Point Charge in the Born-Infeld Electrodynamics

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Abstract

We show that the nonlinear Born-Infeld field equations supplemented by the “dynamical condition” (certain boundary condition for the field along the particle’s trajectory) define perfectly deterministic theory, i.e. particle’s trajectory is determined without any equations of motion. It is a first step towards constructing the consistent theory of point particles interacting with nonlinear electromagnetism.

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1. Introduction

Born-Infeld electrodynamics [1] was proposed in the thirties as an alternative for Maxwell theory (see also [2] for a useful review). Due to the nonlinearity it is very difficult to solve corresponding field equations (even in the absence of a charged matter). Some very specific solutions were found by Pryce [3], however, after the Dirac's paper [4] on a classical electron and the birth of quantum electrodynamics in the forties Born-Infeld theory was totally forgotten for a long time.

Recently, there is a new interest in this theory due to investigations in string theory. It turns out that some very natural objects in this theory, so called D-branes, are described by a kind of nonlinear Born-Infeld action (see e.g. [5]). Moreover, due to the remarkable interest in field and string theory dualities [6], the duality invariance of Born-Infeld electrodynamics was studied in great details [7] (actually this invariance was already observed by Schrödinger [8]).

In this letter, however, we analyse not a string but a classical point charge coupled to the Born-Infeld nonlinear field. Why nonlinear electrodynamics? It is well known that Maxwell electrodynamics when applied to point-like objects is inconsistent (see [9] for the review). This inconsistency originates in the infinite self-energy of the point charge. In the Born-Infeld theory this self-energy is already finite (actually, it was Born’s motivation to find classical solutions representing electrically charged particles with finite self-energy). Therefore, one may hope that in the theory which gives finite value of this quantity it would be possible to describe the particle’s self-interaction in a consistent way. Moreover, the assumption, that the theory is effectively nonlinear in the vicinity of the charged particle is very natural from the physical point of view and this we already learned from quantum electrodynamics (Born tried to make contact with quantum field theory by identifying Born-Infeld Lagrangian as an effective Euler-Heisenberg Lagrangian [10]. It has been shown [11] that the effective Lagrangian can coincide with those of Born and Infeld up to six-photon interaction terms only).

We consider here very specific model of nonlinear theory because, among other nonlinear theories of electromagnetism, Born-Infeld theory possesses very distinguished physical properties [12]. For example it is the only causal spin-1 theory [13] (apart from the Maxwell one). Recently, Born-Infeld electrodynamics was successfully applied [14] as a model for generation of multipole moments of charged particles.

Our aim in the present letter is to describe the dynamics of a point charge interacting with Born-Infeld electromagnetic field. Due to the nonlinearity of the field equations it is impossible to derive separate equations of motion for the charged particle corresponding e.g. to the celebrated Lorentz-Dirac equation in the Maxwell case. Could we, therefore, determine the particle’s trajectory without equations of motion? In this letter we show that it is in fact possible. For this purpose we propose a new approach which was developed in the Maxwell case in [15]. Analysing the interaction between charged particle and nonlinear electromagnetism we show that the conservation of total four-momentum of the composed (particle+field) system is equivalent to the certain boundary condition for the Born-Infeld field which has to be satisfied along the particle’s trajectory. We call
it “dynamical condition” (formula (23)) because, roughly speaking, it replaces particle’s equations of motion. Field equations supplemented by this condition define perfectly deterministic theory, i.e. initial data for the particle and field uniquely determine the evolution of the system.

The same problem was addressed just after the birth of the theory by Feenberg [16] and Pryce [17]. They also used the similar approach, i.e. they considered a conservation law for the total energy-momentum tensor. Therefore, our result has to be confronted with those obtained 60 years ago. In section 6 we show what is the exact relationship between three conditions: Feenberg’s, Pryce’s and ours. It turns out that our condition given by (23) is correct, whereas those of Feenberg and Pryce are not consistent (Feenberg’s is not consistent with the field dynamics and Pryce’s is not sufficient to determine the particle’s dynamics because it uses not well defined quantities).

Finally, we discuss the physical importance and relevance of the ”dynamical condition” (23) in constructing consistent electrodynamics of point-like objects.

