Discovering Quantum Mechanics Once Again

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Abstract:
We expand on a recent development by Hardy, in which quantum mechanics is derived from classical probability theory supplemented by a single new axiom, Hardy’s Axiom 5. Our scenario involves a ‘pretend world’ with a ‘pretend’ Heisenberg who seeks to construct a dynamical theory of probabilities and is lead – seemingly inevitably – to the Principles of Quantum Mechanics.

1 Introduction

In two recent papers [1,2], Hardy shows how classical probability theory morphs directly into quantum theory, complete with full instructions for measurement and interpretation. Hardy’s demonstration employs five axioms. The first four establish the basic classical probability theory in the vocabulary of a generalized Stern-Gerlach apparatus, which is the paradigm of quantum mechanics. Hardy introduces systems of \( N \) states; each of dimension \( K_N \) (= \( N \) classically, \( N^2 \) quantum mechanically); with subsystems \( M \leq N \); and composite systems with \( N = N_A N_B \) and dimension \( K = K_A K_B \); and finally, Hardy introduces his crucial fifth axiom.

Hardy’s Axiom 5 requires that there exist a *continuous* reversible transformation between any two pure states. Hardy emphasizes the marvelous purity of his derivation encompassed in the fact that the key word *continuous* is the sole genetic marker responsible for the profound distinction between classical probability theory and quantum mechanics.

Our purpose here – following Hardy and essential earlier contributions by Caticha [3] – is also to start with classical probability theory but to ‘derive’ quantum mechanics in a less formal, austere and formidable way than
Hardy. We hope to ‘discover’ in a ‘pretend world’ a pathway equivalent to that of Hardy, which might appeal to the more heuristic and intuitive tastes of all those primarily interested in the physical as distinct from the more mathematical aspects of quantum mechanics.

Of course, quantum mechanics must survive all these shenanigans completely unscathed. What these efforts do hope to provide is not new physics, but perhaps a *raison d’être*, an ‘understanding’, of quantum mechanics – an ‘interpretation’ of quantum mechanics – in line with the overwhelming, but not yet and probably never unanimous, consensus expressed by Fuchs and Peres [4] and many others, and still passionately debated [5]: specifically, that quantum mechanics is a theory of information propagation. To say that quantum mechanics is ‘a’ theory is to seriously understate the case. Quantum mechanics will be seen to be ‘the’ fundamental theory of information propagation. The wave function is found to be the necessarily subjective encoding of information – and when information changes, the wave function must be changed accordingly. This interpretation puts an end at last to all quantum paradoxes. They are now seen to be the result of a too literal – even naive – faith in our introduction to the subject via wave mechanical extensions of the classical objective world view.

2 Quantum Mechanics from Information Theory

Here we present a ‘derivation’ of quantum mechanics starting from information theory. We find that it is surprisingly straightforward to reverse the roles of quantum mechanics and information. The logic sequence is no longer

a) the discovery of the Heisenberg matrix mechanics underlying classical mechanics; and then of

b) Schroedinger wave mechanics; followed by

c) the verification of quantum mechanics in many stationary state situations; but

d) still plagued by paradoxical time-dependent situations; necessitating

e) a host of ad hoc ‘interpretations’ of quantum mechanics to accommodate all paradoxes; and finally

f) resolution of all confusion by recognition of the role of the wave-function of quantum mechanics as the fundamental encoding and propagation of subjective information.
In fact the logic sequence can usefully be completely reversed. We start, of course, with the essential advantage of knowing the desired results, and of having the necessary language and mathematical techniques already familiar from almost a century of development of quantum mechanics. We are then able to ‘derive’ quantum mechanics from the requirement that the information entropy of an isolated system should remain unchanged as it evolves in time. We imagine a ‘pretend world’ in which we ‘discover’ quantum mechanics embedded in classical information probability.

We must be directly driven in this ‘pretend’ world by a fundamental positivist philosophy to invent whatever tools are necessary to keep moving and – hopefully – to keep making progress. These tools – which are dramatically suggested, even demanded, by the formalism – include unitary operators, hermitian generators, commutator brackets; all operating in a Hilbert space; then canonical commutation relations of $q$’s and their newly introduced $p$’s; then Heisenberg; then Schrödinger; then classical mechanics and eventually even the Lagrangian and the Principle of Least Action! Information theory explains the previously inexplicable: Why a Principle of Least Action?

