Assessing the number of atoms in a Rydberg-blockaded mesoscopic ensemble

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The dipole blockade of multiple Rydberg excitations in mesoscopic atomic ensembles allows the implementation of various quantum information tasks using collective states of cold, trapped atoms. Precise coherent manipulations of the collective ground and single Rydberg excitation states of an ensemble require the knowledge of the number of atoms with small uncertainty. We present an efficient method to acquire such information by interrogating the atomic ensemble with resonant pulses while monitoring the resulting Rydberg excitations. We show that after several such steps accompanied by feedback the number of atoms in an ensemble can be assessed with high accuracy. This will facilitate the realization of high fidelity quantum gates, long term storage of quantum information and deterministic sources of single photons with Rydberg-blockaded atomic ensembles.

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Atoms in high-lying Rydberg states strongly interact with each other via the dipole-dipole or van der Waals potential \cite{1,2}. This long-range interaction is the basis for many quantum information processing schemes \cite{2} including quantum gates between individual atoms \cite{3,4,5} and photons \cite{6–9}. The interaction-induced level shifts and photons \cite{6–9}. The interaction-induced level shifts can suppress resonant optical excitation of more than one Rydberg atom within a certain blockade volume containing tens or hundreds of atoms \cite{2,10}. In a seminal paper Lukin et al. \cite{11} proposed to employ the transition between the collective ground and single Rydberg excitation states of an atomic ensemble to perform various quantum information tasks: Collection of atoms confined in a blockade volume can behave as a two-level quantum system—a qubit—and can be used for controlled manipulation of collective spin excitations as well as on-demand generation of single or few photon pulses \cite{12,13}. Several related schemes for scalable quantum information processing with Rydberg-blockaded atomic ensembles were proposed \cite{2} while recent experiments have demonstrated (partially) coherent oscillations of a single Rydberg excitation \cite{14} and the generation of entanglement between a single photon and an atomic ensemble \cite{15}.

When $N$ atoms are symmetrically (uniformly) excited by a coherent laser field, the transition amplitude between the collective ground and single-excitation states of the ensemble is enhanced by a factor of $\sqrt{N}$ relative to the transition amplitude $\Omega$ (Rabi frequency) for a single atom. Hence, an uncertainty $\Delta N$ in the atom number translates into an uncertainty in the collective Rabi frequency which would result in uncontrolled errors during the ensemble manipulations, as detailed below. Furthermore, collective Rydberg excitations are typically stored by mapping onto long-lived spin excitations of trapped atoms. During the storage, the short-range interactions between the atoms in different internal states lead to accumulation of a relative phase between the collective states which is proportional to the number of atoms $N$ and the storage time $\tau$. Hence, again, the atom number uncertainty $\Delta N$ leads to a random relative phase which would reduce the storage fidelity or limit its time.

The above arguments attest to the necessity of determining the actual number of atoms $N(a)$ with sufficient precision in order to achieve high-fidelity quantum gates, storage of quantum information and single photon generation with a Rydberg-blockaded atomic ensemble. Here we present an efficient method to assess $N(a)$ with small uncertainty. The method is based on interrogating the atomic ensemble by a sequence of resonant pulses of proper area, each pulse followed by a (projective) measurement of Rydberg excitation and a feedback to adjust the area of the next pulse. We show that after only a few such steps, we gain enough information on the actual number of atoms so as to perform quantum gate operations with improved fidelity, while after several tens of pulses—and only a few detection events—the resulting gate infidelity drops below the threshold for fault-tolerant quantum computation \cite{16}. Moreover, our algorithm performs well even for imperfect detection of Rydberg atoms.

Before describing the method, let us reexamine resonant laser excitation of a collection of $N$ atoms to the strongly interacting Rydberg state. We denote by $|g\rangle$ and $|r\rangle$ the ground and Rydberg states of individual atoms and assume a spatially uniform laser field. The corresponding atom-field interaction Hamiltonian reads $\mathcal{V}_{af} = \hbar \sum_j \frac{1}{2} \Omega (|r_j\rangle\langle g_j| + |g_j\rangle\langle r_j|)$. The laser field couples the collective ground state $|G\rangle = |g_1, g_2, \ldots, g_N\rangle$ to the symmetric single Rydberg excitation state $|R\rangle = \frac{1}{\sqrt{N!}} \sum_j |g_1, \ldots, r_j, \ldots, g_N\rangle$ with the collective Rabi frequency $\frac{\hbar}{\sqrt{2}} \mathcal{V}_{af} |G\rangle = \sqrt{N}\Omega$. Applying a laser pulse with the single-atom pulse area $\theta = \int \Omega \, dt$ transforms the initial state $|G\rangle$ as

