Mass spectroscopy and strong decays of excited open charm $D_J$ mesons using relativistic Dirac formalism

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Abstract The mass spectra for the heavy-light ($cq$; $q = u$ or $d$) charmed mesons are computed based on a relativistic framework. The low-lying 1P states are found to be in an excellent agreement with the PDG reported values. Using the computed mass spectra and following effective Lagrangian approach based on heavy quark and chiral symmetry, the OZI allowed two body strong decays are computed. The computed decay rates, ratios and branching fractions allow us to identify the proper spin-parity assignments of the newly observed charm states. Accordingly, we could identify $D_J(2560)$ as $^1S_0$, $D_J^*(2680)$ as $^2S_1$, $D_J(2740)$ as $^1D_2$, $D_J^*(2760)$ as $^3D_3$, $D_J^*(3000)$ as $^2P_0$, $D_J(3000)$ as $^1P_1$ and $D_J^*(3000)$ as $^3F_2$ open charm states. The effective coupling constants, $g_T$, $g_H$, $g_Y$, $g_S$ and $g_z$ extracted from the present study are found to be in accordance with the reported values. These coupling constants would be useful in further investigations. We found $D^{*+}\pi^-$ as a favorable channel for the experimental search of the missing $^1F_3$ state.

1 Introduction

During the past decade, numerous excited charmed states have been observed by the experimental groups $LHCb$ and $BABAR$ [1–3]. These achievements provoked great interest in experimental as well as theoretical studies of charmed mesons. In recent past $LHCb$ has employed the Dalitz Plot (DP) technique to analyse the contributing amplitudes in decay channel $B^- \to D^+\pi^-\pi^-$, where charmed states were reconstructed through $D^+ \to K^-\pi^+\pi^+$ decay process. This analysis was based on data collected by the $LHCb$ detector during 2011 and 2012 when the $pp$ collision center of mass energy was 7 TeV and 8 TeV, respectively. Their study summarises the resonant contribution coming from $D^*_2(2460)^0$, $D^*_1(2680)^0$, $D^*_2(2760)^0$ and $D^*_2(3000)^0$ states [1]. Their measured masses and widths are listed in Table 1.

In 2013, $LHCb$ has studied the $D^+\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$ channel invariant mass spectra and enriched the spectrum of charmed mesons. They have reported the resonances $D_0^*(2420)$ in the $D^+\pi^-$ final state and the resonance $D_2^*(2460)$ in the $D^+\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$ final states [2]. Moreover, two natural parity resonances: $D_0^{*+}(2650)$ and $D_2^{*+}(2760)$ and two unnatural parity resonances: $D_1^{*+}(2580)$ and $D_2^{*+}(2740)$ were also observed in $D^{*+}\pi^-$ mass spectra. The $LHCb$ collaboration has tentatively assigned $D_0^{*+}(2650)$ as $2S(1^-)$, $D_2^{*+}(2580)$ as $2S(0^-)$, $D_2^{*+}(2760)$ as $1D(1^-)$ and $D_1(2740)$ as $1D(2^-)$ states. The $D_2^{*+}(3000)$ resonance was observed in $D^{*+}\pi^-$ final states with unnatural parity. The states $D_0^{*+}(3000)$ and $D_2^{*+}(3000)$ were recorded in the $D^+\pi^-$ and $D^0\pi^+$ spectra. The mass and width of these resonances are collected in Table 2.

The $BABAR$ collaboration has observed four resonances $D_0^*(2550)$, $D_0^{*0}(2600)$, $D_0^{*+}(2760)$ and isospin partners $D_0^{*+}(2600)$, $D_2^{*+}(2760)$ in the inclusive production of the $D^+\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$ systems at SLAC PEP-II asymmetric-energy collider, in 2010 [3]. After analysing the helicity distribution, the $BABAR$ collaboration suggested that $D_0^*(2550)$, $D_0^{*0}(2600)$ might be the radial excited states 2$S$ and $D_0^*(2750)$, $D_0^{*+}(2760)$ might belongs to D - wave states. Available details are collected in Table 3.

Although several $D$ mesonic states are reported experimentally, their experimental spin-parity assignments are still an open problem in charm sector. However, analysis of their decay modes provides us a way out to extract the information regarding their internal structure and dynamics. In many cases, theoretical analysis of the mass spectra and decay properties are simultaneously required to assign their spin-parity ($J^P$) quantum numbers. Such a theoretical model also allows suitable testing ground for the newly observed states. The popular non-relativistic Schrödinger treatment is quite con-
Table 1. Charged resonances observed by LHCb Collaboration in the DP analysis of $B^- \rightarrow D^{\ast -}\pi^-\pi^-$ decays [1]

| Resonance     | Mass (in MeV) | Width (in MeV) | Spin-parity |
|---------------|---------------|----------------|-------------|
| $D^+_2(2460)^0$ | 2463.7 ± 0.4 ± 0.4 | 47.0 ± 0.8 ± 0.9 | 2$^+$ |
| $D^*_1(2680)^0$ | 2681.1 ± 5.6 ± 4.9 | 186.7 ± 8.5 ± 8.6 | 1$^-$ |
| $D^*_1(2760)^0$ | 2775.5 ± 4.5 ± 4.5 | 95.3 ± 9.6 ± 7.9 | 3$^-$ |
| $D^*_2(3000)^0$ | 3214 ± 29 ± 33 | 186 ± 38 ± 34 | 2$^+$ |

Table 2. Charged resonances observed from the LHCb analysis of inclusive $D^{\ast \pm}\pi^0$ production [2]

| Resonance     | Mass (in MeV) | Width (in MeV) | Spin-parity |
|---------------|---------------|----------------|-------------|
| $D^0_f(2580)$  | 2579.5 ± 3.4 ± 5.5 | 177.5 ± 17.7 ± 46.0 | UN |
| $D^0_f(2650)$  | 2649.2 ± 3.5 ± 3.5 | 140.2 ± 17.1 ± 18.6 | N |
| $D^0_f(2740)$  | 2737.0 ± 3.5 ± 11.2 | 73.2 ± 13.4 ± 25.0 | UN |
| $D^0_f(2760)$  | 2761.1 ± 5.1 ± 6.5 | 74.4 ± 4.3 ± 37.0 | N |
| $D^0_f(2760)$  | 2760.1 ± 1.1 ± 3.7 | 74.4 ± 3.4 ± 19.1 | N |
| $D^0_f(2760)$  | 2771.7 ± 1.7 ± 3.8 | 66.7 ± 6.6 ± 10.5 | N |
| $D^0_f(3000)$  | 2971.8 ± 8.7 | 1881 ± 44.8 | UN |
| $D^0_f(3000)$  | 3008.1 ± 4.0 | 1105 ± 11.5 | N |

$UN$ unnatural, $N$ natural

Table 3. Charged resonances observed by BABAR Collaboration [3]

| Resonance     | Mass (in MeV) | Width (in MeV) | Spin-parity |
|---------------|---------------|----------------|-------------|
| $D^0_f(2550)$  | 2539.4 ± 4.5 ± 6.8 | 130 ± 12 ± 13 | 0$^-$ |
| $D^0_f(2600)$  | 2608.7 ± 2.4 ± 2.5 | 93 ± 6 ± 13 | N |
| $D^0_f(2750)$  | 2752.4 ± 1.7 ± 2.7 | 71 ± 6 ± 11 | N |
| $D^0_f(2760)$  | 2763.3 ± 2.3 ± 2.3 | 60.9 ± 5.1 ± 3.6 | N |

Sustent for the quantitative study of quarkonium but not appropriate for mesonic systems consisting a heavy-light flavour quarks. For the hadrons consisting a heavy quark, $QCD$ exhibits additional symmetries within the limit that heavy quark mass $m_Q$ becomes infinite compared with the typical $QCD$ scale [4]. This symmetry can be realised systematically within relativistic Dirac equation employing the equally mixed scalar plus vector potential. This framework allows to make satisfactory predictions on excited charmed mesons. Hence, for the present study, we consider relativistic Dirac equation with equally mixed four-vector plus scalar power-law confinement potential for the single particle bound state energy of the quark and anti-quark. Using these single particle energies along with their $jj$ coupling, the massess of the charmed mesons are computed. We fix the potential parameters for the known ground state and predict the masses of the radial and orbital excited states.

Several models are available in literature dealing with decay properties of heavy-light mesons apart from their mass spectra [5–10]. The well known $^3P_0$ model or quark pair creation model is very effective in strong decay study of mesons [11–13]. This model was initially proposed by L. Micu [14] and later developed by Y. Le and collaborators [15–17]. The relativized quark model spectroscopy incorporating $^3P_0$ model for strong decays [5] supports the $D^0_f(2550)$ and $D^0_f(2600)$ as the radial excitation states. In their study they also identified $D^0_{13}(2760)$ and $D^0_{13}(2760)$ as the $^3D_1$ and $^3D_3$ states. But quoted that identification of $D^0_f(2750)$ state requires further measurements. They concluded that the favourable assignment for states $D^0_{13}(3000)$ and $D^0_{13}(3000)$ as $D^0_{13}(1^3F_3)$ and $D(3^1S_0)$, respectively. The $^3P_0$ is a simplified version of the complicated theory. So, predictability of this model is not accurate. Also, the oscillator parameter affects the shape of wave-functions significantly in $^3P_0$ model. More discussion regarding the analysis of uncertainties in $^3P_0$ predictions can be available in Refs. [18–20]. Another approach to predict the strong decays of heavy-light mesons is based on heavy quark effective theory where P. Colangelo et al. has proposed the framework to classify the hadrons in doublets involving the channels with a light final pseudoscalar mesons [21–24]. The work exploring the heavy quark effective theory for charmed mesons is covered in Refs. [25,26] by Wang. Very, recently Colangelo et al. [27] incorporated other channels with a light final vector mesons into their previous study. We believe that in the case of relativistic Dirac model mass predictions as an input, Colangelo et al.’s approach is more consistent as compared to $^3P_0$ model. In spite of the fact that Heavy quark effective theory contains many unknown phe-
The heavy-light mesons are composed of one heavy quark and other light quark, so the properties of heavy-light mesons are significantly influenced by relativistic effects. Such systems can be systematically emphasised within relativistic Dirac framework. The form of the average flavour potential) are the phenomenological parameters which are fixed for the ground state.

