Excitation of atoms in an optical lattice driven by polychromatic amplitude modulation

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Abstract: We investigate the multiphoton process between different Bloch states in an amplitude modulated optical lattice. In the experiment, we perform the modulation with more than one frequency components, which includes a high degree of freedom and provides a flexible way to coherently control quantum states. Based on the study of single frequency modulation, we investigate the collaborative effect of different frequency components in two aspects. Through double frequency modulations, the spectrums of excitation rates for different lattice depths are measured. Moreover, interference between two separated excitation paths is shown, emphasizing the influence of modulation phases when two modulation frequencies are commensurate. Finally, we demonstrate the application of the double frequency modulation to design a large-momentum-transfer beam splitter. The beam splitter is easy in practice and would not introduce phase shift between two arms.

OCIS codes: (020.1475) Bose-Einstein condensates; (020.1670) Coherent optical effects; (270.0270) Quantum optics.

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Normally, studies of amplitude modulated lattices are based on single frequency modulations that only involve processes with one photon emission or absorption. During a polychromatic modulation, not only the amplitudes, but also the phases of different frequency components in the modulation can be controlled independently, providing a more flexible way to coherently manipulate quantum states. In this paper, we coherently transfer atoms from the ground state \( s \) to the high excited \( g \) band via a double frequency modulation. The peaks of transfer rate with

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different lattice depths are measured experimentally, while influence of the modulation phase is demonstrated by performing an interference between two independent paths of excitations. These experiments completely investigated how frequencies and phases of different modulation frequency components would influence the excitation between Bloch states. Furthermore, we show an application of the double frequency modulation to build a large-momentum-transfer (LMT) beam splitter. Comparing with LMT beam splitters based on Bloch oscillation [8,15] or high order Bragg scattering [16], our method is easy in practise and would not introduce phase shift between two separated atom clouds.

The paper is organized as follows: In Sections 2 and 3 the polychromatic modulation theory based on Floquet method and our experimental system are introduced respectively. In Section 4, we study the effects of the modulation phase for the single frequency modulation. In Section 5 collaborative effect of different frequency components is studied, demonstrate the effect of both modulation frequencies and phases. Section 6 presents a way to realize a LMT beam splitter. Discussion and conclusion are in Section 7.

2. Theory for optical lattice with polychromatic modulation

For an atom in an amplitude modulated lattice system along \( \hat{x} \) axis, as schematically shown in Fig. 1, the time dependent Hamiltonian can be written as

\[
H(t) = \frac{p_x^2}{2M} + V_0 \cos^2(k_Lx) + \sum_i V_i \cos(\omega_i t + \phi_i) \cos^2(k_Lx).
\]  

(1)

The first term in the right hand side is kinetic energy with \( M \) the atom’s mass and \( p_x \) its momentum along the \( x \) direction. The second term represents optical lattice without the modulation. \( V_0 \) is the constant part of the lattice depth, and the wave vector is \( k_L = 2\pi/\lambda \) with \( \lambda \) the laser wavelength. The last term expresses the amplitude modulation with the modulation amplitude \( V_i \), the frequency \( \omega_i \) and the phase \( \phi_i \) of each frequency components.

Typically, time-periodic systems are described by Floquet’s theorem [17]. In our system, each of the modulation frequencies \( \omega_i \) gives a period \( T_i = 2\pi/\omega_i \) and the Hamiltonian has a period of \( H(t + T) = H(t) \), with \( T \) the lowest common multiple of \( T_i \). By defining a time-evolution operator for one period as \( \hat{U}(T) \), solutions to this problem must satisfy

\[
|\psi_{q,\alpha}(t + T)\rangle = \hat{U}(T)|\psi_{q,\alpha}(t)\rangle = e^{i\epsilon_{q,\alpha}T}|\psi_{q,\alpha}(t)\rangle,
\]  

(2)

and the Floquet states \( |u_{q,\alpha}\rangle \) are defined as \( |\psi_{q,\alpha}\rangle = e^{i\epsilon_{q,\alpha}t}|u_{q,\alpha}\rangle \) with the quasi-momentum \( q \), the band index \( \alpha \) and quasi-energy \( \epsilon_{q,\alpha} \), which leads to

\[
(H(t) - i\partial_t)|u_{q,\alpha}\rangle = H_0|u_{q,\alpha}\rangle = \epsilon_{q,\alpha}|u_{q,\alpha}\rangle.
\]  

