An approximate calculation of the S-lay method offshore pipeline

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Abstract. The problem of the curvilinear form of the pipeline laid from the pipe-laying vessel to the bottom of the water area and the internal forces associated with this form in the pipeline are considered in the article. The assessment of some existing research methods of the issue under consideration that can lead to the conclusion of its relevance is given. To solve the problem, we propose an original approximate nonlinear differential equation of the second order, compiled for the convenience of its application in relative quantities. A fairly simple algorithm for the numerical solution of this equation in Wolfram Mathematica is developed, a specific numerical example is considered. The results suggest that the proposed method of the stress-strain state analysis of the offshore oil and gas pipeline during its laying can be very useful in practical application.

1. Introduction

The design and construction of pipelines in the Arctic region requires a reliable but at the same time simple mathematical method of calculation. On the Arctic shelf in the condition of the ice absence pipeline-laying is carried out by the S-lay method [1, 2]. Successive extension of the pipe in the horizontal position with subsequent descent along the stinger leads to the appearance of a curved part of the pipeline between the placement point on the bottom soil and the tensioner in the form of the S-shaped structure [3-5]. The relevance of the study of this issue is currently confirmed by the studies [6, 7]. The calculation method of the pipeline in the process of its laying is determined by the normative documents [8, 9] and is explained by the design scheme, in Fig. 1.

The pipeline descends from the stinger of the pipe-laying vessel in point A at angle \(\alpha\) to the horizon and lies on the bottom of the water area in point O. Unknown are the curvilinear form of the pipeline from point O to point A, the length \(l\) of the laying zone, as well as the force boundary conditions shown in red in the diagram. The depth of the water area \(H\), the angle \(\alpha\) of the pipeline descent from the stinger, the angle \(\beta\) of the non-horizontal bottom of the water area and the reduced running load \(q\) can be considered as given. In addition, there is a natural limitation for the bending moment \(M_A\), which cannot be greater than the maximum permissible value for this pipeline, which determines the radius \(R\) of the stinger curvature. In this calculation scheme, it is also assumed that the pipeline curvature in point O is zero (to the left of it the pipeline is straight). The main task is to find the internal forces \(M\), \(Q\) and \(N\) in the sections of the pipeline, on which its strength depends.
Since the curvature $\kappa = d\theta/ds$ of the flat curved axis of the pipeline is associated with the bending moment well known in the mechanics of the deformable solid by the ratio $\kappa = M/EJ$, the natural basic equation for the solution of this problem is the differential ratio

$$\frac{d\theta}{ds} = \frac{M}{EJ},$$

assuming its solution in the coordinates $\theta$ and $s$. This is how it is presented in the standard [8]. It is obvious that the domain $S (0 \leq s \leq S)$ in the formulation of the problem is not defined, and this creates significant computational problems for obtaining practical results. They are manifested in the fact that the solution has to be sought by implementing in each step of the iterative process the finite difference method for the system of differential equations. In addition, in [8] it is assumed that the contact of the pipeline with the stinger occurs along the entire length of the stinger, and this creates additional difficulties for the formation of boundary conditions.

To solve a similar problem in [10], an approximate differential equation of the curved axis of the rod is used, which is in the coordinates Fig.1 is as follows:

$$\frac{d^2y}{dx^2} = -\frac{M}{EJ}.$$  \hspace{1cm} (2)

Here, for the bending moment, the sign rule is such that the positive moment produces a convex curve. With the help of this differential equation, it is impossible to adequately solve the problem of the shape of the curved axis of the pipeline, since the error inherent in it is equal to the ratio of the real angle of inclination of the tangent to the angle $45^\circ$. According to the diagram in Fig.1 it can be seen that this ratio can be close to one, i.e. the error can reach 100%. From this it follows that equation (2) for the solution of the problem is not suitable. Instead, you need to use the so-called exact differential equation

$$\frac{y''}{(1+(y')^2)^{1.5}} = -\frac{M}{EJ},$$ \hspace{1cm} (3)

which is a direct consequence of the equation (1). This latter equation will be used further to solve the problem of the curved axis of the pipeline.

