Parameter Estimation in Mathematical Modelling for Photovoltaic Panel

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Abstract. The demand for solar photovoltaic (PV) system is growing rapidly driven by new technology and strong economies of scale. PV systems directly convert solar energy into electricity without release any pollution into the environment and deplete natural resources. PV technology has matured and its reliability keeps improving. However, PV system is more expensive to produce than conventional sources of energy due in part to the cost of manufacturing PV devices and in part to the conversion efficiencies of the equipment. Besides, important attention in designing, developing and installing the PV systems is time-consuming. In this paper, we propose a mathematical model to predict the PV systems behaviour and performance by considering the plausible factors. The factors accept in this model are solar irradiance and manufacturers' information for the type of PV panel. A case study at the eastern part of Peninsular Malaysia was conducted to examine the effect of factors on the performance of PV. Through the comparative analysis, the results have a good agreement.

1. Introduction

Solar energy is available directly from the sunshine. The sunlight will convert to electricity by solar cells in environmentally friendly. The solar energy is a renewable resource which available in local. This will reduce environmental impacts and also reduces our dependence on imported oil. Electricity is one of the most essential assets in our lives because it does not only increase the economic and social development but also improve human health and welfare as well [1].

Malaysia is one of the countries that is currently experiencing economic growth and rapid social development. Therefore, electricity demand has risen sharply. Beginning 1981, Malaysia has taken the momentous step to introduce the Four Fuel Diversification Policy which focuses on renewable energy (RE) which is consisting of solar photovoltaic (PV), biomass, small hydro and biogas. Before the establishment of the Sustainable Energy Development Authority (SEDA) in 2011, RE was less than 0.2% of the total electricity generation mix. With strong efforts driven by the NREPAP and Feed-in Tariff (FiT), the RE mix has increased to 2% (estimated 3,171 GWh) in 2015, with a compounded annual growth rate of 59% from 2006. The government of Malaysia is empowering renewable energy in order to ensure resilient and sustainable economic production for a long time [2].
Solar PV constitutes the largest portion of installed capacity at 69%, largely driven by the FiT-supported Solar Rooftop Programme, which was launched in 2013. As of 2013, FiT has achieved a total reduction of 432,161 carbon dioxide equivalent in greenhouse gas emissions. By 2020, total RE installed capacity is targeted to reach 2,080 MW, representing 11% of the total installed capacity in Peninsular Malaysia and Sabah [3]. Solar PV is the most popular RE source among FiT applicants due to its ease of installation and design compared to other RE sources of power generation. Solar PV is quiet and visually unobtrusive for small-scale solar plants. It can utilize unused space on rooftops of an existing building. The homeowners can add modules depends on energy usage and financial. Therefore, the technology for solar PV will always be improved and renewed to ensure productivity in a good performance.

Most of PV systems are developed based on circuitry methods and limited research to develop the system based on numerical methods [4-5]. Various studies have been conducted to enhance the system in a small or large scale or acceptance of the system in the community. Due to optimizing the cost, simulations come the best way of designing a good PV model system. The simulation study provides a strong foundation and more effective before the actual model can be developed. Accurate knowledge of solar cell parameters from experimental data is of vital importance for the design of solar cells and for the estimates of their performance. Thus, different solar cell models have been developed to describe their electrical behaviour, but the electrical equivalent circuit is a convenient and common way in most simulation studies. Numerical methods are exploited to model the non-linear PV system characteristics. The implicit non-linear equations that govern the mathematical model of PV system will be developed and simulated in this study.

[6] clarified that solar PV system consists the combination PV arrays which are the group of PV panels. PV panel comprises several photovoltaic cells connected in series and/or parallel. Each cell is depicted from several simple electrical circuits to see the PV system in general. The different solar cell models were introduced by [7] for single-diode model, [8] for the double-diode model and [9] for the triple-diode model. The specification will be discussed specifically in the next section.

