Testing a model for the puzzling spin 0 mesons

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After a brief historical discussion of meson quantum numbers, we examine the possibility of additional internal meson structure. Experimental tests of this structure using the semi-leptonic decays of the $D_{s}^{+}$ (1968) meson are discussed.

§1. Further evolution of the Sakata Model

The Sakata model\textsuperscript{1} attempted to explain the “zoo” of strongly interacting “elementary particles” emerging in the 1950’s by postulating that they were composites of the three low-lying spin 1/2 fermions:

$$p, n, \Lambda.$$

In this picture, mesons were considered to be objects like $p \bar{n}$. This enables one to construct schematically all the observed hadrons. It also suggests a natural SU(3) “flavor” symmetry, first studied\textsuperscript{2} at Nagoya.

A few years later the three fundamental hadronic fields were replaced by the fractionally charged quarks:

$$u, d, s.$$

Mesons are now considered to be objects like $u \bar{d}$.

It should be remarked that in addition to his profound insights into elementary particle physics, Soichi Sakata championed a “democratic” style of organization for physics research and education.

Moving forward, we note that three more quarks were found during the great years for discovery between 1974 and 1995. This brings the total picture to:

$$u, d, s, c, b, t.$$

It is now easier to describe the quarks as:

$$q_{a}, \quad a = 1 \cdots 6$$

and raises the question of whether any more will be found at LHC.

Of course, during this period it was also discovered that the strong dynamics is described by an “SU(3) color” gauge theory so we must add a color index:

$$q_{aA}, \quad a = 1 \cdots 6, \quad A = 1 \cdots 3.$$

But we are still not done. If we regard this symbol as representing a massless Fermi-Dirac field, we know that the left and right handed projections enter differently.
into the unified electroweak theory. Thus, we distinguish the two by agreeing to leave
the left index alone and putting a dot on the right index.

In this language, a spin zero meson made of a quark and an antiquark can be
schematically described as:

\[ M^a_b = (q_b A_q) \gamma_4 \frac{1 + \gamma_5}{2} q_a A_q. \]  \hspace{1cm} (1.1)

Using a matrix notation, the decomposition in terms of scalar and pseudoscalar
fields is:

\[ M = S + i\phi, \quad M^\dagger = S - i\phi. \] \hspace{1cm} (1.2)

If all six quarks were massless, the symmetry of the color gauge theory La-
grangian would be:

\[ SU(6)_L \times SU(6)_R \times U(1)_{VECT}. \]

Actually, the first three quarks are relatively light so the reduced symmetry
\[ SU(3)_L \times SU(3)_R \times U(1)_{VECT}, \]
while spontaneously broken, forms the basis of the
chiral perturbation scheme which is successful at low energies.

§2. Considering the spin 0 mesons

By counting, there are nine light (i.e. made as quark-antiquark composites from
the three lightest flavors) pseudoscalar mesons and nine light scalar mesons. We
learned at this conference that Sakata recommended considering physics problems
from two different perspectives.

Theorist’s perspective:

There are eight zero mass pseudoscalar Nambu-Goldstone bosons and one heav-
ier pseudoscalar boson (since the axial U(1) is intrinsically broken by the axial
anomaly and thus can’t be spontaneously broken).

In order to make chiral perturbation theory calculations, which automatically
recover the “current algebra” theorems as a starting point, it is convenient to neglect
the scalar mesons. This is elegantly done by “integrating them out”. That has given
rise to a belief that the nine scalar mesons should have infinite mass (so integrating
them out would be rigorous) or at least should be very heavy.

Experimentalist’s perspective:

Look at the data!

