A Thread-Local Semantics and Efficient Static Analyses for Race Free Programs

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Abstract Data race free (DRF) programs constitute an important class of concurrent programs. In this paper we provide a framework for designing and proving the correctness of data flow analyses that target this class of programs. These analyses are in the same spirit as the “sync-CFG” analysis proposed in earlier literature. To achieve this, we first propose a novel concrete semantics for DRF programs, called \( L \)-DRF that is thread-local in nature – each thread operates on its own copy of the data state. We show that abstractions of our semantics allow us to reduce the analysis of DRF programs to a sequential analysis. This aids in rapidly porting existing sequential analyses to sound and scalable analyses for DRF programs. Next, we parameterize \( L \)-DRF with a partitioning of the program variables into “regions” which are accessed atomically. Abstractions of the region-parameterized semantics yield more precise analyses for region-race free concurrent programs. We instantiate these abstractions to devise efficient relational analyses for race free programs, which we have implemented in a prototype tool called RATCOP.

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On the benchmarks, RATCOP was able to prove up to 65% of the assertions, in comparison to 25% proved by our baseline. Moreover, in a comparative study with a recent concurrent static analyzer, RATCOP was up to 5 orders of magnitude faster.

**Keywords** Abstract Interpretation · Concurrent Programs · Static Analysis · Data-race freedom

### 1 Introduction

Our aim in this work is to provide a framework for developing data-flow analyses which specifically target the class of data race free (DRF) concurrent programs. DRF programs constitute an important class of concurrent programs, as most programmers strive to write race free code. There are a couple of reasons why programmers do so. Firstly, even assuming sequential consistency (SC) semantics, a racy program often leads to undesirable effects like atomicity violations. Secondly, under the prevalent “SC-for-DRF” policy only DRF programs are guaranteed to have sequentially consistent execution behaviors in many weak memory models [1,6,24]. Non-DRF programs do not have this guarantee; for example the Java Memory Model [24] gives some weak guarantees, while the C++ semantics [6] gives essentially no guarantees, for the execution semantics of racy programs. Thus ensuring that a racy program does something useful is a difficult job for a programmer. For these and other reasons, programmers tend to write race free programs. There is thus a large code base of DRF programs that can benefit from data-flow analysis techniques that leverage the property of race-freedom to provide analyses that run efficiently.

The starting point of this work is the “sync-CFG” style of statically analyzing DRF programs, proposed in [11]. The analysis here essentially runs a sequential analysis on each thread, communicating data-flow facts between threads only via “synchronization edges”, that go from a release statement in one thread to the corresponding acquire statement in another thread. The analysis thus runs on the control-flow graphs (CFGs) of the threads, augmented with synchronization edges, as shown in the center of Fig. 2, which explains the name for this style of analysis. The analysis computes data flow facts about the value of a variable that are sound only at points where that variable is relevant, in that it is read or written to at that location. The analysis thus trades unsoundness of facts at irrelevant points for the efficiency gained by restricting interference between threads to points of synchronization alone.

However, the analysis in [11] suffers from some drawbacks. Firstly, the analysis is intrinsically a “value-set” analysis, which can only keep track of the set of values each variable can assume, and not the relationships between variables. Any naive attempt to extend the analysis to a more precise relational one quickly leads to unsoundness. The second issue is to do with the technique for establishing soundness. A convenient way to prove soundness of an analysis is to show that it is a consistent abstraction [9] of a canonical analysis, like the collecting semantics for sequential programs [9] or the interleaving semantics for concurrent programs [22]. For this one typically makes use of the “local” sufficient conditions for consistent abstraction given in [9]. However, for a sync-CFG-based analysis, it
appears difficult to use this route to show it to be a consistent abstraction of the standard interleaving semantics. This is largely due to the thread-local nature of the states and the unsoundness at irrelevant points, which makes it difficult to come up with natural abstraction and concretization functions that form a Galois connection. Instead, one needs to resort to an intricate argument, as done in [11], which essentially shows that in the least fixed point of the analysis, every write to a variable will flow to a read of that variable via a happens-before path (that is guaranteed to exist by the property of race-freedom). Thus, while one can argue soundness of abstractions of the value-set analysis by demonstrating a consistent abstraction with the latter, to argue soundness of any other proposed sync-CFG style analysis (in particular one that uses a more precise domain than value-sets), one would have to work out a similar involved proof as in [11].

Towards addressing these issues, we propose a framework that facilitates the design of different sync-CFG analyses with varying degrees of precision and efficiency. The foundation of this framework is a novel thread-local semantics for DRF programs, which can play the role of a “most precise” analysis which other sync-CFG analyses can be shown to be consistent abstractions of. This semantics, which we call L-DRF [30], is similar to the interleaving semantics of concurrent programs, but keeps thread-local (or per-thread) copies of the shared state. Intuitively, our semantics works as follows. Apart from its local copy of the shared data state, each thread also maintains a per-variable version count, which is incremented whenever it writes to the variable. The exchange of information between threads is via buffers, associated with release program points in the program. When a thread releases a lock, it stores its local data state to the corresponding buffer, along with the version counts of the variables. As a result, the buffer of a release point records both the local data state and the variable versions, as they were, when the release was last executed. When some thread subsequently acquires the lock, it compares its per-variable version count with those in the buffers pertaining to release points associated with the lock. The thread then copies over the valuation (and the version) of a variable to its local state, if it is newer in some buffer (as indicated by a higher version count). The value of a shared variable in the local state of a thread may be “stale”, in that the variable has subsequently been updated by another thread but has not yet been reflected here. The L-DRF semantics leverages the race freedom property to ensure that the value of a variable is correct in the local state at program points where it is relevant (read or written to). It thus captures the essence of a sync-CFG analysis. The L-DRF semantics is also of independent interest, since it can be viewed as an alternative characterization of the behavior of data race free programs.

The analysis induced by the L-DRF semantics is shown to be sound for DRF programs. In addition, the analysis is, in some sense, the most precise sync-CFG analysis one can hope for: at every point in a thread, the relevant part of the thread-local copy of the shared state is guaranteed to arise in some execution of the program.

Using the L-DRF semantics as a basis, we now propose several precise and efficient relational sync-CFG analyses. The soundness of these analyses all follow immediately, since they can easily be shown to be consistent abstractions of L-DRF. The key idea behind obtaining a sound relational analysis is suggested by the L-DRF analysis: we preserve variable correlations within a thread, whereas
at each acquire point, we apply a mix operator on the abstract values. The mix operation essentially amounts to forgetting all correlations between the variables.

While these analyses allow maintaining fully-relational properties within thread-local states, communicating information over cross-thread edges loses all correlations due to the mix operation. To improve precision further, we refine the $\mathbb{L}$-DRF semantics to take into account data regions. Technically, we introduce the notion of region race freedom and develop the $\mathbb{L}$-RegDRF semantics: the programmer can partition the program variables into “regions” that should be accessed atomically. A program is region race free if it does not contain conflicting accesses to variables in the same region, that are unordered by the happens-before relation. The classical notion of data race freedom is a special case of region race freedom, where each region consists of a single variable. Techniques to determine whether a program is race free can be naturally extended to determine region race freedom (see Sec. 7). $\mathbb{L}$-RegDRF refines $\mathbb{L}$-DRF by taking into account the atomic nature of accesses that the program makes to variables in the same region. For programs which are free from region-races, $\mathbb{L}$-RegDRF produces executions which are indistinguishable, with respect to reads of the regions, from the ones produced by $\mathbb{L}$-DRF. By leveraging the $\mathbb{L}$-RegDRF semantics as a starting point, we obtain more precise sequential analyses that track relational properties within regions and across threads. This is obtained by refining the granularity of the mix operator from single variables to regions.

We have implemented the new relational analyses (based on $\mathbb{L}$-DRF and $\mathbb{L}$-RegDRF) in a prototype analyzer called RATCOP, and provide a thorough empirical evaluation in Sec. 8. We show that RATCOP attains a precision of up to 65% on a subset of race-free programs from the SV-COMP15 suite. This subset contains programs which have interesting relational invariants. In contrast, an interval based value-set analysis derived from [11] (which we use as our baseline) was able to prove only 25% of the assertions. On a separate set of experiments, RATCOP turns out to be nearly 5 orders of magnitude faster than an existing state-of-the-art abstract interpretation based tool.

The rest of this paper is organized as follows. In the next section we give an overview of our thread-local semantics and the associated analyses. In Sec. 3 we define our programming language and its standard interleaving semantics. Sec. 4 contains the L-DRF semantics and the proof of its soundness and completeness vis-a-vis the standard semantics. We then introduce some analyses inspired by the L-DRF semantics, and formally show how we can prove their soundness by showing them to be a consistent abstraction of the $\mathbb{L}$-DRF semantics. In Sec. 7 we introduce our region-based analysis. In Sec. 8 we describe the implementation of our analyses, and experimental evaluation. We conclude in Sec. 9 with related work and discussion.

2 Overview

We illustrate the $\mathbb{L}$-DRF semantics, and its sequential abstractions, on the simple program in Fig. 1. We assume that all variables are shared and are initialized to 0. The threads access $x$ and $y$ only after acquiring lock $m$. The program is free from data races.
Thread t₁() {
1: acquire(m);
2: x := y;
3: x++;  
4: y++;
5: assert(x=y);
6: release(m);
7: }

Thread t₂() {
8: z++;
9: assert(z=1);
10: acquire(m);
11: assert(x=y);
12: release(m);
13: }

Fig. 1 A simple race free multi-threaded program. The variables x, y and z are shared and initialized to 0.

Fig. 2 shows the sync-CFG representation of the program (the control-flow graphs of the threads have been made implicit to improve clarity) in the center. The columns to the left and right show data flow facts obtained using three different analyses based on the L-DRF semantics, which we will describe later.

A state in the L-DRF semantics keeps track of the following components: a location map pc mapping each thread to the location of the next command to be executed, a lock map μ which maps each lock to the thread holding it,
a local environment (variable to value map) $\Theta$ for each thread, and a function $\Lambda$ which maps each buffer (associated with each program location following a release command) to an environment. Every release point of each lock $m$ has an associated buffer, where a thread stores a copy of its local environment when it executes the corresponding release instruction. In the environments, each variable $x$ has a version count associated with it which, along any execution $\pi$, essentially associates this valuation of $x$ with a unique prior write to it in $\pi$. As an example, the “versioned” environment $\langle x \mapsto 1^2, y \mapsto 1^1, z \mapsto 0^0 \rangle$, obtained at some point in an execution $\pi$, says that $x$ has the value 1 by the second write to $x$, $y$ has the value 1 by the first write to $y$ in $\pi$, and $z$ has not been written to. An execution is an interleaving of commands from different threads. Consider an execution of the program in Fig. 1 where, after a certain number of interleaved steps, we have the state

$$pc : t_1 \mapsto 6, t_2 \mapsto 10$$

$$\Theta(t_1) : x \mapsto 1^2, y \mapsto 1^1, z \mapsto 0^0$$

$$\Theta(t_2) : x \mapsto 0^0, y \mapsto 0^0, z \mapsto 1^1$$

$$\mu : m \mapsto t_1$$

$$\Lambda : 7 \mapsto \perp, 13 \mapsto \perp$$

The release buffers are all empty as no thread has executed a release yet. Note that the values (and versions) of $x$ and $y$ in $\Theta(t_2)$ (similarly for $z$ in $t_1$) are stale, as they do not have the latest value of these variables which were updated by another thread. Next, $t_1$ can execute the release at line 6, thereby setting $\mu(m) = \perp$ and storing its current local versioned environment to $\Lambda(7)$. Now $t_2$ can execute the acquire at line 10. In doing so, the following state changes take place. As usual, the $pc$ is updated to say that $t_2$ is now at line 11, and the lock map is updated to say that $t_2$ now holds lock $m$. Additionally $t_2$ “imports” the most up-to-date values (and versions) of $x$ and $y$ from the release buffer $\Lambda(7)$. We call this inter-thread join operation a mix. This results in its local state becoming $\langle x \mapsto 1^2, y \mapsto 1^1, z \mapsto 1^1 \rangle$ (the valuations of $x$ and $y$ are pulled in from the buffer, while the valuation of $z$ in $t_2$’s local state persists). The state thus becomes

$$pc : t_1 \mapsto 7, t_2 \mapsto 11$$

$$\Theta(t_1) : x \mapsto 1^2, y \mapsto 1^1, z \mapsto 0^0$$

$$\Theta(t_2) : x \mapsto 1^2, y \mapsto 1^1, z \mapsto 1^1$$

$$\mu : m \mapsto t_2$$

$$\Lambda(7) : x \mapsto 1^2, y \mapsto 1^1, z \mapsto 0^0$$

$$\Lambda(13) : \perp$$

We note that the values of $x$ and $y$ in $\Theta(t_2)$ are no longer stale: the L-DRF semantics leverages race freedom to ensure that the values of $x$ and $y$ are correct when they are read at line 11.

Roughly, we obtain sequential data-flow abstractions of the L-DRF semantics via the following steps:
- Provide a data abstraction of sets of environments.
- Define the state to be a map from locations to these abstract data values.
- Compute the sync-CFG representation of the program by drawing inter-thread edges which connect releases and acquires of the same lock (as shown in the center of Fig. 2).
- Define an abstract mix operation which soundly approximates the “import” step outlined earlier.
- Analyze the program as if it was a sequential program, with inter-thread join points (the acquire's) using the mix operator.

The analysis in [11] is precisely such a sequential abstraction, where the abstract data values are abstractions of value-sets (variables mapped to sets of values). Value sets do not track correlations between variables, and only allow coarse abstractions like Intervals [8]. The mix operator, in this case, turns out to be the standard join (union of value-sets). For the program of Fig. 1, the interval based value-set analysis, shown in the column “Value-Set” in Fig. 2 only manages to prove the assertion at line 9.

A more precise relational abstraction of L-DRF which we call Rel can be obtained by keeping track of a set of environments at each point. Fig. 2 shows (in the column “Rel”) the results of such an analysis implemented using convex polyhedra [10]. The resulting analysis is more precise than the interval analysis, being able to prove the assertions at lines 5 and 9. However, in this case, the mix must forget the correlations among variables in the incoming states: it essentially treats them as value sets. This is essential for soundness. Thus, even though the acquire at line 10 obtains the fact that $x = y$ from the buffer at 7, and the incoming fact from 9 also has $x = y$, it fails to maintain this correlation after the mix. Consequently, it fails to prove the assertion at line 11.

Finally, one can exploit the fact that $x$ and $y$ form a data “region” in that they are protected by the same lock. The variable $z$ constitutes a region by itself. As we show in later in Sec. 7 the program is region race free for this particular region definition. One can parameterize the L-DRF semantics with this region definition, to yield the L-RegDRF semantics. The resulting analysis called RegRel maintains relational information as in the Rel analysis, but has a more precise mix operator which preserves relational facts that hold within a region. Since both the incoming facts at line 10 satisfy $x = y$, the mix preserves this fact, and the analysis is able to prove the assertion at line 11.

Note that in all the three analyses, we are guaranteed to compute sound facts for variables only at points where they are accessed. For example, all three analyses claim that $x$ and $y$ are both 0 at line 9, which is clearly wrong. However, we note that $x$ and $y$ are not accessed at this point. This loss of soundness at “irrelevant” points helps us gain efficiency in the analysis by not having to propagate all interferences from one thread to all points of another thread. We also point out that in Fig. 2 the inter-thread edges add a spurious loop in the sync-CFG (and, therefore, in the analysis of the program), which prevents us from computing an upper bound for the values of $x$ and $y$. We show in Sec. 5.5 how we can appropriately abstract the versions to avoid some of these spurious loops.
3 Programming Language and Semantics

In this section we introduce the programming language we use to describe multi-threaded programs, and describe the standard interleaving semantics for programs in this language.

3.1 Preliminaries

We begin by introducing some of the mathematical notation we will use in this paper. We denote the set of natural numbers \( \{0, 1, \ldots\} \) by \( \mathbb{N} \). We use \( \rightarrow \) and \( \Rightarrow \) to denote total and partial functions, respectively. We use "\( \perp \)" to denote an undefined value, which we assume is included in every domain under consideration. We denote the length of a finite sequence of elements \( \pi \) by \( |\pi| \), and the \( i \)-th element of \( \pi \), for \( 0 \leq i < |\pi| \), by \( \pi_i \). For a function \( f : A \rightarrow B \), we denote by \( \text{dom}(f) \) its domain \( A \), and for \( a \in A \) and \( b \in B \), we write \( f[a \mapsto b] \) to denote the function \( f' : A \rightarrow B \) such that \( f'(x) = b \) if \( x = a \), and \( f(x) \) otherwise. For a pair of elements \( \langle \phi, \nu \rangle \), we write \( \langle \phi, \nu \rangle.1 \) to denote the first component \( \phi \), and \( \langle \phi, \nu \rangle.2 \) to denote the second component \( \nu \), of the pair \( \langle \phi, \nu \rangle \).

