Effect of limited near-wall inlet data on the direct numerical simulation of turbulent channel flow

K. Ezhilsabareesh\textsuperscript{1}, Callum Atkinson\textsuperscript{1}, Adrian Lozano-Duran\textsuperscript{2}, Peter J. Schimd\textsuperscript{3}, Javier Jimenez\textsuperscript{4}, Julio Soria\textsuperscript{1}

\textsuperscript{1} Laboratory for Turbulence Research in Aerospace and Combustion, Department of Mechanical and Aerospace Engineering, Monash University, Victoria 3800, Australia
\textsuperscript{2} Center for Turbulence Research, Stanford University, CA 94305, USA
\textsuperscript{3} Department of Mathematics, Imperial College London, London, UK
\textsuperscript{4} E.T.S. Ingenieros Aeronauticos, Pl. Cardenal Cisneros 3, 28040 Madrid, Spain

E-mail: callum.atkinson@monash.edu

Abstract. Direct numerical simulation (DNS) of turbulent flows require a large computational domain and a long simulation time to capture and evolve the large-scale structures and attain a statistically stationary state. In contrast, experimental measurements can relatively easily capture the large-scale structures, but struggle to resolve the dissipative flow scales. This study investigates the spatial extent required for the DNS of a turbulent channel flow to recover turbulent fluctuations and energy when using experimental inlet data which is typically unable to capture fluctuations down to the viscous sub-layer. Synthetic experimental fields from streamwise periodic channel flow DNS at $Re_{\tau} = 180$ are used as an inlet for a channel flow DNS with inlet-outlet boundary conditions. The effect of limited near-wall data at the inlet is examined by removing the near-wall energy and fluctuations in all but the zeroth Fourier mode. The influence of limited near-wall data on the convergence of mean and streamwise fluctuating velocity profiles is less significant when the fluctuations are removed at the inlet up to $y^+ = 5$. However, the spanwise fluctuations are slightly weakened. The spanwise energy spectra suggest that at 1/16 of the domain length ($x/h \approx \pi/4$) the flow scales are recovered. When the fluctuations are removed up to $y^+ = 17$ or greater, recovery of full range of flow scales requires a domain larger than $x/h = 4\pi$.

1. Introduction

The ability to accurately observe and obtain a complete understanding of wall-bounded turbulent flows is impeded by our inability to simulate flows at the high Reynolds numbers that are encountered in most industrial and natural flows. Experiments have less limitation on the Reynolds number and can relatively easily capture large scale information at discrete locations in a flow. However, experimental measurements suffer from low spatial resolution and an inability to resolve the smallest scales that are ultimately responsible for the dissipation of kinetic energy \cite{1}. This contrasts with direct numerical simulations (DNS), where large scales require a large computational domain and much longer simulation times to reach a statistically stationary state, while small scales adjust themselves relatively quickly to perturbations \cite{2}. In the case of spatially evolving flows, DNS also requires streamwise extent beyond the targeted analysis domain to minimize transients due to recycling of inlet conditions \cite{3}.

The DNS of turbulent channel flow carried out by Kim et al. \cite{4} shows that long streamwise
wavelengths exist in channels and correlations decay beyond $x/h \approx 4$ in the streamwise direction, where $h$ is the channel half-width. The streamwise periodicity can be justified if the largest eddies in the flow are included in the computational domain and the turbulence fluctuations are uncorrelated at the half domain period [4]. Abe et al. [5] found that the streamwise domain size has negligible effects on the mean flow variables and second-order moments but from the energy spectra they noted that the effects of large-scale structure in the outer region require a large computational domain. In smaller domains, the streamwise extents are not sufficient for the very large-scale structures to break up or meander before they are recycled back to the inlet. Adrian and Jimenez [6] reported that the large-scale structures in the smaller domain are virtually infinitely long but their interaction with smaller scales is correctly represented. In experiments, the small-scales are difficult to resolve because of low spatial resolution and the near-wall fluctuations are often overestimated due to measurement noise introduced by the effect of large displacement gradients across the interrogation region [7]. With an increase in Reynolds number the scale separation also increases and the measurement of small scales becomes more complicated.

