We have studied the production of the $\eta_c$ charmonium state, at the Large Hadron Collider (LHC) in the framework of Non-Relativistic Quantum Chromodynamics (NRQCD) using heavy-quark symmetry. We find that NRQCD predicts a large production cross-section for this resonance at the LHC even after taking account the small branching ratio of $\eta_c$ into two photons. We show that it will be possible to test NRQCD through its predictions for $\eta_c$, with the statistics that will be achieved at the early stage of the LHC, running at a center of mass energy of 7 TeV with an integrated luminosity of 100 pb$^{-1}$.

Non-Relativistic Quantum Chromodynamics (NRQCD) is an effective theory that has been extensively used to study the production and decay of quarkonia. NRQCD is derived from the QCD Lagrangian by neglecting all states of momenta much larger than the heavy quark mass, $M_Q$ and to account for this exclusion by adding new interaction terms in the effective Lagrangian. It is then possible to expand the quarkonium state in-terms of $v$, the relative velocity of the heavy quarks in the bound state. In this expansion, the $Q\overline{Q}$ pair in the intermediate state can be in either colour-singlet or colour-octet configurations denoted by $Q\overline{Q}[2S+1L_J^{[1,8]}]$. However, the colour-octet $Q\overline{Q}$ state evolves non-perturbatively into a physical colour-singlet state by emission of one or more soft gluons. The cross section for production of a quarkonium state $H$ can be factorised as:

$$\sigma(H) = \sum_{n=\{s,s,L,J\}} \frac{F_n}{M_Q^{d_n-4}} \langle O_n^{H(2S+1L_J)} \rangle,$$

where $F_n$'s are the short-distance coefficients and $O_n$ are operators of naive dimension $d_n$, describing the long-distance effects. These non-perturbative matrix elements are guaranteed to be energy-independent due to the NRQCD factorization formula, so that they may be extracted at a given energy and used to predict quarkonium cross-sections at other energies.
Before the effective theory approach of NRQCD was developed, the colour-singlet model (CSM) [2, 3] was used to analyze the production of quarkonia, where the $Q\bar{Q}$ state produced in the short-distance process was assumed to be a colour-singlet. However, it was pointed out in Ref. [4] that contributions from the colour-octet operators are significant in describing the phenomenology of large-$p_T$ $P$-state charmonium production at the Tevatron [5]. In Refs. [6, 7] the complete set of short-distance coefficients in NRQCD needed to study $J/\psi$ and $\chi$ production was calculated and compared with the data from Tevatron [1]. These NRQCD calculations gave a good description of the shapes of the $p_T$ distributions of the charmonium resonances at the Tevatron but the normalization of these distributions was not predicted in NRQCD i.e. the non-perturbative matrix elements which determined the normalization had to be obtained by a fit to the data. Independent tests of the effective theory approach were, therefore, necessary to determine the validity of the approach and, indeed, various proposals were made [10] to test NRQCD. But several of these proposals are not for large-$p_T$ quarkonium production and the validity of NRQCD factorization at low-$p_T$ is suspect.

One interesting test of NRQCD comes from the study of the polarization of $J/\psi$'s at large-$p_T$ [11] which primarily comes from a fragmentation-like processes where a single gluon splits into a $Q\bar{Q}$ pair which inherits the transverse polarization of the gluon. The heavy-quark symmetry of NRQCD then comes into play in protecting this transverse polarization in the non-perturbative evolution of the $Q\bar{Q}$ pair into a $J/\psi$. The large-$p_T$ $J/\psi$ is, therefore, strongly transversely polarized. This is not true at even moderately low $p_T$ where the $J/\psi$ is essentially unpolarized. The $p_T$ dependence of the polarization is, therefore, a very good test of the theory [12].

The CDF experiment has measured the $p_T$-dependence of the polarization and they find no evidence for any transverse polarization at large $p_T$ [13], which seems to indicate a dramatic failure of the theory. Inspite of the successful prediction of the production cross-sections of the various charmonium resonances it may well be that the effective theory is missing out on some aspect of the physics of quarkonium formation. Alternatively, it could be that the charm quark is too light to be treated in NRQCD. On the other hand, polarization measurements are usually fraught with problems and it may well be that the problem is elsewhere. Finally, because the colour-singlet channel predicts unpolarized $J/\psi$'s, there have been attempts to increase up the colour-singlet contribution to the production processes by invoking Reggeized gluons [14] or enhanced effects of higher-order QCD corrections in the singlet channel [15, 16]. For reviews of the current status of these calculations and their experimental consequences, see Refs. [17, 18].

