RARE KAON DECAY $K^+ \rightarrow \pi^+ \nu \bar{\nu}$
IN SU($3)_C \otimes$ SU($3)_L \otimes U(1)_N$ MODELS

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Abstract

The rare kaon decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is considered in the framework of the models based on the SU($3)_C \otimes$ SU($3)_L \otimes U(1)_N$ (3 - 3 - 1) gauge group. It is shown that a lower bound of the $Z'$ mass in the 3 - 3 - 1 model with right-handed neutrinos at a value of 3 TeV is derived, while that in the minimal version – 1.7 TeV.

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I. Introduction

The kaon is the lightest hadron having a non-zero strangeness quantum number. Due to the weak interactions the kaon decays weakly into states with zero strangeness, containing pions, photons and/or leptons. The physics of kaons has played a major role in the development of particle physics. The concept of strangeness, with its implications for the quark model, the discovery of $P$ and $CP$ violation and the GIM mechanism have all emerged from the study of K mesons. Today, rare kaon decays continue to be an active field of investigation [1]. Flavour changing neutral currents (FCNC) are completely suppressed at the tree level by the GIM mechanism in the standard model (SM). In second or higher order interactions, this suppression is not complete due to the different quark masses [2].

The first experimental evidence for atmospheric neutrino oscillations and consequently a non-zero neutrino mass observed at the SuperKamiokande Collaboration calls for the SM extension. Among the possible models, those based on the SU($3)_C \otimes$ SU($3)_L \otimes U(1)_N$ (3 – 3 – 1) gauge group [3][4], contain a number of intriguing features: First, the models predict three families of quarks and leptons if the anomaly free condition on SU($3)_L \otimes U(1)_N$ and the
QCD asymptotic freedom are imposed. Second, the Peccei-Quinn symmetry naturally occurs 
 in these models [7]. The third interesting point is that one generation of quarks is treated 
differently from two others. This could lead to a natural explanation for the unbalancingly 
heavy top quark. This family nonuniversality leads also to the FCNC by the Z' currents at 
the tree level [8, 9]. Finally, the 3 - 3 - 1 models predict new physics at a scale only slightly 
above the SM scale (a few TeVs) [8–11].

In this work we consider the implications of main two 3 - 3 - 1 models for the rare 
\( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) decay, and our aim is to get a bound on the Z' mass.

II. The rare kaon decay \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) in 3 - 3 - 1 models

A. The decay in 3 – 3 – 1 model with right-handed neutrinos

We first recapitulate the basic elements of the model. The leptons in this model are 
arranged into triplets, with the third member being a right-handed neutrino [5, 6]:

\[
f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (\nu'_L)^a \end{pmatrix} \sim (1, 3, -1/3), e_R^a \sim (1, 1, -1),
\]

where \( a = 1, 2, 3 \) is the family index.

The first two families of quarks are in antitriplets and the third one is in a triplet:

\[
Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ D_{iL} \end{pmatrix} \sim (3, 3, 0),
\]

\( u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), \ i = 1, 2, \)

\[
Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3),
\]

\( u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3). \)

The gauge bosons in this model are the photon (A), Z, Z', W\( ^\pm \), Y\( ^\pm \) and complex neutral 
bosons \( X^0 \) and \( X'^0 \),

\[
\sqrt{2} W^+_{\mu} = W^1_{\mu} - iW^2_{\mu}, \sqrt{2} Y^- = W^6_{\mu} - iW^7_{\mu},
\]

\[
\sqrt{2} X^0_{\mu} = W^4_{\mu} - iW^5_{\mu},
\]

\[
A_{\mu} = s_W W^3_{\mu} + c_W \left( -\frac{t_W}{\sqrt{3}} W^8_{\mu} + \sqrt{1 - \frac{t^2_W}{3}} B_{\mu} \right),
\]

\[
Z_{\mu} = c_W W^3_{\mu} - s_W \left( -\frac{t_W}{\sqrt{3}} W^8_{\mu} + \sqrt{1 - \frac{t^2_W}{3}} B_{\mu} \right),
\]

\[
Z'_{\mu} = \sqrt{1 - \frac{t^2_W}{3}} W^8_{\mu} + \frac{t_W}{\sqrt{3}} B_{\mu},
\]
where we use the following notations: \( s_W \equiv \sin \theta_W \) and \( t_W \equiv \tan \theta_W \). The physical states are a mixture of \( Z \) and \( Z' \):

\[
Z_1 = Z \cos \phi - Z' \sin \phi, \\
Z_2 = Z \sin \phi + Z' \cos \phi,
\]

where \( \phi \) is a mixing angle.

