Long-term temporal variations in the areas of sunspot groups

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Abstract

Recently, Javaraiah (2012a) analyzed the combined Greenwich and Solar Optical Observing Network (SOON) sunspot group data during the period 1874–2011 and studied variations in the annual numbers (counts) of the small (maximum area $A_M < 100$ millionth of solar hemisphere, msh), large ($100 \leq A_M < 300$ msh), and big ($A_M \geq 300$ msh) sunspot groups. Here that analysis is extended and studied variations in the mean maximum sizes (the mean values of maximum areas) of the aforementioned three classes of sunspot groups and also their combination. It is found that there is no significant correlation between the mean maximum size of any class of sunspot groups and the International Sunspot Number ($R_Z$), probably due to in a given time interval small sunspot groups/sunspots outnumber the large ones. A pattern of an approximate 9-year period cycle is seen in the variations of the mean maximum sizes of the large and the big sunspot groups during a solar cycle. On long-time scales it is found that there exists a strong 130 or more years cycle in the variation of the mean size of the small sunspot groups, whereas there is a hint on the existence of $\approx 44$-year cycles in the variations of the mean maximum sizes of the large and the big sunspot groups. During the decline phase of cycle 23, there was a scarcity in the sunspot groups whose $A_M \leq 37$ msh, which may be related to the slow growth of sunspot groups during this period. During the minimum between cycles 23 and 24 may be due to the presence of a number of small sunspot groups whose $A_M > 37$ was larger than that of whose $A_M \leq 37$, the relatively large size coronal holes were present at low-latitudes and the total solar irradiance was very low.

Keywords: solar magnetic field, solar activity, solar cycle

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1. Introduction

The study of variations in solar activity is important for understanding the mechanism behind the solar activity, and because of solar activity affects space weather and it may also have a contribution in Earth’s climate change (e.g., Hathaway, 2010; Ahluwalia & Ygbuhay, 2012). Although it is well believed that the solar cycle results from generation of strong toroidal magnetic fields in the Sun, with a main period of about 22-years, by combined effects of convection and differential rotation in the Sun (e.g., Dikpati & Gilman, 2006), it is not known exactly what causes solar cycle. Because of the long record of observations, sunspot measurements constitute a primary source of information to better understand the level and nature of solar activity. It is well-known that the dynamic properties, such as rotation rate and meridional motion, of sunspot groups depend on their sizes. This may imply the dynamics of the Sun’s subsurface layers in which the different size sunspot groups anchor/generated (Howard, 1996; Javaraiah & Gokhale, 1997; Javaraiah, 1999; Hiremath, 2002; Sivaraman et al., 2003, 2007, 2010; Sivaraman & Gokhale, 2004). Therefore, studies on the variations in the sizes of sunspot groups are important, which may provide important clues on the underlying mechanism of solar activity and its variations. In some studies it is found that the sunspot area distribution does not depend on the Zürich or International Sunspot Number (RZ) (e.g., Babij, Efimenko, and Lozitsky, 2011). Recently, we have analyzed the combined Greenwich and SOON sunspot group data during the period 1874–2011 and studied variations in the annual numbers (counts) of the small, large, and big sunspot groups, by classifying the sunspot groups on the basis of their maximum areas (Javaraiah, 2012a; hereafter Paper I). Although, it is found that the number of the sunspot groups in each class shows the 11-year periodic variation of solar cycle, some noticeable differences are seen in the variations of the numbers of the sunspot group in different classes. In the present paper we extended that analysis and studied long-term variations in the mean maximum size of each of the three classes of sunspot groups and also that of all the sunspot groups in the combined class of the three classes.

In the next section we will describe the data and the method of analysis. In Section 3 we will present the results and in Section 4 we will present conclusions and a brief discussion.
2. Data and Analysis

Here the data and the method of analysis are the same as in Paper I. We have used the combined Greenwich and SOON sunspot group data during the period May-1874 to May-2011 (taken from http://solarscience.msfc.nasa.gov/greenwch.shtml). These data include the observation time (the Greenwich data contain the date with the fraction of a day, in the SOON data the fraction is rounded to 0.5 day), heliographic latitude and longitude, central meridian distance (CMD), and corrected umbra and whole-spot areas (in msh), etc., of sunspot groups for each day of observation. The positions of the sunspot groups are geometrical positions of the centers of the groups.

