A Real-time Method on Robot with Continuous Trajectory for Low-drift Odometry and Mapping

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Abstract. This paper mainly proposes a real-time method on the robot with a continuous trajectory for low-drift odometry and mapping, by using range measurements from a 3D laser scanner, but without any other external reference. The common problem of the classical simultaneous localization and mapping (SLAM) method based on lidar is that the estimation of position and attitude is prone to jump discontinuity. It is mainly caused by the vulnerable performance of the optimization solver while it is in a complex scene. By adding kinematic and dynamic constraints into optimization function, our method achieves a lower mean square error (MSE) compare with the original one, and it has been tested in an outdoor experiment with only 3.5% MSE in yaw relative to state of the art.

1. Introduction to related work
The simultaneous localization and mapping (SLAM), that is when the moving platform is in an unknown environment, gradually build the environment map and determine its relative position in the environment at the same time. SLAM problem is one of the classic and vital problems in robot field, and it is also one of the necessary conditions for the robot to realize autonomous motion. This problem was raised by Hugh Durran Whyte and John J. Leonard [1] based on the work recorded by Smith, self and Cheeseman in the literature [2]. After more than 20 years of development, researchers all over the world have solved the SLAM problem of a single platform in various theoretical forms, and have been successfully applied in autonomous driving, crewless aerial vehicles and other fields [3].

Among all the SLAM methods, they could be distinguished by the kind of sensors that is adopted, such as the SLAM algorithm based on the distance sensor [4] and the SLAM algorithm based on the visual sensor [5]. The SLAM algorithm based on range sensor usually refers to the laser SLAM algorithm using single-laser lidar or multi-laser lidar. This algorithm can directly obtain the environmental depth information; thus, it has a better performance in speed and accuracy, but its disadvantage is high cost, which is not suitable for the commercial development of robots. Usually, the imaging principle is cumbersome in calibration, with high computational complexity and low accuracy. According to the different location methods used, SLAM algorithm can be divided into SLAM Based on filtering method [6] and slam based on graph optimization method. The core idea of the former one is to obtain the position and attitude estimation through the inertial measurement unit (IMU) with high measurement accuracy and high sampling frequency, and construct SLAM problem as a filtering problem. These methods mainly focus on how to minimize the noise through the Extended Kalman Filter (EKF) or particle filter. On the contrary, the basic idea of the methods based on graph optimization is: the robot's pose at different times is represented as nodes, and the spatial constraint relationship
between nodes is represented as edges. This constraint relationship can be obtained through odometer or observation, so the SLAM problem can be transformed into a robot pose problem, seeking the optimal solution of the state in the transformation space.

Up to now, as a method to solve SLAM problem with lidar, Zhang Ji's work [7] (low drive and real-time lidar odometry and mapping, LOAM) has been the best one in efficiency and accuracy. This method proposes a solution that can achieve low drift and low complexity at the same time and does not need high precision ranging and inertial measurement. His core idea is making positioning and mapping to work at different frequencies: one is to perform high-frequency odometer but low-precision motion estimation, the other is to perform map matching and calibration odometer at lower frequencies. By combining these two algorithms, it makes high precision and real-time laser odometer. His method has been evaluated by indoor and outdoor experiments as well as the KITTI odometry benchmark.

2. The problems in state of the art
The common problem of the classical simultaneous localization and mapping (SLAM) method based on lidar is that the estimation of position and attitude is prone to jump discontinuity.

2.1. Phenomena of the problem
In the case that the trace of the moving platform is in a continuous and smooth track, while the estimation of the attitude and position is still discontinuous in the process of slam (the red arrows shown in Figure 1 and Figure 2, their start points represents the positions, and the endpoints represent the x-direction of the platform in every sample time). The apparent error is also directly reflected in the reconstructed 3D scene, which is revealed respectively by the wrong intersection (Figure 1) and discontinuity (Figure 1) of point cloud distribution.

