Spatial and temporal scales of force and torque acting on wall-mounted spherical particles in open channel flow

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Data from direct numerical simulation of open channel flow over a geometrically rough wall at a bulk Reynolds number of Re_b = 2900, generated by Chan-Braun et al. 1 [“Force and torque acting on particles in a transitionally rough open-channel flow”, J. Fluid Mech. 684, 441–474 (2011), 10.1017/jfm.2011.311] are further analysed with respect to the time and length scales of force and torque acting on the wall-mounted spheres. For the two sizes of spheres in a square arrangement (11 and 49 wall units in diameter, yielding hydraulically smooth and transitionally rough flow, respectively), the spatial structure of drag, lift and spanwise torque is investigated. The auto-correlation and spectra in time as well as the space-time correlation and convection velocities are presented and discussed. It is found that the statistics of spanwise particle torque are similar to those of shear stress at a smooth wall. Particle drag and lift are shown to differ from spanwise particle torque, exhibiting considerably smaller time and length scales; the convection velocities of drag and lift are somewhat larger than those of spanwise torque. Furthermore, correlations between the flow field and particle-related quantities are presented. The spatial structure of the correlation between streamwise velocity and drag/spanwise torque features elongated shapes reminiscent of buffer-layer streaks. The correlation between the pressure field and the particle drag exhibits two opposite-signed bulges on the upstream and downstream sides of a particle.

Keywords: turbulence, rough wall, particle forces, immersed boundary, direct numerical simulation

I. INTRODUCTION

The present research is motivated by the problem of sediment erosion in open channel flows such as rivers. Other applications of processes involving sediment erosion can be found e.g. in aeolian transport (dune formation), in the chemical industry (pneumatic conveying) and in biological systems (blood flow, flow through the respiratory tract). In order to be able to understand such complex systems a number of fundamental questions remain open. At the core of the problem is the interaction between a turbulent flow field and sediment particles. On the one hand turbulent flow induces a hydrodynamic force and torque on a particle which can lead to the onset of particle motion, while on the other hand the sediment alters the turbulence structure by acting as a rough wall boundary. Despite a long tradition of research on this subject a detailed description of the mechanisms responsible for the onset of erosion in turbulent flow is currently not available.

The fundamental physics of the fluid-particle interaction have commonly been studied in simplified systems, for example by considering mono-sized spheres instead of naturally shaped gravel. Some researchers assume that the sediment particles are fixed in space and, using measurements of the hydrodynamic forces, attempt to analyze the processes eventually leading to erosion. Data describing the force acting on submerged particles as part of a rough wall are in fact relatively scarce 2–5. Recent studies present approximations of lift and drag on cubes, spheres and naturally shaped stones by local pressure measurements 6–9. In Ref. 1 high-fidelity data of open channel flow over an array of wall-mounted spheres was generated by means of direct numerical simulation. The focus in that study, where two particle sizes were simulated (one leading to hydraulically smooth flow, one to transitionally rough flow), was on the characterization of force and torque acting on the particles by means of an analysis of single-point and single-time statistics. It was shown that some aspects of the statistics of particle torque (the shape of the probability density function

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and the magnitude of its moments) can be explained by an analogy to the force/torque acting on a surface element of comparable size in an equivalent flow over a smooth wall. The success of this simplified model indicates that the spheres are acting as a filter with respect to the size of the flow scales which can effectively generate torque fluctuations. In Ref. 1, however, several aspects of the interaction between the turbulent flow and the wall-mounted particles have not been addressed: the particle force and torque data was not analyzed with respect to its spatial and temporal correlation; the relation between force/torque acting on the particles and the surrounding flow field was not investigated.

The structure of wall-bounded turbulent flow in general has been extensively studied in the past. The role of coherent flow structures, in particular, has received considerable attention\textsuperscript{10–13} as it has greatly improved our understanding of the underlying flow dynamics. The particular effect of roughness on wall-bounded flows has been reviewed in Ref. 14. The similarity between smooth-wall flows and those over rough walls – when considering regions away from the wall – has been investigated in detail in recent years\textsuperscript{15–20}. On the other hand it is now generally accepted that the flow is strongly affected by wall-roughness within a wall-distance of several multiples of the roughness height.

For smooth walls, several studies have tackled the problem of the structure and propagation speed of the wall shear-stress and the wall pressure\textsuperscript{21–28}. For flows over rough walls, similarly precise data on the spatial structure and propagation speed of the signature of flow quantities at the wall (e.g. the forces acting on the roughness elements) are missing in the literature. Therefore, a considerable uncertainty exists regarding the flow structures which contribute to the generation of these unsteady forces.

With the objective to provide additional insight into the interaction between turbulent flow and near-wall particles we determine in the present work the spatial structure and the temporal characteristics of the hydrodynamic force and torque acting on wall-mounted spherical particles in open channel flow. For this purpose we further analyze the data of Ref. 1. Since the particle force and torque are generated by the time-varying velocity and pressure fields around the immersed objects, the force and torque signals should reflect some of the spatio-temporal characteristics of the turbulent flow in their vicinity. As a consequence, it can be expected that the particle force/torque and the flow field are substantially correlated. Here we compute and analyze this correlation in an attempt to reveal the flow structures which significantly contribute to the generation of particle force and torque. Both aspects of the process (the spatio-temporal correlation of particle force/torque and the correlation between force/torque and the surrounding flow field) are difficult to investigate in laboratory experiments. The present contribution fills this gap by providing previously unavailable data which contributes to a better understanding of the action of turbulent flow upon complex shaped elements of a solid boundary.

The present article is structured as follows. In § II the setup of the simulations is described together with a brief discussion of the numerical method. In § III the structure of the force and torque acting on the particles is presented. Two-point correlations are discussed in § III A, temporal auto-correlations are discussed in § III B, and spatio-temporal correlations and convection velocities are discussed in § III C. In § IV correlations between the flow field and the force/torque acting on the particles are presented. Conclusions are given in § V.

### II. FLOW CONFIGURATION AND NOTATION

The flow configuration is identical to the one described in Ref. 1 and consists of pressure-driven open channel flow over a geometrically rough wall. The wall is formed by one layer of fixed spheres packed in a square arrangement above a rigid wall located at $y = 0$. The number of particles in this layer is henceforth

| Case | $U_{bh}/u_c$ | $Re_b$ | $Re_v$ | $D^+/D$ | $D/h$ | $D/H$ | $\Delta_x^+$ | $N_p$ | $\tau_c U_{bh}/H$ | $N_{t1}$ | $N_{t2}$ |
|------|---------------|--------|--------|--------|-------|-------|-------------|------|------------------|--------|--------|
| F10  | 15.2          | 2870   | 188    | 10.7   | 0.057 | 14    | 0.77        | 9216 | 120              | 12750  | 67     |
| F50  | 12.2          | 2880   | 235    | 49.3   | 0.210 | 46    | 1.07        | 1024 | 120              | 16500  | 114    |
denoted as $N_p$. This rigid wall is additionally roughened by spherical caps (cf. figure 1 in Ref. 1). The flow is studied through direct numerical simulation of the incompressible Navier-Stokes equations. The equations are discretised in space by a second-order finite-difference scheme on a staggered, equidistant grid. In time a three-step Runge-Kutta scheme is employed for the convective terms, while the viscous terms are treated by a Crank-Nicolson scheme. A fractional-step method is used to enforce the divergence-free constraint. The particles forming the rough wall are represented through an immersed boundary method.

