Measuring the viscosity of dark matter with strongly lensed gravitational waves

Shuo Cao1, Jingzhao Qi2, Marek Biesiada1,3, Tonghua Liu1, Jin Li4, Zong-Hong Zhu1‡

1 Department of Astronomy, Beijing Normal University, 100875, Beijing, China; caoshuo@bnu.edu.cn; zhuzh@bnu.edu.cn
2 Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China;
3 National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland; Marek.Biesiada@mcbj.gov.pl
4 Department of physics, Chongqing University, 400044 Chongqing, China

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ABSTRACT

Based on the strongly lensed gravitational waves (GWs) from compact binary coalescence, we propose a new strategy to examine the fluid shear viscosity of dark matter (DM) in the gravitational wave domain, i.e., whether a GW experiences the damping effect when it propagates in DM fluid with non-zero shear viscosity. By assuming that the dark matter self-scatterings are efficient enough for the hydrodynamic description to be valid, our results demonstrate that future ground-based Einstein Telescope (ET) and satellite GW observatory (Big Bang Observer; BBO) may succeed in detecting any dark matter self-interactions at the scales of galaxies and clusters.

Key words: dark matter — gravitational waves — gravitational lensing: strong

1 INTRODUCTION

It is well known that in the past years have witnessed a new era of gravitational wave astronomy, with the first direct detection of gravitational wave (GW) sources from inspiraling and merging binary black holes (BH) (Abbott et al. 2016a) and neutron stars (NS) (Abbott et al. 2017c). These GW signals, together with their electromagnetic (EM) counterparts can be used as standard sirens providing the luminosity distances ($D_L$) in a direct way, not relying on the cosmic distance ladder (Schutz 1986). Currently, the measurements of luminosity distance by the advanced LIGO and Virgo detectors are affected by large uncertainties. However, as discussed in Gupta et al. (2019) the accuracy of the third generation of GW detectors will reach 1–3% accuracy up to 300 Mpc, much smaller than velocity of host galaxies. This would allow for independent calibration of distances to SNe Ia. Moreover, extending the GW frequency window by space-borne detectors to decihertz band, where the coalescences of binary white dwarfs are detectable, would allow to measure luminosity distance up to 100 Mpc with 1% accuracy (Maselli et al. 2020; Zou et al. 2020). Therefore, multi-messenger observations can put strong constraints on the Hubble constant inferred from low redshift sources. There also exist some interesting possibilities of testing the speed of GWs by measuring the time of arrival delays between photons and GWs over cosmological distances (Nishizawa 2014; Li et al. 2016), the equivalence principle by using the Shapiro effect (Will 2014; Kahya & Desai 2016), and the Lorentz invariance violation (Kostelecky & Mewes 2016).

From theoretical point of view, it has been widely recognized that the absorption and dispersion of GWs in the Universe (filled with a perfect fluid) should be neglected (Ehlers et al. 1996). However, some ideas invoked in the context of dark matter (DM) can challenge this point of view. For instance, many authors have investigated the possibility of gravitational waves disappearing into dark sector (Foot & Vagnozzi 2016), i.e., the so-called U(1)$_D$ charged dark matter with the generation of dark cosmological magnetic fields and efficient graviton-dark photon conversion (Masaki & Soda 2018). On the other hand, if the Universe contains some non-ideal fluids, the fluid energy-momentum tensor might contain a non-zero shear viscosity term $\eta$ (Hawking 1969). Within this approach viscosity provides a damping rate $\beta \equiv 16\pi G\eta$ with which GWs would be dissipated by matter (Madsen 1973; Prasanna 1999). Although the collisionless cold DM paradigm can successfully account for the observations of large scale structure (Bahcall et al. 1997), recent progress made in N-body simulations concerning the small-scale structures has demonstrated its strong conflict with observations on dwarfs (Oh et al. 2011), low surface brightness (LSB) galaxies (Kuzio et al. 2008) and clusters (Newman et al. 2013). In particular, recent works of Graziani et al. (2020); Marassi et al. (2019) should be mentioned in this context, since they provided the properties (SFR) and evolution (metallicity evolution) of dwarf galaxies different from large-scale simulations. More importantly, if DM can be treated as a non-ideal fluid, the DM self-interaction (SI) can generate the cosmological shear vis-
cosy (Goswami et al. 2017; Atreya et al. 2018) and explain the small-scale structure problems of the Universe (Spergel & Steinhardt 2000). It was recently found that dark matter self-interactions may also serve as a negative pressure source, which constitutes another possible mechanism to explain the accelerated expansion of the Universe (Zimdahl et al. 2001; Cao et al. 2011). In this Letter, we propose that the combination of unlensed and lensed GW signals would enable quantitative measurements of GW absorption and dissipation at much higher redshifts ($z \sim 5$), which creates a valuable opportunity to test the propagation of GWs through the cosmic dark matter.