We will make use of the standard notion of labelled transition systems to describe the semantics we will give to our programs. A Labelled Transition System (LTS) is a structure \( L = (S, \Gamma, s_0, \rightarrow) \), where \( S \) is a set of states, \( \Gamma \) is a set of transition labels, \( s_0 \in S \) is the initial state, and \( \rightarrow \subseteq S \times \Gamma \times S \) is the (labelled) transition relation. We sometimes write a transition \( t = \langle s, l, s' \rangle \) as \( s \rightarrow_l s' \).

An execution of an LTS \( L = (S, \Gamma, s_0, \rightarrow) \), is a finite sequence of transitions \( \pi = t_1, t_2, \ldots, t_n \) \( (n \geq 0) \) from \( \rightarrow \), such that there exists a sequence of states \( q_0, q_1, \ldots, q_n \) from \( S \), with \( q_0 = s_0 \) and \( t_i = (q_{i-1}, l_i, q_i) \) for each \( 1 \leq i \leq n \). Whenever convenient we will also represent an execution like \( \pi \) above as an interleaved sequence of the form

\[
q_0 \rightarrow_{l_1} q_1 \rightarrow_{l_2} \cdots \rightarrow_{l_n} q_n.
\]

We also define \( \text{Reach}(L) \) to be the set of states reachable by an execution of \( L \). Thus

\[
\text{Reach}(L) = \{ s \in S \mid \exists \text{ an execution } q_0 \rightarrow_{l_1} \cdots \rightarrow_{l_n} q_n \text{ with } s = q_n \}.
\]

3.2 Programming Language

We consider a simple multi-threaded programming language where each program has a fixed number of static threads. There is no dynamic memory allocation, no dynamic creation of threads and no procedure calls. A program has a finite number of variables \( V \) and locks \( M \) which are shared by the threads of the program. We denote by \( V \) the set of values that the program variables can assume. In this work we will take \( V \) to be simply the set of integers.

Each thread in the program is a control-flow graph in which each edge is labelled by a basic statement (or command) over the set of variables \( V \) and locks \( M \). We allow a small set of basic commands over \( V \) and \( M \), which we denote by \( \text{cmd}_{V,M} \), as shown in Tab. \[1\]. For generality, we refrain from defining the syntax of the expressions \( e \) and boolean conditions \( b \).
Table 1 The set of program commands $cmd_{V,M}$ over variables $V$ and locks $M$

| Type   | Syntax | Description                                      |
|--------|--------|--------------------------------------------------|
| Assignment | $x := e$ | Assigns the value of expression $e$ to variable $x \in V$ |
| Assume  | assume(b) | Blocks execution if condition $b$ does not hold |
| Acquire | acquire(m) | Acquires lock $m \in M$, provided $m$ is not held by any thread |
| Release | release(m) | Releases lock $m \in M$, provided the executing thread holds $m$ |

Formally, we represent a multi-threaded program as a tuple $P = (V, M, T)$ where

- $V$ is a finite set of program variables
- $M$ is a finite set of locks
- $T$ is a finite set of thread identifiers. Each thread $t \in T$ has an associated control-flow graph of the form $G_t = (L_t, ent_t, inst_t)$ where
  - $L_t$ is a finite set of locations of thread $t$
  - $ent_t \subseteq L_t$ is the entry location of thread $t$
  - $inst_t \subseteq L_t \times cmd_{V,M} \times L_t$ is a finite set of instructions of thread $t$.

Some definitions related to threads will be useful going forward. We denote by $L_P = \bigcup_{t \in T} L_t$ the disjoint union of the thread locations. We denote by $ent_P$ the set $\{ent_t \mid t \in T\}$ of all entry locations of $P$. Henceforth, whenever $P$ is clear from the context we will drop the subscript $P$ from $L_P$ and its decorations. For a location $n \in L$, we denote by $tid(n)$ the thread $t$ which contains location $n$. We denote the set of instructions of $P$ by $inst_P = \bigcup_{t \in T} inst_t$. For an instruction $i \in inst_t$, we will also write $tid(i)$ to mean the thread $t$ containing $i$. For an instruction $i = (n_s, c, n_t)$, we call $n_s$ the source location, and $n_t$ the target location of $i$. We expect instructions pertaining to acquire() and release() commands to have unique source and target locations. Let $L^rel_t$ be the set of program locations in thread $t$ which are the target of a release() instruction. We refer to $L^rel_t$ as $t$’s post-release points and denote the set of release points in the program by $L^rel = \bigcup_{t \in T} L^rel_t$. Similarly, we define $t$’s pre-acquire points, denoted $L^{acq}_t$, and denote a program’s acquire points by $L^{acq} = \bigcup_{t \in T} L^{acq}_t$. We denote the sets of post-release and pre-acquire points pertaining to operations on lock $m$ by $L^rel_m$ and $L^{acq}_m$, respectively.

We denote the set of commands appearing in program $P$ by $cmd(P)$. We consider an assignment $x := e$ to be a write-access to $x$, and as a read-access to every variable that appears in the expression $e$. Similarly, an assume(b) statement is considered a read-access to every variable that occurs in the boolean condition $b$.

We illustrate these definitions for the example program from Fig. 1. Here $V = \{x, y, z\}$, $M = \{m\}$, and $T = \{t_1, t_2\}$. Some example instructions in this program are $(2, x := y, 3)$ and $(10, acquire(m), 11)$. The set $L_{t_1}$ of program locations in thread $t_1$ is $\{1, 2, 3, 4, 5, 6, 7\}$, while $tid(8) = t_2$. In this program, the set $L^rel_{t_2}$ of post-release points in $t_2$, is $\{13\}$. The set of post-release points of the whole program $L^rel$ is $\{7, 13\}$. The set of pre-acquire points of the whole program $L^{acq}$ is $\{1, 10\}$. Since this program has a single lock, $m$, $L^{acq}_m = \{7, 13\}$ and $L^{acq}_m = \{1, 10\}$.

Many other standard commands can be expressed using the basic commands in our language. A goto instruction from program location $l$ to $l'$ can be simulated by...
the instruction \(<l, \text{assume}(\text{true}), l')\). Constructs like \(\text{if} \) and \(\text{while} \) can be simulated using \text{assume} \ statements in a standard way.

3.3 Interleaving Semantics

We now define the standard interleaving semantics of a multi-threaded program. We first introduce some notation that will be useful in the sequel. Given a program \(P = (V, M, T)\), an \textit{environment} for \(P\) is a valuation \(\phi : V \rightarrow V\), which assigns values in \(V\) to the variables of \(P\). We denote by \(\text{Env}_P\) the set of all environments for \(P\). A \textit{lock map} for \(P\) is a partial map \(\mu : M \rightarrow T\) which assigns to each lock the thread that holds it (if such a thread exists). We denote by \(\text{LM}_P\) the set of lock maps for \(P\). Finally, a \textit{program counter} for \(P\) is a map \(\text{pc} : T \rightarrow \mathcal{L}_P\) which assigns a location to each thread in \(P\), such that for each \(t \in T\), \(\text{pc}(t) \in \mathcal{L}_t\). We denote by \(\text{PC}_P\) the set of program counters of \(P\). As usual, whenever \(P\) is clear from the context we will drop the subscript \(P\) from these symbols. Fig. 3 summarizes the semantic domains, and the meta-variables ranging over them, that we will make use of in this section and subsequently.

\begin{align*}
x, y & \in V \quad \text{Variable identifiers} \\
n & \in M \quad \text{Lock identifiers} \\
t & \in T \quad \text{Thread identifiers} \\
v & \in \mathcal{L} \quad \text{Program locations} \\
v & \in V \quad \text{Values} \\
r & \in R \quad \text{Region identifiers} \\
\phi & \in \text{Env} \quad \text{Environments} \\
\mu & \in \text{LM} \quad \text{Lock map} \\
\text{pc} & \in \text{PC} \quad \text{Program counters} \\
u & \in \text{Ver} \quad \text{Variable versions} \\
ve & \in \text{VE} \quad \text{Versioned environments} \\
s & = (pc, \mu, \phi) \in S \equiv \text{PC} \times \text{LM} \times \text{Env} \quad \text{Standard States} \\
\sigma & = (pc, \mu, \Theta, \Lambda) \in \Sigma \equiv \text{PC} \times \text{LM} \times (T \rightarrow \text{VE}) \times (\mathcal{L}_{rel} \rightarrow \text{VE}) \quad \text{Thread-Local States}
\end{align*}

\begin{figure}[h]
\centering
\begin{tabular}{|l|}
\hline
\textbf{3.3 Interleaving Semantics} \\
\hline
\end{tabular}
\caption{Some of the semantic domains associated with a program \(P = (V, M, T)\).}
\end{figure}

Let us fix a program \(P = (V, M, T)\). We define the interleaving semantics of \(P\) using an LTS \(L^\text{S}_P = (S, T, s_{\text{ent}}, TR^\text{S}_P)\) whose components are defined below. The set of states \(S\) is \(\text{PC} \times \text{LM} \times \text{Env}\). Thus each state is of the form \(<pc, \mu, \phi>\), where \(pc\) is a program counter, \(\mu\) is a lock map, and \(\phi\) is an environment for \(P\). The transition labels come from the set \(T\) of thread identifiers of \(P\). The initial state \(s_{\text{ent}}\) is \(<\lambda t. \text{ent}_t, \lambda m. \bot, \lambda x. 0>\). Thus, in \(s_{\text{ent}}\), every thread is at its entry program location, no thread holds a lock, and all the variables are initialized to zero.

\textit{Transition Relation.} The transition relation \(TR^\text{S}_P\) is the union of the transition relations \(TR^\text{S}_i\) induced by each instruction \(i\) in \(\text{inst}_P\). We elaborate on this below.

The transition relation for each instruction depends on the command associated with it. Intuitively, the semantics of the program commands are as follows. An assignment \(x := e\) command updates the value of the variable \(x\) according to the (possibly non-deterministic) expression \(e\). An \text{assume}(b) command generates transitions only from states in which the (deterministic) boolean interpretation
of the condition $b$ is true. An acquire($m$) command executed by thread $t$ sets $\mu(m) = t$, provided the lock $m$ is not held by any other thread. A release($m$) command executed by thread $t$ sets $\mu(m) = \bot$ provided $t$ holds $m$.

It will be convenient to first define a notation for the evaluation of expressions. The evaluation of an expression $e$, in an environment $\phi$, is a value in $V$. We denote this value by $\llbracket e \rrbracket_\phi$. The interpretation of a boolean condition $b$, in an environment $\phi$, is a boolean value true or false, and we denote this value by $\llbracket b \rrbracket_\phi$.

For an instruction $\iota = (n, c, n')$ in $\text{inst}_P$, with $\text{tid}(\iota) = t$, we define $TR^S_{t,\iota}$ as the set of all transitions $\langle \langle \langle pc, \mu, \phi \rangle, t, (pc', \mu', \phi') \rangle \rangle$ such that $pc(t) = n$, $pc' = pc[t \mapsto n']$ and the following additional conditions are satisfied:

- If $c$ is a command of the form $x := e$ then $\mu' = \mu$, and $\phi' = \phi[x \mapsto \llbracket e \rrbracket_\phi]$.
- If $c$ is a command of the form assume($b$) then $\mu' = \mu$, $\llbracket b \rrbracket_\phi = \text{true}$, and $\phi' = \phi$.
- If $c$ is a command of the form acquire($m$) then $\mu(m) = \bot$, $\mu' = \mu[m \mapsto t]$, and $\phi' = \phi$.
- If $c$ is a command of the form release($m$) then $\mu(m) = t$, $\mu' = \mu[m \mapsto \bot]$, and $\phi' = \phi$.

For a transition $\tau$ caused by an instruction $\iota = (n, c, n')$ in $\text{inst}_t$, we denote by $tid(\tau)$ the thread $t$, by $\text{instr}(\tau)$ the instruction $\iota$, and by $cmd(\tau)$ the command $c$. The transition relation $TR^S_P$ can now be defined as:

$$TR^S_P = \bigcup_{\iota \in \text{inst}_P} TR^S_{t,\iota}.$$

**Executions.** An execution of the program $P$ in the interleaving semantics is simply an execution of the LTS $L^S_P$. When dealing with executions in the interleaving semantics, we will denote the transition relation $TR^S_P$ by $\Rightarrow^S$. We denote by $\text{Reach}^S(P)$ the set $\text{Reach}(L^S_P)$, namely the set of reachable states in the standard interleaving semantics of $P$.

Fig. 4 depicts an execution of the program in Fig. 1 in the interleaving semantics. To keep it simple we show only the sequence of program instructions (from top to bottom), and the thread they belong to (column $t_1$ or $t_2$). The states along the execution can be inferred by the standard semantics of the commands. The other annotations in the figure will be explained in Sec. 3.4.

### 3.4 Data Races and the Happens-Before Relation

Now that we have formally defined the standard interleaving semantics, we are in a position to formally define what constitutes a data race. A standard way to formalize the notion of data race freedom (DRF), is to use the happens-before relation $\Rightarrow^P$ induced by executions.

For a given execution of the program $P$ in the standard interleaving semantics, the happens-before relation is defined as the reflexive and transitive closure of the program-order and synchronizes-with relations, formalized below.

**Definition 1 (Program order)** Let $\pi$ be an execution of $P$. Transition $\pi_i$ is related to the transition $\pi_j$, according to the program-order relation in $\pi$, denoted by $\pi_i \xrightarrow{PO} \pi_j$, if

$$j = \min \{ k \mid i < k < |\pi| \land \text{tid}(\pi_k) = \text{tid}(\pi_i) \}.$$
Fig. 4 A typical execution of the program in Fig. 1 with two threads, according to the standard interleaving semantics. Time flows from the top to the bottom. Instructions ordered by program-order are annotated as po. The release executed by $t_1$ and the acquire executed by $t_2$ are related by synchronizes-with, and is annotated as $sw$. The write of $x$ in thread $t_1$, and its subsequent read in thread $t_2$, are connected by a happens-before path, comprising $po$ and $sw$ annotated edges.

That is, $\pi_i$ and $\pi_j$ are successive executions, in $\pi$, of instructions by the same thread.

The transitions related by program-order in Fig. 4 are marked with $po$.

**Definition 2 (Synchronizes-with)** Let $\pi$ be an execution of $P$. Transition $\pi_i$ is related in $\pi$, by the synchronizes-with relation, to the transition $\pi_j$, denoted by $\pi_i \xrightarrow{sw} \pi_j$, if $cmd(\pi_i) = \text{release}(m)$ for some lock $m$, and

$$j = \min\{k \mid i < k < |\pi| \land cmd(\pi_k) = \text{acquire}(m)\}.$$

That is, $\pi_i$ is a release of lock $m$ in $\pi$, and $\pi_j$ is a subsequent acquire of $m$, and there are no intervening acquires of $m$.

The transitions related by synchronizes-with in Fig. 4 are marked with $sw$.

**Definition 3 (Happens before)** The happens-before relation pertaining to an execution $\pi$ of $P$, denoted by $\xrightarrow{hb}$, is the reflexive and transitive closure of the union of the program-order and synchronizes-with relations induced by the execution $\pi$.

---

1 Strictly speaking, the various relations we define are between indices $\{0, \ldots, |\pi| - 1\}$ of an execution, and not transitions, so we should have written, e.g., $i \xrightarrow{po} j$ instead of $\pi_i \xrightarrow{po} \pi_j$. We use the informal latter notation, for readability.
Note that transitions executed by the same thread are always related by program-order, and are thus always related according to the happens-before relation.

**Definition 4 (Data Race)** Let \( \pi \) be an execution of \( P \). Transitions \( \pi_i \) and \( \pi_j \), in \( \pi \), constitute a racing pair, or a data-race, if the following conditions are satisfied:

1. \( \text{cmd}(\pi_i) \) and \( \text{cmd}(\pi_j) \) are conflicting accesses to a variable \( x \) (i.e. they both access the variable \( x \), and at least one of them is a write-access), and
2. neither \( \pi_i \overset{hb}{\rightarrow} \pi_j \) nor \( \pi_j \overset{hb}{\rightarrow} \pi_i \) holds.

