A predictive mirror twin Higgs with small $Z_2$ breaking

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ABSTRACT: The twin Higgs mechanism is a solution to the little hierarchy problem in which the top partner is neutral under the Standard Model (SM) gauge group. The simplest mirror twin Higgs (MTH) model — where a $Z_2$ symmetry copies each SM particle — has too many relativistic degrees of freedom to be consistent with cosmological observations. We demonstrate that MTH models can have an observationally viable cosmology if the twin mass spectrum leads to twin neutrino decoupling before the SM and twin QCD phase transitions. Our solution requires the twin photon to have a mass of $\sim 20$ MeV and kinetically mix with the SM photon to mediate entropy transfer from the twin sector to the SM. This twin photon can be robustly discovered or excluded by future experiments. Additionally, the residual twin degrees of freedom present in the early Universe in this scenario would be detectable by future observations of the cosmic microwave background.

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1 Introduction

The disparity between the electroweak scale and the Planck scale is one of the most outstanding problems in particle physics (see, e.g., [1]). Explanations have been provided by both supersymmetry and the compositeness of the Higgs, where the electroweak scale originates from a supersymmetry breaking scale [2–5] or a composite scale [6, 7]. Without fine-tuning parameters, these classes of solutions generically predict the existence of a partner to the top quark that is colored and as light as the electroweak scale. Such a particle has not been observed at the Large Hadron Collider (LHC), providing a strong lower bound on its mass, typically around 1 TeV [8–11]. In order to accommodate this bound, these kinds of theories require fine-tuning of their parameters to fix the electroweak scale. The need for this fine-tuning is called the little hierarchy problem.

The twin Higgs mechanism [12] addresses this problem. The mechanism is based on a $Z_2$ symmetry that introduces a copy of the Standard Model (SM) particles which we call twin particles, and an approximate global symmetry of the scalar potential. After the twin Higgs obtains a vacuum expectation value (VEV), $f$, the SM Higgs becomes a pseudo-Nambu-Goldstone boson, protecting the Higgs mass from quantum corrections up to the scale $\Lambda_{TH} \approx 4\pi f$. The top quark partner is now twin colored and not easily produced at the LHC, thereby solving the little hierarchy problem. The twin Higgs mechanism is readily incorporated into solutions of the full hierarchy problem; for instance, supersymmetric [13–19] and composite [20–27] realizations of the idea have been explored.
While the twin Higgs mechanism is theoretically appealing, it is difficult to reconcile with cosmological observations. The simplest realization of this scenario is the mirror twin Higgs (MTH) model where the $Z_2$ symmetry is a fundamental symmetry (as opposed to an emergent symmetry). Twin particles thermalize with SM particles via Higgs exchange in the early Universe. The fundamental $Z_2$ symmetry predicts that the entropy of light twin particles is eventually transferred into twin photons and twin neutrinos, which behave as extra radiation components. During epochs when the Universe is radiation dominated, these extra radiation components contribute appreciably to the expansion of the Universe. The expansion rate depends on the energy density in relativistic species, which is typically parameterized in relation to the photon energy density as

$$\rho_r = \left(1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right) \rho_\gamma,$$

(1.1)

where $N_{\text{eff}}$ is the effective number of (light) neutrino species, the factor of $7/8$ comes from Fermi-Dirac statistics, and the factor of $(4/11)^{4/3}$ comes from the fact that electron-positron pairs annihilate after SM neutrino decoupling and heat the photons. SM neutrinos are still partially in thermal equilibrium with the rest of the thermal bath when the electron-positron pairs start to annihilate, which yields a predicted SM value of $N_{\text{eff}} \approx 3.046$ [28, 29]. Meanwhile, in the MTH model, the additional number of relativistic species (twin photons and twin neutrinos) modify the SM prediction by an amount $\Delta N_{\text{eff}} \sim 5.6$ [30]. Big bang nucleosynthesis (BBN) and anisotropies in the cosmic microwave background (CMB) are both exquisitely sensitive to the expansion history during epochs when the energy density in radiation was non-negligible, and provide independent measurements of $N_{\text{eff}}$. The BBN measurement of $N_{\text{eff}}$ (including the observed helium and deuterium abundances) is $2.85 \pm 0.28$ [31]. Meanwhile, the Planck 2018 measurement of $N_{\text{eff}}$ (from $TT$, $TE$, and $EE$ power spectra combined with lensing and baryon acoustic oscillations) is $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ at 95% confidence [32]. Both of these measurements indicate that the MTH scenario is excluded at high significance.

A number of ways to reduce the twin contribution to $N_{\text{eff}}$ have been explored. For instance, the fraternal twin Higgs (FTH) mechanism [33] lacks the first and second generations of twin fermions and also lacks a twin photon. The single twin neutrino yields $\Delta N_{\text{eff}} \approx 0.075$ which is still consistent with observations [34]. However, given the lack of $Z_2$ symmetry, the proximity of the top Yukawa, SU(2) gauge, and SU(3) gauge couplings in the SM and twin sectors should be addressed. One could also make the twin neutrinos heavy [35, 36] and even the twin photon heavy without affecting naturalness [37, 38], while expanding the possibilities for twin dark matter candidates [39, 40]. Asymmetric entropy production after the twin and SM sectors decouple is another way to diminish $N_{\text{eff}}$ [30, 36, 41–43]. Its effects on the matter power spectrum could also be seen by future large scale structure observations [44]. Refs. [35, 45] investigate the Minimal MTH where the twin Yukawa couplings are raised, which reduces $\Delta N_{\text{eff}}$ because there are few twin degrees of freedom when the SM and twin sectors decouple from each other. The (nearly) massless twin photons and neutrinos still contribute appreciably to $N_{\text{eff}}$, which is at least 3.3 and in slight tension with the Planck measurement.
In this paper, we consider a MTH model with a fundamental $\mathbb{Z}_2$ symmetry at high energies which is preserved as much as possible at the electroweak scale. As is shown in ref. [35], it is mandatory to increase the twin Yukawa couplings (except for the twin top) to suppress $\Delta N_{\text{eff}}$. Note that the contribution of twin Yukawa couplings $\gtrsim 0.1$ to the Higgs mass squared does not reintroduce fine-tuning below $\Lambda_{\text{TH}}$. To build on the models explored in refs. [35, 45], we give the twin photon a Stueckelberg mass. This allows entropy from the twin QCD phase transition to transfer into the SM via decaying twin photons, thereby minimizing $\Delta N_{\text{eff}}$ while achieving a minimal $\mathbb{Z}_2$ breaking.

