Dynamic Optimization For Heterogeneous Powered Wireless Multimedia Sensor Networks With Correlated Sources and Network Coding

Abstract—The energy consumption in wireless multimedia sensor networks (WMSN) is much greater than that in traditional wireless sensor networks. Thus, it is a huge challenge to remain the perpetual operation for WMSN. In this paper, we propose a new heterogeneous energy supply model for WMSN through the coexistence of renewable energy and electricity grid. We address to cross-layer optimization for the multiple multicast with distributed source coding and intra-session network coding in heterogeneous powered wireless multimedia sensor networks (HPWMSN) with correlated sources. The aim is to achieve the optimal reconstruct distortion at sinks and the minimal cost of purchasing electricity from electricity grid. Based on the Lyapunov drift-plus-penalty with perturbation technique and dual decomposition technique, we propose a fully distributed dynamic cross-layer algorithm, including multicast routing, source rate control, network coding, session scheduling and energy management, only requiring knowledge of the instantaneous system state. The explicit trade-off between the optimization objective and queue backlog is theoretically proven. Finally, the simulation results verify the theoretic claims.

Index Terms—Wireless multimedia sensor networks, heterogeneous energy, cross-layer optimization, Lyapunov optimization, network coding, distributed source coding.

I. INTRODUCTION

Wireless Multimedia Sensor Networks (WMSN) are composed of a large number of heterogeneous multimedia sensors, interconnected through wireless medium to capture video and audio streams, still images, and scalar sensor data, and transmits them to the sink(s) through the wireless manner. WMSN has a lot of potential new applications, for example, multimedia surveillance, traffic avoidance and control, health monitoring, environment monitoring, automated manufacturing processes, and industrial process control [1].

Similar to wireless sensor networks (WSN), the sensor nodes in WMSN are traditionally battery-operated, where the battery is usually limited and non-rechargeable. As the multimedia applications in WMSN require high transmission rate and extensive processing, the energy consumption of WMSN is much greater than that of WSN. Moreover, the sensors may be placed in remote and dangerous areas, the replacement of sensors is unrealistic and impossible. Hence, the network lifetime is an important issue for the operation of the most applications in WMSN, and an efficient power management should be proposed to extend the network lifetime. Recently, a large number of works focus on the energy efficiency for WMSN with the limited battery capacity [2] [3]. Although the energy efficiency techniques increase the sensor’s lifetime, the network lifetime still has an upper bound due to the limited battery capacity. In order to solve this problem, the renewable energy technique is applied to power the sensors via solar panels or wind-powered generators [4]. However, the renewable energy technique exiting some shortcomings, such as low recharging rate and time-varying profile of energy replenishment process, which can not guarantee to provide the perpetual operation for WMSN. As the electricity grid can provide persistent power input, the coexistence of renewable energy and electricity grid, called Heterogeneous Power (HP) is expected to achieve the infinite network lifetime [5].

Since the sensors in MWSN are always densely deployed in the most application scenarios, and the sensors generally have large monitoring range. For example, the nearby camera sensors’s monitoring range may be overlapped, and the multimedia information flows generated by them have a certain degree of spatial correlation and redundancy [6]. By removing the correlation and redundancy of the multimedia data, the amount of data transmitted in the network can be reduced, thereby reducing the energy consumption and improving the energy efficiency for MWSN. Distributed source coding (DSC) is one of the techniques that can remove the correlation and redundancy of multiple correlated sources, without communication between the sources. By modeling the correlation between multiple sources at the decoder side, DSC is able to shift the computation burden in sources to the joint decoder [7] [8]. Thus, DSC provides appropriate frameworks for WMSN with complexity-constrained sensors and powerful sink(s). It is more important to use DSC in HPWMSN, since DSC enables multiple correlated sources to trade energy resources among them.

Network coding (NC) can combine various traffic flows or packets into a single packet in the intermediate node via simple algebraic operations, and then forwards it through one or more outgoing links [9]. Thus, NC has the potential of achieving substantial throughput and power efficiency gains in wireless networks. NC applied in WMSN can save a lot of energy consumption, and then prolong the lifetime of WMSN.

In this paper, we address a discrete-time stochastic cross-layer optimization problem with joint DSC and NC for HP-WMSN to achieve the high energy efficiency and the desirable network performance. The key contribution are summarized as follows:

- Since most multimedia applications in WMSN require extensive data processing, we propose the overall energy consumption model to include multiple energy consumptions due to data processing, transmission and reception. In order to obtain potentially infinite lifetime, we propose the new energy heterogenous supply model through the coexistence of renewable energy and electricity grid.
Sensors in WMSN can be powered by renewable energy, or electricity grid or both.

- We address to cross-layer optimization for the multiple multicast with DSC and NC in HPWMSN with correlated sources. We formulate a discrete-time stochastic cross-layer optimization problem with the goal of minimizing the time-average utility of reconstruct distortion at sinks and minimizing the cost of purchasing energy from electricity grid subject to network strong stability, link capacity, and data/energy availability constraints. This will involves all the layer of the network, such as the transmission layer, the network layer, especially the physical layer and medium access layer.

- Based the Lyapunov drift-plus-penalty with perturbation technique, we transfer the stochastic optimization problem into a deterministic optimization problem. By Lagrange dual decomposition, the problem is decoupled into five subproblems, including energy harvesting and battery charging/discharging problem, source rate control problem, distortion control problem, information/physical flow rate and power control problem, and session scheduling. Finally, a fully distributed iterative algorithm is designed.

- We analyze the performance of the proposed distributed algorithm, and show that a control parameter $V$ enables an explicit trade-off between the average objective value and queue backlog. Specifically, our algorithm can achieve a time average objective value that is within $O(1/V)$ of the optimal objective for any $V > 0$, while ensuring that the average queue backlog is $O(V)$. Finally, the simulation results verify the theoretic claims.

Throughout this paper, we use the following notations. We denote the probability $\Pr(A)$ of an event $A$. For a random variable $X$, $\mathbb{E}[X]$ represents its expected value, $\mathbb{E}[X | A]$ represents its expected value conditioned on event $A$. $1_A$ is the indicator function for an event $A$, it is 1 if $A$ occurs and is 0 otherwise. $[x]^+ = \max(x, 0)$.

The remainder of the paper is organized as follows. The related works are given in Section II. In Section III, we give the system model and problem formulation. In Section IV, we present the distributed cross-layer optimization algorithm. In Section V, we present the performance analysis of our proposed algorithm. Simulation results are given in Section VI. Concluding remarks are provided in Section VI.

II. RELATED WORKS

A. Distributed Source Coding

Using Slepian-Wolf model and a joint entropy coding model, Cristescu et al. in [10] propose a minimum cost of data transmission optimization problem in WSNs, where the sources are spatially correlated. Wang et al. in [11] propose a methodology for cross-layer optimization between routing and DSC in WSNs. In order to meet the data transmission latency and QoS requirements, Khoshroo et al. in [12] jointly consider the resource allocation, channel coding, and DSC. Cheng et al. in [13] proposed a hierarchical DSC coding scheme to completely remove inter-node data correlation and redundancy.

Wang et al. in [14] jointly consider the optimization of Slepian-Wolf (SW) source coding and transmission rates to obtain the maximum source rate over a Gaussian multiple access channel. However, these works do not consider the link capacity constraint, which may cause congestion and deteriorate the network performance. Hence, Yuen et al. in [15] introduce the fixed link capacity constraint. They propose a distributed algorithm of joint rate allocation and transmission structure in WSN with correlated data. In practice, the link capacity depends on the power allocated to the link and the channel state. Ramamoorthy in [16] introduce the unfixed link capacity constraints, and consider a minimum cost optimization problem in WSN with multiple correlated sources. He et al. in [17] firstly focus on the network lifetime maximization that jointly considers routing, power control, and link-layer random access.

As far as we know, almost all works usually ignore the energy consumption of the data processing. However, in some applications, the energy consumption of data processing may be comparable to the energy consumption of data transmission. Hence, Tapparello et al. in [18] consider the joint energy allocation for communication module and processing module together in the multihop WSN scenario. They show that through applying DSC, the sensor with sufficient energy can ease the transmission requirements of correlated data on nearby node with low energy. This characteristic of DSC is very suitable for energy-harvesting networks. In [19], Stankovic et al. in [19] apply DSC to improve the network performance, consider the multimedia multicast over heterogeneous wireless-wireline networks, and propose a network-aware cross-layer optimization problem. In [8], Puri et al. apply distributed video coding to model the correlation between multiple sources at the decoder side together with channel codes. To the end, the encoder is simple, and the computational complexity is shifted to the decoder. Hence DSC is a promising technique for WMSN.

B. Network Coding

The network coding techniques have been widely used in WSN. Chachulski et al. [20] exploit the broadcast nature of wireless transmission and propose a random network coding scheme. Li et al. in [21] consider multirate multicast with network coding over wireless video networks, to optimize the video quality and network performance. Lin et al. in [22] propose an approximate optimization scheme to jointly optimize the link scheduling, rate control, and flow allocation problems for multicast with intraflow network coding in multirate multichannel wireless mesh networks. Ho et al. in [23] design a random linear network coding (RLNC) for wireless transmission. Chen et al. in [24] study the multicast flow control based network coding for wired networks and wireless networks. Rajawat et al. in [25] consider the random network coding for slotted wireless multihop networks. These works show that NC can obtain throughput gain and save a lot of energy and thus prolong the network lifetime. However, these works usually assume the data transmission over link is free of interference, which weakens the applicability of NC.
technique. Hence, some works address to NC in interference-limited networks. Xi et al. in [27] propose a framework for the minimum-cost optimization problem in interference-limited wireless networks, and jointly consider the power allocation, network coding and multicast. NC and DSC are jointly considered in [28] and [29]. Given a time-varying networks, Ho et al. in [28] propose a dynamic algorithms for the problem of multiple multicast sessions with in-session network coding. [29] jointly consider the adaptive DSC, channel coding, NC and power control with QoS constraint for Co-Channel Interference (CCI)-limited wireless networks. However, they do not take the energy constraint into consideration, which is one of the most important constraint in WMSN.

