APPLICATIONS OF THE SUPERFLAVOR SYMMETRY TO
HEAVY BARYON-ANTIBARYON PAIR PRODUCTION
IN ELECTRON-POSITRON COLLISION

Xin-Heng Guo $^{a,b}$, Hong-Ying Jin $^d$ and Xue-Qian Li $^{c,d,e}$

a: Institute of Physics, Academia Sinica, Taipei, Taiwan, China.
   b: Institute of High Energy Physics,
   P.O.Box 918-4, Beijing 100039, China
   c: The China Center for Advanced Science and Technology, (CCAST),
   (World Laboratory)
   P.O.Box 8730, Beijing, 100080, China.
   d: High Energy Group, ICTP, P.O. Box 586, Trieste, 34100, Italy
   e: Department of Physics,
   Nankai University, Tianjin, 300071, China.

Abstract
The baryons containing two heavy quarks and a light quark are believed to have the diquark-
quark structure and the diquark is composed of the two heavy quarks which is a spin-0 or
spin-1 object. The superflavor symmetry can associate productions of such heavy baryon-
antibaryon pair with the heavy meson productions. The whole scenario is presented in some
details in this work, and the observation prospect in future experiments is discussed.

This work was partly supported by the National Natural Science Foundation of China (NSFC)
Applications of the Superflavor Symmetry to Heavy Baryon-Antibaryon Pair Production in Electron-Positron Collisions

I. Introduction

To evaluate the hadronic exclusive processes is very difficult, because the hadronization is fully non-perturbative and, so far there is not a trustworthy way to deal with it from any underlying principles. The meson case has been studied for many years and remarkable progress is achieved. Since there is a good amount of data on meson production and decay, some relatively reasonable theories have been developed, for example the QCD sum rules [1], chiral Lagrangian [2] and the potential model [3] etc.. By contrast, the baryon case is much more complicated and obscure. It is not only because there are three valence quarks in baryons, while only a quark-antiquark pair in mesons, but also lack of data and available effective theories. It is known that applications of the QCD sum rules to the baryon case is much harder than to the meson case.

However, study on baryon physics can much enrich our knowledge of hadron structure and the mechanisms in the production and decay processes.

Fortunately, the heavy quark effective theory provides a way to appropriately simplify the evaluation of the hadronic matrix elements [4][5]. In the effective theory extra symmetries \( SU_s(2) \times SU_f(2) \) manifest and the non-perturbative effects are attributed into the well-defined Isgur-Wise function \( \xi(v \cdot v') \), where \( v \) and \( v' \) are the four-velocities of the concerned heavy quarks. In the heavy quark effective theory \( P_\mu = m_Q v_\mu + p_\mu \) where \( p \) is the residual momentum and of the \( \Lambda_{QCD} \) energy scale. Under the heavy quark limit, \( P_\mu = m_Q v_\mu \). The higher order \( 1/m_Q \) corrections to some processes have also been considered [6].

In this work we study the processes of \( e^+e^- \rightarrow X_s\bar{X}_{s'} \) where \( X \) stands for baryons containing two heavy quarks and the subscripts \( s \) and \( s' \) denote their spins. Concretely, we evaluate

\[
e^+e^- \rightarrow X_{1/2}^S\bar{X}_{1/2}^S, \ X_{1/2}^A\bar{X}_{1/2}^A, \ X_{3/2}^A\bar{X}_{3/2}^A, \ X_{1/2}^A\bar{X}_{3/2}^A \]

separately, where the superscript \( S \) and \( A \) describe the diquark spin-status (see below).

The baryon which contains two heavy quarks is believed to have the diquark structure, namely the two heavy quarks constitute a relatively stable object, i.e., the diquark, while the left-over light quark moves in the color field induced by the diquark. In this scenario, the three-body problem becomes a simpler two-body problem, therefore a theoretical evaluation on its properties is simplified. Falk et al., [7] investigated the heavy quark fragmentation
of baryons containing two heavy quarks in this framework. Since both $Q$ and $Q'$ are heavy, they are bound into diquark with a radius much less than $1/\Lambda_{QCD}$, in the heavy quark limit, the diquark is a point-like color-triplet object when seen by the light quark system, one can expect that the deviation from this picture is proportional to $(\Lambda_{QCD}/m_Q)^n$ with $n \geq 1$. Combining a light quark with this diquark makes a baryon.

In this picture, the structure is in close analog to the heavy meson case where a heavy quark is bound with a light quark into a color singlet. The only difference is the color factor. According to the authors of ref.\[7\], it is an overall normalization factor $\sqrt{3}/2$. In fact, the source of the confinement in the potential model is not clear yet, for example, whether it is a scalar or vector confinement is still in dispute. So in our work in addition to the color factor we set a free parameter $\beta_1$ to the confinement $\kappa r$ term while keeping the Coulomb piece corrected by the pure color factor, i.e., $\beta_2 = 0.5$, since it is caused by the single-gluon exchange. In later numerical evaluation we choose two typical values for $\beta_1$ (the notations will be defined in detail in next section).

Due to the analogue of heavy meson and baryon in the diquark picture, we may employ the superflavor symmetry to evaluate the baryon-antibaryon production. The superflavor symmetry was established by Georgi and Wise \[8\] for interchanging the heavy quark and scalar degrees of freedom, while Carone \[9\] generalized it to the symmetry of interchanging heavy quark and vector degrees of freedom.

Obviously, the ground state diquark can be either a color triplet scalar or vector. They have different effective vertex form factors, so the resultant production cross sections of the baryon-antibaryon pairs which are composed of scalar or vector diquark are also different. In this work, we study the cross sections of various baryon-antibaryon pair production and compare them with the heavy meson production rates which were estimated by several authors in the heavy quark effective theory \[10\].

The general forms for the production amplitudes have been given by Georogi, Wise and Carone. However, to evaluate the concrete processes, one need to derive the form factors for the effective vertices in $e^+e^- \rightarrow \chi\chi^*$ where $\chi$ is a scalar or vector diquark in a reliable framework. In ref.\[7\], the authors determined that in a hydrogen-like potential, the radial wavefunction $R(0)$ is proportional to $(C_F\alpha_s)^{3/2}$ and then the diquark production form factor is associated with the zero-point wavefunction of $J/\psi$ as

$$|R_{cc}(0)|^2 \approx |R_{\psi}(0)|^2/8.$$  

In our work, the situation is somewhat different, namely, we are dealing with exclusive processes compared to ref.\[7\] where only inclusive processes are involved. Thus we employ
the B-S equation to calculate the vertex form factors under some reasonable approximations.

