Non-Cooperative Game Based Power Allocation for Energy and Spectrum Efficient Downlink NOMA HetNets

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This work has been supported in part by the Support Center for Advanced Telecommunications Technology Research Foundation, JSPS KAKENHI (JP17K06427), the research project “Super-HETs, Empowering 5G HetNets for better Performance” funded by NTRA Egypt, Kyushu University Platform of Inter/Transdisciplinary Energy Research (Q-PIT), and Suez Canal University.

ABSTRACT In this paper, the tradeoff between spectrum efficiency (SE) and energy efficiency (EE) is investigated in terms of interference management and power allocation for heterogeneous networks (HetNets) with non-orthogonal multiple access (NOMA). The EE and SE tradeoff is modeled as a multi-objective problem (MOP) under the maximum power and quality of service (QoS) constraints, which is non-convex. The MOP is relaxed into a convex single objective problem (SOP) by adopting a weighted sum strategy with the hypograph transformation. The SOP is solved in two steps. In the first step, we propose a power allocation technique based on non-cooperative (NC) game for EE and SE in NOMA HetNets. In the proposed NC game, the macro base station (MBS) and the small BSs (SBSs) compete with an equal priority in order to optimize their transmit powers towards maximizing the weighted sum of SE and EE. In the second step, a closed-form formula is proposed to control the power allocated to users taking into account both QoS constraint and successive interference cancellation (SIC) condition. From simulations, the proposed technique can, in some dedicated settings, considerably improve the tradeoff between EE and SE over conventional techniques.

INDEX TERMS Heterogeneous networks (HetNets), energy efficiency (EE), spectrum efficiency (SE), interference mitigation, power control (PC), non-cooperative (NC) game.

I. INTRODUCTION
Towards more efficient communication systems, spectrum-efficient (SE) and energy-efficient (EE) cellular systems need to be maximized to meet the critical demand for high data rates while saving energy for the green communication objective. Two notable wireless technologies can be deployed to achieve this end. The first one is the heterogeneous networks (HetNets), where the SE can be achieved by deploying small-cell (SCs) tiers with a short-range small base station (SBS) under the coverage of a macro-cell (MC) tier with a powerful macro BS (MBS) [1]. The second technology is the non-orthogonal multiple access (NOMA), where users are multiplexed on the same time/frequency/space resources while distinguishing them by allocating different power levels to users according to several criteria including quality of service (QoS) or relative channel gains [2], [3]. NOMA improves SE and EE at the cost of inter-user interference. However, due to the resource sharing among different tiers, HetNets with NOMA suffers from co-tier and cross-tier interference [4]–[7], while acquiring the advantages of NOMA HetNets depends on mitigating these types of interference.

One way to manage both co-tier and cross-tier interference is to control the allocated power to the adopted BSs in NOMA HetNets such that each BS transmits a proper power level to sustain its users’ QoS without causing excessive interference to users of other cells [8], [9]. However, the power control problem can itself be a dilemma since adjusting the transmitted power has a contradictory effect on both SE and EE [10], [11]. On one hand, increasing the transmitted power from one of the BSs has a positive impact on its
spectral efficiency (i.e., SE) as it reduces the probability of outage and allows for higher-order modulation to be utilized. However, interference on the other cells caused by increasing the transmitted power has a negative impact on energy efficiency (i.e., EE), which needs to be improved to conform to the green communication objective. Consequently, a tradeoff exists such that the sacrificing SE can be reflected as an EE gain or vice versa [12]. In other words, the allocated power to the system cannot be optimized to improve both EE and SE simultaneously. A Pareto optimal solution for the power control problem can be attained by solving a multi-objective problem (MOP) that maximizes both SE and EE [10], [12]. However, the MOP that represents SE-EE tradeoff is non-convex and nonlinear problem, which is very computationally costly to be solved in the MOP form. One way to solve the EE-SE tradeoff problem is to relax the original MOP into a single objective problem (SOP) using the weighted sum strategy, through which the direction of the optimization problem is dynamically changed according to the application demands or surrounding circumstances. Among different techniques for PC, utilizing the non-cooperative (NC) game can reduce the complexity of the PC without significantly increasing the required signaling sharing overhead among different cells. The NC game based PC has been proposed to maximize only EE in [8] or only SE in [9]. However, the NC game has not been adopted before to jointly maximize both EE and SE.

In this paper, we propose a low-complex and a fast convergence power control (PC) technique based on the NC game to maximize both SE and EE (NC-EE-SE) in HetNets with NOMA. Unlike the existing works, the proposed technique considers a general NOMA-HetNets model, where both co-tier and cross-tier interference can be managed by jointly optimizing the allocated power to MBS and a general number of SBSs with a general number of NOMA users. The proposed algorithm is able to find a better tradeoff point that improves EE and SE in comparison with the state-of-the-arts PC based NC game techniques. The main contributions of this paper are summarized as follows:

- We formulate the tradeoff between SE and EE in terms of MOP, where SE and EE are jointly maximized in HetNets with NOMA. In the formulated MOP, the allocated power to the MBS and all SBSs are optimized under the maximum power and minimum QoS constraints, which is non-convex. The non-convexity of the MOP is relaxed into a convex SOP by applying the weighted sum strategy with hypograph transformation.
- The formulated SOP problem is solved in two steps. In the first step, a PC technique is proposed based on the non-cooperative (NC) game. In the proposed NC game, SBSs and MBS fairly compete for optimizing the price of their transmitted power to maximize the weighted sum of SE and EE simultaneously and independently. In the second step, a closed-form formula is proposed to control the power allocated to NOMA users at each BS independently. The proposed formula considers both QoS constraint and successive interference cancellation (SIC) condition simultaneously.
- Through the numerical results, the tradeoff is confirmed such that the EE improvement is achieved at the cost of SE or vice versa. The proposed algorithm is compared with the upper and lower limits of the NC algorithms in [8], [9]. Our results show that the proposed technique can, in some dedicated settings, considerably improve the tradeoff between EE and SE over conventional techniques.

