Either late autumn this year or latest early next year LHC should have results with 2–3 times the current data which might give first clues on the couplings of the light narrow resonance. A strategy for measuring deviations from the Standard Model can be based on using the “full” Standard Model, including all available QCD and electroweak higher-order corrections, and supplement it with $d = 6$ local operators. Their Wilson coefficients are assumed to be small enough that they can be treated at leading order. Examples of the connection of local operators with BSM Lagrangians are presented as well as a discussion of Lagrangians with/without decoupling of heavy degrees of freedom. The whole strategy is critically reviewed in the light of internal consistency.

Keywords: Feynman diagrams, loop calculations, radiative corrections, effective Lagrangian, Higgs physics

PACS: 11.15.Bt, 12.38.Bx, 13.85.Lg, 14.80.Bn, 14.80.Cp
Contents

1 Introduction 1

2 \( \mathcal{L}_{SM} \): definitions 2

3 Simplified effective Lagrangian 4
   3.1 From the Lagrangian to the \( S \)-matrix ................................. 5
   3.2 Nature of \( d = 6 \) operators ........................................... 8
      3.2.1 Insertion of \( d = 6 \) operators in loops ............................ 9
      3.2.2 Admissible operators ............................................. 11
   3.3 Effective theory and renormalization ................................... 13

4 Higgs vertices 15

5 \( Z \) couplings 17

6 Partial decay widths 17
   6.1 \( gg \rightarrow H \) .............................................................. 20
   6.2 Simplified scenario ....................................................... 21
   6.3 BSM Lagrangians .......................................................... 21
   6.4 MSSM ................................................................. 24
   6.5 Decoupling .............................................................. 25
   6.6 Mixing ................................................................. 27

7 Decays into 4-fermions 27

8 Double Higgs production 32

9 Perturbative unitarity 32

10 Conclusions 34

A Appendix: The ghost Lagrangian 35

B Appendix: Polarization vectors 35
1 Introduction

An interesting question is how present and future experiments will be able to probe the couplings of the Higgs boson at a high level of precision, see Ref. [1] for a discussion. There is a wide variety of beyond the Standard Model (BSM) theories where the Higgs couplings differs from the Standard Model (SM) ones by less that 10%, as discussed in Ref. [2]. Among many papers dealing with the subject we quote those in Refs. [3–7]. For the most recent update on the subject we refer to the work of Refs. [8–10]. Interim recommendations to explore the coupling structure of a Higgs-like particle can be found in Ref. [11].

In this work, following Independence Day [12,13], we imagine that there is a huge space of theories, represented by local and renormalizable Lagrangians where SM is only one point. A possible strategy to look for deviations from the SM is the following:

• we take the SM as the theory of “light” degrees of freedom, i.e. \( d = 4 \) operators

• we simulate the unknown extension of the SM by the most general set of \( d = 6 \) operators that are obtained by integrating out the heavy degrees of freedom (we also assume no sensitivity to operators with \( d \geq 8 \) at LHC). This is equivalent to say that the BSM theory is unknown or matching is too difficult to carry out, so we write the most general set of interactions consistent with symmetries. The effective theory contains an infinite number of operators but only a finite number is needed for present (LHC) precision.

With enough statistics it should be possible to fit \( a_i \), the Wilson coefficients of the \( d = 6 \) operators, and there are two possibilities: a) they are close to zero (where zero = SM) or b) they are not. Option a) tell us that NLO corrections (or the residual theoretical uncertainty at NNLO level) and the \( a_i \) coefficients are small, and the SM is actually a minimum in our Lagrangian space or very close to it. This will explain nothing but it is internally consistent. Option b) raises serious problems since the effect of local operators is large and they cannot be included only at LO, but inserting operators in SM loops creates even more problems.

In case it is option b) we should move in the Lagrangian space and adopt a new renormalizable Lagrangian with the virtue of making zero that specific (large) Wilson coefficient \( a_i \); local operators are then redefined w.r.t. the new Lagrangian. Of course there will be more Lagrangians projecting into the same set of operators but still we could see how our new choice handles the rest of the data.

In principle, there will be a blurred arrow in our space of Lagrangians, and we should simply focus the arrow. Without invoking the explicit example of Supersymmetry this is the so-called inverse problem introduced in Refs. [14,15]: if LHC finds evidence for physics beyond the SM, how can one determine the underlying theory?

It is worth noting that this question is highly difficult to receive a complete answer at the LHC. The main goal will be to identify the structure of the effective Lagrangian (i.e. the different scalings of the various \( d = 6 \) operators) and to derive qualitative information on new physics; the question of the ultraviolet (UV) completion cannot be answered unless there is sensitivity to \( d > 6 \) operators. Therefore, we are looking for a relatively modest goal on the road to understand if the effective theory can be UV completed (bottom-up approach with no obvious embedding).

To set up our definitions of an enlarged theory we have to specify the concept of Higgs fields: it is the set of scalar fields that break electroweak symmetry (EW) by developing a vacuum expectation value (VEV). What we are looking for is evidence of SM Higgs properties or deviations from the SM behavior; in the latter case one has to understand consistency with other EW symmetry-breaking frameworks. Alternatively we can consider scenarios with more scalar fields, that are not Higgs fields (Higgs partners); the problem with more VEVs, or one VEV different from \( (T, Y) = (\frac{1}{2}, 1) \) (\( T \) is isospin and \( Y \) is hypercharge), is partially
related to the rho-parameter [16] which at tree-level is given by

\[ \rho_{\text{LO}} = \frac{1}{2} \sum_i \frac{c_i \left| v_i \right|^2 + r_i u_i^2}{\sum Y_i^2 \left| v_i \right|^2}, \quad c_i = T_i (T_i + 1) - Y_i^2, \quad r_i = T_i (T_i + 1), \]  

(1)

where the sum is over all Higgs fields, \( v_i (u_i) \) gives the VEV of a complex (real) Higgs field with hypercharge \( Y_i \) and weak-isospin \( T_i \). Our considerations will be presented in Section 6.5. The experimental limit on \( \rho - 1 \) are rather stringent. For a complete discussion of models respecting custodial symmetry we refer to Ref. [17].

In this paper we do not discuss questions related to spin, mass or CP quantum numbers but only couplings. In particular we discuss couplings to vector bosons since they control the unitarity behavior of longitudinal \( VV \)-scattering at high energy [18] (automatic in the SM). We also discuss the effects of Higgs partners and of Higgs self-couplings in Section 8.

For a better illustration of our approach we observe that a consistent effective theory, defined by

\[ \mathcal{L} = \mathcal{L}_4 + \sum_{n=4}^{N_4} \sum_{i=1}^{N_i} \frac{a_n^d}{\Lambda^{d-n}} \phi_i^{(d-n)} \]  

(2)

has arbitrary Wilson coefficients \( a_n^d \) which, however, give the leading amplitudes in an exactly unitary S-matrix at energies far below \( \Lambda \). The theory is non-renormalizable, which means that an infinite number of higher operators must be included. Nevertheless there is a consistent expansion of amplitudes in power of \( E/\Lambda \). Our goal will be to understand the \( d = 6 \) operators as a first step towards an UV completion, possibly a weakly-coupled one i.e. one where weakly-coupled new physics opens up around \( \Lambda \) and restores unitarity (a different scenario, classicalization, has been proposed in Refs. [19,20]).

In other words, the question is: can we classify the low-energy (LHC) observables that determine the road to UV completion? Note that there is a claim in the literature [21,22] that the coefficients \( a_i \) must be positive to have an UV completion which respects the usual axioms of S-matrix theory. In particular, in the work of Ref. [22] it is shown that UV completion is encoded in the sign of the scattering amplitude for longitudinal vector-bosons and that weakly-coupled UV completion requires a positive sign. Constraints on the sign of the couplings in an effective Higgs Lagrangian using prime principles have been derived in Ref. [23].

In Section 2, we present the SM Lagrangian, and in Section 3 we introduce the effective Lagrangian. We discuss Higgs vertices in Section 4 and \( Z \) vertices in Section 5. Section 6 gives the relevant partial decay widths of the H boson. In Section 7 we list the various \( H \rightarrow 4f \) decays. Double Higgs production is discussed in Section 8. We discuss perturbative unitarity in Section 9. We give our conclusions in Section 10.

2 \( \mathcal{L}_{\text{SM}} \): definitions

In this Section we collect all definitions that are needed to write the SM Lagrangian [24]. The scalar field \( K \) (with hypercharge 1/2) is defined by

\[ K = \frac{1}{\sqrt{2}} \left( \begin{array}{c} H + 2 \frac{M_g}{\sqrt{2}} + i \phi^0 \\ \sqrt{2} i \phi^- \end{array} \right) \]

H is the custodial singlet in \((2_L \otimes 2_R) = 1 \oplus 3\). Charge conjugation gives \( K^c_i = e_{ij} K_j^f \), or

\[ K^c = -\frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2} i \phi^+ \\ H + 2 \frac{M_g}{\sqrt{2}} - i \phi^0 \end{array} \right) \]
The covariant derivative $D_\mu$ is

$$D_\mu K = \left( \partial_\mu - i \frac{g_0}{2} B_\mu^a \tau_a - i \frac{g}{2} g_1 B_\mu^0 \right) K$$

(3)

with $g_1 = -s_\theta/c_\theta$ and where $\tau^a$ are Pauli matrices while $s_\theta (c_\theta)$ is the sine(cosine) of the weak-mixing angle. Furthermore

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (B_\mu^1 \mp i B_\mu^2), \quad Z_\mu = c_\theta B_\mu^3 - s_\theta B_\mu^0, \quad A_\mu = s_\theta B_\mu^3 + c_\theta B_\mu^0. \quad (4)$$

Here $a, b, \cdots = 1, \ldots, 3$. The dual tensor is defined by

$$F_{\mu \nu}^a = \epsilon_{\mu \nu \alpha \beta} F^{\alpha \beta}. \quad (6)$$

Furthermore, for the QCD part we introduce

$$G_{\mu \nu}^a = \partial_\mu g^a_\nu - \partial_\nu g^a_\mu + g_s f^{abc} g^b_\mu g^c_\nu. \quad (7)$$

Here $a, b, \cdots = 1, \ldots, 8$ and the $f$ are the $SU(3)$ structure constants. Finally, we introduce fermions,

$$\psi_L = \begin{pmatrix} 1 \\ b \end{pmatrix}_L, \quad f_{LR} = \frac{1}{2} (1 \pm \gamma^5) f$$

and their covariant derivatives

$$D_\mu \psi_L = (\partial_\mu + g B_\mu t_i) \psi_L, \quad i = 0, \ldots, 3$$

$$T^a = \frac{i}{2} \tau^a, \quad T^0 = -\frac{i}{2} g_2 I,$$

(8)

$$D_\mu \psi_R = (\partial_\mu + g B_\mu t_i) \psi_R, \quad t^a = 0,$$

(9)

$$t^0 = -\frac{i}{2} \begin{pmatrix} g_3 & 0 \\ 0 & g_4 \end{pmatrix}$$

with $g_i = -s_\theta/c_\theta \lambda_i$ and

$$\lambda_2 = 1 - 2 Q_u, \quad \lambda_3 = -2 Q_u, \quad \lambda_4 = -2 Q_d. \quad (10)$$

The Standard Model Lagrangian is the sum of several terms:

$$\mathcal{L}_{SM} = \mathcal{L}_{YM} + \mathcal{L}_K + \mathcal{L}_{gf} + \mathcal{L}_{FP} + \mathcal{L}_f$$

(11)

i.e., Yang-Mills, scalar, gauge-fixing, Faddeev-Popov ghosts and fermions. Furthermore, for a proper treatment of the neutral sector of the SM, we introduce a new coupling constant $g$, defined by the relation

$$g_0 = g (1 + g^2 \Gamma), \quad (12)$$

where $\Gamma$ is fixed by the request that the Z – A transition is zero at $p^2 = 0$, see Ref. [25]. The scalar Lagrangian is given by

$$\mathcal{L}_K = - (D_\mu K)^+ D_\mu K - \mu^2 K^+ K - \frac{1}{2} \lambda (K^+ K)^2.$$  

(13)

We will work in the $\beta_\hbar$-scheme [25], where

$$\mu^2 = \beta_\hbar - 2 \frac{\lambda}{g^2} M^2, \quad \lambda = \frac{1}{4} g^2 \frac{M^2}{\hbar M^2}.$$  

(14)

Furthermore, we introduce $v = \sqrt{2} M / g$, and fix $\beta_\hbar$ order-by-order in perturbation theory by requiring $< 0 | H | 0 > = 0$. 

3
Table 1: A selection of relevant \( d = 6 \) operators

\[
\begin{align*}
\mathcal{O}_K &= -\frac{g^3}{f} (K^\dagger K)^3 \\
\mathcal{O}^1_K &= g^2 (K^\dagger K) (D_\mu K)^\dagger D_\mu K \\
\mathcal{O}^3_K &= g^2 (K^\dagger D_\mu K) \left[(D_\mu K)^\dagger K\right] \\
\mathcal{O}^4_K &= i g^2 (D_\mu K)^\dagger \tau_a D_\mu K F_{\mu\nu}^a \\
\mathcal{O}^1_V &= g \left(K^\dagger K - \nu^2\right) F_{\mu\nu}^a F_{\mu\nu}^a \\
\mathcal{O}^3_V &= g K^\dagger \tau_a K F_{\mu\nu}^a F_{\mu\nu}^a \\
\mathcal{O}^2_V &= g \left(K^\dagger K - \nu^2\right) F_{\mu\nu}^0 F_{\mu\nu}^0 \\
\mathcal{O}^1_\tau &= g \left(K^\dagger K - \nu^2\right) \tilde{F}_{\mu\nu}^a \tilde{F}_{\mu\nu}^a \\
\mathcal{O}^3_\tau &= g K^\dagger \tau_a K \tilde{F}_{\mu\nu}^a F_{\mu\nu}^0 \\
\mathcal{O}^2_\tau &= g^2 \left(K^\dagger K - \nu^2\right) \bar{\psi}_L K^c b_R + \text{h. c.} \\
\mathcal{O}^3_\tau &= \bar{\psi}_L D_\mu b_R D_\mu K^c + \text{h. c.}
\end{align*}
\]

Table 2: Alternative single-fermionic-current \( d = 6 \) operators

\[
\begin{align*}
\mathcal{O}^5_1 &= \left(K^\dagger D_\mu K\right) \bar{\psi}_L \gamma^\mu \psi_L + \text{h. c.} \\
\mathcal{O}^6_1 &= \left(K^\dagger D_\mu K\right) \bar{\psi}_R \gamma^\mu \psi_R + \text{h. c.} \\
\mathcal{O}^7_1 &= \left(K^\dagger \tau_a D_\mu K\right) \bar{\psi}_L \gamma^\mu \gamma^\alpha \psi_L + \text{h. c.}
\end{align*}
\]

3 Simplified effective Lagrangian

Our minimal list of \( d = 6 \) operators is based on the work of Refs. [26,27] and of Refs. [28–31] (see also Refs. [32,33], Ref. [34], Refs. [35–39], Refs. [40–42] and Ref. [43]) and is given in Table 1.

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{\alpha_i}{\Lambda^2} \mathcal{O}^{d=6}_i, \quad (15)
\]

The structure of the \( d = 6 \) operators is chosen in such a way that, with \( \beta_0 = 0 \), no term proportional to \( 1/g \) will appear in the Lagrangian (a part from irrelevant constant terms). Operators containing \( \tilde{F}_{\mu\nu}^a \) are CP-odd, the remaining ones are CP-even.

Additional operators not included in Table 1 have been considered in Eq. (4) of Ref. [38] and are given in Table 2. In certain models their effect can be comparable to the one of \( \mathcal{O}_g \). However, they do not contribute to the \( H\bar{q}q \) vertex, as explained in Ref. [38], because the vector current \( \bar{\tau}q^\mu q \) is conserved. For a complete list of \( d = 6 \) operators (other than the four-fermion ones) we refer to Table 2 of Ref. [27].

For the single-fermionic-current operators we have adopted the (simplified) choice of Ref. [30], discarding the chromomagnetic dipole moment operator, which affects the process \( gg \to \bar{t}t \); in general it is known how to remove derivatives acting on the spinors using integration-by-parts. For a complete classification we refer again to Table 2 of Ref. [27] where there are 13 operators of dimension six involving single-currents of quark fields.

If one restricts the analysis to the calculation of on-shell matrix elements then there are linear combinations of operators that vanish by the Equations-Of-Motion (EOM). Under this assumption there are
redundant operators, e.g. $O_{K}^1$, which can be expressed in terms of a $d = 4$ operator $(K^+ K)^2$, of $O_{K}, O_{\partial K}$ and of higher dimensional Yukawa interactions involving $\bar{\psi} \psi$ and three $K$-fields, i.e. $O_{K}^{1,2}$. Since we are working with unstable particles, the use of EOM should be taken with due caution; indeed, only $S$-matrix elements will be the same for equivalent operators but not the Green’s functions.

It has been pointed out in Ref. [44] that, even if the $S$-matrix elements cannot distinguish between two equivalent operators $O$ and $O'$, there is a large quantitative difference whether the underlying theory can generate $O'$ or not. It is equally reasonable not to eliminate redundant operators and, eventually, exploit redundancy to check $S$-matrix elements. If one eliminates them, whenever the Higgs boson is taken on-shell and the full set of $d = 6$ operators of Ref. [27] is used, the presence of $O_{K}^1$ is redundant, and one should set $a_{K}^1 = 0$. Strictly speaking, the last statement only applies to single-Higgs processes; the argument is simple (see Appendix. D of Ref. [44]), given a theory with a Lagrangian $L[\varphi]$ consider an effective Lagrangian $L_{\text{eff}} = L + g O + g' O'$ where $\bar{O} - O' = F[\varphi] \delta L / \delta \varphi$, and $F$ is some local functional of $\varphi$. The effect of $O'$ on $L_{\text{eff}} = L + g O$ is to shift $g \rightarrow g + g'$ and to replace $\varphi \rightarrow \varphi + g' F$ and $F$ contains terms with several fields, Q.E.D.

The effective Lagrangian of Eq.(15) is one possible way of parametrizing deviations of the Higgs couplings to SM particles; if confirmed, these deviations require new physics models that are the ultraviolet completion of the set of $d = 6$ operators. However, there are specific assumptions in considering Eq.(15), namely decoupling of heavy degrees of freedom is assumed and absence of mass-mixing of the new heavy scalars with the SM Higgs doublet.

