The dark matter interpretation of the 511 keV line

Céline Bœhm
LAPTH, CNRS, 9 chemin de Bellevue, BP 110, 74941
Annecy-Le-Vieux, France
E-mail: celine.boehm@cern.ch

New Journal of Physics 11 (2009) 105009 (20pp)
Received 16 April 2009
Published 16 October 2009
Online at http://www.njp.org/
doi:10.1088/1367-2630/11/10/105009

Abstract. The 511 keV high-precision map obtained by the INTEGRAL/SPI experiment suggested a surprisingly large amount of positrons in our galaxy with respect to ‘naive’ astrophysical expectations. Although an astrophysical origin is not excluded, this signal may shed light on dark matter. Here we discuss a possible MeV dark matter interpretation of the 511 keV line and point out a possible way of validating the MeV dark matter scenario. In particular, we show that the number of electrons generated by MeV annihilating particles would be large enough to lead to a visible Sunyaev–Zel’dovich (SZ) effect signature in clusters of galaxies if their dark matter halo profile behaves as $\rho(r) \propto 1/r^\gamma$ with $\gamma > 1$ in the inner part (i.e. on distances corresponding to a sub-arcsecond resolution for an experiment dedicated to SZ measurement).
1. Introduction

It is very well known, in both astrophysics and particle physics, that 511 keV photons are a tracer of low-energy electron–positron annihilation, that is—in an astrophysical context—an indication of the presence of anti-matter in the observed region. Yet, such an observation becomes meaningful only if one is able to also measure accurately the associated flux and simultaneously probe that the 511 keV photons do originate from positronium formation. To probe the existence of positronium, one needs to observe the two positronium states: (i) the ortho-positronium that forms with a probability $P = 2/3$ and decays into a gamma ray continuum and (ii) the para-positronium that forms with a probability $P = 1/3$ and decays into two photons, thus producing an emission line at an energy $E = m_e = 511$ keV. If one is able to measure, in addition, the associated flux in the region of observation, it then becomes possible to estimate the number density of low-energy positrons in this region and determine with precision the region of positronium formation. This is precisely what INTEGRAL/SPI [1, 2] has enabled us to achieve for our galaxy.

Although, several experiments had indicated the presence of anti-matter [3]–[11] in the past, there was doubt on the positron spatial distribution. By mapping the 511 keV line with this precision, INTEGRAL/SPI has revealed that most of the low-energy positrons were located in the inner part of our galaxy, in an almost spherical region (centred on the centre of the galaxy) of about ten degrees of diameter.

1 The spatial extension depends on the distribution that is considered to perform the likelihood analysis.
Even though the presence of positrons in the centre of our galaxy is not a big surprise, their spatial distribution and the flux associated with the 511 keV emission appears rather puzzling. Indeed, most of the astrophysical sources that are known as positron emitters are expected to lie in the disc of the galaxy. It is therefore difficult to explain how positrons emitted by such astrophysical sources could be distributed in a spherical halo surrounding the galactic disc. Also, the 511 keV flux appears to be anomalously high (suggesting a large amount of positrons) and is therefore difficult to explain. Explaining the so-called bulk-to-disc ratio (B/D), i.e. why the positrons (and, hence, the 511 keV line emission) are located in a spherical halo instead of being distributed in the disc, turns out to be extremely challenging.

The astrophysical sources that could nevertheless satisfy the B/D constraint are old galactic populations, such as low mass x-ray binaries (LMXB) and type Ia supernovae (SNIa) [2, 12]. Unfortunately, in both cases, one has to make strong hypothesis. LMXB require that the positrons emitted in the disc escape into the bulge, while SNIa need a positron escape fraction and an explosion rate that are large enough for maintaining a steady flux. More generally, it was concluded that eight (or more) point sources could explain the diffuse emission as observed by SPI [2]. However, Weidenspointner et al [13] have not found any evidence of significant emission from point sources in the galactic centre as yet.

