Stick-Slip Motion of the Wigner Solid on Liquid Helium

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We present time-resolved transport measurements of a Wigner solid (WS) on the surface of liquid Helium confined in a micron-scale channel. At rest, the WS is ‘dressed’ by a cloud of quantised capillary waves (ripplons). Under a driving force, we find that repeated WS-ripplon decoupling leads to stick-slip current oscillations, the frequency of which can be tuned by adjusting the temperature, pressing electric field, or electron density. The WS on liquid He is a promising system for the study of polaron-like decoupling dynamics.

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Surface-state electrons (SSEs) on liquid helium substrates (Fig. 1) form a model Coulomb system[1], the ground state of which remains the clearest example of the classical Wigner solid (WS)[2]. The WS is dressed by a cloud of ripples, the Bragg scattering of which from the electron lattice gives rise to a commensurate deformation of the He surface known as the dimple lattice (DL)[3–6]. The WS-DL system is analogous to polaron states in which electrons are dressed by a cloud of virtual phonons, or lattice deformation[7–9]. On liquid He individual electrons do not perform self-trapping; rather, the DL appears as a consequence of the long-range electron ordering. Also, the lattice constant of the WS (and so the DL) is no less than 100 nm and a continuum description of the medium (liquid He) is applicable. Despite these differences, the decoupling dynamics of the WS and polaron systems exhibit strong similarities[10, 11], although the rate depends on the different binding energies; polaron decoupling in a GaAs crystal occurs in the femtosecond range[12], whereas the decoupling of the WS from the DL takes place on nanosecond timescales. It is therefore more straightforward to observe the real-time decoupling for the WS than for polarons.

In contrast with other systems, the coupling of the WS with the DL is of a dynamical nature, due to a resonant interaction between the WS and ripples emitted by the moving electron lattice. According to hydrodynamic theory, when the WS is at rest the depth of the surface dimples $\xi_0$ can be estimated as

$$\xi_0 \approx -\frac{\sqrt{3}eE_z}{8\pi^2\sigma}e^{-W},$$

where $e$, $E_z$, $\sigma$ and $e^{-W}$ are the elementary charge, perpendicular pressing electric field, surface tension coefficient of liquid He and the self-consistent Debye-Waller factor, respectively[12]. Typically, $\xi_0 \approx 10^{-12} \text{ m}$. However, when the velocity of the WS-DL system approaches the phase velocity of ripples of wavevector equal to the WS periodicity, $v_{BC}$, constructive interference resonantly deepens the DL. As a result, the resistive force exerted on the WS increases dramatically. This is called the Bragg-Cherenkov (BC) effect[14–16]. The decoupling of the WS therefore occurs from this dynamically pinned state and, hence, under strongly nonequilibrium conditions. So far, however, very little is known about this decoupling process. It is an interesting problem concerning strongly correlated systems far from equilibrium, and also from a hydrodynamics point of view.

The WS decoupling from the DL was first observed in Corbino conductivity measurements under perpendicular magnetic fields[17]. The decoupling was signalled by an abrupt jump in conductivity on sweeping the amplitude or frequency of the sinusoidal driving potential, or the magnetic field, or so on[18]. In such experiments, magnetic fields are necessary because an electric field large enough to cause the decoupling cannot be applied to the WS due to its high mobility. The electric current then flows azimuthally, whereas the electric field acts in the radial direction. The Hall angle is almost 90 degrees, and hence, the azimuthal current or velocity of the SSEs is extremely difficult to determine. Furthermore, the electric field is spatially inhomogeneous and time varying. These ambiguities make analysis of the decoupling process difficult.

