AN OSCILLATION CRITERION FOR DELAY DIFFERENTIAL EQUATIONS WITH SEVERAL NON-MONOTONE ARGUMENTS

H. AKCA, G. E. CHATZARAKIS*, AND I. P. STAVROULAKIS

Abstract. The oscillatory behavior of the solutions to a differential equation with several non-monotone delay arguments and non-negative coefficients is studied. A new sufficient oscillation condition, involving lim sup, is obtained. An example illustrating the significance of the result is also given.

Keywords: differential equation, non-monotone delay argument, oscillatory solutions, nonoscillatory solutions.

2010 Mathematics Subject Classification: 34K11, 34K06.

1. INTRODUCTION

The paper deals with the differential equation with several non-monotone delay arguments of the form

\[ x'(t) + \sum_{i=1}^{m} p_i(t)x(\tau_i(t)) = 0, \quad \forall t \geq 0, \]  

(1.1)

where \( p_i, 1 \leq i \leq m, \) are functions of nonnegative real numbers, and \( \tau_i, 1 \leq i \leq m, \) are non-monotone functions of positive real numbers such that

\[ \tau_i(t) < t, \quad t \geq 0 \quad \text{and} \quad \lim_{t \to \infty} \tau_i(t) = \infty, \quad 1 \leq i \leq m. \]  

(1.2)

Let \( T_0 \in [0, +\infty), \) \( \tau(t) = \min\{\tau_i(t) : i = 1, \ldots, m\} \) and \( \tau(-1)(t) = \sup\{s : \tau(s) \leq t\}. \) By a solution of the equation (1.1) we understand a function \( x \in C([T_0, +\infty); \mathbb{R}), \) continuously differentiable on \( [\tau(-1)(T_0), +\infty) \) and that satisfies (1.1) for \( t \geq \tau(-1)(T_0). \)

A solution \( x(t) \) of (1.1) is oscillatory, if it is neither eventually positive nor eventually negative. If there exists an eventually positive or an eventually negative solution, the equation is nonoscillatory. An equation is oscillatory if all its solutions oscillate.

The problem of establishing sufficient conditions for the oscillation of all solutions of equation (1.1) has been the subject of many investigations. See, for example, [2, 3, 5–13, 15,17,18] and the references cited therein. Most of these papers concern the special case where the arguments are nondecreasing, while a small number of these papers are dealing with the general case where the arguments are non-monotone. See, for example, [2,3,16] and the references cited therein. For the general oscillation theory of differential equations the reader is referred to the monographs [1, 4, 14].

*Corresponding author : George E. Chatzarakis; email address: geaxatz@otenet.gr; gea.xatz@aspete.gr; tel. +30-210-2896774; Greece.
In 1978 Ladde [13] and in 1982 Ladas and Stavroulakis [12] proved that if

$$\liminf_{t \to \infty} \int_{\tau_{\max}(t)}^{t} \sum_{i=1}^{m} p_i(s) \, ds > \frac{1}{e},$$

(1.3)

where \(\tau_{\max}(t) = \max_{1 \leq i \leq m} \{\tau_i(t)\}\), then all solutions of (1.1) oscillate.

In 1984, Hunt and Yorke [7] proved that if

$$\liminf_{t \to \infty} \sum_{i=1}^{m} p_i(t) (t - \tau_i(t)) > \frac{1}{e},$$

(1.4)

then all solutions of (1.1) oscillate.

When \(m = 1\), that is in the special case of the equation

$$x'(t) + p(t)x(\tau(t)) = 0, \quad \forall t \geq 0,$$

(1.1')

in 1991, Kwong [11], proved that if

$$0 < \alpha := \liminf_{t \to \infty} \int_{\tau(t)}^{t} p(s) \, ds \leq \frac{1}{e},$$

$$\tau(t) \text{ is decreasing} \quad \text{and} \quad \limsup_{t \to \infty} \int_{\tau(t)}^{t} p(s) \, ds > \frac{1 + \ln \lambda_0}{\lambda_0},$$

(1.5)

where \(\lambda_0\) is the smaller root of the equation \(\lambda = e^{\alpha \lambda}\), then all solutions of (1.1)’ oscillate.