Mathematical modelling of PV module is continuously updated to enable researchers to have a better understanding of PV module performance. Thus, this proposed mathematical model focused on the parameters that affect the accuracy of the PV system performance. The result will show the flow remains stable and capable of supplying electricity depending on the requirement of the load. This model can be used as a reference for the improvement of the PV system performance in the future.

2. Mathematical Model of Photovoltaic Cells

2.1. Single-Diode Model

[6] derived an analytical expression to extract the parameters of the solar cell single-diode model using experimental data. Figure 1 shows the ideal photovoltaic cell circuit with a diode (D), a shunt resistance (R_{SH}) and a series resistance (R_s). The diode and shunt resistance is in parallel with the current source. The current source known as photocurrent (I_{PV}) is generated when the cell is in the exposure of sunlight. I_{PV} is linearly changed when solar irradiation varies and the cell temperature is constant. The I_D is the current which flows through the parallel diode. I_{SH} is the shunt current flown due to the presence of R_{SH}. 

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The equation for the output current $I_o$ is given by:

$$I_o = I_{PV} - I_D - I_{SH}$$  \hspace{1cm} (1)$$

with

$$I_{PV} = I_{SC} \left( 1 + \frac{R_S}{R_{SH}} \right) + I_{RC} \left( \exp \left\{ \frac{I_{SC} R_S}{a V_T} - 1 \right\} \right) , \quad I_D = I_{RC} \left[ \exp \left\{ \frac{V + IR_S}{a V_T} \right\} - 1 \right] , \quad I_{SH} = \frac{V + IR_S}{R_{SH}}$$

and

$$V_T = \frac{kT}{q} \quad \text{The definition of } I_D \text{ is the Shockley diode equation, } I_{RC} \text{ is the reverse saturation current of the diode, } q \text{ is the electron charge } (1.60217646 \times 10^{-19} \text{C}) \text{, } k \text{ is the Boltzmann constant } 1.3806503 \times 10^{-23} \text{ J/K} \text{, } T \text{ is the ambient temperature and } a \text{ is the diode ideal constant.}$$

Equation (1) will be;

$$I_o = I_{SC} \left( 1 + \frac{R_S}{R_{SH}} \right) + I_{RC} \left( \exp \left( \frac{I_{SC} R_S}{n V_T} - 1 \right) \right) - I_{RC} \left[ \exp \left( \frac{V + IR_S}{a V_T} \right) - 1 \right] - \frac{V + IR_S}{R_{SH}} \quad \text{(2)}$$

2.2. Double Diode Model

[8] improve the derivation of [6] by introducing the two-diode model of the photovoltaic (PV) cell which is fast becoming a viable alternative for PV simulation. This can be attributed to its superiority in predicting the $I$-$V$ characteristics over a wide range of solar irradiance ($G$) and temperature ($T$) variations. The double-diode model is to make the single-diode model more accurately by considering the combined effects of carriers.

The equation for the output current $I_o$ in this double diode model given by;

$$I_o = I_{PV} - I_D - I_{D1} - I_{SH}$$  \hspace{1cm} (3)$$

with

$$I_D = I_{RC1} \left[ \exp \left( \frac{V + IR_S}{a_1 V_T} \right) - 1 \right] \quad \text{and} \quad I_{D1} = I_{RC2} \left[ \exp \left( \frac{V + IR_S}{a_2 V_T} \right) - 1 \right], \quad a_1 \text{ and } a_2 \text{ represent ideality factor for diodes } D \text{ and } D_1; \quad I_{RC1} \text{ and } I_{RC2} \text{ are the reverse saturation current of the diodes.}$$

Equation (3) becomes;
\[ I_o = I_{SC} \left( 1 + \frac{R_S}{R_{SH}} \right) + I_{RC} \left( \exp \left( \frac{I_{SC}R_S}{aVT} \right) - 1 \right) - I_{RC1} \left( \exp \left( \frac{V + IR_S}{a_1VT} \right) - 1 \right) - I_{RC2} \left( \exp \left( \frac{V + IR_S}{a_2VT} \right) - 1 \right) - \frac{V + IR_S}{R_{SH}} \] \tag{4}

2.3. Triple-Diode Model

[9] introduced the three-diode circuit model due to difficulty in analyzing the properties of small size PV cell caused by large leakage current through peripheries in a real model of PV cell. This model was accepted for analysing the performance of PV cell accurately. Figure 2 shows the design of three diodes in the circuit.