FIG. 1. (Color online) Schematic depiction of electrons on the surface of liquid helium. (a) Above $T_m$ electrons move freely in the 2D plane above the surface. (b) Below $T_m$ the Wigner solid is dressed by capillary waves, giving rise to the dimple lattice.
FIG. 2. (Color online) (a) Schematic circuit diagram of the dc resistance measurement. The SSE in the CM form an electrical circuit by the capacitances $C_l$ and $C_r$. The current $I_{dc}$ flows through the CM in response to varying $V_{lr}$, resulting in the transfer of charge $Q(t)$ from the left to the right reservoir, which accumulates in $C_l$ and $C_r$. The capacitive coupling to the Guard electrode is omitted for simplicity. (b) False-colour scanning electron micrograph of a sample identical to those used in the experiment. The Left and Right Reservoirs each contain 25 microchannels of length 0.7 mm connected in parallel.

Recently, SSEs in capillary-condensed microchannel devices have been found to show the decoupling phenomena without applying magnetic fields. In such devices the electric current is homogeneous but is typically measured using a sinusoidal driving voltage. The transport measurement is then inevitably complicated by the non-linear response of the WS during each ac cycle. In this Letter we report simultaneous measurements of the WS velocity and driving electric field by employing a linear sweep of the driving potential. That is, we performed the first time-resolved transport measurement of a quasi-1D WS confined in a microchannel. We demonstrate that, under a driving potential, repeated ripplon dressing and decoupling causes stick-slip motion of the WS and so oscillations of the electron velocity. Our experiment allows the control and quantitative analysis of decoupling dynamics of the WS.

The device used in the experiment is shown in Fig. 2(b). Our sample is electrons floating on a helium surface in a microchannel 7.5 μm wide, 100 μm long and 2.2 μm deep. We denote this the central microchannel (CM). Two large arrays of microchannels, which act as electron reservoirs, are attached to the CM. The SSE in the reservoirs are capacitively coupled to the external circuit by $C_l$ and $C_r$, as shown schematically in Fig. 2(a). To measure SSE transport through the CM, the voltage $V_{lr}$ applied to the electrode beneath the He in the left reservoir (Left Reservoir electrode) was ramped from 0 to +50 mV in a time $t_r$. As $V_{lr}$ is varied, charge moves between the reservoirs. After the ramp period, the charge distribution achieves a new equilibrium state and the current stops. To determine the charge accumulated in $C_l$ and $C_r$, and so the SSE current $I_{dc}$ passing through the CM, the current flowing in the electrode beneath the He in the right reservoir (Right Reservoir electrode, $V_{rr} = 0$ V) was recorded. To tune the electrostatic confinement for SSEs in the CM, voltages $V_{bg}$ and $V_{sg}$ were applied to the electrode beneath the He in the CM (Bottom Gate electrode) and the electrode in the plane of the He surface (Split Gate electrode), respectively. The voltage on the Guard electrode was -0.2 V.

The current measurement was performed using a current preamplifier and a digital storage oscilloscope. The measurement was averaged over several thousand cycles. A small current component due to cross-talk between the electrical cables was subtracted from the measurement. The bandwidth of the preamplifier was 200 kHz, which leads to the smoothing of some of the transport features shown here but does not detract from the essential physics. Finite-element modelling (FEM) was used to calculate the average areal electron density $n_s$ (m$^{-2}$), the electrostatic potential of the electron system $V_e$ (which depends on $n_s$), the effective width $w_e$ of the charge sheet in the CM, the linear electron density $n_l$ (m$^{-1}$), and the number of electron rows $N_y$, for varying bias conditions.

