Joule-Thomson Expansion and Heat Engine of the FRW Universe

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We investigate the thermodynamics of FRW (Friedmann-Robertson-Walker) universe in the extended phase space. We generalize the unified first law with a cosmological constant Λ by using the Misner-Sharp energy. We treat the cosmological constant as the thermodynamic pressure of the system, and derive thermodynamic equation of state $P = P(V, T)$ for the FRW universe. To clarify our general result, we present two applications of this thermodynamic equation of state, including Joule-Thomson expansions and efficiency of the Carnot heat engines. These investigations lead to physical insights of the evolution of the universe in view of thermodynamics.

I. INTRODUCTION

Friedmann-Robertson-Walker (FRW) universe is a dynamical space-time. It has been widely accepted that the FRW universe has thermodynamics embodied on the apparent horizon, whose area plays the role of the entropy, and whose surface gravity behaves as temperature. In spirit of Jacobson’s derivation [1] of the Einstein field equations from the Clausius relation, Cai and Kim [2] first investigated the Friedmann equations of the FRW universe on the apparent horizon, whose thermodynamics is associated with the unified first law [3]. A similar connection between the Friedmann equations and the first law of thermodynamics has also been discovered in alternative theories of gravity [4, 5]. Researches on the thermodynamics of FRW universe shed light on inherent relationship between thermodynamics and gravity. It is a quite universal and fundamental finding that would be helpful to decode the nature and the quantization of gravity.

However, to our knowledge, the thermodynamics of the FRW universe has not been investigated deeply and extensively in the situation with a cosmological constant. Some preliminary works have been done in analogy to black holes. Recently, the idea of pressure and volume as well as the thermodynamic equation of state in the extended phase space of AdS black hole thermodynamics has drawn a lot of attention [6–9]. In this framework, the negative cosmological constant Λ is treated as the thermodynamic variable analogous to the pressure $P = -\Lambda/8\pi$ [10, 11]. In [12], the author redstudied the thermodynamic properties of the FRW universe. However, they only considered the case that the apparent horizon is fixed or $\dot{R}_A \approx 0$. Due to dynamical nature of the FRW universe, we shall consider time-dependence of the apparent horizon, which is also related to the Hawking temperature. In this paper, we investigate the varying apparent horizon and generalize the unified first law of the FRW universe with cosmological constants. We treat the cosmological constant as the thermodynamic pressure of the system, and derive the thermodynamic equation of state $P = P(V, T)$ by relating the components of Einstein field equations with Hawking temperature.

Furthermore, we study the Joule-Thomson expansion and the heat engine of the FRW universe as applications of the thermodynamic equation of state. In classical thermodynamics, the Joule-Thomson expansion describes the expansion of gas passing through a throat from a high pressure region to a low pressure region under the fixed enthalpy. Ref [13] creatively applied the famous Joule-Thomson expansion of the van der Waals systems to the charged AdS black hole, and found that there exist the inversion temperature and the inversion pressure. Subsequently, various black holes have been studied, such as Gauss-Bonnet black holes [14], regular (Bardeen)-AdS black holes [15], and Born-Infeld AdS black holes [16]. In this paper, we investigate the Joule-Thomson expansion of the FRW universe in analogy to AdS black holes. In addition, it would be interesting to study the heat engine of the FRW universe. In [17], the author

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proposed a heat engine for the AdS black hole, where the cosmological constant \( \Lambda \) plays the role of thermodynamic pressure. Since then, a great deal of attention has been drawn to this topic with many interesting results [18–22]. In this work, we study the properties of FRW universe as a thermodynamic heat engine and discuss its efficiency in a Carnot cycle.

The organization of this paper is as follows. In Sec.II, we introduce thermodynamics of the FRW universe on the apparent horizon in Einstein gravity without cosmological constant. In Sec.III, we study the unified first law of the FRW universe with a cosmological constant, and further derive the thermodynamic equation of state \( P = P(V, T) \). In Sec.IV, we study the Joule-Thomson expansion and efficiency of the Carnot heat engine for FRW universe as two applications of the thermodynamic equation of state. In Sec.V, we present conclusions and discussion of this paper. Throughout this article, we adopt natural units such that \( c = G = \hbar = k_B = 1 \).

