Broken discs: warp propagation in accretion discs

Christopher J. Nixon⋆ and Andrew R. King
Department of Physics & Astronomy, University of Leicester, Leicester LE1 7RH

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ABSTRACT
We simulate the viscous evolution of an accretion disc around a spinning black hole. In general, any such disc is misaligned, and warped by the Lense–Thirring effect. Unlike previous studies, we use effective viscosities constrained to be consistent with the internal fluid dynamics of the disc. We find that non-linear fluid effects, which reduce the effective viscosities in warped regions, can promote breaking of the disc into two distinct planes. This occurs when the Shakura & Sunyaev dimensionless viscosity parameter $\alpha$ is $\lesssim 0.3$ and the initial angle of misalignment between the disc and hole is $\gtrsim 45^\circ$. The break can be a long-lived feature, propagating outwards in the disc on the usual alignment time-scale, after which the disc is fully co-aligned or counter-aligned with the hole. Such a break in the disc may be significant in systems where we know the inclination of the outer accretion disc to the line of sight, such as some X-ray binaries: the inner disc, and so any jets, may be noticeably misaligned with respect to the orbital plane.

Key words: accretion, accretion discs – black hole physics – galaxies: active – galaxies: evolution – galaxies: jets.

1 INTRODUCTION
In realistic astrophysical situations, the angular momentum of a spinning black hole may be significantly misaligned with respect to a surrounding accretion disc. This is expected in black hole X-ray binaries where the hole may have received a significant kick during formation (Shklovskii 1970; Sutantyo 1978; Arzoumanian, Chernoff & Cordes 2002; Hobbs et al. 2005). It is also likely for accretion on to a supermassive black hole (SMBH) in the centre of a galaxy. The scale of the SMBH is so small compared with the galaxy that any process which drives gas down to the galactic centre is unlikely to have any special orientation with respect to the SMBH spin. The spin direction is set by the SMBH’s accretion history and not by whatever causes the next accretion event. Processes such as star formation, merging of satellite galaxies, gas cloud collisions and other galactic phenomena likely to drive gas inwards are chaotic, so it is reasonable to expect a random spread of angles between the angular momentum of infalling gas and that of the hole (cf. King & Pringle 2006, 2007; King, Pringle & Hofmann 2008). This gas is likely to form a ring around the SMBH which then viscously spreads into a misaligned accretion disc.

The Lense–Thirring (hereafter LT) effect of a spinning black hole causes non-equatorial orbits to precess at rates dependent on the distance from the hole (Lense & Thirring 1918). In an accretion disc, this differential precession causes a warp which propagates through the disc. This is the Bardeen–Pettersson effect (Bardeen & Petterson 1975). In this paper, we restrict our attention to viscous Keplerian accretion discs with negligible self-gravity. Papaloizou & Pringle (1983) linearized the equations of fluid dynamics to derive the first consistent equation governing the evolution of a warped disc of this type, assuming a tilt angle smaller than the disc angular semi-thickness. They introduced two effective kinematic viscosities controlling the angular momentum transport in the disc. In principle, these two quantities depend on the nature of the warp: $\nu_1$ governs the usual radial communication of the component of angular momentum perpendicular to the plane of the disc due to differential rotation (for a flat disc, this is the Shakura & Sunyaev $\alpha$ viscosity), while $\nu_2$ governs the radial communication of the component of angular momentum parallel to the local orbital plane; this acts to flatten tilted rings of gas. Papaloizou & Pringle (1983) also showed that conservation laws require a relation between $\nu_1$ and $\nu_2$. For small warps, this is

$$\nu_2 = \frac{\nu_1}{2\alpha^2}, \quad (1)$$

where $\alpha$ is the usual dimensionless viscosity parameter for discs (Shakura & Sunyaev 1973) assumed $\ll 1$ by Papaloizou & Pringle (1983). This gives

$$\alpha_2 = \frac{1}{2\alpha}, \quad (2)$$

(again for small $\alpha$), where $\alpha_2$ is an equivalent dimensionless viscosity parameter for $\nu_2$.

