Integrated Structure and Semantics for Reo Connectors and Petri Nets

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In this paper, we present an integrated structural and behavioral model of Reo connectors and Petri nets, allowing a direct comparison of the two concurrency models. For this purpose, we introduce a notion of connectors which consist of a number of interconnected, user-defined primitives with fixed behavior. While the structure of connectors resembles hypergraphs, their semantics is given in terms of so-called port automata. We define both models in a categorical setting where composition operations can be elegantly defined and integrated. Specifically, we formalize structural gluings of connectors as pushouts, and joins of port automata as pullbacks. We then define a semantical functor from the connector to the port automata category which preserves this composition. We further show how to encode Reo connectors and Petri nets into this model and indicate applications to dynamic reconfigurations modeled using double pushout graph transformation.

1 Introduction

Reo [1] is a channel-based coordination language which has its main application area in component and service composition. The idea in Reo is to construct complex, so-called connectors out of a set of user-defined primitives, most commonly channels. Among a number of sophisticated features, such as mobility [11], context-dependency [6, 5] and dynamic reconfigurability [14], on a more basic level Reo can be seen also as a model of concurrency. Comparing Reo with Petri nets, the first obvious commonality is the fact that they both use a graph-based model, i.e. their structure can be modeled using typed graphs. Moreover, both models combine control-flow and data-flow aspects. In this paper, we are particularly interested in the concurrency properties of the two models, i.e. parallel or synchronized actions vs. interleaved or mutually excluded actions. To understand the relationship between Reo connectors and Petri nets, we follow an approach in this paper where we map both models to so-called port automata [13], which serve as our common semantical domain. We can thereby gain an integrated view on structure and semantics of Reo connectors and Petri nets and moreover compare both models.

As a motivating example, Fig. 1 depicts a Reo connector, a Petri net and a port automaton, all modeling the same simple protocol. If considering the initial state also as final, the accepted language is \((AB + CD)^+\). Port automata model explicitly synchronization of actions. This is witnessed by the fact that the transitions in the automaton are sets of truly concurrent actions. Such a port automaton transition corresponds to a concurrent firing of transitions in a Petri net, or a synchronized activity on nodes in a Reo connector. This is our starting point for using port automata as a common semantical models for the structural models of Reo connectors and Petri nets. Our general idea is to compose – potentially user-defined – primitives into a graph-structure which we will refer to as connector. While in Reo, these primitives are communication channels, in Petri nets we consider places as primitives. Moreover our

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approach emphasizes compositionality, i.e. the port automata semantics of primitives is predefined, but the semantics of connectors is derived using a join-operation.

We make the following contributions in this paper. We present a model of connectors which combines structural and behavioral descriptions. The structure of connectors resembles hypergraphs and their semantics is defined using port automata. Our most important result is the compositionality of the model in the following sense: a structural gluing of connectors corresponds to a join of the corresponding port automata. For this purpose, we define the categories \texttt{Connector} and \texttt{PortAut}, and a contravariant functor \( \text{Sem} : \text{Connector} \to \text{PortAut}^{\text{op}} \). In categorical terms, our compositionality result means that this functor sends pushouts of connectors to pullbacks of the corresponding port automata, i.e. for connectors \( C_0, C_1 \) and \( C_2 \):

\[
\text{Sem}(C_1 + C_0 C_2) = \text{Sem}(C_1) \times \text{Sem}(C_0) \text{Sem}(C_2).
\]

Furthermore we show how Reo connectors and Petri nets can be modeled directly in our framework. While for Petri nets, compositionality results similar to ours exist already, this paper constitutes the first formal integration of the graph structure and the automata semantics of Reo connectors. Further it is a starting point for synthesis algorithms and in particular for semantics of graph transformation based reconfigurations. Specifically, our composition operation fits into the double pushout approach [8, 10] for graph transformation, which has been used for instance to model reconfigurations of Reo connectors in [14], and of Petri nets in [15].

\textbf{Organization.} The rest of the paper is organized as follows. We start with the semantical model by introducing port automata in Section 2. Based on this, we then define our notion of connectors in Section 3. Section 4 contains our main compositionality result and Section 5 shows how Reo connectors and Petri nets can be encoded in our connector model. Finally, Section 6 contains a discussion and future work, and Section 7 includes related work.

