Lattice QCD determination of states with spin $\frac{5}{2}$ or higher in the spectrum of nucleons

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Energies for excited isospin $\frac{1}{2}$ states that include the nucleon are computed using quenched, anisotropic lattices. Baryon interpolating field operators that are used include nonlocal operators that provide $G_2$ irreducible representations of the octahedral group. The decomposition of spin $\frac{5}{2}$ or higher states is realized for the first time in a lattice QCD calculation. We observe patterns of degenerate energies in the irreducible representations of the octahedral group that correspond to the subduction of the continuum spin $\frac{5}{2}$ or higher.

PACS numbers: 12.38.Gc 21.10.Dr

The theoretical determination of the spectrum of baryon resonances from the fundamental quark and gluon degrees of freedom is an important goal for lattice QCD. To date there have been studies of experimental ground state energies for different baryons [1-3] but only a few results for excited state energies have been reported. [4-9] No clear determination of states with spin $\frac{5}{2}$ or higher has been reported because nonlocal operators have not been used. In this work we find degenerate energies that occur in irreducible representations of the octahedral group corresponding to the subduction of the continuum spin $\frac{5}{2}$ or higher. This provides the first lattice QCD calculation that realizes the decomposition of spins greater than $\frac{3}{2}$.

Lattice correlation functions do not correspond to definite values of total angular momentum. However, they do correspond to definite irreducible representations (irreps) of the octahedral group when the source and sink operators transform accordingly. There are six double-valued irreps of the octahedral group: three for even-parity that are labeled with a $g$ subscript (gerade) and three for odd-parity that are labeled with a $u$ subscript (ungerade). They are: $G_1$, $H_2$, $G_{2g}$, $G_{1u}$, $H_u$, and $G_{2u}$.

Continuum values of total angular momenta are realized in lattice simulations by patterns of degenerate energies in the continuum limit that match the patterns for the subduction of spin $J$ to the double-valued irreps of the octahedral group. These patterns are shown in Table I. For example, a state in one of the $G_2$ irreps is a signal for the subduction of continuum spin $\frac{5}{2}$ or higher.

For spin $\frac{5}{2}$, there must be partner states in the $H$ and $G_2$ irreps that would be degenerate in the continuum limit. For spin $\frac{3}{2}$, there must be partner states in the $G_1$, $H$ and $G_2$ irreps and for spin $\frac{5}{2}$ there must be one partner in the $G_1$ irrep and two in the $H$ irrep. The partner states should be degenerate in the continuum limit where lattice spacing $a \rightarrow 0$.

This paper reports on work to determine the pattern of low-lying states in the isospin $\frac{1}{2}$ spectrum. We carry out an analysis in quenched lattice QCD using a moderately large number of three-quark operators. Work is in progress to use a very large number of operators. [12] Smearing reduces the couplings to short wavelength fluctuations of the theory and provides cleaner determinations of effective energies. This is important when a large array of interpolating field operators is used in order to implement the correlation-matrix method of Refs. [13, 14].

In order to determine suitable operators, including the nonlocal ones that are required for the $G_2$ irreps, we have developed sets of baryon operators that transform according to irreducible representations using an analytical method based on appropriate Clebsch-Gordan coefficients for the octahedral group. [15] An alternative, automated procedure that is developed in Ref. [16] provides very large sets of operators. Both methods provide equivalent results. In this work, we use for positive parity the three-quark operators defined in Tables VI, VII, and X of Ref. [15]. These comprise a complete set of quasilocal operators plus the simplest set of nonlocal operators that have one quark displaced relative to the other two quarks.

