Exoplanetary searches with gravitational microlensing: polarization issues

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Abstract

There are different methods for finding exoplanets such as radial spectral shifts, astrometrical measurements, transits, timing etc. Gravitational microlensing (including pixel-lensing) is among the most promising techniques with the potentiality of detecting Earth-like planets at distances about a few
astronomical units from their host star or near the so-called snow line with a temperature in the range 0 – 100°C on a solid surface of an exoplanet. We emphasize the importance of polarization measurements which can help to resolve degeneracies in theoretical models. In particular, the polarization angle could give additional information about the relative position of the lens with respect to the source.

**Keywords:** Polarimetry; polarization; extra solar planets; microlensing; Stellar atmospheres

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1. Introduction

Already before the discovery of exoplanets Mao & Paczynski (1991) showed how efficient is gravitational microlensing as a tool to search for extrasolar planets, including the low mass ones, even at relatively large distances from their host stars. Later on, observations and simulations gave the opportunity to confirm the robustness of Mao & Paczynski (1991) conclusions. Exoplanets near the snow line may be also detected with this technique as it was shown, for instance, in Fig. 8 presented by Mao (2012). Moreover, in contrast with conventional methods, such as transits and Doppler shift measurements, gravitational microlensing gives a chance to find exoplanets not only in the Milky Way (Beaulieu et al., 2006; Dominik, 2010; Zakharov, 2009, 2011; Zakharov et al., 2010; Wright & Gaudi, 2013; Gaudi, 2012; Mao, 2012; Beaulieu et al., 2013), but also in nearby galaxies, such as the Andromeda galaxy (Ingrosso et al., 2009, 2011), so pixel-lensing towards M31 provides an efficient tool to search for exoplanets and indeed an exoplanet might have been already discovered in the PA-N2-99 event (An et al., 2004; Ingrosso et al., 2009). Since source stars for pixel-lensing towards M31 are basically red giants (and therefore, their typical diameters are comparable to Einstein diameters and the caustic sizes) one has to take into account the source finiteness effect (Pejcha & Heyrovský, 2009). In the case of relatively small size sources, the probability to have features due to binary lens (or planet around star) in the light curves is also small since it is proportional to the caustic area. Giant star sources have large angular sizes and relatively higher probability to touch caustics (Ingrosso et al., 2009).

In the paper we point out an importance of polarization observations for microlensing event candidates to support (or reject) microlensing model and
Figure 1: All exoplanets found with different techniques until November 1, 2013, see http://exoplanetarchive.ipac.caltech.edu/exoplanetplots/.

resolve degeneracies of binary (exoplanetary) microlens models.

2. Exoplanet Searches with Gravitational Microlensing

Since the existence of planets around lens stars leads to the violation of circular symmetry of lens system and, as a result, to the formation of fold and cusp type caustics (Schneider, Ehlers & Falco, 1992; Zakharov, 1995; Petters, Levine & Wambsganss, 2001), one can detect extra peaks in the microlensing light curve due to caustic crossing by the star source as a result of its proper motion.

A list of exoplanets detected with microlensing searches toward the Galactic bulge is given in Table 1 (see, Bennett et al. (2006, 2008); Bennett (2009); Dong et al. (2009a,b); Mao (2012); Kains et al. (2013)). For some planetary systems two probable regions for the planet-to-star distance are given
Table 1: Exoplanets discovered with microlensing. 24 exoplanets have been found in 22 systems, in particular, there are two exoplanets in OGLE-2006-BLG-109L (lines 5,6) and there are two exoplanets in OGLE-2012-BLG-0026 (lines 18,19), see references: [1] Bond et al. (2004); Bennett et al. (2006); [2] Udalski et al. (2005); Dong et al. (2009a); [3] Beaulieu et al. (2006); [4] Gould et al. (2006); [5] Gaudi (2008); [6] Gaudi (2008); [7] Sumi et al. (2010); [8] Bennett et al. (2008); [9] Dong et al. (2009b); [10] Janiczak et al. (2010); [11] Miyake et al. (2011); [12] Batista et al. (2011); [13] Muraki et al. (2011); [14] Yee et al. (2012); [15] Bachelet et al. (2012); [16] Bennett et al. (2012); [17] Kains et al. (2013); [18] Han et al. (2013a); [19] Han et al. (2013b); [20] Han et al. (2013b); [21] Tsapras et al. (2013); Poleski et al. (2013); [22] Furusawa et al. (2013); [23] Choi et al. (2013); [24] Choi et al. (2013).

