Cosmic rays from thermal sources

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Abstract

Energy spectrum of cosmic rays (CR) exhibits power-like behavior with very characteristic "knee" structure. We consider a generalized statistical model for the production process of cosmic rays which accounts for such behavior in a natural way either by assuming existence of temperature fluctuations in the source of CR, or by assuming specific temperature distribution of CR sources. Both possibilities yield the so called Tsallis statistics and lead to the power-like distribution. We argue that the "knee" structure arises as result of abrupt change of fluctuations in the source of CR. Its possible origin is briefly discussed.

Key words: Cosmic rays, energy spectra, thermodynamics in astrophysics

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1 Introduction

The origin of the characteristic features of the energy spectrum of cosmic rays (CR), which has power-like behavior with "knee" structure, remains matter of hot debate (for survey of models proposed to explain the origin of CR see [1]). It could reflect different regimes of diffusive propagation of CR in the Galaxy, but it could also be due to some property of acceleration processes within the source of the CR

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itself. In this second case, a crucial question is whether the sources of CR below the "knee" can also accelerate particles to much higher energies, so that a single population of astrophysical objects can explain the smooth spectrum of cosmic rays, as observed over many orders of magnitude in energy. We address this problem using a generalized statistical model specially adapted to this end. However, our work will concentrate more on the physics of CR than on the generalized statistics, providing therefore some physical explanations to ideas presented already in [2] and [3]. In particular, we shall argue that the observed "knee" structure of the CR energy spectrum arises as result of some abrupt change of fluctuation pattern in the source of CR.

2 Nonextensive statistics and results

We shall start with some basic information on nonextensive statistical mechanics as introduced by Tsallis [4], which have been already successfully applied to a variety of complex physical systems, including CR where, among others, the energy spectrum of cosmic rays have been analyzed from a nonextensive point of view [2,3]. The idea is to maximize the more general entropy measure than the usual Boltzman-Gibbs-Shanon (BGS) entropy, the one which depends on a new additional parameter $q$ and which leads to generalized version of the statistical mechanics. If we optimize, under appropriate constrains, the BGS entropy,

$$S = - \int dE \, P(E) \ln P(E),$$

we obtain the equilibrium distribution in the usual form of exponential distribution,

$$P(E) = \frac{1}{T} \exp \left( -\frac{E}{T} \right).$$

This distribution can alternatively be obtained as the solution of simple differential equation:

$$\frac{dP(E)}{dE} = - \frac{P(E)}{T}. \quad (3)$$

A more general formalism proposed in [4] (and sometimes referred to as nonextensive statistical mechanics) is based on the generalized entropy,

$$S_q = - \int dE \, P^q(E) - \frac{1}{q - 1}. \quad (4)$$
Its maximization under appropriate constrains yields a characteristic power-like distribution (which sometimes is also called $q$-exponential distribution, $\exp_q(\ldots)$):

$$P_q(E) = \frac{2 - q}{T} \exp_q \left( -\frac{E}{T} \right) = \frac{2 - q}{T} \left[ 1 - (1 - q)\frac{E}{T} \right]^{\frac{1}{1-q}}. \quad (5)$$

For $q \to 1$ one recovers the usual exponential distribution (2). This equilibrium distribution can alternatively be obtained by solving the following differential equation

$$\frac{dP(E)}{dE} = -\frac{P_q(E)}{T}. \quad (6)$$

Precisely this equation has been used in [2] to describe the flux $\Phi(E)$ of cosmic rays. However, the values of the temperature obtained there seem to be uncomfortably high.

On the other hand there is growing evidence that nonextensive formalism applies most often to nonequilibrium systems with a stationary state that posses strong fluctuations of the inverse temperature parameter $\beta = 1/T$ [5,6]. In fact, fluctuating $\beta$ according to gamma distribution with variance $Var(\beta)$ results in a power like distribution (5) with nonextensivity parameter $q$ being given by the strength of these fluctuations,

$$q - 1 = \frac{Var(\beta)}{\langle \beta \rangle^2}, \quad (7)$$

distribution (2) is just its limiting case when $q \to 1$. This observation was used in [3] to describe the flux $\Phi(E)$. Although the results were reasonably good the estimated temperature $T \sim 170$ MeV (comparable with the so called Hagedorn temperature known from description of hadronization processes) seems to be, again, overestimated. This was because author insists on description of the whole range of energy spectrum including its very low energy part which, in our opinion, is governed mainly by the geomagnetic cut-off and should be considered separately.

