Thermo-mechanical relaxation of stresses in a glass-metal junction

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Abstract. This work deals with the evaluation of the stress-strain state in an infinite layered cylindrical junction of glass with steel. Structural changes in the glass behavior are described by the Tool-Narayanaswamy-Mazurin-Moynihan model. The relaxation processes in the glass have been presented by the temperature regime depending of the fictive temperature $T_f$, the relaxation time $\tau'$ or the viscosity $\eta$ and the thermal expansion coefficient $\alpha$. The calculations of the stress components in the glass have been made by using the viscoelastic model. The presented model allows to find the optimal annealing conditions for the creation of the glass-metal composite material.

1. Introduction
The investigation of a layered cylindrical soldering of metal with glass (glass-to-metal seal) is associated with the creation and the optimization of the manufacturing mode of the glass-metal composite rod with a glass core [1]. The manufacturing method includes the heating process, the stable phase and the cooling stage. A technology is considered that the soldering metal onto the glass is formed at the temperatures above the glass transition temperature, which the relaxation processes in the glass to metal seals are available, so, it is necessary to construct an algorithm for the calculation of the technological stresses in order to control the annealing mode and the residual stresses. The investigation of the relaxation processes in the glass is actual problem and many phenomenological models for describing of the glass transition kinetics have been presented in paper [2]. In this article, the evaluation of the stress-strain state of the seal is predicted by the Tool-Narayanaswamy model [3]. It is more appropriate way for the description of the experimental data on the vitrification of various amorphous materials, with using the Mazarin’s algorithm for the calculation of the relaxation processes in plate seals [4]. The result of this work is the algorithm for the thermal stress calculations in the cylindrical seals with taking into account the changing structure of the glass and unloading the metal layer due to the relaxation processes in the glass.

2. Mathematical model
The deformation problem of the long two-layered cylinder under the influence of the varying temperature field is considered. The publication [5] has been shown that if the temperature changed uniform then the temperature distribution in the cross-over section of the cylinder was virtually unchangeable for 20 mm cylinders in the diameter. It allows to consider that the...
temperature do not depend on the radius. So, under the assumption that there is a perfect contact between the glass to metal surfaces, the stresses at the junction are generated due to the significant differences in the mechanical characteristic of the joined materials, namely, the thermal expansion coefficients and the Youngs moduli. The dependencies linking the stresses and strains in the theory of thermo-elasticity including the plane strain hypothesis and axial symmetry have been well known [6]:

\[
\begin{align*}
\sigma_r^{(i)}(r,t) &= \lambda^{(i)}(\varepsilon^{(i)}_{\phi}(r,t) + \varepsilon_r^{(i)}(r,t)) + 2\mu^{(i)}\varepsilon_r^{(i)}(r,t) - 2(\lambda^{(i)} + \mu^{(i)})\alpha^{(i)}(T(t) - T_0), \\
\sigma_\phi^{(i)}(r,t) &= \lambda^{(i)}(\varepsilon^{(i)}_{\phi}(r,t) + \varepsilon_r^{(i)}(r,t)) + 2\mu^{(i)}\varepsilon_r^{(i)}(r,t) - 2(\lambda^{(i)} + \mu^{(i)})\alpha^{(i)}(T(t) - T_0), \\
\sigma_z^{(i)}(r,t) &= \lambda^{(i)}(\varepsilon^{(i)}_{\phi}(r,t) + \varepsilon_r^{(i)}(r,t)) - 2\lambda^{(i)}\alpha^{(i)}(T(t) - T_0), \\
\varepsilon_r^{(i)}(r,t) &= \frac{\partial u_r^{(i)}(r,t)}{\partial r}, \quad \varepsilon_\phi^{(i)}(r,t) = \frac{u_r^{(i)}(r,t)}{r}, \\
T(t_0) &= T_0, \quad \sigma^{(i)}(t_0) = 0, \quad \varepsilon^{(i)}(t_0) = 0, \quad u^{(1,2)}(t_0) = 0,
\end{align*}
\]

(1)

where \( i \) is the layer number, the 1\textsuperscript{st} layer is the glass, the 2\textsuperscript{nd} layer is the metal, \( \lambda^{(i)} \) and \( \mu^{(i)} \) are the Lame constants, \( \alpha^{(i)} \) is the thermal expansion coefficients (figure 1). If the glass cylinder in the junction is solid then the stresses in the glass do not depend on the radius.