As in Ref. 1, a coordinate system is used such that $x$, $y$ and $z$ denote the streamwise, wall-normal and spanwise direction respectively; the velocity components in these three coordinate directions are $u$, $v$ and $w$. In the following, normalisation with wall units will be denoted by a superscript +. A prime denotes the fluctuation with respect to an average over wall-parallel planes and time (indicated by angular brackets), e.g. considering a quantity $\phi(x,t)$, its fluctuation is defined as $\phi'(x,t) = \phi(x,t) - \langle \phi(y) \rangle$. The hydrodynamic force vector has the Cartesian components $F_x$, $F_y$ and $F_z$, while the components of the particle torque are denoted as $T_x$, $T_y$ and $T_z$.

The physical and numerical parameters of the simulations are provided in Table I. As in Ref. 1, the value of the friction velocity $u_\tau$ is defined by extrapolation of the total shear stress in the fluid away from the wall down to the location of the virtual wall, which is defined at the plane $y_0 = 0.8D$. The effective channel height is defined as $h = H - y_0$ where $H$ denotes the domain height. Two cases are considered, differing only in the size of the wall-mounted spheres: the spheres’ diameter measures $D/H = 0.055$ in case F10, while it is set to $D/H = 0.18$ in case F50. The corresponding value normalized with wall units is $D^+ = 10.7$ ($D^+ = 49.3$) in case F10 (F50). The bulk velocity based on the domain height is defined as $U_{bh} = 1/H \int_0^H \langle u \rangle dy$ and the bulk velocity based on the effective flow depth is defined as $U_{bh} = 1/h \int_{y_0}^H \langle u \rangle dy$. In the present simulations the bulk velocity $U_{bh}$ was kept constant in time. The chosen values of the Reynolds number defined from the bulk velocity and the domain height, $Re_b = U_{bh} H / \nu$, were practically identical in both cases, measuring 2870 and 2880, respectively. On the other hand the Reynolds number based upon the friction velocity, $Re_\tau = h u_\tau / \nu$, takes a value close to that of an equivalent smooth-wall flow in case F10 ($Re_\tau = 188$), while it is significantly higher in case F50 ($Re_\tau = 235$).

Fig. 1 contains snapshots of the flow fields for both particle sizes, showing iso-surfaces of streamwise velocity fluctuations. In case F10 the spheres are small with respect to the domain dimensions and the scales of turbulent motion. In case F50 the spheres are approximately three times larger when scaled in outer units, but they remain moderately small with respect to the scales of motion. Fig. 2 illustrates the instantaneous particle drag fluctuations in case F10 and case F50. In case F10 (small spheres) the drag is found to correlate over several sphere diameters in the streamwise and the spanwise directions (Fig. 2a). In case F50 the picture is different, and a similarly strong correlation cannot be inferred from Fig. 2b). In both cases, extreme events appear very localised, i.e. events with high particle drag fluctuations coexist next to events with high negative particle drag fluctuations. As will be discussed below this leaves a footprint on the spatial and temporal correlation functions.

### III. Spatial and Temporal Correlations of Particle-Related Quantities

In the following the scales of spanwise torque, drag and lift are studied by correlation functions in time, space and space-time. The correlation function for particle quantities related to an array of spheres in square arrangement can be defined as

$$R_{\phi\psi}(\Delta x, \Delta z, \Delta t) = \frac{1}{N_{i1}N_p} \sum_{i=1}^{N_{i1}} \sum_{l=1}^{N_p} \phi'(x^{(l)}_p, t_i) \psi'(z^{(l)}_p + \Delta x, y^{(l)}_p + \Delta z, t_i + \Delta t),$$

where $\phi'(x^{(l)}_p, t)$ and $\psi'(x^{(l)}_p, t)$ are fluctuations of force or torque components on the $l$th sphere, centred at $x^{(l)}_p$ at time, $t$. Separation in time is denoted by $\Delta t$, separations in the spanwise and the streamwise direction by $\Delta x$ and $\Delta z$, respectively. The subscript $p$ is used to stress the discrete nature of separations in space which can only be multiples of the particle distance. $N_{i1}$ denotes the number of snapshots used in the evaluation of (1), with the values for each case listed in Table I.
A. Spatial correlation of spanwise torque, drag and lift on an array of particles

First the two-dimensional two-point correlation of spanwise torque, drag and lift fluctuations are considered as defined by (1) with $\psi' = \phi' = \{T'_z, F'_x, F'_y\}$ and $\Delta_t = 0$. Fig. 3 shows the correlation coefficient of these quantities as a function of streamwise and spanwise separation, $\Delta_{x,p}$ and $\Delta_{z,p}$ normalised by $D$ and $h$ for both flow cases. For spanwise torque Fig. 3(a,b) reveals a streamwise elongated region of significant positive correlation which expands over several particle diameters and is of the order of the effective domain height $h$ in both flow cases. Additionally a smaller region of negative correlation can be identified centred around $\Delta_{z,p}/h = 0.3$. While in case F10 the region is somewhat elongated in the streamwise direction, in case F50 the negative region is confined to a small circular area. The two-point correlation of drag fluctuations shown in Fig. 3(c,d) differ from those of spanwise torque. In particular, in the spanwise direction an area of significant negative correlation is not visible. Also the region of positive correlation in case F10 is less elongated and exhibits a spanwise contraction around $\Delta_{z,p}/h = 0.3$. In case F50 the two-point correlation exhibits an area of significant negative correlation centred at a streamwise separation of $\Delta_{x,p}/h = 0.3$ which is comparable in size to the area of positive correlation. The two-point correlation of lift is the most confined quantity in both cases (Fig. 3(e,f)); in particular, in case F50 it vanishes after spanwise separations of 2 to 3 particle diameters.

The spatial coherence of spanwise torque, drag and lift can be further investigated by considering one-dimensional correlations along the coordinate axes. In this spirit, Fig. 4 (Fig. 5) shows the same two-point correlations as a function of streamwise (spanwise) separation at zero spanwise (streamwise) separation.
Two scalings of the spatial separation are shown, using $h$ and $D$ as length scales. Additionally the two-point correlation of the wall shear-stress $\tau'_{12} = \mu \partial u'/\partial y\big|_{y=0}$ in an equivalent flow with a smooth wall ($Re_b = 2900$, $Re_{\tau} = 183$) is included in the figures as a function of $\Delta x_p/h$ for reference. Fig. 4 reveals that, when scaled by $h$, the two-point correlation of spanwise torque fluctuation in case F10 overlaps with the one from the shear stress component $\tau'_{12}$ in the smooth wall reference case. In contrast, the two-point correlation of spanwise torque in case F50 is significantly lower. However, the difference between the curves is small when scaled in viscous units (plot omitted). As discussed before with respect to Fig. 3, the two-point correlation of drag and lift fluctuations differ from the one of spanwise torque fluctuation, decreasing more rapidly with streamwise separations. In case F10 the two-point correlations of drag and lift fluctuations remain positive for the range of streamwise separations shown. Here, the two-point correlation of drag exhibits a local minimum located at about $\Delta x_p/h = 0.3$ after which the two-point correlation coefficient first increases and then slowly approaches zero (Fig. 4a). The two-point correlation coefficient of lift rapidly decays and is essentially zero for $\Delta x_p/h > 0.5$. In case F50 (Fig. 4b) the two-point correlation coefficients of drag and lift exhibit negative values at streamwise separations of one and two particle diameters and are close to zero for larger separations.

