Constraints on $\Sigma$ Nucleus Dynamics from Dirac Phenomenology of $\Sigma^-$ Atoms

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Abstract

Strong interaction level shifts and widths in $\Sigma^-$ atoms are analyzed by using a $\Sigma$ nucleus optical potential constructed within the relativistic mean field approach. The analysis leads to potentials with a repulsive real part in the nuclear interior. The data are sufficient to establish the size of the isovector meson–hyperon coupling. Implications to $\Sigma$ hypernuclei are discussed.

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I. INTRODUCTION

The relativistic mean field (RMF) approximation [1] is an interesting alternative to more conventional approaches to the nuclear many-body problem. Numerous studies have demonstrated the usefulness of the RMF approach in describing the behavior of nucleons in the nuclear medium. Recent RMF calculations of Λ- and multi Λ hypernuclei [2-4] have shown that it can be successfully extended to the more general baryon-nucleus systems [5-9]. Hyperon-nucleus scattering has been studied as well [10]. In these works, the meson–hyperon coupling constants were either fitted to known hypernuclear spectra or obtained from the parameters of a nucleon shell model by using the constituent quark model.

Calculations of Σ-hypernuclei [11] have often been based on analyses of Σ⁻ atomic data [12,13] in terms of attractive and absorptive Σ-nucleus potentials, yielding in the nuclear interior depth values

\[
- \text{Re} V_{\Sigma_{\text{opt}}}^\Sigma(0) \approx 25-30 \text{ MeV}, \tag{1}
\]

\[
- \text{Im} V_{\Sigma_{\text{opt}}}^\Sigma(0) \approx 10-15 \text{ MeV}. \tag{2}
\]

However, a considerably shallower potential, particularly for \( \text{Re} V_{\Sigma_{\text{opt}}}^\Sigma \), is indicated by fitting to Σ-hypernuclear spectra from \((K^-,\pi^+)\) reactions [14]. Furthermore, a very recent phenomenological analysis of level shifts and widths in Σ⁻ atoms by Batty et al. [15] suggests that \( \text{Re} V_{\Sigma_{\text{opt}}}^\Sigma \) is attractive only at the nuclear surface, changing into a repulsive potential as the density increases in the nuclear interior. Although the magnitude of the repulsive component cannot be determined unambiguously by the atomic data, the small attractive pocket of such potentials does not provide sufficient binding to form Σ-hypernuclei. This conclusion has been independently arrived at by Harada [16], folding a soft-core repulsive plus long-range attractive ΣN interaction into the nuclear density.

In view of the above seemingly conflicting observations, it is topical to use the RMF approach directly for constructing the Σ-nucleus optical potential by fitting to Σ⁻ atomic data. The aims of the present work are:
i) to check whether or not the RMF approach can reproduce the atomic data;

ii) to establish constraints imposed by fitting to the data on the scalar and vector meson couplings to a Σ hyperon;

iii) to establish the size of the isovector-vector meson-hyperon coupling;

iv) to investigate the sign of the Σ-nucleus potential in the nuclear interior and whether or not Σ hypernuclear bound states are expected to exist.

The paper is organized as follows. Sect. 2 presents updated fits of a phenomenological density dependent (DD) optical potential. In sect. 3 we introduce the RMF optical potential for Σ, discuss the constraints imposed by the atomic data on its form and present results. These RMF results are compared with results of other approaches in sect. 4 which also offers a discussion and summary of our conclusions.

II. PHENOMENOLOGICAL DENSITY DEPENDENT OPTICAL POTENTIAL

Before comparing the Σ-nucleus potentials based on the RMF approach to those derived using the phenomenological analysis, we took the intermediate step of replacing the “macroscopic” nuclear densities (such as 2-parameter Fermi distributions) used in the previous phenomenological analysis [15] by more realistic “microscopic” densities. The motivation for this replacement is twofold: first, we wish to use densities that are more consistent with the resulting RMF densities, and second, it is desirable to use densities that are as appropriate as possible outside the nuclear surface, for discussing Σ− atoms.