Several methods have been proposed to generate correct inflow conditions. Jarrin et al. [8] generated synthetic eddies in the inlet section by a shape function representing the spatial and temporal characteristics of each eddy. Klien et al. [9] and di Mare et al. [10] used digital filtering techniques to generate a correlated field from random data. Fukami et al. [11] proposed the machine-learned inflow generator. It requires an additional computational cost to generate the spatiotemporal data of the target flow to train the machine learning models. These models may not represent the real physics when extrapolated to higher Reynolds number.

This study investigates the spatial extent required for the DNS of a turbulent channel flow to recover turbulent fluctuations and energy when they are removed from the inlet plane. The motivation for this study is to establish the quality of experimental measurements required as inflow conditions of turbulent channel flow DNS as a means of reducing the computational domain required for wall-bounded DNS. The synthetic experimental fields from streamwise periodic channel flow DNS at $Re_\tau = 180$ are used as an inlet into a channel flow DNS with inlet-outlet boundary conditions. The effect of limited near-wall data at the inlet is examined by removing the energy and fluctuations in all but the zeroth Fourier mode, to replicate the absence of near-wall data that generally results from finite interrogation window size and surface laser reflections. The streamwise extent required to recover the missing inlet information is determined through comparison of statistics and instantaneous velocity fields with fully resolved periodic DNS.

2. Numerical Methodolgy
A fractional-step method [12] is used to solve the incompressible Navier-stokes equations with a staggered, second-order, finite-differences scheme. The equations are advanced in time using a third-order Runge-Kutta scheme [13]. The channel flow is simulated with the number of grid points $N_x$, $N_y$, and $N_z$ in the streamwise, wall-normal and spanwise directions respectively, with the size of computational domain in the respective directions denoted by $L_x$, $L_y$ and $L_z$. The turbulent channel flow code has been validated in previous studies [14]. The simulations were performed for a frictional Reynolds number, $Re_\tau = 180$ with the grid properties listed in Table 1. The grid spacing in the streamwise ($\Delta x^+$) and spanwise directions ($\Delta z^+$) are constant. The minimum wall-normal grid spacing is located near the wall ($\Delta y_{\text{min}}^+$) and the maximum wall-normal grid spacing at the channel centreline ($\Delta y_{\text{max}}^+$) [15]. Grid spacing is given in viscous units, where they are non-dimensionalised by the length scale $\nu/u_\tau$ and denoted by the superscript $+$. The constant time-step size $\Delta t^+ = \Delta t_{\text{edd}}^+ = h/u_\tau$ over which the statistics are accumulated are given in Table 1. The baseline case is a streamwise periodic channel flow DNS. The mean velocity and root-mean-squared
Table 1: DNS numerical details

| Reτ   | (Lx, Ly, Lz) | (Nx, Ny, Nz) | (∆x⁺, ∆y⁺, ∆y⁺, ∆z⁺) | ∆t⁺ | h/ut⁺ |
|-------|--------------|--------------|------------------------|------|-------|
| 180   | (4π, 2, 2π)  | (258, 152, 130) | (7.88, 0.37, 3.37, 3.93) | 0.08 | 160   |
| 180   | (8π, 2, 2π)  | (516, 152, 130) | (7.88, 0.37, 3.37, 3.93) | 0.08 | -     |

Figure 1: (a) Mean velocity profile for baseline case (blue dots) and DNS of [16] (dashed red line) in viscous units. (b) Root-mean-squared fluctuating velocities for baseline case an DNS of [16] (dashed red line).

(RMS) statistics for the fluctuating velocity of the baseline case are plotted in Fig. 1 and all of the statistical profiles are consistent with the DNS of [16].