![Figure 2. Triple-Diode Model](image)

The equation derived from the Figure 3 is;

\[ I = I_{PV} - I_D - I_{D1} - I_{D2} - I_{SH} \] \tag{5}

\[ I_{D2} = I_{RC3} \left\{ \exp \left( \frac{V + IR_S}{a_3VT} \right) - 1 \right\} \]

with \( a_3 \) as ideal factor for diode \( D_2 \) and \( I_{RC3} \) as the reverse saturation current of the diode.

Equation (5) becomes;

\[ I_o = I_{PV} - I_{RC1} \left\{ \exp \left( \frac{V + IR_S}{a_1VT} \right) - 1 \right\} - I_{RC2} \left\{ \exp \left( \frac{V + IR_S}{a_2VT} \right) - 1 \right\} - I_{RC3} \left\{ \exp \left( \frac{V + IR_S}{a_3VT} \right) - 1 \right\} - \frac{V + IR_S}{R_{SH}} \] \tag{6}

The single-diode model with 5-parameters becomes previous researchers’ choice in modelling PV systems \[4\] and it led to more accurate current estimations \[10\].

3. The Current-Voltage (I-V) and Power-Voltage (P-V) Characteristics Curve

Current-Voltage (I-V) and Power-Voltage (P-V) characteristic curves are generally used to describe the performance of output current \( (I_o) \) and output power \( (P) \) which are produced by PV panel.
Generally, the curve represents the combinations of current, voltage and power at which the string could be operated or 'loaded' if the solar irradiance and cell temperature could be held constant.

The curve of \( (I - V) \) and \( (P - V) \) characteristics for PV cell show a detailed description of its solar energy conversion ability and efficiency. This curve is determining the device’s output performance and solar efficiency. \( I_{SC} \) is the maximum current that the cell can provide and it occurs when the cells are short-circuited. \( V_{OC} \) is the maximum voltage that exists between the cells’ terminals and is obtained when there is no load connected across them. \( I_{MPP} \) and \( V_{MPP} \) are being produced by the cell. It is called as \( P_{MAX} \) is product of \( I_{MPP} \) and \( V_{MPP} \). Whereas \( I_{SC} \), \( V_{OC} \), \( I_{MPP} \), \( V_{MPP} \) and \( P_{MAX} \) are given by the manufacturer. Figure 3 described the plotting point of \( I_{SC} \), \( V_{OC} \), \( I_{MPP} \), \( V_{MPP} \) and \( P_{MAX} \).

![Figure 3. A typical current, voltage and power curve](image)

4. Parameter Estimation

This study focused on single diode model of 5-parameters. The assumption made in this case are \( I_{PV} \) is influencing by solar irradiance \( (G) \) [7], [11], [12]. While the resistances affected due to ambient temperature \( (T) \) [13]. This is not parallel with [11] which assumed the leakage current through the resistance are independent of temperature and solar irradiance. Figure 4 shows no significant relationship between temperature and solar irradiance. The value of \( R^2 = 0.3864 \) shows in graph meant the weak relationship between temperature and solar irradiance. This finding denies the photocurrent \( (I_{PV}) \) is influenced by the temperature mentioned in [12].

