Cache-Aided Interference Management in Partially Connected Wireless Networks

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Abstract—Cache-aided communication is emerging as a new topic in wireless networks. Previous works have shown that caching in interference networks can change the interference topology by changing the information flow and hence facilitate advanced interference management. This paper studies the gain of caching in partially connected interference networks where each receiver can only communicate with a subset of transmitters. The performance is characterized by an information-theoretic metric, normalized delivery time (NDT). We obtain an order-optimal NDT for the \((K + L - 1) \times K\) partially connected linear interference network with any number of receivers \(K\), any receiver connectivity \(L \leq K\), and with caches equipped at all transmitters and receivers. The cache placement phase adopts a file splitting strategy tailor-made for the partial receiver connectivity. Via the aid of virtual receivers, the proposed delivery strategy exploits coded multicasting gain by XOR combining and transmitter coordination gain by interference alignment. In the special case when \(L\) is a divisor of \(K\), our NDT results are directly applicable to \(K \times K\) partially connected circular interference networks.

I. INTRODUCTION

Caching is a novel solution to enhance the communication efficiency by exploiting the increasingly rich storage resource in wireless networks. It can significantly alleviate network congestion and reduce user access latency by pre-fetching popular video contents at femto base stations or pushing directly to mobile devices during off-peak times [1], [2]. The promise of wireless caching mainly owes to the invention of coded caching in an information-theoretic framework [3]. The notion of coded caching is to cache non-identical subfiles among different receivers so as to provide coded multicasting opportunities thanks to the broadcast nature of wireless media.

Recently, the gain of caching is studied in wireless interference networks (IN) where caches are equipped at transmitters [4]–[6]. The authors in [4] showed that the original interference channel can be turned into more favorable channels including X channel and broadcast channel by proper file splitting and placement. The total degrees of freedom (DoF) of the channel can thus be improved compared with naive caching. The authors in [5] studied the cloud and cache aided wireless network, and characterized the tradeoff between cache storage size and content delivery time. The authors in [6] characterized a similar storage-latency tradeoff in a partially connected IN where the base stations have no access to channel state information beyond the network connectivity. These works [4]–[6] show that caching at the transmitter side in IN can induce transmitter coordination or cooperation for interference management.

More recently, the gain of caching in wireless IN with caches equipped at all transmitters and receivers is investigated [7]–[11]. Our previous works [8] characterized the storage-latency tradeoff in a general IN with any number of transmitters, any number of receivers, and at any feasible cache size region. It reveals that, with the cooperative Tx/Rx coded caching strategy in [7], the network topology can be turned into a new class of channels, namely, cooperative X-multicast channels. The network is thus able to leverage opportunistically transmitter cooperation gain (via interference management), coded multicasting gain (via XOR combining) and receiver local caching gain. The work [11] extends the study in [8] to a MIMO network where each node is equipped with multiple antennas. In [9], by separating the physical layer and the network layer, the authors obtained an order-optimal approximation of system DoF through interference alignment for arbitrary number of transmitters and receivers but with restricted cache size region. The authors in [10], on the other hand, analyzed the standard sum DoF by one-shot linear interference neutralization schemes.

Note that all these studies in [7]–[11] assumed a fully connected IN, where all the transmitters can communicate with all the receivers with independent and identically distributed (i.i.d) fading channels. In practical wireless networks, considering the natural path loss due to radio propagation and signal attenuation due to blocking objects, some links are inevitably weaker than others. This scenario is typically approximated as partially connected IN in the literature [6], [12], [13], where each receiver can only communicate with a subset of transmitters. The existing works [6], [12] only studied caching at either transmitter side or receiver side. It is thus of both theoretical and practical importance to investigate caching at both sides in partially connected INs.

In this paper, we consider a partially connected linear IN with caches equipped at all transmitters and receivers. The consider linear network topology can well model many

This work is supported by the National Natural Science Foundation of China under grants 61571299, 61329101, and 61521062.
practical communication systems such as highway roadside communications and railway wayside communications. We aim to characterize the storage-latency tradeoff using the normalized delivery time (NDT) as adopted in [5], [8], [11]. We first present a new file splitting and caching strategy for the cache placement phase, which specifically takes the partial receiver connectivity into account. In the delivery phase, to make the design and analysis more tractable, we introduce virtual receivers which have caches and partial connectivity but do not send any content request. We then exploit both coded multicasting gain through XOR combining and transmitter coordination gain through interference alignment in the considered partially connected network. It is shown that the achievable NDT depends on the receiver connectivity, but not the total number of receivers. It is within a multiplicative gap of 2 to a cut-set like theoretical lower bound of NDT. In special cases, the NDT results obtained for the partially connected linear IN can be directly applied to partially connected circular IN as well as fully connected IN.

Notations: \([K]\) denotes the set \(\{1, 2, \ldots, K\}\), and \([K]^+\) denotes the set \(\{0, 1, 2, \ldots, K\}\). \(\mathcal{C} \sim \mathcal{N}(0, 1)\) denotes the Gaussian distribution with zero mean and unit variance.

