The role of the measurement process in resolving the gauge ambiguity of the effective gravitational potential is reexamined. The motion of a classical point-like particle in the field of an arbitrary linear source, and in the field of another point-like particle is investigated. It is shown that in both cases the value of the gravitational field read off from the one-loop effective action of the testing particle depends on the Feynman weighting parameter. The found dependence is essential in that it persists in the expression for the gravitational potential. This result disproves the general conjecture about gauge independence of the effective equations of motion of classical point-like particles.

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I. INTRODUCTION

The so-called gauge dependence problem is probably the main obstacle for a straightforward physical interpretation of calculations in the theory of effective fields. Formulated in terms of the quantum fields averages, this theory provides general means for analyzing radiative corrections to various physical processes including those outside the scope of the standard scattering theory. In short, the problem consists in the following. The results of calculations in the effective field formalism are generally ambiguous because of an arbitrariness in the choice of gauge conditions fixing the gauge freedom (closely related to this is the parameterization dependence problem originating from the freedom in the choice of dynamical variables in terms of which the theory is quantized). Formal features of the gauge dependence problem in the case of gravity are quite similar to those in the Yang-Mills theories. From the point of view of the Batalin-Vilkovisky formalism [1], for instance, a change of the gauge conditions induces an anti-canonical transformation of the effective action in both cases [2]. From the physical point of view, however, interpretations of this problem are quite different. Unlike the case of Yang-Mills theory, the notion of a gauge in the theory of gravitation is directly related to the properties of physical spacetime. Namely, arbitrariness in the choice of gauge conditions corresponds to the arbitrariness in the choice of the reference frame, i.e., of the way various spacetime points are identified by means of the reference bodies. Accordingly, the gauge dependence problem in quantum gravity is actually the question of whether a change of the gauge can be interpreted as a deformation of the reference frame, and vice versa.

The notion of the reference frame is closely related to the notion of measurement, and as such it has an essentially classical meaning. Therefore, the freedom in the choice of reference frame in quantum gravity is the same as in classical theory. Thus, the above question is whether quantization of the gravitational field introduces an ambiguity into the correspondence between gauge conditions and reference frames. This question is rather nontrivial because of the following circumstance. In quantum theory, the gauge fixing procedure, e.g., in the functional integral approach, is formulated in terms of the integration variables, rather than expectation values used in the discussion of physical issues such as transitions between different reference frames. Detailed analysis shows that this rather indirect connection between gauge conditions and reference frames is indeed unambiguous as far as contributions of zero order in the Planck constant $\hbar$ are considered [3, 4]. More precisely, this connection turns out to be the same as in classical theory up to a spacetime diffeomorphism.

At the next order in $\hbar$, things turn out to be more complicated. As was emphasized in Ref. [5], a consistent treatment of the gauge dependence of $O(\hbar)$-corrections to the effective gravitational field requires an explicit introduction of a measuring device into consideration. In the process of measurement, the value of the gravitational field is affected by the measuring device. Even if the mass of the measuring apparatus is infinitely small, and therefore, so is its contribution to the gravitational field, it cannot be neglected nevertheless. The point is that the value of the effective
gravitational field is read off from the effective action of the measuring apparatus (or from its effective equations of motion), which is also small, while the relative value of the $O(h)$-corrections to the gravitational field is mass-independent. Thus, the $O(h)$ contribution of the device–graviton interaction to the effective device action turns out to be of the same order as that of the graviton interaction with the external source of the gravitational field. It was shown in Ref. [8] by an explicit calculation that in the course of derivation of the effective equations of motion of the measuring device, the gauge-dependent parts of the two contributions cancel each other.

In connection with this result, we would like to note the following. The gauge dependence cancellation was demonstrated in Ref. [8] in a very special case when the source of the gravitational field as well as the measuring device are point-like non-relativistic particles. To the best of our knowledge, it has never been verified under more general conditions. Nevertheless, the statement about eventual gauge independence of the effective equations of motion of a point-like measuring device has been used in a number of later developments as a well established result. In particular, it has been assumed in investigation of the graviton corrections to the Maxwell equations [6], as well as particle dynamics in the Robertson Walker spacetime [7]. At the same time, it was shown in Ref. [8] that in the case when the role of the measuring device is played by a classical scalar field, its effective equations of motion are still gauge dependent. This raises a question about factual conditions under which the gauge dependence cancellation takes places. The purpose of this paper is to reexamine the role of the measurement using point-like classical particles in solving the gauge dependence problem. The reason we turn back to this case is that although the class of gauge conditions considered in Ref. [8] is general enough to assert that the found cancellation of gauge dependence is not accidental, the whole consideration of Ref. [8] was carried out in the framework of the background field method. As is well known, the gauge-fixed quantum action in this case possesses an additional symmetry connected with the gauge transformations of the background field. It was mentioned already in Ref. [8] that this symmetry reduces the number of diagrams contributing to the gauge dependent part of the effective device action. The question of whether the gauge dependence cancellation is a byproduct of the background field method prompts one to go beyond this method. From the formal point of view, this means nothing but a discrete change of the gauge conditions from the background to the ordinary ones. If the effective device action is really gauge independent, it must be invariant under this change. Below, we will follow the general method of calculating the gauge dependent part of the effective device action, developed in Ref. [8] (thereafter referred to as I). This method has a number of advantages. First of all, it makes an explicit evaluation of the mean gravitational field unnecessary. Second, in the case when the device contribution to the mean gravitational field is small, which is of primary interest, it allows one to avoid the necessity of performing the Legendre transformation from the field sources to the mean fields, required in constructing the effective device action. Finally, the use of the Slavnov identities makes the structure of various contributions to the gauge dependent part of the effective device action most transparent and easily dealt with. In connection with the latter we would like to emphasize that the reasons in favor of using the Slavnov identities lie actually far beyond the matter of convenience. If the gauge dependence cancellation in the effective equations of motion is a fundamental property of the classical device indeed, there must be a formal mathematical reason underlying this cancellation. Such reason is naturally expected to have its roots in the original gauge invariance of the classical theory, expressed in the quantum domain by the Slavnov identities. As we will see, the use of the Slavnov identities reveals an unfortunate fact that the gauge dependent contributions of the two types discussed above tend to add up, at least partially, rather than subtract.

Our paper is organized as follows. In Sec. II we consider the motion of a point-like apparatus in the gravitational field produced by an arbitrary source linear in the field. We find no gauge cancellation in this case. To show that the non-linearity of realistic sources does not change this conclusion, the special case of a point-like classical source is investigated in Sec. III. For this purpose, we generalize the original method developed in I to the nonlinear case by introducing an auxiliary source for the BRST-variation of the matter action, and then find the contribution of the nonlinear graviton-source interaction to the gauge dependent part of the effective device action. The results of the work are discussed in Sec. IV.

As a rule, we employ condensed notation throughout this paper, and omit the signs of spacetime integrals, implying that the integration is done along with summation over repeated indices; integrals along the world lines of the point particles are written out explicitly. Also, left derivatives with respect to odd variables are used. The dimensional regularization of all divergent quantities is assumed.