Pringle (1992) derived an evolution equation valid for larger warps expressing global angular momentum conservation for a disc.
composed of rings of gas with no internal degrees of freedom. Pringle (1992) went on to develop a numerical technique for integrating this equation over rings of gas to study the viscous evolution of a time-dependent warped disc. Lodato & Pringle (2006) used this to study the Bardeen–Petterson effect. They assumed constant effective viscosities, related as expected for small-amplitude warps (equation 1). This treatment suggests that a disc warp must always straighten itself out on a time-scale shorter than the accretion time-scale (assumed only the LT torque and the internal disc torques, that is, no other external torques), because equation (1) implies \( \nu_2 \gg \nu_1 \) for typical values of \( \alpha \). The realignment process enhances the accretion rate by inducing extra dissipation (Lodato & Pringle 2006).

The analysis of Pringle (1992) did not allow for any internal degrees of freedom within the gas rings, and was described as a ‘naive approach’ in Papaloizou & Pringle (1983). However, Ogilvie (1999) derived equations of motion directly from the full 3D fluid dynamical equations with an assumed isotropic viscosity. This confirmed that the equations of Pringle (1992) are formally valid only when two differences are taken into account (or can be safely neglected). The first is that the Pringle (1992) equations omit a torque causing rings to precess if they are tilted with respect to their neighbours. In a viscous Keplerian disc where \( \alpha \ll 1 \) but not close to zero, this is suggestive, since the evolution of a misaligned circumbinary disc is qualitatively similar to that of a misaligned disc around a spin-misaligned black hole (Nixon, King & Pringle 2011b). SPH simulations by Lodato & Price (2010) also found disc breaking. They assumed the presence of a large-amplitude warp and followed its viscous development. Their results agreed with those expected for the constrained viscosities derived in Ogilvie (1999), and showed that the disc could break if the warp amplitude was large enough (note that this paper did not attempt to show that the LT precession could produce such a warp).

In this paper, we explore disc evolution under the LT effect with general warp amplitudes, and the effective viscosities constrained as in Ogilvie (1999). We explore the evolution for a range of \( \alpha \) values and different degrees of misalignment between the disc and hole. We aim to discover if the internal torques in an accretion disc can sustain a smooth warped configuration under the LT torque, or whether instead the disc breaks into distinct planes.

### 2 NUMERICAL METHOD

#### 2.1 Evolution equation

The problem of disc breaking is complex, and to make progress we adopt the simplest approach. We use a 1D Eulerian ring code of the type described in Pringle (1992) and used in Lodato & Pringle (2006). Our code has been adapted to include the third effective viscosity and the non-linear fluid dynamical constraints found by Ogilvie (1999). Here ‘1D’ means that the properties assigned to each ring depend only on its radius. Section 2.1 of Lodato & Pringle (2006) gives a detailed discussion of the validity of this approach. The main restriction is that the evolution of a disc warp must be diffusive rather than wave like. Papaloizou & Pringle (1983) show that this holds if \( \alpha > H/R \), which is usually satisfied for accretion discs around compact objects.

Our 1D ring code describes the evolution of warped accretion discs by evolving the angular momentum density vector of each ring \( \mathbf{L}(R, t) \) through the equation

\[
\frac{d\mathbf{L}}{dt} = \frac{1}{R} \frac{\partial}{\partial R} \left\{ \left( \frac{\partial}{\partial R} [v_1 \Sigma (\Omega/R^3)] \right) \mathbf{L} \right\} + \frac{1}{R} \frac{\partial}{\partial R} \left( \frac{1}{2} v_2 R |\mathbf{L}| \frac{\partial \Omega}{\partial R} \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( \frac{1}{2} v_2 R^3 \Omega \frac{\partial |\mathbf{L}|}{\partial R} (\Omega/R^3) + v_1 \left( \frac{R \Omega}{\Omega - \Omega_x} \right) \right) \mathbf{L} \right\} + \frac{1}{R} \frac{\partial}{\partial R} \left( v_3 R |\mathbf{L}| \ell \times \frac{\partial \Omega}{\partial R} \right) + \Omega_x \times \mathbf{L},
\]

