On the CPT-even Lorentz-breaking extension of the QED

T. Mariz,¹ R. V. Maluf,²,∗ J. R. Nascimento,³,† and A. Yu. Petrov³,‡

¹Instituto de Física, Universidade Federal de Alagoas,
   57072-270, Maceió, Alagoas, Brazil

²Departamento de Física, Universidade Federal do Ceará (UFC),
   Caixa Postal 6030, 60455-760, Fortaleza, CE, Brazil

³Departamento de Física, Universidade Federal da Paraíba
   Caixa Postal 5008, 58051-970, João Pessoa, Paraíba, Brazil

Abstract

In this paper, we describe the CPT-even Lorentz-breaking extension of QED in four and five dimensions and generate perturbatively the aether-like term for the gauge field which is finite in five dimensions within the dimensional regularization, but divergent in four dimensions.

∗Electronic address: r.v.mul@fisica.ufc.br
†Electronic address: jroberto@fisica.ufpb.br
‡Electronic address: petrov@fisica.ufpb.br
§Electronic address: tmariz@fis.ufal.br
I. INTRODUCTION

The possible violation of the Lorentz symmetry is intensively discussed now. A typical manner of its introduction is based on an extension of the corresponding action by new terms involving constant vector or tensor fields which explicitly break the Lorentz symmetry by introducing the privileged direction of the space-time. Many examples of such additive terms are presented in [1, 2]. This matter naturally calls the interest to study of different issues related to such models, both at the classical and at the quantum levels. Certainly, extensions of gauge field theories are of especial interest. It was shown that in the tree approximations, such theories display highly nontrivial effects such as birefringence of waves and rotation of the plane of polarization in the vacuum (see f.e. [3, 4]).

In the quantum level, these theories exhibit even more interesting properties. The paradigmatic example is the QED with the Carroll-Field-Jackiw (CFJ) term which is known to be gauge invariant. The CFJ term can arise as a quantum correction from a Lorentz-breaking extended QED with an axial coupling, and, despite the corresponding Feynman diagram is superficially divergent, this term is finite but ambiguous (an incomplete list of references on the CFJ term is given by [6]). These studies certainly call the interest to consideration of quantum properties of other Lorentz-breaking extensions of the QED. In particular, the studies of the CPT-even Lorentz-breaking extensions of the QED are of particular importance. At the classical level, many issues related to such extensions, especially exact solutions and dispersion relations, were studied in [7], and the paper [8] opened an interest in these theories from the viewpoint of the extra dimension concept. Further, in [9], the CPT-even terms were shown to arise as quantum corrections in a CPT-odd extended QED with a nonminimal coupling. Moreover, they turn out to be finite.

While the theory considered in [9] is, first, CPT-odd, second, involving nonrenormalizable couplings (another interesting studies on quantum corrections in non-renormalizable Lorentz-breaking extensions of QED are presented in [10]), the natural question is – whether the CPT-even contributions can be generated from an essentially CPT-even Lorentz-breaking extension of the QED? In this paper, we use CPT-even terms intro-
duced in [8], with the resulting theory is essentially renormalizable in the four-dimensional space-time. Some preliminary results on renormalization of this model, including the lower-order renormalization constants, were obtained already in [1] (it is worth to mention now other studies on renormalization of Lorentz-breaking theories [11]), however, it is certainly interesting to obtain the next order aether-like counterterms. This allows us to generate the aether-like terms in the QED. We also generate these results to the five-dimensional case where the aether-like contributions are explicitly finite within the framework of the dimensional regularization.

The structure of the paper looks like follows. In section 2, we introduce the renormalizable CPT-even extension of the QED. In section 3, we calculate the one-loop contributions to the two-point function of the gauge field in four dimensions. In section 4, we generalize our calculations to five dimensions where the results are finite. Finally, in the Summary, we discuss our results.

II. THE MODEL

We start with the following extended QED with a CPT-even Lorentz-breaking term (see f.e. [1]), given by

\[ S_{\psi} = \int d^{4}x \bar{\psi} \left[ i/\partial - m + i u_{\mu}u_{\nu} \gamma_{\mu} \partial_{\nu} - eiA_{\nu} \right] \psi, \] (1)

where \( c_{\mu\nu} = u_{\mu}u_{\nu} \) and \( g^{\mu\nu} = \text{diag}(1, -1, -1, -1) \). We can ensure the smallness of \( c_{\mu\nu} \) requiring that \( |u_{\mu}| \ll 1 \) for any \( \mu \). We can explicitly write the action (1) as follows:

\[ S_{\psi} = \int d^{4}x \bar{\psi} \left[ i\phi - m + i u_{\mu}u_{\nu} \gamma_{\mu} \partial_{\nu} - e\phi - eu_{\mu}u_{\nu} \gamma_{\mu} A_{\nu} \right] \psi. \] (2)

The essential feature of this theory is its renormalizability in four dimensions. Indeed, all constants in the theory are dimensionless.

