Studying the microlenses mass function from statistical analysis of the caustic concentration.

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Abstract. The statistical distribution of caustic crossings by the images of a lensed quasar depends on the properties of the distribution of microlenses in the lens galaxy. We use a procedure based in Inverse Polygon Mapping to easily identify the critical and caustic curves generated by a distribution of stars in the lens galaxy. We analyze the statistical distributions of the number of caustic crossings by a pixel size source for several projected mass densities and different mass distributions. We compare the results of simulations with theoretical binomial distributions. Finally we apply this method to the study of the stellar mass distribution in the lens galaxy of QSO 2237+0305.

1. Introduction.
Fluctuations induced by microlensing in the observed light curves of quasars contain information about the lensing objects (masses, density), about the unresolved source structure (size of the continuum or broad line regions of the quasar, brightness profile of the quasar) and about the lens system (transversal velocity). So from a comparison between observed and simulated quasar microlensing one can obtain information about the physical parameters of interest. These conclusions can only be done in a statistical sense [1].

Thus, simulating magnification maps for some values of the physical parameters of interest we can try to calculate the probability that microlensing magnification takes a value determined from observations. This statistical analysis faces a problem: experimental errors and sources of variability other than microlensing can significantly affect the data and results.

To solve this problem we propose to simplify it reducing the microlensing effect to a series of discrete events, the caustic crossings. If the source size is small enough (X-ray emitting source) each caustic crossing will appear as a single event. In addition, caustic crossings are events of high magnification (little affected by measurement errors), and they are difficult to mistake with other kind of variability.
2. Statistical analysis of the caustics concentration based on caustic crossing counts.

In figure 1 we can see a magnification map. The straight line corresponds to the path of a pixel size source. In the same figure we have represented the source’s light curve. Whenever the source crosses a caustic a large increase in the magnification that only can be associated with the phenomenon of caustic crossing is produced. That is, for a source of this size the light curve would provide direct information of an observable very interesting from a statistical point of view: the number of caustic crossings. One source of this size corresponds, in practice, to the X-ray emission.

The distribution of caustics in the magnification maps depends on the characteristics of the distribution of stars and dark matter in the lensing galaxy. In this work, we study how does the existence of a range of masses in the distribution of stars affect to the caustics concentration.

The statistical analysis of the caustics spatial distribution is based on the following steps: (i) simulate magnification maps for different densities of matter and different mass distribution, (ii) identify the caustic curves, (iii) count the number of caustics detected in a one-dimensional window of certain size in pixels for each axis, (iv) estimate the probability of detecting a caustic in a pixel for each axis and (v) compare the experimental distributions obtained in simulations with the theoretical binomial distribution.

We have used the method of Inverse Polygon Mapping [2] to carry out steps (i) and (ii).

3. Application to QSO 2237+0305
We have simulated magnification and caustic maps for the four images of QSO 2237+0305. We have considered two extreme cases in the stars mass distribution: stars distributed in a range of masses (from 0.01 to 1 solar masses with an m^{-1} density law; hypothesis I) and simple distribution of identical stars of one solar mass (hypothesis II).

We use the same simulation for images A and B because their convergence and shear are similar.
Looking at Figure 2 we can see that for the four images the number of caustics is larger when the microlenses are distributed in a range of masses.

The probabilities of detecting a caustic in a pixel for each axis are shown in table 1:

|       | P_X  | P_Y  |
|-------|------|------|
| A and B (hypothesis II) | 0.00664 | 0.01204 |
| A and B (hypothesis I)  | 0.01088 | 0.01745 |
| C (hypothesis II)       | 0.00515 | 0.00895 |
| C (hypothesis I)        | 0.00844 | 0.01440 |
| D (hypothesis II)       | 0.00688 | 0.01100 |
| D (hypothesis I)        | 0.02126 | 0.03135 |

We observe that the probability is higher when the stars are distributed in a range of masses than in the single mass case. The highest probability corresponds to image D when the stars are distributed in a range of masses.

Figure 3. Image D, comparisons between experimental (dotted line) and theoretical
(continuous line) probability distributions in windows of (from left to right) 4, 40, 200 and 400 pixels. Top panels correspond to hypothesis II and bottom panels to hypothesis I. X axis in red, Y axis in Blue.

We have counted the number of caustics detected in one-dimensional windows of 4, 40, 200 and 400 pixels for each axis (1 pixel corresponds to 0.012 Einstein radii for a solar mass star) and we have calculated experimental and theoretical binomial probability distributions. Comparing the different distributions (Figure 3) we have derived the following results for image D:

- Differences between probability distributions are very significant. For example considering a 400 pixel size window for the X axis, the peak and centroid of the number count distribution correspond to 6 or 7 detections when the stars are distributed in a range of masses but only to one detection in the case of 1 solar mass stars.
- From an experimental point of view, a single measure of the number of caustics detected in a window of 400 pixels may be sufficient to distinguish between hypotheses I and II.
- To constrain the stellar mass distribution, the transversal velocity of the quasar (used to define the observing window in suitable Einstein radii units) is needed. Reciprocally, for a given distribution of stellar masses, the transversal velocity may be estimated from the caustic crossing distribution.

4. Conclusions.

Our study reveals that caustic crossing statistics can be significantly affected by the microlenses mass function. For the QSO 2237+0305D image, the detection of even a relatively small number of events (about ten) may allow to constrain the star mass distribution in the lens galaxy.

References
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