II. EFFECTIVE GRAVITATIONAL FIELD OF A LINEAR SOURCE

In this section, we consider the motion of a point particle in the gravitational field produced by a matter source linear in the graviton field operator, i.e., $T^{\mu\nu}h_{\mu\nu}$, where $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ is the deviation of the spacetime metric $g_{\mu\nu}$.
from the flat Minkowski metric $\eta_{\mu\nu}$, and the source $T^{\mu\nu}$ a function of the spacetime coordinates, satisfying
\begin{equation}
\frac{\partial T^{\mu\nu}}{\partial x^\mu} = 0,
\end{equation}
and arbitrary otherwise. We write the ordinary derivatives in Eq. (1) instead of covariant ones implying that all calculations throughout this paper are carried out in the linear approximation with respect to $T^{\mu\nu}$ (note that the external gravitational field is $O(T)$.) The mass $m$ of the particle is assumed sufficiently small so as to neglect its contribution to the mean gravitational field $\langle h_{\mu\nu} \rangle$. The value of the effective gravitational field measured by the particle is read off from its effective equations of motion
\begin{equation}
\frac{\delta \Gamma_m(x(\tau), h(\tau))}{\delta x^\mu(\tau)} = 0,
\end{equation}
where $\Gamma_m$ denotes the effective action of the particle, which is a function of the particles’ spacetime position $x(\tau) \equiv \{x^\mu(\tau)\}$ parameterized by its proper time $\tau$ built from the flat metric, $d\tau^2 = \eta_{\mu\nu}dx^\mu dx^\nu$, and of the mean gravitational field $h(\tau) \equiv \{h_{\mu\nu}(x(\tau))\}$ at the point $x(\tau)$; the angle brackets denoting the operation of averaging are omitted for brevity. The question of gauge (in)dependence of the measured gravitational field is thus the question of gauge (in)dependence of its effective action $\Gamma_m$.

The gauge dependent part of $\Gamma_m$ is the sum of two different contributions. One of them comes from an explicit gauge dependence of the effective action $\Gamma[x, h]$, while the other – from an implicit gauge dependence of the mean field $h = h(T)$. As was shown in I, the full derivative of the effective device action with respect to a gauge parameter $\zeta$, which takes into account both contributions, can be written as
\begin{equation}
\frac{d\Gamma_m}{d\zeta} = \frac{\partial W_m}{\partial \zeta},
\end{equation}
where $W_m$ is the device contribution to the generating functional of connected Green functions, $W$. The latter is defined by
\begin{equation}
e^{iW} = \int dhdCd\bar{C} \exp\{i(\Sigma + \bar{\alpha}_\beta C_\alpha + \bar{C}_\alpha \beta + T^{\mu\nu}h_{\mu\nu})\},
\end{equation}
where
\begin{equation}
\Sigma = S_{FP} + \int d^4x K^{\mu\nu}D^\alpha_{\mu\nu}C_\alpha - \int d^4x \frac{L^2}{2} f^{\alpha\gamma \beta} C_\alpha C_\beta + J \int d^4x \delta S_m \delta h_{\mu\nu} D^\alpha_{\mu\nu} C_\alpha,
\end{equation}
$S_{FP}$ is the Faddeev-Popov quantum action,
\begin{equation}
S_{FP} = S + S_m + S_{gf} + \int d^4x \bar{C}_\beta F_{\beta}^{\alpha\mu\nu} D^\alpha_{\mu\nu} C_\alpha;
\end{equation}
$S, S_m$ are the action functionals for the gravitational field and the measuring apparatus, respectively,\footnote{Our notation is $R_{\mu\nu} \equiv R_{\alpha\mu\nu}^\alpha = \partial_\alpha \Gamma_{\mu\nu}^\alpha - \cdots$, $R \equiv R_{\mu\nu} g^{\mu\nu}$, $g \equiv \det g_{\mu\nu}$, $g_{\mu\nu} = \text{sgn}(+, -, -, -)$. We use units in which $c = \hbar = 16\pi G = 1$, $G$ being the Newton gravitational constant. Indices are raised and lowered with the help of Minkowski metric $\eta_{\mu\nu}$.}
\begin{equation}
S = -\int d^4x \sqrt{-g} R,
\end{equation}
\begin{equation}
S_m = -m \int \sqrt{g_{\mu\nu}dx^\mu dx^\nu} = -m \int d\tau \sqrt{1 + h_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}},
\end{equation}
while $S_{gf}$ is the gauge-fixing term
\begin{equation}
S_{gf} = \int d^4x \bar{F}_{\alpha} F_{\alpha}^{\alpha} \frac{2\xi}{2},
\end{equation}
where $\xi$ is a parameter weighting the gauge conditions $F_\alpha$ specified below; the c-functions $K^{\mu\nu}(x)$ (odd) and $L^\alpha(x)$ (even) are sources for the Becchi-Rouet-Stora-Tyutin (BRST) transformations of the gravitational field and the Faddeev-Popov ghost field $C_\alpha$, respectively; $D$, $\bar{D}$ are the generators of gauge transformations of the variables $h, x$

\[
\delta h_{\mu\nu} = \xi^\alpha \partial_{\nu} h_{\mu\alpha} + (\eta_{\alpha\mu} + h_{\alpha\mu}) \partial_{\nu} \xi^\alpha + (\eta_{\nu\alpha} + h_{\nu\alpha}) \partial_{\mu} \xi^\alpha \equiv D_{\mu\nu}^\alpha \xi^\alpha, \\
\delta x^\mu = -\xi^\mu \equiv \bar{D}_{\mu} \xi^\alpha,
\]

where $\xi^\alpha$ are infinitesimal gauge functions; following I, we introduced into $\Sigma$ the constant source $J$ (odd) for the BRST-variation of the apparatus action; finally, $\beta$ and $\bar{\beta}$ are the ordinary sources for the Faddeev-Popov ghost–anti-ghost fields, $C$ and $\bar{C}$, respectively.

As was mentioned above, the gauge dependence problem is in fact the question whether a change of the gauge conditions can be unambiguously interpreted as a deformation of the reference frame. Following I, this question will be examined below in the case when the deformation is induced by a variation of the gauge parameter $\xi$. This parameter plays the role of a weight of the set $F_\alpha$ of the gauge conditions, and is a potential source of gauge ambiguity in the values of observable quantities. In classical theory, the gravitational field is $\xi$-independent. At the $h^0$-order of quantum theory, variations of this parameter induce spacetime diffeomorphisms, thus preserving the $\xi$-independence of observables.

