Electroweak radiative corrections to neutrino–nucleon scattering at NuTeV

Kwangwoo Park
Department of Physics, Southern Methodist University, Dallas, TX 75275, USA

Ulrich Bau
Department of Physics, SUNY at Buffalo, Buffalo, NY 14260, USA

Doreen Wackeroth
Department of Physics, SUNY at Buffalo, Buffalo, NY 14260, USA
Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), Universität Karlsruhe, D-76128 Karlsruhe, Germany

A dedicated effort by both the experimental and theoretical communities is crucial for achieving a precise determination of Standard Model parameters such as the W mass ($M_W$). $M_W$ is measured directly at the CERN LEP2 $e^+e^-$ and the Fermilab Tevatron $p\bar{p}$ colliders, resulting in a precision of $\delta M_W/M_W = 0.03\%$ [1]. A complementary $M_W$ measurement is provided by the NuTeV collaboration [2, 3], which extract $\sin^2\theta_W$, and thus $M_W$, from the ratio of deep-inelastic neutral and charged-current neutrino(anti-neutrino)-Nucleon ($\nuN(\bar{\nu}N)$) scattering cross sections. However, their result differs from direct measurements performed at LEP2 and the Tevatron by about three standard deviations [2, 3]. Possible sources for the origin of this discrepancy have been extensively studied in the literature (see, e.g., [6]), among them the impact of electroweak radiative corrections [2, 3]. Here we provide first (preliminary) results of a new calculation of electroweak $O(\alpha)$ corrections with emphasis on the effects of non-zero muon and charm quark masses. We find non-negligible shifts in $\sin^2\theta_W$ due to these mass effects but more detailed studies including detector resolution effects are needed to determine their impact on $M_W$ as extracted by the NuTeV collaboration.

I. INTRODUCTION

The Standard Model (SM) represents the best current understanding of electroweak and strong interactions of elementary particles. In recent years it has been impressively confirmed experimentally through the precise determination of $W$ and $Z$ boson properties at the CERN LEP and the Stanford Linear $e^+e^-$ colliders, and the discovery of the top quark at the Fermilab Tevatron $p\bar{p}$ collider.

A precise measurement of $M_W$ does not only provide a further precisely known SM input parameter, but significantly improves the indirect limit on the Higgs-boson mass obtained by comparing SM predictions with electroweak precision data as illustrated in Fig. 1.

A measurement of $M_W$ can also be extracted from a measurement of the sine squared of the weak mixing angle, $\sin^2\theta_W$, via the well-known relation between the $W$ and $Z$ boson masses, $M_W^2 = M_Z^2(1 - \sin^2\theta_W)$. The NuTeV collaboration extracts $M_W$ from the ratio of neutral and charged-current neutrino and antineutrino cross sections [2, 3]. Their results differ from direct measurements performed at LEP2 and the Tevatron by about $3\sigma$ [2, 3] as shown in Fig. 2.

Much effort, both experimentally and theoretically, has gone into understanding this discrepancy. These efforts include studies of QCD corrections, parton distribution functions [4, 5], and nuclear structure (see, e.g., [6] for an overview). However, the impact of electroweak radiative corrections has not been fully studied yet. In the extraction of $M_W$ from NuTeV data, only part of the electroweak corrections have been included [10]. Since then the complete calculation of these corrections has been made available in the literature [7, 8, 9], but a realistic, experimental study of their impact on the NuTeV measurement on $M_W$ has not been performed yet.

In order to remedy this situation we calculated the complete $O(\alpha)$ contribution to neutrino–nucleon scattering including the full muon and charm-quark mass dependence, which has been neglected in previous studies. Here we present first preliminary results of this new calculation with emphasis on the above-mentioned mass effects. A detailed study, also taking into account more realistic detector resolution effects, is in progress [11].

