Real-time pixel-level polarization modulation using polarized-spatial light modulator based on phase vectorization

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Abstract:
Vector polarization induced by the change of scalar phase has been far beyond our understanding about the relationship between polarization and phase in classical optics due to the entanglement of inherent polarized modes of light beam. To overcome this limitation, we establish this miraculous relationship by the principle of phase vectorization that can transform three particular phases into linear, circular and elliptical polarization, respectively. Polarized-spatial light modulator based on the principle of phase vectorization can therefore be realized using a phase-only spatial light modulator, which not only enables pixel-level polarization manipulation of light beam in a real-time dynamic way, but also retains complete phase control simultaneously. This work demonstrates the fundamental phase-to-polarization link and declares the creation of polarized-spatial light modulator, which will facilitate extensive developments in optics and pave ways for the era of vector optics.
Introduction

Amplitude, phase and polarization characterize a light beam in classical optics. The wavefront information of light beam are carried by amplitude and phase while polarization refers to the inherent oscillation of electric field. Throughout the development of optics, every breakthrough regarding the interaction between these three natures not only deepens our knowledge, but also opens new era of optics, thereby leading to the flourish of optics. The renowned Malus law demonstrates that polarization modulation of light beam can turn into amplitude adjustment with the aid of a polarizer, thereby establishing the polarization-to-amplitude link. Academically speaking, the polarization-to-amplitude link has deepened our understanding on the behavior of light beam.

The mutual relationship between amplitude and phase is the cornerstone of scalar optics, which can be established by wavefront modulation techniques [1-4]. In past centuries, the world has witnessed great successes of scalar optics. The arise of optical research areas, including optical communications [5,6], optical storage [7-9], optical display [10,11], optical lithography [12], optical imaging [13-15], have huge impacts on our life. However, as a stage in the development of optics, the limitations of scalar optics are gradually being realized. For example, the capacity of free-space communication cannot be enhanced by the technique of orbital angular momentum multiplexing anymore until the vector polarized modes are employed [16, 17]. By focusing cylindrical vector beam, diffraction limit can be broken through, which is considered to be impossible in the framework of scalar optics [18,19]. Lithography polarization optimization becomes the core technique to enhance the resolution of optical lithography [20,21]. With no doubt, the future will forge ahead into the era of vector optics from the time of scalar optics and nowadays is only its dawn. The key to open this era requires a polarized-spatial light modulator (polarized-SLM) that can modify the polarization state of light beam with resolution down to a single pixel in a real-time dynamic way.

The polarization-to-phase link demonstrated by Pancharatnam-Berry (PB) phase in 1987 brings the hope of addressing successfully this issue [22]. Polarization convertor based on PB phase can convert a right circularly polarized beam into a left circularly one along with an additional phase and vice versa. Therefore, a spatial-variant linearly polarized beam can be obtained by passing through a linearly polarized beam. Because of the potential of phase and polarization manipulation, the polarization-to-phase link has received special attentions and inspired the researches of metasurface [23-25], Q-plate [26-28] and so on. However, due to the fixed structure of polarization convertors, dynamic polarization manipulation is considered to be impossible so far. Worse still, the principle of PB phase, in essence,
hinders the coexistence of circular and linear polarization within one whole light beam simultaneously, thereby making full polarization modulation impossible. Both fatal flaws imply that the polarization-to-phase link cannot meet the requirements of polarized-SLM.

According to the above three pairs of relationship in classical optics, one can find that only vector polarization can link to scalar phase and amplitude, but not vice versa. However, if the inverse correlation of polarization-to-phase link can be established, the scalar phase of light beam can entirely be vectorized into its vector polarization. The adjustment of polarization is no longer dependent on the structure of polarized convertor, but only relies on the phase of light beam. In this case, pixelate devices, such as phase-only SLM, can be utilized to form a polarized-SLM, which enables pixel-level polarization modulation of light beam in a real-time dynamic way. Undoubtedly, establishing the phase-to-polarization link is an effective way to tackle both fatal flaws of polarization-to-phase link, but still far beyond our understanding on the vector polarization and scalar phase of light beam.

Here, we establish the fourth fundamental relationship in classical optics, namely the phase-to-polarization link, by the principle of phase vectorization. As the inverse process of PB phase, phase vectorization is realized by extracting the inherent desired polarized mode from light beam with different polarization responses in a filter system. Based on phase vectorization, the polarization state of light beam is merely determined by the scalar phase. Thus, polarized-SLM based on phase vectorization can not only access a fully polarization modulation in a real-time dynamic way, but also retains a completed phase control of light beam, which is qualified as the key for vector optics.
Results

Phase vectorization using vortex vector beam

Phase vectorization represents the phase-to-polarization link indicated by arrow D in Supplementary Figure 1, which is capable to vectorize the scalar phase of light beam into its vector polarization. As demonstrated in supplementary Note 1, there are two critical conditions for the realization of phase vectorization. One is that the light beam possesses inherent different polarization responses of left and right circularly polarized mode; another is that the undesired circularly polarized mode must be eliminated without affecting the desired one. Normally, both left and right circularly polarized mode in Supplementary Equation 2, 3 are tangled with each other during propagation in free space. For this reason, extracting one of polarized modes from light beam is always thought to be a hopeless task.

Although one cannot achieve a direct separation of both circularly polarized modes within a light beam, an indirective way can be realized by focusing a suitable vector light beam. Specifically, some vector light beams can divide into a left and a right circularly polarized mode with different position in the focal region after focusing by an objective lens. Using a filter system, one of polarized modes can easily be obtained by eliminating another one, thereby permitting the realization of phase vectorization. Here, we take vortex vector beam (VVB) as example to vectorize the scalar phase into vector polarization, which can be considered as a special case of the generalized phase vectorization depicted in Supplementary Note 1. In this case, \( f(\varphi, \theta) = m\varphi \) in Supplementary Equation 1 and the light beam turns into a \( m \) order VVB, which can be written as [16]

\[
E_v = \exp(i m \varphi)\langle R \rangle + \exp(-i m \varphi)\langle L \rangle
\]

where \( \langle L \rangle = [1 \ i] \) and \( \langle R \rangle = [1 \ -i] \) denote left and right circularly polarized mode, respectively. \( \varphi \) is the azimuthal angle. When modulating by the phase \( \phi = \pm (m\varphi - \beta) \), \( m \) order VVB in Eq. (1) is transformed into

\[
E_{vl} = \exp[i (2m\varphi - \beta)]\langle R \rangle + \exp(-i \beta)\langle L \rangle \tag{2}
\]

\[
E_{vr} = \exp[-i (2m\varphi - \beta)]\langle L \rangle + \exp(i \beta)\langle R \rangle \tag{3}
\]

