Binary Choice Models with High-Dimensional Individual and Time Fixed Effects

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Empirical economists are often deterred from the application of binary choice models with fixed effects mainly for two reasons: the incidental parameter bias and the computational challenge in (moderately) large data sets. We show how both issues can be alleviated in the context of binary choice models with individual and time fixed effects. Thanks to several bias-corrections proposed by Fernández-Val and Weidner (2016), the incidental parameter bias can be reduced substantially. In order to make the estimation feasible even in panels with many fixed effects, we develop an efficient software routine, embedded in the R-package alpaca, that combines these corrections with an approach called method of alternating projections. Further, we contribute to the existing literature by conducting extensive simulation experiments in large and even unbalanced panel settings. Finally, we estimate a dynamic probit model, to study the inter-temporal labor force participation of women in Germany.

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1. INTRODUCTION

Panel data models are widely used in econometrics because they allow to control for different sorts of unobserved heterogeneity, such as individual and time specific effects. One popular specification are fixed effects models that treat these unobserved effects as parameters to be estimated. Because including an additional individual or time-period also increases the number of model parameters, these unobserved effects are also called incidental parameters. A crucial challenge of non-linear fixed effects estimators is their inconsistency if $N \to \infty$ and $T$ is held fixed, known as the incidental parameter problem (IPP) tracing back to Neyman and Scott (1948). The problem arises because only a fixed number of observations contributes to the identification of one specific unobserved effect, resulting in potentially noisy estimates, which in turn carry over to the structural parameter estimates (see Arellano and Hahn (2007) and Fernández-Val and Weidner (2016, 2018a)).

In the early stage of panel data econometrics, panels consist of a relatively small number of observations per individual. This strand of literature has been concerned to derive fixed $T$ consistent estimators. For binary choice with individual fixed effects, the so-called conditional logit estimator has been proposed for static and dynamic models tracing back to Rasch (1960), Andersen (1970), Chamberlain (1980), and Honoré and Kyriazidou (2000). However, it is not possible to derive fixed $T$ consistent fixed effects estimators for all kind of models, e.g. the probit model. Another shortcoming of all conditional logit estimators is that they preclude the estimation of partial effects (see Arellano and Hahn (2007) and Fernández-Val and Weidner (2018a)).

For these reasons, among others, and also motivated by the rising availability of large scale panel data, a growing literature now focuses on large $T$ asymptotics, since the seminal paper of Phillips and Moon (1999). If $N, T \to \infty$, IPP becomes an asymptotic bias problem, which is easier to deal with than an inconsistency problem. The large $T$ literature proposes bias-corrections to obtain an estimator that has only a small bias relative to its dispersion. In the meantime, there are several different approaches to construct bias-corrected fixed effects estimators for several non-linear models with different type of error structure. We refer the reader to Arellano and Hahn (2007) and Fernández-Val and Weidner (2018a) for detailed overviews. In this article we focus on bias-corrections proposed by Fernández-Val and Weidner (2016) that are suitable for binary choice models with individual and time fixed effects.

Another seemingly challenge in non-linear fixed effects models is the computation in the presence of high-dimensional fixed effects. One-way error component models are easy to handle, thanks to the partitioned inverse formula (see Chamberlain (1980) and Greene (2002)),...
or an approach introduced as pseudo-demeaning by Stammann, Heiss, and McFadden (2016). Also the estimation of multiple fixed effects in non-linear panel models is feasible, using algorithms such as Guimarães and Portugal (2010) and Stammann (2018).

This article complements the works of Stammann, Heiss, and McFadden (2016) and Hinz, Wanner, and Stammann (2019), who both provide guidance to the estimation of bias-corrected binary choice models in the presence of high-dimensional fixed effects. Whereas the former focuses on one-way fixed effects models, the latter is concerned with special two- and three-way error components commonly used in international trade. We contribute to the existing literature by offering new insights that facilitate and validate the usage of binary choice models with individual and time fixed effects in empirical applications. First of all, we show how the computational obstacle, that often precludes the application of bias-corrections, can be tackled by combining them with the method of alternating projections (MAP). Until now, the aforementioned bias-corrections are only provided by Cruz-Gonzalez, Fernández-Val, and Weidner (2017) in a Stata routine, which is not adapted to large panel data. In order to make the application of bias-corrections more attractive or even feasible in larger panel structures, we have included it in our R-package `alpaca`.¹ This accelerated routine allows us to conduct extensive simulation experiments, in terms of estimators and panel dimensions. More precisely, we study the properties of several analytical and split-panel jackknife bias-corrected estimators. Further, we also include different patterns of unbalancedness in our analysis, because many real word data sets are unbalanced by nature. Our findings suggest that analytical bias-corrections should be preferred to split-panel jackknife approaches. Additionally, we question the use of (bias-corrected) linear probability models, despite their popularity. Finally, we provide an extended empirical example using a large panel data set drawn from the German Socio Economic Panel (see Wagner, Frick, and Schupp (2007)) to investigate the inter-temporal labor force participation of 10,712 women between 1984 and 2013.

The rest of the article is organized as follows. Section 2 introduces the model and different bias-corrections. Section 3 demonstrates how to handle high-dimensional fixed effects. Section 4 provides results of extensive simulation experiments. Section 5 applies the different bias-corrected estimators to an empirical example from labor economics. Finally section 6 concludes.

¹ We add the corresponding subroutines for the bias-correction in the next update (version 0.3).
2. BIAS-CORRECTIONS FOR FIXED EFFECTS BINARY CHOICE MODELS

2.1. Fixed Effects Binary Choice Models and the Incidental Parameters Problem

The availability of rich panel data sets offers several advantages to researchers, compared to pure cross-sections or time series (see chapter 1.2 in Hsiao (2014) for a comprehensive list of advantages). One major concern in econometrics is that unobservables correlate with the explanatory variables and thus invalidate the findings of the analysis. A popular strategy to mitigate this concern is to assume that the unobservables, driving the inconsistency of the results, are additive separable individual and/or time specific constants that are allowed to be arbitrarily correlated with the explanatory variables. So called fixed effects models are designed to take this kind of unobserved heterogeneity into account.

Next, we introduce the fixed effects binary choice model, which can be derived from a latent variable model with additive separable two-way error component. Let

\[ y^*_{it} = x'_{it} \beta + \alpha_i + \gamma_t + e_{it}, \]

be the latent variable, where \( i = 1, \ldots, N \) and \( t = 1, \ldots, T \) are individual and time specific identifiers, \( x_{it} \) is a \( J \)-dimensional vector of explanatory variables, \( \beta \) are the corresponding parameters, and \( e_{it} \) is an idiosyncratic error term. Note that \( x_{it} \) might also include predetermined variables. Further, let \( \alpha_i \) and \( \gamma_t \) denote unobserved individual and time specific heterogeneity, respectively. Throughout the paper, we call \( \beta \) the structural and \( \phi = (\alpha, \gamma) \) the incidental parameters. However, instead of the latent variable, we only observe \( y_{it} = 1 \) if \( y^*_{it} \geq 0 \) and \( y_{it} = 0 \) otherwise, which leads to the non-linear nature of this model.

The most popular way to derive an parametric estimator for fixed effects binary choice models is the principle of maximum likelihood. Suppose the idiosyncratic error term is drawn independently from a specific symmetric distribution. Then

\[ l_{it}(\beta, \alpha, \gamma) = y_{it} \log(F_{it}) + (1 - y_{it}) \log(1 - F_{it}), \]

is the log-likelihood contribution of individual \( i \) at time \( t \), where \( F_{it} \) is the cumulative distribution function of the idiosyncratic error term evaluated at \( \eta_{it} = x'_{it} \beta + \alpha_i + \gamma_t \). Note that in the literature of generalized linear models (GLMs), \( \eta_{it} \) is known as the linear predictor. Common choices for \( F_{it} \) are the standard normal, the logistic, and the complementary log-log distribution. The corresponding maximum likelihood estimator is

\[ \hat{\theta} = (\hat{\beta}, \hat{\alpha}, \hat{\gamma}) = \arg \max_{\beta, \alpha, \gamma} \log L(\beta, \alpha, \gamma), \]
where
\[ \mathcal{L}(\beta, \alpha, \gamma) = \sum_{i}^{N} \sum_{t}^{T} l_{it}(\beta, \alpha_{i}, \gamma_{t}). \]

Contrary to standard least-squares problems, (1) does not have a closed form solution and thus has to be solved numerically. Note that binary choice models with fixed effects can be estimated using any available standard software routine by adding indicators for each individual and time period to the list of explanatory variables. However, if \( N \) and \( T \) increases this approach quickly becomes very time consuming or even infeasible.

Besides some computational obstacles, fixed effects estimators also suffer from the so-called incidental parameters problem (IPP) tracing back to Neyman and Scott (1948). In order to get an intuition of IPP suppose that \( T \) is small. In this case only a few observations per individual contribute to the estimation of \( \alpha \). The same logic applies to \( \gamma \), if \( N \) is small. Thus the estimation error of the incidental parameters can be very severe. Due to the non-linear nature of binary choice models, the estimation error carries over to \( \hat{\beta} \) which is known as IPP (see among others Arellano and Hahn (2007) and Fernández-Val and Weidner (2018a)). In order to deal with this problem, several bias-corrected estimators have been proposed (see among others Hahn and Newey (2004), Fernández-Val (2009), Dhaene and Jochmans (2015), Fernández-Val and Weidner (2016), and Kim and Sun (2016)).

Next, we briefly describe the key findings of Fernández-Val and Weidner (2016), who developed bias-corrected estimators for non-linear models with two-way error component. The authors show that under certain conditions, most notably additive separability and concavity, the fixed effects estimator \( \hat{\beta} \) has an asymptotic expansion as \( N, T \to \infty \) with respect to the asymptotic sequence \( N/T \to \kappa^2 \). The expansion yields the following asymptotic distribution of \( \hat{\beta} \):
\[
\sqrt{NT}(\hat{\beta} - \beta) \xrightarrow{d} \mathcal{N}(\kappa \mathbf{B}_{\infty}^{\beta} + \kappa^{-1} \mathbf{C}_{\infty}^{\beta}, \mathbf{W}_{\infty}),
\]
where \( \mathbf{B}_{\infty}^{\beta} \) and \( \mathbf{C}_{\infty}^{\beta} \) are asymptotic bias terms stemming from the inclusion of individual and time specific fixed effects and \( \mathbf{W}_{\infty} \) is the Hessian of the concentrated log-likelihood:
\[
\mathcal{L}^*(\beta) = \max_{\alpha, \gamma} \sum_{i}^{N} \sum_{t}^{T} l_{it}(\beta, \alpha_{i}, \gamma_{t}).
\]

Despite that \( \hat{\beta} \) is consistent (plim\( N,T \to \infty \)\( \hat{\beta} = \beta \)), the derived distribution reveals an asymptotic bias. As noted by Fernández-Val and Weidner (2016, 2018a), this bias can lead to severe consequences for inference even in moderately large panels.

Often researchers are not directly interested in estimates of \( \beta \), but rather in so-called partial effects. Let \( \Delta_{itj} \) denote the partial effect of a change in \( x_{itj} \) corresponding to individual
$i$ at time $t$, where $x_{itj}$ is the $j$-th element in $x_{it}$. This yields

$$
\Delta_{itj} = \beta_j \partial_\eta F_{it}
$$

(3)

for continuous and

$$
\Delta_{itj} = F_{it}|_{x_{itj}=1} - F_{it}|_{x_{itj}=0}
$$

(4)

for binary variables, where $\partial_\eta F_{it}$ is the first-order partial derivative of $F_{it}$ with respect to $\eta_{it}$. Because $\Delta_{itj}$ is most likely different across individuals and time periods, a common strategy is to compute an average such that $\delta_j = \frac{1}{NT} \sum_i \sum_t \Delta_{itj}$. This quantity is known as the average partial effect of a change in $x_{itj}$.