$$|G\rangle \to \cos \left( \frac{1}{2} \sqrt{N} \theta \right) |G\rangle + i \sin \left( \frac{1}{2} \sqrt{N} \theta \right) |R\rangle,$$

and similarly for the initial $|R\rangle$ (with the replacement
\( \langle G \rangle \leftrightarrow |R\rangle \). Throughout this paper, we assume that multiple Rydberg excitations are strongly suppressed by the dipole blockade \( 2 \times 10^{-14} \).

We can estimate the average fidelity of performing quantum gates with Rydberg-blockaded atomic ensembles in a realistic experiment. Usually, upon preparing an ensemble of atoms in a trap, the experimentalist does not know their actual number, but he can assign to it a random variable \( N \) with a certain probability distribution \( P(N) \) and a mean \( \langle N \rangle \). It is then natural to take as the most probable number of atoms \( N^{(p)} = \langle N \rangle \).

Consider, e.g., the NOT (or SWAP) gate which swaps the states \( |G\rangle \) and \( |R\rangle \). To within a trivial \( \frac{\pi}{2} \) phase shift of the final state, the SWAP is realized by applying a collective \( \pi \)-pulse \( \sqrt{N^{(p)}} \theta = \pi \). If the actual number of atoms \( N^{(a)} = \langle N \rangle \), then \( |G\rangle \rightarrow i|R\rangle \) and \( |R\rangle \rightarrow i|G\rangle \) [cf. Eq. (1)]. But in general the number of atoms is uncertain with the standard deviation \( \Delta N \). Defining the fidelity as \( F = \langle |\Psi^{(a)}\rangle |\Psi^{(a)}\rangle \rangle \), where \( |\Psi^{(a)}\rangle \rangle \) is the ideal target state and \( |\Psi^{(a)}\rangle \) is the actually obtained one, we have

\[
F = \sin \left( \frac{\pi}{2} \sqrt{\frac{N^{(a)}}{N^{(p)}}} \right) .
\]

To estimate the average fidelity, we may replace \( N^{(a)} \rightarrow \langle N \rangle \pm \Delta N \) assuming \( \Delta N \ll \langle N \rangle \), which leads to \( F \approx \cos \left( \frac{\Delta N}{\langle N \rangle} \right) \). Then the average infidelity, i.e., deviation of \( F \) from unity, scales as \( 1 - F \approx \frac{\pi^2}{32} \left( \frac{\Delta N}{\langle N \rangle} \right)^2 \). It is easy to verify that the infidelity for any single-qubit rotation gate scales quadratically with the uncertainty \( \Delta N \). Typically, the probability distribution of the atom number is Poissonian \( P(N) = P_{\text{Poisson}}(N) = e^{-\langle N \rangle} \langle N \rangle^N / N! \) with the variance \( \langle N \rangle^2 = \langle N \rangle^2 = \langle N \rangle \). Then the average infidelity reduces to \( 1 - F \approx \frac{\pi^2}{32} \frac{\Delta N}{\langle N \rangle} \) and to have a small error of, e.g., \( 1 - F \lesssim 10^{-4} \) requires \( \langle N \rangle \gtrsim 3000 \) atoms, which is an impractically large number for few \( \mu \text{m} \) sized trap (to guarantee the blockade).

Next, once a Rydberg excitation \( |r\rangle \) is created, it is transferred to a lower spin-flip state \( |s\rangle \), which maps the collective state \( |R\rangle \) onto the metastable state \( |S\rangle = \sqrt{\sum_j |g, \ldots, s, \ldots, g\rangle} \). Cold atoms in a tight trap interact with each other, albeit weakly, with the strength \( U_{\mu \nu} \) determined by the (state-dependent) \( s \)-wave scattering length and trap geometry [14]. The interaction energies of states \( |G\rangle \) and \( |S\rangle \) are then given by \( E_G = \frac{1}{2} U_{gg} \langle N-1 \rangle \) and \( E_S = \frac{1}{2} U_{gg} \langle N-1 \rangle \langle N-2 \rangle + U_{gs} \langle N-1 \rangle \), and their difference is \( E_G - E_S = \delta U \langle N-1 \rangle \), where \( \delta U \equiv U_{gg} - U_{gs} \).

