Correlation of damage index and natural frequencies of SMRF steel structures

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Abstract. Damage Index quantifies the damage level on a scale of 0 to 1, which represents undamaged to severe damage. The decrease of natural frequency is an indication that damage to the structure is ongoing. The study of damage index and its correlation to the change of natural frequency was performed on three low-rise SMRF steel structures, two stories with one and two bays and six stories with two bays structures. The index was determined according to the Park Ang equation. The study was conducted numerically by using OpenSees software. A validation model based on an experimental study conducted by other researchers was carried out to ensure the FE model could represent the actual structure. Semi-cyclic pushover analysis was performed to obtain the parameters needed to calculate the index. It is found that severe damage with the damage index close to 1 causes stiffness degradation to the structure, which is resulting in a decrease of natural frequency. This study helps to understand the structural behavior and makes damage assessment using natural frequency usable in structural health monitoring.

1. Introduction
Performance-based is a design approach of a high-rise building located in moderate to high seismic zone. The approach let the damage to be occurred at non-structural and structural components due to severe earthquakes. The damages are purposely located at particular places to ensure that the damage does not let the building collapse, which is known as strong column weak beam. According to FEMA 356, 2000, structural performances are categorized on four levels, which are operational level, immediate occupancy, life safety, and collapse prevention. That performance reveals the damage level from light to severe damages. The building should be designed to meet the performance level.

When the earthquake hit the structural building, the intensity of damage should be quantified to identify structural health condition. Damage index is one of the methods to quantify damage that is proposed by Park and Ang [1]. The index is represented on a scale of 0 - 1, where 0 represents the structure’s condition that is not damaged (in its elastic range), and 1 describes the structural failure. The number can be used to assess vulnerability and to evaluate structural performance. An earlier study by Sentosa [2] found that damage index values increase as cracks start to form. This is also accompanied by a decrease in natural frequency.

Numerous amounts of research have been done to study the damage index of structures, but most of them were RC structures. This is because the damage is more obvious, which is indicated by the crack and crushing of the concrete. Studies of damage index on various steel structures such as eccentric-braced frame, concentric-braced frame, and ordinary moment-resisting frame have been conducted [3] – [5]. The study found that the change of natural frequency relates to the damage of the structure. The results agreed with the previous study conducted by Sentosa [2] on RC structures. None of those studies has been conducted on a special moment-resisting frame (SMRF). Reduced Beam Section (RBS) is part of SMRF, which is one of the lateral load resisting systems of a steel structure. The beam is designed as a weaker part to locate damage known as the plastic hinge, a place of energy dissipation. RBS is part
of SMRF. This research aimed to study the damage index of RBS and the correlation with a natural frequency.

Research about RBS was started after the Northridge earthquake in 1994; the concern lies in the detailing of the beams, columns, and joints to withstand flexural, axial, and shear as a result of the number of inelastic displacement cycles when an earthquake occurs. New detailing was proposed to move the location of the dissipative zone away from columns and joints. AISC-341 adopts a strong column weak beam design approach [6]. The aim is to achieve multi-story side-sway mechanisms, which are dominated by the formation of plastic hinges in the beams, and to avoid a single-story mechanism that forms plastic hinges at the top and bottom of the column on one floor. When this single-story mechanism happens, most of the structural drift will form on this floor and result in a very large P-delta at that location [7].

This research examined the effect of lateral load on the behavior of SMRF steel structures. The structural damages were quantified as damage index and the results were analyzed and associated with the changes in natural frequencies. Then, the relationship between the changes of damage index with the natural frequency of the building was identified. Damage of the structure was determined based on structural response analysis. The equation proposed by Park & Ang was adopted to determine the index. Structural damages lead to stiffness degradation and hence affect the natural frequency.

