Even-odd effect in the thermopower and strongly enhanced thermoelectric efficiency for superconducting single-electron transistors

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Abstract

It is well-known that the transport properties of single-electron transistors with a superconducting island and normal-conducting leads (NSN SET) may depend on whether or not there is a single quasiparticle on the island. This parity effect has pronounced consequences for the linear transport properties. Here we analyze the thermopower of NSN SET with and without parity effect, for entirely realistic values of device parameters. Besides a marked dependence of the thermopower on the superconducting gap $\Delta$ we observe an enhancement in the parity regime which is accompanied by a dramatic increase of the thermoelectric figure of merit $ZT$. The latter can be explained within a simple re-interpretation of $ZT$ in terms of averages and variances of transport energies.

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I. INTRODUCTION

The properties of transport through small conducting islands have been investigated extensively during the past years. Electric current in such devices flows due to tunneling of single electrons and is subject to the so-called Coulomb blockade effect [1] which is characterized by a new energy scale, the capacitive charging energy $E_C$ of the island (see below). In the recent past, substantial attention has been devoted also to thermoelectric effects in single-electron devices [2–17]. Single-electron devices are interesting candidates for thermoelectric applications as it has long been known that dimensional reduction of the electron dynamics may lead to an enhanced thermoelectric efficiency [18, 19].

While an immense amount of work has been done to investigate thermopower for quantum dots, surprisingly little is known about the thermoelectric effects in single-electron transistors (SET) with superconducting electrodes. In particular, SET with superconducting islands are interesting as they may exhibit the parity effect where a single unpaired quasiparticle determines the macroscopic thermodynamic properties of the island electrode [20, 21] as well as the current-voltage characteristics of SET with normal-conducting electrodes and a superconducting island (NSN SET) [22–24]. The parity effect can be observed below a crossover temperature $T^* \approx \Delta/8$ for typical system parameters (here $\Delta$ is the energy gap of the superconductor). One may expect that the peculiar combination of properties like several competing energy scales ($E_C, \Delta, T^*$) and the presence of a singularity in the quasiparticle spectrum gives rise to interesting behavior in the thermoelectric response of such systems.

In Ref. [11] the thermopower for an NSN SET was studied for $\Delta < E_C$ and $T > T^*$. Even for this regime without parity effect, interesting oscillations of the thermopower as a function of the electrostatic island potential and their strong dependence on the ratio $\Delta/E_C$ were predicted. In this article, we investigate the thermopower of NSN SET for temperatures below the crossover temperature. We find that the interplay of energy scales, Coulomb-blockade and parity effects, and the peculiarity of the electronic spectrum lead to a rich variety of features in the thermopower $S$. Most intriguingly, however, for certain gate voltages this system displays a dramatic enhancement of the thermoelectric efficiency quantified by the figure of merit $ZT$.

This paper is organized as follows. First we introduce the setup and the theoretical
methods that are used to describe transport in such systems. Subsequently we briefly review the parity effect in SET with superconducting islands. We then turn to describe the results for the thermopower, and to interpret them in terms of average transport energies. Finally we discuss the surprising results for the figure of merit $ZT$ which we explain in the frame of a simple re-interpretation of this quantity.

II. THE TRANSISTOR SETUP AND MASTER EQUATION

In an NSN SET, a superconducting island with a small electrostatic capacitance $C$ is connected via tunnel junctions to two normal-conducting leads (cf. Fig. 1). The corresponding charging energy $E_C \equiv e^2/(2C) \gg k_BT$ is large compared to the temperatures under consideration (here, $e > 0$ denotes the elementary charge). The conductances of the tunnel junctions are assumed to be small compared to $e^2/h$, so sequential tunneling dominates and cotunneling effects may be neglected.

FIG. 1. The NSN SET consists of a superconducting island (S) which is coupled to two normal-conducting leads (N) via tunnel barriers. The electrostatic potential of the island can be controlled by the gate voltage $V_g$. There may flow a current through the system due to the bias voltage $V$ or a temperature difference $\Delta T = T_l - T_r$ between the two leads. In order to detect the thermopower $S = -V/\Delta T$ as a function of the gate voltage $V_g$ the bias $V$ is adjusted such that the corresponding current exactly cancels the current which arises due to the temperature difference.
The electrostatic potential of the island can be controlled by means of an external potential \( n_x \propto V_g \) due to the gate voltage \( V_g \), and the electrostatic energy of the setup with \( n \) excess electrons on the island can be expressed as

\[
E_n(n_x) = E_C(n^2 - 2nn_x) .
\]  

The energy cost for adding a single electron to the island is \( u_n(n_x) = E_{n+1}(n_x) - E_n(n_x) \) while to add two electrons, an energy \( E_{n+2}(n_x) - E_n(n_x) = u_{n+1}(n_x) + u_n(n_x) \) is required.