We should mention that Dirac equation [30,32]
\[ \psi(x) = \gamma \left[ \frac{1}{2} \gamma^\mu \bar{\alpha}_\mu - V(r) - m \right] \psi(x) \]  

The independent quark wave function \( \psi(\vec{r}) \) satisfies the Dirac equation [30,32]
\[ [\gamma^0 E - \vec{\gamma}.\vec{P} - m - V(r)] \psi(\vec{r}) = 0. \]
where \( E \) is the individual quark binding energy.

The solution to the independent-quark wave function (normalised) can be written as [31,32,34,35]
\[ \psi_{nlj}(r) = \left( \begin{array}{c} \psi_{nlj}^{(+)}(r) \\ \psi_{nlj}^{(-)}(r) \end{array} \right) \]

where
\[ \psi_{nlj}^{(+)}(\vec{r}) = N_{nlj} \left( \frac{ig(r)/r}{(\sigma.\hat{r})f(r)/r} \right) \chi_{ljm}(\hat{r}) \]  
\[ \psi_{nlj}^{(-)}(\vec{r}) = N_{nlj} \left( \frac{i(\sigma.\hat{r})f(r)/r}{g(r)/r} \right) (-1)^{j+m_l-1} \chi_{ljm}(\hat{r}) \]
\[ N_{nlj} \]

and \( N_{nlj} \) is the overall normalization constant [31,32,34,36]. The two spinors \( \chi_{ljm} \) are the eigenfunctions of the spin operators and explicitly written as [34]
\[ \chi_{\frac{1}{2}\frac{1}{2}} = \left( \begin{array}{c} 1 \\ 0 \end{array} \right), \chi_{\frac{1}{2}\frac{1}{2}} = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \]

The reduced radial parts \( g(r) \) and \( f(r) \) can be found to satisfy the following equations [6,31,32,35]
\[ \frac{d^2 g(r)}{dr^2} + \left[ (E_D + m_q)(E_D - m_q - V(r)) - \frac{\kappa(\kappa + 1)}{r^2} \right] g(r) = 0 \]
\[ \frac{d^2 f(r)}{dr^2} + \left[ (E_D + m_q)(E_D - m_q - V(r)) - \frac{\kappa(\kappa - 1)}{r^2} \right] f(r) = 0 \]

with the definition of quantum number \( \kappa \) as [34]
\[ \kappa = \left\{ \begin{array}{ll} - (\ell + 1) & \text{for } j = \ell + \frac{1}{2} \\ \ell & \text{for } j = \ell - \frac{1}{2} \end{array} \right. \]

Takin the form of \( V(r) \) as given in Eq. (1) and introducing a dimensionless variable \( \rho = \frac{r}{r_0} \) with an arbitrary scale factor [31,32,36,37]
\[ r_0 = \left( m_q + E_D \right)^{\frac{1}{2}}, \]
Eqs. (9) and (10) reduces to the Schrödinger form [31,32,36–38]
\[ \frac{d^2 g(\rho)}{d\rho^2} + \left[ \varepsilon - \rho^{\ell} - \frac{\kappa(\kappa + 1)}{\rho^2} \right] g(\rho) = 0 \]
\[ \frac{d^2 f(\rho)}{d\rho^2} + \left[ \varepsilon - \rho^{\ell} - \frac{\kappa(\kappa - 1)}{\rho^2} \right] f(\rho) = 0 \]
Table 4 The input parameters for the D mesons

| Parameter                      | Value                                    |
|--------------------------------|------------------------------------------|
| Quark mass (in GeV)            | $m_{u,d} = 0.003$ and $m_c = 1.27$       |
| Potential strength ($\lambda$) | $0.0352(2n + l + 1)^{0.5}$ GeV$^{0.5}$   |
| $V_0$                          | $-0.0001$ GeV                            |
| $\sigma (j - j$ coupling strength) | $0.0946$ GeV$^3$                        |

The solutions $g(\rho)$ and $f(\rho)$ are normalized to

$$
\int_0^\infty (f_2^2(\rho) + g_2^2(\rho)) \, d\rho = 1.
$$

(16)

The wavefunctions for heavy-light charmed meson now can be constructed as the symmetric combinations of the positive energy solution $\psi_Q$ and the negative energy solution $\psi_{\bar{Q}}$ of Eqs. (5) and (6) and the corresponding mass of the $Q\bar{Q}$ system can be written as

$$
M_{Q\bar{Q}} = E_D^Q + E_D^{\bar{Q}}
$$

(17)

where $E_D^Q$ and $E_D^{\bar{Q}}$ are obtained from (15) which include the centrifugal repulsion of the centre of mass also.

The mass of the state $M_{2S+1L_J}$ is obtained by adding the contributions from spin–spin, spin–orbit and tensor interactions of the confined one-gluon exchange potential (COGEP) between quark and anti-quark to $M_{Q\bar{Q}}$. Thus, we write

$$
M_{2S+1L_J} = M_{Q\bar{Q}} (n_1l_1j_1, n_2l_2j_2) + \langle V_{Q\bar{Q}}^{lj_1j_2} \rangle + \langle V_{Q\bar{Q}}^{LS} \rangle + \langle V_{Q\bar{Q}}^T \rangle
$$

(18)

The spin–spin coupling part is defined from COGEP as

$$
\langle V_{Q\bar{Q}}^{lj_1j_2} \rangle = \frac{\sigma \langle j_1j_2JM|\hat{j}_1\hat{j}_2|j_1j_2JM \rangle}{(E_Q + m_Q)(E_{\bar{Q}} + m_{\bar{Q}})}
$$

(19)

where $\sigma$ represents the $jj$ coupling constant. The angular brackets appearing in the term $\langle j_1j_2JM|\hat{j}_1\hat{j}_2|j_1j_2JM \rangle$ contains square of Clebsch–Gordan coefficients.

The tensor and spin–orbit parts of confined one-gluon exchange potential (COGEP) [39,40] have the form

$$
\langle V_{Q\bar{Q}}^T \rangle = -\frac{\alpha_s}{4} \frac{N_Q^2N_{\bar{Q}}^2}{(E_Q + m_Q)(E_{\bar{Q}} + m_{\bar{Q}})}
$$

and

$$
\langle V_{Q\bar{Q}}^{LS} \rangle = \lambda_Q\lambda_{\bar{Q}}\left(\frac{D^J_1(r) - D^J_1(r)}{3} \right) S_{Q\bar{Q}}
$$

(20)

where $N_D, N_{\bar{Q}}$ are the normalization constants of quark–antiquark wavefunctions, $S_{Q\bar{Q}} = [3(\sigma_Q\hat{r})(\sigma_{\bar{Q}}\hat{r}) - \sigma_Q\sigma_{\bar{Q}}]$ and $\hat{r} = \hat{r}_Q - \hat{r}_{\bar{Q}}$ is the unit vector in the direction of $\hat{r}$ and

$$
V_{Q\bar{Q}}^{LS}(r) = \frac{\alpha_s}{4} \frac{N_Q^2N_{\bar{Q}}^2}{(E_Q + m_Q)(E_{\bar{Q}} + m_{\bar{Q}})} \frac{\lambda_Q\lambda_{\bar{Q}}}{2} \left[ [\hat{r} \times (\hat{p}_{Q\bar{Q}} - \hat{p}_Q)].(\sigma_Q + \sigma_{\bar{Q}}) \right] [D^J_0(r) + 2D^J_1(r)] + [\hat{r} \times (\hat{p}_{Q\bar{Q}} + \hat{p}_Q).(-\sigma_Q - \sigma_{\bar{Q}})] [D^J_0(r) - D^J_1(r)]
$$

(21)

where $\alpha_s$ is the running strong coupling constant expressed as

$$
\alpha_s = \frac{4\pi}{(11 - 2\frac{2}{3} n_f) \log \left( \frac{M^2_{Q\bar{Q}}}{\Lambda^2_{QCD}} \right)}
$$

(22)

$n_f$ is number of flavours and for charmed mesons we take it as 3. The term $\langle \lambda_Q\lambda_{\bar{Q}} \rangle = -\frac{4}{3}$ is the color factor and independent of the flavour contents of the quarks [41].

The confined gluon propagators are given by [39,40,42] as

$$
D_0(r) = \left( \frac{\alpha_1}{r} + \frac{\alpha_2}{r} \right) \exp(-r^2c_0^2/2)
$$

(23)

and

$$
D_1(r) = \frac{\gamma}{r} \exp(-r^2c_1^2/2)
$$

(24)

with $\alpha_1 = 1.035, \alpha_2 = 0.3977$ GeV, $c_0 = 0.3418$ GeV, $\gamma = 0.8639$ and $c_1 = 0.4123$ GeV GeV as in the previous study [32,42,43]. Other model parameters employed in the present study are listed in Table 4. Results obtained from the present study are tabulated in Tables 5, 6 and 7. The present results are compared with the available average experimental values reported by PDG [33].

