Research and Simulation Analysis of Automobile Vibration System

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Abstract. Based on the analysis of the vehicle vibration system and considering the effects of
the body's roll, pitch and seat on the vibration, this paper establishes an 11-degree-of-freedom
vibration model of the vehicle. Then, with the help of MATLAB/SIMULIK, the vertical
vibration of the vehicle is simulated, which considers the asymmetry of the road surface
excitation and the lag effect of the rear wheel input. After optimizing the suspension parameters,
this paper compares the seat acceleration and the dynamic deflection of the suspension before
and after optimization. The results show that the model established in this paper has a certain
effect on improving the riding comfort of the vehicle.

1. Introduction

When the vehicle is moving, the unevenness of the road surface and the vehicle components such as
wheels and transmission system will excite the vibration of the vehicle, and then cause the vibration of
the human-seat system through the suspension system. The research on vibration problem of automobile
system under random road surface excitation is an important basis for improving automobile riding
comfort[1].

In this paper, based on the study of the automobile vibration system, the 11-degree-of-freedom
dynamic model of the automobile vibration system is established, and the road surface excitation is used
as the basic input of the vibration system. The final outputs of the system are the rms values of the seat
acceleration and the dynamic deflection of the suspension. And they are used as the the riding comfort
evaluation indexes to optimize the suspension parameters.

2. Eleven-degree-of-freedom vehicle vibration model

The commonly used model of the automobile vibration system is the 7-degree-of-freedom vibration
model, which considers the body's roll and pitch movements, but there are still deficiencies: It does not
consider the effect of the seat on the vibration acceleration. In order to more fully reflect the vertical
vibration of the vehicle, this paper established an 11-degree-of-freedom vehicle vibration model. The
input of the entire vehicle vibration model is road surface excitation. The left-right asymmetry of the
road surface excitation and the hysteresis of the rear wheel input are considered. In addition, on the basis
of considering the impact of the vehicle's roll and pitch motion on the body vibration, the role of the seat
is also considered, which more fully and truly reflects the actual situation of vehicle vibration[2]. The
three-dimensional model of 11-degree-of-freedom vehicle vibration is shown in Figure 1.
The meanings of the symbols in the figure are as follows: \(m_i, k_i, c_i\) \((i=1,2,3,4)\) respectively represent the mass of four seats and the driver or occupants, seat stiffness, seat damping. \(m_s, J_b, l_b\) respectively represent the mass of the vehicle body, the moment of inertia in the direction \(\theta\) (pitch inertia), and the moment of inertia in the direction \(\phi\) (roll inertia). \(m_i, k_i, c_i\) \((i=1,2,3,4)\) respectively represent the unsprung mass, suspension stiffness and damping. \(b_i, \theta, \phi\) respectively represent the vertical displacement of the body, the body pitch angle, the body roll angle. \(z_s(i=1,2,3,4)\) represents the vertical displacement of the four seats. \(z_m(i=1,2,3,4)\) represents the vertical displacement of the four connection points of the body and the seats. \(z_n, \theta, \phi\) respectively represent the vertical displacement of the body, the body pitch angle, the body roll angle. \(z_s(i=1,2,3,4)\) represents the vertical displacement of the four connection points of the body and the suspension. \(z_r(i=1,2,3,4)\) represents the vertical displacement of 4 wheels, and \(z_r(i=1,2,3,4)\) represents the road roughness input of 4 wheels.

According to the geometric relationship of the above figure,
\[
\begin{align*}
z_{b1} &= z_b - l_f \theta + b_f \phi, \quad z_{n1} = z_b - l_i \theta + b_i \phi \\
z_{b2} &= z_b - l_f \theta - b_f \phi, \quad z_{n2} = z_b - l_i \theta - b_i \phi \\
z_{b3} &= z_b + l_f \theta + b_f \phi, \quad z_{n3} = z_b + l_i \theta + b_i \phi \\
z_{b4} &= z_b + l_f \theta - b_f \phi, \quad z_{n4} = z_b + l_i \theta - b_i \phi
\end{align*}
\]  \(1\)

