Optimization of the flight program of a passenger aircraft taking into account operational limitations and the influence of climatic factors.

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Abstract. The paper presents the results of optimization of the flight program for a passenger aircraft. In mathematical modeling, real operational limitations and possible changes in atmospheric parameters were taken into account. The mathematical model is characterized by the description of the flight of an airplane as a material point and the description of a gas turbine engine by a model of the first level of accuracy. The following operational limitations were considered: the possibility of cruising at certain levels, the restriction on the vertical speed with a decrease, the possibility of transition to the next echelon with an available stock of thrust of 20%. The optimization of the flight program is based on the criterion of the minimum amount of fuel expended for the flight, at a given range. Calculations were conducted for several standards of air temperature change, (depending on the climatic zone). A comparative analysis of the obtained optimization results is carried out, the degree of influence of the change in atmospheric conditions is estimated. When comparing flights in different climatic zones, the fuel costs were compared for a flight program optimized for the given atmospheric conditions and for a flight program optimized for use in the mathematical model of the International Standard Atmosphere (ISA).

1. Introduction
The problem on optimizing the flight program of passenger aircraft has not lost its relevance for quite a long time [1-22]. In the present work we investigate how atmospheric conditions influence on the optimization results. We also estimate possible fuel losses under the flight according to the flight programs, optimal for the ISA conditions, but not optimal for specific climatic conditions.

Traditional approaches are used for simulating the flight of a subsonic aircraft in the vertical plane [1], [2]. The calculation algorithm considers features of flight sections: take-off, primary climb, main climb, etc. For each part of the flight, in accordance with the accepted practice, we generate our own set of differential equations for an aircraft motion (describing only the motion of the center of mass without taking into account the moment equations) typical for the given flight mode. In the equations of motion, we do not neglect by the angles of attack.

Engine performances are calculated in a separate subroutine, which is called within the numerical method for solving a set of differential equations. The engine model corresponds to the first level [3]. Performances of engine’s units are presented in the form of mathematical models of zero level
(generalized approximation relationships) [3]. Transient modes of engine operation are not modeled. The thermodynamic properties of the working fluid are determined in accordance with the algorithms presented in [3], [4].

2. Mathematical problem

Optimization of the flight program for the acceleration-climbing segment is carried out by using the efficiency criterion: the minimum fuel consumed in this part of the flight [1], [16]. This efficiency criterion is a functional in the optimal control problem:

\[ m_{fc} = \int_{H_{e1}}^{H_{e2}} F \, dH_e \rightarrow \min \]

The numerical solution of the optimal control problem is reduced to the problem on minimizing Florov’s function at each calculation point of the flight program for the main climb part of the flight [1], [2]:

\[ F = \frac{G_f \cdot i}{n \cdot V} \]

where \( G_f \) is the fuel consumed by one engine, which depends on the altitude, flight velocity, atmospheric conditions, the current weight of the aircraft and its aerodynamic qualities, \( i \) - is the number of engines, \( V \) - is the flight velocity, \( H \) - is the flight altitude, \( n \) - is the horizontal overload

\[ n = \frac{P (\cos(\alpha + \varphi_0) i - X)}{m_a g} \]

\( \varphi_0 \) - is the angle of the engine installation in the vertical plane with respect to the wing chord, \( \alpha \) - is the angle of attack, \( m_a \) - the aircraft’s mass, \( g = 9.81, \text{m/c} \) – is the acceleration of gravity, \( X = C_x \rho V^2 \text{S} \) - is the aerodynamic drag, \( P \) - the thrust of one engine.

This function is none other than the derivative of the corresponding integral optimization criterion for energy altitude \( \left( H_e = \frac{V^2}{2 \cdot g} + H \right) \), in particular, \( F = \frac{dm_{fc}}{dH_e} \). The values of the energy height of the beginning and the end of the flight section are strictly fixed.