\section{Port automata}

Port automata are an operational model for connectors and have been mainly studied in the context of Reo. They are an abstraction of so-called constraint automata [3] which is the quasi-standard semantics of Reo. Port automata describe the synchronization on sets of ports, depending on the internal state of the connector. The model abstracts from both the direction and content of data flow. For a proper modeling of data we refer to the constraint automata model.
In this paper, we present port automata in a categorical setting, i.e., we consider them as objects in a category which we will denote with \( \text{PortAut} \). We now give the definition for port automata.

**Definition 1** (Port automaton). A port automaton \( A = (Q, N, \rightarrow, i) \) consists of a set of states \( Q \), a set of port names \( N \), a transition relation \( \rightarrow \subseteq Q \times 2^N \times Q \) and an initial state \( i \in Q \).

We denote transitions often as \( q \xrightarrow{S} p \) with \( q, p \in Q \) and \( S \subseteq N \). The interpretation is that there is concurrent activity at the ports \( S \) and no activity at the rest of the ports \( N \setminus S \). The model permits \( \tau \)-transitions, namely whenever \( S = \emptyset \). Hence, there can be silent steps without any action. In the following we define a notion of port automata morphism.

**Definition 2** (Port automata morphism). A morphism of port automata \( f : A_1 \rightarrow A_2 \) is a pair of functions \( f = (f_Q, f_N) \) with \( f_Q : Q_1 \rightarrow Q_2 \) and \( f_N : N_2 \rightarrow N_1 \), such that: \( f_Q(i_1) = i_2 \) and for all transitions \( q \xrightarrow{S_1} p \) in \( A_1 \) there exists a transition \( f_Q(q) \xrightarrow{S_2} f_Q(p) \) in \( A_2 \) with
\[
f_N(N_2) \cap S_1 = f_N(S_2). \tag{1}
\]

Port automata morphisms can be seen as a kind of simulation. The definition uses a function for relating the states of the automata instead of a relation, which one might expect for a simulation of automata. However, in our categorical context, especially when mapping connector morphisms to (inverse) simulations, this definition is sufficient and easier to handle. Note further that the port names are mapped in the opposite direction and that condition (1) defines \( S_2 \) as the restricted preimage of \( S_1 \). The following example illustrates this notion of automata morphisms.

**Example 1.** An example of a port automata morphism is depicted in Fig. 2. States \( q_0, q_2 \) are both mapped to \( p_0 \), and \( q_1 \) is mapped to \( p_1 \). The port names function is the inclusion map in the opposite direction. The transition via \( \{B, C\} \) in the source corresponds to the transition via \( \{B\} \), and \( \{C\} \) to the \( \tau \)-step in the target automaton.

![Figure 2: A morphism of port automata.](image)

Note that if the port name map \( f_N \) is the identity, a morphism also gives rise to a language inclusion. If there is no confusion, we abuse notation and write \( f \) for both \( f_Q \) and \( f_N \). If there is a morphism between two port automata \( A_0 \) and \( A_1 \), we may also write \( A_0 \succeq A_1 \) for short. Similarly, if there exists a (categorical) isomorphism, we denote this by \( A_0 \cong A_1 \). Note that this notion of behavioral equivalence is stronger than usual definitions, e.g. using bisimulations.

Composition and identity of port automata morphisms are defined componentwise in \( \text{Set} \). The resulting category of port automata is denoted by \( \text{PortAut} \). The port automaton with one state, an empty port names set and a \( \tau \)-transition is the final object in this category, denoted by \( 1 \). At this point, we already make use of our categorical setting and define composition of port automata using pullbacks.
Theorem 1 (Pullbacks of port automata). The category \textbf{PortAut} has pullbacks and they can be constructed componentwise in \textbf{Set}. For a cospan \( A_1 \rightarrow A_0 \leftarrow A_2 \), the pullback object is \( A_3 = (Q_3, N_3, \rightarrow_3, i_3) \) where

- \( Q_3 = Q_1 \times_{Q_0} Q_2 \) (pullback in \textbf{Set})
- \( N_3 = N_1 + N_0 \times N_2 \) (pushout in \textbf{Set})
- \( i_3 = \langle i_1, i_2 \rangle \)
- if \( q_1 \xrightarrow{S_1} p_1 \) and \( q_2 \xrightarrow{S_2} p_2 \), such that
  \[
  g_1(S_1) \cap g_2(N_2) = g_2(S_2) \cap g_1(N_1)
  \]
  then \( \langle q_1, q_2 \rangle \xrightarrow{g_1(S_1) \cup g_2(S_2)} \langle p_1, p_2 \rangle \) in \( A_3 \).