According to the suspension motion characteristics and Newton's second law, the matrix form of the system's motion equation can be written:
\[
MZ + CZ + KZ = M\ddot{z} + C\dot{z} + Kz, \quad (2)
\]
\[
M = \text{diag}\{m_1, m_2, m_3, m_4, m_b, J_b, l_b, m_s, m_s, m_s\}, \quad (3)
\]
\[ C = \begin{bmatrix} C_1 & C_2 \\ \\ C_3 & C_4 \end{bmatrix}, \quad K = \begin{bmatrix} K_1 & K_2 \\ \\ K_3 & K_4 \end{bmatrix}, \]

\[ C_1 = \begin{bmatrix} -c_{11} & l_{c_{12}} & -b_{c_{13}} & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 \\ 0 & 0 & c_{33} & 0 \\ 0 & 0 & 0 & c_{44} \end{bmatrix}, \quad K_1 = \begin{bmatrix} k_{11} & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 \\ 0 & 0 & k_{33} & 0 \\ 0 & 0 & 0 & k_{44} \end{bmatrix}. \]

\[ C_2 = \begin{bmatrix} -c_{12} & l_{c_{12}} & -b_{c_{13}} & 0 & 0 & 0 & 0 \\ -c_{13} & -l_{c_{13}} & -b_{c_{13}} & 0 & 0 & 0 & 0 \\ -c_{14} & -l_{c_{14}} & -b_{c_{13}} & b_{c_{14}} & 0 & 0 & 0 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -k_{12} & l_{k_{12}} & b_{k_{13}} & 0 & 0 & 0 & 0 \\ -k_{13} & -l_{k_{13}} & -b_{k_{13}} & 0 & 0 & 0 & 0 \\ -k_{14} & -l_{k_{14}} & b_{k_{14}} & b_{k_{14}} & 0 & 0 & 0 \end{bmatrix}. \]

\[ C_3 = \begin{bmatrix} -c_{11} & -c_{12} & -c_{13} & -c_{14} \\ l_{c_{11}} & l_{c_{12}} & -l_{c_{13}} & -l_{c_{14}} \\ -b_{c_{11}} & b_{c_{12}} & -b_{c_{13}} & b_{c_{14}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad K_3 = \begin{bmatrix} -k_{11} & -k_{12} & -k_{13} & -k_{14} \\ l_{k_{11}} & l_{k_{12}} & -l_{k_{13}} & -l_{k_{14}} \\ -b_{k_{11}} & b_{k_{12}} & -b_{k_{13}} & b_{k_{14}} \\ 0 & 0 & 0 & 0 \end{bmatrix}. \]

\[ C_4 = \begin{bmatrix} 2 (c_1 + c_2) & 2 (l_{c_{12}} - l_{c_{12}}) & 0 & -c_1 & -c_2 & -c_3 & -c_4 \\ 2 (l_{c_{12}} - l_{c_{12}}) & 2 (l_{c_{13}} - l_{c_{13}}) & 0 & l_{c_{11}} & l_{c_{12}} & -l_{c_{13}} & -l_{c_{14}} \end{bmatrix}. \]

\[ K_4 = \begin{bmatrix} 2(k_1 + k_2) & 2(l_{k_{12}} - l_{k_{12}}) & 0 & -k_1 & -k_2 & -k_3 & -k_4 \\ 2(l_{k_{12}} - l_{k_{12}}) & 2(l_{k_{13}} + l_{k_{13}}) & 0 & l_{k_{11}} & l_{k_{12}} & -l_{k_{13}} & -l_{k_{14}} \end{bmatrix}. \]
3. Simulation analysis and parameters optimization

3.1. Establishment of the simulation model of "road-vehicle" vibration system

When the vehicle is moving, the road roughness is the basic input of the vehicle vibration system. In this paper, the white noise module in SIMULINK is used to generate road excitation[3]. According to random vibration theory,

$$q(t) = -2\pi n_1 v q(t) + 2\pi n_0 \sqrt{G_q(n_0)} w(t)$$

(11)

Where $$n_1$$ is the lower cut-off spatial frequency, the value is 0.011 m\(^{-1}\). $$v$$ is the moving speed of the vehicle. This paper uses two values of 60km/h and 120km/h. $$q(t)$$ is the random elevation displacement of the road surface, m. In the vehicle vibration model established in this paper, $$z_r(t)$$ is used to represent this variable. $$n_0$$ is the reference spatial frequency, and its value is 0.1 m\(^{-1}\). $$w(t)$$ is the Gauss white noise with an average value of 0. $$G_q(n_0)$$ is called pavement unevenness coefficient, which is the pavement power spectral density at the reference spatial frequency $$n_0$$. Pavement unevenness is graded according to this parameter. In this paper, grade B pavement is selected. According to relevant standards, its value is \(64 \times 10^{-6}\) m\(^3\).