In this situation, it is worthwhile looking for other tests of NRQCD which successfully navigate between low-$p_T$ and polarization. The heavy-quark symmetry of NRQCD provides a set of relations between non-perturbative parameters of different resonances so a measurement of a given state yields information on the non-perturbative parameter of another state related to the former by heavy-quark symmetry. This fact has been exploited to study $h_c$ production at the Tevatron [19] and, more recently, at the LHC [20]. Similarly $\eta_c$ production at the Tevatron has also been studied [21]. In this paper, we study the production of $\eta_c$ at the LHC.

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1See also Ref. [8]. For a detailed review of quarkonium production see Ref. [9].
The Fock space expansion of the physical $\eta_c$, which is a $^1S_0$ ($J^{PC} = 0^{-+}$) state, is:

$$|\eta_c\rangle = O(1) \left| Q\overline{Q}[^1S_0] \right| + O(v^2) \left| Q\overline{Q}[^1P_1] \right| + O(v^4) \left| Q\overline{Q}[^3S_1] \right| + \cdots.$$  \hspace{1cm} (2)

In the above expansion the colour-singlet $^1S_0$ state contributes at $O(1)$. As the $P$-state production is itself down by factor of $O(v^2)$ both the colour-octet $^1P_1$ and $^3S_1$ channels effectively contribute at the same order. The colour-octet state $^1P_1$ ($^3S_1$) becomes a physical $\eta_c$ by emitting a gluon in an E1 (M1) transition. Keeping terms up-to $O(\alpha_s^2 v^r)$ the $\eta_c$ production cross section can be parameterized as:

$$\sigma(\eta_c) = \frac{F_1[^1S_0]}{M^2} \langle 0| O_{\eta_c}[^1S_0] |0\rangle + \frac{F_8[^1P_1]}{M^4} \langle 0| O_{\eta_c}[^1P_1] |0\rangle + \frac{F_8[^3S_1]}{M^2} \langle 0| O_{\eta_c}[^3S_1] |0\rangle ,$$  \hspace{1cm} (3)

where the coefficients, $F$’s, are the cross sections for the production of $c\bar{c}$ pair in the respective angular momentum and colour states. The differential cross section for $c\bar{c}$ pair production with specific angular momentum and colour states at the LHC is given by:

$$\frac{d\sigma}{dp_T} (pp \to c\bar{c} [{}^{2S+1}L^j_{\eta_c}] X) =$$

$$\sum \int dy \int dx_1 x_1 G_{a/p}(x_1) x_2 G_{b/p}(x_2) \frac{4p_T}{2x_1 - \tau} \frac{d\sigma}{dt} (ab \to c\bar{c} [{}^{2S+1}L^j_{\eta_c}] d) ,$$  \hspace{1cm} (4)

where the summation is over the partons ($a$ and $b$), the final state $Q\overline{Q}$ is in the $[^1S_0]$, $[^1P_1]$, $[^3S_1]$ states and $G_{a/p}, G_{b/p}$ are the distributions of partons $a$ and $b$ in the protons and $x_1, x_2$ are the respective momentum they carry. $x_2$ is related with $x_1$ as:

$$x_2 = \frac{x_1}{2x_1 - \tau} e^{-\tau} ,$$  \hspace{1cm} (5)

where $\tau = \sqrt{x_T^2 + 4\tau} \equiv 2M_T/\sqrt{s}$ with $x_T = 2p_T/\sqrt{s}$ and $\tau = M^2/s$. Here $\sqrt{s}$ is the center-of-mass energy, $M$ is the mass of the resonance and $y$ is the rapidity at which the resonance is produced. The subprocesses contributing to Eq.(4) are:

$$g\ g \rightarrow Q\overline{Q}[{}^{2S+1}L^j_{\eta_c}] g,$$

$$g\ q(\bar{q}) \rightarrow Q\overline{Q}[{}^{2S+1}L^j_{\eta_c}] q(\bar{q}),$$

$$q\ \bar{q} \rightarrow Q\overline{Q}[{}^{2S+1}L^j_{\eta_c}] g.$$  \hspace{1cm} (6)

The matrix elements for the subprocesses corresponding to $F_1[^1S_0]$ and $F_8[^3S_1]$ are listed in Refs. \cite{7,22}. The remaining coefficient $F_8[^1P_1]$ has been calculated and used in \cite{21} to analyze $\eta_c$ production at the Tevatron.