Fig. 5 shows the two-point correlation as a function of spanwise separation $\Delta z_p$ again with two axes showing the scaling by $D$ and by $h$. It is found that in both cases (F10 and F50), the two-point correlation of spanwise torque fluctuations overlaps with the two-point correlation of the smooth wall shear stress. The curves rapidly drop towards a local minimum with negative correlation coefficients. In all cases the spanwise torque is essentially uncorrelated for spanwise separations $\Delta z_p/h > 0.6$. In contrast to spanwise torque, the two-point correlation of drag on particles decreases monotonically with spanwise separation and is essentially zero for $\Delta z_p/D > 5$. In case F10 the two-point correlation of lift fluctuation decreases faster in spanwise direction than drag or spanwise torque. In case F50, on the other hand, the two-point correlations of lift and spanwise torque nearly coincide, both exhibiting a significantly more rapid initial decorrelation than particle drag. However, this does not imply that drag and lift might not be related to similar flow structures, which will be discussed in more detail in § III B 2 below.
FIG. 3. Two-point correlation, \( R_{\phi\phi}(\Delta x,p)/\sigma^{2}_{\phi} \), of particle torque and force fluctuation as a function of space lag, \( \Delta x,p \) and \( \Delta z,p \) scaled by \( h \) and by \( D \). The panels show results of case F10 (a,c,e) and case F50 (b,d,f). Spanwise torque fluctuation, \( T'_z \) (a,b), drag fluctuation, \( F'_x \) (c,d), and lift fluctuations, \( F'_y \) (e,f). The shading shows values from \(-0.2\) to \(1\) from white to black. Lines show iso-contour at \(0.15\) (---) and \(-0.15\) (– – –).

FIG. 4. Two-point correlation, \( R_{\phi\phi}(\Delta x,p)/\sigma^{2}_{\phi} \), of drag (\( F'_x, \square \)), lift (\( F'_y, \triangledown \)) and spanwise torque (\( T'_z, \lozenge \)) fluctuations on a particle for streamwise separations \( \Delta x,p \) in case of case F10 (a) and case F50 (b). As a reference, the continuous line shows the two-point correlation of shear stress fluctuation \( \tau'_{12} = \mu \partial u'/\partial y|_{y=0} \) at a smooth wall as a function of \( \Delta x,p/h \).
FIG. 5. Two-point correlation, $R_{\phi\phi}(\Delta_{z,p})/\sigma_\phi^2$, of drag ($F_z'$, □), lift ($F_y'$, ▽) and spanwise torque ($T_z'$, ◻) fluctuations on a particle for spanwise separations $\Delta_{z,p}$ in case of case F10 (a) and case F50 (b). As a reference, the continuous line shows the two-point correlation of shear stress fluctuation $\tau'_{12} = \mu \partial u'/\partial y|_{y=0}$ at a smooth wall as a function of $\Delta_{z,p}/h$.

B. Temporal correlation of force and torque on a particle

1. Auto-correlation of drag, lift and spanwise torque

In the following the auto-correlations in time at zero spatial separation are considered (i.e. $\phi = \psi$ and $\Delta_{x,p} = \Delta_{y,p} = 0$ in equation 1). Fig. 6(a) shows the auto-correlation in time of spanwise torque fluctuation, $T_z'$, for case F10 and F50 in comparison with the auto-correlation of $\tau'_{12}$ in the smooth-wall reference simulation. As in the case of the two-point correlation, it is found that the temporal auto-correlation of spanwise torque almost overlaps with the auto-correlation of the smooth-wall shear stress $\tau'_{12}$ in case F10. The auto-correlation of spanwise torque in case F50 exhibits smaller values for a separation in time scaled in outer flow units (Fig. 6a). When scaled in inner flow units the difference is smaller. This is reflected by the integral time scale which is defined as the integral of the auto-correlation, here computed in the range $T_{\Delta_{z,p}}$, defined by the integral of the absolute value of the auto-correlation, and the temporal Taylor micro-scale, $\tau_{\lambda}$, defined by the auto-correlation function at zero separation.

In turbulent wall-bounded flows the convection velocity, $U_c$, is sometimes used to relate length scales to time scales. To this aim, it is often assumed that the flow is not significantly modified as it is convected downstream with a constant convection velocity, which is commonly referred to as Taylor’s frozen turbulence hypothesis. A detailed discussion on the issue can be found in del Alamo and Jiménez and the references therein. Here we have applied Taylor’s hypothesis to the spatial correlation data discussed in the previous section in order to compute an approximate temporal auto-correlation. To this aim we use the value of the convection velocity as defined in Eq. (2) and further discussed in § III C below. The resulting auto-correlations for the spanwise torque are also included in Fig. 6(a). It can be observed that the agreement between the actual auto-correlations and the corresponding data obtained from the spatial correlations via Taylor’s hypothesis is satisfactory, in particular for small separation times. Thus, in the present cases the Taylor approximation appears to be appropriate to relate scales of spanwise torque in time to the corresponding spatial scale, implying that in both cases the flow structures relevant for spanwise torque indeed vary more slowly than the convective time scale.

Similar conclusions as from the auto-correlations in time, $R_{\phi\phi}(\Delta_t)$, discussed above can be drawn from the temporal spectra, $\hat{\mathcal{R}}_{\phi\phi}(\omega)$, where $\omega$ is the frequency. In case of infinite or periodic time signals, $\hat{\mathcal{R}}_{\phi\phi}(\omega)$ can be defined as the Fourier transform of the auto-correlation or alternatively as $\hat{\mathcal{R}}_{\phi\phi}(\omega) = \hat{\phi}(\omega)\hat{\phi}^*(\omega)$, where
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\[ \frac{R_{\phi\phi}}{\sigma_\phi^2} \]

\[ \Delta t \frac{U_{bh}}{h} \]

\[ \frac{R_{\phi\phi}}{\sigma_\phi^2} \]

\[ \Delta t \frac{U_{bh}}{h} \]

\[ \frac{R_{\phi\phi}}{\sigma_\phi^2} \]

\[ \Delta t \frac{U_{bh}}{h} \]

FIG. 6. Auto-correlation of (a) spanwise torque fluctuations, \( T'_z \), (b) drag fluctuations, \( F'_x \) and (c) lift fluctuations, \( F'_y \). Lines show the temporal auto-correlation of case F10 (\(-\text{---}\)) and case F50 (\(-\text{--\--}\)) as a function of the time lag, \( \Delta t \). Additionally, the auto-correlation of the shear stress fluctuation \( \tau_{12} = \mu \frac{\partial u'}{\partial y}|_{y=0} \) in the smooth wall reference simulation (\(-\cdot\text{---}\)) is provided as a reference in (a) and (b). Symbols show the approximate auto-correlation computed from the spatial correlations in case F10 (\(\circ\)) and F50 (\(\nabla\)) by applying a Taylor approximation using the convection velocity \( U_c \), as discussed in § III C. In all figures the bulk time is used to normalize the temporal separation.

\[ \hat{\phi}(\omega) \] is the Fourier transform of \( \phi(t) \), and \( \hat{\phi}^*(\omega) \) the conjugate complex of \( \hat{\phi} \). In the present case, however, the time signals are finite and non-periodic, and \( \hat{R}_{\phi\phi}(\omega) \) is approximated by the method of Welch\(^{32}\) with 50% overlap and applying a Hamming window\(^{33}\) to the original signal \( \phi \), prior of transferring it into spectral space. Fig. 7 provides the pre-multiplied spectra \( \omega \hat{R}_{\phi\phi}(\omega) \) normalised with the integral of the spectra over all frequencies, shown as a function of the period, \( T = 2\pi/\omega \). Concerning spanwise torque fluctuations (Fig. 7a), the majority of contributions stems from periods roughly centered around \( T U_{bh}/h = 10 \) in case
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\[ \frac{\omega}{\int \phi \phi(\omega) d\omega} \]

\[ \frac{\omega}{\int \phi \phi(\omega) d\omega} \]

\[ \frac{\omega}{\int \phi \phi(\omega) d\omega} \]

FIG. 7. Pre-multiplied spectrum \( \kappa_\phi \phi \phi \) of (a) spanwise torque fluctuations, \( T'_z \), (b) drag fluctuations, \( F'_x \) and (c) lift fluctuations, \( F'_y \), shown as a function of the period \( T \) normalised by the bulk time scale. Line styles are as in Fig. 6.

