Research Article

Experimental Investigation of Water–Sand Mixed Fluid Initiation and Migration in Porous Skeleton during Water and Sand Inrush

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Water–sand inrush is one of the most serious disasters for mining in China. The evaluation of the occurrence and development of a high-concentration water and sand mixed fluid is an important issue for mining in China. In this study, contraposing to the 3 phases of water–sand inrush, three kinds of experiments are designed for the investigation of initiation, development, and occurrence of the disaster. A new sand–water transport testing system is setup to perform the tests. The results show that there are two key points in the disaster: (1) sand particle incipient motion and (2) porous skeleton structural instability. The incipient motion of sand grains is accompanied with the phenomena of volumetric dilatation and granular fluidization. The critical velocity of the incipient motion of the water–sand mixed fluid is significantly affected by the particle size and external stress. The interaction between water and sand grains is the key factor affecting the motion characteristics of water–sand mixture. When the hydraulic conditions exceed the threshold, the water and sand grains are mutually promoted, and the aquifer skeleton becomes unstable. Furthermore, during the water–sand inrush, the curves of volumetric flow rates of sand and water, respectively, for different samples manifest as two distinct waveforms.

1. Introduction

Water–sand inrush is a most serious mine disaster in western China, particularly in the Shaanxi province, the major energy resource centre of China [1, 2]. The typical geological characteristics of the coal mines in this area provide the conditions for the occurrence of water–sand inrush. According to the statistics data on the scenario of the water–sand inrush occurring in western China, accidents during coal mining extremely affect over 47.5% of the estimated reserves, equivalent to an excess of 100 billion tons [3]. These disasters cause serious damage, enormous financial loss, and numerous casualties. For example, when a water–sand inrush occurred at the Longde coal mine in 2012 [4], the total subsidence volume reached 16 350 m3 and the direct economic losses exceeded 100 million yuan. Another typical example is that of a highly serious water–sand inrush accident occurrence at the Ciyaowan coal mine of the Shenfu ore district in 1990 [6]. In this accident, the instantaneous inflow water volume reached up to 200 m3/h, and the 306-meter long underground mining face was buried such that the average buried thickness of the sediment exceeded 2 m.

In addition to the threat to the safety of mining operations, water–sand intrusion also causes serious environmental issues [7, 8]. Therefore, over the past few years, simulation and laboratory investigations of the effects of a water–sand inrush on the hydrogeological characteristics, mechanism properties, and mining environment have been performed extensively [1, 9–12]. The results of these investigations show that during the process of a water and sand inrush, the sand–water mixed fluid can migrate through the cracks and broken rock mass and swarm into the working areas. Following this,
the groundwater is discharged without the solids, and solid grains remain in the goaf and bury the equipment. The characteristics of the aquifer combined with the mining-induced strata failure [9, 13] and inherent geological structures of a coalfield [2, 14] such as shallow mining depths, thick coal seams, and overlying loose quaternary Aeolian sand near the ground surface are the main factors that cause accidents [12, 15]. As shown in Figure 1, a crushed rock mass (porous skeleton) in a caved zone is connecting an aquifer and a goaf and creating an effective water–sand inrush channel. The incipient motion of saturated sand grains and migration features of the sand–water mixture could be the determining factors of water–sand intrusion accidents.

Until now, the studies on the water–sand inrush mechanism have been mainly focused on either the influence of the overburden failure or the qualitative analysis of the seepage evolution [12, 16–21]. The permeability characteristics of a granular rock mass during the water inrush process have been widely studied [22–29]; however, such a high concentration of particle migration during a sand and water inrush involving a mixed fluid cannot be regarded as a single-phase fluid [30, 31], being significantly different from a water inrush. Therefore, in addition to the factors of the hydraulic features of the geological structure, the groundwater seepage characteristic is also a key issue to consider in the incipient motion and migration of a water and sand mixed fluid.

At present, the existing test equipment and method cannot separate water and sand mixtures completely, and it is difficult to measure the real-time water/sand single-phase velocity accurately. There is lack of quantitative and scientific research on the initiation and development process of a water–sand inrush. In this study, a custom-built high-speed water–sand transfer experimental system with a water–sand separation velocimeter (WPV) was adopted to investigate the flow characteristics of a water–sand mixed fluid. Three sets of laboratory tests were designed and performed, aimed at revealing the occurrence, development process, and evolution feature of a water–sand inrush. The effective factors of the incipient motion of the sand grains in a porous skeleton, characteristics of the fine particle loss in the porous skeleton, and characteristics of the migration of the high-concentration water–sand mixed fluid migration were analysed based on the test data obtained in the laboratory. From the perspective of hydraulics, two necessary conditions were proposed for the occurrence of a water–sand inrush disaster. The dominant relationship between water and sand and rheological feature of a water and sand mixture fluid were revealed.