Negative-parity operators are obtained by applying the charge-conjugation transformation to the positive-parity operators. A three-quark operator that transforms according to irrep $\Lambda$ and row $\lambda$ of the octahedral group is related by charge conjugation to an operator that transforms according to irrep $\Lambda^c$ and row $\lambda^c$ and has opposite

| $\Lambda$ | $J = \frac{1}{2}$ | $\frac{3}{2}$ | $\frac{5}{2}$ | $\frac{7}{2}$ | $\frac{9}{2}$ | $\frac{11}{2}$ |
|----------|-----------------|-------------|-------------|-------------|-------------|-------------|
| $G_1$    | 1               | 0           | 0           | 1           | 1           | 1           |
| $H$      | 0               | 1           | 1           | 1           | 2           | 2           |
| $G_2$    | 0               | 0           | 1           | 1           | 0           | 1           |
of the effective energies and the use of PCT symmetry increases the statistics. In addition, we use anisotropic lattices with temporal lattice spacing \( a_t \) one-third of the spatial lattice spacing \( a_s \). The finer spacing increases the number of time-slices for extraction of energies before the signal for a high-energy state decays to the level of the noise.

In this work anisotropic lattices with two different volumes are used: 239 gauge field configurations are used for a \( 16^3 \times 64 \) lattice and 167 configurations are used for a \( 24^3 \times 64 \) lattice. Because of the use of PCT symmetry, the size of the statistical ensemble is effectively doubled. Gauge-field configurations are generated using the anisotropic, unimproved Wilson gauge action [18–21] in the quenched approximation with \( \beta = 6.1 \) and the pion mass is 490 MeV. For both lattices, the temporal lattice spacing corresponds to \( a_t^{-1} = 6.65(1) \) GeV [8] as determined from the string tension. Calculations are performed using the Chroma software. [22] Further details of the action and lattice parameters will be presented elsewhere.

We have extracted energies for isospin \( \frac{1}{2} \) and \( \frac{3}{2} \) channels by diagonalizing matrices of correlation functions formed from three-quark operators that share the same octahedral symmetry. Starting with a large number of operators, we first eliminate operators that have less influence on the determination of the effective energies and the eigenvectors \( \mathbf{v}^{(n)} \) of the low-lying states. Once a set of good operators is obtained, we form final matrices of correlation functions, diagonalize them and extract effective energies. This procedure yields solid results for energies of low-lying states and is more efficient than diagonalizing matrices of the largest dimension. We have obtained 17 energies for nucleonic states and 11 energies for delta baryon states. In this paper, we focus on the nucleon channel and the determination of spin.

Our lattice results for the spectrum are shown in Fig. 1. For positive parity, the nucleon state is very well determined at lattice energy \( a_t E = 0.193(3) \). It is an isolated \( G_{11} \) state corresponding to spin \( \frac{1}{2} \). Well above the nucleon is a group of three degenerate states (within errors) in the \( G_{11g}, H_g \) and \( G_{2g} \) irreps. The results on both lattice volumes are essentially the same, although the lowest \( G_{2g} \) state in the smaller volume has a larger error.

Energies of \( G_2 \) states are the most difficult to extract because the plateaus are relatively short-lived, signal-to-noise ratios are large, and the number of operators that we have used for the matrices of correlation functions is limited. The limitation on the number of operators is because we restrict the operators used in this work to ones that can be constructed from one-link displacements. In work that is in progress, more varied types of operators are used to obtain very larger numbers of \( G_2 \) operators, which is essential for determining states with higher energies.

Figure 2 shows effective masses obtained for the lowest energy \( G_{11g}, H_g \) and \( G_{2g} \) states. The horizontal lines indicate the time ranges used to fit the principal eigenvalues
$a^{(n)}(t, t_0)$ by a single exponential form. The resulting mean energy value and the error range are indicated by the lines.

Although there are substantial discretization errors with the quark action that is used, and they could contribute differently in the different irreps, clear patterns in the degeneracies emerge. Focusing on the group of three positive-parity states near lattice energy $a_tE = 0.36$ in Fig. 1, two interpretations are possible. A.) The group consists of a spin $\frac{1}{2}$ state and a spin $\frac{3}{2}$ state that are accidentally degenerate. In this case the $G_{1g}$ state corresponds to spin $\frac{1}{2}$ and the $H_g$ and $G_{2g}$ partner states correspond to the subduction of spin $\frac{3}{2}$. B.) The group consists of a single state with the degenerate $G_{1g}$, $H_g$ and $G_{2g}$ partner states corresponding to the subduction of spin $\frac{7}{2}$.