A current may flow in the device if a bias voltage \( \Delta V = V_r - V_l \) and/or a temperature difference \( \Delta T = T_l - T_r \) is applied. Throughout this work we consider the linear-response regime, that is, \(|\Delta V/E_C| \ll 1 \) and \(|\Delta T/T| \ll 1 \).

Linear transport of charge and heat is conveniently described in terms of the equations

\[
\begin{pmatrix}
I_e \\
I_q
\end{pmatrix} =
\begin{pmatrix}
G_V & G_T \\
M & K
\end{pmatrix}
\begin{pmatrix}
\Delta V \\
\Delta T
\end{pmatrix}
\]  

which relate the charge and heat current response, \( I_e \) and \( I_q \), to an applied potential and temperature difference. Here, \( G_V \) is the linear (charge) conductance, and the thermal conductance \( \kappa \) is defined via \( I_q = \kappa \Delta T \) for \( I_e = 0 \), i.e., \( \kappa = K - G_V T S^2 \). The relevant quantity for the thermoelectric response is the thermopower \( S \), given by \( S = -\Delta V/\Delta T = G_T/G_V \).

In order to calculate the the conductances \( G_V \), \( G_T \) and the thermopower \( S \) we employ a master-equation formalism. According to the orthodox theory \[1\] charge and heat current through the system can be written as

\[
I_e = -e \sum_n \sum_{j=1,2} P_n \left[ \Gamma_{r}^{n \rightarrow n-j} - \Gamma_{r}^{n \rightarrow n+j} \right]
\]  

\[
I_q = \sum_n \sum_{j=1,2} P_n \left[ \varphi_{r}^{n \rightarrow n-j} - \varphi_{r}^{n \rightarrow n+j} \right]
\]  

where \( P_n \) is the stationary probability for finding \( n \) excess electrons on the island, \( \Gamma_{r}^{n \rightarrow n-j} \) is the tunneling rate of \( j \) electrons from the island to the right lead, and \( \Gamma_{r}^{n \rightarrow n+j} \) denotes the tunneling rate of \( j \) electrons from the right lead to the island. Correspondingly, \( \varphi_{r}^{n \rightarrow n-j} \) is the energy transfer rate in \( j \)-electron tunneling to the right lead whereas \( \varphi_{r}^{n \rightarrow n-j} \) denotes the energy transfer rate in a \( j \)-electron tunneling event from the right lead to the island. We consider only sequential tunneling and neglect co-tunneling events.

In Eqs. \[3\],\[4\] we have taken into account the possibility of single-electron tunneling \((j = 1)\) and coherent two-electron tunneling \((j = 2)\) \[25\]. The rates for the latter process
are given by
\[ \Gamma_{i}^{n \rightarrow n \pm \epsilon_{i}^{(\pm 2)}} = \frac{G_{A,i}}{e^{2}} \frac{\epsilon_{i}^{(\pm 2)}}{\exp(\epsilon_{i}^{(\pm 2)}/k_{B}T_{i})} - 1, \quad i = l, r \]  
(5)
where \( G_{A,l}, G_{A,r} \) are the Andreev conductances in the left and the right junction and \( \epsilon_{i}^{(\pm 2)} \), \( \epsilon_{r}^{(\pm 2)} \) are the energies which are dissipated in a two-electron transfer, e.g., \( \epsilon_{l}^{(+2)} = u_{n}(n_{x}) + u_{n+1}(n_{x}) - 2eV_{l} \). The energy transfer rate for two-electron tunneling is then obtained as
\[ q_{i}^{n \rightarrow n \pm \epsilon_{i}^{(\pm 2)}} = \epsilon_{i}^{(\pm 2)} \Gamma_{i}^{n \rightarrow n \pm \epsilon_{i}^{(\pm 2)}} \]  
(6)
where the reference point is the Fermi level of the leads.