The synthetic inlet data generated from the baseline case is fed into the inlet-outflow DNS case at every time step as illustrated in Fig. 2. A convective boundary condition is applied at the domain exit. The initial field of the inlet-fed DNS is populated with the temporal mean of the synthetic inlet and the statistics are accumulated over 10 eddy-turnover times after the initial transients are washed out and the simulation becomes statistically stationary. To validate the methodology for the inlet-fed DNS the fully resolved inlet data from the baseline simulation was used and the statistics were compared with the baseline case and the DNS of [16], velocity profiles are plotted in Fig. 3, which shows that the mean velocity and fluctuating velocity profiles of inlet-fed DNS are identical with the baseline case. The pre-multiplied spanwise spectra of the streamwise velocity (k_z φ_{uu}) as a function of y at various streamwise locations are also compared between the inlet-fed DNS case and the baseline case and are presented in Fig. 4. It shows the inlet-fed DNS is able to maintain the energy across all the scales.

3. Results and discussion

Three cases were considered where the instantaneous velocity components are replaced with the mean velocity up to y⁺ = 5, 17 and 35. At every time step, the inlet plane from the baseline case is truncated and replaced with the mean flow below the indicated heights and fed into the inlet-fed DNS case. The initial field for these cases is populated with the mean flow and evolved until simulation reaches a statistically stationary state. The low-order turbulence statistics such as mean velocity and fluctuating velocity profiles as a function of y⁺ at different streamwise
Figure 2: Arrangements of the computational domains

Figure 3: (a) Mean velocity profile. Full resolution inlet-fed DNS case (blue dots); Baseline case (black line) and DNS of [16] (dashed red line). (b) Root-mean-squared fluctuating velocity profile.

Figure 4: The pre-multiplied spanwise spectra of the streamwise velocity, $k_z \phi_{uu}$, as a function of $y$ (Averaged in streamwise direction). The contours are 0.1 and 1.0 of the maximum value of $\phi_{uu}$, Baseline case (black dotted line), Inlet-fed DNS case (contour).
3.1. The statistical velocity profiles

In Fig. 5 the mean velocity and fluctuating velocity profiles are shown for inlet-fed DNS case with mean flow up to $y^+ = 5$ at the inlet. The use of the inlet without fluctuations up to $y^+ = 5$ has no effects on the downstream mean velocity and limited effect on the streamwise fluctuating velocity profiles, however the spanwise fluctuations are weakened. The mean velocity and fluctuating velocity profiles of the inlet-fed DNS case with mean flow up to $y^+ = 17$ at the inlet are presented in Fig. 6. The inlet without fluctuations up to $y^+ = 17$ has negligible effects on the downstream mean velocity profile as shown in Fig. 6a. In Fig. 6b, the velocity fluctuations at the start of the domain are removed. Downstream of the inlet, the streamwise and wall-normal fluctuations are evolved and collapse with the baseline case. However, the spanwise fluctuation is weakened.

The fluctuating components are replaced with the mean flow up to $y^+ = 35$ at the inlet of inlet-fed DNS and the results are presented in Fig. 7, streamwise extent is doubled ($L_x = 8\pi$) for this case. Fig. 7a shows the lack of convergence of the mean velocity profile in the inner region. In Fig. 7b, the fluctuating velocity profiles shows that the fluctuations are removed up to $y^+ = 35$ at the inlet and the streamwise fluctuations are recovered and become stronger than that of the baseline as the flow propagates downstream, even over an extended domain of $x/h = 8\pi$. However, while the streamwise fluctuations are energized, the spanwise fluctuation is weakened. Even at $x/h = 8\pi$, the streamwise fluctuations near the centreline of the channel are underpredicted.

The $u_{rms}$ contours of inlet-fed DNS are presented in Fig. 8. Fig. 8a shows the flow recovered quickly when the fluctuations are removed up to $y^+ = 5$. The inlet without fluctuations up to $y^+ = 17$ corresponds to the disruption of the near-wall peak and the inlet without fluctuations up to $y^+ = 35$ has removed much of the near-wall cycle [17]. Fig. 8b and 8c shows the recovery of near-wall peak but also a drop in the fluctuations in the outer-flow that lags behind the modification to the inlet at $x/h \approx 2\pi$ for $y^+ = 17$ and for $y^+ = 35$ it gradually propagates away from the wall due to the disruption of the near-wall cycle.
Figure 6: The statistical velocity profiles of inlet-fed DNS with mean flow up to \( y^+ = 17 \) at inlet (a) Mean velocity profile. Inlet-fed DNS case (Averaged over time and spanwise direction at \( x/h \) location); Baseline case (black line) and DNS of Hoyas and Jimenez [16] (dashed red line) in viscous units. (b) Root-mean-squared fluctuating velocity profiles. (\( u_{rms}: \bullet, v_{rms}: \ast, w_{rms}: \times \))