To see whether this is so at the first order in $\hbar$, we have to calculate, according to Eq. (2), the partial derivative $\partial W_m / \partial \xi$. This quantity can be conveniently found using the Slavnov identity for the functional $W$, modified by adding a source term $Y \int d^4 x \, C_\alpha F_\alpha$, $Y$ being a new constant odd parameter. Denoting the modified generating functional of connected Green functions by $\mathcal{W}$, we thus have

\[
e^{\mathcal{W}} = \int dh dC d\bar{C} \exp \{ i(\Sigma + Y \bar{C}^\alpha F_\alpha + \bar{\beta}^\alpha C_\alpha + \bar{\beta}^\alpha C_\alpha + T^{\mu\nu} h_{\mu\nu}) \}.
\]

Except for the last term, the functional $\Sigma$ is invariant under the following BRST transformation

\[
\delta h_{\mu\nu} = D_{\mu\nu}^\alpha C_\alpha \xi, \\
\delta C_\gamma = -\frac{1}{2} f^{\alpha\beta} C_\alpha C_\beta \xi, \\
\delta \bar{C}^\alpha = \frac{1}{\xi} F^\alpha, \\
\delta x^\mu = \bar{D}_{\mu} C^\alpha \xi,
\]

where $\lambda$ is a constant (odd) parameter. On the other hand, the $J$-term is invariant under the “quantum” BRST-transformation described by Eqs. (11–13). Taking into account these facts, it is not difficult to derive the following Slavnov identity for the functional $\mathcal{W}$ (see Appendix of I)

\[
T^{\mu\nu} \delta \mathcal{W} \delta K_{\mu\nu} - \bar{\beta}^\gamma \delta \mathcal{W} \delta L^\alpha - \frac{1}{2} \beta^\gamma C^{\alpha,\mu\nu} \frac{\delta \mathcal{W}}{\partial T^{\mu\nu}} + \frac{\delta \mathcal{W}}{\partial J} - Y \beta_\alpha \frac{\delta \mathcal{W}}{\partial \beta_\alpha} - 2Y \xi \frac{\partial \mathcal{W}}{\partial \xi} = 0.
\]

Setting $L = \beta = \bar{\beta} = 0$ in Eq. (15), and collecting terms proportional to $Y$ yields

\[
2\xi \frac{\partial W}{\partial \xi} = -T^{\mu\nu} \frac{\delta W}{\delta K_{\mu\nu}} - \frac{\partial W}{\partial J},
\]

where $W, \bar{W}$ are defined by

\[
\mathcal{W} = W + Y \bar{W},
\]

and the sources $K^{\mu\nu}, J$ are also set equal to zero after differentiation. Finally, extracting the device contribution in Eq. (16), we obtain the Slavnov identity we are looking for

\[
2\xi \frac{\partial W_m}{\partial \xi} = -T^{\mu\nu} \frac{\delta W_m}{\delta K_{\mu\nu}} - \frac{\partial W_m}{\partial J}.
\]

---

2 To avoid appearance of the second derivatives of $x^\mu(\tau)$ in the effective apparatus action, we have written this variation with respect to $h_{\mu\nu}$, rather than $x^\mu$ (as in I). In view of the gauge invariance of the action $S_m$, this amounts simply to the change $J \rightarrow -J$. Accordingly, this change is made in the Slavnov identity.
Let us proceed to evaluation of the right hand side of Eq. \(17\). In the linear approximation with respect to the external field \(T^{\mu\nu}\), the one-loop contributions of the first and second term are given by the diagrams pictured in Figs. 4 and 2, respectively. Some of these diagrams are zeros identically. Indeed, those pictured collectively in Fig. 4(c) vanish in view of the conservation law, Eq. \(11\). Furthermore, the loop in the diagrams 2(e) and 2(g) has zero external momentum flow. Since this loop involves only massless propagators, its dimensionally regularized value is zero [in the diagram of Fig. 4(e), there is also the ghost propagator at zero momentum attached to the loop, leading to a 0/0-type indefiniteness. However, it can be easily resolved to give zero (see I for more detail)].

Calculation of the remaining diagrams can be simplified using the ordinary Slavnov identities as follows. Neglecting device contribution in Eq. \(13\), differentiating it twice with respect to \(\beta_\alpha\) and \(T^{\mu\nu}\), and setting all the sources equal to zero yields

\[
\frac{\delta^2 W}{\delta \beta_\alpha \delta K^{\mu\nu}} - \frac{1}{\xi} F^{\alpha,\lambda \sigma} \frac{\delta^2 W}{\delta T^{\sigma\lambda \delta T^{\mu\nu}}} = 0. \tag{18}
\]

At the tree level, it reads

\[
\frac{1}{\xi} F^{\alpha,\mu\nu} G_{\mu\nu\sigma\lambda} = D^{(0)\alpha}_{\sigma\lambda} \tilde{G}_\beta, \tag{19}
\]

and is easily verified with the help of explicit expressions for the graviton and ghost propagators, \(G_{\mu\nu\sigma\lambda}, \tilde{G}_\beta\), given below [see Eqs. (22), (23)]. This identity allows us to substitute the graviton propagator going out of the \(Y\)-vertex by the ghost propagator. Furthermore, after having done this substitution, the triple graviton vertex appearing in the diagrams 4(c), 2(a,f) can be conveniently expressed through the second variation of the gravity action with the help of the identity

\[
\frac{\delta^3 S}{\delta h_{\mu\nu} \delta h_{\sigma\lambda} \delta h_{\rho\tau}} \bigg|_{h=0} \frac{\delta^2 S}{\delta h_{\mu\nu} \delta h_{\rho\tau}} \bigg|_{h=0} \frac{\delta D^{\alpha}_{\mu\nu}}{\delta h_{\sigma\lambda}} \bigg|_{h=0} \frac{\delta^2 S}{\delta h_{\mu\nu} \delta h_{\sigma\lambda}} \frac{\delta D\alpha_{\mu\nu}}{\delta h_{\rho\tau}} = 0, \tag{20}
\]

obtained by double differentiating the basic identity

\[
\frac{\delta S}{\delta h_{\mu\nu}} D^{\alpha}_{\mu\nu} = 0. \tag{21}
\]

Next, as in I, it can be shown using the one-loop identity \(18\) that the sum of diagrams 2(c),(d) is equal to the diagram 4(d), so that it is sufficient to find the contribution of the latter. Finally, applying identity \(19\) to the graviton line going out from the \(Y\)-vertex in the diagrams of Fig. 2(b), one sees that diagrams of this type do not contribute because of the energy-momentum conservation \(11\).

The building blocks of the diagrams in the momentum space are defined as follows. The second variation of the Einstein action

\[
S^{\mu\nu \sigma\lambda}(k) = \left\{ \frac{1}{4} (\eta^{\mu\sigma} \eta^{\nu\lambda} + \eta^{\mu\lambda} \eta^{\nu\sigma} - 2 \eta^{\mu\nu} \eta^{\sigma\lambda}) k^2 + \frac{1}{2} (\eta^{\sigma\lambda} \eta^{\mu\nu} k^2 + \eta^{\mu\sigma} \eta^{\nu\lambda} k^2) - \frac{1}{4} (\eta^{\sigma\lambda} k^2 k^\nu + \eta^{\mu\nu} k^\sigma k^\lambda) + \frac{1}{2} \left( \eta^{\mu\sigma} k^\lambda k^\nu + \eta^{\nu\lambda} k^\sigma k^\mu + \eta^{\sigma\lambda} k^\mu k^\nu \right) \right\}. \tag{22}
\]

Below, we choose the gauge conditions to be

\[
F_\alpha = \partial^\mu h_{\mu\alpha} - \frac{1}{2} \partial_\alpha h, \quad h \equiv \eta^{\mu\nu} h_{\mu\nu}. \tag{23}
\]

