The ccbar Pentaquarks by a Quark Model

Sachiko Takeuchi\textsuperscript{1,3,4} and Makoto Takizawa\textsuperscript{2,4,5}

1 Japan College of Social Work, Kiyose, Tokyo 204-8555, Japan
2 Showa Pharmaceutical University, Machida, Tokyo 194-8543, Japan
3 Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka, 567-0047, Japan
4 Theoretical Research Division, Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan
5 J-PARC Branch, KEK Theory Center, IPNS, KEK, Tokai, Ibaraki, 319-1106, Japan
E-mail: s.takeuchi@jcsw.ac.jp

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Recent LHCb experiments have shown us that there are two resonances in the $J/\psi p$ channel in the $\Lambda_b$ decay, whose spin and parity are most probably $(3/2^- 5/2^+)$ or $(3/2^+ 5/2^-)$. In this work, we investigate the $I(J^P) = (1/2^- 3/2^+ 5/2^-)$ ccbar pentaquark states by employing the quark cluster model. It is found that the color-octet isospin-$1/2$ spin-$3/2$ uud configuration gives an attraction to such five-quark systems. This configuration together with the color-octet c$\bar{c}$ pair gives structures around the $\Sigma^{(*)} D^{(*)}$ thresholds: one bound state, two resonances, and one large cusp are found in the uudc$\bar{c}$ negative parity channels. We argue that these resonances and cusp may correspond to, or combine to form, the negative parity pentaquark peak observed by LHCb.

KEYWORDS: hidden-charm pentaquark; color-octet baryon; exotic hadron; multiquark hadron; baryon-meson scattering

1. Introduction

In 2015, two candidates of the new exotic baryons, $P_c(4380)$ and $P_c(4450)$, had been reported by LHCb. They are observed in the $\Lambda_b^0 \rightarrow J/\psi pK^-\bar{K}$ decay. The $P_c(4380)$ has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV while $P_c(4450)$ has a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. The most favorable set of the spin parity for the lower and the higher peaks is $J^P = (3/2^-, 5/2^+)$, but $(3/2^+, 5/2^-)$ or $(5/2^-, 3/2^-)$ are also acceptable according to their analysis \cite{2}. Their configuration is considered to be $uudc\bar{c}$: a hidden-charm pentaquark of the isospin $1/2$.

In this work, we discuss the negative-parity $uudc\bar{c}$ pentaquarks \cite{1}. They are considered to couple to baryon-meson states, and their feature is observed in the short range properties of such baryon-meson states. For this purpose, we employ the quark cluster model, which successfully explained the short range part of the baryon-baryon interaction and the structure of the light-flavored pentaquark $\Lambda(1405)$ \cite{3,4}. Recent lattice QCD results are found to give similar short range potentials to those of the quark cluster model for the baryon-baryon interaction \cite{5}.

Let us first discuss possible configurations of $uud$ quarks in the $uudc\bar{c}$ pentaquarks. These three light quarks can be color-singlet or color-octet. So, when the orbital configuration is totally symmetric, the $uud$ configuration in the $uudc\bar{c}$ systems can be totally symmetric (56-plet) or mixed symmetric (70-plet) in the flavor-spin $SU_{f,\sigma}(6)$ space accordingly. They are classified as:

$$56_{fr} = 8_f \times 2_{\sigma} + 10_f \times 4_{\sigma}, \quad 70_{fr} = 1_f \times 2_{\sigma} + 8_f \times 2_{\sigma} + 8_f \times 4_{\sigma} + 10_f \times 2_{\sigma}. \quad (1)$$

The color-singlet $uud$ systems correspond to the usual 56-plet baryons, whereas the color-octet ones correspond to the 70-plet systems. Since the present work concerns systems of the isospin $1/2$ and the strangeness zero, the configurations of the three light quarks correspond to one of the following three:
Table I. The classification of the isospin-$\frac{1}{2}$ negative parity $qqq\bar{c}\bar{c}$ states. The $uud$ spin ($s_q$), color ($c$), CMI of the five quark systems at the heavy quark limit ($\langle O_{\text{cmi}}\rangle_{\text{5q}}^{(HQ)}$), the possible five quark spin with the multiplicity ($J$), the lowest $S$-wave threshold ($T$) and the CMI contribution to the threshold energy ($\langle O_{\text{cmi}}\rangle_{\text{T}}^{(HQ)}$) are listed.