F10 and around \( TU_{bh}/h = 5 \) in case F50, in both cases corresponding to approximately \( Tu^2/\nu = 100 \). An approximate match between the spectra in both flow cases is reached when inner scaling is used (plot omitted).

Turning now to the auto-correlations of drag and lift forces on a particle, \( F'_x \) and \( F'_y \) (Fig. 6), it is found that they differ between case F10 and case F50. In case F50 the curves of drag and lift fluctuations exhibit pronounced local minima of negative value (drag: \( -0.25 \) at \( \Delta t U_{bh}/h = 0.55 \), lift: \( -0.18 \) at \( \Delta t U_{bh}/h = 0.57 \)). Conversely, in case F10 the auto-correlation of drag has a clear local minimum, albeit with a positive correlation value (0.20 at \( \Delta t U_{bh}/h = 0.47 \)). For the auto-correlation of lift in case F10 there exists no visible
local minimum. In order to quantify the short-time behavior of the auto-correlation functions one can refer to the temporal Taylor micro-scale \( \tau_i \) provided in Table II. It can be observed that the temporal Taylor micro scales of spanwise torque are larger than those of drag and lift. The temporal Taylor micro scales of drag and lift compare best between cases F10 and F50 when scaled in outer flow units, i.e. \( \tau_u U_{bh}/h = 0.24 \pm 0.01 \). When scaled in inner units the time scales of drag and lift are approximately 50% larger in case F50 than in case F10. In contrast to the temporal Taylor micro-scale, the integral time scales \( \tau \) and \( \tau_{abs} \) are smaller in case F50 than in case F10 revealing a shortening of the largest time scales in the time signals.

The pre-multiplied temporal spectra of particle drag and lift (Fig. 7b,c) differ significantly from those of spanwise torque on the particles (cf. Fig. 7a). In particular, the largest spectral contributions to force fluctuations are shifted to smaller periods. It is found that in case F10 the spectrum of drag is bi-modal, exhibiting a second (weaker) local maximum at larger periods (\( T U_{bh}/h \approx 10 \)), coinciding with the range of maximum contribution to the spectrum of the smooth wall shear stress component \( \tau_{22} \). It also corresponds to the range of temporal periods where the torque spectrum has its peak. This second peak is not present in case F50, neither is it present for any case in the spectrum of particle lift.

Finally, Fig. 7 also includes the drag and lift reconstructed from spatial data by way of Taylor’s hypothesis. In case F10, both spectra agree over the entire range of periods. In case F50 the agreement for large periods is similarly good, whereas a pile-up of energy in the smallest periods is exhibited by the spectrum based upon the data converted from the spatial correlations. This results from the finite size of particles yielding a spatially discrete nature of the force and torque data. As a consequence, the smallest wavelength that can be resolved in space is limited to the distance between the particle centres and, upon conversion to temporal data, the smallest resolvable period is \( T_{min U_{bh}}/h = (2D/U_x)(U_{bh}/h) \). In particular this yields \( T_{min U_{bh}}/h \approx 0.16 \) in case F10 and 0.8 in case F50. In case F50 this leads to an aliasing of the contribution to force fluctuations in the small periods for the spectra in space as seen in Fig. 7.

2. Cross-correlation between drag and lift

Here we are addressing the question whether drag and lift are correlated in time and, therefore, related to similar flow structures. Fig. 8 presents the temporal cross-correlation of drag and lift defined by (1) with \( \phi = F_x, \psi = F_y \) and \( \Delta x, p = \Delta z, p = 0 \). For \( \Delta t = 0 \), the cross-correlation of drag and lift fluctuations is small, i.e. \( R_{F_x F_y}(\Delta t)/\sigma^2_x \sigma^2_y = 0.23 \) (−0.06) in case F10 (F50). However, a significant correlation occurs at a finite temporal separation. The cross-correlation reaches a maximum value of \( R_{F_x F_y}(\Delta t)/\sigma^2_x \sigma^2_y = 0.55 \) (0.45) for a separation in time of \( \Delta t U_{bh}/h = 0.19 \) (0.24) in case F10 (F50). Thus, on average lift fluctuations follow drag fluctuations of equal sign with a time lag which approximately matches the values of the Taylor micro-scale related to drag and lift (cf. Table II). The cross-correlation coefficient in Fig. 8 reaches a local minimum of value −0.03 (−0.49) for a negative separation in time of \( \Delta t U_{bh}/h = -0.17 \) (−0.20) in case F10 (F50). While in case F10 the correlation is essentially zero at the local minimum, in case F50 it reaches significant negative values, indicating that drag and lift fluctuations are on average of opposite sign at a separation in time of \( \Delta t U_{bh}/h = -0.20 \). As the separation in time takes increasingly large negative values the correlation becomes positive once more, then decaying to zero from above.

In addition to the results of case F10 and F50, the results of two experimental studies are presented in Fig. 8. The first data set corresponds to one of the experiments of Hofland 34 (his figure 6.5a), who indirectly measured drag and lift by the difference of two simultaneous pressure measurements. These measurements were taken on both sides (upstream and downstream) of a cube with a protrusion height of 3300 viscous units at \( Re_b = 1.3 \cdot 10^5 \). The ratio of channel width to open channel height was 3.0 and the ratio of cube

| case | quantity | \( T_z \) | \( F_x \) | \( F_y \) | quantity | \( T_z \) | \( F_x \) | \( F_y \) |
|------|---------|---------|---------|---------|---------|---------|---------|---------|
| F10  | \( \tau_u u_x^2/\nu \) | 21.9    | 10.8    | 6.1     | \( \tau U_{bh}/h \) | 1.77    | 0.87    | 0.50    |
| F50  | \( \tau_u u_x^2/\nu \) | 19.1    | 2.6     | 1.9     | \( \tau U_{bh}/h \) | 1.01    | 0.14    | 0.10    |
| F50  | \( \tau_{abs} u_x^2/\nu \) | 19.1    | 6.3     | 5.2     | \( \tau_{abs} U_{bh}/h \) | 1.01    | 0.34    | 0.28    |
| F10  | \( \tau_u u_y^2/\nu \) | 10.3    | 2.9     | 3.1     | \( \tau U_{bh}/h \) | 0.83    | 0.24    | 0.25    |
| F50  | \( \tau_u u_y^2/\nu \) | 11.4    | 4.5     | 4.5     | \( \tau U_{bh}/h \) | 0.61    | 0.24    | 0.24    |

TABLE II. Integral time scale \( \tau_i \) (integrated over 100 viscous time units) and temporal Taylor micro-scale \( \tau \) of particle torque and force in case F10 and F50. Note that the integral of the absolute value of the respective auto-correlation, denoted as \( \tau_{abs} \), is also given in case F50.
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\[ R_{\psi \psi}(\Delta t) / (\sigma^2_{\psi x} \sigma^2_{\psi y}) \]

