Field theory with coherent states for many-body problems with specified particle- and symmetry- quantum numbers

(Non-relativistic electrons in a central potential and an external magnetic field.)

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Abstract

Coherent states and coherent state path integrals are applied to a many-body problem for non-relativistic electrons in a central potential and an external magnetic field; however, in addition to previous problems of coherent state path integrals, we definitely fix the particle number and the other conserved symmetry quantum numbers to specific values. We determine the maximal commuting set of symmetry quantum numbers in terms of second quantized field operators which are restricted by corresponding delta functions to fixed, specific values in a trace representation of anti-commuting coherent states. One has to distinguish between one-particle and two-particle operators which are all transformed to a normal ordering. After calculation of the delta functions for one-particle operators with exponential integrals from the Dirac identity, we perform an anomalous doubling within the coherent state representation for the delta functions of the two-particle parts so that a Hubbard-Stratonovich transformation (HST) with 'hinge' fields can be taken for corresponding self-energies with a subsequent coset decomposition. Therefore, the remaining field theory of the Coulomb problem only consists of scalar self-energy densities and coset matrices with the anomalous field degrees of freedom. This construction allows for the obvious separation to self-energy densities with a saddle point approximation which are to be inserted as definite values into the remaining field theory of coset matrices. The extension of the given field theory is also briefly described for an ensemble average over the external magnetic field so that mean eigenvalue densities and eigenvalue correlations can be obtained from coherent state representations of delta functions with fixed, maximal commuting sets of symmetry quantum numbers in a trace of anti-commuting fields for further HST’s and coset decompositions of self-energies.

Keywords: many electron atoms, coherent state path integral, nonlinear sigma model, many-particle physics.

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1 Introduction

1.1 Traces of delta functions of maximal-commuting, second-quantized operator-sets

Many phenomena of many-body physics can conveniently be represented by coherent states and their corresponding coherent state path integrals \[1\]-\[4\]. Coherent state path integrals have been mainly applied for problems with varying, unspecified particle numbers where a Hamiltonian of second quantized operators in the ‘\(d\)’ dimensional case can be transferred to a time-ordered path integral for a ‘\((d + 1)\)’ dimensional, classical system. These coherent state path integrals therefore allow various classical approximations, both in the bosonic and fermionic case, and can be further changed by a Hubbard-Stratonovich transformation (HST) into self-energy matrices with anomalous doubled pairing of fields so that one can perform a coset decomposition into density related and ‘Nambu-doubled’ parts \[5\]-\[10\].

However, in the present article we also use coherent states for a many electron atom with a fixed number of electrons and specifically fixed symmetry quantum numbers. This is accomplished by a coherent state representation for the trace over delta functions for a definite number of electrons and for specific symmetry quantum numbers which are originally given in terms of second quantized field operators. We calculate the density of states \(g(E, l, s; n_0, l_z, s_z)\) \[(1.1)\] for a many electron atom in the infinite nuclear mass approximation and take the maximal commuting set of symmetry operators for the problem

\[
g(E, l, s; n_0, l_z, s_z) = \text{Coherent state path integral of the trace of}
\]

\[
:= \text{Tr} \left[ \delta \left( \mathbf{h}_{s_z} - \mathbf{S}_{z} \left( \hat{\psi}^+, \hat{\psi} \right) \right) \delta \left( \mathbf{h}_{l_z} - \mathbf{L}_{z} \left( \hat{\psi}^+, \hat{\psi} \right) \right) \delta \left( n_0 - \mathbf{N} \left( \hat{\psi}^+, \hat{\psi} \right) \right) \times \right.
\]

\[
\times \delta \left( \mathbf{h}^2 s(s + 1) \mathbf{S} \left( \hat{\psi}^+, \hat{\psi} \right) \mathbf{S}^{\dagger} \left( \hat{\psi}^+, \hat{\psi} \right) \delta \left( \mathbf{h}^2 l(l + 1) \mathbf{L} \left( \hat{\psi}^+, \hat{\psi} \right) \mathbf{L}^{\dagger} \left( \hat{\psi}^+, \hat{\psi} \right) \delta \left( E - \mathbf{H} \left( \hat{\psi}^+, \hat{\psi}; B_z \right) \right) \right]
\]

In later sections we have to distinguish between delta functions of one-particle operators \(\mathbf{N} \left( \hat{\psi}^+, \hat{\psi} \right)\), \(\mathbf{L}_{z} \left( \hat{\psi}^+, \hat{\psi} \right)\), \(\mathbf{S}_{z} \left( \hat{\psi}^+, \hat{\psi} \right)\) and two-particle operators \(\mathbf{H} \left( \hat{\psi}^+, \hat{\psi}; B_z \right)\), \(\mathbf{L} \left( \hat{\psi}^+, \hat{\psi} \right) \mathbf{L}^{\dagger} \left( \hat{\psi}^+, \hat{\psi} \right)\), \(\mathbf{S} \left( \hat{\psi}^+, \hat{\psi} \right) \mathbf{S}^{\dagger} \left( \hat{\psi}^+, \hat{\psi} \right)\) with the quantum numbers \(n_0, h_{l_z}, h_{s_z}\) and \(E, h^2 l(l + 1), h^2 s(s + 1)\), respectively. We note that relation \[(1.1)\] defines a quantum field theory of second quantized operators where the delta functions constrain the maximal commuting set of symmetry operators to those numbers which are anticipated by ordinary quantum mechanics with a Fock space. It is furthermore possible to determine with coherent state path integrals a mean eigenvalue density \[(1.2)\] of a disordered system for fixed symmetry quantum numbers within an ensemble average, e. g. with a Gaussian distribution for an external magnetic field \(B_z\). Moreover, one can extend the coherent state representation for specific symmetry numbers to the computation of eigenvalue correlations \[(1.3)\] of a disordered system with an ensemble average, as e. g. also with a Gaussian distribution of an external magnetic field

\[
\rho(E, l, s; n_0, l_z, s_z) = \text{Tr} \left[ \delta \left( \mathbf{h}_{s_z} - \mathbf{S}_{z} \left( \hat{\psi}^+, \hat{\psi} \right) \right) \delta \left( \mathbf{h}_{l_z} - \mathbf{L}_{z} \left( \hat{\psi}^+, \hat{\psi} \right) \right) \delta \left( n_0 - \mathbf{N} \left( \hat{\psi}^+, \hat{\psi} \right) \right) \times \right.
\]

\[
\times \delta \left( \mathbf{h}^2 s(s + 1) \mathbf{S} \left( \hat{\psi}^+, \hat{\psi} \right) \mathbf{S}^{\dagger} \left( \hat{\psi}^+, \hat{\psi} \right) \delta \left( \mathbf{h}^2 l(l + 1) \mathbf{L} \left( \hat{\psi}^+, \hat{\psi} \right) \mathbf{L}^{\dagger} \left( \hat{\psi}^+, \hat{\psi} \right) \right] \times
\]

\[
\times \left. \delta \left( E - \mathbf{H} \left( \hat{\psi}^+, \hat{\psi}; B_z \right) \right) \right]
\]

In the following we describe the considered Hamiltonian for a many electron atom with a rigid position of the nucleus at the coordinate origin in terms of second quantized Fermi operators \[(1.4)\], labelled by the coordinate vector \(\vec{x}\) and the electron spin \(s = \uparrow, \downarrow\) which are also abbreviated into the common labels of vectors \(\vec{y} = (\vec{x}, s = \uparrow, \downarrow)\), \(\vec{y}' = (\vec{x}, s' = \uparrow, \downarrow)\), \(\vec{y}_1 = (\vec{x}_1, s_1 = \uparrow, \downarrow)\), etc. . The sum \[(1.5)\] over the field operators is normalized by a volume \(V(d)\) which gives rise to a total number of discrete, spatial points \(N_x\) from the underlying, smallest volume element \((\Delta x)^d\). In the remainder the usual summation convention with normalization \(1/N_x\) according to \[(1.5)\] is always implied for the repeated occurrence of
indices $\vec{x}$, $s = \uparrow, \downarrow$ or the combined form $\vec{y}$ in order to attain an abbreviated kind of equations and relations

$$\psi_{\vec{x},s} \psi_{\vec{x},s}^\dagger \quad ; \quad s = \uparrow, \downarrow \quad ; \quad \vec{x} = \{x^{(1)}, x^{(2)}, x^{(3)}\} \quad ; \quad (d = 3);$$

$$\vec{y} = ( \vec{x} = \{x^{(1)}, x^{(2)}, x^{(3)}\} , \quad s = \{\uparrow, \downarrow\});$$

$$\sum_{\vec{x}} \ldots = \int_{V(d)} (\Delta x)^d \quad \ldots = \sum_{\vec{x}} \frac{1}{N_x} \ldots \quad N_x = \frac{V(d)}{(\Delta x)^d};$$

$$\sum_{\vec{y}} \ldots = \sum_{\vec{x},s = \uparrow, \downarrow} \ldots = \sum_{\vec{x},s=\uparrow,\downarrow} \int_{V(d)} (\Delta x)^d \quad \ldots = \sum_{\vec{x},s=\uparrow,\downarrow} \frac{1}{N_x} \ldots ;$$

$$m_e/M_{\text{nucl.}} \rightarrow 0 .$$

The Hamiltonian $\hat{H}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z)$ (1.6) consists of the one-particle part $\hat{H}^{(1)}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z)$ (1.7) with an external magnetic field $B_z$ and the repulsive Coulomb interaction $\hat{V}_c(\hat{\psi}^{\dagger}, \hat{\psi})$ among the electrons. The standard one-particle Hamiltonian has density related terms of bilinear, second quantized fields $\hat{\psi}_{\vec{x},s} \hat{\psi}_{\vec{x},s}$ with one-particle matrix elements $\hat{H}^{(1)}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z)$ (1.8). The latter one-particle matrix elements (1.8) contain the kinetic energy with an external magnetic field $B_z$ (1.9) in symmetric gauge and the Coulomb attraction of the nucleus with total charge $z_0 e$ at the coordinate origin. Furthermore, the coupling of the electron’s magnetic moment to the external magnetic field is added with inclusion of the Pauli matrix $(\vec{\sigma})_{st,s'} = (\vec{\sigma}_3)_{st,s'}$ for the external magnetic field $B_z$ in z-direction

$$\hat{H}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z) = \hat{H}^{(1)}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z) + \hat{V}_c(\hat{\psi}^{\dagger}, \hat{\psi}) ;$$

$$\hat{H}^{(1)}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z) = \sum_{\vec{x},s,s'} \hat{\psi}_{\vec{x},s}^{\dagger} \left[ \left( \frac{\vec{p}^2 + e\vec{A}(\vec{x})}{2 m_e} - \frac{1}{4 \pi \varepsilon_0} \frac{z_0 e^2}{|\vec{x}| + k_{\text{nucl.}}} \right) \delta_{s's} + \mu_B B_z \left( \begin{array}{c} 1 \\ -1 \end{array} \right)_{s's} \right] \hat{\psi}_{\vec{x},s} ;$$

$$\hat{H}^{(1)}_{\vec{x},s;\vec{x}',s'}(B_z) = \left[ \left( - \frac{\vec{B}_z \cdot \vec{\partial}}{2 m_e} ; \vec{\partial} \right) + \frac{e B_z}{2 m_e} \left( \vec{x}' \times \frac{\vec{B}_z}{1 + \varepsilon_0} \right) + \frac{e^2 B_z^2}{8 m_e} \left( x'^2 + y'^2 \right) \right] \delta_{s's} + \mu_B B_z \left( \begin{array}{c} 1 \\ -1 \end{array} \right)_{s's} \delta_{\vec{x}',\vec{x}} N_x ;$$

$$\vec{A}(\vec{x}) = \left( B_z/2 \right) \left( -y, x, 0 \right)^T ; \quad \mu_B = e \hbar / (2 m_e) ; \quad (e > 0) .$$

The Coulomb repulsion $\hat{V}_c(\hat{\psi}^{\dagger}, \hat{\psi})$ of the electrons is described in relation (1.10) where one can also introduce an inverse screening length $k_c$ as for the nuclear screening with $k_{\text{nucl}}$. of (1.7-1.8) in order to regularize the Coulomb potential

$$\hat{V}_c(\hat{\psi}^{\dagger}, \hat{\psi}) = \sum_{\vec{x},s,s',s'} \hat{\psi}_{\vec{x},s}^{\dagger} \hat{\psi}_{\vec{x}',s'}^{\dagger} \frac{e^2}{4 \pi \varepsilon_0} \frac{1}{|\vec{x} - \vec{x}'| + k_c} \hat{\psi}_{\vec{x}',s'} \hat{\psi}_{\vec{x},s} .$$

The rotational symmetry of the electrons in the central potential and the interaction with an external magnetic field in absence of a spin-orbit coupling imply the following, maximal commuting set of symmetry operators in second quantized form

$$\hat{H}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z) , \hat{L}(\hat{\psi}^{\dagger}, \hat{\psi}) , \hat{S}(\hat{\psi}^{\dagger}, \hat{\psi}) , \hat{N}(\hat{\psi}^{\dagger}, \hat{\psi}) , \hat{L}_z(\hat{\psi}^{\dagger}, \hat{\psi}) , \hat{S}_z(\hat{\psi}^{\dagger}, \hat{\psi}) \rightarrow \quad (\hat{1.11})$$

$$\rightarrow \quad \text{maximal commuting set of second quantized operators according to the symmetries} .$$

In the remaining of this section the various commutator relations are computed in order to verify the given set (1.11) of maximal commuting symmetry operators. Aside from the Hamiltonian as a two-particle operator, there appear the
one-particle operators \( \hat{\mathbf{S}}^{(k=1,2,3)}(\hat{\psi}^\dagger, \hat{\psi}) \) (1.12), with matrix elements \( \hat{\mathbf{S}}^{(k)}_{x',s';x,s} \) (1.13), \( \hat{\mathbf{S}}^{(k)}_{x';s';x,s} \) (1.18), which are introduced for the total number \( \hat{N}(\hat{\psi}^\dagger, \hat{\psi}) \) (1.14) of electrons and the total orbital angular momentum \( \hat{L}_z(\hat{\psi}^\dagger, \hat{\psi}) \) (1.16) and spin momentum \( \hat{S}_z(\hat{\psi}^\dagger, \hat{\psi}) \) (1.10) in z-direction. However, the squares of absolute values \( \hat{\mathbf{S}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}) \) (1.19) of orbital \( \hat{L}(\hat{\psi}^\dagger, \hat{\psi}) \cdot \hat{L}(\hat{\psi}^\dagger, \hat{\psi}) \) and spin angular momentum \( \hat{S}(\hat{\psi}^\dagger, \hat{\psi}) \cdot \hat{S}(\hat{\psi}^\dagger, \hat{\psi}) \) contain two-particle parts which therefore require transformations analogous to the Hamiltonian

\[
\text{one-particle operators} \quad \hat{\mathbf{S}}^{(k=1,2,3)}(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{x',s'} \hat{\psi}^\dagger_{x',s'} \hat{\mathbf{S}}^{(k)}_{x',s';x,s} \hat{\psi}_{x,s} \quad (k = 1, 2, 3); \\
\hat{\mathbf{S}}^{(k=1)}(\hat{\psi}^\dagger, \hat{\psi}) = \hat{\mathbf{N}}(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{x,s} \hat{\psi}^\dagger_x \hat{\psi}_{x,s} \rightarrow \hat{\mathbf{S}}^{(k=1)}_{x',s';x,s} = \mathbf{N}_x \delta_{x',x} \delta_{s',s}; \\
\hat{\mathbf{S}}^{(k=2)}(\hat{\psi}^\dagger, \hat{\psi}) = \hat{\mathbf{L}}_z(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{x,s} \hat{\psi}^\dagger_{x,s} (\hat{x} \times \hat{\mathbf{p}})_z \hat{\psi}_{x,s} \rightarrow \hat{\mathbf{S}}^{(k=2)}_{x',s';x,s} = (\hat{x}' \times \hat{\mathbf{p}}')_z \mathbf{N}_x \delta_{s',s}; \\
\hat{\mathbf{S}}^{(k=3)}(\hat{\psi}^\dagger, \hat{\psi}) = \hat{\mathbf{S}}_z(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{x,s,s'} \hat{\psi}^\dagger_{x,s} \frac{\hbar}{2} \left( \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right) \hat{\psi}_{s,s'} \rightarrow \hat{\mathbf{S}}^{(k=3)}_{x',s';x,s} = \frac{\hbar}{2} \left( \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right) \mathbf{N}_x \delta_{x',x} \delta_{s',s}; \\
\hat{\mathbf{S}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}) = \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{k=1}^{k=2} \hat{\mathbf{S}}_k(\hat{\psi}^\dagger, \hat{\psi}) = \hat{\mathbf{S}}^{(k=1)}(\hat{\psi}^\dagger, \hat{\psi}) \cdot \hat{\mathbf{S}}^{(k=2)}(\hat{\psi}^\dagger, \hat{\psi}) \quad (k = 1, 2). \\
\text{two-particle operators} \quad \hat{\mathbf{S}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}) = \hat{\mathbf{S}}^{(k=1)}(\hat{\psi}^\dagger, \hat{\psi}) \cdot \hat{\mathbf{S}}^{(k=2)}(\hat{\psi}^\dagger, \hat{\psi}) \quad (k = 1, 2). \tag{1.19}
\]