Then the graviton propagator, \(G_{\mu\nu\alpha\beta}\), and the ghost propagator, \(\tilde{G}_\alpha\), defined by

\[
(S + S_{\text{gf}})_{\mu\nu \sigma\lambda} G_{\sigma\lambda \alpha\beta} = -\delta_{\alpha\beta} \delta^{\mu\nu}; \quad \delta^{\mu\nu}_{\alpha\beta} = \frac{1}{2} (\delta^{\alpha\beta} + \delta^{\mu\nu} \delta^{\alpha\beta}),
\]

\[
F^{\alpha,\mu\nu} D^{(0)\beta}_{\mu\nu} \tilde{G}_{\beta} = -\delta_{\alpha},
\]

take the form

\[
G_{\mu\nu\alpha\lambda}(k) = - (\eta_{\mu\sigma} \eta_{\nu\lambda} + \eta_{\mu\lambda} \eta_{\nu\sigma} - \eta_{\mu\nu} \eta_{\sigma\lambda}) \frac{1}{k^2}
\]

\[
- (\xi - 1)(\eta_{\mu\sigma} k_{\nu\lambda} + \eta_{\mu\lambda} k_{\nu\sigma} + \eta_{\nu\sigma} k_{\mu\lambda} + \eta_{\nu\lambda} k_{\mu\sigma}) \frac{1}{k^4}, \tag{22}
\]
and
\[ G^\alpha_\beta = \frac{\delta^\alpha_\beta}{k^2}, \tag{23} \]
respectively. Finally, the graviton-apparatus vertices are generated by expanding the apparatus action in powers of \( h_{\mu\nu} \)
\[
S_m = -m \int d\tau \left[ 1 + \frac{1}{2} h_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \frac{1}{8} h_{\mu\nu} h_{\alpha\beta} \dot{x}^\mu \dot{x}^\alpha \dot{x}^\beta + \cdots \right]
\equiv -m \int d\tau + \int \text{d}^4 y h_{\mu\nu}(y) T^\mu_\nu(y) + \frac{1}{2} \int d^4 y h_{\mu\nu}(y) h_{\alpha\beta}(y) Q^\mu_\alpha^\nu_\beta(y) + \cdots, \tag{24}
\]
where
\[
T^\mu_\nu(y) = -\frac{m}{2} \int d\tau \dot{x}^\mu \dot{x}^\nu \delta^4(y - x(\tau)),
\]
\[
Q^\mu_\alpha^\nu_\beta(y) = \frac{m}{4} \int d\tau \dot{x}^\mu \dot{x}^\nu \dot{x}^\alpha \dot{x}^\beta \delta^4(y - x(\tau)), \quad \dot{x}^\mu \equiv \frac{dx^\mu(\tau)}{d\tau}.
\]
To avoid appearance of the second derivatives of \( x^\mu(\tau) \) in the effective device action, an integration by parts is to be performed in the diagrams (a)–(f). After all the above transformations, the analytical expressions of the diagrams to be calculated take the form:
\[
\mathcal{J}_j = \int \frac{d^4 p}{(2\pi)^4} I_j(p),
\]
\[
I_{1(a)}(p) = i \mu^\epsilon \int \frac{d^4 k}{(2\pi)^4} T^\mu_\nu(p) Q^\tau_\rho^\sigma_\lambda(p) G^\rho_\sigma_\lambda(k) \tilde{G}^\beta_\gamma_\delta(p + k) \times \left\{ k_\alpha \delta^\sigma_\rho - \delta^\sigma_\rho (p_\alpha + k_\alpha) - \delta^\sigma_\rho (p_\nu + k_\nu) \right\} \{ \eta_{\sigma\gamma}(p_\lambda + k_\lambda) + \eta_{\lambda\gamma}(p_\sigma + k_\sigma) \}, \tag{25}
\]
\[
I_{1(b)}(p) = -i \mu^\epsilon \int \frac{d^4 k}{(2\pi)^4} T^\mu_\nu(p) T^\tau_\rho_\sigma_\lambda(p) G^\rho_\sigma_\lambda(k) \tilde{G}^\beta_\gamma_\delta(p + k) \times \left\{ (p^2 + k^2) \eta^\tau_\rho - (p^\tau + k^\tau) \eta_\rho^\tau - (p_\rho + k_\rho) \eta^\rho_\tau \right\} \{ \eta_{\sigma\gamma}(p_\lambda + k_\lambda) + \eta_{\lambda\gamma}(p_\sigma + k_\sigma) \}, \tag{26}
\]
\[
I_{1(c)}(p) = -i \mu^\epsilon \int \frac{d^4 k}{(2\pi)^4} T^\mu_\nu(p) T^\tau_\rho_\omega_\sigma_\lambda(p) G^\rho_\sigma_\lambda(k) \tilde{G}^\beta_\gamma_\delta(p + k) \times \left( S^\sigma_\omega \tau_\rho(p) \left\{ -k_\gamma \delta_{\sigma\lambda} + \delta_{\gamma\rho} (p_\lambda + k_\lambda) + \delta_{\lambda\rho} (p_\sigma + k_\sigma) \right\} + S^\sigma_\omega \chi_\rho(k) \left\{ -p_\sigma \delta_{\tau\omega} + \delta_\tau_\rho (p_\lambda + k_\lambda) + \delta_{\lambda\rho} (p_\sigma + k_\sigma) \right\} \right) \times G_{\chi_\sigma_\rho_\omega}(p) \{ \eta_{\sigma\gamma}(p_\lambda + k_\lambda) + \eta_{\lambda\gamma}(p_\sigma + k_\sigma) \} \}, \tag{27}
\]
\[
I_{1(d)}(p) = -i \mu^\epsilon \int \frac{d^4 k}{(2\pi)^4} T^\mu_\nu(p) T^\tau_\rho_\omega_\sigma_\lambda(p) G^\rho_\sigma_\lambda(k) \tilde{G}^\beta_\gamma_\delta(p) \times \left\{ k_\alpha \delta^\sigma_\rho - \delta^\sigma_\rho (p_\alpha + k_\alpha) - \delta^\sigma_\rho (p_\nu + k_\nu) \right\} \left\{ -k_\gamma \delta_{\tau\omega} + \delta_{\tau\rho} (p_\lambda + k_\lambda) + \delta_{\lambda\rho} (p_\sigma + k_\sigma) \right\} \times \left\{ (p^2 + k^2) \eta^\tau_\rho - (p^\tau + k^\tau) \eta_\rho^\tau - (p_\rho + k_\rho) \eta^\rho_\tau \right\} \{ \eta_{\sigma\gamma}(p_\lambda + k_\lambda) + \eta_{\lambda\gamma}(p_\sigma + k_\sigma) \}, \tag{28}
\]
\[
I_{2(a)}(p) = -\mu^2 \int \frac{d^{d-\varepsilon}k}{(2\pi)^d} T_{m}^{\xi\eta}(p) \mathcal{G}_{\alpha\beta}(p) \tilde{G}_{\alpha}^\beta(p + k) \xi \tilde{G}_{\gamma}^\gamma(p + k) \times \left( S^{\sigma\lambda} \tau^\mu \left\{ -\kappa_\lambda \delta^{\theta}_\sigma \delta^{\eta}_\theta + \delta^{\chi}_\lambda (p_\lambda + k_\lambda) + \delta^{\chi}_\sigma \delta^{\eta}_\gamma (p_\sigma + k_\sigma) \right\} \right) \\
+ S^{\sigma\lambda} \times \chi^\theta \left( -p_\gamma \delta^{\tau^\rho}_\sigma \delta^{\eta^\rho}_\gamma (p_\lambda + k_\lambda) + \delta^{\tau^\rho}_\gamma (p_\sigma + k_\sigma) \right) \right) \times h_{\tau\rho}(p) \mathcal{G}_{\chi\varphi}(k) \left\{ p^\mu \eta^{\mu\nu} - p^\mu \eta^{\nu\mu} - p^\nu \eta^{\mu\mu} \right\} \times \left\{ k_\alpha \delta^{\rho\nu}_\mu - \delta^{\rho\nu}_\mu (p_\nu + k_\nu) - \delta^{\rho\mu}_\nu (p_\mu + k_\mu) \right\},
\]
\[
I_{2(b)}(p) = -\mu^2 \int \frac{d^{d-\varepsilon}k}{(2\pi)^d} T_{m}^{\xi\eta}(p) \mathcal{G}_{\alpha\beta}(p) \tilde{G}_{\beta}^\alpha(p + k) \tilde{G}_{\alpha}^\beta(k) \tilde{G}_{\mu\rho}(p) \times \left\{ \frac{1}{2} \left\{ (p^\beta + k^\beta) \eta^{\tau^\rho} - (p^\tau + k^\tau) \eta^{\rho^\tau} - (p^\rho + k^\rho) \eta^{\beta^\tau} \right\} \right\} \times \left\{ -p_\gamma h_{\tau\rho}(p) - h_{\gamma\rho}(p) k_\tau \right\} \left\{ p^\mu \eta^{\mu\nu} - p^\mu \eta^{\nu\mu} - p^\nu \eta^{\mu\mu} \right\} \times \left\{ k_\alpha \delta^{\rho\nu}_\mu - \delta^{\rho\nu}_\mu (p_\nu + k_\nu) - \delta^{\rho\mu}_\nu (p_\mu + k_\mu) \right\},
\]
\[
I_{2(c)}(p) = -\mu^2 \int \frac{d^{d-\varepsilon}k}{(2\pi)^d} \tilde{G}_{\alpha}^\beta(k) \tilde{G}_{\alpha}^\beta(p + k) \tilde{G}_{\beta}^\alpha(k) \tilde{G}_{\mu\rho}(p) \times \left\{ \frac{1}{2} \left\{ (p^\beta + k^\beta) \eta^{\tau^\rho} - (p^\tau + k^\tau) \eta^{\rho^\tau} - (p^\rho + k^\rho) \eta^{\beta^\tau} \right\} \right\} \times \left\{ -p_\gamma h_{\tau\rho}(p) - h_{\gamma\rho}(p) k_\tau \right\} \left\{ p^\mu \eta^{\mu\nu} - p^\mu \eta^{\nu\mu} - p^\nu \eta^{\mu\mu} \right\} \times \left\{ k_\alpha \delta^{\rho\nu}_\mu - \delta^{\rho\nu}_\mu (p_\nu + k_\nu) - \delta^{\rho\mu}_\nu (p_\mu + k_\mu) \right\},
\]
where \( h_{\mu\nu}(p) \) stands for the tree value of the gravitational field produced by \( T^{\mu\nu} \) (in the linear approximation),