| $s_q$ | $c$ | $\langle O_{\text{cmi}}\rangle_{\text{5q}}^{(HQ)}$ | $J$ | $T$ | $\langle O_{\text{cmi}}\rangle_{\text{T}}^{(HQ)}$ |
|-------|-----|---------------------------------|-----|-----|---------------------------------|
| $[q^3\frac{1}{2}]$ | $\frac{1}{2}$ | $-8$ | $\frac{1}{2}$ | $N\eta_c, N\phi\eta$ | $-8$ |
| $[q^3\frac{3}{2}]$ | $\frac{1}{2}$ | $8$ | $\frac{1}{2}$ | $\Lambda_c\bar{D}^*$ | $-8$ |
| $[q^3\frac{3}{2}]$ | $\frac{1}{2}$ | $8$ | $\frac{1}{2}$ | $\Sigma_c^{(*)}\bar{D}^*$ | $\frac{1}{2}$ |

(a) color-singlet spin-$\frac{1}{2}$ baryon in ($8_f \times 2_r$), namely, nucleon, (b) color-octet spin-$\frac{1}{2}$ $q^3$ in ($8_f \times 2_r$), and (c) color-octet spin-$\frac{1}{2}$ $q^3$ in ($8_f \times 4_r$). In the following, we denote each of them by $[q^3\frac{1}{2}]$, $[q^3\frac{3}{2}]$, and $[q^3\frac{3}{2}]$, respectively. Since the spin of the $c\bar{c}$ pair is either 0 or 1, the total spin of the $uudc\bar{c}$ systems is either $\frac{1}{2}$ (5-fold), $\frac{3}{2}$ (4-fold), or $\frac{5}{2}$ (1-fold). (See Table I.)

In Table I, we list the color magnetic interaction (CMI) evaluated by the $uud$ part of the five-quark system, ($\langle O_{\text{cmi}}\rangle_{\text{5q}}^{(HQ)}$), which corresponds to the CMI contribution to the five-quark system at the heavy quark limit. The lowest $S$-wave thresholds ($T$) are also shown together with the CMI contribution to the threshold energy ($\langle O_{\text{cmi}}\rangle_{\text{T}}^{(HQ)}$). As seen from the table, $\langle O_{\text{cmi}}\rangle_{\text{5q}}^{(HQ)}$ is smaller than $\langle O_{\text{cmi}}\rangle_{\text{T}}^{(HQ)}$ for the $[q^3\frac{3}{2}]$ configuration; which means that CMI is attractive in this configuration. Since $uudc\bar{c}$ is color-singlet as a whole, the system of the color-octet $uud$ with the color-octet $c\bar{c}$ can be observed as $\Lambda_c\bar{D}^*$ or $\Sigma_c^{(*)}\bar{D}^*$ baryon meson states, where each of the hadrons is color-singlet. The above CMI contribution is expected to be seen as an attraction in the $\Sigma_c^{(*)}\bar{D}^*$ baryon meson channels. We argue that this attraction may cause the one of the observed peaks by LHCb.

2. Model

The model Hamiltonian, $H_q$, consists of the central term, $H_c$, and the color spin term, $V_{\text{cmi}}$. The $H_c$ consists of the kinetic term, $K$, the confinement term, $V_{\text{conf}}$, and the color Coulomb term, $V_{\text{coul}}$:

$$H_q = H_c + V_{\text{cmi}}, \quad H_c = K + V_{\text{conf}} + V_{\text{coul}}.$$  \hspace{1cm} (2)

Both of the $V_{\text{coul}}$ and $V_{\text{cmi}}$ terms come from the effective one-gluon exchange interaction between the quarks.

