Correspondence between DBI-essence and Modified Chaplygin Gas and the Generalized Second Law of Thermodynamics

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Abstract In this work, we have considered the DBI-essence dark energy model in FRW Universe. We have found the exact solutions of potential, warped brane tension and DBI scalar field. We also calculate the statefinder parameters for our model that make it distinguishable among numerous dark energy models. Moreover, we establish correspondence between DBI-essence and modified Chaplygin gas (MCG) and hence reconstruct the potential and warped brane tension. By this reconstruction, we observe that DBI scalar field and potential increase and warped brane tension decreases during evolution of the Universe. Finally, we investigate the validity of the generalized second law (GSL) of thermodynamics in the presence of DBI-essence and modified Chaplygin gas. It is observed that the GSL breaks down for DBI-essence model but GSL always satisfied for MCG model.

Keywords Dark energy; Chaplygin gas; quintessence; phantom energy.

1 Introduction

Observations of Type Ia supernovae (SNIa) indicate that currently the observable Universe is undergoing an accelerating expansion [Riess et al. 1998]. This cosmic acceleration has also been confirmed by numerous observations of large scale structure (LSS) [Tegmark et al. 2004] and measurements of the cosmic microwave background (CMB) anisotropy [Bennett et al. 2003]. The cause for this cosmic acceleration is generally dubbed as “dark energy”, a mysterious exotic energy which generates large negative pressure, whose energy density is dominated the Universe (for a review see e.g. [Copeland et al. 2006]). The astrophysical nature of dark energy is that it does not cluster at any scale unlike normal baryonic matter which form structures. The combined analysis of cosmological observations suggests that the Universe is spatially flat, and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. The nature of dark energy as well as its cosmological origin remain enigmatic at present. The future of the Universe crucially depends on the nature of dark energy: if it is quintessence (having equation of state parameter EoS \(w < -1/3\)) then the energy density of quintessence dilutes with the expansion and the acceleration will be replaced by deceleration in far future; if the cause of cosmic acceleration is cosmological constant (\(w = -1\)) then the Universe will accelerate forever since its constant energy density provides a continuous source of vacuum energy to produce acceleration; however if the dark energy is phantom energy (\(w < -1\)) then acceleration of the Universe will convert into super-acceleration in far future which will eventually destroy every gravitationally stable structure in the Universe [Zhang 2005].

In recent years, the thermodynamics of the accelerating Universe has got much attention and numerous interesting results are obtained. In particular, the generalized second law (GSL) of thermodynamics has been widely studied in the cosmological context; the law states that the entropy of a closed isolated system along with the entropy of its boundary is always an increasing function of time. It needs to be stressed that this law has some informal proofs but on several instances in cosmology, it is violated [Wall 2009]. Fur-
thermore, the validity of GSL crucially depends on the choice of boundary of the FRW Universe: for instance the GSL is respected if the FRW boundary is the dynamical apparent horizon (Jamil et al. 2010) but it is conditionally valid if the boundary is future event horizon (Sadjadi & Jamil 2010). In this work, we choose the boundary as the future event horizon and show that the GSL is violated in all the cases studied in this paper.

The main motivation and the organization of this work are as follows: In section II, we introduce basic equations and solutions for DBI-essence model. Two particular solutions are found and the scalar field and corresponding potentials are analyzed. In section III, we calculate statefinder parameters for DBI dark energy and its nature are investigated during evolution of the universe. In section IV, we develop correspondence between DBI-essence and modified Chaplygin gas and reconstruct the potential and warped brane tension as well as dynamics of scalar field are analyzed for two types of solutions. In section V, we study GSL in the presence of MCG and DBI-essence and examine the validity of GSL during evolution of the universe bounded by the event horizon. Final section is devoted to the discussion.