We postpone a more detailed discussion of non-decoupling effects to Section 6.5; here we note that Eq.(15) comprises all heavy physics effects at scales below $\Lambda$, and in a decoupling scenario $\Lambda$ is the mass of the additional, heavy, degree of freedom. A typical non-decoupling scenario is given by the inclusion of a scalar triplet; here higher dimensional operators are not suppressed by inverse powers of the triplet mass. It is considerably more difficult to construct a perfectly sensible low-energy effective theory in the non-decoupling scenario and the construction is model dependent, e.g. it has been shown in Ref. [45] that (in the heavy triplet case) $\Lambda$ is related to the ratio of the renormalized triplet VEV to the renormalized doublet VEV. Therefore, additional work is needed in handling models showing a non-decoupling behavior, e.g. looking for the presence of alternative large parameters.

3.1 From the Lagrangian to the $S$-matrix

There are several technical points that deserve a careful comment when we construct $S$-matrix elements from the Lagrangian of Eq.(15).

- **Field-scaling, parameter re-definition**

We perform field re-definitions so that all kinetic and mass terms in the Lagrangian of Eq.(15) have the canonical normalization. First we define

$$
H = \Pi \left[ 1 + \frac{M^2}{\Lambda^2} \left( a_K^1 + a_K^2 + 2 a_{\partial K} \right) \right],
$$

$$
\phi^0 = \tilde{\phi}^0 \left[ 1 + \frac{M^2}{\Lambda^2} \left( a_K^1 + a_K^3 \right) \right], \quad \phi^\pm = \tilde{\phi}^\pm \left[ 1 + \frac{M^2}{\Lambda^2} a_K^1 \right],
$$

then we introduce new parameters,

$$
M = \overline{M} \left( 1 + \frac{M^2}{\Lambda^2} a_K^1 \right), \quad c_\theta = \tilde{c}_\theta \left( 1 - \frac{M^2}{\Lambda^2} a_K^3 \right).
$$
Finally we rescale again the fields
\[ Z_\mu = Z_\mu \left( 1 - 4 \frac{M^2}{\Lambda^2} \hat{s}_\theta \hat{c}_\theta a^3_\nu \right) \quad A_\mu = A_\mu \left( 1 + 4 \frac{M^2}{\Lambda^2} \hat{s}_\theta \hat{c}_\theta a^3_\nu \right), \] (19)

redefine the weak-mixing angle as
\[ \hat{c}_\theta = \hat{c}_\theta \left( 1 - 4 \frac{M^2}{\Lambda^2} \hat{s}_\theta \hat{c}_\theta a^3_\nu \right), \] (20)

and introduce Higgs parameters
\[ M^2_H = \left[ 1 + \frac{M^2}{\Lambda^2} \left( a_1^1 + a_3^3 + 2 a_1^3 \right) \right] M^2_H - 16 \frac{M^4}{\Lambda^2} a_{K}, \] (21)
\[ \beta_H = \left[ 1 - \frac{M^2}{\Lambda^2} \left( 2 a_1^1 + a_3^3 + 2 a_1^3 \right) \right] \beta_H - 4 \frac{M^4}{\Lambda^2} a_{K}. \] (22)

It is worth noting that a different definition of \( O_i^f \), i.e.
\[ O_1^f = g^3 K^{+} \bar{\psi}_L K \bar{t}, \quad O_2^f = g^3 K^{+} \bar{\psi}_L K \bar{c} \bar{b} + \text{h. c.}, \] (23)
requires a re-definition of the \( t-b \) bare masses,
\[ \bar{M}_t = M_t - 2 \sqrt{2} \frac{M^2}{\Lambda^2} a_t^1, \quad \bar{M}_b = M_b - 2 \sqrt{2} \frac{M^2}{\Lambda^2} a_t^2. \] (24)

In the option of Eq.(1) the Hff-Yukawa couplings are
\[ \mathcal{L}_{Hff} = \mathcal{L}_{Hff}^{SM} - \frac{1}{2} \frac{M^2}{\Lambda^2} \left[ g \frac{M_t}{M} \left( a_3^3 + 2 a_1^3 \right) - 4 \sqrt{2} a_t^1 \right] \bar{H} \bar{t} \\
- \frac{1}{2} \frac{M^2}{\Lambda^2} \left[ g \frac{M_b}{M} \left( a_3^3 + 2 a_1^3 \right) - 4 \sqrt{2} a_t^2 \right] \bar{H} \bar{b} \] (25)
while, following Eq.(23) and Eq.(24), we obtain
\[ \mathcal{L}_{Hff} = \mathcal{L}_{Hff}^{SM} - \frac{1}{2} g \frac{M_{t,b} M}{\Lambda^2} \left( a_3^3 + 2 a_1^3 a_1^1 \right) \bar{H} \bar{t} - \frac{1}{2} g \frac{M_{t,b} M}{\Lambda^2} \left( a_3^3 + 2 a_1^3 a_1^1 \right) \bar{H} \bar{b}. \] (26)

\textbf{gauge-fixing term}

We define a modified gauge-fixing term for the W, Z-fields,
\[ C^\pm = - \partial_\mu W_\mu^\pm + \xi_\mu M \phi^\pm, \quad C_Z = - \partial_\mu Z_\mu + \xi_Z \frac{M}{c_b} \phi^0 \] (27)
where the gauge-parameters are
\[ \xi_\mu = 1 - 2 \frac{M^2}{\Lambda^2} a_{K}^1, \quad \xi_Z = 1 - 2 \frac{M^2}{\Lambda^2} \left( a_{K}^1 + a_{K}^3 \right). \] (28)
It is straightforward to show that
\[ C^\pm = -\partial_\mu W^\pm_\mu + \bar{M} \phi^\pm, \quad C_Z = -\frac{1}{\xi_Z} \partial_\mu Z_\mu + \bar{\xi}_Z \frac{\bar{M}}{c_\theta} \phi^0, \] (29)
where the gauge parameter is
\[ \bar{\xi}_Z = 1 + 4 \frac{\bar{M}^2}{\Lambda^2} \bar{s}_\theta \bar{c}_\theta a^3. \] (30)
Note that the photon gauge-fixing term remains unchanged, i.e.
\[ C_A = -\partial_\mu \bar{A}_\mu. \] (31)

With our choice for the scaling factors, the parameter redefinition and the form of the gauge-fixing term it follows that the part of the Lagrangian which is quadratic in the (bosonic) fields reads:
\[
\mathcal{L}^{\text{bos}}_2 = -\partial_\mu W^+_\mu \partial_\mu W^- - \bar{M}^2 W^+_\mu W^- - \frac{1}{2} \partial_\mu Z_\nu \partial_\mu Z_\nu - \frac{1}{2} \frac{\bar{M}^2}{c_\theta} Z_\mu Z_\mu - \frac{1}{2} \partial_\mu \bar{A}_\nu \partial_\mu \bar{A}_\nu \\
- \frac{1}{2} \partial_\mu \bar{H} \partial_\mu \bar{H} - \frac{1}{2} \bar{M}_1^2 \bar{H}^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \bar{M}^2 \phi^+ \phi^- - \frac{1}{2} \partial_\mu \bar{\phi}^0 \partial_\mu \bar{\phi}^0 - \frac{1}{2} \bar{\xi}_Z \frac{\bar{M}^2}{c_\theta} (\bar{\phi}^0)^2 \\
+ 4 \frac{\bar{M}^2}{\Lambda^2} \bar{c}_\theta \bar{s}_\theta \left( d_V^\gamma \left( \partial_\mu \bar{Z}_\nu \partial_\nu \bar{Z}_\mu - \partial_\mu \bar{A}_\nu \partial_\nu \bar{A}_\mu \right) - 2 \epsilon^{\mu \nu \alpha \beta} d_V^\gamma \left( \partial_\mu \bar{Z}_\nu \partial_\alpha \bar{Z}_\beta - \partial_\mu \bar{A}_\nu \partial_\alpha \bar{A}_\beta \right) \right) \\
+ 4 (1 - 2 \bar{s}_\theta^2) \frac{\bar{M}^2}{\Lambda^2} \left[ d_V^\gamma \left( \partial_\mu \bar{Z}_\nu \partial_\nu \bar{A}_\mu - \partial_\mu \bar{Z}_\nu \partial_\nu \bar{A}_\mu \right) + 2 \epsilon^{\mu \nu \alpha \beta} d_V^\gamma \partial_\nu \bar{Z}_\mu \partial_\alpha \bar{A}_\beta \right] 
\] (32)

Therefore, kinetic and mass terms are SM-like, and the bare parameter is \( \mathcal{O}(1/\Lambda^2) \), by construction.

**Dyson resummed propagators**

Dyson resummed propagators are crucial for discussing several issues, from renormalization to Ward-Stravnov-Taylor (WST) identities [46–48]. Consider the W or Z self-energy; in the SM we have
\[ \Pi^{VV}_{\mu \nu} (p^2) = \Pi^{VV}_0 (p^2) \delta_{\mu \nu} + \Pi^{VV}_1 (p^2) p_\mu p_\nu. \] (33)
Once \( d = 6 \) operators are added the W Dyson resummed propagator remains unchanged, i.e.
\[
\bar{\Lambda}^{WW}_{\mu \nu} = \frac{\delta_{\mu \nu}}{p^2 + \bar{M}^2 - \Pi^{WW}_0} + \frac{\Pi^{WW}_1 p_\mu p_\nu}{\left( p^2 + \bar{M}^2 - \Pi^{WW}_0 \right) \left( p^2 + \bar{M}^2 - \Pi^{WW}_0 - \Pi^{WW}_1 \right) p^2}, \] (34)
while the Z propagator changes as follows:
\[ \Pi^{ZZ} \rightarrow \Pi^{ZZ}_1 + 4 \frac{\bar{M}^2}{\Lambda^2} \bar{s}_\theta \bar{c}_\theta a^3. \] (35)

For the \( \bar{\phi} \) propagators we get
\[
\bar{\Lambda}^{\bar{\phi}^0}_{\bar{\phi}^0} (p^2) = \frac{1}{p^2 + \bar{M}^2 \frac{\bar{M}^2}{c_\theta}}, \quad \bar{\Lambda}^{\bar{\phi}^+ \phi^-} (p^2) = \frac{1}{p^2 + \bar{M}^2}. \] (36)
with the gauge parameter defined in Eq.(30).
Figure 1: Example of Ward-Salvnov-Taylor identity; the grey circle denotes insertion of $d = 6$ operators, black circles denote the replacement of the polarization vector by $i$ times the momentum flowing inwards. $Z$ and $\bar{\phi}^0$ lines represent Dyson resummed propagators.

- **WST identities**

With the Feynman rules developed above we can prove WST identities; we show an example in Figure 1 where one should take into account that all lines must be on-shell otherwise there are additional terms involving FP-ghost lines, i.e. BRST-invariance requires also effective operators involving ghost-fields.

- **Wave-function factors**

Due to the rescaling of the fields each external leg in a $S$-matrix element has to be multiplied by a factor; the argument is general, given a Lagrangian

$$L = Z - 2\phi \Delta^{-1}\phi + J\phi$$

we have to normalize the source $J$ in such a way that the residue of the two-point $S$-matrix element is one; therefore we fix $J \rightarrow Z^{-1} J$ for the $S$-matrix element containing one external $\phi$-line. We define $Z_i = 1 + \delta Z_i$ and obtain

$$\delta Z_H = \frac{M^2}{\Lambda^2} \left( a_K^1 + a_K^3 + 2 a_K \theta \right), \quad \delta Z_{\phi^0} = \frac{M^2}{\Lambda^2} \left( a_K^1 + a_K^3 \right), \quad \delta Z_\phi = \frac{M^2}{\Lambda^2} a_K^1,$$

$$\delta Z_W = 0, \quad \delta Z_Z = -4 \frac{M^2}{\Lambda^2} \frac{\hat{s}_\theta}{\hat{c}_\theta} a_3^3, \quad \delta Z_A = 4 \frac{M^2}{\Lambda^2} \frac{\hat{s}_\theta}{\hat{c}_\theta} a_3^3.$$  

(38)

3.2 Nature of $d = 6$ operators

The $d = 6$ operators are supposed to arise from a local Lagrangian, containing heavy degrees of freedom, once the latter are integrated out. Of course, the correspondence Lagrangians $\rightarrow$ effective operators is not bijective since many different Lagrangians can give raise to the same operator. Nevertheless these operators are of two different origins [49]:

- $T$-operators are those that arise from the tree-level exchange of some heavy degree of freedom
- $L$-operators are those that arise from loops of heavy degrees of freedom.

The $L$-operators are usually not included in the analysis. The accuracy at which results should be presented is given by

$$\mathcal{M} = \mathcal{M}_{5M}^{LO} + \mathcal{M}_{5M}^{NLO} + \mathcal{M}_{d=6}^{LO},$$

(40)

where LO means the first order in perturbation theory where the amplitude receive a contribution. To be precise, if the underlying theory is weakly-coupled operators containing field-strength tensors cannot be
\( T \)-operators, and their Wilson coefficients are \( 1/(16 \pi^2) \) suppressed. If only \( T \)-operators (coefficients of \( O(1) \)) are included only 14 out of 34 entries in Table 2 of Ref. [27] are relevant. There is another caveat: \( d = 6 \), \( L \)-operators have Wilson coefficients \( \sim 1/(16 \pi^2) \) and \( d = 8 \), \( T \)-operators are \( \sim v^2/\Lambda^2 \); therefore, below \( \approx 3 \) TeV one should include both of them or none of them.

### 3.2.1 Insertion of \( d = 6 \) operators in loops

The question remains on insertion of \( d = 6 \) operators in SM loop diagrams. This is better discussed in terms of a concrete example: consider the Lagrangian [29]

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{1}{2} M_S^2 S^2 + \mu_S K^\dagger K S,
\]

where \( S \) is a heavy (scalar) singlet. The interaction is

\[
\mathcal{L}_{\text{int}} = \frac{1}{2} \mu_S \left( H^2 + \phi^0 \phi^0 + 2 \phi^+ \phi^- \right) S.
\]

In the limit \( M_S \to \infty \) we have

\[
\mathcal{L} \to \mathcal{L}_{\text{LO}} + \frac{\mu_S^2}{M_S^2} K^\dagger K + \frac{\mu_S^2}{M_S^4} \mathcal{O}_{\partial K}.
\]

The \( d = 4 \) operator in Eq.(43) can be absorbed through a parameter redefinition, and we are left with a contribution to the \( d = 6 \) operator \( \mathcal{O}_{\partial K} \). Clearly, \( \mathcal{O}_{\partial K} \) (as well as \( \mathcal{O}_K, \mathcal{O}_K^1 \) and \( \mathcal{O}_K^3 \)) is a \( T \)-operator [29,50,49].

![Figure 2](image)

Figure 2: The three-point function \( H^3 \) with the insertion of the \( \mathcal{O}_{\partial K} \) operator (left) and the same contribution in the full Lagrangian of Eq.(41).

Imagine we want to compute the \( H^3 \) Green function: we analyze the ultraviolet (UV) behavior of the two diagrams in Figure 2. In the effective theory (left diagram) there is an UV divergence and one option would be to subtract it by introducing counterterms in \( \mathcal{L}_{d=6} \). However, this shows how the insertion of local operators of higher dimensionality in SM diagrams is not really consistent since, in the full theory, the corresponding diagram is not divergent. If we introduce \( \Lambda^2 = \mu_S^2/m_S^4 \) the diagram behaves like \( \Lambda^{-2} \ln \Lambda \), i.e. the divergence is controlled by the heavy mass. From this point of view it is important to stress that one should avoid using a cutoff procedure with the dimensionful parameter \( \Lambda \). Computing with dimensional regularization gives a different pole structure reflecting the different counterterms in the full and effective theory. This difference is independent of infrared (IR) physics, since both theories have the same IR behavior. As we have seen, there can also be logarithmic dependence on \( \Lambda \); if these logarithms are included they must be summed.
To give an example we consider again the two diagrams of Figure 2 with \( s = -(p_1 + p_2)^2 \). In the effective theory the insertion of \( \partial \partial \mathcal{K} \) (left diagram in Figure 2) produces

\[
I_{\text{eff}} = \frac{3}{4} g \frac{\mathcal{M}^2_H}{\mathcal{M}^2_A} \mu_R^2 \int d^n q \frac{(q + p_1)^2}{(q^2 + \mathcal{M}^2_H) \left( (q + p_1 + p_2)^2 + \mathcal{M}^2_H \right)},
\]

(44)

where \( n = 4 - \epsilon \). Suppose that we use a cut-off regularization, the integral is \( \mathcal{O} (1) \) for \( \Lambda \to \infty \) but the same is true for all integrals containing the insertion of \( \partial^n \partial \mathcal{K} \) operators; therefore, all these diagrams are of the same order and cannot be neglected. In dimensional regularization (DR) we obtain

\[
I_{\text{eff}}^{\text{DR}} = \frac{3}{4} g \frac{\mathcal{M}^2_H}{\mathcal{M}^2_A} \left[ \left( \frac{1}{2} s - 3 \mathcal{M}^2_H \right) \left( \frac{1}{\bar{\epsilon}} - \ln \frac{\mu^2_R}{s} \right) + \text{finite part} \right].
\]

(45)

where \( 1/\bar{\epsilon} = 2/(4 - n) - \gamma - \ln \pi \) and \( \mu_R \) is the renormalization scale. In principle, we could add counterterms (in the \( \overline{\text{MS}} \) scheme) to remove the UV pole and make a choice for the scale, \( \mu_R \), which minimizes the remaining logarithms in the UV finite part. After subtracting the UV pole we can say that the insertion of a \( d = 6 \) operator produces a result

\[
I_{\text{ren}}^{d=6} \sim \frac{1}{\Lambda^2} \ln \mu_R.
\]

(46)

The insertion of a \( d = 8 \) operator, always working in dimensional regularization, gives

\[
I_{\text{ren}}^{d=8} \sim \frac{1}{\Lambda^4} \ln \mu_R,
\]

(47)

etc. Note that, with cutoff regularization, both integrals would be of \( \mathcal{O} (1) \). Note that in a mass-independent scheme like \( \overline{\text{MS}} \) the conditions for the decoupling theorem are not satisfied. Furthermore, the logarithms of the renormalization mass may become large. In principle, the problem can be solved but the solution requires matching conditions (for a discussion see Ref. [51]).

