Response time of a normal-superconductor hybrid system under the step-like pulse bias

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The response of a quantum dot coupled with one normal lead and a superconductor lead driven by a step-like pulse bias \(V_L\) is studied using the non-equilibrium Green function method. In the linear pulse bias regime, the responses of the upwards and downwards bias are symmetric. In this regime the turn-on time and turn-off time are much slower than that of the normal system due to the Andreev reflection. On the other hand, for the large pulse bias \(V_L\), the instantaneous current exhibits oscillatory behaviors with the frequency \(h\Omega = qV_L\). The turn on/off times are in (or shorter than) the scale of \(1/V_L\), so they are faster for the larger bias \(V_L\). In addition, the responses for the upwards and downwards bias are asymmetric at large \(V_L\). The turn-on time is larger than the turn-off time but the relaxation time depends only on the coupling strength \(\Gamma\) and it is much smaller than the turn-on/off times for the large bias \(V_L\).

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I. INTRODUCTION

In the past two decades, nanoscopic physics has developed significantly and becomes an active field of condensed-matter physics. The quantum transport property also becomes one of the most interesting phenomena in nanoscopic physics because of the possibility of designing and fabricating artificial setups in the nanometer scale. Based on the transport physics in nanoscopic system, a rich field for basic and applied research is opened. Furthermore, the time dependent nanoscopic transport, in which the external time dependent fields drives the electrons tunnel through a nanoscopic system, has received increasing attention in recent years. The main feature of the transport in the nanometer scale is that the electron keeps the phase coherence when traversing through the device. While the external time dependent field affects the phase factor of the incident electron differently in different parts of the system. If the external time dependent field is sinusoidal (e.g. microwave radiation), an electron can tunnel through the system by emitting or absorbing photons giving rise to the photon-assisted tunnelling (PAT). Electron transport with PAT has been extensively investigated for various systems, such as single or two coupled quantum dot (QD) Kondo regime, hybrid system and so on. For transient transport, one of the most interesting issues is how fast a device turn on or turn off a current. With the development of the molecular devices, there is clearly a need to technologically provide a particular viable switching device. Indeed, some recent experimental and theoretical works have already begun to study the response of ac signals of the molecular devices. Consequently, a step or pulsed ac signals are the simplest choice, since it can provide a less ambiguous measure of time scales. For this reason, the pulsed field was studied in a variety of systems, including Kondo regime a single QD or nano structure.

So far, the study of response of pulsed bias is only focused on normal nanostructures. Since the interplay between nanoscopic physics and the physics of superconductivity has made the hybrid structure a very fruitful research field, it will be interesting to study the dynamic response a hybrid structure with a superconductor lead where the Andreev reflection is present near the normal-superconductor (N-S) interface. Indeed, there are many interesting phenomena in the N-S hybrid systems. First of all, because there exists an energy gap \(\Delta\) in the superconductor, an incident electron from the normal side with energy \(\epsilon\) inside the gap \(\Delta\) can not tunnel into the superconductor. But the tunneling can occur via a two-particle process, in which the incident electron is reflected as a hole with the energy \(-\epsilon\). At the same time, a Cooper pair is created in the superconductor region. This is the Andreev reflection.\(^{15}\) Secondly, for the superconductor-normal region-superconductor (S-N-S) system, Andreev bound states form in normal region due to the Andreev reflections at N-S interfaces. These bound states exist in pairs, and a Josephson supercurrent can flow through the S-N-S system which is carried by the Andreev bound state.\(^{16}\) Thirdly, when the S-N-S device is under an external dc bias \(V\), an ac current with frequency \(\omega = 2|e|V\) appears. The time-average current versus bias \(V\) exhibit the subharmonic gap structure when \(eV < 2\Delta\).

In this paper, we explore the effect of Andreev reflection on the ac response of hybrid system. Specifically, we investigate ac response of a quantum dot (QD) with a single energy level \(\epsilon_0\) connected by a normal and a superconductor lead (N-QD-S). For simplicity, we consider a large QD so that the intra-dot electron-electron (e-e) is weak and can be neglected. The transient transport is driven by a pulsed bias potential \(W(t)\). For simplicity, the ac pulsed bias is only added in the left lead, and we set \(W_R(t) = 0\). We consider two different pulsed biases: (i) upwards pulse...
with \( W_L(t) = 0 \) for \( t < 0 \) and \( W_L(t) = V_L \) otherwise. (ii) downwards pulse with \( W_L(t) = V_L \) when \( t < 0 \) and \( W_L(t) = 0 \) otherwise. For normal structures, Wingreen et al. presented a general formula for the current driven by the time dependent external fields by using the non-equilibrium Green function (NEGF) method. With this general formula the time dependent current driven by the ac pulse can be calculated. For hybrid structures, the system is in steady state at \( t = 0 \) and the current is time independent. At \( t = 0 \), bias is abruptly turned on for the upwards pulse case or turned off for the downwards pulse case. After that, the system begins to relax and the Andreev reflection plays an important role in the relaxation process. Finally, the system enters into a new steady state. We find that, the relaxation time depends on the coupling strength and is slower in the N-QD-S system (named hybrid system hereafter) than in the N-QD-N system (named normal system hereafter). In the linear bias regime, the rising and falling processes are symmetric so that the turn-on time is same as the turn-off time. In this regime, the Andreev reflection is important. As a result, the instantaneous current shows a clear increase (decrease) before reaching the new steady state for the downwards (upwards) pulse. For the large bias case, the time dependent current oscillates with the frequency \( \omega = qV_L \). In this regime, the upwards and the downwards processes are asymmetric and the turn-on time is much larger than the turn-off time. In this nonlinear regime, the Andreev process is negligible and the current in the hybrid system is close to that of the normal system.