In order to appreciate the latter point, we have repeated the “notch test” of ref. [15] in a modified form to see which range of radii has a significant effect on the atomic levels. The sensitivity of the data to the potential around the radius $R_N$ is obtained by multiplying the best-fit potential by a factor:

$$f = 1 - d \exp[-(\frac{r - R_N}{a_N})^2]$$  \hspace{1cm} (3)
representing a “notch” in the potential around \( r = R_N \) spread approximately over \( \pm a_N \) (in fm), whose relative depth is \( d \). By varying the depth \( d \) and observing the changes in \( \chi^2 \), the sensitivity (defined as the values of \( d \) that cause \( \chi^2 \) to increase by one unit) can be determined. For \( a_N \) the value of 0.5 fm was chosen. By scanning over \( R_N \), the radial region where the fit to the data is sensitive to the potential is determined. Figure 1 shows the results obtained, using “microscopic” densities as described below, by scanning \( R_N \) in steps of a nuclear diffuseness \( a_0 \), i.e., \( R_N = R_0 + a_0 \Delta \), with \( R_0 = 1.1 A^{1/3} \) fm and \( a_0 = 0.5 \) fm. It is seen that the radial region which is “sampled” by the data is outside of the nuclear surface. This updates the similar results shown in Fig. 3 of ref. [15] using “macroscopic” densities.

The method chosen to generate “microscopic” nuclear densities was to fill in single particle (SP) levels in Woods-Saxon potentials separately for protons and neutrons. The radius parameter of the potential was adjusted to reproduce the \( rms \) radius of the charge distribution (after folding in the finite size of the proton). The binding energy of the least bound particle was set equal to the corresponding separation energy. For \( N > Z \) nuclei the \( rms \) radius of the neutron density distribution was chosen to be slightly larger than that of the protons. By using this method, rather realistic density distributions are obtained in the region of the exponential fall-off. These densities are less reliable in the nuclear interior, but the \( \Sigma^- \) atom data are not sensitive to this region as is clearly seen from Fig. 1.

Several of the fits of ref. [15] have been repeated using these SP densities. The \( \Sigma^- \) nucleus potential is written as

\[
2\mu V_{opt}^\Sigma(r) = -4\pi(1 + \frac{\mu}{m}) \left\{ \left[ b_0 + B_0\left(\frac{\rho(r)}{\rho(0)}\right)^\alpha\right] \rho(r) + \left[ b_1 + B_1\left(\frac{\rho(r)}{\rho(0)}\right)^\alpha\right] \delta\rho(r) \right\}
\]

(4)

where \( \mu \) is the \( \Sigma \)-nucleus reduced mass, \( m \) is the mass of the nucleon and \( \rho(r) = \rho_n(r) + \rho_p(r) \) is the nuclear density distribution normalized to the number of nucleons \( A \). The isovector density distribution is given by \( \delta\rho(r) = \rho_n(r) - \rho_p(r) \). The parameters \( b \) and \( B \) are given in units of fm and are obtained from fits to the \( \Sigma^- \) atom data.

For the density-independent \( t_{eff}\rho \) potential \((B_0 = 0, B_1 = 0)\), the results obtained with the SP densities were very similar to those obtained before [15], namely, values of \( \chi^2 \) per
degree of freedom, $\chi^2/F$, were about 2.4 and the isovector part of the potential was not determined by the data. With the introduction of density dependence into the potential, substantial improvement in the fit to the data is observed. The best-fit values of $\chi^2$ obtained with SP densities are significantly lower than those obtained with the schematic parameterization of the nuclear densities [15]. Moreover, the real part of the isovector potential is now found to be reasonably well-determined, presumably because the SP densities represent more reliably the differences between protons and neutrons outside of the nuclear surface.

Results of two DD fits are summarized in table 1. Potential I is the analog of potential D or A’ of ref. [15], where the density dependence is introduced only into the real isoscalar part via the parameter $\text{Re}B_0$. It is assumed throughout that $\text{Im}b_1=\text{Im}b_0$ so that $\Sigma^-$ particles are absorbed only on protons. All four parameters varied in the fit are phenomenological and it is seen that they are reasonably well-determined. The DD exponent $\alpha$ was held fixed during the parameter search and its value was subsequently varied between 0.1 and 1.1 and fits repeated. Potential II is analogous to the potential of ref. [15] for which the linear density terms were held fixed at values suggested by One Boson Exchange (OBE) models: $b_0=1.2+i0.45$ fm, $b_1=-0.45-i0.45$ fm. In this case, three adjustable parameters, namely, the complex $B_0$ and $\text{Re}B_1$ ($\text{Im}B_1=\text{Im}B_0$), were obtained from fits to the data. In both cases the fits are excellent and in both cases the variation of $\chi^2$ with $\alpha$ was weak. The values shown in table 1 are typical examples and $\alpha$ could not be determined uniquely in the range of 0.4 to 0.9.