For the electron-positron annihilation processes, there may be two intermediate vector-bosons, the photon and $Z^0$-boson. In the present work we are discussing the processes near the production thresholds of the heavy b- and c-baryons, the concerned energy $\sqrt{s}$ is much lower than the $Z^0$-mass, the propagator $\frac{1}{s-M^2_Z}$ is much more suppressed than $\frac{1}{s}$, so that the contribution of $Z^0$-boson as intermediate boson can be neglected. This is consistent with the situation for the energy ranges of the proposed charm-tau factory and the B-factory. In the following we only consider the electromagnetic interaction vertices.

The paper is organized as following. After this introduction, we briefly discuss the diquark-quark structure of baryons, in the third section we give a detailed derivation of the form factors of the effective vertices, in the fourth section, in terms of the superflavor symmetry, we obtain the baryon production cross sections of $e^+e^- \rightarrow X\bar{X}$ where $X$ contains two heavy quarks. Finally the last section is devoted to numerical evaluation and discussions.

II. The baryon quark structure

In this work, we only concern the baryons which contain two heavy quarks and the two heavy quarks consist of a color-triplet diquark. The baryon quark structure has been discussed by some authors [11] [12]. Meanwhile the diquark structure of baryons has also been studied for quite a long time[13]. One may believe that there is a dynamical mechanism which binds two quarks into a diquark system, but not only due to some group theory tricks.

For a all-light quark system or one heavy and two light quark system, because the light quarks are relativistic and the binding energy is not too large, whether the diquark exists as a whole object is suspicious, at least the spin interactions between the soft gluons and the diquark do not decouple. In the previous work, we studied the baryon which was a system of one heavy and two light quarks and suggested that the heavy one attracts one of the light quarks to constitute a diquark and another moves around [14], but in that scenario the spin interaction between the diquark and the light system does not decouple, so makes the application of HQET not as reliable as the case in this work.

By contrast, in the heavy quark effective theory, the diquark structure of the baryons containing two heavy quarks appears more realistic and convincing. The two heavy quarks constitute a stable diquark, at least at the heavy quark limit. In this scenario, since $b$ and $c$ are isospin singlets, so by group theory, one can construct seven different baryons which contain a heavy diquark and a light quark.

$$X_{[bc]}(1/2, 1/2) = \frac{1}{3}e_{ijk}[b_i^\uparrow c_j^\downarrow q_k^\uparrow - \frac{1}{2}b_i^\uparrow c_j^\downarrow q_k^\downarrow + q_k^\downarrow c_j^\downarrow - \frac{1}{2} q_k^\downarrow c_j^\downarrow] |0>$$  \ (1)
where the \(q_i\)'s are creation operators of quarks, \(i, j, k\) are the color indices, the subscripts \(cc\), \(bc\) and \(bb\) denote the heavy quark constituents in the baryon, due to the Pauli principle, \(cc\) and \(bb\) can only be in the spin-1 state while \(bc\) can be in either spin-1 or spin-0 states, namely diquarks \(bb\) and \(cc\) are color triplet spin-1 diquarks while \(bc\) can be color triplet spin-1 \(\chi_{bc1}\) or scalar \(\chi_{bc0}\). In this work we only concern the ground state baryons, so the baryons can only be spin-1/2 and 3/2. The excited baryon states were discussed by Körner et al. [12].

As a matter of fact, when the two quarks are heavy, we have the superflavor symmetry which relates the low energy matrix elements of heavy mesons and baryons with two heavy quarks. The production of heavy diquarks from a virtual photon has effective form factors \(g_A\) or \(g_S\) (see section III) which is factorized out of the low energy matrix elements. We derive these form factors for the effective production vertex from the B-S equation.

### III. Derivation of the effective vertices for production of color triplet spin-1 and -0 diquarks

The physical picture for the production of color triplet spin-1 or -0 diquarks is similar to that of \(Z^0\) decay into charmonium via charm quark fragmentation described by several authors [15] and the inclusive diquark production was also estimated [16]. Later Falk et al., employed this picture to describe inclusive baryon production with the diquark picture. Here in this work we are going to deal with the exclusive processes of baryon production in \(e^+e^-\) collisions with a similar approach.

It is noted that for applying the HQET the diquark should be of a point-like structure, the reason is that all nonperturbative effects are attributed into a well-defined Isgur-Wise function, therefore the necessary condition is that the diquark is seen by the light quark as point-like. However, it by no means demands that electromagnetic current would see a point-like structureless object, by contraries, there is a complicated structure, but derivable
in the framework of perturbative QCD theory, so the virtual photon would "see" an effective vertex and deriving it is the task of this section.

The Feynman diagrams are shown in Fig.1 where the off-shell photon produces a pair of heavy quark-antiquark, then an off-shell gluon is emitted from one leg and turns into another pair of heavy quark-antiquark. The produced quarks and antiquarks are bound into a diquark and an antidiquark. Finally, they pick up a light quark and a light antiquark respectively to constitute a baryon-antibaryon pair. Since it is very difficult to pick up a heavy quark from the sea, the contribution from the current which couples to the light quark-antiquark can be ignored. It is also noticed that if \( Q \) and \( Q' \) are different heavy quarks (\( b \) and \( c \) explicitly), there are four topologically distinct Feynman diagrams corresponding to (a) through (d) in Fig.1, while \( Q \) and \( Q' \) are the same quarks (\( bb \) or \( cc \)), there only two diagrams (a) and (b) exist.

Since the emitted gluon turns to heavy quark-antiquark, the energy scale for this process is large, so the gluon is hard. At this energy scale, the perturbative QCD reliably applies. Therefore the form factors derived in the framework of perturbative QCD make sense. Hence in our case, one can employ the perturbative QCD confidently and only needs to consider the leading order Feynman diagrams which are shown in Fig.1.

In this section in terms of the B-S equation [17], we derive the form factors of the effective vertices for the diquark-antidiquark production and then we evaluate their numerical results.