Figure 7: The statistical velocity profiles of inlet-fed DNS with mean flow up to \( y^+ = 35 \) at inlet (a) Mean velocity profile. Inlet-fed DNS case (Averaged over time and spanwise direction at \( x/h \) location); Baseline case (black line) and DNS of Hoyas and Jimenez [16] (dashed red line) in viscous units. (b) Root-mean-squared fluctuating velocity profiles. (\( u_{rms}: \bullet, v_{rms}: \ast, w_{rms}: \times \))

3.2. Spanwise pre-multiplied energy spectra

In Fig. 9, the energy spectra for the inlet-fed DNS case with mean flow up to \( y^+ = 5 \) at the inlet is presented. The flow scales near the wall are removed at the inlet of the domain as shown in Fig. 9a. The flow evolved downstream of inlet and at 1/16 of the domain \( (x/h \approx \pi/4) \), the flow scales are recovered, and the inlet-fed DNS is able to maintain the energy at resolved scales as seen in Fig. 9b. This suggests that 1/16 of the domain length \( (x/h \approx \pi/4) \) is sufficient to recover the flow scales for the inlet with no fluctuations up to \( y^+ = 5 \). Fig. 10 shows the energy spectra for the inlet-fed DNS case with mean flow up to \( y^+ = 17 \) at the inlet. The flow scales near the wall are removed at the inlet of the domain as shown in Fig. 10a. In Fig. 10b, the large scales are resolved at half of the domain \( (x/h \approx 2\pi) \) and the position of the peak collapse...
Figure 8: $u_{rms}$ contours of Inlet-fed DNS. Mean flow at the inlet up to a) $y^+ = 5$, b) $y^+ = 17$ and c) $y^+ = 35$.

with the baseline case, however, the near-wall flow scales are not recovered completely. Near the exit of the domain ($x/h \approx 4\pi$), the flow has fully relaxed, and the inlet-fed DNS is able to maintain the flow scales as observed in Fig. 10c, but the the magnitude of the peak is higher than the baseline case. The energy spectra for inlet-fed DNS with mean flow up to $y^+ = 35$ is shown in Fig. 11. The fluctuations at the inlet are removed up to $y^+ = 35$ as shown in Fig. 11a. In Fig. 11b, at half of the domain ($x/h \approx 4\pi$), the flow scales in the near-wall region is not recovered completely whereas in the inlet without fluctuations up to $y^+ = 17$, the small scales are recovered at $x/h \approx 4\pi$. Fig. 11c shows the flow scales are recovered near the exit of the domain $x/h \approx 8\pi$. The position of the peak collapse with the baseline case, however, the magnitude of the peak is higher than the baseline case.
Figure 9: The pre-multiplied spanwise spectra of the streamwise velocity, $k_z\phi_{uu}$, as a function of $y$ at various streamwise locations, mean flow up to $y^+ = 5$ at the inlet. (a) $x/h \approx 0$, (b) $x/h \approx \pi/4$. The contours are 0.1 and 1.0 of the maximum value of $k_z\phi_{uu}$, Baseline case (black dotted line), Inlet-fed DNS case (contour).