Imposing further sampling conditions, Fernández-Val and Weidner (2016) derive the asymptotic distribution of the average partial effects estimator $\hat{\delta}$. Given that the average partial effects are computed based on any bias corrected estimator of $\beta$, the expansion yields the following asymptotic distribution of $\hat{\delta}$:

$$
\frac{r(\hat{\delta} - \delta - T^{-1}B_\infty^\delta - N^{-1}C_\infty^\delta)}{d} \overset{\mathcal{N}}{\longrightarrow} (0, \overline{V}_\infty^\delta),
$$

where $r$ is a convergence rate and $\overline{V}_\infty^\delta$ is the asymptotic variance. Again, $B_\infty^\delta$ and $C_\infty^\delta$ are asymptotic bias terms stemming from the inclusion of individual and time specific fixed effects. Thus similar to $\hat{\beta}$ there is an asymptotic bias in the distribution of $\hat{\delta}$.

Fernández-Val and Weidner (2016) also present different approaches that can be used to construct bias-corrected estimators. In the next subsection, we describe how to compute those for structural parameters and average partial effects of fixed effects binary choice models.

2.2. Asymptotic Bias-Corrections

Before we present the different bias-corrected estimators for fixed effects binary choice models proposed by Fernández-Val and Weidner (2016), we introduce some notation. Let $\partial_\eta \hat{G}_{it}$ and $\partial_\eta^2 \hat{G}_{it}$ denote the first- and second-order partial derivative of an arbitrary function $G_{it}$ with respect to $\eta_{it}$ evaluated at its sample analogue. For instance, we denote $\hat{\eta}_{it} = x_{it}' \hat{\beta} + \hat{\alpha}_i + \hat{\gamma}_t$ as the sample analogue of $\eta_{it}$. Further let $\partial_\eta \hat{I}_{it} = \hat{H}_{it}(y_{it} - \hat{F}_{it})$, $\hat{\omega}_{it} = \hat{H}_{it}\partial_\eta \hat{F}_{it}$, and $\hat{H}_{it} = \partial_\eta \hat{F}_{it}/(\hat{F}_{it}(1-\hat{F}_{it}))$. Finally, we define the residual projection $\hat{M} = \mathbb{1}_{NT} - \hat{P} = \mathbb{1}_{NT} - \mathbf{D}(\mathbf{D}'\hat{\Omega})^{-1}\mathbf{D}'\hat{\Omega}$, where $\mathbf{D}$ is a sparse indicator matrix arising from dummy encoding of individual and time identifiers, $\mathbf{X}$ is a matrix of explanatory variables, and $\hat{\Omega}$ is a diagonal matrix with $\text{diag}(\hat{\Omega}) = \hat{\omega}$.²

² We provide explicit expressions for logit and probit models in table 13 in the appendix.
Throughout the paper we distinguish between two types of bias-corrections: analytical and re-sampling. The latter uses jackknife or bootstrap techniques to construct consistent estimators of the bias terms, whereas the analytical correction relies on explicit expressions. A general expression for the bias-corrected estimator of the structural parameter is

$$\tilde{\beta} = \hat{\beta} - \hat{b}^\beta,$$  

(5)

where $\hat{b}^\beta$ is a consistent estimator of the composite bias term such that

$$\sqrt{NT}(\tilde{\beta} - \beta) \overset{d}{\rightarrow} \mathcal{N}(0, \hat{W}_\infty^{-1}).$$

Next, we describe the first-order analytical bias-correction proposed by Fernández-Val and Weidner (2016). The corresponding estimator of the composite bias term is

$$\hat{b}_{abc}^\beta = \hat{W}^{-1} (\hat{B}^\beta + \hat{C}^\beta),$$

where

$$\hat{B}^\beta = -\frac{1}{2} \sum_{t=1}^{N} \frac{T}{T+1} \sum_{i=1}^{T} \hat{H}_{it} \frac{\partial^2 \hat{F}_{it}(\hat{m}X)_{it}}{\partial \eta^2} + \sum_{l=1}^{L} \sum_{i=1}^{T} \frac{T}{T+1} \frac{\partial \hat{F}_{it}(\hat{m}X)_{it}}{\partial \omega_i\hat{t}^{l-1}} \hat{\omega}_{it}(\hat{m}X)_{it},$$

$$\hat{C}^\beta = -\frac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{N} \frac{\partial^2 \hat{F}_{it}(\hat{m}X)_{it}}{\partial \omega_i},$$

$$\hat{W} = \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\omega}_{it}(\hat{m}X)_{it}(\hat{m}X)_{it}^\prime.$$ 

Note that $\hat{W}$ is the Hessian of (2) evaluated at $\hat{\beta}$ after convergence, $L$ is a bandwidth parameter proposed by Hahn and Kuersteiner (2011) required for the estimation of spectral densities, and $T/(T-L)$ is a finite sample adjustment. If all explanatory variables are strictly exogenous, we can set $L = 0$ and the second term in $\hat{B}^\beta$ drops out, leading to symmetric bias terms. If not, Fernández-Val and Weidner (2016, 2018a) suggest to do a sensitivity analysis reporting estimates for $L \in \{1, \ldots, 4\}$. The authors also note that the analytical bias-corrected estimator can be further iterated. More precisely, for a given $\tilde{\beta}$, we can compute $\hat{b}_{abc}^\beta$ and update $\tilde{\beta}$ again and again. Although the asymptotic distribution of (5) is not affected by the iteration, its finite-sample performance might improve (see among others Arellano and Hahn (2007)).

Fernández-Val and Weidner (2016) also extend the split-panel jackknife bias-correction of Dhaene and Jochmans (2015) to non-linear models with two-way error component. The idea is to split the panel into smaller sub panels and use those to form an estimator of the
composite bias term. Those sub panels are extracted as blocks, to maintain the dependency structure of the panel. Next, we describe two estimators of the bias term that are based on different splitting strategies to generate sub panels. The first one is described in Fernández-Val and Weidner (2016). Let
\[ \hat{b}_{spj1}^\beta = 2 \hat{\beta} - \hat{\beta}_N - \hat{\beta}_T \tag{6} \]
be an estimator of the composite bias term, where
\[ \hat{\beta}_N = \frac{1}{2} \left( \hat{\beta}_{\{i \leq \lceil N/2 \rceil\}} + \hat{\beta}_{\{i \geq \lceil N/2 + 1 \rceil\}} \right), \]
\[ \hat{\beta}_T = \frac{1}{2} \left( \hat{\beta}_{\{t \leq \lceil T/2 \rceil\}} + \hat{\beta}_{\{t \geq \lfloor T/2 + 1 \rfloor\}} \right), \]
\( \lceil \cdot \rceil \) and \( \lfloor \cdot \rfloor \) are floor and ceiling functions, and the subscript in curly brackets indicates the corresponding sub panel. For instance, \( \{i \leq \lceil N/2 \rceil\} \) means that we only use the first half of all individuals in the sample to compute \( \hat{\beta} \). Cruz-Gonzalez, Fernández-Val, and Weidner (2017) propose another splitting strategy. The corresponding estimator of the composite bias term is
\[ \hat{b}_{spj2}^\beta = \hat{\beta} - \hat{\beta}_{NT} \tag{7} \]
where
\[ \hat{\beta}_{NT} = \frac{1}{4} \left( \hat{\beta}_{\{i \leq \lceil N/2 \rceil; t \leq \lceil T/2 \rceil\}} + \hat{\beta}_{\{i \leq \lceil N/2 \rceil; t \geq \lfloor T/2 + 1 \rfloor\}} + \hat{\beta}_{\{i \geq \lceil N/2 + 1 \rceil; t \leq \lceil T/2 \rceil\}} + \hat{\beta}_{\{i \geq \lceil N/2 + 1 \rceil; t \geq \lfloor T/2 + 1 \rfloor\}} \right). \]
Contrary to the first strategy, the panel is split simultaneously along both dimensions. Thus \( \{i \leq \lceil N/2 \rceil; t \leq \lceil T/2 \rceil\} \) indicates that \( \hat{\beta} \) is computed based on the first half of all individuals in the first half of all time periods. The second splitting strategy is computationally less intense because the four sub panels are smaller. However this strategy might lead to larger dispersion compared to the first one. Also note that the split-panel jackknife bias-correction additionally requires unconditional homogeneity (see assumption 4.3 in Fernández-Val and Weidner (2016) for details) in contrast to analytical bias-corrections. For instance, this condition rules out time-trends or structural breaks in the explanatory variables. Intuitively, if the sub panels stem from very different data generating processes (e.g. due to non-stationarity), this will result in a poor estimate of the bias term because the sub panel estimates are very different from each other (see Dhaene and Jochmans (2015) and Fernández-Val and Weidner (2016, 2018a)).

So far the bias-correction is applied at the level of the estimator. Fernández-Val and Weidner (2016) also show how to apply the analytical correction at the level of score. The corresponding bias-corrected estimator can be obtained by solving the following system of
equations

\[
(\widehat{\Omega} \mathbf{X}(\widehat{\beta}))' \hat{\Omega}(\widehat{\beta}) \hat{v}(\widehat{\beta}) = \hat{B} + \hat{C}
\]

for \( \hat{\beta} \) using any non-linear solver. Note that the left hand side is the gradient of (2) evaluated at \( \hat{\beta} \). The authors also suggest a continuously updated score correction by replacing \( \hat{B} \) and \( \hat{C} \) with \( \hat{B}(\hat{\beta}) \) and \( \hat{C}(\hat{\beta}) \), respectively.

Additionally Fernández-Val and Weidner (2016) derive bias corrections for average partial effects. Let \( \hat{\delta} = (NT)^{-1} \sum_i \sum_t \hat{\Delta}_{it} \), where \( \hat{\Delta}_{it} \) is the sample analogue of (3) or (4). Similar to the structural parameters, a bias-corrected estimator for the average partial effects is

\[
\tilde{\delta} = \hat{\delta} - \hat{b} \delta,
\]

where \( \hat{b} \delta \) is a consistent estimator of the composite bias term such that

\[
r(\tilde{\delta} - \delta) \xrightarrow{d} \mathcal{N}(0, \sqrt{\hat{V} \delta})
\]

and thus \( \tilde{\delta} \) is asymptotically unbiased. Again, we can either use analytical expressions to construct a consistent estimator of the composite bias term or we can use re-sampling methods. Because the adjustment of the different splitting strategies to average partial effects is generic and straightforward, we omit it for brevity.

Next, we describe the analytical bias-corrected estimator of the averaged partial effects proposed by Fernández-Val and Weidner (2016, 2018b). Given that \( \tilde{\delta} \) and \( \hat{\Delta} \) are constructed from bias-corrected estimates of \( \beta \), the analytical estimator of the composite bias term is

\[
\hat{b} \delta = (NT)^{-1} \left( \hat{B} + \hat{C} \right),
\]

where

\[
\hat{B} = \frac{1}{2} \sum_{i=1}^{N} \sum_{t=1}^{T} -\hat{H}_{it} \partial_{\eta^2} \hat{F}_{it}(\hat{P} \hat{\Psi})_{it} + \partial_{\eta^2} \hat{\Delta}_{it} + \sum_{l=1}^{L} (T/(T-l)) \sum_{i=1}^{T} \partial_{\eta^2} \hat{\Delta}_{it} - \hat{\phi}_{it} \hat{\delta}(\hat{M} \hat{\Psi})_{it}
\]

and

\[
\hat{C} = \frac{1}{2} \sum_{i=1}^{N} \sum_{t=1}^{T} -\hat{H}_{it} \partial_{\eta^2} \hat{F}_{it}(\hat{P} \hat{\Psi})_{it} + \partial_{\eta^2} \hat{\Delta}_{it} - \hat{\phi}_{it} \hat{\delta}(\hat{M} \hat{\Psi})_{it}
\]

and \( \hat{\Psi}_{it} = \partial_{\eta} \hat{\Delta}_{it} / \hat{\phi}_{it} \). Under the assumption that \( \{\alpha_i\}_{N} \) and \( \{\gamma_l\}_{T} \) are independent sequences, the authors also derive the following estimator for the covariance \( \sqrt{\hat{V} \delta} \):

\[
\sqrt{\hat{V} \delta} = \frac{r^2}{N^2 T^2} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \hat{\Delta}_{it} \hat{\Delta}_{is} + \sum_{j \neq i}^{N} \sum_{t=1}^{T} \hat{\Delta}_{it} \hat{\Delta}_{jt} + \sum_{t=1}^{T} \hat{\Gamma}_{it} \hat{\Gamma}_{jt} + 2 \sum_{s > t}^{T} \hat{\Delta}_{it} \hat{\Gamma}_{is} \right),
\]

3. The expression is simplified using the fact that \( \hat{M} \) is idempotent.
where
\[
\tilde{\Gamma}_{it} = \left( \sum_{i=1}^{N} \sum_{t=1}^{T} \partial_\beta \tilde{\Delta}_{it} - (\hat{\Phi}X)_{it} \partial_\eta \tilde{\Delta}_{it} \right)^t \hat{W}^{-1} (\hat{\Omega}X)_{it} \partial_{\bar{\omega}} (\hat{\Omega} \hat{\nu})_{it} - (\hat{\Phi})_{it} \partial_\eta \hat{f}_{it},
\]
\[
\tilde{\Delta}_{it} = \hat{\Delta}_{it} - \hat{\delta}, \text{ and } \partial_\beta \hat{\Delta}_{it} \text{ is the first-order partial derivative of } \Delta_{it} \text{ with respect to } \beta \text{ evaluated at its sample analogue. Note that the first two terms take into account the variation induced by estimating sample instead of population means, the third term captures variation due to parameter estimation also known as the delta method, and the last term is a covariance between both sources of variation that can be dropped if all explanatory variables are assumed to be strictly exogenous.}

However, the estimation of binary choice models with two-way error component is computationally challenging even in moderately large panel data sets. The same issue applies to the computation of \(\hat{\Omega}\) and \(\hat{\Phi}\) that are needed for the analytical bias-corrections. In the next section, we describe two algorithms that tackle these problems.

