Improvement and Application of the Softened Strut-and-Tie Model

Guoxi Fan¹, Debin Wang², Yuhong Diao³, Huaishuai Shang⁴, Xiaocheng Tang⁵, Hai Sun¹

¹School of Engineering, Ocean University of China, Qingdao, Shandong, 266100, China
²School of Civil and Safety Engineering, Dalian Jiaotong University, Dalian, Liaoning, 116028, China;
³Shandong Zhuoyuan Architectural Design Co.Ltd, Qingdao, Shandong, 266011, China;
⁴School of Civil Engineering, Qingdao University of Technology, Qingdao, Shandong, 266033, China;
⁵School of Civil Engineering and Architecture, Northeast Electric Power University, Jilin, Jilin, 132012, China

Abstract. Previous experimental researches indicate that reinforced concrete beam-column joints play an important role in the mechanical properties of moment resisting frame structures, so as to require proper design. The aims of this paper are to predict the joint carrying capacity and cracks development theoretically. Thus, a rational model needs to be developed. Based on the former considerations, the softened strut-and-tie model is selected to be introduced and analyzed. Four adjustments including modifications of the depth of the diagonal strut, the inclination angle of diagonal compression strut, the smeared stress of mild steel bars embedded in concrete, as well as the softening coefficient are made. After that, the carrying capacity of beam-column joint and cracks development are predicted using the improved softened strut-and-tie model. Based on the test results, it is not difficult to find that the improved softened strut-and-tie model can be used to predict the joint carrying capacity and cracks development with sufficient accuracy.

1. Introduction
At present, numerous theoretical models have been proposed to investigate the mechanical properties of beam-column joints. Paulay [1] pointed out that 2 kinds of shear resistant mechanisms including diagonal strut mechanism and truss mechanism existed in the joint core area. However, this model does not consider the constraint effect of stirrups on the concrete in the joint core area. Subsequently, the softened truss model was proposed which was one of five kinds of models belonging to the reinforced concrete unified theory, and satisfied equilibrium, compatibility and constitutive laws. Hsu [2] proposed the rotating angle softened truss model, while Pang and Hsu [3] proposed the fixed angle softened truss model. Ji [4] pointed out that the rotating angle softened truss model could not explain the existence of shear stress due to aggregate interlock and dowel action of reinforcement bars, and was no longer suitable for higher accuracy structure calculation using the constitutive relation of bare reinforcement bars. In regard to the fixed angle softened truss model, a problem was ignored that the direction of cracks was not only related with the external stress, but also related with the ratio of two perpendicular directions of reinforcement bars. In contrast, the softened strut-and-tie model is often
termed as a rational model for determining the shear carrying capacity of beam-column joints [5, 6]. The softened strut-and-tie model originates from the strut-and-tie concept and satisfies equilibrium, compatibility, and constitutive laws of cracked reinforced concrete, which has been proposed for determining the shear strength of beam-column joints.

Among those models, the softened strut-and-tie model can be used as a tool to clarify the roles of different parameters, besides the particular use in the strength prediction of discontinuity regions. The aims of this paper are to predict the joint carrying capacity and cracks development theoretically. Based on the former considerations, four adjustments for the softened strut-and-tie model are made to get better predicting of the test results. And the application of improved beam-column joint model is deeply discussed.

2. Modification of the softened strut-and-tie model

The softened strut-and-tie model consists of the diagonal, horizontal, and vertical mechanisms. On the basis of the softened strut-and-tie model derived previously, Hwang and Lee [7] proposed a simple predicting procedure, which dealt with the carrying capacity prediction of reinforced concrete discontinuity regions failing in diagonal compressions without getting lost in trivialities. This simplified approach is a useful and practical tool for determining the shear carrying capacity of the discontinuity regions failing in diagonal compressions, especially for that of beam-column joints.