Proof sketch. It is sufficient to show that the componentwise construction of \( g_1, g_2 \) and \( h \) yields valid morphisms of port automata, i.e. that condition (1) holds. A detailed proof is given in the appendix. \( \square \)

Example 2. An example of a port automata pullback is depicted in Fig. 3. The state maps are indicated by the indices, e.g. \( p_0 \) is mapped to \( q_0 \) and \( p_1, p'_1, p_2 \) are all mapped to \( q_{12} \). The resulting automaton on the bottom right is the automaton from our previous example in Fig. 1. Note that it actually includes more states which are not shown here because they are unreachable.

We use the default notation for pullbacks of port automata, i.e. \( A_3 = A_1 \times_{A_0} A_2 \). This notion of composition generalizes the join-operation in [13] for port automata and in [3] for constraint automata since it allows a composition along a common interface automaton. In the traditional approaches, automata are joined only along a common set of port names. Moreover, the categorical construction using pullbacks includes the morphisms into the original automata and thereby relates them with the result using simu-
A and the port name map must be compatible with the nodes map. It is worth mentioning at this point, that
get connector. Moreover, for all mapped primitives there must exist simulations in the opposite direction,

\textbf{Definition 3 (Connector).} A connector \( \mathcal{C} = (\mathcal{A}, \mathcal{N}) \) consists of a set of port automata \( \mathcal{A} \) and a set of nodes \( \mathcal{N} \), such that \( \mathcal{N} \subseteq \mathcal{N} \) for all \( A = (Q, N, \rightarrow, i) \in \mathcal{A} \).

Port names can now be interpreted as nodes and the port automata as edges in a hypergraph. We will refer to the port automata in a connector as \textit{primitives}. As mentioned already, the idea is to construct arbitrarily complex connectors out of a fixed class of primitives, e.g. the set of standard channels in Reo.

\textbf{Definition 4 (Connector morphism).} A connector morphism \( f : \mathcal{C}_1 \rightarrow \mathcal{C}_2 \) is a pair of functions \( f = (f_{\mathcal{A}}, f_{\mathcal{N}}) \) such that for all \( A = (Q, N, \rightarrow, i) \in \mathcal{A}_1 \) there exists a port automata morphism \( f_A : f_{\mathcal{A}}(A) \rightarrow A \) with \( f_N(A) = f_{\mathcal{N}}(N) \).

A connector morphism consists of a map of nodes and a map of primitives from the source to the target connector. Moreover, for all mapped primitives there must exist simulations in the opposite direction, and the port name map must be compatible with the nodes map. It is worth mentioning at this point, that
the existence of an inverse simulation has the consequence that a primitive can in principle be mapped to
primitive with potentially different interface (port name sets) and behavior (port automaton itself). Due
to this property, connector morphisms permit a refinement of primitives.

Composition and identity of connector morphisms are again defined componentwise in \( \text{Set} \). We
denote the category of connectors and their morphisms as \( \text{Connector} \). We use pushouts to compose
connectors. This makes the approach particularly interesting for applying algebraic graph transformation
techniques for modeling reconfigurations (cf. \[14, 12\]).

**Theorem 3** (Pushouts of connectors). The category \( \text{Connector} \) has pushouts. For a span of connectors
\( \mathcal{C}_1 \leftarrow \mathcal{C}_0 \rightarrow \mathcal{C}_2 \) the pushout object is given by \( \mathcal{C}_3 = (\mathcal{A}_3, \mathcal{N}_3) \) with

\[
\begin{align*}
\mathcal{C}_0 & \xrightarrow{f_1} \mathcal{C}_1 \\
\mathcal{C}_1 & \xrightarrow{g_1} \mathcal{C}_2 \\
\mathcal{C}_2 & \xrightarrow{h_1} \mathcal{C}_3
\end{align*}
\]

