Frontiers the Physics of Dense Matter for Neutron Stars

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Abstract. Neutron stars are an excellent laboratory for nuclear physics. They probe the nucleon-nucleon interaction, the structure of nuclei, and the nature of dense QCD in ways which complement current experimental efforts. This article very briefly summarizes some of the current frontiers in neutron stars and dense matter with an emphasis on how our understanding might be improved in the near future.

1. Neutron Stars
From the point of view of nuclear theory, a simplified description of neutron stars is as follows (see also Fig. 1). The outermost region, the atmosphere, consists of elements ranging between hydrogen and iron depending on the neutron star age, its history, and the composition of any accreted material. In the outer crust, electrons form a degenerate Fermi sea and nuclei form a Coulomb lattice. As one proceeds deeper into the neutron star, the nuclei become more and more neutron rich. In the inner crust, the energetically favored state is one in which some of the neutrons leave the nuclei and form a degenerate Fermi gas (the “neutron drip”). Finally in the core of the neutron star, the Coulomb lattice gives way to a strongly-interacting fluid of neutrons protons and electrons.

The inner core may contain deconfined quark matter, hyperons, or Bose condensates of pions or kaons. All of the possibilities which involve fermions may also include superconductivity. Most importantly, the most relevant degrees of freedom for matter in the neutron star core may not be directly related to the same degrees of freedom which we are accustomed to.

Fig. 1 also summarizes modern estimates of basic neutron star properties. Observed spin frequencies range between 0.1 and 720 Hz, but there are likely many slowly-rotating sources which we do not observe. The lower limit in mass is around 1 $M_\odot$; the limitation coming from the core-collapse supernova mechanism. The maximum mass must be larger than about 2 $M_\odot$.

Presuming, for the moment, that systematic uncertainties from neutron star radius measurements are small enough to be ignored, one can estimate probable ranges for other structural quantities. The radius is likely between 10 and 13 km, the moment of inertia between 50 and 200 $M_\odot$ km$^2$, the central baryon density is $0.6 - 1.3$ fm$^{-3}$, the central energy density is $500 - 1600$ MeV/fm$^3$, the thickness of the crust is $0.4 - 2.0$ km and the tidal deformability is between $(0.2 - 6) \times 10^{36}$ g cm$^2$ s$^2$ [1].
Figure 1. A cartoon representation of a neutron star with some basic structural quantities.

2. Radius Constraints and Systematic Uncertainties

The two principal methods for obtaining radius constraints are from thermal emission in quiescent low-mass X-ray binaries and from photospheric radius expansion (PRE) X-ray bursts. Unfortunately, there are several systematic uncertainties which currently plague neutron star radius constraints. The best way to attack these uncertainties is to obtain more data; we look forward to contributions from NICER, Athena+, LIGO, GAIA, LOFT, Astrosat, and Astro-H. On the theory side, there are several possible fronts which might enable progress. It would be helpful to understand more about how heat is transported across the neutron star surface, especially in the presence of magnetic fields. Also, better X-ray burst models may improve our understanding of how PRE X-ray bursts behave with time (e.g. see Ref. [2]). Finally, a more complete understanding of the diversity of X-ray burst observations might provide important insights (e.g. see Ref. [3]).

Because the observational data is still somewhat sparse, the problem of determining neutron star radii is underconstrained; there are more parameters than data points. To handle this, we have employed Bayesian inference (beginning with Ref. [4]). We have found that disentangling some of the possible systematics is possible by combining (i) radius constraints from observational data, (ii) nuclear theory calculations of neutron-rich matter, and (iii) a statistical analysis employing Bayes factors to compare different models. For example, using Bayes factors Ref. [5] found that radius data from quiescent low-mass X-ray binaries was better explained with alternative models for X-ray absorption and by allowing some of the objects to have helium, rather than hydrogen, atmospheres (see Fig. 2).
Figure 2. The left panel shows radius constraints from quiescent low-mass X-ray binaries assuming Hydrogen atmospheres. The right panel shows radius constraints from the same neutron stars allowing for helium atmospheres and an alternative model for X-ray absorption in between the neutron star and the observer. Taken from Ref. [5]. The results from the left panel are unlikely to be correct, as it is difficult to trace a physical mass-radius curve between all the observations.