(3)
For a special case of single frequency modulation, considering the time and coordinate periodicity of the Floquet states \(|\alpha, \omega\rangle\), by a Fourier transformation we can study the problem with a set of basis \(|v_{lm}\rangle = e^{i(2\hbar k_L x - m\omega t)}\) in an extended Hilbert space \(\mathcal{H} = \mathcal{H}_R \otimes \mathcal{F}\). Where \(\mathcal{H}_R\) is the Hilbert space and \(\mathcal{F}\) is the space of all functions with periodic \(T\). By integration in one period, we can turn to a time-independent system.

\[
A_{lm,l'm'} = \frac{1}{T} \int_0^T |v_{l'm'}\rangle H_0(v_{lm}) dt.
\] (4)

To get rid of the phase \(\phi_1\), we perform an unitary transformation \(U_{lm,lm} = e^{i\phi v_{1}}\) to the Hamiltonian by \(A' = U^{-1}AU\). There are three kinds of terms in the matrix of \(A'\). The diagonal terms \(A'_{lm,lm} = (2\hbar k_L)^2/2M + m\hbar \omega_1\) are the energy of momentum states shifted by absorbing or emitting Floquet photons with energy \(\hbar \omega_1\). The terms \(A'_{lm,\pm1,l'm} = V_0/4\) show stationary component of the lattice would coupling atoms with momentum difference \(2\hbar k_L\). In addition, \(A'_{lm,\pm1,l'm}= V_1/8\) terms show the time modulation of the lattice, which would coupling two momentum states separated by \(2\hbar k_L\) while absorbing or emitting one Floquet photon.

When the modulation frequency is near-resonant with the energy difference between two specific bands and far detuned from the others, we can get an effective Hamiltonian by a rotating wave approximation.

\[
H_{R} = \begin{pmatrix}
E_{\alpha} & V_1 e^{i\phi} \Omega_{\alpha\beta} & V_4 e^{i\phi} \Omega_{\alpha\beta} \\
V_1 e^{-i\phi} \Omega_{\alpha\beta} & V_1 e^{-i\phi} \Omega_{\alpha\beta} & V_1 e^{i\phi} \Omega_{\alpha\beta} \\
E_{\beta} - \hbar \omega_1 & V_1 e^{i\phi} \Omega_{\alpha\beta} & E_{\beta} - \hbar \omega_1
\end{pmatrix},
\] (5)

where \(E_{\alpha}\) and \(E_{\beta}\) are the energy of Bloch states \(|\alpha\rangle, |\beta\rangle\) in the system without modulation. Coupling constant between two states is \(\Omega_{\alpha\beta} = \langle \alpha | \cos^2(k_L x) | \beta \rangle / 4\).

The extension to polychromatic driven is straightforward. For example, a double frequency modulation induced two-photon process between \(s-g\) band is described by an effective Hamiltonian \(H_{sg}\) as:

\[
H_{sg} = \begin{pmatrix}
E_s & e^{i\phi} V_1 \Omega_{sd} & e^{i\phi} V_2 \Omega_{sd} & 0 & 0 & 0 \\
e^{-i\phi} V_1 \Omega_{sd} & E_d - \hbar \omega_1 & 0 & e^{i\phi} V_2 \Omega_{dg} & 0 & 0 \\
e^{-i\phi} V_2 \Omega_{sd} & 0 & E_d - \hbar \omega_2 & e^{i\phi} V_1 \Omega_{dg} & 0 & 0 \\
0 & e^{-i\phi} V_1 \Omega_{dg} & e^{-i\phi} V_2 \Omega_{dg} & E_g - \hbar (\omega_1 + \omega_2) & 0 & 0 \\
0 & 0 & e^{-i\phi} V_2 \Omega_{dg} & 0 & E_g - 2\hbar \omega_1 & 0 \\
0 & 0 & 0 & 0 & E_g - 2\hbar \omega_2 & E_g - 2\hbar \omega_2
\end{pmatrix}.
\] (6)

