Reliability Estimation of Aero-engine Based on Mixed Weibull Distribution Model

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Abstract. Aero-engine is a complex mechanical electronic system, based on analysis of reliability of mechanical electronic system, Weibull distribution model has an irreplaceable role. Till now, only two-parameter Weibull distribution model and three-parameter Weibull distribution are widely used. Due to diversity of engine failure modes, there is a big error with single Weibull distribution model. By contrast, a variety of engine failure modes can be taken into account with mixed Weibull distribution model, so it is a good statistical analysis model. Except the concept of dynamic weight coefficient, in order to make reliability estimation result more accurately, three-parameter correlation coefficient optimization method is applied to enhance Weibull distribution model, thus precision of mixed distribution reliability model is improved greatly. All of these are advantageous to popularize Weibull distribution model in engineering applications.

1. Introduction

In the field of aviation maintenance, Weibull distribution model is a very powerful analytical tool which changes three parameters. Weibull distribution can change into exponential distribution, normal distribution and lognormal distribution. So it has a strong ability to adapt, and thus has strong and precise ability for aviation engine failure data.

For a long time, aircraft engine performance engineers assess reliability of engine fleet with three-parameter Weibull distribution model. Civil aviation engine is a complex mechanical electronic systems, in many cases, engine failure modes are not a single one, and engine failure data are often in a mixed state [1]; In addition, with the airlines to further improve engine fleet management, three-parameter Weibull distribution model is difficult to meet the requirements of accuracy for engine reliability evaluation. Therefore, some academics use multiple mixed three parameter Weibull distribution model to fit engine failure data, but determination of weight coefficient for mixed Weibull distribution model has not been effectively resolved.

In a word, engine failure data is often based on a variety of failure modes [1], it is difficult for three-parameter Weibull distribution model to meet precision requirement, therefore, mixed Weibull distribution model is used for aero-engine reliability estimation.

Dynamic weight coefficient of mixed Weibull distribution model reflects the weight of some failure modes about the aero-engine, and the importance of this child distribution model in mixed distribution model, it also can reflect the importance and authority of the failure data. Weight coefficient affects...
precision of reliability evaluation directly. Previous weight coefficient is based on subjective or objective method, this paper adopts the method based on subjective and objective comprehensive empowerment [2] to determine weight coefficient, relative to previous weight determination method, it is more scientific and reasonable.

2. An Overview of Mixed Weibull Distribution Model
Number of failure modes in aircraft engine fleet is m, and failure data of every failure mode obey Weibull distribution. Probability density functions of each failure mode is
\[ f_i(t), f_2(t), \ldots, f_m(t) \]

Reliability function and failure probability density function
\[ R(t) = \sum_{i=1}^{m} \left[ p_i \cdot R_i(t) \right], \left( \sum_{i=1}^{m} p_i = 1 \right) \]
\[ f(t) = \sum_{i=1}^{m} p_i \cdot f_i(t), (i = 1, 2, \ldots, m) \]

3. Weight Coefficient of the Model
Now, engine performance engineers often use three-parameter Weibull distribution model to estimate reliability of engine fleet. But civil aviation engine is a complex mechanical electronic system, and in many cases, failure modes are not single. Therefore, engine failure data are often in mixed state [1]; further, some academics have used three-parameter mixed Weibull distribution model to fitting engine failure data. But weight coefficient haven't been solved effectively.

To solve the problem, concept of dynamic weight coefficient is put forward. With the method, weight of certain type failure mode for aero-engine can be revealed, the importance and authority of failure data can also be revealed. Precision of reliability evaluation can be affected directly by weight coefficient. Previous weight coefficient is based on the method of subjective or objective, this paper adopts the method based on subjective and objective comprehensive empowerment [2] to determine weight coefficient [3], it is more scientific and reasonable.

In consideration of aero-engine failure modes [4] (subjective) and engine failure data (objective), dynamic weight coefficient is used as weight coefficient of mixed Weibull distribution.

3.1. The choice of aero-engine failure mode
First of all, engine reliability [5] is effected differently by each engine failure mode. Therefore, in order to characterize proportion of each engine failure mode, importance of failure modes is chosen as the first evaluation indicator of mixed weight coefficient.

Secondly, engine failure data used in Weibull model come from engine working on wing, due to existence of the subjective and objective factors such as different flight conditions, these failure data exist problems inevitably, such as unstable. Therefore, in order to make stable and objective failure data in mixed Weibull distribution model to occupy a large proportion, reliability of failure data is chosen as the second evaluation indicator.

