Late-time tail of a coupled scalar field in the background of a black hole with a global monopole

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Abstract

Using the technique of spectral decomposition, we investigated the late-time tails of massless and massive coupled scalar fields in the background of a black hole with a global monopole. We found that due to existence of the coupling between the scalar and gravitational fields, the massless scalar field decay faster at timelike infinity \( i_+ \), and so does the massive one in the intermediate late time. But the asymptotically late-time tail for the massive scalar field is not affected and its decay rate still is \( t^{-5/6} \).

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I. INTRODUCTION

It is well known that the dynamical evolution of field perturbations on a black hole consists roughly of three stages \[1\]. The first one is an initial wave burst coming directly from the source of perturbation and is dependent on the initial form of original wave field. The second one involves the damped oscillations named quasinormal modes, which are dependent only on the structure of the background spacetime and irrelevant of the initial perturbation. The last one is the power-law tail behavior of the field caused by backscattering of the gravitational field at late time.

The study of the late-time behaviors of the fields outside black holes has attracted a lot of attention for a long time \[2\]-\[27\]. Price \[4\] found that in the Schwarzschild black hole spacetime the late-time behaviors of the massless scalar, gravitational and electromagnetic perturbations fields for a fixed $r$ are dominated by the factor $t^{-(2l+3)}$. In other static spacetimes, it is found that the late-time tails of these massless perturbations also possess the same decay rate \[5\]-\[13\]. Comparing with the massless case, the late-time tail of the massive scalar field has quite different characteristic. For example, it is dominated by the oscillatory inverse power-law form $t^{-(l+3/2)} \sin \mu t$ in the intermediate late time \[14\]-\[20\]. The similar properties of the late-time behaviors of these perturbations were found in the rotational background metrics \[21\]-\[25\].

However, it is possible that the late-time behaviors of the perturbations have some distinct properties in some special case. For example, Jing \[26\] \[27\] recently considered the evolution of the massive Dirac field in the Schwarzschild black hole and of the massive charged Dirac field in the Reissner-Nordström black hole, and found that they are dominated by decaying tails without any oscillation, which is quite different from that of the scalar fields. Yu \[28\] studied the late-time tail of the scalar field in the black hole spacetime with a global monopole \[29\] and found that due to present of a solid angle deficit, the massive scalar fields decay faster in the intermediate times.

It is well known that the coupling between the scalar and gravitational fields plays some important roles in the black hole physics. In Ref. \[30\] \[31\], we found the couple factor affects the asymptotic quasinormal frequencies in the dilaton black hole spacetimes. Then it is interesting to ask the question whether the couple factor influence on the late-time tail of the scalar field. Obviously, in the Schwarzschild, Reissner-Nordström, Kerr, and Kerr-Newman black holes, the couple constant does not affect these late-time behaviors because in which the Ricci scalar curvatures are zero and then the couple terms vanish in the effective potentials. For the dilaton black hole spacetimes, although the Ricci scalar curvatures are not equal to zero, the couple
factor still does not affect the late-time behaviors because the couple term appears in the higher order term $O(1/r^3)$. However, Brady [32] found that in the Schwarzschild de Sitter spacetime the late-time behaviors of the coupled scalar field depend on the couple constant $\xi$.

Our purpose in this paper is to extend the initial works of Yu [28] and Brady [32]. We consider the late-time tails of the massless and massive coupled scalar fields in the background of a black hole with a global monopole [29] and obtain some new results.

The plan of this paper is as follows. In Sec.II we present the evolution equation of the coupled scalar field in the background of a black hole with a global monopole and introduce the black hole Green’s function using the spectral decomposition method [33]. In Sec.III and Sec.IV, we study the late-time behavior of the massless and massive coupled scalar field in the spacetime with a global monopole, respectively. In Sec.V we make a summary and some discussions.