### 2.1 Energy spectrum

For relativistic particles (where the rest mass $m$ can be neglected) the energy $E \sim p$ and the density of states of an ideal gas in three dimensions is given by $\omega(E) \propto E^2$. The flux $\Phi(E)$ can be then obtained straightforwardly from $P(E)$ and reads

$$\Phi(E) = N_0 E^2 P(E), \quad (8)$$

where $N_0$ is normalization factor. For $E >> T$ we have power spectrum $\Phi(E) \propto E^{-\gamma}$ with the slope parameter $\gamma$ which in terms of parameter $q$ introduced above is
\( \gamma = (3 - 2q)/(q - 1) \). In the case of CR this spectrum has the shape of broken power law \( E^{-\gamma} \) which changes pattern in the region named as "knee" with the slope \( \gamma_1 \simeq 2.7 \) at energies below \( \sim 10^{15} \) eV and \( \gamma_2 \simeq 3.1 \) at energies between the "knee" and the highest measurable energies \( E \sim 10^{18} \) eV. In the language of the nonextensivity parameters it would mean that \( q_1 = 1.213 \) before and \( q_2 = 1.196 \) after the "knee", i.e., one can argue then that at the "knee" one witnesses the change of fluctuation pattern.

2.2 Temperature fluctuations

As mentioned above the special role in converting exponential distribution to its \( q \)-exponential counterpart play fluctuations of the scale parameter provided in the form of gamma function. There are at least two scenarios leading to gamma distribution in \( \beta \) mentioned above (here \( \mu^{-1} = \beta_0(q - 1) \) and \( \nu^{-1} = q - 1 \)):

\[
f(\beta) = \frac{\mu}{\Gamma(\nu)} (\mu \beta)^{\nu-1} \exp(-\beta \mu), \tag{9}
\]

(a) temperature distribution of sources;
(b) temperature fluctuations in small parts of a source.

In what concerns the first possibility notice that gamma distribution (9) is the most probable outcome of the maximalization of Shannon information entropy (1) under constraints that \( \int f(\beta) d\beta = 1 \), \( \langle \beta \rangle = \beta_0 \) and (because distribution we are looking for is one sided, i.e., defined only for \( \beta > 0 \)) that \( \langle \ln(\beta) \rangle = \ln(\nu \beta_0) \). However, this possibility is rather unlikely because in this case one expects that there is some cut temperature \( T_{\text{cut}} \) such that \( T < T_{\text{cut}} \), which would result in very characteristic rapid break in the CR energy spectrum, not observed in experiment.

To illustrate the second possibility let us suppose that one has thermodynamic system, different small parts of which have locally different temperatures, i.e., its temperature understand in the usual way fluctuates. Let \( \xi(t) \) describes the stochastic changes of temperature in time and let it be defined by the white Gaussian noise (\( \langle \xi(t) \rangle = 0 \) and \( \langle \xi(t)\xi(t + \Delta t) \rangle = 2D\delta(\Delta t) \)). The inevitable exchange of heat which takes place between the selected regions of our system leads ultimately to the equalization of temperature. As we have advocated in [3], the corresponding process of the heat conductance leads eventually to the gamma distribution \( f(\beta) \) (9) with variance (7) related to the specific heat capacity \( C_V \) of the material composing this system by

\[
q = 1 + \frac{1}{C_V}. \tag{10}
\]
The change of fluctuation pattern in the "knee" region mentioned before would therefore correspond in this case to abrupt change in the heat capacity of the order of $C_2/C_1 = 1.09$.

Fig. 1. Example of the possible modification of gamma distribution needed to describe fluctuation of temperature which could lead to description of the "knee region" of the CR spectra. At $T = 10^9$ MeV parameter $q$ changes from $q = 1.214$ (it would result in dotted line) to $q = 1.2$ (in this case curve changes slope and continues as full line). Such break represents some abrupt change in the fluctuation pattern, which we attempt to explain here.