Recently, Braverman, Chatzarakis and Stavroulakis [2], established the following theorem in the general case that the arguments \(\tau_i(t)\), \(1 \leq i \leq m\) are non-monotone.

**Theorem 1.** Assume that \(p_i(t) \geq 0, \ 1 \leq i \leq m\),

$$h(t) = \max_{1 \leq i \leq m} h_i(t), \quad \text{where} \quad h_i(t) = \sup_{0 \leq s \leq t} \tau_i(s), \quad t \geq 0,$$

(1.6)

and \(a_r(t, s), \ r \in \mathbb{N}\) are defined as

$$a_1(t, s) := \exp \left\{ \int_{s}^{t} \sum_{i=1}^{m} p_i(\zeta) \, d\zeta \right\}, \quad a_{r+1}(t, s) := \exp \left\{ \int_{s}^{t} \sum_{i=1}^{m} p_i(\zeta)a_r(\zeta, \tau_i(\zeta)) \, d\zeta \right\}.$$

(1.7)

If for some \(r \in \mathbb{N}\)

$$\limsup_{t \to \infty} \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(\zeta)a_r(h(t), \tau_i(\zeta)) \, d\zeta > 1,$$

(1.8)

or

$$0 < \alpha := \liminf_{t \to \infty} \int_{h(t)}^{t} p_i(s) \, ds \leq \frac{1}{e},$$

and

$$\limsup_{t \to \infty} \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(\zeta)a_r(h(t), \tau_i(\zeta)) \, d\zeta > 1 - \frac{1 - \alpha - \sqrt{1 - 2\alpha - \alpha^2}}{2},$$

(1.9)

then all solutions of (1.1) oscillate.

An oscillation criterion involving \(\limsup\), which essentially improves the above results is established. An example illustrating the result is also given.
2. MAIN RESULT

The proof of our main result is essentially based on the following lemmas.

**Lemma 1.** [2, Lemma 1] Assume that \( x(t) \) is a positive solution of (1.1) and \( a_r(t, s) \) are defined by (1.7). Then
\[
x(t)a_r(t, s) \leq x(s), \quad t \geq s \geq 0.
\] (2.1)

**Lemma 2.** [cf. 8] Assume that \( x(t) \) is a positive solution of (1.1), and
\[
0 < \alpha := \liminf_{t \to \infty} \int_t^\infty \sum_{i=1}^m p_i(s)ds \leq \frac{1}{e},
\] (2.2)
where \( \tau(t) = \max_{1 \leq i \leq m} \tau_i(t) \). Then we have
\[
\liminf_{t \to \infty} \frac{x(h(t))}{x(t)} \geq \lambda_0,
\] (2.3)
where \( h(t) \) is defined by (1.6) and \( \lambda_0 \) is the smaller root of the equation \( e^{\alpha \lambda} = \lambda \).

**Proof:** Assume that \( x(t) \) is an eventually positive solution of (1.1). Then there exists \( t_1 > 0 \) such that \( x(t), \ x(\tau_i(t)) > 0 \), for all \( t \geq t_1 \). Thus, from (1.1) we have
\[
x'(t) = -\sum_{i=1}^m p_i(t)x(\tau_i(t)) \leq 0, \quad \text{for all } t \geq t_1,
\]
which means that \( x(t) \) is an eventually nonincreasing function of positive numbers.

