Random walk of magnetic field lines for different values of the energy-range spectral index

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Abstract

An analytical nonlinear description of field-line wandering in partially statistically magnetic systems was proposed recently. In this article we investigate the influence of the wave-spectrum in the energy-range onto field line random walk by applying this formulation. It is demonstrated that in all considered cases we clearly obtain a superdiffusive behaviour of the field-lines. If the energy-range spectral index exceeds unity a free-streaming behaviour of the field-lines can be found for all relevant length-scales of turbulence. Since the superdiffusive results obtained for the slab model are exact, it seems that superdiffusion is the normal behavior of field line wandering.

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I. INTRODUCTION

Understanding turbulence is an issue of major importance in space physics and astrophysics; see, e.g., in Refs. [1, 2, 3, 4, 5, 6]. It has been demonstrated in several articles that stochastic wandering of magnetic field-lines directly influences the transport of charged cosmic rays (see e.g. [7, 8, 9, 10, 11, 12, 13, 14, 15]). Several theories have been developed to describe field-line random-walk (FLRW) analytically. The classic work of Jokipii (see [16]), for instance, employed a quasilinear approach for FLRW. In this theory the unperturbed field-lines are used to describe field-line wandering by using a perturbation method. It has often been stated that this approach is correct in the limit of weak turbulence where it is assumed that the turbulent fields are much weaker than the uniform mean field ($\delta B_i \ll B_0$).

To achieve a more reliable and general description of field-line wandering Matthaeus et al. (see [17]) developed a nonperturbative statistical approach by combining certain assumptions about the properties of the field-lines (e.g. Gaussian statistics) with a diffusion model. More precisely, in the Matthaeus et al. theory of field-line wandering it has explicitly been assumed that field-line wandering behaves diffusively.

An improved theory for FLRW, which is essentially a generalization of the theory of Matthaeus et al., was recently developed by Shalchi & Kourakis (see [18]). By explicitly assuming a diffusive behavior of the field-lines, the Matthaeus et al. theory can be obtained from the Shalchi & Kourakis approach as a special limit. However, it has also been demonstrated in [18] that for slab/2D turbulence geometry, the field-lines behave superdiffusively. Thus, the Matthaeus et al. theory cannot be applied for slab/2D composite geometry. As also demonstrated in [18], quasilinear theory is only correct for pure slab geometry or for small length scales.

In most past studies a constant spectrum in the energy-range has been assumed (in this case the energy-range spectral index is equal to zero). It is the purpose of this article to explore different values of the energy-range spectral index. The layout of this article goes as follows. In Section 2, we discuss different forms of the wave-spectrum which are appropriate for solar wind turbulence. In Section 3, we calculate the FLRW for pure slab geometry for different values of the energy-range spectral index by applying the exact formulation for field-line wandering. In Section 4, we employ the nonlinear theory of Shalchi & Kourakis for FLRW, in order to deduce an analytic form for the field-line MSD for pure 2D turbulence.
These results can easily be combined with the pure 2D result to describe field-line wandering in the slab/2D composite model (Section 5). In Section 6 we summarize our new results.

II. DIFFERENT FORMS OF THE WAVE-SPECTRUM

In [19] a two-component turbulence model has been proposed as a realistic model for solar wind turbulence. In this model we describe the turbulent fields as a superposition of a slab model ($\vec{k} \parallel \vec{B}_0$) and a 2D model ($\vec{k} \perp \vec{B}_0$). In this case the $xx$–component of the correlation tensor can be written as

$$P_{xx}(\vec{k}) = P_{xx}^{slab}(\vec{k}) + P_{xx}^{2D}(\vec{k})$$

(1)

with the slab contribution

$$P_{xx}^{slab}(\vec{k}) = g^{slab}(k_{\parallel}) \frac{\delta(k_{\perp})}{k_{\perp}}$$

(2)

and the 2D contribution

$$P_{xx}^{2D}(\vec{k}) = g^{2D}(k_{\perp}) \frac{\delta(k_{\parallel})}{k_{\perp}} \left[ 1 - \frac{k_{\perp}^2}{k_{\parallel}^2} \right].$$

(3)

