TWO RESULTS CONCERNING CP VIOLATION FOR J = 0 MESONS.

B. Machet

Laboratoire de Physique Théorique et Hautes Energies,
Universités Pierre et Marie Curie (Paris 6) et Denis Diderot (Paris 7);
Unité associée au CNRS URA 280.

Abstract: I show, in the framework of a SU(2)_L × U(1) gauge theory for J = 0 mesons expressed as scalar or pseudoscalar di-quark fields, that:
- the existence, at the fermionic level, of a complex mixing matrix of the Kobayashi-Maskawa type is not a sufficient condition for electroweak mass eigenstates to be different from CP eigenstates;
- unitarity constrains this phenomenon to arise from an admixture of states with different parities, and present experiments probing “indirect” CP violation are likely to be interpreted with P violation only.

PACS: 11.15.-q 11.30.Er 12.15.-y 12.60.-i 14.40.-n
1 Introduction.

The only yet observed phenomenon of \( CP \) violation \([1] \) \([2]\), called “indirect” \( CP \) violation \([3]\), is that some electroweak mass eigenstates are not \( CP \) eigenstates. The unavoidable presence, for a number of generations \( N/2 \geq 3 \), of a complex number among the entries of the quark mixing matrix \([4]\) is the preferred mechanism to trigger it in the framework of a \( SU(2)_L \times U(1) \) electroweak gauge theory for quarks \([5]\). Other possibilities need enlarging the scalar sector of the model \([6]\).

We shall deal here with mesons only, and stick to their interpretation as composite di-quark fields, first proposed by Gell-Mann \([7]\) for the case of three flavours; the success of the \( SU(3) \) classification of the corresponding eigenstates was next extended to \( N \) flavours, with a similar role played by the diagonal subgroup of the chiral \( U(N)_L \times U(N)_R \) group. The importance of the latter and of its breaking, specially as far as strong interactions are concerned, was put forward long ago \([8]\); we shall refer to the corresponding eigenstates as the “flavour” or “strong” eigenstates (strong interactions are considered to be flavour independent).

The quarks being in the fundamental representation of \( U(N) \), one is naturally led to consider mesons as \( N \times N \) matrices \([9]\); each of them is given in addition a quantum number \((+1)\) or \((-1)\) when acted upon by the parity-changing operator \( \mathcal{P} \), such that their total number \((2N^2)\) of degrees of freedom matches the one of scalar and pseudoscalar \( J = 0 \) mesons. The action on them of the \( U(N) \times U(N) \) generators, which are \( N \times N \) matrices, too, is defined inside the associative algebra that they form. Fermions can then be forgotten, though the group action as defined in \([9]\) can be easily recovered by acting with the chiral group on both fermionic components of the mesonic wave function and introducing the appropriate “left” and “right” projectors, with a \( \gamma_5 \) matrix, respectively for the generators of \( U(N)_L \) and \( U(N)_R \).

The extension of the Glashow-Salam-Weinberg model \([5]\) to \( J = 0 \) mesons that I proposed in \([9]\) is thus a \( SU(2)_L \times U(1) \) gauge theory of matrices. As the action of the gauge group can only be defined if its generators are also \( N \times N \) matrices, it is considered as a subgroup of the chiral group. Its orientation within the latter has to be compatible with the customary action of the electroweak group on fermions, and is determined by a unitary \( N/2 \times N/2 \) matrix which is nothing else than the Cabibbo-Kobayashi-Maskawa mixing matrix \( K \) \([10] \) \([4]\).

The \( SU(2)_L \) generators are \([4]\)

\[
T^3_L = \frac{1}{2} \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad T^+_L = \begin{pmatrix} 0 & K \\ 0 & 0 \end{pmatrix}, \quad T^-_L = \begin{pmatrix} 0 & 0 \\ K^\dagger & 0 \end{pmatrix}, \quad (1)
\]

and act trivially on the \( N \)-vector of quarks

\[
\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}.
\]

\( I \) is the \( N/2 \times N/2 \) identity matrix.

