Equal Sinusoidal Division of Array Manifold Matrix Based Direction-of-Arrival Estimation for Dual-functional Radar-communication

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Abstract. In view of the large computational complexity and poor real-time performance of \( \ell_1 \) norm convex optimization process for DOA estimation based on compressed sensing sparse reconstruction, the SAMP greedy algorithm is used to replace the \( \ell_1 \) norm to obtain the approximate solution of \( \ell_0 \) norm optimization in DOA estimation. The array manifold matrix is divided by equal sinusoidal, which satisfies the MIP criterion better. An iterative regularization sparsity adaptive matching pursuit (IR-SAMP) algorithm is proposed, which makes the SAMP algorithm better approximate the signal sparsity and reduces the estimation error. IR-SAMP algorithm uses regularization method and backtracking screening, and eliminates the inappropriate atoms in the backtracking stage, so as to better approximate the signal source sparsity. Simulation results show that the proposed IR-SAMP algorithm in single snapshot DOA estimation under equal sinusoidal sparse division is better. At the same time, IR-SAMP algorithm has low computational complexity, as for coherent signal source, which also has a better estimation effect.

1. Introduction
This DIRECTION-OF-ARRIVAL (DOA) estimation of array signals is one of the main research fields in array signal processing. How to realize DOA estimation of real-time, high-precision and low-cost signal source has been a continuous research direction in array signal processing. In recent years, the compressed sensing (CS) [1] and sparse representation theory proposed in the field of information theory and processing provide a new parameter estimation method. The DOA estimation based on compressed sensing sparse reconstruction has been widely studied. In [2], time-domain compressive sampling is performed on received signals by array to reduce the number of time-domain sampling points received by the array. However, this method requires that the received signals of the array be compressible signals of known form.

Malioutov D and Cetin M first proposed the \( \ell_1 \)-SVD algorithm [3] in 2005 for DOA estimation, which opened the way for the study of compressed sensing DOA estimation[4]. DOA estimation based on greedy algorithm has also attracted the attention of researchers [5]. Literature proposed an improved OMP algorithm for DOA estimation, but the algorithm required a known number of sources. In [6], a SMP greedy algorithm was proposed for DOA estimation under single snapshot. By changing antenna’s radiation pattern and has proven to improve doa estimates[7].

In view of the above problems, this paper firstly applies sparsity adaptive matching pursuit (SAMP) [8] algorithm to DOA estimation. At the same time, an iterative regularization sparsity adaptive matching pursuit (IR-SAMP) algorithm based on SAMP algorithm is proposed in this paper. Simulation
results show that the proposed IR-SAMP algorithm in single snapshot DOA estimation under equal sinusoidal sparse division is better than the OMP algorithm, SAMP algorithm and is better than MUSIC algorithm, \( l_1 \)-SVD algorithm in multi-snapshot. At the same time, IR-SAMP algorithm has low computational complexity, as for coherent signal source, which also has a better estimation effect.

2. Mathematical model
Consider a uniform linear array (ULA) consisting of identical and omnidirectional elements shown in Figure 1. Assume that field narrowband signals \( \{ s_k(t) \} \), \( k = 1, 2, \ldots, K \), impinge on the array from direction angles \( \theta = [\theta_1, \theta_2, \ldots, \theta_K] \). The complex envelope of the signal can be expressed by the following formula

\[
\begin{align*}
\mathbf{s}(t) &= A(t) e^{j\omega t + j\phi(t)} \\
\mathbf{s}(t-\tau) &= A(t-\tau) e^{j\omega(t-\tau) + j\phi(t-\tau)}
\end{align*}
\]

where \( A(t) \), \( \omega \), \( \phi(t) \) are the amplitude, frequency and phase of the received narrowband signals respectively. For narrowband far field signals, the following formula holds:

\[
\mathbf{s}(t) \approx A(t) e^{j\omega t + j\phi(t)} \quad \mathbf{s}(t-\tau) \approx A(t-\tau) e^{j\omega(t-\tau) + j\phi(t-\tau)}
\]