The B-S equation of a diquark can be written in the following form

\[
\chi_P(p) = S_1(\lambda_1 P + p) \int G(P, p, q) \chi_P(q) \frac{d^4q}{(2\pi)^4} S_2(\lambda_2 P - p),
\]

where \( S_i (i = 1, 2) \) are the propagators of quark 1 and quark 2 in the diquark respectively and \( G(P, p, q) \) is the reductive kernel, \( \lambda_1 = \frac{m_1}{m_1 + m_2} \), \( \lambda_2 = \frac{m_2}{m_1 + m_2} \), and \( m_1, m_2 \) are the quark masses. \( P \) is the total momentum of the diquark and can be expressed as \( P = Mv \) where \( M \) is the mass of the diquark and \( v \) is its four-velocity.

Using the relation

\[
S_j(p) = i\left[ \frac{\Lambda_j^+(p_t)}{p_t - W_j + i\epsilon} + \frac{\Lambda_j^-(p_t)}{p_t + W_j - i\epsilon} \right]^j \quad (j = 1, 2)
\]

where \( p_t = p \cdot v, p_t = p - p_tv, W_j = \sqrt{|p_t|^2 + m_j^2} \) and \( \Lambda_j^\pm(p_t) = \frac{W_j \pm \sqrt{(-p_t + m_j)^2}}{2W_j} \), eq.(8) can be expressed explicitly as

\[
\chi_P^{++}(p) = \frac{-\Lambda_1^+(p_t)^4}{\lambda_1 M + p_t - W_1 + i\epsilon} \int G(P, p, q)[\chi^{++}(q) + \chi^{--}(q)] \frac{d^4q}{(2\pi)^4}
\]

\[
\frac{\hat{\Lambda}_1^-(p_t)}{p_t + W_2 - \lambda_2 M - i\epsilon}
\]

5
\[
\chi_P^{-}(p) = \frac{-\Lambda_1^{-}(pt)\not\! p}{\lambda_1 M + pt + W_1 - i\epsilon} \int G(P,p,q) [\chi^{++}(q) + \chi^{--}(q)] \frac{d^4q}{(2\pi)^4}
\]

where \(\chi_P^{\pm}(p) = \Lambda_1^{\pm}(pt) \chi_P^{\pm}(p) \Lambda_1^{\pm}(pt)\).

In the non-relativistic approximation, which applies to the low-lying states of the twoheavy-quark system, \(\chi_P^{-}\) is small and negligible at the first order and \(\Lambda_1^{+}(pt) \approx \frac{1 + \not\! p}{2}\), \(\Lambda_2^{+}(pt) \approx \frac{1 + \not\! p}{2}\).

So for a scalar or an axial vector diquark, the B-S wavefunction can be written in the forms

\[
\chi_P^S(p) = \frac{1 + \not\! p}{2} \sqrt{2M} \phi(p), \quad \text{or} \quad \chi_P^A(p) = \frac{1 + \not\! p}{2} \sqrt{2M} \gamma_5 \phi(p).
\]

The superscript S and A denote the scalar and axial vector respectively and \(\eta\) is the polarization vector of the vector-diquark.

Now we assume the kernel \(G\) to have a form

\[
- iG = 1 \otimes 1 V_1 + \not\! p \otimes \not\! V_2
\]

where

\[
V_1(p,q) = \frac{8\pi \beta_1 \kappa}{(|pt - q| + \mu^2)^2} - (2\pi)^3 \delta^3(pt - q) \int \frac{8\pi \beta_1 \kappa}{(k^2 + \mu^2)^2} \frac{d^3k}{(2\pi)^3}
\]

and

\[
V_2(p,q) = - \frac{16\pi \beta_2 \alpha_s}{3(|pt - q| + \mu^2)}
\]

The parameters \(\beta_1\) and \(\beta_2\) are different for the various color states. For mesons, \(\beta_1 = 1, \beta_2 = 1\), while for color-triplet diquarks, \(\beta_2\) is directly associated to the color factor caused by the single-gluon exchange, so should be 0.5, in contrast, \(\beta_1\) which is related to the linear confinement, as aforementioned, cannot be determined so far and we just take it as a free parameter in numerical evaluations. As a matter of fact, later we pick up two typical values 0.5 and 1 for \(\beta_1\) for demonstrating the influence of the color factor. The parameters \(\kappa\) and \(\alpha_s\) are well determined by fitting experimental data of heavy meson spectra. From the heavy meson experimental data, \(\kappa = 0.18, \alpha_s = 0.4\). Then, we solve the integral equation

\[
\tilde{\phi}(pt) = \frac{-1}{M - W_1 - W_2} \int (V_1 - V_2) \tilde{\phi}(qt) \frac{d^3q}{(2\pi)^3}
\]

(13)

to obtain the B-S wave function by numerical calculation, where \(\tilde{\phi}(pt) = \int \phi(p) \frac{d^3p}{2\pi^3}\).

In general, the matrix elements of the diquark-antidiquark production from electromagnetic interaction can be factorized as

\[
< M^S(v') M^{S*}(v) | J^\mu | 0 > = M f_1(v \cdot v') (v' - v)^\mu,
\]
\[ < M^A(v', \eta') M^{*A}(v, \eta) | J^\mu | 0 > = M[f_2(v \cdot v') \eta \cdot \eta(v'_\mu - v_\mu) + f_3(v \cdot v')(\eta \cdot v' \eta'_\mu - \eta \cdot v_\mu)]. \] (14)

On the other hand, the effective matrix elements of Fig.1 can be expressed by the B-S wave function in following

\[ < MM^* | J | 0 > = \int Tr[\chi^M_\mu(p')(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4)\chi^{M*}_\mu(p)] \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4} \] (15)

where \( J = \sum_{k=1}^2 \hbar k \Gamma_k h_k \), \( M^* \) is the anti-particle of \( M \) and \( \chi^S_\mu = \frac{1}{2}\sqrt{2M}\phi(p) \), \( \chi^{A}_\mu = \frac{1}{2}\gamma^5\phi(p) \) for scalar- and axial vector-state respectively. The \( \Omega_i(i=1,..,4) \) are defined as

\[
\begin{align*}
\Omega_1 &= -\frac{g^2}{4} s_1^2 \lambda_1 S_1\gamma^\mu \gamma^\nu(p'_1 - p - q), \\
\Omega_2 &= -\frac{g^2}{4} s_1^2 \lambda_2 \gamma^\mu \gamma^\nu(p_1 + q) \gamma_\mu D(p'_1 - p - q), \\
\Omega_3 &= -\frac{g^2}{4} s_1^2 \lambda_2 \gamma^\mu \gamma^\nu(p'_2 - q) \Gamma_2 D(p'_2 - p - q), \\
\Omega_4 &= -\frac{g^2}{4} s_1^2 \lambda_2 \gamma^\mu \gamma^\nu \Gamma_2 S_2(p_2 + q) \gamma_\mu D(p'_2 - p - q),
\end{align*}
\]

respectively, where \( g^\mu\nu D \) is the propagator of the gluon, \( g_s \) is the coupling constant of strong interaction, \( p'_i \) and \( p_i(i=1,2) \) are

\[
\begin{align*}
p'_1 &= \lambda_1 Mv' + p', & p'_2 &= \lambda_2 Mv' - p' \\
p_1 &= -\lambda_1 Mv - p, & p_2 &= -\lambda_2 Mv + p
\end{align*}
\]

respectively.

