A statistical method for luminosity monitoring in high energy collider experiments

João Bastos, João Carvalho, Michael Schmelling

1 LIP-Coimbra, Univ. de Coimbra, P-3004-516 Coimbra, Portugal
2 Max-Planck-Institut für Kernphysik, D-69029 Heidelberg, Germany

Abstract

A statistical method which uses a combination of two subdetectors to monitor the luminosity in high energy interactions is presented. To illustrate its performance, this method was applied to random triggered minimum bias data collected in the commissioning period of the HERA-B experiment in spring 2000. It is found that luminosity estimates with an intrinsic systematic error of 3% can be obtained.

1 Introduction

The precise determination of the luminosity of experimental data is required for absolute cross section measurements. Luminosity $l$ is defined as the proportionality factor between interaction rate and cross section for the process under consideration. The integrated luminosity $L$ relates cross section $\sigma$ and interaction count $N$ for a time interval $T$, i.e.

$$l(t) = \frac{1}{\sigma} \frac{dN}{dt} \quad \text{and thus} \quad L = \int_0^T l(t) dt = \frac{N}{\sigma}. \quad (1)$$

Given the cross section $\sigma$ for a particular process, such as the inelastic cross section in high energy hadronic interactions, the determination of the integrated luminosity for a given data set is equivalent to determining the number of interactions of that process. For the following we will focus on collider experiments, where a bunched beam produces events with a well defined time structure, and where the number of interactions per bunch crossing will fluctuate statistically.

As a first approach, determining the number of interactions could be accomplished by simply counting the number of reconstructed primary vertices in the
data. To achieve that, the vertex reconstruction efficiencies must be known. Additionally, two neighboring vertices may be merged by the reconstruction package while others may be split. Therefore, the probabilities for these processes must also be calculated, which is often difficult and introduces poorly known systematic errors. An alternative technique consists in extracting the total number of interactions from inclusive quantities in the reconstructed data which are proportional to the number of primary collisions in an event, such as the number of hits or the total energy deposition. Obviously, the main difficulty associated with this approach is the need for an absolute calibration, that is, the average signal for a single interaction must be known.

In a different approach (the so called “statistical method”), a poissonian distribution for the number of interactions per bunch crossing is assumed and the average number of interactions is extracted from the number of empty events in the data sample. The advantage of this method is that nothing about the average signal for a single interaction has to be known or assumed. However, the acceptances for tagging non-empty events must be estimated and the occurrence of noise events, which may be tagged as non-empty, must be taken into account.

In this paper a method for determining the integrated luminosity by counting the fraction of empty events simultaneously in two subdetectors is proposed. With this procedure the detector acceptances for a single interaction and the fraction of noise events can in principle be obtained from data, relaxing the dependence on Monte Carlo simulations to derive these quantities and the introduction of systematic errors which are difficult to estimate.

This paper is organized as follows. In the next section, an expression for the probability to observe an empty event is derived, assuming that the distribution of the number of interactions per bunch crossing follows Poisson statistics but allowing also for non-negligible rate fluctuations. Based on this, in Section 3, the Two-System Statistical Method (TSSM) is introduced. It is shown how counting the fraction of empty events in either of two subdetectors simultaneously in both allows to determine the acceptance of both subdetectors and the mean number of interactions in the data. In Section 4, this procedure is applied to minimum bias events collected in the commissioning period of the HERA-B experiment \[1\] in spring 2000. The conclusions are presented in Section 5.

### 2 Counting Empty Events

A particle collider usually has circulating beams with many bunches contributing to the observed interaction rate. Since the individual bunch currents, in
general, can differ by significant amounts, the following analysis is formulated for an ensemble of distinguishable bunches. This entails a slight complication of the formalism, but, as will become clear later, gains a lot of information which can be exploited in the analysis.

