Development of a Simpler but Accurate Free Gas Deviation Factor for Fractured (Dual-Porosity) Volumetric Gas Reservoir Fluid

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Authors’ contributions

This work was carried out in collaboration between both authors. Authors AJA and OAF designed the study. Author AJA performed the statistical analysis and wrote the protocol and the first draft of the manuscript under the supervision of author OAF. Both authors managed the literature searches, the analyses of the study, read and approved the final manuscript.

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ABSTRACT

Gas compressibility factor, also known as gas deviation factor or Z-factor, is a thermodynamic correction factor which describes the deviation of a real gas from ideal gas behaviour. The free gas Z-factor in the Material Balance Equation (MBE) of single-porosity gas reservoirs with insignificant rock (matrix) compaction (after pressure depletion) does not reflect cases in low-permeability gas reservoirs having remarkable rock compaction. Through gas MBE modifications, previous researchers developed Z-factors for dual-porosity (fractured) low permeability gas reservoirs by incorporating gas desorption; however, their approaches create complexity for routine calculations. Therefore this study was designed with the purpose of deriving a free gas Z-factor for single-porosity low-permeability gas reservoirs and further modifying it for more simplicity and accuracy in a dual-porosity scenario. The free gas Z-factor derived for single-porosity low-permeability gas reservoirs is expressed as: 

\[ Z^* = Z \cdot \left(1 - \left(\frac{C_{W_1}S_{W_1} + C_{matrix}}{S_{g_1}}\right) \Delta P \right)^{-1} \]

where \( Z \), \( C_{W_1} \), \( S_{W_1} \), \( C_{matrix} \), \( S_{g_1} \) and \( \Delta P \) are single-porosity Z-factor without rock compaction at pressure \( P \), water

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compressibility, initial water saturation, matrix compressibility, initial gas saturation and pressure depletion, respectively. However, the developed dual porosity free gas Z-factor model incorporates ratio of dual porosity to initial matrix porosity, and it is expressed as: 

\[ Z'' = Z \cdot \left(1 - \frac{\phi_{\text{frac}}}{\phi_{\text{mat}}} \right) \left(\frac{c_{\text{water}} + c_{\text{matrix}}}{S_g}\right) - \Delta P \]

where \( \phi_{\text{mat}} \) and \( \phi_{\text{frac}} \) are initial matrix porosity and fracture porosity, respectively. The Z-factor model was graphically and statistically correlated with an existing free gas Z-factor model for dual porosity reservoirs. For all the hydraulically fractured shale gas formations considered, the correlations yield R² values of 1.000.

**Keywords:** Single-porosity; dual-porosity; free gas; Z-factor; hydraulically fractured gas reservoirs.

### 1. INTRODUCTION

The concept of an ideal gas is based on the assumptions that the molecule’s kinetic energy is more remarkable than the potential energy due to intermolecular attractive and repulsive forces (because the collision between the particles are assumed to be elastic), and that the size of the molecules is insignificant compared to the empty space between them. This is usually attained at higher temperature and lower pressure. However, a gas behaves more like a real gas at lower temperature and higher pressure where the gas is getting close to a phase change, and the kinetic theory and the relative size assumptions do not hold.

Gas compressibility factor, also known as gas deviation factor or Z-factor, is a thermodynamic correction factor which describes the deviation of a real gas from ideal gas behaviour. Z-factor values are usually obtained through experiments or by calculation from equation of state (EoS) featuring pressure \( P \), volume \( V \), temperature \( T \) and number of moles \( n \) [1-3]. Gas compressibility \( c \) should not be confused with the compressibility factor (Z-factor). The compressibility of a substance is defined as the change in volume per unit volume per unit change in pressure.

Z-factor determination through laboratory PVT analyses are sometimes expensive and time consuming. Hence, charts, correlations and other mathematical methods are often used to predict Z-factor because they are much easier and faster than equations of state thus save cost. Standing and Katz [4] presented a generalized natural gas Z-factor chart as a function of the pseudo-reduced pressure and pseudo-reduced temperature. The ranges considered are 1.05 ≤ \( T_{pr} \) ≤ 3.0 and 0 ≤ \( P_{pr} \) ≤ 15.

For the purpose of improving accuracy, many empirical equations have been fit to the original Standing and Katz chart. These correlations are: Wichert and Aziz correlation [5] (that considers effect of \( \text{H}_2\text{S} \) and \( \text{CO}_2 \) contaminants), Hall and Yarborough correlation [6] (that offers an accurate representation of the Standing and Katz chart), Beggs and Brils correlation [7], Dranchuk-Purvis-Robinson correlation [8], Dranchuk and Abou-Kassem [9] (that fits an eleven-constant EoS to the Standing and Katz data, and extrapolated this correlation to higher reduced pressures of the range \( 0 \leq P_{pr} \leq 20 \)) and Takacs correlation [10].

Based on Takacs [10] comparison of eight correlations representing the Standing and Katz chart [5], the Hall and Yarborough [6], and the Dranchuk and Abou-Kassem [9] equations give the most accurate representation for a wide range of temperatures and pressures. Both equations are valid for \( 1 \leq T_{pr} \leq 3 \) and \( 0.2 \leq P_{pr} \leq 25 \text{ to } 30 \). The Hall and Yarborough or Dranchuk and Abou-Kassem equation is recommended for the evaluation of the Z-factor of most natural gases. For sour gas, gas deviation factor can be calculated using Piper et al. correlation [11], and Elsharkawy and Elkamel correlation [12].

The correlations developed for calculating pseudo-critical properties (pseudo-critical pressure and pseudo-critical temperature) are Standing correlation [13] developed only for low molecular weight natural gases with minor amount of non-hydrocarbon gases; Sutton correlation [14] for a wide range of natural gas (it considers high molecular weight natural gases which are rich in heptane plus with minor concentration of carbon dioxide and nitrogen, and no hydrogen sulphide); Wichert-Aziz and Carr-Kobayashi-Burrows correction methods [15] applicable for mixture with impurities such as \( \text{N}_2 \), \( \text{CO}_2 \), \( \text{H}_2\text{S} \); Elsharkawy et al. correlation [16] suitable for retrograde gases (gas condensates); and Guo and Ghahambor [17] correlation valid for
H$_2$S < 3%, N$_2$ < 5%, and total content of inorganic compounds less than 7%.

Standing [13] correlation gives consideration to both dry and wet hydrocarbon gases. For dry hydrocarbon gases ($y_g < 0.75$) the correlation is stated as:

$$P_{pc} = 667 + 15.0y_g - 37.5y_g^2$$  \hspace{1cm} (1)

$$T_{pc} = 168 + 325y_g - 12.5y_g^2$$  \hspace{1cm} (2)

For wet hydrocarbon gases ($y_g \geq 0.75$) the correlation is stated as:

$$P_{pc} = 706 + 51.7y_g - 11.1y_g^2$$  \hspace{1cm} (3)

$$T_{pc} = 187 + 330y_g - 71.5y_g^2$$  \hspace{1cm} (4)

where $P_{pc}$ is in psi, $T_{pc}$ is in °R and $y_g$ is the gas specific gravity which is expressed as the ratio of gas molecular weight to air molecular weight.

Sutton correlation [14] is valid for 0.57 < $y_g$ < 1.68 based on 264 different gas samples. Pseudo-critical pressure and pseudo-critical temperature in Sutton correlation are stated as:

$$P_{pc} = 756.8 - 131.0y_g - 3.6y_g^2$$  \hspace{1cm} (5)

$$T_{pc} = 169.2 + 349.5y_g - 74.0$$  \hspace{1cm} (6)

Given the size of the database used in its development, Sutton’s correlation is a good representative for all gas reservoirs [18].

