Why the Salpeter screening formula cannot be applied in the Sun

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ABSTRACT

In a recent paper, Bahcall et al. (2000) list various new approaches to the problem of screening of nuclear reactions in stellar plasma and assert that they are all wrong or irrelevant. Except for two, all approaches mentioned by Bahcall et al. assume the mean field approximation. The two exceptions are Carraro et al. (1988) and Shaviv & Shaviv (2000a). While Carraro et al. (1988) paper is discussed shortly and refuted by Bahcall et al. (2000) the Shaviv & Shaviv (2000a) paper is not discussed and refuted only by association. However, the association is totally unfounded because Shaviv & Shaviv (2000a) have shown that kinetic equations must be used to solve the screening problem and that the mean field approximation is inadequate for this problem. They also showed that the Carraro et al. (1988) approach is erroneous.

Therefore we summarize here the method of $S^2$ and their main result. We contrast the kinetic equations method with the mean field approximation and expose the different assumptions and omissions in each method.

1. Introduction

In view of the controversy about how to calculate the screening and the new paper by Bahcall et al. (2000) we feel that an explanation of the method of Shaviv & Shaviv (2000a) (hereafter $S^2$) and in particular juxtaposition of it with the mean field methods is appropriate. The issue of this paper is not whether the screening resolves or not the solar neutrino problem. This paper is about how to calculate the screening irrespective of the consequences to the solar neutrino problem. It may very well be that the new screening aggravates the classical solar neutrino problem and enhances the discrepancy between the prediction of the standard solar neutrino model and the experiments.
The plasma correction to the rate of the solar nuclear reactions affects the theoretical prediction of the solar neutrino fluxes and consequently the predicted counting rate in the various undergoing experiments to detect solar neutrinos. Therefore, as accurate values as possible are needed for evaluation of the nuclear reaction rates. As is well known, if all attempts to explain the solar neutrino discrepancy between theory and observations fail, then one of the suggested explanations is neutrino oscillations. In this case, the accurate derivation of the parameters of the solution, like the oscillation length, would depend on the exact prediction of the neutrino fluxes in the classical theory. As plain as day, before a new theory is invented it is crucial to calculate correctly all effects created by the ‘classical’ theory.

The paper by Bahcall et al. (2000) enumerates various attempts to derive the screening correction to the rate of nuclear reactions in stellar plasma. The paper gives reference to $S^2$ in the context of dynamic screening and then does not discuss the paper, the method or the results, but instead discusses the dynamic screening of Carraro et al. (1988). As a matter of fact, $S^2$ show in detail that Carraro et al. (1988) are wrong in using the approach of a test particle instead of a particle in thermodynamic equilibrium. Neither the effect $S^2$ discuss, nor their kinetic approach, are mentioned in the Bahcall et al. (2000) paper but the impression the reader is provoked to adopt is that it is identical to Carraro et al. and hence equally wrong (see section 3.1 of their paper). This is far from being the truth and the purpose of this paper is therefore

- To show in what way the method of $S^2$ is new and different from the mean field methods.
- To explain what is the plasma effect $S^2$ discuss and why it is a natural consequence of an equilibrium state.
- Explain why the assumption of mean field does not apply to the screening in the Sun.

We first discuss the premises of the standard treatment of screening and then enumerate the assumptions of the $S^2$ method, as well as the $S^2$ definition of the screening from first principles and finally compare the various sets of tacit assumptions in each approach.