III. SYSTEM MODEL

We consider a general interconnected multi-hop WSN that operates over time slots \( t \in \mathcal{T} = \{0, 1, 2, \ldots\} \). WSN is modeled by a direct graph \( \mathcal{G} = (\mathcal{N}, \mathcal{L}) \). \( \mathcal{N} = \mathcal{N}_H \cup \mathcal{N}_G \cup \mathcal{N}_M = \{1, 2, 3, \ldots, N\} \) denotes the set of sensor nodes in the network, \( \mathcal{N}_H \) is the set of EH nodes powered by renewable energy, \( \mathcal{N}_G \) is the set of EG nodes powered by electricity grid, and \( \mathcal{N}_M \) is the set of ME nodes powered by both renewable energy and electricity grid, respectively. \( \mathcal{N}_s = \{1, 2, \ldots, N_s\} \subset \mathcal{N} \) denotes the set of all source nodes which measure the information source(s), therein, we note that the sources are continuous and correlated. \( \mathcal{N}_d = \{1, 2, \ldots, N_d\} \subset \mathcal{N} \) denotes the set of all sink nodes. Let \( \mathcal{F} = \{1, 2, \ldots, F\} \) denote the set of sessions in the network, and we assume the sessions are multicast, i.e., for all \( f \in \mathcal{F} \), the session \( f \) has at least one source and one sink. Therefore, we define \( \mathcal{N}_f^s = \{1, 2, 3, \ldots, N_s^f\} \) and \( \mathcal{N}_f^d = \{1, 2, 3, \ldots, N_d^f\} \) as the set of session \( f \)'s sources and sinks, respectively. \( \mathcal{N}_f^s \subset \mathcal{N}_s \), \( \mathcal{N}_f^d \subset \mathcal{N}_d \). The source node transmits the data to the corresponding sink node through multi-hop routing. We use \( \mathcal{O}(n) \) to denote the set of nodes \( m \) with \( (n, b) \in \mathcal{L} \), and \( \mathcal{I}(n) \) to denote the set of nodes \( a \) with \( (a, n) \in \mathcal{L} \). \( \mathcal{L} = \{(n, b), n, b \in \mathcal{N}\} \ b \in \mathcal{O}(n) \) represents the set of communication links.

Fig. 1 describes the composition of a single node system, which is divided into two sub-systems, i.e., sensor node sub-system and energy supply sub-system. Sensor node sub-system includes data sensing/processing and data transmission/reception. Their energy consumption is given in the subsection III-D. The energy supply sub-system is detailed described in the subsection III-E.

A. Network and Coding Model

In our network, in order to achieve the optimal throughput when transmit multiple multicast sessions, we apply the network coding technology. Note that the sink nodes only receive fractional sessions, combing different sessions together brings difficulty for data reconstruction, hence, we assume the network coding is limited within each session.

Let \( x_{nb}^f(t) \) denote the physical flow rate of the session \( f \) over link \( (n, b) \), and \( x_{nb}^{fsd}(t) \) denote the information flow rate of the session \( f \) over link \( (n, b) \) from source \( s \in \mathcal{N}_f^s \) to sink \( d \in \mathcal{N}_f^d \), where \( b \in \mathcal{O}(n) \). Via network coding, the flows for different destinations of a multicast session can be coded together so that they can share the network capacity, while the actual physical flow over each link need only to be the maximum of the individual destinations’ flows, i.e.,

\[
\tilde{x}_{nb}^{fsd}(t) \leq x_{nb}^f(t), \ f \in \mathcal{F}, \ s \in \mathcal{N}_f^s, \ d \in \mathcal{N}_f^d, \ b \in \mathcal{O}(n) \tag{1}
\]

B. Distributed Lossy Coding

For continuous sources, we consider the lossy source coding which allows a reconstruction distortion. In our model, the distributed lossy coding can be described as the follows.

At every time slot \( t \), for each session \( f \), the node \( n \in \mathcal{N}_f^s \) measures data from the environment and compresses it before transmission. We assume the node \( n \in \mathcal{N}_f^s \) multicasts data to all the sinks of session \( f \) at rate \( r_{nf}(t) \) with distortion \( D_{nf}(t) \). Consider any subset \( \mathcal{N}_f^d \subseteq \mathcal{N}_f^d \), for each node \( n \in \mathcal{N}_f^d \) we have:

\[
\sum_{n \in \mathcal{N}_f^d} r_{nf}(t) \geq H\left(\frac{\mathcal{N}_f^d}{|\mathcal{N}_f^d|} - \frac{\mathcal{N}_f^d}{|\mathcal{N}_f^d|}\right) - \log((2\pi e)^{\frac{1}{2}}) |\mathcal{N}_f^d| \prod_{n \in \mathcal{N}_f^d} D_{nf}(t) \tag{2}
\]

where \( H\left(\frac{\mathcal{N}_f^d}{|\mathcal{N}_f^d|} - \frac{\mathcal{N}_f^d}{|\mathcal{N}_f^d|}\right) \) represents the entropy of \( \mathcal{N}_f^d \) conditioned on \( \mathcal{N}_f^d \).

We also assume that

\[
0 \leq r_{nf}(t) \leq R_{\text{max}}, \forall f \in \mathcal{F}, n \in \mathcal{N}_f^s \tag{3}
\]

and

\[
D_{\text{min}} \leq D_{nf}(t) \leq D_{\text{max}}, \forall f \in \mathcal{F}, n \in \mathcal{N}_f^s \tag{4}
\]

C. Random Access and Data Transmission

We assume that the links in the network may interfere with each other when they transmit data simultaneously, we also assume each node cannot transmit or receive at the same time, and define \( q_n(t) \) as the transmission probability of node \( n \) at slot \( t \), with

\[
0 \leq q_n(t) \leq 1, \forall n \in \mathcal{N}. \tag{5}
\]
When node \( n \) is selected to transmit data at slot \( t \), the probability of choosing outgoing link \((n,b), b \in O(n)\) is \( q_{nb}(t) \), with \( \sum_{b \in O(n)} q_{nb}(t) = q_n(t) \).

Then, we introduce the transmission probability matrix \( \gamma(t) = \{q_{nb}(t), (n,b) \in L|0 \leq q_{nb}(t) \leq 1\} \). Summary, the link \((n,b)\) can transmit data successfully at time slot \( t \) with probability

\[
\alpha_{nb}(t) = q_{nb}(t) \prod_{a \in \mathcal{I}(n)} (1 - q_{an}(t))(1 - \sum_{c \in \mathcal{O}(b)} q_{ac}(t)),
\]

The probability of no transmission of \( n \) The probability of no transmission of \( b \)

We define \( \gamma_n(t) \) to denote the signal to interference plus noise ratio (SINR) of link \((n,b)\), which can be calculated by the following:

\[
\gamma_{nb}(t) = \frac{S_{nb}^C(t) p_{nb}^T(t) / N_{nb}}{\sum_{a \in \mathcal{I}(n), (a,c) \in L} S_{ac}^C(t) p_{ac}^T(t) q_{ac}(t)},
\]

where \( \mathcal{I}_{n,b} \) represents the set of nodes whose transmission can interfere the transmission over link \((n,b)\), \( S_n^C(t) \) represents the link fading coefficient from \( n \) to \( b \), and \( N_{nb} \) represents the noise which is assumed to be constant. We assume that \( S_n^C(t) \) may be time varying and i.i.d. at every slot. Denote \( S_n^C(t) = \{S_n^C(t), (n,b) \in L\} \) as the network channel state matrix, taking non-negative values from a finite but arbitrarily large set \( S^C \).

The link capacity is defined as the following:

\[
C_{nb}(t) = BW \alpha_{nb}(t) \log(\gamma_{nb}(t))
\]

where \( BW \) is the bandwidth.

Because of the total rates of all sessions cannot exceed the link capacity, so, the following constraints must be met:

\[
0 \leq \sum_{f \in F} x_{nb}^f(t) \leq C_{nb}(t), \forall n \in \mathcal{N}, \forall b \in O(n)
\]

Without loss of generality, we assume that for all time over all links under any power allocation matrix and any channel state, there exists some finite constant \( X_{\text{max}} \) such that

\[
0 \leq \sum_{f \in F} x_{nb}^f(t) \leq X_{\text{max}}
\]

D. Energy Consumption Model

The energy consumption model is divided into three parts: data sensing/processing, data transmission and data reception. Therein, we define \( \bar{P}_S^f \) and \( \bar{P}_R^f \) as the cost of per data sensing and reception, respectively.

When node \( n \in \mathcal{N}_f \) achieves the compressed data rate at \( r_n^f(t) \), we assume the energy consumed is \( \bar{P}_S^f r_n^f(t) \), which is linear. When node \( n \) receives session \( f \) from its neighbor node \( a \in \mathcal{I}_n \) at rate \( x_{an}^f(t) \), the energy required is \( \bar{P}_R^f x_{an}^f(t) \).

Above all, the total energy consumption \( p_n^{\text{Total}}(t) \) of node \( n \) at slot \( t \) is:

\[
p_n^{\text{Total}}(t) = \sum_{f \in F} C_n^f \bar{P}_S^f r_n^f(t) + \sum_{b \in O(n)} \bar{P}_R^f x_{nb}^f(t)
\]

E. Energy Supply Model

First, we describe the energy supply model of ME node shown in Fig[1]. Each ME node is equipped with a battery. As depicted in Fig[1], the harvested energy \( e_n(t) \) at time \( t \) for ME node \( n \) is stored in the battery. On the other hand, the energy supplied by the electricity grid at time \( t \) for ME node \( n \) is denoted by \( y_n(t) \). Some of this grid energy is used to charge the battery at a charging rate \( g_n(t) \), while the remaining energy \( y_n(t) - g_n(t) \) is used to supply the operation of ME node \( n \).

