Poisson Shifted Gompertz Distribution: Properties and Applications

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Abstract: A novel distribution using Poisson-Generating family of distribution with parent distribution as shifted Gompertz distribution called Poisson shifted Gompertz distribution with relevant properties has been introduced. The estimation of unknown parameters is carried out via established methods including Maximum likelihood estimation (MLE). R software is applied for computational purposes. The application of the proposed model has been illustrated considering a real set of data and investigated the goodness-of-fit attained by the Poisson shifted Gompertz model through different graphical methods and test statistics where better fit was observed for the set of real data.

Keywords: Estimation method, LSE, MLE, Poisson-Generating family, Shifted Gompertz distribution

1. INTRODUCTION

In the statistical literature it has been noticed that the many life-time distributions have been generated but the real data sets related to engineering, life sciences, biology, hydrology do not present a better fit in these models So, the generation of new modified models appears to be necessary to deal with the problems in these fields. For achieving a better fit for the data we encounter in survival analysis different distributions are created making changes to the baseline distribution.\ The extended family Poisson-Weibull distribution, introduced by (Bereta,at al, 2011) demonstrates failure rate functions with decreasing and increasing nature, also exponential-Poisson distribution, presented by (Kus, 2007) with zero truncated Poisson distribution and exponential distribution compounded together. Exponential Poisson distribution’s CDF is, 

\[ G(t;\alpha,\theta) = \frac{e^{-\alpha t} - \exp\left(-\theta \left(1 - e^{-\alpha t}\right)\right)}{1 - e^{-\alpha t}} \]

\[ ; t > 0, (\alpha, \theta) > 0 \]

Similarly Louzada-Neto et al., (2011) introduced Poisson-exponential having two parameters via Bayesian approach. Alkarni and Oraby (2012) have presented Poisson family class obtained via a lifetime distribution and truncated Poisson distribution compounded together

The Poisson family’s CDF is as follows,

\[ W(y;\beta,\omega) = \frac{1 - \exp\left(-\beta \left[1 - G(y;\omega)\right]\right)}{1 - e^{-\beta}} ; \beta > 0 \]  (1.1)

And its corresponding PDF can be expressed as

\[ w(y;\beta,\omega) = \frac{\beta g(y;\omega) \exp\left(-\beta \left[1 - G(y;\omega)\right]\right)}{1 - e^{-\beta}} ; \beta > 0 \]  (1.2)

Where \(\omega\) the parameter is space and \(g(t;\omega)\) and \(G(t;\omega)\) are the PDF and CDF.

Employing same approach the Poisson Weibull power series class of distributions was given by (Morais & Barreto-Souza, 2011). Exponentiated Weibull–Poisson model with four parameters with increasing, decreasing, bathtub-shaped, and unimodal failure rate has been presented by (Mahmoudi & Sepahdari, 2013) generated compounding exponentiated Weibull and Poisson distributions. Weibull–Poisson distribution is introduced by (Lu & Shi, 2012). Further Kaviyaras and Fawaz (2017) made an extensive study on Weibull–Poisson distribution through a reliability sampling plan. Kyurkchiev et al. (2018) used the exponentiated exponential-Poisson as the software reliability model. Joshi & Kumar (2020) presented Poisson exponential power distribution and used different estimation methods to estimate the model parameter. Chaudhary & Kumar (2020) have introduced a new distribution using Poisson-G family called Poisson inverse NHE distribution. Chaudhary & Kumar(2020) introduced a new distribution generated by using the Poisson-G-family with parent distribution as NHE distribution named Poisson NHE distribution. Chaudhary & Kumar(2020) used Markov chain Monte Carlo (MCMC) method is used to estimate the parameters of the Gompertz extension distribution based on a complete sampleJoshi & Kumar (2020) have introduced a new model using Gompertz distribution called
Lindley Gompertz distribution which is more flexible than Gompertz distribution. In this paper we have taken base distribution of shifted Gompertz (Bemmaor, 1992) the PDF and CDF of shifted Gompertz model can be expressed as
\[ g(x) = \beta \exp(-\beta x + \alpha e^{-\beta x}) \left\{ 1 + \alpha (1-e^{-\beta x}) \right\} \]
\[ ; x \geq 0, (\alpha \beta) > 0 \]  
\[ \text{(1.3)} \]
\[ G(x) = (1-e^{-\beta x}) \exp\left\{ -\alpha e^{-\beta x} \right\} ; x \geq 0, (\alpha, \beta) > 0 \]  
\[ \text{(1.4)} \]

