A solution to the $\mu$ problem in the supersymmetric unparticle physics

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Recently, conceptually new physics beyond the Standard Model has been proposed by Georgi, where a new physics sector becomes conformal and provides “unparticle” which couples to the Standard Model sector through higher dimensional operators in low energy effective theory. Among several possibilities, we focus on operators involving the (scalar) unparticle, Higgs and the gauge bosons. Once the Higgs develops the vacuum expectation value (VEV), the conformal symmetry is broken and as a result, the mixing between the unparticle and the Higgs boson emerges. In this paper, we consider the unparticle as a hidden sector of supersymmetry (SUSY) breaking, and give some phenomenological consequences of this scenario. The result shows that there is a possibility for the unparticle as a hidden sector in SUSY breaking sector, and can provide a solution to the $\mu$ problem in SUSY models.

INTRODUCTION

In spite of the success of the Standard Model (SM) in describing all the existing experimental data, the Higgs boson, which is responsible for the electroweak symmetry breaking, has not yet been directly observed, and is one of the main targets at the CERN Large Hadron Collider (LHC). At the LHC, the main production process of Higgs boson is through gluon fusion, and if Higgs boson is light, say $m_h \lesssim 150$ GeV, the primary discovery mode is through its decay into two photons. In the SM, these processes occur only at the loop level and Higgs boson couples with gluons and photons very weakly.

A certain class of new physics models includes a scalar field which is singlet under the SM gauge group. In general, such a scalar field can mix with the Higgs boson and also can directly couple with gluons and photons through higher dimensional operators with a cutoff in effective low energy theory. Even if the cutoff scale is very high, say, 100-1000 TeV, the couplings with gluons and photons can be comparable to or even larger than those of the Higgs boson induced only at the loop level in the SM. This fact implies that if such a new physics exists, it potentially has an impact on Higgs boson phenomenology at the LHC. In other words, such a new physics may be observed together with the discovery of Higgs boson.

As one of such models, in this letter, we investigate a new physics recently proposed by Georgi [1], which is described in terms of “unparticle” provided by a hidden conformal sector in low energy effective theory. A concrete example of unparticle physics recently proposed by Georgi [1], which is described in terms of “unparticle” provided by a hidden conformal sector in low energy effective theory. A concrete example of unparticle physics has been matched to the unparticle operator $\mathcal{U}$ with dimension $d_{\mathcal{U}}$, in low energy effective theory, we have the operator of the form,

$$\mathcal{L} = \frac{c_n}{M^{d_{\mathcal{UV}}+n-4}} \mathcal{U} \mathcal{O}_\text{UV} \mathcal{O}_\text{SM},$$

(1)

where $c_n$ is a dimensionless constant, and $M$ is the energy scale characterizing the new physics. This new physics sector is assumed to become conformal at a energy $\Lambda_{\mathcal{U}}$, and the operator $\mathcal{O}_\text{UV}$ flows to the unparticle operator $\mathcal{U}$ with dimension $d_{\mathcal{U}}$. In low energy effective theory, we have the operator of the form,

$$\mathcal{L} = \frac{c_n}{M^{d_{\mathcal{UV}}+n-4}} \mathcal{U} \mathcal{O}_\text{SM} = \frac{1}{\Lambda^{d_{\mathcal{U}}+n-4}} \mathcal{U} \mathcal{O}_\text{SM},$$

(2)

where the dimension of the unparticle $\mathcal{U}$ have been matched by $\Lambda_{\mathcal{U}}$ which is induced in the dimensional transmutation, and $\Lambda$ is the (effective) cutoff scale of low energy effective theory. In this paper, we consider only the scalar unparticle.