Shale gas processing is often guided by gas compositions across a field [19]. Therefore, ethane and propane levels should be considered in choosing either Standing correlation [13] or Sutton correlation [14] for shale gas pseudo-critical properties evaluation.

Many research works have been done in determining Z-factor experimentally or evaluating it through different Z-factor correlations and other mathematical methods. Adeeye and Olamigoke [20] used Artificial Neural Networks (ANN) in predicting Z-factor. The ANN model correlates gas compressibility factor as a function of reservoir temperature and dew point pressure. ANN was applied to the 40 raw data sets in the range of 105-226 °F and 2445-4843 psia for temperature, and pressure, respectively.

Also, Obuba et al. [21] presented a new natural gas Z-factor correlation for Niger Delta gas fields by developing gas properties databank from twenty-two laboratory gas PVT reports from the fields (comprising dry gas, solution gas, rich CO$_2$ gas and rich condensate gas reservoirs). The developed correlation yielded better match than the existing correlations considered.

Using samples of gas mixtures from high-pressure-high-temperature (HPHT) gas reservoirs in the Niger Delta region of Nigeria, Azubuike et al. [22] carried out laboratory measurement of Z-factors. Most of the Z-factor correlations studied were found to overestimate the gas compressibility factor at HPHT. However, Beggs and Brills correlation [7] was reported to predict better than other correlations with a mean relative error of -4.77 and absolute error of 7.187.

Hamada [23] studied the effect of non-hydrocarbon components on Z-factor values using correlations. Gas pseudo-critical temperature was found to decrease with increase of N$_2$ and H$_2$S. Also, some deviations in Z-factor estimation were observed when using Stewart mixing rule and Kay’s mixing rule for gas reservoirs containing C7+. However, these deviations were negligible when correlation considering non-hydrocarbon was used.

Also, Mamedov et al. [24] presented an analytical model for evaluating the Z-factor of real gases such as H$_2$, N$_2$, He, CO$_2$, CH$_4$ and air at various temperatures using the second virial coefficient with a Lennard–Jones (12-6) potential. Results indicated good match with the data in the literature.

Akinsete and Omotosho [25] modeled two-phase gas Z-factor for gas-condensate reservoir using Artificial Neural Network. The two-phase gas Z-factor obtained in ANN method gave the closest value to the observed two-phase gas Z-factor from laboratory work.

### 1.1 Basis for Z-Factor Modification for Dual-Porosity Gas Reservoirs

The Z-factors evaluated experimentally or through different Z-factor empirical correlations and other mathematical methods represent cases of gas flow in single-porosity natural gas reservoirs with insignificant pore compaction after pressure depletion. The Z-factor does not reflect the remarkable rock compaction phenomenon in low-permeability natural gas reservoirs with single-porosity and dual-porosity.
1.2 Z-Factor Modification for Dual-Porosity Gas Reservoirs using Gas Material Balance Equation (MBE)

Schilthuis [26] presented an equation called material balance equation (MBE), formulated to consider the reservoir as a single tank characterised by homogeneous rock properties. The equation keeps inventory of all materials entering, leaving and accumulating in the reservoir and was developed as a volume balance in which the underground withdrawal in the form of observed cumulative production is equal to the expansion of the fluids in the reservoir due to pressure depletion. MBE thereafter became a basic tool often used by reservoir engineers to interpret and predict reservoir performance. When appropriately used, MBE can be utilised to evaluate initial hydrocarbon volumes in place, forecast reservoir production performance, and predict oil and gas recovery under different types of primary driving mechanisms [27].

If there is negligible rock compaction in a single-porosity volumetric reservoir (no water influx or water production), the traditional gas MBE is expressed as:

$$\frac{P}{Z} = \frac{P_i}{Z_i} (1 - \frac{G_p}{G})$$  \hspace{1cm} (7)

i.e.

$$\frac{G_p}{G} = 1 - \frac{P_i}{Z_i} \frac{P}{Z}$$  \hspace{1cm} (8)

where $G_p$ is cumulative gas produced (scf), $G$ is free gas initially in place (scf), $P$ is reservoir pressure, $Z$ is gas deviation factor (Z-factor), and subscript $i$ denoted condition before expansion.

Researchers that previously worked on modifying Z-factor in single-porosity gas MBE, for dual-porosity gas reservoirs, include King [28], Aguilera [29], Moghadam et al. [30], and Duarte et al. [31].

King [28] modified the MBE for original gas-in-place (OGIP) evaluation and future performance prediction for wells in coaled and Devonian shale reservoirs. In the approach, equilibrium condition was assumed for free and adsorbed gases. Also, gas desorption from the matrix blocks to the fracture system was assumed to be in pseudo-steady state. King’s method works just like the traditional MBE, where the straight line plot of $P/Z$ versus cumulative production $G_p$ is used in estimating OGIP.

The MBE presented by King [28] is expressed as:

$$\left(\frac{G}{Z}\right)_{\text{Total Gas}} = 1 - \frac{P_i}{Z_i} \frac{P}{Z} \left(\frac{P}{Z}\right)$$  \hspace{1cm} (9)

where $Z' = Z/\left((1 - (1 - \omega)C + \omega \omega) \Delta p\right)$ is the Aguilera [29] dual-porosity free gas Z-factor, $Z$ is single-porosity Z-factor at pressure $P$ without pore compaction $\omega_f$ is OGIP (free gas) fraction within the fractures, $(1 - \omega_f)$ is OGIP (free gas) fraction within the matrix blocks, $\Delta p$ is change in pressure, $C' = \frac{C_{wm} + C_{W}S_{wm}}{1 - S_{wm}}$, $C'' = \frac{C_{fr} + C_{W}S_{Wf}}{1 - S_{Wf}}$, $C_w$ is water compressibility, $S_{wm}$ is water saturation in the matrix and $S_{Wf}$ is water saturation in the fracture system. The details of the derivation of Aguilera [29] dual-porosity free gas Z-factor is shown in Appendix.

Moghadam et al. [30] improved on the MBE presented by King [28] and obtained a normalised compressibility factor expressed as:

$$Z'' = Z \left(\frac{Z_i}{Z_i'}\right)$$  \hspace{1cm} (11)
The plot of $P/Z_*$ versus $G_p$ has resemblance with the traditional $P/Z$ versus $G_p$. The modified material balance equation was reported to be applicable to all kinds of gas reservoirs i.e., unconventional, over-pressure and water-driven. Cumulative gas production $G_p$ can thus be evaluated as:

$$G_{p\text{Total}} = \frac{\phi V_g Z_0}{B_p} \left( \frac{P_i}{Z} - \frac{P}{Z^*} \right)$$

(12)

The estimated values of $G_p$ were then correlated with $G_p$ values obtained from shale gas production analysis (SGPA) by plotting $P/Z$ versus $G_p$, and $G_p$ versus time. The results showed good match.