2.3 Heat capacity

Can one expect something of this kind to happen in the astrophysical environment of the CR? In what follows we shall argue that, indeed, one can. Let us first notice that subject of temperature fluctuations in astrophysics is much-discussed problem nowadays. Its effect on the temperatures empirically derived from the spectroscopic observation was first investigated in [7] whereas in [8,9,10,11] it was shown that temperature fluctuations in photoionized nebulae have great importance to all abundance determinations in such objects. It means that discussion of the heat capacity or, equivalently, the behavior of parameter $q$ defining the energy spectrum, is fully justified.

Let us concentrate therefore on the problem of heat capacity of astrophysical objects, in particular in neutron stars. In such objects one observes the following feature. The total specific heat of their crust, $C$, is the sum of contributions from the relativistic degenerate electrons, from the ions and from degenerate neutrons. In the temperature that can be reached in the crust of an acreating neutron star (which is of the order of $T \sim 5 \cdot 10^8$ K and is below the Debaye temperature $T_D \sim 5 \cdot 10^9$ K) we have
\(C_{\text{ion}} < C_e < C_n\). When the temperature drops below the critical value \(T = T_C\) the neutrons become superfluid and their heat capacity \(C_{sf}^n\) increases \([12,13]\),

\[
\frac{C_{sf}^n}{C_n} \simeq 3.15 \frac{T_C}{T} \exp \left(-1.76 \frac{T_C}{T}\right) \cdot \left[2.5 - 1.66 \left(\frac{T}{T_C}\right) + 3.68 \left(\frac{T}{T_C}\right)^2\right].
\] (11)

At \(T \sim 0.7T_C\) we have \(C_{sf}^n \sim 1.1C_n\) what corresponds to the changes of spectral index by \(\Delta \gamma \sim 0.5\). To summarize: one witnesses here the abrupt change in the heat capacity at some temperature.

Fig. 2. CR energy spectra fitted by simple Tsallis distribution (left panel) and distribution using specially adopted fluctuation function example of which is presented in Fig. 1 (right panel). Notice the total inadequacy of simple exponential (Boltzman, denoted by BG here) distribution in description of these spectra.

Suppose now that we take seriously the conjecture expressed by eq. (10) that \(C_V\) is directly connected with the parameter \(q\). It means then that, the usual fluctuation pattern given by gamma distribution \([9]\) should be replaced by it slightly modified version shown in Fig. 1, which is characterized by two nonextensivity parameters, \(q_1\) and \(q_2\). This change is assumed to be abrupt and taking place at some temperature \(T_{cut}\) and it differs our proposition from what was proposed in \([2]\), which from our point of view, corresponds to some form of composition of two (suitably normalized) such gamma functions with different \(q\) each. Following our proposition one obtains the following flux of CR:

\[
\Phi(E) = N_0 E^2 \cdot \left[P_{q_1}(E) - \alpha_1(E)P_{q_1}(E) + \alpha_2(E)P_{q_2}(E)\right],
\] (12)

where \(P_q(E)\) is given by eq. \([5]\) and

\[
\alpha_i = \Gamma \left[\frac{1}{q_i - 1}, \frac{1}{(q_i - 1)T_{cut}/T}\right] / \Gamma \left(\frac{1}{q_i - 1}\right).
\]
Our results are presented in Fig. 2. The "knee" region is reproduced very well, however, the price to be paid is the need of suitable choice of energy at which fluctuation pattern changes (cf. Fig. 1).

