Elastic $\alpha$ transfer in the $^{16}\text{O}+^{12}\text{C}$ scattering and its impact on the nuclear rainbow

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Abstract. Elastic $^{16}\text{O}+^{12}\text{C}$ scattering is known to exhibit the nuclear rainbow pattern at incident energies $E_{\text{lab}} \gtrsim 200$ MeV, with the Airy structure of the far-side scattering cross section clearly seen at medium and large angles. Such a rainbow pattern is well described by the deep real optical potential (OP) given by the double-folding model (DFM). At lower energies, the extensive elastic $^{16}\text{O}+^{12}\text{C}$ scattering data show consistently that the nuclear rainbow pattern at backward angles is deteriorated by an oscillating enhancement of elastic cross section that is difficult to describe in the conventional optical model (OM). Given a significant $\alpha$ spectroscopic factor predicted for the dissociation $^{16}\text{O} \rightarrow \alpha+^{12}\text{C}$ by the shell model and $\alpha$-cluster models, the contribution of the elastic $\alpha$ transfer (or the core-core exchange) to the elastic $^{16}\text{O}+^{12}\text{C}$ scattering should not be negligible and is expected to account for the enhanced elastic cross section at backward angles. To reveal the impact of the elastic $\alpha$ transfer, a systematic coupled reaction channels analysis of the elastic $^{16}\text{O}+^{12}\text{C}$ scattering has been performed, with the coupling between the elastic scattering and elastic $\alpha$ transfer channels treated explicitly, using the real OP given by the DFM. We found that the elastic $\alpha$ transfer enhances the near-side scattering significantly at backward angles, giving rise to an oscillating distortion of the smooth Airy structure. The dynamic polarization of the OP by the coupling between the elastic scattering and elastic $\alpha$ transfer channels can be effectively taken into account in the OM calculation by an angular-momentum (or parity) dependent potential added to the imaginary OP, as suggested by Frahn and Hussein 40 years ago.

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1 Introduction

Although elastic heavy-ion (HI) scattering is usually dominated by the strong absorption [12], some light HI and $\alpha$-nucleus systems are quite weakly absorbing so that the nuclear rainbow pattern survives at medium and large scattering angles. The observation of the nuclear rainbow allows the determination of the real nucleus-nucleus optical potential (OP) with much less ambiguity (see the topical review [3] for more details). The nuclear rainbow originates from the far-side scattering [4], and usually is associated with a broad Airy oscillation [5]. The observation of the Airy minima, in particular, the first Airy minimum $A_1$ that is followed by a shoulder-like bump is essential for the identification of the nuclear rainbow [35]. The far-side scattering pattern of the nuclear rainbow can be revealed by the decomposition of the elastic scattering amplitude into the internal component that penetrates the Coulomb + centrifugal barrier well into the interior of the real OP, and the barrier component that is reflected from the barrier [78]. An alternative interpretation of the far-side scattering is the decomposition of the elastic scattering amplitude into the near-side and far-side components [8] which is referred to and discussed throughout this work. In any case, the observation of the nuclear rainbow pattern provides very important database for the mean-field study of the refractive nucleus-nucleus scattering.

The recent folding model analysis [13] of elastic $^{12}\text{C}+^{12}\text{C}$ and $^{16}\text{O}+^{12}\text{C}$ scattering has pointed out a range of refractive energies ($10 \lesssim E \lesssim 40$ MeV/nucleon for the incident $^{12}\text{C}$ and $^{16}\text{O}$ ions), where the nuclear rainbow pattern can be clearly identified. At lower energies, the rainbow shoulders following the Airy minima are located closer to backward angles, and are deteriorated by the Mott interference in the symmetric $^{12}\text{C}+^{12}\text{C}$ and $^{16}\text{O}+^{16}\text{O}$ systems or by the elastic $\alpha$ transfer in the $^{16}\text{O}+^{12}\text{C}$ system. On the other hand, the broad Airy structure is moving to forward angles with the energies increasing above the refractive range, and the nuclear rainbow pattern is destroyed by the interference of the near-side and far-side scatterings that leads to the well-known Fraunhofer oscillation.

