Electromagnetic vortex beam dynamics in degenerate electron-positron astrophysical plasmas

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Abstract For degenerate astrophysical electron-positron plasmas we have considered dynamics of electromagnetic beams carrying angular momentum. It is found for arbitrary level of degeneracy such a beam having the power exceeding a certain critical value breaks up into many filaments, eventually leading to the formation of stable spatial solitons keeping zero field in the center of the structure.

Keywords

1 Introduction

A wide class of astrophysical objects emit the electromagnetic (EM) waves in a broad frequency range. These waves interacting with matter surrounding the objects might lead to non-trivial observational features \cite{Carroll2010, Shapiro1971}. It is established that under certain conditions radiation generated in astrophysical environment can acquire orbital angular momentum (twisted radiation). A series of papers has been dedicated to the study of different aspects of the mentioned problem. In particular, \cite{Harwit} has considered the role of twisted radiation in different aspects: influence of the Kerr black hole on angular properties of radiation; search for extraterrestrial intelligence; considering astrophysical masers as probes for inhomogeneities of the interstellar medium and the study of pulsars and quasars. It is shown that in case of the Kerr black hole the photon orbital angular momentum might be generated by the partial "absorption" of the black hole's angular momentum. This effect has been used to measure the rate of rotation of the central black hole of the active galactic nuclei (AGN) M87 \cite{Tamburini}. Recently, non-linear dynamics of EM vortex solitons in the magnetospheric electron-positron-ion relativistic plasmas of AGN has been investigated \cite{Berezhiani}. Peculiarities of generation of vortex solitonary structures and their instability was studied and possible observational signatures has been discussed.

One should emphasize that electron-positron (e-p) pair plasma can be generated in a wide class of compact astrophysical objects: pulsars \cite{Sturrock1971}, AGN \cite{Guilbert1983}, and gamma ray bursts (GRB) \cite{Aksenov}. Under certain conditions the gravitational collapse results in charge separation, which might lead to extremely efficient pair creation characterized by density degeneracy with the e-p number density of the order of $10^{30}$--$10^{37}$ cm$^{-3}$ \cite{Aksenov, Han}. For this case the average distance between the closest particles becomes smaller than the de Broglie wavelength and the Fermi energy, $\epsilon_F = \hbar^2 (3\pi^2 n_0)^{2/3} / 2m_e$ is larger than the interaction energy, $\epsilon^2 n_0^{1/3}$, making the gas ideal \cite{Landau}. Here we use the following notations: $\hbar$ is the Planck’s constant, $n_0$ denotes the electron/positron number density and $e$ and $m_e$ are the electron’s charge and mass respectively. One can straightforwardly show that the Fermi gas is ideal if the following condition is satisfied: $n_0 >> 8m_e^2 e^6 / (9\pi^4 \hbar^6) \simeq 6.3 \times 10^{22}$ cm$^{-3}$. For even higher particle densities the Fermi energy becomes relativistic, $\epsilon_F = m_e c^2 \left( 1 + R_0^2 \right)^{1/2} - 1$, where $R_0 = (n_0 / n_e)^{1/3}$ and $n_e = m_e^2 c^3 / (3\pi^2 \hbar^3) \simeq 5.9 \times 10^{29}$ cm$^{-3}$ is the critical number density. The pair annihilation time-scale

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for such densities is small compared to the plasma oscillation periods, and therefore collective phenomena has enough time to develop (Berezhiani et al. 2013).

Different aspects of EM radiation propagating in highly degenerate electron-ion as well as e-p plasma has been investigated in a series of papers (Berezhiani et al. 2014; Goshadze et al. 2019; Berezhiani et al. 2015; Haas & Kourakis 2015; Mikaberidze et al. 2015).

In the present paper we consider non-linear dynamics of EM beams carrying orbital angular momentum propagating in highly degenerate relativistic e-p plasmas.

The paper is organised in the following way: in Sec. 2 we introduce the equations governing the self-guiding regime in the astrophysical e-p plasmas, numerically solve them and obtain results and in Sec. 3 we summarise them.

