A formula for charmonium suppression

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In this work a formula for charmonium suppression obtained by Matsui in 1989 is analytically generalized for the case of complex $c\bar{c}$ potential described by a 3-dimensional and isotropic time-dependent harmonic oscillator (THO). It is suggested that under certain conditions the formula can be applied to describe $J/\psi$ suppression in heavy-ion collisions at CERN-SPS, RHIC, and LHC with the advantage of analytical tractability.

I. INTRODUCTION

The modification of the charmonium production cross section has been studied using a schematic 3-dimensional harmonic oscillator for the intermediate and final $c\bar{c}$ pair in \cite{1}. In that reference the distorted wave Born approximation was used for the two-gluon fusion model and suppression ratios were calculated. In the present paper, we consider a 3-dimensional THO with a complex and continuous time dependent frequency. For such a generalization, we derive the suppression ratio for charmonia states and present a formula for $J/\psi$ suppression including feed-down contributions.

II. QUANTUM MECHANICAL EVOLUTION OF THE $c\bar{c}$ STATE

The Charmonium suppression ratio was defined as a ratio of two cross sections by the expression $S_{\psi}(t) = \frac{\sigma(2g\rightarrow\psi)}{\sigma_0(2g\rightarrow\psi)}$ and was calculated explicitly in Ref. \cite{1}. From Eqs. (2.22) and (4.17) of that paper the survival probability for the s-wave can be written in the following form

\begin{verbatim}[*Electronic address: pena@ift.uni.wroc.pl]
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\end{verbatim}
\[ S_\psi(t) = \left| \lim_{t \to 0} \int_0^\infty dr \, r^2 \, \psi(r) \, U_{cc}(r,t) \right|^2. \]  

(1)

### III. TIME EVOLUTION OPERATOR FOR THE THO MODEL

We make use of the standard path integral approach in order to calculate the time evolution operator \( U_{cc}(r,t) \). We start by considering a 3-dimensional isotropic THO model with the Hamiltonian \( H = \frac{p^2}{2m} + \frac{\mu}{4} \omega^2(\tau) \, r^2(\tau) \), where \( r \) is the \( cc \) separation and the complex function of time \( \omega(\tau) \) enters in the classical equation of motion for the heavy pair as

\[ \ddot{r}(\tau) + \omega^2(\tau) \, r(\tau) = 0. \]  

(2)

The general solution of equation (2) is a linear combination given by

\[ r(\tau) = \rho(\tau) \left[ A \cos \gamma(\tau) + B \sin \gamma(\tau) \right], \]

where \( \gamma(\tau) = \int_0^\tau dt' \frac{1}{\rho(t')} \). Replacing these definitions into (2), clearly leads to the following Ermakov equation

\[ \ddot{\rho}(\tau) + \omega^2(\tau) \, \rho(\tau) - \frac{1}{\rho^3(\tau)} = 0. \]  

(3)

If \( \tau \in [0, t] \) then A and B can be easily obtained from the initial conditions as

\[ A = \frac{r(0)}{\rho(0)}, \quad B = \frac{1}{\sin \gamma(t)} \left[ \frac{r(t)}{\rho(t)} - \frac{r(0)}{\rho(0)} \cos \gamma(t) \right]. \]  

(4)

Where we have used that \( \gamma(0) = 0 \). By replacing A and B in the general solution, we obtain \( r(\tau) \) and \( \dot{r}(\tau) \). For a THO the classical action \( s_{cl} \) and the fluctuation factor \( F(t) \) in the 3-dimensional isotropic space are defined in Ref. [3]. We calculate here their relationship with Ermakov function as\(^1\)

\[ s_{cl} = \frac{\mu}{2} \left( r(t) \, \dot{r}(t) - r(0) \, \dot{r}(0) \right) \]

\[ = \frac{\mu}{2} \frac{1}{\sin \gamma(t)} \times \left[ r(t)^2 \left( \dot{\gamma}(t) \cos \gamma(t) + \frac{\dot{\rho}(t)}{\rho(t)} \sin \gamma(t) \right) \right. \]

\[ + r(0)^2 \left( \dot{\gamma}(0) \cos \gamma(t) - \frac{\dot{\rho}(0)}{\rho(0)} \sin \gamma(t) \right) - r(t) \, r(0) \left( \frac{\rho(t)}{\rho(0)} \dot{\gamma}(t) + \frac{\rho(0)}{\rho(t)} \dot{\gamma}(0) \right) \right]. \]  