The first step is to introduce the entropy of a macro-ensemble

$$S = -\text{trace } \{P \ln P\}$$

where $P$ is the probability of a micro-ensemble. None of these quantities will be fully specified but rather they will be explored. $S$ is chosen to have the appropriate limiting values: $S = 0$ if $P = 1$ for the simplest situation of a ‘pure state’. Otherwise $S \geq 0$, and $S = \ln N$ for a completely random mixture of $N$ such simplest situations each with probability $P = 1/N$.

The ‘trace’ is a dimensional reduction by summing over all internal coordinates labeling the micro-ensembles which contribute to the macro-ensemble. We are obviously cribbing the fundamental role of the density matrix and the implicit role of macro- and micro-ensembles.

The next step is the stuff of legend: We imagine a brilliant 24 year old on a solitary vacation on a rain-swept rocky shore, daydreaming about the dynamical problem of calculating the probabilities $P$ rather than just assigning them, as we progress from thermodynamics to the next more fundamental theoretical level of statistical mechanics and ultimately to a fully dynamical theory. Perhaps from analogy with Maxwell’s dynamical equations for field
amplitudes rather than for positive field energy densities, our young genius decides to:
a) factorize the probability $P$ into the suggestive form

$$P \rightarrow \Psi^2 = \Psi \times \Psi.$$  

This form has the virtue that $P \geq 0$ for arbitrary real $\Psi$. Factorization leads to dynamical quantities satisfying the anticipated requirements of variational calculus where the virtual variations in $\Psi - \delta \Psi$ – are unconstrained by the requirement that $P \geq 0$. Here we have to borrow heavily from classical mechanics and the requirements on classical dynamical variables that they be continuous and differentiable. This motivates us to search for an analytic dynamical theory for the probabilities $P$, but that is impossible. The normalization of the probabilities – $\text{tr}P = 1$ – is a holonomic constraint (i.e., an equality constraining the candidate generalized coordinates $P$) in Goldstein’s classification [6], and is manageable in the context of classical mechanics simply by eliminating one $P$. However, the positivity condition on the probabilities – $P \geq 0$ – as an inequality is a nonholonomic constraint which “there is no general way of attacking”.

A factorizable probability is the especially simple case of a pure state but it is an immensely instructive warm-up exercise. Proceeding,
b) we require $\Psi$ to be more than just $\sqrt{P}$. We require $\Psi$ to be analytic and differentiable in its variables, and therefore to have the possibility of sign-changes where $P = 0$.
c) The invariance of $P$ and $S$ under a simple sign-change of $\Psi$ is further extended by allowing $\Psi$ to be complex so $P \rightarrow \Psi \Psi^*$. Why? Why complex numbers? Our basic response is: Why not? Our only requirement was that $P \geq 0$, so to choose $\Psi$ real was to overconstrain it. With complex $\Psi$
d) the full invariance group of $P$ and $S$ is the group of all unitary transformations $U$ on $\Psi$, with $U^\dagger U = 1$

$$\Psi \rightarrow \Psi' = U\Psi,$$

and the merit of allowing $\Psi$ to be complex becomes evident.

Without complex $\Psi$, the invariance group of $P$ and $S$ is just the trivial change of sign $\Psi \rightarrow -\Psi$, and our search for a dynamical theory of $P$ comes to a grinding halt. With complex $\Psi$ we are on familiar ground. $\Psi$ is the
state function for the macro-ensemble of the density matrix, here reduced (trivialized) to the simplest pure-state micro-ensemble. We will relax these restrictions soon.

e) For the system to evolve smoothly in time without change of entropy of information, requires the time evolution of the state function $\Psi$ to be a unitary transformation

$$|\Psi(t = 0) > \Rightarrow |\Psi(t) > = U(t)|\Psi(0) > \equiv e^{-iHt}|\Psi(0) >$$

(going over to the familiar Dirac notation). The invariance of the entropy is made clear in this way for our simplest system of a single micro-ensemble, and is maintained for a mixture of noninteracting micro-ensembles. Caticha [see 3] enforces this requirement by requiring the Hilbert space as the “uniquely natural” choice. Caticha formalizes this development, but we proceed with our ‘pretend’ discovery scenario.