The elastic $^{12}\text{C}+^{12}\text{C}$ and $^{16}\text{O}+^{16}\text{O}$ scattering is strongly refractive and favorable for the observation of nuclear rainbow, like the pronounced primary rainbow observed in the elastic $^{16}\text{O}+^{16}\text{O}$ scattering at $E_{\text{lab}} = 350$ MeV [14]. How-
ever, the Airy structure of these systems is destroyed at scattering angles $\theta_{\text{c.m.}} \gtrsim 90^\circ$ because the boson symmetry of two identical nuclei gives rise to a rapidly oscillating elastic cross section there. For this reason, the asymmetric $^{16}\text{O}+^{12}\text{C}$ system was considered as a good candidate for the study of nuclear rainbow [12]. Numerous experiments have been performed so far to measure the elastic $^{16}\text{O}+^{12}\text{C}$ scattering with high-precision, over a wide range of energies ($E_{\text{lab}} \approx 20-1503$ MeV) and a broad angular region (up to $\theta_{\text{c.m.}} > 130^\circ$ at low energies) [13,14,15,16,17,18,19,20,21]. Very essential for the present study are the measured for the optical model (OM) and folding model studies of the elastic data [20]. The optical model (OM) and folding model studies of the elastic data measured for the $^{12}\text{C}+^{12}\text{C}$, $^{16}\text{O}+^{16}\text{O}$, and $^{16}\text{O}+^{12}\text{C}$ systems [10,22,23,24,25,26,27] have shown unambiguously the nuclear rainbow pattern, especially, the evolution of the Airy structure with the energy. However, the elastic $^{16}\text{O}+^{12}\text{C}$ data measured at low energies ($E_{\text{lab}} \lesssim 132$ MeV) show that the nuclear rainbow pattern is substantially deteriorated by a quick oscillation of elastic cross section at backward angles.

![Figure 1](image.png)

**Fig. 1.** Kinematical illustration of the elastic scattering and elastic $\alpha$ transfer processes in the $^{16}\text{O}+^{12}\text{C}$ system.

The enhanced oscillatory cross section at backward angles, known as the “anomalous large angle scattering” (ALAS) was observed for different light HI systems at low energies [28]. All these data show consistently that the surface of the dinuclear system is more transparent to result on the modified elastic $S$-matrix that contains a parity dependent component. While the oscillation of the low-energy elastic $^{12}\text{C}+^{12}\text{C}$ or $^{16}\text{O}+^{16}\text{O}$ cross section at backward angles is caused mainly by the boson symmetry of two identical nuclei [10,25,26], the direct and indirect $\alpha$ transfer was shown recently as the main physics origin of the oscillating enhancement of the elastic $^{16}\text{O}+^{12}\text{C}$ cross section at backward angles [52]. In general, the OM analysis of low-energy elastic $^{16}\text{O}+^{12}\text{C}$ data is more difficult compared to other light HI systems [28], and an $\ell$-dependent term was often added to the complex OP, which was suggested by von Oertzen and Bohlen [33] as necessary to account for the core-core exchange or elastic $\alpha$ transfer between $^{16}\text{O}$ and $^{12}\text{C}$ (see Fig. 1). Within the OM using standard parity-independent Woods-Saxon (WS) potential, one could obtain a good description of the elastic $^{16}\text{O}+^{12}\text{C}$ data at low energies only if an extremely small diffuseness of the imaginary Woods-Saxon (WS) potential is used [20]. Such an abrupt shape of the absorptive WS potential is drastically different from the global OP established for the $^{16}\text{O}+^{12}\text{C}$ system [2].

The direct $\alpha$ transfer in the elastic $^{16}\text{O}+^{12}\text{C}$ scattering at low energies was studied in the distorted wave Born approximation (DWBA) [34,35], and a good DWBA description of elastic $^{16}\text{O}+^{12}\text{C}$ data was obtained with the $\alpha$ transfer amplitude added to the elastic scattering amplitude. Some scenarios of the $\alpha$ transfer in elastic $^{16}\text{O}+^{12}\text{C}$ scattering were considered within the coupled reaction channel (CRC) formalism [36,37], which is the most appropriate method to study transfer reactions. A detailed CRC analysis of the $\alpha$ transfer in the elastic $^{16}\text{O}+^{12}\text{C}$ scattering at low energies was done recently [32], where the multistep coupling of the elastic scattering channel to the inelastic scattering, direct and indirect $\alpha$ transfer channels was treated explicitly, and a good CRC description of the data was obtained using the $\alpha$ spectroscopic factors predicted by the large-scale shell-model (SM) calculation [39].