2. Field dynamics

The Born-Infeld nonlinear electrodynamics [1] is based on the following Lagrangian (we use the Heaviside-Lorentz system of units with the velocity of light $c = 1$):

$$L_{BI} := \sqrt{-\det(b\eta_{\mu\nu})} - \sqrt{-\det(b\eta_{\mu\nu} + F_{\mu\nu})}$$

$$= b^2 \left( 1 - \sqrt{1 - 2b^{-2}S - b^{-4}P^2} \right),$$  \hspace{1cm} (1)

where $\eta_{\mu\nu}$ denotes the Minkowski metric with the signature ($- , +, +, +$) (the theory can be formulated in a general covariant way, however, in this paper we will consider only the flat Minkowski space-time). The standard Lorentz invariants $S$ and $P$ are defined by: $S = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ and $P = -\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$ ($\tilde{F}^{\mu\nu}$ denotes the dual tensor). The arbitrary parameter “b” has a dimension of a field strength (Born and Infeld called it the absolute field) and it measures the nonlinearity of the theory. In the limit $b \to \infty$ the Lagrangian $L_{BI}$ tends to the Maxwell Lagrangian $S$.

Adding to (1) the standard electromagnetic interaction term “$j^\mu A_\mu$” we may derive the inhomogeneous field equations

$$\partial_\mu G^{\mu\nu} = -j^\nu,$$  \hspace{1cm} (2)

where $G^{\mu\nu} := -2\partial L_{BI}/\partial F_{\mu\nu}$. Equations (2) have formally the same form as Maxwell equations. What makes the theory effectively nonlinear are the constitutive relations, i.e. relations between inductions ($D, B$) and intensities ($E, H$):

$$D(E, B) := \frac{\partial L_{BI}}{\partial E} = \frac{E + b^{-2}(EB)B}{\sqrt{1 - b^{-2}(E^2 - B^2) - b^{-4}(EB)^2}},$$

$$H(E, B) := -\frac{\partial L_{BI}}{\partial B} = \frac{B - b^{-2}(EB)E}{\sqrt{1 - b^{-2}(E^2 - B^2) - b^{-4}(EB)^2}}.$$  \hspace{1cm} (3)

(4)
In the Maxwell case we have simply $\mathbf{D} = \mathbf{E}$ and $\mathbf{H} = \mathbf{B}$.

Fields $\mathbf{D}$ and $\mathbf{B}$ play in our analysis important role (they serve as a Cauchy data for the field evolution). Therefore, it is desirable to express $\mathbf{E}$ and $\mathbf{H}$ in terms of the Cauchy data $\mathbf{D}$ and $\mathbf{B}$. Using (3) and (4) one easily gets:

$$\mathbf{E}(\mathbf{D}, \mathbf{B}) = \frac{1}{b^2 R} \left[ (b^2 + \mathbf{B}^2) \mathbf{D} - (\mathbf{DB}) \mathbf{B} \right] ,$$

(5)

$$\mathbf{H}(\mathbf{D}, \mathbf{B}) = \frac{1}{b^2 R} \left[ (b^2 + \mathbf{D}^2) \mathbf{B} - (\mathbf{DB}) \mathbf{D} \right] ,$$

(6)

with $R := \sqrt{1 + b^{-2}(\mathbf{D}^2 + \mathbf{B}^2) + b^{-4}(\mathbf{D} \times \mathbf{B})^2}$.

Now, let us assume that the external current $j^\mu$ in (2) is produced by a point-like particle moving along the time-like trajectory $\zeta$ parameterized by its proper time $\tau$, i.e. $j^\mu(x) = e \int_{-\infty}^{\infty} d\tau \delta(x - \zeta(\tau)) w^\mu(\tau)$. This system is very complicated to analyse. In particular, contrary to the Maxwell case, we do not know the general solution to the inhomogeneous Born-Infeld field equations. Therefore, following [15], we propose the following “trick”. Instead of solving distribution equations (2) on the entire Minkowski space-time $\mathcal{M}$ let us treat them as a boundary problem in the region $\mathcal{M}_\zeta := \mathcal{M} - \{\zeta\}$, i.e. outside the trajectory. In order to well pose the problem we have to find an appropriate boundary condition which has to be satisfied along $\zeta$, i.e. on the boundary $\mathcal{M}_\zeta$.