Thus, optimal values \( H \) and \( V \) for each energy altitude (the argument in the system of motion control) is determined by the condition of minimizing the derivative of the corresponding efficiency criterion with respect to the energy altitude, under the constraints:

\[ H_{k+1} \geq H_k, \quad V_{k+1} \geq V_k. \]

Optimization of the flight program for the cruising flight segment is carried out by using the efficiency criterion: the minimal fuel \( m_{fcr} \) consumed at this part of the flight, since the flight is considered for a given range. This efficiency criterion is a functional for the optimal control problem:

\[ m_{fcr} = \int_{L_1}^{L_2} q(V_c, H_{cr}) \, dL \rightarrow \min. \]

The solution of the optimal control problem is reduced to the problem on minimizing the fuel consumption per kilometer at each design point of the cruising:
3.

It is not reasonable to change the velocity during the cruise flight, since on the one hand the acceleration during flight reduces the comfort, on the other hand, the desire to shorten the flight time remains sufficiently important, in connection with which the maximum permissible constants Mach number \( = 0.8 \ldots 0.85 \) (in terms of moderate aerodynamic drag at subsonic flight velocities) are selected.

The following operational limitations must be considered:
- flight can only be performed at the given discrete altitudes (echelons) \([5]\),
- the transition to the next echelon is possible with a stock of engine thrust of at least 20% of the maximum mode, and its reasonability is determined by the smaller fuel consumption per kilometer at the next echelon with respect to the current altitude,
- possible cruising altitude of flight ranges from 10650 up to 13700 m,
- the Mach number of the flight at the cruising site is 0.8.

If we consider the long-distance flights, the problem on minimizing the fuel consumption for the whole flight can be separated on to subtasks to minimize fuel consumption at different parts of the flight \([1], [2]\).

The descent-deceleration section is optimized by the same way as for the acceleration-climbing part of the flight, taking into account the restrictions on the vertical descent velocity (not more than 7 m / sec), and also the restrictions written as: \( H_{k+1} \leq H_k \), \( V_{k+1} \leq V_k \).

The aircraft parameters typical for medium- and long-haul airplanes (high aerodynamic quality of about 20 ... 22 for the cruising flight, two-contour engines with a degree of two-contouring of about 50\% in cruise flight, sufficiently high takeoff weight - 142.88 tons) were chosen for modeling. The aircraft version selected according to its mass, thrust and aerodynamic performances is close to the aircraft Boing-767-200. The system of vertical echeloning is taken for aircrafts being in the airspace of the Russian Federation with reduced intervals (flight for track angles ranging from 0 up to 179 degrees is considered) \([5]\). The flight for a range of 9500 km is examined. The thrust is not considered as an optimization parameter, it is assumed that the engine control law (nominal mode) is set in the climbing part of the flight, and the engine control law (flight small thrust) is also set at the descending section, which is associated with safety requirements and comfort of the flight.

The equations of motion in the mentioned sections of the flight are presented in details in \([1], [2], [16-19]\). Here we give briefly:
- set of equations for the main acceleration-climb section, and for descending-decelerating segment:

\[
\begin{align*}
\frac{dm_a}{dH_e} &= -G_f \cdot i \frac{m_a g}{(P \cdot i \cdot \cos(\alpha + \varphi_0) - X) \cdot V} \\
\frac{dt}{dH_e} &= \frac{m_a g}{(P \cdot i \cdot \cos(\alpha + \varphi_0) - X) \cdot V} \\
\frac{dL}{dH_e} &= \frac{m_a g \cdot \cos \theta}{P \cdot i \cdot \cos(\alpha + \varphi_0) - X}
\end{align*}
\]

where \( L \) - is the flight range, \( t \) - is the flight time, \( \theta \) - is the trajectory angle;
- set of equations for the cruising flight segment:
The required thrust and the required angle of attack are determined from the set of algebraic equations:

\[
\begin{aligned}
\frac{dm_a}{dL} &= -\frac{G_f \cdot i}{V} \\
\frac{dt}{dL} &= \frac{1}{V} \\
\end{aligned}
\]

The temperature and pressure deviation from the accepted ISA effects first of all to the mode of engine operation and to the aerodynamic parameters during the flight due to the possible change in air density. The following standards are considered for different climatic conditions: № 0 - the ISA for the Earth, № 1 - the minimal for arctic conditions, № 2 - the minimal for tropical and moderate conditions, № 3 - maximal for moderate and arctic conditions, № 4 - the maximal intercontinental ICAO, № 5 - the maximal for tropical conditions, № 6 - the maximal for tropical conditions (100% humidity).