However, Duarte et al. [31] incorporated gas desorption into the Aguiler [29] MBE for fractured gas reservoirs to yield:

$$\left( \frac{G_p}{G_{\text{Total Gas}}} \right)_{\text{Total Gas}} = 1 - \frac{P/Z}{P/Z_i}$$

(13)

where

$$Z = Z_{p} [1 - \omega_n - (\omega_n C_m + \omega_n C_J) \Delta P + \omega_n \frac{\rho_b}{3150 P_i (1 - \omega_m) V_{ads}}]^{-1}$$

(14)

and $\omega_n = \frac{OGIP_n}{OGIP_{\text{Total}}}$ is the ratio of the adsorbed gas to the original gas-in-place, $\omega_m = \frac{OGIP_m}{OGIP_{\text{Total}}}$ is the fraction of the original gas-in-place that is initially stored within the matrix pores, $\omega_f = \frac{OGIP_f}{OGIP_{\text{Total}}}$ is the fraction of the original gas-in-place that is initially stored within the fracture network, $\rho_b$ is shale bulk density (g/cm$^3$), $\phi$ is porosity and $V_{ads}$ is the adsorbed gas volume. Duarte et al. [31] evaluated $V_{ads}$ using the Langmuir adsorption isotherm $V_{ads} = \frac{P}{P_i + P}$, where $P_i$ is Langmuir volume (scf/ton), $P$ is average reservoir pressure (psia) and $P_i$ is Langmuir pressure (psia).

It is thus observed that King [28], Moghadam et al. [30] and Duarte et al. [31] developed Z-factors into which gas desorption was lumped, rendering them complex for routine calculations because cumulative free gas production should feature Z-factor while cumulative gas desorption should feature adsorption isotherm. However, Aguiler [29] developed a dual-porosity free gas Z-factor that incorporates OGIP fractions within fractures and matrix pores.

Total gas production is the sum of cumulative free gas production and cumulative gas desorption, i.e., $G_{p\text{Total}} = G_{p\text{free}} + G_{p\text{desorbed}}$.

Therefore, in this work, free gas Z-factor is derived for single-porosity reservoirs with remarkable pore compaction, and it is further modified to a simpler but accurate dual-porosity free gas Z-factor that is statistically correlated with Aguiler [29] free gas Z-factor.

2. METHODOLOGY

2.1 Evaluation of Z-Factor

Three shale gas formations A, B and C are considered. As stated earlier, the choice of using either Standing correlation [13] or Sutton correlation [14] for shale gas pseudo-critical properties evaluation depends on ethane and propane levels consideration.

The pseudo-reduced pressure $P_{pr}$ and pseudo-reduced temperature $T_{pr}$ are then calculated as $P_{pr} = \frac{P}{P_c}$ and $T_{pr} = \frac{T}{T_c}$, respectively. Thereafter, the single-porosity Z-factors at different pressure levels without pore compaction are evaluated using the Dranchuk-Abou-Kassem eleven-constant equation of state (EoS) [9] expressed as:

$$Z = 1 + C_1(T_{pr}) \cdot \rho_r + C_2(T_{pr}) \cdot \rho_r^2 - C_3(T_{pr}) \cdot \rho_r^2 - C_4(T_{pr}) \cdot \rho_r^2$$

(15)

where

$$\rho_r = \frac{0.277 T_{pr}}{Z_{pr}}$$

(16)

$$C_1(T_{pr}) = A_1 + A_2 \frac{\rho_r}{T_{pr}} + A_3 \frac{\rho_r^2}{T_{pr}} + A_4 \frac{\rho_r^3}{T_{pr}} + A_5 \frac{\rho_r^4}{T_{pr}}$$

(17)

$$C_2(T_{pr}) = A_6 + A_7 \frac{\rho_r}{T_{pr}} + A_8 \frac{\rho_r^2}{T_{pr}}$$

(18)

$$C_3(T_{pr}) = A_9 \frac{\rho_r}{T_{pr}} + A_{10} \frac{\rho_r^2}{T_{pr}}$$

(19)

$$C_4(T_{pr}, T_{pr}) = A_{11} \left( 1 + A_{12} \rho_r^2 \frac{\rho_r^2}{T_{pr}} \right) \exp \left( -A_{13} \rho_r^2 \right)$$

(20)

where $A_1 = 0.3265$, $A_2 = -0.700$, $A_3 = -0.5339$, $A_4 = 0.01569$, $A_5 = -0.05165$, $A_6 = 0.5475$, $A_7 = -0.7361$, $A_8 = 0.1844$, $A_9 = 0.1056$, $A_{10} = 0.6134$, and $A_{11} = 0.7210$.

The Dranchuk-Abou-Kassem EoS [9] must be solved iteratively since the Z-factor is on both sides of the equation. This task could be executed using the Newton-Raphson iteration method. Rearranging Equation 15 for this purpose yields:
For volumetric gas reservoirs (no water influx or gas saturation), the MBE is as follows:

\[ F(Z) = Z - \left[ 1 + C_1(\rho_{pr}) \cdot \rho_r + C_2(\rho_{pr}) \cdot \rho_r^2 - C_3(\rho_{pr}) \cdot \rho_r^2 \right] \cdot \frac{\rho_r \cdot S_{pr}}{\rho_{pr} \cdot S_{pr}} \tag{21} \]

where

\[ \frac{\partial F(Z)}{\partial Z} = 1 + C_1(\rho_{pr}) \cdot \frac{\rho_r^2}{2} + 2C_2(\rho_{pr}) \cdot \rho_r^2 - 5C_3(\rho_{pr}) \cdot \rho_r^2 \left[ \frac{1 + A_{11} \rho_r^2 - (A_{11} \rho_r^2)^2}{(1 + A_{11} \rho_r^2^2)} \right] \exp(-A_{11} \rho_r^2) \] \tag{22}

The steps involved in evaluating Z are highlighted as follows:

1. Use Standing and Katz Z-factor \( Z_{SK} \) as initial guess \( Z^i \)
2. Evaluate the new improved estimate of \( Z \) as:

\[ Z^{i+1} = Z^i - \frac{F(Z^i)}{F'(Z^i)} \tag{23} \]

where

\[ F'(Z^i) = \frac{\partial F(Z^i)}{\partial Z} \] \tag{24}

Repeat step 2 several times \( n \) until the error i.e. \( \text{abs}(Z^n - Z^{n+1}) \approx 0 \), such that \( Z^n = Z^{n+1} \) at a convergence criterion of \( 10^{-4} \).

The Newton-Raphson iteration method of determining the single-porosity Z-factors at different pressure levels without pore compaction was executed through a MAPPLE program.

### 2.2 Derivation of Free Gas Z-Factor for Single Porosity Gas Reservoirs with Rock Compaction

For single-porosity (no fracture) volumetric gas reservoirs (i.e. no water influx or water production), the MBE is as follows:

\[ G_{mat} \left( B_g - B_{g_{\text{si}}} \right) + \left( G_{mat} B_{g_{\text{si}}} \right) \frac{\left( C_{\text{water}} + C_{\text{matrix}} \right)}{S_{g_{\text{si}}}} \Delta P + W_e = G_p B_g + W_p B_w \tag{25} \]

where \( G_{mat} \) is the OGIP in the matrix, \( B_{g_{\text{si}}} \) is initial formation volume factor, \( B_g \) is formation volume factor after pressure depletion \( \Delta P \), \( C_{\text{matrix}} \) is matrix compressibility, \( C_w \) is water compressibility, \( S_{w_{\text{si}}} \) is initial water saturation, \( C_{\text{matrix}} \) is matrix compressibility and \( S_{g_{\text{si}}} \) is initial gas saturation.