II. RELATED WORKS

The interference problem has been widely studied in HetNets in terms of maximizing the SE by properly controlling the allocated power to the SBSs and MBS as in [5], [13]. Authors in [5] propose a distributed PC algorithm with a user scheduling scheme, while authors in [13] propose a PC algorithm based on the compressive sensing theory to improve the SE of HetNets. Game theory is also utilized for SE based PC in HetNets as in [4], [7], [9], where the many-to-one matching game for PC is proposed in [4], and the leader-follower Stackelberg game is proposed in [7]. In addition, a non-cooperative game based PC is proposed in [9], where the game is performed only between the MBS and one SBS (not all SBSs) that has the worst channel condition user. In [14], a joint transmission coordinated multi-point (JT-CoMP) scheme is designed for NOMA HetNets, where SE is maximized by allowing users to benefit from multi-connectivity of CoMP. Then, a mixed-integer monotonic optimization and sequential programming is proposed to solve the PC problem. In [15], the joint optimization of user association and power control is formulated as a mixed integer programming problem for SE maximization, where Lagrange duality theory is applied to solve the formulated problem. However, the literature [4], [7], [9], [14], [15] investigate the PC only for SE maximization and do not include EE in the problem formulation. In other words, although these PC techniques can guarantee the minimum QoS for all users, they are not energy efficient.

On the other hand, maximizing EE by controlling the allocated power for interference management in HetNets has

| Notation | definition |
|----------|------------|
| $X$ | Vectors are denoted by boldface letters. |
| $SC_i$, $SBS_i$ | The set of SCs and the number of SCs. |
| $MU_i$, $SU_i$ | The $i^{th}$ SC and the $i^{th}$ SBS. |
| $N_{MU_i}$, $N_{SU_i}$ | The sets of MC’s users and SC’s users. |
| $N_{MU_i}$, $SU_i$ | Number of MUs and SUs per SC. |
| $\alpha_{i,n}$ | Power allocation coefficients for $MU_i$ and $SU_i$, respectively. |
| $(\cdot)^T$, $(\cdot)^H$ | Transpose, conjugate transpose, and cardinality operators. |
| $\leq$, $\geq$, $\#(\cdot)$ | Absolute operator, and excluding ‘b’ from ‘a’ |
| $\mathbb{R}^a \times b$, $\mathbb{C}^a \times b$ | Real, and complex fields of dimension $a \times b$. |

TABLE 1: Definition of notations.
been studied in literature [8], [16]–[24]. A Dinkelbach based method is utilized to solve the HetNets EE maximization problem in [16]–[18], where the Dinkelbach is combined with PC time switching control in [16] and combined with the Lagrange dual decomposition (LDD) method to obtain a closed form expressions for the optimal PC in [17]. Moreover, Dinkelbach is joined with PC non-cooperative game in [18]. Also, the PC based non-cooperative game is also proposed in [8], where the MBS and SBS are competing to maximize their EE. Moreover, in [19], the Stackelberg game is utilized for EE maximization with frequency allocation optimization. Authors in [20] maximize the EE by jointly considering PC with interference alignment (IA). Moreover, a particle swarm optimization (PSO) is proposed in [21], while a convolution neural network-based scheme is proposed in [22] for EE based PC in HetNets. In [23], a sequential quadratic programming (SQP) is utilized to estimate the optimal power while maximizing EE with QoS in NOMA HetNets. In [24], the joint problem of PC and user association to maximize the EE is formulated as a fractional programming problem. However, for [8], [16]–[24], the objective function of the maximization problem considers only the EE while the QoS is considered as a constraint.

To jointly optimize SE and EE, multi-objective optimization algorithms are needed through which we can find the optimal tradeoff points, i.e., Pareto-optimal solution. Since the priority for SE and EE is the same, the weighted sum is an appropriate strategy to convert the MOP into an SOP as in [10]–[12], [25], [26]. Authors in [25] propose a Dinkelbach-based iterative approach to maximize the weighted sum EE-SE problem in OFDM single-cell systems. Authors in [10] propose a dual Lagrangian based PC algorithm to maximize the weighted-sum EE-SE problem among SCs only for co-tier interference management. However, [10] does not consider the cross-tier interference. In [11], the dual Lagrangian algorithm is utilized in HetNet that operates in the reverse time division duplex (RTDD) to avoid the cross-tier interference. Thus, in [11], the cross-tier interference does not contribute to the PC problem. In [12], the authors propose a Levenberg-Marquardt based PC algorithm to maximize the weighted sum problem while a fractional frequency reuse is utilized. However, in [12], the co-tier interference is not considered while optimizing the allocated power to the BSs. Authors in [26] divide the weighted-sum problem into several subproblems that can be solved separately using the concave-convex procedure (CCCP). However, [26] also does not consider co-tier interference. In [27], the stochastic geometry is employed to model and analyze a spectrum-aware energy efficiency HetNets cognitive D2D communication. However, NOMA is not included in the analysis carried out in [27]. From the above literature, we can conclude that the tradeoff between SE and EE in NOMA HetNets has not been investigated sufficiently, where both co-tier and cross-tier interference contribute to the PC problem. Besides, the complexity in the above EE-SE algorithms is still high. Thus, more investigation needs to be achieved.