II. DESCRIPTION OF THE SYSTEM AND BLACK HOLE GREEN’S FUNCTION

The metric of the background of a black hole with a global monopole is [29]

$$ds^2 = -(1 - 8\pi G \eta_0^2 - \frac{2Gm}{R})d\tau^2 + (1 - 8\pi G \eta_0^2 - \frac{2Gm}{R})^{-1}dR^2 + R^2(d\theta^2 + \sin^2 \theta d\varphi^2),$$

(2.1)

where $m$ is the mass parameter of the black hole and $\eta_0$ is the symmetry breaking scale when the monopole is formed. Introducing the coordinate transformations

$$t = (1 - 8\pi G \eta_0^2)^{\frac{1}{2}}\tau, \quad r \to (1 - 8\pi G \eta_0^2)^{-\frac{1}{2}}R,$$

(2.2)

and new parameters

$$M = (1 - 8\pi G \eta_0^2)^{-\frac{3}{2}}m, \quad b = (1 - 8\pi G \eta_0^2),$$

(2.3)

then metric (2.1) can be rewritten as

$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + b^2(d\theta^2 + \sin^2 \theta d\varphi^2).$$

(2.4)

It is a static and spherically symmetric metric with an additional solid angle deficit ($\Delta = 4\pi b = 32\pi G \eta_0^3$).

The Klein-Gordon equation for a coupled scalar field with mass $\mu$ is

$$\frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi) - (\mu^2 + \xi R)\phi = 0,$$

(2.5)

where $\phi$ is the scalar field and $R$ is the Ricci scalar curvature. The coupling between the scalar field and the gravitational field is represented by the term $\xi R\phi$, where $\xi$ is a coupling constant.
Decomposing the field into $\phi = \sum_{l,m} \frac{\psi(t,r)}{r} Y(\theta, \varphi)$ and introducing the “tortoise coordinate” change

$$r_* = r + 2M \ln \frac{r - 2M}{2M},$$

we obtain a wave equation

$$\psi_{tt} - \psi_{r_* r_*} + V \psi = 0,$$

with the effective potential

$$V = \left( 1 - \frac{2M}{r} \right) \left[ \frac{l(l+1)}{br^2} + \frac{2M}{r^3} + \mu^2 + \xi R \right],$$

where the Ricci scalar curvature is given by

$$R = \frac{2(1-b)}{br^2}.$$

The time evolution of a wave field $\Psi(r_*, t)$ described by Eq. (2.7) follows form

$$\Psi(r_*, t) = \int [G(r_*, r'_*; t) \partial_t \Psi(r'_*, 0) + \partial_t G(r_*, r'_*; t) \Psi(r'_*, 0)] dr'_*,$$

where the retarded Green’s function $G(r_*, r'_*; t)$ is defined by

$$\left[ \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + V(r) \right] G(r_*, r'_*; t) = \delta(t) \delta(r_* - r'_*).$$

The causality condition gives us the initial condition $G(r_*, r'_*; t) = 0$ for $t < 0$. In order to get $G(r_*, r'_*; t)$, we use the Fourier transform

$$\tilde{G}(r_*, r'_*; \omega) = \int_{-\infty}^{\infty} G(r_*, r'_*; t) e^{i\omega t} dt.$$

The Fourier transform is analytic in the upper half $\omega$-plane, and corresponding inversion formula is given by

$$G(r_*, r'_*; t) = \frac{1}{2\pi} \int_{-\infty - i c}^{\infty + i c} \tilde{G}(r_*, r'_*; \omega) e^{-i\omega t} d\omega,$$

where $c$ is some positive constant. We define two auxiliary functions $\tilde{\Psi}_1(r_*, \omega)$ and $\tilde{\Psi}_2(r_*, \omega)$ which are linearly independent solutions to the homogeneous equation

$$\left[ \frac{d^2}{dr_*^2} + \omega^2 - V[r(x)] \right] \tilde{\Psi}_i(r_*, \omega) = 0, \quad i = 1, 2.$$