For the start let us assume that the distribution of the number of interactions per bunch crossing follows Poisson statistics. If the average number of interaction produced by bunch number $i$ is $\mu_i$, the probability that there are $n$ interactions in an event from this bunch crossing is

$$P(n, i) = \frac{\mu_i^n}{n!} e^{-\mu_i}. \tag{2}$$

Now suppose that a certain subdetector is used to count the number of empty events in the data set. An event is tagged as being empty if a quantity associated with this subdetector (e.g. hits, tracks, energy deposition, etc.) is below a specified threshold value. This value represents a compromise between a large efficiency for tagging non-empty interactions and an effective exclusion of noisy “events” in which no interaction has occurred. The probability to observe an empty event in this system is

$$P(0, i) = (1 - q) \sum_{n=0}^{\infty} (1 - a^{(n)}) P(n, i), \tag{3}$$

where $a^{(n)}$ is the acceptance, or efficiency, to tag an event with $n$ interactions as non-empty and $q$ is the probability to observe an event due to noise in the subdetector or to background (i.e. beam gas interactions). If the probability to pass the tagging threshold is independent of the number of primary interactions, which to a good approximation is valid if the threshold is set such that a single interaction has a large probability to exceed it, then $a^{(n)}$ can be approximated by

$$a^{(n)} = 1 - (1 - a)^n \tag{4}$$

where $a \equiv a^{(1)}$ is the efficiency to tag a single interaction as non-empty. Substituting Eq. (2) and (3) into Eq. (4) one gets

$$P(0, i) = (1 - q)e^{-a\mu_i}. \tag{5}$$

However, some bunches may suffer from rate instabilities so that Eq. (2) does no longer describe the interaction multiplicities correctly. In this case, the average number of interactions $\mu_i$ is no longer constant but it fluctuates by a random amount $\nu_i$ around its central value, $\mu_i \rightarrow \mu_i + \nu_i$. With $g(\nu_i)$ the probability density function of those fluctuations, the probability to observe
an empty event becomes

\[ P(0, i) = (1 - q) \int_{-\mu_i}^{\infty} e^{-a(\mu_i + \nu_i)} g(\nu_i) d\nu_i \]  

(6)

Assuming further that the fluctuations around \( \mu_i \) are Gaussian distributed, with zero average \( \langle \nu_i \rangle = 0 \) and standard deviation \( \sigma_i \ll \mu_i \), Eq. (6) can be integrated analytically to yield

\[ P(0, i) = (1 - q) \exp \left( -a \mu_i + \frac{1}{2} a^2 \sigma_i^2 \right) . \]  

(7)

One sees that rate fluctuations enter as second order effects, i.e. as long as they are small the assumption of poissonian distribution for interaction multiplicities is a good approximation. Large rate fluctuations, however, have a sizeable impact and have to be taken into account in the analysis.

3 The Two-System Statistical Method

Now let us consider two subdetectors or combinations of subdetectors, which will be denoted by “system 1” and “system 2”. According to Eq. (7) the probabilities \( p_k, k = \{1, 2\} \) to observe an empty event in either of the two systems are

\[ p_{ki} \equiv P(0, i)_k = (1 - q_k) \exp \left( -a_k \mu_i + \frac{1}{2} a_k^2 \sigma_i^2 \right) \]  

(8)

where \( q_k \) is the probability to record an event due to background or noise in system \( k \) and \( a_k \) is the efficiency to tag single interactions in this system. If the two systems are independent the probability \( p_0 \) to observe an empty event simultaneously in both subdetectors is given by an analogous expression

\[ p_{0i} = (1 - q_0) \exp \left( -a_0 \mu_i + \frac{1}{2} a_0^2 \sigma_i^2 \right) , \]  

(9)

where \( q_0 = q_1 + q_2 - q_1 q_2 \) and \( a_0 = a_1 + a_2 - a_1 a_2 \).