In previous studies the forms

$$g^{slab}(k_{\parallel}) = \frac{C(\nu)}{2\pi} l_{slab} \delta B^2_{slab} (1 + k_{\parallel}^2 l_{slab}^2)^{-\nu}$$

(4)

for the slab wave-spectrum, and

$$g^{2D}(k_{\perp}) = \frac{2C(\nu)}{\pi} l_{2D} \delta B^2_{2D} (1 + k_{\perp}^2 l_{2D}^2)^{-\nu}$$

(5)

for the 2D wave-spectrum were used. The energy-range of the spectrum is defined for $k_{\parallel} \ll l_{slab}^{-1}$ and $k_{\perp} \ll l_{2D}^{-1}$. Clearly, both spectrum forms are constant in the energy-range. However, as discussed in several previous articles (see e.g. [20]), we find in heliospheric observations a steeper spectrum (according to [20] the energy-range spectral index - cf. [6] below - should be $q = 1.07$). In the following we deduce and discuss analytical forms of the wave spectrum for slab and 2D turbulence models.
case | Normalization constants $c_i$
---|---
$0 < q < 1$ | $c_1 := \left(\frac{4}{1-q} + \frac{4}{2\nu-1}\right)^{-1}$
$q = 1$ | $c_2 := \frac{1}{3} \left(\ln\left(\frac{1}{k_{\text{min}} l_{\text{slab}}}\right)\right)^{-1}$
$1 < q < 2$ | $c_3 := \frac{q-1}{4} \left(k_{\text{min}} l_{\text{slab}}\right)^{-q-1}$

TABLE I: The exact values of the various normalization constants $c_i$ ($i = 1, 2, 3$) are provided. These expressions are correct for $k_{\text{min}} l_{\text{slab}} \ll 1$.

A. General form of the slab wavespectrum

According to solar wind observations, the following form of the spectrum should be appropriate:

$$g_{\text{slab}}(k_\parallel) = \frac{c_i}{2\pi} l_{\text{slab}} \delta B_{\text{slab}}^2$$

$$\times \begin{cases} 
0 & \text{if } k_\parallel < k_{\text{min}} \\
(k_\parallel l_{\text{slab}})^{-q} & \text{if } k_{\text{min}} \leq k_\parallel \leq l_{\text{slab}}^{-1} \\
(k_\parallel l_{\text{slab}})^{-2\nu} & \text{if } l_{\text{slab}}^{-1} < k_\parallel.
\end{cases}$$

(6)

Here we defined the slab-bendover-scale $l_{\text{slab}}$, the strength of the turbulent field $\delta B_{\text{slab}}^2$, and the inertial-range spectral index $2\nu$. $k_{\text{min}}$ indicates the smallest wave-number which might be related to the bulk plasma length scale $L$ via $k_{\text{min}} \sim L^{-1}$. We have also introduced the energy-range spectral index $q$. By taking into account the normalization condition

$$\delta B_{\text{slab}}^2 = \delta B_x^2 + \delta B_y^2 = \int d^3k \left[P_{xx}^{\text{slab}}(\vec{k}) + P_{yy}^{\text{slab}}(\vec{k})\right]$$

(7)

we find for the normalization constant $c_i$ the values shown in Table I. The values shown there are valid if the condition $k_{\text{min}} l_{\text{slab}} \ll 1$ is fulfilled.
Normalization constants

case | Normalization constants $d_i$
--- | ---
$0 < q < 1$ | $d_1 := \left( \frac{1}{1-q} + \frac{1}{2\nu-1} \right)^{-1}$
$q = 1$ | $d_2 := \left( \ln \left( \frac{1}{k_{\text{min}} l_{2D}} \right) \right)^{-1}$
$1 < q < 2$ | $d_3 := (q - 1) \left( k_{\text{min}} l_{2D} \right)^{q-1}$

TABLE II: The exact values of the various normalization constants $d_i$ $(i = 1, 2, 3)$ are provided. These expressions are correct for $k_{\text{min}} l_{2D} \ll 1$.