It turns out that the simplest way to analyse this problem is to use the reference frame which is Fermi-transported along $\zeta$ (in this frame the particle is always at rest). The general discussion of the accelerated frame can be found in [18]. Obviously, the theory is perfectly Lorentz invariant, however, the use of this special frame considerably simplifies our analysis. Let $(y^\mu)$ denotes the standard Lorentz coordinates in a fixed laboratory frame. At each point $y^\mu(\tau) \in \zeta$ let $\Sigma_\tau$ denotes the 3-dimensional hyperplane orthogonal to $\zeta$. Choose on one $\Sigma_\tau$, say $\Sigma_\tau_0$, the system of cartesian coordinates $(x^k)$, such that the particle is located at its origin, and transport it (via the Fermi-transport) to all other $\Sigma_\tau$. This way we obtain a system $(x^\mu) = (x^0 = \tau, x^k)$ of “co-moving” coordinates in a neighbourhood of $\zeta$. Obviously, it is not a global system because different $\Sigma$’s may intersect. Nevertheless, we will use it globally to describe the evolution of the electromagnetic field from one $\Sigma_\tau$ to another (for the hyperbolic theory this is well defined problem).

The Born-Infeld field equations have in this frame the following form (see [13] for the discussion in the Maxwell case):

$$\dot{\mathbf{D}} = \nabla \times (N \mathbf{H}) ,$$

(7)

$$\dot{\mathbf{B}} = -\nabla \times (N \mathbf{E}) ,$$

(8)

where $N = 1 + a^k x_k$ (it is the lapse function corresponding to the Minkowski metric rewritten in the co-moving frame) and $a^k$ stands for the rest-frame particle’s acceleration. Equations (7) and (8) have to be supplemented by the constraints: $\nabla \mathbf{D} = 0$ and $\nabla \mathbf{B} = 0$ (note that in $\mathcal{M}$ we have $\nabla \mathbf{D} = e\delta_0$).
3. Asymptotic conditions

It is well known ([1] - [2]) that \( E \) and \( B \) fields are bounded for \( r \to 0 \), whereas \( D \) and \( H \) vary as \( r^{-2} \) (\( r \) stands for the radial coordinate, i.e. \( r^2 = x^k x_k \)). Therefore, one could think that the standard Lorentz equations of motion can be applied in this case. However, despite the fact that \( E \) and \( B \) are bounded, they are not regular in \( r = 0 \) and the Lorentz force \( e(E + v \times B) \) is not well defined. Let us formally write the following expansions:

\[
E(r) = \sum_{n=0}^{\infty} r^n E_n , \quad B(r) = \sum_{n=0}^{\infty} r^n B_n , \\
D(r) = \sum_{n=-2}^{\infty} r^n D_n , \quad H(r) = \sum_{n=-2}^{\infty} r^n H_n ,
\]

where the vectors \( E_n \) etc. do not depend on \( r \). Let us carefully analyse the behaviour of the fields for \( r \to 0 \) in the co-moving frame. In the Maxwell case \( D_{\text{Maxwell}} = \frac{eA}{4\pi} \frac{r}{r} \). Now, the \( D_{-2} \) term may have much more general form:

\[
D_{-2} = \frac{eA}{4\pi} \frac{r}{r} ,
\]

(9)

where, due to the Gauss law, the monopole part of the \( r \)-independent function \( A \) equals 1. Observe, that due to (7), \( H_{-2} \) term would have produced an \( r^{-3} \) term in \( \dot{D} \), which has to vanish. Therefore, \( H_{-2} = 0 \). On the other hand, from (8) it follows that \( H_{-2} = 0 \) if and only if \( B_0 = 0 \). Therefore, \( B \) behaves at least like \( r \). This information together with (8) imply the following constraints on \( E \):

\[
\nabla N \times E_{(0)} + \nabla \times rE_{(1)} = 0 .
\]

(10)

Now, from (3) one has \( E_{(0)} = \frac{ber}{|e|} r \) and, therefore

\[
r \nabla \times rE_{(1)} = -\frac{be}{|e|} a \times r .
\]

(11)

The above equation provides the constraint on the transversal part \( E_{T_{(1)}} \) of \( E_{(1)} \). Due to \( \nabla rE_{T_{(1)}} = 0 \), the transversal part is uniquely given by:

\[
E_{T_{(1)}} = \frac{be}{4|e|} \left( 3a - r^{-2}(ar)r \right) .
\]

(12)

This way we have proved the following

Theorem 1 Any regular solution of Born-Infeld field equations with point-like external current satisfies (12).