![Figure 4. Solar Irradiance in February 2016](image)
Figure 5 shows the value of resistances that influence by an environmental factor. The calculation of series resistance \( R_S \) and shunt resistance \( R_{SH} \) are using the proposed mathematical modelling Equation (7) and Equation (8). As shown in the figure, the value of resistance is slowly changes with respect to temperature. PV model is used in this calculation is ART265-60-4-8B0. The value of both resistance increased when the temperature increase.

\[
R_{SH} = \frac{(N_S I_{SC} R_S - V_{OC})(e^{qA/B} - B(e^C - 1))(D - (qN_S I_{SC}))}{((N_S^2 I_{MP} a k T)(e^C - 1))(D - (qN_S I_{SC}))}
\] (7)

with \( A = V_{MPP} + N_S I_{MP} R_S \), \( B = a k T N_S \), \( C = q V_{OC} / a k T N_S \) and \( D = V_{MPP} - N_S I_{MP} R_S \)

\[
R_S = \frac{a k T}{q I_{MPP}} \ln \left[ e^C - \frac{I_{MP}}{I_{SC}} \right] - \frac{V_{MPP}}{N_S I_{MPP}}
\] (8)

Although researchers might suffer from premature convergence in the case of improper selection of control parameters and initial conditions, the result proofed the better findings.

5. Proposed Mathematical Model

The focus of this study was on a single diode model for PV cell. In practice, PV cells are connected in series into PV module and these PV modules then are connected in series or parallel to form PV arrays for generating more electricity depending on the load’s need. The performances of PV modules affect PV arrays too.

The mathematical model is important because it has the ability to predict the PV cells behaviour and performance by considering the plausible factors. In this study, the factors affecting the performance of the PV module are \( G \) and \( T \) [12], [14], [15], and [16]. These factors will influencing \( I_{PV} \), \( I_{RC} \), \( R_S \), \( R_{SH} \) and \( a \). The others factor should be considered is fitted parameter given by manufacturer.

An accurate estimation of performance for PV module is the main goal by minimal in making the assumption. The previous study agreed that \( I_{RC} \) as represent the characteristics of a component that caused the accuracy of \( I \). Thus, the estimation of \( I_{RC} \) must be considered with the environmental
and working condition for having more accurate solution. The proposed $I_{rc}$ is formulated as Equation (9). While, the mathematical model for $a$, developed by implanted by band gap and temperature as given as in Equation (10).

$$I_{rc} = \left( \frac{I_{sc}}{G_n} \right)^3 \exp\left( \frac{E_g}{K_iT_n} - \frac{E_g}{K_iT} \right)$$

(9)

$$a = \frac{E_g - qV_{oc}}{kT \ln \left( \frac{I_{mpp}}{I_{sc}} \right)}$$

(10)

The photocurrent ($I_{pv}$) is an electric current produced by a photoelectric effect but it is not counted as output current due to leakage that representing by diode and resistances showed in Figure 1, measuring of combination between environmental factors and specification of photovoltaic module [13]. The proposed model for $I_{pv}$ is formulated as given;

$$I_{pv} = \left( \frac{I_{sc}}{G_n} \right)\left( 1 + K_i(T - T_n) \right)$$

(11)

Substitute Equation (9) and (11) to Equation (1), then:

$$I_o = \left( \frac{I_{sc}}{G_n} \right)\left( 1 + K_i(T - T_n) \right) - \left( \frac{I_{sc}}{G_n} \right)^3 \exp\left( \frac{E_g}{K_iT_n} - \frac{E_g}{K_iT} \right) \left[ \exp\left( \frac{V + N_sIR_s}{N_sakT/q} \right) - 1 \right] - \frac{V + N_sIR_s}{N_sR_{sh}}$$

(12)

The next task in this study is set for our MATLAB programming to calculate the measured and predicted current productions and efficiencies for different PV modules. The program is developed in script files in the MATLAB environment and the result will discuss in next section.

6. Result and Discussion

In modelling the PV system, the input variables are solar irradiance and temperature, which are under the category of weather data that are always changing with respect to time. On the other hand, the output variables are current, $I$, voltage, $V$, and power, $P$. Subsequently, we can then obtain the characteristic of $I - V$ and $P - V$ curve. A bit bit changing in the input value will affect the output. This is the reason why we need an accurate model for the PV system [12]. A general mathematical description of $I - V$ output characteristics for a PV cell has been studied over the past four decades.