From the parameter values in table 1 it is seen that whereas the potentials are attractive at low densities, they become repulsive as the density approaches nuclear matter densities, a feature that had already been observed [15]. Figure 2 shows the real and imaginary potentials for Si, Ca and Pb near the nuclear surface. It is seen that there is a small attractive pocket near the surface and that the real potential becomes repulsive toward the interior. The imaginary potential is purely absorptive; similar results for $\text{Re}V^\Sigma_{opt}$ are obtained if $\text{Im}V^\Sigma_{opt}$ is made to saturate at nuclear matter density [17]. It should be mentioned that with the limited data available and the very different quality of various data, the isovector part of
the potential is determined solely by the experimental results for $\Sigma^-$ atomic Pb.

### III. CONSTRUCTION OF THE RMF OPTICAL POTENTIAL

The RMF formalism describes baryons as Dirac spinors coupled to scalar ($\sigma$) and vector ($\omega, \rho$) meson fields. The underlying RMF model used here is based on the Lagrangian density of the form:

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\Sigma, \quad (5)$$

$$\mathcal{L}_\Sigma = \bar{\Psi}_\Sigma \left[ i \gamma_\mu \partial^\mu - g_{\omega \Sigma} \gamma_\mu \omega^\mu - (M_\Sigma + g_{\sigma \Sigma} \sigma) \right] \Psi_\Sigma + \mathcal{L}_{\rho \Sigma} + \mathcal{L}_{A \Sigma}, \quad (6)$$

$$\mathcal{L}_{\rho \Sigma} + \mathcal{L}_{A \Sigma} = -\bar{\Sigma}_{ij} \left( \frac{g_{\rho \Sigma}}{2} \gamma_\mu \Theta^\mu_{jk} + \frac{e}{2} \gamma_\mu A^\mu (\tau_3)_{jk} \right) \Sigma_{ki}, \quad (7)$$

where

$$\Sigma = \begin{pmatrix} \Psi_{\Sigma^0} & \sqrt{2} \Psi_{\Sigma^+} \\ \sqrt{2} \Psi_{\Sigma^-} & -\Psi_{\Sigma^0} \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (8)$$

and

$$\Theta^\mu = \begin{pmatrix} \rho^\mu_0 & \sqrt{2} \rho^\mu_+ \\ \sqrt{2} \rho^\mu_- & -\rho^\mu_0 \end{pmatrix}. \quad (9)$$

The standard form of $\mathcal{L}_N$ can be found elsewhere [1]. The Lagrangian density $\mathcal{L}_\Sigma$ includes interactions of a $\Sigma$ hyperon with the isoscalar ($\sigma, \omega$) and isovector ($\rho$) meson fields, as well as with the photon generating in leading order the charged $\Sigma$ Coulomb field due to the nuclear charge distribution. The Euler-Lagrange equations lead to a coupled system of equations of motion for both baryon and meson fields. For the $\Sigma$ particle, and retaining only the timelike component $\mu = 0$ for the vector fields in the mean (meson) field approximation, the resulting Dirac equation acquires the form:

$$\left[ -i \alpha \cdot \nabla + V(\mathbf{r}) + \beta (M + S(\mathbf{r})) + V_{\text{Coulomb}}(\mathbf{r}) \right] \psi(\mathbf{r}) = E \psi(\mathbf{r}), \quad (10)$$
where
\[ S(r) = g_{\sigma\Sigma}\sigma(r) \quad \text{and} \quad V(r) = g_{\omega\Sigma}\omega(r) + g_{\rho\Sigma}I_3\rho_0(r) \] (11)
are the scalar and vector potentials, respectively. In eq. (11), \( I_3 = +1, 0, -1 \) for \( \Sigma^+ \), \( \Sigma^0 \), \( \Sigma^- \), respectively. More details about the equations of motion and their solution are given in ref. [18].

