Coherent versus Incoherent Transport 
inan Layered Doped Mott Insulators

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Abstract

There exist strong experimental evidences for the dimensional cross-over from two to three dimensions as La$_{2-x}$Sr$_x$CuO$_4$ compounds are overdoped. In this paper we describe the dimensional cross-over of the layered correlated metal in the gauge theory framework. In particular, we obtain the anomalous exponent $3/2$ for the temperature dependence of resistivity observed in overdoped La$_{2-x}$Sr$_x$CuO$_4$.

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I. INTRODUCTION

The normal state properties of high-$T_c$ compounds are anomalous. In particular, at optimal doping in-plane resistivity $\rho_{ab}$ decreases linearly with temperature [7], while out-of-plane resistivity $\rho_c$ increases with temperature. The different character of in-plane and out-of-plane transport reflects the layered structure of the cuprates and indicates the hopping character of the interlayer transport. It is generally believed that cuprates evolve into Fermi liquids as doping increases.

However, the systematic studies of transport properties of overdoped La$_{2-x}$Sr$_x$CuO$_4$ [1–4] have shown a deviation from the Fermi liquid as well as from the optimally doped compounds. The temperature dependence of resistivity was found to be $T^\alpha$ with an exponent close to 1.5 [1]. In addition in the overdoped regime ($x \geq 0.25$) both in-plane and out-of-plane resistivity show similar, although anisotropic, temperature dependence: $\rho_c/\rho_{ab}$, which is in the order of 50-100, is almost independent of temperature and is not far from the value predicted by the band structure calculation ($\approx 25$) [1,11]. This is to be compared with the anisotropy of the order of 500-1000 at optimal doping $x \approx 0.15$ near $T_c$. This forces us to conclude that there exists a dimensional cross-over from two dimensional anomalous (strange) metal at optimal doping to three dimensional anisotropic anomalous metal in overdoped case.

The dimensional crossover is observed while varying the doping. In fact, it is also a crossover in temperature. At sufficiently low temperature $T < T_d(x)$, unless it is cut off by the superconducting transition, any layered material is essentially three dimensional. This means, in particular, that there is only one transport time $\tau_{tr}^{-1}$ that determines the temperature dependence of in and out-of-plane resistivity, so that the ratio $\rho_c/\rho_{ab}$ does not depend on temperature. In this case temperature generally increases scattering and resistivity. We may call this type of transport coherent. However, at higher temperature $T > T_d(x)$ all relaxation times may be shorter than the interlayer hopping time, thus the out-of-plane conductivity is determined by one particle tunneling.
Eventually, the temperature scale $T_d(x)$ which determines the crossover in La$_{2-x}$Sr$_x$CuO$_4$ strongly depends on the doping $x \sim (0.15 - 0.35)$. The overdoped La$_{2-x}$Sr$_x$CuO$_4$ most likely lies in the low temperature coherent regime $T < T_d(x)$, while the optimally doped La$_{2-x}$Sr$_x$CuO$_4$ lies in the high temperature two dimensional regime $T > T_d(x)$.

We consider the overdoped cuprates as an intermediate metallic state which interpolates two dimensional anomalous metal \cite{14} and the conventional three dimensional metal. The nearly temperature independent anisotropy ratio $\rho_{ab}/\rho_c$ implies the common scattering mechanism for in-plane and out-of-plane charge transports. We propose that three dimensional anisotropic gauge theory may be a suitable model to describe the charge transport experiments in the overdoped cuprates as well as the optimally doped cuprates in an unified way.

At $T > T_d(x)$, when the out-of-plane transport is due to one particle tunneling, we face the question of whether an electron is a quasi-particle or not. If it is, the out-of-plane conductivity is proportional to the one particle relaxation time $\tau$ and still decreases with temperature. In the case there is no difference between $\tau_{tr}$ and $\tau$ and the ratio $\rho_c/\rho_{ab}$ still does not depend on temperature. If, however, electron is not quasi-particle due to strong interaction, (i.e. its Green function does not possess a pole), a new time $\tau_c$ of decay to true quasiparticles comes into play. We call $\tau_c$ the coherence time. If the interlayer hopping time is longer than the coherence time, the electron decays into components during the hopping. We refer to this type of transport as incoherent. A feature of this incoherent transport is that the temperature increases the out-of-plane mobility and decreases resistivity. This situation having no analogue in Fermi liquid is considered in this paper \cite{12}.

Among theories proposed for the anomalous normal states of cuprates near optimal doping, the gauge theory \cite{14,17,18} highly emphasizes retarded scattering by the chirality fluctuations provided by infinitely strong on-site repulsion. In particular it gives $T$-linear in-plane resistivity in two dimensional regime. In this paper, we extend the gauge theory to describe out-of-plane transport and the crossover between coherent and incoherent regimes.
We also note that diverse models captured other mechanisms of scattering of the peculiar out-of-plane transport of cuprates [20–27].

Summarizing, we show that at \( T \geq T_d(x) \) (two dimensional regime, optimal doping), in addition to \( \rho_{ab} \propto T \), the gauge theory gives incoherent out-of-plane resistivity

\[
\rho_c \propto \frac{1}{\sqrt{T}},
\]  
whereas at \( T \leq T_d(x) \) (three dimensional regime)

\[
\rho_{ab} \propto \rho_c \propto T^{3/2}
\]  

The exponent 3/2 has been observed in La\(_{2-x}\)Sr\(_x\)CuO\(_4\) [12].

