Left-right asymmetries, the weak mixing angle, and new physics

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The goal of this article is to outline the advantages of the measurement of left-right asymmetries in lepton-lepton ($l^- l^- \rightarrow l^- l^-$) scattering for performing precision measurements of $\sin^2 \theta_W$ and the discovery of “new physics”.

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I. INTRODUCTION

It is well known that the observables in the lepton-lepton scattering have less uncertainties than in the lepton-hadron or hadron-hadron cases. This is because gauge models only specify the lepton-quark or quark-quark vertices, and some parton model assumptions of hadron structure must be invoked to relate the lepton-quark and quark-quark interactions with the lepton-hadron and hadron-hadron ones, and this implies in introducing some uncertainties in the calculation.

In particular, the appealing features for studying the parity-violating asymmetries between the scattering of left- and right-handed polarized electrons on a variety of fixed targets ($e^- e^-, e^- \mu^-$) were pointed out some years ago by Derman and Marciano [1], and they were systematically studied in lepton-lepton scattering for both fixed target ($e^- e^-, e^- \mu^-$) and collider ($e^- e^-, e^- \mu^-, \mu^- \mu^-$) experiments in Refs. [2–6].

The left-right asymmetry is defined as

$$A_{RL}(ll \rightarrow ll) = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L},$$

(1)

where $d\sigma_R(L)$ is the differential cross section for one right (left)-handed lepton $l$ scattering on an unpolarized lepton $l$. Another interesting possibility is the case when both leptons are polarized. We can define an asymmetry $A_{RL;RL}$ in which one beam is always in the same polarization state, say right-handed, and the other is either right- or left-handed polarized (similarly we can define $A_{L;LR}$):

$$A_{R;RL} = \frac{d\sigma_{RR} - d\sigma_{RL}}{d\sigma_{RR} + d\sigma_{RL}},$$

$$A_{L;RL} = \frac{d\sigma_{LR} - d\sigma_{LL}}{d\sigma_{LL} + d\sigma_{LR}}.$$  

(2)

We can define also an asymmetry when one incident particle is right- handed and the other is left-handed and the final states are right- and left or left- and right-handed:

$$A_{RL;RL,LR} = \frac{d\sigma_{RL;RL} - d\sigma_{RL;LR}}{d\sigma_{RL;RL} + d\sigma_{RL;LR}},$$

(3)

or similarly, $A_{L;RL,LR}$. All of these asymmetries can be calculated for both fixed target and colliders experiments. For more details on the notation see Ref. [5].

The appealing features of this type of measurement, following [1], are: (1) The asymmetry is manifestly parity violating: therefore its measurement determines the electron’s parity violation weak neutral current interaction with the target or with the other beam in the case of colliders. (2) Because the effect investigated is due to the interference between the weak and electromagnetic amplitudes, the asymmetry is proportional to $G_F$ and hence larger than the usual weak interaction effects which are $O(G_F^2)$. (3) Since the asymmetry is a ratio, uncertainties (theoretical and experimental) which are common to both the numerator and denominator cancel out and therefore we can use them to perform precision measurements in the context of the standard model [7] (see section II). (4) Finally, this kind of interference measurements determines the relative sign between the weak and electromagnetic interactions. Unified gauge theories give unique predictions for this algebraic sign; therefore their determination provides an additional check on various models- i.e. some models may predict the correct magnitude for the asymmetries, but the wrong sign! (see section III). For fixed target experiments, in $e^- e^-$ scattering a very intense polarized electron beam inciding
on an unpolarized hydrogen target while in $e^-\mu^-$ scattering it is considered an inciding unpolarized muon beam on polarized electron target.