This paper aims to present a model which can provide better fit to the data we encounter in lifetime distribution. In Section 2 Poisson inverted Lomax distribution with its statistical and mathematical properties has been presented. In section 3 we discuss the estimation models. In Section 4 using a real dataset, parameter’s estimated values and their corresponding asymptotic confidence intervals and Fisher information matrix is given and the different test criteria to assess the potentiality of the proposed model is discussed. In section 5 we give conclusion

II. THE POISSON SHIFTED GOMPERTZ DISTRIBUTION

A new distribution Poisson shifted Gompertz (PSG) distribution is presented using the Poisson-G family defined by (Alkarni and Oraby, 2012). The PDF and CDF of PSG distribution is obtained by taking (1.3) and (1.4) as baseline distribution and is given by
\[ f(x) = \frac{\beta e^{-\beta x}}{(1-e^{-\gamma})} \exp(-\beta x - \alpha e^{-\beta x})[1 + \alpha (1-e^{-\beta x})] \exp[-\lambda (1-(1-e^{-\gamma})) \exp(-\alpha e^{-\beta x})] \]
\[ (2.1) \]
\[ F(x) = \frac{1}{(1-e^{-\gamma})} \left\{ 1-\exp[-\lambda (1-(1-e^{-\gamma})) \exp(-\alpha e^{-\beta x})] \right\} \]
\[ ; x \geq 0, (\alpha, \beta, \lambda) > 0 \]  
\[ \text{(2.2)} \]
PSG distribution’s reliability function is
\[ S(x) = 1 - F(x) = \frac{\exp[-\lambda (1-(1-e^{-\beta x})) \exp(-\alpha e^{-\beta x})]}{(1-e^{-\gamma})} \exp(-\lambda) \]
\[ ; x \geq 0, (\alpha, \beta, \lambda) > 0 \]  
\[ \text{(2.3)} \]
Hazard rate function(HRF) of PSG distribution
\[ h(x) = \frac{f(x)}{S(x)} = \frac{\beta e^{-\beta x}}{(1-e^{-\gamma})} \exp(-\beta x - \alpha e^{-\beta x})[1 + \alpha (1-e^{-\beta x})] \exp[-\lambda (1-(1-e^{-\gamma})) \exp(-\alpha e^{-\beta x})] \exp(-\lambda (1-(1-e^{-\gamma})) \exp(-\alpha e^{-\beta x})) \exp(-\lambda) \]
\[ (2.4) \]

Figure 1 illustrates the PSG distribution’s curves of the PDF and HRF where PDF’s curve displayed variety of shapes. The PSG model’s hazard rate function (HRF) is observed to be flexible as it exhibited different shapes like reverse bathtub, increasing--decreasing, increasing for different values of parameters.

![Figure 1](image)

### Skewness and Kurtosis:

The coefficient of skewness of PSG distribution can be obtained as
\[ S_k(B) = \frac{Q(3/4) + Q(1/4) - 2Q(1/2)}{Q(3/4) - Q(1/4)} \quad \text{and} \]

The coefficient of kurtosis given by (Moors, 1988) is

\[ K_x(M) = \frac{Q(0.875) - Q(0.625) + Q(0.375) - Q(0.125)}{Q(0.75) - Q(0.25)} \]

### III. METHODS OF PARAMETER ESTIMATION

Here parameter estimation of the unknown parameter is done with brief explanation of the estimation method.

#### 3.1. Maximum Likelihood Estimation (MLE) method

Consider a random sample denoted by \( \mathbf{x} = (x_1, \ldots, x_n) \) an n-sample size from PSG(\( \alpha, \beta, \lambda \)) then the log likelihood function is,

\[
l = n \ln \alpha + n \ln \beta - \beta \sum_{i=1}^{n} x_i - \alpha \sum_{i=1}^{n} e^{-\beta x_i} + \sum_{i=1}^{n} \ln(1 + \alpha (1 - e^{-\beta x_i})) - \sum_{i=1}^{n} \lambda [1 - (1 - e^{-\beta x_i})] \exp(-\alpha e^{-\beta x_i})
\]

(3.1.1)