We use heavy quark spin-symmetry to obtain the values of $\langle O_n^{H}\rangle$’s from the experimentally predicted values $\langle O_n^{I/\psi}\rangle$’s. Using this symmetry the $\langle O_n^{H}\rangle$’s are related as:

$$\langle 0| O_{\eta_c}[^1S_0] |0\rangle = \frac{1}{3} \langle 0| O_{I/\psi}[^3S_1] |0\rangle (1 + O(v^2)),$$

$$\langle 0| O_{\eta_c}[^1P_1] |0\rangle = \langle 0| O_{I/\psi}[^3P_0] |0\rangle (1 + O(v^2)),$$

$$\langle 0| O_{\eta_c}[^3S_1] |0\rangle = \langle 0| O_{I/\psi}[^1S_0] |0\rangle (1 + O(v^2)).$$  \hspace{1cm} (7)
We use the predicted value of the singlet matrix elements listed in Ref. [6] ($C_1 \equiv \langle 0 | O_{J/\psi}^{[3]S_1} | 0 \rangle = 1.2 \text{ GeV}^3$) and the octet matrix elements extracted from the CDF $J/\psi$ data [7] (i.e. $C_2 + C_3 \equiv \frac{\langle 0 | O_{J/\psi}^{[3]P_0} | 0 \rangle}{M_T^2} + \frac{3}{2} \langle 0 | O_{J/\psi}^{[1]S_0} | 0 \rangle = (2.2 \pm 0.5) \times 10^{-2} \text{ GeV}^3$). It is to be noted that only a linear combination $C_2 + C_3$ can be extracted from the CDF data as the shapes of $C_2$ and $C_3$ contributions to the $J/\psi p_T$-distribution are almost identical. Hence, for predicting the $\eta_c$ rate we assume that either $C_2$ or $C_3$ saturates the sum so that our predictions indicate the band within which we expect the experimental value of the $\eta_c$ production cross-section to lie. More explicitly, we consider the two extreme cases where in the first case, the maximum possible contribution is from the $3S_1$ channel and none from the $1P_1$ channel whereas in the second case the $1P_1$ contributes its maximum while the $3S_1$ channel does not contribute.

Figure 1: $d\sigma/dp_T$ (in nb/GeV) for $\eta_c$ production (after folding in with $\text{Br}(\eta_c \rightarrow \gamma\gamma) = 3.0 \times 10^{-4}$) in $pp$ collisions at $\sqrt{s} = 7$ TeV and 14 TeV with $-2 \leq y \leq 2$.

In Fig. 1 we have displayed the differential cross section $Bd\sigma/dp_T$ as a function of $p_T$ at two different center-of-mass energies, viz. $\sqrt{s} = 7$ and 14 TeV respectively, where $B$ is the $\eta_c \rightarrow \gamma\gamma$ branching ratio ($B = 3 \times 10^{-4}$). We have used CTEQ 5L LO parton densities [23] evolved to a scale $Q = M_T$. In both (a) and (b) of Fig. 1, we have curves marked I and II where I is the sum of the colour-singlet and the $3S_1$ contributions and II is, likewise, the sum of the colour-singlet and the $1P_1$ contribution. In addition, we also display the colour-singlet curve in both the figures, to bring out the fact that the octet contributions overwhelmingly dominate the cross-section.

To account for the experimental threshold in $p_T$ for the photons which is about 10 GeV, we use a lower-$p_T$ cut of 20 GeV on the $\eta_c$ to calculate the integrated cross-sections. For the LHC running at $\sqrt{s} = 7$ TeV, with an integrated luminosity ($\mathcal{L}$) of 100 pb$^{-1}$, we find that the number of events in the $\gamma\gamma$ channel from the singlet $1S_0^{[1]}$ is about 40 while the number of events from $3S_1^{[8]}$ is about $10^6$ when $1P_1$ contribution is neglected, while the number of events from $1P_1^{[8]}$ is about 37660 when $3S_1^{[8]}$ contribution is absent. For the
case of $\sqrt{s} = 14$ TeV and $L = 100$ pb$^{-1}$, the respective numbers would be 100, $2.8 \times 10^6$ and $10^5$. Thus the minimum $\eta_c \to \gamma\gamma$ events at the LHC running at $\sqrt{s} = 7$ and 14 TeV will be 37700 and $10^5$ respectively.