\(^1\)This construction of course requires an even number \( N \) of flavours.
The $U(1)$ generator satisfies the Gell-Mann-Nishijima relation (written in its “chiral” form)

$$\left(\gamma_L, \gamma_R\right) = \left(Q_L, Q_R\right) - \left(T^3_L, 0\right),$$

and the customary electric charge operator

$$Q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix},$$

yields back the usual expressions for the “left” and “right” hypercharges

$$\gamma_L = \frac{1}{6} I, \quad \gamma_R = Q_R.$$  (5)

$Q$ turns out to be the “third” generator of the custodial $SU(2)_V$ symmetry uncovered in [9].

The electroweak eigenstates can be classified into two types of quadruplets, respectively “even” and “odd” by the parity changing operator $P$. Both write

$$\Phi(D) = \left(M^\nu, M^{\mu\nu}, M^+, M^-\right)(D)$$

$$= \begin{array}{cc}
\frac{1}{\sqrt{2}} \begin{pmatrix} D & 0 \\ 0 & \bar{K}^\dagger D K \end{pmatrix}, & \frac{i}{\sqrt{2}} \begin{pmatrix} D & 0 \\ 0 & -\bar{K}^\dagger D K \end{pmatrix}, \\
& \begin{pmatrix} 0 & D K \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ \bar{K}^\dagger D & 0 \end{pmatrix}\end{array},$$

where $D$ is a real $N/2 \times N/2$ matrix. That the entries $M^+$ and $M^-$ are, up to a sign, hermitian conjugate (i.e. charge conjugate) requires that the $D$’s are restricted to symmetric or antisymmetric matrices. Because of the presence of an “$i$” for the for $M^{3,\pm}$ and not for $M^0$, the quadruplets always mix entries of different behaviour by hermitian (charge) conjugation, and are consequently not hermitian representations.

Each of them is the sum of two doublets of $SU(2)_L$, and also the sum of one singlet plus one triplet of the custodial diagonal $SU(2)_V$. The $P$-even and $P$-odd quadruplets do not transform in the same way by $SU(2)_L$ (the Latin indices $i, j, k$ run from 1 to 3); for $P$-even quadruplets, one has

$$T^i_{L} \cdot M^j_{P\text{even}} = -\frac{i}{2} \left(\epsilon_{ijk} M^k_{P\text{even}} + \delta_{ij} M^0_{P\text{even}}\right),$$

$$T^i_{L} \cdot M^0_{P\text{even}} = \frac{i}{2} M^i_{P\text{even}};$$

while $P$-odd quadruplets transform according to

$$T^i_{L} \cdot M^j_{P\text{odd}} = -\frac{i}{2} \left(\epsilon_{ijk} M^k_{P\text{odd}} - \delta_{ij} M^0_{P\text{odd}}\right),$$

$$T^i_{L} \cdot M^0_{P\text{odd}} = -\frac{i}{2} M^i_{P\text{odd}},$$

and only representations transforming alike, $P$-even or $P$-odd, can be linearly mixed. The (diagonal) charge operator acts indifferently on both types of representations by:

$$Q \cdot M^i = -i \epsilon_{ij3} M^j,$$

$$Q \cdot M^0 = 0.$$  (9)

The misalignment of “strong” and electroweak eigenstates, resulting from the one of the electroweak group with respect to the chiral group, is conspicuous from the presence of the mixing matrix in the definition (6).
By adding or subtracting eqs. (7) and (8), and defining scalar \( S \) and pseudoscalar \( P \) fields by
\[
(M_{P_{\text{even}}} + M_{P_{\text{odd}}}) = S, \tag{10}
\]
and
\[
(M_{P_{\text{even}}} - M_{P_{\text{odd}}}) = P, \tag{11}
\]
one finds two new types of stable quadruplets which include objects of different parities, but which now correspond to a given \( CP \) quantum number, depending in particular whether \( D \) is a symmetric or skew-symmetric matrix \([9]\)
\[
(M^0, \vec{M}) = (S^0, \vec{P}), \tag{12}
\]
and
\[
(M^0, \vec{M}) = (P^0, \vec{S}); \tag{13}
\]
they transform in the same way by the gauge group, according to eq. (7), and thus can be linearly mixed. As they span the whole space of \( J = 0 \) mesons too, this last property makes them specially convenient to build an electroweak gauge theory.