The Greenwich data (May-1874 to December-1976) have been compiled from the majority of the white light photographs which were secured at the Royal Greenwich Observatory and at the Royal Observatory, Cape of Good Hope. The gaps in their observations were filled with photographs from other observatories, Cape Town, South Africa; Kodaikanal, India; and Mauritius. The SOON data (January-1977 to May-2011) include measurements made by the United States Air Force (USAF) based on sunspot drawings obtained by a network of the observatories in Boulder, Hawaii, and so on. David Hathaway scrutinized the Greenwich and SOON data and produced a reliable continuous data series from 1874 up to date. In the case of SOON data, we increased the corrected whole-spot areas by a factor of 1.4. David Hathaway found this correction was necessary to have a combined homogeneous Greenwich and SOON data. However, the agreement between these two data sets may be still not 100% [Hathaway & Choudhary, 2008]. The combined Greenwich and SOON sunspot group data are the largest available, reliable data that include the positions and the areas of sunspot groups.

We have used the data on only those sunspot groups whose birth and death occurred within a disk passage. That is, we have not used the sunspot groups whose central meridian distance $|\text{CMD}| > 75^\circ$ in any day of their respective life times. This reduces the foreshortening effect and helps to obtain the maximum area of a sunspot group unambiguously.

If $A_1, A_2, \ldots, A_n$ denote the areas (corrected for the foreshortening effect) of all the sunspots in a sunspot group observed at times $t_1, t_2, \ldots, t_n$ during the life time of the sunspot group $T = t_n - t_1$ (days), then the maximum area is defined as follows:

$$A_M = \max(A_1, A_2, \ldots, A_n),$$ (1)
where \( n = 2, 3, \ldots \). Each appearance of a recurrent group is treated as an independent group. Thus, \( T \leq 12 \) days. We have used here only the sunspot groups which had \( T \geq 1 \) day (SOON data do not contain the data on the sunspot groups whose \( T < 1 \) day).

On the basis of \( A_M \) values we have classified sunspot groups into three classes as small sunspot groups (SSGs: \( A_M < 100 \) msh), large sunspot groups (LSGs: \( 100 \geq A_M < 300 \) msh), and big sunspot groups (BSGs: \( A_M \geq 300 \) msh). We determined the mean maximum sizes (sum of the maximum areas divided by the number of groups) \( \bar{A}_{SSG}, \bar{A}_{LSG}, \) and \( \bar{A}_{BSG} \) of SSGs, LSGs, and BSGs respectively, for each year during the period 1874–2011 as follows:

\[
\bar{A}_{SSG} = \frac{1}{\text{NSG}} \sum_{i=1}^{\text{NSG}} A_{M,i}, \quad \bar{A}_{LSG} = \frac{1}{\text{NLG}} \sum_{j=1}^{\text{NLG}} A_{M,j}, \quad \text{and} \quad \bar{A}_{BSG} = \frac{1}{\text{NBG}} \sum_{k=1}^{\text{NBG}} A_{M,k},
\]

where NSG, NLG, and NBG are the numbers (counts) of SSGs, LSGs, and BSGs, respectively, in a given year (NSG > NLG > NBG). The mean maximum size (\( \bar{A}_{ASG} \)) of all sunspot groups (ASGs) of the same year is calculated as follows:

\[
\bar{A}_{ASG} = \frac{1}{\text{NSG} + \text{NLG} + \text{NBG}} (\text{NSG} \times \bar{A}_{SSG} + \text{NLG} \times \bar{A}_{LSG} + \text{NBG} \times \bar{A}_{BSG}).
\]

We have also calculated the mean maximum size of each of the above classes of sunspot groups over a solar cycle (\( \bar{A}_{WCN} \), the sum of the yearly means divided by the number of years of a cycle) as follows:

\[
\bar{A}_{WCN} = \frac{1}{L_{WCN}} \sum_{i=1}^{L_{WCN}} \bar{A}_{c,i},
\]

where WCN and \( L_{WCN} \) represent the Waldmeier cycle number and length (in years) of a cycle, respectively; \( c \) represents any of the above classes of sunspot groups, and \( i \) is a year of the corresponding cycle. The uncertainty (standard error) in \( \bar{A}_{WCN} \) is also determined.