Now, let us derive the Feynman rules for this theory. The kinetic term for the spinor field is

\[ S_{\text{kin}} = \int d^{4}x \bar{\psi} \left[ i\phi - m + i u_{\mu}u_{\nu} \gamma_{\mu} \partial_{\nu} \right] \psi. \] (3)
The corresponding propagator is
\[
\langle \psi(p) \bar{\psi}(-p) \rangle \equiv S(p) = \frac{i}{\slashed{p} - m + u^\mu u^\nu \gamma_\mu p_\nu}.
\] (4)

Within this paper, we will adopt the perturbative expansion for the free propagator:
\[
\frac{i}{\slashed{p} - m + u^\mu u^\nu \gamma_\mu p_\nu} \simeq \frac{i}{\slashed{p} - m} + \frac{i}{\slashed{p} - m} (iu^\mu u^\nu \gamma_\mu p_\nu) \frac{i}{\slashed{p} - m}.
\] (5)

Interaction term
\[
S_{kin} = \int d^4x \bar{\psi} \left[ -eA^\mu - eu^\mu u^\nu \gamma_\mu A^\nu \right] \psi = \int d^4x \bar{\psi} \left[ -e \gamma_\mu (\eta^{\mu\nu} + u^\mu u^\nu) A_\nu \right] \psi.
\] (6)

Vertices:
\[
V_1 = -e \bar{\psi} \gamma^\mu \psi A_\mu,
\]
(7)
\[
V_2 = -e \bar{\psi} \gamma_\nu u^\mu u^\nu A_\mu.
\] (8)

Graphically, these Feynman rules can be presented by the Fig. 1.

They will be used to introduce one-loop Feynman diagrams.

III. RADIATIVE CORRECTIONS AND INDUCED TERMS

The fermionic determinant in our theory can be read off from the generating functional:
\[
Z[A_\mu] = \int D\bar{\psi} D\psi e^{i \int d^4x \mathcal{L}_\psi} = e^{i S_{\text{eff}}},
\] (9)
so that, by integrating over the fermions, we obtain the one-loop effective action

\[ S_{\text{eff}} = -i \text{Tr} \ln(\not{\!p} - m + u^{\mu}u^{\nu}\gamma_{\mu}p_{\nu} - e\not{\!A} - eu^{\mu}u^{\nu}\gamma_{\mu}A_{\nu}). \]  

(10)

Here, Tr stands for the trace over the Dirac matrices, as well as the trace over the integration in momentum and coordinate spaces.

In order to single out the quadratic terms in \( A_{\mu} \) of the effective action, we initially rewrite the expression (10) as

\[ S_{\text{eff}} = S_{\text{eff}}^{(0)} + \sum_{n=1}^{\infty} S_{\text{eff}}^{(n)}, \]

(11)

where \( S_{\text{eff}}^{(0)} = -i \text{Tr} \ln(\not{\!p} - m + u^{\mu}u^{\nu}\gamma_{\mu}p_{\nu}) \) and

\[ S_{\text{eff}}^{(n)} = \frac{i}{n} \text{Tr} \left[ \frac{1}{\not{\!p} - m + u^{\mu}u^{\nu}\gamma_{\mu}p_{\nu}} (e\not{\!A} + eu^{\mu}u^{\nu}\gamma_{\mu}A_{\nu}) \right]^{n}. \]

(12)

Then, after evaluating the trace over the coordinate space, by using the commutation relation \( A_{\mu}(x)G_{c}(p) = G_{c}(p - i\partial)A_{\mu}(x) \) and the completeness relation of the momentum space, for the quadratic action \( S_{\text{eff}}^{(2)} \), we have

\[ S_{\text{eff}}^{(2)} = \frac{1}{2} \int d^{4}x \Pi^{\mu\nu} A_{\mu}A_{\nu}, \]

(13)

where

\[ \Pi^{\mu\nu} = ie^{2} \int \frac{d^{4}p}{(2\pi)^{4}} \text{tr} G_{c}(p)\gamma^{\mu}G_{c}(p - i\partial)\gamma^{\nu}, \]

(14)

with

\[ G_{c}(p) = \frac{1}{\not{\!p} - m + u^{\mu}u^{\nu}\gamma_{\mu}p_{\nu}} \]

(15)

being the Feynman propagator. Note that the derivative contained in \( \Pi^{\mu\nu} \) acts only on the first gauge field \( A_{\mu} \), in Eq. (13).