III. HETNET SYSTEM MODEL

Fig. 1 shows the downlink two-tier NOMA HetNets considered in this paper, where a set of $N_{SC}$ SCs tiers, denoted by $SC \triangleq \{1, \ldots, N_{SC}\}$, each with a single-antenna SBS are uniformly distributed under the coverage of a single MC of MBS with single-antenna. Regarding the resource allocation per time/frequency slot, the MBS and each SBS serves a set of $N_{SU}$ small users (SUs) and $N_{MU}$ macro users (MUs) denoted by $SU \triangleq \{1, \ldots, N_{SU}\}$ and $MU \triangleq \{1, \ldots, N_{MU}\}$, respectively. NOMA is adopted in both MC and SCs, where users with the worst channel conditions are decoded first and then sequentially subtracted from the received signal\(^1\) [7], [28]. For system simplicity, we assume that all users in a given BS are grouped into one NOMA cluster\(^2\). All BSs are proposed to reuse the same time/frequency resources. Also, the channel state information (CSI) between the user and their BS, and that between the user and interfering BSs are assumed to be shared with a central control unit (CCU) that allocates the power to BSs. A predetermined steps of fixed user association and user pairing are assumed. Notations used in this paper are summarized in Table I.

A. MATHEMATICAL SIGNAL MODELLING

Let us consider $x_n^{[M]} = \sum_{n=1}^{N_{MU}} x_n^{[M]}$ and $x_i^{[S]} = \sum_{n=1}^{N_{SU}} x_i^{[S]}$, $x_n^{[M]}$ and $x_i^{[S]}$ are the transmitted superimposed NOMA signal from the MBS and the SBS, respectively, where $x_n^{[M]} = \alpha_n^{[M]} p^{[M]} s_n^{[M]}$ and $x_i^{[S]} = \alpha_i^{[S]} p_i^{[S]} s_i^{[S]}$ are the transmitted signal to MU$_n$ and SU$_i$, respectively. A fractions of $\alpha_n^{[M]}$ and $\alpha_i^{[S]}$ from the MBS’s power and SBS’s power, $p^{[M]}$ and $p_i^{[S]}$ are assigned to MU$_n$ and SU$_i$, respectively, while $s_n^{[M]}$ and $s_i^{[S]}$ are the message signals to MU$_n$ and SU$_i$, respectively.

\(^1\)In this work, we assume that a perfect SIC detection is carried out at the receiver sides, which provides an upper bound in terms of the achieved data rates.

\(^2\)Single-cluster NOMA provides a benchmark performance of the proposed algorithm for the EE-SE tradeoff. Also, the proposed algorithm is straightforwardly compatible with any clustering technique [29]–[31].
By considering $i \in SC$, and $n \in SU$, the received signal at the SU$_{i,n}$, $y_{i,n}^S$, can be written as:

$$
y_{i,n}^S = h_{i,n}^S p_i^S x_{i,n}^S + \sum_{j=1, j \neq i}^{N_{SU}} h_{j,n}^S p_j^S x_{j,n}^S + \sum_{k=1}^{N_{MU}} g_{k,n}^S x_k^M + z_{i,n}^S
$$

where $h_{i,n}^S$, $g_{j,n}^S$, and $g_{k,n}^M$ are the channel coefficients that between SBS$_i$ and SU$_{i,n}$, the co-channel coefficients that between SBS$_j$ and SU$_{i,n}$, and the cross-channel coefficients that between MBS and SU$_{i,n}$, respectively. $z_{i,n}^S$ is the additive white Gaussian noise (AWGN) at SU$_{i,n}$ with variance $\sigma^2$. Similarly, by assuming $n \in MU$, the received signal at the MU$_{i,n}$, $y_{i,n}^M$, can be written as:

$$
y_{i,n}^M = h_{i,n}^M p_i^M x_{i,n}^M + \sum_{j=1, j \neq i}^{N_{MU}} h_{j,n}^M p_j^M x_{j,n}^M + \sum_{k=1}^{N_{SU}} g_{k,n}^M x_k^S + z_{i,n}^M
$$

where $h_{i,n}^M$, and $g_{j,n}^M$, are the channel coefficients that between MBS and its MU$_{i,n}$, and the cross-channel coefficients that between SBS$_j$ and MU$_{i,n}$, respectively. $z_{i,n}^M$ is the AWGN at MU$_{i,n}$. From equations (1) and (2), since the MC and the SCs share the same resources, three distinct kinds of interference exist. Inter-user interference occurs among users in the same cell due to the non-orthogonal multiplexing of NOMA. SUs experience co-tier interference from other SBSs, while both SUs and MUs are affected by a cross-tier interference from MBS and SBSs, respectively.