For volumetric gas reservoirs (no water influx or water production), the MBE is expressed as:

\[ G_{mat} \left( B_g - B_{g_{\text{si}}} \right) + \left( G_{mat} B_{g_{\text{si}}} \right) \frac{\left( C_{\text{water}} + C_{\text{matrix}} \right)}{S_{g_{\text{si}}}} \Delta P = G_p B_g \tag{26} \]

\[ 1 - \frac{B_{g_{\text{si}}}}{B_g} + \left( \frac{B_{g_{\text{si}}}}{B_g} \right) \frac{\left( C_{\text{water}} + C_{\text{matrix}} \right)}{S_{g_{\text{si}}}} \Delta P = \frac{G_{mat}}{G_{mat}} \tag{27} \]

\[ \frac{G_{mat}}{G_{mat}} = 1 - \frac{B_{g_{\text{si}}}}{B_g} \left[ 1 - \left( \frac{C_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \Delta P \right) \right] \tag{28} \]

But

\[ \frac{B_{g_{\text{si}}}}{B_g} = \frac{Z - P \Delta P}{P \cdot Z} \tag{29} \]

Hence,

\[ \left( \frac{S_{g_{\text{si}}}}{S_{g_{\text{si}}}} \right)_{\text{free}} = 1 - \frac{P \Delta P}{P \cdot Z_i} \left[ 1 - \left( \frac{C_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \Delta P \right) \right] \tag{30} \]

Equation 30 is the MBE developed for single-porosity gas reservoirs with rock compaction after pressure depletion, and the corresponding Z-factor is:

\[ Z = Z \cdot \left[ 1 - \left( \frac{C_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \Delta P \right) \right]^{-1} \tag{31} \]

where \( Z \) is single-porosity Z-factor at pressure \( P \) without pore compaction and \( \left( \frac{C_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \Delta P \right) \) is the single-porosity matrix pore volume compressibility. It is expressed here as \( C_{\text{matrix}}^{\text{water}} \).

### 2.3 Modification of Single-Porosity Free Gas Z-Factor to Dual-Porosity Free Gas Z-Factor

In this work, a simpler but accurate approach of developing dual porosity free gas Z-factor in MBE (that is different from the Aguilera [29] method) is considered. Incorporating the ratio of dual porosity to initial matrix porosity into single-porosity gas MBE yields dual-porosity MBE. The approach is as follows:

With reference to Equation 26, for dual porosity gas reservoirs,

\[ G \left( B_g - B_{g_{\text{si}}} \right) + \left( \frac{\text{Dual Porosity Initial Matrix Porosity}}{\text{Dual Porosity Initial Matrix Porosity}} \right) \frac{\left( C_{\text{water}} + C_{\text{matrix}} \right)}{S_{g_{\text{si}}}} \cdot B_g \cdot \Delta P = G_p \cdot B_g \tag{32} \]

\[ \left( B_{g_{\text{si}}} \right) + \left( \frac{\text{Dual Porosity Initial Matrix Porosity}}{\text{Dual Porosity Initial Matrix Porosity}} \right) \frac{\left( C_{\text{water}} + C_{\text{matrix}} \right)}{S_{g_{\text{si}}}} \cdot B_g \cdot \Delta P = \frac{Q_{g_{\text{si}}}}{c} \tag{33} \]

\[ \left( B_{g_{\text{si}}} \right) + \frac{Q_{g_{\text{si}}}}{c} \frac{\left( 1 - \left( \frac{S_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \right) \Delta P \right)}{\left( \frac{S_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \right) \Delta P} = \frac{Q_{g_{\text{si}}}}{c} \tag{34} \]

\[ \frac{Q_{g_{\text{si}}}}{c} = 1 - \frac{B_{g_{\text{si}}}}{B_g} + \frac{B_{g_{\text{si}}}}{B_g} \left( \frac{1 - \left( \frac{S_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \right) \Delta P}{\left( \frac{S_{\text{water}} + C_{\text{matrix}}}{S_{g_{\text{si}}}} \right) \Delta P} \right) \tag{35} \]
2.4 Generalisation of the Developed Dual-Porosity Free Gas Z-Factor

Aguilera [29] dual-porosity free gas Z-factors is expressed as:

\[ Z'' = Z \left\{ 1 - \left(1 - \omega_f \right) C_{\text{mat}}^* + \omega_f C_{\text{frac}}^* \right\} \Delta P \]  \hspace{1cm} (38)

where \( Z \) is single-porosity Z-factor at pressure \( P \) without pore compaction, \( \omega_f \) is fraction of OGIP (free gas) in the matrix system and \( \left(1 - \omega_f \right) \) is fraction of OGIP (free gas) in the matrix, \( \Delta P \) is pressure depletion, matrix pore volume compressibility \( C_{\text{mat}}^* \) is expressed as:

\[ \left(\frac{c_w s_{\text{mat}} + c_{\text{frac}}}{s_{\text{g}}}ight) \] and fracture volume compressibility \( C_{\text{frac}}^* \) is expressed as:

\[ \left(\frac{c_w s_{\text{g}} + c_{\text{frac}}}{s_{\text{g}}}ight) \]

However, from the concept of porosity, Aguilera [29] dual-porosity Z-factor is expressed in this work as:

\[ Z^* = Z \left\{ 1 - \left(\frac{\phi_{\text{frac}}}{\phi_{\text{frac}} + \phi_{\text{mat}}}\right) C_{\text{mat}}^* + \left(\frac{\phi_{\text{frac}}}{\phi_{\text{frac}} + \phi_{\text{mat}}}\right) C_{\text{frac}}^* \right\} \Delta P \]  \hspace{1cm} (39)

where \( \phi_{\text{frac}} \) is fracture porosity, \( \phi_{\text{mat}} \) is matrix porosity after fracturing and \( \phi_{\text{mat}}^* \) is initial matrix porosity.

During hydraulic fractures stabilisation and porosity/permeability maintenance before pressure depletion, fracture volume compressibility \( C_{\text{frac}}^* \) is sustained at a lower level that corresponds to injection water compressibility \( c_{\text{frac}} \). Hence, \( C_{\text{frac}}^* \) is considered to be equal to \( c_{\text{frac}} \).

For the developed dual porosity free gas Z-factor to be generalised for hydraulically-fractured gas reservoirs, it is correlated with Aguilera [29] dual-porosity free gas Z-factor, and the qualities of fit were graphically and statistically assessed.

3. RESULTS AND DISCUSSION

Sampled gas compositions for shale formations A, B and C are shown in Tables 1, 2 and 3 respectively. The gas compositions in the Tables had been normalised to the reported compounds by Bullin and Krouskop [19], and Hill et al. [32].

### Table 1. Gas composition for shale formation A

| Well | Gas Composition (%) | C1 | C2 | C3 | CO2 | N2 |
|------|---------------------|----|----|----|-----|----|
| 1    | 79.4                | 16.1| 4.0|    | 0.1 | 0.4|
| 2    | 82.1                | 14.0| 3.5|    | 0.1 | 0.3|
| 3    | 83.8                | 12.0| 3.0|    | 0.9 | 0.3|
| 4    | 95.5                | 3.0 | 1.0|    | 0.3 | 0.2|

Source: Bullin and Krouskop [19]; Hill et al. [32]

### Table 2. Gas composition for shale formation B

| Well | Gas Composition (%) | C1 | C2 | C3 | CO2 | N2 |
|------|---------------------|----|----|----|-----|----|
|      |                     | 95.0| 0.1| 0  | 4.8 | 0.1|

Average

Source: Bullin and Krouskop [19]; Hill et al. [32]

### Table 3. Gas composition for shale formation C

| Well | Gas Composition (%) | C1 | C2 | C3 | CO2 | N2 |
|------|---------------------|----|----|----|-----|----|
| 1    | 80.3                | 8.1 | 2.3| 1.4| 7.9 |
| 2    | 81.2                | 11.8| 5.2| 0.3| 1.5 |
| 3    | 91.8                | 4.4 | 0.4| 2.3| 1.1 |
| 4    | 93.7                | 2.6 | 0.0| 2.7| 1.0 |

Source: Bullin and Krouskop [19]; Hill et al. [32]

In this work, Sutton correlation [14] (0.57 < \( \gamma_g < 1.68 \)) is used in calculating pseudo-critical pressure \( P_{pc} \) and pseudo-critical temperature \( T_{pc} \) for shale formation A having average of 85.2% C1 see Table 1. However, Standing correlation [13] (\( \gamma_g < 0.75 \)) is used in calculating \( P_{pc} \) and \( T_{pc} \) for shale formation B having average of 95.0% C1 see Table 2 and shale formation C having average of 86.75% C1 see Table 3.