For the zero order in \( c_{\mu\nu} \), we have

\[ \Pi_{0}^{\mu\nu} = ie^{2} \text{tr} \int \frac{d^{4}p}{(2\pi)^{4}} S(p)\gamma^{\mu}S(p - k)\gamma^{\nu}, \]

(16)

which yields a paradigmatic result of the usual QED presented in textbooks, see f.e. [12]:

\[ \Pi_{0}^{\mu\nu} = \left( \frac{e^{2}}{6\pi^{2}e'} + A_{0}(\eta) \right) (k^{\mu}k^{\nu} - k^{2}g^{\mu\nu}), \]

(17)
where \( \frac{1}{\epsilon} = \frac{1}{4} - \ln \frac{m}{\mu'} \), with \( \epsilon = 4 - D \) and \( \mu'^2 = 4\pi\mu^2e^{-\gamma-i\pi} \), and

\[
A_0(\eta) = \frac{\epsilon^2}{36\pi^2k^4} \left[ k^4 \left( 6\eta \tan^{-1} \eta + 5 \right) - 12k^2m^2 \left( \eta \tan^{-1} \eta - 1 \right) - 48m^4\eta \tan^{-1} \eta \right],
\]

with \( \eta^2 = \frac{k^2}{4m^2 - k^2} \). Then, for small \( k^2 \), we have

\[
\Pi_0^{\mu\nu} = \left( \frac{\epsilon^2}{6\pi^2\epsilon} + \frac{\epsilon^2 k^2}{60\pi^2 m^2} \right) \left( k^\mu k^\nu - k^2 g^{\mu\nu} \right) + \cdots.
\]

(19)

From the graphical viewpoint, the first-order Lorentz-breaking corrections are depicted at Fig. 2.

![Diagram](image)

**FIG. 2**: First-order Lorentz-breaking contributions.

Explicitly, the contribution of the first order in \( c_{\mu\nu} = u_\mu u_\nu \), is (see also [1]): \( \Pi_1^{\mu\nu} = \Pi_{1,1}^{\mu\nu} + \Pi_{1,2}^{\mu\nu} + \Pi_{1,3}^{\mu\nu} + \Pi_{1,4}^{\mu\nu} \), with

\[
\Pi_{1,1}^{\mu\nu} = -ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) u^\alpha u^\beta \gamma_{\alpha\beta} S(p) \gamma^\mu S(p-k) \gamma^\nu,
\]

(20)

\[
\Pi_{1,2}^{\mu\nu} = -ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) \gamma^\mu S(p-k) u^\alpha u^\beta \gamma_{\alpha\beta} (p-k) \gamma^\nu,
\]

(21)

\[
\Pi_{1,3}^{\mu\nu} = ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) u^\alpha \gamma_{\alpha\nu} S(p-k) \gamma^\mu,
\]

(22)

\[
\Pi_{1,4}^{\mu\nu} = ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) \gamma^\mu S(p-k) u^\alpha u^\nu \gamma_{\alpha\nu}.
\]

(23)

The total result is

\[
\Pi_1^{\mu\nu} = \frac{e^2}{6\pi^2\epsilon^2} \left[ \left( u^2 k^2 - 2(u \cdot k)^2 \right) g^{\mu\nu} + 2u^\mu \left( k^\nu (u \cdot k) - k^2 u^\nu \right) + k^\mu \left( 2u^\nu (u \cdot k) - u^2 k^\nu \right) + A_1(\eta) \right. \\
+ \left. B_1(\eta) \right]
\]

(24)
where
\[
A_1(\eta) = \frac{e^2}{36\pi^2 k^2 \eta} \left[ k^2 (5\eta - 6 \tan^{-1} \eta) + 12m^2 \eta - \tan^{-1} \eta \right],
\]
\[
B_1(\eta) = \frac{e^2\eta}{9\pi^2 k^3} \left[ k^2 m^2 (6 \tan^{-1} \eta - 7\eta) + k^4 \left( \eta - 3 \tan^{-1} \eta \right) + 12m^4 (\eta - \tan^{-1} \eta) \right],
\]
\[
C_1(\eta) = \frac{e^2\eta}{6\pi^2 k^6} \left[ k^2 \eta (k^2 + 2m^2) + 24m^4 (\tan^{-1} \eta - \eta) \right].
\]

