The critical state line of nonplastic tailings

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Abstract: The probability of failure of tailing dams and associated risks demand improvements in engineering practice. The critical state line provides a robust framework for the characterization of mine tailings. New experimental data for nonplastic platinum tailings and a large database for tailings and nonplastic soils (grain size between 2 and 300 μm) show that the critical state parameters for nonplastic tailings follow the same trends as nonplastic soils as a function of particle-scale characteristics and extreme void ratios. Critical state lines determined for extreme tailings gradations underestimate the range of critical state parameters that may be encountered in a tailings dam; in fact, mixtures with intermediate fines content exhibit the densest granular packing at critical state. The minimum void ratio e_min captures the underlying role of particle shape and grain size distribution on granular packing and emerges as a valuable index property to inform sampling strategies for the assessment of spatial variability. Mineralogy does not significantly affect the intercept I_e0, but it does affect the slope λ. The friction coefficients M of tailings are similar to those of other nonplastic soils; while mineralogy does not have a significant effect on friction, more angular grains lead to higher friction coefficients.

Key words: tailings, critical state line, minimum void ratio, nonplastic soils.

Introduction

The historical failure rate of tailing dams and the associated risks demand improvements in material characterization, the design of the containment structure, construction practices, and monitoring technology (Santamarina et al. 2019). The critical state line (CSL) provides a robust frame of reference for the assessment of mine tailings (Carrera et al. 2011; Castro et al. 1982; Fourie and Papageorgiou 2001; Jefferies and Been 2015; Li et al. 2018; Vermeulen 2001). The CSL swings through the confinement–shear–volume space (p′−q−e) and generates two projections. The q−p′ projection is the linear Coulomb strength model

\[ q = M p' \]

where the M factor captures the frictional strength. The e−p′ projection (Fig. 1) follows a semi-logarithmic trend

\[ e = I_e - \lambda \log[p'/p^*] \]

where e is the void ratio, λ is an arbitrary reference stress, and the intercept \( I_e \) is the void ratio when \( p' = p^* \). This study adopts \( p^* = 100 \text{ kPa} \) to anchor the linear approximation in eq. 2 around stress levels that are relevant to field conditions. Although the reference stress \( p^* \) does not affect the critical state line (CSL) in itself, it does affect the strength of the correlations between \( I_e \) and other parameters (Torres-Cruz 2019). This manuscript adopts the classical definitions of the mean effective stress \( p' = (\sigma'_{1} + \sigma'_{2} + \sigma'_{3})/3 \) and deviator stress \( q = \sigma'_{1} - \sigma'_{3} \).

There is strong lateral and vertical variability in grain size distribution within tailings dams (Carrera et al. 2011; Fourie and Papageorgiou 2001; Li 2017). Therefore, the proper selection of tailings samples is crucial to critical state characterization. One approach is to test the extreme, coarsest and finest, gradations in the deposit (Jefferies and Been 2015). However, this approach may be misleading as illustrated by multiple critical state studies of sand–silt mixtures (Papadopoulou and Tika 2008; Rahman and Lo 2008; Thevanayagam et al. 2002; Yang et al. 2006; Zlatovic and Ishihara 1995); the critical state line (CSL) shifts downwards (a
Li and Coop 2019; this work trends between critical state parameters (nonplastic tailings and 132 nonplastic soils), and seeks to identify (CSL) of nonplastic platinum tailings, analyzes results in the con-

iron tailings can exhibit considerable plasticity). The scatter in the trends between critical state parameters and

ebetween the slope $\lambda$ and the gap between extreme void ratios ($e_{\text{max}} - e_{\text{min}}$), i.e., the volumetric compression potential (Cho et al. 2006; Cubrinovski and Ishihara 2000), and (ii) between $\lambda$ and $e_{\text{min}}$, i.e., $e_{\text{min}}$ alone can provide some information on volumetric compressibility (Torres-Cruz 2019). Overall, these trends capture the underlying role of particle shape and grain size distribution on granular packing and extreme void ratios $e_{\text{max}}$ and $e_{\text{min}}$ (Cho et al. 2006; Torres-Cruz 2019). Traditionally, extreme void ratios are used to characterize coarse-grained soils, i.e., retained on sieve #200. However, they can be equally useful to characterize the behavior of nonplastic silts (Lade et al. 1998; Carrera et al. 2011; Park and Santamarina 2017; Li and Coop 2019; Torres-Cruz 2019).