Reservoir data for shale formation A is shown in Table 4.
3.1 Variation of Natural Gas Compressibility Factor with Pressure for Shale Formation A

Reservoir temperature $T$ of 175 °F (i.e. 635 °R) and a natural gas gravity $\gamma_g$ of 0.65 are considered for shale formation A. Based on gas compositions (see Table 1), applying Sutton correlation [14] ($0.57 < \gamma_g < 1.68$) yields a pseudo-critical pressure $P_{pc}$ of 670.13 psi (see Equation 5), pseudo-critical temperature $T_{pc}$ of 365.11 °R (see Equation 6) and a pseudo-reduced temperature $T_{pr}$ of 1.7392.

Standing and Katz Z-factors $Z_{SK}$ are used as initial guesses in evaluating Dranchuk-Abou-Kassem Z-factors $Z_{DAK}$ (see Equation 15) indicated as $Z$ in Table 5. The pressure range considered for shale formation A is: $0 \leq P \leq 3,500$ psig. The initial gas formation volume factor of shale formation A:

$$B_{g_i} = \left(\frac{P_{pc}}{P_i}\right)^{\frac{T}{T_i}}$$  \hspace{1cm} (40)

is evaluated as $4.6313 \times 10^{-3}$ rcf/scf.

With $\phi_{frac} = 0$, single-porosity Z-factor with pore compaction is evaluated as:

$$Z^* = Z\{1 - (1.5429E - 6)\Delta P\}^{-1}$$  \hspace{1cm} (41)

With $\phi_{frac} = 0.04$, Aguilera [27] dual-porosity Z-factor with pore compaction is evaluated as:

$$Z'' = Z\{1 - (2.1479E - 6)\Delta P\}^{-1}$$  \hspace{1cm} (42)

and the modified dual-porosity Z-factor with pore compaction is evaluated as:

$$Z^{**} = Z\{1 - (2.0983E - 6)\Delta P\}^{-1}$$  \hspace{1cm} (43)

The variations of the Z-factors with pressure depletion for shale formation A are shown in Table 5 and Fig. 1. It is observed that the single-porosity Z-factor, evaluated for shale formation A using the Dranchuk-Abou-Kassem eleven-constant equation of state (EoS) [9], decreases from 1.0000 (at $P = 0$), reaches its minimum value around 2,300 psig (i.e. around $P_{pr} = 3.4322$) and thereafter rises. The trend conforms to Standing and Katz chart within the pressure range considered but the higher decimal points yielded more accuracy than the Standing and Katz chart. Correlating the modified dual-porosity Z-factor with Aguilera [29] dual porosity Z-factor yields a $R^2$ value of 1.000.

| Table 4. Reservoir data for shale formation A |
|------------------------------------------------|
| Parameter | Symbol | Value | Unit |
|-----------|--------|-------|------|
| Matrix porosity | $\phi_{mat}$ | 0.10 | - |
| before fracturing | | | |
| Fracture porosity | $\phi_{frac}$ | 0.04 | - |
| Initial gas saturation | $S_g$ | 0.70 | - |
| Initial water saturation | $S_{w_i}$ | 0.30 | - |
| Water compressibility | $c_w$ | $3.6 \times 10^6$ | psi$^{-1}$ |
| Rock matrix compressibility | $c_{matrix}$ | $4.0 \times 10^{12}$ | psi$^{-1}$ |

*Source: SPE Formation Evaluation [33]; US Department of Energy [34]; Soeder [35]*

Fig. 1. Variation of natural gas $Z$, $Z^*$, $Z''$ and $Z^{**}$ with pressure for shale formation A based on $\phi_{frac}$ of 0.04

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3.2 Variation of Natural Gas Compressibility Factor with Pressure for Shale Formation B

Reservoir temperature $T$ of 300°F (i.e. 760°F) and a natural gas gravity $\gamma_g$ of 0.65 are considered for shale formation B. Based on gas compositions (see Table 2), applying Standing [13] correlation ($\gamma_g < 0.75$) for dry gas yields a pseudo-critical pressure $P_{pc}$ of 660.91 psi (see Equation 1), pseudo-critical temperature $T_{pc}$ of 373.97 °R (see Equation 2) and a pseudo-reduced temperature $T_{pr}$ of 2.0322.

Standing and Katz Z-factors $Z_{SK}$ are used as initial guesses in evaluating Dranchuk-Abou-Kassem Z-factors $Z_{DAX}$ (see Equation 15) indicated as $Z$ in Table 7. The pressure range considered for shale formation B is: $0 \leq P \leq 12,000$ psi. The initial gas formation volume factor of shale formation B:

$$B_g = \left( \frac{P_{pc}}{T_{pc}} \right) \frac{DAX}{P}$$

is evaluated as $2.1653 \times 10^{-3}$ rcf/scf.

With $\phi_{frac} = 0$, single-porosity Z-factor with pore compaction is evaluated as:

$$Z^* = Z \left( 1 - (1.5429E - 6) \Delta P \right)^{-1}$$

With $\phi_{frac} = 0.04$, Aguilera [27] dual-porosity Z-factor with pore compaction is evaluated as:

$$Z^- = Z \left( 1 - (2.2196E - 6) \Delta P \right)^{-1}$$

and the modified dual-porosity Z-factor with pore compaction is evaluated as:

$$Z^+ = Z \left( 1 - (2.2073E - 6) \Delta P \right)^{-1}$$

The variations of the Z-factors with pressure depletion for shale formation B are shown in Table 7 and Fig. 2. Also, the single-porosity Z-factor calculated for shale formation B was found to decrease from 1.0000 (at $P = 0$), reaches its minimum value around 2,250 psi (i.e. around $P_{pr} = 3.4044$) and thereafter rises. As usual, the trend conforms to Standing and Katz chart within the pressure range considered but the higher decimal points showed more accuracy than the Standing and Katz chart. Correlating the modified dual-porosity Z-factor with Aguilera [29] dual porosity Z-factor yields a $R^2$ value of 1.000.

Reservoir data for shale formation C is shown in Table 8.

3.3 Variation of Barnett Shale Gas Compressibility Factor with Pressure

Reservoir temperature $T$ of 180 °F (i.e. 640 °R) and a natural gas gravity $\gamma_g$ of 0.65 are
considered for shale formation C. Based on gas compositions (see Table 3), applying Standing correlation \[13\] \(r_y < 0.75\) for dry gas yields a pseudo-critical pressure \(P_{pc}\) of 660.91 psi (see Equation 1), pseudo-critical temperature \(T_{pc}\) of 373.97 °R (see Equation 2) and a pseudo-reduced temperature \(T_{pr}\) of 1.71140.