Observe, that in the Maxwell case we can derive very similar formula, namely

\[
E_{(-1)} = -\frac{e}{8\pi} \left( a + r^{-2}(ar)r \right) .
\]

(13)
One may easily check that any regular (retarded or advanced) solution of Maxwell equations satisfies (13) (cf. [15]).

Observe, that (12), according to our “boundary philosophy”, may be interpreted as a boundary condition for $E$ on $\partial M_\zeta$. Due to the hyperbolicity of (2) one may prove

Theorem 2  The mixed (initial-boundary) value problem for the Born-Infeld equations in $M_\zeta$ with (12) playing the role of boundary condition on $\partial M_\zeta$ has the unique solution.

4. Particle’s dynamics

Up to now our charged particle served only as the point-like external current for the nonlinear field dynamics. Now, we would like to keep field and particle’s degrees of freedom at the same footing, i.e. we shall consider a particle as a dynamical object. Of course field equations alone are not sufficient to uniquely determine the evolution of the composed (particle + field) system. Therefore, we impose the conservation law of the total four-momentum as the additional equation in the theory.

This point may be further clarified on the level of the boundary condition (12). Choosing particle’s position $q$ and velocity $v$ as the Cauchy data for the particle’s dynamics let us observe that despite the fact that the time derivatives ($\dot{D}, \dot{B}, \dot{q}, \dot{v}$) of the Cauchy data are uniquely determined by the data themselves, the evolution of the composed system is not uniquely determined. Indeed, $\dot{D}$ and $\dot{B}$ are given by the field equations, $\dot{q} = v$ and $\dot{v}$ may be calculated from (12). Nevertheless, the initial value problem is not well posed: keeping the same initial data, particle’s trajectory can be modified almost at will. This is due to the fact, that now (12) plays no longer the role of boundary condition because we use it to as a dynamical equation to determine $a^k$. Therefore a new boundary condition is necessary. We show that this missing condition is implied by the conservation law of the total four-momentum for the “particle + field” system.

The co-moving components of the total four-momentum are given by:

\begin{align*}
P^0(\tau) &= m - \int_{\Sigma_\tau} T^0_0 d^3x , \quad (14) \\
P_k(\tau) &= \int_{\Sigma_\tau} NT^0_k d^3x , \quad (15)
\end{align*}

where $T^\mu_\nu$ denotes the symmetric energy-momentum tensor of the Born-Infeld field:

\[ T^\mu_\nu := \delta^\mu_\nu L_{BI} - \frac{\partial L_{BI}}{\partial F^\mu_\lambda} F^\lambda_\nu - \frac{\partial L_{BI}}{\partial P} F^\mu_\lambda \tilde{F}^\lambda_\nu . \]

Using (1) one easily gets:

\begin{align*}
T^{00} &= b^2 \left( \sqrt{1 + b^{-2}(D^2 + B^2)} + b^{-4}(D \times B)^2 - 1 \right) , \\
T^{0k} &= (D \times B)^k , \\
T^{kl} &= \delta^{kl} (ED + HB - T^{00}) - (E^k D^l + H^k B^l) ,
\end{align*}
with \( \mathbf{E} \) and \( \mathbf{H} \) given by (5) and (6) respectively.

The factor \( N \) in (15) is necessary because only the co-vector \( Ndx^0 \) and not \( dx^0 \) is constant on \( \Sigma_\tau \) (cf. [18]). This factor is absent in (14) because the “upper 0” introduces additional \( N^{-1} \)-factor. The “\( m \)” in (14) denotes particle’s mass. We stress that we do not perform any mass renormalization. The particle’s self-energy is finite and it is already contained in the field energy \( \int T^{00} \). Due to the nonlinearity of the theory there is no way to separate this self-energy from the total field energy (this separation is possible in the Maxwell theory and it enables us to perform mass renormalization, i.e. to include the infinite self-energy into \( m \)).