2. Problem statement and solution

From the design scheme in Fig. 1 the following expression for the bending moment follows:

$$M(x) = M_A - Q_A \cos \alpha \cdot (l - x) - Q_A \sin \alpha \cdot (H - y) +$$
\[ +N_A \cos \alpha \cdot (H - y) - N_A \sin \alpha \cdot (l - x) + \int_x^l q \, ds (u - x). \quad (4) \]

If this expression is substituted into the differential equation (3), then the integral term in it, which is the bending moment in the section \( x \) of the reduced distributed load \( q \), will create almost insurmountable difficulties for its solution. In order to simplify the situation, this integral term can be represented as following:

\[ \int_x^l q \, ds (u - x) = q (S - s) \cdot 0.5 (l - x). \quad (5) \]

Here, the value \( (S - s) \) is the length of the pipeline from the current point with abscissa \( x \) to point A. This curvilinear length \( (S - s) \) can be expressed in terms of the horizontal length \( (l - x) \) by multiplying the latter by some dimensionless numerical parameter \( \alpha \):

\[ s = a (l - x). \quad (6) \]

The value of this parameter is variable along the length of the pipeline and according to the scheme in Fig.1, obviously more than one. It can be estimated approximately by the value slightly higher than the ratio of the hypotenuse of the rectangle with sides \( (l - x) \) and \( (H - y) \) to the cathetus \( (l - x) \).

The expression (4) with respect to (5) and (6) takes the form

\[ M(x) = M_A - Q_A \left( l - x \right) \cos \alpha + (H - y) \sin \alpha \] + \[ +N_A (H - y) \cos \alpha - (l - x) \sin \alpha \] + \[ +0.5qa(l - x)^2. \quad (7) \]

Assuming, as already mentioned, that the curvature of the pipeline at the point of contact with the ground is zero, i.e. using the initial condition

\[ M(x)|_{x=0} = 0. \quad (8) \]

we express the bending moment at the point of descent of the pipeline from the stinger through the remaining forces at this point and the linear load on the pipeline:

\[ M_A = Q_A (l \cos \alpha + H \sin \alpha) - N_A (H \cos \alpha - l \sin \alpha) - 0.5qla^2. \quad (9) \]

The forces \( QA \) and \( NA \) determine the force interaction of the pipeline with the pipe-laying vessel. It is clear that the vertical component of this interaction will always be present, and the horizontal component \( XA \) can be controlled by the ship’s engine. Suppose that the most economical mode is one, in which this component will stretch the pipeline, as shown in the diagram in Fig. 1. Taking this condition, we obtain the ratio

\[ N_A \cos \alpha - Q_A \sin \alpha = X_A. \quad (10) \]

Directly from here follows

\[ N_A = \frac{M_A \cos \alpha}{l} + \frac{X_A}{l} \tan \alpha. \quad (11) \]

Further substitution (11) in (9) leads to the following expression:

\[ M_A = \frac{Q_A l}{\cos \alpha} - X_A (H - l \tan \alpha) - 0.5qla^2, \quad (12) \]

where

\[ Q_A = \frac{M_A \cos \alpha}{l} + \frac{X_A}{l} (H \cos \alpha - l \sin \alpha) + 0.5ql \cos \alpha. \quad (13) \]

Next, substituting (13) into (11), we obtain

\[ N_A = \frac{M_A \sin \alpha}{l} + \frac{X_A}{l} (\frac{H}{T} \sin \alpha + \cos \alpha) + 0.5ql \sin \alpha. \quad (14) \]

Using the relations (13) and (14), the expression (7) is obtained in the following form:

\[ M(x) = M_A \frac{x}{l} - X_A H \left( \frac{y}{H} - \frac{x}{l} \right) - 0.5aql^2 \frac{y}{H} \left( 1 - \frac{x}{l} \right). \quad (15) \]

Next, the expression for the moment (15) is substituted into the differential equation (3), cancelling off the denominator on the left:

\[ y'' = \left[ - \frac{M_A \cos \alpha}{EJ} \frac{x}{l} + \frac{X_A H}{EJ} \left( \frac{y}{H} - \frac{x}{l} \right) + 0.5aql^2 \frac{y}{H} \left( 1 - \frac{x}{l} \right) \right] \frac{1}{1 + (y')^2} \left( 1 + (y')^2 \right)^{1.5}. \quad (16) \]