By differentiating (3.1.1) w.r.t. \( \beta, \lambda, \alpha \) we obtain

\[
\frac{\partial l}{\partial \alpha} = n + \sum_{i=1}^{n} \log(1 + \beta / x_i) - \lambda \sum_{i=1}^{n} (1 + \beta / x_i)^{-\alpha} \ln(1 + \beta / x_i)
\]

\[
\frac{\partial l}{\partial \beta} = n + \sum_{i=1}^{n} \frac{1}{x_i (1 + \beta x_i)} + \lambda \sum_{i=1}^{n} \frac{1}{x_i (1 + \beta x_i)^{a+1}}
\]

\[
\frac{\partial l}{\partial \lambda} = n e^{-\lambda} - \sum_{i=1}^{n} (1 + \beta / x_i)
\]

Solving these non-linear functions for \( (\alpha, \beta, \lambda) \) by equating to zero we will obtain the ML estimators of the PSG distribution. The computer softwares like R, Mathematica, Matlab etc can be used to solve them manually. Let \( \varphi = (\alpha, \beta, \lambda) \) parameter vector and consider MLE of \( \varphi \)

as \( \hat{\varphi} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda}) \), then normal distribution is followed by \( \hat{\varphi} - \varphi \rightarrow N_3 \left[ 0, (A(\varphi))^{-1} \right] \), where \( A(\varphi) \) is the information matrix of Fisher obtained by,

\[
A = \begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix}
\]

where

\[
A_{11} = \frac{\partial^2 l}{\partial \alpha^2}, \quad A_{12} = \frac{\partial^2 l}{\partial \alpha \partial \beta}, \quad A_{13} = \frac{\partial^2 l}{\partial \alpha \partial \lambda}
\]

\[
A_{21} = \frac{\partial^2 l}{\partial \beta \partial \alpha}, \quad A_{22} = \frac{\partial^2 l}{\partial \beta^2}, \quad A_{23} = \frac{\partial^2 l}{\partial \beta \partial \lambda}
\]

\[
A_{31} = \frac{\partial^2 l}{\partial \lambda \partial \alpha}, \quad A_{32} = \frac{\partial^2 l}{\partial \beta \partial \lambda}, \quad A_{33} = \frac{\partial^2 l}{\partial \lambda^2}
\]

MLE having an asymptotic variance \( (A(\varphi))^{-1} \) is practically useless with \( \varphi \) unknown so putting estimated parameters value we approximate asymptotic variance. Via the algorithm of Newton-Raphson, maximization of likelihood gives the observed information matrix and the var-cov matrix is,

\[
(A(\varphi))^{-1} = \begin{pmatrix}
V(\hat{\alpha}) & \text{cov}(\hat{\alpha}, \hat{\beta}) & \text{cov}(\hat{\alpha}, \hat{\lambda}) \\
\text{cov}(\hat{\alpha}, \hat{\beta}) & V(\hat{\beta}) & \text{cov}(\hat{\beta}, \hat{\lambda}) \\
\text{cov}(\hat{\alpha}, \hat{\lambda}) & \text{cov}(\hat{\beta}, \hat{\lambda}) & V(\hat{\lambda})
\end{pmatrix}
\]

Thus for \( \alpha, \beta, \lambda \), using MLEs' asymptotic normality, approximate 100(1-\( \alpha \))% confidence intervals is given as \( \hat{\alpha} \pm Z_{\alpha/2} SE(\hat{\alpha}) \), \( \hat{\beta} \pm Z_{\alpha/2} SE(\hat{\beta}) \) and, \( \hat{\lambda} \pm Z_{\alpha/2} SE(\hat{\lambda}) \)

#### 3.2. LSE method

The another estimation method we have used is least-square estimation to estimate PSG distribution’s \( \alpha, \beta, \lambda \) which is calculated with minimization of

\[
B(X; \alpha, \beta, \lambda) = \sum_{i=1}^{n} \left[ F(X_i) - \frac{i}{n+1} \right]^2
\]

w.r.t \( \lambda, \alpha \) and \( \beta \).

From a distribution function \( F(.) \), suppose \( F(X_i) \) represents ordered random variables \( (X_{(1)} < X_{(2)} < \ldots < X_{(n)}) \)’s CDF and random sample is denoted as \( X_1, X_2, \ldots, X_n \) with “n” size. The LSEs of the unknown parameters \( \hat{\alpha}, \hat{\beta}, \text{and} \hat{\lambda} \) is acquired with minimization of

\[
B(X; \alpha, \beta, \lambda) = \sum_{i=1}^{n} \left[ \frac{1}{1 - \exp(-\alpha e^{-\beta x_i})} \right]^{1 - \left[ 1 - (1 - \exp(-\alpha e^{-\beta x_i})) \right]} - \frac{i}{n+1}
\]

w.r.t \( \beta, \lambda \) and \( \alpha \).