II. WARMUP: THE THERMODYNAMICS OF THE FRW UNIVERSE ON APPARENT HORIZON

In this section, we will make an introduction of the apparent horizon, Hawking temperature and unified first law of FRW universe in absence of cosmological constant. In an isotropic coordinate system \( x^\mu = (t, r, \theta, \varphi) \), the line element is

\[
ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right],
\]

(2.1)

where \( a(t) \) is scale factor describing the evolution of the universe, \( k = +1, 0, -1 \) are the spatial curvatures corresponding to a spherical, flat and hyperbolic universe, respectively. If we introduce the areal radius \( R(t, r) \equiv a(t) r \), the metric (2.1) can be rewritten as:

\[
ds^2 = h_{ij} dx^i dx^j + R^2(t, r) (d\theta^2 + \sin^2 \theta d\varphi^2),
\]

(2.2)

where \( h = \det(h_{ij}) \). For FRW universe, one can obtain surface gravity of the apparent horizon

\[
\kappa = \frac{-1}{2\sqrt{-h}} \partial_i \left( \sqrt{-h} h^{ij} \partial_j R \right),
\]

(2.5)

where \( h = \det(h_{ij}) \). For FRW universe, one can obtain surface gravity of the apparent horizon

\[
\kappa = -\frac{1}{R_A} \left( 1 - \frac{\dot{R}_A}{2HR_A} \right).
\]

(2.6)

For dynamical apparent horizon, one can see that the surface gravity is determined not only by the Hubble parameter and the apparent horizon radius, but also the variation rate of the apparent horizon. In this paper, we are only interested in the case \( \kappa < 0 \). Therefore, the Hawking temperature on the apparent horizon is [23]

\[
T \equiv \frac{|\kappa|}{2\pi} = \frac{1}{2\pi R_A} \left( 1 - \frac{\dot{R}_A}{2HR_A} \right).
\]

(2.7)
If we assume that the apparent horizon $R_A$ changes very slowly, i.e., $\dot{R}_A/(2HR_A) \ll 1$, the Hawking temperature can be written as

$$T \approx \frac{1}{2\pi R_A}.$$  \hfill (2.8)

However, $\dot{R}_A$ is not necessarily a small quantity. This is one of the key points of our discussion, and we use Eq.(2.7) to perform the following calculations.

In fact, the surface of apparent horizon acts as a boundary of the thermal system of the FRW universe, it is time dependence. Let $dR_A$ be an infinitesimal change in radius of the apparent horizon, which will appears a small change $dV$ in the volume $V$ of the expanding universe. Naturally, it forms two spherical thermal system of space-time with radius $R_A$ and $R_A + dR_A$. These two thermal states of the space-time that satisfies Einstein field equations with a common source of fluid. Therefore, to clarify the connection between the Einstein field equations and thermal quantities, we study the unified first law of thermodynamics of the FRW universe. The Misner-Sharp energy is defined by [24]

$$E = \frac{R}{2} (1 - h^{ij}\partial_i R \partial_j R).$$  \hfill (2.9)

The work density and energy-supply are defined as

$$W = -\frac{1}{2} h^{ij} T_{ij}, \quad \Psi_i = T^j_i \partial_j R + W \partial_i R,$$  \hfill (2.10)

where $T_{ij}$ is the projection of the energy-momentum tensor $T_{\mu\nu}$ of the matter in the $(t, r)$ directions. Differentiating the Misner-Sharp energy and using the above two quantities, one finds that the $(t, t)$ component of the Einstein field equations can be written as [3]

$$dE = A\Psi + WdV,$$  \hfill (2.11)

where $A = 4\pi R^2$ and $V = 4\pi R^3/3$ are area and thermodynamic volume of the 3-dimensional sphere with radius $R$, which is just the Hayward’s unified first law of thermodynamics [25, 26]. One sees that the Misner-Sharp energy is just the total internal energy within radius $R$ in Einstein gravity without cosmological constant. For the FRW universe, the energy momentum tensor of the perfect fluid is given by