II. SYSTEM MODEL

A. Partially Connected Linear Interference Model

We consider a \((K + L - 1) \times K\) partially connected linear IN, where there are \(K + L - 1\) transmitters, indexed by \(\{0, 1, \ldots, K + L - 2\}\), \(K\) receivers, indexed by \(\{0, 1, \ldots, K - 1\}\), and each receiver \(i\) is connected to \(L\) consecutive transmitters \(\{i, i+1, \ldots, i+L-1\}\), with \(L \leq K\). \(L\) is referred to as receiver connectivity. Fig. 1 shows an example with \(K = 4\) and \(L = 3\). Let \(\mathcal{T}_i \triangleq \{i, i+1, \ldots, i+L-1\}\), for \(i \in [K-1]^+\), denote the set of transmitters connected to receiver \(i\). Let \(\mathcal{R}_j \triangleq \{j, j-1, \ldots, j-L+1\} \cap [K-1]^+\), for \(j \in [K+L-2]^+\), denote the set of receivers connected to transmitter \(j\). Each node is equipped with a cache memory of finite size, and has single antenna. The communication at each time slot \(t\) over this network is modeled by

\[
Y_i(t) = \sum_{j \in \mathcal{T}_i} h_{ij}(t)X_j(t) + Z_i(t), i \in \{0, 1, \ldots, K - 1\},
\]

where \(Y_i(t) \in \mathbb{C}\) denotes the received signal at receiver \(i\), \(X_j(t) \in \mathbb{C}\) denotes the transmitted signal at transmitter \(j\), \(h_{ij}(t) \in \mathbb{C}\) denotes the channel coefficient from transmitter \(j\) to receiver \(i\) which is time-variant and i.i.d distributed from some continuous distribution, and \(Z_i(t)\) denotes the noise at receiver \(i\) distributed as \(\mathcal{C} \sim \mathcal{N}(0, 1)\).

Note that this network model is an extension of the \(K \times K\) linear IN [13], where we have \(L - 1\) more transmitters \(\{K, K+1, \ldots, K+L-2\}\), to maintain the constant connectivity of \(L\) for all receivers. Two constant constraints are maintained in the linear network topology. A constant receiver connectivity and the linear network topology are crucial in this work to make the cache-aided interference management tractable.

This network model is also highly correlated to the \(K \times K\) partially connected circular IN where each receiver \(i\) is connected to \(L\) circulants transmitters denoted as \(\mathcal{T}_i^c \triangleq \{i, i+1, \ldots, i+L-1\} \mod K\). In specific, if we merge the transmitters \(j\) and \(K+j\) for each \(j\) with \(0 \leq j \leq L-2\), in the linear network, then the merged node \((j, K+j)\) can be equivalent to the transmitter \(j\) in the circular network. This is because the connected receiver set of transmitter \(j\) in the circular network, denoted as \(\mathcal{R}_j^c \triangleq \{j-L+1, j-L+2, \ldots, j\} \mod K\), is the union of the connected receiver sets of transmitters \(j\) and \(K+j\) in the linear network, i.e., \(\mathcal{R}_j^c = \mathcal{R}_j \cup \mathcal{R}_{K+j}\). The original channel coefficients \(h_{ij}(t) (i \in \mathcal{R}_j)\) and \(h_{i,K+j}(t) (i \in \mathcal{R}_{K+j})\) can thus be viewed as the channel coefficients \(h_{ij}(t) (i \in \mathcal{R}_j^c)\) in the circular network. In the special case when \(L = K\), the partially connected circular IN reduces to the fully connected IN as in [8], [11]. We shall apply our analysis on caching for the partially connected linear network to both partially connected circular network and fully connected network in Section VII.

B. Cache Model

Consider a database consisting of \(N\) files \((N \geq K)\), denoted by \(\{W_0, W_1, \ldots, W_{N-1}\}\), each of size \(F\) bits. Each transmitter and receiver can cache at most \(M_T F\) bits and \(M_R F\) bits, respectively, where \(M_T, M_R \leq N\). The normalized cache sizes at each transmitter and receiver are defined, respectively, as

\[
\mu_T \triangleq \frac{M_T}{N}, \quad \mu_R \triangleq \frac{M_R}{N}.
\]

We focus on the cache size region \(\mu_T \geq \frac{1}{K} \geq \mu_R \geq 0\). This means that the accumulated cache memories at any \(L\) transmitters are large enough to collectively store the entire database.

The cache-aided system operates in two phases, cache placement and content delivery. During the cache placement phase, each transmitter \(j\) designs a caching function \(\phi_j\), mapping the \(N\) files in the database to its local cached content \(U_j \triangleq \phi_j(W_0, W_1, \ldots, W_{N-1})\). Each receiver \(i\) also designs a caching function \(\psi_i\), mapping the \(N\) files to its local cached content \(V_i \triangleq \psi_i(W_0, W_1, \ldots, W_{N-1})\). The caching functions \(\{\phi_j, \psi_i\}\) allow for arbitrary coding among files.