\[
h_{\mu\nu}(p) = G_{\mu\nu,\alpha\beta}(p) T^{\alpha\beta}(p) ,
\]
\( \mu \) – arbitrary mass scale, and \( \varepsilon = 4 - d \), \( d \) being the dimensionality of spacetime.

As in Refs. 5 and I, we restrict ourselves to the calculation of the leading quantum corrections in the low-energy limit. To this end, it is sufficient to find the ultraviolet divergences of the above Feynman integrals. The logarithmic part of the integrals, representing the leading contribution in the present case, can then be obtained by substituting

\[
\frac{1}{\varepsilon} \rightarrow \frac{1}{2} \ln \left( \frac{-p^2}{\mu^2} \right).
\]

To find the ultraviolet divergences, one has to expand the integrands in powers of the momentum transfer, \( p_\mu \), and to collect terms proportional to \( k^{-4} \). Obviously, expansion of the ghost propagators \( \tilde{G}_{\alpha}^\beta(p + k) \) is needed only. Since the degree of divergence of the above integrals is \( \leq 3 \), it is sufficient to write

\[
\tilde{G}_{\alpha}^\beta(p + k) = \frac{\delta_{\alpha}^\beta}{k^2} \left( 1 - \frac{2(pk)}{k^2} - \frac{p^2}{k^2} + \frac{4(pk)^2}{k^4} + \frac{4(pk)p^2}{k^4} - \frac{8(pk)^3}{k^6} + O \left( \frac{p^4}{k^4} \right) \right).
\]

The tensor multiplication as well as integration over angles in the \( k \)-space can be easily performed using the tensor package 12 for the REDUCE system. Thus, we find

\[
I_{1,\text{log}}(p) = \frac{m_\xi}{16\pi^2} \ln \left( \frac{-p^2}{\mu^2} \right) \int d\tau \exp(i p_\mu x_\mu(\tau)) J_3(\tau, p),
\]

(34)
\begin{align*}
J_{1(a)}(\tau, p) &= \frac{1}{24} \left\{ (2\xi + 1)T(p) + 4(\xi - 1)\dot{x}^\mu \dot{x}^\nu T_{\mu\nu}(p) \right\}, \\
J_{1(b)}(\tau, p) &= \frac{1}{3} \left\{ T(p) - (\xi - 2)\frac{(\dot{x}^\mu p_\mu)^2}{p^2}T(p) \right\}, \\
J_{1(c)}(\tau, p) &= -\frac{1}{6} \left\{ (2\xi - 1)T(p) + 10\xi \frac{(\dot{x}^\mu p_\mu)^2}{p^2}T(p) + 2(2\xi + 1)\dot{x}^\mu \dot{x}^\nu T_{\mu\nu}(p) \right\}, \\
J_{1(d)}(\tau, p) &= -\frac{(\xi - 1)(\dot{x}^\mu p_\mu)^2}{3p^2}T(p), \\
J_{2(a)}(\tau, p) &= -\frac{2(2\xi + 1)(\dot{x}^\mu p_\mu)^2}{3p^2}T(p), \\
J_{2(b)}(\tau, p) &= \frac{(\dot{x}^\mu p_\mu)^2}{2p^2}T(p), \\
J_{2(c)}(\tau, p) &= \frac{1}{12} \left\{ 2T(p) + 4\dot{x}^\mu \dot{x}^\nu T_{\mu\nu}(p) + \frac{(\dot{x}^\mu p_\mu)^2}{p^2}T(p) + 2\frac{(\dot{x}^\mu p_\mu)^2}{p^2}\dot{x}^\mu \dot{x}^\nu T_{\mu\nu}(p) \right\}, \\
J_{2(d)}(\tau, p) &= -\frac{1}{24} \left\{ (6\xi - 1)T(p) - 14\frac{(\dot{x}^\mu p_\mu)^2}{p^2}T(p) + 4 \left[ (3\xi + 1) - \frac{(\dot{x}^\mu p_\mu)^2}{p^2} \right] \dot{x}^\mu \dot{x}^\nu T_{\mu\nu}(p) \right\},
\end{align*}

