On fermion masses and mixing in a model with $A_4$ symmetry

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Abstract

In a recently proposed multi-Higgs extension of the standard model in which discrete symmetries, $A_4$ and $Z_3$ are imposed we show that, after accommodating the fermion masses and the mixing matrices in the charged currents, the mixing matrices in the neutral currents induced by neutral scalars are numerically obtained. However, the flavor changing neutral currents are under control mainly by mixing and/or mass suppressions in the neutral scalar sector.

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I. INTRODUCTION

One of the main motivations to go beyond the standard model is to have some hints about the flavor problem, that is, to understand the pattern of fermion masses and mixing. Most electroweak models have mass matrices of the form $M_{\alpha\beta} = \sum_i (\Gamma_i)_{\alpha\beta} \langle \Phi_i^0 \rangle$, where the $\Gamma_i$s are, for Dirac fermions, arbitrary complex dimensionless $3 \times 3$ matrices, and $\langle \Phi_i^0 \rangle$ denotes the vacuum expectation values (VEVs) of the neutral scalar fields in the model. For Majorana fermions, the $\Gamma_i$s are complex symmetric matrices. The mixing matrix and the mass pattern in each charge sector depend on the structure of the respective $\Gamma_i$s. It is well known that explicit and predictive forms of these matrices can be obtained by imposing flavor symmetries. In particular, most of the ansatze for the fermion mass matrices are of the form \cite{1}

$$M_Q = \begin{pmatrix} E_Q & D_Q & 0 \\ D_Q^* & C_Q & B_Q \\ 0 & B_Q^* & A_Q \end{pmatrix}, \quad M'_Q = \begin{pmatrix} 0 & D_Q & 0 \\ D_Q^* & C_Q & B_Q \\ 0 & B_Q^* & A_Q \end{pmatrix}, \quad (1)$$

and other textures which have in common the fact that the zeros appear in symmetric entries. Recently, it was proposed an extension of the electroweak model with $A_4$ and $Z_3$ discrete symmetries, in which the quark mass matrices have the following texture \cite{2}

$$M_Q = \begin{pmatrix} A_Q & 0 & D_Q \\ E_Q & B_Q & 0 \\ 0 & F_Q & C_Q \end{pmatrix}, \quad (2)$$

where the zeros occur in non-symmetrical entries.

II. QUARK MASSES AND MIXING IN MODEL WITH $A_4$ SYMMETRY

We consider a model with $G_{SM} \otimes A_4 \otimes Z_3 \otimes Z'_3 \otimes Z''_3$ symmetry, where $G_{SM}$ is the standard model (SM) gauge symmetry, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. In \cite{2} quarks and leptons were considered briefly. Here we show a more detailed study in both sectors. Let us begin by considering quarks. The model needs twelve $SU(2)$ doublets, four triplets of the $A_4$ symmetry: $H \equiv (H_1, H_2, H_3)$, and $\Phi \equiv (\Phi_1, \Phi_2, \Phi_3)$ for the masses of the $u$-quarks; $H' \equiv (H'_1, H'_2, H'_3)$ and $\Phi' \equiv (\Phi'_1, \Phi'_2, \Phi'_3)$ for the masses of the $d$-quarks. We also introduce a complex scalar singlet $\zeta$ \cite{2}.
Fermion weak eigenstates transform under \((A_4, Z_3, Z'_3, Z''_3)\) as:

\[
Q_L \equiv (Q_{1L}, Q_{2L}, Q_{3L}) \sim (3, \omega^2, 1, 1); \quad U_R \equiv (u_{2R}, u_{3R}, u_{1R}) \sim (3, 1, 1, \omega);
\]

\[
D_R \equiv (d_{2R}, d_{3R}, d_{1R}) \sim (3, 1, \omega, \omega),
\]

in which \(\omega = e^{2\pi i/3}\), and \(Q_{iL} = (d_i u_i)^T, \ i = 1, 2, 3\). All fermion fields in (3) are symmetry eigenstates. Notice that, assuming the usual ordering of \(Q_{iL}\), the \(A_4\) symmetry will allow, after its breaking, to distinguish among the right-handed quark components. The scalar fields transform under the same symmetries as:

\[
H \sim (3, \omega, 1, \omega), \quad H' \sim (3, \omega^2, \omega, \omega^2), \quad \Phi \sim (3, \omega, \omega, \omega),
\]

\[
\Phi' \sim (3, \omega^2, 1, \omega), \quad \zeta \sim (1, 1, \omega, 1).
\]

The notation in the scalar sector is slightly different from that in [2].