Fig. 1: 6 \times 4 partially connected linear IN with receiver connectivity \(L = 3\).
In the delivery phase, each receiver \( i \) requests a file \( W_d \). We denote \( \mathbf{d} \triangleq (d_i)_{i=0}^{K-1} \in [N-1]^{+K} \) as the demand vector. Each transmitter \( j \) has an encoding function \( \Lambda_j \) to map itscached content \( U_j \), receiver demand \( \mathbf{d} \), and network-wide channel realization \( \mathbf{H} = [h_{ij}(t)_{i\in[K-1]^+,j\in[R],t\in[T]}] \) to the signal \((X_j(t))_{t=1}^{T} \triangleq \Lambda_j(U_j, \mathbf{d}, \mathbf{H})\), where \( T \) is the block length of the code. Note that \( T \) may depend on the receiver demand \( \mathbf{d} \) and channel realization \( \mathbf{H} \). Each codeword \( (X_j(t))_{t=1}^{T} \) has an average transmit power constraint \( P \). Each receiver \( i \) has a decoding function \( \Gamma_i \) to decode \( W_{d_i} \triangleq \Gamma_i(V_i, (Y_i(t))_{t=1}^{T}, \mathbf{H}, \mathbf{d}) \) of its desired file \( W_d \) using its cached content \( V_i \), received signal \( (Y_i(t))_{t=1}^{T} \), channel realization \( \mathbf{H} \), and demand \( \mathbf{d} \). The worst-case error probability is

\[
P_e = \max_{\mathbf{d} \in [N-1]^{+K}} \max_{i \in \{0,1,\ldots,K-1\}} P(\hat{W}_d \neq W_d).
\]

A given caching and coding scheme \( \{\phi_j, \psi_i, \Lambda_j, \Gamma_i\} \) is said to be feasible if \( P_e \to 0 \) for almost all channel realizations when \( F \to \infty \).

### C. Performance Metric

**Definition 1** (5). The normalized delivery time (NDT) for a given feasible caching and coding scheme is defined as

\[
\tau(\mu_R, \mu_T) \triangleq \lim_{P \to \infty} \lim_{T \to \infty} \sup_{\mathbf{d} \in [N-1]^{+K}} \frac{\max T}{F / \log P}.
\]

Moreover, the minimum NDT at given normalized cache sizes \( \mu_T \) and \( \mu_R \) is defined as

\[
\tau^*(\mu_R, \mu_T) = \inf\{\tau(\mu_R, \mu_T) : (\mu_R, \mu_T) \text{ is achievable}\}.
\]

**Remark 1** (8). Let \( R \) denote the worst-case traffic load per user with respect to the file size \( F \). The per-user capacity of the network in the high SNR region is approximately given by \((d \cdot \log P + o(\log P))\), where \( d \) is the per-user DoF. Then, by Definition 1, NDT can be expressed more conveniently as

\[
\tau = R / d,
\]

which suggests that NDT characterizes the delivery time of the actual traffic load \( R \) at a transmission rate specified by DoF \( d \).

### III. File Splitting and Cache Placement

In this section, we present a new file splitting and cache placement strategy, which is modified from the novel scheme in (8) by taking the partial network connectivity into account.

Given that \( \mu_T \geq 1 / L \), we first split each file \( W_n \), for \( n \in [N-1]^+ \), into \( L \) equal-sized subfiles, denoted by \( \{W_{n,p}\}_{p=0}^{L-1} \). Each transmitter \( j \), for \( j \in [K + L - 2]^+ \), caches \( \{W_{n,p}\}_{n=0}^{N-1} \) with \( p = j \text{ mod } L \). Such transmitter cache placement ensures that each receiver can access the whole database through its connected \( L \) consecutive transmitters. Then, we consider the cache placement at the receiver side. Each subfile \( W_{n,p} \) is further split into \( 2^p \) subfiles with possibly different size, denoted by \( \{W_{n,p,q}\} \), where \( Q \subseteq [L-1]^+ \). Each subfile \( W_{n,p,q} \) is then cached at receiver set \( \mathcal{R}_q \triangleq \{i : (i \text{ mod } L) \in Q\} \). By this cache placement strategy, the cached contents at the transmitters indexed by \( \{j, j + L, j + 2L, \ldots\} \) are the same, and the cached contents at the receivers indexed by \( \{i, i + L, i + 2L, \ldots\} \) are also the same. For example, we have \( U_0 = U_3, U_1 = U_4, U_2 = U_5 \), and \( V_0 = V_3 \) in the \( 6 \times 4 \) network shown in Fig. 1. We further assume that the sizes of the subfiles \( \{W_{n,p,q}\} \) with the same cardinality of \( Q \) are equal. Denote the size of \( W_{n,p,q} \) as \( a_r \) bits, where \( r = |Q| \) and \( a_r \) is the file splitting ratio. For example, the subfile \( W_{n,1,0,1} \) is cached in transmitters \{1, 4\} and receivers \{0, 1, 3\} in the \( 6 \times 4 \) network in Fig. 1 has a size of \( a_2F \) bits. By this caching approach, the splitting ratios \( \{a_r\} \) must satisfy the following constraints:

\[
\begin{aligned}
L \sum_{r=0}^{L-1} \left(\begin{array}{c}L \\ r\end{array}\right) a_r &= 1, \\
L \sum_{r=1}^{L-1} \left(\begin{array}{c}L-1 \\ r-1\end{array}\right) a_r &\leq \mu_R.
\end{aligned}
\]

Constraint (2) comes from the constraint of file size, and constraint (3) comes from the receiver cache size limit.

