Numerical Analysis on Target Strength of the Filled Spherical Elastic Shell

Bing Jia\textsuperscript{a}, Jihui Wang\textsuperscript{b}, Guijuan Li\textsuperscript{c}, Yunfei Chen\textsuperscript{d}

Science and Technology on Underwater Test and Control Laboratory, Dalian, Liaoning, China

\textsuperscript{a}jbobin@sina.com, \textsuperscript{b}wangjihuihc@163.com, \textsuperscript{c}18641198261@163.com, \textsuperscript{d}yunfeilut@163.com

Abstract: In this paper, the acoustic target strength of spherical shells in water with internal fillers is calculated by using the normal series expansion method, and the effects of metal and non-spherical shell materials, size factors and different fillers on the target strength of spherical shells are analyzed. The results show that for different elastic shell parameters, the influence of the acoustic characteristics of the internal filler on the target strength is quite different. For non-metallic sound transmission spherical shells filled with fluid, only when the sound velocity difference between the filler and the water medium is large, the target strength is greatly improved compared with the corresponding rigid sphere, and the shell size parameters have little effect on the target strength. However, for metal spherical shells with internal fillers, the material and size factors of the shell have a greater impact on its target strength, and the sensitivity to sound velocity mismatch inside and outside the shell is relatively small.

1. Introduction

The prediction of target intensity can be understood as solving the scattered acoustic field in three-dimensional fluid space, which satisfies the surface boundary conditions, wave equation and radiation conditions. In theory, target intensity prediction is to solve a mathematical and physical problem, which is essentially a problem of acoustic scattering. At present, the theoretical solutions of acoustic scattering problems mainly include integral equation method, Rayleigh normal series solution caused by separation of variables method, Sommerfeld-Watson integral transformation method\cite{1}, resonance scattering theory and singular point expansion theory.

Rayleigh was the first scholar to study the scattering of a sphere. He proposed an approximate solution to the scattering sound field when the wavelength is much larger than the diameter of a sphere\cite{2}. Junger first used the thin shell theory to study the acoustic scattering of air-filled elastic shells\cite{3}. Goodman and Stern have studied the plane wave acoustic scattering of submerged spherical shells with precise elastodynamics theory, and derived Rayleigh normal series solution of scattering sound field of spherical shells filled with fluid in the same fluid\cite{4}. J.A. Fawcett gave the solution of acoustic field of elastic shell partially filled with fluid\cite{5}. Gregory Kaduchak and Charles M. Loeffler studied the relationship between material parameters and target strength of liquid filled spherical shells in water, and gave some test results \cite{6}. In this paper, the separation variable method is used to calculate the target strength of spherical shells in internal and external fluid media, and the effects of the acoustic characteristics of metal spherical shells, non-metallic sound transmission spherical shells, dimensions
and different fillers on the target strength of spherical shells are calculated and analyzed. The research of this paper will serve the design of underwater acoustic scattering body.

2. Computing method

The method of separating variables is a classical method for solving wave equation. The scattered wave is expressed as Rayleigh normal series solution of basic wave function. The expansion coefficients of series are obtained according to boundary conditions[7] and incident wave, and then the scattered wave solution is obtained.

2.1. Normal mode solutions of target strength of rigid sphere

The exact solution of rigid spherical scattering is often regarded as a standard problem. Scattering field of rigid sphere obtained from expansion coefficient:

\[
\phi_n(r, \theta) = \sum_{n=0}^{\infty} j_n(ka) \frac{h_n^{(1)}(ka)}{h_n^{(2)}(ka)} P_n(\cos \theta) h_n^{(1)}(kr) \]

(1)

Definition of shape function:

\[
f(x, \theta) = \frac{2r}{a} e^{-ax} \frac{\phi(x, \theta)}{\phi(x)}
\]

(2)

According to the definition of target strength in far field, the relationship between the shape function of the sphere and the target strength can be obtained, as shown below:

\[
TS = 20 \log \left| \frac{a}{r} f(x, \theta) \right|
\]

(3)

\[
= 20 \log \left| \frac{a}{r} \sum_{n=0}^{\infty} \frac{(2n+1)}{r} j_n(x) P_n(\cos \theta) \right|
\]

(4)

2.2. Normal mode solutions of target strength of elastic sphere shell

There is an elastic spherical shell immersed in fluid, as shown in Figure 1. The outer radius of the spherical shell is \(r\), the thickness is \(h\). The velocity of compression wave is represented as \(C_l\), and the velocity of shear wave is represented as \(C_t\).

