Mathematical simulation of heat transfer in an elliptic channel depending on the tangential momentum accommodation coefficient

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Abstract. This paper considers a flow of rarefied gas with a specular-diffuse reflection of gas molecules from a model of a elliptical cross section of channel walls in the transition regime. We assume that the channel maintains a constant pressure gradient. Heat flow is obtained as a function of the tangential momentum accommodation coefficient and the ratio between the semiaxes of the ellipse in the whole range of the Knudsen number. Similar results presented in the literature are compared.

1. Introduction

The nanoelectronic systems open a new area in the rarefied gas dynamics. Indeed, rarefied gas flows through channels with different cross-sections are very important in practice [1]. Many industrial apparatuses such as nanotubes or vacuum equipment are the examples of devices involving the gas flow at an arbitrary Knudsen number. In the general case, the mathematical simulation of internal rarefied gas flows has to be based on solving the Boltzmann kinetic equation or model kinetic equations [2]. In view of the fact that in experimental investigational various conditions of the interaction between rarefied gas molecules and the channel walls cannot be always reproduced, the mathematical simulation methods have assumed great importance in investigating the transfer processes in the case of incomplete accommodation of rarefied gas molecules by the channel walls. Moreover, the use of mathematical simulation makes it possible to considerably reduce the amount of experimental investigations in studying internal rarefied gas flows [3]. One of the approaches applied for describing the interaction of gas molecules with surfaces in a flow is the application of the Maxwell mirror-diffuse reflection model [3]. In [4–6], the results were obtained using this model for the circle channel cross section. However, the wide diversity of the channel forms used in practice requires a further development of the existing approach so that it would be possible to calculate gas flows through channels of arbitrary form, e.g. ellipse, rectangle etc. Heat and mass transfer problems in an elliptical tube [1], [7–9] and a rectangular channel [10–12] were solved using the diffuse reflection model. In this work we present a heat flow study in an elliptical channel based on the Maxwell mirror-diffuse reflection model. The kinetics of the process is described by the Williams kinetic equation. The mirror-diffuse reflection of gas molecules from the channel surface is considered using the approach...
proposed in [13]. The heat flow due to the pressure gradient is calculated over the whole range of the Knudsen number and in a wide range of the pipe section aspect ratio.

2. Materials and methods

Let us consider a channel whose cross section is an ellipse with semiaxes $a'$ and $b'$. Suppose in the channel a constant pressure gradient directed along its axis is maintained. Let us direct the $Ox'$ and $Oy'$ axes of the Cartesian system of coordinates along the ellipse semiaxes and $Oz'$ in the direction opposite to the temperature gradient. In the chosen system of coordinates, the Williams equation is written as [8]

\[
\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} = \frac{\omega}{\gamma l_g} (f_\ast - f),
\]

where $\omega = |v - u(r')|$; $v$ is the velocity of gas molecules; $u(r')$ is the mass velocity of gas molecules; $r'$ and $m$ are radius vector and mass of gas molecules, respectively; $l_g$ is the mean free path of gas molecules; $k_B$ is the Boltzmann constant; $\gamma = 5\sqrt{\pi}/4$. Parameters $n_\ast$, $T_\ast$ and $u_\ast$, which included in function (2), are determined from the condition that the model collision integral satisfies the particle number, momentum, and energy conservation laws as follows [2].

As a boundary condition on the walls of the channel, we use the Maxwell specular-diffuse reflection model [3]

\[
\begin{align*}
\frac{\partial f^+}{\partial \nu} (r'_T, v) &= (1 - \alpha) f^- (r'_T, v - 2n(v\nu)) + \alpha f^\prime_T (r'_T, v), \quad vn > 0, \\
f^\prime_T (r'_T, v) &= n(r'_T) \left( \frac{m}{2\pi k_B T_\ast} \right)^{3/2} \exp \left( -\frac{m}{2k_B T_\ast} (v - u_\ast)^2 \right),
\end{align*}
\]

Here, $f^+ (r'_T, v)$ and $f^- (r'_T, v)$ are the distribution functions of the gas molecules reflected from the channel walls and incident on the walls, respectively, $r'_T$ is the dimensionless radius-vector of the points on the channel walls, $n$ is the vector of the normal to the wall directed toward the gas, $\alpha$ is the tangential momentum accommodation coefficient, and $f^\prime_T (r'_T, v)$ is the locally equilibrium distribution function.