## B. General form of the 2D wavespectrum

For the 2D spectrum we can adopt the same form for the spectrum as used in the last subsection for the slab spectrum:

$$g^{2D}(k_\perp) = \frac{d_i}{2\pi} l_{2D} \delta B^{2D}_{2D}$$

$$\times \begin{cases} 
0 & \text{if } k_\perp < k_{\text{min}} \\
(k_\perp l_{2D})^{-q} & \text{if } k_{\text{min}} \leq k_\perp \leq l_{2D}^{-1} \\
(k_\perp l_{2D})^{-2\nu} & \text{if } l_{2D}^{-1} < k_\perp.
\end{cases}$$

(8)

Here we used the 2D-bendover-scale $l_{2D}$, the strength of the turbulent field $\delta B^{2D}_{2D}$, and the inertial-range spectral index $2\nu$. $k_{\text{min}}$ indicates again the smallest wave-number and $q$ is again the energy-range spectral index. Fulfilling the normalization condition

$$\delta B^{2D}_{2D} = \delta B^2_x + \delta B^2_y = \int d^3 k \left[ P^{2D}_{xx}(\vec{k}) + P^{2D}_{yy}(\vec{k}) \right]$$

we find for the normalization constant $d_i$ the values shown in table II. In the following we consider different values of the energy-range spectral index $q$ and calculate the field-line mean square deviation analytically for pure-slab, pure-2D, and two-component turbulence.

## III. FLRW FOR PURE-SLAB TURBULENCE

As shown in several previous papers (e.g. [18]) the field-line mean square deviation can be calculated exactly for pure slab geometry. For standard forms of the wave spectrum, where $q = 0$, we find the classical diffusive result: $\langle (\Delta x)^2 \rangle = 2\kappa_{FL} |z|$ (see e.g. [16, 17]), with the field-line diffusion coefficient $\kappa_{FL}$. Shalchi & Kourakis (see [18]) derived the following...
ordinary differential equation (ODE) for the mean square deviation and slab geometry

\[
\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle = \frac{2}{B_0^2} \int d^3k \ P_{xx}^{slab}(\vec{k}) \cos(k_{\parallel}z)
\]

\[
= \frac{8\pi}{B_0^2} \int_0^{\infty} dk_{\parallel} \ g_{slab}^{slab}(k_{\parallel}) \cos(k_{\parallel}z).
\]  

(10)

For the wave spectrum of Eq. (6) this becomes

\[
\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle \approx 4c_i l_{slab}^{1-q} \delta B_{slab}^2 \frac{B_0^2}{B_0^2} \int_{k_{\text{min}}}^{l_{slab}^{-1}} dk_{\parallel} \ k_{\parallel}^{-q} \cos(k_{\parallel}z) + \ldots
\]  

(11)

It can easily be proven that the contribution of the inertial-range \((k_{\parallel} \geq l_{slab}^{-1})\) is much smaller than the contribution of the energy-range \((k_{\parallel} \leq l_{slab}^{-1})\), and was thus neglected in the right-hand side (rhs) of Eq. (11). Furthermore, the upper limit of the \(k_{\parallel}\)-integral can be extended to infinity. Here \(q > 0\) is assumed, for convergence.

Taking into account the relation

\[
\int_u^{\infty} dx \ x^{\mu-1} \cos x = \frac{1}{2} \left[ e^{-i\pi \mu/2} \Gamma(\mu, +iu) + e^{+i\pi \mu/2} \Gamma(\mu, -iu) \right],
\]  

(12)

according to Gradshteyn & Ryzhik (see [21], page 430, Eq. 3.761.7 therein), for \(\mu < 1\) (implying here \(q > 0\)), where we have employed the incomplete Gamma function \(\Gamma(\mu, x) = \int_x^{\infty} dt \ t^{\mu-1} e^{-t}\) (see Eq. 8.35 in the latter reference), and approximating \(\Gamma(\mu, x)\), for small values of the argument \(x\), as

\[
\Gamma(\mu, x \ll 1) \approx \Gamma(\mu) \left[ 1 - \frac{x^\mu}{\mu \Gamma(\mu)} \right]
\]  

(13)

(see Eq. 8.354.2 in the same reference) we find

\[
\int_u^{\infty} dx \ x^{\mu-1} \cos x \approx \Gamma(\mu) \cos \left( \frac{\pi \mu}{2} \right) - \frac{1}{\mu} u^\mu.
\]  

(14)