Here, we call the first and second term in Eq. (2, 3) as undesired and desired polarized mode, respectively. After focusing by an objective lens, the undesired polarized modes \( \exp[i (2m\varphi - \beta)]\langle R \rangle \) and \( \exp[-i (2m\varphi - \beta)]\langle L \rangle \) are located at the periphery of focal plane while the desired ones
exp(−iβ)⟨L⟩ and exp(iβ)⟨R⟩ are at the center [See Supplementary Figure 3]. The bigger the order $m$, the larger the distance between the undesired and desired polarized mode. Once the distance enlarges sufficiently, the undesired polarized modes can be eliminated using a pinhole and only the desired ones are retained. Thus, the polarized modes after the pinhole can be simplified as

$$E_{pl} = \exp(−iβ)⟨L⟩$$

$$E_{pr} = \exp(iβ)⟨R⟩$$

Equation 4, 5 demonstrate that the polarized modes $E_{pl}$ and $E_{pr}$ are merely determined by the phase $\phi = ±(m\varphi − \beta)$. After reconstructed by an objective lens, both reconstruction polarizations of $E_{pl}$ and $E_{pr}$ link with the phase of VVB directly [See Supplementary Note 2: Part 2]. Without loss of generality, one can simply extent this link into a generalized form, namely phase vectorization. That is, the phase in Eq. (6) can be vectorized into vector polarization in Eq. (7).

$$\phi = \text{Phase}[\cos \varphi_0 \exp[i(m\varphi − \beta)] + \sin \varphi_0 \exp[−i(m\varphi − \beta)]]$$

$$E = \cos \varphi_0 \exp(−i\beta)⟨L⟩ + \sin \varphi_0 \exp(\beta)⟨R⟩$$

In Equation 6, the parameter $\varphi_0$ is the weight factor that adjusts the proportion of $⟨L⟩$ and $⟨R⟩$. As shown in Fig. 1, the phases $\phi_{1,2} = ±(m\varphi − \beta)$ can be vectorized into left and right circular polarization, respectively. As for linear polarization, there is a one-to-one correspondence between the parameter $\beta$ of $\phi_3 = \text{Phase}[\cos(m\varphi − \beta)]$ and polarization direction. Thus, a radially polarized beam is easily realized by the phase $\phi_3$ with $\beta = \varphi$. Note that the phases $\phi_{1,2} = ±(m\varphi − \beta)$ are obtained with $\varphi_0 = 0.5\pi$ in Eq. (6), while $\phi_3 = \text{Phase}[\cos(m\varphi − \beta)]$ is the result of Eq. (6) with $\varphi_0 = 0.25\pi$. For other $\varphi_0$, the phase in Eq. (6) can be vectorized into elliptical polarization accordingly.

**Polarized-SLM based on phase vectorization**

In the following experiment, we establish a polarized-SLM based on phase vectorization using $m=30$ order VVB. Figure 2 presents the schematic of polarized-SLM, the simplification of which is shown in Supplementary Figure 2. As shown in Fig.2, polarized-SLM is composed of a phase control system (green dot box) and a filter system (blue dot box). In the former system, a collimated incident linearly polarized beam with wavelength 633nm propagating along the optical axis passes through a phase-only SLM and two lenses (L3, L4) before it is converted into a $m=30$ order VVB by vortex polarizer (VP). VP can be easily manufactured using the technique of Q-plate [27]. Figure 2(a, b) are the light intensities of
$m=30$ order VVB without and with a polarizer indicated by the purple arrow. $L_3$ ($f_3=150$ mm) and $L_4$ ($f_4=150$ mm) compose a 4$f$-system that makes the phase coded in the phase-only SLM and VP conjugate. Thus, the modulated VVB in Eq. (2, 3) can easily be obtained by coding the phase $\phi = \pm (m \phi - \beta)$ in the phase-only SLM, where $m=30$.

In the latter system, the modulated VVB in Eq. (2, 3) is divided into left and right circularly polarized modes with different position in the focal region after focusing by the objective lens $OL_1$. Specifically, the undesired polarized modes $\exp\left(i(2m\phi - \beta)\right)$ and $\exp\left[-i(2m\phi - \beta)\right]$ locate at the outer ring with the topologic charge of $\pm 60$ while the desired polarized modes $\exp\left(-i\beta\right)$ and $\exp\left(i\beta\right)$ are in the position of geometric focus of $OL_1$ [See Supplementary Note 2]. The distance between both pairs of polarized modes is 671$\mu$m. After passing through a pinhole with radius 400$\mu$m, the undesired polarized modes are eliminated and only the desired ones can further be recovered by the subsequent objective lens $OL_2$. Since the desired polarized modes in the focal region of $OL_1$ only depend on the phase of VVB, the polarization from $OL_2$ possesses a direct link with the phase of VVB as well. That is, the vector polarization can entirely be manipulated at will by merely the phase of VVB. Here, both numerical apertures (NA) of $OL_1$, $OL_2$ are 0.01, and the red arrow represents the propagation direction of light beam.

Figure 3 presents the experiment result of phase vectorization shown in Fig. 1, the theoretical result of which is shown in Supplementary Figure 4. The phases $\phi = \pm (m \phi - \beta)$ with $m=30$ and $\beta = 0$ in Fig. 3(a, c) are vectorized into left and right circular polarization, respectively, both of which can be distinguished from each other using a quarter-wave plate indicated by blue arrow and a polarizer indicated by purple arrow, as shown in Fig. 3(b, c, d, f, g, h). In term of linear polarization, light beam with on-demand polarization direction can be realized by the phase $\phi = \text{Phase}[\cos(m \phi - \beta)]$. After passing through a horizontal polarizer (purple arrow), different polarization direction leads to different light intensities, which is determined by the parameter $\beta$. Here, $\beta=0, 0.25\pi, 0.5\pi, 0.75\pi$ in Fig. 3(m, n, o, p), respectively. The varieties of light intensities in Fig. 3(m, n, o, p) imply that the polarization direction has a one-to-one correspondence with the parameter $\beta$, as shown in Fig. 1. Thus, radial polarization is realized with $\beta=\phi$, see Supplementary Figure 7 (a-c). Although it is beyond imagination, the inverse process of PB phase, namely phase vectorization, is realized using high order VVB. Note that the hollow shape of vector beam from $OL_2$ is attributed to the slight deviation between the
conjugated planes of phase-only SLM and VP, which does not affect the validity of polarized-SLM.

**Pixel-level modulation using polarized-SLM**

Phase vectorization establishes the phase-to-polarization link that enables the manipulation of polarization through the phase of VVB directly. Generally, the phase of VVB can be pixelized by the phase-only SLM in Fig. 2. Pixelate phase gives rise to pixelate polarization, thereby permitting the individual adjustment of polarization in each pixel, as shown in Fig. 4.

