The dependence of strange hadron multiplicities on the speed of hadronization

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Abstract

Hadron multiplicities are calculated in the ALCOR model [1, 2, 3] for the Pb+Pb collisions at CERN SPS energy. Considering the newest experimental results, we display our prediction obtained from the ALCOR model for stable hadrons including strange baryons and anti-baryons.

1 Introduction

With the presentation of this talk we would like to raise the question how a theory can be disproved. To reply this question we have to point out that quantitative predictions of a model must be published before the corresponding experimental data are known. In this spirit we published our predictions for the Pb+Pb collision in the proceedings of Strange Matter '96 [3]. Now it turned out that the simple ALCOR model, which described excellently the S+S results [2, 3] fails to account for all the recently measured data in Pb+Pb collision, presented at this conference [4, 5]. Therefore at least one of the assumptions underlying the original ALCOR model has to be modified. In particular searching for the cause of our disagreement on the $\Omega^+/\Omega^-$ ratio we found that not all hadronization processes are as sudden as it would verify the coalescence approximation of the underlying evolution.

Since a long time the strange particles are considered to be the important messengers of the happenings in the heavy ion reactions. This has two aspects: i) the amount of created $S\bar{S}$ pairs and ii) the ratio of the multiplicities of different strange hadrons. While

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the first question rather belongs to the area of the collision models, rehadronization models search the answers to the second problem.

In the early years it was assumed that by whatever mechanism, the colliding nuclei will stop on each other to produce a quark-gluon plasma. It was assumed to exist in thermal and chemical equilibrium. At the next level of theoretical sophistication it was assumed to hadronize in a very slow quasi-equilibrium process. When compared to the pure hadronic scenario, this leads to a strongly different multiplicity distribution. Then we were happy that we had a nice signature for the quark-gluon plasma.

In the last six years, however, different nucleon-nucleon collision models were introduced, which claimed that they can describe the ultrarelativistic heavy ion reactions by purely hadron-hadron collisions. Unfortunately, the attention was too much focused on these Monte Carlo hadron collision programs. There were only few attempts to discuss the hadronization using the concepts of quasi-stationary thermodynamics. We started, on the other hand, a completely out of equilibrium hadronization approach. The first model was constructed in 1994 [1], and I was happy to hear:

"Finally, there is a real need for more sophisticated dynamical models of the transition from quark-gluon plasma to hadronic gas."

This was said in the lecture of B. Müller at the QM'95 conference in Monterey, in January 1995.

In the present work we discuss principles of the fast hadronization model, ALCOR, and compare its predictions for the multiplicities of strange hadrons with experimental data. Eventually we address the possibility, that the very recently measured $\Omega^+/\Omega^-$ ratio still can agree with the fast hadronization scenario for mesons assuming a time-dependent screening length in the quark matter.

First we show how ALCOR emerges from requiring such a solution of the rate equations for quarks, antiquarks and diquarks, by which all free quarks and antiquarks vanish after hadronization.

The basic philosophy of ALCOR consists of several assumptions about the physical conditions prior to hadronization:

- the initial state is a quark-antiquark plasma (and not a quark-gluon plasma),
- the constituent particles are dressed having in-medium masses,
- the momentum distributions of valence quarks and antiquarks correspond to a thermal distribution having longitudinal and transversal flow, too,
there are microscopic descriptions which yield cross section for the process $q + 7 \rightarrow \text{hadrons}$, and the produced hadrons are immediately in the hadron phase (like in case of a Mott transition),

the momentum distribution of produced first generation hadrons are determined by the valence quark momentum distribution and by the kinematics of the microprocesses,

the question, whether the system will have a long enough lifetime for allowing further interactions between hadrons before break up to change the momentum distribution is left open.

In the original formulation of the ALCOR model we used the picture of quark redistribution among the final hadrons for determining internal parameters, the $b$ factors. Now we generalize this concept by relating these factors to the time-integral average of products of different species numbers.