2. Experimental Equipment

2.1. Experimental Setup. Figure 2 illustrates the connection and principle of the testing system. The apparatus consists of four main parts as follows:

Test Cell. The test cell of the apparatus mainly consists of a cylindrical tube composed of an organic glass with an inner diameter of 60 mm, wall thickness of 7 mm, and strength of 2 MPa, as shown in Figure 3(a), and cone-shaped joints, as shown in Figure 3(b), which permit the visualization of the testing process. The tube is sealed with the cone-shaped reducing pipes at both ends, which serve as diffusers to create uniform flow conditions. Two porous plates (thickness of 4 mm and bore diameter of 2 or 4 mm, Figure 3(c)) are fastened to the bellbottom of the reducing pipes. To avoid the effects of air on the experiments, the following measures are adopted. Before starting the experiments, a vacuum pump (Figure 2(b)) is first used to remove the air from the inside of the sample.

Water Supply Cell. The water supply system is composed of a plunger-type constant-pressure water pump powered by an air compressor and a plunger-type constant-flow water pump powered by a servo motor. The constant-pressure water pump had a volume of 12 L and could provide a maximum water pressure of 2 MPa. Concurrently, the upstream pressure of the samples could be precisely controlled with a precision of 0.005 MPa by adjusting the air pressure regulator. In addition, the constant-flow water pump provided a stable water flow of 0–3 L/min.

Data Measuring Cell. A custom-built WPV is developed for the flow velocity measurement, as shown in Figure 4, which can realize real-time and accurate measurement of the single-phase flow velocity of the water and sand mixed fluid. The water pressure at the upstream and downstream ends of the sample was measured by a pressure sensor (Q/JL015-2009, Longchi Xinda Electronics, Beijing, China) in the range of 0–2.38 MPa ± 0.1% of the full scale (FS). The speed measuring device included two electronic balances (TCL 12000, Gottingen, Germany) set in the range of 0–12,000 g ± 0.1% FS and a large beaker.

Data Recording Cell. During the test, a data acquisition system in the computer, as shown in Figure 2(a), is used for recording and storing the experimental data every 3 s.

2.2. Velocity Measurement Principle. The key component of the experimental system is the WPV (see Figure 4). It is comprised of two electronic scales and two vessels. Vessel A named, as the constant-volume liquid cup, has an overflow vent on the top and a honeycomb duct at the water inlet. Vessel B, named as the liquid cup, has a funnel-shaped water inlet and no outlet. The test procedure and working principle of the velocimeter are as follows:

(i) Firstly, distilled water is poured into the constant-volume liquid cup until it overflows from the vent. The outer wall of vessel A is dried and then placed on balance A

(ii) Secondly, the dry empty vessel B is placed on balance B. It is ensured that the water inlet of vessel B is placed directly below the overflow nozzle of vessel A

(iii) Thirdly, when the water and sand flow initiates through the sample, initially the mixed fluid enters vessel A, then the sand particles are filtered out, and pure water falls into vessel B

As shown in Figure 4(b), in vessel A, the coarse sand granules sink down to the bottom, and the fine particles float above it. A filter screen is fastened to the bottom of the
overflow pipe to prevent the sand particles from being flushed out of vessel A. The mixed fluid displaces an equal volume of pure water in vessel B through the overflow nozzle. Electronic scale A reads the difference in the masses of the sand and the water it replaces, and electronic scale B reads the mass of the water with the same volume as the mixed fluid, i.e.,

\[
\begin{align*}
M_{A,j} &= m_{s,j} - m_{ds,j} \quad (j = 1, 2, \cdots, n), \\
M_{B,j} &= m_{w,j} + m_{ds,j} \quad (j = 1, 2, \cdots, n),
\end{align*}
\]

where \( M_{A,j} \) is the record of electronic scale A, \( M_{B,j} \) is the record of electronic scale B, \( m_{s,j} \) represents the mass of the sand flowing out of the sample (SFO), \( m_{ds,j} \) represents the mass of the water displaced by the sand, and \( m_{w,j} \) is the mass of the water flowing through the sample (WFT). The volume of the sand \( (V_{s,j}) \) is equal to the volume of the displaced water \( (V_{ds,j}) \); therefore, the total volumes of the sand and water can, respectively, be calculated by

\[
\begin{align*}
V_{s,j} &= \frac{M_{A,j}}{\rho_s - \rho_w} \quad (j = 1, 2, \cdots, n), \\
V_{w,j} &= \frac{M_{B,j}}{\rho_w} - \frac{M_{A,j}}{\rho_s - \rho_w} \quad (j = 1, 2, \cdots, n).
\end{align*}
\]

