Low-energy Electro-weak Reactions

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Abstract. Chiral effective field theory (EFT) provides a systematic and controlled approach to low-energy nuclear physics. Here, we use chiral EFT to calculate low-energy weak Gamow-Teller transitions. We put special emphasis on the role of two-body (2b) weak currents within the nucleus, and discuss their applications in predicting physical observables.

1. Introduction
Electro-weak nuclear reactions at low energies, which are the subject of this contribution, have an important role in many physical scenarios. They are the microscopic motor of astrophysical phenomena, such as solar fusion, core collapse supernovae and the nucleosynthesis within. In addition, these reactions are used to study nuclear structure and dynamics. Moreover, low-energy β-decays and weak transitions provide a window into high-energy physics. For example, superallowed decays allow high precision tests of the Standard Model, and 0νββ decays probe the nature of neutrinos, their hierarchy and mass. High accuracy theoretical, as well as experimental understanding of the nuclear response is a key ingredient needed to accomplish these tasks.

However, the non-perturbative character of QCD at low energies makes a theoretical calculation of the nuclear response extremely challenging. In the last two decades, chiral effective field theory of QCD at low energies was developed, enabling a construction of a nuclear Lagrangian that is consistent with QCD. Chiral EFT provides a systematic basis for nuclear forces and consistent electroweak currents [1, 2], where pion couplings contribute both to the electroweak axial current and to nuclear interactions. This is already seen at leading order: the axial constant, gA, determines the axial one-body (1b) current and the one-pion-exchange potential. Two-body (2b) currents, also known as meson-exchange currents, enter at higher order, just like three-nucleon (3N) forces [1]. As shown in Fig. 1, the leading axial contributions are due to long-range one-pion-exchange and short-range parts [2], with couplings c3, c4 and cD, which also enter the leading 3N (and subleading NN) forces [1, 3].

In this contribution we present some recent studies of chiral currents and their consistency with nuclear forces. We emphasize the importance of using chiral 2b currents in weak Gamow-Teller transitions in nuclei.

2. Gamow-Teller transitions
At low energies, the coupling of weak probes to a nucleus is given by the current-current interaction,

\[ H_W = \frac{G_F}{\sqrt{2}} \int d^3r e^{-ip\cdot r} j_{L\mu} \tilde{J}^\mu_L + \text{h.c.}, \]

where \( G_F \) is the Fermi constant, \( p \) the momentum transferred from nucleons to leptons, and \( j_{L\mu} \) the leptonic current of an electron.
coupled to a left-handed electron neutrino. This approximation creates a separation between the leptons and baryons, which is a great simplification of the problem. It suggests that only the symmetry of the interaction is needed to determine the structure of the current. For example, any axial probe, regardless of its fundamental origin, would couple to the same current within the nucleus, which is the Noether current derived from the axial symmetry of the chiral Lagrangian, which we will consider up to next-to-next-to-leading order (N³LO) \[2\].

The nuclear current \(J_{i,1b}^0\) is organized in an expansion in powers of momentum \(Q \sim m_\pi\) over a breakdown scale \(\Lambda_\pi \sim 500\) MeV. To order \(Q^2\) (and also \(Q^3\) in this counting), the 1b current, \(J_{i,1b}^0(\mathbf{r}) = \sum_{i=1}^{A} \tau_i \left[ \delta^{00} J_{i,1b}^0 - \delta^{ik} J_{i,1b}^k \right] \delta(\mathbf{r} - \mathbf{r}_i)\), has temporal and spatial parts in momentum space \[2\]:

\[
J_{i,1b}^0(p^2) = g_V(p^2) - g_A \frac{\mathbf{P} \cdot \mathbf{\sigma}_i}{2m} + g_P(p^2) \frac{E(\mathbf{p} \cdot \mathbf{\sigma}_i)}{2m},
\]

\[
J_{i,1b}(p^2) = g_A(p^2) \mathbf{\sigma}_i - g_P(p^2) \frac{\mathbf{P}(\mathbf{p} \cdot \mathbf{\sigma}_i)}{2m} + i(g_M + g_V) \frac{\mathbf{\sigma}_i \times \mathbf{P}}{2m} - \frac{g_V}{2m},
\]