\section{Hadronization channels}

In general we consider many possible hadronization channels. Noting the reaction channel index by $\nu$, the (anti)quark flavor index by $i, j, k$ the diquark flavor by $(ij)$ a meson made from quark $i$ and antiquark $j$ by $[ij]$ and finally a baryon made from quark $i$ and diquark $(jk)$ by $[ijk]$, we arrive at reactions in different reaction channels $\nu$. They can be classified by the following types, creating primarily mesons, (anti)diquarks and (anti)baryons:

\begin{equation}
\begin{aligned}
m_{i}^{\nu}q_{i} + \overline{m}_{j}^{\nu}q_{j} \rightarrow m_{[ij]}^{\nu}M_{ij} + m_{(ij)}^{\nu}D_{ij} + m_{(ij)}^{\nu}\overline{D}_{ij}
\end{aligned}
\end{equation}

with cross section $\sigma_{i + j \rightarrow [ij]}^{\nu}$,

\begin{equation}
\begin{aligned}
d_{i}^{\nu}q_{i} + d_{j}^{\nu}q_{j} \rightarrow d_{[ij]}^{\nu}M_{ij} + d_{(ij)}^{\nu}D_{ij}
\end{aligned}
\end{equation}

with cross section $\sigma_{i + j \rightarrow (ij)}^{\nu}$,

\begin{equation}
\begin{aligned}
b_{i}^{\nu}q_{i} + b_{(jk)}^{\nu}D_{(jk)} \rightarrow b_{[ij]}^{\nu}M_{ij} + b_{(ijk)}^{\nu}B_{ijk}
\end{aligned}
\end{equation}

with cross section $\sigma_{i + (jk) \rightarrow [ijk]}^{\nu}$. The $m_{i}^{\nu}$, $d_{i}^{\nu}$, and $b_{i}^{\nu}$ are the appropriate stoichiometric coefficients. The processes for antihadrons are obtained due to interchanging quarks and antiquarks, the cross sections we consider are the same.
3 Effective rates

Using the above stoichiometric coefficients and cross sections we define the following effective rates for number changing reactions:

quark rates

\[ Q_{ij} = \sum_{\nu} m_{\nu i}^{\nu} \langle \sigma_{i+\nu \rightarrow [ij]}^{\nu} \rangle, \quad Q_{ij} = \sum_{\nu} d_{\nu i}^{\nu} \langle \sigma_{i+\nu \rightarrow (ij)}^{\nu} \rangle, \]

\[ Q_{ijk} = \sum_{\nu} b_{\nu i}^{\nu} \langle \sigma_{i+(jk) \rightarrow [ijk]}^{\nu} \rangle, \] \hspace{1cm} (4)

diquark rates

\[ D_{jk} = \sum_{\nu} m_{\nu j}^{\nu} \langle \sigma_{j+(Ik) \rightarrow (jk)}^{\nu} \rangle, \quad D_{jk} = \sum_{\nu} d_{\nu j}^{\nu} \langle \sigma_{j+k \rightarrow (jk)}^{\nu} \rangle, \]

\[ D_{ijk} = \sum_{\nu} b_{\nu j}^{\nu} \langle \sigma_{i+(jk) \rightarrow [ijk]}^{\nu} \rangle, \] \hspace{1cm} (5)

meson rates

\[ M_{ij} = \sum_{\nu} m_{\nu i}^{\nu} \langle \sigma_{i+\nu \rightarrow [ij]}^{\nu} \rangle, \quad M_{ij} = \sum_{\nu} d_{\nu i}^{\nu} \langle \sigma_{i+\nu \rightarrow (ij)}^{\nu} \rangle, \]

\[ M_{ijk} = \sum_{\nu} b_{\nu i}^{\nu} \langle \sigma_{i+(jk) \rightarrow [ijk]}^{\nu} \rangle, \] \hspace{1cm} (6)

and baryon rates

\[ B_{ijk} = \sum_{\nu} b_{\nu i}^{\nu} \langle \sigma_{i+(jk) \rightarrow [ijk]}^{\nu} \rangle. \] \hspace{1cm} (7)

Many of these rates can be zero in a given model, specified by a set of cross sections \( \sigma^{\nu} \). Using the above notations one can derive the following rate equations for quarks

\[ V \frac{dN_{i}}{dt} = -\sum_{j} Q_{ij}N_{i}N_{j} - \sum_{j} Q_{(ij)}N_{i}N_{j} - \sum_{jk} Q_{[ijk]}N_{i}N_{(jk)}, \] \hspace{1cm} (8)

for diquarks

\[ V \frac{dN_{(jk)}}{dt} = D_{(jk)}N_{j}N_{k} + \sum_{l} D_{[jk]l}N_{j}N_{l} - \sum_{i} D_{[ijk]}N_{i}N_{(jk)} \] \hspace{1cm} (9)

for mesons

\[ V \frac{dN_{[ij]}}{dt} = M_{[ij]}N_{i}N_{j} + M_{(ij)}N_{i}N_{j} + \sum_{k} M_{[ijk]}N_{i}N_{(jk)} \] \hspace{1cm} (10)

and finally for baryons

\[ V \frac{dN_{[ijk]}}{dt} = B_{[ijk]}N_{i}N_{(jk)} + \text{permutations}. \] \hspace{1cm} (11)

