Two-loop QCD corrections to $Wb\bar{b}$ production at hadron colliders

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with Simon Badger and Simone Zoia

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Precise prediction for the LHC

⇒ QCD corrections are important at the LHC

\[ d\sigma = d\sigma^{\text{LO}} + \left( d\sigma^{\text{NLO}} + d\sigma^{\text{NNLO}} \right) + \ldots \]

10–30% \hspace{1cm} 1–10%

NNLO frontier: 2 to 3 scattering

- \( pp \to j jj \): \( R_{3/2}, m_{jjj} \Rightarrow \alpha_s \) determination at multi-TeV range
- \( pp \to \gamma\gamma j \): background to Higgs \( p_T \), signal/background interference effects
- \( pp \to Hjj \): Higgs \( p_T \), background to VBF (probes Higgs coupling)
- \( pp \to Vjj \): Vector boson \( p_T \), \( W^+/W^- \) ratios, multiplicity scaling
- \( pp \to VVj \): background for new physics
NNLO cross sections for $2 \rightarrow 3$ processes

loop amplitude = $\sum$ (rational coefficients) $\times$ (integral/special functions)

finite remainder = loop amplitude $-$ poles
Massive progress in massless 2-loop 5-particle scattering

- All 2-loop 5-particle integrals are known, talk by Vasily

- Many 2-loop 5-particle QCD amplitudes known analytically, talks by Federico, Herschel, Vasily

  Leading colour $\Rightarrow 5g, 2q3g, 4q1g, 2q3\gamma, 2q1g2\gamma$
  
  Full colour $\Rightarrow 5g\text{ all-plus}, 2q1g2\gamma$

- NNLO QCD calculations for $2 \rightarrow 3$ processes, talk by Rene

  $pp \rightarrow \gamma\gamma\gamma$ [Chawdhry,Czakon,Mitov,Poncelet(2019)][Kallweit,Sotnikov,Wiesemann(2020)]

  $pp \rightarrow \gamma\gamma j$ [Chawdhry,Czakon,Mitov,Poncelet(2021)]
Scattering with an off-shell leg

$pp \to H + 2j$

$pp \to W/Z + 2j$

$pp \to W/Z + \gamma j$

- rich potential phenomenology
- massless internal particles, focus on QCD corrections
- high algebraic and analytic complexity
  - $\Rightarrow$ six independent variables
  - $\Rightarrow$ 3 square roots

Bayu Hartanto (Cambridge)
**A first look: two-loop $W + 4$ parton amplitudes**

Numerical evaluation of leading colour $q\bar{Q}Qq'\nu\ell$ and $qg\bar{q}'\nu\ell$ helicity amplitudes at two loops

| Feynman diagrams | integrand reduction | IBP reduction | finite-field sampling |
|-------------------|---------------------|---------------|-----------------------|

- Full solutions of the master integrals were not available back then
- Unknown MIs are evaluated numerically using pySecDec/Fiesta

\[
I\left(\begin{array}{c}
\frac{6}{1} \frac{k_2}{3} \frac{k_1}{4} \\
\frac{1}{3} \frac{1}{2} \frac{1}{2}
\end{array}\right) \left[\langle 4|k_2|p_{56}|4\rangle \mu_{11}\right] \sim \mathcal{O}(\epsilon)
\]

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I\left(\begin{array}{c}
\frac{6}{1} \frac{k_2}{3} \frac{k_1}{4} \\
\frac{1}{3} \frac{1}{2} \frac{1}{2}
\end{array}\right) \left[\text{tr} - (1(k_1 - p_1)(k_1 - p_{12})3)(4|k_2|p_{56}|4)\right] \sim \mathcal{O}(1)
\]

- Use momentum twistor parametrisation for $2 \rightarrow 4$ massless scattering: 8 variables
- Coefficient of master integrals are computed numerically over finite fields
Two-loop master integrals for 5-point 1-mass process

