To the Modeling of multilayer Thin Prismatic Bodies

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Abstract. Proceeding from three-dimensional formulations of initial boundary value problems of the three-dimensional linear micropolar theory of thermoelasticity similar formulations of initial boundary value problems for the theory of multilayer thermoelastic thin bodies are obtained. The initial boundary value problems for thin bodies were also obtained in the moments with respect to systems of orthogonal polynomials. We consider some particular cases of formulations of initial boundary value problems. In particular, the statements of the initial-boundary value problems of the fifth approximation of the micropolar theory of five-layer thin prismatic bodies are considered in more detail.

1. Introduction
The new parametrization in the case of a one-layered thin body is described in detail in [1–3]. Various representations of the equations of motion, the heat influx, the constitutive relations of physical and heat content were given for the new body domain parametrization. There also has been given the definition of the $k$th order moment of a certain quantity with respect to an orthonormal polynomial system. We obtained the expressions in moments of first- and second-order partial derivatives of a certain tensor field and they were also done for some important expressions required for constructing different variants of the theory of thin body. We got the various variants of the equations of motion in moments with respect to the orthogonal polynomial systems.

2. Representations of the motion equations
These equations have the following form [1, 2, 4–7]

\begin{equation}
\mathbf{g}^P_N \mathbf{P}^M + \partial_3 \mathbf{P}^3 + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad \mathbf{g}^P_N \mathbf{M}^M + \partial_3 \mathbf{M}^3 + \mathbf{C} \mathbf{=} 2 \otimes \mathbf{P} + \rho \mathbf{m} = \mathbf{J} \partial_t^2 \mathbf{\varphi},
\end{equation}

where $\mathbf{g}^P_N$ and $\mathbf{g}^{ij}$ are the components of the unit tensor of the second rank. The definition of inner $r$-product and the problems related to it are considered in [1,2,4–9].
The representation of the heat influx equation has the following form [10]

\[ -g^P N_p q^M \frac{\partial q}{\partial t} + \rho q - T_0 \frac{d}{dt}(a \otimes \mathbf{P} + d \otimes \mathbf{\mu}) + W^* = \rho c_p \partial_T T \]  \hspace{1cm} (2)

where \( \mathbf{q} \) is the vector of exterior heat influx, \( q \) is the mass heat influx, \( T \) is the temperature, \( a, d \) are the tensors of heat extension, \( \mathbf{P} \neq \mathbf{P}^T \) is the stress tensor, \( \mathbf{\mu} \neq \mathbf{\mu}^T \) is the couple stress tensor, \( W^* \) is the scattering function, \( \rho \) is the medium density, and \( c_p \) is the heat capacity under a constant pressure.

It is seen that Eqs. (1) and (2) contain infinitely many summands. Therefore, they cannot be used in practice. Naturally, we need to consider approximate equations with finitely many summands.

2.1. Representations of constitutive relations of physical and heat content

Let us represent the constitutive relations of physical content under non-isothermal processes in the linear micropolar elasticity theory and the corresponding representation for the Fourier heat conduction law under the new parametrization of the thin body domain in the approximation of order \( s \)

\[ \mathbf{P}(s) = A^{\otimes} \otimes \mathbf{p}(s)^M \mathbf{N}_p \mathbf{u} + r^3 \partial_3 \mathbf{u} + B^{\otimes} \otimes \mathbf{p}(s)^M \mathbf{N}_p \mathbf{v} + r^3 \partial_3 \mathbf{v} - A^{\otimes} \otimes C \otimes \varphi - b \vartheta, \]

\[ \mathbf{\mu}(s) = C^{\otimes} \otimes \mathbf{p}(s)^M \mathbf{N}_p \mathbf{u} + r^3 \partial_3 \mathbf{u} + D^{\otimes} \otimes \mathbf{p}(s)^M \mathbf{N}_p \mathbf{v} + r^3 \partial_3 \mathbf{v} - C^{\otimes} \otimes C \otimes \varphi - \beta \vartheta, \]

\[ \mathbf{q}(s) = -A^{\otimes} \mathbf{p}(s)^M \mathbf{N}_p T - A^{\otimes} \vartheta^3 \partial_3 T, \quad \mathbf{A} \cdot \mathbf{m} = A^{\otimes} \mathbf{m}, \quad \mathbf{g}^P(n) = \sum_{n=0}^{\infty} A^{\otimes} \mathbf{P}^{(3^p)}(x^3)^n, \]

