to the state of both lenses at once.

\[
\text{(|>) : Para l p1 x } \rightarrow \text{Para x p2 r } \rightarrow
\]

\[
\begin{array}{c}
\text{Para l (p1 } * \text{ p2) r}
\end{array}
\]

\[
\begin{array}{c}
\text{Q } \times \text{ Q'} \\
\text{P } \times \text{ P'}
\end{array}
\quad
\begin{array}{c}
\text{X} \\
\text{T}
\end{array}
\quad
\begin{array}{c}
\rightarrow \text{Y} \\
\rightarrow \text{S}
\end{array}
\quad
\begin{array}{c}
\rightarrow \text{Z} \\
\rightarrow \text{R}
\end{array}
\]

\[
\text{Q' } \times \text{ Q'} \\
\text{P' } \times \text{ P'}
\]

2.2.3. The state lens. Using ‘corner wires’ (see Figure 5a) we can expose the top boundary as the focus of our lens for querying and updating. In our server interpretation, the top boundary will play the role of state parameters, that is why we call this lens \texttt{State}.

\[
\text{state} : \text{(b : Boundary)} \rightarrow \text{Para ((), () \ b \ b)}
\]

\[
\text{state (MkB p q)} = \text{MkLens \ snd \ (const \ ((),()))}
\]

2.2.4. Reparametrisation. Reparametrisation (Figure 5b) allows changing the boundary along the top using a plain lens, this is the extra dimension along which we can compose lenses.

\[
\text{reparam : Para \ l \ p \ r \ \rightarrow \ Lens \ p' \ p \ \rightarrow \ Para \ l \ p' \ r}
\]

2.2.5. Pre-composition (\texttt{\ll}). Pre-composition can be implemented in terms of 2.2.1, 2.2.2, and 2.2.4. It allows changing the left boundary using a plain lens. This will be used to pre-process incoming requests before handling them.

\[
\text{\\lll : Lens \ l \ x \ \rightarrow \ Para \ x \ p \ r \ \rightarrow \ Para \ l \ p \ r}
\]

2.2.6. Post-composition (\texttt{\gg}). Similarly with post-composition, it allows changing the right boundary using a plain lens: This will be used to implement new endpoints from existing ones by focusing on deeper parts of a resource.
3. Resources as parametrised lenses

Our first contribution consists in relating lenses and resources. We defined a resource on a server as being a pair of endpoints: one GET endpoint to retrieve data and a POST endpoint to update the same data. This section will explain how we go from pairs of such endpoints to server implementation using lenses. We start by recapping the properties of HTTP we are interested in modeling, then explain how those properties can be captured as plain lenses and finally we move on to implementing servers using parametrised lenses.

3.1. HTTP. Servers communicate through HTTP\cite{FGM99}, where requests are sent from the client to the server and the server responds with the content asked. The use of lenses and resources is inspired by RESTful servers architecture \cite{Fie00} which is widely used in commercial software. HTTP requests are composed of many parts: HTTP method, URI, protocol version, headers, and body. For our purposes, we are going to direct our attention to HTTP methods, URI, and request body. Here are the three things to look out for:

3.1.1. HTTP Methods. HTTP methods have different properties, we have laid them out in the following table:

| Method | Side effects | Idempotent | Description |
|--------|--------------|------------|-------------|
| GET    | NO           | YES        | Obtain a value from the server. |
| POST   | YES          | NO         | Change some data on the server. |
| DELETE | YES          | YES        | Remove some data from the server. |
| PUT    | YES          | YES        | Replace some data on the server. |
| PATCH  | YES          | NO         | Append some data on the server. |

The table shows that endpoints that use HTTP methods such as PUT, GET and DELETE are meant to be idempotent, but others do not have to.
3.1.2. URI. Each resource can be accessed through its URI which traditionally represented the path to the file on the server’s filesystem. Nowadays, due to the dynamic nature of web pages, servers parse the URI and use it to decide which resource to return irrespective of its path in the filesystem and usually from a database. Additionally, URLs can carry data using captures e.g: /user/:id/name

This means that paths such as /user/3/name are routed through this api. But /user/book will not, because “book” is not an ID. In this paper we are going to write down the type that we expect our captures to have, this way, if we expect an Int as id, we write /user/Int:id/name. URI and captures being such a fundamental feature of HTTP requests, our server interpretation must have a way to represent them.

3.1.3. Request body. The request body is an arbitrary blob of data that is parsed by the server. In recent years, this data is often serialised to JSON. GET requests are special because their request body is not meant to carry any information.

3.2. Servers as functions. After getting familiar with the conditions under which a server must operate we can start building abstractions to represent them. A functional programmer’s way to do this would be to lay out the functions which operate a server and identify their types and their properties, so we are going to do just that for the endpoints of a server.

We are going to focus on GET and POST endpoints since POST can implement every other HTTP method by restricting it. And GET requests are the only ones that do not perform a side effect on the server and do not carry a request body. If we recall the lens laws from Section 2 we see that the put-put law ensures that performing an update twice in a row produces the same result as updating once, encoding idempotency of update. This suggests that lawful lenses encode PUT or DELETE requests while unlawful lenses encode POST or PATCH requests.

3.2.1. GET endpoints for a client. From the point of view of a client, GET endpoints take as argument the content of an HTTP request and return some resource from the server.

getEndpoint : Request -> Response

More precisely, the arguments that the GET endpoint has access to are the URI, since the body is supposed to carry no information whatsoever.

getEndpoint : uri -> Response

Finally, the Reponse type is a bit too broad, each endpoint returns a single value which will be serialised in the body of the response. The rest of the response will be built the same way for each endpoint. After refactoring the common parts we are left with:

getEndpoint : Serialisable resultGet => uri -> resultGet
3.2.2. POST endpoints for a client. Unlike GET endpoints, POST endpoints make essential use of their request body and also modify the state of the server, so while the overall picture \texttt{postEndpoint: Request -> Response} is the same, refining the types results in the following:

\texttt{postEndpoint : Parsable body => Serialisable resultPost => uri -> body -> resultPost}

We have not addressed the fact that this function can change the state of the server, but from the point of view of the client, this is irrelevant, the client only cares about the result given and will trust that the server performed the side effect as expected.

