Color Glass Condensate and BFKL dynamics
in deep inelastic scattering at small \( x \)

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Abstract. The proton structure function \( F_2(x, Q^2) \) for \( x \leq 10^{-2} \) and \( 0.045 \leq Q^2 \leq 45 \) GeV\(^2\), measured in the deep inelastic scattering at HERA, can be well described within the framework of the Color Glass Condensate.

1. Introduction

There has been a surge of theoretical and experimental interest in the “Color Glass Condensate (CGC)” which appears in the new perturbative regime of QCD relevant for high energy scattering [1]. This new state is characterized by high density gluons whose longitudinal momenta are very small compared to the total momentum of the parent hadron (such gluons are called “small-\( x \)” gluons since the ratio of the momenta is denoted as \( x \)). The gluon density is typically as large as \( O(1/\alpha_s) \), and cannot be too large (“saturated”) so that the unitarity of physical cross sections is ensured.

Recent rapid theoretical progress in understanding the physics of CGC is triggered by the experiments currently investigated at HERA (DESY) and RHIC (BNL). The relevant process at HERA is the deep inelastic scattering (DIS) of an electron off a proton, while at RHIC it is more complicated Au-Au or deuteron-Au collision. These two apparently different experiments (different in complexity and energy) are nevertheless closely related to each other in the context of CGC through the “universality” of the hadron/nucleus wavefunctions or the gluon distributions. Here we mean differently by the word “universality” than in the usual sense. Namely, in the saturated regime at high energy, the gluon distributions of the proton and nucleus are both described by the same function of the ratio of the transverse momentum of gluons \( k_\perp \) to the saturation momentum \( Q_s \), which is the (inverse of) typical transverse size of gluons (for more explanations, see for example, Refs. [2, 3]). Therefore, one will be able to ‘translate’ the HERA physics for protons into the RHIC physics for gold nuclei. Furthermore, the saturation scales in two experiments are accidentally of the same order because of its particular dependencies upon \( x \) and the atomic number \( A \), i.e., \( Q_s^2(A, x) \propto A^{1/3}(1/x)^{A} \sim (A/x)^{0.3} \). Indeed, at HERA, \( A = 1 \) and \( x \sim 10^{-4} \) while at RHIC, \( A \approx 200 \) and typically \( x \sim 10^{-2} \). This fact also strengthens the importance of understanding the HERA physics in relation to the RHIC physics. In this talk, I will show that the recent HERA data [4] at small-\( x \) with not so large \( Q^2 \) is consistent with the current picture of the CGC (for more detail, see Ref. [5]), which, according to the discussion above, suggests the importance of CGC at RHIC.
2. DIS at small $x$ and previous fits with saturation models

DIS at small-$x$ looks simple in the “dipole picture” which leads to an intuitively understandable factorization formula. The scattering between the virtual photon $\gamma^*$ (emitted from the projectile electron) and the proton is seen as the dissociation of $\gamma^*$ into a quark-antiquark pair (the “color dipole”) followed by the interaction of this dipole with the color fields in the proton. Then one can write the $F_2$ structure function as

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}} \sum_{T,L} \int dz \, d^2 r \, |\Psi_{T,L}(z, r, Q^2)|^2 \sigma_{\text{dipole}}(x, r),$$

where, $\Psi_{T,L}$ are the light-cone wavefunctions of $\gamma^*$ with transverse, or longitudinal, polarization, and $\sigma_{\text{dipole}}(x, r)$ is the cross-section for dipole–proton scattering (for a dipole of transverse size $r$), containing all the information about hadronic interactions such as the unitarization or saturation effects.

A simple parametrization for $\sigma_{\text{dipole}}(x, r)$ which has qualitatively plausible behaviors like color transparency and saturation effects was first proposed by Golec-Biernat and Wüsthoff (GBW) \[6\]. They used a very simple function $\sigma_{\text{dipole}}(x, r) = \sigma_0 \left(1 - e^{-r^2 Q^2_s(x)/4}\right)$ with only three parameters $\sigma_0$ (a hadronic cross-section), $x_0$ and $\lambda$ for the saturation momentum $Q^2_s(x) = (x_0/x)^\lambda$ GeV$^2$, and managed to provide rather good fits to the (old) HERA data for $x \leq 10^{-2}$ and all $Q^2$. This success was quite impressive by itself since it suggested the relevance of saturation physics in the HERA data, but at the same time required more serious theoretical effort towards understanding the HERA data with the saturation picture better rooted in QCD. In fact, there is no kinematical regime in which the GBW model can be (strictly) justified from QCD, and the GBW model must be improved with the information of QCD, or replaced by other QCD-based parametrization. So far, there are several attempts to improve the GBW model \[7, 8\], but they mostly focused upon improving the behavior of the fit at large $Q^2$, by including DGLAP dynamics. On the other hand, we know that the BFKL dynamics, rather than DGLAP, should be the right physics in the transition regime towards saturation. This BFKL physics was not addressed so far, and we will focus on the regime with not too large $Q^2$ where the BFKL and saturation physics should be more relevant, and will present a new analysis of the HERA data, which is rather orthogonal to the previous attempts to improve the GBW model.