Taking the hermitian conjugate of any representation \( \Phi \) swaps the relative sign between \( M^0 \) and \( \vec{M} \); as a consequence, \( \Phi_{P_{\text{even}}}^\dagger \) transforms by \( SU(2)_L \) as would formally do a \( P \)-odd representation, and vice-versa; on the other hand, the quadruplets \([8]\) are also representations of of \( SU(2)_R \), the action of which is obtained by swapping eqs. \((7) \) and \((8) \); so, the hermitian conjugate of a given representation of \( SU(2)_L \) is a representation of \( SU(2)_R \) with the same law of transformation, and vice-versa. The same result holds for any (complex) linear representation \( U \) of quadruplets transforming alike by the gauge group.

The link with usually defined \( J = 0 \) “strong” mesonic eigenstates proceeds as follows: consider for example the case \( N = 4 \), for which \( K \) shrinks back to the Cabibbo mixing matrix; the pseudoscalar \( \pi^+ \) meson is represented in our notation, up to a scaling factor (see below), by the matrix
\[
\Pi^+ = \begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix}, \tag{14}
\]
since, sandwiched between two 4-vectors \( \Psi \) of quarks \([2]\), it gives
\[
\Psi \Pi^+ \Psi = \bar{u}d, \tag{15}
\]
which indeed corresponds, according to the classification by flavour \( SU(4) \), to the \((+1)\) charged pion. One identifies similarly the other strong pseudoscalar mesons, for example \( K^+ = \bar{u}s \), \( D^+ = \bar{c}d \), \( D_s^+ = \bar{c}s \). So, for example, with the scaling that has to be introduced (see \([11]\), \([12]\) \([9]\), where I show that it leads in particular to the correct leptonic decays), the pseudoscalar entry \( P^+ \) with charge \((+1)\)
\[
P^+ = i \begin{pmatrix}
c_\theta & s_\theta \\
-s_\theta & c_\theta
\end{pmatrix} \tag{16}
\]
corresponding to the matrix
\[
\mathbb{D}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\]
represents the following linear combination of pseudoscalar mesons
\[
\mathcal{P}^+(\mathbb{D}_1) = i\frac{f}{\langle H \rangle} \left( c_0 (\pi^+ + D_s^+) + s_0 (K^+ - D^+) \right),
\]
where \( f \) is the leptonic decay constant of the mesons, that we consider to be the same for all of them, and \( H \) is the Higgs boson (see the remark at the end of Appendix A).

2 Quadratic invariants.

To every representation is associated a quadratic expression invariant by the electroweak gauge group \( SU(2)_L \times U(1) \)
\[
\mathcal{I} = (M^0, \bar{M}) \otimes (M^0, \bar{M}) = M^0 \otimes M^0 + \bar{M} \otimes \bar{M};
\]
the “\( \otimes \)” product is a tensor product, not the usual multiplication of matrices and means the product of fields as functions of space-time; \( \bar{M} \otimes \bar{M} \) stands for \( \sum_{i=1,2,3} M_i \otimes M_i \).

For the relevant cases \( N = 2, 4, 6 \), there exists a set of \( \mathbb{D} \) matrices (see appendix A) such that the algebraic sum (specified below) of invariants extended over all representations defined by \( \mathbb{D} \)
\[
\mathcal{I} = \frac{1}{2} \left( \sum_{\text{symmetric } \mathbb{D}} - \sum_{\text{skew-symmetric } \mathbb{D}} \right) \left( (S^0, \bar{P}) (\mathbb{D}) \otimes (S^0, \bar{P}) (\mathbb{D}) - (P^0, \bar{S}) (\mathbb{D}) \otimes (P^0, \bar{S}) (\mathbb{D}) \right)
\]
\[
= \frac{1}{4} \left( \sum_{\text{symmetric } \mathbb{D}} - \sum_{\text{skew-symmetric } \mathbb{D}} \right) \left( \Phi_{\mathbb{P}_{\text{even}}} (\mathbb{D}) \otimes \Phi_{\mathbb{P}_{\text{odd}}} (\mathbb{D}) + \Phi_{\mathbb{P}_{\text{odd}}} (\mathbb{D}) \otimes \Phi_{\mathbb{P}_{\text{even}}} (\mathbb{D}) \right)
\]
is diagonal both in the electroweak basis and in the basis of strong eigenstates: in the latter basis, all terms are normalized alike to \((+1)\) (including the sign). Note that two “\(-\)” signs occur in eq. (20):
- the first between the \((\mathbb{P}^0, \bar{S})\) and \((\mathbb{S}^0, \bar{P})\) quadruplets, because, as seen on eq. (13), the \( \mathbb{P}^0 \) entry of the former has no “\( i \)” factor, while the \( \bar{P} \)’s of the latter do have one; as we define all pseudoscalars without an “\( i \)” (like \( \pi^+ = \bar{u}d \)), a \((\pm i)\) relative factor has to be introduced between the two types of representations, yielding a “\(-\)” sign in eq. (20);
- the second for the representations corresponding to skew-symmetric \( \mathbb{D} \) matrices, which have an opposite behaviour by charge conjugation (\( i.e. \) hermitian conjugation) as compared to the ones with symmetric \( \mathbb{D} \)’s.