### 3. Computation in Large Panel Data

Recently Stammann (2018) presented a feasible and fast algorithm to estimate generalized linear models with a multi-way error component. We briefly review the algorithm for binary choice models with individual and time fixed effects and show how parts of the estimation algorithm can be used to accelerate analytical bias-corrections.

Remember, (1) has no closed form solution and thus has to be solved numerically with an iterative algorithm. Using Newton’s method, the update step in iteration \(r\) is

\[
(\hat{\theta}_{r+1} - \hat{\theta}_r) = (Z' \hat{\Omega} Z)^{-1} Z' \hat{\Omega} \hat{\nu},
\]

where \(Z = (X, D)\), \(\hat{\nu}_{it} = (y_{it} - \hat{F}_{it})/\partial_\eta \hat{F}_{it}\), and \(\hat{\theta} = (\hat{\beta}, \hat{\phi})\). Because increasing the number of observations also increases the rank of \(D\), the computation of the update step quickly becomes infeasible. Fortunately, a closer look reveals that (8) is essentially the solution of the following weighted least-squares problem:

\[
\hat{\nu} = X(\beta_{r+1} - \beta_r) + D(\phi_{r+1} - \phi_r) + u,
\]

where \(\hat{\Omega}\) is the corresponding weighting matrix. The normal equations of (9) are

\[
X' \hat{\Omega} X (\beta_{r+1} - \beta_r) + X' \hat{\Omega} D (\phi_{r+1} - \phi_r) = X' \hat{\Omega} \hat{\nu},
\]

\[
D' \hat{\Omega} X (\beta_{r+1} - \beta_r) + D' \hat{\Omega} D (\phi_{r+1} - \phi_r) = D' \hat{\Omega} \hat{\nu}.
\]
Re-arranging (11) yields
\[ \mathbf{D}(\phi_{r+1} - \phi_r) = \mathbf{\hat{P}}(\mathbf{v} - \mathbf{X}(\hat{\beta}_{r+1} - \hat{\beta}_r)) . \] (12)

Substituting (12) in (10) and exploiting that \( \mathbf{\hat{M}} \) is idempotent reveals that
\[ (\hat{\beta}_{r+1} - \hat{\beta}_r) = \left( (\mathbf{\hat{M}}\mathbf{X})'\mathbf{\hat{\Omega}}(\mathbf{\hat{M}}\mathbf{X}) \right)^{-1} (\mathbf{\hat{M}}\mathbf{X})'\mathbf{\hat{\Omega}}(\mathbf{\hat{M}}\mathbf{\hat{v}}) \]
is the weighted least-squares solution of
\[ \mathbf{\hat{M}}\mathbf{\hat{v}} = \mathbf{\hat{M}}\mathbf{X}(\hat{\beta}_{r+1} - \hat{\beta}_r) + \mathbf{u} . \] (13)

Thus similar to the linear model, we can separate the estimation of the structural from the incidental parameters.\(^4\)

However, we also need to update \( \hat{\mathbf{v}} \) and \( \mathbf{\hat{\Omega}} \) in each iteration. Both are functions of the linear predictor \( \hat{\eta} \), which is a function of the incidental parameters as well. Either we need to use a numerical solver to find estimates of the incidental parameters for a given \( \hat{\beta} \), which can be very computationally demanding, or we need to find a way to update the linear predictor itself. Fortunately, \( \hat{\eta} \) can be updated quite easily using already computed quantities. From the linear fixed effects model it is well known that the residuals of (9) and (13) are equal (see Gaure (2013b)). Some rearrangements and substituting yields
\[ (\hat{\eta}_{r+1} - \hat{\eta}_r) = \hat{\mathbf{v}} - \mathbf{\hat{M}}\mathbf{\hat{v}} + \mathbf{\hat{M}}\mathbf{X}(\hat{\beta}_{r+1} - \hat{\beta}_r) . \]

Summing up, the entire algorithm can be sketched as follows:

**Definition.** Newton’s Method

- **Initialize** \( \hat{\beta} \) and \( \hat{\eta} \); repeat the following steps until convergence

  - **Step 1:** Given \( \hat{\eta} \) compute \( \hat{\mathbf{v}} \) and \( \mathbf{\hat{\Omega}} \)
  - **Step 2:** Given \( \hat{\mathbf{v}} \) and \( \mathbf{\hat{\Omega}} \) update \( \hat{\beta} \)
  - **Step 3:** Given \( \hat{\beta} \) update \( \hat{\eta} \)

So far we have re-arranged the optimization problem such that it abstains from the estimation of potentially many incidental parameters. Unfortunately, a remaining challenge is

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\(^4\)Note that Stammann (2018) proposes an additional valid residual projection. Let \( \mathbf{\hat{M}} = \mathbf{1}_{NT} - \mathbf{\hat{P}} = \mathbf{1}_{NT} - \mathbf{D}(\mathbf{D}\mathbf{D})^{-1}\mathbf{D}' \), where \( \mathbf{D} = \mathbf{\Omega}^{1/2} \mathbf{D} \). An estimate of \( (\beta_{r+1} - \beta_r) \) can be obtained by regressing \( \mathbf{\hat{M}}\hat{\mathbf{v}} \) on \( \mathbf{\hat{M}}\mathbf{X} \), where \( \hat{\mathbf{v}} = \mathbf{\hat{\Omega}}^{1/2}\hat{\mathbf{v}} \) and \( \mathbf{X} = \mathbf{\hat{\Omega}}^{1/2}\mathbf{X} \). Thus \( \mathbf{\hat{M}}\mathbf{\hat{v}} = \mathbf{\hat{M}}\hat{\mathbf{v}} \) and \( \mathbf{\hat{\Omega}}^{1/2}\mathbf{\hat{M}}\mathbf{X} = \mathbf{\hat{M}}\mathbf{X} \). During extensive studies in the development of our R-package *alpaca*, we did not find any projection to be superior in terms of computation time. In this article, we use \( \mathbf{\hat{M}} \) because it is in line with notation used in Fernández-Val and Weidner (2016, 2018a).
the computation of \( \hat{M} \) itself. Because the residual projection is of dimension \((NT \times NT)\), the computation and storage quickly becomes infeasible. In case of a one-way error component, \( \hat{M}v \) is essentially a weighted within-transformation applied to an arbitrary vector \( v \). Thus instead of computing a large and sparse residual projection, it is more efficient to compute \( \hat{M}v \) directly by subtracting weighted group means from \( v \). Throughout the article we refer to any \( \hat{M}v \) as centered variable. However, because \( \hat{M} \) loses its sparse structure for models with a multi-way error component, we cannot derive a simple scalar expression for the general weighted within transformation of these cases.

Guimarães and Portugal (2010) and Gaure (2013b) propose another approach to obtain centered variables in the context of linear models. Combining the results of Neumann (1949) and Halperin (1962), they suggest an iterative procedure known as the method of alternating projections (MAP), which results in an arbitrary close approximation of the within transformation. MAP exploits the fact that, in case of one-way error component models, the computation of any centered variable translates into a simple scalar expression. Gaure (2013b) gives a detailed theoretical foundation of this approach in the context of linear models. Stammann (2018) shows how to extend MAP to GLMs.

In order to get an intuition how MAP works, we briefly describe an algorithm for GLMs with a two-way error component. Let \( D = (D_1, D_2) \), where \( D_1 \) and \( D_2 \) are sub matrices indicating individuals and time periods, respectively. Further we introduce the following centered variables \( \hat{M}_k v = \mathbb{1}_{NT} - D_k (D_k' \hat{\Omega} D_k)^{-1} D_k' \hat{\Omega} v \), where \( k \in \{1, 2\} \). Note that \( \hat{M}_k \) eliminates \( D_k \) and the corresponding incidental parameters from (9). The appropriate scalar expressions for the weighted within transformations are

\[
(\hat{M}_1 v)_{it} = v_{it} - \frac{\sum_{t=1}^{T} \hat{\omega}_{it} v_{it}}{\sum_{t=1}^{T} \hat{\omega}_{it}}
\]

and

\[
(\hat{M}_2 v)_{it} = v_{it} - \frac{\sum_{i=1}^{N} \hat{\omega}_{it} v_{it}}{\sum_{i=1}^{N} \hat{\omega}_{it}}
\]

The MAP algorithm for an arbitrary vector \( v \) can be described as follows:

**Definition. Method of Alternating Projections (Halperin)**

Initialize \( \hat{v} = v \); repeat the following steps until convergence

**Step 1:** Compute \( \hat{M}_1 \hat{v} \) and update \( \hat{v} \) such that \( \hat{v} = \hat{M}_1 \hat{v} \)

**Step 2:** Compute \( \hat{M}_2 \hat{v} \) and update \( \hat{v} \) such that \( \hat{v} = \hat{M}_2 \hat{v} \)

Because the algorithm only needs to evaluate scalar expressions, it is memory efficient and quite fast. Further, given an appropriate tolerance level, it returns an arbitrary close ap-
proximation to $\hat{\mathbf{v}}$, that can be used to accelerate Newton’s method as well as the analytical bias-correction (see Stammann (2018)).

Next, we give a short impression about the capabilities of the two algorithms presented here. For this we estimate an uncorrected probit model with 2,000 individuals, 52 time periods, and three explanatory variables using different R commands. More precisely, we use `feglm()` provided in our R-package `alpaca`, which is based on the algorithms described in this section, and compare it to `speedglm()` and `glm()` provided in `speedglm` (Enea (2017)) and base R (R Core Team (2019)), respectively. Our routine requires roughly half a second to estimate the model, whereas `speedglm()` and `glm()` need 22 and 1,120 seconds.

In summary, we have presented two algorithms that help to speed up the computation of binary choice models with two-way error components. In the next subsequent sections, we use both algorithms in an extensive simulation study and an empirical example from labor economics.

4. Simulation Experiments

We want to analyze the finite sample behavior of different uncorrected and bias-corrected fixed effects estimators for binary choice models. The quantities of interest are the structural parameters and average partial effects. Beside the different non-linear estimators introduced in this article, we additionally consider the linear probability model as an alternative estimator of the average partial effects. We restrict ourselves to the analysis of dynamic models, because the properties of quantities with respect to exogenous regressors are similar in static and dynamic designs (see Fernández-Val and Weidner (2016)).

Next, we describe all estimators analyzed in this simulation study. Besides the uncorrected probit estimator (MLE), we consider four different analytical bias-corrections for the structural parameters. Two of them correct the estimator itself, whereas the others are obtained by minimizing modified score equations. ABC1 is the analytical bias-correction analyzed by Fernández-Val and Weidner (2016, 2018a). ABC2 is essentially ABC1, but additionally iterated until convergence. Arellano and Hahn (2007) refer to this approach as infinitely repeated analytical bias-correction. ABC3 and ABC4 are the score-corrected estimators. They only differ in that ABC4 updates the bias terms in each iteration of the non-linear solver, whereas ABC3 treats them as fixed. The analytical bias-corrected estimators of the average partial effects are labeled analogously. Further, we consider two split-panel jackknife bias-corrected estimators that differ in their splitting strategy. SPJ1 and SPJ2 refer to the strategies used in (6) and (7), respectively. Finally, we use the analytical bias-corrected estimator for dynamic linear fixed effects models proposed by Nickell (1981), Hahn
and Kuersteiner (2002), Hahn and Moon (2006), and Fernández-Val and Weidner (2018a). Throughout the article, we denote the bias-corrected linear probability model as LPM.