Here we have introduced the hermitian generator of infinitesimal time translations $H = H^\dagger$. This is by definition the Hamiltonian, but we have not imposed anything except hermiticity in the new information based dynamics. The familiar Hamiltonian dynamics will be a product of the information theory. We have also kept to the simplest possible case by taking $H$ to be time independent.

Next,

f) the Heisenberg equations of motion follow immediately from

$$< x(t) > = \text{tr}\{x(0)P(t)\}$$

$$= \text{tr}\{x(0)U|\Psi(0)><\Psi(0)|U^\dagger\}$$

$$= \text{tr}\{U^\dagger x(0)U|\Psi(0)><\Psi(0)|\},$$

so

$$x(t) = U^\dagger x(0)U$$

and

$$\dot{x}(t) = iU^\dagger[H, x(0)]_{CB}U$$

$$= i[H, x(t)]_{CB}.$$

This is the familiar Heisenberg matrix equation of motion involving the commutator bracket of $x(t)$ with the generator of time translations $H$. 
Proceeding \textit{ab initio} from a Principle of Stationary Information Entropy seems logically possible, but we forgo the exercise by identifying $H$ with a non-trivial Hamiltonian and introducing for each generalized coordinate $x$ the canonically conjugate operator $p$ which diagonalizes the commutation relations; then we get the structure of Heisenberg matrix mechanics; and from that the Schroedinger derivative representation of the commutation relations, and wave mechanics; and finally the classical limit of commutator brackets as Poisson brackets. So we could also even ‘discover’ classical mechanics in this inverted logic.

The Hamiltonian is paramount in this formulation, as it is in ordinary non-relativistic quantum mechanics. It remains to find the Lagrangian and the Action Principle, both of which seem somewhat contrived and \textit{ad hoc} in the usual derivation of non-relativistic quantum mechanics. Helmholtz and Gibbs \cite{7} have showed that the stationary minimum of the free energy $A=E-TS+PV$ \cite{8} fulfills the role of Lagrangian in reversible chemical reactions. Extending the invariance group of the entropy to the Lorentz group will be useful also.

What can be said when the probability $P$ is not simply factorizable? The density matrix \cite{9} must then be written as a sum over micro-ensembles $\Psi_j$ each weighted with its own positive probability $P_j$. (Keep in mind that the $P_j$ are now real numbers satisfying $P_j \geq 0$ and $\sum_j P_j = 1$.)

$$\rho(t = 0) = \sum_j \{|\Psi_j > P_j < \Psi_j|\}$$

in the usual notation. All the above equations are \textit{required} to survive as

\begin{align*}
<x(t)> & \equiv \text{tr} \{U^\dagger x(0)U\rho(0)\} \\
<\dot{x}(t)> & = i \text{tr} \{U^\dagger [H, x(0)]_{CB}U\rho(0)\} \\
& = i \text{tr} \{[H, x(t)]_{CB}\rho(0)\}.
\end{align*}

The conditions for this survival are profound: we require a Hilbert space of the $\Psi$’s and the corresponding operator or matrix representation of the $U$’s. We introduced the notation somewhat gratuitously in the above ‘pure’-case, but now the necessity of the full Hilbert space formalism is clear. If we were to make any progress at all in our ‘pretend world’ in the non-trivial
‘mixed’-case we would have had to invent Hilbert space and operator $U$’s at this point.

What is a Hilbert space? It is the complex vector space of the eigenvectors $|\Psi_j\rangle$ of the hermitian operator $H$, so

$$H|\Psi_j\rangle = E_j|\Psi_j\rangle,$$

which have the quadratic norm

$$<\Psi_j|\Psi_k> = \delta(j, k)$$

and so are orthogonal. They are complete

$$\sum_j |\Psi_j\rangle <\Psi_j| = 1,$$

where 1 must be ‘interpreted’. We are confronted by the seemingly absurd, but in fact profound, Clintonesque question: What is 1?