With these fermion and scalar fields, and the discrete symmetries above, we have the leading Yukawa interactions in the quark sector

\[
-L_Q = \left[ \left( h_u [\overline{Q}_L \tilde{H}]_A + h'_u \frac{\zeta}{\Lambda} [\overline{Q}_L \Phi]_B \right) U_R \right]_1^1 + \left[ \left( h_d [\overline{Q}_L H']_A + h'_d \frac{\zeta}{\Lambda} [\overline{Q}_L \Phi']_B \right) D_R \right]_1^1 + \text{H.c.}
\]

where \(\tilde{\phi} = \varepsilon \phi^*\), with \(\varepsilon = i\sigma_2\) (\(\sigma_2\) being the usual Pauli matrix); \([X Y]_{A,B}\) means the appropriate product, and \([X Y]_1\) denotes the singlet state, see Ref. [3] and references therein. Instead of the \(A\) and \(B\) triplet representations we can use the symmetric, \(s\), and antisymmetric, \(a\), ones. In fact, \(A = s + a\) and \(B = s - a\). We choose to have some fields which products are in \(A\) (or \(s\)), and others in \(B\) (or \(a\)). We assume this, firstly, because in global symmetries it is not mandatory to use all the product representation allowed by the symmetry and, secondly, because that choice may be explained by an underlying still unknown dynamics. Notice that the \(A_4\) symmetry imposes only two dimensionless Yukawa couplings in each charge sector. We assume that \(V/\Lambda \ll \mathcal{O}(1)\), where \(V\) denotes any VEV of the model and \(\Lambda\) is an energy scale characterizing an unknown physics. Notice that the model has flavor changing neutral currents (FCNC) since several Higgs doublets contribute to the masses of a given charge sector [4].

The renormalizable interactions in (5) induce diagonal interactions in the weak basis
which explicitly read

\[- \mathcal{L}_Q = h_u \left( \bar{Q}_{2L} \tilde{H}_3 u_{2R} + \bar{Q}_{3L} \tilde{H}_1 u_{3R} + \bar{Q}_{1L} \tilde{H}_2 u_{1R} \right) + h_d \left( \bar{Q}_{2L} H_d^* d_{2R} + \bar{Q}_{3L} H_d^* d_{3R} + \bar{Q}_{1L} H_d^* d_{1R} \right) + H.c. \]  

(6)

All scalar doublets are of the form \((x_i^+ x_i^0)^T\). On the other hand, the non-renormalizable interactions \([5]\) are written explicitly as

\[- \mathcal{L}_{Q}^{nr} = h_u' v_u \left[ \bar{Q}_{3L} \tilde{\Phi}_3 u_{2R} + \bar{Q}_{1L} \tilde{\Phi}_1 u_{3R} + \bar{Q}_{2L} \tilde{\Phi}_1 u_{1R} \right] + h_d' v_d \left[ \bar{Q}_{3L} \tilde{\Phi}_d' d_{2R} + \bar{Q}_{1L} \tilde{\Phi}_d' d_{3R} + \bar{Q}_{2L} \tilde{\Phi}_d' d_{1R} \right] + H.c. \]  

(7)

The mass matrices obtained from Eqs. \((6)\) and \((7)\) are

\[ M_u \approx h_u \begin{pmatrix} v_2 & 0 & a_u v_{\phi_3} \\ a_u v_{\phi_1} & v_3 & 0 \\ 0 & a_u v_{\phi_2} & v_1 \end{pmatrix} + H.c., \]  

(8)

for the \(u\)-type quarks, and

\[ M_d \approx h_d \begin{pmatrix} v_2' & 0 & a_d v_{\phi_1}' \\ a_d v_{\phi_1}' & v_3' & 0 \\ 0 & a_d v_{\phi_2}' & v_1' \end{pmatrix} + H.c., \]  

(9)

for the \(d\)-type quarks. The texture of these matrices are different from those of other multi-Higgs models like the private Higgs in \([5]\), and we stress that they are a consequence of the \(A_4\) symmetry and, mainly, of the choice of the representation \(A\) or \(B\). Above we have defined \(a_u = \frac{h_u' v_u}{h_u v_u} \) and \(a_d = \frac{h_d' v_d}{h_d v_d} \) for the \(2/3\) and \(-1/3\) charged quarks, respectively, and we have denoted \(\langle h_i^0 \rangle = v_i, \langle \phi_i^0 \rangle = v_{\phi_i}, \langle h_i^0' \rangle = v_i', \langle \phi_i^0' \rangle = v_{\phi_i}'\), and \(\langle \zeta \rangle = v_\zeta\). For the sake of simplicity, all parameters in Eqs. \((8)\) and \((9)\) have been considered real. Notice that since each charged sector has its private VEVs, these matrices are independent from each other.