where \( T \equiv T_{\mu\nu}\eta_{\mu\nu} \). When we used the identity \( \eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = 1 \), and changed \( p_\mu \rightarrow -p_\mu \) in the expressions for \( J_{1(a-f)} \). Doubling the contribution of the diagram \( \Pi^{(d)} \) and summing up, we finally obtain the following expression for the full \( \xi \)-derivative of the effective device action

\[
\frac{dT^{\text{loop}}}{d\xi} = \frac{m}{32\pi^2} \int d\tau \int \frac{d^3 p}{(2\pi)^3} \exp\{ip_\mu x^\mu(\tau)\} \ln \left( -\frac{p^2}{\mu^2} \right) \left\{ \frac{2\xi - 3}{4}T(p) + \frac{3\xi + 1}{3} \dot{x}^\mu \dot{x}^\nu T_{\mu\nu}(p) + \frac{24\xi - 11}{6} \frac{(\dot{x}^\mu p_\mu)^2}{p^2}T(p) - \frac{1}{3} \frac{(\dot{x}^\mu p_\mu)^2}{p^2} \dot{x}^\mu \dot{x}^\nu T_{\mu\nu}(p) \right\}. \tag{35}
\]

Comparison of the right hand side of this equation with the linear term in the expansion of the apparatus action, Eq. \( \Pi^{(d)} \), shows that the \( \xi \)-dependent part of the effective gravitational field cannot be represented in the form \( D_{\mu\nu}^{(0)} \Xi_\alpha \) with some \( \Xi_\alpha \). That implies an ambiguity in the values of physical quantities is most clearly seen considering the particular case of a static source and non-relativistic testing particle. In this case, one has \( T_{\mu\nu}(p) = 2\pi \delta(p^0)\Theta_{\mu\nu}(p), \dot{x}^\mu \approx \delta_0^\mu, \dot{x}^\mu p_\mu \approx 0 \). Using the formula

\[
\int \frac{d^3 p}{(2\pi)^3} \exp[ipr] \ln p^2 = -\frac{1}{2\pi r^3}, \quad r \equiv |r|,
\]

and restoring the ordinary units yields

\[
\frac{dT^{\text{loop}}}{d\xi} = -\frac{G^2 a}{\pi c^5} \int dt \int d^3 r \frac{m \Xi_{\mu\nu}(r)}{|r - x(\tau)|^3} \left\{ (2\xi - 3)\eta_{\mu\nu} + 4 \left( \xi + \frac{1}{3} \right) \delta_0^\mu \delta_0^\nu \right\}, \tag{36}
\]

\[ \Xi_{\mu\nu}(r) = \int \frac{d^3 p}{(2\pi)^3} \Theta_{\mu\nu}(p) \exp(-ipr). \]

It follows from Eq. \( \Pi^{(d)} \) that the value of the static gravitational potential, \( V \), measured by observing the motion of a non-relativistic point particle, is \( \xi \)-dependent:

\[
\frac{dV}{d\xi} = \frac{G^2 a}{\pi c^5} \int d^3 r \frac{\Xi_{\mu\nu}(r)}{|r - x(\tau)|^3} \left\{ (2\xi - 3)\eta_{\mu\nu} + 4 \left( \xi + \frac{1}{3} \right) \delta_0^\mu \delta_0^\nu \right\}.
\]

Thus, as in the case of scalar field, an explicit introduction of the classical measuring apparatus does not remove the gauge ambiguity of the effective gravitational potential. One might think, however, that this is because of the artificial form of the source for the gravitational field. The point is that this source is not invariant with respect to spacetime diffeomorphisms. As we mentioned in the Introduction, the reason for the gauge dependence cancellation, if any, is expected to have its roots in the gauge invariance of the initial classical theory. In the next section, therefore, we turn to the investigation of a realistic source nonlinear in the gravitational field.
III. EFFECTIVE GRAVITATIONAL FIELD OF A POINT-LIKE PARTICLE

In this section, we will consider the special case when the gravitational field is produced by a point-like classical particle with mass $M$. This is the setting of Ref. [5], except that we use the ordinary gauge conditions instead of the background ones. The method used in the preceding section does not apply to this case because of the nonlinearity of the source. However, it can be readily extended to take into account this nonlinearity as follows.

Let us introduce, analogously to the source $J$, a constant source $\beta$ (odd) for the BRST-variation of the source particle action

$$\beta \int d^4x \frac{\delta S_M}{\delta h_{\mu\nu}} D^\alpha_{\mu\nu} C^\alpha,$$

where $S_M$ is the source particle action

$$S_m = -M \int \sqrt{g} \frac{\partial^2}{\partial \beta \partial \gamma} \frac{d^4 \beta}{d^4 \gamma} = -M \int d\theta \sqrt{1 + h_{\mu\nu} \frac{d^2 \rho}{d\theta}} \rho, \quad d\theta^2 = \eta_{\mu\nu} dz^\mu dz^\nu. \quad (37)$$

Expanding this action in powers of $h_{\mu\nu}$ generates the vertices of the source-graviton interaction:

$$S_M = -M \int d\theta \left[ 1 + \frac{1}{2} h_{\mu\nu} \frac{d^2}{d\theta} \frac{d^2}{d\theta} - \frac{1}{8} h_{\alpha\beta} \frac{d^2}{d\theta} \frac{d^2}{d\theta} + \cdots \right]$$

$$\equiv -M \int d\theta + \int d^4 y h_{\mu\nu}(y) T^{\mu\nu}_M(y) + \frac{1}{2} \int d^4 y h_{\mu\nu}(y) h_{\alpha\beta}(y) Q_M^{\mu\nu\alpha\beta}(y) + \cdots, \quad (38)$$

where

$$T^{\mu\nu}_M(y) = -\frac{M}{2} \int d\theta \frac{d^2}{d\theta} \frac{d^2}{d\theta} \delta^4(y - z(\theta)), \quad Q_M^{\mu\nu\alpha\beta}(y) = \frac{M}{4} \int d\theta \frac{d^2}{d\theta} \frac{d^2}{d\theta} \delta^4(y - z(\theta)), \quad \hat{\mu} \equiv \frac{d^2}{d\theta} \frac{d^2}{d\theta}.$$

The $\Sigma$-functional entering the generating functional of Green functions [10] thus takes the form

$$\Sigma = S_{FP} + \int d^4x \ K^{\mu\nu} D^\alpha_{\mu\nu} C^\alpha - \int d^4x \ \frac{L^4}{2} f_{\beta\gamma} \partial^\alpha \partial^\gamma C^\alpha + J \int d^4x \frac{\delta S_m}{\delta h_{\mu\nu}} D^\alpha_{\mu\nu} C^\alpha + \beta \int d^4x \frac{\delta S_M}{\delta h_{\mu\nu}} D^\alpha_{\mu\nu} C^\alpha. \quad (39)$$

In this new formulation, the source $T^{\mu\nu}$ entering the generating functional is an auxiliary quantity set equal to zero (together with all other sources) in the end of calculations, and no longer plays the role of the source of the gravitational field being measured.