Considering the asymmetry of the left and right road surfaces, this paper established two road surface models, which are used as the inputs of the left and right wheels of the vehicle. Besides, compared with the front wheels, there is a certain lag in the road surface excitation of the rear wheels. The lag time is:

$$\Delta t = \frac{l_f + l_r}{v}$$

(12)

According to equations (11) and (12), the following pavement models are established in SIMULINK:
Where $Gain1 = Gain10 = 2\pi n_0 \sqrt{G_q (n_0) v}$, $Gain2 = Gain11 = -2\pi n_0 v$.

After the establishment of pavement model, the equation (3) needs to be transformed into the form of state equation to facilitate the construction of the simulation model. This paper ignores the tire damping[4]. After transformation, the equation is:

$$\ddot{Z} = -M^{-1}C\dot{Z} - M^{-1}KZ + M^{-1}K_rZ.$$  \hspace{1cm} (13)

According to equation (14), a simulation model of an 11-degree-of-freedom vehicle vibration system can be established in SIMULINK, as shown in the following figure:

3.2. Simulation analysis and parameters optimization at 60km/h
The parameters of a vehicle are shown in Table 1.
Table 1. Parameters of a vehicle

| $m_b$ (kg) | $m_l$ (kg) | $m_3$ (kg) | $m_{sl}$ (kg) | $m_{s3}$ (kg) | $J_b$ (kg·m²) | $I_b$ (kg·m²) | $k_l$ (kN/m) |
|------------|------------|------------|---------------|---------------|---------------|---------------|--------------|
| 1680       | 40.5       | 45.4       | 60            | 80            | 2440          | 380           | 19           |

| $k_3$ (kN/m) | $k_{s1}$ (kN/m) | $k_{s3}$ (kN/m) | $c_1$ (kN·s/m) | $c_3$ (kN·s/m) | $c_{s1}$ (kN·s/m) | $c_{s3}$ (kN·s/m) | $k_{i1}$ (kN/m) | $k_{i3}$ (kN/m) | $l_f$ (m) |
|--------------|-----------------|-----------------|----------------|----------------|-------------------|-------------------|----------------|----------------|----------|
| 24           | 7               | 9               | 1.5            | 2              | 0.7               | 192               |               |               | 1.25     |

| $l_r$ (m) | $b_r$ (m) | $b_l$ (m) | $l_1$ (m) | $l_2$ (m) | $b_1$ (m) | $b_2$ (m) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1.51      | 0.74      | 0.74      | 0.33      | 0.54      | 0.54      | 0.54      |

In this paper, $k_1 = k_2, k_3 = k_4, c_1 = c_2, c_3 = c_4, k_{s1} = k_{s2}, k_{s3} = k_{s4}, c_{s1} = c_{s2} = c_{s3} = c_{s4}, m_l = m_2, m_3 = m_{s2}, k_i = k_{i2} = k_{i3} = k_{i4}.

Based on the parameters given by Table 1, this paper optimizes the stiffness of the suspension of the vehicle to reduce the vibration of the seats and improve the riding comfort. The objective function is

$$ Y = \min (\ddot{Z}) $$  \hspace{1cm} (14)

The design variable is

$$ X = [k_1, k_3]^T $$  \hspace{1cm} (15)

In order to improve ride comfort, the frequency of the front suspension is usually lower than that of the rear suspension. At the same time, the static deflection of the suspension is between 0.10m and 0.35m. Therefore, the constraints[5] are

$$ \frac{k_i}{k_3} \leq \frac{m_{sl}}{m_{s3}} \leq \frac{m_{sl}^g}{0.35} \leq k_i \leq \frac{m_{sl}^g}{0.10} $$  \hspace{1cm} (16)

$$ \frac{m_{s3}^g}{0.35} \leq k_i \leq \frac{m_{s3}^g}{0.10} $$

Where $m_{sl}, m_{s3}$ represent the sprung masses of the left front and left rear suspensions respectively, which can be calculated as follows.

$$ m_{sl} = \frac{1}{2} \left[ m_b \frac{I_f}{I_f + I_r} + \left( \sum_{i=1}^{k} m_{si} \right) \frac{l_f + l_i}{l_f + l_i} + \left( \sum_{i=1}^{k} m_{si} \right) \frac{l_i - l_f}{l_f + l_i} \right] $$

$$ m_{s3} = \frac{1}{2} \left[ m_b \frac{I_f}{I_f + l_i} + \left( \sum_{i=1}^{k} m_{si} \right) \frac{l_f - l_i}{l_f + l_i} + \left( \sum_{i=1}^{k} m_{si} \right) \frac{l_i + l_f}{l_f + l_i} \right] $$  \hspace{1cm} (17)