Then, it can be deduced that \( \phi(t-\tau) \approx \phi(t) \), \( k = 1, 2, \ldots, K \). The received signal, \( y_m(t) \), by the \( m \) th array element is given as follows:

\[
y_m(t) = \sum_{k=1}^{K} e^{-j2\pi f_d \sin \theta_k/c} s_k(t) + n_m(t)
\]

where \( f_d = (m-1) \sin \theta_k / c \) denotes the delay of the \( k \) th signal source to the \( m \) th array element with respect to the reference element, and \( c \) the velocity of propagation. According to Eq.(2), at the moment \( t \), the signals received by \( M \) array elements can be expressed as:

\[
y(t) = \mathbf{y}(t) = A(t) \mathbf{s}(t) + \mathbf{n}(t)
\]

where \( \mathbf{y}(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T \) denotes \( M \)-dimension data vector of a single snapshot received by the array, \( \mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T \) the corresponding signal vector, \( \mathbf{n}(t) \) is a \( M \)-dimension noise vector, \( A = [a_1, \ldots, a_K] \) is the \( M \times K \) array manifold matrix, whose \( i \) th column is the \( i \) th signal array steering vector:

\[
a_i = [1, e^{-j2\pi d\sin \theta_i/c}, \ldots, e^{-j2\pi (M-1)d\sin \theta_i/c}]^T
\]

According to the above analysis, each column of the array manifold matrix corresponds to the spatial position of a signal source. Using (4), it can be seen that array manifold matrix of a uniform linear array has the Vandermonde structure.

\[
\begin{align*}
\mathbf{A} &\triangleq A(t) \\
\mathbf{a}_i &\triangleq [a_1, a_2, \ldots, a_K]
\end{align*}
\]

2.1. Equal angle division of array manifold matrix
The array manifold matrix is divided based on equal angle division, that is, the search angle space \((-90^\circ, 90^\circ)\) is divided into \( Q \) angles, \( \theta_n = -\frac{\pi}{2} + \frac{n-1}{Q-1} \pi, n = 1, 2, \ldots Q \), at this time, array manifold matrix can be expressed as

\[
\mathbf{A}_n = [\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_Q]
\]
Where \( \mathbf{a} \), can represent as follows
\[
\mathbf{a}_i = \left[ 1, e^{-j\pi \sin \theta}, \ldots, e^{-j(M-1)\pi \sin \theta} \right]^T
\]

Here \( i=1,2,\ldots,Q \). Let \( \mathbf{a}_i, \mathbf{a}_j \) denote two column vectors in manifold matrix \( \mathbf{A}_i \). The matrix \( \mathbf{A}_i \) is normalized to get \( \mathbf{A}_i \), \( \mathbf{A}_i = \left[ \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_Q \right] \). When the redundant dictionary is established by equal angle division, the coefficient of any two column vectors in redundant dictionary is
\[
\mu(\mathbf{A}_i) = \mu(\mathbf{a}_i, \mathbf{a}_j) = \frac{\mathbf{a}_i^H \mathbf{a}_j}{\left| \mathbf{a}_i^H \mathbf{a}_j \right|}
\]

Where \( M \) represents the number of array elements, \( Q = \sin \theta - \sin \theta_0 \). It can be seen from the above equation that the value of \( \mu(\mathbf{A}_i) \) is mainly determined by \( M \) and \( Q \). As shown in Figure 2(a), the correlation value between any two column vectors in array manifold matrix divided by equal angle division is simulated, and in the simulation, \( Q = 181 \). The X and Y axes in the simulation diagram represent the first column to \( Q \)th column of the array manifold matrix, the Z axis denotes correlation value.

### 2.2 Equal sinusoidal division of array manifold matrix

The array manifold matrix is divided based on equal sinusoidal division. Each column of the array manifold matrix established by the equal sinusoidal division can be expressed as
\[
\sin \theta_n = -1 + \frac{2(n-1)}{Q-1}, \quad n = 1,2,\ldots,Q
\]
So the array manifold matrix can be represented as \( \mathbf{A}_i = \left[ \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_Q \right] \), \( \mathbf{a}_i \) can represent as follows
\[
\mathbf{a}_i = \left[ 1, e^{-j\frac{2\pi}{Q} (n-1)}, \ldots, e^{-j\frac{2\pi}{Q} (M-1)} \right]^T
\]

where \( i=1,2,\ldots,Q \). Let \( \mathbf{a}_i, \mathbf{a}_j \) denote two column vectors in manifold matrix \( \mathbf{A}_i \). First, the matrix \( \mathbf{A}_i \) is normalized to get \( \mathbf{A}_i \), \( \mathbf{A}_i = \left[ \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_Q \right] \). When the redundant dictionary is established by equal sinusoidal division, the coefficient of any two column vectors in redundant dictionary is
\[
\mu(\mathbf{A}_i) = \frac{\mathbf{a}_i^H \mathbf{a}_j}{\mathbf{a}_i^H \mathbf{a}_j} = \frac{\sin(M \pi Q (i,j))}{\|M \sin(\pi Q (i,j))\|}
\]

![Figure 2](image-url)
vectors in array manifold matrix divided by equal sinusoidal division is simulated, and in the simulation, \( Q = 181 \).