![Figure 1. Local jump of attitude estimation in the transition phase of adjacent frames.](image1)

![Figure 2. Local jump of position estimation in the transition phase of adjacent frames.](image2)

2.2. Analysis of the problem
The odometer mainly estimates the motion of the rigid body between two adjacent frames, which is the different term of the position and attitude and could also be considered as the estimation of velocity term and angular velocity term. While the process of mapping aims at the estimation of the position and attitude themselves. These two processes split the motion process, sacrificing the continuity of the motion estimation. That is the direct cause of discontinuous motion state estimation, especially in the complex outdoor scene, the performance of the non-linear optimization solver is reduced. As for the continuity of motion state, Figure 3 and Figure 4 show the principle that should be obeyed by the direction of velocity and the direction of displacement, according to the mean value theory.
3. The new method with kinematic and dynamic constraints

3.1. Mathematical description of motion constraints

To solve the problem from the view of motion continuity constraint, we propose two underlying assumptions:

**Assumption 1:** In the course of manoeuvring, the drift and lateral motion are ignored, and the instantaneous velocity direction is generally consistent with the forward direction of the platform coordinate system.

**Assumption 2:** Between two adjacent frames, it is approximately assumed that the platform is uniformly accelerating in the ground reference system, and the uniformly accelerating trajectory must be a first-order derivative continuous convex trajectory (the straight line is also considered as a convex trajectory).

From these two assumptions, we further propose a lemma:

**Lemma:** Suppose there is a first-order continuous convex trajectory segment in space, \( S(t) = (x(t), y(t), z(t)), t \in [t_a, t_b] \). If the trajectory comes from a moving platform satisfying **assumption 1** and **assumption 2**, then the displacement vector \( \overline{AB} = S(t_b) - S(t_a) \) should satisfy \( (\overline{AB} \times \dot{S}(t_a))^\top (\overline{AB} \times \dot{S}(t_b))^\top < 0 \).

**Proof:** In the \( x^*y^*z^* \) plane composed of constant acceleration velocity \( \alpha \) and in-point speed \( \dot{s}^*(t_a) \). The trajectory of motion is a parabola. According to Cauchy mean value theorem of a continuously differentiable function, there must exist a \( \xi \in [t_a, t_b] \), s.t. the parabola \( S^*(t) = (x^*(t), y^*(t)) \) abides by

\[
\frac{x^*(t_b) - x^*(t_a)}{y^*(t_b) - y^*(t_a)} = \frac{x^*(\xi)}{y^*(\xi)}
\]

And because of the monotone continuity of the first derivative of a convex function, there should be

\[
\frac{x^*(t_a) - x^*(\xi)}{y^*(t_a) - y^*(\xi)} \cdot \frac{x^*(t_b) - x^*(\xi)}{y^*(t_b) - y^*(\xi)} < 0
\]

Represent by vector operation, and the positive and negative direction of shaving is represented by the operation of cross multiplication symbol \( \times \). We can directly obtain the constraints given by lemma, so far the proof is completed.

3.2. Redefinition of optimization objective function
For the case that should be avoided as much as possible in Figure 3, the motion constraint is added to the optimization objective function, to avoid the occurrence of attitude jump during the estimation of pose and attitude. Use unit vector $\bar{\mu}(t_k)$ defined by $\bar{\mu}_k(t_k) = \text{col}(1,0,0)$ represents the forward direction at $t_k$ in the body-coordinate system. In this coordinate system, the observed forward direction at $t_{k+1}$ is represented by $\bar{\mu}_k(t_{k+1})$, which is defined by $\bar{\mu}_k(t_k) = R_x(r_x)R_y(r_y)R_z(r_z)\text{col}(1,0,0)$. Similarly, the observed displacement direction is unit vector $\delta P$, defined as $1/|\tau| \cdot \text{col}(t_x, t_y, t_z)$. Then, map the constraints in lemma from $[-1,1]$ to $[1, +\infty)$ by tangent function, and represented by $\kappa(T_k)$:

$$\kappa(T_k) = \tan\left(\frac{\pi}{2}\left(\delta P \times \bar{\mu}_k(t_k)\right)^T \cdot \left(\delta P \times \bar{\mu}_k(t_{k+1})\right) + 3\right)$$  

(3)

Then the optimization objective function can be rewritten as:

$$T_{[k + 1|k]} = \arg \min \kappa(T_k) ||d(T_k)||_2$$  

(4)

For $\kappa(T_k)$, $J_k = \partial \kappa(T_k)/\partial T_k$ is used to express its Jacobian matrix. In order to distinguish, the Jacobian matrix of the vector function $d$ is represented by $I_d$. Then the First order linear expansion of Formula above is

$$\delta T_k = \arg \min_{T_k \in \mathbb{R}^6} \left(\kappa(T_k) + J_k \delta T_k\right) ||I_d \delta T_k + d(T_k)||^2 + \mu_k ||\delta T_k||^2$$  