3.3. Resources as lenses. Earlier, we defined pairs of endpoints viewing and updating the same data as a resource. Placing the two functions describing our resource side by side yields something very close to a lens:

\texttt{getEndpoint : Serialisable resultGet => Parsable uri => uri -> resultGet}

\texttt{postEndpoint : Parsable body => Serialisable resultPost => uri -> body -> resultPost}

In fact, if we remove the constraints we see in Figure 6 that \texttt{uri -> resultGet} forms the forward part of a lens, while \texttt{uri -> body -> resultPost} is the backward part.

If we assume our server is composed of two endpoints that permit us to view and update a single user, our lens would look like this:

\begin{center}
\begin{tikzpicture}

\node (get) at (0,0) {\texttt{get endpoint}};
\node (post) at (0,-1) {\texttt{post endpoint}};
\node (result) at (-1,0) {result get};
\node (uri) at (-1,-1) {uri};
\node (body) at (0,-1) {body};
\node (resultPost) at (1,-1) {resultPost};

\draw (get) -- (result);
\draw (post) -- (resultPost);
\draw (uri) -- (get);
\draw (body) -- (post);
\end{tikzpicture}
\end{center}

With this setup, creating a pair of endpoints exposing the \texttt{Address} resource can be done by composing it with a lens \((\text{User, User}) \rightarrow (\text{Address, Address})\):

\begin{center}
\begin{tikzpicture}

\node (Int) at (0,0) {\texttt{Int}};
\node (User) at (1,0) {\texttt{User}};
\node (Address) at (2,0) {\texttt{Address}};
\node (post) at (0,-1) {\texttt{post endpoint}};
\node (User) at (0,-1.5) {\texttt{User}};
\node (Address) at (1,-1.5) {\texttt{Address}};

\draw (Int) -- (User);
\draw (User) -- (Address);
\draw (post) -- (Address);
\end{tikzpicture}
\end{center}
We observe that the uri of this lens is a type \texttt{Int}, we will develop the tools to write uri as types in Section 3.4.1.

3.4. \textbf{Pairs of Endpoints as Parameteried lenses.} We’ve explained how servers \textit{look} like lenses from the perspective of the client but this does not explain how to \textit{implement} servers. From the previous section, we know we could, in principle, build endpoints out of existing ones by extending their boundaries with lenses, but how do we implement the endpoints to begin with?

If we look at the pair of functions encoding the access to a resource, we see that we are unable to implement our server:

\begin{verbatim}
getEndpoint : uri -> responseGET
postEndpoint : uri -> requestBody -> responsePOST
\end{verbatim}

A \texttt{GET} endpoint cannot rely solely on the \texttt{URI} to serve the relevant information to the client, it needs access to a \textit{state}. Similarly, a \texttt{POST} endpoint needs to both access the state \textit{and} modify it to correctly implement its functionality. To take into account the state we need to access and modify we are going to add a type parameter to our endpoints:

\begin{verbatim}
getEndpoint : (inputState, uri) -> responseGET
postEndpoint : (inputState, uri) -> requestBody -> (outputState, responsePOST)
\end{verbatim}

In essence, if the client sees a resource as a plain lens \((X, S) \rightarrow (Y, R)\), the server sees it as a lens parametrised over the state \((P, Q)\) in which it operates: \((P, Q) \times (X, S) \rightarrow (Y, R)\). We thus end up with a complete definition of server:

\begin{center}
\begin{tikzpicture}
    \node (s) at (0,0) {state in \hspace{1em} state out}
    \node (u) at (0,-1) {uri \hspace{1em} \rightarrow \hspace{1em} \rightarrow \hspace{1em} result get
    \node (r) at (0,-2) {response post \hspace{1em} \rightarrow \hspace{1em} \rightarrow \hspace{1em} request body
    \end{tikzpicture}
\end{center}

Before we move on to the implementation of our server, we need to explain how this is a good enough representation of servers. First of all, parametrised lenses capture resources and state updates using their top boundary. And using \textit{post-composition} we can focus on deeper parts of a resource. Secondly, lawful lenses represent idempotent resources that can be called with a \texttt{PUT} request for their updates. Finally, unlawful lenses represent all other server endpoints that are not necessarily resources but that we still need to represent.
But if post-composition represents focusing on a deeper part of the resource, what do other lens operations such as pre-composition and parallel composition do for us? In what follows we explain how to use them for URI manipulation.

3.4.1. **URIs as products.** URIs represent paths to a resource, traditionally from a filesystem, but with modern servers, they allow routing requests. Because we use URIs to route requests, the chief property they have for us is that they are **parsable**. What is more, they are **structured** into path components, which can either be **captures** or **path components**.

We decided to use right-nested products to represent URIs as types. This way, a \((b \ast (c \ast d))\) represents the uri a/b/c/d. With this representation, we can encode captures as **types** and path components as **singleton** types, such as:

```haskell
data Str : String -> Type where
  MkS : (str : String) -> Str str
```

This data declaration ensures the type \(\text{Str } "hello"\) only has a single inhabitant \(\text{MkS } "hello"\).

We ensure that each path is parsable by requiring that each capture is parsable. Singleton strings are trivially parsable using an equality check.

3.4.2. **Projections as path extensions.** Using pre-composition we can compose a lens to the left of an endpoint that will perform some pre-processing on its left boundary. Because we are interested in performing operations such as declaring an endpoint as living under a higher directory level, we want to model **URI extension** as a lens.