To illustrate these definitions, consider the execution of the program of Fig. 1, shown in Fig. 4. The program-order relation between transitions is shown using edges marked \( \text{po} \), while the synchronizes-with relation is shown using edges marked \( \text{sw} \). For example, the transitions where \( t_1 \) executes \( x := y \) and the one where \( t_1 \) executes \( x++ \) are related by program-order. The transition where \( t_1 \) releases the lock \( m \), and the subsequent transition where \( t_2 \) acquires \( m \), are related by the synchronizes-with relation. There is a happens-before path, namely the path comprising \( \text{po} \) and \( \text{sw} \) annotated edges in Fig. 4, between the write to \( x \) by \( t_1 \), and the subsequent read of \( x \) by \( t_2 \). Note that even though the instruction \( x := y \) is executed by \( t_1 \) before \( t_2 \) executes \( z++ \) in the execution in Fig. 4, these two instructions are not related by happens-before. Consider, for a moment, if \( t_2 \) did not have the \( \text{acquire}(m) \) instruction. Then, the transitions made by \( t_1 \) could never be happens-before related to the ones in \( t_2 \) (due to the absence of \( \text{sw} \) edges). In particular, the write to \( x \) by \( t_1 \) would not be happens-before ordered with the read of \( x \) in \( t_2 \), and we would have a data race in the execution.

A program in which every execution is free from data races is said to be data race free. The program in Fig. 1 is an example of such a race free program.

We say an instruction \( \iota \) in \( P \) is racy if there is an execution \( \pi \) of \( P \) in which two transitions \( \pi_i \) and \( \pi_j \) are involved in a race and \( \text{instr}(\pi_i) = \iota \).

We can now define the notion of the set of variables “owned” by a thread at one of its locations. We say variable \( x \) is owned by a thread \( t \) at a location \( n \in \mathcal{L}_t \), in program \( P \), if the introduction of a read of \( x \) at location \( n \) is not racy. In other words, if we introduce the instruction \( \iota \) with command \( \text{assume}(x == x) \) at point \( n \) in \( t \), to get the program \( P' \), then instruction \( \iota \) is not racy in \( P' \). For example, in the program of Fig. 1 at location 3, thread \( t_1 \) owns the variables \( x \) and \( y \). However it does not own the variable \( z \) at location 3, since a read of \( z \) introduced at this point would be racy (it would race with the write to \( z \) at line 8 in \( t_2 \)).

4 The Thread-Local Semantics \( L \)-DRF

In this section, we introduce a novel semantics for the class of data race free programs, which we refer to as the \( L \)-DRF semantics [30]. The “\( L \)” highlights the fact that the semantics is thread-local in nature, while \( DRF \) emphasizes that we deal exclusively with data race free programs. The \( L \)-DRF semantics paves the way towards devising efficient “thread-local” data flow analyses for race free concurrent programs. Like the standard interleaving semantics we saw in Sec. 3.3, we present the \( L \)-DRF semantics of a program as a labeled transition system. We then prove that the \( L \)-DRF semantics is sound and complete with respect to the
standard semantics, in the sense that for each execution of the program in the standard semantics, there is an “equivalent” execution in the $L$-DRF semantics, and vice versa.

4.1 The L-DRF Semantics

Our thread-local semantics, like the standard one defined in Sec. 3.3, is based on the interleaving of transitions made by different threads, and the use of a lock map to coordinate the use of locks. However, unlike the standard semantics, where the threads share access to a single global environment, in the L-DRF semantics, every thread has its own local environment which it uses to evaluate conditions and perform assignments.

Threads exchange information through release buffers: every post-release point $n \in L^rel_t$ of a thread $t$ is associated with a buffer $\Lambda(n)$ which records a snapshot of $t$’s local environment the last time $t$ ended up at the program point $n$. Recall that this happens right after $t$ executes the instruction $\langle n', \text{release}(m), n \rangle \in \text{inst}_P$. When a thread $t'$ subsequently acquires the lock $m$, it updates its local environment using the snapshots stored in all the buffers pertaining to the release of $m$.

To ensure that $t$ updates its environment such that the value of every variable is up-to-date, every thread maintains its own version map $\nu : V \rightarrow \mathbb{N}$, which associates a count to each variable. A thread increments $\nu(x)$ whenever it writes to $x$. Along any execution, the version $\nu(x)$, for $x \in V$, in the version map $\nu$ of thread $t$, associates a unique prior write with this particular valuation of $x$. It also reflects the total number of write accesses made (across threads) to $x$ to obtain the value of $x$ stored in the map. A thread stores both its local environment and version map in the buffer after releasing a lock $m$. When a thread subsequently acquires lock $m$, it copies from the release buffers at $L^rel_m$ the most up-to-date value (according to the version numbers) of every variable. We prove that for data race free programs, there can be only one such value. If the version of $x$ is the local state of $t$ is higher than the versions of $x$ in the associated release buffers, then the value of $x$ in the local state persists.

Let us fix a concurrent race free program $P = (V, M, T)$. As in Sec. 3.3, we define the L-DRF semantics of $P$ in terms of a labeled transition system $L_P^L = (\Sigma, T, \sigma_{ent}, T_{TR}^L_P)$ whose components we define below.

**States.** A state $\sigma \in \Sigma$ in the L-DRF semantics of $P$ is a tuple $\langle pc, \mu, \Theta, \Lambda \rangle$, where $pc$ and $\mu$ are the program counter and lock map, as in the standard interleaving semantics (Sec. 3.3). A versioned environment is a pair $\langle \phi, \nu \rangle$, where $\phi \in Env$ is an environment and $\nu : V \rightarrow \mathbb{N}$ is a version map, which assigns a version count to each variable. We denote by $VE_P$ (or just $VE$ when $P$ is clear from the context) the set of versioned environments of program $P$. The local environment map $\Theta : T \rightarrow VE$ maps every thread to a local versioned environment, and the release buffer map $\Lambda : L^rel \rightarrow VE$ records the snapshots of versioned environments stored in buffers associated with post-release points.

---

2 Recall that $L^rel_t$ is the set of all post-release points in the thread $t$.
3 Recall that $L^rel$ is the set of all post-release points in the program associated with the release of lock $m$. 
Initial State. The initial state $\sigma_{\text{ent}}$ is defined to be

$$\sigma_{\text{ent}} = (\lambda t. \text{ent}_t, \lambda m. \bot, \lambda t. \text{ve}_{\text{ent}}, \lambda l \in L^{rel}. \text{ve}_{\text{ent}})$$

where $\text{ve}_{\text{ent}} = (\lambda x. 0, \lambda x. 0)$. Thus, in $\sigma_{\text{ent}}$, every thread is at its entry program location, no thread holds a lock, and all the thread-local versioned environments have all the variables and versions initialized to 0. The release buffers are also initialized to the versioned environment where all variable values and versions are 0.

Transition Relation. The transition relation $TR^P \subseteq \Sigma \times T \times \Sigma$ captures the interleaving nature of the L-DRF semantics of $P$. Like the interleaving semantics in Sec. 3.3, $TR^P$ is the union of the transition relations $TR^i$ induced by each instruction $i \in \text{inst}_P$.

For an instruction $i = (n, c, n')$ in $\text{inst}_P$, with $\text{tid}(i) = t$, we define $TR^i_n$ as the set of all transitions $(\langle \text{pc}, \mu, \Theta, \Lambda \rangle, t, \langle \text{pc'}, \mu', \Theta', \Lambda' \rangle)$ such that $\text{pc}(t) = n$, $\text{pc'} = \text{pc}[t \mapsto n']$ and the following additional conditions are satisfied:

- **Assignment.** If $c$ is a command of the form $x := e$ then $\mu' = \mu$, and $\Theta' = \Theta[t \mapsto \langle \phi', \nu' \rangle]$, where $\phi'$ and $\nu'$ are given as follows. Let $\Theta(t) = \langle \phi, \nu \rangle$. Then $\phi' = \phi[x \mapsto [e] \phi]$, and $\nu' = \nu[x \mapsto \nu(x) + 1]$. For subsequent use, we define the interpretation of an assignment statement $x := e$ on a versioned environment $\langle \phi, \nu \rangle$, denoted $[x := e]_\Theta(\langle \phi, \nu \rangle)$, to be $\langle \phi', \nu' \rangle$, where $\phi' = \phi[x \mapsto [e] \phi]$ and $\nu' = \nu[x \mapsto \nu(x) + 1]$.

- **Assume.** If $c$ is an assume statement of the form $\text{assume}(b)$, then $\mu' = \mu$, and $\Theta' = \Theta$. $\Lambda' = \Lambda$, and $[b]_\Theta(\Theta(t))$ is true. Here by $[b]_\Theta(\langle \phi, \nu \rangle)$ we simply mean $[b]_\phi$.

We note that for instructions which execute either assignment or assume commands, the executing thread accesses and modifies only its own local versioned environment.

- **Acquire.** An acquire($m$) command, executed by a thread $t$, has the same effect on the lock map component as in the standard semantics (see Sec. 3.3). In addition, it updates the versioned environment $\Theta(t)$ based on the contents of the relevant release buffers. The release buffers relevant to a thread when it acquires $m$ are the ones at $L^{rel}_m$.

We define an auxiliary function $\text{updEnv}$ to update the value of each $x \in V$ (along with its version) in $\Theta(t)$, by taking its value from a snapshot stored at a relevant buffer which has the highest version of $x$, if the latter version is higher than $(\Theta(t), 2)(x)$. If the version of $x$ is highest in $(\Theta(t), 2)(x)$, then $t$ simply retains this value. Finding the most up-to-date (value, version) pairs for a variable $x$ from a set of versioned environments is the job of the auxiliary function $\text{take}_e$. We will separately prove (in Lemma 3) that all reachable L-DRF states are admissible in that in any two component versioned environments (i.e. the thread local versioned environments or release buffers of the state), if the versions for a variable coincide, then so must their values. Thus if $\langle \phi, \nu \rangle$ and $\langle \phi', \nu' \rangle$ are two versioned environments in the components of a reachable state, then for each variable $x$, $\nu(x) = \nu'(x) \implies \phi(x) = \phi'(x)$.

Given a set of versioned environments $Y$, we define $\text{take}_e(Y)$ to be the set of (value,version) pairs $\langle v, m \rangle$ such that there exists a versioned environment
\langle \phi, \nu \rangle \text{ in } Y \text{ with } \phi(x) = v \text{ and } \nu(x) = m, \text{ and } m \text{ is the highest version of } x \text{ among the versioned environments in } Y \text{ (i.e. } \nu(x) \geq \nu'(x) \text{ for each } \langle \phi', \nu' \rangle \text{ in } Y). 

Given a versioned environment \( ve \) and a set of versioned environments \( X \), we define \( \text{updEnv}(ve, X) \) to be the set of versioned environments \( \langle \phi', \nu' \rangle \) such that for each variable \( x \in \mathcal{V} \), \( \langle \phi'(x), \nu'(x) \rangle \in \text{take}_x(\{ve\} \cup X) \).

We can now define the transition induced by an acquire command. If \( c \) is an acquire statement of the form \text{acquire}(m), \text{ then } \mu[m] = \bot, \mu'[m \mapsto t], \Theta' = \Theta[t \mapsto ve'], \text{ and } \Lambda' = \Lambda, \text{ where } ve' = \text{updEnv}(\Theta(t), \Lambda^\ast) \text{ and } \Lambda^\ast = \{\Lambda(n'') | n'' \in \mathcal{L}_n^m\} \text{ is the set of versioned environments relevant to } m. \text{ As an example, consider again the execution of the program of Fig. 1, as shown in Fig. 4. When thread } t_2 \text{ executes the acquire(m) instruction, the condition of the relevant buffers and the thread local state of } t_2 \text{ is shown in Fig. 5. The figure also outlines the operation of the functions } \text{take}_x, \text{take}_y \text{ and } \text{take}_z, \text{ and finally the operation of the function } \text{updEnv}. \text{ The transition relation } TR_L^P \text{ of program } P \text{ according to the } L\text{-DRF semantics, is the union of the set of all possible transitions generated by its instructions. Formally, } TR_L^P = \bigcup_{i \in \text{inst}_P} TR_L^i. \text{ This completes the description of the labelled transition system } L_L^P \text{ capturing the } L\text{-DRF semantics. An execution of program } P \text{ in the } L\text{-DRF semantics is sim-} \text{- Release. If } c \text{ is a release statement of the form } \text{release}(m), \text{ then } \mu[m] = t, \mu'[m \mapsto \bot], \Theta' = \Theta, \text{ and } \Lambda' = \Lambda[n' \mapsto \Theta(t)]. \text{ Thus an instruction } i \text{ pertaining to a } \text{release}(m) \text{ command has the same effect on the lock map component of the state in the } L\text{-DRF semantics that it has in the standard semantics (See Sec. 3.3). In addition, it stores the local versioned environment of thread } t (= \text{tid}(i)), \Theta(t), \text{ in the buffer associated with the post-release point of the executed } \text{release}(m) \text{ instruction.} \text{ The superscripts indicate the versions.} \text{ Fig. 5 Operation of the functions } \text{take}_x, \text{take}_y, \text{take}_z \text{ and } \text{updEnv} \text{ when } t_2 \text{ acquires } m \text{ in the execution of the program of Fig. 1, as shown in Fig. 4.} \text{ The transition relation } TR_L^P \text{ of program } P \text{ according to the } L\text{-DRF semantics, is the union of the set of all possible transitions generated by its instructions. Formally, } TR_L^P = \bigcup_{i \in \text{inst}_P} TR_L^i. \text{ This completes the description of the labelled transition system } L_L^P \text{ capturing the } L\text{-DRF semantics. An execution of program } P \text{ in the } L\text{-DRF semantics is sim-}
ply an execution of the transition system $L_P^L$. When dealing with executions in the $L$-DRF semantics, we will denote the transition relation $TR^L_P$ by $\Rightarrow^L$. We denote by $\text{Reach}^L(P)$ the set of reachable states in this semantics, namely $\text{Reach}(L^L_P)$.

4.2 Soundness and Completeness of $L$-DRF

In this section, we show that for the class of data race free programs, the thread local semantics $L$-DRF is sound and complete with respect to the standard interleaving semantics. Intuitively, the $L$-DRF and the standard semantics are “equivalent” in the sense that for each execution of a program $P$ in the standard semantics, one can find a corresponding execution in the $L$-DRF semantics which coincides with the values read from the variables. Likewise, every execution of program $P$ in the $L$-DRF semantics has a corresponding execution in the standard semantics.

Let us fix a race free program $P = (V, M, T)$. To formalize the above claim, we first define a function which extracts a state in the interleaving semantics from a state in the $L$-DRF semantics.

**Definition 5 (Extraction Function $\chi$)** The extraction function $\chi : \Sigma \rightarrow S$ is defined for admissible states (see Sec. 4.1) in $\Sigma$ as follows:

$$\chi((pc, \mu, \Theta, \Lambda)) = (pc, \mu, \phi),$$

where $\phi$ is defined as follows. For each $x \in V$, $\phi(x) = v$, provided there exists a version value $m$, with $\langle v, m \rangle \in \text{take}_x(\bigcup_{t \in T} \Theta(t))$. The function $\chi$ thus preserves the values of the program counters and the lock map, while it takes the value of a variable $x$ from the thread which has the maximal version count for $x$ in its local environment. The map $\chi$ is clearly well-defined for admissible states.

The function $\chi$ can be extended to executions in the $L$-DRF semantics, in the following sense. Given an execution $\hat{\pi} = \sigma_0 \Rightarrow^L_1 \ldots \Rightarrow^L_n \sigma_n$ of program $P$ in the $L$-DRF semantics, and an execution $\pi = s_0 \Rightarrow S_1 \ldots \Rightarrow S_l s_l$ of $P$ in the standard semantics, we say $\pi = \chi(\hat{\pi})$ if $l = n$ and for each $i : 0 \leq i \leq n$, $s_i = \chi(\sigma_i)$.

**Theorem 1 (Completeness)** For any execution $\pi$ of $P$ in the standard interleaving semantics, there exists an execution $\hat{\pi}$ of $P$ in the $L$-DRF semantics such that $\chi(\hat{\pi}) = \pi$.

**Theorem 2 (Soundness)** For any execution $\hat{\pi}$ of $P$ in the $L$-DRF semantics, there is an execution $\pi$ in the standard interleaving semantics of $P$, with $\pi = \chi(\hat{\pi})$.

In order to prove Theorem 1 and Theorem 2 we need to establish a few intermediate results.

**Lemma 1** In any execution $\hat{\pi}$ in the $L$-DRF semantics of $P$, the version of any variable $x \in V$, in any component versioned environment of any state $\sigma$ in $\hat{\pi}$, is bounded by the total number of writes to $x$ preceding it.

