Heavy quark symmetry in strong decays of P-wave heavy-light mesons

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Heavy quark symmetry and its breaking in strong decays of P-wave heavy-light mesons are examined within the EHQ’s method. The consistence of theory with experiments indicates that the breaking of heavy quark symmetry in the two-body strong decays of these mesons is not large. The relation between the EHQ’s method and the 3P0 model is investigated, and the phenomenological transition strengths $F_{j_0 l_0}^{h_q l}(0)$ of the P-wave heavy-light mesons within the EHQ method are analytically derived within the 3P0 model.

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I. INTRODUCTION

Heavy quark symmetry and the physics behind it were observed in heavy-light systems 30 years ago, heavy quark effective theory (HQET) has been established and successfully applied in many processes. The heavy quark symmetry in strong decays has also been investigated.

In Ref. [4], some consequences of the heavy quark symmetry for heavy-quark spectroscopy, and in particular for strong decay widths, have been obtained. The obtained amplitudes for strong decays of heavy mesons were proportional to sums of the products of four Clebsch-Gordan coefficients. Through a consideration of heavy quark symmetry, implications about the spectroscopy of the light degrees of freedom are discovered. However, only relationships between the heavy-quark systems involving given states of these degrees of freedom could be predicted.

In Ref. [5], a concise formula for two-body strong decay rates has been proposed based on heavy quark symmetry. In the formula, there is a $6-j$ symbol and a phenomenological transition strength $F_{j_0 l_0}^{h_q l}(0)$. The $6-j$ symbol exhibits the heavy quark symmetry in the strong decays of heavy-light mesons. The transition strength $F_{j_0 l_0}^{h_q l}(0)$ is a phenomenological parameter fitted by experimental data. To fix the $F_{j_0 l_0}^{h_q l}(0)$, K mesons were treated as heavy-light systems in Ref. [5].

The transition strength $F_{j_0 l_0}^{h_q l}(0)$ can also be obtained in theory. In Ref. [6], the transition strength was constructed from a relativistic chiral quark model. In Ref. [7], the transition strength derived within the 3P0 model was employed for the analysis, but the detail was omitted. In this paper, the derivation will be presented in detail.

In experiment, more and more highly excited heavy-light mesons (with one heavy $c$ or $b$ quark/anti-quark and one light $u$, $d$ or $s$ anti-quark/quark) have been observed [8]. Among the highly excited mesons, the $P-$wave mesons have often been concentrated on. In this paper, the $P-$wave heavy-light mesons with the following spin, parity $J^P$ and total angular momenta of the light degrees of freedom $j_l$: $0^+$ ($j_l = 1/2$), $1^+$ ($j_l = 1/2$), $1^+$ ($j_l = 3/2$) and $2^+$ ($j_l = 3/2$), will be denoted with $A_0^*$, $A_1^*$, $A_2^*$, respectively. There is no mixing between $A_0^*$ and $A_2^*$, but there exists possible mixing between $A_1^*$ and $A_2^*$. When there exists mixing, it is more difficult to identify the states. How to identify the mixing of these states is also an important urgent topic.

Now let us have a glimpse at those $P-$wave heavy-light mesons. $D_0^*(2400)$ and $D_{s0}^*(2317)$ are identified with the $A_0^*$ D and $D_s$, respectively. $B_0^*$ or $B_{s0}^*$ has not been observed. $D_2^*(2460)$ and $D_{s2}^*(2573)$ are supposed the $A_2^*$ D and $D_s$, and $B_{s2}^*(5747)$ and $B_{s2}^*(5840)$ are believed the $A_2^*$ B and $B_s$, respectively.

As for the $J^P = 1^+$ axial vector mesons ($A_1^*$, $A_1$ or the mixtures of them), $D_{11}(2420)$ was observed by Belle collaboration [9] but not confirmed by any other collaboration. $D_{11}(2430)$, $D_{s1}(2460)$ and $D_{s1}(2536)$ were observed by different collaborations [9], where $D_{11}(2430)$ and $D_{s1}(2536)$ were supposed the $A_1^*$ D and $D_s$, respectively [10]. $B_{12}(5721)$ and $B_{s2}(5830)$ were also observed by different collaborations [9], but $B_{12}^*(5732)$ was only observed by L3 Collaboration [11].