The rest of this paper is organized as follows: In Sec.II, the theoretical formula for calculating the time dependent current in N-QD-S system is presented. To understand the numerical results, the current away from the current at \( t = 0 \) is expanded to the first order in the external bias. In Sec.III, we show the numerical results along with some discussions. Finally, the brief summary is given in Sec. IV.

II. THEORETICAL FORMULA

Considering a hybrid system that consists of a QD coupled to a normal metal lead and a superconductor lead with the external time dependent bias potential \( W_L(t) \) that is added only on the left normal lead. The Hamiltonian of the system is written as follows:

\[
H = H_L + H_R + H_D + H_T
\]

where \( H_L \) and \( H_R \) describe the left normal lead and the right superconductor lead, respectively. \( H_D \) is Hamiltonian of the isolated central QD, and the \( H_T \) couples the left and right leads to the QD. They can be written in the following forms:

\[
H_L = \sum_{k,\sigma} (\epsilon_{L,k} + W_L(t)) C_{L,k\sigma}^\dagger C_{L,k\sigma}
\]

\[
H_R = \sum_{k,\sigma} \epsilon_{R,k\sigma} C_{R,k\sigma}^\dagger C_{R,k\sigma} + \sum_k [\Delta C_{R,k\uparrow}^\dagger C_{R,\downarrow k\uparrow}^\dagger + \Delta C_{R,\downarrow k\downarrow}^\dagger C_{R,k\downarrow}^\dagger]
\]

\[
H_D = \sum_\sigma \epsilon_0 d_\sigma^\dagger d_\sigma
\]

\[
H_T = \sum_{k,\sigma,\alpha} t_{k,\alpha} C_{\alpha,k\sigma}^\dagger d_\sigma + h.c.,
\]

where \( \alpha = L, R \). The operator \( d_\sigma \) and \( C_{\alpha,k\sigma} \) destroy an electron with spin \( \sigma \) in the QD and in the left or right lead, respectively. For simplicity, we only consider a single level in the QD and neglect intradot electron-electron Coulomb interaction. Under the adiabatic approximation, the time-dependent bias potential can be included in the single electron energy \( \epsilon_{L,R}(t) \). We separate \( \epsilon_{L,R}(t) \) into two parts: \( \epsilon_{L,R} \) and \( W_L(t) \), where \( \epsilon_{L,R} \) is the time-independent single electron energy and \( W_L(t) \) is a time dependent part from the external time dependent bias potential. In this paper, \( W_L(t) \) is the step-like pulse with two different forms: (i) upwards pulse with \( W_L(t) = 0 \) when \( t < 0 \) and \( W_L(t) = V_L \) otherwise, (ii) downwards pulse with \( W_L(t) = V_L \) when \( t < 0 \) and \( W_L(t) = 0 \) otherwise. These two types of pulse describe the system abruptly turned on or turned off at time \( t = 0 \). \( \Delta \) in the Hamiltonian \( H_R \) is the superconducting energy gap. We assume that \( \Delta \) is a real parameter by selecting a special phase of the superconductor lead in our calculation. Due to the existence of the superconducting lead, it is convenient to introduce the Nambu representation. In the Nambu representation, the Fermi energy of the left normal lead is set at the superconducting condensate and for the spin down electron the energy is negative and is viewed as the hole. So, the Hamiltonian in Eqs. (2) can be rewritten in the matrix form:

\[
H_L = \sum_k \Psi_{L,k}^\dagger \begin{pmatrix} \epsilon_{L,k} + W_L(t) & 0 \\ 0 & -\epsilon_{L,-k} - W_L(t) \end{pmatrix} \Psi_{L,k}
\]

\[
H_R = \sum_k \Psi_{R,k}^\dagger \begin{pmatrix} \epsilon_{R,k} & \Delta \\ \Delta & -\epsilon_{R,-k} \end{pmatrix} \Psi_{R,k}
\]