The effective Hamiltonian \(H_{sg}\) is constructed by means of nearly degenerate perturbation technique [18], in which we include six nearly degenerate states considering four main processes in the excitation. The six states are \(|E_s\rangle\) the \(s\) band, \(|E_d - \hbar \omega_1\rangle\), \(|E_d - \hbar \omega_2\rangle\) the \(d\) band dressed by Floquet photon \(\omega_1\) or \(\omega_2\) and \(|E_g - \hbar (\omega_1 + \omega_2)\rangle\), \(|E_g - 2\hbar \omega_1\rangle\) and \(|E_g - 2\hbar \omega_2\rangle\) the \(g\) band dressed by two Floquet photons. Using this basis a general state \((v_1, v_2, v_3, v_4, v_5, v_6)^T\) gives complex coefficient of the six dressed states. Population of Bloch states \(s\) is \(|v_1|^2\), while population on \(g\) band is \(|v_4 e^{i(\omega_1 + \omega_2)t} + v_5 e^{2i\omega_1 t} + v_6 e^{2i\omega_2 t}|^2\), given by coherent superposition of all \(g\) band states dressed with different Floquet photons. Solution of the model consists with the time dependent Schrödinger equation and the effective model provides us a better understanding of the multiphoton process. However, in the calculation more states associated with higher order processes could be included to get a more accurate result, especially when the modulation amplitude is large.
Fig. 2. Left side is the calculated Floquet spectra of a single frequency driven system, with parameters $V_0 = 5E_r, V_1 = 0.5E_r, \hbar\omega_1 = 5E_r$. In the figure the first seven bands are presented. The heavy lines depict states maximally overlapping with the $s$ (blue), $p$ (green) and $d$ (red) Bloch bands respectively. Right side shows the details of two Floquet bands most overlapping with $s$ and $d$ bands. The two bands are separated by a band gap $E_F$ at $q = 0$.

3. Experimental system

Our experiment begins with a quasi-pure condensate of typically $1.5 \times 10^5$ $^{87}\text{Rb}$ atoms in the $|F = 2, m_F = 2\rangle$ hyperfine ground state, produced in a combined potential of a single-beam optical dipole trap and a quadrupole magnetic trap. The trapping frequencies are $\omega_x = 2\pi \times 28\text{Hz}, \omega_y = 2\pi \times 60\text{Hz}, \omega_z = 2\pi \times 70\text{Hz}$. The optical lattice is formed by a retro-reflected red detuned laser beam, with lattice constant $a = \lambda / 2 = 426\text{nm}$ focused to a waist of $110\mu\text{m}$. Density of atoms in our system is less than $5 \times 10^{13} \text{cm}^{-3}$, and the mean-field interaction can be omitted to capture the main physical mechanism in the excitation [19, 20].

The lattice depth is calibrated by Kapitza-Dirac scattering and the modulation of lattice depth is controlled by an acousto-optic modulator(AOM). The modulation amplitudes $V_i$ and phases $\phi_i$ are generated from a signal generator and the intensity of lattice laser is monitored by a photodetector. Experimental results are absorption images taken after $28\text{ms}$ time-of-flight (TOF). Occupation number at different momentum states $|2lh_{k_L}\rangle$ $(l$ is integral momentum index) can be given from TOF images by $n_l = N_l / N$, with $N_l$ the atom number at momentum state $|2lh_{k_L}\rangle$ and $N$ the total atom number. The initial state for experiment is prepared non-adiabatically with numerically designed sequence of lattice pulses [21, 22]. For experimental convenience, the lattice pulses are carried out with the same depth as the constant part of the modulated lattice potential $V_0$. Typically, each of the pulses and the subsequent intervals are lasting for no more than $25\mu\text{s}$, and the whole process can be finished within $60\mu\text{s}$, thus the loading time is greatly reduced comparing with traditional adiabatic loading method.

4. The initial phase effect in single frequency modulation

Single frequency modulation can be seen as the basis of polychromatic driven. In this part, we present the preparation of a Floquet state in the single frequency driven system, which is shown to be highly related to the modulation phase.