This is not so easy. The starting point is a partial wave analysis of pi-pi scattering
in the \( I = J = 0 \) channel. The real part, \( R_0^0(\sqrt{s}) \) is plotted both in Fig. 1a and Fig.
1b, the experimental results corresponding to the points with error bars. Certainly
the pattern is a complicated one. The fits shown by the solid lines (M. Harada, F.
Sannino, J.S.)\(^3\) include the particular contributions:

a) chiral Lagrangian background
b) rho meson background
c) broad scalar (sigma) meson around 550MeV
d) “f₀(980)” candidate scalar meson (In Fig. 1a it is assumed that this meson decays completely into two pions while in Fig. 1b some decay into $K + \bar{K}$ is allowed.

A number of other groups have found similar results.

For orientation, note that the relevant “current algebra” theorem for this scattering gives the initial slope at threshold. It says nothing about the detailed pattern away from threshold. For fitting the experimental pattern away from threshold it turns out that the rather light broad sigma state is crucial; without the sigma, the very large rho exchange contribution would force the amplitude quite soon above the unitarity bound, $R^0_0 < 1/2$.

Similar treatments of the pi-$K$ scattering and the pi-eta scattering have produced evidence for a spin 0 scalar strange meson multiplet (kappa) and agreement with the experimental determination of another scalar resonance with $I = 1$, the $a_0(980)$. That finally yields a putative full nonet of scalars:

$$
I = 0 : \quad m(\sigma) = 550 \text{ MeV} \\
I = 1/2 : \quad m(\kappa) = 800 \text{ MeV} \\
I = 1 : \quad m(a_0) = 980 \text{ MeV} \\
I = 0 : \quad m(f_0) = 980 \text{ MeV}
$$

(2.1)

This may be compared with the (most standard) vector meson nonet:

$$
I = 0 : \quad m(\omega) = 783 \text{ MeV} \quad \bar{n}\bar{n} \\
I = 1 : \quad m(\rho) = 776 \text{ MeV} \quad \bar{n}\bar{n} \\
I = 1/2 : \quad m(K^*) = 980 \text{ MeV} \quad n\bar{s} \\
I = 0 : \quad m(\phi) = 1020 \text{ MeV} \quad s\bar{s}
$$

(2.2)

In this vector meson case, the usual quark-antiquark composition of each state is also displayed (s stands for a strange quark and n stands for either a u or a d
quark). For the standard vector meson nonet, the masses increase from the almost degenerate $I = 0$ and $I = 1$ particles to the lone $I = 0$ particle $\phi$. Basically, this is due to the strange quark being much heavier than the non-strange quarks $u$ and $d$. However, for the $J = 0$ scalar nonet candidates illustrated, the mass dependence is seen to be exactly reversed! A long time ago Jaffe [7] argued that a nonet made of two quarks plus two antiquarks would have this reversed behavior (it simply reflects the SU(3) Clebsch Gordon coefficient $\epsilon_{abc}$ needed to couple two quarks to an antiquark). We shall make use of this effect here.

§3. Linear sigma models

In order to check the pi-pi, pi-K and pi-eta scattering results discussed above, D. Black, A.H. Fariborz, S. Moussa, S. Nasri and JS [8] recalculated them in a relatively simple three flavor linear sigma model using the field $M = S + i\phi$ mentioned above. In that model the unitarization of the partial wave amplitudes was accomplished using the K-matrix approach. This approach has the “advantage” that it does not introduce any additional parameters. It was found that the same form of the complicated amplitude $R_0^0$ could be obtained. Similarly reasonable descriptions of the scattering partial waves involving the $\kappa$ and $a_0$ scalar states could also be obtained. But, by construction, both the pseudoscalar and scalar nonets start out as quark-antiquark objects in the model.

To add confusion, there are some observed scalar and pseudoscalar states which are not accommodated in this model. Also the lighter scalar masses are much lower than where they are expected to be according to the reasonable non-relativistic quark model. In that model, the lowest mass nonets (below 1 GeV) are the pseudoscalars and the vectors. The next highest nonets (somewhat above 1 GeV) are the scalars, tensors and two axial vectors (with different $C$ properties).