Given the $\alpha$ transfer in the elastic $^{16}\text{O}+^{12}\text{C}$ scattering now well established, we focus in the present paper on how the $\alpha$ transfer affects the nuclear rainbow pattern in the elastic $^{16}\text{O}+^{12}\text{C}$ cross section at large scattering angles.

### 2 Optical model description of the elastic $^{16}\text{O}+^{12}\text{C}$ scattering and nuclear rainbow

A realistic choice of the complex OP for elastic $^{16}\text{O}+^{12}\text{C}$ scattering is a prerequisite for both the identification of nuclear rainbow and CRC study of the nonelastic reaction channels. In particular, the Fraunhofer oscillation at forward angles is formed entirely by elastic scattering, and a properly chosen OP for the $^{16}\text{O}+^{12}\text{C}$ system should reproduce the elastic data at small angles as accurately as possible. With the energy increasing to about 10 to 40 MeV/nucleon, the nuclear rainbow pattern becomes well observable, and the most pronounced rainbow pattern associated with the first Airy minimum $A_1$ was identified in the elastic $^{16}\text{O}+^{12}\text{C}$ data measured at $E_{\text{lab}} = 200$ MeV [17]. Such data were often used to validate different theoretical models of the nucleus-nucleus OP, like the double-folding model (DFM) that calculates the real OP using the realistic wave functions of two colliding nuclei and effective nucleon-nucleon interaction between the projectile- and target nucleons [10,22,23]. In the present work, the real OP given by the DFM is used in both the OM and
CRC calculations. At low energies the imaginary OP is due to a few nonelastic channels and is parametrized in the standard WS form. The details of the DFM calculation of the real OP, Coulomb potential, and the WS parameters of the imaginary OP are given in Ref. [32]. The obtained OM description of the elastic $^{16}$O+$^{12}$C scattering data measured at incident energies of 115.9, 124 MeV [20], and 132 MeV [16, 17] is shown in Fig. 2.

To show the Airy structure of the nuclear rainbow pattern we decomposed the elastic scattering amplitude into the near-side and far-side components using Fuller’s method [9]. Namely, by splitting the Legendre function $P_l(\cos \theta)$ into two waves scattered at $\theta$ but running in the opposite directions around the scattering center, the elastic scattering amplitude $f_{ES}(\theta)$ can be expressed in terms of the near-side ($f_N$) and far-side ($f_F$) components as

$$f_{ES}(\theta) = f_N(\theta) + f_F(\theta) = \frac{i}{2k} \sum_{\ell} (2\ell + 1) A_{\ell} \times \left[ \tilde{Q}_{\ell}^{(-)}(\cos \theta) + \tilde{Q}_{\ell}^{(+)}(\cos \theta) \right],$$

where $\tilde{Q}_{\ell}^{(\pm)}(\cos \theta) = \frac{1}{2} \left[ P_{\ell}(\cos \theta) \pm \frac{2i}{\pi} Q_{\ell}(\cos \theta) \right]$, and $Q_{\ell}(\cos \theta)$ is the Legendre function of the second kind. $f_N(\theta)$ represents the wave deflected to the direction of $\theta$ on the near side of the scattering center, and $f_F(\theta)$ represents the wave traveling on the opposite, far side of the scattering center to the same angle $\theta$. Thus, $f_N(\theta)$ accounts mainly for the diffractive scattering that occurs at the surface, and $f_F(\theta)$ accounts for the refractive scattering that penetrates more into the interior of the dinuclear system. The far-side scattering cross sections given by the best OM fit to the elastic $^{16}$O+$^{12}$C data measured at $E_{\text{lab}} = 115.9$, 124, and 132 MeV are shown as dotted lines in Figs. 2 and the broad rainbow shoulders following the second (A2) and third (A3) Airy minima are clearly seen. At these low energies, the first Airy minimum A1 is rather weak and located at backward angles (on the “dark” side of rainbow). Because the refractive Airy structure of the far-side scattering is frequently obscured by the absorption, the OM calculation was done also with a strength of the imaginary WS potential reduced by 50%, and the far-side cross sections (see the dashed lines in Fig. 2) show clearly the broad Airy oscillation pattern of nuclear rainbow which was established earlier in the extensive OM analyses of elastic $^{16}$O+$^{12}$C data over a wide range of energies [10, 17]. All the OM calculations were done using the code ECIS97 written by Raynal [38].