2 METHOD AND SIMULATED DATA

Up to now, LIGO and Virgo Collaborations have released the measurements of $D_{L,\text{obs}}$ for eleven GW sources. However, it should be stressed that the luminosity distances $D_{L,\text{obs}}$ were derived from the observed strain $h(t)$ and frequency evolution $f(t)$ under the assumption that the Universe is filled only with perfect fluids. DM self-interaction manifesting as effective shear viscosity of DM component would modify this inference. Namely, as demonstrated in Goswami et al. (2017), Lu et al. (2018), the strain of GW signals at the co-moving distance of $D$, which then propagate through the viscous fluid would be modified to

$$h_{\text{a,visc}} = h_{\text{a}} e^{-\beta D/2},$$

where $\alpha$ index denotes (+, x) polarizations of the GW. Considering the inverse relation between the amplitude of GW waveform and the luminosity distance ($D_L$), let us define the effective luminosity distance inferred from the GW signal propagating through the viscous fluid

$$D_{L,\text{eff}}(z, \beta) = D_L(z)e^{\beta D(z)/2}.$$  \hfill (2)

Once we have independent measurements of the luminosity distance $D_L(z)$ of the source, for example from the EM counterpart, one can use the $D_{L,\text{eff}}(z)$ derived from the GWs to estimate the damping rate $\beta$ and thus the shear viscosity $\eta$ of DM fluid. Such strategy is however hard to realize. We propose another approach by using strongly lensed GW accompanied by EM signal to test the DM damping effect on unlensed GW signals. The main advantage of GW strong lensing over strongly lensed quasars and SNe Ia (Cao et al. 2017; 2018), is that the time delay between multiple images can be determined with unprecedented accuracy (Liao et al. 2017).

In the general framework of strong gravitational lensing, for a coalescing compact binary system located at the position of $\beta$, the time delay between its detected GW signals $\theta_i$ and $\theta_j$ can be written as (Treu et al. 2010)

$$\Delta t_{i,j} = \frac{D_L(z_i)}{c} \Delta \phi_{i,j},$$

where $\Delta \phi_{i,j}$ is the Fermat potential difference determined by the source position and two-dimensional lensing potential $\psi$. Here the time-delay distance is defined as

$$D_{\Delta t}(z_i, z_j) \equiv \frac{D_A(z_i)D_A(z_j)}{D_A(z_i, z_j)},$$

a combination of three angular diameter distances between the observer ($z = 0$), the lens ($z_l = z_l$) and the source ($z = z_s$). From Eq. (3) one can see that $D_{\Delta t}$ can be estimated from time delays, provided one is able to reconstruct the Fermat potential. Such reconstruction is possible if optical images of the lensing systems are available. Because the speed of GWs has already been confirmed to be equal to the speed of light with fractional accuracy of $10^{-16}$, time delays derived from GW are not affected by DM damping effect. Therefore the measured $D_{\Delta t}^{\text{lensed}}(z_i, z_j)$ can be tested against $D_{\Delta t}^{\text{unlensed}}(z_i, z_l, z_s)$ derived from effective luminosity distances of GW signals originating at redshifts close to $z_i, z_j$. More precisely, the angular diameter distance and the luminosity distance are related as $D_A(z) = D_L(z)/(1+z)^2$. Moreover, in a flat universe one has $(1+z)D_A(z_i, z_j) = (1+z)D_A(z_i) - (1+z)D_A(z_j)$, based on which one could derive the time delay distance as

$$D_{\Delta t} = \frac{D_L(z_i)D_L(z_j)}{(1+z)^2D_L(z_i) - (1+z)(1+z)D_L(z_j)}.$$  \hfill (5)