Let the Wronskian be

$$W(\omega) = W(\tilde{\Psi}_1, \tilde{\Psi}_2) = \tilde{\Psi}_1 \dot{\tilde{\Psi}}_2 - \dot{\tilde{\Psi}}_1 \tilde{\Psi}_2,$$
and using the solutions $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$, the black hole Green’s function can be constructed as

$$
\tilde{G}(r_*, r_*'; \omega) = -\frac{1}{W(\omega)} \left\{ \begin{array}{ll}
\tilde{\Psi}_1(r_*, \omega)\tilde{\Psi}_2(r_*', \omega), & r_* < r_*'; \\
\tilde{\Psi}_1(r_*', \omega)\tilde{\Psi}_2(r_*, \omega), & r_* > r_*'.
\end{array} \right.
$$

(2.16)

To calculate $G(r_*, r_*'; t)$ using Eq. (2.13), one must close the contour of integration into the lower half of the complex frequency plane. It is by now well known that there exists a branch cut in $\tilde{\Psi}_2$ placed along the negative imaginary $\omega$-axis and the contribution to late-time tail comes from the integral of $\tilde{G}(r_*, r_*'; \omega)$ around this branch cut which is denoted by $G_C(r_*, r_*'; t)$. Thus, in the study of late-time evolution of an external field, we just consider $G_C(r_*, r_*'; t)$.

III. LATE-TIME BEHAVIOR OF THE MASSLESS COUPLED SCALAR FIELD

Let us now discuss the late-time behaviors of the massless coupled scalar fields. It is well known that the late-time behavior of massless perturbation fields is determined by the backscattering from asymptotically far regions and the leading contribution to the Green’s function comes from the low-frequency parts. Thus, we can study the asymptotic late-time behavior of the field by adopting the low-frequency approximation. Neglecting terms of order $O((\frac{M}{r})^2)$ and higher term, we can expand the wave equation (2.14) for the massless scalar field as a power series in $M/r$

$$\left[ \frac{d^2}{dr^2} + \omega^2 + \frac{4M\omega^2}{r} - \frac{l(l+1) + 2\xi(1-b)}{br^2} \right] \zeta(r, \omega) = 0,$$

(3.1)

where $\zeta(r, \omega) = \sqrt{1 - \frac{2M}{r} \tilde{\Psi}}$. Let us now introduce a second auxiliary field $\Phi(z)$ defined by

$$\zeta(r, \omega) = r^{\rho+\frac{1}{2}} e^{-\frac{z}{2}} \Phi(z),$$

(3.2)

where

$$z = -2i\omega r, \quad \rho = \sqrt{\frac{l(l+1) + 2\xi(1-b)}{b} + \frac{1}{4}}.$$

(3.3)

Then the equation becomes

$$z \frac{d^2 \Phi}{dz^2} + (1 + 2\rho - z) \frac{d \Phi}{dz} - \left( \frac{1}{2} + \rho - 2iM\omega \right) \Phi = 0.$$

(3.4)

The two basic solutions of this equation required in order to build the black hole Green’s function are

$$\tilde{\Psi}_1 = Ae^{i\omega r} r^{\frac{1}{2} + \rho} M(\frac{1}{2} + \rho - 2iM\omega, 1 + 2\rho, -2i\omega r),$$

(3.5)

$$\tilde{\Psi}_2 = Be^{i\omega r} r^{\frac{1}{2} + \rho} U(\frac{1}{2} + \rho - 2iM\omega, 1 + 2\rho, -2i\omega r),$$

(3.6)
where $A$ and $B$ are normalization constants. $M(a, b, z)$ and $U(a, b, z)$ are the two standard solutions to the confluent hypergeometric equation. Since $U(a, b, z)$ is a many-valued function, there will be a cut in $\tilde{\Psi}_2$. According to Eq. (2.13), one finds that the branch cut contribution to the Green’s function is

$$G^C(r_*, r'_*, t) = \frac{1}{2\pi} \int_0^{-i\infty} \tilde{\Psi}_1(r'_*, \omega) \left[ \frac{\tilde{\Psi}_2(r_*, \omega e^{2\pi i})}{W(\omega e^{2\pi i})} - \frac{\tilde{\Psi}_2(r_*, \omega)}{W(\omega)} \right] e^{-i\omega t} d\omega. \quad (3.7)$$