We can easily observe that for small \(k^2\), we obtain
\[
\Pi^{\mu\nu}_1 = \frac{e^2}{6\pi^2} \left\{ \left( u^2 k^2 - 2(u \cdot k)^2 \right) g^{\mu\nu} + 2u^\mu \left( k^\nu(u \cdot k) - k^2 u^\nu \right) + k^\mu \left( 2u^\nu(u \cdot k) - u^2 k^\nu \right) \right\}
\]
\[
+ \frac{e^2}{60\pi^2 m^2} \left\{ k^\mu \left[ 2k^2 u^\nu(u \cdot k) + k^\nu \left( 2(u \cdot k)^2 - u^2 k^2 \right) \right] + k^2 \left[ \left( u^2 k^2 - 4(u \cdot k)^2 \right) g^{\mu\nu} + 2u^\mu \left( k^\nu(u \cdot k) - k^2 u^\nu \right) \right] \right\} + \cdots.
\]

It is clear that this self-energy tensor is transversal. Manifestly, the corresponding divergent contribution to the effective action is
\[
S^{(2,1)}_{\text{eff}} = \int d^4x \frac{e^2}{6\pi^2} \left( \frac{u^2}{4} F_{\mu\nu} F^{\mu\nu} - u^\mu F_{\mu\nu} u_\lambda F^{\lambda\nu} \right),
\]
which replays the structures of Maxwell term and the aether term. Now, let us perform the next step which naturally consists in calculating of the second-order aether-like quantum correction which never was consider earlier. For the second order in \(e_{\mu\nu}\),
\[
\Pi_{2,1}^{\mu\nu} = \Pi_{2,1}^{\mu\nu} + \Pi_{2,2}^{\mu\nu} + \Pi_{2,3}^{\mu\nu} + \Pi_{2,4}^{\mu\nu} + \Pi_{2,5}^{\mu\nu} + \Pi_{2,6}^{\mu\nu} + \Pi_{2,7}^{\mu\nu} + \Pi_{2,8}^{\mu\nu},
\]
with
\[
\Pi_{2,1}^{\mu\nu} = ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) u^\alpha u^\beta \gamma_\alpha p_\beta S(p) u^\gamma u^\delta \gamma_\delta p_\gamma S(p) \gamma^\mu S(p-k) \gamma^\nu,
\]
\[
\Pi_{2,2}^{\mu\nu} = ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) u^\alpha u^\beta \gamma_\alpha p_\beta S(p) \gamma^\mu S(p-k) u^\gamma u^\delta \gamma_\delta (p-k) S(p-k) \gamma^\nu,
\]
\[
\Pi_{2,3}^{\mu\nu} = ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) \gamma^\mu S(p-k) u^\alpha u^\beta \gamma_\alpha (p-k) S(p-k) u^\gamma u^\delta \gamma_\delta (p-k) S(p-k) \gamma^\nu,
\]
\[
\Pi_{2,4}^{\mu\nu} = -ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) u^\alpha u^\beta \gamma_\alpha p_\beta S(p) u^\gamma u^\delta \gamma_\delta S(p-k) \gamma^\nu,
\]
\[
\Pi_{2,5}^{\mu\nu} = -ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) u^\gamma u^\delta \gamma_\delta S(p-k) u^\alpha u^\beta \gamma_\alpha (p-k) S(p-k) \gamma^\nu,
\]
\[
\Pi_{2,6}^{\mu\nu} = -ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p) u^\alpha u^\beta \gamma_\alpha p_\beta S(p) \gamma^\mu S(p-k) u^\gamma u^\nu \gamma_\nu.
\]
\[ \Pi_{\mu\nu}^{2,7} = -ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)\gamma^\mu S(p - k)u^\alpha u^\beta \gamma_\alpha(p - k)\beta S(p - k)u^\gamma u^\gamma, \]  
(36)

\[ \Pi_{\mu\nu}^{2,8} = ie^2 \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)u^\alpha u^\mu \gamma_\alpha S(p - k)u^\beta u^\nu \gamma_\beta. \]  
(37)

From the graphical viewpoint, the second-order Lorentz-breaking corrections are depicted at Fig. 3.

