There might be superluminal particles in nature *

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Abstract

Based on experimental discovery that the mass-square of neutrino is negative, a quantum equation for superluminal neutrino is proposed in comparison with Dirac equation and Dirac equation with imaginary mass. A virtual particle may also be viewed as superluminal one. Both the basic symmetry of space-time inversion and the maximum violation of space-inversion symmetry are emphasized.

Key words: ordinary neutrinos; neutrino mass and mixing; non-standard-model neutrinos

1 INTRODUCTION

Nothing can travel faster than light. Is this statement as true now as it ever was? In 2000 there were two experiments showing superluminal propagation of microwave [1] or laser pulse [2]. Yet physicists believe that the law of physics have remained intact [3]. However, the experimental discovery of negative mass square of neutrino in recent years [4],

\begin{equation}
E^2 = c^2 p^2 + m^2 c^4,
\end{equation}

\begin{equation}
m^2(\nu_e) = -2.5 \pm 3.3 eV^2,
\end{equation}

though far from accurate, does strongly hint that neutrino might be a particle moving faster than light. Actually, rewriting (1) as

\begin{equation}
E^2 = c^2 p^2 - m^2_s c^4
\end{equation}

and using the quantum relations \( E = \hbar \omega \) and \( p = \hbar k \), one easily derives the kinematic relation for superluminal particle (also named as tachyon in literature) as follows \((m_s \text{ is called "proper mass"}):

\begin{equation}
p = \tilde{m} u = \frac{m_s u}{\sqrt{u^2 c^2 - 1}}, \quad E = \tilde{m} c^2 = \frac{m_s c^2}{\sqrt{u^2 c^2 - 1}}.
\end{equation}

Here the velocity of particle \( u \) is identified with the group velocity \( u_g = \frac{d\omega}{dk} \) of wave. To derive (3), a quantum equation for neutrino was proposed in Ref. [5,6] (see also [7]). In this paper, we will compare three kinds of equation—Dirac equation and two "superluminal" equations and discuss relevant problems.

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2 Dirac equation versus superluminal equation

The Dirac equation for fermion with rest mass $m_0$ reads

$$i\hbar \frac{\partial}{\partial t}\psi = i\hbar \vec{\alpha} \cdot \nabla \psi + \beta m_0 c^2 \psi$$

(5)

where $\psi$ is a four-component spinor wave function and $\alpha_i$ and $\beta$ are $4 \times 4$ matrices. In Dirac representation, they are expressed as

$$\psi = \left( \begin{array}{c} \varphi \\ \chi \end{array} \right), \quad \alpha_i = \left( \begin{array}{cc} 0 & \sigma_i \\ \sigma_i & 0 \end{array} \right), \quad \beta = \left( \begin{array}{cc} I & 0 \\ 0 & -I \end{array} \right)$$

(6)

where $\varphi$ and $\chi$ are two-component spinors, $\sigma_i$ are Pauli matrices. Now we perform a unitary transformation as

$$\psi \rightarrow U \psi = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array} \right) \psi = \psi' = \left( \begin{array}{c} \xi \\ \eta \end{array} \right).$$

(7)

Let the matrix $U$ act on Eq.(5) from the left, due to noncommutativity between $U$ and $\alpha_i$ (or $\beta$), we find Dirac equation in ”Weyl representation” with

$$\psi = \left( \begin{array}{c} \xi \\ \eta \end{array} \right), \quad \alpha_i^{(W)} = \left( \begin{array}{cc} \sigma_i & 0 \\ 0 & -\sigma_i \end{array} \right), \quad \beta^{(W)} = \left( \begin{array}{cc} 0 & I \\ I & 0 \end{array} \right)$$

(8)

Note that, however, the representation transformation leads to important change in physical interpretation. While $\varphi$ and $\chi$ in (6) represent the ”hidden particle and antiparticle fields” in a particle state$^{[8]}$, $\xi$ and $\eta$ in (8) characterize the ”hidden left-handed and right-handed rotating fields” in a particle with 100% left-handed helicity explicitly.