We use the dynamic model design of Fernández-Val and Weidner (2016) and generate

\[ y_{it} = \mathbf{1}\left[ \rho y_{it-1} + \beta x_{it} + \alpha_i + \gamma_t \geq \epsilon_{it} \right], \]
\[ y_{i0} = \mathbf{1}\left[ \beta x_{i0} + \alpha_i + \gamma_0 \geq \epsilon_{i0} \right], \]

where \( i = 1, \ldots, N, t = s_i, \ldots, T_i \), and \( \mathbf{1}\left[ \cdot \right] \) is an indicator function. We generate \( \alpha_i \sim \text{iid. } \mathcal{N}(0, 1/16) \), \( \gamma_t \sim \text{iid. } \mathcal{N}(0, 1/16) \), and \( \epsilon_{it} \sim \text{iid. } \mathcal{N}(0, 1) \). Furthermore, we assume that the exogenous regressor follows an AR-1 process:

\[ x_{it} = 0.5 x_{it-1} + \alpha_i + \gamma_t + \nu_{it}, \quad \nu_{it} \sim \text{iid. } \mathcal{N}(0, 0.5) \]

and \( x_{i0} \sim \text{iid. } \mathcal{N}(0, 1) \). The corresponding structural parameters are \( \rho = 0.5 \) and \( \beta = 1 \).

Contrary to Fernández-Val and Weidner (2016), we analyze three different panel structures and use sample sizes that better reflect commonly used real world data sets (more individuals than time periods). More precisely, the first structure is a balanced panel, whereas the others mimic different patterns of randomly missing observations. In order to describe the different patterns, we introduce two types of individuals: type 1 and type 2. Let \( N_1 \) and \( N_2 \) denote the number of type 1 and type 2, such that \( N = N_1 + N_2 \). Further, we assume that type 1 and type 2 are observed for \( T_1 \) and \( T_2 \) consecutive time periods, respectively. In the first pattern, the time series of both types starts at \( t = 1 \), but type 1 leaves the panel at an earlier point of time, such that \( T_1 < T_2 \). The second pattern is identical in the sense that type 2 is observed longer than type 1. However, the time series of any type 1 does not necessary start at \( t = 1 \). Instead an initial period is chosen randomly for each type 1 such that \( t = s_i, \ldots, s_i + T_1 \), where \( s_i \) is sampled with equal probability from \( \{0, 1, \ldots, T_2 - T_1\} \). Figure 1 provides a graphical illustration for both of the missing data patterns. Further, we generate panel data sets of different sizes. In case of balanced data \( N = 200 \) and \( T_i = T \in \{10, 15, 20, 25, 30\} \), whereas in case of unbalanced data \( \{N_1, N_2\} \in \{\{100, 100\}, \{150, 50\}\} \) and \( \{T_1, T_2\} \in \{\{10, 30\}, \{15, 25\}\} \). The different combinations of \( \{N_1, N_2\} \) and \( \{T_1, T_2\} \) allow to analyze how the finite sample properties of the estimators are affected by the average number of individuals (\( \bar{N} \)) and time periods (\( \bar{T} \)). An overview of the

| \( N_1 \) | \( N_2 \) | \( T_1 \) | \( T_2 \) | \( \bar{N} \) | \( \bar{T} \) |
|-------|-------|-------|-------|-------|-------|
| 100   | 100   | 10    | 30    | 133   | 20    |
| 100   | 100   | 15    | 25    | 160   | 20    |
| 150   | 50    | 10    | 30    | 100   | 15    |
| 150   | 50    | 15    | 25    | 140   | 17.5  |

Table 1: Combinations - Unbalanced Panel Data
different combinations is given in table 1.

In order to analyze the finite-sample properties and ensure comparability, we follow Fernández-Val and Weidner (2016) and compute the following statistics: biases, standard deviations (SD), root mean squared errors (RMSE), average ratios of standard errors and standard deviations (SE/SD), and empirical coverage probabilities of 95% confidence intervals (CP .95). Throughout this article we report biases, SD, and RMSE in percentage relative to the truth. The average partial effects are computed using (3) and (4). Additionally, we analyze the size when testing $H_0 : \rho = 0.5 \land \beta = 1$ using a Wald test and a nominal level of 5 %. We also consider different choices of the bandwidth parameter for the analytical bias-corrections, $L \in \{1, 2, 3, 4\}$. All results are based on 1,000 replications and R version 3.5.3 (R Core Team (2019)). A complete summary of all statistics can be found in the supplementary material.  

Table 2 reports the relative biases of different analytical bias-corrected estimators of the structural parameters and average partial effects along with different choices of the bandwidth parameter. For brevity, we only present results for balanced panels and $T \in \{10, 30\}$. The relative biases of estimators corresponding to the predetermined variable are more severe than their exogenous counterpart. As expected, all corrections reduce a larger fraction

5. Additionally, we use the lfe package of Gaure (2013a) for the estimation of linear probability models and the non-linear equations solver (nleqslv) provided by Hasselman (2018) for the score-corrected analytical bias-corrections.

6. We also report results of a static data generating process and different designs of the exogenous regressor following Fernández-Val and Weidner (2016). Additionally, we provide a replication of the authors simulation study. https://github.com/dczarnowske?tab=repositories.
| Coefficients | APE |
|--------------|-----|
| L = 1 | L = 2 | L = 3 | L = 4 | L = 1 | L = 2 | L = 3 | L = 4 |
|---|---|---|---|---|---|---|---|
| Lagged Dependent Variable | Lagged Dependent Variable |
| ABC1 | -7.97 | -9.64 | -18.42 | -27.68 | -12.24 | -13.73 | -21.91 | -30.61 |
| ABC2 | -13.02 | -13.09 | -20.57 | -28.89 | -17.97 | -18.03 | -24.91 | -32.61 |
| ABC3 | -11.97 | -13.48 | -21.42 | -29.90 | -15.93 | -17.31 | -24.81 | -32.85 |
| ABC4 | -12.49 | -12.43 | -19.62 | -27.86 | -16.57 | -16.50 | -23.27 | -31.02 |
| Exogenous Regressor | Exogenous Regressor |
| ABC1 | 1.08 | 0.91 | 1.08 | 1.25 | -3.18 | -3.18 | -2.32 | -1.44 |
| ABC2 | 4.34 | 4.22 | 4.30 | 4.30 | -0.82 | -0.97 | -0.31 | 0.37 |
| ABC3 | 1.28 | 1.18 | 1.55 | 1.91 | -2.62 | -2.59 | -1.72 | -0.83 |
| ABC4 | 1.65 | 1.50 | 1.84 | 2.05 | -2.35 | -2.54 | -1.77 | -0.98 |
| N = 200, T = 10 |
|---|---|---|---|---|---|---|---|
| Lagged Dependent Variable | Lagged Dependent Variable |
| ABC1 | -3.20 | -1.27 | -1.84 | -2.69 | -4.11 | -2.20 | -2.76 | -3.60 |
| ABC2 | -3.92 | -1.91 | -2.38 | -3.16 | -4.98 | -3.00 | -3.47 | -4.24 |
| ABC3 | -3.56 | -1.69 | -2.24 | -3.07 | -4.46 | -2.60 | -3.14 | -3.97 |
| ABC4 | -3.82 | -1.82 | -2.28 | -3.06 | -4.74 | -2.75 | -3.21 | -3.99 |
| Exogenous Regressor | Exogenous Regressor |
| ABC1 | 0.52 | 0.40 | 0.43 | 0.48 | -0.38 | -0.61 | -0.54 | -0.44 |
| ABC2 | 0.94 | 0.84 | 0.87 | 0.92 | -0.06 | -0.29 | -0.24 | -0.15 |
| ABC3 | 0.53 | 0.40 | 0.44 | 0.49 | -0.33 | -0.56 | -0.49 | -0.39 |
| ABC4 | 0.60 | 0.46 | 0.51 | 0.57 | -0.26 | -0.51 | -0.45 | -0.35 |
of the bias as \( T \) increases. Further, the differences between the estimators appear most in case of \( T = 10 \), where \( ABC2-ABC4 \) are clearly dominated by \( ABC1 \). This also holds for \( T = 30 \), but the differences in relative biases become negligible small. If we additionally take into account that the other bias-corrections are much more computationally demanding, \( ABC1 \) is clearly preferable. Additionally, we find that values of \( L \in \{1, 2\} \) are the most appropriate bandwidth choices for our chosen panel sizes.

Next, we want to compare the two different split-panel jackknife estimators described in this article. Again for brevity we restrict ourselves to the case of balanced panels and note that we find the same for unbalanced panels. The results are reported in table 3. Similar to the analytical correction, the bias reduction improves as \( T \) increases. We find almost identical properties of both estimators which is remarkably, because we would expect that the splitting strategy of \( SPJ2 \) leads to higher dispersion due to the use of smaller sub panels. Only for estimators of the structural parameters and \( T = 10 \), we observe that the relative bias and dispersion of \( SPJ1 \) is slightly lower. For the average partial effects, we observe that the properties of both estimators are indistinguishable irrespective of the sample size. Also note that \( SPJ2 \) is computationally less demanding, because the model is re-estimated on smaller sub panels.

In the following, we focus on comparing the small sample properties of \( MLE, ABC1, SPJ1, \) and \( LPM \). Parentheses indicate the corresponding choice of the bandwidth parameter. Table 4 and 5 report the results based on balanced panel data sets. First, we find that the

|                | Coefficients | APE                   |
|----------------|--------------|-----------------------|
|                |              | Lagged Dependent Variable |                      |
| N = 200, T = 10| 19.38        | 23.09                 | -12.38               |
| N = 200, T = 15| 0.56         | 15.54                 | 1.09                 |
| N = 200, T = 20| 3.51         | 12.59                 | 3.88                 |
| N = 200, T = 25| 0.42         | 10.73                 | 0.65                 |
| N = 200, T = 30| 1.27         | 9.54                  | 1.45                 |
|                |              |                       | -12.31               |
|                |              |                       | 21.02                |
|                |              | Exogenous Regressor   |                      |
| N = 200, T = 10| -7.40        | 9.36                  | -10.07               |
| N = 200, T = 15| -1.14        | 5.80                  | -2.05                |
| N = 200, T = 20| -1.72        | 4.60                  | -2.39                |
| N = 200, T = 25| -0.60        | 4.04                  | -1.01                |
| N = 200, T = 30| -0.83        | 3.57                  | -1.14                |