Here we will show, numerically, that with the mass matrices \((8)\) and \((9)\) it is possible to accommodate the observed masses and the mixing matrices in the quark sector. These mass matrices are diagonalized by bi-unitary transformations, \(V_L^U M_u V_L^{U*} = \hat{M}_u\) and \(V_L^D M_d V_L^{D*} = \hat{M}_d\), respectively, with \(\hat{M}_u = diag(m_u, m_c, m_t)\) and \(\hat{M}_d = diag(m_d, m_s, m_b)\). The change of basis is \(q_{iL(R)} = (V_{L(R)}^Q)^\dagger q_{\alpha L(R)}\), \(q_{\alpha}\) denotes the quark mass eigenstates of the respective charge sector, \(q_{\alpha} = u, c, t\) for quarks with electric charge \(2/3\) and \(q_{\alpha} = d, s, b\) for quarks
with electric charge $-1/3$. We obtain the unitary $V_{L,R}^{U,D}$ matrices, by solving the matrix equations:

$$V_L D M_D M_D^+ V_L^{D\dagger} = \text{diag}(m_d^2, m_s^2, m_b^2) = (\hat{M}_D)^2, \quad V_R^D M_D^+ M_D V_R^{D\dagger} = (\hat{M}_D)^2,$$

$$V_L^U M_U M_U^+ V_L^{U\dagger} = \text{diag}(m_u^2, m_c^2, m_t^2) = (\hat{M}_U)^2, \quad V_R^U M_U^+ M_U V_R^{U\dagger} = (\hat{M}_U)^2,$$

$$V_Q^Q V_L^Q = 1, \quad V_R^Q V_R^Q = 1, \quad Q = U, D,$$

(10)

using as input parameters those VEVs, $a_q$ and $h_q$, appearing in $M_U, M_D$, which give the observed quark masses at an appropriate energy scale (the $Z$ mass in the present case) and the Cabibbo-Kobayashi-Maskawa quark–mixing matrix, defined as $V_{\text{CKM}} = V_L^U V_L^{D\dagger}$. The latter one is such that, at least in the context of the SM, the mixing between the first two families is almost scale independent and for the third family $V_{cb}$ and $V_{ub}$ change at the level of 13-16% between $m_t$ and $10^{15}$ GeV\cite{7}. Here, therefore the $V_{\text{CKM}}$ will be considered scale independent.

Since the mass matrices (8) and (9) are predictions of the model, they are valid at the energies at which all symmetries of the model are realized, i.e., at the electroweak scale. For this reason, we use the running quark masses at $\mu = M_Z$, taken from Ref.\cite{5}, for light quarks (in MeV): $m_u = 1.27^{+0.50}_{-0.42}$, $m_d = 2.90^{+1.24}_{-1.19}$, $m_s = 55^{+16}_{-15}$; and for heavy quarks (in GeV): $m_c = 0.619 \pm 0.084$, $m_b = 2.89 \pm 0.09$, $m_t = 171.7 \pm 3.0$.

In order to obtain the values of the $V_{L,R}^{U,D}$ matrix elements within an interval, we find two sets of values of the input parameters which give the quark masses and the CKM entries within the experimental errors. For instance, using: 1) $h_D = 0.1$ and $a_D = 0.11$ and (all VEVs are given in GeV) $v' \bar{1} = 28.9$, $v' \bar{2} = 0.44$, $v' \bar{3} = 0.03$, $v' \phi_1 = 1.95$, $v' \phi_2 = 0.03$, and $v' \phi_3 = 8.1$, and 2) $h_D = 0.1$ and $a_D = 0.2$ and $v' \bar{1} = 29.8$, $v' \bar{2} = 0.54$, $v' \bar{3} = 0.03$, $v' \phi_1 = 0.35$, $v' \phi_2 = 0.01$, and $v' \phi_3 = 2.9$, we obtain the following values for the masses in the $d$-quark sector: $m_d = (2.70 - 2.97)$ MeV, $m_s = (48.95 - 54.4)$ MeV, and $m_b = (2.89 - 2.98)$ GeV. For the $u$-quark sector, we use 1) $h_U = 1.11$, $a_U = 0.2$, and the VEVs $v_1 = 153$, $v_2 = 0.54$, $v_3 = 0.001125$, $v_\phi_1 = 0.08875$, $v_\phi_2 = 0.03555$, $v_\phi_3 = 57.2355$; 2) $h_U = 1.11$, $a_U = 0.13$, and the VEVs $v_1 = 153$, $v_2 = 0.531$, $v_3 = 0.00108$, $v_\phi_1 = 1.2048$, $v_\phi_2 = 0.3199$, $v_\phi_3 = 70.783$, we obtain: $m_u = (1.27 - 1.93)$ MeV, $m_c = (598 - 613)$ GeV, and $m_t = (170.305 - 170.137)$ GeV. In spite that some VEVs are small, this does not imply necessarily the existence of light scalars. See Ref.\cite{8} where the case of three Higgs scalar doublets with $A_4$ symmetry were considered in details.
The matrices $V_{L,R}^{U,D}$ can be obtained by solving (10) with values for the parameters considered above. An example of numerical $V_{L}^{U,D}$ matrices obtained is given by:

