Automatic optimization of working modes of a gas centrifuge for isotope separation

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Abstract

The adaptive algorithms for the automatic search for the optimum separative power of a single gas centrifuge (GC) depending on the pressure of the working gas near the wall of the working chamber are studied. It is demonstrated that the proposed control system is insensitive to the displacement of the found optimum of the separative power, a change in the shape of the extremum, including deformation of the "plateau" in the maximum region, as well as the additive errors of the technical means measuring the GC separation ability.

1. Introduction

The typical dependence of the separative power $\delta U$ of a model Iguassu gas centrifuge (GC) with 1 m long on the working substance pressure near its working chamber wall is discussed in [1]. It was shown in [2] that the dependence $\delta U(P)$ for the longer GCs with a 2-meter rotor in some working modes can have two markedly different maxima and outlines corresponding to pressure $P_1=35$ mmHg and $P_2=350$ mmHg (see Figure 1).

It is quite obvious that technologically the working mode corresponding to higher pressure and smoother maximum is expedient. The high pressure maximum for the Iguassu GC is located on a wide "plateau" from $P_1 = 200$ mmHg to $P_2 = 500$ mmHg, where the optimal separative power is caused by both thermal and mechanical drives simultaneously slightly in the above range of pressure and is changes.

Such dependence of the separation ability of the pressure is convenient for solving the process control problem which can be considered as an indicator of the optimal separation working mode of the GC.
Figure 1. Dependence of the separative power on the wall pressure $\delta U(P)$ for 2 meter long GC. The dependence approximation in the vicinity of the second smoother maximum is specially highlighted.

In the simulation of $\delta U(P)$, the following assumption have been accepted: according to [1], the extremum in the vicinity of $P = 350$ mmHg can be “stretched” to a plateau of unknown width with respect to the parameter $P$ at pressures greater than $P_2$, but a characteristic significant rate of change of the $\delta U(P)$ dependence is assumed to the left of the extremum.

This allows, as a sign of the optimal mode of the GC, to take the value $d\delta U/dP = \gamma > 0$ close to the extremum, to the left of the extremum, corresponding to the small value of the angle $\gamma > 0$ of the tangent to the $\delta U(P)$ function (see Figure 2) in the vicinity of the extremum, i.e. small “non-reached to the extremum.”

The examples of successful and cost-effective solutions for the optimization problems of various technological processes with this kind of the quality indicator one can find in [3-5].

Figure 2. Possible positions of the $\delta U(P)$ dependence in the coordinates $\{\delta U; P\}$. $\gamma$ is a sign of the optimal working mode of the Iguassu GC.

When automation is operating, significant deviations from a given small value of $d\delta U/dP = \gamma > 0$ are unlikely. Therefore, in a neighborhood of the extremum of the $\delta U(P)$ dependence is approximated by the quadratic function as follows: $\delta U(P) = \alpha(P - 350)^2 + 16.75$; $P$ in mmHg, $\alpha = 1.5 \times 10^{-2}$, $\delta U$ in SWU.

The control task consists in determining and tracking the specified value of $d\delta U/dP = \gamma > 0$ of the characteristic $\delta U(P)$, the predetermined state and shape of which in coordinates $\{\delta U; P\}$ is not known.

The problem is solved by using the self-tuning step-by-step algorithms for automatic search for a specified value of a partial derivative of a nonlinear static characteristic [3-8].
For operation of such systems, the existence of a nonlinear static characteristic of \( \delta U(P) \) in the range of possible changes in the parameters of the process, for which a given \( d\delta U/dP = \gamma > 0 \) correspond to a specified and close to the maximum separative power.

2. Control law
The operation of the automatic search algorithm is based on determining the direction of movement of the search coordinate \( P(t) \) to the operating point \( \{ \delta U, P \} \), where \( d\delta U/dP = \gamma > 0 \) or in the finite differences \( \Delta P_n = P_n - P_{n-1} \); where \( \Delta P_n \) is the amplitude of the search step of duration \( T, n = 1, 2, ..., \infty, \Delta\delta U_n = \delta U_n - \delta U_{n-1} \) is the increment of the output coordinate of the object, in the logic suggested in [3]:

\[
\begin{align*}
\text{Sign} \, [\Delta\delta U_n \Delta P_n - \theta] &< 0 & \to (\Delta P_{n+1} > 0) \\
\text{Sign} \, [\Delta\delta U_n \Delta P_n - \theta] &> 0 & \to (\Delta P_{n+1} < 0)
\end{align*}
\]

\( \theta = \Delta P^2 \gamma \) and \( P_{n+1} = \sum_{n=1}^{\infty} \Delta P_n \)

The procedure of multiplication \( (\Delta\delta U_n \times \Delta P_n) \) in the logic invented in [1] is similar to division \( (\Delta\delta U/P \Delta P_n) \), but it avoids the possible problems of “division by zero” in the modeling and practical implementation of the algorithm.