The kinetic part of the \( SU(2)_L \times U(1) \) Lagrangian for \( J = 0 \) mesons is built from the same combination (20) of invariants, now used for the covariant derivatives of the fields with respect to the gauge group; it is thus diagonal in both the strong and electroweak basis, too.

Other invariants can be built like tensor products of two representations transforming alike by the gauge group: two \( \mathbb{P} \)-odd or two \( \mathbb{P} \)-even, two \((\mathbb{S}^0, \bar{P})\), two \((\mathbb{P}^0, \bar{S})\), or one \((\mathbb{S}^0, \bar{P})\) and one \((\mathbb{P}^0, \bar{S})\); for example such is
\[
\mathcal{I}_{12} = (S^0, \bar{P}) (\mathbb{D}_1) \otimes (P^0, \bar{S}) (\mathbb{D}_2) = S^0 (\mathbb{D}_1) \otimes P^0 (\mathbb{D}_2) + \bar{P} (\mathbb{D}_1) \otimes \bar{S} (\mathbb{D}_2).
\]

\(^2\)Eq. (20) specifies eq. (25) of [1], in which the “\(-\)” signs were not explicitly written.
According to the remark made in the previous section, all the above expressions are also invariant by the action of $SU(2)_R$.

They naturally enter the mass terms in the Lagrangian, and there are a priori as many $(N^2/2)$ independent mass scales as there are independent representations. Introduced in a gauge invariant way, they share with the leptonic case the same arbitrariness; the ratios of mesonic masses have here the same status as the one between the muon and the electron. Note that we have given a purely electroweak origin to the mass splittings, since, from the diagonalization property of eq. (20), equal electroweak mass terms also correspond to equal mass terms for strong eigenstates.

2.1 The basic property of the quadratic invariants.

The quadratic $SU(2)_L$ invariants are not a priori self conjugate expressions and have consequently no definite property by hermitian conjugation; in particular, the one associated with a given representation $U$ is $U \otimes U$ and not $U \otimes U^\dagger$ (we have seen in the previous section that $U$ and $U^\dagger$ do not transform alike by the gauge group).

As far as one only deals with representations of the type of eqs. (6,12,13), it has no consequence since each of their entries has a well defined behaviour by hermitian conjugation: the associated quadratic invariants are then always hermitian. But electroweak mass eigenstates are in general (complex) linear combinations of them with, consequently, no definite behaviour by hermitian (charge) conjugation.

3 Two results concerning $CP$ violation.

Let us use the invariants associated to the $N^2/4$ quadruplets (12) and $N^2/4$ quadruplets (13), which all transform by (7), to construct a $SU(2)_L \times U(1)$ gauge Lagrangian for the $2N^2$ scalar and pseudoscalar $J = 0$ mesons.

3.1 A first result.

Unitarity compels this Lagrangian to be hermitian, in particular its quadratic part.

Suppose that it has been diagonalized and let us restrict for the sake of simplicity to a subsystem of two non-degenerate electroweak mass eigenstates $U$ and $V$; they are in general complex linear combinations of quadruplets (12) and (13), and transform by $SU(2)_L$ according to (7). $\mathcal{L}$ writes, for example

$$\mathcal{L} = \frac{1}{2} (\partial_\mu U \otimes \partial^\mu U - \partial_\mu V \otimes \partial^\mu V - m^2_U U \otimes U + m^2_V V \otimes V + \cdots).$$

(22)

with $m^2_U \neq m^2_V$.