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + pg_{\mu\nu},$$  \hfill (2.12)

where $u^\mu$ denotes the four-velocity of the fluid, which reads $u^\mu = 1/\sqrt{-g_{tt}} \delta^\mu_t$. Here $\rho$ and $p$ are the energy density and pressure, respectively. For perfect fluids one has

$$W = -\frac{1}{2} (T^t_t + T^r_r) = \frac{1}{2}(\rho - p), \quad \Psi = \Psi_i dx^i = \frac{1}{2}(\rho + p)(-HR dt + adr).$$  \hfill (2.13)

One finds that on the apparent horizon, Eq.(2.11) is equal to

$$dE = \frac{\kappa}{8\pi} dA + WdV = -TdS + WdV,$$  \hfill (2.14)

where the expression of the temperature (2.7) and the Bekenstein-Hawking entropy

$$S = \frac{A}{4} = \frac{\pi R_A^2}{4}$$  \hfill (2.15)

have been used. The minus sign before $TdS$ in (2.14) arises from the treatment that surface gravity $\kappa$ is negative, while the corresponding temperature $T$ should be positive [4, 27]. The work term $WdV$ on the apparent horizon can be regarded as the work done due to the change of the apparent horizon, while the energy-supply is just the total energy flow across the apparent horizon.

III. THE UNIFIED FIRST LAW AND THERMODYNAMIC EQUATION OF STATE WITH A COSMOLOGICAL CONSTANT

In this section, we derive the unified first law and the equation of state with the cosmological constant $\Lambda$ being treated as the thermodynamic pressure in the FRW universe. We first obtain unified first law by using the Misner-Sharp
energy and Einstein field equations of the FRW universe. The Misner-Sharp energy plays an important role in the unified first law, and is related with the structure of the space-time and the Einstein field equations.

Note that the original form (2.9) for the Misner-Sharp energy is applicable for Einstein gravity without cosmological constant in four dimensions. In addition, a generalization of the Misner-Sharp energy in presence of the cosmological constant $\Lambda$ has also been considered in Einstein gravity as well as the Einstein-Gauss-Bonnet gravity, $f(R)$ gravity, etc.

In FRW universe, the Misner-Sharp energy with $\Lambda$ in Einstein gravity is obtained from [28–30]

\[
M = \frac{R}{2} \left[ -\frac{\Lambda}{3} R^2 + R^2 \left( H^2 + \frac{k}{a^2} \right) \right].
\]  

The Einstein field equations can be written as

\[
G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} ,
\]

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor. For the FRW metric (2.2) and energy-momentum tensor (2.12), the $(t, t)$ component of the Einstein field equation can be written as [31]

\[
H^2 + \frac{k}{a^2} = \frac{8\pi}{3} \rho + \frac{\Lambda}{3},
\]

which is just the Friedmann equation. On the other hand, from the energy-momentum conservation law of matter fields $\nabla_\nu T^\mu_\nu = 0$, one finds the continuity equation [32]

\[
\dot{\rho} + 3H(\rho + p) = 0.
\]

By differentiation of the Misner-Sharp energy (3.1) and combination with Eqs. (2.11), (3.3) and (3.4), we have

\[
dM = d \left\{ \frac{R}{2} \left[ -\frac{\Lambda}{3} R^2 + R^2 \left( H^2 + \frac{k}{a^2} \right) \right] \right\}
= R^3 H \left( \dot{H} - \frac{k}{a^2} \right) dt + \frac{R^2}{2} \left[ 3 \left( H^2 + \frac{k}{a^2} \right) - \Lambda \right] dR - \frac{R^3}{6} d\Lambda
= -4\pi R^3 (\rho + p) dt + 4\pi R^2 \rho dR - \frac{R^3}{6} d\Lambda
= A\Psi + WdV - \frac{R^3}{6} d\Lambda.
\]

Equation (3.5) is the unified first law with a $\Lambda$, from which we also derive the Friedmann equation.