where \(E = E_i - E_i'\), \(\mathbf{p} = \mathbf{p}_i - \mathbf{p}_i'\), and \(\mathbf{P} = \mathbf{p}_i + \mathbf{p}_i'\). \(g_V(p^2), g_A(p^2), g_P(p^2),\) and \(g_M(p^2)\) are the vector (V), axial (A), pseudo-scalar (P), and magnetic (M) couplings. In chiral EFT, the \(p\) dependence is due to loop corrections and pion propagators, to order \(Q^2\): \(g_{V,A}(p^2) = g_{V,A}(1 - 2\frac{m^2}{\Lambda^2_{V,A}})\), with \(g_V = 1, \Lambda_V = 850\) MeV, \(\Lambda_A = 2\sqrt{3}/r_A = 1040\) MeV; \(g_P(p^2) = \frac{2g_{\pi NN} F_\pi}{m_\pi^2 + p^2} - 4 g_A(p^2)^2 m_\pi^2 / \Lambda_A^2\), and \(g_M = \mu_p - \mu_n = 3.70\), with pion decay constant \(F_\pi = 92.4\) MeV, \(m_\pi = 138.04\) MeV, and \(g_{\pi NN} = 13.05\) \[4\].

At leading order \(Q^0\), only the momentum-independent \(g_A\) and \(g_V\) terms contribute. They give rise to low-momentum-transfer Gamow-Teller and Fermi decays. The former has the known form, \(E_{11}^1|_{LO} = \frac{\sqrt{14}}{\sqrt{62}} \sum_{i=1}^{A} \sigma_i \tau_i^-\).

Among the \(Q^2\) terms, form-factor-type (ff) contributions and the \(g_P\) part of \(J_{i,1b}\) dominate. The remaining \(Q^2\) terms are odd under parity, so they require either a \(P\)-wave lepton or another odd-parity term to connect to \(0^+\) states, common for ground states. In such cases, the \(\mathbf{P}\) and \(E\) terms in Eqs. (1) and (2) can be neglected, and only the term with the large \(g_M + g_V = 4.70\) is kept.

At order \(Q^3\), 2b currents enter in chiral EFT \[2\]. These include vector spatial, axial temporal, and axial spatial parts\(^1\). The first two are odd under parity, and therefore can be neglected in couplings to \(0^+\) ground states. In addition, within chiral EFT the vector spatial current is conserved, satisfying the CVC hypothesis, thus at low-energies their contribution can be calculated via the Siegfert theorem. As a result, the dominant weak 2b currents include an axial

\(^1\) Vector temporal parts do not contribute at this order \[2\].
Table 1. Calculated $^3$H, $^3$He and $^4$He g.s. energies (MeV) and point-proton root-mean-squared radii (fm)

|       | $^3$H | $^3$He | $^4$He |
|-------|-------|-------|-------|
|       | $E_{\text{g.s.}}$ | $(r_p^2)^{1/2}$ | $E_{\text{g.s.}}$ | $(r_p^2)^{1/2}$ | $E_{\text{g.s.}}$ | $(r_p^2)^{1/2}$ |
| $NN$  | -7.852(4) | 1.651(5) | -7.124(4) | 1.847(5) | -25.39(1) | 1.515(2) |
| $NN+NNN$ | -8.473(4) | 1.605(5) | -7.727(4) | 1.786(5) | -28.50(2) | 1.461(2) |
| Expt. | -8.482 | 1.60 | -7.718 | 1.77 | -28.296 | 1.467(13) |

spatial component, $J_{2b}^{\text{axial}} = \sum_{i<j}^{A} J_{ij}$, with [2]

\[
J_{12} = -\frac{g_A}{F_{\pi}^2} \left[ 2d_1 (\sigma_1 \tau_1^- + \sigma_2 \tau_2^-) + d_2 \sigma_x \tau_x^- \right] \\
- \frac{g_A}{2F_{\pi}^2} \frac{1}{m_{\pi}^2 + k^2} \left[ \left( c_4 + \frac{1}{4m} \right) k \times (\sigma_x \times k) \tau_x^- \right] \\
+ 4c_3 k \cdot (\sigma_1 \tau_1^- + \sigma_2 \tau_2^-) k - \frac{i}{2m} k \cdot (\sigma_1 - \sigma_2) q \tau_x^-, \tag{3}
\]

where $\tau_x^- = (\tau_1 \times \tau_2)^-$ and the same for $\sigma_x$, $k = \frac{1}{2} (p_2 - p_2 + p_1 + p_1)$ and $q = \frac{1}{2} (p_1 + p_1 - p_2 - p_2)$.