Ironically, the gauge theory suggests a different physical mechanism which also gives \( \rho_{ab} \propto T^{3/2} \) (but incoherent \( \rho_c \)) in two dimensional regime. At high temperature the scattering by the chirality fluctuations becomes inelastic. The inelasticity changes the linear temperature dependence of the resistivity to \( T^{3/2} \). Although it is unlikely that this regime is achieved in La\(_{2-x}\)Sr\(_x\)CuO\(_4\), we discuss this mechanism in Sec.V.

II. THE GAUGE MODELS OF NORMAL STATES

Strong on-site Coulomb repulsion forbids double occupations and imposes the constraint \( \sum_{\alpha} c_{n,\alpha}^\dagger(r)c_{n,\alpha}(r) \leq 1 \) (\( r \) are coordinates on a layer and \( n \) labels the layers). The gauge field is a tool to deal with this constraint. The constraint can be implemented by representing an electronic operator \( c_{n,\alpha}(r) \) by the product of a fictitious spinon \( f_{\alpha,n}(r) \) and a holon \( b_n^\dagger(r) \) that keeps track of vacant sites: \( \sum_{\alpha} f_{\alpha,n}^\dagger(r)f_{\alpha,n}(r) + b_n^\dagger(r)b_n(r) = 1 \). One of them is a fermion, while the other is a boson. An accepted phenomenological model for each layer that captures a vector character of the interaction has the form [13–19]

\[
H_{2D} = \int d^2r \left[ \sum_{\alpha} f_{\alpha,n}^\dagger(r) \left(-a_0 - \mu_f - \frac{1}{2m_F}(\nabla - ia)^2\right)f_{\alpha}(r) \\
+ b_n^\dagger(r) \left(-a_0 - \mu_b - \frac{1}{2m_B}(\nabla - ia)^2\right)b(r) \right]
\]  

(2.1)
A small interaction between layers can be represented by adding an interlayer hopping term

$$H^\perp = t_\perp \sum_n \int d^2r \left[ c_{n,\alpha}(r)c_{n+1,\alpha}^\dagger(r) + \text{h.c} \right]$$  \hspace{1cm} (2.2)

We neglect the interlayer magnetic exchange in La$_{2-x}$Sr$_x$CuO$_4$ since it is smaller than the intralayer magnetism by a factor of $10^{-5}$ (see e.g. \[8\]). We use this model (2.1), (2.2) to describe the incoherent out-of-plane transport in the optimally doped two dimensional regime, i.e. at $T > T_d(x)$, where the interlayer hopping $t_\perp$ is the smallest energy scale.

At $T < T_d(x)$, to which we believe the overdoped La$_{2-x}$Sr$_x$CuO$_4$ belongs, the system is assumed to be three dimensional and we employ a different model which is an anisotropic generalization of the two dimensional gauge theory (2.1):

$$H_{3D} = \int d^3r \left[ f^\dagger_\alpha(r) \left( \frac{(-i\nabla - \mathbf{a})^2}{2m_B^\parallel} + \frac{(-i\partial_z - a_z)^2}{2m_F^\parallel} - \mu_f - a_0 \right) f_\alpha(r) ight. \\
+ \left. b^\dagger(r) \left( \frac{(-i\nabla - \mathbf{a})^2}{2m_B^\perp} + \frac{(-i\partial_z - a_z)^2}{2m_B^\perp} - \mu_b - a_0 \right) b(r) \right]$$  \hspace{1cm} (2.3)

In layered materials the inter-plane masses $m_B^\perp, m_F^\perp$ is much larger than the in-plane masses $m_B^\parallel, m_F^\parallel$.

A few comments are in order. The microscopic basis of the 2D model (2.1) is weak, nevertheless the model has attractive universal features. This model has been derived by different authors \[16,18\] from different physical assumptions: In \[16\] a strong short range magnetic exchange was essential, whereas in \[18\] no magnetic exchange was assumed at all.

In \[16\] $m_F^\parallel$ is determined by the magnetic exchange $J$ and $m_B^\parallel$ stands for the hopping $t$. In \[18\], both $m_F^\parallel, m_B^\parallel$ are determined by the hopping. In both cases, the model (2.1) captures the physics of scattering by chirality fluctuations, namely by magnetic polarization produced by mobile dopants. If one is interested in how these non-local retarded processes contribute to the normal state transport, it may be a good idea to treat spinon mass $m_F^\parallel$ and holon mass $m_B^\parallel$ as phenomenological parameters.

It is even more so for the 3D model (2.3). The 3D model (2.3) is suggested by transport properties of overdoped La$_{2-x}$Sr$_x$CuO$_4$ \[1-4\]. We have failed, however, in justi-
ing this model in a quantitatively microscopic way for the range of parameters known for La$_{2-x}$Sr$_x$CuO$_4$.

Another comment is that, although two models (2.1, 2.2) and (2.3) are different, they equivalently describe the dimensional crossover of in plane transport. They are essentially different, however, as far as out-of-plane transport is concerned: while both models give the same result for \( \rho_c \) in 3D regime, i.e. at \( T < T_d \), they give different \( \rho_c \) at \( T > T_d \). The reason for this is that the 3D model (2.3) neglects fluctuations of the amplitude of effective electronic hopping in-plane as well as between different layers but stresses the fluctuations of their phases. This is a correct approximation at low \( T \). In contrast, the 2D model (2.1, 2.2) neglects the variations of in-plane hopping amplitude, but takes into account the fluctuations of the out-of-plane amplitude which become important at high temperature.