Let us introduce the following dimensionless quantities $x' = x'/b'$, $y' = y'/b'$, $z = z'/b'$, $C = \beta^{1/2} v$, where $\beta = m/(2k_B T_0)$. Assuming that the temperature is constant and the pressure gradient in the channel is small, i.e.

\[
G_p = \frac{1}{p_0} \frac{dp}{dz}, \quad |G_p| \ll 1,
\]

we have $p(z) = p_0 (1 + G_p z)$. In this case, the solution of the problem can be obtained in a linearized form. We expand expression (4) into a series in the small parameter $G_p$, considering that $p(z) = n(z) k_B T$. Restricting ourselves to the linear terms in the expansion we arrive at the following expression for the locally equilibrium distribution function

\[
f^\prime_T (z, C) = f_0(C) \left( 1 + G_p z \right),
\]

where $f_0(C) = n_0(\beta/\pi)^{3/2} \exp (-C^2)$ is the absolute Maxwellian. We linearize the distribution function with respect to $f^\prime_T (z, C)$. In view of expansion (5), we obtain

\[
f(r, C) = f_0(C) \left( 1 + G_p z + h(x, y, C) \right).
\]
The function \( f_\ast(r, C) \) is written as

\[
f_\ast(r, C) = f_0(C)(1 + h_\ast(r, C)), \tag{7}
\]

\[
h_\ast(r, C) = \frac{\delta n_\ast}{n_0} + 2CU_\ast + \left(C^2 - \frac{3}{2}\right) \frac{\delta T_\ast}{T_0}.
\]

Substituting (6) and (7) into (1), we arrive at the equation

\[
\left(C_x \frac{\partial h}{\partial x} + C_y \frac{\partial h}{\partial y} + G_p\right) \gamma KnC_z + Ch(x, y, C) =
\]

\[
= \frac{C}{2\pi} \int C' \exp(-C'^2)k(C, C')C'_2 h(x, y, C') d^3C', \tag{8}
\]

where \( k(C, C') = 1 + 3CC'/2 + (C^2 - 2) (C'^2 - 2)/2 \).

A solution of the equation (8) is sought in the form

\[
h(x, y, C) = \gamma KnG_pC_zZ(x, y, C_x, C_y).	ag{9}
\]

Substituting (9) into (8) and (3) yields an equation in function \( Z(x, y, C_x, C_y) \)

\[
C_x \frac{\partial Z}{\partial x} \gamma Kn + C_y \frac{\partial Z}{\partial y} \gamma Kn + CZ(x, y, C_x, C_y) + 1 =
\]

\[
= \frac{3C}{4\pi} \int C' \exp(-C'^2)C'^2 Z(x, y, C'_x, C'_y) d^3C', \tag{10}
\]

with the boundary condition

\[
Z(x, y, C_x, C_y) = (1 - \alpha)Z(x, y, C^*_x, C^*_y), \quad C^* = C - 2n(nC), \quad nC > 0. \tag{11}
\]

To find a solution to equation (10), it is necessary to select appropriate functions from a set of functions that depend on the molecular velocity magnitude and compose a scalar product with weight \( g(C) = C^5 \exp(-C^2) \) on their basis as follows:

\[
(f_1, f_2) = \int_0^{+\infty} g(C)f_1(C)f_2(C)dC. \tag{12}
\]

We take orthogonal functions \( e_1 = 1 \) and \( e_2 = 1/C - 3\sqrt{\pi}/8 \) (here, orthogonality is meant as the vanishing of the scalar product (12)) and expand function \( Z(x, y, C_x, C_y) \) in the orthogonal functions:

\[
Z(x, y, C_x, C_y) = Z_1(x, y, \varphi, \theta) + \left(\frac{1}{C} - \frac{3\sqrt{\pi}}{8}\right) Z_2(x, y, \varphi, \theta). \tag{13}
\]