The scatter in the trends between critical state parameters and extreme void ratios may be too high for reliable predictions of $I_{100}$ and $\lambda$. However, these correlations suggest the possibility that an easily measured index property could assist in assessing the potential spatial variability of critical state parameters in nonplastic tailings, such as those that result from the extraction of gold, platinum, copper, and iron (Bedin et al. 2012; Li et al. 2018; Li 2017; Li and Coop 2019; this work — Note: kimberlite and some types of iron tailings can exhibit considerable plasticity).

This study explores the determination of the critical state line (CSL) of nonplastic platinum tailings, analyzes results in the context of a large database of CSLs compiled from the literature (25 nonplastic tailings and 132 nonplastic soils), and seeks to identify trends between critical state parameters ($I_{100}$, $\lambda$, and $M$) and index properties that can be used to readily assess spatial variability in tailings dams.

Triaxial testing program

Materials and methods: platinum tailings

Platinum tailings were sourced from the upstream beach of an upstream spigotted tailings dam located in the North West Province of South Africa. The tailings are nonplastic, angular-shaped (Fig. 2), and have a complex mineralogy dominated by enstatite (27%), bytontite (27%), chromite (21%), and hornblende (8%), with traces (<5%) of diopside, epidote, talc, and biotite, among other minerals (Amponsah-Dacosta 2017).

The laboratory investigation centers around six mixtures prepared with pre-sieved tailings fractions. The mixtures have a fines content (FC) that ranges from 10% to 98% (Fig. 3 and Table 1 —

Fig. 1. Typical critical state line (CSL) and its idealization in $e - \log_{10}(p^*)$ space. The choice of $I_{100}$ avoids extrapolation effects. [Colour online.]

reduction in $I_{\rho}$ as the silt content increases from 0% to ~30%, but it shifts upwards (an increase in $I_{\rho}$) as the silt content exceeds 30%. Therefore, analyses based on the two extreme gradations underestimate the range of critical states present at a nonplastic tailings dam.

Previous studies report a correlation between the intercept $I_{100}$ and the minimum void ratio $e_{\text{min}}$ for nonplastic soils (Cho et al. 2006; Torres-Cruz 2019). In addition, there is some correlation (i) between the slope $\lambda$ and the gap between extreme void ratios ($e_{\text{max}} - e_{\text{min}}$), i.e., the volumetric compression potential (Cho et al. 2006; Cubrinovski and Ishihara 2000), and (ii) between $\lambda$ and $e_{\text{min}}$, i.e., $e_{\text{min}}$ alone can provide some information on volumetric compressibility (Torres-Cruz 2019). Overall, these trends capture the underlying role of particle shape and grain size distribution on granular packing and extreme void ratios $e_{\text{max}}$ and $e_{\text{min}}$ (Cho et al. 2006; Torres-Cruz 2019). Traditionally, extreme void ratios are used to characterize coarse-grained soils, i.e., retained on sieve #200. However, they can be equally useful to characterize the behavior of nonplastic silts (Lade et al. 1998; Carrera et al. 2011; Park and Santamarina 2017; Li and Coop 2019; Torres-Cruz 2019).

The scatter in the trends between critical state parameters and extreme void ratios may be too high for reliable predictions of $I_{100}$ and $\lambda$. However, these correlations suggest the possibility that an easily measured index property could assist in assessing the potential spatial variability of critical state parameters in nonplastic tailings, such as those that result from the extraction of gold, platinum, copper, and iron (Bedin et al. 2012; Li et al. 2018; Li 2017; Li and Coop 2019; this work — Note: kimberlite and some types of iron tailings can exhibit considerable plasticity).

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Platinum tailings were sourced from the upper beach of an upstream spigotted tailings dam located in the North West Province of South Africa. The tailings are nonplastic, angular-shaped (Fig. 2), and have a complex mineralogy dominated by enstatite (27%), bytontite (27%), chromite (21%), and hornblende (8%), with traces (<5%) of diopside, epidote, talc, and biotite, among other minerals (Amponsah-Dacosta 2017).