2.2 Strong decays of heavy-light mesons

The underlying dynamics of the hadrons involving single heavy quark can be understood systematically by considering the heavy quark mass limit tends to infinity ($m_Q \to \infty$) formalised in a heavy quark effective theory (HQET). This HQET limit allows to classify heavy-light ($Q\bar{q}$) mesons based upon the decoupling of the heavy quark spin $S_Q$ from the total angular momentum $J$ of the light degrees of freedom. Here, $Q$ corresponds to $c$ quark while $q$ refers to light quarks $u$ and $d$. The heavy quark spin $S_Q$ and total angular momentum of light d.o.f $J$ are separately conserved in strong interactions. This leads to classification of heavy mesons in doublets as per the different value of $J$. Each doublet here has two states, spin partners having total spin $J = J \pm \frac{1}{2}$ and parity $P = (-1)^{J+1}$.
Table 5 S-wave D meson ($c\bar{q}, q = u, d$) spectrum (in MeV)

| State ($s_J^p$)  | $M_{Q\bar{q}}$ (MeV) | $\langle V_{J1/2} \rangle$ | Present | Experiment |
|------------------|----------------------|---------------------------|---------|------------|
| $1^3S_1(1^-)$    | 1983.34              | 8.99                      | 2002.33 | $D^*(2010)$ (2010.26 ± 0.05) | … |
| $1^1S_0(0^-)$    | 1899.66              | −26.98                    | 1872.68 | $D$ (1867.83 ± 0.05)          | … |
| $2^3S_1(1^-)$    | 2651.19              | 4.55                      | 2655.75 | ?$D^*$ (2600) (2623 ± 12)    | … |
| $2^1S_0(0^-)$    | 2556.74              | −13.67                    | 2543.07 | ?$D$ (2550) (2564 ± 20)      | … |
| $3^3S_1(1^-)$    | 3140.05              | 3.23                      | 3143.28 | …                        | … |
| $3^1S_0(0^-)$    | 3054.00              | −9.69                     | 3044.31 | …                        | … |
| $4^3S_1(1^-)$    | 3639.83              | 2.43                      | 3642.26 | …                        | … |
| $4^1S_0(0^-)$    | 3558.06              | −7.29                     | 3550.76 | …                        | … |
| $5^3S_1(1^-)$    | 4114.29              | 1.93                      | 4116.22 | …                        | … |
| $5^1S_0(0^-)$    | 4036.00              | −5.80                     | 4030.19 | …                        | … |
| $6^3S_1(1^-)$    | 4569.18              | 1.59                      | 4570.78 | …                        | … |
| $6^1S_0(0^-)$    | 4493.77              | −4.79                     | 4488.98 | …                        | … |

where $l$ is the orbital angular momentum of light d.o.f. and $s_{\hat{J}} = \hat{l} + \hat{s}_{\hat{q}}$, $\hat{s}_{\hat{q}}$ being the spin of light antiquark.

For the lowest lying states $l = 0$ (S-wave states of the quark model), $s_J^p = \frac{1}{2}^-$ and doublet consists of two states along with spin-parity $J^P = (0^-, 1^-)$ represented by $(P, P^*)$. For $l = 1$ (P-wave states) one can write $s_J^p = \frac{1}{2}^+$ and $s_J^p = \frac{3}{2}^+$. The two doublets can be denoted as $J^P = (0^+, 1^+)$ with the members as $(P_0^*, P_1^*)$ and $J^P = (1^+, 2^+)$ with the members as $(P_1, P_2^*)$. Similarly, $l = 2$ (D-wave states) corresponds to $s_J^p = \frac{3}{2}^-$ and $s_J^p = \frac{5}{2}^-$. Here, one doublet is expressed as $J^P = (1^-, 2^-)$ consisting members as $(P_1^*, P_2)$ and another is $J^P = (2^-, 3^-)$ with $(P_1^*, P_2^*)$.

The matrix elements for the different channels are given by

$$\langle \mu \nu | \gamma_5 P_{\mu \nu} | \gamma_5 Q \bar{q} \rangle = \langle \mu \nu | \gamma_5 P_{\mu \nu} | \gamma_5 \bar{s} \bar{q} \rangle \times \left[ g^{\mu \nu} - \gamma_5 (g^{\mu \nu} + i\epsilon^{\mu \nu}) \right] \left( \frac{1}{3} \right)$$  \hspace{1cm} (28)

where $H_a (P, P^*)$ describes S-wave mesons; $S_a (P_0^*, P_1^*)$ and $T_a (P_1, P_2^*)$ associated with P-wave mesons; $X_a (P_1^*, P_2)$
| State ($s_l J^P$) | $M_{Q\bar{q}}$ (MeV) | $\langle V_{1/2} \rangle$ (MeV) | $\langle V_T \rangle$ (MeV) | $\langle V_{LS} \rangle$ (MeV) | Present | Experiment |
|------------------|----------------------|-----------------|-----------------|-----------------|--------|------------|
| $1^3 P_2(\frac{3}{2}^+)$ | 2458.43 | -8.053 | 16.60 | -1.84 | 2465.1 | \(D_2(2460) \ (2460.7 \pm 0.4)\) | \(D_2^*(2460) \ (2463.7 \pm 0.4)\) |
| $1^3 P_1(\frac{1}{2}^+)$ | 2458.43 | -26.84 | -16.60 | 9.19 | 2424.2 | \(D_1(2420) \ (2420.8 \pm 0.5)\) |
| $1^3 P_0(\frac{1}{2}^+)$ | 2458.43 | -16.11 | -33.20 | -18.37 | 2390.7 | \(D_0(2300) \ (2349 \pm 7)\) |
| $1^3 P_1(\frac{1}{2}^+)$ | 2353.82 | 65.26 | 0 | 0 | 2419.1 | \(D_1(2430) \ (2427 \pm 40)\) |
| $2^3 P_2(\frac{1}{2}^+)$ | 2995.54 | -5.32 | 11.99 | -1.33 | 3000.90 | \(\ldots\) |
| $2^3 P_1(\frac{1}{2}^+)$ | 2995.54 | -17.33 | -11.99 | 6.64 | 2972.50 | \(\ldots\) |
| $2^3 P_0(\frac{1}{2}^+)$ | 2995.54 | -10.64 | -23.99 | -13.28 | 2947.60 | \(\ldots\) |
| $2^1 P_1(\frac{1}{2}^+)$ | 2900.25 | 40.09 | 0 | 0 | 2940.13 | \(\ldots\) |
| $3^3 P_2(\frac{1}{2}^+)$ | 3498.17 | -3.93 | 9.55 | -1.06 | 3502.6 | \(\ldots\) |
| $3^3 P_1(\frac{1}{2}^+)$ | 3498.17 | -13.11 | -9.55 | 5.29 | 3480.8 | \(\ldots\) |
| $3^3 P_0(\frac{1}{2}^+)$ | 3498.17 | -7.86 | -19.09 | -10.58 | 3460.6 | \(\ldots\) |
| $3^3 P_1(\frac{1}{2}^+)$ | 3409.38 | 28.67 | 0 | 0 | 3449.5 | \(\ldots\) |
| $4^3 P_2(\frac{1}{2}^+)$ | 3975.79 | -3.09 | 7.99 | -0.89 | 3979.8 | \(\ldots\) |
| $4^3 P_1(\frac{1}{2}^+)$ | 3975.79 | -10.82 | -7.99 | 4.43 | 3961.9 | \(\ldots\) |
| $4^3 P_0(\frac{1}{2}^+)$ | 3975.79 | -6.18 | -15.98 | -8.87 | 3944.8 | \(\ldots\) |
| $4^1 P_1(\frac{1}{2}^+)$ | 3891.87 | 22.11 | 0 | 0 | 3914.0 | \(\ldots\) |
| $5^3 P_2(\frac{1}{2}^+)$ | 4433.77 | -2.53 | 6.90 | -0.77 | 4437.4 | \(\ldots\) |
| $5^3 P_1(\frac{1}{2}^+)$ | 4433.77 | -8.43 | -6.90 | 3.83 | 4422.3 | \(\ldots\) |
| $5^3 P_0(\frac{1}{2}^+)$ | 4433.77 | -5.06 | -13.80 | -7.67 | 4407.3 | \(\ldots\) |
| $5^1 P_1(\frac{3}{2}^+)$ | 4353.51 | 17.85 | 0 | 0 | 4371.4 | \(\ldots\) |
Table 7  D-wave and F-wave $D$ meson ($q\bar{q}, q = u, d$) spectrum (in MeV)