2 Basic equations and solutions for DBI-essence

Note that a simple scalar field, a ‘quintessence field’, is a suitable candidate as an alternative to the ‘cosmological constant’ Ratra & Peebles (1988). The dynamics of the scalar fields depend on the scalar potentials. However, scalar fields with inverse power-law potentials have attracted lot of research interests since in this case the equations of motion yield attractor solutions. It ensures that the late time behavior of the Universe is independent to the choice of arbitrary initial conditions. However, this exquisite behavior of the quintessence field comes at a price of extreme fine-tuning of the cosmological parameters. This problem of fine-tuning can be resolved if the quintessence field is modeled via approaches beyond the standard model of particle physics, for instance, string theory (Ibrar 2007). In this paper, we proceed with a scalar field model where the kinetic term is non-canonical. Such non-canonical terms in the Lagrangian generally appear in the Brane-world gravity Kachru et al. (2003). Here, the kinetic term has a Dirac-Born-Infeld (DBI) form. Physically, this originates from the fact that the action of the system is proportional to the volume traced out by the Brane during its motion. This volume is given by the square-root of the induced metric which automatically leads to a DBI kinetic term Martin & Yamaguchi (2008).

The action of the Dirac-Born-Infeld (DBI) scalar field $\phi$ can be written as (choosing $8\pi G = c = 1$)

$$S_{DBI} = -\int d^4x \sqrt{-g} \left[ T(\phi) \sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} - T(\phi) + V(\phi) \right],$$

(1)

where $V(\phi)$ is the self-interacting potential and $T(\phi)$ is the warped brane tension. In the later analysis, we shall determine exact forms of these two functions. Now let us consider the matter content of the Universe is composed of DBI type dark energy scalar field. The background spacetime is the spatially flat Friedmann-Robertson-Walker (FRW), for which the Einstein field equations are

$$3H^2 = \rho,$$

(2)

$$2\dot{H} = -(\rho + p),$$

(3)

where $H(= \frac{\dot{a}}{a})$ is the Hubble parameter. Notice that $\dot{H} > 0$ produces super-acceleration while $\dot{H} = 0$ corresponds to accelerated expansion due to cosmological constant.

The energy density and pressure of the scalar field are respectively given by

$$\rho = \rho_\phi = (\gamma - 1)T(\phi) + V(\phi),$$

(4)

$$p = p_\phi = \frac{\gamma - 1}{\gamma} T(\phi) - V(\phi),$$

(5)

where the quantity $\gamma$ is reminiscent from the usual relativistic Lorentz factor and is given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}}}.$$

(6)

From above expression (6), we observe that $T(\phi) > \dot{\phi}^2$ and $\gamma > 1$ always. From (4) and (5) we have $\rho_\phi + p_\phi = \frac{\gamma^2 - 1}{\gamma} T(\phi)$ which is always $> 0$. From energy conservation equation, we have the wave equation for $\phi$ as

$$\ddot{\phi} - \frac{3T'(\phi)}{2T(\phi)} \dot{\phi}^2 + T'(\phi) + \frac{3}{\gamma^2} \frac{\dot{a}}{a} \phi + \frac{1}{\gamma^3} [V'(\phi) - T'(\phi)] = 0.$$

(7)

Now we consider two cases: (I) $\gamma = \text{constant}$ and (II) $\gamma \neq \text{constant}$. 


where, \( a = a_0 t^\beta \), \( \beta = \frac{1}{3\gamma}[2(m-n)+(2n-1)\gamma^3] \), \( a_0 = \left( \frac{3\gamma C \sqrt{(\gamma-1)n+m}}{\sqrt{3}[2(m-n)+(2n-1)\gamma^3]} \right)^\beta \), \( \phi_1 = \frac{C}{a_0} \) with \( C \) and \( \phi_0 \) are constants. From (10) and (11), we see that \( T(\phi) \) and \( V(\phi) \) always decrease as \( \phi \) increases.

For this solution, the deceleration parameter \( q \) becomes,

\[
q = -\frac{\ddot{a}}{\dot{a}^2} = -1 + \frac{1}{\beta}
\]

For acceleration of the Universe, \( q \) must be negative i.e. \( \beta > 1 \) and hence \( 2(m-n)+(2n-1)\gamma^3 > 3\gamma \).