It was found in Ref. [1], by exploiting scale invariance of the unparticle, the phase space for an unparticle operator with the scale dimension $d_{\mathcal{U}}$ and momentum $p$ is the same as the phase space for $d_{\mathcal{U}}$ invisible massless particles,

$$d\Phi_{\mathcal{U}}(p) = A_{d_{\mathcal{U}}} \theta(p^0) \theta(p^2) (p^2)^{d_{\mathcal{U}}-2} \frac{d^4p}{(2\pi)^4},$$

(3)

where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^2}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}}+rac{1}{2})}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}.$$
Also, based on the argument on the scale invariance, the (scalar) propagator for the unparticle was suggested to be
\[
\frac{\Lambda_{dt}}{2 \sin(\pi d_{lt}) (p^2)^{2-d_{lt}} e^{-i(d_{lt}-2)\pi}.
\]

Because of its unusual mass dimension, unparticle wave function behaves as \(\sim (p^2)^{d_{lt}}\) (in the case of scalar unparticle).

**UNPARTICLE AND THE HIGGS SECTOR**

First, we begin with a brief review of our previous work on the Higgs phenomenology in the unparticle physics [3]. Among several possibilities, we will focus on the operators which include the unparticle and the Higgs sector,
\[
\mathcal{L} = \frac{1}{\Lambda_{dt}^{n-4}} \mathcal{U} \mathcal{O}_{SM}(H^\dagger H) + \frac{1}{\Lambda_{dt}^{n-4}} \mathcal{U}^2 \mathcal{O}_{SM}(H^\dagger H),
\]
where \(H\) is the Standard Model Higgs doublet and \(\mathcal{O}_{SM}(H^\dagger H)\) is the Standard Model operator as a function of the gauge invariant bi-linear of the Higgs doublet. Once the Higgs doublet develops the VEV, the tadpole term for the unparticle operator is induced,
\[
\mathcal{L}_{\tilde{U}} = \Lambda_{\tilde{U}}^{4-d_{lt}} \mathcal{U},
\]
and the conformal symmetry in the new physics sector is broken [5]. Here, \(\Lambda_{\tilde{U}}^{4-d_{lt}} = \langle \mathcal{O}_{SM} \rangle / \Lambda_{dt}^{n-4}\) is the conformal symmetry breaking scale. At the same time, we have the interaction terms between the unparticle and the physical Standard Model Higgs boson \((h)\) such as (up to \(\mathcal{O}(1)\) coefficients)
\[
\mathcal{L}_{Higgs} = \frac{\Lambda_{\tilde{U}}^{4-d_{lt}}}{v} \mathcal{U} h + \frac{\Lambda_{\tilde{U}}^{4-d_{lt}}}{v^2} \mathcal{U} h^2 + \frac{\Lambda_{\tilde{U}}^{4-2d_{lt}}}{v^2} \mathcal{U}^2 h + \cdots,
\]
where \(v = 246\) GeV is the Higgs VEV. In order not to cause a drastic change or instability in the Higgs potential, the scale of the conformal symmetry breaking should naturally be smaller than the Higgs VEV, \(\Lambda_{\tilde{U}} \lesssim v\). When we define the ‘mass’ of the unparticle as a coefficient of the second derivative of the Lagrangian with respect to the unparticle, \(\mathcal{U}\), then the mass of the unparticle can be obtained in the following form,
\[
m_{\tilde{U}}^2 = \Lambda_{\tilde{U}}^{4-2d_{lt}}.
\]
As operators between the unparticle and the Standard Model sector, we consider
\[
\mathcal{L}_{\tilde{U}} = -\frac{\lambda_y}{4} \mathcal{U}^4 + \frac{\lambda_t}{4} \mathcal{U} \mathcal{O}_{SM} + \frac{\lambda_t}{4} \mathcal{O}_{SM} \mathcal{O}_{SM} - \frac{\lambda_t}{4} \mathcal{U} \mathcal{O}_{SM} \mathcal{O}_{SM} F_{\mu\nu} F_{\mu\nu},
\]
where we took into account of the two possible relative signs of the coefficients, \(\lambda_y = \pm 1\) and \(\lambda_t = \pm 1\). We will see that these operators are the most important ones relevant to the Higgs phenomenology.