Table 3: Bias and Dispersion (in %) - Split-Panel Jackknife Bias-Corrections
### Table 4: Finite Sample Properties - Balanced - Lagged Dependent Variable

|                | Coefficients | APE          |
|----------------|--------------|--------------|
|                | Bias  SD  RMSE SE/SD | CP .95  Bias  SD  RMSE SE/SD | CP .95 |
| N = 200; T = 10|              |              |              |
| MLE            | -64 18 67 0.95 0.04 | -71 15 73 1.06 0.01 |
| ABC1 (1)       | -8 16 18 1.11 0.95 | -12 17 21 1.10 0.91 |
| ABC1 (2)       | -10 17 20 1.02 0.93 | -14 18 23 1.01 0.89 |
| SPJ1           | 19 23 30 0.76 0.74 | -12 21 24 0.92 0.87 |
| LPM (1)        |              | 5 18 19 0.94 0.92 |
| LPM (2)        |              | 7 20 21 0.86 0.89 |
| N = 200; T = 15|              |              |              |
| MLE            | -42 14 44 0.97 0.14 | -49 13 51 1.01 0.04 |
| ABC1 (1)       | -5 13 14 1.07 0.95 | -8 14 16 1.02 0.92 |
| ABC1 (2)       | -4 13 14 1.01 0.95 | -6 15 16 0.97 0.92 |
| SPJ1           | 1 16 16 0.88 0.92 | -10 16 19 0.89 0.85 |
| LPM (1)        |              | 10 15 18 0.91 0.86 |
| LPM (2)        |              | 13 16 21 0.86 0.79 |
| N = 200; T = 20|              |              |              |
| MLE            | -31 12 33 0.98 0.24 | -37 11 39 0.99 0.09 |
| ABC1 (1)       | -4 11 12 1.06 0.94 | -6 12 13 1.01 0.92 |
| ABC1 (2)       | -2 11 12 1.01 0.95 | -4 12 13 0.97 0.93 |
| SPJ1           | 4 13 13 0.92 0.92 | -3 14 14 0.90 0.91 |
| LPM (1)        |              | 12 12 18 0.94 0.80 |
| LPM (2)        |              | 15 13 20 0.90 0.72 |
| N = 200; T = 25|              |              |              |
| MLE            | -24 10 26 0.99 0.34 | -30 10 32 1.00 0.16 |
| ABC1 (1)       | -3 10 10 1.05 0.94 | -4 10 11 1.02 0.94 |
| ABC1 (2)       | -1 10 10 1.01 0.95 | -2 11 11 0.98 0.94 |
| SPJ1           | 0 11 11 0.95 0.94 | -3 11 12 0.94 0.92 |
| LPM (1)        |              | 14 11 18 0.91 0.70 |
| LPM (2)        |              | 17 12 21 0.88 0.62 |
| N = 200; T = 30|              |              |              |
| MLE            | -21 9 23 1.01 0.39 | -26 9 27 1.00 0.20 |
| ABC1 (1)       | -3 9 9 1.07 0.95 | -4 10 10 1.02 0.94 |
| ABC1 (2)       | -1 9 9 1.04 0.95 | -2 10 10 1.00 0.94 |
| SPJ1           | 1 10 10 0.97 0.93 | -1 10 10 0.94 0.93 |
| LPM (1)        |              | 15 10 18 0.93 0.66 |
| LPM (2)        |              | 17 10 20 0.91 0.57 |
## Table 5: Finite Sample Properties - Balanced - Exogenous Regressor

|                | Coefficients |                   |                 | APE  |                   |                 |
|----------------|--------------|-------------------|-----------------|------|-------------------|-----------------|
|                | Bias         | SD                | RMSE            | SE/SD| CP .95            | Bias            |
| N = 200; T = 10|              |                   |                 |      |                   |                 |
| MLE            | 22           | 8                 | 23              | 0.85 | 0.14              | 3               | 7               | 7               | 1.08            | 0.93            |
| ABC1 (1)       | 1            | 6                 | 7               | 1.08 | 0.96              | -3              | 7               | 8               | 1.01            | 0.94            |
| ABC1 (2)       | 1            | 7                 | 7               | 1.07 | 0.96              | -3              | 7               | 8               | 1.01            | 0.94            |
| SPJ1           | -7           | 9                 | 12              | 0.75 | 0.73              | 6               | 8               | 10              | 0.92            | 0.84            |
| LPM (1)        |              |                   |                 |      |                   | 0               | 6               | 6               | 0.81            | 0.88            |
| LPM (2)        |              |                   |                 |      |                   | 0               | 6               | 6               | 0.81            | 0.88            |
|                |              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| N = 200; T = 15|              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| MLE            | 14           | 6                 | 15              | 0.94 | 0.21              | 3               | 5               | 6               | 1.00            | 0.90            |
| ABC1 (1)       | 1            | 5                 | 5               | 1.10 | 0.97              | -2              | 5               | 6               | 0.96            | 0.94            |
| ABC1 (2)       | 1            | 5                 | 5               | 1.10 | 0.97              | -2              | 5               | 6               | 0.96            | 0.93            |
| SPJ1           | -1           | 6                 | 6               | 0.90 | 0.91              | 2               | 5               | 6               | 0.94            | 0.89            |
| LPM (1)        |              |                   |                 |      |                   | 0               | 5               | 5               | 0.81            | 0.89            |
| LPM (2)        |              |                   |                 |      |                   | 0               | 5               | 5               | 0.81            | 0.89            |
|                |              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| N = 200; T = 20|              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| MLE            | 11           | 5                 | 12              | 0.93 | 0.28              | 2               | 4               | 5               | 0.97            | 0.90            |
| ABC1 (1)       | 1            | 4                 | 4               | 1.05 | 0.96              | -1              | 5               | 5               | 0.94            | 0.94            |
| ABC1 (2)       | 1            | 4                 | 4               | 1.04 | 0.96              | -1              | 5               | 5               | 0.94            | 0.93            |
| SPJ1           | -2           | 5                 | 5               | 0.95 | 0.91              | 1               | 5               | 5               | 0.96            | 0.93            |
| LPM (1)        |              |                   |                 |      |                   | -0              | 4               | 4               | 0.80            | 0.89            |
| LPM (2)        |              |                   |                 |      |                   | -0              | 4               | 4               | 0.80            | 0.88            |
|                |              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| N = 200; T = 25|              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| MLE            | 8            | 4                 | 9               | 0.96 | 0.39              | 2               | 4               | 4               | 0.92            | 0.91            |
| ABC1 (1)       | 1            | 4                 | 4               | 1.05 | 0.96              | -1              | 4               | 4               | 0.90            | 0.92            |
| ABC1 (2)       | 0            | 4                 | 4               | 1.05 | 0.96              | -1              | 4               | 4               | 0.90            | 0.92            |
| SPJ1           | -1           | 4                 | 4               | 0.94 | 0.93              | 1               | 4               | 4               | 0.89            | 0.92            |
| LPM (1)        |              |                   |                 |      |                   | -0              | 4               | 4               | 0.74            | 0.85            |
| LPM (2)        |              |                   |                 |      |                   | -0              | 4               | 4               | 0.74            | 0.85            |
|                |              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| N = 200; T = 30|              |                   |                 |      |                   |                 |                 |                 |                 |                 |
| MLE            | 7            | 4                 | 8               | 0.95 | 0.47              | 2               | 4               | 4               | 0.96            | 0.90            |
| ABC1 (1)       | 1            | 3                 | 3               | 1.03 | 0.96              | -0              | 4               | 4               | 0.94            | 0.93            |
| ABC1 (2)       | 0            | 3                 | 3               | 1.03 | 0.95              | -1              | 4               | 4               | 0.94            | 0.93            |
| SPJ1           | -1           | 4                 | 4               | 0.96 | 0.93              | 1               | 4               | 4               | 0.95            | 0.94            |
| LPM (1)        |              |                   |                 |      |                   | -0              | 4               | 4               | 0.75            | 0.85            |
| LPM (2)        |              |                   |                 |      |                   | 0               | 4               | 4               | 0.75            | 0.85            |
properties of the estimators that refer to effect of the predetermined variable are worse than those that are related to the exogenous regressor. For instance, we observe larger relative biases and dispersion as well as coverage probabilities further away from their nominal level. The relative distortion we find in the coefficients is also reflected in the estimates of the average partial effects. That is contrary to the results we observe with regard to the average partial effects of the exogenous regressor, where we can only find negligibly small relative biases.\footnote{Hahn and Newey (2004), Fernández-Val (2009), and Fernández-Val and Weidner (2016) also find only small biases in average partial effects of the exogenous regressor.} Generally, the bias corrections work as expected. They reduce the relative biases and improve the coverage probabilities. As in Fernández-Val and Weidner (2016), the properties of \textit{SPJ1} are worse than those of \textit{ABC1}. Another interesting insight can be learned from \textit{LPM}. In case of the exogenous regressor, the estimators do not show any distortion, but valid inference is questionable, because standard errors are underestimated and coverage probabilities are lower than their nominal level. For the predetermined variable, there is also the curiosity that the relative bias increases in $T$.\footnote{In order to ensure that this is not due to a weird programming error, we add a small simulation study in the appendix. Here we apply the bias-corrected estimator to a standard data generating process for dynamic linear fixed effects models.}

Next, we analyze how the different patterns of unbalancedness affect the properties of the estimators. Our results, summarized in table 6–9, support the conjecture of Fernández-Val and Weidner (2018a) that the asymptotic distribution of \textit{MLE}, in case of randomly missing observations, depends on $N$ and $T$. This also applies to the properties of \textit{ABC1}. Whereas the missing data pattern does not matter for \textit{ABC1} and \textit{LPM}, it affects the properties of \textit{SPJ1}. In particular, \textit{SPJ1} cannot handle pattern 1 well. This can be seen, for example, from the fact that the reduction in distortion is worsening and the dispersion is also increasing. An intuitive explanation is that the splitting strategy leads to sub panels of widely differing sizes. This effect is not that strong in pattern 2, but the performance is still worse than in the balanced case.

Table 10 reports the sizes of a Wald test. Whereas the sizes of a test based on \textit{MLE} are heavily distorted, using bias-corrected estimators to construct test-statistics brings them closer to their nominal level. \textit{ABC1} strictly dominates \textit{SPJ1} as its sizes are always close to their nominal level.

Finally, we can conclude that the different analytical corrections (\textit{ABC1–ABC4}) and the different splitting strategies ((6) and (7)) work similarly well with each other. Further, we find that \textit{ABC} is preferable to \textit{SPJ} and \textit{LPM}. From a practical point of view, the latter have an advantage in that they are relatively easy to implement, but this is associated with performance losses. Firstly, \textit{SPJ} has a higher distortion than \textit{ABC}, which is particularly
|                   | Coefficients |               |               |               | APE          |               |               |               |               |
|-------------------|--------------|---------------|---------------|---------------|--------------|---------------|---------------|---------------|---------------|
|                   | Bias         | SD            | RMSE          | SE/SD         | CP .95      | Bias          | SD            | RMSE          | SE/SD         | CP .95      |
| MLE               | -30          | 12            | 32            | 1.01          | 0.28        | -36           | 11            | 38            | 1.03          | 0.10        |
| ABC1 (1)          | -5           | 11            | 12            | 1.09          | 0.95        | -6            | 11            | 13            | 1.05          | 0.92        |
| ABC1 (2)          | -3           | 11            | 11            | 1.05          | 0.95        | -5            | 12            | 13            | 1.01          | 0.93        |
| SPJ1              | -14          | 13            | 19            | 0.93          | 0.76        | -19           | 13            | 23            | 0.93          | 0.64        |
| LPM (1)           |              |               |               |               |             | 13            | 12            | 18            | 0.96          | 0.79        |
| LPM (2)           |              |               |               |               |             | 16            | 13            | 21            | 0.93          | 0.70        |

|                   | Coefficients |               |               |               | APE          |               |               |               |               |
|                   | Bias         | SD            | RMSE          | SE/SD         | CP .95      | Bias          | SD            | RMSE          | SE/SD         | CP .95      |
| MLE               | -30          | 12            | 32            | 0.99          | 0.26        | -37           | 11            | 38            | 1.01          | 0.10        |
| ABC1 (1)          | -4           | 11            | 11            | 1.07          | 0.94        | -5            | 12            | 13            | 1.02          | 0.93        |
| ABC1 (2)          | -2           | 11            | 11            | 1.03          | 0.94        | -3            | 12            | 13            | 0.98          | 0.93        |
| SPJ1              | -0           | 13            | 13            | 0.91          | 0.93        | -6            | 13            | 15            | 0.92          | 0.89        |
| LPM (1)           |              |               |               |               |             | 13            | 12            | 18            | 0.94          | 0.78        |
| LPM (2)           |              |               |               |               |             | 16            | 13            | 21            | 0.90          | 0.69        |

|                   | Coefficients |               |               |               | APE          |               |               |               |               |
|                   | Bias         | SD            | RMSE          | SE/SD         | CP .95      | Bias          | SD            | RMSE          | SE/SD         | CP .95      |
| MLE               | -40          | 14            | 43            | 1.00          | 0.17        | -48           | 12            | 50            | 1.05          | 0.03        |
| ABC1 (1)          | -6           | 12            | 14            | 1.10          | 0.94        | -9            | 13            | 16            | 1.08          | 0.92        |
| ABC1 (2)          | -6           | 13            | 14            | 1.04          | 0.94        | -8            | 14            | 16            | 1.02          | 0.91        |
| SPJ1              | -32          | 16            | 36            | 0.85          | 0.39        | -38           | 15            | 41            | 0.90          | 0.24        |
| LPM (1)           |              |               |               |               |             | 11            | 14            | 18            | 0.95          | 0.86        |
| LPM (2)           |              |               |               |               |             | 14            | 15            | 20            | 0.90          | 0.80        |

|                   | Coefficients |               |               |               | APE          |               |               |               |               |
|                   | Bias         | SD            | RMSE          | SE/SD         | CP .95      | Bias          | SD            | RMSE          | SE/SD         | CP .95      |
| MLE               | -35          | 13            | 37            | 0.93          | 0.21        | -42           | 13            | 44            | 0.95          | 0.08        |
| ABC1 (1)          | -5           | 12            | 13            | 1.02          | 0.94        | -6            | 13            | 15            | 0.96          | 0.91        |
| ABC1 (2)          | -3           | 13            | 13            | 0.97          | 0.94        | -5            | 14            | 15            | 0.92          | 0.91        |
| SPJ1              | 8            | 17            | 19            | 0.72          | 0.82        | -4            | 16            | 17            | 0.80          | 0.88        |
| LPM (1)           |              |               |               |               |             | 12            | 14            | 18            | 0.89          | 0.81        |
| LPM (2)           |              |               |               |               |             | 15            | 15            | 21            | 0.85          | 0.75        |