To derive the Slavnov identities for the functional $\Sigma$, we perform the “quantum” BRST change of integration variables in the functional integral [10], and obtain the following Slavnov identity

$$\int d\theta dC d\hat{C} \bar{C} \frac{\delta S_m}{\delta h_{\mu\nu}} D^\alpha_{\mu\nu} C^\alpha + \frac{\delta S_M}{\delta h_{\mu\nu}} D^\alpha_{\mu\nu} C^\alpha + Y \bar{C}^\alpha F^{\mu\nu}_{\alpha\beta} D^\beta_{\mu\nu} C^\alpha + \frac{F^2}{\xi} F_{\alpha\beta} + T^{\mu\nu} D^\alpha_{\mu\nu} C^\alpha$$

$$- \frac{\bar{\beta}^{\alpha\beta}}{2} f_{\alpha\gamma} C^\beta_{\gamma} C^\alpha - \beta^{\alpha\beta} F^{\alpha\beta} \xi \exp \{ i (\Sigma + Y F_{\alpha\beta} \bar{C}^\alpha + \bar{\beta}^{\alpha\beta} C^\alpha + \bar{C}^\alpha \beta^\alpha + T^{\mu\nu} h_{\mu\nu} ) \} = 0. \quad (40)$$

The first two terms in the square brackets will be replaced by the derivatives with respect to the sources $J$ and $\beta$, respectively. The third term can be transformed using the ghost equation of motion

$$\int d\theta dC d\hat{C} \left[ F_{\gamma}^{\mu\nu} D^\alpha_{\mu\nu} C^\alpha - Y F_{\gamma} + \beta_\gamma \right]$$

$$\times \exp \{ i (\Sigma + Y F_{\alpha\beta} \bar{C}^\alpha + \bar{\beta}^{\alpha\beta} C^\alpha + \bar{C}^\alpha \beta^\alpha + T^{\mu\nu} h_{\mu\nu} ) \} = 0,$$

obtained by performing a shift $\hat{C} \rightarrow \hat{C} + \delta \hat{C}$ of integration variables in the functional integral [10]. Differentiating this equation with respect to $\beta_\gamma$ gives

$$Y \int d\theta dC d\hat{C} \left[ i \bar{C}^{\gamma} F_{\gamma}^{\mu\nu} D^\alpha_{\mu\nu} C^\alpha + \beta_\gamma \frac{\delta}{\delta \beta_\gamma} \right] \exp \{ \cdots \} = 0,$$
where the use of the property \( Y^2 = 0 \) has been made, and the expression \( \delta \beta_\gamma / \delta \beta_\alpha \sim \delta^4(0) \), equal to zero in the dimensional regularization, is omitted. Expressing the remaining terms as derivatives with respect to the BRST-sources, and introducing the generating functional of connected Green functions yields

\[
T^{\mu\nu} \frac{\delta \mathcal{W}}{\delta K^{\mu\nu}} - \beta_\alpha \frac{\delta \mathcal{W}}{\delta \Lambda^\alpha} - \frac{1}{\xi} \frac{\delta \mathcal{W}}{\delta T^{\alpha\mu\nu}} \frac{\partial}{\partial \delta J} + \frac{\partial}{\partial \delta J} - Y \beta_\alpha \frac{\delta \mathcal{W}}{\delta \beta_\alpha} - 2 Y \xi \frac{\partial \mathcal{W}}{\partial \xi} = 0. \tag{41}
\]

Doing as in the preceding section, we obtain from Eq. (41) the following equation for the full \( \xi \)-derivative of the effective action:

\[
2 \xi \frac{d \Gamma_m}{d \xi} = - \frac{\partial \mathcal{W}_m}{\partial \delta J} - \frac{\partial \mathcal{W}_m}{\partial \delta J}. \tag{42}
\]

Evaluation of the right hand side of Eq. (42) proceeds in the same way as in the case of a linear source, except that there appear additional diagrams containing the \( Q_M \)-vertex. Namely, contribution of the second term is represented by the diagrams of Fig. 2 with the substitution \( T^{\mu\nu} \rightarrow T^{\mu\nu}_M \), and three new diagrams pictured in Fig. 3. As before, it is not difficult to show that the sum of diagrams \( 2(c,d) \) and \( 3(c) \) is equal to the diagram \( 1(d) \) in which the \( K \)-vertex is substituted by the \( \beta \)-vertex. Indeed, neglecting device contribution, differentiating Eq. (41) with respect to \( \beta_\alpha \), and setting all the sources equal to zero gives

\[
\frac{1}{\xi} \frac{\delta \mathcal{W}}{\delta T^{\alpha\mu\nu}} = \frac{\delta \mathcal{W}}{\delta \beta_\alpha} . \tag{43}
\]

In the linear approximation with respect to the source \( M \), the left hand side of this identity is represented by the sum of the vertical parts of diagrams \( 2(c,d), 3(c) \), while the right hand side – by the horizontal part of \( 1(d) \) (with \( K \rightarrow \beta \)). It remains only to apply Eq. (19) to the vertical part of the latter diagram.

The analytical expressions of the other two diagrams in Fig. 3 have the form

\[
\mathcal{J}_j = \int \frac{d^4 p}{(2\pi)^4} J_j(p) ,
\]

where

\[
I_{3(a)}(p) = - i \mu^* \int \frac{d^{4-\varepsilon} k}{(2\pi)^4} T^{\gamma\mu\nu}_M(p) p_\nu Q_M^{\gamma\mu\nu}(p) G_T^{\gamma\rho \theta}(k) \tilde{G}^{\gamma\rho \theta}_M(k) Q_M^{\gamma\rho \theta}(k) \xi \tilde{G}^{\gamma\beta\gamma}(p + k) \times \{ \eta_{\gamma \eta} (p_\Lambda + k_\Lambda) + \eta_{\gamma \Lambda} (p_\sigma + k_\sigma) \} \{ \eta^{\Lambda \beta} - p^{\Lambda \beta} - p^{\gamma \beta} \} \times \{ \eta_{\alpha \beta} \delta^{\gamma \rho} - \delta^{\gamma \rho} (p_\rho + k_\rho) - \delta^{\gamma \rho} (p_\mu + k_\mu) \} ,
\]

\[
I_{3(b)}(p) = i \mu^* \int \frac{d^{4-\varepsilon} k}{(2\pi)^4} (p) Q_M^{\gamma\beta\rho\gamma}(p) \tilde{G}^{\beta\rho\gamma}_M(p + k) \xi \tilde{G}^{\beta\rho\gamma}(p + k) \times G^{\gamma\rho \theta \omega}(k) \{ \eta_{\gamma \eta} (p_\Lambda + k_\Lambda) + \eta_{\gamma \Lambda} (p_\sigma + k_\sigma) \} \times \{ T^{\mu\nu}_M(p) [ k_\rho \delta^\gamma_{\mu \rho} - \delta^\gamma_{\mu \rho} (p_\mu + k_\mu) - \delta^\gamma_{\mu \rho} (p_\mu + k_\mu) ] - Q^{\gamma\beta\rho\gamma}_M(p) [ \eta_{\mu \alpha} (p_\rho + k_\rho) + \eta_{\mu \alpha} (p_\mu + k_\mu) ] \} .
\]