Two simplest examples are useful to keep in mind:
a) The simplest is the two dimensional space of a spin-$\frac{1}{2}$ particle with

$$S_x = \frac{\sigma_x}{2}, \text{ etc. for } y, z;$$

$$H = \Delta S_z$$

with two eigenstates

$$|\Psi_1\rangle \equiv \alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\Psi_2\rangle \equiv \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$  

Orthonormalization and completeness are apparent. Complex representations could equally well have been chosen, e.g., the eigenstates of $\sigma_x$ or $\sigma_y$ instead of those for $\sigma_z$. The complex vector space in this simplest case is spanned by two 2-dimensional orthonormal and complete basis-vectors, such as $\alpha$ and $\beta$.
b) A less trivial example is the free particle in three unconstrained dimensions. The Hamiltonian is

$$H = \frac{p^2}{2m}$$
and we choose simultaneous energy and momentum eigenstates

\[ |\Psi_\vec{p}(\vec{x})> = e^{i\vec{p}\cdot\vec{x}}. \]

These have continuum orthonormalization

\[ \sum_{\vec{x}} <\Psi_{\vec{p}}(\vec{x})|\Psi_{\vec{k}}(\vec{x})> = \int d^3x e^{i(\vec{k}-\vec{p})\cdot\vec{x}} \]

\[ = (2\pi)^3 \delta^3(\vec{k} - \vec{p}), \]

and the very similar completeness relation

\[ \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}(\vec{x}-\vec{y})} = \delta^3(\vec{x} - \vec{y}). \]

In this case, the complex vector space is spanned by a mind-boggling array of basis-vectors: there is a triple-infinity of orthonormal and complete basis-vectors labeled by \( \vec{p} \), each with a triple-infinity of components labeled by \( \vec{x} \) (or vice versa).

We conclude that a Principle of Stationary Information Entropy, coupled with some inspired (but \textit{a posteriori} inevitable) requirements during the analysis, can completely reverse the logical structure of quantum mechanics. With this approach, there is no ‘interpretation’ of quantum mechanics; nor need there be any hesitation or delay in this ‘pretend world’ of making all the usual applications. A Hilbert space is required. Heisenberg matrix-commutator mechanics is required. We can imagine that the concept of generalized coordinates and their conjugate momenta would naturally occur, even without classical mechanics, as an optimal minimum set of non-commuting variables defined to diagonalize the commutation relations in a standard way. Schroedinger’s differential representation would be next and with it, all of wave mechanics and its intuitive (but occasionally misleading) guidance. And – as now – we can imagine a compelling route even to classical physics, but with a deeper understanding of the Principle of Least Action.

3 Adding Structure

Now we have to ask: What is \( \Psi_j \)? and what is \( \sum_j \)? In fact, what is \( j \)?
Our answers to these questions must satisfy the requirement that “Ψ is the complex resolution of unity” so we have completeness

\[ 1 = \sum_j |\Psi_j><\Psi_j|, \]

and orthogonality

\[ <\Psi_j|\Psi_k> = \delta(j, k). \]

The density matrix is constructed as

\[ \rho = \sum_j |\Psi_j > P_j <\Psi_j| \]

with \( P_j \geq 0 \) and \( \sum_j P_j = 1 \). Our target structure will be ordinary quantum mechanics which will make its appearance in the old-fashioned but explicit and intuitive Fock-representation.

The simplest case has dimension \( D = 2 \) and \( \Psi_j \)'s which are two states specified by the elementary Pauli spinors. It is a simple but instructive example and proves to be a faithful guide to any level of complexity. We have the standard representation

\[ |\Psi_1 > \equiv \alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\Psi_2 > \equiv \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \]

The two dimensional (real-)resolution of unity is

\[ 1 = |\Psi_1><\Psi_1| + |\Psi_2><\Psi_2| = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \]

and the density matrix is

\[ \rho = |\Psi_1 > P_1 <\Psi_1| + |\Psi_2 > P_2 <\Psi_2| = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}. \]

The unitary time evolution of these states which leaves \( \rho \) unchanged is generated by the no-interaction Hamiltonian

\[ H_0 = \frac{\Delta}{2} \sigma_z = \frac{\Delta}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]
An interaction between the two states would involve $\sigma_x$ and/or $\sigma_y$ and would induce continuous transformations between the two states, which were chosen to diagonalize the initial density matrix. Hardy [1,2] identifies the existence of these continuous transformations between basis states as the essential distinction between classical probability theory and quantum mechanics. It is seen to arise here from the extension of the ‘resolutions of unity’ – suggested by the requirement that the sought-for conjectured classical dynamical variables ($\sim \sqrt{P}$) satisfy only holonomic constraints – to the ‘complex resolutions of unity’. These must be included for the sake of logical completeness.