Similar to ref. [19], we ignore the dependence of \( p' \) and \( p \) in \( \Omega_i(i=1,..,4) \), then for electromagnetic current we find

\[
\begin{align*}
f_1 &= (Q_1 E(m_1, m_2) + Q_2 E(m_2, m_1)), \\
f_2 &= (-Q_1 f(m_1, m_2) - Q_2 f(m_2, m_1)), \\
f_3 &= (Q_1 g(m_1, m_2) + Q_2 g(m_2, m_1)),
\end{align*}
\]

where \( Q_i \) is the charge of quark \( i \) (\( i=1,2 \)) respectively and

\[
\begin{align*}
E(m_1, m_2) &= \left[ \frac{m_1}{M} + \lambda_1 - 2\lambda_2(1 + v \cdot v') \right] h(m_1, m_2) \\
&\approx 2[\lambda_1 - \lambda_2(1 + v \cdot v')] h(m_1, m_2) \\
E(m_2, m_1) &= \left[ \frac{m_2}{M} + \lambda_2 - 2\lambda_1(1 + v \cdot v') \right] h(m_2, m_1) \\
&\approx 2[\lambda_2 - \lambda_1(1 + v \cdot v')] h(m_2, m_1) \\
f(m_1, m_2) &= \left( \frac{m_1}{M} + \lambda_1 \right) h(m_1, m_2) \\
&\approx 2\lambda_1 h(m_1, m_2), \\
f(m_2, m_1) &= \left( \frac{m_2}{M} + \lambda_2 \right) h(m_2, m_1)
\end{align*}
\]

7
\[ \approx 2\lambda_2 h(m_2, m_1) \]
\[ g(m_1, m_2) = \left( \frac{m_1}{M} + \lambda_2 + 1 \right) h(m_1, m_2) \]
\[ \approx 2h(m_1, m_2), \]
\[ g(m_2, m_1) = \left( \frac{m_2}{M} + \lambda_1 + 1 \right) h(m_2, m_1) \]
\[ \approx 2h(m_2, m_1), \]
\[ h(m_1, m_2) = \frac{1}{M^3 \lambda^2_2 (1 + v \cdot v')(1 + \lambda^2_1 - \lambda^2_2 + 2\lambda_2 v \cdot v')} \frac{2g_s^2(4m_2^2)F^2}{3} \]
\[ \approx \frac{1}{3(1 + v \cdot v')^2} \frac{g_s^2(4m_2^2)F^2}{m_2^3}, \]
\[ h(m_2, m_1) = \frac{1}{M^3 \lambda^2_1 (1 + v \cdot v')(1 + \lambda^2_1 - \lambda^2_2 + 2\lambda_1 v \cdot v')} \frac{2g_s^2(4m_1^2)F^2}{3} \]
\[ \approx \frac{1}{3(1 + v \cdot v')^2} \frac{g_s^2(4m_1^2)F^2}{m_1^3} \]
\[ F = \int \phi(p) \frac{d^4p}{(2\pi)^4}. \tag{19} \]

For obtaining the final results of the above expressions, the approximation \( M = m_1 + m_2 \) is taken.

The numerical results are shown in table 1. The masses of b-quark and c-quark are chosen to be \( m_b = 5.02 \text{ GeV}, m_c = 1.58 \text{ GeV} \) which are determined in ref. [20], \( \beta_1 \) is to be 0.5 or 1.0 for a comparison.

| \( \beta_1 \) | 0.5 | 1   | 0.5 | 1   | 0.5 | 1   |
|----------------|-----|-----|-----|-----|-----|-----|
| \( m_1(\text{GeV}) \) | 5.02 | 5.02 | 5.02 | 5.02 | 5.02 | 5.02 |
| \( Q_1 \)  | -1/3 | -1/3 | -1/3 | -1/3 | 2/3  | 2/3  |
| \( m_2(\text{GeV}) \) | 5.02 | 5.02 | 5.02 | 5.02 | 5.02 | 5.02 |
| \( Q_2 \)  | -1/3 | -1/3 | -1/3 | -1/3 | 2/3  | 2/3  |
| \( M(\text{GeV}) \) | 10.08 | 10.17 | 6.76 | 6.89 | 3.40 | 3.57 |
| \( F(\text{GeV}^3/2) \) | 0.287 | 0.341 | 0.177 | 0.226 | 0.128 | 0.169 |

Table 1
The concerned numerical values where \( \alpha_s = 0.4, \kappa = 0.18 \text{ GeV}^2, m_b = 5.02 \text{ GeV}, m_c = 1.58 \text{ GeV}. \)

The forms of the flavor currents to produce the diquark pairs we will use are the following

\[ J_A^\lambda = i g_s (\chi^+ \partial^\lambda \chi - \partial^\lambda \chi^+ \chi), \tag{20} \]
\[ J_A^\lambda = -i g_A [A^\mu \partial_\mu A^\lambda - (\partial_\mu A^\lambda)^+ A^\mu - \alpha (A^\mu \partial^\lambda A_\mu - (\partial^\lambda A_\mu)^+ A_\mu)], \tag{21} \]
where $J_3^\lambda$ is the current for scalar and $J_A^\lambda$ is for axial vector $g_S$ and $g_A$ are the effective vertex form factors and $a$ is a parameter which can be found to be the minus ration of $f_2$ over $f_3$ in eq.(14).

