Mathematical representation of pressure regulator with variable characteristic

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Abstract

One of major pressure regulator features is its static characteristic. It is necessary to choose parameters fulfilling static characteristic requirements while designing a regulator. The article observes possible requirements imposed on static characteristic and shows mathematical description of balance of pressure regulator with variable characteristic. Static characteristics are plotted for some of parameters being varied and others fixed. Influence of parameters is analyzed on the base of obtained data and recommendations for its choosing are given.

Introduction

Variable-displacement pumps with pressure regulator became widespread in application for hydraulic servos of portable machines and other industrial fields. Mathematical modeling of such devices is an important problem \cite{1}-\cite{9} as well as of hydraulic servo and other devices included into it \cite{10}-\cite{17}.

Choo regulator is a complex and problematic task \cite{18}-\cite{20}, therefore influence of each the parameters should be considered and trends should be revealed.

Existing hydrosystems working at constant pressure require pressure level to be changed according to operation performed by technological hydraulic hardware. We will use NP-96 high pressure pump arrangement for analytical research. The pump is a variable-displacement plunger pump, which has a functional ability of regulator pressure setup correction using service liquid under pressure generated by control unit under pressure formed by control unit.

This pump design shaft considers that the bar rotates between two roller bearings and it is connected with driving prings by splines while adjustable angle cam plate is fixed in body frame on half axles. Adjustable angle cam plate is controlled by control plunger which is reacted against by a spring. Service liquid under reduced pressure is supplied to control plunger. Reduced pressure level depends on control valve position. The liquid is driven to draining channel through a plunger hole. Pump delivery pressure is applied to one face of control valve, while spring force and regulator setting correcting pressure is applied to the other one. Control valve position in steady mode provides equality of liquid flow through filling and draining openings between valve and shell casings edges. Pressure under control plunger is constant, control plunger effort is equal to spring effort, and adjustable angle cam plate position provide pump supply and pressure required by technological equipment.

While regulator setting correcting pressure increases, control valve moves and closes service liquid supply under control plunger. Pressure under plunger decreases, angle of cam plate grows, while pump supply and pressure also grow. Growth of pump supply pressure stops control valve movement.
Pressure under plunger becomes stable but less than the initial one. Control plunger effort is equal to spring effort. Adjustable angle cam plate is set in a position providing required pressure behind the pump.

**Mathematical model of the regulator.**

![Figure 1. Principal scheme of the regulator.](image)

Piping balance equation:

\[
\frac{c_2}{R_y}(y_0 + y) - \frac{T_{hy}}{R_y} = p_y \cdot A_2
\]  

Where \( c_2 \) — control piping spring stiffness, \( y_0 \) — initial spring prepressure, \( y \) — piping position coordinate, \( T_{hy} \) — adjustable angle cam plate moment, \( R_y \) — distance between plate and actuating cylinder steering axes, \( p_y \) — control pressure, \( A_2 \) — control piping end face area.

Initial spring prepressure must compensate force applied to the piping from plate side:

\[
\frac{c_2}{R_y} \cdot y_0 = \frac{T_{hy}}{R_y}  
\]  

We get

\[
c_2 \cdot y = p_y \cdot A_2  
\]  

Valve balance equation:

\[
c_1 (x_0 + x) + p_{ex} \cdot A_{ey} = p_s \cdot A_{ef}
\]
Where, \( c_1 \)- stiffness of spring affecting on valve, \\
\( x_0 \)- initial spring prepressure, \\
\( x \)- displacement between valve and neutral point, \\
\( p_{ex} \)- external pressure, \\
\( A_0 \)- area to which external pressure is applied, \\
\( p_s \)- supply pressure, \\
\( A_{vf} \)- valve end face area.

Connection between \( p_y \) and \( p_s \):

\[
p_s = p_y \cdot \frac{(A_{sl} + f_d)^2 + A_{s}^2}{A_s^2}
\]

(5)

Where:

\( A_{sl} = b \cdot X_{sl} \), \\
\( A_s = b \cdot X_s \),

\( b \)- summarized width of distributor windows

\( f_d \)- area of throttle draining control chamber.

Following model of valve geometry is used:

\[
S + 2R + x + S + 2R + x \sqrt{(
\delta + 2R)^2 + 0.25}\cdot
\]

\[
X_s = -2R + \sqrt{\delta + 2R^2}
\]

(6)

\[
S + 2R - x + S + 2R - x \sqrt{(
\delta + 2R)^2 + 0.25}\cdot
\]

\[
X_{sl} = -2R + \sqrt{\delta + 2R^2}
\]

(7)

Where:

\( R \)- valve edge radius, \\
\( \delta \)- radial gap of valve, \\
\( S \)- valve edges overlap.