To verify the pixel-level polarization modulation of polarized-SLM, we present two experimental results of polarization manipulation in Fig. 5, 6. Both corresponding theoretical results can be found in Supplementary Figure 5, 6, respectively. Firstly, we take an arbitrary pixel-like zone as example to manipulate the polarization locally. Figure 5 presents three vector light beams, which are all composed of a pixel-like zone and background. As shown in Fig. 5 (b), light beam with y linear polarization in the pixel-like zone and x linear polarization in the background can be obtained by the phase in Fig. 5(a). The polarization difference between the pixel-like zone and background is distinguished using the polarizer (purple arrow) in Fig. 5 (c, d), respectively. Likewise, the phase in Fig. 5(e) can be vectorized into circularly polarized light beam in Fig. 5(f), where the pixel-like zone and background are left and right circular polarization, respectively. As shown in Fig. 5(g, h), one can extract one from another using a quarter-wave plate (blue arrow) along with a polarizer (purple arrow). Light beam in Fig. 5 (j) can be considered as the combination of circular (pixel-like zone) and linear polarization (background), which is created by the phase in Fig. 5(i). By passing through the quarter-wave plate (blue arrow) and polarizer (purple arrow), one can find that the pixel-like zone is right circular polarization while the background is x linear polarization, see Fig. 5 (k, l). The coexistence of circular and linear polarization within one whole light beam demonstrates the full polarization modulation of polarized-SLM, which is one of superior advantages of phase vectorization over the polarization-to-phase link.

Figure 5 not only verifies the full polarization modulation of polarized-SLM, but also implies the essence of phase vectorization. According to Eq. (6, 7), linear polarization can be obtained by the phase \( \phi = \text{Phase}[\cos(m\phi - \beta)] \), where the parameter \( \beta \) controls the polarization direction. As shown in Fig. 5(m), all phases for the creation of linear polarization have one identical binary phase structure, which is determined by the order \( m \) of VVB. However, different \( \beta \) induces a relative phase displacement in Fig. 5(m), which further causes a polarization change locally in Fig. 5(b, c, d). Similarly, left and right circular polarization are corresponding to the vortex phases \( \phi = \pm(m\phi - \beta) \), respectively. Both vortex phases also have the same phase structure except the inverse topological charge of \( \pm m \). In this case, the
modification of left and right circular polarization is no longer dependent on the parameter $\beta$, but the sign of topological charge $\pm m$. The relative displacement of phase structure caused by $\beta$ can only imposes an additional phase on circular polarization. Thus, the essence of phase vectorization is to transfer the polarization change into three particular phases: binary phase $\phi = \text{Phase}[\cos(m\phi - \beta)]$ for linear polarization, vortex phase $\phi = \pm (m\phi - \beta)$ for circular polarization, and the combination of both for elliptical polarization. Note that elliptical polarization is the combination of circular and linear polarization.

Since the above three particular phases can be pixelated by the phase-only SLM in Fig. 2, the pixel-like zone in Fig. 5 can shrink to one single pixel. If only the phase in the pixel is corresponding to the polarization state of VVB, one can always obtain a desired polarization in a real-time dynamic way. Figure 6 presents the experimental result of pixel-level polarization modulation using polarized-SLM. As shown in Fig. 6(a, d, g), the phases of VVB possess four pixel-like zones, namely A, B, C, D zone. Their corresponding polarization states are shown in Fig. 6(j, k, l, m), which can be expressed as

$$P_n = \begin{bmatrix} \cos[n\phi + (n-1)\pi/4] \\ \sin[n\phi + (n-1)\pi/4] \end{bmatrix}$$

where $n=1,2,3,4$ stand for A, B, C, D zone, respectively. The background of light beams in Fig. 6(b, e, h) are linear polarization, which are adjusted to be orthogonal with the polarizer by the parameter $\beta$ of the phase $\phi = \text{Phase}[\cos(m\phi - \beta)]$. Here, the polarization directions of background light beams (yellow arrow) in Fig. 6(c, f, i) are $0.5\pi$, $0.75\pi$, $0$, respectively. Their corresponding polarizers (purple arrow) are $0$, $0.25\pi$, $0.5\pi$, respectively. Owing to the orthogonality between the background light beams and the polarizers, the background light beams are always eliminated by the polarizers, thereby providing a clear display of petal-like patterns of A, B, C, D zone. As shown in Fig. 6(c, f, i), the petals with the number of $2n$ are rotated along with the polarizer in A, B, C, D zone, which are coincident with the theoretical result of A, B, C, D zones in Supplementary Figure 6. Besides, we also generate other vector beams in Supplementary Note 3 by patterning the phase in polarized-SLM.

**Discussion**

**Technical discussion of polarized-SLM**

As the core of polarized-SLM in Fig. 2, phase vectorization is capable to vectorize the scalar phase into vector polarization. In this way, pixel-level polarization modulation can be realized in a real-time dynamic way. Supposed there is an additional phase $\psi$ superposed with the phase $\phi$ in Eq. (6). The
entire phase of VVB can therefore be expressed as $\Omega = \phi + \psi$. According to the essence of phase vectorization, the first term $\phi$ stands for three particular phases, namely binary phase, vortex phases, and the combination of both, which are corresponding to linear, circular, and elliptical polarization, respectively. The second term $\psi$ represents the pure phase modulation of light beam. Once the polarization state is determined by the first term $\phi$, the light beam is modulated by the phase $\psi$. Therefore, polarized-SLM can not only access full polarization manipulation, but also retains complete phase adjustment of light beam.

Although polarized-SLM enables both polarization and phase modulation simultaneously, the active area is determined by the pinhole. On the premise of eliminating the undesired polarized mode, the pinhole must be sufficiently large so that there is enough space for the phase and polarization modulation of desired polarized mode. There are two methods to increase the size of pinhole [See Supplement Note 2]. One is to enlarge the order $m$ of VVB. For example, if $m=30$, $\sigma = 10$ in Supplementary Equations 14, the inner desired and outer undesired polarized modes are 136.22μm and 563.24μm away from the geometric focus of OL1. In this case, the size of pinhole is 400μm. If $m=60$, $\sigma = 10$, the inner desired and outer undesired polarized modes are 136.22μm and 1186.88μm away from the geometric focus of OL1, and the size of pinhole can be adjusted to 800μm. Larger $m$ leads to bigger size of pinhole. Benefitting from the mature technique of Q-plate [7, 8], the order $m$ of VVB can reach as high as 126. In this case, the inner and outer polarized modes are 136.22μm and 2537.06μm away from the geometric focus of OL1. Thus, the pinhole can be set to an inconceivable size with 2000μm. Another method is based on the conversion of polarized mode with the aid of half-waveplate (HWP). In this case, the topological charge difference between the undesired and desired polarized mode remains the same, namely $\Delta l = |2m|$. Since the pinhole is placed between the inner desired and outer undesired polarized mode, the size of pinhole must be enlarged as the topological charge of both desired and undesired polarized mode increase, thereby providing a large active area of desired polarized mode. Combining both methods, sufficient room can always be obtained for the polarization and phase modulation of polarized-SLM.

Energy efficiency is another factor affected by the pinhole. When modulating by the phase in Eq. (6), $m$ order VVB is divided into two parts, namely the desired and undesired polarized mode. Both polarized modes are of equal energy. In the process of phase vectorization, the undesired polarized mode is always eliminated by the pinhole while the desired one is retained. Thus, the overall energy efficiency
of polarized-SLM is 50%, which is almost ten times as high as that of the methods based on interference principle (normally at the level of a few percent) [30,31].