When plane waves incident on a spherical shell, the expression of incident plane waves in polar coordinates is that the wave equation satisfies the boundary conditions at the spherical boundary, solve the equation, the expression of scattered sound field is as below[8]:

\[
P_s(r, \theta) = P_0 \sum_{n=0}^{\infty} (2n+1) j_n(ka) h_n^{(1)}(kr) P_n(\cos \theta)
\]

(4)

The far-field sound pressure expression is as follows:

\[
P_f(r, \theta) = P_0 \sum_{n=0}^{\infty} (2n+1) j_n(ka) P_n(\cos \theta) e^{-jkr} \frac{e^{-jkr}}{r}
\]

(5)
When the far-field sound pressure is reduced to a distance of 1 m from the center of the spherical shell, the target strength of the spherical shell is expressed as follows:

\[
TS = 20 \log \left( \frac{P(r, \theta)}{P(r, \theta)_{\text{ref}}}, 1 \text{m} \right)
\]

(6)

3. Simulation calculation and analysis

3.1. Calculation mode

In the simulation, we only considered the transmission and scattering of sound waves in spherical shells and filling materials. The interaction between the two media has less effect on sound waves and is not considered in the calculation. The outer radius of the spherical shell is 0.5m. The inner part is filled with liquid and the outer part is water medium. As shown in Table 1, the acoustic characteristics parameters of metal material aluminum and non-metal material natural rubber are respectively shown, while those of air, water, brine, alcohol and carbon tetrachloride are shown in Table 2. In the simulation, the parameters used are the frequency range of 50 Hz-10 kHz.

| Table 1. Acoustic parameters of spherical shell materials |
|----------------------------------------------------------|
| Materials  | Cl[\text{m/s}] | Ct[\text{m/s}] | Density [\text{kg/m}^3] |
| Aluminum   | 6260          | 3080          | 2700                   |
| gum rubber | 1418          | 210           | 970                    |

| Table 2. Acoustic parameters of fillers in spherical shells |
|-------------------------------------------------------------|
| Fluid            | Cl[\text{m/s}] | Density [\text{kg/m}^3] |
| Air              | 331           | 0.0013                 |
| Water            | 1500          | 1000                   |
| Saline water     | 1702          | 1131                   |
| Alcohol          | 1168          | 789                    |
| CCL4             | 935           | 1590                   |

3.2 Calculation results and analysis

When the coordinate origin is in the center of the circle and plane wave is incident, the field point is located 1000 meters away from the center of the sphere (monostatic).

![Figure 2. Target strength of a rigid sphere of radius 0.5 m](image)
As shown in Figure 2, when the radius of the rigid sphere $a$ is 0.5m and $K_a >> 1$, $K$ is the number of sound waves, the target strength is about -12dB. Using the method above, the spherical shell material fixed to aluminum, we change the filling material and calculate TS, the results are shown in Figure 3.

![Figure 3](image)

**Figure 3.** Target strength of three different internal fillers for Aluminum Spherical Shells

As shown in Figure 3 (a), there is no significant difference in the effect of target strength enhancement when air, water and CCl4 medium are filled in the aluminum spherical shell with a radius of 0.5m and a thickness of 0.1m.

As shown in Figure 3 (b), when the aluminum spherical shell with a radius of 0.5m and a thickness of 0.05m is filled with CCl4, the target intensity near 6kHz is higher in the whole frequency band because of the great difference between the sound speed and water. Compared with the rigid sphere with the same size, the maximum enhancement is about 20dB in the broader band. When water and air are filled in the interior, the target strength fluctuates sharply, the elastic echo of aluminum spherical shell is obvious, and the target strength is greatly enhanced at a specific frequency.

As shown in Figure 3 (c), when the aluminum spherical shell with a radius of 0.5m and a thickness of 0.005m is filled with CCl4, the target strength is relatively high in the 3-10kHz and is about 0dB in the broader band, which is obviously enhanced compared with the rigid sphere with the same size. At frequencies below 1 kHz, when the internal filler is air, the target strength increases greatly, but only slightly at other frequencies. For the internal filling water medium, it has no effect on target strength enhancement.