If the cosmological constant is treated as the thermodynamic pressure

\[
P = -\frac{\Lambda}{8\pi},
\]

then Eq.(3.5) becomes,

\[
dM = A\Psi + WdV + VdP.
\]

Therefore, the Misner-Sharp energy can be regarded as the thermodynamic enthalpy $M = \mathcal{H} \equiv E + PV$ in an extended phase space. At the apparent horizon, one gets the first law of thermodynamics

\[
dM = \frac{\kappa}{8\pi} dA + WdV + VdP = -TdS + WdV + VdP.
\]

Now, we derive the thermodynamic equation of state of the FRW universe. Regarding the apparent horizon (2.3), from (3.3), one further gets

\[
\rho = \frac{3}{8\pi R_A^2} - \frac{\Lambda}{8\pi}.
\]

From (2.4), (3.4) and (3.9), $p$ is retrieved as

\[
p = -\frac{3}{8\pi R_A^2} + \frac{\Lambda}{8\pi} + \frac{\dot{R}_A}{4\pi H R_A^2}.
\]
Note that the variation rate of the apparent horizon in (3.10) is obtained by solving (2.7). Then in the general framework of perfect fluid \( p = \omega \rho \), from (3.9) and (3.10) combined with (3.6), one expresses the temperature \( T \) as a function of \( P \) and \( R_A \) in the following form,

\[
T = -(\omega + 1) PR_A + \frac{1 - 3\omega}{8\pi R_A},
\]

(3.11)

where \( R_A = (\frac{3V}{4\pi})^{1/3} \). When \( \omega = -1 \), the temperature is \( T = 1/(2\pi R_A) \), which corresponds to \( \dot{R}_A = 0 \). In this case, the temperature \( T \) is independent of \( P \), and there is no thermodynamic equation of state. When \( \omega \neq -1 \), we can express the thermodynamic pressure as

\[
P = -\frac{1}{\omega + 1} \left( \frac{T}{R_A} + \frac{3\omega - 1}{8\pi R_A^2} \right).
\]

(3.12)

The minus sign of the first term in (3.12) means that this is a special thermodynamic system. Usually it is positive such as ideal gas or van der Waals system even black holes. In the present work, we only focus on the fluid with \(-1 < \omega \leq 1/3\), which is derived from the cosmological observations. Figure 1 shows the thermodynamic pressure \( P \) with respect to the apparent horizon \( R_A \) for the FRW universe.

FIG. 1: The thermodynamic pressure \( P \) as a function of the apparent horizon \( R_A \) for the FRW universe when \( \omega = 0 \). The values of \( T \) are marked on the curves.

Note that, the figure is plotted for the case of the fluid with \( \omega = 0 \). According to Eq.(3.12), we conclude that there is no \( P - V \) criticality for the FRW universe. Once the thermodynamic equation of state is obtained, we can discuss its thermodynamic properties as the universe evolves.

IV. APPLICATIONS OF THE THERMODYNAMIC EQUATION OF STATE

In this section, we demonstrate applications of the thermodynamic equation of state (3.12) in studies of the Joule-Thomson expansion and heat engine.

A. Joule-Thomson Expansion

In this subsection, we study the Joule-Thomson expansion for the FRW universe, obtaining the Joule-Thomson coefficient, and further demonstrate whether inversion temperature and inversion pressure exist or not. The Joule-Thomson expansion is an interesting physical process with important feature that the temperature changes with pressure while keeping the enthalpy fixed during this expansion process [13]. Since the Misner-Sharp energy \( M \) with cosmological constant in Einstein gravity could be interpreted as enthalpy in the extended phase space, we consider that \( M \) remains constant during the expansion process. The Joule-Thomson coefficient \( \mu \) is defined by

\[
\mu = \left( \frac{\partial T}{\partial P} \right)_{\mathcal{H}}.
\]

(4.1)
The sign of \( \mu \) is determined by cooling or heating occurred in the thermodynamic system. \( \mu > 0 \) means that the system remains cooling, and the temperature of the system decreases. On the contrary, \( \mu < 0 \) means that the system remains heating. Therefore, the cooling-heating inversion points lie at \( \mu = 0 \), and the temperature of the thermodynamic system at that point is named as inversion temperature \( T_i \). When the temperature of the system is exactly \( T_i \), the corresponding pressure is the inversion pressure \( P_i \), which defines a special point called the inversion point \((T_i, P_i)\).

