Long-Range Correlations Between Transmitted and Reflected Fluxes of Electromagnetic Waves

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Abstract. We study the long-range spatial correlations between intensity fluctuations in speckles formed by multiply scattered light. The correlation function between intensity fluctuations at the opposite boundaries of the slab are analyzed under the conditions of circular polarization memory. It shown that, until the scattered light is depolarized completely, the polarization and scalar contributions to the correlation function are of the same order of magnitude. As the slab thickness increases, their ratio falls off in inverse proportion to the thickness.

1. Introduction
Long-range correlations between the wave fields scattered in a disordered medium are responsible for numerous mesoscopic effects [1], in particular, for correlations between transmitted and reflected speckle patterns (see, e.g., [2, 3]). The corresponding spatial correlation function was calculated recently [4] within the scalar wave approximation without regard to the polarization state of scattered waves. Such an approach is not valid under the conditions of the polarization memory effect [5–11] where the depolarization of light occurs deep in the diffusive regime, and the scalar and polarization contributions to the correlation function are of the same order of magnitude.

The pronounced effect of circular polarization memory has been discovered recently [9–11] for resonant Mie particles with high refractive index. In this case the polarization contribution to intensity correlations through a disordered slab should be observed up to the optical thickness of the order of a few tens.

Below we study the intensity correlation function for two points lying on the input and output boundaries of diffusive-scattering medium. For the slab of great optical thickness $L \gg l_{tr}$, where $l_{tr}$ is the transport mean free path, the intensity fluctuations at the opposite boundaries of the slab are negatively correlated at spatial difference $\rho < L$. As $\rho$ increases, the correlation function changes its sign. The polarization contribution to the correlation function, as compared to the scalar one, changes its sign at smaller values of $\rho$ and is characterized by a greater amplitude of positive correlations. This contribution remains noticeable even though the transmitted light is completely depolarized.
2. General relations

The correlation function between the local values of irradiance (or, for brevity, the correlation function of intensity fluctuations) is defined as

\[
C_{RT}(z_f^{(1)}, z_f^{(2)}|\rho = r - r_1) = \int d\mathbf{n} \int d\mathbf{n}_1 (\mathbf{n}\mathbf{n}_0) (\mathbf{n}_1\mathbf{n}_0)
\]

\[
\langle I(r_1, \mathbf{n})I(r_2, \mathbf{n}_1) \rangle - \langle I(r_1, \mathbf{n}) \rangle \langle I(r_2, \mathbf{n}_1) \rangle
\]

(1)

where \(z_f\) denotes position of the output boundary of the slab (\(z_f = 0\) and \(z_f = L\) in reflection and transmission, respectively), \(\mathbf{n}_{ext}\) is the external normal to the output boundary, \(I(r, \mathbf{n})\) is the intensity of radiation propagating in direction \(\mathbf{n} = k/k_0\) (\(k\) is the wave vector, \(k_0 = 2\pi/\lambda\)) at point \(r\) \[12\] and \(\mathbf{r}_f\) in reflection. Brackets \(\langle \ldots \rangle\) appearing in Eq.(1) denote averaging over disorder.

In addition to the correlation function, it is convenient to introduce the spatial spectrum of intensity fluctuations \[12\]

\[
M_{RT}(z_f^{(1)}, z_f^{(2)}, \mathbf{q}) = \int d\rho \exp(-i\mathbf{q}\rho)C(z_f, \rho)
\]

(2)

Behavior of the spectrum at low spatial frequencies \(q\) is related directly to the long-range asymptotics of the correlation function and is sensitive to the regime of wave propagation in the medium. The value of the spectrum at \(q = 0\) determines the variance of total transmission \(T\) or reflection \(R\) coefficients \[13, 14\]

\[
\langle (\delta T)^2 \rangle = \frac{1}{A} M(z_f^{(1)} = z_f^{(2)} = L, \mathbf{q} = 0), \quad \langle (\delta R)^2 \rangle = \frac{1}{A} M(z_f^{(1)} = z_f^{(2)} = 0, \mathbf{q} = 0)
\]

(3)

and also the covariance between these coefficients

\[
\langle (\delta T)(\delta R) \rangle = \frac{1}{A} M(z_f^{(1)} = L, z_f^{(2)} = 0, \mathbf{q} = 0)
\]

(4)

where \(A\) is the area of the slab surface, \(T\) is the ratio of the total transmitted flux to the incident one \((R\) is defined analogously).