Figure 10: The pre-multiplied spanwise spectra of the streamwise velocity, $k_z\phi_{uu}$, as a function of $y$ at various streamwise locations, mean flow up to $y^+ = 17$ at the inlet. (a) $x/h \approx 0$, (b) $x/h \approx \pi$ and (c) $x/h \approx 4\pi$. The contours are 0.1 and 1.0 of the maximum value of $k_z\phi_{uu}$, Baseline case (black dotted line), Inlet-fed DNS case (contour).
Figure 11: The pre-multiplied spanwise spectra of the streamwise velocity, $k_z \phi_{uu}$, as a function of $y$ at various streamwise locations, mean flow up to $y^+ = 35$ at the inlet. (a) $x/h \approx 0$, (b) $x/h \approx 4\pi$ and (c) $x/h \approx 8\pi$. The contours are 0.1 and 1.0 of the maximum value of $k_z \phi_{uu}$, Baseline case (black dotted line), Inlet-fed DNS case (contour).

3.3. Velocity correlations
The velocity correlations are plotted in Fig. 12 to compare the structure characteristics between the inlet-fed DNS and baseline case. The correlation between two fluctuating velocity $u_i$ and $u_j$ is defined as

$$\rho_{ij} = \frac{\langle u_i(x, y)u_j(x, y)\rangle}{\sigma_{u_i}\sigma_{u_j}}\quad (1)$$

where $u_i$ and $u_j$ are fluctuating velocities of inlet-fed DNS and baseline case respectively. $\sigma$ is the standard deviation. The correlation $\rho_{ij}$ is averaged over time and spanwise direction. In Fig. 12a, the fluctuations are removed at the inlet up to $y^+ = 5$. It shows the Inlet-fed DNS represents the same structure as the baseline case. Fig. 12b shows the flow structures are not maintained when the fluctuations are removed at the inlet up to $y^+ = 17$. The correlation between the inlet-fed DNS and baseline case is weakened further when the fluctuations are removed at the inlet up to $y^+ = 35$ as shown in Fig. 12c. The statistics of the flow are almost recovered as shown in Fig. 6 and 7, however, the reduced correlation between the two cases shows that the removal of the near-wall peak is enough to remove correlation in near-wall and outer flow as it evolves downstream. This effect is even larger on the wall-normal fluctuations. This indicates that the inlet-fed DNS may not represent the same evolution of large scale structures present in
the inlet when the near-wall peak is removed. A scale by scale comparison of the inlet-fed DNS and baseline case will be required to explore this further.

4. Conclusion
In this study, the effect of limited near-wall data on the turbulence statistics and spanwise energy spectra of the streamwise velocity were investigated for inlet-fed DNS. The influence of limited near-wall data on the convergence of mean and streamwise fluctuating velocity profiles is less significant when the fluctuations are removed at the inlet up to $y^+ = 5$. However the spanwise fluctuations are weakened. The inlet without fluctuations up to $y^+ = 17$ has negligible effect on the mean velocity profile and the streamwise fluctuations are evolved downstream of the inlet. The streamwise fluctuations are over predicted and the redistribution of turbulent kinetic energy to the spanwise fluctuation is weakened when the fluctuations are removed at the inlet up to $y^+ = 35$ and therefore shows a lack of convergence. The spanwise energy spectra suggest that at 1/16 of the domain length ($x/h \approx \pi/4$) the flow scales are recovered. At half of the domain ($x/h \approx 2\pi$), the flow scales in the inner region are not recovered completely when the fluctuations are removed at the inlet up to $y^+ = 17$. Near the exit of the domain ($x/h \approx 4\pi$), the inlet-fed DNS is able to maintain the flow scales but the peak of the energy spectra is higher than the baseline case. This suggests that the streamwise extent $L_x = 4\pi$ is not sufficient. When the fluctuations are removed at the inlet up to $y^+ = 35$, the flow scales are not fully recovered near the exit.
of the domain \( x/h \approx 8\pi \), and the peak of the energy spectra is higher than the baseline case. The inlet without fluctuations up to \( y^+ = 17 \) corresponds to the distruption of the near-wall peak and much of the near-wall cycle is removed for inlet without fluctuations up to \( y^+ = 35 \). This suggests that despite maintaining the zero modes and hence the mean shear stress in the inlet, failing to capture the structure of the near-wall cycle results in a significant deficit in the production of turbulent kinetic energy which ultimately leads to a decrease in the turbulent fluctuation throughout the channel, until the near-wall cycle is re-established. The correlation between the inlet-fed DNS and baseline case indicates that the inlet-fed DNS while recovering the energy and statistics of the flows, is only able to maintain the same instantaneous structure of baseline when the near-wall peak is captured by the inlet. While this is not necessarily a requirement for the use of experimental data as an appropriate inlet to a DNS, it does indicate that maintaining the near-wall structure is important if one intends the flow to maintain the same evolution.