For each particular nucleus we constructed the Schrödinger equivalent (SE) \( \Sigma \)-nucleus potential from the scalar (attractive) and vector (repulsive) Dirac potentials. The SE potential \( V_{SE}(r) \) was then used as a real part of the optical potential in the calculations of \( \Sigma^- \) atoms:
\[ \text{Re}V^\Sigma_{\text{opt}}(r) \equiv V_{SE}(r) = S(r) + \frac{E V(r)}{M_{\Sigma}} + \frac{(S^2(r) - V^2(r))}{2M_{\Sigma}} \] (12)
It is to be stressed here that the isovector part of the real potential comes out quite naturally in the RMF approach as one allows for coupling the \( \Sigma \) with the \( \rho \) field (see eq.(11) for \( V(r) \)).

The imaginary part of the \( \Sigma \)-nucleus optical potential describes the conversion of \( \Sigma^- \) to \( \Lambda \) via the reaction \( \Sigma^- p \rightarrow \Lambda n \). The expression for the imaginary part of the potential (and consequently the \( \Sigma \)-hypernuclear width) was derived by Gal and Dover [13] and Dabrowski and Rozynek [19]. The values of the well-depth, \(-\text{Im}V^\Sigma_{\text{opt}}(0)\), were estimated to be between 11.6 and 14 MeV in nuclear matter. For finite nuclei, a reduction from these estimates should be expected. In the present work a phenomenological imaginary potential of the form
\[ \text{Im}V^\Sigma_{\text{opt}}(r) = t\rho_p(r) \] (13)
was used, where the proton density \( \rho_p \) was calculated in the RMF model and \( t \) was taken as a parameter to be determined by fitting the atomic data. The use of \( \rho_p \) in the construction of the imaginary potential reflects the fact that the \( \Sigma^- \) conversion takes place exclusively on protons.

For \( L_N \) the linear (L) parametrization of Horowitz and Serot [18] was used as well as the nonlinear (NL) parametrization \( NL1 \) of Reinhard et al. [20]. For \( L_\Sigma \) the hyperon couplings
were characterized as in previous works [2-10] via the coupling constant ratios $\alpha_i = \frac{g_{i\Sigma}}{g_{iN}}$, ($i = \sigma, \omega, \rho$). In the case of the vector meson coupling ratio $\alpha_\omega$ three values, namely $1/3, 2/3$ and 1, were used. The choice of $\alpha_\omega = 1/3$ was inspired by a number of early hypernuclear calculations [2,21-24] in which the strength of a hyperon (mainly $\Lambda$) coupling used was between $1/3$ and 0.4. On the other hand, the $2/3$ ratio follows from the constituent quark model. Finally, for $\alpha_\omega = 1$, the equality $g_{\omega\Sigma} = g_{\omega N}$ is motivated by a recent QCD sum rule evaluation for $\Sigma$ hyperons in nuclear matter [25].

For each of the above choices of $\alpha_\omega$ the ratio $\alpha_\sigma$ together with $t$ from eq. (13) were fitted to the experimental atomic shift and width in Si. This particular atom was singled out due to its relatively accurate shift and width data among $N=Z$ (isoscalar) core nuclei. The isovector part of the optical potential was then included and the ratio $\alpha_\rho$ was determined by fitting the shift and width in Pb while holding the isoscalar parametrization fixed. In such a way, we found for each RMF model, whether L or NL, three sets of parameters that give, by construction, an excellent fit for Si and Pb. Typical results are illustrated in Fig. 3 for $\Sigma$ nucleus NL optical potentials in Si. When going from $\alpha_\omega = 1/3$ to 1, $\text{Re}V_{\Sigma \text{opt}}$ changes from attraction to repulsion in the nuclear interior. In addition, the imaginary part $\text{Im}V_{\Sigma \text{opt}}$ becomes more absorptive. The same holds for the linear model which, for a given value of $\alpha_\omega$, predicts more attraction for $\text{Re}V_{\Sigma \text{opt}}$ and less absorption for $\text{Im}V_{\Sigma \text{opt}}$ than the NL parameterization yields, as is shown for $\alpha_\omega = 2/3$ in Fig. 4 where $\Sigma$ optical potentials for Pb are compared with each other. We observed that the $\rho - \Sigma$ coupling is determined unambiguously by the fit to the Pb data, with a value $\alpha_\rho \approx 2/3$ in all cases.