With the full theory at our disposal we compute

\[
I_{\text{full}} = -\frac{3}{2} g \frac{\mathcal{M}^2_H \mu_S^2}{M_S} \int d^n q \frac{1}{(q^2 + \mathcal{M}^2_H) \left( (q + p_1)^2 + M_S^2 \right) \left( (q + p_1 + p_2)^2 + \mathcal{M}^2_H \right)}.
\]

(48)

Working (for simplicity) with \( \mathcal{M}_H^2 \ll s \ll M_S^2 \) we obtain

\[
I_{\text{full}} = \frac{3}{2} g \frac{\mathcal{M}_H^2 \mu_S^2}{M_S} \left[ \zeta (2) - \text{Li}_2 \left( 1 + \frac{s + i0}{M_S^2} \right) \right].
\]

(49)

We can identify \( \Lambda = M_S^2 / \mu_S \), expand in \( s / M_S^2 \), and obtain

\[
I_{\text{full}} = -\frac{3}{2} g \frac{\mathcal{M}_H^2 \mu_S^2}{M_S^2} \left[ 1 - \frac{1}{4} \frac{s}{M_S^2} + \left( 1 - \frac{1}{2} \frac{s}{M_S^2} \right) \ln \frac{s - i0}{M_S^2} + \mathcal{O} \left( \frac{s^2}{M_S^4} \right) \right].
\]

(50)

The first term in \( I_{\text{full}} \) reproduces the \( d = 4 \) operator of Eq.(43) while the second term corresponds to the \( d = 6 \), \( \partial \partial \mathcal{K} \) operator. There is no UV divergence in \( I_{\text{full}} \) and the logarithm is uniquely fixed.
An alternative way to understand the two different approaches is the following: we start from Eq.(48) and expand in the integrand

\[
\frac{1}{(q + p)^2 + M_S^2} = \frac{1}{M_S^2} \left[1 - \frac{(q + p_1)^2}{M_S^2} + \cdots \right],
\]

which is equivalent to inserting \( d \geq 4 \) operators or introduce Feynman parameters:

\[
J = \int d^nq \frac{1}{q^2 \left((q + p_1)^2 + M_S^2\right) (q + p_1 + p_2)^2}
\]

\[
= \int_0^1 dx \int_0^x dy \left[M_S^2 (x - y) - sy (1 - x)\right]^{-1}
\]

use a Mellin-Barnes representation, and expand as follows (\( M_S \to \infty \)):

\[
J = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} dv \frac{M_S^2}{(M_S^2)^{-1}} (-s)^{-v} \int_0^1 dx \int_0^x dy B(v, 1 - v) (x - y)^{v-1} y^{-v} (1 - x)^{-v}
\]

\[
= \frac{1}{2\pi i} \frac{1}{M_S^2} \int_{-\infty}^{+\infty} dv \frac{\Gamma^2(s) \Gamma^2(1 - s)}{1 - s} \left(-\frac{M_S^2}{s}\right)^v.
\]

Here \( B(x, y) \) is the Euler beta-function. Using the well know Laurent and Taylor expansions of the Euler gamma-function we obtain the result summing over the poles at \( s = -n \):

\[
J = \sum_{n=0}^{\infty} \frac{1}{n+1} \left[-\frac{(-s)^n}{(M_S^2)^{n+1}} + \frac{1}{n+1} + \ln \left(-\frac{M_S^2}{s}\right)\right].
\]

The result is manifestly UV finite, term-by-term, and has the correct structure of logarithms.

### 3.2.2 Admissible operators

Missing a candidate for the BSM Lagrangian, we will not deal with renormalization of composite operators; therefore, we will not include local operators in loops. To be more precise we will use the following set of rules:

1. operators altering the UV power-counting of a SM diagram are non-admissible

2. operators that do not change the UV power-counting are admissible only in a very specific case: we say that a set of SM diagrams is UV-scalable w.r.t. a combination of \( d = 6 \) operators if

   - their sum is UV finite
   - all diagrams in the set are scaled by the same combination of \( d = 6 \) operators.

To explain with one specific example, let us consider the HWW vertex with off-shell lines and no wavefunction factor inserted:

\[
V_{\text{HWW}}^{\mu\nu} = -g \frac{M}{\Lambda^2} \left[1 + \left(a_k^3 - 2a_k^2 + 2a_k \partial_k\right) \frac{M^2}{\Lambda^2}\right] \delta^{\mu\nu} + a_k^4 \frac{M}{\Lambda^2} p.a \delta^{\mu\nu}
\]

\[
+ 8 a_k^1 \frac{M}{\Lambda^2} T^{\mu\nu} - a_k^4 \frac{M}{\Lambda^2} \left(p_1^\mu p_1^\nu + 2p_1^\mu p_2^\nu + p_2^\mu p_2^\nu\right) + 16 a_k^1 \frac{M}{\Lambda^2} e^{\alpha\beta\mu\nu} p_{1\alpha} p_{2\beta}.
\]
with $T^{\mu \nu} = p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu \nu}$ and $P = p_1 + p_2$. Consider the one-loop diagram contributing to H → γγ containing a W loop: the operators $\mathcal{O}_K^d$ and $\mathcal{O}_C^1$, $\mathcal{O}_C^1_{UV}$ change the UV power-counting of the original SM diagram and are non-admissible.

In the one-loop (bosonic) amplitude for H → γγ there are three different contributions, a W-loop, a charged φ-loop and a mixed W − φ loop. We find

$$V_{WW}^\gamma = ig \left[ 1 + \left( a_K^2 - 2a_K^1 - 2a_K \partial \right) \frac{M_W^2}{\Lambda_s^2} \right] p_1^\gamma + ia_K \frac{p_1^2 - p_2^2}{\Lambda^2} p_1^\gamma$$

$$+ \frac{i}{2} g \left[ 1 + \left( 3a_K^1 - 2a_K \partial \right) \frac{M_W^2}{\Lambda^2} \right] p_2^\gamma - ia_K \frac{p_1^2 - p_2^2}{\Lambda^2} p_2^\gamma$$

$$V_{H\phi} = -\frac{1}{2} g M_H \left[ 1 + \left( a_K^2 - 2a_K^1 - 2a_K \partial \right) \frac{M_\phi^2}{\Lambda^2} \right] + g M a_K \left( \frac{p_1^2 - p_1^2 + p_2^2}{\Lambda^2} \right.$$}

\[ + g M \left( \frac{a_K^2 - a_K^1 - 2a_K \partial}{\Lambda^2} \right)^2 \left( \frac{p_1^2 - p_1^2 + p_2^2}{\Lambda^2} \right) \right] \tag{56}

It is straightforward to conclude that the SM one-loop, bosonic, amplitude for H → γγ with on-shell Higgs line is UV-scalable w.r.t. the combination

$$C_{bos} = \frac{M^2}{\Lambda^2} \left( a_K^2 - 2a_K^1 + 2a_K \partial \right), \tag{57}$$

which could be admissible. However, in the one-loop amplitude we also have FP-ghost loops with vertices (see Eq.(201))

$$V_{H\phi} = -\frac{1}{2} g M \left[ 1 + \left( a_K^2 - 2a_K^1 + 2a_K \partial \right) \frac{M_\phi^2}{\Lambda^2} \right]. \tag{58}$$

Therefore the bosonic component is only UV-scalable w.r.t. the combination

$$C_{bos}^1 = \frac{M^2}{\Lambda^2} \left( a_K^2 + 2a_K \partial \right). \tag{59}$$

Similarly, we consider the γWW, γWφ, γφφ and γX±X± vertices, which also appear in the one-loop bosonic amplitude for H → γγ, and conclude that the latter is UV-scalable w.r.t. the combination

$$C_{bos}^2 = \frac{M^2}{\Lambda^2} \left( \frac{4a_K^2 a_1^3 + 4a_K a_1^3}{\Lambda^2} \right), \tag{60}$$

which is also admissible. Obviously, the wave-function factors of Eqs.(38)–(39) are also admissible. To be more precise, the one-loop bosonic amplitude for H → γγ is made of three different families of diagrams, shown in Figure 3. We find that the γγWW, γγWφ and γγφφ vertices are all UV-scalable w.r.t. $2C_{bos}^2$. Furthermore, the vertex γHWφ is UV-scalable w.r.t. $C_{bos}^1 + C_{bos}^2$. The underlying algebra is such that the quadrilinear vertex with two γs is equivalent to the square of the trilinear vertex with one γ (to $O(1/\Lambda^2)$) and the quadrilinear vertex with one H is equivalent (to the same order) to the product of the two trilinear vertices, with a γ and with a H. As a consequence, there is a non-trivial scaling factor which is admissible, not spoiling the UV behavior.

The fermionic amplitude for H → γγ contains a top-quark loop and a bottom-quark loop. The top contribution is UV-scalable w.r.t. the combination

$$C_{fer} = \frac{1}{2} g \frac{M_t}{M} \left[ 1 + \left( a_K^2 + 2a_K \partial \right) \frac{M_t^2}{\Lambda^2} \right] + \frac{1}{4} \frac{M^2}{\Lambda^2} \left[ 2a_1^3 + \frac{P \cdot P - M_1^2}{M^2} a_1^3 \right]. \tag{61}$$
Figure 3: The three families of diagrams contributing to the bosonic amplitude for $H \rightarrow \gamma\gamma$; $W/\phi$ denotes a $W$-line or a $\phi$-line. $X^\pm$ denotes a FP-ghost line

while for the bottom-quark we have

$$C_{\text{fer}}^b = -\frac{1}{2} g \frac{M_b}{M} \left[ 1 + \left( a^3_K + 2a_2K \right) \frac{M^2}{\Lambda^2} \right] + \frac{1}{4 \sqrt{2}} \frac{M^2}{\Lambda^2} \left[ 2a^2 + \frac{P \cdot P - 2M_b^2}{M^2} a_4 \right].$$

(62)

One example of $L$-operator is given by in Figure 4 with contributions from heavy colored scalar fields transforming in a $(C, T, Y)$ representation of $SU(3) \otimes SU(2) \otimes U(1)$, e.g. the $(8, 2, 1/2)$ representation of [52–55]. Since the additional colored scalar (weak-isospin) doublet contains also an electrically charged scalar (and two neutral scalars) it will contribute to the decay $H \rightarrow \gamma\gamma$ [56]. As long as the scalars are in a representation $(C, T, Y)$ such that $\bar{C} \otimes C \ni 8$ there will also be a contribution to gluon fusion.

![Figure 4: Example of diagram giving a contribution to the $d = 6$ operator of type $L$. Solid lines represent colored scalar fields, e.g. transforming in the $(8, 2, \frac{1}{2})$ representation of $SU(3) \otimes SU(2) \otimes U(1)$.](image)

3.3 Effective theory and renormalization

There are two conceptual frameworks to discuss renormalization and effective theories. In one case we are only interested in setting up an expansion in power of $E/\Lambda$ where $\Lambda$ is the cutoff and $E$ is the scale relevant for a given set of processes.

Counterterms are introduced to remove UV-divergences and, in presence of $d > 4$ operators, an infinite number of them is required. However, once the requested precision of the calculation is fixed only a limited number of term is needed.

This is not the goal for Higgs physics where we want to search for new physics without committing to a particular extension of the SM. The effective theory should simply capture the low-energy effects of the underlying, BSM, theory and must be replaced by a new one when $E$ is approaching $\Lambda$, where it should be discarded. Having this difference in mind we proceed in discussing renormalization.

The processes $gg \rightarrow H$ and $H \rightarrow \gamma\gamma$ are special in the sense that there is no tree-level coupling and therefore the NLO (one-loop) amplitude is UV finite. This is not the case for other processes, i.e. $H \rightarrow \bar{b}b$
etc. In general we will have

$$\mathcal{A} = f(\{a_{AC}\}) \left[ A_{LO}(\{p_0\}) + A_{NLO}(\{p_0\}) \right] + A_{AC}(\{a_{AC}, p_0\}),$$

(63)

where \(\{p_0\}\) is the set of bare parameters (masses and couplings), \(\{a_{AC}\}\) a set of effective parameters; furthermore \(A_{LO}(A_{NLO})\) is the LO(NLO) SM amplitude. Since \(A_{NLO}\) contains UV divergences we introduce counterterms

$$p_0 = p_{\text{ren}} + \delta Z_p,$$

(64)

where \(p_{\text{ren}}\) is the renormalized parameter and \(\delta Z_p\) contains counterterms. If \(A'\) denotes the derivative of the amplitude w.r.t. parameters we obtain

$$\mathcal{A} = f(\{a_{AC}\}) \left[ A_{LO}(\{p_{\text{ren}}\}) + A'_{LO}(\{p_{\text{ren}}\}) \otimes \{Z_p\} + A_{NLO}(\{p_{\text{ren}}\}) \right] + A_{AC}(\{a_{AC}, p_{\text{ren}}\}).$$

(65)

The combination

$$A'_{LO}(\{p_{\text{ren}}\}) \otimes \{Z_p\} + A_{NLO}(\{p_{\text{ren}}\})$$

(66)

is now UV finite. Note that we have replaced \(p_0 \rightarrow p_{\text{ren}}\) in \(A_{AC}\) because in the full theory \(A_{AC}\) is of the same order of \(A_{NLO}\), i.e. renormalization of \(a_{AC}\) can only be discussed in the context of the full theory. In a sense, the \(a_{AC}\) parameters are already the renormalized ones.

There is a final step in the procedure, finite renormalization, where we have to relate renormalized parameters to physical quantities (e.g. \(e^2 = g^2 \hat{s}_0 = \alpha/(4 \pi)\)),

$$p_{\text{ren}} = p_{\text{exp}} + F(\{p_{\text{exp}}\}).$$

(67)

This substitution induces another shift in the amplitude

$$A_{LO}(\{p_{\text{exp}}\}) \rightarrow A_{LO}(\{p_{\text{exp}}\}) + A'_{LO}(\{p_{\text{exp}}\}) F(\{p_{\text{exp}}\}),$$

(68)

with \(p_{\text{ren}} = p_{\text{exp}}\) in both \(A_{NLO}\) and \(A_{AC}\). This set of replacements completely defines our renormalization procedure.

A subtle point is the following: in the process \(H \rightarrow \gamma\gamma\) we have a bosonic component of \(A_{LO}\) and a fermionic one and both are UV finite. Therefore, as long as all tree-vertices in the bosonic part are scaled with the same factor, we would like to have

$$\mathcal{A} = f_{\text{bos}}(\{a_{AC}\}) A_{LO}^{\text{bos}}(\{p_0\}) + f_{\text{fer}}(\{a_{AC}\}) A_{LO}^{\text{fer}}(\{p_0\}) + A_{AC}(\{a_{AC}, p_0\}).$$

(69)

LO implies one-loop diagrams where the splitting bosonic-fermionic has a meaning. Once we try to go to NLO (i.e. two-loops) the splitting is not definable and renormalization is requested, i.e. one has to insert counterterms in the one-loop diagrams. Clearly, an arbitrary scaling of the two LO components kills two-loop UV finiteness (at least in the electroweak sector). The two-loop electroweak corrections to \(H \rightarrow \gamma\gamma\) are \(-1.65\%\) at \(M_H = 125 \text{ GeV} [57]\), therefore neglecting them is tolerable but the internal inconsistency remains. The effect on \(gg \rightarrow H\) is larger, \(\mathcal{O}(5\%)\).

To conclude this section we compare the BSM scenario with heavy degrees of freedom and the SM one in the limit of infinitely massless top-quark. In this case we have a coupling \(Hgg\) of the form

$$\mathcal{L}_f = -\frac{1}{4} \lambda_{\text{SM}} H G^a_{\mu\nu} G^a_{\mu\nu},$$

(70)

where \(\lambda_{\text{SM}}\) has inverse mass dimension. The important point is that \(\lambda_{\text{SM}}\) is computed by matching the effective theory to the full SM [58–69]. Even more important is the fact that \(\lambda_{\text{SM}}\) in the effective theory is the
renormalized one, with its renormalization constant computed to all orders [70]. Therefore, the logical steps are: first renormalization in the full theory, then construction of the effective one.

To be more precise we consider a theory with both light and heavy particles; the Lagrangian is \( \mathcal{L}(m) \) where \( m \) is the mass of the heavy degree of freedom. Next, we introduce the corresponding \( \mathcal{L}_{\text{eff}} \), the effective theory valid up to a scale \( \Lambda = m \). We renormalize the two theories, say in the \( \overline{\text{MS}} \)-scheme (taking care that loop-integration and heavy limit are operations that do not commute), and impose matching conditions among renormalized “light” 1PI Green’s functions

\[
\Gamma^R_{\text{full}}(\mu) = \Gamma^R_{\text{eff}}(\mu), \quad \mu \leq m. \tag{71}
\]

For the case where the full theory is the SM and \( m = M_H \) the whole procedure has been developed in Ref. [71].