Figure 1 depicts the relationships between the air temperature and altitude for all mentioned standards. The atmospheric pressure variation with height is calculated from the following assumptions: hydrostatics equations: \( \frac{dp}{dH} = -\rho g \), where \( \rho \) - is the air density, \( g \) - is the acceleration of gravity, and equations of state for ideal gas: \( p = R \rho T \), where \( R \) - is the gas constant for air. From these relationships we obtain: \( \ln p(H) = \ln p_0 - \frac{1}{R_0} \int_0^H \frac{g \, dh}{T(h)} \), where \( p_0 \) - is the pressure at sea level.

Figure 2 depicts pressure curves corresponding to all examined standards.

**Figure 1.** The relationship between temperature and altitude
3. Numerical methods used in algorithms

The set of differential equations is solved numerically by predictor-corrector method (Adams method), where the first steps are calculated by using the fourth-order Runge-Kutta method.

For solving a set of algebraic nonlinear equations (in the module for calculating engine performances) a modified Newton method is used. One equation (for determining the required angle of attack in cruising flight) is solved by the dichotomy method.

For determining the thermodynamic properties of the working fluid (in the module for calculating the engine’s performances), an iterative method is used for solving one algebraic equation.

Information on aerodynamic parameters is incorporated into the calculation program in the form of data array. Intermediate values are calculated by linear interpolation. The moment of transition to the next echelon in cruise flight is determined at each integration step by checking fuel consumption per kilometer at the current and next flight echelons and by checking whether the thrust margin is sufficient.

4. Results of Calculations

Figures 3 and 4 depict optimal flight trajectories for different atmospheric conditions.
Figure 4. Optimization of the cruise flight segment for 3 standards of temperature variation with altitude: №№ 2, 4, 5

Figures 5 and 6 depict optimal flight programs for flight sections: climbing and descending with a standard pressure value at sea level (101325 Pa). Figures 7 and 8 depict the required angles of attack for ensuring optimal flight programs in these areas.

Figure 5. Optimal flight programs for climbing in the form of $H = f(V)$

Figure 6. Optimum descending programs in the form of $H = f(V)$

Figure 7. Required angles of attack during acceleration-climb segment
Figure 8. Required angles of attack under descending-decelerating segment

Figure 9 depicts fuel consumption for flying under various flight programs that are optimal for specific climatic conditions, as well as for the flight program according to the atmospheric parameters corresponding to the International Standard Atmosphere, but taking place under other climatic conditions.

Figure 9. Fuel consumption for flight under different climatic conditions and different flight programs.

5. Conclusions
The results of calculations are typical for an airplane with good aerodynamic quality and sufficient thrust of engines (turbojet by-pass engine with by-pass degree of about 5), which makes it possible to achieve a primary cruise altitude higher than 10,000 m. Under other parameters of the aircraft and of the engines the quantitative results can be differed from the obtained ones.

The air temperature influences greatly on to the results of flight program optimization. The climbing segment of flight for tropical climates should be more abrupt than it is for moderate or arctic climate. For the tropical climate, higher echelons of cruising flight become preferable. A similar situation takes place for optimal flight programs under descending segment of flight.

If the flight is performed without considering the real temperature distribution over the altitude, the fuel consumption for flight can be much more than for the flight with optimum flight programs and by considering the climatic conditions. Such fuel losses in cold climate can run up to 7.5% (up to 3 tons), while in tropical climate the fuel losses can be 4.5% (up to 2 tons).

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