where \( v_1, v_2 \) and \( v_3 \) are the effective viscosities, \( \Omega_x(R) \) is the local azimuthal angular velocity, \( \Sigma(R) \) is the disc surface density, \( \Omega(R) \) is the unit angular momentum vector and \( R \) is the spherical radius coordinate. Ogilvie (1999) used the equations of fluid dynamics to determine the relation between the effective viscosities for a general warp amplitude. In contrast to an assumption of constant effective viscosities, this analysis suggests that these quantities drop in a warped region, making it much harder for the disc to straighten any twisted rings of gas. Indeed, if the warp becomes large enough the disc may break into distinct planes with only a tenuous connection. We expect this to occur in large warps, as non-linear corrections to the fluid flow are significant for warp amplitudes \( |\psi| \gtrsim \alpha \) (Ogilvie 1999).

There have been several theoretical investigations which show evidence of breaking discs. Larwood & Papaloizou (1997) report an example in smoothed particle hydrodynamics (SPH) simulations of a circumbinary disc with a strong radial density decrease. This is suggestive, since the evolution of a misaligned circumbinary disc is qualitatively similar to that of a misaligned disc around a spinning black hole (Nixon, King & Pringle 2011b). SPH simulations by Lodato & Price (2010) also found disc breaking. They assumed the presence of a large-amplitude warp and followed its viscous development. Their results agreed with those expected for the constrained viscosities derived in Ogilvie (1999), and showed that the disc could break if the warp amplitude was large enough (note that this paper did not attempt to show that the LT precession could produce such a warp).

In this paper, we explore disc evolution under the LT effect with general warp amplitudes, and the effective viscosities constrained as in Ogilvie (1999). We explore the evolution for a range of \( \alpha \) values and different degrees of misalignment between the disc and hole. We aim to discover if the internal torques in an accretion disc can sustain a smooth warped configuration under the LT torque, or whether instead the disc breaks into distinct planes.
The fifth term is the LT torque arising from gravitomagnetic interaction of the disc with the spinning hole. This induces precessions in the disc orbits, and hence causes the angular momentum vector for each ring to precess around the spin vector for the black hole with a frequency

$$\Omega_p = \frac{2GJ_0}{c^2R^3},$$

(4)

with $c$ the speed of light and $G$ the gravitational constant. The magnitude of $J_0$ is given by $J_0 = a\mathcal{M}(GM/c^2)$, where $a$ is the dimensionless spin parameter and $M$ is the mass of the hole (Kumar & Pringle 1985). We include the back reaction on the hole angular momentum vector $J_h$ by summing the effect of the precessions over all radii, giving the torque on the hole as

$$\frac{dJ_h}{dt} = -2\pi \int \Omega_p \times LR \, dR.$$ 

(5)

Since $\Omega_p \propto J_h$, this equation makes it clear that the magnitude of $J_h$ is conserved and hence $J_h$ can only precess on a sphere, simply because the LT torque has no component in the direction of $J_h$ for any disc structure. Equivalently, the LT torque induces only precessions in the disc, so that the reaction back on the hole is simply a sum of precessions, which is itself a precession.

We integrate both equations (3) and (5) using a simple forward Euler method (Pringle 1992) on a logarithmically spaced grid. However, we adopt a different implementation of the boundary conditions. Rather than implementing a sink over a few grid cells, we simply remove all of the angular momentum that reaches the innermost and outermost cells at the end of each integration step. This allows angular momentum (and hence mass) to flow freely across the boundary, giving the usual $\Sigma = 0$, torque-free, accreting boundary condition. This is the form adopted in Lodato & Price (2010). We note that this boundary condition does not conserve angular momentum exactly. However, this small effect does not affect the inclination of any of the rings and hence does not alter our results while the warp is propagating through the computational domain. Once the warp reaches the outer boundary, it flows freely off the computational domain and the disc is flattened (i.e. we do not impose a boundary condition at the outer edge of the disc to maintain the disc warp).