Following the discussion in Sec. 2, Fig. 2 depicts a typical quasi-energy spectrum of the single frequency driven system, which is obtained by direct diagonalization of $A'$ at various quasi-momentum $q$. The same calculation also gives eigenvectors in the extended Hilbert space. The spectrum exhibits a complex structure as a result of the periodically repetition of high excited bands.
Fig. 3. Time evolution of $n_l$ measured from the experiments with initial modulation phase (a1) $\phi = -\pi/2$ and (a2) $\phi = \pi/2$. Time averaged fraction $\langle n_l \rangle$ are also shown for (b1) $\phi = -\pi/2$ and (b2) $\phi = \pi/2$ respectively. $n_0$ is shown with black dots comparing to the numerical simulation in solid lines. $n_1$ and $n_{-1}$ are shown in average with red circles the corresponding numerical result is shown in dashed lines. Each point is averaged by three experiments and the error bars indicate the standard deviation.

To connect our calculation with the experiments, we should project the Floquet states $|u_{q,\alpha}\rangle$ into momentum space. The occupation number at momentum states $|2\hbar k_L\rangle$ and its phase can be given by a summation of all the Fourier components with the same $l$ as $c_l(t)|2\hbar k_L\rangle = \sum_m e^{-im\phi_1} \tau \nu_{lm} |2\hbar k_L\rangle$, where $\nu_{lm} = \langle v_{lm}|u_{q,\alpha}\rangle$ is coefficient of the Floquet state. $\tau$ is related to the modulation phase $\phi_1$ and holding time $t$ as $\tau = \frac{\phi_1 + 2\pi}{\pi} T$. It is also useful to define the overlapping between the Floquet state $|u_{q,\alpha}\rangle$ and a Bloch state $|n_{q}\rangle$ of the undriven potential as $P = \int_0^T |\sum_l c_l(t) |n_{q}\rangle|^2 dt$. Property of the Floquet band is typically characterized by its most overlapping Bloch band [23]. Without loss of generality, our experiments are restricted to quasi-momentum $q = 0$, and energy gap between Bloch bands $\alpha$ and $\beta$ are written as $\hbar \omega_{\alpha\beta}$. The technique can also be applied in systems with acceleration [12], which brings out phenomena different from our study.

When modulation phase $\phi_1$ is given, a Floquet state can be projected into the momentum space, and such a state can be prepared by carrying out two lattice pulses with numerically designed pulse sequence [21, 22]. In the fast loading process, lattice depth of the two pulses is kept constant. For a target state, the fidelity of a prepared state can be given numerically for different pulse durations and time intervals, and the optimized pulse sequence is obtained by finding the maximum loading fidelity. Throughout this method we can get a loading fidelity of more than 95% experimentally.

With the initial modulation phase $\phi_1 = -\pi/2$, a pure Floquet state is loaded by lattice pulses.
Therefore, we chose modulation can be neglected, only the phase difference between two modulations is important.

band components of the Floquet state. However, with a nearly pure single frequency modulation shows that parameters $V_0$, $V_1$, $V_2$, $\omega_1$, $\omega_2$, $\phi_1$, $\phi_2$. During the experiments, we mainly focus on the influence of modulation frequencies and phases, while keeping other parameters constant.

5.1. Spectrum of two-photon excitation

The single frequency modulation shows that $\phi_1$ is related to relative phase between $s$ and $d$ band components of the Floquet state. However, with a nearly pure $s$ band, the phase of single modulation can be neglected, only the phase difference between two modulations is important. Therefore, we chose $\phi_1 = \pi$ while leaving $\phi_2$ variable to control the relative phase between two modulations.

For $s$-$g$ band coupling through a two-photon absorption process, the sum of the two modulation frequencies is chosen as $\omega_1 + \omega_2 = \omega_0$. This process is well described by Eq. (6), in...
which we have considered different pumping paths, as schematically shown in Fig. 4(a). There are two cases which would benefit the excitation process.

**Case 1:** Resonant two-photon process. When $\omega_1 = \omega_{sd}$ or $\omega_2 = \omega_{sd}$, atoms are transferred from $|s\rangle$ to $|g\rangle$ with the assistance of $d$ band as an intermediate band.

**Case 2:** Equal frequency two-photon process. When $\omega_1 = \omega_2 = \omega_{sg}/2$, two modulations with the same frequency can be added together, and the coupling strength of the process is doubled.

With resonance condition $\omega_1 = \omega_{sd}$ in **Case 1**, when two modulation amplitudes are chosen as $V_1 \Omega_{sd} = V_2 \Omega_{dg}$ the transfer rate would show a maximum resonant peak. And we keep this modulation amplitudes while sweeping $\omega_1$. Modulation phase is chosen from numerical simulation to get a maximum transfer.