| #  | Star Mass ($M_\odot$) | Planet Mass | Star–planet Separation (AU) | Reference |
|----|----------------------|-------------|-----------------------------|-----------|
| 1  | $0.63^{+0.07}_{-0.09}$ | $830^{+250}_{-190}M_\odot$ | $4.3^{+2.5}_{-0.8}$ | [1] |
| 2  | $0.46 \pm 0.04$      | $(1100 \pm 100)M_\odot$  | $(3.6 \pm 0.2)$          | [2] |
| 3  | $0.22^{+0.21}_{-0.11}$ | $5.5^{+5.5}_{-2.5}M_\odot$ | $2.6^{+1.5}_{-0.6}$     | [3] |
| 4  | $0.49^{+0.23}_{-0.29}$ | $13^{+6.6}_{-8.6}M_\odot$  | $2.7^{+1.7}_{-1.4}$    | [4] |
| 5  | $0.51^{+0.05}_{-0.04}$ | $(230 \pm 19)M_\odot$     | $(2.3 \pm 0.5)$        | [5] |
| 6  | $0.51^{+0.05}_{-0.04}$ | $(86 \pm 7)M_\odot$      | $4.5^{+2.1}_{-1.0}$     | [6] |
| 7  | $0.64^{+0.21}_{-0.26}$ | $20^{+7}_{-5}M_\odot$     | $3.3^{+1.4}_{-0.8}$     | [7] |
| 8  | $0.084^{+0.015}_{-0.012}$ | $3.2^{+5.2}_{-1.8}M_\odot$ | $0.66^{+0.19}_{-0.14}$ | [8] |
| 9  | $0.30^{+0.19}_{-0.12}$ | $260.54^{+165.22}_{-101.85}M_\odot$ | $0.72^{+0.38}_{-0.16}/6.5^{+3.2}_{-1.2}$ | [9] |
| 10 | $0.67 \pm 0.14$      | $28^{+58}_{-23}M_\odot$  | $1.4^{+0.7}_{-0.3}$     | [10] |
| 11 | $0.38^{+0.34}_{-0.18}$ | $50^{+24}_{-18}M_\odot$  | $2.4^{+1.2}_{-0.6}$     | [11] |
| 12 | $0.19^{+0.30}_{-0.12}$ | $2.6^{+1.2}_{-1.6}M_J$   | $1.8^{+0.9}_{-0.7}$     | [12] |
| 13 | $0.56 \pm 0.09$      | $10.4 \pm 1.7M_\odot$   | $3.2^{+1.9}_{-0.5}$     | [13] |
| 14 | $0.44^{+0.27}_{-0.17}$ | $2.4^{+1.2}_{-1.0}M_J$   | $1.0 \pm 0.1/3.5 \pm 0.5$ | [14] |
| 15 | $0.67^{+0.33}_{-0.13}$ | $1.5^{+0.8}_{-0.3}M_J$   | $2^{+3}_{-1}$           | [15] |
| 16 | $0.75^{+0.33}_{-0.41}$ | $3.7 \pm 2.1M_J$        | $8.3^{+4.5}_{-2.7}$     | [16] |
| 17 | $0.26 \pm 0.11$      | $0.53 \pm 0.21M_J$      | $2.72 \pm 0.75/1.50 \pm 0.50$ | [17] |
| 18 | $0.82 \pm 0.13$      | $0.11 \pm 0.02M_J$      | $3.82 \pm 0.30$        | [18] |
| 19 | $0.82 \pm 0.13$      | $0.53 \pm 0.21M_J$      | $4.63 \pm 0.37$        | [19] |
| 20 | $0.022 \pm 0.002$    | $1.88 \pm 0.19M_J$      | $0.88 \pm 0.03$        | [20] |
| 21 | $0.44 \pm 0.07$      | $2.73 \pm 0.43M_J$      | $3.45 \pm 0.26$        | [21] |
| 22 | $0.11 \pm 0.01$      | $9.2 \pm 2.2M_\odot$    | $0.92 \pm 0.16$        | [22] |
| 23 | $0.025 \pm 0.001$    | $9.4 \pm 0.5M_J$        | $0.19 \pm 0.01$        | [23] |
| 24 | $0.018 \pm 0.001$    | $7.2 \pm 0.5M_J$        | $0.31 \pm 0.01$        | [24] |