Standing and Katz Z-factors \(Z_{SK}\) are used as initial guesses in evaluating Dranchuk-About-Kassem Z-factors \(Z_{DAK}\) (see Equation 15) indicated as Z in Table 9.

The pressure range considered for shale formation C is: \(0 \leq P \leq 3,900\) psig. The initial gas formation volume factor of shale formation C:

\[B_i = \left( \frac{P_{sc}}{P_{pc}} \right) Z_i^{T_i} \quad \text{(48)}\]

is evaluated as \(4.2614 \times 10^{-3}\) rcf/scf.

With \(\phi_{frac} = 0\), single-porosity Z-factor with pore compaction is evaluated as:

\[Z^* = Z(1 - (1.5429E - 6) \Delta P)^{-1} \quad \text{(49)}\]

With \(\phi_{frac} = 0.02\), Aguilera [27] dual-porosity Z-factor with pore compaction is evaluated as:

\[Z^- = Z(1 - (2.1847E - 6) \Delta P)^{-1} \quad \text{(50)}\]

and the modified dual-porosity Z-factor with pore compaction is evaluated as:

\[Z^{* -} = Z(1 - (2.1978E - 6) \Delta P)^{-1} \quad \text{(51)}\]

The variations of the Z-factors with pressure depletion for shale formation B are shown in Table 9 and Fig. 3. The single-porosity Z-factor evaluated for shale formation C was found to decrease from 1.0000 (at \(P = 0\)), reaches its minimum value around 2,300 psig (i.e. around \(P_{pr} = 3.4800\)) and thereafter rises. The trend conforms to Standing and Katz chart within the pressure range considered but the higher decimal points showed more accuracy than the Standing and Katz chart. Correlating the modified dual-porosity Z-factor with Aguilera [29] dual porosity Z-factor yields a \(R^2\) value of 1.000.

### Table 6. Reservoir data for shale formation B

| Parameter                  | Symbol | Value | Unit       |
|----------------------------|--------|-------|------------|
| Matrix porosity before fracturing | \(\phi_{mat}\) | 0.085 | -          |
| Fracture porosity          | \(\phi_{frac}\) | 0.04  | -          |
| Initial gas saturation     | \(S_{gi}\) | 0.70  | -          |
| Initial water saturation   | \(S_{wi}\) | 0.30  | -          |
| Water compressibility      | \(C_w\) | \(3.6 \times 10^{-6}\) | psi\(^{-1}\) |
| Rock matrix compressibility| \(C_{matrix}\) | \(3.0 \times 10^{-12}\) | psi\(^{-1}\) |

Source: US Department of Energy [34]; Male et al. [36]

![Fig. 2. Variations of natural gas Z, Z*, Z'' and Z*** with pressure for shale formation B based on \(\phi_{frac}\) of 0.04](image)
Table 7. Variations of natural gas $Z$, $Z^\prime$, $Z^\prime\prime$ and $Z^\prime\prime$ with pressure for shale formation B based on $\phi_{frac}$ of 0.04

| (psig) | $P_{pr}$ | $Z$ | $\Delta P$ | $Z^\prime$ | $Z^\prime\prime$ | (Aguilera) | (Developed) |
|--------|----------|-----|------------|------------|----------------|------------|-------------|
| 12000  | 18.1568  | 1.2094 | 0          | 1.2094     | 1.2094        | 1.2094     | 1.2094      |
| 11250  | 17.0220  | 1.1931 | 750        | 1.1945     | 1.1951        | 1.1951     | 1.1951      |
| 10500  | 15.8872  | 1.1780 | 1500       | 1.1807     | 1.1819        | 1.1819     | 1.1819      |
| 9750   | 14.7524  | 1.1607 | 2250       | 1.1647     | 1.1665        | 1.1665     | 1.1665      |
| 9000   | 13.6176  | 1.1415 | 3000       | 1.1468     | 1.1492        | 1.1492     | 1.1491      |
| 8508   | 12.8732  | 1.1296 | 3492       | 1.1357     | 1.1384        | 1.1384     | 1.1384      |
| 8250   | 12.4824  | 1.1205 | 3750       | 1.1270     | 1.1299        | 1.1299     | 1.1298      |
| 7500   | 11.3480  | 1.0983 | 4500       | 1.1060     | 1.1094        | 1.1094     | 1.1093      |
| 6750   | 10.2132  | 1.0754 | 5250       | 1.0842     | 1.0881        | 1.0881     | 1.0880      |
| 6000   | 9.0784   | 1.0541 | 6000       | 1.0639     | 1.0683        | 1.0683     | 1.0682      |
| 5250   | 7.9436   | 1.0297 | 6750       | 1.0405     | 1.0454        | 1.0454     | 1.0453      |
| 4500   | 6.8088   | 1.0069 | 7500       | 1.0187     | 1.0239        | 1.0239     | 1.0238      |
| 3750   | 5.6740   | 0.9818 | 8250       | 0.9945     | 1.0001        | 1.0001     | 1.0000      |
| 3000   | 4.5392   | 0.9562 | 9000       | 0.9697     | 0.9757        | 0.9757     | 0.9756      |
| 2250   | 3.4044   | 0.9436 | 9750       | 0.9580     | 0.9645        | 0.9645     | 0.9644      |
| 1500   | 2.2696   | 0.9467 | 10500      | 0.9623     | 0.9693        | 0.9693     | 0.9692      |
| 750    | 1.1348   | 0.9661 | 11250      | 0.9832     | 0.9908        | 0.9908     | 0.9909      |
| 375    | 0.5674   | 0.9815 | 11625      | 0.9994     | 1.0075        | 1.0075     | 1.0073      |
| 0      | 0        | 1.0000 | 12000      | 1.0189     | 1.0274        | 1.0274     | 1.0273      |

Table 8. Reservoir data for shale formation C

| Parameter                        | Symbol | Value       | Unit |
|---------------------------------|--------|-------------|------|
| Matrix porosity before fracturing | $\phi_{mat}$ | 0.045       | -    |
| Fracture porosity               | $\phi_{frac}$ | 0.02       | -    |
| Initial gas saturation          | $S_g$ | 0.70        | -    |
| Initial water saturation        | $S_w$ | 0.30        | -    |
| Water compressibility            | $c_w$ | $3.6 \times 10^{-6}$ | psi$^{-1}$ |
| Rock matrix compressibility      | $c_{matrix}$ | $3.0 \times 10^{-12}$ | psi$^{-1}$ |

Source: US Department of Energy [34]; Bowker [37]