2. Methodology

The earthquake induces lateral and vertical excitation to the building. Hence, the building experiences a back-and-forth movement as if the cyclic load is applied. Severe earthquake initiates large deformation beyond the yielding stage where the damage appears. As mentioned earlier, the damage index quantifies the damage level. The value ranges from 0 - 1, where 0 and 1 indicate undamaged and collapsed structures, respectively. The damage index proposed by Park & Ang is a linear combination of failures caused by excessive deformation and the effects of repeated cyclic loading. The formula of damage index (D) is presented in equation 1.

\[ D = \frac{\delta_M}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \]  

(1)

\( \delta_M \) and \( \delta_u \): the maximum deformation due to earthquake monotonic loading,

\( \delta_u \) and \( Q_y \): yield displacement and yield strength

\( \int dE \): absorbed hysteretic energy

\( \beta \): non-negative coefficient, for RC = 0.05, steel = 0.025

The natural frequency is the main dynamic parameter of a structure which depends on its mass (m) and stiffness (k). Damage reduced the stiffness and decreased the natural frequency accordingly. Stiffness reduction is resulting from accumulated stiffness degradation of structural elements. The value of a building's natural frequency is a unique parameter because each building has different stiffness and mass, depending on the configuration and the structural material.

2.1 Validation Model

The validation model was carried out based on an experimental study conducted by Avgerinou [8] on two stories building, as shown in Figure 1. The building is categorized as SMRF, which employed Reduced Beam Section (RBS). The RBS increases ductility by weakening the beam flange at the intended location to let the plastic hinge form. The RBS property is presented in Figure 2 and Table 1 which refers to Eurocode 8-3 [9]. The test found that crack initiation and local buckling occurred in the flange of the RBS zone.
The validation model was performed using OpenSees and SAP2000 software. Both software were employed to study the damage index of SMRF. Modeling in OpenSees referred to a model which was created by Laura Eads [10]. Concentrated plasticity was employed to describe the RBS zone. It consists of primary nodes of the structure as well as additional nodes for the panel zone. Beams and columns were defined as elastic elements and zero-length elements were used to define the plastic hinges. The panel zone was constructed by forming twelve elastic elements known as the scissors model. It required rotational springs with predefined parameters [11,12] to represent the nonlinear behavior of the structure. The spring was defined as bilinear behavior, while the panel zone was defined as trilinear behavior [13].

SAP2000 was employed to determine the natural frequency after the structure experienced damage. Materials, section properties, and hinges were defined similarly to the model built in OpenSees. RBS was defined as the reduction in the beam flange area. Plastic hinges were defined similarly to rotational spring parameters used in OpenSees. Rigid end factor was used to describe the stiffness of the beam-column joints. The value was taken as zero for steel structures since it was assumed that the joint experiences a large shear deformation in the panel zone.

Figure 3 shows the results of the validation model. The result of cyclic loading obtained from OpenSees is close to the experiment and Abaqus modeling conducted by Avgerinou [8]. Results from SAP2000 model due to monotonic load was agree with the OpenSees result. To ensure the validity in the elastic stage, the envelope from cyclic loading obtained from the experiment and OpenSees results were compared, and as can be seen the initial stiffness was very close. Based on these figures, it can be concluded that the FE models built by both software could represent the experiment very well. Similarities in behavior were detected between the FE model and the experiment, and hence the FE model can be used for further study.

2.2 Parametric Test

The study was conducted on three different structural lays out. The first structure is validated structure which is the 2 story - 1 span structure as shown in Figure 1. The other two structures are 2 stories - 2 bay structure and the 6 stories – 2 bay structures as shown in Figure 4. All frames are taken from
All structures have different RBS detail. The 2 story – 2 bay uses circular and drill hole RBS whereas the 6 story - 2 spans used RBS with 30% cutting of the flange area. Pushover analysis was carried out by applying monotonic and semi-cyclic loading. The load and monitored displacement were located at the roof. The monotonic pushover was aimed to obtain the ultimate deformation, whereas the semi-cyclic was meant to obtain the maximum deformation and the dissipated energy in each cycle. The data were used to determine the damage index. SAP2000 was employed to obtain the natural frequency of each cycle. Hence, the relationship between changes in natural frequency and the damage index could be examined from each loading cycle.

3. Result and Analysis
The results of pushover analysis by applying monotonic and semi-cyclic load are presented in Figure 5 and Figure 6. The semi-cyclic pushover analysis was performed after the monotonic analysis was done. The magnitude of load applied on semi-cyclic was equal to the maximum base shear obtained from the monotonic results. The semi-cyclic was conducted in three cycles and the geometric change of steel structure at each cycle was then modeled in SAP2000 to find its corresponding natural frequency. Hence, the damage index was determined at each point of the semi-cyclic loading results.