Comparing eqs. (18) (19) (20) and (21) one obtains

$$g_{bb}^A = \frac{4e}{9(1 + v \cdot v')^2} \frac{4\pi \alpha_s(4m_h^2)}{m_b^2} F_{bb}^2$$  \hspace{1cm} (22)

$$g_{bc}^S = \frac{2e}{9(1 + v \cdot v')^2} \left[ \frac{m_b}{m_b + m_c} \frac{4\pi \alpha_s(4m_h^2)}{m_c^3} - \frac{2m_c}{m_b + m_c} \frac{4\pi \alpha_s(4m_h^2)}{m_b^3} \right] F_{bc}^2$$  \hspace{1cm} (23)

$$g_{bc}^A = \frac{2e}{9(1 + v \cdot v')^2} \left[ \frac{m_b}{m_b + m_c} \frac{4\pi \alpha_s(4m_h^2)}{m_c^3} - \frac{2m_c}{m_b + m_c} \frac{4\pi \alpha_s(4m_h^2)}{m_b^3} \right] F_{bc}^2$$  \hspace{1cm} (24)

$$g_{cc}^A = \frac{-8e}{9(1 + v \cdot v')^2} \frac{4\pi \alpha_s(4m_h^2)}{m_c^3} F_{cc}^2$$  \hspace{1cm} (25)

where $\alpha_s(M^2)$ is the running coupling constant of QCD. It can be seen from eqs. (18) (19) (20) and (21) that $a=0.5$ for $bb$ and $cc$ cases while $a=1$ for $bc$ diquark.

IV. The superflavor symmetry and the production cross sections

The superflavor symmetry interchanges the heavy quark and spin-0 or spin-1 degrees of freedom, so can determine relations between their hadronic matrix elements. For the scalar case the fields of the heavy quark and the scalar can be put together into a five-component column vector with a given velocity,

$$\Psi_{\nu} = \begin{pmatrix} h_\nu^+ \\ \chi_\nu \end{pmatrix}$$

where $h_\nu^+$ is the heavy quark spinor, $\psi h_\nu^+ = h_\nu^+$ and $\chi_\nu$ is the heavy scalar field, while for the spin-1 case, the wavefunction becomes an 8-component column vector,

$$\Psi_{\nu} = \begin{pmatrix} h_\nu^+ \\ A^\mu_\nu \end{pmatrix}$$

where $A^\mu_\nu$ is a heavy axial-vector field with a constraint $v_\nu A^\mu_\nu = 0$. One can write the wavefunctions of meson and baryon corresponding to the heavy quark and scalar $\chi_\nu$ or vector $A^\mu_\nu$ diquarks as

$$\Psi_H(v) = \begin{pmatrix} \sqrt{m_h/2} \gamma_5 (1 - \not{v}) \\ 0 \end{pmatrix} \hspace{1cm} \Psi_{H^*}(v) = \begin{pmatrix} \sqrt{m_h/2} (1 - \gamma_5) \\ 0 \end{pmatrix}$$

and

$$\Psi_{\chi_S}^{1/2}(v) = \begin{pmatrix} 0 \\ U^T C / \sqrt{2m_h} \end{pmatrix} \hspace{1cm} \Psi_{\chi_A}^{1/2}(v) = \begin{pmatrix} 0 \\ 1/\sqrt{6m_A} \end{pmatrix} (U^T C \sigma_{\beta \gamma} v_\beta \gamma_5)$$
\[ \Psi_{\chi A}^{3/2}(v) = \frac{1}{\sqrt{2m_A}} \begin{pmatrix} 0 \\ U^{\mu T} C \end{pmatrix} \]

where \( U \) is the spinor of baryons and \( C \) is the charge conjugation operator, \( U^\mu \) is the Rarita-Schwinger spinor-vector wavefunction satisfying constraints \( v_\mu U^\mu = 0 \) and \( \gamma_\mu U^\mu = 0 \).

Below we will give the concrete forms for the production amplitudes for

\[ e^+ e^- \to H\bar{H}, H^*\bar{H}^*, X^*_S X_S, X_V(1/2)\bar{X}_V(1/2), X_V(3/2)\bar{X}_V(3/2) \text{ and } X_V(1/2)\bar{X}_V(3/2), \]

where \( H, H^*, X_S, X_A(1/2) \) and \( X_A(3/2) \) denote the meson, vector-meson, spin-1/2 baryon with the diquark being a scalar, spin-1/2 baryon with the spin-1 diquark and spin-3/2 baryon respectively.

The meson production rate was calculated [10] as

\[ < H(v')\bar{H}(v)|J_\lambda|0 > = f_+(-v \cdot v')(P_H - \bar{P_H})_\lambda. \] (26)

In the approach given by Georgi and Wise, [8]

\[ < H(v')\bar{H}(v)|\bar{h}\gamma_\lambda h|0 > = \xi(-v \cdot v')m_h(v' - v)_\lambda, \] (27)
\[ < H^*(v')\bar{H}^*(v)|\bar{h}\gamma_\lambda h|0 > = -\xi(-v \cdot v')[(\epsilon^* \cdot \epsilon)(v' - v)_\lambda + (\epsilon^* \cdot v)\epsilon_\lambda - (\epsilon \cdot v')\epsilon^*], \] (28)

where \( \xi(v \cdot v') \) is the Isgur-Wise function with the normalization \( \xi(1) = 1 \).

Similarly for the baryon case,

\[ < X_S(v')\bar{X}_S(v)|J_3^\pm|0 > = g_S\xi(-v \cdot v')\frac{1}{2}(v' - v)^3\bar{U}^V \] (29)
\[ < X_A^{1/2}\bar{X}_A^{1/2}|J_3^\pm|0 > = \frac{1}{6}g_A\xi(-v \cdot v')a(2 - v \cdot v')(v' - v)^3\bar{U}^V \]
\[ + (1 - v'v)\bar{U}^V(2\gamma^\lambda + v^\lambda - v'^\lambda)V \] (30)
\[ < X_A^{3/2}\bar{X}_A^{3/2}|J_3^\pm|0 > = \frac{1}{2}g_A\xi(-v \cdot v')[v \cdot v')(v' - v)^3\bar{U}^\mu V^\mu + \bar{U}^\lambda v^\alpha V^\alpha - v_\alpha\bar{U}^\alpha V^\lambda, \] (31)
\[ < X_A^{1/2}\bar{X}_A^{1/2}|J_3^\pm|0 > = \frac{1}{2\sqrt{3}}g_A\xi(-v \cdot v')[(1 - v \cdot v')\bar{U}^\gamma_5 V^\lambda + \bar{U}^\gamma_5(\gamma^\lambda - av^\lambda + (1 - a)v'^\lambda)v_\alpha V^\alpha, \] (32)

where \( U \) and \( V \) are spinors of the baryon and antibaryon, the value of \( a \) has been discussed above.