At a temperature range where the effects of Bose condensation are irrelevant, the chirality fluctuations are small and can be treated perturbatively. The tendency to condense is suppressed by the gauge interaction and strong on-site repulsion: the holons are hard core bosons. These effects are beyond the perturbation theory and have remained obscure. As the result we do not know the low temperature bound of the perturbation theory. Of course the upper estimate of the bound is given by the mean field value of the bose condensation temperature \( T_0 \sim x/m_B \). For cuprates \( T_0 \) is too high (around 1500 K). In fact, this boson condensation problem makes an application of the gauge theory to cuprates questionable unless the interactions eliminate the condensation, thus drag down the perturbation theory to much lower temperature \[14,19\].

The strategy of the perturbative calculation of the transport in the gauge theory is well known \[13,14,16–18\]. Let us assign electric charge to, say, the fermions. Then one may find the spinon and holon currents produced by external electromagnetic field \( A_{\nu}^{\text{ext}} \) and the gauge field:

\[
 j_F^\mu = \Pi_{\mu\nu}^F (a_\nu + A_{\nu}^{\text{ext}}), \quad j_B^\mu = \Pi_{\mu\nu}^B a_\nu,
\]

where \( \Pi_{\mu\nu}^{F,B}(k,\omega) \) is a free fermionic (bosonic) polarization operator. An infinite on-site
repulsion, implemented by the gauge field, renders the spinon current to be opposite to the holon current \( j^F_\mu = - j^B_\mu \). This allows us to find electromagnetic current as a response to the external electromagnetic field \( j_\mu = j^F_\mu - j^B_\mu \). The physical conductivity \( \sigma_{\mu\nu} = \omega^{-1} \Pi_{\mu\nu}^{\text{phys}}(k = 0, \omega) \) is given by the combination rule

\[
\left( \Pi_{\mu\nu}^{\text{phys}}(k, \omega) \right)^{-1} = \left( \Pi_{\mu\nu}^{F}(k, \omega) \right)^{-1} + \left( \Pi_{\mu\nu}^{B}(k, \omega) \right)^{-1}
\]

At low temperature the fermionic contribution is smaller than the bosonic one \([13, 14, 18]\). This is, roughly, due to the temperature dependence of the number of bosons at a given chemical potential. As a result, at low temperature the conductivity is determined by bosonic transport relaxation time

\[
\sigma_{ab} \approx \frac{x e^2}{m_B} \tau_{tr}, \quad \sigma_{c} \approx \frac{x e^2}{m_B} \tau_{tr} \tag{2.4}
\]

The scalar component of gauge field is short-ranged due to Debye screening while the unscreened transverse part of vector potential produces anomalously strong scattering. In the next section we calculate transport time for anisotropic 3D gauge model \((2.3)\). The answer is summarized in Eq.\((1.2)\).

To describe the incoherent out-of-plane transport in optimally doped case, we employ a different approach. In this case the interlayer tunneling is a perturbation of the 2D model \((2.1, 2.2)\). In the lowest order of \( t^2 \) the Kubo formula gives,

\[
\sigma^{(0)}_c = 2 e^2 t^2 \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \frac{d^2 p}{(2\pi)^2} \left( -\frac{\partial n_F(\epsilon)}{\partial \epsilon} \right) \left[ -\frac{1}{\pi} \text{Im} G^R(\epsilon, p) \right]^2, \tag{2.5}
\]

where \( G^R(x - y) \) is the retarded Green function of two dimensional electrons in a layer.

In Fermi liquid where one particle Green function is characterized by a relaxation time \( \tau \), Eq.\((2.5)\) gives \( \sigma^{(0)}_c \sim e^2 t^2 m \tau \), and thus \( \rho_c \) is proportional to \( \rho_{ab} \). It appears to be metallic and coherent even though \( t^2 \) is the lowest energy.

The situation is very different in the gauge theory as well as in any other theory where electrons is not quasiparticles, i.e. their Green function do not possess a quasi-particle pole
In this case electron decay into "spinon" and "holon" and does not constitute stable excitations. Spinons and holons themselves are coupled by the gauge field and are not true quasi-particles, either. However, at $T > T_0$ the gauge coupling is weak, so that in the first approximation the electron Green function is simply a product of non-interacting fermion and boson Green function. A short range decay of the bosonic Green function in a layer destroys the coherence between electrons on different layers. Our results for the incoherent regime is summarized in Eq.(1.1) and the calculations are presented in Sec.IV.

The first step of the computation of conductivity in 3D gauge theory (2.3) is to determine the propagators of the gauge fields. Since the gauge field is a Lagrangian multiplier, its dynamics emerges entirely from the polarization produced by bosons and fermions. Perturbatively, it is given by fermionic and bosonic loops

$$\Pi_{\mu\nu}(k,\omega) = \Pi_{\mu\nu}^F(k,\omega) + \Pi_{\mu\nu}^B(k,\omega)$$

(2.6)

The propagators of the gauge fields in the transverse are the inverse of polarization operator.

$$\langle A_\mu(k,\omega) A_\nu(-k,-\omega) \rangle = D_{\mu\nu}(k,\omega) = (\Pi_{\mu\nu})^{-1}(k,\omega)$$

(2.7)

As in the 2D case the fermionic contribution is the larger one, so that only the transversal component of $\Pi_{\mu\nu}^F(k,\omega)$ are needed. Due to the uniaxial symmetry the matrices $D_{ij}$, $\Pi_{ij}$ ($i,j = x,y,z$) can be parametrized by two elements ($D_\parallel, D_\perp$) ($\Pi_\parallel, \Pi_\perp$), respectively.

