I. INTRODUCTION

The density in the core of a neutron star (NS) surpasses the nuclear saturation density \( n_0 \sim 2 \times 10^{14} \text{ g cm}^{-3} \) and even the highest density that can be achieved in terrestrial experiments. It is argued that strangeness in the form of hyperons, meson condensates, or even deconfined quark matter may appear at such high densities, affecting several NS observable properties. The behavior of ultra-dense NS matter is still largely unknown and therefore NSs provide a natural laboratory to study nuclear matter under extreme conditions such as high magnetic field and rotation.

The NS macroscopic observables relate to the microscopic NS physics through the pressure density relationship or the equation of state (EoS). The two main approaches used to model nuclear EoS are (i) microscopic or \( \text{ab-initio} \) and (ii) phenomenological relativistic mean field models. Yet another approach is to construct empirical fits rather than microphysics-based EoS. Where the high-density behavior of EoS is described by polytropic models (pressure is proportional to a power of density) either using piece-wise polytropes or using the spectral decomposition method. One can also infer the NS EoS from observations using a nonparametric description of NS EoS or can even use a hybrid approach to describe the NS EoS.

On the observational side, NSs are observed at multiple wavelengths in current generation electromagnetic telescopes. The masses of these objects are typically well measured in binary systems, whereas the measurement of radius involves larger uncertainty. However, the recently launched NICER (Neutron Star Interior Composition Explorer) mission has improved radius measurement and can further improve the estimation of the NS radius upto 5% addition to GWs, and hence opened a new window in multi-messenger astronomy.

The tidal deformation of the stars in a BNS contributes to phase of the GW signal starting at the fifth Post Newtonian (PN) order. In general, tidal deformability (for electric type) of order \( \mathcal{O} \) appears at \( 2l+1 \) Post Newtonian (PN) order. The major contribution comes...
from the quadrupolar tides, followed by higher-order octupolar ($A_3$) and hexadecapolar ($A_4$) terms \[18 \div 21\]. Recent waveform models have been updated to consider the effect of the higher order tidal parameters $A_3$, $A_4$ and also the magnetic type deformation (quadrupolar magnetic $\Sigma_q$) in the GW phase \[24 \div 25\]. In most analyses, the tides are considered adiabatic, i.e. the GW frequency is much lower than the NS resonant oscillation mode frequency. However, recent efforts largely support the excitation of NS fundamental modes (f-modes) in the late inspiral \[26 \div 31\]. In a frequency domain model, the dynamical tidal contribution to the GW phase due to the excitation of f-modes appears at 8 PN order \[18 \div 32\] and depends upon the f-mode frequency. Since the additional tidal deformability parameters and the f-mode dynamical phase contribute at high PN orders, they are often dropped while constructing waveform approximants. However, the f-mode dynamical tidal correction has been shown to significantly affect the inference of NS properties from a binary due to its resonance behaviour \[33 \div 34\].

Previous works have suggested that there exist EoS-insensitive universal relations (UR) between $\Lambda_2$ and each of $A_3$, $A_4$, and $\Sigma_q$ (hereafter referred as ‘multipole Love’ relation) \[35 \div 37\]. Similarly, URs exist between the f-mode frequency and tidal parameters \[35\] (hereafter referred as ‘f-Love’ relation). This allows the additional contribution due to higher order tidal parameters or f-mode parameters to be considered without having to actually sample over them in a parameter estimation run, reducing the search parameter space. Additionally, there exists UR involving stellar compactness and quadrupole tidal parameters, which can be used to infer NS radius from the measured mass and quadrupole tidal parameters from a BNS event \[39\]. In this work, we will only be interested in the URs mentioned above. There also exist URs involving the combination of $\Lambda_2$ and mass ratio $q$ of the binaries, known as ‘binary Love relations,’ as well as URs involving NS spin which reduces effort in measuring the NS tidal parameter (and NS radius) in a BNS event \[40 \div 46\].

The existing URs are employed by considering a few theoretical EoS models and some of the selective EoSs are now incompatible with the current astrophysical constraints \[35 \div 37\]. Also, the existing URs do not consider the EoS uncertainties resulting from the nuclear parameters. In this work, we improve the multipole Love and f-Love URs relevant for GW astronomy and investigate the impact of the updated URs by analyzing the BNS event GW170817 and performing several injection and recovery studies with the future GW detector configurations. This work is organized in the following way. In section \[II\] we discuss the choices of Eos, followed by the description of the methodology to solve for multipole tidal parameters and to solve for the NS f-mode characteristics. We compile our results in Section \[III\] and summarise our conclusions in Section \[IV\].

\[\text{II. METHOD}\]

A. Choices of Equations of State

The EoS is essentially the relation between the pressure ($p$) and density ($\rho$) (or energy density $\epsilon$), i.e., $p = p(\rho)$. As described in Section \[I\], different physical motivations develop a diverse family of EoSs depending upon the physical descriptions of the NS matter. In this work, we consider a wide range of EoSs based upon different physical descriptions. They are discussed below:

a. Relativistic Mean Field (RMF) Models: RMF models are phenomenological models where baryon-baryon interaction is mediated via exchange of mesons. The Lagrangian density describes the interaction between baryons through the exchange of mesons. The complete description of the Lagrangian density for nucleonic ($\nu p\mu$) and nucleon-hyperon ($\nu p\mu Y$) mattered NS, and hence the EoS within the RMF model considered in this work can be found in \[18\]. The parameters of the RMF model are calibrated to the nuclear and hypernuclear parameters at saturation: nuclear saturation density ($n_0$), the binding energy per nucleon ($E/A$ or $E_{\text{sat}}$), incompressibility ($K$), the effective nucleon mass ($m^*$), symmetry energy ($J$) and slope of symmetry energy ($L$) at saturation. Hyperon coupling constants are fixed using hyperon nucleon potential depths ($U_Y$) or using symmetry properties. We consider the total uncertainty ranges in the saturation parameters resulting from nuclear and hypernuclear experiments, summarized in Table \[IV\] \[49\].

b. Selective EoSs: Along with the RMF models with nucleonic and hyperonic mattered EoSs, we consider many realistic EoSs. The realistic EoSs are taken either from CompOSE database \[51 \div 53\] or from LALSimulation \[54\]. The considered realistic EoSs are APR4, APR3 \[18 \div 55\], SLy4 \[56\], BL \[57\], DD2(GPPVA) \[58\], SRO-APR \[59\], BSk22 \[60 \div 61\], WFF1 \[62\], MPA1 \[63\]. We also consider the Soft and Stiff EoS from \[4\]. In Figure \[LA\] through the Stiff-EoS terminates at earlier energy density, it is sufficient to reach the maximum stable NS mass the EoS model can reproduce \[5\]. To capture the hypothesis of a deconfined quark phase in the interior of the NS core, we account for some realistic hybrid EoSs containing the quark phase in the interior. The selective hybrid EoS models are BFH-D \[64 \div 65\], KBH (QHC21 − AT) \[66\], JJ-VQCD \[67\] and OOS-DD2(FRG) \[68\]. We do not consider the EoSs regarding quark stars only, as they

\[\text{\[1\]} \text{Recently a new EoS insensitive approach involving the NS mass and tidal deformability is proposed to constrain NS properties from a GW event \[17\].} \]

\[\text{\[2\]} \text{In Section \[III\] we discuss the choices of Eos, followed by the description of the methodology to solve for multipole tidal parameters and to solve for the NS f-mode characteristics. We compile our results in Section \[IV\] and summarise our conclusions in Section \[V\].} \]

\[\text{\[II\]} \text{We discuss the choices of Eos, followed by the description of the methodology to solve for multipole tidal parameters and to solve for the NS f-mode characteristics. We compile our results in Section \[III\] and summarise our conclusions in Section \[IV\].} \]
might deviate from the universal behavior and leave them for a separate investigation [35, 69].