Table 6: Properties - Unbalanced 1 - Lagged Dependent Variable

N₁ = 100; N₂ = 100; T₁ = 10; T₂ = 30

N₁ = 100; N₂ = 100; T₁ = 15; T₂ = 25

N₁ = 150; N₂ = 50; T₁ = 10; T₂ = 30

N₁ = 150; N₂ = 50; T₁ = 15; T₂ = 25
Table 7: Properties - Unbalanced 1 - Exogenous Regressor

| Coefficients | APE          |
|--------------|--------------|
|              | Bias | SD  | RMSE | SE/SD | CP .95 | Bias | SD  | RMSE | SE/SD | CP .95 |
| N₁ = 100; N₂ = 100; T₁ = 10; T₂ = 30 |
| MLE          | 11   | 5   | 12   | 0.89  | 0.35   | 2    | 4   | 5    | 0.97  | 0.92   |
| ABC1 (1)     | 1    | 4   | 4    | 1.00  | 0.95   | -1   | 4   | 5    | 0.95  | 0.95   |
| ABC1 (2)     | 1    | 4   | 4    | 1.00  | 0.96   | -1   | 5   | 5    | 0.95  | 0.94   |
| SPJ1         | 3    | 5   | 6    | 0.87  | 0.85   | 1    | 5   | 5    | 0.90  | 0.91   |
| LPM (1)      |      |     |      |       |        | -1   | 4   | 4    | 0.80  | 0.89   |
| LPM (2)      |      |     |      |       |        | -1   | 4   | 4    | 0.80  | 0.88   |

| N₁ = 100; N₂ = 100; T₁ = 15; T₂ = 25 |
| MLE          | 11   | 5   | 12   | 0.95  | 0.31   | 2    | 4   | 5    | 1.01  | 0.91   |
| ABC1 (1)     | 1    | 4   | 4    | 1.06  | 0.96   | -1   | 4   | 4    | 0.98  | 0.94   |
| ABC1 (2)     | 1    | 4   | 4    | 1.06  | 0.96   | -1   | 4   | 4    | 0.98  | 0.94   |
| SPJ1         | 0    | 5   | 5    | 0.86  | 0.91   | 3    | 5   | 6    | 0.90  | 0.84   |
| LPM (1)      |      |     |      |       |        | -0   | 4   | 4    | 0.83  | 0.88   |
| LPM (2)      |      |     |      |       |        | -0   | 4   | 4    | 0.83  | 0.89   |

| N₁ = 150; N₂ = 50; T₁ = 10; T₂ = 30 |
| MLE          | 15   | 6   | 16   | 0.88  | 0.21   | 2    | 5   | 5    | 1.01  | 0.92   |
| ABC1 (1)     | 1    | 5   | 5    | 1.03  | 0.95   | -1   | 5   | 5    | 0.99  | 0.95   |
| ABC1 (2)     | 1    | 5   | 5    | 1.03  | 0.95   | -2   | 5   | 5    | 0.98  | 0.94   |
| SPJ1         | 8    | 7   | 11   | 0.79  | 0.63   | 3    | 6   | 7    | 0.87  | 0.87   |
| LPM (1)      |      |     |      |       |        | -0   | 5   | 5    | 0.81  | 0.88   |
| LPM (2)      |      |     |      |       |        | -0   | 5   | 5    | 0.80  | 0.87   |

| N₁ = 150; N₂ = 50; T₁ = 15; T₂ = 25 |
| MLE          | 12   | 5   | 14   | 0.91  | 0.27   | 2    | 5   | 5    | 0.96  | 0.90   |
| ABC1 (1)     | 1    | 5   | 5    | 1.05  | 0.97   | -1   | 5   | 5    | 0.94  | 0.93   |
| ABC1 (2)     | 1    | 5   | 5    | 1.05  | 0.97   | -1   | 5   | 5    | 0.93  | 0.93   |
| SPJ1         | -2   | 7   | 7    | 0.71  | 0.82   | 6    | 6   | 8    | 0.81  | 0.69   |
| LPM (1)      |      |     |      |       |        | -0   | 5   | 5    | 0.79  | 0.88   |
| LPM (2)      |      |     |      |       |        | -0   | 5   | 5    | 0.79  | 0.88   |
Table 8: Properties - Unbalanced 2 - Lagged Dependent Variable

| Coefficients | APE |
|--------------|-----|
|              | Bias | SD  | RMSE | SE/SD | CP .95 | Bias | SD  | RMSE | SE/SD | CP .95 |
| N₁ = 100; N₂ = 100; T₁ = 10; T₂ = 30 |
| MLE          | -30  | 12  | 32   | 1.00  | 0.26   | -37  | 11  | 39   | 1.02  | 0.10  |
| ABC1 (1)     | -5   | 11  | 12   | 1.07  | 0.95   | -7   | 12  | 13   | 1.04  | 0.92  |
| ABC1 (2)     | -4   | 11  | 12   | 1.03  | 0.95   | -6   | 12  | 13   | 1.01  | 0.92  |
| SPJ1         | -9   | 12  | 15   | 0.94  | 0.87   | -14  | 13  | 19   | 0.94  | 0.77  |
| LPM (1)      |      |     |      |       |        | 13   | 12  | 18   | 0.94  | 0.78  |
| LPM (2)      |      |     |      |       |        | 16   | 13  | 20   | 0.91  | 0.70  |

| N₁ = 100; N₂ = 100; T₁ = 15; T₂ = 25 |
| MLE          | -30  | 11  | 32   | 1.01  | 0.24   | -37  | 11  | 39   | 1.02  | 0.10  |
| ABC1 (1)     | -4   | 11  | 11   | 1.10  | 0.96   | -6   | 12  | 13   | 1.03  | 0.94  |
| ABC1 (2)     | -2   | 11  | 11   | 1.05  | 0.96   | -4   | 12  | 13   | 0.99  | 0.94  |
| SPJ1         | -1   | 13  | 12   | 0.94  | 0.94   | -7   | 13  | 15   | 0.92  | 0.88  |
| LPM (1)      |      |     |      |       |        | 13   | 12  | 18   | 0.94  | 0.79  |
| LPM (2)      |      |     |      |       |        | 16   | 13  | 20   | 0.90  | 0.71  |

| N₁ = 150; N₂ = 50; T₁ = 10; T₂ = 30 |
| MLE          | -39  | 13  | 42   | 0.95  | 0.19   | -47  | 13  | 49   | 1.00  | 0.06  |
| ABC1 (1)     | -6   | 13  | 14   | 1.05  | 0.93   | -8   | 14  | 16   | 1.02  | 0.91  |
| ABC1 (2)     | -5   | 14  | 15   | 0.99  | 0.93   | -7   | 14  | 16   | 0.98  | 0.90  |
| SPJ1         | -19  | 15  | 24   | 0.91  | 0.70   | -26  | 15  | 30   | 0.93  | 0.51  |
| LPM (1)      |      |     |      |       |        | 12   | 15  | 19   | 0.92  | 0.85  |
| LPM (2)      |      |     |      |       |        | 15   | 16  | 22   | 0.87  | 0.78  |

| N₁ = 150; N₂ = 50; T₁ = 15; T₂ = 25 |
| MLE          | -35  | 12  | 37   | 1.00  | 0.19   | -42  | 12  | 44   | 1.03  | 0.06  |
| ABC1 (1)     | -5   | 11  | 12   | 1.09  | 0.95   | -7   | 12  | 14   | 1.05  | 0.93  |
| ABC1 (2)     | -3   | 12  | 12   | 1.04  | 0.96   | -5   | 13  | 14   | 1.01  | 0.93  |
| SPJ1         | -3   | 14  | 14   | 0.92  | 0.93   | -10  | 14  | 17   | 0.92  | 0.86  |
| LPM (1)      |      |     |      |       |        | 11   | 13  | 17   | 0.95  | 0.83  |
| LPM (2)      |      |     |      |       |        | 14   | 14  | 20   | 0.91  | 0.76  |
Table 9: Properties - Unbalanced 2 - Exogenous Regressor

|            |       |       |       |       |       |       |       |       |       |       |       |       |       |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|            | Bias  | SD    | RMSE  | SE/SD | CP .95| Bias  | SD    | RMSE  | SE/SD | CP .95| Bias  | SD    | RMSE  | SE/SD | CP .95|
| N1 = 100; N2 = 100; T1 = 10; T2 = 30 |
| MLE        | 11    | 5     | 12    | 0.95  | 0.30  | 2     | 4     | 5     | 0.99  | 0.91  | 2     | 4     | 5     | 0.97  | 0.92  |
| ABC1 (1)   | 1     | 4     | 4     | 1.06  | 0.95  | -1    | 4     | 4     | 0.98  | 0.95  | -1    | 4     | 4     | 0.97  | 0.94  |
| ABC1 (2)   | 1     | 4     | 4     | 1.06  | 0.96  | -1    | 4     | 4     | 0.97  | 0.94  | -1    | 4     | 4     | 0.97  | 0.92  |
| SPJ1       | 2     | 5     | 5     | 0.96  | 0.91  | 2     | 4     | 5     | 0.97  | 0.92  | -1    | 4     | 4     | 0.84  | 0.90  |
| LPM (1)    | -0    | 4     | 4     | 0.83  | 0.90  | -0    | 4     | 4     | 0.83  | 0.90  | -0    | 4     | 4     | 0.83  | 0.90  |
| LPM (2)    | -0    | 4     | 4     | 0.83  | 0.90  | -0    | 4     | 4     | 0.83  | 0.90  | -0    | 4     | 4     | 0.83  | 0.90  |

| N1 = 100; N2 = 100; T1 = 15; T2 = 25 |
| MLE        | 11    | 5     | 12    | 0.92  | 0.32  | 2     | 4     | 5     | 0.97  | 0.90  | 2     | 4     | 5     | 0.97  | 0.93  |
| ABC1 (1)   | 1     | 4     | 4     | 1.03  | 0.96  | -1    | 4     | 4     | 0.95  | 0.93  | -1    | 4     | 4     | 0.95  | 0.93  |
| ABC1 (2)   | 0     | 4     | 4     | 1.03  | 0.96  | -1    | 4     | 4     | 0.95  | 0.93  | -1    | 4     | 5     | 0.93  | 0.91  |
| SPJ1       | 0     | 5     | 5     | 0.89  | 0.93  | 2     | 5     | 5     | 0.93  | 0.91  | -1    | 5     | 5     | 0.89  | 0.89  |
| LPM (1)    | -0    | 4     | 4     | 0.80  | 0.88  | -0    | 4     | 4     | 0.80  | 0.88  | -0    | 4     | 4     | 0.80  | 0.88  |
| LPM (2)    | -0    | 4     | 4     | 0.80  | 0.88  | -0    | 4     | 4     | 0.80  | 0.88  | -0    | 4     | 4     | 0.80  | 0.88  |