Hermiticity yields the two following equations, coming respectively from the kinetic and mass terms

$$\begin{cases} 
(U \otimes U - V \otimes V)^\dagger = U \otimes U - V \otimes V, \\
(m^2_U U \otimes U - m^2_V V \otimes V)^\dagger = m^2_U U \otimes U - m^2_V V \otimes V,
\end{cases}$$

(23)

which, if we reject complex values of the (mass)$^2$, entail

$$U = \pm U^\dagger, \quad V = \pm V^\dagger;$$

(24)

$^3$The hermitian combination (24), used to build the kinetic terms, is special in this respect too.
unitarity thus requires that the electroweak mass eigenstates be also \( C \) eigenstates.

Consequence: if electroweak mass eigenstates are observed not to be \( CP \) eigenstates, they can only be mixtures of states with different parities.

### 3.2 A second result.

Suppose that we have a complex mixing matrix \( \mathbb{K} \); the following Lagrangian for \( J = 0 \) mesons, where the sum is extended to all representations defined by eqs. (12,13,6), is nevertheless hermitian, \((D_\mu \text{ is the covariant derivative with respect to } SU(2)_L \times U(1))\)

\[
\mathcal{L} = \frac{1}{2} \sum_{\text{symmetric } D} \left( D_\mu (S^0, \bar{S})(D) \otimes D^\mu (S^0, \bar{S})(D) - \bar{D}_\mu (S^0, \bar{S})(D) \otimes (S^0, \bar{S})(D) \right) \\
- \frac{1}{2} \sum_{\text{skew-symmetric } D} \left( D_\mu (P^0, \bar{S})(D) \otimes D^\mu (P^0, \bar{S})(D) - \bar{D}_\mu (P^0, \bar{S})(D) \otimes (P^0, \bar{S})(D) \right) \\
- \frac{1}{2} \sum_{\text{symmetric } D} \left( D_\mu (S^0, \bar{S})(D) \otimes D^\mu (S^0, \bar{S})(D) - \bar{D}_\mu (S^0, \bar{S})(D) \otimes (S^0, \bar{S})(D) \right) \\
- \frac{1}{2} \sum_{\text{skew-symmetric } D} \left( D_\mu (P^0, \bar{S})(D) \otimes D^\mu (P^0, \bar{S})(D) - \bar{D}_\mu (P^0, \bar{S})(D) \otimes (P^0, \bar{S})(D) \right),
\]

(25)

and its mass eigenstates, being the \((S^0, \bar{P})\) and \((P^0, \bar{S})\) representations given by eqs. (12,13) are \( CP \) eigenstates \([\text{3}]\). It is of course straightforward to also build hermitian \( SU(2)_L \times U(1) \) invariant quartic terms.

Consequence: The existence of a complex phase in the mixing matrix for quarks is not a sufficient condition for the existence of electroweak mass eigenstates for \( J = 0 \) mesons different from \( CP \) eigenstates.

### 4 Conclusion.

Until we observe direct \( CP \) violation \([\text{3}]\), and if we stick to a \( SU(2)_L \times U(1) \) gauge theory of particles, the origin of observed features of \( CP \) violation for \( J = 0 \) mesons transforming like composite di-quark fields by the chiral group \( U(N)_L \times U(N)_R \) should be looked for into a mixture of scalar and pseudoscalar states, and be interpreted as a simple effect of parity violation at the mesonic level.
Appendix

A Diagonalizing eq. (20) in the basis of strong eigenstates: a choice of $D$ matrices.

The property is most simply verified for the “non-rotated” $SU(2)_L \times U(1)$ group and representations [3].

A.1 $N = 2$.

Trivial case: $D$ is a number.

A.2 $N = 4$.

The four $2 \times 2$ $D$ matrices (3 symmetric and 1 skew-symmetric) can be taken as

\[
D_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad D_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad D_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\]

A.3 $N = 6$.