due to the planet and star-lens parameter degeneracy (Dominik, 1999; Bennett, 2009), see rows 9, 14, 17 in Table 1. Reports about these discoveries
were published by Bond et al. (2004); Udalski et al. (2005); Beaulieu et al. (2006); Gould et al. (2006); Gaudi (2008); Bennett et al. (2008); Dong et al. (2009a,b); Janczak et al. (2010); Miyake et al. (2011); Batista et al. (2011); Muraki et al. (2011); Yee et al. (2012); Bachelet et al. (2012); Bennett et al. (2012); Kains et al. (2013); Han et al. (2013a,b); Tsapras et al. (2013); Poleski et al. (2013); Furusawa et al. (2013); Choi et al. (2013). In these searches we have a continuous transition from massive exoplanets to brown dwarfs, since an analysis of the anomalous microlensing event, MOA-2010-BLG-073 has been done by Street et al. (2013), where the primary of the lens is an M-dwarf with $M_{L1} = 0.16 \pm 0.03 M_{\odot}$ while the companion has $M_{L2} = 11.0 \pm 2.0 M_{J}$, at a perpendicular distance around $1.21 \pm 0.16$ AU from the host star, so the low mass component of the system is near a boundary between planets and brown dwarfs.

It is remarkable that the first exoplanet has been discovered by the MOA-I collaboration with only a 0.6 m telescope (Bond et al., 2004; Bennett, 2009). This microlensing event was also detected by the OGLE collaboration, but the MOA observations with a larger field of view CCD, made about 5 exposures per night for each of their fields. This was an important advantage and shows that even observations with modest facilities (around 1 meter telescope size and even smaller) can give a crucial contribution in such discoveries. Until now four super-Earth exoplanets (with masses about $10 M_{\oplus}$) have been discovered by microlensing (see Table 1 and Fig. 1), showing that this technique is very efficient in detecting Earth mass exoplanets at a few AU from their host stars, since a significant fraction of all exoplanets discovered with different techniques and located in the region near the so-called snow line (or the habitable zone) found with gravitational microlensing. Some of these exoplanets are among the lightest exoplanets see lines 3 and 8 in Table 1. For comparison, Doppler shift measurements help to detect an Earth-mass planet orbiting our neighbor star a Centauri B. The planet has an orbital period of 3.236 days and is about 0.04 AU from the star (Dumusque et al., 2012). Recently, a sub-Mercury size exoplanet Kepler-37b has been discovered with a transit technique (Barclay et al., 2013). It means that the existence of

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1According to the definition of a "planet" done by the working group of the International Astronomical Union on February 28, 2003 has the following statement: "... Objects with true masses below the limiting mass for thermonuclear fusion of deuterium (currently calculated to be 13 Jupiter masses for objects of solar metallicity) that orbit stars or stellar remnants are "planets" (no matter how they formed)."
cool rocky planets is a common phenomenon in the Universe (Beaulieu et al., 2006; Dominik, 2006; Dominik et al., 2006). Moreover, recently, Cassan et al. (2012) claimed that around 17% of stars host Jupiter-mass planets $(0.3-10 \, M_J)$, cool Neptunes $(10-30 \, M_{\oplus})$ and super-Earths $(10-30 \, M_{\oplus})$ have relative abundances per star in the Milky Way such as 52% and 62%, respectively. Analysis of Kepler space telescope data also shows that a significant fraction of all stars has to have exoplanets (Fressin et al., 2013).