### IV. Content Delivery

In the delivery phase, without loss of generality, we assume that receiver \( i \) desires file \( W_i \), for \( 0 \leq i \leq K - 1 \). Due to its local cache, receiver \( i \) only needs subfiles:

\[
W_i^{\text{need}} \triangleq \{W_{i,p,q} : p \in [L-1]^+, Q \subseteq [L-1]^+ \setminus \{i \text{ mod } L\}\}
\]

We first present our delivery scheme through an example, and then proceed to the general algorithm.

**A. Example 1 (6 × 4 Network)**

Consider the \( 6 \times 4 \) network with receiver connectivity \( L = 3 \) shown in Fig. 1. We divide the total subfiles to be delivered \( \{W_{n,p,q}\}_{n=0}^{6} \) into 3 groups according to the size of \( Q \), with each group having the same \( |Q| \). Each group of subfiles is delivered individually in the time division manner. Here, we take the group of subfiles with \( r = |Q| = 1 \) as an example to illustrate the delivery scheme. These subfiles are shown in Table 1 where \( A_{p,q} \triangleq W_{0,p,q}, B_{p,q} \triangleq W_{1,p,q}, C_{p,q} \triangleq W_{2,p,q}, D_{p,q} \triangleq W_{3,p,q} \), for notation simplicity.

To make the problem more tractable, we transform the channel into an expanded partially connected linear IN by introducing virtual receivers. Generate virtual receivers indexed by \( \{-2, -1, 4, 5\} \) as shown in Fig. 2 where receivers \( \{-2, 4\} \) cache the same subfiles as in receiver 1, receivers \( \{-1, 5\} \) cache the same subfiles as in receiver 2. Note that each virtual receiver does not send any content request. In such expanded network, each subfile desired by one actual receiver is always cached in one (virtual) receiver that connects to the same transmitter as this actual receiver. Coded multicasting gain via XOR combining can thus be exploited among these two receivers. In specific, according to Table 1 each transmitter can generate coded messages as follows:

\[
\text{Tx 0} : \begin{cases}
W_{t,[1]}^{[-2,0]} &\triangleq A_{0,[1]} \text{ needed by Rx 0 (and -2)}, \\
W_{t,-1}^{[-1,0]} &\triangleq A_{0,[2]} \text{ needed by Rx 0 (and -1)},
\end{cases}
\]
TABLE I: Subfiles to be delivered with $r = |Q| = 1$

| Desired by Rx 0 | Coded at Tx 0 | Desired by Rx 1 | Coded at Tx 1 | Desired by Rx 2 | Coded at Tx 2 | Desired by Rx 3 | Coded at Tx 3 | Desired by Rx 4 | Coded at Tx 4 | Desired by Rx 5 | Coded at Tx 5 |
|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|
| $A_{0},(1)$   | $A_{1},(1)$  | $B_{1},(2)$    | $B_{2},(0)$  | $C_{2},(0)$    | $D_{2},(1)$  | $W_{1}^{a_{1},a_{2}}$ | $W_{2}^{a_{1},a_{2}}$ | $W_{3}^{a_{1},a_{2}}$ | $W_{4}^{a_{1},a_{2}}$ | $W_{5}^{a_{1},a_{2}}$ | $W_{6}^{a_{1},a_{2}}$ |

Fig. 2: Coded message flow of the subfile delivery with $r = 1$ in the $6 \times 4$ network, where circles denote the multicast group of the message with the same color.

Fig. 3: Signal space for messages with $r = 1$ in the $6 \times 4$ network.

undesired messages. We need to design beamforming vectors to align the undesired messages in a same subspace at each receiver. Denote $V_{x}^{(t_{1},t_{2})}$ as the transmit beamforming vector of message $W_{t_{j}}^{(t_{1},t_{2})}$ and $H_{ij}$ as the channel realization between transmitter $j$ and receiver $i$. The interference alignment conditions on each actual receiver $r_{0}$, $r_{1}$, $r_{2}$, and $r_{3}$ are as follows:

$\text{span}(H_{01}v_{1}^{-1,1}) = \text{span}(H_{02}v_{2}^{1,2})$, $\text{span}(H_{11}v_{1}^{-1,0}) = \text{span}(H_{12}v_{2}^{0,2}) = \text{span}(H_{13}v_{3}^{2,3})$, $\text{span}(H_{22}v_{2}^{0,1}) = \text{span}(H_{23}v_{3}^{1,3}) = \text{span}(H_{24}v_{4}^{3,4})$, $\text{span}(H_{33}v_{3}^{1,2}) = \text{span}(H_{34}v_{4}^{2,4})$.

Using asymptotic interference alignment, the beamforming vectors can be designed such that each receiver can decode its six desired messages, each taking up one dimension, while its undesired messages are aligned together at another dimension. The signal space is sketched in Fig. 5. Since the total messages received at each receiver take up seven dimensions, six of which are taken by desired messages, each receiver can thus achieve per-user DoF of 6/7. The detailed proof is in Appendix A.

Since each receiver desires six messages, each with splitting ratio $a_{1}$, the NDT of this group is $\tau = \frac{6a_{1}}{6/7} = 7a_{1}$.