\[
H_D = \Phi^\dagger \begin{pmatrix} \epsilon_0 & 0 \\ 0 & -\epsilon_0 \end{pmatrix} \Phi
\]

\[
H_T = \sum_{k,\alpha} \Psi_{k,\alpha}^\dagger \begin{pmatrix} t_{k,\alpha} & 0 \\ 0 & -t_{k,\alpha}^\dagger \end{pmatrix} \Phi + H.C.,
\]

where

\[
\Psi_{\alpha,k} = \begin{pmatrix} C_{\alpha,k\uparrow} \\ C_{\alpha,-k\downarrow} \end{pmatrix}, \quad \Phi = \begin{pmatrix} d_\uparrow \\ d_\downarrow^\dagger \end{pmatrix}.
\]

The current from the left lead to the QD can be calculated from the evolution of the number operator of the electrons in the left lead, \( N_{L,\uparrow(t)} = \).
\[ J_L(t) = -2qR \int_{-\infty}^{t} dt' \{ [G^<(t', t')] \Sigma_L^<(t', t) + G^<(t', t') \Sigma_L^>(t', t) + G^>(t', t') \Sigma_L^<(t', t) \}. \]

Here the Green function \( G^{r,a}(t, t') \) and the self-energy \( \Sigma^{r,a}(t, t') \) are all two dimensional matrices in the Nambu representation. Since the spin up and spin down are symmetric in the Hamiltonian, the current contributed by the electrons with spin up is same as the current by the spin down electrons. Consequently, the current is given by:

\[ J_L(t) = -4qR \int_{-\infty}^{t} dt' \{ [G^<(t', t')] \Sigma_L^<(t', t) + G^<(t', t') \Sigma_L^>(t', t) \}. \]

where \( \Sigma_L^>(t, t') = \Sigma_L^<(t, t') \). Here the Green function \( G^r < t and the self-energy \( \Sigma^r < a \) are the 2 unit matrix. Note that only \( G^<(t, t') \) is needed in the Eq. (6). By using the Keldysh equation \( G^r = G^r \Sigma^r G^a \) with the self-energies obtained in the appendix, the Green function \( G^r(t, t) \) can be solved:

\[ G^r(t, t) = \sum_{\sigma} \int dt_1 \int dt_2 G^r(t, t_1) \Sigma^r_{\sigma}(t_1, t_2) G^a(t_2, t) \]

\[ = i \int d\omega f(\omega) G^r(\omega) \tilde{\Gamma}(\omega) G^a(\omega) + \]

\[ i \sum_{\sigma} \int d\omega f(\omega) A_{L,\sigma}(\omega, t) s_\sigma \Gamma_{R,\sigma}(\omega) A_{\sigma}^+(\omega, t), \]

\[ \Gamma_{R}(\omega) = \theta(\omega - \Delta) \frac{\Gamma_R}{\sqrt{\omega^2 - \Delta^2}} \left( \begin{array}{c} |\omega| \\ \Delta \end{array} \right), \]

and

\[ s_\uparrow = \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} \right), \quad s_\downarrow = \left( \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array} \right), \]

\[ A_{L,\sigma}(\omega, t) = \int_{-\infty}^{t} dt_1 G^r(t, t_1) e^{i\omega(t-t_1) + i\sigma \int_{t_1}^{t} dt_2 W_L(t_2)}. \]

The Green functions \( G^{r,a}(\omega) \) in Eq. (7) are the Fourier transformation of \( G^{r,a}(t, t') \) with \( G^{r,a}(\omega) = \int dt (t - t') e^{i\omega(t-t')} G^{r,a}(t, t') \). Notice that in the present system the retarded and advanced Green functions \( G^{r,a}(t, t') \) are still the function of the time difference \( t - t' \), although there exists the time dependent bias \( W_L(t) \), since \( G^r(\omega) \) can be obtained from Dyson equation:

\[ G^r(\omega) = \left[ \omega - \hat{H}_{dot} - \Sigma_{\sigma}^r - \Sigma_{\sigma}^R \right]^{-1} = \frac{1}{Det} \times \left( \begin{array}{c} B_{11} \\ i\omega \Gamma R \beta' / 2 \\ B_{22} \end{array} \right) \]

where \( B_{11} = \omega + \epsilon_0 + i\Gamma_L / 2 + i\Gamma_R \beta / 2, B_{22} = \omega - \epsilon_0 + i\Gamma_L / 2 + i\Gamma_R \beta / 2, \beta = \Delta / \sqrt{\omega^2 - \Delta^2}, \beta' = \omega / \sqrt{\omega^2 - \Delta^2}, \)

\[ Det = B_{11}B_{22} + (\Gamma_R \beta)^2 / 4, \]

and \( \nu = 1 \) for \( \omega > -\Delta \) and \( \nu = -1 \) otherwise. In the above derivation, the wide-band limit has been used and \( \Gamma_0 \) are assumed independent of \( \omega \). It also is worth mentioning that the Green function \( G^{r,a}(\omega) \) is not affected by the time-dependent bias potential \( W_L(t) \).

