High efficient superresolution combination filter with twin LCD spatial light modulators

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Abstract: A comparative study of pupil filters for transverse superresolution is presented in this article. We propose to combine the advantages of amplitude and phase filters in one complex filter that performs better than either phase or amplitude filters designed so far. The performance here refers to having a smaller spot size along with higher peak to side lobe intensity ratio. Using numerical simulation the limitations of phase and amplitude filters are assessed. The experimental verification of the designed combination filter is performed with two LCD spatial light modulators used for displaying separately the phase and amplitude part of the filter. Results obtained from this setup confirm the simulation.

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1. Introduction

Superresolution has been demonstrated both computationally as well as experimentally using pupil filters [1-4]. These filters in general manipulate either amplitude or phase transmittance at the exit pupil of the optical system. Either of the filters can be used to reduce the central core of the classical point spread intensity distribution depending on the system’s requirement and complexity. The ‘efficiency’ of a filter which is the reduced spot size achieved at a particular Strehl ratio has been the primary concern for applications like in astronomy where the energy requirements are stringent. For applications which use lasers it is only the reduced spot size.

In recent years, phase filters have gained importance because of their higher efficiency compared to the amplitude filters [5]. But the main disadvantage of these filters is the increase in the side lobe intensity with the decrease of spot size, which enhances the artifacts and misrepresentations in the images obtained. Not much research has been done to reduce the side lobe intensity while keeping the reduced spot size constant [6]. Ref. 6 deals with the design of amplitude-only filters for superresolution with improved central peak to side lobe intensity ratio in comparison to phase filters.

In this paper, we show by simulation the advantages and disadvantages of amplitude and phase filters. We use the merit function \( \Gamma \), which is defined as the ratio of the peak intensity at the center of the diffraction pattern to the peak intensity of the first side lobe. When the spot size is reduced to 65% and below, the phase filters produce side lobe intensities as high as half the central peak intensity with a Strehl ratio (S) of 0.2 and below. For amplitude filters, the peak to side lobe intensity ratio (\( \Gamma \)) can be kept above 7 but the Strehl ratio is as low as \( 10^{-4} \) to \( 10^{-6} \). Because of this both types of filters are impractical to generate superresolution below 70% of the spot size. Therefore, we design a combination filter which incorporates the advantages of both types of filters to produce a high efficient filter with higher \( \Gamma \).

The experimental verification is performed with two twisted nematic liquid crystal Spatial Light Modulators (SLMs) combined together in a 4F architecture. The experimental results are shown to match the simulation.

2. Theory and simulation

The field amplitude \( U \) at the focal plane can be expressed as [7]

\[
U(v, u) = \int_{\rho} P(\rho) J_0(u \rho) \exp(\imath u \rho^2 / 2) \rho \, d\rho
\]

\( P(\rho) \) is the pupil function which is radially symmetric in pupil coordinate \( \rho \), while \( v \) and \( u \) are the radial and axial optical coordinates given by:

\[
v = (2\pi / \lambda) r \sin \alpha
\]

\[
u = 4(2\pi / \lambda) z \sin^2(\alpha / 2)
\]

Here \( \sin \alpha \) denotes the numerical aperture whereas \( r \) and \( z \) denote the radial and axial distances from the focal point. In the focal plane the amplitude is the Hankel transform of the pupil function, which from Eq. (1) can be written as

\[
U(v,0) = \int_{\rho} P(\rho) J_0(v \rho) \rho \, d\rho
\]

We consider a complex pupil function \( P(\rho) \),

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\[ P(p) = g(p) \exp(\imath f(p)) \quad (5) \]

where \( g(p) \) is the amplitude function and \( f(p) \) is the phase function. \( f(p) = \) constant produces amplitude-only filters while \( g(p) = \) constant generates phase-only filters.

The amplitude function we chose is a polynomial with even orders of the radial coordinate \( p \)

\[ g(p) = \sum_{n=0}^{k} a_{2n} p^{2n} \quad (6) \]

although a general polynomial expansion with both even and odd powers can be used for \( g(p) \). One can optimize the function with both even and odd terms by truncating \( J_0 \) to 20-30 orders of \( \nu \) or by polynomial approximation of \( J_0 \) in a particular range of \( \nu \), for example in the interval \( \nu = [0,15] \), since for most filters the intensity outside this interval is negligible. We observed during the simulation that the improvement in the performance of the filter with amplitude functions with or without odd order is less than 1% for \( k = 6 \).