II. SOME DETAILS OF THE CALCULATION

Our calculation of the complete $O(\alpha)$ corrections to the neutral-current (NC) and charged-current (CC) $\nuN(\bar{\nu}N)$ scattering processes (the tree-level Feynman
FIG. 1: The SM prediction for $M_W$ with dependence on the top-quark mass ($M_t$) and Higgs boson mass ($M_H$), resulting in the shaded band, is compared with the experimental values of $M_W$ and $M_t$ (solid ellipse) and an indirect measurement from all electroweak precision data (dotted ellipse) [1]. Present values of $M_W$, and $M_t$ favor a relatively light SM Higgs boson, while the NuTeV value of $M_W (= 80.136 \pm 0.084 \text{ GeV})$ [2, 3] prefers a much higher Higgs boson mass.

FIG. 2: Direct (Tevatron, LEP2, and NuTeV) and indirect measurements of $M_W$. The NuTeV value of $M_W$ differs from the world average value by about $3\sigma$ [1].

diagrams are shown in Fig. 3) follows closely the treatment of s-channel $W$ and $Z$ production at hadron colliders of [12, 13]. The $O(\alpha)$ corrections consist of the full set of electroweak one-loop diagrams and real photon radiation from both the external charged fermion legs and the internal $W$ boson in the CC process. As usual, they exhibit UV and IR divergences. UV divergences are canceled by including the counterterms of the on-shell renormalization scheme [14, 15]. By applying the two-cutoff phase-space-slicing method [16], we extract the soft and collinear singularities from the real photonic corrections. We use fermion masses and a fictitious photon mass as regulators for the soft and collinear singularities. The photon mass dependence cancels in the sum of virtual and real soft-photon radiation, but mass singularities of the form $\log(t/m_\gamma^2)$ may survive, which arise when the photon is emitted collinear with the charged fermion. In the case of final-state photon radiation, in inclusive observables these mass singularities cancel. However, mass singularities connected to initial-state photon radiation survive in general. These need to be absorbed in the parton distribution functions (PDF), which can be done in analogy to gluon radiation in QCD. Finally, the numerical phase space integration was done using Monte Carlo integration techniques based on the Vegas algorithm [17].

After convolution with the quark PDFs, the predictions for the hadronic, electroweak (EW) next-to-leading order (NLO) cross section for $\nu N$ scattering is obtained as follows ($j = NC, CC$):

$$d\sigma_j^\nu(E_\nu) = \sum_i \int dx q_i(x, Q^2) \left( d\hat{\sigma}_{0,(j)}^\nu + d\hat{\sigma}_{v+s}^j \right) + \sum_i \int_{z_1}^{z_2(x)} dz \frac{1}{z} q_i \left( \frac{z}{z_1}, Q^2 \right) d\hat{\sigma}_h^j$$

where the parton level cross section consists of the tree-level cross section, virtual, soft and collinear $O(\alpha)$ contributions (including the PDF counterterms) and the real hard photon radiation contribution.

A. Fermion-mass effects

We performed the calculation with and without including fermion-mass effects and are considering the following two cases:

case 1: All external fermions are considered to be massless and we only keep non-zero fermion masses as regulators of the collinear singularities.
case 2: The full muon and charm-quark mass dependence is taken into account, but light external fermions are treated as in the first case.

Fermion-mass effects in EW radiative corrections may not be numerically negligible in this process, since the relevant parton-level energy scale \( \hat{t} \) and the fermion masses can be of the same order of magnitude. This is illustrated below for the example of the \( W \) self-energy correction \((\hat{\Sigma}_W)\) to the CC process shown in Fig. 4. Its contribution to the one-loop corrected matrix element at \( \mathcal{O}(\alpha) \) reads

\[
\mathcal{M}_{\text{virt}}^{CC} = -\frac{e^4}{8s^2_W (q^2 - M_W^2)} \left[ \overline{u}_{\rho} \gamma^\rho (1 - \gamma_5) u_\nu \right] \left[ \overline{u}_{\beta} \gamma^\beta (1 - \gamma_5) u_\alpha \right].
\]  

(2)