4 Higgs vertices

We are now in the position of writing the complete expression for vertices. There are different level of implementation and accuracy. We start with LO-inspired accuracy where the SM vertices are at LO and the tensor structure of the vertices is the same as the LO SM one but every coefficient coming from the effective Lagrangian is kept. Next we go to LO-improved accuracy where extra tensor structures from the effective Lagrangian is included. Finally there is an NLO-inspired accuracy where the SM components are at NLO but contributions from \( d = 6 \) operators are included only under the constraint that they do not spoil UV-finiteness. With the introduction of the following tensors we obtain

\[
T^{\mu \nu} = p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu \nu}, \quad p^{\mu \nu} = p_1^\mu p_2^\nu + 2 p_1^\nu p_2^\mu + p_2^\mu p_1^\nu, \quad E^{\mu \nu} = \varepsilon^{\alpha \beta \mu \nu} p_1^\alpha p_2^\beta. \tag{72}
\]

\[
HAA \quad 8 \frac{M}{\Lambda^2} \left( \hat{s}_\theta a_1^1 + \hat{c}_\theta a_2^1 + \hat{c}_\theta \hat{s}_\theta a_3^1 \right) T^{\mu \nu} + 16 \frac{M}{\Lambda^2} \left( \hat{s}_\theta a_1^2 + \hat{c}_\theta a_2^2 - \hat{c}_\theta \hat{s}_\theta a_3^2 \right) E^{\mu \nu} \tag{73}
\]

\[
HZZ \quad -g \frac{M}{\hat{c}_\theta} \left[ 1 - \left( 2 a_1^1 + a_3^1 - 2 a_2^1 \right) \frac{M^2}{\Lambda^2} \right] \delta^{\mu \nu} + \frac{M}{\Lambda^2} \frac{M_H}{\hat{c}_\theta} \left( \hat{s}_\theta a_1^2 - \hat{c}_\theta a_2^2 \right) \delta^{\mu \nu} + 8 \frac{M}{\Lambda^2} \left( \hat{c}_\theta a_1^1 + \hat{s}_\theta a_2^1 - \hat{c}_\theta \hat{s}_\theta a_3^1 \right) T^{\mu \nu} + 16 \frac{M}{\Lambda^2} \left( \hat{c}_\theta a_1^2 + \hat{s}_\theta a_2^2 - \hat{c}_\theta \hat{s}_\theta a_3^2 \right) E^{\mu \nu} + \frac{M}{\Lambda^2} \hat{c}_\theta \left( \hat{s}_\theta a_1^2 - \hat{c}_\theta a_2^2 \right) p^{\mu \nu} \tag{74}
\]
\[
\begin{align*}
&-\left(\hat{c}_\theta a_K^5 + \hat{s}_\theta a_K^4\right) \frac{M}{\Lambda^2 \hat{c}_\theta} T^{\mu\nu} \\
&+ 8 \frac{M}{\Lambda^2} \left[g \left(1 - 2 \hat{s}_\theta^2\right) a^3_{cV} + 2 \hat{c}_\theta \hat{s}_\theta \left(a^1_{cV} - a^3_{cV}\right)\right] E^{\mu\nu} \\
&- \frac{M}{\Lambda^2 \hat{c}_\theta} \left(\hat{c}_\theta a_K^5 + \hat{s}_\theta a_K^4\right) P_1^\mu P_1^\nu \quad (75)
\end{align*}
\]

\[
\begin{align*}
&-g\frac{M}{\Lambda^2} \left[1 - \left(2 a_K^1 - a_K^3 - 2 a_{\beta K}\right) \frac{M^2}{\Lambda^2}\right] \delta^{\mu\nu} \\
&+ 8 \frac{M}{\Lambda^2} a^1_{aV} T^{\mu\nu} + \frac{M}{\Lambda^2} a^4_{aK} (P \cdot P \delta^{\mu\nu} - P^{\mu\nu}) \\
&+ 16 \frac{M}{\Lambda^2} a^4_{aV} E^{\mu\nu} \quad (76)
\end{align*}
\]

\[
\begin{align*}
&8 \frac{M}{\Lambda^2} a_{g} \delta^{a,b} T^{\mu\nu} \quad (77)
\end{align*}
\]

\[
\begin{align*}
&-\frac{1}{2} \delta \frac{M_t}{M} \left[1 + \left(a_K^3 + 2 a_{\beta K}\right) \frac{M^2}{\Lambda^2}\right] \\
&+ 2 \sqrt{2} \frac{M^2}{\Lambda^2} a^4_t + \frac{1}{4 \sqrt{2} \Lambda^2} \frac{p^2}{a^4_t} \quad (78)
\end{align*}
\]

Similarly for \(f = b\) we have

\[
\begin{align*}
H(P) \rightarrow \bar{b}(p_1) + b(p_2) &= -\frac{1}{2} g \frac{M_b}{M} \left[1 + \left(a_K^3 + 2 a_{\beta K}\right) \frac{M^2}{\Lambda^2}\right] \\
&+ 2 \sqrt{2} \frac{M^2}{\Lambda^2} a^4_t + \frac{1}{4 \sqrt{2} \Lambda^2} \frac{p^2}{a^4_t} \quad (79)
\end{align*}
\]
5 Z couplings

The $Z f \bar{f}$ vertex can be parametrized as follows:
\[
\frac{ig}{2\hat{c}_\theta} \rho_t \gamma^\mu \left[ I_{3f} (1 + \gamma^5) - 2 Q_t \kappa_t \hat{s}_\theta^2 \right],
\]
where $I_{3f}$ is the third component of isospin and $Q_t = -1$, $Q_\nu = 0$, $Q_u = 2/3$ and $Q_d = -1/3$. The anomalous part reads as follows:
\[
\Delta \rho_f = \frac{M^2}{\Lambda^2} \left[ a^3 K - 32 Q_t I_{3f} (1 - \hat{s}_\theta) \hat{c}_\theta a^3 V \right],
\]
\[
\Delta \kappa_f = 2 \frac{M^2}{\Lambda^2} \hat{c}_\theta^2 \left[ a^3 K + 4 \left[ 1 + 4 Q_t I_{3f} \hat{s}_\theta (1 - \hat{s}_\theta) \right] \hat{s}_\theta \hat{c}_\theta a^3 V \right]
\]

6 Partial decay widths

In this Section we compute the partial decay widths of the Higgs boson for the most relevant channels: first we introduces the dimensionless coupling
\[
g_6 = \frac{1}{G_F A^2} = 0.085736 \left( \frac{TeV}{\Lambda} \right)^2
\]
which parametrizes deviations from the SM results. Furthermore, we introduce new couplings
\[
g a^1_V = A^1_V, \quad g a^2_V = A^2_V, \quad g^2 a^3_V = A^3_V, \quad g a^R = A^R
\]
\[
g^2 a^1_K = A^1_K, \quad g^2 a^3_K = A^3_K, \quad g^2 a^R_K = A^R_K,
\]
\[
g a^1_t = \frac{1}{4 \sqrt{2} \tilde{M}_t} A^1_t, \quad g a^2_t = \frac{1}{4 \sqrt{2} \tilde{M}_b} A^2_t,
\]
and express all amplitudes in terms of a SM-component (eventually scaled by the effect of $d = 6$ operators) and by a contact component, as shown in Figure 5. We introduce an auxiliary coefficient,
\[
A^0_K = A^1_K + 2 \frac{A^3_K}{\hat{s}_\theta} + 4 A^R_K.
\]

Figure 5: Amplitude for a two-body decay of the Higgs boson (dash line) including LO+NLO SM contributions with a sum over all one-loop diagrams (i); SM diagrams are eventually multiplied by a universal scaling from $d = 6$ operators (black circle); the grey circle represents a contact term.

We will now show results for various decay processes.
For $H \to \gamma\gamma$ the SM amplitude reads

$$
\mathcal{M}_{SM} = F_{SM} \left( \delta^{\mu\nu} + 2 \frac{p_1^\mu p_2^\nu}{M_H^2} \right) e_H (p_1) e_\gamma (p_2)
$$

(87)

where

$$
F_{SM} = -g M W + \frac{M_f}{2} F_0 + \frac{1}{2} g M_b b_{SM}.
$$

(88)

$$
F^W_{SM} = 6 + \frac{M_H^2}{M^2} + 6 \left( \frac{M_H^2}{M^2} - 2 \right) C_0 \left( -M_{H}^2, 0, 0; \frac{M}{M}, \frac{M}{M}, \frac{M}{M} \right),
$$

(89)

$$
F^t_{SM} = -8 - 4 \left( \frac{M_H^2}{M^2} - 4 M_t^2 \right) C_0 \left( -M_{H}^2, 0, 0; M_t, M_t, M_t \right),
$$

(90)

etc. The $C_0$ function is given by

$$
C_0 \left( -M^2, 0, 0; m, m, m \right) = -\frac{1}{2 M^2} \ln^2 \frac{(1-x)^{1/2} + 1}{(1-x)^{1/2} - 1},
$$

where $x = 4 (m^2 - i 0) / M^2$. Note that there is no need to split the result for this $C_0$-function into the two regions $x > 1$ and $x \leq 1$ since the $i 0$ prescription uniquely defines the analytic continuation. We find

$$
\mathcal{M}_{H \to \gamma\gamma} = 4 \sqrt{2} g_{\gamma} \left\{ \left[ O_4^W F_{SM}^W + 3 Q^2 F_{SM}^t + 3 Q^2 F_{SM}^b \right] + F_{AC} \right\}
$$

(91)

where the scaling factors are given by

$$
C^\gamma_W = \frac{1}{4} M^2 \left\{ 1 + \frac{g_6}{4 \sqrt{2}} \left[ 8 A^\gamma_\lambda \bar{c}_{\theta} \left( \bar{s}_{\theta} + \frac{1}{\bar{s}_{\theta}} \right) + A^0_\lambda \right] \right\}
$$

(92)

for the $W$-loop and

$$
C^\gamma_t = \frac{1}{8} M_t^2 \left\{ 1 + \frac{g_6}{4 \sqrt{2}} \left[ 8 A^\gamma_\lambda \bar{c}_{\theta} \left( \bar{s}_{\theta} + \frac{1}{\bar{s}_{\theta}} \right) + A^0_\lambda - A^1_\lambda \right] \right\}
$$

(93)

$$
C^\gamma_b = \frac{1}{8} M_b^2 \left\{ 1 + \frac{g_6}{4 \sqrt{2}} \left[ 8 A^\gamma_\lambda \bar{c}_{\theta} \left( \bar{s}_{\theta} + \frac{1}{\bar{s}_{\theta}} \right) + A^0_\lambda - A^2_\lambda \right] \right\}
$$

(94)

for the quark loops.

The amplitude is the sum of the $W$, $t$ and $b$ SM components, each scaled by some combination of Wilson coefficients, and of a contact term. The latter is $O \left( g_6 \right)$ while the rest of the corrections is $O \left( \frac{g_6}{4} \right)$. However, one should remember that $O^\lambda_\gamma$ are operators of $L$-type, i.e. they arise from loop correction in the complete theory. Therefore, the corresponding coefficients are expected to be very small although this is only an argument about naturalness without a specific quantitative counterpart (a part from a $1/(16 \pi^2)$ factor from loop integration).

The result for $H \to gg$ follows straightforwardly.
• $H \to gg$

The result for $H \to gg$ is straightforward. Including also the $b$-loop we obtain

$$\mathcal{M}_{H \to gg} = (4 \sqrt{2} G_F)^{1/2} \left[ -\frac{\alpha_s(M_H)}{\pi} \left(C_{gg}^{t} F_{SM}^t + C_{gg}^{b} F_{SM}^b\right) + \frac{g_6}{\sqrt{2} M_H^2} A_g \right]$$

(95)

where the scaling of the quark components is given by

$$C_{gg}^{t} = \frac{1}{16} M_t^2 \left[ 1 + \frac{g_6}{4 \sqrt{2}} \left(A_0^0 - A_1^1\right)\right]$$

(96)

$$C_{gg}^{b} = \frac{1}{16} M_b^2 \left[ 1 + \frac{g_6}{4 \sqrt{2}} \left(A_0^0 - A_2^2\right)\right]$$

(97)

• $H \to b\bar{b}$

For the $H \to b\bar{b}$ amplitude we have to examine again if the are UV-scalable diagrams. In this case there is a tree-level amplitude, and renormalization is required. At NLO there are two type of diagrams, the abelian ones involving the $H\bar{t}t$ vertex and the non-abelian ones involving a $HVV$ ($HV\phi$, $H\phi\phi$) vertex. Therefore we have to search for the unique combination that multiply all the vertices, which is

$$\frac{4}{\Lambda^2} M_\phi^2 a_\partial K.$$  

(98)

The SM amplitude reads as follows:

$$\mathcal{M}_{H \to b\bar{b}}^{SM} = g_3 M_b F_{SM}^{H \to b\bar{b}} \bar{u}(p_2) v(p_1).$$

(99)

The expression for $F_{SM}^{H \to b\bar{b}}$ can be found in Section 5.9.4 of Ref. [24]. Renormalization and QCD corrections are discussed in Section (11.2–11.4) of Ref. [24]. The complete amplitude for $H \to b\bar{b}$ is

$$\mathcal{M}_{H \to b\bar{b}} = \left(4 \sqrt{2} G_F\right)^{1/2} M_b \bar{u}(p_2) v(p_1) \left\{ \frac{G_F M_\phi^2}{\pi^2} C_{b} F_{SM}^{H \to b\bar{b}} ight. \right.$$  

$$\left. + \frac{g_6}{128 \sqrt{2}} \left[ \frac{M_H^2}{M_\phi^2} A_4^t - 16 \left( A_0^t + A_1^t + A_2^t + 6 A_3^t\right)\right] \right\}.$$  

(100)

$$C_{b} = \frac{1}{2 \sqrt{2}} \left[ 1 + \frac{g_6}{4 \sqrt{2}} \left(A_0^0 + A_2^2 + 6A_3^3\right)\right].$$  

(101)

In the SM, NLO corrections to the amplitude include a QED part so that, technically speaking, the process is $H \to b\bar{b}(\gamma)$, i.e. real corrections are added. There is also a contribution from the $d = 6$ operators

$$\frac{ig}{3 \sqrt{2} A^2} \hat{s}_\theta A_t^t \bar{b} \gamma^5 b \bar{\phi} \partial_\mu \phi,$$

(102)

which, however, is not infrared divergent and will not be included.

In this Section we have considered partial decay widths of the SM Higgs boson; in the SM, the common belief is that (for a light Higgs boson) the product of on-shell production cross-section (say in gluon-gluon fusion) and branching ratios (zero-width approximation or ZWA) reproduces the correct result to great accuracy. The work of Ref. [72] shows the inadequacy of ZWA for a light Higgs boson signal at the level of 5%. Therefore, one should always implement the results of this Section within a consistent off-shell formulation of the problem.
6.1 \(gg \to H\)

In ZWA the inclusive cross section for the production of the SM Higgs boson in hadronic collisions can be written as

\[
\sigma \left(s, M_H^2\right) = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{i/h_1} (x_1, \mu_F^2) f_{j/h_2} (x_2, \mu_F^2) \times \\
\times \int_0^1 dz \delta \left(z - \frac{M_H^2}{x_1 x_2}\right) z \sigma^{(0)} G_{ij} \left(z; \alpha_s(\mu_F^2), M_R^2/\mu_R^2; M_R^2/\mu_R^2\right),
\]

(103)

where \(\sqrt{s}\) is the center-of-mass energy and \(\mu_F\) and \(\mu_R\) stand for factorization and renormalization scales.

In Eq.(103) the partonic cross section for the sub-process \(ij \to H + X\), with \(i(j) = g, q, \bar{q}\), has been convoluted with the parton densities \(f_{i/h}\) for the colliding hadrons \(h_1\) and \(h_2\). The Born factor \(\sigma^{(0)}\) reads

\[
\sigma^{(0)} = \frac{G_F}{288\sqrt{2}\pi} \left| \sum_{q=t,b} \mathcal{M}_q^{SM} \right|^2,
\]

(104)

where \(G_F\) is the Fermi-coupling constant; the amplitude is generalized to

\[
\mathcal{M} = \sum_{q=t,b} c_{i}^{gg} \mathcal{M}_q^{SM} + \mathcal{M}_i^{AC},
\]

(105)

where the last term is induced by the operator \(\delta_g\), and where the scaling factors are

\[
c_t^{gg} = 1 + \frac{\alpha_s}{4\sqrt{2}} \left( A_t^0 - A_t^1 \right) \quad c_b^{gg} = 1 + \frac{\alpha_s}{4\sqrt{2}} \left( A_b^0 - A_b^2 \right).
\]

(106)

Since \(\mathcal{M}_t^{SM}\) and \(\mathcal{M}_b^{SM}\) are separately UV finite it is possible to include NLO(NNLO) QCD corrections even in presence of anomalous scaling factors. The coefficient functions \(G_{ij}\) of Eq.(103) can be computed in QCD through a perturbative expansion in the strong-coupling constant \(\alpha_s\),

\[
G_{ij} \left(z; \alpha_s(\mu_R^2), M_R^2/\mu_R^2; M_R^2/\mu_R^2\right) = \alpha_s^2(\mu_R^2) \sum_{n=0}^{\infty} \left( \frac{\alpha_s(\mu_R^2)}{\pi} \right)^n G_{ij}^{(n)} \left(z; M_R^2/\mu_R^2; M_R^2/\mu_R^2\right),
\]

(107)

with a scale-independent LO contribution given by

\[
G_{ij}^{(0)}(z) = \delta_{ig} \delta_{jg} \delta (1 - z).
\]

(108)

The NLO QCD coefficients have been computed in Ref. [73], keeping the exact \(M_t\) and \(M_b\) dependence. NNLO results have been derived in Ref. [74] in the large \(M_t\) limit (see Ref. [75] for the NLO case); analytical expressions can be found in Ref. [64]. The accuracy of these fixed-order computations has been improved with soft-gluon resummed calculations [76–78].

QCD corrections cannot be implemented in the additive part of Eq.(105). To do that one needs a model for \(\delta_g\), as done in Section 2 of Ref. [54] where the SM is extended to included colored scalars, so that one has

\[
\mathcal{M} = \sum_q \mathcal{M}_q + \sum_S \mathcal{M}_S,
\]

(109)

where fermions and scalars transform according to some \((C, \, T, \, Y)\) representation of \(SU(3) \otimes SU(2) \otimes U(1)\), as long as \(\mathcal{C} \otimes C \ni 8\). Complete QCD corrections for fermion and scalar amplitudes have been computed in Ref. [54].
6.2 Simplified scenario

If we restrict the scenario to bosonic \( T \)-operators only \((a_K, a_{1,3}^K, \text{and } a_{3K})\) the scaling factors are:

\[
C_W^{\gamma\gamma} = C_t^{\gamma\gamma} = C_b^{\gamma\gamma} = \frac{1}{4} \overline{M}^2 \left[ 1 + \frac{g_6}{4\sqrt{2}} A_K^0 \right]
\]

\[
C_t^{gg} = \frac{1}{16} M_t^2 \left[ 1 + \frac{g_6}{4\sqrt{2}} A_K^0 \right] \quad C_b^{gg} = \frac{1}{16} M_b^2 \left[ 1 + \frac{g_6}{4\sqrt{2}} A_K^0 \right]
\]

\[
C_5^b = \frac{1}{2\sqrt{2}} \left[ 1 + \frac{g_6}{4\sqrt{2}} \left( A_K^1 + A_K^3 + 6A_0^\Delta \right) \right].
\]

The contact terms are all zero but

\[
A_{H\rightarrow b\bar{b}}^{CI} = -\frac{g_6}{16\sqrt{2}} M_b \left( A_3^3 + 2A_0^\Delta \right).
\]

In this case it is not possible to differentiate bosonic loops from quark loops.