Considering the property of the first law of thermodynamics of FRW universe, we find a fine method to derive the Joule-Thomson coefficient \( \mu \). On the apparent horizon, the \((3.1)\) is rewritten as

\[
M = \frac{R_A}{2} + \frac{4\pi R_A^3}{3}P.\tag{4.2}
\]

From \((4.2)\), the pressure \( P \) can be rewritten as a function of \( M \) and \( R_A \),

\[
P(M, R_A) = \frac{3(2M - R_A)}{8\pi R_A^3}.\tag{4.3}
\]

Then substituting \( P(M, R_A) \) into the temperature \((3.11)\), one writes the temperature as a function of \( M \) and \( R_A \) as follows

\[
T(M, R_A) = \frac{2R_A - 3M(\omega + 1)}{4\pi R_A^2}.\tag{4.4}
\]

By using \((4.1)\) combined with Eqs.\((4.3)\) and \((4.4)\), we obtain the Joule-Thomson coefficient of the FRW universe as a function of \( T \) and \( R_A \),

\[
\mu = \frac{(\partial T/\partial R_A)_M}{(\partial P/\partial R_A)_M} = \frac{2R_A[R_A - 3M(\omega + 1)]}{3(3M - R_A)}.\tag{4.5}
\]

We see from the above equation that the Joule-Thomson coefficient is always negative while \( \omega = 0 \), suggesting that the FRW universe is always in the heating stage, i.e., there is no inversion temperature and inversion pressure. The Joule-Thomson expansion occurs in the isenthalpic process. Thus it is significant to study the isenthalpic curves of the FRW universe. Figure 2 shows the corresponding diagram of isenthalpics with the fluid \( \omega = 0 \), which is based on the Eqs.\((4.3), (4.4)\).

![FIG. 2: The isenthalpic curves of the FRW universe for \( \omega = 0 \). The slope changing curves are the isenthalps for \( M = 5, 6 \).](image)

When \( \omega \neq 0 \), there exist both a divergence point and an inversion point at each different value of \( \omega \) and the influence of FRW universe on Joule-Thomson coefficient is clear. Setting \( \mu = 0 \), and combined with \((4.4)\), we get the inversion temperature

\[
T_i = \frac{1}{12\pi M(\omega + 1)}.\tag{4.6}
\]

Combined with \((4.3)\), we get the inversion pressure

\[
P_i = -\frac{1 + 3\omega}{72\pi M^2(\omega + 1)^3}.\tag{4.7}
\]
We see from (4.7) that, the inversion pressure is positive for \(-1 < \omega < -1/3\), and negative for \(\omega \in (-1/3, 0) \cup (0, 1/3)\). From (4.6) and (4.7), one expresses the inversion temperature as a function of inversion pressure given by

\[
T_i = \sqrt{-\frac{P_i(\omega + 1)}{2\pi(3\omega + 1)}}.
\]  

(4.8)

Clearly, we see that the inversion temperature for a given inversion pressure tends to increase monotonically with given \(\omega\).

In the followings, we plot the inversion and isenthalpic curves of the FRW universe for fluids with \(\omega \in (-1, 0) \cup (0, 1/3)\) in \(P - T\) plane by fixing the \(M\), and determine the cooling-heating region. Figure 3 shows the inversion and isenthalpic curves, which are plotted by Eqs.(4.3), (4.4) and (4.8).

At first sight the inversion curve intersects with the maximum points of isenthalpic curves, and it would separate the plane into the cooling and heating regions. At that point the Joule-Thomson coefficient vanishes, which indicates that the cooling-heating transition coincides at the maximum points of isenthalpic curves. In fact, the inversion curve acts as a boundary between the cooling and heating regions, and cooling (heating) does not occur on the inversion curve. Therefore, we can distinguish between the cooling and heating region by checking the sign of the slope of the isenthalpic curves. The positive sign of slope stands for the cooling region and the minus for the heating region. We conclude that the temperature and pressure are different for different values of \(\omega\) and \(M\). The inversion point moves from the positive pressure to the direction of negative pressure when the \(\omega\) changes from \(-1\) to \(1/3\). The temperature rises when the pressure decreases in the heating region. By contrast, the temperature is reduces as the decreasing of the pressure in the cooling region. Moreover, the cooling-heating region shrinks as \(M\) grows.
B. Heat Engine for the FRW Universe