In addition, some energy from the battery is discharged at a discharging rate \( d_n(t) \) to supplement the energy drawn from the electricity grid so as to meet the energy demand of the node. Thus, the total energy consumption \( p_n^{\text{Total}}(t) \) of a ME node \( n \) at slot \( t \) is determined by the energy supplied by the electricity grid, and the energy discharged from the battery, therefore expressed as

\[
p_n^{\text{Total}}(t) = y_n(t) - g_n(t) + d_n(t)
\]

which is equivalent to

\[
y_n(t) \geq g_n(t) - d_n(t) + p_n^{\text{Total}}(t), \forall n \in \mathcal{N}_G \cup \mathcal{N}_M.
\]

1) Energy queue: We assume each ME-node \( n \) knows its own current energy availability \( E_n^M(t) \) denoting the energy queue size for \( n \in \mathcal{N}_M \) at time slot \( t \). We define \( E_M(t) = (E_n^M(t), n \in \mathcal{N}_M) \) over time slots \( t \in \mathcal{T} \) as the vector of the energy queue sizes. For ME node \( n \), the energy queuing dynamic equation is

\[
E_n^M(t + 1) = E_n^M(t) + e_n(t) - d_n(t) - g_n(t)
\]

with \( E_n^M(0) = 0 \). In any time slot \( t \), the total energy consumption at ME node \( n \) must satisfy the following energy-availability constraint:

\[
E_n^M(t) \geq d_n(t), \forall n \in \mathcal{N}_M.
\]

2) Two special cases: In fact, both EH node and EG node are two special cases of ME node.

For EH node, \( y_n(t) = 0, g_n(t) = 0 \), and so \( p_n^{\text{Total}}(t) = d_n(t) \). Thus, the energy queueing dynamic equation at EH node \( n \) is changed from (13) into:

\[
E_n^H(t + 1) = E_n^H(t) + e_n(t) - p_n^{\text{Total}}(t), n \in \mathcal{N}_H
\]

with \( E_n^H(0) = 0 \). The energy-availability constraint is changed from (14) into:

\[
E_n^H(t) \geq p_n^{\text{Total}}(t), \forall n \in \mathcal{N}_H.
\]

For EG node, \( e_n(t) = 0 \). Thus, the energy queueing dynamic equation at EG node \( n \) is changed from (13) into:

\[
E_n^G(t + 1) = E_n^G(t) + g_n(t) - d_n(t), \quad n \in \mathcal{N}_G
\]
with \( E_n^G(0) = 0 \). The energy-availability constraint is changed from (14) into:

\[
E_n^G(t) \geq d_n(t), \quad \forall n \in N_G.
\]

(18)

We define \( E_H(t) = (E_n^H(t), n \in N_H) \) and \( E_G(t) = (E_n^G(t), n \in N_G) \) over time slots \( t \in T \) as the vector of the energy queue sizes for all EH nodes and that for all EG nodes, respectively.

3) Some constraints: We assume the available amount of harvesting energy at slot \( t \) is \( h_n(t) \) with \( h_n(t) \leq h_{\max} \) for all \( t \). The amount of actually harvested energy \( e_n(t) \) at slot \( t \), should satisfy

\[
0 \leq e_n(t) \leq h_n(t), \forall n \in N_H \cup N_M,
\]

(19)

where \( h_n(t) \) is randomly varying over time slots in an i.i.d. fashion according to a potentially unknown distribution and taking non-negative values from a finite but arbitrarily large set \( S^H \). We define the harvestable energy vector \( S(t) = (h_n(t), n \in N_H \cup N_M) \), called the harvestable energy state.

The charging rate \( g_n(t) \) of the battery of node \( n \) at slot \( t \) should satisfy:

\[
0 \leq g_n(t) \leq g_{\max}, \forall n \in N_G \cup N_M,
\]

(20)

with some finite \( g_{\max} \).

The discharging rate \( d_n(t) \) of the battery of node \( n \) at slot \( t \) should satisfy:

\[
0 \leq d_n(t) \leq d_{\max}, \forall n \in N_G \cup N_M,
\]

(21)

with some finite \( d_{\max} \).

The energy supplied by the electricity grid \( y_n(t) \) of the node \( n \) at slot \( t \) should satisfy:

\[
0 \leq y_n(t) \leq y_{\max}, \forall n \in N_G \cup N_M,
\]

(22)

with some finite \( y_{\max} \).

Moreover, at any time slot \( t \), we also assume the total energy volume stored in battery on node \( n \in N_H \) is limited by the weight perturbation \( \theta_n^H \) introduced in section IV, the reason will be explained in Theorem 1, thus the following inequations must be satisfied

\[
E_n^H(t) + e_n(t) \leq \theta_n^H, \forall n \in N_H
\]

(23)

F. Electricity Price Model

At time slot \( t \), the cost of purchasing one unit electricity from the electricity grid at node \( n \in N_G \cup N_M \) is characterized by the function \( P_n^G(t) \), which may be with respect to the energy \( y_n(t) \) supplied by EG and the electricity price state variable \( S_n^G(t) \). We assume \( S_n^G(t) \) is an i.i.d. process and takes non-negative values from a finite but arbitrarily large set \( S^G \). Denote \( S(t) = \{S_n^G(t), n \in N_G \cup N_M\} \) as the electricity price vector. Therein, we assume that \( P_n^G(t) \) is a function of both \( S_n^G(t) \) and \( y_n(t) \), i.e.,

\[
P_n^G(t) = P_n^G(S_n^G(t), y_n(t))
\]

Note that the dependence of \( P_n^G(t) \) on \( S_n^G(t) \) and \( y_n(t) \) is implicit for notational convenience in the sequel. For each \( S_n^G(t) \), \( P_n^G(t) \) is assumed to be a increasing and continuous convex function of \( y_n(t) \).

G. Data Queue Model

For \( f \in F \) at node \( n \), we use \( Q_n^{f,d}(t) \) to denote the data backlog of the \( f \)-th session from source \( s \in N_s^f \) to sink \( d \in N_d^f \) at slot \( t \). We define \( Q(t) = (Q_n^{f,d}(t), n \in N, f \in F, s \in N_s^f, d \in N_d^f) \) over time slots \( t \in T \). Then, the data queueing dynamic equation at the network layer is

\[
Q_n^{f,d}(t+1) = Q_n^{f,d}(t) - \sum_{b \in G(n)} z_n^{f,b}(t)
\]

\[
+ \sum_{a \in I(n)} z_n^{a}(t) + 1_{f \in F_n} r_n^{f}(t).
\]

(24)

with \( Q_n^{f,d}(0) = 0 \). Also, the following data-availability constraint at the network layer should be satisfied:

\[
0 \leq \sum_{b \in G(n)} z_n^{f,b}(t) \leq Q_n^{f,d}(t),
\]

\[
\forall n \in N, f \in F, s \in N_s^f, d \in N_d^f.
\]

(25)

To ensure the network is strongly stable, the following inequation must be satisfied:

\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{n \in N} \sum_{f \in F} \sum_{s \in N_s^f} \sum_{d \in N_d^f} \mathbb{E}\{Q_n^{f,d}(t)\} < \infty
\]

(26)

H. Optimization Problem

For each session \( f \), at each node \( n \in N_s^f \), we assume the utility is \( U_n^f(D_n^f(t)) \) with the corresponding distortion \( D_n^f(t) \). We assume the \( U_n^f(\cdot) \) is decreasing and concave, So, our goal is to design a full distributed algorithm that achieves the optimal trade-off between the time-average utility of the distortion and the time-average cost of energy consumption in electricity grid subject to all of the constraints described above.

Mathematically, we will address the stochastic optimization problem \( \textbf{P1} \) as follows:

\[
\begin{align*}
\text{maximize} & \quad \bar{O} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{O(t)\} \\
\text{subject to} & \quad (1) - (6), (9), (10), (12), (14), (16), (18), (19) - (23), (25), (26)
\end{align*}
\]

(27)

with the queuing dynamics (13) for \( \forall n \in N_M, \quad (15) \) for \( \forall n \in N_H, \quad (17) \) for \( \forall n \in N_G, \quad (24) \) for \( \forall n \in N, \forall f \in F, s \in N_s^f, d \in N_d^f \).
IV. DISTRIBUTED CROSS-LAYER OPTIMIZATION ALGORITHM

In this section, we assume the rand access probabilities are known a prior, so we will only determine the energy harvesting, the energy purchasing and battery discharge/charge, energy allocation, routing and scheduling decisions.

We will propose a fully distributed algorithm which makes greedy decisions at each time slot without requiring any statistical knowledge of the harvestable energy states, of the electricity price states and of the channel states.

A. Lyapunov optimization

First, we introduce the weight perturbation $\theta_{eH} = \left(\hat{\theta}_{eH}^n, n \in N_H\right)$, $\theta_{eM} = \left(\hat{\theta}_{eM}^n, n \in N_M\right)$ and $\theta_{eG} = \left(\hat{\theta}_{eG}^n, n \in N_G\right)$. Then we define the network state at time slot $t$ as $Z(t) = (S^G(t), S^H(t), S^G(t), Q(t), E_M(t), E_H(t), E_G(t))$. Define the Lyapunov function as

$$L(t) = \frac{1}{2} \sum_{n \in N} \sum_{f \in F} \sum_{s \in S^G} \sum_{d \in D^G} \left(Q_{ind}(t)\right)^2 + \frac{1}{2} \sum_{n \in N_H} \left(E^H_n(t) - \theta_{eH}^n\right)^2 + \frac{1}{2} \sum_{n \in N_M} \left(E^M_n(t) - \theta_{eM}^n\right)^2 + \frac{1}{2} \sum_{n \in N_G} \left(E^G_n(t) - \theta_{eG}^n\right)^2 \tag{28}$$

Remark 3.1 From the above equation (28), we can see that when minimizing the Lyapunov function $L(t)$, we push the queue backlog towards the corresponding perturbed variable value. So, as long as we choose appropriate perturbed variables, the constraint (14), (16) and (18) will always be satisfied. The detailed proof will be given in the next section. Thus, we can get rid of this constraint in the sequel.

Now define the drift-plus-penalty as

$$\Delta_V(t) = E \left(L(t+1) - L(t) - VO(t)\right|Z(t)), \text{ where } V \text{ is a non-negative weight, which can be tuned to control } \Omega \text{ arbitrarily close to the optimum with a corresponding tradeoff in average queue size. Next, the upper bound of } \Delta_V(t) \text{ is given in}$$

$$\Delta_V(t) \leq B + E \left(\hat{\Delta}_V(t)\right|Z(t)), \tag{29}$$

where $\hat{\Delta}_V(t)$ is shown in (30),

$$B = \sum_{n \in N} \sum_{f \in F} \sum_{s \in S^G} \sum_{d \in D^G} B_Q + \sum_{n \in N_H} B^H_E + \sum_{n \in N_M} B^M_E + \sum_{n \in N_G} B^G_E,$$

and $l_{max}$ denotes the largest number of the outgoing/ incoming links that any node in the network can have. $B_Q = \frac{3}{2} l_{max} X_{max}^2 + \frac{1}{2} P_{max}^2$, $B^H_E = \frac{1}{2} l_{max}^2 + \frac{1}{2} (P_{max}^2)$, $B^M_E = \frac{1}{2} l_{max}^2 + \frac{1}{2} X_{max}^2$, $B^G_E = \frac{1}{2} (g_{max}^n)^2 + \frac{1}{2} (d_{max}^n)^2$, $P_{max}^T = \sum_{f \in F} \sum_{n \in N^f} \hat{P}^f_{n, max} R_{max} + \sum_{n \in N^T} \hat{P}^T_{n, max} X_{max}$.