B. General Network

Now, we proceed to consider the delivery of subfiles in the general $(K + L - 1) \times K$ partially connected network. We divide the total subfiles to be delivered, $\{W_{\text{need}}\}^{K-1}_{i=0}$, into $L$ groups according to the size of $Q$ of the subfiles. Each group $r$, for $r = |Q| \in [L-1]^{+}$, contains $KL(L-r-1)$ subfiles. Each group of subfiles is delivered individually in the time division manner. In what follows, we present the detailed delivery strategy for an arbitrary group $r$, for $r \in [L-1]^{+}$.

Similar to Example 1, to ensure that each transmitter is connected to $L$ receivers, we introduce virtual receivers to transform the network into the expanded partially connected network.
linear IN. Each virtual receiver adopts the same cache placement strategy as an actual receiver, but does not send any content request. In the expanded network, each subfile desired by one actual receiver is cached in $r$ (virtual) receivers connecting to the same transmitter as this actual receiver. Coded multicasting gain via XOR combining can thus be exploited among the $r + 1$ receivers. The detailed generation of the expanded network and coded multicast messages is shown in Algorithm 1.

**Algorithm 1** Network expansion and coded message generation in the $(K + L - 1) \times K$ partially connected linear IN

1. Generate virtual receivers $\{-L + 1, -L + 2, \ldots, -1\} \cup \{K, K + 1, \ldots, K + L - 2\}$, each virtual receiver $i$ is connected to transmitters $\{i, i + 1, \ldots, i + L - 1\} \cap [K + L - 2]$. Denote $\mathcal{R}_i^L \triangleq \{j - L + 1, j - L + 2, \ldots, j\}$ as the set of (virtual) receivers connected to transmitter $j$.
2. Virtual receiver $i \in \{-L + 1, -L + 2, \ldots, -1\}$ caches the same subfiles as in actual receiver $i + L$, and virtual receiver $i \in \{K, K + 1, \ldots, K + L - 2\}$ caches the same subfiles as in actual receiver $i - L$.
3. For $j = 0, 1, \ldots, K + L - 2$ do
4. For $\mathcal{R} \subseteq \mathcal{R}_j^L$, $|\mathcal{R}| = r + 1$ do
5. If $\mathcal{R} \cap [K - 1]^+ \neq \emptyset$, $|\mathcal{R}| = r + 1$ then
6. Transmitter $j$ generates coded message $\mathcal{W}_{j,\mathcal{R}} = \bigoplus_{i \in \mathcal{R} \cap [K - 1]^+} \mathcal{W}_{i,j,\mathcal{R} \setminus \{i\}}$, where $j = j \mod L$, $i = i \mod L$, $\mathcal{R} = \mathcal{R} \mod L$.
7. End if
8. End for
9. End for

Based on Algorithm 1, the network topology is converted into the expanded partially connected X-multicast channel with multicast size $r + 1$, where each transmitter $j$ has an independent coded message $\mathcal{W}_{j,\mathcal{R}}$ intended to the actual receivers in multicast group $\mathcal{R}$ satisfying

$$\mathcal{R} \subseteq \mathcal{R}_j^L, \mathcal{R} \cap [K - 1]^+ \neq \emptyset, |\mathcal{R}| = r + 1.$$ 

By using a novel interference management scheme, the DoF per actual user of this channel is given in the following lemma.

**Lemma 1.** The achievable per-user DoF of the expanded $(K + L - 1) \times K$ partially connected X-multicast channel with receiver connectivity $L$ and multicast size $r + 1$ is

$$d = \frac{L}{L + \frac{L - r}{r + 1}}. \quad (4)$$

**Sketch of the proof:** Divide all multicast messages into $\binom{L}{r + 1}$ sets according to their intended receiver multicast groups, similar to Example 1. Then design precoding matrices to align the messages from a same set in a same subspace at each undesired receiver using asymptotic interference alignment. The detailed proof is in Appendix A.

The per-user DoF in (4) appears the same as the per-user DoF of an $L \times L$ fully connected X-multicast channel with multicast size $r + 1$ in [9] Theorem 2. This is because by our proposed interference management scheme, each actual receiver in the expanded partially connected X-multicast channel sees an equivalent $L \times L$ fully connected X-multicast channel, independent of $K$. Our scheme is similar to [9], but the message grouping for alignment is different due to the difference in message flow and channel topology.

**Remark 2.** The purpose of introducing virtual receivers during the delivery phase is two-fold. The first is to unify the notation of coded messages in line 6 of Algorithm 1. The second is to convert the channel during the delivery phase into an expanded partially connected X-multicast channel, for which the DoF analysis is more tractable as given in Lemma 1. The virtual receivers however do not affect the actual DoF results of the original unexpanded network, since they do not send any content request, neither intend to decode any message.

Since each receiver desires $L\binom{L - 1}{r}$ subfiles, each with splitting ratio $\alpha_r$, the NDT of group $r$ is, by Remark 1

$$\tau_r = \frac{L(L - 1)}{d} \alpha_r = \left[ L\left(\frac{L - 1}{r}\right) + \left(\frac{L - 1}{r + 1}\right) \right] \alpha_r. \quad (5)$$

Summing up the NDTs of all groups yields the total achievable NDT of the network as

$$\tau = \sum_{r=0}^{L-1} \left[ L\left(\frac{L - 1}{r}\right) + \left(\frac{L - 1}{r + 1}\right) \right] \alpha_r. \quad (6)$$

V. CACHING OPTIMIZATION

In this section, we study the optimization of the file splitting ratios to minimize the total NDT (6) subject to the constraints (3) and (4). This can be formulated as a linear programming problem, which leads to our main findings in the following Theorem.