For the unification of the sought solution the equation (16) is driven to dimensionless coordinates:

\[ \frac{x}{l} = \xi, \quad \frac{y}{H} \equiv \eta. \quad (17) \]

The derivatives of the required function \( y \) in the new variables \( \xi \) and \( \eta \) according to (17) will look as follows:
In dimensionless coordinates, the equation (16) takes the form
\[ \eta'' = \left[ 0.5a \frac{q^2}{EJ} H \xi (1 - \xi) + \frac{X_\alpha^2}{EJ} (\eta - \xi) - \frac{M_\alpha l}{EJ} H \xi \right] \left[ 1 + \left( \frac{H}{l} \right)^2 (\eta')^2 \right]^{1.5}. \]  
(19)

Here, the power and geometric parameters are also logically driven to dimensionless form:
\[ \frac{q^2}{EJ} \equiv \kappa; \quad \frac{X_\alpha^2}{EJ} \equiv \chi; \quad \frac{M_\alpha l}{EJ} \equiv m; \quad \frac{1}{H} \equiv \lambda. \]  
(20)

After that the equation (19) will look as follows:
\[ \eta'' = [0.5a \kappa \lambda \xi (1 - \xi) + \chi (\eta - \xi) - m \lambda \xi][1 + (\eta')^2 / \lambda^2]^{1.5}. \]  
(21)

This form is most convenient for the numerical solution, which should be carried out taking into account the natural initial conditions
\[ \eta\big|_{\xi=0} = 0; \quad \eta'\big|_{\xi=0} = \lambda \tan \beta. \]  
(22)

In equation (21), the parameters \( \kappa, \chi \) and \( m \) characterize the external force action on the pipeline. The relative bending moment \( m \) at point A is limited by the flexural bearing capacity of the pipeline and can therefore be assigned as a known initial datum. The relative distributed load \( \kappa \) that loads the pipeline against the surface can also be assigned a priori; however, it should be assigned, perhaps, a smaller value, since it is the main force, to which the pipeline should resist. The horizontal tension parameter \( \chi \) is technological and is determined by the running capabilities of the pipe-laying vessel.

The geometric parameter \( a \) can be assigned with sufficient accuracy from the approximate ratio
\[ \sqrt{1^2 + H^2} \approx 1, \]
from which follows
\[ a \approx \sqrt{1 + 1/\lambda^2}. \]  
(23)

Under these conditions, the only freely variable parameter is \( \lambda \). When solving the differential equation (21), it should be chosen in such a way that the boundary condition at the top, at point A, is satisfied:
\[ \eta\big|_{\xi=1} = 1. \]  
(24)

The numerical solution of equation (21) is convenient to perform in Wolfram Mathematica. It is obtained in the form of the interpolation function \( \eta(\xi) \), as a result of numerical differentiation of which the interpolation functions \( \eta(\xi) \) and \( \eta'(\xi) \) are also formed. Further, the interpolation function for the bending moment is derived from the relations (3) and (18):
\[ M(\xi) = -\frac{EJ \eta''(\xi)}{H \lambda^2 (1 + (\eta(\xi)/\lambda)^2)^{1.5}}. \]  
(25)

The transverse force in the cross-sections of the pipe is obtained by the usual differentiation of the bending moment:
\[ Q(\xi) = -\frac{1}{\lambda^2 \sin \alpha} \frac{dM(\xi)}{d\xi}. \]  
(26)

For longitudinal force according to the diagram in Fig.1 an approximate ratio can be obtained:
\[ N(\xi) \approx N_A - q \sin \alpha (S - s). \]  
(27)

Hence, taking into account the relations (6), (17) and (23) for \( N(\xi) \), the following formula is obtained:
\[ N(\xi) \approx N_A - qH(1 - \xi). \]  
(28)

