Experimental noise-resistant Bell-inequality violations for polarization-entangled photons

Fabio A. Bovino and Giuseppe Castagnoli
Elsag Spa, Via Puccini 2-16154 Genova, Italy

Adán Cabello
Departamento de Física Aplicada II, Universidad de Sevilla, 41012 Sevilla, Spain

Antía Lamas-Linares
Quantum Information Technology Lab, Physics Department, National University of Singapore, 2 Science Drive 3, 117542 Singapore

(Dated: April 1, 2022)

We experimentally demonstrate that violations of Bell’s inequalities for two-photon polarization-entangled states with colored noise are extremely robust, whereas this is not the case for states with white noise. Controlling the amount of noise by using the timing compensation scheme introduced by Kim et al. [Phys. Rev. A 67, 010301(R) (2003)], we have observed violations even for states with very high noise, in excellent agreement with the predictions of Cabello et al. [Phys. Rev. A 72, 052112 (2005)].

PACS numbers: 03.65.Ud, 03.67.Pp, 42.50.-p

I. INTRODUCTION

Entanglement, “the characteristic trait of quantum mechanics” [1], is central in Einstein, Podolsky, and Rosen’s (EPR’s) argument of incompleteness of quantum mechanics [2] and in Bell’s proof that quantum mechanics is incompatible with EPR’s local realistic view of the world [3]. This debate stimulated the search for sources of entangled states and the first experiments on the violation of Bell’s inequalities [5]. Nowadays, however, the role of quantum entanglement is more ubiquitous. Entanglement is considered a physical resource and a key ingredient for quantum information processing and quantum computation [3, 5, 8, 9]. The challenge now for the development of quantum information technologies is having reliable and efficient sources to produce, distribute, and detect entangled states. Although sources of entanglement have been described and demonstrated in many branches of physics, so far the most common way to distribute entanglement is by means of pairs of photons. The most reliable source of entanglement between photons is the spontaneous parametric down conversion (SPDC) process [10, 11]. The importance of having an accurate description of the distributed entangled states created in SPDC processes is therefore clear.

The violation of Bell’s inequalities provides a basic tool with which to detect entanglement [12]. In realistic applications, where pure entangled states become mixed states due to different types of noise, violations of Bell’s inequalities provide a method to characterize the robustness of the entanglement against noise. For this purpose, different methods for creating two-photon polarization mixed states have been proposed, analyzed, and tested [13, 14, 15].

It has been recently pointed out [16] that a colorless noise model is not the best choice for describing states produced in type II SPDC, but that a more realistic description is given by an alternative one parameter model where a maximally entangled state is mixed with decoherence terms in a preferred polarization basis. It turns out to be that this distinction between colorless and colored noise is crucial when we look for maximal violations of Bell’s inequalities for bipartite entangled states.

In real applications, an example of colored noise could be due to the first order polarization mode dispersion (PMD) phenomena because of birefringence in optical fibers. There are different manifestations of PMD depending on the view taken: in the frequency domain, one sees, for a fixed input polarization, a change with frequency \( \omega \) of the output polarization; in the time domain, one observes a mean time delay of a pulse traversing the fiber which is a function of the polarization of the input pulse.

Poole and Wagner [17] discovered that there exists special orthogonal pairs of polarization at the input and the output of the fiber called the PSPs. Light launched in a PSP does not change polarization at the output to first order in \( \omega \). These PSPs have group delays \( \tau_g \), which are the maximum and minimum mean time delays of the time domain view.

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Some insight into the PMD problem can be made simply by contemplating a piece of polarization-maintaining fiber. Its PSPs are the polarizations along the principal axes of birefringence of the fiber. Let us suppose that a polarization entangled state is launched in this kind of fiber: after the propagation the state will be affected by
colored noise due to the different group velocity experienced by photons in the birefringent medium.