4-point sub-topologies known from $pp \to V_1^* V_2^*$

- [Gehrman,Remiddi(2000)]
- [Henn,Melnikov,Smirnov(2014)]
- [Gehrmann,von Manteuffel,Tancredi(2015)]

All planar 2-loop integrals are available

- [Papadopoulos, Tomassini, Wever(2015)]
- [Papadopulos, Wever(2019)]
- [Abreu, Ita, Moriello, Page, Tschernow, Zeng(2020)]
- [Canko, Papadopoulos, Syrrakos(2020)]

Non-planar integrals in progress

Talks by Ben & Costas
Leading colour $Wb\bar{b}$ amplitude

$$\bar{d}(p_1) + u(p_2) \to b(p_3) + \bar{b}(p_4) + W^+(p_5)$$

- colour decomposition at leading colour → only planar contribution

$$A^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W) \sim g_s^6 g_W N_c^2 \delta_{i_1}^{\bar{i}_4} \delta_{i_3}^{\bar{i}_2} A^{(2)}(1_{\bar{d}}, 2_u, 3_b, 4_{\bar{b}}, 5_W)$$

- massless $b$ quarks, $p_3^2 = p_4^2 = 0$

- onshell $W$ boson

$$p_5^2 = m_W^2, \quad \sum_\lambda \varepsilon^\mu_\lambda(p_5, \lambda) \varepsilon_\nu(p_5, \lambda) = -g^{\mu\nu} + \frac{p_5^\mu p_5^\nu}{m_W^2}$$

Invariants:

$$s_{12} = (p_1 + p_2)^2, \quad s_{23} = (p_2 - p_3)^2, \quad s_{34} = (p_3 + p_4)^2,$$

$$s_{45} = (p_4 + p_5)^2, \quad s_{15} = (p_1 - p_5)^2, \quad s_5 = p_5^2,$$

$$\text{tr}_5 = 4i\varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma.$$
Integrand construction

Feynman diagrams generated using QGRAF [Nogueira(1993)]

\[
\begin{array}{c}
\begin{aligned}
&\bar{d} \rightarrow W^+ u \\
&\bar{b} \rightarrow b \\
&\bar{d} \rightarrow W^+ u
\end{aligned}
\end{array}
\]

\[
\begin{array}{c}
\begin{aligned}
&\bar{b} \rightarrow b \\
&\bar{d} \rightarrow W^+ u
\end{aligned}
\end{array}
\]

\[
\begin{array}{c}
\begin{aligned}
&u \rightarrow W^+ \bar{b} \\
&\bar{d} \rightarrow b \\
&\bar{b} \rightarrow \bar{b}
\end{aligned}
\end{array}
\]

\[
\begin{array}{c}
\begin{aligned}
&u \rightarrow W^+ \bar{b} \\
&\bar{d} \rightarrow b \\
&\bar{b} \rightarrow \bar{b}
\end{aligned}
\end{array}
\]

MATHEMATICA+FORM to process the numerator topologies and interfere with tree level

\[
M^{(2)} = \sum_{\text{spin}} A^{(0)*} A^{(2)} = M^{(2)}_{\text{even}} + \text{tr}_5 M^{(2)}_{\text{odd}}
\]

Numerators containing: \(\text{tr}(\cdots)\) and \(\text{tr}(\cdots \gamma_5 \cdots \gamma_5 \cdots)\) ⇒ anti-commuting \(\gamma_5\) prescription

\(\text{tr}(\cdots \gamma_5 \cdots)\) ⇒ Larin’s prescription [Larin(1993)]

Amplitudes in terms of scalar integrals

\[
M^{(2)}_k(\{p\}) = \sum_i c_{k,i}(\epsilon, \{p\}) \mathcal{I}_{k,i}(\epsilon, \{p\}), \quad k \in \{\text{even, odd}\}
\]
Integration-by-parts (IBP) reduction to master integrals

- Map each topology to a set of maximal topologies
- IBP systems for $T_1 - T_{10}$

$$M_k^{(2)}(\{p\}, \epsilon) = \sum_i c_{k,i}(\{p\}, \epsilon) I_k,i(\{p\}, \epsilon)$$