\( m_{s,j} \) and \( m_{w,j} \) can, respectively, be obtained as

\[
\begin{align*}
m_{s,j} &= V_{s,j} \cdot \rho_s = \frac{M_{A,j}}{\rho_s - \rho_w} \cdot \rho_s \quad (j = 1, 2, \cdots, n), \\
m_{w,j} &= V_{w,j} \cdot \rho_w = \left( \frac{M_{B,j}}{\rho_w} - \frac{M_{A,j}}{\rho_s - \rho_w} \right) \cdot \rho_w \quad (j = 1, 2, \cdots, n).
\end{align*}
\]

where \( V_{s,j} \) is the volume of SFO and \( V_{w,j} \) is the volume of WFT; therefore, the volumetric flow rate can be determined as follows:

\[
\begin{align*}
v_{s,j} &= \frac{V_{s,j}}{t_j} = \frac{M_{A,j}}{(\rho_s - \rho_w) t_j} \quad (j = 1, 2, \cdots, n), \\
v_{w,j} &= \frac{V_{w,j}}{t_j} = \frac{M_{B,j}}{\rho_w t_j} - \frac{M_{A,j}}{(\rho_s - \rho_w) t_j} \quad (j = 1, 2, \cdots, n),
\end{align*}
\]

where \( v_{s,j} \) is the average volumetric flow rate of SFO and \( v_{w,j} \) is the average volumetric flow rate of WFT, \( \text{cm}^3/s \). \( \rho_s \) is the density of the sand particles and \( \rho_w \) is the density of distilled water \( (\rho_w = 1 \text{ g/cm}^3) \). During the test, the data are recording and storing every 3 seconds, and the masses measured at the time intervals read \( M_{A,1}, M_{A,2}, \cdots, M_{A,n} \) and \( M_{B,1}, M_{B,2}, \cdots, M_{B,n} \). As the duration of the test is 155 seconds for the longest, the maximum \( n \) value is 85.

3. Materials and Methods

3.1. Experimental Materials. In the experimental research, natural Aeolian sand was adopted as the main research object. The Aeolian sand was collected from the Xiojihan coal mine in the Yulin mining area, Shaanxi province of China. By the sieve analysis, the size distribution of Aeolian sand particles is presented in Figure 5. In the original Aeolian sand samples, the particles with a diameter of less than 0.6 mm accounted for 98.8% of the total, and the residual were impurities.

The river sand, quartz sand, and steel balls are used as auxiliary experimental material in the tests. Using a vibratory sieve shaker, the Aeolian sand river sand particles are separated into six groups according to their sizes ranging from 0.075 mm to 2.36 mm, as shown in Figures 6(a)–6(f). The quartz sand has a particle size of 4.75–9.5 mm, as shown in Figure 6(g), and the steel spheres are 14 mm in diameter, as shown in Figure 6(h).
Figure 2: Continued.
3.2. Preliminary Tests. A set of preliminary tests were carried out, in order to observe sand particle motion characters and determine the follow-up test program. Natural Aeolian sand, with a particle size of less than 0.6 mm, was adopted in this test. Saturated original Aeolian sand is filled in the test tube layer by layer with the height of 3.5 cm and 12 cm, respectively. The initial hydraulic gradient value was 0.1, and the hydraulic gradient was increased slowly by 0.1 each time. Figure 7 illustrates the evolution of movement for natural Aeolian sand column under water, with a hydraulic gradient value of 1.0 and height of 12 cm. It can be observed that, when the hydraulic gradient threshold of particle incipient motion is achieved, a volume dilation of the sand column is forming, and the sand grains undergo fluidization. In 5 seconds, a dominant channel for the granular sand migration is formed along the smooth tube inner wall. The finer sand particles flow upward along the channel, with the larger particles settling. In 20 seconds, the volume of the sand column reaches maximum. Apparently, this is similar to the behaviour of the dilatation effect of a solid mass; the movement of a sand swarm is accompanied by volume dilation [32], which may be attributed to the nonnegligible frictional drag between sand particles. In addition, it can be found from the tests that, with the same grain composition, the higher the hydraulic gradient value, the greater the expanded volume of the sand column. The time required for the volume of the sand column to reach maximum depends on the initial height of the sand column, the size of the grains, and the gradation of sand sample. This issue will be analysed in depth in subsequent research.

3.3. Experimental Procedure. The occurrence of a water–sand inrush disaster is a complex physical process, which can be identified in three stages [33]: (i) the “sand particle incipient motion” stage, at this stage, the sand particles in the subsurface system are activated by groundwater seepage and leads to instability seepage; (ii) the “sand particles migration” stage, during this stage, movable sand particles transport in the aquifer, leaving behind a coarse porous skeleton, moreover, with the increase of the amount of mobile sand, the skeleton porosity increases; (iii) the “high-concentration water–sand flow” stage, at this stage, the hydraulic gradient threshold of sand failure is reached and water–sand inrush is formed. According to the three different stages of water–sand inrush, three kinds of experiments were designed, including (1) sand particle incipient motion tests, (2) sand particle migration in porous skeleton, and (3) water–sand mixture flow tests. Figure 8 presents the sketch of samples of the three experiments to be tested.