Obviously, \( \mathcal{P}^0 \) and \( \mathcal{P}_k \) are not conserved, i.e. they depend upon \( \tau \). Conserved is the corresponding four-momentum in the laboratory frame. Therefore, one has to transform \( \mathcal{P}^0 \) and \( \mathcal{P}_k \) to the laboratory frame and compute the time derivatives using field equations. However, there is a simpler way to implement the conservation of momentum in our system. Note, that the corresponding four-momentum in the laboratory frame is conserved iff \( \mathcal{P}^0 \) and \( \mathcal{P}_k \) are Fermi-transported along \( \zeta \) (cf. [18]). Therefore

\[
\dot{\mathcal{P}}^0 = -a_k \mathcal{P}^k, \quad \dot{\mathcal{P}}^k = -a^k \mathcal{P}^0.
\]

On the other hand, it is easy to show that

\[
\partial_\alpha T^\alpha_0 = N a^k T^0_k, \quad \partial_\alpha (N T^\alpha_k) = a_k T^0_0.
\]

In the laboratory frame (\( a^k = 0 \) and \( N = 1 \)) these formulae reduce to the simple conservation law \( \partial_\mu T^\mu_\nu = 0 \). Now, using (17), we compute \( \dot{\mathcal{P}}^0 \) and \( \dot{\mathcal{P}}^k \):

\[
\dot{\mathcal{P}}^0 = -\int_{\Sigma^0} \partial_0 T^0_0 d^3x = -\lim_{\epsilon \to 0} \int_{S(\epsilon)} T^\perp_0 d\sigma - a^k \int_{\Sigma} N T^0_k d^3x = -a_k \mathcal{P}^k,
\]

because the surface integral vanishes in the limit \( \epsilon \to 0 \) due to the asymptotic condition \( \mathbf{B}(0) = 0 \). In (13), \( S(\epsilon) \) denotes 2-sphere with radius \( \epsilon \) centered at the particle’s position, “\( \perp \)” denotes the component perpendicular to \( S(\epsilon) \) and \( \Sigma^0 = \Sigma \cap \mathcal{M}_\zeta \equiv \Sigma - \{0\} \), where, for simplicity, we skipped the subscript \( \tau \). In the same way

\[
\dot{\mathcal{P}}_k = \int_{\Sigma^0} \partial_0 (N T^0_k) d^3x = \lim_{\epsilon \to 0} \int_{S(\epsilon)} N T^\perp_k d\sigma + a^k \int_{\Sigma} T^0_0 d^3x.
\]

Now, the boundary term does not vanish. Using asymptotic conditions it is easy to show that

\[
\lim_{\epsilon \to 0} \int_{S(\epsilon)} N T^\perp_k d\sigma = -\frac{|e|}{4\pi} \int_{S(1)} \frac{x_k}{r} A d\sigma = -\frac{|e|}{3} A_k,
\]

where \( A_k \) is the dipole part of \( A \), i.e. \( \text{DP}(A) =: A_k x^k/r \). Finally,

\[
\dot{\mathcal{P}}_k = -a_k \mathcal{P}^0 - \left( \frac{|e|}{3} A_k - m a_k \right).
\]
Comparing (21) with (16) we obtain:

\[ ma_k = \frac{|e|b}{3} A_k . \]  

(22)

The above equation looks formally like a standard Newton equation. However, it could not be interpreted as the Newton equation because its r.h.s. is not \textit{a priori} given (it must be calculated from field equations).

5. Dynamical condition

To correctly interpret (22) we have to take into account (12). Now, calculating \( a^k \) in terms of \( E^T_{(1)} \) and inserting into (22) we obtain a relation between \( E^T_{(1)} \) and \( D_{(-2)} \). Due to (12) the radial component \( (E^T_{(1)})^r = (be/2|e|r)(a^k x_k) \) and, therefore, \( a^k \) equals to the dipole part of \( (2e/b|e|)(E^T_{(1)})^r \). Moreover, from (3), \( dp(|e|A) = (4\pi e/|e|)DP((D_{(-2)})^r) \). Therefore, (12) and (22) lead to

\[ DP \left( \frac{2m}{b} (E^T_{(1)})^r - \frac{4\pi b}{3} (D_{(-2)})^r \right) = 0 . \]

