Migration of a solid and arbitrarily-shaped particle near a plane slipping wall

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Abstract. This work is concerned with the rigid-body migration of a solid and arbitrary-shaped particle immersed in a Newtonian liquid in vicinity of a plane, motionless and impermeable wall where a Navier slip condition [1] holds. The net hydrodynamic force and torque exerted on the moving particle are obtained by appealing to a new boundary elements approach which makes use of a specific Green tensor recently determined elsewhere [2]. The advocated technique results in the treatment of a Fredholm boundary-integral equation of the first kind on the particle surface and, by contrast to earlier works in this field, makes it possible to cope with non-spherical particles. The proposed numerical implementation is benchmarked against results obtained for a sphere by using the bipolar coordinates. Preliminary new results for the friction coefficients of an non-spheroidal ellipsoid are also reported and compared with those for a volume-equivalent sphere. The variations of the friction coefficients with the slip length are analogous for both particles.

1. Introduction

The effective viscosity of a suspension bounded by a solid wall not only depends upon the shape and volume fraction of the immersed solid particles but also upon the shape and nature of the bounding wall. For sufficiently dilute suspensions it is possible to restrict, at the leading order, the problem to a single particle interacting with the wall. For applications, three different types of motionless walls are encountered:

(i) The quite usual solid and no-slipping wall at which one requires the fluid velocity to vanish.

(ii) The free surface at which one requires zero normal velocity and tangential stress components.

(iii) The case of a slipping and impermeable wall for which one usually employs the so-called Navier [1] boundary conditions of zero normal velocity and another relation between the non-zero tangential velocity and stress (see (3)).

Although the particle may either have a slipping or no-slipping surface, we restrict our attention to a no-slipping particle and direct the reader to [3] for the interesting case of a slipping sphere interacting with a slipping plane wall. In addition, we assume the boundary to be a plane wall (at least at the particle micro-scale). However, one should note that non-flat boundaries might be handled as well (as already done in Case (i)) but this challenging task would result in tremendous efforts too much involved to be treated here. In contrast to Cases (i)-(ii), Case
(iii) received a little attention in the literature (see, for instance, [4, 5, 6]) but recently attracted again attention because of experimental evidence [7]. Consequently, [8, 9] recently treated the migration of a spherical and solid particle in the vicinity of a plane, impermeable and slipping wall (Case (iii)) by employing the bipolar coordinates approach, as earlier done in [5, 6]. In these works [8, 9] the sphere experiences a prescribed translation or rotation parallel with the slipping wall or is freely-suspended in a prescribed ambient linear shear flow tangent to the wall. In essence the bipolar coordinates method is solely able to deal with a spherical particle and, to the authors’ very best knowledge, there is unfortunately no available work coping with the challenging case of arbitrarily-shaped particles. This work therefore introduces a new approach valid for a non-spherical particle. It also reports preliminary computations for the friction coefficients of a non-spheroidal ellipsoid which are seen to differ from the ones of a volume-equivalent sphere.

2. Governing problem

We consider, as depicted in Fig. 1, a solid and arbitrarily-shaped particle \( \mathcal{P} \), with attached point \( O' \) and smooth surface \( S \), suspended in a Newtonian liquid with uniform density \( \rho_f \) and viscosity \( \mu > 0 \) occupying the \( x_3 > 0 \) half-space domain. We adopt Cartesian coordinates \((O, x_1, x_2, x_3)\) with origin \( O \) located on the bounding \( x_3 = 0 \) plane wall \( \Sigma \) and assume a particle rigid-body motion with prescribed translational velocity \( U \) (here the velocity of the attached point \( O' \)) and angular velocity \( \Omega \). By viscosity, the particle migration induces a liquid flow with velocity \( u \) and pressure \( p \) in the liquid domain \( \mathcal{D} \). Denoting by \( a \) and \( V \) the particle typical length-scale and the reference flow velocity we define the flow Reynolds number \( \text{Re} \) as \( \text{Re} = \rho_f V a / \mu \). Assuming a no-slip particle surface \( S \), negligible inertial effects (i.e. \( \text{Re} \ll 1 \)) and a quiescent liquid far from the particle, the flow \((u, p)\) is governed by the following Stokes equations, far-field behaviour and boundary condition