We keep the $\mathbb{Z}_2$ breaking as minimal as possible and do not consider $\mathbb{Z}_2$ breaking gauge couplings. This not only motivated by minimality, but also by the theory of flavor. The hierarchy of the Yukawa couplings in the SM is one of its great mysteries which can be explained by introducing some fields whose values control the Yukawa couplings, like in the Froggatt-Nielsen mechanism [46]. In such a mechanism, it is possible that the field controlling the SM Yukawa couplings spontaneously takes different values from its twin counterpart which sets the twin Yukawa couplings [45]. This scheme naturally maintains $y_t \sim y'_t$ necessary for the twin Higgs mechanism. We could also introduce moduli fields whose values control the gauge couplings perhaps motivated by string theory, but these would differ from the Yukawa-setting fields in the Froggatt-Nielsen mechanism in that these moduli would not be motivated by low-energy, known SM problems.

We assume that the twin neutrinos are effectively massless, as is the case in the SM, motivated by the following observation. Let us assume that the neutrino mass originates from a see-saw mechanism [47–50]. We may raise the twin neutrino masses by smaller twin right-handed neutrino masses. Raising the twin neutrino masses to a level where they do not contribute to $N_{\text{eff}}$ requires significant $\mathbb{Z}_2$ breaking right-handed neutrino masses, in contrast to the situation described above, and care must be taken to avoid spoiling the twin Higgs mechanism. We may instead raise the twin neutrino masses by a larger yukawa coupling of the twin right-handed neutrino to the twin left-handed neutrino and the twin Higgs. Let us consider a well-motivated benchmark point of thermal leptogenesis [51], which requires the SM yukawa coupling $y_N$ to satisfy $y_N^2 > 10^{-5}$ [52, 53]. Then even if the twin $y_N = O(1)$ and $f/v = 10$, the mirror neutrino mass is at the most $0.1 \text{eV} \cdot 10^5 \cdot (10)^2 = \text{MeV}$, which is not large enough to evade cosmological bounds. For $f/v \gg 10$, even this thermal leptogenesis benchmark has twin neutrinos with masses much greater than an MeV. Thus, the twin neutrinos may no longer contribute significantly to $\Delta N_{\text{eff}}$. However, the non-zero mirror photon mass we consider is still useful as it allows the energy density of twin photons to efficiently transfer to the SM before the SM neutrinos decouple, thus preventing a problematic contribution of the twin photon itself to $\Delta N_{\text{eff}}$.

We consider a concrete example where all the charged twin fermion masses are several tens of GeV and where the twin photon has a Stueckelberg mass around 20 MeV. The high mass scale of the twin fermions leads to twin neutrino decoupling before the SM and twin QCD phase transitions. The twin and visible sectors are in thermal contact via kinetic mixing between the twin and SM photons so that entropy can transfer from the twin sector to the SM. The twin and SM QCD phase transitions and SM annihilations heat the SM neutrinos relative to the decoupled twin neutrinos, diluting the twin neutrino contribution.
to $N_{\text{eff}}$. The cosmic timeline of this scenario is illustrated schematically in figure 1. As we will see, with an 18 MeV twin photon, $\Delta N_{\text{eff}}$ may be as small as $\sim 0.10$.

The rest of this paper is organized as follows. In section 2, we review the twin Higgs mechanism and determine the necessary MTH mass spectrum for our proposed scenario. We then calculate the twin contributions to $N_{\text{eff}}$ in section 3 and the effects on the Helium mass fraction in section 4. Figure 3 is the culmination of these calculations which predicts $m_{\gamma'}$, $N_{\text{eff}}$, and the Helium mass fraction for our MTH model. We discuss implications of upcoming experiments and observations and conclude in section 5.

2 Twin sector

2.1 Twin Higgs mechanism with $Z_2$ breaking

The MTH model consists of a twin sector that is related to the SM by a $Z_2$ symmetry at a scale above the SM electroweak scale. In particular, the twin sector has a copy of the SM gauge group, $U(1)^' \times SU(2)^' \times SU(3)^'$, with respective couplings ($g_1^', g_2^', g_3^'$) and a doublet $H^'$ under this SU(2)$'$ which is the twin Higgs. Throughout this paper, superscripts $'$ on SM particles or quantities indicate their twin sector counterparts. An accidental, approximate SU(4) global symmetry in the full Higgs sector is spontaneously broken when the twin Higgs doublet acquires a VEV $f$. The SM Higgs is identified as one of the pseudo-Nambu-Goldstone bosons from the SU(4) breaking whose mass is protected from quadratic divergences by the $Z_2$ symmetry up to the cutoff $\Lambda_{TH} \approx 4\pi f$. The SM Higgs doublet acquires its measured VEV $v$.

The required fine-tuning (F.T.) of the parameters to obtain the SM electroweak scale $v$ from the twin one $f$ is

$$\text{F.T.} = 2 \frac{v^2}{f^2}. \quad (2.1)$$
The Higgs observed at the LHC has properties that are consistent with the SM Higgs, which places a limit on the ratio of the Higgs VEVs $f \gtrsim 3v$ [54]. Thus, tuning in Twin Higgs models is always greater than 20%. In this work, we require that our MTH model does not result in tuning greater than 1% and therefore, that $f/v \lesssim 14$. Requiring our MTH model to be consistent with the latest Planck results yields a lower bound of $f/v \gtrsim 10$, as we find below (see figure 3). In a supersymmetric UV completion of the twin Higgs model with an SU(4) symmetric potential from an $F$ term, fine-tuning of a few percent is already required [15], and $f/v \gtrsim 10$ does not introduce additional fine-tuning. The same is true for a $D$ term model with a high mediation scale of the supersymmetry breaking [18].

The Yukawa couplings of the twin and SM sectors may be written succinctly as

\[ \mathcal{L}_{\text{Yuk}} \supset \sum_f -y_f f_R H f_L - y_{f'} f'_R H f'_L. \]  

(2.2)

As mentioned, we assume a hard breaking of the $Z_2$ in these Yukawas so that $y_f \neq y_{f'}$ (except for the top Yukawas). In fact, the models we consider have $y_{f'} > y_f$ for all fermions besides the top quarks. We assume that the twin neutrinos are still light and can be treated as dark radiation. We also assume that the gauge coupling constants preserve the $Z_2$ symmetry up to the quantum correction from $Z_2$-breaking fermion masses, which raises the twin QCD scale. We introduce a Stueckelberg mass for the twin photon.