3 CRC description of the elastic alpha transfer $^{12}$C ($^{16}$O, $^{12}$C)$^{16}$O

Given the core-core symmetry of the $^{16}$O+$^{12}$C system, the elastic $\alpha$ transfer from $^{16}$O to $^{12}$C leads to the final state that is indistinguishable from that of pure elastic scattering (see Fig. 1). Therefore, the total elastic amplitude must be a coherent sum of the elastic scattering (ES) amplitude $f_{ES}$ and elastic transfer (ET) amplitude $f_{ET}$. The interference between $f_{ES}$ and $f_{ET}$ was shown by the recent multichannel CRC analysis of elastic $^{16}$O+$^{12}$C scattering [32] to give rise to an enhanced oscillating elastic cross section at large angles, using the $\alpha$ spectroscopic factors predicted by the large-scale SM calculation [39] and 4o cluster model of $^{16}$O [40]. Because the multichannel coupling of the indirect reaction channels to the elastic $^{16}$O+$^{12}$C scattering channel can be accounted for by using an effective $\alpha$ spectroscopic factor adjusted to the best CRC fit to elastic data, taking into account the direct $\alpha$ transfer only [32], it is sufficient to focus on the coupling between the elastic scattering $^{12}$C ($^{16}$O,$^{12}$C)$^{16}$O and direct $\alpha$ transfer $^{12}$C ($^{16}$O,$^{12}$C)$^{16}$O channels in the present study. Thus, the
CRC equations for the two considered channels are solved using the code Fresco written by Thompson \[11\]

\[
(E_a - T_a - U_a)\chi_a = [(a|W_b|b) + (a|b)(T_b + U_b - E_b)]\chi_b
\]

\[
(E_b - T_b - U_b)\chi_b = [(b|W_a|a) + (b|a)(T_a + U_a - E_a)]\chi_a
\]

where \(|a| \equiv |^{16}\text{O},^{12}\text{C}\) and \(|b| \equiv |^{12}\text{C},^{16}\text{O}\), with the interchange of projectile and target; \(\chi_a\) and \(\chi_b\) are the scattering wave functions of the elastic scattering and \(\alpha\) transfer channels, respectively, and \(U_a\) and \(U_b\) are the corresponding OP's. Due to the identity of two channels, all post form formulas are equivalent to the prior ones and transfer interaction is determined \[12,43\] as

\[
W_a = W_b = V_{\alpha+^{12}\text{C}} + (U_{^{12}\text{C}+^{12}\text{C}} - U_{^{16}\text{O}+^{12}\text{C}}),
\]

where \((U_{^{12}\text{C}+^{12}\text{C}} - U_{^{16}\text{O}+^{12}\text{C}})\) is the remnant term and \(V_{\alpha+^{12}\text{C}}\) is the binding potential of the \(\alpha\) cluster in \(^{16}\text{O}\). Further details of the CRC calculation can be found in Ref. \[52\]. An important input for the CRC calculation is the dinuclear overlap \(^{(12)^6}\text{C}(^{16}\text{O})\) that is directly proportional to the \(\alpha\) spectroscopic factor \(S_\alpha\). The solutions \(\chi_a\) and \(\chi_b\) of the CRC equations are used to determine \(f_{\text{ES}}\) and \(f_{\text{ET}}\), and the total elastic \(^{16}\text{O}+^{12}\text{C}\) cross section is obtained as

\[
\frac{d\sigma(\theta)}{d\Omega} = |f(\theta)|^2 = |f_{\text{ES}}(\theta) + f_{\text{ET}}(\pi - \theta)|^2,
\]