In the steady-state automatic search mode, the operation of the algorithm is characterized by a classical self-oscillating process at the point \( \{ \delta U_n; P_n \} \) where \( \delta U_n/P_n=\gamma > 0 \), duration is \( 4T \) and search motions with the amplitude \( \pm\Delta P \) is applicable to the variable \( P_n \) [3].

The duration and the amplitude for the self-oscillation process can be reduced by 2 times due to the addition of extra reverse conditions (2) compatible with algorithm (1), and separate for search steps \( \Delta P_n \) of different signs [3].

\[
\begin{align*}
(\Delta P_n > 0, \Delta\delta U_n > \theta) & \to (\Delta P_{n+1} > 0), \\
(\Delta P_n > 0, \Delta\delta U_n < \theta) & \to (\Delta P_{n+1} < 0), \\
(\Delta P_n < 0, -\Delta\delta U_n < -\theta) & \to (\Delta P_{n+1} > 0), \\
(\Delta P_n < 0, -\Delta\delta U_n > \theta) & \to (\Delta P_{n+1} < 0),
\end{align*}
\]

where \( \theta; \beta \) are the constants for changing the reverse conditions \( \Delta P_n \) in the vicinity of the search \( \gamma: \theta \) for \( \Delta P > 0; -\beta \) for \( \Delta P < 0; \theta > \vert -\beta \vert \).

The structure of the automatic search system \( \delta U/\Delta P = \gamma \) is shown in Figure 3

![Scheme of automatic optimization of the separative power of a single GC on the wall pressure.](image)

3. Model of the object
The model \( \delta U(P) = \alpha (P - 350)^2 + 16.75 [SWU] \) in terms of Simulink MATLAB [9] is built on the basis of Produkt3 multiplication blocks, gain Gain2 \( \div 1.5 \times 10^{-5} [SWU/(mmHg)^2] \) and Constant5 \( \div 16.75 [SWU] \) (see Figure 4).
4. Model of the system

The shifts of the initial position (pos. 1) $\delta U(P) = \alpha(P - 350)^2 + 16.75$ to the state (pos. 2) $\delta U(P) = \alpha(P - 300)^2 + 16.75$, and then (pos. 3) $\delta U(P) = \alpha(P - 400)^2 + 16.75$ and (pos. 4) $\delta U(P) = \alpha(P - 400)^2 + 15.75$ implemented by setting the parameters of the blocks Step, Step1, Step2 with setting the amplitudes and the synchronous start of action in time of the blocks “Step ...” (Figure 5, left side).

The system algorithm (the right side of Figure 5) is built as the digital one in the terms of Simulink MATLAB. The time of the quantization period is taken as the duration of the search step $T = 1$ in the automatic search algorithm.

Calculation of the first difference between the “input” $\Delta P_n$ and the “output” $\delta U_n$ of the extreme characteristic $\delta U$ is carried out by delay elements of $T$ (on one “step”) with $W(z) = 1/z$, where $z$ is the argument of “Z transformations” which is the famous mathematical apparatus for the analysis of the digital automatic systems.