(5)

However, the second-order term of $\delta T$ appears when the above equation is calculated according to the optimality condition, which brings great difficulty to judge the extreme value condition. If the second-order term is ignored, the above equation becomes:

$$\delta T_k = \arg \min_{T_k \in \mathbb{R}^6} \left(\kappa(T_k) ||J_k \delta T_k + d(T_k)||^2 + (\kappa(T_k) + J_k \delta T_k)||d(T_k)||^2 + \mu_k ||\delta T_k||^2\right)$$  

(6)

Find the vector derivative of $\delta T$ for the above formula, and take the extreme value condition, then we could obtain:

$$||d||^2 J_k + \kappa(2J_d^T d + 2J_d I_d) + \mu_k ||\delta T_k|| = 0$$  

(7)

So, we can get the iterative solution term, which is

$$\delta T_k = -(2\kappa J_d^T + \mu_k I)^{-1}(||d||^2 J_k + 2\kappa J_d^T)$$  

(8)

4. An experiment in outdoor scene

In the experimental design, as shown in Table 1, the experimental scene is selected near a railway track, which can more clearly reflect whether there will be a jump of motion estimation in position and attitude from the reconstructed three-dimensional scene.

| Table 1. Experiment condition about the original and the improved algorithm. |
|-----------------|-----------------|-----------------|
| Condition       | Original Loam   | Improved Loam   |
| Sensor          | Lidar           | MPU6050         |
| IMU             | RFans-16        |                 |
| Height Range    | 2m ± 0.4m above the ground |                 |
| Velocity Range  | 2.5m/s ± 1m/s   |                 |
| Pitch Range     | ±10°            | ±10°            |
| Yaw Range       | ±30°            | ±30°            |
| Roll Range      | ±5°             | ±5°             |
| Motion constraint | No              | Yes             |

If there is a jump, the generated railway must be discontinuous, otherwise, it should be continuous and smooth. The real site is a track as shown in Figure 5. Furthermore the reconstructed scene has been represented in Figure 6, from which we could find the point cloud is evenly distributed, which reflects
the shape characteristics of all kinds of features, especially the smooth and continuous rail and smooth platform of the station.

Figure 5. The experimental environment

Figure 6. Using the improved slam method based on motion constraints to reconstruct the environment.

We take the attitude angle offered by IMU as the reference value, and the lidar, as well as the IMU, have been unified in the same body coordinate system through external parameter calibration at the beginning. Considering the performance of the selected IMU, it needs to be interpolated and filtered in order to be used as the truth value of attitude. To distinguish these two performances of the attitude in table 2, we named our method as improved loam while state of the art as original loam.

Table 2. Algorithm performance comparison in attitude estimation.

|        | MSE       | Original Loam | Improved Loam | Relative MSE |
|--------|-----------|---------------|---------------|--------------|
| Pitch  | 0.0624    | 0.07433       | -             |              |
| Roll   | 0.1485    | 0.0261        | 17.5%         |              |
| Yaw    | 2.716     | 0.0952        | 3.5%          |              |

In Figure 7, compared with the pitching angle state, both the original loam algorithm and the improved loam algorithm are quite different from the reference truth value obtained by IMU, and the residual value is increasing, with the maximum residual value reaching 0.2 rad.

In Figure 8, compared with the roll angle state, the residual between the improved loam algorithm and the reference truth value of the state obtained by IMU is large at the beginning, and then gradually reduces to the maximum residual value of 0.025 rad with time, while the original loam algorithm has several large jumps, and the maximum difference of the jump reaches 0.4 rad.

In Figure 9, comparing the heading angle state, the difference among IMU, loam and improved loam is within tolerance.

Figure 7. The estimated Pitch from IMU, improved Loam and original Loam.
5. Conclusion
In this paper, an improved optimization method for continuous motion constraint is proposed to solve the problem of discontinuous attitude state jump in the process of online real-time estimation of motion attitude for outdoor complex scenes by using the classical laser radar SLAM algorithm. In this paper, the reason of the present jump of attitude estimation is discussed, and the continuous motion constraint of convex trajectory property and the improved iterative optimization algorithm are given.

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