So, the situation concerning the spin 0 chiral partners seems to call for clarification. For this purpose, D. Black et al [8] also proposed that there might be two chiral spin 0 nonets - one of quark-antiquark type ($M$) and the other of two quark-two antiquark type ($M' = S' + i\phi'$) and that they be allowed to mix with each other. This mixing is expected to lead to level repulsion which could make the lighter scalars even lighter and the heavier ones even heavier.

What would the schematic structure of the “four quark” chiral nonet, $M'$ look like? Assuming that $M'$ has the same chiral transformation property as $M$, there are three possibilities:

“Molecular” type:

$$M^{(2)\hat{b}}_a = \epsilon_{abc}^{\hat{b}\hat{c}\hat{e}} \left( M^\dagger \right)^{\hat{c}} \left( M^\dagger \right)^{\hat{e}}_{\hat{f}}$$

(3.1)

Color triplet diquark - anti diquark type:

$$M^{(3)\hat{f}}_g = (L^A)^\dagger R^f A,$$

$$M^{(4)\hat{f}}_a = \epsilon_{abc}^{\hat{b}\hat{c}\hat{e}} \left( M^\dagger \right)^{\hat{c}} \left( M^\dagger \right)^{\hat{e}}_{\hat{f}}.$$
where,
\[ L^{gE} = \epsilon^{gab} \epsilon^{EAB} q_{aA}^{-1} \frac{1 + \gamma_5}{2} q_{bB} \]
\[ R^{gE} = \epsilon^{gab} \epsilon^{EAB} q_{aA}^{-1} \frac{1 - \gamma_5}{2} q_{bB} \]

Color sextet diquark - anti diquark type:
\[ M^{(4)f} = (L^{g}_{\mu\nu,AB})^\dagger R^{f}_{\mu\nu,AB} \quad , \]

where,
\[ L^{g}_{\mu\nu,AB} = \epsilon^{gab} q_{aA}^{-1} \frac{1 + \gamma_5}{2} q_{bB} \]
\[ R^{g}_{\mu\nu,AB} = \epsilon^{gab} q_{aA}^{-1} \frac{1 - \gamma_5}{2} q_{bB} \]

The distinction between molecular and diquark-antidiquark pieces is not fundamental because of the Fierz identity:
\[ 8M^{(2)b} = 2M^{(3)b} - M^{(4)b} \]

For the purpose of constructing a generalized linear sigma model containing both quark-antiquark and diquark-antidiquark mesons we just need to assume that some unspecified linear combination of \( M^{(2)}, M^{(3)} \) and \( M^{(4)} \) is bound and may be designated as \( M' \). Note that \( M \) and \( M' \) have different behaviors under the singlet axial transformations \( U(1)_A \) even though they transform in the same way under \( SU(3)_L \times SU(3)_R \).

\section*{§4. \( M - M' \) linear sigma model}

This complicated model has a number of aspects and was further discussed in a series of papers\cite{9,10}. See also\cite{17}. Similar perspectives are discussed in the papers\cite{18}.

We do not make any a priori assumptions about what are the quark-antiquark and two quark-two antiquark contents of the 18 scalar and 18 pseudoscalar states which emerge but let the model, with some experimental inputs, tell us the answer.

The pieces of the “toy” model Lagrangian include:

A) kinetic terms:
\[- \frac{1}{2} Tr(\partial_\mu M \partial^\mu M^\dagger) - \frac{1}{2} Tr(\partial_\mu M' \partial^\mu M'^\dagger),\]

B) symmetry breaking quark mass terms:
\[- 2 Tr(AS), \]

where \( A = diag(A_1, A_2, A_3) \) are proportional to the three light quark masses.

C) chiral invariant interaction terms:
\[ -c_2 \text{Tr}(MM^\dagger) + c_3^2 \text{Tr}(MM^\dagger MM^\dagger) + d_2 \text{Tr}(M'M'^\dagger) + c_3^3 (\epsilon_{abc} e^{def} M_a^b M_c^d M_f^e + h.c.)) \]

(There are about 20 renormalizable terms, we just kept those with 8 or less underlying quarks.)