The conditions (1) of the search procedure are realized by the Produkt3 blocks and the Sign operator, and the additional logic (2) of the improved automatic search algorithm $\gamma$ is implemented by the Sign2, Switch1 blocks with the settings Constant6 (“$\theta$”) and Constant5 (“$\beta$”). When “$\theta$”=“$\beta$”, the classical automatic search algorithm $\delta U_n/\Delta P_n = \gamma > 0$ is executed. The search action $P_n$ is formed by a discrete integrator $W(z) = T/(z - 1)$. The value of a single “step” for $\Delta P_n$ is adjusted by the transmission coefficient “Gai4”. Visualization of the system is carried out by two-axis XY Graph2 and Skope oscilloscopes.
5. System tuning
The steady-state self-oscillatory mode of the automatic search system is carried out in a small neighborhood of a given value \( \Delta \delta U / \Delta P = \gamma > 0 \) - to the left of the extremum. For the “nominal” \( \delta U(P) \) (Figure 2, Figure 3, pos. 1), it is assumed that \( \gamma = 0.001 \). When choosing \( \Delta P = 20 \) [mmHg], the quantity \( \theta = \gamma \Delta P^2 = 0.4 \) [\( \delta U \times \text{mmHg} \)], and \( \beta = 0 < \theta \) [3].

When the system tuning up, the correct operation of the algorithm (1) for \( \beta = 0 \) is initially applied. Next, the logical conditions (2) are connected (the shaded right blocks in Figure 4 and the conditions \( \theta = 0.4, \beta = 0 \) are assigned.

6. Results of modeling
Figure 6 illustrates the process of automatic search for a given value of \( \gamma = 0.001 \) at the quasi-stationary states of the extreme characteristic \( \delta U(P) \) in pos.1 → pos.2 → pos.3 → pos.4 by the classical automatic search algorithm (1), which implements the standard limit cycle of duration 4\( T \) with deviations of \( \pm \Delta P_n \) relatively to \( P_n \) in the vicinity of which \( \gamma = 0.001 \).

![Figure 6](image_url)

This is demonstration of the classic automatic search algorithm for \( \gamma = 0.001 \) at displacement of \( \delta U(P) \); \( \Delta P_n = 20 \) [mmHg]. The centers of the self-oscillating mode of the search coordinate \( P_n \) correspond to the coordinates \( \{ \delta U; P \} \) where \( \gamma = 0 \).

It is presented in Figure 6:
- \( \Delta t = 0 \) …20; it is the nominal regime for \( \delta U(P) \) pos.1. Search of the extremum from the initial conditions.
- \( P = 100 \) mmHg. Self-oscillating steady-state search with amplitude \( |\Delta P_n| = \pm 20 \) [mmHg] in the neighborhood of \( \{ \delta U; P \} \), where \( \gamma = 0.001 \).
- \( \Delta t = (20-40) \); The dependence \( \delta U(P) \) is biased to pos. 2 to the left of the optimal state of the process pos.1. The algorithm “finds” the center of the self-oscillating regime in the vicinity of \( \{ \delta U; P \} \) where \( \gamma = 1 \).
- \( t = (40-45) \); The dependence \( \delta U(P) \) is biased to pos.3, to the right from the optimal state of the extremum. The algorithm “finds” the center of the self-oscillating regime in the vicinity of \( \{ \delta U; P \} \) where \( \gamma = 0.001 \).
\( t = 45 \); the automatic search process with a parallel shift of the \( \delta U \) from pos.3 to pos.4 along the \( \delta U \) coordinate due to the technological conditions or additive errors of the \( \delta U \) limit means. The absolute value of the extremum decreased by \( \Delta \delta U = 0.1 \) without changing the coordinate of the extremum along the \( P \) axis.

In all considered situations, high-precision stabilization of “shortage to extremum” is determined, which is determined by a given angle \( \gamma \) the tangent to the static characteristic \( \delta U(P) \).

A very insignificant relative error of stabilization for a given slope angle \( \Delta \delta U / \Delta P = \gamma \) for the displacements \( \delta U(P) \) is related to the discrete nature of the finite differences and nonlinear elements in the self-oscillating system, as a result of which the center of the self-oscillating regime can vary slightly depending on the initial conditions. These errors decrease with decreasing amplitude of the search step.

It should not be forgotten that the adaptive control system under consideration belongs to the class of nonlinear digital ones with a large period of time quantization and coarse level quantization.

Figure 7 illustrates the automatic search and tracking of the maximum \( \delta U(P) \) by the automatic search algorithm (1) with additional logic (2) under conditions which are similar to presented in Figure 5.

![Figure 7](image)

Figure 7. Illustration of automatic search for \( \gamma = 0.001 \) by the algorithm (2) at the displacements \( \delta U(P) \) under the condition \( \Delta P = 20 \, [\text{mmHg}] \)

The quality indicators of the automatic search for the modified algorithm relatively that shown in Figure 6 are almost 2 times better in the maximum deviation from the coordinate \( \{ \delta U; P \} \), where \( \gamma = 0.001 \), the duration and amplitude of the limit cycle in the neighborhood of \( \gamma \).

