Has the neutral double hypernucleus $^4\Lambda\Lambda n$ been observed?

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Abstract. The BNL-AGS E906 experiment was the first fully electronic experiment to produce and study double hypernuclei with large statistics. Unfortunately, the interpretation of the measured $\pi^-\pi^-$ momentum correlation is still blurry because the hypothesized production of $^3\Lambda H+^4\Lambda H$ pairs remains questionable. We show, that neither a scenario where the hypernuclei are produced after the capture of a stopped $\Xi^-$ by a $^9\text{Be}$ nucleus nor interactions of energetic $\Xi^-$ with $^9\text{Be}$ nuclei in the target material can produce a sufficient amount of such pairs. We have therefore explored the conjecture that decays of the $^4\Lambda\Lambda n$ may be responsible for the observed structure. Indeed, the inclusion of $^4\Lambda\Lambda n$ with a two-body $\pi^-$ branching ratio of 50% in the statistical multifragmentation model allows to describe the E906 data remarkably well.

1. The E906 puzzle
In 2001 the AGS-E885/E906 collaboration announced the first 'mass production' of about 30 $^4\Lambda\Lambda H$ events based on $\sim 10^4$ stopped $\Xi^-$ in a $^9\text{Be}$ target [1]. The $\Lambda\Lambda$-hypernuclei were identified through the sequential weak decay via $\pi^-$ emission after a $(K^-K^+)$ reaction deposited two units of strangeness in a $^9\text{Be}$ target. Coincidences between two negative pions helped to trace the sequential decay of a $\Lambda\Lambda$-hypernucleus or pairs of single hypernuclei, so called twin hypernuclei. Two dominant structures were observed in the correlated $\pi^-\pi^-$ momenta at $(p_{\pi^-H}, p_{\pi^+L}) = (133,114)\text{ MeV}/c$ and at $(114,104)\text{ MeV}/c$. For the $(133,114)\text{ MeV}/c$ structure, the interpretation in terms of $^3\Lambda H+^4\Lambda H$ twins seems inevitable. However, the $^3\Lambda H+^4\Lambda H+t$ mass lies above the initial mass $m_0 = m(\Xi^-) + m(^9\text{Be})$ and can therefore not be produced in a $\Xi^-\text{stopped}+^9\text{Be}$ compound nucleus production scheme.

In view of this puzzling situation, a production process different from stopped $\Xi^-$ capture seems to be necessary in order to explain the singular structure at $(133,114)\text{ MeV}/c$ in terms of $^3\Lambda H+^4\Lambda H$ twins. In Ref. [1] it has been suggested that energetic $\Xi^- + ^9\text{Be}$ nuclear reactions may lead to highly excited systems which allow decays into $^3\Lambda H+^4\Lambda H$ pairs. As an alternative, A. Gal conjectured in a talk at the HYP-2003 conference that the sequential weak decay of $^4\Lambda\Lambda n \rightarrow \pi^-+^3\Lambda H$ (see also Tab. 1 in [5]) might also cause an enhancement around $(133,114)\text{ MeV}/c$ [10, 16].
Figure 1. Relative production probability of double (left) and twin hypernuclei (right) for a primary $^{10}_{\Lambda\Lambda}$Li* nucleus as a function of its excitation energy. The hypothetical neutral $^{4}_{\Lambda}$n [17, 11] was not included in these calculations. Figure taken from [4].

2. Why is the capture of stopped $\Xi^-$ no solution?
In order to illustrate the role of the excitation energy in systems formed after the capture of $\Xi^-$ by $^9$Be, we constructed the excitation function for all possible decay channels. The calculations were done within a micro-canonical decay model [14]. Fig. 1 shows the production probability of possible pairs of twin hypernuclei (right part) and double hypernuclei (left part) as a function of the assumed excitation energy of the $^{10}_{\Lambda\Lambda}$Li* system. The figure demonstrates that the excitation functions of all important decay channels are rather flat for excitation energies between about 25 and 30 MeV. As a consequence, the results of the simulations will only depend weakly on whether the $\Xi^-$ conversion happens from an atomic bound state with $B_{\Xi^-} \approx 0$ MeV or from a nuclear bound state at $B_{\Xi^-} \approx 4$ MeV. In the following we assume $B_{\Xi^-} = 0$ MeV and thus fix the initial excitation energy of the $^{10}_{\Lambda\Lambda}$Li* system at 29.5 MeV.

