International Conference on Industrial Engineering, ICIE 2016

Designing of Optimal Annular Nozzles with Multiphase Flows

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Abstract

The problem of profiling of optimal (i.e. securing maximum value of thrust) configurations for annular nozzles of the external expansion with a multiphase medium is considered. Direct methods of a calculus of variations are applied to the solution of a problem in view. The variational problem of search of an optimal annular nozzle configuration is reduced to a problem of a nonlinear programming. Optimal configurations of annular nozzles for different working mediums and operating conditions were built as a result of optimization. The result may be used for searching of the optimal configuration of different gas-dynamic devices.

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Peer-review under responsibility of the organizing committee of ICIE 2016

Keywords: optimal annular nozzles; multiphase flows; variational problem.

1. Introduction

One of the major problems occurred at designing of the annular nozzles is problem of profiling optimal (i.e. securing maximum value of thrust) configurations of annular nozzle with a multiphase working medium [1]. The configuration of this nozzle presented in Fig. 1. Search of an optimal configuration of an annular nozzle is carried out by the solution of a variational problem which can be realized by means of different methods, for example, by a method of a control contour or method of Lagrangian multipliers. However, as noted by A.N. Kraiko [2], in some cases application of such methods is very complex, laborious and does not guarantee success. In this connection to one of possible decisions can become application of direct methods of a calculus of variations [1, 3]. In variational problems of gas dynamics the essence of these methods consists in the following.

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Each smooth section required optimal generatrix nozzles is approximated by linear combinations of some known functions with beforehand unknowns coefficients. Existence of additional isoperimetrical conditions of geometrical character, defined the given restrictions on geometry of the optimized nozzle, imposes some connections on coefficients, reducing number of free parameters. All free parameters indicated as \( c_1, \ldots, c_n \), generate \( n \)-dimensional space in which in direct methods the minimum or maximum of the optimized functional \( J \) being a function \( c_1, \ldots, c_n \) is searched. Transition from a functional to a function represents effect of replacement of the smooth sections generatrix final combinations of approximating functions.

Considered variational problem at which statement the number of optimized parameters includes geometrical characteristics of a nozzle (thus parameters of a condensed phase and isentropic index \( k \), and also the have available pressure differential in the nozzle \( \Delta P \) are considered as stationary values), is reduced to a problem of nonlinear programming [3]. Basic elements of the given approach are direct calculations of a field of flow and a method of search of an extremum of functions of many variables that makes it applicable to all gas dynamic problems for which methods of calculation of a field of flow including for annular nozzles with multiphase flows are known.

2. Main part

2.1. Procedure of optimization and statement of variation problem

The procedure of optimization of a geometrical configuration of an annular nozzle represents a combination of analytical methods of construction of an optimized functional and the representation geometrical profile of an annular nozzle, with methods of search of the extremum of function being a function several variables, and direct calculations of a field of flow with the help of computational methods. Profiling optimal on trust annular nozzle on is carried out in conditions of the set restrictions on its geometrical characteristics.

The arbitrary stationary axisymmetric flow of gas in the nozzle is considered. Let it is required to construct a nozzle contour \( y = \xi(x) \) supplying extremum to a functional:

\[
J = \int_A^B \Phi(x, \xi, \xi', u_1, \ldots, u_n) \, dx
\]

where \( \Phi \) – a known function, \( \{u_i\} (i = 1, \ldots, n) \) – a system of the functions, satisfying to flow equations; \( A \) and \( B \) - initial and final points of a nozzle contour. The stroke designated derivatives on \( x \) along a nozzle contour.

Following isoperimetric conditions are considered:

\[
K_j = \int_A^B G_j(x, \xi, \xi') \, dx, \quad j = 1, \ldots, m
\]
where $G_j(x, \xi, \xi')$ and $K_j$ - known functions and constants.

The required optimal contour can be determined as follows:

$$y'(x) = \xi_o(x) + \Delta \xi(x), \ y(y_o) = y_o,$$  \hspace{1cm} (3)

where $\xi_o(x)$ - known function, $x_o$ - the initial point of a contour, value $\Delta \xi(x)$ is approximated by a segment of series:

$$\Delta \xi(x) = \sum_{k=0}^{l} c_k \phi_k(x),$$  \hspace{1cm} (4)

where $\{\phi_k\}$ - a system linearly independent basis functions, $c_k$ - coefficients. Then for a contour, the given as (4), we shall have:

$$J = J(c_1,\ldots,c_r), \ r < l,$$  \hspace{1cm} (5)

and $r$ it is selected so that $l-r$ coefficients in (3) it was possible to satisfy to isoperimetric conditions (2).