The effects of heavy quark symmetry and its breaking in the mass spectrum and in the weak processes have been comprehensively explored. However, the heavy quark symmetry and its breaking in the strong decays of the highly excited heavy-light mesons have seldom been examined. With more and more data of the heavy-light mesons accumulated, it is possible to examine the heavy quark symmetry and its breaking in these mesons.

The paper is organized as follows. In Sec.II, the two-body strong decays of $P-$wave heavy-light mesons are studied to examine the heavy quark symmetry. In Sec.III, we present the detail of deriving the phenomenological transition strengths $F_{j_0 l_0}^{h_q l}(0)$ of the P-wave heavy-light mesons within the 3P0 model. Our conclusions and discussions are included in Sec.IV.

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STRONG DECAYS OF HEAVY-LIGHT MESONS

II. HEAVY QUARK SYMMETRY IN THE STRONG DECAYS OF HEAVY-LIGHT MESONS

For convenience, here and in what follows the notation \( nL(J^P, j_q) \) is used to label the heavy-light mesons. When an excited heavy-light meson \( H \), characterized by \( nL(J^P, j_q) \), decays to a heavy-light meson \( H' \) (n'L(J'\(^P\), j'\(_q\)) and a light hadron \( h \) with spin \( s_h \) and orbital angular momentum \( l \) relative to \( H' \), the two-body strong decay width (the EHQ’s formula) is

\[
\Gamma_{H \rightarrow H' h} = \zeta \left( C_{j_h, j_q, j'_q}^{s_q, j'_q, j'} \right)^2 \mathcal{F}_{j_h, j, j_q}(0) p^{2l+1} \exp(-p^2/6j^2),
\]

where

\[
C_{j_h, j_q, j'_q}^{s_q, j'_q, j'} = \sqrt{(2J + 1)(2j + 1)} \left\{ \begin{array}{c} s_q \  j'_q \ j' \\ j_h \ j \ j_q \end{array} \right\}
\]

is a normalized \( 6 - j \) Wigner coefficient and \( \mathcal{F}_{j_h, j, j_q}(0) \) is a phenomenological parameter (transition strength), and \( p \) is the momentum of decay products in the rest frame of \( H \). The \( 6 - j \) symbol of the coefficients \( \mathcal{C} \) exhibits the heavy quark symmetry in the strong decays of heavy-light mesons. The allowed transitions between two doublets are governed by transition strength \( \mathcal{F}_{j_h, j, j_q}(0) \).

The flavor factor \( \zeta \) corresponding to some strong decay channels of \( D, D_s, B \) and \( B_s \) mesons have been computed and presented in Table I.

The EHQ’s method is based on the consideration of heavy quark symmetry, therefore any deviation of theoretical predictions from experiments will provide an insight into the heavy quark symmetry and its breaking of the heavy-light mesons.

In the original Eq. (1), the transition strength was fixed by K mesons. In our computation, \( D_s^*(2400) \) and \( D_s^*(2460) \) are used as the experimental inputs. In HQET, mesons of \( S \) doublet decay to \( H \) doublet and a light meson through an s-wave, mesons of the \( T \) doublet decay to \( H \) doublet and a light meson through a d-wave. Accordingly, the transition strength \( \mathcal{F}_{1/2, 1/2}^{1/2, 1/2}(0) \) (s-wave) and \( \mathcal{F}_{1/2, 1/2}^{1/2, 1/2}(0) \) (d-wave) are fixed at 0.267 and 1.07 GeV\(^{-1}\), respectively. In terms of these fixed transition strengths, the decay widths of all possible \( P \)-wave heavy-light mesons have been calculated and presented in the second column of Table II.

As shown in the next section, the transition strength \( \mathcal{F}_{0, 0}^{J=0, J=0}(0) \) (s-wave) and \( \mathcal{F}_{2, 2}^{J=2, J=2}(0) \) (d-wave) can also be obtained within the \( 3P_0 \) model. The corresponding decay widths of these \( P \)-wave heavy-light mesons are presented in the third column of Table II.

The electromagnetic interaction and \( SU(2) \) isospin symmetry breaking are believed to be two possible mechanisms for the isospin-violates decays of \( D_{s0}^*(2317)^\pm \) and \( D_{s2}^*(2460)^\pm \). The decay widths given by these two ways are listed in Table II. For all these \( P \)-wave heavy-light mesons in Table II, it is easy to observe that theoretical predictions based on heavy quark symmetry agree well with experiments in most cases. For the decay width of \( B_s^0(5670)^0 \), the predicted result seems much larger than the experimental one.

