Light hadron spectrum in three-flavor QCD with $O(a)$-improved Wilson quark action

CP-PACS and JLQCD Collaborations: T. Kaneko$^a$, S. Aoki$^b$, M. Fukugita$^c$, S. Hashimoto$^d$
K-I. Ishikawa$^d$, T. Ishikawa$^d$, N. Ishizuka$^{b,d}$, Y. Iwasaki$^{b,d}$, K. Kanaya$^{a,b}$, Y. Kuramashi$^a$, M. Okawa$^e$
N. Taniguchi$^b$, N. Tsutsui$^a$, A. Ukawa$^{b,d}$, T. Yoshi$^{e,b}$

$^a$High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan
$^b$Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan
$^c$Institute for Cosmic Ray Research, University of Tokyo, Kashiwa 277-8582, Japan
$^d$Center for Computational Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan
$^e$Department of Physics, Hiroshima University, Higashi-Hiroshima, Hiroshima 739-8526, Japan

We report on a calculation of the light hadron spectrum and quark masses in three-flavor dynamical QCD using the non-perturbatively $O(a)$-improved Wilson quark action and a renormalization-group improved gauge action. Simulations are carried out on a $16^3 \times 32$ lattice at $\beta = 1.9$, where $a^{-1} \approx 2$ GeV, with 6 $ud$ quark masses corresponding to $m_{\pi}/m_{\rho} \simeq 0.64 - 0.77$ and 2 $s$ quark masses close to the physical value. We observe that the inclusion of dynamical strange quark brings the lattice QCD meson spectrum to good agreement with experiment. Dynamical strange quarks also lead to a reduction of the $uds$ quark masses by about 15%.

1. Introduction

As a joint collaboration of CP-PACS and JLQCD, we have initiated a project for the precise measurement of the light hadron spectrum and the quark mass with three-flavor full QCD. The systematic deviation of the quenched lattice spectrum from experiment found in [1] is significantly reduced with the inclusion of dynamical up and down quarks [2,3]. We now extend the work to three flavors.

We adopt the renormalization-group improved gauge action and the $O(a)$-improved Wilson quark action with the non-perturbatively improved clover coefficient $c_{SW}$ obtained in [4]. Simulations are carried out at $\beta = 1.9$, at the scale $a^{-1} = 2.05(5)$ GeV, on a $16^3 \times 32$ lattice using a Polynomial Hybrid Monte Carlo algorithm developed in [5]. Six values of the $ud$ quark mass, corresponding to $m_{PS,LL}/m_{V,LL} \simeq 0.64 - 0.77^1$, are used to extrapolate to the physical value $m_s/m_{\rho} \approx 0.18$. Two values are taken for the $s$ quark mass, corresponding to $m_{PS,SS}/m_{V,SS} \simeq 0.72$ and 0.77. They are close to the semi-empirical value $m_{\eta}/m_{\phi} = 0.68$ from chiral perturbation theory, so that a short extrapolation is enough to get to the physical value. 3000 HMC trajectories are accumulated at each simulation parameter.

2. meson masses

We focus on the meson spectrum since our lattice size of $La \approx 1.6$ fm is too small for baryons [6].

Fig. 1 shows the $J$ parameter [4] defined by

$$J = \frac{m_{V,LS} - m_{V,LL}}{m_{PS,LS} - m_{PS,LL}} \frac{m_{V,LS}}{m_{PS,LS}}$$

(1)

This is compared with the previous results in two-flavor and quenched QCD [2], which are evaluated for the physical kaons $m_{PS}/m_{V} = m_{K}/m_{K^*}$, as $J = m_{V}(dm_{V}/dm_{PS})$ after taking the chiral limit spectively.

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$^1$In this report, subscripts LL, LS and SS represent the light-light, light-strange and strange-strange mesons, respectively.
for sea quarks. We see that $J$ in three-flavor QCD stays at values systematically higher than in quenched and two-flavor QCD, and is consistent with experiment.

