Research on Formation Keeping of Multi-rotor UAVs Based on Improved Virtual Structure Method

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Abstract. Multi-UAV cooperative formation flight (CFF) can greatly improve the damage resistance and mission completion efficiency of single UAV, so it has become a research hotspot in recent years. The entire process of multi-UAV formation flight can be decomposed into target tracking, formation keeping, obstacle avoidance and collision avoidance. The research of formation keeping mainly uses the leader-follower or virtual structure method. The former has the problem of single point of failure, and the formation error will be amplified step by step. The latter is difficult to select virtual leader, and is hard to adjust the formation adaptively with limited communication conditions. Based on the virtual structure and multicast method, this paper proposes a recursive fusion strategy which reduces the requirement of communication frequency between UAVs, increases the adaptability of formation to single point of failure, and improves the accuracy and adaptability of formation keeping to a certain extent. On this basis, the cascade PID model is used to design the formation controller. Finally, the effectiveness of the strategy is verified by using MATLAB.

Keywords. Cooperative formation flight; virtual structure; formation keeping.

1. Introduction

In recent years, single UAV have extensive and in-depth applications in military and civil fields. However, the limitations of single UAV begin to appear, such as low anti damage, difficult to improve the efficiency of task execution and so on. Cooperative formation flight of multi-UAV has made up for the lack of a single UAV to a great extent. How to realize self-organized intelligent real-time formation flying of multiple drones has become a research hotspot in recent years.

Multi-UAV Cooperative Formation Flight (CFF) is to arrange drones with autonomous flight functions according to the designed three-dimensional space structure, so that they keep a stable formation during the flight process, and can change the formation shape according to mission needs and environmental changes [1-3]. Commonly used cooperative control algorithms are: leader-follower, virtual structure, behavior-based method. Among them, the leader-follower method is the most commonly used and mature strategy. The method is simple and easy to implement. But the disadvantage is that the formation is too dependent on the leader UAV, and the problem of formation error amplification is obvious [4-5]. The advantage of virtual structure method is that the formation keeping accuracy is high, but it requires high communication and airborne computing ability [6-7]. Behavior-based method has good flexibility and robustness, but the accuracy of formation keeping is low, and it is difficult to describe mathematically [8-10].

This paper studies based on the virtual structure method. However, when the formation error is large, the problem of insufficient mobility is very obvious. Once encountering unexpected conditions...
such as strong winds or changes in mission, the selection of the virtual leader UAV becomes very difficult.

In order to achieve multi-rotor UAVs formation flight in three-dimensional space, this paper proposes a fusion recursive strategy based on multicast and virtual structure method to update the formation reference state, and the effectiveness of the strategy is verified by simulation.

2. Multi-rotor UAVs Formation System Modeling

2.1. Problem Description

For a multi-UAV formation, assuming that there are n UAVs in the formation, the n UAVs need to form a predetermined formation and fly stably under the initial conditions of large formation error. This article makes the following settings for the research content:

(1) The research object of this paper is limited to short-range formation, that is, any UAV in the formation is within the communication range of other UAVs.

(2) There is a good communication environment during the formation flight.

(3) This article is limited to the multi-rotor UAVs model, which requires the UAV to be able to hover, retreat and other actions, and each UAV can quickly track the target motion command.

(4) A route is designed before the formation flight.

2.2. Coordinate System Definition

Ground inertial coordinate system: The origin point is taken as a fixed point on the ground. X axis refers to east, Y axis refers to north, and the Z axis perpendicular to the XY plane points to the sky direction. Let I denote the ground inertial coordinate system.

Heading coordinate system: The origin of this coordinate system is taken as the mass center of the UAV, with the horizontal speed direction of the UAV as the Y axis direction, the Z axis is vertical to the local horizontal plane, and the X axis is perpendicular to the YZ plane and meets the right-hand coordinate system.

Formation coordinate system: The coordinate system used to describe the relative positional relationship of UAVs in the formation is a heading coordinate system centered on the virtual leader UAV. Let b denote this coordinate system.

2.3. Relative Kinematic Modeling

This paper designs a strategy for formation keeping of a multi-UAV formation based on the virtual structure method. Figure 1 shows the relative motion relationship in multi-UAV formation. In the formation structure, it is assumed that a virtual leader UAV $L$ is used as a reference for the entire formation status information. The formation is defined by the distance between each UAV and the virtual leader point on the formation coordinate axis. Defined as follows:

$$\rho^d = \begin{pmatrix} x_{iL}^d & y_{iL}^d & z_{iL}^d \\ \vdots & \vdots & \vdots \\ x_{nL}^d & y_{nL}^d & z_{nL}^d \end{pmatrix}$$

(1)

$\rho_{iL}^d$ is the distance between UAV $i$ and the virtual leader UAV $L$. Set the position vector of UAV $i$ and the virtual leader UAV $L$ in the formation under $I$ coordinate system to be $P^i_I = (x_i, y_i, z_i)$ and $P^i_L = (x_L, y_L, z_L)$. The actual relative position vector is $\rho^i_{L}$, which satisfies

$$\rho^i_{L} = P^i_I - P^i_L$$

(2)

Using the coordinate conversion matrix $C_{ib}$ to convert the above relationship from the $I$ coordinate system to the $b$ coordinate system, we get:
The ultimate goal of formation control is to make $\rho_{ia}^b = \rho_{ia}^d$.