By applying this formula onto Eq. (11) one gets

\[
\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle = 4c_i l_{slab}^{1-q} \delta B_{slab}^2 \frac{B_0^2}{B_0^2} z^{-q-1}
\]

\[
\times \left[ \Gamma(1-q) \sin \left( \frac{\pi q}{2} \right) + \frac{1}{q-1} \right] (zk_{\text{min}})^{1-q}. \]  

(15)

The result can easily be integrated to obtain

\[
\langle (\Delta x)^2 \rangle = \frac{4c_i}{q(q+1)} l_{slab}^{1-q} \delta B_{slab}^2 \frac{B_0^2}{B_0^2} z^{q+1}
\]

\[
\times \left[ \Gamma(1-q) \sin \left( \frac{\pi q}{2} \right) + \frac{q(q+1)}{2(q-1)} \right] (zk_{\text{min}})^{1-q}. \]  

(16)
This expression is valid for $0 < q < 1$ and for $1 < q < 2$. For $q = 1$, Eq. (11) can be directly evaluated and we find a logarithmic behavior of the MSD. In the following, we shall further simplify Eq. (16), by distinguishing the ranges $0 < q < 1$ and $1 < q < 2$. We stress that the are interested in the large $z$ range, although we note that the condition $z k_{\text{min}} = \epsilon \ll 1$ is assumed to hold everywhere (since $k_{\text{min}}$ is related to the inverse size of the plasma “box”). We therefore retain the definition of the small parameter $\epsilon$, whose polynomial contribution may be singled out, for order of magnitude estimates.

A. Smooth spectrum form: the case $0 < q < 1$

In this case the first term in the rhs of Eq. (16) is dominant and we obtain

$$\langle (\Delta x)^2 \rangle \approx \frac{4c_1}{q(q+1)l_{\text{slab}}^{1-q}z^{q+1}} \frac{\delta B_{\text{slab}}^2}{B_0^2} \Gamma (1-q) \sin \left( \frac{\pi q}{2} \right) \sim z^{q+1}, \quad (17)$$

while a contribution $\sim O(\epsilon^{1-q})$ within the brackets in (16) is omitted. In general the mean square deviation of the field-lines has the form $\langle (\Delta x)^2 \rangle = az^b$. According to Eq. (17) we find for the slab model and for the values of the energy-range spectral index considered the characteristic exponent

$$b = q + 1. \quad (18)$$

It is obvious that we obtain superdiffusion ($1 < b < 2$) for $0 < q < 1$.

B. Steep spectrum form: the case $1 < q < 2$

In this case the second term in the rhs of Eq. (16) is dominant (of the order $\epsilon^{1-q} \gg 1$) and one gets

$$\langle (\Delta x)^2 \rangle = \frac{z^2 \delta B_{\text{slab}}^2}{2} \frac{B_{\text{slab}}^2}{B_0^2}. \quad (19)$$

This result if formally the same as the initial free-streaming result which can be found for small $z$ values (see e.g. [18]).
IV. FLRW FOR PURE 2D TURBULENCE

In this Section, we shall follow the nonlinear formalism for FLRW proposed by Shalchi & Kourakis ([18]). According to the results therein we have for pure 2D turbulence

\[
\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle = \frac{2\pi}{B_0^2} \int_0^\infty dk_\perp g^{2D}(k_\perp) e^{-\frac{1}{2} \langle (\Delta x)^2 \rangle k_\perp^2}.
\] (20)

With the spectrum of Eq. (8) we find

\[
\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle \approx d_i l_{2D}^{1-q} \delta B_{2D}^2 B_0^2 \int_{l_{2D}}^{l_{2D}^1} dk_\perp k_\perp^{-q} e^{-\frac{1}{2} \langle (\Delta x)^2 \rangle k_\perp^2} + \ldots
\] (21)

The detailed calculation (limited to the case \( q = 0 \)) was carried out in the latter reference, where a superdiffusive behavior of the form \( \langle (\Delta x)^2 \rangle \sim z^{4/3} \) was obtained. Our purpose in the following is to extend that result, for a general form of the wave spectrum.