**Proof** In $\hat{\pi}$, the only transitions which can increment the version of variable $x$ pertain to instructions containing commands which write to $x$, of the form $x := e$. Instructions containing other commands (assume, acquire and release) only make
copies of existing version counts. If there are \( n \) such transitions containing instructions writing to \( x \) in \( \hat{\pi} \), and the initial version count of \( x \) is 0 in all the component versioned environments of the initial state \( \sigma_{init} \), the version of \( x \), in any component versioned environment of any state \( \sigma \) in \( \hat{\pi} \) can be at most \( n \).

**Lemma 2** Let \( \hat{\pi} = \langle pc_0, \mu_0, \Theta_0, A_0 \rangle \Rightarrow^{L_1} \ldots \Rightarrow^{L_N} \langle pc_N, \mu_N, \Theta_N, A_N \rangle \) be an execution in the L-DRF semantics of program \( P \). Let 

\[
\tau_j = \langle pc_{j-1}, \mu_{j-1}, \Theta_{j-1}, A_{j-1} \rangle \Rightarrow^{L_j} \langle pc_j, \mu_j, \Theta_j, A_j \rangle
\]

be a transition in \( \hat{\pi} \) which contains an access (read or write) to the variable \( x \). Suppose there is a prior write to \( x \) in \( \hat{\pi} \), and let 

\[
\tau_i = \langle pc_{i-1}, \mu_{i-1}, \Theta_{i-1}, A_{i-1} \rangle \Rightarrow^{L_i} \langle pc_i, \mu_i, \Theta_i, A_i \rangle
\]

be the last transition, prior to \( \tau_j \), which contains an assignment to \( x \). Then,

\[
(\Theta_j(t_j)(x)) \leq (\Theta_i(t_i)(x)).
\]

In other words, the version of \( x \) in \( \Theta(t_j) \) is no less than the version of \( x \) in the local state of \( t_i \) post the write at \( \tau_i \).

**Fig. 6** A typical execution of a program \( P \) in the L-DRF semantics. The solid arrows represent the interleaved execution of the instructions from different threads. The dotted arrows denote the happens-before path induced by this execution. The figure marks the sections of the happens-before path which are program-order related (po), and the transitions related by synchronizes-with (sw).

**Proof** Fig. 6 provides a pictorial description of the situation we are considering. We lift the notion of a happens-before path, which we defined for the interleaving semantics, in a natural way to L-DRF executions. The sequence of transitions in \( \hat{\pi} \) can also be viewed as an standard execution, and the resulting happens-before path in \( \hat{\pi} \) contains the same sequence of transitions as the happens-before path in
the execution in the standard interleaving semantics. Since \( \tau_i \) and \( \tau_j \) are conflicting accesses to the variable \( x \), and since the program \( P \) is assumed to be free from races, we have \( \tau_i \xrightarrow{hb} \tau_j \) (indicated by the path comprising dotted arrows in Fig. 6).

Let \( \rho \) be such a happens-before path between \( \tau_i \) and \( \tau_j \), excluding both \( \tau_i \) and \( \tau_j \). If \( \rho \) is of 0 length, then \( \tau_i \) must immediately follow \( \tau_j \) in the same thread, and the lemma clearly holds. Suppose \( \rho \) is of length at least one, and consider a transition \( \tau_k \) in \( \rho \). By induction on the position \( n \) of \( \tau_k \) in \( \rho \), we claim that \( (\Theta_k(t_k),\mu)(x) \geq (\Theta(t_i),\mu)(x) \).

**Base Case.** If \( n = 1 \), then \( \tau_i \) and \( \tau_k \) must be related by program order, which implies \( t_i = t_k \) and \( k = i + 1 \). Thus clearly \( (\Theta(t_k),\mu)(x) \geq (\Theta(t_i),\mu)(x) \).

**Inductive Case.** Assume that the hypothesis holds for all transitions at positions less than or equal to \( n \) in \( \rho \), and let us suppose \( \tau_k \) occurs at position \( n + 1 \) in \( \rho \). Let the \( n \)-th transition in \( \rho \) be

\[
(\tau_n = (pc_{u-1},\mu_{u-1},\Theta_{u-1},A_{u-1}) \Rightarrow_{t_n} (pc_u,\mu_u,\Theta_u, A_u)).
\]

There are two possible cases here. Either \( \tau_u \xrightarrow{po} \tau_k \), and consequently \( t_u = t_k \). In this case too, clearly \( (\Theta(t_k),\mu)(x) \geq (\Theta(t_u),\mu)(x) \), which, by the induction hypothesis, is greater than or equal to \( (\Theta(t_i),\mu)(x) \). Hence this case is taken care of.

On the other hand, if \( \tau_u \xrightarrow{aw} \tau_k \), then \( \tau_u \) must be the release of some lock \( m \), and \( \tau_k \) must be the acquire of \( m \). By the L-DRF semantics of acquire, thread \( t_k \) will observe the buffer associated with the release command of \( \tau_u \). Consequently, \( (\Theta_k(t_k),\mu)(x) \geq (\Theta_u(t_u),\mu)(x) \), by the semantics of the acquire command and \( (\Theta_u(t_u),\mu)(x) \geq (\Theta(t_i),\mu)(x) \), by the induction hypothesis. Thus, the hypothesis holds in this case as well. This proves the claim.

The lemma now follows directly from the claim. \( \square \)

**Lemma 3** Let \( \hat{\pi} = (pc_0,\mu_0,\Theta_0, A_0) \Rightarrow_{t_1} \cdots \Rightarrow_{t_N} (pc_N,\mu_N,\Theta_N, A_N) \) be an execution in the L-DRF semantics of program \( P \), and let the \( i \)-th transition in the execution be

\[
(\tau_i = (pc_{i-1},\mu_{i-1},\Theta_{i-1},A_{i-1}) \Rightarrow_{t_i} (pc_i,\mu_i,\Theta_i, A_i))
\]

Consider a transition \( \tau_k \) with \( cmd(\tau_k) \)\(^\dag\) being an assignment to a variable \( x \). Then

\[
(\Theta_k(t_k),\mu)(x) \geq \{i : i \leq k \text{ and } cmd(\tau_i) \text{ is a} \text{ an assignment to } x\}
\]

That is, in the post-state of an assignment to a variable \( x \) by thread \( t \), the version of \( x \) in the local versioned environment of \( t \) equals the total number of writes made to \( x \) till that point.

**Proof** We prove the lemma by induction on \( k \).

**Base Case.** If \( k = 1 \), then clearly \( (\Theta_k(t_k),\mu)(x) \geq 1 \), and we are done.

**Inductive Case.** Let \( k = n + 1 \) and assume the lemma holds for all earlier writes to \( x \) in \( \hat{\pi} \). Let the last write to \( x \), prior to \( \tau_{n+1} \), be in the transition \( \tau_i \). By the induction hypothesis,

\( \dag \) By abuse of notation we use \( cmd(\tau) \) to denote the command of the instruction \( \epsilon \) causing the transition \( \tau \).
\[(\Theta_i(t_i).2)(x) = |\{j : j \leq i \land cmd(\tau_j)\text{ is an assignment to } x\}| = w \text{ (say)}\]

We now infer the following:
\[
(\Theta_n(t_{n+1}).2)(x) \geq w \quad \text{from Lemma 2 and}
\]
\[
(\Theta_n(t_{n+1}).2)(x) \leq w \quad \text{from Lemma 1}
\]

Therefore \((\Theta_n(t_{n+1}).2)(x) = w\). Since \(\tau_{n+1}\) increments the version of \(x\) in \(\Theta_{n+1}(t_{n+1})\), we have
\[
(\Theta_{n+1}(t_{n+1}).2)(x) = (\Theta_n(t_{n+1}).2)(x) + 1
= w + 1
= |\{j : j \leq i \land cmd(\tau_j)\text{ is an assignment to } x\}| + 1
= |\{j : j \leq n + 1 \land cmd(\tau_j)\text{ is an assignment to } x\}|.
\]

This completes the proof of the lemma. \(\square\)

**Corollary 1** Let \(\hat{\pi} = \sigma_0 \Rightarrow^L_{t_1} \ldots \Rightarrow^L_{t_N} \sigma_N\) be an execution in the L-DRF semantics of program \(P\). Let \(\sigma_i = \langle pc_i, \mu_i, \Theta_i, \Lambda_i \rangle\), and let the \(i\)-th transition in the execution be
\[
\tau_i = \sigma_{i-1} \Rightarrow^L_{t_i} \sigma_i.
\]

Suppose \(\tau_k\) contains an access (read or write) to the variable \(x\). Let \(m\) be the highest version count of \(x\) among all component versioned environments in \(\sigma_{k-1}\).

Then \((\Theta_{k-1}(t_k).2)(x) = m\). In other words, whenever a thread accesses a variable \(x\), the version of \(x\) is the highest in its local versioned environment.

**Proof** Suppose \(\tau_k\) is the first write to \(x\) in \(\hat{\pi}\). Then by Lemma 1 \(\Theta_{k-1}(t) = 0\) for each \(t \in T\), and we are done. Otherwise, let there be \(m \geq 1\) earlier writes to \(x\) before \(\tau_k\), and let \(\tau_i\) be the last such write. Then by Lemma 1 \((\Theta_{k-1}(t).2)(x) \leq m\) for each \(t \in T\), and also \((\Lambda_{k-1}(n).2)(x) \leq m\) for each \(n \in L^{rel}\). Further, by Lemma 3 \((\Theta(t_i).2)(x) = m\), and by Lemma 2 \((\Theta_{k-1}(t_k).2)(x) \geq m\). Hence \((\Theta_{k-1}(t_k).2)(x) = m\), and we have the corollary. \(\square\)

The next Lemma proves that the L-DRF semantics generates only admissible states.

**Lemma 4** Let \(\hat{\pi} = \sigma_{ent} \Rightarrow^L_{t_1} \ldots \Rightarrow^L_{t_N} \sigma_N\) be an execution of \(P\) in the L-DRF semantics. Then, for any \(\sigma_k\), with two component versioned environments (in thread local states or buffers) \(\langle \phi_1, \nu_2 \rangle\) and \(\langle \phi_2, \nu_2 \rangle\), and any variable \(x \in V\), if \(\nu_1(x) = \nu_2(x)\), then \(\phi_1(x) = \phi_2(x)\).

**Proof** We prove the lemma using induction on the position \(k\) in \(\hat{\pi}\). Let the \(i\)-th transition in \(\hat{\pi}\) be \(\tau_i = \sigma_{i-1} \Rightarrow^L_{t_i} \sigma_i\), and let each \(\sigma_0\) be \(\langle pc_0, \mu_0, \Theta_0, \Lambda_0 \rangle\).

**Base Case** When \(k = 0\), we have \(\sigma_k = \sigma_{ent}\). Since all versions and values are 0, the hypothesis clearly holds.
**Inductive Case.** Let us assume that for all $k \leq n$ the claim of the lemma holds, and consider $k = n + 1$. We consider the different cases for $cmd(\tau_{n+1})$. If $cmd(\tau_{n+1})$ is either an *assume* or a *release* statement, then the claim clearly holds since by assumption it holds for $\sigma_n$ and these commands do not alter any versions or values in going from $\sigma_n$ to $\sigma_{n+1}$.

If $cmd(\tau_{n+1})$ is of the form $\text{acquire}(m)$, then $t_{n+1}$ updates its local versioned environment based on its local versioned environment and the versioned environments at relevant buffers. By the induction hypothesis, the versioned environments in $\sigma_n$ satisfy the property of the lemma. By the semantics of the acquire command, $t_{n+1}$ copies over the version and the valuation of $x$ from one such $ve$ (including, possibly, $t_{n+1}$’s local versioned environment) in $\sigma_n$. Thus, the hypothesis also holds for $\sigma_{n+1}$ in this case.

If $cmd(\tau_{n+1})$ is of the form $x := e$, then $t_{n+1}$ updates the version and valuation of $x$ in its local versioned environment. By Lemma 1 in any component versioned environment $\langle \phi, \nu \rangle$ in $\sigma_n$, we must have

$$\phi(x) \leq m,$$

where $m$ is the total number of writes to $x$ preceding $\tau_{n+1}$. By Lemma 3

$$(\Theta_{n+1}(t_{n+1}), 2)(x) = m + 1.$$

This implies that for any component versioned environment $\langle \phi', \nu' \rangle$ in $\sigma_{n+1}$, other than the local versioned environment of $t_{n+1}$,

$$\nu'(x) < (\Theta_{n+1}(t_{n+1}), 2)(x).$$

Since none of the other versioned environments is modified, the claim of the lemma continues to hold for $\sigma_{n+1}$. This completes the proof the lemma. □

We now proceed to prove the completeness and soundness results (Theorem 1 and Theorem 2).

**Proof (Completeness, Theorem 1)** We first outline the idea behind the proof, using Fig. 7. For any trace $\pi$ of $P$ in the interleaving semantics, we obtain a corresponding trace $\hat{\pi}$ in the $L$-DRF semantics by taking the same interleaving of instructions from the threads. Our inductive hypothesis is that every $N$ length standard interleaving execution has a corresponding $N$ length $L$-DRF execution. We now consider a $N + 1$ length execution $\pi$ in the standard interleaving semantics, and we show that there exists a state $\sigma_{n+1}$, using which we can extend the $N$ length $L$-DRF trace to create a $N + 1$ length trace which is $\chi$-equivalent to $\pi$.

We prove the result using induction on the length of the execution. Let $P(N)$ denote the following hypothesis. For any trace

$$\pi = \langle pc_0, \phi_0, \mu_0 \rangle \Rightarrow^S \tau_1 \ldots \Rightarrow^S \tau_N \langle pc_N, \phi_N, \mu_N \rangle$$

of program $P$ in the standard semantics, there exists a trace

$$\hat{\pi} = \langle pc_0, \mu_0, \Theta_0, A_0 \rangle \Rightarrow^L \tau_1 \ldots \Rightarrow^L \tau_N \langle pc_N, \mu_N, \Theta_N, A_N \rangle$$

in the $L$-DRF semantics such that $\chi(\hat{\pi}) = \pi$.

We outline the inductive arguments.
The inductive proof obligation for Completeness. If we hypothesize that every \( n \) length trace \( \pi \) of program \( P \) in the standard semantics has an equivalent trace \( \hat{\pi} \) in \( \mathbb{L} \)-DRF semantics, and if we can extend the trace \( \pi \) by a single step to reach state \( s_{n+1} \), then there exists a state \( \sigma_{n+1} \), with \( \chi(\sigma_{n+1}) = s_{n+1} \), by which we can extend the \( \hat{\pi} \) trace by a single step as well.

**Base Case.** For \( N = 0 \), the execution \( \pi \) contains the single state \( s_{\text{ent}} \). The length 0 \( \mathbb{L} \)-DRF execution contains the single state \( \sigma_{\text{ent}} \). Since \( \chi(\sigma_{\text{ent}}) = s_{\text{ent}} \), \( P(0) \) holds.

**Inductive Case.** Assume that \( P(k) \) holds for all executions of length \( k \), where \( 0 \leq k \leq n \). We prove that \( P(n+1) \) holds. Consider a \( n+1 \) length execution

\[
\pi = \langle pc_0, \phi_0, \mu_0 \rangle \Rightarrow^S_1 \cdots \Rightarrow^S_n \langle pc_{n+1}, \phi_{n+1}, \mu_{n+1} \rangle
\]

of program \( P \) in the interleaving semantics. Let the instruction corresponding to the last transition in \( \pi \) be \( \langle l, c, l' \rangle \). We denote by \( \pi[1 \ldots n] \) the \( n \)-length prefix of \( \pi \).

By the induction hypothesis, there exists a trace

\[
\hat{\pi}' = \langle pc_0, \mu_0, \Theta_0, \Lambda_0 \rangle \Rightarrow^S_1 \cdots \Rightarrow^S_{n-1} \langle pc_n, \mu_n, \Theta_n, \Lambda_n \rangle
\]

of length \( n \) in the \( \mathbb{L} \)-DRF semantics, such that \( \pi[1 \ldots n] = \chi(\hat{\pi}') \). Note that this implies that

\[
\chi(\sigma_n) = s_n \tag{1}
\]

where \( \sigma_n = \langle pc_n, \mu_n, \Theta_n, \Lambda_n \rangle \) and \( s_n = \langle pc_n, \mu_n, \phi_n \rangle \).