- \( \mathcal{A}_3 = \mathcal{A}_1 \cup_{\mathcal{A}_0} \mathcal{A}_2 \) (pushout in \( \text{Set} \))
- \( \mathcal{N}_3 = \mathcal{N}_1 \cup_{\mathcal{N}_0} \mathcal{N}_2 \) (pushout in \( \text{Set} \))
- for all \( A_0 \in \mathcal{A}_0 \), \( A_1 = f_1(A_0) \) and \( A_2 = f_2(A_0) \):
  \[
  A_3 = A_1 \times_{\mathcal{A}_0} A_2 \in \mathcal{A}_3 \quad \text{(pullback in PortAut)}
  \]
- for all \( A_j \in \mathcal{A}_j \setminus f_j(\mathcal{A}_0), \ j \in \{1, 2\} : \)
  \[
  A_3 = A_j \in \mathcal{A}_3
  \]

**Proof.** Due to the componentwise construction in \( \text{Set} \) and \( \text{PortAut} \) again we have to show only that the
construction yields a valid connector \( \mathcal{C}_3 \) and valid connector morphisms \( g_1, g_2 \) and \( h \). The connector \( \mathcal{C}_3 \)
is valid since

\[
N_3 = N_1 +_{\mathcal{N}_0} N_2 \subseteq N_1 +_{\mathcal{N}_0} N_2 = \mathcal{N}_3
\]
in case (3) and \( N_3 = N_j \subseteq \mathcal{N}_j = \mathcal{N}_3 \) in case (4). Moreover, for every \( A \in \mathcal{A}_j \) there exists a port automata
morphism \( g_A : g_j(A) \rightarrow A \). In case (4) it is the identity and in (3) it is the projection of the pullback.
Since the port name maps in \( g_A \) and \( g_j \) are both constructed as the injections into \( N_3 \) and \( \mathcal{N}_3 \) respectively,
\( g_A(N) = g_j(N) \) holds as well. Hence, \( g_1, g_2 \) are valid connector morphisms. Validity of \( h \) can be shown
analogously.

**Example 3.** A pushout of Reo connectors in purely structural notation is depicted in Fig. 4. In this
notation, nodes (which correspond to port names) are depicted as filled circles. The port automata
semantics for the different channel types are given in Fig. 5. So-called FIFO channels are asynchronous
channels with a buffer of size one. They are represented as arrows with a rectangle in the middle. There
are in fact two versions of this channel type: with and without a token, respectively called FullFIFO
and EmptyFIFO. Circles with a cross denote two dual primitives: the Router (left of the FullFIFO)
and the Merger (right of it). Both have in total three ports and the same semantics (cf. Fig. 5 for their
port automata semantics).

Note that we abused notation in this example, in the sense that there are two hidden nodes between
the Router, the FullFIFO and the Merger. The resulting connector on the bottom right is the initial
example from Fig. 1. It consists in total of five primitives (three FIFOs, one Merger and one Router)
and six nodes (A-D, plus two hidden ones).
4 Compositional Semantics

In this section we show how to compute the port automaton for a connector using its primitives’ semantics. We extend this mapping to a functor and show compositionality.

Remark 1. In the following definitions we use the fact that the product is associative and commutative, i.e. there exist natural isomorphisms $(A \times B) \times C \cong A \times (B \times C)$ and $A \times B \cong B \times A$.

Definition 5 (Connector semantics). Given a connector $\mathcal{C} = (\mathcal{A}, \mathcal{N})$ with $\mathcal{A} = \{A_1, \ldots, A_n\}$ and $A_j = (Q_j, N_j, \rightarrow_j, i_j)$ for $j \in \{1, \ldots, n\}$. We define

$$\text{Sem}(\mathcal{C}) = (Q_1 \times \ldots \times Q_n, N, \rightarrow, \langle i_1, \ldots, i_n \rangle)$$

where $\rightarrow$ is given by:

$$\forall j, k \in \{1, \ldots, n\}: q_j \xrightarrow{S_j} p_j, \quad q_k \xrightarrow{S_k} p_k, \quad S_j \cap N_k = S_k \cap N_j$$

(5)

Note that the nodes of the connector become the port names of the resulting port automaton. The definition further implies that all actions on a (shared) node are synchronized. This corresponds to so-called Hoare-style synchronizations, as opposed to Milner-style synchronizations where exactly one input end is synchronized with one output end. We extend now the given connector semantics to a functor $\mathcal{F} \text{em} : \mathcal{C} \rightarrow \mathcal{P} \text{ortAut}$.