The interactions among fermions and \( Z_1, Z_2 \) are given by

\[
\mathcal{L}^{NC} = \frac{g}{2c_W} \left\{ \tilde{f} \gamma^\mu [a_{1L}(f)(1 - \gamma_5) + a_{1R}(f)(1 + \gamma_5)] f Z_1^\mu \\
+ \tilde{f} \gamma^\mu [a_{2L}(f)(1 - \gamma_5) + a_{2R}(f)(1 + \gamma_5)] f Z_2^\mu \right\},
\]

where

\[
a_{1L,R}(f) = \cos \phi \left[ T^3(f_{L,R}) - s_W^2 Q(f) \right] \\
- c_W^2 \left[ \frac{3N(f_{L,R})}{(3 - 4s_W^2)^{1/2}} - \frac{(3 - 4s_W^2)^{1/2}}{2c_W^2} \right] \sin \phi,
\]

\[
a_{2L,R}(f) = c_W^2 \left[ \frac{3N(f_{L,R})}{(3 - 4s_W^2)^{1/2}} - \frac{(3 - 4s_W^2)^{1/2}}{2c_W^2} \right] \cos \phi \\
+ \sin \phi \left[ T^3(f_{L,R}) - s_W^2 Q(f) \right],
\]

where \( T^3(f) \) and \( Q(f) \) are, respectively, the third component of the weak isospin and the charge of the fermion \( f \). The mixing angle \( \phi \) is constrained to be very small \( 8 \times 10^{-3} \leq \phi \leq 1.8 \times 10^{-4} \) and can therefore be neglected.

Because one family of left-handed quarks is treated differently from the other two, the \( N \) charges for left-handed quarks are also different (see Eq. (3)). Therefore, the flavour-changing neutral current \( Z' \) occurs through a mismatch between weak and mass eigenstates. We diagonalize mass matrices by three biunitary transformations

\[
U_L' = V_L^U U_L, \quad U_R' = V_R^U U_R, \\
D_L' = V_L^D D_L, \quad D_R' = V_R^D D_R,
\]

where \( U \equiv (u, c, t)^T \), and \( D \equiv (d, s, b)^T \). The usual Cabibbo-Kobayashi-Maskawa matrix is given by

\[
V_{CKM} = V_L^{U+} V_L^D.
\]

Using unitarity of the \( V^D \) and \( V^U \) matrices, we obtain flavour-changing neutral interactions

\[
\mathcal{L}_{ds}^{NC} = \frac{g c_W}{2 \sqrt{3 - 4s_W^2}} \left[ V^D_{Lis} V^D_{Lid} \right] \bar{d}_L \gamma^\mu s_L Z_\mu',
\]

where \( i \) denotes the number of “different” quark family i.e. the SU(3)$_L$ quark triplet. It was shown in Ref. [9] that \( i \) must be equal to 3, i.e. the third family of quarks must be different from the first two.
We consider the decay
\[ K^+(p1) \to \pi^+(p2) \nu(k1) \bar{\nu}(k2), \tag{10} \]
where the symbols in parentheses stand for the momenta of the particles. The one-loop effective SM Lagrangian for this process was calculated by Inami and Lim and other authors \[2\].

Due to family nonuniversality in the 3 - 3 - 1 models, the decay can be mediated by the \( Z' \) at the tree level. The Feynman diagram contributing to the considered decay is depicted in Fig. 1

![Feynman diagram](image)

Figure 1: Feynman diagram for \( K^+ \to \pi^+ \nu \bar{\nu} \)
in the 3 3 1 models

The decay amplitude is given by
\[ \mathcal{M}(K^+ \to \pi^+ \nu \bar{\nu}) = \frac{G_F m_W^2}{\sqrt{2} M_{Z'}^2} V^D_{Lbd} V^D_{Lbs} \langle \pi^+(p2)|\bar{s}_L \gamma_\mu d_L|K^+(p1)\rangle \bar{\nu}_L(k1) \gamma^\mu \nu_L(k2), \tag{11} \]

where \( m_W \) and \( M_{Z'} \) stand for masses of the \( W \) and \( Z' \) bosons, respectively.

