The effect of single-particle space-momentum angle distribution on two-pion HBT correlation in high energy heavy-ion collisions

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Abstract. Using several source models, we analyze the transverse momentum dependence of HBT radii. The results show that the single-particle space-momentum angle distribution plays an important role in the transverse momentum dependence of HBT radii. In a cylinder source, we use several formulas to describe the transverse momentum dependence of HBT radii and the single pion space-momentum angle distribution. We also make a numerical connection between them in the transverse plane.

Keywords: HBT Radii, Transverse Momentum Dependence, Space-Momentum Angle Distribution
1. Introduction

A new state of matter has been found in the Relativistic Heavy Ion Collider, which is called Quark-Gluon-Plasma (QGP)\cite{1, 2, 3}. It is a strongly interacting partonic matter under extreme temperature and energy density formed by deconfined quarks and gluons. This state is similar to the early time of the universe after the big bang\cite{4}. It aroused people’s a lot of interest. A powerful tool in studying the mechanism of particle production in hot QCD matter is the two-pion intensity interferometry. The interferometry analyses were first shown by Hanbury Brown and Twiss to measure the angular diameter of stars in the 1950s\cite{5}, so the method is named as HBT method. Then G. Goldhaber, S. Goldhaber, W. Lee and A. Pais extend this method in $p+p$ collisions\cite{6}. After that, the two-pion interferometry has made great efforts in high energy heavy ion collisions, and the method is also developed and improved a lot. For example, the HBT radii parameters may locate the Critical End Point (CEP) in QCD phase digram\cite{7}, and the multi-pion interferometry has been used in high-energy heavy-ion collisions, as an extension of two-pion interferometry\cite{8, 9, 10}.

Many collaborations use HBT method to analyze different collisions in different energies\cite{11, 12, 13, 14, 15}. Most of them show the phenomenon of transverse momentum or transverse mass dependence of HBT radii. We think the single-particle space-momentum angle distribution takes a main role in transverse momentum dependence of HBT radii, and this distribution is caused by flow. In this paper, we discuss the effect of single-particle space-momentum angle distribution of HBT radii in several sources. Then find a connection between the single-particle space-momentum angle distribution and the transverse momentum dependence of HBT radii, in transverse plane.

This paper is structured as follows. Sec. 2 briefly introduces the CRAB code and the method used to calculate the HBT radii. In Sec. 3, we calculate the HBT radii for pion in different sources. In Sec. 4, a numerical connection has been built between the single-particle space-momentum angle distribution and the transverse momentum dependence of $R_o$, $R_s$. Finally, we summarize our conclusions in Sec. 4.

2. CRAB code and methodology

In this paper, we use Correlation After Burner (Crab) code to read the phase-space information of generated pions and calculate the two-pion correlation functions\cite{16}. The code is based on the formula

$$C(q, K) = 1 + \frac{\int d^4x_1 d^4x_2 S_1(x_1, p_2)S_2(x_2, p_2)|\psi_{rel}|^2}{\int d^4x_1 d^4x_2 S_1(x_1, p_2)S_2(x_2, p_2)}, \quad (1)$$

where $q = p_1 - p_2$, $K = (p_1 + p_2)/2$, and $\psi_{rel}$ is the two particle wave function. In further discussion, we neglect the Coulomb interaction and strong interactions between pions. The correlation functions can be calculated in different $P_T$ bins by changing the kinematic cuts in the fitter of the CRAB code. And the information of single pion can
also be got in the calculation. Then we use it to analyses the space-momentum angle $\Delta \varphi$ distribution.

We usually use the ‘out-side-long’ (o-s-l) coordinate system in HBT research. The long direction is along the beam direction, and the transverse plane is perpendicular to the long direction. In the transverse plane, the momentum direction of pair particles is the out direction. And the direction, which is perpendicular to the out direction, is called side direction.

In this paper, when calculating the HBT correlation function, the rapidity range is always set to $-0.5 < \eta < 0.5$. We show an example of correlation functions of a Gaussian source in Figure 1. It is in $q_o$ and $q_s$ directions, and $-3 < q_t < 3$ MeV/c.