c. Spectral Decomposition: To span the EoSs with empirical fit formalism, we consider the four parameters spectral decomposition method developed in [11]. In spectral decomposition, the adiabatic index (Γ) of the EoS is spectrally decomposed onto a set of polynomial basis functions and expressed as,

\[ \Gamma(p) = \exp \left( \sum_k \gamma_k \left[ \ln(p/p_0) \right]^k \right) \] (1)

where \( \gamma_k \) is the expansion coefficient and \( p_0 \) is the reference pressure where the high-density EoS is stitched to the low-density crustal EoS. The EoS can then be generated by integrating the relation,

\[ \frac{d\epsilon}{dp} = \frac{\epsilon + p}{p\Gamma(p)} \] (2)

which can be reduced to,

\[ \epsilon(p) = \epsilon(p_0) + \frac{1}{\mu(p)} \int_{p_0}^{p} \frac{\mu(p')}{\Gamma(p')} dp' \] (3)

where,

\[ \mu(p) = \exp \left( - \int_{p_0}^{p} \frac{dp'}{p'\Gamma(p')} \right) \]

We fix the low-density EoS to SLy EoS [70] and stitch the high-density EoS at a density below half of the saturation density such that the NS macroscopic properties will not be affected significantly. We generate spectral decomposed EoSs as implemented in LALSimulation [54] and consider the ranges for spectral indices \( (\gamma_k) \) from [20, 21].

Before proceeding with any further calculations, we ensure that each EoS satisfies the required physical conditions, such as thermodynamic stability \( (dp/d\epsilon > 0) \), causality \( (\sqrt{dp/d\epsilon} \leq 1) \) and the monotonic behaviour of pressure \( (dp/d\rho > 0) \) and \( d\epsilon/d\rho > 0 \). We additionally impose the constraint that the EoS must be able to produce a \( 2M_\odot \) stable NS and the tidal deformability of a \( 1.4M_\odot \) is less than 800 (i.e., \( \Lambda_{1.4M_\odot} \leq 800 \)) [20, 21]. EoSs and the corresponding mass-radius relations used in this work are displayed in Figure [1a] and Figure [1b] respectively.

### Table I: Range of nuclear and hypernuclear saturation parameters considered in this work. Meson and nucleon masses are fixed at \( m_\pi = 550 \text{ MeV}, m_\omega = 783 \text{ MeV}, m_\eta = 770 \text{ MeV}, m_{\sigma} = 975 \text{ MeV}, m_\phi = 1020 \text{ MeV} \) and \( m_N = 939 \text{ MeV} \). Masses of the hyperons are fixed from [50]. Note that we fix the \( A \) hyperon potential depth \( (U_A = -30 \text{ MeV}) \).

| Model | \( n_0 \) (\( \text{fm}^{-3} \)) | \( E_{\text{sat}} \) (MeV) | \( K \) (MeV) | \( J \) (MeV) | \( L \) (MeV) | \( m^*/m_N \) | \( U_S \) MeV | \( U_\Xi \) MeV |
|-------|-----------------|-----------------|--------|--------|--------|--------------|--------|--------|
| RMF [48] | [0.14, 0.17] | [-16.5, -15.5] | [200, 300] | [28, 34] | [40, 70] | [0.55, 0.75] | [0, +40] | [-40,0] |

### B. Macroscopic Structure and Tidal Deformabilities

For a given EoS, NS mass \( (M) \) and radius \( (R) \) are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations. The vanishing of pressure at the surface of NS \( (p(R) = 0) \) provides the stellar radius \( R \) and the stellar mass \( M = m(R) \). TOV equations corresponding to a static and spherically symmetric metric [4] are summarised in Eq. [7] [74, 75].

\[ ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\lambda(r)r^2}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \] (4)

\[ \frac{dm(r)}{dr} = 4\pi r^2\epsilon(r) \] ,

\[ \frac{dp(r)}{dr} = -\left[\epsilon(r) + \left(\frac{m(r) + 4\pi r^3p(r)}{r(r - 2m(r))}\right)\right] \] ,

\[ \frac{d\Phi(r)}{dr} = \left(\frac{-1}{\epsilon(r) + p(r)}\right) \frac{dp}{dr} \] ,

\[ e^{2\lambda(r)} = \left(\frac{r}{r - 2m(r)}\right) \] .

The NS can be tidally deformed in a binary system due to mutual gravitational interaction. The tidal field can be decomposed to an electric \( (\mathcal{E}_{ij}) \) and magnetic \( (\mathcal{M}_{ij}) \) component, leading to the induction of mass multipole moment \( (Q_{ij}) \) and current multipole moment \( (S_{ij}) \). The gravitoelectric tidal deformability \( (\lambda_l) \) and gravitomagnetic tidal deformability \( (\sigma_l) \) of order \( l \) can be defined as [70].

\[ Q_{ij} = \lambda_l \mathcal{E}_{ij} \] ,

\[ S_{ij} = \sigma_l \mathcal{M}_{ij} \] (6)

\[ k_l = \frac{(2l - 1)!!}{2} \lambda_l \] ,

\[ j_l = 4(2l - 1)!! \sigma_l \] .

The electric Love number \( (k_l) \) and magnetic Love number \( (j_l) \) relate to the tidal deformability parameters as given in Eq. [7]. The valuable parameters that can be determined from a GW signal are dimensionless tidal deformability. The dimensionless gravitoelectric tidal deformability parameter \( (\Lambda_l) \) and gravitomagnetic tidal deformability parameter \( (\Sigma_l) \) can be expressed using the corresponding tidal Love numbers and stellar compactness \( (C = M/R) \) as [77]:

\[ \Lambda_l = \frac{k_l}{C} \] ,

\[ \Sigma_l = \frac{j_l}{C} \] .
FIG. 1: (a) EoSs used in this work. (b) Mass radius relation corresponding to the EoSs used in this work. For each EoS, the corresponding M-R relationship is shown up to the maximum possible stable mass. Horizontal bands correspond to masses $M = 2.072^{+0.067}_{-0.066} M_\odot$ of PSR J0740+6620 and $M = 2.01^{+0.04}_{-0.05} M_\odot$ of PSR J0348+0432. The mass radius estimates of the two companion neutron stars in the merger event GW170817 are shown by the shaded area labeled with GW170817 M1 (M2). [8]

For computing the electric and magnetic Love numbers, one needs to integrate additional set of differential equations along with the TOV equations. We follow the methodology developed in [77] to solve for the Love numbers. We test our numerical scheme by reproducing the methodology developed in [77] to solve for the Love numbers [77] along with the TOV equations. We follow the one needs to integrate additional set of differential equations for computing the electric and magnetic Love numbers, which one has only outgoing wave solution to the Zerilli’s wave equation [82] is integrated to far away from the star. Then a search is carried out for the complex f-mode frequency ($\omega = 2\pi f + i\tau_f$) for which one has only outgoing wave solution to the Zerilli’s equation at infinity. The real part of $\omega$ represents the f-mode angular frequency, and the imaginary part represents the damping time. For finding the mode characteristics, we use the numerical methods developed in our previous work [78].