In this context, it is worth investigating other possibilities. In particular, in what follows, we shall discuss the possibility that the 511 keV emission originates from dark matter (DM).

2. Origins of positrons in DM models

In the simplest model, both annihilating and decaying DM are meant to produce positrons. The number density of such positrons, however, depend on the DM characteristics (DM mass and annihilation cross section or decay rate). Since DM particles have not been detected, predicting the number of positrons from DM annihilation or decay is unfortunately a speculative game. Nevertheless, it is based on important requirements such as the DM lifetime and/or relic density that has been measured very precisely by Wilkinson Microwave Anisotropy Probe (WMAP) assuming Friedmann–Robertson–Walker (FRW) cosmology [14].

A common feature in all models of massive DM particles, nevertheless, is that the DM number density is proportional to the DM energy density divided by the DM mass \( n_{dm} = \frac{\rho_{dm}}{m_{dm}} \). In other words, the DM number density is fully correlated with the DM halo profile of the galaxy.

Also, if DM has a thermal origin, it is easy to show that its number density must decrease by several orders of magnitude in order to explain the observed DM relic density (and fit the value of the DM cosmological parameter measured by WMAP experiment). A solution to this problem is to assume that DM either annihilates or decays into standard model particles which eventually thermalize with the thermal bath.

Within these assumptions, one expects DM to produce positrons in the early Universe and in any other places where the DM number density is relatively high (like, in particular, the centre of our galaxy). This feature actually provides a very powerful prediction. Indeed, this means that the number density of positrons produced by DM annihilation or decay tracks either the square of the DM number density, that is, \( n_{dm}^2 \) (in the annihilating case) or one power of the number density, that is, \( n_{dm} \), in the decaying case. In other words, if most of the 511 keV line originates from DM, one should observe that the positron number density must follow the DM energy density.
To make predictions and determine whether DM can explain the morphology of the 511 keV line as mapped by INTEGRAL/SPI, we must therefore discuss the shape (spatial and energy distribution) of the DM halo profile.

2.1. DM spatial distribution

It has been commonly accepted by the community from the late 1970s that the anomalous behaviour of flat rotation curves of spiral galaxies could be explained in the presence of a DM component. Although, for the Milky Way, there is a general consensus that this dark material must be distributed in a halo surrounding the galactic disc, the characteristics of the DM spatial and energy density distributions are still under debate.

For example, although halos are meant to be slightly triaxial at formation [15], their final shape may be more spherical due to the many astrophysical processes that take place in a galaxy during its evolution. It was shown, for example, that the inclusion of gas cooling tends to make halos spherical [16]. Yet, this may not be a generic feature as some galaxies do exhibit triaxial properties [17].

In fact, the situation for the Milky Way itself is rather confusing. Observations of the Sagittarius tidal stream have been interpreted as favouring a spherical halo [18, 19], but other analyses indicate that this could also be compatible with either an oblate [20] or prolate [21] DM halo. These triaxiality effects would remain nevertheless quite small.

2.2. DM density halo profile

The DM energy density ($\rho(r)$, with $r$ being the distance to the centre of the galaxy) is also very controversial. It was first described as a power law profile ($\rho(r) \propto r^{-\gamma}$) with an index ranging from 2 to 2.25 [22, 23]. However, work based on numerical N-body simulations favour $\gamma < 2$ at small radii ($r < r_0$) and $\gamma \approx 3$ at large radii. More precisely, typical values of $\gamma$ at small radii range from 1 ([24], hereafter referred to as NFW) to 1.5 ([25] hereafter referred to as M99).

However, the prediction remains unclear since, when baryonic physics is added, one obtains different kinds of profiles: adiabatic contraction effects of the DM component are due to the presence of baryons [26] and the existence of a black hole at the centre of the galaxy [27] tends to increase the central DM energy density, leading to a very steep profile in the innermost regions.