As pointed in the introduction, \( A_1' \) (1\(P(1^+, 1/2)\)) and \( A_2 \) (1\(P(1^+, 3/2)\)) may mix with each other. In fact, there exists evidence that the axial vector mesons \( D_s^+(2460)^\pm \) and \( D_{s2}^+(2536)^\pm \) are mixtures of \( A_1' \) and \( A_2 \). The s-channel decay of \( D_{s1}^+(2536)^\pm \) has been observed and the measured branching ratio is

\[
\frac{\Gamma((D^*(2010)^+K^0_{s-\text{wave}}))}{\Gamma(D^*(2010)^+K^0)} \approx 0.72 \pm 0.05 \pm 0.01
\]
Therefore, it is very possible that
\[
\begin{pmatrix}
D_{s1}(2460)^+ \\
D_{s1}(2536)^+
\end{pmatrix}
= \begin{pmatrix}
\cos \varphi & -\sin \varphi \\
\sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
1P(1^+, \frac{3}{2}) \\
1P(1^+, \frac{1}{2})
\end{pmatrix},
\]
where
\[
\varphi = \phi + 35.3^\circ.
\]

In existing literature, there is not yet a consensus on the mixing angle \(\phi\) or \(\varphi\). In Ref. \[13\], the mixing angle \(\phi\) are determined at 2.9^0 \sim 8.6^0\) and 0.9^0 \sim 2.6^0\) for D and B systems, respectively. In a hadron loops model, the mixing angle was found at \(\varphi \approx 29^0 \sim 35^0\) [14].

When a unitary rotation between the bases of \(Q\) mesons \((J^2, j_1^2, S^2, J_z)\) and \(\phi\) mesons \((J^2, L^2, S^2, J_z)\) is performed, \(\phi \approx 0.3^0 \sim 6.3^0\) [16].

In Refs. \[3, 17\], the mixing between the 2S and the 1D \(D_s\) is investigated and the mixing angle is fixed. In a similar way, the mixing angle in Eq. (3) can be determined as follows.

In our process, the mixing angle \(\phi\) is treated as a free variable and fixed by the comparison of theoretical prediction with experiments. In Fig. 1, the dependence of two branch ratios, \(\Gamma(D^{*0}K^+) / \Gamma(D^{*+}K^0)\) and \(\Gamma(D^{*+}K^0)_s / \Gamma(D^{*+}K^0)\), on the mixing angle \(\phi\) is presented. In plotting the red lines and the blue lines, the predicted masses of pure \(1^3P_1\) and \(1^1P_1\) in Ref. 18 and Ref. 19, respectively, are employed and \(F_{J_n, l_n}^{h_n, l_n}(0)\) is fixed by experimental data. The mixing angles determined by two different masses inputs are used as a reasonable boundaries of the mixing angle. Obviously, \(D_{s1}(2536)^\pm\) is \(1P(1^+, \frac{3}{2})\) dominant. In Fig. 2, similar figures are plotted with \(F_{j_n, l_n}^{h_n, l_n}(0)\) obtained within the \(3^3P_0\) model.

In addition to the decay widths, some branching ratios of the heavy-light mesons have been measured. Accordingly, these ratios are calculated in terms of Eq. (1). All the theoretical and experimental results are presented in Table III. Obviously, theoretical predictions of the branching ratios agree quite well with experiments.

In summary, theoretical prediction of the strong decays widths and branching ratios based on heavy quark symmetry is consistent with experiment, which indicates that the breaking of heavy quark symmetry in the two-body strong decays of these \(D_s\) is not large.

**III. TRANSITION STRENGTH \(F_{j_n, l_n}^{h_n, l_n}(0)\) IN THE \(3^3P_0\) MODEL**

The \(3^3P_0\) model, also known as the quark-pair creation model, was originally introduced by Micu [21] and further developed by Le Yaouanc et al., [22]. It has been extensively employed to study the strong decay of hadrons. The key idea of the \(3^3P_0\) decay model is the assumption of the creation of a \(0^{++}(3^3P_0)\) quark-antiquark pair from the vacuum [21].