2. The standard treatment of screening

All treatments of the screening (except for Carraro et al. 1988 and $S^2$) are based on the mean field approximation. The mean field is the average field a particle feels in the plasma.
The average is calculated over a thermodynamically long times. According to the ergodic hypothesis this field is equal to the average field calculated in a snapshot with the average taken over all particles in the system. Is this approximation valid for the solar conditions? The Debye radius in the solar core is about \(0.87 < r >\), where \(< r >\) is the mean interparticle distance, so that \(N_D\), the mean number of particles in a Debye sphere, is 2-5 (depending on the charge of the ion). The mean field approximations treats this number as a fixed and very large number which does not fluctuate (because it is assumes that \(1/\sqrt{N} \ll 1\)) so as to obtain the mean field potential. The first claim of \(S^2\) is that because of the smallness of \(N_D\) the mean field approximation is a poor approximation for reaction kinetics. (cf Montgomery & Tidman 1964 for a discussion of the breakdown of the cluster expansion when \(N_D\) is small)

Why do the fluctuations matter? When the relevant property is constant with the kinetic energy like the potential energy, the averaging over the fluctuations is equivalent to averaging over a constant mean potential. But when a phenomenon is sensitive to the energy like the Coulomb barrier penetration, the average over fluctuations is not equivalent to penetration with the average energy.

The Bahcall et al. (2000) refers to the screening in the kinetic approach a la Clayton (1968) where particles react via a mean potential. However, we mention in passing that one can discuss the screening effect through a change in the number density of particles due to the correlation (cf Ichimaru (1994)) without considering the interaction directly.

3. The treatment of \(S^2\)

\(S^2\) assume neither the mean field nor that the amount of energy transferred in a ion-ion scattering is the mean potential energy per ion, but instead start from first principles. They defined the plasma effect during a scattering of two ions and calculated it directly without any additional assumption about the long time average field.

The formalism to include the plasma effect on the nuclear rate assumes that the particles are free at infinity and the only interaction is through the bare pair potential. In other words, the classical stellar nuclear reaction theory defines the screening energy as follows. The total energy of a pair (ignoring the surrounding plasma) is given as:

\[
E_{\text{pair}}^{\text{bin}} = E_{\text{kin},1} + E_{\text{kin},2} + \frac{Z_1 Z_2 e^2}{r_{12}}. \tag{1}
\]

The screening energy, that is the energy the incoming pair gains from the plasma, is then given by:

\[
E_{\text{scr}} = \Delta E_{\text{bin}}^{\text{pair}} = E_{\text{bin},e}^{\text{pair}} - E_{\text{bin},f}^{\text{pair}} \tag{2}
\]
where index c means close and index f means far away. Equation 2 compares directly the energy of the pair when it is close and when it is far away. The calculation should proceed therefore, as follows: calculate the evolution of all particles in the system. For every particle, find the nearest particle and declare it as ‘mates’. Evaluate now the dynamic evolution from the identification moment as a pair of approaching particles, through the approach and until the pair separates a given distance. Once the particles moved away calculate Eq. 2. In this way the screening is calculated directly from first principles without any additional assumptions.

Clearly, when the reaction takes place in vacuum $E_{\text{scr}} \equiv 0$. However, any effect the plasma has on the approaching pair will appear in $E_{\text{scr}}$. We stress that Eq. 2 correlates between the distance of closest approach and far away and not between the ‘initial’ and ‘final’ scattering states. According to the classical Salpeter or any mean field theory, there is no difference between the above two: the energy gained from the plasma by the approaching particles is returned by the separating particles. The balance is maintained per each collision.

In summary, the mean potential energy per particle, $E_{\text{pot}}$, namely the long time average of the potential energy, does not depend on the absolute or the relative kinetic energy of the particle. This classical result of statistical mechanics is manifested clearly in the calculations of $S^2$. However, the energy gained (or lost) from the plasma by a scattering pair as given by Eq. 2 may depend on the relative kinetic energy.

It is clear that in order to solve Eq. 2 for particles in the plasma one needs a proper kinetic treatment and not a mean field approach. $S^2$ found that in view of the complexity of the problem (see also later) it is advantageous to look for a method that handles the problem from first principles and without any additional assumptions or approximations. Such a method is the Molecular Dynamics method (MD).