We now estimate the average fidelity of storage of a quantum state in a coherent superposition of \( |G\rangle \) and \( |S\rangle \). Unless \( U_{gg} = U_{gs} \) (\( \delta U = 0 \)), the uncertainty \( \Delta N \) in the total atom number results in the accumulation during time \( \tau \) of a random relative phase \( \Delta \phi = \Delta N \delta U \tau / 2 \) between states \( |G\rangle \) and \( |S\rangle \). Upon averaging over all qubit states, the storage fidelity is \( F = \sqrt{1 + \cos(\Delta \phi)} / 2 \); and for \( \Delta \phi \ll 1 \) the infidelity \( 1 - F \approx (\Delta \phi)^2 / 8 \) scales again quadratically with the uncertainty \( \Delta N \). For the Poissonian distribution of the atom number, we then have \( 1 - F \approx \langle N \rangle (\delta U \tau)^2 / 8 \), i.e., small error for longer storage times requires now a smaller mean number of atoms \( \langle N \rangle \). As an example, with \( \langle N \rangle \approx 200 \) and \( \delta U / 2 \tau \approx 1 - 10 \text{ Hz} \), the infidelity \( 1 - F \lesssim 10^{-4} \) limits the storage time to \( \tau \lesssim 0.1 \text{ ms} \).

Having discussed the detrimental effects of the atom number uncertainty, we now turn to the description of a protocol to determine with high accuracy the actual number of atoms \( N^{(a)} \). This is done in a finite number of steps, each step consisting of (i) deducing the most probable number of atoms \( N^{(p)} \) from the probability distribution \( P(N) \), (ii) applying to the atomic ensemble in state \(|G\rangle \) a collective pulse of area \( \sqrt{N^{(p)}} \theta = 2\pi n \) (\( m \in \mathbb{Z} \)), (iii) measuring the Rydberg excitation and updating \( P(N) \) using the Bayes’ rule. Specifically, before each step \( i \), there is a probability distribution \( P_{i-1}(N) \) of the atom number which reflects our current state of knowledge—or ignorance, for that matter—about the actual number of atoms. As the most probable atom number, we take \( N^{(p)}_{i-1} \) which maximizes \( P_{i-1}(N) \). We then set the collective pulse area \( \sqrt{N^{(p)}_{i-1}} \theta \) to be a multiple \( (m) \) of \( 2\pi \), i.e., \( \theta_i = 2m\pi / \sqrt{N^{(p)}_{i-1}} \). After the pulse, we perform projective measurement of the Rydberg excitation of the ensemble, which may be accompanied by either discarding the atom in state \(|r\rangle \) or recycling it back to state \(|g\rangle \). If the result of the measurement is negative, the updated probability distribution becomes \( P_i(N) = \delta_i(N) P_{i-1}(N) / S_i \) \( \forall \) \( N \), where \( \delta_i(N) = \cos^2 \left( \frac{N\theta_i}{\sqrt{N^{(p)}_{i-1}}} \right) \) is the probability of no Rydberg excitation [cf. Eq. (1)]; if the measurement does yield a Rydberg excitation, then the updated probability distribution becomes \( P_i(N) = \sigma_i(N) P_{i-1}(N) / S_i \) \( \forall \) \( N \) (if the Rydberg atom is transferred back to state \(|g\rangle \) [14]) or \( P_i(N) = \sigma_i(N+1) P_{i-1}(N+1) / S_i \) \( \forall \) \( N \) (if the Rydberg atom is removed) [12], where \( \sigma_i(N) = \sin^2 \left( \frac{N\theta_i}{\sqrt{N^{(p)}_{i-1}}} \right) \) is the probability of Rydberg excitation [Eq. (1)] and \( S_i \) renormalizes \( P_i(N) \). In turn, in the (numerical or laboratory) experiment, the outcome of the measurement is determined by the actual—but unknown—number of atoms \( N^{(a)} \) through the actual probability of Rydberg excitation

\[
\sigma_i^{(a)}(N) = \sin^2 \left( m\pi \sqrt{\frac{N^{(a)}_{i-1}}{N^{(p)}_{i-1}}} \right) .
\]