As discussed before, once the Higgs doublet develops the VEV, the conformal symmetry is broken in the new physics sector, providing the tadpole term in Eq. (7). Once such a tadpole term is induced, the unparticle will subsequently develop the VEV [5, 7] whose order is naturally the same as the scale of the conformal symmetry breaking,
\[
\langle \mathcal{U} \rangle = (c \Lambda_{\tilde{U}})^{d_{lt}}.
\]

Here we have introduced a numerical factor \(c\), which can be \(c = \mathcal{O}(0.1) - \mathcal{O}(1)\), depending on the naturalness criteria. Through this conformal symmetry breaking, parameters in the model are severely constrained by the current precision measurements. We follow the discussion in Ref. [5]. From Eq. (9), the VEV of the unparticle leads to the modification of the photon kinetic term,
\[
\mathcal{L} = -\frac{1}{4} \left[ 1 \pm \frac{\langle \mathcal{U} \rangle}{\Lambda_{\tilde{U}}} \right] F_{\mu\nu} F^{\mu\nu},
\]
which can be interpreted as a threshold correction in the gauge coupling evolution across the scale \(\langle \mathcal{U} \rangle^{1/d_{lt}}\). The evolution of the fine structure constant from zero energy to the Z-pole is consistent with the Standard Model prediction, and the largest uncertainty arises from the fine structure constant measured at the Z-pole [14].
\[
\bar{\alpha}^{-1}(M_Z) = 127.918 \pm 0.019.
\]
This uncertainty (in the \(\overline{\text{MS}}\) scheme) can be converted to the constraint,
\[
\epsilon = \frac{\langle \mathcal{U} \rangle}{\Lambda_{\tilde{U}}} \lesssim 1.4 \times 10^{-4}.
\]
This provides a lower bound on the effective cutoff scale. For \(d_{lt} \simeq 1\) and \(\Lambda_{\tilde{U}} \simeq v\) we find
\[
\Lambda \gtrsim c \times 1000 \text{ TeV},
\]
This is a very severe constraint on the scale of new physics, for example, \(\Lambda \gtrsim 100 \text{ TeV}\) for \(c \gtrsim 0.1\).

**SUPERSYMMETRIC UNPARTICLE**

A supersymmetric extension of the original unparticle physics has been proposed by [15]. We begin by reviewing the scenario of the supersymmetric unparticle and give some further details.

The unparticle which has originally been introduced as a sort of scalar operator is now extended to be a chiral multiplet in order to fit with the supersymmetric theory. Explicitly speaking, it is written as
\[
\mathcal{U} = \mathcal{U} + \sqrt{2} \theta \alpha \alpha \mathcal{U}^\alpha + \theta^2 F_{\mu\nu},
\]
where \(\alpha\) is a spinor index, and we write the same notation for the scalar component of the unparticle chiral multiplet as the...
chiral multiplet itself. Then the interaction or the superpotential between the unparticle and the MSSM sector is, in general, given in the same way as the non-supersymmetric unparticle:

$$\mathcal{L} = \int d^2\theta \left( \frac{1}{N_{dU} + n - 3} \mathcal{U} \mathcal{O}_{\text{MSSM}} + h.c. \right)$$  \hspace{1cm} (14)

**SUSY QCD as a natural candidate of the unparticle**

The most promising example of the SUSY unparticle is given by the SUSY QCD based on $SU(N_c)$ gauge symmetry with $N_f$ flavors [16], which is a natural SUSY extension of the Banks-Zaks model [2]. This correspondence has already been noted in the literature [5]. We take $\frac{3}{2} N_c \leq N_f \leq 3 N_c$ so that the unparticle SUSY QCD is in the conformal window. We denote the chiral superfields for the $N_f$ flavors by $Q_i$ and $\bar{Q}_j$ ($i,j = 1 \ldots N_f$). $Q_i$ transforms as a fundamental representation of $SU(N_c)$ and $\bar{Q}_j$ transforms as an anti-fundamental representation. In general, SUSY QCD in the conformal window ($\frac{3}{2} N_c \leq N_f \leq 3 N_c$) flows to a strongly coupled conformal fixed point in the infrared (IR). At the fixed point the theory has a dual description (Seiberg dual) with a gauge group $SU(N_f - N_c)$. $N_f$ dual-quark superfields ($Q_i$, $\bar{Q}_j$), a gauge singlet meson superfield $M_{ij}$ (transforming in the bifundamental representation of the $SU(N_f) \times SU(N_f)$ flavor symmetry, and the superpotential is given by

$$W = \bar{Q}^i M_{ij} Q^j.$$  \hspace{1cm} (15)