It can easily be demonstrated that the inertial-range of the spectrum yields a negligible contribution in the rhs of (21) and was thus here neglected. The integral from the energy-range, extending the upper limit to infinity (\( l_{2D}^{-1} \rightarrow \infty \)), can be expressed by Gamma functions

\[
\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle = \frac{d_i l_{2D}^{1-q} \delta B_{2D}^2 B_0^2}{2} \left( \frac{\langle (\Delta x)^2 \rangle}{2} \right)^{(q-1)/2} \times \left[ \Gamma \left( \frac{1-q}{2} \right) + \Gamma \left( \frac{1-q}{2}, \frac{1}{2} \langle (\Delta x)^2 \rangle k_{\min}^2 \right) \right].
\] (22)

Assuming that \( \langle (\Delta x)^2 \rangle k_{\min}^2 \ll 1 \) (i.e., the field-line MSD cannot exceed the maximum turbulence square length scale \( k_{\min}^{-2} \)), and using Eq. (13) we find

\[
\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle \approx d_i l_{2D}^{1-q} \delta B_{2D}^2 B_0^2 \left( \frac{\langle (\Delta x)^2 \rangle}{2} \right)^{(q-1)/2} \times \left[ \Gamma \left( \frac{1-q}{2} \right) + \frac{1}{q-1} \left( \frac{\langle (\Delta x)^2 \rangle k_{\min}^2}{2} \right)^{(1-q)/2} \right].
\] (23)

The formula can be applied so long as \( 0 < q < 2 \), except for \( q = 1 \). In the latter case, Eq. (21) can be directly evaluated and we find a logarithmic behavior of the MSD. In the following, we shall further simplify Eq. (23), separately considering the cases \( 0 \leq q < 1 \) and for \( 1 < q < 2 \). The relation \( \epsilon' = \langle (\Delta x)^2 \rangle k_{\min}^2 \ll 1 \) is assumed to hold everywhere.
A. The case $0 < q < 1$

In this case the first term in Eq. (23) is dominant

$$\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle \approx d_1 l_{2D}^{1-q} \frac{\delta B_{2D}^2}{B_0^2} \left( \frac{\langle (\Delta x)^2 \rangle}{2} \right)^{(q-1)/2} \Gamma \left( \frac{1-q}{2} \right).$$

By making the ansatz $\langle (\Delta x)^2 \rangle = az^b$ we can solve this ODE analytically. It can easily be demonstrated that

$$b = \frac{4}{3 - q}$$

and

$$a = \left[ d_1 l_{2D}^{1-q} \frac{\delta B_{2D}^2}{B_0^2} 2^{(1-q)/2} \frac{(3-q)^2}{4(1+q)} \Gamma \left( \frac{1-q}{2} \right) \right]^{2/(3-q)}.$$

Obviously we find

$$\frac{4}{3} < b < 2$$

which is interpreted as superdiffusion. A diffusive behavior ($b = 1$) cannot be obtained. Interestingly, even for $q = 0$, one finds $b = 4/3$ (see in [18]).

B. The case $1 < q < 2$

In this case the second term within brackets in Eq. (23) is dominant (of the order $\sim \epsilon^{(1-q)/2} \gg 1$; see above) and we have

$$\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle = \frac{d_3}{q-1} \frac{\delta B_{2D}^2}{B_0^2} (l_{2D} k_{\min})^{1-q}.$$

By using Table II for $d_3$ this can be simplified to

$$\frac{d^2}{dz^2} \langle (\Delta x)^2 \rangle = \frac{\delta B_{2D}^2}{B_0^2},$$

and we finally find

$$\langle (\Delta x)^2 \rangle = \frac{z^2 \delta B_{2D}^2}{2 B_0^2}$$

which is again the initial free-streaming (parabolic MSD) result.

V. FLRW FOR SLAB/2D COMPOSITE GEOMETRY

According to cosmic observations, it is more realistic than plainly adopting a pure-slab or pure-2D model, to consider a 20% slab/80% 2D composite model (see e.g. [19]). In this
case, one rigorously obtains a 2nd-order ODE [cf. (10), (20)], whose RHS is the sum of the slab and 2D contributions, the relative weight of which is determined by the corresponding turbulence strength, i.e., $\delta B_{\text{slab}}^2/\delta B^2$ and $\delta B_{2D}^2/\delta B^2$.

We shall now attempt to evaluate the asymptotic behavior of the field-line MSD in this hybrid (composite) model.