![Figure 1. HBT correlation function in $q_o$ and $q_s$ directions for a Gaussian source.](image)

The HBT correlation function of the Gaussian form can be written as

$$C(q, K) = 1 + \lambda \exp[-q_o^2 R_o^2(K) - q_s^2 R_s^2(K) - q_t^2 R_t^2(K)],$$

where $\lambda$ is coherence parameter. The $R_i^2$ can be expressed as

$$R_s^2 = \langle r_s^2 \rangle,$$

$$R_o^2 = \langle (r_o - \beta_ot)^2 \rangle - \langle r_o \rangle^2,$$

$$R_t^2 = \langle (r_1 - \beta_lt)^2 \rangle - \langle r_1 \rangle^2,$$

here the average notation is defined as

$$\langle \xi \rangle = \frac{\int d^4x \xi S(x, p)}{\int d^4x S(x, p)}.$$

We can calculate the HBT radii by using equation (2) to fit the HBT correlation function which is generated from CRAB code.
3. Transverse momentum dependence of HBT radii

We think the \( p \) and \( r \) angle distribution, also called \( \Delta \varphi \) angle distribution, can directly cause the transverse momentum \( P_t \) dependence. We will use several source models to prove our thoughts.

Firstly, we use a Gaussian source to generate data of pions. The space and freeze out time of pions are accorded with Gaussian distribution, and the momentum follow the Boltzmann distribution. The emission function can be written as

\[
S(x, p) = A p^2 \exp \left( -\frac{\sqrt{p^2 + m^2}}{T} \right) \exp \left( -\frac{r^2}{2R^2} - \frac{t^2}{2(\Delta t)^2} \right),
\]

(7)

where we always set source size \( R = 6.0 \text{ fm} \), temperature \( T = 100 \text{ MeV} \), and mass of pions \( m = 139.58 \text{ MeV}/c^2 \). Then use CRAB code to calculate the HBT correlation functions of pions. After that, we use equation (2) to fit the correlation functions in different \( P_t \) bins, and there are 9 bins in \( 125 \text{ MeV}/c < P_t < 625 \text{ MeV}/c \). The transverse momentum dependence of HBT radii are shown in Figure 2.

![Figure 2. Transverse momentum dependence of HBT radii for a Gaussian source.](image)

In Figure 2(a), the life time of source \( \Delta t = 0 \text{ fm/c} \), so all the pions freeze out at same time. One can see that HBT radii coincide with each other, and they are almost equal to the source radii. There is no transverse momentum dependence of HBT radii. The changes of \( P_t \) can not affect the HBT radii. While \( \Delta t = 6 \text{ fm/c} \), as shown in Figure 2(b), the value of \( R_o \) increases a lot, and it changes with the \( P_t \). There is only a little changes of \( R_l \) and \( R_s \). Therefore, the lifetime of source has a great influence on the the values of \( R_o \). Then we show the HBT radii by changing the value of \( \Delta t \) in Figure 3.

In Figure 3, the Gaussian source radius is still set to 6 fm, and the transverse momentum range is 125-625 MeV/c. We can see the increase of \( R_o \) at higher lifetime \( \Delta t \) of source. And because of the rapidity cut is \(-0.5 < \eta < 0.5 \), \( R_l \) only changes a little. There is barely no changes of \( R_s \). Since we have already know the emission function of
Gaussian source, by using equation (3)-(7), the HBT radius can be expressed as

\[ R_s^2 = \langle r_s^2 \rangle, \]
\[ R_o^2 = \langle r_o^2 \rangle + \langle \beta_o \rangle^2 \langle (\Delta t)^2 \rangle, \]
\[ R_l^2 = \langle r_l^2 \rangle + \langle \beta_l \rangle^2 \langle (\Delta t)^2 \rangle, \]

here, \( r \) and \( \beta \) are space coordinate and velocity of single particle. Therefore we can reduce the influence of the lifetime of source by decreasing the value of \( \Delta t \). When \( \Delta t = 0 \) fm/c in Gaussian source, no matter how the \( P_T \) changes, there is no appearance of the \( P_T \) dependence. And there is no correlation between the momentum and space of single particle.