**Theorem 1. (Achievable NDT)** For the cache-aided $(K + L - 1) \times K$ partially connected linear interference network, an achievable NDT is given by the optimal solution of the following linear programming (LP) problem:

$$\tau^*(\mu_R, \mu_T) \leq \tau_{ub} \triangleq \min_{\alpha_r} \sum_{r=0}^{L-1} \left[ L\left(\frac{L - 1}{r}\right) + \left(\frac{L - 1}{r + 1}\right) \right] \alpha_r$$

s.t. \(\alpha_r \geq 0, \alpha_r = 1 - \mu_R \geq \frac{1}{3}\). \(\mu_R \leq \frac{1}{3}\), \(\frac{1}{3} \leq \mu_R \leq 1\).

The LP problem in Theorem 1 can be solved efficiently by linear equation substitutions and other manipulations. A closed-form and optimal solution for the $6 \times 4$ network with $L = 3$ in Example 1 is given in the following corollary.

**Corollary 1.** The achievable NDT for the $6 \times 4$ network with $L = 3$ is

$$\tau_{ub} = \begin{cases} \frac{5}{3} - 8\mu_R, & 0 \leq \mu_R < \frac{1}{3} \\ \frac{1}{3} - 3\mu_R, & \frac{1}{3} \leq \mu_R \leq \frac{4}{3} \\ 1 - \mu_R, & \frac{4}{3} \leq \mu_R \leq 1 \end{cases}.$$
Corollary 2.

with optimal (not necessarily unique) file splitting ratios \( \{a_0^*, a_1^*, a_2^*, a_3^*\} \) given by

\[
\begin{align*}
0 \leq \mu_R < \frac{1}{3} : & \quad a_0^* = \frac{1}{3} - \mu_R, a_1^* = \frac{2}{3}, a_2^* = a_3^* = 0 \\
\frac{1}{3} \leq \mu_R < \frac{2}{3} : & \quad a_0^* = 0, a_1^* = \frac{2}{3} - \frac{3\mu_R}{2}, a_2^* = \frac{1-3\mu_R}{2}, a_3^* = 0 \\
\mu_R \leq 1 : & \quad a_0^* = a_1^* = 0, a_2^* = \frac{1-2\mu_R}{3}, a_3^* = -\frac{2+3\mu_R}{3}.
\end{align*}
\]

When \( \mu_R = 1/L \), for \( l \in [L]^+ \), which is called the integer point in \( K \), one feasible solution for the LP problem in Theorem 1 is \( a_l^* = 1/(L\binom{l}{1}) \) and others being 0, and the corresponding achievable NDT is

\[
\tau_{ab} = \frac{L\binom{l-1}{1} + L\binom{l-1}{1}}{L\binom{l}{1}} = \left(1 - \frac{1}{1 + L\mu_R}\right)^{-1}(1 - \mu_R).
\]

The expression in (7) suggests that the proposed scheme achieves the receiver local caching gain of \( (1 - \mu_R) \) and a combined coded multicasting and transmitter coordination gain of \( (1 - \frac{1}{1 + L\mu_R}) \).

VI. CONVERSE

In this section, we present a lower bound of the minimum NDT, followed by the discussion on the optimality of the achievable scheme presented in the previous sections.

**Theorem 2.** (Lower Bound of NDT) For the cache-aided \( (K + L - 1) \times K \) partially connected linear interference network, the minimum NDT is lower-bounded by:

\[
\tau^* (\mu_R, \mu_T) \geq 1 - \mu_R.
\]

The proof of Theorem 2 is based on a cut-set argument at each receiver, similar to that in [3] and thus ignored.

**Corollary 2.** The multiplicative gap between the achievable upper bound and the theoretical lower bound of the minimum NDT for the considered network is less than 2.

**Proof.** Let \( g \) denote the multiplicative gap. Given any cache sizes \( \mu_R, \mu_T (\mu_T \geq \frac{1}{L}) \), we have

\[
g = \min_{a_r} \frac{\sum_{r=0}^{L-1} L\binom{l-1}{1} + L\binom{l-1}{1}}{1 - \mu_R} a_r (9) \leq \frac{2L-1}{L} \leq 2.
\]

Here, (9a) is obtained by setting \( a_L = \frac{\mu_T}{L} \), \( a_0 = \frac{1-\mu_R}{L} \), and \( a_r = 0 \) for \( r \in [L-1] \).

**Remark 3.** Note that the lower bound in Theorem 2 allows for arbitrary coding among files in the cache placement. However, our achievable cache placement scheme only involves file splitting without any inter-file or intra-file coding. This implies that inter-file or intra-file coding can at most reduce NDT by two times in the network.

**Remark 4.** In the proposed caching scheme, each transmitter only caches \( NF \) bits and does not fully utilize its cache storage of \( MTF \) bits. Transmitter cooperation gain is thus not fully exploited, in contrast to [8]. However, in certain cases, the obtained NDT is still optimal. For instance, in the 6 \( \times \) 4 network with \( L = 3 \), the achievable NDT is optimal when \( \frac{1}{3} \leq \mu_R \leq 1 \) by Corollary 1 and Theorem 2. In general cases, Corollary 2 implies that transmitter cooperation gain cannot provide any gain more than a constant factor of 2.