The color flavor spin part of the $q^3$ or $q\bar{q}$ wave functions is taken as a conventional way [6]. The orbital wave function of the mesons, $\phi_M$, and that of the baryons, $\phi_B$, are written by Gaussian with a size parameter $b$, $\phi(r, b)$:

$$\phi_M(r_M) = \phi(r_{12}, \frac{x_0}{\sqrt{\mu_{12}}}), \quad \phi_B(r_B) = \phi(r_{12}, \frac{x_0}{\sqrt{\mu_{12}}})\phi(r_{12-3}, \frac{x_0}{\sqrt{\mu_{12-3}}}),$$  \hspace{1cm} (3)

where the reduced masses, $\mu_{12}$ and $\mu_{12-3}$, correspond to the Jacobi coordinates, $r_{12}$ and $r_{12-3}$. We assume that the size parameter of the orbital motion can be approximated by $b = x_0/\sqrt{m}$ and minimize the central part of the Hamiltonian, $H_c$, against $x_0$ for each of the flavor sets: $u\bar{c}$, $c\bar{c}$, $uud$, $udc$. For the baryons, this means that the ratio of the size parameters is kept to a certain mass ratio; e.g., $b_{uc}/b_{ud}$ in $\Lambda_c$ or $\Sigma_c$ is equal to $\sqrt{\mu_{uc}/\mu_{uc}}$.

We employ the resonating group method (RGM) in order to solve the five-quark systems. The wave function of the five quark system, $\Psi$, consists of the $q^3$ baryon and the $q\bar{q}$ meson with the relative
3. Results

where the three-body operator, $H$, is obtained from the equation of motion for the quarks, ($NJ\psi$ channel). By integrating out the internal wave function of the hadrons, the RGM wave function $\chi$ [3, 4]:

$$\Psi = \sum_v c^v \mathcal{A}_v \langle \psi^v_B(r_B)\psi^v_M(r_M)\chi^v(R) \rangle,$$

where $\mathcal{A}_v$ stands for the quark antisymmetrization which operates on the four quarks, and $v$ for the baryon-meson channel. By integrating out the internal wave function of the hadrons, the RGM equation can be obtained from the equation of motion for the quarks, $(H_q - E)\Psi = 0$, as

$$\sum_v \int (H^{v\nu'} - EN^{v\nu'})\chi^{v'} = 0,$$

where $H^{v\nu'}$ and $N^{v\nu'}$ are the hamiltonian and the normalization kernels.

In order to investigate the nature of the resonance states as well as the bound states, We define a three-body operator, $P^{cs}$, to extract the $uud$ color $c$, spin $s_q$, orbital $(0s)^3$ component:

$$P^{csq} = P^{c123}_q + P^{c124}_q + P^{c134}_q + P^{c234}_q$$

(6)

$$P^{cijk}_q = \langle uud; cs_q(0s)^3\rangle\langle uud; cs_{s}(0s)^3\rangle$$

(7)

$$\langle P^{csq} \rangle = \sum_{v\nu} \int \int \int d\mathbf{r} \langle \psi^v_B(\mathbf{r}_B)\psi^v_M(\mathbf{r}_M)\chi^v(\mathbf{R}) \rangle \sum_{ijk} P^{cijk}_q \mathcal{A}_v \langle \psi^v_B(\mathbf{r}_B)\psi^v_M(\mathbf{r}_M)\chi^v(\mathbf{R}) \rangle.$$

(8)

3. Results

It is found that a very shallow bound state appears in the $\Sigma^++\bar{D}^*$ $J = \frac{3}{2}^-$ system, in which the $uud$ is in the $[q^3s^3\frac{3}{2}]$ configuration. As seen from the scattering phase shifts shown in Figs. 1 (a) and (b), there is one sharp resonance in the $\Lambda_c\bar{D}$ channel of the $J = \frac{1}{2}^-$ system, while one sharp resonance and one strong cusp are found in the $\Lambda_c\bar{D}^*$ channel of the $J = \frac{3}{2}^-$ system. The number of these structures, one in $J = \frac{1}{2}^-$, two in $\frac{3}{2}^-$, one in $\frac{5}{2}^-$, corresponds exactly to the number of the multiplicity for the $[q^3s^3\frac{3}{2}]$ configuration shown in Table 1.