Violations of Bell’s inequalities for two-photon polarization-entangled states with colored noise are extremely robust against noise, whereas this is not the case for states with white noise \[^{10}\]. From the experimental point of view, there exists some evidence supporting this predicted behavior in experiments with two-qutrit states and low noise \[^{15}\]. The aim of this paper is to experimentally test the predictions of \[^{10}\] for the case of two-qubit states for the full range of values of the parameter characterizing the amount of noise.

SPDC may be viewed as a three-photon coherent process: a noncentrosymmetric crystal is illuminated by a pump laser intense enough to stimulate nonlinear effects. The second order interaction results in the annihilation of a pump photon and the creation of two down converted photons, entangled in space-time or, equivalently, in wave number-frequency \[^{14}\]. Specifically, in type II SPDC the incident pump is split in a pair of orthogonally polarized photons.

Phase-matching by itself does not guarantee entanglement because, if we restrict our attention to the photons found in the intersections of the cones made by the extraordinary (e) and ordinary (o) rays exiting the crystal, the two down-conversion alternatives \(|\alpha_1\rangle|e_2\rangle \) and \(|e_1\rangle|o_2\rangle\) are not indistinguishable. The indistinguishability is achieved by the use of different kinds of compensation schemes. These schemes, however, are typically not perfect. For this reason, while correlations are very strong in the natural basis of the crystal, the same is not true for the maximally conjugated basis. A realistic description of the states produced in type II SPDC is given by a one parameter model, where a pure state is mixed with decoherence terms in a preferred polarization basis. These states with colored noise can be expressed as

\[
\rho_C = \frac{1}{2} \left( |\alpha_1\rangle|o_2\rangle + |e_1\rangle|e_2\rangle | \right),
\]

where

\[
|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|\alpha_1\rangle|o_2\rangle + |e_1\rangle|e_2\rangle).
\]

The parameter \(p\) is the probability of creating the Bell state \(|\Phi^+\rangle\).

If we are interested in maximal violations of the most general two-party two-output Bell inequality, the Clauser-Horne-Shimony-Holt (CHSH) inequality \[^{11}\], given by

\[
|\beta| \leq 2,
\]

where the Bell operator is

\[
\beta = \langle \hat{A}_0 \hat{B}_0 \rangle + \langle \hat{A}_0 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_0 \rangle - \langle \hat{A}_1 \hat{B}_1 \rangle,
\]

then, it is sufficient to consider the following local observables:

\[
\hat{A}_0 = \sigma_z,
\]

\[
\hat{A}_1 = \cos (2\theta) \sigma_x + \sin (2\theta) \sigma_z,
\]

\[
\hat{B}_0 = \cos (2\phi) \sigma_z + \sin (2\phi) \sigma_x,
\]

\[
\hat{B}_1 = \cos (2\phi - 2\theta) \sigma_x + \sin (2\phi - 2\theta) \sigma_z,
\]

where \(\sigma_z\) and \(\sigma_x\) denote the usual Pauli matrices.

For \(\rho_C\) states \[^{11}\] and the local observables \[^{5} - 8\], the Bell operator is

\[
\beta(p, \theta, \phi) = \cos (2\phi) \left[ (1 + p) \sin^2 (2\theta) + 2 \cos (2\theta) \right] + \sin (2\phi) (1 + p) \sin (2\theta) \left[ 1 - \cos (2\theta) \right].
\]

The maximum value of \(\beta(p, \theta, \phi)\) depends on \(p\) in a complex way (for a similar calculation, see \[^{10}\]). The interesting points are that \(\rho_C\) states violate the CHSH inequality for any \(p\), and that the maximum violations occur for local observables which depend on \(p\) \[^{10}\].