Using the relation as follows

$$\tilde{\Psi}_1(r_*, \omega e^{2\pi i}) = \tilde{\Psi}_1(r_*, \omega),$$

$$\tilde{\Psi}_2(r_*, \omega e^{2\pi i}) = \frac{B}{A} \frac{2\pi i e^{-\pi(2\rho+1)i}}{\Gamma(2\rho + 1)\Gamma(\frac{1}{2} - \rho - 2i\omega M)} \tilde{\Psi}_1(r_*, \omega) + e^{-2\pi(2\rho+1)i} \tilde{\Psi}_2(r_*, \omega), \quad (3.8)$$

we obtain

$$W(\omega) = \frac{AB \Gamma(2\rho + 1)(-i)^{-(2\rho+2)}(2\omega)^{-2\rho}}{\Gamma(\frac{1}{2} + \rho - 2i\omega M)}, \quad (3.9)$$

and

$$W(\omega e^{2\pi i}) = e^{-2\pi i(2\rho+1)}W(\omega). \quad (3.10)$$

In the low frequency approximation, the Green’s function can be expressed as

$$G^C(r_*, r'_*, t) = \frac{2^p M \Gamma(\rho + \frac{1}{2})^2 (-i)^{-2\rho}}{\pi A^2 [2(2\rho+1)]^2} \times \int_0^{-i\infty} \tilde{\Psi}_1(r_*, \omega) \tilde{\Psi}_1(r'_*, \omega) \omega^{2\rho+1} e^{-i\omega t} d\omega. \quad (3.11)$$

Let us now consider the asymptotic behavior at timelike infinity $i_+$. As we described before, the late-time behavior of the field arises from the low-frequency contribution to the Green’s function. Thus, the effective contribution to the integral in Eq. (3.11) should come from $|\omega| = O(1/t)$. In the condition that $|\omega| = O(1/t)$, we have

$$\tilde{\Psi}_1(r_*, \omega) \simeq Ar_*^{\rho+1/2}, \quad \tilde{\Psi}_1(r'_*, \omega) \simeq Ar'_*^{\rho+1/2}. \quad (3.12)$$

Substituting them into Eq. (3.11) and performing the integration, we find the asymptotic behavior of the massless scalar field at timelike infinity is described by

$$G^C(r_*, r'_*, t) = \frac{2^p M \Gamma(\rho + \frac{1}{2})^2 (-1)^{2\rho-1} \Gamma(2\rho + 2)}{\pi \Gamma(2\rho + 1)^2} (r'_* r_*)^{\rho+1/2} t^{-2\rho-2}. \quad (3.13)$$

Thus, for a coupled massless scalar field, the late-time tails are dominated not only by the multiple moment $l$ and the symmetry breaking scale $\eta_0$, but also by the coupling constant $\xi$. The presence of $\xi$ makes the field decay more rapidly. It implies that the interaction between the matter and gravitation fields plays an important role in the late-time evolutions of the matter fields in the background of a black hole with a global monopole.
IV. LATE-TIME BEHAVIOR OF THE MASSIVE COUPLED SCALAR FIELD

For the massive case, we also just evaluate the $G^c(r_*, r'_*; t)$ as in the massless one. But there exist some slightly differences in the massive scalar field case. The integral of the Green’s function $G(r_*, r'_*; t)$ around the branch cut performs in the interval $-\mu \leq \omega \leq \mu$ \[^{[14]}\] rather than along the total negative imaginary $\omega$-axis. Assuming that both the observer and the initial data are situated far away from the black hole so that $r \gg M$, we can expand the wave equation \((2.14)\) as a power series in $M/r$ and obtain [neglecting terms of order $O((M/r)^2)$ and higher]

$$\left[ \frac{d^2}{dr^2} + \omega^2 - \mu^2 + \frac{4M\omega^2 - 2M\mu^2}{r} - \frac{l(l+1) + 2\xi(1-b)}{br^2} \right] \zeta(r, \omega) = 0, \quad (4.1)$$