6.3 BSM Lagrangians

By BSM Lagrangians we mean those Lagrangians containing new, heavy degrees of freedom that can produce \( d = 6 \) operators when the heavy particles are integrated out. One of the most important questions is about the sign of the Wilson coefficients \( a_i \) in Eq. (15), i.e. to find the set of coefficients such that

\[
\{ a_i \mid a_i > 0 \} \subset \{ \mathcal{L}_+ \}.
\]

Before entering the discussion on BSM Lagrangian we recall few, well-known, facts about tree-level custodial symmetry. The SM Higgs potential is invariant under \( SO(4); \) furthermore, \( SO(4) \sim SU(2)_L \otimes SU(2)_R \) and the Higgs VEV breaks it down to the diagonal subgroup \( SU(2)_V \). It is an approximate symmetry since the \( U(1)_Y \) is a subgroup of \( SU(2)_R \) and only that subgroup is gauged. Furthermore, the Yukawa interactions are only invariant under \( SU(2)_L \otimes U(1)_Y \) and not under \( SU(2)_L \otimes SU(2)_R \) and therefore not under the custodial subgroup. Therefore, if we require a new CP-even scalar, which is also in a custodial representation of the group, the \( W/Z \)-bosons can only couple to a singlet or a 5-plet, as discussed in Ref. [79]. If \( \left( N_L, N_R \right) \) denotes a representation of \( SU(2)_L \otimes SU(2)_R \), the usual Higgs doublet scalar is a \((2, \bar{2})\), while the \((3, \bar{3}) = 1 \oplus 3 \oplus 5\) contains the Higgs-Kibble ghosts (the 3), a real triplet (with \( Y = 2 \)) and a complex triplet (with \( Y = 0 \)). The Georgi - Machaceck model [80] has EWSB from both a \((2, \bar{2})\) and a \((3, \bar{3})\).

To introduce the discussion on BSM Lagrangians we define the following quantity:

\[
\Delta C = g_6 A_K^0.
\]

Assuming \( A_\gamma^3 = 0 \) and requiring that the coupling \( HW^+W^- \) in the decay \( H \rightarrow \gamma\gamma \) has the standard value, i.e. that

\[
C_W^{\gamma\gamma} = C_W^{\gamma\gamma}_{\text{SM}}
\]

we obtain the condition \( \Delta C = 0 \). We now examine different models and explicitly compute the corresponding value for \( \Delta C \). At the same time we address the question of models allowing for non-standard coupling \( H \bar{t}t \) in the loop for \( H \rightarrow \gamma\gamma \).

In general, the basis for a representation of \( SU(2) \) can be characterized [81] as a tensor field

\[
\psi_{i_1 \cdots i_n} \rightarrow G_{i_1 j_1} \cdots G_{i_n j_n} \psi_{j_1 \cdots j_n},
\]

\[
(117)
\]
where \( G \) are \( SU(2) \)-matrices. An irreducible representation of spin \( n/2 \) is characterized by a totally symmetric field with \( n \) indices. The hermitian conjugate \( \psi_{i_1 \ldots i_n}^\dagger \) transforms according to the complex conjugate representation, \( \psi_{i_1 \ldots i_n}^\dagger \) and indices can be lowered using the metric tensor \( e_{ij} \). To define the covariant derivative we introduce

\[
\begin{align*}
I_{i_1 \ldots i_n}^\dagger & = \prod_{r=1}^{n} \delta^j_{i_r} \\
D_{i_1 \ldots i_n}^\dagger & = \prod_{r=1}^{n-1} \delta^j_{i_r} \left(-\frac{i}{2} \tau_a\right) I_{i_1 \ldots i_n}^\dagger \prod_{i_r=1+1}^{n} \delta^j_{i_r}, \\
U_{i_1 \ldots i_n}^\dagger & = \prod_{r=1}^{n-1} \delta^j_{i_r} \left(\frac{i}{2} \tau_a\right) I_{i_1 \ldots i_n}^\dagger \prod_{i_r=1+1}^{n} \delta^j_{i_r}.
\end{align*}
\]

The covariant derivative is

\[
(D_\mu \psi)^{j_1 \ldots j_n}_{i_1 \ldots i_n} = \left\{ I_{i_1 \ldots i_n}^\dagger \eta_{j_1 \ldots j_n} + g W^a_{\mu} I_{i_1 \ldots i_n}^\dagger \sum_{i_l=1}^{n} D_{i_1 \ldots i_l}^\dagger \eta_{j_1 \ldots j_n} \right\} \psi^{j_1 \ldots j_n}_{i_1 \ldots i_n}.
\]

where \( a = 0, \ldots, 3 \), \( W^{1,2,3}_\mu = B^{1,2,3}_\mu \) and \( W^{0}_\mu = g_1 B^{0}_\mu \).

Here are a few examples of BSM Lagrangians.

- **Example 1**

Consider the following Lagrangian [29]:

\[
\mathcal{L}_1 = \mathcal{L}_{\text{SM}} + \mathcal{L}_3
\]

\[
\mathcal{L}_3 = -\frac{1}{2} \partial_\mu S \partial_\mu S - \frac{1}{2} M^2 S^2 + \mu_S K^+ K S - \frac{1}{2} (D_\mu \eta)^a (D_\mu \eta)^a - \frac{1}{2} M^2 \eta^a \eta^a + \mu_T K^+ \tau_a K \eta^a - (D_\mu \xi)^a (D_\mu \xi)^a - M^2 \xi^a \xi^a + \left[ \mu_\xi K^+ \tau_a K \xi^a \right] + \text{h.c.}
\]

where \( S \) is a scalar singlet and \( \eta, \xi \) are scalar triplets with different hypercharge, see Refs. [16,82]. To be more precise, \( \eta, \xi \) can be written as a complex symmetric tensor of rank two and \( \xi \) as a traceless tensor. In our case we introduce

\[
X_S = \frac{\mu_S^2}{G_F M^2_S}, \quad X_\eta = \frac{\mu_\eta^2}{G_F M^2_\eta}, \quad X_\xi = \frac{|\mu_\xi|^2}{G_F M^2_\eta}.
\]

Projecting onto the \( d = 6 \) operators we obtain

\[
\Delta C = -2 \left[ \left( \frac{2}{s^2_\theta} - 1 \right) X_\eta - 2 X_S - 2 \left( 1 + \frac{2}{s^2_\theta} \right) X_\xi \right].
\]

The scenario of Eq.(116) has a solution

\[
X_S = \left( \frac{1}{s^2_\theta} - \frac{1}{2} \right) X_\eta - \left( 1 + \frac{2}{s^2_\theta} \right) X_\xi
\]

which requires the condition

\[
X_\eta \geq \frac{2 + \frac{s^2_\theta}{2 - s^2_\theta}}{2 - s^2_\theta} X_\xi
\]
• Example 2

Alternatively, we could consider a Lagrangian [29]

\[ \mathcal{L}_2 = \mathcal{L}_{SM} + \mathcal{L}_v \]  

\[ \mathcal{L}_v = \frac{-1}{4} V_{\mu\nu} V_{\mu\nu} - \frac{1}{2} M_S^2 V_{\mu\nu} V_{\mu\nu} - i g V V_{\mu} \left[ (D_{\mu} K)^\dagger K - K^\dagger D_{\mu} K \right] \]

\[-\frac{1}{4} U_{\mu\nu} U^a_{\mu\nu} - \frac{1}{2} M_U^2 U^a_{\mu\nu} - \frac{i}{2} g U V_{\mu} \left[ (D_{\mu} K)^\dagger \tau_a K - K^\dagger \tau_a D_{\mu} K \right], \]

which contains \( I = 0 \) and \( I = 1 \) new vector fields; introducing

\[ X_{VU} = \frac{g_{V,U}^2}{G_F M_{U,V}} \]  

we obtain that the scenario of Eq.(116) requires

\[ X_U = 8 \frac{c^2}{s^2} PX_V \]

• Example 3

A mixture of vector and scalar fields [29], e.g.

\[ \mathcal{L}_3 = \mathcal{L}_{SM} + \mathcal{L}_{sv} \]  

\[ \mathcal{L}_{sv} = -\frac{1}{2} \partial_{\mu} S \partial_{\mu} S - \frac{1}{2} M_S^2 S^2 + \mu_S K^\dagger KS - \mu_{VS} V_{\mu} \partial_{\mu} S, \]

\[-\frac{1}{4} V_{\mu\nu} V_{\mu\nu} - \frac{1}{2} M_V^2 V_{\mu\nu} V_{\mu\nu} - i g V V_{\mu} \left[ (D_{\mu} K)^\dagger K - K^\dagger D_{\mu} K \right] \]

\[-\frac{1}{4} U_{\mu\nu} U^a_{\mu\nu} - \frac{1}{2} M_U^2 U^a_{\mu\nu} V_{\mu} V_{\mu} - \frac{i}{2} g U V_{\mu} \tau_a K - K^\dagger \tau_a D_{\mu} K \]

(132)

gives

\[ \Delta C = \frac{1}{2} X_U - 4 \frac{c^2}{s^2} X_V + 4 X_S \left( 1 - \frac{\mu_{VS}^2}{M_V^2} \right). \]

The scenario of Eq.(116) requires large values for \( \mu_{VS} \). When we include all scalar and vector fields, Eq.(116) is satisfied by

\[ X_U = 8 \frac{c^2}{s^2} PX_V \]

\[ X_U = 4 \left[ \frac{2}{s^2} c^2 X_V + X_n - 6 X_S - 2 X_S \left( 1 - \frac{\mu_{VS}^2}{M_V^2} \right) \right] \]

• Example 4

In order to differentiate the bosonic amplitude from the fermionic one we need \( \phi_f^{1,2} \). One way to introduce them is to consider an additional Lagrangian,

\[ \mathcal{L}_4 = \mathcal{L}_{SM} + \mathcal{L}_\chi \]  

(135)
where $\chi$ is a doublet

$$
\mathcal{L}_\chi = -\frac{1}{2} (D_\mu \chi)^\dagger D_\mu \chi - \frac{1}{2} M_\chi^2 \chi^\dagger \chi + \left[ \lambda \chi (K^\dagger K) (K^\dagger \chi) + \text{h.c.} \right] \\
+ \left[ Y_\chi \bar{\psi}_L \chi^\dagger t_R + y_\chi \bar{\psi}_L \chi b_R + \text{h.c.} \right],
$$

(136)

which would produce $a^f_1$ of the order of $(Y_\chi \lambda \chi)/M_\chi^2$.

Finally, we examine the possibility of a non-zero $F_{AC}$ in Eq.(91). This requires $\mathcal{O}_V$ operators. One option is to include colored scalar fields [56] but we could also include a real triplet [80,82,83]

$$
\xi^\dagger = (\xi^-, \xi^0, \xi^+) 
$$

(137)

with hypercharge $Y = 0$. The Lagrangian reads as follows

$$
\mathcal{L}_\xi = - (D_\mu \xi)^\dagger D_\mu \xi - M_\xi^2 \xi^\dagger \xi + \lambda \xi (K^\dagger K) \xi^\dagger \xi, 
$$

(138)

with covariant derivative $D_\mu = \partial_\mu - i g B_\mu T_a$ and

$$
T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \\
T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}
$$

which gives the following couplings:

$$
i s_\theta A_\mu \left( \xi^+ \partial_\mu \xi^- - \xi^- \partial_\mu \xi^+ \right) - 4M_\xi \frac{g}{g} H^0 \xi^+, \\
- g^2 s^2_\theta A_\mu A_\mu \xi^+ \xi^- 
$$

(139)

and produces a loop of $\xi$ scalars in the $H\gamma\gamma$ coupling.

Additional examples of BSM Lagrangians can be found in Refs. [84,85] and in Ref. [86]. General studies can also be found in Refs. [87,88,8].

6.4 MSSM

In this paper we assume that the starting point in comparing theory with data is the SM. Another choice could be to start from the Minimal-Supersymmetric Standard Model (MSSM); in this case all the amplitudes should be replaced, e.g.

$$
\mathcal{M}_{SM}(H \to \gamma\gamma) \to \mathcal{M}_{MSSM}(h \to \gamma\gamma) \\
= \mathcal{M}_{SM}(h \to \gamma\gamma) + g_{hH^+H} \frac{M_{H^+}^2}{M_{H^0}^2} A_0 \left( \tau_{H^+} \right) \\
+ \sum_f N_f^f Q_f^2 g_{hH^+H} \frac{M_{H^+}^2}{M_f^2} A_0 \left( \tau_{H^+} \right) + \sum_i g_{bH^+_i H^0_i} \frac{M_{H^+_i}^2}{M_{H^0_i}^2} A^i_2 \left( \tau_{H^+_i} \right)
$$

(140)

with $\tau_i = M_{H^+_i}^2/(4M_f^2)$ and

$$
A_2(\tau) = \frac{2}{\tau^2} \left[ \tau + (\tau - 1) f(\tau) \right], \\
A_0(\tau) = -\frac{1}{\tau^2} \left[ \tau - f(\tau) \right],
$$

(141)

and

$$
f(\tau) = -\frac{1}{4} \ln^2 \frac{\sqrt{1-\tau^{-1}}+1}{\sqrt{1-\tau^{-1}}-1}.
$$

(142)
Here $g_{hXX}$ is the coupling of $h$ to $X = \{H^\pm, \tilde{f}, \chi^\pm\}$.

Given the number of free parameters in the MSSM that are relevant for Higgs phenomenology, the present experimental information will clearly not be sufficient to fit the MSSM parameters and a further set of Wilson coefficients for $d = 6$ operators.

An alternative option would be to integrate out the heavy MSSM Higgses (since Buchmüller - Wyler basis only has a single Higgs field). By squaring the corresponding MSSM interaction Lagrangian and contracting the propagators in all possible ways the coefficients will be calculable.

### 6.5 Decoupling

In this Section we study the problem of decoupling of high degrees of freedom by considering again the decay $H \to \gamma \gamma$. To be fully general we assume the existence of heavy fermions and scalar that transform according to generic $R_f$ and $R_S$ representations of $SU(3)$ [56]. The BSM amplitude is based on couplings

$$H_{ff} = \frac{1}{2} g \lambda_f \frac{M_f}{M_W}, \quad H S^{+} S^{-} = g \lambda_S \frac{\mu_S^2}{M_W}.$$ (143)

where the $\lambda$s are numerical coefficients (model dependent) and $\mu_S$ has the dimension of a mass. The $H S^{+} S^{-}$ vertex follows from the following choice of the potential:

$$V = V_{\text{SM}} + 2 \left( M_S^2 - \lambda_S \mu_S^2 \right) \text{Tr} S^+ S + g^2 \lambda_S \frac{\mu_S^2}{M^2} (K^+ K) \text{Tr} S^+ S + \cdots$$ (144)

where $S = S^a T_a$ ($T_a$ are the generators in the $R_S$ representation) and where the trace is over color and $SU(2)$ indices of the field $S$,

$$S = \frac{1}{\sqrt{2}} \left( S^a + i S^3_a \right)$$

and where $Q$ is the electric charge of the particle and $N^c$ is the color factor. In the SM we have

$$\lambda_f = 1, \quad \lambda_S = 0, \quad N_f^c = 3 \quad \text{and} \quad R_f = 3.$$ (146)

The amplitudes are

$$\mathcal{M}_f = \frac{2}{\tau_f} \left[ \tau_f + (\tau_f - 1) f (\tau_f) \right], \quad \mathcal{M}_S = -\frac{1}{\tau_s} \left[ \tau_s - f (\tau_s) \right]$$ (147)

with $\tau_i = M_i^2/(4 M_i^2)$. In the limit $M_i \to \infty$ we have

$$f (\tau_i) = \tau_i + \frac{\tau_i^2}{3} + O (\tau_i^2).$$ (148)

The limit $\tau_i \to 0$ gives

$$\mathcal{M}_{\text{BSM}} (H \to \gamma \gamma) \to \frac{4}{3} N_f^c \lambda_f Q^2_f + \frac{1}{3} N_S^c \lambda_S Q^2_S \frac{\mu_S^2}{M_S^2}$$ (149)
showing decoupling for $PS$. As stated in Ref. [56] there is decoupling in the theory when $\nu = \sqrt{2} M/g \ll M_Z$; therefore, colored scalars disappear from the low energy physics as their mass increases (Appelquist-Carazzone “decoupling theorem” [89]). However, the same is not true for fermions, as shown in Eq.(149). We repeat here the argument of Ref. [90]: for a given amplitude involving a massive degree of freedom (with mass $m$), in the limit $m \to \infty$ we will distinguish decoupling $A \sim 1/m^2$ (or more), screening $A \to$ constant (or $\ln m^2$) and enhancement $A \sim m^2$ (or more). Any Feynman diagram contributing to the process has dimension one; however, the total amplitude must be proportional to $\rho = p_1^\mu p_2^\nu - p_1 \cdot p_2 \delta^{\mu\nu}$ because of gauge invariance. For any fermion $f$ the Yukawa coupling is proportional to $m_f/M_W$ and $T$ has dimension two; therefore, the asymptotic behavior of any diagram must be proportional to $T/m_f$ when $m_f \to \infty$. The part of the diagram, which is not proportional to $T$, will cancel in the total because of gauge invariance (all higher powers of $m_f$ will go away and this explains the presence of huge cancellations in the total amplitude). At LO there is only one Yukawa coupling as in NLO(NNLO) QCD where one add only gluon lines, so there is screening.

It is worth noting, once again, that electroweak NLO corrections change the scenario: there are diagrams with three Yukawa couplings, therefore giving the net $m_f^2$ behavior predicted in [91], so there is enhancement and, at two-loop level, it goes at most with $m_f^2$. At the moment, the NLO electroweak corrections for heavy scalar are missing and no conclusion can be drawn on decoupling at NLO.

In conclusion the decoupling theorem [89] holds in theories where masses and couplings are independent. In all theories where masses are generated by spontaneous symmetry breaking the theorem does not hold in general. Another typical example is given by the inclusion of a Higgs triplet: if the triplet develops a vacuum expectation value $v_\xi(v_\eta)$ then the $\rho$-parameter deviates from unity at the tree-level [92,93] with

$$
\rho_{\text{LO}} = 1 - 2\sqrt{2} G_F v_\xi^2, \quad \text{for} \quad Y = 1
$$

$$
\rho_{\text{LO}} = 1 + 2\sqrt{2} G_F v_\eta^2, \quad \text{for} \quad Y = 0.
$$

(150)

We will not discuss details of renormalization but one should always remember that whenever $\rho \neq 1$ at tree-level quadratic power-like contribution to $\Delta \rho$ are absorbed by renormalization of the new parameters of the model and $\rho$ is not a measure of the custodial symmetry breaking [94]. Alternatively we could impose custodial symmetry, $v_\xi = v_\eta$, in a model with both triplets; an example is found in Ref. [80] containing $SU(2)_L \otimes SU(2)_R$ multiplets.