VII. APPLICATION TO PARTIALLY CONNECTED CIRCULAR NETWORKS AND FULLY CONNECTED NETWORKS

As mentioned in Section II-A, our partially connected linear IN has strong connection with the circular IN and the fully connected IN. In this section, we apply our caching and delivery scheme to these two network models.

Recall that if we merge transmitter pair \( (j, K + j) \), \( \forall j \in [L-2]^+ \) in the partially connected \( (K + L - 1) \times K \) linear IN, we will arrive at a partially connected \( K \times K \) circular IN. To make the two network models completely identical, the cached content on the merged node should remain the same as before merging. By the proposed cache placement strategy in Section III, the cached contents in the transmitter pair \( (j, K + j) \) of the linear network will be identical when \( L = d \) is a divisor of \( K \). Thus, under the condition that \( L = d \) is a divisor of \( K \), the achievable NDT in Theorem 1 is directly applicable to the partially connected \( K \times K \) circular IN.

In the special case when \( L = K \), the circular IN becomes the fully connected IN. In particular, by letting \( L = K = 1 \) of Lemma 1, the achievable per-user DoF of the X-multicast channel is consistent with (9 Theorem 2).

**APPENDIX A: PROOF OF LEMMA 1**

Consider the \( (K + L - 1) \times K \) partially connected X-multicast channel with receiver connectivity \( L \) and multicast size \( r + 1 \). Each transmitter \( j \) has an independent message, denoted as \( W_{ij}^{R} \), aimed for the actual receivers in each of its connected receiver multicast group \( R \) satisfying

\[
R \subseteq R_j^c, R \cap [K - 1]^+ \neq \emptyset, |R| = r + 1.
\]

We divide all the messages into \( \binom{L}{r+1} \) message sets. Each message set is denoted by a unique tuple \( (q_0, q_1, \ldots, q_r) \), where \( q_i \in \mathbb{Z} \) for \( i \in [r]^+ \) and \( 0 \leq q_0 < q_1 < \cdots < q_r \leq L - 1 \). Messages in an arbitrary message set \( Q = (q_0, q_1, \ldots, q_r) \) are given by

\[
W_Q = \left\{ W_{ij}^R \mid j \in [K + L - 2]^+, R \cap [K - 1]^+ \neq \emptyset, R \subseteq R_j^c, R \mod L = Q \right\}
\]

From (10), it can be seen that each transmitter has at most one message in each set. Messages in the same set will be aligned in a same subspace at undesired receivers. This interference management idea can also be found similarly in [9], [14], [15], but differs in message partition scheme.

We use a \( T_n = L\binom{l-1}{1}n(K+L-1)(L-r-1) + \binom{l-1}{1}(n+1)(K+L-1)(L-r-1) \) symbol extension, where \( n \) is a positive integer, and the channel between transmitter \( j \) and receiver \( i \), \( j \in T_i \), becomes a \( T_n \times T_n \) diagonal matrix \( H_{ij} \) whose diagonal elements \( h_{ij}(\tau) (1 \leq \tau \leq T_n) \) are i.i.d. distributed in some continuous distribution. We encode each message \( W_{ij}^R \) into a \( n(K+L-1)(L-r-1) \times 1 \) symbol vector \( x_{ij}^R = [x_{ij, m}]_{1 \leq m \leq n(K+L-1)(L-r-1)} \), and each symbol \( x_{ij, m}^R \) is
beamformed along a $T_n \times 1$ vector $v^{R}_{t_j,m}$. Then, the codeword of message $W_{ij}^{R}$ is

$$y_i = \sum_{j \in T_i} \sum_{R \subseteq R^c_i, |R| = r+1} v^{R}_{t_j,m} x^{R}_{t_j,m}.$$  

The received signal at an arbitrary actual receiver $i$, for $i \in [K - 1]^+$, is given by (neglecting the noise)

$$y_i = \sum_{j \in T_i} \sum_{R \subseteq R^c_i, |R| = r+1} v^{R}_{t_j,m} x^{R}_{t_j,m}.$$  

(11)

Note that symbols $\{x^{R}_{t_j,m}\}$ in (11) satisfying $i \notin R$ are undesired by receiver $i$. Thus, we need to align their received signal vectors $\{\mathbf{H}_{ij} v^{R}_{t_j,m}\}$ in the same direction at receiver $i$.

In specific, symbols generated from the same message set (10) are aligned together. This idea is similar to [9], but the detailed message grouping and design of beamforming vectors are different.