In the scalar wave approximation the spectrum of intensity fluctuations was calculated in Refs. \[13, 14\]. Extension of results \[13, 14\] with allowance for the polarization state of the incident light was performed in Refs. \[15, 16\]. Within the circular representation \[17, 18\], the spectrum of transmitted (or reflected) intensity can be written as \[16\]

\[
M(z_f^{(1)} = z_f^{(2)}, \mathbf{q}) = \left(\frac{2\pi}{k_0}\right)^2 n_0 \int_0^L \int d\mathbf{n} \int d\mathbf{n}'
\]

\[
|I_q(z_f|z, \mathbf{n}) - I_q(z_f|z, \mathbf{n}')|^2 T^m(z, \mathbf{n})d_{mn}(\mathbf{n}, \mathbf{n}')I_n(z, \mathbf{n}')
\]

(5)

Quantities \(I_m\) \((m = \pm 0, \pm 2)\) are expressed in terms of the Stokes parameters \(I, Q, U\) and \(V\)

\[
\hat{I} = \frac{1}{\sqrt{2}} (I_2, I_0, I_{-2}) = \frac{1}{2} (Q - iU, I - V, I + V, Q + iU),
\]

(6)

and are subject to the vector radiative transfer equation \[17, 18\]

\[
\left\{ \mathbf{n} \frac{\partial}{\partial r} + n_0\sigma_{tot} \right\} I_m(r, \mathbf{n}) = n_0 \int d\mathbf{n}'d_{mn}(\mathbf{n}, \mathbf{n}')I_n(r, \mathbf{n}')
\]

(7)
where \( \sigma_{\text{tot}} = \sigma + \sigma_a \) is the total cross section, \( \sigma \) and \( \sigma_a \) are the cross sections of elastic scattering and absorption, respectively. Matrix \( d_{mn}(n, n') \) entering into Eqs.(5) and (7) is the single-scattering phase matrix in the circular representation [18, 19]

\[
d_{mn} = \begin{pmatrix}
    a_e e^{2i\chi_+} & b_e e^{-2i\beta} & b_e e^{-2i\gamma} & a_e e^{2i\chi_-} \\
    b_e e^{2i\beta} & a_e & a_e & b_e e^{2i\gamma} \\
    b_e e^{-2i\gamma} & a_e & a_e & b_e e^{2i\beta} \\
    a_e e^{-2i\chi_-} & b_e e^{2i\gamma} & b_e e^{2i\beta} & a_e e^{-2i\chi_+}
\end{pmatrix}
\]

(8)

where angles \( \beta, \beta' \) and \( \chi_\pm = \pi - (\beta \pm \beta') \) are defined in [19], \( a_\pm = (a_1 \pm a_2) / 2 \), quantities \( a_1, a_2 \) and \( b_1 \) are the elements of single scattering matrix (see, e.g., [18]).

Expression (5) can easily be generalized to the case of correlations between the reflected and transmitted fluxes (in this case the values of \( z_f^{(1)} \) and \( z_f^{(2)} \) are different from each other). To do this, the product of “outgoing” propagators which enters into Eq.(5) should be modified as follows

\[
M_{RT}(q) = \left( \frac{2\pi}{k_0} \right)^2 n_0 \int_0^L dz \int d\mathbf{n} \int d\mathbf{n}' \left( I_q \left( z_f^{(1)} = 0 \mid z, \mathbf{n} \right) - I_q \left( z_f^{(1)} = 0 \mid z, \mathbf{n}' \right) \right)
\]

(9)

Equations (5) and (9) can be combined, to give the spectrum of total (transmitted plus reflected) intensity fluctuations \( M_{\text{tot}}(q) = M_R(q) + M_T(q) + 2M_{RT}(q) \).

As has been shown in Refs. [15, 16] the polarization of light can reveal itself in the intensity correlations only under the conditions of the polarization memory effect [6–11, 19]. This effect is observable for circularly polarized light provided that the scattering matrix elements \( a_1 \) and \( a_2 \) are very close to each other (e.g., for microspheres with high refractive index in the vicinity of the first Kerker point [9–11, 20] or for large (compared to the wavelength of light) weakly refracting particles [6–8, 19]).