II. DESCRIPTION OF THE EXPERIMENT

To test this predicted behavior of the colored mixed states in the laboratory we use the timing compensation

![Image](image_url)
scheme introduced by Kim et al. [20, 21, 22] described in Fig. 1. In arm 1 of the interferometer, a half wave plate (HWP) rotates the polarization 90°, so that the natural emission \(|\psi_1\rangle|\psi_2\rangle, |\psi_1\rangle|\psi_2\rangle \rangle \), becomes \(|\psi_1\rangle|\psi_2\rangle, |\psi_1\rangle|\psi_2\rangle \rangle \). The state after the first polarization beamsplitter (PBS) is

$$\langle \Psi \rangle = C \int_{-L}^{0} dz \int d\nu_p \mathcal{E}(\nu_p) e^{i\lambda \nu_p z} \int d\nu \nu e^{-iD_G \nu z} x a_{1\nu}^{+} (\nu + \frac{\Omega_p + \nu_p}{2}) a_{2\nu}^{+} (-\nu + \frac{\Omega_p + \nu_p}{2}),$$

where \( \mathcal{E}(\nu_p) \) describes the spectral distribution of the pump field and \( \Omega_p \) is the central wavelength, and

$$\Lambda_p = \frac{1}{\nu_p (\Omega_p)} - \frac{1}{2} \left( \frac{1}{u_\nu (\Omega_p / 2)} + \frac{1}{u_e (\Omega_p / 2)} \right),$$

$$D_G = \frac{1}{u_\nu (\Omega_p / 2)} - \frac{1}{u_e (\Omega_p / 2)},$$

where \( u_\nu (\Omega_p), u_e (\Omega_p / 2), \) and \( u_e (\Omega_p / 2) \) are, respectively, the group velocities of the pump, the \( \nu \)-photon, and the \( e \)-photon inside the L-long crystal. The density matrix (obtained by tracing over the nonpolarization degrees of freedom), can be expressed as

$$\rho = p(\tau) |\Phi^+\rangle\langle \Phi^+ | + \frac{1 - p(\tau)}{2} (|oo\rangle\langle oo | + |ee\rangle\langle ee |),$$

where

$$p(\tau) = \mathcal{F} \left( \frac{\tau}{D_G L} \right) \left( 1 - 2 \left| \frac{\tau}{D_G L} \right| \right) e^{-2\sigma_p^2 (\Lambda_p \tau / D_G L)^2},$$

being

$$\mathcal{F}(x) = \begin{cases} 1 & \text{if } |x| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

and \( \sigma_p \) is the bandwidth of the pump laser. Therefore, by changing the parameter \( \tau \), which is related to the optical delay introduced by the “trombone” in arm 2 of the interferometer, it is possible to control the degree of colored noise of the state. For \( \tau = 0 \) we ideally obtain the pure state \( |\Phi^+\rangle \).

In the experimental setup (see Fig. 1), a 3 mm long beta-barium borate crystal, cut for a type II phase-matching, is pumped by a train of UV (\( \Omega_p = 410 \text{ nm} \)) ultrafast (120 fs) pulses generated by the second harmonic of a Ti:sapphire laser. SPDC photon pairs at 820 nm (\( \Omega_p / 2 \)) are generated with an emission angle of 3°. After passing through the interferometer, the photons are coupled by lenses into single-mode fibers. Coupling efficiency has been optimized by a proper engineering of the pump and the collecting mode in experimental conditions. Dichroic mirrors are placed in front of the fiber couplers to reduce stray light due to pump scattering. HWP before the fiber coupler, together with fiber-integrated polarizing beamsplitters (PBSs), project photons in the polarization basis \( |s(2\alpha)\rangle = |s \rangle |\alpha\rangle + \sin(\alpha)|e\rangle, |s^\perp(2\alpha)\rangle = |s \rangle |H\rangle - \cos(\alpha)|V\rangle \). Photons are detected by single photon counters (Perkin-Elmer SPCM-AQR-14).