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Note: fines are <75 μm). This range of FC covers field observations (from 33% to 95%; Torres-Cruz 2016). This study follows two methods to measure $e_{\text{min}}$: the standard ASTM D1557 (ASTM 2002a) and a nonstandard method that benefits from a small sample size (similar to that proposed by Lade et al. 1998, as described in Torres-Cruz 2016). Although not applicable to FC > 15%, this study involved the ASTM D4254 (ASTM 2000) to measure $e_{\text{max}}$ due to a lack of a standardized alternative. Figure 4 presents $e_{\text{max}}$ and $e_{\text{min}}$ values for the six mixtures and shows similar trends for $e_{\text{min}}$ values gathered with the two methods. Clearly, the extreme specimens (FC = 10% and 98%) exhibit the highest $e_{\text{max}}$ and $e_{\text{min}}$. Conversely, the lowest values correspond to mixtures with 30% ≤ FC ≤ 47%, in agreement with other studies of sand–silt mixtures (Papadopoulou and Tika 2008; Rahman and Lo 2008; Thevanayagam et al. 2002; Yang et al. 2006; Zlatović and Ishihara 1995; Park and Santamarina 2017).

Based on these results, critical state testing focused on three mixtures with distinctly different $e_{\text{min}}$ values: FC = 10%, 30%, and 81%. The preparation of triaxial test specimens (70 mm diameter, 141 mm height) involved moist tamping to achieve loose contractive specimens (Ishihara 1993) to minimize strain localization (Jefferies and Been 2015 — Note: ends were not lubricated as non-uniform radial strains may remain; refer to Rees 2010). Air-CO₂ replacement prior to water injection and back-pressure produced high degrees of saturation; in fact, all specimens exhibited a Skempton’s parameter $B ≥ 0.96$. The stress path consisted of isotropic consolidation applied in a single stage followed by monotonous shearing, either under drained (CD) or undrained (CU) conditions. The displacement-controlled loading frame applied the deviatoric stress at constant cell pressure. Water content measurements at the end of the test enabled void ratio determinations (refer to Verdugo and Ishihara 1996). Table 2 presents a summary of the testing program (additional experimental details in Torres-Cruz 2016).

Results

Figure 5 illustrates typical response curves obtained for platinum tailings specimens subjected to drained and undrained loading in terms of deviatoric stress $q$, pore pressure $u$, and void ratio $e$ vs. vertical strain $u$. The hyperbolic model fitted to the pre-peak portion of the deviatoric stress vs. axial strain curve $q$-$e$, corrected early seating and misalignment effects and identified the true start of loading (Bishop and Henkel 1957).

On the other end of the response curves, most specimens do not reach stable pore pressure $u$ (CU tests) or void ratio $e$ (CD tests) within the strain level attainable in triaxial tests (Fig. 5). Several authors have suggested extrapolation procedures that allow for improved estimates of critical state conditions (Li 2017; Murthy et al. 2007). The following criteria were implemented in this study (Fig. 6 — refer to Carrera et al. 2011): (Fig. 6a) extrapolate $q/p'$ to $\delta u/\delta e = 0$ for undrained tests, (Fig. 6b) extrapolate $q$-$p'$ trends to asymptotic $M$, and (Fig. 6c) extrapolate void ratio to the critical state condition of zero dilatancy $\delta u/\delta e = 0$ for drained tests. Table 2 summarizes the critical state values determined for all tests.

Figure 7 shows the critical states for all CD and CU tests for the three selected mixtures. The CSLs on the $e$-$p'$ projection follow the semi-log linearization and are distinctly different for the three mixtures (Fig. 7a). More importantly, the intercept $I_{100}$ is lowest for the mixture with fines content (FC) ≤ 30%, in agreement with extreme void ratio trends (Fig. 4). Figure 7b shows the $q$-$p'$ projections of the CSLs; the computed $M$ values for the three mixtures fall within a narrow range of $M = 1.27 ± 0.02$. This is consistent with previous results that show that $M$ is largely independent of grain size distribution (Bandini and Coop 2011; Carrera et al. 2011; Li et al. 2015). Table 1 lists the values of $I_{100}$, $\lambda$, and $M$ for each mixture. Distinctly different $e$-$p'$ projections but indistinguishable $q$-$p'$ projections imply that the dilative tendency of a speci-
men at an initial $e_0$ and $p'_0$ will depend on its gradation; yet, gradation will not affect the stress ratio the specimen will reach at critical state.