For our initial purpose we present the well measured semileptonic decay \( K^+(p1) \to \pi^0(p2)e^+(k1)\nu(k2) \). The tree-level amplitude for this process can be written as
\[ \mathcal{M}(K^+ \to \pi^0 e^+ \nu) = \frac{G_F}{\sqrt{2}} V^*_{us} \langle \pi^0(p2)|\bar{s}_L \gamma_\mu u_L|K^+(p1)\rangle \bar{\nu}_e(k1) \gamma^\mu e_L(k2). \tag{12} \]

The isospin symmetry relates hadronic matrix elements in (11) to (12) to a very good precision \[12\]
\[ \langle \pi^+(p2)|\bar{s}_L \gamma_\mu d_L|K^+(p1)\rangle = \sqrt{2} \langle \pi^0(p2)|\bar{s}_L \gamma_\mu u_L|K^+(p1)\rangle. \tag{13} \]

Neglecting differences in the phase space of two considered decays occurring because \( m_{\pi^+} \neq m_{\pi^0} \) and \( m_e \neq 0 \), we sum over three neutrino flavours and obtain
\[ \frac{B_{r \text{hnn}}(K^+ \to \pi^+ \nu \bar{\nu})}{B_r(K^+ \to \pi^0 e^+ \nu)} = 6 \left( \frac{m_W^2}{M_{Z'}^2} \right)^2 \frac{\left| V^D_{Lbd} V^D_{Lbs} \right|^2}{\left| V^*_{us} \right|^2}, \tag{14} \]

where the symbol \( r \text{hnn} \) added to the branching ratio indicates the case under consideration.

We now apply the simple Fritzsch \[13\] scheme as
\[ V^D_{ij} \approx \left( \frac{m_i}{m_j} \right)^{1/2}, \quad i < j. \tag{15} \]
Inserting (13) into (14), we obtain

\[ Br^{\text{rhn}}(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = 6 \left( \frac{m_W^2}{M_{Z'}^2} \right)^2 \left( \frac{m_d m_s}{m_d^2 V_{us}^2} \right) Br(K^+ \rightarrow \pi^0 e^+ \nu). \] (16)

In Fig. 2 we plot \( Br^{\text{rhn}} \) as a function of \( M_{Z'} \), using the data \[ m_W = 80.41 \text{ GeV}, \ |V_{us}| = 0.2196, \ m_d = 7 \text{ MeV}, \]
\[ m_s = 115 \text{ MeV}, \ m_b = 4.3 \text{ GeV}, \ Br(K^+ \rightarrow \pi^0 e^+ \nu) = 4.42 \times 10^{-2}. \] (17)

The horizontal lines are the upper \( (4.9 \times 10^{-10}) \) and the lower \( (0.3 \times 10^{-10}) \) experimental data \[ 15\].

From the figure we see that a lower bound on the \( Z' \) mass is in a range from 2.3 TeV to 4.35 TeV. This bound is approximately twice bigger than that derived from the mass difference of the kaon mixing system \( \Delta m_K \). We thus arrive at the previous conclusion again: for the \( Z' \) mass to be relatively low, the third family of quarks must be different from the other two.

B. The decay in minimal 3 – 3 – 1 model

This model treats the leptons as \( SU(3)_L \) antitriplets \[ 4, 10 \], with the third element being the antilepton (the name of this version comes from the fact that no new leptons are introduced)

\[ f^a_L = \begin{pmatrix} e^a_L \\ -\nu^a_L \end{pmatrix} \sim (1, \bar{3}, 0). \] (18)