The Eq. (5) merely shows how the time delay distance could be calculated in terms of luminosity distances. The true, unaffected value of $D_{\Delta t}$ (by DM damping effect on the waveforms) would be obtained from accurate time delays of lensed GW signals (note that no template fitting is needed for this purpose), combined with Fermat potential reconstruction following dedicated examination of the lensing system in the optical. However, the statistics of unlensed GW events would be much richer than lensed ones and one would be able to find the unlensed events matched to $z_i$ and $z_j$ of the GW strong lensing system. In this way one would be able to reconstruct the time delay distance of lensing system using measured luminosity distances of unlensed standard sirens, i.e. the effective luminosity distances which are affected by viscous DM damping (through Eq. (2)).

In this work, we focus on GW signals from binary neutron stars (BNS) covering the redshift range of $z \sim 5$, which could be detected in the future using the third-generation ground-based technology of the Einstein Telescope (ET) (Taylor & Gair 2012) and the second-generation technology of space-borne Big Bang Observer (BBO) (Cutler & Holz 2009). It should be mentioned that in the meantime, another space-borne interferometer, DECI-hertz Interferometer Gravitational wave Observatory (DECIGO) has also been proposed (Kawamura et al. 2011). It will be sensitive to GWs at mHz to 100 Hz and capable of observing GWs from the binaries which would also be targets of ground-based detectors, such as LIGO/Virgo and the ET (Kawamura et al. 2014). Different strategies should be applied to derive the redshift of GW sources: one is identifying electromagnetic counterparts like short $\gamma$ ray burst (SGRB) and the other is uniquely determining the host galaxy in the optical (by LSST) and coordinated optical/infrared campaign, benefit from BBO’s unprecedented angular resolution. We perform Monte Carlo simulations to create the unlensed and lensed samples. For the unlensed GW events, the observations of these standard sirens are simulated as follows: (1) Instead of the specific fitting form of BNS merger rate extensively used in the literature (Cai et al. 2016; Qi et al. 2013), we apply the redshift-dependent merger rate of double compact objects, based on the conservative SFR function from (Dominik et al. 2013), to characterize the probability distribution function of $10^4$ GW sources in the red-

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shift range of $0 < z < 5$. The review of cosmic history of star formation can be found in Madau & Dickinson (2014), where the reliable SFR estimates from the observations are provided. In this analysis we use, however the results of Dominik et al. (2013) obtained with the population synthesis code StarTrack, for the sake of consistency with the strong lensing predictions made using this model.

(2) The mass of neutron stars in the BNS systems, whose position is randomly sampled in the interval of $\theta \in [0, \pi]$, will be sampled from uniform distribution of $m \in [1, 2] M_\odot$. Such methodology follows the recent analysis of both unlensed and lensed BNS events in the framework of Einstein telescope (Piörkowska et al. 2014; Cai et al. 2016). (3) Different error strategies will be implemented to our simulation of luminosity distance $D_{L,\text{eff}}(z)$ in ground-based Einstein Telescope (ET) and space-based Big Bang Observer (BBO). On the one hand, for the ET which is very sensitive in the frequency range of $1 - 10^9$ Hz, the amplitude of GW signals produced by inspiralling binaries could be derived in the Fourier space, focusing on the post-Newtonian and stationary phase approximation: (Zhao et al. 2011)

$$A = \frac{1}{D_{L,\text{eff}}(z)} \sqrt{\frac{2}{\nu^3}} (1 + \cos^2(\nu)) \sqrt{\pi} \nu^2 \cos^2(\nu) \times \sqrt{5/90} \nu^{-7/6} M_\odot^{1/6},$$