Equation (3) includes contributions from the one-pion-exchange $c_3, c_4$ parts and from the short-range couplings $d_1, d_2$, where due to the Pauli principle only the combination $d_1 + 2d_2 = c_D / (g_A \Lambda_{\chi})$ enters [with $\Lambda_{\chi} = 700$ MeV], see Fig 1. This result is extremely interesting as it connects the low-energy constants (LECs) determining the strength of the meson exchange current (MEC) contact term to $c_D$, which is one of the two unknown low-energy constants, $c_D$ and $c_E$, that determine the 3N force up to $N^3$LO, and do not contribute to 2N force. This relation shows that one can use a weak observable, e.g., an empirical extraction of $(E_1^A)_{\text{emp}}$ from a decay rate, as a constraint for the determination of $c_D$ and $c_E$.

3. Using triton $\beta$-decay to determine the 3N force

We use the relation between the short range contribution to the current and the 3N force to fully constrain the Lagrangian in systems with up to 3-body, and predict $^4$He properties. The calculation steps are as follows: (i) calculate the $^3$H and $^3$He g.s. wave functions by solving the Schrödinger equation for three nucleons interacting via the chiral $NN$ potential at $N^3$LO of Ref. [5] and the $NNN$ interaction at $N^2$LO [6]; (ii) determine for which $c_D$ values along the trajectory the calculated reduced matrix element of the $E_1^A$ operator (at $N^3$LO) reproduces $(E_1^A)_{\text{emp}}$ extracted from $^3$H $\beta$-decay. We find that these two constraints are uncorrelated, producing a stringent determination of the LECs: $c_D = -0.2 \pm 0.1$ and $c_E = -0.205 \pm 0.015$ [3].

With this calibration of $c_D$ and $c_E$, for this potential, in principle, any other calculation is a prediction! In Table 1 a collection of $A = 3$ and 4 data is given, obtained with and without inclusion of the $NNN$ force for $c_D = -0.2$ ($c_E = -0.205$). The $NN+NNN$ predictions for the $^4$He are in good agreement with measurement.

To this important conclusion, one can add the results of few additional tests, appearing in Fig. 2, that are aimed to probe the axial correlations in the nucleus, by analyzing the sensitivity of the triton half life to $NNN$ force and/or MEC:

- The fundamental importance of the axial two-body currents. By suppressing the MEC, in the whole investigated $c_D$-$c_E$ range, the calculations underpredict $(E_1^A)_{\text{emp}}$ by about 2%.

Within the MEC, one finds a large cancellation between the long-range one-pion-exchange term (corresponding to the left diagram in Fig. 1), and the contact term (second diagram from the left Fig. 1).
Figure 2. The ratio $\langle E_A^1 \rangle_{\text{th}}/\langle E_A^1 \rangle_{\text{emp}}$ using the N$^3$LO $NN$ potential [5] with and without the N$^2$LO $NNN$ interaction [6], and the axial current with and without MEC. The shaded area is twice the experimental uncertainty. Also shown: results for the phenomenological AV18 potential (with $\Lambda = 500$ MeV imposed in the current), and for the N$^3$LO $NN$ potential of Epelbaum et al. [7] (with $\Lambda = 450, 600$ MeV, and a 700 MeV spectral-function cutoff in the two-pion exchange), which should be compared with the full calculation (black solid line). The line denoted by $d_R = 0$ includes only the long range contribution to the MEC, neglecting the contact term contribution. Figure taken from [3].

- A negligible effect of the $NNN$ force on this observable.
- Weak dependence on the specific character of the force.

These features, which might be unique to s-shell nuclei, entail that the determination of $c_D$ and $c_E$ obtained in this work is robust.