D) terms to mock up the U(1) axial anomaly:

\[ c_3 [\gamma_1 \ln(\text{det}M/\text{det}M^\dagger) + (1 - \gamma_1)\ln((\text{Tr}(MM^\dagger)/\text{Tr}(M'M'^\dagger))) \]

Terms of both the detM and Tr(MM\dagger) types appear in the 3-flavor 't Hooft-type instanton calculation.

E) The needed vacuum values (assuming isospin invariance) are:

\[ < S_1^1 > , < S_2^2 > , < S_3^3 > , < S_1'^1 > , < S_3'^3 > \]

F) The following 8 inputs are used:

\[
\begin{align*}
F_\pi &= < S_1^1 > + < S_2^2 > = 131\text{MeV}, \quad A_3/A_1 = 20 - 30, \\
m[a_0(980)] &= 984.7 \pm 1.2\text{MeV}, \quad m[a_0(1450)] = 1474 \pm 19\text{MeV}, \\
m_\pi &= 137\text{MeV}, \quad m[\pi(1300)] = 1300 \pm 100\text{MeV},
\end{align*}
\]

and in addition: two mass parameters for the four I=0 η's.

§5. Predicted meson properties in the M – M' model

In the M – M' model there are eight different pseudoscalar isomultiplets. Their tree level masses are displayed in the table below and are clearly a mixture of the input masses and a few predictions. However, all eight of the “two quark” vs. “four quark” percentages are predicted by the model. Not surprisingly, the lower mass particles of each isospin turn out to be dominantly of quark-antiquark type. Note that all four of the I = 0 particles (the η's) mix with each other to some extent. Of course there are enough pseudoscalars to fill two nonets.

| State | \(\bar{q}q\)% | \(\bar{q}qqq\)% | \(m\) (GeV) |
|-------|--------------|----------------|-------------|
| \(\pi\) | 85 | 15 | 0.137 |
| \(\pi'\) | 15 | 85 | 1.215 |
| \(K\) | 86 | 14 | 0.515 |
| \(K'\) | 14 | 86 | 1.195 |
| \(\eta_1\) | 89 | 11 | 0.553 |
| \(\eta_2\) | 78 | 22 | 0.982 |
| \(\eta_3\) | 32 | 68 | 1.225 |
| \(\eta_4\) | 1 | 99 | 1.794 |

Table I. Typical predicted properties of pseudoscalar states: \(\bar{q}q\) percentage (2nd column), \(\bar{q}qqq\) (3rd column) and masses (last column).
The properties of the scalar mesons in the $M - M'$ model are displayed in the table below. In this case the only mass inputs were for the $a$ and $a'$ isotriplets; all the other masses and the two quark vs. four quark percentages are predictions of the model. Here the situation is opposite to that of the pseudoscalars. The lower lying states are predominantly two quark - two antiquark (or “four quark”) type. For example, the lighter isovector, $a$ is 76 per cent “four quark” while the heavier isovector, $a'$ is 24 per cent “four quark”.

The famous $\sigma = \sigma_1$ is 40 percent quark-antiquark and 60 per cent “four quark”. The $f_0(980)$ is 95 percent of “four quark” type.

Note that these masses are “tree level” ones. For the $\sigma$, the unitarity corrections (which involve computing the $\pi\pi$ scattering amplitude in the model) reduced the predicted mass to 477 MeV.

| State | $qq\%$ | $qqqq\%$ | $m$ (GeV) |
|-------|---------|-----------|-----------|
| $a$   | 24      | 76        | 0.984     |
| $a'$  | 76      | 24        | 1.474     |
| $\kappa$ | 8      | 92        | 1.067     |
| $\kappa'$ | 92     | 8         | 1.624     |
| $\sigma_1$ | 40     | 60        | 0.742     |
| $\sigma_2$ | 5      | 95        | 1.085     |
| $\sigma_3$ | 63     | 37        | 1.493     |
| $\sigma_4$ | 93     | 7         | 1.783     |

Table II. Typical predicted properties of scalar states: $qq$ percentage (2nd column), $qqqq$ (3rd column) and masses (last column).