### 2.2 Twin photon

A crucial requirement for entropy dilution is that the twin photon is able to mediate the transfer of entropy from the twin sector to the SM via the kinetic mixing,

\[ \mathcal{L}_{\gamma'\gamma} = \frac{\epsilon}{2} F_{\mu\nu}' F^\mu\nu, \]  

(2.3)

where $\epsilon$ is the mixing strength between the SM photon and the twin photon, which have field strengths of $F_{\mu\nu}$ and $F_{\mu\nu}'$ respectively. Efficient transfer of entropy is guaranteed as long as the twin photons are thermalized with the SM bath. The twin photons must be massive enough for their decays to proceed in the forward direction at MeV-scale temperatures in order to deplete their number density before BBN. This requirement is satisfied if the twin photon is heavier than a few MeV.

In the $1-10$ MeV twin photon mass range, terrestrial and supernova constraints [55–57] require $\epsilon \lesssim 10^{-11}$, which is too small to thermalize the twin and SM sectors. As shown in figure 2, larger kinetic mixing is allowed for slightly larger twin photon masses, with constraints from beam dump searches [58] and $\alpha + g_\mu$ measurements [59] restricting some of the parameter space. We thus consider twin photon masses above 18 MeV with values of the kinetic mixing in the range that is allowed by these constraints. The remaining parameter space in $\epsilon$ can be explored with LDMX [60], Mu3e [61], SeaQuest [62], HPS [63], SHiP [64], FASER [65], and NA62 [66], so this model has considerable discovery potential.

Without introducing tuning greater than 1%, our MTH model is only consistent with cosmological observations for $m_{\gamma'} \lesssim 27$ MeV, hence the range of $m_{\gamma'}$ plotted (see figure 3).

The allowed values of kinetic mixing shown in figure 2 are more than adequate to thermalize the twin and SM sectors. At high temperatures relative to the twin photon and
Figure 2. Existing and projected constraints on dark photon parameter space. Our MTH model is viable for all values of $\epsilon$ currently unconstrained in the mass range $18$ MeV $\leq m_{\gamma'} \leq 27$ MeV. The beam dump constraints are from the compilation [58], while the $\alpha + g_\epsilon$ constraint is from [59]. The lines are projected constraints from LDMX [60], Mu3e [61], SeaQuest [62], HPS [63], SHiP [64], FASER [65], and NA62 [66]. Dashed lines would rule out the space below the line and solid lines would constrain the space above.

SM fermion masses, thermalization occurs primarily through $2 \rightarrow 2$ scatters. For example, the rate for $\gamma' e \rightarrow \gamma e$ for temperatures much larger than $m_{\gamma'}$ is roughly

$$\Gamma_{\gamma' e \rightarrow \gamma e} \approx \frac{3\zeta(3)}{8\pi^3} \epsilon^2 \alpha^2 T,$$

(2.4)

where $\alpha$ is the usual SM fine structure constant. Throughout this paper, $T$ refers to the temperature of the SM photon bath and all SM constants are taken from [67]. This rate is greater than the Hubble rate for all $T$ in the range $m_{\gamma'} \ll T \lesssim 400$ GeV for the smallest $\epsilon^2 \sim 10^{-9}$ we can consider. For $T \lesssim m_{\gamma'}$, twin photon decays into electron-positron pairs become more efficient. The rest-frame rate for a kinetically mixed twin photon to decay to SM electron-positron pairs is

$$\Gamma_{\gamma' \rightarrow e^+ e^-} \approx \frac{\epsilon^2 \alpha(2m_e^2 + m_{\gamma'}^2)}{3m_{\gamma'}}.$$

(2.5)

However, this rate gets suppressed by a factor of $\sim m_{\gamma'}/T$ to account for time dilation at temperatures comparable to or larger than $m_{\gamma'}$. Comparing this rate to the Hubble rate, we find that decays become efficient at mediating entropy transfer below $T \sim 8$ GeV for the smallest $m_{\gamma'} = 18$ MeV and $\epsilon^2 \sim 10^{-9}$ we can consider. We conclude that the twin photon can transfer entropy efficiently to the SM for $T \lesssim 400$ GeV for the available parameter space shown in figure 2.

There is an additional, nontrivial requirement on the available twin photon parameter space in figure 2: since the twin $Z$ mass eigenstate contains some of the twin photon gauge eigenstate, as discussed in appendix A, the twin $Z$ and SM photon mix. This allows SM fermions to thermally couple to twin neutrinos through elastic scattering and annihilation.
For temperatures much larger than the participating SM fermion masses, the cross sections for annihilations $f \bar{f} \to \nu'\nu'$ and elastic scatters $\nu'f \to \nu'f$ are comparable and roughly

$$
\sigma_{\nu'f} \approx \frac{16\pi}{3} \frac{\epsilon^2 Q_f^2 \alpha^2}{\cos^4 \theta_W} \frac{T^2}{m_Z^2},
$$

The total rate for both annihilations and elastic scatters from all SM charged fermions but the top is

$$
\Gamma_{\nu'f} \approx \frac{640\zeta(3)}{3\pi} \frac{\epsilon^2 \alpha^2}{\cos^4 \theta_W} \frac{T^5}{m_Z^2}.
$$

Ideally, the earliest the twin neutrinos can decouple from the bath is before the SM bottom-antibottom pairs annihilate in our scenario. This rate is smaller than the Hubble rate at $T = m_b$ if

$$
\epsilon \lesssim 10^{-3} \left( \frac{f/v}{10} \right)^2,
$$

which is satisfied by the entire parameter space in figure 2 for our models in which $f/v \gtrsim 10$. Therefore, the effective $Z' - \gamma$ mixing does not re-thermalize the twin neutrinos via scattering with SM fermions.

### 2.3 Charged twin fermions

In our setup, the twin neutrinos should decouple from the bath as early as possible. Subsequent QCD phase transitions and SM particle annihilations then raise the temperature of the SM neutrinos relative to the twin neutrinos as much as possible, thus minimizing the twin neutrino contribution to $N_{\text{eff}}$. We consider both the best and next-best scenarios in which the twin neutrinos decouple before the SM bottom-antibottom and SM tau-antitau pairs annihilate, respectively. As we show in section 4, the best scenario has the largest parameter space consistent with cosmological observations and naturalness (see figure 3), whereas the next-best scenario requires less $Z_2$-breaking.