4. Extracting the weak structure of the nucleon from $\mu^-$ capture on $^3$He [8]

In the previous section it was shown that for a weak transition in the 3-nucleon sector, the specific character of the force had a minor effect on the reaction rate. We extrapolate this understanding to the muon capture on $^3$He which results in a triton, $\mu^- + ^3\text{He} \rightarrow \nu_\mu + ^3\text{H}$. We study this reaction in a hybrid approach: the calculation uses the chiral EFT based weak current, combined with the phenomenological nucleon-nucleon potential Argonne v$\text{18}$ (AV18) [9] augmented by the Urbana IX (UIX) [10] three nucleon force. This allows a study of the cutoff variation of the observable.

$\mu$ capture on $^3$He has an accurately measured rate $\Gamma(\mu^- + ^3\text{He} \rightarrow \nu_\mu + ^3\text{H})^{\text{exp}} = 1496(4)$ Hz, i.e. a $\pm0.3\%$ precision [11]. Combining with the sizable momentum transfer in ordinary muon capture (OMC), that enhances the effect of the induced-pseudoscalar form factor, allows its study. In addition, one can use OMC to study second class currents [12].

The calculation process demands fixing the single contact unknown LEC, which determines the weak axial current. We do this by reproducing the experimental triton half-life, for various cutoff values for the chiral current. The results for the nuclear matrix element of the muon capture process show a 9% effect due to the MEC contribution, and a small 0.3% effect due to the cutoff dependence. It is worthwhile noting that the relative contribution of the MEC to this process is almost three times bigger than the MEC contribution to the triton half life. An
extremely weak cutoff dependence shows that the essential physics is captured in the chiral EFT operators.

The final prediction for the capture rate is

$$\Gamma = 1499(2)A(3)_{NM}(5)t(6)_{RC} \text{Hz},$$

where the first error is due to the Heavy Baryon chiral perturbation theory (HBχPT) cutoff, the second is due to uncertainties in the extrapolation of the form factors to finite momentum transfer, and in the choice of the specific nuclear model, the third error is related to the uncertainty in the triton half life, and the last error is due to theoretical uncertainty in the electroweak radiative corrections calculated for nuclei [13]. This sums to a total error estimate of about 1%, which overlaps with the uncertainty.

The conservative error estimation still allows rather interesting conclusions. First, one notices that the calculated capture rate agrees with the experimental measurement $\Gamma(\mu^- +^3\text{He} \to \nu\mu + ^3\text{H})_{\text{exp}} = 1496(4) \text{Hz}$. Thus, one concludes that EFT accurately predicts the capture rate.

However, the most interesting result concerns the weak form factors of the nucleon. In order to constrain the induced pseudoscalar and second class form factors, we take the following approach. In each case, we set all the other form factors to their nominal value, and change this form factor in a way which keeps an overlap between the experimental rate and the theoretically allowed rate. The nominal value of the form factor is set to reproduce the experimental measurement. The resulting constraint on the induced pseudoscalar form factor is:

$$g_P(q^2 = -0.954m^2_\mu) = 8.13 \pm 0.6,$$

in very good agreement with the HBχPT prediction of Ref. [14].

A second conclusion concerns the contribution of second class currents. While the current constraint on the axial G-parity breaking term is rather weak, this calculation puts the tightest limit on CVC, constraining it to $m_F/S/F_V = (0.5 \pm 2) \times 10^{-4}$, which is consistent with CVC. One notes that this is already close to the regime of CVC breaking, predicted by χPT [14].

A recent calculation in the framework of the previous section, i.e., taking currents and forces from the same chiral EFT Lagrangian, has reached similar results [15].

5. The in-medium behavior of the axial constant
Surprisingly, key aspects of β decays remain a puzzle. In particular, when calculations of Gamow-Teller (GT) transitions of the spin–isospin-lowering operator $g_A \sigma \tau^-$ are confronted with experiment, some degree of renormalization, or “quenching” $q$, of the axial coupling $g^A_{\text{eff}} = q g_A$ is needed. Compared to the single-nucleon value $g_A = 1.2695(29)$, the GT term seems to be weaker in nuclei. This was first conjectured in studies of β-decay rates, with a typical $q \approx 0.75$ in many-body calculations [16, 17]. In view of the significant effect on weak reaction rates, it is no surprise that this suppression has been the target of many theoretical works. It is also a major uncertainty for 0νββ decay nuclear matrix elements (NMEs), which probe larger momentum transfers of the order of the pion mass, $p \sim m_\pi$, where the renormalization could be different. Here we revisit this puzzle based on chiral EFT currents.