Substituting \( G^{r,a}(t, t) \) [in Eq. (7)] and the self-energies \( \Sigma^{r,a}(t, t) \) (in appendix) into Eq. (6), the time-dependent current \( J_L(t) \) is obtained straightforwardly. Similar to work in the normal system by Wingreen, Jauho, and Meir, the current \( J_L(t) \) can also be split into two terms \( J_L^{\text{in}}(t) \) and \( J_L^{\text{out}}(t) \):

\[ J_L^{\text{in}}(t) = 4q \int d\omega f(\omega) \text{Im} \{ \Gamma_{L}[A_{L\uparrow}(\omega, t)] \} \]

\[ J_L^{\text{out}}(t) = -2q \int d\omega f(\omega) \text{Re} \{ \Gamma_{L}(\omega) G^r(\omega) G^a(\omega) + \sum_{\sigma} A_{L,\sigma}(\omega, t) s_\sigma \Gamma_{R,\sigma}(\omega) A_{\sigma}^+(\omega, t) \}. \]

The above formulations [Eqs. (9,11)] for calculating the current are valid for any time-dependent bias \( W_L(t) \). In the following, two special cases for upwards and downwards pulses \( W_L(t) \) are substituted into these formulations to obtain \( A_{L\uparrow}(\epsilon, t) \) [Eq. (9)] and then the currents \( J_L^{\text{in}}(t) \) and \( J_L^{\text{out}}(t) \) [Eqs. (11)] for the downwards pulse with \( W_L(t < 0) = V_L \) and \( W_L(t > 0) = 0, A_{L\uparrow}(\epsilon, t) \) is found to be:

\[ A_{L\uparrow}(\omega, t < 0) = G^r(\omega + V_L) \]

\[ A_{L\uparrow}(\omega, t > 0) = G^r(\omega) + \int \frac{dE}{2\pi i} e^{-i(E - \omega) t} G^r(E) \]

\[ \frac{1}{E - \omega - V_L - i0^+} - \frac{1}{E - \omega - i0^+} \]

For the upward pulse with \( W_L(t < 0) = 0 \) and \( W_L(t > 0) = V_L, A_{L\uparrow}(\epsilon, t) \) is:

\[ A_{L\uparrow}(\omega, t < 0) = G^r(\omega) \]

\[ A_{L\uparrow}(\omega, t > 0) = G^r(\omega + V_L) - \int \frac{dE}{2\pi i} e^{-i(E - \omega - V_L) t} G^r(E) \]

\[ \frac{1}{E - \omega - V_L - i0^+} - \frac{1}{E - \omega - i0^+} \]
rewrite $A_{LD,U,s}(t,t)$ for $t > 0$ in the following form by using the residue theorem:

$$A_{LD,1}(\omega,t > 0) = G^r(\omega) + e^{-iVLt} \int_t^\infty d\tau e^{i(\omega+VL)\tau}G^r(\tau) - \int_t^\infty d\tau e^{i\omega\tau}G^r(\tau),$$

$$A_{LU,1}(\omega,t > 0) = G^r(\omega + VL) + e^{iVLt} \int_t^\infty d\tau e^{i\omega\tau}G^r(\tau) - \int_t^\infty d\tau e^{i(\omega+VL)\tau}G^r(\tau).$$

(14)

The expressions of $A_{L,\pm}(\omega,t)$ are similar to that of $A_{LD,1}(\omega,t)$ and can be obtained from Eq. (14) by changing $VL$ to $-VL$. After solving $G^r(\omega)$ and $A_{LD}(\omega,t)$, the currents $J_{in}^L(t)$ and $J_{out}^L(t)$ Eq. (11) can be calculated straightforwardly. In the limits $t \leq 0$ and $t \to \infty$, the system is in the steady state. $A_{LD,\sigma}(\omega,t)$ and $A_{LU,\sigma}(\omega,t)$ in Eq. (14) then reduce to the value of the steady state in these two limits and so is the current $J_L(t)$. For example, for the downward pulse, $A_{LD,\sigma}(\omega,t) = G^r(\omega + \sigma VL)$ for $t \to 0$, and $A_{LD,\sigma}(\omega,t) = G^r(\omega)$ when $t \to \infty$. Furthermore, the current $J_L(t)$ reduces to the one of the steady case with dc bias $VL$ when $t \leq 0$, and is zero when $t \to \infty$. On the other hand, for the upwards pulse, the current $J_L(t)$ is zero when $t \leq 0$, and is same with the steady state current with the dc bias $VL$ in $t \to \infty$ limit.