After using the Lyapunov optimization, our optimization problem (27) is changed into minimizing $\Delta_V(t)$ i.e., P2 as follows:

$$\text{minimize } \hat{\Delta}_V(t), \tag{31}$$

subject to \begin{align*}
\text{(1)} - \text{(4)}, \text{(6)}, \text{(9)}, \text{(10)}, \text{(12)}, \text{(19)} - \text{(23)}, \text{(25)}.
\end{align*}

As our optimization problem (31) is non-convex, first, we will show that it can be an equivalent convex problem by using the log change. First, let $\tilde{p}_{nb}^T(t) = \log(\tilde{p}_{nb}^T(t))$, the definition (11) of $p_{total}^T(t)$ is changed into

$$p_{total}^T(t) = \sum_{f \in F} \sum_{n \in N^f} \tilde{P}_{total}^S f_n(t) + \sum_{b \in O(n)} e_{nb}^T(t) \tag{32}$$

and the constraint (6) changes as follows

$$\sum_{b \in O(n)} e_{nb}^T(t) \leq p_{max}^T, \tag{33}$$

and the constraint (9) is changed into the following:

$$\sum_{f \in F} \sum_{n \in N^f} \tilde{a}_{nb}^f(t) \leq \tilde{C}_{nb}(t), \forall n \in N, \forall b \in O(n) \tag{34}$$

where $\tilde{C}_{nb}(t) = BW \rho_{nb}(t) \log(\tilde{\gamma}_{nb}(t))$, with

$$\tilde{\gamma}_{nb}(t) = \frac{N_{nb} + \sum_{a \in I_n \cup \{a,m\} \in E} S_{nb}^G(t) e_{nb}^G(t) q_{am}(t)}{S_{nb}^G(t) e_{nb}^G(t) q_{am}(t)}, \tag{35}$$

Then, plugging (32) into (30), and rearranging all terms of the righthand side (RHS) in (30), the $\Delta_V(t)$ is changed into $\tilde{\Delta}_V(t)$. Finally, our optimization problem P2 is changed into the equivalent problem P3 as follows:

$$\text{minimize } \tilde{\Delta}_V(t), \tag{37}$$

subject to \begin{align*}
\text{(1)} - \text{(4)}, \text{(10)}, \text{(12)}, \text{(19)} - \text{(23)}, \text{(25)}, \text{(35)}, \text{(36)}.
\end{align*}

Above all, we have the following Theorem 1: Assume the random access probabilities are known a prior and the battery capacity of EH nodes have an upper bound $\theta_{eH}^n$. By using the log change for (8), the optimization problem P3 is convex.

Proof: First, we show the object function in (37) is convex. Due to the function of $e(t), r(t), g(t), d(t), x(t)$ and $\tilde{x}(t)$ is linear, hence, we only have to show the function of $D(t), g(t)$ and $p^T(t)$ is convex. By assuming the the battery capacity of EH nodes have an upper bound $\theta_{eH}^n$, we have that $E_{H}^n(t) - \theta_{eH}^n < 0, \forall n \in N_H$. Due to the the electricity price $p_{total}^T(t)$ is assumed to be a increasing and continuous convex function of $\gamma_{nb}(t)$ for each given $S_{nb}^G(t)$, the $U_{n}^T (\cdot)$ is concave and the convexity of $e_{nb}(t)$, we can get the conclusion that our object function is a convex function of $\chi(t)$.
\[
\tilde{\Delta}_V(t) = \sum_{n \in N^H} (E_n^H(t) - \theta_n^H) e_n(t) + \sum_{n \in N_M} (E_n^M(t) - \theta_n^M) e_n(t) \\
+ \sum_{n \in N_G} \left[(E_n^G(t) - \theta_n^G) (g_n(t) - d_n(t)) + V(1 - \omega_1)\omega_2 P_n^G(t) y_n(t)\right] \\
+ \sum_{n \in N_M} \left[(E_n^M(t) - \theta_n^M) (g_n(t) - d_n(t)) + V(1 - \omega_1)\omega_2 P_n^G(t) y_n(t)\right]
\]

\[
\tilde{\Delta}_V(t) = \sum_{n \in N^H} (E_n^H(t) - \theta_n^H) e_n(t) + \sum_{n \in N_M} (E_n^M(t) - \theta_n^M) e_n(t) \\
+ \sum_{n \in N_G} \left[(E_n^G(t) - \theta_n^G) (g_n(t) - d_n(t)) + V(1 - \omega_1)\omega_2 P_n^G(t) y_n(t)\right] \\
+ \sum_{n \in N_M} \left[(E_n^M(t) - \theta_n^M) (g_n(t) - d_n(t)) + V(1 - \omega_1)\omega_2 P_n^G(t) y_n(t)\right]
\]

Second, we show the constraints in (37) are convex. We can easily have the constraints (1), (4), (10), (12), (13), (18), (19), (23), (25) are convex, and due to the convexity of \( e^{\beta_{n,d}(t)} \), we can have (12), (33) are convex. Next,

\[
\log (\gamma_{nb}(t)) = \log S_{nb}^G(t) + \tilde{p}_T(t) \\
- \log \left( N_{nb} + \sum_{a \in L_a,b (a,m) \in L} S_{am}^G(t) q_{am}(t) e^{\tilde{T}_{am}(t)} \right)
\]

it is not difficult to prove that \( \log (\gamma_{nb}(t)) \) is a strictly concave function of a logarithmically transformed power vector \( \tilde{p}_T(t) \) [30], so (34) is convex.

B. Framework of

The framework of our algorithm is summarized in TABLE I.

C. Dual decomposition

Therein, we apply the dual decomposition to solve problem P3.

As the object is to minimize the function of \( y(t) \), and the function is convex, so the constraint (12) is equivalent to the following inequation

\[
y_n(t) \geq g_n(t) - d_n(t) + p_n^{Total}(t).
\]
Thus, the problem P3 is changed into P4:

\[
\begin{align*}
& \text{minimize} & & \mathcal{L}(\chi(t), \lambda(t), \rho(t)) \\
& \text{subject to} & & \frac{1}{2} \sum_{n \in \mathcal{N}_H} \sum_{m \in \mathcal{N}_m^f} |\sum_{n \in \mathcal{N}_H} |\sum_{m \in \mathcal{N}_m^f} \rho_m^f(t) \\
& & & - \sum_{d \in \mathcal{N}_d^f} Q_n^{f,d}(t) + A_n(t) \tilde{P}_n^f r_n^f(t),
\end{align*}
\]

where \(\mathcal{L}(\chi(t), \lambda(t), \rho(t))\) is the Lagrangian function of problem P3.

\[
\begin{align*}
\mathcal{L}(\chi(t), \lambda(t), \rho(t)) &= \tilde{\Delta}_V(t) \\
&+ \sum_{n \in \mathcal{N}_H} \lambda_n(t) \left( g_n(t) - d_n(t) + p_n^T(t) - y_n(t) \right) \\
&+ \sum_{f \in \mathcal{F}} \sum_{m=1}^{2N_f^f-1} \rho_m^f(t) \left[ H \left( \Delta_m^f \Delta_s^f - \Delta_m^s \Delta_s^f \right) - \sum_{n \in \mathcal{N}_m^f} r_n^f(t) \\
&- \log((2\pi e)^{\frac{N_m^f}{2}}) \prod_{n \in \mathcal{N}_m^f} D_n^f(t) \right] \\
&- \log((2\pi e)^{\frac{N_f^s}{2}}) \prod_{n \in \mathcal{N}_m^s} D_n^s(t) \\
\end{align*}
\]

(41)

Therein, we note that for each session \(f\), there exits \(2N_f^f - 1\) subsets \(\Delta_m^f\) of its source set \(\mathcal{N}_m^f\).

Plugging (39) into (41), we have

\[
\begin{align*}
\mathcal{L}(\chi(t), \lambda(t), \rho(t)) &= \mathcal{L}_1(e(t), g(t), \lambda(t)) \\
&+ \mathcal{L}_2(d(t), y(t), \lambda(t)) + \mathcal{L}_3(g(t), d(t), y(t), \lambda(t)) \\
&- \mathcal{L}_4(D(t), \lambda(t)) - \mathcal{L}_5(r(t), \lambda(t), \rho(t)) \\
&- \mathcal{L}_6(\chi(t), \tilde{x}(t), \tilde{P}(t), \lambda(t))
\end{align*}
\]

(42)

where

\[
\begin{align*}
\mathcal{L}_1(e(t), g(t), \lambda(t)) &= \sum_{n \in \mathcal{N}_H} \left[ (E_n^H(t) - \theta_n^H) e_n(t) + \lambda_n(t) \right] \\
&+ \left( E_n^M(t) - \theta_n^M \right) e_n(t),
\end{align*}
\]

(43)

\[
\begin{align*}
\mathcal{L}_2(d(t), y(t), \lambda(t)) &= \sum_{n \in \mathcal{N}_H} \left[ (V(1 - \omega_1) \varpi_2 P_n^G(t) - \lambda_n) \ y_n(t) \\
&- (E_n^M(t) - \theta_n^M + \lambda_n) \ d_n(t) \right]
\end{align*}
\]

(44)

\[
\begin{align*}
\mathcal{L}_3(g(t), d(t), y(t), \lambda(t)) &= \sum_{n \in \mathcal{N}_H} \left[ (E_n^G(t) - \theta_n^G + \lambda_n) \ (g_n(t) - d_n(t)) \\
&+ (V(1 - \omega_1) \varpi_2 P_n^G(t) - \lambda_n) \ y_n(t), \right]
\end{align*}
\]