For both scenarios, the temperature of twin neutrino decoupling determines the appropriate twin fermion mass spectrum, since elastic scattering off twin fermions is the process that keeps the twin neutrinos in equilibrium at the lowest temperatures. Scattering processes are more important than fermion-antifermion pair annihilations at temperatures below the twin fermion mass since annihilations are suppressed by a relative factor of $e^{-m_f/T}$. The elastic scattering rate is

$$
\Gamma_{f'\nu'} \approx \frac{4(3 + 3 \cdot 5)}{\pi} G_F^2 T^2 v^4 \left( \frac{m_{f'}T}{f^4} \right)^3 e^{-m_{f'}/T}.
$$

Requiring the elastic scattering rate in eq. (2.9) to be less than the Hubble rate at $T = m_b$ imposes $m_{f'} \gtrsim 76$ GeV ($m_{f'} \gtrsim 70$ GeV) for $f/v = 10$ ($f/v = 14$). Thus, for this best scenario where decoupling occurs before $T = m_b$, we set $m_{f'} = 80$ GeV for all charged twin fermions besides the twin top. This $Z_2$ symmetry breaking is small enough to not ruin the MTH mechanism.
In the next-best scenario, the twin neutrinos decouple before the SM tau-antitau pairs annihilate. Requiring the elastic scattering rate in eq. (2.9) to be less than the Hubble rate at $T = m_\tau$ imposes $m_{f'} \gtrsim 27$ GeV ($m_{f'} \gtrsim 24$ GeV) for $f/v = 10$ ($f/v = 14$). Thus, for this next-best scenario, we set $m_{f'} = 30$ GeV for all charged twin fermions besides the twin top. The primary motivation for this next-best scenario is that the $Z_2$ symmetry breaking is even smaller than in the best scenario. Since $m_{f'} = 30$ GeV, we must additionally consider the Higgs decaying invisibly to twin fermions. The LHC does not probe our predicted Higgs invisible decay rate or reduced Higgs signal strength since we only consider $f/v \lesssim 10$. However, our predictions for both of these observables fall within the projected capabilities of future colliders such as the ILC [68], giving another future test of this more $Z_2$-symmetric benchmark. See appendix B for more details.

2.4 Twin gluons

After the charged twin fermions leave the bath, it is still possible for the twin neutrinos to be coupled to the twin gluons. Using the method in ref. [69], we find that the lowest dimension operator which conserves lepton number and allows twin gluon-neutrino scattering is

$$L \supset \frac{1}{(4\pi)^4} \frac{1}{m_{q'}^2} \frac{1}{m_{Z'}^2} G_{\mu\nu}^a D_\rho G^{a\rho\nu} \gamma^\rho \gamma^\mu',$$

(2.10)

where $G_{\mu\nu}^a$ is the field strength for the twin gluons. Thus, the elastic scattering rate is

$$\Gamma_{\nu'g' \rightarrow \nu'g'} \approx \frac{1}{(4\pi)^3} \frac{1}{m_{q'}^4} \frac{1}{m_{Z'}^4} T^9.$$  

(2.11)

For the best scenario, requiring that the rate in eq. (2.11) is less than the Hubble rate at $T = m_b$ yields

$$m_{q'} \gtrsim 2 \left( \frac{f/v}{10} \right) \text{GeV},$$

(2.12)

which is easily satisfied because $m_{f'} = 80$ GeV. For the next-best scenario, requiring the rate in eq. (2.11) is less than the Hubble rate at $T = m_\tau$ yields an even more trivially satisfied condition for our benchmark $m_{f'} = 30$ GeV. Thus, $\nu' - g'$ scattering does not re-thermalize the twin neutrinos.

In order for the entropy in the twin gluons to be transferred to the SM via twin photons, we require that the twin photons and gluons stay in equilibrium after the twin quarks leave the thermal bath and as the twin QCD phase transition is proceeding. Integrating out the heavy twin quarks, the twin gluons and photons are coupled at lowest order by the dimension-8 operators [70]

$$L_{F'F'G'G'} = \frac{\alpha' \alpha'_S}{180} \left( \sum_i Q_i^2 \right) \left[ 28 F_{\mu\nu} F_{\nu\lambda} G^{a\lambda} \sigma_{\mu\nu} G^{a\mu} G^{a\nu} + 14 F_{\mu\nu} F_{\lambda\sigma} G^{a\lambda} \sigma_{\mu\nu} G^{a\mu} \right]$$

$$- 10 \left( F^{a\mu} G^{a_{\mu\nu}} \right) \left( F^{a}_{\alpha\beta} G^{a}_{\alpha\beta} \right) - 5 \left( F^{a}_{\mu\nu} F^{a}_{\mu\nu} \right) \left( G^{a}_{\alpha\beta} G^{a}_{\alpha\beta} \right),$$

(2.13)
where $\alpha', \alpha'_S$ are the twin U(1)$_{\text{EM}}'$ and SU(3)$_{\text{c}}'$ fine structure constants. We sum over the twin quark charges-squared (aside from the top, which contributes negligibly). We require that the $2 \to 2$ scattering rate provided by this coupling is faster than the Hubble rate at the twin QCD phase transition

$$H|_{\Lambda'_{\text{QCD}}} \lesssim 0.01 \left( \frac{\alpha' \alpha'_S}{m_{q'}^2} \right)^2 \Lambda'_{\text{QCD}}^6 \implies m_{q'} \lesssim 100 \left( \frac{\Lambda'_{\text{QCD}}}{2 \text{ GeV}} \right)^{7/8} \text{ GeV.} \quad (2.14)$$

Here we take $\alpha'_S (\Lambda'_{\text{QCD}}) = 4\pi$, but the upper bound on $m_{q'}$ weakly depends on the value. For $f/v = 10$ and $m_{q'} = 30$–80 GeV, we find $\Lambda'_{\text{QCD}} = 1.8$–2.5 GeV. This condition is satisfied by $m_{q'} = 80$ GeV ($m_{q'} = 30$ GeV) in the best (next-best) scenario. Hence, the twin photons and twin gluons are in equilibrium throughout the twin QCD phase transition and the entropy is transferred to the twin photons and therefore the SM bath efficiently. We obtain the same conclusion by computing the decay rate of twin glueballs into a pair of twin photons.

3 Twin contributions to $\Delta N_{\text{eff}}$

We have established that the twin neutrinos decouple before the SM bottom-antibottom pairs leave the bath in the best-case scenario. The particles in the twin and SM thermal bath after twin neutrino decoupling are:

- SM and twin gluons and photons
- all SM quarks, except the top
- all SM leptons.

In the next-best scenario, the twin neutrinos decouple before the SM tau-antitau pairs annihilate and the particles in the bath after twin neutrino decoupling are the same except for the absent SM bottoms. We also established that thermalization of the SM and twin baths is guaranteed by twin photons for all temperatures $T \lesssim 400$ GeV.

