Cosmic ray protons in the energy range $10^{16} - 10^{18.5}$ eV: stochastic gyroresonant acceleration in hypernova shocks?

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ABSTRACT
The hypernovae (HNe) associated with Gamma-ray Bursts (GRBs) may have a fairly steep energy-velocity distribution, i.e., $E(\geq \beta) \propto \beta^{-q}$ for $q < 2$ and $\beta \geq \beta_0$, where $\beta$ is the velocity of the material and $\beta_0 \sim 0.1$ is the velocity of the slowest ejecta of the HN explosion, both in units of the speed of light ($c$). The cosmic ray protons above the second knee but below the ankle may be accelerated by the HN shocks in the velocity range of $\beta \sim \beta_0 - 4\beta_0$. When $\beta \leq 4\beta_0$, the radius of the shock front to the central engine is very large and the medium decelerating the HN outflow is very likely to be homogeneous. With this argument, we show that for $q \sim 1.7$, as inferred from the optical modelling of SN 2003lw, the stochastic gyroresonant acceleration model can account for the spectrum change of high energy protons around the second knee. The self-magnetized shock acceleration model, however, yields a too much steep spectrum that is inconsistent with the observation unless, the medium surrounding the HN is a free wind holding up to a (unrealistic large) radius $\sim 1 - 10$ kpc or alternatively the particle acceleration mainly occurs in a narrow “dense” shell that terminates the free wind at a radius $\sim 10^{19}$ cm.

Key words: acceleration of particles – cosmic rays – Gamma Rays: bursts – supernovae: general – supernova remnants

1 INTRODUCTION
One of the most widely suggested sources of cosmic rays (CRs) is the supernova (SN) remnants (see Hillas 2005 for a review). Since 2001, more and more researchers have noted that hypernovae (HNe), in particular those associated with gamma-ray bursts (GRBs), may play an important role in interpreting the CR spectrum above its first knee$^1$, i.e., $\sim 3 \times 10^{15}$ eV (Dermer 2001a, 2001b; Erlykin, Wibig and Wolfendale 2001; Sveshnikova 2003; Wick, Dermer and Atoyan 2004; Wang et al. 2007; Budnik et al. 2008). This is reasonable since the average velocity and the total kinetic energy of HN outflows are much larger than that of normal SNe. So is the energy of accelerated particles. A reliable interpretation of CR spectrum up to $\sim 10^{18}$ eV thus should take into account the different energies and types among SNe (Sveshnikova 2003).

How to accelerate protons up to an energy $\sim 10^{19}$ eV in the HN blast waves? Dermer (2001b) suggested that the gyroresonant stochastic acceleration might play such a role (see Fig.10 of Dermer 2001a for a quantitative plot). Other authors (Wang et al. 2007; Budnik et al. 2008; Erlykin et al. 2001) considered the self-magnetized acceleration model put forward by Bell & Lucek (2001), in which the magnetic field of the upstream has been significantly amplified by CRs. Considering the energy distribution of the HN outflow$^2$, Wang et al. (2007) and Budnik et al. (2008) suggested that with the second model the CR proton spectrum steepening around the second (first) knee could be reproduced. In this work, we point out one potential limit of such an interpretation and show that the gyroresonant stochastic acceleration model does not suffer from that problem.

This paper is arranged as follows. In section 2 we discuss the energy-velocity distribution of HN outflows and the medium profile surrounding the HN outflows. We find that for HN outflows owning a fairly steep energy-velocity distribution, when $\beta_0 \leq \beta \leq 4\beta_0$ that may play the main role in

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$^1$ The second knee and the ankle in the CR spectrum are at $\sim 3 \times 10^{17}$ eV and $\sim 3 \times 10^{18}$ eV, respectively.

$^2$ See Berezhko & Völk (2004) and Ptuskin & Zirakashvili (2005) for the influence of the energy distribution of normal SN outflows on the spectrum of accelerated CRs.
accelerating the CR protons above the second knee but below the ankle (see the discussions below eq.(8) and eq.(16)), the radius of the shock front to the central engine is very large and the medium decelerating the HN outflow is likely to be homogeneous. In section 3 we calculate the change of the CR spectrum around the second knee which is caused by the energy-velocity distribution of HN outflows, and compare the results with the CR spectrum observation so as to constrain the models. In Section 4, we summarize our results with some discussions.