2 Theory and discussion

In this section we introduce the equations governing the process of self-guiding, we numerically solve them and discuss the obtained results.

By using the reductive perturbation method (Berezhiani & Shatashvili 2014) that the dynamics of EM beams propagating in degenerate e-p plasma is governed by the nonlinear Schrodinger equation (NSE) with saturating nonlinearity which in dimensionless form reads

\[ 2i \frac{\partial A}{\partial z} + \nabla^2_A + f(|A|^2) A = 0 \tag{1} \]

with

\[ f(|A|^2) = 1 - \frac{(1 - |A|^2)^{3/2}}{(1 - |A|^2 d)^{1/2}} \tag{2} \]

Here \( A \) is the slowly varying amplitude of the circularly polarized vector potential

\[ \frac{eA}{m_e c^2 R_0} = (1/2) (\hat{x} + i \hat{y}) A \exp(-i \omega_0 t - k_0 z) + c.c. \tag{3} \]

where \( \hat{x} \) and \( \hat{y} \) are unite vectors directed across the propagation of the EM beam \( z \). The field frequency \( \omega_0 \) and wave vector \( k_0 \) satisfy the dispersion relation

\[ \omega_0^2 = k_0^2 c^2 + 2 \omega_c^2 / \Gamma_0 \text{ , } \omega_c = (4 \pi e^2 n_0 / m_e)^{1/2} \text{ - is the plasma frequency and } \Gamma_0 = (1 + R_0^2)^{1/2} \text{ is the generalized relativistic factor where } R_0 = (n_0 / n_c)^{1/3}. \]

The dimensionless coordinates reads as \( z = (2 \omega_c / c \omega_0 \Gamma_0) z \), \( r_\perp = (\sqrt{2} \omega_e / c \sqrt{\Gamma_0}) r_\perp \) and the operator \( \nabla^2_A \) is the Laplacian in the \( r_\perp = (x, y) \) plane. In Eq. (2) \( d = R_0^2 / (1 + R_0^2) \ll 1 \) measures a level of degeneracy. For the weakly degenerate case \( R_0 << 1 \) \( d \approx R_0^2 \) while for the relativistic degeneracy \( R_0 >> 1 \) \( d \rightarrow 1 \).

In the system of Eqs. (1)-(2) it is assumed that plasma is highly transparent \( \omega_e / \omega_0 << 1 \) and \( \lambda << L_\perp << L_\parallel \) where \( \lambda \approx 2 \pi c / \omega_0 \) is the wavelength of the EM radiation, \( L_\perp \) and \( L_\parallel \) are the characteristic longitudinal and transverse spatial dimensions of the EM beam. This system describes the dynamics of strong amplitude narrow EM beams in e-p plasma with the arbitrary strength of degeneracy.

In an unmagnetized e-p plasma the EM pressure is equal for both the electrons and positrons and consequently modification of plasma density takes place without producing the charge separation. For the normalized electron/positron density \( N = N / n_0 \) we have the following expression

\[ N = \left( \frac{1 - |A|^2}{1 - |A|^2 d} \right)^{1/2} \tag{4} \]

From Eq.(3) it follows that our considerations remain valid provided that \( |A| < 1 \); the plasma density decreases in the area of the EM field localization and if at a certain point of this area one has \( |A| \rightarrow 1 \), then the plasma density becomes zero \( (N \rightarrow 0) \). Therefore, at that point the cavitation takes place (Berezhiani et al. 2021). Here we would like to remark that the condition \( |A| < 1 \) does not necessarily imply that the strength of the EM field is relativistic. In dimensions this condition reads as \( e |A| / (m_e c^2) < R_0 \) and for a weakly degenerate case \( R_0 << 1 \) the cavitation can take place even if the strength of the EM field is weakly relativistic. At this end we would like to emphasize that values of the parameter \( R_0 \) are bounded from below and can not be taken to be zero. Indeed, for the degenerate plasma the average energy of e-p particle interaction should be less than the Fermi energy. This condition implies that plasma density should be \( n_0 \geq e^6 m_e^2 / h^6 = 6.7 \times 10^{24} cm^{-3} \) and consequently \( R_0 >> 0.02 \) \( (d >> 4 \times 10^{-4}) \).

