DOA estimation based on density-tapered thinned array

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Abstract. In this paper, several classical DOA estimation methods are applied to density-tapered thinned array. The density-tapered thinned array has the advantage that there is no grating lobe in the whole angle scanning range, and the unambiguous angle estimation of the target is obtained. The experimental results show that the classical DOA estimation method can be applied directly to the density-tapered thinned array to obtain satisfactory results, which lays a foundation for the practical application of the density-tapered thinned array. In addition, the effects of sparsity and number of elements on DOA estimation are analyzed, and the advantages of density-tapered thinned array over uniform arrays are verified.

1. Introduction

Thinned antenna arrays can significantly reduce the number of receiving and sending units of large-aperture arrays, and accurate target angle resolution can be achieved by signal processing. It is especially suitable for high-resolution working environments where device weight is restricted, cost is constrained.

Thinned array refers to the antenna array whose elements are limited on uniform grids, which can be regarded as the result of reducing some elements in a uniform full array[1]. Many array design methods have been used to design thinned arrays, such as density-tapered method[2, 3], population optimization algorithm[4, 5], convex optimization method[6], compressed sensing method [7] and so on. Among them, the thinned array designed by density-tapered method can obtain low peak sidelobe level (PSLL) and narrow half power beam width (HPBW) when the excitation amplitude of array elements is the same, so it has the potential advantage of being compatible with existing array signal processing algorithms.

The DOA estimation is sometimes called spatial spectrum estimation technology. As people realize the similarity between spatial spectrum estimation and time-domain signal spectrum estimation, some high-resolution spectrum estimation methods, such as MUSIC algorithm [8] and linear prediction method[9], are applied to spatial spectrum estimation, which greatly improves the accuracy of angle estimation. The research on DOA estimation of nonuniform array mainly focuses on coprime arrays[10], but the processing method is complex.

Considering that there is little research on density-tapered thinned array signal processing at present, this paper first describes the mature technologies that we have validated for density-tapered array signal processing, and then uses these techniques to conduct simulation experiments on thinned arrays. The results show that these signal processing methods are effective for density-tapered thinned arrays.
2. Classical DOA estimation algorithm

2.1. Signal model

Assuming that the array works in the far-field and narrow-band condition, the wavelength of the signal is \( \lambda \), then the array manifold vector affected by the array structure can be expressed as

\[
v(k) = \left[ e^{-jk |p|}, e^{-jk |p_e|}, \ldots, e^{-jk |p|} \right] T
\]

Where \( k = \frac{2\pi}{\lambda} \) is defined as wave number. \( T \) stands for transpose symbol, and the \( a = [-\sin \theta \cos \varphi, -\sin \theta \sin \varphi, -\cos \varphi] T \), is the unit direction vector, \( p_e \) is the position vector of the \( n \)-th array element. \( N \) is the total number of elements. For linear arrays, the array manifold vector can be reduced to

\[
v(\theta) = \left[ e^{j\frac{2\pi}{\lambda} N \sin \theta}, e^{j\frac{2\pi}{\lambda} N - 1 \sin \theta}, \ldots, e^{j\frac{2\pi}{\lambda} N \sin \theta} \right] T
\]

Where \( \theta \) is the angle between the incoming wave direction and the normal direction of the array. The signal sampling value received by the array can be expressed as

\[
X(k) = f(k) \times v_n(\theta) + N(k) \quad k = 1, 2, \ldots, K
\]

Where \( X \) and \( N \) are vectors of size \( N \times 1 \), \( f(k) \) is a complex number containing amplitude and phase information of a signal, and \( K \) is the number of samples per channel. The sampling correlation matrix of array output signals can be expressed as

\[
C_{ss} = \frac{1}{K} \sum_{k=1}^{K} X(k)X(k) H
\]

In the following content, we will use this sampling covariance matrix for DOA estimation. It should be noted that this paper studies the DOA estimation of thinned linear array. Because thinned array lacks some array elements compared with uniform array, there are a large number of zeros in \( X \). In order to ensure that the covariance matrix is nonsingular matrix, it needs to be corrected by diagonal loading technology as follow

\[
C_s = C_{ss} + \sigma^2 I
\]

2.2. Algorithm

This section briefly introduces several classical DOA estimation methods [9]. The simplest DOA estimation method is the beam search method, which is to search in a certain angle range after forming the beam through conventional beamforming to obtain the estimated value of beam power at a specific angle. It can be expressed as

\[
P_b = v^H(\theta)C_v(\theta)
\]

In practical application, assuming that the number of signals \( D \) is known, the angles of \( D \) peaks of \( P_b \) are selected as the DOA estimation result.

Capon method is to apply MVDR beamformer to DOA estimation. The MVDR beamformer can form spikes in the direction of the signal. The spatial spectrum estimation results are expressed by the formula

\[
P_c = (v^H(\theta)C_v^{-1}v_n(\theta))^{-1}
\]

Similarly, the estimation result of DOA can be obtained by finding \( D \) peaks.
MUSIC algorithm uses the orthogonality of signal subspace and noise subspace to estimate the direction of arrival of signal. First, the covariance matrix of the array received signal is expressed as

$$C_x = \sum_{i=1}^{N} \lambda_i \phi_i \phi_i^H$$

(8)

Then, the eigenvalues are arranged from large to small:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_D > \lambda_{D+1} = \cdots = \sigma_w^2$$

(9)