With the renormalized \( W \) self energy \( \hat{\Sigma}_W \) being decomposed in transverse and longitudinal parts, \( \hat{\Sigma}_W = \left( g_{\rho\sigma} - \frac{q_{\rho} q_{\sigma}}{q^2} \right) \hat{\Sigma}_T^{\rho\sigma} + \frac{q_{\rho} q_{\sigma}}{q^2} \hat{\Sigma}_L^{\rho\sigma} \), one finds the following contribution to the NLO matrix element squared (up to terms of \( \mathcal{O}(m_f^2/M_W^2) \)):

\[
2 \text{Re} \mathcal{M}_{LO}^{CC} \mathcal{M}_{\text{virt}}^{CC} = -\frac{4e^4}{s^2_W (\hat{t} - M_W^2)} \left[ p_1 \cdot p_2 p_3 \cdot p_4 \text{Re} \hat{\Sigma}_T^{\rho\sigma} \right] + m_4^2 \left( m_2^2 p_1 \cdot p_3 - m_3^2 p_1 \cdot p_2 \right) \frac{\text{Re} \left( \hat{\Sigma}_L^{\rho\sigma} - \hat{\Sigma}_T^{\rho\sigma} \right)}{4\hat{t}}.
\]

(3)

If we consider massless fermions for the external legs (case 1), the second term in Eq. (3) vanishes. However, for massive fermions (case 2) the longitudinal two–point function contributes to the physical cross section. Since we work in the Feynman–t Hooft gauge, we also had to include the contributions from the would-be Goldstone bosons, which are not explicitly shown here. In the \( s \)-channel \( W \) production process such as gauge-boson production in Drell-Yan processes at the Tevatron and the LHC, \( \hat{t} \) is replaced with \( \hat{s} \), so that the second term is usually negligible. In \( t \)-channel deep inelastic scattering, however, fermion-mass effects deserve a closer investigation, especially in the small \( \hat{t} \) region, which corresponds to a small momentum fraction \( x \). Note that similar effects also arise from vertex and box corrections.

In case of real photon radiation amplitude-level fermion-mass effects only arise in the CC \( \nu \bar{\nu} \) scattering process (the matrix elements to real photon radiation in the NC process of cases 1 and 2 are identical). The Feynman diagrams for the \( W^\pm \) exchange contribution are shown in Fig. 5 and the corresponding matrix element \( \mathcal{M}_{\text{virt}}^{CC} \) can be written in terms of \( U(1) \)-conserved leptonic and hadronic currents as follows \( (\epsilon_{\rho}(k) \) denotes the photon polarization vector):

\[
M_{\epsilon \rho}^{CC} = \frac{e^3}{8s^2_w} \left[ \mathcal{M}_{\text{had}}^{CC,\rho} \left( \hat{t} - M_\nu^2 \right) + \mathcal{M}_{\text{lept}}^{CC,\rho} \left( \hat{t} - M_W^2 - 2k \cdot q \right) \right] \epsilon_{\rho}(k),
\]

(4)

with

\[
\mathcal{M}_{\text{had}}^{CC,\rho} = \left[ \overline{\nu}_{\rho} \gamma_{\rho} \gamma_L u_{\nu} \right] \left[ \overline{\nu}_{\tau} \left( \Gamma_{\tau \rho}^{\mu} + \Gamma_{\tau \nu}^{\mu} \right) \gamma_L u_{\nu} \right],
\]

\[
\mathcal{M}_{\text{lept}}^{CC,\rho} = \left[ \overline{\nu}_{\rho} \left( \Gamma_{\nu}^{\mu \rho} + \Gamma_{\nu}^{\mu \mu} \right) \gamma_L u_{\nu} \right] \left[ \overline{\nu}_{\tau} \gamma_L u_{\nu} \right] - \mathcal{J}_m^{\epsilon \rho},
\]

where \( \gamma_L = 1 - \gamma_5 \) and,

\[
\Gamma_{\nu}^{\mu \rho} = Q_3 \gamma^\rho \frac{p_2^\rho - k \cdot q}{-k \cdot p_3}, \quad \Gamma_{\nu}^{\mu \mu} = Q_3 \frac{p_2^\rho + \gamma^\rho k^\rho/2}{-k \cdot q},
\]

\[
\Gamma_{\nu}^{\mu \rho} = Q_4 \frac{p_4^\rho + \gamma^\rho k^\rho/2}{k \cdot q}, \quad \Gamma_{\nu}^{\mu \mu} = Q_4 \frac{\gamma^\rho q^\rho - \gamma^\rho k^\rho - g_{\rho \mu}}{k \cdot q}, \quad \Gamma_{\nu}^{\mu \rho} = \Gamma_{\nu}^{\mu \mu}.
\]

FIG. 4: Feynman diagrams for self-energy corrections to the NC and CC \( \nu \bar{\nu} \) production processes.