FIG. 8. Cross-correlation coefficient of drag and lift fluctuations, \( R_{\psi \psi}(\Delta t) / (\sigma^2_{\psi x} \sigma^2_{\psi y}) \), as a function of separation in time \( \Delta t U_{bh}/h \) in case F10 (---) and case F50 (-- -- --). Additionally, the figure provides the results of the indirect measurements of Hofland \(^{34}\) (his Fig. 6.5(a), lowest protrusion) for a cube within natural gravel with a height of 3300\( \delta_v \) at \( Re_b = 1.3 \cdot 10^5 \), (-----) and the direct measurements of Dwivedi \(^{35}\) (his Fig. 7.23(a), zero protrusion) of spheres with \( D^+ = 3100 \) at \( Re_b = 1.68 \cdot 10^5 \) (---).

| \( U_c/U_{bh} \) | \( T_z' \) | \( F_x' \) | \( F_y' \) | \( U_c/u_r \) | \( T_z' \) | \( F_x' \) | \( F_y' \) |
|----------------|--------|--------|--------|----------------|--------|--------|--------|
| F10            | 0.62   | 0.71   | 0.67   | F10            | 9.5    | 10.8   | 10.2   |
| F50            | 0.54   | 0.58   | 0.61   | F50            | 6.7    | 7.2    | 7.5    |

TABLE III. Convection velocities, \( U_c \), of force and torque fluctuations on particles in case F10 and F50 normalised by \( U_{bh} \) and \( u_r \).

C. Space-time correlation and convection velocities of spanwise torque, drag and lift

This section focuses on the convection velocities of force and torque fluctuations as a result of turbulent flow structures. The definition of the convection velocity used in the present context requires a streamwise-space/time correlation of force or torque fluctuations, which in physical space can be defined by (1) with \( \phi = \psi \) and \( \Delta z,p = 0 \). For reasons of efficiency the space-time correlations were not computed as above in physical space but in spectral space by employing the method of Welch \(^{32}\) in time and a Fourier transform in the streamwise direction (cf. §III B 1). The space-time correlation in spectral space is denoted by \( \hat{R}_{\phi\psi}(\kappa^x, \omega) \), as a function of streamwise wave number, \( \kappa^x \), and frequency, \( \omega \).

Several definitions of the convection velocity in turbulent shear flows have been proposed\(^{26,27}\). It should be noted, that a convection velocity of a flow field component cannot be defined unambiguously as scales with different lengths might travel at different speeds. Here, the convection velocity, \( U_c \), of a particle-related quantity (force, torque) is defined as a weighted average of the convection velocity \( u_c \) related to a given wave.
FIG. 9. Auto-correlation of different particle-related quantities in case F10 (a,c,e) and case F50 (b,d,f). Graphs (a,b) show spanwise torque fluctuations, (c,d) show drag fluctuations, and (e,f) show lift fluctuations. The correlation (normalised by its integral value) is plotted as a function of frequency, $\omega h/U_{bh}$, and streamwise wave number, $\kappa_x p h$. Lines show iso-contours at values of [0.001 0.01] (———). The symbols (+) mark $\omega_c(\kappa_x p h)$ as defined in the text (only every 4th point is shown). In all panels the slope of the dash-dotted lines corresponds to $-U_c$, where $U_c$ is the convection velocity defined in (2).
number, $\kappa_p^x$ (Ref. 27) as

$$U_c = -\frac{\int_{\kappa_p} \int_\omega u_c \left| \tilde{R}_{\phi \psi}(\kappa_p^x, \omega) \right| (\kappa_p^x)^2 d\omega d\kappa_p}{\int_{\kappa_p} \int_\omega |\tilde{R}_{\phi \psi}(\kappa_p^x, \omega)| (\kappa_p^x)^2 d\omega d\kappa_p}.$$  

(2)

This average weights the convection velocities of each wave number with the energy contained in the respective wave number. Thus the higher energetic wave numbers contribute more to the global convection velocity defined by (2). For a given streamwise wavenumber, $\kappa_p^x$, del Alamo and Jiménez$^{27}$ propose to define $u_c$ by the centre of mass of the profile $\tilde{R}_{\phi \psi}(\kappa_p^x, \omega)$ denoted as $\omega_c$ in the following, i.e. $u_c = -\omega_c(\kappa_p^x)/\kappa_p^x$.

However, in the present case such a definition leads to a strong bias due to the aliasing of energy in the large wave numbers (small wave lengths) as discussed in the context of Fig. 7. The aliasing results in the existence of contour lines at positive and negative frequencies for wave numbers of large magnitude in Fig. 9(d). In order to overcome the bias due to the aliasing, $\omega_c(\kappa_p^x)$ is defined by the centre of mass of the one-sided frequency spectrum in the present work, i.e. only the range for which $-\text{sgn}(\kappa_p^x)\omega > 0$ is considered. This leads to a bias for very small wave numbers (which do not contribute much to the global convection velocity) but reduces the otherwise dominant bias from large wave numbers.

The above definitions can be illustrated with the aid of Fig. 9 which shows the space-time correlation of drag, lift and spanwise torque fluctuation in spectral space. The figure contains parts of the information discussed in the previous chapters, i.e. the pre-multiplied spectra in time and in streamwise direction are obtained by summation over the horizontal and vertical axes, respectively, in Fig. 9. Therefore the discussion in the following is limited to new aspects. The symbols “+” in Fig. 9 show $\omega_c(\kappa_p^x)$ as defined above. As can be seen, the definition of $\omega_c(\kappa_p^x)$ results in an approximately linear variation of $\omega_c(\kappa_p^x)$ and qualitatively agrees with the expectations.

The convection velocities $U_c$, which are obtained by (2) are illustrated in form of straight dash-dotted lines (with slope $-U_c$) in Fig. 9, and the corresponding numerical values are given in Table III. Let us first consider case F10, where the roughness elements are small enough to yield a hydraulically smooth flow. In this case the values of the convection velocity are smallest for $T_{bh}$ ($U_c/U_{bh} = 0.62$, i.e. $U_c/u_\tau = 9.5$) and largest for $F_{2}^{x}$ ($U_c/U_{bh} = 0.71$, $U_c/u_\tau = 10.8$). The values are in the range which is commonly reported for the convection velocity of flow quantities in smooth wall flows. For example, for smooth wall channel flow del Alamo and Jiménez$^{27}$ reported values of $U_c/U_{bh}$ in the range of 0.5 to 0.8 and 0.6 to 0.8 for the streamwise and spanwise velocity fluctuations close to the wall, respectively. Other studies in smooth-walled channels and boundary layers have reported similar values$^{30,36}$. The convection velocity of shear stress fluctuations and pressure fluctuations in smooth-wall channel flow have been studied by Jeon et al.$^{25}$ The authors report values of 9.6 for the streamwise shear stress and 13.1 for the pressure fluctuations. The present results for the convection velocity of force and torque fluctuations follow a similar trend. Both drag and lift are by definition influenced by pressure fluctuations, and their convection velocity is found to be higher than the one for spanwise torque to which pressure does not contribute.

Next, let us compare the convection velocity of the transitionally rough flow case F50 to the above results for F10. Table III shows that the convection velocities of spanwise torque, drag and lift decrease from case F10 to case F50. In particular, $U_c/U_{bh}$ ($U_c/u_\tau$) in case F50 is on average 13% (30%) smaller than in case F10 for the particle properties above. As in case F10 the values of $U_c/U_{bh}$ and $U_c/u_\tau$ in case F50 are smallest for $T_{bh}^{y}$. Interestingly, in case F50 the convection velocity of lift exceeds the one of drag. Data on the convection velocities of flow in the vicinity of a rough wall is scarce. From the simulations and the analysis of Flores and Jiménez$^{27}$ it can be deduced that the convection velocity tends to decrease due to the influence of wall roughness. In the experiments of Krogstad and Antonia$^{38}$ a reduction by 18% of the smooth-wall value for the convection velocity was assumed in the rough wall case.