Table 9. Variations of natural gas $Z$, $Z^\prime$, $Z^\prime\prime$ and $Z^\prime\prime$ with pressure for shale formation C based on $\phi_{frac}$ of 0.02

| $P$(psig) | $P_{pr}$ | $Z$ | $\Delta P$(psig) | $Z^\prime$ | $Z^\prime\prime$ (Aguilera) | $Z^\prime\prime$ (Developed) |
|-----------|----------|-----|-----------------|------------|-----------------------------|-----------------------------|
| 3900      | 5.9009   | 0.9186 | 0              | 0.9186     | 0.9186                      | 0.9186                      |
| 3600      | 5.4470   | 0.9020 | 300             | 0.9024     | 0.9026                      | 0.9026                      |
| 3300      | 4.9931   | 0.8878 | 600             | 0.8886     | 0.8890                      | 0.8890                      |
| 3050      | 4.6148   | 0.8784 | 850             | 0.8795     | 0.8800                      | 0.8800                      |
| 2800      | 4.2366   | 0.8714 | 1100            | 0.8729     | 0.8735                      | 0.8735                      |
| 2550      | 3.8583   | 0.8671 | 1350            | 0.8689     | 0.8697                      | 0.8697                      |
| 2300      | 3.4800   | 0.8660 | 1600            | 0.8681     | 0.8690                      | 0.8690                      |
| 2088      | 3.1593   | 0.8676 | 1812            | 0.8700     | 0.8710                      | 0.8711                      |
| 1800      | 2.7235   | 0.8738 | 2100            | 0.8766     | 0.8778                      | 0.8779                      |
| 1550      | 2.3452   | 0.8830 | 2350            | 0.8862     | 0.8876                      | 0.8876                      |
| 1300      | 1.9670   | 0.8955 | 2600            | 0.8991     | 0.9006                      | 0.9006                      |
| 1050      | 1.5887   | 0.9110 | 2850            | 0.9150     | 0.9167                      | 0.9167                      |
| 800       | 1.2104   | 0.9292 | 3100            | 0.9337     | 0.9355                      | 0.9356                      |
| 550       | 0.8322   | 0.9495 | 3350            | 0.9544     | 0.9565                      | 0.9565                      |
| 300       | 0.4539   | 0.9717 | 3600            | 0.9771     | 0.9794                      | 0.9794                      |
| 150       | 0.2270   | 0.9856 | 3750            | 0.9913     | 0.9937                      | 0.9938                      |
| 0         | 0        | 1.0000 | 3900            | 1.0060     | 1.0086                      | 1.0086                      |
4. CONCLUSIONS

The, free gas Z-factor in the Material Balance Equation (MBE) of single-porosity gas reservoirs with insignificant rock (matrix) compaction (after pressure depletion) does not reflect cases in low-permeability gas reservoirs having remarkable rock compaction. Therefore, in this work, a free gas Z-factor for single-porosity reservoirs with remarkable pore compaction (such as low-permeability gas reservoirs) was derived and it is expressed as: 

\[ Z^* = Z \left( 1 - \frac{c_w s_w + c_{matrix}}{s_{g_1}} \Delta P \right)^{-\frac{1}{T}} \]

where \( Z \), \( c_w \), \( s_w \), \( c_{matrix} \), and \( s_{g_1} \) are single-porosity Z-factor at \( P \) without rock compaction, water compressibility, initial water saturation, matrix compressibility and initial gas saturation, respectively.

However, \( Z^* \) was further modified to a simpler but accurate dual-porosity free gas Z-factor which incorporates ratio of dual porosity to initial matrix porosity, and it is expressed as: 

\[ Z^{**} = Z \left( 1 - \frac{\phi_{frac} + \phi_{frac}}{\phi_{mat}} \frac{c_w s_w + c_{matrix}}{s_{g_1}} \Delta P \right)^{-\frac{1}{T}} \]

where \( \phi_{mat} \) and \( \phi_{frac} \) are initial matrix porosity and fracture porosity respectively.

It is observed that the single-porosity Z-factor, evaluated for shale formations considered using the Dranchuk-Abou-Kassem eleven-constant equation of state (EoS) decreases from 1.0000 (at \( P = 0 \)), reaches a minimum value and thereafter rises. The trend conforms to Standing and Katz chart within the pressure (and pseudo-pressure) range considered but the higher decimal points yielded more accuracy than the Standing and Katz chart.

The developed dual porosity free gas Z-factor was graphically and statistically correlated with Aguilera [29] dual porosity free gas Z-factor. For all the hydraulically fractured shale gas formations considered, the correlations yield \( R^2 \) values of 1.000.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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REFERENCES

1. McQuarrie DA, Simon JD. Molecular Thermodynamics. University Science Books. 1999;55.
2. Zucker RD, Biblarz O. Fundamentals of Gas Dynamics. Second Edition, Wiley Books. 2002; 327.
3. Cengel YA, Boles MA. Thermodynamics: An Engineering Approach. Eighth Edition. McGraw-Hill Education. 2015;139-140.
4. Standing M.B. Katz D.L. Density of Natural Gases. Trans. AIME. 1942;146:140-149.
5. Wilhert E. Aziz K. Calculation of Z’s for Sour Gases. Hydrocarbon Processing. 1972;51:5.
6. Hall K.R. Yarbrugh L. A. New Equation of State for Z-Factor Calculations. Oil and Gas. 1973; 82.
7. Beggs HD, Brills JP. Two-Phase Flow in Pipes. University of Tulsa, Tulsa, OK. USA; 1974.
8. Dranchuk PM, Purvis RA. Robinson D.B. Computer Calculation of Natural Gas Compressibility Factors using the Standing and Katz Correlation. Institute of Petroleum Technical Series. 1974; IP74-008.
9. Dranchuk PM, Abou-Kassem JH. Calculation of Z-Factors for Natural Gases using Equations of State. Journal of Canadian Petroleum Technology. July-September. 1975;14(3):34.
10. Takacs G. Comparisons Made for Computer Z-Factor Calculations. Oil and Gas Journal. 1976; 64-66.
11. Piper LD, Jr. McCain WD, Jr. Corredor JH. Compressibility Factors for Naturally Occurring Petroleum Gases. Paper SPE. 1993;26668.
12. Elsharkawy A.M, Elkamel A. The Accuracy of Predicting Compressibility Factor for Sour Natural Gases. Petroleum Science and Technology. 2001;19(5-6):711-731.
13. Standing M.B. Volumetric and Phase Behaviour of Oil Field Hydrocarbon Systems. 9th Edition. Society of Petroleum Engineers of AIME. Dallas, TX. USA; 1981.
14. Sutton RP. Compressibility Factors for High Molecular Weight Reservoir Gases. Paper SPE 14265 presented at 60th Annual Technical Conference and Exhibition of the SPE. Las Vegas. USA. September. 1985;22-25.
15. Ahmed T. Equation of State and PVT Analysis. 2nd Ed. Boston: Gulf Publishing Company; 2007. DOI: https://doi.org/10.1016C2013-0-15511-0.
16. Elsharkawy AM, Hashem YK. Alikhan A.A. Compressibility Factor for Sour Natural Gas Condensate Reservoirs. Paper SPE. 2000;59702.
17. Guo B, Ghalambor A. Natural Gas Engineering Handbook. Houston, TX. Gulf Publishing Company. USA; 2005.
18. Blaisingame TA. Review Materiaiks: Properties of Reservoir Fluids. Texas A&M University; 1998.
19. Bullin K. Krouskop P. Composition Variety Complicates Processing Plans for US Shale Gas. Annual Forum of Gas Processors Association. Houston, Texas. USA; 2008.
20. Adeleye J, Olamigoke O. Prediction of Gas Compressibility Factor using Artificial Neural Network. 2nd UMaT Biennial International Mining and Mineral Conference, Tarkwa, Ghana; 2012.
21. Obuba J, Ikiesnikimama SS, Ubani CE, Ekeke IC. Natural Gas Compressibility Factor Correlation Evaluation for Niger Delta Gas Fields. OSR Journal of Electrical and Electronics Engineering. 2013;6(4):01-10.
22. Azubuike II, Ikiesnikimama SS, Orodu OD. Natural Gas Compressibility Factor Measurement and Evaluation for High Pressure High Temperature Gas Reservoirs. International Journal of Scientific and Engineering Research. 2016;7(7):1173-1181.
23. Hamada GM. Effect of Non-Hydrocarbon Components on Gas Compressibility Factor Values and Correlations. Journal of Applied Biotechnology and Bioengineering. 2017;2(4):124-134.
24. Mamedov BA, Somuncu E, Askarov IM. Theoretical Assessment of Compressibility Factor of Gases by using Second Virial Coefficient. Zeitschrift fur Naturforschung: A Journal of Physical Sciences. December. 2017;73.2:121-125.
25. Akinsete OO, Omotosho AA. Modeling of Two-Phase Gas Deviation Factor for Gas-Condensate Reservoir using Artificial Neural Network. Advances in Research. 2018;14(1):1-8.
26. Schilthuis R. Active Oil and Reservoir Energy. Trans. AIME. 1936;118:33.
27. Ahmed T. Reservoir Engineering Handbook. 2nd Ed. Boston: Gulf Professional Publishing; 1989.
28. King G.R. Material Balance Techniques for Coal Seam and Devonian Shale Gas Reservoirs, SPE Paper 20730 presented at the 65th Annual Technical Conference.
and Exhibition. New Orleans, Louisiana. USA. September. 1990;23-26.