**Insights from the database**

**Data sources**

The following analyses take into consideration the critical state lines (CSLs) of the three platinum mixtures described above, 25 tailings reported in the literature (Table 3), and 132 nonplastic soils collected from published studies (including natural soils and material from rock crushers; Torres-Cruz 2019). Figure 8 illustrates the range in particle size distributions — between 2 and 500 $\mu$m — for tailings in the database.

**Uncertainty in $T_{100}$ and $\lambda$**

The least squares solution identifies model parameters by minimizing the sum of squared errors $SSE = \sum (e_m - e_p)^2$ between mea-
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### Table 1. Properties of mixtures of platinum tailings.

| FC (%) | d50 (μm) | C_u | C_c | e_min | e_max | G_s | G_c | ρ_m/cm³ | qmin | qmax | G_s/cm³ | qmin | G_s/cm³ | qmax | M |
|--------|---------|-----|-----|-------|-------|-----|-----|---------|------|------|---------|------|---------|------|----|
| 10     | 106     | 1.3 | 3.59 | 0.70  | 1.08  | 1.05| 0.90 | 0.59    | 0.90 | 0.90 | 0.59    | 0.90 | 0.90   | 0.59 | 0.90|
| 20     | 188     | 3.4 | 3.48 | 0.52  | 0.58  | 0.93| 0.81 | 0.58    | 0.81 | 0.81 | 0.58    | 0.81 | 0.81   | 0.58 | 0.81|
| 47     | 81      | 3.7 | 3.48 | 0.52  | 0.58  | 0.93| 0.81 | 0.58    | 0.81 | 0.81 | 0.58    | 0.81 | 0.81   | 0.58 | 0.81|
| 64     | 62      | 3.0 | 3.54 | 0.59  | 0.61  | 0.93| 0.81 | 0.58    | 0.81 | 0.81 | 0.58    | 0.81 | 0.81   | 0.58 | 0.81|
| 81     | 52      | 2.8 | 3.59 | 0.63  | 0.66  | 1.12| 0.92 | 0.66    | 0.92 | 0.92 | 0.66    | 0.92 | 0.92   | 0.66 | 0.92|
| 98     | 45      | 2.6 | 3.61 | 0.65  | 0.68  | 1.18| 0.92 | 0.68    | 0.92 | 0.92 | 0.68    | 0.92 | 0.92   | 0.68 | 0.92|

Notes:
- Coefficient of uniformity (d50/d10).
- Specific gravity, ASTM D854 (ASTM 2002a).
- Nonstandard procedure.
- ASTM D1557 (ASTM 2002a).
- ASTM D4254 (ASTM 2000).

### Table 2. Critical state values and end of test conditions of triaxial tests on platinum tailings.

| FC (e_min) | Test ID | Test type | e | p' (kPa) | q | M | End of test conditiona |
|------------|--------|-----------|---|----------|---|---|------------------------|
| Low 10% (0.70) | 10-1   | CU        | 1.020 | 177 | 216 | 1.22 | Critical state       |
|             | 10-2   | CU        | 1.040 | 145 | 174 | 1.20 | Critical state       |
|             | 10-3b  | CD        | 1.018 | 328 | 384 | 1.17 | Critical state       |
|             | 10-4   | CU        | 1.040 | 105 | 131 | 1.25 | Critical state       |
|             | 10-5   | CD        | 0.992 | 469 | 612 | 1.30 | Critical state       |
|             | 10-6b  | CU        | 1.056 | 95 | 117 | 1.23 | Critical state       |
| Medium 30% (0.53) | 30-1   | CU        | 0.755 | 540 | 701 | 1.30 | Critical state       |
|             | 30-2   | CU        | 0.813 | 90 | 118 | 1.31 | Critical state       |
|             | 30-3   | CU        | 0.806 | 138 | 178 | 1.29 | Critical state       |
|             | 30-4   | CU        | 0.812 | 144 | 187 | 1.29 | Critical state       |
|             | 30-5   | CU        | 0.808 | 77 | 101 | 1.30 | Critical state       |
|             | 30-6b  | CD        | 0.802 | 285 | 368 | 1.29 | Critical state       |
|             | 30-7b  | CD        | 0.773 | 502 | 648 | 1.29 | Critical state       |
| High 81% (0.63) | 81-1b  | CU        | 0.885 | 326 | 408 | 1.25 | Critical state       |
|             | 81-2b  | CU        | 0.889 | 335 | 419 | 1.25 | Critical state       |
|             | 81-3b  | CU        | 0.891 | 333 | 415 | 1.24 | Critical state       |
|             | 81-4   | CU        | 0.916 | 119 | 146 | 1.23 | Critical state       |
|             | 81-5   | CD        | 0.879 | 448 | 568 | 1.27 | Critical state       |
|             | 81-6b  | CU        | 0.914 | 175 | 215 | 1.23 | Critical state       |
|             | 81-7   | CD        | 0.892 | 239 | 303 | 1.27 | Critical state       |

