Radiative Processes in Quark-Gluon Plasma

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Abstract

The spectrum of emitted gluons from the process \( gg \rightarrow ggg \) has been evaluated by relaxing some of the approximations used in earlier works. The formula obtained in the present work has been applied to several physical quantities. A general expression for the dead cone of gluons radiated by virtual partons has been derived. It is observed that the suppression caused by the high virtuality is overwhelmingly large as compared to that on account of conventional dead-cone of heavy quarks.

1 Introduction

Radiative processes like \( gg \rightarrow ggg \) are of particular importance in the study of quark gluon plasma (QGP) expected to be formed in heavy ion collisions (HIC) at ultra-relativistic energies. The process, \( g + g \rightarrow g + g + g \) has drawn particular attention in view of its importance for the chemical equilibration, energy loss of gluons in QGP, evaluation of transport coefficients of the gluonic plasma etc. There has been recent attempts [1, 2] to generalize the Gunion-Bertsch (GB) formula [3] for gluon radiation from light partons. Our aim is to find correction terms to GB formula relaxing earlier approximations and to see the effects of the correction terms on (i) equilibration rate and (ii) the energy loss of gluons in gluonic plasma. Also, it is expected that the radiated soft gluon spectrum emitted from heavy quarks will be suppressed compared to that emitted from light quarks [4]. A new suppression mechanism due to high virtuality of quarks has been proposed and it is shown that gluon emitted from virtual quarks, irrespective of their masses, are exposed to this suppression. The dead cone due to virtuality may play a crucial role in explaining the observed similar suppression patterns of light and heavy quarks jets in heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) [5].

2 Correction to GB Spectrum

Following [6], the matrix element for \( gg \rightarrow ggg \), \( |M_{gg \rightarrow ggg}|^2 \) after simplification can be written as [7]:

\[
|M_{gg \rightarrow ggg}|^2 = 12g^2|M_{gg \rightarrow gg}|^2 \frac{1}{k_2^2} \times [(1 + \frac{t}{2s} + \frac{5t^2}{2s^2} - \frac{t^3}{s^3})]
\]

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Figure 1: (Color online) Temperature variation of the ratio of the equili-

bration rate (inverse of the time scale) obtained in the present work (solid 

line), Ref. [1] (dashed line), and [2] (dot-dashed) normalized by the GB 

value for the process gg → ggg.

\[- \left( \frac{3}{2\sqrt{s}} + \frac{4t}{s\sqrt{s}} - \frac{3t^2}{2s^2\sqrt{s}} \right) k_\perp^{2} \]

\[+ \left( \frac{5}{2s} + \frac{t}{2s^2} + \frac{5t^2}{s^3} \right) k_\perp^{2}, \tag{1}\]

where \( |M_{gg\rightarrow gg}|^2 = (9/2)q^4s^2/t^2 \), \( s = (k_1 + k_2)^2 \), \( t = (k_1 - k_3)^2 \), \( u = (k_1 - 

k_4)^2 \), \( k_\perp \) is the transverse momentum of the radiated gluon. \( g = \sqrt{4\pi\alpha_s} \) 

is the color charge, and \( \alpha_s \) is the strong coupling. Effect of the modified 

gluon distribution on energy loss of gluons in gluonic plasma and on its 
equilibration rate has been discussed in later sections.

3 Dead-cone due to virtuality

For the demonstration of suppression of soft gluon radiation due to virtual-

ity, we take up the \( e^+e^- \rightarrow Q\bar{Q}g \) process, where \( Q \) is quark. The spectrum 
of the soft gluons emitted by the virtual quarks can be shown to be [8]:

\[F = \omega^2(2R_{43} - R_{44} - R_{33})\]

\[= 4\beta^2 \left( \frac{V^4}{\omega E^2} + \frac{4V^2}{\omega E} + 4\sin^2\theta \right. \]

\[\left. \left( \frac{V^4}{\omega E^2} + \frac{2V^2}{\omega E} + 4(1 - \beta^2\cos^2\theta) \right)^2 \right), \tag{2}\]

where we assume that external quarks are on the verge of being on-shell 
so that Diracs equation can be applied. \( V \) is the ‘virtuality parameter’ 
defined by the equation, \( V^2 = q^2 - m_Q^2 \) where \( q^2 \) is four-momentum square 
of external virtual particles, \( q^2 = m_Q^2 \) implies \( V = 0 \), i.e. the particle 
becomes on-shell. We can show that the spectrum is that of gluons emitted 
from on-shell quarks when \( V = 0 \). \( \omega \) is the energy of the soft gluon emitted.
at angle $\theta$ with the parent quark whose velocity is $\beta$ and energy is $E$. We replace the virtuality by $V = \sqrt{q^2 - m^2} = \sqrt{E^2 - p^2 - m^2}$, where $p = \beta E$ and define $F_{RH\theta} = F(E = 1.5 GeV, \theta)/F(E = 1.5 GeV, \theta = 0)$ for heavy quarks. A similar quantity $F_{RL\theta} = F(E = 3 GeV, \theta)/F(E = 3 GeV, \theta = 0)$ is defined for light quarks. Similarly $F_{RHE} = F(E, \theta = \pi/4)/F(E = 100 GeV, \theta = \pi/4)$ and $F_{RLE} = F(E, \theta = \pi/4)/F(E = 100 GeV, \theta = \pi/4)$ are two quantities just by choosing proper $F$(spectrum) for heavy or light quarks.

4 Results and Discussion

The relative energy loss ($\Delta E_R$), defined by energy loss of gluons ($\Delta E$) normalized by the corresponding value obtained from GB approximation and relative equilibration rate($\Gamma_R$) of fast gluons, defined in the same way, are calculated and compared with [1, 2]. We observe that with the correction terms the value of $\Delta E_R$ is enhanced by about 40% and 20% for $T = 300$ MeV and 400 MeV, respectively, compared to the $\Delta E_R$ obtained from the spectra of Refs. [1] and [2]. Such differences may have important consequences on the heavy-ion phenomenology at RHIC and LHC collision energies.

Soft gluon spectrum ($\omega = 30$ MeV) radiated by virtual light quarks, $F_{RL\theta}$ with low virtuality ($E = 3$ GeV) varies differently (see Fig. 3) with $\theta$ from the corresponding quantity, $F_{RH\theta}$ for virtual heavy quarks (see Fig. 1). This is obvious because for low virtuality the light partons are not subjected to any dead cone suppression at $\theta = 0$ and $\pi$ unlike heavy quarks. But the spectrum is similarly suppressed with energy for heavy or light quarks. Since energy is measure of virtuality in our formalism, we say that highly virtual quarks, heavy or light are always exposed to similar suppression.
In summary, we have calculated the correction to Gunion-Bertsch spectrum relaxing earlier approximations and calculated energy loss as well as equilibration rate of fast gluons passing through gluonic plasma. We propose a new radiative soft gluon suppression due to virtuality of parent quark. For high virtuality of quarks, gluons emitted from them are exposed to similar radiative suppression.

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Figure 4: (Colour online) The variation of (a) $F_{RH\theta}$ with $\theta$ and (b) $F_{RHE}$ with $E$ for virtual heavy quarks.

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