Nonlinear angle control of a sectioned airfoil by using shape memory alloys

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Abstract. The present work illustrates an application of shape memory alloys and nonlinear controller applied to the active angular control of a sectioned airfoil. The main objective of the proposed control system is to modify the shape of the profile based on a reference angle. The change of the sectioned airfoil angle is resultant by the effect of shape memory of the alloy due to heating of the wire caused by an electric current that changes its temperature by Joule effect. Considering the presence of plant’s nonlinear effects, especially in the mathematical model of the alloy, this work proposes the application of an on-off control system.

1 Introduction

The observation of flight in nature has motivated the human desire to fly, and ultimately the development of aircraft. The designs of the first flying machines were relatively crude and even today nature has much to teach us and continuously inspires research.

In just a century, engineers built aircrafts that can travel above the sound speed, cross the circumference of the earth without refueling and even cross the atmosphere to the space. The most modern aircraft are able to rapidly change shape to transition from efficient cruise to aggressive maneuvering and precision descents. There are specific geometric features required (for example: curvature of the wings) that allow to achieve maximum flight efficiency successfully [1].

Incorporating morphing structures into aircraft is not a new idea. In fact, the first powered aircraft to take flight, the Wright Flyer, was based on a wing design intended to smoothly deform or morph. In the last couple of decades, advances in materials have made it feasible to create robust morphing aerospace structures that allow the aircraft to fly for a variety of flight regimes [1]. The ability of a wing surface to change its geometry during flight has interested researchers and designers over the years. An adaptive wing diminishes the compromises required to insure the operation of the airplane in multiple flight conditions ([2]-[6]).

Several mechanisms were proposed to create an adaptive wing [2]. The main difficulty of trying to recreate artificially such system is to find a lightweight actuator, as efficient as the muscles, able to perform such and still deform considerably. A synthetic material that has this characteristic is the Shape Memory Alloy (SMA) [7]. This material is capable of converting thermal energy into mechanical energy and once deformed the material can return to its original shape by heating. There are a large number of research works carried out in recent years focusing on the use of shape memory alloys in aeronautical wings adaptive ([8]-[10]). It proves the scientific and technological relevance of this issue.

However, because it is a relatively new topic of study, which are being continuously incorporated technological developments and new possibilities for practical applications, much research effort still has to be done. For this purpose, the control of aeronautic adaptive profiles should be developed, including, particularly, the application of control techniques based on artificial intelligence for adaptive wings.

Due to the presence of nonlinear effects, especially in the mathematical model of the SMA [7], this paper proposes an angular control system by using an on-off controller. One obvious issue is that system exhibits highly nonlinear behavior and some of the system parameters are unknown and/or environment-dependent. On-off control has the advantages of reducing the design complexity for control problems untreated by classical techniques [8, 9].

This work focuses on the design and numerical results of active angular control of a particular sectioned airfoil actuated by a pair of SMA wires. The profile consists of a NACA-0012 model and has approximately 500 mm chord. By using a pair of SMAs as actuators, an on-off controller is designed to control the profile angle actively.

The organization of the paper is as follows. Next section presents a description of the system used for the angular control of the proposed sectioned airfoil with a pair of SMA wires. Then the mathematical model for the
system is presented. The theoretical concepts and the main features of used on-off controller are described. Numerical simulations tests with on-off controller are carried out in order to verify the effectiveness of the proposed control system. Finally, the concluding remarks are presented.

2 System description

Figure 1 shows a schematic sectioned airfoil with a pair of SMA wires. The profile consists of a NACA-0012 model and has approximately 500 mm chord. By using a pair of SMAs as actuators, an on-off controller is designed to control the profile angle actively.

![Fig. 1. Schematic drawing of the sectioned airfoil with SMA wires.](image)

As shown in Fig. 2, when one of the wires is heated, it contracts and induces tension in the other SMA wire and the airfoil profile (parts P3 and P4) rotates around point O.

![Fig. 2. Behavior of the sectioned airfoil when (a) SMA 1 is heated and (b) SMA 2 is heated.](image)

When the second wire is heated, its contraction will result in the extension of the opposing SMA actuator mechanically. Thus it recovers the strain and moves the sectioned airfoil in the opposite direction. The position of the profile is determined by heating and cooling the two SWA wires. By heating one of the wires in an initially deformed state, a reduction in wire length due to transformation strain in the SMA will create a relative rotation between airfoil parts (P3 and P4) while straining the opposing, cooling wire. After the actuation is complete and the desired angular position is reached, no additional energy is required to maintain the deformed shape. The purpose of the system is to force the rotation angle \( \theta \) of the profile to follow the desired trajectory \( \theta_d \).