In (13), we passed to the spherical coordinates in the velocity space, \( C_x = C \cos \varphi \sin \theta, \quad C_y = C \sin \varphi \sin \theta, \quad C_z = C \cos \theta \). Substituting expansion (13) in (10) and (11), we come to a set of equations

\[
\left(\cos \varphi \frac{\partial Z_1}{\partial x} + \sin \varphi \frac{\partial Z_1}{\partial y}\right) \sin \theta \gamma Kn + Z_1(x, y, \varphi, \theta) + \frac{3\sqrt{\pi}}{8} =
\]

\[
= \frac{3}{4\pi} \int_0^\pi \cos^2 \theta' \sin \theta' d\theta' \int_0^{2\pi} Z_1(x, y, \varphi', \theta') d\varphi',
\]

\[
\int_0^{2\pi} Z_1(x, y, \varphi', \theta') d\varphi',
\]
Therefore, to find a solution to the problem, we must find function $t$, etc., and $t$ is the moment of the last collision with the surface, $t$ score of these collisions in the opposite direction of the molecule movement. We assume, that $W$ is determined by the equation

$$W = \frac{\partial Z_2}{\partial y} \sin \theta \gamma Kn + Z_2(x, y, \varphi, \theta) + 1 = 0,$$

with the boundary conditions

$$Z_i(x_T, y_T, \varphi, \theta) = (1 - \alpha)Z_i(x_T, y_T, \varphi^*, \theta), \quad b^2 x_T \cos \varphi + y_T a^2 \sin \varphi < 0, \quad i = 1, 2;$$

$$\cos \varphi^* = \frac{\cos \varphi (a^4 y_T^2 - b^4 x_T^2)}{a^4 y_T^2 + b^4 x_T^2} - 2a^2 b^2 x_T y_T \sin \varphi,$$

$$\sin \varphi^* = -\sin \varphi (a^4 y_T^2 - b^4 x_T^2) + 2a^2 b^2 x_T y_T \cos \varphi.$$

With regard to the statistical meaning of the distribution function, the nonzero component of the heat flow vector is given by [3]

$$q_z(x, y) = \frac{m}{2} \int (v_z - u_z(x, y))|v - u(x, y)|^2 f(r', v)d^3v = \frac{p_0}{\beta^{1/2}} q_z(x, y).$$

Here, function $q_z(x, y)$ is the dimensional $z$-component of the heat flow vector. It is given by

$$q_z(x, y) = \pi^{-3/2} \int \exp \left(-C^2\right) C_z \left(C^2 - \frac{5}{2}\right) h(r, C)d^3C =$$

$$= -\frac{G \gamma Kn}{4\pi^{3/2}} \int_0^\pi \cos^2 \theta \sin \theta d\theta \int_0^{2\pi} Z_2(x, y, \varphi, \theta) d\varphi. \quad (18)$$

From (18) it follows that function $Z_1(x, y, \varphi, \theta)$ does not contribute to the heat flow vector. Therefore, to find a solution to the problem, we must find function $Z_2(x, y, \varphi, \theta)$ from (14) with boundary condition (15). Let us introduce the following function $W(x, y, \varphi, \theta) \equiv Z_2(x, y, \varphi, \theta)$. The variation of $W(x, y, \varphi, \theta)$ along a trajectory (characteristic) [13]

$$dr_\perp = c_\perp dt,$$

is determined by the equation

$$\frac{\partial W}{\partial r_\perp} c_\perp \gamma Kn + W(x, y, \varphi, \theta) + 1 = 0. \quad (20)$$

Here, $r_\perp = (x, y)$, $c_\perp = (\sin \theta \cos \varphi, \sin \theta \sin \varphi)$ are the components of the radius vector $r$ and velocity $c = C/C$ of a molecule, respectively, in the plane normal to the $Oz$ axis.

The equation (20) is written as

$$dW = -\frac{1}{\gamma Kn} (W(x, y, \varphi, \theta) + 1) dt. \quad (21)$$

Let us consider the process of collision of gas molecules with the channel walls. We will keep score of these collisions in the opposite direction of the molecule movement. We assume, that $t_1$ is the moment of the last collision with the surface, $t_2$ is the moment of the previous collision, etc., and $t_0$ is time upcoming collision, $T_n = t_n - t_{n+1}$ ($n = 1, 2, \ldots$).