$\Lambda_l = \frac{2}{(2l+1)!!} \frac{k_l}{C^{2l+1}},$

$\Sigma_l = \frac{1}{4(2l+1)!!} \frac{j_l}{C^{2l+1}}.$

(8)

For computing the electric and magnetic Love numbers, one needs to integrate additional set of differential equations along with the TOV equations. We follow the methodology developed in [77] to solve for the Love numbers. We test our numerical scheme by reproducing the methodology developed in [77] to solve for the Love numbers. We provide the electric tidal deformability up to order $\ell = 4$ and magnetic tidal deformability up to order $\ell = 3$.

C. Finding f-mode oscillation characteristics

A neutron star can have several quasi-normal modes depending upon the restoring force. Mode characteristics (frequency and damping time) contain information about the NS interior. Hence, the determination of mode parameters can be employed to constrain the NS interior or EoS. In our previous work [78], we had shown that depending upon the EoS, the Cowling approximation can overestimate the quadrupolar f-mode frequency up to about 30%. As recent efforts are going on to improve the gravitational waveform models with consideration of excitation of f-modes which depends on the mode frequency [32], one should consider the general relativistic formalism to find the mode characteristics. We use the direct integration method developed in [78–81] to find the NS f-mode frequency. In short, the coupled equations for perturbed metric and fluid variables are integrated within the NS interior with appropriate boundary conditions [80]. Outside of the NS, fluid variables are set to zero, and then the Zerilli’s wave equation [82] is integrated to far away from the star. Then a search is carried out for the complex f-mode frequency ($\omega = 2\pi f + i\tau_f$) for which one has only outgoing wave solution to the Zerilli’s equation at infinity. The real part of $\omega$ represents the f-mode angular frequency, and the imaginary part represents the damping time. For finding the mode characteristics, we use the numerical methods developed in our previous work [78].

III. RESULTS

A. Multipole Love and f-Love Relations

The universality among higher-order electric tidal deformability parameters ($\Lambda_{2 \ell \ell}$) or the magnetic deformability parameter ($\Sigma_{2 \ell \ell}$) with the quadrupolar tidal deformability parameter ($\Lambda_{22}$) were first discussed in [35] [40] with few selective EoSs. The universality of compactness ($C$) with $\Lambda_2$ initially introduced in [39] and the UR involving f-mode frequency and tidal deformability parameters are introduced in [35]. Original URs from
where,

$$ P = \sum_{k=0}^{6} a_k \ln(\Lambda_2)^k $$

and

$$ P = \{\ln(A_3), \ln(A_4), \ln(\Sigma_2), \ln(\Sigma_3), C, M\omega_2, M\omega_3, M\omega_4\} $$

In recent works [43, 83], the universal relations for electric type deformability were updated by considering the phenomenological piece-wise polytropic EoSs. In different waveform models, the correction on the tidal phase includes the electric tidal parameter up to order $\ell \leq 4$ and for magnetic deformation up to order $\ell \leq 3$. Hence, we provide the universal relations for $A_3$ and $A_4$ with $A_2$ for electric type, whereas for magnetic type, we provide the universal relation of $\Sigma_2$ and $\Sigma_3$ with $A_2$. From the tidal deformability and mass obtained from a binary system, one can infer the NS radius ($R$) by using the EoS-independent relation that exists between stellar compactness ($C = M/R$) and tidal deformability parameter $\Sigma$. There are EoS-independent relations, which involve the tidal deformability of both binary NSs and binary parameters (like $q = m_1/m_2$). The universal relation involving the binary parameters and the $C - \Lambda_2$ relation is also used to infer the NS radii [40, 41, 43, 87]. Though binary relations reduce the parameter space of BNS search parameters, the inferred NS parameters are model dependent [88].

As discussed in Section II A, we consider tentatively 6000 EoSs originating from different physical motivations. We fix the lower mass for an EoS by imposing the constraint resulting from the maximum rotation pulsar PSR J1748-2446 [83, 86] (also notice the rotation limiting curve in Figure 11a). The maximum mass for an EoS is fixed at the maximum stable NS mass the EoS can produce. We then use the method described in Section II A to obtain the multipole tidal parameters ($\Lambda_\ell$ and $\Sigma_\ell$). We obtain the multipole Love relations by solving the NS properties (with tidal parameters) for $\sim 1.2 \times 10^6$ neutron stars. We produce the universal relation for multipole Love relations as a polynomial fit [9] as introduced in [35]. Note that the original fit from Yagi [35] was a 4th order polynomial which was then updated to a 6th order polynomial for a larger data set in [83]. In our data set, we notice that by updating the polynomial from a quartic polynomial to a 5th order polynomial, the resulting goodness of fit improved significantly and then did not improve significantly after increasing the order of the polynomial. However, we provide the universal relations with a 6th order polynomial to be consistent with the updated relations. Universal relations $A_3 - \Lambda_2$, $A_4 - \Lambda_2$, $\Sigma_2 - \Lambda_2$ and $\Sigma_3 - \Lambda_2$ are displayed in Figures 2a, 2b, 3a, and 3b, respectively. The fit parameters for the multipole Love relations [9] involving tidal parameters obtained from this work and other works are tabulated in Table I. Similarly, for $C - \Lambda_2$ relation we provide the fit parameters in Table III and displayed the relation in Figure 9. Our multipole universal relations are valid for $2.3 \leq \Lambda_2 \leq 4 \times 10^4$ which is the essential range for GW astronomy (mostly in the range $\leq 10^4$) and for $C - \Lambda_2$, the relation is valid for $0.08 \leq C \leq 0.33$.

Although the detection of f-modes from BNS would only be possible with third generation detectors [31], the impact on the inferred NS parameters can be seen in the A+ detector configurations [31, 33]. In the frequency domain, the phase correction due to the dynamical excitation of f-modes depends solely on the mode frequency [15, 20, 22, 21]. The universality of mass scaled f-mode angular frequency (i.e., $M\omega$) with the tidal parameter (A) was first studied in the work of Chan et al. [35], where the universality was explained by using the universal behavior of f-mode frequency with the moment of inertia ($I$), and the $I - Love - Q$ relations [38, 44]. To avoid the numerical instabilities, we fix the lower mass $\sim 1M_\odot$ for each EoS despite the rotation limit while solving for the f-mode frequency. The proposed quartic polynomial fit of [35] is updated to a 5th order fit in [73, 92]. We provide the polynomial fit up to 6th order [9] to be consistent with other multipole universal relations. Our f-Love relations are valid for $2.3 \leq \Lambda_2 \leq 4 \times 10^4$.