We can see from Fig. [1] that the shapes of the $^3S_1^{[8]}$ and the $^1P_1^{[8]}$ contributions to the $p_T$-distribution is different and may allow the non-perturbative matrix elements $C_2$ and $C_3$ to be individually determined. We have noted earlier that the $J/\psi$ cross-section does not provide such a separation. But it is also pertinent to note that the integrated cross-sections are also very sensitive to the the values of $C_2$ and $C_3$ and so the measurement of the integrated cross-section alone may provide this discriminatory ability.

Figure 2: Variation of total cross section with respect to chosen minimum $p_T$-cut for $\eta_c$ production (after folding in with Br($\eta_c \to \gamma\gamma$) = $3.0 \times 10^{-4}$) in pp collisions at $\sqrt{s} = 7$ TeV and 14 TeV with $-2 \leq y \leq 2$.

In Fig. 2 we have displayed the effect of increasing the $p_T$-cut on the magnitude of the integrated cross-section. As explained earlier, the cut on $p_T$ of the $\eta_c$ will be determined by the minimum $p_T$ threshold that the experiments use to trigger on the photons, for which the usual choice is 10 GeV. In case the experiments use a larger cut on the $p_T$ of the photons in order to improve the quality of their signal, the $p_T$ cut on the $\eta_c$ will be correspondingly higher. We see, from Fig. 2, that even when the cut on the minimum $p_T$ is as large as 50 GeV, the cross-section is substantial.

We also have analyzed the effect on the cross-section of the variation of the QCD scale, the parton densities and the non-perturbative matrix elements. Table 1 shows the variation in the minimum and maximum number of $\eta_c$ events with scale $Q$, expected at the LHC running at $\sqrt{s} = 7$ and 14 TeV. We find that the cross section decreases by about 25-30% changing the scale from $Q = M_T$ to $Q = 2M_T$ and it increases by 40-50% for the scale choice $Q = M_T/2$ instead of $Q = M_T$. We have checked that the cross section decreases by about 10-20% if we use MRST LO densities [24] instead of CTEQ5L LO [23] densities. Since the heavy-quark symmetry is an approximate symmetry we can expect about 30% variation in the values of the non-perturbative matrix elements we have used.
Table 1: Minimum and maximum number of $\eta_c$ events expected at the LHC for an integrated luminosity of 100 pb$^{-1}$.

| $\sqrt{s}$ | $Q = M_T/2$ | $Q = M_T$ | $Q = 2 M_T$ |
|------------|-------------|-----------|-------------|
| 7 TeV      | $5.6 \times 10^4 - 1.5 \times 10^6$ | $3.8 \times 10^4 - 10^6$ | $2.6 \times 10^4 - 7.3 \times 10^5$ |
| 14 TeV     | $1.5 \times 10^5 - 3.9 \times 10^6$ | $10^5 - 2.8 \times 10^6$ | $7.6 \times 10^4 - 2.1 \times 10^6$ |

All through we have considered only the direct production of $\eta_c$ at the LHC. However, an additional contribution to $\eta_c$ signal comes from the decays of $J/\psi$. We have estimated that this additional contribution to the signal coming from $J/\psi$ decays can change our predictions by about 1% as $\text{Br}(J/\psi \to \eta_c \gamma) \sim O(10^{-1})$ and the $J/\psi$ production cross section is expected to be of same order of $\eta_c$ production cross section.

We would like to remark that such an analysis may also be carried out for the bottomonium resonance $\eta_b$. The corresponding non-perturbative parameters in that case, however, are very poorly determined and suffer from large errors.

In conclusion, the heavy-quark symmetry of NRQCD allows us to make predictions for $\eta_c$ production at the LHC. Measurements of the integrated cross-section and the $p_T$ distribution of $\eta_c$ at the LHC will provide a very good test of NRQCD. We show that NRQCD predicts a large cross-section for $\eta_c$ at the LHC even at $\sqrt{s} = 7$ TeV and so this prediction is easily testable.

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