Thanks to our representation of URIs as products we can achieve this by taking a URI and nesting it to the right of a product:

```haskell
prependPath : String -> (uri : Type) -> Type
prependPath str path = Str str \ast path
```

Similarly, to add a capture to a URI we nest the existing path to the right of the type of our capture:

```haskell
-- Carrier is just a wrapper around types that are parsable
prependType : (ty : Type) -> Parsable ty => (uri : Type) -> Type
prependType ty path = Carrier ty \ast path
```

We implement the extension of paths with an adapter that has the extended path as its left boundary and the projected path as its right boundary. Figure 7 shows a server extended by such an adapter.

We abstract this operation with the operator \((/):\)

```haskell
(/) : (str : String) -> Lens (X, S) (Y, R) ->
    Lens (Str str \ast X, S) (X, Y)
```
3.4.3. Captures as parallel composition. If we want to add a URI capture to an existing server we have to use parallel composition on our endpoint with an identity lens that will carry the capture to the lens that will make use of it. In the following diagram, we make use of a state lens \((s)\) that exposes the state as a right boundary. We then use parallel composition with an identity adapter that forwards the URI capture to the right boundary of our state lens. Then we post-compose the server with a lens \((op)\) that performs lookup and insert operations. Additionally, we pre-compose this server with an adapter \((p)\) which converts the left boundary into a valid URI and a response body.

Because this careful threading of string diagrams is quite repetitive and error-prone, we’ve abstracted this operation of carrying URI captures under the operator \((::/)\).

-- Carrier carries around the evidence that ‘ty’ is Parsable

\((::/) : (ty : Type) \rightarrow\) Parsable\(ty \Rightarrow\) Lens \((X, S) (Y, R) \rightarrow\)

\(\text{Lens} (\text{Carrier} ty * X, S) (\text{Carrier} ty * Y, R)\)

Combined with our previous operator for path extensions, this results in a familiar syntax to build path components: "user" / Int ::/ "Todo" / ...

3.5. Server implementation and limitations. In this section, we explain at a high level how a collection of lenses can be translated into a running server, and how parametrised lenses remain limited.

Because lenses represent resources and resources are pairs of endpoints, Each lens generates a GET and POST endpoint which behave like data accessors. That is, the forward part will
implement the GET endpoint, and the backward part the POST endpoint. Given a list of those lenses, we can collect all the endpoints and their implementations and give them to a server engine. This engine will wait for requests, parse them using the parsers from the list of lenses provided, and handle the request using the forward/backward part (depending on if the request was POST or GET). Finally, the result will be sent back to the client and its internal state will be updated.

This definition of servers is able to model URI parsing, GET and POST endpoints, and create new endpoints by composing lenses with existing ones. However, we observe two limitations:

- All our lenses need to operate in the same state, otherwise we do not know how to update our state. But in principle, we would like to combine two lenses that operate in two different environments.
- We can combine our lenses in a list to build a server, but once in a list form, we cannot use lens operations to further extend and compose our server with other servers and endpoints.

While in practice this does not stop us from running servers, it does fall short of our goal to find a data structure that explains server composition in its entirety. Having to rely on an external program to perform routing and being unable to combine servers of different states is insufficient.

Both those limitations suggest that we are missing a composition operation that gives us the choice of which lens to run. This is traditionally achieved with a coproduct: $A + B$.

Attempting to implement a coproduct on lenses will shed light on the next steps to take:

```
coproduct : Lens a b -> Lens x y -> Lens (a + x) (b + y)
coproduct (Lens get1 set1) (Lens get2 set2) =
    Lens (bimap get1 get2)
        (\(input : a.\pi1 + x.\pi1)\), (arg : b.\pi2 + y.\pi2) => ?what
```

It turns out that we cannot implement this operator on the backward part of the lens because we only have access to functions $set1 : a.\pi1 \rightarrow b.\pi2 \rightarrow a.\pi1$ and $set2 : x.\pi1 \rightarrow y.\pi2 \rightarrow x.\pi2$. Unfortunately, the type signature we need to implement allows for inputs of type $a.\pi1$ and $y.\pi2$ to be available simultaneously, even if we do not have access to a function $a.\pi1 \rightarrow y.\pi2 \rightarrow a.\pi2 + x.\pi2$.

But if were able to constrain the type of $arg$ to precisely correspond to the lens taken by the input, then we could ensure that $arg$ has type $b.\pi2$ whenever $input$ is a Left value.

This property can only be achieved by introducing dependent types in our lenses. Which is the topic of the next section.
4. Dependent (parametrised) lenses

Much like Section 2, we define lenses and their parametrised counterparts, however, we are going to replace our boundaries with dependent boundaries. This extension of lenses toward dependent lenses should provide us with the missing coproduct operator on lenses.

Previously our definition of lenses looked like this:

```
record Lens (l, r : Boundary) where
    constructor MkLens
    view : l.π₁ -> r.π₁
    update : l.π₁ -> r.π₂ -> l.π₂
```

We replace our Boundary type by a dependent boundary `(x : Type, x -> Type)`

```
record DBoundary where
    π₁ : Type
    π₂ : π₁ -> Type
public export
record DLens (l, r : DBoundary) where
    view : l.π₁ -> r.π₁
    update : (v : l.π₁) -> r.π₂ (view v) -> l.π₂
```

This program does not typecheck because `l.π₂` requires an argument of type `l.π₁` and similarly `r.π₂` expects a value of type `r.π₁`. We can easily find a value of type `l.π₁`, it’s `v`, but we need a little bit more gymnastic to find a value of `r.π₁` and we do this by making a call to `view` in the type of `update`. Our corrected definition looks like so:

```
record DLens (l, r : DBoundary) where
    view : l.π₁ -> r.π₁
    update : (v : l.π₁) -> r.π₂ (view v) -> l.π₂
```