For discussing the influence of single-particle angle distribution on the transverse momentum dependence of HBT radii, we introduce another source which is called space-momentum angle correlation source. The emission function can be written as

\[ S(x, p) = A p^2 \exp \left( -\frac{\sqrt{p^2 + m^2}}{T} \right) \exp \left( -\frac{r^2}{2R^2} - \frac{t^2}{2(\Delta t)^2} \right) w(\Delta \varphi), \]

where \( \Delta \varphi \) is the single-pion space-momentum angle at freeze-out time. By changing the formula of \( w(\Delta \varphi) \), we can change the angle \( \Delta \varphi \) distribution. If \( w(\Delta \varphi) = 1 \), the source become a Gaussian source, and the \( \Delta \varphi \) value is totally random between \( 0 - \pi \). Here, the function \( w \) in equation (11) can be written as

\[ w(\Delta \varphi) = \begin{cases} 
0 & \alpha < \Delta \varphi \leq \pi \\
1 & 0 \leq \Delta \varphi \leq \alpha 
\end{cases}, \]

where \( \alpha \) is a given value. This function means that, only the pions whose angle \( \Delta \varphi \) smaller than the \( \alpha \) value can exist. The radius of source is \( R = 6 \) fm, and the lifetime is \( \Delta t = 0 \) fm/c. Then we plot Figure 4.
In Figure 4(a), $\alpha = \frac{\pi}{2}$. $R_o$ values are lower than $R_s$ and $R_l$, and the HBT radii are all become straight lines. There is barely any appearance of $P_T$ dependence. In Figure 4(b), $\alpha = \frac{\pi}{3}$. Comparing with the Figure 4(a), the value of $R_s$ and $R_l$ become smaller. The value of $R_o$ only changes little. The present work shows that, the $\Delta \varphi$ distribution can affect the value of HBT radii. Further more, we can give the HBT radii changes by the $\cos \alpha$ value, which is shown in Figure 5.

In Figure 5, the transverse momentum range is 125-625 MeV/c. One can see that HBT radii have regular changes. When $-1 < \cos \alpha < 0$, the $R_o$ values decrease with the increase of $\cos \alpha$, while $R_s$ and $R_l$ only have little changes. When $0 < \cos \alpha < 1$, there is almost no changes of $R_o$, while $R_s$ and $R_l$ decrease with the increase of $\cos \alpha$. The different space-momentum angle $\Delta \varphi$ distributions corresponding to different HBT
radii. Therefore, if we can control the $\Delta \varphi$ angle distribution, we can reproduce the $P_T$ dependence phenomenon.

We use homogeneous expansion source to calculate the HBT radii in different $P_T$ sections, then we use space-momentum angle correlation source to rebuild this phenomenon. The homogeneous expansion source is based on the Gaussian source. Every pion has given an expansion velocity $\beta$ along the $r$ direction. And the momentum is generated by using Lorentz transformation. The emission function can be written as

$$S = A p^2 \exp \left( -\frac{\gamma E - \gamma \beta p}{T} \right) \exp \left( -\frac{r^2}{2R^2} - \frac{t^2}{2(\Delta t)^2} \right),$$  \hspace{1cm} (13)

where $\gamma = 1/\sqrt{1 - \beta^2}$ is the Lorentz factor, still $R = 6$ fm and the $\Delta t = 0$ fm/c. After using it to generate data, we use CRAB code to calculate correlations in different $P_T$ sections. Meanwhile, we can also get the phase-space information of pions in different $P_T$ sections. The equation to fit the normalized space-momentum angle distribution can be written as

$$f(\Delta \varphi) = c_1 \exp(c_2 \cos(\Delta \varphi)).$$  \hspace{1cm} (14)

Where, $c_1$ and $c_2$ are fit parameters. One fit process is shown in Figure 6.

![Figure 6](image)

**Figure 6.** Fit ($\Delta \varphi$) process for a homogeneous expansion source. The expansion speed is $\beta = 0.9$ and the transverse momentum range is 125-175 MeV/c. The black dots are normalized numbers of pions, and the red line is the fit line.