(2)

where the elastic \(\alpha\) transfer amplitude at the angle \((\pi - \theta)\) is coherently added to the elastic scattering amplitude at \(\theta\) in the center-of-mass (c.m.) frame as shown kinematically in Fig. 1. The two amplitudes can be expressed in terms of the partial wave expansion as

\[
f_{\text{ES}}(\theta) = f_R(\theta) + \frac{1}{2ik} \sum_\ell (2\ell + 1)e^{2i\sigma_\ell} (S_\ell^{\text{ES}} - 1) P_\ell(\cos \theta),
\]

(3)

where \(f_R(\theta)\) and \(\sigma_\ell\) are the Rutherford scattering amplitude and Rutherford phase shift, respectively, which are available in the analytical form \[12\].

\[
f_{\text{ET}}(\theta) = \frac{1}{2ik} \sum_\ell (2\ell + 1)e^{2i\sigma_\ell} S_\ell^{\text{ET}} P_\ell(\cos(\pi - \theta)),
\]

\[
= \frac{1}{2ik} \sum_\ell (2\ell + 1)e^{2i\sigma_\ell} S_\ell^{\text{ET}} (-1)^\ell P_\ell(\cos \theta).
\]

(4)

The total elastic amplitude is then obtained as

\[
f(\theta) = f_R(\theta) + \frac{1}{2ik} \sum_\ell (2\ell + 1)e^{2i\sigma_\ell} (S_\ell - 1) P_\ell(\cos \theta),
\]

(5)

where \(S_\ell = S_\ell^{\text{ES}} + (-1)^\ell S_\ell^{\text{ET}}\).

We have thus obtained the total elastic amplitude in the same partial wave expansion as the purely elastic scattering amplitude \[3\], with an angular-momentum or parity dependent contribution from the elastic \(\alpha\) transfer added to the elastic scattering \(S\)-matrix. The interference between these two terms in the total \(S\)-matrix \[10\] gives rise naturally to an oscillation of elastic cross section at large angles, similar to that caused by the Mott oscillation observed in the elastic scattering of two identical nuclei, like \(^{12}\text{C}+^{12}\text{C}\) or \(^{16}\text{O}+^{16}\text{O}\) \[10\]. We note that the ALAS calculation by adding an angular-momentum or parity dependent term to the total OP \[23,53\]. In particular, the dynamic coupling between the elastic scattering and elastic \(\alpha\) transfer channels was suggested by Frahn and Hussein \[29,30,31\] to lead to an effective coupling potential that depends explicitly on the angular momentum. Moreover, it was also shown by these authors \[30\] that the contribution of such dynamic \(f\)-dependent coupling potential can be represented by a modified elastic \(S\)-matrix that contains an \(f\)-dependent component like that in Eq. \(\text{(1)}\).

The total elastic \(^{16}\text{O}+^{12}\text{C}\) cross sections given by the two-channel CRC calculation are compared with elastic \(^{16}\text{O}+^{12}\text{C}\) data measured at \(E_{\text{lab}} = 115.9, 124\) MeV \[29\], and 132 MeV \[16,17\] in Fig. 3, and one can see that the enhanced oscillating elastic cross sections at backward angles are due entirely to the elastic \(\alpha\) transfer \(^{12}\text{C}(^{16}\text{O},^{12}\text{C})^{16}\text{O}\) process. The back coupling of the direct \(\alpha\) transfer at backward angles to the purely elastic \(^{16}\text{O}+^{12}\text{C}\) scattering at
forward angles turned out to be quite weak, and the OP used in the OM calculation discussed in Sec. 2 was used also in the two-channel CRC calculation. It is remarkable that the good CRC description of elastic $^{16}$O+$^{12}$C cross section at backward angles at three energies has been reached consistently with the $\alpha$ spectroscopic factor $S_\alpha \approx 1.96$, in agreement with those deduced earlier from the DWBA and two-channel CRC calculations of elastic $^{16}$O+$^{12}$C scattering at low energies [35,36]. Although such a value of the $\alpha$ spectroscopic factor is larger than that predicted by the SM calculation [29] or 4$\alpha$ cluster model [40], it can be considered as the effective $S_\alpha$ that accounts for the elastic $\alpha$ transfer in the two-channel approximation. In fact, the comprehensive CRC calculation of elastic $^{16}$O+$^{12}$C scattering including explicitly up to 10 reaction channels of both the direct and indirect (multistep) $\alpha$ transfers [32] accounts equally well for the elastic data measured at backward angles, using $S_\alpha$ predicted by the SM calculation [39]. It is, thus, sufficient to investigate the impact by the elastic $\alpha$ transfer on nuclear rainbow based on the results of the two-channel CRC calculation only.