Entropy in the bath is given by

$$s = \frac{2\pi^2}{45} g_{*s} T^3, \quad (3.1)$$

where $g_{*s}(T)$ tracks the effective number of relativistic degrees of freedom. After particles in the twin and SM sectors annihilate or decay, their entropy cascades down to lighter species that are still coupled. Conservation of entropy then requires that the relative temperature between the twin neutrinos and the thermal bath is

$$\left( \frac{T_{\nu}}{T_b} \right)^3 = \frac{g_{*s}}{g_{*s,0}}, \quad (3.2)$$

where $g_{*s,0}$ is the effective number of degrees of freedom still in the thermal bath just after twin neutrino decoupling and $g_{*s}$ is the effective number of degrees of freedom at some
later time. At the time of SM neutrino decoupling, \( T_{\nu}^{\text{dec}} \approx 2.7 \text{ MeV} \) \([71]\), \( g_{\ast s} = 43/4 \). Meanwhile, given the degrees of freedom listed above, \( g_{\ast s,0} = 421/4 \) in the best scenario and \( g_{\ast s,0} = 379/4 \) in the next-best. The smallest possible contribution to \( \Delta N_{\text{eff}} \) from twin neutrinos occurs when they do not receive any entropy injections after decoupling from the twin bath. Assuming this happens and using the definition in eq. (1.1), the contribution of twin neutrinos to \( N_{\text{eff}} \) is

\[
\Delta N_{\text{eff}}^{\nu^\prime, \text{min.}} = 3 \left( \frac{43/4}{421/4} \right)^{4/3} \approx 0.14
\]

for the best scenario and 0.16 for the next-best. Thus, both scenarios seem allowed by the latest Planck results \([32]\) which give \( N_{\text{eff}} = N_{\text{eff}} - 3.046 < 0.284 \) at 95\% confidence.

However, twin photons decay into twin neutrinos since the twin photon mass eigenstate has a small amount of the twin \( Z \) gauge eigenstate (see appendix A). Thus, the \( \Delta N_{\text{eff}}^{\nu^\prime, \text{min.}} \) in eq. (3.3) is never attainable in practice. To account for this reheating of the decoupled twin neutrinos, we must solve their energy density Boltzmann equation

\[
\partial_t \rho_{\nu^\prime} + 4H \rho_{\nu^\prime} = m_{\gamma^\prime} \Gamma_{\gamma^\prime \rightarrow \nu^\prime \nu^\prime} n_{\gamma^\prime}^{\text{eq}}(T),
\]

where \( \rho_{\nu^\prime} \) is the total energy in all 3 twin neutrino species and \( \Gamma_{\gamma^\prime \rightarrow \nu^\prime \nu^\prime} \) is the total decay rate of \( \gamma^\prime \) into any of the twin neutrino pairs, given by eq. (A.8) in appendix A. The twin photons are in chemical and kinetic equilibrium with the SM with the number density

\[
n_{\gamma^\prime}^{\text{eq}}(T) = \frac{3m_{\gamma^\prime}^2}{2\pi^2} K_2(m_{\gamma^\prime}/T).
\]

We ignore the back reaction and neutrino Pauli blocking in eq. (3.4) since the number density of twin neutrinos is small in order for \( \Delta N_{\text{eff}} \) to be consistent with observations. By neglecting inverse decays, we overestimate \( \rho_{\nu^\prime} \) and therefore overestimate the twin contribution to \( \Delta N_{\text{eff}} \). With the change of variables \( \rho_{\nu^\prime} \equiv s^{4/3} y \), eq. (3.4) simplifies to

\[
\frac{\partial y}{\partial T} = -\frac{m_{\gamma^\prime} \Gamma_{\gamma^\prime \rightarrow \nu^\prime \nu^\prime} n_{\gamma^\prime}^{\text{eq}}(T)}{3H s^{4/3}} \left( \frac{3}{T} + \frac{\partial g_{\ast s}}{\partial T} \frac{1}{g_{\ast s}} \right).
\]

For the range of \( m_{\gamma^\prime} \) we consider around tens of MeV, we find that \( \frac{\partial g_{\ast s}}{\partial T} \frac{1}{g_{\ast s}} \ll \frac{3}{T} \). We integrate (3.6) to find

\[
y(T) - y(T_0) = \frac{c_y \Gamma_{\gamma^\prime \rightarrow \nu^\prime \nu^\prime} M_{\text{Pl}}}{m_{\gamma^\prime}^2} \int_{m_{\gamma^\prime}/T_0}^{m_{\gamma^\prime}/T} dx \frac{x^4 K_2(x)}{\sqrt{g_{\ast s}} g_{\ast s}^{4/3}} = \frac{c_y c_{\text{int}} \Gamma_{\gamma^\prime \rightarrow \nu^\prime \nu^\prime} M_{\text{Pl}}}{m_{\gamma^\prime}^2},
\]

where

\[
x \equiv \frac{m_{\gamma^\prime}}{T}, \quad c_y \equiv \frac{9\sqrt{10}}{2\pi^3 \left( \frac{2\pi^2}{45} \right)^{4/3}}, \quad \text{and} \quad c_{\text{int}} = 0.26
\]

is the value of the dimensionless integral. We evaluate the integral from \( x_1 = 1/5 \) to \( x_f = 10 \) because it effectively converges over this domain and the twin photons have all but left the bath by \( x_f \). The integral doesn’t change appreciably over our range of \( m_{\gamma^\prime} \).
We thus find the final energy density, \( \rho_{\nu'} \),
\[
\rho_{\nu'} (T_f) = s^{4/3} y |_{T_f} = T_f^4 \left( \frac{\pi^2}{30} \frac{3}{8} \frac{7}{6} \left( \frac{g_{ss}|_{T_f}}{g_{ss}|_{T_{\nu'}^{\text{dec}}}} \right)^{4/3} + \left( \frac{2\pi^2}{45} \frac{g_{ss}|_{T_f}}{m_{\gamma'}} \right)^{4/3} \frac{c_y \Gamma_{\gamma'\to\nu'\nu'}}{m_{\gamma'}^2} \frac{M_{\text{Pl}}}{c_{\text{int}}} \right). \tag{3.9}
\]