The $\langle P^{csq} \rangle$'s are shown in Figs. 2 (a) and (b). The size of $[q^3s^3\frac{3}{2}]$ configuration enhances at the resonance or the cusp energies. This feature is more clearly found in the $J = \frac{3}{2}^-$ system. There, the attraction originally forms a bound state each in the $\Sigma_3^+\bar{D}$ and in the $\Sigma_3^+\bar{D}^*$, which become a resonance and a cusp in the $\Lambda_c\bar{D}^*$ channel by the channel coupling. At these resonance and cusp energies, the
The factor $\langle P \rangle$ to find the $uud(0s)^3$ configuration in the scattering wave function with the initial $NJ/\psi$ channel in the $S$-wave $uud\overline{c}c J(I^P)=1/2(1^-)$ channel (Fig. a) and that of the $1/2(3^-)$ channel (Fig. b). The solid line stands for the factor to find in the color octet spin $3/2$, the dashed line for the color-octet spin $1/2$, and the dot-dashed line for the color singlet spin $1/2$. (color online)

short range part of the wave function enhances and the proportion of $[q^38\frac{3}{2}]$ also enhances. These resonance and cusp indeed have the $[q^38\frac{3}{2}]$ configuration at the short range part. As for the $J = \frac{1}{2}$ system, the size of this configuration is much smaller than that of the $\frac{3}{2}$ system. Without the coupling to the $\Lambda_cD$ channel, however, the resonance becomes a bound state of the $\Sigma_cD$ channel, and the $[q^38\frac{3}{2}]$ component is 0.7 of the whole $uud(0s)^3$ component of that bound state. This configuration plays an important role to make the resonance also for the $J = \frac{1}{2}$ system though the mixing of the $\Lambda_cD$ channel reduces its size. The above situation shows us that $[q^38\frac{3}{2}]$, the $uud$ color-octet spin $\frac{3}{2}$ configuration, which may be called as a ‘color-octet $uud$ baryon,’ causes these resonances.

In order to compare our results to the experimental spectra, it will be necessary to include the effects of the meson-exchange in the long range baryon-meson interaction. We would like to argue, however, these resonances and cusp may correspond to, or combine to form, the negative parity pentaquark peak observed by LHCb.

4. Summary

The $I(J^P) = \frac{1}{2}(\frac{3}{2}^-), \frac{1}{2}(\frac{5}{2}^-)$, and $\frac{1}{2}(\frac{7}{2}^-)$ $uud\overline{c}c$ systems are investigated by the quark cluster model. It is shown that the color-octet isospin-$\frac{1}{2}$ spin-$\frac{1}{2}$ $uud$ configuration gains attraction from the color magnetic interaction. The $uud\overline{c}c$ states with this configuration cause structures around the $\Sigma_c(\ast\overline{D}c^\ast)$ thresholds. We have found one bound state in $\frac{1}{2}(\frac{5}{2}^-)$, one resonance and a cusp in $\frac{1}{2}(\frac{3}{2}^-)$, and one resonance in $\frac{1}{2}(\frac{7}{2}^-)$ in the negative parity channels, which may be the origin of the negative parity $P_c$ peak observed in the $\Lambda_b$ decay.

References
[1] A part of this work has been discussed in S. Takeuchi and M. Takizawa, arXiv:1608.05475 [hep-ph].
[2] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 115, 072001 (2015).
[3] M. Oka, K. Shimizu and K. Yazaki, Prog. Theor. Phys. Suppl. 137, 1 (2000).
[4] S. Takeuchi and K. Shimizu, Phys. Rev. C 76, 035204 (2007).
[5] K. Sasaki et al. [HAL QCD Collaboration], Prog. Theor. Exp. Phys. 2015, 113B01 (2015).
[6] See, for example, K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).
[7] Y. Yamaguchi and E. Santopinto, arXiv:1606.08330 [hep-ph].