3. Neutron-rich Matter
The description of matter at low densities is currently best done in terms of neutron and proton degrees of freedom. Because we are not directly employing the QCD Lagrangian, essentially all methods are in some sense, “phenomenological”. Approaches to describe nuclei and nucleonic matter fall on a continuum between models which are as close as possible to QCD and describe only a limited set of data and models which are farther from QCD yet describe a larger data set. In either case, it is critical to quantify the precision and accuracy with which a particular model can reproduce its target data set (and make predictions beyond that data set). This kind of “uncertainty quantification” has been recently enabled by the wide availability of computing resources. In fact, arguably most of the progress in the past decade has not been in developing new many-body methods, but rather in improving the ways in which uncertainties in old methods are quantified.

The best descriptions of homogeneous nucleonic matter come from quantum Monte Carlo (QMC) and from several many-body methods applied to interactions based on chiral effective theory. QMC typically employs nearly-local potentials constructed in coordinate space. The advantage of this method is that it is exact, given a particular potential. The disadvantage is its large computational cost. It has been difficult to find local interactions which accurately describe laboratory nuclei which do not also create bound neutron matter, which is not observed. This is a strong indication that the isospin properties of the potential is not well-known [6]. This is important for neutron stars, which are so isospin asymmetric. A full uncertainty analysis has not been done in QMC because of the large computational cost: it may yet be possible to find a nearly-local potential which faithfully reproduces the scattering phase shifts and the properties of light nuclei while also properly describing low-density neutron matter. Another important frontier is the development of a nearly local interaction directly from chiral effective theory (see Ref. [7]).

Chiral effective theory (CET) is useful in large part because the pion is so much less massive
than the nucleon. Thus treating pion exchange accurately is sufficient to describe most of the low-momentum properties of nuclei and nucleonic matter (see e.g. the reviews in Refs. [8, 9]). CET-based interactions have been employed to describe nuclei with several different many-body methods, including no-core shell model and coupled cluster methods (e.g. Ref. [10]). For homogeneous nucleonic matter, many-body perturbation theory has also found success (e.g. Ref. [11]). While it is generally believed that CET-based interactions can be trusted up to the nuclear saturation density, quantifying the accuracy is difficult (see e.g. Ref. [12] for recent progress on that front). In addition, it seems unclear at which density more degrees of freedom (like the Δ resonance) might give important contributions.

QMC and CET-based methods give relatively similar predictions for neutron-rich matter up to the nuclear saturation density. These methods tend to predict neutron star radii between 10 and 13 km [1] unless there is a strong phase transition to exotic matter just above the saturation density. An example of this from Ref. [13] is given in Fig. 3, where equations of state based on QMC calculations at lower densities and phenomenological models of hadronic matter (polytropes) or quark matter at high densities give neutron star radii between 10 and 12.5 km.
4. Nuclei for Neutron Stars

None of these methods can yet describe the heaviest nuclei without additional approximations (though progress has been made, see Ref. [16]). Thus for heavy nuclei, the mean-field approximation is still the dominant method in use. Skyrme models [17, 18] employ a phenomenological zero-range two-nucleon interaction and three-body force to generate a non-relativistic Hamiltonian. This Hamiltonian, in turn, can be thought of as an energy density functional. These energy density functionals are employed in the Hartree-Fock approximation to describe nuclei (see e.g. Ref. [19, 20]).

While Skyrme forces have dominated the field for almost 60 years, it is well known that the isospin part of the bulk and gradient contributions to the energy density is not well under control (e.g. Ref. [21]). In addition, the tensor part of the nuclear force (not typically included in Skyrme calculations) likely also plays a role. Uncertainty quantification was pioneered in Ref. [22], which computed the uncertainty in the parameters from the fit to nuclear binding energies and charge radii. These uncertainties can be propagated to help understand the theoretical uncertainties related to the nuclei in the crust.