We determined the variations in \( \bar{A}_{SSG}, \bar{A}_{LSG}, \bar{A}_{BSG}, \bar{A}_{ASG}, \) and \( \bar{A}_{WCN} \).
3. Results

Fig. 1 shows the variations in $\tilde{A}_{SSG}$, $\tilde{A}_{LSG}$, $\tilde{A}_{BSG}$, and $\tilde{A}_{ASG}$ (i.e., annual variations in the mean maximum sizes of SSGs, LSGs, BSGs, and ASGs, cf., Eqs. (2) and (3)) during the period 1874–2011 (a value equal to zero implies the absent of sunspot groups in the corresponding class). In the same figure we have also shown the variation in the annual mean $R_Z$, whose values are taken from the website, [ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/INTERNATIONAL/yearly/YEAR.PLT]). As can be seen in this figure the mean maximum size of the sunspot groups in each class varies considerably, but the patterns of the variations considerably differ with the patterns of the corresponding 11-year cycles in $R_Z$. There is no significant correlation between the mean maximum size of any class of sunspot groups and $R_Z$ (in the case of big groups the value of the correlation coefficient is largest and equal to 0.315). The mean maximum size of the sunspot groups in any class does not have a 11-year solar cycle pattern. Figs. 2 and 3 show the comparisons of $\tilde{A}_{SSG}$ and $\tilde{A}_{LSG}$ with $R_Z$ during different solar cycles, using superposed epochs related to the respective maximum epochs of the cycles (corresponding figures of $\tilde{A}_{BSG}$ and $\tilde{A}_{ASG}$ are not shown). As can be seen in Fig. 3, there is a pattern of an approximate 9-year cycle in $\tilde{A}_{LSG}$. The similar pattern is also found in the case of $\tilde{A}_{BSG}$, but in $\tilde{A}_{SSG}$ it cannot be seen clearly (see Fig. 2). No clear such pattern seen in $\tilde{A}_{ASG}$ also. That is, in both of these cases the variations during different solar cycles are highly different. The mean variation (represented by the solid curve) is statistically highly insignificant, i.e. the errors bars are too large.

Figs. 4(a)–4(d) show the variations in $\tilde{A}_{WCN}$ (cf., Eq. (4)) of the sunspot groups in each class (i.e., cycle-to-cycle variations in the mean maximum sizes of SSGs, LSGs, BSGs and ASGs). As can be seen in this figure $\tilde{A}_{WCN}$ of SSGs varies systematically on a time scale of about 12 cycles, suggesting the existence of a strong 130 or more years cycle in $\tilde{A}_{WCN}$, whose minimum was taken place during cycles 15–16. The long-term variations in $\tilde{A}_{WCN}$ of LSGs and BSGs suggest the existence of $\approx 44$-year cycles in these parameters, but the variations are not well defined (the error bars are large). A 44-year periodicity was seen in sunspot activity (Javaraiah, 2008). From cycle 12 to 20 the pattern of the variation in $\tilde{A}_{WCN}$ of ASGs (see Fig. 4(d)) is largely same as the corresponding variations in $\tilde{A}_{WCN}$ of SSGs, whereas it is drastically different during the last three cycles in which it seems the sunspot groups were relatively small (this could be due the areas of sunspot groups of SOON
data may not be increased adequately, see Section 3). The sunspot groups in cycles 12 and 17 were relatively large, whereas in cycles 14 and 20 (relatively small and long cycles) they were relatively small, suggesting the existence of a 55–60 year cycle in $A_{\text{WCN}}$ of ASGs.

Incidentally, the values of $A_{\text{WCN}}$ of SSGs of cycles 12 and 23 are equal (see Fig. 4(a)). In the case of cycle 23, the high value of $A_{\text{WCN}}$ of SSGs was largely contributed by the sunspot groups in the decline phase, whereas in case of cycle 12 the high value of $A_{\text{WCN}}$ of SSGs was largely contributed by the sunspot groups in the rise phase (see Fig. 1). In fact, in the former case the dominance in the mean maximum sizes of SSGs was continued from the decline phase of cycle 11. During this period of cycle 11 the mean maximum sizes of all the three classes of sunspot groups were relatively larger, whereas in the case of the decline phase of cycle 23 only the mean maximum sizes of SSGs were relatively larger. That is, during this period of cycle 23 (and some extent also during the rise of cycle 12) there was a scarcity in the sunspot groups whose areas $\leq 37$ msh. This can be seen in Fig. 5, in which solar cycle variations in the annual numbers of the small sunspot groups whose $A_M \leq 37$ msh and $37 < A_M \leq 100$ msh are shown. During 2000–2006 the values that are represented by the red squares (connected by dotted curves) in 5(a) are lower than the corresponding values in 5(b). Fig. 6 shows the cycle-to-cycle variations in the mean annual numbers of these two classes of the small sunspot groups. As can be see in this figure in cycle 23 on the average the annual mean number of sunspot groups whose $A_M \leq 37$ msh is relatively less than those of whose $37 < A_M \leq 100$ msh.