The lightest nucleus that undergoes a β-decay is the triton. However, the theory cannot be checked in the triton since its half-life is used to calibrate the strength of the MEC. The lightest nucleus that can provide a test to the theory is thus $^6\text{He}$ ($J^\pi = 0^+$), an unstable nucleus, which undergoes a β decay with a half-life $\tau_{1/2} = 806.7 \pm 1.5$ msec to the ground state of $^6\text{Li}$ ($J^\pi = 1^+$) [18]. As this is a 6-body problem, we use a soft nuclear potential, JISP16. This potential leads to an underbinding of about 80keV for the $^3\text{He}$ and 120keV for the triton, where we calibrate the one unknown LEC determining the current. One calculates binding energies of 28.70MeV for $^6\text{He}$ and 31.46MeV for $^6\text{Li}$. These results underestimate the experimental energies by about
Incorporating the χPT based contributions to the weak-current one can calculate the full $^6$He-$^6$Li GT matrix-element, as a function of the cutoff imposed in the weak current:

$$|\text{GT}[^6\text{He}]|_{\text{theo}} = 2.198(1)\Lambda(2)N(4)I[^6\text{He}]= 2.198 \pm 0.007.$$  \hfill (6)

The first error is the cutoff variation dependence, the second is numerical, the third is due to uncertainties in the triton half-life, and the last is due to uncertainties in $g_A$. This should be compared to the experimental matrix-element $|\text{GT}[^6\text{He}]|_{\text{exp}} = 2.161 \pm 0.005$. Thus, the theory overpredicts GT by about 1.7%, which is reasonable considering the approximation in using a phenomenological, pure 2-body, potential.

For heavier nuclei a microscopic calculation is still out of reach. We thus study the impact of chiral 2b currents in nuclei at the normal-ordered 1b level by summing the second nucleon over occupied states in a spin and isospin symmetric reference state or core. Taking a Fermi gas approximation for the core and neglecting tensor-like terms ($\mathbf{k} \cdot \mathbf{\sigma} \mathbf{k} - \frac{1}{3}k^2\mathbf{\sigma}\mathbf{\tau}^-$), we obtain the normal-ordered 1b current [19]:

$$J_{\text{eff}}^{1,2b} = -g_A\mathbf{\sigma}_i\tau_i^- \frac{\rho}{F^2} \left[ \frac{c_D}{g_A A^{\frac{1}{2}}} \mathbf{p}^2 + \frac{2}{3} c_3 \frac{4m^2}{m_p^2} + I(\rho, P) \left( \frac{1}{3} (2c_4 - c_3) + \frac{1}{6m} \right) \right],$$  \hfill (7)

where $\rho = 2k_F^2/(3\pi^2)$ is the density of the reference state, $k_F$ the corresponding Fermi momentum, and $I(\rho, P)$ is due to the summation in the exchange term. The effective 1b current $J_{\text{eff}}^{1,2b}$ only contributes to the GT operator and can be included as a correction to the $g_A(p^2)$ part of the 1b current, Eq. (2). This demonstrates that chiral 2b currents naturally contribute to the quenching of GT transitions. Compared to light nuclei, their contributions are amplified...
because of the larger nucleon momenta. A quenching of $p \approx 0$ GT transitions is predicted, as well as the momentum transfer dependence, as seen in Fig. 3.

This formalism can be used to evolve the chiral EFT understanding various phenomena, including the prediction of $0\nu\beta\beta$ decay rates. Indeed, such a calculation of the $0\nu\beta\beta$ decay operator based on chiral EFT currents shows that 2b contributions to the nuclear matrix elements are significant, and range from $-35\%$ to $10\%$ [19]. Another recent application, which will not be covered in this contribution, is to the scattering of a weakly interacting massive particle (WIMP) candidate on nuclei [20].

6. Summary
We showed that chiral EFT enables parameter free calculations of electro-weak Gamow Teller transitions, that can be used in various fields: from constraining the nuclear forces, through a deeper understanding of in-medium nuclear effects, and up to predicting weak reactions for the use of experiments studying the limits of the Standard Model.

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