Thus one can obtain the cross section for the productions as

\[ \sigma = \frac{m_e^2}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{M}{E_1} \frac{d^3p_2}{(2\pi)^3} \frac{M}{E_2} (2\pi)^4 \delta(p + p' - p_1 - p_2)|T|^2. \] (33)

It is noticed that here for the cross section evaluation the normalization is \( \bar{U}U = 1 \).
Without losing generality, we set \( p = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \hat{z}) \) as the momentum of the electron, and \( p' = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \hat{z}) \) as the momentum of positron \( e^+ \). \( p_1 = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{-4M^2}, \hat{n}) \) is the momentum of the outgoing baryon, and in the heavy quark limit \( Mv' = p_1 \), while \( p_2 = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{-4M^2}, \hat{n}) \) is the momentum of the outgoing antibaryon, and in the heavy quark limit \( Mv = p_2 \). \( \hat{n} \) is an arbitrary three-dimensional unit vector as \( \hat{n}^2 = 1 \) and \( \hat{n} \cdot \hat{z} = \cos \theta \). In the formulae, we denote the mass of the heavy baryon as \( M \) and that of the electron as \( m_e \).

\[
\sigma (e^+e^- \to X_SX_S) = \frac{\pi^2 g_S^2}{48\pi s^2 \sqrt{s} M^2} \sqrt{(s - 4M^2)}|\xi(-\omega)|^2 s(\frac{s}{2} - 2M^2)(s - 4M^2), \quad (34)
\]

\[
\sigma (e^+e^- \to X_A^{1/2}X_A^{1/2}) = \frac{\pi^2 g_A^2}{3456\pi s^2 \sqrt{s} M^6} \sqrt{(s - 4M^2)}|\xi(-\omega)|^2 [64(2a + 1)^2 M^8 + 32(-8a^2 - 2a + 5)M^6 s + 24(4a^2 - 2a - 3)M^4 s^2 + 4(-4a^2 + 5a + 1)M^2 s^3 + (a - 1)^2 s^4], \quad (35)
\]

\[
\sigma (e^+e^- \to X_A^{3/2}X_A^{3/2}) = \frac{\pi^2 g_A^2}{1728\pi s^2 \sqrt{s} M^6} \sqrt{(s - 4M^2)}|\xi(-\omega)|^2 [32M^6 + 16(a^2 - 2a)M^4 s + 2(4a^2 + 8a - 3)M^2 s^2 + (a - 1)^2 s^4], \quad (37)
\]

where \( \omega = v \cdot v' \), \( g_S \) and \( g_A \) are the effective vertex form factors and derived in last section and again \( a=1 \) for bc diquark and \( a=0.5 \) for bb or cc diquark. Moreover, the differential cross sections \( d\sigma/d\cos\theta \) are given in the appendix. It is noted that the differential cross sections can be grouped into the transverse piece multiplying \( (1 + \cos^2 \theta) \) and the longitudinal one multiplying \( \sin^2 \theta \) respectively \[21\].

In comparison with the meson production case \( e^+e^- \to D\bar{D} \), whose cross section is

\[
\sigma (e^+e^- \to D\bar{D}) = \frac{\pi^2 g'_{D}^2}{6\pi s^2 \sqrt{s}} |\xi(-\omega)|^2 (\frac{s}{4} - M_D^2)^3/2
\]

where \( g'_{D} = \frac{2}{3} g \) for the charm-meson case, one can immediately obtain the ratios.

In next section, we will discuss numerical results.

V. Numerical results and discussions

Since so far, the baryons which contain two heavy quarks have not been detected in experiments yet, one cannot determine their masses precisely. However, in the heavy quark theory, their masses are very close to the sum of the two heavy quarks, because the binding
energy is of the $\Lambda_{QCD}$ scale which is much smaller than the heavy quark mass. Moreover, in the heavy quark effective theory there is an extra symmetry $SU_s(2) \times SU_f(2)$, the spin splitting is at $1/M$ order. Therefore, in the numerical evaluation, we simply use

$$M \approx m_{Q_1} + m_{Q_2} + \Lambda$$

while the binding energy $\Lambda$ is calculated in terms of the B-S equation.

The values calculated in terms of the B-S equation are $M_{bb} = 10.17$ GeV, $M_{bc} = 6.89$ GeV and $M_{cc} = 3.57$ GeV. In Tables 2,3 and 4 shown in Appendix B we give the numerical values of the cross sections corresponding to $X_{cc}, X_{bc}$ and $X_{bb}$ respectively in a range of $\sqrt{s}$ above the threshold which corresponds to $\omega$ from 1 to 2. In Table 5 we show the results for $\sigma(e^+e^- \rightarrow D\overline{D})$ for comparison.

All non-perturbative QCD effects associated with light quarks are attributed to the Isgur-Wise function. It is noted that the cross sections are proportional to the Isgur-Wise function which is at a negative argument region, $\xi(-\omega)$. Because a transition from Q to Q$'$ is at the s-channel which is the time-like region, so the argument is positive $v \cdot v'$, in contrast, the production of pair $Q\overline{Q}'$ is at the t-channel which is the space-like region and the argument is $-v \cdot v'$. The Isgur-Wise function is normalized to $\xi(1) = 1$ and can be expanded at small $\omega = v \cdot v'$[22], but since the negative $\omega$–values are far apart from 1, so an extrapolation is not legitimate, and to our knowledge it has not been evaluated at the space-like regions. It is understood that the experiments to obtain the cross section of meson-anti-meson (for instance $e^+e^- \rightarrow D\overline{D}$) should be much easier than those for $X_{cc}, X_{bc}$ and $X_{bb}$ and actually BEPC is just working at this energy region. If the cross section for meson anti-meson is measured, with help of the superflavor symmetry scenario, the results can be associated with that for heavy baryon production, so we may elude the troublesome point since the $\xi(-\omega)$ can be cancelled for baryon and meson cases if they have the same $\omega = s/2M^2 - 1$. Therefore, if we know the cross section for, say $D\overline{D}$ at some $\omega$ we can give the predictions for $X_{cc}, X_{bc}$ and $X_{bb}$ at the same $\omega$. The easiest regions are near the production thresholds where $\omega$ is near 1. In this case $\xi(-\omega)$ is not far away from the value at $\omega = 1$, hence $\xi(-\omega)$ can be cancelled in baryon and meson cases.