### 2.2 Generating the constrained effective viscosities

We use the effective viscosities derived by Ogilvie (1999) for a locally isothermal disc. These are constrained to be consistent with the internal fluid dynamics of the disc, assuming an isotropic physical viscosity. The physics of these constraints is simply local conservation of mass and angular momentum in the disc.

The effective viscosities which describe the transport of angular momentum in a warped disc take the usual form of a Shakura & Sunyaev disc viscosity. The constraints determine the non-linear coefficients ($Q_1$, $Q_2$, $Q_3$) of the effective viscosities (cf. equations 10, 11 and 12, respectively). These coefficients all decrease in magnitude in the presence of a strong warp, where the disc rings are highly inclined to each other. Communication of angular momentum is reduced because the rings of gas are no longer in perfect contact. We expect angular momentum to be communicated through perturbations of the particles from circular orbits. The rate of communication, and hence the magnitude of the viscosities, therefore depends on how easily these particles can interact. This is harder for particles on inclined orbits, and once these reach a critical inclination this can only happen where the rings cross. We note that for inviscid discs, which we do not consider here, this interpretation is no longer valid (see Ogilvie 1999 for further details).

We now detail the calculation of the effective viscosities from the disc properties. The notation in Ogilvie (1999) differs from that in Pringle (1992). Our equation (3) and equation (122) from Ogilvie (1999) relate the effective viscosities ($v_1$, $v_2$, and $v_3$) to the non-linear viscosity coefficients ($Q_1$, $Q_2$, and $Q_3$) by

$$v_1 = \frac{Q_1\mathcal{J}\Omega^2}{\Sigma R^2 \pi},$$

(6)

$$v_2 = \frac{2Q_2\mathcal{J}\Omega}{\Sigma},$$

(7)

and

$$v_3 = \frac{Q_3\mathcal{J}\Omega}{\Sigma},$$

(8)

respectively, where $\mathcal{J}$ is given by

$$\mathcal{J} = \frac{1}{2\pi} \int_0^{2\pi} J \, d\phi = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \rho^2 \, d\rho \, dz \, d\phi.$$ 

(9)

Assuming $\Sigma$ constant in azimuth, and a locally isothermal disc with sound speed $c_s$, we can write this as $\mathcal{J} \propto \Sigma c_s^2 / \Omega^2$. For a point-mass potential with $\Omega = (GM/R^3)^{1/2}$, we get after some algebra

$$v_1 = \frac{2}{3}Q_1 \left[ (H/R)^2 R^2 \Omega \right],$$

(10)

$$v_2 = 2Q_2 \left[ (H/R)^2 R^2 \Omega \right],$$

(11)

and

$$v_3 = Q_3 \left[ (H/R)^2 R^2 \Omega \right],$$

(12)

where $H = c_s / \Omega$ is the disc thickness.

For simplicity, we assume that $(H/R)^2 R^2 \Omega = \text{constant}.$

(13)

This removes the radial dependence of the viscosities, making them functions only of $|\psi|$ and $\alpha$ and so allowing a direct comparison with the usual steady-state disc, where $c_s \propto R^{-3/4}.$ This compares with the usual steady-state disc, where $c_s \propto R^{-3/8}$ (e.g. Frank, King & Raine 2002).

We note that for a disc without a warp, $v_1$ reduces to exactly the usual $\alpha$ disc viscosity (Shakura & Sunyaev 1973), since

$$v_1 = \alpha c_s H = \alpha (H/R)^2 R^2 \Omega$$

(14)

and

$$Q_1(|\psi| = 0) = -3\alpha/2.$$ 

(15)

The effective viscosities are now fixed by the non-linear coefficients $Q_1(\alpha, |\psi|), Q_2(\alpha, |\psi|)$ and $Q_3(\alpha, |\psi|).$ We calculate these coefficients in the same way as described in Ogilvie (1999) with $\Gamma = 1$ and $\alpha_0 = 0$ using the code kindly provided by Gordon Ogilvie. Fig. 1 shows the coefficients plotted against the warp amplitude for $\alpha = 0.2$.