The DLens data type is also known in the litterature as *container morphism* [AAG03, AAGM03]. With this in mind we replace our boundaries by Containers and settle on this definition:

```
record Container where
    shape : Type
    position : shape -> Type

record DLens (l, r : Container) where
    constructor MkDLens
    view : l.shape -> r.shape
    update : (v : l.shape) -> r.position (view v) -> l.position v
```
The intuition of lenses as container morphisms suggests that, if we think of containers as type descriptors and morphisms between them as maps between types, then lenses convert from one type to the other and the implementation of the lens performs the transformation between terms described by the containers. However, the analogy breaks down when we look at the forward part and the backward part of the lens as two endpoints modifying different kinds of data. This curious disconnect will be revisited in future work (see Section 7). For now, we are going to consider dependent lenses as lenses with additional structure, we start by reiterating the operators we use on containers for dependent lens composition.

4.1. **Operations on Containers.** To combine our servers we need to understand how to combine their boundaries, for this we quickly summarize the operations on containers as seen in [AAG03]

4.1.1. *Multiplication* (*). This operator performs a product on the shapes but a coproduct on the positions.

\[
(*) : (c1, c2 : \text{Container}) \rightarrow \text{Container}
\]

\[
(*) \text{ c1 c2} = \text{MkCont}
\]

\[
(c1.\text{shp} * c2.\text{shp})
\]

\[
(\lambda \text{x} \Rightarrow c1.\text{pos x.fst} + c2.\text{pos x.snd})
\]

4.1.2. *Addition* (+). This operator performs a coproduct on the shapes and, depending on which shape we are looking at, returns the position associated with the first or second container. This represents a choice of containers.

\[
(+) : (c1, c2 : \text{Container}) \rightarrow \text{Container}
\]

\[
(+) \text{ c1 c2} = \text{MkCont}
\]

\[
(c1.\text{shp} + c2.\text{shp})
\]

\[
(\text{either c1.pos c2.pos})
\]

4.1.3. *Tensor*. This operator performs a product on both the shapes and positions. This represents a parallel composition of containers.

\[
tensor : (c1, c2 : \text{Container}) \rightarrow \text{Container}
\]

\[
tensor \text{ c1 c2} = \text{MkCont}
\]

\[
(c1.\text{shp} * c2.\text{shp})
\]

\[
(\lambda \text{x : c1.shp * c2.shp} \Rightarrow
\]

\[
c1.\text{pos x.pi1} * c2.\text{pos x.pi2})
\]

4.1.4. *Const*. This operator builds a container using the same type for the shapes and the positions.
4.2. **Dependent Parametrised Lenses.** Because dependent lenses come equipped with the same composition and parallel composition operations as plain lenses, we elude them and jump straight into their parametrised variants, which we use for servers.

Just like in Section 2.2, we are going to extend our notion of dependent lenses by adding a *parameter* to our lenses, this parameter will be the top boundary of our lenses which will represent the state in our server. We define dependent parametrised lenses by using the tensor product to combine the top boundary and the left boundary:

\[
D_{\text{Para}} : (l, p, r : \text{Container}) \rightarrow \text{Type}
\]

\[
D_{\text{Para}} l \ p \ r = \text{DLens} (l \ '\text{tensor}' \ p) \ r
\]

Just like before we unlock several operations on dependent parametrised lenses:

4.2.1. **Reparametrisation.** Just like plain parametrised lenses, we can modify the top boundary using a dependent lens.

\[
\text{reparam} : D_{\text{Para}} l \ p \ r \rightarrow \text{DLens} p' \ p \rightarrow D_{\text{Para}} l \ p' \ r
\]

\[
\text{reparam} \ \text{lens} \ \text{para} = \text{MkDLens}
\]

\[
(\text{lens.view} \ . \ \text{mapSnd} \ \text{para.view})
\]

\[
(\langle x, y \rangle \Rightarrow \text{mapSnd} \ (\text{para.update} \ y)
\]

\[
. \ \text{lens.update} \ (x, \ \text{para.view} \ y))
\]

4.2.2. **Sequential composition.** Just like with plain parametrised lenses, the top boundaries are tensored.

\[
\text{associate} : a \ast (b \ast c) \rightarrow (a \ast b) \ast c
\]

\[
\text{-- we use a different operator to differentiate with}
\]

\[
\text{-- composition of dependent lenses.}
\]

\[
(|>) : D_{\text{Para}} l \ p \ x \rightarrow D_{\text{Para}} x \ q \ r \rightarrow D_{\text{Para}} l \ (p \ '\text{tensor}' \ q) \ r
\]

\[
(|>) \ (\text{MkDLens} \ v1 \ u1) \ (\text{MkDLens} \ v2 \ u2) = \text{MkDLens}
\]

\[
(v2 \ . \ \text{mapFst} \ v1 \ . \ \text{associate})
\]

\[
(\langle x, (y, z)\rangle, \ \text{arg} \Rightarrow \text{let}
\]

\[
\{ \ (v1, \ \text{st1}) = \ u2 \ (v1 \ (x, y), \ z) \ \text{arg}
\]

\[
; \ (v2, \ \text{st2}) = \ u1 \ (x, y) \ v1
\]

\[
\} \ \text{in} \ (v2, \ \text{st2}, \ \text{st1})
\]
4.2.3. **Pre-composition** (\(\langle\langle\rangle\rangle\)) and **Post-composition** (\(\langle\rangle\rangle\)). Those operations are implemented in terms of composition and re-parametrisation.