FIG. 5: Feynman diagrams for real photon radiation in the CC \( \nu \bar{\nu} \) scattering process. Shown here are only \( W^\pm \) exchange diagrams.
FIG. 6: Feynman diagrams for the mixed $W^\pm - \phi^\pm$ exchange contribution to real photon radiation in the CC $\nu N$ scattering process, and the Feynman rules for the $\phi^\pm f f'$ coupling.

\[
\begin{align*}
\mu : & \frac{-ie}{2\sqrt{2}s_WM_W} (m_4(1 - \gamma_5) - m_1(1 + \gamma_5)) \\
\nu : & \frac{ie}{2\sqrt{2}s_WM_W} (m_3(1 - \gamma_5) - m_2(1 + \gamma_5))
\end{align*}
\]

where the subscripts, I, II, III, and IV correspond to the diagrams shown in Fig. 5. The fermion-mass dependence of $\mathcal{M}_r^{CC}$ described by $\mathcal{J}_m^\rho$ vanishes when the contribution of the mixed $\phi^\pm - W^\pm$ exchange diagrams shown in Fig. 6 is included. The only surviving fermion-mass dependence at the amplitude-level is due to the $\phi^\pm$ exchange diagrams which are, however, suppressed by $O(m_f^2/M_W^2)$.

**B. Treatment of numerical instabilities**

It is well-known (see, e.g., Ref. [2]) that the EW NLO cross section to the NC process suffers from a numerical instability at small values of $\hat{t}$ owing to photon exchange diagrams such as shown in Fig. 4 and Fig. 7. As a remedy of this kind of instability, we apply a Taylor expansion around small $\hat{t}$. In Fig. 7 we illustrate the stability of this expansion.

Box corrections exhibit numerical instabilities originating from vanishing Gram determinants at small kinematic variables when the standard Passarino-Veltman reduction formalism [18] is employed to determine the coefficient functions of vector and tensor four-point integrals. Especially, the crossed box contribution in the low $x$-region suffers from these instabilities which can be traced back to numerical unstable coefficients of three-point integrals. In the following we present a reduction formalism which yields stable results as illustrated in Fig. 8 for case 1 (see also, e.g., [21] for an alternate solution). In phase-space regions of small kinematic variables, the coefficients...
of the three-point vector and tensor integral

\[
C^{\mu}, C_{\mu\nu} = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^Dq \frac{q^{\mu}q^{\nu}q^{\nu}}{[q^2-m_0^2][(q+p_1)^2-m_1^2][(q+p_2)^2-m_2^2]},
\]

defined as

\[
C^{\mu} = p_1^\mu C_1 + p_2^\mu C_2,
\]
\[
C_{\mu\nu} = \epsilon_{\mu\nu\rho} C_{00} + p_1^\rho C_{11} + p_2^\rho C_{12} + p_3^\rho p_4^\nu C_{21} + p_3^\mu p_4^\nu C_{22},
\]
can be approximated in terms of two-point functions as follows