In the experiments, we sweep the frequency $\omega_1$ for different lattice depths $V_0 = 5E_r$, $10E_r$ and $14E_r$. Population on momentum states $\pm 4\hbar k_L$ measured from the experiment are shown with rectangles in Fig. 5, comparing with the theoretical calculation shown in solid curves. Within the lattice depth we considered, $g$ band is greatly concentrated on $|\pm 4\hbar k_L\rangle$ momentum states, thus $n_{\pm 2}$ can reflect transfer rate to $g$ band.

Figure 5(a) shows the case of $V_0 = 5E_r$. For the lattice depth we have $\Omega_{sd}/\Omega_{dg} = 1.11$, correspondingly the modulation amplitudes are chosen as $V_1 = 1.4E_r$, $V_2 = 1.6E_r$, and $t = 300\mu s$. The holding time $t$ may be chosen shorter if the maximum of numerical simulation is
Fig. 6. The excited population on g band shows the interference between two paths. (a) Population transferred to \( n_{\pm 2} \) is shown in black dots with error bars. The dashed line shows theoretical simulation for comparison. (b1)-(b4) \( V_L \) for different phases.

reached at an earlier time. In the figure, two peaks appear at \( \omega_1 = \omega_{sd} \) and \( \omega_2 = \omega_{sd} \) which follows case 1 we have discussed. And there the central peak at frequency \( \omega_1 = \omega_2 \) following case 2 is much lower than two peaks for case 1.

Figure 5(b) shows the spectrum with \( V_0 = 10E_r \). With the increasing of \( V_0 \), the energy difference \( \omega_{sd} \) is getting closer to \( \omega_{dg} \), and three peaks are overlapping. Comparing with \( V_0 = 5E_r \) the central peak is much higher, because the process of case 2 is also near resonance with d band.

For \( V_0 = 14E_r \) only one peak would be measured in the spectrum as shown in Fig. 5(c), which means case1 and case2 are fulfilled simultaneously. Under this condition, the coupling between s and g band is also greatly enhanced. Modulation amplitudes are \( V_1 = V_2 = 2.5E_r \), for the two modulation frequencies are the same at case1 and can’t be distinguished.

The experimentally detected peaks are governed by two cases, which are within the description of Eq. (6). Thus in the numerical simulation we can neglect higher order processes of emission and absorption of Floquet photons. The discrepancy between experimental result and the theoretical simulation is probably due to the influence of interaction and initial momentum distribution of the condensate. These effects would destroy coherency during the modulation, and the measured population of excited state would be lower than the maximum value in the theoretical simulation.

5.2. The role of modulation phases

When \( \omega_1 \) and \( \omega_2 \) are incommensurate, the influence of relative phase is not prominent because there is only one path for the pumping process and no states could interfere with each other. However, the effect of modulation phases would be more pronounced when two modulation frequencies are commensurate.

As shown in Fig. 4(b), when \( 2\omega_1 = \omega_2 \), relative phase of modulations can be shown by performing a one-photon process simultaneously with a two-photon process. We choose the frequency \( \omega_1 = \omega_{sg}/2 \) at central peak of \( V_0 = 10E_r \) lattice and the second modulation is performed with \( \omega_2 = \omega_{sg} \). Modulation amplitudes are \( V_1 = V_2 = 2.5E_r \) and \( t = 500\mu s \). Similar to the process described by Eq. (6), in this problem we consider the interference between two states \( |E_g - 2h\omega_1 \rangle \) and \( |E_g - \hbar\omega_2 \rangle \). During the two-photon process with \( \phi_1 = \pi \), phase of state \( |E_g - 2h\omega_1 \rangle \) remains zero, while \( \phi_2 \) determine phase of state \( |E_g - \hbar\omega_2 \rangle \) as \( \phi_2 - \pi \). Relative phase of two modulations is defined as \( (\phi_2 - \pi) - \frac{\pi}{2}(\phi_1 - \pi) = \phi_2 - \pi \), and the interference can be changed from constructive to destructive with different \( \phi_2 \).
Figure 6(a) depicts the population transferred to $g$ band with the phase of one-photon process changing by $2\pi$. Figure 6(b) shows how the depth of lattice is varying with time for four different modulation phases. With $\phi_2 = \pi$, the population transferred to $|g\rangle$ through one-photon process and through two-photon process are in phase and the transfer rate is enhanced. When $\phi_2$ is increasing, relative phase of two processes are deviating from zero, and the population on $|g\rangle$ would decrease. In the case of $\phi_2 = 0$ the two process are out of phase, and only few atoms can be transferred to $g$ band. Further increasing of $\phi_2$ would increase the measured population, and when $\phi_2$ is changed by $2\pi$ the population on $g$ band reaches the maximum value again.