-- Pre-composition
\[
\langle\langle\rangle\rangle : \text{DLens } l \ l' \rightarrow \text{DPara } l' \ p \ r \rightarrow \text{DPara } l \ p \ r
\]

-- Post-composition
\[
\langle\rangle\rangle : \text{DPara } l \ p \ r' \rightarrow \text{DLens } r' \ r \rightarrow \text{DPara } l \ p \ r
\]

4.2.4. **Parallel composition.** Parallel composition is inherited from parallel composition of dependent lenses, composed with re-associating the left boundary.

reassoc : \((a \ast x) \ast (b \ast y) \rightarrow (a \ast b) \ast (x \ast y)\)

parallel : \(\text{DPara } a \ p \ b \rightarrow \text{DPara } x \ q \ y \rightarrow\)
\[
\text{DPara } (a \ast \text{tensor } x) \ (p \ast \text{tensor } q) \ (b \ast \text{tensor } y)
\]
parallel l r = MkDLens reassoc (\(((a, x), (b, y)) \rightarrow \text{reassoc})
\]
\|
\>
parallel l r -- This is the 'parallel' operator
|-- from DLens and \(\triangleright\triangleright\) is sequential
|-- composition from DPara

4.2.5. **External choice** (+\&\&+). This is the ‘coproduct-like’ operator we were looking for; it allows a client to choose which lens to run. Its implementation relies on the bifunctorial nature of (+) and (*).

distributive : \((s + s') \ast (p \ast p') \rightarrow (s \ast p) + (s' \ast p')\)

distChoice : \(\text{DPara } l \ p \ r \rightarrow \text{DPara } x \ q \ y \rightarrow\)
\[
\text{DPara } (l + x) \ (p \ast q) \ (r + y)
\]
distChoice (MkDLens v1 u1) (MkDLens v2 u2) = MkDLens
\[
(bimap v1 v2 \ . \ \text{distributive})
\]
\[
\langle\text{case } \{ ((\text{Right } v), (p, q)) \rightarrow \text{mapSnd Right } . u2 (v, q)
\.
; ((\text{Left } v), (p, q)) \rightarrow \text{mapSnd Left } . u1 (v, p)\})\rangle
\]

Parameters of both lenses need to be available in order to be ready to handle either input. We notice that, in the backward part, both p and q are available but we only use the one that corresponds to choice of the input. We start using parallax diagrams to demonstrate the choice of two lenses:
Because this is the crux of our implementation of servers as lenses we take some time to carefully explain how to read this result.

Looking at the diagram, an incoming request will pick which endpoint to call by providing a value of type X or W. For the purposes of this explanation, let us assume that the client calls a GET endpoint with a uri X. From this, the forward part of the lens in the background will be run and return a value of type Y, which will be sent back as a response.

If the client calls the server with a POST request, and a uri X, the backward part will be run. But before it does, the type of the backward part needs to be computed. That is because depending on which endpoints we call, we expect to parse different request bodies. To find out the type of the request body, we run the forward part with our value \( a : X \) to obtain a value \( b : \text{Left} \ Y \), which we use to compute the type of the request body. Computing this type amounts to evaluating the function \( \text{either} \ R \ T \ b \), because we got a \( \text{Left} \ Y \), the request body is expected to be \( R \). We attempt to parse the request body as a value of type \( R \) and if successful, we finally run the backward part with arguments \( a \) and request body of type \( R \). With the result of the backward part, we update the state with a value of type \( Q \ p.\pi 1 \) and send a response back of type \( S \).

To summarise, if the client provides an input of type X, the lens in the background of the diagram is run, otherwise, the lens in the foreground is run.

4.2.6. **Clone choice (&&&)**. Clone choice is our final operator and is defined by reparameterisation using operators \( \text{dup} : x \to x \times x \) and \( \text{dia} : x + x \to x \). This operation allows the choice of two lenses to share the same parametr.

\[
\text{cloneChoice} : \text{DPara} \ l \ p \ r \to \text{DPara} \ x \ p \ y \to \\
\text{DPara} \ (l + x) \ p \ (r + y)
\]

\[
\text{cloneChoice} \ l1 \ l2 = \\
\text{reparam} \ (\text{extChoice} \ l1 \ l2) \ (\text{MkDLens} \ \text{dup} \ (\_ \to \text{dia}))
\]

Just like external choice, clone choice allows the client to pick which endpoints to call using either X or W and depending on the choice, the corresponding lens is run. The difference is that the state between those two lenses is now shared.
5. Servers as dependent parametrised lenses

Previously we saw that parametrised lenses were not powerful enough to build entire servers because they lacked a coproduct, since dependent parametrised lenses provide this coproduct we are updating our definitions to make use of it.

In what follows, because there is no meaningful distinction between endpoints, resources, and servers, we are going to refer to all of them as “servers”. Similarly, because each dependent parametrised lens corresponds to a server, we are going to refer to them as Server in code samples rather than DPara.

5.1. External choice for state composition. The first operator we will use is extChoice, which allows combining two resources to form a server.

The external choice perfectly captures combining two endpoints into a server by performing a co-product on the left boundary of the lens, giving the client the choice of which endpoint to call. The states are combined with a product to ensure they both are available regardless of which endpoint is called, this can be seen in the implementation of the backward part of the lens:

```haskell
-- p and q are both available because we don’t know
-- if we will receive a ‘Left v’ or a ‘Right v’
\case (Right v, (p, q)) => mapSnd Right . u2 (v, q)
    (Left v, (p, q)) => mapSnd Left . u1 (v, p))
```

As a concrete example, Figure 9 shows that if we have a server with state Dict ID User and another one with state Dict ID Todo, combining both results in a server that works with state Dict ID User * Dict ID Todo

As a welcomed side effect, this buys us the ability to run each of those servers concurrently since the states will not share any data.