Acknowledgements
The support of the Australian Research Council (ARC) and the computational resources provided through NCMAS by the Australian National Computational Infrastructure and the Pawsey Supercomputing Centre, and PRACE supported by the European Union, for this work is gratefully acknowledged. This work was also funded in part by the Coturb program of the European Research Council. We are grateful to Luca Guastoni for his careful reviewing of the manuscript.

References
[1] Atkinson C, Coudert S, Foucaut J, Stanislas M, and Soria J. The accuracy of tomographic particle image velocimetry for measurements of a turbulent boundary layer. *Exp. Fluids*, 50:1031–1056, 2011.
[2] Sillero J, Jiménez J, and Moser R D. Two-point statistics for turbulent boundary layers and channels at Reynolds numbers up to \( \delta^+ = 2000 \). *Phys. Fluids*, 26, 2014.
[3] Lund T S, Wu X, and Squires K D. Generation of turbulent inflow data for spatially-developing boundary layer simulations. *J. Comput. Phys.*, 140:233–258, 1998.
[4] Kim J, Moin P, and Moser R. Turbulence statistics in fully developed channel flow at low Reynolds number. *J. Fluid Mech.*, 177:133–166, 1987.
[5] Abe H, Kawamura H, and Matsuo Y. Direct numerical simulation of a fully developed turbulent channel flow with respect to the Reynolds number dependence. *J. Fluid Eng.–T. ASME*, 123:382–393, 2001.
[6] Lozano-Durán A and Jiménez J. Effect of the computational domain on direct simulations of turbulent channels up to \( Re_\tau = 4200 \). *Phys. Fluids*, 26:1–7, 2014.
[7] Atkinson C, Buchmann NA, Amili O, and Soria J. On the appropriate filtering of PIV measurements of turbulent shear flows. *Exp. Fluids*, 55, 2014.
[8] N Jarrin, S Benhamadouche, D Laurence, and R Prosser. A synthetic-eddy-method for generating inflow conditions for large-eddy simulations. *Int. J. Heat Fluid Flow*, 27:585–593, 2006.
[9] M Klein, A Sadiki, and J Janicka. A digital filter based generation of inflow data for spatially developing direct numerical or large eddy simulations. *J. Comput. Phys.*, 186:652–665, 2003.
[10] L Di Mare, M Klein, WP Jones, and J Janicka. Synthetic turbulence inflow conditions for large-eddy simulation. *Phys. Fluids*, 18:025107, 2006.
[11] Fukami K, Nabae Y, Kawai K, and Fukagata K. Synthetic turbulent inflow generator using machine learning. *Phys. Rev. Fluids*, 4:064603, 2019.
[12] Kim J and Moin P. Application of a fractional-step method to incompressible Navier–Stokes equations. *J. Comput. Phys.*, 59:308–323, 1985.
[13] Wray A. A. Minimal storage time-advancement schemes for spectral methods. *NASA Tech. Rep.*, #MS 202, 1990.
[14] Lozano-Durán A and Bae H J. Turbulent channel with slip boundaries as a benchmark for subgrid-scale models in LES. *Center Turb. Res.–Ann. Res. Briefs*, 2016.
[15] Orlandi P. Fluid flow phenomena: A numerical toolkit. *Kluwer*, 2000.
[16] Hoyas S and Jiménez J. Reynolds number effects on the Reynolds-stress budgets in turbulent channels. *Phys. Fluids*, 20:1–9, 2008.
[17] Jiménez J and Pinelli A. The autonomous cycle of near-wall turbulence. *J. Fluid Mech.*, 389:335–359, 1999.