For efficient NOMA signal detection under the presence of interference, the decoding order has to be in the ascending channel gain order of the users’ normalized channel gain, as explained in [4], [5]. The normalized channel gain is defined as the channel gain-to-the-noise, cross-tier, and co-tier interference, and can be expressed as

$$
\kappa_{i,n}^S = \frac{\left| h_{i,n}^S \right|^2}{\sum_{j=1}^{N_{SC}} \left| f_{j,n}^S \right|^2 p_j^S + \sum_{j=1}^{N_{MU}} \left| g_{j,n}^M \right|^2 p_j^M + \sigma^2}.
$$

and

$$
\kappa_n^M = \frac{\left| h_{i,n}^M \right|^2}{\sum_{j=1}^{N_{SU}} \left| g_{j,n}^S \right|^2 p_j^S + \sigma^2}.
$$

where $\kappa_{i,n}^S$ and $\kappa_n^M$ are the normalized channel gain for SU$_{i,n}$ and MU$_{n}$, respectively. In this work, we assume that the normalized channel gain order for MC and SC$_i$ are $\kappa_1^S \geq \kappa_2^S \geq \cdots \geq \kappa_{N_{SC}}^S$ and $\kappa_1^M \geq \kappa_2^M \geq \cdots \geq \kappa_{N_{MU}}^M$, respectively, where $n = N_{MU}$ and $n = N_{SU}$ correspond to users with the worst channel condition in MC and SC, respectively.

Based on $\kappa_{i,n}^S$ and $\kappa_n^M$, the signal to interference plus noise power ratio (SINR) for SU$_{i,n}$ and MU$_{n}$ can be, respectively, expressed as

$$
\gamma_{i,n}^S = \frac{p_i^S \kappa_{i,n}^S}{\sum_{l=1}^{N_{SU}} p_l^S \kappa_{l,n}^S + 1},
$$

and

$$
\gamma_n^M = \frac{p_n^M \kappa_n^M}{\sum_{l=1}^{N_{MU}} p_l^M \kappa_{l,n}^M + 1},
$$

where $\gamma_{i,n}^S$ and $\gamma_n^M$ are the SINR at SU$_{i,n}$ and MU$_{n}$ respectively. Consequently, the sum rates of SC$_i$ and MC can be, respectively, calculated from

$$
R_{SC_i} = \sum_{n=1}^{N_{SU}} r_{i,n}^S = \sum_{n=1}^{N_{SU}} \log_2(1 + \gamma_{i,n}^S),
$$

and

$$
R_{MC} = \sum_{n=1}^{N_{MU}} r_n^M = \sum_{n=1}^{N_{MU}} \log_2(1 + \gamma_n^M),
$$

respectively, where $r_{i,n}^S$ and $r_n^M$ are the individual data rates for SU$_{i,n}$ and MU$_{n}$, respectively.

### B. PROBLEM FORMULATION

In NOMA HetNets, although the increase in the transmitted power from the BS of a dedicated cell will increase its sum-rate (i.e., SE), it affects the other cells negatively through interference as long as increasing the consumed energy (i.e., EE). In contrast, the decrease in the transmitted power from the BS will save its consumed energy. However, users may fail to reach their required QoS. The utilized problem in this work investigates the trade-off that exists between SE and EE in NOMA HetNets. Our goal is to maximize both SE and EE by adequately allocating power to SBSs and MBS, while considering the maximum power and the QoS constraints. Due to the contradiction between SE and EE, MOP is formulated to maximize both as in [10].

SE reflects how the available spectrum is efficiently utilized in terms of the achieved data-rate over an assigned bandwidth, $B$. Thus, the SE for the SC$_i$ and the MC can be given, respectively, as

$$
SE_{SC_i} = \frac{R_{SC_i}}{B}, \quad \forall i \in SC
$$

""
where the constraints \( C_2^{[S]} \) and \( C_2^{[M]} \) are introduced to guarantee that the power allocated to SBSs and the MBS do not exceed the maximum transmitting power, \( P_{\text{max}}^{[S]} \) and \( P_{\text{max}}^{[M]} \), respectively. The constraints \( C_2^{[S]} \) and \( C_2^{[M]} \) ensure that the minimum data rate for SU and MU do not fall below a predefined threshold values \( r_{\text{th}}^{[S]} \) and \( r_{\text{th}}^{[M]} \), respectively.