| N1 = 150; N2 = 50; T1 = 10; T2 = 30 |
| MLE        | 14    | 6     | 16    | 0.91  | 0.23  | 2     | 5     | 5     | 1.01  | 0.93  | 2     | 5     | 5     | 1.01  | 0.93  |
| ABC1 (1)   | 1     | 5     | 5     | 1.06  | 0.96  | -1    | 5     | 5     | 1.01  | 0.94  | -1    | 5     | 5     | 1.01  | 0.95  |
| ABC1 (2)   | 1     | 5     | 5     | 1.06  | 0.96  | -2    | 5     | 5     | 1.01  | 0.95  | -2    | 5     | 5     | 1.01  | 0.95  |
| SPJ1       | 5     | 6     | 8     | 0.91  | 0.80  | 3     | 5     | 6     | 0.97  | 0.89  | -1    | 5     | 5     | 0.86  | 0.89  |
| LPM (1)    | -1    | 5     | 5     | 0.86  | 0.89  | -1    | 5     | 5     | 0.86  | 0.89  | -1    | 5     | 5     | 0.86  | 0.89  |
| LPM (2)    | -1    | 5     | 5     | 0.86  | 0.89  | -1    | 5     | 5     | 0.86  | 0.89  | -1    | 5     | 5     | 0.86  | 0.89  |

| N1 = 150; N2 = 50; T1 = 15; T2 = 25 |
| MLE        | 12    | 5     | 13    | 0.93  | 0.27  | 2     | 5     | 5     | 0.96  | 0.90  | 2     | 5     | 5     | 0.96  | 0.90  |
| ABC1 (1)   | 1     | 4     | 4     | 1.07  | 0.96  | -1    | 5     | 5     | 0.94  | 0.94  | -1    | 5     | 5     | 0.94  | 0.94  |
| ABC1 (2)   | 0     | 4     | 4     | 1.06  | 0.96  | -1    | 5     | 5     | 0.94  | 0.93  | -1    | 5     | 5     | 0.94  | 0.93  |
| SPJ1       | 0     | 5     | 5     | 0.92  | 0.93  | 3     | 5     | 5     | 0.92  | 0.89  | -1    | 5     | 5     | 0.81  | 0.89  |
| LPM (1)    | -0    | 5     | 5     | 0.81  | 0.89  | -0    | 5     | 5     | 0.81  | 0.89  | -0    | 5     | 5     | 0.81  | 0.89  |
| LPM (2)    | -0    | 5     | 5     | 0.81  | 0.89  | -0    | 5     | 5     | 0.81  | 0.89  | -0    | 5     | 5     | 0.81  | 0.89  |
Table 10: Size of a Wald Test - Nominal Level = 5 %

|            | MLE | ABC1 | SPJ1 |
|------------|-----|------|------|
|            | L = 1 | L = 2 |      |
| Balanced   |      |      |      |
| N = 200; T = 10 | 0.99  | 0.04 | 0.07  | 0.37  |
| N = 200; T = 15 | 0.96  | 0.04 | 0.04  | 0.10  |
| N = 200; T = 20 | 0.90  | 0.04 | 0.04  | 0.10  |
| N = 200; T = 25 | 0.80  | 0.05 | 0.05  | 0.08  |
| N = 200; T = 30 | 0.76  | 0.04 | 0.04  | 0.08  |
| Unbalanced 1 |      |      |      |
| N₁ = 100; N₂ = 100; T₁ = 10; T₂ = 30 | 0.86  | 0.05 | 0.05  | 0.25  |
| N₁ = 150; N₂ = 50; T₁ = 10; T₂ = 30 | 0.94  | 0.05 | 0.06  | 0.69  |
| N₁ = 150; N₂ = 50; T₁ = 15; T₂ = 25 | 0.91  | 0.05 | 0.05  | 0.25  |
| Unbalanced 2 |      |      |      |
| N₁ = 100; N₂ = 100; T₁ = 10; T₂ = 30 | 0.88  | 0.05 | 0.05  | 0.14  |
| N₁ = 150; N₂ = 50; T₁ = 10; T₂ = 30 | 0.93  | 0.04 | 0.04  | 0.33  |
| N₁ = 150; N₂ = 50; T₁ = 15; T₂ = 25 | 0.93  | 0.04 | 0.04  | 0.08  |

severe in the case of unbalanced panels. On the other hand, the inference based on LPM is questionable.

In the next section, we apply MLE, ABC1, SPJ1, and LPM to an empirical example of labor economics where we investigate the inter-temporal labor force participation of 10,712 women between 1984 and 2013.

5. Empirical Illustration

In the following, we illustrate one possible area of application by analyzing the inter-temporal labor-force participation of women using longitudinal data from the German Socio Economic Panel (GSOEP). More precisely, we want to examine how fertility decisions and the presence of non-labor income jointly affect women’s participation decisions in the labor market.

For a long time labor economists are concerned with fertility decisions being endogenous due to correlation with multiple unobserved variables. Most studies use cross-sectional data along with an instrumental variable strategy to deal with this problem (see among others Angrist and Evans (1998)). However, the availability of comprehensive panel data sets offers new reliefs to researchers. For instance, Heckman and MaCurdy (1980, 1982), Hyslop (1999), and Carro (2007) use panel data from the Panel Study of Income Dynamics (PSID), which allows them to tackle this omitted variables problem by controlling for individual specific
unobserved effects.

We use an empirical strategy similar to Hyslop (1999) and estimate the following dynamic binary choice model:

\[ y_{it} = 1 \left[ \rho y_{it-1} + x'_{it} \beta + z'_{it} \pi + \alpha_i + \gamma_t + e_{it} \geq 0 \right] \]

where \( i = 1, \ldots, N \) and \( t = s_i, \ldots, T_i \) are individual and time specific identifiers, \( y_{it} \) is an indicator equal to one if woman \( i \) is in labor-force at time period \( t \), \( x_{it} \) and \( z_{it} \) are vectors of explanatory and further control variables, \( \gamma \), \( \beta \), and \( \pi \) are the corresponding parameters, and \( e_{it} \) is an idiosyncratic error term assumed to be independent and identically distributed standard normal. More precisely, we consider the following explanatory variables: number of children in different age groups, non-labor income, and an indicator that is equal to one if a birth occurs in the next time period. Further controls are martial status, regional identifier, number of children between zero and one in the previous period, and number of other household members. Additionally, we include individual and time specific intercepts to control for unobserved heterogeneity. For instance, \( \alpha_i \) captures individual specific taste for labor and permanent income, whereas \( \gamma_t \) controls for the business cycle and other time specific shifts in preferences.

For our analysis, we extract an unbalanced panel data set of 10,712 women from the GSOEP.\(^9\) Because we want to estimate a dynamic model of labor supply, we restrict the sample to women between 16 and 65 that are observed consecutively for at least five years and do not receive any retirement income. A woman is assumed to participate in labor-force if she has positive income from individual labor and works at least 52 hours a year. Further, a proxy for transitory non-labor income is constructed from post-government household income minus woman’s individual labor earnings. Note that all income variables are converted to constant 2010 EURO and that labor earnings are reported before taxes. Thus we additionally correct labor income by a household specific tax rate. In order to make income comparable between different household sizes, we use an equivalence scale proposed by Buhmann et al. (1988). More precisely, we divide the transitory non-labor income by the square root of household members. In order to analyze whether the effect of transitory non-labor income on participation decisions differs across groups, we define the following three income classes: lower, middle, and upper. A woman belongs to the lower class if she has a non-labor income of less than 11,278 EURO at her disposal. Contrary a woman is in the upper income class if she has more than 56,391 EURO available. Women in between this interval belong to the middle class. Those numbers are equal to 60% and 300% of the annual median equiva-

\(^9\) More precisely, we use the $PEQUV$-File from 1984–2013 (version 30 of the GSOEP).
The class distinction is taken from the Armuts- und Reichtumsbericht of the federal government. Further, we follow Grabka (2014) and construct regional identifiers. Therefore, the federal states are grouped in four geographic regions (north, south, west, and east) which allows us to control for regional differences in preferences for labor.

Table 11: Descriptive Statistics

|                      | Full   |       | Always |       | Never  |       | Movers |       |
|----------------------|--------|-------|--------|-------|--------|-------|--------|-------|
|                      | Mean   | SD    | Mean   | SD    | Mean   | SD    | Mean   | SD    |
| Participation        | 0.72   | 0.45  | 1.00   | 0.00  | 0.00   | 0.00  | 0.65   | 0.48  |
| Age                  | 40.10  | 11.76 | 42.47  | 10.38 | 46.83  | 12.73 | 37.40  | 11.66 |
| Married              | 0.66   | 0.47  | 0.64   | 0.48  | 0.85   | 0.36  | 0.65   | 0.48  |
| Middle Class         | 0.44   | 0.50  | 0.42   | 0.49  | 0.45   | 0.50  | 0.45   | 0.50  |
| Upper Class          | 0.01   | 0.09  | 0.01   | 0.09  | 0.01   | 0.11  | 0.01   | 0.08  |
| North                | 0.13   | 0.34  | 0.13   | 0.33  | 0.16   | 0.37  | 0.13   | 0.34  |
| East                 | 0.22   | 0.41  | 0.27   | 0.44  | 0.08   | 0.27  | 0.21   | 0.41  |
| South                | 0.36   | 0.48  | 0.35   | 0.48  | 0.35   | 0.48  | 0.37   | 0.48  |
| #Children 0-1        | 0.04   | 0.21  | 0.02   | 0.13  | 0.05   | 0.22  | 0.06   | 0.25  |
| #Children 2-4        | 0.12   | 0.35  | 0.05   | 0.23  | 0.13   | 0.39  | 0.16   | 0.41  |
| #Children 5-18       | 0.68   | 0.94  | 0.53   | 0.81  | 0.74   | 1.10  | 0.76   | 0.98  |
| #HH older            | 2.27   | 0.86  | 2.18   | 0.81  | 2.55   | 0.97  | 2.28   | 0.87  |
| Birth_{t+1}          | 0.03   | 0.18  | 0.01   | 0.12  | 0.03   | 0.17  | 0.04   | 0.21  |
| #Observations        | 127736 |       | 46398  |       | 11644  |       | 69694  |       |
| #Individuals (N)     | 10712  |       | 4220   |       | 1146   |       | 5346   |       |
| Avg. Duration (\(\bar{T}\)) | 11.92  |       | 10.99  |       | 10.16  |       | 13.04  |       |

Source: GSOEP 1984–2013

The descriptive statistics of our data set are reported in table 11. The average participation rate is 72% in the full sample and 62% for women who change their labor-force participation decision at least once. We refer to the latter group as movers. Further, the group of women who never participate is the smallest and most different from the other groups. On average, this group is older, more likely to be married, and prefers to live in the west instead of the east. Interestingly, women who always participate have less children and live in smaller households. Note that identification in fixed effects probit models is solely based on the group of movers, which consist of 5,346 women observed for roughly 13 time periods on average.

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10. https://www.destatis.de/DE/Themen/Gesellschaft-Umwelt/Einkommen-Konsum-Lebensbedingungen/Lebensbedingungen-Armutsgefaehrdung/Tabellen/einkommensverteilung-silc.html
11. https://www.armuts-und-reichtumsbericht.de/
12. North: Schleswig-Holstein, Hamburg, Lower-Saxony, Bremen; South: Hessen, Baden-Wuerttemberg, Bavaria; North-Rhine-Westfalia, Rheinland-Pfalz, Saarland; Berlin, Brandenburg, Mecklenburg-Vorpommern, Saxony, Saxony-Anhalt, Thuringia.
|                        | MLE  | ABC1 | SPJ1 | LPM  |
|------------------------|------|------|------|------|
|                        | L = 1 | L = 2 | L = 1 | L = 2 |
| Participation\(t-1\)  | 0.216 | 0.276 | 0.284 | 0.283 | 0.502 | 0.531 |
|                        | (0.036) | (0.038) | (0.039) | (0.039) | (0.003) | (0.003) |
| Middle Class           | -0.016 | -0.015 | -0.016 | -0.014 | -0.014 | -0.015 |
|                        | (0.007) | (0.007) | (0.007) | (0.007) | (0.002) | (0.002) |
| Upper Class            | -0.050 | -0.049 | -0.047 | -0.054 | -0.042 | -0.042 |
|                        | (0.076) | (0.089) | (0.094) | (0.098) | (0.013) | (0.013) |
| #Children 0-1          | -0.202 | -0.200 | -0.196 | -0.204 | -0.303 | -0.302 |
|                        | (0.035) | (0.030) | (0.029) | (0.030) | (0.004) | (0.004) |
| #Children 2-4          | -0.043 | -0.035 | -0.033 | -0.042 | -0.041 | -0.039 |
|                        | (0.009) | (0.008) | (0.007) | (0.008) | (0.003) | (0.003) |
| #Children 5-18         | -0.010 | -0.008 | -0.007 | -0.006 | -0.008 | -0.007 |
|                        | (0.003) | (0.003) | (0.003) | (0.003) | (0.001) | (0.001) |
| Birth\(t+1\)          | -0.065 | -0.069 | -0.067 | -0.073 | -0.085 | -0.085 |
|                        | (0.016) | (0.015) | (0.015) | (0.016) | (0.005) | (0.005) |

Note: Standard errors in parentheses; additional covariates: married, regional identifiers, number of children between zero and one in the previous period, and number of household members above 18; estimates relative to lower income class.
Source: GSOEP 1984–2013

Table 12 reports estimates of the average partial effects obtained by different fixed effects probit estimators and a bias-corrected linear probability model.\(^\text{13}\) The labels are identical to the ones used in the simulation experiments. All results are intuitive and in line with the theoretical model of Hyslop (1999). We find strong positive state-dependence and negative effects with respect to transitory non-labor income, number of children, and expectations about future fertility. Remarkably, the estimated average partial effects obtained from dynamic probit models are all very close to each other. An exception is the state dependence, which ranges from roughly 0.22 up to 0.28. All effects are significant at the 5% level, except being in the upper income class. The estimates of the bias-corrected linear probability models are also very close to their non-linear counterpart. Two exceptions are the average partial effects of the lagged dependent variable and number of children between zero and one. However the standard errors obtained by the linear probability models are unreasonable low. Overall, these findings are in line with the results of our simulation experiments.