Also, by a similar procedure as in the proof of Lemma 2.1.1 [4], we have
\[
\liminf_{t \to \infty} \int_{h(t)}^t \sum_{i=1}^m p_i(s)ds = \liminf_{t \to \infty} \int_{\tau(t)}^t \sum_{i=1}^m p_i(s)ds = \alpha.
\] (2.4)
In view of this, for any \( \varepsilon \in (0, \alpha) \), there exists \( t_\varepsilon \in \mathbb{R}_+ \) such that
\[
\int_{h(t)}^t \sum_{i=1}^m p_i(s)ds \geq \alpha - \varepsilon \quad \text{for } t \geq t_\varepsilon \geq t_1.
\] (2.5)
We will show that
\[
\liminf_{t \to \infty} \frac{x(h(t))}{x(t)} \geq \lambda_0(\varepsilon),
\] (2.6)
where \( \lambda_0(\varepsilon) \) is the smaller root of the equation
\[
e^{(\alpha-\varepsilon)\lambda} = \lambda.
\]
Assume, for the sake of contradiction, that (2.6) is not satisfied. Then there exists \( \varepsilon_0 > 0 \) such that
\[
\frac{e^{(\alpha-\varepsilon)\gamma}}{\gamma} \geq 1 + \varepsilon_0,
\] (2.7)
where
\[
\gamma = \liminf_{t \to \infty} \frac{x(h(t))}{x(t)} < \lambda_0(\varepsilon).
\]
On the other hand, for any \( \delta > 0 \) there exists \( t_\delta \) such that
\[
\frac{x(h(t))}{x(t)} \geq \gamma - \delta \quad \text{for } t \geq t_\delta.
\]
Dividing (1.1) by \( x(t) \) we obtain
\[
\frac{x'(t)}{x(t)} = \sum_{i=1}^{m} p_i(t) \frac{x(\tau_i(t))}{x(t)} \geq \sum_{i=1}^{m} p_i(t) \frac{x(h(t))}{x(t)} \geq (\gamma - \delta) \sum_{i=1}^{m} p_i(t).
\]
Integrating last inequality from \( h(t) \) to \( t \) for sufficiently large \( t \), and taking into account (2.5), we have
\[
- \int_{h(t)}^{t} \frac{x'(s)}{x(s)} ds \geq (\gamma - \delta) \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(s)ds \geq (\gamma - \delta) (\alpha - \varepsilon),
\]
or
\[
\frac{x(h(t))}{x(t)} \geq e^{(\alpha - \varepsilon)(\gamma - \delta)} \quad \text{for large } t.
\]
Therefore
\[
\gamma = \liminf_{t \to \infty} \frac{x(h(t))}{x(t)} \geq e^{(\alpha - \varepsilon)(\gamma - \delta)}
\]
which implies
\[
\gamma \geq e^{(\alpha - \varepsilon)\gamma}.
\]
This contradicts (2.7) and therefore (2.6) is true. Thus, as \( \varepsilon \to 0 \), (2.6) implies (2.3). The proof of the lemma is complete. \( \square \)

**Remark 1.** If \( \alpha > 1/e \) then equation \( \lambda = e^{\alpha \lambda} \) has no real roots. In this case, lemma is inappropriate since (1.1) does not have nonoscillatory solutions at all.

**Theorem 2.** Assume that (2.2) holds and for some \( r \in \mathbb{N} \)
\[
\limsup_{t \to \infty} \int_{t}^{\tau} \sum_{i=1}^{m} p_i(\zeta) a_r(h(\zeta), \tau_i(\zeta)) \ d\zeta > \frac{1 + \ln \lambda_0}{\lambda_0}, \quad (2.8)
\]
where \( h(t) \) is defined by (1.6), \( a_r(t,s) \) is defined by (1.7), and \( \lambda_0 \) is the smaller root of the equation \( \lambda = e^{\alpha \lambda} \). Then all solutions of (1.1) oscillate.

**Proof.** Assume, for the sake of contradiction, that there exists a nonoscillatory solution \( x(t) \) of (1.1). Since \( -x(t) \) is also a solution of (1.1), we can confine our discussion only to the case where the solution \( x(t) \) is eventually positive. Then there exists \( t_1 > 0 \) such that \( x(t) \), \( x(\tau_i(t)) \) > 0, for all \( t \geq t_1 \). Thus, from (1.1) we have
\[
x'(t) = - \sum_{i=1}^{m} p_i(t) x(\tau_i(t)) \leq 0, \quad \text{for all } t \geq t_1,
\]
which means that \( x(t) \) is an eventually nonincreasing function of positive numbers.