Theorem 4 (Semantics functor). Let $f = (f_{\mathcal{A}}, f_{\mathcal{N}}) : \mathcal{C}_1 \rightarrow \mathcal{C}_2$ a connector morphism, $\mathcal{F} \text{em}(\mathcal{C}_1) = (\mathcal{D}_1, \mathcal{N}_1, \rightarrow_1, i_1)$ and $\mathcal{F} \text{em}(\mathcal{C}_2) = (\mathcal{D}_2, \mathcal{N}_2, \rightarrow_2, i_2)$ where $\mathcal{C}_1 = (\mathcal{A}_1, \mathcal{N}_1)$ with $\mathcal{A}_1 = (A_1, \ldots, A_n)$ and $f_{A_j} = (f_{Q_j}, f_{N_j}) : f_{\mathcal{A}}(A_j) \rightarrow A_j$. Let

$$f_{\mathcal{D}} = f_{Q_1} \times \ldots \times f_{Q_n} : Q \rightarrow \mathcal{D}_1,$$

then there also exists a projection $\pi_Q : \mathcal{D}_2 \rightarrow Q$. Defining $\mathcal{F} \text{em}(f) : \mathcal{F} \text{em}(\mathcal{C}_2) \rightarrow \mathcal{F} \text{em}(\mathcal{C}_1)$ as $\mathcal{F} \text{em}(f) = (f_{\mathcal{D}} \circ \pi_Q, f_{\mathcal{N}})$ gives rise to a contravariant functor $\mathcal{F} \text{em} : \mathcal{C} \rightarrow \mathcal{P} \text{ortAut}^\text{op}$.
Proof. $Q$ is the product of the state sets of those primitives in $\mathcal{C}_2$ that are in the image of $f_\sigma$, and $\pi_Q$ the projection to the product of the state sets of these reached primitives. The states map of $\mathcal{I}em(f)$ preserves the transitions of $\mathcal{I}em(\mathcal{C}_2)$, since both $\pi_Q$ and $f_\sigma$ do. Hence, $\mathcal{I}em(f)$ is a valid port automata morphism. For showing that $\mathcal{I}em$ is a functor we observe that composition is preserved: $\mathcal{I}em(g \circ f) = \mathcal{I}em((g_\sigma, g_\pi) \circ (f_\sigma, f_\pi)) = \mathcal{I}em(g_\sigma \circ f_\sigma, g_\pi \circ f_\pi) = (f_\sigma \circ g_\sigma, f_\pi \circ g_\pi) = (f_\sigma \circ f_\pi) \circ (g_\sigma, g_\pi)$. □

This result in particular shows that a (structural) morphism of connectors corresponds to an inverse simulation on the semantical level. We now phrase our main result, i.e., the compositionality of the port automata semantics for connectors.

**Theorem 5** (Compositionality of semantics). The functor $\mathcal{I}em$ maps pushouts of connectors to pullbacks of port automata, i.e.

$$\mathcal{I}em(\mathcal{C}_1 +_{\mathcal{C}_0} \mathcal{C}_2) = \mathcal{I}em(\mathcal{C}_1) \times \mathcal{I}em(\mathcal{C}_0) \mathcal{I}em(\mathcal{C}_2).$$

Proof. Both in $\textbf{Connector}$ and $\textbf{PortAut}$ the port name sets are composed using pushouts in $\textbf{Set}$, and $\mathcal{I}em$ preserves these sets. Hence, the port names are correctly mapped. The primitives in the connector pushout $\mathcal{C}_3 = \mathcal{C}_1 +_{\mathcal{C}_0} \mathcal{C}_2$ are either of the form $A_1 \times_{A_0} A_2$ (case (3)) or $A_j = A_1 \times_{A_1} 1$ with $j \in \{1, 2\}$ (case (4)). The primitives’ state sets are of the same form, i.e. they can be all written as pullbacks. $\mathcal{I}em$ sends these state sets to their product. Now, since