We translate this energy density into the corresponding contribution to \( \Delta N_{\text{eff}} \). At \( T_f \), the energy density in a single SM neutrino is just \( \frac{7}{3} \frac{g_{ss}|_{T_f}}{T^{4}_{f}} \). Even though the SM neutrinos may have decoupled before \( T_f = \frac{m_{\nu'}}{10} \), they are still at the same temperature as the SM bath since electron-positron pairs do not start to annihilate in the forward direction until \( T < 1 \text{ MeV} \) and the smallest \( T_f \) we consider is \( T_f = \frac{18 \text{ MeV}}{10} = 1.8 \text{ MeV} \). Taking the ratio of the final twin-neutrino energy density from (3.9) to a single SM neutrino’s, we find
\[
\Delta N_{\gamma'}_{\text{eff}} = 3 \left( \frac{g_{ss}|_{T_f}}{g_{ss}|_{T_{\nu'}^{\text{dec}}}} \right)^{4/3} + \left( \frac{4}{11 + 2g_{ss}(T_{\nu'}^{\text{dec}})} \right)^{1/3} \frac{540\sqrt{10}c_{\text{int}} \Gamma_{\gamma'\to\nu'\nu'}M_{\text{Pl}}}{7\pi^5 m_{\gamma'}^2}. \tag{3.10}
\]

This simplifies to the result in eq. (3.3), in the limit \( \Gamma_{\gamma'\to\nu'\nu'} \to 0 \).

There is still an appreciable number density of twin photons in the SM thermal bath when the SM neutrinos decouple. These twin photons subsequently decay to electron-positron pairs with which they are in equilibrium. This causes the SM neutrinos to be cooler than usual relative to the SM photons. Thus, the twin photons contribute negatively to \( \Delta N_{\text{eff}} \), denoted by \( \Delta N_{\gamma'} \). Using entropy conservation at \( T_{\nu'}^{\text{dec}} \) and \( T_f \), we find:
\[
\frac{T_{\nu}}{T} = \left( \frac{g_{ss}(T_f)}{g_{ss}(T_{\nu'}^{\text{dec}})} \right)^{1/3} = \left( \frac{4}{11 + 2g_{ss}(T_{\nu'}^{\text{dec}})} \right)^{1/3}. \tag{3.11}
\]

Comparing the energy density at this reduced temperature to the definition of \( N_{\text{eff}} \) in eq. (1.1), we find
\[
\Delta N_{\gamma'}_{\text{eff}} = 3 \cdot \left( \frac{11}{11 + 2g_{ss}(T_{\nu'}^{\text{dec}})} \right)^{4/3} - 3. \tag{3.12}
\]

Of course, SM neutrinos do not decouple instantaneously at 2.7 MeV. Some of the entropy transfer from these dark photon decays into SM electron-positron pairs will eventually move into SM neutrinos so that their temperature relative to the SM photons is not quite as small as in (3.11). This should not introduce more than a 10\% error in our \( \Delta N_{\gamma'} \) calculation. Combining the \( \Delta N_{\text{eff}} \) contribution in (3.12) with the contribution in (3.10), we arrive at our final change to \( N_{\text{eff}} \)
\[
\Delta N_{\text{eff}} = \Delta N_{\gamma'}_{\text{eff}} + \Delta N_{\gamma'}_{\text{eff}}. \tag{3.13}
\]

### 4 The helium mass fraction

For twin photon masses as light as 18 MeV to be consistent with measurements of \( N_{\text{eff}} \), the negative contribution to \( \Delta N_{\text{eff}} \) from \( \gamma' \) decay in eq. (3.12) is critical. This change in
the ratio between SM photon and neutrino temperatures occurs close to the time of BBN and thus may affect the primordial Helium mass fraction $Y_P$, which has been measured to be $Y_P = 0.2449 \pm 0.0040$ [72]. This observable is sensitive not only to the expansion rate at BBN but also to the weak interaction rates, which are themselves dependent on the electron neutrino temperature relative to the photon bath. Since the decaying twin photons alter this ratio of temperatures, we must ensure our prediction for $Y_P$ is consistent with measurement.

Our analysis relies on the numerical results from ref. [73] which uses a modification of the publicly available AlterBBN code [74, 75]. Ref. [73] calculates cosmological observables as a function of the number of degrees of freedom that are relativistic at recombination (besides SM photons) as well as the effective temperature of those degrees of freedom. They refer to these degrees of freedom as neutrinos since they include SM neutrinos. But, since they vary both the number, $N_\nu$, and temperature, $T_\nu$, of these degrees of freedom, their parameterization subsumes our situation in which we change the temperature of SM neutrinos and have extra relativistic degrees of freedom at BBN. We calculate $T_\nu$ relative to its usual temperature in the SM using eq. (3.11)

$$\frac{T_\nu}{T_{\nu,\text{SM}}} = \left( \frac{1}{1 + \frac{2}{\Pi} g^\nu \left( T_\nu^{\text{dec}} \right)} \right)^{1/3}.\tag{4.1}$$

$N_\nu$ as defined in ref. [73] is related to $N_{\text{eff}}$ by

$$N_{\text{eff}} T_{\nu,\text{SM}}^4 = N_\nu T_\nu^4,\tag{4.2}$$

since the $N_{\text{eff}}$ which appears in eq. (1.1) is inferred from measurements of the total energy density at BBN and recombination. From this relation, we find

$$N_\nu = 3.046 + \Delta N_{\text{eff}} \left( \frac{T_{\nu,\text{SM}}}{T_\nu} \right)^4,\tag{4.3}$$

where the first term is the contribution from SM neutrinos [28, 29] and the second term from twin neutrinos, as in eq. (3.10). With eqs. (4.1) and (4.3), we use the results of ref. [73] to calculate $Y_P$.

The left panel of figure 3 shows contours of $m_{\gamma'}$ and $f/v$ on the $N_{\text{eff}}$-$Y_P$ plane for the best scenario in which we set $m_{\gamma'} = 80$ GeV so that the twin neutrinos decouple before the SM bottom-antibottom pairs annihilate. Additionally, we include the $1\sigma$ and $2\sigma$ containment from Planck [32] as dark and light blue regions, respectively, resulting in a slim parameter space where both the cosmology and naturalness of these models is reasonable. For the lightest twin photon we can consider, $m_{\gamma'} = 18$ MeV, the data require that $f/v > 10$. For larger $m_{\gamma'}$, larger $f/v$ are necessary to suppress the twin photon decays to twin neutrinos. In order to have a twin photon as heavy as $m_{\gamma'} = 27$ MeV, $f/v > 14$ is required. The smallest $\Delta N_{\text{eff}}$ we can achieve is 0.10 and corresponds to $m_{\gamma'} = 18$ MeV and $f/v = 14$.