2 THE HYPERNOVA OUTFLOW: ENERGY-VELOCITY DISTRIBUTION AND THE MEDIUM IT EXPANDS INTO

2.1 Energy-velocity distribution of hypernova outflows

HNe, especially those of which associated with GRBs/XRFs, are distinguished for the broad lines in their spectra, indicating very high expansion velocity of the ejecta. The modelling of optical light curves and spectra, in principle, can reconstruct the energy-velocity distribution of the outflows. However, no reliable constraint can be given on the \( \beta > 0.3 \) outflow by optical even if that part had some optical depth since the current optical modelling is not fully relativistic (J. S. Deng 2008, private communication). In SN 1998bw, SN 2003dh, SN 2003lw and SN 2006aj, strong photospheric velocity evolution (the inner the outflow, the smaller the velocity) are evident (Hjorth et al. 2003; Mazzali et al. 2006; Soderberg et al. 2006). The optical modelling of SN 2003lw gave that material moving faster than \((0.1 - 0.2)c\) were \((1.4 - 0.1)\, M_\odot\), respectively (Mazzali et al. 2006), implying a fairly steep initial kinetic energy distribution \( E(\geq \Gamma \beta) \propto (\Gamma \beta)^{-1.7} \), where \( \Gamma = (1 - \beta^2)^{-1/2} \). But in other events, no result has been published. Soderberg et al. (2006) constrained the kinetic energy profile of HN outflows in a more speculative way. They used optical spectral data to probe the slowest ejecta in supernova explosions and employed the radio observation to trace the fastest component of the outflow. They then took these two data points to estimate the energy-velocity distribution. Their results may be biased because the fast moving material identified by radio observation might be the decelerated GRB/XRF ejecta rather than the fastest component of the main SN explosion. If so, it is not a continuous distribution of matter between the two data points (Soderberg et al. 2006; Xu et al. 2008).

Fairly speaking, observationally so far we do not have a reliable estimate of the initial kinetic energy-velocity distribution of (most) HN outflows in the velocity range of \( \beta \sim 0.1 - 0.5 \). Theoretically the standard hydrodynamic collapse of a massive star (Tan, Matzner & McKee 2001) results in a kinetic energy profile of the SN explosion \( E(\geq \Gamma \beta) \propto (\Gamma \beta)^{-5.2} \). Such a steep function, however, is inconsistent with the constraint from the optical data of SN 2003lw (Mazzali et al. 2006), for which a rough estimate gives \( E(\geq \Gamma \beta) \propto (\Gamma \beta)^{-1.7} \). Motivated by this fact, we assume that all HNe associated with GRBs have a fairly steep energy distribution, which is generally written as \( E(\geq \Gamma \beta) = A(\Gamma \beta/\Gamma_0 \beta_0)^{-q} \) for \( \beta < 0.5 \), where \( \Gamma_0 = (1 - \beta_0^2)^{-1/2} \). For SN998bw/GRB980425, SN2003dh/GRB030329, SN2003lw/GRB031203, and SN2006aj/GRB060218, optical modelling suggests \( A \sim 0.2 - 6 \times 10^{32}\text{erg} \) and \( \Gamma_0 \beta_0 \sim 0.04 \) - 0.1 (Soderberg et al. 2006). The parameter \( q \), however, is not reliably determined in most cases. For simplicity, we approximate \( E(\geq \Gamma \beta) = A(\Gamma \beta/\Gamma_0 \beta_0)^{-q} \) as \( E(\geq \beta) = A(\beta/\beta_0)^{-q} \) for \( \Gamma \sim 1 \).