### 3 Simulations

We use the method described above to simulate the warping of a misaligned accretion disc under the LT effect. We set up our initially planar but misaligned disc on a logarithmically spaced grid from $R_{\text{in}} = 10R_{\text{isco}}$ to $R_{\text{out}} = 1000R_{\text{isco}}$ using 100 grid cells (where $R_{\text{isco}} = 6GM/c^2$ is the radius of the innermost stable circular orbit around
3.1 Does the disc break?

We perform several simulations for $0.2 \leq \alpha \leq 0.5$ to determine the various non-linear responses of the disc to warping. We do not simulate $\alpha < 0.2$ as this is where the viscosity $\nu_1$ is predicted to become negative in strong warps. We simulate a range of $\theta_0$ from $10^\circ$ to $60^\circ$. This allows us to explore the two regimes: $|\psi| \leq \alpha$, where non-linear effects should be negligible, and $|\psi| \geq \alpha$, where non-linear effects should be important. Our setup corresponds to $J_\psi/J_b \approx 0.02$, with $R_s/R_0$ varying between $-0.95$ and $-1.20$ as $\alpha$ changes. Note that we evaluate equation (13) at $R_0$ with $H/R = 10^{-3}$.

We perform these simulations for two cases. In the first, we use constant effective viscosities. This acts as a control so we can isolate the effect of introducing non-linear dynamics in the second set of simulations, where we use the full effective viscosities. The constant effective viscosities are simply calculated using the method described in Section 2.2 with $|\psi| = 0$.

For the simulations involving constant effective viscosities, we expect to recover the Bardeen–Petterson effect as seen by several other authors (e.g. Bardeen & Petterson 1975; Pringle 1992; Scheuer & Feiler 1996; Lodato & Pringle 2006, etc.). However, the simulations involving the full non-linear effective viscosities should modify this somewhat.

3.1.1 $\theta = 10^\circ$

In Fig. 2, we show the disc structures for simulations with $\theta_0 = 10^\circ$ and the constant effective viscosities at 0.01, 0.1 and $1\tau_{\text{isc}}$. Here we define the viscous time-scale as $t_{\text{isc}} = R_0^2/\nu_1$ with $\nu_1$ evaluated for $|\psi| = 0$ and $\alpha$ as defined in the particular case. Fig. 2 shows the behaviour for different values of the disc viscosity parameter $\alpha$. In this case, the behaviour is similar for all simulations, with lower $\alpha$ aligning in a shorter fraction of the viscous time (although in reality more slowly because the viscous time-scale is longer).

In Fig. 3, we show the same simulations as in Fig. 2. However, this time we use the full effective viscosities. We expect the evolution here to be similar, as the warp is never able to achieve a large amplitude, making the viscosities similar in both cases. This is indeed the case, as there is no visible difference between Figs 2 and 3.

3.1.2 $\theta = 30^\circ$

In Figs 4 and 5, we show the disc structures for simulations with $\theta_0 = 30^\circ$. Again there is very little difference between the disc structures obtained using the constant effective viscosities and those with the full effective viscosities. However, the disc profile steepens in the early stages of the $\alpha = 0.2$ simulation. This suggests that for $\theta = 30^\circ$ we would have to go to $\alpha \ll 0.2$ to get disc breaking.

3.1.3 $\theta = 45^\circ$

In Figs 6 and 7, we show the disc structures for simulations with $\theta_0 = 45^\circ$. There is now a noticeable difference between the constant effective viscosity and full effective viscosity simulations. Even for large $\alpha$ the disc profile is steepened and for $\alpha \sim 0.2$ the disc breaks into two distinct planes. This break persists for more than a tenth of the viscous time-scale for the disc. The disc aligns when the break propagates out to the edge of our grid which occurs at $t = 0.13\tau_{\text{isc}}$.