Concerning the property of a maximal commuting set of symmetry operators, we attain relations (1.20,1.21) for one-particle and two-particle operators (including the Hamiltonian) and therefore attest through the given commutators the validity of maximal chosen symmetries

\[
\left[ \hat{\mathbf{S}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}), \hat{\mathbf{S}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}) \right] = \sum_{x',s';s} \hat{\psi}^\dagger_{x',s'} \left[ \hat{\mathbf{S}}^{(k)}_{x',s';x,s}, \hat{\mathbf{S}}^{(k)}_{x,s';x',s} \right] \hat{\psi}_{x,s} = 0; \tag{1.20}
\]

\[
\left\{ \hat{\psi}_{x,s}, \hat{\psi}^\dagger_{x',s'} \right\} = \mathbf{N}_x \delta_{x,x'} \delta_{s,s'} \quad \left\{ \hat{\psi}_{x,s}, \hat{\psi}^\dagger_{x',s'} \right\} = 0; \tag{1.21}
\]

\[
\left[ \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}), \hat{\mathbf{S}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}) \right] = \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}) \cdot \hat{\mathbf{S}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}) = 0; \tag{1.22}
\]

\[
\left[ \hat{\psi}_{x,s}^\dagger \right] \left[ \hat{\psi}^\dagger(\hat{\psi}^\dagger, \hat{\psi}) \right] = \hat{\psi}_x \hat{\psi}_s; \tag{1.23}
\]

\[
\hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}) = \sum_{x,s} \hat{\psi}^\dagger_x \left( \hat{x} \times \hat{\mathbf{p}} \right)_k \hat{\psi}_s \tag{1.24}
\]

\[
\left[ \hat{\mathbf{S}}_k(\hat{\psi}^\dagger, \hat{\psi}), \hat{\psi}^\dagger(\hat{\psi}^\dagger, \hat{\psi}) \right] = 0; \tag{1.25}
\]

\[
\left[ \hat{\mathbf{S}}_k(\hat{\psi}^\dagger, \hat{\psi}), \hat{\psi}^\dagger(\hat{\psi}^\dagger, \hat{\psi}) \right] = 0; \tag{1.26}
\]

\[
\left[ \hat{\mathbf{N}}(\hat{\psi}^\dagger, \hat{\psi}), \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}) \right] = \left[ \hat{\mathbf{N}}(\hat{\psi}^\dagger, \hat{\psi}), \hat{\mathbf{S}}_k(\hat{\psi}^\dagger, \hat{\psi}) \right] = 0; \tag{1.27}
\]

\[
\left[ \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}), \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}) \right] = 0; \tag{1.28}
\]

\[
\left[ \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}), \hat{\mathbf{L}}_k(\hat{\psi}^\dagger, \hat{\psi}) \right] = 0; \tag{1.29}
\]

The commutators (1.20,1.21) result from the fundamental anti-commutators (1.22,1.30) of second quantization which we further apply to determine the two-particle content (1.31,1.32) of the absolute values of the orbital \( \hat{\mathbf{S}}^{(k=1)}(\hat{\psi}^\dagger, \hat{\psi}) = \).

1 INTRODUCTION
\[ \hat{L}(\psi^\dagger, \psi) \cdot \hat{L}(\psi^\dagger, \psi) = 1.33 \text{(1.31)} \]

This equation represents the product of the matrix elements of the Hamiltonian \( \hat{L} \) with itself, indicating a relationship between the primed and unprimed coordinate labels.

**Traces of delta functions of maximal-commuting, second-quantized operator-sets**

Apart from the matrix elements \((\hat{L} \cdot \hat{L})_{\vec{x},\vec{x}}\), \((\hat{S} \cdot \hat{S})_{\vec{s}}\) of the one-particle ingredients, we point out the two-particle parts of the absolute values of the two angular momentum types, similar and analogous to the total Hamiltonian \(1.32\) 

\[ \hat{\mathbf{S}}(\kappa=0)(\psi^\dagger, \psi) = \hat{H}(\psi^\dagger, \psi; B_z) \]

as one-particle elements \(\psi(\vec{x}_1, \vec{y}_1; B_z) = \hat{H}(\psi^\dagger, \psi; B_z) \) and two-particle matrix elements \(1.42\) 

\[ \begin{align*}
\{ \psi_{x,s}, \psi_{x',s'}^\dagger \}_+ &= N_x \delta_{x,x'} \delta_{s,s'} ; \\
\hat{\mathbf{S}}(\kappa=1)(\psi^\dagger, \psi) &= \hat{L}(\psi^\dagger, \psi) \cdot \hat{L}(\psi^\dagger, \psi) = \sum_{\vec{x}_1,\vec{x}_2,\vec{s}_1,\vec{s}_2} \psi_{x_1,s_1,\vec{x}_1} \psi_{x_2,s_1,\vec{x}_2} ; \\
\hat{\mathbf{S}}(\kappa=2)(\psi^\dagger, \psi) &= \hat{S}(\psi^\dagger, \psi) \cdot \hat{S}(\psi^\dagger, \psi) = \sum_{\vec{x}_1,\vec{x}_2,\vec{s}_1,\vec{s}_2} \psi_{x_1,s_1,\vec{x}_1} \psi_{x_2,s_1,\vec{x}_2} ; \\
\{ \psi_{x,s}, \psi_{x',s'}^\dagger \}_+ &= N_x \delta_{x,x'} \delta_{s,s'} + 3 \hbar^2 \delta_{s,s'} ; \\
\hat{\mathbf{S}}(\kappa=0)(\psi^\dagger, \psi) &= \hat{H}(\psi^\dagger, \psi; B_z) = \hat{\mathbf{S}}(\kappa=1)(\psi^\dagger, \psi) ; \\
\psi(\vec{s}, \vec{x}, \vec{y}; B_z) &= \psi(\vec{s}, \vec{x}, \vec{y}) \cdot \psi(\vec{s}, \vec{x}, \vec{y}) = \hat{H}(\psi^\dagger, \psi; B_z) ; \\
\hat{\mathbf{S}}(\kappa=0)(\psi^\dagger, \psi) &= \hat{H}(\psi^\dagger, \psi; B_z) = \hat{\mathbf{S}}(\kappa=1)(\psi^\dagger, \psi) ; \\
\psi(\vec{s}, \vec{x}, \vec{y}; B_z) &= \psi(\vec{s}, \vec{x}, \vec{y}) \cdot \psi(\vec{s}, \vec{x}, \vec{y}) = \hat{H}(\psi^\dagger, \psi; B_z) .
\end{align*} \]
In this introductory section we have described the maximal commuting set of symmetry operators in terms of their commutators and have grouped the operator set into one-particle and two-particle classes so that the given definitions allow collective, abbreviated treatment for just two types of operators instead of a total of six different operators.

1.2 Coherent state representation of delta functions within trace operations

Aside from the appropriate choice of the maximal commuting set of symmetry operators, we use the Dirac identity [1.43] in order to represent the delta functions of second quantized Fermi operators as part of the inverse of the corresponding symmetry operators. Since one has to distinguish between one-particle \( \hat{S}^{(k)}(\psi^\dagger, \psi) \) and two-particle operators \( \hat{\gamma}^{(\kappa)}(\psi^\dagger, \psi) \) in later transformations, the analogous Dirac identities are listed separately in relations \( 1.43, 1.45 \) and \( 1.46, 1.49 \), respectively. Note that the non-hermitian, imaginary increments \( \mp \varepsilon^{(k)}_\kappa \) and \( \mp \varepsilon^{(\kappa)} \) have to take the physical dimensions as their one-particle or two-particle operators or their chosen, corresponding quantum numbers \( 1.45 \) and \( 1.49 \).

\[
\begin{align*}
\lim_{\varepsilon^{(k)}_+ \to 0_+} \frac{1}{\varepsilon^{(k)}_+ - \sigma^{(k)}} &= \frac{\pm \pi \, \delta(\sigma^{(k)} - \hat{S}^{(k)}(\psi^\dagger, \psi))}{\sigma^{(k)} - \hat{S}^{(k)}(\psi^\dagger, \psi)} ; \\
\hat{S}^{(k)}(\psi^\dagger, \psi) &= \left[ \sum_{k=1}^{3} \frac{1}{k-1} \int \frac{d\psi^{(k)}}{2\pi \hbar} \exp \left\{ -i \varepsilon^{(k)}_\kappa \frac{\psi^{(k)}}{\hbar} \right\} \right] ;
\end{align*}
\]

Since the inverted operators of the Dirac identities \( 1.43, 1.49 \) can be obtained from integration of exponentials with generalized 'time' parameters \( t^{(k)}_{\kappa} / \hbar, \eta^{(k)}_{\kappa} / \hbar \) having the inverted physical dimensions as their operators or quantum numbers, we can represent the delta functions of one-particle and two-particle operators with relations \( 1.50 \) and \( 1.52 \) where we additionally list the delta function \( 1.51 \) of the Hamilton operator for further reference and transformations of the other two-particle operators. In order to eliminate the principal values of the Dirac identities, one has to consider two separate integration branches \( p_k = \pm , \eta_k = \pm \) under inclusion of metric signs \( \eta_{pk} = \pm , \eta_{\kappa} = \pm \) for every, one-particle and two-particle observable \( \hat{S}^{(k)}(\psi^\dagger, \psi), \hat{\gamma}^{(\kappa)}(\psi^\dagger, \psi) \) so that their delta functions are only left in correspondence. However, we emphasize again the imaginary values \( -i \varepsilon^{(k)}_{pk}, -i \varepsilon^{(\kappa)}_{\kappa} \) of the limit process within the Dirac identities which have to be adapted to their metric signs \( \eta^{(k)}_{pk}, \eta^{(\kappa)}_{\kappa} \) in such a manner that convergent integrals follow for infinitely increasing upper integration boundaries \( T^{(k)} \to +\infty, \, T^{(\kappa)} \to +\infty \). Furthermore, a kind of 'time'-ordering \( \exp \{ \ldots \} \) of the exponential step operators has to be included where one starts out from smaller, generalized 'time'-parameters on the right-hand side to larger 'time'-values on the left-hand side.

\[
\begin{align*}
\delta(\sigma^{(k)} - \hat{S}^{(k)}(\psi^\dagger, \psi)) &= \sum_{p_k = \pm} \left. \lim_{\varepsilon^{(k)}_+ \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{d\psi^{(k)}}{2\pi \hbar} \exp \left\{ -i \varepsilon^{(k)}_\kappa \frac{\psi^{(k)}}{\hbar} \right\} \right\} ; \\
\delta(E - \hat{H}(\psi^\dagger, \psi; B_z)) &= \sum_{q_k = \pm} \left. \lim_{\varepsilon^{(k)}_+ \to 0} \lim_{T^{(\kappa)} \to +\infty} \int_0^{T^{(\kappa)}} \frac{d\psi^{(\kappa)}}{2\pi \hbar} \exp \left\{ -i \varepsilon^{(\kappa)}_{\kappa} \frac{\psi^{(\kappa)}}{\hbar} \right\} \right\}.
\end{align*}
\]
\[ \delta(v^{(\kappa)} - \hat{\Delta}^{(\kappa)}(\hat{\psi}^{\dagger}, \hat{\psi})) \] 

The notation of the generalized time variables \( t^{(k)}_k / \hbar \), \( \eta^{(k)}_k / \hbar \) of one-particle operators \( \hat{\Delta}^{(k)}(\hat{\psi}^{\dagger}, \hat{\psi}) \) follows according to the upper indices \((k=1,2,3)\) and \((\kappa=0,1,2)\) which specify the chosen symmetry operators as \( \hat{N}(\hat{\psi}^{\dagger}, \hat{\psi}) \) \[ \hat{L}_x(\hat{\psi}^{\dagger}, \hat{\psi}) \] \[ \hat{S}_z(\hat{\psi}^{\dagger}, \hat{\psi}) \] or as two-particle operators \( \hat{H}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z) \) \[ \hat{L}(\hat{\psi}^{\dagger}, \hat{\psi}) \hat{L}(\hat{\psi}^{\dagger}, \hat{\psi}) \] \[ \hat{S}(\hat{\psi}^{\dagger}, \hat{\psi}) \cdot \hat{S}(\hat{\psi}^{\dagger}, \hat{\psi}) \] respectively. Since one has to remove the principal values within the Dirac identity by subtraction for the remaining delta function of the chosen symmetry operator, we have to introduce two different, separate integration branches for the generalized time variables \( t^{(k)}_k / \hbar \), \( \eta^{(k)}_k / \hbar \) which are denoted by the additional lower indices \( p_k = \pm, \eta = \pm \) with respective metric signs ' \( \eta^{(k)}_{p_k} = \pm \)' \( \eta^{(k)}_{\eta} = \pm \). It remains to transform the exponential integrand within the representation of the delta functions of operators to coherent state path integrals. In order to abbreviate notations, we specify again the total density of states \[ \text{(1.61)} \] in terms of one-particle and two-particle parts with additional separation of the Hamiltonian \( \hat{H}^{(\kappa=0=E)}(\hat{\psi}^{\dagger}, \hat{\psi}) \) for further reference and transformations.\

\[ g(E, v^{(\kappa=1,2)}; \theta^{(k=1,2,3)} = \text{Tr} \left[ \prod_{k=1}^{3} \delta \left( \theta^{(k)} - \hat{\Delta}^{(k)}(\hat{\psi}^{\dagger}, \hat{\psi}) \right) \right] \delta \left( v^{(\kappa)} - \hat{\Delta}^{(\kappa)}(\hat{\psi}^{\dagger}, \hat{\psi}) \right) \delta \left( E - \hat{H}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z) \right) \] 