Having determined the isoscalar as well as isovector parameters of $V_{\Sigma \text{opt}}$ by fitting the Si and Pb data we constructed SE optical potentials for all nuclei for which $\Sigma^-$ atomic data exist. Whereas for $\alpha_\omega = 2/3$ and 1 we obtained reasonable values of $\chi^2$ without any further adjustments, we failed to get below $\chi^2 = 26.5$ for $\alpha_\omega = 1/3$. The results for the linear model are presented in Table 2. It is to be noted that we did not aim strictly at the lowest $\chi^2$ value but merely to get $\chi^2 \approx 23$, which is a typical value in previous DD fits [15]. Consequently, only the value of $\chi^2$ for the potential L3 from Table 2 represents a true minimum for this
particular parametrization. From this point of view a comparison of $\chi^2$ values listed in Table 2 might even be too favorable to the L3 potential.

Although the $\Sigma^-$ atomic data are of a poor quality and, moreover, determine the shape of $V_{opt}^\Sigma$ only around the surface and outside the nucleus as demonstrated by the “notch” test (Fig. 1), these data nevertheless significantly constrain the RMF parametrization of the $\Sigma$ nucleus optical potential. Not only that the value of the $\rho$-$\Sigma$ coupling ratio $\alpha_\rho \approx 2/3$ holds unambiguously for all the parametrizations used, but in addition, a weak $\omega$-$\Sigma$ coupling is almost certainly ruled out by fitting to the $\Sigma^-$ atom data. Consequently, the data from $\Sigma^-$ atoms imply, within the RMF approach, a $\Sigma$-nucleus optical potential with a repulsion in the nuclear interior and a shallow attractive pocket outside the nuclear surface. For some light nuclei (Si, S, Mg), in contrast to Fig. 4 for Pb, the linear model with $\alpha_\omega = 2/3$ predicts that the repulsion in the interior goes back into a second shallow attractive pocket (less than 3.5 MeV) at the origin, but this obviously cannot be tested by using $\Sigma^-$ atom data.

IV. DISCUSSION AND SUMMARY

In this work we have improved on the phenomenological analysis [15] of $\Sigma^-$ atoms in terms of a $\Sigma$ nucleus DD optical potential by using nuclear density distributions which account more realistically for the nuclear surface region and outside of it. The earlier work [15] established that the isoscalar component of $V_{opt}^\Sigma$ changes in the nuclear surface region from attraction to repulsion as one penetrates the nuclear interior, and that the isovector component is undetermined by the data. While the isoscalar component, in the present work, remains qualitatively unchanged, the introduction of these more realistic densities has a pronounced effect on the extraction of the isovector component of $V_{opt}^\Sigma$, which for $\Sigma^-$ is reliably determined to be repulsive. It should be recalled that this extraction hinges almost exclusively on the shift and width data for Pb [26] which are the most accurate among all available $\Sigma^-$ atom data to date, not just the $N > Z$ data subset. More precise measurements, particularly for other $N > Z$ atoms, are highly desirable.
We have also confirmed the results of ref. [15] that $V_{\Sigma_{\text{opt}}}^\Sigma$ is determined by the $\Sigma^-$ atom data only outside the nucleus. Nevertheless, the novel feature of $V_{\Sigma_{\text{opt}}}^\Sigma$ becoming repulsive is found to occur at a region of space outside the nuclear radius, still where $V_{\Sigma_{\text{opt}}}^\Sigma$ is determined, even if not very accurately, by the $\Sigma^-$ atom data. To have a glimpse into the nuclear interior, a theoretical model is necessary, and in the present work we have used RMF as a working hypothesis. Although the RMF approach was applied before [7, 8], for studying the binding of $\Sigma$ hypernuclei, these studies were not constrained by any sound phenomenology since no $\Sigma$ hypernuclear bound state has ever been clearly established [11], with the exception perhaps of a $J^p = 0^+, I = 1/2^- / 2^+$ $^4_\Sigma$He bound state [27] which is a too light system to be used as input to RMF calculations. Furthermore, recent searches for $\Sigma$ hypernuclear peaks, some of which had been reported on the basis of limited statistics, have yielded negative result [28, 29]. This leaves $\Sigma^-$ atom data as the sole source of any possible $\Sigma$ hypernuclear phenomenology.