In a SN explosion, the larger the velocity, the outer the ejecta. When the fast component is decelerated by the medium, the slower part will catch up with the decelerating shock front. As a result, the total kinetic energy of the shocked medium increases and the deceleration of the shock is suppressed. In the quasi-similar evolution phase of the HN shock, the fastest component has swept enough medium and has got decelerated. Significant part of the initial kinetic energy of the HN material \( E(\geq \beta) \) has been used to accelerate the medium to a velocity \( \beta \). So when we talk about the CR acceleration in the blast wave, the \( E(\geq \beta) \) mentioned there actually represents the total kinetic energy of the shocked medium moving with a velocity \( \beta \). For a medium taking the profile \( n \propto R^{-k} \), the rest mass swept by the HN blast wave is \( M_{\text{med}} = \int_0^\infty 4\pi \rho R^2 dR \propto R^{k-2} \). For \( \beta > \beta_0 \), conservation of energy gives \( E(\geq \beta) \approx M_{\text{med}} \beta^2 / 2 \), i.e.,

\[
\beta^{-q+2} \propto R^{1-k}.
\]

With the relation that \( R \sim \beta t \), the dynamics of the HN outflow is described by (for \( \beta > \beta_0 \))

\[
\beta \propto t^{-\frac{1}{2(q+2)}}.
\]

In the next section we’ll show how \( q \) and \( k \) influence the CR spectrum.

2.2 The medium the hypernova outflow expands into

As shown in Eq. (2), the dynamics of a HN shock depends on the medium profile sensitively. Here we review the medium profiles of all four GRB-HN events, based on the GRB and/or HN afterglow modeling. For GRB 980425, the medium is Wind like (i.e., \( k \sim 2 \)). The afterglow modeling favors an unusual small \( A_* \sim 0.01 - 0.04 \) (Li & Chevalier 1999; Waxman 2004), where \( A_* \equiv (\dot{M}/10^{-5} M_\odot \text{ yr}^{-1})(v_\infty/10^8 \text{ cm s}^{-1})^{-1} \), \( \dot{M} \) is the mass loss rate of the progenitor, and \( v_\infty \) is the velocity of the stellar wind. For GRB 030329, the circumburst medium is found to be homogeneous (i.e., \( k \sim 0 \)), as shown in many independent investigations (Frail et al. 2005; Pihlström et al. 2007; van der Horst et al. 2008; Xue et al. 2008). For GRB 031203, after modeling the radio data, Soderberg et al. (2004) got a constant \( n \sim 0.6 \text{ cm}^{-3} \). For GRB 060218, the high quality radio data supports the homogeneous medium model

\[3\] There is an additional evidence for such a conclusion. With an electron energy distribution index \( p \sim 2.6 \) inferred from the radio data, the X-ray spectrum \( F_\nu \propto \nu^{-0.8} \) (Soderberg et al. 2004) suggests that the X-ray emission is below the cooling frequency of the forward shock. For a free-wind medium, the late-time (\( t > 1 \text{ day} \)) X-ray light curve should drop with the time as \( t^{-1.7} \), deviating from the detected \( t^{-1} \) decline (e.g., Ramirez-Ruiz et al. 2005) significantly. If the medium is ISM-like, the expected late-time decline is \( t^{-1.2} \) consistent with the data. In this scenario, the early time (\( t < 1 \text{ day} \)) X-ray flattening, as those well detected
with \( n \sim 100 \text{ cm}^{-3} \) (Fan, Piran & Xu 2006; Soderberg et al. 2006). As such, we have no compelling evidence for a Wind-like medium surrounding most GRBs, even for those associated with HNe. The physical reason is not clear, yet. A post-common envelope binary merger model (e.g., Fryer, Rockefeller & Young 2006) or a fast motion of the Wolf-Rayet star relative to the ISM (van Marle et al. 2006) may be able to solve this puzzle. 