We can extend this simplest example in a multitude of directions.

a) The first extension is almost trivial: generalize the 2-D $SU(2)$ example above to other similar groups including the familiar $O(3)$ and $SU(3)$.

b) A second extension is to the interesting dynamical problems which arise when the density matrix of two (or more) a priori independent systems is defined in the direct product space of the two systems as

$$\rho(1,2) = \rho(1) \otimes \rho(2).$$

This requires basis representations

$$\Psi(1,2) = \Psi(1) \otimes \Psi(2)$$

and the Hamiltonian

$$H(1,2) = H_0(1) \bigoplus H_0(2) \bigoplus H_{int}(1,2)$$

which dynamically couples the two systems. Such dual – but isolated – systems still fall short of a model for the full measurement process [10]. We should not be surprised or disappointed. Standard quantum mechanics is our limited goal.

A third extension required for continuous variables

c) like $\vec{x}$ and $\vec{p}$ is somewhat different but is directly suggested by the $SU(2)$ example: it involves judicious replacements of sums by integrals and Kronecker $\delta_K(j,j')$ by Dirac $\delta_D(p-p')$. For example, the ‘complex resolution of unity’ for a probability distribution defined in momentum space now becomes

$$1 \Rightarrow (2\pi)^3 \delta^3(p-k) = \int d^3x e^{i(p-k)\cdot \vec{x}}$$
and

\[ \Psi_j(p) \Rightarrow e^{ip \cdot \vec{x}}. \]

One might well ask: Where did the plane wave solutions come from in this ‘pretend world’? Of course, we have the answer from quantum mechanics as we have been taught it; but how would we arrive at this result starting with a stationary probability, factored into ‘complex resolutions of unity’?

We must start with a dimensionality chosen for the problem at hand. This is where physics, judgement, and discovery enters. The next step is to require the time evolution operator to be unitary, and the generator to be a hermitian Hamiltonian operator whose commutator with \( x \) is the velocity \( v \). We can follow two paths here: The easiest one is to take classical mechanics as already known, and simply use the Hamiltonian \( H = \frac{p^2}{2m} \). Then the commutation relation requires \( p = \frac{\hbar \nabla}{i} \), and the choice of plane waves as ‘resolvent’ functions is dictated by the resulting Hilbert space. Again there are judgements to be made, and we choose a rectangular basis which diagonalizes the linear momenta. This choice replaces all operators by their eigenvalues.

A second path suggests itself, as mentioned above, of deducing classical mechanics \textit{ab initio} from the factorization requirement, and diagonalizing the commutation relations. In this way, we are lead to ‘discover’ the generalized momentum \( p_x \) conjugate to the generalized coordinate \( x \). We are limited in this way to a unit metric in generalizing sums to integrals, but we do get a toehold and could subsequently change to a basis of angular momentum eigenstates for example.

Traditional quantum calculations are quite conveniently made in this density function representation. We give a brief heuristic sketch of transitions between basis states induced by a non-diagonal interaction Hamiltonian \( H' \). The initial density matrix evolves in time to

\begin{align*}
\rho(0) & \equiv |\Psi_j><\Psi_j| \\
\rightarrow \rho(t) & = e^{-iH't}\rho(0)e^{+iH't} \\
\rightarrow \sum_m |\Psi_m > P_m(t) <\Psi_m|.
\end{align*}
The probability $P_k(t)$ at time $t$ of a state $k \neq j$ is

$$P_k(t) = \text{tr}|\Psi_k><\Psi_k|\rho(t)$$

$$= |<\Psi_k|e^{-iH't}|\Psi_j>|^2$$

$$\simeq t^2 |<\Psi_k|H'|\Psi_j>|^2,$$

proportional in first order to the absolute square of the matrix element of the perturbing interaction Hamiltonian. The factor $t^2$ requires some care in application to realistic energy conserving transitions but we won’t pursue that. We should perhaps better recover the Schroedinger equation directly to evaluate

$$e^{-iH't}|\Psi_j>$$

in terms of the energy eigenstates of the full Hamiltonian. In either case, we return directly to the usual quantum results.

4 Concluding Remarks

Let us summarize and reiterate what has been done. We start with the information entropy of a macro-ensemble

$$S = -\sum_j \{P_j \ln P_j\} \geq 0,$$

where $j$ enumerates the micro-ensembles occurring in the macro-ensemble with probability $P_j$. $P_j$ must satisfy the obvious restrictions

$$0 \leq P_j \leq 1 \text{ and } \sum_j P_j = 1.$$

We then embark on a ‘pretend’-journey of ‘discovery’ resulting in quantum mechanics.