### B. Error Analysis

We analyze the errors and compare the URs in the range $\Lambda_2 \leq 10^4$ as required for GW astronomy. Our $A_3 - \Lambda_2$ relation holds a maximum error of 11%, with 90% of the errors are below 7%. In comparison, the original fit from Yagi [35] holds a maximum error of 16% with 90% of the errors below 10%, and the updated $A_3 - \Lambda_2$ UR from Godzieba et al. [43] holds a maximum error of 23% with 90% of the errors below 16%. On similar lines, the $A_4 - \Lambda_2$ relation developed in this work holds a maximum error of 24% with 90% of the errors below 13%, whereas, the original fit from Yagi holds a maximum error of 35% with 90% of the errors below 20%, and the UR from [43] holds a maximum error $\sim 44%$ with 90% of the errors below 30%. For $50 \leq \Lambda_2 \leq 10^4$, the URs developed in this work behave quite similarly to the URs developed in [43] and also in this range, the updated URs, as well as the URs from [43] account less error compared to the original fits from Yagi [35]. However, in the complete range of $\Lambda_2$, our relation have lower error bounds (see Figure 9 in Appendix A). For $\Lambda_2 \leq 10^4$, our $\Sigma_2 - \Lambda_2$ fit holds a maximum error

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2 The systematic errors that could be occurred in the inferred NS properties due to the choice of binary Love relation or $C - \Lambda_2$ UR are recently discussed in [15, 83].
FIG. 2: Showing Universal relations of higher order tidal deformabilities with $\Lambda_2$ along with the relative errors corresponding to fit relation (a) for $\Lambda_3$ and (b) for $\Lambda_4$.

FIG. 3: Showing Universal relations of tidal deformabilities of magnetic type with $\Lambda_2$ along with the relative errors corresponding to fit relation (a) for $\Sigma_2$ and (b) for $\Sigma_3$.

The original $\Sigma_2 - \Lambda_2$ fit from Yagi [37] holds a maximum error of 32% with 90% of the errors below 28%. Although we notice a similar behavior among the $\Sigma_2 - \Lambda_2$ relations from this work and the original fit from Yagi [37] in the range $3 \leq \Lambda_2 \leq 100$, our fit performs much better for $\Lambda_2 > 100$. Our $\Sigma_3 - \Lambda_2$ fit holds a maximum error of 12% with 90% of the errors below 5%. Our $C - \Lambda_2$ relation holds a maximum error $\sim 4\%$ with 90% of the errors below 2.5%. The original fit of Maselli et al. [39] holds a maximum error of 11% with 90% of the errors below 8%. We notice that our $C - \Lambda_2$ relation always accounts for smaller errors than the other URs in all ranges of $\Lambda_2 \leq 10^4$.

Also, we notice that the universal relation developed in this work involving the quadrupolar $f$-mode parameter has a maximum error of 0.4%. In the original work of Chan et al. [38], the universality of $M_{\omega \ell} - \Lambda_\ell$ has given for $\ell = \ell'$ (though the relation with $\ell \neq \ell'$ are plotted) arguing that the relation with $\ell \neq \ell'$ can introduce a maximum error. However, in our data set, we notice that the relation $M_{\omega \ell} - \Lambda_\ell$ holds a maximum error $\sim 1.5\%$ (in the original fit, the error can reach up to 10%) and $M_{\omega 3} - \Lambda_2$ has a maximum error $\sim 3\%$ which indicates that the relations are tight enough to obtain $\omega_{\ell \geq 2}$ from $\Lambda_2$. The advantages of providing the universal relations for $M_{\omega \ell \geq 2}$ with $\Lambda_2$ are (i) one can avoid the use of $\Lambda_{\ell \geq 3} - \Lambda_2$ relation which can involve up to 15-20% error for further use of $M_{\omega \ell} - \Lambda_\ell$ relation (ii) further the relation $M_{\omega \ell \geq 2} - \Lambda_2$ can be directly implemented in the waveform modeling as $\Lambda_2$ is the dominant parameter contributing to the GW phase. We display the f-Love relations for $M_{\omega 2} - \Lambda_2$, $M_{\omega 3} - \Lambda_2$ and $M_{\omega 4} - \Lambda_2$ in Figures 5a to 5c respectively. The fit parameters for $M_{\omega \ell} - \Lambda_2$ relations corresponding to polynomial fit (9) along with relations from Chan et al. [38] are tabulated in Table IV.
Additionally, we terminate the waveform at either the spin-orbit correction or the spin-spin correction to the waveform model. We also consider the spin-orbit correction at 8PN. We also consider the spin-orbit correction to the waveform model. Additionally, we terminate the waveform at either the contact frequency or the frequency corresponding to stable circular orbit \( f_{ISCO} = [6^{3/2} \pi (m_1 + m_2)]^{-1} \), depending on which one is smaller.

We perform a Bayesian parameter estimation of GW170817 strain data \(^{[96]}\) with the noise curve given in \(^{[96]}\), using the dynamic nested sampler \texttt{dynesty} \(^{[97]}\) as implemented in the parameter estimation package \texttt{bilby} \(^{[98]}\). To speed up the likelihood evaluation, we use the heterodyned likelihood calculation \(^{[99]}\) (also referred to as “relative binning”) as implemented in \texttt{bilby} (we use “relative binning” for the BNS event GW170817, where later in Section \(\text{[70]}\) we use the nested sampling algorithm \texttt{dynesty} as implemented in python package \texttt{bilby} and not “relative binning”). We use uniform priors on chirp mass \( M_c \in [1.18, 1.21] M_{\odot} \), mass ratio \( q \in [0.5, 1] \), individual spin magnitudes \( |\chi_i| \in [0, 0.05] \), individual quadrupolar tidal deformability parameter \( \Lambda_{2,1} \in [5, 5000] \), power law prior on the luminosity distance \( d_L \in [1, 80] \) Mpc and sine in the binary inclination angle \( \theta_{in} \) (angle between angular momentum and line of sight). We fix the right ascension (ra) and declination (dec) to -0.4081 radian and ra = 3.446 radian respectively.

We perform Bayesian parameter estimation with three different approximants for the waveform.

1. TaylorF2 waveform model with adiabatic tidal effect from quadrupolar tidal deformability \( \Lambda_{2} \): hereafter referred as TF2,

2. further we add the adiabatic tidal correction from \( \Lambda_{3} \) and \( \Sigma_{2} \) : this is named as TF2_{\Lambda_{3}, \Sigma_{2}} and

\(^{[97]}\)https://www.gw-openscience.org/events/GW170817/
To see the impact of universal relations, we consider the previously developed relation from [35, 37, 38] one family of universal relations (mentioned as ‘Yagi,’ if only multipole Love relation in the figures and tables) and the universal relations developed in this work as another family (mentioned as ‘This work’ in the figures and tables).

The important parameters inferred with different waveform model corrections and with different universal relations are tabulated in Table V. The inferred NS radius of pri-

3. then we add the quadrupolar f-mode dynamical tide (‘fntidal’ phase from [32]) and mention it as TF2A3,S2,t.

For TF2A3,S2,t waveform model, we find that using the URs developed in this work (i.e. with TF2A3,S2,t ) predicts a higher median for A by 6% (or by ~ 23 units) than when URs from Yagi [35] and Chan et al. [38] are used. Also TF2A3,S2,t predicts a higher 90% upper bound on A by 6.7% (by 50 units) compared to TF2Yagi+Chan A3,S2,t.