Despite these discrepancies, all studies nevertheless agree that the DM halo extends further the visible matter. They, in addition, demonstrate that halos of both galaxies and cluster of galaxies share some universal properties. More specifically, assuming spherical halos, it was shown that they could be well described by a function

$$\rho(r) = \frac{\rho_0}{(r/r_0)^\gamma \left[1 + (r/r_0)\right]^{(\beta - \gamma)/\alpha}},$$

where $\rho_0$ and $r_0$ are the characteristic energy density and radius of the halo and $\alpha, \beta, \gamma$ are the required parameters to reproduce the properties of the object under study.

In principle, observations should solve the theoretical discrepancies regarding the exact value of $\alpha, \beta, \gamma, r_0$ and $\rho_0$. However, in some cases, they are controversial themselves. For example, based on the measurement of the micro lensing optical depth towards the galactic centre by the MACHO experiment [28], Binney and Evans [29] concluded that the Milky Way...
halo profile had a core ($\gamma \sim 0.3$). With the new measurement, one obtains a steeper profile, although this is actually debated [30].

Since the characterization of the inner slope of the Milky Way halo profile appears to be difficult, it may be instructive to look at other spiral galaxies than the Milky Way. Surprisingly enough, such observations seem to rather favour core profiles [31], thus conflicting with theoretical expectations of the shape of our DM halo.

Observations of dwarf and low surface brightness galaxies also contribute to the controversy [32]–[34]. Analyses of these objects show that steep (NFW-like) profiles may be consistent with observational data once the effects of inclination, non-circular orbits and triaxiality of the DM halos are taken into account [35]. However, other studies obtain different conclusions [36, 37].

Hence, although the overall shape of our DM halo is known, the inner slope remains to be determined. The general consensus is that the DM halo profile is well approximated by a power law. However, this could be questioned again. For example, recently, analysis showed that the DM halo profile of the Milky Way could be divided into two parts: an inner and an outer profiles. As we will see, this aspect (and the detailed characteristics of our DM halo) is of crucial importance when one investigates the possibility that DM is at the origin of the 511 keV line.

3. Predictions

From now on, and given the discussion above, we shall assume that the DM halo profile is well approximated by a power law $\rho(r) \propto 1/r^\gamma$ at small radii $r < r_0$. Within this approximation, one naïvely expects the DM (and, hence, the positron) number density to be the largest near the galactic centre.

As a result, in both annihilating and decaying cases, positrons originating from DM annihilation or decay should be localized (if one neglects their propagation) in the inner galaxy (depending on the exact detail of the DM halo profile). The positrons, produced in the region where there are already many (galactic) electrons, can then form positronium atoms with the electrons and produce a 511 keV line.

Also, neglecting the possible triaxial effects (and positron propagation issue), one expects the positrons to be distributed (at least at production) in a sphere surrounding the galactic disc, accordingly to the spherical shape of the Milky Way profile.

Hence, naïvely, one expects that the 511 keV emission line resulting from all the above assumptions are very bright in a spherical region that basically corresponds to the galactic centre (and more precisely to the ‘stellar’ bulge where there are many galactic electrons) and fainter outside.

However, as mentioned previously, we made several assumptions. First of all, we neglected low-energy positron propagation. This is a difficult problem that has been discussed several times in the literature. Unfortunately, contradictory results have been published, hence this is still a debated issue (e.g. [38]–[40]). Also, due to the aperture mask on SPI, one has to make sure that the map of positrons produced by DM annihilation or decay does indeed reproduce the map of 511 keV photons that has been observed by SPI. Hence, a likelihood analysis that takes into account SPI characteristics and a proper modelization of the background (at 511 keV energies) is necessary.
Table 1. Maximum log-likelihood ratio (MLR) and total flux (normalized to $10^{-3}$ cm$^{-2}$ s$^{-1}$) for decaying DM with decaying rate $\Gamma_d$ (normalized to $10^{-20}$ s$^{-1}$), annihilating DM with velocity-independent cross section $\sigma_v \simeq a$, velocity-dependent cross section $\sigma_v \simeq b v^2$ (both normalized to $10^{-26}$ cm$^3$ s$^{-1}$), and for various DM halo profiles: ISO (isothermal), BE (Binney–Evans), NFW (Navarro–Frenk–White) and M99 (Moore). The best fit to SPI data (morphology of the 511 keV line) corresponds to a model with velocity-independent cross section and an NFW Milky Way DM halo profile. The value of the annihilation cross section is then derived by comparing the flux thus obtained with observation.