A feature of the space-time correlation deserves further explanation, namely, the contributions of small streamwise wavenumbers to lift fluctuations at zero frequency in case F10 which appear in Fig. 9(e) in the form of two needle-like, horizontal excursions of the lowest-valued contour at zero frequency. The lift force acting on the particles, when averaged in time (not over the particles), $\langle F_{2}^{(i)}(x^{(i)},\omega^{(i)})_{l}\rangle$, is not fully converged in the present data-set; instead it varies with a a standard deviation of 0.03 $F_{R}$.

IV. CORRELATION BETWEEN PARTICLE-RELATED QUANTITIES AND THE FLOW FIELD

The purpose of the present section is to infer the flow structures which significantly contribute to the particle force and torque statistics. To this end we have computed correlation functions between the flow
field (either $u$, $v$, $w$ or $p$) on one hand, and the particle-related quantities (the six components of force and torque) on the other. It is beyond the scope of the present paper to discuss the correlations between all possible combinations of these quantities. Our selection is based upon the following considerations. The net particle drag force $F_z$ consists of two contributions, one from viscous stresses (which mainly stem from the streamwise velocity field) and one from the pressure. Therefore, we choose to discuss first the correlations between $F_z$ and the streamwise velocity fluctuations $u'$, and secondly those between $F_z$ and the fluctuating pressure field $p'$. Concerning the torque, it is of interest to analyze its correlation with the velocity field, since − by definition − this quantity is not affected by the pressure field. Finally, we are omitting the correlations involving the lift force $F_y$, since they are found to be similar to the corresponding correlations involving drag, and since we have found that drag and lift are significantly correlated with a lag in time (cf. figure 8).

The correlation function between the fluctuation of a scalar flow quantity $\phi$ and a quantity $\psi(t)$ associated with the $l$th particle is defined as

$$R_{\phi\psi}(\Delta_x, y, \Delta_z) = \frac{1}{N_{l2}N_p} \sum_{i=1}^{N_{l2}} \sum_{l=1}^{N_p} \phi(x_p^{(l)}, t_i) \psi(x_p^{(l)}, t_i).$$

(3)

Note two main differences with respect to definition (1): first, the resulting correlation function $R_{\phi\psi}$ is a three-dimensional field; second, the correlation function is not restricted to a discrete grid related to the inter-particle spacing, but to the finite-difference grid (which is considerably finer). The sums in (3) run over the number of particles $N_p$ and over a number of snapshots $N_{l2}$ distributed over the observation interval (cf. Table I). The statistical convergence of a correlation can be judged by deviations from the symmetry (or anti-symmetry) with respect to $\Delta_x$ or the $\Delta_z$-axis and is found to be satisfactory for the correlations shown.

The maximum absolute values of the considered correlations, $|R_{\phi\psi}|_{\text{max}}$, are given in Table IV. Here, $|R_{\phi\psi}|_{\text{max}}$ is normalised by $u_\tau$ and $\rho_f u_\tau^2$ as a characteristic scale for the velocity and pressure fluctuations, respectively, and by the standard deviation of the respective force or torque fluctuation. Note that under the normalization used, the values of $R_{\phi\psi}$ reported do not correspond to correlation coefficients, i.e. $|R_{\phi\psi}|_{\text{max}}$ is not bounded from above by unity. Instead, the upper bound is given by the maximum of the standard deviation of the flow quantity $\phi'$: e.g. under the current normalisation the correlation between $F_z$ and $u'$ is bounded by the maximum of $u_{\text{rms}}/u_\tau$.

Figures 10 to 12 present the correlation of particle drag fluctuation, $F_z'$, with the streamwise fluid velocity fluctuations, $u'$, in case F10 and case F50. Fig. 10 illustrates the correlations in form of iso-surfaces at $\pm 0.15 |R_{\phi\psi}|_{\text{max}}$, with lengths scaled by $h$. Additionally, Fig. 11 and Fig. 12 display the correlation in form of iso-contours of value $\pm 0.15 |R_{\phi\psi}|_{\text{max}}$ in cross-sections of zero streamwise and spanwise separation, respectively. In Fig. 11 the axes are normalised by $h$, in figure 12 the axes are normalised by $D$. All figures exhibit positive values of the correlation in the vicinity of the particle centre. This reveals that on average structures of positive (negative) $u'$ in the vicinity of a particle relate to positive (negative) drag fluctuations.

The surfaces visualized by the chosen isovalue are of similar size when scaled with $h$ (cf. Fig. 10 and Fig. 11) and differ between cases F10 and F50 when scaled in $D$ (cf. Fig. 12). In the streamwise direction the positively-valued isosurface extends over approximately $5.5h$ ($4.5h$) in case F10 (F50), cf. Fig. 11a, which corresponds to approximately $100D$ ($22D$), cf. Fig. 12a. In both cases the positively-valued isosurface of the correlation between drag and the streamwise velocity extends over a substantial fraction of the channel height, while larger wall distances are reached in case F50 (up to $(y - y_0)/h = 0.4$ in case F10 and $(y - y_0)/h = 0.6$ in case F50). In the latter case, the isosurface appears to be somewhat lifted away from the wall.
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FIG. 10. Iso-surfaces of correlation, $R_{\phi\psi}(\Delta_x, y, \Delta_z)/|R_{\phi\psi}|_{\text{max}}$, of the streamwise particle force fluctuation, $F'_x$, and streamwise velocity fluctuation, $u'$ at values 0.15 (light) and −0.15 (dark). Panels show case F10 (a) and case F50 (b).

FIG. 11. Correlation, $R_{\phi\psi}(\Delta_x, y, \Delta_z)/|R_{\phi\psi}|_{\text{max}}$, of the streamwise particle force fluctuation, $F'_x$, and streamwise velocity fluctuation, $u'$. (a) Plane at zero spanwise shift, $\Delta_z = 0$, as function of $(y - y_0)/h$ and streamwise shift $\Delta_x/h$; (b) plane at zero streamwise shift, $\Delta_x = 0$, as function $(y - y_0)/h$ and spanwise shift $\Delta_z/h$. Red and blue coloured lines show 0.15 and −0.15, respectively, of the maximum amplitude in each case. Continuous (dashed) lines indicate results for case F10 (F50).

FIG. 12. As Fig. 11 but axes scaled with particle diameter $D$. 
The uplifted shape of the correlation function in case F50 is intriguing. Krogstad and Antonia reported that roughness alters flow structures close to the roughness surface in that they are more inclined with respect to the horizontal axis, i.e. the angle of the two-point correlation function was 10° in their considered boundary layer above a smooth wall, and 38° in the boundary layer above a rough wall. This is thought to be a local effect, limited to the roughness layer at a wall-distance up to five times the roughness scale and thus does not contradict Townsends wall similarity hypothesis. On the other hand, it has been observed that the large-scale structures of the streamwise velocity are damped in case F50 compared to the case of a smooth wall. These two observations might be related to the lower correlation for $\Delta_x/h > 1.5h$ and $(y - y_0)/h < 0.4$ in case F50 (cf. figure 11a) as compared to case F10.