Besides the solar irradiance and temperature the manufacturer’s specification has been considered this proposed model. The model’s specifications for ART265-60-4-8B0 Monocrystalline Solar Module from Suniva Artisun as listed in Table 1. The following parameters are used in the experiments and the common parameters involved in the proposed mathematical modelling. It shows when the parameter
changed then the proposed mathematical modelling still produce the expected result nicely. This is discussed in all the insights generated by Matlab. This proposed model is in a good argument for any solar irradiance and ambient temperature.

| Table 1. ART265-60-4-8B0 Specification. |
|------------------------------------------|
| **Maximum Power** ($P_{MAX}$)            | 265 W               |
| **Maximum Power Voltage** ($V_{MPP}$)    | 30.5 V              |
| **Maximum Power Current** ($I_{MPP}$)    | 8.69 A              |
| **Short-circuit current** ($I_{SC}$)      | 9.22 A              |
| **Open-circuit voltage** ($V_{OC}$)      | 38.1 V              |
| **The temperature coefficient of short-circuit current** ($K_f$) | 0.0006 /C |

Figure 6 and Figure 7 show the $I$-$V$ and $P$-$V$ characteristics for various solar irradiance level at $G = 1000\text{W/m}^2, 800\text{W/m}^2$ and $600\text{W/m}^2$. The curves plotted respectively using the proposed mathematical model for different hardware photovoltaic models. It can be seen that different solar irradiance is given a different graph. Small different in solar irradiance will give big different in $I$-$V$ curve and $P$-$V$ curves. This is due to our mathematical model that compute, and a simultaneously. The graph is consistent with the experimental data. This indicates that the actual behaviour of the solar cell can be accurately described by the proposed mathematical modelling.

![Figure 6. $I$-$V$ Characteristics with various solar irradiance](image)

A decrease in solar irradiance will cause a decrease in efficiency and output power of solar cell [17], [16], [12]. Solar irradiance increase will directly be affected in producing $I_{PV}$, Equation 5 had calculated and proven the highest $I_{PV}$ most influenced by the value of $G$.

Figure 8 and Figure 9 show the $I$-$V$ curves and $P$-$V$ curves respectively using the proposed mathematical model for different hardware photovoltaic models for various temperature level at $T = 25^\circ C, 26.5^\circ C, 27.5^\circ C$ and It can be seen that different temperature is given a different graph. Small differences in temperature will give big
different in $I$-$V$ curve and $P$-$V$ curves. This is due to our mathematical model that compute $R_s$, $R_{sh}$ and $a$ simultaneously. The graph is consistent with the experimental data. This indicates that the actual behaviour of the solar cell can be accurately described by the proposed mathematical modelling.

**Figure 7.** $P$-$V$ Characteristics with various solar irradiance.

**Figure 8.** $I$-$V$ Characteristics with various temperature

**Figure 9.** $P$-$V$ Characteristics with various temperature
All results in Figure 8 and Figure 9 are paralleled with [18], [17], and [15]. Determination of $R_s$ and $R_{SH}$ by computing the model has proven the resistances are strictly increased with respect to temperature. In this study, the resistances are reacted with the ambient temperature and influence the $I-V$ and $P-V$ characteristics. An increase in temperature will cause a decrease in efficiency and output power of solar cell [18], [15] and [17]

7. Conclusion

This study focused in improving the mathematical modelling by computing the values of ideality diode factor, $a$, series resistance, $R_s$ and shunt resistance, $R_{SH}$. Most established mathematical models set a certain value for $I_{PV}$, $R_{SH}$, $R_s$ as the initial data. In order to improve the performance of solar photovoltaic, a model for $I_{PV}$, $R_s$ and $R_{SH}$ are developed. The values for $I_{PV}$, $R_s$ and $R_{SH}$ were computed simultaneously using our mathematical formulation that takes into account the factors of ambient temperature, $T$ and solar irradiance $G$. Then, we validated the mathematical model by comparing the calculated data with the experimental data manufacturer’s datasheet. It is found that our mathematical model consistent with the data for any environmental condition.

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