(6)

where $D_{L,\text{eff}}(z)$ is the effective luminosity distance of the source defined above (affected by DM damping), $F_{\nu, \nu}$ represent the beam-pattern functions depending on the position of GW source ($\theta, \phi$) and the polarization angle ($\alpha$), while $\nu$ denotes the inclination of the binary’s orbital plane. Note that a criterion of $\nu < 20^\circ$ is implemented in our simulations, considering that SGRB is emitted in a narrow cone (Nakar 2007). Focusing on a binary system consists of component masses $m_1$ and $m_2$, one could define the chirp mass as $M_c = M_\odot^{3/5} / 2$, with the total mass of $M = m_1 + m_2$ and the symmetric mass ratio of $\nu = m_1 m_2 / M^2$. Given the GW waveform, the combined signal-to-noise ratio (SNR) of three ET interferometers is given by $\rho_{\text{net}} = \sqrt{\sum_{i=1}^{3} (H(i), H(i))}$, the square root of the inner product of the Fourier strain (in this work we take $\rho_{\text{net}} = 8$ as the detector’s threshold). In order to characterize the sensitivity of ET on the strain of GW signals, we turn to Zhao et al. (2011) for the explicit expression of the one-side noise power spectral density (PSD), $S_n(f)$. Now the instrumental error on the luminosity distance could be estimated at the level of $\sigma_{D_{L,\text{eff}}} / \rho_{\text{net}} \simeq 2D_{L,\text{eff}} / \rho_{\text{net}}$, if the effective luminosity distance is uncorrelated with other parameters of GW sources and detectors (Cai et al. 2010). For the BBO, we determine the corresponding distance uncertainties from the results shown in Fig. 4 of Cutler & Holz (2009), which may be reduced to 1% percent accuracy due to the fundamentally self-calibrating capability of space-based detectors at lower frequencies ($10^{-3} - 1$ Hz). On the hand, the second source of systematic error on $D_{L,\text{eff}}(z)$ comes from the weak-lensing effect, which is quantified as $\sigma_{D_{L,\text{eff}}} / D_{L,\text{eff}} = 0.05 z$ for ET in the present analysis (Sathyaprakash et al. 2010). For BBO, we use the fitting formula of Cutler & Holz (2009) to estimate the weak lensing uncertainty of the effective distance: $\sigma_{D_{L,\text{eff}}} / D_{L,\text{eff}} = 0.04 z$, which is much larger than the instrumental uncertainties arising from the detectors.