\textbf{Case II}: \( \gamma \neq \) constant. Let us assume, \( \gamma = \tilde{\phi}^{-2} \), so from (6) we have \( T(\phi) = \frac{\dot{\phi}^2}{1-\tilde{\phi}^2} > \dot{\phi}^2 \). Since \( \gamma > 1 \), so that we have \( \dot{\phi}^2 < 1 \). Let us also assume \( V(\phi) = T(\phi) \).

In this case, we have the solutions:

\[
\dot{\phi}^2 = \sqrt{1 + \frac{1}{3\log \frac{a_0}{a}}}.
\]

\[
V(\phi) = T(\phi) = 3\log \frac{a}{a_0} \times \sqrt{1 + \frac{1}{3\log \frac{a_0}{a}}}.
\]

where \( a_0 \) is the integration constant and the expression for deceleration parameter \( q \) as

\[
q = -1 - \frac{1}{2\log \frac{a_0}{a}}.
\]

From (13) and (14), we see that the solution is valid for \( a > a_0 e^{\frac{a}{a}} \). For acceleration of the Universe, \( q \) must be negative i.e, \( a > a_0 e^{\frac{a}{a}} \).

\textbf{3 Statefinder diagnostics for DBI-essence}

Since there are various candidates for the dark energy model, we often face with the problem of discriminating between them, which were solved by introducing statefinder parameters (Sahni et al. 2003). These statefinder diagnostic pair i.e., \( \{r, s\} \) parameters are of the form:

\[
r = \frac{\ddot{a}}{aH^2} = 1 + \frac{9}{2} \left( 1 + \frac{p_\phi}{\rho_\phi} \right) \frac{\partial \rho_\phi}{\partial p_\phi},
\]

\[
s = \frac{r - 1}{3} + \frac{1}{2} \left( 1 + \frac{\rho_\phi}{\rho_\phi} \right) \frac{\partial \rho_\phi}{\partial p_\phi}.
\]

These parameters are dimensionless and allow us to characterize the properties of dark energy in a model independent manner. The statefinder is dimensionless and is constructed from the scale factor of the Universe and its time derivatives only. The parameter \( r \) forms the next step in the hierarchy of geometrical cosmological parameters after \( H \) and \( q \). For cosmological constant with a fixed equation of state \( (w = -1) \) and a fixed Newton’s gravitational constant, we have \( \{1, 0\} \). Moreover \( \{1, 1\} \) represents the standard cold dark matter model containing no radiation while Einstein static Universe corresponds to \( \{\infty, -\infty\} \) (Debnath 2003). In literature, the diagnostic pair is analyzed for various dark energy candidates including holographic dark energy (Zhang & Wu 2007), agegraphic dark energy (Wei & Cai 2007), quintessence (Zhang 2005), dilaton dark energy (Huang et al. 2008), Yang-Mills dark energy (Zhang 2008), viscous dark energy (Hu & Meng 2006), interacting dark energy (Zimdahl & Pavon 2004), tachyon (Shao & Gui 2008), modified Chaplygin gas (Charaborty & Debnath 2007) and \( f(R) \) gravity (Setare & Jamil 2011) to name a few.

In Case I, we have the expressions of \( r \) and \( s \) as \( r = \frac{(3\gamma-2)(\gamma-1)}{2\gamma \beta} \) and \( s = \frac{2}{3\gamma} \), which are constants.

In Case II, we have found the relation between density and pressure as

\[
p_\phi = -\rho_\phi + 1,
\]

and the relation between \( r \) and \( s \) as

\[
s = \frac{2(1-r)}{7+2r}.
\]

The behavior of the parameters \( r, s \) in (19) is shown in Fig.1. From the figure, we have seen that \( s \) decreases from some negative value to \( -\infty \) as \( r \) increases upto a certain stage but they obey negative sign. After that \( s \) also decreases from \( +\infty \) to some negative value as \( r \) increases from negative label to positive label during evolution of the Universe.
4 Relation between DBI-essence and Modified Chaplygin Gas