For circularly polarized light Eq.(5) is simplified, as only two components of the Stokes vector make a contribution to that equation,

\[
I^*_m(z, \mathbf{n})d_{mn}(\mathbf{n}, \mathbf{n}')I_n(z, \mathbf{n}') = a_1 (\mathbf{n} \mathbf{n}') I(z, \mu)I(z, \mu') + a_2 (\mathbf{n} \mathbf{n}') V(z, \mu)V(z, \mu').
\]

(10)

According to Eq.(10) the spatial spectra (5) and (9) can be presented as the sum of the scalar contribution and the polarization one.

3. Intensity correlations in diffusive transport of circularly polarized light

In the diffusive approximation the intensity propagators entering into (9) and (10) can be expanded in directions \( \mathbf{n} \) [12]. Then the spectrum (9) takes the form

\[
M_{RT}(q) = \frac{l_{tc}}{48\pi^2 k_0^2} \int_0^L dz \left[ \frac{\partial}{\partial z} I_q \left( z_f^{(1)} = 0 \mid z \right) \right] \left[ \frac{\partial}{\partial z} I_q \left( z_f^{(2)} = L \mid z \right) \right] + q^2 I_q \left( z_f^{(1)} = 0 \mid z \right) I_q \left( z_f^{(2)} = L \mid z \right) [I^2(z) + V^2(z)]
\]

(11)

Substituting the intensity propagators \( I_q (z_f \mid z), I(z) \) and \( V(z) \) calculated in the diffusive approximation [16] to Eq.(11) we obtain the spectrum in the form

\[
M_{RT} = M_{RT}^{(I)} + M_{RT}^{(V)} = \frac{9\pi l_{tc}}{k_0^2 L} \left( F_{RT}(qL, 0) + F_{RT} \left( qL, \frac{L}{l_{\text{circ}}} \right) \right)
\]

(12)
where $l_{tr}$ and $l_{circ}$ are the transport mean free path and the depolarization length for circularly polarized light (see, e.g., [7,10]), and

$$F_{RT}(x, y) = -\left(\frac{x}{\sinh x}\right)^2 \coth^2 y \left(\frac{\sinh x y - \tanh y \cosh x}{x^2 - y^2} - \frac{\sinh x}{x} \frac{1}{\sinh^2 y}\right)$$

(13)

Figure 1. Spatial spectrum of intensity fluctuations. Scalar and polarization contributions are shown by solid and dashed lines, respectively. The upper and middle curves correspond to the reflection and transmission geometries, the lower curves describe correlations between the reflected and transmitted fluxes. The polarization contribution is calculated for $L = 5 l_{circ}$.

The $q$-dependence of the scalar and polarization contributions to the spectrum $M_{RT}$ is illustrated in Fig. 1. The both contributions are negative, which means anti-correlation between the transmitted and reflected fluxes. The spectra $M_T$ and $M_R$ are also shown in Fig. 1 for comparison. Due to the total flux conservation the equality $M_R = M_T = -M_{RT}$ is fulfilled at $q = 0$.

Figure 2. Correlation function between transmitted and reflected intensity fluctuations $\nu_{RT} = \nu_{RT}(x, 0) + \nu_{RT}(x, y)$ ($y = 1$ solid red line, $y = 3$ dashed blue line, $y = 7$ dash-dotted green line). The scalar contribution $\nu_{RT}(x, 0)$ is shown by dotted black line. The curves are normalized to the value of $|\nu_{RT}(0, 0)| = 0.5104$. 
From Eq. (12) it follows that the intensity-intensity correlation function can be written as

\[ C_{RT} = C_{Rt}^{(l)} + C_{Rt}^{(V)} = \frac{9\pi l_{tr}}{k_0^2 L^3} \left[ \nu_{RT} \left( \frac{p}{L}, 0 \right) + \nu_{RT} \left( \frac{p}{L}, \frac{L}{l_{\text{circ}}} \right) \right] \] (14)

where

\[ \nu_{RT} (x, y) = -\frac{\Phi_1 (x)}{\sinh^2 y} + \coth^2 y [\Phi_2 (x, y) - y \tanh y \Phi_3 (x, y)] , \] (15)

\[ \Phi_1 (x) = \frac{1}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \pi^2 n^2 K_0 (\pi n x) , \quad \Phi_2 (x, y) = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \pi^4 n^4}{\pi^2 n^2 + y^2} K_0 (\pi n x) , \]

\[ \Phi_3 (x, y) (x, y) = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \pi^2 n^2}{\pi^2 n^2 + y^2} \left[ \pi n x K_1 (\pi n x) - \frac{\pi^2 n^2 + y^2}{\pi^2 n^2 + y^2} K_0 (\pi n x) \right] \] (16)

Functions \( K_n (z) \) appearing in Eq. (16) are the modified Bessel functions.