The rates for single-electron transitions are given by a sum of two contributions. On the one hand, we have the standard expressions for tunneling, e.g., from a normal to a superconductor \[26\]
\[ \Gamma_{i}(\epsilon) = \frac{G_{i}}{e^{2}} \frac{1}{2} \int_{\Delta}^{\infty} dE \frac{E}{\sqrt{E^{2} - \Delta^{2}}} [f_{i}(E - \epsilon)f_{isl}(-E) + f_{i}(-E - \epsilon)f_{isl}(E)], \quad i = l, r \]  
(7)
(where \( f_{i}(x) = 1/(1 + \exp(x/T_{i})) \) denotes the Fermi function with the appropriate temperature \( T_{l}, T_{r}, \) or \( T \) for the leads or the island, respectively). On the other hand, there is the escape rate of a single unpaired quasiparticle whose energy equals that of \( \Delta \) \[24, 28\]
\[ \gamma_{i}(\epsilon) = \frac{G_{i}}{e^{2}} \frac{1}{2\nu_{isl}} (1 - f_{i}(\Delta + \epsilon)) \]  
(8)
with the normal-electron density of states per spin on the island \( \nu_{isl} \). Note that there is also a corresponding recombination rate. In many cases these escape rates can be neglected, however, at very low temperatures they may exceed the subgap tunneling rate originating from thermally excited quasiparticles in the superconductor. In that case, they produces a different macroscopic behavior of the system, depending on whether the total charge number on the island is even or odd. Thus we have
\[ \Gamma_{i}^{n \rightarrow n \pm \epsilon_{i}^{(\pm 1)}} = \begin{cases} \Gamma_{i}(\epsilon_{i}^{(\pm 1)}) & \text{for } n \text{ even} \\ \Gamma_{i}(\epsilon_{i}^{(\pm 1)}) + \gamma(\epsilon_{i}^{(\pm 1)}) & \text{for } n \text{ odd} \end{cases} \]  
(9)
where, e.g., \( \epsilon_{l}^{(+1)} = u_{n}(n_{x}) - eV_{l} \). Finally, the corresponding energy transfer rates for single-electron tunneling are found from
\[ \Gamma_{i}^{q}(\epsilon) = \frac{G_{i}}{e^{2}} \frac{1}{2} \int_{\Delta}^{\infty} dE \frac{E}{\sqrt{E^{2} - \Delta^{2}}} [(E - \epsilon)f_{i}(E - \epsilon)f_{isl}(-E) + (-E - \epsilon)f_{i}(-E - \epsilon)f_{isl}(E)] \]
and

\[ \gamma^q_i(\epsilon) = \frac{G_i \Delta + \epsilon}{e^2 \nu_{isl}} (1 - f_i(\Delta + \epsilon)) \]

(where again the reference point is the lead Fermi level) such that we have

\[ q_i^{n\rightarrow n\pm 1}(\epsilon_i^{(\pm 1)}) = \begin{cases} 
\Gamma^q_i(\epsilon_i^{(\pm 1)}) & \text{for } n \text{ even} \\
\Gamma^q_i(\epsilon_i^{(\pm 1)}) + \gamma^q(\epsilon_i^{(\pm 1)}) & \text{for } n \text{ odd}
\end{cases} \]

In order to calculate the currents (3), (4) we solve the stationary master equation for the probabilities \( P_n \)

\[ \frac{\partial P_n}{\partial t} = 0 = \sum_{k \neq n} P_k \Gamma^{k\rightarrow n} - P_n \Gamma^{n\rightarrow k} \]

for an applied (small) bias voltage \( \Delta V \) or a (small) temperature difference \( \Delta T \). If a voltage or temperature difference is applied the island is, strictly speaking, in a non-equilibrium state. For the given physical situation it is reasonable to neglect the non-equilibrium part of the distribution function (which we have already done by writing the transfer rates in the form above). For the temperature \( T \) of the island we assume the arithmetic mean \( T = (T_l + T_r)/2 \). We note that there is an independent test of the numerical calculation by checking the Onsager relation \( G_T = M/T \) for the coefficients in Eq. (2) which is obeyed to a high accuracy by our method.

### III. THE PARITY EFFECT

As mentioned above, the parity effect arises as a macroscopic manifestation of the parity of the electron number in a superconductor, i.e., different behavior depending on whether the total electron number is even or odd. The effect was predicted in Ref. [20] and first observed by Tinkham et. al. [21].