$$D_{xx} = \hat{k}_y^2 D_\parallel + \hat{k}_z^2 D_\perp, \quad D_{yy} = \hat{k}_x^2 D_\parallel + \hat{k}_z^2 D_\perp, \quad D_{zz} = (\hat{k}_x^2 + \hat{k}_y^2) D_\perp$$

$$D_{xy} = -\hat{k}_x \hat{k}_y D_\parallel, \quad D_{xz} = -\hat{k}_x \hat{k}_z D_\perp, \quad D_{yz} = -\hat{k}_y \hat{k}_z D_\perp,$$

(2.8)

where

$$D_\parallel(\omega,k) = \frac{\Pi_\parallel k_x^2 + \Pi_\perp k_z^2 - \Pi_\parallel k_z^2}{\Pi_\perp (\Pi_\parallel k_x^2 + \Pi_\perp k_z^2)}, \quad D_\perp(\omega,k) = \Pi_\perp^{-1}$$

(2.9)

and $\hat{k} = k/|k|$ is a unit wave vector along $k$, and $k_\parallel^2$ is an in-plane momentum. We assume that for typical momentum transfer $k_z \sim (m_F^2 T_0)^{1/2}$, $k_\parallel \sim (m_0^2 T^{1/2})$ the following relation holds $(m_F^2 d)^{-1} k_z \ll v_F k_\parallel$, where $d$ is the inter-layer distance. Then,
where $\chi_{\parallel} \propto 1/m_{\parallel F}$, $\chi_{\perp} \propto 1/m_{\perp F}$ are the components of the diamagnetic susceptibilities, and $p_F$ and $v_F$ are the Fermi momentum and velocity of the two dimensional Fermi surface. The imaginary parts of the fermion loop are given by the Landau damping:

$$\text{Im} \Pi^{R}_{ij}(\omega, k) = -\frac{2\pi}{m_F} \int \frac{d^3 p}{(2\pi)^3} v_i(p + k) v_j(p) \left( -\frac{\partial n_F(\xi_p)}{\partial \xi_p} \right) \delta(\omega + \xi_p - \xi_{p+k})$$

$$\approx -2\pi m_{\parallel F} \omega \int \frac{d\theta}{2\pi} \frac{d p_z}{2\pi} v_{F,i} v_{F,j} \delta(\xi_{p_F+k})$$

(2.11)

where $v_i(p) = \frac{\partial \epsilon}{\partial p_i}$, $\xi_p = \epsilon_p - \mu_F$, $v_F = v(p_F)$ is the Fermi velocity, $\theta$ is an angle between $v_F$ and $k$, $p_F \equiv m_{\parallel F} v_F$ and the integration over $p_z$ is limited by the inverse inter-layer distance $\pi/d$. Employing that $(m_{\perp F}^d)^{-1} k_z \ll v_F k_{\parallel}$ we find that $v_F$ is almost perpendicular to $k_{\parallel}$. Under these conditions the Landau damping is similar to the 2D case. At low $\omega < v_F k_{\parallel}$ we have

$$\text{Im} \Pi^{R}_{yy}(\omega, k) = -\frac{p_F}{\pi d} \frac{\omega}{k_{\parallel}}$$

$$\text{Im} \Pi^{R}_{zz}(\omega, k) = -\left( \frac{m_{\parallel F}}{2d p_F m_F} \right)^2 \frac{p_F}{\pi d} \frac{\omega}{k_{\parallel}}$$

(2.12)

III. ANISOTROPIC COHERENT TRANSPORT

In this section we calculate transport time in terms of the anisotropic 3D gauge theory (2.3). The calculation of the conductivity of the system interacting via gauge forces is peculiar. To obtain the conductivity one must sum up the leading corrections to the vertex and Green function of the polarization operator. However, they are connected by the Ward identity. This connection implements the gauge invariance of interaction. Moreover in 2D the corrections to the Green function and to the vertex diverge, although taken all together, they give a finite result. Naively it looks like there exists a difference between transport relaxation time and one particle relaxation time determined by the decay of one particle
Green function. In fact, in our model those relaxation times are identical if one takes a proper gauge invariant definition of the one particle relaxation time, namely as a decay of the gauge invariant Green function
\[ G_{\text{inv}}(x) = \langle b(x) \exp(i \int_0^x a_i dx^i) b^\dagger(0) \rangle, \]
being calculated on the mass shell. Then the tail factor \( \exp(i \int_0^x a_i dx^i) \) takes care of the vertex corrections. At small and smooth gauge field, the gauge invariant Green function does not depend on the path of the tail.

Assuming that the gauge field is in the equilibrium, the relaxation time of bosons scattered by the gauge field in the second order of the gauge field is [14,18]:
\[
\tau_{\text{tr}}^{-1}(p) \sim \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \text{Im} \left( \langle v_p \times \hat{k} \cdot B(\omega, k) \rangle \langle v_p \times \hat{k} \cdot B(\omega, k) \rangle \right) \times \left( 1 + n_B(\omega) \right) \left( 1 + n_B(\xi_p + k) \right) \delta(\xi_p - \xi_{p+k} - \omega) |p|^{-2} \quad (3.1)
\]
, where \( \xi_p = \epsilon_p - \mu_B \). The ”magnetic field” \( B = \nabla \times A \) is a chirality:
\[
\langle (v_p \times \hat{k} \cdot B(\omega, k)) (v_p \times \hat{k} \cdot B(\omega, k)) \rangle = |k|^2 v_p^i v_p^j D_{ij}(\omega, k) \quad (3.2)
\]
The factor \( k^2 \) in the in the above expression comes from the tail and guarantees the convergence of the scattering by soft chirality fluctuations.