3 Mathematical model

The mathematical model of the system is composed of a thermal model, a phase transformation model, and a description of the system dynamics.

3.1 Thermal model

SMA actuators are most commonly used in the wire form and the phase transformation is commonly induced by electrical heating. In the conventional method of actuation, each SMA wire is continuously heated by electric current.

A common model of the heat transfer associated with electrical heating (also known as Joule heating) of the wire is [11]:

\[
(\rho A) c_p \frac{dT(t)}{dt} = \frac{V^2}{R_w} - h_c A_c [T(t) - T_w]
\]

where \( \rho \) is the density of the shape memory material, \( A \) is the cross-sectional area \( (A = \pi d^2 / 4, \text{ where } d \text{ is the diameter of the wire}) \), \( c_p \) is the specific heat of the wire and \( R_w \) is the resistance per unit length of the material. The parameter \( h_c \) is the heat transfer coefficient and \( A_c \) is the circumferential area of the unit length of wire \( (A = \pi d) \). The ambient temperature is denoted \( T_w \) and the electric voltage \( V \) across the SMA wire is the control variable of the system.

Assuming that the voltage is constant and the initial temperature is equal to the ambient temperature, the solution to this differential equation (1) is

\[
T(t) = T_w + \frac{1}{R_w h_c A_c} \left( 1 - e^{-t/\tau_h} \right) V^2
\]

where \( \tau_h = \frac{\rho A c_p}{h_c A_c} \) is defined as time constant associated with the heat transfer process.

The temperature increase causes the SMA wire to go through the phase transformation, resulting in contraction of the wire. When cooled, the wire returns or stretches back to the original length. The amount of contraction or stretch is determined by the martensite fraction, which can be obtained from the phase transformation model described below.

3.2 Phase transformation

The SMA model used in the numerical simulations is derived by [12]. This particular model was chosen due to excellent accuracy between the simulation and experimental results, which clearly justify the use of the model for describing the transformation between martensite (M) and austenite (A) phases [7]. This model replicates the behavior of the SMA at the phenomenological level. During heating, the transformation occurs from martensite to austenite, and during the cooling phase, the opposite transformation occurs.

The SMA constitutive model defines the thermomechanical characteristics of the material, i.e. the effect of the temperature on the stress as the SMA...
undergoes phase transformation. The relationships between stress (σ), strain (ε), temperature and martensite fraction within the SMA wire during phase transformation (ξ) can be defined as [12]:

\[ σ = D\ddot{ε} + Θ\dot{ε} + Ωξ \]  

(3)

where D is Young’s modulus of the alloy, Θ is the thermo-elastic factor, and Ω is the phase transformation coefficient and ξ = ξS + ξT is decomposed further into a summation of two variables: ξs is the fraction of stress-induced martensite in the material and ξT is the fraction of temperature-induced martensite in the material.

[12] demonstrated that despite the thermo-elastic effect one can obtain the following relation based on Eq. (3), considering εL as the maximum strain that can recovered through the transformation phase:

\[ σ = D(ξ)[ε - ε_L ξ] \]  

(4)

where the actual Young’s modulus D(ξ) is assumed to be a linear function of the martensite fraction: D(ξ) = DA + ξ(DM - DA), and DA and DM are, respectively, the elastic modulus in the austenite and martensite state.

According [12], the transformation equations also require modification to account for the transformation between the different types of martensite. The kinetic law for conversion from martensite to austenite is:

For \( T > M_s \) and \( C_A(T - A_f) < σ < C_A(T - A_s) \):

\[ \dot{ξ} = \frac{1}{2}\left\{ \cos\left[ a_A\left(T - A_s - \frac{σ}{C_A}\right) + 1\right] + 1 \right\}, \]

(5)

where \( a_A = \frac{π}{M_s - M_f} \), \( a_A = \frac{π}{A_f - A_s} \), \( ξ_{S0} \) and \( ξ_{T0} \) are the initial martensite fractions; \( A_s \) and \( A_f \) are the initial and final temperature of austenite transformation, respectively; \( M_s \) and \( M_f \) are the initial and final temperature of martensite transformation, respectively.