![Diagram](image)

**FIG. 3: Second-order Lorentz-breaking corrections.**

Then, the total result is

\[ \Pi_{\mu\nu}^2 = \frac{e^2 u^2}{6\pi^2 e'} \left[ ((u \cdot k)^2 - u^2 k^2) g_{\mu\nu} + u^\mu \left( k^2 u^\nu - k^\nu (u \cdot k) \right) + k^\mu \left( u^2 k^\nu - u^\nu (u \cdot k) \right) \right] + A_2(\eta) u^4 \left( k^2 g_{\mu\nu} - k^\mu k^\nu \right) + B_2(\eta) (u \cdot k)^2 \left[ 4k^2 u^\mu u^\nu + k^\mu \left( u^2 k^\nu - 4u^\nu (u \cdot k) \right) \right] + C_2(\eta) (u \cdot k)^4 g_{\mu\nu} + D_2(\eta) u^2 (u \cdot k)^2 g_{\mu\nu} + E_2(\eta) u^2 u^\mu u^\nu + F_2(\eta) u^\mu k^\nu (u \cdot k)^3 + G_2(\eta) u^2 u^\nu k^\mu (u \cdot k) + H_2(\eta) u^2 u^\mu k^\nu (u \cdot k) + I_2(\eta) k^\mu k^\nu (u \cdot k)^4, \]  
(38)

where

\[ A_2(\eta) = \frac{e^2}{36\pi^2 k^4 \eta} \left[ 4k^2 m^2 (3 \tan^{-1} \eta - 2\eta^3) + k^4 (5\eta^3 + 6 \tan^{-1} \eta) - 48m^4 \eta^3 \right], \]  
(39)

\[ B_2(\eta) = \frac{e^2}{12\pi^2 k^6} \left( 6k^2 m^2 + k^4 - 24m^4 \eta \tan^{-1} \eta \right), \]  
(40)

\[ C_2(\eta) = \frac{e^2 \eta^2}{6\pi^2 k^8} \left\{ 48m^6 \eta \tan^{-1} \eta - k^2 \left[ -4k^2 m^2 + k^4 + 12m^4 (2\eta \tan^{-1} \eta + 1) \right] \right\}, \]  
(41)

\[ D_2(\eta) = \frac{e^2}{18\pi^2 k^4} \left[ -3k^2 m^2 (2\eta \tan^{-1} \eta + 1) + k^4 (3\eta \tan^{-1} \eta + 1) + 12m^4 \eta \tan^{-1} \eta \right], \]  
(42)

\[ E_2(\eta) = \frac{e^2}{36\pi^2 k^2} \left[ -12k^2 m^2 (\eta \tan^{-1} \eta - 1) + k^4 (6\eta \tan^{-1} \eta + 5) - 48m^4 \eta \tan^{-1} \eta \right], \]  
(43)
Thus, for small $k$, the renormalization constant for the Maxwell term is

$$F_2(\eta) = \frac{e^2 \eta}{3\pi^2 k^4} \left[ k^2 \eta \left( k^2 + 2m^2 \right) + 24m^4 \left( \tan^{-1} \eta - \eta \right) \right], \quad (44)$$

$$G_2(\eta) = \frac{e^2}{36\pi^2 k^4} \left[ 12k^2m^2 \left( \eta \tan^{-1} \eta - 1 \right) - k^4 \left( 6\eta \tan^{-1} \eta + 5 \right) + 48m^4 \eta \tan^{-1} \eta \right] \quad (45)$$

$$H_2(\eta) = \frac{e^2}{36\pi^2 k^4 \eta} \left[ 4k^2m^2 \left( 3 \tan^{-1} \eta - 2\eta^3 \right) + k^4 \left( 5\eta^3 + 6 \tan^{-1} \eta \right) - 48m^4 \eta^3 \right], \quad (46)$$

$$I_2(\eta) = -\frac{e^2 \eta^2}{6\pi^2 k^4} \left[ -12k^2m^4 \left( 6\eta \tan^{-1} \eta + 5 \right) + 8k^4m^2 + k^6 + 240m^6 \eta \tan^{-1} \eta \right]. \quad (47)$$

Thus, for small $k^2$, we obtain

$$\Pi_2^{\mu \nu} = \frac{e^2 u^2}{6\pi^2 \epsilon^2} \left[ \left( u \cdot k \right)^2 - u^2 k^2 \right] g^{\mu \nu} + u^\nu \left( k^2 u^\nu - k^\nu \left( u \cdot k \right) \right) + k^\mu \left( u^2 k^\nu - u^\nu \left( u \cdot k \right) \right) \right] \quad (48)$$

$$- \frac{e^2}{60\pi^2 m^2} \left\{ \left( u^4 k^4 - 2u^2 k^2 \left( u \cdot k \right)^2 + 4 \left( u \cdot k \right)^4 \right) g^{\mu \nu} + k^\mu \left( u^2 k^\nu \left( u \cdot k \right)^2 - u^2 k^2 \right) \right. \right.$$ 

$$\left. + u^\nu \left( u \cdot k \right) \left( u^2 k^2 - 4 \left( u \cdot k \right)^2 \right) \right\} - u^\mu \left( 4 \left( u \cdot k \right)^2 - u^2 k^2 \right) \left( k^\nu \left( u \cdot k \right) - k^2 u^\nu \right) \right\} + \cdots. \quad (49)$$