Our final conclusions are based on the results obtained by the different fixed effects probit estimators. First, we detect strong serial persistence in womens’ participation decisions. A woman who has currently a job increases her probability to participate in the future by

\(^\text{13}\) We also report estimates of the structural parameters in table 14 in the appendix.
22-28 percentage points. Second, we find that women only respond weakly to changes in transitory non-labor income. More precisely, being in the middle class reduces the participation probability by roughly two percentage points. The reduction associated with belonging to the upper income class is stronger (five percentage points), but not significantly different from zero. Finally, the number of children reduces the likelihood of current participation decision significantly. As expected, the effect is negative and declining in age of children. Each additional child between zero and one reduces current participation probability by roughly 20 percentage points. For children older than four, the reduction is only one percentage point. The results presented in this illustration are largely consistent with Hyslop (1999). However, contrary to him, we find that future birth always negatively affects current participation decision across different models, which might confirm the author’s perfect foresight assumption with respect to life-cycle fertility decisions.

6. Conclusion

This article addresses two problems of binary choice models with individual and time fixed effects and thus offers an attractive alternative to conventional approaches, such as conditional logit estimators (Rasch (1960), Andersen (1970), Chamberlain (1980), and Honoré and Kyriazidou (2000)), correlated random effect models (Mundlak (1978), Heckman (1981a, 1981b), and Wooldridge (2005)), and (bias-corrected) linear probability models (Nickell (1981), Hahn and Kuersteiner (2002), Hahn and Moon (2006), and Fernández-Val and Weidner (2018a)). First of all, fixed effects binary choice models suffer from the well-known incidental parameter problem, leading to potentially severe biases in estimates of structural parameters and partial effects. This issue can however be alleviated, by using bias-corrections. In the case of two-way error-components, Fernández-Val and Weidner (2016) offer suitable corrections, which reduce the biases substantially. The second challenge is the computational burden arising from the need to estimate the nuisance parameters in non-linear fixed effects models. Thus we extend a recently developed efficient algorithm for the maximum likelihood estimation of multi-way fixed effects models proposed by Stammann (2018) with several bias-corrections of Fernández-Val and Weidner (2016), to broaden their applicability. In order to encourage the usage of bias-corrected binary choice models, we embedded the aforementioned routine in our R-package alpaca. Extensive simulation experiments demonstrate desirable properties of the bias-corrected estimators even in unbalanced panels.

An empirical example from labor economics gives a first impression about the applicability of bias-corrections in large panel data sets. Although we focus on binary choice models,
it is straightforward to apply the same acceleration technique and bias-corrections to other
generalized linear models, e.g. poisson models. Also note that the bias-corrections proposed
by Fernández-Val and Weidner (2016) are not limited to classical panel structures as consid-
ered in this article. For instance, Cruz-Gonzalez, Fernández-Val, and Weidner (2017) apply
the same correction to cross-sectional data of bilateral trade flows.

Other related research projects dealing with bias-corrections in large panel data include
Weidner and Zylkin (2018) and Hinz, Wanner, and Stamann (2019), who adapt and extend
the bias-corrections of Fernández-Val and Weidner (2016) to special two- and three-way error
components that are particularly relevant in the context of international trade. Whereas
the former deal with pseudo-poisson gravity models, the latter treat (dynamic) binary choice
models.

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A. COMPUTATIONALLY EFFICIENT Offset Algorithm

In this section we want to introduce another algorithm that can be useful in the context of bias-corrections. Suppose we have bias-corrected the structural parameter estimates. In some situations we want to re-estimate our model given $\hat{\beta}$ is already known. This type of algorithm is often called offset tracing back to Nelder and Wedderburn (1972).

In the following we derive an offset algorithm that can be combined with MAP and thus is fast and memory efficient as well. The maximization problem in (1) can re-formulated as

$$\hat{\phi} = (\hat{a}, \hat{\gamma}) = \arg\max_{\alpha, \gamma} \sum_{i}^{N} \sum_{t}^{T} l_{it}(\hat{\beta}, \alpha_i, \gamma_t),$$

(14)

where $\hat{\beta}$ is assumed to be known. For instance, $\hat{\beta}$ can be the bias-corrected estimates. This yields the following update in iteration $r$:

$$\hat{\phi}_{r+1} = (D'\hat{\Omega}D)^{-1}D'\hat{\Omega}(\hat{\nu} + \hat{\eta}_r - X\hat{\beta}).$$

(15)

Note that (15) is the iterative re-weighted least-squares formulation of generalized linear models instead of the classical Newton one used in (8). Let us denote $\hat{q} = (\hat{\nu} + \hat{\eta}_r - X\hat{\beta})$. Again, (15) is essentially the weighted least-squares solution of $\hat{q} = D\phi + e$ given some weighting matrix $\hat{\Omega}$. Applying the same logic as used to derive Newton’s method above, results in the following update of the linear predictor in iteration $r$:

$$\hat{\eta}_{r+1} = \hat{q} - \hat{M}\hat{q} + X\hat{\beta}.$$  

(16)

Note that the linear predictor is enough to compute standard errors, partial effects, or other quantities of interest. The entire offset algorithm can be summarized as follows:

**Definition.** Newton’s Method (Offset)

Given $\hat{\beta}$; initialize $\hat{\eta}$; repeat the following steps until convergence

Step 1: Given $\hat{\eta}$ compute $\hat{q}$ and $\hat{\Omega}$

Step 2: Given $\hat{q}$ and $\hat{\Omega}$ update $\hat{\eta}$
B. ADDITIONAL TABLES

| Table 13: Derivatives for Logit and Probit Models |
|--------------------------------------------------|
| $F_{it}$ | $(1 + \exp(-\eta_{it}))^{-1}$ | $\Phi(\eta_{it})$ |
| $\partial_{\eta}F_{it}$ | $F_{it}(1 - F_{it})$ | $\phi(\eta_{it})$ |
| $\partial_{\eta}^2F_{it}$ | $\partial_{\eta}F_{it}(1 - 2F_{it})$ | $-\eta_{it}\phi(\eta_{it})$ |
| $\partial_{\eta}^3F_{it}$ | $\partial_{\eta}F_{it}((1 - 2F_{it})^2 - 2\eta_{it}\phi(\eta_{it}))$ | $(\eta_{it}^2 - 1)\phi(\eta_{it})$ |

**Note:** $\Phi(\cdot)$ and $\phi(\cdot)$ are the cumulative distribution and probability density function of the standard normal distribution.

| Table 14: Labor-Force Participation - Coefficients |
|--------------------------------------------------|
| MLE | ABC1 | SPJ1 |
|------|------|------|
| Participation$_{t-1}$ | 1.383 | 1.524 | 1.596 | 1.615 |
| | (0.014) | (0.014) | (0.014) | (0.014) |
| Middle Class | -0.141 | -0.120 | -0.126 | -0.114 |
| | (0.020) | (0.020) | (0.020) | (0.020) |
| Upper Class | -0.421 | -0.361 | -0.355 | -0.397 |
| | (0.107) | (0.107) | (0.107) | (0.107) |
| #Children 0-1 | -1.791 | -1.567 | -1.553 | -1.664 |
| | (0.033) | (0.033) | (0.033) | (0.033) |
| #Children 2-4 | -0.381 | -0.275 | -0.265 | -0.334 |
| | (0.023) | (0.023) | (0.023) | (0.023) |
| #Children 5-18 | -0.090 | -0.060 | -0.055 | -0.043 |
| | (0.011) | (0.011) | (0.011) | (0.011) |
| Birth$_{t+1}$ | -0.546 | -0.502 | -0.493 | -0.561 |
| | (0.034) | (0.034) | (0.034) | (0.034) |

**Note:** Standard errors in parentheses; additional covariates: married, regional identifiers, number of children between zero and one in the previous period, and number of household members above 18; estimates relative to lower income class.
C. Further Simulation Experiments (Linear Model)

In order to demonstrate that the analytical bias-corrections for linear models work as intended, we adjust the dynamic data generating process used in this article to linear models. More precisely, we change the data generating process to

\[ y_{it} = \rho y_{i,t-1} + \beta x_{it} + \alpha_i + \gamma_t + \epsilon_{it}, \]

\[ y_{i0} = \beta x_{i0} + \alpha_i + \gamma_0 + \epsilon_{i0}, \]

and everything else remains unchanged. \textit{LM} and \textit{BC} denote the uncorrected and bias-corrected fixed effects estimator. Values in parentheses indicate the bandwidth choice.

\textbf{Table 15: Finite Sample Properties - Linear Models - Balanced}

|            | Coefficients (\(\hat{\rho}\)) | Coefficients (\(\hat{\beta}\)) |
|------------|---------------------------------|---------------------------------|
|            | Bias   | SD     | RMSE | SE/SD | CP .95 | Bias   | SD     | RMSE | SE/SD | CP .95 |
| \textit{LM} | -11    | 3      | 11   | 0.98  | 0.01   | 3      | 3      | 4     | 1.00  | 0.84   |
| \textit{BC (1)} | -5     | 2      | 6    | 1.01  | 0.44   | 1      | 3      | 3     | 0.96  | 0.92   |
| \textit{BC (2)} | -3     | 3      | 4    | 0.99  | 0.81   | 0      | 3      | 3     | 0.95  | 0.92   |

|            | Coefficients (\(\hat{\rho}\)) | Coefficients (\(\hat{\beta}\)) |
|------------|---------------------------------|---------------------------------|
|            | Bias   | SD     | RMSE | SE/SD | CP .95 | Bias   | SD     | RMSE | SE/SD | CP .95 |
| \textit{LM} | -7     | 2      | 7    | 0.97  | 0.07   | 2      | 2      | 3     | 0.98  | 0.84   |
| \textit{BC (1)} | -3     | 2      | 4    | 0.99  | 0.60   | 1      | 2      | 2     | 0.99  | 0.93   |
| \textit{BC (2)} | -2     | 2      | 3    | 0.97  | 0.82   | 0      | 2      | 2     | 0.99  | 0.94   |

|            | Coefficients (\(\hat{\rho}\)) | Coefficients (\(\hat{\beta}\)) |
|------------|---------------------------------|---------------------------------|
|            | Bias   | SD     | RMSE | SE/SD | CP .95 | Bias   | SD     | RMSE | SE/SD | CP .95 |
| \textit{LM} | -5     | 2      | 6    | 0.99  | 0.13   | 2      | 2      | 2     | 1.02  | 0.87   |
| \textit{BC (1)} | -3     | 2      | 3    | 1.00  | 0.68   | 1      | 2      | 2     | 1.02  | 0.94   |
| \textit{BC (2)} | -1     | 2      | 2    | 1.00  | 0.88   | 0      | 2      | 2     | 1.02  | 0.95   |