What is the Molecular Dynamic method? One takes a system of N positive (the protons) and N negative (the electrons) particles, assumes a pure Coulomb interaction between every two particles (p-p, p-e and e-e) and solves the 3N second order Newton’s equation of motion. The MD method does not assume anything about a mean field or smoothing field etc. Nor does it assume a long time average potential for the scattering of any two particles. In the MD method the scattering of each pair with all the interaction of the particles around it is exactly followed. To have a decent representation of the various effects N must be large. In the case of $S^2$ $N = 10^5$ was used. (Some authors apply screened potential. However, $S^2$ apply a simple $1/r$ potential).
3.1. The new effect

Consider a pair of mutually scattering particles with a given relative kinetic energy between them. The basic findings of $S^2$ is that when the relative kinetic energy of the pair is low it gains energy from the plasma as they approach each other. On the other hand, particles with large relative kinetic energy lose energy to the plasma as they approach each other. Gain or loss are used in terms of the relative kinetic energy of the pair and not in absolute terms. If one takes a random pair of approaching particles with a given relative kinetic energy, it may lose or gain energy. However, the average for a given relative kinetic energy is positive for low relative kinetic energies and negative for high relative kinetic energies. (*In this respect it is exactly like the action of dynamic friction in a cluster of stars.*)

Of course, on the average the energy gained/lost by the scattering particles when summed overall particles in the plasma, must vanish. The balance is over all particles not per collision.

The physical explanation of the effect is simple. When the relative energy of the scattering particle is lower than the mean thermal energy of the particles in the screening cloud, it gains energy upon penetrating into the cloud. However, when the relative kinetic energy is higher than the mean thermal energy, the pair loses energy by penetrating into the cloud of each other. One could predict the existence of such an effect without going into the long calculations. However, the amount of energy exchange and the energy of turn over from gain to loss must be obtained from calculations.

We note at this point that none of the various treatments of screening (for reference see Bahcall et al. 2000) satisfy the condition of overall energy balance explicitly. In other words, how a pair of scattering protons, which gains energy from a cloud of 3 particles upon approaching each other, returns this energy to the cloud as they separate? The classical treatments assume implicitly a detailed balance, namely each approaching pair, which gains energy from the plasma, must return it to the plasma upon separation (which is apparently good for $N_D \to \infty$). If this assumption is dropped from the mean field approximation, then one has to explain how the approaching particles return the gained energy to the plasma.

3.2. The MD method reproduce the statistical mechanics results

Without fail the MD reproduces all standard statistical mechanics results obtained after a long time average. The effect found in $S^2$ does not violate thermodynamics nor does it need a new or special assumption about thermodynamics or screening. It is a simple consequence
of the global balance in the plasma. If some particles gain energy from the plasma others must lose energy to the plasma.

The MD calculation of $S^2$ reproduces the following standard statistical mechanics results:

- The particles obey a Maxwell-Boltzmann distribution. The calculation starts from an arbitrary initial distribution in the phases space and relaxes after an initial time to a very accurate Maxwell-Boltzmann distribution. The distribution is reproduced over many orders of magnitudes in number of particles.
- The long time average of the potential energy per particle does not depend on the absolute kinetic energy of the particle.
- The average force acting on the particle vanishes but the root mean square does not and as a matter of fact is very large and does not depend on the kinetic energy of the particle.
- The potential energy of the particle does not depend on the mass of the particle. The potential energy per particle does not change when the mass of the particles changes.
- The long time average potential between two ions, say protons, is the Debye Hückel potential and the potential energy per particle is close to $\Gamma$, the plasma parameter.
- All thermodynamic properties like mean kinetic energy, mean potential energy per particle etc are in agreement with statistical mechanics.
- The power spectrum of the fluctuations does not show any scale and obey a power law.
- The distribution of the potential energy per particle as seen in a snapshot (space average) is properly obtained.