**Theoretical impact of phase vectorization**

In the following, we would like to discuss the theoretical impact of phase vectorization in classical optics. Generally, polarization manipulation always requires two orthogonally polarized beams with controllable phases. For example, one can achieve full polarization adjustment of light beam by using a metasurface with effective birefringence, where the phases of two electric components $E_x$ and $E_y$ can be adjusted at will [23]. Using interferometric configuration [29-31], light beam with arbitrary polarization can be realized by superposing left and right circular polarized beams with different phases. Unlike this conventional principle, phase vectorization is not the result of superposing two orthogonally polarized light beams, but a direct control on the electric field of light beam by vectorizing the scalar phase into its vector polarization. Thus, polarized-SLM does not require the complicate algorithm, high interferometrically precise alignment, expensive and difficult fabrication process. From this point of view, the principle of phase vectorization provides entirely new knowledge about polarization manipulation.

Besides, phase vectorization unifies amplitude, phase and polarization as a whole. There are heretofore three relationships between amplitude, phase and polarization in classical optics, namely the mutual link between phase and amplitude, the polarization-to-amplitude link and the polarization-to-phase link. These three links imply that only the vector polarization can link to scalar phase and amplitude, but not the opposite. That is, these three properties of light beam are relatively independent. Phase vectorization establishes the unimaginable phase-to-polarization link by transforming all polarization states into three particular phases. Combining the above three relationships with the principle of phase vectorization, one can predict that the phase, amplitude and polarization of light beam can be adjusted simultaneously by merely the phase. For this reason, the phase-to-polarization link can be considered as the last piece of unifying amplitude, phase and polarization.

In conclusion, we have theoretically and experimentally demonstrated the phase-to-polarization link by the principle of phase vectorization. Phase vectorization realized by the phase modulation of $m=30$ order VVB in a filter system can vectorize three particular phases, namely binary phase, vortex phase, and the combination of both, into linear, circular and elliptical polarization, respectively. Using a phase-only SLM, polarized-SLM is established based on phase vectorization, which enables both pixel-level polarization and phase manipulation of light beam in a real-time dynamic way. In theory, phase
vectorization not only renews our understanding on the relationship between scalar phase and vector polarization, but also makes it possible to manipulate the amplitude, phase and polarization by merely the phase, which opens a new avenue for the development of full-SLM. Besides, as a key for the era of vector optics, real-time pixel-level polarization modulation using polarized-SLM offers promising applications in many scientific studies, such as optical communication [17], optical lithography [20, 21].

**Data Availability**

All data supporting the findings of this study are available from the corresponding author on request.
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Author contributions

X. Weng conceived of the research and designed the experiments. G. Sui, X. Dong and X. Weng performed the experiments. L. Liu, Q. Song and X. Weng analyzed all of the data. X. Weng and X. Gao co-wrote the paper, and J. Qu offered advice regarding its development. X. Gao, J. Qu and S. Zhuang directed the entire project. All authors discussed the results and contributed to the manuscript.

Competing interests statement

A provisional patent application (PCT/CN2020/082764) has been filed on the subject of this work.
**Figures**

**Figure 1** Schematic of phase vectorization. The phase $\phi_{1,2} = \pm (m\varphi - \beta)$ can be vectorized into left and right circular polarization, respectively, while linear polarization can be obtained by the phase $\phi = \text{Phase} \{\cos (m\varphi - \beta)\}$, $\beta$ indicates the polarization direction. For example, a radially polarized beam can be realized by the phase $\phi$ with $\beta = \varphi$.

**Figure 2** Polarized-SLM based on phase vectorization. Phase control system (green dot box) and filter system (blue dot box) compose an entire polarized-SLM. In the former system, a collimated incident $x$ linearly polarized beam with wavelength 633nm propagating along the optical axis passes through a phase-only SLM and two lenses ($L_3$, $L_4$) before it is converted into a $m=30$ order VVB by vortex polarizer (VP). VP can be manufactured using the technique of Q-plate. $L_3$ ($f_3=150$ mm) and $L_4$ ($f_4=150$ mm) compose a 4f-system that makes the phase coded in the phase-only SLM and VP conjugate. (a, b) are the light intensities of $m=30$ order VVB without and with a polarizer (purple arrow), respectively. The modulated VVB output from the first system is divided into a desired and an undesired polarized mode. After passing through the filter system, the undesired one is eliminated by the pinhole (PH) in the focal region of objective lens OL$_1$, and only the desired one is retained. Being reconstructing by the objective lens OL$_2$, the desired polarized mode transforms into the desired polarization, which is recorded using a CCD.
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**Figure 3** Experiment result of phase vectorization in Fig. 1. The vortex phases $\phi = \pm (m\phi - \beta)$ with $m = 30$ (a, e) are vectorized into left and right circular polarization, respectively. Both circular polarizations are analyzed by observing the light intensities (b, c, d, f, g, h) passing through a quarter waveplate with the fast axis indicated by blue arrow and a polarizer indicated by purple arrow. Linear polarization is realized by the phase $\phi = \text{Phase}[\cos(m\phi - \beta)]$, where $\beta = 0, 0.25\pi, 0.5\pi, 0.75\pi$ in (m, n, o, p), respectively. After passing through the polarizer (purple arrow), linear polarization with different direction exhibits different amplitude of light intensity, see (i, j, k, l). All light intensities of light beams are normalized to a unit value, which are indicated by the color bar (q). The color bar (r) shows the phase scale of (a, e, m-p).

**Figure 4** Schematic of pixelate polarization modulation realized by pixelate phase. a, b, c, d denote four phases of light beam. Their corresponding polarizations are shown in A, B, C, D, respectively, which are indicated by the black arrow.
Figure 5  Polarization modulation in arbitrary pixel-like zone of light beam. The light beam (b, f, j) are composed of a pixel-like zone and background, which are created by vectorizing the phase in (a, e, i), respectively. By observing the light intensities (c, d, g, h, k, l) passing through the polarizer (purple arrow) and the quarter waveplate (blue arrow), the polarization states of background and pixel-like zone are (b) x and y linear polarization; (f) right and left circular polarization; (j) x linear and right circular polarization, respectively. All light intensities of light beams are normalized to a unit value, which are indicated by the color bar (p). The color bar (q) shows the phase scale of (a, e, i).

Figure 6  Pixel-level polarization modulation using polarized-SLM. The light beams (b, e, h) created by the phases (a, d, g) are composed of background and four pixel-like zones, namely A, B, C, D zone. The polarizations of A, B, C, D zone are shown in (j, k, l, m), respectively, which can be expressed as Eq. (6). The polarizations of background (yellow arrow) are adjusted to be orthogonal with the polarizers (purple arrow). When passing through the polarizers, the background light beams are always eliminated, thereby providing a clear display of petal-like patterns in A, B, C, D zone (c, f, i). All light intensities of light beams are normalized to a unit value, which are indicated by the color bar (o). The color bar (p) shows the phase scale of (a, d, g).
Supplementary Information

Real-time pixel-level polarization modulation using polarized-spatial light modulator based on phase vectorization

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Supplementary Figure 1 The relationship between amplitude, phase and polarization of light. Arrow A denotes the mutual relationship between amplitude and phase; Arrow B indicated the polarization-to-phase link; Arrow C denotes the polarization-to-amplitude link; Arrow D represent the phase-to-polarization link.