\[ \begin{align*}
\text{IV. PROPOSED ALGORITHM} \\
\text{The MOPs in (13) and (14) are non-convex and nonlinear problems, which are very computationally costly to be solved in this form. In this section, we try to find a sub-optimum solution for the power allocation problem represented by the MOPs in (13) and (14). First, the MOPs in (13) and (14) is reformulated into an SOP using the weighted sum strategy. Then, the SOP problem of power allocation is solved in two stages. In the first stage, the power is allocated to HetNet SBSs and MBs through a non-cooperative game based technique for a near-optimum solution. In the second stage, the power is allocated to NOMA users independently at each cell considering both QoS constraint and SIC condition.} \\
\text{A. PROPOSED WEIGHTED-SUM SOP OF EE AND SE} \\
\text{Maximizing SE and EE are both important for HetNets. Thus, the weighted sum strategy is considered a proper choice to model the above tradeoff in an SOP, where the EE and SE are weighted summed. A dedicated balance between EE and SE can be achieved based on the system requirements by adapting the weights. For example, giving more weights to SE is important during the peak hours to serve more users, while giving more priority to EE is preferable at the off-peak time to reduce the consumed energy [10]. The MOP (13) can be modeled as an SOP as follow} \\
\max_{\pi^n_{[i]}} \quad w_s \cdot EE_{SC_i} + (1 - w_s) \cdot SE_{SC_i} \\
\text{s.t.} \quad C_1^{[S]} : \sum_{n=1}^{N_M} p_{i,n}^{[S]} \leq P_{\text{max}}^{[S]} \\
\quad C_2^{[S]} : r_{i,n}^{[S]} \geq r_{\text{th}}^{[S]} , \quad \forall n \\
\quad C_2^{[M]} : r_{i,n}^{[M]} \geq r_{\text{th}}^{[M]} , \quad \forall n \\
\text{where the constraints} \ C_1^{[S]} \text{ and} \ C_1^{[M]} \text{ are introduced to guarantee that the power allocated to SBSs and the MBS do not exceed the maximum transmitting power,} \ P_{\text{max}}^{[S]} \text{ and} \ P_{\text{max}}^{[M]} \text{, respectively. The constraints} \ C_2^{[S]} \text{ and} \ C_2^{[M]} \text{ ensure that the minimum data rate for SU and MU do not fall below a predefined threshold values} \ r_{\text{th}}^{[S]} \text{ and} \ r_{\text{th}}^{[M]} \text{, respectively.} \\
\end{align*} \]
B. EXISTENCE OF NASH EQUILIBRIUM (NE) IN THE PROPOSED NC GAME

Since each BS jointly affects others through co/cross-tier interference, the PC problem can be modeled as a game \( G(N, S, U) \), where

- \( N \) is the set of all BSs (i.e., MBS and \( S_{SC} \) SBSs) that represents the game players.
- \( S = P_1[S] \times \cdots \times P_{NSC}[S] \times P[M] \) is the space of the transmit power, where \( P_1[S] \) and \( P[M] \) are the available action space for the SBSs and MBS, respectively.
- \( U = \{U[M], U[S]|i \in SC\} \) is the set of utility of each deployed BS, where

\[
U_i[S](P) = w_s EE_{SC,i} + (1 - w_s) SE_{SC,i}, \quad (19a)
\]

\[
U_i[M](P) = w_m EE_{MC} + (1 - w_m) SE_{MC}, \quad (19b)
\]

where \( P = [p_1[S], \ldots, p_{NSC}[S], p[M]] \) is the concatenated power vector that contains the power of all BSs. In the non-cooperative game, nash equilibrium (NE) is the point that gives a stable outcome of a game, such that all the players with conflict interests are satisfied, and no player wants to deviate. In other words, NE satisfies \( U_i[S](P^*) > U_i[S](P), \forall i \in SC \), and \( U_i[M](P^*) > U_i[M](P), \forall P \neq P^* \). Since \( U_i[S](P) \) and \( U_i[M](P) \) have been proved in [12] to be concave functions of \( p_i[S] \) and \( p_i[M] \), respectively, the game \( G \) has a unique NE point.

C. ALLOCATING POWER TO HETNET BSS BASED ON THE NC GAME

In this step, the power allocated to SBSs and MBS is controlled based on the proposed NC game based technique. In the proposed NC game, the competing players, SBSs and MBS, choose their actions toward maximizing the SE and EE simultaneously and independently. Consequently, each BS (i.e., SBSs or MBS) has the opportunity to maximize its SOP of joint EE and SE by considering the power transmitted from the other BSs as a constant. We can reach the game equilibrium by intersecting the solutions of the SOP problems (17), for \( \forall i \in SC \), and (18).

To find an expression for the power of the SBSs, \( p_i[S] \), we need first to solve its equivalent unconstrained Lagrangian equation, \( L_i[S] \), in (20) (shown at the beginning of the next page). Equation (20) is solved by considering \( p_i[M], p_j[S], \forall j \in SC \setminus i \), as a constant (i.e., constant co-tier and cross-tier interference). In that case, (20) is convex w.r.t. \( p_i[S] \) [7], [33], where the parameters \( \lambda_i[S], \mu_i[S], \beta_i[S] \) are the Lagrangian multipliers (LMs) related to the SOP of the SBSs. By assuming that \( U_{i,N_SU} \) is the user with the worst channel condition within the SC, we can find an expression for \( p_i[S] \) by substituting \( \xi_i[S] \) with its value in (3), and then taking the first derivative of \( L_i[S] \) w.r.t. the \( p_i[S] \) for \( n = N_{SU} \). Thus, we can obtain equation (21).

\[
\frac{\partial L_i[S]}{\partial p_i[N_{SU}]} = \frac{1 - w_s + \beta_i[S] w_s}{h_i[N_{SU}]^2} \Bigg[ (h_i[N_{SU}]^2 p_i[S] + \sum_{j=1}^{NSC} f_{j,i,N_{SU}}^S p_j[S] + g_{i,N_{SU}}^S p_{[M]} + \sigma^2) \ln 2
\]

\[
- \beta_i[S] \xi_i[S] \xi_{EE_i} - \lambda_i[S] + \mu_i[S] h_i[N_{SU}]^2 \Bigg]. \quad (21)
\]