We show that there exists a state \( \sigma_{n+1} = \langle pc_{n+1}, \mu_{n+1}, \Theta_{n+1}, \Lambda_{n+1} \rangle \) in the \( \mathbb{L} \)-DRF semantics, such that \( \chi(\sigma_{n+1}) = s_{n+1} \) and \( \sigma_n \Rightarrow^S_{t_{n+1}} \sigma_{n+1} \) via the same instruction \( \langle l, c, l' \rangle \) used in the transition \( s_n \Rightarrow^S_{t_{n+1}} s_{n+1} \). Let \( \hat{\pi} \) be the resulting \( \mathbb{L} \)-DRF execution \( \sigma_0 \Rightarrow^L_1 \cdots \Rightarrow^L_{t_{n+1}-1} \sigma_n \Rightarrow^L_{t_{n+1}} \sigma_{n+1} \). Then this would prove that \( \hat{\pi} \) satisfies the property \( \chi(\hat{\pi}) = \pi \). We show this proof obligation diagrammatically in Fig. 7.

We note that since \( \langle l, c, l' \rangle \) is the last instruction in \( \pi \), we must have:

\[
pc_n(t_{n+1}) = l \\
pc_{n+1}(t_{n+1}) = l'.
\]
Note that, by construction, the pc and μ components of σ_{n+1} and s_{n+1} are made equal. Thus the components pc_{n+1} and μ_{n+1} of σ_{n+1} are already fixed, and it remains to define Θ_{n+1} and Λ_{n+1} appropriately. We now case split on the command c.

- c = acquire(m): We define the components of state σ_{n+1} as follows:
  \[ Θ_{n+1} = Θ_n[t_n+1 \mapsto \text{updEnv}(Θ_n(t_n+1), Λ_n)] \]
  \[ Λ_{n+1} = Λ_n. \]

Since the lock maps in both σ_n and s_n are the same, the lock acquisition succeeds from σ_n as well. By the L-DRF semantics of acquire, σ_n \Rightarrow_{t_n+1}^l σ_{n+1}. Since the acquire does not change the maximum version, and the corresponding value, of each x \in V between σ_n and σ_{n+1}, we have χ(σ_{n+1}) = s_{n+1}. Thus P(n+1) holds in this case.

- c = release(m): We define the components of state σ_{n+1} as follows:
  \[ Θ_{n+1} = Θ_n \]
  \[ Λ_{n+1} = Λ_n[t_n' \mapsto Θ_n(t_n+1)]. \]

Once again the lock release must succeed from σ_n as well. By the L-DRF semantics of release, σ_n \Rightarrow_{t_n+1}^l σ_{n+1}. Since the release does not change the maximum version, and the corresponding value, of each x \in V between σ_n and σ_{n+1}, we have χ(σ_{n+1}) = s_{n+1}. Thus P(n+1) holds in this case as well.

- c = assume(b): We define the components of state σ_{n+1} as follows:
  \[ Θ_{n+1} = Θ_n \]
  \[ Λ_{n+1} = Λ_n. \]

Consider an arbitrary variable x that is read in the condition b. By Corollary 1 in σ_n, the version of x is highest in Θ_n(t_n+1). Given that by the induction hypothesis χ(σ_n) = s_n, this implies that for any such variable x, φ_n(x) = (Θ_n(t_n+1))_x(x). Hence, it follows that [b][φ_n = [b][l](Θ_n(t_n+1))).

Since, by assumption, s_n \Rightarrow_{t_n+1}^l s_{n+1}, it follows that σ_n \Rightarrow_{t_n+1}^l σ_{n+1}. Since the assume does not alter the maximum version, and the corresponding value, of each x \in V between σ_n and σ_{n+1}, we have χ(σ_{n+1}) = s_{n+1}. Thus P(n+1) holds in this case as well.

- c = x := e: We define the components of state σ_{n+1} as follows:
  \[ Θ_{n+1} = Θ_n[t_n+1 \mapsto \langle φ', ν' \rangle] \]
  \[ Λ_{n+1} = Λ_n. \]

where φ' and ν' are defined as follows. Let Θ_n(t_n+1) be \langle φ, ν \rangle. Then
  \[ φ' = φ[x \mapsto φ_{n+1}(x)] \]
  \[ ν' = ν[x \mapsto ν(x) + 1]. \]
Consider an arbitrary variable $y$ that is read in the expression $e$. By Corollary 1 in $\sigma_n$, the version of $y$ is highest in $\Theta_n(t_{n+1})$. This implies that for any such variable $y \in \mathcal{V}$,

$$
\phi_n(y) = \phi(y) \\
\implies [e]\phi_n = [e]_\Theta(\Theta_n(t_{n+1})) \\
\implies \phi_{n+1}(x) = (\Theta_{n+1}(t_{n+1}).1)(x)
$$

Coupled with the definition of $\nu'$, this proves that

$$
\langle \phi', \nu' \rangle = [x := e]_\Theta(\Theta_n(t_{n+1}))
$$

which allows us to conclude that $\sigma_n \Rightarrow^t \sigma_{n+1}$. By Lemma 3 and the construction of $\sigma_{n+1}$, the version of $x$ is highest in $\Theta_{n+1}(t_{n+1})$, among all other component versioned environments of $\sigma_{n+1}$. This, coupled with the fact that no other versions are modified, lets us conclude that $\chi(\sigma_{n+1}) = s_{n+1}$. Consequently, $P(n+1)$ holds here as well.

This completes the induction argument, and hence the lemma. $\square$

![Fig. 8](image-url) The inductive proof obligation for Soundness. If we hypothesize that every $n$ length execution $\hat{\pi}$ of program $P$ in the L-DRF semantics has an equivalent execution $\pi$ in the standard semantics, and if we can extend the execution $\hat{\pi}$ by a single step to reach state $\sigma_{n+1}$, then there exists a state $s_{n+1}$, with $\chi(\sigma_{n+1}) = s_{n+1}$, by which we can extend the execution $\pi$ by a single step as well.

**Proof (Soundness, Theorem 2)** We outline the proof idea using Fig. 8. Here the situation is the inverse of that in Fig. 7. Given any execution $\hat{\pi}$ in the L-DRF semantics of $P$, we show that the sequence of states induced by the $\chi$-map, is a valid execution of $P$ in the interleaving semantics. Our induction hypothesis is on the length $n$ of the L-DRF execution. When we consider a $n + 1$ length L-DRF execution $\hat{\pi}'$, we know there exists an execution $\pi$ in the interleaving semantics corresponding to the $n$ length prefix of $\hat{\pi}'$. We show that we can extend $\pi$ by using
χ(σ_{n+1}) in order to obtain an n + 1 length execution in the interleaving semantics, which is χ related to ˆπ'.

Consider an execution

\[ ˆπ = σ_{ent} \Rightarrow_{t_1}^{L} \cdots \Rightarrow_{t_N}^{L} σ_N \]

in the L-DRF semantics of program P. We define a sequence of states of P in the standard semantics

\[ π = s_{ent} \Rightarrow_{t_1}^{S} \cdots \Rightarrow_{t_N}^{S} s_N, \]

where for each i : 0 ≤ i ≤ N, \( s_i = χ(σ_i) \), and claim this to be a valid execution of P in the standard semantics. For each i, let \( σ_i = (pc_i, μ_i, Θ_i, A_i) \) and \( s_i = (pc_i, μ_i, φ_i) \). We prove the claim by induction on the length N of the execution \( π \).

**Base Case.** If \( N = 0 \), the execution ˆπ contains the single state \( σ_0 = σ_{ent} \). Since \( χ(σ_0) = s_0 = s_{ent} \), we have that \( π \) is a valid length 0 execution of P in the standard semantics.

**Inductive Case.** Assume that the claim holds for all L-DRF executions of length \( n \). Let \( \pi[1 \ldots n] \) denotes the n length prefix of the execution ˆπ, then by the induction hypothesis,

\[ s_0 \Rightarrow_{t_1}^{S} \cdots \Rightarrow_{t_n}^{S} s_n \]

is a valid execution of P in the interleaving semantics. We show that \( s_n \Rightarrow_{t_{n+1}}^{S} s_{n+1} \), where \( s_{n+1} = χ(σ_{n+1}) \), using the same instruction in the corresponding transition of ˆπ. We show the proof obligation diagrammatically in Fig. 7.

We case split on cmd(τ_{n+1}), where \( τ_{n+1} \) is the last transition in π.

If cmd(τ_{n+1}) is either an acquire or a release, then since the location maps and lock maps are identical in both \( s_n \) and \( σ_n \), the lock acquisition (or release) is enabled from \( s_n \). Moreover, since neither of the commands alter the versions between \( σ_n \) and \( σ_{n+1} \), we have \( φ_n = φ_{n+1} \). Thus, \( s_n ⇒_{t_{n+1}}^{S} s_{n+1} \), and the claim holds in this case.

If cmd(τ_{n+1}) is assume(b), then, by Corollary 1 the version of any variable \( x \) read in the condition \( b \) is highest in \( Θ_n(t_{n+1}) \). Moreover, since \( χ(σ_n) = s_n \), for any variable \( x \) accessed in the condition \( b \), we must have

\[ φ_n(x) = (Θ_n(t_{n+1}).1)(x). \]

This implies that \([b]φ_n = [b]_L(Θ_n(t_{n+1})). Thus, \( s_n ⇒_{t_{n+1}}^{S} s_{n+1} \) and the claim holds in this case too.

Finally, we consider the case when cmd(τ_{n+1}) is an assignment statement of the form \( x := e \). In a manner analogous to the case of the assume earlier, we can prove that \([e]φ_n = [e]_L(Θ_n(t_{n+1})). By, Lemma 3 the version of x in \( σ_{n+1} \) is highest in \( Θ_{n+1}(t_{n+1}) \). Thus,

\[ φ_{n+1}(x) = (Θ_{n+1}(t_{n+1}).1)(x). \]

Since the assignment command is always enabled, and the above facts hold, we obtain that \( s_n ⇒_{t_{n+1}}^{S} s_{n+1} \) is a valid transition, and we are done.

This completes the proof of the claim, and hence the theorem follows. □
An important corollary of the proofs of these theorems is that the L-DRF semantics is both sound and precise (vis-a-vis the standard semantics) in a relational sense, provided we restrict our attention to variables owned by a thread at a program point. For environments φ and φ′ and a subset of variables V of V, we use the notation φ =_V φ′ to mean that φ and φ′ agree on the values of variables in V; i.e. for all x ∈ V we have φ(x) = φ′(x).

**Corollary 2** Let P be a race-free program as above. Consider a thread t ∈ T and a point n ∈ L_t. Let V ⊆ V be the set of variables owned by t at n. Then

1. If ⟨pc, μ, φ⟩ is a reachable state in the standard interleaving semantics of P, with pc(t) = n, then there exists a reachable state in the L-DRF semantics of the form ⟨pc, μ, Θ, Λ⟩, with Θ(t).1 =_V φ.
2. Conversely, if ⟨pc, μ, Θ, Λ⟩ is a reachable state in the L-DRF semantics of P, with pc(t) = n, then there exists a reachable state in the standard semantics of the form ⟨pc, μ, φ⟩, with Θ(t).1 =_V φ.

**Proof** We prove the two parts separately.

1. Since s = ⟨pc, μ, φ′⟩ is a reachable state in the interleaving semantics, there is an execution π in the standard semantics that ends at s. By the completeness proof, there exists an execution ˇπ of the L-DRF semantics ending in a state σ, with π = χ(ˇπ). It follows that σ must be of the form ⟨pc, μ, Θ, Λ⟩ with s = χ(σ). Further, it follows from Corollary 1 that for each variable x ∈ V, the version of x must be highest in t. It now follows that φ′ =_V Θ(t).φ.

2. If σ = ⟨pc, μ, Θ, Λ⟩ is a state in Reach^L(P), then there must exist an execution ˇπ = σ_{ext} ⇒_1^T ... ⇒_T^T σ of P in the L-DRF semantics. By Theorem 2, there exists an execution σ_{ext} ⇒_1^T ... ⇒_T^T s of P in the standard semantics, with χ(σ) = s. Thus s is of the form ⟨pc, μ, φ⟩ for some environment φ. Once again, it follows from Corollary 1 that the version of each x ∈ V is highest in Θ(t), among all component versioned environments in σ. By the construction of the function χ, it follows that for each variable x ∈ V, φ(x) = (Θ(t).1)(x).

**Remark** Until now we assumed that buffers associated with every post-release point in L^rel are “relevant” to each pre-acquire point in L^pred. That is, for a post-release point n, if we take G(n) to be the set of pre-acquire points for which n is relevant, then so far we have assumed that G(n) = L^pred. However, if no (standard) execution of the program P contains a transition τ_n (with the target location being n) which synchronizes-with a transition τ_i (with source location n' ∈ L^pred), then Theorem 1 (as well as Theorem 2) holds even if we remove n' from G(n). This is true because in race-free programs, conflicting accesses are ordered by the happens-before relation. Thus, if the most up-to-date value of a variable accessed by t was written by another thread t', then in between these accesses there must be a (sequence of) synchronization operations starting at a lock released by t' and ending at a lock acquired by t. This refinement of the set G based on the above observation can be used to improve the precision of the analyses derived from L-DRF, as it reduces the set of possible release points an acquire can observe.

5 Abstract Analyses based on L-DRF

In this section we introduce and illustrate a few static program analyses which are based on the sync-CFG representation of a program and are, in turn, derived from
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the L-DRF semantics. We also reason about the correctness of such analyses using the notion of consistent abstractions. We begin by adapting the standard notion of abstract interpretation [9] to our setting, and recalling the theory of consistent abstractions.

5.1 Abstract Interpretation of programs

Let us fix a program $P = (\mathcal{V}, \mathcal{M}, T)$ for the rest of this section.

An abstract interpretation (or data-flow analysis) of $P$ is a structure of the form $A = (D, \leq, d_0, F)$ where

- $D$ is the set of abstract states and $\leq$ represents a partial ordering over $D$.
- $(D, \leq)$ forms a complete lattice. We denote the join (least upper bound) in this lattice by $\sqcup$, or simply $\sqcup$ when the ordering is clear from the context.
- $d_0 \in D$ is the initial abstract state.
- $F : \text{inst}_P \to (D \to D)$ associates a transfer function $F(\iota)$ with each instruction $\iota$ of $P$. In what follows, we will write $F(\iota)$ instead of $F(\iota)$ for ease of presentation.

We require each transfer function $F(\iota)$ to be monotonic, in that whenever $d \leq d'$ we have $F(\iota)(d) \leq F(\iota)(d')$.

An abstract interpretation $A = (D, \leq, d_0, F)$ of $P$ induces a “global” transfer function $F : D \to D$, given by

$$ F(d) = d_0 \sqcup \bigcup_{\iota \in \text{inst}_P} F(\iota)(d). $$

This transfer function can also be seen to be monotonic. By the Knaster-Tarski theorem [33], $F$ has a least fixed point (LFP) in $D$, and we define this to be the “semantics” or “meaning” associated to $P$ by the interpretation $A$, and denote it as $[P]_A$. Formally,

$$ [P]_A \triangleq \text{LFP}(F). $$

Given two analyses $C = (D, \leq, d_0, F)$ and $A = (D', \leq', d'_0, F')$ for $P$, we say $A$ is a consistent abstraction of $C$ if there exists functions $\alpha : D \to D'$ (called the abstraction function), and $\gamma : D' \to D$ (called the concretization function), such that:

1. $\alpha$ and $\gamma$ form a Galois connection, which entails the following:
   (a) $\alpha$ and $\gamma$ are monotonic
   (b) $\alpha$ and $\gamma$ satisfy the following conditions
      - $\forall d \in D : \gamma(\alpha(d)) \geq d$
      - $\forall d' \in D' : \alpha(\gamma(d')) = d'$
2. $\alpha([P]_C) \leq' [P]_A$ (or, equivalently, $[P]_C \leq \gamma([P]_A)$).