Now let’s perform a unitary transformation on Dirac equation (5) in Weyl representation:

$$\psi = \left( \begin{array}{c} \xi \\ \eta \end{array} \right) \rightarrow U_s \psi = \left( \begin{array}{cc} i & 0 \\ 0 & 1 \end{array} \right) \psi = \psi_s = \left( \begin{array}{c} \xi_s \\ \eta_s \end{array} \right).$$

(9)

After setting

$$m_s = -im_0,$$

(10)

we obtain an equation with real proper mass $m_s$:

$$i\hbar \frac{\partial}{\partial t} \xi_s = i\hbar \vec{\sigma} \cdot \nabla \xi_s - m_s c^2 \eta_s,$$

$$i\hbar \frac{\partial}{\partial t} \eta_s = -i\hbar \vec{\sigma} \cdot \nabla \eta_s + m_s c^2 \xi_s.$$
\[ \psi_s = \begin{pmatrix} \varphi_s \\ \chi_s \end{pmatrix} = U \begin{pmatrix} \xi_s \\ \eta_s \end{pmatrix}, \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}. \quad (13) \]

Note that the matrix \( \beta_s \) and its counterpart in Eq. (11) are antihermitian.

On the other hand, if we directly set \( m_0 = im_s \) in Eq. (5) to get the ”Dirac equation with imaginary mass” as

\[ i\hbar \frac{\partial}{\partial t} \psi^{(i)} = i\hbar \vec{\alpha} \cdot \nabla \psi^{(i)} + \beta_s^{(i)} m_s e^2 \psi^{(i)}, \quad (14) \]

then we have

\[ \psi^{(i)} = \begin{pmatrix} \varphi^{(i)} \\ \chi^{(i)} \end{pmatrix}, \quad \beta^{(i)}_s = \begin{pmatrix} iI & 0 \\ 0 & -iI \end{pmatrix}. \quad (15) \]

The fact that equation (14) is wrong can easily be seen by its plane-wave solution when \( p \rightarrow 0 \), yielding a decoupling solution:

\[ \varphi^{(i)} \sim e^{-i(m_s t)} \sim e^{m_s t}, \quad \chi^{(i)} \sim e^{-i(-m_s t)} \sim e^{-m_s t} \quad (16) \]

which show the violation of unitarity. By contrast, such kind of meaningless solution is definitely excluded in Eq. (12). The sharp difference is stemming from the following fact. Three kinds of Eqs. (5), (12) and (14), all respect the basic symmetry: under the space-time inversion \((\vec{x} \rightarrow -\vec{x}, t \rightarrow -t)\) [8]

\[ \varphi(-\vec{x}, -t) \rightarrow \chi(\vec{x}, t), \quad \chi(-\vec{x}, -t) \rightarrow \varphi(\vec{x}, t), \quad (17) \]

the theory remains invariant (while a concrete solution of particle transforms into that of antiparticle).

However, we should consider a smaller symmetry—the space-inversion \((\vec{x} \rightarrow -\vec{x}, t \rightarrow t)\):

\[ \xi(-\vec{x}, t) \rightarrow \eta(\vec{x}, t), \quad \eta(-\vec{x}, t) \rightarrow \xi(\vec{x}, t). \quad (18) \]

Then we see that Eqs. (5) and (14) remain invariant whereas Eq. (12) (or (11)) fails to do so. In other words, our new superluminal equation reflects the maximum parity violation, a property exactly explaining the permanent helicity of neutrino—while \( \nu_L \) and \( \nu_R \) are physically realized, \( \nu_R \) and \( \nu_L \) are forbidden strictly—as verified by experiments [9,10].

3 Virtual particle as a superluminal particle

It is no surprise to see solution like (16) when Eq. (15) contains an imaginary mass \( im_s \) explicitly. Actually, in quantum mechanics, we often endow an imaginary part of mass \( m \) to wave function for describing an unstable particle:

\[ m \rightarrow m - \frac{\Gamma}{2}, \quad \Gamma = \hbar/\tau, \quad |\psi|^2 \sim e^{-t/\tau} \quad (19) \]

with \( \tau \) being the decay lifetime. Of course, the unitarity of wave function is destroyed.