By comparing the experimental results and the calculation results of beam-column joints at home and abroad, it is found that the following problems exist in the softened strut-and-tie model. (1) Test results show that, because the neglect of $a_b$ (the height of concrete compression zone of beam section) in computing $a_s$ (the height of effective cross section of diagonal compression strut) is assumed, the proposed model underestimates the joint shear carrying capacity for the cases with lower axial loads [6]. (2) The average value of the reduction factor of concrete strength estimated by the simplified method is lower than that computed by the general method [7]. (3) The column axial load plays a beneficial role in the shear carrying capacity of beam-column joints in a certain range, because it increases the height of effective cross section of diagonal compression strut. But on the contrary, the column axial load provides an adverse effect on the joint shear carrying capacity due to the early crushed of the concrete in the core area of the joint when the column axial load is too large. The softened strut-and-tie model does not consider the adverse effect of the larger column axial load. (4) The smeared stress of mild steel bars embedded in concrete $f_s$ is lower than the yield stress of the bare steel bars $f_y$ [8]. However, the stress-strain relationship of the bare steel bars is adopted in the softened strut-and-tie model. In view of the above problems, four adjustments for the softened strut-and-tie model are made.

(1) The relative height of concrete compression zone of beam section can be obtained by the ductile design control criterion of reinforced concrete structures, which is approximate 0.2 [9].

(2) The inclination angle $\theta$ of diagonal compression strut can be calculated by equation (1) for the range of axial compression ratio less than 0.1. Otherwise, it is defined as equation (2).

\[
\theta = \tan^{-1}\left(\frac{h^c}{h^c + \frac{b}{3}c^h}\right) 
\]

\[
\theta = \tan^{-1}\left(\frac{h^c}{h^c + \frac{2a}{3}}\right) 
\]

(3) The smeared stress of mild steel bars embedded in concrete $f_s$ is lower than the yield stress of the bare steel bars $f_y$ [8]. It is expressed as:

\[
f_s = f_y (0.93 - 2B)
\]

\[
B = \frac{(f_{cr} / f_s)^{1.5}}{\rho} = \frac{(0.31\sqrt{f_{cr} / f_s})^{1.5}}{\rho}
\]
where $\rho$ is the reinforcement steel ratio (limited to a minimum of 0.25%).

(4) The average value of $\zeta$ estimated by the simplified method is lower than that computed by the general method [7]. So the softening coefficient can be computed by equation (5).

$$\zeta = \frac{3.65}{\sqrt{f_c}} \leq 0.52$$

(5)

3. Application of improved beam-column joint model

3.1 Prediction of the carrying capacity of beam-column joints

Based on the test results of references [10-16], 19 specimens are selected to verify the adaptability of the improved beam-column joint model. The specimens selected encompass a wide range of material properties, reinforcement detailing and cross section geometry. Figure 1 shows that satisfactory results are obtained for the comparison of measured and computed joint shear carrying capacity. After previously mentioned adjustments made, the average ratio of the carrying capacity of beam-column joints computed by the softened strut-and-tie model and experimental results is 0.8847, which is very close to the reduction factor. On the other hand, it also proves the correctness of the adjustments mentioned above. Thus, it is not difficult to find that the improved softened strut-and-tie model can be used to predict the joint carrying capacity with sufficient accuracy.

![Figure 1. Correlation of experimental and predicted joint shear carrying capacity.](image)

3.2 Prediction of cracks development of beam-column joints

The prediction process of cracks development can be determined under the premise of known material properties, reinforcement detailing, axial compression ratio and cross section geometry. Based on the value of $\theta$ (equation (1)) and the geometric relation, the inclination angle $\alpha$ of flat strut and the ratio of tensile force provided by the horizontal tie to the joint horizontal shear force $\gamma_h$ can be obtained according to equation (6) and (7), respectively.

$$\tan \alpha = \frac{1}{2} \tan \theta$$

$$\gamma_h = \frac{2 \tan \theta - 1}{3}$$

Under the action of horizontal shear $V_{jh}$, the compression force of the flat strut $D_1$, the tensile force of the horizontal tie $F_h$ and the compression force of the diagonal strut $D$ can be calculated according to equations (8), (9) and (10), respectively.

$$D_1 = \frac{F_h}{\cos \alpha}$$
The effective area of the diagonal strut $A_{str}$ is defined as:

$$A_{str} = a_s \times b_s$$

(11)

where $b_s$ is the width of the effective cross section of diagonal strut, which is equal to the effective width of the beam-column joint. $a_s$ is the depth of the effective cross section of diagonal strut, which can be determined as:

$$a_s = \sqrt{a_b^2 + a_c^2}$$

(12)