Supplementary Figure 2 Simplification of polarized-SLM based on phase vectorization. Polarized-SLM can be simplified into a filter system, which is composed of a focusing system and a reconstruction system. After passing through polarized-SLM, pixelate phase of $m$ order VVB can be vectorized into pixelate polarization. PH denotes a pinhole; OL$_1$ and OL$_2$ are two objective lenses, both numerical aperture of which are 0.01. $\theta$ is the convergent angle of OL$_1$. 
**Supplementary Figure 3** Spatial separation of desired and undesired polarized mode in the focal region of OL. (a) The light intensities of polarized mode with different topological charge along x axis; (b, c) The focal light intensity of VVB obtained by the modulation of the phase $\phi = m\phi - \beta$ with $m=30$ and (b) $\beta = 0$; (c) $\beta = 5\phi$. DPH: Desired polarized mode located at the center; UDPH: Undesired polarized mode located at the outer ring; PH: pinhole placed between DPH and UDPH.

**Supplementary Figure 4** Theoretical result of Figure 3. The vortex phases $\phi = \pm (m\phi - \beta)$ with $m=30$ and $\beta=0$ (a, e) are vectorized into left and right circular polarization, respectively, while linear polarizations are realized by the phases $\phi = \text{Phase}[\cos(m\phi - \beta)]$ with $\beta=0, 0.25\pi, 0.5\pi, 0.75\pi$ in (m, n, o, p), respectively. (b-d, f-h, i-l) are the light intensities passing through the polarizer (purple arrow) and quarter waveplate (blue arrow), which are normalized to a unit value indicated by the color bar (q). The color bar (r) shows the phase scale of (a, e, m-p).
Supplementary Figure 5 Theoretical result of Figure 5. The light beams in (b, f, j) are created by the phases in (a, e, i), respectively. (m, n, o) are the desired polarized modes in the focal region of OL₁, which are further reconstructed by OL₂ to form the desired polarizations in (b, f, j), respectively. (c, d, g, h, k, l) are the light intensities of light beam in (b, f, j) passing through the polarizer indicated by purple arrow and quarter waveplate indicated by blue arrow. All light intensities of light beams are normalized to a unit value, which are indicated by the color bar (p). The color bar (q) shows the phase scale of (a, e, i).

Supplementary Figure 6 Theoretical result of Figure 6. The light beams in (b, e, h) are created by the phases in (a, d, g), respectively. (j, k, l) are the desired polarized modes in the focal region of OL₁, which are reconstructed by OL₂ to form the desired polarizations in (b, e, h), respectively. (c, f, i) are the light intensities of light beam in (b, e, h) passing through the polarizer (purple arrow). Since the polarizations of background indicated by yellow arrow are adjusted to be orthogonal with the polarizers (purple arrow), the light intensities of background are always eliminated and the polarizations of A, B, C, D zone can be demonstrated by the petal-like patterns, see (c, f, i). All light intensities of light beams are normalized to a unit value, which are indicated by the color bar (m). The color bar (n) shows the phase scale of (a, d, g).
Supplementary Figure 7 Arbitrary order VVB. (b) 1 order VVB; (e) 2 order VVB; (i) 3 order VVB; (c, f, j) are their corresponding light intensities passing through the polarizer indicated by purple arrow. (a, d, h) are the needed phases $\phi = \text{Phase}[\cos((m\phi - \beta))]$ with $m=30$, (a) $\beta = \phi$; (d) $\beta = 2\phi$; (h) $\beta = 3\phi$. All light intensities of light beams are normalized to a unit value, which are indicated by the color bar (l). The color bar (k) shows the phase scale of (a, d, h).

Supplementary Figure 8 Vector beam with multiple zones. (b, f) are the light beams with three polarized zones, namely A, B, C zone, which are realized by the phases in (a, e), respectively. After passing through the polarizers indicated by purple arrow, the light intensities in (c, d, g, h) demonstrate that the polarizations of A, B, C zone are consistent with that of Supplementary Equation 26, 27, respectively. All light intensities of light beam are normalized to a unit value, which are indicated by the color bar (i). The color bar (j) shows the phase scale of (a, e).
Supplementary Figure 9 Special vector beam. The phases (a, b, c, d) are vectorized into the special vector beams (e, f, g, h), respectively. (i, j, k, l) are their corresponding light intensities passing through the polarizer indicated by purple arrow. All light intensities of light beams are normalized to a unit value, which are indicated by the color bar (n). The color bar (m) shows the phase scale of (a-d).
Supplementary Note 1: The principle of phase vectorization

In classical optics, there are so far three pairs of relationship between amplitude, phase and polarization of light beam. As shown in Supplementary Figure 1, arrow A indicates the mutual relationship between amplitude and phase, which can link with each other by wavefront modulation techniques [1-4]. The Malus law establishes the polarization-to-amplitude link with the aid of a polarizer denoted by arrow C. The polarization-to-phase link indicated by arrow B is demonstrated by Pancharatnam-Berry (PB) phase in 1987[5]. From these three links, one can link the vector polarization to the scalar phase and amplitude, the reverse, however, does not apply. If the inverse correlation of PB phase can be established, the scalar phase of light beam can entirely be vectorized into vector polarization. Here we call this inverse process of arrow B as phase vectorization, namely the phase-to-polarization link indicated by arrow D.

To establish the phase-to-polarization link, the light beam must possess different polarization responses. Generally, such light beam has a generalized form, the electric field of which can be expressed as

$$E_v = \exp[i f(\varphi, \theta)]\langle R \rangle + \exp[-i f(\varphi, \theta)]\langle L \rangle$$  \hspace{1cm} (1)

where $\langle L \rangle=[1 \quad i]$ and $\langle R \rangle=[1 \quad -i]$ denote left and right circularly polarized mode, respectively. $f(\varphi, \theta)$ represents a generalized phase function, $\theta$ and $\varphi$ are the convergence and azimuthal angle of the focusing system in Suplemental Figure 2, respectively.