By setting the \( \frac{\partial L_i[S]}{\partial p_i[N_{SU}]} = 0 \), the optimal value for \( p_i[S] \), \( \forall i \in SC \), can be given as

\[
p_i[S] = \frac{1 - w_s + \beta_i[S] w_s}{h_i[N_{SU}]^2} \Bigg[ \sum_{j=1}^{NSC} f_{j,i,N_{SU}}^S p_j[S] + g_{i,N_{SU}}^S p_{[M]} + \sigma^2 \ln 2
\]

\[
- \beta_i[S] \xi_i[S] \xi_{EE_i} - \lambda_i[S] + \mu_i[S] h_i[N_{SU}]^2 \Bigg]. \quad (22)
\]

Similarly, to find an expression for the power of the MBS, \( p_{[M]} \), we need to solve its equivalent unconstrained Lagrangian equation in (23), \( L_{[M]} \), by considering \( p_i[S], \forall i \), as a constant (i.e., constant cross-tier interference). Thus, (23) is convex w.r.t. \( p_{[M]} \), where the parameters \( \lambda_{[M]}, \mu_{[M]} \) are the LMs related to the SOP of the MBS. By assuming that \( M_{N_{MU}} \) is the user with the worst channel condition within the MC, we can find an expression for \( p_{[M]} \) by replace \( \kappa_{[M]} \) with its value in (4), and then taking the first derivative of \( L_{[M]} \) w.r.t. the \( p_{[N_{MU}]} \) for \( n = N_{MU} \). Thus, we can obtain equation (24).

\[
\frac{\partial L_{[M]}}{\partial p_{N_{MU}}} = \frac{(1 - w_m + \beta_{[M]} w_m) h_{[M]}_{N_{MU}}^M}{h_{[M]}_{N_{MU}}^M} \Bigg[ (h_{[M]}_{N_{MU}}^M)^2 p_{[M]} + \sum_{j=1}^{NSC} g_{j,i,N_{MU}}^M p_j[S] + \sigma^2) \ln 2
\]

\[
- \beta_{[M]} \xi_{EE} - \lambda_{[M]} + \mu_{[M]} h_{[M]}_{N_{MU}}^M \Bigg]. \quad (24)
\]

By setting the \( \frac{\partial L_{[M]}}{\partial p_{N_{MU}}} = 0 \), the optimal value for \( p_{[M]} \) can be expressed by

\[
p_{[M]} = \frac{(1 - w_m + \beta_{[M]} w_m) h_{[M]}_{N_{MU}}^M}{h_{[M]}_{N_{MU}}^M} \bigg[ \beta_{[M]} \xi_{EE} - \lambda_{[M]} + \mu_{[M]} h_{[M]}_{N_{MU}}^M \bigg] \ln 2
\]

\[
- \sum_{j=1}^{NSC} g_{j,i,N_{MU}}^M p_j[S] - \sigma^2 \bigg]. \quad (25)
\]

The set of equations consisting of (22), \( \forall i \in SC \), and (25) are deterministic since they are a linear function of each
other. Thus, by substitutionally solving this set of equations, an equilibrium solution can be obtained. Moreover, in (22) and (25), we consider the channel and the co/cross-channel of the user with the worst channel condition in each cell since if we ensure the minimum rate for the worst channel condition user, we can provide higher rate than the minimum rate for other users within the cell. Besides, the proposed NC game dramatically decreases the signaling needed to be shared among cells since we need to share only the channel and the co/cross-channel of the worst user in each cell with the CCU.

On the other hand, the equilibrium level is affected by the values given to the LMs. One of the simplest and fastest ways to find optimum values for these multipliers is to utilize one of the metaheuristic algorithms. In this work, we adopt the differential evolution (DE) algorithm proposed in [34] as a fast convergence algorithm to find near-optimum values for the LMs. Algorithm 1 presents the proposed non-cooperative game for joint EE and SE and the details regarding the updating based DE for the LMs, where $T_{\text{max}}$ is the maximum number of iterations.

### D. ALLOCATING POWER TO NOMA USERS

In this step, the power allocated to each NOMA macro and small user, $p_n^{[M]}$ and $p_{i,n}^{[S]}$, is optimized while considering both the QoS constraint and SIC condition. By optimizing the power allocated to the SBSs and MBS using the NC-EE-SE algorithm, the cross-tier and co-tier interference can be treated as constant when estimating the allocated power to NOMA users. Thus, the values of $p_n^{[M]}$ and $p_{i,n}^{[S]}$ can be independently estimated. To satisfy a required QoS, the power allocated to NOMA users should guarantee that the users’ SINRs do not fall below a threshold value. The SINR threshold at the SU$_{i,n}$ can be expressed as:

$$
\gamma_{i,n}^{[S]} \geq \theta_{i,n}^{[S]} \frac{p_{i,n}^{[S]} \kappa_{i,n}^{[S]}}{\sum_{l=1}^{n-1} p_{i,l}^{[S]} \kappa_{i,n}^{[S]} + 1},
$$

(26)

where $\theta_{i,n}^{[S]} = 2^{\gamma_{i,n}^{[S]}} - 1$ is the SINR threshold value at SU$_{i,n}$. Using simple manipulations, Eq. (26) can be reformulated as

$$
\begin{align*}
\gamma_{i,n}^{[S]} &\leq \theta_{i,n}^{[S]} \frac{p_{i,n}^{[S]} \kappa_{i,n}^{[S]}}{\sum_{l=1}^{n-1} p_{i,l}^{[S]} \kappa_{i,n}^{[S]} + 1}, \\
\frac{\gamma_{i,n}^{[S]} p_{i,n}^{[S]} \kappa_{i,n}^{[S]}}{\sum_{l=1}^{n-1} p_{i,l}^{[S]} \kappa_{i,n}^{[S]} + 1} &\leq \theta_{i,n}^{[S]}, \\
\end{align*}
$$

