Analysis of parameters for technological equipment of parallel kinematics based on rods of variable length for processing accuracy assurance

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Abstract. A new classification of parallel kinematics mechanisms on symmetry coefficient, being proportional to mechanism stiffness and accuracy of the processing product using the technological equipment under study, is proposed. A new version of the Stewart platform with a high symmetry coefficient is presented for analysis. The workspace of the mechanism under study is described, this space being a complex solid figure. The workspace end points are reached by the center of the mobile platform which moves in parallel related to the base plate. Parameters affecting the processing accuracy, namely the static and dynamic stiffness, natural vibration frequencies are determined. The capability assessment of the mechanism operation under various loads, taking into account resonance phenomena at different points of the workspace, was conducted. The study proved that stiffness and therefore, processing accuracy with the use of the above mentioned mechanisms are comparable with the stiffness and accuracy of medium-sized series-produced machines.

1. Introduction
Mechanisms of parallel kinematics [1, 3] based on the rods of variable length have been widely used in different technical fields: in operation units of robots, in machine-tools [6, 10], coordinate measuring machines [4], as training devices and simulators [3]. Compared to standard sequential operation mechanisms, parallel mechanisms are of larger stiffness at lower materials intensity and high positioning accuracy [6] as well as larger workspace as opposed to technological equipment with the conventional layout. In modern production the application of technological equipment with multi-drive mechanisms of parallel kinematics is quite restricted. This is due to insufficient previous study of this equipment, namely there is no classification of structures on stiffness parameter, on workspace parameters, static and dynamic characteristics and, therefore, on processing accuracy [5,9]. A specific feature of such system is that particular accuracy values correspond to each point of the workspace due to various stiffness and dynamic characteristics.

2. Problem Statement
Parallel kinematics mechanisms are called multi-movement mechanisms consisting of a mobile platform and base plate, connected at least by two parallel serial kinematic chains [1]. These kinematic chains are called mechanism legs. The investigation tasks are to identify mechanism structures with high symmetry coefficient, to study workspace parameters for the parallel kinematics mechanism in terms of stiffness as the parameter which greatly affects the accuracy of positioning and processing in machine tool industry. Another task is an experimental study of static, dynamic and vibrational characteristics to use them as output data in mathematical modeling of parallel kinematics spatial mechanisms by the parameter of accuracy. The study of dynamic characteristics [11] allows taking
into account such structural elements as the mass of actuated parts and work-in-process pieces. Solving dynamic problems, it is necessary to derive equations to describe mechanism behavior with allowance for mass - inertial properties of its elements, in other words in the construction of the dynamic model. These equations allow the operation of equipment under various modes to be modeled. The input data for mathematical modeling are the parameters obtained by the full-scale specimen examination [12].

3. Theoretical Basis
Multi-drive mechanisms of a rod type, mechanisms based on the Stewart platform [1, 2, 3] in particular, are widely used in metal-cutting machines, measuring machines and robotics. There is not much technical literature on theoretical and experimental studies in the field of these mechanisms. Recently, an acute problem has been the classification of these mechanisms in terms of any parameters. In a number of works [5] multi-drive mechanisms were considered, where drivers were symmetrical in space.

It is proposed to synthesize the mechanisms on stiffness parameter [7, 8], these mechanisms being used in machine tool technology. The dynamic properties of the mechanism apart from the external power action and inertial characteristics are determined by kinematic parameters of the mechanism among other factors. Kinematic parameters of such mechanisms are determined by the geometry of the structure, in other words the number, linear dimensions, limit angular span in joints and positional relationship of kinematic pairs.

Optimality of these structures will be estimated by the parameter $S_i$ which is conditionally called the i-th group coefficient of symmetry:

$$S_i = \frac{(K_{n,i})_2}{(K_{n,i})_1},$$

where $(K_{n,i})_1$ and $(K_{n,i})_2$ are the number of mechanism structures of the i-th group (table 1).

The analysis of the works [7, 8] shows that the most symmetrical group is the group $e_{14}$ which is not inferior in its symmetry ($S_{14} = 3.2$) to the group $e_{13}$ (the Stewart Platform, $S_{13} = 4$). The groups $s_5$ and $e_3$ are slightly inferior. These groups are not six-degree-of-freedom mechanisms. They are unit elements for the six-drive mechanism assembly. And as separate mechanisms they are widely used in the form of three degree-of-freedom manipulators. Kinematic and dynamics properties of the structures $e_{13}$ (the Stewart platform), $e_5$ and $e_3$ are described thoroughly in [6, 10, 11]. So, investigation of the poorly studied six-drive group $e_{14}$ with high symmetry and, hence, stiffness is of great interest. The analysis of this system enables the degree of conformity of this structure with the requirements to automated machine-building production to be evaluated.