Actually, a free wind medium, supposed to surround the progenitor, is unlikely to be able to keep such a profile up to the radius 

\[
R_{\text{dec}}(\beta_0) \sim 3 \times 10^{22} \left( \frac{M_0}{10M_\odot} \right) A_{\text{s, -1}}^{-1} \text{ cm},
\]

where \( M_0 \) is the rest mass of the GRB-associated HN ejecta, and \( R_{\text{dec}} \) is the deceleration radius. This is because during their evolution, massive stars lose a major fraction of their mass in the form of a stellar wind. The interaction between this stellar wind and the surrounding interstellar medium creates a circumstellar bubble (e.g., Wijers 2001; Ramirez-Ruiz et al. 2001; Dai & Wu 2003; Chevalier et al. 2004; van Marle et al. 2006). The analytical calculation suggests that the free wind of a Wolf-Rayet star usually terminates at 

(4)

\[
R_\text{t} = 5.7 \times 10^{18} \left( \frac{v_\text{w}}{10^3 \text{ km s}^{-1}} \right)^{1/2} \left( \frac{p}{10^6 \text{ cm}^2 \text{ s}^{-1}} \right)^{1/2} \text{ cm}.
\]

where \( p \) is the pressure in the shocked wind and \( k \) is the Boltzmann constant. This is confirmed by observations of Wolf-Rayet nebulae, such as NGC 6888 and RCW 58, which also have radii of the order of a few pc (Gruendl et al. 2000). Here we take the numerical example given in Figure 1 of Chevalier et al. (2004) to show that the HN outflow is mainly decelerated in the ISM-like medium region. In their numerical example, \( n \sim 0.5 R_{\odot}^{-2} \text{ cm}^{-3} \) for \( R_{\odot} < 1.2 \). The total mass of the free wind medium is thus \( \sim 7 \times 10^{-3} M_\odot \ll M_0 \). Here and throughout this work, the convention \( Q_x = Q/10^x \) has been adopted in cgs units.

We therefore conclude that the medium is most likely to be ISM-like at the radius where the HN outflow has been decelerated to \( \beta < 4 \beta_0 \). This can be also understood as follows. One can infer from eq.(1) that for \( k = 2 \) the outflow component with a \( \beta > \beta_0 \) will decelerate at a radius \( R_{\text{dec}}(\beta) \approx (\beta/\beta_0)^{-(2+k)} R_{\text{dec}}(\beta_0) \). So for \( \beta \sim 4 \beta_0 \), and \( q \sim 2 \), 

\[
R_{\text{dec}}(4\beta_0) \sim 4 \times 10^{-3} R_{\text{dec}}(\beta_0) \sim R_\text{t}.
\]

### 3 Spectrum of Cosmic Ray Protons: Observation and Interpretation

#### 3.1 A new CR proton component in the energy range of \( 10^{18} \) to \( 10^{18.5} \text{ eV} \)

The spectrum of protons steepens suddenly at the first knee by a factor of 

\( \Delta \gamma(1) \sim -2.1 \).

In view that the spectra of heavier particles would steepen at higher energies, the likely interpretation of the steepening in Swift GRBs, can be attributed to an energy injection from the central engine.

of all CRs at the first knee is the sudden decline of the light particles such as H and He (see Hillas 2005; Hörandel 2008 and the references therein).

The proton CR spectrum before and after the second knee, after subtracting the modeled “galactic” component, can be roughly estimated as (Ulrich et al. 2004; Antoni et al. 2005; Hillas 2005; Hörandel 2008)

\[
dN_{\text{CR}}/dE_{\text{CR}} \propto \begin{cases} 
E_{\text{CR}}^{-2.4} & \text{for } 0.1 < E_{\text{CR}, 17} < 3, \\
E_{\text{CR}}^{-3.3} & \text{for } 3 < E_{\text{CR}, 17} < 30,
\end{cases}
\]

which indicates the factor of spectral steepening is \( \Delta \gamma(II) \sim -0.9 \). Above the ankle, the CR spectrum changes to \( E_{\text{CR}}^{-2.7} \), so the factor of flattening is \( \Delta \gamma(III) \sim 0.6 \).

The interpretations of spectral changes at the second knee and at the ankle are much less clear. Hillas (2005) interpreted them as a result of an extragalactic component with a spectrum \( \propto E_{\text{CR}}^{-2.2} \) suffering losses by the interaction between cosmological microwave background radiation and starlight. In this work, we consider the detected spectral change around the second knee is due to the energy-velocity distribution of HN outflows.

#### 3.2 Theoretical interpretation

##### 3.2.1 Self-magnetized shock acceleration model

In this model, the magnetic field of the upstream is assumed to be amplified significantly by the CRs themselves (e.g., Bell & Lucek 2001).