A sufficient condition for consistent abstraction, that can be checked “locally” for each instruction, was proposed in [9]:

**Theorem 3** ([9]) Let $C = (D, \leq, d_0, F)$ and $A = (D', \leq', d'_0, F')$ be analyses for $P$. A sufficient condition for $A$ to be a consistent abstraction of $C$ is that there exist maps $\alpha : D \to D'$, and $\gamma : D' \to D$, which satisfy:

1. $\alpha$ and $\gamma$ form a Galois connection,
2. for each $i \in \text{inst}_P$, $F'_i$ safely approximates $F_i$, in that
   \[
   \forall d \in D : \alpha(F_i(d)) \leq' F'_i(\alpha(d)),
   \]
3. and $\alpha(d_0) \leq' d'_0$.  
\[\square\]

5.2 Collecting Analyses

The interleaving semantics of Sec. 3.3 induces a “collecting” analysis of $P$,
\[
\mathcal{A}^\delta = (P(S), \subseteq, \{s_{\text{ent}}\}, F^\delta),
\]
where, for any instruction $i \in \text{inst}_P$, with $\text{tid}(i) = t$ say, and for any subset $X \subseteq S$, $F^\delta_i(X) = \{s' | \exists s \in X \text{ with } s \Rightarrow^\delta s'\}$. It turns out that the LFP of this analysis is exactly the reachable set of states in the transition system $L^\delta_P$:
\[
[P]_{\mathcal{A}^\delta} = \text{Reach}(L^\delta_P).
\]

In a similar way, the $L$-DRF semantics of Sec. 4 induces a collecting analysis $\mathcal{A}^L$ given by
\[
\mathcal{A}^L = (P(\Sigma), \subseteq, \{\sigma_{\text{ent}}\}, F^L),
\]
where, for any instruction $i \in \text{inst}_P$, with $\text{tid}(i) = t$ say, and for any subset $X \subseteq \Sigma$, $F^L_i(X) = \{\sigma | \sigma \in X \text{ with } \sigma \Rightarrow^L \sigma'\}$. Once again, the LFP of this analysis can be seen to coincide with the reachable set of states in the transition system $L^L_P$ of Sec. 4 for the L-DRF semantics:
\[
[P]_{\mathcal{A}^L} = \text{Reach}(L^L_P).
\]

5.3 Sync-CFG based analyses

We now introduce the class of sync-CFG based analyses, so called because they analyze concurrent programs using their “sync-CFG”. The sync-CFG representation of a concurrent program $P$ comprises the control flow graphs of each static thread code, augmented with synchronizes-with edges between synchronization operations (like releases and acquires of the same lock). Each thread operates on local copies of the data states, and communication between the threads is limited to synchronization points alone. Such an analysis was first introduced in [11], while analyses similar in spirit have been proposed in the literature (for example the thread-modular shape analysis of [16]).

A sync-CFG differs from the standard “product-graph” representation of concurrent programs in two important ways:

1. The sync-CFG contains nodes corresponding to each control location in the concurrent program $P$. In contrast, the product graph contains nodes corresponding to every possible combination of control locations in $P$.
2. Each execution of $P$ corresponds to some path in its product graph representation. A sync-CFG does not maintain such a property in general. On the other hand, a key property maintained by the sync-CFG is that for each execution of $P$, every happens-before path induced by the execution corresponds to some path in the sync-CFG.
As an example, consider again the program in Fig. 1. The sync-CFG representation of the program is given on the left in Fig. 9 (also shown in the center of Fig. 2). On the other hand, an excerpt of the far larger product-graph of this program is shown on the right of the same figure. As one may expect, any analysis based on the product graph would be intractable for large programs.

Fig. 9 The sync-CFG representation of the program of Fig. 1 is presented on the left. On the right is an excerpt of the standard product graph representation of the same program.

More precisely, we say an abstract interpretation $\mathcal{A}$ of a program $P$ is a sync-CFG based analysis if:

1. The domain of abstract states of $\mathcal{A}$ is of the form $L_P \rightarrow D'$. Thus the domain associates an abstract fact from $D'$ with each location in $P$.
2. The transfer function for each instruction $\iota = (n, c, n')$ depends only on the abstract fact at $n$ for commands other than $\text{acquire}()$, while for $\text{acquire}()$ commands the transfer function depends on the abstract facts at $n$ and associated $\text{release}()$ points.

The soundness of the facts computed by a sync-CFG based analysis needs to be qualified. The abstract fact computed by the analysis at each program point may not be an over-approximation of the set of concrete (interleaving) states arising at that point. However, the facts are sound as long as they are interpreted in the window of variables owned by the thread at that point (cf. Sec. 3.4). This property of soundness of sync-CFG analyses was hitherto proved by a direct and somewhat involved argument that the LFP of the analysis will over-approximate the owned portion of the concrete state along an execution [11, 16]. In particular, it appears difficult to argue soundness by showing that the analysis is a consistent abstraction of the standard interleaving semantics.

Instead, we give a way of arguing soundness of sync-CFG-based analyses by showing them to be consistent abstractions of the $L$-DRF semantics. In this sense, the $L$-DRF semantics is a kind of canonical or reference analysis for sync-CFG based analyses. We elaborate on this in Sec. 5.6. Before that, however, we outline several sync-CFG based analyses, as examples, which can be derived from the $L$-DRF semantics.
Thread \texttt{t1}() { 
1: \texttt{acquire(1)};
2: \texttt{x := y};
3: \texttt{x++;}
4: \texttt{y++;}
5: \texttt{release(1)};
6: }

Thread \texttt{t2}() { 
7: \texttt{acquire(1)};
8: \texttt{x++;}
9: \texttt{y++;}
10: \texttt{release(1)};
11: }

Fig. 10 A simple race-free program on which we illustrate the analyses \textit{VRel}, \textit{Rel} and \textit{ValSet}. All the variables are shared.

5.4 Some Sync-CFG based analyses induced by L-DRF

We introduce and illustrate some sync-CFG analyses that are derived from the L-DRF semantics. We call these analyses (in decreasing order of precision) \textit{VRel} (for “Versioned Relational”), \textit{Rel} (for “Relational”) and \textit{ValSet} (for “Value Set” \cite{11}). We will use the race free program in Fig. 10 as an example to illustrate these analyses.

5.4.1 The \textit{VRel} analysis

The \textit{VRel} analysis keeps track of sets of versioned environments at each program point. The abstract states are functions mapping program locations to sets of environments, ordered by point-wise inclusion. We call these states cartesian, since they now lose the correlation between thread locations in the program counter.

We define \textit{VRel} = ($\mathcal{L} \rightarrow \mathcal{P}(VE), \preceq, d_0^{VRel}, F^{VRel}$), where

- $f \preceq g$ iff for each $n \in \mathcal{L}$ we have $f(n) \subseteq g(n)$.
- The initial abstract state is
  
  \[ d_0^{VRel} = \lambda n. \begin{cases} \{ve_{ent}\} & \text{if } n \in ent_P \\ \emptyset & \text{otherwise.} \end{cases} \]

  Here $ve_{ent}$ is the versioned environment $\langle \lambda x.0, \lambda x.0 \rangle$.
- The transfer function $F^{VRel}_i$, for an instruction $inst = (n, c, n')$ of $P$ is given by
  
  \[ F^{VRel}_i = \lambda f. (f \sqcup \preceq f') \]

  where $f'$ is defined based on the command $c$ as follows. If $c$ is an assignment command $x := e$, 

  \[ f'(l) = \begin{cases} [x := e]_L(f(n)) & \text{if } l = n' \\ \emptyset & \text{otherwise.} \end{cases} \]

  By $[c]_L(f(n))$ we mean the application of the semantics of the command $c$, $[c]_L$, pointwise on the set of versioned environments $f(n)$. The case when $c$ is an \texttt{assume}(b) command is handled similarly.

  When $c$ is an \texttt{acquire}(m) command, we define

  \[ f'(l) = \begin{cases} \bigcup_{ve \in f(n)} \text{UpdEnv}(ve, X) & \text{if } l = n' \\ \emptyset & \text{otherwise,} \end{cases} \]
where \( X = \bigcup_{n \in L} f(n) \).

Interestingly, the effect of release commands in the cartesian semantics is the same as \texttt{skip}: This is because the abstraction neither tracks ownership of locks nor explicitly manipulates the contents of buffers. Thus when \( c \) is a release command, we define

\[
f'(l) = \begin{cases} f(n) & \text{if } l = n' \\ \emptyset & \text{otherwise} \end{cases}
\]

\textbf{Remark 2} We note here that we have chosen to define the transfer function in the form of \( F_\times = \lambda d. (d \sqcup d') \) instead of simply \( F_\times = \lambda d. d' \). This is because (a) it is easy to see that the LFP of the analyses coincide in both forms, and (b) the latter form will be convenient for showing the sufficient conditions for consistent abstraction in Sec. 6.

Fig. 12 shows a sequence of instructions from the program in Fig. 10, along with the abstract states obtained by running the \textit{VRel} analysis along this path. This is shown in the column marked \textit{VRel}. We show only the state at the relevant locations of the active thread along the execution. The leftmost column shows the \( \mathbb{L} - DRF \) states along the execution. Each \( \mathbb{L} - DRF \) state shown has four rows corresponding to the location counter, the local state of the thread \( t_1 \), the local state of thread \( t_2 \), and finally the contents of the release buffers. We ignore the lock maps here. It is instructive to see how the \textit{VRel} analysis over-approximates the \( \mathbb{L} - DRF \) analysis at each step along the execution path. The abstraction map here maps a set of \( \mathbb{L} \)-DRF states \( X \) to a set of versioned environments \( Y_n \) at point \( n \) in a thread \( t \), which contains the thread-local versioned environments of \( t \) in the states of \( X \) where thread \( t \) is a point \( n \). Finally, Fig. 13 shows the fixed point solutions of the three analyses we consider here, for the program of Fig. 10. The leftmost columns on the two sides of the program show the values for the \textit{VRel} analysis, with version tags abstracted away.

5.4.2 The Rel Analysis

We now define the \textit{Rel} analysis, which abstracts the \textit{VRel} analysis by abstracting away the version numbers. This is a more practicable analysis, and is one of the analyses we focus on subsequently in our experiments.

We define \( Rel = (A_\times, \subseteq_\times, a_\times^{ent}, F_\times) \), where

- The set of abstracts states is \( \mathbb{L} \rightarrow \mathcal{P}(Env) \), which we call \( A_\times \), and we range over it using the meta-variable \( a_\times \).
- We have \( a_\times \subseteq_\times a'_\times \) iff \( \forall n \in \mathbb{L} \) we have \( a_\times(n) \subseteq a'_\times(n) \).
- The initial abstract state is

\[
a_\times^{ent} = \lambda n. \begin{cases} \{\lambda x.0\} & \text{if } n \in ent_P \\ \emptyset & \text{otherwise} \end{cases}
\]

The initial state thus maps the entry location of every thread to the set containing the single environment, where all the variables are initialized to 0. Every other program location is mapped to the empty set.

- The transfer function \( F_\times \), for an instruction \( inst = (n, c, n') \) of \( P \) is given as follows. We define

\[
F_\times = \lambda a_\times.(a_\times \sqcup a'_\times),
\]
where $a'_x$ is defined as follows.

When $c$ is an assignment command $x := e$, we define

$$a'_x(l) = \begin{cases} [x := c]_{\mathbb{B}}(a_x(n)) & \text{if } l = n' \\ \emptyset & \text{otherwise.} \end{cases}$$

Here $[c]_{\mathbb{B}}$ is the interpretation of the command $c$ according to the standard semantics, assumed to apply pointwise on a set of environments. The case of an assume command is defined similarly.

When $c$ is a release command, we have

$$a'_x(l) = \begin{cases} a_x(n) & \text{if } l = n' \\ \emptyset & \text{otherwise,} \end{cases}$$

More directly,

$$F \times \iota = \lambda a \times a \times \{ n \mapsto a \times n \}.$$ 

When $c$ is an acquire$(m)$ command, we define

$$a'_x(l) = \begin{cases} E_{\text{mix}} & \text{if } l = n' \\ \emptyset & \text{otherwise,} \end{cases}$$

where

$$E_{\text{mix}} = \text{mix}(a_x(n') \cup \bigcup \{ a_x(\bar{n}) \mid \bar{n} \in \mathcal{L}_a^{rel} \land n \in G(\bar{n}) \}),$$

and

$$\text{mix} : \mathcal{P}(\mathbb{E}) \rightarrow \mathcal{P}(\mathbb{E}) \equiv \lambda B \times \{ \phi' \mid \forall x \in \mathcal{V}, \exists \phi \in B \times : \phi'(x) = \phi(x) \}.$$ 

In other words, the $\text{mix}$ returns a cartesian product of the input states. Note that as a result of abstracting away the version numbers, a thread cannot determine the most up-to-date value of a variable, and thus conservatively picks any possible value found either in its own local environment or in a relevant release buffer. Fig. 11 illustrates the operation of the $\text{mix}$ function on two arbitrary input environments.

We denote the LFP of the $\text{Rel}$ analysis for program $P$ by $[P]_\times$. 

![Fig. 11 Illustrating the $\text{mix}$ on a set of containing two environments $\phi_1$ and $\phi_2$. Observe that the invariant $x = y$ holds in the input environments. However, since this $\text{mix}$ operates at the granularity of single variables, the correlation is lost in the output states.](image-url)
5.4.3 The ValSet Analysis

The ValSet analysis of \([11]\) can be obtained as an abstraction of the Rel analysis. The abstract domain of the ValSet analysis is of the form \(L \rightarrow VS\), where VS is the “value-set” domain which maps each program variable to a set of values, that is, \(VS : V \rightarrow P(V)\).

We define \(ValSet = (L \rightarrow VS, \subseteq, s_0^{ValSet}, P^{ValSet})\) where

\(- s \subseteq s' \) iff \(\forall n \in \text{locs} \) we have \(s(n)(x) \subseteq s'(n)(x)\).
The initial abstract state is 
\[ s_0^{\text{ValSet}} = \lambda n. \begin{cases} 
\lambda x. \{0\} & \text{if } n \in \text{ent} \\
\lambda x. \emptyset & \text{otherwise.} 
\end{cases} \]

The transfer function \( F^{\text{ValSet}} \) can be defined via the transfer function \( F^\times \) of the \( \text{Rel} \) analysis. Let us define the value-set abstraction function \( \alpha_{\text{vs}} : \mathcal{A}_\times \rightarrow (\mathcal{L} \rightarrow \mathcal{VS}) \) as 
\[ \alpha_{\text{vs}}(a_\times) = \lambda n. (\lambda x. \{v \mid \exists \phi \in a_\times(n) : \phi(x) = v\}) , \]
and the value-set concretization function \( \gamma_{\text{VS}} : (\mathcal{L} \rightarrow \mathcal{VS}) \rightarrow \mathcal{A}_\times \) as 
\[ \gamma_{\text{VS}}(s) = \lambda n. \{\phi \mid \forall x \in V : \phi(x) \in s(n)(x)\} . \]

The transfer function of the \( \text{ValSet} \) analysis for an instruction \( \iota \) can now be defined as \( F^{\text{ValSet}}(s) = \alpha_{\text{VS}}(F^\times(\gamma_{\text{VS}}(s))) \).

In the \( \text{ValSet} \) analysis, the abstract mix operator reduces to the standard value-set join operation (which takes a component wise union of the value-sets).

The abstract state of the \( \text{ValSet} \) analysis along the example execution is shown in the third column of Fig. 12 and the fixed point solution in the third column of Fig. 13.

As one can see from Fig. 13, the analysis \( \text{VRel} \) computes the most precise facts – it is able to establish the equality between \( x \) and \( y \) prior to the \( \text{release()} \) command in both the threads. The \( \text{Rel} \) analysis loses this correlation after the \( \text{acquire()} \) command in thread \( t_2 \). Lastly, the \( \text{ValSet} \) analysis fails to establish any useful relation between \( x \) and \( y \).

5.5 Other abstractions of L-DRF

We can improve upon \( \text{Rel} \) in a practicable way by not forgetting the versions entirely. We augment \( \mathcal{A}_\times \) with “recency” information based on the versions as
follows. For a set $C$ of states of the L-DRF semantics, define $\text{recent}(C)$ to be the set of threads $t \in T$ such that there exists a state $(pc, \mu, \Theta, \Lambda) \in C$, and $x \in V$, such that $(\Theta(t).2)(x) \geq (\Theta(t').2)(x)$ for each $t' \in T$. In other words, $\text{recent}(C)$ is the set of threads which contain the most up-to-date value of some variable $x$. This additional information can now be used to improve the precision of $\text{mix}$.

![Fig. 14](image)

A simple race-free program to demonstrate the benefit of using thread-identifiers in the abstract state. In the normal setting, the synchronizes-with edges create a cycle in the program, and it is not possible to derive an upper bound on the value of $x$. However, if we track thread-identifiers in the state, thread $t_1$ observes that any state it receives from $t_2$ is tagged with the set $\{t_1\}$, and thus $t_1$ can safely drop the data flow facts.