3.1.4 $\theta = 60^\circ$

In Figs 8 and 9, we show the disc structures for simulations with $\theta_0 = 60^\circ$. There is again a noticeable difference between the constant effective viscosity and full effective viscosity simulations. At $t = 0.01\tau_{\text{isc}}$, the disc profile is steepened noticeably more than in the $\theta = 45^\circ$ case and by $t = 0.1\tau_{\text{isc}}$ all of the simulations have maintained a break. We can also see that at $t = 0.01\tau_{\text{isc}}$ in the constant effective viscosity simulation with $\alpha = 0.2$ it is possible for the disc to break without the non-linear effects.

These simulations suggest two main results of using the full effective viscosities. First, discs can break for higher values of $\alpha$, and secondly, discs can break for lower inclination angles.
Figure 2. Disc structures for the constant effective viscosity simulations, given by plotting the tilt angle $\theta$ between the disc and the hole against the log of the radius. The initial misalignment of the disc is $\theta_0 = 10^\circ$. In each plot, the legend gives the value of $\alpha$ used in the simulation. From the left-hand to right-hand side, the panels correspond to $t = 0.01, 0.1$ and $1$ viscous times after the start of the calculation. Note that in the latter case the warp has moved to the edge of our grid and the disc aligns.

Figure 3. Disc structures for the full effective viscosity simulations, given by plotting the tilt angle between the disc and the hole against the log of the radius. The initial misalignment of the disc is $\theta_0 = 10^\circ$. In each plot, the legend gives the value of $\alpha$ used in the simulation. From the left-hand to right-hand side, the panels correspond to $t = 0.01, 0.1$ and $1$ viscous times after the start of the calculation. When compared to Fig. 2 this figure shows the negligible effect of the full effective viscosities when the misalignment between the disc and hole is small.

Figure 4. Disc structures for the constant effective viscosity simulations as in Fig. 2, but this time for initial misalignment $\theta = 30^\circ$. From the left-hand to right-hand side, the panels correspond to $t = 0.01, 0.1$ and $1$ viscous times after the start of the calculation.

Figure 5. Disc structures for the full effective viscosity simulations as in Fig. 3, but this time for initial misalignment $\theta = 30^\circ$. From the left-hand to right-hand side, the panels correspond to $t = 0.01, 0.1$ and $1$ viscous times after the start of the calculation. When compared to Fig. 4 this figure shows that even for a $30^\circ$ misalignment angle the effect of the full effective viscosities can be small; however, there is a steepening of the disc profile at early times for the $\alpha = 0.2$ simulations. This suggests that for the non-linear effects to be important at $\theta_0 = 30^\circ$ we need $\alpha \ll 0.2$. 

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Disc structures for the constant effective viscosity simulations as in Fig. 2, but this time for initial misalignment \( \theta = 45^\circ \). From the left-hand to right-hand side, the panels correspond to \( t = 0.01, 0.1 \) and 1 viscous times after the start of the calculation.

Disc structures for the full effective viscosity simulations as in Fig. 3, but this time for initial misalignment \( \theta = 45^\circ \). From the left-hand to right-hand side, the panels correspond to \( t = 0.01, 0.1 \) and 1 viscous times after the start of the calculation. When compared to Fig. 6 this figure shows that the non-linear effects can significantly influence the evolution of the disc. Here the profile is noticeably steepened at \( t = 0.01 t_{\text{visc}} \) for all \( \alpha \). By \( t = 0.1 t_{\text{visc}} \), the disc break is only maintained for small \( \alpha \). For \( \alpha = 0.2 \), the break propagates outwards in the disc until reaching the outer edge of the grid (at \( t = 0.13 t_{\text{visc}} \)) at which point the disc aligns. For larger \( \alpha \), the disc is smoothed before it aligns.