| State ($s\,J^P$) | $M_{Q\bar{q}}$ | $(V_{J12})$ | $(V_T)$ | $(V_{L5})$ | Present | Experiment |
|------------------|----------------|------------|--------|----------|---------|------------|
|                  |                |            |        |          |         | PDG [33]   | $LHCb$ [1] | $LHCb$ [2] |
| $1^3D_3(\frac{1}{2}^-)$ | 2771.05       | −30.46     | 28.03  | −2.21    | 2766.4  | $D_s^+(2750)(2763.5 \pm 3.4)$ | $D_s^+(2760)(2775.5 \pm 4.5)$ | $D_s^+(2760)(2760.1 \pm 1.1)$ |
| $1^3D_2(\frac{1}{2}^-)$ | 2771.05       | −27.29     | −14.01 | 7.75     | 2737.5  | $D_J(2740)(2737.0 \pm 3.5)$   | $D_J(2740)(2737.0 \pm 3.5)$   | $D_J(2740)(2737.0 \pm 3.5)$   |
| $1^3D_1(\frac{1}{2}^-)$ | 2771.05       | −8.72      | −42.04 | −7.75    | 2712.53 | ...                                    | ...                                    | ...                                    |
| $1^1D_2(\frac{1}{2}^-)$ | 2671.12       | 54.22      | 0      | 0        | 2725.34 | $D_J(2740)(2737.0 \pm 3.5)$   | $D_J(2740)(2737.0 \pm 3.5)$   | $D_J(2740)(2737.0 \pm 3.5)$   |
| $2^3D_3(\frac{1}{2}^-)$ | 3354.29       | −20.83     | 21.55  | −1.70    | 3353.3  | ...                                    | ...                                    | ...                                    |
| $2^3D_2(\frac{1}{2}^-)$ | 3354.29       | −18.67     | −10.77 | 5.96     | 3330.82 | ...                                    | ...                                    | ...                                    |
| $2^3D_1(\frac{1}{2}^-)$ | 3354.29       | −5.97      | −32.31 | −5.96    | 3310.04 | ...                                    | ...                                    | ...                                    |
| $2^1D_2(\frac{1}{2}^-)$ | 3258.33       | 35.49      | 0      | 0        | 3293.82 | ...                                    | ...                                    | ...                                    |
| $3^3D_3(\frac{1}{2}^-)$ | 3835.63       | −16.16     | 17.66  | −1.40    | 3835.73 | ...                                    | ...                                    | ...                                    |
| $3^3D_2(\frac{1}{2}^-)$ | 3835.63       | −14.48     | −8.83  | 4.89     | 3817.21 | ...                                    | ...                                    | ...                                    |
| $3^3D_1(\frac{1}{2}^-)$ | 3835.63       | −4.63      | −26.48 | −4.89    | 3799.62 | ...                                    | ...                                    | ...                                    |
| $3^1D_2(\frac{1}{2}^-)$ | 3746.04       | 22.78      | 0      | 0        | 3772.9  | ...                                    | ...                                    | ...                                    |
| $4^3D_3(\frac{1}{2}^-)$ | 4297.05       | −13.14     | 15.04  | −1.19    | 4297.74 | ...                                    | ...                                    | ...                                    |
| $4^3D_2(\frac{1}{2}^-)$ | 4297.05       | −11.77     | −7.51  | 4.17     | 4281.93 | ...                                    | ...                                    | ...                                    |
| $4^3D_1(\frac{1}{2}^-)$ | 4297.05       | −3.76      | −22.53 | −4.17    | 4266.6  | ...                                    | ...                                    | ...                                    |
| $4^1D_2(\frac{1}{2}^-)$ | 4212.28       | 21.43      | 0      | 0        | 4233.71 | ...                                    | ...                                    | ...                                    |
| $1^3F_5(\frac{1}{2}^+)$ | 3230.46       | −11.18     | 35.48  | −2.18    | 3252.55 | ...                                    | ...                                    | ...                                    |
| $1^3F_3(\frac{1}{2}^+)$ | 3230.46       | −16.97     | −11.81 | 6.54     | 3208.21 | ...                                    | ...                                    | ...                                    |
| $1^3F_2(\frac{3}{2}^+)$ | 3230.46       | −5.38      | −47.26 | −5.23    | 3172.58 | ...                                    | ...                                    | ...                                    |
| $1^1F_3(\frac{3}{2}^+)$ | 3125.76       | 38.73      | 0      | 0        | 3164.49 | ...                                    | ...                                    | ...                                    |
and $Y_a (P'_a, P^+_a)$ represents D-wave mesons; $Z_a (P^+_a, P_3)$ and $R_a (P'_a, P^+_a)$ related with F-wave mesons. Here, $a = u, d$ or $s$ is $SU(3)$ light flavour index and $v$ corresponds to meson four-velocity. For radial quantum numbers $n = 1, 2, 3, 4, \ldots$, heavy mesons with the same heavy flavour holds identical parity, time-reversal, charge conjugation properties and only differ in the mass. So, it is possible to combine them into effective fields $H_a, H'_a, H''_a \ldots S_a, S'_a, S''_a \ldots$ where $i, i', i''$ indicates $n = 2, 3, 4 \ldots$ states. Following this analogy, for example, for radial excitation $n = 2$ one can represent the states by tilde ($\tilde{P}, \tilde{P}^+$).

The definition $\xi = e^{i\frac{\Delta}{\Lambda}}$ introduce octet of light pseudoscalar mesons in the theory through the matrix

$$\mathcal{M} = \left( \begin{array}{cc} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^0 \frac{K^0}{\sqrt{2}} \\ \pi^- \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \frac{K^0}{\sqrt{3}} \end{array} \right).$$

containing light pseudoscalar $\pi, K$ and $\eta$ fields. We take $f_\pi = 130$ MeV for calculations.

At the leading order approximation in the heavy quark mass and light quark momentum, the interaction Lagrangian terms $\mathcal{L}_H, \mathcal{L}_S, \mathcal{L}_T, \mathcal{L}_X, \mathcal{L}_Y, \mathcal{L}_Z$ and $\mathcal{L}_R$ can be described as [44]

$$\mathcal{L}_H = g_H \left( \hat{H}_a \hat{H}_b \gamma_{\mu} \gamma_{5} A_{\mu}^{\leftrightarrow} \right)$$

$$\mathcal{L}_S = g_S \left( \hat{H}_a \gamma_{\mu} \gamma_{5} A_{\mu}^{\leftrightarrow} \right) + H.c.$$  \hspace{1cm} (33)

At the leading order approximation in the heavy quark mass and light quark momentum, the interaction Lagrangian terms $\mathcal{L}_H, \mathcal{L}_S, \mathcal{L}_T, \mathcal{L}_X, \mathcal{L}_Y, \mathcal{L}_Z$ and $\mathcal{L}_R$ can be described as [44]

$$\mathcal{L}_H = g_H \left( \hat{H}_a \hat{H}_b \gamma_{\mu} \gamma_{5} A_{\mu}^{\leftrightarrow} \right)$$

$$\mathcal{L}_S = g_S \left( \hat{H}_a \gamma_{\mu} \gamma_{5} A_{\mu}^{\leftrightarrow} \right) + H.c.$$  \hspace{1cm} (33)

$$\mathcal{L}_T = \frac{g_T}{\Lambda} \left( \hat{H}_a T_{\mu}^{\leftrightarrow} \left( i D_{\mu} A + i \mathcal{D} A_{\mu} \right) \gamma_{5} \right) + H.c.$$  \hspace{1cm} (34)

The vector and axial-vector currents are defined as

$$V_{\mu} = \frac{1}{2} (\xi^\dagger \partial_{\mu} \xi + \xi \partial_{\mu} \xi^\dagger)$$  \hspace{1cm} (39)
The values of the coefficients $C_p$ involved are different for different pseudoscalar mesons: $C_{π^+} = C_{K^+} = 1$, $C_{π^0} = 1/2$, $C_{η} = 3/2$ [24]. The strong coupling constants and their notations depends upon the radial quantum number $n$. For transitions within $n_i = n_f = 1$ notations are $g_{H,S,T,X,Y,Z,R}$ while for $n_i = 2$ and $n_f = 1$ they are $g_{H,S,T,X,Y,Z,R}^2$. Higher order loop corrections are eliminated to bypass the introduction of new coupling constants. For the present study we adopt the approximation $A_{1} = i \frac{\partial}{\partial M}$. If the momenta of the emitted light pseudoscalar mesons are not very small, the additional terms can be added to introduce unknown coupling constants. Moreover, the spin and the flavour violation correction having the order of $O(\frac{1}{M^2})$ to the heavy quark limit could be sizable, and in that case introduction of new coupling constants to the theory may not cancel out in the ratios of the decay widths. But we expect that their contribution would be much less than the leading order contributions. In decay rates the leading order unknown coupling constants can be theoretically predicted or can be evaluated from the available experimental data of decay widths. Successful predictions on coupling constants based on the QCD sum rules [21] and Lattice QCD [45] are found in literature. The numerical values of the meson masses used as input parameters for present calculations are $M_{D^{*+}} = 2010.26$ MeV, $M_{D^{**+}} = 2112.2$ MeV, $M_{D^{*0}} = 2006.85$ MeV, $M_{D^+} = 1869.65$ MeV, $M_{D_s^+} = 1968.34$ MeV, $M_{D_s^0} = 1864.83$ MeV, $M_{\eta^0} = 134.97$ MeV, $M_{\eta^-} = 139.57$ MeV, $M_{K^+} = 493.67$ MeV, $M_{\eta} = 547.86$ MeV [33].