According to Refs. \[ \text{(1.54)} \] to \[ \text{(1.59)} \], we straightforwardly transform the integral representations \[ \text{(1.50)} \] to \[ \text{(1.52)} \] of delta functions within the trace to coherent state path integrals. The Grassmann fields \( \chi_{\xi,s}, \chi_{\xi,s}^\dagger \) \[ \text{(1.54)} \] to \[ \text{(1.56)} \] with their anti-commuting integrations \[ \text{(1.56)} \] replace the fermionic annihilation and creation operators \( \hat{\psi}_{\xi,s}, \hat{\psi}_{\xi,s}^\dagger \) as eigenvalues of coherent states and are used to split the total product of second quantized delta functions to separate factors. They therefore act as source fields for the separate factors of delta functions of one-particle and two-particle operators. In consequence we list the combined definitions of the Grassmann fields \( \chi_{\xi,s}, \chi_{\xi,s}^\dagger \) \[ \text{(1.54)} \] to \[ \text{(1.57)} \] and \( \xi_{\xi,s}, \xi_{\xi,s}^\dagger \) \[ \text{(1.58)} \] to \[ \text{(1.60)} \] (for one-particle and two-particle components) with their overcompleteness relations \[ \text{(1.57)} \] to \[ \text{(1.60)} \] together with the factoring of delta functions \[ \text{(1.61)} \] within the coherent state trace representation of the density of states \( g(E, v^{(\kappa=1,2)}; \theta^{(k=1,2,3)} = \text{Tr} \left[ \prod_{k=1}^{3} \delta \left( \theta^{(k)} - \hat{\Delta}^{(k)}(\hat{\psi}^{\dagger}, \hat{\psi}) \right) \right] \delta \left( v^{(\kappa)} - \hat{\Delta}^{(\kappa)}(\hat{\psi}^{\dagger}, \hat{\psi}) \right) \delta \left( E - \hat{H}(\hat{\psi}^{\dagger}, \hat{\psi}; B_z) \right) \] 

\[ \begin{align*} 
\hat{\psi}^{\dagger}_{\xi,s}|0\rangle &= \sqrt{\mathcal{N}_x} \ldots , n_{\xi,s} = 1, \ldots ; & \langle 0|\hat{\psi}_{\xi,s} = \langle \ldots , n_{\xi,s} = 1, \ldots | \sqrt{\mathcal{N}_x} ; \\
|\chi\rangle &= \exp \left\{ \sum_{\xi,s} \chi_{\xi,s} \hat{\psi}^{\dagger}_{\xi,s} \right\} |0\rangle ; & \hat{\psi}_{\xi,s} |\chi\rangle = \chi_{\xi,s} |\chi\rangle ; & \langle \chi| \hat{\psi}^{\dagger}_{\xi,s} = \langle \chi| \chi_{\xi,s} ; \\
\int d\chi_{\xi,s} \chi \chi^\dagger & = \mathcal{N}_x \delta_{\xi,s} \delta_{ss'} ; & \int d\chi^\dagger_{\xi,s} \chi^\dagger \chi^\dagger = \mathcal{N}_x \delta_{\xi,s} \delta_{ss'} ;
\end{align*} \] 

\[ \begin{align*} 
1 &= \int d[\chi, \chi] \exp \left\{ -\sum_{\xi,s} \chi^\dagger_{\xi,s} \chi_{\xi,s} \right\} |\chi\rangle \langle \chi| ; & d[\chi, \chi] = \prod_{\xi,s} \frac{d\chi^\dagger_{\xi,s} d\chi_{\xi,s}}{\mathcal{N}_x} ; \\
|\xi\rangle &= \exp \left\{ \sum_{\xi,s} \xi^\dagger_{\xi,s} \hat{\psi}^{\dagger}_{\xi,s} \right\} |0\rangle ; & \hat{\psi}^\dagger_{\xi,s} |\xi\rangle = \xi_{\xi,s} |\xi\rangle ; & \langle \xi| \hat{\psi}^\dagger_{\xi,s} = \langle \xi| \xi_{\xi,s} ; \\
\int d\xi_{\xi,s} \xi \xi^\dagger & = \mathcal{N}_x \delta_{\xi,s} \delta_{ss'} ; & \int d\xi_{\xi,s}^\dagger \xi^\dagger \xi^\dagger = \mathcal{N}_x \delta_{\xi,s} \delta_{ss'} ;
\end{align*} \] 

\[ \begin{align*} 
1 &= \int d[\xi, \xi] \exp \left\{ -\sum_{\xi,s} \xi^\dagger_{\xi,s} \xi_{\xi,s} \right\} |\xi\rangle \langle \xi| ; & d[\xi, \xi] = \prod_{\xi,s} \frac{d\xi^\dagger_{\xi,s} d\xi_{\xi,s}}{\mathcal{N}_x} ;
\end{align*} \]
Since we regard a purely fermionic system of electrons, the Grassmann fields have to fulfill anti-periodic boundary conditions following from the overcompleteness relation for the trace operation.

1.3 The remaining field theory of coset matrices derived in subsequent sections

Since the field theory of coset matrices follows after several transformations with involved appearance, we briefly describe the result in advance. We do not omit the precise generalized time steps following from the co-homology representation of exponentials from the Dirac identity for delta functions in order to testify the exactness of coherent state path integrals with exact non-hermitian parts \( \psi_{(k)}(\varphi_{(k)} + \Delta \varphi_{(k)}) \ldots \psi_{(k)}(\varphi_{(k)}) \) instead of approximating, more appealing hermitian kinds \( \psi_{(k)}(\varphi_{(k)}) \ldots \psi_{(k)}(\varphi_{(k)}) \). This exactness of coherent state path integrals also holds for the anomalous doubling of Fermi fields with further precise time steps within a coset decomposition for the self-energies after a HST transformation. We even keep the precise, exact generalized time steps constrained by the normal ordering of the second quantized field operators in the final form of the field theory with remaining coset degrees of freedom which is listed in subsequent equation. Since one has to split the evolution generators into infinitesimals, generalized time development steps, one has to regard additional time parameters \( \tau_{(k)}^{(k)} \) and \( \theta_{(k)}^{(k)} \) within the exponentials of corresponding one-particle and two-particle parts

\[
\vartheta_{(k)}^{(k)} - i \varepsilon_{(k)}^{(k)} \}
\]

\[
\times \left\{ - \frac{i}{2\hbar} \right\}
\]

\[
\times \left\{ \exp \left\{ \frac{1}{\hbar} \nabla_{(k)}^{(k)} \right\}
\]

\[
\times \left\{ \exp \left\{ \frac{1}{\hbar} \nabla_{(k)}^{(k)} \right\}
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\times \left\{ \exp \left\{ \frac{1}{\hbar} \nabla_{(k)}^{(k)} \right\}
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\times \left\{ \exp \left\{ \frac{1}{\hbar} \nabla_{(k)}^{(k)} \right\}
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\times \left\{ \exp \left\{ \frac{1}{\hbar} \nabla_{(k)}^{(k)} \right\}
\]

\[
\times \left\{ \exp \left\{ \frac{1}{\hbar} \nabla_{(k)}^{(k)} \right\}
\]

\[
\times \left\{ \exp \left\{ \frac{1}{\hbar} \nabla_{(k)}^{(k)} \right\}
\]}
of the three one-particle operators (particle number, z-components of orbital and spin angular momentum) and the three 'interacting' two-particle parts with the total Hamilton operator and the absolute values of the orbital and spin angular momentum. The various details, how to attain the given structure with further definitions and precise time steps, are specified in the remainder of this article.

2 Transformation of the delta functions composed of one-particle operators

2.1 Field theory for delta functions with second quantized operators

Although the case of one-particle operators $\hat{S}^{(k)}(\hat{\psi}^+, \hat{\psi})$ can be directly computed from the exponentials of bilinear, density related, second quantized Fermi operators, we outline the calculation of the one-particle operators within Grassmann-valued matrix elements of delta functions, separated from the total density of state relation \(1.01 \text{[1.02]}. \) Since one-particle, density related operators do not allow for anomalous terms as $\langle \hat{\psi}_{x,s} \hat{\psi}_{x',s'}^{\dagger} \rangle$, the calculation simplifies considerably. We define the anti-commuting fields \(2.1 \text{[2.2]}\) and overcompleteness relation \(2.2 \text{[2.4]}\) to be inserted at the various time development steps of exponential operators. Moreover, we abbreviate the corresponding, Grassmann-valued, integration measures, which frequently occur in the transformation to coherent state path integrals, by relations \(2.0 \text{[2.7]} \text{[3]}\).

$$\langle \hat{\psi}(\tau_{pk}) \rangle = \exp \left\{ \sum_{x,s} \psi_{x,s}(\tau_{pk})^* \psi_{x,s}(\tau_{pk}) \right\} |0 \rangle$$ \hspace{1cm} (2.1)

$$\hat{\psi}_{x,s} \psi(\tau_{pk}) = \psi_{x,s}(\tau_{pk})^* \psi_{x,s}(\tau_{pk}) : \langle \psi(\tau_{pk}) \rangle \psi_{x,s}(\tau_{pk})$$ \hspace{1cm} (2.2)

$$\int d\psi_{x,s}(\tau_{pk}) \psi_{x',s'}(\tau_{pk})^* = N_x \delta_{x,x'} \delta_{s,s'} \delta(\tau_{pk}^k - \tau_{pk}^r);$$ \hspace{1cm} (2.3)

$$\int d\psi_{x,s}^*(\tau_{pk}) \psi_{x',s'}(\tau_{pk})^* = N_x \delta_{x,x'} \delta_{s,s'} \delta(\tau_{pk}^k - \tau_{pk}^r);$$ \hspace{1cm} (2.4)

$$1 = \int d[\psi^*(\tau_{pk}), \psi(\tau_{pk})] \exp \left\{ - \sum_{x,s} \psi_{x,s}^*(\tau_{pk}) \psi_{x,s}(\tau_{pk}) \right\} \langle \psi(\tau_{pk}) \rangle \langle \psi(\tau_{pk}) \rangle;$$ \hspace{1cm} (2.5)

$$d[\psi^*(\tau_{pk}), \psi(\tau_{pk})] = \prod_{x,s} \frac{d\psi_{x,s}^*(\tau_{pk}) \psi_{x,s}(\tau_{pk})}{N_x}.$$ \hspace{1cm} (2.6)

$$d[\psi^*(\tau_{pk}), \psi(\tau_{pk})] = \prod_{0 \leq \tau_{pk}^k \leq \tau_{pk}^r} \frac{d\psi_{x,s}^*(\tau_{pk}) \psi_{x,s}(\tau_{pk})}{N_x}.$$ \hspace{1cm} (2.7)

After we have introduced the overcompleteness relation \(2.5 \text{[2.5]}; \) at the various, sequentially ordered, exponential 'time' development steps of relations \(1.01 \text{[1.01]}\), we finally achieve the coherent state path integral \(2.8 \text{[2.8]; with the fermionic source fields $\chi_{x,s}^{(k+1)}$, which couple to the fields $\chi_{x,s}^{(k)}$, $\tau_{pk}^k = 0$ and $\chi_{x,s}^{(k)}, \tau_{pk}^r = t_{pk}^r$ within the action $A^{(k)}(\chi^{(k+1)}), \chi^{(k)}).$\)

According to the wanted preciseness of our exact discretization, we have additionally to incorporate a remaining Gaussian field part for zero time labels into the source action $A^{(k)}(\chi^{(k+1)}), \chi^{(k)}).$ \hspace{1cm} (2.8)

$$\langle \chi^{(k+1)} | \delta(o^{(k)} - \hat{S}^{(k)}(\hat{\psi}^+, \hat{\psi})) | \chi^{(k)} \rangle = \sum_{p_{k} = \pm} \lim_{|p_{k}^e| \rightarrow 0} \lim_{T^{(k)} \rightarrow +\infty} \int_0^{T^{(k)}} \frac{dt_{pk}}{2\pi \hbar} \frac{d\psi^*(\tau_{pk}), \psi(\tau_{pk})}{N_x} \times$$ \hspace{1cm} (2.8)

$$\times \exp \left\{ - \frac{i}{\hbar} \int_0^{\tau_{pk} - \Delta \tau^{(k)}} \mathcal{F}^{(k)}(\tau_{pk}) \left( \sum_{x,s} \psi_{x,s}^*(\tau_{pk} + \Delta \tau^{(k)}) (\tau_{pk} - \tau_{pk}^e) \right) \right\} \times$$ \hspace{1cm} (2.8)

$$\times \frac{\sum_{x,s} \psi_{x,s}^*(\tau_{pk} + \Delta \tau^{(k)}) (\tau_{pk} - \tau_{pk}^e)}{\Delta \tau^{(k)}}.$$ \hspace{1cm} (2.8)

\[\text{[3]}\text{The various, dimensionless Kronecker deltas of generalized 'time' parameters $\tau_{pk}, \tau_{pk}^e$ are symbolized by the notation $\delta(\tau_{pk} - \tau_{pk}^e)$ and correspond to the analogous delta function of time $\tau_{pk} - \tau_{pk}^e$ without the inverted 'time' interval $(\Delta \tau^{(k)})^{-1}$.}\]
\[
- \sum_{g \neq g'} \psi_{g'}^* (r_{p_k} + \Delta r^{(k)} - \eta_{p_k}) \eta_{p_k}^{(k)} \hat{\Delta}_{g,g'} \psi_g (r_{p_k}^{(1)}) \right\} \times \\
\exp \left\{ - \frac{t}{\hbar} \eta_{p_k}^{(k)} \left( \sigma^{(k)} - \imath \varepsilon_{p_k}^{(k)} \right) \right\};
\]

\[
\exp \left\{ \hat{A}^{(k)} \left( \chi^{(k+1)*}, \chi^{(k)} \right) \right\} = \exp \left\{ - \sum_{g} \psi_{g}^{*} (r_{p_k}^{(1)} = 0) \psi_{g} (r_{p_k}^{(k)}) \right\};
\]

\[
\hat{A}^{(k)} \left( \chi^{(k+1)*}, \chi^{(k)} \right) = \sum_{g} \left( \chi_{g}^{(k+1)*} \psi_{g} (r_{p_k}^{(k)}) - \sum_{g} \chi_{g}^{(k+1)*} \psi_{g} (r_{p_k}^{(k)}) \right); \\
\psi_{g,x} (r_{p_k}^{(k)}) = \psi_{g,x} (r_{p_k}^{(k)}) = \psi_{g,x} \left( r_{p_k}^{(k)} = t_{p_k}^{(k)} \right); \\
N_{p_k}^{(k)} = \frac{\eta_{p_k}^{(k)}}{\Delta r^{(k)}}. 
\]

Each summation with a generalized time integration in (2.8) consists of fermionic fields with generalized time points between \( r_{p_k}^{(k)} = 0 \) and \( r_{p_k}^{(k)} = t_{p_k}^{(k)} \), of discrete intervals \( \Delta r^{(k)} \) for the one-particle matrix elements \( \hat{\Sigma}_{x,x}^{(k)} \).

Since the matrix elements of the delta function in (2.8) are only composed of exponentials with bilinear Grassmann fields, the latter can be integrated out to obtain the determinant of the characteristic one-particle matrix \( \hat{M}_{x,x}^{(k,p_k)} \).