Cycle 23 was unusually long (about 12.4 years) and followed by the unusually deep and prolonged minimum. When the Sun reached the minimum activity between cycles 22 and 23 in 1996, it seems the low-latitude coronal holes were almost completely absent, whereas in 2007 and 2008, in spite of the extremely low sunspot activity, coronal holes at low-latitudes were still relatively large (e.g. de Toma, 2012). The occurrence of BSGs and LSGs stopped in 2004 and 2006, respectively, whereas the occurrence of SSGs continued (see Fig. 1, also see Javaraiah, 2012a). Fig. 7 shows the plot of yearly mean sizes of SSGs during the preceding minimum epochs of cycles 12–23 and around the minimum between cycles 23 and 24 (i.e., in years 2006, 2007, 2008 and 2009) versus the corresponding years. As can be seen in this figure in 2007 the mean size of SSGs whose $37 < A_M \leq 100$ msh is relatively large, and in 2008 the overall mean size of all SSGs are relatively large. In 2007 the mean sizes of both the two classes of SSGs are relatively larger than corre-
sponding values of in 2008, whereas in the case of the combined class of SSGs the value at 2008 is relatively large (this is because the number of sunspot groups with \( A_M \leq 37 \) are relatively less in 2008). The presence of relatively large coronal holes at low-latitudes in 2007 and 2008 may be related to the presence of relatively large size SSGs in these years. Since the sunspots block the Sun’s radiative output, the existence of the relatively large size SSGs in the minimum between cycles 23 and 24 (the evolution rate of SSGs is also relatively low, [Javaraiah, 2012b]) may be also responsible for the existence of very low total solar irradiance during this period [Fröhlich, 2009].

4. Conclusions and Discussion

From the above analyses of a large set of sunspot group data the following conclusions can be drawn:

1. There is no significant correlation between the yearly mean maximum size of the sunspot groups in any class and \( R_Z \).
2. A pattern of \( \approx 9 \)-year cycle is seen in the variations of the mean maximum sizes of LSGs and BSGs with phases of solar cycles.
3. There exists a strong 130 or more years cycle in the variation of the mean maximum size of SSGs, whereas there is a hint on the existence of \( \approx 44 \)-year cycles in the variations of the mean maximum sizes of LSGs and BSGs.
4. During the decline phase of cycle 23, there was a scarcity in the sunspot groups whose areas \( \leq 37 \) msh.
5. During the minimum between cycles 23 and 24 may be due to the presence of a number of small sunspot groups whose \( A_M > 37 \) was larger than that of whose \( A_M \leq 37 \), the relatively large size coronal holes were present at low-latitudes and the total solar irradiance was very low.

It is believed that the area of a sunspot or a sunspot group has a better physical significance than \( R_Z \) because the area is a better measure (proxy) of solar magnetic flux than \( R_Z \) (Dikpati & Gilman, 2006); an area of 130 msh (1 msh \( \approx 3 \times 10^6 \) km\(^2\)) corresponds approximately to \( 10^{22} \) Mx (maxwell) (Wang & Sheeley, 1989). The annual sum of the areas of sunspot groups reasonably well correlate to the annual average \( R_Z \) (Javaraiah, 2012b). The sum of the areas of sunspots and sunspot groups may be representing the total magnetic flux of the sunspots and sunspot groups rather than \( R_Z \). It may be worth to note
that $R_Z$ is not weighted with the areas of sunspots and sunspot groups, i.e. it gives an equal weight to all sunspots and to all sunspot groups regardless of their actual size (e.g. Clette & Lefevre, 2012). The yearly mean $R_Z$ is determined from the corresponding monthly means, i.e., the sum of monthly means divided by number of months (it seems to be always 12, in spite of in some months the values of the sums are zero, i.e. absent of sunspots and sunspot groups). Here the yearly mean size of sunspot groups is directly determined (cf., Eq. (2)). That is, there is a difference in the methods of determinations of the yearly mean size of sunspot groups and the yearly mean of $R_Z$. The annual number of ASGs reasonably well correlate to $R_Z$ (the variations in NSG, NLG and NBG considerably differ with that of $R_Z$ during some solar cycles, see Figs. 2–4 in Javaraiah, 2012a). The sum of the areas of any class of sunspot groups reasonably correlate to $R_Z$. The good correlations of both the number and the sum of the areas of ASGs with $R_Z$ implies that in a given time interval (year) the areas of a large number of sunspot groups are not drastically different (a few or no outliers). It is well-known that the relative frequency of small sunspot groups is larger during solar minimum than during solar maximum (Mandrykina, 1974). However, the smallest regions dominate the global flux emergence rate (Gokhale & Sivaraman, 1981; Zirin, 1987), and small sunspots and small sunspot groups outnumber the large ones (Clette & Lefèvre, 2012; Javaraiah, 2012a). This reduces the overall mean sizes of the sunspots groups in the maximum years (for example, there exits an high anticorrelation between the variations that are represented by the solid curves in Figs. 4(a) and 6). All these together may be constitute a possible reason for there is no statistical significant correlation between the yearly mean maximum size of sunspot groups in any class and $R_Z$ (conclusion 1 above). May be for the same reason the annual mean daily rates of growth and decay of sunspot groups do not correlate to the annual $R_Z$ (Javaraiah, 2011), whereas the annual rates of the growth and decay of sunspot groups very well correlate to the annual $R_Z$ (Javaraiah, 2012b).