Thus, the variational problem of search of optimal geometry of a nozzle contour under the given conditions is reduced to a problem of search of a point $(c_1,\ldots,c_r)$ in which the value of a function $J$ is extreme. Methods of nonlinear programming are applied to a searching of extremum of function. Components of a gradient of a function $J$ are calculated under formulas:

$$\frac{\partial J}{\partial c_k} \approx \frac{J(c_1,\ldots,c_k + \Delta c_k,\ldots,c_r) - J(c_1,\ldots,c_k,\ldots,c_r)}{\Delta c_k},$$  \hspace{1cm} (6)

$$\frac{\partial J}{\partial c_k} \approx \frac{J(c_1,\ldots,c_k - \Delta c_k,\ldots,c_r) - J(c_1,\ldots,c_k,\ldots,c_r)}{2\Delta c_k},$$  \hspace{1cm} (7)

in which the function $J$ is determined after calculation of a field of flow for a nozzle contour, the given as (3).

In this research, such approach is applied to search of optimal annular nozzles of different configurations with the multiphase working medium, profiled in conditions of different restrictions on overall dimensions. The main attention is given to a problem of profiling of an optimal configuration of an annular nozzle of external expansion (without an external contour in supersonic field of nozzle) with a shortcut center body (Fig. 2), implementing maximum thrust at the given restrictions on geometrical characteristics of the nozzle.

As of the main criterion of optimization, the thrust coefficient is used:

$$K_T = \frac{R}{F_t \cdot P_o},$$  \hspace{1cm} (8)

where $R$ – thrust of the nozzle, $F_t$ – the square of throat nozzle, $P_o$ – total pressure on an entrance in the nozzle.

At statement or variational problem for the investigated nozzle, the thrust coefficient can be recorded as:

$$K_T = K_{Tr} + \alpha \int_A P(x, y(x); \Theta, dF + K_{T\Omega}$$  \hspace{1cm} (9)

where $K_{Tr}$ – a thrust coefficient, created by a flow in throat nozzle, $A, B$ – initial and final points of a profile of a
center body, $\Theta_*$ – angle of inclination of a plane of annular throat to a axis of the nozzle, $P(x,y(x);\Theta_*)$ – pressure distribution along the center body, $\alpha$ – dimension a multiplier, $K_{T,0}$ – a thrust coefficient, created of butt of a shortcut center body.

![Fig. 2. A configuration of an annular nozzle for optimization](image)

The integration is carried out on the area of a projection of a surface of a center body on a plane $X = \text{const}$.

The contour of a center body is divided into segments with some allocation of clusters and the smooth fulfilment between them. Therefore, the variational problem of a determination of a function $y(x)$ and the angle $\Theta_*$, ensuring a maximum of a thrust coefficient in conditions of the given dimensional restrictions, is reduced to a problem of a searching of extremum of a function several variables:

$$K_T = f\left(\Theta_*, c_1, c_2, ..., c_n\right),$$  \hspace{1cm} (10)

where $c_i$ – coefficients, defined a shape of a center body, which can be solved by methods of nonlinear programming.

Direct calculations of a field of flow are carried out for a multiphase medium with monodisperse and polydisperse condensed phase.

2.2. Mathematical modelling of flows in annular nozzles

Two-phase monodisperse flow in an annular nozzle is described by equations set of a axisymmetric flow of a two-phase mixture in the integral form \([1,4]\), permitting to realize the «transparent» calculation without preliminary selection of discontinuity in computational field.

Calculation of a field of flow is carried out by the relaxation method with use of the computational algorithm base on the scheme of Godunov–Kolgan \([1,5-7]\). Boundary conditions of set of equations are received as follows:

on rigid walls the impermeability condition $n_v = 0$ is accepted;

on input subsonic boundary: entropy $S=\text{const}$, total enthalpy $H_o = \text{const}$; allocation of angle of inclination of velocity vector $\Theta_* = \Theta_*(x, y)$.

The output boundary is selected so that normal to boundary the component of a velocity would be supersonic, in this case statements of boundary conditions it is not required.

The two-phase flow is characterized by essential interaction of gas and condensed phases among themselves. Calculation of interaction of particles of a condensed phase with gas is carried out with the help of the exchange terms of equations considering an exchange of an impulse and energy between considered phases. For the description of exchange terms in a two-phase monodisperse medium the ratios offered in research efforts \([4,8]\) for flows in Laval nozzles were used. According to this approach, parameters of viscous interaction of gas and particles, and also heat exchange parameters between gas and particles are considered. For gas and particles on input boundary, the balance condition is realized.
Flow in an annular nozzle is «mixed», that is in various subareas of the solution of a problem of an equation of a gas phase can belong to the elliptical or hyperbolic type that results in necessity to apply different methods for the solution of a problem in each of them. Complexity of the solution of a problem is caused by that boundaries of the indicated subareas are beforehand unknown. Therefore, to calculate parameters of the «mixed» flow rationally also to apply a relaxation method, as well as at modeling of flows of perfect gas.