| Profile | Total flux | Inner flux | MLR |
|---------|------------|------------|-----|
| $\Gamma_d$ |           |            |     |
| ISO     | 6.82 ± 0.58 | 2.95 ± 0.25 | 135.2 |
| BE      | 7.23 ± 0.57 | 3.18 ± 0.25 | 167.3 |
| NFW     | 77.36 ± 0.46 | 3.53 ± 0.22 | 261.2 |
| M99     | 6.86 ± 0.37 | 3.48 ± 0.19 | 332.0 |
| $\sigma_v \simeq a$ | | | |
| ISO     | 5.55 ± 0.33 | 3.40 ± 0.20 | 282.8 |
| BE      | 4.98 ± 0.27 | 3.16 ± 0.17 | 353.6 |
| NFW     | 2.49 ± 0.11 | 1.95 ± 0.09 | 459.9 |
| M99     | 0.83 ± 0.04 | 0.76 ± 0.04 | 339.2 |
| $\sigma_v \simeq b v^2$ | | | |
| ISO     | 6.00 ± 0.38 | 3.46 ± 0.22 | 258.3 |
| BE      | 5.76 ± 0.32 | 3.40 ± 0.19 | 305.7 |
| NFW     | 3.61 ± 0.18 | 2.57 ± 0.13 | 422.4 |
| M99     | 1.57 ± 0.07 | 1.32 ± 0.06 | 430.0 |

After analysis (see table 1), it was found that, with the first year SPI data, decaying DM is totally ruled out. Annihilating particles do fit the observations but the DM halo profile must correspond to an NFW profile with $\gamma \sim 1$. This is actually a strong prediction which, in fact, seems to be in disagreement with faraway galaxies. Yet, it is difficult to conclude whether this observation excludes the DM solution since the astrophysical processes that may have shaped the DM halo profile in our galaxy could differ from that in other galaxies.

Another important conclusion drawn from the likelihood analysis is that the cross section should be mostly independent of the DM velocity (it should be an a-term). This excludes $Z'$ exchange cross section as the main origin of the 511 keV line but also forbids fermionic DM candidates (they would need to exchange charged particles lighter than 100 GeV. However, such particles have been excluded by the LEP and TEV ATRON constraints). In contrast, this favours scalar DM particles for which the annihilation cross section is expected to be velocity-independent and equal to

$$\sigma_{v_{511}} \simeq 2 \times 10^{-30} \text{ cm}^3 \text{ s}^{-1} m_{\text{dm}}^2 \text{ MeV}^{-2},$$

assuming interaction properties that are compatible with LEP and TEV ATRON analysis.
4. Models

Reference [41] was the first paper to relate an existing model with SPI observation. However, after this paper, there has been a lot of papers proposing new particle physics models to explain the 511 keV line emission, see for example [42]–[45]. Some are based on decaying particles, others on annihilating particles. However, most of the models are excluded by the results displayed in table 1.

4.1. Light annihilating particles

Thermal annihilating DM particles are often assumed to be very heavy [46], with a mass between 1 GeV and 1 TeV. This is because the relic density argument, based on the Boltzmann equation, indicates that the DM (whatever its nature or mass is) can represent about a fourth of the critical energy density of the Universe if its annihilation cross section is about $10^{-26}$–$10^{-27}$ cm$^3$ s$^{-1}$. In a particle physics model (such as massive neutrinos), where the annihilation cross section is given by

$$\sigma v \sim \frac{m_{dm}^2}{m_W^4},$$

this translates into a lower limit on the DM mass of about 1 GeV called the Hut–Lee–Weinberg limit.