The application of the RMF calculational scheme to $\Sigma^-$ atom data in terms of three coupling-constant ratios ($\alpha_\omega, \alpha_\sigma, \alpha_\rho$) has been successful in showing that very good quality fits, reproducing these data, can be made. The larger $\alpha_\omega$ is (in the range 0 to 1), the better is the fit. Of these fits the best ones, in the range $\alpha_\omega \sim 2/3$ to 1, indeed produce SE $V_{\Sigma_{\text{opt}}}^\Sigma$ with a volume repulsion in the nuclear interior. The $\alpha_\omega = 1$ fit is an excellent one, reaching almost as low a $\chi^2$ value as those produced by the completely phenomenological DD optical potential fits of table 1. The observation that using relatively high values of $\alpha_\omega$ is distinctly superior to using relatively low values of $\alpha_\omega$ within the RMF $\Sigma$-nuclear dynamics is in contrast to the situation in applying the RMF approach to $\Lambda$ hypernuclei [2, 3, 6]. There, as stressed in ref. [30], a wider range of values for $\alpha_\omega$ is roughly equally acceptable, since for any given value of $\alpha_\omega$, the constraint of a $\Lambda$-nucleus potential well depth of about 28 MeV attraction [31] can be satisfied by deriving an appropriate value for $\alpha_\sigma$. In the absence of $\Sigma$ hypernuclear data, no analogous constraint is operative for RMF $\Sigma$-nuclear applications. The volume repulsion obtained in this work for the isoscalar component of $V_{\Sigma_{\text{opt}}}^\Sigma$ in fact precludes binding for $\Sigma$ hypernuclei, at least for those based on core nuclei with a
small neutron excess, \((N - Z)/A \ll 1\).

It is gratifying to conclude that both approaches, the phenomenological one and the RMF approach, agree with each other, for a comparable degree of fit, in producing an isoscalar repulsion in the nuclear interior and, for \(\Sigma^-\), an added isovector repulsion. We stress that this isovector repulsion, with \(\alpha_\rho \sim 2/3\), was derived independently of the values assumed by the isoscalar coupling constants ratios. We recall that the common assumption of a pure \(F\) coupling for the \(\rho\) meson in SU(3), or equivalently of a universal \(\rho\) coupling to isospin, gives \(\alpha_\rho = 1\), a value which is rather different from that determined by fitting to the \(\Sigma^-\) atom data. However, it is well known that in the OBE approach which unlike the RMF approach does use SU(3) guidelines for connecting the \(S = -1\) sector to the \(S = 0\) sector, the nuclear isovector single-particle potential \(V_1^{(N)}\) is generated largely by other meson contributions than that due to a direct \(\rho\) exchange \[32\]. In the RMF approach, the \(\rho\) field is the only agency through which \(V_1^{(N)}\) or \(V_1^{(\Sigma)}\) can be generated, so that the statement \(\alpha_\rho \sim 2/3\) is to be construed just as implying that \(V_1^{(\Sigma)} \sim (2/3)V_1^{(N)}\), where the baryon-nucleus isovector potential contribution is defined by

\[
V_1^{(B)} \tilde{t}_B \cdot \vec{T}/A , \quad \tilde{t}_B = \frac{\frac{1}{2}\vec{\tau}}{\vec{T}} \quad \text{(14)}
\]

and \(\vec{T}\) is the nuclear isospin operator with \(T_3 = (Z - N)/2\). Since the nuclear isovector potential close to zero energy is estimated \[33\] as \(V_1^{(N)} \sim 120\) MeV, our estimate for \(V_1^{(\Sigma)}\), based on fitting \(\Sigma^-\) atomic Pb, is \(V_1^{(\Sigma)} \sim 80\) MeV. This value may be compared with the value \(V_1^{(\Sigma)} \sim 60\) MeV \[20\], or \(V_1^{(\Sigma)} \sim 55\) MeV \[32\], both estimates following Model D of the Nijmegen group \[34\] in agreement with the estimate derived from the phenomenology of \(^{12}\)C\((K^-, \pi^\pm)\) reactions \[35\].