Fig. 10 shows that in both cases regions of negative correlation values exist. The shapes of the negative-valued iso-surfaces are similar between those two cases, the one of case F10 being slightly more elongated. In both cases the negative-valued iso-surface intersects little with the plane at $\Delta_x = 0$, which leads to the small contours at $-0.15 |R_{\phi\psi}|_{\text{max}}$ observed in Fig. 11(b). The centre of the negative-valued region is located downstream of the particle position and at a spanwise distance of approximately $\Delta_z/h = \pm 0.3$. This corresponds to approximately 55 (70) wall units in case F10 (F50) and compares well to the average distance between high-speed and low-speed streaks commonly found in smooth wall flows.

Please observe that the shape of the visualized isosurfaces of the correlation ($F_x, u'$) agrees relatively well for cases F10 and F50, while the respective maxima of the correlations differ by roughly a factor of three from case F50 to case F10 under the presently chosen normalization using the friction velocity and the standard deviation of drag (cf. Table IV).

A clear difference between the characteristics of the correlations in case F10 and case F50 is the pronounced drop around the origin visible in Fig. 10(b) and in the contour lines in Fig. 11(a). A drop is also present in case F10, however to a much smaller extent.

Figs. 13 and 14 show the correlation between spanwise torque fluctuations acting on the particle, $T_z'$, and streamwise fluid velocity fluctuations, $u'$. From Fig. 13 it can be observed that the visualized iso-surface...
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\[ (y - y_0)/h \]

\[ \Delta x/h \]

\[ (y - y_0)/h \]

\[ \Delta z/h \]

FIG. 15. Iso-surfaces of correlation, \( R_{\phi\psi}(\Delta_x, y, \Delta_z)/|R_{\phi\psi}|_{\text{max}} \), of the streamwise particle force fluctuation, \( F'_{\phi} \), and pressure fluctuation, \( p' \), at values 0.15 (light) and -0.15 (dark). Panels show case F10 (a) and case F50 (b).

FIG. 16. Correlation, \( R_{\phi\psi}(\Delta_x, y, \Delta_z)/|R_{\phi\psi}|_{\text{max}} \), of the streamwise particle force fluctuation, \( F'_{\phi} \), and pressure fluctuation, \( p' \), at zero spanwise shift, \( \Delta_z = 0 \). Line styles as in Fig. 11.

of the correlation has a similar shape as the correlation of \( F'_{\phi} \) and \( u' \) shown in Fig. 10, albeit at opposite sign. The shown iso-surfaces are again elongated in the streamwise direction and of sizes much larger than the particle diameter. Therefore, structures of positive (negative) \( u' \) relate on average to negative (positive) spanwise torque fluctuations on a particle. Likewise, the regions of negative correlation are flanked by smaller streamwise elongated regions of correlation with a positive sign. Again, the identified region in figure 14(a) shortens in its streamwise extent from case F10 to case F50 and appears to be more inclined to the \( x \)-axis in case F50. The main difference between the correlation \((T'_{\phi}, u')\) (Fig. 14) and the correlation \((F'_{\phi}, u')\) (Fig. 11) is the absence of the drop around the origin in the former.

FIG. 15 shows the correlation of particle drag fluctuations, \( F'_{x} \), with the fluctuating pressure field, \( p' \). Fig. 16(a) shows the corresponding iso-contours at zero spanwise separation, i.e. \( \Delta_z = 0 \), in outer length scales. In both flow cases, we observe two bulges corresponding to iso-surfaces of opposite sign located upstream (positive correlation value) and downstream of the particle (negative value). In contrast to the previously presented correlations, the observed shapes are not streamwise elongated and do not present significant differences between flow cases F10 and F50. Therefore, it is found that, on average, a negative (positive) pressure gradient across the particle relates to positive (negative) drag fluctuations. Note, that in the three-dimensional time-averaged three-dimensional pressure field around a particle a similar formation of high and low values upstream and downstream of the particle exists as a consequence of the mean streamline curvature very close to the roughness elements\(^1\). The dimensions of the present iso-surfaces, however, are significantly larger than the particle diameter, e.g. \( 10D \) (3D) in wall-normal direction for case F10 (F50) and therefore do not stem from the time-averaged pressure field around the particles. Note that the bulge-like iso-surfaces found here for the correlation \((F'_{x}, u')\) are approximately located in the region where a notable drop (case F50) in the correlation amplitudes was observed when correlating the same force component \((F'_{x})\) with the streamwise velocity \((u')\) (cf. Fig. 11).

V. CONCLUSION

We have further analyzed the results of the direct numerical simulation of open channel flow over a geometrically rough wall of Ref. 1, focusing on the spatial and temporal structure of the hydrodynamic force
and torque acting on the wall-mounted spheres as well as on their correlation with the surrounding flow field.

In the simulations of Ref. 1 two flow cases were considered. In the first one, F10, the spheres are small and the flow regime is hydraulically smooth. Here it has been shown that the spatial and the temporal statistics of the spanwise torque acting on the spheres agree well with those of the shear stress on a smooth wall. In the second case, F50, the spheres are approximately three times larger, leading to a transitionally rough flow regime. It is found that in this case the similarity between the spatial and temporal characteristics of the spanwise torque acting on the wall-mounted spheres and those of the shear stress on a smooth wall is less pronounced.

The spatial and temporal structure of drag presents discrepancies with the structure of the shear stress on a smooth wall already for case F10. The differences are more evident in case F50, as can be expected. The auto-correlations of drag present local minima for both cases, F10 and F50, minima which are not present in the correlation of shear stress on a smooth wall. Also for the lift, for case F50, a local minimum is present.

Lift and drag are correlated with a shift in time. This result appears to be very robust, since we have compared the temporal cross-correlation from the two flow cases considered in the present study to experimental data obtained at different flow conditions (higher Reynolds number) and different roughness geometries (cube positioned in natural gravel, spheres in hexagonal packing). The temporal location of the local extrema of the cross-correlation between lift and drag agrees well in all cases, when scaled with the bulk velocity and the flow depth. Some differences are observed in the amplitude of the extrema, which might be linked to the differences between the setups and the way measurements were performed.

By using the convection velocity of the force (or torque) fluctuations, it has been possible to convert, using the Taylor hypothesis, the spatial two-point correlation into a temporal auto-correlation. It has been shown that the Taylor hypothesis works relatively well in both flow cases. The convection velocities computed in the present study follow trends reported in previous studies. First, it has been shown in flow over smooth walls that pressure fluctuations travel somewhat faster than shear stress fluctuations. In the present case, drag and lift fluctuations, which are by definition influenced by pressure fluctuations, have been found to travel somewhat faster than torque fluctuations, which are unaffected by pressure. Second, it can be expected that convection velocities close to the rough wall become smaller when increasing the roughness effect. Consistently, we have observed a reduction of the convection velocity from case F10 (hydraulically smooth) to F50 (transitionally rough).

Finally, the analysis of the correlation between the streamwise velocity field and the drag/spanwise torque acting on the particles has revealed that the spatial structure of such correlation functions is reminiscent of buffer-layer streaks. For both flow cases, the correlation between the streamwise velocity fluctuations and the drag/spanwise-torque presents an elongated shape, which scales with the flow depth and not with the size of the particle. In case F50, where the effect of roughness is more significant than in case F10, the selected isosurfaces are somewhat lifted away from the wall compared to case F10. Additionally, in case F50 the correlation between the streamwise velocity fluctuation and drag presents a drop above the particle which is not present in the correlation between the streamwise velocity fluctuation and torque. The correlation between the pressure fluctuations and the drag has also been discussed. It has been shown that this correlation is not elongated and consists of two short bulges located upstream and downstream of the particle.

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Scales of force and torque on wall-mounted spheres in channel flow

1. C. Chan-Braun, M. García-Villalba, and M. Uhlmann, “Force and torque acting on particles in a transitionally rough open-channel flow,” J. Fluid Mech. 684, 441–474 (2011).