3.3.1. Sand Particle Incipient Motion Tests. In this set of tests, a 3-component grade Aeolian sand was adopted; at the same time, three groups of river sands were also selected in order to analyse the effect of particle size on the critical parameters. The sketch of the sample to be tested is shown in Figure 8(a), the sand specimen is filled in the bottom part of the test tube layer by layer, with a height of 12 cm. Steel balls are filled in the upside, with the height of 8 cm. Considering the stress environment of the subsurface aquifer during the water–sand inrush, an axial stress was loaded on the sample. The design values of the axial stress were 0, 0.1, 0.5, 1.0,
1.5, and 1.8 MPa, respectively. On the one hand, loading axial stress is in order to compact the sand specimen to be tested; on the other hand, it is to compare the initiation characteristics of the sand particles under different compressive stress. It is worth to mention that, at first, we chose the quartz sand as the porous skeleton; however, some of the quartz sand particles were crushed when the axial pressure was over 0.5 MPa, which leads to a configuration changing in the porous skeleton. This has a nonnegligible influence on the accuracy of the experiment. Thus, we chose steel balls instead of the quartz sand. In view of blocking, probability will significantly increase, when the ratio of the hole diameter to the maximum sand diameter is smaller than 4 [34]. A porous plate, with the larger hole of 4 mm and the smaller hole of 2 mm as shown in Figure 3(c), is placed between them only for the specimens with particle sizes of less than 0.6 mm. The purpose is to prevent the steel balls from falling into the fine particles before the test begins. The sample to be tested was thoroughly saturated with distilled water before the test begins. The test procedure consisted of applying an upward hydraulic flow at prerequisite pressure levels. The increments of hydraulic gradient were kept between 0.1 and 0.3.

### 3.3.2. Sand Particle Migration in Porous Skeleton Tests

In this set of tests, white quartz was used as the porous skeleton to produce a significant colour contrast with the yellow Aeolian sand, making it convenient to observe the movement of the...
Aeolian sand during the experiment. Before the tests, the Aeolian sand and quartz were thoroughly saturated. Figure 8(b) shows the sample sketch. Firstly, fill the quartz sand layer by layer into the test tube to ensure the quartz skeleton uniform. Then, fasten two porous plates to the ends of the quartz skeleton. \( \phi_q = 49.31\% \) is the porosity of the quartz skeleton. At last, fill the Aeolian sand grains into the interstitial voids of the quartz skeleton until it is completely filled. \( \phi_o = 30.94\% \) is the initial porosity of the Aeolian-quartz sample. Water was pumped from the bottom of the test tube, which flowed upwards through the sample, before flowing into the WPV device. The upstream water pressures were 0.1 MPa, 0.2 MPa, 0.275 MPa, 0.375 MPa, and 0.45 MPa, respectively; the corresponding water pressure differences were 0.095, 0.17, 0.23, 0.25, and 0.377 MPa, respectively.

The continuous flow may cause the sand particle to move out of the aquifer skeleton, which result in an increase in the porosity. Using Equation (3), the mass of SFO \( (m_{s,j}) \) can be calculated, based on the cumulative mass of sand loss. At every 3 seconds, the porosity \( \phi_j \) and the porosity increase rate \( \phi'_j \) can, respectively, be calculated by

\[
\phi_j = \frac{Ah - (m_i/q_q + m_{j-1}/P_a - m_{s,j}/P_a)}{A\cdot h} \quad (j = 1, 2, \cdots, n),
\]

\[
\phi'_j = \frac{\phi_j - \phi_{j-1}}{3} \quad (j = 1, 2, \cdots, n),
\]

where \( A \) is the cross-section of the test tube, \( h \) is the height of the sample, \( m_i = 808 \text{ kg} \) and \( q_q = 2.82 \text{ g/cm}^3 \) are the mass and density of the quartz gravel, respectively, \( m_{j-1} = 272 \text{ kg} \) is the mass of the Aeolian sand in the pores of the quartz
skeleton, and \( \rho_a = 2.62 \text{ g/cm}^3 \) is the density of natural Aeolian sand.

3.3.3. Water–Sand Mixture Flow Tests. In the current experiment, ten groups of water–sand mixture flow tests were carried out. As shown in Figure 8(c), the test tube, with the height of 32 cm, was filled with saturated Aeolian sand. Water was injected from the bottom of the sample and flow upward, with the hydraulic gradients, \( i \), being 1.14, 1.66, 2.17, 2.69, 3.20, 3.72, 4.23, 4.75, 5.26, and 5.78, respectively. The concentration of the water–sand mixture, \( C_j \) (g/cm\(^3\)) can be calculated as

\[
C_j = \frac{m_{w,j}}{V_{w,j}} (j = 1, 2, \ldots, n). \tag{9}
\]

The details of each sample composition are summarized in Table 1.

4. Results and Discussion

4.1. Effect of Particle Diameter and Axial Stress on Sand Grain Incipient Motion

4.1.1. Effect of Particle Diameter on Sand Grain Incipient Motion. The incipient motion criterion of particles on a sedimentary bed can be theoretically solved according to the Shields curve [34–36]. And the criteria for the incipient motion of near-bed sediment have been well established and studied by several authors [37, 38]. However, due to the difference in motion space and stress state, the incipient motion of sand particles in the subsurface aquifer is significantly different from the former.