§ 6. Experimental information on scalar mesons

Typically, it comes from partial wave analyses of scattering processes. Another source arises from Dalitz analyses of multiparticle final states in non-leptonic weak decays.

Recently, the CLEO Collaboration obtained a simple neat determination of the mass and the width of the $f_0(980)$ [$\sigma_2$ in the notation above] from the semi-leptonic decay of a charmed meson:

$$D_s^+(1968) \rightarrow f_0(980) + e^+ + \nu_e.$$  

This process corresponds to the “quark” picture in Fig. 2.

§ 7. Predicting some semi-leptonic decay widths

It seems interesting that in addition to the decay into what we called $\sigma_2$, there should also exist decays into $\sigma_1$, $\sigma_3$ and $\sigma_4$ which are easily predictable in the model. All four of these particles arise as mixtures of the standard basis states
We denote $f$ as the four component vector: $(f_1, f_2, f_3, f_4)$. For typical values of the model’s input parameters the mass eigenstates, $\sigma_i$, make up a four vector, $\sigma = L^{-1}_0 f$ with,

\[
(L^{-1}_0) = \begin{pmatrix}
0.601 & 0.199 & 0.600 & 0.489 \\
-0.107 & 0.189 & 0.643 & -0.735 \\
0.790 & -0.050 & -0.391 & -0.470 \\
0.062 & -0.960 & 0.272 & -0.019
\end{pmatrix}
\]

(7.2)

The physical states are identified, with nominal mass values, as

\[
\sigma = \begin{pmatrix}
f_0(600) \\
f_0(980) \\
f_0(1370) \\
f_0(1800)
\end{pmatrix}
\]

(7.3)

Similarly, there are four predictions of the model for decays of the $D_s^+$ into the pseudoscalar singlets $\eta_i +$ leptons.

The required hadronic information consists of the vector (for decays into the $\eta_i$ and the axial vector (for decays into the $\sigma_i$) Noether currents of the model. For the three flavor model these currents take the well known forms:

\[
V^{b}_{\mu a} (total) = V^{b}_{\mu a} + V'^{b}_{\mu a}, \\
A^{b}_{\mu a} (total) = A^{b}_{\mu a} + A'^{b}_{\mu a}.
\]

(7.4)

\[
V^{b}_{\mu a} = i\phi_c^{\leftrightarrow} \partial^b_{\mu c} \phi_c^b + i\tilde{S}^{\leftrightarrow}_{a} \partial^b_{\mu} \tilde{S}_{a}^b + i(\alpha_a - \alpha_b)\partial^b_{\mu} \tilde{S}_{a}^b,
\]
\[ A_{\mu a}^b = S_a^c \bar{\partial}_\mu \phi_c^b - \bar{\phi}_a^c \partial_\mu S_c^b + (\alpha_a + \alpha_b) \partial_\mu \phi_c^b, \quad (7.5) \]

\[ V_{\mu a}^{tb} = i\phi_a^c \bar{\partial}_\mu \phi_c^b + i\bar{S}_a^c \bar{\partial}_\mu \bar{S}_c^b + i(\beta_a - \beta_b) \partial_\mu \bar{S}_a^b, \quad (7.6) \]

Note that \( \alpha_a = \langle S_a^a \rangle \) and \( \beta_a = \langle S_a^a \rangle \). Also \( \bar{S}_b^a = S_b^a - \langle S_b^a \rangle \).