The circles in the left part of Fig. 2 show the relative probabilities for the production of particle stable twin and double hypernuclei in the E906 experiment after the capture and conversion of a stopped $\Xi^-$, i.e. $\Xi^- + ^9$Be $\rightarrow ^{10}_{\Lambda\Lambda}$Li* with $B_{\Xi^-} = 0$ MeV. Ground state(s) and excited state(s) - if they exist - have been added. Weak decay probabilities are not taken into account in this graph. Furthermore, neutral hypernuclei were not yet included as possible decay channels. The area of the circles in Fig. 2 is proportional to the production probabilities. The centers of the circles are fixed by the pion momenta for mesonic two-body decays. Obviously, there is no conventional strong pionic decay channel appearing which could produce a bump at $(133,114)$ MeV/c.

The right part of Fig. 2 shows the result of the statistical multifragmentation model (SMM) calculations now taking into account the pionic two-body branching ratios and the momentum resolution of 3.5 MeV/c of the E906 experiment. The binning of the momentum scale is the same as in the experimental matrix of E906. More details on the SMM simulations can be found in Refs. [5, 4]. The dashed rectangle highlights the region of the enhancement seen by the E906 experiment. Even after weak decays, no accumulation of events in the region of interest is discernible.

3. Can $\Xi^-/^9$Be reactions solve the problem?
The $(K^- K^+)$ reaction produces $\Xi^-$ hyperons with a broad momentum spectrum. Only $\Xi^-$ with momenta below approximately 500 MeV/c can be stopped with high efficiency in a target
Figure 2. Left: Momenta of two correlated pions measured by the E906 collaboration (grey squares) [1]. Overlaid are production yields of various twin pairs (blue), double hypernuclei including their excited states (green) and double hypernuclei with a stable ground state only (red). Right: SMM production probabilities. For more details see text. Figures taken from [5].

material. More energetic $\Xi^-$ hyperons will decay in flight or will perform hadronic interactions which eventually may produce hypernuclei as well.

The black histogram in the left panel of Fig. 3 shows the momentum distribution of $\Xi^-$ hyperons produced in the $(K^-,K^+)$ reactions of E906. This distribution was obtained from a fit to the free $\Xi^-$ energy spectrum of E906 [15] and a transformation to a momentum scale. These $\Xi^-$ hyperons were propagated in the beryllium target by means of a GEANT simulation. The fraction of $\Xi^-$ decaying in flight (red), stopping in the target (green) and undergoing reactions (blue) are shown.

With 2.83%, the reaction probability is similar to the probability of stopped $\Xi^-$. In the following, the dashed blue distribution in Fig. 3 will be folded with predictions of Giessen Boltzmann-Uehling-Uhlenbeck transport model (GiBUU) simulations [6, 9] in order to estimate the contribution by secondary $\Xi^-+^9$Be nuclear interactions to the production of $^3_\Lambda^4H+^4_\Lambda^4H$ twins.

The right part of Fig. 3 shows the predicted production probability for different $S=-2$ event types as a function of the $\Xi^-$ momentum. Across the complete momentum range, two free $\Lambda$ particles or a free $\Lambda$ plus a single $\Lambda$-hypernucleus ($SH+\Lambda$) are the dominating channels. Only at low momenta, where the stopping probability of $\Xi^-$ hyperons is also large (see Fig. 3, left), double hypernuclei (DH) have a similar probability to be produced (gray triangles). The production of $^3_\Lambda^4H+^4_\Lambda^4H$ twins (green triangles) in particular reaches only a probability per interaction of the order of $10^{-4}$.