As in ordinary nuclear physics, the \(\Sigma\)-nucleus isovector potential cancels partly (for charged \(\Sigma^\pm\)) the \(\Sigma\) Coulomb potential due to the nuclear charge distribution. For \(\Sigma^+\), we have checked that the attractive symmetry-energy contribution due to \(V_1^{(\Sigma)}\) generally does not overcome the repulsive isoscalar contribution plus the repulsive Coulomb energy, so that it is unlikely to bind a \(\Sigma^+\) in nuclei. For \(\Sigma^0\), where no symmetry energy or Coulomb energy
contribute, binding is precluded by the volume repulsion of the isoscalar $\Sigma$ nucleus potential. These considerations have to be modified in very light hypernuclei where symmetry energy dominates over the Coulomb interaction (plus other charge-dependent effects due mostly to the mass differences within the $\Sigma$ charge triplet), leading to a better description of $\Sigma$ hypernuclei in terms of isospin eigenstates rather than charged $\Sigma$ states [35]. The only $\Sigma$ hypernuclear system for which this dominance has been established by a detailed calculation [36] is $^4_2$He where the $\Sigma$-nucleus isovector potential, represented as a Lane term of the form (14), is responsible for the binding of the $I = 1/2, 0^+$ state. Without this Lane term, $^4_2$He would be unbound.

For $\Sigma^-$, the attractive Coulomb potential due to the nuclear charge distribution gives rise to an infinite set of $\Sigma^-$ atomic states. Some of these states, however, for high $Z$ nuclear cores, may be called nuclear states inasmuch as the corresponding wavefunctions are localized within the nucleus or in its immediate surface region. The energies and wavefunctions of these “nuclear” states generated by $V_C$ can be approximated by noting that for a uniform charge distribution of a total charge $Ze$ and radius $R$, $V_C(r < R)$ is a harmonic oscillator potential

$$V_C = -V_0 + \frac{1}{2} M\omega^2 r^2,$$  

(15)

$$V_0 = \frac{3}{2} Z \alpha \frac{\hbar c}{R}, \quad \hbar \omega = \sqrt{\frac{2 V_0 \hbar c}{3 M c^2 R}},$$  

(16)

with a depth $V_0$ that grows as $A^{2/3}$ and an essentially $A$-independent oscillator spacing $\hbar \omega$, $\hbar \omega = 3.25 \pm 0.03$ MeV between $^{40}$Ca and $^{208}$Pb for $R = [(5/3) < r^2 >]^{1/2}$ in terms of the nuclear charge rms radius $< r^2 >^{1/2}$. For example, $R = 4.49$ fm for $^{40}$Ca and $R = 7.10$ fm for $^{208}$Pb. Requiring the rms radius of the $\Sigma^-$ harmonic oscillator wavefunction to be less than or equal to $R$, one gets a series of nuclear radius values $R_N$

$$R_0 = 3.87 \text{ fm}, \quad R_1 = 5.00 \text{ fm}, \quad R_2 = 5.92 \text{ fm}, \quad R_3 = 6.71 \text{ fm}, \quad R_4 = 7.42 \text{ fm},$$  

(17)

such that for $R_N < R$, the states of the first major shells up to and including $N$ are “nuclear”. For $^{208}$Pb, this suggest that these $\Sigma$ harmonic oscillator states belonging to $N = 0, 1, 2, 3$
are “nuclear”. This is confirmed by a numerical evaluation using the precise $V_C$, yielding the following binding energies

$$1s : 20.35 \text{ MeV} \quad (18)$$

$$1p : 16.92 \text{ MeV} \quad (19)$$

$$1d : 13.63 \text{ MeV}, \quad 2s : 13.83 \text{ MeV} \quad (20)$$

$$1f : 10.58 \text{ MeV}, \quad 2p : 11.11 \text{ MeV} \quad (21)$$

Including $V_{\Sigma}^{\Sigma}$ (L with $\alpha_\omega = 2/3$) in the binding-energy calculation, we find that these levels are displaced by 2 to 7.5 MeV to lower binding energies and also acquire strong-interaction widths. The $1s$ level is very broad, about 36 MeV, whereas the widths of the other levels are relatively small and vary between 1.2 MeV to 2.6 MeV. The wavefunctions of these other levels are “pushed” by $V_{\Sigma}^{\Sigma}$ outside the nucleus. These states may be considered a special case of “Coulomb assisted states”, a concept put forward in ref. [36]. A reaction which would excite preferentially a single $\ell$ value for $\Sigma^{-}$ in $^{208}\text{Pb}$, could locate these “nuclear” states for $\ell = 1, 2, 3$. Their location will give a valuable information on the nature (attractive or repulsive) of $V_{\Sigma}^{\Sigma}$ in this mass region. However, since these states are “pushed” outside the nucleus, the cross sections to excite them by a nuclear reaction are not expected to be sizable. The chances of establishing a meaningful $\Sigma$ hypernuclear spectroscopy, therefore, are not particularly encouraging at present.