§8. Noether currents in the four flavor case

Clearly, an extension is needed to accommodate the charmed \( D_s^+ \) particle required to calculate the desired semi-leptonic decay rates. Also, it at first seems dubious to consider the fourth heavy quark in the same Lagrangian as the three light quarks. However for the present calculation, only the Noether currents are needed and these depend only on the kinetic terms which are independent of mass related parameters.

So, one’s first thought is to just sum 1 - 4 instead of 1 - 3 in the currents above. However there is still a problem with this simple extension. A fourth quark flavor will not allow the construction of a two quark-two antiquark state which has the same chiral \( SU(4) \) transformation property as the one quark - one antiquark state with which it is supposed to mix. For example, trying a “molecule” form would result in

\[ M_{ag}^{(2)bh} = \epsilon_{agdf} \epsilon^{bhcf} (M^\dagger)_f^e (M^\dagger)_e^d. \]

But, instead of transforming under \( SU(4)_L \times SU(4)_R \) as \( (4, \bar{4}) \) it transforms as \( (6, \bar{6}) \) owing to the two sets of antisymmetric indices which appear. It may be shown\(^{14}\) that 3 flavors are special in allowing the kind of mixing which preserves the underlying chiral symmetry.

Thus, we assume that there are no two quark - two antiquark components for the mesons containing a charm quark. The kinetic terms for the model may then be written as:

\[ L = -\frac{1}{2} Tr^A (\partial_\mu M^\dagger M) - \frac{1}{2} Tr^B (\partial_\mu M^M \dagger M^M \dagger), \quad (8.1) \]

where the meaning of the superscript on the trace symbol is that the first term should be summed over the heavy quark index as well as the three light indices. This stands in contrast to the second term which is just summed over the three light quark indices pertaining to the two quark - two antiquark field \( M' \). Since the Noether currents are sensitive only to these kinetic terms in the model, the vector and axial vector currents with flavor indices 1 through 3 in this model are just the same as in Eq.\(^{(7.4)}\) above. However if either or both flavor indices take on the value 4 (referring to the heavy flavor) the current will only have contributions from the field \( M \). This should be clarified by the following example,
\[ V^{a}_{\mu 4}(\text{total}) = V^{a}_{\mu 4} = i\phi^{a}_{c} \partial_{\mu} \phi^{a}_{c} + iS^{a}_{c} \partial_{\mu} S^{a}_{c}. \]
\[ A^{a}_{\mu 4}(\text{total}) = A^{a}_{\mu 4} = S^{a}_{c} \partial_{\mu} \phi^{a}_{c} - \phi^{a}_{c} \partial_{\mu} S^{a}_{c}. \]

(8.2)

Here the unspecified indices can run from 1 to 4.

Note that the currents do not contain any unknowns; their normalization is given by the component which is the electric current (i.e. “conserved vector current” hypothesis). Then the unintegrated decay widths into any of the four isoscalar 0+ mesons or four isoscalar 0− mesons is given by,

\[ \frac{d\Gamma}{dq} = \frac{G^{2}_{F}|V_{cs}|^{2}}{12\pi^{3}} \left\{ \frac{\langle (R_{0})_{2i} \rangle^{2}}{\langle (L_{0})_{2i} \rangle^{2}} \right\} m(D_{s})|q^{4}| q_{0}. \]

(8.3)

where \( q_{\mu} \) is the final meson four momentum and \( V_{cs} \) is the Kobayashi Maskawa matrix element. \( R_{0} \) is the pseudoscalar analog of \( L_{0} \) introduced in Eq. (7.2) for the scalars.