The formation keeping scheme based on the virtual structure method, because there is no physical leader UAV, can avoid the single point of failure. There are two main types of processing for virtual leader in traditional algorithms, but both of these methods have obvious problems. The former has low adaptability and robustness, and the latter’s equal-weight average can easily amplify the effect of outliers. In order to solve this problem, this paper proposes a recursive fusion strategy based on multicast to update the virtual leader status.

3. Multi-rotor UAVs Formation Keeping Control Scheme

Based on the assumptions made in the paper, the communication environment during formation flight is good, and any UAV is within the communication range of other UAVs, which can ensure that the information sent by any UAV can be obtained by other UAVs in time. For any UAV, in the formation, the method of calculating the virtual leader is as follows:

1. It may be assumed that at time $t_n$ UAVi knows the motion status and control command of the virtual leader of time $t_{n-1}$ as equation (5). Then the virtual leader public reference state at time $t_n$ can be calculated and recorded as $\xi_{L-a}(t_n)$ which is the same for any UAVi.

\[
\begin{aligned}
\xi_{L-a}(t_n) &= (P_L(t_{n-1}), \psi_{L}(t_{n-1})) \\
\end{aligned}
\]

2. Calculate the private reference position $\xi_{L-b}(t_n)$ of the virtual leader based on the current position $P_i(t_n)$ of UAVi and the formation matrix $C^d$, where C is the rotation matrix from I to b.

\[
P_{L-b}(t_n) = P(t_n) - C^T \rho_{L}^d
\]

3. The virtual leader public reference position $P_{L-a}(t_n)$ and the private reference position $P_{L-b}(t_n)$ are weighted and merged to obtain the final virtual leader reference position $P_L(t_n)$ at time $t_n$.

\[
\begin{aligned}
\end{aligned}
\]
tn, where \( f(\Theta) \) is the trust degree determined by \( P_{L,a}(t_n) \), \( P_{L,b}(t_n) \) and \( \rho_{it}^d \), namely the weight coefficient. Figure 2 indicates that these two positions may be different. At the same time, the heading angle is always equal to the angle \( \psi_j \) of the current flight segment in the design route, and \( j \) is the current route segment.

\[
\begin{align*}
    P_L(t_n) &= (1 - f(\Theta))P_{L,b}(t_n) + f(\Theta)P_{L,a}(t_n) \\
    \psi_L(t_n) &= \psi_j
\end{align*}
\]

(4) The position \( P_L(t_n) \) of virtual leader calculated by this method is likely to fluctuate greatly, which is easy to cause the formation to deviate from the predetermined trajectory. Therefore, it is necessary to project it in the direction of the route on the basis of \( P_L(t_n) \). So far, the latest status of virtual long machine has been obtained.

![Figure 2. Virtual leader computing.](image)

When the communication status is good, UAV\(_i\) will release the current virtual leader status to the outside and the publishing method is multicast. In the form of multicast, when the source host is required to send information to multi-point targets, only one piece of data needs to be sent. The target address is the multicast group address, and other UAVs in the formation can received a copy of the data as long as they belong to the group. This method reduces the frequency of transmission frequency of single UAV, and avoids the problems of repeated data transmission and excessive bandwidth consumption in the case of unicast.

For the entire formation, only one UAV can send virtual leader status information to the multicast address at a time, and other UAVs are in receiving state. This method can ensure that the reference standards of all UAVs are consistent, and the formation error will not be amplified in multiple stages. At the same time, the source host needs to be rotated in the formation UAVs. That is, assuming a formation with n UAVs, the state update frequency of the virtual leader is specified as N. The state of the virtual leader is released by UAV\(_1\) at first, followed by UAV\(_i\) (\( i=2,3,\ldots,n \)). The frequency for each UAV to release the reference status of the virtual leader is \( N/n \). After \( n/N \) seconds, the state of the virtual leader will be the weighted result of the state of all UAVs, which makes the backward UAVs accelerate and the leading UAVs decelerate to achieve the purpose of adaptive formation adjustment. This method can also solve the problem of single point of failure. For example, during the flight, UAV\(_i\) malfunctioned and could not fly or communicate normally. Other UAVs only need to time after receiving the information sent by the drone UAV\(_{i-1}\). If no information from UAV\(_i\) is received within \( 2/N \) seconds, UAV\(_{i+1}\) will skip UAV\(_i\) and starts to send information. UAV\(_i\) is directly excluded in the subsequent communication cycle until UAV\(_i\) returns to normal.
3.2. Formation Keeping Controller