A three zone binary phase function is chosen for \( f(p) \) with phases set to zero in the central and outer zone and to \( \pi \) in the intermediate zone. The performance of the 3 zone binary phase filter matches the performance of more general continuous phase filters designed in ref. 10 and 11, as is shown in the following.

Initially the optimization is performed to understand the behavior of these two functions separately to assess their properties. The optimization is performed using the nonlinear programming method of MATLAB’s constrained optimization toolbox [9] and the function chosen is ‘fmincon’ which suites our requirement. Since we want to reduce the spot size the objective function for the MATLAB function is written to get \( \nu \) for the first zero of Eq. (4) for each chosen set of \( a_{2n} \)'s. The function ‘fmincon’ minimizes this value for a given constrained set. This set consists of restrictions on the Strehl ratio and side lobe intensity. The side lobe intensity is evaluated by solving \( dU \) for the first zero. An additional constraint to be included, is given by

\[ 0 \leq P(p) \leq 1 \quad \text{for each set of } a_{2n} \text{'s} \quad (7) \]

The simulation results on 3 zone phase filters can be grouped into two: the objective function is written (i) for minimization of spot size and (ii) for reducing the side lobe intensity while keeping the spot size constant. In the first group the assumed 3 zones are always forced into 2 zone phase filters which provide the minimum spot size for a given Strehl ratio. The second group remained a 3 zone filter with higher \( \Gamma \). For the case of amplitude filter the objective function is written to minimize only the spot size since the nature of the amplitude filter is to absorb the energy unlike phase filters where the energy lost in the central core tends to accumulate in the side lobes [3, 8]. We consider only \( k = 6 \) since we have shown that \( k > 6 \) produces no higher effect on optimization [6]. This is because the optimum shape of the function is well approached with \( k = 6 \), and further increase in \( k \) will not change the shape. If the optimum shape included a sharp edge like a discontinuity, \( k > 6 \) would be required, but then it were easier to represent and optimize \( g(p) \) by multiple zones.

The optimized values for three types of filters are plotted in Fig. 1 and 2. The relative spot size \( G \) refers to the ratio of the spot size obtained with and without the filter. It can be seen from Fig. 1 that both phase filters achieve \( G < 60% \) in contrast to the amplitude filter. Over the entire range the 2 zone filter reaches its optimum \( G \) value at a higher Strehl ratio than the 3 zone filter. Even though the amplitude filter reaches a particular \( G \) value at lower Strehl ratio, it can be observed from Fig. 2 that it always has a higher \( \Gamma \) than the phase filters.
Fig. 1. Strehl Ratio $S$ versus relative spot size $G$ for three filter types.

Moreover, for amplitude filters, $\Gamma$ can be improved at all $G$ values by modifying the objective function to minimize the side lobe intensity at a given $G$ value [6]. The parameters of the filters are provided in Table 1. The parameter $p_1$ of the 2 zone phase filter indicates the radius of the first zone where the phase is arbitrary but constant and the outer zone having a phase difference of $\pi$ radians from the inner zone. Similarly, for the 3 zone phase filter, $p_1$ indicates the radial extent of the innermost zone and $(p_2 - p_1)$ is the extent of the intermediate zone. The intermediate zone transmits light with a phase difference of $\pi$ radians from that of the adjacent two zones.

From Fig. 1 and 2, it can be concluded that even though amplitude filters have a higher $\Gamma$ they cannot be used in practice for producing $G < 70\%$ since the $S$ values are in the range $10^{-2}$-$10^{-6}$ and they tend to become thin circular slits like a 2 zone blocking filter.
Table 1. Parameter of the filters shown in Fig. 1 and 2.

| G (%) | 2 zone phase filters | 3 zone phase filters | Continuous amplitude filters |
|-------|---------------------|----------------------|-----------------------------|
|       | pl | pl | pl2 | a0 | a2 | a4 | a6 | a8 | a10 | a12 |
| 94    | 0.16 | 0.41 | 0.47 | 0.52 | 2.70 | -8.98 | 9.99 | 2.34 | -8.31 | 2.67 |
| 90    | 0.23 | 0.34 | 0.44 | 0.40 | -0.84 | 12.12 | -28.28 | 18.87 | 7.04 | -8.30 |
| 85    | 0.29 | 0.30 | 0.45 | 0.32 | -0.35 | 3.37 | -4.84 | 3.16 | 2.53 | -3.34 |
| 81    | 0.34 | 0.33 | 0.52 | 0.57 | -4.30 | 10.00 | -0.95 | -3.12 | -7.11 | 5.92 |
| 76    | 0.38 | 0.27 | 0.51 | 0.31 | -2.00 | 1.45 | 8.31 | -3.01 | -10.00 | 5.91 |
| 72    | 0.43 | 0.27 | 0.55 | 0.00 | 0.23 | -1.87 | 1.83 | 12.04 | -15.71 | 4.42 |
| 67    | 0.48 | 0.24 | 0.56 | 2 zone blocking filter with boundary at pl = 0.87 |
| 61    | 0.53 | 0.23 | 0.60 | 2 zone blocking filter with boundary at pl = 0.98 |
| 53    | 0.58 | 0.23 | 0.65 | Not possible even at Strehl ratio S = 10-6 |