Because there are $D$ signals, we regard the first $D$ eigenvalues as the eigenvalues of the signal subspace, their corresponding eigenvectors as the eigenvectors of the signal subspace, and the remaining eigenvectors constitute the noise subspace.

$$U_N = [\phi_{D+1} \phi_{D+2} \cdots \phi_N]$$

(10)

Finally, the spatial spectrum estimation result of MUSIC algorithm can be expressed as:

$$P_{\text{music}} = (v^H(\theta) U_N U_N^H v(\theta))^{-1}$$

(11)

![Figure 1. Structure of the one-step linear predictor.](image)

Based on the principle of Wiener filter, linear prediction (LP) obtains the estimation of the input signal by adjusting the weight $a_n$. When applied to DOA estimation, the system function of the whole filter is obtained by $a_n$, and then the spatial spectrum of the signal is estimated by the system function. Figure 1 shows the structure of the one-step linear predictor. Assuming that the order of the predictor is $M-1$, the canonical equation can be expressed as

$$
\begin{bmatrix}
    r(0) & r(1) & \cdots & r(M-1) \\
    r^*(1) & r(0) & \cdots & r(M-2) \\
    \vdots & \vdots & \ddots & \vdots \\
    r^*(M-1) & r^*(M-2) & \cdots & r(0)
\end{bmatrix}
\begin{bmatrix}
    a_{M-1} \\
    a_{M-2} \\
    \vdots \\
    a_1 \\
    1
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    0 \\
    \vdots \\
    0 \\
    \delta_{M-1}
\end{bmatrix}
$$

(12)

Where the autocorrelation coefficient $r(m) = E\{X_n X_{n+m}\}$, $\delta_{M-1}$ is related to the prediction error. Thus, the system function can be expressed as

$$H(z) = 1 + a_1 z^{-1} + \cdots + a_{M-1} z^{-(M-1)}$$

(13)

The linear predictor can actually be regarded as a whitening filter, and the output $e_n$ can be regarded as white noise. Therefore, the input spatial spectrum can be regarded as the reciprocal of the system function. Finally, the spatial spectrum estimation result of LP method can be expressed as

$$P_z = |H(z)|^2$$

(14)
Where \( z = \exp \left( j \frac{2\pi d}{\lambda \cos \zeta} \right) \), \( \zeta = 90^\circ - \theta \). By calculating the values of formula (14) at different angles, the peaks are selected as the DOA estimation results.

3. Simulation experiment

In reference [3], a method of designing thinned array using density-tapered principle is proposed. The core idea of the density-tapered method is to simulate amplitude weighting by changing the density of array element. The area with high density has higher equivalent amplitude than that with low density. At the end of this section, DOA estimation will be carried out based on the array designed in reference [3].

Firstly, we select a uniform linear array with 20 elements to investigate the DOA estimation algorithm mentioned above. There are signals from five directions, with angles of 85°, 90°, 90.5°, 92° and 95° respectively, and have the same power. The signal-to-noise ratio is 10dB, and the DOA estimation results are shown in figure 2.

![Figure 2. The DOA estimation results of uniform linear array with 20 elements.](image)

In this example, the angular resolution of the uniform linear array with 20 elements is about \( 110 / 20 = 5.5^\circ \), so the spatial spectrum obtained by beam search method can not distinguish these five targets in theory. The azure line in figure 2 verifies our speculation. By observing the green, red and dark blue curves in the figure, it can be seen that the spatial spectrum curve calculated by the Capon method has only three peaks, which can not separate the targets in the directions of 90°, 90.5° and 92°. The effect of linear prediction method is better than the Capon method, and four targets are distinguished, but the two nearest targets are still not distinguished. MUSIC algorithm achieved the best effect among them, obtained five peaks, and accurately estimated the DOA of the signal.

Next, we expand the number of array elements to 256 and still use the uniform linear array. The DOA estimation results are shown in figure 3. It can be seen that due to the increase of the number of array elements, the array aperture becomes larger, resulting in the increase of the angular resolution of the array, which greatly improves the DOA performance of the beam search method, but the estimation accuracy is still lower than that of other algorithms. The results of Capon, MUSIC and linear prediction methods are close.
Finally, we will use the method in reference [3] to design a density-tapered thinned array with 750 grids and 256 elements. The four DOA estimation methods mentioned above are applied to the DOA estimation with thinned linear array. The simulation results are shown in figure 4.

It can be seen that after the array is thinned, due to the larger aperture, the DOA estimation performance of the beam search method has also been improved, and all the four methods can achieve good DOA estimation results. This also verifies that the density-tapered thinned array has good adaptability to the four DOA estimation methods, and this is an advantage that coprime arrays do not have. Different from figure 3, the spatial spectrum estimation curve obtained by the beam search method in figure 4 no longer has the shape similar to the Singh function, because the beam pattern obtained by the conventional beamforming of the density-tapered thinned array does not have the shape similar to the Singh function.
4. Conclusion
Because the beam search method uses the conventional beam to search the space, its angular resolution will not exceed the HPBW of the pattern. When the array aperture is small, it can not distinguish two targets close in angle. Other DOA estimation methods break through the limitation of HPBW, but their performance is also different. In general, the DOA estimation performance can be ranked from high to low as: Music > linear prediction > Capon > beam search.

The density-tapered thinned array is well compatible with the classical DOA estimation methods, which is of great positive significance for the practical application of the array. The research on the compatibility of density-tapered thinned array with classical adaptive beamforming algorithm has been included in the research plan of the next stage.

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