$$
V_{L}^{U} = \begin{pmatrix}
0.03290 & 0.28285 & -(0.95902 \rightarrow 0.99946) & -(0.00246 \rightarrow 0.01678) \\
-(0.95733 \rightarrow 0.99667) & -(0.0330 \rightarrow 0.28334) & 0.05680 \rightarrow 0.07457 \\
0.05923 \rightarrow 0.07461 & (0.3 \rightarrow 2) \times 10^{-7} & 0.99721 \rightarrow 0.99824
\end{pmatrix},
$$

$$
V_{L}^{D} = \begin{pmatrix}
0.12895 \rightarrow 0.25492 & -(0.96693 \rightarrow 0.99165) & -(0.00251 \rightarrow 0.00786) \\
-(0.96647 \rightarrow 0.99146) & -(0.12896 \rightarrow 0.25504) & 0.0193 \rightarrow 0.0298 \\
0.01946 \rightarrow 0.03082 & (2 \rightarrow 9) \times 10^{-7} & 0.99952 \rightarrow 0.99981
\end{pmatrix}, \tag{11}
$$

Using these matrices and the definition $V_{CKM} = V_{L}^{U} V_{L}^{D\dagger}$, we obtain

$$
|V_{CKM}| = \begin{pmatrix}
0.9748 - 0.9875 & 0.1571 - 0.2230 & 0.0014 - 0.0113 \\
0.1574 - 0.2227 & 0.9739 - 0.9868 & 0.0381 - 0.0438 \\
0.0051 - 0.0111 & 0.0394 - 0.0424 & 0.9990 - 0.9992
\end{pmatrix}. \tag{12}
$$

We compare this matrix with the global fit of the magnitude of the $V_{CKM}$ elements given in Eq. (11.27) of PDG [9]

$$
|V_{CKM}^{PDG}| = \begin{pmatrix}
0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\
0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.010^{+0.0011}_{-0.0007} \\
0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045}
\end{pmatrix}. \tag{13}
$$

Considering the elements of $V_{CKM}^{PDG}$, we note that the $V_{CKM}$ in Eq. (12) has all its entries within 1-σ but the $V_{td}$, which is within 1.7-σ. We recall that at present there are several discrepancies between experiments and the standard model at the tree level that are about 3σ standard deviations [10]. Notice that we are not considering CP violation in our analysis. This is because in our framework the CP violation issue is more complicated than in the SM case since in our model there are more CP violation phases. Although we can perform the usual phase redefinition in the left-handed fields of the charged currents, in order to obtain the full, single–phase, $V_{CKM}$ matrix, many CP violating phases will still be present in the Yukawa interaction Lagrangian. We find that this subject deserves a separate study.
In the same way we have obtained, from (10), the respective $V_{U,D}^{R}$ matrices:

$$
V_{U}^{R} = \begin{pmatrix}
-(0.00091 \rightarrow 0.00371) & -(0.99999 \rightarrow 1.) & 0.00005 \rightarrow 0.00027 \\
0.99999 \rightarrow 1. & 0.00091 \rightarrow 0.00372 & -(0.0002 \rightarrow 0.00026) \\
0.0002 \rightarrow 0.00026 & 0.00004 \rightarrow 0.00027 & 1.
\end{pmatrix}
$$

$$
V_{D}^{R} = \begin{pmatrix}
0.00047 \rightarrow 0.01661 & -(0.99986 \rightarrow 0.99997) & 0.00006 \rightarrow 0.00011 \\
-(0.99986 \rightarrow 0.99997) & -(0.00703 \rightarrow 0.01661) & 0.00035 \rightarrow 0.00047 \\
0.00035 \rightarrow 0.00047 & 0.00007 \rightarrow 0.00011 & 1.
\end{pmatrix}
$$

(14)

The matrices above will appear in flavor changing neutral currents in the Yukawa interactions.

Notice that the mass matrices in Eqs. (8) and (9) have the following parameters (assuming all of them to be real): $h_{U}, a_{U}, h_{D}, a_{D}$ and twelve VEVs. It means 16 real free parameters to explain 12 mixing angles of four matrices: $V_{U,D}^{U,D}$ (or one of $V_{L}^{U,D}$ and $V_{CKM}$) and $V_{R}^{U,D}$. However, if $CP$ violation is allowed, it is necessary to chose a weak basis [11, 12] in order to eliminate some extra phases which may be difficult to measure.

**FCNC suppression.** There are flavor changing neutral currents (FCNC) effects in both quark sectors. Here we will consider only some of these sort of effects which occurs at the tree level. For instance, from (6) the renormalizable interactions of the doublets with the $d$-type quarks (in the weak basis), are given by $h_{D}(\bar{d}_{2L}s_{2R}\varphi_{3}^{0} + \bar{d}_{3L}d_{3R}\varphi_{1}^{0} + \bar{d}_{4L}d_{4R}\varphi_{2}^{0})$. The other three doublets have interactions, which are suppressed by the scale $\Lambda$, will not be considered.