It is interesting to note that, although this self-energy tensor is also transversal, it differs from \[(24)\], since the Maxwell term and the aether term enter this contribution with weights different from \[(24)\]. The purely divergent contribution to the effective action from this sector is

$$S_{\text{eff}}^{(2,2)} = \int d^4x \frac{e^2}{6\pi^2 \epsilon^2} \left( -\frac{u^4}{4} F_{\mu \nu} F^{\mu \nu} + \frac{u^2}{2} u^\mu F_{\mu \nu} u_\lambda F^{\lambda \nu} \right). \quad (49)$$

So, we succeeded to find aether-like one-loop divergences. Finally, the complete one-loop contribution to the two-point function is given by the sum of \[(17, 24, 38)\]. Thus, the modified renormalization constant for the Maxwell term is

$$Z_3 = 1 - \frac{e^2}{6\pi^2 \epsilon} \left( 1 - u^2 + u^4 \right). \quad (50)$$

To introduce renormalization of the aether term in a consistent way, one should introduce this term already at the tree level. For example, if we suggest the tree-level aether term to look like \[(8)\]

$$\mathcal{L}_{\text{aether}} = -\frac{1}{2} u^\mu F_{\mu \nu} u_\lambda F^{\lambda \nu}, \quad (51)$$

we will find the renormalization constant for this term to be

$$Z_{\text{aether}} = 1 - \frac{e^2}{6\pi^2 \epsilon} (2 - u^2). \quad (52)$$
This situation essentially differs from the case when the aether term is generated on the base of the CPT-odd magnetic coupling \[9\] where the aether term turns out to be finite and regularization dependent.

IV. FIVE-DIMENSIONAL GENERALIZATION

We start with the following model with a CPT-even Lorentz-breaking term, given by

\[
S = \int d^5x \bar{\psi}(i\not{\partial} - m + iu^\mu u^\nu \gamma_\mu \partial_\nu - eA_\mu - eA_\mu \gamma_\nu u^\mu u^\nu) \psi, \tag{53}
\]

where now the indices run from 0 to 4. By its essence, this model can be treated as one with a modified metric \(\tilde{g}^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu\) so that

\[
S = \int d^5x \bar{\psi}[\tilde{g}^{\mu\nu}(i\gamma_\mu \partial_\nu - e\gamma_\mu A_\nu) - m] \psi. \tag{54}
\]

The Feynman rules are the same as above, presented at Fig. 1. So, again we consider just the same graphs depicted at Figs. 2 and 3. We carry the calculations within the framework of the dimensional regularization which ensures the finiteness of all one-loop results. Since we mostly will be interested in the five-dimensional results, after the calculations we must put \(D = 5\).

Actually, here we proceed with the calculation of \([10]\) in a five-dimensional space-time. So, in this section we adopt the calculations carried out in the section III, for the five-dimensional space-time.

To calculate the matrix traces one should remind that in five-dimensional space the gamma matrices are \(4 \times 4\) with \(\gamma^0 \ldots \gamma^3\) being the same as in four dimensions, and the five-dimensional \(\gamma^4\) coinciding with the four-dimensional chirality matrix \(\gamma_5\) (indeed, \(\gamma_5\) anticommutes with each \(\gamma^\mu\), \(\mu = 0 \ldots 3\), and \((\gamma_5)^2 = -1\)). The gamma matrices defined in this way satisfy the definition \(\\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\), with \(\eta^{\mu\nu} = diag(+ - \ldots -)\).

Also, we can verify that the four-dimensional relation for the trace of the product \(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma\) is valid also in five space-time dimensions

\[
\begin{align*}
\text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) &= 4(\eta^{\mu\nu} \eta^{\rho\sigma} - \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho}), \\
\text{tr}(\gamma^\mu \gamma^\nu) &= 4\eta^{\mu\nu}. \tag{55}
\end{align*}
\]
Carrying out the Wick rotation and proceeding with a quite long calculation following
the same lines as in the previous sections, that is, it involves integrations over momenta,
summing up all terms, taking into account only the contributions of the second order in
external momenta, and finally returning to coordinate representation, we arrive at the
following final result for the zero order in $u^\mu u^\nu$,

$$\Pi_0^{\mu\nu} = A_0(\xi) \left( k^\mu k^\nu - k^2 g^{\mu\nu} \right),$$  \hspace{1cm} (56)

where

$$A_0(\xi) = \frac{e^2}{256\pi^2 \sqrt{k^6}} \left( 16\sqrt{k^2} m^3 + 8k^2 m^2 \log \xi - 12\sqrt{k^6} m - 3k^4 \log \xi + 16m^4 \log \xi \right),$$  \hspace{1cm} (57)

with $\xi = \frac{2m + \sqrt{k^2}}{2m - \sqrt{k^2}}$. Then, for small $k^2$, we have

$$\Pi_0^{\mu\nu} = -\frac{e^2 m}{12\pi^2} \left( k^\mu k^\nu - k^2 g^{\mu\nu} \right) + \cdots.$$  \hspace{1cm} (58)