Based on the system of Eq.(1) in (Berezhiani & Shatashvili 2016) the authors demonstrated that under well defined conditions e-p plasma supports existence of fundamental (nodules) stable 2D solitonic structures for arbitrary values of the degeneracy parameter \( d \). The EM beam with above certain critical power can be trapped in self-guiding regime of propagation and subsequently formation solitonic structures takes place.

At first we consider 2D solitary wave solutions carrying vortices. On assuming that solutions in polar
coordinates are of the form $A = A_0 \exp(n \theta + ikz)$, where $r = \sqrt{x^2 + y^2}$, $\theta$ is the polar angle, Eq.(1) reduces to an ordinary differential equation the real valued amplitude $A_0$

\[
\frac{d^2 A_0}{dr^2} + \frac{1}{r} \frac{dA_0}{dr} - kA_0 - \frac{m^2}{r^2} A_0 + A_0 \left(1 - \frac{(1 - A_0^2)^{3/2}}{(1 - A_0^2d)^{3/2}}\right) = 0
\]

(5)

where $A_0$ is the real valued amplitude, $k$ is nonlinear propagation constant, and $m (\neq 0)$ is an integer known as the topological charge of the vortex.

For $0 < k < 1$ Eq.(4) admits an infinity of localized discrete bound states $A_{0n}(r)$ with $A_{0n}(0) = 0$ and $A_{0n}(r \to \infty) = 0$ with with the following asymptotic behavior: $A_{0n}(r \to 0) \sim r^n$ and $A_{0n}(r \to \infty) \sim \exp(-r \sqrt{k})/\sqrt{r}$. Here $n$ denotes number of zeros of eigenfunction for $r \neq 0$. In what follows we consider the lowest order, the ground state solutions, which have the node at the origin $r = 0$, reaches a maximum, and then monotonically decrease with increasing $r$. Such solutions are obtained numerically applying the shooting code for different level of degeneracy parameter $d$.

For $m = 1$ in Fig. 1 we present the profiles of ground state solutions versus the propagation constant, $k$, for the strongly degenerate case $d = 0.5$. Similar profiles of the solitons can be contained for arbitrary values of $d$ ($< 1$). The maxima of the field amplitudes $A_m << 1$ for $k \to 0$ while for $k \to k_{cr}(d) < 1$, $A_m \to 1$, i.e. the plasma cavitation takes place. Here $k_{cr}(d)$ depends on the level of degeneracy $d$, and, for instance $k_{cr}(0.01) \approx 0.44$, $k_{cr}(0.5) \approx 0.37$. Similar behavior of the solutions can be obtained for vortices with higher charge ($m = 2, 3, ...$); corresponding figures are not displayed here.

The power of single charged ($m = 1$) EM beam trapped in the vortex soliton modes ($P = 2\pi \int_0^\infty r A_0^2 dr$) is a growing function of $k$. Dependence of $P$ on the propagation constant $k$ for different level of plasma degeneracy $d = 0.01; 0.5; 0.9$ is exhibited in Fig.2. For $k \to 0$ ($A_m \to 0$), $P \to P_{cr}(d)$ where $P_{cr}(d)$ is a critical power. In case of weak degeneracy ($d = 0.01$) the critical power $P_{cr}(0) \approx 32.1$ while for relativistic degeneracy cases $P_{cr}(0.5) \approx 38.6$, $P_{cr}(0.9) \approx 46.1$. Thus, the necessary condition to form the vortex soliton is that the EM beam power must exceed $P_{cr}(d)$ while an upper bound of the power is related to the plasma cavitation that is taking place at $k \to k_{cr} (A_m \approx 1)$.

In dimensional units the critical power is expressed as

\[
P_d \approx 0.17 (\omega/\omega_e)^2 \Gamma_0 R_0^2 P_{cr} (d) \text{ [GW]}
\]

(6)
Here we show the contours of constant intensity beams for \( m = 1, 2 \) and \( d = 0.5 \).