Most of the remaining models for heavy nuclei can be traced back to phenomenological covariant Lagrangians which treat nucleons as interacting via the exchange of mesons; these are the so-called “relativistic mean-field (RMF) models” [23, 24]. Traditionally, the meson-nucleon couplings are constant with density, but allowing these couplings to be density-dependent (e.g. Ref. [25]) gives more parameters which can be fit to experiment. It is, as yet, unclear if RMF models can give results which are fully competitive with Skyrme energy density functionals. Fig. 4 shows typical results for Skyrme (NRAPR) and RMF-based models (RAPR) which were simultaneously fit to doubly-magic nuclei and the neutron matter calculation from Ref. [14].

It is important to note that the most critical systematic uncertainty remains unknown: the systematic uncertainty in various nuclear structure quantities as a result of the fact that we may have the wrong energy density functional or wrong Hamiltonian (assuming that correlations beyond the mean-field approximation are already included in the functional). A basic estimate of this systematic error can be made by comparing Skyrme and RMF models, and these types of comparisons have a long history [26, 27]. However, these comparisons do not account for the fact that different parameterizations are fit to different data sets with different uncertainties assumed for the data points. A fully consistent and even-handed comparison of the two types of model has yet to be performed. In addition, both Skyrme and RMF models are fit to nuclear data, but then extrapolated to the higher densities that are achieved in neutron star interiors. It is (obviously) unclear if this extrapolation is physically accurate.

Another frontier for improving the description of heavy nuclei is better optimization of the data set to which a model is fit to (see the discussion in Ref. [28]). It may be possible, for example, to obtain better constraints on the nuclear symmetry energy by fitting to a data set which is more tightly correlated to isospin properties. Fitting to the surface thickness of the proton density distribution may be better than simply fitting to the charge radius\(^1\).

Impressive results, pioneered by Refs. [29], on reproducing nuclear binding energies have from liquid-drop type fits combined with shell energies computed from the Strutinsky method. An alternative was published in the same year in Ref. [30], which gives better binding energies but may have a poorer description of isospin-asymmetric nuclei. These phenomenological fits are impressive, and although they do not give much insight into QCD, they have been useful in astrophysical simulations where exotic binding energies are required. However, uncertainty quantification in these sorts of methods has not generally been performed at the same level which the Skyrme model [31]. A notable exception is Ref. [32], made possible because the code to generate the masses in Ref. [30] was made publicly available.

\(^1\) Priv. comm. with Stefan Typel
Figure 4. Skyrme- and RMF-based predictions for the proton and neutron density distributions in $^{208}$Pb, from Ref. [27], based on fitting charge radii, binding energies, and the neutron matter calculation from Ref. [14]. Also plotted are results in a Thomas-Fermi-based “semi-infinite nuclear matter” (SINM) approximation (with no shell effects).

5. Matter Above the Saturation Density
It has not yet been possible to determine the composition of the neutron star directly from the QCD Lagrangian. For this reason, there are a plethora of phenomenological models of dense matter based on a small set of constraints. Assuming the strange quark hypothesis is false, then all models must, at the very least, ensure that no exotic matter is present at nuclear densities and that the mass-radius curve must generate a 2 M$_\odot$ neutron star. Neutron star matter must also be hydrodynamically stable ($dP/d\varepsilon > 0$) and the speed of sound must be less than the speed of light ($dP/d\varepsilon < 1$ in units where $\hbar = c = 1$). Under this small constraints, a huge number of papers have generated mass-radius curves from the plethora of possible models.

The most promising frontier for probing the composition of the core is to use neutron star observations other than mass and radius. Among these are: (i) the cooling of isolated neutron stars [33], (ii) the cooling of the crust of a low-mass X-ray binary which was recently accreting but entered a quiescent (non-accreting) period, (iii) X-ray bursts and superbursts, and (iv) neutron star oscillations including r-modes [34, 35] and magnetar flare oscillations [36]. All of these observations are potentially sensitive to the core composition.