By Lemma 2, inequality (2.3) is fulfilled. Therefore
\[
\frac{x(h(t))}{x(t)} > \lambda_0 - \varepsilon, \quad \text{for all } t \geq t_2 \geq t_1, \quad (2.9)
\]
where \( \varepsilon \) is an arbitrary real number with \( 0 < \varepsilon < \lambda_0 \). Thus, there exists a \( t^* \in (h(t), t) \) such that
\[
\frac{x(h(t))}{x(t^*)} = \lambda_0 - \varepsilon, \quad \text{for all } t \geq t_2.
\]
Integrating (1.1) from \( t^* \) to \( t \) and using Lemma 1, we have
\[
x(t) - x(t^*) + x(h(t)) \int_{t^*}^{t} \sum_{i=1}^{m} p_i(\zeta) a_r(h(\zeta), \tau_i(\zeta)) d\zeta \leq 0.
\]
Hence
\[ \int_{t}^{t^*} \sum_{i=1}^{m} p_i(\xi) a_r(h(t), \tau_i(\xi)) d\xi \leq \frac{x(t^*)}{x(h(t))}, \]
or
\[ \int_{t}^{t^*} \sum_{i=1}^{m} p_i(\xi) a_r(h(\xi), \tau_i(\xi)) d\xi \leq \frac{x(t^*)}{x(h(t))}, \]
which, in view of (2.10), gives
\[ \int_{t}^{t^*} \sum_{i=1}^{m} p_i(\xi) a_r(h(\xi), \tau_i(\xi)) d\xi \leq \frac{1}{\lambda_0 - \varepsilon}. \] (2.11)

Dividing (1.1) by \( x(t) \), integrating from \( h(t) \) to \( t^* \) and using Lemma 1, we have
\[ - \int_{h(t)}^{t^*} \frac{x'(\xi)}{x(\xi)} d\xi \geq \int_{h(t)}^{t^*} \sum_{i=1}^{m} p_i(\xi) \frac{x(h(\xi))}{x(\xi)} a_r(h(\xi), \tau_i(\xi)) d\xi. \] (2.12)

Taking into account the fact that \( \xi \geq h(t) \) the inequality (2.9) guarantees that
\[ \frac{x(h(\xi))}{x(\xi)} > \lambda_0 - \varepsilon, \quad \text{for all } \xi \geq h(t) \geq t_2. \] (2.13)

In view of this, (2.12) gives
\[ - \int_{h(t)}^{t^*} \frac{x'(\xi)}{x(\xi)} d\xi > (\lambda_0 - \varepsilon) \int_{h(t)}^{t^*} \sum_{i=1}^{m} p_i(\xi) a_r(h(\xi), \tau_i(\xi)) d\xi, \]
or
\[ \int_{h(t)}^{t^*} \sum_{i=1}^{m} p_i(\xi) a_r(h(\xi), \tau_i(\xi)) d\xi \leq - \frac{1}{\lambda_0 - \varepsilon} \int_{h(t)}^{t^*} \frac{x'(\xi)}{x(\xi)} d\xi = \frac{1}{\lambda_0 - \varepsilon} \ln \frac{x(h(t))}{x(t^*)}. \]
i.e.,
\[ \int_{h(t)}^{t^*} \sum_{i=1}^{m} p_i(\xi) a_r(h(\xi), \tau_i(\xi)) d\xi \leq \frac{\ln(\lambda_0 - \varepsilon)}{\lambda_0 - \varepsilon}. \] (2.14)

Combining the inequalities (2.11) and (2.14), we have
\[ \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(\xi) a_r(h(\xi), \tau_i(\xi)) d\xi \leq \frac{1}{\lambda_0 - \varepsilon} + \frac{\ln(\lambda_0 - \varepsilon)}{\lambda_0 - \varepsilon}. \]

The last inequality holds true for all real numbers \( \varepsilon \) with \( 0 < \varepsilon < \lambda_0 \). Hence, for \( \varepsilon \to 0 \), we have
\[ \limsup_{t \to \infty} \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(\xi) a_r(h(\xi), \tau_i(\xi)) d\xi \leq \frac{1 + \ln \lambda_0}{\lambda_0}, \]
which contradicts (2.8). The proof of the theorem is complete. \( \square \)

**Example 1.** Consider the delay differential equation
\[ x'(t) + \frac{27}{200} x(\tau_1(t)) + \frac{27}{200} x(\tau_2(t)) = 0, \quad t \geq 0, \] (2.15)
with
\[ \tau_1(t) = \begin{cases} t - 1, & \text{if } t \in [3k, 3k + 1] \\ -3t + 12k + 3, & \text{if } t \in [3k + 1, 3k + 2] \\ 5t - 12k - 13, & \text{if } t \in [3k + 2, 3k + 3] \end{cases} \]
and \( \tau_2(t) = \tau_1(t) - 0.1, \quad k \in \mathbb{N}_0, \)
where $\mathbb{N}_0$ is the set of non-negative integers.