Differentiation of (3.2.2) w.r.t the unknown parameters \( (\beta, \lambda \text{ and } \alpha) \) we get

\[
\frac{\partial B}{\partial \alpha} = -2 \lambda \sum_{i=1}^{n} \left[ \frac{1}{1 - (1 - \exp(-\alpha e^{-\beta x_i}))} - \frac{i}{n+1} \right] Z(x_i) \exp(-\alpha e^{-\beta x_i}) - \beta x_i) (1 - (1 - \exp(-\beta x_i))
\]

\[
\frac{\partial B}{\partial \beta} = 2 \alpha \lambda \sum_{i=1}^{n} \left[ \frac{1}{1 - (1 - \exp(-\alpha e^{-\beta x_i}))} - \frac{i}{n+1} \right] Z(x_i) x_i \exp(-\alpha e^{-\beta x_i}) - \beta x_i) (1 - (1 - \exp(-\beta x_i)) + 1]
\]
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\[ \frac{\partial B}{\partial \lambda} = 2 \sum_{i=1}^{n} \left[ \frac{1}{(1-e^{-\lambda})} \left\{ 1 - Z(x_i) \right\} - \frac{i}{n+1} \right] \]

\[ = \frac{1}{(1-e^{-\lambda})} \left\{ Z(x_i)[1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i})] + \frac{e^{-\lambda}}{(1-e^{-\lambda})^2}(1-Z(x_i)) \right\} \]

Where

\[ Z(x_i) = \exp[-\lambda(1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i}))] \]

Likewise weighted LSEs is given with minimization of

\[ B(X; \alpha, \beta, \lambda) = \sum_{i=1}^{n} w_i \left[ F(X(i)) - \frac{i}{n+1} \right] \]

w.r.t \( \beta, \lambda \) and \( \alpha \).

Here weights is given as

\[ w_i = \frac{1}{\text{Var}(X(i))} \left( \frac{(n+1)^2}{i(n-i+1)} \right) \]

Minimization of the following equation

\[ b(X; \alpha, \beta, \lambda) = \sum_{i=1}^{n} \left[ \frac{1}{(1-e^{-\lambda})} \left\{ 1 - \exp[-\lambda(1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i}))] \right\} - \frac{i}{n+1} \right]^2 \]

w.r.t. \( \alpha, \beta \) and \( \lambda \) gives the unknown parameter’s weighted least square estimators

3.3. CVME estimation method

The Cramer–Von-Mises estimators of unknown parameter can be attained with minimization of

\[ C(X; \alpha, \beta, \lambda) = \frac{1}{12n} \sum_{i=1}^{n} \left[ F(X(i)) - \frac{2i-1}{2n} \right]^2 \]

Differentiation of (3.3.1) w.r.t \( \lambda, \beta \) and \( \alpha \), following can be obtained

\[ \frac{\partial C}{\partial \alpha} = 2 \sum_{i=1}^{n} \left( 1 - Z(x_i) \right) \frac{e^{-\lambda}}{(1-e^{-\lambda})} \left( \frac{1}{(1-e^{-\lambda})} \right) \left( \frac{1}{(1-e^{-\lambda})^2} \right) \]

\[ \frac{\partial C}{\partial \beta} = 2 \sum_{i=1}^{n} \left( 1 - Z(x_i) \right) \frac{e^{-\lambda}}{(1-e^{-\lambda})} \left( \frac{1}{(1-e^{-\lambda})} \right) \left( \frac{1}{(1-e^{-\lambda})^2} \right) \]

Where \( Z(x_i) = \exp[-\lambda(1-(1-e^{-\beta x_i})\exp(-\alpha e^{-\beta x_i}))] \)

After solving non-linear equations

\[ \frac{\partial C}{\partial \alpha} = 0, \frac{\partial C}{\partial \beta} = 0 \text{ and } \frac{\partial C}{\partial \lambda} = 0 \]

CVM estimators can be obtained

IV. APPLICATION TO A REAL DATASET

Here, we illustrated the PSG distribution’s applicability and suitability using a real set of data (Bader & Priest, 1982). Following data was composed of single carbon fibers’ tensile strength (GPA) where the fibers’ gauge lengths is 10mm and size of sample is 63.