![Figure 4](image)

**Figure 4** (a) 2 story – 2 bay structure; (b) 6 story – 2 bay structure

![Figure 5](image)

**Figure 5** Monotonic pushover analysis of (a) 2 story – 1 bay structure; (b) 2 story – 2 bay structure; (c) 6 story – 2 bay structure
3.1 Damage Index

The damage index was calculated at three stages. First, at the initial stage when the structure has no damage. Second, the yield stage when the structure experienced yielding. The third was calculated at each peak load based on the semi-cyclic loading curve. Referred to equation 1 the index needs the maximum deformation ($\delta_M$) and the absorbed energy. The maximum deformation was obtained from the displacement of each peak of the semi-cycle results, whereas the absorbed energy was equal to the loop area of each cycle based on the semi-cyclic pushover curve. Meanwhile, the maximum deformation value due to the monotonic lateral load ($\delta_u$) was the deformation of the structure before the collapse. The other parameter was the yield stress ($Q_y$) which was obtained from the monotonic loading curve. The summary of those values is presented in Table 2 and the corresponding damage index of three structures.

| Parameters | 2 Story – 1 Bay | 2 Story – 2 Bay | 6 Story – 2 Bay |
|------------|----------------|----------------|----------------|
| $Q_y$      | 59.9 kN        | 121.3 kN       | 1650 kN        |
| $\delta_u$| 232 mm         | 226 mm         | 877 mm         |
| Initial    | 0              | 0              | 0              |
| Yield      | 83             | 83             | 200            |
| $\delta_M$| Cycle 1 140 mm | Cycle 2 185 mm | Cycle 3 232 mm |
| Initial    | 0              | 0              | 0              |
| Yield      | 0              | 0              | 0              |
| $\int dE$  | Cycle 1 55.9 kNmm | Cycle 2 2486.9 kNmm | Cycle 3 447792 kNmm |
| Initial    | 0              | 0              | 0              |
| Yield      | 0.358          | 0.367          | 0.23           |
| $DI_{PA}$  | Cycle 1 0.604 - | Cycle 2 0.878 - | Cycle 3 1.073 |
| Cycle 3    | 1              | 1              | 1              |

As presented in Table 2, as the structural damage increases the damage index value is close to one. Structural damage is a function of $\delta_M$ and $\int dE$, which depend on loading history, while other parameters such as $\beta$, $\delta_u$, and $Q_y$ are independent of loading history. The value of $\frac{\delta_M}{\delta_u}$ tremendously affects the damage index value. If the value of $\delta_M$ approaches $\delta_u$, the damage index is close to one, which indicates that the structure is near collapse. As shown in Figure 6, there was no visible looping in the semi-cyclic

Figure 6 Semi-cyclic pushover analysis of (a) 2 story – 1 bay structure; (b) 2 story – 2 bay structure; (c) 6 story – 2 bay structure
curve of all structures; this caused an insignificant $\int dE$ value - affected the damage index value around 0.01. The small $\int dE$ value makes $\frac{\beta}{Q_y \delta_u} \int dE$ does not significantly affect the amount of the damage index.

3.2 Natural Frequency

SAP2000 was employed to identify two modal parameters, which were the natural frequency and the mode shapes of three structures. Based on the geometric model imported from OpenSees results, pushover analysis was carried out to obtain a monotonic curve and steps representing structural damage. The steps were selected to model the damage from nodal deformation and hinges conditions. Moment-rotation of the hinges was modified using partial fixity, which was obtained from the SAP2000 hinge states. When loaded, displacement occurs in the structure; therefore, the deformation of each node that occurs in each loading cycle needs to be considered in constructing the damage model of the structure.

**Table 3** Nodes deformation 2 story – 1 bay structure after cycle 1

| Story | Node | Disp. [mm] | Story | Node | Disp. [mm] |
|-------|------|------------|-------|------|------------|
| 1     | 5    | 23.9225    | 6     | 46.326|
| 10    |      |            | 14    |      |            |
| 16    |      | 23.9292    | 18    |      | 46.3231    |
| 9     |      |            | 13    |      |            |
| 8     |      |            | 12    |      |            |
| 15    |      | 23.9224    | 17    |      | 46.3258    |
| 7     |      |            | 11    |      |            |
| 2     |      | 23.9225    | 3     |      | 46.326    |