However, something is not right, if we combine two servers that work in the same state, the state parameter is duplicated after external choice, as you see in Figure 10. Performing a
state update through the POST endpoint of one server will not reflect the change in the other server. To correct this, we will use cloneChoice.

5.2. Clone choice for endpoint composition. cloneChoice combines two servers that operate in the same environment. This will fix our issue with extChoice which duplicates the state when combining two servers with the same state. With cloneChoice the state is now shared between the two endpoints in such a way that if a change is performed on one of
the states, it will be reflected in the state of the other server, Figure 11 illustrates the result of `cloneChoice` on two servers sharing the same state.

5.3. Request routing. Previously in Section 3.5, we had to rely on an external mechanism, our list of request handlers, to route our incoming requests to the corresponding lens. Now that we can implement our entire server as a single lens, we can rely on the parsing function of its input boundary to perform routing.

We saw in Section 3.4 how to use composition and tensor for path extension and captures. Those operations perform a coproduct and a product on the input boundaries, and because our servers require their input boundary to be parsable, the resulting boundary will remain parsable.

We use parser combinators [HM99] which are closed over products and coproducts, through functions typically called `sequence` and `alternative` respectfully:

```haskell
alternative : Parser a -> Parser b -> Parser (a + b)
```

```haskell
sequence : Parser a -> Parser b -> Parser (a * b)
```

Because we use an `interface`\(^1\) to enforce the parsable constraint of the left boundary we can automatically derive implementation for products and coproducts of parsers:

```haskell
Parsable a => Parsable b => Parsable (a * b) where
    parse = sequence (parse {t=a}) (parser {=b})
```

```haskell
Parsable a => Parsable b => Parsable (a + b) where
    parse = alternative (parse {t=a}) (parse {t=b})
```

We use a similar technique to enforce serialization constraints on the response types. This way, any composition of two valid servers with external choice or parallel composition results in a valid server.

5.4. State management. Previously, we were limited to servers whose resources all worked in the same state. We have lifted this limitation by using containers as boundaries. This upgrade comes with a caveat: The engine performing state updates for the server is now unable to override its previous state with the new state. This is because given a top boundary `st`, the engine that runs the server expects an initial state of type `p : st.π1`, but the result of running the backward part of our server lens results in a value of type `st.π2 p`. We can clearly see this inconsistency in the type of the backward part:

\(^1\)Interfaces are similar to typeclasses in haskell.
update : (v : p.π1 * l.π1) → r.π2 (view v) → p.π2 v.π1 * l.π2 v.π2

This is problematic for our state update because when we update our internal state we perform a side effect of the sort:

```haskell
-- pseudo code
handleRequest : Request → IO ()
handleRequest req =
  do currentState <- get state
     (newState, result) <- handleRequest req currentState
     put newState -- update the state here
     response result
```

We overcome this issue by requiring every top boundary of a server to carry an action of the shapes on the positions of our container. We write this in terms of an interface on containers:

```haskell
interface Action (c : Container) where
  act : (v : c.shape) → c.positions v → c.shape
```

Just like with parsing we provide implementations for products, coproducts, and tensor which will ensure closure over the action with regard to server composition:

```haskell
Action c1 => Action c2 => Action (c1 `tensor` c2) where
  act (s1, s2) (p1, p2) = (act s1 p1, act s2 p2)

Action c1 => Action c2 => Action (c1 + c2) where
  act (Left shape) p = Left (act shape p)
  act (Right shape) p = Right (act shape p)

Action c1 => Action c2 => Action (c1 * c2) where
  act (s1, s2) (Left x) = (act s1 x, s2)
  act (s1, s2) (Right x) = (s1, act s2 x)
```

Using our action we can patch up our previous implementation to make it typecheck.

```haskell
-- pseudo code
handleRequest : Request → IO ()
handleRequest req = do
  overallState <- get state
  (newState, result) <- handleRequest req overallState
```
5.5. **Implementing a dependent lens as a server.** The definitions we gave culminate in this section where we explain how to convert a single lens to a fully functional webserver. To summarise, a server is any dependent parametrised lens \((P, Q) \times (X, S) \rightarrow (Y, R)\) such that \(X\) and \(R\) are *Parsable*, \(S\) and \(Y\) are *Serialisable* and \(P\) and \(Q\) have an *Action*. In Figure 12, we update the diagram in Section 3 to capture the fact that each boundary depends on which lens was picked by the client when calling the server.

From this, we can re-use the same architecture from Section 3 but we only need to generate 2 request handlers, one for POST and one for GET requests, the rest is managed by the implementation of the lens.

In the following program, we achieved our goal to represent a server as a single lens. The type informs us of the conditions under which this abstraction makes sense.

```haskell
-- ‘ISerialisable’ and ‘IParsable’ are dependent version of
-- ‘Parsable’ and ‘Serialisable’
toHandler' : Parsable l.shape => IParsable r.position =>
    ISerialisable l.position => Serialisable r.shape =>
    Action p => Default p.shape =>
    Server l p r -> IO ()
toHandler' (MkDLens view update) =
    runServer (generateHandlers view update) def
```

We see that every lens gives rise to a server as long as its top left and bottom right boundary are *Parsable*, and it’s top right and bottom left boundary are *Serialisable*. Additionally, the state must have an accompanying *Action* and the product of states must have a *Default* value which acts as the initial state. *runServer* is our *engine* that waits for requests and provides responses.
6. The recombine library

The previous constructions have been implemented in Idris in a library we call RECOMBINE. This library allows you to write lenses using a small DSL and instantiate them into a running server. Using the building blocks we presented earlier we can write, extend and abstract over entire servers. To demonstrate the library we showcase 3 examples of servers with varying levels of complexity:

- A calculator.
- An IoT server that controls devices.
- A todo App.

Those examples will culminate in a combined server that hosts all three servers at once.