We estimate radii of the components of BNS event GW170817 through C – A2 UR using the mass and A2 posterior of the components, i.e., (m1, A2,1), and (m2, A2,2). For the posterior obtained using the previous URs [35, 37, 38], we use the C – A2 UR from Maselli et al. [39] to infer the NSs radii. However, for the posteriors obtained using the URs from this work, we infer NSs radii using the C – A2 relation developed in this work. Inferred NS radii of the components of GW170817 resulting from different C – A2 URs are tabulated in Table V. The inferred NS radius of primary component (R1) and for the secondary component (R2) of GW170817 are displayed in Figures 7a and 7b respectively. Looking at Figure 7 and Table V one can conclude the following.

| Work         | Relation | a0         | a1         | a2         | a3         | a4         | a5         | a6         |
|--------------|----------|------------|------------|------------|------------|------------|------------|------------|
| Chan et al.  | Mω2 – A3 2.245 × 10^{-3} | -6.836 × 10^{-3} | -4.196 × 10^{-3} | 5.215 × 10^{-4} | -1.857 × 10^{-6} | - | - |
| This Work    | Mω2 – A3 2.245 × 10^{-3} | -1.500 × 10^{-2} | -1.412 × 10^{-3} | 1.832 × 10^{-4} | -5.561 × 10^{-6} | - | - |
|              | Mω3 – A3 2.401 × 10^{-1} | -5.164 × 10^{-2} | -5.897 × 10^{-4} | 8.695 × 10^{-5} | -2.368 × 10^{-6} | - | - |

TABLE IV: Values of the fit parameters for mass scaled angular frequency (Mω) and A2 found in this work for the given equation (9).
We notice that with the same posterior \((m_1, \Lambda_2, 1)\), changing the \(C - \Lambda_2\) UR from Maselli et al. \[39\] to the \(C - \Lambda_2\) UR developed in this work (labeled as TF2This Work) in Figures 7a and 7b, the median for \(R_1\) increases by \(\sim 200\) m (for secondary component the median of \(R_2\) increases by 250 m due to change of \(C - \Lambda_2\) relation), whereas the 90% upper bound on \(R_1\) increase by \(\sim 750\) m using \(C - \Lambda_2\) relation from this work comparing to the URs from Maselli et al. \[39\] (for the upper bound of \(R_2\) the difference is \(\sim 600\) m).

Now, using the \((m_1, \Lambda_2, 1)\) or \((m_2, \Lambda_2, 2)\) distribution from TF2\(_{\Lambda_3, \Sigma_2}\) model (i.e., with adding the \(\Lambda_3\) and \(\Sigma_2\) effect) the median of \(R_1\) or \(R_2\) does not change significantly comparing to the radii of TF2 model. For waveform model with f-mode corrections and with previously developed universal relations i.e., from \[35, 37, 39\] (labeled as TF2\(_{\Lambda_3, \Sigma_2, f}\), in Figures 7a and 7b), predicts a lower median for \(R_1\) by \(\sim 400\) m (or median of \(R_2\) drops by \(\sim 400\) m) comparing to the NS radii inferred with TF2\(_{\Lambda_3, \Sigma_2, f}\) or TF2\(_{\text{Maselli}}\) model.

Changing the URs to the URs from this work with f-mode dynamical corrections effect, i.e., with TF2This Work the median of \(R_1\) (even for \(R_2\)) decreases by 300 m comparing to the NS radii informed with TF2This Work or TF2This Work waveform models. For TF2\(_{\Lambda_3, \Sigma_2, f}\) model, comparing the URs developed in this work with that developed in the previous works, we find that updated URs predict a higher median for both \(R_1\) and \(R_2\) by 300-400 m. Inclusion of the f-mode dynamical tidal phase, the 90% upper bound on NS radii drops by 400-800 m compared to the NS radii estimated by waveform models without the f-mode dynamical phase.

### D. Injection Studies

For the BNS event GW170817, the higher frequency region is mainly dominated by noise and does not show any significant impact regarding the f-mode or due to the change of URs \[31, 43, 91\]. However, it is anticipated that with the upgraded sensitivity in the future LIGO-VIRGO runs (with A+ configuration) or even in next-generation detectors (ET or CE), the dynamical f-mode tidal effect or even the change in URs can have a significant impact on the NS properties inferred from BNS events. Recently Pratten et al. \[33\] concluded that ignoring the dynamical tidal phase can overestimate the NS radii by \(\sim 10\)%. In \[33, 34\], it has been discussed that the impact of the dynamical tide on the inferred tidal parameter depends upon the choice of EoS. Hence, we investigate the impact of the URs in addition to the f-mode dynamical tidal effect (as the literature suggests...
that this has a dominant impact) and see if our conclusion depends upon the choices of the EoSs.

We consider the detector configurations for A+ and ET similar to that of \(^{33}\), i.e., 2 detectors from LIGO (H1 and L1) and VIRGO with the A+ design sensitivity \(^{101}\)\(^{103}\) as anticipated for the fifth observing run (O5) and the third generation (3G) Einstein telescope (ET) with ET-D sensitivity \(^{102}\)\(^{104}\). We inject the simulated BNS waveform using the TF2\(_{\Lambda_3, \Sigma_2, \ell}\) waveform model: i.e., including the tidal correction from additional tidal parameter \(\Lambda_3, \Sigma_2\) and \(\ell\)-mode dynamical tide \(^{35}\). We assume the NSs are nonspinning and the orbits are quasi-circular, i.e., we ignore the individual spins and orbital eccentricity \(^{35}\). As in a BNS system the tidal information is mostly contained in the in-spiral phase, we focus on the inspiral waveform only and truncate the waveform at a frequency that is the minimum among the contact frequency \(f_{\text{contact}}\) and \(f_{\text{ISCO}}\), i.e., \(f_{\text{max}} = \text{Minimum}(f_{\text{contact}}, f_{\text{ISCO}})\). The injected waveform starts at a minimum frequency \(f_{\text{min}} = 20\) Hz.

We recover the BNS parameters from injected signals with and without the dynamical tide: a model with adiabatic tidal correction including the additional multipole Love parameters, TF2\(_{\Lambda_3, \Sigma_2, \ell}\) and another with both adiabatic and dynamical tidal corrections (TF2\(_{\Lambda_3, \Sigma_2, \ell, \ell}\)). During recovery, the URs from \(^{35}\)\(^{37}\)\(^{38}\) are kept as one family, and the URs developed in this work as of a different family whereas, for TF2\(_{\Lambda_3, \Sigma_2, \ell}\) model we use the URs developed in this work to recover \(\Lambda_3\) and \(\Sigma_2\) from \(\Lambda_2\). During injection, for \(\Lambda_3, \Sigma_2,\) and \(\Lambda_2\), we use their actual values corresponding to each EoS and use the UR only while recovering the parameters. We perform the Bayesian parameter estimation using GW data inference package Bilby_Pipe with nested sampler dynasty as implemented in Billy \(^{38}\).