3.3. What MD has that mean field does not

What does the Molecular Dynamic contain that the mean field is missing:

- The exact energy exchange between the two approaching ions and each massive ion in the cloud around the interacting pair is followed. The same is true of the energy exchange of the approaching pair and the electrons. Thus, while the pair of approaching particles gains/loses energy, the energy lost/gained by the other particles is fully
accounted for. In the mean field approach the energy lost by the cloud of particles composing the mean field is not accounted for. The implicit assumption is that the mass of the potential is infinite.

- The recoil of each particle composing the ‘effective potential’ upon the approach of the interacting pair of ions is fully accounted for in the MD method. In the mean field approach the tacit assumption is that the potential has an infinite mass and does not recoil (or lose/gain energy during the approach).

- The MD secure automatically an overall detailed balance of the energy exchange between the plasma and the scattering pair. Some pairs lose energy while others gain, but the sum over the entire system vanishes. In the mean field one assumes a detailed balance, namely while a pair of ions gains energy from the plasma as they approach each other, it loses exactly this energy as it moves away. But it is difficult to see how energy gained from a cloud of 2-5 particles during the approach is exactly returned to the cloud during the separation, or is it returned elsewhere?

- As the number of particles in the Debye sphere is very small, the MD treats properly the third body interaction as well as all higher orders. In the mean field, it is always the effect of the ‘many’ particles composing the mean spherical field.

- On the one hand the instantaneous potential energy of all particles is not identical (it fluctuates) and there is a distribution of potential energies. On the other hand, the Coulomb penetration is very not linear with kinetic energy. Consequently, taking the Coulomb penetration at the value of the mean field may be very inaccurate.

- The mean field approach assumes that the energy gained by the approaching pair (and later returned to the plasma) is always the mean potential energy of an ion in the plasma. The MD calculates the energy exchange exactly without recourse to any assumption.

- The method of Molecular Dynamics is equivalent to summing the ladder (interaction) diagrams to all orders.

4. Conclusions

4.1. General conclusions

- \( S^2 \) applied Molecular Dynamics to calculate from first principles the effect of the plasma on a pair of reacting particles. The numerical method is equivalent to summation of
The interaction diagrams to all orders.

- The MD reproduces all results of statistical mechanics to a high accuracy.

- The basic result of $S^2$ is that the mean energy exchange with the plasma is positive for pairs with low relative kinetic energy (the pair gains energy) and negative for high relative energy pairs (the pair loses energy). The sum overall scatterings vanishes for each specie in the plasma.

- The interaction of the plasma with the scattering particles is dynamic and must be derived from a proper kinematic considerations.

- The spread in the energy exchange of the particles with the plasma is large and must be taken into account in calculating the effective Coulomb penetration.

- The pair-plasma energy exchange in a single scattering is mainly between the massive pair and the massive ion component. The contribution of the electrons in a single collision is calculated by the MD method but is effectively negligible. Of course, over thermodynamic times there is an energy exchange between the massive ions and the light electrons.

- The mean field approximation ceases to be valid under the conditions prevailing in stellar cores in general and in the Sun in particular. Even using only the ion contribution to the mean field is not sufficient since few body interactions cannot be neglected.

- Averaging the effect of the plasma into a mean long term potential is not appropriate for handling very non linear functions, like the Coulomb barrier penetration.

- The penetration factor calculation must include the spread in the energy exchange.

### 4.2. Could the screening affect the prediction of the solar neutrino flux?

While it is not the purpose of this paper to discuss this particular problem we would like to comment, as an epilogue, as follows: all numerical tests about the effect of the plasma assumed a mean field approximation where all reaction are either enhanced or not affected (for a review see Dzitko & Turck-Chièze(1995). Shaviv & Shaviv (2000b) show that (a) due to fluctuations the problem is more complicated than just assumed by the approach using a ‘corrected’ penetration factor. (b) Some reactions are enhanced and others are suppressed depending on the relation of the Gamow energy to the turnover energy (from gains to losses) and the width of the screening distribution. Finally, assessment of the effect is carried out and will be published in due course. The effect found so far is far from being negligible.
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