In the small pulse bias $VL$ limits, we can expand $A_{LD}(\omega,t > 0)$ to the first order of $VL$ as: $A_{LD,\sigma}(\omega,t > 0) = A_{L,\sigma}(\omega,t) + A_{LU,\sigma}(\omega,t)VL$. $A_{L,\sigma}(\omega,t > 0)$ can be expressed as:

$$A_{L,\sigma}(\omega,t > 0) = -i\sigma \int_0^\infty d\tau e^{i\omega\tau}G^r(\tau) - \sigma \int_0^\infty d\tau e^{i\omega\tau}G^r(\tau)$$

(15)

From Eq. (15), we can see that $A_{LU,\sigma}(\omega,t) = -A_{L,\sigma}(\omega,t)$. This means that the upwards pulse and downwards pulse induce the same relaxation process in the small pulse bias $VL$ limits, except that the currents deduced from them are relaxed in the opposite direction. Finally, the currents $J_{in}^L(t)$ and $J_{out}^L(t)$ in small $VL$ limits can also be expanded as: $J_{in}^{in/out}(t) = J_{in/out}(0) + X^{in/out}(t)VL$. Here $X^{in/out}(t)$ is the first order expansion coefficient with the respect to $VL$, and $X^{in/out}(t)$ is expressed as:

$$X^{in}(t) = 4q \int \frac{d\omega}{2\pi} \text{Im} f(\omega) \Gamma_L \{A_{L,1}(\omega,t)\}_{11}$$

$$X^{out}(t) = -2q \int \frac{d\omega}{2\pi} \text{Re} f(\omega) \Gamma_L \sum_{\sigma} \{A_{L,\sigma}(\omega,t)\}_{s,\sigma} \Gamma_L [A_{L,\sigma}(\omega,t)]_{11}$$

(16)

FIG. 1: (Color on line) The first order expansion coefficient $X(t)$ of the current $J_L(t) - J_L(0)$ vs. the time $t$ for the downwards and upwards pulse bias case in the hybrid N-QD-S system (a) and the normal N-QD-N system (b). The parameters are: $\Gamma = 1$, $\delta \Gamma = 0$, $\Delta = 15$, $\epsilon_0 = 0$.

III. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculation, we set temperature to zero. In fact, finite temperature only makes the current curve more smooth and does not affect main features. We focus on the weak coupling case with $\Gamma_L/R \ll \Delta$ and set $\Gamma = \Gamma_L + \Gamma_R = 1$ as energy unit. The energy gap of the superconductor is $\Delta = 15$. The energy level $\epsilon_0$ in the central region is assumed to be zero which is same to the right Fermi level. Because at $t \leq 0$ the system is in the steady state and the current is time independent, so we only plot the current $J_L(t)$ and the related quantities for $t \geq 0$ in the following discussion.

First of all, we study the small pulse bias $VL$ limit, in which the instantaneous current $J_L(t)$ can be expanded as: $J_L(t) = J_L(0) + X(t)VL$, and we also take the symmetric barriers, i.e., $\delta \Gamma = \Gamma_L - \Gamma_R = 0$. The first-order expansion parameters $X^{in/out}(t)$ of the currents $J_{in}^L(t)$ and $J_{out}^L(t)$ versus the time $t$ are plotted in Fig. 1. Here the indices $U$ and $D$ denote the upwards and downwards pulses, respectively. For comparison, we also show the corresponding parameters $X^{U,D,in/out}(t)$ for the normal system in Fig. 1(b). From Fig. 1, we can see that the expanding parameters $X(t)$ for the upwards and downwards pulses are symmetric, i.e. $X^{U,D,in/out}(t) = -X^{D,in/out}(t)$. It means that in the small $VL$ limit (i.e. the linear regime), the current turned off or turned on by the downwards or upwards pulses in exactly the same manner with the same time scale for both normal system and hybrid system. In other words, the case of the downwards pulse is the reversal process of the upwards pulse. So in the following, we use the upwards pulse as an example in the linear region.