2.4 Acceleration

As one can see in Fig. 2 we have obtained agreement with data for the whole range of CR energy spectrum but the price paid for this is, again, apparently too high value of the temperature parameter used, \( T = 100 \text{ MeV} \). We have prone therefore to the same kind of criticism as we have applied to previous attempts in this field \[2,3\]. The possible way out of this dilemma is to argue that the break in the original spectrum and connected with it phase transition occurs actually at much slower energies and that resultant spectrum is then accelerated to the observed energies - for example by the magneto-hydrodynamical turbulence and/or shock discontinuities (i.e., by the so called DSA mechanism, cf. [19]). The simplest version of this mechanism, as discussed in [20], implies that distribution function after the shock, \( f_{\text{after}}(p) \), is related to the original distribution before the shock, \( f_{\text{before}}(p) \) in the following way:

\[
  f_{\text{after}}(p) = \frac{b}{p^b} \int_{p_{\text{min}}}^{p} dp' \left(p' (b-1) f_{\text{before}}(p') \right), \tag{13}
\]

where \( b = 3r/(r - 1) \). Here \( p \) denotes the particle momentum, \( r = \rho_2/\rho_1 \) describes the compression of densities across the shock and \( p_{\text{min}} \) denotes the minimal value of momenta. DSA mechanism transforms a \( \delta(p - p_0) \) spectrum of relativistic particles in a power-like spectrum of the type \( f(p) \propto p^{-b} \). For example, if one has initial spectrum of the form \( \propto p^{-c} \) which encounters a shock with strength given by \( b \) then eq. (13) shows that:

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1 Two remarks are in order at this point. First, the nucleon superfluidity was predicted already in [14] and today pulsar glitches provide strong observational support for this hypothesis [15]. Nucleon superfluidity arises from the formation of Cooper pairs of fermions (actually in [16] also quark superfluidity from cooling neutron stars were investigated). Continuous formation and breaking of the Cooper pairs takes place slightly below \( T = T_C \) (critical temperature \( T_C \) is in the order \( 10^9 - 10^{10} \text{ K} \)). The other, neutron stars are born extremely hot in supernova explosions, with interior temperatures around \( T \sim 10^{12} \text{ K} \). Already within a day, the temperature in the central region of the neutron star will drop down to \( \sim 10^9 - 10^{10} \text{ K} \) and will reach \( 10^7 \text{ K} \) in about 100 years [17]. The first measurements of the temperature of a neutron star interior (core temperature of the Vela pulsar \( T \sim 10^8 \text{ K} \), while the core temperature of PSR B0659+14 and Geminga exceeds \( 2 \cdot 10^8 \text{ K} \) allow to determine the critical temperature \( T_C \sim 7.5 \cdot 10^9 \text{ K} \) [18].
in the case of $b > c$ (which corresponds to the initial spectrum being softer than it would result from a $\delta$-function injected into the shock) one has $f_{after} \propto p^{-c}$, i.e., the acceleration does not change the shape of the spectrum;

in the case of $b < c$ (i.e., for the steep initial spectrum) one has $f_{after} \propto p^{-b}$, which coincides with result of injection of a $\delta$-function into the shock.

It can be then shown from eq. (13) that when an ensemble of shocks is encountered, the shape of the spectrum should be given by the strongest shock [20].

That such scenario is \textit{a priori} plausible in the case considered here can be seen by the following argumentation. If the energy growth is given by

$$\frac{\partial E}{\partial t} = a + bE,$$

then from master equation

$$\frac{\partial P(E)}{\partial t} = -c P(E)$$

one gets the evolution equation

$$\frac{\partial P(E)}{\partial E} = -c P(E) \frac{\partial t}{\partial E} = -\frac{P(E)}{T(E)}$$

in the form of eq.(3) with energy dependent temperature parameter:

$$T(E) = \frac{a + bE}{c} = \frac{T_0 + (q-1)E}{q},$$

where we have used: $c = T_0^{-1}$, $a = q^{-1}$ and $b = (1 - q^{-1})T_0^{-1}$. Energy dependent temperature parameter $T(E)$ in this form immediately leads to the energy distribution given by eq.(5). Notice that $q^{-1}$ plays here the role of the weight with which we select the constant (thermal) and linear (accelerating) terms in the equation describing the growth of energy. It means therefore that Tsallis distribution preserves its structure when subjected to the aforementioned acceleration scheme. The problem, which remains to be solved is whether the broken spectrum with the "knee" structure can be suitably transformed in the same way. In particular, whether the "knee" structure is preserved and how its position before the acceleration process is related to that actually observed.