Disc structures for the constant effective viscosity simulations as in Fig. 2, but this time for initial misalignment \( \theta = 60^\circ \). From the left-hand to right-hand side, the panels correspond to \( t = 0.01, 0.1 \) and 1 viscous times after the start of the calculation.

Disc structures for the full effective viscosity simulations as in Fig. 3, but this time for initial misalignment \( \theta = 60^\circ \). From the left-hand to right-hand side, the panels correspond to \( t = 0.01, 0.1 \) and 1 viscous times after the start of the calculation. When compared to Fig. 8 this figure shows that the non-linear effects can significantly influence the evolution of the disc. Initially the disc profile is significantly steepened for all \( \alpha \) and the disc breaks for all \( \alpha \). This break propagates outwards in the disc until reaching the outer edge of the grid at which point the disc aligns.
There are two features of the simulations which we have not discussed so far. The first is the wiggles in the disc at the top of the warp in Figs 7 and 9. In these simulations, the disc appears to try to break into more than just two distinct planes. It may be that for smaller $\alpha$ or larger $|\psi|$ the disc can break into more than two planes. However, we deem this beyond the scope of the present investigation.

Secondly, the torques on the disc in the strong warp regions generate rapid mass flow rates into the ring inside the warp. This results in prominent spikes in $\Sigma(R)$ inside regions with large $|\psi|$. Increasing resolution only resolves these spikes better. We therefore leave the investigation of this effect to future work, which will require a different numerical method.

4 DISCUSSION

We have simulated the evolution of a misaligned accretion event on to a spinning black hole. Many previous simulations of this type assumed that the effective viscosities governing the angular momentum transport were constant. Here we have used effective viscosities constrained to be consistent with the internal fluid dynamics of the disc. We compare these results with those of equivalent simulations which assume constant effective viscosities. In the latter case, we find that for small enough $\alpha$ the precession induced by the LT effect can cause the disc to break into two distinct planes if the misalignment is large enough. We also find that if we allow non-linear effects (Ogilvie 1999) the disc can break for higher values of $\alpha$ and smaller inclination angles. This suggests that non-linear effects significantly change the evolution of strongly warped discs. The main effect is in reducing the magnitude of the effective viscosities in the warped region, noticeably reducing $v_{\eta_1}$ more than $v_2$. This reduces the disc’s ability to smooth out the warp induced by the LT torque and so prevent a break. This is essentially a modified form of the Bardeen–Petterson effect, where the warp is steepened by the weakened disc response.

For small values of $\alpha$, $v_2 \gg v_1$. Then the effective viscosity that tries to flatten tilted rings of gas is much stronger than the effective viscosity responsible for transporting angular momentum radially. However, the $v_2$ and $v_2$ torques responsible for communicating angular momentum are proportional to $\ell$ and $\partial \ell/\partial R$, respectively. Thus, to break the disc we also need a large warp. As $|\psi|$ increases, the $v_2$ torque becomes comparatively stronger that the $v_1$ torque. When the discrepancy between the magnitudes of the two torques is large enough, the disc is effectively unable to communicate radially. As the differential precession induced by the LT effect acts faster at smaller radii, this allows the vertical effective viscosity ($v_2$) to flatten each ring in turn into the plane of the spinning black hole. To maintain a smooth (small-amplitude) warp, the disc must be able to communicate radially on a time-scale comparable to the flattening of rings into the plane of the black hole. When this does not hold, the disc breaks.

For $\alpha \sim 1$, the disc can efficiently maintain a smooth warp. However, for $\alpha \sim 0.2$–0.4 and a large misalignment, we show there is a strong tendency for the disc to break under an external torque such as the LT torque. We see no reason for this behaviour to change when $\alpha \leq 0.1$. However, there may be other non-linear effects which become important here. The next step is a full 3D hydrodynamical numerical investigation to determine the disc response in this regime (e.g. Nelson & Papaloizou 2000; Lodato & Price 2010).