On the other hand, the neutral scalar weak eigenstates $\varphi_{0}^{\alpha}$ are linear combinations of the neutral scalar mass eigenstates, $h_{n}^{0}$, i.e., $\varphi_{0}^{\alpha} = \sum_{n} U_{\alpha n} h_{n}^{0}$, $\alpha, n = 1, 2, 3$. This is in fact a simplification, since the model has several doublets, but it may be assumed (for the sake of simplicity) that in each charge sector there are three doublets which give the more important effects. It means that the mixing matrices in the full scalar sector may be almost block diagonal, with each one related to a given fermion charge sector.

The suppression of FCNC can be obtained at least with a reasonable fine tuning in the mixing parameters or by considering heavy enough the charged and neutral scalars. We consider some examples just for illustrating this point. The contributions of the neutral scalars to the $\Delta M_{K}$ put the strongest constraint on some of the parameters of the model. From (6), the renormalizable Yukawa interactions between the $d$ and $s$ quarks and a given
neutral (pseudo)scalar, denoted by \( h_n^0(A_n^0) \), are given by

\[
-\mathcal{L}_{ds} = h_D \sum_n (\bar{d}_L K_n s_R + \bar{s}_L K_n' d_R) (h_n^0 + iA_n^0) + \text{H.c.} \quad (15)
\]

where we have assumed \( h_D \) real and defined

\[
K_n = (V_L^D)^*_{sd}(V_R^D)_{ss} U_{3n} + (V_L^D)^*_{bd}(V_R^D)_{bs} U_{1n} + (V_L^D)^*_{dd}(V_R^D)_{ds} U_{2n} \approx -0.01U_{3n} + 0.16U_{2n} + 10^{-9}U_{1n},
\]

\[
K_n' = (V_L^D)^*_{ss}(V_R^D)_{sd} U_{3n} + (V_L^D)^*_{bs}(V_R^D)_{bd} U_{1n} + (V_L^D)^*_{ds}(V_R^D)_{dd} U_{2n} \approx 0.01U_{3n} - 0.01U_{2n} + 10^{-8}U_{1n},
\]

where we have used the values of the \( V_L^D \) and \( V_R^D \) matrix elements given in (11) and (14), respectively.

Thus, the interactions in (15) can be rewritten as

\[
-\mathcal{L}_{sd} = \frac{h_D^2}{2} \left\{ [(K_n + K_n')^2 d\bar{s}) + (K_n - K_n')(\bar{d}\gamma_5 s)](h_n^0 + iA_n^0)]
+ [(K_n^* + K_n'^* d\bar{s}) - (K_n^* - K_n'^*)(\bar{s}\gamma_5 d)](h_n^0 + iA_n^0)^*, \quad (17)
\]

and the effective Hamiltonian contributing to \( K^0 \leftrightarrow \bar{K}^0 \) transition is given by

\[
\mathcal{H}^{\Delta S=2}_{\text{eff}}|_{\text{scalars}} = \sum_n \frac{h_D^2}{4m_n^2} [(K_n^* + K_n'^*)^2 (d\bar{s})^2 + (K_n^* - K_n'^*)^2 (\bar{s}\gamma_5 d)^2] \quad (18)
\]

In the vacuum insertion approximation \( \langle \bar{K}^0 |(\bar{s}d)^2|K^0 \rangle = 2M_K f_K^2/3 \). Using (up to some phases) \( \{13, 14\} \)

\[
\langle K^0 |(\bar{s}d)^2|K^0 \rangle = -\frac{f_K^2 M_K}{12} \left[ 1 - \frac{M_K^2}{(m_s + m_d)^2} \right],
\]

\[
\langle \bar{K}^0 |(\bar{s}\gamma_5 d)^2|K^0 \rangle = \frac{f_K^2 M_K}{12} \left[ 1 - 11 \frac{M_K^2}{(m_s + m_d)^2} \right], \quad (19)
\]

we obtain the following extra contributions to \( \Delta M_K \) due to the neutral scalar

\[
\Delta M_K|_{\text{scalars}} = 2\text{Re}(\bar{K}^0|\mathcal{H}_{\text{eff}}^{\Delta S=2}|_{\text{scalars}}|K^0) = \text{Re} \sum_n \zeta_{sd} n^2 2M_K f_K^2, \quad (20)
\]

where

\[
\text{Re} \sum_n \zeta_{sd} n^2 = \frac{h_D^2}{8} \sum_n \frac{1}{m_n^2} \text{Re} \left\{ -(K_n^* + K_n'^*)^2 \left[ 1 - \frac{M_K^2}{(m_s + m_d)^2} \right] + (K_n^* - K_n'^*)^2 \left[ 1 - 11 \frac{M_K^2}{(m_s + m_d)^2} \right] \right\}
\approx -\text{Re} \sum_n \frac{7.52(U_{3n}^* + U_{3n})U_{2n}^* + 0.75U_{2n}^2}{(m_n/100\text{GeV})^2} 10^{-5} \text{GeV}^{-2}, \quad (21)
\]
where we have used only the main contributions in Eq.