Results of transient current measurements are shown in Fig. 3 for $t_r = 20$ μs. $V_{bg} = -0.2$ V. Generally, current flows in the CM in response to the $V_{lr}$ ramp. However, the transport is strongly influenced by $V_{bg}$, the value of which determines the electron density in the CM. The
The typical electron spacing is 300 nm, and therefore the electron liquid is limited by the BC scattering but, as the potential difference builds up during the phase. For $I_{dc}$ from the DL, this releases the force and the electron liquid is pinned to the DL and, due to BC scattering, the electron thermal motion on the WS decoupling can be determined unambiguously. Approaching the WS melting point, thermal fluctuations in the electron positions should degrade the DL, leading to a decrease in the decoupling threshold force. This in turn should lead to a decrease in the time taken for the decoupling to occur in our transport measurements. In Fig. 4(a) we show $I_{dc}$ against time $t$ at different temperatures from 640 mK to 800 mK, as indicated on the plot. Here $t_{r} = 80 \mu s$, $V_{bg} = 1.2$ V and $V_{ag} = −1$ V. Each data set is shifted by 1 nA for clarity. For $T = 730$ mK, we plot the expected $I_{BC}$ (dashed line). (b) $E_{z}$ against $t$ for the data shown in (a) for $T = 640$ mK and 800 mK. (c) $E_{max}$ against $T$, for the data shown in (a). The dashed line is a guide to the eye. The solid/dotted line indicates the value of $E_{max}$ expected for a dimple depth $\xi_{0}$. $T_m = 1.02$ K is indicated by the arrow.

Because the SSE system is capacitively coupled to both the Reservoir and Guard electrodes, the measurement of the current registered on the Right Reservoir electrode does not give the true current flowing in the CM. The average charge detected by integrating the current measurements shown in Fig. 3 is $Q_{av} = 1.58 \times 10^{5}\text{e}$. The FEM analysis gives the expected value $Q_{FEM} = 2.03 \times 10^{5}\text{e}$. The deficit is due to the displacement current flowing into the Guard electrode. In Figs. 3 and 4, the current measurement is therefore corrected by the factor $Q_{FEM}/Q_{av} = 1.28$.

BC scattering followed by WS-DL decoupling results in an abrupt jump in the WS conductivity with increasing driving force. The influence of BC scattering on the WS velocity in the CM can be estimated using the periodicity of the WS along the channel, $a = N_{y}/n_{x}$. The limiting velocity is given by $v_{BC} = (\sigma k/\rho)^{1/2}$, where the wavevector $k = 2\pi/a$ and $\rho$ is density of liquid He. The typical electron spacing is 300 nm, and therefore $kh \gg 1$. In this ‘thick-film’ limit the dispersion relation for ripplons on bulk liquid He $\omega^{2} = (\sigma/\rho)k^{3}$ is applicable. The BC scattering-limited current was calculated as $I_{BC} = n_{x} e v_{BC}$ for the bias conditions in Figs. 3 and 4.

On comparing $I_{BC}$ with the measurements in Fig. 3, it is clear that when $V_{bg}$ is sufficiently large the WS is pinned to the DL and, due to BC scattering, $I_{dc} = I_{BC}$. Then, because $I_{dc}$ is limited, the potential difference between the left and right reservoirs builds up during the $V_{lr}$ ramp and the current continues to flow after the ramp phase. For $V_{bg} = 1.5 \text{ V}$ the electron velocity is initially limited by the BC scattering but, as the potential difference builds, suddenly increases when the electrons decouple from the DL. This releases the force and the electron velocity quickly returns to $v_{BC}$. For $V_{bg} = 0.5 \text{ V}$ the driving force always overcomes the weaker pinning and the current exceeds $I_{BC}$.

The stronger pinning of the WS with increasing $V_{bg}$ is expected, as $n_{s}$ (and so $\Gamma$) and $E_{s}$ both increase with $V_{bg}$. However, by keeping the electrode bias conditions fixed and changing temperature, the influence of the electron thermal motion on the WS decoupling can be determined unambiguously. Approaching the WS melting point, thermal fluctuations in the electron positions should degrade the DL, leading to a decrease in the decoupling threshold force. This in turn should lead to a decrease in the time taken for the decoupling to occur in our transport measurements. In Fig. 4(a) we show $I_{dc}$ against time $t$ at different temperatures. Here $t_{r} = 80 \mu s$. The Split-Gate voltage was $V_{ag} = −1 \text{ V}$ which reduces $n_{s}$ and $w_{e}$. This increases the resistance of the SSE system $R$ and thereby slows the system response to the $V_{lr}$ ramp (see Fig. 2(a)). During the ramp phase, multiple decoupling events occur, at regular intervals. After each decoupling event $I_{dc}$ returns to $I_{BC}$ and the stick-slip cycle is repeated. For increasing $T$ (decreasing $\Gamma$) the stick-slip period decreases as the time taken to reach the decoupling threshold force is reduced.