With Eq. (2), we have the radius of the forward shock front as \( R \propto \frac{k}{\beta R} \). The maximum energy accelerated by the forward shock can be estimated by (Bell & Lucek 2001; Berezhko & Völk 2004; Ptuskin & Zirakashvili 2005)

\[
E_{\text{max}}(k, q) \sim Z\beta eBR \propto t_{\text{dec}}^{-k/q-1/2} \propto \beta^{-4-k/q-1/2},
\]

where \( B \propto \beta R^{-k/2} \) is the magnetic field in the upstream of the shock, which is on the same order of that of the shocked medium. In the ISM case (i.e., \( k = 0 \)), we have

\[
E_{\text{max}}(0, q) \propto \beta^{4-q}/3,
\]

while in the WIND case (i.e., \( k = 2 \)),

\[
E_{\text{max}}(2, q) \propto \beta^{2}.
\]

Here we do not present the numerical coefficient of \( E_{\text{max}}(k, q) \) because Wang et al. (2007) and Budnik et al. (2008) have already shown that for typical parameters, \( \beta \sim \beta_\odot \sim 0.1 \) is high enough to accelerate protons up to \( \sim 10^{17} \text{ eV} \) regardless of \( k \). In a stellar wind medium, the HN shock front with \( \beta \sim 4 \beta_0 \) can accelerate protons up to \( \sim 3 \times 10^{18} \text{ eV} \). Therefore the CR protons above the second knee but below the ankle are mainly accelerated by the HN shock in the velocity range of \( \beta_0 - 4 \beta_0 \).

To get an estimate of the spectrum of the accelerated particle, following Berezhko & Völk (2004) and Ptuskin & Zirakashvili (2005) we assume: (1) The particles with an energy \( E_{\text{max}} \) escape the shock immediately; (2) The total energy of the accelerated particles at an energy \( E_{\text{CR}} = E_{\text{max}}(\beta) \) is proportional to \( E(\geq \beta) \). In view of the relations \( E(\geq \beta) \propto \left[ E_{\text{max}}(\beta) \right]^{-3q/(4-q)} \) for \( k = 0 \) and \( E(\geq \beta) \propto \left[ E_{\text{max}}(\beta) \right]^{-q/2} \) for \( k = 2 \), we have
\[
\frac{dN}{dE_{\gamma}} \propto \begin{cases} 
E_{\gamma}^{-2+\frac{3\beta}{2}} & \text{for } k = 0, \\
E_{\gamma}^{-2+\frac{3\beta}{2}} & \text{for } k = 2, 
\end{cases}
\]

where \( \delta \approx 0.4 \) is introduced to account for the proton spectrum in the energy range of \( 10^{16} - 3 \times 10^{17} \text{ eV} \). As \( \beta \lesssim \beta_s \), \( E(\geq \beta) \propto \beta^2 \) if the energy loss of the HN shock is ignorable. The accelerated proton spectrum should be \( \propto E_{\gamma}^{-2+\beta} \). This answers why there comes a spectrum change around the second knee if \( E_{\text{max}}(\beta_s) \approx 3 \times 10^{17} \text{ eV} \).

With a \( \delta = 0 \), to match the detected proton spectrum \( dN/dE_{\gamma} \propto E_{\gamma}^{-3.3} \), one has to have \( q \approx 2.6 \), which is very close to that of SN 2003lw and SN 1998bw reported in Soderberg et al.\(^4\) (2006). Therefore Wang et al. (2007) concluded that the self-magnetized shock acceleration model could account for the spectrum data. However, a few puzzles have to be solved before accepting this argument: (I) If \( \delta = 0 \), some novel effects are needed to interpret why the proton spectrum departs from \( E_{\gamma}^{-3.3} \) significantly in the \( 10^{16} - 3 \times 10^{17} \text{ eV} \) range. The authors also need to explain why these effects, if any, disappeared in the \( 3 \times 10^{17} - 10^{18} \text{ eV} \) range. (II) A wind profile holding to a radius \( \sim 1 - 10 \text{ kpc} \) is crucial for their argument. If the medium is ISM-like when the outflow gets decelerated to \( \beta < 0.4 \), Wang et al. (2007)'s approach would yield a spectrum

\[
\frac{dN}{dE_{\gamma}} \propto E_{\gamma}^{-5}
\]

for \( q \approx 2 \), which is too steep to be consistent with the data. We take this puzzle as a potential limit of their interpretation.