Without solving this set of time-dependent differential equations, what we eventually do, we can estimate time-averages assuming that they are proportional to the product of
corresponding initial numbers of the reacting species. First we integrate the rate equations for quarks, yielding

\[
V \int_0^\tau \frac{dN_i(t)}{dt} \, dt = - \sum_j Q_{[ij]} \int_0^\tau N_i(t)N_j(t) \, dt \\
- \sum_j Q_{(ij)} \int_0^\tau N_i(t)N_j(t) \, dt - \sum_{jk} Q_{[ijk]} \int_0^\tau N_i(t)N_{(jk)}(t) \, dt.
\] (12)

Now we replace the time integrals of products by values scaled by the initial numbers of (anti)quark species

\[
\frac{V}{\tau} N_i(0) = + \sum_j Q_{[ij]} b_i N_i(0) \bar{b}_j \bar{N}_j(0) \\
+ \sum_j Q_{(ij)} b_i N_i(0) b_j N_j(0) + \sum_{jk} Q_{[ijk]} b_i N_i(0) N_{(jk)}^{\text{eff}}.
\] (13)

The coefficients \(b_i\) are then internal variables of the model which can be determined by solving algebraic equations. This system of equations were closed without the diquark contribution; their effect is taken into account by using their effective number, \(N_{(jk)}^{\text{eff}}\), which represents the time integral in the original problem.

In order to obtain \(N_{(jk)}^{\text{eff}}\) we integrate the rate equation for diquarks, yielding

\[
\frac{V}{\tau} [N_{(jk)}(\tau) - N_{(jk)}(0)] = +D_{(jk)} b_j N_j(0) b_k N_k(0) \\
+ \sum_l D_{[jkl]} b_j N_j(0) \bar{b}_l \bar{N}_l(0) - \sum_i D_{[ijk]} b_i N_i(0) N_{(jk)}(t^*)
\] (14)

Let us now assume, that the number of diquarks is zero both at \(t = 0\) and at \(t = \tau\). Then from the above equation one expresses \(N_{(jk)}^{\text{eff}}\) as

\[
N_{(jk)}(t^*) = \frac{D_{(jk)} b_j N_j(0) b_k N_k(0) + \sum_l D_{[jkl]} b_j N_j(0) \bar{b}_l \bar{N}_l(0)}{\sum D_{[jkl]} b_i N_i(0)}
\] (15)

Substituting this expression into equation(16), we can determine the \(b_i\) factors, since there are just as many independent equation then as many \(b_i\) factors we have. Knowing the \(b_i\) factors, we can integrate the differential equations for the mesons and baryons, and thus their number after hadronization is obtained.

The number of produced mesons becomes

\[
N_{[ij]}(\tau) = \frac{\tau}{V} [M_{[ij]} N_i(0) \bar{N}_j(0) \bar{b}_i \bar{b}_j \\
+ M_{(ij)} N_i(0) N_j(0) b_j b_j + \sum_k M_{[ijk]} N_i(0) b_i N_{(jk)}^{\text{eff}}]
\] (16)

and the number of produced baryons:

\[
N_{[ijk]}(\tau) = \frac{\tau}{V} [B_{[ijk]} N_i(0) b_i N_{(jk)}^{\text{eff}} + \text{permutations}].
\] (17)
Finally we note that in order to resemble a coalescence model we define

\[ N_{(jk)}^{\text{eff}} = g_B N_j(0)N_k(0)b_jb_k, \]  

and call \( g_B \) baryon suppression factor. Now every produced hadron number is combined from initial quark number products.