2.614, 2.616, 2.624, 2.659, 3.030, 3.139, 3.145, 3.220, 3.223, 3.235, 2.996, 3.243, 3.264, 3.272, 3.294, 3.332, 2.675, 2.937, 2.937, 1.901, 2.132, 2.203, 2.228, 4.225, 4.395, 5.0202738, 2.740, 2.856, 2.575, 2.917, 2.928, 4.024, 4.230, 2.257, 2.361, 2.977, 2.397, 2.396, 2.454, 2.445, 2.518, 2.474, 2.618, 2.522, 3.346, 2.525, 3.408, 3.377, 2.532, 3.493, 3.435, 3.501, 3.554, 3.537, 3.125, 3.628, 3.562, 3.871, 3.852, 3.886, 3.971, 4.027.

By utilizing R software (R Core Team, 2020) of the optim() function, we have calculated the MLEs of PSG distribution by maximizing the likelihood function (3.1.1) (Mailund, 2017) where Log-Likelihood’s values was obtained as \( l = -56.5282 \). For \( \alpha, \beta, \) and \( \lambda \) MLE’s with their standard errors (SE) has been illustrated in Table 1.

| Parameter | MLE | SE |
|-----------|-----|----|
| alpha     | 13.5877 | 7.2056 |
| beta      | 2.0139  | 0.1749  |
| theta     | 18.8875 | 4.4254  |

Log-likelihood function’s plot for \( \alpha, \beta \) and \( \lambda \) has been illustrated in Figure 2 and found that the ML estimates can be calculated uniquely.

Figure 2. For the parameters \( \alpha, \beta \) and \( \lambda \) plots of log-likelihood function
We have presented the graph of K-S plot and Q-Q plot in Figure 3 and where PSG distribution is observed to fit the data very well.

Figure 3. The plots of Q-Q (upper panel) and K-S (lower panel) of the PSG distribution.

With the help of MLE, LSE and CVE method estimated parameters values and their negative log-likelihood, and AIC criterion in Table 2.

Table 2  
| Method of Estimation | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\lambda}$ | -LL | AIC |
|----------------------|--------------|-------------|-------------|------|------|
| MLE                  | 13.5877      | 2.0139      | 18.8875     | 56.5282 | 119.0565 |
| LSE                  | 15.9309      | 1.8403      | 10.2438     | 57.3261 | 120.6522 |
| CVE                  | 16.7643      | 1.8753      | 10.7990     | 57.0725 | 120.1449 |

The KS, W and $A^2$ statistic with their corresponding p-value of MLE, LSE and CVE estimates we have presented in Table 3.

Table 3  
| Method of Estimation | A$^2$(p-value) | W(p-value) | KS(p-value) |
|----------------------|---------------|------------|-------------|
| MLE                  | 0.3632(0.8839)| 0.0704(0.7509) | 0.0883(0.7102) |
| LSE                  | 0.3878(0.8603)| 0.0522(0.8647) | 0.0645(0.9560) |
| CVE                  | 0.3521(0.8942)| 0.0512(0.8710) | 0.0674(0.9370) |

Here we have presented Poisson shifted Gompertz model’s applicability using a real dataset used by earlier researchers. To compare the potentiality of the proposed distribution, following distribution models are taken.

I. Exponentiated Exponential Poisson (EEP):  
The probability density function of EEP (Ristić & Nadarajah, 2014) is  
\[
f(x) = \frac{\alpha \beta \lambda}{1 - e^{-\lambda x}} e^{-\beta x - \frac{\lambda x}{1-e^{-\lambda x}}} x^{\alpha - 1} \exp \left\{-\lambda \left(1 - e^{-\beta x}\right)^{\alpha} \right\}, \quad x > 0, \alpha > 0, \lambda > 0
\]

II. Poisson–exponential distribution (PE)  
The PE distribution’s PDF was defined by (Louzada-Neto et al., 2011) also it was used by (Rodrigues et al., 2018) is  
\[
f(x) = \frac{\beta \lambda}{1 - e^{-\lambda x}} e^{-\beta x} \exp(-\lambda e^{-\beta x}), \quad \beta > 0, \lambda > 0, x > 0
\]