The right panel of figure 3 is equivalent for the next-best scenario in which we set $m_{\gamma'} = 30$ GeV so that the twin neutrinos decouple before the SM tau-antitau pairs annihilate.
Figure 3. Contours of constant $m_{\gamma'}$ (solid) and $f/v$ (dashed) on the $N_{\text{eff}}$-$Y_P$ plane assuming $m_{\gamma'} = 80$ (left) and $30$ (right) GeV for non-top, charged twin fermions. The dark and light blue regions are respectively the 1σ and 2σ containment from Planck [32]. They combine the Planck TT, TE, and EE+lowE+lensing+BAO data with the $Y_P$ bounds from [72]. Twin photons lighter than 18 MeV are constrained by experiments and $f/v \geq 14$ requires fine-tuning greater than 1%.

Again, the lightest twin photon $m_{\gamma'} = 18$ MeV requires $f/v \gtrsim 10$, but the largest $m_{\gamma'}$ consistent with cosmological data when $f/v = 14$ is 25 MeV. The smallest $\Delta N_{\text{eff}}$ we can achieve for this scenario is 0.12 and again corresponds to $m_{\gamma'} = 18$ MeV and $f/v = 14$.

5 Discussion

In this paper, we have considered a new way to mitigate the $N_{\text{eff}}$ problem of MTH models. While other works have considered lifting twin Yukawa couplings as we have done here, we have additionally given the twin photon a mass. This greatly reduces $N_{\text{eff}}$ by allowing all of the entropy transferred after the twin neutrinos decouple to eventually go into the SM bath instead of staying in the twin photons. In the best scenario, all charged twin fermions (besides the twin top) have $m_{\gamma'} = 80$ GeV. For this spectrum, the twin neutrinos decouple before the SM bottom-antibottom pairs leave the bath which yields our smallest possible $\Delta N_{\text{eff}} = 0.10$ when $m_{\gamma'} = 18$ MeV and $f/v = 14$. We have carefully accounted for the effects of the twin spectrum not only on $N_{\text{eff}}$ but also on $Y_P$ when determining the viability of our model. We also considered the next-best scenario in which $m_{\gamma'} = 30$ GeV so that the twin neutrinos decouple before the SM tau-antitau pairs leave the bath. For this scenario, the smallest possible $\Delta N_{\text{eff}} = 0.12$ corresponds to $m_{\gamma'} = 18$ MeV and $f/v = 14$. One simple generalization of both mass benchmarks would be to allow a smaller twin hypercharge gauge coupling. This would decrease the rate of twin photon decays into twin neutrinos and therefore $\Delta N_{\text{eff}}$. However, as our motivation has been to maintain minimal $Z_2$ breaking, we do not pursue this further here.

CMB stage 3 experiments [76–79] are projected to reach a sensitivity of $\Delta N_{\text{eff}} \sim 0.06$, while stage 4 experiments have a target $\Delta N_{\text{eff}} = 0.027$ [80]. Figure 3 shows that $\Delta N_{\text{eff}} \gtrsim 0.10$ and $\Delta N_{\text{eff}} \gtrsim 0.12$ in our heavier and lighter MTH models, making them imminently discoverable by current and future observations. Current experimental constraints and naturalness considerations allow $m_{\gamma'} \in [18, 27]$ MeV with a kinetic mixing $\epsilon \sim O(10^{-4})$. Interestingly, this parameter space is also imminently discoverable by a host of proposed
experiments, as shown in figure 2. Whether from CMB light or dark-photon light, we will soon know if our MTH model is viable and accurately predicts an observable $\Delta N_{\text{eff}}$ and massive dark photon.

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A $\gamma' \to \bar{\nu}'\nu'$ decays

The rate of $\gamma' \to \bar{\nu}'\nu'$ depends on the amount of $\gamma' - Z'$ mixing. The relevant parts of the twin Lagrangian are

$$\mathcal{L}^\text{twin} \supset -\frac{1}{4} (W'_{\mu\nu})^2 - \frac{1}{4} (B'_{\mu\nu})^2 + \frac{1}{2} m_D^2 B'_{\mu}^2 + \frac{1}{2} m_{Z'}^2 Z'^2,$$

where $m_D$ is the mass of the twin hyper charge gauge boson. Using the weak-angle rotation, we find these terms may be written as

$$\mathcal{L}^\text{twin} \supset -\frac{1}{4} (Z'_{\mu\nu})^2 - \frac{1}{4} (F'_{\mu\nu})^2 + \frac{1}{2} (Z'_\mu A'_\mu) \begin{pmatrix} m_{Z'}^2 + s_{W'} m_D^2 & -s_{W'} c_{W'} m_D^2 \\ -s_{W'} c_{W'} m_D^2 & c_{W'} m_D^2 \end{pmatrix} \begin{pmatrix} Z'_\mu \\ A'_\mu \end{pmatrix},$$

where $c_{W'} \equiv \cos \theta_{W'}$ and $s_{W'} \equiv \sin \theta_{W'}$. When $m_D^2 = 0$, $(Z', A')$ is just the normal twin mass basis. The eigenvalues of the symmetric mass-squared matrix in (A.2) are

$$m_{Z'}^2 = m_D^2 c_{W'}, -\mathcal{O} \left( \frac{m_D^2}{m_{Z'}^2} \right), \quad m_{Z'}^2 = m_{Z'}^2 + \mathcal{O} \left( \frac{m_D^2}{m_{Z'}^2} \right).$$

The mass matrix is rotated to the mass basis $(\tilde{Z}'_\mu, \tilde{A}'_\mu)$ via

$$\begin{pmatrix} Z'_\mu \\ A'_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \tilde{Z}'_\mu \\ \tilde{A}'_\mu \end{pmatrix},$$

where $\cos \theta = 1 - \mathcal{O} \left( m_D^4 / m_{Z'}^4 \right)$ and

$$\sin \theta = s_{W'} c_{W'} \frac{m_D^2}{m_{Z'}^2} + \mathcal{O} \left( \frac{m_D^4}{m_{Z'}^4} \right) = \frac{s_{W'} m_Z^2}{c_{W'} m_{Z'}^2} + \mathcal{O} \left( \frac{m_D^4}{m_{Z'}^4} \right).$$


The mass eigenstate twin photon $\gamma'$ has a small mixing with the gauge eigenstate $Z'$ given by

$$\sin \theta = \frac{s_{W'}}{c_{W'}} \frac{m_{\gamma'}}{m_{Z'}}.$$  \hfill (A.6)

The decay rate of the $Z$ boson to a single generation of neutrinos in the SM

$$\Gamma_{Z\to \nu\nu} = \frac{\alpha M_Z}{24 s_W^2 c_W^2},$$  \hfill (A.7)

where we neglected the $\nu$ masses. Since the twin photon mixes with the twin $Z$, the total decay rate is

$$\Gamma_{\gamma'\to \nu'\nu'} = \frac{\alpha' m_{\gamma'}}{8 s_W^2 c_W^2} \sin^2 \theta = \frac{\alpha' \sin^2 \theta}{m_{\gamma'}^2} = \frac{\alpha' m_{\gamma'}^5}{2\pi g_2^2 (g_2^2 + g_1^2) f^4}.$$  \hfill (A.8)

To minimize $Z_2$-breaking, we take $\alpha' = \alpha$, $\cos \theta_W = \cos \theta_W$, and $m_{Z'} = f/v \cdot m_Z$.