where \( A, B, D \) are material tensors of the fourth rank, \( \vartheta \) is the temperature overfall, \( b = A^{\otimes} a + B^{\otimes} d, \beta = C^{\otimes} a + D^{\otimes} d, \) which are called the tensors of thermomechanical properties. It should be noted that if we consider a body with center of symmetry [10–12], then \( B = 0, C = 0 \), and in this case, the constitutive relations presented above are simplified. The Fourier heat conduction law [12, 13] has the form \( \mathbf{q} = -A \cdot \nabla T \), where the second-rank positive-definite tensor \( A \) is called the heat conduction tensor.

3. Systems of equations of motion in moments for multilayer thin bodies with one small size

Let us write systems of equations of motion in moments with respect to Legendre polynomial systems for multilayer thin bodies with one small size of the approximations \( (0, N) \) and \( (1, N) \) taking into account only the boundary conditions of physical content on the frontal surfaces, which can be obtained by using the corresponding systems of equations from [2, 4, 6, 14–16]

\[
\begin{align*}
\{ \nabla_{(k)} P_{(k)}^T - g^T_{(k)} & \left[\frac{1}{2}(k + 1) \sum_{p=0}^{(k)} \left( \frac{(k)}{\alpha} \right) \right] - (2k + 1) \sum_{p=0}^{(k)} \left[ 1 - (-1)^{k+1} \right] \frac{\left( \frac{(k)}{\alpha} \right)}{\alpha} \} + (2k + 1) \left[ \frac{g^T_{(k)} P_{(k)}^T}{\alpha} + (-1)^K \frac{g^T_{(k)} P_{(k)}^T}{\alpha} \right] \} + \rho \mathbf{F} = \rho \partial_T \mathbf{u}, \\
\{ \mathbf{P} \Rightarrow \mathbf{\mu} \} + C^{\otimes} \mathbf{P} + \rho \mathbf{m} = J \cdot \partial_3 \mathbf{\mu}, \quad k = 0, 1, \quad \alpha = 0, 1, 2.
\end{align*}
\]
\[
\begin{align*}
\left\{ \nabla \mathbf{P}_{\alpha}^{(k)} & = \frac{1}{2} \left( g_{\alpha M}^{(k)} - g_{\alpha M}^{(k)} \right) \left( \frac{k}{2k-1} \nabla p_{\alpha}^{(k)} + \nabla \mathbf{P}_{\alpha}^{(k)} \right) + \frac{k + 1}{2k + 3} \nabla p_{\alpha}^{(k+1)} - \\
-(2k + 1) \sum_{p=0}^{k} \left[ 1 - (-1)^{k+p} \mathbf{P}_{\alpha}^{(p)} - \mathbf{g}_{\alpha M}^{(k)} \sum_{p=0}^{k} \mathbf{P}_{\alpha}^{(p)} \right] + \\
+(g_{\alpha M}^{(k)} - g_{\alpha M}^{(k)}) \left[ \frac{(k-1)k}{2(2k-1)} \mathbf{P}_{\alpha}^{M} + k \mathbf{P}_{\alpha}^{M} - \frac{(k+1)(k+2)}{2(2k+3)} \mathbf{P}_{\alpha}^{M} - (2k + 1) \sum_{p=0}^{k} \mathbf{P}_{\alpha}^{(p)} \right] + \\
+(2k + 1) \left[ \mathbf{g}_{\alpha M}^{(k)} (3k^2 + 3k + 1) \mathbf{P}_{\alpha}^{(k+1)} + \mathbf{g}_{\alpha M}^{(k)} (3k^2 + 3k + 1) \mathbf{P}_{\alpha}^{(k+1)} \right] + \mathbf{p} \mathbf{F} = \rho \mathbf{g}_{\alpha}^{(k)} \mathbf{u}_{\alpha}. \end{align*}
\]

\{ \mathbf{P} \Rightarrow \mathbf{u} \} + C \begin{array}{c}
\mathbf{g}_{\alpha}^{(k)} \mathbf{P}_{\alpha}^{(k+1)} + \mathbf{g}_{\alpha}^{(k)} \mathbf{P}_{\alpha}^{(k+1)} \mathbf{u}_{\alpha}^{(k+1)} = 0, \quad \alpha = 0, N, \quad \alpha = 0, N. \end{array}