As with one-particle part \( \hat{\Sigma}_{x,x}^{(k)} \) only has a non-vanishing main-diagonal and a nonzero, first lower sub-diagonal in the 'time' indices, the determinant entirely reduces to the value one so that we have left with the propagator (2.17) for the source fields \( \chi_{x,x}^{(k+1)*} \).

\[
\langle \chi^{(k+1)} | \delta (\sigma^{(k)} - \hat{\Sigma}^{(k)} (\hat{\psi}^{1}, \hat{\psi})) | \chi^{(k)} \rangle = \\
\sum_{p_k=\pm} \lim_{|\varepsilon^{(k)}| \to 0} \lim_{T^{(k)} \to -\infty} \int_{0}^{T^{(k)}} \frac{d\varepsilon^{(k)}}{2\pi \hbar} \times \det \left[ \hat{M}_{x,x}^{(k,p_k)} \right] \times \exp \left\{ - \frac{t}{\hbar} \eta_{p_k}^{(k)} \left( \sigma^{(k)} - \imath \varepsilon_{p_k}^{(k)} \right) \right\};
\]

\[
\hat{M}_{x,x}^{(k,p_k)} \left( r_{p_k}^{(k)} \right) = \delta_{x,x} \hat{\Sigma}_{x,x}^{(k)} \left( r_{p_k}^{(k)} = 0 \right); \\
\hat{M}_{x,x}^{(k,p_k)} \left( \tau_{p_k}^{(k)} \right) = \hat{\Sigma}_{x,x}^{(k)} \left( \tau_{p_k}^{(k)} = t_{p_k}^{(k)} \right); \\
N_{p_k}^{(k)} = \frac{\eta_{p_k}^{(k)}}{\Delta r^{(k)}}. 
\]

As we conduct the continuum limit \( N_{p_k}^{(k)} \to \infty \) of infinitesimal time steps \( \Delta \tau^{(k)} \), we accomplish the exponential (2.18) of one-particle matrix elements \( \hat{\Sigma}_{x,x}^{(k)} \).
After substitution of the determinant and the propagator with (2.16) and (2.17, 2.18), the delta function with source fields \( \chi_{x,s}^{(k+1)\ast} \), \( \chi_{x,s}^{(k)} \) simplifies to relation (2.19) with integrations over the two separate branches '\( p_k = \pm \)' with metric signs '\( \eta_{p_k}^{(k)} \)'. The integrations over the generalized time '\( T^{(k)} / \hbar \)' result in a Fourier transformation for the generalized frequency '\( \epsilon^{(k)} \)' of the source action with fields \( \chi_{x,s}^{(k+1)\ast} \), \( \chi_{x,s}^{(k)} \) which couple to the 'time' '\( T^{(k)} / \hbar \)' varying exponential with one-particle matrix elements (1.13, 1.14) of the original second quantized, symmetry operators

\[
\langle \chi^{(k+1)} | \delta(\alpha^{(k)} - \tilde{\Delta}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})) | \chi^{(k)} \rangle = \sum_{p_k = \pm} \lim_{|\epsilon^{(k)}_{p_k}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{dt^{(k)}}{2\pi \hbar} \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} (\alpha^{(k)} - \epsilon^{(k)}_{p_k}) \right\} \times (2.19)
\]

\[
= \sum_{p_k = \pm} \lim_{|\epsilon^{(k)}_{p_k}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{dt^{(k)}}{2\pi \hbar} \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} (\alpha^{(k)} - \epsilon^{(k)}_{p_k}) \right\} \sum_{\mathcal{N}_x} \chi^{(k+1)\ast}_{x',s'} \left( \exp \left\{ \frac{i}{\hbar} \eta^{(k)}_{p_k} \frac{1}{\mathcal{N}_x} \tilde{\Delta}^{(k)}_{x',s';s',s'} \right\} \right) \chi^{(k)}_{x,s}
\]

In the case of the one-particle operators \( \hat{N}(\hat{\psi}^\dagger, \hat{\psi}), \hat{S}_x^{(1)}(\hat{\psi}^\dagger, \hat{\psi}) \), the Fourier transformations can be easily performed because the corresponding source actions only consist of a single frequency

\[
\langle \chi^{(2)} | \delta(n_0 - \tilde{N}(\hat{\psi}^\dagger, \hat{\psi})) | \chi^{(1)} \rangle = \left( \frac{\sum_{\hat{\psi}} \chi^{(2)}_{\hat{\psi}} \chi^{(1)}_{\hat{\psi}}}{n_0!} \right)^n ; \quad n_0 \in \mathbb{N} ;
\]

\[
\langle \chi^{(3)} | \delta(\hbar s_z - \tilde{S}_z(\hat{\psi}^\dagger, \hat{\psi})) | \chi^{(2)} \rangle = \sum_{p_k = \pm} \lim_{|\epsilon^{(k)}_{p_k}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{dt^{(k)}}{2\pi \hbar} \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} (\alpha^{(k)} - \epsilon^{(k)}_{p_k}) \right\} \times (2.21)
\]

\[
= \sum_{p_k = \pm} \lim_{|\epsilon^{(k)}_{p_k}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{dt^{(k)}}{2\pi \hbar} \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} (\alpha^{(k)} - \epsilon^{(k)}_{p_k}) \right\} \sum_{\mathcal{N}_x} \chi^{(4)\ast}_{x,1} \chi^{(3)}_{x,1} \exp \left\{ \frac{i}{\hbar} \eta^{(k)}_{p_k} \frac{\tilde{S}_z(\hat{\psi}^\dagger, \hat{\psi})}{\mathcal{N}_x} \right\} \exp \left\{ \frac{i}{\hbar} \eta^{(k)}_{p_k} (\alpha^{(k)} - \epsilon^{(k)}_{p_k}) \right\}
\]

\[
= \sum_{p_k = \pm} \lim_{|\epsilon^{(k)}_{p_k}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{dt^{(k)}}{2\pi \hbar} \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} (\alpha^{(k)} - \epsilon^{(k)}_{p_k}) \right\} \sum_{\mathcal{N}_x} \chi^{(4)\ast}_{x,1} \chi^{(3)}_{x,1} \exp \left\{ \frac{i}{\hbar} \eta^{(k)}_{p_k} (\alpha^{(k)} - \epsilon^{(k)}_{p_k}) \right\}
\]

\[
= \sum_{n=0}^{+\infty} \frac{1}{n!} \sum_{n_1=0}^{n} \left( \sum_{x} \chi^{(4)\ast}_{x,1} \chi^{(3)}_{x,1} \right) \left( \sum_{x} \chi^{(4)\ast}_{x,1} \chi^{(3)}_{x,1} \right) \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} \frac{\tilde{S}_z(\hat{\psi}^\dagger, \hat{\psi})}{\mathcal{N}_x} \right\}
\]

\[
= \frac{1}{\Delta T^{(k)}} \sum_{n=0}^{+\infty} \frac{1}{n! (n_1 - 1)!} \left( \sum_{x} \chi^{(4)\ast}_{x,1} \chi^{(3)}_{x,1} \right)^{n_1 - s_z - \frac{1}{2}} \left( \sum_{x} \chi^{(4)\ast}_{x,1} \chi^{(3)}_{x,1} \right)^{n_1 - s_z - \frac{1}{2}} ; \quad (s_z = \pm \frac{1}{2})
\]

whereas the orbital angular momentum component \( \hat{L}_z(\hat{\psi}^\dagger, \hat{\psi}) \) has an infinite number '\( m \)' ('\(-l \leq m \leq +l\) , \( l \in [0, \infty)\)) of different frequencies with varying coefficients following from the spin summation and radial space integration of the prevailing source fields. However, as we define coefficients \( \eta^{(l)}_{m} \) (2.22) with Heaviside function \( \theta(l + \frac{1}{2} - |m|) \), one can also describe the resulting summation (2.22) with Kronecker deltas \( \delta(l \cdot n_{-l_0} \cdot (-l_0) + \ldots + n_m \cdot m + \ldots + n_{+l_0} \cdot (+l_0)) \) which restrict the total z-orbital angular momentum to those combinations having value '\( \hbar l_z \)'

\[
\langle \chi^{(3)} | \delta(\hbar l_z - \hat{L}_z(\hat{\psi}^\dagger, \hat{\psi})) | \chi^{(2)} \rangle = \sum_{p_k = \pm} \lim_{|\epsilon^{(k)}_{p_k}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{dt^{(k)}}{2\pi \hbar} \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} \frac{\tilde{L}_z(\hat{\psi}^\dagger, \hat{\psi})}{\mathcal{N}_x} \right\} \times (2.22)
\]

\[
= \sum_{p_k = \pm} \lim_{|\epsilon^{(k)}_{p_k}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{dt^{(k)}}{2\pi \hbar} \exp \left\{ - \frac{i}{\hbar} \eta^{(k)}_{p_k} \frac{\tilde{L}_z(\hat{\psi}^\dagger, \hat{\psi})}{\mathcal{N}_x} \right\}
\]
\[ \times \exp \left\{ \sum_{l=0}^{+\infty} \sum_{m=-l}^{+l} \left( \int_0^R \frac{d|x|}{V(d)} \frac{|x|}{2} \sum_{s} \chi_{[t_l,m,s]}^{(2)} \chi_{[-t_l,m,s]}^{(2*)} \right) \exp \left\{ i \eta_{t_p2}^{(2)} \right\} m \cdot t_{p2}^{(2)} \right\} = \]

\[ = \sum_{n=0}^{+\infty} \frac{1}{\Delta \tau^{(2)}} \sum_{n_{-n_0} + \ldots + n_{m} + \ldots + n_{+n_0} = n} \delta(l_{n-n_0} \cdot (-l_0) + \ldots + n_m \cdot m + \ldots + n_{+n_0} \cdot (+l_0)) \times \]

\[ \times \left( \bar{c}_{(l_0)}^{(l_0)} \right)^{n_{-l_0}} \ldots \left( \bar{c}_{m}^{(l_0)} \right)^{n_m} \ldots \left( \bar{c}_{+l_0}^{(l_0)} \right)^{n_{+l_0}} ; \]

\[ \bar{c}_{m}^{(l_0)} = \frac{1}{\Delta \tau^{(2)}} \sum_{l=0}^{+l_0} \left( \int_0^R \frac{d|x|}{V(d)} \frac{|x|}{2} \sum_{s} \chi_{[t_l,m,s]}^{(2)} \chi_{[-t_l,m,s]}^{(2*)} \right) \theta(l + \frac{1}{2} - |m|) ; \quad (2.23) \]

\[ \chi_{[t_l,m,s]}^{(2)} = \int_0^{2\pi} d\beta \int_0^{2\pi} d\alpha \sin(\alpha) Y_{l,m}(\alpha,\beta) \chi_{[t_l,m,s]}^{(2)} ; \]

\[ \chi_{[t_l,m,s]}^{(3*)} = \int_0^{2\pi} d\beta \int_0^{2\pi} d\alpha \sin(\alpha) Y_{l,m}(\alpha,\beta) \chi_{[-t_l,m,s]}^{(3*)} ; \quad (2.24) \]

\[ \bar{x} = |x| \left\{ \sin(\alpha) \cos(\beta), \sin(\alpha) \sin(\beta), \cos(\alpha) \right\} . \quad (2.25) \]

### 3 Transformation of the delta functions composed of two-particle operators

#### 3.1 HST to self-energies with anomalous doubled pairs

Various references infer that coherent state path integrals depend on the chosen, underlying kind of discrete time grid so that one can only attain unreliable results, following from transformations of these. However, we have already pointed out in Refs. how to conduct a coset decomposition within coherent state path integrals in an exact, appropriate manner from second quantized Hamiltonians in normal order. The fundamental point of a correct discrete time grid of normal ordered Hamiltonians consists of two different kinds of anomalous doubled fields \( \psi_{2,s}^{(1)}(\varphi_{q,s}^{(\kappa)}) \), \( \psi_{2,s}^{(a)}(\varphi_{q,s}^{(\kappa)}) \) with two different kinds of hermitian conjugation '†' and '‡'.

\[ (1) : \text{'equal time', anomalous-doubled field} : (0 \leq \varphi_{q,s}^{(\kappa)} \leq \eta^{(\kappa)} - \Delta \varphi^{(\kappa)}) \]

\[ \psi_{2,s}^{(1)}(\varphi_{q,s}^{(\kappa)}) = \begin{pmatrix} \psi_{2,s}^{(\varphi_{q,s}^{(\kappa)} + \Delta \varphi^{(\kappa)})} & \psi_{2,s}^{(\varphi_{q,s}^{(\kappa)})} \end{pmatrix}^{T} ; \quad (\kappa = 0, 1, 2) ; \quad (\kappa = 0 \equiv E) ; \]

\[ (2) : \text{'hermitian-conjugation †' of 'equal time', anomalous-doubled field} : \]

\[ \psi_{2,s}^{(1)}(\varphi_{q,s}^{(\kappa)}) = \begin{pmatrix} \psi_{2,s}^{(\varphi_{q,s}^{(\kappa)})} & \psi_{2,s}^{(\varphi_{q,s}^{(\kappa)} + \Delta \varphi^{(\kappa)})} \end{pmatrix}^{T} ; \quad (\kappa = 0, 1, 2) ; \quad (\kappa = 0 \equiv E) ; \]

As we insert overcomplete sets of coherent states between the generalized 'time' steps \( \Delta \varphi^{(\kappa)} \) for the 'time' development \( \varphi_{q,s}^{(\kappa)} \) with exponential operators, one has to take into account the sole appearance of field combinations \( \psi_{2,s}^{(\varphi_{q,s}^{(\kappa)} + \Delta \varphi^{(\kappa)})} \) and \( \psi_{2,s}^{(\varphi_{q,s}^{(\kappa)})} \).
\[ \Delta \theta^{(c)} \cdots \psi_{x,s}(\hat{\theta}^{(c)}_{q_n}) \text{ instead of the more appealing hermitian form } \psi^*_{x,s}(\hat{\theta}^{(c)}_{q_n}) \cdots \psi_{x,s}(\hat{\theta}^{(c)}_{q_n}) \text{ so that the complex conjugated field } \psi^*_{x,s,\sigma'}(\hat{\theta}^{(c)}_{q_n}+\Delta \theta^{(c)}) \text{ always occurs a particular time step } \Delta \theta^{(c)} \text{ later than its corresponding field } \psi_{x,s}(\hat{\theta}^{(c)}_{q_n}) \text{.} \]

In order to preserve this property of the time shifted '\( \Delta \theta^{(c)} \)', complex conjugated fields \( \psi^*_{x,s}(\hat{\theta}^{(c)}_{q_n}+\Delta \theta^{(c)}) \) under a coset decomposition, we have to use the 'time shifted' '\( \Delta \theta^{(c)} \)' anomalous doubled field \( \tilde{\Psi}^{(c)(1/2)}_{x,s}(\hat{\theta}^{(c)}_{q_n}) \) \((3.3)\), additionally marked by the symbol '\( \tilde{\cdot} \)', with its particular hermitian conjugation \( \psi^* \) \((3.4)\), having the time shift correction \( \Delta \theta^{(c)} \) of the anomalous doubling has to be introduced, due to the anti-commutativity of the Fermi fields.

\[ \tilde{\Psi}^{(c)(1/2)}_{x,s}(\hat{\theta}^{(c)}_{q_n}) = \frac{1}{2} \left( \psi^*_{x,s}(\hat{\theta}^{(c)}_{q_n}+\Delta \theta^{(c)}) \psi_{x,s}(\hat{\theta}^{(c)}_{q_n}) \right) \]

\[ \tilde{\Psi}^{(c)(1/2)}_{x,s}(\hat{\theta}^{(c)}_{q_n}) = \frac{1}{2} \left( \psi^*_{x,s}(\hat{\theta}^{(c)}_{q_n}) \tilde{\psi}^*_{x,s}(\hat{\theta}^{(c)}_{q_n}) \right) ; \]

\[ \tilde{\psi}^*_{x,s}(\hat{\theta}^{(c)}_{q_n}) = \delta_{ab} \delta_{ab} \left( \frac{1}{a} \right) . \]