In the program shown in Fig. 14, thread $t_1$ writes to $x$, while holding the lock $m$, whereas thread $t_2$ reads from $x$ while holding $m$. In the usual sync-CFG setting, the synchronizes-with edges creates a cycle in the program graph. Thus, the data flow facts propagate back and forth between the threads, and the analysis, in this example, fails to derive an upper bound for the value of $x$. In the recency based analysis, the data flow fact comprises elements from $A_x$, as well as a set $S$ of thread-identifiers that overapproximate the recency information. Whenever a thread writes to a variable, it adds its identifier to $S$. Other commands do not affect $S$. In the example, $t_1$ adds its identifier to $S$, and this is propagated to $t_2$. However, since $t_2$ does not write to $x$, the set $S$ is propagated back, unaltered, to $t_1$. The thread $t_1$ now finds that the incoming data flow fact contains a singleton $S$, with its own thread-identifier, which indicates it is receiving a stale fact. This allows the thread to safely drop the data flow fact along an incoming sync-edge, thereby breaking the cycle. An abstract analysis based on thread-identifiers can, in fact, prove an upper bound for $x$.

5.6 Soundness of Sync-CFG analyses

Consider a sync-CFG analysis $A$ for program $P$. We can prove the “soundness” of $A$, in the sense defined in Sec. 5.3 with respect to the interleaving semantics, by showing $A$ to be a consistent abstraction of the L-DRF analysis via an abstraction map $\alpha$ and concretization map $\gamma$. Simply put, the set of environments computed by the sync-CFG analysis $A$ at location $n$ in thread $t$, is guaranteed to be a safe approximation of the actual concrete (standard) states arising whenever thread $t$ is at location $n$, provided we restrict our attention to the sub-environments on the set of variables owned by $t$ at $n$. We state this more formally below.

**Theorem 4** Let $A$ be a sync-CFG analysis of a race free program $P$. Suppose that $A$ has been shown to be a consistent abstraction of the L-DRF analysis, via
an abstraction map $\alpha$ and concretization map $\gamma$. Let $t \in T$ and $n \in L_t$, and let $V$ be the set of variables owned by $t$ at location $n$. Let $s = \langle pc, \mu, \phi \rangle$ be a reachable state of the interleaving semantics, with $pc(t) = n$. Then there exists a state $\sigma = \langle pc, \mu, \Theta, \Lambda \rangle$ in $\gamma([P]_A)$ with $\phi = V(\Theta(t), 1)$.

**Proof** The proof is immediate since, by Corollary 2 there is a reachable state $\sigma$ of the L-DRF semantics which coincides with $s$, modulo the restriction to $V$. The fact that $A$ is a consistent abstraction of L-DRF says that the $\gamma$ image of its LFP must contain the state $\sigma$. 

For example, the facts about $x$ and $y$ inferred by each of the three analyses in Fig. 13 at point 4 is sound (since both $x$ and $y$ are owned by $t_1$ at these points). However at point 1, the inferred facts may not be sound (and in fact they are not), since $x$ and $y$ are not owned at point 1.

6 Soundness of Rel analysis

In this section we show that the Rel analysis is a consistent abstraction of the $A^L$ analysis based on L-DRF.

**Claim** For any program $P$, the analysis Rel is a consistent abstraction of the $A^L$ analysis for $P$.

**Proof** Consider a program $P = (V, M, T)$. We will make use of the definitions of the analysis $A^L$ from Sec. 5.2 and Rel from Sec. 5.4.2 and we refer the reader to them. To show that Rel is a consistent abstraction of $A^L$, it suffices (by Theorem 3) to exhibit an abstraction function $\alpha_x$ and a concretization function $\gamma_x$ satisfying the conditions of Theorem 3.

The abstraction function $\alpha_x$ maps a set of L-DRF states $C \subseteq \Sigma$ to an abstract state $\alpha_x \in A_x$. The abstract value $\alpha_x(C)(n)$ contains the collection of $t$’s environments (where $t = \text{tid}(n)$) coming from any state $\sigma \in C$ where $t$ is at location $n$. In addition, if $n$ is a post-release point, $\alpha_x(C)(n)$ also contains the contents of the buffer $A(n)$ for each state $\sigma \in C$. We define $\alpha_x : \mathcal{P}(\Sigma) \rightarrow A_x$, given by:

$$\alpha_x(C) = \lambda n. \{ \phi | \{ pc, \mu, \Theta, \Lambda \} \in C \land \text{tid}(n) = t \land pc(t) = n \land \Theta(t) = \phi \land \nu \} \cup \{ \phi | \{ pc, \mu, \Theta, \Lambda \} \in C \land n \in L^{rel} \land A(n) = \phi \land \nu \}.$$ 

The concretization function $\gamma_x$ maps a cartesian state $a_x$ to a set of L-DRF states $C$ in which the local state of a thread $t$, when $t$ is at program point $n \in L_t$, comes from $a_x(n)$ and the contents of the release buffer pertaining to the post-release location $n \in L^{rel}$ also comes from $a_x(n)$. We define $\gamma_x : A_x \rightarrow \mathcal{P}(\Sigma)$ given by:

$$\gamma_x(a_x) = \left\{ \{ pc, \mu, \Theta, \Lambda \} \in \Sigma | \forall t \in T : \Theta(t) = \phi \land \nu \in a_x(pc(t)) \land \forall n \in L^{rel} : A(n) = \phi \land \nu \in a_x(n) \right\}.$$ 

Let $X \subseteq \Sigma$ be a set of states of $P$ in the L-DRF semantics. Let $\epsilon = (n, c, n')$ be an instruction in $P$, with $\text{tid}(n) = t$. Let $X' = F^L_{\epsilon}(X) = \{ \sigma' | \exists \sigma \in X, \sigma \Rightarrow^L_{\epsilon} \sigma' \}$. 


Further, let \( a_\times = \alpha_\times(X) \) and \( a'_\times = F^\times_n(a_\times) \). Then we need to show that
\[
\alpha_\times(X') \subseteq a'_\times.
\] (2)

This is depicted in Fig. 15.

We observe that for each \( \sigma' = \langle pc', \mu', \Theta', \Lambda' \rangle \) in \( X' \) we have \( pc'(t) = n' \), and there exists a state \( \sigma = \langle pc, \mu, \Theta, \Lambda \rangle \) in \( X \) such that \( pc(t) = n, pc' = pc[t \mapsto n'] \), and for each \( t' \neq t \) we have \( \Theta'(t') = \Theta(t') \). Further, every environment \( \phi' \) that occurs in \( \Theta(t') \) where \( t' \neq t \) is already present in \( a_\times \). This is because (a) it is present in \( \sigma \) and \( a_\times \) ensures that it is present in the appropriate location in \( a_\times \); and (b) by the definition of the transfer function \( F^\times_n \), every environment at location \( l \) in \( a_\times \) is also at location \( l \) in \( a'_\times \). Thus to show that (2) holds, it suffices to show for an arbitrary \( \sigma' = \langle pc', \mu', \Theta', \Lambda' \rangle \) that the environments in \( \Theta(t) \) and \( \Lambda' \) are present in the appropriate locations \( (n' \text{ and release points, respectively}) \) in \( a'_\times \).

Let us fix an \( \sigma' = \langle pc', \mu', \Theta', \Lambda' \rangle \in X' \) and a \( \sigma = \langle pc, \mu, \Theta, \Lambda \rangle \in X \) as above. We now show this subclaim for each command \( c \).

**Assignment.** When \( c \) is an assignment of the form \( x := e \). Let \( \Theta(t) = \langle \phi', \nu' \rangle \). Then \( \phi' = [x := e] \phi \), where \( \Theta(t) = \langle \phi, \nu \rangle \), for some \( \nu \). Now \( \phi \in a_\times(n) \), and by the definition of \( F^\times_n \), also in \( a'_\times(n') \).

Further, since \( \Lambda' = \Lambda \), its environments are all included in \( a_\times \) and hence also in \( a'_\times \).

The case of assume commands is handled similarly.

**Release.** Recall that in this case \( a'_\times = a_\times[n' \mapsto (a_\times(n') \cup a_\times(n))] \). Now \( \phi' = \phi \) and therefore \( \phi' \in a_\times(n') \). Also, \( \Lambda' = \Lambda[n' \mapsto \langle \phi, \nu \rangle] \). But \( \phi \) already belongs to \( a_\times(n') \).

**Acquire.** In this case, \( \Theta(t) \) chooses to take the value of a variable \( x \) in the thread-local environment of \( t \), from the versioned environment \( ve \) in some relevant buffer, or the existing thread-local environment of \( t \). By the construction of \( a_\times \), if \( ve \) was chosen from some post-release point \( \bar{n} \), then this environment is guaranteed to exist in \( a_\times(n) \). Likewise, if \( ve \) is simply the thread-local versioned environment of \( t \), then the environment would be in \( a_\times(n) \). Since, by the semantics of the acquire...
in the Rel analysis, all the environments at all such $\bar{n}$, and the environment at $n$, is taken into account in the mix, and since this operation is performed for each variable $x \in \mathcal{V}$, we have $\Theta'(t).1 \in a_x(n')$.

This completes the proof of (2) and hence of the Claim. \qed

From Theorem 4, it now follows that the facts inferred by the Rel analysis about the owned set of variables at each location in a program $P$, are indeed sound.

7 A Region-Parameterized version of L-DRF

In this section, we introduce a refined notion of data race freedom, based on data regions, and derive from it a more precise abstract analysis capable of transferring some relational information between threads at synchronization points. The objective is to modify the L-DRF semantics such that the abstract mix operates at a granularity higher than individual variables.

7.1 Why do we need another semantics?

Fig. 11, which illustrates the operation of mix, also highlights the key issue with the L-DRF semantics: any abstract analysis derived from the L-DRF semantics must make use of an abstract mix which operates at the granularity of individual variables. Thus, even though two variables may be related in the input environments to mix (like $x = y$ in Fig. 11), the function must necessarily forget their correlation after the mixing. This is essential for soundness. This is the reason that prevents us from proving the assertion $x = y$ at line 11 in the motivating example in Fig. 2. Even though the acquire($m$) in $t_2$ obtains the fact $x = y$ from both its input edges, it fails to maintain this correlation post the mix.

While the VRel analysis we saw in Sec. 5.4 had a mix operator which did better for the program in Fig. 10 – it preserved the correlation between $x$ and $y$ after the mix in thread $t_2$ – the analysis is not practicable (it does not provide an abstraction of the versions, which may grow in an unbounded fashion).

Our solution is to make use of user-defined regions. Essentially, regions are a user-defined partitioning of the set of program variables. We call each partition a region $r$, denote the set of regions as $R$, and the region of a variable $x$ by $rg(x)$.

The semantics precisely tracks correlations between variables within regions across inter-thread communication, while abstracting away the correlations between variables across regions. This partitioning is based on the semantics of the program: developers often write code where a group of variables forms a logical cluster. Often, some invariant holds on the variables within this cluster at specific program points. Since we make this partitioning explicit in the semantics, with suitable abstractions the tracked correlations can improve the precision of the abstract analyses for programs which conform to the notion of race freedom defined below.
7.2 Region Race Freedom

We present a refinement of the standard notion of data race freedom by ensuring that variables residing in the same region are manipulated atomically across threads. A region-level data race occurs when two concurrent threads access variables from the same region (not necessarily the same variable), with at least one access being a write, and the accesses are devoid of any ordering constraints.

A command \( x := e \) constitutes a write access to the region \( \text{rg}(x) \), and a read access of every region \( \text{rg}(y) \), for each variable \( y \) appearing in the expression \( e \).

Similarly, a command \( \text{assume}(b) \) constitutes a read access of every region \( \text{rg}(y) \), for each variable \( y \) appearing in the condition \( b \). We are now in a position to introduce our notion of region level races.

**Definition 6 (Region-level races)** Let \( P \) be a program and let \( R \) be a region partitioning of \( P \). An execution \( \pi \) of \( P \), in the standard interleaving semantics, has a region-level race if there exists \( 0 \leq i < j < |\pi| \), such that \( c(\pi_i) \) and \( c(\pi_j) \) both access variables in region \( r \in R \), at least one access is a write, and it is not the case that \( \pi_i 
rightarrow_{hb} \pi_j \).

The problem of checking for region races can be reduced to the problem of checking for data races as follows. We introduce a fresh variable \( X_r \) for each region \( r \in R \). We now transform the input program \( P \) to a program \( P' \) with the following additions. We assume without loss of generality that \( \text{assume()} \) statements in only reference thread-local variables. For example, we replace \( \text{assume}(x < y) \) by the statements "\( l_x := x; \ l_y := y; \text{assume}(l_x < l_y) \)".

- We precede every assignment statement \( x := e \), where \( r_w \) is the region which is written to, and \( r_1, \ldots, r_n \) are the regions read, with a sequence of instructions \( X_{r_1} := X_{r_1}; \ldots; X_{r_n} := X_{r_n}; \).
- Statements of the form \( \text{assume}(b) \) do not need to be changed because \( b \) refers only to thread-private variables.
- The \text{acquire} and \text{release} statements do not involve the access of any variable.

Thus, they remain unmodified.

Note that these modifications do not alter the semantics of the original program (for each trace of \( P \) there is a corresponding trace in \( P' \), and vice versa). We now check for data races on the \( X_r \) variables.

7.3 The L-RegDRF semantics

The region-based version of L-DRF semantics, which we call here the L-RegDRF semantics, is obtained via a simple change to the L-DRF semantics: a write-access to a variable \( x \) leads to incrementing the version of every variable that resides in \( x \)'s region. In other words, the semantics of the assignment command, \([x := e] : VE \rightarrow VE\), is defined as follows:

\[ [x := e] \langle \phi, \nu \rangle = \langle \phi', \nu' \rangle \]

where \( \phi' = \phi[x \mapsto [e] \phi] \), and \( \nu' \) is given by:

\[ \nu'(y) = \begin{cases} 
\nu'(y) + 1 & \text{if } \text{rg}(y) = \text{rg}(x), \\
\nu'(y) & \text{otherwise.}
\end{cases} \]
It is not difficult to see that the versions of Theorems 1 and 2 hold for the completeness and soundness of the $L$-RegDRF semantics vis-a-vis the standard interleaving semantics, for programs that are region-race free. Hence, we can analyze such programs using abstractions of $L$-RegDRF and obtain sound results with respect to the standard interleaving semantics (Sec. 3.3).

7.4 Thread-Local Abstractions of the $L$-RegDRF Semantics

The cartesian abstractions defined in Sec. 5 can be extended to accommodate regions in a natural way. The only difference lies in the definition of the \textit{mix} operation, which now operates at the granularity of \textit{regions}, rather than variables:

$$\text{mix} : \mathcal{P}(\mathcal{E}) \rightarrow \mathcal{P}(\mathcal{E}) \overset{\text{def}}{=} \lambda B \times \{ \phi' \mid \forall r \in R, \exists \phi \in B_r \text{ s.t. } \forall x \in V \text{ s.t. } \text{rg}(x) = r \text{ we have } \phi'(x) = \phi(x) \}.$$  

Mixing environments at the granularity of regions is permitted because the $L$-RegDRF semantics ensures that all the variables in the same region have the same version. Thus, their most up-to-date values reside in either the thread’s local environment or in one of the release buffers. As before, we can obtain an effective analysis using any sequential abstraction, provided that the abstract domain supports the (more precise) region based \textit{mix} operator.

7.5 Illustrative Example

We illustrate the effect of the regions using some small examples. Consider again the situation in Fig. 11. Recall that even though the input environments maintained $x = y$, the \textit{mix} was unable to preserve this correlation because it operated at the granularity of individual variables. However, when \textit{mix} is made aware of the region definitions, it maintains the correlation between variables within a region. Thus, in Fig. 16, the invariant $x = y$ continues to hold in the output state.

Returning to the program in Fig. 2, consider the situation at the \texttt{acquire} at line 10 (illustrated in Fig. 17). It receives the invariant $x = y$ from both its input branches. The \textit{mix} in the $Rel$ abstraction of $L$-DRF only outputs the correct bounds for the variables, and forgets the correlation between $x$ and $y$. However, the region-aware \textit{mix} preserves this invariant, which enables the region-aware version of $Rel$ derived from $L$-RegDRF, which we call $RegRel$, to prove the assertion at line 11.

8 Implementation and Experiments

8.1 RATCOP: Relational Analysis Tool for COncurrent Programs

In this section, we perform a thorough empirical evaluation of our analyses using a prototype analyzer which we have developed, called RATCOP \cite{29}, for the static intra-procedural analysis of race-free concurrent Java programs. RATCOP

\footnote{The project artifacts are available at https://bitbucket.org/suvam/ratcop}
 Thread-Local Analyses

Fig. 16 Illustrating the operation of *mix* when it is aware of regions. In this example, with the regions being \(\langle \{x, y\}, \{z\} \rangle\), the function maintains the correlation between \(x\) and \(y\) in the output.

Fig. 17 The improved precision of the region aware *mix* derived from the \(L\)-RegDRF semantics allows it to prove the additional assertion at line 11 in Fig. 2.