The local observables \( \hat{A}_0, \hat{A}_1, \hat{B}_0, \) and \( \hat{B}_1 \) can be rewritten for the chosen polarization basis \( \{|s(2\alpha)\rangle, |s^\perp(2\alpha)\rangle \} \) as

$$\hat{A}_{0,1}(\alpha) = \langle s(2\alpha) | s(2\alpha) \rangle - |s^\perp(2\alpha) \rangle \langle s^\perp(2\alpha) \rangle, \quad (16)$$

$$\hat{B}_{0,1}(\beta) = \langle s(2\beta) | s(2\beta) \rangle - |s^\perp(2\beta) \rangle \langle s^\perp(2\beta) \rangle, \quad (17)$$

and the correlation function \( \langle \hat{A}_{0,1}(\alpha) \hat{B}_{0,1}(\beta) \rangle \) can be expressed in terms of coincidence detection probabilities \( p_{x_{a,y_{b}}}(\alpha, \beta) \) as

$$\langle \hat{A}_{0,1}(\alpha) \hat{B}_{0,1}(\beta) \rangle = p_{+\alpha, +\beta}(\alpha, \beta) - p_{-\alpha, -\beta}(\alpha, \beta), \quad (18)$$

where \( x, y = +, - \) are the two outputs of the integrated PBS and \( p_{x_{a,y_{b}}}(\alpha, \beta) \) are expressed in terms of coincident counts,

$$p_{x_{a,y_{b}}}(\alpha, \beta) = N_{x_{a,y_{b}}}(\alpha, \beta) / \left( N_{+\alpha, +\beta}(\alpha, \beta) + N_{-\alpha, -\beta}(\alpha, \beta) \right) \quad (19)$$

where \( N_{x_{a,y_{b}}}(\alpha, \beta) \) is the number of coincidences measured by the pair of detectors \( x_{a}, y_{b} \) in the polarization basis described above.

The experimental results are presented in Figs. 2, 3. In Fig. 2 we present the maximum violations of the CHSH inequality [i.e., the maximum values of the Bell operator \( \beta \) given by Fig. 4] for 11 values of the parameter \( p \) characterizing the amount of noise and the corresponding local parameters giving those maximum values. There is a very good qualitative and quantitative agreement between the experimental data and the theoretical predictions. Specifically, we observe violations of the CHSH inequality even for low values of \( p \). Note, however, that the experimental maximum violations of the CHSH stands slightly below the theoretical predictions. This effect is due to a little amount of white noise imputable to experimental imperfections. This residual white noise is visible in the tomographic reconstructions of the density matrices for three values of \( p \) (1.0, 0.6, and 0.0) presented in Fig. 3.

In addition, in Fig. 4 we present three contour plots representing the experimental reconstruction of the Bell operator \( \beta \) for three values of \( p \) (1.0, 0.6, and 0.0).

III. CONCLUSIONS

Summing up, we have performed a range of Bell’s inequality tests and tomographic analysis on two-photon
polarization states created by type II SPDC processes affected by a controlled colored noise. We have shown that these states violate the CHSH inequality, even the very noise ones, in excellent agreement with the theoretical predictions in [16]. Our results should have a direct relevance to the problems of controlling, manipulating and mapping quantum states for quantum technologies. Specifically, this description should be useful for optimizing protocols for the distillation of Bell states [23] and for quantifying the security of realistic quantum key distribution schemes based on pairs of polarization-entangled photons [24].

FIG. 2: Maximum values of the Bell operator for any set of measurements for different values of $p$ (above) and the corresponding local angles giving those maximum values (below). In both cases dashed lines represent the theoretical predictions of the model.

FIG. 3: Tomographic reconstructions of the density matrices for three different values of $p$. The tomographic reconstruction of the two extremal cases ($p = 1$ and $p = 0$) and one intermediate case with a moderate amount of noise ($p = 0.6$) independently show that the colored noise model is a good description of the produced states.

FIG. 4: Experimental reconstruction of the Bell operator $\beta$ for three different values of $p$.

Acknowledgments

These experiments were carried out in the Quantum Optics Laboratory of Elsag Spa, Genova. F.A.B. acknowledge support from EC-FET Project No. QAP-2005-015848. A.C. acknowledges support from Projects No. FIS2005-07689 and No. FQM-239. A.L.-L. acknowledges support from Grant No. R-144-000-137-112.
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