This lower limit is actually compatible with the results obtained in supersymmetry (more precisely in the Minimal Supersymmetric Standard Model (MSSM), if one considers the lightest neutralino) as well as in Kaluza–Klein theories.

However, if one relaxes the assumption that DM is made of fermionic particles with electroweak interactions and assumes, instead, scalar DM particles (or even fermionic DM particles coupled to a new light weakly coupled gauge boson $Z'$), one obtains that the relic density argument enables DM particles as light as a few MeV (or even lighter, in fact, in some cases).

This is particularly true if the DM candidate is actually a scalar particle non-chirally coupled to standard model fermions via the exchange of a heavy fermion $F$. Such particles were introduced in 2002 [47] to illustrate a new damping effect found in [48] (namely the generalization to the Silk damping in the case of non-vanishing DM–neutrino couplings).

In [49], it was shown that the model that gives the best fit to SPI data is a scalar DM candidate coupled to both a $Z'$ gauge boson (weakly interacting with couplings to electron of about $z_e = 7 \times 10^{-5}$ (m$_{Z'}$/MeV) $\sqrt{(\delta a_e/10^{-11})}$) and a heavy fermion. The $Z'$ exchange enables one to obtain the correct relic density, while the $F$ exchange ensures the right 511 keV flux. Nonetheless, there are alternative scenarios where the relic density criterion is ensured via scalar annihilation into Majorana neutrinos. Also, there are scenarios in which DM annihilates via the exchange of a Higgs [43].

4.2. Light decaying DM

Decaying particles (e.g. [50]–[54]) have also been proposed subsequently to the INTEGRAL/SPI map. Yet, according to the SPI likelihood analysis, they do not provide a good fit to the data.
Scenarios in which DM both annihilates and decays have been also investigated in [55]. For some parameters, the DM mass can be as small as a few MeV, which enables one to establish a connection with the 511 keV line. The main limitations of these scenarios come from the relic density argument and gamma ray constraints.

Also, WIMPs collisional (potentially long-lived) excited states have been proposed as possible positron emitters (see e.g. [42, 56, 57]). These particles would decay into a lighter particle, corresponding to the DM ground state, and pair-produce simultaneously electrons and positrons. This scenario requires that the excited and ground states are almost mass degenerated (the mass difference should be at most a few MeV). This scenario may also explain other observables such as PAMELA positron excess [58].

4.3. Other types of particle physics candidates

In principle, one cannot exclude the possibility that, in our Universe and even in our galaxy, there is a small fraction of exotic objects such as topological defects (e.g. cosmic strings) or Q-balls.

Superconducting strings could emit particles (including electrons and positrons) depending on their interactions with the plasma in the Milky Way while at the same time having a negligible energy density (and therefore not contributing to the DM in our Milky Way and in the Universe). Such a scenario has been proposed to explain the 511 keV line in [45, 59]. As shown in these references, the details of the positron emission depend on the characteristics of the strings and on the galactic magnetic field.

Another possible scenario to explain the 511 keV line is the presence of supersymmetric Q-balls [60]. These objects may have a long enough lifetime and yet a small energy density that might enable them to be present in our galaxy and to explain the 511 keV line. Q-balls can be depicted as a stable localized field configuration, whose stability is guaranteed by a conserved charge $Q$ associated with a $U(1)$ symmetry. For example $Q$ could be the electric charge.

Some authors also introduce superconducting DM [61] or compact composite objects [62]. These macroscopic objects are meant to form during the QCD phase transition and could be schematically depicted as ‘quark’ balls. They would introduce a link between the DM and baryonic energy densities nowadays and eventually explain why these two quantities are about the same order of magnitude. However, the ability of such a scenario to explain the 511 keV line faces the criticisms displayed in [63].

Other authors introduced MeV milli-charged particles [64]. The claim is that particles with such a mass could have the correct relic density and the required characteristics to explain the 511 keV line, while not being excluded by particle physics experiments as yet.