As far as the triplet contribution to $H \to \gamma\gamma f$ is concerned it is also known [94] that decoupling occurs only for special values of the mixing angles in the triplet sector.

An important tool in studying decoupling of heavy degrees of freedom is given by the $m$-theorem, proved in Ref. [95]: the theorem gives sufficient conditions for a loop integral to vanish in the large $m$-limit. For the one-loop case it concerns

$$
I = m^\alpha \int d^4 q \frac{P(q)}{\prod_i \left(k_i^2 + m_i^2\right)^{\omega_i}},
$$

(151)

where

$$
k_i = q + \sum_{j=1}^N \lambda_{ij} p_j, \quad m_i = 0 \text{ or } m,
$$

(152)

$P(q)$ is a monomial in the components of $q$, $\{p\}$ are the external momenta and $\alpha$ is an arbitrary real number. Let $\omega$ be the IR degree of $I$ at zero external momenta; we define

$$
d = \dim I, \quad \Omega = \min\{0, \omega\}.
$$

(153)

If $I$ is both UV and IR convergent and $d < \Omega$ then $I \to 0$ when $m \to \infty$. 26
In conclusion one should say that BSM Lagrangians can be also classified according to decoupling. Thus the strategy can be summarized as follows: first, fix benchmark models to parametrize deviations from the SM, then search for

\[
\text{benchmark models } \left\{ \begin{array}{c}
ed = 6 \text{ operators} \\
\in \{ \mathcal{L}_{\text{dec}} \} \\
\in \{ \mathcal{L}_{\text{non-dec}} \}
\end{array} \right. 
\]

### 6.6 Mixing

There is one assumption in Eq.(15) and in its interpretation in terms of ultraviolet completions: the absence of mass mixing of the new heavy scalars with the SM Higgs doublet. Presence of mixings changes the scenario; consider for instance a model with two doublets and \( Y = 1/2 \) (THDM), \( \phi_1 \) and \( \phi_2 \). These doublets are first rotated, with an angle \( \beta \), to the Georgi-Higgs basis and successively a mixing-angle \( \alpha \) diagonalizes the mass matrix for the CP-even states, \( h \) and \( H \). The SM-like Higgs boson is denoted by \( h \) while the VEV of \( H \) is zero. The couplings of \( h \) to SM particles are almost the same of a SM Higgs boson with the same mass (at LO) only if we assume \( \sin(\beta - \alpha) = 1 \). Therefore, interpreting large deviations in the couplings within a THDM should be done only after relaxing this assumption.

The case of triplet-like scalars is even more complex; in the simplest case of a triplet with \( Y = 1 \) there are four mixing angles, all of them entering the coefficients of

\[
\frac{1}{\tau_S} [\tau_S - f(\tau_S)] 
\]

in the amplitude for \( h \to \gamma\gamma \) (where \( S = H^+, H^{++} \)) and giving the couplings \( hhH^- \) and \( hH^+H^- \), where \( h \) is the SM-like Higgs boson. Only in a very special case, requiring also zero VEV for the triplet, these couplings assume the simplified form

\[
\begin{align*}
c_{hH^+H^-} &= 2 \frac{M_2^-}{v}, \\
c_{hH^{++}H^{--}} &= 2 \frac{M_2^{++}}{v},
\end{align*}
\]

where \( v \) is the SM Higgs VEV. Furthermore, decoupling of the charged Higgs partners depends on the mixing angles and it is the exception not the rule.

### 7 Decays into 4-fermions

With a light Higgs boson the decay \( H \to VV \) is not open, and one should consider the full \( H \to 4f \) channel. In order to understand how the calculation can be organized we start with \( H \to ZZ \) where both \( Zs \) are real and on-shell.

- **\( H \to ZZ \)**

The SM amplitude is

\[
\mathcal{M}^{\mu\nu}_{\text{SM}} = -g \frac{M}{\sqrt{2} \theta} \left\{ \left[ F_{\text{SM,LO}}^{\text{SM,LO}} + \frac{g^2}{16\pi^2} F_{\text{SM,NLO}}^{\text{SM,NLO}} \right] \delta^{\mu\nu} + \frac{g^2}{16\pi^2} F_{\text{T}}^{\text{SM,NLO}} T^{\mu\nu} \right\},
\]

where

\[
T^{\mu\nu} = \frac{p_1^\mu p_2^\nu}{p_1 \cdot p_2} - \delta^{\mu\nu}.
\]
We introduce auxiliary coefficients

\[ A_\pm^\pm = A_\pm^3 \hat{s}_\theta \pm A_\pm^4 \hat{\bar{c}}_\theta, \quad \hat{A}_K^0 = A_K^1 + A_K^3 + 2A_{\partial K}. \]  

(158)

The full amplitude reads as follows

\[ \mathcal{M}^{\mu\nu} = 2^{5/4} G_F^{1/2} (\mathcal{M}_D S^{\mu\nu} + \mathcal{M}_T T^{\mu\nu}), \]

(159)

\[ \mathcal{M}_D = \frac{g_6 M^2}{\sqrt{2} \hat{c}_\theta^3} \left[ \left( 8A_V^3 \hat{s}_\theta \hat{c}_\theta + A_K^1 - 4A_{\partial K} \right) \hat{c}_\theta + 2A_K^4 \right] \]

\[ - \frac{M^2}{\hat{c}_\theta^3} F_{SM,LO}^{\mathcal{M}_D,LO} \left[ 1 - \frac{g_6}{4 \sqrt{2}} \left( 8A_V^3 \hat{s}_\theta \hat{c}_\theta - \hat{A}_K^0 \right) \right] \]

\[ - \frac{G_F M^4}{2 \sqrt{2} \hat{c}_\theta^3} \pi^2 F_{SM,NLO}^{\mathcal{M}_D,NLO} \left[ 1 - \frac{g_6}{4 \sqrt{2}} \left( 8A_V^3 \hat{s}_\theta \hat{c}_\theta - \hat{A}_K^0 \right) \right] \]

(160)

\[ \mathcal{M}_T = -\frac{g_6}{2 \sqrt{2}} M_H^2 \left[ A_V^3 \hat{c}_\theta \hat{s}_\theta - A_V^2 \hat{s}_\theta^2 - A_V^1 \hat{c}_\theta^2 - \frac{1}{4} A_K^1 \right] \]

\[ - \frac{G_F M^4}{2 \sqrt{2} \hat{c}_\theta^3} \pi^2 F_{SM,NLO}^{\mathcal{M}_T,LO} \left[ 1 - \frac{g_6}{4 \sqrt{2}} \left( 8A_V^3 \hat{s}_\theta \hat{c}_\theta - \hat{A}_K^0 \right) \right] \]

(161)

Following the same strategy we consider the case \( M_H < 2M_Z \). The process to consider is then

\[ \bullet \ H \rightarrow Z\bar{t}f \]

which means \( H \rightarrow ZZ^* \rightarrow Z\bar{t}f \). If we work at LO and only include local operators proportional to the tree-level HZZ coupling, then the SM amplitude is multiplied by a factor

\[ \mathcal{M}_{LO}^Z = \mathcal{M}_{LO}^{SM} \left( 1 + \frac{g_6}{4 \sqrt{2}} \kappa_Z^Z \right) \]

(162)

where the correction w.r.t. the SM is given by

\[ \kappa_Z^Z = \frac{M_H^2}{M^2} \hat{c}_\theta^2 \left( \hat{c}_\theta A_K^4 - \hat{s}_\theta A_K^5 \right) - A_K^4 + 4A_{\partial K} - 4\hat{s}_\theta \hat{c}_\theta A_V^3 \]

(163)

Therefore we can use (at LO), the SM result and write

\[ \Gamma (H \rightarrow ZZ^*) = \sum_f \Gamma (H \rightarrow ZZ^* \rightarrow Z\bar{t}f) = \left( 1 + \frac{g_6}{2 \sqrt{2}} \kappa_Z^Z \right) \Gamma_{SM} (H \rightarrow ZZ^*), \]

(164)

where the SM partial width is

\[ \Gamma_{SM} (H \rightarrow ZZ^*) = \frac{G_F^2 M_0^4}{64 \pi^3} M_H F \left( \frac{M_0^2}{M_H^2} \right) \left( 7 - \frac{40}{9} \hat{s}_\theta^2 + \frac{190}{9} \hat{s}_\theta^4 \right). \]

(165)

\( F \) is the three-body decay phase-space integral,

\[ F(x) = (x-1) \left( \frac{47}{2} x - \frac{13}{2} + \frac{1}{x} \right) + \frac{3}{2} \left( 1 - 6x + 4x^2 \right) \ln x \]

\[ + 3 \left[ 1 - 8x + 20x^2 \right] \arccos \left( \frac{1}{2} \frac{3x - 1}{x^{3/2}} \right). \]

(166)

Note that this result cannot be extended beyond LO.
Similarly to the previous case we have a correction factor

\[ \kappa_W = \frac{M_W^2}{M_T^2} A_K^4 - A_K^2 + 2 A_K^2 + 4 A_{6K}, \]  

(167)

and the partial decay width can we written as follows:

\[ \Gamma_{SM} (H \rightarrow WW^*) = \frac{3 G_F^2 M_W^4}{32 \pi^3} M_H F \left( \frac{M_T^2}{M_H^2} \right). \]  

(168)

Taking the ratio we obtain

\[ R_{ZW} = \frac{\Gamma (H \rightarrow ZZ^*)}{\Gamma (H \rightarrow WW^*)} = R_{ZW}^{SM} \left( 1 - \frac{g_6}{2 \sqrt{2}} r_{ZW} \right) \]  

(169)

where the correction factor is

\[ r_{ZW} = 2 A_K^3 + 4 \hat{\delta}_\theta \hat{c}_\theta A_V^3 + \frac{M_H^2}{M_T^2} \hat{\delta}_\theta (\hat{\delta}_\theta A_K^4 + \hat{\delta}_\theta A_K^5). \]  

(170)

However, if we want to deal with the whole process without approximations the final state is 4-fermions (say ee\(\mu\mu\)) and the SM amplitude has also non-factorizable contributions (e.g. pentagons).

\[ \mathcal{M}_{SM} = \mathcal{M}_{c}^{\mu\nu} (p_1, p_2) \Delta_{\mu\alpha} (p_1) \Delta_{\nu\beta} (p_2) J^\alpha (q_1, k_1) J^\beta (q_2, k_2) + \mathcal{M}_{nfc} (p_1, p_2), \]  

(171)

where \( J \) is the fermionic current

\[ J^\mu (q, k) = g \bar{u}(q) \gamma^\mu (\nu_f + a_f \gamma^5) v(k), \quad p = q + k. \]  

(172)

Furthermore, \( \Delta^{\mu\nu} (p) \) is the Z propagator and \( \mathcal{M}_{nfc} \) collects all diagrams that are not doubly (Z) resonant.

The question is: can we extract informations on \( F_{SM,LO} = \frac{g^2}{16 \pi^2} F_{SM,LO}, \quad F_{SM,NLO} = \frac{g^2}{16 \pi^2} F_{SM,NLO}, \)  

or deviations from the two SM structures from the decay \( H \rightarrow 4f? \)

The form of the Z propagator depends on the choice of gauge but, as long as the fermion current is conserved all differences are irrelevant. With the polarization vectors of Appendix B one obtains

\[ \sum_{\lambda = -1, +1} e_{\mu}(p, \lambda) e_{\nu}^\ast (p, \lambda) = \delta_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2} - \epsilon_{L, \mu}(p) \epsilon_{L, \nu}(p). \]  

(174)

and we can safely replace the \( \delta^{\mu\nu} \) in the propagator with a sum over polarizations, even for off-shell Zs. Using Eq.(174) we replace

\[ \Delta^{\mu\nu} (p) \rightarrow \sum_{\lambda} e_{\mu}(p, \lambda) e_{\nu}^\ast (p, \lambda) \Delta (p^2), \quad \Delta (p^2) = \frac{1}{s + M_0^2}, \]  

(175)
with $\overline{\mathbb{M}}_B^2 = \overline{\mathbb{M}}^2/\tilde{\mathbb{c}}_B^2$, $p^2 = -s$ and

$$e_\mu(p,0) = e_\mu^L(p), \quad e_\mu(p,\pm 1) = e_\mu^H(p,\pm 1).$$ (176)

We introduce the following matrices

$$P_{ij} = \left[ \mathcal{M}_D \delta^{\mu\nu} + \mathcal{M}_T T^{\mu\nu} \right] e_\mu(p_1,i) e_\nu(p_2,j),$$ (177)

$$D_{ij}(p) = \sum_{\text{spin}} E_i(p) E_j^\dagger(p), \quad E_i(p) = J^{\mu} (q,k) e_\nu^*(p,i)$$ (178)

where $i,j = -1,0,+1$ and $p = q+k$. We obtain

$$\sum_{\text{spin}} |\mathcal{M}_{ij}|^2 = \sum_{ijkl} P_{ij} P_{kl}^\dagger D_{ik}(p_1) D_{jl}(p_2) |\Delta(s_1) \Delta(s_2)|^2 = \sum_{ijkl} A_{ijkl} |\Delta(s_1) \Delta(s_2)|^2$$

$$= \left[ \sum_{i} A_{i iii} + \sum_{ij} A_{i j i j} + \sum_{ijkl} A_{ijkl} \right] |\Delta(s_1) \Delta(s_2)|^2. $$ (179)

where $\mathcal{M}$ is the matrix element comprising all factorizable contributions, not only the SM ones. $A_{i iii}$ gives informations on $H$ decaying into two $Z$ of the same helicity ($0,0$ etc.), $A_{i j i j}$ on mixed helicities ($0,1$ etc.) while the third term gives the interference. Therefore

$$A_i = A_{i iii}, \quad A_{ij} = A_{ij ii},$$ (180)

are good candidates to define pseudo-observables. The final step is achieved through the realization that pseudo-observables are defined in one-point of phase-space and the choice must respect gauge invariance [96]. The amplitude in Eq.(179) has the general structure

$$\mathcal{M}_{ij} = \sum_{ij} a_{ij} (s,s_1,s_2,\ldots) \Delta(s_1) \Delta(s_2)$$

$$= \sum_{ij} a_{ij} (s_H,s_Z,s_Z \ldots) \Delta(s_1) \Delta(s_2) + N(s,s_1,s_2,\ldots),$$ (181)

where $N$ denotes the remainder of the double expansion around $s_{1,2} = s_Z, s = -(p_1+p_2)^2$ and

$$\Delta(s) = \frac{1}{s-s_Z}, $$ (182)

$s_H,s_Z$ being the $H,Z$ complex poles. Therefore, we define pseudo-observables

$$\Gamma_i = \int d\Phi_{1\rightarrow 4} \sum_{\text{spin}} |a_{ii} (s_H,s_Z,s_Z \ldots) \Delta(s_1) \Delta(s_2)|^2, $$ (183)

with similar definitions for $\Gamma_{ij}$. Since the problem is extracting pseudo-observables, analytic continuation is performed only after integration over all variables but $s_1,s_2$. Nevertheless, if one wants to introduce cuts on differential distributions alternative algorithms must be introduced, see Ref. [97].

The matrices $D,E$ are given by:

$$P_{00} = -\frac{1}{2} (s_1s_2)^{-1/2} z_H \left[ \mathcal{M}_D - 4 \frac{s_1s_2}{z_H^2} \mathcal{M}_T \right],$$
\[ P_{++} = P_{--} = -i \left( \frac{N_L}{s_{12}N_1 N_L^1} \right)^{1/2} (\mathcal{M}_D - \mathcal{M}_T) \varepsilon (k_1, k_2, q_1, q_2), \]
\[ P_{+-} = P_{-+} = \frac{1}{8} \left( s_{12} N_1 N_1^1 \right)^{-1/2} (\mathcal{M}_D - \mathcal{M}_T) \left\{ 2 s_{12} s_{45} - \left[ s_{46} s_{56} - s_h z_H \right] z_H \right\}, \] (184)

where we have introduced
\[ s_{ij} = s_i + s_j, \quad z_H = s_H - s_1 - s_2, \quad \varepsilon (k_1, k_2, q_1, q_2) = \varepsilon_{\mu \nu \alpha \beta} k_{1 \mu} k_{2 \nu} q_{1 \alpha} q_{2 \beta}. \] (185)

The elements of the $D$-matrix are given by
\[ D_{00}(p_1) = 2 \left( V_+^2 + V_-^2 \right) \left[ 4 + \frac{1}{N_L} \left( 2 s_1 s_2 - (s_{34}^2 + s_{56}^2) \right) \right] \]
\[ D_{-+}(p_1) = D_{++}(p_1) = \left( V_+^2 + V_-^2 \right) \frac{1}{N_L} \left\{ (s_{34} + s_{56}) s_{56} \right. \]
\[ \left. + \frac{1}{4 N_L} \left[ 2 (s_1^2 s_2^2 - (s_{34}^2 - 2 s_1 s_2) s_{56}^2) - (s_{34} - s_{56})^2 s_1 s_2 - (s_{34} + s_{56})^2 s_{56}^2 \right] \right\} \]
\[ D_{0-}(p_1) = \frac{i}{\sqrt{2 N_L N_1^1}} (V_+^2 - V_-^2) \left[ 2 N_L - \frac{1}{2} (s_{34} - s_{56})^2 \right] \]
\[ \left. + \frac{i}{\sqrt{N_1^1}} \left( V_+^2 + V_-^2 \right) \left\{ 4 s_{56} + \frac{1}{N_L} \left[ (s_{34} + s_{56}) (s_1 s_2 - s_{56}^2) - 2 (s_{34} - s_{56}) s_{56} \right] \right\} \right] \]
\[ D_{0+}(p_1) = \frac{i}{\sqrt{2 N_L N_1^1}} (V_+^2 - V_-^2) \left[ 2 N_L - \frac{1}{2} (s_{34} - s_{56})^2 \right] \]
\[ \left. - \frac{i}{\sqrt{2 N_1^1}} (V_+^2 + V_-^2) \left\{ 4 s_{56} + \frac{1}{N_L} \left[ (s_{34} + s_{56}) (s_1 s_2 - s_{56}^2) - 2 (s_{34} - s_{56}) s_{56} \right] \right\} \right] \]
(186)

and similarly for $D_{ij}(p_2)$.