\[
\mu \nabla^2 u = \nabla p \quad \text{and} \quad \nabla \cdot u = 0 \quad \text{in} \quad \mathcal{D}, \quad (u, p) \rightarrow (0, 0) \quad \text{as} \quad |OM| \rightarrow \infty, \quad (1)
\]

\[
u \nabla^2 u = \nabla p \quad \text{and} \quad \nabla \cdot u = 0 \quad \text{in} \quad \mathcal{D}, \quad (u, p) \rightarrow (0, 0) \quad \text{as} \quad |OM| \rightarrow \infty, \quad (1)
\]

\[
\begin{align*}
\nabla^2 u &= \nabla p, \\
\nabla \cdot u &= 0 \quad \text{in} \quad \mathcal{D}, \\
(u, p) &\rightarrow (0, 0) \quad \text{as} \quad |OM| \rightarrow \infty \quad \text{as} \quad |OM| \rightarrow \infty,
\end{align*}
\]

(1)

Of course, one has to supplement (1)-(2) with relevant boundary conditions on the plane wall \( \Sigma \). Here, \( \Sigma \) is assumed to be a motionless, impermeable and slipping surface characterized by its prescribed slip length \( \lambda > 0 \) introduced in [1]. Therefore, we impose on \( \Sigma \) the famous and so-called Navier slip conditions

\[
\begin{align*}
\mathbf{u} \cdot \mathbf{e}_3 &= 0, \\
\mathbf{u} \cdot \mathbf{e}_1 &= \lambda \frac{\partial u \cdot \mathbf{e}_1}{\partial x_3} \quad \text{and} \\
\mathbf{u} \cdot \mathbf{e}_2 &= \lambda \frac{\partial u \cdot \mathbf{e}_2}{\partial x_3} \quad \text{at} \quad x_3 = 0.
\end{align*}
\]

(3)

\[
\begin{align*}
\mathbf{n} \cdot \mathbf{u} &= 0, \\
\mathbf{n} \cdot \mathbf{p} &= 0 \quad \text{on} \quad \Sigma, \\
\mathbf{n} \cdot \mathbf{u} &= 0, \\
\mathbf{n} \cdot \mathbf{p} &= 0 \quad \text{on} \quad \Sigma.
\end{align*}
\]

(3)
Calculating the flow field \((u, p)\) as solution of (1)-(3) and its stress tensor \(\sigma\) in the entire liquid domain for arbitrary solid particle (shape and location), rigid-body \((U, \Omega)\) migration and wall slip length \(\lambda > 0\) is a very challenging issue of great importance for applications. Of course, once this is achieved it is subsequently possible to calculate the net force \(F\) and torque \(\Gamma\) (about \(O')\) applied on the particle. Indeed, if the smooth particle surface \(S\) has unit normal \(n\) pointing into the liquid (as shown in Fig. 1) these quantities read
\[
F_h = \int_S \sigma \cdot n dS, \quad \Gamma_h = \int_S O'M \land \sigma \cdot n dS.
\]

3. Relevant boundary formulation and boundary-integral equations
In this section we present a boundary elements approach that provides a solution of the problem (1)-(3) for arbitrarily-shaped particles.

3.1. Usual Green tensor and boundary formulation
Let us introduce, as illustrated in Fig. 2, a so-called pole \(y\) in the \(y_3 > 0\) half-space. We then consider for \(k = 1, 2, 3\) Stokes flows with pressure \(p^{(k)}\), velocity \(v^{(k)}\) due to a concentrated point force of strength \(e_k\) placed at the selected pole \(y\), i.e. obeying
\[
\mu \nabla^2 v^{(k)} = \nabla p^{(k)} - \delta_{3d}(x - y)e_k, \quad \nabla \cdot v^{(k)} = 0 \text{ in the } x_3 > 0 \text{ half-space}
\]
(5)
\[
(u^{(k)}, p^{(k)}) \to (0, 0) \text{ as } |OM| \to \infty.
\]
(6)
with \(\delta_{3d}(x - y) = \delta_{3d}(x_1 - y_1)\delta_2(x_2 - y_2)\delta_3(x_3 - y_3)\) and \(\delta_3\) the Dirac pseudo-function. Those flows permit one to define a so-called second-rank Green tensor \(G\), with Cartesian components \(G_{jk}(x, y) = v^{(k)}(x, y) \cdot e_j\). One should note that, because there is no prescribed boundary conditions on the \(x_3 = 0\) plane, the flows \((v^{(k)}, p^{(k)})\) and the related Green tensor \(G\) are not unique. However, at that stage the basic point is that the velocity \(u\) solution to (1) admits in the entire liquid domain \(D\) and for any selected Green tensor \(G\) the following basic integral representation [10]
\[
u(x) \cdot e_j = -\int_{S \cup \Sigma} \left\{e_i \cdot \nabla \cdot e_j \cdot n(y) G_{ij}(y, x) - [u(y) \cdot e_i] T_{ijk}(y, x) [n(y) \cdot e_k] \right\} dS(y), \quad x \in D
\]
(7)
where we set \(n = e_3\) on the surface \(\Sigma\) and the third-rank tensor \(T = T_{ijk}(y, x) e_i \otimes e_j \otimes e_k\) is the stress tensor associated with \(G\).
3.2. Specific Green tensor and resulting boundary formulation