Let’s investigate whether a specific wind-bubble can solve this puzzle. We assume that the free wind profile is terminated at a radius \( R_t \approx 10^{19} \text{ cm} \) and is followed by an ISM-like shell. Suppose that the shell is so massive that the deceleration of the whole HN outflow occurs at \( R - R_t \approx \text{const} \), we have \( E_{\text{max}} \propto Z\beta_e B R \propto \beta B \). If the shell is not dense enough to form a strong reverse shock, i.e., the forward shock velocity decreases continually rather than abruptly, then \( B \propto \beta^{1/2} \). As a result, we have \( E_{\text{max}} \propto \beta^{2} n^{1/2} \) and \( dN/dE_{\gamma} \propto E_{\gamma}^{-2+\frac{3\beta}{2}} \), provided that the CR protons in the energy range of \( \sim 3 \times (10^{17}, 10^{18}) \text{ eV} \) are mainly accelerated in the shocked shell. Though such a possibility is attractive, the request that the reverse shock does not form is hard to satisfy. This is because at a radius \( \sim R_t \approx 10^{19} \text{ cm} \) the number density of the wind medium \( n_w \approx 3 \times 10^{-4} \text{ cm}^{-3} A_{\text{wind}}^{-1} R_{19}^{-2} \). On the other hand, the assumption that \( 4\pi R_t^2 n_t n_p \sim M_{\text{ej}} \) requires that \( n_t \approx 1 \text{ cm}^{-3} (M_{\text{ej}} / 10^{57}) R_{19}^{-3} \). So we have a density contrast \( n_t/n_p \approx 10^{5} \). The forward shock expanding into the dense shell will have a pressure \( \propto \beta^2 n_t m_p \rho^2/3 \), which is much higher than that of the shocked wind medium \( \sim \beta^2 n_p m_p \rho^2/3 \). A pressure balance will be established by a strong reverse shock penetrating into the shocked wind medium. Therefore the forward shock velocity is much smaller than \( \beta \) [shocked wind medium] and can not accelerate protons to an energy \( \sim 10^{18} \text{ eV} \). The reverse shock with a velocity \( \beta_r \approx \beta \) [shocked wind medium] plausibly plays a more important role in accelerating high energy CR protons. The

3.2.2 GYRORESONANT STOCHASTIC ACCELERATION MODEL

The maximum energy-gain rate due to the stochastic Fermi acceleration for marginally relativistic shock can be estimated as (Dermer 2001b)

\[
\frac{dE_{\gamma}}{dR} \approx \frac{\varepsilon_{\text{turb}}(v - 1)}{2^{3/2}} \frac{Ze B v^2 (2^{1/2} E_{CR})}{Z e B f_{\Delta R} R^3} v^{-1},
\]

where \( Z \) is the atomic number, \( \varepsilon_{\text{turb}} \) is the ratio of plasma turbulence to the shock energy density, \( B \approx 0.4 n^{1/2} \varepsilon_{\text{turb}}^{-1/2} \) Gauss, \( f_{\Delta} \approx 1/12 \) is the ratio of the width of the swept medium by the shock to \( R \) (Dermer & Humi 2001), and \( v \) is the spectrum index of the turbulence (\( v = 5/3 \) for Kolmogorov turbulence and 3/2 for Kraichnan turbulence).