The above two criteria based on subjective and objective evaluation indicator are shown in table 1.

| score      | importance of engine failure mode | reliability of failure data                                      |
|------------|-----------------------------------|-----------------------------------------------------------------|
| [0.75~1]   | reflect engine failure directly   | engine wear and tear or failure is prone and number > 8         |
| [0.5~0.75] | reflect engine failure more clearly| engine wear and tear or malfunction occurs more easily and number > 4 |
[0.25~0.5) reflect engine failure generally engine wear and tear or failure is not easy to occur and number < 4
[0~0.25) reflect engine failure obviously less engine wear and tear or malfunction occurs and number < 4

3.2. Determination of dynamic weight coefficient

From the above two indicators, evaluate typical failure modes respectively such as compressor and turbine rotor blade failure, compressor disk and turbine disc failure, shaft and bearing failure, gear failure [3], and then base on the principle of entropy weight method to determine dynamic weight coefficient of each failure mode in mixed Weibull distribution model. With this approach, weight coefficient of mixed Weibull distribution model presents dynamic, to overcome disadvantages bases on subjective or objective method.

According to table 1, estimate the \( j \)th engine failure mode, to get estimation \( z_y \) corresponding the \( i \)th evaluation index, the eigenvalues matrix for evaluation indicators is

\[
Z = (z_y)_{m \times n}
\]

(3)

\( m \) is the number of evaluation indicators in dynamic weight coefficient, as shown in table 1 there is only two evaluation indexes: importance of engine failure modes and reliability of engine failure data, therefore, \( m=2 \), \( n \) is the number of failure modes for a given engine failure data, and \( n \) values differently with dynamic change of failure data.

The number of engine failure modes \( n \) is not fixed, engine failure modes and proportion of each one is likely to be different reflected by different failure data, at this point, Weibull probability paper is used to determine the value of \( n \).

First of all, by Eq. 4 [6], \( \hat{\gamma} \) is calculated. And then every data plus or minus \( \hat{\gamma} \) (minus when \( \hat{\gamma} \) is a positive value, and vice versa), so as to make failure data on Weibull probability paper presents for piecewise linear with inflection point, it is not hard to find the number of inflection point on piecewise linear, the number of inflection point plus 1 is the \( n \) value, namely the number of failure mode.

\[
\hat{\gamma} = t_2 - \frac{(t_3 - t_1)(t_2 - t_1)}{(t_2 - t_1) - (t_2 - t_1)}
\]

(4)

\( t_1 \) is the first time for engine failure, \( t_3 \) is the last time for engine failure, \( t_2 \) is the Y coordinates taking midpoint of \( t_1 \) and \( t_3 \).

In addition, in most cases, by above method can obtain the piecewise linear on Weibull probability paper and different slope of each line (\( \hat{\beta} \)), this is due to different \( \hat{\beta} \) represents different failure mode [7]. Bases on this, engine failure modes can be obtained from different \( \hat{\beta} \), and dynamic weight coefficient of mixed Weibull distribution model can be calculated according to table 1.

By principle of entropy weight method [8], entropy for the \( i \)th evaluation indicator [9]

\[
H_i = -k \sum_{j=1}^{n} f_{ij} \ln f_{ij}, \quad i = 1, 2
\]

(5)

Thereinto, \( k = \frac{1}{\ln n} \); \( f_{ij} = \frac{z_{ij}}{\sum_{j=1}^{n} z_{ij}} \). While \( f_{ij} = 0 \), \( \ln f_{ij} = 0 \).

Then entropy weight of the \( i \)th evaluation indicator
\[ \omega_i = \frac{1 - H_i}{2 - \sum_{i=1}^{2} H_i} \]  
(6)

Dynamic weight coefficient corresponding to the jth failure mode

\[ p_j = \frac{\sum_{i=1}^{2} \omega_i z_{ij}}{\sum_{j=1}^{2} \sum_{i=1}^{2} \omega_i z_{ij}} \]  
(7)

Dynamic weight coefficient of each failure mode can be obtained

\[ p = [p_1, p_2, \cdots, p_j, \cdots, p_n] \]  
(8)

Sum up, processes are shown in figure 1.

- **Determine the engine failure mode**
- **According to table 1, Z = \( z_{ij} \)_{mn}**
  
  \[ f_y = \frac{z_{ij}}{\sum_{j=1}^{x} z_y}; \quad k = \frac{1}{\ln n} \]
  
- **Calculate entropy of the ith evaluation indicator**, \( H_i = -k \sum_{j=1}^{n} f_y \ln f_y; i = 1, 2, \cdots, m \)
- **Entropy weight of the ith evaluation indicator**, \( \omega_i = \frac{1 - H_i}{m - \sum_{i=1}^{m} H_i} \)
  
  \[ p_j = \frac{\sum_{i=1}^{m} \omega_i z_{ij}}{\sum_{j=1}^{m} \sum_{i=1}^{m} \omega_i z_{ij}} \]