Similar narrow-band current oscillations, also attributed to WS sliding, have been observed for degenerate electron systems in the WS regime. However, for such cases little is known about the nature of the collective electron ground states or the mechanisms by which pinning forces are overcome. Qualitatively similar phenomena are observed in charge/spin density wave systems. But, contrary to those systems, the present system does not have a pinning mechanism due to irregularities. In our experiments, the quantitative understanding of the BC scattering mechanism allows us to clearly demonstrate the link between WS sliding and the appearance of spontaneous current oscillations.

Stick-slip motion typically results in a saw-tooth-type force-velocity profile. For the data shown in Fig. 4(a), to calculate the time-varying electric field along
the CM, $E_x(t)$, we integrate $I_{dc}$ at each value of $t$ to obtain the charge accumulated in the right reservoir $Q(t)$. Analysis of the circuit diagram in Fig. 2(a) gives $E_x(t) = |V_{tr}(t) - V_{r} - 2Q(t)/C|/l$, where $l = 100$ μm and $C = C_1 = C_r = 1.01$ pF is given by the measurement of $Q_{av}$. In Fig. 4(b) we show the dependence of $E_x$ on $t$, at $T = 800$ and $640$ mK. The driving field builds steadily when the WS is pinned by the DL and then drops rapidly as the WS decouples. The maximum value of the driving field $E_x^{max}$ is higher at lower temperature, as expected. For this measurement, the FEM gives $n_s = 2.04 \times 10^{13}$ m$^{-2}$, for which the melting temperature $T_m = 1.02$ K. As shown in Fig. 4(c), $E_x^{max}$ extrapolates close to this value, as the decoupling force becomes zero when the WS melts.

Assuming a sinusoidal surface profile, equating the components of the forces acting on each electron in the direction of motion yields the expression $k\xi_{th} = E_x^{max}/E_z$, where $\xi_{th}$ is the dimple depth at the decoupling threshold. The $E_x^{max}$ expected for the static DL ($\xi_{th} = \xi_0$) is shown in Fig. 4(c). (Note that the dependence of $\xi_0$ on $T$ is weak, without taking the WS melting into account). As expected, the experimental values of $E_x^{max}$ are larger than those for the static DL, as a consequence of the resonant deepening of the DL due to BC scattering. For $T = 0.64$ K, we estimate $E_x^{max}/E_z = 1.4 \times 10^{-3}$, $a = 240$ nm, and so $\xi_{th} = 5.2 \times 10^{-11}$ m. This value exceeds the static dimple depth $\xi_0 \approx 2 \times 10^{-12}$ m, and is consistent with other measurements[19].

For the bulk 2D WS the magnitude of the WS-DL coupling is estimated by $eE_x\xi_0$ (Eq. (1)). Since $E_x$ and $W$ are functions of $V_{bg}$, the decoupling threshold depends on $V_{bg}$, yet the dependence should be monotonous and smooth. In Fig. 5 we show $I_{dc}$ against $t$, recorded for $V_{bg}$ values between 0.85 and 1.10 V. Here $t_r = 80$ μs, $V_{bg} = -1$ V and $T = 0.6$ K. The time values at which $I_{dc}$ reaches its peak value, $t_p$, are plotted in the inset. Although the general trend, the increase of the decoupling threshold, is as expected, it is remarkably tortuous. We ascribe this effect to successive structural transitions of electron row formation. The numbers of electron rows along the quasi-1D channel, as calculated by the FEM, are indicated in the inset of Fig. 5. Clear correspondence is observed. Previous (ac) measurements have shown a similar modulation effect due to the reduced positional order of the WS at each structural boundary[21, 22]. Further details of the influence of confinement on the WS ordering will be reported elsewhere.