Heat is a form of energy that can be transferred from one body to another or from one part of a body to another part of it. Heat engine is an important topic in thermodynamics. It converts heat into mechanical energy by doing work, and absorbs heat from hot region and releases it to the cold region. In the followings, we will explore the Carnot heat engine for the FRW universe when \( P > 0 \) in detail. Figure 4 shows the corresponding diagram of Carnot cycle for the FRW universe.

\[
\begin{align*}
T_H \text{ and } T_C & \text{ represent temperatures of the hot and cold regions respectively with } T_H > T_C. \text{ The heats are absorbed from the hot region (state 1 to state 2) and released to the cold region (state 3 to state 4), which are denoted as } Q_H \text{ and } Q_C \text{ respectively. The total work done in this cycle is } W = Q_H - Q_C. \text{ Thus, the efficiency of the heat engine is defined by the ratio of total work and the amount of heat absorbed from the hot region, i.e. } \eta = \frac{W}{Q_H}. \text{ Using (3.11) with the entropy (2.15), the heat absorbed from the hot region is}
\end{align*}
\]

\[
Q_H = \int_{S_1}^{S_2} T_H dS = \pi T_H (R_{A2}^2 - R_{A1}^2). \tag{4.9}
\]

Similarly, the heat released to the cold region is

\[
Q_C = -\int_{S_3}^{S_4} T_C dS = \pi T_C (R_{A3}^2 - R_{A4}^2). \tag{4.10}
\]

In this case, the two adiabatic lines are just equal volume (or \( R_A \) lines, so \( R_{A1} = R_{A4}, R_{A2} = R_{A3} \), and the total work is

\[
W = Q_H - Q_C = \pi (T_H - T_C)(R_{A2}^2 - R_{A1}^2). \tag{4.11}
\]

The maximum efficiency of the Carnot engine becomes

\[
\eta_C = \frac{W}{Q_H} = 1 - \frac{T_C}{T_H}. \tag{4.12}
\]

One infers from the result (4.12) that \( 0 < \eta_P < 1 \) because \( T_H > T_C \).

It is worth mentioning that the total work of the Carnot heat engine consists of two parts, the work done by the thermodynamic pressure and the work done by the fluid (matter part). Now let’s discuss the work done by the
We have discussed the apparent horizon and its thermodynamics, and derived the unified first law by using the Misner-Sharp energy and Einstein field equations with cosmological constants. We have derived the thermodynamic equation of state (3.12), where the pressure increases as the temperature decreases, so the Carnot cycle is anti-clockwise. We also get the work done by the fluid (matter part),

\[ W_{matter} = W - W_p = \pi(T_H - T_C)(R_{A2}^2 - R_{A1}^2)(1 + \frac{2}{1 + \omega}). \]  
(4.17)

The efficiency of the fluid becomes

\[ \eta_{matter} = \frac{W_{matter}}{Q_H} = (1 + \frac{2}{1 + \omega}) \left(1 - \frac{T_C}{T_H}\right). \]  
(4.18)

The efficiency of the matter part in (4.18) is always positive for the fluid with \(-1 < \omega \leq 1/3\), and larger than the total efficiency obtained in (4.12).

V. CONCLUSIONS AND DISCUSSION

In this article, we have studied the thermodynamic properties of the FRW universe with a cosmological constant \( \Lambda \). We have discussed the apparent horizon and its thermodynamics, and derived the unified first law by using the Misner-Sharp energy and Einstein field equations with cosmological constants. We have derived the thermodynamic equation of state \( P = P(R_A, T) \) by treating the cosmological constant as the thermodynamic pressure of the system, and found that there is no \( P - V \) criticality.