(45)

\[
\begin{align*}
\mathcal{L}_4(D(t), \lambda(t)) &= \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{N}_f^f} \left[ V \varpi_1 U_n^f \left( D_n^f(t) \right) \right] \\
&+ \log \left( D_n^f(t) \right) \sum_{m,n \in \mathcal{N}_m^f} \rho_m^f(t)
\end{align*}
\]

(46)

\[
\begin{align*}
\mathcal{L}_5(r(t), \lambda(t), \rho(t)) &= \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{N}_f^f} \sum_{m,n \in \mathcal{N}_m^f} \rho_m^f(t) \\
&- \sum_{d \in \mathcal{N}_d^f} Q_n^{f,d}(t) + A_n(t) \tilde{P}_n^f r_n^f(t),
\end{align*}
\]

(47)

with

\[
\begin{align*}
A_n(t) &= \Delta \mathbf{1}_{n \in \mathcal{N}_H} \left( E_n^H(t) - \theta_n^H \right) - \mathbf{1}_{n \in \mathcal{N}_G \cup \mathcal{N}_M} \lambda_n \\
&\mathcal{L}_6(\chi(t), \tilde{x}(t), \tilde{P}(t), \lambda(t)) \\
&= \sum_{n \in \mathcal{N}_H} \sum_{b \in \mathcal{O}(n)} \sum_{f \in \mathcal{F}} \sum_{d \in \mathcal{N}_d^f} \tilde{x}_{nb}^d(t) \left( Q_n^{f,d}(t) - Q_b^{f,d}(t) \right) \\
&- \sum_{n \in \mathcal{N}_H} \sum_{b \in \mathcal{O}(n)} \sum_{f \in \mathcal{F}} A_b(t) \tilde{P}_n^f \tilde{x}_{nb}^d(t) + A_n(t) \tilde{P}_n^f \tilde{x}_{nb}^d(t)
\end{align*}
\]

(49)

Let \(\mathcal{D}(\lambda(t), \rho(t))\) be the solution of P4, the dual problem of P4 is

\[
\max_{\lambda(t) \geq 0, \rho(t) \geq 0} \mathcal{D}(\lambda(t), \rho(t))
\]

(50)

and can be solved by a gradient projection method, i.e.,

\[
\begin{align*}
&\lambda_n(t_{i+1}) = \left[ \lambda_n(t_i) + \kappa_\lambda(t_i) \frac{\partial \mathcal{L}(\chi(t_i), \lambda(t_i), \rho(t_i))}{\partial \lambda_n(t_i)} \right]^{-1} \\
&\rho_m^f(t_{i+1}) = \left[ \rho_m^f(t_i) + \kappa_\rho(t_i) \frac{\partial \mathcal{L}(\chi(t_i), \lambda(t_i), \rho(t_i))}{\partial \rho_m^f(t_i)} \right]^{-1}
\end{align*}
\]

(51) \hspace{1cm} \hspace{1cm} (52)

where

\[
\frac{\partial \mathcal{L}(\chi(t_i), \lambda(t_i), \rho(t_i))}{\partial \lambda_n(t_i)} = g_n(t) - d_n(t) + \sum_{f \in \mathcal{F}} 1_{n \in \mathcal{N}_f^f} \tilde{P}_n^f r_n^f(t) \\
+ \sum_{b \in \mathcal{O}(n)} \sum_{a \in \mathcal{T}(n)} \sum_{f \in \mathcal{F}} \varpi_0^f \tilde{x}_{nb}^d(t) - y_n(t)
\]

(53)

\[
\frac{\partial \mathcal{L}(\chi(t_i), \lambda(t_i), \rho(t_i))}{\partial \rho_m^f(t_i)} = \left( \Delta_m^f \Delta_s^f - \Delta_m^s \Delta_s^f \right) - \log \left( (2\pi e)^{\frac{N_m^f}{2}} \prod_{n \in \mathcal{N}_m^f} D_n^f(t) \right) \\
- \sum_{n \in \mathcal{N}_m^f} r_n^f(t)
\]

(54)

where \(t_i\) represents the iteration number at time slot \(t\), \(\kappa_\lambda(t_i)\) and \(\kappa_\rho(t_i)\) are the step sizes.

D. Solving P4

Our problem P4 can be divided into five independent subproblems, in the following subsections, we will give the full subscripts.

1. Energy harvesting on EH node For each EH node \(n \in \mathcal{N}_H\), combining the first term of the RHS of (42) with
the constraint (19) and (23), we have the optimization problem of $e_n(t)$ as follows:

$$\begin{align*}
\text{minimize} & \quad (E_n^H(t) - \theta_n^H) e_n(t) \\
\text{subject to} & \quad 0 \leq e_n(t) \leq h_n(t) \\
& \quad E_n^H(t) + e_n(t) \leq \theta_n^H
\end{align*}$$

(56)

Remark 3.2 (56) indicates that all the incoming energy is stored if there is enough room in the energy buffer according to the limitation imposed by $\theta_n^H$, and otherwise it stores all the energy that it can, filling up the battery size of $\theta_n^H$. Hence, $E_n^H(t) < \theta_n^H$ for all $t$, which means that our algorithm can be implemented with finite energy storage capacity $\theta_n^H$ at node $n \in \mathcal{N}_H$.

(2) Energy harvesting and Battery charging on ME nodes Combining the $L_1$ with the constraints (19) and (20), for each ME node $n \in \mathcal{N}_M$, we have the optimization problem of $e_n(t)$ and $g_n(t)$ as follows:

$$\begin{align*}
\text{minimize} & \quad (E_n^M(t) - \theta_n^M + \lambda_n) g_n(t) \\
& \quad + (E_n^d(t) - \theta_n^M) e_n(t) \\
\text{subject to} & \quad 0 \leq e_n(t) \leq h_n(t) \\
& \quad 0 \leq g_n(t) \leq g_n^{\text{max}}
\end{align*}$$

(57)

(3) Energy purchase and Battery discharging on ME nodes Combining the $L_2$ with the constraints (21) and (22), for each ME node $n \in \mathcal{N}_M$, we have the optimization problem of $d_n(t)$ and $y_n(t)$ as follows:

$$\begin{align*}
\text{minimize} & \quad (V(1 - \omega_2)x_2P_n^G(t) - \lambda_n) y_n(t) \\
& \quad - (E_n^M(t) - \theta_n^M + \lambda_n) d_n(t) \\
\text{subject to} & \quad 0 \leq d_n(t) \leq d_n^{\text{max}} \\
& \quad 0 \leq y_n(t) \leq y_n^{\text{max}}
\end{align*}$$

(58)

(4) Energy purchase and Battery charge/discharge on EG node Combining the $L_3$ with the constraints (20) and (22) for each node $n \in \mathcal{N}_G$, we get the optimization problem of $y_n(t)$, $g_n(t)$ and $d_n(t)$ as follows:

$$\begin{align*}
\text{minimize} & \quad (E_n^G(t) - \theta_n^G + \lambda_n) (y_n(t) - d_n(t)) \\
& \quad + (V(1 - \omega_2)x_2P_n^G(t) - \lambda_n) y_n(t) \\
\text{subject to} & \quad 0 \leq y_n(t) \leq y_n^{\text{max}} \\
& \quad 0 \leq d_n(t) \leq d_n^{\text{max}} \\
& \quad 0 \leq y_n(t) \leq y_n^{\text{max}}
\end{align*}$$

(59)

(5) Distortion optimization Combining the $L_4$ with the constraints (4), for each session $f \in \mathcal{F}$, at node $n \in \mathcal{N}_S$, we get the optimization problem of $D_n^f(t)$ as follows:

$$\begin{align*}
\text{maximize} & \quad V \omega_1U_n^f(D_n^f(t)) + \log(D_n^f(t)) \\
\text{subject to} & \quad D_{\text{min}} \leq D_n^f(t) \leq D_{\text{max}}
\end{align*}$$

(60)

(6) Source rate control Combining the $L_5$ with the constraints (3), for each session $f \in \mathcal{F}$, at node $n \in \mathcal{N}_S$, we get the optimization problem of $r_n^f(t)$ as follows:

$$\begin{align*}
\text{maximize} & \quad r_n^f(t) \\
\text{subject to} & \quad 0 \leq r_n^f(t) \leq R_{\text{max}}
\end{align*}$$

(61)

(7) Information rate, Physical rate and Power allocation We have the optimization problem of $\bar{x}(t), \bar{p}(t)$ and $\bar{p}^T(t)$ as follows:

$$\begin{align*}
\text{maximize} & \quad \mathcal{L}_6(\bar{x}(t), \bar{p}(t), \bar{p}^T(t), \lambda(t)) \\
\text{subject to} & \quad \bar{x}_{\text{nb}}^f(t) \leq x_{\text{nb}}^f(t), \quad \forall n \in \mathcal{N}, b \in \mathcal{O}(n), \\
& \quad f \in \mathcal{F}, s \in \mathcal{N}_f^d, d \in \mathcal{N}_d^d \\
& \quad \sum_{b \in \mathcal{O}(n)} \bar{x}_{\text{nb}}^f(t) \leq Q_{\text{nb}}^f(t), \forall n \in \mathcal{N}, f \in \mathcal{F}, s \in \mathcal{N}_f^d, d \in \mathcal{N}_d^d \\
& \quad \sum_{s \in \mathcal{F}} (Q_{\text{nb}}^f(t) - Q_{\text{nb}}^f(t)) + A_n(t)e_{\text{nb}}^T(t) \\
& \quad \forall n \in \mathcal{N}, f \in \mathcal{F}, s \in \mathcal{N}_f^d, d \in \mathcal{N}_d^d \\
& \quad \sum_{b \in \mathcal{O}(n)} e_{\text{nb}}^T(t) \leq P_{\text{max}}, \forall n \in \mathcal{N}
\end{align*}$$

(62)

Next, we replace the variable $\bar{x}_{\text{nb}}^f(t)$ with the variable $x_{\text{nb}}^f(t)$ due to the constraint (63), we have

$$\begin{align*}
\text{maximize} & \quad \mathcal{L}_6(\bar{x}(t), \bar{p}(t), \lambda(t)) \\
= & \sum_{n \in \mathcal{N}_b \in \mathcal{O}(n)} \sum_{f \in \mathcal{F}} (w_{\text{nb}}^f(t) x_{\text{nb}}^f(t) + A_n(t)e_{\text{nb}}^T(t))
\end{align*}$$

(64)

where

$$w_{\text{nb}}^f(t) = \sum_{s \in \mathcal{N}_f^d} (Q_{\text{nb}}^f(t) - Q_{\text{nb}}^f(t)) + A_n(t)e_{\text{nb}}^T(t)$$

(65)

We define $W_{\text{nb}}^f(t) \Delta = \left[ w_{\text{nb}}^f(t) - \sum_{s \in \mathcal{N}_f^d} \epsilon \right]^{+}$,

where $\epsilon = l_{\text{max}}X + R_{\text{max}}$. Note that the data-availability constraint (25) is always satisfied by introducing $\epsilon$, we will show it in Part D of Theorem 2. Thus, we can get rid of this constraint.