Table 1 summarizes the production probabilities for various channels in stopped $\Xi^-$ reactions and $\Xi^-+^9$Be interactions. The probabilities are normalized to the total number of produced $\Xi^-$ in the $(K^--K^+)$ reaction, i.e. the black histogram in Fig. 3.

Concerning the production via stopped $\Xi^-$ hyperons, a comparable rate estimate has to take into account not only the $\Xi^-$ production and stopping probability (green histogram in Fig. 3) but also the capture probability, the $p\Xi^- \rightarrow \Lambda\Lambda$ conversion and finally the fragmentation processes. The latter is treated by SMM. However, the other two missing pieces – capture and conversion – require additional considerations.

Several theoretical estimates for the capture and sticking probability exist which can be used
Figure 3. Left: The black histogram represents a fit to the momentum distribution of $\Xi^{-}$ in the E906 experiment which was deduced from the measured $K^{+}$ distribution (cf. Ref. [15]). The fraction of $\Xi^{-}$ decaying in flight in the beryllium target is given by the red histogram. $\Xi^{-}$-hyperons triggering a nuclear reaction in the target are represented by the blue histograms. Right: Production probability of different $S = -2$ final states predicted by the GiBUU code [9] for $\Xi^{-} + ^{9}$Be reactions as a function of the $\Xi^{-}$ momentum. For these calculations the YN-interaction model of Fujiwara et al. [8] was used. Figures taken from [4].

Table 1. Probability per produced $\Xi^{-}$ for different $S = -2$ channels by stopped $\Xi^{-}$ hyperons and by nuclear $\Xi^{-} + ^{9}$Be reactions. Here, SH (DH) stands for single (double) hypernucleus, respectively. In case of the stopped $\Xi^{-}$ process a capture $\times$ conversion probability for producing excited $\Lambda\Lambda$ nuclear systems of 5% was taken into account. The probabilities in the last 5 columns are multiplied by $10^{4}$.

| process | probability [%] | model | $\Lambda+\Lambda$ | $\Lambda+SH$ | $SH+SH$ | DH | $\Lambda^{++}$ |
|----------|----------------|-------|-------------------|--------------|---------|-----|----------------|
| stopped $\Xi^{-}$ | 4.65 $\times$ 0.05 | SMM $^{10}_{\Lambda\Lambda}$LI* | 0.525 | 6.365 | 6.659 | 9.699 | 0 |
| $\Xi^{-} + ^{9}$Be | 2.83 | GiBUU | 37.2 | 20.9 | 0.070 | 3.23 | 0.016 |

as a guidance. Existing experimental data from earlier emulsion experiments also provide an estimate for these two factors [2].

- Within a doorway picture for the $\Xi^{-} + p \rightarrow \Lambda\Lambda$ conversion [18, 19], the $\Lambda\Lambda$ sticking probability, i.e. the summed production rate per $\Xi^{-}$ for $\Lambda\Lambda$ and twin $\Lambda$-hypernuclei, is predicted to be about 5%.

- Hirata et al. studied the formation of single, twin and double hypernuclei after $\Xi^{-}$ absorption at rest on $^{12}$C with a microscopic transport model [12]. They found a probability of around 10% for double hyperfragment production and about 1% for twin production. This provides a lower limit of 11% for the combined capture $\times$ conversion probability.

- In [13, 20] the transition amplitudes from $\Xi^{-} + ^{12}$C atomic states and $\Xi^{-} + ^{12}$C hypernuclear states to final $\Lambda\Lambda$ states were calculated. Folding these numbers for the different $\Xi^{-}$-atomic/nuclear states with calculations of the atomic cascade for carbon [3], one finds a total capture $\times$ conversion probability of a few percent depending on $\Xi^{-}$ potentials.

- Based on the number of twin (TH) and double (DH) $\Lambda$-hypernuclei observed by the KEK E176 collaboration one can estimate a lower limit of 4.8% for the capture $\times$ conversion $\times$ (TH or DH) in light nuclei [2]. According to our statistical model, in 70% of all decays a
twin or a double hypernucleus is formed. From this we can estimate a lower limit for the capture $\times$ conversion probability of about 7%.