A. The case $0 < q < 1$

In this case we can combine Eqs. (15) and (24) into:

$$
\frac{d^2}{dz^2} \left\langle (\Delta x)^2 \right\rangle = 4c_1 l_{\text{slab}}^{1-q} z^{q-1} \frac{\delta B_{\text{slab}}^2}{B_0^2} \Gamma (1 - q) \sin \left( \frac{\pi q}{2} \right)
$$

$$
+ d_1 l_{2D}^{1-q} \frac{\delta B_{2D}^2}{B_0^2} \left( \left\langle (\Delta x)^2 \right\rangle \right)^{(q-1)/2} \Gamma \left( \frac{1 - q}{2} \right),
$$

where negligible contributions were omitted in the rhs. Obviously this equation has the form

$$
\frac{d^2}{dz^2} \left\langle (\Delta x)^2 \right\rangle = \alpha z^{q-1} + \beta \left\langle (\Delta x)^2 \right\rangle^{(q-1)/2}.
$$

By applying the ansatz $\left\langle (\Delta x)^2 \right\rangle = az^b$ we find

$$
ab(b - 1) z^{b-2} = \alpha z^{q-1} + \beta a^{(q-1)/2} z^{b(q-1)/2}
$$

where the definitions of $\alpha$ and $\beta$ are obvious. It is straightforward to prove that, since $b < 2$, the second term in the rhs is dominant for $z \to \infty$. The slab contribution can therefore be neglected, so we can use Eqs. (25) and (26) also within the two-component model.

B. The case $1 < q < 2$

In this case we can simply add the two contributions (Eqs. (19) and (30)):

$$
\left\langle (\Delta x)^2 \right\rangle = \frac{z^2 \delta B^2}{2 B_0^2}
$$

where we have set

$$
\delta B^2 = \delta B_{\text{slab}}^2 + \delta B_{2D}^2.
$$
TABLE III: In this table, the results obtained for the parameters $a$ and $b$, having adopted the form $\langle (\Delta x)^2 \rangle = az^b$ for the field-line mean square deviation, are presented. In all (but one) cases, we find either superdiffusion ($1 < q < 2$) or free-streaming ($q = 2$) of the field-lines. Diffusion ($q = 1$) can only be found for slab geometry and $q = 0$.

VI. SUMMARY AND CONCLUSION

We have investigated the random walk of magnetic field-lines for a more general spectrum, than the one employed in previous works. By exploring pure slab, pure 2D, and two-component turbulence models, we have calculated the field-line mean square deviation by applying the analytical description for FLRW proposed by Shalchi & Kourakis (see [18]). A superdiffusive behaviour is found in all cases considered. In Table III the results obtained in this article are summarized. The only case where one obtains diffusion is for pure slab geometry and $q = 0$. As shown in this article the energy-range spectral index is a key-input parameter if FLRW is described.

In the two-component turbulence model, which has been considered as a realistic model for solar wind turbulence (see [19]), we already find a weakly superdiffusive behavior if $q = 0$. For larger values of $q$ we have $\langle (\Delta x)^2 \rangle \sim z^{4/(3-q)}$. If the energy-range spectral index exceeds unity we find the same solution as in the initial free-streaming regime. Obviously, the energy-range spectral index has a very strong influence on FLRW behavior.

From a theoretical point of view the results for pure slab geometry deduced in Section 3 are very interesting and important because of two reasons:

- for pure slab turbulence the parameter $\langle (\Delta x)^2 \rangle$ can be calculated exactly. No theory nor any ad hoc assumption have to be applied.
• In all cases except \( q = 0 \) we find superdiffusion of FLRW.

Since in reality 20% of the fluctuations can be represented by slab modes (see [19]) it is self-evident to assume that superdiffusion and not (classical or Markovian) diffusion is the regular case in astrophysical turbulence.

If we merge from pure slab geometry to the slab/2D composite model a (nonlinear) theory has to be applied and an exact description of FLRW in no longer possible. By applying the ODE deduced by Shalchi & Kourakis (see [18]) we have shown that the superdiffusivity becomes even stronger in comparison to the pure slab results.

It must be the subject of future work to apply these new results on realistic systems, such as solar wind turbulence. An important example is perpendicular transport of charged cosmic rays which is directly controlled by the FLRW, since charged particles are tied to magnetic field-lines (see [15]).

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