Transmission Power Allocation Component For each node, find any $f^* \in \arg\max_f W_{\text{nb}}^f(t)$. Define $W_{\text{nb}}^f(t) = \max_f W_{\text{nb}}^f(t)$ as the corresponding optimal weight of link $(n, b)$. Observe the current channel state $S(t)$, and select the transmission powers $\bar{p}^T(t)$ by solving the following optimization problem:

$$\begin{align*}
\text{maximize} & \quad \bar{p}(t) \\
\text{subject to} & \quad \sum_{b \in \mathcal{O}(n)} e_{\text{nb}}^T(t) \leq P_{\text{max}}, \forall n \in \mathcal{N}
\end{align*}$$

(66)

Session Scheduling The data of session $f^*$ is selected for routing over link $(n, b)$ whenever $W_{\text{nb}}^f(t) > 0$. That is, if $W_{\text{nb}}^f(t) > 0$, set $x_{\text{nb}}^f(t) = \bar{C}_{\text{nb}}(\bar{p}^T, S(t))$. Next, for each session $f \in \mathcal{F}$, if $f = f^*$ and $Q_{\text{nb}}^f(t) - Q_{\text{nb}}^f(t) + A_n(t)e_{\text{nb}}^T(t) > 0$, let $x_{\text{nb}}^f(t) = \bar{C}_{\text{nb}}(\bar{p}^T, S(t))$.
Remark 3.3: To distributively solve the problem (66), we propose a distributed iterative algorithm based on block coordinate descent (BCD) method.

V. PERFORMANCE ANALYSIS

Now, we analyze the performance of our proposed algorithm. To start with, we assume that there exists \( \delta > 0 \) such that

\[
C_{nb}(p^T(t), s(t)) \leq \delta BW p_{nb}^T(t), \forall n \in N, \forall b \in O(n).
\]

We define \( \tilde{p}_s = \min_f \tilde{p}_{sf}, \tilde{p}_R = \min_n \tilde{p}_R^n, \sigma = \min\{ \tilde{p}_s, \tilde{p}_R, 1 \}, \) and

\[
\beta = \frac{1}{\sigma} \sup_{D_{\min} \leq D_n(t) \leq D_{\max}} \frac{U_n^f(D_n(t)) - U_n^f(D_{\max})}{\log(D_{\max}) - \log(D_n(t))}
\]

Theorem 2: The perturbed variables are chosen as follows:

\[
\theta_n^H = \max\{ \frac{1}{\sigma} \delta BW, n \in N_H \}
\]

\[
\theta_n^G = \frac{1}{\sigma} \beta + d_n^{\max}, n \in N_G
\]

\[
\theta_n^M = \frac{1}{\sigma} \beta + d_n^{\max}, n \in N_M
\]

Then, implementing the algorithm with any fixed parameter \( V > 0 \) for all time slots, we have the following performance guarantees:

(A). Suppose the initial data queues and the initial energy queues satisfy:

\[
Q_{n,s}^{i,sd}(0) \leq \beta V + R_n, s \in N_s, f \in N_f, d \in N_d^{i,s}(71)
\]

\[
E_n^H(0) \leq \theta_n^H, n \in N_H
\]

\[
E_n^G(0) \leq \theta_n^G + g_n^{\max}, n \in N_G
\]

\[
E_n^M(0) \leq \theta_n^M + g_n^{\max} + h_n^{\max}, n \in N_M
\]

then, the data queues and the energy queues of all nodes for all time slots are always bounded as

\[
Q_{n,s}^{i,sd}(t) \leq \beta V + R_n, s \in N_s, f \in N_f, d \in N_d^{i,s}(75)
\]

\[
E_n^H(t) \leq \theta_n^H, n \in N_H
\]

\[
E_n^G(t) \leq \theta_n^G + g_n^{\max}, n \in N_G
\]

\[
E_n^M(t) \leq \theta_n^M + g_n^{\max} + h_n^{\max}, n \in N_M
\]

(B). The objective function value of the problem P1 achieved by the proposed algorithm satisfies the bound

\[
\overline{O} \geq O^* - \frac{\tilde{B}}{V}
\]

where \( O^* \) is the optimal value of the problem P1, and \( \tilde{B} = B + FN_n N_d e^{l_{\max}} X_{\max} \).

(C). When node \( n \in N_H \) allocates nonzero power for data sensing, data transmission and/or data reception, we have:

\[
E_n^H(t) \geq P_{n,\text{max}}, n \in N_H
\]

When the battery on node \( n \in N_G \cup N_M \) is discharging, we have:

\[
E_n^G(t) \geq d_n^{\max}, n \in N_G
\]

VI. SIMULATION RESULTS

In this section, we provide some numerical examples to verify the performance of our algorithm. We consider the network topology in Fig[2] which has 6 nodes, 7 links. Moreover, we assume there is only one multicast session, the nodes \{A, B\} are the two correlated sources and the nodes \{E, F\} are the sinks. Throughout, the form of the rate utility function is set as \( U_n^f(D_n(t)) = \log(1 + D_n(t)) \). The form of the electricity cost function is set as \( P_n^f(t) = S_n^G(t) \).

A. Default simulation setting

Set \( N_H = \{ A, D \}, N_G = \{ B \}, N_M = \{ C \} \) as the default node scenario. Set several default values as in TABLE II:

| Entry | Value |
|-------|-------|
| \( S_{\min} \) | 0.5 |
| \( S_{\max} \) | 1 |

Therein, we assume the \( S_n^G(t) = d_n^{\max} \). The energy-harvesting vector \( S_n^H(t) \) has independent entries that are uniformly distributed in \([0, 10]\) at ME nodes and \([0, 50]\) at EH nodes. The electricity price vector \( S_n^G(t) \) has independent entries that are uniformly distributed in \([S_{\min}^{G}, S_{\max}^{G}] \) with \( S_{\min}^{G} = 0.5, S_{\max}^{G} = 1 \) as default values.

The initial queue sizes and the queue upper bounds are set according to Theorem 2. Therein, we set all the initial queue

Fig. 2. Network topology.

\[
E_n^M(t) \geq d_n^{\max}, n \in N_M
\]
sized to be zero. According to (68)-(70) in Theorem 2, we set $\theta_n^H = 224V + 38, \theta_n^G = 56V + 15, \theta_n^M = 56V + 15$. According to (75)-(78), for $\forall n, f, s \in N^d_f, d \in N^d_f$, each data queue $Q^{f,s}$ has an upper bound $2.8V + 10$, and the battery buffer sizes of each EH node, of each EG node, and of each ME node are set to be $224V + 38, 56V + 30$ and $56V + 40$, respectively.

**B. Algorithm performance evaluation**

We simulate $V = [50, 100, 200, 300, 500, 1000, 2000, 3500]$. In all simulations, the simulation time is $7 \times 10^4$ time slots.

We first examine the effect of parameter $V$ we described in Theorem 2, which can achieve a tradeoff $(1/V, 1/V)$ between queue sizes and the gap from the optimal value. In Fig. 3(a) describes the gap from the optimal value, and we can see that as $V$ increases, the time average optimization objective value keep increasing and converge to very close to the optimum. This confirms the results of (79). (b) describes the data queue sizes, we see that as $V$ increases, the queue size keeps increasing. (c)-(e) describes the energy queue sizes, we observe that the queue size increases as $V$ increases. These confirm the results of (75)-(78).

Fig 4 shows three data queue processes and three energy queue processes under $V = 100$. (a) shows the data queue processes of node $A$ from source node $A$ to sink node $E$, node $B$ from source node $B$ to sink node $F$ and node $C$ from source node $A$ to sink node $F$, respectively. (b) shows the energy queue processes for EH node $A$, EG node $B$ and ME node $C$, respectively. It can be seen that the queue sizes are all bounded with upper bound given in Theorem 2.

**VII. CONCLUSIONS**

It is a huge challenge to remain the perpetual operation for WMSN, due to the extensive energy consumption. In this paper, we consider the coexistence of renewable energy and grid power supply for WMSN. We consider multiple energy consumptions, and heterogeneous energy supplies in the system model, and formulate a discrete-time stochastic cross-layer optimization problem for the multiple multicast with DSC and NC to achieve the optimal reconstruct distortion at sinks and the minimal cost of purchasing electricity from electricity grid. Based on the Lyapunov drift-plus-penalty with perturbation technique and dual decomposition technique, we propose a fully distributed and low-complexity cross-layer algorithm only requiring knowledge of the instantaneous system state. The theoretic proof and the simulation results show that a parameter $V$ enables an explicit trade-off between the optimization objective and queue backlog. In the future, we are interested in delay reduction by modifying the queueing disciplines.

**APPENDIX A**

**PROOF OF PART (A) IN THEOREM 2**

For $t = 0$, we can easily have (75), then we assume (75) is hold at time slot $t$, next we will show that it holds at $t + 1$. 

![Fig. 3. Verification of Theorem 1.](image1)

![Fig. 4. Detailed verification of the queueing bounds.](image2)
Case 1: If queue $Q_{n}^{sd}(t)$ doesn’t receive any data at time $t$, we have $Q_{n}^{sd}(t+1) \leq Q_{n}^{sd}(t) \leq \beta V + R_{\text{max}}$.