### 3 Results and discussion

The mass spectra of the charmed mesons ($c\bar{q}$); $q = u$ or $d$ are computed by employing the relativistic Dirac framework and listed in Tables 5, 6 and 7. The states $D$ (1867.83 ± 0.05) and $D^*$ (2010.26 ± 0.05) are well-established ground state of $D$ mesons. Our estimation of the mass of $1^{1}S_0$ is 1872.68 MeV and for $1^{3}S_1$ is 2002.33 MeV; which is found to be in close agreement with reported mass of $D$ and $D^*$ (2010) [33]. However, if we incorporate the uncertainty of $\Delta m_c = 1.5\%$ as per $m_c = 1.27\pm 0.02$ GeV [33] results into just 1% variation in the predicted $D$ meson mass for $1^{3}S_1$ state and that for $1^{1}S_0$ state. Here, as up and down quark mass and their uncertainty are negligible (few MeV only) we don’t consider it in the uncertainty estimations. The other parameters $\lambda$, $V_0$ and $\sigma$ are the optimized potential parameters which are fixed to yield the experimental ground state masses. The variation of the parameter $\lambda$ (potential strength) of 5% and 10% keeping other two fixed leads to the uncertainty of less than 1% and 2% in mass of $1S$ state. Also, The variation of the parameter $\sigma$ (coupling strength) of 5% and 10% keeping other two fixed gives uncertainty of less than 0.07% and 0.09% in mass of $1S$
state. The third parameter \( V_0 \) variation results into negligible variations in the mass of 1S state. So, the variations are not much compared to the optimized value of these parameters. However, the value of \( \lambda \) is slightly more sensitive than that of \( V_0 \) and \( \sigma \). The detailed results on the sensitivity of charm quark mass and these model parameters are presented Table 16 and 17 of Appendix B.

The states \( D_2^*(2460) \), \( D_1^*(2420) \), \( D_0^*(2400) \) and \( D_1^*(2430) \) are also well defined \( J^P \) multiplets. Our predictions for these states are in excellent agreement with PDG listed values [33]. With the successful description of 1S and 1P multiplets, we are able to assign proper \( J^P \) values of the newly observed states by LHCb Collaboration [1,2] \( D_J^*(2580) \), \( D_{J^*}^*(2650) \), \( D_J^*(2740) \), \( D_J^*(2760) \), \( D_J^*(3000) \), \( D_{J^*}^*(3000) \) and \( D_{J^*}^*(3000) \). Further to check the reliability of the present formalism in terms of the strong decays we have computed the ratio \( \Gamma(D^+ \pi^-)/\Gamma(D^{*+} \pi^-) \) for the well-established \( D_{J^*}^*(2460) \) state. The transitions of the \( 1P_3 \) multiplets \( (1^3 P_2, 1^3 P_1) \) to ground state charm meson by emitting a pseudoscalar meson are described by the coupling constant \( g_T \). The partial decay widths in terms of the \( g_T \) are listed in Table 8 for \( D_{J^*}^*(2460) \). In Table 9 the computed ratio, \( \Gamma(D_{J^*}^*(2460) \to D^{*+} \pi^-)/\Gamma(D_{J^*}^*(2460) \to D^{*+} \pi^-) \) is compared with those reported by various experimental groups. The average experimental value found to be \( 2.35 \pm 0.6 \) which is in good agreement with the value of 2.26 predicted by the present study. This indicates the usefulness of the present formalism in the prediction of strong decay widths. From the present study, we found \( g_T = 0.40 \pm 0.003 \) which is in excellent agreement with 0.43 \( \pm 0.05 \) reported by in [46] and 0.43 \( \pm 0.01 \) reported by [26]. The details of computing the uncertainty in the estimation of coupling constants are shown in Appendix C.

The LHCb [2] and BABAR [3] have observed states around the mass region 2550 MeV with unnatural parity and 2600 MeV with natural parity with different labels. But the mass and the widths of these states are so close making them to be considered having the same \( J^P \) values. Also, their properties and mass range make them suitable candidates for radial excited (2S) states of the charm mesons. In the case of \( D(2550) \), the measured total width by LHCb [2] and BABAR [3] contain very large error bar. The mass difference of LHCb [2] and BABAR [3] is roughly 40 MeV (see Table 5). From our predictions the mass of \( 2^1 S_0 \) is 2543.07 MeV which is in good agreement with the PDG average value of 2564 \( \pm 20 \) MeV. Similarly for the \( D(2600) \) our predicted mass is 2655 MeV. The mass and total width difference between LHCb [2] and BABAR [3] is around again 40 MeV and 47 MeV with large error bars, respectively. The latest effort of LHCb has reported \( D_{1}^*(2680) \) with 2681.1 \( \pm 5.6 \) MeV mass and 186.7 \( \pm 8.5 \) MeV width. Hence, we identify the \( D(2580) \) and \( D_{J^*}^*(2650) \) as \( 2^1 S_0 \) and \( 2^3 S_1 \) respectively. The partial decay widths in terms of the effective coupling constants are listed in Table 8 for \( D_{J}^*(2580) \) and \( D_{J}^*(2650) \) using our computed masses of \( 2^1 S_0 \) and \( 2^1 S_0 \). The total decay width \( \Gamma(2580) = 1101.39 \sqrt{g_H^2} \) MeV and \( \Gamma(2650) = 2627.78 \sqrt{g_H^2} \) MeV and can be compared with the experimentally measured width to extract the effective coupling constant. Considering \( D_{J}^*(2580) \), \( D_{J}^*(2650) \) as radial excited states the average effective coupling constant \( \bar{g}_H \) is deduced as \( 0.31 \pm 0.017 \) which is close to 0.28 \( \pm 0.01 \) reported by [26] but double the value of 0.14 \( \pm 0.03 \) reported by Ref [46]. This, radial excited state can also decay to 1P states emitting the light pseudoscalar mesons (\( \pi, \eta, K \)). Their incorporation requires additional introduction of the unknown coupling constants into the theory. So, to avoid complexity we do not consider them here.

BABAR and LHCb also reported many states falling within the mass region between 2740 and 2800 MeV [2, 3, 47, 48]. These observed states can be arranged into natural and unnatural parity states. The natural parity states are grouped with designation \( D_J^*(2760) \) (2763.5 \( \pm 3.4 \) MeV) and unnatural parity states are labeled as \( D_J^*(2740) \) (2737 \( \pm 3.5 \) MeV). This mass range is predicted to be close to 1D and 2P multiplets [5]. The quantum numbers of these states as 2P multiplets are found to be inconsistent with LHCb [47, 48] measurements. Assuming both of them to be spin partners of each other, the possible spin-parity assignments, for \( D_J^*(2760) \) are \( 1^3 D_1(\frac{1}{2}^+) \) and \( 1^3 D_3(\frac{3}{2}^-) \) while for \( D_J^*(2740) \) are \( (1^1 D_2)(\frac{1}{2}^+) \) and \( (1^3 D_2)(\frac{3}{2}^-) \). The predicted masses from the present study for \( 1^3 D_3 \) (2766.4 MeV) and \( 1^3 D_2 \) (2737.5 MeV) are close to reported mass of \( D_{J}^*(2760) \) and \( D_J^*(2740) \). The mass predicted for states \( 1^3 D_1 \) (2712.53 MeV) and \( 1^1 D_2 \) (2725.34 MeV) are also in good agreement with respect to reported mass of \( D_{J}^*(2760) \) and \( D_J^*(2740) \). For both the cases \( J^P \) value for \( D_J^*(2740) \) is \( 2^- \). The partial decay widths in terms of the effective coupling constants are listed in Table 8 for \( D_{J}^*(2760) \) and \( D_J^*(2740) \) for both possible assignments. The \( D^+ \pi^- \) mode found to be dominant in both \( 1^3 D_1 \) and \( 1^3 D_3 \). The ratio, \( \frac{\Gamma(D^+ \pi^-)}{\Gamma(D^{*+} \pi^-)} \) for \( 1^3 D_1 \) is found to be 4.05 whereas for \( 1^3 D_3 \) it is 1.93. These suggests that \( D^+ \pi^- \) mode is more dominant in \( 1^3 D_1 \). The BABAR Collaboration has observed \( D^*(2760) \) signal in \( D^+ \pi^- \) mode very close in masss to the \( D(2750) \) signal observed in \( D^{*+} \pi^- \) [3]. The state \( D_{J}^*(2760) \) was observed in both \( D^{*+} \pi^- \) and \( D^+ \pi^- \) decay modes [2]. The LHCb has reported two \( D(2760) \) states: One assigned as \( 1^- \) [49] and other as \( 3^- \) [48] in different analysis. For the case of \( 1^- \) assignment, the reported mass and total width are \( 2781 \pm 18 \pm 11 \pm 6 \) MeV and 177 \pm 32 \pm 20 \pm 7 \) MeV respectively. The three quoted errors are statistical, experimental systematic and model uncertainties, respectively. While in the case of \( 3^- \) assignment (\( m = 2798 \pm 7 \pm 1 \pm 7 \) MeV ; \( \Gamma = 105 \pm 18 \pm 6 \pm 23 \) (Isobar) and \( m = 2802 \pm 11 \pm 10 \pm 3 \))
Table 8 The strong decay widths (in MeV) for charmed resonance with possible spin-parity assignments. The ratio is calculated from $\Gamma(D_j^+ \rightarrow D^{++}\pi^-)$. Fraction (in%) represents the percentage of the partial decay width with respect to total decay width.