6.1. Calculator. Our calculator API has one endpoint per operation, each operation takes two arguments and returns a result without accessing or modifying any state. This is a simple example of a pure server running without state. Here is the API we aim to implement:

```
/add/Int:n1/Int:n2
/sub/Int:n1/Int:n2
/mul/Int:n1/Int:n2
/div/Int:n1/Int:n2
```

Using our DSL we can start with the `add` endpoint:

```
adddition : Server (MkCont (Int * Int) unit) (Const ()) (MkCont Int unit)
adddition = GetLens add
```

This endpoint accepts two integer numbers and returns another Int. It is implemented using the smart constructor `GetLens` which only populates the forward part and uses an identity as the backward part. We extend this server on its left boundary to have it live under the `/add` prefix:

```
addEndpoint : Server (MkCont (Str "add" * Int * Int) unit) (Const ()) (MkCont Int unit)
addEndpoint = "add" / adddition
```

We define `sub`, `mul`, `div` similarity so we end up with the following declarations:

```
subEndpoint : Server (Str "sub" * Int * Int, unit) ? ?
```
mulEndpoint : Server (Str "mul" * Int * Int, unit) ? ?
divEndpoint : Server (Str "div" * Int * Int, unit) ? ?

We use ? (question mark) in the type to indicate that we let Idris infer the type. We leave them off except for the first one to document how the type signature carries the API information. We build the server by using the cloneChoice operator (&&&):

calculator : Server (Str "add" * Int * Int
+ Str "sub" * Int * Int
+ Str "mul" * Int * Int
+ Str "div" * Int * Int) ? ?
calculator = addEndpoint &&& subEndpoint
&&& mulEndpoint &&& divEndpoint

One of the benefits of this approach is that the entire API of the server is visible in the type of the server and is computed along with its implementation.

6.2. IOT server. This server aims to showcase the ability of the framework to be extended with lenses that focus on deeper parts of a resource. We exemplify this by means of a home automation server whose state is the product of booleans which represent the state of various devices: \( \text{bool} \times (\text{bool} \times \text{bool}) \).

As a product of types, we can use lenses \( \text{fst} : ((a, b), (a, b)) \rightarrow (a, a) \) and \( \text{snd} : ((a, b), (a, b)) \rightarrow (b, b) \) to access and update each element. Before we start, here is the API we want to obtain:

GET /boiler \rightarrow\text{Bool}
POST /boiler {\text{Bool}} \rightarrow\text{Bool}
GET /lights/1 \rightarrow\text{Bool}
POST /lights/1 {\text{Bool}} \rightarrow\text{Bool}
GET /lights/1 \rightarrow\text{Bool}
POST /lights/2 {\text{Bool}} \rightarrow\text{Bool}

The type of the body expected for POST requests is written in curly braces.

The first step is to declare an endpoint that views and updates the state:

stateEndpoint : Server (Const ()) (Const HomeState) (Const HomeState)
stateEndpoint = State HomeState

In order to access and modify the boiler, we need to post-compose a lens that focuses on the first component of the product. We also need to pre-compose a lens that parses the /boiler URI. We combine both operations in this definition:
boilerEndpoint : Server (MkCont (Str "boiler") unit)
               (Const HomeState)
               (Const Bool)
boilerEndpoint = "boiler" / stateEndpoint => fst

We explicitly wrote the type signature to demonstrate how, for resources, the boundaries are Const containers.

To implement the endpoints for lights we create a parent endpoint that exposes the lights, and we post-compose it with fst and snd lenses to give access to each light individually.

lights : Server ?? ??
lights = stateEndpoint => snd

iotServer = Server ?? ??
iotServer = "boiler" / stateEndpoint => fst
       && "lights" / ("1" / lights => fst
       && "2" / lights => snd)

We leave off the types completely in this example, this show how the framework gives the choice of either declaring the API in advance and let the types guide the implementation, or write the implementation and check the API is suitable after typechecking.

6.3. **Todo app.** A Todo App is a perfect example that demonstrates how to implement a real server with practical functionality. It shows how abandoning the lens laws still results in a practical and composable server. Here is the API we set out to write:

**GET** /all/userId:Nat -> List Todo
**POST** /add/userId:Nat {Todo} -> ()

We implement this server by using GetLens defined previously and PostLens, its POST counterpart.

getTodos : Server (MkCont ? (const ())),
           (Const ServerState)
           (MkCont (List Todo) (const ()))
getTodos = "all" / Nat :/
           (GetLens (\(st, userId) =>
                    fromMaybe [] . lookup st userId))

postTodo : Server ? (Const ServerState) ?
postTodo = "add" / Nat :/
           (PostLens (\(st, userId), todo =>
update userId (todo :: st))

-- The server is the composition of the previous two endpoints
todoServer : Server ? (Const ServerState) ?
todoServer = getTodos &&& postTodo

6.4. Combining multiple servers. Because the external choice of servers results in a valid server, we expect to be able to combine multiple servers of drastically different nature together. Here is how we combine the three servers we just defined into one:

combinedServer : Server ? (Const ( ) * Const ServerState * Const HomeState) ?
combinedServer = "todo" / todoServer
   + &&& +
   "calculator" / calculator
   + &&& +
   "iot" / iotServer

We explicitly write down the type of the state, to demonstrate that it is correctly composed using (*) on containers.

7. Conclusion and future work

We found a similarity in the way we represent data access using lenses and the way servers expose their API. This approach motivated the use of parametrised lenses, dependent lenses, and dependent parametrised lenses, which turned out to be the missing abstraction to explain server composition that handles state composition, routing, and nested state updates.