Case 2: If queue $Q_{n}^{sd}(t)$ receives the endogenous data from other nodes $a \in \mathcal{I}(n)$, we can get from the session scheduling component that $Q_{n}^{sd}(t) - Q_{n}^{sd}(t) + A_{n}(t)P_{R} - \epsilon \geq 0$. By plugging the definition of $\epsilon$, and due to $A_{n}(t) \leq 0$ and $P_{R} > 0$, we have

$$Q_{n}^{sd}(t) \leq Q_{n}^{sd}(t) - (R_{\text{max}} + l_{\text{max}}X_{\text{max}}). \quad (84)$$

Plugging (75) into (84), we have

$$Q_{n}^{sd}(t) \leq \beta V + R_{\text{max}} - (R_{\text{max}} + l_{\text{max}}X_{\text{max}}) = \beta V - l_{\text{max}}X_{\text{max}}. \quad (85)$$

At every slot, the node can receive the amount of session $f$ data from source $s \in \mathcal{N}_{I}$ to sink $d \in \mathcal{N}_{f}$ at most $R_{\text{max}} + l_{\text{max}}X_{\text{max}}$. So

$$Q_{n}^{sd}(t+1) \leq Q_{n}^{sd}(t) + l_{\text{max}}X_{\text{max}} + R_{\text{max}}. \quad (86)$$

Combining (85) and (86), we have $Q_{n}^{sd}(t+1) \leq \beta V + r_{f}^{\text{max}}$.

Case 3: Lemma 1: For any vector $\lambda^{\ast}, \rho^{\ast}$ that maximizing (50), we have

$$\sum_{m : n \in \Delta_{f_{n}}^{m}} \rho_{n}^{\ast}(t) \leq \beta V \quad (87)$$

$$\lambda_{n}^{\ast} \leq \frac{1}{\sigma} \beta V \quad (88)$$

Proof: please see Appendix E.

If queue $Q_{n}^{sd}(t)$ only receives the new local data, in other words, the node $n = s$, and exits an optimal solution $r^{\ast} > 0$ of (51), we have the following:

$$\sum_{m : n \in \Delta_{f_{n}}^{m}} \rho_{n}^{\ast}(t) - \sum_{d \in \mathcal{N}_{d}} Q_{n}^{sd}(t) + A_{n}(t)P_{d}^{S} \geq 0$$

From (87), we get

$$\sum_{d \in \mathcal{N}_{d}} Q_{n}^{sd}(t) \leq \sum_{m : n \in \Delta_{f_{n}}^{m}} \rho_{n}^{\ast}(t) + A_{n}(t)P_{d}^{S} \leq \beta V \quad (89)$$

i.e., $Q_{n}^{sd}(t) \leq \beta V$. Hence $Q_{n}^{sd}(t+1) \leq Q_{n}^{sd}(t) + R_{\text{max}} \leq \beta V + R_{\text{max}}$.

To sum up the above, we complete the proof of (75).

From Remark 3.2, we can have (76).

From (58), we obtain that if $E_{n}^{G}(t) \leq \theta_{n}^{G} - \lambda_{n}$, i.e., $E_{n}^{G}(t) \leq \theta_{n}^{G}$, the EG node is charging, so $E_{n}^{G}(t+1) \leq E_{n}^{G}(t) + g_{n}^{\text{max}} \leq \theta_{n}^{G} + g_{n}^{\text{max}}$.

From (77), we obtain that if $E_{n}^{M}(t) \leq \theta_{n}^{M} - \lambda_{n}$, i.e., $E_{n}^{M}(t) \leq \theta_{n}^{M}$, the ME node is charging, so $E_{n}^{M}(t+1) \leq E_{n}^{M}(t) + g_{n}^{\text{max}} + h_{n}^{\text{max}} \leq \theta_{n}^{G} + g_{n}^{\text{max}} + h_{n}^{\text{max}}$.

So, this completes the proof of Theorem 2(A). □

APPENDIX B

PROOF OF PART (B) IN THEOREM 2

Before giving the proof of Part (B) in Theorem 2, we give Lemma 2 and its proof. At time $t$, we denote $\alpha_{n, f_{n}}^{Q}(t)$, $\alpha_{n, f_{n}}^{H}(t)$, $\alpha_{n, f_{n}}^{G}(t)$, $\alpha_{n, f_{n}}^{M}(t)$ and $\rho_{n, f_{n}}^{Q}(t)$, $\rho_{n, f_{n}}^{H}(t)$, $\rho_{n, f_{n}}^{G}(t)$, $\rho_{n, f_{n}}^{M}(t)$ as the input and output of the queue $Q_{n}^{sd}(t)$, $E_{n}^{H}(t)$, $E_{n}^{G}(t)$ and $E_{n}^{M}(t)$ for $\forall f \in \mathcal{F}, s \in \mathcal{N}_{f}^{I}, d \in \mathcal{N}_{f}^{I}, n \in \mathcal{N}, b \in \mathcal{O}(n)$, respectively.

Lemma 2: We assume $\phi^{\ast}$ is the optimal solution to the following problem (90) and the $z(t)$ takes non-negative values from a finite but arbitrarily large set $\mathcal{Z}$, where $Pr(z(t) = z_{i}) = \pi_{z_{i}}$, and $\mathcal{K} = \{1, 2, \cdots, \mathcal{N}^{2} + 2\}$. Then we have an upper bound for the optimization problem (27), i.e., $VO^{\ast} \leq \phi^{\ast}$.

Proof of Lemma 2: For any stable policy, for any queue, the time average input rate cannot exceed the time average output rate. From constraints (14), (16), (18), (25), we have that, for any queue, the input rate should be more than the output rate, so the time average input rate is equal to the time average output rate. By using Caratheodory’s theorem (see, for example, (31)), and comparing the problem (27) with (90), we conclude that $VO^{\ast}$ is one feasible solution to (90), so $VO^{\ast} \leq \phi^{\ast}$. □

Now, we begin to give the proof of Part (B) in Theorem 2. We can see from the problem (27) that, after using Lyapunov optimization, our optimization problem is to minimize $\hat{\Delta}_{V}(t)$. However, what we actually solve in the section IV is to maximize $\Delta_{V}^{\ast}(t) + \epsilon \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{N}_{f}^{I}} \sum_{b \in \mathcal{O}(n)} \sum_{d \in \mathcal{N}_{f}^{I}} x_{f_{n}, b}(t)$ by introducing $\sigma$, where $\Delta_{V}^{\ast}(t)$ is the function $\Delta_{V}(t)$ minimized under our proposed algorithm. Then, we have

$$\Delta_{V}^{\ast}(t) + \epsilon \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{N}_{f}^{I}} \sum_{b \in \mathcal{O}(n)} \sum_{d \in \mathcal{N}_{f}^{I}} x_{f_{n}, b}(t) \leq \Delta_{V}^{F}(t) + \epsilon \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{N}_{f}^{I}} \sum_{b \in \mathcal{O}(n)} \sum_{d \in \mathcal{N}_{f}^{I}} x_{f_{n}, b}(t) \leq N_{f}N_{nd}\lambda_{\text{max}}X_{\text{max}},$$

where $\Delta_{V}^{F}(t) \leq \Delta_{V}^{\ast}(t) + N_{f}N_{nd}\lambda_{\text{max}}X_{\text{max}}$, when $F$ represents any other feasible policy. Compared to (90), we have that $-E\{\Delta_{V}^{F}(t)\} = \phi^{\ast}$. Then, we have

$$\Delta(t) = -VE\{O(t)\} \leq \hat{B} - \phi^{\ast} \quad (91)$$

where $\hat{B} = B + N_{f}N_{nd}\lambda_{\text{max}}X_{\text{max}}$. Based on the conclusion of Lemma 1, i.e., $VO^{\ast} \leq \phi^{\ast}$, we have

Then,

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} E[\Delta(t) - VE\{O(t)\}] \leq \hat{B} - VO^{\ast}$$

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} E[L(t+1) - L(t) - VO(t)] \leq \hat{B} - VO^{\ast}$$

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{L(T) - L(0)\} - \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{O(t)\} \leq \hat{B} - VO^{\ast}$$
maximize \( V \sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} O \left( z_i, \chi_k^{(z_i)} \right) \) 
subject to 
\[
\sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} Q_{n,f,sd} \left( z_i, \chi_k^{(z_i)} \right) = \sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} \mu_{n,f,sd} \left( z_i, \chi_k^{(z_i)} \right),
\]
\[
\sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} e^{H} \left( z_i, \chi_k^{(z_i)} \right) = \sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} \mu_{n,f,sd} \left( z_i, \chi_k^{(z_i)} \right),
\]
\[
\sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} e^{G} \left( z_i, \chi_k^{(z_i)} \right) = \sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} e^{H} \left( z_i, \chi_k^{(z_i)} \right),
\]
\[
\sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} e^{M} \left( z_i, \chi_k^{(z_i)} \right) = \sum_{z_i \in \mathcal{Z}} \pi_{z_i} \sum_{k \in \mathcal{K}} a_k^{(z_i)} e^{G} \left( z_i, \chi_k^{(z_i)} \right),
\]
\[
\sum_{n,k \in \mathcal{N}_f^s} \gamma_{n,k} \geq H \left( \mathcal{N}_f^s \mathcal{N}_d^f \right) - \log((2\pi e)^{\mathcal{N}_f^s} \prod_{n \in \mathcal{N}_f^s} D_{n,k}^f), \quad \forall n \in \mathcal{N}_f^s
\]
\[
0 \leq x_{f,sd}^{(z_i)} \leq \bar{R}_m, \quad D_{n,k} \leq D_{n,k}^f \leq D_{n,k}^max, \quad \forall n \in \mathcal{N}_f^s
\]
\[
x_{f,sd}^{(z_i)} \leq x_{f,sd}^{(z_i)} + \sum_{f \in \mathcal{F}} x_{f,sd}^{(z_i)} + c_{F}^{(z_i)} \sum_{b \in \mathcal{O}(n)} T_{s}^{(z_i)} \leq F_{n,k} \quad \forall n \in \mathcal{N}_d^f
\]
\[
0 \leq g_{n,k}^{max} \leq g_{n,k} \leq g_{n,k}^{max}, \quad 0 \leq y_{n,k}^{max} \leq y_{n,k}, \quad \forall n \in \mathcal{N}_d^f \cup \mathcal{N}_d^f
\]
\[
y_{n,k} = g_{n,k} - d_{n,k} + \Pi_{n,k} + \Pi_{n,k} \quad \forall n \in \mathcal{N}_d^f \cup \mathcal{N}_d^f
\]
\[
0 \leq \epsilon_{n,k} \quad \forall n \in \mathcal{N}_d^f \cup \mathcal{N}_d^f
\]
\[
\sum_{k \in \mathcal{K}} a_k^{(z_i)} = 1, \quad a_k^{(z_i)} \geq 0, \quad \chi_k^{(z_i)} \in \chi_i,
\]
\[
\forall k, z_i, n, f, s \in \mathcal{N}_f^f, d \in \mathcal{N}_d^f
\]

Thus, we have
\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \{ O(t) \} \geq O^* - \frac{\bar{B}}{V}, \quad (92)
\]
So, this completes the proof of Theorem 2(B). □

APPENDIX C
PROOF OF PART (C) IN THEOREM 2

In order to prove (80), we first assume that \( E_H^m (t) < P_{T,n,m} \), when node \( n \in \mathcal{N}_d^f \) allocates nonzero power for data sensing, compression and transmission.