According to the above requirements, this paper sets the moving speed of the vehicle to 60km/h and simulates the vehicle model to obtain the curves of the seat vertical acceleration and suspension dynamic deflection with time. The results of before and after optimization are compared. The simulation time is 50s with 10 ~ 40s for plotting.
At a speed of 60km/h, the rms values of seat acceleration and dynamic deflection of each suspension are shown in Table 2.
Table 2. Comparison of each output before and after optimization at 60km/h

| Stiffness (kN/m) | Driver seat acceleration (m/s²) | Right rear seat acceleration (m/s²) | Left front deflection (mm) | Right front deflection (mm) | Left rear deflection (mm) | Right rear deflection (mm) |
|------------------|---------------------------------|-------------------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| $k_1 = 19$       | 0.4254                          | 0.4835                              | 7.2                       | 7.0                       | 5.4                       | 5.1                       |
| $k_1 = 24$       |                                 |                                     |                           |                           |                           |                           |
| $k_1 = 15$       | 0.3738                          | 0.4209                              | 7.5                       | 7.1                       | 5.4                       | 5.2                       |
| $k_1 = 20$       |                                 |                                     |                           |                           |                           |                           |

As can be seen from Table 2, on the basis of almost no change in the dynamic deflection of the suspension, the optimized driver seat acceleration and right rear passenger seat acceleration are significantly reduced. Therefore, the optimization effectively improves the riding comfort of the vehicle.

3.3. Simulation analysis and parameters optimization at 120km/h

In order to verify the correctness of the model and study the impact of different moving speeds on vehicle vibration, this paper also makes a simulation analysis with a moving speed of 120km/h. The simulation conditions other than speed are the same as Section 3.2, and the results are as follows:

Figure 10. Comparison of driver seat acceleration before and after optimization.
Figure 11. Comparison of right rear seat acceleration before and after optimization.
Figure 12. Comparison of left front dynamic deflection before and after optimization.
Figure 13. Comparison of right front dynamic deflection before and after optimization.
At a speed of 120km/h, the rms values of seat acceleration and dynamic deflection of each suspension are shown in Table 3.

| Stiffness (kN/m) | Driver seat acceleration (m/s²) | Right rear seat acceleration (m/s²) | Left front deflection (mm) | Right front deflection (mm) | Left rear deflection (mm) | Right rear deflection (mm) |
|-----------------|-------------------------------|------------------------------------|---------------------------|---------------------------|-------------------------|--------------------------|
| $k_1$ = 19      | 0.5777                        | 0.6638                             | 9.7                       | 9.5                       | 7.3                     | 7.0                      |
| $k_1$ = 24      | 0.5066                        | 0.5773                             | 10.0                      | 9.6                       | 7.4                     | 7.0                      |
| $k_1$ = 15      |                               |                                    |                           |                           |                         |                          |
| $k_1$ = 20      |                               |                                    |                           |                           |                         |                          |

It can be seen from Table 2 and Table 3 that the simulation results of 120km/h and 60km/h show the same regular pattern, so the model established in this paper is correct. At the same time, as the moving speed increases, the seat acceleration also increases, and the riding comfort of the vehicle decreases.

4. Conclusion
The 11-degree-of-freedom vehicle vibration model established in this paper considers not only the effect of seats on vibration, but also the left-right asymmetry of the road surface and the input hysteresis of the rear wheels. The model more fully and truly reflects the actual situation of vehicle vibration. With the help of MATLAB/SIMULINK, this paper builds the simulation model and optimizes the suspension parameters, and obtains the comparison charts of the vehicle vibration system simulation curves before and after optimization. This has a certain effect on adjusting parameters in the design stage and improving vehicle riding comfort for vehicle’s designers.

References
[1] Dixon, J. (2009) Suspension Geometry and Computation. A John Wiley and Sons, Ltd, Publication, New Jersey.
[2] Li, C.F., Wang, D.G., Liu, J., Wen, B.C. (2009) Study and Application of 11-DOF Nonlinear Ride Comfort Model for Vehicle. Journal of Northeastern University(Natural Science), 30(06): 857-860.
[3] Chen, J.P., Chen, W.W., Zhu, H., Zhu, M.F. (2010) Modeling and Simulation on Stochastic Road Surface Irregularity Based on Matlab/SIMULINK. Transactions of the Chinese Society for Agricultural Machinery, 41(03): 11-15.

[4] Wu, L.T. (2015) Modeling and Simulating of Automobile Ride Comfort. Journal of Hebei United University (Natural Science Edition), 37(02): 127-132.

[5] Yang, J., Dong, M.M. (2018) Research on Vibration of Automobile Suspension Design. EDP Sciences, 153.