Entire workflow on finite fields

$$M_k^{(2)}(\{p\}, \epsilon) = \sum_i d_{k,i}(\{p\}, \epsilon) MI_k,i(\{p\}, \epsilon)$$

$k \in \{\text{even, odd}\}$

- IBP reduction directly to canonical MIs [Abreu, et al. (2020)]
- IBP systems generated with LiteRed [Lee (2012)], solved with FiniteFlow [Peraro (2019)] using Laporta algorithm [Laporta (2000)]
- IBP tables known numerically at each value of $(\{p\}, \epsilon)$
- Numerically compute $d_{k,i}$ over finite fields
### Plugging in the master integrals

1. [Abreu, Ita, Moriello, Page, Tschernow, Zeng (2020)] talk by Ben
   - Planar alphabet identified (58 letters, 3 square-roots), canonical DEs derived
   - Integrate DEs numerically using generalised series expansions [Moriello (2019)]

2. [Canko, Papadopoulos, Syrrakos (2020)] talks by Costas, Nikolaos
   - Construct Simplified Differential Equations (SDEs) using known canonical basis
   - Analytic solutions in term of Goncharov PolyLogarithms (GPLs)

⇒ analytically reconstructing MI coefficients is still too complicated
⇒ GPLs not linearly independent: no analytic pole cancellations

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A basis of special functions

- use the components of the $\epsilon$-expansion of the MIs as special functions

$$MI_i(s) = \sum_{w \geq 0} \epsilon^w MI_i^{(w)}(s)$$

- starting from canonical DEs [Abreu, et al. (2020)] write MIs in terms of Chen’s iterated integrals [Chen (1977)] for example:

$$MI_i^{(2)} = [w_1, w_2]_{s_0} + [w_1, w_3]_{s_0} + \cdots + t_{c_j}^{(2)}(s_0)$$

where

$$[w_1, \ldots, w_n]_{s_0}(s) = \int_{\gamma} d \log w_i(s') [w_i, \ldots, w_{i-1}]_{s_0}(s')$$

- Use GPL expressions [Canko, et al. (2020)] [Syrrakos (2020)] + PSLQ algorithm to prepare the boundary values

- Shuffle algebra to remove products of lower-weight functions + linear algebra to extract linearly independent functions talk by Vasily

$$\left\{ MI_i^{(w)}(s) \right\} \implies \left\{ f_i^{(w)}(s) \right\}$$
Reconstructing the finite remainders

\[ M_k^{(2)}(\{p\}, \epsilon) = \sum_i c_{k,i}(\{p\}, \epsilon) \mathcal{I}_{k,i}(\{p\}, \epsilon) \]

\[ \downarrow \text{IBP reduction} \]

\[ M_k^{(2)}(\{p\}, \epsilon) = \sum_i d_{k,i}(\{p\}, \epsilon) \mathcal{M}_{k,i}(\{p\}, \epsilon) \]

\[ \downarrow \text{map to special function basis} \]

\[ \downarrow \text{subtract UV/IR poles} \]

\[ \downarrow \epsilon \text{ expansion} \]

\[ F_k^{(2)}(\{p\}) = \sum_i e_{k,i}(\{p\}) m_{k,i}(\{f\}) + \mathcal{O}(\epsilon) \]

\[ k \in \{\text{even, odd}\} \]

- **Finite remainders**

\[ F_k^{(2)} = M_k^{(2)} - \sum_{j=1}^{2} I^{(j)} M_k^{(2-j)} \]

\[ I^{(L)} \rightarrow L\text{-loop universal UV/IR poles} \]

- **Numerically compute** \( e_{k,i} \) **over finite fields**

- **Analytic pole cancellation**

- **Drop in polynomial complexities for** \( e_{k,i} \)

- **Reconstruct analytic expressions of** \( e_{k,i} \) **from several numerical evaluations** [Peraro(2016)]
Reconstructing the finite remainders

\[ F^{(2)}_k(p) = \sum_i e_{k,i}(p) m_{k,i}(f) + \mathcal{O}(\epsilon), \quad k \in \{\text{even, odd}\} \]