The solution of the equation (21) has the form

$$W_n(t) = A_n \exp \left(\frac{-t - t_n}{\gamma Kn}\right) - 1, \quad t_n \leq t \leq t_{n-1}, \quad n = 1, 2, \ldots \quad (22)$$
At the point \( t = t_n \) of reflection (from any surface), the function (22) is discontinuous:

\[
W_n(t_n + 0) = (1 - \alpha)W_{n+1}(t_n - 0). \tag{23}
\]

The plus and minus sign denotes the limits (with respect to the time of flight) immediately after and before a reflection, respectively, of the function \( W_n(t) \) at the reflection point \( t_n \).

At the point \( t = t_1 + 0 \), we have

\[
W_1(t_1 + 0) = A_1 - 1. \tag{24}
\]

Since \( t_1 - 0 = t_2 + T_1 \), we obtain

\[
W_2(t_1 - 0) = A_2 \exp \left( -\frac{T_1}{\gamma Kn} \right) - 1.
\]

From (23) it follows that \( W_1(t_1 + 0) = (1 - \alpha)W_2(t_1 - 0) \). Then we can find the constant \( A_1 \) as

\[
A_1 = (1 - \alpha) \left( A_2 \exp \left( -\frac{T_1}{\gamma Kn} \right) - 1 \right) + 1. \tag{25}
\]

Similarly, we express \( W_2(t_2 + 0) \) in terms of \( W_3(t_2 - 0) \) and so on and arrive at a recurrence relation for \( A_n \) as

\[
A_n = (1 - \alpha) \left( A_{n+1} \exp \left( -\frac{T_n}{\gamma Kn} \right) - 1 \right) + 1, \quad n = 2, 3, \ldots \tag{26}
\]

Substituting (26) into (25), we obtain

\[
A_1 = 1 + (1 - \alpha) \left( \exp \left( -\frac{T_1}{\gamma Kn} \right) - 1 \right) + (1 - \alpha)^2 \left( \exp \left( -\frac{T_2}{\gamma Kn} \right) - 1 \right) \exp \left( -\frac{T_1}{\gamma Kn} \right) + \ldots
\]

For a channel with a circular cross section \( (T_n = T \) because of the flow symmetry), we arrive at an expression for \( A_1 \) in terms of the sum of the infinitely decreasing geometric progression

\[
A_1 = 1 + \frac{(1 - \alpha) \left( \exp \left( -\frac{T}{\gamma Kn} \right) - 1 \right)}{1 - (1 - \alpha) \exp \left( -\frac{T}{\gamma Kn} \right)}. \tag{28}
\]

The parameter \( t - t_1 \) can be related to the coordinates of the point \( (r_\perp, c_\perp) \) in the phase and velocity spaces by the conditions (15) and (19) as

\[
t - t_1 = \frac{x - x_{\Gamma,1}}{\sin \theta \cos \varphi}, \quad x_{\Gamma,1} = x_{\Gamma}(x, y, \varphi)
\]

\[
x_{\Gamma} = \frac{a^2 \sin \varphi(x \sin \varphi - y \cos \varphi) - ab \cos \varphi \sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi} - (y \cos \varphi - x \sin \varphi)^2}{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}.
\]
Considering the point on the interval \([t_{n+1}, t_n]\) and taking \(T_n = t_n - t_{n+1}\) and (29) into account, we find the parameter \(T_n\)

\[
T_n = \frac{x_{\Gamma,n} - x_{\Gamma,n+1}}{\sin \theta \cos \varphi_n}.
\]