To see the biases due to the choices of EoSs, we consider three different EoSs with different stiffness spanning from soft to stiff: Soft-EoS from \(^{[5]}\), an intermediate EoS

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### Table V: Median and 90% symmetric credible interval of recovered GW170817 parameters: source frame chirp mass \(\mathcal{M}_c^{\text{source}} [M_\odot]\), effective spin \(\chi_{\text{eff}}\), binary inclination angle \(\theta_{\text{inj}}\), \(\Lambda, \delta\Lambda\). For mass ratio \(q\), the upper bound 1 is restricted by the prior so we note the 90\% symmetric credible lower bound with the upper bound as 1. The different URs used are labeled in the superscript corresponding to each model (e.g., TF2\(_{\Lambda_3, \Sigma_2, \ell}\) means that the URs used for \(\Lambda_3 - \Lambda_2\) and \(\Sigma_2 - \Lambda_2\) are those given by Yagi \(^{35}\)\(^{37}\) and for \(\mathcal{M}_c - \Lambda_2\) the UR relation from \(^{35}\) is used whereas, models labeled with ‘This Work’ in the superscript use the URs developed in this work.) For the reduced tidal parameter \(\tilde{\Lambda}\) we also provide the 90\% highest-probability-density (HPD) credible regions. We provide the Bayes factor for different model corrections computed against TF2 model.

| Parameters   | TF2 | TF2\(_{\Lambda_3, \Sigma_2, \ell}\) | TF2\(_{\Lambda_3, \Sigma_2, \ell}^{\text{This Work}}\) | TF2\(_{\Lambda_3, \Sigma_2, \ell}^{\text{Yagi + Chan}}\) | TF2\(_{\Lambda_3, \Sigma_2, \ell}^{\text{This Work}}\) |
|--------------|-----|-------------------------------|---------------------------------|---------------------------------|---------------------------------|
| \(\mathcal{M}_c^{\text{source}} [M_\odot]\) | 1.1869\(^{+0.0038}_{-0.0019}\) | 1.1870\(^{+0.0037}_{-0.0020}\) | 1.1869\(^{+0.0038}_{-0.0019}\) | 1.1869\(^{+0.0038}_{-0.0019}\) |
| \(q\)       | (0.72, 1.00) | (0.72, 1.00) | (0.71, 1.00) | (0.71, 1.00) |
| \(\chi_{\text{eff}}\) | 0.00\(^{+0.01}_{-0.01}\) | 0.00\(^{+0.01}_{-0.01}\) | 0.00\(^{+0.01}_{-0.01}\) | 0.00\(^{+0.01}_{-0.01}\) |
| \(d_L [\text{Mpc}]\) | 40\(^{+7}_{-14}\) | 39\(^{+7}_{-14}\) | 40\(^{+7}_{-14}\) | 39\(^{+7}_{-14}\) |
| \(\theta_{\text{inj}} [\text{rad}]\) | 2.55\(^{+0.42}_{-0.46}\) | 2.54\(^{+0.42}_{-0.45}\) | 2.56\(^{+0.42}_{-0.46}\) | 2.55\(^{+0.43}_{-0.45}\) |
| \(\tilde{\Lambda} (\text{HPD})\) | 425\(^{+557}_{-296}\) | 419\(^{+522}_{-282}\) | 418\(^{+572}_{-295}\) | 377\(^{+389}_{-267}\) |
| \(\tilde{\Lambda} (\text{Sym})\) | 425\(^{+659}_{-247}\) | 419\(^{+645}_{-246}\) | 418\(^{+638}_{-253}\) | 377\(^{+475}_{-244}\) |
| \(\delta\tilde{\Lambda}\) | 6\(^{+242}_{-225}\) | 7\(^{+226}_{-216}\) | 7\(^{+231}_{-221}\) | 8\(^{+201}_{-185}\) |
| \(R_1 [\text{km}] (\text{HPD})\) | 12.00\(^{+2.91}_{-3.19}\) | 11.99\(^{+2.84}_{-3.22}\) | - | 11.59\(^{+2.83}_{-2.99}\) |
| \(R_1 [\text{km}] (\text{This Work})\) | 12.19\(^{+3.48}_{-3.12}\) | - | 12.19\(^{+3.14}_{-3.37}\) | - |
| \(R_2 [\text{km}] (\text{HPD})\) | 11.08\(^{+3.15}_{-3.01}\) | 11.07\(^{+2.98}_{-2.78}\) | - | 10.67\(^{+2.67}_{-2.79}\) |
| \(R_2 [\text{km}] (\text{This Work})\) | 11.32\(^{+3.58}_{-3.18}\) | - | 11.29\(^{+3.43}_{-3.13}\) | - |
| \(\ln (\mathcal{M}_c^{\mathcal{M}} f_{\text{TF2}})\) | 0 | -0.03 | -0.12 | -0.63 |

\(^1\)https://dcc.ligo.org/LIGO-T2000012/public

\(^2\)https://dcc.ligo.org/LIGO-T1500293/public
For APR4 \cite{10} and the Stiff-EoS \cite{11}, we perform three different injection and recover studies: one injection with source frame chirp mass that of the BNS event GW170817, $M_{\text{source}}^{\text{GW170817}} = 1.186 M_\odot$ and other 2 events with $M_{\text{source}} = \pm 10\% M_{\text{source}}^{\text{GW170817}}$, i.e., the considered events have varying the source frame chirp mass $M_{\text{source}} = \{0.1067, 1.186, 1.304\} M_\odot$ and mass ratio $q = 0.855 \pm 5\%$. We include Gaussian noise in our analysis. Following the arguments from \cite{33}, we focus on the nearby sources (with luminosity distance $\leq 150$ Mpc) and fix the distance and source position during the recovery (which is based upon the assumption that the BNS events can be associated with electromagnetic (EM) counterparts). The priors are uniform in $M_c$, uniform in symmetric mass ratio $\eta = m_1 m_2 / (m_1 + m_2)$ and uniform priors in $\Lambda$ and $\delta \Lambda$. We choose the detector configurations and injection parameters similar to \cite{33}, and check our numerical method regarding the implementation of the dynamical tide by reproducing results from \cite{33} for APR4 EoS and considered mass range.

We tabulate the important recovered source frame NS parameters for different injected events along with the injected parameters for Soft-EoS, APR4, and Stiff-EoS in Tables VI to VIII respectively. We provide the deviation of the median of recovered $\Lambda$ from the injected value along with a check or cross mark indicating whether or not the injected value of $\Lambda$ is recovered within the symmetric 90% credible interval during parameter estimation.

For A+ detector configuration, the one-dimensional marginalized posterior distribution of recovered reduced tidal parameter $\tilde{\Lambda}$ for different events with Soft-EoS, APR4 and Stiff-EoS are displayed in Figures 8a to 8c respectively. For injections with Soft-EoS, although the ignorance of $\ell$-mode dynamical tide can overestimate the $\Lambda$ up to 10-20\% compare to injected values, expect for $M_c = 1.067$ the injected value is well recovered within 90\% credible of recovered posterior even ignoring the $\ell$-mode dynamical tide. Switching the EoS to the intermediate APR4 and ignoring the dynamical tide in recovery waveform, the injected $\Lambda$ is only recovered for $M_c = 1.304$ (with a 10\% deviation in the median compared to the injected value) within 90\% credible region. For injections with Stiff-EoS, we never recover the injected $\Lambda$ by ignoring the dynamical tide in the recovery waveform, irrespective of the mass ranges considered in this work. However, all the injection parameters are well recovered by considering the $\ell$-mode dynamical tides in the recovery waveform. Additionally, we notice that both the set of URs, i.e., URs from \cite{35, 37, 38} and the updated URs in this work perform similarly. They both recover the injected parameters very well. The maximum deviation found in the median due to the change of URs is $\sim 5\%$.