Notes:
- A Sign convention: positive for contraction and negative for dilation.
- Extrapolation scheme applied.
- Average M value of other tests done on the same mixture.

### Fig. 4. Extreme void ratios vs. fines content of platinum tailings.

![Graph showing extreme void ratios vs. fines content](https://www.nrcresearchpress.com/kb/48950.png)

where N is the number of data points, and subindices stand for m = measured and avg = average (Navidi 2015). The normalizations NSE_m = SE_m/Range(λ) and NSE_f_m = SE_f_m/Range(Γ_100) facilitate the analysis. Figure 10 compares the normalized standard errors NSE_m and NSE_f_m computed for 91 CSLs with known e-p' data points. The normalized error of the slope NSE_e is 3 to 12 times...
levels of interest, typically that do not exhibit cementation or crushing at effective stress tailings and nonplastic soils. The database is dominated by soils.

Critical state parameters and index properties

Previous studies showed that particle-scale characteristics such as shape and grain size distribution affect index properties and critical state parameters (Torres-Cruz 2019; Cho et al. 2006; Cubrinovski and Ishihara 2000). This section explores the correlation between index properties and critical state parameters for tailings and nonplastic soils. The database is dominated by soils that do not exhibit cementation or crushing at effective stress levels of interest, typically $\alpha' < 1$ MPa (see Jung et al. 2012 for the effect of cementation on critical state).

Figure 11 shows that tailings fall along the same $\Gamma_{100}$ vs. $\epsilon_{\text{min}}$ and $\Gamma_{100}$ vs. $\epsilon_{\text{max}}$ trends of other nonplastic soils. There is no clear clustering when the data are discriminated by mineralogy (not shown here). Furthermore, the intercept $\Gamma_{100}$ is

- higher than the minimum void ratio for 96% of the database; the overall trend is $\Gamma_{100} \approx 1.4 \epsilon_{\text{min}}$; thus, the spatial variability of $\epsilon_{\text{min}}$ is a good indicator of potential spatial variability of $\Gamma_{100}$ in nonplastic tailings
- lower than the maximum void ratio for 94% of the database; typically $\Gamma_{100} \approx 0.8 \epsilon_{\text{max}}$.

Note that the spread in $\epsilon_{\text{max}}$ is much higher than that in $\epsilon_{\text{min}}$ ($1.2 \leq \epsilon_{\text{max}}/\epsilon_{\text{min}} \leq 2.2$, for 90% of the database). This hinders correlations that involve $(\epsilon_{\text{max}} - \epsilon_{\text{min}})$. Figure 12a shows that tailings populate the same area as other nonplastic soils in the $\lambda$ vs. $\epsilon_{\text{min}}$ space. The range of possible slope values $\lambda$ increases with $\epsilon_{\text{min}}$. In other words, the potential contractiveness at critical state decreases for soils with low $\epsilon_{\text{min}}$; in fact $\lambda < (\epsilon_{\text{min}}/3)$ for 95% of the database. High hardness soils (e.g., quartz and silica) exhibit lower slopes ($\lambda$), mostly in the range of $\lambda < 0.10$ (Fig. 12b). Therefore, hardness affects contractiveness at critical state even though most of the data corresponds to intermediate stress levels where marked crushing is unlikely (all but one of the CSLs are defined for $p' < 4$ MPa). The scatter in Fig. 12 reflects the complex interactions among grain size distribution, particle shape, and mineralogy. The scatter also reflects the limited invertibility of the slope (refer to Figs. 9 and 10). There is no trend or clustering in plots of $\lambda$ versus void ratio gap $(\epsilon_{\text{max}} - \epsilon_{\text{min}})$; however, note that the contraction in one log cycle $\lambda$ is smaller than $(\epsilon_{\text{max}} - \epsilon_{\text{min}})$ for the entire database.