Our goal is to find a dynamical theory governing the system. We rule out the probabilities $P_j$ themselves as candidate fundamental variables of such a dynamical theory. The reason is familiar from classical mechanics: the inequality $P_j \geq 0$ is a non-holonomic constraint and “there is no general way” of attacking such problems. This leads us to factorize the probabilities ultimately to

$$P_j \rightarrow \Psi_j^2 \text{ and further to } \rightarrow \Psi_j\Psi_j^*.$$
a) Why factorization? To satisfy the non-holonomic constraint identically.
b) Why complex factors? This generalization is obviously permissible and thus a logical necessity.
c) What factors? The answer to this question builds upon experience with the simplest system imaginable: the 2-dimensional complex space of spin-$\frac{1}{2}$. Finally we can conclude that the $\Psi_j$'s are a complete orthonormal set of complex basis vectors in the Hilbert space generated by the Hamiltonian describing the dynamics of the system under consideration.
d) What dynamics? To preserve the entropy of a macro-ensemble, we require the time dependence of the $\Psi_j$ to be a unitary transformation generated by the hermitian time evolution operator (i.e., the Hamiltonian) of the system. The actual choice of the Hamiltonian is not made by the quantum theory per se but becomes an act of creative judgement subject only to the achievement of interesting results.
e) Elementary considerations lead directly to the Heisenberg equations of motion involving the commutator with the Hamiltonian.
f) The $\Psi_j(t)$ satisfy the Schroedinger equation and in the usual way constitute the required Hilbert space vectors.

We could continue in this way, or we could return to ordinary quantum mechanics with the – perhaps not so new – understanding of the wave functions as 'complex resolutions of unity' – i.e., projection operators – onto each particular micro-ensemble in the macro-ensemble under consideration.

What has been gained is not a new quantum mechanics, but a reason for the existence of the old one. The existence of quantum mechanics is necessary in order for there to be a fundamental dynamical theory governing the elementary probabilities in the Shannon Information Entropy. In addition, this derivation justifies the subjective interpretation of the wave function as the encoding of information. Even further, such fundamental entities as the Heisenberg matrix equations of motion appear naturally, suggesting the fundamental commutation relations diagonalized by the introduction of the momentum canonically conjugate to each independent coordinate; and even the derivation of classical mechanics from this quantum mechanics, and an alternative to the Principle of Least Action; all from a Principle of Stationary Entropy.
As advertized, our development is intended to be a ‘pretend’ voyage of discovery, not a mathematical derivation of quantum mechanics. Caticha [3] points out the many logical shortcomings in our too-facile assumptions, some of which we describe below. First, however, let us point out that the possibilities employed in our scenario do exist as a path through the maze. Our specific choice of complex numbers, Hilbert spaces, and unitary transformations turns out to be sufficient and self-consistent but not proven to be necessary. Caticha [3] points out the possibility of Clifford algebras [11] of real vectors inter alia. These do have a possible presence in extensions of quantum mechanics to include Weyl spinors and Grassmann variables, but the onus so far has been on the conformability of these structures with the pre-existing structure of quantum mechanics.

Our philosophy is a pragmatic positivist one. In this view, every exception is to be viewed not as a barrier to progress, but as an opportunity. It is an appeal to a Correspondence Principle. At the same time, we have to acknowledge the possibility that some arcana really are really mysterious.

5 Acknowledgements

I am deeply indebted to Lucien Hardy’s monumental tome [1] which I now feel more fully able to appreciate. The earlier fundamental work by Ariel Caticha [3] was particularly valuable, as was the popular account in PHYSICS TODAY by Chris Fuchs and Asher Peres [4]. In addition, Asher Peres has kindly brought to my attention his similar scenario [12] in which probabilities $P_{mn}$ are shown to be absolute squares of unitary matrices $U_{mn}$, which are defined by generalized Stern-Gerlach devices.

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The first four axioms, in brief: (1) define probability as the relative frequency of occurrence in an ensemble; (2) require simplicity in terms of minimum degrees of freedom $K$ for given dimensionality $N$; (3) have subsystems obeying the same rules; so (4) composite systems can be directly composed of subsystems.
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