In a traditional study of internal erosion in granular soils, the hydraulic gradient \( i \) is often adopted as the critical criterion, which was proposed by Terzahi (1922) firstly, and widely used by many scholars in engineering research [39–41]. However, it is difficult to determine the hydraulic gradient of the damaged area in the underground engineering. Thus, in the current tests, the threshold velocity of the river sand and Aeolian sand were measured, and the critical particle Reynolds number (\( \text{Re}_c \)) were calculated [42], as shown in Table 2.

\[
\text{Re}_c = \frac{u_c d_m}{\nu}, \tag{10}
\]

where \( u_c \) is critical shear or friction velocity, being equal to the critical velocity in porous medium; \( d_m \) is mean particle diameter; and \( \nu \) is kinematic viscosity. The fitting relationship between critical Reynolds number and average grain diameter is shown in Figure 9.

The external environment of sand particle incipient motion during water–sand inrush is similar to that of piping initiation; it can be described as fine sand particle initial movement by a subsurface water flow or seepage [39, 43]. Thus, the forces acting on the mobile sand particles in the specimen column can be determined following the analysis of Fleshman and Rice [43]. As a vertical hydraulic gradient is imposed on the sand specimen, the forces acting on the sand particles include weight (FW), buoyant force (FB), seepage force (FS), viscous shear forces (FV), and intergranular forces (FI). Their research suggests that as the size of voids increases, seepage force and viscous shear forces all decrease.

As shown in Figure 9, the critical incipient Reynolds number increases in a power-law relationship with the average particle diameter under various external stress. This can be attributed to the fact that, as the size of the particle increases, given a larger void, it leads to a decrease in FS and FV. At the same time, as the particle size increases, the weight of the particle (FW) increases, and the more energy the particle needs for incipient motion. Consequently, the larger of the particle size, the higher the critical Reynolds number. In addition, in Figure 9, \( \text{Re}_c = a d_m^2 \) can be obtained, and the goodness-of-fit \( R^2 \) of the fitting curve ranges from 0.988 to 0.993. The axial stress is 0 MPa, 0.1 MPa, 0.5 MPa, 1.0 MPa, 1.5 MPa, and 1.8 MPa, respectively; the corresponding constant coefficient \( a \) is 0.04061, 0.04312, 0.04526, 0.04656, 0.04884, and 0.05191, respectively. It is clear that the coefficient \( a \) increases with the increasing of axial stress. Furthermore, the critical Reynolds number with larger particle size is more sensitive to axial stress. This will be explained in Section 4.1.2.
4.1.2. Effect of Axial Stress on Sand Grain Incipient Motion. 

The intergranular forces are caused by two distinct physical origins: (1) intergrain contacts between kinetic sand particles and (2) intergrain contacts between kinetic sand particle and skeleton granular. According to Table 2, it also can be found that the critical Reynolds number increases with the increasing axial stress. This is because the axial stress makes the sand swarm more compact and to a certain extent reduces the void volume among particles. As a result, the two kinds of intergranular forces mentioned above are increased. Therefore, a larger external stress denotes higher critical Reynolds number for sand grain incipient motion. Furthermore, Figure 10 depicts the effect of axial stress on critical Reynolds number. Through the image, as mentioned above, the river sand samples are more sensitive to the external stress than the Aeolian sand samples. This result is caused by the river sand particle being rougher than the Aeolian sand particle in the current tests. The particle with rougher surfaces generated greater intergranular force, leading to the increase of critical Reynolds number for particle incipient motion.

Figure 8: Sketch of the sample to be tested for (a) sand particle incipient motion tests, (b) sand particles migration in porous skeleton tests, and (c) water–sand mixture flow tests.
4.2. Characteristics of Water–Sand Mixture Migration in Quartz Gravel Skeleton

4.2.1. Water and Sand Flow Rate Evolution. The continuous flow may cause the accumulation in the quantitative mechanism of mobile sand, to such an extent that the sand skeleton of the aquifer becomes unstable. In the second experiment, the evolution of volumetric flow rates of the Aeolian sand ($v_s$) and water ($v_w$) are calculated using Equations (5) and (6), as shown in Table 3.

Figure 11 presents the evolution of (a) sand and (b) water flow rates with time for B1–B5 samples. As shown in Figure 11(a), in the beginning of the test, the sand flow rate increases rapidly, reaches the maximum value in 6 seconds, and then gradually decreases. It can be observed that the transportation of sand particles in the porosity skeleton is a nonlinear process. Owing to the sand grain continuous flow out of the test tube, the porosity of the sample keeps increasing. In the first 6 seconds, the sand flow rate rises sharply. However, under the same water pressure difference, the higher the porosity, the lower the interstitial water flow rate, leading to a decrease of the particle flushing ability. Therefore, as the test progresses, the sand flow rate of all the samples increases first and then decreases.

In Figure 11(b), the water flow rate shows obvious fluctuations. The greater the water pressure difference, the more significant the increase or decrease in flow rate. On the one hand, the continuous water flow causes the movable sand particles in the upper part of the sample to flow out of the test tube, while the sand particles in the lower part of the sample fill in the upper part, resulting in the local porosity continuous change. On the other hand, the migration of sand grains causes changes in the structure of the porous skeleton, which in turn causes changes in permeability. Thus, there is a character of significant fluctuations in the water flow rate.