To commence the procedure, we assume a certain mean number of atoms \( \langle N \rangle \) and a width \( W \) of the atom number distribution \( P_0(N) \). For definiteness, we take as a seed the Poissonian distribution \( P_0(N) = P_{\text{Poisson}}(N) \) (with FWHM \( W = 2\sqrt{\ln(2)\langle N \rangle} \)), but any other distribution with a well-defined range (and no long wings) will do as
well. For the first step, the most probable atom number is \( N_{(p)}^{(a)} \approx \langle N \rangle \). The positive integer constant \( m \) is then chosen so as to maximize the detection probability \( \sigma(N) \) at the edges of the initial distribution, e.g., at \( N \approx \langle N \rangle \pm \frac{3}{2}W \), which leads to \( m = \lfloor 2\langle N \rangle / 3W \rfloor = \lfloor \sqrt{\langle N \rangle}/3.53 \rfloor \), where \( \lfloor \ldots \rfloor \) denotes the floor function.

In the adaptive (feedback) scheme described above, no Rydberg excitation at a given step \( i \) narrows the probability distribution \( P_i(N) \) around the perceived atom number \( N_{(p)}^{(a)} \), while a detection of Rydberg excitation drastically reshapes \( P_i(N) \) by opening a hole around \( N_{(p)}^{(a)} \) which leads to a new \( N_{(p)}^{(a)} \) for the following step. Observe now that the actual probability of Rydberg excitation in Eq. (3) is appreciable when the perceived \( N_{(p)}^{(a)} \) and actual \( N_{(a)}^{(a)} \) atom numbers differ significantly, while \( \sigma_{(a)}^{(a)} \) is small when \( N_{(p)}^{(a)} \approx N_{(a)}^{(a)} \), and \( \sigma_{(a)}^{(a)} = 0 \) and remains so if our guess is correct, \( N_{(p)}^{(a)} = N_{(a)}^{(a)} \). Hence, if the initial difference \( N_{(p)}^{(a)} - N_{(a)}^{(a)} \) is large, \( N_{(p)}^{(a)} \) will rapidly change during the first several steps, but once it is close to \( N_{(a)}^{(a)} \), its further approach to \( N_{(a)}^{(a)} \) will slow down. One may optimize the search of the atom number by readjusting \( m \) at each step, but for simplicity we fix \( m \) from the beginning, which still leads to a rapid improvement of fidelity, as attested below.

To demonstrate the algorithm, in Fig. 1(a) we show the results of a single trajectory simulation for several steps of the stochastic process. In the numerical experiment, the occurrence of Rydberg excitations and their detection is determined via Monte Carlo procedure. Namely, at every step we draw a uniform random number \( q \in [0,1] \) and compare it with the current probability \( \sigma_{(a)}^{(a)} \); if \( q < \sigma_{(a)}^{(a)} \) the ensemble is in the excited state \( |g\rangle \), otherwise it is in the ground state \( |G\rangle \). Assuming for now a perfect detector, whenever a Rydberg atom is produced, it is detected and removed from the ensemble, which ends up in state \( |G\rangle \) but with one atom less, \( N_{(a)}^{(a)} \rightarrow N_{(a)}^{(a)} - 1 \). The algorithm works equally well if the Rydberg atom \( |p\rangle \) is recycled back to the ground state \( |g\rangle \), which leaves \( N_{(a)}^{(a)} \) unchanged. But since we deal with \( N_{(a)}^{(a)} \gg 1 \) and create only a few excitations, the difference between the two approaches is inconsequential. In practice, detecting the Rydberg atom via state-selective ionization accompanied by its removal is perhaps experimentally easier [18] and it does not heat the atomic ensemble.

Our benchmark is the infidelity \( 1 - F \) of the SWAP gate on the transition \( |G\rangle \rightarrow |R\rangle \) executed with the perceived atom number \( N_{(p)}^{(a)} \) [cf. Eq. (2)]. This is shown in the inset of Fig. 1(b). Clearly, once \( N_{(p)}^{(a)} \) is close to the actual atom number \( N_{(a)}^{(a)} \), the infidelity is very small.