III. Exponential power (EP) distribution:  
EP distribution’s PDF (Smith & Bain, 1975) is  
\[
f(x) = \frac{1}{\theta} \left(1 - e^{-\frac{x}{\theta}}\right)^{\alpha - 1} \exp \left\{-\frac{x}{\theta}\left(1 - e^{-\frac{x}{\theta}}\right)^{\alpha} \right\}
\]

\[
; (\alpha, \lambda) > 0, \quad x \geq 0.0
\]

IV. LINDLEY-EXPONENTIAL (LE) DISTRIBUTION:  
LE (Bhatti, 2015)’s PDF is  
\[
f_{LE}(x) = \lambda \left(\frac{\theta^2}{1 + \theta}\right) e^{-\alpha x} \left(1 - e^{-\alpha x}\right)^{\theta - 1} \left[1 - \ln \left(1 - e^{-\alpha x}\right)\right]
\]

\[
; \lambda, \theta > 0, x > 0
\]

V. Weibull Extension Model:  
The PDF of WE model (Tang et al., 2003) with three parameters $\left(\alpha, \beta, \lambda\right)$ is  
\[
f_{WE}(x; \alpha, \beta, \lambda) = \frac{\lambda \beta}{\alpha} x^{\alpha - 1} \exp \left\{\frac{-x}{\alpha}\right\} \exp \left\{-\lambda \alpha \exp \left\{\frac{-x}{\alpha}\right\} - 1\right\}
\]

\[
; x > 0, \quad \alpha > 0, \beta > 0 \text{ and } \lambda > 0
\]
We have illustrated the Bayesian information criterion (BIC), Akaike information criterion (AIC), Hannan-Quinn information criterion (HQIC) and Corrected Akaike information criterion (CAIC) for the evaluation of the applicability of the PSG distribution in Table 4.

| Model | AIC   | -LL    | CAIC   | BIC   | HQIC  |
|-------|-------|--------|--------|-------|-------|
| PSG   | 119.0565 | 56.3282 | 119.4633 | 125.4859 | 121.5852 |
| EEP   | 120.1261 | 57.0630 | 120.5328 | 126.5555 | 122.6548 |
| PE    | 118.4105 | 57.2052 | 118.6105 | 122.6967 | 120.0963 |
| LE    | 119.9929 | 57.9964 | 120.1929 | 124.2792 | 121.6787 |
| WE    | 129.9731 | 61.9865 | 130.3798 | 136.4025 | 132.5018 |
| EP    | 142.6598 | 69.3299 | 142.8533 | 146.9461 | 144.3456 |

Table 4

AIC, Log-likelihood (LL), CAIC BIC, and HQIC

We have presented the plot of goodness-of-fit of PSG distribution and some selected distributions are in Figure 4.

![Figure 4. Empirical distribution function with estimated distribution function (upper panel) and The PDF along with histogram for distribution models taken (lower panel) of PSG distribution.](image)

To compare the PSG distribution’s goodness of fit among different models, different values of goodness of fit statistics are presented in Table 5 where the test statistics for model purposed was observed to have low value also the p-value was higher. Thus conclusion that PSG distribution shows better fit with more reliability and consistency in results among others taken for comparison.

| Model | KS(p-value) | AD(p-value) | CVM(p-value) |
|-------|-------------|-------------|--------------|
| PSG   | 0.0883(0.7102) | 0.3632(0.8839) |              |
| EEP   | 0.0907(0.6784) | 0.4002(0.8480) |              |
| PE    | 0.0635(0.9613) | 0.4044(0.8438) |              |
| LE    | 0.0919(0.6613) | 0.4613(0.7859) |              |
| WE    | 0.0879(0.7148) | 0.9381(0.3911) |              |
| EP    | 0.1443(0.1452) | 2.3516(0.0595) |              |

Table 5
The goodness-of-fit statistics and their corresponding p-value

V. CONCLUSION

A new distribution named Poisson shifted Gompertz distribution is introduced. A comprehensive study of some distributional characteristics of the new distribution is presented build a clearer picture for the distribution purposed. Parameter estimation is carried out with MLE along with CVME and LSE. The curves of the PDF of the proposed distribution have shown that its shape is increasing-decreasing and right skewed and flexible for modeling real-life data. Also, the graph of the hazard function is monotonically increasing or constant or reverse j-shaped according to model parameters’ value. The proposed distribution’s applicability and suitability has been evaluated by considering a real set of data and the results exposed that the proposed distribution is much flexible as compared to some other fitted distributions.

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