where $\zeta(r, \omega) = \sqrt{1 - \frac{2M}{r}} \tilde{\Psi}$. The equation can be rewritten as the confluent hypergeometric equation

$$zd^2\Phi dz^2 + (1 + 2\rho - z) \frac{d\Phi}{dz} - \left( \frac{1}{2} + \rho - \lambda \right) \Phi = 0, \quad (4.2)$$

with

$$z = 2\sqrt{\mu^2 - \omega^2}r = 2\varpi r, \quad \lambda = \frac{M\mu^2}{\varpi} - 2M\varpi, \quad \rho = \sqrt{\frac{l(l+1) + 2\xi(1-b)}{b}} + \frac{1}{4}, \quad (4.3)$$

and the two solutions required in order to build the Green’s function are given by (for $|\omega| \ll \mu$)

$$\tilde{\Psi}_1 = A'M_{\lambda,\rho}(2\varpi r) = A'e^{-\varpi r}(2\varpi r)^{\frac{1}{2}+\rho}M\left(\frac{1}{2} + \rho - \lambda, 1 + 2\rho, 2\varpi r\right), \quad (4.4)$$

$$\tilde{\Psi}_2 = B'W_{\lambda,\rho}(2\varpi r) = B'e^{-\varpi r}(2\varpi r)^{\frac{1}{2}+\rho}U\left(\frac{1}{2} + \rho - \lambda, 1 + 2\rho, 2\varpi r\right), \quad (4.5)$$

where $A'$ and $B'$ are normalization constants.

Making use of Eq.\((2.13)\), one finds that the contribution originated from the branch cut to the Green’s function is given by

$$G^c(r_*, r'_*; t) = \frac{1}{2\pi} \int_{-\mu}^{\mu} \left[ \tilde{\Psi}_1(r'_*, \omega e^{i\pi}) \tilde{\Psi}_2(r_*, \omega e^{i\pi}) \frac{W(\omega e^{i\pi})}{W(\omega)} - \tilde{\Psi}_1(r'_*, \omega) \tilde{\Psi}_2(r_*, \omega) \frac{W(\omega)}{W(\omega)} \right] e^{-i\omega t} d\omega = \frac{1}{2\pi} \int_{-\mu}^{\mu} F(\varpi) e^{-i\omega t} d\omega. \quad (4.6)$$

Using the following relation

$$W_{\lambda,\rho}(2\varpi r) = \frac{\Gamma(-2\rho)}{\Gamma(\frac{1}{2} - \rho - \lambda)} M_{\lambda,\rho}(2\varpi r) + \frac{\Gamma(2\rho)}{\Gamma(\frac{1}{2} + \rho - \lambda)} M_{\lambda,-\rho}(2\varpi r), \quad (4.7)$$

$$M_{\lambda,\rho}(e^{\pi}2\varpi r) = e^{(\frac{1}{2}+\rho)i\pi} M_{\lambda,-\rho}(2\varpi r), \quad (4.8)$$
we find

\[ W(\varpi e^{i\pi}) = -W(\varpi) = A'B' \frac{\Gamma(2\rho)}{\Gamma(\frac{1}{2} + \rho - \lambda)} 4\rho \varpi, \quad (4.9) \]

and

\[ F(\varpi) = \frac{1}{4\rho \varpi} [ M_{\lambda,\rho}(2\varpi r'_*) M_{\lambda,-\rho}(2\varpi r_*) - M_{-\lambda,\rho}(2\varpi r'_*) M_{-\lambda,-\rho}(2\varpi r_*) ] + \frac{1}{4\rho \varpi} \times \]

\[ \frac{\Gamma(-2\rho) \Gamma(\frac{1}{2} + \rho - \lambda)}{\Gamma(2\rho) \Gamma(\frac{1}{2} - \rho)} [ M_{\lambda,\rho}(2\varpi r'_*) M_{\lambda,\rho}(2\varpi r_*) - e^{2(\rho+1)i\pi} M_{-\lambda,\rho}(2\varpi r'_*) M_{-\lambda,\rho}(2\varpi r_*) ] \quad (4.10) \]