When modulating by the phase $\phi = \pm [f(\theta, \varphi) - \beta]$, the light beam in Supplementary Equation 1 is transformed into

$$E_v = \exp[i (2 f(\theta, \varphi) - \beta)]\langle R \rangle + \exp(-i \beta)\langle L \rangle$$  \hspace{1cm} (2)

$$E_v = \exp[-i (2 f(\theta, \varphi) - \beta)]\langle L \rangle + \exp(i \beta)\langle R \rangle$$  \hspace{1cm} (3)

Supplementary Equation 2, 3 imply that the change of phase gives rise to different polarization response of $\exp(-i \beta)\langle L \rangle$ and $\exp(i \beta)\langle R \rangle$ except the undesired polarized modes $\exp[i (2 f(\theta, \varphi) - \beta)]\langle R \rangle$ and $\exp[-i (2 f(\theta, \varphi) - \beta)]\langle L \rangle$. Suppose that both undesired polarized modes can be eliminated, the light beams in Supplementary Equation 2, 3 can be simplified as

$$E_l = \exp(-i \beta)\langle L \rangle$$  \hspace{1cm} (4)

$$E_r = \exp(i \beta)\langle R \rangle$$  \hspace{1cm} (5)
According to Supplementary Equation 4, 5, one can easily obtain the phase-to-polarization link, by which the scalar phase in Supplementary Equation 6 can be vectorized into the vector polarization in Supplementary Equation 7.

\[
\phi = \text{Phase}[\cos \varphi_0 \exp[i (f(\theta, \varphi) - \beta)] + \sin \varphi_0 \exp[-i (f(\theta, \varphi) - \beta)]]
\]

\[
E = \cos \varphi_0 \exp(-i\beta) \langle L \rangle + \sin \varphi_0 \exp(i\beta) \langle R \rangle
\]

In Supplementary Equation 6, the parameter \( \varphi_0 \) is the weight factor that adjusts the proportion of \( \langle L \rangle \) and \( \langle R \rangle \). For \( \varphi_0 = 0 \), \( \langle L \rangle \) and \( \langle R \rangle \) can be obtained, respectively. For \( \varphi_0 = 0.25\pi \), the phase in Supplementary Equation 6 can be simplified as \( \phi = \text{Phase}[\cos(f(\theta, \varphi) - \beta)] \), thereby creating a linearly polarized beam with the polarization direction \( \beta \), which can be expressed as

\[
E_\beta = \cos \beta \langle x \rangle + \sin \beta \langle y \rangle
\]

Here, \( \langle x \rangle = [1 \ 0] \) and \( \langle y \rangle = [0 \ 1] \) denote x- and y- linearly polarized mode, respectively. For other \( \varphi_0 \), the phase in Supplementary Equation 6 can be vectorized into elliptical polarization accordingly.

It should be emphasized that there are two critical conditions for the realization of phase vectorization. One is that the light beam possesses inherent different polarization responses of left and right circularly polarized mode; another is that the undesired circularly polarized modes of \( \exp[i(2f(\theta, \varphi) - \beta)] \langle R \rangle \) and \( \exp[-i(2f(\theta, \varphi) - \beta)] \langle L \rangle \) must be eliminated. If only both above conditions are satisfied, phase vectorization can be achieved, and the scalar phase in Supplementary Equation 6 can link with the vector polarization in Supplementary Equation 7 directly.
Supplementary Note 2: Theoretical principle of polarized-SLM

In this note, we present the entire theoretical principle of polarized-SLM and all the theoretical results of the experiments in the main text. As shown in Supplementary Figure 2, polarized-SLM in Fig. 2 can be simplified into one single filter system, which is composed of a focusing system and a reconstruction system. Here, PH denotes the pinhole; OL₁ and OL₂ are two objective lenses.

Part 1: Focusing system

In the focusing system, the incident \( m=30 \) order VVB is composed of a right and a left circularly polarized mode with inverse vortex [See Equation 1]. When modulating by the phase \( \phi = \pm (m \varphi - \beta) \), VVB exhibits different polarization responses, thereby dividing into desired and undesired polarized modes [See Equations 2, 3]. Here, we demonstrate the spatial separation of desired and undesired polarized modes in the focal region of OL₁.

Based on the Debye vectorial diffractive theory, the electric fields near the focus can be expressed as [6]

\[
E_f = \mathcal{F}\left\{ P(\rho)T_p l_o(\theta)\cos^{3/2}\theta ME, \exp(-ik_zz) / \cos \theta \right\}
\]

where \( \theta \) is the convergent angle; and the maximum convergent angle \( \alpha = \arcsin(NA/n) \), \( NA \) is the numerical aperture of OL₁, and \( n \) is the refractive index in the focusing space. \( k_z = k \cos \theta \) is the \( z \) component of wavevector \( k = 2n\pi / \lambda \), where \( \lambda \) is the wavelength of the incident VVB. \( \mathcal{F} \) denotes the Fourier transform. \( P(\rho) \) represents the aperture of OL₁, which can be expressed as

\[
P(\rho) = \begin{cases} 
1 & 0 < \rho < R \\
0 & \text{otherwise}
\end{cases}
\]

where \( R \) is the radius of OL₁. \( T_p = \exp(i\phi) \), where the phase \( \phi \) can be found in Equation 6. \( l_o(\theta) \) denotes the electric amplitude of the incident VVB. Owing to the slight deviation between the conjugated plane of phase-only SLM and VP, the output vector beam from polarized-SLM is hollow shape. In this case, \( l_o(\theta) \) can be approximately expressed as

\[
l_o(\theta) = \begin{cases} 
0 & 0 < \sin \theta / NA < 0.2 \\
1 & 0.2 \leq \sin \theta / NA < 1
\end{cases}
\]
In Supplementary Equation 9, \( E \) represents the polarization state of VVB. For the sake of simplicity, \( m=30 \) order VVB in Equation 1 is simplified into the superposition of \( x \) and \( y \) polarized mode, which can be expressed as

\[
E_i = \cos(m\phi)\langle x \rangle + \sin(m\phi)\langle y \rangle
\]

where \( \langle x \rangle \) and \( \langle y \rangle \) denote \( x \) and \( y \) linearly polarized mode, respectively. \( M \) denotes a polarization transformation matrix caused by OL1, which can be expressed as [6]

\[
M = \begin{bmatrix}
\cos^2 \phi \cos \theta + \sin^2 \phi & (\cos \theta - 1) \sin \phi \cos \phi & -\sin \theta \cos \phi \\
(\cos \theta - 1) \sin \phi \cos \phi & \cos^2 \phi + \sin^2 \phi \cos \theta & -\sin \theta \sin \phi \\
\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta 
\end{bmatrix}
\]

Eventually, the focal light intensity of VVB can be obtained using \( I = |E_f|^2 \).

**Spatial separation of desired and undesired polarized modes**

In the following simulations, \( NA=0.01, n=1 \). The unit of length in all figures is the wavelength \( \lambda \), and the light intensity is normalized to the unit value. Since the absolute value of topological charges in Equation 2, 3 are equal for the desired and undesired polarized modes, the polarized modes in Equation 2 have identical positions with their counterparts in Equation 3. Here, we only take the light beam in Equation 2 as example to investigate the spatial separation of desired and undesired polarized mode in the focal region of OL1.