### 3. Example of calculation of the pipeline

The above calculation method was tested on a specific example with reference to the conditions of the Baydaratskaya Bay. The calculation is performed in Wolfram Mathematica for the pipeline with the following initial data. The depth of the water area \( H = 50 \) m; modulus of elasticity of the metal pipe \( E = 2 \cdot 10^5 \) MPa; inner radius of the pipe \( r = 0.50 \) m; outer radius of the pipe \( R = 0.51 \) m; normative resistance of the metal pipe \( [\xi] = 300 \) MPa. The value of the uniformly distributed load \( q \) is taken as the fraction of the upward force acting in the water on the unfilled pipeline, and in the example this fraction was 0.1. The bending moment in the pipe on the stinger \( M_A \) is assumed to be 0.9 of the maximum permissible \( M_{\text{m}} \). The horizontal tension force of the pipeline by \( X_A \) vessel is adopted 30 kN.
First, the program is presented, in which the numerical value of the parameter $\lambda$, is selected to ensure the fulfillment of the boundary condition (24).

\[
H = 50.0; \\
E1 = 20000000000.0; \quad r = 0.50; \quad R = 0.51; \\
\delta = R - r; \\
J = 3.14*\delta*r^3; \\
\sigma_d = 300000000.0; \\
Md = 3.14*\delta*r^2*\sigma_d; \\
EJ = E1*J; \\
\rho = 77000.0; \\
\rho в = 10000.0; \\
A = 2*3.14*\delta*r; \\
A_{обр} = 3.14*r^2; \\
q в = \rho в*A_{обр} - \rho*А; \\
kq = 0.1; \\
q = kq*qв; \\
\lambda = 4.58; \\
l = \lambda*H; \\
a = \sqrt{1.0 + 1/\lambda^2}; \\
k = 0.9; \\
Ma = k*Md; \\
Xa = 30000.; \\
\kappa = q*l^3/EJ; \\
\chi = Xa*l^2/EJ; \\
m = Ma*l/EJ; \\
sol = NDSolve[\{\eta'[\xi] == (0.5*a*k*\lambda*\xi*(1.0 - \xi) + \chi*(\eta[\xi] - \xi) - m*\lambda*\xi)*(1.0 + \\
(\eta[\xi])^2/\lambda^2)^1.5, \quad \eta[0] == 0, \quad \eta'[0] == 0, \quad \eta, \quad \xi, \quad 0, \quad 1.0\}]; \\
Plot[\eta[\xi]/.sol, \{\xi, \quad 0, \quad 1.0\}]
\]

The main thing in this part of the program is the penultimate operator performing the numerical solution of the equation (21), which is the interpolation function $\eta(\xi)$. It is printed in the form of a graph, by the form of which the condition is checked (24). In this example, this occurred at $\lambda = 4.58$ (in the text of the program highlighted in red), and the graph of the interpolation function looks like it is shown in Fig.2.

Then the first two derivatives of the interpolation function $\eta(\xi)$ are formed and the bending moment as the function of the relative coordinate $\xi$ is calculated according to the ratio (25).

\[
\eta_1 = \eta'[\xi]/.sol;
\]
\[ \eta_2 = \eta'[\xi] / \text{sol}; \]
\[ M = -EJ \ast \eta_2 / (H \ast \lambda^2 \ast (1 + (\eta_1 / \lambda)^2)^{1.5}) \]

\[ Q = \frac{1}{\lambda \ast H} \ast (\partial_\xi (-EJ \ast \eta_2 / (H \ast \lambda^2 \ast (1 + (\eta_1 / \lambda)^2)^{1.5}))) / \text{sol}; \]
\[ Q_a = Q / \xi \rightarrow 1; \]
\[ N_a = \frac{1}{\lambda} \ast (\sqrt{\lambda^2 + 1} \ast Xa + Qa); \]
\[ N = N_a - q \ast H \ast (1 - \xi) \]

The graphs of these efforts look as shown in Fig. 4.
4. Conclusion
A relatively simple differential equation (21) of the curved axis of the pipeline laid from the pipe-laying vessel to the bottom of the water area is obtained. A simple algorithm for the numerical solution of this equation in Wolfram Mathematica is proposed, which inevitably leads to the real result in the form of internal forces in the sections of the pipeline. The algorithm is implemented for a specific numerical example, for which graphical and numerical results are presented. Thus, it is possible to consider that the effective, simple for implementation and practically significant method of calculation of the pipelines laid from the water area is developed.

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