A. the intermediate late-time behavior

Let us now focus on the intermediate late-time behavior of the massive scalar field. That is the tail in the range \( M \ll r \ll t \ll M/(\mu M)^2 \). In this time scale, it is very easy for us to verify that the dominant contribution to the integral in the Green’s function \( G^C(r_*, r'_*; t) \) arises from the frequency range \( \varpi = O(\sqrt{\mu/t}) \), or equivalently \( \lambda \ll 1 \). From the massive scalar field equation (4.1), we know that \( \lambda \) originates from the \( 1/r \) term which describes from the effect of backscattering off the spacetime curvature. Thus, the parameter \( \lambda \ll 1 \) means that the backscattering off the curvature from the asymptotically far regions is negligible. Therefore, we have in this case

\[ F(\varpi) \approx \frac{1 + e^{2(\rho+1)i\pi}}{4\rho \varpi} \frac{\Gamma(-2\rho) \Gamma(\frac{1}{2} + \rho)}{\Gamma(2\rho) \Gamma(\frac{1}{2} - \rho)} M_{0,\rho}(2\varpi r'_*) M_{0,\rho}(2\varpi r_*) \quad (4.11) \]

In order to calculate the intermediate late-time behavior of the massive scalar field at a fixed radius (where \( r'_*, r_* \ll t \)), we can make use of the limit \( \varpi r \ll 1 \) and the property \( M(a, b, z) \approx 1 \) as \( z \) approaches zero. Then we find that Eq. (4.11) can be approximated as

\[ F(\varpi) \approx \frac{1 + e^{2(\rho+1)i\pi}}{4\rho 2^{-2\rho-1}} \frac{\Gamma(-2\rho) \Gamma(\frac{1}{2} + \rho)}{\Gamma(2\rho) \Gamma(\frac{1}{2} - \rho)} (r'_* r_*)^{\frac{1}{2} + \rho} \varpi^{2\rho}. \quad (4.12) \]

Thus, in the limit \( t \gg \mu^{-1} \), the Green’s function \( G^C(r_*, r'_*; t) \) becomes

\[ G^C(r_*, r'_*; t) = \frac{(1 + e^{2(\rho+1)i\pi}) \Gamma(-2\rho) \Gamma(\frac{1}{2} + \rho) \Gamma(1 + \rho) \mu^\rho}{\pi \rho 2^{-2\rho-2} \Gamma(2\rho) \Gamma(\frac{1}{2} - \rho)} \]

\[ \times (r'_* r_*)^{\frac{1}{2} + \rho} t^{-\rho-1} \cos [\mu t - \frac{\pi (\rho + 1)}{2}]. \quad (4.13) \]

We find the intermediate late-time behavior of a coupled massive scalar field is dominated by an oscillatory inverse power-law tail which decays slower than the massless case. As the massless case, the power-law tail of a coupled massive field at a fixed radius in the intermediate time also depends on the multiple number of the wave modes, the symmetry breaking scale \( \eta_0 \) and the coupling between the scalar and gravitational fields. Furthermore, we find that the coupled scalar field decays faster than the non-coupled one \([28]\) in this black hole spacetime.
B. the asymptotical late-time behavior

We now discuss the asymptotical late-time tail of a coupled massive scalar field. Since the asymptotic tail behavior is caused by a resonance backscattering of spacetime curvature at very late times $\mu t \gg 1/(\mu M)^2$, it is expected that the inverse power-law decay is replaced by another pattern of decay. In this case the backscattering from asymptotically far regions is important because that the parameter $\lambda$ can not be negligible. When $\lambda \gg 1$, we find

$$M_{\pm, \pm}(2\mathcal{M}r) \approx \Gamma(1 \pm 2\rho)(2\mathcal{M}r)^{\frac{1}{2}}(\pm \lambda)^{\pm \rho}J_{\pm 2\rho}(\sqrt{\pm \alpha r}), \quad (4.14)$$