**Table 4** Nodes deformation 2 story – 1 bay structure after cycle 2

| Story | Node | Disp. [mm] | Story | Node | Disp. [mm] |
|-------|------|------------|-------|------|------------|
| 1     | 5    | 49.7317    | 6     |      | 97.686   |
| 10    |      |            | 14    |      |            |
| 16    |      | 49.7275    | 18    |      | 97.683   |
| 9     |      |            | 13    |      |            |
| 8     |      |            | 12    |      |            |
| 15    |      | 49.7316    | 17    |      | 97.6858  |
| 7     |      |            | 11    |      |            |
| 2     |      | 49.7317    | 3     |      | 97.686   |
Modal analysis was performed to obtain the modal parameters of the examined structure. In the end, five natural frequencies were obtained, which were the initial stage, the yield stage, cycle 1, cycle 2, and cycle 3. The results are shown in Figure 7.

| Story | Node | Disp. [mm] | Story | Node | Disp. [mm] |
|-------|------|------------|-------|------|------------|
| 5     | 77.6023 | 6       | 153.606 |
| 10    |        | 14      |       | 13    | |
| 16    | 77.5985 | 18      | 153.603 |
| 9     |        | 13      |       | 12    | |
| 15    | 77.6021 | 17      | 153.606 |
| 7     |        | 11      |       | 10    | |
| 2     | 77.6023 | 3       | 153.606 |

Table 5 Nodes deformation 2 story – 1 bay structure after cycle 3

Figure 9 Plastic hinge 2 story – 1 bay structure cycle 3

Figure 10 Comparison of natural frequency

Figure 11 Damage index versus natural frequency of (a) 2 story – 1 bay structure (b) 2 story – 2 bay structure (c) 6 story – 2 bay structure
As presented in Figure 7, the natural frequency decreases when the structure deforms. The results agree well with the theory that the frequency corresponds to the stiffness of the structural mass. As for steel structures, there is no change of mass since the failure mode is usually related to the frame's behavior. A deformed structure will experience a decrease in stiffness; therefore, the deformation will continue to accumulate until the structure finally collapses. The 6-story 2-span structure has the smallest natural frequency - 3.9 Hz. This is due to higher stories having a greater period where it is inversely proportional to the natural frequency. Hence, the higher stories, the smaller its natural frequency.

3.3 Correlation between Damage Index and Natural Frequency

Figure 8 presents the correlation between damage index and the natural frequency. The x-axis represents the base shear in kN, and the y-axis shows the ratio of the natural frequency to the initial condition and the damage index. The y-axis has a similar value with a scale of 0 – 1. Hence, the natural frequency of the entire structure in each cycle can be compared to the damage index. As can be seen, the damage index is inversely proportional to the natural frequency. The critical limit shown in Figure 8 is defined as the condition when the damage index increases drastically while the natural frequency does not decrease significantly [2]. It is marked with a red dot line. For both 2-story structures, the failure occurs when the structure has passed its ultimate strength, then there is a gradual decrease in the structure's ability to withstand lateral forces until it finally collapses. Meanwhile, the 6-story 2-spans structure collapses when it reaches its ultimate strength. Therefore, the location of the critical limit is different from the other two structures.

4. Conclusion

OpenSees and SAP2000 were used to determine the Park-Ang damage index and natural frequency of three steel SMRF structures. Pushover analysis was conducted to obtain the parameters needed to determine the index. The results found that the index increase along with the deformation of the structure, and the damage continues to accumulate and approaches the value of 1. The parameter \( \frac{\delta_M}{\delta_u} \) significantly affects the damage index. If the value of \( \delta_M \) approaches \( \delta_u \), the damage index closes to 1 where the structure near collapses. There was no visible looping in the semi-cyclic curve for all structures; this caused the \( \int dE \) value to be insignificant and affected the damage index value of around 0.01. The natural frequency of the initial condition of 2 stories with 1 bay and two bays are 7.9 Hz and 8.73, respectively. The 6-story has the lowest natural frequency, which was 3.9 Hz. The natural frequency increase with the number of spans and decrease with the number of levels.

The damage index is inversely proportional to the natural frequency. This occurs because the natural frequency depends on structural stiffness and mass. The decrease in structural stiffness causes a lower natural frequency. Greater deformation causes the index to increase by about 20% - 30% between the examined cycles, while the natural frequency significantly decreased after passing through the yield point on the 2-story 1-span structure, and at peak 1 to peak 2 on the 2-story 2-span structure and 6-story 2-span structure.

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