By (1.6), we see that

$$h_1(t) := \sup_{s \leq t} \tau_1(s) = \begin{cases} 
    t - 1, & \text{if } t \in [3k, 3k + 1] \\
    3k, & \text{if } t \in [3k + 1, 3k + 2.6] \\
    5t - 12k - 13, & \text{if } t \in [3k + 2.6, 3k + 3]
\end{cases}$$

and consequently

$$h(t) = \max_{1 \leq i \leq 2} \{h_i(t)\} = h_1(t).$$

Observe that the function $f : \mathbb{R}_0 \to \mathbb{R}_+$ defined as $f_r(t) = \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(\zeta) a_r(h(\zeta), \tau_i(\zeta)) \, d\zeta$ attains its maximum at $t = 3k + 2.6$, $k \in \mathbb{N}_0$, for every $r \in \mathbb{N}$. Specifically,

$$f_1(t = 3k + 2.6) = \int_{3k}^{3k+2.6} \sum_{i=1}^{2} p_i(\zeta) a_1(h(\zeta), \tau_1(\zeta)) \, d\zeta$$

$$= \int_{3k}^{3k+2} [p_1(\zeta) a_1(h(\zeta), \tau_1(\zeta)) + p_2(\zeta) a_1(h(\zeta), \tau_2(\zeta))] \, d\zeta$$

$$+ \int_{3k+1}^{3k+2.6} [p_1(\zeta) a_1(h(\zeta), \tau_1(\zeta)) + p_2(\zeta) a_1(h(\zeta), \tau_2(\zeta))] \, d\zeta$$

$$+ \int_{3k+2}^{3k+2.6} [p_1(\zeta) a_1(h(\zeta), \tau_1(\zeta)) + p_2(\zeta) a_1(h(\zeta), \tau_2(\zeta))] \, d\zeta$$

where

$$\int_{3k}^{3k+1} p_1(\zeta) a_1(h(\zeta), \tau_1(\zeta)) \, d\zeta = 0.135$$

$$\int_{3k}^{3k+1} p_2(\zeta) a_1(h(\zeta), \tau_2(\zeta)) \, d\zeta \simeq 0.138695$$

$$\int_{3k+1}^{3k+2} p_1(\zeta) a_1(h(\zeta), \tau_1(\zeta)) \, d\zeta \simeq 0.207985$$

$$\int_{3k+1}^{3k+2} p_2(\zeta) a_1(h(\zeta), \tau_2(\zeta)) \, d\zeta \simeq 0.213677$$

$$\int_{3k+2}^{3k+2.6} p_1(\zeta) a_1(h(\zeta), \tau_1(\zeta)) \, d\zeta \simeq 0.124791$$

$$\int_{3k+2}^{3k+2.6} p_2(\zeta) a_1(h(\zeta), \tau_2(\zeta)) \, d\zeta \simeq 0.128206$$

Thus

$$\limsup_{t \to \infty} f_1(t) = \limsup_{t \to \infty} \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(\zeta) a_1(h(\zeta), \tau_i(\zeta)) \, d\zeta \simeq 0.948354.$$ 

Now, we see that

$$\alpha = \liminf_{t \to \infty} \int_{\tau(t)}^{t} \sum_{i=1}^{m} p_i(s) \, ds = \frac{27}{100} \liminf_{t \to \infty} (t - \tau(t)) = 0.27 < \frac{1}{e},$$

$$\liminf_{t \to \infty} \sum_{i=1}^{m} p_i(t) (t - \tau_i(t)) = \frac{27}{200} \cdot 1 + \frac{27}{200} \cdot 1.1 = 0.2835 < \frac{1}{e},$$

and

$$\limsup_{t \to \infty} \int_{h(t)}^{t} \sum_{i=1}^{m} p_i(\zeta) a_r(h(\zeta), \tau_i(\zeta)) \, d\zeta \simeq 0.988865 < 1.$$
that is, none of the conditions (1.3), (1.4) and (1.8) is satisfied.

