Motivated by recent experimental results reporting anomalous drag resistance behavior in dilute bilayer two-dimensional (2D) hole systems in the presence of a magnetic field parallel to the 2D plane, we have carried out a many-body Fermi liquid theory calculation of bilayer magnetodrag comparing it to the corresponding single layer magnetoresistance. In qualitative agreement with experiment we find relatively similar behavior in our calculated magnetodrag and magnetoresistance arising from the physical effects of screening being similarly modified ("suppressed") by carrier spin polarization (at "low" field) and the conductivity effective mass being similarly modified ("enhanced") by strong magneto-orbital correction (at "high" fields) in both cases. We critically discuss agreement and disagreement between our theory and the experimental results, concluding that the magnetodrag data are qualitatively consistent with the Fermi liquid theory.

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Much attention has recently focused on low-density 2D systems in semiconductor structures [1–7] where carrier transport properties may be strongly affected by interaction effects. In particular, low temperature transport [1,2], magnetotransport [3,4], drag [5,6], and magnetodrag [7] properties have recently been studied in low-density electron [1,3,6] and hole [2,4,5,7], single-layer [1–4] and bilayer [5–7] systems, providing a great deal of detailed quantitative information on the temperature, density, and magnetic field dependent 2D resistivity \( \rho(T, n, B) \) and 2D drag-resistivity \( \rho_D(T, n, B) \) behavior. The externally applied magnetic field being discussed throughout this work and in the relevant experimental references [1–7] is an ‘in-plane’ magnetic field \( B \) applied parallel to the 2D layer.) A very recent experimental work [7] by Pillarsetty et. al. reports some striking qualitative resemblance between the 2D bilayer drag \( \rho_D \) and the corresponding single-layer resistivity \( \rho \) as a function of the applied parallel field \( B \) in a low density low disordered 2D GaAs hole system. Since the physical mechanisms underlying \( \rho \) and \( \rho_D \) are generally thought to be qualitatively different at low temperatures, the experimental observations of ref. 7 take on important qualitative significance. In particular, \( \rho \) in high-mobility 2D systems at low temperatures is entirely due to scattering by random charged impurity centers whereas \( \rho_D \) at low temperatures arises entirely from inter-layer electron-electron scattering. (Electron-phonon scattering makes negligible contributions to both \( \rho \) and \( \rho_D \) at low temperature [1,2].) Since electron-electron scattering does not directly contribute to \( \rho \) in translationally invariant 2D semiconductor systems, the reported [7] qualitative similarity between the observed \( \rho \) and \( \rho_D \) behaviors in its magnetic field dependence presents a significant theoretical challenge. Since it is manifestly obvious that electron-impurity scattering can at best play an unimportant and indirect secondary role [8] in determining the interlayer drag resistance, the experimental observation of ref. 7 raises very serious fundamental questions regarding our understanding of the nature of the ground state of a low-density 2D carrier system. We note that electron-electron interaction induced umklapp scattering, which could contribute to the single-layer resistivity (since umklapp processes do not conserve momentum), is completely irrelevant in 2D semiconductor structures where all the electronic physics occurs essentially at the zone-center \( \Gamma \) point in the effective mass approximation sense (and the real lattice structure does not play any role).

In view of the considerable fundamental significance of the issues raised by the experimental observations, we present in this Letter a careful theoretical calculation of both \( \rho(B) \) and \( \rho_D(B) \) in a low-density 2D carrier system within the canonical many-body Fermi liquid theory that has earlier been found to be successful in providing a reasonable qualitative (and perhaps even semi-quantitative) description of the approximate temperature and density dependence of \( \rho \) [9] and \( \rho_D \) [10] at low temperatures and densities in the absence of any applied in-plane magnetic field. We note that the zero-field temperature and carrier density dependence of 2D resistivity \( \rho(T, n) \) and 2D drag resistivity \( \rho_D(T, n) \) in the absence of any external magnetic field are certainly very different as one would expect on the basis of \( \rho \) and \( \rho_D \) being determined by different scattering processes: \( \rho \) by screened charged impurity scattering and \( \rho_D \) by interlayer electron-electron scattering. For example, \( \rho \) shows [1–4] an approximate linear increase with \( T \) at low temperatures as is expected [9] for screened Coulomb impurity scattering and \( \rho_D \) shows [5,8,10] an approximate quadratic increase with \( T \) at low temperatures as is expected for electron-electron scattering. (Similarly the carrier density dependence of \( \rho \) and \( \rho_D \) are also very different at \( B = 0 \).) The question therefore naturally arises why the in-plane magnetic field dependences of \( \rho(B) \) and \( \rho_D(B) \) reported in ref. [7] show qualitative similarities.