Note that Eqs. (3) and (4) are deduced by using the recurrence relations for Legendre polynomials. Also, note that \( \mathbf{P}_{\alpha}^{(1)} \) and \( \mathbf{P}_{\alpha+1}^{(1)} \) (\( \alpha = 1, K-1 \)) are stress vectors (couple stresses) of interaction between the layers \( \alpha \) and \( \alpha + 1 \), which act on the surfaces \( S_{\alpha}^{(1)} \) and \( S_{\alpha+1}^{(1)} \) respectively, and \( \mathbf{P}_{\alpha}^{(1)} \) (\( \alpha \)) and \( \mathbf{P}_{K}^{(1)} \) (\( \alpha \)) are given stress vectors (couple stresses) on the frontal surfaces \( S_{\alpha}^{(1)} \) and \( S_{K}^{(1)} \), respectively. The systems of equations of heat influx of approximations \((0, N)\) and \((1, N)\), and also the constitutive relation of heat content for multilayer thin bodies are obtained in full analogy with (3)–(4). Therefore, for brevity, we do not dwell on them.

To help the reader to understand this work, we refer him to [2, 4–6, 10, 14, 17], where for the theories of one-layer thin body with one small size and two small sizes, and also for theory of multilayer constructions with the use of the Legendre and Chebyshev polynomial systems, many analogous problems are presented in detail.

### 3.1. Systems of equations in moments of the displacement vector with respect to Legendre polynomial systems for multilayer thin bodies with one small size

Let us write down the systems of equations of zero approximation in moments for the displacement vector for the classical theory, respectively

\[
\frac{\mathbf{A}}{\alpha} \cdot \nabla_{i} \nabla_{j} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} + \left( \frac{\mathbf{A}}{\alpha} \frac{3}{\alpha} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} + \frac{\mathbf{A}}{\alpha} \frac{3}{\alpha} \right) \cdot \nabla_{i} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} + \frac{\mathbf{A}}{\alpha} \frac{3}{\alpha} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} - \rho \mathbf{F}_{\alpha} = \rho \mathbf{g}_{\alpha}^{(k)} \mathbf{u}_{\alpha}, \quad k \geq 0, \quad \alpha = 0, K,
\]

where \( \frac{\mathbf{A}}{\alpha} \cdot \nabla \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} + \frac{\mathbf{A}}{\alpha} \frac{3}{\alpha} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} + \frac{\mathbf{A}}{\alpha} \frac{3}{\alpha} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} = \begin{array}{c}
\mathbf{b}^{\alpha} \cdot \nabla_{i} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} + \mathbf{b}^{\alpha} \frac{3}{\alpha} \frac{\mathbf{P}_{\alpha}^{(k)}}{\alpha} + \rho \mathbf{F}_{\alpha} = \rho \mathbf{g}_{\alpha}^{(k)} \mathbf{u}_{\alpha}, \quad k \geq 0, \quad \alpha = 0, K,
\end{array}
\]

\begin{align*}
\frac{(k)}{u} &= (2k + 1) \sum_{p=0}^{(k+2p+1)} \frac{2k + 1}{2} \left[ u \right] (u) + \frac{2k + 1}{2} \left[ 1 - (-1)^{k+p} \right] \left[ u \right] (u) + \\
(\frac{k}{u}) &= 4(2k + 1) \left( 2k + 3 \right) \left[ (k + 2) u + 2(k + 5) \left[ (k + 4) u + 3(2k + 7) \left[ (k + 6) u + \ldots \right] \right] \right) = \\
(\frac{k}{u}) &= (k + 1) \left[ (k + 1) u + (k + 1) \left( u \right) + (-1) \left[ (k + 1) u \right] \right] + (\frac{k}{u}) + \\
(\frac{k}{u}) &= 4(2k + 1) \left( (k + 2) u + 2(k + 3) \left[ (k + 4) u + 3(2k + 5) \left[ (k + 6) u + \ldots \right] \right] \right).
\end{align*}

Here \( (\frac{\partial u}{\partial x}) = (\frac{\partial u}{\partial x}) \left|_{x=0}, (\frac{\partial u}{\partial x}) = (\frac{\partial u}{\partial x}) \left|_{x=1}, u = u \right|_{x=0} \text{ and } u = u \right|_{x=1}.\)
Taking into account the expressions for \( \frac{(k_1)}{\alpha} \) and \( \frac{(k_2)}{\alpha} \), which are obtained from (6) (see also (9)) when \( \textbf{u} \) is replaced by \( \textbf{u} \) from (5) we obtain various representations of the equations of different approximations in moments of the displacement vector with respect to Legendre polynomials.