The existence of approximate 90-year (Gleissberg cycle) and 160–270 year (Schöne, 1983) periodicities in sunspot activity is known. In Paper I these periodicities were seen (visualized) in the variations of the numbers (counts) of LSGs/BSGs and SSGs (see also Fig. 5), respectively. The existence of approximate 130-year periodicity (which is seen here in the variations of the mean maximum size of SSGs, conclusion 3 above) in some solar activity indices is also known. In addition, the period of the well-known Gleissberg cycle is reported to be very wide, 60–140 years (Nagovitsyn, 1997).
The aforementioned all periodicities seem to be related to the subharmonics of the 22-year solar magnetic cycle (Attolini et al., 1990). Gokhale & Javaraiah (1990, 1992, 1995) and Gokhale et al. (1992), from the spherical Fourier analysis of a large set of sunspot group data, found that solar variability may be the resultant of the superpositions of global modes of solar magneto-hydrodynamic oscillations. Jose (1965) connected ≈ 180-year period of solar magnetic activity to the 179-year period of the Sun’s motion around the solar system barycenter. Javaraiah (2005) found that violations of the Gnevyshev and Ohl rule or G-O rule (Gnevyshev & Ohl, 1948) in sunspot cycles may be related to the Sun’s retrograde motion around the solar system barycenter, and he has suggested a role of the Sun’s spin-orbit coupling in solar variability. Nevertheless, the origin of long-term variations in solar cycle is still an unsolved problem (e.g., Tan, 2011).

The large/big/strong sunspot groups/magnetic-field-regions may be associated with a deep dynamo mechanism, whereas the small/weak sunspot groups/magnetic-field-regions may be associated with a near surface dynamo mechanism (Gilman & Foukal, 1979; Javaraiah & Gokhale, 1997; Hiremath, 2002; Sivaraman et al., 2003; Sivaraman & Gokhale, 2004; Meunier, 2005; Pevtsov et al., 2011). The small sunspot groups may be the fragmented or the branched parts of the large/big sunspot groups (Javaraiah, 2003). The 130-year and 44-year periodicities seen in the variations of the mean maximum sizes of SSGs and LSGs/BSGs (conclusion 3 above) may be related to the near surface (local) and deep (global) dynamo mechanisms, respectively.