The perturbation theory is valid only at temperature where the effects of Bose condensation are negligible. Therefore the factor \( n_B(\xi_{p+k}) \) can be neglected. For the scattering of fermion Eq.(3.1) remains the same, except that the factor \( 1 + n_B(\xi_{p+k}) \) is replaced by \( 1 - n_F(\xi_{p+k}) \). According to the Sec.II, the transport relaxation time of bosons (2.4) dominates over the fermionic one and determines the conductivity.

At low temperature, the scattering is elastic. This means that the gauge fluctuations are damped if the frequency \( \omega^* \sim \chi^{-1}(k_\parallel)^3 \) (See \( \Pi_{\parallel}(k, \omega) \) in (2.10), \( \gamma = p_F/(\pi d) \) ) exceeds temperature. This happens at \( T < T_{\text{in}} \equiv (\gamma/\chi)^2 (m_B^\parallel)^{-3} \) (the opposite, inelastic case is discussed in sec.V ). The out-of-plane component of the gauge field is damped at even higher frequency \( (m_B^\perp/m_B^\parallel) \omega^* \). This implies that one may take into account only the static chirality fluctuations. In static approximation Eq.(3.1) takes the form:
\[
\tau_{\text{tr}}^{-1}(p) \sim T \int \frac{d^3k}{(2\pi)^3} \frac{|k|^2}{|p|^2} v_p^i v_p^j D_{ij}(0, k) \delta(\xi_p - \xi_{p+k}) \quad (3.3)
\]
To obtain the conductivity, the momentum dependent transport time $\tau_{tr}^{-1}(p)$ has to be averaged over the momentum $p$ with the Boltzmann distribution. The sole effect of the averaging is to replace momentum by its thermal value: $p_i^2 \sim m_B^i T$, $v_i^2 \sim T/m_B^i$. Therefore, $p_\parallel^2 \ll p_\perp^2$, $v_\parallel^2 \gg v_\perp^2$.

Due to the above anisotropy $k_z^2 \gg k_\parallel^2$ holds and under this condition Eq.(3.2) simplifies:

$$|k|^2 v_\parallel^i v_\parallel^j D_{ij}(0,k) \approx \frac{v_\parallel^2}{\chi^\parallel k_\parallel^2 + \chi_\perp k_z^2} \left( k_z^2 + \frac{\chi^\parallel k_\parallel^2}{\chi_\perp} \right)$$  \hspace{1cm} (3.4)

, where we kept only the term proportional to $v_\parallel^2$, neglecting term proportional to $v_\perp^2$.

The next step is the integration over the angle between $v_\parallel$ and $k_\parallel$, which gives $(v_\parallel k_\parallel)^{-1}$.

The last integration over $k_\parallel$, $k_z$ and the thermal averaging over $p$ yield $\tau_{tr}^{-1} = \tau_\parallel^{-1} + \tau_\perp^{-1}$, where $\tau_\parallel^{-1}$ and $\tau_\perp^{-1}$ are given by:

$$\tau_\parallel^{-1} \sim T\sqrt{m_B^\parallel T \over \chi^\parallel m_B^\parallel}$$  \hspace{1cm} (3.5)

$$\tau_\perp^{-1} \sim T\sqrt{m_B^\perp T \over \chi_\perp m_B^\perp}$$  \hspace{1cm} (3.6)

The essential difference of the above result with two dimensional one is an extra factor $\sqrt{T}$, which originates from the density of states. We acknowledge that $T^{3/2}$ dependence of the resistivity in the context of gauge theory was mentioned in [17].

There is a simple way to understand Eqs. (3.5) and (3.6): $\tau_\perp$, $\tau_\parallel$ contain the static chirality fluctuations $\langle B^2_x(k) \rangle \sim \langle B^2_y(k) \rangle \approx T/\chi_\perp$, $\langle B^2_z(k) \rangle \approx T/\chi_\parallel$, the projected area onto the xy- plane and yz-plane of the contour composed of the path of a boson in a unit time: $S_{yz} \sim (m_B^\parallel m_B^\perp)^{-1/2}$, $S_{xy} \sim (m_B^\parallel)^{-1}$ and the density of states in the parallel and transverse directions: $(m_B^\parallel T)^{1/2}$, $(m_B^\perp T)^{1/2}$. Considering the products of three factors, $\tau_\perp^{-1}$ and $\tau_\parallel^{-1}$ can be obtained, respectively.

As discussed in the introduction, the anisotropic gauge theory (2.3) is assumed to be valid at temperature below the dimensional crossover temperature. However, the 3D theory can give an upper limit for the crossover temperature $T_d^\perp$. Interlayer relaxation rate $\tau_\perp^{-1}$
increases with the anisotropy. When it reaches the interlayer hopping amplitude $t_\perp$, the kinetic equation and, as a consequence, Eq.(3.5,3.6) are no longer valid. Thus the out-of-plane conductivity reverses its temperature behaviour (see a footnote in introduction). It is likely that at $T = T_d^\perp$ the out-of-plane wavelength $1/p_\perp \sim (m_B^\perp T)^{-1}$ reaches the interlayer distance $d$. Then, the condition $\tau_\perp^{-1} \sim t_\perp$ gives a temperature scale of the crossover $T_d^\perp = t_\perp m_B^\perp d \chi_\perp$. If the value of $m_B^\perp$ can be identified with $(t_\perp d^2)^{-1}$, $T_d^\perp \sim \chi_\perp/d$.

On the contrary $\tau_\parallel^{-1}$ is smooth through dimensional cross-over, and Eq.(3.5) is still valid for in-plane transport. The only difference is that the integration over $p_z$ have to be cut off by the inverse interlayer distance $1/d$. Therefore, at $T > (2m_B^\perp d^2)^{-1}$:

$$\tau_\parallel^{-1} \sim \frac{T}{\chi_\parallel m_B^\perp d}$$

(3.7)

This is the well-known $T$-linear in-plane resistivity in two dimensional limit. Note that in this limit $\chi_\parallel d = \chi^{(2D)}$.