However, even though phase filters have lower G values, they cannot be used in practice since G values below 2 enhance artifacts in the image.

Therefore, to produce low G values with high G we propose a combination filter in which the optimization takes care of higher side lobe produced by the 3 zone phase filter with a continuous amplitude filter. First, a 3 zone filter is designed to produce a lower G value than targeted. The side lobe intensities created by this procedure are reduced in the next step by introducing an amplitude type filter. The constraint on G is relaxed to the targeted value (64% in our example). This procedure helps the amplitude filter to attain a better functional form than just introducing it only for the sake of reducing the side lobe intensity. In comparison to ref. 11 our choice of both the filter function and approach is not only more general but is also backed by a more comprehensive reasoning.

Figure 3 shows the point spread function of the complex combination filter in comparison with the 2 and 3-zone binary phase filters.

![Figure 3](attachment:image.png)

Fig. 3. Point spread functions with 2 zone phase, 3 zone phase and complex combination filter.
The optimized complex combination filter has the component 3 zone phase filter with zone boundaries at 0.3 and 0.7, and the amplitude filter with \( a_n's \) given by \([0.306, -0.277, 1.105, 0.695, 0.113, -0.344, -0.628]\) resulting in \( G = 64\% \) and \( \Gamma \sim 4 \). For this \( G \) value the continuous phase filter in ref. 8 has produced \( \Gamma \sim 2 \). The parameters of the phase filters in Fig. 3 are \( \rho_1 = 0.505 \) for the 2 zone filter, and \( \rho_1 = 0.20 \) and \( \rho_2 = 0.56 \) for the 3 zone filter.

The complex filter offers the maximum suppression of side lobe intensity at the given \( G \) value of 64%. The numerical estimations of total energy reveals that the ratio of central core to the side lobe energies for the 2 zone and 3 zone phase filters in Fig. 3 are about 1:8 as compared to 1:4 for the complex combination filter. This shows that there is less energy spread outside the central core for the complex combination filter than for the phase filters at this spot size.

3. Experiment

The optimized phase and amplitude filters of the complex combination filter are mapped on two LCD Spatial Light Modulators as shown in Fig. 4. We use high spatial resolution (SVGA) LCDs for this purpose, the Sony LCX016AL having 832 x 624 pixels with a pixel pitch of 32\( \mu \)m and 1.3 inch diagonal size. The wavelength used was 532 nm. In general a twisted nematic LCD varies both phase and amplitude of the transmitted light together but not independently. The procedure to implement either phase-only or amplitude-only SLM is to use proper orientation angles of polarizers and \( \lambda/4 \) plates used for sandwiching the LCD. This procedure has been elaborately presented in references \([12-14]\). The characteristic behaviors of our SLMs are shown in Fig. 5 and 6, respectively. The total phase-only change achieved with this LCD is approximately \( \pi \) rad. For the amplitude-only SLM case, an intensity variation of up to 96% has been found.
combined effect of these two filters is shown in Fig. 7(b). The spot size attained is 990 µm which is about 64% and $\Gamma = 4$.

Fig. 5. The characteristic behavior of phase-only LCD SLM.

Fig. 6. The characteristic behavior of amplitude-only LCD SLM.
4. Conclusion

We presented through simulation the limitations of applicability of phase-only and amplitude-only pupil filters for superresolved spot sizes. We proposed a complex combination filter, which combines positive aspects of both types of filters to produce high superresolution with high peak to side lobe intensity ratio and also lower spread of energy in the side lobes. The designed complex filter has been implemented on two LCD SLMs for the separate display of amplitude and phase part. The results obtained with the experimental setup confirm the simulation, and it can be concluded that a complex filter has more potential in attaining a highly efficient superresolution.