### B Higgs invisible decays and signal strength

In Twin Higgs models, the SM-like Higgs we observe decays to invisible twin particles because the SM-like Higgs, $h$, is a mixture of both the physical SM Higgs, $h_{\text{phys}}$, and the physical twin Higgs, $h'_{\text{phys}}$:

$$h = \cos (v/f) h_{\text{phys}} + \sin (v/f) h'_{\text{phys}} \approx \left(1 - 1/2 (v/f)^2\right) h_{\text{phys}} + v/f \cdot h'_{\text{phys}},$$  \hfill (B.1)

where the approximation in the second line is valid for the $f/v \gtrsim 10$ we consider. Twin fermions couple to the twin Higgs with coupling $g_{\nu'}/\sqrt{2} = m_{\nu'}/f$. The total SM-like Higgs decay rate to “invisible” twin fermions is

$$\Gamma_h^\text{inv} = \frac{m_h v^4}{8\pi f^4} \left(1 - \left(\frac{2m_{\nu'}}{m_h}\right)^2\right)^{3/2} \left[N_{\nu} \frac{m_{\nu'}^2}{v^2} + 3N_{q'} \left(\frac{m_{q'}}{v}\right)^2 \left(1 + \frac{5.67}{\pi} \alpha_{q'}(m_h)\right)\right].$$  \hfill (B.2)
where $N_q$ is the number of twin quarks that the Higgs can decay into and $N_l$ is the number of twin leptons it can decay into. While the tree-level rate is sufficiently accurate for twin leptons, we must include twin QCD radiative and running-quark-mass corrections in the decay rate into twin quarks. We set $\alpha_S (m_h) = \alpha_S (m_h) = 0.112 \ [67]$, as is roughly required by the TH mechanism. The running quark mass to leading order is

$$m_q^{\text{eq}} (m_h) = m_q^{\text{eq}} \left( 1 - \frac{\alpha_S (m_h)}{\pi} \left( \frac{4}{3} + \log \frac{m_h^2}{m_q^2} \right) \right).$$

The total decay width of the SM Higgs with $m_h = 125 \text{ GeV}$ is $\Gamma_{h}^{\text{SM}} = 4.07 \times 10^{-3} \text{ GeV}$, with a relative uncertainty of $\approx 4\%$ both up and down [81]. Thus, we only require our own theoretical uncertainties in $\Gamma_{h}^{\text{inv}}$ to be less than $\approx 10\%$. Note that the total Higgs decay rate, $\Gamma_h$, is related to the total Higgs decay rate in the SM, $\Gamma_h^{\text{SM}}$, via $\Gamma_h = (1 - (v/f)^2) \Gamma_h^{\text{SM}} + \Gamma_h^{\text{inv}}$. Thus, we find the Higgs-to-invisible branching ratio

$$\text{BR} (h \rightarrow \text{inv}) = \frac{\Gamma_{h}^{\text{inv}}}{\Gamma_h} = \frac{\Gamma_{h}^{\text{inv}}}{(1 - (v/f)^2) \Gamma_h^{\text{SM}} + \Gamma_{h}^{\text{inv}}} = \left( 1 + \frac{(1 - (v/f)^2) \Gamma_h^{\text{SM}}}{\Gamma_{h}^{\text{inv}}} \right)^{-1},$$

where $\Gamma_{h}^{\text{inv}}$ is given by eq. (B.2). We require the branching ratio to anything in the twin sector to total less than $0.25 \ [67]$. Figure 4 demonstrates that our light twin benchmark with $m_f = 30 \text{ GeV}$ and $f/v \gtrsim 10$ is well below the current invisible branching ratio bound. The 250 GeV ILC will be able to probe Higgs invisible decays down to $0.3\% \ [68]$. Incredibly, the ILC will therefore be able to probe the entire $f/v$ parameter space for our light twin benchmark.

In addition to evading the current limit on the Higgs-to-invisible branching ratio, we also need our light twin benchmark to satisfy bounds on the Higgs signal strength. We define the Higgs signal strength as [82]

$$\mu = \frac{\sigma \times \text{BR}}{(\sigma \times \text{BR})_{\text{SM}}}. \tag{B.5}$$

Since the SM-like Higgs is not quite the SM Higgs, any cross section which yields a single Higgs in the final states will be suppressed by the same amount, namely

$$\frac{\sigma}{\sigma_{\text{SM}}} = 1 - (v/f)^2 \tag{B.6}.$$ 

Additionally, the Higgs branching ratio for any Higgs decay to SM particles, $h \rightarrow f$, will be reduced. The Higgs decay rate itself will be reduced by the same factor as the production cross section $\Gamma_{h \rightarrow f} = (1 - (v/f)^2) \Gamma_{h \rightarrow f}^{\text{SM}}$. Thus, the branching ratio is

$$\text{BR} (h \rightarrow f) = \frac{(1 - (v/f)^2) \Gamma_{h \rightarrow f}^{\text{SM}}}{\Gamma_h}, \tag{B.7}$$

Combining eq.’s (B.5) to (B.7), we find

$$\mu = \left( 1 - (v/f)^2 \right) (1 - \text{BR} (h \rightarrow \text{inv})). \tag{B.8}$$
The most up-to-date bounds on the signal strength $\mu$ come from ref. [82]. We don’t use the global signal strength they report below eq. (2) because they combine many inaccurate channels to arrive at their global fit. Instead, we take the result for the $gg \rightarrow h_{\text{phys}} (0\text{-jet})$ from the top of figure (9). We require our light twin benchmark to satisfy $\mu \geq 0.8$. Figure 4 demonstrates that our parameter space easily avoids this current bound.

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