Of the nine gauge bosons \( W^a(a = 1, 2, ..., 8) \) and \( B \) of \( SU(3)_L \) and \( U(1)_N \), four are light: the photon \( (A) \), \( Z \) and \( W^\pm \). The remaining five correspond to new heavy gauge bosons \( Z' \) and \( Y^\pm \) and the doubly charged bileptons \( X^{\pm\pm} \). They are expressed in terms of \( W^a \) and \( B \) as \[ 10 \]:

\[ \sqrt{2} W^\mu_+ = W^\mu_1 - iW^\mu_2, \quad \sqrt{2} Y^\mu_+ = W^\mu_6 - iW^\mu_7, \]
\[ \sqrt{2} X^{\mu+} = W^\mu_4 - iW^\mu_5, \]
\[ A_\mu = s_W W^\mu_2 + c_W \left( \sqrt{3} t_W W^\mu_8 + \sqrt{1 - 3 t_W^2} B_\mu \right), \]
\[ Z^\mu = c_W W^\mu_3 - s_W \left( \sqrt{3} t_W W^\mu_8 + \sqrt{1 - 3 t_W^2} B_\mu \right), \]
\[ Z^\mu_\prime = -\sqrt{1 - 3 t_W^2} W^\mu_8 + \sqrt{3} t_W B_\mu. \] (19)

As before, the physical states are a mixture of \( Z \) and \( Z' \),

\[ Z_1 = Z \cos \phi - Z' \sin \phi, \]
\[ Z_2 = Z \sin \phi + Z' \cos \phi, \]
and the mixing angle $\phi$ is also bounded to be very small. We can therefore assume $\phi \approx 0$. Applying Eq. (4.4) in [10] we obtain the interactions among the $Z'$ and neutrinos,

$$a'_L(\nu) = -a'_A(\nu) = \frac{1}{2\sqrt{3}} \sqrt{1 - 4s_W^2}. \quad (20)$$

One necessary vertex, namely the FCNC is given in [8],

$$\mathcal{L}_{ds}^{NC} = \frac{g_{CW}}{2\sqrt{3}(1 - 4s_W^2)} \left[V_{Lbd}^* V_{Lis}^D \bar{d}_L \gamma^\mu s_L Z'_\mu \right]. \quad (21)$$

Combining (20) and (21) we obtain the decay amplitude

$$\mathcal{M}^{\text{min}}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{1}{3} \frac{G_{F} m_W^2}{\sqrt{2} M_{Z'}} \left[V_{Lbd}^* V_{Lis}^D \langle \pi^+(p2) | \bar{s}_L \gamma_\mu d_L | K^+(p1) \rangle \right] \bar{\nu}_L(k1) \gamma^\mu \nu_L(k2). \quad (22)$$

From Eq. (22) it is straightforward to obtain

$$\frac{Br^{\text{min}}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{Br(K^+ \rightarrow \pi^0 e^+ \nu)} = \frac{2}{3} \left(\frac{m_W^2}{M_{Z'}^2}\right)^2 \left(\frac{|V_{Lbd}^* V_{Lis}^D|^2}{|V_{us}^*|^2}\right). \quad (23)$$

As in the previous section, in Fig. 2 we also plot $Br^{\text{min}}$ as a function of $M_{Z'}$. As a consequence, the lower bound on the $Z'$ mass is in a range from 1.25 TeV to 2.45 TeV. This bound is bigger than the one derived from the mass difference of the kaon mixing system $\Delta m_K$ (see Dumm D. G. et al in [8]). For the $Z'$ mass to be relatively low, the third family of quarks must be different from the other two. It is worth mentioning that the branching ratio is not sensitive to the value of $\sin^2 \theta_W$, while the expression of $\Delta m_K$ in the minimal version is very sensitive due to a factor $\frac{1}{1 - 4s_W^2}$.

III. Conclusions

We have considered the rare kaon decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the 3 - 3 - 1 models at the tree level. It was shown that in the model involving right-handed neutrinos, the decay width is by about one order bigger than that in the minimal version. As a result, we obtained bounds on the $Z'$ mass in the range from 2.3 TeV to 4.3 TeV in the model with right-handed neutrinos and from 1.2 TeV to 2.4 TeV in the minimal version. There is a point worth noting: these mass limits are in agreement with recent analysis [10] showing that there are indications of the $Z'$ in electroweak precision data. We do hope that the new experimental data from the Collaborations at BNL and Fermilab will bring new indications on the Extra neutral gauge boson $Z'$ - one of the best motivated extension of the SM.

In this work, we consider only the CP conserving kaon decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Implications for the CP violating K and B decays are subjects of future studies.

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Figure 2: Branching ratio ($Br$) as a function of $M_{Z'}$. 