According to the two different kinds \((3.1.3.2)\) and \((3.3.3.4)\) of anomalous doubling, we have two different kinds of dyadic products \((3.7)\) and \((3.8)\). As we have already described in Ref. \textbf{11}, the anomalous doubled self-energy matrix has to comply with the symmetries \((3.9.3.10)\) of the dyadic product of the equal time anomalous doubled field and its equal time hermitian conjugation whereas the anomalous doubled density matrices have to follow from the dyadic product \((3.11.3.12)\) of a time shifted anomalous doubled field with its time-shift corrected, hermitian conjugated field. In correspondence to Ref. \textbf{11}, we briefly list the two kinds of dyadic products of anomalous doubled fields, which can be regarded as the appropriate order parameter matrices of the coset decomposition, and additionally define the transposition \((3.13.3.16)\), trace \((3.14.3.17)\) and hermitian conjugation \((3.15.3.18)\) of these two kinds of doubled ordered parameter matrices for the self-energy \((3.7.3.9)\) and the density matrix \((3.8.3.11)\), respectively.
We can straightforwardly generalize the coherent state path integral of the one-particle case to the delta functions of the operators $\hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})$ to the delta functions of the operators $\hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})$. Since the operators $\hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})$ finally contain the second quantized Fermi operators only in normal order as the one-particle operators, we achieve the coherent state path integral with the additional interaction part of four anti-commuting fields

$$d[\psi^*(\hat{\psi}^{(k)}_{q_\kappa}), \psi(\hat{\psi}^{(k)}_{q_\kappa})] = \prod_{\{\hat{x}, s\}, \{\hat{y}, s\}} \frac{d\psi^*_{\hat{x}, s}(\hat{\psi}^{(k)}_{q_\kappa})}{\mathcal{N}_x} d\psi_{\hat{x}, s}(\hat{\psi}^{(k)}_{q_\kappa});$$

$$d[\psi^*(\hat{\psi}^{(k)}), \psi(\hat{\psi}^{(k)})] = \prod_{0 \leq |\hat{\psi}^{(k)}| \leq |\epsilon^{(k)}_{\kappa}|} \prod_{\{\hat{x}, s\}, \{\hat{y}, s\}} \frac{d\psi^*_{\hat{x}, s}(\hat{\psi}^{(k)})}{\mathcal{N}_x} d\psi_{\hat{x}, s}(\hat{\psi}^{(k)});$$

$$\langle \xi^{(k+1)} | \delta(\psi^{(k)} - \hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})) | \xi^{(k)} \rangle =$$

$$= \sum_{\kappa_\kappa = \pm} \lim_{|\epsilon^{(k)}_{\kappa}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{d\psi^{(k)}_{q_\kappa}}{2\pi} \langle \xi^{(k+1)} | \exp \left\{-i \epsilon^{(k)}_{\kappa} \frac{t^{(k)}_{q_\kappa}}{\hbar} \left( \psi^{(k)} - \hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi}) - i \epsilon^{(k)}_{\kappa} \right) \right\} | \xi^{(k)} \rangle$$

$$= \sum_{\kappa_\kappa = \pm} \lim_{|\epsilon^{(k)}_{\kappa}| \to 0} \lim_{T^{(k)} \to +\infty} \int_0^{T^{(k)}} \frac{d\psi^{(k)}_{q_\kappa}}{2\pi} d[\psi^{(k)}_{q_\kappa}, \psi^{(k)}_{q_\kappa}] \exp \left\{-i \epsilon^{(k)}_{\kappa} \frac{t^{(k)}_{q_\kappa}}{\hbar} \left( \psi^{(k)} - i \epsilon^{(k)}_{\kappa} \right) \right\} \times$$

$$\times \exp \left\{-i \hbar \int_0^{t^{(k)}_{q_\kappa}} d\psi^{(k)}_{q_\kappa} \left( \sum_{\kappa} \psi^*_\kappa \psi^{(k)}_{\kappa} + \Delta \hat{\psi}^{(k)} \right) \right\} \times$$

$$\times \exp \left\{ \frac{1}{\hbar} \int_0^{t^{(k)}_{q_\kappa}} d\psi^{(k)}_{q_\kappa} \left( \sum_{\kappa} \psi^*_\kappa \psi^{(k)}_{\kappa} \right) \right\} \times$$

$$\times \exp \left\{ \frac{1}{\hbar} \int_0^{t^{(k)}_{q_\kappa}} d\psi^{(k)}_{q_\kappa} \left( \sum_{\kappa} \psi^*_\kappa \psi^{(k)}_{\kappa} \right) \right\} \times$$

Apart from the two-particle part, the coherent state path integral with $\hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})$ encompasses the analogous terms as e. g. the source action $\mathcal{A}^{(k)}(\xi^{(k+1)}_{\kappa}, \xi^{(k)}_{\kappa})$ with additional Gaussian term for the 'zero' time fields in order to achieve the precise discretization so that each field from the overcomplete sets is exactly contained in $\hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})$ within the normal ordered operators $\hat{\mathcal{F}}^{(k)}(\hat{\psi}^\dagger, \hat{\psi})$ of the total Hamiltonian and the absolute values of orbital and spin angular momentum

$$\exp \left\{ \mathcal{A}^{(k)}(\xi^{(k+1)}_{\kappa}, \xi^{(k)}_{\kappa}) \right\} = \exp \left\{ \mathcal{A}^{(k)}(\xi^{(k+1)}_{\kappa}, \xi^{(k)}_{\kappa}) - \sum_{\kappa} \psi^*_\kappa \psi^{(k)}_{\kappa} = 0 \right\} \psi^{(k)}_{\kappa} = 0 \right\};$$

$$\mathcal{A}^{(k)}(\xi^{(k+1)}_{\kappa}, \xi^{(k)}_{\kappa}) = \sum_{\kappa} \left( \psi^*_\kappa \psi^{(k)}_{\kappa} = 0 \right) \xi^{(k)}_{\kappa};$$
3.1 HST to self-energies with anomalous doubled pairs

\[ \psi_{x,s}(\theta_{qa}^{(k)}) \in \psi_{x,s}(\theta_{qa}^{(k)}) = \Delta \theta^{(k)}, \ldots, \psi_{x,s}(\theta_{qa}^{(k)}) = t_{qa}^{(k)}; \]

\[ \mathcal{Y}_{h,q}^{(k)} = t_{qa}^{(k)}/\Delta \theta^{(k)}. \]

However, the two-particle interaction term of four Fermi fields allows for nontrivial anomalous pairings of fields, as e.g. in the order parameters \[ \mathcal{Y}_{h,q}^{(k)} \] from the dyadic products, so that we have also to take the anomalous doubling of the one-particle parts of \[ \mathcal{Y}_{h,q}^{(k)} \] which is combined into the anomalous doubled, one-particle Hamiltonian \[ \tilde{H}_{x,s|k}^{(k)}(\theta_{qa}^{(k)}|\theta_{qa}^{(k)}) \]

\[ \mathcal{Y}_{h,q}^{(k)} \]

with one-particle matrix elements \[ \tilde{H}_{x,s|k}^{(k)}(\theta_{qa}^{(k)}|\theta_{qa}^{(k)}) \] and Kronecker \[ \delta(\theta_{qa}^{(k)}|\theta_{qa}^{(k)}) \] of generalized time variables \[ \theta_{qa}^{(k)} ; \]

\[ \mathcal{Y}_{h,q}^{(k)} \]

In advance we emphasize that one has to take into account an important, further correction which concerns the precise discretization of anomalous doubled operators as the doubled Hamiltonian \[ \tilde{H}_{x,s|k}^{(k)}(\theta_{qa}^{(k)}|\theta_{qa}^{(k)}) \] and the anomalous doubled self-energies. In order to preserve the exact discretization for a normal ordered operator, we have to define the extended, anomalous doubled fields \[ \tilde{\Psi}_{x}^{0}(\theta_{qa}^{(k)}) \]

\[ \mathcal{Y}_{h,q}^{(k)} \]

with a further change of the time shift corrections in the hermitian conjugation \[ \dagger. \]

We label the discrete steps of generalized time variables \[ \theta_{qa}^{(k)} \] by \[ \Delta \theta^{(k)} \cdot n_{qa}^{(k)} \]

\[ \mathcal{Y}_{h,q}^{(k)} \]

and the corresponding end point \[ t_{qa}^{(k)} = \Delta \theta^{(k)} \cdot n_{qa}^{(k)} \] in order to present the precise form of anomalous doubled fields \[ \tilde{\Psi}_{x}^{0}(\theta_{qa}^{(k)}) \]

\[ \mathcal{Y}_{h,q}^{(k)} \]

for an exact discretization and an exact coset decomposition (Note that we have introduced a different symbol \[ \tilde{\Psi}_{x}^{0}(\theta_{qa}^{(k)}) \] above \[ \tilde{\Psi}_{x}^{0}(\theta_{qa}^{(k)}) \] to be distinguished from \[ \varepsilon \] in \[ \tilde{\Psi}_{x}^{0}(\theta_{qa}^{(k)}) \] and \[ \tilde{\Psi}_{x}^{0}(\theta_{qa}^{(k)}) \] of eqs. \[ \mathcal{Y}_{h,q}^{(k)} \]

\[ \mathcal{Y}_{h,q}^{(k)} \]

\[ \mathcal{Y}_{h,q}^{(k)} \]
we accomplish an anti-symmetric matrix \((\mathbf{\hat{g}}(\mathbf{g}))^{\alpha\beta}\) so that bilinear, anomalous doubled fields \((\mathbf{\hat{g}}(\mathbf{g}))^{\alpha\beta}\) can be removed by integration in later steps of the following HST. This explains the particular, involved appearance of the one-particle Hamiltonian matrix \(\mathbf{\hat{S}}(\mathbf{g})^{\alpha\beta}\) \((\mathbf{\hat{g}}(\mathbf{g}))^{\alpha\beta}\) \((\mathbf{\hat{g}}(\mathbf{g}))^{\alpha\beta}\) \((\mathbf{\hat{g}}(\mathbf{g}))^{\alpha\beta}\) \((\mathbf{\hat{g}}(\mathbf{g}))^{\alpha\beta}\). Note that the matrices \((\hat{\mathbf{g}}(\mathbf{g}))^{\alpha\beta}\), \((\hat{\mathbf{S}}(\mathbf{g}))^{\alpha\beta}\) have
their respective form with Pauli matrix \(\tilde{\sigma}_1\) and 'Nambu' metric \(\tilde{S}^{ba}\) along the main diagonal, but differ by the additional unit matrix \(\tilde{1}\) in the uppermost left corner and lowest right corner just outside the range of fields \(\tilde{\Psi}^{a}_{\tilde{y}}(\vartheta_{q_{\tilde{a}}}^{(s)}), \tilde{\Psi}^{a}_{\tilde{y}}(\vartheta_{q_{\tilde{a}}}^{(s)})\)

with further adaption for the fields \(\tilde{\Psi}^{a}_{\tilde{y}}(\vartheta_{q_{\tilde{a}}}^{(s)}), \tilde{\Psi}^{a}_{\tilde{y}}(\vartheta_{q_{\tilde{a}}}^{(s)})\) having extra entries \(\psi_{\tilde{y}}^0(\vartheta_{q_{\tilde{a}}}^{(s)}), \psi_{\tilde{y}}^0(\vartheta_{q_{\tilde{a}}}^{(s)})\) at the beginning and end of their arrays. We define these various, involved appearing matrices and arrays in order to testify by their appearance matrices and arrays in order to testify by their usage the possibility of exact discrete time grids for a coset decomposition enforced from the original normal ordering of operators!

Similarly to relations (3.32, 3.27), we perform the anomalous doubling in the quartic interaction of Fermi fields so that two factors of \(\frac{1}{2}\) have to be incorporated (cf. Refs. [9, 10]) with the time shift correction \(\Delta \vartheta_{\tilde{y}}^{(s)}\) in the complex conjugated parts and have to consider the metric \(\tilde{S}_{\tilde{y}}^{ab}\) of the anomalous doubling which has further to be factorized into \(\tilde{I} \cdot \tilde{I}\) (3.33, 3.36) for the application of the Weyl unitary trick in the coset decompositions [13, 14]. As we have already pointed out, the two-particle matrix elements \(\tilde{\Psi}^{a}_{\tilde{y}}(\vartheta_{q_{\tilde{a}}}^{(s)}), \tilde{\Psi}^{a}_{\tilde{y}}(\vartheta_{q_{\tilde{a}}}^{(s)})\) separate into two independent factors for each orbital and spin angular momentum component \(i = 1, 2, 3\) whereas we have a 'true' interaction \(V_{[\tilde{y}_{\tilde{1}} - \tilde{x}_{\tilde{1}}]}^{(i=0)}\) of Coulomb repulsion (3.37) between primed \(\tilde{x}_{\tilde{1}}\) and unprimed \(\tilde{x}_{\tilde{1}}\) parts of two-particle matrix elements. This different behaviour between the Hamiltonian part \(\tilde{H}(\vartheta^{(s)}; \vartheta, \vartheta) = \tilde{H}(\vartheta^{(s)}; \vartheta, \vartheta)\) and the absolute values \(\tilde{\Psi}^{(s)}(\vartheta_{\tilde{y}}^{(s)}; \vartheta_{\tilde{y}}^{(s)}) = \tilde{L}(\vartheta_{\tilde{y}}^{(s)}; \vartheta_{\tilde{y}}^{(s)})\) of orbital and spin angular momentum is regarded by the prevailing, adapted matrix elements \(\tilde{S}^{(s);a=b}_{\tilde{x}_{\tilde{2}}, \tilde{x}_{\tilde{1}}; \tilde{x}_{\tilde{1}}, \tilde{x}_{\tilde{1}}}\) (3.37), (3.38, 3.39), which are also doubled in anomalous kind (3.40) for the Fermi fields and which accompany the matrix tensor \(\tilde{S}^{ab}\) for the anti-commuting property

\[
\tilde{S}^{ab} = \delta_{ab} \text{ diag} (\tilde{S}^{11} = \tilde{1}; \tilde{S}^{22} = \tilde{1}); \\
\tilde{S} = \tilde{I}; \\
\tilde{I}^{ab} = \delta_{ab} \text{ diag} (\tilde{I}^{11} = \tilde{1}; \tilde{I}^{22} = \tilde{1}); \\
(3.34)
\]

The above relations (3.34) with (3.25, 3.26) allow to transform the two-particle delta functions (3.21) to their anomalous doubled form. This is first determined for the Hamiltonian delta function (3.31) because one can straightforwardly
follow subsequent steps in correspondence to Refs. [7][8][10][11], due to the 'true' interaction term \( \mathfrak{S} \) without factoring into primed and unprimed labels. According to the considerations for an exact discrete time grid with eqs. \( (3.2.2)\text{,} (3.3.2) \), we point out the occurrence of the two anomalous doubled fields \( \tilde{\Psi}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E), \tilde{\Psi}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) \) with matrix \( \mathfrak{S} \) having entries with \( \tilde{\Psi}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) \) \( (\tilde{\tilde{\Psi}}_E) \cdot (\tilde{\tilde{\Psi}}_E) \)

\[
\langle \xi^{(1)} \rangle \delta(E - \tilde{\mathcal{H}}(\tilde{\tilde{\Psi}}_E, \tilde{\tilde{\Psi}}_E)) \| \xi^{(0)} \rangle = \\
= \sum_{q E = \pm} \lim_{|q E| \to 0} \int_{T(E)}^{T(E) + \infty} \frac{d \omega(E)}{2 \pi \hbar} \langle \xi^{(1)} \rangle \| \exp \left\{ - \frac{i \omega(E)}{\hbar} \left( E - \tilde{\mathcal{H}}(\tilde{\tilde{\Psi}}_E, \tilde{\tilde{\Psi}}_E) - i \epsilon^{(E)}_q \right) \right\} \| \xi^{(0)} \rangle
\]

\[
= \sum_{q E = \pm} \lim_{|q E| \to 0} \int_{T(E)}^{T(E) + \infty} \frac{d \omega(E)}{2 \pi \hbar} d \omega(E)^T \left\{ \epsilon^{(E)}_q \right\} \| \exp \left\{ - \frac{i \omega(E)^T \frac{1}{h} \left( E - i \epsilon^{(E)}_q \right) \right\} \| \xi^{(0)} \rangle
\]

\[
\times \exp \left\{ - \frac{1}{2} \int_{0}^{T(E) + \infty} \int_{0}^{T(E) + \infty} \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \right\} \| \xi^{(0)} \rangle
\]

\[
\times \exp \left\{ \frac{1}{2} \int_{0}^{T(E) + \infty} \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \right\} \| \xi^{(0)} \rangle
\]