The relative contractiveness $R_c$ shows the position of the CSL between two extreme density conditions (Verdugo and Ishihara 1996):

$$ R_c = \frac{(\epsilon_{\text{max}})_{100} - \Gamma_{100}}{(\epsilon_{\text{max}})_{100} - (\epsilon_{\text{min}})_{100}} $$

where $(\epsilon_{\text{max}})_{100}$ and $(\epsilon_{\text{min}})_{100}$ are the void ratios attained by isotropically compressing the soil to 100 kPa from its lowest $\epsilon_{\text{max}}$ and densest $\epsilon_{\text{min}}$ conditions. The values of $(\epsilon_{\text{max}})_{100}$ and $(\epsilon_{\text{min}})_{100}$ are unknown for most soils; therefore, the estimate of $R_c$ uses $\epsilon_{\text{max}}$ and $\epsilon_{\text{min}}$. The computed relative contractiveness values for the database spans the full $R_c = 0$ to 1 range (not shown here). Tailings cannot be distinguished from other nonplastic soils. There is a slight tendency for rounded soils to exhibit lower $R_c$ than angular soils, and there is a weak inverse trend between $R_c$ and median particle size $D_{50}$.

The database allows us to explore correlations with particle size. In particular:

- The maximum void ratio $\epsilon_{\text{max}}$ is independent of particle size while $D_{50} > 100 \mu$m, but it increases for finer particles, probably due to electrostatic interactions (Fig. 13a).
- The minimum void ratio $\epsilon_{\text{min}}$ and the intercept $\Gamma_{100}$ are independent of particle size for all entries in the database ($10 \mu$m < $D_{50}$ < 2000 $\mu$m; Figs. 13b and 13c); indeed, the effect of electrostatic interactions diminishes for the high energy conditions that prevail in $\epsilon_{\text{min}}$ testing and under $p' = 100$ kPa (Santamarina 2003).

Figure 13d presents the friction parameter $M$ vs. the median grain size $D_{50}$. The $M$ value varies widely for tailings from $M = 1.1$ to 1.8, in a similar range to nonplastic soils. The associated variation in critical state friction angle is from $\phi_c = 27.7^\circ$ to 43.8$^\circ$ (for reference, $M = 0.9$ or $\phi_c = 23^\circ$ for glass beads). Previous studies have shown that mineralogy and particle angularity largely define the value of $M$ (Cho et al. 2006; Sadrekarimi and Olson 2011; Li et al. 2012).
However, the database shows substantial overlap; the $M$ value of angular soils is $14\%$ larger than that for rounded soils (Fig. 13d), and the data do not reveal any distinct effect of mineralogy on $M$.

Figures 11, 12, and 13 are based on published data produced by a large number of laboratories around the world. Therefore, there are potential differences in test protocols and data interpretation, including inconsistent assessment of particle shape and mineralogy. Then, apparent discrepancies between these results and previously reported observations from focused studies highlight the need for consistent assessment and reporting of soil properties.

Conclusions

This study explored critical state parameters for South African platinum tailings, identified critical state data for 25 other tailings, and considered additional data for 132 nonplastic soils previously reported in the literature. Inherent limitations in the strain levels attainable in triaxial tests hindered the determination of critical state; therefore, robust extrapolation strategies helped define consistent critical states for both drained and undrained tests.

The complete dataset allows the comparison of tailings with other nonplastic soils and the identification of trends between critical state parameters and index properties. Salient conclusions follow:
1. Data of void ratio vs. confinement $e-p'$ at critical state extracted from triaxial test data allow the estimation of the intercept $\Gamma_{100}$ at $p' = 100$ kPa with better accuracy than the estimation of the slope $\lambda$.