The \( \rho \)-dependence of the correlation function (14) is illustrated in Fig. 2. From Fig. 2 it follows that the intensity fluctuations at the opposite boundaries of the slab are negatively correlated at \( \rho < L \). As \( \rho \) increases, the correlation function changes its sign. As compared to the scalar contribution, the resulting correlation function changes its sign at smaller values of \( \rho \) and is characterized by a greater amplitude of positive correlations. The asymptotic behavior of the correlation function \( C_{RT} \) at \( \rho \gg L \) is identical to that of \( C_T, C_R \).

Our conclusions regarding the scalar contribution are in qualitative agreement with those of Ref. [4] where the density-density type of correlations were studied (contrary to the current-current one considered here).

4. Discussion
The results presented above give insight into the spatial correlations between the transmitted flux and the reflected one, including the impact of the polarization memory effect on the correlations.

Intensity fluctuations at the opposite boundaries of a disordered slab are always negatively correlated. This is explained by the total flux conservation \( \langle (\delta R + \delta T)^2 \rangle = 0 \) [13, 14], resulting in the anti-correlation effect for the covariance \( \langle \delta T \delta R \rangle = -\langle (\delta R)^2 \rangle + \langle (\delta T)^2 \rangle /2 \) and, as a consequence, both for the angular-resolved intensity correlations (see, e.g., [2, 3]) and for fluctuations of the local values of irradiance considered above.

The polarization contribution to the intensity correlation function is rather noticeable provided that the circular polarization decreases slowly as compared to the flux randomization over directions of propagation, \( l_{tr} < l_{\text{circ}} \). In this case the relative value of the covariance \( \langle \delta T \delta R \rangle / \langle T \rangle \langle R \rangle \) can be written in the form

\[ \frac{\langle \delta T \delta R \rangle}{\langle T \rangle \langle R \rangle} = -\frac{3^{3/2}}{2N} \left( 1 + \frac{\sinh 2y - 2y}{4 y \sinh^2 y} \right) \] (17)

where \( N = Ak_0^2 / (2\pi) \) is the number of propagating modes and \( y = L / l_{\text{circ}} \). The scalar contribution to Eq. (17) is \( L \)-independent, while the polarization one decreases with increasing \( L \). As long as the scattered light remains polarized, \( L < l_{\text{circ}} \), the scalar and polarization contributions to Eq. (17) are of the same order of magnitude. As the slab thickness \( L \) increases, the polarization term in Eq. (17) decreases as \( l_{\text{circ}} / L \).
For a thick slab, the correlations between intensity fluctuations at the opposite points of the slab boundaries fall off as $1/L^3$ with $L$ (see Eq.(14) at $\rho = 0$). In the limit $L > l_{circ}$

$$C_{RT}(\rho = 0) = -\frac{9\pi l_{tr}}{k_0^3 L^5} \left(0.51 + 1.94 \frac{l_{circ}}{L}\right)$$

(18)

Ratio between the polarization contribution and the scalar one decreases with $L$ as $l_{circ}/L$ while the degree of circular polarization $V/I$ proves to be exponentially small, $V/I \sim \exp(-L/l_{circ})$.

5. Conclusions

In conclusion, we have developed an analytical theory of long-range intensity correlations in speckles arising in multiple scattering of polarized light. Under the conditions of circular polarization memory, the correlation function has been presented as a sum of the contributions from the intensity and from the basic mode of circular polarization. We have shown that the polarization contribution to mesoscopic reflection-transmission fluctuations remains noticeable even for thick samples where the polarization of transmitted light vanishes. The results obtained above give a rather complete insight into the manifestation of the vector nature of light in mesoscopic fluctuations and can be useful for studies of optical properties of disordered media.

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