For injection and recover studies with ET, we display the distributions of recovered $\tilde{\Lambda}$ for different considered scenarios in Figure 10 of Appendix B. It is interesting to conclude that with the ET, we do not recover the injected value of $\tilde{\Lambda}$ within the 90% credible interval by ignoring the dynamical tide in the recovery waveform irrespective of the considered masses and EoSs. This indicates that the $\ell$-mode dynamical tide has much higher impact on 3G detectors compared to A+ configuration (this is expected as the $\ell$-mode dynamical effect dominates at high frequency and 3G detectors are more sensitive in high-frequency regimes compared to A+ configurations). Though the choice of URs changes the median of recovered posterior of $\Lambda$ only by $\leq 5\%$, the distributions get slightly more affected with ET compared to A+ configuration depending upon the choice of URs. For the Stiff-EoS \cite{11}, we barely recover the injected $\Lambda$ using the previous URs from \cite{35, 37, 38}. The biases in the posteriors due to the choices of URs increase with the increase of the mass or the stiffness of the EoS (mostly with the injections corresponding to lower $\Lambda$ value). Also, use of updated URs recovers the injected $\Lambda$ more efficiently compared to existing URs (e.g, see Figure 10), which may be due to the better performance of our URs in the lower $\Lambda_2$ values compared to the existing URs \cite{35, 37, 38}.

Our results regarding the dominant effect of the $\ell$-mode dynamical tide at low mass NS and the increasing impact with increasing the stiffness of the EoS are consistent with the results from \cite{33}. This can be explained in the following way: the dynamical tidal phase is $\propto \Lambda_2 / (M_\omega)^2$ \cite{92, 93} and a Stiff-EoS corresponds to

\[\times 10^{-3}\]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{The one-dimensional marginalized posterior distributions of $\Lambda$, recovered with the different model corrections and different URs.}
\end{figure}
larger $\Lambda_2$ and lower $M_{\omega_2}$ indicating a higher dynamical tidal effect (even for a fixed EoS with increasing mass, $\Lambda_2$ decreases hence $M_{\omega_2}$ increases (see, Figure 5a) with decreasing the impact of dynamical tide).

IV. CONCLUSIONS

We update the multipole Love universal relation and f-Love universal relations by considering a wide range of EoSs originating from different physical motivations and applying astrophysical constraints. We consider the uncertainties in the nuclear and hyper-nuclear parameters by considering the state-of-art relativistic mean field model to describe the NS matter. In addition, we consider 15 other realistic EoSs, including three hybrid-quark matter EoSs and a sample of polytropic EoSs described by the spectral decomposition method. We provide the updated URs for electric type tidal parameter $\Lambda_\ell - \Lambda_2$ for $\ell \leq 4$ (see Table III) and for magnetic tidal

FIG. 7: (a) The radius of the primary component ($m_1$, heavier one) of the binary, estimated through $C - \Lambda_2$ URs from the tidal parameter ($\Lambda_2$,) and mass distribution ($m_1$). Different URs used are labelled in the superscript to the name of waveform models.

(b) Same as Figure 7a but for the lighter component ($m_2$).

| Parameters | Injection | $A_+$ | $A_-$ |
|------------|-----------|-------|-------|
| $M_{\text{source}}$ [$M_\odot$] | 1.067 | 1.067 | 1.067 |
| $q$ | 0.855 | 0.855 | 0.855 |
| $\delta \Lambda$ | 15.428 | 15.428 | 15.428 |
| $\Lambda$ | 144.84 | 144.84 | 144.84 |
| $\Delta M_\Lambda$ [%] | - | 9.77 | 9.77 |

TABLE VI: Median and 90% symmetric credible interval of recovered source parameters ($M_{\text{c}}$, $q$, $\Lambda$, $\delta \Lambda$) for different events in $A_+$ and ET configuration. All the events are injected with TF2$A_3, z_2$ waveform model. The injection parameters correspond to the Soft-EoS. We also tabulate the deviation of the median of reduced tidal parameter $\Lambda$ from the injection values ($\Delta M_\Lambda$). If the injected value of $\Lambda$ is recovered with in the 90% credible region we give a check mark, else a cross mark is mentioned.
deformation $\Sigma - \Lambda_3$ (for $\ell \leq 3$, see Table II) as required to connect the higher order tidal parameter with the quadrupolar tidal parameter $\Lambda_2$ related to the correction in GW models. The original fits for the tidal parameters can be found in [35, 37]; note that in [37] the UR is only given for $\Sigma_2 - \Lambda_2$. Over the range $2 \leq \Lambda_2 \leq 4^4$, our multipole Love URs $\Lambda_3 - \Lambda_2$, $\Lambda_4 - \Lambda_2$, $\Sigma_2 - \Lambda_2$ and $\Sigma_3 - \Lambda_2$ hold maximum error of 11%, 24%, 3% and 12% respectively.

Further, unlike the original work of Chan et al. [38], where the URs for $M\omega_\ell - \Lambda_\ell$ are given only for $\ell = \ell'$ (arguing that $\ell \neq \ell'$ can introduce large error) we provide the URs for $M\omega_\ell$ with $\Lambda_2$ irrespective that $\ell=2$ or not (see Table IV). Our updated URs $M\omega_2 - \Lambda_2$, $M\omega_3 - \Lambda_2$ and $M\omega_4 - \Lambda_2$ fits have a maximum error of 0.4%, 1.5% and 3% respectively (in the original fits from Chan et al. [35] the maximum errors are 1.5%, 10%, 15%). Furthermore, we update the $C-\Lambda_2$ UR useful for estimating the NS radius from the mass and quadrupolar tidal parameter posterior obtained from the GW observational events involving NSs. Our updated $C-\Lambda_2$ relation holds a maximum error $\leq 5\%$ (see Table III).

To see the effect of URs, we analyze the BNS event GW170817 with inspiral only frequency domain waveform model TaylorF2 and also adding different waveform model corrections or changing the URs. An investigation of summarized points of Section III C leads one to conclude the following: (1) Including the higher order $\Lambda_3$ and magnetic $\Sigma_2$ tidal effect to the GW phase, have no significant effect on the estimated tidal parameter, (2) by the change of URs for multipole Love relation (i.e., for $\Lambda_3$ and $\Sigma_2$) with TF2$_{\Lambda_2, \Sigma_2}$ waveform model from the URs of Yagi [35] to the URs developed in this work, we do not notice any significant change in $\Lambda$ (although our

| Parameters | Injection | A+ | ET |
|------------|-----------|----|----|
| $\mathcal{M}_t$ [M$_{\odot}$] | 1.067 | 1.067 | 1.067 |
| $\alpha$ | 0.855 | 0.855 | 0.855 |
| $\delta \Lambda$ | 62.439 | 62.439 | 62.439 |
| $\Delta M_\Lambda$ [%] | 590.35 | 590.35 | 590.35 |

| Parameters | Injection | A+ | ET |
|------------|-----------|----|----|
| $\mathcal{M}_t$ [M$_{\odot}$] | 1.186 | 1.186 | 1.186 |
| $\alpha$ | 0.855 | 0.855 | 0.855 |
| $\delta \Lambda$ | 37.390 | 37.390 | 37.390 |
| $\Delta M_\Lambda$ [%] | 313.94 | 313.94 | 313.94 |

| Parameters | Injection | A+ | ET |
|------------|-----------|----|----|
| $\mathcal{M}_t$ [M$_{\odot}$] | 1.304 | 1.304 | 1.304 |
| $\alpha$ | 0.855 | 0.855 | 0.855 |
| $\delta \Lambda$ | 23.114 | 23.114 | 23.114 |
| $\Delta M_\Lambda$ [%] | 168.01 | 168.01 | 168.01 |

