Nonequilibrium description of dilepton production in heavy ion reactions

B Schenke and C Greiner
Institut für Theoretische Physik, Johann Wolfgang Goethe – Universität Frankfurt, Max–von–Laue–Straße 1, D–60438 Frankfurt am Main, Germany
E-mail: schenke@th.physik.uni-frankfurt.de

Abstract. Time dependent medium modifications of low mass vector mesons are investigated within a nonequilibrium quantum field theoretical description on the basis of the Kadanoff-Baym equations. Time scales for the adaption of the spectral properties to changing self energies are given and, under use of a model for the fireball evolution, nonequilibrium dilepton yields from the decay of $\rho$- and $\omega$-mesons are calculated. Comparison of the results with calculations that assume instantaneous adaption to the changing medium show that the consideration of memory effects is important.

1. Introduction and Motivation
High energy heavy ion reactions allow for studying strongly interacting matter under extreme conditions as high densities and temperatures. One of the main objectives is the creation and identification of new states of matter, most notably the quark-gluon plasma (QGP). Photons and dileptons do not undergo strong interactions and thus may carry undistorted information especially on the early hot and dense phases of the fireball because the production rates increase rapidly with temperature. Photon spectra are a suitable observable for the temperature whereas dileptons have encoded additional dynamic information via their invariant mass. Particularly in the low mass region, dileptons couple directly to the light vector mesons and reflect their mass distribution. They are thus considered the prime observable in studying mass (de-)generation related to restoration of the spontaneously broken chiral symmetry. In order to draw conclusions from the data (eg. CERES [1], NA60 [2] or HADES) a precise theoretical description of the medium effects has to be established. Therefore it is necessary to consider the fact that especially during the early stages of a heavy ion reaction the system is out of equilibrium. We account for that by using a nonequilibrium quantum field theoretical description, based on the Kadanoff-Baym equations [3]. We simulate modifications of the light vector mesons in heavy ion collisions by introducing a certain time dependence of the self energies, which then allows for analyzing the dynamics of the mesons’ spectral properties as well as that of the resulting dilepton rates. To give a quantitative description of the resulting retardation we introduce time scales which characterize the memory of the spectral function and can be compared to typical time scales in heavy ion reactions, which allows for judging whether changes are adiabatic for the spectral function or whether memory effects are important. Dilepton yields are calculated for constant temperature and volume and furthermore by convolving the dynamic rates with a fireball model employing a Bjorken like expansion. Comparison to the quantities computed in the Markovian
limit, where all meson properties adjust to the medium instantaneously, will reveal directly in which cases and to what extend the assumption of adiabaticity is reasonable.

2. The nonequilibrium production rate
We utilize the Schwinger-Keldysh formalism in order to derive the dynamic non-equilibrium rate of produced electron-positron pairs, coming from the decay of light vector mesons via virtual photons in a spatially homogeneous system (details in [4]).

Projecting on the particle number in the electron propagator \( G^< \) and using the equations of motion for \( G^< \), the Kadanoff-Baym equations, we find the production rate of electrons for a homogeneous, yet time dependent system, to read

\[
\partial_\tau N(p, \tau) = 2 \text{Im} \left[ \text{Tr} \left\{ \frac{\not{p} + m}{2E_p} \int_{t_0}^{\tau} dt \left( \Sigma^<(p, \tau, \bar{t}) e^{iE_p(\tau - \bar{t})} \right) \right\} \right],
\]

with the electron self energy \( \Sigma \) and \( p_0 = E_p \). We used the free electron propagator because the electrons are not expected to interact with the medium after being produced due to their long mean free path. The medium effects enter via the dressing of the virtual photon propagator in the electron self energy (see Fig. 1).

\[
\begin{array}{c}
\Pi \\
1 \\
2 \\
G^0
\end{array}
\]

Figure 1. The electron self energy \( \Sigma(1, 2) \). \( \Pi \) is the self energy of the virtual photon.

We have

\[
i \Sigma^<(p, t_1, t_2) = -e^2 \gamma_\mu \left( \int \frac{d^3k}{(2\pi)^3} D^<_{\gamma \mu \nu}(k, t_1, t_2) G^0_\nu(p - k, t_1, t_2) \right) \gamma_\nu,
\]

with the virtual photon propagator \( D_\gamma \) and the momentum of the virtual photon \( k \). On inserting this self energy and defining \( p^+ = k - p \) as the four-momentum of the positron and \( p^- = p \) as that of the electron, Eq. (1) becomes

\[
E_+ E_- \frac{dR}{d^3p^+ d^3p^-}(\tau) = \frac{2e^2}{(2\pi)^6} \mathcal{L}_{\mu\nu} \text{Re} \left[ \int_{t_0}^{\tau} d\bar{t} \int D^<_{\gamma \mu \nu}(k, \tau, \bar{t}) e^{i(E_+ + E_-)(\tau - \bar{t})} \right],
\]