To summarize this part: stochastic mechanisms of acceleration of CR particles (like acceleration on the fronts of shock waves or Fermi acceleration in turbulent plasmas, both analogous in some sense to Brownian motion) do not change the shape of the production spectra, but, unfortunately, they are not particularly effective, i.e., they do not lead to large increase of energy [21]. The increase of energy per one collision is of the order $\Delta E/E \sim u^2/c^2$, what for the plasma velocity $u \approx 10^6 - 10^7$ cm/s
gives $\Delta E/E \approx 10^{-8}$ and leads to the mean relative increase of energy during the time life of Galaxy ($t \approx 3 \cdot 10^{17}$ s) only by factor $\langle \delta \rangle \approx 3$. Fluctuation on the steep spectrum of accelerated particles result in additional increase of energy. Because of the multiplicative character of acceleration we have log-normal distribution of variable $\delta$, $P(\delta)d\ln\delta = \left(\sqrt{2\pi}\sigma\right)^{-1}\exp\left[-(\ln\delta - \ln\langle\delta\rangle)^2/2\sigma^2\right]d\ln\delta$, what results in shift of the spectrum of source on the energy scale by $\langle\delta\rangle^{(\gamma-1)/\gamma}\exp\left[(\gamma - 1)^2\sigma^2/(2\gamma)\right]$, where $\sigma^2$ is variation of the distribution $P(\delta)$. For $\gamma \approx \langle\delta\rangle \approx \sigma^2 \approx 3$ we can obtain only order of magnitude shift of the energy spectrum.

3 Summary and conclusions

Let us summaries arguments presented above.

- The spectrum of cosmic rays has the shape of broken power law $E^{-\gamma}$, with the slope $\gamma_1 \simeq 2.7$ at energies below $\sim 10^{15}$ eV and $\gamma_2 \simeq 3.1$ at energies between the knee and $E \sim 10^{18}$ eV. This slopes correspond to the nonextensivity parameters (taking into account that $q = (3 + \gamma)/(2 + \gamma))$ $q_1 = 1.213$ and $q_2 = 1.196$, respectively, i.e., to the change of heat capacity of the order of $C_2/C_1 = 1.09$.
- Nonextensive statistics successfully describes the smooth power-law spectrum and traces its origin back to fluctuations of temperature, $f(\beta)$, being given by gamma distribution.
- Out of two possible scenarios leading to such distribution of inverse temperature we prefer the temperature fluctuation in the source rather than the temperature distribution of sources. The point is that in the second case some cut temperature $T_{\text{cut}}$ is expected which would result in the rapid break in the energy spectrum and which is not observed. The temperature $T$ (not essential in the high energy region, $E >> T$) seems to be of the order of MeV, i.e. of the order of the interior stars temperature (if stars are born extremely hot in supernova explosions, with interior temperatures around $T \sim 100$ MeV, already within a day the temperature in the central region of the star will have dropped down to $\sim 0.1 - 1$ MeV and reach the 1 keV in about 100 years). The critical temperature (corresponding to the nucleon superfluidity) is $T_C \sim 0.1 - 1$ MeV. It means then that the origin of changes of the nonextensivity parameters at temperature $T_{\text{cut}} \simeq 10^{15}$ eV $\simeq 10^{19}$ K is still open question.
- The nonextensive formalism leads to production (injection) spectrum and the acceleration processes ($dE/dt \sim E$, which does not change the shape of power spectrum) and allows the shift of this spectrum to high energies. The question of how to connect ”knee” position before and after such acceleration process remains,
Let us close with some very intriguing observation. Independently of the discussion presented above one can notice that the measured CR energy spectrum can be converted using arguments presented above (eq. \ref{eq:10} connecting Tsallis parameter $q$ with the heat capacity) into energy dependence of the heat capacity $C$. The result is shown in Fig. 3. As one can see $C$ acts here as a kind of magnifying glass converting all subtle structures of $\Phi(E)$ into much more pronounced and structured bump. Its importance would parallel long-standing discussion of the origin of the knee-like structure of the CR energy spectrum, but exposed in much more dramatic and visible way. At the moment we can only offer two examples of the possible explanation of this feature. The first is that this effect is due to the change of the effective number of degrees of freedom in the incoming projectiles with energy. Assuming that most of CR consist of protons, which are build from three quarks, one could speculate that each bumps correspond to excitations from single proton to proton plus one quark and two quark structure, after exciting all three quarks one comes back to the original situation (much in the spirit of the changes observed when ice becomes water and this becomes steam - there also corresponding $C$’s show characteristic jump \cite{22}). The other, perhaps more realistic explanation of Fig. 3 is to connect it with the behavior of heat capacity in Fermi liquids. Following \cite{12} the proton heat capacity $C \sim m^*/m$, i.e., it is proportional to the ratio of the effective mass of the proton in the neutron fluid to the mass of the free proton. In the case of a mixture of Fermi liquids the proton effective mass $m^*$ is affected by interactions with neutrons.
and other protons and is given by