There are several frontiers for theoretical work which would enable more close connections between the astronomical observations and the microphysical neutron star properties. Uncertainty quantification is, again, particularly important. Ref. [37] incorrectly concluded that some pulsar glitches could not be explained by superfluidity in the neutron star crust because
they chose a small set of models which did not represent the full uncertainties. Refs \cite{38, 1} showed that this apparent disagreement was not conclusively present (see also Fig. ??).

6. Evidence that Neutrons and Protons are not Enough

In 2007, Ref. \cite{39} found evidence of a neutron star, SAX J1808.4−3658, which could not be explained by a standard neutron star model with neutrons and protons alone. Given an estimate of the time-averaged accretion rate of material onto the neutron star, and assuming a model for the heat generated by deep crustal heating, one should be able to estimate the luminosity of the neutron star in quiescence. Ref. \cite{39} found SAX J1808.4−3658 is particularly underluminous implying cooling from an exotic process or from some exotic composition in this neutron star.

More evidence for exotic matter comes from the speed of sound. We have shown \cite{40} that, because the neutron star maximum mass must be larger than 2 M⊙, there is a strong constraint on the squared speed of sound, c². In particular, c² must be larger than c²/3 for some density inside the neutron star. Since QCD perturbation theory calculations suggest that, at asymptotic densities, c² approaches c²/3 from below, this means that one of two things must happen: either the speed of sound must have a jump discontinuity, or it must change concavity at least twice inside a 2 M⊙ neutron star. This is shown in Fig. 6: the only two options for c² look like the dotted or dashed curves. The solid curve is ruled out. This strongly suggests that either a phase transition occurs in dense matter or some new physical length scale appears in order to explain the observed behavior of the sound speed. With these examples in hand, the time is ripe to
consider carefully the evidence for the composition of dense matter.

7. The Neutron Star Crust
The neutron star crust is an especially exotic system: nuclei which are more neutron-rich than those which can be experimentally observed reside in a multicomponent Coulomb lattice and embedded in a sea of superfluid neutrons with strong interactions. Furthering the complexity, all of these various components are strongly interacting with each other and placed in strong gravity and in large magnetic fields.

If one assumes the Wigner-Seitz approximation, then matter in the crust can also be treated in the mean-field approximation with Skyrme-based or RMF-based interactions (as an example, see Ref. [41]) as with laboratory nuclei. The Wigner-Seitz approximation, however, is known to fail at the largest densities. Often this high-density phase is referred to as nuclear pasta; nuclei are strongly deformed [?] and the crust becomes “frustrated”. This latter state is manifest in the first classical molecular dynamics simulations of the neutron star crust in Ref. [42]. Frustration also has a large impact on the transport properties of neutron star crust matter (shear modulus, thermal conductivity, etc.) as well as the neutrino opacities. The fundamental difficulty in theoretical studies of the crust is that molecular dynamics simulations have a difficult
time correctly describing the quantum nature of the crust (e.g. its superfluidity) but mean-field calculations cannot yet be done on large enough scales to properly treat the long-range Coulomb corrections.

8. Open Source
Computational advances have led to more computationally-based approaches to problems in nuclear structure and nucleonic matter. Thus code development has become an increasingly important part of research. Having several research groups around the world develop identical codes can create large inefficiencies. Open-source computing can make the community more efficient by allowing researchers to build on previous work without having to constantly “reinvent the wheel”. The astronomy community has been more innovative that the nuclear physics community on this front, developing the Astrophysics Source Code Library and a large set of well-developed publicly available codes. Ref. [43] is an example of how publicly available code is playing an important role in current research.

9. Conclusion
Neutron stars continue to be an excellent laboratory for nuclear physics. They are challenging experimentalists, observers and theorists to work together to understand the nature of QCD.

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