\[
C_1 = \alpha B_1^1 + 2\alpha^2 [B_{00}^1+B_{00}^2+(p_2^1-p_1^1)p_2B_{01}^1]
+ \mathcal{O} ((p_2^1)^2, (p_2^2)^2, (p_1^1p_2^1)^2)
\]
\[
C_2 = -\alpha B_2^1 + 2\alpha^2 [B_{00}^1+B_{00}^2+(p_2^1-p_1^1)p_2B_{11}^1]
+ \mathcal{O} ((p_2^1)^2, (p_2^2)^2, (p_1^1p_2^1)^2)
\]
\[
C_{00} = \alpha (B_{00}^1-B_{00}^2) + 2\alpha^2 [(p_2^1-p_1^1)p_2B_{00}^1]
+ (p_2^2-p_1^2)(p_2B_{01}^1) + \mathcal{O} ((p_2^1)^2, (p_2^2)^2, (p_1^1p_2^1)^2)
\]
\[
C_{11} = \alpha (B_{11}^1) + 2\alpha^2 [2B_{00}^1+(p_2^1-p_1^1)p_2B_{11}^1]
+ \mathcal{O} ((p_2^1)^2, (p_2^2)^2, (p_1^1p_2^1)^2)
\]
\[
C_{12} = C_{21} = -\alpha (B_{00}^1+B_{00}^2)
+ \mathcal{O} ((p_2^1)^2, (p_2^2)^2, (p_1^1p_2^1)^2)
\]
\[
C_{22} = -\alpha (B_{11}^1) + 2\alpha^2 [2B_{00}^1+(p_2^1-p_1^1)p_2B_{11}^1]
+ \mathcal{O} ((p_2^1)^2, (p_2^2)^2, (p_1^1p_2^1)^2)
\]

and the scalar three-point integral reads:

\[
C_0 = \alpha (B_{00}^1-B_{00}^2) + 2\alpha^2 [(p_2^1-p_1^1)p_2B_{00}^1]
+ (p_2^2-p_1^2)(p_2B_{01}^1) + \mathcal{O} ((p_2^1)^2, (p_2^2)^2, (p_1^1p_2^1)^2).
\]

Here,

\[
\alpha = \frac{1}{m_1^2-m_2^2-p_1^2-p_2^2}, \quad B_{\mu\nu\ldots} = B_{\mu\nu\ldots}(p_2^2, m_0^2, m_1^2).
\]

The detailed derivation of these expressions can be found in [19]. In Fig. 8 we show a comparison of these two derivations of the $C_{ij}$ functions in the critical phase space region, i.e. at small $x$.

III. NUMERICAL RESULTS

The measured value of $\sin^2\theta_W$ can be extracted from the Paschos-Wolfenstein relation [21]

\[
R = \frac{\sigma_{0,NC}(\nu N \rightarrow \nu X) - \sigma_{NC}(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma_{CC}(\nu N \rightarrow \ell X) - \sigma_{CC}(\bar{\nu} N \rightarrow \ell X)}
= \rho^2 \left( \frac{1}{2} - \sin^2\theta_W \right)
\]

\[R = \frac{\sigma_{0,NC}(\nu N \rightarrow \nu X) - \sigma_{NC}(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma_{CC}(\nu N \rightarrow \ell X) - \sigma_{CC}(\bar{\nu} N \rightarrow \ell X)}
\]

\[= \rho^2 \left( \frac{1}{2} - \sin^2\theta_W \right)
\]

IV. CONCLUSION

Deep-inelastic neutrino–nucleon scattering provides an excellent testing ground for the electroweak SM, complementary to $e^+e^-$ and hadronic colliders. Measurements of electroweak parameters in neutrino–
nucleon scattering are not only comparable in precision but also probe the EW SM at many orders of magnitude in $t = q^2$, i.e. the parton-level momentum transfer in these processes.

The NuTeV collaboration used the calculation of Ref. [10], which is based on a massless fermion approximation, and did not include the entire set of electroweak $O(\alpha)$ corrections. Seventeen years later, a complete calculation of the $O(\alpha)$ corrections to neutrino-nucleon scattering became available [7]. In a follow-up paper [9], leading higher order corrections, i.e. beyond one-loop, have been included as well. In Ref. [3], the discussion focused on the EW input scheme dependence of $\sin^2 \theta_W$ measured in neutrino–nucleon scattering. They concluded that the theoretical uncertainty due to missing higher-order corrections has been underestimated by the NuTeV collaboration, and, thus, is a potential source for at least part of the observed discrepancy.

In this study we focus on another potential source of a theoretical uncertainty, which has not been considered before. i.e. the effects of muon and charm-quark masses in the calculation of electroweak corrections. We calculated the complete electroweak $O(\alpha)$ corrections to neutrino–nucleon scattering with and without taking into account these fermion-mass effects. We studied their impact on $\sin^2 \theta_W$ as extracted from the $\nu N$ scattering cross section by the NuTeV collaboration. We found non-negligible differences in $\sin^2 \theta_W$ when using our calculation with and without considering non-zero muon and charm-quark masses.