For the lensed GW events, one can expect that a considerable catalog of strongly-lensed gravitational waves will be detected, within the observing strategy of future ground-based and space-ground GW detectors. Focusing on the detection of lensed BNS mergers by ET and BBO with their EM counterpart determined by Large Synoptic Survey Telescope (LSST), in this analysis we carry out the simulation approach as follows (Collett 2015; Cao et al. 2019). (1) The redshift distribution of the lensing galaxies is characterized by the modified Schechter function, i.e., the velocity dispersion distribution function and their values are taken from SDSS Data Release 5 of elliptical galaxies (Choi et al. 2007). Meanwhile, a spherically symmetric power-law model ($\rho \sim r^{-\gamma}$) is applied to quantify the mass distribution of each galactic-scale lens, with the average density slope of SLACS lenses ($\gamma = 2.085 \pm 0.02$) as our fiducial lens model (Koopmans et al. 2009). The final results show that for the simulated population of realistic lensing systems, the distributions of the lens redshift ($z_l = 0.8$) and the velocity dispersion ($\sigma_v = 210 \pm 50$ km/s) are well consistent with those of the available LSD lenses. (2) In each simulation, we assume that ET will conservatively detect 10 lensing events (with their EM counterparts) during its successful operation (Ding, Biesiada & Zhu 2013), while $10^4$ such events would be possibly registered by BBO from the observations of $10^4$ BNS systems (Cutler & Holz 2009). Concerning the sampling distribution of the lensed BNS population, we adopt the same approach as in Ding, Biesiada & Zhu (2013) and obtain the sampling distribution of lensed GWs, from the well-known "rest frame rates" calibrated by strong lensing effects. (3) In our simulation of lensed GWs, three ingredients are considered to contribute the uncertainty of the time delay distance. Firstly, compared with traditional strong lensing systems which monitor light curves of quasars and SNe Ia, the time delays measured from lensed GWs can be accurately determined given their negligible uncertainties due to the short duration of GW signals from BNS mergers ($\sim 0.1$ s) (Liao et al. 2017). Secondly, benefit from high-resolution imaging of the EM counterpart (i.e., the host galaxy), the absence of central dazzling active galactic nuclei (AGN) could decrease the fractional uncertainty of Fermat potential difference to the level of $\sim 0.5\%$ (Suyu et al. 2017). Finally, similar to the strong lensing systems in the EM domain, the lensed GWs are still confronted with the light-of-sight (LOS) contamination, which will introduce an additional 1% uncertainty in the Fermat potential reconstruction (Cao et al. 2014).

3 IMPLEMENTATION AND RESULTS

Now we will illustrate what kind of results could be derived from our method, based on the collisionless cold DM ($\eta = 0$) and $\Lambda$CDM cosmology from Planck + WMAP + highL + BAO (Planck Collaboration XIII 2016). The final results are shown in Table 1, based on the simulation process repeated $10^3$ times for each lensed and unlensed GW data set. More specifically, in our analysis the $\beta$ parameter is determined in the framework of Markov Chain Monte Carlo (MCMC) minimizations, with the corresponding $\chi^2$ defined
self-interacting dark matter in galaxies and galaxy clusters.

as the damping rate with uncertainty $\Delta \beta$. The main question to be addressed is: Is such precision high enough to detect possible DM shear viscosity or self-interactions in gravitational wave domain? For our analysis, we have considered two scenarios: one with the DM SI cross sections and average velocity deduced from the fittings to dwarf galaxies, and low surface brightness (LSB) galaxies; and the other with the DM SI cross sections and average velocity deduced from the fittings to galaxy clusters (Kaplinghat et al. 2016). Let us first consider the SI model deduced from the fittings to dwarf galaxies and LSB galaxies (the velocity-dependent cross section is $\sigma_\chi/m_\chi \sim 1$ cm$^2$/g on galaxy scales). When considering the intrinsic uncertainty in the mean collision velocity $\Delta(v) \sim 10^4$ km/s, 10 lensed GWs detected by ET will be able to detect (if any) galactic-scale DM SI cross section at the precision of $\Delta(\sigma_\chi/m_\chi) \sim 10^{-4}$ cm$^2$/g. Focusing on the SI model corresponding to dark matter in galaxy clusters we have $\Delta(\sigma_\chi/m_\chi) \sim 10^{-3}$ cm$^2$/g, considering the intrinsic uncertainty in the mean collision velocity $\Delta(v) \sim 10^3$ km/s. Therefore, ET will also succeed in detecting cluster-scale DM SI at very high confidence level, since the velocity-dependent cross section is $\sigma_\chi/m_\chi \sim 10^{-1}$ cm$^2$/g on cluster scales, consistent with the upper limits from merging clusters (Kaplinghat et al. 2016). In the framework of BBO, the second-generation space-based GW detector, it is expected to detect galaxy-scale DM SI with the precision reaching at even $\Delta(\sigma_\chi/m_\chi) \sim 10^{-6}$ cm$^2$/g. Significant improvements would also be obtained for the cluster-scale case: $\Delta(\sigma_\chi/m_\chi) \sim 10^{-5}$ cm$^2$/g. Therefore, we expect such stringent DM SI constraints from the GW damping can probe the DM self-scattering solution to the small-scale structure problems (Kaplinghat et al. 2016).