In different $P_T$ sections, we can get a series of two parameters $c_1$, $c_2$. In space-momentum angle correlation source, we let

$$w(\Delta \varphi) = f(\Delta \varphi) = c_1 \exp(c_2 \cos(\Delta \varphi)).$$  \hspace{1cm} (15)

After the calculation, the simulation results and the HBT radii calculated by homogeneous expansion source are shown in Figure 7.

In Figure 7(a) and Figure 7(b), one can see that the quality of the simulations are good. And the HBT radii in Figure 7(b) are smaller than the HBT radii in Figure 7(a). It because the expansion speed increases, and the $\Delta \varphi$ angles are all decrease in different $P_T$ sections, than lead the HBT radii decreases. The simulation results indicate that, the source expansion can cause the space-momentum angle $\Delta \varphi$ distribution changing with
transverse momentum. And the different space-momentum angle $\Delta \varphi$ distributions can cause different values of HBT radii, then cause the transverse momentum dependence of HBT radii.

4. Space-momentum angle distribution in transverse plane

We use cylinder expansion source\cite{19} to get the connection between the HBT radii and $\Delta \theta$(angle between $\vec{P}_T$ and $\vec{r}_T$) distribution. It can be written as

$$S(x, p) = AM_T \cosh(\eta - Y) \exp \left( -\frac{mu(x)}{T} \right) \exp \left( -\frac{(\tau - \tau_0)^2}{2(\delta \tau)^2} - \frac{\rho^2 - 2R_g^2}{2(\delta \eta)^2} \right),$$  \hspace{1cm} (16)

where $u(x)$ is the 4-velocity and can be decomposed as

$$u(x) = (\cosh \eta \cosh \eta_T, \sinh \eta_T \vec{e}_T, \sinh \eta \cosh \eta_T),$$  \hspace{1cm} (17)

and $\eta = \frac{1}{2} \ln[(p + z)/(p - z)]$ is the longitudinal flow rapidities. The transverse flow rapidity is defined as

$$\eta_T = \begin{cases} 
\eta_{t\max} \frac{\rho}{R_g}, & \rho < R_g \\
\eta_{t\max}, & \rho \geq R_g 
\end{cases},$$  \hspace{1cm} (18)
The rapidity of the pion is \( Y = \frac{1}{2} \ln[(E + R)/(E - R)] \), and the proper time is \( \tau = \sqrt{t^2 - z^2} \). We set \( T = 100 \) MeV, \( \delta \tau = 0 \) fm/c, \( \tau_0 = 10 \) fm/c, \( R_b = 6.0 \) fm and \( \delta \eta = 3.0 \), and the variable is \( \eta_{\text{max}} \).

Since the CRAB filter is set \(-0.5 < \eta < 0.5\) and \( \delta \tau = 0 \) fm/c, all pions almost freeze out at the same time \((\Delta t < 1.3 \text{fm/c})\). The effect of the source lifetime is negligible. \( \beta_{\text{max}} \) are set as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, then the \( \eta_{\text{max}} \) values are calculated by

\[
\eta_{\text{max}} = \frac{1}{2} \ln \left( \frac{1 + \beta_{\text{max}}}{1 - \beta_{\text{max}}} \right).
\]

We can fit the HBT radii in different \( P_T \) sections by

\[
R = a P_T^b,
\]

where \( a \) and \( b \) are fit parameters, parameter \( b \) describe the strength of \( P_T \) dependence, the lager of \( |b| \), the more obvious phenomenon of \( P_T \) dependence. The distribution of \( \cos(\Delta \theta) \) is divided by the same number of pions cosine value distribution which calculated by random \( P_T \) and \( r_T \), to get the normalized \( \cos(\Delta \theta) \) distribution. We fit normalized \( \cos(\Delta \theta) \) distribution with equation (14), and the fit results are list in Table 1.