The interaction Hamiltonian \(H_I\) relevant to the pair production involves two Dirac quark fields [23]

\[
H_I^{3^3P_0} = \gamma \int d^3k b_k \psi_f(k) \psi_i(k),
\]

where \(m_q\) is the mass of the produced quarks. The dimensionless constant \(\gamma\) is the \(3^3P_0\) pair production coupling constant which can be extracted by fitting the data.
TABLE III: The branching ratios from EHQ’s formula and experiments [8].

| Branching ratios                          | EHQ’s       | Expt.       |
|-------------------------------------------|-------------|-------------|
| $B(D^+(2010)^0 \rightarrow D^0 \pi^+)$   | 2.21        | 2.21 ± 0.05 |
| $B(D^{*+}(2010)^0 \rightarrow D^{*+}\pi^0)$ | 2.21        | 2.15 ± 0.16 |
| $B(D^{*+}(2460)^0 \rightarrow D^{*+}\pi^0)$ | 2.22        | 1.90 ± 1.10 |
| $B(D^{*+}(2460)^+ \rightarrow D^{*+}\pi^0)$ | 0.10        | < 0.33      |
| $B(D^0(2573)^0 \rightarrow D^0 K^+)$    | 0.90        | 1.10 ± 0.50 |

where a color matrix element $\langle \omega_B \omega_C | \omega_{A\omega_0} \rangle$ is 1/3 is included.

In a non-relativistic quark model, the helicity amplitude for a process $A \rightarrow BC$ can be written as [24]

$$ M^{\text{helicity}} = \langle BC | H_l^2 p_{0l} | A \rangle = \sum_{l_i, s_i} W_{l_i}(J_i, L_i, S_i; j_i, l_i, s_i) I_{l_i}(p), \quad (5) $$

The center-of-mass system, the partial overlap integral is

$$ I_{l_i}(p) = \int d^3k \psi_{n_{l_B}, l_B, l_A}^s \psi_{s_{C_{l_C}, s_{C_{l_C}}}}^s \psi_{s_{A_{l_A}}, l_i, s} Y_{l_i} \cdot \cdot \cdot. \quad (6) $$

The partial width is

$$ \Gamma_{A \rightarrow BC} = 2\pi \zeta \frac{M_B M_C}{M_A} \rho $$

$$ \times \sum_{L,S} | \langle JA, j_A | L, 0; S, j_A \rangle |^2 M^{\text{helicity}}^2 $$

$$ = 2\pi \zeta \frac{M_B M_C}{M_A} \rho \sum_{L,S} | M_{L,S} |^2, \quad (7) $$

where the mock-meson masses $\tilde{M}_i$ are explained in detail in Ref. [25]. $\zeta$ is an integral overlap of the flavor wave function

$$ \zeta = \langle \varphi_B \varphi_C | \varphi_A \varphi_0 \rangle $$

where $\varphi_0$ is a flavor singlet of vacuum.

Due to the heavy quark symmetry in the heavy-light systems, the partial wave amplitude in the base $(J^2, j_q, Q_Q)$ can also be written directly as [4, 5]

$$ M_{L,S} = (-1)^{s_q + j_q + j_q' + j_q'} c_{j_q, j_q', L, S} A_R(jh_l, l, j_q, j_q'), \quad (8) $$

where the reduced amplitude $A_R$ is

$$ \delta_{Q_Q, s_Q} \langle h(j_h, j_h) | H_l^2 p_{0l} | \frac{1}{2}, s_Q; j_q, j_q', j_q' \rangle $$

where $\delta_{Q_Q, s_Q}$ is a spin forbiddenness.

| $X(n^{2S+1}L_J)$ | $H(1S_0) + h(1S_0)$ | $H(1S_1) + h(1S_0)$ |
|------------------|----------------------|----------------------|
| $1S_1$           | $-\sqrt{1/3} f_P$    | $\sqrt{2/3} f_P$     |
| $1P_0$           | $f_S$                | $-$                  |
| $1P_1$           | $- \sqrt{2/3} f_S$   | $\sqrt{1/3} f_D$     |
| $1P_2$           | $- \sqrt{1/3} f_P$   | $\sqrt{2/3} f_D$     |

TABLE IV: The decay amplitudes $M_{L,S}$ where a coefficient $1/\sqrt{3} e^{- s^2/12}$ has been omitted [23].