In order to get a handle on rate fluctuations, we now combine the statistical approach with a measurement based on an inclusive quantity, which is insensitive to deviations from a poissonian for the interaction multiplicities. This is achieved by expressing \( \mu_i \) in terms of a bunch dependent inclusive quantity \( \langle n \rangle_i \), which is proportional to the number of interactions per bunch crossing,

\[ \langle n \rangle_i = \tau \mu_i . \]  

(10)

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The parameter $\tau$ is the mean value of the inclusive quantity per interaction within the detector acceptance. Substituting Eq. (10) in Eq. (8) and (9) we have

$$p_{ki} = (1 - q_k) \exp \left( -a_k \frac{\langle n \rangle_i}{\tau} + \frac{1}{2} a_k^2 \sigma_i^2 \right), \quad \text{with} \quad k = \{0, 1, 2\}. \quad (11)$$

If there are no rate fluctuations, $\sigma_i \simeq 0, \forall i$, the global (bunch independent) parameters, $q_k, a_k$ and $\tau$, can be obtained from Eq. (11) by fitting the values of $p_{ki}$ as a function of the observable $\langle n \rangle_i$. Once $\tau$ is known, the average number of interactions $\mu_i$ for every bunch is calculated according to Eq. (10). From $\mu_i$ and the number of recorded events in each of the bunches, the total number of interactions in the data sample and thus the integrated luminosity can be calculated.

In case that rate fluctuations are present for some bunches, those bunches have to be identified and removed from the global fit. This can be achieved by considering the following relation between the probabilities $p_{ki}$,

$$\ln \frac{p_{0i}}{p_{1i}p_{2i}} = a_1 a_2 \left[ \frac{\langle n \rangle_i}{\tau} + \sigma_i^2 \left( 1 - a_0 - \frac{1}{2} a_1 a_2 \right) \right]. \quad (12)$$

In case of negligible rate fluctuations we have a simple linear relation between $\ln(p_{0i}/p_{1i}p_{2i})$ and $\langle n \rangle_i$. Bunches with significant rate fluctuations would deviate from that relation and can be excluded from the global fit. Note that Eq. (12) also has the potential to detect situations where all bunches are subject to rate fluctuations. In this case one has no outlier bunches, but a straight line fit to $\ln(p_{0i}/p_{1i}p_{1i})$ versus $\langle n \rangle_i$ would not pass through the origin, unless $\sigma_i^2$ and $\mu_i$ are proportional.

### 4 Application to HERA-B Minimum Bias Data

To illustrate its properties, the proposed method was applied to minimum bias events, collected with a simple random trigger during the HERA-B commissioning period in spring 2000. Applying the TSSM to real data shows how the considerations that went into its design cope with problems arising under realistic conditions. With respect to HERA-B, please note that the random-trigger based method described in this paper should not be confused with other ones employed by the HERA-B collaboration for luminosity measurements, such as for example the method [2] applied to the interaction-triggered data recorded in 2002/2003.

HERA-B is a large acceptance fixed target experiment that studies the interactions of 920 GeV protons with wire targets placed in the beam halo.
of the HERA storage ring, at DESY. The HERA-B target consists of two stations separated by 4 cm along the beam. Each station comprises four wires of different materials, with dimensions ranging from 0.5 to 1 mm along the beam and from 50 to 100 µm perpendicular to the beam. Each wire can be independently moved inside the beam halo in order to adjust the interaction rate. The reconstruction of primary and secondary vertices is performed by a silicon micro-strip Vertex Detector System. The main tracker is divided into the Inner Tracker, composed of micro-strip gas chambers with gas electron multipliers, and the Outer Tracker made of honeycomb drift cells. Particle identification is performed by a ring imaging Cherenkov detector, an electromagnetic calorimeter and a muon detector.

The runs analysed in the following were taken with four different target materials: carbon, aluminum, titanium and tungsten. The nominal interaction rates ranged from 2 to 20 MHz. In the HERA proton ring there are 220 slots for bunches separated by 96 ns. Usually only 180 of these are filled with protons. These are organized in three groups of 60 bunches, separated by three gaps of 5+5+15 empty slots. In turn, each group is composed of 6 subgroups of 10 contiguous bunches. These subgroups are separated by a single empty slot.