The tool comprises around 4000 lines of Java code, and implements a variety of relational analyses based on the theoretical underpinnings described in earlier sections of this paper. Through command line arguments, each analysis can be made to use any one of the following three numerical abstract domains provided by the Apron library \[19\]: Convex Polyhedra (with support for strict inequalities), Octagons and Intervals. RATCOP also makes use of the Soot \[34\] analysis framework for Java. The tool reuses the code for fixed point computation and the graph data structures in the implementation of \[11\].

The tool takes as input a Java program with assertions marked at appropriate program points. We first checked all the programs in our benchmarks for data races and region races using Chord \[31\]. For detecting region races, we have implemented the translation scheme outlined in Sec. 7.2. RATCOP then performs the necessary static analysis on the program until a fixpoint is reached. Subsequently, the tool automatically tries to prove the assertions using the inferred facts (which translates to checking whether the inferred fact at a program point, projected to the variables owned at that point, implies the assertion condition): if it fails to
prove an assertion, it records the corresponding inferred fact in a log file for manual inspection. Fig. 18 summarizes the set of operations in RATCOP.

As benchmarks, we use a subset of concurrent programs from the SV-COMP 2015 suite [4]. We chose only those programs which we believe have interesting relational invariants. We ported the programs (which are originally in C) to Java and introduced locks appropriately to remove races. We also use a program from [26], which is an abstraction of a producer-consumer scenario. While these programs are not too large, they have challenging invariants to prove, and provide a good test for the precision of the various analyses. We ran the tool in a virtual machine with 16GB RAM and 4 cores. The virtual machine, in turn, ran on a machine with 32GB RAM and a quad-core Intel i7 processor. We evaluated five analyses on the benchmarks. The first four are based on the \textit{Rel} analysis (Sec. 5.4.2), and employ the Octagon numerical abstract domain. The last is based on the \textit{ValSet} analysis (Sec. 5.4.3), and uses the Interval domain. These analyses are named as follows:

1. \textit{RT}: Without regions and thread identifiers.
2. \textit{RT}: With regions, but with no thread identifiers.
3. \textit{RT}: Without regions, but with thread identifiers.
4. \textit{RT}: With regions and thread identifiers.
5. \textit{VS}: The value-set analysis of [11].

In terms of the precision of the abstract domains, the analyses form the following partial order: \textit{VS} \preceq \textit{RT} \preceq \textit{RT} \preceq \textit{RT} \preceq \textit{RT} \preceq \textit{RT} \preceq \textit{RT}. We use \textit{VS} as the baseline.

8.2 Evaluation

\textit{Porting Sequential Analyses to Concurrent Analyses}. For the sequential commands, we performed a lightweight parsing of statements and simply re-use the built-in transformers of Apron. The only operator we needed to define afresh was the abstract \textit{mix}. Since Apron exposes functions to perform each of the constituent steps, implementing the abstract \textit{mix} was straightforward as well.

\footnote{By thread-identifiers we are referring to the abstraction of the versions (recency information) outlined in Remark 5.5.}
*Precision and Efficiency.* Table 2 summarizes the results of the experiments.
| Program       | LOC | Threads | Asserts | √     | Time (ms) | √     | Time (ms) | √     | Time (ms) | √     | Time (ms) | √     | Time (ms) |
|--------------|-----|---------|---------|-------|----------|-------|----------|-------|----------|-------|----------|-------|----------|
| reorder_2    | 106 | 5       | 2       | 0(C)  | 77       | 2(C)  | 43       | 0(C)  | 71       | 2(C)  | 37       | 0     | 25       |
| sigma *      | 118 | 5       | 5       | 0     | 132      | 0     | 138      | 4     | 48       | 4     | 50       | 0     | 506      |
| ssc12        | 98  | 3       | 4       | 4     | 76       | 4     | 90       | 4     | 82       | 4     | 86       | 2     | 25       |
| unverif      | 82  | 3       | 2       | 0     | 115      | 0     | 121      | 0     | 84       | 0     | 86       | 0     | 46       |
| spin2003     | 65  | 3       | 2       | 2     | 6        | 2     | 9        | 2     | 10       | 2     | 10       | 2     | 8        |
| simpleLoop   | 4   | 2       | 2       | 2     | 56       | 2     | 61       | 2     | 57       | 2     | 64       | 0     | 27       |
| simpleLoop5  | 84  | 4       | 1       | 0     | 40       | 0     | 50       | 0     | 31       | 0     | 37       | 0     | 20       |
| doubleLock_p3| 64  | 3       | 1       | 1     | 11       | 1     | 24       | 1     | 16       | 1     | 19       | 1     | 9        |
| fib.Bench_Lon| 82  | 3       | 2       | 0     | 138      | 0     | 118      | 0     | 129      | 0     | 102      | 0     | 55       |
| index01      | 82  | 3       | 2       | 0     | 95       | 0     | 103      | 0     | 123      | 0     | 91       | 0     | 35       |
| twostage_3   | 93  | 2       | 2       | 2     | 1522     | 2     | 1637     | 2     | 1750     | 2     | 1733     | 2     | 719      |
| singleton_   | 59  | 2       | 1       | 1     | 31       | 1     | 29       | 1     | 14       | 1     | 10       | 1     | 28       |
| with_uninit  |     |         |         |       |          |       |          |       |          |       |          |       |          |
| stack        | 85  | 2       | 2       | 0     | 151      | 0     | 175      | 0     | 127      | 0     | 129      | 0     | 71       |
| stack_longer | 85  | 2       | 2       | 0     | 1163     | 0     | 669      | 0     | 1082     | 0     | 1186     | 0     | 597      |
| stack_longest| 85  | 2       | 2       | 0     | 1732     | 0     | 1679     | 0     | 1873     | 0     | 2068     | 0     | 920      |
| sync01 *     | 65  | 2       | 2       | 2     | 7        | 2     | 25       | 2     | 37       | 2     | 33       | 2     | 10       |
| mp2004 *     | 50  | 2       | 4       | 0     | 1401     | 4     | 1890     | 0     | 1784     | 4     | 1913     | 0     | 698      |
| [26] Fig. 3.11| 89  | 2       | 2       | 0     | 39       | 2     | 46       | 0     | 54       | 2     | 36       | 0     | 19       |
| Total        | 1625| 3 (Avg) | 42      | 14    | 361 (Avg)| 22    | 366 (Avg)| 18    | 374 (Avg)| 26    | 406 (Avg)| 10    | 204 (Avg)|

Table 2: Summary of the experiments. Superscript A indicates that the program has an actual bug. (C) indicates the use of Convex Polyhedra as abstract data domain. "**" indicates a program we have altered/weakened the original assertion. The √ column indicates the number of assertions the tool was able to prove.
While all the analyses failed to prove the assertions in `reorder_2`, `RT` and `RT` were able to prove them when they used convex polyhedra instead of octagons. Since none of the analyses track arrays precisely, all of them failed to prove the original assertion in `sigma` (which involves checking a property involving the sum of the array elements). However, `RT` and `RT` correctly detect a potential array out-of-bounds violation in the program. The improved precision is due to the fact that `RT` and `RT` track thread identifiers in the abstract state, which avoids spurious read-write cycles in the analysis of `sigma`. The program `twostage_3` has an actual bug, and the assertions are expected to fail. This program provides a "sanity check" of the soundness of the analyses. Programs marked with "*" contain assertions which we have altered completely and/or weakened. In these cases, the original assertion was either expected to fail or was too precise (possibly requiring a disjunctive domain in order to prove it). In `qv2004`, for example, our modified assertions are of the form $x = y$. `RT` and `RT` perform well in this case, since we can specify a region containing $x$ and $y$, which precisely tracks their correlation across threads. The imprecision in the remaining cases are mostly due to the program requiring disjunctive domains to discharge the assertions, or the presence of spurious write-write cycles which weaken the inferred facts. Abstracting our semantics to handle such cycles is an interesting future work.

Of the total 40 "valid" assertions (excluding the two in `twostage_3`), `RT` is the most precise, being able to prove 65% of them. It is followed by `RT` (55%), `RT` (45%), `RT` (35%) and, lastly, `VS` (25%). Thus, the new analyses derived from $\mathbb{L}$-DRF and $\mathbb{L}$-RegDRF perform significantly better than the value-set analysis of [11]. Moreover, this total order respects the partial ordering between the analyses defined earlier.

With respect to the running times, the maximum time taken, across all the programs, is around 2 seconds, by `RT`. `VS` turns out to be the fastest in general, due to its lightweight abstract domain. `RT` and `RT` are typically slower than `RT` and `RT` respectively. The slowdown can be attributed to the additional tracking of regions by the former analyses. Note that for the program `sigma`, `RT` was both more precise and faster than the baseline `VS`.

### 8.3 Comparison with a recent abstract interpretation based tool.

We also compared the efficiency of RATCOP with that of Batman, a tool implementing the previous state-of-the-art analyses based on abstract interpretation [27, 28] (a discussion on the precision of our analyses against those in [27] is presented in Sec. 9). The basic structure of the benchmark programs for this experiment is as follows: each program defines a set of shared variables. A `main` thread then partitions the set of shared variables, and creates threads which access and modify variables in a unique partition. Thus, the set of memory locations accessed by any two threads is disjoint. In our experiments, each thread simply performed a sequence of writes to a specific set of shared variables. In some sense, these programs represent a "best-case" scenario for concurrent program analyses because there are no interferences between threads. Unlike RATCOP, the Batman tool, in its current form, only supports a small toy language and does not provide the means to automatically check assertions. Thus, for the purposes of this experiment, we only compare the time required to reach a fixpoint in the two
tools. We compare $\text{RT}$ against Batman running with the Octagon domain and the BddApron library \cite{18} (Bm-oct).

| #Threads | RT Time (ms) | Bm-oct Time (ms) |
|----------|-------------|------------------|
| 2        | 61          | 7706             |
| 3        | 86          | 82545            |
| 4        | 138         | 507963           |
| 5        | 194         | 2906585          |
| 6        | 261         | 13995977         |
| 7        | 368         | 53239574         |

Table 3 Running times of RATCOP ($\text{RT}$) and Batman (Bm-oct) on loosely coupled threads. The number of shared variables is fixed at 6.

The running times of the two analyses are given in Table 3. The graph in Fig. 19 plots these running times on a logarithmic scale. In the benchmarks, with increasing number of threads, RATCOP was up to 5 orders of magnitude faster than Bm-oct. The rate of increase in running time was roughly linear for RATCOP, while it was almost exponential for Bm-oct. We believe the reason for this difference in running times is that the analyses in \cite{27,28} compute sound facts at every program point. Thus, as the number of threads increase, these analyses have to account for data flow over an exponential number of context-switch points, which contributes to the slowdown. RATCOP, on the other hand, does not attempt to be sound at all program points. For assertions in thread $t$ which only involve variables in the logical partition of $t$, RATCOP is at least as precise as Batman, since proving such assertions do not require inter-thread reasoning.

Fig. 19 Graphical representation of the data in Table 3 on a logarithmic scale. RATCOP performs exponentially faster, compared to Batman, on this benchmark.
9 Related Work and Discussion

In this paper we have presented a framework for developing intra-procedural data-flow analyses for data race free shared-memory concurrent programs, with a statically fixed number of threads, and with variables having primitive data types.

There is a rich literature on data flow analysis of concurrent programs. We refer the reader to the detailed survey by Rinard [32] which provides details of the main approaches. In this section, we proceed to compare our work with some of the relevant prior approaches.

Degree of Inter-thread Communication. Chugh et al [7] automatically lift a given sequential analysis to a sound analysis for concurrent programs, using a data race detector. However, data-flow facts are not communicated across threads, and this can cause a loss in precision. The work by Mine [25] allows a greater degree of inter-thread communication. Here, the overall analysis can be considered to proceed in rounds of thread-modular analyses. At the end of each round, every thread generates a set of per-thread “interferences” – for each variable $x$, a thread $t$ stores the set of values it writes to $x$ when $t$ was analyzed modularly. In the next iteration, each thread $t' \neq t$ takes into account this interference information from $t$, whenever it reads $x$. This, in turn, generates more interferences for $t'$, and the process continues till fixpoint. Thus, the inter-thread communication is flow insensitive. Unlike our semantics, this analysis is unable to infer relational properties between variables.

Mine [27] presents an abstract interpretation formulation of the rely-guarantee proof paradigm [20, 35], and allows one to derive analyses with varying degrees of inter-thread flow sensitivity. In particular, the work in [25] is shown to be an abstraction of the semantics in [27]. The semantics in [27] involves a nested fixed-point computation, compared to our single fixed-point formulation. The resulting analysis aims to be sound at all program points (e.g., in Fig. 2 the value of $y$ at line 9 in $t_2$), due to which many more interferences will have to be propagated than we do, leading to a less efficient analysis. The times clocked by Batman, in comparison to RATCOP, is testament to this. [27] attempts to retrieve some degree of efficiency by computing “lock invariants”, which are essentially summaries of each critical section. However, to make use of this, the program must be well-synchronized – every access of a shared variable must be protected by a lock, which is a stronger requirement than data race freedom. Moreover, for certain programs, our abstract analyses are more precise. Fig. 20 shows a program which is race free, even though the conflicting accesses to $x$ in lines 2 and 12 are not protected by a common lock. The “lock invariants” in [27] would consider these accesses as potentially racy, and would allow the read at line 12 to observe the write at line 2, thereby being unable to prove the assertion. However, our analyses would ensure that the read only observes the write at line 11, and is able to prove the assertion. [15] presents an operational semantics for concurrent programs, parameterized by a relation. It makes additional assumptions about code regions which are unsynchronized (allowing only read-only shared variables and local variables in such regions). Moreover, it too computes sound facts at every point, resulting in less efficient abstractions. In this sense, De et al [11] strikes a sweet spot: by leveraging the race freedom assumption, the analysis restricts data flow facts to
synchronization points alone, thereby gaining efficiency. However, this work cannot compute relational information either, being based on a cartesian value-set domain.

**Control Flow Representation.** The methods described in [11,12,17] present concurrent data flow algorithms by building specialized concurrent flow graphs. However, the class of analyses they address are restricted -- [12] handles properties expressible as Quantified Regular Expressions, [17] handles reaching definitions, while [11] only handles value-set analyses. While our analyses also makes use of the sync-CFG data structure of [11], the L-DRF and L-RegDRF semantics allows us to use it in conjunction with much more expressive abstract domains. In contrast to our approach, the techniques in [13,14] provide an approach to verifying properties of concurrent programs using data flow graphs, rather than use control flow graphs like we do.

```plaintext
Thread t1() {
  1: acquire(m);
  2: x := 1;
  3: y := 1;
  4: release(m);
}

Thread t2() {
  6: while ( p != 1 ) {
     7: acquire(m);
    8: p := y;
    9: release(m);
  10: }
  11: x := 2;
  12: p := x;
  13: assert (p != 1);
  14: }
```

**Fig. 20** Example demonstrating that a program can be DRF, when the accesses of a global variable (in this case, the write and read of x at lines 11 and 12 respectively) are not directly guarded by any lock.

**Resource Invariants vs. Regions.** A traditional approach to analyzing concurrent programs involves resource invariants associated with every lock (e.g. Gotsman et al [16]). This approach depends on a locking policy where a thread only accesses global data if it holds a protecting lock. In contrast, our approach does not require a particular locking policy (e.g., see Fig. 20), and is based on a parameterized notion of data-race-freedom, which allows to encode locking policies as a particular case. Thus, at the overhead cost of ensuring data race freedom, our new semantics provides greater flexibility to analysis writers. The analysis in [16] also works in a similar spirit as the sync-CFG a selected part of the heap protected by a lock is made accessible to a thread only when it acquires the lock. In contrast, the synchronization edges in a sync-CFG propagate entire data flow facts. The locking policy employed by [16] is stronger than the notion of race freedom, and the class of programs the analysis can handle is a subset of what we handle in this work.

**Region Races.** Our notion of region races is inspired by the notion of high-level data races [3]. The concept of splitting the state space into regions was earlier used in [23], which used these regions to perform shape analysis for concurrent programs. However, that algorithm still performs a full interleaving analysis which results in poor scalability. The notion of variable packing [5] is similar to our notion
of data regions. However, variable packs constitute a purely syntactic grouping of variables, while regions are semantic in nature. A syntactic block may not access all variables in a semantic region, which would result in a region partitioning more refined than what the programmer has in mind, which would result in decreased precision.

As future work, we would like to evaluate the performance of our tool when equipped with disjunctive relational domains. In this work, we do not consider dynamically allocated memory, and extending the L-DRF semantics to account for the heap memory is interesting future work. Abstractions of such a semantics could potentially yield efficient shape analyses for race free concurrent programs.

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