There are also similar contributions induced by the pseudoscalar, $A_n^0$. For illustrative purposes we showed above only the scalar contributions.

Notice that, in the present context, in Eq.

yet and that, independently of the mass eigenstates $h_n^0$, $U_{1n}$ will not be constrained by processes like $\Delta M_K$. In order to be consistent with data Re $\sum_n \zeta_{sd}^n$ must be smaller than the contribution of the SM: i.e., $\text{Re} \zeta_{sd}^{SM} = G_F^2 m_t^2 \text{Re} [(V_{CKM})_{cd}^* (V_{CKM})_{cs}]^2 / 16 \pi^2 \approx 10^{-14} \text{GeV}^{-2}$ (we have used only the dominant contribution of the $c$ quark and $g(m_c/M_W) = 1$). By imposing that $\text{Re} \sum_n \zeta_{sd}^n < \text{Re} \zeta_{sd}^{SM}$, implies, from Eq.

Even if for a given $n$, $m_n = 100$ GeV, there are two different ways for each term in Eq.

to satisfy the constraint: a) all parameters involved are of the order of $10^{-5}$, i.e. $\text{Re} U_{3n} \sim \text{Re} U_{2n} \sim \text{Im} U_{2n} \sim 10^{-5}$, or b) there is a fine tuning among the parameters. For instance, $\text{Re} U_{3n} = 0.01, \text{Re} U_{2n} = 0.01, \text{Im} U_{2n} = 0.045826742$. For heavier neutral scalars the $U_{an}$ matrix elements may be greater, limited only by the unitarity of the matrix.

Similar constraints come from $\Delta M_B$ and $\Delta M_{B_s}$ data and other $\Delta B = 1$ weak processes, like $B_{s,d} \to \mu^+\mu^-$ and $B^+ \to K^+\mu^+\mu^-$ decays. Experimentally, $\Gamma(B^+ \to K^+\mu^+\mu^-)/\Gamma_{\text{total}} < 5.2 \times 10^{-7}$ \cite{9}. In this case, the quark interactions involved are $h_D \bar{b}_L [(V_L^D)^*_{sb} (V_R^D)_{ss'} v_3^{0*} + (V_L^D)^*_{tb} (V_R^D)_{tss'} v_1^{0*} + (V_L^D)^*_{db} (V_R^D)_{ds} v_2^{0*}] s_R + H.c.$, and we recall that, in this model, leptons have their own scalar sector with the Yukawa interactions $g_L[[\mathcal{H}]_A L_R]_1$. Here, $\mathcal{H}$ denotes the triplet of $A_4$ formed by three scalar doublets (see below). For muons it means $g_L \mu_R \hat{h}_3^0$, and the muon mass is given by $m_\mu = g_L \hat{h}_3^0 \equiv g_L v_3$. In the scalar potential the scalar triplet $\mathcal{H}$ mixes with the scalar triplets related to the quark sector only by quartic terms like $\lambda_1 \left[ \mathcal{H}^\dagger \mathcal{H} \right]_A \left[ H'^4 H' \right]_A = \lambda_1 \left[ \hat{H}^1_1 \hat{H}^1_2 \hat{H}^1_3 H'_3 + \hat{H}^1_2 \hat{H}^1_1 \hat{H}^1_3 H'_1 + \hat{H}^1_1 \hat{H}^1_2 \hat{H}^1_3 H'_2 \right]$. This implies terms like $\lambda_1 v'_2 \hat{v}_3$ in the mass matrix in the neutral scalar sector and, in the weak basis a mass insertion implies that the propagator becomes $\lambda_1 v'_2 \hat{v}_3/m^4$, where $m$ denotes a typical value for the neutral scalar masses. As we said before, in that basis, the Higgs scalars coupled to leptons are different from those coupled to quarks. It means that in semi-leptonic decays a detailed analysis may be done only if we consider at least six doublets of scalar Higgs bosons. Here, just for
illustration consider $b \to s\mu\mu$ decay. Working in the weak basis we obtain,

$$\frac{\Gamma(b \to s\mu^+\mu^-)}{\Gamma(b \to c\bar{v}_\mu\mu)} \propto \frac{\lambda_1^2 \Lambda^2}{m_G^2} \frac{|(V^D_L)_d^s|(V^D_R)_d^s|^2}{G_F^2 m^8} m_{\mu^2}^2 \mu^2 \mu^2 = 1.155 \times 10^4 \lambda_1^2/(m/\text{GeV})^8 < 10^{-7},$$

and we have used the mixing matrix elements $(V^D_L)_d^s$ and $(V^D_R)_d^s$ from \([11]\) and \([14]\), respectively. All parameters in \([23]\) but $\lambda_1$ and $m$ are already known. It implies $\lambda_1^2/(m/\text{GeV})^8 < 10^{-11}$ which is valid for a wide range of the parameters. For instance, for $m > 10$ GeV and $\lambda_1^2 \approx 10^{-3}$ the above condition is satisfied.