Observe that the dimensional cross-over of the in-plane and out-of-plane transport stem from different mechanisms and may occur at different temperatures. Nevertheless, if one assumes that all phenomenological parameters of the out-of-plane part of the model are of the same order $(m_B^\perp d^2)^{-1} \sim \chi_\perp/d \sim t_\perp$ the estimate of the crossover temperature is

$$T_d(x) \sim t_\perp$$

(3.8)

Let us discuss the experimental side of the story [1-4]. The experiments have shown that in overdoped La$_{2-x}$Sr$_x$CuO$_4$ $\rho_{ab}$ is proportional to $T^\alpha$, where $1 < \alpha < 2$ [1-4]. Notably in Ref. [1] $\alpha$ was found to be very close to 3/2 for La$_{2-x}$Sr$_x$CuO$_4$ with $x=0.35$ and agrees with the Eq.(1.2).

From the data of Ref. [1] the experimental value of $T_d(x = 0.35)$ can be estimated to be around 800 K. Measurements of the c-axis polarized optical spectrum over doping range $0.1 < x < 0.3$ [4] are consistent with resistivity data. An estimate of $t_\perp$ may be taken from the optical conductivity data. The Drude-like fitting gave $\tau_\perp^{-1} \sim \max(\omega_\alpha^\alpha, T^\alpha)$ with $1 < \alpha < 2$. In [4] the ratio of in-plane and out-of-plane plasma frequencies was also found...
At doping \( x = 0.15 \) and \( x = 0.3 \), \( \omega_{p,\parallel}/\omega_{p,\perp} \approx 30 \) and 10, respectively. These data enable one to get an estimate of \( t_{\perp} \) [8,9]:

\[
\frac{\omega_{p,\perp}}{\omega_{p,\parallel}} = \sqrt{2} \left( \frac{d}{a} \right) \frac{t_{\perp}}{E'_a},
\]

where \( a = 3.79 \text{ Å} \) is the lattice spacing in layer and \( d = 13.21 \text{ Å} \) is the inter-cell distance of \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \). \( E'_a a = \hbar v_F \) and \( v_F \) is the in-plane Fermi velocity [9]. According to [10], the band structure calculations yield \( v_F = 3.1 \times 10^7 \text{ cm/s} \) at \( x = 0.15, x = 0.20 \). Combining all of the above formula and data we obtain a somewhat lower \( t_{\perp} \approx 200 \text{ K} \) for \( x = 0.3 - 0.35 \).

Near the optimal doping [8] \( t_{\perp} \approx 2.4 \text{ meV} = 28 \text{ K} \) at \( x = 0.16 \), which is also obtained from the optical measurements. The superconducting transition temperature at \( x = 0.16 \) is \( T_c(x = 0.16) = 34 \text{ K} \). Thus near optimal doping the dimensional cross-over can be possibly screened by the superconducting transition.

In the overdoped case we may rely on the band theory. The value of hopping amplitudes quoted in Ref. [20] are \( t_\parallel \approx 0.5 \text{ eV} \approx 6,000 \text{ K} \) and \( t_{\perp} \approx 0.05 \text{ eV} \approx 600 \text{ K} \). The band theory value of \( t_{\perp} \) is not very different from the value of the \( t_{\perp} \) obtained above from optical data [1]. This can be expected, since an interaction in overdoped case is not as strong as in the case of optimal doping.

Above estimates of \( T_d(x) \) indicates that there is a room for the three dimensional regime in overdoped \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) \( (x > 0.25) \).

### IV. INCOHERENT TRANSPORT IN OPTIMALY DOPED \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \)

In this section we consider the out-of-plane transport in case the interlayer hopping amplitude \( t_{\perp} \) is the smallest rate: (i) the time of hopping \( t_{\perp}^{-1} \) is longer than in-plane relaxation time and (more importantly) (ii) longer than the characteristic time of all kind of magnetic fluctuations. This case corresponds to the optimally doped \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \).

Under the condition (i) the hopping term [2,2] can be treated as perturbation and under the condition (ii) the approximation which allows us to write the hopping term [2,2] in the
form (2.3) is no longer valid.

The c-axis conductivity was calculated with 2D gauge theory in [15,20]. In [20], \( \rho_c \propto 1/T \) was obtained using the tunneling conductivity formula and in [15] it was found \( \rho_c \propto \sqrt{T} \). We will adopt Kubo formula for the conductivity as in [15].

It is instructive to compare the c-axis conductivity of Fermi liquid with that of (2D) gauge theories. In Fermi liquid electrons in a layer are quasiparticles with some relaxation time and their retarded Green function has a pole in the lower half plane. Then, provided that there is no interlayer scattering, the Eqn.(2.5). yields

\[
\sigma_c^{(0)} \sim e^2 t^2_\perp m_F \tau
\]

(4.1)

Therefore, \( \rho_c \) and \( \rho_{ab} \) have the same temperature dependence.

The situation is different if the electron is not quasi particle. Once the hopping is treated as a perturbation, electron always decays to true quasi-particles during the interlayer tunneling, so the quantum states of the electron in different layers are incoherent. As a result of this incoherence, the out-of-plane transport is blocked and may be relaxed by thermal processes , which is similar to a semiconducting behaviour.