In an even-number superconductor at \( T = 0 \) all electrons near the Fermi level are bound in Cooper pairs and there is not a single unpaired quasiparticle left. This system has an energy gap \( 2\Delta \). On the other hand, adding one electron results in a single quasiparticle excitation which does not have an excitation gap. However, the response to an external field is drastically reduced as it is caused just by a single electron. It is intuitively clear that the difference between the two parities will fade away as soon as there are more thermal quasiparticles. This defines the criterion for a crossover temperature \( T^* \) beyond which even-odd differences disappear for an isolated superconductor such as the island in the NSN SET:
It is the temperature at which there is on average one thermally excite quasiparticle and it is defined by

\[ T^* = \frac{\Delta}{\ln N_{\text{eff}}(T^*)} \]  

(11)

where \( N_{\text{eff}}(T) = \nu_{\text{isl}} \sqrt{2\pi T \Delta} \) can be viewed as an effective number of accessible quasiparticle states at the temperature \( T \). Superconducting islands in SET are made of aluminum, and for typical parameters (cf. Refs. [21–23]) one has \( N_{\text{eff}} \sim 10^4 \) and \( T^* \sim 250 \text{mK} \).

Keeping in mind that an SET has the characteristic energy scale \( E_C \), there arise four interesting transport regimes: For the gap and the charging energy we may have \( \Delta < E_C \) or \( \Delta > E_C \). These two cases may be studied for \( T > T^* \) (no parity effects) or in the parity regime \( T < T^* \). Interestingly, with our numerical method we have a choice for studying the system with parity effects. For computations at high temperatures \( T > T^* \) it does not matter whether or not the two-electron and escape rates are included, they do not give any observable effect. On the other hand, for \( T < T^* \) even-odd differences cannot be observed without including these rates. Therefore, including or not including these rates in calculations below \( T^* \) helps us to identify the contribution due to the 'parity-generating' processes.

IV. THERMOPOWER OF AN NSN SETUP

Before we discuss our results we briefly recall an idea due to Matveev [27] (cf. also [11]) to interpret the thermopower as an average energy \( \langle \xi \rangle \) at which current is transported in the voltage-biased system (under linear transport conditions). This is an intuitive and powerful method which will use throughout our work to interpret our results.

The idea can be understood by taking into account that the current \( I_e \) through a device can be written as \( I_e = -e \int [f_l(\xi) - f_r(\xi)] w(\xi) d\xi \) where \( f_l(\xi) \) and \( f_r(\xi) \) denote the Fermi functions in the left and right lead for the respective potential and temperature (the energies \( \xi \) are taken with respect to the chemical potentials). Here \( w(\xi) \) includes all other quantities such as density of states in the leads and transparency of the device at the energy \( \xi \). By noting that \( G_T = \partial I_q/\partial T_l \) and \( G_V = \partial I_e/\partial V \) one finds

\[ \langle \xi \rangle \equiv \frac{\int \xi (\frac{-\partial f}{\partial \xi}) w(\xi) d\xi}{\int (\frac{-\partial f}{\partial \xi}) w(\xi) d\xi} = \frac{(-e)T G_T}{G_V} \]
and consequently for the thermopower

\[ S = -\frac{\langle \xi \rangle}{eT}. \]  

That is, up to a factor the thermopower measures directly this average energy \( \langle \xi \rangle \).

Let us turn now to our calculations for the thermopower of NSN SET. For the numerics we measure energy and inverse time in units of \( E_C \), and conductance in units of \( e^2 \). Then, the free parameters are the ratios of the conductances \( G_l, G_r, G_{A,l}, G_{A,r} \), and the escape rates \( \gamma_l, \gamma_r \). The realistic parameter values which enter our calculations are: \( G_l = G_r = (50k\Omega)^{-1} \), \( G_{A,l} = G_{A,r} = 5 \cdot 10^{-9} \Omega^{-1} \), \( E_C = 100 \mu eV \), \( \gamma_l = \gamma_r = 10^6 s^{-1} \). The experiments in Ref. [22, 23] have been carried out with device parameters close to these values.