$$(X \times Y) \times (X' \times Y', Z') = (X \times X') \times (Y \times Y') \times (Z \times Z')$$

the state set of the resulting automaton $A_3 = \mathcal{I}em(\mathcal{C}_3)$ is of the form $Q_1 \times Q_0 Q_2$ where $Q_1$ is the state set of $\mathcal{I}em(\mathcal{C}_1)$, and $Q_2$ of $\mathcal{I}em(\mathcal{C}_2)$ and $Q_0$ of $\mathcal{I}em(\mathcal{C}_0)$. Hence, the (initial) states are also correctly mapped. Moreover, the transition structure is preserved, since (5) implies (2) and the port name sets on transitions are in both cases composed by taking their union. □

## 5 Applications

In this section, we show how Reo connectors and Petri nets are modeled by our notion of connectors. This enables us also to do a direct comparison of the two models.

### 5.1 Modeling Reo connectors

Reo connectors are directly modeled by our notion of connectors. The primitives used in this paper are summarized in Fig. 5. It includes in particular the channel types $\text{Sync}$, $\text{EmptyFIFO}$ and $\text{FullFIFO}$. Note also that all primitives explicitly include $\tau$-steps to allow interleavings, i.e. other parts of the connector can fire independently without (observable) activity of such a primitive.

While channels are user-defined entities, Reo defines a fixed semantics for nodes. A node in Reo merges input from all target ends and replicates it to all source ends. This can be seen as a $1:n$ synchronization, as opposed the Hoare-style synchronizations in our framework, where basically all coinciding channel ends (no matter if source or target) are synchronized. As a consequence we have to model the merging explicitly using a primitive. The $\text{Merger}$, denoted by a circle with a cross in Fig. 1, is used for this purpose. It has two source and one target end and is therefore not a channel. We also define the dual of this primitive, called $\text{Router}$. It has the same semantics but its ends are inverse. Note again,
that in the example of Fig. 5 and 4 there are two hidden nodes between the \textit{Router}, the \textit{FullFIFO} and the \textit{Merger}, which are not relevant here.

The pushout diagram in Fig. 4 shows a gluing of two Reo connectors along a common subconnector. Note that this gluing is of a purely structural nature, although – in principle – it could also include a refinement of primitives, i.e., if one of the primitive simulations is not an isomorphism. The port automata corresponding to the connectors in this example are depicted in Fig. 3. As we have shown in Theorem 5, they form a pullback. Note again, that we omitted unreachable states in the result.

### 5.2 Modeling Petri nets

Petri nets can also be modeled directly with our connector notion. As illustrated in the motivating example in Fig. 1, transitions in a Petri net should be interpreted as nodes in this setting. Hence, places become the primitives in the connector model. They are basically unbounded buffers without ordering constraints (as opposed to the \textit{FIFO} channel in Reo).

Formally, the port automaton \( A_p = (Q, N, \to, i) \) for a place \( p \) with \( in(p) \) and \( out(p) \) respectively the sets of incoming and outgoing transitions\(^1\) of \( p \) is defined in the following way:

- \( Q \) is the set of all markings of \( p \), e.g. the natural numbers, or a finite set for places with capacities.
- \( m \xrightarrow{T} m' \) whenever a concurrent firing of the transitions \( T \subseteq in(p) \cup out(p) \) turns the marking \( m \) into \( m' \).
- \( i \in Q \) the initial marking of \( p \).

This encoding works because the transitions in a Petri net also do a basic Hoare-style synchronization. Without giving a proof, we claim that the port automaton \( \mathcal{S}em(N) \) of a Petri net \( N \) correctly models its behavior, in the sense that it has the set of all possible markings of the net as states and transitions that correspond to a concurrent firing of net transitions.

Our notion of connector morphisms requires that the ports of primitives are preserved. Since we interpret places as primitives (and transitions as nodes) our connectors correspond to the following Petri net model:

\[
N = P \xrightarrow{in} \xleftarrow{out} T^\oplus.
\]

Note that in the literature (see e.g. [15]) one often finds a similar but different model of Petri nets, where instead of the maps \( in, out \) functions \( pre, post : T \rightarrow P^\oplus \) are used. However, this only modifies the notion of net morphism, but not the Petri net model itself. A comparison of the two types of Petri net morphisms is out of the scope of this paper.