The above case is true of doped Mott insulator: at sufficiently high temperature electrons decay very fast ( in the time scale of \( 1/J \) or \( 1/t_\parallel \) ) into ”spinons” and ”holons” and don’t constitute stable excitation. At this temperature range the gauge interaction is perturbative and electron Green function is simply a product of fermion and boson Green functions ( recall \( c_\alpha = f_\alpha b_\alpha^\dagger \) )

\[
G_e(x, y) = - \langle f(x) f^\dagger(y) b(y) b^\dagger(x) \rangle \sim G_F(x, y) G_B(y, x)
\]

(4.2)

Therefore, the propagating character of fermion Green function

\[
\langle f(x) f^\dagger(y) \rangle \sim e^{i(|x-y| - v_F(t_x - t_y))} \text{ is blocked by the localized boson Green function} \quad \langle b(y) b^\dagger(x) \rangle \sim \frac{T_0}{\hbar} \exp(-|x-y|^2 m_B T).
\]

The simplest way to evaluate the integral (2.5) in gauge theory is to rewrite it in the form of fermi and bose density-density correlation functions \( \pi_F \) and \( \pi_B \) using the decomposition
\[
\pi_F(i\omega, q) = \sum_x \int_0^\beta d\tau e^{i q \cdot x + i \omega \tau} G_F(x, \tau) G_F(-x, -\tau)
\]

\[
\pi_B(i\omega, q) = \sum_x \int_0^\beta d\tau e^{i q \cdot x + i \omega \tau} G_B(x, \tau) G_B(-x, -\tau)
\]

The superscripts of Green functions denote two layers involved in hopping process (top, bottom). In terms of \(\pi_F, \pi_B\) (2.5) takes the form:

\[
\sigma_c^{(0)} = 2e^2 t_\perp^2 \sum_q \int \frac{d\omega}{2\pi} \left( -\frac{\partial n_B(\omega)}{\partial \omega} \right) \text{Im} \pi_F^R(\omega, q) \text{Im} \pi_B^R(\omega, q) \tag{4.3}
\]

At small frequency and momentum and at \(T > T_0\) the imaginary parts of the polarization operators are

\[
\text{Im} \pi_F^R(\omega, q) = -m_F a^2 \frac{\omega}{v_F |q|}, \quad \text{Im} \pi_B^R(\omega, q) = -\frac{T_0}{T} m_B a^2 \frac{\omega}{v_B |q|}
\]

, where \(a\) is the lattice constant in a layer and \(v_B = (k_B T/m_B)^{1/2}\) is the thermal boson velocity. The momentum integration in Eq.(4.3) is logarithmic and is cut by \(T/v_F\) at lower limit. Due to the exponential decay the bose factor \((\partial n_B(\omega)/\partial \omega \propto e^{-|\omega|/T})\) the frequency integral is convergent at ultra-violet limit. The main contribution to the frequency integral comes from the region \(|\omega| \leq T\), in which \(-\partial n_B(\omega)/\partial \omega \approx T/\omega^2\). The \(\omega^2\) in denominator is cancelled by \(\omega^2\) coming from \(\text{Im} \pi_F^R \text{Im} \pi_B^R\). Thus the frequency integral gives \(T^2\). Rearranging other factors, within logarithmic accuracy, we obtain

\[
\sigma_c^{(0)} = \text{const.} e^2 t_\perp^2 x m_F^2 \sqrt{m_B T} \tag{4.4}
\]

The dimensional crossover to the anisotropic three dimensional regime is complex. In particular it evolves hopping process (2.2) into anisotropic gauge theory (2.3) and requires more sophisticated analysis. Let us just note that two models (2.1, 2.2) and (2.3) are essentially different, so that an estimate of the crossover temperature from the high temperature side may not necessarily coincide with the estimate from the low temperature side. In any case it is very likely that the cross-over temperature in optimally doped cuprates falls below the superconducting transition temperature.
A comment is necessary at this point. In optimal La$_{2-x}$Sr$_x$CuO$_4$, near $T = 300$ K the out-of-plane resistivity stops decreasing and starts to grow with temperature. This up-turn is attributed to the structural transformation from high temperature tetragonal phase to low temperaturer orthorhombic phase [2]. Above this up-turn temperature the c-axis-conductivity is still much lower than the Mott minimal metallic conductivity($\approx 10^2$ s/cm) and can not be considered to be metallic.

V. INELASTIC SCATTERING BY GAUGE FIELDS

Two dimensional gauge theory gives rise to the linear temperature dependence of in-plane resistivity of optimally doped La$_{2-x}$Sr$_x$CuO$_4$ in the regime where scattering is elastic. At sufficiently high temperature inelastic processes change the linear-T behaviour into $T^{3/2}$.

In three dimensional case the scattering by $D_\perp(k,\omega)$ is almost always elastic (see Sec. III), while the scattering by $D_\parallel(k,\omega)$ can be inelastic at high temperature. It turns out that $T_{in}^{3D}$ is very close to $T_{in}^{2D}$. For the three dimensional inelastic regime to be observed the condition $T_{in}^{3D} \leq T_{d}(x)$ should be satisfied. (See the discussion below on the experimental estimate of $T_{in}^{2D}$, $T_{in}^{3D}$). In three dimensional inelastic regime we would have \( \rho_{ab} \propto T^{3/2}(m_B T)^{1/2} \propto T^2 \), so the anomalous exponent 3/2 cannot be explained. Instead we will discuss two dimensional case in detail.