**Figure 1.** Calculating Dynamic Weight Coefficient

Linearly dependent coefficient [6]
\[
I = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}} = \frac{l_{xy}}{\sqrt{l_{xx}l_{yy}}}
\]

\[
l_{xx} = \sum_{i=1}^{n} \left[ \ln(t_i - \gamma) \right]^2 - \frac{1}{n} \left[ \sum_{i=1}^{n} \ln(t_i - \gamma) \right]^2
\]

\[
l_{yy} = \sum_{i=1}^{n} \left[ \ln \left( \frac{1}{1-F(t_i)} \right) \right] \ln(t_i - \gamma) - \frac{1}{n} \left[ \sum_{i=1}^{n} \ln \left( \frac{1}{1-F(t_i)} \right) \right] \ln(t_i - \gamma)
\]

\[
R_e \in [-1,1], \text{ the closer with plus or minus 1, the more accurate reliability assessment is.}
\]

4. Reliability Estimation

Table 2 are failure data for engine fleet:

| serial number | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| time/h        | 3453| 3856| 3894| 4085| 4119| 4541| 4675| 4924| 5025| 5194|
| serial number | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  |
| time/h        | 5462| 5653| 5742| 5794| 6075| 6299| 6429| 6479| 6715| 6851|
| serial number | 21  | 22  | 23  | 24  | 25  | 26  | 27  | 28  | 29  | 30  |
| time/h        | 7392| 7613| 7795| 8161| 8856| 9745| 9846| 9873| 9901| 9975|

First of all, by Eq. 4, \( \hat{\gamma} = 229 \), the above data minus \( \hat{\gamma} \), then make graphical analysis for data using MATLAB software [10,11]. As shown in figure 2, fitting 30 failure data with a straight line is impossible. In fact, there are two lines: (1)3453, 3856, 3894, 4085, 4119; (2) In addition to the first group.

![Weibull Probability Plot](image)

*Figure 2. Weibull Probability Graph*

With the method mentioned in section 2, \( p_1=0.27 \), \( p_2=0.73 \). With correlation coefficient of three-parameters optimization [12], parameters are estimated in table 3 [13].
Table 3. Parameters Estimation of the Models

| model                  | $\hat{\beta}$ | $\hat{\eta}$ | $\hat{\gamma}$ | $r_e$ |
|------------------------|---------------|---------------|----------------|------|
| single Weibull distribution | $\hat{\beta}$=3.88 | $\hat{\eta}$=4967 | $\hat{\gamma}$=4160 | 0.8535 |
| mixed Weibull distribution | $\hat{\beta}_1$=7.78, $\hat{\beta}_2$=1.92 | $\hat{\eta}_1$=4153, $\hat{\eta}_2$=5333 | $\hat{\gamma}_1$=3372, $\hat{\gamma}_2$=4547 | 0.9771 |

Reliability function, probability density function and failure rate function of Weibull distribution model are available, make a curve with Mathematica Software respectively:

![Figure 3. Mixed Weibull Distribution Model](image1)
![Figure 4. Single Weibull Distribution Model](image2)

With extending of engine working time, reliability of its parts declines gradually, and with engine reliability function curve we can determine reliability of engine roughly in a given period of working time. By reliability function curve shown in figure 3, when the engine is just put into use in running-in period, its reliability declines faster, by contrast, curve shown in figure 4 has a larger deviation from actual situation.
Weibull probability density function curve reflects distribution of engine failure interval. Probability density curve shown in figure 5 is more reasonable, according to bathtub curve, engine just put into use is still in the running-in period, so it is in high failure rate, different from curve shown in figure 6 which is close to zero.
Figure 8. Single Weibull Distribution Model

On the whole, engine failure rate is proportional to time on wing flight. Therefore, according to engine reliability curve and failure rate curve, engine performance engineers can determine optimal maintenance interval roughly for preventive maintenance. Under the condition of various failure modes, curve shown in figure 8 is monotone increasing [14], but engineering practical experience shows that curve shown in figure 7 is more reasonable.

5. Conclusion
Table 3 shows that the correlation coefficient of mixed Weibull distribution model is significantly greater than the single one. Thus, for mixed Weibull distribution model, its ability of fitting engine failure data is stronger, so it is more accurate to evaluate engine reliability. At the same time, bases on the method of subjective and objective comprehensive empowerment, to determine dynamic weight coefficient method is effective to avoid deficiency by using single method. Airlines also have gotten good feedback in practical operation, and have proved its good engineering practical value, and I think it should be put into popularization.

Engine failure data on the Weibull probability paper presents as a turning point of the piecewise linear, due to the size of the piecewise linear slope represents the engine different failure modes, accordingly we can determine its failure mode through it [7]. For example, when slope at around 3, often because of low cycle fatigue [7], By optimizing preventive maintenance plan, engineers can optimize the engine maintenance cost effectively in the early stage of loss.

Through this work, we can use failure data of engine fleet to obtain its reliability, with the same way, under given reliability, we can also get the reliable life of some engine fleet. At the same time we can get other characteristics, such as average life expectancy, all of them can reflect the reliability of the engine fleet at the same time. In actual operations of the airlines, it is very meaningful for aero-engine performance engineers.

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