The conclusion 4 above, is consistent with the result that deficiency of the very small sunspots since the activity maximum around 2000 (Lefèvre & Clette, 2011; Clette & Lefèvre, 2012), and it may be related to a very low growth rate of sunspot groups during this period of cycle 23 (Javaraiah, 2011). The area of a large/small sunspot group (magnetic flux) decreases in a fast/slow rate and in fact, it seems the amounts of growth and decay of the magnetic flux of the sunspot groups in a given time interval depends on (proportional to) the total amount of the flux in that interval (Javaraiah, 2012b). The growth of sunspot groups (largely contributed by the emergence of new magnetic flux) may be mostly related to the generation mechanism of solar activity, whereas the decay of sunspot groups may be related to the magnetic flux diffusion and magnetic reconnection processes. The total amount of flux of cycle 23 is relatively small (compared to the recent cycles) and it may be related to the slow growth of sunspot groups during this cycle (relatively less emergence of new magnetic flux, which is in addition due to the low efficiency of dynamo,
my be due to a strong effect of the Coriolis force on the rising/emerging magnetic flux, Fan et al. (1993). The average size of BSGs/LSGs is relatively small in this cycle (see Figs. 4(b) and 4(c)). Kilcik et al. (2011) found that the average area of a sunspot that belongs to a large sunspot group decreased in solar cycle 23, and interpreted this result as the large sunspot groups of cycle 23 could be more complex and may have a large number of small sunspots. The arguments above imply that on the average the sunspot groups of cycle 23 are small due to a low supplement of newly emerging magnetic flux, a contradiction to the aforementioned conclusion of Kilcik et al. (2011). The relatively long length of this cycle may be related to the slow growth rate (and also relatively slow decay/fragmentation of BSGs/LSGs of this cycle because of their relatively small size, Javaraiah, 2012b). The relatively slow rate of the fragmentation may be also contributed for the less number of small sunspot groups in this cycle. It may be interesting to note here that the cycle pair (22, 23) violated the G-O rule mainly due to a large scarcity of SSGs in cycle 23 (Javaraiah, 2012a). All these may be related to the different behavior of the mean meridional motion of the sunspot groups, i.e. during this cycle the mean meridional motion of sunspot groups was high and its behaviour was quite different, south-bound during minimum years and north-bound during maximum years (Javaraiah, 2010).

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Figure 1: Variations in $\bar{A}_{\text{SSG}}, \bar{A}_{\text{LSG}}, \bar{A}_{\text{BSG}},$ and $\bar{A}_{\text{ASG}},$ i.e., the annual mean maximum sizes of the small (cross-dashed curve, green), large (filled circle-solid curve, black), big (triangle-three dotted-dashed curve, red), and all (square-one dotted-dashed curve, blue) spot groups divided by 1, 5, 20, and 4, respectively, during the period 1874–2011 (Note: a value equal to zero implies the absent of sunspot groups in the corresponding class, the data in 1874 and 2011 are incomplete). The open circles connected by the dotted lines represent the values of $R_Z$ divided by 2. The horizontal lines represent the respective mean values. Near the maximum of each cycle the corresponding Waldmeier cycle number is indicated. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Figure 2: The annual mean maximum sizes of the small sunspot groups ($\bar{A}_{SSG}$) versus the year from the maximum epoch of the solar cycle. The blue (dashed curves) and red (dotted curves) colors are used for even- and odd-numbered cycles, respectively. (For the sake of clarity black color is used for cycle 11.) Different symbols are used for different cycles (numbers are given in the parentheses): asterisks (11), pluses (12 and 13), open-circles (14 and 15), crosses (16 and 17), diamonds (18 and 19), triangles (20 and 21), and squares (22 and 23). The filled circles connected by the solid lines represent the mean solar cycle variation determined from the values of annual means. The error bars represent the standard error. There is only one data point at years -6 (beginning of cycle 14), 8 (end of cycle 23) and 9–11 (first three years of cycle 24). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Figure 3: The same as Fig. 2 but for the mean maximum size of large sunspot groups ($\bar{A}_{\text{LSG}}$). The blue square (connected by dashed line) at year $-3$ of cycle 22 and the red square (connected by dotted line) at year $+8$ of cycle 23 can’t be seen because the corresponding values are equal to zero due to the absent of LSGs. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Figure 4: Variations in $\bar{\Delta}_WN$, i.e. cycle-to-cycle variations in the mean maximum sizes (in msh) of (a) small sunspot groups (SSGs), (b) large sunspot groups (LSGs), (c) big sunspot groups (BSGs), and (d) all sunspot groups. The error bars represent the standard error.
Figure 5: Solar cycle variations in the annual numbers of the small sunspot groups whose (a) $A_M \leq 37$ msh and (b) $37 < A_M \leq 100$ msh. The different symbols and colors represent the different cycles, the same as in Figure 2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Figure 6: Cycle-to-cycle variations in the mean annual numbers of small sunspot groups whose $A_M \leq 37$ msh (filled circle-solid curve) and $37 < A_M \leq 100$ msh (open circle-dotted curve).
Figure 7: Plots of the annual mean sizes of SSGs whose $A_M \leq 37$ msh (open circle-dotted curve) and $37 < A_M \leq 100$ msh (filled circle-solid curve), and all SSGs whose $A_M \leq 100$ msh (squares-dashed curve) during the preceding minimum epochs of cycles 12–23 and around the minimum between cycle 23 and 24 versus the corresponding years, 1878, 1889, 1901, 1913, 1923, 1933, 1944, 1954, 1964, 1976, 1986, 1996, 2006, 2007, 2008, and 2009.