Case 1: When there exits energy consumption for data sensing/compression at node \( n \), from (61), if \( E_H^m (t) < P_{T,n,m} \),
\[
\sum_{m:n \in \mathcal{N}_m^{f,sd}} \rho_{m}^{f,s} (t) - \sum_{d \in \mathcal{N}_d^f} Q_n^{f,sd} (t) + A_n (t) \tilde{P}_f^S \leq \beta V - \max \left\{ \frac{1}{\sigma}, \delta BW \right\} N_s N_d \beta V \tilde{P}_f^S
\]
\[
\leq \beta V - \frac{1}{\sigma} N_s N_d \beta V \tilde{P}_f^S \leq 0
\]
from (48) and \( Q_n^{f,sd} (t) \geq 0 \) we have the first inequality, from (87) and (76) we have the second inequality, and from the definition of \( \sigma \) we have the last inequality. The last inequality shows that if \( E_H^m (t) < P_{T,n,m} \), the optimal solution of (61) is zero, which means there exits no energy consumption for data sensing/compression.

Case 2: It is easy to verify that the following inequation holds according to the definition of \( C_{ab} (p^T (t), S (t)) \):
\[
C_{ab} (p^T (t), S (t)) \leq C_{ab} (p^T (t), S (t)) \quad (93)
\]
where \( p^T (t) \) obtained by setting \( p_{n,m}^T (t) \) of \( p^T (t) \) to zero, \( (a, b) \in \mathcal{L} \) and \( (a, b) \neq (n, m) \).

Whenever the link \( (n, m) \) do transmission, for each session \( f \) we get
\[
W_{n,m}^f (t)
\]
\[
= \left[ \sum_{s \in \mathcal{N}_f^s} \sum_{d \in \mathcal{N}_d^f} \left( Q_n^{f,sd} (t) - Q_n^{f,sd} (t) - \epsilon \right) + A_n (t) \tilde{P}_m^R \right]^+
\]
\[
\leq \left[ \sum_{s \in \mathcal{N}_f^s} \sum_{d \in \mathcal{N}_d^f} \left( Q_n^{f,sd} (t) - \epsilon \right) \right]^+
\]
\[
(94)
\]
By plugging the definition of \( \epsilon \) and (75) into (94), we have
\[
W_{n,m}^f (t)
\]
\[
\leq \sum_{s \in \mathcal{N}_f^s} \sum_{d \in \mathcal{N}_d^f} \left[ \beta V + R_{max} - l_{max} X_{max} - R_{max} \right]^+
\]
\[
= N_s N_d \left[ \beta V - l_{max} X_{max} \right]^+
\]
\[
(95)
\]
Moreover, we have
\[
W_{nm}^* (t) \leq N_s N_d [\beta V - l_{\text{max}} X_{\text{max}}]^+ \tag{96}
\]
We assume that \( E_{n}^H (t) < P_{n, \text{max}}^{\text{Total}} \) when node \( n \in \mathcal{N} \) allocates nonzero power for data transmission. Furthermore, we assume that the power allocation control vector \( p^T (t) \) is the optimal solution to (66), and without loss of generality, there exists some \( p_{nm}^* (t) > 0 \). By setting \( p_{nm}^* (t) = 0 \) in \( p^T (t) \), we get another power allocation control vector \( p^T (t) \). We denote \( G (p^T (t), S (t)) \) as the objective function of (65). In this way, we get:
\[
G (p^T (t), S (t)) - G (p^T (t), S (t)) = \sum_{n \in \mathcal{N}} \sum_{b \in O(t)} [C_{nb} (p^T (t), S (t)) - C_{nb} (p^T (t), S (t))] W_{nb}^* (t) + (E_{n}^H (t) - \theta_n^H) p_{nm}^* \tag{97}
\]
From (75), we have \( C_{nb} (p^T (t), S (t)) - C_{nb} (p^T (t), S (t)) \leq 0 \) for \( b \neq m \). So
\[
G (p^T (t), S (t)) - G (p^T (t), S (t)) \leq C_{nm} (p^T (t), S (t)) W_{nm}^* (t) + (E_{n}^H (t) - \theta_n^H) p_{nm}^* \tag{98}
\]
According to our assumption \( E_{n}^H (t) < P_{n, \text{max}}^{\text{Total}} \) and the definition of \( \theta_n^H \) in (68), we have
\[
E_{n}^H (t) - \theta_n^H < P_{n, \text{max}}^{\text{Total}} - \theta_n^H = -\max \left\{ \frac{1}{\sigma}, \delta BW \right\} N_s N_d [\beta V] \tag{99}
\]
Plugging (67), (96) and (99) into (98), we have
\[
G (p^T (t), S) - G (p^T (t), S) \leq \delta BW p_{nm}^* N_s N_d [\beta V - l_{\text{max}} X_{\text{max}}]^+ - \max \left\{ \frac{1}{\sigma}, \delta BW \right\} N_s N_d [\beta V] p_{nm}^* < 0
\]
the last inequality is due to the assumption \( \max \left\{ \frac{1}{\sigma}, \delta BW \right\} \geq \delta BW \). From the last inequality, we can see that if \( E_{n}^H (t) < P_{n, \text{max}}^{\text{Total}} \), the optimal solution to (66) is not the optimal solution to (66).

**Case 3:** When node \( n \) receives data from other node \( a \in \mathcal{I} (n) \) in the network, if \( E_{n}^H (t) < P_{n, \text{max}}^{\text{Total}} \) for each session \( f \) we get
\[
W_{an}^f (t) = \left[ \sum_{s \in \mathcal{N}^f} \sum_{d \in \mathcal{N}^f} (Q_{n}^{\text{sd}} (t) - Q_{n}^{\text{sd}} (t) - \epsilon) + A_n (t) \right]^+ \tag{100}
\]
where
\[
Q_{n}^{\text{sd}} (t) = \max \left\{ \frac{1}{\omega}, \beta V \right\} N_s N_d [\beta V] R_{n}^f \tag{101}
\]
and acceptance, which is opposite with our assumption. So, \( E_{n}^H (t) \geq P_{n, \text{max}}^{\text{Total}} \), which completes the proof of (80).

When the EG node is discharging, from (57), we obtain the following:
\[
E_{n}^H (t) \geq \theta_n^H - \lambda_n \leq \frac{1}{\sigma} \beta V + d_n - \frac{1}{\sigma} \beta V = d_n \tag{102}
\]
where the first inequality is obtained from (69) and (83), the (81) is proved.

The prove of (82) is similar with (81), wherein, we omit it for brief.

Until now, we complete the proof of Theorem 2(C). \( \square \)

**APPENDIX D**

**PROOF OF PART (D) IN THEOREM 2**

For each node \( n \in \mathcal{N} \), when any data of the \( f \)-th session from source \( s \) to sink \( d \) is transmitted to other node, we can get from the session scheduling component that \( Q_{n}^{\text{sd}} (t) = Q_{n}^{\text{sd}} (t) + A_b (t) P_{b}^R - \epsilon > 0 \), i.e., \( Q_{n}^{\text{sd}} (t) > Q_{n}^{\text{sd}} (t) - A_b (t) P_{b}^R + \epsilon > 0 \) From the definition of \( \epsilon \), \( Q_{n}^{\text{sd}} (t) \geq 0 \) and \( A_b (t) \leq 0 \) we have \( Q_{n}^{\text{sd}} (t) > l_{\text{max}} X_{\text{max}} + P_{b}^R \), which completes the proof of Theorem 2(D). \( \square \)

**APPENDIX E**

**PROOF OF LEMMA 1**

We assume \( X^* \) and \( P^* \) are the optimal solution of (53). For (60), let \( D^* \) be the optimal solution, hence, we have
\[
V_1 \sum_{m \in \mathcal{N}^f_m} \rho_m^f (t) \log \left( \frac{U_m^f (D_m^f (t))}{U_m^f (D^*_m (t))} \right) \geq 0
\]
Next, we have
\[
V_1 \sum_{m \in \mathcal{N}^f_m} \rho_m^f (t) \leq V_1 \sum_{m \in \mathcal{N}^f_m} \log \left( \frac{U_m^f (D_m^f (t))}{U_m^f (D^*_m (t))} \right) \leq V_1 \sup_{D_{\text{min}} \leq D_m^f (t) \leq D_{\text{max}}} \left( \log \left( \frac{U_m^f (D_m^f (t))}{U_m^f (D^*_m (t))} \right) \right)
\]
Similarly, for (61) at node \( m \in \mathcal{N}^f \cup \mathcal{N}^A \), let \( x^* \) be the optimal solution, we have
\[
\sum_{m \in \mathcal{N}^f_m} \rho_m^f (t) - \sum_{d \in \mathcal{N}^f_d} Q_{n}^{\text{sd}} (t) - \lambda_n^f \tilde{P}_f^S \geq 0
\]
so
\[
\lambda_n^f \tilde{P}_f^S \leq \sum_{m \in \mathcal{N}^f_m} \rho_m^f (t) - \sum_{d \in \mathcal{N}^f_d} Q_{n}^{\text{sd}} (t)
\]
and from the definition of \( \sigma \), we have \( \lambda_n^f \sigma \leq \beta V \), i.e., \( \lambda_n^f \leq \frac{1}{\sigma} \beta V \).

Above all, we complete the proof of Lemma 1.

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