Pump supply:

\[
Q_s = Q_{max} - k_y \cdot y
\]

(8)

Where:

\( Q_{max} \)- maximal pump supply.

\[
k_y = \frac{Q_{max}}{y_{max}}
\]

(9)

Where \( y_{max} \)- maximal shift of control piping.

Therefore mathematical description of pressure regulator balance is as follows:
\[ c_2 \cdot y = p_y \cdot A_2 \]
\[ c_1 (x_0 + x) + p_{ex} \cdot A_y = p_x \cdot A_f \]
\[ p_x = p_y \cdot \frac{(A_d + f_d)^2 + A_s^2}{A_s^2} \]
\[ S + 2R + x + S + 2R + x \vee \]
\[ (2) \]
\[ S + 2R - x + S + 2R - x \vee \]
\[ (2) \]
\[ \frac{\delta + 2R}{\sqrt{(\delta + 2R)^2 + 0.25 \cdot Q_s}} = Q_{\text{max}} - k \cdot y \]
\[ (\delta + 2R)^2 + 0.25 \cdot A_d = b \cdot \]
\[ -2R + \sqrt{A_s} = b \cdot \]

**Mathematical modeling results**

Equation system (10) was solved with the help of Python programming language. Matplotlib library was used for plotting. Required solution is correspondence \(Q(p)\). Static characteristics for regulator parameters chosen for design calculated for two different values \(p_{ex} = 1\text{MPa}\) and \(p_{ex} = 5.5\text{MPa}\).

![Static characteristic](image.png)

**Fig. 2.** Static characteristic

Following requirements were put on static characteristic of the regulator:
1. Certain position of sloping part of the static characteristic must correspond to certain value of external pressure.
2. Sharp slope angle of sloping part of the characteristic provides almost constant supply pressure level.
3. Slope angle of sloping part must remain sharp at different levels of external pressure.
Let’s consider influence of regulator parameters on static characteristic. To achieve that we will vary one of the parameter’s value while keeping other parameters constant. In order to check correspondence to the 3d requirement put on the regulator static characteristic, we will carry external pressure level, choosing values of $P_{ex} = 1$ MPa and $P_{ex} = 5.5$ MPa. The results are shown on one plot.

$Q_{max}$ - pumping unit parameter defining regulator operating range. This parameter influences the height of horizontal part of the static characteristic.

Let’s consider parameters influencing sloping part of the static characteristic. $R_y$ is reversely proportional to $k_y$. Variations of $k_y$ give us the following plots.

![Fig. 3. Influence of $R_y$ on static characteristic](image)

The plots allow to conclude that decreasing parameter $R_y$ leads to increase of slope angle of static characteristic.

Influence of parameters $c_2$, $\frac{1}{A_p}$ on static characteristic is the same. In order to increase slope of the plot those parameters values should be as low as possible. However, it’s important to note that moment on the regulating unit has to be compensated by increasing values of parameters $c_2$, $R_y$, $y_0$.

Parameters $b$ and $f_d$ also influence the slope. Value of parameter $b$ is limited by technological requirements. The plots show that parameter $b$ should as big as possible. It allows to improve static characteristic and increase speed of control of pressure control piston. However, since the speed of control of pressure control piston often is an initial parameter for regulator design, change of windows width may cause a need to increase control of pressure control piston spring stiffness. It’s not desirable according to previous conclusions regarding static characteristic.

Design process allows to use quite wide range of values of parameter $f_d$. By decreasing the parameter it’s possible to improve static characteristic and compensate conflict of choice of $b$ and $c_2$ parameter values.
Let’s consider parameters influencing initial regulating pressure level. Such parameters are $c_1$, $x_0$, $A_y$, $A_{sf}$. During design process these parameters are defined basing on initial data.
Fig. 6. Influence of $c_1$ on static characteristic

Fig. 7. Influence of $x_0$ on static characteristic
Let’s consider affection of valve microgeometry on statistic characteristics of controller.
Fig. 10. Influence of $R_{on}$ static characteristic

Fig. 11. Influence of $S_{on}$ static characteristic
These parameters are limited by technological requirements. The plots allow to conclude that the parameters values should be as low as possible, but if they are low enough, further decrease doesn’t change static characteristic remarkably.

Conclusions.
Pressure regulator static characteristic changeable by external pressure level was obtained. Represented mathematical representation takes into account affection of microgeometry of valve on static characteristic. Analysis of regulator parameter affection on its static characteristic is held. Also in this article were shown demands on static characteristics and recommendations about its support. Some allowances have been made in this model. For example, moment on piston cam plate changes in a minor way.

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