\[
\frac{m^*}{m} = 1 + \frac{1}{3} D_p \left[ f_{pp}^{1} + \left( \frac{k_{F_n}}{k_{F_p}} \right)^{2} f_{pn}^{1} \right],
\]

where \( D_p \) denotes the density of quasiparticle states at Fermi surface given by wave vectors \( k_{F_n} \) and \( k_{F_p} \) for, respectively, neutrons and protons whereas \( f_{pp}^{1} \) and \( f_{pn}^{1} \) are Landau parameters \([23]\). Fig. 3 can be then interpreted as showing changes of \( C \) with energy in the Fermi liquid. We start with the superfluid liquid with \( C_1 = 4.7 \) (here \( m^* \) represents effective mass for \( pp \) and \( pn \) interactions), when energy increases we stop to see nuclear interactions and \( C_2 = 5.1 \) (with \( m^* \) representing \( pp \) interactions only), finally, for large \( T \), one has Fermi gas with \( C_3 = 5.3 \) and, still further, the usual Fermi liquid \([2]\). Notice that

\[
\frac{1}{3} D_P f_{pp}^{1} = \frac{C_2 - C_3}{C_3} \quad \text{and} \quad \frac{1}{3} D_P \left( \frac{k_{F_n}}{k_{F_p}} \right)^{2} f_{pn}^{1} = \frac{C_1 - C_2}{C_3},
\]

what results in the following relation between Landau parameters,

\[
\left( \frac{k_{F_n}}{k_{F_p}} \right)^{2} \frac{f_{pn}^{1}}{f_{pp}^{1}} = \frac{C_1 - C_2}{C_2 - C_3} = 2.
\]

In the case of a one-component Fermi liquid we have the well known identity, \( m^* / m = 1 + F_{1}^{pp}/3 \), where \( F_{1}^{pp} = D_p f_{pp}^{1} \). From \([20]\) we can see that in two-component Fermi liquid the quantity \( 1 - m^* / m \) is 3 times bigger (this is because parameter \( f_{1} \) which determines interaction between quasiparticles is negative what results in smaller effective mass). From properties of excited states in nuclear matter (\( Pb \) and neighbor nuclei \([25]\)) \( F_{1}^{pp} = -0.5 \pm 0.25 \). If \( F_{1}^{pn} < F_{1}^{pp} < F_{1}^{pp} \) and taking (after \([26]\)) \( F_{1}^{pn} - F_{1}^{pp} = -0.2 \), we can estimate that for neutron-star matter one has \( m^* / m = 1 + F_{1}^{pp} \simeq 1 - 0.4 \pm 0.3 = 0.6 \pm 0.3 \).

We end by saying that the above are just plausible examples, not fully convincing explanation(s). It means then that this problem deserves further scrutiny to be performed elsewhere.

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\[2\] It is worth to remember that fluctuations of temperature we are talking about in this work refer to fluctuations in small region \( V \). For Fermi liquid the heat capacity expressed in units of Boltzmann constant \( k_B \) (i.e., for \( k_B = 1 \)) is of the order \( C \simeq 3 \cdot 10^{35} \text{ cm}^{-3} \) \([24]\). Therefore, taking values of \( C \) estimated from the slope of the primary CR spectra (cf. Fig. \([4]\) one gets that size of the region of fluctuations is \( V \simeq 10^4 \text{ fm}^3 \).
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