However, a more realistic study is needed including a simulation of the detector resolution, for instance, to determine whether these effects can account for part of the NuTeV anomaly. Such a study is currently in progress.

**Acknowledgments**

We are grateful to Kevin McFarland for fruitful discussions and guidance concerning experimental issues. The work of K. P. is supported by the U.S. Department of Energy under grant DE-FG02-04ER41299. This research is also supported by the National Science Foundation under grants No. NSF-PHY-0547564 and NSF-PHY-0757691. The work of D. W. is presently also supported by a DFG Mercator Visiting Professorship.

---

**Table I: Preliminary results obtained with the MRST2004QED PDF set and a cut on $y = -t/s \geq 0.12.$**

| Case | $R^e_C$ | $\delta R^e_{NC}$ | $\delta R^e_{CC}$ | $\Delta \sin^2 \theta_W$ |
|------|---------|----------------|----------------|-----------------|
| 1    | 0.30638 | 0.05277        | -0.0916        | -0.0182         |
| 2    | 0.31477 | 0.05482        | -0.1059        | -0.0258         |

---

[1] The LEP (ALEPH, DELPHI, L3, OPAL), SLD and Tevatron (CDF, D0) Collaborations, LEP Electroweak WG, Tevatron Electroweak WG, SLD Electroweak WG and Heavy Flavour Group, arXiv:0811.4682 [hep-ex], update taken from http://lepewwg.web.cern.ch/LEPEWWG/plots/summer2009.

[2] G. P. Zeller et al. [NuTeV Collaboration], Phys. Rev. Lett. 88, 091802 (2002) [Erratum-ibid. 90, 239902 (2003)] arXiv:hep-ex/0110059.

[3] G. P. Zeller [NuTeV Collaboration], arXiv:hep-ex/0207037.

[4] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C 39, 155 (2005) arXiv:hep-ph/0411040.

[5] R. D. Ball et al. [The NNPDF Collaboration], Nucl. Phys. B 823, 195 (2009) arXiv:0906.1958 [hep-ph].

[6] K. S. Mcfarland and S. O. Moch, arXiv:hep-ph/0306052.

[7] K. P. O. Diener, S. Dittmaier and W. Hollik, Phys. Rev. D 69, 073005 (2004) arXiv:hep-ph/0311036.

[8] A. B. Arbuzov, D. Y. Bardin and L. V. Kalinovskaya, JHEP 0506, 078 (2005) arXiv:hep-ph/0407203.

[9] K. P. Diener, S. Dittmaier and W. Hollik, Phys. Rev. D 72, 093002 (2005) arXiv:hep-ph/0509084.

[10] D. Y. Bardin and V. A. Dokuchaeva, JINR-E2-86-260.

[11] U. Baur, K. McFarland, K. Park, and D. Wackeroth, in preparation.

[12] U. Baur, S. Keller and D. Wackeroth, Phys. Rev. D 59, 013002 (1999) arXiv:hep-ph/9807417.

[13] U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackeroth, Phys. Rev. D 65, 033007 (2002) arXiv:hep-ph/0108274.

[14] M. Hohm, H. Spiesberger and W. Hollik, Fortsch. Phys. 34, 687 (1986).

[15] A. Denner, Fortsch. Phys. 41, 307 (1993) arXiv:0709.1075 [hep-ph].

[16] B. W. Harris and J. F. Owens, Phys. Rev. D 65, 094032 (2002) arXiv:hep-ph/0102128.

[17] G. P. Lepage, J. Comput. Phys. 27, 192 (1978).

[18] G. Passarino and M. J. G. Veltman, Nucl. Phys. B 160, 151 (1979).

[19] K. Park, “Reduction of one loop integrals,” (in preparation).

[20] E. A. Paschos and L. Wolfenstein, Phys. Rev. D 7, 91 (1973).

[21] A. Denner and S. Dittmaier, Nucl. Phys. B 734, 62 (2006) arXiv:hep-ph/0509141.