A. \( \Delta < E_C \)

In Fig. 2 we show the results for calculations of the thermopower with and without parity effects. The functional dependence \( S(n_x) \) in the latter case has been explained analytically in Ref. [11]. As we have \( T \ll \Delta, E_C \) it is sufficient to include two (or at most three) charge states in the considerations in order to understand the behavior of \( S(n_x) \). Here we focus on a brief discussion of the additional features in the parity regime \( T < T^* \). The key to understand the functional dependence of \( S(n_x) \) without even-odd effects is that at the points \( u_n = 0 \), that is, when \( n_x \) takes half-integer values tunneling to the island \( n \to n+1 \) and from the island \( n+1 \to n \) occurs with equal probability. By using simple arguments for the probabilities \( P_n \) and \( P_{n+1} \) the following equation has been derived in Ref. [11]

\[ S = -\frac{1}{eT} \left( u_0(n_x) - \Delta \tanh \left[ \frac{u_0(n_x)}{2T} \right] \right) \]  

which governs the behavior of \( S(n_x) \) in the intervals from \( u_n(n_x) = 0 \) to the zeros of \( S(n_x) \) which are closest to those \( n_x \) values.

By inspecting Fig. 2 we note that the essential difference introduced in the parity regime is that the purple curve is shifted from the half-integer values of \( n_x \) towards the closest even number. As the slope of the curve does not change in this shift, the consequence is that the maximum absolute value of \( S \) increases, compared to the case without parity. It is not difficult to describe this behavior analytically by repeating the arguments that lead to Eq. (13), now taking into account that the recombination rate of quasiparticles on the
FIG. 2. Thermopower $S(n_x)$ of the NSN SET with $\Delta = 0.5E_C$ and temperature $T = 0.03E_C$. The green dashed line represents the thermopower without two-electron tunneling and escape rate and therefore displays no even-odd differences. The purple solid line displays the result including all processes. It shows clear 2e periodicity and an enhancement of the thermopower for certain gate charges.

island is no longer $\propto \exp \left( -\Delta / T \right)$. In this range of $n_x$ there is an unpaired quasiparticle on the island whose recombination rate provides the dominating contribution to the charge current. This rate is $\propto (e^{-\Delta/T} + \frac{1}{2N_{\text{eff}}})$. From this we get a modified relation

$$S = -\frac{1}{eT} \left( u_0(n_x) - \Delta \tanh \left[ \frac{u_0(n_x) + \eta}{2T} \right] \right)$$

with $\eta = T \ln \left[ 1 + \exp \left( \frac{\Delta}{T} + \ln \frac{1}{2N_{\text{eff}}} \right) \right]$. The relation (14) captures the essential features of the purple curve in Fig. 2.

B. $\Delta > E_C$

The green dashed line in Fig. 3 shows an example for the thermopower without parity effects for $\Delta > E_C$. The arguments we have given in the preceding subsection for the thermopower at half-integer values of $n_x$ are valid also in this case. That is, Eq. (13) correctly describes the behavior of the function $S(n_x)$ also in this parameter range. Hence, this equation turns out to be the key for understanding the thermopower in NSN SET, i.e., a device with an electronic spectrum that is gapped around the Fermi energy. We further mention that for the thermopower without parity effects for the average energy close to
integer values of \( n_x \approx n \) one has \( \langle \xi \rangle \approx (u_n + u_{n-1})/2 \) which yields

\[
S \approx -\frac{u_n + u_{n-1}}{2eT}, \quad n_x \approx n.
\] (15)

FIG. 3. Thermopower \( S(n_x) \) of the NSN SET with \( \Delta = 1.2E_C \) and \( T = 0.03E_C \): a) including all tunneling processes (purple solid line), b) without two-electron tunneling and without escape processes (green dashed line). While the green line is \( e \)-periodic the purple curve shows clear \( 2e \)-periodicity and substantial qualitative changes with respect to the case without even-odd effect.

If the gap exceeds the charging energy, two-electron tunneling starts to play a prominent role for the current-voltage characteristics of NSN SET in the parity regime \( T < T^* \) [25].

Around odd-integer values of \( n_x \) there occurs a current peak which is due to a cycle of two-electron tunneling processes. Note that these processes do not have a gap. The thermopower is analogous to that of single-electron tunneling in NNN SET, the only difference is that we have to substitute the charging energy difference for two-electron tunneling, \( \langle \xi \rangle \approx (u_n(n_x) + u_{n-1}(n_x))/2 \) which leads to a thermopower \( S \approx -(u_n + u_{n-1}/(2eT)) \). This result curiously coincides with the one in the absence of parity effects, Eq. (15) although the dominating tunneling process is a different one.