29. Aguilera R. Effect of Fracture Compressibility on Gas-In-Place Calculations of Stress-Sensitive Naturally Fractured Reservoirs. SPE Paper 100451. SPE Reservoir Evaluation and Engineering. 2008;11(02):307-310.

30. Moghadam S, Jeje O, Mattar L. Advanced Gas Material Balance in Simplified Format. Paper PETSOC 2009-149 presented at the 10th Canadian International Petroleum Conference (CICP). Calgary, Alberta. Canada; 2009.

31. Duarte JC, Vinas EC, Ciancaglini M. Material Balance Analysis of Naturally or Artificially Fractured Shale Gas Reservoirs to Maximize Final Recovery. SPE Paper 169480-MS presented at the SPE Latin American and Caribbean Petroleum Engineering Conference. Maracaibo. Venezuela; 2014.

32. Hill RJ, Jarvie DM, Zumberge J, Henry M, Pollastro RM. Oil and Gas Geochemistry and Petroleum Systems of the Fort Worth Basin. AAPG Bulletin. 2007;91(4):445-473. SPE Formation Evaluation. March; 1998.

33. US. Department of Energy, Office of Fossil Energy and National Energy Technology Laboratory. Modern Shale Gas Development in the United States: A Primer. 2009;DE-FG26-04NT15455.

34. Soeder DJ. Resource and Environmental Studies on the Marcellus Shale. Savage River Watershed Association Meeting. Frostburg, MD. National Energy Technology Laboratory. US Dept. of Energy; 2011.

35. Male F, Islam AW, Patzek TW, Ikonnikova S, Browning J, Marder MP. Analysis of Gas Production from Hydraulically Fractured Wells in the Haynesville Shale using Scaling Methods. Journal of Unconventional Oil and Gas Resources. 2015;10:11-17.

36. Bowker KA. Barnett Shale Gas Production, Fort Worth Basin: Issues and Discussion. AAPG Bulletin. 2007;91.4:523-533.
APPENDIX

Derivation of Aguilera (2008) Dual-Porosity Free Gas Z-Factor

For fractured gas reservoirs without adsorption, the derivation of Aguilera (2008) gas MBE is as follows:

For volumetric gas reservoirs (no water influx or water production), the MBE is expressed as:

\[ G_{mat} \left( B_g - B_{g1} \right) + \left( G_{mat} B_{g1} \right) \frac{C G_{sw} + C_{pm}}{S_g} \Delta P = G_{p_{mat}} B_g \] \hspace{1cm} \text{(A.1)}

For the matrix blocks in the dual-porosity system,

\[ G_{mat} \left( B_g - B_{g1} \right) + \left( G_{mat} B_{g1} \right) \frac{C G_{sw} + C_{pm}}{1 - S_{wm}} \Delta P = G_{p_{mat}} B_g \] \hspace{1cm} \text{(A.2)}

where \( G_{mat} \), \( G_{p_{mat}} \) and \( \frac{C G_{sw} + C_{pm}}{1 - S_{wm}} \) are OGIP within the matrix, cumulative gas production from the matrix and matrix pore volume compressibility respectively.

However, for the fractures,

\[ G_{frac} \left( B_g - B_{g1} \right) + \left( G_{frac} B_{g1} \right) \frac{C G_{sw} + C_f}{1 - S_{sw}} \Delta P = G_{p_{frac}} B_g \] \hspace{1cm} \text{(A.3)}

where \( G_{frac} \), \( G_{p_{frac}} \) and \( \frac{C G_{sw} + C_f}{1 - S_{sw}} \) are OGIP within the fractures, cumulative gas production from the fractures and fracture volume compressibility respectively.

Expressing \( \frac{C G_{sw} + C_{pm}}{1 - S_{wm}} \) and \( \frac{C G_{sw} + C_f}{1 - S_{sw}} \) as \( C' \) and \( C'' \) respectively, and adding Equations A.2 and A.3 to obtain dual-porosity gas MBE (i.e. after fracturing) gives:

\[ (G_{mat} + G_{frac}) \left( B_g - B_{g1} \right) + \left( G_{mat} C' + G_{frac} C'' \right) B_g \Delta P = \left( G_{p_{mat}} + G_{p_{frac}} \right) B_g \] \hspace{1cm} \text{(A.4)}

However, \( G' \) (OGIP) is constant, i.e. \( G = G_{mat} + G_{frac} \). Also, \( G_{p_{mat}} + G_{p_{frac}} = G_p \). Hence,

\[ G \left( B_g - B_{g1} \right) + \left( G_{mat} C' + G_{frac} C'' \right) B_g \Delta P = G_p B_g \] \hspace{1cm} \text{(A.5)}

\[ \left( B_g - B_{g1} \right) + \left( \frac{G_{mat}}{G} C' + \frac{G_{frac}}{G} C'' \right) B_g \Delta P = \frac{G_p}{G} B_g \] \hspace{1cm} \text{(A.6)}

\[ \left( B_g - B_{g1} \right) + \left( \left( 1 - \omega_f \right) C' + \omega_f C'' \right) B_{g1} \Delta P = \frac{G_p}{G} B_g \] \hspace{1cm} \text{(A.7)}

where \( \omega_f \) is fraction of OGIP (free gas) in the fracture system and \( 1 - \omega_f \) is fraction of OGIP (free gas) in the matrix.

\[ \left( \frac{G_p}{G} \right)_{free} = 1 - \frac{B_{g1}}{B_g} + \left( \left( 1 - \omega_f \right) C' + \omega_f C'' \right) \frac{B_{g1}}{B_g} \Delta P \] \hspace{1cm} \text{(A.8)}

Hence,

\[ \left( \frac{G_p}{G} \right)_{free} = 1 - \frac{p_f}{p_{gw}} \cdot \left( 1 - \left( 1 - \omega_f \right) C' + \omega_f C'' \right) \Delta P \] \hspace{1cm} \text{(A.9)}
where the modified Z-factor \((Z^*)\) is expressed as:

\[
Z^* = Z \cdot \left\{ 1 - \left[ (1 - \omega_f)C' + \omega_f C'' \right] \cdot \Delta P \right\}^{-1}
\]  

(A.10)

where \(Z\) is single-porosity Z-factor at pressure \(P\) without rock compaction. Equation A.9 is the Aguilera (2008) gas MBE for fractured gas reservoirs without adsorption and Equation A.10 is the Aguilera (2008) dual-porosity Z-factor.

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