Here, it is interesting to find the possible relation between the DBI-essence and the modified Chaplygin gas (MCG) [Benouna 2002]. The MCG best fits with the 3-year WMAP and the SDSS data with the choice of parameters $A = -0.085$ and $\alpha = 1.724$ [Jal 2008] which are improved constraints than the previous ones $-0.35 < A < 0.025$ [Dao-Jun & Zhou 2003]. Recently it is shown that the dynamical attractor for the MCG exists at $\omega_{de} = -1$, hence MCG crosses this value from either side $\omega_{de} > -1$ or $\omega_{de} < -1$, independent to the choice of model parameters [Jing et al 2008]. A generalization of MCG is suggested in [Debnath 2007] by considering $B \equiv B(a) = B_o a^k$, where $k$ and $B_o$ are constants. The MCG is the generalization of generalized Chaplygin gas and matter [Panotopoulos 2008]. Recently, several works on Chaplygin gas [Setare 2007a,b] and other dark energy model like tachyonic field [Setare 2007c, 2009] have been discussed for interacting and non-interacting scenarios of the accelerating universe.

In this section, we will show that, by choosing a proper potential, the DBI-essence can be described by a modified Chaplygin gas at late times. To find the possible relation between the DBI-essence and the modified Chaplygin gas, we set

$$p_\phi = A\rho_\phi - \frac{B}{\rho_\phi^2} , \ (A > 0, \ 0 \leq \alpha \leq 1).$$

From energy conservation equation, we have the solution of $\rho_\phi$ in modified Chaplygin gas as

$$\rho_\phi = \left[ \frac{B}{1 + A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}},$$

where $C$ is an arbitrary positive integration constant.

From equations (4) - (6), we have

$$T(\phi) = \frac{\dot{\phi}^2(\rho_\phi + p_\phi)^2}{(\rho_\phi + p_\phi)^2 - \phi^4},$$

$$V(\phi) = \frac{\dot{\phi}^2 p_\phi - p_\phi(\rho_\phi + p_\phi)}{\dot{\phi}^2 + (\rho_\phi + p_\phi)}.$$  

Now consider the following two cases:

**Case I:** $\gamma = \text{constant}$. From equations (6) and (22), it is easy to seen that the expression of $\dot{\phi}^2$ is

$$\dot{\phi}^2 = \frac{1}{\gamma}(\rho_\phi + p_\phi).$$

From (20) - (24), we get the solutions

$$\phi = \phi_0 - \frac{2}{\sqrt{3\gamma(1 + A)(1 + \alpha)}} \times \tanh^{-1} \left( \frac{\sqrt{C(1 + A) + B a^{3(1+A)(1+\alpha)}}}{\sqrt{C(1 + A)}} \right),$$

$$T(\phi) = \frac{\gamma C(1 + A)}{(\gamma^2 - 1)a^{3(1+A)}} \times \left[ C + \frac{B}{1 + A} a^{3(1+A)(1+\alpha)} \right]^{\frac{1}{1+\alpha}},$$

Fig. 2 It shows the variations of $\phi$ against $a$ respectively in Case I for $A = 1/3, B = 0.5, C = 0.5, \alpha = 0.6, \gamma = 2, \phi_0 = 10.$
Fig. 3  It shows the variations of $T(\phi)$ against $a$ respectively in Case I for $A = 1/3, B = 0.5, C = 0.5, \alpha = 0.6, \gamma = 2, \phi_0 = 10$.

![Graph of T(\phi) vs a](image)

Fig. 4  It shows the variations of $V(\phi)$ against $a$ respectively in Case I for $A = 1/3, B = 0.5, C = 0.5, \alpha = 0.6, \gamma = 2, \phi_0 = 10$.

![Graph of V(\phi) vs a](image)