In fact, just above the threshold of $D\overline{D}$ production, there is a rich spectrum structure with many resonances crowded in a small region, but not for the diquark-antidiquark case in the present work. This is because we are dealing with the baryon production in the leading order of HQET where $m_Q \rightarrow \infty$ we can neglect the resonance structure in our calculations. The reason is that the separation of resonances are related to $1/m_Q$ corrections which are ignored at present in our calculations.
It is noted that the heavy baryon production rates in the tables shown in Appendix B are 5 to 8 orders smaller than the heavy meson production rates and it seems reasonable.

The BEPC is going to be upgraded to higher energies and luminosity and the proposed charm-tau factory is under discussion, meanwhile the B-factory is also under way and their energy is enough to produce pairs of baryon-antibaryon containing two heavy quarks and the luminosity of $10^{33} \text{ cm}^{-2}\text{s}^{-1}$ as proposed can also produce sufficient events. Therefore we suggest our experimental colleagues to explore the heavy baryon-antibaryon production at B- and charm-tau factories.

From eqs. (34) we can see that as $s$ is sufficiently large the cross section beside the factor $|\xi(-\omega)|^2$ will increase with $s$. However, unitarity requires that the cross section decrease in the end. We believe that this can be achieved by the behaviour of $\xi(-\omega)$ at large $\omega$. As mentioned above $\xi(-\omega)$ is not evaluated up to now. In the time-like region several models [23], [24] suggest that $\xi(\omega)$ is suppressed by exponential or higher order $1/\omega$ factors as $\omega$ increases.

Our conclusion is that in terms of the superflavor symmetry, we evaluate the ratio of the production rates of baryon-antibaryon pair which contains two heavy quarks (antiquarks) and that of the meson-antimeson (for example $D\bar{D}$). It is found that this ratio is $10^{-5}$ to $10^{-8}$. One can be optimistic to the measurements in the proposed B- and charm-tau factories.

Acknowledgment

Two of us (Jin and Li) would like to thank the International Center for Theoretical Physics which provides them with a wonderful working atmosphere and excellent library, and part of the work is done over there. One of us (Guo) is indebted to the Institute of Physics for the hospitality during his stay. Li is also indebted to Prof. C. Chang, Dr. Y. Chen and G. Han for helpful discussions.

References

[1] M. Shifman, A. Vainshtein and V. Zakharov, Nucl.Phys. B147 (1979) 385; 448; 519.
[2] H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin Pub.Inc. (1984) Menlo Park; M. Luke, Phys.Rev.Lett. 63 (1989) 1917.

[3] W. Lucha, F. Schöbrel and D. Gromes, Phys.Rept.200 (1991) 127.

[4] N. Isgur and M. Wise, Phys.Lett. 232B (1989) 113.

[5] H. Georgi, Phys.Lett. 240B (1990) 447; Nucl.Phys. B348 (1991) 293.

[6] T. Mannel, W. Roberts and Z. Ryzak, Phys.Lett. 259B (1991) 485; A. Falk, M. Neubert and M. Luke, Nucl.Phys. B388 (1992) 363; A. Falk and M. Neubert, Phys.Rev. D47 (1993) 2965; ibid 2982; Y.-B. Dai, X.-H. Guo and C.-S. Huang, Nucl.Phys. B412 (1994) 277; Y.-B. Dai, X.-H. Guo, C.-S. Huang and M.-Q. Huang, Phys. Rev. D51 (1995) 3532.

[7] A. Falk, M. Luke, M. Savage and M. Wise, Phys.Rev. D49 (1994) 555.

[8] H. Georgi and M. Wise, Phys.Lett. B243 (1990) 279.

[9] C. Carone, Phys.Lett. B253 (1991) 408.

[10] T. Cohen and J. Milana, Phys.Lett. B306 (1993) 134; A. Falk and B. Grinstein, Phys.Lett. B249 (1990) 314.

[11] N. Isgur and M. Wise, Nucl.Phys. 348B (1991) 276; T. Mannel, W. Roberts and Z. Ryzak, Nucl.Phys. 355B (1991) 38; M. Savage and M. Wise, Nucl.Phys. 326B (1989) 15.

[12] J. Körner, D. Pirjol and M. Krämer, Progr. Part. Nucl. Phys. 33 (1994) 787; F. Hussain and G. Thompson and J. Körner, IC/93/314.

[13] J. Richard, Phys.Rept. 212 (1992) 1.

[14] X.-H. Guo and X. Li, Commun.Theor.Phys. 14 (1994) 337.

[15] E. Braaten, K. Cheung and T. Yuan, Phys.Rev. D48 (1993) 4230; 5049; C. Chang and Y. Chen, Phys.Rev. D48 (1993) 4086 ; Y. Chen, Phys.Rev. D48 (1993) 5181 and Ph.D. Thesis, ITP, Academia Sinica (1992).

[16] G. Han, M.S. Thesis, ITP, Academia Sinica, (1994).

[17] H. Bethe and E. Salpeter, Phys.Rev. 82 (1951) 309; Phys.Rev 84 (1951) 1232.

[18] E. Eichten, K. Gottfried, T. Kinoshita, K. Lane and T. Yan, Phys.Rev. D17 (1978) 3090; Phys.Rev. D21 (1980) 203.
[19] G. Guberrina, J. Kühn, R. Peccei and R. Rückl, Nucl. Phys. 174B (1980) 317.

[20] Y. Dai, C. Huang and H. Jin, Phys. Lett. B331 (1994) 174.

[21] J. Körner and M. Kuroda, Phys. Rev. D16 (1977) 2165.

[22] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D 39(1989)799. F.J. Gilman and M.B. Wise, Phys. Rev. D27(1983)1128; M. Neubert and V. Rieckert, Nucl. Phys. B382(1992)97.

[23] X.-H. Guo and P. Kroll, Z. Phys. C59 (1993) 567.