(5)

\[ F(t) = \sqrt[3]{\frac{2\pi i}{\mu^2 \rho(0)}} \left( - \frac{\partial r(t)}{\partial r(0)} \right) = \sqrt[3]{\frac{2\pi i}{\mu \rho(0) \sin \gamma(t)}}. \]  

(6)

\(^1\) We use the notation \( \dot{r}(t) = \frac{dr(\tau)}{d\tau} |_{\tau=t} \) for all functions of time.
The time evolution operator for THO is given exactly by $U(r,t) = F(t) \exp(i\;s\;t)$. In the present context, it will represent the quantum mechanical evolution of a $c\bar{c}$ state for a medium-modified (distorted) interaction up to the time $t$ when it gets projected onto the asymptotic bound state spectrum. Thus we define $U_c(r,t) = U(r,t)$. In fact formula (1) is independent of the initial condition which may be taken as $r(0) = 0$.

IV. THE THO FORMULA FOR CHARMONIUM SUPPRESSION

The ground state of charmonium $J/\psi$ can be identified with the 1s-wave of the harmonic oscillator given by $\psi(r) = \psi(0) \exp\left(\frac{-r^2}{2\;\rho_0}\right)$ with $\rho_0 = \sqrt{\frac{1}{\mu\;\omega}}$ [1]. Thus we integrate the gaussian shape over $r$ appearing in (1) which leads to the following suppression

$$S_{J/\psi}(t) = \left|\frac{\rho(t)}{\rho(0)}\right|^3 \times \cos\gamma(t) + \left(\frac{\dot{\rho}(t)\rho(t)^{-1}}{\dot{\gamma}(t)} + i\;\frac{\omega_\psi}{\dot{\gamma}(t)}\right) \sin\gamma(t)\right|^3.$$  (7)

The formula (7) depends on $\gamma(t)$, the frequency $\omega_\psi$ and the Ermakov function $\rho(t)$. For the case of the charmonium state $\psi'$ we take the 2s-wave given by $\varphi(r) = \frac{2}{\pi} \varphi(0) \left(\frac{3}{2} - \frac{r^2}{\rho_0^2}\right) \exp\left(\frac{-r^2}{2\;\rho_0}\right)$. Applying the formula (1) we obtain

$$S_{\psi'}(t) = S_{J/\psi}(t) \left|1 - \frac{2i\;\omega_\psi \sin\gamma(t)}{i\;\omega_\psi + \frac{\rho(0)}{\rho(t)}} \sin\gamma(t) + \dot{\gamma}(t) \cos\gamma(t)\right|^2.$$  (8)

For the Charmonium state $\chi_c$ we take the 2p-wave given by $\chi(r) = \chi'(0) \; r \; \exp\left(\frac{-r^2}{2\;\rho_0}\right)$. However, in this case there is a contribution of the angular momentum and it was shown in Ref. [1] that for such waves the formula (1) vanishes and the next-to-leading order term in momentum $O(p/m)$ must be considered leading to the expression

$$S_{\chi}(t) = \left|\frac{\int_0^\infty dr \; r^2 \; \chi(r) \; U_{ce}'(r,t)}{\lim_{t \to 0} \int_0^\infty dr \; r^2 \; \chi(r) \; U_{ce}(r,t)}\right|^2 = S_{J/\psi}^2.$$  (9)

with $U_{ce}' = -\frac{\rho}{2\sin\gamma(t)}(\rho(t) \dot{\gamma}(t) \rho(0)^{-1} + \rho(0) \dot{\gamma}(0) \rho(t)^{-1})U_{ce}$. The observable $J/\psi$ suppression ratio is influenced by feed-down from the higher charmonia states and we shall assume the following composition of the total contribution

$$S(t) = 0.6 \; S_{J/\psi}(t) + 0.3 \; S_{\chi}(t) + 0.1 \; S_{\psi'}(t).$$  (10)

The case of no feed-down is described by the expression $S_{no}(t) = S_{J/\psi}(t)$. Since we have already shown that $S_{\chi}(t) < S_{J/\psi}(t)$ and $S_{\psi'}(t) < S_{J/\psi}(t)$ for $S_{J/\psi}(t) < 1$ it is clear that $S(t) < S_{no}(t)$.
V. SUMMARY

We have generalized Matsui’s harmonic oscillator model for charmonium suppression to the case of time-dependent complex oscillator strengths and included the effects of feed-down on the $J/\psi$ suppression ratio. Preliminary results for the comparison with experimental results from CERN SPS and RHIC can be found in [5].

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