At time $t \leq 0$, the driving bias is zero for the upwards case. The system is in equilibrium state so the current $J_L^U$ is zero and $J_{LU}^{U,in}$ and $J_{LU}^{U,out}$ cancel to each other. At $t = 0$, the bias is abruptly switched on. At $t > 0$, the
bias $W_L(t)$ is kept at $V_L$ all along, the electrons with the energy in the bias window begin to traverse through QD. As the time $t$ increases, $J_{L}^{U,in}$ and $J_{L}^{U,out}$ deviate from the initial value ($t = 0$). A net current gradually increases and the device is gradually turned on. As a result, for the time $t$ from 0 to about $0.5(2\pi/\Gamma)$, $X_{L}^{U,in}(t)$ and $X_{L}^{U,out}(t)$ gradually increase (see Fig.1). This increasing process is almost the same for the normal system and the hybrid system. For the normal N-QD-N device, the relaxation process completes near the time $t = 0.5(2\pi/\Gamma)$ and $X_{L}^{U,out}(t)$ is the half of $X_{L}^{U,in}(t)$ at large time. On the other hand, for the hybrid N-QD-S device, the behavior of $X_{L}^{U,in}(t)$ is approximatively the same as that of N-QD-N at large time, but $X_{L}^{U,out}(t)$ begins to decrease when $t > 0.5 (2\pi/\Gamma)$, and it ends to zero at the end of the relaxation process. So the current $J_{L}^{U}(t = \infty)$ for the N-QD-S device is twice as large as that of the N-QD-N device. We interpret these properties as follows. For the normal system, the fact that $X_{L}^{in}(t = \infty)$ is twice of $X_{L}^{out}(t = \infty)$ is because $X_{L}^{in}(t = \infty)$ and $X_{L}^{out}(t = \infty)$ are respectively contributed by the electrons tunnelling from the left lead into the empty QD and from the QD into the empty left lead with the electronic energy $\omega$ between 0 and $V_L$, and in this energy range the distribution of the left lead is $f_L(\omega) = 1$ but the distribution in the QD is $(f_L(\omega) + f_R(\omega))/2 = 1/2$ for $t = \infty$. While for the hybrid N-QD-S system, after the bias is turned on, the Andreev reflection begins to play a role. For $J_{L}^{U,in}$, there is not much difference between the normal and hybrid systems, since the electrons always tunnel from the left lead into the QD in both systems. But for $J_{L}^{U,out}$, instead of reflecting electrons from QD into the left lead in normal system, the Andreev process reflects back the hole out of QD, which makes $J_{L}^{U,out}$ decrease. Note that $T_A$ can be expressed as:

$$T_A = \frac{\Gamma^4}{64\omega^4 + (\Gamma^2 + 4\Gamma^2)^2},$$

in the small bias limit ($\omega \approx 0$) and $\delta \Gamma = 0$, nearly all of the incoming electrons participate in the Andreev reflection. Because of this, $J_{L}^{U,out}(t = \infty)$ goes back to the initial (t=0) value. So $X_{L}^{out}(t)$ decreases to zero at $t = \infty$.

Next, we study the case of large pulse $V_L$. Fig.2(c) and (d) depict the currents $J_{L}^{U,in}$ and $J_{L}^{U,out}$ versus time $t$ for the large pulse strength $V_L = 10$. For comparison, $J_{L}^{U,in}$ and $J_{L}^{U,out}$ for the small pulse strength $V_L = 0.1$ are also plotted in the Fig.2(a) and (b). The currents $J_{L}^{U,in}$ and $J_{L}^{U,out}$ in the large bias case have the following characteristics: (i) In the small bias limit, the relaxation processes of upwards and downwards are symmetric. However, in the large pulsed bias $V_L$ case, they are asymmetric (see Fig.2c and 2d). For larger pulse bias $V_L$, the asymmetry are stronger. (ii) For the large bias case, $J_{L}^{U,in}$ for the upwards pulse oscillates with the frequency $\hbar \Omega = qV_L$, which can be clearly seen in Fig.2c and 2d for $V_L = 10$. At $V_L = 0.1$ the oscillation disappears because $\hbar \Omega = qV_L$ is too small to oscillate before the system is completely relaxed. (iii) $J_{L}^{U,out} (J_{L}^{D,out})$ of hybrid system increases (decreases) in the first and then decreases (increases), and it reaches maximum (minimum) before the current relaxed completely. This is different from the normal system, in which the currents $J_{L}^{U,out}$ and $J_{L}^{D,out}$ are monotonously relaxed into the steady state. (iv) The decreasing (increasing) process of the current $J_{L}^{U,out} (J_{L}^{D,out})$ in the large bias case is much weaker than that of the small bias case (see Fig.2b and d). Because for the large pulse, the energy of the incident electrons $\omega$ is large, then $T_A \ll 1$ from Eq. (17) and the Andreev reflection is weak. So most of the incident electrons participate in the normal reflection. Consequently, $J_{L}^{U,in}$ is humped up (or down) slightly.

Since the currents $J_{L}^{U,in}$ and $J_{L}^{U,out}$ can not be observed independently, in the following we study the total current $J_{L}(t) = J_{L}^{U,in} - J_{L}^{U,out}$ which can be measured in the experiment. Fig.3 shows the current $J_{L}^{U,D}$ driven by the upwards and downwards pulses versus the time $t$ for the different pulse strengths $V_L$. Here the current responses to the upwards and the downwards pulse are symmetric at small linear bias $V_L$ (see inset of Fig.3), but are asymmetric at the large bias $V_L$ (see main of Fig.3). At the large $V_L$, $J_{L}^{U,D}$ oscillates with the frequency $\hbar \Omega = V_L$. On the other hand, $J_{L}^{D}$ always changes slowly regardless of the large and small $V_L$.