**Table 1.** Fit results of normalized \( \cos(\Delta \theta) \) distribution.

| \( \eta_{\text{max}} \) | par | \( P_T \) \( 150-250 \) MeV | 250-350 MeV | 350-450 MeV | 450-600 MeV |
|---|---|---|---|---|---|
| 0.1003 | \( c_1 \) | 0.9994 ± 0.0002 | 0.9990 ± 0.0003 | 0.9981 ± 0.0003 | 0.9976 ± 0.0003 |
| | \( c_2 \) | 0.0339 ± 0.0003 | 0.0496 ± 0.0004 | 0.0614 ± 0.0004 | 0.0852 ± 0.0005 |
| 0.2027 | \( c_1 \) | 0.9985 ± 0.0002 | 0.9974 ± 0.0003 | 0.9954 ± 0.0003 | 0.9924 ± 0.0003 |
| | \( c_2 \) | 0.0663 ± 0.0003 | 0.0990 ± 0.0004 | 0.1299 ± 0.0004 | 0.1621 ± 0.0005 |
| 0.3095 | \( c_1 \) | 0.9969 ± 0.0002 | 0.9937 ± 0.0003 | 0.9905 ± 0.0003 | 0.9851 ± 0.0003 |
| | \( c_2 \) | 0.1043 ± 0.0003 | 0.1485 ± 0.0004 | 0.1903 ± 0.0004 | 0.2316 ± 0.0005 |
| 0.4236 | \( c_1 \) | 0.9940 ± 0.0002 | 0.9887 ± 0.0003 | 0.9836 ± 0.0003 | 0.9775 ± 0.0003 |
| | \( c_2 \) | 0.1443 ± 0.0004 | 0.1990 ± 0.0004 | 0.2446 ± 0.0004 | 0.2842 ± 0.0005 |
| 0.5493 | \( c_1 \) | 0.9890 ± 0.0002 | 0.9829 ± 0.0003 | 0.9766 ± 0.0003 | 0.9718 ± 0.0003 |
| | \( c_2 \) | 0.1914 ± 0.0004 | 0.2467 ± 0.0004 | 0.2893 ± 0.0008 | 0.3186 ± 0.0005 |
| 0.6931 | \( c_1 \) | 0.9817 ± 0.0003 | 0.9735 ± 0.0003 | 0.9683 ± 0.0003 | 0.9631 ± 0.0004 |
| | \( c_2 \) | 0.2450 ± 0.0004 | 0.2945 ± 0.0004 | 0.3270 ± 0.0005 | 0.3480 ± 0.0006 |

From the fit results, with increase of \( P_T \) and \( \eta_{\text{max}} \), \( c_1 \) become smaller and \( c_2 \) become bigger. We find \( c_1 \) and \( c_2 \) can be fitted by

\[
\begin{align*}
    c_1 &= k_1 P_T^{k_1}, \\
    c_2 &= k_2 P_T^{k_2}
\end{align*}
\]

(20) (21)

where \( k \) and \( \kappa \) are fit parameters. These parameters are plotted in Figure 8.

In figure 8 one can see regular changes of parameters. And because of the longitudinal limit, there is barely no changes of \( b_{\text{long}} \). The parameters in out and side directions are basically the same because the source lifetime is small enough. It indicate that there is connection between the HBT radii and the \( \Delta \theta \) distribution. The red lines
Figure 8. Fit parameters in cylinder expansion source. Parameter $b$ from HBT radii fit function $R = ae^{bP_T}$, $\kappa$ and $k$ are from fit function $c = ke^{\kappa P_T}$, red lines are fit lines.
are fit lines and the fit functions are

\begin{align*}
  b(k_1) &= \mu_{11} k_1^{\mu_{12}}, \\
  b(\kappa_1) &= \nu_{11} e^{-\nu_{12} \kappa_1}, \\
  b(k_2) &= \mu_{21} \ln k_2 + \mu_{22}, \\
  b(\kappa_2) &= \nu_{21} \left( \frac{1}{1 + e^{\nu_{22} \kappa_2}} - 1 \right),
\end{align*}

the fit parameters values are shown in Table 2.

| Parameters | \( b_o \) | \( b_s \) |
|------------|-----------|-----------|
| \( c_1 \)  |           |           |
| \( \mu_{11} \) | -0.026 ± 0.001 | -0.021 ± 0.001 |
| \( \mu_{12} \) | -24.3 ± 0.5 | -28 ± 1 |
| \( \nu_{11} \) | -0.034 ± 0.002 | -0.028 ± 0.002 |
| \( \nu_{12} \) | 97 ± 3 | 107 ± 4 |
| \( c_2 \)  |           |           |
| \( \mu_{21} \) | -0.0478 ± 0.0006 | -0.0514 ± 0.0006 |
| \( \mu_{22} \) | -0.418 ± 0.004 | -0.437 ± 0.004 |
| \( \nu_{21} \) | 1.17 ± 0.02 | 1.25 ± 0.02 |
| \( \nu_{22} \) | -3.09 ± 0.03 | -3.21 ± 0.04 |