with \( E_+ = E_p \), \( E_- = E_{k-p} \) and \( \mathcal{L}_{\mu\nu} = p_+^\mu p_-^\nu + p_+^\nu p_-^\mu - g_{\mu\nu}(p^+ p^- + m^2) \). The dynamic information is inherent in the memory integral on the right that runs over all virtual photon occupation numbers \( D^<_{\gamma} \) from the initial time to the present. This way the full nonequilibrium electron production rate at the present time \( \tau \) is determined. Applying the equilibrium properties of \( D^<_{\gamma} \), it can be shown that Eq. (3) is the generalization of the well known thermal production rate for lepton pairs in the stationary case [5, 6]:

\[
E_+ E_- \frac{dR}{d^3p^+ d^3p^-}(\tau) = -\frac{2e^2}{(2\pi)^6} \mathcal{L}_{\mu\nu} \frac{1}{M^4} \frac{1}{e^{\beta E} - 1} \text{Im} \Pi^{\text{eq}}_{\gamma \mu \nu}(k, \tau)
\]
Projection on the virtual photon momentum $k$ and only regarding $\rho$-mesons resting with respect to the medium, i.e., the momentum mode $k = 0$ we find

$$\frac{dN}{d^4\tau d^4k} (\tau, E, k = 0) = \frac{2}{3} \frac{e^2}{(2\pi)^3} \Re \left[ \int_0^\tau d\tau D^\rho_T (k = 0, \tau, \bar{t}) e^{iE(\tau-\bar{t})} \right],$$  \hspace{1cm} (5)

with the transverse photon propagator $D^\rho_T$. We introduce the dynamic medium dependence by dressing this propagator with the medium dependent $\rho$- or $\omega$-meson. This dressing enters with the self energy $\Pi^\rho_T$ via the generalized fluctuation dissipation relation (FDR)

$$D^\rho_T = D^{\rho T}_\text{ret} \otimes \Pi^\rho_T \otimes D^{\rho T}_\text{adv}. \hspace{1cm} (6)$$

Vector meson dominance relates $\Pi^\rho_T$ to the meson propagator

$$\Pi^\rho_T = \frac{e^2}{g^2 m^4} D^\rho_T,$$  \hspace{1cm} (7)

which also follows an FDR:

$$D^\rho_T = D^{\rho T}_\text{ret} \otimes \Sigma^\rho_T \otimes D^{\rho T}_\text{adv}. \hspace{1cm} (8)$$

The retarded and advanced propagators of the vector meson in a spatially homogeneous and isotropic medium fulfill the equation of motion

$$(-\frac{\partial^2}{\partial t^2} - m^2 - k^2) D^{\rho T}_\text{ret} (k, t_1, t_2) - \int_{t_2}^{t_1} dt \tilde{\Sigma}^{\rho T}_\text{ret} (k, t_1, \tilde{t}) D^{\rho T}_\text{ret} (k, \tilde{t}, t_2) = \delta(t_1 - t_2) \hspace{1cm} (9)$$

The dynamic medium evolution is now introduced by hand via a specified time dependent retarded meson self energy $\Sigma^{\rho T}_\text{ret} (\tau, \omega)$ [4]. From that the self energy $\Sigma^\rho_T$, needed for solving Eq. (8), follows by introducing a background temperature of the fireball. The fireball is assumed to generate the time dependent self energy $\Sigma^{\rho T}_\text{ret}$ and, assuming a quasi thermalized system, the $\rho$-meson current-current correlator $\Sigma^\rho_T$ is given via

$$\Sigma^\rho_T (\tau, \omega, k) = 2 i m_B (T(\tau)) \text{Im} \Sigma^{\rho T}_\text{ret} (\tau, \omega, k),$$

which follows from the KMS relation [7], being valid for thermal systems. The latter is a rather strong assumption, but necessary in order to proceed.

3. Nonequilibrium dilepton production from an evolving medium

3.1. Self energies

The medium effects are introduced via a specific evolving self energy of the vector meson. A simple self energy

$$\text{Im} \Sigma^{\text{ret}} (\omega, \tau) = -\omega \Gamma (\tau),$$  \hspace{1cm} (10)

with a $k$- and $\omega$-independent width $\Gamma$, leads to a Breit-Wigner distribution for the spectral function. The time dependence is being accounted for by introduction of the parameter $\tau$. For the $k = 0$ mode, the full self energy for coupling to $J^P = \frac{3}{2}^-$-resonances is given by

$$\text{Im} \Sigma (\tau, \omega, k = 0) = -\frac{\rho(\tau)}{3} \left( \frac{f_{RN\rho}}{m_\rho} \right)^2 \frac{g_I}{\omega^2 + \frac{\Gamma_R(\tau)^2}{4} - E^2)^2 + (\Gamma_R(\tau)^2)^2 - \omega \Gamma (\tau)}.$$

with $E = \sqrt{m_R^2 + k^2} - m_N$ and $m_R$ and $m_N$ the masses of the resonance and the nucleon respectively. $\Gamma_R$ is the width of the resonance and $g_I$ the isospin factor [8].
3.2. Adaption time scales and quantum interference