5 Generalized second law of thermodynamics

Gibbons & Hawking conjectured that event horizon area, including cosmological event horizons, might quite
generally have associated entropy \cite{GibbonsHawking1977}. A prominent example is de Sitter space. We consider the FRW Universe as a thermodynamical system with the future even horizon surface as a boundary of the system, which is a valid assumption \cite{Davis2003}. This horizon has got recent attention since it yields a correct equation of state of dark energy, namely for the holographic dark energy \cite{Jamil2009}. In general, the radius of the event horizon \( R_h \) is not constant but changes with time (or expansion of the Universe). Let \( dR_h \) be an infinitesimal change in the radius of the future event horizon during a time of interval \( dt \). This small displacement \( dR_h \) will produce an infinitesimal change \( dV \) in the volume \( V \) of the event horizon. Each spacetime describing a thermodynamical system and satisfying Einstein’s equations differs infinitesimally in the extensive variables volume, energy and entropy by \( dV, dE \) and \( dS \), respectively, while having the same values for the intensive variables temperature \( T \) and pressure \( p \). Thus, for these two spacetimes describing two thermodynamical states, there must exist some relation among these thermodynamic quantities. To study the generalized second law of thermodynamics through the Universe we deduce the expression for normal entropy using the first law of thermodynamics

\[
T \, dS = p \, dV + dE,
\]

where, \( T, S, p, V \) and \( E \) are respectively temperature, entropy, pressure, volume and internal energy within the event horizon. Here the expression for internal energy can be written as \( E = \rho V \). Now the volume of the sphere is \( V = \frac{4}{3} \pi R_h^3 \), where \( R_h \) is the radius of the event horizon defined by

\[
R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{a^2 H},
\]

which immediately gives

\[
\dot{R}_h = HR_h - 1.
\]

The temperature of the event horizon is \cite{SadjadiJamil2010}

\[
T = \frac{1}{2\pi R_h}.
\]

So using the above relations (33)-(35), equation (32) can be written as

\[
\dot{S} = \frac{8\pi H R_h^2}{T} = 16\pi^2 H R_h^3.
\]

Also the entropy on the event horizon is \cite{Davis2003}

\[
S_h = \frac{\pi R_h^2}{G} = 8\pi^2 R_h^2.
\]
Using (34), (36) and (37), we have the rate of change of total entropy as
\[ \dot{S} + \dot{S}_h = 16\pi^2 R_h (HR_h^2 + HR_h - 1). \]  
(38)
The generalized second law states that total entropy can not be decrease i.e.,
\[ \dot{S} + \dot{S}_h \geq 0, \text{ i.e., } HR_h^2 + HR_h - 1 \geq 0. \]  
(39)

Now we shall examine the validity of GSL of thermodynamics for DBI-essence and modified Chaplygin gas separately.

- **DBI-essence:**

  **Case I:** $\gamma = \text{constant}$: For the DBI solution (8), the radius of the event horizon is
  \[ R_h = \frac{t}{\beta - 1}, \beta > 1 \]  
(40)

  In this case, the rate of change of total entropy becomes
  \[ \dot{S} + \dot{S}_h = -\frac{16\pi^2 t}{(\beta - 1)^3} < 0 \text{ for all } t. \]  
(41)

  So from (41) we see that, generalized second law can not be satisfied in case I for DBI-essence model.

  **Case II:** $\gamma \neq \text{constant}$: For the DBI solutions (13) and (14) and using (2)-(6), (33) and (34), we get the radius of the event horizon as
  \[ R_h = \sqrt{\pi} \frac{a}{a_0} \text{Erfc} \left( \sqrt{\log \frac{a}{a_0}} \right), \]  
(42)

where Erfc represents the complementary error function. The rate of change of total entropy becomes
\[ \dot{S} + \dot{S}_h = 16\pi^2 \frac{a}{a_0} \text{Erfc} \left( \sqrt{\log \frac{a}{a_0}} \right) \]
\[ \times \left[ \sqrt{\pi} \frac{a}{a_0} \sqrt{\log \frac{a}{a_0}} \text{Erfc} \left( \sqrt{\log \frac{a}{a_0}} \right) - 1 \right] \]
\[ + \frac{3\pi}{2} \left( \frac{a}{a_0} \right)^2 \log \frac{a}{a_0} \left( 1 + \frac{1}{3\log \frac{a}{a_0}} - \sqrt{1 + \frac{1}{3\log \frac{a}{a_0}}} \right) \]
\[ \times \left( \text{Erfc} \left( \sqrt{\log \frac{a}{a_0}} \right) \right)^2 \]
(43)

The above expression is very complicated form in $a$. So we have drawn the figure of $\dot{S} + \dot{S}_h$ against $a$ in fig. 8. From the figure, we see that $\dot{S} + \dot{S}_h < 0$ for all values of $a$. Hence we conclude that GSL cannot be satisfied during evolution of the Universe in case II of DBI-essence model.