It should be noted that to close systems of equations (3) and (4), we need to add to them the system of equations of heat influx, constitutive relations, boundary and initial conditions of physical and heat contents in moments of the corresponding approximations, and also the inter-layer contact conditions depending on the connections of neighboring surfaces.

Similarly to the above, for the closure of the equations (5), we need to add to them the system of equations of heat influx and boundary and initial conditions of physical and heat contents in moments of the corresponding approximations, and also the inter-layer contact conditions depending on the connections of neighboring surfaces.

3.2. Quasi-static problems of the micropolar theory of multilayer prismatic bodies in displacements and rotations and in moments of displacement and rotation vectors

Using the rule, set out in [2,4,5,14], to obtain the desired relation of the multilayer thin body from the corresponding relation of monolayer thin body under the new parametrization, the system of equations of the micropolar theory of multilayer prismatic bodies with constant thickness (each layer has a constant thickness) in displacements and rotations can be written as \[18, 19\]

\[
\begin{align*}
\Delta^3_s + A\Delta_s^2 + h^{-2}(3\Delta_s + 2A)\Delta_s^2 - h^{-2}(3\Delta_s + A)\Delta_s^2 + h^{-6}\partial^2_s \textbf{u}_s + S^{**} = 0, \\
\Delta^3_s + (B\Delta_s + A)\Delta_s + h^{-2}(3\Delta_s + 2B)\Delta_s + C\partial^2_s + h^{-4}(3\Delta_s + B)\partial^2_s + h^{-6}\partial^3_s \varphi_s + \textbf{H}^{**} = 0, \quad (7)
\end{align*}
\]

where \( h = \text{const} \) is the \( s \)th layer thickness, \( \Delta_s = g_{\lambda}^s \partial_\lambda \partial_\lambda \). Here we have introduced the notations

\[
\begin{align*}
S^{**} &= \left(\lambda + 2\mu)(\mu + \alpha)(\delta + \beta)\right)_s, \\
H^{**} &= \left(\gamma + 2\delta)(\mu + \alpha)(\delta + \beta)\right)_s, \\
A_s &= \frac{4\alpha \mu}{(\mu + \alpha)(\delta + \beta)} , \\
B_s &= -\frac{4\mu(\gamma + 2\delta) + (\mu + \alpha)(\delta + \beta)}{\gamma + 2\delta) + (\mu + \alpha)(\delta + \beta)} , \\
C_s &= \frac{16\alpha^2 \mu}{(\gamma + 2\delta)(\mu + \alpha)(\delta + \beta)} .
\end{align*}
\]

Applying to the equations (7) the 4th moment operator of any system of orthogonal Legendre polynomials we obtain for the micropolar theory of prismatic bodies of constant thickness the following equations in moments of displacement and rotation vectors:

\[
\begin{align*}
\Delta^3_s + A\Delta_s^2 + h^{-2}(3\Delta_s + 2A)\Delta_s^2 - h^{-2}(3\Delta_s + A)\Delta_s^2 + h^{-6}\partial^2_s \textbf{u}_s + S^{**} = 0, \\
\Delta^3_s + (B\Delta_s + A)\Delta_s + h^{-2}(3\Delta_s + 2B)\Delta_s + C\partial^2_s + h^{-4}(3\Delta_s + B)\partial^2_s + h^{-6}\partial^3_s \varphi_s + \textbf{H}^{**} = 0, \quad (8)
\end{align*}
\]

\( k \geq 0, \ s = 1, K \).