When the disc breaks, it splits into a co-aligned or counter-aligned inner disc and a misaligned outer disc. This break can propagate outwards and (dependent on $\alpha$ and $\theta_0$) may persist until it reaches the outer regions of the disc. At this point, the whole disc is co-aligned or counter-aligned with the hole spin. This suggests that a break in the disc can be a long-lived feature, lasting for roughly the alignment time-scale. We note that although we have not included any simulations of counter-aligning discs, it is already known that for $J_{\ell}/J_{\alpha} \ll 1$ there is a symmetry in the behaviour of discs of this type around $\theta = 90^\circ$ (King et al. 2005; Lodato & Pringle 2006). Therefore, the same behaviour would be seen in discs that are counter-aligning with the black hole (i.e. a misalignment of $120^\circ$ would produce the same dynamics with a counter-aligning disc as the $60^\circ$ case produces here).

For observational purposes, it is important to compare the alignment time-scale $t_{\alpha}$ with the lifetime $t_{life}$ of the system. If $t_{\alpha} > t_{life}$, the centre of the disc is never aligned with its warped outer plane. In an accreting stellar mass black hole binary, this would mean that jets (which propagate along the axis of the central parts of the disc, and so the spin of the black hole) are in general misaligned with respect to the orbital plane. This is seen in the black hole binary GRO J1655–40 (Martin, Tout & Pringle 2008, and references therein). Similarly, the continuum-fitting method for estimating black hole spins (Kulkarni et al. 2011, and references therein) uses the assumption that the black hole spin is aligned with the orbital axis in estimating the radiating disc area and hence the innermost stable circular orbit radius. It therefore implicitly requires $t_{\alpha} < t_{life}$. In our simulations, it is clear that discs are able to remain warped or broken for significant fractions of their viscous time-scale. However, we note that for strong warps there may be other non-linear effects not included in the analysis of Ogilvie (1999) or additional dissipation caused by fluid instabilities (cf. Gammie, Goodman & Ogilvie 2000). These may impact upon the alignment time-scale.

It is also possible that the disc is never able to align fully; for example, an accretion disc in a misaligned black hole binary system may be twisted one way by the LT effect and a different way by the binary torque (e.g. Martin, Pringle & Tout 2009). We therefore suggest careful consideration before assuming that black hole accretion discs are aligned – particularly as Lodato & Pringle (2006) suggest that the accretion rate through warped discs is significantly enhanced, which suggests the possibility that strongly accreting discs are also strongly warped.

Disc breaking can have some important observational consequences. For example, accretion discs in binary systems often have an outer disc aligned to the binary plane. However, the inner disc (and so the direction of any jets) must be aligned with the black hole spin, which may be significantly misaligned with respect to the binary axis. Similarly, breaks in protoplanetary discs may occur through the internal disc response to warping during formation of the disc, rather than being driven by the presence of a planet.

In this paper, we have used a 1D Eulerian ring code to calculate the evolution of misaligned accretion events on to a spinning black hole. We caution that there is still much work to do before we can fully understand the evolution of such a disc. For example, our treatment requires gas to orbit on circular rings, inherently excluding any effects which might break the cylindrical symmetry in the local disc plane. We also assume that the disc responds viscously, with no wave-like behaviour. The method also prevents us from examining exactly what happens in a disc break: by its nature, it occurs over only over a small number of rings and is therefore poorly resolved – however, this does not affect our conclusion that the disc can break. The most frustrating restriction is the difficulty in simulating small enough $\alpha$: the simulations we present here suggest that for small $\alpha$ non-linear fluid effects can be
particularly important. We will address these restrictions in future investigations.

We remark finally that although we have considered alignment effects induced by the LT effect on an accretion disc around a spinning black hole, the torque between a misaligned binary and an external accretion disc has a very similar form (Nixon, King & Pringle 2011b). Thus, we expect similar phenomena to appear in that case. The enhanced accretion rates through the disc due to the dissipation in the warp (Lodato & Pringle 2006) would increase the mass flow rate on to the binary. Therefore, the coalescence timescale for the binary would be reduced further (Nixon et al. 2011a).

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