Now we focus the turn on/off time (or rise/fall time) and the relaxation time (or saturation time). The former describes how fast can a device turn on/off a current, which is necessary to provide a particular viable switching device, and the latter was referred to how fast can the device goes to a new steady state after a bias is abruptly switched on. For the small bias $V_L$, the turn-on time, turn-off time, and the relaxation time are almost same.
regardless of the normal and hybrid systems. However these (turn on/off or relaxation) times for the normal N-QD-N device are much shorter than that of the hybrid N-QD-S device. For the normal device, it has been well turned on or off at $t = 0.2(2\pi/\Gamma)$. But for the hybrid device, the system is turned on or off until $t = 1.0(2\pi/\Gamma)$. On the other hand, for the large bias, the current $J_L(t)$ of the hybrid system has the same character with that of the normal system, so do the turn-on/off time and the relaxation time. Note that these three time scales are not equal now. The turn-on time is the fastest, even faster than the scale $1/V_L(2\pi/\Gamma)$. The turn-off time is in the scale $1/V_L(2\pi/\Gamma)$, which is longer than the turn-on time. The relaxation is $\sim 0.5(2\pi/\Gamma)$, which is the longest and only depends on the coupling strength $\Gamma$. Let us explain why the character of $J_L(t)$ for the normal and hybrid system are the same at large $V_L$ but very different at small $V_L$. Because at the large bias $V_L$, most of the incoming electron have the large energy $\omega$, then $T_A \ll 1$ from Eq. (14) and the Andreev reflection is weak, so the N-QD-S device and the N-QD-N device have the same turn-on/off and relaxation time. But for the small bias $V_L$, the resonant Andreev reflection is dominant in the transport process in the hybrid system, so that the current $J_L(t = \infty)$ of the hybrid system is twice as that of the normal system, and their character of $J_L(t)$ also are very different. So we will only discuss the small pulsed bias $V_L$ case further in the following.

At last, we consider the case of asymmetric barriers (i.e., $\delta \Gamma = \Gamma_L - \Gamma_R \neq 0$) and in the small pulsed bias $V_L$. Because in the small $V_L$, the time-dependent current $J_L(t)$ for the upwards and downwards pulse are symmetric, we only study the upwards case. Fig.4 plots the current $J_L(t)$ versus the time $t$ for the different asymmetric coupling strengths $\delta \Gamma$, and they have the following behaviors: (i) As $\delta \Gamma$ (i.e. $\Gamma_L$) increases, the current $J_L(t)$ rises faster, i.e. the turn-on time is shorter, because electrons with the energy in bias window can tunnel through the left barrier more easily with the larger $\Gamma_L$. This rising process of $J_L(t)$ are nearly same for the normal and hybrid systems. (ii) After the rise of $J_L(t)$ (at $t \approx 0.2(2\pi/\Gamma)$), Andreev reflection begins to dominate and gives rise to different sequent relaxation processes for the normal and hybrid systems. At $\delta \Gamma < 0$, $J_L(t)$ of the hybrid system humps slightly in the relaxation process, which is obviously different from the normal system in which $J_L(t)$ is monotonically relaxed into steady state. When $\delta \Gamma = 0$, $J_L(t)$ of the hybrid system passes a step and increases again. The relaxation time for the hybrid system is much longer than that of the normal system when $\delta \Gamma \leq 0$. When $\delta \Gamma > 0$, the relaxation processes of $J_L(t)$ are similar for the hybrid system and the normal system. These behaviors can be interpreted by combining the density of state (DOS) of the QD with the Andreev reflection possibility $T_A$. In fact, at $\delta \Gamma > 0$, the DOS of the QD in the hybrid system is similar to that of the normal system and $T_A \ll 1$, so that the two systems have the similar turn-on/off and relaxation characteristic. On the other hand, when $\delta \Gamma = 0$ or $\delta \Gamma < 0$, the resonant or the near resonant Andreev reflection occurs, Andreev bound states appears in the QD, and the DOS of the QD is very different from the normal system. This makes the relaxation processes very different for the N-QD-S and N-QD-N systems. (iii) Although $J_L(t)$ for $\delta \Gamma = +a$ and $\delta \Gamma = -a$ ($a$ is an arbitrary real number) experience different rising and relaxation processes, they have the same steady value at $t = \infty$. In fact, in the steady state case and at the small bias $V_L$ limit, the transmission possibility of the normal

![ FIG. 3: (Color on line) The currents $J_L(t)$ vs. the time $t$ for the N-QD-S system (a) and the N-QD-N system (b) with the different pulsed bias $V_L$. Main Figure is for the case of $V_L = 1$ and $V_L = 10$. The case of $V_L = 0.1$ is plotted in inset panel. The curves are labelled as: (1) $J_L^+(V_L = 0.1)$; (2) $J_L^-(V_L = 0.1)$; (3) $J_L^+(V_L = 1.0)$; (4) $J_L^-(V_L = 1.0)$; (5) $J_L^+(V_L = 10)$; (6) $J_L^-(V_L = 10)$. The other parameters are the same as Fig.1.](image)