In specific, consider interference channel matrix set

$$\mathcal{H}_Q = \{ \mathbf{H}_{ij} | j \in [K + L - 2]^+, R \mod L = Q, R \subseteq R^c_i, i \in R^c_i, i \notin R \}.$$  

(12)

for message set $Q$. Note that the channel matrices observed in virtual receivers from their connected transmitters are also assumed to be diagonal and independent with each other and the original channel matrix. The diagonal elements $h_{ij}(\tau)$ ($1 \leq \tau \leq T_n$) of channel matrix between each virtual receiver and its connected transmitter are i.i.d. distributed in some continuous distribution. There are totally $(K + L - 1)(L - r - 1)$ matrices in each matrix set $\mathcal{H}_Q$. Then the beamforming vector $v^{R}_{t_j,m}$ for each message $W_{ij}^{R} \in \mathcal{W}_Q$ is given by a unique vector in vector set

$$\mathcal{V}_Q(n + 1) = \left\{ \prod_{H_{ij} \in \mathcal{H}_Q} (H_{ij})^{\alpha_{ij}} \cdot b_Q | 1 \leq \alpha_{ij} \leq n \right\},$$  

(13)

where $b_Q$ is a $T_n \times 1$ vector $[b_{Q,m}]_{1 \leq m \leq T_n}$, whose elements are i.i.d. chosen from some continuous distribution, and independent with other $b_Q$ for different $Q$.

Then, the received signal at an arbitrary actual receiver $i$, for $i \in [K - 1]^+$, can be rewritten as

$$y_i = \sum_{j \in T_i} \sum_{R \subseteq R^c_i, |R| = r+1} v^{R}_{t_j,m} x^{R}_{t_j,m}.$$  

(14)

For message $W_{ij}^{R} \in \mathcal{W}_Q$ where $i \notin R$ in (14) (which implies $(i \mod L) \notin Q$), it can be seen that $\mathbf{H}_{ij}$ lies in $\mathcal{H}_Q$. Thus, the received vector $\mathbf{H}_{ij} v^{R}_{t_j,m}$ of $x^{R}_{t_j,m}$ lies in vector set $\mathcal{V}_Q(n+1)$. This implies that interference is aligned, i.e., the received vectors of undesired messages $W_{ij}^{R} \in \mathcal{W}_Q$ ($(i \mod L) \notin Q, j \in T_i$) all lie in $\mathcal{V}_Q(n+1)$. The received vectors of desired messages of receiver $i$ are given by the column vectors of the following $T_n \times L^{(L-1)}(n(1)K + L - 1)(L - r - 1)$ matrix:

$$A_{\text{desire}} = \left[ \mathbf{H}_{ij} v^{R}_{t_j,m} | j \in T_i, R \right.$$  

(15)

The received vectors of undesired messages of receiver $i$ lie in the linear space of the vector set:

$$\mathcal{V}_Q(n + 1) = \left\{ \prod_{H_{ij} \in \mathcal{H}_Q} (H_{ij})^{\alpha_{ij}} \cdot b_Q | 1 \leq \alpha_{ij} \leq n + 1 \right\},$$  

(16)

where $Q$ satisfies $(i \mod L) \notin Q$. This is equivalent to the linear space formed by the column vectors of the following $T_n \times (L-1)(n(1)K + L - 1)(L - r - 1)$ matrix:

$$A_{\text{undesire}} = [u]_{n \in \mathcal{V}_Q(n+1)}, \text{ and } Q \text{ satisfies } (i \mod L) \notin Q.$$  

(17)

Next we need to assure that the received $T_n \times T_n$ matrix

$$A = [A_{\text{desire}}, A_{\text{undesire}}]$$

is full rank almost surely.

By using [14] Lemma 1, we only need to prove the elements in the same row of $A$ are different monomials. Consider an arbitrary $\tau$-th row of $A$. It is obvious that elements of messages in different message sets $Q$ differ in $b_{Q,\tau}$. Then consider an arbitrary set $Q ((i \mod L) \in Q)$. In this set, transmitter $j (j \in T_i)$ has a message $W_{ij}^{R} \in \mathcal{W}_Q$ desired by receiver $i$. Note that $\mathbf{H}_{ij} \notin \mathcal{H}_Q$. Therefore, elements of messages from transmitter $j$ have a unique term $h_{ij}(\tau)$ in this set. Finally, consider the message in an arbitrary set $Q ((i \mod L) \notin Q)$ and from transmitter $j$. Its corresponding elements are

$$\{h_{ij}(\tau) \prod_{H_{ip} \in \mathcal{H}_Q} (h_{ip}(\tau))^{\alpha_{ip}} \cdot b_{Q,\tau} | 1 \leq \alpha_{ip} \leq n \},$$

which are different monomials. Similar arguments can be applied to message sets $Q$ satisfying $(i \mod L) \notin Q$. Thus we proved that the elements in the same row of $A$ are different monomials. By [14] Lemma 1, we assure that $A$ is full rank almost surely. Receiver $i$ can successfully decode its desired message, so as the other receivers.

Since each receiver decodes its $L^{(L-1)}$ desired messages, each encoded into $n(1)K + L - 1)(L - r - 1)$ symbols, in $T_n$ symbol extension. Per-user DoF of

$$\text{DoF} = \frac{L^{(L-1)} n(1)K + L - 1)(L - r - 1)}{L^{(L-1)} n(1)K + L - 1)(L - r - 1) + (L-1)(n+1)(K + L - 1)(L - r - 1)}$$
is achieved. Let $n \to \infty$, per-user DoF of $\frac{L^{(L-1)}_{(L-1)^{+}+\left(\frac{L-1}{r}+1\right)}}{L+r+1}$ is achieved. Lemma 1 is proved.

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