According to Refs. [7][8][10][11], we introduce the self-energies \( \tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E), \tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) \) which are further split into density related terms with \( \delta \tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) \) and anomalous paired terms with cospet matrices \( \delta \tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) \) \( (\tilde{\tilde{\Psi}}_E) \)

\[
\tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) = \left( \sigma_{D}^{(0)}(\tilde{x}, \tilde{\tilde{\Psi}}_E) \right) \| \tilde{\tilde{\Psi}}_E \|^2 + \delta \tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E)
\]

\[
\| \tilde{\tilde{\Psi}}_E \|^2 = \left( \sigma_{D}^{(0)}(\tilde{x}, \tilde{\tilde{\Psi}}_E) \right) \| \tilde{\tilde{\Psi}}_E \|^2 + \delta \tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E)
\]

\[
\tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) = \left( \tilde{\tilde{\Sigma}}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E) \right) \| \tilde{\tilde{\Psi}}_E \|^2 + \delta \tilde{\Sigma}_{\tilde{x},\tilde{s}}(\tilde{\tilde{\Psi}}_E)
\]

The self-energies \( (3.3.2)\text{,} (3.3.3) \) allow to conduct the HST transformation of quartic Fermi fields in \( (3.3.5) \) so that bilinear, anomalous doubled Grassmann fields only remain in the one-particle part with matrix elements \( \mathfrak{S} \) having entries with \( \tilde{\tilde{\Psi}}_E \) \( (\tilde{\tilde{\Psi}}_E) \) \( (\tilde{\tilde{\Psi}}_E) \) and in the density matrix \( \mathfrak{S} \) \( (\tilde{\tilde{\Psi}}_E) \) \( (\tilde{\tilde{\Psi}}_E) \) which has exactly the similar form as the anomalous doubled order parameter \( \mathfrak{B}^{(0)} \) \( (\tilde{\tilde{\Psi}}_E) \) \( (\tilde{\tilde{\Psi}}_E) \) \( (\tilde{\tilde{\Psi}}_E) \). We therefore accomplish relation \( (3.3.16) \) for the delta function with the total energy term where various Gaussian integrals of combined self-energies \( (3.3.2)\text{,} (3.3.3) \) replace the quartic interaction part with remaining bilinear integrations of Grassmann fields

\[
\langle \xi^{(1)} \rangle \| \tilde{\mathcal{H}}(\tilde{\tilde{\Psi}}_E, \tilde{\tilde{\Psi}}_E) \rangle \| \xi^{(0)} \rangle
\]

\[
= \sum_{q E = \pm} \lim_{|q E| \to 0} \int_{T(E)}^{T(E) + \infty} \frac{d \omega(E)}{2 \pi \hbar} \langle \xi^{(1)} \rangle \| \exp \left\{ - \frac{i \omega(E)^T \frac{1}{h} \left( E - i \epsilon^{(E)}_q \right) \right\} \| \xi^{(0)} \rangle
\]

\[
\times \exp \left\{ - \frac{1}{2} \int_{0}^{T(E) + \infty} \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \right\} \| \xi^{(0)} \rangle
\]

\[
\times \exp \left\{ \frac{1}{2} \int_{0}^{T(E) + \infty} \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \mathcal{V}^{(0)}(\tilde{x}, \tilde{y}) \right\} \| \xi^{(0)} \rangle
\]
We proceed to transform the delta functions of the absolute values of orbital and spin angular momentum \((\ell = 1, 2)\) instead of \((\kappa = 0 \approx E)\) in analogous manner and define the similar, three component self-energies \(\tilde{\Sigma}_{\ell\ell^\prime q}\), \(\tilde{\Sigma}_{\ell\ell^\prime q}\), \(\sigma_{\ell\ell^\prime q}\) with \(i = 1, 2, 3\); however, the appropriate coset decomposition into block diagonal density terms \(\tilde{\Sigma}_{\ell\ell^\prime q}\), as hinge fields in a SSB, and into anomalous terms with coset matrices \(\tilde{T}_{\ell\ell^\prime q}\) \((\kappa = 1, 2)\) has to be performed for the sum of the three component self-energies \(\tilde{\Sigma}_{\ell\ell^\prime q}\), \(\sigma_{\ell\ell^\prime}\) with \(i = 1, 2, 3\) in order to allow for a possible factoring of the coset matrices within the functional determinant and within the corresponding propagator

\[
\tilde{\Sigma}_{\ell\ell^\prime q} = \delta_{\ell\ell^\prime} \tilde{\Sigma}_{\ell\ell^\prime q} + \delta_{\ell\ell^\prime} \tilde{\Sigma}_{\ell\ell^\prime q} + \delta_{\ell\ell^\prime} \tilde{\Sigma}_{\ell\ell^\prime q} \tag{3.49}
\]

\[
\sum_{i=1}^{J_x} \tilde{\Sigma}_{\ell\ell^\prime q}^{(i)} = \tilde{T}_{\ell\ell^\prime q}^{(i)} \tag{3.50}
\]

\[
\sum_{i=1}^{J_x} \tilde{\Sigma}_{\ell\ell^\prime q}^{(i)} + \delta_{\ell\ell^\prime} \tilde{\Sigma}_{\ell\ell^\prime q}^{(i)} = \tilde{T}_{\ell\ell^\prime q}^{(i)} \tag{3.51}
\]

\[
\sum_{i=1}^{J_x} \tilde{\Sigma}_{\ell\ell^\prime q}^{(i)} + \delta_{\ell\ell^\prime} \tilde{\Sigma}_{\ell\ell^\prime q}^{(i)} = \tilde{T}_{\ell\ell^\prime q}^{(i)} \tag{3.52}
\]

According to the factorization (3.33) of the two-particle matrix elements \((3.33, 3.40)\) for the absolute values of the orbital and spin angular momentum, the self-energy density \(\sigma_{\ell\ell^\prime q}\) does not depend on the spatial vector \(\vec{x}\), but has to take three independent components \(i = 1, 2, 3\) which, however, have to be combined with \(\tilde{\Sigma}_{\ell\ell^\prime q}\), \(\tilde{\Sigma}_{\ell\ell^\prime q}\) for an appropriate coset decomposition with factorization of coset matrices of anomalous terms for the functional determinant and propagator

\[
\langle \xi^{(k+1)} | \delta \{ \phi^{(k)} - \tilde{T}(\xi) \} (\psi^{(k)}, \tilde{\psi}) | \xi^{(k)} \rangle = \tag{3.54}
\]
\[ \sum_{q_n=\pm e^{i\phi(n)}} \lim_{T(n)\to 0} \int_{-\infty}^{\infty} \frac{d\epsilon_n^{(2)}}{2\pi \hbar} \exp \left\{ -i \frac{\epsilon_n^{(2)}}{\hbar} \left( \frac{\psi_n^{(2)}}{\epsilon_n^{(2)}} - \hat{\mathcal{F}}^{(2)}(\psi_n^{(2)}, \bar{\psi}_n^{(2)}) - i \epsilon_n^{(2)} \right) \right\} \left( \epsilon_n^{(2)} \right) \]

\[ \times \exp \left\{ - \frac{1}{2} \int_{-\Delta \theta(n)}^{\Delta \theta(n)} \frac{d\gamma_n^{(2)}}{\gamma_n^{(2)}} \sum_{\bar{y}_n,y_n} \bar{\psi}_n^{(2)} \gamma_n^{(2)} \tilde{S} \gamma_n^{(2)} \psi_n^{(2)} \right\} \exp \left\{ -i \frac{\epsilon_n^{(2)}}{\hbar} \left( \frac{\psi_n^{(2)}}{\epsilon_n^{(2)}} - i \epsilon_n^{(2)} \right) \right\} \times \]

\[ \int d[\sigma^{(i)}(\theta_n^{(2)})] d[\delta \Sigma^{(i)}_{ab}(\theta_n^{(2)})] \exp \left\{ - \frac{1}{2\hbar} \int_{0}^{T(n)_{\theta(n)}} \frac{d\gamma_n^{(2)}}{\gamma_n^{(2)}} \frac{\delta \Sigma^{(i)}_{ab}(\gamma_n^{(2)})}{\gamma_n^{(2)}} \sum_{\bar{y}_n,y_n} \right\}

\[ \sum_{\bar{x}_1/2; \bar{x}_1/2} \left( \delta_{\alpha=\beta} - \delta_{\alpha \neq \beta} \right) \right\} \right\}; \quad (3.55) \]

\[ \delta_{\bar{x}_2; \bar{x}_1} N_s = \sum_{\bar{x}_1} \left( V^{(\bar{x}_1)}_{\bar{x}_2 - \bar{x}_1} - 1 \right) \left( V^{(\bar{x}_1)}_{\bar{x}_2 - \bar{x}_1} + i \epsilon_n^{(2)} \right) \delta_{\bar{x}_2; \bar{x}_1}. \quad (3.57) \]

The above relation [3.54] is the result of HST transformations for the two-particle delta functions which have been combined into similar notations for further reduction to anomalous pairs in following section 3.2.

### 3.2 Coset decomposition of self-energies with anomalous pairs as remaining field degrees of freedom

The delta functions of the two-particle operators \( \hat{\mathcal{F}}^{(\kappa=0,1,2)}(\psi, \bar{\psi}) \) have been changed by HST’s to bilinear, anomalous doubled Fermi fields and self-energy matrices which have been separated into density related and 'Nambu' related parts for anomalous pairs in a coset decomposition. As we take into account the appropriate Jacobian for the coset decomposition from the invariant integration measure with its metric tensor and project onto remaining coset degrees of freedom, we finally succeed in relation [3.55] for the delta functions with two-particle terms after removal of bilinear Fermi fields by their integration. Note that we have adapted notations of the Hamiltonian case \( (\kappa = 0 \approx E) \) to the orbital and spin angular momentum case \( (\kappa = 1, 2) \). Since the angular momentum cases \( (\kappa = 1, 2) \) involve the sum over three independent self-energy components \( i = 1, 2, 3 \), we have to include delta functions [3.55][3.59] which reduce these sums to a single self-energy so that a coset decomposition and projection within the Fermi determinant can be obtained for a factorization of the complete self-energy parts (compare Refs. 9 [11] for calculating the Jacobian).

\[ \Delta^{(\kappa=E)}\left( \hat{T}^{(E)}_{\bar{y}, \bar{y}}(\varphi^{(E)}_{\bar{y}, \bar{y}}); \hat{T}^{(E)}_{\bar{y}, \bar{y}}(\varphi^{(E)}_{\bar{y}, \bar{y}}) \right) = \int \frac{d[\delta \Sigma^{(a)}_{\bar{y}, \bar{y}}(t^{(E)}_{\bar{y}, \bar{y}})]}{\delta \Sigma^{(a)}_{\bar{y}, \bar{y}}(t^{(E)}_{\bar{y}, \bar{y}})} \frac{d[\delta \Sigma^{(b)}_{\bar{y}, \bar{y}}(t^{(E)}_{\bar{y}, \bar{y}})]}{\delta \Sigma^{(b)}_{\bar{y}, \bar{y}}(t^{(E)}_{\bar{y}, \bar{y}})} \| P \left( \delta \lambda^{(E)}_{\bar{y}, \bar{y}} \right) \times \]

\[ \times \exp \left\{ \frac{1}{4\hbar} \int_{0}^{T(n)_{\theta(n)}} \frac{d\gamma_n^{(2)}}{\gamma_n^{(2)}} \frac{\delta \Sigma^{(i)}_{\bar{y}, \bar{y}}(\gamma_n^{(2)})}{\gamma_n^{(2)}} \delta_{\alpha=\beta} - \delta_{\alpha \neq \beta} \right\} \right\}; \quad (3.58) \]

\[ \times \delta \left( \hat{T}(\varphi^{(E)}_{\bar{y}, \bar{y}})_{\bar{y}, \bar{y}} \delta \Sigma^{(a)}_{\bar{y}, \bar{y}}(\varphi^{(E)}_{\bar{y}, \bar{y}})_{\bar{y}, \bar{y}} \right) - \delta \Sigma^{(b)}_{\bar{y}, \bar{y}}(\varphi^{(E)}_{\bar{y}, \bar{y}})_{\bar{y}, \bar{y}} \right\}; \quad (3.59) \]
According to our various definitions (3.42-3.57) and (3.58-3.67), we obtain the path integral (3.68) for the two-particle:

$$\psi_a^{(\kappa)}(\bar{\gamma}^{\kappa}_{\tilde{y}} \bar{\gamma}^{\kappa}_{\tilde{y}}) \times \delta\left(\left(\tilde{T}(\varphi^{(\kappa)}_{\kappa a})\right)^{\alpha \alpha}_{\tilde{y}^{\kappa}_{\tilde{y}}}, \tilde{S}^{a a}_{D,\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \left(\tilde{T}^{-1}(\varphi^{(\kappa)}_{\kappa a})\right)^{\alpha \alpha}_{\tilde{y}^{\kappa}_{\tilde{y}}} - \bar{\psi}^{\kappa}_{\tilde{y}}(\kappa) \hat{\Sigma}^{(\kappa);a}_{\tilde{y}} \right).$$

We can straightforwardly follow Refs. [7] [9] [10] [11] for the coset decomposition of the self-energies after the HST transformations above, but have to specify various matrices for adaption to the anomalous doubled fields $\tilde{\psi}^{\alpha}_{\tilde{y}}(\varphi^{(\kappa)}_{\kappa a})$, $\tilde{\Psi}^{a a}_{\tilde{y}}(\varphi^{(\kappa)}_{\kappa a})$, $\tilde{\psi}^{a a}_{\tilde{y}}(\varphi^{(\kappa)}_{\kappa a})$ and corresponding discrete time grids. Despite of involved appearance this proves that coherent state path integrals definitely allow for exact kinds of discrete time grids taking into account the various limit processes of many-particle theory for the second quantized operators $\tilde{\psi}^{\kappa}_{\tilde{y}}, \tilde{\psi}^{\kappa}_{\tilde{y}}$, where the hermitian conjugated fields should always follow a particular, infinitesimal time step later than their correspondents without complex conjugation, due to the normal ordering.