2. H. A. Einstein and E.-S. A. El-Samni, “Hydrodynamic forces on a rough wall,” Rev. Mod. Phys 21, 520–524 (1949).

3. A. Mollinger and F. Nieuwstadt, “Measurement of the lift force on a particle fixed to the wall in the viscous sublayer of a fully developed turbulent boundary layer,” J. Fluid Mech. 187, 451–466 (1988).

4. A. J. del ‘Alamo and J. Jiménez, “Estimation of turbulent convection velocities and corrections to Taylor’s approximation,” J. Fluid Mech. 640, 5–26 (2009).

5. B. Hofland, J. Battjes, and R. Booij, “Measurement of fluctuating pressures on coarse bed material,” J. Hydraul. Engng 131, 770–781 (2005).

6. B. Hofland and J. Battjes, “Probability density functions of instantaneous drag forces and shear stresses on a bed,” J. Hydraul. Engng 132, 1169–1175 (2000).

7. A. G. Kravchenko, H. Choi, and P. Moin, “On the relation of near-wall streamwise vortices to wall skin friction in turbulent wall flows,” J. Fluid Mech. 136, 756–769 (2010).

8. S. Jeon, H. Choi, J. Y. Yoo, and P. Moin, “Space–time characteristics of the wall shear-stress fluctuations in a low-Reynolds-number channel flow,” J. Fluid Mech. 552, 263–293 (2007).

9. R. J. Volino, M. P. Schultz, and K. A. Flack, “Turbulence structure in rough- and smooth-wall boundary layers,” J. Fluid Mech. 592, 263–293 (2007).

10. M. P. Schultz and K. A. Flack, “Turbulent boundary layers on a systematically varied rough wall,” Phys. Fluids 21, 015104 (2009).

11. J. Hong, J. Katz, and M. P. Schultz, “Near-wall turbulence statistics and flow structures over three-dimensional roughness in a turbulent channel flow,” Journal of Fluid Mechanics 667, 1–37 (2011).

12. J. Monty, J. Allen, K. Lien, and M. Chong, “Modification of the large-scale features of high Reynolds number wall turbulence by passive surface obestractions,” Experiments in Fluids 51, 1755–1763 (2011).

13. H. Choi and P. Moin, “On the relation of near-wall streamwise vortices to wall skin friction in turbulent boundary layers,” Phys. Fluids 5, 3307–3309 (1993).

14. S. K. Robinson, “Coherent motions in the turbulent boundary layer,” Annu. Rev. Fluid Mech. 23, 601–638 (1991).

15. R. L. Panton, “Overview of the self-sustaining mechanisms of wall turbulence,” Progress in aerospace sciences 37, 341–383 (2001).

16. B. J. McKeon and K. R. Sreenivasan, “Scaling and structure in high Reynolds number wall–bounded flows,” Phil. Trans. Royal Soc. A 365, 635–646 (2009).

17. J. Jiménez, “Turbulence near rough walls,” Annu. Rev. Fluid Mech. 36, 173–196 (2004).

18. Y. Wu and K. T. Christensen, “Outer-layer similarity in the presence of a practical rough-wall topography,” Physics of Fluids 19, 085108 (2007).

19. J. Kim, “On the structure of pressure fluctuations in simulated turbulent channel flow,” J. Fluid Mech. 205, 421–451 (1989).

20. A. G. Kravchenko, H. Choi, and P. Moin, “On the relation of near-wall streamwise vortices to wall skin friction in turbulent boundary layers,” Phys. Fluids 5, 3307–3309 (1993).

21. J. Kim and F. Hussain, “Propagation velocity of perturbations in turbulent channel flow,” Phys. Fluids 5, 695–706 (1993).

22. S. K. Robinson, “Coherent motions in the turbulent boundary layer,” Annu. Rev. Fluid Mech. 37, 1–21 (1995).

23. A. Oppenheim and R. Schafer, Discrete-time signal processing (Prentice-Hall, 1989) pp. 447–448.

24. B. Hofland, Rock and Roll, Turbulence-induced damage to granular bed protections, Ph.D. thesis, Delft University of Technology (2005).

25. A. Dwivedi, Mechanics of sediment entrainment, Ph.D. thesis, The University of Auckland (2010).

26. B. Hofland, “Structure of force and torque acting on particles in a transitionally rough open-channel flow,” J. Fluid Mech. 684, 441–474 (2011).

27. R. J. Volino, M. P. Schultz, and K. A. Flack, “Turbulence structure in rough- and smooth-wall boundary layers,” J. Fluid Mech. 592, 263–293 (2007).

28. M. Quadrat and P. Luchini, “Integral space–time scales in turbulent wall flows,” Phys. Fluids 15, 2219–2227 (2003).

29. J. C. del Álamo and J. Jiménez, “Estimation of turbulent convection velocities and corrections to Taylor’s approximation,” J. Fluid Mech. 640, 5–26 (2009).

30. D. Hall, “Measurements of the mean force on a particle near a boundary in turbulent flow,” J. Fluid Mech. 136, 451–466 (1988).

31. A. Dwiwedi, B. Melville, and A. Y. Shamseldin, “Hydrodynamic forces generated on a spherical sediment particle during entrainment,” J. Hydraul. Engng 136, 756–769 (2010).

32. J. Kim and F. Hussain, “Propagation velocity of perturbations in turbulent channel flow,” Phys. Fluids 5, 695–706 (1993).

33. S. K. Robinson, “Coherent motions in the turbulent boundary layer,” Annu. Rev. Fluid Mech. 37, 1–21 (1995).

34. A. Oppenheim and R. Schafer, Discrete-time signal processing (Prentice-Hall, 1989) pp. 447–448.

35. B. Hofland, Rock and Roll, Turbulence-induced damage to granular bed protections, Ph.D. thesis, Delft University of Technology (2005).

36. A. Dwivedi, Mechanics of sediment entrainment, Ph.D. thesis, The University of Auckland (2010).

37. M. Quadrat and P. Luchini, “Integral space–time scales in turbulent wall flows,” Phys. Fluids 15, 2219–2227 (2003).

38. J. C. del Álamo and J. Jiménez, “Estimation of turbulent convection velocities and corrections to Taylor’s approximation,” J. Fluid Mech. 640, 5–26 (2009).

39. D. Hall, “Measurements of the mean force on a particle near a boundary in turbulent flow,” J. Fluid Mech. 136, 451–466 (1988).

40. O. Flores and J. Jiménez, “Effect of wall-boundary disturbances on turbulent channel flows,” J. Fluid Mech. 566, 357–376 (2006).

41. P.-Å. Krogstad and R. A. Antonia, “Structure of turbulent boundary layers on smooth and rough walls,” J. Fluid Mech. 277, 1–21 (1994).

42. S. Nakagawa and T. J. Hanratty, “Particle image velocimetry measurements of flow over a wavy wall,” Physics of Fluids 13, 3504–3507 (2001).

43. R. J. Volino, M. P. Schultz, and K. A. Flack, “Turbulence structure in a boundary layer with two-dimensional roughness,” J. Fluid Mech. 635, 75–101 (2009).
C. R. Smith and S. P. Metzler, “The characteristics of low-speed streaks in the near-wall region of a turbulent boundary layer,” J. Fluid Mech. 129, 27–54 (1983).

J. Kim, P. Moin, and R. Moser, “Turbulence statistics in fully developed channel flow at low Reynolds number,” J. Fluid Mech. 177, 133–166 (1987).