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### Table 1 – Parameters of DD $\Sigma^-$ nucleus potentials. All parameters (except $\alpha$) are in units of fm. Underlined parameters were held fixed during the fits.

| potential | $Reb_0$ | $Imb_0$ | $Reb_1$ | $ReB_0$ | $ImB_0$ | $ReB_1$ | $\alpha$ | $\chi^2$ | $\chi^2/F$ |
|-----------|---------|---------|---------|---------|---------|---------|----------|----------|------------|
| I         | 2.1     | 0.50    | $-1.0$  | $-4.3$  | 0       | 0       | 0.5      | 16.7     | 0.9        |
|           | $\pm 0.8$ | $\pm 0.12$ | $\pm 0.4$ | $\pm 2.7$ |         |         |         |          |            |
| II        | 1.2     | 0.45    | $-0.45$ | $-2.4$  | $-0.18$ | $-1.9$  | 0.8      | 17.8     | 0.9        |
|           | $\pm 0.4$ | $\pm 0.19$ | $\pm 1.4$ |         |         |         |          |          |            |
Table 2 – Parameters of the RMF $\Sigma^-$ nucleus optical potentials. The linear parametrization from ref. [18] was used for the nucleonic sector. $\alpha_i = g_{i\Sigma}/g_{iN}$ ($i = \sigma, \omega, \rho$), and $t$ is defined in eq.(13).

| potential | $\alpha_\omega$ | $\alpha_\sigma$ | $\alpha_\rho$ | $t$ [MeV fm$^3$] | $\chi^2$ |
|-----------|----------------|----------------|--------------|----------------|---------|
| L1        | 1.0            | 0.770          | 2/3          | -400           | 18.1    |
| L2        | 2/3            | 0.544          | 2/3          | -300           | 23.9    |
| L3        | 1/3            | 0.313          | 0.65         | -265           | 26.5    |
FIGURE CAPTIONS

Fig.1: Total $\chi^2$ values as a function of the relative notch depth $d$ (cf. eq.3) superimposed on the best-fit DD optical potential I from Table 1 with $\alpha = 0.5$ (cf. eq.4), for several positions $R_N$ ($R_N = R_0 + a_0\Delta$) expressed via steps $\Delta$ of a nuclear diffuseness, with a radius $R_0 = 1.1A^{1/3}$ fm and a diffuseness $a_0 = 0.5$ fm.

Fig.2: $\text{Re} V_{\Sigma_{\text{opt}}}^\Sigma$ (solid lines) and $\text{Im} V_{\Sigma_{\text{opt}}}^\Sigma$ (dashed lines) as functions of $r$ for the best-fit DD optical potential I (cf. eq.4 and Table 1) for Si (Fig. 2a), Ca (Fig. 2b) and Pb (Fig. 2c). Arrows indicate the position of the corresponding nuclear rms radius.

Fig.3: $\text{Re} V_{\Sigma_{\text{opt}}}^\Sigma$ (solid lines) and $\text{Im} V_{\Sigma_{\text{opt}}}^\Sigma$ (dashed lines) as functions of $r$ for the RMF $\Sigma^-$ optical potential NL in Si. Potentials in Figs. 3a, 3b and 3c correspond to the vector meson coupling ratios $\alpha_\omega = 1/3, 2/3$ and 1, respectively.

Fig.4: Comparison of the linear (L) (Fig. 4a) and nonlinear (NL) (Fig. 4b) RMF $\Sigma^-$ optical potentials in Pb for the isoscalar vector meson coupling ratio $\alpha_\omega = 2/3$. The isovector vector meson coupling ratio is $\alpha_\rho = 2/3$ in both cases.