By comparing Figures 3 (a, b, c), it can be found that the difference of sound velocity between internal and external fluid is not significant for thick metal spherical shells. When the shell becomes thinner, the mismatch of sound velocity between internal and external fluid has a greater effect on target strength enhancement.
The spherical shell material fixed to rubber, we change the filling material and calculate the TS, the results are shown in Figure 4.

![Figure 4](image)

**Figure 4.** Calculations of Target Strength of Natural Rubber Spherical Shells with Three Different Internal Fillers

As shown in Figure 4 (a), the target strength of natural rubber spherical shell with radius r=0.5 mm thick h=0.1 m filled with CCl4 and air medium is higher than that filled with water medium except for 9 frequencies, which indicates that the sound velocity mismatch effect of internal and external fluid medium is obvious. When air is filled in the interior, the effect of internal gas is remarkable. The target strength fluctuates very little in a wider band, and the target strength is enhanced compared with the rigid spherical shell.

As shown in Figure 4 (b), the target strength law of natural rubber spherical shell with radius r=0.5m thick h=0.05m is basically consistent with that of Figure 4 (a). As shown in Figure 4 (c), the target strength of natural rubber spherical shell with radius of 0.5m and thickness of 0.005m is very small when filled with water, which is due to the good sound transmission performance of natural rubber. Compared with Figure 3, the target strength curves of natural rubber spherical shells filled with three kinds of fillers are basically the same, which shows that the matching degree of sound velocity of fluid inside and outside the shell plays a major role in the sound transmission of non-metallic sound transmitting material.

To compare the effects of the shell material on TS, we calculated TS of spherical shells with the same shape structure and filled liquid, but different materials, as shown in Figure 5 and Figure 6.
From Figure 5 and 6, it can be seen that the fillers in the spherical shell are different fluids, the density of these fluids is different, and the sound velocity in the fluid is different, which has an effect on the sound scattering. The mismatch of the sound velocity inside and outside the spherical shell plays an important role in improving the target strength of the spherical shell. Due to the size factor of shell material, the variation law of target strength is different, which should be fully considered in practical engineering application.

4. Conclusions

In this paper, the effects of the material, size and acoustic characteristics of different fillers on the strength of spherical shells are calculated and analyzed. The following conclusions are drawn:

1) The difference of sound velocity between inner and outer fluid of metal thick spherical shell is not significant, but the mismatch of sound velocity between inner and outer fluid of metal thin spherical shell plays a greater role in enhancing its target strength.

2) The matching degree of sound velocity of fluid inside and outside the shell plays a major role in the target strength of non-metallic sound transmission materials.

In addition, parameters such as the microscopic structure of the spherical shell and the temperature of the filling material also affect the TS value. These contents will be studied later.

5. References

[1] Kevin L. Williams and Philip L. Marston. Backscattering from an elastic sphere: Sommerfeld-Watson transformation and experimental confirmation. J.Acoust.Soc.Am.79(3), 1093-1102, 1985
[2] L. Rayleigh. Theory of Sound. Dover Publications, New York, 1945
[3] M.C. Junger. Sound scattering by thin elastic shells. J. Acoust. Soc. Am. 24(4):336-373, 1952
[4] R.R. Goodman and R. Stern. Reflection and transmission of sound by elastic spherical shells J. Acoust. Soc. Am. 34(3):338-344, 1962
[5] J.A. Fawcett. Scattering from a partially fluid-filled, elastic shelled sphere. J. Acoust. Soc. Am. 109(2):508-513, 2001
[6] G. Kaduchak and C. M. Loeffler. Relationship between material parameters and target strength of fluid-filled spherical shells in water: Calculations and observations. IEEE J. Ocean. Eng., vol. 23, no. 1, pp. 26-30, Jan, 1998.
[7] Gaunaurd G.C., Werby M.F. Sound scattering by resonantly exited, fluid-loaded, elastic spherical J. Acoust. Soc. Am., 90(5):2536-2550, 1991
[8] David M. Deveau and Anthony P. Lyons. Fluid-Filled Passive Sonar Calibration Spheres: Design, Modeling, and Measurement. IEEE Journal of Oceanic Engineering, VOL. 34, NO. 1, 2009