As discussed above in Sec. 2, the oscillatory elastic $^{16}$O+$^{12}$C cross sections seen at backward angles could be reproduced in the standard OM calculation only by using an extremely small diffuseness of the WS imaginary OP which enhances the near-side scattering at large angles, and the near-far interference there gives rise to a quick oscillation of elastic cross section [32]. Given the strong impact of the elastic $\alpha$ transfer shown in Fig. 3, we have decomposed the total elastic amplitude [11] into the near-side and far-side components [11], and the results are shown in Fig. 4. One can see that the elastic $\alpha$ transfer leads indeed to the enhanced strength of the near-side scattering at backward angles, and the near-far interference results on the oscillating elastic $^{16}$O+$^{12}$C cross section. At the given scattering angle $\theta$, the elastic $\alpha$ transfer amplitude calculated at the angle $(\pi - \theta)$ is added to the purely elastic scattering amplitude at $\theta$. Therefore, the enhancement of the total elastic $^{16}$O+$^{12}$C cross section at backward angles is in fact caused by the elastic $\alpha$ transfer occurring physically at forward angles, at the surface of two colliding nuclei. This is a strong indication that the $\alpha^{12}$C cluster configuration is likely formed at the surface of the $^{16}$O nucleus. The same conclusion was also drawn from the multichannel CRC analysis of the elastic $^{16}$O+$^{12}$C scattering that included the direct as well as indirect $\alpha$ transfer channels [32]. As a consequence, the smooth Airy pattern of nuclear rainbow in the elastic $^{16}$O+$^{12}$C scattering at low energies is strongly distorted by the near-far interference at backward angles that is due mainly to the elastic $\alpha$ transfer.

We note that the CRC results shown here were obtained with the WS imaginary OP that has a normal diffuseness $\alpha_V \approx 0.5 - 0.6$ fm. Therefore, the abnormal surface absorption of the OP given by an extremely small diffuseness of the WS imaginary OP [29] just mimics the strong dynamic polarization of the OP by the reaction-channel coupling between the purely elastic scattering and elastic $\alpha$ transfer channels.
It was shown explicitly by Frahn and Hussein 40 years ago [29] that the dynamic coupling between the elastic scattering and elastic transfer channels can be effectively taken into account in the OM calculation by adding an ℓ-dependent term to the OP. In fact, the angular-momentum or parity-dependent OP was used earlier to account for the oscillating ALAS pattern at backward angles in the elastic light HI scattering at low energies [23,33]. To further explore this important conclusion we have added to the WS imaginary OP of elastic \(^{16}\text{O}+^{12}\text{C}\) scattering at \(E_{\text{lab}} = 115.9\) and 132 MeV an ℓ-dependent term \(W_\ell(R)\) parametrized in the form suggested in Ref. [29]

\[
W_\ell(R) = (-1)^\ell W_s \frac{\exp[-\beta^2(R - R_s)^2]}{\beta R} \equiv (-1)^\ell W_\ell(R),
\]

(7)