As it stands, (7) shows that it is sufficient to calculate on the entire fluid boundary \( S \cup \Sigma \) the velocity \( \mathbf{u} \) and the surface force \( \mathbf{f} = \sigma \cdot \mathbf{n} \). This is a nice property but unfortunately the plane wall \( \Sigma \) is unbounded. To circumvent this main drawback, the trick consists in adequately selecting a specific Green tensor \( \mathbf{G}^c \) so that the integrals over the boundary \( \Sigma \) vanish in (7). As the reader may easily check by applying the usual reciprocal identity [11] this is achieved by selecting a Green tensor \( \mathbf{G}^c \) obeying the Navier slip condition (3). In other words, the retained Green tensor \( \mathbf{G}^c \) is built by using Stokes flows \( (\mathbf{v}^{(k)}, p^{(k)}) \) governed by (5)-(6) and the Navier slip boundary conditions

\[
\mathbf{v}^{(k)} \cdot \mathbf{e}_3 = 0, \quad \mathbf{v}^{(k)} \cdot \mathbf{e}_1 = \lambda \frac{\partial \mathbf{v}^{(k)} \cdot \mathbf{e}_1}{\partial x_3} \quad \text{and} \quad \mathbf{v}^{(k)} \cdot \mathbf{e}_2 = \lambda \frac{\partial \mathbf{v}^{(k)} \cdot \mathbf{e}_2}{\partial x_3} \quad \text{on} \ \Sigma.
\]

Using (7) for the Green tensor \( \mathbf{G}^c \) and its associated stress tensor \( \mathbf{T}^c \) then provides the more pleasant velocity representation

\[
\mathbf{u}(\mathbf{x}) \cdot \mathbf{e}_j = - \int_S \left\{ \mathbf{e}_i \cdot \sigma \cdot \mathbf{n} \right\}(\mathbf{y}) \mathbf{G}^c_{ij}(\mathbf{y}, \mathbf{x}) - [\mathbf{u}(\mathbf{y}) \cdot \mathbf{e}_j] \mathbf{T}^c_{ijk}(\mathbf{y}, \mathbf{x}) \mathbf{n}(\mathbf{y}) \cdot \mathbf{e}_k \} dS(\mathbf{y}), \quad \mathbf{x} \in \mathcal{D}. \tag{9}
\]

Another improvement is also obtained by noting that we are confining ourselves in this work to a velocity field \( \mathbf{u} \) which is, by virtue of the boundary condition (2), a rigid-body motion on the particle surface. As shown in [12], the second integral on the right-hand side of (9) (the so-called double-layer term) then vanishes! Accordingly, one ends up for the Stokes flow \( (\mathbf{u}, p) \) governed by (1)-(2) with the simple integral representation

\[
\mathbf{u}(\mathbf{x}) \cdot \mathbf{e}_j = - \int_S \left[ \mathbf{e}_i \cdot \sigma \cdot \mathbf{n} \right](\mathbf{y}) \mathbf{G}^c_{ij}(\mathbf{y}, \mathbf{x}) dS(\mathbf{y}), \quad \mathbf{x} \in \mathcal{D} \tag{10}
\]