The case of FCNCs in the $u$-quark sector involves the matrices $V^U_L$ and $V^U_R$. The analysis of the FCNC in this model is similar to that of Ref. \([16]\) in which two doublets were considered.

**The lepton sector.** In the lepton sector the discrete symmetries are the same as in the quark sector. Leptons transform under $A_4 \otimes (Z_3)^3$ as follows:

$$L \equiv (L_e, L_\mu, L_\tau) \sim (3, \omega, \omega^2, 1); \quad l_R \equiv (\mu_R, \tau_R, \epsilon_R) \sim (3, 1, 1, \omega),$$

and the leptophilic scalars transform under those symmetries as

$$H''_L \equiv (H''_1, H''_2, H''_3) \sim (3, \omega, \omega^2, 1), \quad \hat{H} \equiv (\hat{H}_1, \hat{H}_2, \hat{H}_3) \sim (3, \omega, \omega^2, \omega^3),$$

$$\Phi'' \equiv (\Phi''_1, \Phi''_2, \Phi''_3) \sim (3, 1, 1, 1), \quad \mathcal{T} \equiv (\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) \sim (3, \omega^2, \omega, 1).$$

The Lagrangian of the lepton sector is given by (see the notation in \([2]\)):

$$\mathcal{L} = \left( g_l [\mathcal{L} \hat{H}]_A + \frac{g'_l}{\Lambda^2} [\mathcal{L} \hat{H}]_A |\zeta|^2 + \cdots \right) l_R$$

$$+ \frac{1}{\Lambda} \left( f_\nu [\mathcal{L} \nu H''_L^\prime]_A [(L\epsilon \Phi''_L)]_B + \frac{f'_\nu}{\Lambda} [\mathcal{L} \tilde{\epsilon} \epsilon \hat{T}]_B [L\Phi''_L^\prime]_{BC} + \cdots \right) + H.c.,$$

where $g_l, g'_l, f_\nu$ and $f'_\nu$ are dimensionless Yukawa couplings and $\Lambda$ an energy scale which may be, or not, the same as that in the quark sector. The mass matrix for the charged leptons is almost diagonal: $M_l = g_l \text{diag}(\tilde{v}_2, \tilde{v}_3, \tilde{v}_1) + \mathcal{O}(g_l^2 \tilde{v}_2^2 / \Lambda^2)$, where $\langle \hat{H}_i \rangle = \tilde{v}_i$. In this case the renormalizable interactions are dominant, i.e., $g_l \tilde{v}_1 \simeq m_\tau$, $g_l \tilde{v}_3 \simeq m_\mu$ and $g_l \tilde{v}_2 \simeq m_e$. Neglecting the contributions proportional to $g'_l$, the charged lepton mass matrix is diagonal, thus the values of the VEVs are easily obtained: Just for illustrating, by using the central value for the lepton masses (in MeV) at the $Z$ pole scale: $m_e \approx 0.486$, $m_\mu \approx 102.718$ and $m_\tau \approx 1746.24$, hence $\tilde{v}_2 \simeq m_e/g_l$, $\tilde{v}_3 \simeq m_\mu/g_l$, $\tilde{v}_1 \simeq m_\tau/g_l$. 

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On the other hand, the mass matrix for the neutrinos is

\[
M_\nu \approx \begin{pmatrix}
\delta_1 & \frac{v''_1}{v'_1} & \frac{v''_1}{v''_1}
\frac{v''_2}{v'_2} & \frac{v''_2}{v''_2}
\frac{v''_3}{v''_1} & \frac{v''_3}{v''_2}
\frac{v''_4}{v''_1} & \frac{v''_4}{v''_2} & \frac{v''_4}{v''_3}
\frac{v''_5}{v''_1} & \frac{v''_5}{v''_2} & \frac{v''_5}{v''_3} & \frac{v''_5}{v''_4}
\end{pmatrix}
\left(\begin{array}{c}
f_\nu v''_1
\Lambda
\end{array}\right)
\]

(27)
in which \(\delta_i\) is given by

\[
\delta_1 = \frac{f_\nu v_{T3} v_{\phi3} v_\chi}{f_\nu \Lambda v_1 v_\phi}, \quad
\delta_2 = \frac{f_\nu v_{T1} v_\chi}{f_\nu \Lambda v_1}, \quad
\delta_3 = \frac{f_\nu v_{T2} v_{\phi2} v_\chi}{f_\nu \Lambda v_1 v_\phi},
\]

(28)
where \(\langle \chi \rangle = v_\chi, \quad e \langle \Delta^0 \rangle = v_{T_i}\).