We theoretically argue, showing concrete calculated result within the many body Fermi liquid theory, that the qualitative magnetic field dependence of \( \rho(B) \) and \( \rho_D(B) \)
should indeed be similar since the scattering processes controlling the two properties (electron charged impurity scattering for \( \rho \) and electron-electron scattering for \( \rho_D \)) are both screened by the carriers themselves and the dominant behavior in both cases arises primarily from the magnetic field dependence of electronic screening [11] (through the spin polarization process) and (somewhat to a lesser degree) from the magneto-orbital effect [12] (through the modifications of the 2D conductivity effective mass and the confining quasi-2D wave function).

The reported qualitative similarity between magnetodrag and magnetoresistance thus arises from drag and resistance being dominated by screened carrier-carrier scattering and screened carrier-impurity scattering respectively. The fact that long-ranged charged impurity potential is the dominant source of resistive scattering in 2D semiconductor structures (and this long-ranged charged impurity scattering must necessarily be screened by the carriers) is therefore the key reason for the broad qualitative similarity between \( \rho_D(B) \) and \( \rho(B) \) reported in [7].

We start by writing down the zero-field theoretical formulae for \( \rho \) [9] and \( \rho_D \) [10,8] in the many-body Fermi liquid RPA-Boltzmann theory approximation widely used in the literature. The resistivity is given by \( \rho^{-1} = \frac{ne^2(\tau)/m}{2} \), where \( n, m \) are the 2D carrier density and the conductivity effective mass respectively whereas the transport relaxation time \( \tau \) is given by

\[
\frac{1}{\tau(\varepsilon_k)} = \frac{2\pi}{h} \sum_{k'} n_i |u_{ei}(k-k')|^2 (1 - \cos \theta_{kk'}) \delta(\varepsilon_k - \varepsilon_{k'}),
\]

with \( \langle \tau \rangle \) being a thermal average over the carrier energy \( \varepsilon \). Here \( n_i \) is the density of charged impurity centers in the 2D system (including the interface and the insulator), and \( u_{ei}(q) \) is the screened carrier-impurity scattering strength give by \( u(q) = v^*(q)/\epsilon(q) \), where \( \epsilon(q) \) is the single-layer 2D carrier dielectric function. (For details on the derivation and implications of the formula for \( \rho \), see ref. [9].) The drag resistivity is given by

\[
\rho_D = \frac{\hbar^2}{2\pi e^2 n^2 k_B T} \int \frac{d^2 q}{(2\pi)^2} \int \frac{d\omega}{2\pi} \frac{F_1(q,\omega) F_2(q,\omega)}{\sinh^2(\beta\omega/2)},
\]

where \( F_{1,2}(q,\omega) = |u_{12}^*| F_{12}(q,\omega) \) and \( u_{12}^* = v_{12}^* \epsilon(q,\omega) \) is the dynamically screened interlayer Coulomb interaction between layers 1 and 2, and \( \Pi \) is the 2D polarizability. (We consider the so-called balanced situation here with the same carrier density \( n \) in both layers.) Note that the dielectric function \( \epsilon(q,\omega) = 1 - v(q) \Pi(q,\omega) \) entering Eq. (2) is the two component dielectric tensor for the bilayer system [13]. (For details on the drag formula and its implications, see refs. [8,10].)

It is important to emphasize that dielectric screening by the carriers themselves is a key ingredient in determining both \( \rho \) and \( \rho_D \) although the static single layer (scalar) dielectric function \( \epsilon(q) \) determines \( \rho \) through the screened charged impurity scattering whereas the dynamical bilayer (tensor) dielectric function \( \epsilon(q,\omega) \) determines \( \rho_D \) through the screened interlayer Coulomb interaction. At low carrier densities used in ref. [7], the difference between static and dynamic screening is not of any qualitative significance since the effective plasma frequency scale is rather low at low densities. Therefore both \( \rho \) and \( \rho_D \) depend on the carrier dielectric function properties, which is why they have qualitative similar magnetic field dependence as we show and discuss below.