There are a few other ways in which our technique might be implemented. In our approach the traditional strong lensing systems in the EM domain, i.e., strongly lensed quasars and SNe Ia with well-measured time delays (Goobar et al. 2017) could also be used instead of lensed GWs. More importantly, the upcoming LSST accompanied by a long baseline multi-epoch and 10 year z-band observational campaign, will enable the discovery of $10^3$ quasar-elliptical galaxy systems

Table 1. Summary of the constraints obtained from different observations. Type I and II respectively correspond to the two cases of self-interacting dark matter scenario (Tulin & Yu 2017), in which DM self-scattering is efficient for hydrodynamic description to be valid and assigning a Maxwellian distribution for DM particles, the DM SI cross section $\sigma_\chi$ can be related to the shear viscosity $\eta$ as $\eta = \frac{1.18 \times 10^{-6} \times \langle v \rangle}{\sigma_\chi^2 m_\chi}$, where $m_\chi$ and $\langle v \rangle$ are the mass and average velocity of DM particles, respectively (Atreya et al. 2018). It is straightforward to obtain the above relation in terms of the GW damping rate as

$$\frac{\sigma_\chi}{m_\chi} = \frac{6.3 \pi G \langle v \rangle}{\beta}. \quad (8)$$

Figure 1. Fits on the viscosity of dark matter halos with different number of strongly lensed sources in both GW (lensed GWs) and EM domains (lensed SNe Ia and quasars).

\[ \Delta \beta \]

\[ \Delta(\sigma_\chi/m_\chi)(I) \]

\[ \Delta(\sigma_\chi/m_\chi)(II) \]

\[ \Delta \]
and 200 SNe Ia-elliptical galaxy systems with well-measured time-delays (Oguri & Marshall 2010). Following the recent analysis by the TDC (Dobler et al. 2013), the Δt measurements may achieve the precision of ~3% and 1% including systematics. We remark here that the well-characterized spectral sequences of lensed SNe Ia will significantly contribute to the reduction of relative uncertainty from time delay measurements, which has been extensively discussed in the literature (Nugent et al. 2002; Pereira et al. 2013). Meanwhile, the average relative uncertainty of lens modeling with high-resolution imaging will reach to the level of ~3%, due to the inevitable effects of dazzling AGNs in the source center (Suyu et al. 2013). Actually, as can be clearly seen from the results shown in Table I, such combination of strongly lensed SNe Ia or quasars in EM domain will also results in stringent constraints on the viscosity of dark matter halos, similar to the precision obtained from lensed GWs. For a better comparison, Fig. 1 shows the β parameter assessment as a function of sample sizes.

Finally, we need to comment about the technical barriers that could potentially affect precise measurements of DM self-scattering. In our approach based on lensed GW+EM events, a high temporal resolution EM detector is also necessary to monitor the multiple images of the EM event. Qualitatively, one can expect that such issue will be suitably addressed in the future analyses. As was extensively discussed concerning the observational schemes of many on-going and planned GW detectors in the near future (Taylor & Gair 2012), the source location uncertainties of ET will be much smaller than those of the joint detections by Advanced LIGO and Virgo. Spaced based detectors are much more competitive in this respect than ground-based ones. Therefore, considering the strong correlation between gravitational wave and its electromagnetic counterparts (SGRB, etc.), it is surely possible to detect the strongly lensed GW and EM events simultaneously, based on the combination of future GW detectors (both ground-based and space-based detectors) and their follow-up facilities in the EM domain. Summarizing, given the expected wealth of gravitational lensing data in GW and EM domains, we may hopefully obtain precise measurements of the viscosity of dark matter in the early Universe (z ~ 5), which could shed lights on our understanding of dark matter self-interactions at both galaxy and cluster scales (Cao et al. 2011).

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**DATA AVAILABILITY STATEMENTS**

The data underlying this article will be shared on reasonable request to the corresponding author.

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