The above analysis indicates that the interaction between water and movable sand is the key factor affecting the motion characteristics of the water–sand mixture. When the aquifer skeleton is stable, the water and movable sand grains are

| Table 1: Details of the testing samples. |
|-----------------------------------------|
| Sample no. | Particle size (mm) | Porosity | Axial stress, P (MPa) |
|------------|--------------------|----------|---------------------|
| A1         | 0.075-0.15         | 36.09%   | 0, 0.1, 0.5, 1.0, 1.5, and 1.8 for each sample |
| A2         | 0.15-0.3           | 33.57%   |                                   |
| A3         | 0.3-0.6            | 33.98%   |                                   |
| A4         | 0.6-1.0            | 45.32%   |                                   |
| A5         | 1.0-2.0            | 42.36%   |                                   |
| A6         | 2.0-2.36           | 43.36%   |                                   |

| Table 2: Critical Reynolds number of Aeolian sand and river sand particles under various axial stress. |
|---------------------------------------------------------------|
| Sample no. | Particle size (mm) | Critical Reynolds number, $Re_c$ |
|------------|--------------------|---------------------------------|
| A1         | 0.075-0.15         | 0.1888 0.1987 0.2082 0.2135 0.2236 0.2390 |
| A2         | 0.15-0.3           | 0.1028 0.1124 0.1182 0.1199 0.1261 0.1299 |
| A3         | 0.3-0.6            | 0.0212 0.0227 0.0256 0.0373 0.0404 0.0475 |
| A4         | 0.6-1.0            | 0.0015 0.0015 0.00157 0.00165 0.0017 0.00177 |
| A5         | 1.0-2.0            | 1.5817e$^{-4}$ 2.3715e$^{-4}$ 3.1613e$^{-4}$ 5.5328e$^{-4}$ 6.3248e$^{-4}$ 7.1145e$^{-4}$ |
| A6         | 2.0-2.36           | 2.1960e$^{-5}$ 2.2044e$^{-5}$ 2.2629e$^{-5}$ 2.3965e$^{-5}$ 2.4700e$^{-5}$ 2.8975e$^{-5}$ |
mutually restricted, and the flow tends to be stable. On the contrary, when the water and movable sand grains are mutually promoted, the water–sand mixture flow rate continues to increase, and the aquifer skeleton becomes unstable. Therefore, aquifer structure instability is one of the key criterions for the water–sand inrush disaster occurrence.

4.2.2. Porosity Evolution. Using Equations (7) and (8), the porosity \( \phi \) and the porosity increase rate \( \phi' \) can be calculated, respectively. The results are shown in Table 4.

Figure 12 presents the evolution of (a) porosity and (b) change rate of porosity with time, respectively. It can be found that the porosity remains increased with time. Compared with Figure 11(a), the change rate of porosity increases sharply in the first 6 seconds due to the sand flow rate increasing rapidly. Meanwhile, the rate of change in porosity reaches the maximum value at the sixth second, just corresponding to the peak value of the sand flow rate. Then, the increase slows down and shows notable fluctuations and decreases, as shown in Figure 12(b). Furthermore, it can be seen that the higher the water pressure difference, the more significant the change of porosity.

The above phenomenon indicates that, as the movable sand particle transport through the porous skeleton, the pore structure rearranges. At the beginning of the test, water is the dominant phase. As the water flow rate increases, a large number of particles flow with the water flow, and the sand becomes the dominant phase. Meanwhile, a large amount of movable sand particles occupy the flow pathways, leading to the reduction of the water flow rate. However, as the flow rate of water decreases, the particle-carrying ability decreases accordingly [31]. Therefore, some of the movable sand
particles are deposited in the porous skeleton, and the sand flow rate decreases. Subsequently, the water flow once again occupies the flow pathways and becomes the dominant phase, and the water flow rate increases again.

To sum up, when the hydraulic condition reaches the threshold of water–sand inrush, the sand flow rate increases significantly and the change rate of porosity remains at a high level. Otherwise, the sand flow rate decreases with time. It is noteworthy that the water–sand inrush prevention methods should be performed to control the water pressure difference and sand flow rate.

4.3. Character of High-Concentration Water–Sand Mixed Fluid Migration

4.3.1. The Evolution of Solid Concentration. In the current experiment, the evolution of the solid concentration of the water–sand mixture is calculated based on Equation (9). The results for the solid concentration evolution are shown in Figure 13. It should be noted that in the relatively short period of time after the start of the tests (0-40 s), the solid particle concentration changes drastically, while in a longer period of time after the start of the tests (0-40 s), the solid concentration gradually remains constant. Therefore, in order to describe the characteristics of the mixture from a mesomechanic perspective, it would require more sophisticated testing equipment and designing other experiments. Therefore, in this research, we analyse the motion characteristics of the mixture from a macroscopic view, and the behaviour is explained in the sequel.