[24] E. Jenkins, A. Manohar and M.B. Wise, Nucl. Phys. B396 (1993) 38; M. Sadzikowski and K. Zalewski, Z. Phys. C59 (1993) 677.
Appendix A

The differential cross section \( \frac{d\sigma}{d\cos \theta} \) for \( e^+e^- \rightarrow X_s\bar{X}_s \),

(i) For \( e^+e^- \rightarrow X_S\bar{X}_S \),

\[
\frac{d\sigma}{d\cos \theta} = \frac{1}{8\pi s^3} \sqrt{s} e^2 g_A^2 |\xi(-\omega)|^2 \frac{1}{8M^2} s(s - 4M^2)^{5/2}(1 - \cos^2 \theta). \tag{39}
\]

(ii) For \( e^+e^- \rightarrow X_A^{1/2}\bar{X}_A^{1/2} \),

\[
\frac{d\sigma}{d\cos \theta} = \frac{e^2 g_A^2}{4608\pi s^2 \sqrt{s} M^8} \sqrt{(s - 4M^2)} |\xi(-\omega)|^2 [4(16M^4 - 8M^2 s + s^2)(1 + \cos^2 \theta) + \\
64(2a + 1)^2 M^8 + 32(-8a^2 - 2a + 1) M^6 s + 12(8a^2 - 4a - 1) M^4 s^2 + 4(-4a^2 + 5a - 1) M^2 s^3 + \\
(a - 1)^2 s^4)](1 - \cos^2 \theta)]. \tag{40}
\]

(iii) For \( e^+e^- \rightarrow X_A^{3/2}\bar{X}_A^{3/2} \),

\[
\frac{d\sigma}{d\cos \theta} = \frac{e^2 g_A^2}{2304\pi s^2 \sqrt{s} M^4} \sqrt{(s - 4M^2)} |\xi(-\omega)|^2 [s(64M^4 - 28M^2 s + 3s^2)(1 + \cos^2 \theta) + \\
2(288a^2 M^8 + 16(-11a^2 + 3a) M^6 s + 2(25a^2 - 26a + 8) M^4 s^2 + 2(-5a^2 + 9a - 4) M^2 s + \\
(a - 1)^2 s^4)(1 - \cos^2 \theta)]. \tag{41}
\]

(iv) For \( e^+e^- \rightarrow X_A^{1/2}\bar{X}_A^{3/2} \),

\[
\frac{d\sigma}{d\cos \theta} = \frac{e^2 g_A^2}{2304\pi s^2 \sqrt{s} M^4} \sqrt{(s - 4M^2)} |\xi(-\omega)|^2 [(16M^4 - 8M^2 s + s^2)(1 + \cos^2 \theta) + \\
16(a + 1)^2 M^4 - 8(a - 1)^2 M^2 s + (a - 1)^2 s^2)](1 - \cos^2 \theta)]. \tag{42}
\]
Appendix B

Table 2 Cross section for scalar diquark ($10^{-16}$ GeV$^{-2}|\xi(-\omega)|^2$)

| $\omega$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.6 | 1.8 | 2.0 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\sqrt{s}(\text{GeV})$ | 13.8 | 14.1 | 14.5 | 14.8 | 15.1 | 15.7 | 16.3 | 16.9 |
| $\sigma(e^+e^- \rightarrow XX_S)$ | 0 | 2.0 | 7.6 | 14.0 | 19.0 | 23.2 | 20.1 | 15.4 |

Table 3 Cross section for baryon containing axial cc diquark ($10^{-13}$ GeV$^{-2}|\xi(-\omega)|^2$)

| $\omega$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.6 | 1.8 | 2.0 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\sqrt{s}(\text{GeV})$ | 7.14 | 7.32 | 7.49 | 7.66 | 7.82 | 8.14 | 8.45 | 8.75 |
| $\sigma(e^+e^- \rightarrow X_{A,1/2}X_{A,1/2})$ | 0 | 0.6 | 2.8 | 6.0 | 9.7 | 17.0 | 23.1 | 27.5 |
| $\sigma(e^+e^- \rightarrow X_{A,1/2}X_{A,3/2})$ | 0 | 0.2 | 1.0 | 2.3 | 3.8 | 6.9 | 9.7 | 11.9 |
| $\sigma(e^+e^- \rightarrow X_{A,3/2}X_{A,3/2})$ | 0 | 1.3 | 5.8 | 12.5 | 20.3 | 36.1 | 49.5 | 59.4 |

Table 4 Cross section for baryon containing axial bc diquark ($10^{-14}$ GeV$^{-2}|\xi(-\omega)|^2$)

| $\omega$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.6 | 1.8 | 2.0 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\sqrt{s}(\text{GeV})$ | 13.8 | 14.1 | 14.5 | 14.8 | 15.1 | 15.7 | 16.3 | 16.9 |
| $\sigma(e^+e^- \rightarrow X_{A,1/2}X_{A,1/2})$ | 0 | 0.2 | 0.8 | 1.7 | 2.7 | 4.6 | 6.1 | 7.1 |
| $\sigma(e^+e^- \rightarrow X_{A,1/2}X_{A,3/2})$ | 0 | 0.05 | 0.2 | 0.4 | 0.7 | 1.2 | 1.7 | 2.0 |
| $\sigma(e^+e^- \rightarrow X_{A,3/2}X_{A,3/2})$ | 0 | 0.4 | 1.8 | 3.8 | 6.1 | 10.5 | 13.9 | 16.2 |

Table 5 Cross section for baryon containing axial bb diquark ($10^{-16}$ GeV$^{-2}|\xi(-\omega)|^2$)

| $\omega$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.6 | 1.8 | 2.0 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\sqrt{s}(\text{GeV})$ | 20.3 | 20.8 | 21.3 | 21.8 | 22.3 | 23.2 | 24.1 | 24.9 |
| $\sigma(e^+e^- \rightarrow X_{A,1/2}X_{A,1/2})$ | 0 | 0.3 | 1.4 | 3.0 | 4.8 | 8.5 | 11.5 | 13.6 |
| $\sigma(e^+e^- \rightarrow X_{A,1/2}X_{A,3/2})$ | 0 | 0.1 | 0.5 | 1.1 | 1.9 | 3.4 | 4.8 | 5.9 |
| $\sigma(e^+e^- \rightarrow X_{A,3/2}X_{A,3/2})$ | 0 | 0.6 | 2.9 | 6.2 | 10.1 | 17.9 | 24.6 | 29.5 |

Table 6 Cross section for $D\bar{D}$ ($10^{-8}$ GeV$^{-2}|\xi(-\omega)|^2$)

| $\omega$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.6 | 1.8 | 2.0 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\sqrt{s}(\text{GeV})$ | 3.7 | 3.8 | 3.9 | 4.0 | 4.1 | 4.3 | 4.4 | 4.6 |
| $\sigma(e^+e^- \rightarrow D\bar{D})$ | 0 | 1.8 | 4.4 | 7.3 | 10.1 | 15.1 | 19.3 | 22.8 |
Figure Captions

Fig.1, The leading Feynman diagrams where the emitted gluon as the intermediate boson is hard and the form factor of the effective vertices are calculated in the framework of perturbative QCD.