There are some advantages in considering asymmetries in $e^-e^-$ and $e^-\mu^-$ scattering. Firstly, although in fixed target experiment the value of $A_{RL}$ asymmetry is small (see below), the cross sections of these processes are large allowing for a good statistic and, on the other hand, in collider experiments the asymmetry is large but cross sections are small. Secondly, in $e^-\mu^-$ scattering the background contribution from $\mu^-N$ scattering is less severe because one could trigger on a single scattered electron which could not have arisen from a $\mu^-N$ collision. Besides in the $e^-\mu^-$ case that muon beams generally have energies at least one order of magnitude greater than the electron beams. We would like to point out that these asymmetries could be already measured in existing fixed target experiments like E158 and NA47.

On the other hand, we have been interested in these asymmetries in collider experiments since in the future we hope that colliders like the so called Next Linear Collider (NLC) or the International Linear Collider (ILC) could work in the $e^-e^-$ mode and the First Muon Collider (FMC) could work in $\mu^-\mu^-$ modes and hybrid collider could do well in $e^-\mu^-$ scattering.

This paper is organized as follows. The usefulness of the measurements of these asymmetries in collider and fixed target experiments are discussed briefly in section II while “new physics” in section III. Our conclusion are given in the last section.

II. DETERMINATION OF $\sin^2 \theta_W$

At present only a couple of observables differ from the prediction of the standard model at the level of 3 standard deviations: the value of the forward-backward asymmetry $A_{FB}^{e\mu}$ in $e^+e^-$, and the value of $\sin^2 \theta_W(\nu N)$ obtained from the ratios of the charged and neutral currents in neutrino-nucleon scattering. Hence a better measurement of the weak mixing angle in process different from $e^+e^-$ and $\nu N$ scattering has become mandatory. This could be the left-right asymmetries in $e^-e^-$. 

In the standard electroweak model the $A_{RL}(e^-e^-)$ asymmetry at the tree level in fixed target experiment is given by:

$$A_{RL}^{FT,SM}(ee) \approx -\frac{G_F Q^2}{\sqrt{2} \pi \alpha} \frac{1 - y}{1 + y^4 + (1 - y)^4} (1 - 4 \sin^2 \theta_W),$$  

where $Q^2 = -y(2m_e^2 + 2m_e E_{\text{beam}})$, $y = \sin(\theta/2)$. In the Eq. (4) the approximation $m_e^2 \ll Q^2 \ll M_Z^2$ was used, and if $E_{\text{beam}} = 50 \text{ GeV}$, $\theta \approx 90^\circ$ ($y = 1/2$) we obtain the following value:

$$A_{RL}^{FT,SM}(ee) \approx -3 \times 10^{-7}. $$

Marciano and Czarnecki, Ref. [13] have calculated the one loop electroweak radiative corrections and found a rather substantial $40 \pm 3\%$ reduction of the tree level prediction.

We have also verified that $A_{RL}(e^-\mu^-)$, in fixed target experiments, is sensitive to the value of $\sin^2 \theta_W$. In this case the $A_{RL}$ asymmetry, for $m_\mu = 0$, is given by (1):

$$A_{RL}^{FT,SM}(e\mu) \bigg|_{m_\mu = 0} \approx -\frac{8G_F}{\sqrt{2}} \frac{g_V g_A M_W^2}{\pi \alpha M_Z^2} \frac{y s}{1 + (1 - y)^2} (1 - 4 \sin^2 \theta_W).$$

If we consider a fixed target experiment with $E_\mu = 190 \text{ GeV}$, $E_\mu$ is the muon beam energy, a 0.5% change in the $A_{RL}(e^-\mu^-)$ value corresponds to a 0.04% change in $\sin^2 (\theta_W)$, i.e., a change from 0.2315 to 0.2316. This fact show that it could be useful for doing very precise electroweak studies in this kind of experiment too.

More recently it was pointed out in Ref. [1] that the asymmetry $A_{RL}(e^-e^-)$ in colliders experiments can be used to measure $\sin^2 \theta_W$, with $\mu^-\mu^-$ colliders. The results for other values of $E_{\text{CM}}$ are shown in Table I. We see that the weak mixing angle is more sensitive to that asymmetry for the $e^-\mu^-$ scattering.