- **Modified Chaplygin Gas:**

  For Chaplygin gas model, let us assume $\frac{B}{1 + A} a^{3(1 + A)(1 + \alpha)} = Cx$, so that the solution for density (21) reduces to
  \[ \rho = \left( \frac{B}{1 + A} \right)^{\frac{1}{1 + \alpha}} (1 + \frac{1}{x})^{\frac{1}{1 + \alpha}}. \]  
(43)

Using equations (2), (33) and (44), the radius of the event horizon can be expressed as
\[ R_h = \frac{2\sqrt{3}}{1 + 3A} \left( \frac{B}{1 + A} \right)^{\frac{1}{2(1 + \alpha)}} \text{F} \left( \frac{1 + 3A}{6(1 + A)(1 + \alpha)}, \frac{1}{2(1 + \alpha)} \right) \]
\[ \times D - x^{\frac{1 + 3A}{6(1 + A)(1 + \alpha)}} \left[ 1 + \frac{1 + 3A}{6(1 + A)(1 + \alpha)} \right], \]  
(44)

**Fig. 8** It shows $\dot{S} + \dot{S}_h$ against $a$ in DBI model in case II.

**Fig. 9** Fig. 9 shows the figure of $d(S + S_h)/dx$ against $x$ in Modified Chaplygin gas model for $A = 1/3, B = 0.5, \alpha = 0.6$. 


where

\[ D = \left[ \frac{1+3A}{2(1+\alpha)} \right] \frac{1}{x^2} \left( \frac{1+3A}{6(1+A)(1+\alpha)} \right)^{-x} \] (45)

From equation (38), we have the deviation of total entropy as

\[ \frac{d(S + S_h)}{dx} = \frac{32\pi^2 x^{-2}}{(1+\alpha)(1+3A)^2} \left( \frac{B}{1+A} \right)^{\frac{1}{1+\alpha}} \times \left[ D - x^{-\frac{1+3A}{(1+A)(1+\alpha)}} \right] \left[ 6(1+A)(1+\alpha) \right] \]

\[ \times \left[ x^{\frac{1+3A}{(1+A)(1+\alpha)}} \left( \frac{2}{1+A} - \frac{1}{1+x} \right) \right] \]

\[ \times \left[ 1 + \frac{1+3A}{6(1+A)(1+\alpha)} \right] \frac{1}{2} \left[ 1 + \frac{1+3A}{6(1+A)(1+\alpha)} \right] \]

\[ \times \left[ 1 + \frac{1+3A}{6(1+A)(1+\alpha)} \right] \frac{1}{2} \left[ 1 + \frac{1+3A}{6(1+A)(1+\alpha)} \right] \]

\[ \times \left[ \frac{1}{1+A} + \frac{3}{1+x} \right] D \]

\[ - \frac{1+3A}{1+A} x^{\frac{1+3A}{(1+A)(1+\alpha)}} (1+x)^{-\frac{2}{1+\alpha}} \] (46)

The above expression is very complicated form in \( x \). So we have drawn the figure of \( d(S + S_h)/dx \) against \( x \) in fig. 9. From the figure, we see that \( d(S + S_h)/dx > 0 \) for all values of \( x \). Hence we conclude that GSL always satisfied during evolution of the Universe in modified Chaplygin gas model.

### 6 Concluding Remarks

In this work, we have considered the FRW Universe with DBI-essence dark energy model, which is a scalar field having a non-canonical kinetic term. We have found the exact solutions of potential, warped brane tension and DBI scalar field. We also calculate the statefinder parameters for our model that make it distinguishable among numerous dark energy models. From the fig. 1, we have seen that \( s \) decreases from some negative value to \( -\infty \) as \( r \) increases up to a certain stage but they obey negative sign. After that \( s \) also decreases from \( +\infty \) to some negative value as \( r \) increases from negative label to positive label during evolution of the Universe. Moreover, we establish correspondence between DBI-essence and modified Chaplygin gas and hence we have reconstructed the potential and warped brane tension in cases I and II. By this reconstruction and from figs. 2-7, we have seen that DBI scalar field and potential increase and warped brane tension decreases during evolution of the Universe.