From (15)-(17) it follows that

\[
\cos \varphi_1 = \cos \varphi^\ast (x_{\Gamma,1}, y_{\Gamma,1}, \varphi), \quad y_{\Gamma,1} = y_{\Gamma}(x, y, \varphi),
\]

\[
y_{\Gamma} = \frac{b^2 \cos \varphi(y \cos \varphi - x \sin \varphi) - ab \sin \varphi \sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi - (y \cos \varphi - x \sin \varphi)^2}}{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi},
\]

\[
x_{\Gamma,n} = x_{\Gamma}(x_{\Gamma,n-1}, y_{\Gamma,n-1} \cos \varphi_{n-1}), \quad y_{\Gamma,n} = y_{\Gamma}(x_{\Gamma,n-1}, y_{\Gamma,n-1} \cos \varphi_{n-1}),
\]

\[
\cos \varphi_n = \cos \varphi^\ast (x_{\Gamma,n}, y_{\Gamma,n}, \varphi_{n-1}), \quad \sin \varphi_n = \sin \varphi^\ast (x_{\Gamma,n}, y_{\Gamma,n}, \varphi_{n-1}).
\]

Relations (24), (27), (29)-(31) fully determine the function \(W(x, y, \varphi, \theta)\).

Taking into account \(Z_2(x, y, \varphi, \theta) \equiv W_1(x, y, \varphi, \theta)\) on the interval \([t_1, t_0]\) and substituting (22) in (18), we obtain the \(z\)-component of the heat flow vector:

\[
q_z(x, y, \varphi, \theta) = \frac{G_p \gamma K_n}{3 \sqrt{\pi}} \left(1 - \frac{3}{4\pi} \int_0^{\pi} \cos^2 \theta \sin \theta d\theta \int_0^{2\pi} A_1 \exp \left(-\frac{x - x_{\Gamma,1}}{\gamma K_n \cos \varphi \sin \theta} \right) d\varphi \right).
\]

here \(A_1\) is defined by relation (27).

The reduced heat flow can be calculated via the dimensionless component \(q_z(x, y)\) as [8]

\[
J_Q = \frac{8}{ab\sqrt{\pi}} \int_0^a \int_0^{b \sqrt{\frac{x^2 - y^2}{b^2}}} q_z(x, y) dx dy.
\]

3. Results and discussion

Taking into account the four members of the series (27), we calculate \(J_Q/G_p\) values by using formula (33) for various ratios \(a = a'/b'\) and different values \(Kn\). The obtained heat flow values are given in the tables 1 and 2. The table 1 also contains the results obtained in [9] and [14] at the total accommodation of rarefied gas molecules. The difference between the heat flow values presented here and those presented in [9], [14] is due to the fact that the results [9] and [14] were obtained using the numerical integration of the BGK- and S-models of the Boltzmann kinetic equation for the constant molecular collision frequency.

The \(\alpha = 0.9, 0.95\) values, which appeared in table 2, are determined from the condition that tangential momentum accommodation coefficient on the surface of channel untreated in a special way is close to unity. So, for \(Ne\) and \(Ar\), these values of \(\alpha\) are 0.925 ± 0.014 and 0.927 ± 0.028 [15] on glass surface of circular channel, respectively. The accommodation coefficient for \(He\) is smaller than the other gas coefficients (\(\alpha = 0.895 ± 0.004\)). For \(a = 1\), the values of \(J_Q/G_p\) are also found to (33) taking into account (28). For the semiaxis ratios \(a\) given in tables 1 and 2 a decrease in the accommodation coefficient \(\alpha\) leads to an increase in the reduced heat flow through the channel elliptical cross-section, which is confirmed by the conclusions made in [16]. An analysis of the data given in table 2 shows that the largest deviations of the heat flow are observed as the free-molecular regime is approached.
Table 1. Heat flow $J_Q/G_p$ for $\alpha = 1$, various ratios $a = a'/b'$ and different values $Kn$.