2. The critical state parameters for nonplastic tailings fall on the same trends as data for a wide range of nonplastic soils. This suggests that inferences about soil behavior made from nonplastic soils (nontailings) can be reasonably adopted for nonplastic tailings provided that these inferences account for potential differences in particle shape, grain size distribution, and mineralogy.
### Table 3. Properties of tailings compiled from published studies.

| Name (FC) | Symbol | $D_{50}$ (μm) | $C_u$ | $\varepsilon_{\min}$ | $\varepsilon_{\max}$ | $\lambda$ | $\lambda$ | $p'$ range (kPa) | $G_s$ | $\lambda$ | $C_s$ | Mineralogy<sup>a</sup> | Particle shape<sup>b</sup> | Reference |
|-----------|--------|---------------|-------|----------------------|----------------------|---------|---------|------------------|-------|---------|-------|----------------|-----------------|-----------|
| Brazilian Gold (65) | BAU | 65 | 6.9 | 0.90 | 2.00 | 0.89 | 2.06 | 1.33 | 100–780 | 3.10 | Qtz = 27%, Ab = 25%, Chl = 35% | A-SA | Bedin et al. 2012; |
| Brazilian Gold (95) | 10 | 7.7 | 0.68 | 1.20 | 0.89 | 0.17 | 1.41 | 30–900 | 2.89 | — | — | — | — | — | Li et al. 2018 |
| Deixing Copper (95) | DCU | 30 | 5.1 | 0.62 | 1.28 | 0.84 | 0.126 | 1.43 | 25–3700 | 3.75 | Fa = 78%, Mag = 22% | A-SA | Li 2017 |
| Hilton Mines (2.5) | HMS | 200 | — | 0.62 | 1.05 | 0.98 | 0.17 | 1.42 | 50–1000 | 50 | — | — | — | Jefferies and Been 2015 |
| Lornex Copper (7) | HVC | 260 | 2.7 | 0.68 | 1.08 | 1.00 | 1.264 | 1.40 | 70–1300 | 2.68 | Qtz = 100% | A-SA | Castro et al. 1982; Robertson et al. 2000; Wride et al. 2000 |
| Highland Valley Copper (8) | 200 | 2.8 | 0.54 | 1.06 | 0.84 | 0.068 | — | 1–220 | 2.66 | Qtz = 36%, Mca = 27%, Ill = 15%, Fsp = 11% | A-SA | Li 2017 |
| Merriespruit Gold (0) | MER | 130 | 1.9 | 0.74 | 1.22 | 1.11 | 0.061 | 1.24 | 2–200 | 2.7 | — | — | A-SA | Fourie and Papageorgiou 2000; Papageorgiou 2004; Tshabalala 2003 |
| Merriespruit Gold (20) | 120 | 4.7 | 0.70 | 1.33 | 1.10 | 0.157 | 1.17 | 7–80 | 2.7 | — | — | A-SA | Li et al. 2018 |
| Merriespruit Gold (30) | 100 | 30 | 0.58 | 1.33 | 0.90 | 0.073 | 1.12 | 2–130 | 2.7 | — | — | A-SA | Li et al. 2018 |
| Merriespruit Gold (60) | 60 | 25 | 0.66 | 1.83 | 0.76 | 0.024 | 1.19 | 10–125 | 2.7 | — | — | A-SA | Li et al. 2018 |
| Syncrude Oil Sand (3.5) | OIL | 207 | — | 0.54 | 0.90 | 0.73 | 0.065 | 1.33 | 10–620 | 2.64 | — | — | A-SA | Jefferies and Been 2015 |
| Syncrude Mildred Lake Oil Sand (10) | 160 | 2.2 | 0.52 | 0.96 | 0.85 | 0.035 | — | 37–200 | 2.66 | Qtz = 90%, Fsp = 5%, Kln = 5% | SA-SR | Robertson et al. 2000; Wride et al. 2000 |
| Syncrude Oil Sand (12) | 170 | 2.4 | 0.52 | 0.96 | 0.85 | 0.068 | 1.19 | 10–800 | 2.62 | Qtz = 95% | A-SA | Sladen and Hanford 1987 |
| Panzhihua Iron UB (19) | PFE | 220 | 10 | 0.50 | 1.10 | 0.79 | 0.252 | 1.41 | 180–11 000 | 3.37 | Di = 35%, Lab = 30%, Hbl = 15% | A-SA | Li 2017; Li and Coop 2019 |
| Panzhihua Iron MB (68) | 35 | 9.0 | 0.60 | 1.10 | 0.81 | 0.152 | 1.36 | 90–1900 | 3.14 | — | — | A-SA | Li et al. 2018 |
| Panzhihua Iron PO (93) | 25 | 6.7 | 0.70 | 1.22 | 0.76 | 0.185 | 1.40 | 25–2000 | 3.11 | — | — | A-SA | Li et al. 2018 |
| Mizpah Dam Gold (72) | SAU | 30 | 28 | 0.48 | 1.80 | 0.73 | 0.136 | 1.47 | 20–240 | 2.73 | Qtz = 75%, Ms = 9% | A-SA | Vermeulen 2001 |
| Pay Dam Gold (77) | 25 | 22 | 0.64 | 2.10 | 0.70 | 0.176 | 1.40 | 30–200 | 2.74 | — | — | A-SA | Vermeulen 2001 |
| Witwatersrand Gold UB (41) | 90 | 23 | 0.64 | 1.24 | 0.76 | 0.185 | 1.40 | 25–300 | 2.75 | — | — | A-SA | Chang et al. 2011 |
| Witwatersrand Gold MB (56) | 50 | 11 | 0.65 | 0.15 | 0.78 | 0.04 | 1.78 | 40–210 | 2.69 | — | — | A-SA | Chang et al. 2011 |
| Witwatersrand Gold PO (99) | 5 | 3 | 3.00 | 0.00 | 1.62 | 0.185 | 1.40 | 130–330 | 2.75 | — | — | A-SA | Chang et al. 2011 |
| Stava Fluorite (0) | STA | 190 | 2.4 | 0.71 | 0.94 | 0.94 | 0.180 | 1.44 | 60–1100 | 2.72 | Qtz = 78%, Cal = 10% | A-SA | Carrera et al. 2011 |
| Stava Fluorite (30) | 130 | 9.7 | 0.69 | 0.85 | 0.63 | 0.138 | 1.45 | 45–1350 | 2.75 | — | — | A-SA | Carrera et al. 2011 |
| Stava Fluorite (50) | 75 | 10 | 0.48 | 0.76 | 0.58 | 0.061 | 1.44 | 15–770 | 2.78 | — | — | A-SA | Carrera et al. 2011 |
| Stava Fluorite (100) | 25 | 7.8 | 0.75 | 0.93 | 0.71 | 0.081 | 1.38 | 15–1220 | 2.83 | — | — | A-SA | Carrera et al. 2011 |