In the physical spectrum of positive-parity nucleon resonances, the lowest excited state, $N(1440, \frac{1}{2}^+)$, lies below all negative parity states. We do not find a signal for a positive-parity excitation that has lower energy than the negative-parity excitations at this quark mass. The next two excited nucleon states are essentially degenerate, namely, $N(1680, \frac{3}{2}^+)$ and $N(1710, \frac{3}{2}^+)$, each with a width of about 100 MeV. Spin $\frac{7}{2}$ states occur only at significantly higher energy (1990 MeV). Primarily because of the absence of spin $\frac{7}{2}$ in the low-lying spectrum, interpretation A.) of our lattice results is more consistent with the physical pattern of energies and spins. In the absence of $G_2$ operators, previous lattice studies have assumed that states obtained with $H$ irrep operators correspond to continuum spin $\frac{7}{2}$, [4, 6, 7, 9]. With the $G_2$ operators, we find low-lying states in irrep $H$ that are consistent with the subduction of spin $\frac{7}{2}$ or higher.

In the negative-parity spectrum, we also obtain essentially the same results for both lattice volumes. The three lowest states shown on the right half of Fig. 1 are unambiguously identified as follows: the lowest $G_{1u}$ state corresponds to spin $\frac{1}{2}$ and the lowest two $H_u$ states correspond to distinct spin $\frac{3}{2}$ states. Above these is a group of three states with roughly the same lattice energy: $a_tE \approx 0.33$ (within errors). Again there are two possible interpretations. C.) The group consists of a spin $\frac{1}{2}$ state in $G_{1u}$ that is accidentally degenerate with a spin $\frac{3}{2}$ state, the latter having degenerate partner states in $H_u$ and $G_{2u}$. D.) The group consists a spin $\frac{3}{2}$ state having degenerate partner states in $G_{1u}$, $H_u$ and $G_{2u}$.

The pattern of low-lying physical states starts with $N(\frac{1}{2}^+, 1520)$, $N(\frac{1}{2}^+, 1535)$ and $N(\frac{3}{2}^-, 1700)$. These should show up as distinct $H_u$, $G_{1u}$ and $H_u$ states on the lattice, in agreement with the three lowest negative-parity states in Fig. 1. The next physical states include $N(\frac{1}{2}^+, 1680)$ and $N(\frac{5}{2}^-, 1675)$, which essentially are degenerate. They should show up as degenerate $G_{1u}$, $H_u$ and $G_{2u}$ states on the lattice. This pattern of spins is consistent with interpretation C.) of the lattice states at lattice energy $a_tE \approx 0.33$. The pattern of energies of the physical states has $N(\frac{3}{2}^-, 1675)$ a little lower in energy than $N(\frac{5}{2}^-, 1700)$, but the lattice results at lattice spacing 0.1F place the spin $\frac{3}{2}$ state above the spin $\frac{5}{2}$ state. Study of the continuum limit of the lattice spectrum is required in order to resolve these issues.

Because the minimum spin that is contained in the $G_2$ irrep is $\frac{7}{2}$, we have found strong evidence for spin $\frac{7}{2}$ or higher for both parities in our lattice spectra for isospin $\frac{1}{2}$. We also have found evidence for degenerate partner states corresponding to the subduction of spin $\frac{7}{2}$ or higher to the octahedral irreps. The lattice results for the low-lying excited states of isospin $\frac{1}{2}$ provide the
correct number of octahedral states for the subduction of the spins of the low-lying physical states. These results are significant but they are based on the quenched approximation and a 490 MeV pion mass. Similar calculations in full QCD with several pion masses and several lattice spacings are planned and work is under way to calculate the anisotropic gauge configurations.

Acknowledgments

This work was supported by the U.S. National Science Foundation under Award PHY-0354982 and by the U.S. Department of Energy under contracts DE-AC05-06OR23177 and DE-FG02-93ER-40762.

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