| $Kn$ | 1 1[9] | 1[14] | 1.1 | 1.1[9] | 2 2[9] | 10 10[9] |
|------|--------|--------|-----|--------|--------|--------|
| 0.001 | 0.0008 | –      | –   | 0.0008 | –      | 0.0008 |
| 0.010 | 0.0083 | 0.0116 | –   | 0.0083 | –      | 0.0083 |
| 0.100 | 0.0765 | 0.1020 | 0.1018 | 0.1024 | 0.0780 | 0.1044 | 0.0789 | 0.1049 |
| 0.500 | 0.2705 | 0.3027 | 0.2753 | 0.3095 | 0.2981 | 0.1856 | 0.3167 | 0.3492 |
| 1.000 | 0.3881 | 0.3968 | 0.3962 | 0.3983 | 0.4381 | 0.4519 | 0.4676 | 0.5073 | 0.5469 |
| 2.000 | 0.4977 | 0.4784 | 0.4780 | 0.5144 | 0.4953 | 0.6096 | 0.5872 | 0.7392 | 0.6558 |
| 5.000 | 0.6080 | –      | 0.6324 | 0.5896 | 0.7825 | 0.7231 | 1.0610 | 0.8831 |
| 10.00 | 0.6632 | 0.6209 | 0.6206 | 0.6919 | 0.6469 | 0.8746 | 0.8081 | 1.2762 | 1.0645 |
| 100.0 | 0.7376 | 0.7210 | 0.7223 | 0.7725 | 0.7559 | 1.0054 | 0.9770 | 1.6659 | 1.5502 |
| 1000  | 0.7502 | 0.7469 | –    | 0.7863 | –      | 1.0289 | –      | 1.7536 | –      |

Table 2. Heat flow $J_Q/G_p$ for $\alpha = 0.9$, various ratios $a = a'/b'$ and different values $Kn$.

| $Kn$ | 1 1[9] | 1.1 | 2 5 | 10 | 50 |
|------|--------|-----|----|----|----|
| $\alpha = 0.9$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 0.0100 | 0.0083 | 0.0083 | 0.0083 | 0.0083 | 0.0083 | 0.0083 | 0.0083 |
| 0.1000 | 0.0771 | 0.0776 | 0.0787 | 0.0794 | 0.0795 | 0.0796 |
| 0.5000 | 0.2833 | 0.2880 | 0.3089 | 0.3229 | 0.3266 | 0.3276 |
| 1.0000 | 0.4196 | 0.4301 | 0.4806 | 0.5213 | 0.5330 | 0.5379 |
| 2.0000 | 0.5562 | 0.5734 | 0.6674 | 0.7601 | 0.7937 | 0.8129 |
| 5.0000 | 0.7045 | 0.7309 | 0.8843 | 1.0752 | 1.1703 | 1.2456 |
| 10.000 | 0.7834 | 0.8168 | 1.0075 | 1.2696 | 1.4299 | 1.5994 |
| 100.00 | 0.8962 | 0.9380 | 1.1881 | 1.5911 | 1.9053 | 2.5492 |
| 1000  | 0.9148 | 0.9574 | 1.2177 | 1.6533 | 2.0038 | 2.8777 |

| $\alpha = 0.95$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 0.0100 | 0.0083 | 0.0083 | 0.0083 | 0.0083 | 0.0083 | 0.0083 | 0.0083 |
| 0.1000 | 0.0768 | 0.0771 | 0.0782 | 0.0789 | 0.0790 | 0.0791 |
| 1.0000 | 0.4036 | 0.4135 | 0.4656 | 0.5077 | 0.5194 | 0.5262 |
| 2.0000 | 0.5262 | 0.5428 | 0.6376 | 0.7323 | 0.7658 | 0.7845 |
| 5.0000 | 0.6545 | 0.6795 | 0.8312 | 1.0211 | 0.6545 | 1.1851 |
| 10.000 | 0.7208 | 0.7497 | 0.9359 | 1.1971 | 1.3501 | 1.5098 |
| 100.00 | 0.8130 | 0.8514 | 1.0910 | 1.4789 | 1.7736 | 2.3641 |
| 1000.0 | 0.8276 | 0.8665 | 1.1148 | 1.5345 | 1.8631 | 2.6592 |

Profiles of the $z$-component of the heat flow vector $q_z(x,y)/G_p$ in the elliptic channel calculated according to (32) for the accommodation coefficient $\alpha = 0.9$ and the ellipse semiaxis ratios $a = 1.2$ and 10 are presented in figures 1–4. For the channel with the semi-axis ratio $a = 1.2$ the mass velocity profile in the channel is similar in shape with the paraboloid of revolution at $Kn = 1$ (figure 1), which takes place for the cylindrical channel in the free molecular regime.
This figure show that the distribution of the heat flow vector $z$-component has a maximum at the origin. The profiles in figure 3 is close to a plane perpendicular to the axis of the channel, a deviation from which occurs in a thin wall layer (Knudsen layer), which is consistent with the conclusions of [3]. In the case of an increase in the ellipse semi-axis ratio $a$, the mass velocity profile curves at the channel walls, as shown in figures 2 and 4.