In Fig. 1 one can see the number of recorded events as a functions of the bunch number for a run of 500k events taken with a carbon wire target. As can be seen, the data acquisition system samples all bunches very uniformly (even the ones which are nominally empty).

![Figure 1: Number of events recorded with a random trigger as a function of bunch number. Bunches that are nominally empty are also sampled.](image)

At HERA-B inelastic interactions dominate the total visible cross section and therefore they are the natural reference process for luminosity determina-
Elastic scattering events are normally outside the detectors acceptances and contribute marginally to the rate. The inelastic cross section of $pA$ collisions was measured by several fixed target experiments, for a large number of target materials and beam energies [10, 11, 12, 13, 14, 15]. It is found to be approximately independent of the incident particle energy and a power law dependence on the target atomic weight $A$ is well fitted by the experimental data [16]. The inelastic cross section comprises a non-diffractive and a diffractive component. Since for the latter both experimental acceptance and the contribution to the total cross section are small, it is a good approximation to assume that only the non-diffractive component of the inelastic cross section contributes to the luminosity determination. The resulting bias can be estimated by Monte Carlo simulations.

### 4.1 Mean number of tracks per bunch crossing

In Eq. (10) $\mu_i$ was expressed in terms of an inclusive quantity which is proportional to the number of interactions per bunch crossing. In the following we choose this quantity to be the mean number of reconstructed tracks $\langle n_t \rangle$, which, to a good approximation, scales linearly with the number of primary collisions, i.e. $\langle n_t \rangle = \tau \mu_i$, where $\tau$ is the mean number of reconstructed tracks in one interaction. The validity of this assumption was checked with a Monte Carlo simulation based on the FRITIOF 7.02 generator [17] and the subsequent simulation of the HERA-B detector. In order to exclusively select tracks originating from primary interactions, and eliminate non-target related tracks from secondary decays such as $K_s \rightarrow \pi^+\pi^-$ and conversions $\gamma \rightarrow e^+e^-$, the following selection criteria were applied to all tracks in the event. Only tracks containing at least 6 reconstructed hits in the vertex detector (VDS) are accepted. To avoid counting multiply reconstructed tracks (the so-called clones), tracks sharing a VDS segment with a previously accepted track were rejected. Finally, an impact parameter below 1 mm at the primary vertex is required.

The plot on the left of Fig. 2 shows a Monte Carlo simulation for the mean number of reconstructed tracks $\langle n_t \rangle_n$ which satisfy these criteria as a function of the number $n$ of superimposed interactions. One sees that $\langle n_t \rangle_n$ indeed scales linearly with interaction rate up to 4 superimposed interactions, which corresponds to a rate of about 40 MHz. Furthermore, heavier target materials yield higher track multiplicities. The plot on the right of Fig. 2 shows $\langle n_t \rangle_i$ as a function of the bunch number $i$ for the same run considered in Fig. 1. There are remarkable variations in the track multiplicities between different bunches, which clearly indicates distinct contributions to the total rate. In this plot, we can also identify the bunches which are nominally empty and contribute only marginally to the rate.
4.2 Defining the systems

In principle, any subdetector or combination of subdetectors in the experiment can be chosen as a system for counting empty events. In order to minimize the dependence on Monte Carlo simulations, the requirement is a large acceptance for tagging non-empty events, reasonably low noise levels and good stability with time. We have chosen the most stable subdetectors in the data taking period of year 2000. System 1 consists of the vertex detector (VDS). An event is not empty in this system if:

- there is at least 1 reconstructed track satisfying the track selection criteria explained above.