Note that when using the system of Legendre polynomials, expressions for \( (k)u^u \), \( (k)u^{IV} \), \( (k)V^I \), \( \varphi^u \), \( \varphi^{IV} \) and \( \varphi^{VI} \) in (8) are found using [2, 4]

\[
\begin{align*}
\textbf{u}^{(2m)}(x_1, x_2) &= (2n + 1) \sum_{k=1}^{\infty} C_k^{2m-1} 2^{m-1} \sum_{s=1}^{2m-1} (2n + 2k + 2s - 1)^{n+k} \frac{1}{n+k+1} (x_1^2 - x_2^2) \textbf{u}^{(2m)} = \\
&= \frac{2n + 1}{2} \sum_{k=1}^{2m} (-1)^{k+1} \left( \partial_3^{2m-k} \textbf{u} + (-1)^{n+k} \partial_3^{2m-k} \textbf{u} \right) \frac{1}{n+k+1} (1 + \textbf{u}^{(2m)}), \quad (9)
\end{align*}
\]
\[ u^{(k)2m} = (2n + 1) \left( \sum_{k=1}^{n/2-m+1} C_{k+2m-2}^{2m-1} \prod_{s=1}^{2m-1} (2n-2k-2s+3)^{(n-2k-2m+2)} \right) \], \quad n \geq 0, \quad m > 0.

In the relations (8) \( s \) is an index of layers, \( K \) is a number of layers. It should be noted that, as in the case of a single layer prismatic body, so in the case of multilayer prismatic body for each of the equations, obtained after splitting (8), using the method of Vekua [20], we can write an analytic solution. Consequently, for correct statement of problems the boundary conditions in moments and the interlayer contact conditions must be added to the equations (8) (see in [2, 4, 5, 14, 18]). To satisfy the boundary conditions on the frontal surfaces, as well as to describe the interlayer contact conditions, under the simplified method of reducing the three-dimensional problem to the two-dimensional, it is necessary to construct the correcting terms [2, 14]. At the same time the analytical solution of each layer (except the first and last) with the corrective terms can be written so that it satisfies the interlayer contact conditions. For the first (last) layer the analytical solution by means of corrective terms can be represented in such a way that it satisfies the boundary conditions on the inner (outer) surface and the interlayer contact conditions on the outer (inner) surface. Therefore, we suppose that the interlayer contact conditions would be taken into account better if the order of approximation is higher. This is very important in the theory of multilayer structures. We note that the questions considered above are described in some detail in [18] (see also [2, 4–6, 14, 18]).

4. Weld conditions (complete ideal contact conditions)

We assume that a multilayer thin construction consists of \( K \) layers. Denote by \( ^{(+)S} \) and \( ^{(-)S} \) \( (\alpha = 1, K) \) the exterior and inner surfaces of the layer \( \alpha \) \( (\alpha = 1, K) \), respectively and consider several cases of mutual relation of neighboring surfaces \( ^{(+)S} \) \( ^{(-)S} \) \( \alpha + 1 \) \( (\alpha = 1, K-1) \), which are important in practice.

In case of the weld conditions the forces and moments of interaction between the layers \( \alpha \) and \( \alpha + 1 \) \( (\alpha = 1, K-1) \) are unknown. These forces and moments certainly are equal and have opposite directions. Therefore, there additionally arise six unknown functions. However, in the case considered, we have six additional conditions, which express the continuity of displacement vectors and the rotation of welded surface points. In other words, displacement vectors and rotation vectors of contacted surfaces are equal. Denoting the forces and moments of interaction of the contacted surfaces \( ^{(+)S} \) \( ^{(-)S} \) \( \alpha + 1 \) \( (\alpha = 1, K-1) \) by \( ^{(+)P} \), \( ^{(-)P} \), \( ^{(+)u} \), \( ^{(-)u} \), \( ^{(+)\varphi} \), \( ^{(-)\varphi} \) \( (\alpha = 1, K-1) \), respectively, and the displacement and rotation vectors of points of these surfaces by \( ^{(+)u} \), \( ^{(-)u} \), \( ^{(+)\varphi} \), \( ^{(-)\varphi} \) \( (\alpha = 1, K-1) \), we can represent the complete contact conditions in micropolar theory of multilayer thin bodies in the form

\[ ^{(+)P} = ^{(-)P}, \quad ^{(+)u} = ^{(-)u}, \quad ^{(+)\varphi} = ^{(-)\varphi}, \quad \alpha = 1, K-1. \]  (10)

Neglecting the characteristics of micropolar theory (the second and fourth relations) in (10), we obtain the ideal contact conditions for the classical theory (the first and third relations).

Note that of great interest are works on thermomechanics of composite structures under high temperatures [21–25].
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