### 3.3 System dynamics

Figure 3a shows the inertial frame O-XY of the airfoil profile centered at point O. The task consists in forcing the profile to follow a specified angular trajectory. The trajectory lies on the plane defined by the coordinate frame O-xy. Note that this frame is a non-inertial reference frame, i.e. it is moving with the profile (P3 and P4 parts) and its position and orientation are related to the sectioned airfoil’s initial configuration.

![Fig. 3. (a) Inertial (XY) and non-inertial (xy) reference axes and (b) SMA wire connection point positions.](image)

As shown in Fig. 3b, the position vector of points A, B, C and D (SMA wire connections) with respect the frame O-XYZ is defined, respectively, as:

\[ \vec{r}_{OA} = \begin{bmatrix} a_1 \\ a_2 \\ 0 \end{bmatrix} ; \quad \vec{r}_{OB} = \begin{bmatrix} b_1 \\ 0 \\ a_2 \end{bmatrix} ; \quad \vec{r}_{OC} = \begin{bmatrix} c_1 \\ 0 \\ c_2 \end{bmatrix} \]

and

\[ \vec{r}_{OD} = R \begin{bmatrix} d_1 \\ -d_2 \\ 0 \end{bmatrix} \]  

(9)

where R is the rotation matrix between frames (x,y,z) and (X,Y,Z) given by:

\[ R = \begin{bmatrix} \cosθ \sinθ & 0 \\ -\sinθ \cosθ & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  

(10)

Finally, the length of each SMA wire (\( \hat{r}_{AB} \) and \( \hat{r}_{CD} \)) are defined as follows:

\[ Δ_{Tξ} = \frac{1 - ξ_{T0}}{2} \left\{ \cos\left[a_B\left(T - M_f\right)\right] + 1\right\} \]

if \( M_f < T < M_s \) and \( T < T_0 \). Otherwise, \( Δ_{Tξ} = 0 \).

where \( a_B = \frac{π}{M_s - M_f} \), \( a_B = \frac{π}{A_f - A_s} \), \( ξ_{S0} \) and \( ξ_{T0} \) are the initial martensite fractions; \( A_s \) and \( A_f \) are the initial and final temperature of austenite transformation, respectively; \( M_s \) and \( M_f \) are the initial and final temperature of martensite transformation, respectively.
\[ \dot{r}_{AB} = \dot{r}_{AO} + \dot{r}_{OB} = \begin{bmatrix} b_1 \cos \theta - b_2 \sin \theta + a_1 \\ b_1 \sin \theta + b_2 \cos \theta - a_2 \end{bmatrix} \quad (11a) \]

\[ \dot{r}_{CD} = \dot{r}_{CO} + \dot{r}_{OD} = \begin{bmatrix} d_1 \cos \theta + d_2 \sin \theta + c_1 \\ d_1 \sin \theta - d_2 \cos \theta - c_2 \end{bmatrix} \quad (11b) \]

where \( \dot{r}_{AB} \) and \( \dot{r}_{CD} \) are the velocities of points A and C with respect to the frame O-XYZ.

**4 Controller design**

The controller is designed to set the activation of each SMA based on the overall configuration of the sectioned airfoil. Based on the angular error \( E = \theta_d - \theta \), the control system selects the appropriate activation voltage for each SMA wire according to equation below:

\[
V(t) = \begin{cases} V_{\text{SMA1}} = 6.75 V & \text{if } E(t) > 0 \\ V_{\text{SMA2}} = 6.75 V & \text{if } E(t) < 0 \end{cases} \quad (17)
\]

where \( V_{\text{SMA1}} \) and \( V_{\text{SMA2}} \) are the constant electric voltages applied to SMA wires 1 and 2, respectively, by using the on-off controller.

**5 Numerical simulation results**

In order to improve the motion control performance, it is essential to understand the effects of these parameters on the sectioned airfoil motion. Thus, in order to show the effectiveness of the proposed controller, the numerical results from the control of the airfoil at different angular positions are presented. A simple on-off control scheme is used. This controller corresponds to the so-called Band–Bang controller that activates the SMA actuator until the desired angular displacement is reached.