As to the first term in the right hand side of Eq. (42), its contribution is given by Eqs. (25) – (28) with the substitution \( T^{\mu\nu} \rightarrow T^{\mu\nu}_M \), plus the same expressions in which the factor \( T^{\mu\nu}_M D^{\alpha\beta\gamma}_{\mu \nu} \) is changed to \( Q^{\mu\nu\alpha\beta\gamma}_M D^{(0)}_{\mu \nu} \). Distinguishing these additional contributions by a prime, and doing the loop integrals as in the preceding section we find

\[
I_1^{\log}(p) = \frac{m M}{16 \pi^2} \frac{1}{\mu^2} \left( \frac{- \mu^2}{\mu^2} \right) \int d\theta \int d\tau \exp \{ i p_\mu [ x^\mu (\tau) - z^\mu (\theta) ] \} J_j(\tau, \theta, p) , \tag{44}
\]

where

\[
J'_{1(a)}(\tau, \theta, p) = \frac{1}{48} \left\{ (2\xi - 5) (\dot{x}^\mu \dot{z}_\mu)^2 + 2 (2\xi + 1) (\dot{x}^\mu \dot{z}_\mu)^4 \right\} ,
\]

\[
J'_{1(b)}(\tau, \theta, p) = \frac{1}{12} \left\{ -2 (\dot{x}^\mu \dot{z}_\mu)^2 + 1 \right\} ,
\]

\[
J'_{1(c)}(\tau, \theta, p) = \frac{1}{48} \left\{ (2\xi - 3) + 2 (4\xi - 1) (\dot{x}^\mu p_\mu)^2 + 4 (\xi + 1) (\dot{x}^\mu \dot{z}_\mu)^2 \right\} ,
\]

\[
J'_{1(d)}(\tau, \theta, p) = 0 .
\]
\[ I_{(a)}(\tau, \theta, \mu) = \frac{-4\xi + 1}{24} (\dot{x}^\mu p_\mu)^2, \]
\[ I_{(b)}(\tau, \theta, \mu) = \frac{2\xi + 1}{48} \{2(\dot{x}^\mu \dot{z}_\mu)^4 - (\dot{x}^\mu \dot{z}_\mu)^2 - 1\}, \]

and we have taken into account that \( \eta_{\mu\nu} \dot{z}^\nu = 1, \dot{z}^\nu = 0 \). Adding up all the contributions, we finally arrive at the following expression for the \( \xi \)-derivative of the one-loop correction to the effective action of a point testing particle in the gravitational field of a point classical mass

\[
\frac{d\Gamma_{\text{loop}}^{(m)}}{d\xi} = \frac{mM}{32\pi^2} \int d\tau \int d\theta \int \frac{d^4 p}{(2\pi)^4} \exp\{ip_\mu[x^\mu(\tau) - z^\mu(\theta)]\} \ln \left( \frac{-p^2}{\mu^2} \right) \left\{ \left( \frac{\xi}{6} - \frac{1}{4} \right) \right\}. 
\]

In the case when the particle producing gravitational field is at rest, and the testing particle is non-relativistic, one has \( z^\mu(\theta) = (\theta, z_0) \), \( \dot{x}^\mu \dot{z}_\mu \approx 1 \), and an elementary integration gives

\[
\frac{d\Gamma_{\text{loop}}^{(m)}}{d\xi} = m M \frac{3(2\xi - 1)}{256\pi^2} \int dt \int \frac{d^3 p}{(2\pi)^3} \exp\{-ip[\mathbf{x}(t) - z_0]\} \ln \left( \frac{p^2}{\mu^2} \right) 
\]

\[
= -m M \frac{3(2\xi - 1)}{512\pi^3} \int \frac{dt}{|\mathbf{x}(t) - z_0|^3}. 
\]

As in the case of a linear source, the value of the static gravitational potential turns out to be gauge-dependent. In the ordinary units,

\[
\frac{dV}{d\xi} = \frac{3(2\xi - 1) G^2 \hbar M}{2\pi \, c^3 \rho^3}, \quad r = |\mathbf{x} - z_0|. 
\]

This completes exposition of the main results of the work.

**IV. DISCUSSION AND CONCLUSIONS**

We have shown that, as in the case of scalar field considered in I, the \( O(\hbar) \) contribution to the effective action of a point-like particle is gauge dependent, and that this dependence leads to an ambiguity in the values of observables. This was demonstrated in the case of an arbitrary linear source in Sec. II, as well as in the particular case of a nonlinear point-like source in Sec. III. In our opinion, these results are sufficient to conclude that the explicit introduction of the classical measuring apparatus into consideration does not solve the gauge dependence problem. Still, some remarks concerning the whole approach are worth to be made. The method used in this paper allows one to elucidate the relative role of the two factors, explicit and implicit, contributing to the gauge dependent part of the effective device action. Let us consider equation (17) [or Eq. (42) in the case of a nonlinear source]. Clearly, the first term in the right hand side of this equation describes gauge dependence of the mean gravitational field (taking into account contribution of the nonlinear graviton-device interaction), i.e., it represents an implicit contribution to \( \partial \Gamma_n/\partial \xi \), while the second term corresponds to an explicit gauge dependence of \( \Gamma_n \). The general analysis performed in Secs. III III reveals the fact that the two contributions tend to add up, rather than subtract, undermining thereby the very idea of the gauge dependence cancellation. Indeed, as we have shown using the Slavnov identities, contribution of the diagrams (2 c.d) [and (3 c) in the nonlinear case] doubles the contribution of diagram (1 d), rather than cancels it. However, correlation between the rest of diagrams in Figs. 1 2 is more intricate. Namely, it is not difficult to verify that the sum of these diagrams is zero identically provided that the diagrams (1 b) and (2 f) are taken with opposite signs. Analogously, additional contributions found in Sec. III cancel each other if the signs of \( J_{(b)} \) and \( J_{(a,b)} \) are reversed. At the same time, it should be noted that even if the two terms entered the right hand side of Eq. (17) with opposite signs, the action \( \Gamma_n \) would still be \( \xi \)-dependent.  

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3 This conclusion, of course, does not abolish the fact that such an introduction is necessary as far as contributions of the order \( O(\hbar) \) are considered.

4 In the case of a point-like source, considered in Sec. III \( \Gamma_m \) would be \( \xi \)-independent in the non-relativistic limit if the signs of the two terms in Eq. (17) were opposite, but this is a trivial consequence of the symmetry of these terms with respect to the change \( m \leftrightarrow M \) in this limit.
Finally, we would like to note that although the main conclusion of the present paper is the same as in I in the case of scalar field, specific form of the $\xi$-dependence of the effective gravitational field is different. Therefore, one should not exclude the possibility that the resolution of the gauge dependence problem can still be found in the context discussed above by appropriately refining the model of the measurement process.

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FIG. 1: The one-loop contribution of the first term in the right hand side of Eq. (17). Wavy lines represent gravitons, broken lines ghosts, dotted lines the source $T^{\mu\nu}$, solid lines denote the coefficient functions $T_{m}^{\mu\nu}$, $Q_{m}^{\mu\nu\alpha\beta}$ appearing in the expansion of the device action. Diagrams of the type (f) do not contribute in view of the conservation law (1), so we do not picture them in detail.
FIG. 2: The one-loop contribution of the second term in the right hand side of Eq. (17). Diagrams of the type (h) do not contribute because of the energy-momentum conservation.
FIG. 3: Diagrams accounting the nonlinearity of the graviton-source interaction, generated by the second term in the right hand side of Eq. (17).