Computer Algebra Methods in Modeling the Dynamics of Particle Beams in Accelerators

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Abstract. This article is devoted to the methods of numerical and symbolical methods of calculations, which are based on perturbation theory and are implemented in terms of matrix formalism. These methods were used to solve various problems of modeling and optimization in accelerator physics. The basic theoretical principles are verified using methods to neural network technologies, machine learning and others.

1. Introduction

Modern methods and technologies of accelerator physics make a huge contribution in both fundamental and applied problems of science and technology in various directions. It should be noted that the main tool used to model the dynamics of particle beams is traditional numerical modeling. Requirements for efficiency of methods, for simulation results are constantly increasing from both theoretical and practical points of view. It is also necessary notice that practical methods in beam physics base on the various technologies: from both classical methods upto neural network technologies, machine learning, etc. (see, for example, [1, 2, 3]). In this case, the beam is considered as a combination of a huge number of particles. In this case traditional classical numerical methods lead to the need to implement a huge number of computational procedures, which significantly reduces their effectiveness even when using modern computational methods and technologies. The complexity of the objects and processes under consideration leads to the need to develop and implement rigorous formal methods (mathematical and informational) using modern analytical and symbolic methods. This article discusses the methods of symbolic and numerical methods of integration of dynamic processes. The proposed methods are based both on Sophus Lie algebraic methods [4] and on the matrix formalism [5]. The developed algorithms are based on both numerical and symbolic presentations using the special algorithms. The presented approaches allows also realizing efficient computational computational processes and use both parallel and distributed computational systems using methods of both network technologies and machine learning.

2. Formulation of the problem

It should be noted that traditional (classical) numerical methods lead to the need for a huge number of computational procedures, which significantly reduces their computational efficiency even when using modern computational methods and technologies. The complexity of the objects and processes under consideration leads to the need to develop and implement not
only strict formal methods (mathematical and informational), but a wider class of methods, for example, using modern analytical and symbolic methods. The article discusses both numerical and symbolic methods for integrating dynamic processes. The proposed approach allows us to efficiently (from a computational point of view) carry out numerical modeling using both parallel and distributed computing technologies. Modern accelerator systems are extremely complex systems that must provide the necessary evolution of the beam (very often with extremely high accuracy). Particular attention in modern accelerator centers and laboratories is paid to increasing the efficiency of the corresponding processes, both at the modeling stage and at the stage of supporting existing and developing new accelerator complexes. It should be especially noted that the requirements for modern installations lead to the need to develop new effective (primarily from a computational point of view) both mathematical methods and information technologies in order to increase the computational efficiency of the research. Such requirements lead to the need to develop new methods (quite specific), based on the use of both new strict (formal) methods and new software technologies. In this case, special attention should be paid both to problems of increasing the accuracy of computational methods, and to ensuring the necessary correctness of the results of computational procedures (verification stage).

Modern accelerator systems are extremely complex systems that must provide the necessary evolution of the beam. Particular attention in modern acceleration centers and laboratories is paid to increasing the efficiency of the corresponding processes, both at the modeling stage and at the stage of supporting existing and developing new accelerator complexes. It should be noted that the requirements for modern installations lead to the need to develop both new effective (from a computational point of view) mathematical methods and information technologies in order to increase the computational efficiency of implemented studies. In addition, urgent tasks are the tasks of increasing accuracy and ensuring the necessary correctness of the results of computational procedures.

3. The matrix formalism and LEGO objects concepts as basic instruments for computational procedures

Given the need for both the development of new and the modification of existing accelerator systems, special attention is paid to the creation of computationally efficient and correct mathematical and software methods. In addition, special attention is paid to the problems of creating computationally efficient software, as well as supporting the processes of analysis, synthesis and optimization of control systems in order to increase the "quality" of the particle beam. It should be especially noted that the approach proposed in the work is based on the concept of virtual objects, the formation (development) of the organization of the necessary computational procedures, and also on the search for optimal solutions. The concept of the matrix formalism [5] is used as a basic tool for developing mathematical models of accelerator systems.

The corresponding software allows not only to "track" the evolution of the beam, to form the necessary optimal control that provides the specified characteristics of the beam, but also ensures the preservation of basic properties, for example, the properties of symplecticity [5]. The use of such software allows not only to correctly configure the accelerator control system, but also to create databases of specialized elements used, for example, within the framework of the concept of LEGO [6] objects. It should also be noted that in our case (unlike [7]), the concept of LEGO objects is based on a matrix formalism. This allows you to design various options for the accelerator subsystems and its components with the aim of "gluing" objects used in various controls, as well as the formation of controls with the necessary degree of accuracy. It should also be noted that this formalism allows the formation of various complexes of control elements and the use of the obtained data in parallel (distributed) flows in the case of both operations of the same type and distributed in the case of various (e.g., in terms of the volume of computational
flows) operations. Placement in the corresponding databases both the formed objects, and also implemented prototypes of elements of a distributed control system will significantly increase the efficiency of the corresponding computational operations.

4. Neural network methods and the concept of multi-agent systems as computing tools

The possibilities of neural network methods in particle beam physics are fairly well understood. There are a large number of publications that demonstrate the effectiveness of their use, see, for example, [1]. It should also be noted that in modern accelerator systems a particle beam consists of a huge number of particles (from $10^{10}$ to $10^{20}$). We note that, only to describe the evolution of each particle, it is necessary to set at least six parameters. In addition, in many modern transportation systems, it is necessary to take into account not only the interaction of the particle beam with the external (control) field, but also the interaction of the particles with each other. In addition, for dense beams, it is necessary to take into account their self-interaction, because each particle generates its own field, which affects the dynamics of the particle itself. Accounting for similar interactions leads to the need to develop specialized software that can will significantly increase the computational efficiency of the software used. In this paper, to describe the dynamics of particles and their interaction with both an external (control) field and its own field, the concept of matrix formalism is used [5]. It should be noted that this formalism allows not only to use traditional methods of classical mathematics, but also methods of geometric integration, which not only significantly improves the performance of computational procedures, but also use the methods of computer algebra. This allows you to get decisions not only in numerical form, but also in symbolical form.

We should also note that the possibility of obtaining solutions in a symbolic representation (in the form of formulas) allows one to carry out effective parametric studies in order to identify the features of the considered dynamic systems. Moreover, the obtained solutions also allow us to carry out parametric studies in order to identify the characteristics of the dynamics of the considered dynamic processes. It should be noted that the symbolic computational procedures that allow the researcher not only to find satisfactory solutions (including optimal ones), but also to form specific solutions, the existence of which researchers often do not realize.

Note that the representation of solutions in the form of two and three-dimensional using, for example, the package Mathematica [9] ) allows us to find new solutions generated by the processes under study. Moreover, the construction from two up to four dimensional dimensional images of the studied phenomena and processes allows the researcher not only to find optimal (in one sense or another) solutions, but to search for new effective solutions using both stationary and time-varying of control objects. The similar approach can be used both for preliminary modeling in accelerator physics and for for solving optimization problems for in beam physics [10].

The obtained results demonstrated sufficient computational efficiency, which will help to significantly improve not only the quality of particle beam control, but also provide the tools for identifying the features of the systems under consideration, which will allow for targeted monitoring of the beam dynamics, thereby ensuring optimal control characteristics. Further, it is proposed to use enhanced teaching methods, taking into account the use of large volumes of data. The necessary data necessary for this will be generated by high-speed monitoring detectors, using diagnostic systems of accelerators. The necessary verification of the correctness of the obtained solutions is implemented using the generated test (tested) models with known solutions.
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