Fig. 3

where \( P_d \) is the critical power measured in GW. For \( d = 0.01 (R_0 \simeq 0.1) \) and, for instance \( d = 0.5 (R_0 = 1) \) the corresponding densities of plasma are respectively \( n_0 \simeq 5.96 \times 10^{26} \text{cm}^{-3} \) and \( n_0 \simeq 5.96 \times 10^{29} \text{cm}^{-3} \). Since plasma is assumed to be transparent photon energies of EM beam should be in hard X-ray band \((\hbar \omega > (1 \text{KeV} - 29 \text{KeV}))\) while for the critical powers we have \( P_d = (5.4 - 0.6) \text{GW} \).

The stability of the vortex soliton solutions of NSE have been studied in the past for different kinds of saturating nonlinearity (Firth & Skryabin 1997; Skarka et al. 2001). It is well established that though \( dP/dk > 0 \) and the Vakhiton and Kolokolov criteria guarantees stability of solutions against small radial perturbations, for the symmetry breaking azimuthal perturbations the solitons are unstable. The instability causes the breakup of the structure into multiple filaments while number of filaments are usually \( 2m \).

The filaments must conserve total angular momentum \( (\sim mP) \) and also they can not fuse to due to topological reasons. These filaments carry zero topological charges \( (m = 0) \), they can eventually spiral about each other or fly off tangentially to the initial ring generating stable solitonic 2D structures. Our simulations demonstrate that similar scenario of instability development takes place for the degenerate e-p plasma described by Eqs.(1)-(2).

We carried out the numerical simulations to investigate the dynamics of EM vortex beam with parameters and shapes far from ground state vortex solitons. For this purpose an input vortex beam is assumed to be Gaussian \( A(z - 0, x, y) = (x + iy)^m A_1 \exp(-r^2/2D^2) \). The radial profile of beam intensity being zero at \( r = 0 \), reaches a maximum \( A_{\text{max}} = A_1 D^m \exp(m/2) \) at \( r_{\text{max}} = D \sqrt{m} \) and then exponentially decays with increasing \( r \). Here \( D \) is the characteristic width of the structure and \( A_1 \) is a constant which (along with \( D \)) determines the EM beam power \( P = \pi A_1^2 D^{2(m+1)}m! \). For numerical simulations it is comfortable to characterize input beam by its power and amplitude \( A_{\text{max}} < 1 \). If the power of the beam less than the critical one \( P < P_d \) the beam diffracts while for \( P > P_d \) breakup of the structure into filaments takes place for all levels of degeneracy parameter \( (0 < d < 1) \). In Fig. 3 contours of constant intensity of the beams are displayed for \( m = 1, 2 \) and \( d = 0.5 \). For both cases we assume that \( P = 42 \) and \( A_{\text{max}} = 0.3 \) implying that the vacuum diffraction length \( (z_{\text{dif}} = D^2) \) for single and double charges beams are respectively \( z_{\text{dif}} \simeq 55 \) and \( z_{\text{dif}} \simeq 40 \). One can see that in few diffraction length the single charged vortex beam breaks up into two while double charged beam into four filaments. These filaments carry zero topological charge \( (m = 0) \), flying off tangentially
to the initial ring like distribution of EM field intensity with subsequent formation of stable 2D spatial solitons.

3 Conclusion

Critical power of the EM beam carrying angular momentum has been found. It is shown that the vortex soliton solution with power above the critical value turns out to be unstable against symmetry breaking perturbations, leading to the formation of stable soliton solutions with zero topological charge, keeping zero field in the centre of the structure.

We demonstrated that in general, the EM beam with an initial Gaussian shape, having the power less than the critical one diffracts monotonically. However, if the power is overcritical the vortex beam breaks into filaments with the subsequent formation of stable solitary structures with an arbitrary level of degeneracy.

Self-guiding structure which is composed of beamlets are running away tangentially from the initial ring of field distribution, leaving the zero-field at the centre. In the course of propagation these beamlets form stable 2D spatial solitons.
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