which provides in the entire liquid domain the velocity \( \mathbf{u} \) solely in terms of the surface force \( \mathbf{f} = \sigma \cdot \mathbf{n} \) exerted by the Stokes flow \( (\mathbf{u}, p) \) on the particle surface. In practice, the required vector \( \mathbf{f} \) is gained by letting the point \( \mathbf{x} \) in (10) tend onto the surface \( S \). This procedure is valid because, as seen in §3.3, each Cartesian component \( G^c_{ij}(\mathbf{y}, \mathbf{x}) \) is weakly-singular as \( \mathbf{x} \) approaches \( \mathbf{y} \). As a result, one gets the following Fredholm boundary-integral equation of the first kind

\[
[U + \Omega \wedge \mathbf{O}' \mathbf{M}] \cdot \mathbf{e}_j = - \int_S [\mathbf{f} \cdot \mathbf{e}_i](\mathbf{y}) \mathbf{G}^c_{ij}(\mathbf{y}, \mathbf{x}) dS(\mathbf{y}) \quad \text{for} \ \mathbf{x} = \mathbf{OM} \text{ on } S. \tag{11}
\]

In summary, the advocated treatment of the Stokes problem (1)-(3) appeals to the following steps:

(i) Obtain the previously-introduced specific Green tensor \( \mathbf{G}^c \).

(ii) Invert the boundary-integral equation (11) to get the surface force \( \mathbf{f} \) exerted by the flow \( (\mathbf{u}, p) \) on the particle surface.

(iii) Evaluate the net force and torque \( \mathbf{F} \) and \( \mathbf{\Gamma} \) given by (4) from the knowledge of \( \mathbf{f} \).

(iv) Compute, if needed, the velocity field \( \mathbf{u} \) in the liquid domain by employing the nice integral representation (10).

3.3. Determination of the specific Green tensor

The key step for our procedure is the determination of the specific Green tensor \( \mathbf{G}^c \). More precisely, we look at the Cartesian component \( G^c_{jk}(\mathbf{x}, \mathbf{y}) = \mathbf{v}^{(k)}(\mathbf{x}, \mathbf{y}) \cdot \mathbf{e}_j \) for pole \( \mathbf{y} \) and a so-called observation point \( \mathbf{x} \) located in the half-space domain above the plane wall \( \Sigma \) (see Fig. 2) and Stokes flows \( (\mathbf{v}^{(k)}, p^{(k)}) \) fulfilling (5)-(6) and (8). Here we briefly present the results and direct
for further details the reader to [2]. First, we introduce the well-known [12] free-space Oseen-Burgers Green tensor tensor $G_{\text{free-space}}^{\delta k}$ having Cartesian components and associated pressure $p_{\text{free-space}}^{\delta k}(x, y)$

$$G_{\text{free-space}}^{\delta k}(x, y) = \frac{1}{8\pi \mu} \left\{ \frac{\delta_{jk}}{|x - y|} + \frac{[(x - y) \cdot e_j] [e_j - (x - y) \cdot e_k]}{|x - y|^3} \right\},$$

(12)

$$p_{\text{free-space}}^{\delta k}(x, y) = \frac{(x - y) \cdot e_k}{4\pi},$$

(13)

with $\delta$ the Kronecker delta symbol. Then, we use the decompositions

$$G_{\text{free}}^{\delta k}(x, y) = G_{\text{free}}^{\delta k}(x, y) - G_{\text{free}}^{\delta k}(x, y') + w_{\delta k}^{(j)}(x, y),$$

(14)

$$p_{\text{free}}^{\delta k}(x, y) = p_{\text{free}}^{\delta k}(x, y) - p_{\text{free}}^{\delta k}(x, y') + s_k^{(j)}(x, y)$$

(15)

with, as depicted in Fig.2, $y'$ the symmetric of the pole $y$ with respect to the plane slipping wall $\Sigma$. The Fourier transforms of functions $w_{\delta k}^{(j)}$ and $s_k^{(j)}$ are obtained as detailed in [2]. Applying the inverse Fourier transform finally provides $w_{\delta k}^{(j)}$ and $s_k^{(j)}$. Setting $h = y \cdot e_3, x_3 = x \cdot e_3, R_3' = x_3 + h, R_j = (x - y) \cdot e_j$ for $j = 1, 2$ and $\rho = \{R_1^2 + R_2^2\}^{1/2}$ one gets (see [2])

$$\frac{4\pi \mu}{h} w_1^{(3)} = \frac{R_1}{\rho} \left\{ \int_0^\infty \frac{\xi e^{-R_3' \xi} J_1(\rho \xi) d\xi}{1 + 2 \lambda \xi} - \int_0^\infty \frac{\xi^2 e^{-R_3' \xi} J_1(\rho \xi) d\xi}{1 + 2 \lambda \xi} \right\},$$