4.3.2. Volumetric Flow Rates of Sand and Water. For every 3 seconds ($\Delta t$), based on $m_{s,j}$ and $m_{w,j}$, the interval volumetric flow rate of water ($v_{w,j}$) and sand ($v_{s,j}$) can, respectively, be calculated as

$$v_{s,j} = \frac{m_{s,j} - m_{s,j-1}}{\rho_s \cdot \Delta t} (\Delta t = 3, j = 1, 2, \ldots, n),$$

$$v_{w,j} = \frac{m_{w,j} - m_{w,j-1}}{\rho_w \cdot \Delta t} (\Delta t = 3, j = 1, 2, \ldots, n),$$

Figure 14 shows the variations of the water and sand flow rates with time for different samples, respectively. In general, the higher the water pressure, the higher the sand flow rate. Comparing the images of Figure 14, with the increase of hydraulic gradient, a clear increase in the sand flow rate has been witnessed. It can be observed that, for samples C1, C8, C9, and C10 (record as group-1), the volumetric flow rate of sand is significantly higher than that of water, leading to a greater solid concentration (greater than 0.7). While, for samples C2, C3, C4, C5, C6 (record as group-2), and C7, the volumetric flow rate of sand is a little higher than that of water, resulting in a relatively lower solid concentration (between 0.51 and 0.554). Moreover, it is an interesting observation that for all the samples C1 to C10, the curves of the volumetric flow rate of water and sand shows regular fluctuations. It can be seen from Figure 14 that there are two types of distinct waveforms in the curves. For the samples of group-1, the curve of the water flow rate displays an “upward convex” shape, and the curve of sand flow rate displays a “downward concave” shape. By contrast, for the samples of group-2, the flow rate curve shows a “downward concave” shape for water and an “upward convex” shape for sand. Apparently, group-1 samples with waveform-i (see Figure 14) have a significantly higher concentration than that of group-2 with waveform-ii (see Figure 14). This may owe to the rheology of the mixed fluid, e.g., R. Medina et al. indicated that extremely small
Table 3: The evolution of flow rates for (a) Aeolian sand particles and (b) water.

| Time (s) | $v_i$ (cm$^3$/s) | $v_o$ (cm$^3$/s) | $v_i$ (cm$^3$/s) | $v_o$ (cm$^3$/s) | $v_i$ (cm$^3$/s) | $v_o$ (cm$^3$/s) | $v_i$ (cm$^3$/s) | $v_o$ (cm$^3$/s) | $v_i$ (cm$^3$/s) | $v_o$ (cm$^3$/s) |
|----------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|          | 0.095 MPa        | 0.17 MPa         | 0.23 MPa         | 0.25 MPa         | 0.377 MPa        | 0.095 MPa        | 0.17 MPa         | 0.23 MPa         | 0.25 MPa         | 0.377 MPa        |
| 3        | 2.263            | 8.971            | 2.675            | 11.864           | 3.909            | 46.691           | 4.527            | 65.700           | 8.025            | 90.309           |
| 6        | 3.685            | 18.292           | 4.321            | 26.152           | 5.041            | 62.383           | 5.967            | 69.080           | 8.128            | 83.372           |
| 9        | 4.275            | 24.196           | 4.951            | 39.845           | 5.362            | 61.192           | 6.061            | 76.118           | 7.407            | 82.815           |
| 12       | 4.270            | 22.644           | 4.605            | 38.463           | 4.938            | 60.660           | 5.677            | 79.637           | 7.024            | 83.597           |
| 15       | 4.033            | 25.157           | 4.294            | 37.659           | 4.740            | 60.010           | 5.529            | 68.021           | 6.132            | 107.579          |
| 18       | 3.807            | 23.367           | 4.087            | 37.123           | 4.319            | 54.133           | 4.991            | 60.618           | 5.418            | 96.738           |
| 21       | 3.527            | 23.619           | 3.762            | 38.035           | 4.030            | 48.423           | 4.543            | 54.419           | 4.826            | 90.284           |
| 24       | 3.318            | 30.011           | 3.627            | 36.709           | 3.880            | 45.591           | 4.150            | 48.254           | 4.286            | 79.834           |
| 27       | 3.369            | 34.898           | 3.544            | 35.224           | 3.651            | 43.998           | 3.731            | 46.266           | 3.824            | 74.262           |
| 30       | 3.354            | 32.799           | 3.374            | 38.629           | 3.407            | 41.627           | 3.436            | 43.196           | 3.457            | 67.660           |

Figure 11: The evolution in (a) sand flow rates and (b) water flow rates with time for different water pressure.

Table 4: The porosity and porosity change rate data.