Stick-slip motion is common in nature, although complex microscopic processes often determine macroscopic motion[20]. Here, for SSEs in a microchannel confinement, we have demonstrated a quantitative understanding of stick-slip friction at a microscopic level and that the coupling between the WS and the He substrate depends on the positional ordering of the quasi-1D electron system. Recently, quasi-1D cold ion systems comprising less than 10 particles have been used to investigate the influences of dimensionality and commensurability on sliding friction[20]. Our results demonstrate that similar investigations can be conducted with quasi-1D SSE systems, which can contain a much larger number of particles, by measuring decoupling from the DL or from lithographically structured substrates.

In conclusion, we have presented the first demonstration of stick-slip motion of the WS on a liquid He substrate. Our microchannel device allows precise control and quantitative analysis of the WS decoupling, which exhibits basic similarities with polaron dynamics. SSEs on He are a promising system to study stick-slip friction at the nanoscale.

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SUPPLEMENTAL - MATERIALS AND METHODS

The sample was fabricated on a silicon wafer using optical lithography. Two gold layers, each 80 nm thick, were separated by an insulating layer of hard-baked photoresist. The metal layers were patterned to form the different electrodes. The widths of the gaps between the electrodes were approximately 0.5 µm in the lower layer and 3 µm in the upper layer. After lift-off of the second layer, the photoresist that was not covered by metal was removed by etching to form the channel structures. The reservoirs comprised 25 microchannels, each 20 µm wide and 700 µm long, connected in parallel.

The samples were wire bonded to a ceramic chip carrier that was then mounted in a hermetically sealed copper refrigerator. 4He was condensed into the sample cell until the surface of the superfluid was approximately 0.5 mm below the sample, allowing the microchannels to fill by the capillary action of the superfluid film. A 100 ms voltage pulse was applied to a tungsten filament placed several mm above the sample to induce the thermal emission of electrons. The charging procedure was performed at 0.9 K, at which temperature the electrons thermalise rapidly due to collisions with He gas atoms before being trapped on the surface.

Ac transport measurements, in which the driving force applied to the SSE can be made very small, were performed by superimposing a small ac voltage $V_{in}$ on the Left Reservoir electrode and measuring the ac current $I_{ac}$ induced in the Right Reservoir electrode, using the current preamplifier and a lock-in amplifier. The ac frequency was 89.2 kHz.

SUPPLEMENTAL - AC TRANSPORT MEASUREMENTS AND FINITE ELEMENT MODELLING

Before investigating the dc transport, we performed ac transport measurements, and used the results to build an electrostatic model of the device with the aid of finite element modelling (FEM). We first measured the Split-Gate voltage threshold of current flow $V_{sg}^{th}$ for different values of $V_{bg}$. The results of the measurements are shown in Fig. 1(a) for $V_{gu} = -0.2$ V, $V_{lr} = V_{rr} = 0$ V and $V_{in} = 3$ mVpp. For such small driving voltage, the system remains close to equilibrium during the measurement. For increasingly positive $V_{bg}$, a more negative $V_{sg}$ is required to ‘pinch-off’ the current. At this threshold, where electrons enter the CM forming a single row along the centre of the channel, two energetic conditions must be satisfied. Firstly, the potential at the centre of the CM, $V_c$, must be more positive than the electrostatic potential of the electron system $V_e$. Were the electron system to behave as a charge continuum, the satisfaction of this condition alone would result in current flow. However, because the electron system is granular in nature, a second condition arises, that the charging energy required to populate the CM with electrons must be compensated for. We consider that this condition is approximately satisfied when the number of electrons allowed in a square section of the effective CM area $S = w_c^2$ is equal to 1 or, equivalently, that the separation between the electrons along the centre of the CM is comparable to $w_e$.