Investigating which contributions in Eq. (5) come from which times in the past, shows that there are contributions from early times as well as alternating positive and negative contributions. An interpretation of this becomes difficult and it follows that only the time integrated yield is a physical quantity.

In order to quantify the times that the mesons’ spectral properties need to adjust to the evolving medium we change the self energy linearly in time (see Fig. 2).

**Figure 2.** Linear switching off of in-medium effects over a certain time $\Delta \tau$

As a possible characteristic timescale we consider the difference of the final spectral function to the dynamically calculated one at the time where the medium effects are fully turned off, described by the difference in the moment $\int_0^\infty A(\omega)^2 \omega^2 d\omega$ of the two spectral functions or the difference in the peak position and height. All methods lead to similar results [4]. We find an exponentially decreasing difference with increasing duration of the change $\Delta \tau$, from that we extract a time constant $\bar{\tau}$. For the the $\rho$-meson we find a typical timescale of about 3 fm/c. The vector mesons possess a certain memory of the past, and even if they decay outside the medium, they still carry information on the medium in that they were produced. It turns out that $\bar{\tau}$ is proportional to $c/\Gamma^2$ with $c$ lying between 2 and 3.5, depending on $m$ and $\Gamma_1$. The reason for the time scale to be (at least) $2/\Gamma$ is that the spectral function is given by the imaginary part of the retarded propagator, which, representing a probability amplitude, is proportional to $e^{-1/2 \Gamma t}$. Its square, an actual probability, is proportional to $e^{-\Gamma t}$, giving the appropriate decay rate. The time needed by the dilepton rate to follow changes is approximately equal to that of the spectral function.

The quantum mechanical nature of the regarded systems leads to oscillations and negative values in the changing spectral functions, occupation numbers and production rates as well as interferences that one does not get in semi-classical, adiabatic calculations. The rate calculated here possesses the full quantum mechanical information incorporated and contains ”memory” interferences that might cause cancellations - hence the rate has to be able to become negative while the time integrated yield always stays positive as the only observable physical quantity. An intriguing example for the occurring oscillations is shown in Figs. 3 and 4.
3.3. Yields at constant temperature

In order to get a first impression of how the memory effects affect dilepton yields, we investigate yields from systems with a constant size at constant temperature. The results for four different ($\rho$-meson-) scenarios are shown in Fig. 5 below. The largest effect one finds for the mass shift of the $\rho$-meson: The yield is increased by a factor of about 1.8 in the range from 200 to 450 MeV due to memory effects. The other cases show differences, but not as pronounced - we see a minor enhancement of the lower mass tail due to the Bose factor in all cases. In the case of the broadened $\rho$-meson, one can nicely see that the dynamically calculated yield is a broader distribution and that for the case of the coupling to the N(1520) without broadening the resonance peak is stronger, due to the system’s memory of it, and the vacuum peak is shifted further to the right due to memory of the level repulsion effect.

To conclude this section we state that for the $\rho$-meson we found exactly the modifications of the yields that one would expect qualitatively when including memory effects for the constant temperature case.

3.4. Fireball model and dilepton yields

For the lifetime of the hadronic phase of a fireball, created in a heavy ion collision, we have

$$\tau_{\text{fireball}} \simeq \tilde{\tau}$$

Hence the consideration of memory effects is important here. We employ a longitudinal Bjorken expansion for the fireball evolution combined with an accelerating radial flow. The same scenarios as in the constant temperature case are investigated and the results are shown in Fig. 6. The modifications of the yields are enhanced as compared to the constant temperature case. Also the behavior of the regarded quantities after freeze-out is found to be different from that assumed in Markovian calculations. For the mass shift of the $\rho$-meson a factor of two difference between the two calculated yields is found around the in-medium mass - a significant effect. Furthermore we show the yield from the $\omega$-meson, modified to 682 MeV mass and 40 MeV width in the medium, in Fig. 7 where we find about a factor of 2 difference in the range between 682 and 730 MeV.
Figure 5. Comparison of the dynamically calculated (solid) to the Markovian (dashed) dilepton yields from an interval of duration $\Delta \tau = 7.18$ fm/c in that the self energy was changed linearly as indicated in the corresponding figure. Initial quantities have index 1, final ones index 2. The temperature was kept constant at $T = 175$ MeV.