(16)

$$\frac{4\pi \mu}{h} w_2^{(3)} = \frac{R_2}{\rho} \left\{ \int_0^\infty \frac{\xi e^{-R_3' \xi} J_1(\rho \xi) d\xi}{1 + 2 \lambda \xi} - \int_0^\infty \frac{\xi^2 e^{-R_3' \xi} J_1(\rho \xi) d\xi}{1 + 2 \lambda \xi} \right\},$$

(17)

$$\frac{4\pi \mu}{h} w_3^{(3)} = -x_3 \int_0^\infty \frac{\xi^2 e^{-R_3' \xi} J_0(\rho \xi) d\xi}{1 + 2 \lambda \xi},$$

(18)

$$\frac{4\pi \mu}{h} w_1^{(1)} = \frac{2 \lambda + x_3 (1 + 2 \lambda/h)}{\rho^2} \left\{ \int_0^\infty \frac{\xi J_2(\rho \xi) d\xi}{1 + 2 \lambda \xi} \right\},$$

(19)

$$\frac{4\pi \mu}{h} w_2^{(1)} = \frac{R_1 R_2}{\rho^2} \left\{ \int_0^\infty \frac{\xi J_2(\rho \xi) d\xi}{1 + 2 \lambda \xi} \right\},$$

(20)

$$\frac{4\pi \mu}{h} w_3^{(1)} = \frac{R_1}{\rho} \left\{ x_3 (1 + 2 \lambda/h) \right\},$$

(21)

with $J_m$ the Bessel function of positive integer order $m$. The quantities $w_k^{(2)}$ are obtained by switching superscripts or subscripts 1 and 2 in (19)-(21).

### 4. Numerical implementation and preliminary numerical results

This section briefly presents the numerical method. It also provides benchmark tests against the results obtained elsewhere [5, 6, 8, 9] for a spherical particle and reports preliminary numerical results for a non-spherical particle.
4.1. Numerical method and benchmark tests

A collocation method [13, 14] is implemented to numerically invert the boundary-integral equation (11). For a sake of accuracy, we actually use on the particle surface a $N-node$ mesh consisting of $6-node$ curved triangular boundary elements. The discretized counter-part of (11) is a linear system $AX = Y$ with a $3N \times 3N$ dense and non-symmetric influence square matrix $A$ with coefficients calculated as explained in [14]. The reader should note that each integral arising on the right-hand sides of identities (16)-(21) must be accurately computed. For a sake of conciseness the employed method is however not detailed here and the reader is directed to [15] for further details. Finally, the system $AX = Y$ is solved by Gaussian elimination.

Comparisons are made against the results previously-obtained in the literature, using the quite different method of bipolar coordinates [5, 6, 8], for a spherical particle with radius $a$ and center $O'$ such that $OO' = le_3$ and $l > a$. By linearity and also for symmetry reasons, the net force $\mathbf{F}$ and net torque $\mathbf{\Gamma}$ (about the sphere center $O'$) exerted by the Stokes flow about the moving sphere with a rigid-body motion $(\mathbf{U}, \Omega)$ solely depend upon six so-called friction coefficients $f_{33}, c_{33}, f_{11}, c_{12}, f_{21}$ and $c_{22}$ defined as follows

\[
\mathbf{F} = -6\pi\mu a^3 \mathbf{U} \quad \text{and} \quad \mathbf{\Gamma} = 0 \quad \text{for} \quad \mathbf{U} \wedge \mathbf{e}_3 = \Omega = 0, \quad (22)
\]
\[
\mathbf{\Gamma} = -8\pi\mu a^2 c_{33} \mathbf{\Omega} \quad \text{and} \quad \mathbf{F} = 0 \quad \text{for} \quad \mathbf{\Omega} \wedge \mathbf{e}_3 = \mathbf{U} = 0, \quad (23)
\]
\[
\mathbf{F} = -6\pi\mu f_{11} \mathbf{U} \quad \text{and} \quad \mathbf{\Gamma} = -8\pi\mu a^2 c_{12} \mathbf{U} \wedge \mathbf{e}_3 \quad \text{for} \quad \mathbf{U} \wedge \mathbf{e}_1 = \Omega = 0, \quad (24)
\]
\[
\mathbf{\Gamma} = -8\pi\mu a^2 c_{22} \mathbf{\Omega} \quad \text{and} \quad \mathbf{F} = -6\pi\mu a^2 f_{21} \mathbf{\Omega} \wedge \mathbf{e}_3 \quad \text{for} \quad \mathbf{\Omega} \wedge \mathbf{e}_2 = \mathbf{U} = 0. \quad (25)
\]