<sup>a</sup>Ab, albite; Cal, calcite; Chl, chlorite; Di, diopside; Fa, fayalite; Fsp, feldspar; Hbl, hornblende; Ill, illite; Kln, kaolinite; Lab, labradorite; Mag, magnetite; Mca, mica; Ms, muscovite; Prl, pyrophyllite; Qtz, quartz.

<sup>b</sup>A, angular; R, rounded; SA, subangular; SR, subrounded.
3. Mineralogy does not significantly affect the intercept $I_{100}$ and related correlations, but it does affect the slope $\lambda$ even when tests are conducted at intermediate stress levels below crushing. In fact, almost all high hardness soils (quartz and silica) have $\lambda < 0.10$, while soils with other mineralogies and variable hardnesses span the entire range of $\lambda$.

4. Published data and new experimental results show that the intercept $I_{100}$ as well as the value of $e_{\min}$ are lowest for mixtures with intermediate fines content (FC), typically FC $= 30\%$. Therefore, the assessment of field conditions based on extreme gradations underestimates the range of critical state parameters in the tailings dam.

5. The spatial variability of $e_{\min}$ within a tailings deposit anticipates spatial variability in $I_{100}$. Additionally, the range of possible $\lambda$ values increases with $e_{\min}$. These observations can be used to inform sampling strategies.

6. The range of the friction coefficient $M$ of tailings is similar to the range of values exhibited by other nonplastic soils. On average, angular soils exhibit greater $M$ than rounded soils, in agreement with previous studies. Mineralogy and median particle size do not appear to significantly influence $M$.

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