We can see that the non-correlation performance of the array manifold matrix divided by equal sinusoidal is better than the array manifold matrix partitioned at equal angle. Therefore, this paper then divides the array manifold matrix with equal sinusoidal values to carry out compressed sensing DOA estimation.

3. DOA Estimation Based On IR-SAMP Algorithm

The IR-SAMP algorithm is based on SAMP algorithm. The core idea of IR-SAMP is to select atoms based on fixed step size, and use regularization to achieve retrospective screening, which can flexibly remove inappropriate atoms in the backtracking stage. Regularization is a method of classification based on element energy level. Suppose a set \( \Omega \) is composed of several elements \( x_i \), and the subscripts of \( x_i \) correspond to the index set \( I \), that is, \( A = \{x_i | i = 1, 2, \ldots, m \} \), all the elements are grouped by according to the rules \( |x_m| \leq |x_i|, m, n \in I \). The corresponding index set is divided into several groups of subsets \( I_j \).

Finally, select the largest subset of energy subset, this subset is marked as \( I_{\text{max}} \), that is

\[
\left\| v_{\text{max}} \right\| = \max \{\left\| v_j \right\|, j = 1, 2, \ldots, J\}
\]  \hspace{1cm} (10)

Where \( J \) is the number of the total subsets. Through the above regularization process, we can achieve the adaptive selection of the appropriate atoms as alternative atoms.

DOA estimation processing based on the array manifold matrix equal sinusoidal division under single snapshot of IR-SAMP algorithm is summarized as follows:

Input: \( M \times Q \) dimension array manifold matrix \( \mathbf{A} \), \( M \times 1 \) dimension received array data vector \( y \), a fixed increase unit step for step size. Initialization: initial residual \( r_0 = y \), support set \( \Lambda_0 = \emptyset \), step size \( \Gamma = \text{step} \), the index of iteration \( t = 1 \), backtracking stage index \( \text{stage} = 1 \).

1) Calculate the correlation coefficient \( U = \text{abs} \left( \mathbf{A}^H r_{\text{max}} \right) = \text{abs} \{ r_{\text{max}, i} \} , 1 \leq j \leq Q \), and select the \( \Gamma \) largest values from \( U \), the value of the column number corresponding to the matrix \( \mathbf{A} \) forms the candidate set \( J \).

2) Regularize the correlation coefficient of the atoms corresponding to the index value in \( J \) and store the regularized result in the set \( \tilde{\Lambda}_J \).

3) Get candidate set \( F \) by \( F = \Lambda_{J, \text{max}} \cup J \), and \( \tilde{\Lambda}_J = \{a_j | j \in F \} \), \( a_j \) represents \( j \) th column of the matrix \( \mathbf{A} \).

4) Find the least squares solution of the formula \( y = \mathbf{A}_J \hat{s} \). \( \hat{s} = \arg \min \left\{ \| \mathbf{y} - \mathbf{A}_{J} \hat{s} \|_2^2 \right\} = \left( \mathbf{A}_{J}^{H} \mathbf{A}_{J} \right)^{-1} \mathbf{A}_{J}^{H} \mathbf{y} \)

5) Update the residual \( r_{\text{new}} = y - \mathbf{A}_J \left( \mathbf{A}_J^{H} \mathbf{A}_J \right)^{-1} \mathbf{A}_J^{H} y \)

6) If \( \| r_{\text{new}} \| \leq \varepsilon \) (in the simulation experiment, \( \varepsilon = 10^{-9} \)), let \( \Lambda_t = F \) and stop the iteration and go to step 7).

7) If \( \| r_{\text{new}} \| \geq \| r_0 \| \), let \( \text{stage} = \text{stage} + 1 \), and update step size by \( \Gamma = \text{stage} \times \text{step} \) and let \( t = 1 \), and return to step 1) to continue iteration; otherwise let \( \Lambda_t = F \), \( r_t = r_{\text{new}} \), \( t = t + 1 \), if \( t = M \), stop the iteration and go to step 7), otherwise go to step 1).

The reconstructed \( \hat{s} \) at \( \Lambda_{tM} \) is nonzero, the value of which is the result of the last iteration.

Output: \( Q \times 1 \) dimension sparse signal \( \hat{s} \), \( M \times 1 \) dimension residual vector \( r_T \) by \( r_T \).

SAMP algorithm and IR-SAMP algorithm with single snapshot ( ), the computation complexity is much smaller than the traditional subspace algorithm and convex optimization algorithm. However, for the large number of snapshots and the spatial refinement of the grid, the computation complexity of the
The $l_1$-SVD algorithm has surged. For the OMP, SAMP and IR-SAMP algorithms, the computation complexity will increase exponentially, but it is still smaller than that of the traditional subspace algorithm. Therefore, the proposed IR-SAMP algorithm in this paper has certain advantages in terms of computation complexity. The computation complexity of common DOA estimation algorithm and IR-SAMP algorithm proposed in this paper is shown in Table 1.