Supplementary Figure 3 represents the focal light intensity of VVB modulated by the phase \( \phi = m\phi - \beta \) with \( m=30 \) and (b) \( \beta = 0 \); (c) \( \beta = 5\phi \). That is, the topological charges of left and right circularly polarized mode in Equation 2 are 0, 60 for Supplementary Figure 3(b) and 5, 55 for Supplementary Figure 3(c), respectively. Normally, both left and right polarized modes are tangled with each other during propagating in free space. For this reason, the phase-to-polarization link is thought to be impossible. However, when focusing by OL1, different topological charge leads to different position in the focal region, thereby permitting the spatial separation of left and right polarized mode. Specifically, the desired left circularly polarized mode locates at the center while the undesired right circularly polarized mode is in the position of outer ring, as shown in Supplementary Figure 3 (b, c). Thus, one can simply retain the desired polarized mode by eliminating the undesired one using a pinhole, which is the key of phase vectorization.

Note that the pinhole must be placed between the desired and undesired polarized mode so that the undesired one is eliminated without affecting the desired one. Therefore, the active area of desired
polarized mode is determined by the size of pinhole. Generally, larger active area is the better. In Supplementary Figure 3, VVB modulated by the phase $\phi = \pm (m\varphi - \beta)$ with $\beta = \sigma \varphi$ can be expressed as

$$E_x = \exp[i(2m - \sigma)\varphi] \langle R \rangle + \exp(-i\sigma\varphi) \langle L \rangle$$ \hspace{1cm} (14)

$$E_y = \exp[-i(2m - \sigma)\varphi] \langle L \rangle + \exp(i\sigma\varphi) \langle R \rangle$$ \hspace{1cm} (15)

where $\langle L \rangle = [1 \ i]$ and $\langle R \rangle = [1 \ -i]$ denote left and right circularly polarized mode, respectively. As shown in Supplementary Figure 3(a), the larger the difference of topological charge $\Delta l = |2m - \sigma| - |\sigma|$, the farther the distance between the inner and outer polarized mode. Larger distance leads to bigger size of pinhole, thereby increasing the active area of desired polarized mode. There are two methods to enlarge the size of pinhole.

**First method**

When $\sigma > 0$, $\Delta l$ becomes small with the increment of $\sigma$, thus only a relatively small size of pinhole can be obtained. However, when $\sigma \leq 0$, no matter how big $|\sigma|$ is, one can always obtain an invariant $\Delta l = |2m|$. As $|\sigma|$ increases, the topological charges of outer and inner polarized mode enlarge accordingly. For this reason, the size of pinhole must be increased so as not to affect the desired polarized mode. That is, a large active area of desired polarized mode can be achieved. It should be emphasized that the above case is only valid in the condition of $\sigma \leq 0$. Suppose that the invariant $\Delta l = |2m|$ can also be achieved in the condition of $\sigma > 0$, the active area of desired polarized mode is no longer restricted by the pinhole. Accordingly to Supplementary Equation 14,15, the inner desired polarized modes are $\exp(-i|\sigma|\varphi) \langle L \rangle$ and $\exp(i|\sigma|\varphi) \langle R \rangle$ for the case of $\sigma > 0$, while that of $\sigma \leq 0$ are $\exp(i|\sigma|\varphi) \langle L \rangle$ and $\exp(-i|\sigma|\varphi) \langle R \rangle$. Thus, the key to realize an identical $\Delta l = |2m|$ in the condition of $\sigma > 0$ lays on how to convert the desired inner polarized modes $\exp(i|\sigma|\varphi) \langle L \rangle$ and $\exp(-i|\sigma|\varphi) \langle R \rangle$ for the case of $\sigma \leq 0$ into $\exp(-i|\sigma|\varphi) \langle L \rangle$ and $\exp(i|\sigma|\varphi) \langle R \rangle$ for the case of $\sigma > 0$.

This polarization conversion can easily be realized with the aid of one half-waveplate (HWP). When passing through HWP, the desired polarized modes $\exp(i|\sigma|\varphi) \langle L \rangle$ and $\exp(-i|\sigma|\varphi) \langle R \rangle$ for the case of $\sigma \leq 0$ can be expressed as

$$E_{out} = \exp(i|\sigma|\varphi) \mathbf{H} \langle L \rangle$$ \hspace{1cm} (16)
\[ E_{outR} = \exp(-i|\sigma|\varphi)H(R) \]  

(17)

where \( H \) is the Jones matrix of HWP, which can be written as

\[
H = \begin{bmatrix}
\cos 2\psi & \sin 2\psi \\
\sin 2\psi & -\cos 2\psi
\end{bmatrix}
\]

(18)

Here \( \psi=0 \) represents the angle between the fast axis and x-axis. The output polarized modes in Supplementary Equation 16, 17 can further be simplified as

\[
E_{outL} = \exp(i|\sigma|\varphi)\langle R \rangle
\]

(19)

\[
E_{outR} = \exp(-i|\sigma|\varphi)\langle L \rangle
\]

(20)

Supplementary Equation 19, 20 indicate that the desired polarized modes of VVB modulated by \( \phi = \mp(m - \sigma)\varphi \) in the condition of \( \sigma \leq 0 \) can convert into that of VVB modulated by \( \phi = \pm(m - \sigma)\varphi \) in the condition of \( \sigma > 0 \) with the aid of HWP. Since the invariant \( \Delta l = |2m| \) can be obtained by the above polarization conversion, a large active area can be realized for desired polarized mode.

**Second method**

The second method, which is considered as the most directive way, is to increase the order \( m \) of VVB. For example, if \( m = 30, \sigma = 10 \), the desired and undesired polarized modes are 136.22μm and 563.24μm away from the geometric focus of OL1, respectively. Thus, the radius of pinhole can be adjusted to 400μm. If \( m = 60, \sigma = 10 \), the desired and undesired polarized modes are 136.22μm and 1186.88μm away from the geometric focus of OL1, and the radius of pinhole can be increased as large as 800μm. Larger \( m \) leads to bigger size of pinhole. Thanks to the mature technique of Q-plate [7, 8], the order \( m \) of VVB can reach as high as 126. Thus, the desired and undesired polarized modes are 136.22μm and 2537.06μm away from the geometric focus of OL1, and the pinhole can be adjusted to an inconceivable radius with 2000μm. Combining the above two methods, one can always obtain sufficient room for the polarization and phase modulation of polarized-SLM.

**Part 2: Reconstruction system**

Part 1 has demonstrated that \( m \) order VVB modulated by the phase in Equation 6 is spatially separated into left and right circularly polarized mode with different position in the focal region of OL1 [See Supplementary Figure 3]. Thus, the desired polarized mode can be extracted by filtering out the undesired one using a pinhole. After passing through the objective lens OL2, the desired polarized mode is reconstructed, the electric field of which can be expressed as [6]
\[
E = \left( P(\rho \cos^{1/2} \theta) \right)^{-1} M^{-1} \exp(ik_z z) \cos \theta \mathcal{F}^{-1}(E_p)
\]  

where \( \mathcal{F}^{-1} \) denotes the inverse Fourier transform. \( E_p \) is the electric field behind the pinhole in Supplementary Figure 2, namely the desired polarized mode. \( M^{-1} \) is the inverse polarization transformation matrix of the reconstructive lens OL_2, which can be expressed as [6]

\[
M^{-1} = \begin{bmatrix}
\cos^2 \varphi \cos \theta + \sin^2 \varphi & (\cos \theta - 1) \sin \varphi \cos \varphi & \sin \varphi \cos \varphi \\
(\cos \theta - 1) \sin \varphi \cos \varphi & \cos^2 \varphi + \sin^2 \varphi \cos \theta & \sin \varphi \sin \varphi \\
-\sin \theta \cos \varphi & -\sin \varphi \sin \varphi & \cos \theta
\end{bmatrix}
\]

Finally, the light intensity output from OL_2 can be obtained using \( I = |E|^2 \).