| Resonance | State $(s_tJ^P)$ | Decay mode | Width | Ratio | Fraction | Exp (in MeV) |
|-----------|----------------|------------|-------|-------|----------|--------------|
| $D_J(2580)$ | $2^1S_0 (\frac{3}{2}^+)$ | $D^{++}\pi^-$ | $725.18 \; g_H^2$ | 1.00 | 65.84 | |
| | | $D^{++}\pi^-$ | $371.29 \; g_H^2$ | 0.51 | 33.71 | |
| | | $D^{++}\pi^-$ | $492 \; g_H^2$ | 0.00 | 0.44 | |
| | | Total | $1161.39 \; g_H^2$ | | | $177.5 \pm 17.8$ [2] |
| $D_J^*(2650)$ | $2^3S_1 (\frac{1}{2}^-)$ | $D^+\pi^-$ | $631.90 \; g_H^2$ | 0.78 | 24.04 | |
| | | $D^+\pi^-$ | $165.17 \; g_H^2$ | 0.20 | 6.28 | |
| | | $D^+\pi^-$ | $321.17 \; g_H^2$ | 0.39 | 12.22 | |
| | | $D^+\pi^-$ | $164.47 \; g_H^2$ | 0.20 | 6.26 | |
| | | $D^+\pi^-$ | $805.35 \; g_H^2$ | 1.00 | 30.65 | |
| | | $D^+\pi^-$ | $41.82 \; g_H^2$ | 0.05 | 1.59 | |
| | | $D^+\pi^-$ | $409.79 \; g_H^2$ | 0.50 | 15.59 | |
| | | $D^+\pi^-$ | $88.11 \; g_H^2$ | 0.10 | 3.35 | |
| | | Total | $2627.78 \; g_H^2$ | | | $140.2 \pm 17.1$ [2] |
| $D_J^*(2460)$ | $1^3P_2 (\frac{1}{2}^+)$ | $D^+\pi^-$ | $129.75 \; g_H^2$ | 2.25 | 41.32 | |
| | | $D^+\pi^-$ | $67.78 \; g_T^2$ | 1.18 | 23.66 | |
| | | $D^+\pi^-$ | $11.11 \; g_T^2$ | 0.02 | 0.39 | |
| | | $D^+\pi^-$ | $57.43 \; g_T^2$ | 1.00 | 20.05 | |
| | | $D^+\pi^-$ | $30.26 \; g_T^2$ | 0.52 | 10.56 | |
| | | Total | $286.39 \; g_T^2$ | | | $47.0 \pm 0.8$ [1] |
| $D_J^*(2760)$ | $1^3D_1 (\frac{1}{2}^-)$ | $D^+\pi^-$ | $1020.91 \; g_X^2$ | 4.05 | 38.06 | |
| | | $D^+\pi^-$ | $312.76 \; g_X^2$ | 1.24 | 11.66 | |
| | | $D^+\pi^-$ | $522.46 \; g_X^2$ | 2.07 | 19.47 | |
| | | $D^+\pi^-$ | $362.46 \; g_X^2$ | 1.43 | 13.51 | |
| | | $D^+\pi^-$ | $251.99 \; g_X^2$ | 1.00 | 9.39 | |
| | | $D^+\pi^-$ | $30.13 \; g_X^2$ | 0.12 | 1.12 | |
| | | $D^+\pi^-$ | $128.89 \; g_X^2$ | 0.51 | 4.80 | |
| | | $D^+\pi^-$ | $52.48 \; g_X^2$ | 0.21 | 1.95 | |
| | | Total | $2682.08 \; g_X^2$ | | | $74.4 \pm 3.4$ [2] |
| $D_J^*(2760)$ | $1^3D_3 (\frac{1}{2}^-)$ | $D^+\pi^-$ | $179.99 \; g_T^2$ | 1.93 | 38.59 | |
| | | $D^+\pi^-$ | $18.88 \; g_T^2$ | 0.20 | 4.04 | |
| | | $D^+\pi^-$ | $93.14 \; g_T^2$ | 1.00 | 19.97 | |
| | | $D^+\pi^-$ | $25.58 \; g_T^2$ | 0.27 | 5.48 | |
| | | $D^+\pi^-$ | $93.04 \; g_T^2$ | 1.00 | 19.95 | |
| | | $D^+\pi^-$ | $2.35 \; g_T^2$ | 0.02 | 0.50 | |
| | | $D^+\pi^-$ | $48.13 \; g_T^2$ | 0.51 | 10.31 | |
| | | $D^+\pi^-$ | $5.26 \; g_T^2$ | 0.06 | 1.12 | |
| | | Total | $466.37 \; g_T^2$ | | | $74.4 \pm 3.4$ [2] |
| $D_J(2740)$ | $1^3D_2 (\frac{3}{2}^-)$ | $D^+\pi^-$ | $817.76 \; g_X^2$ | 1.00 | 53.41 | |
| | | $D^+\pi^-$ | $111.95 \; g_X^2$ | 0.14 | 7.31 | |
| | | $D^+\pi^-$ | $418.04 \; g_X^2$ | 0.51 | 27.30 | |
| | | $D^+\pi^-$ | $183.30 \; g_X^2$ | 0.22 | 11.97 | |
| | | Total | $1531.06 \; g_X^2$ | | | $73.2 \pm 13.4$ [2] |
The values of ratio \( \frac{\Gamma(D_s^*(2460)^0 \rightarrow D^{*+}\pi^-)}{\Gamma(D_s^*(2460)^0 \rightarrow D^{*-}\pi^+)} \) from various experiments and from present work

| Present | 2.26 |
| BABAR  | 1.47 ± 0.03 ± 0.16 |
| CLEO   | 2.2 ± 0.7 ± 0.6 |
| CLEO   | 2.3 ± 0.8 |
| ARGUS  | 3.0 ± 1.1 ± 1.5 |
| ZEUS   | 2.8 ± 0.8 ± 0.5 |
| Exp. average | 2.35 ± 0.6 |

Looking into the good agreement for 1S, 2S and 1P masses, we believe that for higher excited states, our predictions are reliable. The predicted masses from the present study for 3S1 (3143.28 MeV), 3F2 (3172.58 MeV), 3F4 (3252.55 MeV), 3F6 (3044.31 MeV), 1F3 (3164.49 MeV), 1F4 (3208.21 MeV) are high as compared to 3008.1 ± 4 MeV and 2971.8 ± 8 MeV for \( D_s^* \) (3000) and \( D_s(3000) \). However, the mass of 23P0 (2947.60 MeV) and 21P1 (2940.13 MeV) are comparable whereas mass of 33P2 (3000.90 MeV) and 23P1 (2972.50 MeV) are close to reported mass of \( D_s^* \) (3000) and \( D_s(3000) \). The partial decay widths in terms of the effective coupling constants are listed in Tables 10 and 11 for \( D^* \) (3000) and \( D_s(3000) \) respectively where we have used our computed masses. Among the five possibilities later two can be completely ruled out as 13F4, 21F3 resulting very small decay width and 13F4, 10F3 have very large decay width which is far from the experimentally predicted widths of the \( D_s^* \) (3000) and \( D_s(3000) \) [2]. The state \( D_s^* \) (3000) decays to \( D^{*+}\pi^- \) whereas \( D_s(3000) \) decays to \( D^{*+}\pi^- \) final state [2]. If we identify \( D_s^* \) (3000) as 3S1 then the dominant mode is \( D^{*+}\pi^- \) which is not favoured by experiment. The other modes such as \( D^0\pi^0 \), \( D^0\eta \), \( D^{*+}\eta \) are very small. However, the two remaining possibilities leads to the \( J^P \) of \( D_s^* \) (3000) as \( 1^+ \). If we consider the \( D^* \) (3000) 23P0 then the \( D^*\pi \) mode is completely forbidden and \( D\pi \) mode is dominant. This fact is consistent with the experimental observation. Also, the \( D_s^* \) K\(^-\) and \( D^0\eta \) modes are considerably large which is supported by experimental analysis. Similarly if we assign \( D_s(3000) \) as 21P1, then \( D_s^* \) K\(^-\) and \( D^{0*}\eta \) modes are sufficiently large as compared to 23P1. With all such considerations, we identify \( D_s^* \) (3000) as 23P1 and \( D_s(3000) \) as 21P1. The previous study Ref. [50] supports this assignments for \( D_s^* \) (3000) and \( D_s(3000) \) states while the other studies [18] argues that \( D_s^* \) (3000) could be 13F4 or 13F2 and