Observe, however, that the smaller root of $e^{\alpha \lambda} = \lambda$ is $\lambda_0 = 1.49883$. Thus

$$0.948354 > \frac{1 + \ln \lambda_0}{\lambda_0} \simeq 0.937188.$$ 

That is, condition (2.8) of Theorem 2 is satisfied for $r = 1$, and therefore all solutions of (2.15) oscillate.

Acknowledgement 1. The authors would like to thank both referees for the constructive remarks which improved the presentation of the paper.

REFERENCES

[1] R.P. Agarwal, L. Berezansky, E. Braverman and A. Domoshnitsky, Nonoscillation Theory of Functional Differential Equations with Applications, Springer, New York, 2012.
[2] E. Braverman, G. E. Chatzarakis and I. P. Stavroulakis, Iterative oscillation tests for differential equations with several non-monotone arguments, Adv. Difference Equ., 2016 (in press).
[3] E. Braverman, B. Karpuz, On oscillation of differential and difference equations with non-monotone delays, Appl. Math. Comput., 218 (2011) 3880–3887.
[4] L. H. Erbe, Qingkai Kong and B.G. Zhang, Oscillation Theory for Functional Differential Equations, Marcel Dekker, New York, 1995.
[5] L. H. Erbe and B. G. Zhang, Oscillation of first order linear differential equations with deviating arguments, Differential Integral Equations, 1 (1988), 305-314.
[6] N. Fukagai and T. Kusano, Oscillation theory of first order functional-differential equations with deviating arguments, Ann. Mat. Pura Appl. 136 (1984), 95–117.
[7] B. R. Hunt and J. A. Yorke, When all solutions of $x'(t) = - \sum q_i(t)x(t - T_i(t))$ oscillate, J. Differential Equations 53 (1984), 139–145.
[8] G. Infante, R.Koplatadze and I. P. Stavroulakis, Oscillation criteria for differential equations with several retarded arguments, Funkcial. Ekvac., 58 (2015), 347–364.
[9] R.G. Koplatadze and G. Kvinikadze, On the oscillation of solutions of first order delay differential inequalities and equations, Georgian Math. J., 3 (1994), 675–685.
[10] T. Kusano, On even-order functional-differential equations with advanced and retarded arguments, J. Differential Equations 45 (1982), 75–84.
[11] M. K. Kwong, Oscillation of first-order delay equations, J. Math. Anal. Appl. 156 (1991), 274–286.
[12] G. Ladde and I. P. Stavroulakis, Oscillations caused by several retarded and advanced arguments, J. Differential Equations 44 (1982), 134–152.
[13] G.S. Ladde, Oscillations caused by retarded perturbations of first order linear ordinary differential equations, Atti Acad. Naz. Lincei Rendiconti 63 (1978), 351–359.
[14] G.S. Ladde, V. Lakshmikantham, B.G. Zhang, Oscillation Theory of Differential Equations with Deviating Arguments, Monographs and Textbooks in Pure and Applied Mathematics, vol. 110, Marcel Dekker, Inc., New York, 1987.
[15] H. Onose, Oscillatory properties of the first-order differential inequalities with deviating argument, Funkcial. Ekvac. 26 (1983), 189–195.
[16] I.P. Stavroulakis, Oscillation criteria for delay and difference equations with non-monotone arguments, Appl. Math. Comput. 226 (2014), 661-672.
[17] X.H. Tang, Oscillation of first order delay differential equations with distributed delay, J. Math. Anal. Appl. 289 (2004), 367–378.
[18] D. Zhou, On some problems on oscillation of functional differential equations of first order, J. Shandong University 25 (1990), 434–442.
H. AKCA, G. E. CHATZARAKIS*, AND I. P. STAVROULAKIS

Department of Applied Sciences and Mathematics, College of Arts and Sciences, Abu Dhabi University, Abu Dhabi, UAE
E-mail address: haydar.akca@adu.ac.ae, akcahy@yahoo.com

Department of Electrical and Electronic Engineering Educators, School of Pedagogical and Technological Education (ASPETE), 14121, N. Heraklio, Athens, Greece
E-mail address: geaxatz@otenet.gr, gea.xatz@aspete.gr

Department of Mathematics, University of Ioannina, 451 10 Ioannina, Greece
E-mail address: ipstav@uoi.gr