Table III summarizes the calculations of the predicted widths, for \( D_{s}^{+} \) decays into the four pseudoscalar singlet mesons (\( \eta_{1} = \eta(547), \eta_{2} = \eta(982), \eta_{3} = \eta(1225), \eta_{4} = \eta(1794) \)). Notice that the listed masses, \( m_{i} \) are the “predicted” ones in the present model.

| \( m_{i} \) (MeV) | \( (R_{0})_{2i} \) | \( (q_{max})_{i} \) (MeV) | \( \Gamma_{i} \) (MeV) |
|----------------|----------------|----------------|----------------|
| 553           | 0.661          | 906.20         | 4.14 \times 10^{-11} |
| 982           | 0.512          | 739.00         | 7.16 \times 10^{-12} |
| 1225          | -0.546         | 602.74         | 2.57 \times 10^{-12} |
| 1794          | 0.051          | 166.31         | 2.65 \times 10^{-11} |

Table III. pseudoscalars.

Table IV, with the same conventions, summarizes the calculations of the predicted widths for \( D_{s}^{+} \) decays into the four scalar singlet mesons \( [(\sigma_{1}, \sigma_{2}, \cdots) = (\sigma, f_{0}(980), \cdots)] \) and leptons.

| \( m_{i} \) (MeV) | \( (L_{0})_{2i} \) | \( (q_{max})_{i} \) (MeV) | \( \Gamma_{i} \) (MeV) |
|----------------|----------------|----------------|----------------|
| 477           | 0.199          | 933.23         | 4.56 \times 10^{-12} |
| 1037          | 0.189          | 710.79         | 7.80 \times 10^{-13} |
| 1127          | -0.050         | 661.30         | 3.62 \times 10^{-14} |
| 1735          | -0.960         | 219.21         | 3.85 \times 10^{-14} |

Table IV. scalars.

Experimental data exist for only three of these eight decay modes:

\[ \Gamma(D_{s}^{+} \rightarrow \eta e^{+} \nu_{e}) = (3.5 \pm 0.6) \times 10^{-11} \text{ MeV} \]
\[ \Gamma(D_{s}^{+} \rightarrow \eta^{'} e^{+} \nu_{e}) = (1.29 \pm 0.30) \times 10^{-11} \text{ MeV} \]
\[ \Gamma(D_{s}^{+} \rightarrow f_{0} e^{+} \nu_{e}) = (2.6 \pm 0.4) \times 10^{-12} \text{ MeV} \]

(8.4)
It is encouraging that even though the calculation utilized the simplest model for the current and no arbitrary parameters were introduced, the prediction for the lightest hadronic mode, \( \Gamma(D^+_s \rightarrow \eta e^+\nu_e) \) agrees with the measured value. In the case of the decay \( D^+_s \rightarrow \eta e^+\nu_e \) the predicted width is about 30\% less than the measured value. For the mode \( D^+_s \rightarrow f_0(980)e^+\nu_e \) the predicted value is about one third the measured value. Conceivably, considering the large predicted width into the very broad sigma state centered at 477 MeV, some of the higher mass sigma events might have been counted as \( f_0(980) \) events, which would improve the agreement. It would be very interesting to obtain experimental information about the energy regions relevant to the other five predicted isosinglet modes.

Furthermore, varying the particular choice for the quark mass ratio \( A_3/A_1 \) and the precise mass of the very broad \( II(1300) \) resonance within the allowable ranges can lead to a satisfactory fit to experiment, as shown in.

\[ \text{§9. Summary} \]

The light spin 0 pseudoscalar mesons appear to be of \( q\bar{q} \) type.

The light spin 0 scalar mesons appear to be of \( qq\bar{q}\bar{q} \) type.

Chiral symmetry is a symmetry of massless QCD so these mesons should be reasonably approximated as chiral partners. Then how can we reconcile their different compositions?

Proposed solution: Introduce a chiral \( q\bar{q} \) multiplet and a chiral \( qq\bar{q}\bar{q} \) multiplet. They mix and the lightest pseudoscalars are mainly \( q\bar{q} \) while the lightest scalars are mainly \( qq\bar{q}\bar{q} \).

Semi-leptonic decays of the heavy mesons seem to provide useful experimental information for checking this picture.

\[ \text{§10. Acknowledgements} \]

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