System 2 is a combination of the ring imaging Cherenkov counter (RICH) and the electromagnetic calorimeter (ECAL). An event is considered to be not empty in this system if the following conditions are both fulfilled:

- there are at least 30 reconstructed hits in the RICH.
- the deposited energy in the inner part of the ECAL is above 5 GeV.

In Fig. 3 we can see the distributions of number of tracks satisfying the track selection criteria (a), number of hits in the Cherenkov detector (b) and the total energy deposition in the inner part of the electromagnetic calorimeter (c) for a run taken with a carbon target wire.
Figure 3: Distributions of (a) number of tracks satisfying the track selection criteria, (b) number of hits in the RICH, and (c) energy deposition in inner ECAL, for a run taken with the carbon wire. The inserts are a zoom to the first bins for each distribution.

In Fig. 4 we can find the probabilities $p_{ki}$, $k = \{0, 1, 2\}$, estimated as the fraction of empty events in system 1 (a), the fraction of empty events in system 2 (b) and the fraction of empty events in both systems (c), for the 180 nominally filled bunches. Again, remarkable variations are found between bunches, indicating different contributions to the total interaction rate.

Figure 4: Fraction of empty events as a function of bunch number in (a) system 1; (b) system 2; and (c) both systems.

Figure 5 shows $\ln(1/p_1)$, $\ln(1/p_2)$ and $\ln(p_0/p_1p_2)$ as a function of $\langle n_t \rangle$ for two different runs. Each entry corresponds to one bunch. The top row is for a run taken with the carbon wire and very small rate fluctuations. The bottom row corresponds to a run taken with an aluminum wire and large rate fluctuations. The global parameters are obtained from an unweighted linear fit, performed after the bunches subject to rate fluctuations have been removed from the fit. These bunches are identified according to the constraint
In Table 1 we give the average values over all runs of the efficiencies $a_{1,2}$, the noise-probabilities $q_{1,2}$ and the average number of tracks per interaction $\tau$, obtained from the global fits to the selected runs. It can be seen that the efficiencies for system 1 are typically larger than for system 2. On the other hand, the efficiencies are, within errors, quite similar for all target materials. However, we could expect them to be larger for heavier materials, which yield higher track multiplicities. The probabilities $q_{1,2}$ are similar for runs acquired...
Figure 6: Values of $\ln(p_0/p_1p_2)/\langle n_i \rangle$ as a function of bunch number for a run taken with the carbon wire (left) and the aluminum wire (right).

Table 1: Bunch independent variables obtained from global fits to nominally filled bunches.

|       | $a_1$     | $a_2$     | $q_1$             | $q_2$             | $\tau$          |
|-------|-----------|-----------|-------------------|-------------------|-----------------|
| C     | 0.95 ± 0.02 | 0.86 ± 0.02 | 0.0189 ± 0.0003 | 0.0116 ± 0.0001 | 7.69 ± 0.26    |
| Al    | 0.93 ± 0.02 | 0.83 ± 0.02 | 0.0516 ± 0.0008 | 0.0535 ± 0.0011 | 8.27 ± 0.24    |
| Ti    | 0.97 ± 0.02 | 0.86 ± 0.02 | 0.0212 ± 0.0003 | 0.0142 ± 0.0002 | 9.95 ± 0.25    |
| W     | 0.96 ± 0.06 | 0.87 ± 0.05 | 0.0177 ± 0.0002 | 0.0126 ± 0.0001 | 13.23 ± 0.99   |

with carbon, titanium and tungsten targets, but larger for runs acquired with the aluminum target. This fact may be explained by the large fraction of coasting beam (unbunched protons uniformly distributed under the pulsed bunch structure) which plagues all runs taken with this wire [18]. Furthermore, because the runs taken with the aluminum wire target tend to show large rate instabilities it is natural to speculate if these are related to the presence of coasting beam.