- set \( s_{12} = 1 \)
- Not all \( e_{k,i} \) coefficients independent
  \( \Rightarrow \) find linear relations between coefficients and reconstruct the simpler ones
  \[ \sum_i y_i e_i = 0, \quad y_i \in \mathbb{Q} \]
- \( \Rightarrow \) allow to supply known/candidate coefficients \( \tilde{e}_j \)
  \[ \sum_i y_i e_i + \sum_j \tilde{y}_j \tilde{e}_j = 0, \quad y_i, \tilde{y}_j \in \mathbb{Q} \]
- guess the denominator \( \rightarrow \) from letters [Abreu, et al.(2019), Abreu, et al.(2020)]
- partial fraction in one variable (\( s_{23} \)) and reconstruct in the remaining variables (\( s_{34}, s_{45}, s_{15}, s_5 \))
  \( \Rightarrow \sim 4 \) times speed up \( \Rightarrow 2 \) prime fields needed
- Reconstructed analytic expressions are simplified using \textsc{MultivariateApart} [Heller, von Manteuffel(2021)]
Numerical evaluation

- Only 19 linear combinations of $f_i^{(4)}$ appear in the two-loop finite remainder
  
  $\Rightarrow$ define a new basis $g_i^{(w)}$

  \[
  \left\{ f_i^{(w)}(s) \right\} \mapsto \left\{ g_i^{(w)}(s) \right\}
  \]

- Apply generalised series expansion method directly to the $g_i^{(w)}$ basis

  \[
  \vec{g} = \begin{pmatrix}
  \epsilon^4 g_i^{(4)} \\
  \epsilon^3 g_i^{(3)} \\
  \epsilon^2 g_i^{(2)} \\
  \epsilon g_i^{(1)} \\
  1
  \end{pmatrix}
  \quad \Rightarrow \quad d\vec{g} = \epsilon d\vec{B} \cdot \vec{g}
  \]

- Much simpler than the DEs for the master integrals

- Use generalised series expansion approach [Moriello(2019)]
  as implemented in DIFFEXP [Hidding(2020)] talk by Martijn
Numerical evaluation

Evaluation on a univariate slice of the physical phase space

\[ p_1 = \frac{\sqrt{5}}{2} (1, 0, 0, 1), \]
\[ p_2 = \frac{\sqrt{5}}{2} (1, 0, 0, -1), \]
\[ p_3 = \frac{x_1\sqrt{5}}{2} (1, 1, 0, 0), \]
\[ p_4 = \frac{x_2\sqrt{3}}{2} (1, \cos \theta, -\sin \phi \sin \theta, -\cos \phi \sin \theta), \]
\[ p_5 = \sqrt{5} (1, 0, 0, 0) - p_3 - p_4 \]

\[ s = 1, \ m_W^2 = 0.1, \ \phi = 0.1, \ x_1 = 0.6 \]

- 1100 points → average 260 s/point
- Reasonable evaluation time with basic DIFFEXP setup
- further optimisation is possible

[Abreu,etal(2020)][Becchetti,etal(2020)]
Summary

- First analytic result for 2-loop 5-point amplitude with one massive leg
  \[ u \bar{d} \rightarrow W^+ b \bar{b} \]
- Basis of special functions for leading colour 5-particle amplitudes with 1 off-shell leg up to 2 loops
  - Include \( W \)-boson decay
  - Application to other processes
  - Full colour (need non-planar integrals)
Summary

✓ First analytic result for 2-loop 5-point amplitude with one massive leg
  ⇒ leading colour $u\bar{d} \rightarrow W^+ b\bar{b}$

✓ Basis of special functions for leading colour 5-particle amplitudes with 1 off-shell leg up to 2 loops

✗ Include $W$-boson decay

✗ Application to other processes

✗ Full colour (need non-planar integrals)

THANK YOU!!!