We are going to conclude on a note about dependent types. While lenses from Section 2.2 are good enough to describe single resources, only dependent lenses provides a complete feature set for a server library. Dependent types help tremendously in the user experience of writing servers. Indeed the documentation of the API is readily available while writing the server. And because the boundaries can be inferred from the implementation, the programmer has the choice of either writing the server first and then check its API, or write the API first and then implement a program that serves it. One could conceivably expect a mechanism such as type providers to read a specification file, such as OpenAPI, and generate an API type to implement.

Despite being successful in describing servers and implementing them, this work prompts a lot of questions that we leave for future work, we summarize them here.
While we can derive server APIs and implementation from dependent lenses, and themselves are container morphisms, we should expect more interaction with the different semantics given by the Poly and Cont interpretation of our lens boundaries. For example, we expect to be able to express aspects of server communication using dependent lenses and interpret them as a protocol that handles errors if the protocol is not respected. This research area should prove fruitful when combined with indexed containers \cite{agh15} to represent legal transitions in stateful protocols. This approach could tell us something about implementing stateful servers and link session types with our description of server based on container morphisms. In practice, this would enable the same framework to be reused with a different engine than HTTP and find itself useful for describing UDP servers or Bluetooth communications.

Another unanswered question is the interpretation of dependent parametrised morphisms as an interactive system the categorical cybernetics way \cite{cghr21b}. In it, parametrised lenses represent bidirectional systems with a nesting, interacting controller acting as on the system through the ‘top boundary’. In that interpretation, we see the parameter as guiding the system after it’s been presented with an input, and observing the outcome after a response has been computed. There is an analogy between a server’s routing mechanism guiding a request toward its handler and an agent being guided through the steps of a game. In future work, we plan to merge those two ideas, parameters as performing routing, and polynomials as interactive systems better describe the dynamic nature of servers. In particular, we expect to even better model the interaction between routing and serving content from a storage, making it even more compositional and principled than proposed here. The rough idea is to follow \cite{cglf21} and thinking of URIs as ‘strategies’ for parametrised data accessors on a stored state, thereby flipping the picture we currently have.

Finally, one of the shortcomings of our approach is that we cannot define POST and GET endpoints independently from each other and that we cannot explain middleware as a lens, in particular, some middlewares are allowed to perform state updates including during the handling of GET requests. Using the intuition from containers as “question-answers” mechanisms we are confident we can implement both those ideas.

References

\cite{aag03} Michael Abbott, Thorsten Altenkirch, and Neil Ghani. Categories of containers. In Foundations of Software Science and Computation Structures. FoSSaCS 2003, volume 2620 of Lecture Notes in Computer Science. Springer, 2003.

\cite{aagm03} Michael Abbott, Thorsten Altenkirch, Neil Ghani, and Conor McBride. Derivatives of Containers, volume 2701, page 16–30. Springer Berlin Heidelberg, 2003.
LENSES FOR COMPOSABLE SERVERS

[AGH15] Thorsten Altenkirch, Neil Ghani, Peter Hancock, Conor Mcbride, and Peter Morris. Indexed containers. *Journal of Functional Programming*, 25:e5, 2015.

[BHZ19] Joe Bolt, Jules Hedges, and Philipp Zahn. Bayesian open games. Forthcoming in *Compositionality*, arXiv:1910.03656, 2019.

[Bra13] Edwin Brady. Idris, a general-purpose dependently typed programming language: Design and implementation. *Journal of Functional Programming*, 23(5):552–593, Sep 2013.

[Bra21] Edwin Brady. Idris 2: Quantitative type theory in practice. *arXiv:2104.00480 [cs]*, Apr 2021. arXiv: 2104.00480.

[CEG20] Bryce Clarke, Derek Elkins, Jeremy Gibbons, Fosco Loregian, Bartosz Milewski, Emily Pillmore, and Mario Román. Profunctor optics: A categorical update. *NWPT 2019*, page 47, 2020.

[CGHR21a] Matteo Capucci, Bruno Gavranović, Jules Hedges, and Eigil Fjeldgren Rischel. Towards foundations of categorical cybernetics. In *Proceedings of Applied Category Theory 2021*. EPTCS, 2021. Available at [https://arxiv.org/abs/2105.06763](https://arxiv.org/abs/2105.06763).

[CGHR21b] Matteo Capucci, Bruno Gavranović, Jules Hedges, and Eigil Fjeldgren Rischel. Towards foundations of categorical cybernetics. Forthcoming in *Proceedings of ACT 2021*, arXiv:2105.06332, 2021.

[CGLF21] Matteo Capucci, Neil Ghani, Jérémy Ledent, and Fredrik Nordvall Forsberg. Translating extensive form games to open games with agency. 2021.

[FGM99] R. Fielding, J. Gettys, J. Mogul, H. Frystyk, L. Masinter, P. Leach, and T. Berners-Lee. *Hyper- text Transfer Protocol – HTTP/1.1*. Number RFC2616. Jun 1999.

[FGM07] J. Nathan Foster, Michael B. Greenwald, Jonathan T. Moore, Benjamin C. Pierce, and Alan Schmitt. Combinators for bidirectional tree transformations: A linguistic approach to the view-update problem. *ACM Transactions on Programming Languages and Systems*, 29(3):17, 2007.

[Fie00] Roy Thomas Fielding. *Architectural styles and the design of network-based software architectures*. PhD thesis, University of California, Irvine, 2000. AAI9980887 ISBN-10: 0599871180.

[HM99] Graham Hutton and Erik Meijer. Monadic parser combinators. 09 1999.

[Kme12] Edward Kmett. lens: Lenses, folds and traversals. [https://github.com/ekmett/lens/](https://github.com/ekmett/lens/), 2012.

[Ril18] Mitchell Riley. Categories of optics. Sep 2018.

[War15] Adam Warski. Quicklens. [https://github.com/softwaremill/quicklens](https://github.com/softwaremill/quicklens), 2015.