More difficult case is application in annular nozzles as a working medium of a multiphase medium. Within an offered approach, for mathematical modeling of flows of a multiphase medium in annular nozzles the equations of continuous model recorded for the discrete distribution function (original positions of the applied discrete approach for Laval nozzles are shown in [4,9]) were used.

Parameters subsonic and transonic areas of flow are calculated with the help of a relaxation method with use of the scheme of Godunov–Kolgan, in supersonic area for calculation of parameters of a gas phase Ivanov–Kraiko–Mikhailov modified mid-flight scheme is used. Calculation of interaction of particles of a condensed phase with gas was carried out with the help of the exchange members considering exchange of an impulse and energy between considered phases.

At calculation of a supersonic flow as initial data gas parameters on a sound surface are used. Such parameters can be set on a flat annular sound surface (coinciding with the plane of annular throat of a nozzle) with a uniform allocation of velocity vector, on value equal speeds of sonic, in this case on an acoustical surface it is possible to consider parameters invariable at change of width of minimum annular throat and an angle of its inclination to a nozzle axis. Other way of the representation of initial data for calculation of parameters of supersonic area of flow assumes calculation of parameters subsonic and transonic areas of flow with definition of the form of a sound surface and the representation of the distributed parameters on this surface. In this case changes of geometrical parameters of an annular nozzle during optimization results and to change of geometry of a subsonic part of the nozzle and annular throat that, in turn, results in necessity of recalculation of parameters subsonic and transonic areas of flow and essentially complicates a procedure of optimization, but increases accuracy of the received solution.

2.3. Results of optimization of annular nozzle’s geometrical configuration

For search of an extremum of criterion function various methods [10], such as a cyclical alternating-variable descent method, Rozenbrok’s method (versions of a method with discrete step and with minimization on a direction), allowing to carry out multidimensional search without use derivatives and possessing a sufficient velocity of convergence, a steepest descent method using derivatives at definition of directions of search are used.

Optimization of a geometrical configuration of an annular nozzle with the account to subsonic and transonic areas of flow was carried out for an initial configuration of an annular nozzle with following parameters (dimensionless, divided by value $R_{a}$): relative radius of top point of annular throat $R_{a} = 1$; relative area of annular throat $A_{a} = 0.65$; isentropic exponent $k = 1.25$; length of the nozzle $L_{c} = R_{a}$; pressure differential in the nozzle $p_{a}/p_{n} = 100$.

The profile of a center body was approximated by polynomials of Chebyshev which operational effectiveness is shown in research efforts [3,10] and power-mode polynomials which efficiency for approximating a nozzle contour in a problem of optimization is shown in [11].

On an optimized a profile dimensional restriction on length $L_{c}$ and to radius $R_{c}$ were imposed. Restrictions on ordinate of a final point of a profile of a center body were not imposed.

On Fig. 3 results of optimization of geometry of an annular nozzle with the account and without taking into account subsonic and transonic areas of flow are presented.

The received optimal profile of a center body locates a slightly below, than the profile of a nozzle, received at carrying out direct calculations of a field of flow from a flat sonic surface, angle of inclination of the plane of annular throat was increased. The value $K_{T}$ of the nozzle received with the account subsonic and transonic areas of flow, appeared above $K_{T}$, received without taking into account this area of flow on $\sim 1.2 \%$ ($K_{T} = 1.531$ against $K_{T} = 1.512$), that makes enough considerable change of propulsive performance characteristics. An interesting feature of
received result is that inequality of a flow in throat of the nozzle results in increase of its propulsive performance characteristics.

![Figure 3: The configurations of annular nozzles received under different conditions of profiling](image)

Results of optimization of an initial profile of an annular nozzle and results of its optimization for perfect gas and for a multiphase polydisperse medium with a mass part of a condensed phase $z = 0.3$ are presented on Fig. 4.

In this case, parameters of an initial profile of the center body were equal to both considered variational problems. Results of a mathematical modeling showed the considerable effect of a condensed phase on an optimized profile of the center body of a nozzle, thus optimal for perfect gas and the multiphase medium annular nozzles considerably differ a profile of a end segment of a center body.

![Figure 4: Comparison optimal nozzles for different working mediums (perfect gas and a multiphase medium)](image)

### 3. Conclusion

Optimal configurations of annular nozzles with two-phase monodisperse and multiphase polydisperse working mediums are profiled as a result of the solution of put variational problems and carried out by means of a direct method of optimization of annular nozzles' geometry. Comparison of optimal annular nozzles possesses by higher thrust with two-phase and multiphase-working mediums with an optimal annular nozzle for perfect gas is carried out.

Results of mathematical modeling are received with application of high-efficiency calculations using a supercomputer «TORNADO» of South Ural State University (national research university) with processing power up to 473,6 TFlops.
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