Our lattice size $La \simeq 1.6$ fm is significantly smaller than that used in quenched or two-flavor QCD. Because finite-size effects generally lower the $J$ parameter [3], we conclude that the consistency of $J$ between three-flavor QCD and experiment is not an artifact of finite-size effects.

Chiral extrapolations are carried out using polynomial forms. We fix the lattice spacing and the $ud$ quark mass with $m_\pi$ and $m_\rho$. The strange quark mass is fixed with either $m_K$ ($K$-input) or $m_\phi$ ($\phi$-input).

In Fig. 2 $m_K^*$ is plotted as a function of the lattice spacing. The three-flavor result is higher than that in two-flavor QCD at comparable lattice spacings. The consistency of the three-flavor value with experiment seen already at a finite lattice spacing lends us hope that scaling violation with the use of the non-perturbatively $O(a)$-improved action is indeed small and the consistency with experiment holds in the continuum limit. This should be verified in a future study.

Similar consistency with experiment can also be seen in other meson masses as shown in Fig. 3 where deviations from experiment are plotted. Improvement over the quenched results of 2 is evident, as anticipated from the agreement of the $J$ with experiment.

3. Quark masses
The physical quark masses, $m_{ud}$ and $m_s$, are calculated through the axial-vector Ward identity (AWI): $m_q = \langle \Delta A_4 P \rangle / \langle 2 (PP) \rangle$. The matching to the $\overline{MS}$ scheme is carried out at $\mu = a^{-1}$ using the one-loop $Z$ factor with the tadpole improvement. The renormalized mass is evolved to $\mu = 2$ GeV using the 4-loop $\beta$ function.

The vector Ward identity (VWI) may offer an independent determination of $m_{ud}$ and $m_s$. The critical hopping parameter $K_c$ for $ud$ quarks, defined where the light-light PS meson mass vanishes, depends on $m_s$ due to the lack of chiral symmetry. The two natural choices, $K_c(m_s = 0)$, i.e., $K_c$ in degenerate three-flavor QCD, and

Figure 1. The $J$ parameter in two- and three-flavor QCD (left panel) and in quenched QCD (right panel).

Figure 2. $K^*$ meson mass as a function of lattice spacing.

Figure 3. Deviation of meson spectrum from experiment in three-flavor (filled symbols) and quenched QCD (open symbols).
$K_c(m_s = m_s^{\text{phys}})$ with the physical $m_s$, however, lead to $m_{ud}$ with the sign opposite to each other, indicating a large uncertainty from chiral symmetry breaking at $a^{-1} \simeq 2$ GeV. The determination of the VWI $m_{ud}$ requires a careful study of scaling violation; we focus on the AWI quark mass in this report.

Fig. 4 compares $m_{ud}$ and $m_s$ in three-flavor QCD with our previous results in two-flavor and quenched QCD [1,2]. The discrepancy in $m_s$ between $K$- and $\phi$-inputs in quenched QCD disappears in two- and three-flavor QCD. Note that $m_{ud}$ and $m_s$ decrease with an increasing number of flavors of dynamical quarks.

Taking the AWI mass with the $K$-input as the central value and including the deviation between the $K$- and $\phi$-inputs into the error, we obtain

$\begin{align*}
    m_{ud}^{\text{MS}} &= 2.89(6) \text{ MeV}, \\
    m_s^{\text{MS}} &= 75.6(3.4) \text{ MeV}, \\
    m_s/m_{ud} &= 26.2(1.0)
\end{align*}$

in three-flavor QCD. Dynamical strange quarks decrease both $m_{ud}$ and $m_s$ by about 15%. The ratio $m_s/m_{ud}$ is consistent with the one-loop estimate of chiral perturbation theory 24.4(1.5) [3].

4. Conclusion

Our simulation for three-flavor QCD shows that the inclusion of dynamical strange quarks brings the lattice QCD meson spectrum to good agreement with experiment. A non-negligible reduction is also found for quark masses. These preliminary results are sufficiently encouraging to warrant further detailed and careful investigations with time-consuming three-flavor full QCD.

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