It is clear that in the case of $e^-e^-$ we need to measure $A_{RL}$ with more precision than in the case $e^-\mu^-$ to have the same variation in $\sin^2 \theta_W$ at the energies $\sqrt{s} = 1.0$ TeV, 1.5 TeV and 2 TeV. In the case of $\sqrt{s} = 0.5$ TeV we need the same precision in both experiments. We should stress that the results for $\mu^-\mu^-$ scattering are the same as in the case of $e^-e^-$. 


As we have shown on this section the standard model implies a predictable degree of parity violation in lepton-lepton scattering, ranging from low energy phenomena to high energy, and we can use them to do very precise tests of the model. Of course, such sensitivity implies that a measurement of $A_{RL}$ is also a good probe of “new physics” if it really exists, as we will show on the next section.

### III. NEW PHYSICS

In general the standard model is exceedingly successful in describing leptons, quarks and their interactions, it is in excellent agreement with the worldwide data [17]. However, the necessity to go beyond it, from the experimental point of view, comes at the moment only from neutrino data [18]. If neutrinos are massive then new physics beyond the standard model is needed.

On the other hand, any extension of the standard model implies necessarily the existence of new particles. We can have a rich scalar-boson sector if there are several Higgs-boson multiplets [19] or have more vector and scalar fields in models with a larger gauge symmetry as in the left–right symmetric [20] and in 3-3-1 models [21], or we also can have at the same time more scalar, fermion, and vector particles as in the supersymmetric extensions of the standard model [22], or in the supersymmetric 3-3-1 model [23].

Recently Dimopoulos and Kaplan [24] take up again the Weinberg’s idea [25] that the observed value of the weak mixing angle $\sin^2 \theta_W = 0.231$ suggests indeed an $SU(3)$ electroweak symmetry, for instance models with $SU(3) \otimes SU(2) \otimes U(1)_{X}$ electroweak gauge symmetry at the 1 TeV scale. The point is that in such models $\sin^2 \theta_W (M) = 1/4$, with $M$ being an energy scale of $SU(3)$ breaking. This is very important since it means the possible existence of a new fundamental energy scale which is not related to supersymmetry, neutrino masses, unification, or superstrings [26]. At this energy scale new exotic quarks, doubly charged vector bosons and/or extra dimensions may exist. One way of considering the known quarks with the extra $SU(3)$ electroweak symmetry model is by embedding it in a Pati-Salam-type model [27], with gauge symmetry $SU(4)_c \otimes SU(2)_L' \otimes SU(2)_R' \otimes SU(3)'$, and with the known quarks transforming only through the first three factors [24]. There is a first symmetry breakdown to $SU(3)_c \otimes SU(2)_L' \otimes U(1)'$, and finally to $SU(2)_L' \otimes U(1)$. Another interesting way to solve the introduction of quarks without invoking a $U(1)$ factor as in the 3-3-1 models [21], is to consider $SU(3)_c \otimes SU(3)_W$ models in 5 dimensions [22].

Independently of this interesting fact, we recall that the $SU(3)$ symmetry among the lightest particles i.e., $e^-, e^+, \nu_e$ and $\nu_\mu$ could be the last symmetry involving the known leptons or $SU(4)$ if we add right-handed neutrinos $\nu_\tau$. If we impose this symmetry on the electron sector we must also to impose it upon all the other particles and if we do not want to introduce extra dimensions we must introduce extra exotic charged quarks. In this case it is also possible to have anomaly cancellation only if the number of families is three or a multiple of three [21]. If we accept that the observed value for the weak mixing angle is an indication of a $SU(3)$ symmetry at the TeV level, it means that the respective supersymmetric model has also to have this $SU(3)$ symmetry, and supersymmetry has to be naturally broken at the same scale and it can also be embedded in theories of TeV-gravity [24]. Any way, all those sort of models involve doubly charged vector bosons.