We have also considered total entropy as sum of the entropies of a cosmological event horizon and the entropy of the DBI-essence. We have investigated the validity of the generalized second law (GSL) of thermodynamics in the presence of DBI-essence and modified Chaplygin gas separately. In all of the cases (cases I and II) of DBI-essence model, we have observed, the time derivative of the total entropy is remaining at the negative level during the evolution. This means that the total entropy is a decreasing function of time in the situations considered in this work. So the GSL breaks down for DBI-essence model in both the cases. It is also observed that the GSL always satisfied for MCG model.

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References

Riess A. et al., 1998 Astron. J., 116, 1009
Tegmark M. et al., 2004 Phys. Rev. D 69, 103501
Bennett M. et al., 2003 Astrophys. J. Suppl. 148, 1
Copeland E.J., Sami M., & Tsujikawa T., 2006 Int. J. Mod. Phys. D., 15, 1753
Zhang X., & Wu F-Q., 2007 Phys. Rev. D 76, 023502
Wall A.C., 2009 JHEP 0906, 021
Jamil M., Saridakis E.N., & Setare M.R., 2010 Phys. Rev. D 81, 023007
Sadjadi H.M. & Jamil M., 2010 EPL 92, 69001
Ratra B. & Peebles P.J.E., 1988 Phys. Rev. D 37, 3406
Brax S., [arXiv:0711.2428] [hep-ph]
Kachru S. et al., 2003 JCAP 0310, 013
Martin J. & Yamaguchi M., 2008 Phys. Rev. D 77, 123508
 Sahni V. et al., 2003 JETP Lett. 77, 201
Debnath U., 2008 Class. Quant. Grav. 25, 205019
Zhang X., 2005 Int. J. Mod. Phys. D 14, 1597
Wei H. & Cai R.G., 2007 Phys. Lett. B 655, 1
Huang J.Z et al., 2008 Astrophys. Space Sci. 315, 175
Zhao W., 2008 Int. J. Mod. Phys. D 17, 1245
Zimdahl W. & Pavon D., 2004 Gen. Relativ. Grav. 36, 1483
Hu M. & Meng X.H., 2006 Phys. Lett. B 635, 186
Shao Y. & Gui Y., 2008 Mod. Phys. Lett. A 23, 65
Chakraborty W. & Debnath U., 2007 Mod. Phys. Lett. A 22, 1805
Setare M.R. & Jamil M., 2011 Gen. Rel. Grav. 43, 293
Benaoum H.B., [hep-th/0205140]
Lu J. et al., 2008 Phys. Lett. B 662, 87
Dao-Jun L. & L. Xin-Zhou., 2005 Chin. Phys. Lett. 22, 1600
Jing H. et al., Chin. Phys. Lett. 25, 347
Debnath U., 2007 Astrophys. Space Sci. 312, 295
Barreiro T. & Sen A. A., 2004 Phys. Rev. D 70, 124013
Carturan D. & Finelli F., 2003 Phys. Rev. D 68, 103501
Barreiro et al., 2008, Phys. Rev. D 78, 045030
Makler M. et al., 2003 Phys. Lett. B 555, 1
Kamenshchik A. et al 2001 Phys. Lett. B 511, 265
Panotopoulos G. 2008 Phys. Rev. D 77, 107303
Setare M.R., 2007 Phys. Lett. B 648, 329
Setare M.R., 2007 Phys. Lett. B 654, 1
Setare M.R., 2007 Eur. Phys. J. C 52, 689
Setare M.R., 2007 Phys. Lett. B 653, 116
Setare M.R., 2009 Phys. Lett. B 673, 241
Gibbons G.W. & Hawking S.W., 1977 Phys. Rev. D 15, 2738
Davis T.M. et al., 2003 Class. Quant. Grav. 20, 2753
Jamil M. et al., 2009 Phys. Lett. B 679, 172