![Figure 1](image1.png)  
**Figure 1.** Distribution of the heat flow vector $z$-component in the channel for $a = 1.2$, $\alpha = 0.9$ and $Kn = 1.2$.

![Figure 2](image2.png)  
**Figure 2.** Distribution of the heat flow vector $z$-component in the channel for $a = 10$, $\alpha = 0.9$ and $Kn = 1.1$.

![Figure 3](image3.png)  
**Figure 3.** Distribution of the heat flow vector $z$-component in the channel for $a = 1.2$, $\alpha = 0.9$ and $Kn = 0.1$.

![Figure 4](image4.png)  
**Figure 4.** Distribution of the heat flow vector $z$-component in the channel for $a = 10$, $\alpha = 0.9$ and $Kn = 0.1$.

Profiles of the $z$-component of the heat flow vector $q_z^{(1)} = q_z|_{\alpha=1}$ and $q_z^{(2)} = q_z|_{\alpha=0.9}$ in the elliptic channel at $Kn = 10$ for the ellipse semiaxis ratios $a = 1.2$ and $10$ are presented in figures 5 and 6. As follows from figures 5 and 6 a decrease in the accommodation coefficient leads to an increase in $q_z/G_p$ with the conservation of the profile shape.
For a flow regime close to the free molecular limit, expression (32) can be represented as series in powers of the small parameter $Kn^{-1}$. In this case, retaining the linear expansion terms, we obtain

\[
q_{z,f:m}(x, y) = q_{z,f:m}(x, y)\bigg|_{\alpha=1} + \frac{G_p}{8\sqrt{\pi}} \left(1 - \alpha\right) \int_{0}^{2\pi} T'_1 d\varphi +
\]

\[
+ (1 - \alpha)^2 \int_{0}^{2\pi} T'_2 d\varphi + \ldots \right), \quad T'_n = T_n \sin \theta.
\]

(34)

Here, parameters $T_n$ ($n = 1, 2, \ldots$) are determined from the expressions (30). The function $q_{z,f:m}(x, y)\big|_{\alpha=1}$ defines the $z$-component of the heat flow vector, calculated for $\alpha = 1$ in the free molecular regime, i.e.

\[
q_{z,f:m}(x, y)\big|_{\alpha=1} = \frac{abG_p}{4\sqrt{\pi}} \int_{-\pi/2}^{\pi/2} \sqrt{\frac{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi - (y \cos \varphi - x \sin \varphi)^2}{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}}
\]

(35)

For nearly hydrodynamic flow regimes ($Kn \ll 1$), an analysis of expression (32) yields

\[
J_Q = \frac{5G_p Kn}{6}.
\]

(36)

Thus, for nearly hydrodynamic flow regimes, the relative heat flow is independent of the ratios ellipse semiaxes. This conclusion is confirmed by the results presented in Tables 1 and 2 for $Kn \leq 0.001$, and the expression for $Kn = 0$ in (36) coincides with that given in [16].

4. Conclusion

In the transition regime, the expression for heat flow in an elliptical channel under a constant pressure gradient directed along the symmetry axis of the channel has been derived in the framework of the Maxwell specular-diffuse reflection model. The expression obtained is
numerically analyzed. The values of the heat flow through the cross section of the channel were obtained in a wide range of Knudsen numbers. For various ratios between the semiaxes of the ellipse, the distribution profile for the component of the heat flow vector has been constructed as a function of the tangential momentum accommodation coefficient. It has been shown that the reduced heat flow significantly depends on the tangential momentum accommodation coefficient and in the limiting cases the results obtained in this work pass into similar results for the hydrodynamic and free-molecular regimes.

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