Finally, authors proposed the existence of decaying particles that do not contribute significantly to the DM energy density today [44, 60]. Particles with a lifetime comparable to the age of the Universe and with a small energy density could indeed emit electrons and positrons at a significant rate.

5. Constraints and possible ways to detect MeV DM signatures

The constraint of an NFW profile, obtained in [49], however excludes several candidates in this list if DM is the main source of low energy positrons but there are several important constraints on light DM scenarios, that generally restrict the DM mass that can be considered.
Below are summarized the constraints obtained by assuming that MeV particles were coupled to electrons and, more precisely, at the origin of the 511 keV emission line.

5.1. Final state radiation/inner Bremsstrahlung

Assuming that all positrons and electrons would eventually produce one photon (and neglecting energy losses), Boehm et al [65] obtained that, to avoid an unwanted gamma ray continuum production, the electron–positron production cross section in our galaxy should be smaller than

\[
\sigma v_{\nu e^+e^-} \lesssim 10^{-30} - 10^{-31} m_{dm}^2 \text{ MeV}^{-2} \text{ cm}^3 \text{ s}^{-1},
\]

which is about five orders of magnitude less than the relic density cross section (\(\sigma v_{\text{relic}}\)). Together with the value of the annihilation cross section obtained from [49] to fit the 511 keV line (namely \(\sigma v_{\nu e^+e^-} \sim 2 \times 10^{-30} \text{ cm}^3 \text{ s}^{-1}\)), the relic density and 511 keV line cannot originate from the same particle physics process. This also shows that DM should be relatively light.

However, using an approximate expression of the final state cross section (called, in this case, inner Bremsstrahlung), Beacom et al [66] carried the argument further and obtained an upper limit of 20 MeV on the DM mass. This upper limit was found nevertheless to be 30 MeV in [67] after proper computation of the radiative cross section and by using astrophysical measurements of the gamma ray flux.

5.2. Inflight annihilation

Inflight annihilations actually set the most stringent astrophysical limit on the DM mass (at least if one assumes that DM is the single source of the 511 keV photons).

Beacom and Yuksel [68] thus obtained that DM should be lighter than 3 MeV. However, in [69], the same argument with different assumptions lead to an upper lower limit of 7 MeV. Perhaps a fair estimate is that DM should definitely be lighter than 10 MeV to avoid producing too energetic electrons and positrons while, simultaneously, explaining all the 511 keV line emissions. This is one of the most stringent constraints on this type of scenario.

5.3. Anomalous magnetic moment

MeV annihilating particles give a new contribution to the electron and muon anomalous magnetic moment (the so-called electron and muon g − 2) [47]. By requiring that this contribution is not too large, Ascasibar et al [49] and Boehm and Ascasibar [70] found that the DM mass should not exceed 7 MeV.

5.4. Monochromatic line

A standard signature of DM annihilation is the existence of a monochromatic line at an energy \(E = m_{dm}\). Indeed, if DM pair annihilates into two photons (via box or triangle diagrams) in our galaxy for example, kinematics argument impose that the energy of the photons thus emitted is equal to the DM energy. Since DM is meant to be almost at rest in large-scale structures, the photons thus emitted have an energy that is basically equal to the DM mass. This provides a unique signature of the DM annihilation.
Such a signature has been investigated for MeV particles [71]. Unfortunately, their pair annihilation cross section into two photons is very suppressed. It is about
\[ \sigma v \lesssim 10^{-5} \times \sigma_{511} \ m_{\text{DM}}^{-1} \text{MeV}. \]

This is too small to be detected in future (dedicated) experiments unless, perhaps, one points toward dwarf galaxies where the background is very suppressed. Indeed, in our galaxy and for small DM masses of a few MeV, one could only reach a large significance (\( \Sigma = \text{signal}/\sqrt{\text{background}} \approx 10-20 \)) by using an ideal detector of 1 m\(^2\) and a 10\(^{-3}\) energy resolution.