When the simple harmonic oscillator approximation is employed as the meson space wave functions, the helicity amplitude in Eq. (6) is obtained analytically in the base $(J^2, L^2, S^2, J_z)$. Therefore, a unitary rotation between the base $(J^2, j_q, Q_Q, J_z)$ and $(J^2, L^2, S^2, J_z)$ is usually employed as follows [13, 10]

$$ | J; j_q \rangle = (-1)^{J+L+1} \sqrt{(2S+1)(2j_q+1)} $$

$$ \times \left\{ \begin{array}{c} 1/2 \\
L \\
J \\
j_q \end{array} \right\} | J; S \rangle. \quad (9) $$

In this way, the transition amplitude $M_{L,S}$ of P-wave mesons have been calculated in the base $(J^2, L^2, S^2, J_z)$. The results are presented in Table IV with

$$ f_S = \frac{2}{3} \beta (1 - \frac{2}{9} \beta^2) $$

$$ f_P = \frac{2}{3} \beta^3 $$

$$ f_D = - \frac{2}{3} \beta^2 $$

In terms of Eq. (9), one can get the reduced amplitude $A_R$ easily. Through a comparison of Eq. (1) with Eq. (7), the transition strength $F_{j_q, j_q'}(0)$ can be obtained

$$ F_{j_q, j_q'}^0(0) = G^2 \frac{1}{3} \beta^3 $$

$$ F_{j_q, j_q'}^1(0) = G^2 \frac{1}{3} \beta^3 $$

$$ F_{j_q, j_q'}^2(0) = G^2 \frac{1}{3} \beta^3 $$

where the constant

$$ G = \pi^{1/2} \gamma_2 \frac{2}{3} \frac{M_B M_C}{M_A}. $$

$F_{j_q, j_q'}^0(0)$ depends both on $p$ and $\beta$, $F_{j_q, j_q'}^1(0)$ and $F_{j_q, j_q'}^2(0)$ depend only on $\beta$. Other $F_{j_q, j_q'}^0(0)$ can be obtained in the same way.

Within the $3P_0$ model, $\beta$ was usually fixed by the data of many light mesons. The reasonable value of $\beta$ is often supposed around 0.35 – 0.4 GeV [23].

In the reasonable region of $\beta$, $F_{j_q, j_q'}^0(0)$ fixed by experiments are consistent with $F_{j_q, j_q'}^0(0)$ obtained within the
$^3P_0$ model. If $\beta = 0.38$ GeV is employed as in Ref. 7, the following ratios of transition strengths are obtained

$$\frac{F^{+,+}_{0,0}(0)}{F^{+,+}_{2,2}(0)} = 0.30(\text{EHQ's}) \quad 0.33(3^3P_0)$$

$$\frac{F^{+,+}_{2,2}(0)}{F^{+,+}_{4,4}(0)} = 1.64(\text{EHQ's}) \quad 1.95(3^3P_0)$$

Obviously, the ratios of the transition strength obtained within the $^3P_0$ model are consistent with those fixed by experimental data (EHQ’s method) within uncertainties.

IV. CONCLUSIONS AND DISCUSSIONS

In this work, the formula (Eq. (1)) proposed by Eichten et al. is employed to compute the strong decay of P-wave heavy-light mesons. Theoretical prediction of the decay widths and branching ratios is consistent with experiments. As well known, Eq. (1) is based on the consideration of heavy quark symmetry. Our results indicate that the heavy quark symmetry keeps well in the P-wave heavy-light mesons.

The axial vector mesons $D^*_s(2460)\,^\pm$ and $D_{s1}(2536)\,^\pm$ are mixtures of $A'_1$ and $A_1$ as indicated in Eq. 8. The fitted mixing angle $\phi$ is around $-5.2^\circ - 1.4^\circ$.

In the original work of Eichten et al., the phenomenological transition strength $F^{+\pm}_{j_0,l}(0)$ is fitted by experiments. In this paper, the parameters $F^{+\pm}_{j_0,l}(0)$ of P-wave heavy-light mesons are analytically obtained within the $^3P_0$ model. When the universal width parameter $\beta$ is chosen with 0.38 GeV, the predicted results agree well with those fitted by experiments.

Of course, the $1/m_Q$ corrections in the strong decays has not been taken into account in the method of Eichten et al. How large these corrections are is still not clear. For lack of data, the heavy quark symmetry in strong decays of other higher excited heavy-light mesons have not been examined.

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