In Table. 1 we summarized our results on the ALCOR analysis of the Pb+Pb data. The “experimental data” was obtained directly from the references (see Ref. \[1, 2\]) or we estimated the total multiplicities from the published (preliminary) rapidity spectra (see Ref. \[4, 7\]). We assumed different values for \( g_S \) to indicate differences in strangeness production. Then the total \( h^- \) multiplicity \((\langle h^- \rangle = 680)\) was fixed by parameter \( N_{q,pair} \). Finally we obtained value for the baryon suppression, \( g_B \), from the estimated multiplicity of the \( \Upsilon^0 \), which is \( \{\Upsilon^0\} \approx 8 \). Here we used the usual notation \( \Upsilon^0 = \Sigma^0 + \Lambda^0 \) and \( \Upsilon^0 = \Sigma^0 + \Lambda^0 \) since these two particles can not be distinguished experimentally \[4\].

From Table 1. one can obtain the following conclusions:

- The ALCOR model can reproduce approximately the experimental data. Since these data are preliminary, it was not required a full agreement, immediately.

- The best fit of the ALCOR model is in the region \( g_S = 0.22 - 0.24 \), which is close to the value of \( g_S = 0.255 \) obtained in the S+S collision. However, we did not find any further strangeness enhancement beyond the level obtained in the S+S collision, even it is a little bit less, as it was mentioned in Ref. \[4\]. This is clearly indicated in the \( Kaon/Pion \) ratio. On the other hand the \( K^+, K^- \) and \( K_S^0 \) multiplicities can not be reproduced together, at the same time, we overpredicted the \( K^+ \). Also we have a slight overprediction for the \( Y^0 \).

- Investigating the multi-strange hyperon ratios, one can see that at \( g_S \geq 0.24 \) three of the four ratios were reproduced quite well by the ALCOR, on the other hand the \( \Omega^+/\Omega^- \) ratio is out of the range of the ALCOR predictions by a factor of 2. Since there is a trivial connection among the multi-strange ratios,

\[ \frac{\Omega^+}{\Omega^-} = \frac{\Omega^+}{\Xi^+} \cdot \frac{\Xi^+}{\Xi^-} \cdot \left( \frac{\Omega^-}{\Xi^-} \right)^{-1}, \]  

we conclude that reproducing the ratios on the right hand side with a given error do not automatically reproduce the \( \Omega^+/\Omega^- \) ratio in the left hand side. In fact we obtained a 100 \% disagreement on this ratio.

- We nearly doubled the baryon suppression factor, \( g_B \). (It’s value was \( g_B^0 \) for S+S collision \[2\].) This indicates that the inner microscopical processes changed in the Pb+Pb collision, the presence of a quark matter medium is stronger.
Table 1: Hadron multiplicities are displayed for $Pb+Pb$ collision. The values of the second column were observed experimentally or were estimated from the experimental data by us, we used the notation $\{ \}$. The other columns contain the prediction of the ALCOR model at different strangeness production, $g_S$.

| Pb+Pb | Exp. data | ALCOR |
|-------|-----------|--------|
| $g_S$ | —         | 0.16   | 0.18 |
| $N_{q,\text{pair}}$ | —         | 415    | 408  |
| $g_B/g_B^0$ | —         | 1.6    | 1.7  |
| $h$   | 680 $^a$  | 680    | 680  |
| $K^0_S$ | $\{54\}^{b,c}$ | 41.5  | 46.2 |
| $K^+$ | $\{56\}^c$ | 57.1   | 63.6 |
| $K^-$ | $\{32\}^c$ | 25.9   | 28.7 |
| $p^+ - \bar{p}^-$ | $\{145\}^a$ | 156.6  | 153.5 |
| $Y^0$ | $\{50\}^c$ | 49.0   | 52.9 |
| $\bar{Y}^0$ | $\{8\}^c$ | 8.2    | 8.0  |
| $\Xi^+ / \Xi^-$ | $0.27 \pm 0.05^d$ | 0.39  | 0.36 |
| $\Omega^+ / \Omega^-$ | $0.42 \pm 0.12^d$ | 0.91  | 0.85 |
| $\Omega^- / \Xi^-$ | $0.19 \pm 0.04^d$ | 0.10  | 0.11 |
| $\Omega^+ / \Xi^+$ | $0.30 \pm 0.09^d$ | 0.23  | 0.26 |
| $(K+K)/\pi$ | $0.13^c$ | 0.09  | 0.10 |
| $K^- / \pi^-$ | $0.04$ | 0.04  | 0.05 |
| Meson/Baryon | $4.67$ | 4.72  | 4.77 |

$^a$ From [6].
$^b$ Estimated from [4].
$^c$ Estimated from [4].
$^d$ From [5].

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