We have carried out a thoroughly nontrivial generalization of the above theories for \( \rho \) and \( \rho_D \) to the finite in-plane magnetic field situation \( \rho(B), \rho_D(B) \). Details will be provided elsewhere [14], but here we mention the main physical effects of the applied field for \( \rho \) and \( \rho_D \). The applied field has two completely different physical effects through its coupling to carrier spin ("magneto-spin") [11] and orbital ("magneto-orbital") dynamics [12]. The magneto-spin effect arises from field-induced carrier spin polarization due to the Zeeman coupling, and saturates at a density dependent saturation field \( B_s \) when the carrier system is fully spin polarized (i.e. the magneto-spin effect exists only for \( B \leq B_s \)). The magneto-orbital effect [12] arises from the orbital coupling of the in-plane magnetic field to the transverse dimension due to the quasi-2D nature of the 2D layer and the magneto-orbital effect is therefore monotonically increasing with increasing magnetic field since this orbital coupling is important only when the magnetic length \( l = (\pi / eB)^{1/2} \) is smaller than the quasi-2D width of the 2D system. Thus one important qualitative difference between the magneto-spin and the orbital effect is that the spin effect is essentially a "weak-field" effect lasting only upto the saturation field \( B_s \) whereas the magneto-orbital effect increases monotonically with increasing field.

The magneto-spin mechanism itself has two distinct effects: Suppression of screening due to spin polarization [11] as the spin degeneracy decreases from 2 (at \( B = 0 \)) to 1 (at \( B \geq B_s \)) and the increase of the effective 2D Fermi surface as the value of the 2D Fermi wave vector \( k_F \) increases by a factor of \( \sqrt{2} \) with \( B \) increasing from zero to \( B_s \) due to the lifting of the spin degeneracy. Similarly, the magneto-orbital mechanism also has two distinct effects: The increase of the transport effective mass in the direction perpendicular to the magnetic field direction and the field-induced intersubband scattering among the quasi-2D subband — both of these are only operational at relatively high fields when \( l < a \) where \( a \) is the average transverse width of the carrier wave function. It is important to realize that three of these four field induced effects (spin polarization induced screening suppression, and both of the magneto-orbital effects) always produce positive magnetoresistance whereas the Fermi surface effect (which is significant only at high carrier densities where \( 2k_F \gg q_{TF}, q_{TT} \) being the screening wave vector)
always produces a negative magnetoresistance. For the hole-doped low-density samples of ref. [7], the Fermi surface (i.e. $k_F \to \sqrt{2}k_F$ as $B \to B_s$) effect is negligible since the system is in the strong screening $q_{TF} \gg 2k_F$ limit.

The combination and the interplay of magneto-spin and magneto-orbital effects are quite complex and sensitive to the parameter \((n, T, B)\) details, but a few general comments can still be made: (1) At low carrier densities [7] of interest to us, the static and dynamic screening operational respectively in $\rho(B)$ and $\rho_D(B)$ behave similarly, and therefore the spin-polarization induced screening effect is qualitatively similar for $\rho(B)$ and $\rho_D(B)$; (2) since field-induced magneto-spin effect operates only for $B \leq B_s$, both $\rho(B)$ and $\rho_D(B)$ manifest a cusp-type structure at $B = B_s$ where spins are completely polarized; (3) the maximum theoretically allowed magnetoresistance $\rho(B)/\rho(0)$ and magnetodrag $\rho_D(B)/\rho_D(0)$ corrections arising from the spin polarization induced magneto-screening mechanism are factors of 4 and 16 respectively since screening itself could be suppressed at most by a factor of 2 due to spin polarization effect (and Eqs. 1 and 2 respectively for $\rho$ and $\rho_D$ come with the second and the fourth power of the spin degeneracy); (4) the main magneto-orbital effect for the relatively narrow p-GaAs quantum well systems (width $\sim 150$ Å) used in ref. [7] is the enhanced conductivity mass at higher magnetic field values — the condition $l \ll 150$ Å necessary for strong magneto-orbital correction is satisfied for $B \gg 4$ T whereas the spin polarization saturation field $B_s$ for the low hole densities used in ref. [7] is $B_s \sim 3 - 6$ T; thus the magneto-spin effects dominate for $B$ upto $3 - 6$ T whereas the magneto-orbital effects dominate at higher fields; (5) the magneto-orbital effects are “similar” in both cases since both $\rho$ and $\rho_D$ are proportional to the field-dependent effective mass (which increases quadratically with the applied field). We mention that these five features are in excellent qualitative agreement with experimental results [7].