We present two applications to explain the thermodynamic properties of the FRW universe: the Joule-Thomson expansion and the heat engine. For the first application, we have derived the Joule-Thomson coefficient, and have found that it is always negative for matter dominated universe with \( \omega = 0 \), which means that there is no inversion point for the FRW universe and the universe will always be heating in this situation. For \( \omega \neq 0 \), we have plotted the inversion and isenthalpic curves of the FRW universe in the \( P - T \) plane, and have shown the cooling and heating regions for some values of \( \omega \) and \( M \). In the second application, we treat the expansion of the FRW universe as a thermodynamic heat engine. For the Carnot heat engine, we have obtained the total work, the work done by the thermodynamic pressure and the work done by the fluid, and have calculated the corresponding efficiencies. We have found that the total efficiency satisfies \( 0 < \eta < 1 \), but the efficiency of the thermodynamic pressure is negative for \(-1 < \omega < 1/3\), which originates from the unusual thermodynamic equation of state.

The assumption that the cosmological constant could be interpreted as the thermodynamic pressure and the investigations of the thermodynamic properties for the FRW universe may lead to more physical insights. It would be interesting to extend our methods to various theories of gravity, and if there is a phase transition, it can deepen our understanding of the evolution of the universe. Indeed, under some modified theory of gravity, there may be a critical behavior, and the physics behind it should be discussed in future works.
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[1] T. Jacobson, “Thermodynamics of space-time: The Einstein equation of state”, Phys. Rev. Lett. 75, 1260-1263 (1995), [arXiv:gr-qc/9504004].
[2] R. G. Cai and S. P. Kim, “First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe”, JHEP 02, 050 (2005), [arXiv:hep-th/0501055].
[3] R. G. Cai and L. M. Cao, “Unified first law and thermodynamics of apparent horizon in FRW universe”, Phys. Rev. D 75, 064008 (2007), [arXiv:gr-qc/0611071].
[4] M. Akbar and R. G. Cai, “Thermodynamic Behavior of Friedmann Equations at Apparent Horizon of FRW Universe”, Phys. Rev. D 75, 084003 (2007), [arXiv:hep-th/0609128].
[5] M. Akbar and R. G. Cai, “Thermodynamic Behavior of Field Equations for f(R) Gravity”, Phys. Lett. B 648, 243-248 (2007), [arXiv:gr-qc/0612089].
[6] D. Kubiznak and R. B. Mann, “P-V criticality of charged AdS black holes”, JHEP 07, 033 (2012), [arXiv:hep-th/1205.0559].
[7] J. Xu, L. M. Cao, and Y. P. Hu, “P-V criticality in the extended phase space of black holes in massive gravity”, Phys. Rev. D 91, no.12, 124033 (2015), [arXiv:gr-qc/1506.03578].
[8] D. Kubiznak, R. B. Mann and M. Teo, “Black hole chemistry: thermodynamics with Lambda”, Class. Quant. Grav. 34, no.6, 063001 (2017), [arXiv:hep-th/1608.06147].
[9] Y. P. Hu, H. A. Zeng, Z. M. Jiang, and H. Zhang, “P-V criticality in the extended phase space of black holes in Einstein-Horndeski gravity”, Phys. Rev. D 100, no.8, 084004 (2019), [arXiv:gr-qc/1812.09938].
[10] D. Kastor, S. Ray, and J. Traschen, “Enthalpy and the Mechanics of AdS Black Holes”, Class. Quant. Grav. 26, 195011 (2009), [arXiv:hep-th/0904.2765].
[11] B. P. Dolan, “The cosmological constant and the black hole equation of state”, Class. Quant. Grav. 28, 125020 (2011), [arXiv:gr-qc/1008.5023].