From the propagator of the gauge field $D^R(\omega,k) = (\chi k^2 - i \gamma \omega/k)^{-1}$ it follows that the energy transfer $\omega$ scales like $\omega_k = \chi \gamma^{-1} k^3$. At finite temperature the boson energy is typically of order $T$. Thus the typical momentum transfer in the scattering of boson by gauge field is $(m_B T)^{1/2}$. As a result the typical energy transfer in scattering would be $\omega^* \sim \chi/\gamma (m_B T)^{3/2}$. This is larger than the thermal energy of scattered bosons, i.e. $T$ at $T > T_{in} \sim (\frac{2}{\chi})^2 \frac{1}{m_B}$ and at this temperature the $\omega$ dependence of the propagators has to be taken into account. This inelasticity softens the infrared singularity of scattering, thus leads to the less singular temperature dependence of resistivity.

In 2D case the Eq.(3.1) reads:
After angular integration it becomes

\[ \tau_{tr}^{-1} \approx \frac{1}{v_p m_B^2 \gamma} \int_0^{p_f} k^3 dk \int d\omega \frac{\omega}{(\omega_k)^2 + \omega^2} (1 + n_B(\omega/T)) \] (5.2)

At low temperature \( T < T_{in} \), \( \omega \approx \omega_k^* \ll T \) thus \( n_B(\omega/T) \sim T/\omega \). Then the frequency integral is finite and it gives \( (\omega_k^*)^{-1} \). The remaining momentum integration gives T-linear transport time \( \tau_{tr}^{-1} \sim T/(\chi m_B) \) \[14,18\]. Note that the transport time is independent of Landau damping parameter \( \gamma \), which is not the case in inelastic regime.

At high temperature \( T > T_{in} \), \( \omega \sim \omega_k^* \gg T \) thus \( n_B(\omega/T) \ll 1 \). Now the frequency integral is the order of \( \log \Lambda \), where \( \Lambda \) is some high frequency cut-off. The momentum integral gives \( p^4 \). Combining all factors and replacing the boson momentum by its thermal value \( (m_B T)^{1/2} \) we obtain in inelastic limit,

\[ \tau_{tr}^{-1} \approx \frac{p^3}{m_B \gamma} \propto \frac{T^{3/2} m_B^{1/2}}{\gamma} \] (5.3)

The value of \( T_{in} \) is very sensitive to \( m_B \) and can hardly be estimated from the available experimental data. The slope of T-linear resistivity at optimal doping \( (\approx 1.0 \, \mu \Omega \, cm/K) \) gives \( \chi^{2D} \) to be around 500 K. The resistivity data of overdoped La\(_{2-x}\)Sr\(_x\)CuO\(_4\) suggest that \( \chi^{3D}_\parallel \) is the same order as \( \chi^{2D} \). The one-loop value of the damping \( \gamma \) is order of 1 \[17\].

The estimates of \( m_B \) which enters into \( T_{in} \) vary appreciably depending on the kinds of experiments. The optical conductivity measurements \[8\] provides the value of \( m_B \) at high energy : \( m_B \approx 2 m_e \), which is almost independent of doping. Especially \( m_B \approx (2 \sim 3)m_e \) is almost independent of the probe energy scale in overdoped range. From another side the magnetic susceptibility data provides the value of \( m_B \) at low energy : \( m_B \approx 15 m_e \) near optimal doping \[3,14\]. These estimates of \( m_B \) makes the estimate of \( T_{in} \) range from 500 K (the susceptibility data) to \( 10^5 \) K (the optical data).

If \( T_d(x) \leq T_{in} \) for some doping range the following behavior of the resistivities are possible.
\[ \rho_{ab} \sim \rho_c \propto T^{3/2}, \quad \text{for} \quad T \leq T_d \leq T_{in} \]
\[ \rho_{ab} \propto T, \quad \rho_c \propto T^{-1/2} \quad \text{for} \quad T_d \leq T \leq T_{in} \]
\[ \rho_{ab} \propto T^{3/2}, \quad \text{for} \quad T_{d} \leq T_{in} \leq T \] (5.4)

If one accepts the lower estimate of \( T_{in} \) one may exploit the inelastic mechanism in order to explain \( T^{3/2} \) behaviour. If \( T_d(x) \geq T_{in} \)

\[ \rho_{ab} \sim \rho_c \propto T^{3/2}, \quad \text{for} \quad T \leq T_{in} \leq T_d \]
\[ \rho_{ab} \sim \rho_c \propto T^2, \quad \text{for} \quad T_{in} \leq T \leq T_d \]
\[ \rho_{ab} \propto T^{3/2}, \quad \text{for} \quad T_{in} \leq T_d \leq T \] (5.5)

In fact the optical estimate, which is close to the band theory value, seems more realistic. This means that the inelastic regime is very likely irrelevant for the overdoped cuprates.

**VI. CONCLUSION**

We adopted the gauge theory of normal states of doped Mott insulator to explain anomalous transport phenomena observed in overdoped cuprates. We assumed that \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) interpolates between layered and anisotropic anomalous metal for the doping range \( x \sim 0.15 - 0.35 \) and still does not evolve into the ordinary metallic behaviour. We attempted to describe the dimensional crossover of the anomalous metal in temperature. The crossover of the out-of-plane transport is peculiar: due to strong interaction electrons do not constitute an elementary excitation and decay into other particles during the interlayer tunneling. As a result, the character of the out-of-plane transport may change from coherent to incoherent and that of the out-of-plane resistivity changes from metallic to semiconductor-like behaviour. In addition we discussed another crossover between elastic and inelastic scattering as temperature increases. The theory provides an unified approach in understanding the variety of temperature behaviours of the in-plane and out-of-plane resistivity of cooper oxides in wide ranges of doping and temperature. The results qualitatively agree with the available experimental data.
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