Since the mass matrix for the charged leptons is, for practical purposes, diagonal, which implies \(U_{L,R}^T = 1\), the mixing matrix PMNS, is defined as \(V_{PMNS} = U_\nu \equiv U\), i.e., is obtained directly from the diagonalization of the neutrino mass matrix.

In the neutrino sector the \(V_{PMNS}\) matrix compatible with experimental data is, at the 3\(\sigma\) level [17],

\[
|U|_{3\sigma} = \begin{pmatrix}
0.77 - 0.86 & 0.50 - 0.63 & 0.00 - 0.22
0.22 - 0.56 & 0.44 - 0.73 & 0.57 - 0.80
0.21 - 0.55 & 0.40 - 0.71 & 0.59 - 0.82
\end{pmatrix},
\]

(29)
We use a different strategy from that used with quarks. Imposing that the matrix \(U\) satisfies the equation \(U^T M_\nu U = \hat{M}_\nu = \text{diag}(m_1, m_2, m_3)\), where \(M_\nu\) is given by the matrix in (27), we found numerical values for all the parameters in this matrix. Next, we found the square mass differences. For example, using the lower limit of the entries in (29), and the values of \(\Lambda = 1\) TeV, \(v_\chi = 2\) GeV and \(v_{T1} = v_{T2} = 2v_{T3} = 1.25\) GeV and (in eV) \(v_1 = 4, v_2 = 10^5, v_3 = 0.04, v_{\phi1} = 40450, v_{\phi2} = 48000, v_{\phi3} = 34850\), we obtain (in eV)

\[
\hat{M}_\nu \approx \begin{pmatrix}
0.03485 & \sim 0 & 0
\sim 0 & 0.0337083 & \sim 0
0 & \sim 0 & 0.06
\end{pmatrix},
\]

(30)
where \(\sim 0\) means entries smaller than \(10^{-7}\) eV. From (30), the mass squared differences (in eV\(^2\)) obtained are

\[
\Delta m^2_{21} = 8 \times 10^{-5}, \quad \Delta m^2_{31} = 2.5 \times 10^{-3},
\]

(31)
which agree within 1\(\sigma\) with the values at the \(\mu = Z\) given in Ref. [7]. The neutrino sector in the model differs from that of the private Higgs of [18] but, both models can accommodate a
nonzero $\theta_{13}$ angle. Notice that if we assume that $L(H'') = L(T) = -2$, and the other scalars having $L = 0$, the interactions in $[26]$ conserve the lepton number. However, there is no Majoron since the $A_4$ and $Z_3$ symmetries allow terms in the potential which break explicitly the lepton number. For instance, $[H''^\dagger \Phi]_1[\hat{H}^\dagger H]_1$, $[\hat{H}^\dagger e \Phi']_1[H' e H'']_1$, $[H' H'']_1[\hat{H}^\dagger e H']_1$. As in the quark sector, there are also FCNCs in the lepton sector but they will be considered elsewhere.

**Conclusion.** Motivated by the different mass scales in the quark and lepton sector we propose a model in which each charge sector has its own Higgs scalars which acquires appropriate VEVs. In this case, in order to induce the respective fermion masses, the Yukawa couplings do not need to have a large hierarchy among them, i.e., all of them may be of the same order of magnitude [2]. The hierarchy is translated to the values of the VEVs but these could be, in principle, explained by the minimization of the scalar potential. In fact, it was shown in [8] that this is indeed the case, for three scalar doublets.

Matrices like those in (1) are written in terms of dimensionless parameters. This is because in the context of the SM, where these *ansatze* have been usually studied, the mass scale is already determined by the VEV $v_{SM} = (G_F/2\sqrt{2})^{1/2}$ but in multi-Higgs models only $\sqrt{\sum_i v_i^2}$ which satisfies this constrained. This is the case of the model in Ref. [2] that we are considering here, and all VEVs are considered parameters to be fixed only by the fermion masses in each charge sector. We recall that we are not considering physical phases, but $V_{U,D}^{L,R}$ may have the number of phases that are allowed for an arbitrary unitary matrix. We allow that in the $V_{CKM}$ matrix only one physical $CP$ violating phase to survive after redefining the quark fields.

Another concern is if the suppression of the FCNC effect at low energies is stable up to higher energies. The answer is yes, at least for the case of a general two Higgs doublet model [19]. Moreover, QCD corrections [20] have also to be considered at the next-to-leading order [21, 22]. This has been done in the 2HDM in the context of natural flavor conservation and minimal flavour violation in Ref. [23]. A similar analysis in the context of models with at least three scalar doublets with or without extra symmetries, say $A_4, S_3$, will be considered elsewhere.
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