The experimental value of the mass and the total decay width of \( D_s^* \) (3000) is 3008.1 ± 4.0 MeV and 110.5 ± 11.5 MeV [2]. The measured mass of the state \( D_s(3000) \) is 2971.8 ± 8.7 MeV and width is 188.1 ± 44.8 MeV [2]. From the present analysis we may assign these states as the 3S, 2P or 1F multiplets. Considering both of them to be spin partners then the following five assignments of the \( J^P \) for natural parity state \( D_s^* \) (3000) and unnatural parity state \( D_s(3000) \) are possible. 
$D_j(3000)$ could be 1 $F_2$ or 3 $P_j$. Ref. [7] suggests $D_j^*(3000)$ is a $1^+ P_2$ and $D_j(3000)$ is a $2^+ F_2$ state. All these possibilities also can not be ignored and more precise measurements are needed from the experimental side to clarify the puzzles in $D(3000)$ states. Considering $D_j^*(3000)$ and $D_j(3000)$ as $2P(0^+,1^+_1)g$ multiplets, the predicted value of the effective coupling constant $\tilde{g}_F$ found to be 0.10 ± 0.015 which would be useful for the future investigations on heavy-light systems. The numerical values of the decay widths and BRs emitting the light vector meson channels such as $D_0$, $D_1 K^*$ and $D_0\omega$ can also be incorporated to make precise predictions based on the heavy quark effective theory framework [27]. Recently, the $LHCb$ group has observed a state labelled as $D_j^*(3000)$ with mass 3214 ± 29 ± 33 MeV and total width 186 ± 38 ± 34 MeV [1]. The labelling of this newly observed state in $B^-\rightarrow D^+\pi^-\pi^-$ decays and previously reported state $D_j^*(3000)$ are the same. However, the $LHCb$ collaboration assigned $J^P = 2^+$ for $D_j^*(3000)$ while $D_j^*(3000)$ has a natural parity. The mass energy difference between $D_j^*(3000)$ and $D_j^*(3000)$ is 206 MeV which suggests that they both are different states [51]. The mass range 3214 ± 29 ± 33 MeV likely to fall in the mass spectra of 2P, 3P and 1F multiplets. From the present study mass of $1^3 F_4$ is close to 3214 ± 29 ± 33 MeV but the quantum numbers are not consistent with this assignment and for $3^3 P_2$ the mass difference is more than 240 MeV. So, the two most probable assignments for $D_j^*(3000)$ are $1^3 F_2$ or $2^3 P_2$. From the present study the mass of $1^3 F_2$ is 3172.58 MeV while the mass of $3^3 P_2$ is 3000.90 MeV. Allowed decay channels for $D_j^*(3000)$ are given in terms of coupling constant $g_2$ and $\tilde{g}_T$ in Table 12. The two different possibilities bring out different decay widths. The ratio $R_{j0}^{10}$ of various partial widths and branching fractions can be useful in the identification of the new signals with predicted states in experiments. Using the partial widths such ratios are defined as [27]

\[
R_{jk}^{10} = \frac{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi_0)}{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi^-)} \frac{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi^-)}{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi^-)} \frac{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi^-)}{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi^-)}
\]

The ratios for both the possibilities are presented in Table 13. It can be noted that the ratio $R_{k}^{20}$ has less sensitivity to the 2 $P$ and 1 $P$ identification while $R_{k}^{20}$ is highly sensitive. In both the cases $D^{*-\pi^-}$ mode is found to be dominant. So, to get more insights we also considered the ratio $\frac{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi^-)}{\Gamma(D_j^{10}(3000) \rightarrow D_j^{10} \pi^-)}$ for $1^3 F_2$ and $2^3 P_2$ states. This ratio is found to be 0.95 for $2^3 P_2$ while it is 0.38 for $1^3 F_2$. This indicates that $D^{*+\pi^-}$ mode is more dominant in $2^3 P_2$ as compared to $1^3 F_2$. On the

### Table 10

The strong decay widths (in MeV) for resonance $D_j^*(3000)$ with possible spin-parity assignments

| Decay mode     | $3^1 S_0(1^-)$ (in $\tilde{g}_F$ MeV) | $2^1 P_1(1^1^-)$ (in $\tilde{g}_F$ MeV) | $2^1 P_1(1^+)$ (in $\tilde{s}_F$ MeV) | $1^1 F_1(1^3^-)$ (in $\tilde{g}_F$ MeV) | $1^1 F_3(1^3^-)$ (in $\tilde{g}_F$ MeV) |
|---------------|--------------------------------------|---------------------------------------|--------------------------------------|---------------------------------------|---------------------------------------|
| $D^+\pi^-$    | 1918.58                              | 4076.82                               | 1959.61                              | 2209.78                               | 10139.80                             |
| $D^+K^-$      | 1262.91                              | 3247.92                               | 767.10                               | 1048.74                               | 3198.69                              |
| $D^0\pi^0$    | 966.27                               | 2054.66                               | 996.27                               | 1125.38                               | 5191.89                              |
| $D^0\eta$     | 950.14                               | 2629.14                               | 689.21                               | 1005.76                               | 3210.22                              |
| $D^{*+}\pi^-$ | 3140.96                              | ...                                   | 1854.71                              | 857.50                                | 6552.91                              |
| $D^{*+}K^-$   | 1804.16                              | ...                                   | 512.26                               | 324.23                                | 1521.38                              |
| $D^{*0}\pi^0$ | 1581.56                              | ...                                   | 942.13                               | 435.86                                | 3347.20                              |
| $D^{*0}\eta$  | 1425.63                              | ...                                   | 517.46                               | 343.56                                | 1708.31                              |
| Total         | 13050.21                             | 12008.54                              | 8238.75                              | 7350.81                               | 34870.40                             |

### Table 11

The strong decay widths (in MeV) for resonance $D_j(3000)$ with possible spin-parity assignments

| Decay mode     | $3^1 S_0(1^0)$ (in $\tilde{g}_F$ MeV) | $2^1 P_1(1^1^-)$ (in $\tilde{g}_F$ MeV) | $2^1 P_1(1^+)$ (in $\tilde{s}_F$ MeV) | $1^1 F_1(1^3^-)$ (in $\tilde{g}_F$ MeV) | $1^1 F_3(1^3^-)$ (in $\tilde{g}_F$ MeV) |
|---------------|--------------------------------------|---------------------------------------|--------------------------------------|---------------------------------------|---------------------------------------|
| $D^{*+}\pi^-$ | 3822.46                              | 3078.66                               | 2742.03                              | 2059.98                               | 9000.80                              |
| $D^{*+}K^-$   | 1934.80                              | 2179.07                               | 692.41                               | 768.04                                | 1869.22                              |
| $D^{*0}\pi^0$ | 1926.77                              | 1550.91                               | 1393.86                              | 1047.26                               | 4603.02                              |
| $D^{*0}\eta$  | 1857.50                              | 1892.95                               | 718.41                               | 818.25                                | 2171.57                              |
| Total         | 9271.73                              | 8701.59                               | 5546.77                              | 4693.52                               | 17645.3                              |
branching fraction 44.18% for the decay mode $D^{*+}\pi^−$. For the other modes, the contribution is suppressed. If we consider the ratio $gZ = 0.12$, then the total decay width of $^{1}F_3$ found to be 67.15 MeV. Thus, the total width of $^{1}F_3$ is narrower than its spin partner $^{13}F_2$ ($Γ = 110.5 ± 11.5$ MeV). The branching fraction 44.18% for the decay mode $D^{*+}\pi^−$ suggests as an appropriate channel for the experimental search of $^{1}F_3$. Finally, we conclude that the predicted masses of several states of charmed mesons are in good agreement with the experimental values. And the mass spectroscopy is found to be consistent in predictions of strong decays of various $D$-mesonic states.

| Decay mode | $2^3P_2(\frac{1}{2}^+)_{(\pi^0 \eta)}$ | Ratio | Fraction |
|------------|-----------------|--------|---------|
| $D^+\pi^−$ | 1959.61         | 1.06   | 23.78   |
| $D^+_sK^−$ | 767.10          | 0.41   | 9.31    |
| $D^0\pi^0$ | 966.27          | 0.53   | 12.09   |
| $D^0\eta$  | 689.21          | 0.37   | 8.36    |
| $D^{*+}\pi^−$ | 1854.71      | 1      | 22.78   |
| $D^+_sK^−$ | 512.26          | 0.28   | 6.22    |
| $D^{0}\pi^0$ | 942.13         | 0.51   | 11.44   |
| $D^{0}\eta$  | 517.46          | 0.28   | 6.28    |

**Table 13** Various ratios for the two different identification of the state $D^{0}_s(3000)$

| Ratios  | $2^3P_2(\frac{1}{2}^+)_{(\pi^0 \eta)}$ | $1^3F_2(\frac{1}{2}^+)$ |
|---------|-----------------|-----------------|
| $R_x^0$ | 0.95            | 0.38            |
| $R_K^0$ | 0.26            | 0.31            |
| $R_{K}^{0\pi}$ | 0.17          | 0.09            |
| $R_{\eta}$ | 0.23            | 0.30            |
| $R_{\eta}^{0\pi}$ | 0.17          | 0.10            |

In this article, we obtained the mass spectra of the heavy-light charmed mesons within the framework of the relativistic formalism. Our predicted masses are in excellent agreement for the well established low-lying states of charmed mesons. Further, incorporating the heavy quark effective theory at the leading order approximation, we obtained the strong decays of the experimentally observed states and identified $D_J(2560)$ as $2^3S_0$, $D_J^*(2680)$ as $2^3S_1$, $J_1(2740)$ as $3^3D_2$, $D_J^*(2760)$ as $3^3D_3$, $J_1(3000)$ as $4^3P_1$ and $D^{*}_s(3000)$ as $3^3F_2$ open charm excited states. The effective coupling constants $\tilde{g}_T$, $\tilde{g}_\eta$, $\tilde{g}_\pi$, $\tilde{g}_S$ and $gZ$ are also computed. We found $D^{*+}\pi^−$ channel is most suitable mode for experimental search for $^{1}F_3$. The computed decay widths and masses for higher excited states would be useful for the identification of future experimental observation of higher open charm states at BABAR, LHCb, BESIII.

**Acknowledgements** We acknowledge the partial support from DST-SERB, India through the major research project: (SERB/F/8749/2015-16).

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors’ comment: All data generated or analyzed during this study are included in this published article and the paper has no associated data as it is theoretical work.]