With a slow drift \( \delta U(P) \) (see Figure 8), the system monitors a given value \( \gamma = 0.001 \) with features characteristic for the discrete nature of the search procedure, in particular, the asymmetry of dynamic processes at the displacement of \( \delta U(P) \) with different speed and direction in coordinates \( \{ \delta U; P \} \).
Figure 8. Demonstration of the system operation at a slow displacement \( \delta U(P) \) along the \( P \) axis within \( \pm 50 \text{ mm Hg} \) from the nominal value at \( P=350 \text{ [mm Hg]} \) according to the harmonic law \( P=50 \sin(0.00628t) \) under the initial conditions \( P = 150 \text{ [mm Hg]} \).

Nevertheless, the tracking accuracy \( \gamma = 0.001 \), chosen very close to the maximum \( \delta U(P) \) in the simulated technological situation, as is clearly shown in Figure 9, high enough. The maximum deviation from the extremum \( \delta U(P)=16.75 \text{ [SWU]} \) is not more than \( \Delta \delta U = 0.03 \), and the average deviation characterizing the technological mode of operation of the separation unit is not more than \( \Delta \delta U \approx 0.01 \) (i.e. 0.06% of the maximum value of \( \delta U = 16.75 \)).

Figure 9. Evaluation of the tracking accuracy of the optimum sign \( \Delta \delta U/\Delta P_n=\gamma =0.001 \) with a displacement \( \delta U(P) \) along the \( P \) axis within 50 [mm Hg] from the nominal state at \( P = 350 \text{ [mm Hg]} \) according to the harmonic law \( P(t) = 50 \sin(0.00628t) \). The value “0” along the ordinate axis corresponds to the extremum \( \delta U(P) = 16.75 \text{ [SWU]} \).
With a decrease in $\Delta P$, the metrological characteristics of stabilization of the optimal mode of the separation installation are improved. The practical limit is the ability to distinguish the sign of the change of $\Delta \delta U_n$ by a small search step $\pm \Delta P_n$.

As shown in [1], the location and shape of $\delta U(P)$ depend in addition to pressure $P$, varying temperatures, and other factors. Therefore, the problem arises when the quality indicator is the multidimensional extremum $\delta U(P)$ as a function of 3 or more parameters, the solution of which is possible by using search procedures of a multiparameter extremum.

Conclusions
1. The principle of negative feedback is unacceptable for the automatic stabilization of the separative power $\delta U$ of the Iguassu GC as a function of pressure $P$ due to displacements of the extreme dependence $\delta U(P)$, changes in its shape up to the formation of a “plateau” in the vicinity of the extremum during the normal operation of the separation process.
2. As a sign of the optimal regime, the given value of the derivative $d\delta U(P)/dP = \gamma > 0$ of the static characteristic $\delta U(P)$ (the tangent angle) is close to zero, which corresponds to the extremum of $\delta U(P)$. In this case, the appearance of a plateau in the region of the extremum $\delta U(P)$ insignificantly changes the indication of the separation ability of the GC, which is close to the maximum in the specific technological conditions.
3. The adaptive search algorithms for a given value of the derivative $d\delta U(P)/dP = \gamma > 0$ of the static characteristic $\delta U(P)$ (the tangent angle to $\delta U(P)$ in terms of finite differences are applied.
4. The adaptive search system automatically stabilizes the optimal mode of the separation process with an accuracy of hundredths of a percent of deviation from the maximum value of $\delta U(P)$ under the conditions of displacement and change of the $\delta U(P)$ shape.
5. The search control algorithm is insensitive to the additive errors of the measurement sensor of $\delta U$ or since there are none of such errors in the signal $\Delta \delta U_n = \delta U_n - \delta U_{n-1}$. The use of the non-zero scale of the $\delta U$ meter is a reserve for improving the accuracy of the system.
6. The control algorithm is implemented by the standard functional resources of the general industrial automatic controllers with a freely programmable structure.
7. The control system may be switched on periodically, depending on the specific technological situation. The automatic search algorithm can be performed interactively by an operator.

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