Clearly, that if angular sizes of source stars are comparable with corresponding angular impact parameters and Einstein–Chwolson angles then light curves for such sources are different from the standard Paczyński light curve and gravitational lensing could be colorful since one has limb darkening and color distribution for extended background stars (Witt & Mao, 1994; Witt, 1995; Bogdanov & Cherepashchuk, 1995a,b, 1996). The extended source effects in gravitational microlensing enable studying the stellar atmospheres through their limb-darkening profiles and by modelling their microlensed spectra, see Loeb & Sasselov (1995); Sasselov (1996); Alcock et al. (1997); Sasselov (1998); Heyrovský et al. (2000); Cassan et al. (2004, 2006); Thurl et al. (2006); Zub et al. (2011) and references therein for details.

Pixel-lensing towards M31 may provide an efficient tool to search for exoplanets in that galaxy (Chung et al., 2006; Kim et al., 2007; Ingrosso et al., 2009), and indeed an exoplanet might be already discovered in the PA-N2-99 event (Ingrosso et al., 2009). Since source stars for pixel-lensing towards M31 are basically red giants (and therefore, their typical diameters are comparable to Einstein diameters and the caustic sizes) one has to take into account the source finiteness effect, similarly to microlensing in quasars (Agol & Krolik, 1999; Popović et al., 2006; Jovanović et al., 2008; Zakharov, 2009). As it is well known the amplifications for a finite source and for a point-like source are different because there is a gradient of amplification in respect of a source area. If the source size is rather small, the probability to produce features of binary lens (or planet around star) is proportional to the caustic area. However, giant stars have large angular sizes and relatively higher probability to touch planetary caustics (see Ingrosso et al. (2009), for more details).

### 3. Polarization curves for microlens systems with exoplanets

For extended sources, the importance of polarization measurements was pointed out by Bogdanov et al. (1996) for point-like lens and by Agol (1996) for binary lens (see also, Ignace, Bjorkman & Bryce (2006)). For point-like
lens polarization could reach 0.1% while for binary lens it could reach a few percent since the magnification gradient is much greater near caustics. It has been shown that polarization measurements could resolve degeneracies in theoretical models of microlensing events (Agol, 1996). Calculations of polarization curves for microlensing events with features in the light curves induced by the presence of an exoplanet and observed towards the Galactic bulge have been done (Ingrosso et al., 2012, 2013). We use simple polarization and limb darkening models developed by Chandrasekhar (1950), however, improved models are also developed taking into account radiative transfer in spectral lines, see for instance simulation results developed for Sun (Stenflo, 2006). Here we emphasize that measurements of then polarization angle could give additional information about the gravitational microlensing model.\(^2\) If polarization measurements are possible, in principle, one could measure polarization as a function of direction for an orientation of polarimeter and an instant for microlensing event. If a polarimeter is fixed one has an additional function of time to explain observational data, but if a polarimeter could be rotated, polarization is an additional function of direction at each instant. Such an information could help to resolve degeneracies and confirm (or disprove) microlensing models for observed phenomena. For instance, for a point-like lens the direction for the maximal polarization at the instant when an amplification is also maximal (which is perpendicular to the line connecting star and lens) may allow to infer the direction of lens proper motion, thus allowing to eventually pinpoint the lens in following observations. Even in the case of binary lens, the orientation of polarization vector corresponds to the orientation of the fold caustic (or more correctly to the tangent vector to the fold caustic at the intersection point with the path of source), provided the source size is small enough.