Usualy we have several particles contributing to a given observable, and for this reason it is very difficult to identify their contributions in both the usual and exotic processes. In some models [21][22][23][24] the contributions of the scalar-bosons are not suppressed by the fermion masses and they can have the same strength of the fermion–vector-boson coupling. However, for $e^-e^-$ and $\mu^-\mu^-$ scattering, in the $s$-channel, the contributions to the $A_{RL}$ asymmetry of doubly charged scalars, like $H'^-$, cancel out. Hence these parity violating asymmetries are only sensitive to the doubly charged vector bosons. In view of this, we will discuss in this section the role of $A_{RL}$ asymmetry in finding signal for “new physics”: doubly charged and neutral vector bosons, $U_{\mu}^-$ and $Z'$, respectively [22].

In Refs. [1][19] it was noted that the left-right asymmetries in the lepton–lepton diagonal scattering are quite sensible to doubly-charged vector boson $U_l$ contributions. The main result obtained is

$$A_{RL;SU}^U(l^-l^- \rightarrow l^-l^-) \approx -A_{RL;SU}^U(l^-l^- \rightarrow l^-l^-),$$

where $l = e, \mu$.

In the model of the Ref. [21] there is also a $Z'$ neutral vector boson which couples with leptons. It was shown in Ref. [3] that for the non-diagonal scattering $(e^-\mu^-)$ the asymmetries are sensible to the existence of an extra neutral vector-boson $Z'$, because

$$A_{RL;SU}^{CO;ESM}(e^-\mu^- \rightarrow e^-\mu^-) \neq A_{RL;SU}^{CO;ESM+Z'}(e^-\mu^- \rightarrow e^-\mu^-).$$
Hence, both $U^{++}$ and $Z'$ vector bosons can be potentially discovered in these sort of processes by measuring the left-right asymmetries. Here we should stress that although the results about the $U^{++}$ and $Z'$ were obtained in the context of the 3-3-1 model they are still valid in the supersymmetric version of the latter model [23]. The reason is that the lagrangian of the interaction between leptons and gauge bosons are the same in both models [33].

Finally, we want to stress that the $e^-e^-$ collider is ideal for discovering and studying selectrons. This will allow to do precise measurements of the neutralino masses and the opportunity to observe and analyze cascade decays of the selectron [34,35].

IV. CONCLUSIONS

We should remember that left-right asymmetries have played key roles in establishing the validity of the standard model. They will continue to provide valuable tools during the next generation of colliders doing precision studies of the standard model and even more exciting is the possible direct detection of new phenomena such as bileptons, $Z'$s, supersymmetry, and others. Here in this work we have reviewed how the asymmetries, defined in Ref. [4] can be used to perform this kind of study.

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Table I. Dependence of the $A_{RL}$ asymmetry on the $\sin^2 \theta_W$.

| $\sin^2 \theta_W$ | $A_{CO; ESM}^{A_{RL}}(ee^-)$ | $A_{CO; ESM}^{A_{RL}}(e\mu^-)$ |
|-------------------|-----------------|-----------------|
|                   | $E_{CM} = 0.5$ TeV | $E_{CM} = 1.0$ TeV |
| 0.23073           | -0.29536         | -0.01789         |
| 0.23063           | -0.29545         | -0.01801         |
|                   | $E_{CM} = 1.5$ TeV | $E_{CM} = 2$ TeV |
| 0.23073           | -0.33309         | -0.01977         |
| 0.23063           | -0.33319         | -0.01989         |
| 0.23073           | -0.34786         | -0.02016         |
| 0.23063           | -0.34796         | -0.02028         |
| 0.23073           | -0.35574         | -0.02030         |
| 0.23063           | -0.35583         | -0.02043         |

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FIG. 1. The $A_{RL}$ asymmetry as a function of $\sin^2 \theta_W$ for $e^-e^-$ and $e^-\mu^-$ collider experiments and $E_{CM} = \sqrt{s} = 0.5\text{TeV}$. 