The result of Eq.(179) does not include non-factorizable diagrams. To include them we will follow the work of Ref. [98] where standard matrix elements (SME) are introduced (see Eq. (3.1) and Eq. (3.2) of Ref. [98]); they are made of products of
\[ \Gamma^{i, \sigma}_{\mu} = \frac{1}{2} \bar{u}(q_i) \gamma_{\mu} \left( 1 + \sigma \gamma^5 \right) v(k_i), \quad \Gamma^{i, \sigma}_{\mu \nu \alpha \beta} = \frac{1}{2} \bar{u}(q_i) \gamma_\mu \gamma_\nu \gamma_\alpha \left( 1 + \sigma \gamma^5 \right) v(k_i), \] (187)

with $\sigma = \pm 1$ and $i = 1, 2$. For example one has
\[ \mathcal{M}^{12, \sigma}_{\mu} = \Gamma^{1, \sigma \mu}_{\mu}, \quad \Gamma^{i, \sigma}_{\mu \nu \alpha \beta} = \frac{1}{2} \bar{u}(q_i) \gamma_{\mu} \gamma_{\nu} \gamma_{\alpha} \left( 1 + \sigma \gamma^5 \right) v(k_i), \] (188)

eq
The non-factorizable amplitude becomes a sum
\[ \mathcal{M}_{\text{NFC}} = \sum_i F^{12, \sigma}_{i} \mathcal{M}^{12, \sigma}_{i}, \] (189)

where the $F$ are Lorentz invariant form-factors computed up to NLO but excluding those that are double resonant; the full answer follows by adding this amplitude to $\mathcal{M}_{\text{fc}}$. Note that
\[ J^{(i)}(q_i, k_i) = g \left( v_\ell + a_\ell \right) \Gamma^{i, +}_{\mu} + g \left( v_\ell - a_\ell \right) \Gamma^{i, -}_{\mu}. \] (190)
8 Double Higgs production

A non-zero value of $a_g$ gives a contribution also to the ggHH vertex, contributing to double Higgs production, $gg \to HH$ (see also Ref. [99]).

$$\mu a \quad ggHH \quad 4 a_g g_F g_6 T_{\mu \nu} G^{a,b}. \quad (191)$$

An additional contribution to double-Higgs production comes from the HHH vertex where $p_1, p_2, p_3$ are the momenta of the outgoing bosons with $p_1 + p_2 + p_3 = 0$. There are also quartic couplings

$$\mu a \quad HHH \quad -3 \left( \sqrt{2} G_F \right)^{1/2} \left\{ 1 + \frac{g_6}{12 \sqrt{2}} \left[ 3 A_K^3 + 6 A_K + 32 \frac{M_H^2}{M} A_K \right. \right.$$  

$$\left. - 2 \sum_{i=1}^3 \frac{p_i^2}{M_H} A_K^i \right\}, \quad (192)$$

$$\mu a \quad HHHH \quad -3 \sqrt{2} G_F M_H^2 \left[ 1 + \frac{g_6}{2 \sqrt{2}} \left( A_K^0 + 32 \frac{M^2}{M_H^2} A_K \right) \right] \quad (193)$$

9 Perturbative unitarity

In this section we study constraints from perturbative unitarity. With no informations on the Higgs boson mass there are two different scenarios in $V_L V_L \to V_L V_L$ scattering:

1. $M_W^2, M_Z^2 \ll M_H^2 \ll s$

2. $M_W^2, M_Z^2 \ll s \ll M_H^2$

Assuming a light Higgs boson we analyze a new option,

- $M_W^2, M_Z^2, M_H^2 \ll s$. 

32
The SM result is well-known, given

\[
\frac{d}{dt} \sigma_{V_L V_L \rightarrow V_L V_L} = \left| \frac{T(s,t)}{16 \pi s^2} \right|^2, \quad T_{LO}^0 = \frac{1}{16 \pi s} \int_{-\infty}^0 dt T_{LO}
\]

we derive

\[
T_{LO}^0 \left( W_L^+ W_L^- \rightarrow W_L^+ W_L^- \right) \sim -\frac{G_F M_H^2}{4\sqrt{2} \pi}, \quad s \rightarrow \infty
\]

with a critical mass

\[
\left| T_{LO}^0 (M_H = M_t) \right| = 1, \quad M_t^2 = \frac{4}{3} \sqrt{2} \pi G_F^{-1}.
\]

Anomalous couplings violate perturbative unitarity. However, one has to be careful in formulating the problem: the region of interest is

- \( M_W^2, M_Z^2, M_H^2 \ll s \ll \Lambda^2 \).

When \( s \) approaches \( \Lambda^2 \) the effective theory must be replaced by the complete renormalizable, unitary Lagrangian and it makes no sense to study the limit \( s \rightarrow \infty \) in the effective theory (for a discussion see Ref. [100]). To summarize, anomalous vertices with ad hoc (scale-dependent) form-factors are frequently used but one should remember that they cannot be put down to an effective Lagrangian.

However, it is well known that heavy degrees of freedom may induce effects of delayed unitarity cancellation in the intermediate region and these effects could easily be detectable [101]. Without using the equivalence theorem, we compute

\[
T_{SM+AC}^0 = \frac{1}{16 \pi \lambda (s, M^2, M^2)} \int_{s+4M^2}^{\lambda^2} dt T_{SM+AC} \left( W_L^+ W_L^- \rightarrow W_L^+ W_L^- \right),
\]

with a cut \( t_0 > \lambda^2/s \) to avoid the Coulomb pole. Longitudinal polarization vectors are defined as follows [102,103]

\[
e^L_\mu(p_1) = \frac{2}{M s \beta_M} (p_1 \cdot p_2 p_{1\mu} + M^2 p_{2\mu}) \quad e^L_\mu(p_2) = \frac{2}{M s \beta_M} (p_1 \cdot p_2 p_{2\mu} + M^2 p_{1\mu})
\]

\[
e^L_\mu(p_3) = \frac{2}{M s \beta_M} (p_3 \cdot p_4 p_{3\mu} + M^2 p_{4\mu}) \quad e^L_\mu(p_4) = \frac{2}{M s \beta_M} (p_3 \cdot p_4 p_{4\mu} + M^2 p_{3\mu}),
\]

with \( \beta_M^2 = 1 - \frac{4 M^2}{s} \). In the limit \( M_W^2, M_Z^2, M_H^2 \ll s \ll \Lambda^2 \) we obtain the following result

\[
T_{SM+AC}^0 = -\frac{1}{6} \left( 2 + 5 t_0 - t_0^2 \right) (1-t_0) \hat{c}_\beta \hat{s}_\beta \left( 1 - 2 \hat{s}_\theta \right) \frac{G_F s^2}{\pi M^2} A_V^3 g_6
\]

\[
+ \left\{ \frac{1}{32} (1-t_0)^2 \left( A_K^3 + A_K^3 - A_K^3 - 6 \frac{\hat{s}_\theta}{\hat{c}_\theta} A_V^3 \right) \right\} \frac{G_F s}{\pi} g_6
\]

\[
+ \left[ \frac{1}{8} \left( 11 + 10 t_0 - 13 t_0^2 \right) - 2 (1+2 t_0 - 2 t_0^2) \right] \hat{c}_\beta \hat{s}_\beta A_V^3 \left( \frac{\hat{s}_\theta}{\hat{c}_\theta} A_K^3 \right) \frac{G_F s}{\pi} g_6
\]

\[
+ \frac{3}{16 \sqrt{2} (1-t_0)^2} \left( \frac{\hat{s}_\theta}{\hat{c}_\theta} A_K^3 \right) \frac{\sqrt{2} G_F^3 M s}{\pi} g_6 + \mathcal{O} \left( s^0 \right).
\]

As expected the SM part contributes to the constant part while the part proportional to \( g_6 \) has positive powers of \( s \) (up to power two). The leading behavior is controlled by the \( \hat{c}_V^2 \) operator.
10 Conclusions

We have described possible deviations from the Standard Model parametrized in terms of effective $d = 6$ operators made of Higgs, gauge and fermion fields, without making the hypothesis that the new physics shows up in the Higgs sector. Furthermore, we allow effective operators generated at tree level and by loops of heavy particles.

In this paper we have discussed the implementation of effective Lagrangians with emphasis on renormalization. Examples of Lagrangians producing the $d = 6$ operators have been shown and we have discussed both the decoupling and the non-decoupling scenarios. In agreement with the work of Ref. [100] we have been following the effective field theory approach which is cleaner than that of anomalous couplings. An effective field theory is the low-energy approximation ($E \ll \Lambda$) to the new physics and it is only useful up to $E \approx \Lambda$: above $\Lambda$ it should be replaced by a new effective theory, parametrizing the low-energy effects at a yet higher scale.

Effective theories should not be considered beyond their UV cutoff, although this is often done in the literature with the introduction of methods for unitarizing the model, e.g. form-factors are introduced; this requires specific assumptions and cannot be formulated in terms of an effective Lagrangian.

There are many scenarios, e.g. an interesting one (see Ref. [87]) has no new charged fermions and only new bosons. This would unambiguously rule out a large class of BSM theories. There are also scenarios with new physics which will be extremely difficult to distinguish from minimal SM, e.g. see Ref. [104]. However, the analysis of all possible options should not be done hiding uncertainties or the bias from discovering using the minimum $p$-value. Opportunities for precision measurements and BSM sensitivity have been recently described in Ref. [105] and in Ref. [106].

One final comment is needed: the strategy described in the Introduction amounts to search for deviations around a minimum which we assume to be SM. If measured deviations will be large, we will face a problem of interpretation: indeed, consider the ratio

$$R = \frac{g_{hVV}^W}{c_2^2 g_{hZZ}},$$

(200)

where $g_{hVV}$ is the tree-level coupling of the scalar resonance $h$ to $VV$; if $h = H$, the SM Higgs boson, then $R = 1$. Assume that $R_{\text{exp}}$ turns out to be close to $-1/2$, this will be hard to interpret in terms of a weakly coupled theory and it becomes questionable to trust predictions from an effective Lagrangian, based on $d = 6$ operators, with Wilson coefficients of that size; to state it differently, $d = 8$ operators and insertion of $d = 6$ operators in SM loops are all equally important. However, some anomalous value of $R_{\text{exp}}$ could very well be close to another weakly coupled theory; for instance, the $h_5$ of the Georgi - Machaceck model [80] has $R = -1/2$. Starting from the new weakly-coupled Lagrangian will allow us to trust the prediction. Of course, one would like to be as model-independent as possible without repeating the fit for many different starting points; however, there are only very few representations of $SU(2)_L \otimes SU(2)_R$ that respect custodial symmetry, and they should be included in a more comprehensive analysis.

A recent note by ATLAS Collaboration [107], using data taken in 2011 and 2012, reports that, within the current statistical uncertainties, no significant deviations from the Standard Model couplings are observed.

Acknowledgments

We gratefully acknowledge several important discussions with A. David, M. Duehrssen, C. Grojean, M. Spira, G. Weiglein and the LHC Higgs Cross Section Working Group.
A Appendix: The ghost Lagrangian

In this Appendix we give the explicit expression for the Faddeev-Popov ghost Lagrangian.

\[ \mathcal{L}_{FP} = \mathcal{L} + \frac{1}{2} \sum_{\lambda} \mathcal{L}_{A_{\lambda}} \]

\[ = \mathcal{L} + \frac{1}{2} \mathcal{M} \sum_{\lambda} A_{\lambda}^2 + \mathcal{M} \sum_{\lambda} A_{\lambda} \frac{\partial^2 A_{\lambda}}{\partial^2 x^2} - \frac{2}{\mathcal{M}^2} \sum_{\lambda} \frac{\partial^2 A_{\lambda}}{\partial^2 x^2} \]

\[ - \frac{1}{2} \int \left[ \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right)^2 - \frac{1}{2} \mathcal{M} \right] \frac{\partial^2 \phi}{\partial^2 x^2} + \int \left[ \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right)^2 - \frac{1}{2} \mathcal{M} \right] \frac{\partial^2 \phi}{\partial^2 x^2} \]

\[ + \frac{1}{2} \int \left[ \mathcal{M} \frac{\partial^2}{\partial^2 x^2} \right] \phi + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

\[ + \frac{1}{12} \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} - \frac{\phi}{\mathcal{M}^2} \right) \]

B Appendix: Polarization vectors

A convenient choice for the polarizations in \( H \rightarrow \mathcal{V} \mathcal{V} \) is the following:

\[ e_{\perp \mu} (p_1) = -N_1 \left( p_1 \cdot p_2 p_1 p_2 \right), \quad e_{\perp \mu} (p_2) = -N_2 \left( p_1 \cdot p_2 p_2 p_1 \right), \quad (202) \]

where \( N_1,2 \) are the normalizations, \( p_i^2 = -s_i \), and

\[ e_{\perp \mu} (p_1, \lambda) = \frac{1}{\sqrt{2}} \left( n_{\mu} (p_1) + i \lambda n_{\mu} (p_1) \right), \quad n_{\mu} (p_1) = (s_i)^{-1/2} \epsilon_{\mu \alpha \beta \gamma} n^\alpha (p_1) e^\gamma_{\perp \mu} (p_1) p_1^\rho, \quad (203) \]

\[ n_{\mu} (p_1) = i N_1 \epsilon_{\mu \alpha \beta \gamma} k_{\perp 1}^\alpha P_1^\beta P_2^\rho, \quad n_{\mu} (p_2) = i N_2 \epsilon_{\mu \alpha \beta \gamma} k_{\perp 2}^\alpha P_2^\beta P_1^\rho, \quad (204) \]

With this choice one obtains

\[ \sum_{\lambda = -1,1} e_{\perp \mu} (p, \lambda) e_{\perp \nu} (p, \lambda) = \delta_{\mu \nu} - \frac{p_\mu p_\nu}{p^2} - e_{\perp \mu} (p) e_{\perp \nu} (p). \quad (205) \]
Using $P \rightarrow p_1 + p_{2} \rightarrow q_1 + k_1 + q_2 + k_2$ we define

\begin{align*}
p_1 \cdot p_1 &= -s_1 & p_2 \cdot p_2 &= -s_2 & p_1 \cdot p_2 &= \frac{1}{2} (s_1 + s_2 - s_H) \\
q_1 \cdot q_1 &= 0 & q_2 \cdot q_2 &= 0 & k_1 \cdot k_1 &= 0 & k_2 \cdot k_2 &= 0 \\
q_1 \cdot k_1 &= -\frac{1}{2} s_1 & q_1 \cdot q_2 &= -\frac{1}{2} s_3 & q_1 \cdot k_2 &= -\frac{1}{2} s_4 \\
k_1 \cdot q_2 &= -\frac{1}{2} s_5 & k_1 \cdot k_2 &= -\frac{1}{2} s_6 & q_2 \cdot k_2 &= -\frac{1}{2} s_2,
\end{align*}

where we allow for an off-shell Higgs boson, $P^2 = -s_H$. We derive

\begin{align*}
N^i_L &= (s_i N_L)^{-1/2}, & N^2_L &= \frac{1}{4} \lambda (s_H, s_1, s_2),
\end{align*}

where $\lambda$ is the Källen function. Furthermore,

\begin{align*}
N^2_{1 \perp} &= \frac{1}{4} \left[ (s_5 + s_6) (s_3 + s_4) - s_1 s_2 \right], \\
N^2_{2 \perp} &= \frac{1}{4} \left[ (s_4 + s_6) (s_3 + s_5) - s_1 s_2 \right].
\end{align*}
References

[1] M. E. Peskin, *Comparison of LHC and ILC Capabilities for Higgs Boson Coupling Measurements*, arXiv:1207.2516 [hep-ph].

[2] R. S. Gupta, H. Rzehak, and J. D. Wells, *How well do we need to measure Higgs boson couplings?*, arXiv:1206.3560 [hep-ph].

[3] D. Zeppenfeld, R. Kinnunen, A. Nikitenko, and E. Richter-Was, *Measuring Higgs boson couplings at the LHC*, Phys. Rev. D62 (2000) 013009, arXiv:hep-ph/0002036.

[4] A. Belyaev and L. Reina, *pp → t̅H,H → τ⁺τ⁻: Toward a model independent determination of the Higgs boson couplings at the LHC*, JHEP 0208 (2002) 041, arXiv:hep-ph/0205270 [hep-ph].

[5] M. Duhrssen, S. Heinemeyer, H. Logan, D. Rainwater, G. Weiglein, et al., *Extracting Higgs boson couplings from CERN LHC data*, Phys.Rev. D70 (2004) 113009, arXiv:hep-ph/0406323 [hep-ph].

[6] R. Lafaye, T. Plehn, M. Rauch, D. Zerwas, and M. Duhrssen, *Measuring the Higgs Sector*, JHEP 0908 (2009) 009, arXiv:0904.3866 [hep-ph].

[7] M. Klute, R. Lafaye, T. Plehn, M. Rauch, and D. Zerwas, *Measuring Higgs Couplings from LHC Data*, arXiv:1205.2699 [hep-ph].

[8] J. R. Espinosa, M. Muhlleitner, C. Grojean, and M. Trott, *Probing for Invisible Higgs Decays with Global Fits*, arXiv:1205.6790 [hep-ph].

[9] J. Espinosa, C. Grojean, M. Muhlleitner, and M. Trott, *First Glimpses at Higgs’ face*, arXiv:1207.1717 [hep-ph].

[10] J. Espinosa, C. Grojean, M. Muhlleitner, and M. Trott, *Fingerprinting Higgs Suspects at the LHC*, JHEP 1205 (2012) 097, arXiv:1202.3697 [hep-ph].

[11] A. David, A. Denner, M. Duehrssen, M. Grazzini, et al., *LHC HXSWG interim recommendations to explore the coupling structure of a Higgs-like particle*, arXiv:1209.0040 [hep-ph].

[12] ATLAS Collaboration, G. Aad et al., *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, arXiv:1207.7214 [hep-ex].

[13] CMS Collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 12 GeV with the CMS experiment at the LHC*, Phys.Lett.B (2012) , arXiv:1207.7235 [hep-ex].

[14] N. Arkani-Hamed, G. L. Kane, J. Thaler, and L.-T. Wang, *Supersymmetry and the LHC inverse problem*, JHEP 0608 (2006) 070, arXiv:hep-ph/0512190 [hep-ph].