In the following we reduce the path integral (3.54) to the anomalous doubled one-particle Hamiltonian with self-energy density $\sigma^{(i)}(i \tilde{x}, \varphi^{(\kappa)}_{\kappa a})$ and gradient term (3.67) of the coset matrices $\tilde{T}^{1-bb}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a})$, $\tilde{T}^{a a}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a})$ which are similarly constructed as the matrices $(\tilde{T}^{i b}_{\tilde{y}^{\kappa}_{\tilde{y}}}, \tilde{S}^{b a}_{\tilde{y}^{\kappa}_{\tilde{y}}})$ with matrix entries $\tilde{T}^{1-bb}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a})$, $\tilde{T}^{a a}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) (0 \leq \varphi^{(\kappa)}_{\kappa a}, \varphi^{(\kappa)}_{\kappa a} \leq \tilde{t}^{(\kappa)}_{\kappa} - \Delta \tilde{t}^{(\kappa)}_{\kappa})$ along the block diagonal instead of $\tilde{x}^{(\kappa)}_{\kappa}, \tilde{S}^{a a}_{\tilde{y}}$, but with additional unit matrix $\tilde{1}$ in the uppermost left corner and lowest right corner with adaption to the range of generalized time labels for the fields (3.28-3.29).

$$\Delta \varphi^{(\kappa)}_{\kappa a} \delta \tilde{T}^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \left(\tilde{S}^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \right) = \tilde{\delta}^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \left(\tilde{S}^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \right) + \frac{i}{\hbar} \Delta \varphi^{(\kappa)}_{\kappa a} \delta \tilde{T}^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \sum_{\tilde{y}^{\kappa}_{\tilde{y}}} \delta^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}} \left(\tilde{T}^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \right) \left(\tilde{S}^{(i)}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a}) \right) ;$$

(3.60)

$$\sigma^{(i)}_{D}(\tilde{x}, \varphi^{(\kappa)}_{\kappa a}) : 0 \leq \varphi^{(\kappa)}_{\kappa a} \leq \tilde{t}^{(\kappa)}_{\kappa} - \Delta \tilde{t}^{(\kappa)}_{\kappa} ;$$

(3.61)

$$i = i_{\kappa}, \ldots, j_{\kappa} ; \quad (\kappa = 0 ; i_{0} = 0 ; i = 0) ;$$

(3.62)

$$i = i_{\kappa}, \ldots, j_{\kappa} ; \quad (\kappa = 1 ; i_{1} = 1 ; j_{2} = 3 ; i = 1, 2, 3) ;$$

(3.63)

$$i = i_{\kappa}, \ldots, j_{\kappa} ; \quad (\kappa = 2 ; i_{2} = 1 ; j_{3} = 3 ; i = 1, 2, 3) ;$$

(3.64)

$$\sigma^{(i)}_{D}(\tilde{x}, \varphi^{(\kappa)}_{\kappa a}) : \text{diagonal in spin space and 'Nambu' space} ;$$

(3.65)

$$\sigma^{(i)}_{D}(\tilde{x}, \varphi^{(\kappa)}_{\kappa a}) : \text{dependence on spatial coordinates} ;$$

(3.66)

$$\sigma^{(i)}_{D}(\tilde{x}, \varphi^{(\kappa)}_{\kappa a}) : \text{due to factorization of quartic interaction in two density parts} ;$$

(3.67)

According to our various definitions (3.42-3.57) and (3.58-3.67), we obtain the path integral (3.68) for the two-particle delta functions which has the particular form (3.69-3.70) with Grassmann fields $\tilde{\Psi}$ and anti-symmetric matrix $M^i_j$. (The integration intervals $d\varphi^{(\kappa)}_{\kappa a}$, $d\varphi^{(\kappa)}_{\kappa a}$ in the sixth line of (3.68) are absorbed into the matrix $\tilde{X}^{a a}_{\tilde{y}^{\kappa}_{\tilde{y}}}(\varphi^{(\kappa)}_{\kappa a})|\varphi^{(\kappa)}_{\kappa a}|^{a a}_{D} \right) ;$$

(3.71)

$$\langle \xi^{(a+1)} | \delta(\varphi^{(\kappa)}_{\kappa a} - \tilde{\Phi}^{(\kappa)}_{\kappa a}) (\hat{\psi}^{t}, \hat{\psi}) | \xi^{(\kappa)} \rangle =$$

(3.68)

$$= \sum_{\psi^{(\kappa)}_{\kappa a} = \pm} \lim_{\epsilon^{(\kappa)}_{\kappa a} \to - \infty} \int_{0}^{T^{(a+1)}} \frac{d\varphi^{(\kappa)}_{\kappa a}}{2\pi \hbar} \exp \left\{ - \frac{\epsilon^{(\kappa)}_{\kappa a} T^{(a+1)}}{\hbar} \left( \varphi^{(\kappa)}_{\kappa a} - \tilde{\Phi}^{(\kappa)}_{\kappa a}(\hat{\psi}^{t}, \hat{\psi}) - \epsilon^{(\kappa)}_{\kappa a} \right) \right\} \xi^{(\kappa)} \right\} \xi^{(\kappa)} \right\} \times$$

$$\lim_{\epsilon^{(\kappa)}_{\kappa a} \to + \infty} \int_{0}^{T^{(a+1)}} \frac{d\varphi^{(\kappa)}_{\kappa a}}{2\pi \hbar} \exp \left\{ - \frac{\epsilon^{(\kappa)}_{\kappa a} T^{(a+1)}}{\hbar} \left( \varphi^{(\kappa)}_{\kappa a} - \epsilon^{(\kappa)}_{\kappa a} \right) \right\} \times$$

$$\lim_{\epsilon^{(\kappa)}_{\kappa a} \to - \infty} \int_{0}^{T^{(a+1)}} \frac{d\varphi^{(\kappa)}_{\kappa a}}{2\pi \hbar} \exp \left\{ - \frac{\epsilon^{(\kappa)}_{\kappa a} T^{(a+1)}}{\hbar} \left( \varphi^{(\kappa)}_{\kappa a} - \epsilon^{(\kappa)}_{\kappa a} \right) \right\} \times$$
The relevant kind of Grassmann integration of bilinear fields (3.71) analogous to (3.69–3.70) can be applied

\[
\mathcal{N}^{ba}_{g^i : g^j} (\theta^{(r)}_{q^n}, \theta^{(r)}_{q^n}) = \hat{S} \Delta^{(r)} \tilde{\phi}^{(r)}_{g^i : g^j} (\theta^{(r)}_{q^n}, \hat{S} (\theta^{(r)}_{q^n}), \sigma^{(i) \dagger}_D) + \hat{I} \sum_{i : j} \tilde{T}^{ba}_{g^i : g^j} (\theta^{(r)}_{q^n}) \delta \Sigma^{ba}_{D : H_{i} : H_{j}} (\theta^{(r)}_{q^n}) \delta (\theta^{(r)}_{q^n}) \hat{N}^{i, i}_{g^i : g^j} (\theta^{(r)}_{q^n}) \hat{I}.
\]

The relevant kind of Grassmann integration of bilinear fields (3.71) has to be computed in analogy to (3.69–3.70), but with additional coupling to the anti-commuting fields $\xi^{(c)}_g, \xi^{(c)+1)}_g$. Similarly to eq. (3.68), we absorb the integration intervals $d\theta^{(c)}_{q^n}, d\theta^{(c)+1)}_{q^n}$ into the matrix $\mathcal{N}^{ba}_{g^i : g^j} (\theta^{(c)}_{q^n}, \theta^{(c)+1)}_{q^n})$. (This abbreviation is also used in further transformation steps till the end of this section, especially concerning the matrix $\mathcal{N}^{ba}_{g^i : g^j} (\theta^{(c)}_{q^n}, \theta^{(c)+1)}_{q^n})$ (3.73)).

\[
\tilde{\gamma}^{(c)} (\mathcal{N}^{ba}_{g^i : g^j}) = \int d[\psi^{(c)} (t_{q^n}), \psi^{(c)+1)} (t_{q^n})] \times
\]

\[
\times \exp \left\{ - \frac{1}{2} \int_{-\Delta \theta^{(c)}}^{t_{q^n}} \tilde{T}_{g^i : g^j}^{ba} (\theta^{(c)}_{q^n}) \left( (\tilde{\gamma})^{ba}_{g^i : g^j} \mathcal{N}^{ba}_{g^i : g^j} (\theta^{(c)}_{q^n}, \theta^{(c)+1)}_{q^n}) \right) \right\}
\times \exp \left\{ \sum_{g} \left( \psi^{(c)}_{g} (\theta^{(c)}_{q^n}) = 0 \right) \xi^{(c)}_g + \xi^{(c)+1)}_g \psi^{(c)+1)}_g (\theta^{(c)+1)}_{q^n}) \right\}.
\]
Therefore, we define the 'source' fields $\tilde{\Xi}^a_{\bar{y}}(\varphi^{(\kappa)}_{\nu_n})$ with adaption to the time grid of fields $\tilde{\Psi}^a_{\bar{y}}(\varphi^{(\kappa)}_{\nu_n})$ (3.77) by

$$
\tilde{\Xi}^a_{\bar{y}}(\varphi^{(\kappa)}_{\nu_n}) = \left( \begin{array}{c} 2 \xi_{\bar{y}}(\varphi^{(\kappa)}_{\nu_n}) \\ \xi_{\bar{y}}(\varphi^{(\kappa)}_{\nu_n}) \\ \vdots \\ -\xi_{\bar{y}}(\varphi^{(\kappa)}_{\nu_n}) \end{array} \right)^T ,
$$

so that the bilinear Grassmann fields can be removed by integration in (3.68-3.70) corresponding to the general relation (3.69-3.70).

\[ \mathcal{J}^{(\kappa)}[\tilde{\mathcal{N}}^{(\kappa)ba}_{\bar{y}'; \bar{y}}] = \int d[\psi^*(t_{\nu_n}), \psi(t_{\nu_n})] \times \]

$$
\times \exp \left\{ -\frac{1}{2} \int_{-\Delta \varphi^{(\kappa)}}^{t_{\nu_n}} \int_{-\Delta \varphi^{(\kappa)}}^{t_{\nu_n}} \sum_{\bar{y}, \bar{y}'} \tilde{\psi}_{\bar{y}'}^T (\varphi^{(\kappa)}_{\nu_n}) \gamma_{0} \tilde{\psi}_{\bar{y}} (\varphi^{(\kappa)}_{\nu_n}) - \int_{-\Delta \varphi^{(\kappa)}}^{t_{\nu_n}} \sum_{\bar{y}, \bar{y}'} \tilde{\psi}_{\bar{y}'}^T (\varphi^{(\kappa)}_{\nu_n}) \gamma_{0} \tilde{\psi}_{\bar{y}} (\varphi^{(\kappa)}_{\nu_n}) \right\} =
$$

$$
= \left\{ \text{DET} \left( (\tilde{\tau}_1)^{bb' \bar{y}'; \bar{y}} \gamma_{0} (\varphi^{(\kappa)}_{\nu_n}) \gamma_{0} (\varphi^{(\kappa)}_{\nu_n}) \right) \right\}^{1/2} \exp \left\{ -\frac{1}{2} \int_{-\Delta \varphi^{(\kappa)}}^{t_{\nu_n}} \int_{-\Delta \varphi^{(\kappa)}}^{t_{\nu_n}} \sum_{\bar{y}, \bar{y}'} \tilde{\psi}_{\bar{y}'}^T (\varphi^{(\kappa)}_{\nu_n}) \gamma_{0} \tilde{\psi}_{\bar{y}} (\varphi^{(\kappa)}_{\nu_n}) \right\}.
$$

It remains to project the matrix $\tilde{\mathcal{N}}^{(\kappa)ba}_{\bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n})$ (3.77) onto the coset degrees of freedom for the anomalous doubled pairs of fields. Therefore, we factor the coset matrices $\tilde{T}^{bb' \bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n})$, $\tilde{T}_{\bar{y}'; \bar{y}}^{-1; a_1 a}(\varphi^{(\kappa)}_{\nu_n})$ outside of the matrix $\tilde{\mathcal{N}}^{(\kappa)ba}_{\bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n})$ (3.77) in order to achieve following relation (3.79) for (3.77).

\[ \hat{\mathcal{N}}^{(\kappa)ba}_{\bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n}) = \left( \begin{array}{c} \tilde{T}^{bb' \bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n}) \end{array} \right) I^{-1} \times \]

$$
\times \left( \begin{array}{c} \Delta \varphi^{(\kappa)}_{\nu_n} \\ \hat{\Delta}_{\nu_n} \end{array} \right) \gamma_{0} \left( \begin{array}{c} \varphi^{(\kappa)}_{\nu_n} \\ \varphi^{(\kappa)}_{\nu_n} \end{array} \right) \tilde{S} (\kappa; i) \cdot \sigma^{(i)} \right) + \Delta \varphi^{(\kappa)}_{\nu_n} \left( \begin{array}{c} \varphi^{(\kappa)}_{\nu_n} \\ \varphi^{(\kappa)}_{\nu_n} \end{array} \right) \tilde{S} (\kappa; i) \cdot \sigma^{(i)} \right) \right) +
$$

$$
+ \int \delta_{\kappa}^{(\kappa)} \frac{\delta_{\kappa}^{(\kappa)}}{\tilde{\mathcal{N}}^{(\kappa)ba}_{\bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n})} \left( \begin{array}{c} \varphi^{(\kappa)}_{\nu_n} \\ \varphi^{(\kappa)}_{\nu_n} \end{array} \right) \right) \times I^{-1} \tilde{T}_{\bar{y}'; \bar{y}}^{-1; a_1 a}(\varphi^{(\kappa)}_{\nu_n}) \left( \begin{array}{c} \varphi^{(\kappa)}_{\nu_n} \\ \varphi^{(\kappa)}_{\nu_n} \end{array} \right) \right).
$$

According to (3.69-3.70), the anti-symmetric part of $(\tilde{\tau}_1)^{bb' \bar{y}'; \bar{y}}(\gamma_{0} (\varphi^{(\kappa)}_{\nu_n}))$ (3.72-3.73) only remains after the integration over the bilinear anti-commuting fields so that the block diagonal self-energy densities $\hat{S}^{(\kappa)ba}_{\bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n})$ vanish in the integral $\tilde{\mathcal{J}}^{(\kappa)}[\tilde{\mathcal{N}}^{(\kappa)ba}_{\bar{y}'; \bar{y}}]$ (3.76) and act as 'hinge' fields in a spontaneous symmetry breaking with a coset decomposition. Corresponding to the Weyl unitary trick, we can factor out coset matrices $\tilde{T}^{ba}_{\bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n})$

\[ \hat{\mathcal{J}} = \left( \begin{array}{c} \hat{T}^{bb' \bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n}) \end{array} \right) I^{-1} \]

$$
\rightarrow \text{projection onto coset space} !
$$

$$
\tilde{S}^{ba}_{\bar{y}'; \bar{y}} \mathcal{N} \rightarrow \sum_{\bar{y}' \bar{y}} \tilde{T}^{bb' \bar{y}'; \bar{y}}(\varphi^{(\kappa)}_{\nu_n}) \tilde{S}^{ba}_{\bar{y}'; \bar{y}} \tilde{T}_{\bar{y}'; \bar{y}}^{ba_{1} a}(\varphi^{(\kappa)}_{\nu_n}) \varphi^{(\kappa)}_{\nu_n} ,
$$

(3.82)
and finally succeed in eq. \eqref{eq:3.84} for the bilinear integration of Grassmann fields with matrix $\hat{N}^{ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) | \vec{y}_{\kappa}^{(\kappa)} \rangle$ \eqref{eq:3.81} which solely contains the anomalous pairs as remaining degrees of freedom in the coset matrices $\hat{T}^{-1}_{\vec{y}'\vec{y}}(\theta_{\kappa q})$, $\hat{T}^{ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q})$

\[
\gamma^{(\kappa)}[\hat{N}^{(\kappa)ba}_{\vec{y}'\vec{y}}] = \left\{ \text{DET} \left( \hat{N}^{ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) | \vec{y}_{\kappa}^{(\kappa)} \rangle \right) \right\}^{1/2} \times 
\exp \left\{ - \frac{1}{2} \int_{-\Delta\gamma^{(\kappa)}}^{\Delta\gamma^{(\kappa)}} \int_{-\Delta\gamma^{(\kappa)}}^{\Delta\gamma^{(\kappa)}} \sum_{\vec{y}',\vec{y}'} N_{\vec{y}'\vec{y}'} \hat{T}^{-1}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) \hat{N}^{1-ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) | \vec{y}_{\kappa}^{(\kappa)} \rangle \hat{y}_{\kappa}^{(\kappa)} \right\};
\end{align}

\[
\hat{N}^{ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) | \vec{y}_{\kappa}^{(\kappa)} \rangle = \Delta\theta^{(\kappa)} \delta\chi^{(\kappa)ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) | \vec{y}_{\kappa}^{(\kappa)} \rangle \hat{S}^{(\kappa)} \cdot \sigma^{(\kappa)} + \Delta\theta^{(\kappa)} \delta\chi^{(\kappa)ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) | \vec{y}_{\kappa}^{(\kappa)} \rangle \hat{S}^{(\kappa)} \cdot \sigma^{(\kappa)} . \tag{3.84}
\]