**Table 1. Classification of stiff drive mechanisms.**

| $e_i$ | $S_i$ |
|------|-------|
| $e_1$ | 1.33  |
| $e_2$ | 1.33  |
| $e_3$ | 1.6   |
| $e_4$ | 1.6   |
| $e_5$ | 1.6   |
| $e_6$ | 1.6   |
| $e_7$ | 1.6   |
| $e_8$ | 1.6   |
| $e_9$ | 1.6   |
| $e_{10}$ | 1.78 |
| $e_{11}$ | 2     |
| $e_{12}$ | 2.29  |
| $e_{13}$ | 3.2   |
| $e_{14}$ | 4     |

The stiffness of the parallel kinematics mechanism is directly proportional to the symmetry coefficient [7, 8]. The symmetry and stiffness of the mechanism as well as the Stewart platform are explained by the geometrical arrangement of the structure drives.

Firstly, there are three V-shape links with one node conditionally converging (in fact each drive has its kinematic link with platforms), and this increases the structural stability.
Secondly, spaced through the angle of 120° in terms of stability, they present a stiff structure in the form of a triangle on each platform.

Thirdly, it is sufficient to investigate only one V-shape node in making static and dynamic calculation, and this simplifies the calculations.

The structure shown in Figure 1 is a full-value analogue of the Stewart platform, though it has its specific kinematic and dynamic properties. So, this structure of a mechanism proves to be independent, other than the Stewart mechanism.

![Figure 1. Kinematic diagram of the structure e_{14} (n=6, m=6).](image)

3.1. Work range experimental research

To study the work range, stiffness, free and induced vibrations, the model of parallel kinematics mechanism with the structure e_{14} (n=6, m=6) was constructed [12].

![Figure 2. Mechanism under testing.](image)

The mechanism under testing in Figure 2 consists of a mobile platform 1 and fixed base plate 2, six screw leveling legs (rods) 3 linked together by the joints with the possibility to vary length.
Translation and angular movements are possible due to the leg length variation and rotations in two-degree-of-freedom rotating Hooke’s joints. In the base central position the mechanism has three vertical planes of symmetry A-A, B-B, C-C, being set at the angle of 120° relative to each other and crossed at one point (Figure 3).

Let us consider the layout of the workspace when the platform moves in the plane A-A. The graphical layout of the work range is stepwise: the platform movement direction is selected, the number of legs is defined, and variation in their length ensures the intended movement. Then the length of the legs is measured (2) and the mobile platform center coordinates are determined (3), the work range curves being plotted in these coordinates. The center coordinates of the mobile platform in absolute coordinate system are determined by the probe (1) being placed in the center hole of the platform. By fixing its length, which corresponds to the coordinate along the axis OX and varying the lengths of legs until it contacts the base plate (4), we determine the center coordinate of the mobile platform along the axis OZ. Based on the measurements of the work range, its curve in one coordinate plane is plotted on the coordinate grid (Fig. 4). The results will be similar for the planes B-B and C-C of the mechanism layout.

3.2. Developing a mathematical model for the dynamic system
The computational model of the dynamic system requires some assumptions [13]. Under these assumptions vibrations along six coordinate directions are examined. The expression of kinetic energy for the system is described by the equation:
The potential energy of the mechanism is defined as the strain energy of six legs and is represented by the expression:

\[ \sum_{i=1}^{6} \frac{1}{2} I_i \dot{q}_i^2 = \sum_{i=1}^{6} 0.5 \cdot c_i \cdot \Delta S_i^2, \]

where \( c \) is the stiffness of the \( i \)-th rod; \( \Delta \) is the strain of the \( i \)-th rod, \( i \) is the number of rods.

### 3.3. Dynamic system vibrations of the mechanism

To solve the differential equations for system vibrations, let us write them in the form of matrix:

\[ \| k \| \cdot \{ \dot{q} \} + \| k \| \cdot \{ q \} = \{ P(t) \} \]

where \( \| k \| \) is the matrix of inertia coefficient; \( \| k \| \) is the matrix of stiffness coefficients; \( \{ P(t) \} \) is the matrix of generalized disturbing forces.

The solution of the non-homogeneous differential equations system (4) is sought in the form:

\[ q_{g,h} = q_{g,h} + q_{p,n} \]

where: \( q_{g,h} \) is the general solution of the homogeneous differential equations system, \( q_{p,n} \) is the particular solution of the non-homogeneous differential equations system.

The general solution of the homogeneous differential equations system \( q_{g,h} \) describes natural vibration of the system, and it is of interest in studying resonance phenomena of the system and the dependence of the system natural frequency on various parameters of this system.

The solution of the system of particular non-homogeneous differential equations allows the amplitudes of the induced vibrations for the computational model elements to be determined on generalized coordinates. The solution of the given system can be of interest in studying vibrations of the computational model elements due to the action of periodic disturbing force.

### 3.4. Studying natural vibrations

To study the system on the possibility of resonance phenomena under various parameters of the system, let us determine dependencies of natural frequencies on various parameters (mass, leg stiffness, mobile platform projection, leg diameter). The study is based on the solution of the general homogeneous equations using the MATLAB package [13]. The experimental results (data on leg stiffening parameters is performed in the end positions of the mobile platform workspace (points 1, 2, 3, 4 in Figure 4), as far as the system has the smallest stiffness in these positions, and, hence, minimum accuracy. Platform loading was performed using a jack with tension and compression force of the value \( P = 0.2 \ldots 1.2 \) kN, controlled by dynamometer ДОСМ-3-02, the movement being recorded by the indicating gage. Table 2 shows the results of stiffness parameters compared to the Stewart platform.