where $\alpha = 8\lambda \varpi$. Consequently, Eq.$(4.11)$ can be expressed as

$$F(\varpi) = \frac{\Gamma(1 + 2\rho)^2 \Gamma(-2\rho) \Gamma(\frac{1}{2} + \rho - \lambda) r^*_s r_*}{2\rho \Gamma(2\rho)\Gamma(\frac{1}{2} - \rho - \lambda)} \lambda^{-2\rho}[J_{2\rho}(\sqrt{\pm \alpha r^*_s})J_{2\rho}(\sqrt{\pm \alpha r_*)}$$

$$+ I_{2\rho}(\sqrt{\pm \alpha r^*_s})I_{2\rho}(\sqrt{\pm \alpha r_*)} + \frac{\Gamma(1 + 2\rho) \Gamma(1 - 2\rho) r^*_s r_*}{2\rho}$$

$$[J_{2\rho}(\sqrt{\pm \alpha r^*_s})J_{-2\rho}(\sqrt{\pm \alpha r_*)} - I_{2\rho}(\sqrt{\pm \alpha r^*_s})I_{2\rho}(\sqrt{\pm \alpha r_*)}], \quad (4.15)$$

where $I_{2\rho}$ is the modified Bessel function. It is obvious that the late-time tail arising from the second term has a form $t^{-1}$. Now let us discuss the late-time behavior come from the first term. To calculate conveniently, we define

$$L = \frac{\Gamma(1 + 2\rho)^2 \Gamma(-2\rho) r^*_s r_*}{2\rho \Gamma(2\rho)}[J_{2\rho}(\sqrt{\pm \alpha r^*_s})J_{2\rho}(\sqrt{\pm \alpha r_*)} + I_{2\rho}(\sqrt{\pm \alpha r^*_s})I_{2\rho}(\sqrt{\pm \alpha r_*)}]. \quad (4.16)$$

The contribution of the first term in Eq.$(4.15)$ to the Green’s function can be approximated as

$$G^C_1(r^*_s, r'_s; t) = \frac{L}{2\pi} \int_{\mu}^{\mu} \frac{1}{1 + (-1)^{2\rho} e^{-i2\pi \lambda}} e^{i(2\pi \lambda - \omega t)} d\omega. \quad (4.17)$$

Making use of the saddle-point integration [14], we can obtain the asymptotic tail arising from the first term is $\sim t^{-5/6}$ and it dominates over the tail from the second term. Therefore, we obtain $G^C(r^*_s, r'_s; t) \sim t^{-5/6}$. Obviously, the asymptotic late-time tail of a coupled massive scalar field is an oscillatory tail with the decay rate of $t^{-5/6}$, which agrees with that of the non-coupled massive scalar field and can be regarded as a quite general feature for the late-time decay of massive scalar field.

V. SUMMARY AND DISCUSSION

In summary, we have studied analytically the late-time behaviors of the coupled massless and massive scalar fields in the background of a black hole with a global monopole. We find that both the asymptotic late-time tail of the coupled massless scalar field at timelike infinity
and the intermediate late-time tail of the coupled massive scalar field at a fixed radius depend not only on the multiple moment $l$ and the symmetry breaking scale $\eta_0$, but also on the couple constant $\xi$ between the scalar and gravitational fields. We note that the larger the couple constant $\xi$, the faster the decay of the scalar fields. When $b = 1$, our result returns that of in the general Schwarzschild spacetime. The couple constant $\xi$ disappears in the late time behavior because that the Ricci scalar curvature becomes zero. When $b = 0$, the background spacetime can not described by Barriola-Vilenkin metric \((2.1)\). Thus, the late-time behaviors of external fields in this case need to be investigated in the future.

Comparing our result with that of in the Schwarzschild de Sitter spacetime \([32]\), we find that in the Schwarzschild de Sitter spacetime, although the late-time behavior of the massless field depends on the value of the coupling constant $\xi$, the decay factor is independent of $\xi$ when $\xi > \xi_c$ (where $\xi_c$ is a critical value) \([32]\). However, in the background of a black hole with a global monopole, we find that the coupling constant always plays role in the intermediate late-time decay of the coupled scalar field.

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