Dermer (2001b) took a \( \beta \approx \text{const} \), integrated eq.(11) over \( R \), then got \( E_{\text{max}}(R) \). However, currently \( \beta \) evolves with \( R \). As shown below, the smaller the radius, the larger the \( \beta \) and the higher the \( E_{\text{max}} \). Very energetic CRs can be accelerated at early times but can not be accelerated continually because of the adiabatic cooling. Taking into account the adiabatic cooling effect, eq.(11) takes the new form

\[
\frac{dE_{\gamma}}{dR} \approx \frac{\varepsilon_{\text{turb}}(v - 1)}{2^{3/2}} \frac{Ze B v^2 (2^{1/2} E_{CR})}{Z e B f_{\Delta} R^3} v^{-1} - \frac{E_{\gamma}}{R} \]

Now \( E_{\text{max}} \) can be estimated by setting \( \frac{dE_{\gamma}}{dR} = 0 \), then we have

\[
E_{\text{max}} \approx \frac{|\varepsilon_{\text{turb}}(v - 1)\beta|^1/(2-v) \frac{Ze B f_{\Delta} R^3}{2 f_{\Delta} \sqrt{2}}}
\]

**ISM medium.** In this case, we have

\[
E_{\text{max}}(\text{ISM}) \approx Z n_0 (2^{1/2})^{3/2} R_{19}
\]

\[
\{ \begin{array}{l}
10^{16} \text{ eV} (\frac{\varepsilon_{\text{turb}}}{0.8})^{3/2} B_{12}^{1/2} (12 f_{\Delta})^{-2} \quad (v = 5/3), \\
10^{17} \text{ eV} (\frac{\varepsilon_{\text{turb}}}{0.8})^{3/2} B_{12}^{1/2} (12 f_{\Delta})^{-1} \quad (v = 3/2).
\end{array}
\]

The energy conservation \( 4\pi R^2 \beta^2 n_p e^2/3 \approx E(>\beta) \) yields \( R \approx 10^{19} \text{ cm} A_{12}^{1/3} \beta^{-7/3} (v + 2/3) n_0^{-1/3} \). Combining with Eq. (14), we have

\[
E_{\text{max}}(\text{ISM}) \propto \beta^{-5/3v+(2-v)} \]

i.e., \( E_{\text{max}}(\text{ISM}) \propto \beta^{-7/3} \) for \( v = 3/2 \) and \( \propto \beta^{-10/3} \) for \( v = 5/3 \), both are sensitive to \( \beta \).

**WIND medium.** In the termination wind shock model, the stellar wind profile may hold up to a distance \( \sim 10^{18} \text{ cm} \) (e.g., Chevalier et al. 2004). In this case, \( n = 3 \times 10^{15} A_{12} R_{-2}^{-2} \text{ cm}^{-3} \). Now \( B_0 \approx 0.2 A_{12}^{1/2} B_{-1}^{1/2} \) Gauss and \( E_{\text{max}}(\text{wind}) \approx Z A_{12}^{1/2} A_{-1}^{1/2} B_{-1} \)

\[
\{ \begin{array}{l}
2 \times 10^{15} \text{ eV} (\frac{\varepsilon_{\text{turb}}}{0.8})^{3/2} B_{12}^{1/2} (12 f_{\Delta})^{-2} \quad (v = 5/3), \\
2 \times 10^{16} \text{ eV} (\frac{\varepsilon_{\text{turb}}}{0.8})^{3/2} B_{12}^{1/2} (12 f_{\Delta})^{-1} \quad (v = 3/2).
\end{array}
\]

\(^4\) Please see section 2.1 for the discussion of uncertainty of the \( q \) obtained in their way.
As shown in Eqs. (14) and (16), for $v_{\text{turb}} \sim 0.5$ and $v = (3/2, 5/3)$, at $\beta \sim \beta_0 \sim 0.1$, we have $E_{\text{max}} \sim (10^{17}, 10^{16})Z$ eV. Below we focus on the case of $v = 3/2$, because in the case of $v = 5/3$ the request of $E_{\text{max}}(\beta_0) \sim 3 \times 10^7$ eV is more difficult to satisfy. For $\beta \sim 0.5$, the stochastic gyroresonant acceleration is able to accelerate protons to $\sim 10^{19}$ eV (see also Dermer 2001a). The accelerated particle spectrum is thus ($v = 3/2$)

$$\frac{dN}{dE_{\text{CR}}} \propto \begin{cases} E_{\text{CR}}^{-(2+q)/(7-q)} & \text{for } k = 0, \\
E_{\text{CR}}^{-(2+q)} & \text{for } k = 2. \end{cases}$$

(17)

As shown in section 2.1, the main deceleration of the HN outflow is very likely to be in an homogenous medium. The accelerated protons have a spectrum $dN/dE_{\text{CR}} \propto E_{\text{CR}}^{-(2+q)/(7-q)}$. To match the observation $\Delta \gamma(\Pi) \approx -3q/(7-q) \sim -0.9$, we need $q \sim 1.6$.