where \(W_\ell(R)\) is often discussed as the Majorana potential [11], which accounts for the core-core symmetry of a light HI system like \(^{16}\text{O}+^{12}\text{C}\). The best OM fit to the elastic \(^{16}\text{O}+^{12}\text{C}\) data at \(E_{\text{lab}} = 115.9\) and 132 MeV over the whole angular range has been achieved with \(R_s \approx 5.55\) fm, so that the ℓ-dependent contribution \(\Omega(\ell)\) of the OP is peaked at the surface. The remaining parameters depend slightly on energy, \(W_s \approx 1.834\) MeV and \(\beta \approx 0.775\) fm at \(E_{\text{lab}} = 115.9\) MeV, and \(W_s \approx 1.885\) MeV and \(\beta \approx 0.806\) fm at \(E_{\text{lab}} = 132\) MeV. The enhanced oscillating elastic \(^{16}\text{O}+^{12}\text{C}\) cross sections at backward angles are now well reproduced by the OM calculation using the ℓ-dependent OP (see Fig. 5), in about the same way as the two-channel CRC calculation discussed in Sec. 2. Thus, the dynamic polarization of the OP by the reaction-channel coupling between the purely elastic scattering and direct elastic \(\alpha\) transfer channels can be effectively accounted for by the ℓ-dependent potential \(\Omega(\ell)\) added to the imaginary OP in the standard one-channel OM calculation.

To illustrate the dynamic impact of the ℓ-dependent potential \(\Omega(\ell)\), we have plotted in Fig. 6 the elastic S-matrix elements given by both the ℓ-independent and ℓ-dependent OP’s in comparison with the elastic S-matrix given by the two-channel CRC calculation. The S-matrix given by the ℓ-dependent OP turns out to have the same zigzag behavior at small partial waves \(\ell\) as that of the total elastic S-matrix obtained from the two-channel CRC calculation. In fact, such an odd-even staggering in the ℓ dependence of the total elastic S-matrix is due to the ℓ-dependent contribution from the elastic \(\alpha\) transfer amplitude in Eq. (9). The OM results shown in Figs. 5 and 6 also indicate that the use of an ℓ-dependent OP might be necessary in the one-channel OM description of the elastic light HI scattering at low energies, to effectively account for the dynamic polarization of the OP by the nonelastic reaction channels (see more detail in a recent review by Mackintosh [14]).

We note that the symmetric exchange of the two \(^{12}\text{C}\) cores in the elastic \(^{16}\text{O}+^{12}\text{C}\) scattering has been recently confirmed [33] to give rise to the parity dependence of the OP. Namely, the S-matrix generated by the CRC calculation of elastic \(^{16}\text{O}+^{12}\text{C}\) scattering [32] was used as the input for the iterative-perturbative (IP) inversion method [15] to obtain the effective local OP that contains both the (ℓ-independent) Wigner and (ℓ-dependent) Majorana terms. The results of the IP inversion were obtained with high precision and a strong Majorana term has been deduced for the total OP of the \(^{16}\text{O}+^{12}\text{C}\) system, which is a direct estimation of the ℓ-dependent contribution by the core exchange or \(\alpha\) transfer. It is of interest for the present study to compare the best-fit Majorana potential [12] with the imaginary part of the inverted Majorana potential [14], and results plotted in Fig. 7 show indeed that
the dynamic polarization of the OP by the core exchange or elastic transfer can be effectively taken into account by the $\ell$-dependent potential in the Gaussian form suggested by Frahn and Hussein [29]. It is interesting that the results of the two independent approaches shown in Fig. 7 suggest clearly that the elastic transfer occurs mainly at $R \approx 5.4$ fm, i.e., at the surface of the $^{16}\text{O}^{+12}\text{C}$ system.

4 Summary

The effect of the elastic transfer on the nuclear rainbow pattern in the $^{16}\text{O}^{+12}\text{C}$ system has been investigated in the two-channel CRC analysis of elastic $^{16}\text{O}^{+12}\text{C}$ scattering, taking explicitly into account the coupling between the purely elastic scattering and direct elastic transfer channels. The elastic transfer was found to enhance strongly the near-side scattering cross section at backward angles, giving rise to an oscillating distortion of the smooth Airy structure of nuclear rainbow.

The one-channel OM was shown to give about the same good description of elastic $^{16}\text{O}^{+12}\text{C}$ data over the whole angular range as that given by the CRC calculation if the dynamic polarization of the OP by the coupling between the elastic scattering and transfer channels is effectively taken into account by an $\ell$-dependent Majorana potential in the Gaussian form suggested 40 years ago by Frahn and Hussein [29]. This result validates the use of the $\ell$-dependent OP in the extensive OM analysis of elastic scattering data measured at low energies for those light HI systems that have the core-core symmetry, where the elastic nucleon- or cluster transfer can be significant.

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