Generally, there are three kinds of polarization, including left and right circular polarization, linear polarization. Note that elliptical polarization is the combination of circular and linear polarization. Supplementary Figure 4 presents the theoretical results of Figure 3, which demonstrates that these three polarizations can link with three particular phases using phase vectorization. Specifically, forward and reverse vortex phase can be vectorized into left and right circular polarization, respectively, while binary phase is corresponding to linear polarization, the polarized direction of which is adjusted by the relative displacement of phase structure.

To explain the principle of phase vectorization more clearly, we calculate the light intensities behind the pinhole \( I = |E_p|^2 \) and their corresponding reconstruction polarizations from OL_2 in Supplementary Figure 5, 6. The theoretical predictions in Supplementary Figure 5, 6 are consistent with the experimental results in Figure 5, 6, respectively. Taking the reconstruction polarization in Supplementary Figure 5 (b) as example. As demonstrated in Part 1, the undesired polarized mode locates at the outer ring which is far away from the inner desired one. Thus, the undesired polarized mode is eliminated by the pinhole, and only the desired polarized mode is retained, see Supplementary Figure 5(m). Since the desired polarized mode is the result of focusing the VVB with the phase in Supplementary Figure 5(a), the desired polarized mode has a one-to-one correspondence with the phase of VVB, which further establishes the direct link between the phase and the reconstruction polarization in Supplementary Figure 5 (b). In this way, the phase of VVB can be vectorized into arbitrary polarization.

As shown in Supplementary Figure 2, the phase of VVB can be pixelated by phase-only SLM. Pixelate polarization can be realized in a real-time dynamic way by the polarized-SLM accordingly. To
demonstrate this, we present the theoretical results of Figure 6 in Supplementary Figure 6. As shown in Supplementary Figure 6, there are four pixel-like zones named A, B, C, D zone in the light beam, each size of which is the same as that of Supplementary Figure 5. Supplementary Figure 6 (a, d, g) are their corresponding phases of VVB. In these four zones, one can even create four different order VVB [See Equation 8], which verifies that pixelate phase manipulation of VVB gives rise to pixelate polarization adjustment of light beam. Here, Supplementary Figure 6 (j, k, l) are the light intensities behind the pinhole, which are utilized to reconstruct the polarizations in Supplementary Figure 6 (b, e, h), respectively.
Supplementary Note 3: Arbitrary vector light beam

This note presents three examples of vector light beam created using polarized-SLM. According to Equation 6, the phase for linear polarization can be simplified as

$$\phi = \text{Phase}\left\{ \cos \left( (m \varphi - \beta) \right) \right\}$$  \hspace{1cm} (23)

where \( m = 30 \) is the order of VVB; \( \beta \) denotes the angle between the polarization direction and x axis. The corresponding linear polarization can be written as

$$E = \cos \beta \langle x \rangle + \sin \beta \langle y \rangle$$  \hspace{1cm} (24)

Here, \( \langle x \rangle = [1 \ 0] \) and \( \langle y \rangle = [0 \ 1] \) denote x- and y- linearly polarized mode, respectively. Since the parameter \( \beta \) can be adjusted at will using the phase-only SLM in Fig. 2, the phase in Supplementary Equation 23 can be vectorized into a linearly polarized beam with designed polarization distribution.

**Case 1: Arbitrary order VVB**

In this case, the parameter \( \beta = n \varphi \), and polarization in each pixel is adjusted to the direction of \( \beta = n \varphi \). Thus, the phases in Supplementary Figure 7 (a, d, h) can be vectorized into \( n \) order VVBs, which can be expressed as

$$E = \cos n \varphi \langle x \rangle + \sin n \varphi \langle y \rangle$$  \hspace{1cm} (25)

Here, \( n = 1, 2, 3 \) in Supplementary Figure 7 (b, e, i), respectively. Supplementary Figure 7(c, f, j) are their corresponding light intensities passing through a polarizer (purple arrow).

**Case 2: Vector beam with multiple zones**

In this case, the parameter \( \beta \) is divided into three zones, namely A, B, C zone shown in Supplementary Figure 8 (a, e). Polarization in each zone can be manipulated individually using polarized-SLM.

Supplementary Figure 8 (b) presents the experimental result of light beam created by the phase in Supplementary Figure 8(a), the polarization state of which can be expressed as

$$E = \begin{cases} 
VVB_{n=1} & \text{Azone: } 0 < r \leq R / 3 \\
VVB_{n=2} & \text{Bzone: } R / 3 < r \leq 2R / 3 \\
VVB_{n=3} & \text{Czone: } 2R / 3 < r \leq R 
\end{cases}$$  \hspace{1cm} (26)

Here, \( VVB_n \) denotes VVB with the order \( n = 1, 2, 3 \) for A, B, C zone, respectively. Supplementary Figure 8(c, d) are the light intensities of A, B, C zone passing through the polarizer indicated by purple arrow.
Likewise, Supplementary Figure 8(f) presents the experimental result of light beam created by the phase in Supplementary Figure 8(e), the polarization state of which can be expressed as

$$E = \begin{cases} 
LP_{\beta=0} & \text{Azone} : \ 0 < r \leq R / 3 \\
LP_{\beta=-0.25\pi} & \text{Bzone} : R / 3 < r \leq 2R / 3 \\
LP_{\beta=-0.5\pi} & \text{Czone} : 2R / 3 < r \leq R 
\end{cases}$$ (27)

Here, $LP_{\beta}$ denotes a linearly polarized light with the polarization direction of $\beta$. Supplementary Figure 8(g, h) are the light intensities of A, B, C zone passing through the polarizer indicated by purple arrow.

**Case 3: Special vector beam**

In this case, the phases in Supplementary Figure 9(a, b, c, d) are vectorized into special vector beams in Supplementary Figure 9(e, f, g, h), respectively, the polarized state of which can be expressed as

$$E = \cos \beta \langle x \rangle + \sin \beta \langle y \rangle$$ (28)

where the parameter $\beta = 2\eta \pi \sin \theta / NA + \delta \phi$ with $\eta = 2$, $NA = 0.01$ and (a) $\delta = 0$; (b) $\delta = -1$; (c) $\delta = -2$; (d) $\delta = -3$, respectively. Supplementary Figure 9(i, j, k, l) are the light intensities of special vector beam passing through the polarizer indicated by purple arrow.
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