A connection has been made between the \( P_T \) dependence of HBT radii and the \( \cos(\Delta \theta) \) distribution by equation (22)-(25) and Table 2 in a cylinder source. When we get a serious of data of HBT radii in different \( P_T \) sections, if all the pions freeze out almost in the same time, we can describe the space-momentum angle distribution in different \( P_T \) sections in transverse plane, and vice versa. Because we limit the life time of source, the eight parameters of \( b_o \) are similar to the parameters of \( b_s \). If the source life time increase, \( R_o \) will also increase, the parameters of \( b_o \) are no longer suitable. And if we change the model, the value of parameters will also change.

5. Conclusions

By using a lot of source models, we analyze the effect of source life time and single-particle space-momentum angle distribution on HBT radii. In middle rapidity section, \( R_o \) is sensitive to the source lifetime. \( R_o \) increase a lot at higher \( \Delta t \). Further more, the HBT radii are also sensitive to the single-particle space-momentum angle distribution. By controlling the angle distribution, HBT radii will have regular changes. The collective expansion of the source leads the changes of the single-particle space-momentum angle distribution with different \( P_T \), then causes the changes of HBT radii, at last, creates the transverse momentum dependence of HBT radii. In transverse plane of a cylinder expansion source, a numerical connection between the transverse momentum dependence of HBT radii and the the single-particle space-momentum angle distribution has been created. The parameters will change with different sources. If the parameters
are settled, we can describe the single-particle space-momentum angle distribution by the transverse momentum dependence of HBT radii.

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References

[1] Cabibbo N and Parisi G 1975 Physics Letters B 59 67 – 69
[2] Back B, Baker M, Ballintijn M et al. 2005 Nuclear Physics A 757 28 – 101
[3] Shi S 2009 Nuclear Physics A 830 187c – 190c
[4] Satz H 2001 Nuclear Physics B - Proceedings Supplements 94 204 – 218
[5] Brown R H and Twiss R 1954 The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 45 663–682
[6] Goldhaber G, Goldhaber S, Lee W and Pais A 1960 Phys. Rev. C 120(1) 300–312
[7] Lacey R A 2015 Phys. Rev. Lett. 114(14) 142301
[8] Biyajima M 1981 Prog. Theor. Phys 66(5) 1378–1388
[9] Liu Y M, Beavis D, Chu S Y, Fung S Y, Keane D, VanDalen G and Vient M 1986 Phys. Rev. C 34(5) 1667–1672
[10] Bary G, Ru P and Zhang W N 2018 Journal of Physics G: Nuclear and Particle Physics 45 065102
[11] Aamodt K, Quintana A A, Adamov D et al. 2011 Physics Letters B 696 328 – 337
[12] Adams J, Aggarwal M M, Ahammed Z et al. (STAR Collaboration) 2005 Phys. Rev. C 71(4) 044906
[13] Kniege S and (for the NA49 Collaboration) 2004 Journal of Physics G: Nuclear and Particle Physics 30 S1073–S1077
[14] Aamodt K, Abrahantes Quintana A, Adamová D et al. (ALICE Collaboration) 2011 Phys. Rev. D 84(11) 112004
[15] Adamczyk L, Adkins J K, Agakishiev G et al. (STAR Collaboration) 2015 Phys. Rev. C 92(1) 014904
[16] Pratt S 2006 Crab version 3.0 https://web.pa.msu.edu/people/pratts/freecodes/crab/home.html
[17] Alexander G 2003 Reports on Progress in Physics 66 481–522
[18] Heinz U and Jacak B V 1999 Annual Review of Nuclear and Particle Science 49 529–579
[19] Wiedemann U A, Scotto P and Heinz U 1996 Phys. Rev. C 53(2) 918–931