### 3.5. Experimental study of static stiffness

The definition of stiffening parameters is performed in the end positions of the mobile platform workspace (points 1, 2, 3, 4 in Figure 4), as far as the system has the smallest stiffness in these positions, and, hence, minimum accuracy. Platform loading was performed using a jack with tension and compression force of the value \( P = 0.2 \ldots 1.2 \) kN, controlled by dynamometer ДОСМ-3-02, the movement being recorded by the indicating gage. Table 2 shows the results of stiffness parameters compared to the Stewart platform.

### 3.6. Experimental study of dynamic parameters
Let us define the natural frequencies, forms and decrements of the mechanism vibration. Vibration analyzer "Diana 2M" is used for this analysis.

The process of dynamic testing of the mechanism (Figure 5) with the method of induced vibrations was performed by the step change of the eccentric exciter rotational speed. The readings are recorded by the acceleration gage followed by data processing. To determine disturbing frequency and periodic, it is essential to consider time realization with time signaling.

Figure 5. Diagram of the eccentric vibration exciter.

4. Experimental Results

The performed studies allowed to establish that the equipment of parallel kinematics based on rod of variable length with the high symmetry coefficient possesses high stiffness. This coefficient is comparable to the stiffness and accuracy of medium-sized series-produced machines.

Table 2. Comparison of the mechanism under testing with the Stewart platform under the force of loading of 120 kg.

| Point of the work range (Fig. 4) | Compression | Tension |
|---------------------------------|-------------|---------|
|                                 | Stewart platform | Structure $e_{14}$ | Stewart platform | Structure $e_{14}$ |
|                                 | $H_{\text{max}}, \text{mm}$ | $H_{\text{max}}, \text{mm}$ | $H_{\text{max}}, \text{mm}$ | $H_{\text{max}}, \text{mm}$ |
| Point 1                         | 0.3         | 3.7     | 0.28       | 3.43         |
| Point 2                         | 0.28        | 3.2     | 0.27       | 3.06         |
| Point 3                         | 1.01        | 1.37    | 1.17       | 1.85         |
| Point 4                         | 0.77        | 1.87    | 0.97       | 1.69         |

Figure 6. Platform vibration graph.
Having obtained the time realization, it is possible to determine the required frequency and periodic. In Figure 7 the periodic of frequency is $T = 1/n \approx 0.038\text{sec.}$, $f = 1/T \approx 26$ Hz, where $n$ is the number of waves per 1 second, $f$ is the disturbing frequency.

![Figure 7. Frequency spectrum.](image)

An important component for mechanical vibrations within the time domain is the dependence of their amplitudes on time. A physical characteristic of mechanical vibrations within the time domain is their frequency spectrum. The time domain having been transformed with the fast Fourier transform (FFT), we obtain the frequency spectrum (Figure 7), where the first harmonic at the frequency of the disturbing force of 25.7 Hz is seen (above we have defined it as $\approx 26$ Hz) as well as all higher harmonics and the value of the disturbing force amplitude. To determine the natural vibration frequencies and logarithmic decrement of the structure under study, it is subjected to a shock load and it results in damping vibrations (Figure 8).

![Figure 8. Waveform under the shock load. Platform in the position of maximum stiffness.](image)

The resulting wave having been processed, the frequency and natural period are determined. In Figure 7 the natural period of the mechanism under the maximum stiffness equals $T = 0.1/24 \approx 0.00416$ sec, where 0.1 sec. is the time interval corresponding to the twenty-four waves, and the natural frequency is $f = 1/T = 1/0.00416 \approx 240$ Hz.
5. Discussion
It is possible to determine the value of the natural frequency and amplitude if the FFT subjects to time realization shown in Figure 7. As a result we obtain the frequency spectrum with clearly defined spike at the resonance frequency of 67.97 Hz (above we have defined it as ≈ 70 Hz) and the value of frequency amplitude. Resonance frequencies $f_{res}$ and corresponding logarithmic decrements of vibrations $\delta = \pi \times \Delta f / f_{res} = 0.1$ are defined on amplitude frequency response, where $\Delta f$ is the width of resonance peak at the level $2^{0.5}$ from its largest value. Thus, the range of natural frequency variation of $70 – 240$ Hz and logarithmic decrement of damping were determined.

6. Conclusion
- The stiffness of mechanism carrying system depends on the geometric parameters, position of the executive element in the work range. The further away from the symmetry axis it is, the smaller is stiffness. The largest stiffness of the system is exhibited vertically.
- The size of the work range is significantly influenced by the constraints imposed by the angular movements in joints and linear length variation of the legs. The smaller the legs are, the smaller are the overall dimensions of the work range.
- The detected natural frequencies enable us to determine the applicability of this mechanism in particular operating conditions and evaluate the possibility of various on-load operations with allowance for resonance.
- At different positions of a mobile platform it possesses the largest vibration stability and the least error from vibration displacements along the axis Z. More careful calculations are required for vibrations along the axes X and Y and torsion vibrations.

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