(27)

Furthermore, in the existence of interference or noise, to decode a signal of a NOMA user from the superimposed signal of another user, a minimum power difference is required such that

$$
(\alpha_{i,n}^{[S]} - \sum_{l=1}^{n-1} \alpha_{i,l}^{[S]} p_{i,l}^{[S]} \kappa_{i,n}^{[S]} - \kappa_{i,n}^{[S]} - 1 \geq \delta_{\text{diff}}^{[S]},
$$

(28)

where $\delta_{\text{diff}}^{[S]}$ denotes the minimum signal power to noise difference among SUs. Equation (28) shows the mandatory condition to achieve successful SIC, i.e., the decoded and the remaining undetectable signals should be accurately distinguished. Also, by simple manipulations, Eq. (28) can reformulated as

$$
\begin{align*}
\sum_{l=1}^{n-1} p_{i,l}^{[S]} &\leq \frac{\delta_{\text{diff}}^{[S]}}{\kappa_{i,n}^{[S]} - 1}, \\
\sum_{l=1}^{n-1} p_{i,l}^{[S]} &\leq \frac{\delta_{\text{diff}}^{[S]}}{\kappa_{i,n}^{[S]} - 1}. \\
\end{align*}
$$

(29)

By changing the inequality condition in (27) and (29) to equality, the minimum power allocated to the SU$_{i,n}$ can be calculated as

$$
\begin{align*}
p_{i,n}^{[S]} &\geq \frac{\theta_{i,n}^{[S]} \kappa_{i,n}^{[S]} \delta_{\text{diff}}^{[S]} - \kappa_{i,n}^{[S]} - 1}{\kappa_{i,n}^{[S]} + \delta_{\text{diff}}^{[S]} - 1}, \\
p_{i,n}^{[S]} &\geq \frac{\theta_{i,n}^{[S]} \kappa_{i,n}^{[S]} \delta_{\text{diff}}^{[S]} - \kappa_{i,n}^{[S]} - 1}{\kappa_{i,n}^{[S]} + \theta_{i,n}^{[S]} - 1}. \\
\end{align*}
$$

(30)
Thus, Eq. (30) represents the minimum power allocated to each NOMA SU to satisfy both QoS constraint and SIC condition, simultaneously. Moreover, the number of clustered NOMA SUs per SC that can access the same time/frequency resource, \( N_{SU} \), is constrained by the condition \( C_i^{[M]} \). Similarly, the minimum power allocation to the \( p_{i}^{[M]} \) can be calculated as

\[
\hat{p}_{i}^{[M]} = \frac{\theta_{i}^{[M]} (\kappa_{n}^{-1} \delta_{i}^{[M]} - \kappa_{n-1}^{-1})}{\kappa_{n}^{-1} \delta_{n-1}^{[M]} (\theta_{i}^{[M]} - 1)},
\]

where \( \delta_{i}^{[M]} \) is the minimum signal power to noise difference among MUs and \( \theta_{i}^{[M]} \) is SINR threshold at \( M_{U} \). Equation (30) represents the minimum power should allocated to each NOMA MU\( _{n} \) to satisfy both QoS constraint and SIC condition. Also, the number of clustered NOMA MUs that can access the same time/frequency resource, \( N_{MU} \), is constrained by the condition \( C_i^{[M]} \).

E. COMPLEXITY ANALYSIS

The computational complexity of the proposed NC-EE-SE technique is bounded by the complexity of computing \( p_{i}^{[S]} \) and \( p_{i}^{[M]} \) from (22) and (25), respectively. Since Eqs. (22) and (25) contain only summation operators, the overall computational complexity of the proposed NC-EE-SE per iteration is upper bounded by \( O(N_{SC}) \). The bottleneck complexity orders of different algorithms are listed in Table 2. It is obvious from Table 2 that the proposed algorithm has the same complexity as NC-SE and NC-EE, and much lower complexity than the other compared techniques.

V. NUMERICAL RESULTS AND DISCUSSION

A. SIMULATION PARAMETERS

The assumed simulation parameters are listed in Table 3. Two non-overlapped SCs\(^3\) are uniformly adopted under the coverage of MC. Each BS (i.e., MBS or SBSs) serves only two NOMA users per time/frequency resources; one is called a cell-center user (i.e., \( n = 1 \)), while the other is called a cell-edge user (i.e., \( n = 2 \)). SU\(_{i,1}\) and MU\(_{1}\) are randomly distributed over an area of ranges \( d_{i,1}^{[S]} \) and \( d_{1}^{[M]} \) far from the center of their BSs, respectively, while SU\(_{i,2}\) and MU\(_{2}\) are randomly spread over an area of ranges \( d_{i,2}^{[S]} \) and \( d_{1}^{[M]} \), respectively. Moreover, the channel coefficients are randomly produced by the multiplication of the free space path loss and the Rayleigh fading with zero mean and unit variance as in [35].