### Table 1. DOA estimation complexity of several algorithms

| Estimation algorithm | Computation complexity |
|----------------------|------------------------|
| MUSIC                | $O(M^2 L + M^3 + (M^2 + M)Q)$ |
| ESPRIT               | $O((2 \log_2 M + 4/3)M^3)$ |
| L1-SVD               | $O(M^2 L + M^3 + M^3 Q)$ |
| OMP                  | $O((KQ + K(K + 1)/2)ML)$ |
| SAMP                 | $O(MQL)$               |
| IR-SAMP              | $O(MQL)$               |

4. **Experimental Simulation**

#### 4.1. DOA estimation of incoherent signal source

Assuming that there are two far field incoherent signal sources with angles of incidence of 20° and 50° respectively, the number of array elements in a uniform linear array $M = 12$ and the search angle space ($-90°, 90°$) is divided into 181 parts with equal sine value, SNR=5dB. To analyze the proposed DOA estimation performance of the IR-SAMP algorithm under equal sinusoidal division, this paper compares the DOA estimation algorithms of OMP and SAMP, and true DOA value of the signal source, as shown in Figure 3.

It can be seen that the DOA estimation effect of the OMP algorithm is slightly lower than that of the above two algorithms, and the DOA estimation effect of the IR-SAMP algorithm is superior to the SAMP algorithm. The root mean square error (RMSE) for DOA estimation is

$$RMSE = \frac{1}{K} \sum_{k=1}^{K} \left[ \frac{1}{N} \sum_{n=1}^{N} \left( \hat{\theta}_n - \theta_k \right)^2 \right]$$

where $N$ denotes the number of Monte Carlo experiments, $K$ is the number of signals, namely the sparsity of signal $s$, $\hat{\theta}_n$ denotes the DOA estimation value of the $k$th signal source in the $n$th Monte Carlo experiment, $\theta_k$ represents real DOA value of the $k$th signal source.

As we can see from the results of Figure 4, the RMSEs of the DOA estimation of all three algorithms decrease gradually as the SNR increases. The DOA estimation accuracy of the IR-SAMP algorithm is significantly higher than that of the SAMP and OMP algorithms when the SNR is -10~2 dB. When the SNR is -2~10 dB, the accuracy of DOA estimation of the three algorithms is relatively close.

In order to further analyze the DOA estimation accuracy of the proposed IR-SAMP algorithm under equal sinusoidal division. This paper then simulates the RMSE-SNR curve of the OMP, SAMP, IR-SAMP algorithms under single snapshot and $l_1$-SVD, MUSIC algorithms for multi-snapshot ($l_1$-SVD: snapshot=5; MUSIC: snapshot=15).
As we can see from Figure 5, the MUSIC algorithm has a slightly better estimation accuracy when the SNR is 5~7dB. When SNR is 7~13 dB, the DOA estimation accuracy of the IR-SAMP algorithm is significantly higher than that of the other four algorithms. When SNR is greater than 15 dB, the DOA estimation accuracy of the five

4.2. DOA estimation of coherent signal source

In order to verify the performance of proposed IR-SAMP algorithm of DOA estimation for coherent signal sources, two coherent far field signal sources are considered, whose incident angles are 20° and 50°, respectively, and SNR is 5 dB. The estimation results are shown in Figure 6, the IR-SAMP algorithm proposed in this paper is still valid when the coherent signal source exists in space, and the estimation result is close to the real source angle. And the estimation effect of OMP and SAMP algorithms is similar to IR-SAMP, but the MUSIC algorithm has failed, which causes more pseudo peaks, and the effect of MUSIC algorithm has plummeted.

Obviously, in Figure 7, for the coherent signal source, the MUSIC algorithm has failed and the RMSE value is the largest. For the proposed IR-SAMP algorithm, the DOA estimation of the coherent signal source is still valid and the estimation error is smaller than the other algorithms.

5. Conclusion

The SAMP greedy algorithm is proposed to replace the \( l_1 \) norm to approximate the \( l_2 \) norm optimization problem of DOA estimation satisfies the MIP criterion better. At the same time, based on the SAMP algorithm, this paper further proposes the IR-SAMP algorithm. Simulation results show that the
The proposed IR-SAMP algorithm in single snapshot DOA estimation under equal sinusoidal sparse division is better. At the same time, IR-SAMP algorithm has low computational complexity, as for coherent signal source, which also has a better estimation effect. The DOA estimation of array signals is one of the main research fields in array signal processing, so it’s significant to realize DOA estimation of real-time, high-precision and low-cost signal source.

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