In Figs. 1 – 4 we show our calculated results for $\rho$ and $\rho_D$ within the RPA-Boltzmann Fermi liquid theory. Our theory incorporates all realistic effects [9-11] with the charged impurity density ($n_i$) determining $\rho$ as the only unknown free parameter. Our results in Figs. 1 and 2, where $\rho(B)$ and $\rho_D(B)$ are shown for different temperatures and different densities, respectively, bear excellent qualitative resemblance to the corresponding experimental results in ref. [7]. We are not claiming quantitative agreement with experiment by any means since our theory is necessarily approximate at the low carrier densities used in ref. [7] since no exact description of correlation effects at low densities exists for interacting quantum Coulomb system of interest here. The qualitative agreement between theory and experiment is, however, obviously apparent even on a casual comparison between our Figs. 1 and 2 and the corresponding Figs. 1 and 2 in ref. [7]. In particular, both theory and experiment
manifest qualitatively similar, but by no means identical, behaviors in $\rho(B)$ and $\rho_D(B)$, arising, as argued above, from the magneto-spin and magneto-orbital effects.

It is worthwhile to theoretically consider the orbital and the spin effects separately. (Experimentally this cannot, of course, be done but one could get some approximate idea about the relative behavior of the magneto-spin and the magneto-orbital effects in $\rho(B)$ and $\rho_D(B)$ by concentrating on the ‘low’ $B$ ($< B_s$) and the ‘high’ $B$ ($> B_s$) regimes, respectively.) In Fig. 3 we show the calculated $\rho(B)$ and $\rho_D(B)$ including only the magneto-spin or only the magneto-orbital effects. Again the importance of the ‘low-field’ magneto-spin and the ‘high-field’ magneto-orbital effects on both $\rho(B)$ and $\rho_D(B)$ are manifestly obvious in our theoretical results.

Finally in Fig. 4 we present some clear-cut theoretical predictions for the temperature dependence of bilayer magnetodrag $\rho_D(B; n, T)$ in the presence of the in-plane magnetic field $B$. In particular, we fit the temperature-dependence of $\rho_D$ at a fixed low density to approximate power law behaviors: $\rho_D(T) \sim T^\alpha$ with the magnetic field dependent exponents $\alpha(B)$ indicating the nature of the temperature dependence of magnetodrag at various magnetic field values. The striking theoretical prediction, which stands out in Fig. 4, is that $\alpha$ manifests a very strong magnetic field dependence with $\alpha(B)$ decreasing from a low-field value of about 2.3 ($B < B_s$) to a high-field value of about 1.8 ($B > B_s$). This sharp drop in the temperature exponent of magnetodrag is a direct consequence of the strong suppression in magneto-screening arising from the carrier spin polarization induced by the in-plane magnetic field. (Note that these exponents are ‘effective’ exponents and not exact exponents.) Our approximate analysis of the experimental data [7] indicate that our theoretical results for $\alpha$ shown in Fig. 4 are in excellent qualitative (and reasonable quantitative) agreement with ref. [7], where $\alpha$ changes from around 2.5 for $B \sim 0$ to about 1.3 for large $B$.

We conclude by emphasizing that our Fermi liquid theory based detailed calculations are in excellent qualitative agreement with the experimentally observed magnetodrag data [7], and therefore more exotic non-Fermi liquid theory [15] descriptions (which cannot typically produce quantitative results as shown in our Figs. 1–4) seem unnecessary. The ‘smoking gun’ breakdown of the Fermi liquid description of bilayer drag experiment would be the observation of a drag resistance which remains finite as $T \to 0$ since within the Fermi liquid theory $\rho_D(T = 0) = 0$. All existing experimental data seem to be consistent with the Fermi liquid conclusion that $\rho_D(T \to 0) \to 0$.

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