The above formula may be simplified if we make the following observation: a charged particle introduces the characteristic length \( \lambda_0 := e^2/6\pi m \) into the theory. For example, in the Maxwell case \( \lambda_0 \) appears in the Lorentz-Dirac equation: \( a^\mu = \frac{e}{m} F^{\mu\nu}_{ext} u_\nu + \lambda_0 (\dot{a}^\mu - a^\mu u^\mu) \). The “\( b \)” parameter in the Born-Infeld theory introduces a new scale \( r_0 := \sqrt{|e|/4\pi b} \). Using \( \lambda_0 \) and \( r_0 \) the last formula may be rewritten as

\[ DP \left( 4r_0^4 (E^T_{(1)})^r - \lambda_0 (D_{(-2)})^r \right) = 0 . \]  

(23)

Therefore, we finally proved that the conservation of the total four-momentum is equivalent to the boundary condition (23) for the Born-Infeld field along \( \zeta \). We call (23) the “dynamical condition” for the electrodynamics of a point charge. The main result of this letter consists in the following

\textbf{Theorem 3} Born-Infeld field equations supplemented by the dynamical condition (23) define perfectly deterministic theory, i.e. initial data for field and particle uniquely determine the entire evolution of the system.

6. Comparison with previous results

Now, we compare (23) with the results of [16] and [17]. Both authors used the model of a purely electromagnetical particle (in the “spirit” of Einstein’s approach to the unitary field theory). However, if we put \( m = 0 \) in (22) we can compare their results with ours.

Feenberg claimed that the energy and momentum are automatically conserved due to the field equations and he proposed “a new dynamical condition which appears to be
singled out from all other possible conditions by its compelling simplicity”. However, his conjecture is not true. It turns out that the first integral in the r.h.s. of (28) in [16] does not vanish (as claimed by Feenberg) but equals exactly to the r.h.s. of ours (20). The crucial observation in evaluating this integral is the asymptotic behaviour of the \( D \) field given by (9). If one uses instead of (9) the Coulomb field (i.e. \( A = 1 \)) this integral vanishes.

Pryce’s conclusion is the same as ours, i.e. conservation law for energy and momentum imposes the unique condition for the dynamics of charged particles. In his approach one has to evaluate the same boundary integral as Feenberg’s (28) and ours (20) (actually this integral defines the force acting on a charge). In [17] it is given by the first term in the r.h.s. of (5.2). Now, to calculate the force he replaced the point particle by the extended one and obtained very suggestive (5.12). Obviously, (5.12) defines a force for any extended charge distribution. Pryce claimed that his integral in (5.2) is given by the point-particle limit of (5.12). However, it could not be true, because (5.12) is not well defined in this limit: \( E \) and \( B \) are not regular at \( x = 0 \). Therefore, his dynamical condition is also not well defined.

We evaluated the surface integral in (20) without any use of an extended particle’s model. What is crucial for our approach is a thorough asymptotic analysis of the fields in the vicinity of a charge. This analysis enables one to calculate (20) using only field equations outside the particle’s trajectory. In our opinion this “boundary philosophy” is the only consistent way to solve this problem.

7. Concluding remarks

Let us now briefly discuss the physical importance of (23). It turns out that the dynamics of the “particle + field” system based on (23) may be described by an infinite-dimensional Hamiltonian system. Both Lagrangian and Hamiltonian formulation of the above theory will be presented in the next paper. In this letter we consider only one particle case. However, our result may be generalized to many particles interacting with nonlinear electromagnetism.

At this point the most interesting question arises: is the theory based on (23) consistent? We stress that the Theorem 3 does not guarantie the consistency of the theory. The analogous theorem may be proved in the Maxwell case [14], nevertheless, Maxwell electrodynamics of a point charge is not consistent. To answer this question we need a precise notion of consistency. There is a very natural definition of consistency based on the canonical structure of the theory. We show in the next paper that according to this definition Born-Infeld electrodynamics of a point charge is consistent.

It turns out that due to the duality invariance of the Born-Infeld electrodynamics [4] it is possible to describe in the same way the dynamics of magnetic monopoles. This problem will be considered elsewhere.

There are several open questions. In [12] (see also [19]) the Born-Infeld electrodynamics was generalised to the non-abelian gauge theories. It would be interesting to apply the
approach based on (23) also in this case. Of course one has to ask about the quantum version of this theory. This problem is very difficult. Very little is known about quantum aspects of the Born-Infeld electrodynamics. Up to our knowledge only 2 dimensional model was studied in the sixties [20] and recently in [21].

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