Formation keeping requires $\rho_{il}^i = \rho_{il}^d$, that is, the relative position between any $UAV_i$ and the virtual leader is equal to the corresponding value in the formation matrix. Meanwhile, black point and red point in figure 2 are now overlapped. It can be seen from section 3.1 that this condition is equivalent to the following formula for any $UAV_i$.

$$P_{L-b}(t_n) = P_{L-a}(t_n) = P_L(t_n)$$

(7)

Therefore, the PID controller can be designed through different mixed errors $e_i$ to calculate the $UAV$ control commands, as shown in the following formula.

$$u_i = K_pe_i + K_i \int e_i dt + K_D \frac{d}{dt} e_i$$

(8)

4. Analysis of Simulation Results

In the simulation, four multi-rotor UAVs are designed to fly in formation to verify the effectiveness of the virtual leader status update strategy. The starting point of the designed route is at (0,0,0), the end point is at (70,70,20). The initial position of the virtual leader is (0,0,0), the heading always points to the target point, the update frequency of the virtual leader is 10 Hz. The initial positions of the UAV in the ground inertial coordinate system are $\rho_1^i=(0,10,0)$, $\rho_2^i=(-20,20,0)$, $\rho_3^i=(-5,15,-5)$, $\rho_4^i=(0,-5,-10)$ and the formation position in the formation coordinate system are $\rho_1^d=(10,0,0)$, $\rho_2^d=(0,10,0)$, $\rho_3^d=(0,-10,0)$, $\rho_4^d=(-10,0,0)$.

The simulation results are as follows.

Figure 3. Projection of UAV trajectory on XY plane.

Figure 4. Projection of UAV trajectory on XZ plane.

It can be seen from figures 3 and 4 that the four multi-rotor UAVs can finally form the expected formation shape.

The virtual leader status update strategy based on multicast communication mentioned in the article can meet the requirements of short-range multi-rotor formation flight. Figure 5 is the position-time diagram of the virtual leader on the x-axis. Virtual leader almost converges to the destination after 50s.

It can be found that due to the large formation error of the UAV in the early stage of the formation flight, the position difference of the virtual leader calculated by each UAV is relatively large. According to the virtual leader update strategy, in figure 6 the position of the virtual leader fluctuates in the x direction. This fluctuation slows down the speed of the UAV which is ahead of its desired position, and accelerate the UAV which is behind the desired position. This hastened the formation.
generation and as the formation develops, the fluctuations gradually decreased and eventually stabilized.

**Figure 5.** Virtual leader x-t trajectory. **Figure 6.** Partial enlarged drawing of virtual leader x-t trajectory.

Figure 7 is the position of UAV2 in the formation coordinate system, which is fluctuated within a small range. Due to the chaotic formation at this time, the position of the virtual leader fluctuates back and forth, and the multicast-based virtual leader update strategy in this paper will circularly refer to the position information of all UAVs, so the coordinates of any UAVi in formation coordinate system will show a wave trend of N-cycle.

Figure 8 reflects the position error between the simulated trajectory of each UAV and the expected trajectory. It can be seen from figure 8 that the UAV with larger error will approach the target position at a faster speed, and the UAV with smaller error will slowly approaching the target position. At the same time, it can be found that the impact of UAVs with large formation errors on the formation will not be reflected suddenly, but caused by a slow trend. It can be seen that the formation error around 20s is always less than 1m, indicating that the formation has reached the expected formation shape, which shows the effectiveness of the virtual leader update strategy proposed in this paper.

**Figure 7.** position of UAV2 in b system. **Figure 8.** Position error of UAVs in b system.

5. Conclusion
This paper proposes a strategy for updating the status of virtual leader UAV based on multicast and virtual structure method. According to the status of each UAV in the formation, the position of the virtual leader on the predetermined route is adjusted. This method has the following advantages:
(1) Reduce the requirement for the frequency of information transmission, which can save the bandwidth occupied by single-machine communication.

(2) Improve formation accuracy. The status of any UAV can be reflected in the state of the virtual leader to varying degrees, which can make the movement of the entire formation adaptively adjusted.

(3) Make the formation flight not too dependent on the integrity of a certain aircraft, avoiding the single point of failure problem occurred in the traditional formation strategy.

(4) Have the ability to adaptively adjust the trust of any UAV and make the formation movement more stable.

Simulation results show that the above method can make multi-UAV formations form a predetermined formation and keep stable flight.

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