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Table 14 The strong decay widths (in MeV) for $1^F_{23/2}^+$. Fraction (in %) represents the percentage of the partial decay width with respect to total decay width.

| Decay channel | Decay width | Branching fraction |
|---------------|-------------|--------------------|
| $1^F_{23/2}^+$ |             |                    |
| $D^{*+}\pi^-$  | 2059.98 $g_2^2$ | 44.18              |
| $D^{*+}\pi^0$  | 1032.15 $g_2^2$  | 22.13              |
| $D^{*+}\eta$   | 803.36 $g_2^2$   | 17.23              |
| $D^{*+}K^-$    | 767.57 $g_2^2$   | 16.46              |
| Total          |             | 67.15              |

Table 16 The effect on $1S$ mass (in MeV) due to uncertainty in charm quark mass ($m_c$).

| $m_c$ | % variation in $m_c$ | State | Mass   | % variation in mass |
|-------|----------------------|-------|--------|--------------------|
| 1.29  | 1.5                  | $1^3S_1$ | 2021.46 | 0.94               |
|       |                      | $1^1S_0$ | 1892.56 | 1.05               |
| 1.25  | 1.5                  | $1^3S_1$ | 1983.75 | 0.93               |
|       |                      | $1^1S_0$ | 1853.45 | 1.03               |

Appendix A: Numerical single particle Dirac confinement energy of the quarks and antiquarks

The Dirac confinement energy ($E_D$) is extracted from Eq. (15) for quark($Q$) and anti-quark($\bar{q}$) in accordance with Eqs. (5), (6) and (13), (14). These values for $Q$ and $\bar{q}$ are tabulated in Table 15.

Table 15 The Dirac confinement energy (in GeV) of Quark (c) and anti-quark (u or d) for few ground and excited states.

| State | Spin triplet | Spin singlet |
|-------|--------------|--------------|
|       | $E_D^Q$      | $E_D^\bar{q}$ |
| $1S$  | 1.5261       | 0.4672       |
| $2S$  | 1.8123       | 0.8389       |
| $1P$  | 1.7264       | 0.7321       |
| $2P$  | 1.9687       | 1.0268       |
| $1D$  | 1.8664       | 0.9047       |
| $1F$  | 2.0719       | 1.4726       |
| $E_D^\bar{q}$ | 1.4498 | 0.3562 |
| $E_D^\bar{q}$ | 1.7281 | 0.7342 |
| $E_D^\bar{q}$ | 1.6348 | 0.6144 |
| $E_D^\bar{q}$ | 1.8818 | 0.9232 |
| $E_D^\bar{q}$ | 1.7765 | 0.7947 |
| $E_D^\bar{q}$ | 1.9758 | 1.0351 |

Appendix B: Sensitivity calculation of the model parameters

We have used $c$ quark mass $= 1.27$ GeV while the current quark mass for $u$ and $d$ as 0.003 GeV in the present study. This gives the mass of the $D$ meson for $1^3S_1 = 2002.33$ MeV and for $1^1S_0 = 1872.68$ MeV with other optimized model parameters as listed in Table 4. The PDG listed mass of $c$ quark is $1.27 \pm 0.02$ GeV. While the mass of $u$ and $d$ quark is $2.16^{+0.49}_{-0.26}$ MeV and $4.67^{+0.48}_{-0.17}$ MeV; respectively. In order to test the sensitivity of the parameters, we have calculated the mass of $1S$ state incorporating error bars in mass parameter of charm quark only as up and down quark mass and their uncertainty are negligible (few MeV only).

By considering the charm quark mass, $m_c = 1.27 + 0.02$ MeV as the upper bound and $m_c = 1.27 - 0.02$ MeV as the lower bound, the ground state masses of $D-$ mesons for $(1^3S_1, 1^1S_0)$ states are computed. The results are given in Table 16. As one can see from Table 16 by recomputing the mass of $D-$ mesons by taking $m_c = 1.29$ GeV, the upper bound uncertainty of just 1.5 %, we find $D$ meson mass for $1^3S_1$ state as 2021.46 MeV and that for $1^1S_0 = 1892.56$ MeV with a variation of about 1 %. Similar computation using $m_c = 1.25$ GeV, (with the lower bound uncertainty of just 1.5 %) results into prediction of $D$ meson mass for $1^3S_1$ state as 1983.75 MeV and that for $1^1S_0 = 1853.45$ MeV with a variation of about 1 %.

The other model parameters $\lambda$, $V_0$ and $\sigma$ are the optimized potential parameters which are fixed for known ground state. Below we present the effect on $1S$ mass due to an assumed 5% and 10% variations in the parameters keeping other two fixed. The results are shown in Table 17. One can see from Table 17 that the variations are not much compared to the optimized value of these parameters used in the present study. However, the value of $\lambda$ is slightly more sensitive than that of $V_0$ and $\sigma$. 
Table 17 The effect on 1S mass (in MeV) after % change in the potential parameters

| Change in $\lambda$ keeping $V_0$ fixed | % variation in mass | Change in $V_0$ keeping $\lambda$ and $\sigma$ fixed | % variation in mass | Change in $\sigma$ keeping $\lambda$ and $V_0$ fixed | % variation in mass |
|----------------------------------------|---------------------|----------------------------------|-------------------|----------------------------------|-------------------|
| $5\%$                                  |                     |                                  |                   |                                  |                   |
| $1^{3}S_1$                             | 1982.43             | 0.99                             | 2002.32           | $0.49 \times 10^{-3}$            | 2001.88           | 0.02             |
| $1^{1}S_0$                             | 1854.37             | 0.97                             | 1872.67           | $0.53 \times 10^{-3}$            | 1874.03           | 0.07             |
| $10\%$                                 |                     |                                  |                   |                                  |                   |                  |
| $1^{3}S_1$                             | 1962.07             | 2.01                             | 2002.31           | $0.99 \times 10^{-3}$            | 2001.43           | 0.04             |
| $1^{1}S_0$                             | 1835.58             | 1.98                             | 1872.66           | $1.06 \times 10^{-3}$            | 1874.38           | 0.09             |

Appendix C: Illustration of uncertainty estimation in the coupling constants

In general the total decay width from the present study (say $X \times g_i^2$; $i = T, H, Y, S, Z$) can be compared with experimentally measured width (say $\Gamma_{Exp}$) to extract respective coupling constants.

The uncertainty in the estimations of the strong coupling constants as appear in Table 8 Column 4 due to uncertainty in the experimental width as shown in Table 8 Column 7 are computed as below.

$$X \times g_i^2 = \Gamma_{Exp}$$  \hspace{1cm} (C.1)

$$g_i^2 = \frac{\Gamma_{Exp}}{X}$$  \hspace{1cm} (C.2)

Taking the log and the derivative on both the sides we get,

$$\delta g_i = \frac{1}{2} \left[ \frac{\delta \Gamma_{Exp}}{\Gamma_{Exp}} - \frac{\delta X}{X} \right] \times g_i$$  \hspace{1cm} (C.3)

Here, the $\delta \Gamma_{Exp}$ is the uncertainty in $\Gamma_{Exp}$ and $\delta X$ that in $X$. In the present case $\delta X = 0$ results into

$$\delta g_i = \frac{1}{2} \left[ \frac{\delta \Gamma_{Exp}}{\Gamma_{Exp}} \right] \times g_i$$  \hspace{1cm} (C.4)

As a sample case, consider the total strong decay width $\Gamma(2460) = 286.39 g_T^2$ MeV from Table 8 and is equated to the experimentally measured width of $\Gamma(2460) = 47.0 \pm 0.8$ MeV, we get

$$(282.39) g_T^2 = 47.0 \pm 0.8$$

$$g_T^2 = \frac{47.0}{286.39} \Rightarrow g_T = 0.40$$

and $\delta \Gamma_{Exp} = \pm 0.8$ MeV, then using Eq. (C.4)

$$\delta g_T = \frac{1}{2} \left( \frac{\pm 0.8}{47.0} \right) \times 0.40$$

$$= \pm 0.0034$$

$$g_T = 0.40 \pm 0.003$$

The same process is followed for $g_Z$.

For computing the $g_H$ coupling constant we have two relations as shown in Table 8. It follows that, the total decay width $\Gamma(2580) = 1101.39 g_H^2 = 177.5 \pm 17.8$ MeV leads to

$$\delta g_H = \frac{1}{2} \left( \pm \frac{17.8}{177.5} \right) \times 0.40$$

$$= \pm 0.0200$$

$g_H = 0.40 \pm 0.020$

we refer it here as, $g_{H1} = 0.40 \pm 0.020$

Similarly from the width of $\Gamma(2650) = 2627.78 g_H^2 = 140.2 \pm 17.1$ MeV leads to

$$\delta g_H = \frac{1}{2} \left( \pm \frac{17.1}{140.2} \right) \times 0.23$$

$$= \pm 0.0140$$

$g_H = 0.23 \pm 0.014$

We refer it as, $g_{H2} = 0.23 \pm 0.014$ Then the average $g_H$ is obtained as

$$g_H = \frac{g_{H1} + g_{H2}}{2}$$

$$= 0.40 + 0.23$$

$$= 0.31$$  \hspace{1cm} (C.5)
and average uncertainty in this case is computed as the root mean square values
\[
\delta g_{\tilde{H}} = \sqrt{\frac{(\delta g_{\tilde{H}1})^2 + (\delta g_{\tilde{H}2})^2}{2}}
\]
\[
\delta g_{\tilde{Y}} = \sqrt{\frac{(0.0100)^2 + (0.0140)^2}{2}} = \pm 0.0172
\]

Thus we obtain \(g_{\tilde{H}} = 0.31 \pm 0.017\).

The same method is followed for \(g_Y\) and \(g_{\tilde{S}}\). The estimated uncertainty in \(g_i\) for \(i = T, \tilde{H}, Y, \tilde{S}, Z\) are given in Table 18.

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