![ FIG. 4: (Color on line) The current $J_L^+(V_L) = 0.1$ with the different asymmetric coupling strength $\delta \Gamma$. The panel (a) and (b) are for the N-QD-S system and the N-QD-N system, respectively. The other parameters are same to the Fig.1.](image)
N-QD-N device is:

\[ T(\omega) = \frac{\Gamma^2 - \delta\Gamma^2}{4\omega^2 + \Gamma^2}, \]

and the Andreev reflection possibility of the hybrid N-QD-S device is:\textsuperscript{22}

\[ T_A(\omega) = \frac{(\Gamma^2 - \delta\Gamma^2)^2}{4(\omega^2 + \Gamma\delta\Gamma)^2 + (\Gamma^2 - \delta\Gamma^2)^2}, \]

with the current expressions \( J_L = -2q \int \frac{d\omega}{2\pi} (f(\omega - V_L) - f(\omega))T(\omega) \) and \( J_R = -2q \int \frac{d\omega}{2\pi} (f(\omega + V_L) - f(\omega))T_A(\omega) \), respectively. Here \( T \) and \( T_A \) are the same for \( \pm \delta\Gamma \) when \( \omega = 0 \), consequently \( J_L(t = \infty) \) also are same for \( \pm \delta\Gamma \).

IV. CONCLUSIONS

In summary, we have studied the dynamic response of current to the external upwards or downwards pulsed bias for the hybrid N-QD-S system. In the small bias \( V_L \) limit, the turn-on/off time and the relaxation process for the upwards and the downwards pulse bias are symmetric. Comparing with the normal N-QD-N system, the Andreev reflection dominates the transport process. This makes the turn-on/off time much longer and new steady state current almost doubled. For the asymmetric barriers, the transport properties of the hybrid N-QD-S system are nearly same with the normal N-QD-N system when \( \Gamma_L \geq \Gamma_R \). On the other hand, while \( \Gamma_L < \Gamma_R \) the current humps in the relaxation process which reflects the properties of the superconductor. Beyond the linear bias regime, the rising process for upwards bias and the falling process for downwards bias become more and more asymmetric with the increasing bias \( V_L \). The turn-on time is faster than the turn-off time, and the current versus the time \( t \) oscillates with the frequency \( \hbar \Omega = V_L \).

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APPENDIX

\[ \Sigma_{L,\sigma}^>(t', t) = \sum_{k, L} t_{k, L}^r g_{k, \sigma, L}(t', t) t_{k, L} = -\frac{i}{2} i\Gamma_L \delta(t' - t) \]

\[ \Sigma_{L,\uparrow}^<(t', t) = \sum_{k, L} t_{k, L}^r g_{k, \uparrow, L}(t', t) t_{k, L} \]

\[ = i \int \frac{d\omega}{2\pi} f(\omega) \Gamma_L e^{-i\omega(t'-t)} f'_{t} dt_{1} W_L(t_1) \]

\[ \Sigma_{L,\downarrow}^<(t', t) = \sum_{k, L} t_{k, L}^r g_{k, \downarrow, L}(t', t) t_{k, L} \]

\[ = i \int \frac{d\omega}{2\pi} (1 - f(\omega)) \Gamma_L e^{i\omega(t'-t)} + i f'_{t} dt_{1} W_L(t_1) \]

\[ = i \int \frac{d\omega}{2\pi} f(\omega) \Gamma_L e^{-i\omega(t'-t)} + i f'_{t} dt_{1} W_L(t_1) \]

\[ \Sigma_{R}^>(\omega) = -\frac{i}{\nu} \frac{\Gamma_R}{\sqrt{\omega^2 - \Delta^2}} \begin{pmatrix} \omega & \Delta \\ \Delta & \omega \end{pmatrix} \]

\[ \Sigma_{R}^<(\omega) = \frac{\Gamma_R}{\nu} \frac{1}{\sqrt{\omega^2 - \Delta^2}} \begin{pmatrix} \omega & \Delta \\ \Delta & \omega \end{pmatrix} \]

where \( \Gamma_R = 2\pi |t_{k, R}|^2 \rho_R^N \), \( \Delta \) is the energy gap of the superconductor lead, and \( \nu = 1 \) for \( \omega > -\Delta \) and \( \nu = -1 \) otherwise.

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1 The turn on/off time describes how fast can a device turn on/off a current, which is also named rise/fall time in the Ref.(10). While the relaxation time was referred to how fast can the device go to a new steady state after a bias is abruptly switched on, it is also named saturation time in the Ref.(11).

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