As we discuss in [4], for a "semi-static" expanding fireball model the shown calculations for the momentum mode $k = 0$ can be extrapolated to higher momentum modes up to a certain maximal transverse momentum $k_{\text{cut}}$.

$$\left. \frac{dN}{dMdy} \right|_{k_{\perp} < k_{\text{cut}}, y=0} = M \int_{0}^{k_{\text{cut}}} d^{2}k_{\perp} \left( \frac{dN}{dE^{3}k} \right)_{k_{\parallel}=0, k_{\perp}} \approx M \left( \frac{dN}{dE^{3}k} \right)_{k_{\parallel}=0, k_{\perp}=0} \pi k_{\text{cut}}^{2}. \quad (13)$$

This shows that also for measurable finite momentum intervals the calculated influence of finite memory on dilepton yields is to be expected and our primary message remains valid.

This section can be closed by stating that inclusion of the fireball evolution even leads to an enhancement of the differences between the differently calculated yields and it becomes clear that for the realistic situation of a heavy ion collision memory effects are to be considered when it comes to predicting medium modifications of particles.
Figure 6. Comparison of the dynamically calculated (solid) to the Markovian (dashed) final dilepton yields where the self energy was changed linearly over an interval of duration $\Delta \tau = 7.18$ fm/c as indicated in the corresponding figure (index 1 for initial, 2 for final quantities). The time dependent rate was then convoluted with the temperature and volume of the fireball. Dotted lines show yields from unmodified (vacuum) $\rho$-mesons.

4. Summary and Conclusions
In the present work we introduced a method to calculate dilepton production rates within a non-equilibrium field theory formalism, based on the real time approach of Schwinger and Keldysh. We investigated possible medium modifications of the $\rho$ and $\omega$ meson in a fireball created in a heavy ion collision. We considered mass shifts, broadening and coupling to resonances. Special attention was put to possible retardation effects concerning the off-shell evolution.

The timescale on that the spectral function adjusts to changes in the self energy was found to be proportional to the inverse vacuum width of the meson $\Gamma_2$ like $c/\Gamma_2$, with $c$ ranging from 2 to above 3. Further dependence on the in-medium width as well as on the size of the medium modification is present.

The full quantum field theoretical treatment leads to oscillations in all mentioned quantities when changes in the self energy are performed. This oscillatory behavior reveals the quantum mechanical character of the many particle system, present in the investigated heavy ion reaction. The oscillations potentially cancel when the rate is integrated over time such that the measurable dilepton yield is always positive.

Comparison of dynamically calculated yields with those calculated assuming adiabaticity
Figure 7. Comparison of the dynamically calculated (solid) to the Markovian (dashed) final dilepton yields for the $\omega$-meson. The self energy was changed linearly over an interval of duration $\Delta \tau = 7.18 \text{ fm}/c$ as indicated in the figure (index 1 for initial, 2 for final quantities). The time dependent rate was then convoluted with the temperature and volume of the fireball.

reveals differences. About a factor of 2 difference was found within the invariant mass range of 250 to 500 MeV for mass shifts predicted using Brown-Rho scaling for the $\rho$-meson in a fireball at SpS energies (158 AGeV). This is the range where CERES measures an increased dilepton yield as compared to calculations assuming the $\rho$’s vacuum shape. Similar results were found for the coupling of the $\rho$-meson to resonance-hole pairs and the modified $\omega$-meson. Our findings show that exact treatment of medium modifications, as necessary in order to draw conclusions from eg. CERES [1], recent NA60 [2] or HADES data, requires the consideration of memory effects.

References
[1] Agakichiev G et al 1995 Phys. Rev. Lett. 75 1272
[2] Shahoyan et al 2005 Study of dimuon production in indium indium collisions with the NA60 experiment Preprint hep-ex/0505049
[3] Kadanoff L P and Baym G 1962 Quantum Statistical Mechanics (New York: Benjamin)
[4] Schenke B and Greiner C 2005 Dilepton production from hot hadronic matter in nonequilibrium Phys. Rev. C (in print) Preprint hep-ph/0509026
[5] Weldon H A 1990 Reformulation of finite temperature dilepton production Phys. Rev. D 42 2384-2387
[6] Gale C and Kapusta J I 1991 Vector dominance model at finite temperature Nucl. Phys. B 357 65-89
[7] Greiner C and Leupold S 1998 Stochastic interpretation of Kadanoff-Baym equations and their relation to Langevin processes Annals Phys. 270 328-390 (Preprint hep-ph/9802312)
[8] Post M and Leupold S and Mosel U 2004 Hadronic spectral functions in nuclear matter Nucl. Phys. A 741 81-148 (Preprint nucl-th/0309085)