One should note that using the reciprocal identity [10, 11] and the above definitions gives the relation $f_{21} = 4c_{12}/3$ so that one ends up with five coefficients.

Computed values of $f_{11}, c_{12}, f_{21}$ and $c_{22}$ are compared with the predictions of [6, 8] for different $N-node$ meshes on the sphere surface in Tab. 1. Clearly, there is a nice agreement between the bipolar coordinates method and the advocated and quite different boundary elements approach.

**Table 1.** Computed friction coefficients $f_{11}, c_{12}, f_{21}$ and $c_{22}$ for $l/a = 1.5$ and $\lambda = 1$ versus the number $N$ of nodal points put on the sphere surface.

| $N$  | $f_{11}$      | $c_{12}$     | $f_{21}$    | $c_{22}$    |
|------|---------------|--------------|-------------|-------------|
| 74   | 1.28628       | -0.026335    | -0.034849   | 1.03624     |
| 242  | 1.28981       | -0.026200    | -0.035023   | 1.03560     |
| 1058 | 1.29020       | -0.026285    | -0.035054   | 1.03548     |
| [6, 8]| 1.29023       | -0.026293    | -0.035057   | 1.03548     |

In a similar fashion, comparisons with [5] for the friction coefficients $f_{33}$ and $c_{33}$ are given in Tab. 2. One should note that values given in [5] have been actually corrected by coding the equations obtained in [5] using a 16-digit accuracy. Again, a nice convergence towards the (corrected) values obtained by the bipolar coordinates procedure is observed.

**Table 2.** Computed friction coefficients $f_{33}$ and $c_{33}$ for $l/a = 1.543$ and $\lambda = 2.5$ versus the number $N$ of nodal point spread on the sphere surface.

| $N$  | $f_{33}$      | $c_{33}$     |
|------|---------------|--------------|
| 74   | 2.11545       | 0.99321      |
| 242  | 2.12474       | 0.98947      |
| 1058 | 2.12572       | 0.98974      |
| [4]  | 2.11843       | 0.98835      |
| [4]Corrected | 2.12568 | 0.98935 |
4.2. Numerical results for a solid ellipsoidal particle

By contrast to the bipolar coordinates approach restricted to a spherical particle, the advocated boundary method is able to cope with non-spherical particles. It thus permits us to investigate to which extent the force and torque exerted on a solid particle moving near the slipping wall depend upon the particle shape. This is achieved in the present work by comparing the results for the sphere with radius \( a \) and for a volume-equivalent ellipsoidal particle with center of volume \( O' \) and semi-axis \( a_i \) parallel with \( e_i \) for \( i = 1, 2, 3 \). More precisely, we take \( a_1 = 1.2a, a_2 = 1/a_1, a_3 = a \) and set \( OO' = l e_3 \) with \( l > a_3 = a \). Accordingly, for given \( l/a \) the particle-wall gap is \( l - a \) both for the sphere and the ellipsoid. For the selected ellipsoid orientation the relations (22)-(25) still hold with new friction coefficients \( f_{33}, c_{33}, f_{11}, c_{12}, f_{21} \) and \( c_{22} \) for the ellipsoid. One should note that additional friction coefficients, obtained for \( U \wedge e_2 = \Omega = 0 \) and for \( \Omega \wedge e_1 = U = 0 \), are found for the ellipsoid. Here we however confine our comparisons to the coefficients \( f_{33}, c_{33}, f_{11}, c_{22}, f_{21} = 4c_{12}/3 \) which are computed, versus \( l/a \) and for different slip lengths \( \lambda \), both for the sphere and the ellipsoid using \( N = 1058 \) collocation points on each particle surface.