We then used a finite element modelling software package[20] to solve Poisson’s equation for the electrostatic potential $\phi$ at all points in the CM cross section, according to the geometry and bias conditions of the sample. Without electrons, the potential at the centre of the CM was given by $V_c = \alpha V_{bg} + \beta V_{sg}$, where $\alpha$ and $\beta$ are coupling constants. (The influence of the other electrodes is negligible as they are strongly screened.) The electron system was then modelled as a continuous charge sheet of width $w_e$ and potential $V_e$. The width was determined, for varying electrode bias conditions, by iteratively calculating the value of $w_e$ for which the electric field at the edge of the sheet, in the plane of the helium surface, was zero. The surface charge density distribution across the electron sheet $n_s(y)$ was then found by measuring the difference between the electric field above and below the electron sheet, using the expression

$$\frac{\partial \phi}{\partial z}\bigg|_{above} - \varepsilon_H \frac{\partial \phi}{\partial z}\bigg|_{below} = -\frac{en_s(y)}{\varepsilon_0},$$

where $\varepsilon_0$ is the permittivity of free space and $\varepsilon_H = 1.056$ is the dielectric constant of liquid He. The average areal density $n_s$ was given by the average value of $n_s(y)$ whilst the linear electron density $n_l$ in the CM was evaluated numerically using the expression

$$n_l = \int_{-\infty}^{\infty} n_s(y)dy.$$

The number of electron rows in the CM, $N_y$, was then estimated as $N_y = \sqrt{w_e m}$. Given our fabrication procedure, we expect a channel depth of approximately 2 µm. Using a channel depth of 2.2 µm, for which $\alpha = 0.6$ and $\beta = 0.4$, and the value $V_c = -0.13$ V we found an excellent agreement between the voltage values for which our model predicts $N_y = 1$ and the current threshold line, as shown in Fig. 1(a).

In Fig. 1(b) we show the dependence of $I_{ac}$ on $V_{bg}$ for two values of $V_{in}$. Here $T = 0.6$ K and the melting of the classical WS occurs for $n_s \approx 7.1 \times 10^8$ cm$^{-2}$. Our FEM model gives this value for $n_s$ when $V_{bg} = 0.18$ V. For $V_{in} = 2$ mVpp, the current drops abruptly close to this value as the WS couples with the DL. (In this regime, we also note the appearance of current oscillations as the
FIG. 6. Results of ac transport measurements. (a) \(V_{th}^{sg}\) against \(V_{bg}\) for \(V_{lr} = V_{rr} = 0\) V, \(V_{gu} = -0.2\) V, \(V_{in} = 3\) mV\(_{pp}\) and \(T = 0.6\) K. The solid line shows the result of FEM modelling for \(N_y = 1\). (b) \(I_{ac}\) against \(V_{bg}\) for \(V_{gu} = V_{sg} = -0.2\) V and \(V_{in} = 2\) and 20 mV\(_{pp}\). The axes for the two measurements are scaled by a factor of 10 to allow clear comparison. The transition between the electron liquid (EL) and WS phases occurs when \(V_{bg} \approx 0.18\) V. For \(V_{in} = 20\) mV\(_{pp}\) the WS ‘slides’ along the Helium surface before becoming pinned at \(V_{bg} = 1.30\) V.

number of electron rows in the CM, and so the commensurability of the electron lattice with the confinement, is modulated. These effects will be described in detail elsewhere). For \(V_{in} = 20\) mV\(_{pp}\), the current remains high as the WS slides along the Helium surface, before the pinning with the DL occurs at \(V_{bg} = 1.30\) V. In the sliding WS regime a current peak is observed immediately before the SSE are pinned by the DL. For \(V_{in} = 20\) mV\(_{pp}\), the root-mean-square rate of change of voltage is 3.9 mV/\(\mu\)s, comparable with the 2.5 mV/\(\mu\)s used in Fig. 3 in the main text. We therefore conclude that the current peak observed in the ac transport measurement arises due to WS sliding during each ac voltage cycle, resulting in a large signal component recorded at the ac reference frequency. Other resonances weakly observed in the ac measurement at lower values of \(V_{bg}\) occur when multiple sliding events occur during each cycle.