This is surprisingly close to the value $\sim 1.7$ that is inferred from the optical modeling of SN 2003lw. Detailed optical modeling of more GRB-associated HN explosions is highly needed to better constrain $q$ and then confirm or rule out our interpretation.

If GRB-associated HNe expand into a Wind bubble-like medium, a flatter CR spectrum in the higher energy range would appear. At a small radius (say, $< 10^{18}$ cm), the medium is free Wind-like and the accelerated particle spectrum is $\propto E_{\text{CR}}^{-(2+q)/(7-q)}$, which then gets steepened by a factor of $q(2+q)/(7-q) \sim 0.4$ for $q \sim 1.6$ after entering the ISM-like medium. Such a flattening seems not enough to match the observation $\Delta \gamma(\Pi) \sim 0.6$. So CRs above the ankle may be mainly from AGNs, as indicated by the recent analysis of the correlation of the highest-energy CRs with nearby extragalactic objects by the Pierre Auger Collaboration (Abraham et al. 2007).

The rate of local GRB-associated HNe only accounts for $\sim (0.1 - 0.5)\%$ of that for all local SNe (Della Valle 2006; Soderberg 2007). The typical energy of these HNe, however, is tens times larger than that of the normal SNe. Roughly, we expect that a fraction $\sim 10\%$ of CR protons at 3 PeV could be attributed to GRB-associated HNe. It is enough to match the observation (Ulrich et al. 2004; Horandel 2008). So the CR proton spectrum in the energy range of $10^{16} - 10^{18.5}$ eV may be quantitatively interpreted.

4 DISCUSSION AND SUMMARY

The particle acceleration in marginally relativistic HN shocks are discussed. The GRB-associated HN outflows are assumed to have a fairly steep energy distribution against their velocities, i.e., $E(\geq \beta) \propto \beta^{-q}$ for $q \sim 1.7$, as inferred from the optical modelling of SN 2003lw (see section 2.1 for details). A significant fraction of a HN’s kinetic energy is carried by the material moving with a velocity $> \beta_0(\sim 0.1)$, driving an energetic shock wave into the surrounding medium. The cosmic ray protons above the second knee but below the ankle may be accelerated by the HN shocks in the velocity range of $\beta \sim (1 - \delta)\beta_0$. To satisfy this velocity bound, the HN outflows associated with GRBs must have reached a very large radius where the surrounding medium is very likely to be ISM-like (see section 2.2 for details). With this argument, the self-magnetized shock acceleration model adopted in Wang et al. (2007) would yield a very steep spectrum that is inconsistent with the observation unless the medium surrounding the HN is a free wind holding up to a radius $R_{\text{dec}} \sim 10$ kpc ($M_{\odot}/10M_{\odot})A_{\star}^{-1}$. Such a request seems difficult to satisfy. A highly speculative solution is that the particle acceleration mainly occurs in a narrow “dense” shell that terminates the free wind at a radius $\sim 10^{19}$ cm (see the last paragraph of section 3.2.1 for details).

In this work, we find that for $q \sim 1.6$, the stochastic gyroresonant acceleration model can account for the spectrum change of high energy protons around the second knee (see section 3.2.2 for details). As a consequence, the stochastic gyroresonant acceleration mechanism in relativistic GRB forward shock may account for part of the ultra-high energy CRs ($\sim 10^{20}$ eV), as suggested in Dermer (2001b, 2007) and Dermer & Humi (2001). A typical

$$q \approx -\frac{7\Delta \gamma(\Pi)}{3 - \Delta \gamma(\Pi)} \sim 1.6 \text{ for } \Delta \gamma(\Pi) \sim -0.9,$$

if confirmed in future optical modelling of the GRB-associated HN explosions, will be a crucial evidence for our current speculation.

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