![Figure 3. Compared friction coefficients \( f_{33} \) for a sphere (dashed lines) and the ellipsoid (solid lines) for different slip lengths: from top to bottom (○) for \( \lambda/a = 0 \), (□) for \( \lambda/a = 0.7 \), (■) for \( \lambda/a = 2 \), (●) for \( \lambda/a = 5 \).](image)

Computed coefficient \( f_{33} \) (obtained when the particle translates normal to the slipping wall) is plotted, both for the ellipsoid and the volume-equivalent sphere, in Fig. 3 versus the separation parameter \( l/a \) for different normalized slip lengths \( \lambda/a \). It is seen that \( f_{33} \) is nearly the same for the two particles: it increases as the particle approaches the slipping wall and also decreases as the normalized slip length \( \lambda/a \) increases. One should also note that \( f_{33} > 1 \) whatever the particle location and the ratio \( \lambda/a \) (this is also the case for a free surface obtained by letting \( \lambda/a \) tend to infinity).

The coefficient \( f_{11} \) is shown in Fig. 4 again versus the separation parameter \( l/a \) and for different normalized slip lengths \( \lambda/a \). For the ellipsoid this friction coefficient is always weaker than for the equivalent sphere. For both particles \( f_{11} \) decreases as \( \lambda/a \) increases for a prescribed
location \( l/a \). In contrast to the previously-discussed case of \( f_{11} \), note that for \( \lambda/a \) sufficiently large the function \( f_{11} \) does not monotonically decrease as \( l/a \) increases (see the curves associated with the value \( \lambda/a = 5 \)). This means, not surprisingly, that a larger slip reduces the friction for a particle translating parallel to a close slipping wall.

![Figure 4](image)

**Figure 4.** Compared friction coefficients \( f_{11} \) for a sphere (dashed lines) and the ellipsoid (solid lines) for different slip lengths: from top to bottom (○) for \( \lambda/a = 0 \), (□) for \( \lambda/a = 0.7 \), (■) for \( \lambda/a = 2 \), (●) for \( \lambda/a = 5 \).

Fig 5. displays the friction coefficients \( c_{22} \) versus \( l/a \) for different values of \( \lambda/a \). This coefficient is larger than unity, behaves as \( f_{33} \) for each particle and is clearly shape-dependent. The coefficient \( c_{33} \) addressed in Fig. 6 is also larger for the ellipsoid. When \( \lambda/a \) is large enough \( c_{33} \) decreases with \( l/a \) and might be even less than unity for a particle sufficiently close to the wall (this arises in Fig. 6 for the sphere but would also be observed for the ellipsoid for \( \lambda/a \) exceeding a critical value larger than 5).

As seen in Fig. 7, the coefficient \( c_{12} \) is of weak magnitude and, depending upon the value of \( \lambda/a \), either increases or decreases as the particle approaches the slipping wall. For a given particle location note that \( c_{12} \) vanishes and becomes negative for a critical ratio \( \lambda/a \) which strongly depends on the particle shape.

**5. Conclusions**

A new boundary elements approach consisting of the treatment of one Fredholm boundary-integral equation of the first kind on the particle surface has been proposed, implemented and benchmarked against results available for a spherical particle. The advocated procedure appeals to a specific Green tensor obtained in [2] and holds for arbitrary-shaped particles. Preliminary comparisons for a sphere and a volume-equivalent non-spheroidal ellipsoid reveal that all friction coefficients show the same trends versus the slip length for both particles while some of the
Figure 5. Compared friction coefficients $c_{22}$ for a sphere (dashed lines) and the ellipsoid (solid lines) for different slip lengths: from top to bottom ($\circ$) for $\lambda/a = 0$, (□) for $\lambda/a = 0.7$, (■) for $\lambda/a = 2$, (●) for $\lambda/a = 5$.

coefficients are deeply sensitive to the particle shape.

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Figure 6. Compared friction coefficients $c_{33}$ for a sphere (dashed lines) and the ellipsoid (solid lines) for different slip lengths: from top to bottom (○) for $\lambda/a = 0$, (□) for $\lambda/a = 0.7$, (■) for $\lambda/a = 2$, (●) for $\lambda/a = 5$.

Figure 7. Compared friction coefficients $c_{12}$ for a sphere (dashed lines) and the ellipsoid (solid lines) for different slip lengths: from top to bottom (○) for $\lambda/a = 0$, (□) for $\lambda/a = 0.7$, (■) for $\lambda/a = 2$, (●) for $\lambda/a = 5$. 