The single atom Hamiltonian in Eq. (1) can well explain the excitation in our system. However, for the 500$\mu$s modulation, decoherence from mean-field interaction and the momentum distribution becomes significant, thus it is necessary to carry out a simulation based on the time-dependent Gross-Pitaevskii equation,

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_L(x,t) + \frac{1}{2}m\omega_x^2 x^2 + g |\psi|^2 \right] \psi,$$

where $g$ is the parameter of interaction. In the simulation, we consider both the distribution of initial momentum and the mean-field interaction which would deduce the measured excitation rate. Dashed line in Fig. 6(a) gives the result of simulation which fits well with the experiment. Similarly, the relative phase can also be changed by $\phi_1$ of the two-photon process, which gives a period of $\pi$.

The collaborative effect of different frequency components is studied in two aspects, demonstrate the effect of both modulation frequencies and phases of different frequency components. The usefulness of this study is not limited to double frequency modulation, within these two aspects, collaborate effect of more frequency components can also be well understood.

6. Application in LMT beam splitter

The double frequency modulation can also be applied to realize a LMT beam splitter by pumping atoms to higher momentum states, which is useful in experiments of atomic interferometry [24]. According to the resonant condition we have discussed, a preferred choice is $V_0 = 14E_r$ with the modulation frequency $\omega_1 = \omega_{sd} = \omega_{dg}$. The other modulation frequency is chosen as $\omega_2 = \omega_{gi}$ (with $i$ the 6th excited Bloch band) to get a distribution at $\pm 6\hbar k_L$.

We begin with a condensate, the lattice is suddenly turn on, modulated with $V_1 = V_2 = 2.5E_r$ and the preferable phases are found numerically. A TOF image of experimental result is shown in Fig. 7(a). Figure 7(b) shows the population of atoms on momentum states $| \pm 6\hbar k_L \rangle$ where we
have subtracted the thermal gas. Near 80% of the atoms are coherently transferred into $\pm 6\hbar k L$ momentum states within 160$\mu$s, and the maximum lattice depth needed is below $V_0 + V_1 + V_2 = 24E_r$. Comparing with a LMT beam splitter based on high order Bragg scattering [16], our method needs a much lower lattice depth and is easy in practice. Furthermore, the process is symmetric for both sides, and would not introduce phase shift between two separated atom clouds.

A momentum splitting of $12\hbar k L$ is not the limit of the amplitude modulation. More frequency components or another subsequent modulation can be introduced to reach a larger momentum splitting. For example, after the double frequency modulation, by preforming another single frequency modulation resonance with the energy difference between $|\pm 6\hbar k L\rangle$ and $|\pm 8\hbar k L\rangle$ momentum states, the atoms can be transferred to $|\pm 8\hbar k L\rangle$ coherently.

7. Discussion and conclusion

Following Floquet-Bloch theory we have presented an experimental preparation of a Floquet state and study its property. The non-adiabatic loading method would take much less time and greatly reduce the heating problem. The loading method could also be extended to systems with a shaken lattice [7] or a combined modulation [6, 23], which provides a several millisecond longer lifetime for condensate in the experimental study on areas including the detection of Floquet topological states [17, 25, 26].

In conclusion, based on the study of single frequency modulation, we investigate double frequency modulation in detail. Experimental observations show that different modulation frequency components would influence each other in two ways. When two frequencies are resonant with subsequent excitation processes, the modulation can induce a resonant two photon process which can effectively transfer atoms to higher excited bands. With specific modulation frequencies, interference between a one-photon path and a two-photon path is observed, revealing influence of modulation phases when two frequencies are commensurate. The quantum interference can be used to enhance the excitation rate or destruct unwanted excitations. Using the technique of double frequency modulation, we also demonstrate an efficient way to realize a LMT beam splitter with low lattice depth. Our study provides a more flexible way to coherently control quantum states through an optical lattice.

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