In Fig. 2, the light curve, the polarization curve and the polarization angle are shown for the OGLE-2005-BLG-169 event, where a binary system formed by a main sequence star with mass \(M_\odot \sim 0.5 \, M_\odot\) and a Neptune-like exoplanet with mass about 13 \(M_\oplus\) is expected from the light curve analysis (Gould et al., 2006). The event parameters are \(t_E = 42.27\) days, \(u_0 = 1.24 \times 10^{-3}, b = 1.0198, q = 8.6 \times 10^{-5}, \alpha = 117.0\) deg, \(\rho_* = 4.4 \times 10^{-4}\), where \(t_E, u_0, b, q, \alpha, \rho_*\) are the Einstein time, the impact parameter, the pro-

\(^2\)We call polarization angle the angle which corresponds to a direction with the maximal polarization.
jected distance of the exoplanet to the host star, the binary component mass ratio, the angle formed by the source trajectory and the separation vector between the lenses, and the source star size, respectively (all distances are given in $R_E$ units). The effect of the source transiting the caustic (see Gould et al. (2006) is clearly visible both in the polarization curve (see the middle panel in Fig. 2) and in the flip of the polarization angle (see the bottom panel). We would like to stress that the high peak magnification ($A \simeq 800$) of the OGLE-2005-BLG-169 event leading to $I$-magnitude of the source about 13 mag at the maximum gives the opportunity to measure the polarization signal for such kind of events by using present available facilities. In this case, polarization measurements might give additional information about the caustic structure, thus potentially allowing to distinguish among different models of exoplanetary systems. Recently, Gould et al. (2013) found that a variable gi-
ant star source mimics exoplanetary signatures in the MOA-2010-BLG-523S event. In this respect, we emphasize that polarization measurements may be helpful in distinguishing exoplanetary features from other effects in the light curves.

The polarization curve and the polarization angle for the MOA-2008-BLG-310Lb event is shown in Fig. 3. For this event it was expected the existence of a sub-Saturn exoplanet with mass $m = 74 \pm 17 \, M_{\oplus}$ (Janczak et al., 2010). The event parameters are $t_E = 11.14$ days, $u_0 = 3. \times 10^{-3}$, $b = 1.085$, $q = 3.31 \times 10^{-4}$, $\alpha = 69.33$ deg, $\rho_* = 4.93 \times 10^{-3}$. In particular, the event is characterized by large finite source effect since $\rho_*/u_0 > 1$, leading to polarization features similar to those of single lens events. Nevertheless, in this case we can see the variability in the polarization signal that arises when the source touches the first fold caustic at $t_1 \simeq t_0 - 0.07$ days, the
source enters the primary lens at $t_2 \simeq t_0 - t_E \sqrt{\rho_2^2 - u_0^2}$ days, the source exits the primary lens at $t_3 \simeq t_0 + t_E \sqrt{\rho_2^2 - u_0^2}$ days and touches the second fold caustic $t_4 \simeq t_0 + 0.09$ days (see also Fig. 4 in paper by Janczak et al. (2010)).

4. Conclusions

Now there are campaigns of wide field observations by Optical Gravitational Lensing Experiment (OGLE) (Udalski, 2003) and Microlensing Observations in Astrophysics (MOA) (Bond et al., 2001) and a couple of follow-up observations, including MicroFUN$^3$ and PLANET$^4$. It is important to note that small size (even less than one meter diameter) telescopes acting in follow-up campaigns contributed in discoveries of light Earth-like exoplanets and it is a nice illustration that a great science can be done with modest facilities. As it was shown by Ingrosso et al. (2012) polarization measurements are very perspective to remove uncertainties in exoplanet system determination and they give an extra proof for a conventional gravitational microlens model with suspected exoplanets. Moreover, an orientation of polarization angle near the maximum of polarizations (and light) curves gives information on direction of proper motion in respect to gravitational microlens system which could include exoplanet. Such an information could be important for possible further observations of the gravitational lens system in future.

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$^4$http://planet.iap.fr/.
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