As we insert the integration term \eqref{eq:3.83} with matrix \eqref{eq:3.81} into the original path integral \eqref{eq:4.68} \eqref{eq:4.71}, we attain relation \eqref{eq:3.85} for the delta functions of the two-particle operators

\[
\langle \xi^{(\kappa+1)} | \delta(\mathbf{q}^{(\kappa)} - \mathbf{\hat{y}}^{(\kappa)}(\hat{\psi}_{\kappa}^{+}, \hat{\psi}_{\kappa}^{\dagger})) | \xi^{(\kappa)} \rangle =
\]

\[
= \sum_{\kappa=\pm} \lim_{\kappa(\kappa) \rightarrow 0} \lim_{\kappa(\kappa) \rightarrow +\infty} \int_{T(\kappa)}^{T(\kappa)} \frac{dt^{(\kappa)}}{2\pi \hbar} \langle \xi^{(\kappa+1)} | \exp \left\{ - i \frac{\epsilon^{(\kappa)}}{\hbar} \left( \mathbf{q}^{(\kappa)} - \mathbf{\hat{y}}^{(\kappa)}(\hat{\psi}_{\kappa}^{+}, \hat{\psi}_{\kappa}^{\dagger}) - \epsilon^{(\kappa)} \right) \right\} | \xi^{(\kappa)} \rangle \times
\]

\[
\exp \left\{ - \frac{1}{2} \int_{-\Delta\theta^{(\kappa)}}^{\Delta\theta^{(\kappa)}} \int_{-\Delta\theta^{(\kappa)}}^{\Delta\theta^{(\kappa)}} \sum_{\vec{y}',\vec{y}'} N_{\vec{y}'\vec{y}'} \hat{T}^{-1}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) \hat{N}^{1-ba}_{\vec{y}'\vec{y}}(\theta_{\kappa q}) | \vec{y}_{\kappa}^{(\kappa)} \rangle \hat{y}_{\kappa}^{(\kappa)} \right\}.
\]
In order to remove the source fields, we define the source vector field $\Upsilon_{\vec{y}}$ (3.89) and the $6 \times 6$ matrix $\hat{B}_{\vec{y};\vec{y}'}$ (3.91) with propagator terms of one-particle (3.92) and two-particle parts (3.93) so that one obtains the determinant $\det \left( i \delta_{\vec{y};\vec{y}'} - \hat{B}_{\vec{y};\vec{y}'} \right)$ (3.90) after integration over the anti-commuting source fields

$$
\Upsilon_{\vec{y}} = \left( \lambda^{(3)}_{\vec{y}}, \lambda^{(2)}_{\vec{y}}, \lambda^{(1)}_{\vec{y}}, \xi^{(2)}_{\vec{y}}, \xi^{(1)}_{\vec{y}}, \phi^{(0)}_{\vec{y}} \right)^T \quad (3.89)
$$

$$
\mathcal{J}[\chi^{(k)}_{\vec{y}}; \xi^{(n)}_{\vec{y}}] = \prod_{k=1}^{3} \prod_{n=0}^{2} \int d[\chi^{(k)}_{\vec{x},\vec{s}}] d[\xi^{(n)}_{\vec{x},\vec{s}}] d[\delta_{\chi^{(k)}_{\vec{x},\vec{s}}}] \exp \left\{- \sum_{\vec{y},\vec{z}} N_{\vec{x}} \Upsilon_{\vec{x},\vec{y}} \left( i \delta_{\vec{y};\vec{y}'} - \hat{B}_{\vec{y};\vec{y}'} \right) \Upsilon_{\vec{y}} \right\} \quad (3.90)
$$

$$
\hat{B}_{\vec{y};\vec{y}'} = \begin{pmatrix}
0 & \exp \{ \Omega^{(2)} \} \\
0 & 0 & N^{-1}(2') \\
0 & 0 & 0 & N^{-1}(1') \\
-\exp \{ \Omega^{(3)} \} & 0 & 0 & 0
\end{pmatrix} \quad (3.91)
$$

$$
\mathcal{M}^{-1}(\kappa') = M^{-1,0a}_{\vec{y};\vec{y}'} \left( \psi^{(n)}_{\vec{y}} \right) - \Delta \phi^{(n)} ; \quad (b=a=1) \quad (3.92)
$$

The determinant (3.90) with its one-particle and two-particle entries (3.92) (3.93) can be further simplified into a trace-logarithm expansion of period 6' so that the integral relation $\mathcal{J}[\chi^{(k)}_{\vec{y}}; \xi^{(n)}_{\vec{y}}]$ (3.91) reduces to the trace-logarithm term with weight 1/6 over the trace-logarithm operating acting onto the product of the three one-particle elements (3.92) and of the three two-particle elements (3.93)

$$
\mathcal{J}[\chi^{(k)}_{\vec{y}}; \xi^{(n)}_{\vec{y}}] = \det \left( i \delta_{\vec{y};\vec{y}'} - \hat{B}_{\vec{y};\vec{y}'} \right) = \exp \left\{ \text{tr} \ln \left( i \delta_{\vec{y};\vec{y}'} - \hat{B}_{\vec{y};\vec{y}'} \right) \right\} = \exp \left\{ - \sum_{n=1}^{\infty} \frac{1}{n} \text{tr} \left[ \hat{B}^{n} \right] \right\} \quad (3.94)
$$

$$
= \sum_{n=-6}^{\infty} \frac{(-1)^{n+1}}{6n} \text{tr}_{\vec{x},\vec{s}} \left[ \left( \exp \left\{ \frac{\varepsilon^{(1)}}{\hbar} n^{(1)} p^{(1)} \right\} \frac{1}{N_{\vec{s}}} \mathcal{S}_{\vec{s};\vec{y}'} \right) \frac{1}{\hat{g}_{\vec{y};\vec{y}'} \hat{g}_{\vec{y}';\vec{y}''}} \left( \exp \left\{ \frac{\varepsilon^{(2)}}{\hbar} n^{(2)} p^{(2)} \right\} \frac{1}{N_{\vec{s}}} \mathcal{S}_{\vec{s};\vec{y}'} \right) \hat{g}_{\vec{y}';\vec{y}''} \right] \quad (3.94)
$$

$$
= \exp \left\{ \frac{1}{6} \text{tr}_{\vec{x},\vec{s}} \left( i \delta_{\vec{y};\vec{y}'} + \left( \exp \left\{ \frac{\varepsilon^{(1)}}{\hbar} n^{(1)} p^{(1)} \right\} \frac{1}{N_{\vec{s}}} \mathcal{S}_{\vec{s};\vec{y}'} \right) \frac{1}{\hat{g}_{\vec{y};\vec{y}'} \hat{g}_{\vec{y}';\vec{y}''}} \left( \exp \left\{ \frac{\varepsilon^{(2)}}{\hbar} n^{(2)} p^{(2)} \right\} \frac{1}{N_{\vec{s}}} \mathcal{S}_{\vec{s};\vec{y}'} \right) \hat{g}_{\vec{y}';\vec{y}''} \right\} \quad (3.94)
$$
As we replace the Grassmann integration of source fields in \((3.87)\) by \((3.94)\), the trace over the delta functions of the maximal commuting set of symmetry operators finally achieves the form \((3.93)\) with the coset matrices as the remaining field degrees of freedom.

The self-energy density fields \(\sigma_D^{(i)}(\vec{x},\vartheta_q^{(c)})\) are to be determined from a saddle point approximation \((3.96)\) and stay as coefficients in a gradient expansion \((3.99)-(3.97)\) with the coset matrices for the anomalous doubled pairs of fields.

\[
\sigma_D^{(i)}(\vec{x},\vartheta_q^{(c)}) \rightarrow \Re(\sigma_D^{(i)}(\vec{x},\vartheta_q^{(c)})) + i \Im(\sigma_D^{(i)}(\vec{x},\vartheta_q^{(c)})); \quad \text{(Sign of imaginary part from the saddle point approximation has to comply with the corresponding part within delta functional for convergence!)}
\]

\[
\sigma_D^{(i)}(\vec{x},\vartheta_q^{(c)};\vartheta_q^{(a)}) = \sigma_D^{(i)}(\vartheta_q^{(c)};\vartheta_q^{(a)}); \quad \text{(angular momentum cases \(\kappa = 1, 2\) are independent of the spatial vector \(\vec{x}\)!)}
\]

\[
\mathcal{M}_{\vec{y}^1;\vec{y}^2}^{ba}(\vartheta_q^{(a)};\vartheta_q^{(c)}) = \Delta \vartheta^{(a)} \mathcal{M}_{\vec{y}^1;\vec{y}^2}^{ba}(\vartheta_q^{(a)};\vartheta_q^{(c)};\vartheta_q^{(s);i}) \Delta \vartheta_q^{(c);i} - \sigma_D^{(i)} \Delta \vartheta_q^{(c);i} \mathcal{M}_{\vec{y}^1;\vec{y}^2}^{ba}(\vartheta_q^{(a)};\vartheta_q^{(c)};\vartheta_q^{(s);i}) + \Delta \vartheta_q^{(c);i} \mathcal{M}_{\vec{y}^1;\vec{y}^2}^{ba}(\vartheta_q^{(a)};\vartheta_q^{(c)};\vartheta_q^{(s);i}) \sigma_D^{(i)}.
\]  

\section{Summary and conclusions}

Despite of a probably intricate appearance of various transformations, it is the aim of this article to verify the exactness of coherent state path integrals with precise discrete time grids for the various limit processes of many particle theory. Important steps are the introduction of two different anomalous doubled fields in \((3.25)-(3.29)\) with further extension in \((3.29)-(3.31)\). One has also to adapt the 'Nambu' doubled one-particle Hamiltonians \(\hat{H}^{(1,2)}_{\vec{y}^1;\vec{y}^2;\vec{y}^3}(\vartheta_q^{(a)};\vartheta_q^{(c)};\vartheta_q^{(s);i})\) and metric tensors \((\vec{t}_1), (\vec{S})\) \((3.32)\) in order to preserve the exact generalized 'time steps' from insertion of overcomplete sets of anti-commuting fields. Furthermore, we have proved the applicability of coherent states and coherent state path integrals, which are usually anticipated with varying particle numbers, to problems with definite, specific symmetry quantum numbers. Instead of a coordinate space-spin representation \(\hat{y} = (\vec{x}, s = \pm 1, +)\) \((1.4)-(1.5)\), one can also choose the angular momentum representation with various abbreviations of angular momentum labels in order to regard the inherent rotational symmetries of the chosen problem of a many electron atom in a central potential and an external magnetic field. Details of the coset decomposition for fermionic field degrees of freedom are already described in Refs. \((7,9)\) \((10)\) and especially in \((11)\) so that the generator \(\hat{Y}^{a\neq b}(\vartheta_q^{(c)})\) of the coset matrices \(\hat{T}^{ab}_{\vec{y}^1;\vec{y}^2;\vec{y}^3}(\vartheta_q^{(c)}) = (\exp(-\hat{Y}^{a\neq b}_{\vec{y}^1;\vec{y}^2;\vec{y}^3}(\vartheta_q^{(c)})) )^{ab}_{\vec{y}^1;\vec{y}^2;\vec{y}^3}\) is composed of the anti-symmetric, complex-valued sub-generators \(\hat{X}^{a\neq b}_{\vec{y}^1;\vec{y}^2}(\vartheta_q^{(c)})\), \(\hat{X}^{a\neq b}_{\vec{y}^1;\vec{y}^2}(\vartheta_q^{(c)})\) in the off-diagonal blocks \(a \neq b\) of the 'Nambu' doubled space for the anomalous doubled pairs of fields. The given approach of a coherent-state-trace representation of
delta functions with second quantized field operators can also be extended to configurations of molecules with various
nucleons where one achieves inclusion of all relativistic corrections by choosing the total set of relativistically invariant
one-particle and two-particle symmetry quantum numbers and corresponding operators. Since the various precise time
steps of normal ordered operators with field combinations \( \psi_{\mathbf{y}'}(\vartheta(\kappa) + \Delta \vartheta(\kappa)) \ldots \psi(\vartheta(\kappa)) \) may involve intricate appearance
of equations, it is of common use to simplify relations to a hermitian kind \( \psi_{\mathbf{y}'}(\vartheta(\kappa)) \ldots \psi(\vartheta(\kappa)) \) without explicit emphasis
of the limit process \( \Delta \vartheta(\kappa) \to 0 \) for normal ordered, second quantized operators in many-body theory (compare [1]-[4]).
However, as one restricts to a purely classical field theory from variation of anti-commuting fields or coset matrices within
exponentials of the corresponding functional, it is possible to neglect the various limit processes \( \Delta \vartheta(\kappa) \to 0 \) for the classical
field equations and to regard the fields \( \psi_{\mathbf{y}'}(\vartheta(\kappa)) \) and \( \psi(\vartheta(\kappa)) \) within a Hamiltonian density as equivalent [11].

References

[1] J.W. Negele and H. Orland, "Quantum Many-Particle Systems", (Addison-Wesley, Reading, MA, 1988)

[2] T. Kashiwa, Y. Ohnuki and M. Suzuki, "Path Integral Methods", (Oxford Science Publications, Clarendon Press,
Oxford 1997)

[3] N. Nagaosa, "Quantum Field Theory in Condensed Matter Physics", (Texts and Monographs in Physics, Springer-
Verlag, Berlin, Heidelberg, 1999)

[4] N. Nagaosa, "Quantum Field Theory in Strongly Correlated Electronic Systems", (Texts and Monographs in Physics,
Springer-Verlag, Berlin, Heidelberg, 1999)

[5] Y. Nambu, Phys. Rev. Lett. 4 (1960), 380

[6] B. Mieck, "Nonequilibrium ultrashort laser pulse phenomena described by Fermion coherent states", Physica A 269
(1999), 455-475

[7] B. Mieck, "Nonlinear sigma model for a condensate composed of fermionic atoms", Physica A 358 (2005), 347-365

[8] B. Mieck, Rep. Math. Phys. 47 (No. 1) (2000), 139

[9] B. Mieck, "Coherent state path integral and super-symmetry for condensates composed of bosonic and fermionic
atoms", Fortschr. Phys. ("Progress of Physics") 55 (No. 9-10) (2007), 989-1120; (cond-mat/0702223)

[10] B. Mieck, "Ensemble averaged coherent state path integral for disordered bosons with a repulsive interaction (Deriva-
tion of mean field equations)", Fortschr. Phys. 55 (No. 9-10) (2007), 951-988; (cond-mat/0611416)

[11] B. Mieck, "Coherent state path integral and nonlinear sigma model for a condensate composed of fermions with precise,
discrete steps in the time development and with transformations from Euclidean path integrations to spherical field
variables", (prepared for a submission)

[12] H. Kleinert, "Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets", (World
Scientific, 3rd edition (sections 7.7 to 7.9), London, Singapore, reprinted 2005)

[13] H. Weyl, "The Classical Groups (Their Invariants and Representations)", (Princeton Landmarks in Mathematics and
Physics, Princeton University Press, Princeton, New Jersey, 1997)

[14] Bo-Yu Hou and Bo-Yuan Hou, "Differential Geometry for Physicists", (Advanced Series on Theoretical Physical
Science, Vol. 6, World Scientific, Singapore, New Jersey, 1997)