3. The CGC fit \[5\]

We restrict ourselves to the kinematical range where one expects important high density effects — namely, $x \leq 10^{-2}$ and $Q^2 < 50$ GeV$^2$ —, and show that the data in this range are consistent with our present understanding of CGC (BFKL evolution and saturation). The upper limit on $Q^2$ has been chosen large enough to include a significant number of “perturbative” data points, but low enough to justify the emphasis on BFKL, rather than DGLAP, evolution. Within this kinematical range, we shall provide a reasonable fit (which we call the “CGC fit”) to the new HERA data for $F_2$ based on a simple, analytic, formula for the dipole scattering amplitude.
Figure 1. The $F_2$ structure function as a function of $x$ in bins of $Q^2$ for $Q^2 \leq 15$ GeV$^2$ (left) and for $Q^2 > 15$ GeV$^2$ (right). The experimental points are the latest data from the H1 and ZEUS collaborations \[4\]. The full line shows the result of the CGC fit with $N_0 = 0.7$ to the ZEUS data for $x \leq 10^{-2}$ and $Q^2 \leq 45$ GeV$^2$. The dashed line shows the predictions of the pure BFKL part of the fit (no saturation). In the bins with $Q^2 \geq 60$ GeV$^2$, the CGC fit is extrapolated outside of the range of the fit.

The dipole cross-section in the CGC fit reads

$$
\sigma_{\text{dipole}}(x, r) = 2\pi R^2 N(rQ_s, Y),
$$

where the dipole scattering amplitude $N(rQ_s, Y)$ is constructed by smooth interpolation between two limiting solutions to the non-linear evolution equations in QCD \[9, 10\]: the solution to the BFKL equation with saturation boundary \[11, 12, 13\] for small dipole sizes, $r \ll 1/Q_s(x)$, and the Levin-Tuchin law \[14, 3\] for larger dipoles, $r \gg 1/Q_s(x)$. Namely,

1. $N(rQ_s, Y) = N_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \ln(2/rQ_s)\right)}$ for $rQ_s \leq 2$,
2. $N(rQ_s, Y) = 1 - e^{-a\ln^2(brQ_s)}$ for $rQ_s > 2$, 

where $Y = \ln(1/x)$, $Q_s \equiv Q_s(x) = (x_0/x)^{\lambda/2}$ GeV, and we have defined $Q_s$ in such a way that $N(Q_s, Y) = N_0$ for $rQ_s = 2$. The coefficients $a$ and $b$ are determined uniquely from the continuity of $N(Q_s, Y)$ at $rQ_s = 2$. In the first line, $\gamma_s = 0.63$ (or more strictly, $1 - \gamma_s$) is the anomalous dimension, and $\kappa = \chi''(\gamma_s)/\chi'(\gamma_s) \approx 9.9$ is the diffusion coefficient. The anomalous dimension gives the geometric scaling \[15, 11, 12\], while the second ”diffusion” term in the power (the term depending upon $\kappa$) brings in scaling violations. The overall factor $N_0$ is ambiguous, reflecting an ambiguity in the definition of $Q_s$. But the results of the fit do not change largely by changing $N_0$. The saturation exponent $\lambda$ is computable in QCD (known up to the renormalization-group-improved NLO BFKL) \[11, 12, 13\], but we treat $\lambda$ as a free parameter since the results are sensitive to its precise value. We work with three quarks of equal mass $m_q$ and use the same photon wavefunctions $\Psi_{T,L}$ as in Refs. \[6, 7, 8\]. Thus, the only free parameters of the CGC fit are $R$, $x_0$ and $\lambda$, which are the same as in the GBW model ($\sigma_0 = 2\pi R^2$).
The CGC fit has been performed for the $F_2$ data at ZEUS [4] with $x \leq 10^{-2}$ and $Q^2$ between 0.045 and 45 GeV$^2$ (156 data points). In Fig. 1, the results of the fit are plotted against the data for $N_0 = 0.7$ and $m_q = 140$ MeV. The three parameters are determined to be $R = 0.641$ fm, $x_0 = 0.267 \times 10^{-4}$ and $\lambda = 0.253$ with $\chi^2$/d.o.f. = 0.81. We also show (with dashed line) the prediction of the BFKL calculation without saturation, as obtained by extrapolating the formula in the first line of Eq. (2) to arbitrarily large $rQ_s$. This pure BFKL fit shows a too strong increase with $1/x$ at small $Q^2$. On the other hand, the complete fit, including saturation, works remarkably well even at the lowest values of $Q^2$ that we have included. Note also that the value $\lambda = 0.25$ determined from the fit is in good agreement with the theoretical result [13]. We have done the fit separately with the pure scaling part, and found that the fit becomes worse. This means that the “diffusion” term which violates the geometric scaling is crucial to fit the HERA data. This is not surprising because this term effectively changes the anomalous dimension from its BFKL value $\gamma_s = 0.63$ (for relatively large dipole sizes $\lesssim 1/Q_s$) to the DGLAP value $\gamma = 1$ (for small dipole sizes), and thus partially simulates the DGLAP dynamics. However, as is evident from the figure, there is a deviation between the CGC fit and the data at high $Q^2$ and not so small $x$. This is again not surprising simply because this regime is outside the range of validity of the CGC fit, which does not include the DGLAP physics (in its right form) nor valence quark dynamics.

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