| Time (s) | Porosity $\phi$ (%) | Porosity change rate $\dot{\phi}$ (min$^{-1}$) |
|----------|---------------------|-----------------------------------------------|
|          | 0.095 MPa           | 0.17 MPa           | 0.23 MPa           | 0.25 MPa           | 0.377 MPa          | 0.095 MPa           | 0.17 MPa           | 0.23 MPa           | 0.25 MPa           | 0.377 MPa          |
| 0        | 30.94               | 30.94               | 30.94               | 30.94               | 30.94               | 0                  | 0                  | 0                  | 0                  | 0                 |
| 3        | 32.1388             | 32.35723            | 33.01252            | 33.34017            | 35.19683            | 0.23976            | 0.28345            | 0.4145             | 0.48003            | 0.85137           |
| 6        | 34.84917            | 35.52447            | 36.28898            | 37.27191            | 39.56543            | 0.54207            | 0.63345            | 0.65529            | 0.78635            | 0.87372           |
| 9        | 37.74518            | 38.82093            | 39.47622            | 40.58838            | 42.73267            | 0.5792             | 0.65929            | 0.63745            | 0.66329            | 0.63345           |
| 12       | 40.00229            | 40.714              | 41.42209            | 42.99112            | 45.85072            | 0.45142            | 0.37861            | 0.38917            | 0.48055            | 0.62361           |
| 15       | 41.64052            | 42.33221            | 43.51718            | 45.61228            | 47.21049            | 0.32765            | 0.32364            | 0.41902            | 0.52423            | 0.27195           |
| 18       | 43.06032            | 43.95405            | 44.69271            | 46.83366            | 48.19343            | 0.28396            | 0.32437            | 0.2351             | 0.24428            | 0.19659           |
| 21       | 44.04325            | 44.91697            | 45.90991            | 47.81659            | 48.86873            | 0.19659            | 0.19259            | 0.24344            | 0.19659            | 0.13506           |
| 24       | 45.02619            | 46.33677            | 47.41308            | 48.56109            | 49.13885            | 0.19659            | 0.28396            | 0.30063            | 0.1489             | 0.05402           |
| 27       | 47.03208            | 47.86578            | 48.38018            | 48.75951            | 49.20638            | 0.40118            | 0.3058             | 0.19342            | 0.03968            | 0.01351           |
| 30       | 48.7395             | 48.48872            | 49.02034            | 49.17636            | 49.28558            | 0.34148            | 0.19659            | 0.12803            | 0.08337            | 0.01584           |
changes in the solid concentration can result in large variations in the velocity [44].

However, the physical origins of this fluctuation phenomenon are not well understood. We propose that this behaviour can be explained following the research of Haw [36]. If we consider the mixed fluid as a two-fluid system, i.e., a sand particulate fluid and water fluid, at high concentrations, where the interactions between the particles are nonnegligible, then it may lead to a rheological separation. During which some sand particles jam and form a solid that
Figure 14: Continued.
resists the pressure drop and stopping flow. Meanwhile, the water continues to flow through the pores of the jammed particles. Hence, there is an increase flow rate of water relative to sand, resulting in a reduction of sand particle volume fraction and a decrease in the volumetric flow rate of sand. Until the water flow rate reaches the peak, FS and FV increase with the increasing water flow rate, leading to the jammed particles flowing again. Subsequently, the volume fraction and the volumetric flow rate of sand all increase, resulting in a decrease of the water flow rate. Therefore, from a macroscopic perspective, the flow rate of water and sand presents the characteristics of alternating fluctuations (as shown in Figure 14). While, explaining the above phenomenon from a meso-mechanical perspective would require more sophisticated testing equipment and designing other experiments, we will discuss this in depth in the next article.

5. Conclusions

(1) Water–sand inrush disaster is a dynamic evolutionary process, and three phases of the disaster were identified. There are two critical points for the water–sand inrush disaster occurrence: firstly, the critical point of sand particle incipient motion and secondly, the critical point of aquifer structural instability.

(2) The threshold of sand particle incipient motion in the aquifer skeleton was affected by the particle size and external stress. The sample with a larger particle size and higher axial stress demonstrates a higher critical Reynolds number. The particles with larger size are more sensitive to the external stress than those with smaller particles. The relationship between the Reynolds number and mean particle size was obtained

$$\text{Re}_c = a d_m^5.$$  \hspace{1cm} (12)

(3) During the water–sand mixture transport in the aquifer skeleton, the interaction mechanism between water and sand particles is the critical issue for aquifer structure instability. When the hydraulic condition reaches the threshold of the water–sand inrush,
the water and sand grains are mutually promoted, and the aquifer skeleton becomes unstable. On the contrary, when the water and sand grains are mutually restricted, the aquifer skeleton tends to be stable.

(4) When the water–sand inrush disaster occurs, solid concentration remains at a relatively high level, for instance above 0.5 or above 0.7. The motion feature of the mixed fluid was greatly dominated by the rheological property of the sand fluid. Two types of waveforms in the curve of volumetric flow rates for sand and water, respectively, were experimentally observed.

Data Availability

The experiment data used to support the findings of this study are included within the supplementary information file.

Conflicts of Interest

There is no conflict of interest regarding the publication of this paper.

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Supplementary Materials

Table 1: water and sand flow rates for the tests of sand particles migration in sand–quartz-filled bed. Table 2: mass of SFO and porosity for the tests of sand particles migration in sand–quartz-filled bed. Table 3: change rate of porosity for the tests of sand particles migration in sand–quartz-filled bed. Table 4: sand flow rate for the tests of water–sand mixed fluid migration. Table 5: water flow rate for the tests of water–sand mixed fluid migration. Table 6: solid particle concentration for the tests of water–sand mixed fluid migration. (Supplementary Materials)

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