Hence, it is hopeless to set \( m_0 = im_s \) directly in Dirac equation for describing a stable superluminal particle. However, we often discuss the ”virtual particle” in covariant perturbation theory in the sense that its momentum \( p \) and energy \( E \) can vary independently and so are not subjected to the constraint \( E^2 - p^2 = m_0^2 \). Both \( E^2 > p^2 \) and \( E^2 < p^2 \) cases can occur. We might as well say that a virtual particle could be superluminal but it is unstable too. A stable superluminal particle can only be realized as \( \nu_L \) or \( \nu_R \) ensured by the violation of parity symmetry.
4 Maximum parity violation displays the beauty of nature

Usually, a symmetry, i.e., an invariance under some transformation yields an affirmative guarantee for a theory and its violation seems to spoil the validity of the theory. Now we see that for a meaningful discrete symmetry like space-inversion, what happens in nature is either keeping intact (for subluminal particle) or being violated to maximum (for superluminal neutrino). In the latter case the violation must reach its maximum because the constraint of larger symmetry—the space-time invariance—must be held at the same time. [In the first line of Eq. (11), the coefficient of $m_s$ must be fixed as $(-1)$ to ensure the invariance of transformation (17) together with the violation of transformation (18).] We might as well look at the maximum violation of parity as some "antisymmetry" (rather than "asymmetry") which also provides affirmative guarantee for the validity of superluminal theory for neutrino. In fact, the normal consequence of violation of hermitian property being the violation of unitarity (as shown by (16)) is now recast into a strange realization—one would be unstable solutions for a same momentum, two of them ($\nu_R$ and $\bar{\nu}_L$) are eliminated whereas other two ($\nu_L$ and $\bar{\nu}_R$) are stabilized.

5 Summary and discussion

We can have three kinds of Dirac-type equation:

$$i\hbar \frac{\partial}{\partial t} \psi = ic\hbar \vec{\alpha} \cdot \nabla \psi + \beta mc^2 \psi \quad (20)$$

as listed in the following table.

Finally, two remarks are in order:

(a) According to our present understanding, no boson but fermion can be superluminal as long as the parity symmetry is violated to maximum. Most likely, one kind of known particles, the neutrino, is just a superluminal particle, a tachyon with spin 1/2.

(b) The physical implication of nonhermitian transformation $U_s$ amounts to an extra phase difference $\pi/2$ between $\xi_s$ and $\eta_s$ in comparison with that in subluminal case. Hence we see once again that it is the phase which plays a dominant role in quantum mechanics.

(c) The origin (or mechanism) of mass generation has been remained in physics as a mystery for hundred years. It is time to say something now. According to detailed analysis in Ref. [6], the finite and changeable mass of a fermion is not due to one kind of excitation which varies continuously inside but a manifestation of coherent cancellation effect between two fields rotating with opposite helicities implicitly, either one of them can be excited infinitely in essence. It is precisely this fantastic effect explains the amazing result that a tachyon’s energy (mass) approaches zero when its velocity $u$ increases to infinity (as shown in Eq. (4)).

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| property | Dirac equation | Superluminal equation | Dirac equation with imaginary mass |
|----------|----------------|-----------------------|-----------------------------------|
| $m$ (mass) | $m_0$ (rest mass) | $m_s$ (proper mass) | $m_s$ |
| Dirac representation | $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, $\beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, $\beta_s^{(i)} = \begin{pmatrix} iI & 0 \\ 0 & -iI \end{pmatrix}$ | hermitian matrix | antihermitian matrix |
| Weyl representation | $\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$, $\hat{\alpha} = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}$ | $\beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $\beta_s = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$, $\beta_s^{(i)} = \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix}$ | hermitian matrix | antihermitian matrix |
| | | | |
| hermitian property of theory | yes | no | no |
| unitarity of theory | yes | yes | no |
| invariance under space-time inversion | yes | yes | yes |
| (basic symmetry) | | | |
| invariance under space inversion (parity) | yes | no | yes |
| physical meaning | subluminal particles (electron, etc.) | describing possibly the (stable) superluminal neutrino $\nu_L$ and $\bar{\nu}_R$ (with $\nu_R$ and $\bar{\nu}_L$ forbidden) | unstable superluminal particle without physical meaning in reality |