### 5.5. Sunyaev–Zel’dovich (SZ) effect

During the last three decades, there have been many attempts to use astrophysical observations for constraining various DM candidates and, more generally, to eliminate particle physics scenarios beyond the standard model. Among them, there is a claim by Colafrancesco in 2004 that neutralinos with a total annihilation cross section (times relative velocity of 10\(^{-26}\) cm\(^3\) s\(^{-1}\)) should be heavier than 10 GeV so as not to generate a visible SZ effect.

The relativistic SZ (RSZ) effect induced by DM is based on the production of relativistic electrons in clusters of galaxies by the DM. By interacting with the cosmic microwave background (CMB) photons that travel through the cluster, these hot electrons distort the primordial blackbody spectrum. The importance of the deviation depends mostly on the number density of hot electrons injected in the cluster. Since the latter depends on the DM energy distribution \( \rho_{\text{DM}} \) (which actually varies with the distance to the centre), one expects that the observable signal depends on the resolution of CMB experiments.

In the subsections below, we clarify the formalism to compute the SZ effect generated by DM and give (original) analytical expressions to estimate the magnitude of the effect. Finally, we demonstrate that the signal that is expected is basically impossible to detect, unless one finds a cluster with a DM halo profile that is much steeper than that observed in clusters so far.

#### 5.5.1. Blackbody spectrum deviation in the presence of relativistic electrons.

In [72], it was demonstrated that the RSZ effect can be written as
\[ \Delta I_\nu(E_k) = - \int \frac{d^3\tilde{p}}{(2\pi)^3} f_\nu(E_p, \bar{x}) \int d\Omega_{\nu'} \left( 4v_{\text{rel}} \right) \frac{d\sigma}{d\Omega_{\nu'}} \times \left\{ I_\nu^0(E_k) - \frac{I_\nu^0(t E_k)}{t^3} \right\}, \]

where \( v_{\text{rel}} \equiv (1 - \beta \mu) \) is the relative velocity between the incoming particles, \( \mu \) is the cosine of the angle between \( \tilde{p} \) and \( \tilde{k} \), \( \beta \) is the velocity in units of \( c \) of the electron of energy \( E_p \), and \( d\sigma/d\Omega_{\nu'} \) is the differential Compton cross section for an outgoing photon within a solid angle \( d\Omega_{\nu'} \). The term \( I_\nu^0(E_k) \) is the (conventional) blackbody spectrum at an energy \( E = E_k \), and \( t = E_k'/E_k \) is the shift in energy.

For shortening our notation, we denote by \( G(p, E_k) \) the integral
\[ G(p, E_k) = \int d\Omega_{\nu'} \left( 4v_{\text{rel}} \right) \frac{d\sigma}{d\Omega_{\nu'}} \left\{ I_\nu^0(E_k) - \frac{I_\nu^0(t E_k)}{t^3} \right\}. \]

The latter can be written as
\[ G(p, E_k) = \sigma_T \ G_0(p, E_k), \]

where \( \sigma_T \) is the Thomson scattering cross section.

*New Journal of Physics* 11 (2009) 105009 (http://www.njp.org/)
Since an experiment has a finite angular resolution, the relevant quantity to use when making predictions is the average value of $\Delta I_{\gamma}(E_k)$ over the solid angle $\Delta \Omega_{\text{res}}$ carried by the experimental resolution, that is

$$\Delta I_{\gamma}^{\text{obs}}(E_k) = \frac{1}{4\pi} \int d\Omega_{\text{res}} \Delta I_{\gamma}(E_k),$$

(1)

where the dependence in $\Omega_{\text{res}}$ of the intensity is hidden in the spatial dependence of the electron population described by $f_x(p, \vec{x})$. More precisely, if the electron density is spherical in the cluster, then it only depends on the distance $r = |\vec{x} - \vec{x}_{cc}|$ to the cluster centre. This radius $r$ can be related to the angle $\psi_{\text{res}}$ scanning the resolution range through the relation

$$r = \sqrt{l^2 + d^2 - 2dl \cos \psi_{\text{res}},}$$

where $d$ is the distance of the observer to the cluster centre and $l$ denotes the line of sight.