Now, in the first order in $u^\mu u^\nu$ we find:

$$\Pi_1^{\mu\nu} = A_1(\xi) \left[ u^2 k^2 g^{\mu\nu} + 2u^\mu \left( k^\nu (u \cdot k) - k^2 u^\nu \right) + k^\mu \left( 2u^\nu (u \cdot k) - u^2 k^\nu \right) \right] + B_1(\xi)(u \cdot k)^2 g^{\mu\nu} + C_1(\xi)(u \cdot k)^2 k^\mu k^\nu,$$  \hspace{1cm} (59)

where

$$A_1(\xi) = \frac{e^2}{256\pi^2 \sqrt{k^6}} \left( 16\sqrt{k^2} m^3 - 8k^2 m^2 \log \xi - 12\sqrt{k^6} m + 3k^4 \log \xi - 16m^4 \log \xi \right),$$  \hspace{1cm} (60)

$$B_1(\xi) = \frac{e^2}{256\pi^2 \sqrt{k^6}} \left( 16\sqrt{k^2} m^3 + 8k^2 m^2 \log \xi + 36\sqrt{k^6} m - 9k^4 \log \xi - 16m^4 \log \xi \right),$$  \hspace{1cm} (61)

$$C_1(\xi) = \frac{e^2}{256\pi^2 \sqrt{k^6}} \left( -48\sqrt{k^2} m^3 + 8k^2 m^2 \log \xi - 12\sqrt{k^6} m + 3k^4 \log \xi + 48m^4 \log \xi \right).$$  \hspace{1cm} (62)

We can easily observe that for small $k^2$, we obtain the following contribution to the self-energy tensor of the second order in $u^\mu$:

$$\Pi_1^{\mu\nu} = \frac{e^2 m}{12\pi^2} \left[ -2u^\mu k^\nu (u \cdot k) - 2u^\nu k^\mu (u \cdot k) - u^2 k^2 g^{\mu\nu} + u^2 k^\mu k^\nu + 2k^2 u^\mu u^\nu \
+ 2(u \cdot k)^2 g^{\mu\nu} \right] + \cdots.$$  \hspace{1cm} (63)
Finally, for the next (second) order in $u^\mu u^\nu$, one obtains the following contribution to the self-energy tensor:

$$
\Pi_2^{\mu\nu} = A_2(\xi)u^4 (k^\mu k^\nu - k^2 g^{\mu\nu}) + B_2(\xi)(u \cdot k)^2 \left[ 4k^2 u^\mu u^\nu + k^\mu (u^2 k^\nu - 4u^b(u \cdot k)) \right] \\
+ C_2(\xi)(u \cdot k)^4 g^{ab} + D_2(\xi)u^2(u \cdot k)^2 g^{\mu\nu} + E_2(\xi)u^2 u^\mu u^\nu + F_2(\xi)u^\mu k^\nu (u \cdot k)^3 \\
+ G_2(\xi)u^2 u^\nu k^\mu (u \cdot k) + H_2(\xi)u^2 u^\mu k^\nu (u \cdot k) + I_2(\xi)k^\mu k^\nu (u \cdot k)^4,
$$

where

$$
A_2(\xi) = \frac{e^2}{256\pi^2 \sqrt{k^6}} \left( 16\sqrt{k^2} m^3 - 8k^2 m^2 \log \xi - 12\sqrt{k^6} m + 3k^4 \log \xi - 16m^4 \log \xi \right),
$$

$$
B_2(\xi) = \frac{e^2}{512\pi^2 \sqrt{k^{10}}} \left( 48\sqrt{k^2} m^3 - 8k^2 m^2 \log \xi + 12\sqrt{k^6} m - 3k^4 \log \xi - 48m^4 \log \xi \right),
$$

$$
C_2(\xi) = \frac{e^2}{512\pi^2 \sqrt{k^{10}} (k^2 - 4m^2)} \left( 192\sqrt{k^2} m^5 + 80k^2 m^4 \log \xi - 64\sqrt{k^6} m^3 \\
+ 28k^4 m^2 \log \xi + 36\sqrt{k^{10}} m - 9k^6 \log \xi - 192m^6 \log \xi \right),
$$