The ultimate carrying capacity of the diagonal strut $D_c$ and the ultimate carrying capacity of the horizontal tie $F_{hs}$ can be obtained by material strength, geometry dimension of the cross section and equation (13).

$$a_c = (0.25 + 0.85 \frac{N}{A_s f_c}) h_c$$

(13)

If the contrast result is $D_c \geq F_{hs}$, the cracking carrying capacity of the diagonal compression strut $D_{cr}$ and the yielding carrying capacity of the horizontal tie $F_{hy}$ should be determined by equations (14)-(16).

$$f_{cr} = 0.31 \sqrt{f_c}$$

(14)

$$D_{cr} = f_{cr} \times A_{str}$$

(15)

$$F_{hy} = 2 \times (1 + 0.5 \times n) \times f_{yb} \times A_s$$

(16)

where $f_{cr}$ is the cracking strength of concrete. $f_c$ is the compressive strength of concrete cylinder. $n$ represents the number of transverse reinforcements in the joint core area, in addition to the transverse reinforcements within the center half of the joint core. $f_{yb}$ is the yield stress of transverse reinforcement. $A_s$ is the sectional area of transverse reinforcement. Then, the sequence order of the generation of cracks in the diagonal compression strut and the yield of transverse reinforcements can be determined. If the contrast result is $D_c < F_{hs}$, the following conclusion can be draw that cracks occur in the diagonal compression strut before the yielding of transverse reinforcements. Under the action of horizontal shear $V_{jh}$, the position of the first crack can be estimated by the carrying capacity of the diagonal compression strut and the flat strut.

Take the specimen of reference [17] for example, the calculation results are shown in table 1. By comparison, it can be found that the contrast result is $D_c \geq F_{hs}$. In addition, the horizontal tie bears the greater horizontal shear force before yielding, so that the horizontal tie yielding before the diagonal strut failure. According to the calculation results, it is not difficult to find that cracks occur in the area of the strut before transverse reinforcement yielding. Under the action of horizontal shear $V_{jh}$, the pressure beared by the flat strut is greater than that beared by the diagonal strut. However, the softened strut-and-tie model provides that the cracking carrying capacity of the flat strut is smaller than that of the diagonal strut due to the smaller effective cross sectional area. Therefore, the first crack occurs in the flat strut region. Test results show when the first crack occurs, the strain of transverse reinforcement within the center half of the joint core is $660.5 \times 10^{-6}$, which is less than the yielding strain of transverse reinforcement ($1433.3 \times 10^{-6}$). And the first crack occurs in the flat strut region, as shown in figure 2. It also proves the adaptability of the improved strut-and-tie model in predicting cracks development, on the other hand.
Table 1. Calculation results of improved softened strut-and-tie model.

| θ (°) | α (°) | γh | Dh | Fh | D | Dh (kN) | Fh (kN) | Dc (kN) | Fhy (kN) |
|-------|-------|-----|-----|-----|---|---------|---------|---------|----------|
| 55.9  | 36.4  | 0.65 | 0.81 Vjh | 0.65 Vjh | 0.62 Vjh | 985.99 | 196.12 | 69.92   | 169.36   |

Figure 2. Test results of cracks development.

4. Conclusions

Reinforced concrete beam-column joints play an important role in the mechanical properties of moment resisting frame structures, so as to require proper design. The main objective of this paper is to predict the joint carrying capacity and cracks development theoretically. For this purpose, the softened strut-and-tie model is selected to be introduced and analyzed. Based on the analytical results present in this paper, the following conclusions can be drawn:

1. Four adjustments for the softened strut-and-tie model including modifications of the depth of the diagonal strut, the inclination angle $\theta$ of diagonal compression strut, the smeared stress of mild steel bars embedded in concrete, as well as the softening coefficient are made. It has been proved by test data that predicted results by the improved softened strut-and-tie model are consistent with the test data and conservative. The average carrying capacity ratio of predicted results by the improved model and experimental results is 0.8847.

2. Based on the test results, it is not difficult to find that the improved softened strut-and-tie model can be used to predict the joint carrying capacity with sufficient accuracy. Test results also show that the improved strut-and-tie model can be used to predict cracks development.

5. References

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