Consistent with these estimates, we assume a $\Xi^-$ capture $\times$ p conversion probability of 5% and describe the subsequent decay of the excited $\Lambda\Lambda$ pre-fragment by SMM [14].

Comparing both production schemes in Tab. 1, it is apparent that $\Xi^-+^9\text{Be}$ interactions may strongly contribute to the production of free $\Lambda$ particles. The production of pairs of one free $\Lambda$ hyperon with one single $\Lambda$-hypernucleus and of $\Lambda\Lambda$-hypernuclei are comparable to the yield from stopped $\Xi^-$ reactions as well. However, the production of twin hypernuclei (SH+SH) is two orders of magnitude less likely compared to the yield of twins or double hypernuclei from the stopped $\Xi^-$ process. Neither stopped $\Xi^-$ hyperons nor energetic $\Xi^-+^9\text{Be}$ interactions are able to produce $^3\Lambda+^4\text{H}$ pairs in such a large amount to be comparable to the production of other twin hypernuclei or double hypernuclei. We therefore conclude, that interactions of quasi-free produced $\Xi^-$ hyperons within the $^9\text{Be}$ target can not solve the E906 puzzle.

4. Could a neutral $\Lambda_4\Lambda n$ nucleus solve the puzzle?

The $\Lambda_4\Lambda n$ is expected to be unbound [7]. Nevertheless, in order to test this hypothesis, we implemented the $\Lambda_4\Lambda n$ as a possible decay product in the SMM model. To fix its mass, we assumed a binding energy of $B_{\Lambda\Lambda} = 3\text{ MeV}$.

As before, we consider a $\Xi^-$ capture and conversion process at rest as the principle production scheme, thus forming a $^{10}\text{Li}^*$ system with 29.5 MeV excitation energy. Fig. 4 shows the momentum correlations of produced pion pairs for two pionic two-body branching ratios. This plot should be compared to the right panel of Fig. 2, where no neutral hypernuclei were allowed.

Figure 4. Momenta of two correlated pions predicted by the statistical decay model with a pionic two-body branching ratio of 25% (left) and 50% (right) which includes the production and decay of hypothetical neutral $\Lambda_4\Lambda n$ hypernuclei. Figures taken from [5].

For the pionic decay $\Lambda_4\Lambda n \rightarrow \pi^-+^4\text{H}$ a two-body branching ratio of $\Gamma_{2B}/\Gamma_{\text{tot}} = 25\%$ was adopted as the default value (Fig. 4, left). Compared to Fig. 2, a clear enhancement in the region of interest marked by the rectangle is visible. Increasing the branching ratio to 50% (Fig. 4, right) produces a local structure which resembles the experimental observation of E906 in Fig. 2, left.
5. Conclusion
The E906 experiment was the first fully electronic experiment to produce and study double hypernuclei with large statistics. Unfortunately, the interpretation of the measured $\pi^-\pi^-$ momentum correlation is still blurry because the hypothesized production of $^3\Lambda H + ^4\Lambda H$ pairs remains questionable. Neither a scenario where the double hypernuclei are produced after capture of a stopped $\Xi^-$ nor interactions of energetic $\Xi^-$ with $^9\text{Be}$ nuclei in the target can produce a sufficient amount of such pairs. We have therefore explored the conjecture that decays of the $^4\Lambda\Lambda n$ may be responsible for the observed structure. Considering the production of a bound $^4\Lambda\Lambda n$ with a two-body $\pi^-$ branching ratio of 50% in the statistical multifragmentation model, describes the measured data of the E906 experiment remarkably well. Nevertheless, this does not constitute a direct proof for the existence of a bound neutral double hypernucleus $^4\Lambda\Lambda n$. We hope that this work will stimulate further discussions and experimental activities which – finally – may help to solve the E906 puzzle.

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