[12] U. Debnath, “Thermodynamics of FRW Universe: Heat Engine”, Phys. Lett. B 810, 135807 (2020), [arXiv:gr-qc/2010.02102].
[13] Ö. Ökcü and E. Aydıner, “Joule–Thomson expansion of the charged AdS black holes”, Eur. Phys. J. C 77, no.1, 24 (2017), [arXiv:gr-qc/1611.06327].
[14] S. Q. Lan, “Joule-Thomson expansion of charged Gauss-Bonnet black holes in AdS space”, Phys. Rev. D 98, no.8, 084014 (2018), [arXiv:gr-qc/1805.05817].
[15] J. Pu, S. Guo, Q. Q. Jiang, and X. T. Zu, “Joule-Thomson expansion of the regular(Bardeen)-AdS black hole”, Chin. Phys. C 44, no.3, 035102 (2020), [arXiv:gr-qc/1905.02318].
[16] S. Bi, M. Du, J. Tao, and F. Yao, “Joule-Thomson expansion of Born-Infeld AdS black holes”, Chin. Phys. C 45, no.2, 025109 (2021), [arXiv:gr-qc/2006.08920].
[17] C. V. Johnson, “Holographic Heat Engines”, Class. Quant. Grav. 31, 205002 (2014), [arXiv:hep-th/1404.5982].
[18] C. V. Johnson, “Born-Infeld AdS black holes as heat engines”, Class. Quant. Grav. 33, no.13, 135001 (2016), [arXiv:hep-th/1512.01746].
[19] C. V. Johnson, “An Exact Efficiency Formula for Holographic Heat Engines”, Entropy 18, 120 (2016), [arXiv:hep-th/1602.02838].
[20] C. V. Johnson, “Taub–Bolt heat engines”, Class. Quant. Grav. 35, no.4, 045001 (2018), [arXiv:hep-th/1705.04855].
[21] H. Liu and X. H. Meng, “Effects of dark energy on the efficiency of charged AdS black holes as heat engines”, Eur. Phys. J. C 77, no.8, 556 (2017), [arXiv:hep-th/1704.04363].
[22] J. X. Mo, F. Liang, and G. Q. Li, “Heat engine in the three-dimensional spacetime”, JHEP 03, 010 (2017), [arXiv:gr-qc/1701.00883]. J. X. Mo and G. Q. Li, “Holographic Heat engine within the framework of massive gravity”, JHEP 05, 122 (2018) JHEP 05, 122 (2018), [arXiv:hep-th/1707.01235]; J. X. Mo and S. Q. Lan, “Phase transition and heat engine efficiency of phantom AdS black holes”, Eur. Phys. J. C 78, no.8, 666 (2018), [arXiv:gr-qc/1803.02491].
[23] R. G. Cai, L. M. Cao, and Y. P. Hu, “Hawking Radiation of Apparent Horizon in a FRW Universe”, Class. Quant. Grav. 26, 155018 (2009), [arXiv:hep-th/0809.1554].
[24] C.W. Misner and D.H. Sharp Phys. Rev. 136 (1964) B571.
[25] S. A. Hayward, “General laws of black hole dynamics”, Phys. Rev. D 49, 6467-6474 (1994).
[26] S. A. Hayward, “Unified first law of black hole dynamics and relativistic thermodynamics”, Class. Quant. Grav. 15, 3147-3162 (1998), [arXiv:gr-qc/9710089].
[27] B. P. Dolan, D. Kastor, D. Kubiznak, R. B. Mann, and J. Traschen, “Thermodynamic Volumes and Isoperimetric Inequalities for de Sitter Black Holes”, Phys. Rev. D 87, no.10, 104017 (2013), [arXiv:hep-th/1301.5926].
[28] H. Maeda and M. Nozawa, “Generalized Misner-Sharp quasi-local mass in Einstein-Gauss-Bonnet gravity”, Phys. Rev. D 77 (2008), 064031, [arXiv:hep-th/0709.1199].

[29] R. G. Cai, L. M. Cao, Y. P. Hu, and N. Ohta, “Generalized Misner-Sharp Energy in f(R) Gravity”, Phys. Rev. D 80 (2009), 104016, [arXiv:hep-th/0910.2387].

[30] Y. P. Hu and H. Zhang, “Misner-Sharp Mass and the Unified First Law in Massive Gravity,” Phys. Rev. D 92 (2015) no.2, 024006 [arXiv:1502.00069 [hep-th]].

[31] T. Ha, Y. Huang, Q. Ma, K. D. Pechan, T. J. Renner, Z. Wu, and A. Wang, “Classification of the FRW universe with a cosmological constant and a perfect fluid of the equation of state $p = \omega \rho$”, Gen. Rel. Grav. 44, 1433-1458 (2012), [arXiv:physics.pop-ph/0905.0396].

[32] H. Zhang, Y. p. Hu and Y. Zhang, “Towards a sound massive cosmology,” Phys. Dark Univ. 23 (2019), 100257 [arXiv:1901.09331 [gr-qc]].