**Figure 1.** Glass-to-metal seal geometry.

The main characteristic of the structural changes (of the time and temperature) for the glass at the viscoelastic behavior in accordance with the Tool-Narayanaswamy-Mazurin-Moynihan (TNMM) model is the fictive temperature [3,4,7]

\[
T_f(t) = T(t) - \int_0^t \left( \frac{dT}{dt} \right) M[\xi(t) - \xi(t')] dt',
\]

(2)

\[
\xi(t) = \int_0^t \left( \frac{dt'}{\tau'} \right),
\]

(3)

\[
M(\xi) = \exp \left(-\xi^b \right),
\]

(4)

where \( M \) is the relaxation kernel, \( \xi \) is the non-dimensional reduced time depending on the relaxation time \( \tau' \) for the property at arbitrary thermal history, \( b \) is the kinetic model parameter.
The relaxation time can be determined as linearly depending on the viscosity in form [4]

\[
\tau' = \frac{\eta(T, T_f)}{K},
\]

\[
\log \eta(T, T_f) = \log \eta(T_0) + (T_f^{-1} - T_0^{-1})B_e + (T_f^{-1} - T_f^{-1})B_m,
\]

where \( K \) is the structural relaxation characteristic, \( \eta(T_0) \) and \( T_0 \) are the initial values of the viscous and temperature. The initial temperature must satisfy the condition of metastable equilibrium, \( B_e \) and \( B_m \) are the parameters characterizing of the temperature dependencies of the viscosity at the equilibrium and frozen glass structure.

Equations (2) – (5), by using successive approximation algorithm, allow to calculate the fictive temperature \( T_f \) in every time step. The glass thermal expansion coefficient \( \alpha^{(1)} \) depends on the temperature and can be determined by similarly specific heat [6] in form

\[
\alpha^{(1)} = \alpha_g^{(1)} + (\alpha_m^{(1)} - \alpha_g^{(1)}) \frac{dT_f}{dT},
\]

where \( \alpha_m^{(1)} \) and \( \alpha_g^{(1)} \) are the thermal expansion coefficients of metastable liquid phase and glass, respectively. The relaxation processes in the glass are determined by equations (2)–(6). If the stress relaxation is taken into account then it is proposed to use the discrete calculation algorithm presented in [3]. For each \( n \) time step, formulas for the residual stress calculations \( \sigma^{(1)}_{\text{res}}(t) \) in the glass could be presented as

\[
\sigma^{(1)}_{\text{res}, r}(t_n) = \sigma^{(1)}_r(t_n) - \sigma^{(1)}_{\text{rel}, r}(t_n), \quad \sigma^{(1)}_{\text{res}, \phi}(t_n) = \sigma^{(1)}_\phi(t_n) - \sigma^{(1)}_{\text{rel}, \phi}(t_n),
\]

where \( \sigma^{(1)}(t_n) \) are the elastic stresses found at the solution of the problem (1), \( \sigma^{(1)}_{\text{rel}}(t_n) \) are the stresses relaxation by \( n \) time step and which are expressed as

\[
\sigma^{(1)}_{\text{rel}}(t_n) = \sum_{i=0}^{n-1} (1 - M \xi(t_n) - \xi(t_i)) \Delta \sigma^{(1)}(t_i),
\]

\[
\Delta \sigma^{(1)}(t_0) = 0, \quad \Delta \sigma^{(1)}(t_n) = \sigma^{(1)}(t_n) - \Delta \sigma^{(1)}(t_{n-1}).
\]

The solution of the problem (1) in elasticity theory is characterized by the integration constants which are also included in the calculations of the formulas (7) and (8) in the relaxation and residual stresses. The determination of these constants occurs at each time step from the boundary conditions on the free surface and the contact region

\[
\begin{align*}
\sigma^{(1)}_{\text{res}, r}(t_n)|_{r=r_1} &= \sigma_r^{(2)}(t_n)|_{r=r_1}, \\
\varepsilon_r^{(1)}(t_n)|_{r=r_1} &= \varepsilon_r^{(2)}(t_n)|_{r=r_1}, \\
\sigma_r^{(2)}(t_n)|_{r=r_2} &= f(t_n).
\end{align*}
\]

The first boundary condition (9) allows to take into account the relaxation processes in the glass without breaking the ideal contact conditions in the junction.