In Fig. 1(b) we show the mean difference \( \langle N_{(a)}^{(a)} - N_{(p)}^{(a)} \rangle \) between the actual \( N_{(a)}^{(a)} \) and deduced \( N_{(p)}^{(a)} \) number of atoms after \( i \) steps, as obtained from many independent realizations of the algorithm. With increasing \( i \), this difference and its statistical dispersion are decreasing, while on average less than three Rydberg excitation events occur by \( i = 40 \), mainly during the first several steps. The corresponding mean infidelity \( 1 - \langle F \rangle \) of the SWAP gate drops from the initial \( 1 - \langle F \rangle > 10^{-2} \) to \( 2 \times 10^{-4} \) with the uncertainly \( \Delta F \simeq 3 \times 10^{-4} \).

In a laboratory experiment, the detector measuring the Rydberg excitations is never perfect—the best detection efficiency of individual Rydberg atoms achieved so far is \( \eta \approx 0.75 \) [19]. In our numerical simulations, we can account for finite detection efficiency \( \eta \leq 1 \) by reducing the probability of detecting the Rydberg excitation by the corresponding factor; at each step, however, the probability of producing a Rydberg excitation is still given by \( \sigma_{(a)}^{(a)} \). More precisely, once a Rydberg excitation is produced \( (q_{(p)} < \sigma_{(a)}^{(a)}) \), we draw another random number \( q_{\eta} \in [0,1] \) and compare it to \( \eta \); if \( q_{\eta} < \eta \) the Rydberg atom is detected; otherwise it is not and we proceed as if it were not created, but still assume that any Rydberg atom is removed from the ensemble after each step.

In Fig. 2 we show the mean difference \( \langle N_{(a)}^{(a)} - N_{(p)}^{(a)} \rangle \)
Before closing, let us briefly survey the practical side of our work. In the current experiments \cite{14,15}, the uncertainty in the atom number $\Delta N$ is perhaps a lesser evil than an imperfect blockade of multiple Rydberg excitations. The latter problem can be overcome by using higher-lying (stronger interacting) Rydberg states and tighter confinement of the atoms. Single Rydberg excitations transferred to a lower metastable state can then be used for on-demand generation of single photons, which is accomplished by stimulated Raman emission requiring sizable optical depth (OD) of the ensemble \cite{20}. In a blockade volume of several $\mu$m size, $\langle N \rangle \gtrsim 200$ atoms are needed for OD $\gtrsim 5$. Larger $\langle N \rangle$ is better, both for attaining larger OD and smaller relative uncertainty $\Delta N / \langle N \rangle^2$, but too high atom densities $\rho_{at} \gtrsim 10^{13-14}$ cm$^{-3}$ usually lead to excessive collisional decoherence and losses due to the three-body recombination. Hence, we are constrained to deal with the ensembles of a few hundred atoms $\langle N \rangle$ and large associated uncertainty $\Delta N$.

Our method to accurately assess the number of atoms is quite efficient, requiring $i = 20 - 30$ resonant pulses and measurements of Rydberg excitations. With the typical single-atom Rabi frequency $\Omega$ in the MHz range, each pulse duration is $t_p \approx 1 \mu$s, while the Rydberg atom detection through ionization would take $t_d \sim 10 \mu$s. The complete protocol can then be accomplished in less than a ms, yielding the actual number of atoms $N^{(a)}$ with a small uncertainty $\Delta N < \sqrt{N^{(a)}}$. We note that small variations in the single-atom pulse area $\Delta \theta < \frac{\Delta N}{N^{(a)}}$, due to uncertainty in $\Omega$ or $t_p$, do not significantly affect the performance of our algorithm and the fidelity of subsequent quantum gates.

The algorithm we have used to deduce the actual number of atoms is quite unique—we have explored other, somewhat less efficient methods too. Yet, the general concept of interrogating the atomic ensemble by appropriate pulses in order to extract the atom number with reduced uncertainty applies to other approaches as well.

To summarize, we have exposed the detrimental effects of the atom number uncertainty on the fidelity of quantum gates with the Rydberg-blockaded atomic ensembles. We have proposed and analyzed an experimentally realistic method to reduce this uncertainty and thereby increase the fidelity of collective gate operations and storage of quantum information in mesoscopic atomic ensembles. Accurate determination of the atom number will also facilitate deterministic generation of single photons \cite{12,13} and extraction of single Rydberg atoms or ions \cite{21} from mesoscopic atomic ensembles. We note the recent proposals \cite{22,23} employing adiabatic passage to reliably accomplish the above tasks in few-atom systems. Preparing number-squeezed clouds of cold atoms \cite{24} may find further applications in precision metrology and studies of Bose-Einstein condensation.
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