$$
D_2(\xi) = \frac{e^2}{512\pi^2 \sqrt{k^6}} \left( -16\sqrt{k^2} m^3 - 8k^2 m^2 \log \xi - 36\sqrt{k^6} m + 9k^4 \log \xi + 16m^4 \log \xi \right),
$$

$$
E_2(\xi) = \frac{e^2}{256\pi^2 \sqrt{k^2}} \left( 16\sqrt{k^2} m^3 - 8k^2 m^2 \log \xi - 12\sqrt{k^6} m + 3k^4 \log \xi - 16m^4 \log \xi \right),
$$

$$
F_2(\xi) = \frac{e^2}{128\pi^2 \sqrt{k^{10}}} \left( -48\sqrt{k^2} m^3 + 8k^2 m^2 \log \xi - 12\sqrt{k^6} m + 3k^4 \log \xi + 48m^4 \log \xi \right),
$$

$$
G_2(\xi) = H_2(\xi) = -E_2(\xi)
$$

$$
I_2(\xi) = -\frac{e^2}{512\pi^2 \sqrt{k^{10}} (k^2 - 4m^2)} \left( 960\sqrt{k^2} m^5 + 144k^2 m^4 \log \xi - 64\sqrt{k^6} m^3 \\
+ 12k^4 m^2 \log \xi - 12\sqrt{k^{10}} m + 3k^6 \log \xi - 960m^6 \log \xi \right).
$$

Thus, for small $k^2$, we obtain

$$
\Pi_2^{\mu\nu} = \frac{e^2 m}{12\pi^2} \left[ u^4 (k^2 g^{\mu\nu} - k^\mu k^\nu) + u^2 (u^\mu k^\nu (u \cdot k) + u^\nu k^\mu (u \cdot k) - (u \cdot k)^2 g^{\mu\nu} - k^2 u^\mu u^\nu) \right].
$$
The contribution to the effective action generated by $\Pi_{1}^{\mu\nu}$ and $\Pi_{2}^{\mu\nu}$ are, respectively:

$$\Sigma_{1} = \frac{1}{2} A_{\mu} \Pi_{1}^{\mu\nu} A_{\nu} = \frac{me^{2}}{24\pi^{2}} [2u_{\mu} F^{\mu\lambda} u_{\nu} F^{\nu\lambda} - \frac{1}{2} u^{2} F_{\mu\nu} F^{\mu\nu}] ;$$

$$\Sigma_{2} = \frac{1}{2} A_{a} \Pi_{1}^{ab} A_{b} = \frac{me^{2} u^{2}}{12\pi^{2}} [-u_{\mu} F^{\mu\nu} u_{\lambda} F^{\lambda\nu} + \frac{1}{2} u^{2} F_{\mu\nu} F^{\mu\nu}] .$$

Each of these terms, being finite due to the well-known properties of the dimensional regularization in an odd-dimensional space-time, evidently reproduces the desired structure of the aether term whose different properties were discussed in [8]. The sum of these terms is a complete Lorentz-breaking-based contribution to the Maxwell term and the aether term.

V. SUMMARY

We considered the CPT-even Lorentz-breaking extension of the QED. We succeeded to show that the aether term naturally arises as a quantum correction. In the four-dimensional case it diverges which requires an introduction of corresponding counterterms, both in Maxwell and in aether sectors, so, in principle, one needs to introduce the aether term already at the tree level (for a detailed discussion of renormalization in Lorentz-breaking theories, see [11]). However, the theory extended in this way is clearly renormalizable since both the coupling and the Lorentz-breaking vector are dimensionless, while in the five-dimensional case the aether term is finite due to magic of the dimensional regularization. Nevertheless, in other regularizations it evidently diverges. In principle, in five dimensions we can treat the gauge field as a purely external one, therefore in this case the perturbative series closes in one loop, with no renormalization problem in a purely gauge sector. We expect that these results probably can play an important role within the extra dimension context [8]. It is interesting to note that for the light-like $u_{\mu}$, only the lower order in $u_{\mu}$ yields nontrivial contributions both to the Maxwell and to the aether terms.

The importance of our results consists in the fact that in the four-dimensional space-time our theory is all-loop renormalizable, just as the electrodynamics with the CFJ term. Another important feature of our theory consists in arising of an effective metric, therefore the space-time turns out to be the affine one, as it occurs for the CPT-even Lorentz-
breaking theories \cite{13}. It is natural to expect that it can open the way to implement the Lorentz symmetry breaking within the supergravity.

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