**Table VII:** Same as Table VI but the injections are done with APR4 EoS.

| Parameters | Injection | A+ | ET |
|------------|-----------|----|----|
| $\mathcal{M}_t$ [M$_{\odot}$] | 1.067 | 1.067 | 1.067 |
| $\alpha$ | 0.855 | 0.855 | 0.855 |
| $\delta \Lambda$ | 62.439 | 62.439 | 62.439 |
| $\Delta M_\Lambda$ [%] | 590.35 | 590.35 | 590.35 |

| Parameters | Injection | A+ | ET |
|------------|-----------|----|----|
| $\mathcal{M}_t$ [M$_{\odot}$] | 1.186 | 1.186 | 1.186 |
| $\alpha$ | 0.855 | 0.855 | 0.855 |
| $\delta \Lambda$ | 37.390 | 37.390 | 37.390 |
| $\Delta M_\Lambda$ [%] | 313.94 | 313.94 | 313.94 |

| Parameters | Injection | A+ | ET |
|------------|-----------|----|----|
| $\mathcal{M}_t$ [M$_{\odot}$] | 1.304 | 1.304 | 1.304 |
| $\alpha$ | 0.855 | 0.855 | 0.855 |
| $\delta \Lambda$ | 23.114 | 23.114 | 23.114 |
| $\Delta M_\Lambda$ [%] | 168.01 | 168.01 | 168.01 |

**Table VIII:** Same as Table VI but the injections are done with Stiff EoS.
URs favor a lower value for the lower bound of $\tilde{\Lambda}$ and a higher value for the 90% upper bound). (3) Additionally, the inclusion of f-mode dynamical tidal correction in the waveform model puts a tighter constraint on the range of $\tilde{\Lambda}$ by decreasing the 90% upper bound by 16-22% compared to $\tilde{\Lambda}$ estimated with waveform model phase considering only adiabatic tidal phase. Also, the inclusion of f-mode dynamical tide favors a lower median for $\tilde{\Lambda}$ (by 6-10%) compared to other waveform model correction scenarios. Including f-mode dynamical tidal phase, the updated URs predict a higher median and higher 90% upper bound on $\tilde{\Lambda}$ (by ~6%) compared to the URs from [35, 37, 38].

We estimate the NS radii for the components of GW170817 using the estimated mass ($m$) and $\Lambda_2$ posteriors through $C - \Lambda_2$ relations. Our updated $C - \Lambda_2$ UR predicts a higher median (also higher upper bound) for $R_1$ (or $R_2$) by 200-250m compared to use of $C - \Lambda_2$ relation from Maselli et al. [39]. Further adding the effect from $\Lambda_3$ and $\Sigma_2$ to the waveform phase or changing the multipole Love relation, we did not notice any significant change in the inferred NS radii. However considering the

FIG. 8: Histogram and Posterior distribution of recovered $\tilde{\Lambda}$ for different injection events with A+ detector configuration corresponding to the (a) Soft-EoS, (b) APR4 EoS and (c) Stiff-EoS . The injected values are shown with black dashed lines.
dynamical f-mode tidal phase, the median of both $R_1$ and $R_2$ drops by $\sim 300-400$m compared with the radii estimated with waveform without dynamical f-mode phase (the 90% upper bounds on radii drop by 400-800m depending upon the choice of URs). Our updated URs predicts higher radii with TF2$_{A_3,\Sigma_2\ell_4}$ compared to the URs from [55] [57],[59]. We also notice that adding f-mode dynamical tidal phase largely support that $\Lambda \leq 300$ as reported in [21] [56] with waveform models with numerical relativity (NR)-tuned tidal effects.

In particular, recent gravitational waveform modelling developments include the dynamical tidal correction due to the excitation of NS f-modes [31] [62] [103] [104]. Recently in [31], by considering the state of the art effective one body (EOB) model TEOBResumS, it has shown that due to the noise dominance in the higher frequency range of GW170817, both waveform model with and without f-mode dynamics behave similarly. Comparing the Bayes factors from Table V for different model, we also notice that the waveform models with f-mode dynamical correction behaves similar to the waveform models without considering f-mode dynamical tides. The NS properties reported in this work for BNS event GW170817 can be further enhanced by adding the spin-quadrupole [105], spin-tidal effect [106] and spin correction to the f-mode dynamical phase or even by adding eccentricity.

We perform injection and recovery studies with A+ sensitivity and ET by considering three different EoSs with different stiffness. In agreement with [33], we notice that ignoring the f-mode dynamical tide can overestimate the median $\Lambda$ by $\sim 10-20\%$ depending upon the EoS and the dynamical tide has a significant impact for low mass NSs, which further enhanced with increasing the stiffness of the EoS. Additionally, here we show that the effect of re-calibrating the URs is subdominant to neglecting f-mode dynamical tides. Although the median of the recovered tidal parameters with a different set of URs differs only by $\sim 5\%$, the distribution of $\Lambda$ gets slightly more affected with ET compared to A+ configuration. In ET, the injected $\Lambda$ is recovered more efficiently by using the updated URs of this work compared to existing URs. Although the difference between URs is not statistically discernible on a per-event basis, it is potentially important while combining constraints from many events.

Our conclusions regarding the injection and recovery studies may change by considering the additional effect resulting from spin and also with the spin correction to the f-mode dynamical phase. Although we ignore the spin and eccentricity, it was suggested that spin and eccentricity further enhance the excitation of f-modes in binary [30]. However, the corrections to the f-mode dynamical tide due to spin and eccentricity are still matter of investigation and recent efforts are going on this direction [30] [103] [104] [107].

The detection of f-mode characteristics or the post-merger peak frequency $f_{\text{peak}}$ can be used to infer the NS EoS, presence of phase transition or even the NS interior composition [48] [78] [108] [115]. Although the detection of f-modes needs the third generation detector sensitivity [34] [91], the impact is significant for next observing runs [33]. The detection of $f_{\text{peak}}$ becomes more likely in the next observing runs or even with a third generation detector if the merger happens near a supermassive black hole [114]. In Pradhan et al. [78], we have shown that the plane of f-mode frequency and $\Lambda_2$ obtained from NSs in binary can put insights regarding the presence of hyperons in the NS interior. Detection of f-mode frequency can further be used to constrain nuclear parameters [78] [92] [116] [117]. An important breakthrough may also come with the launch of an optimised GW detector to study post-merger nuclear physics in the frequency range 2–4 kHz, as proposed by the ARC Centre of Excellence for Gravitational Wave Discovery in Australia: Neutron Star Extreme Matter Observatory (NEMO) [118].

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Appendix A: Comparing URs

We compare different URs by comparing the relative errors corresponding to different URs. The relative errors
over the range of $\Lambda_2 < 10^4$ for $\Lambda_3, \Lambda_4, \Sigma_2, \Sigma_3, C$ and $M\omega_2$ resulting from different URs are displayed in Figure 9.

Appendix B: Distribution of $\tilde{\Lambda}$ of simulated events in ET configuration

As described in Section [17D], we perform the injection and recovery studies with the third generation Einstein Telescope (ET) detector with the proposed ET-D sensitivity. We display the distribution of recovered $\tilde{\Lambda}$ in ET sensitivity in Figure 10 (similar to Figure 8).

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