The mean number of tracks per interaction $\tau$ increases, as expected, with the atomic weight of the target material. This dependence is usually parameterized by a power law of the atomic weight: $\tau \propto A^\beta$. If we fit the values of $\tau$ as a function of the target atomic weight $A$, we obtain $\beta = 0.20 ± 0.02$, which is statistically compatible with the result $\beta = 0.18 ± 0.02$ obtained in an independent study employing the HERA-B vertex detector [19].

Once $\tau$ is known, the average number of interactions per bunch crossing $\mu_i$ can be calculated according to Eq. (10). Figure 7 shows the values of $\mu_i$ for all bunches, for a run taken with the carbon target wire (a) and with the
aluminum target wire (b). First, it is noteworthy that bunches contribute quite differently to the rate. In the run taken with aluminum target wire we can see a large contribution of nominally empty bunches to the total rate. This behavior can be observed in other runs taken with aluminum wire and, again, this can be explained by the large fraction of coasting beam which is present in all runs taken with this wire.

\[
N_{\text{int}} = \sum_{i=1}^{220} N_i \mu_i,
\]

where \(N_i\) is the total number of events due to bunch \(i\). From \(N_{\text{int}}\) the luminosity is obtained using Eq. (11) and the inelastic cross sections published in Ref. [16].

4.3 Systematic uncertainties

Because the final states of proton-nucleus interactions sample a large phase-space, certain event topologies may be outside the acceptance of both subdetectors, leading to systematic uncertainties in the measured luminosity. Events which are not seen by both systems do not contribute to any inefficiency as inferred by the TSSM, and thus lead to an overestimate of the true acceptance.

The systematic uncertainties of the statistical method were studied with a toy Monte Carlo based on the interaction model MINT [20] and a coarse simulation of the HERA-B detector based on angular acceptance cuts, some rough
estimates for the track finding efficiencies plus some assumption about noise and smearing in the RICH and ECAL. The impact on the measured luminosity of diffractive contributions, rate fluctuations by ±20%, target materials covering the range from Carbon to Tungsten and nominal interaction rates varying by a factor ±e were considered. It is found that for a detector such as HERA-B, there is a small bias on the luminosity estimate from the TSSM. Assuming that the reference cross section is the total inelastic cross section, the luminosity estimate is between 3% and 6% too small. Using instead only the non-diffractive inelastic cross section as a reference, the results are between 1% and 6% too high. Taking conservatively the larger of the two ranges and correcting for the average bias, we conclude that the intrinsic systematic error of the TSSM is around 3%. Note that this figure does not include systematic uncertainties due to imperfect knowledge of the contributing cross sections.

5 Conclusions

A statistical method to measure the integrated luminosity of high energy interactions at collider experiments was presented. The method starts from the assumption that the number of interactions in a random triggered event follows Poisson statistics. Then, two large acceptance subdetectors of the experiment are considered. Counting the fraction of empty events in either of the two subdetectors and simultaneously in both, as function of the bunch crossing numbers, allows to infer the acceptance of the two subdetectors, noise contributions and total number of interactions from the data alone, thereby reducing the dependence of the analysis on Monte Carlo simulations. Introducing also information from an inclusive quantity, the method was implemented such that a bias due to rate fluctuations, which tend to spoil the assumption of Poisson statistics for the interaction multiplicity of a given bunch, can be avoided. This method was applied to random triggered minimum bias data collected in the commissioning period of the HERA-B experiment in spring 2000. Without correcting the luminosity estimates for the bias caused by those parts of the cross section which are not seen by either of the two subsystems considered, the TSSM would have an intrinsic systematic error of 6%. For more hermetic detectors and at higher energies even smaller uncertainties can be expected. Correcting for the bias, the intrinsic systematic error of the method drops to 3%.
Acknowledgment

We would like to thank our colleagues from the HERA-B collaboration for many useful discussions and their support in using HERA-B data to illustrate the method. This work was supported by the Max-Planck Society and Fundação para a Ciência e Tecnologia. One of us (JB) was covered by grant BD/16272/98.

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