3. Results and discussion

Calculations of the stress-strain state have been made for the cylindrical soldering of CH1 glass with steel 20. Material parameters using in the calculations have been taken as \( r_1 = 4 \cdot 10^{-6} \) m, \( r_2 = 5 \cdot 10^{-6} \) m, \( T(t_0) = 600 \) °C, \( \eta(t_0) = 10^{10.25} \), \( B_e = 28726.85 \) °C, \( B_m = 13726.85 \) °C, \( K = 10^{10.7} \), \( \alpha_m^{(1)} = 210 \cdot 10^{-7} \) 1/°C, \( \alpha_g^{(1)} = 52 \cdot 10^{-7} \) 1/°C, \( b = 0.65 \), \( E_1 = 0.67 \cdot 10^8 \) MPa,
\( \alpha^{(2)} = 0.6 \cdot 10^{-6} \cdot T^{0.148} \) \( 1/\degree C \) \[8\] and \( E_2 = 2 \cdot 10^5 \) MPa. The temperature regime was slow cooling at a rate of 5 \( \degree C/min \) from 600 \( \degree C \) to 20 \( \degree C \).

The analysis of the calculated results for the selected parameters showed that at the beginning of the cooling process the structural glass temperature coincided with the outside temperature and then stabilized (see figure 2), i.e. at the beginning there were changes in the melting glass structure, then the melt transformed into the glassy state and the glass structure became frozen. At this time, the relaxation time \( \tau' \) was significantly increasing, respectively, the relaxation processes in the glass were almost stopping. For example, regarding the stress relaxation at the temperature 600 \( \degree C \) the relaxation time \( \tau' = 1 \text{ sec} \) and at the temperature 550 \( \degree C \) the relaxation time \( \tau' = 41 \text{ min} \).

**Figure 2.** a) Time dependencies of the structural and outer temperatures of CH1 glass at a cooling rate of 5 \( \degree C/min \). b) Time dependency of the relaxation time of glass at a cooling rate of 5 \( \degree C/min \). Here, \( T(t) \) is the outer temperature; \( T_f(t) \) is the structural temperature; \( \tau' \) is the relaxation time.

**Figure 3.** Temperature dependencies of thermal expansion coefficients (CTE).

The analysis of changes in the thermal expansion coefficients (figure 3) shows that, at cooling, the glass is exposed to the tensile stresses from 600 \( \degree C \) to 564.5 \( \degree C \), there are no stresses in the
Composition at next temperature and after that, the problem could be actually solved without taking into account the relaxation processes in the glass, which almost do not affect the residual stresses (figures 4b and 5b).

At the beginning of the cooling process, the relaxation processes in the calculations significantly affects the technological stresses (figures 4a and 5a). Note that, the temperature corresponding zero stress state (at slow cooling or heating) determines a possibility to solve the problem without consideration the stress relaxation into the junction. The junction could be reheated again without appearing of the additional stresses higher this temperature. Therefore, in order to make a layered cylindrical soldering of glass with metal, cooling at the temperatures above the temperature of the intersection curves (figure 3) should be slow, with taking into account time required for the stress relaxation and after this temperature, the cooling rate could be increased significantly.

**Figure 4.** The distribution of the radial stresses in the glass-to-metal seal: a) the radial stresses at several temperature points; b) the radial stresses at temperature $T = 590$ °C.

**Figure 5.** The distribution of the tangential stresses in the glass-to-metal seal: a) the tangential stresses at several temperature points; b) the tangential stresses at temperature $T = 590$ °C.
4. Conclusion
The algorithm allows to describe the stress-strain state of the cylindrical junction, to simulate the optimal technological regime for the production of the junction with the specified residual stresses without the solution of the inverse problem. The simplicity of the algorithm allows to include the additional conditions considering the special mechanical properties of joined materials. For example, influence the temperature on the elastic modulus or the yield strength and the adhesion to the glass and metal surfaces.

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