\(^1\)This can be seen as the dual of the usual notion of \textit{pre}- and \textit{post}-sets of Petri net transitions.
5.3 Comparing Reo connectors and Petri nets

As made evident in this paper, one can compare the basic version of Reo nodes, which does only a primitive synchronization, with the transition concept in Petri nets. On the other hand, primitives in our framework correspond to the places in a Petri net and the channels, mergers, routers etc. in a Reo connector. Reo is more expressive in the sense that it allows unbuffered primitives, such as synchronous communication channels. In Petri nets, the primitives, i.e., the places are always buffered. From our point of view, this is the most important difference between Reo connectors and Petri nets: while in Petri nets synchronizations happen always locally at transitions, in Reo synchronous primitives can be used to propagate synchrony through the connector. Other features of Reo, such as context-dependency and priority go beyond the focus of this paper.

5.4 Modeling Reconfigurations

Graph rewriting techniques, such as the double pushout (DPO) approach [8], are a powerful tool for modeling rule-based reconfigurations. As a motivating example we return to the pushout of Reo connectors in Fig. 4. This diagram can be interpreted as a reconfiguration in the following sense. The upper two connectors together with the morphism between them is interpreted as a (structural) reconfiguration rule. An application of this rule creates a new EmptyFIFO between the nodes A and B. The bottom two connectors can be regarded as an application of this rule. In the bottom left is the connector before, and in the bottom right after the rule application.

In our approach we can now perform this reconfiguration directly on the corresponding automata. This becomes particularly interesting when executing such a Reo connector as a state machine and reconfiguring it at runtime. A prototypic implementation of this approach exists already. A typical question in such scenarios is what the state of the connector after a reconfiguration is, and whether it is actually valid. We can make this clear in the corresponding automata pullback in Fig. 3. For instance, if the connector before the reconfiguration is in state $r_{1}$, we can see that in the connector after the reconfiguration the state $s_{1}$ is mapped to $r_{1}$ by the constructed morphism. Thus, we can use this morphism to identify the state after a reconfiguration step. However, we can see also in this example that $r_{2}'$ has a preimage in the target automaton that is an unreachable state. This indicates that a reconfiguration in this state produces an invalid system state. In our example first B and then C fired, and then the reconfiguration was performed. The problem here is essentially that at this point there are two tokens in the connector.

6 Conclusions and Future Work

We have presented an integrated structural and behavioral model of connectors and showed compositionality with respect to gluing constructions. We have then shown how Reo connectors and Petri nets can be modeled in this framework.

As future work, we would like to consider the traditional model of simulations for port automata morphisms, i.e. instead of functions we want to use a notion of upward-closed relations for relating the states. With this change, the model will cover a wider class of connector morphisms. Moreover, we are interested in further properties of the semantical functor.
7 Related work

Padberg et al. provide compositional semantics of Petri nets in [15]. Their results are based on pre/post-net morphisms and a marking graph semantics, and they cover a wider class of Petri nets. Moreover, the authors show preservation of general colimits, as opposed to our work were we consider only pushouts.

A wide range of automata semantics for Petri nets exist. For instance, Droste and Shortt consider so-called automata with concurrency relations in [9], which are more restrictive than port automata. Essentially, a concurrent firing of two net transitions always implies the existence of an interleaved execution of the two net-transitions (parallel independence). The authors show that there is a coreflection between the category of Petri nets and automata with concurrency. Pushouts or general colimits are not considered. They further also use a non-standard notion of net morphisms.

A compositional automata semantics for Reo, called constraint automata, is given by Baier et al. in [3]. Our port automata are an abstraction of constraint automata. The main difference is the used notion of compositionality. In [3], with compositionality the authors mean that the semantics of a connector can be computed out of the semantics of its constituent primitives. However, our notion of compositionality really combines the structural level with the semantical, in the sense that we show how a gluing of connectors corresponds to a join operation of their behaviors. In particular, we generalize the join operation of [3] by allowing to join two automata along a common interface automaton.