The nine $3 \times 3$ $D$ matrices (6 symmetric and 3 skew-symmetric), can be taken as

\[
D_1 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D_2 = \frac{2}{\sqrt{3}} \begin{pmatrix} \sin \alpha & 0 & 0 \\ 0 & \sin(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \sin(\alpha \mp \frac{2\pi}{3}) \end{pmatrix}, \quad D_3 = \frac{2}{\sqrt{3}} \begin{pmatrix} \cos \alpha & 0 & 0 \\ 0 & \cos(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \cos(\alpha \mp \frac{2\pi}{3}) \end{pmatrix},
\]

\[
D_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad D_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad D_6 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D_7 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D_8 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_9 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}.
\]

(27)

where $\alpha$ is an arbitrary phase.

**Remark:** as $D_1$ is the only matrix with a non vanishing trace, $S^0(D_1)$ is the only neutral scalar matrix with the same property; we take it as the Higgs boson.
Considering that it is the only scalar with a non-vanishing vacuum expectation value prevents the occurrence of a hierarchy problem [13].

This last property is tantamount, in the “quark language”, to taking the same value for all condensates \( \langle \bar{q}_i q_i \rangle, i = 1 \cdots N \), in agreement with the flavour independence of “strong interactions” between fermions, supposedly at the origin of this phenomenon in the traditional framework. As the spectrum of mesons is, in the present model, disconnected from a hierarchy between quark condensates (see section 2), it is not affected by our choice of a single Higgs boson.
References

[1] J. H. CHRISTENSON, J. W. CRONIN, J. W. FITCH and R. TURLAY: “Evidence for the 2π decay of the K_0 meson”, Phys. Rev. Lett. 13 (1964) 138;
V. L. FITCH: “The discovery of charge-conjugation parity asymmetry”, Rev. Mod. Phys. 53 (1981) 367;
J. W. CRONIN: “CP symmetry violation - the search for its origin”, Rev. Mod. Phys. 53 (1981) 373.

[2] H. ALBRECHT et al. (ARGUS collaboration): “Observation of B^0 - B^0 mixing”, Phys. Lett. B 245 (1987), 245.

[3] see for example:
Y. NIR: “CP Violation”, Lectures given at 20th Annual SLAC Summer Institute on Particle Physics: The Third Family and the Physics of Flavor (School: Jul 13-24, Topical Conference: Jul 22-24, Symposium on Tau Physics: Jul 24), Stanford, CA, 13-24 Jul 1992. Published in SLAC Summer Inst.1992:81-136 (QCD161:S76:1992)

[4] M. KOBAYASHI and T. MASKAWA: “CP-Violation in the Renormalizable Theory of Weak Interactions”, Prog. Theor. Phys. 49 (1973) 652.

[5] S. L. GLASHOW: Nucl. Phys. 22 (1961) 579;
A. SALAM: in “Elementary Particle Theory: Relativistic Groups and Analyticity” (Nobel symposium No 8), edited by N. Svartholm (Almquist and Wiksell, Stockholm 1968);
S. WEINBERG: “A model of leptons”, Phys. Rev. Lett. 19 (1967) 1264.

[6] T. D. LEE: “A Theory of Spontaneous T Violation”, Phys. Rev. D 8 (1973) 1226;
S. WEINBERG: “Gauge Theory of CP Nonconservation”, Phys. Rev. Lett. 37 (1976) 657.

[7] M. GELL-MANN: “A schematic model of baryons and mesons”, Phys. Lett. 8 (1964) 214.

[8] S. L. ADLER and R. F. DASHEN: “Current Algebra and Application to Particle Physics”, (Benjamin, 1968);
B. W. LEE: “Chiral Dynamics”, (Gordon Breach, 1972).

[9] B. MACHET: “Chiral scalar fields, custodial symmetry in electroweak SU(2)_L \times U(1) and the quantization of the electric charge”, Phys. Lett. B 385 (1996) 198-208.

[10] N. CABIBBO: “Unitary symmetry and leptonic decays”, Phys. Rev. Lett. 10 (1963) 531.

[11] B. MACHET: “Some aspects of pion physics in a dynamically broken abelian gauge theory”, Mod. Phys. Lett. A 9 (1994) 3053-3062.

[12] B. MACHET: “Comments on the Standard Model of electroweak interactions”, Int. J. Mod. Phys. A 11 (1996) 29-63.

[13] E. GILDENER and S. WEINBERG: “Symmetry breaking and scalar bosons”, Phys. Rev. D 13 (1976) 3333;
E. GILDENER: “Gauge-symmetry hierarchies”, Phys. Rev. D 14 (1976) 1667.