In the vicinity of the half-integer values of \( n_x \) we observe the analogous effect to the case \( \Delta < E_C \): The curve gets shifted towards the closest even-integer \( n_x \) without changing the slope. Scrutiny of the dominating current-carrying processes, e.g., for \( n_x < 1/2 \) reveals that also here the largest rate is due to quasiparticle recombination on the island. However, if \( T < T^* \) there are essentially no thermal quasiparticles. There is only a single unpaired electron
which is left from the pair-breaking tunneling off the island. The quantitative description of this recombination process leads in full analogy to the conclusion of the previous subsection, namely that Eq. (14) describes this shift of the curve. The essence also here is that the current is due to tunneling of a single unpaired quasiparticle.

V. FIGURE OF MERIT \( ZT \)

Once we have studied the thermopower of the NSN SET it is an interesting question to investigate the thermoelectric efficiency of this device. This efficiency is quantified by

\[
ZT = \frac{G_V S^2 T}{\kappa} = \frac{G_V S^2 T}{\kappa_e + \kappa_l} \tag{16}
\]

where \( \kappa_e \) and \( \kappa_l \) denote the electronic and lattice contribution to the heat conductance of the device. Our work focuses on a regime of extremely low temperatures for which \( \kappa_l \) is very small, hence we neglect it and \( \kappa = \kappa_e \). We compute \( \kappa \) from the heat current \( I_q \) according to Eqs. (2)–(4).

An example of the results is shown in Fig. 4. For \( \Delta \sim E_C \) we observe huge values of \( ZT \) which is rather uncommon, keeping in mind that typical values for materials reach the order of 1, or for quantum dots the order of \( 10^2 \). The question is why such high values are
possible for this system. A quick answer might be that, given that superconductors are bad heat conductors, it could be expected that by including the superconducting island $ZT$ of a single-electron transistor would be enhanced. Let us try to give a more quantitative answer which takes into account our observations from the previous section.

By transferring Matveev’s idea for the interpretation of $S$ as an average transport energy of the electrons according to Eq. (12) we can derive an expression that illuminates the meaning of $ZT$. In analogy with Eq. (12) we obtain $K = \langle \xi^2 \rangle G_V/(e^2 T)$. This leads, together with Eq. (16) and $\kappa = K - G_V T S^2$, to the new relation

$$ZT = \frac{\langle \xi \rangle^2}{\langle \xi^2 \rangle - \langle \xi \rangle^2}.$$  

(17)

Indeed, all additional factors cancel and $ZT$ turns out to be the ratio of the squared average transport energy and the variance of that energy. This relation clearly indicates the strategy that needs to be used in order to increase $ZT$: The current-carrying electrons should be far from the Fermi energy while their energetic distribution should be as narrow as possible. This corroborates also the conclusion of Ref. [19] that a $\delta$ function in the spectrum is favorable for a high $ZT$ value.

Let us now apply Eq. (17) to the NSN SET. We have already discussed that for the $n_x$ values where the strong enhancement of $ZT$ is found, the current is carried by a single unpaired quasiparticle. The maximum energy of this quasiparticle is $\sim \Delta$ while its energy distribution is rather narrow: it is just given by the temperature $T$. Hence we expect $ZT \sim \Delta^2/T^2$. In fact, this estimate has the correct order of magnitude.

VI. DISCUSSION

We have investigated the thermopower for a single-electron transistor with a superconducting island for arbitrary ratios $\Delta/E_C$ and for temperatures both above and below the crossover temperature for parity effects $T^*$. The results show the expected parity effects also in the functional dependence $S(n_x)$ of the thermopower on the gate charge $n_x$, in particular $2e$ periodicity. We have provided a discussion of the essential features of this functional dependence in terms of Eqs. (14), (15). It is remarkable that the basis for this discussion is Eq. (13) which was found already in Ref. [11].

Apart from the thermopower we have also calculated the thermoelectric figure of merit
Unexpectedly we have found a strong enhancement of $ZT$ compared to common values of this quantity. In order to understand our findings we have given a new interpretation of $ZT$ in terms of Matveev’s idea to represent thermoelectric quantities as moments of the energy distribution for the current-carrying electrons. It shows that large values of $ZT$ can be obtained if the dominant transport mechanism occurs far from the Fermi level, and at the same time, has a narrow distribution in its energies. Clearly, the NSN SET is a system where these conditions can be achieved.

VII. ACKNOWLEDGMENTS

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