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A Proofs

Theorem 1. Let \( j \in \{1, 2\} \). The pullback morphisms \( g_j = (g_{j,Q}, g_{j,N}) \) consist of the projections \( g_{j,Q} : Q_3 \rightarrow Q_j \) and the injections \( g_{j,N} : N_j \rightarrow N_3 \). We will denote both of them with \( g_j \) if there is no confusion. We have to show first that \( g_1 \) and \( g_2 \) are in fact \textit{PortAut}-morphisms. Condition (1) in the morphism definition reads for \( g_j \):

\[
g_j(N_j) \cap (g_1(S_1) \cup g_2(S_2)) = g_j(S_j)
\]

We show this here only for \( j = 1 \), since the other case is analogously:

\[
g_1(N_1) \cap (g_1(S_1) \cup g_2(S_2)) = (g_1(N_1) \cap g_1(S_1)) \cup (g_1(N_1) \cap g_2(S_2)) = g_1(S_1) \cup (g_2(N_2) \cap g_1(S_1)) = g_1(S_1) \cap (g_1(S_1) \cap g_2(S_2)) = g_1(S_1)
\]

Now, the arrow \( h : A_3 \rightarrow X \) exists and is unique due to the componentwise construction in \textit{Set}. What is left to show is that \( h \) is also a valid \textit{PortAut}-morphism. We know for all \( q \xrightarrow{N} p \) in \( X \) there exist transitions \( h_j(q) \xrightarrow{S_j} h_j(p) \) in \( A_j \) with

\[
h_j(N_j) \cap N = h_j(S_j)
\]

since the \( h_j \) are by assumption valid morphisms. Moreover we know that \( h \) maps a state \( q \in Q_X \) in the automaton \( X \) to the state \( h(q) = \langle h_1(q), h_2(q) \rangle \) in the pushout object \( A_3 \). Now we have to show that there exists a transition \( h(q) \xrightarrow{S_3} h(p) \) in \( A_3 \) with

\[
h(N_3) \cap N = h(S_3).
\]
We construct $S_3$ in the following way:

$$
\begin{align*}
   h(N_3) \cap N & = h(g_1(N_1) \cup g_2(N_2)) \cap N \\
   & = (h \circ g_1(N_1) \cup h \circ g_2(N_2)) \cap N \\
   & = (h_1(N_1) \cup h_2(N_2)) \cap N \\
   & = (h_1(N_1) \cap N) \cup (h_2(N_2) \cap N) \\
   & = h_1(S_1) \cup h_2(S_2) \\
   & = h \circ g_1(S_1) \cup h \circ g_2(S_2) \\
   & = h(g_1(S_1) \cup g_2(S_2))
\end{align*}
$$

We have constructed $S_3 = g_1(S_1) \cup g_2(S_2)$ and it fulfills the required property. The last step is to show that this transition in fact exists in $A_3$, which means that (2) holds. Recall that $g_1$ and $g_2$ are valid morphisms:

$$g_1(N_1) \cap S_3 = g_1(S_1) \quad \text{and} \quad g_2(N_2) \cap S_3 = g_2(S_2).$$

We can follow that:

- $(g_1(N_1) \cap S_3) \cup g_2(S_2) = g_1(S_1) \cup g_2(S_2)$
- $(g_2(N_2) \cap S_3) \cup g_1(S_1) = g_2(S_2) \cup g_1(S_1)$

and unify both equations:

$$
\begin{align*}
   (g_1(N_1) \cap S_3) \cup g_2(S_2) & = (g_2(N_2) \cap S_3) \cup g_1(S_1) \\
   (g_1(N_1) \cap (g_1(S_1) \cup g_2(S_2))) \cup g_2(S_2) & = (g_2(N_2) \cap (g_1(S_1) \cup g_2(S_2))) \cup g_1(S_1) \\
   (g_1(N_1) \cup g_2(S_2)) \cap (g_1(S_1) \cup g_2(S_2)) & = (g_2(N_2) \cup g_1(S_1)) \cap (g_1(S_1) \cup g_2(S_2)) \\
   (g_1(N_1) \cup g_1(S_1)) \cap g_2(S_2) & = (g_2(N_2) \cup g_2(S_2)) \cap g_1(S_1) \\
   g_1(N_1) \cap g_2(S_2) & = g_2(N_2) \cap g_1(S_1)
\end{align*}
$$

and we have shown (2).