Simulation results compare the performance in terms of the achieved EE and SE among techniques; 1) NC-SE [9], where only SE is taken into account, 2) NC-EE [8], where only EE is taken into account, and 3) the proposed NC-EE-SE scheme \#1. In these techniques, the same power is allocated to all SBSs based on the user with the worst channel condition among all SBSs. NC-SE [9] and NC-EE [8] techniques can be considered as the optimum performance of non-cooperative game based SE or EE for the case of the same allocated power to all SBSs. In addition, the comparison includes the results of 4) the proposed NC-EE-SE scheme \#2, where the SBSs are allocated with different power by solving (22), \( \forall i \in \text{SC} \), to manage the co-tier interference as long as the cross-tier interference. Also, we assume that \( w_{m} = w_{s} = w \), and \( \kappa_{i}^{[S]} = \kappa_{i}^{[E]} = \kappa \).

B. SIMULATION RESULTS

The performance of the proposed NC-EE-SE algorithm is compared with the state of the art algorithms in terms of total EE \( (EE_{MC} + \sum_{i=1}^{N_{SC}} EE_{SC}) \), and total SE \( (SE_{MC} + \sum_{i=1}^{N_{SC}} SE_{SC}) \), at different values of signal-to-noise ratio (SNR) in Figs. 2(a) and 2(b), respectively.

---

\(^3\)In this paper, we choose \( N_{SC} = 2 \) as in [9]. However, the proposed algorithm is general for the case of \( N_{SC} > 2 \).
Fig. 2 shows that the proposed NC-SE-EE provides higher SE and EE than the conventional OMA and NOMA. Also, Fig. 2 shows that by tuning the parameters $\xi$, the proposed NC-SE-EE, for both Sch#1 and Sch#2, can give comparable approaches.

For different values of the tradeoff balancing parameter $w$, the performance of the proposed NC-EE-SE is investigated in terms of the total EE and the total SE versus the SNR in Figs. 3(a) and 3(b), respectively. In general, increasing $w$ from 0 to 1 means that the NC game will allocate the power so as to improve EE and sacrifice the SE. However, by choosing appropriate values for $w$ and $\xi$, we can get a much higher EE without significantly losing in the SE. Also, it appears from Fig. 3 that by controlling the power of each SBS separately using the proposed scheme #2, we can improve the EE over scheme #1 with almost the same SE. It is also worth noting that at $w = 1$ (i.e., only EE is taken into account), the proposed scheme #2 can improve the EE over the NC-EE [8] without sacrificing in the SE.

The results for $\xi = 0$, $w = 0$, where SE is only taken into account is shown in Fig. 4. It is obvious that the proposed scheme #2 can improve the EE over NC-SE [9] with the same SE level. The results in Figs. 3 and 4 confirm that by adequately allocating the power at each BS below the maximum power, we can find some points where the decrease in the signal power is compensated by the reduction in the level of interference to sustain the users’ QoS while preserving the emitted energy.

The effect of increasing $w$ on the EE and SE of the proposed algorithm for different $\xi$ values is shown in Figs. 5(a) and 5(a), respectively. Algorithms NC-EE and NC-SE do not depend on the $w$. Increasing $w$ will objective the game more towards the EE and far from the SE. In other words, by increasing $w$, the NC game will decrease the power in order to improve the EE. Also, it is obvious from Fig. 5 that increasing $\xi$ will direct the game more in the direction of the EE rather than SE.
outage probabilities of the SU$_{i,n}$ for the proposed NC-EE-SE is plotted in Fig.6(a) and Fig.6(b) against baseline approaches. The outage probability in Fig. 6 is plotted according to equations (28) and (29) in [13]. From Fig. 6, it is clear that the outage performance of the proposed NC-EE-SE is better than NC-EE and comparable to NC-SE and the exhaustive search technique. Also, the outage performance can be improved as $\xi$ and $w$ goes to zero. In other words, the proposed NC-SE-EE can give acceptable outage performance as long as the value of SE threshold $r_{th}^{[S]}$ is feasible and compatible with the value of the EE threshold, $\xi$.

Fig.7 shows the convergence behavior of the NC-EE-SE. It can be observed that adopting the DE algorithm to obtain optimum values for the LMs forces the allocated power to each BS to reach its stable status after a limited number of iterations. Moreover, the convergence is guaranteed, even for $N_{SC} > 2$, as long as the maximum power and QoS threshold values are feasible. Accordingly, the proposed NC-EE-SE is cost-efficient in terms of convergence time in addition to the hardware complexity.

VI. CONCLUSION

In this work, the SE/EE tradeoff in NOMA HetNets has been studied in terms of interference management and PC. The SE/EE tradeoff has been modeled as a non-convex MOP. The MOP has been relaxed into a convex SOP by adopting the weighted sum strategy and the hypograph transformation. Then, a non-cooperative game based technique, NC-EE-SE, has been proposed to allocate the power to the SBSs and MBS in a competitive manner to jointly maximize their EE.
and SE based on the system requirements. Then, a closed-form formula has been proposed to control the power allocated to NOMA users taking into account both QoS and SIC condition. From the discussed results, properly choosing the balancing parameters and the EE threshold value can improve the tradeoff between EE and SE. MIMO can provide extra degree of freedom that can be useful for our NOMA-HetNets systems in terms of accommodating more users or mitigating part of the interference, which will be considered in our future work.

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