The meson superfield $M_{ij}$ in the dual description corresponds to the gauge invariant composite $\bar{Q}Q$ of the original theory. In regards to the unparticle physics, SUSY QCD allows us to determine some parameters in an explicit way. The conformal dimension of the meson superfield $M_{ij}$ is fixed by the $R$-symmetry:

$$d_{UV} = d_M = 3 \frac{N_c - N_f}{N_f}.$$  \hspace{1cm} (16)

It has to be noted that $1 \leq d_{UV} \leq 2$ in the conformal window ($\frac{3}{2} N_c \leq N_f \leq 3 N_c$).

**A solution to the $\mu$ problem**

Now, we can go to the discussion of the unparticle physics in the SUSY sector. Providing a coupling with Higgs sector for the unparticle with non-vanishing VEV can provide a natural solution to the $\mu$ problem in the MSSM exactly the same way as in the Next to minimal supersymmetric Standard Model (NMSSM). Given the following superpotential for the unparticle in connection with the Higgs sector:

$$\mathcal{L} = k_{\mu} \int d^4\theta \frac{1}{X_{dU}} \mathcal{U} \mathcal{H}_u \mathcal{H}_d + h.c.$$  \hspace{1cm} (17)

Here, the $\mu$ term is generated when the unparticle develops the VEV,

$$\mu = k_{\mu} \frac{F_{dU}}{X_{dU}}.$$  \hspace{1cm} (18)

This provide a viable and interesting solution to the $\mu$ problem in the MSSM since the origin of the VEV of unparticle is related to the conformal symmetry breaking and the scale of it could be as much as higher than the weak scale in contrast to the NMSSM cases.

Interesting point in our scenario of unparticle physics as a source of $\mu$ term is that the overall scale of a supersymmetric mass parameter $\mu$ is determined by the scale of the conformal symmetry breaking and the dimension of the unparticle operator, $d_{dU}$. For instance, if we take $k_{\mu} \sim O(1)$ and $\Lambda = 10$ TeV in order for obtaining $\mu \sim 100$ GeV, the required value of the unparticle $F$-term is of order, $\sqrt{F_{dU}} \sim 1$ TeV for $d_{dU} = 1$ and $F_{dU}^{1/3} \sim 10$ TeV for $d_{dU} \sim 2$, which are relatively low compared to the case of usual gravity mediation since the cutoff scale in this case becomes Planck scale.

On the other hand, generating a sizable $B_{\mu}$ term is a on the same footing problem as the $\mu$ problem in the MSSM. In our scenario, in which the $\mu$ term is generated via the conformal symmetry breaking VEV of the unparticle, the $B_{\mu}$ term can be written in the following manner:

$$\mathcal{L} = \int d^4\theta \frac{1}{X_{dU}} \mathcal{U} \mathcal{H}_u \mathcal{H}_d + h.c.$$  \hspace{1cm} (19)

Natural electroweak symmetry breaking requires the scale of $B_{\mu}$ term of order, $B_{\mu} \sim \mu^2$. This is indeed true in our scenario of unparticle physics as a source of both $\mu$ and $B_{\mu}$ terms if $k_{\mu} \sim k_{B}$. It is always the case in minimal supergravity with additional singlet, which is used to generate $\mu$ term, as in the Giudice-Masiero mechanism.

**SUMMARY**

In conclusion, we have considered the unparticle physics focusing on the Higgs phenomenology. Considering the interactions between the unparticle and Higgs boson, unparticle can acquires the VEV, whose mass scale is determined as a consequence of the conformal symmetry breaking. The result shows that there is a possibility for the unparticle as a hidden sector in SUSY breaking sector, and can provide a solution to the $\mu$ problem in SUSY models.

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