**Fig. 5. Overall control block diagram for the sectioned airfoil profile.**

Table 1 shows the main parameters of the system. The SMA wire parameters were obtained from the specifications provided by the manufacturer (Nitinol Company). The geometric parameters were directly measured from the actual system. Based on the models developed and the parameter values obtained, a numerical code was built in Visual C++ to simulate the motion of the sectioned airfoil profile.
Table 1. Parameters used for the system model.

| Parameter | Value       | Parameter | Value       |
|-----------|-------------|-----------|-------------|
| \(d\)     | \(5.08 \times 10^{-4} \text{m}\) | \(D_k\)   | 67 GPa      |
| \(\rho\)  | 6450 kg/m³ | \(D_m\)   | 26 GPa      |
| \(A\)     | 2.02 \times 10^{-4} \text{m}² | \(C_1\)   | 13 MPa/°C   |
| \(A_s\)   | 1.6 \times 10^{-3} \text{m}² | \(C_m\)   | 8 MPa/°C    |
| \(h_c\)   | 70 W/Km²   | \(M_e\)   | 50°C        |
| \(T_{e0}\)| 25°C       | \(M_f\)   | 40°C        |
| \(J\)     | 1.44 \times 10^{-3} \text{kgm²} | \(A_e\)   | 55°C        |
| \(\alpha\) | 2 \times 10^{-3} \text{kgm²/s} | \(A_f\)   | 70°C        |
| \(a_1, c_1\) | 54 \times 10^{-3} \text{m} | \(\sigma_{1T}^{ag}\) | 100 MPa |
| \(a_2, c_2\) | 17 \times 10^{-3} \text{m} | \(\sigma_{2T}^{ag}\) | 170 MPa |
| \(b_1, d_1\) | 50 \times 10^{-3} \text{m} | \(R_w\)   | 3.75 \Omega/m |
| \(b_2, d_2\) | 13 \times 10^{-3} \text{m} | \(\epsilon_p\) | 837 J/kgK |

Due to the SMA actuator stroke limitations as well as mechanical constraints, the range of motion of the airfoil profile is given by: \([\theta_{\text{min}}, \theta_{\text{max}}] = [-15°, 10°]\). Thus, the maximum strains \(\epsilon^L\) for both SMA wires (Eqs. 13a and 13b) are given respectively by: \(\epsilon^A_{\text{AB}} = 0.0593\) and \(\epsilon^L_{\text{CD}} = 0.0703\).

The differential equation (16) is solved by using the subroutine \(rk4\) given in C++ library which implements a simple Runge–Kutta method for an initial value problem. The time step used in solving the differential equation is \(dt = 0.1\) s. The sectioned airfoil is initially stretched out with the minimal angular position \((\theta_0 = -15°)\). Therefore, at \(t = 0\), the material for \(\text{SMA 1}\) has no stress-induced or temperature-induced martensite: \(\xi_0^1 = 0\) and \(\xi_{T0}^1 = 0\), and the material for \(\text{SMA 2}\) is assumed to be at a state of zero stress and zero strain \((\xi_0^2 = 1\) and \(\xi_{T0}^2 = 0)\).

The task consists in forcing the parts \(P3\) and \(P4\) to follow a specified angular trajectory \((\theta_d)\). The objective of the subsequent tracking tests is to verify the ability of the proposed on-off controller to follow commanded trajectories. Figure 6 compares the closed-loop control simulation results obtained from the on-off controller when two angular position step inputs \((10°\) and \(0°)\) were given as command signals \((\theta_d)\).

Note that the return to the desired angular position \(\theta = 0°\) from the overshoot is not too fast, because the on-off controller must rely on cooling of the SMA actuator to return to the desired position.

According to Fig. 6, there is an initial overshoot due to the thermal inertia from the hysteresis in the SMA, which causes a sudden contraction of the SMA wire when its temperature is increased to the austenite phase as shown in Figs. 7 and 8.

![Fig. 6. Angular response of the sectioned airfoil using on-off controllers.](image)

![Fig. 7. Temperature responses of the SMA wires using on-off controller.](image)

![Fig. 8. Fraction of martensite of the SMA wires using on-off controller.](image)

![Fig. 9. Control voltage applied to the SMAs actuators using on-off controller.](image)
The on-off control capability results in better control performance in terms of fast response and small control voltages (see Fig. 9).

These plots show that the on-off controller is able to track the angular trajectory of the sectioned airfoil profile very well.

6 Conclusions

An on-off controller has been developed to control the angular displacements of a sectioned airfoil containing a pair of SMA wires.

The proposed controller activates each one of the wires using a simple rule along with the angular position error. The control strategy required only minimal information on the environment. In order to design the proposed controller, a dynamic model of the system was used to perform a numerical parametric analysis. The simulation results confirmed that the on-off controller is able to track the angular trajectory of the sectioned airfoil profile very well in a numerical step response test.

Acknowledgments

The authors would like to thank the CNPq Brazilian Research Agency and FAPEMIG through INCT-EIE for the financial support of the reported research.

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