[15] N. Arkani-Hamed, P. Schuster, N. Toro, J. Thaler, L.-T. Wang, et al., *MARMOSET: The Path from LHC Data to the New Standard Model via On-Shell Effective Theories*, arXiv:hep-ph/0703088 [HEP-PH].

[16] D. Ross and M. Veltman, *Neutral Currents in Neutrino Experiments*, Nucl.Phys. B95 (1975) 135.
[17] I. Low and J. Lykken, *Revealing the electroweak properties of a new scalar resonance*, JHEP **1010** (2010) 053, arXiv:1005.0872 [hep-ph].

[18] G. Passarino, *WW scattering and perturbative unitarity*, Nucl.Phys. **B343** (1990) 31–59.

[19] G. Dvali, G. F. Giudice, C. Gomez, and A. Kehagias, *UV-Completion by Classicalization*, JHEP **1108** (2011) 108, arXiv:1010.1415 [hep-ph].

[20] G. Dvali and D. Pirtskhalava, *Dynamics of Unitarization by Classicalization*, Phys.Lett. **B699** (2011) 78–86, arXiv:1011.0114 [hep-ph].

[21] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, *Causality, analyticity and an IR obstruction to UV completion*, JHEP **0610** (2006) 014, arXiv:hep-th/0602178 [hep-th].

[22] G. Dvali, A. Franca, and C. Gomez, *Road Signs for UV-Completion*, arXiv:1204.6388 [hep-th].

[23] I. Low, R. Rattazzi, and A. Vichi, *Theoretical Constraints on the Higgs Effective Couplings*, JHEP **1004** (2010) 126, arXiv:0907.5413 [hep-ph].

[24] D. Y. Bardin and G. Passarino, *The standard model in the making: Precision study of the electroweak interactions*, .

[25] S. Actis, A. Ferroglia, M. Passera, and G. Passarino, *Two-Loop Renormalization in the Standard Model. Part I: Prolegomena*, Nucl.Phys. **B777** (2007) 1–34, arXiv:hep-ph/0612122 [hep-ph].

[26] W. Buchmuller and D. Wyler, *Effective Lagrangian Analysis of New Interactions and Flavor Conservation*, Nucl.Phys. **B268** (1986) 621.

[27] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, JHEP **1010** (2010) 085, arXiv:1008.4884 [hep-ph].

[28] F. Bonnet, T. Ota, M. Rauch, and W. Winter, *Interpretation of precision tests in the Higgs sector in terms of physics beyond the Standard Model*, arXiv:1207.4599 [hep-ph].

[29] F. Bonnet, M. Gavela, T. Ota, and W. Winter, *Anomalous Higgs couplings at the LHC, and their theoretical interpretation*, Phys.Rev. **D85** (2012) 035016, arXiv:1105.5140 [hep-ph].

[30] S. Kanemura and K. Tsumura, *Effects of the anomalous Higgs couplings on the Higgs boson production at the Large Hadron Collider*, Eur.Phys.J. **C63** (2009) 11–21, arXiv:0810.0433 [hep-ph].

[31] J. Horejsi and K. Kampf, *Contribution of dimension-six bosonic operators to $H \to \gamma\gamma$ at one loop level*, Mod.Phys.Lett. **A19** (2004) 1681–1694, arXiv:hep-ph/0402147 [hep-ph].

[32] K. Hagiwara, R. Szalapski, and D. Zeppenfeld, *Anomalous Higgs boson production and decay*, Phys.Lett. **B318** (1993) 155–162, arXiv:hep-ph/9308347 [hep-ph].

[33] V. Hankele, G. Klamke, D. Zeppenfeld, and T. Figy, *Anomalous Higgs boson couplings in vector boson fusion at the CERN LHC*, Phys. Rev. **D74** (2006) 095001, arXiv:hep-ph/0609075.

[34] C. Anastasiou, S. Buhler, F. Herzog, and A. Lazopoulos, *Total cross-section for Higgs boson hadroproduction with anomalous Standard Model interactions*, arXiv:1107.0683 [hep-ph].
[35] T. Corbett, O. Eboli, J. Gonzalez-Fraile, and M. Gonzalez-Garcia, Constraining anomalous Higgs interactions, arXiv:1207.1344 [hep-ph].

[36] Y.-H. Qi, Y.-P. Kuang, B.-J. Liu, and B. Zhang, Anomalous gauge couplings of the Higgs boson at the CERN LHC: Semileptonic mode in WW scatterings, Phys.Rev. D79 (2009) 055010, arXiv:0811.3099 [hep-ph].

[37] K. Hasegawa, N. Kurahashi, C. Lim, and K. Tanabe, Anomalous Higgs Interactions in Gauge-Higgs Unification, arXiv:1201.5001 [hep-ph].

[38] C.Degrande, J. Gerard, C. Grojean, F. Maltoni, and G. Servant, Probing top-Higgs non-standard interactions at the LHC, arXiv:1205.1065 [hep-ph].

[39] A. Azatov, R. Contino, D. Del Re, J. Galloway, M. Grassi, et al., Determining Higgs couplings with a model-independent analysis of $H \rightarrow \gamma\gamma$, JHEP 1206 (2012) 134, arXiv:1204.4817 [hep-ph].

[40] M. Gonzalez-Garcia, Anomalous Higgs couplings, Int.J.Mod.Phys. A14 (1999) 3121–3156, arXiv:hep-ph/9902321 [hep-ph].

[41] O. J. Eboli, M. Gonzalez-Garcia, S. Lietti, and S. Novaes, Bounds on Higgs and gauge boson interactions from LEP-2 data, Phys.Lett. B434 (1998) 340–346, arXiv:hep-ph/9802408 [hep-ph].

[42] V. Barger, T. Han, P. Langacker, B. McElrath, and P. Zerwas, Effects of genuine dimension-six Higgs operators, Phys.Rev. D67 (2003) 115001, arXiv:hep-ph/0301097 [hep-ph].

[43] F. del Aguila, J. de Blas, and M. Perez-Victoria, Electroweak Limits on General New Vector Bosons, JHEP 1009 (2010) 033, arXiv:1005.3998 [hep-ph].

[44] J. Wudka, Electroweak effective Lagrangians, Int.J.Mod.Phys. A9 (1994) 2301–2362, arXiv:hep-ph/9406205 [hep-ph].

[45] R. S. Chivukula, N. D. Christensen, and E. H. Simmons, Low-energy effective theory, unitarity, and non-decoupling behavior in a model with heavy Higgs-triplet fields, Phys.Rev. D77 (2008) 035001, arXiv:0712.0546 [hep-ph].

[46] M. Veltman, Generalized ward identities and yang-mills fields, Nucl.Phys. B21 (1970) 288–302.

[47] J. Taylor, Ward Identities and Charge Renormalization of the Yang-Mills Field, Nucl.Phys. B33 (1971) 436–444.

[48] A. Slavnov, Ward Identities in Gauge Theories, Theor.Math.Phys. 10 (1972) 99–107.

[49] C. Arzt, M. Einhorn, and J. Wudka, Patterns of deviation from the standard model, Nucl.Phys. B433 (1995) 41–66, arXiv:hep-ph/9405214 [hep-ph].

[50] C. Arzt, Reduced effective Lagrangians, Phys.Lett. B342 (1995) 189–195, arXiv:hep-ph/9304230 [hep-ph].

[51] A. V. Manohar, Effective field theories, arXiv:hep-ph/9606222 [hep-ph].
[52] A. V. Manohar and M. B. Wise, Flavor changing neutral currents, an extended scalar sector, and the Higgs production rate at the CERN LHC, Phys.Rev. D74 (2006) 035009, arXiv:hep-ph/0606172 [hep-ph].

[53] A. V. Manohar and M. B. Wise, Modifications to the properties of the Higgs boson, Phys.Lett. B636 (2006) 107–113, arXiv:hep-ph/0601212 [hep-ph].

[54] R. Bonciani, G. Degrassi, and A. Vicini, Scalar particle contribution to Higgs production via gluon fusion at NLO, JHEP 11 (2007) 095, arXiv:0709.4227 [hep-ph].

[55] G. D. Kribs and A. Martin, Enhanced di-Higgs Production through Light Colored Scalars, arXiv:1207.4496 [hep-ph].

[56] U. Aglietti, R. Bonciani, G. Degrassi, and A. Vicini, Analytic results for virtual QCD corrections to Higgs production and decay, JHEP 01 (2007) 021, arXiv:hep-ph/0611266.

[57] S. Actis, G. Passarino, C. Sturm, and S. Uccirati, NNLO computational techniques: the cases $H \rightarrow \gamma \gamma$ and $H \rightarrow gg$, Nucl. Phys. B811 (2009) 182–273, arXiv:0809.3667 [hep-ph].

[58] K. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Hadronic Higgs decay to order $\alpha_s^4$, Phys. Rev. Lett. 79 (1997) 353–356, arXiv:hep-ph/9705240 [hep-ph].

[59] S. Catani, D. de Florian, and M. Grazzini, Higgs production in hadron collisions: Soft and virtual QCD corrections at NNLO, JHEP 05 (2001) 025, arXiv:hep-ph/0102227.

[60] R. V. Harlander and W. B. Kilgore, Soft and virtual corrections to $pp \rightarrow H + X$ at NNLO, Phys. Rev. D64 (2001) 013015, arXiv:hep-ph/0102241.

[61] R. V. Harlander and W. B. Kilgore, Next-to-next-to-leading order Higgs production at hadron colliders, Phys. Rev. Lett. 88 (2002) 201801, arXiv:hep-ph/0201206.

[62] R. V. Harlander and W. B. Kilgore, Production of a pseudoscalar Higgs boson at hadron colliders at next-to-next-to leading order, JHEP 0210 (2002) 017, arXiv:hep-ph/0208096 [hep-ph].

[63] R. V. Harlander and W. B. Kilgore, Higgs boson production in bottom quark fusion at next-to-next-to leading order, Phys. Rev. D68 (2003) 013001, arXiv:hep-ph/0304035 [hep-ph].

[64] C. Anastasiou and K. Melnikov, Higgs boson production at hadron colliders in NNLO QCD, Nucl. Phys. B646 (2002) 220–256, arXiv:hep-ph/0207004.

[65] C. Anastasiou and K. Melnikov, Pseudoscalar Higgs boson production at hadron colliders in NNLO QCD, Phys. Rev. D67 (2003) 037501, arXiv:hep-ph/0208115 [hep-ph].

[66] S. Catani and B. Webber, Resummed C parameter distribution in $e^+ e^-$ annihilation, Phys.Lett. B427 (1998) 377–384, arXiv:hep-ph/9801350 [hep-ph].

[67] G. F. Sterman and M. E. Tejeda-Yeomans, Multiloop amplitudes and resummation, Phys.Lett. B552 (2003) 48–56, arXiv:hep-ph/0210130 [hep-ph].

[68] B. A. Kniehl and M. Spira, Low-energy theorems in Higgs physics, Z.Phys. C69 (1995) 77–88, arXiv:hep-ph/9505225 [hep-ph].
[69] K. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Decoupling relations to $O(\alpha_s^3)$ and their connection to low-energy theorems, Nucl.Phys. B510 (1998) 61–87, arXiv:hep-ph/9708255 [hep-ph].

[70] V. Spiridonov and K. Chetyrkin, Nonleading mass corrections and renormalization of the operators $m\psi\bar{\psi}$ and $g^2(\mu\nu)$, Sov.J.Nucl.Phys. 47 (1988) 522–527.

[71] M. J. Herrero and E. Ruiz Morales, Nondecoupling effects of the SM higgs boson to one loop, Nucl.Phys. B437 (1995) 319–355, arXiv:hep-ph/9411207 [hep-ph].

[72] N. Kauer and G. Passarino, Inadequacy of zero-width approximation for a light Higgs boson signal, JHEP 1208 (2012) 116, arXiv:1206.4803 [hep-ph].

[73] M. Spira, A. Djouadi, D. Graudenz, and P. M. Zerwas, Higgs boson production at the LHC, Nucl. Phys. B453 (1995) 17–82, hep-ph/9504378.

[74] R. V. Harlander, Virtual corrections to $gg \rightarrow H$ to two loops in the heavy top limit, Phys. Lett. B492 (2000) 74–80, arXiv:hep-ph/0007289.

[75] S. Dawson, Radiative corrections to Higgs boson production, Nucl. Phys. B359 (1991) 283–300.

[76] S. Catani, D. de Florian, M. Grazzini, and P. Nason, Soft-gluon resummation for Higgs boson production at hadron colliders, JHEP 07 (2003) 028, hep-ph/0306211.

[77] S. Moch and A. Vogt, Higher-order soft corrections to lepton pair and Higgs boson production, Phys. Lett. B631 (2005) 48–57, hep-ph/0508265.

[78] E. Laenen and L. Magnea, Threshold resummation for electroweak annihilation from DIS data, Phys. Lett. B632 (2006) 270–276, hep-ph/0508284.

[79] I. Low, J. Lykken, and G. Shaughnessy, Have We Observed the Higgs (Imposter)?, arXiv:1207.1093 [hep-ph].

[80] H. Georgi and M. Machacek, Doubly charged Higgs bosons, Nucl.Phys. B262 (1985) 463.

[81] M. Einhorn, D. Jones, and M. Veltman, Heavy Particles and the rho Parameter in the Standard Model, Nucl.Phys. B191 (1981) 146.

[82] G. Passarino, The Interplay between the Top Quark MAss and the Structure of the Higgs System, Phys.Lett. B231 (1989) 458.

[83] H. E. Logan and M.-A. Roy, Higgs couplings in a model with triplets, Phys.Rev. D82 (2010) 115011, arXiv:1008.4869 [hep-ph].

[84] N. Craig and S. Thomas, Exclusive Signals of an Extended Higgs Sector, arXiv:1207.4835 [hep-ph].

[85] A. Alves, A. Dias, E. R. Barreto, C. S. Pires, F. S. Queiroz et al., Explaining the Higgs Decays at the LHC with an Extended Electroweak Model, arXiv:1207.3699 [hep-ph].

[86] K. Yagyu, Studies on Extended Higgs Sectors as a Probe of New Physics Beyond the Standard Model, arXiv:1204.0424 [hep-ph].
[87] N. Arkani-Hamed, K. Blum, R. T. D’Agnolo, and J. Fan, 2:1 for Naturalness at the LHC?, arXiv:1207.4482 [hep-ph].

[88] D. Carmi, A. Falkowski, E. Kuflik, T. Volansky, and J. Zupan, Higgs After the Discovery: A Status Report, arXiv:1207.1718 [hep-ph].

[89] T. Appelquist and J. Carazzone, Infrared Singularities and Massive Fields, Phys.Rev. D11 (1975) 2856.

[90] G. Passarino, C. Sturm, and S. Uccirati, Complete Electroweak Corrections to Higgs production in a Standard Model with four generations at the LHC, Phys.Lett. B706 (2011) 195–199, arXiv:1108.2025 [hep-ph].

[91] A. Djouadi and P. Gambino, Leading electroweak correction to Higgs boson production at proton colliders, Phys. Rev. Lett. 73 (1994) 2528–2531, hep-ph/9406432.

[92] G. Passarino, Radiative corrections to the Rho parameter versus the top quark mass, Phys.Lett. B247 (1990) 587–592.

[93] M. Aoki, S. Kanemura, M. Kikuchi, and K. Yagyu, Renormalization of the Higgs sector in the triplet model, arXiv:1204.1951 [hep-ph].

[94] S. Kanemura and K. Yagyu, Radiative corrections to electroweak parameters in the Higgs triplet model and implication with the recent Higgs boson searches, Phys.Rev. D85 (2012) 115009, arXiv:1201.6287 [hep-ph].

[95] G. Giavarini, C. Martin, and F. Ruiz Ruiz, Chern-Simons theory as the large mass limit of topologically massive Yang-Mills theory, Nucl.Phys. B381 (1992) 222–280, arXiv:hep-th/9206007 [hep-th].

[96] G. Passarino, C. Sturm, and S. Uccirati, Higgs pseudo-observables, second Riemann sheet and all that, Nucl. Phys. B834 (2010) 77–115, arXiv:1001.3360 [hep-ph].

[97] S. Goria, G. Passarino, and D. Rosco, The Higgs Boson Lineshape, Nucl.Phys. B864 (2012) 530–579, arXiv:1112.5517 [hep-ph].

[98] A. Bredenstein, A. Denner, S. Dittmaier, and M. M. Weber, Precise predictions for the Higgs-boson decay $H \to WW/ZZ \to 4$ leptons, Phys. Rev. D74 (2006) 013004, arXiv:hep-ph/0604011.

[99] R. Contino, M. Ghezzi, M. Moretti, G. Panico, F. Piccinini, et al., Anomalous Couplings in Double Higgs Production, JHEP 1208 (2012) 154, arXiv:1205.5444 [hep-ph].

[100] C. Degrande, N. Greiner, W. Kilian, O. Mattelaer, H. Mebane, et al., Effective Field Theory: A Modern Approach to Anomalous Couplings, arXiv:1205.4231 [hep-ph].

[101] C.-r. Ahn, M. E. Peskin, B. Lynn, and S. B. Selipsky, Delayed Unitarity Cancellation and heavy Particle Effects in $e^+e^- \to W^+W^-$, Nucl.Phys. B309 (1988) 221.

[102] G. Passarino, Indirect measurement of vector boson scattering at high-energies, Phys.Lett. B183 (1987) 375.

[103] G. Passarino, Helicity Formalism for Transition Amplitudes, Phys.Rev. D28 (1983) 2867.
[104] S. Dawson and E. Furlan, *A Higgs Conundrum with Vector Fermions*, Phys. Rev. **D86** (2012) 015021, arXiv:1205.4733 [hep-ph].

[105] M. L. Mangano and J. Rojo, *Cross Section Ratios between different CM energies at the LHC: opportunities for precision measurements and BSM sensitivity*, JHEP **1208** (2012) 010, arXiv:1206.3557 [hep-ph].

[106] A. Djouadi, *Precision Higgs coupling measurements at the LHC through ratios of production cross sections*, arXiv:1208.3436 [hep-ph].

[107] Coupling Properties of the New Particle Observed at ?126 GeV with the ATLAS detector at the LHC, Tech. Rep. ATLAS-CONF-2012-127, CERN, Geneva, Sep, 2012.