We now have to specify the electron momentum distribution $f_e(p)$. This requires determining the number density of electrons (and positrons) produced by DM annihilation or decay.

5.5.2. Electron distribution. Neglecting convection and reacceleration processes, the transport equation becomes

$$\frac{\partial}{\partial t}(b(E) N(E)) = Q(E, r),$$

(2)

where $b(E)$ are the electron losses and $Q(E, r)$ represents the source of the relativistic electrons, that is—in the present case—the DM annihilations or decays. This source term can therefore be written as

$$Q(E, r) = Q^{(n)}_{\text{dm}} n_{\text{dm}}^{n}(r) X(E),$$

where $Q^{(n=1)}_{\text{dm}} = \Gamma_{\text{dm}}$ is the decaying rate corresponding to the decaying DM, $Q^{(n=2)}_{\text{dm}} = \sigma v$ is the annihilation cross section times the relative velocity (in the centre of mass frame) of annihilating particles and where the function $X(E)$, which defines the energy dependence of the source term, can be written as

$$X^{n}(E) = \delta(E - m_{\text{dm}}),$$

if the electrons are produced directly by the DM annihilations/decay and

$$X^{n}(E) = BR(e^+e^-) \left( \frac{E}{E_0} \right)^{-m} \Theta(m_{\text{dm}} - E)$$

or

$$X^{n}(E) = BR(e^+e^-) \left( \frac{E}{E_0} \right)^{-m} e^{-aE}$$

(with $BR(e^+e^-)$ being the branching ratio into electrons).

Note that we assume that $Q^{(n)}_{\text{dm}}$ has no dependence in energy (which is, in principle, true since the DM is meant to be at rest in the halo). Writing this energy density as

$$\rho_{\text{dm}}(r) = \rho_{0(\gamma)} g_{\text{dm}}(r),$$

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with $\rho_0(\gamma)$ a normalization and

$$g_{\text{dm}}(r) = \frac{1}{(r/r_0)^\alpha \left(1 + (r/r_0)^\alpha\right)^{(\beta-\gamma)/\alpha}}$$

the DM halo profile (written as a universal function with $\gamma = 0, 1, 3/2, 2$ for, respectively, a cored, NFW, Moore, Jaffe DM halo profile), we obtain that

$$Q(E, r) = Q^{(n)}_{\text{dm}} \left[\frac{\rho_0(\gamma)}{m_{\text{dm}}}\right]^n [g_{\text{dm}}(r)]^n \chi(E),$$

leading to

$$N(E, r) = Q^{(n)}_{\text{dm}} \left[\frac{\rho_0(\gamma)}{m_{\text{dm}}}\right]^n [g_{\text{dm}}(r)]^n \frac{b(E)}{E} \int E' \chi^{\gamma, p}(E) \, dE'.$$

Using the definition $n_e = \int d^3 p f_e(p)$, we finally find the electron energy distribution

$$f_e(p) = \frac{1}{pE} N(E, r) = \frac{Q^{(n)}_{\text{dm}}}{pE} \left[\frac{\rho_0(\gamma)}{m_{\text{dm}}}\right]^n [g_{\text{dm}}(r)]^n \frac{b(E)}{E} \int E' \chi^{\gamma, p}(E).$$

The inverse Compton, synchrotron, Coulomb energy losses ($b(e) = dE/dt$) are given in many references, including [73] (where one can use the relations $\gamma = \frac{E}{m_e c^2}$ and $b(E) = b(\gamma) \times (m_e c^2)$). The minimal energy that can be reached by the electrons and positrons after losses can be obtained by comparing the energy loss timescale with the age of the cluster.

We can now compute the expected deviation to the blackbody spectrum induced by relativistic electrons:

$$\Delta I^\text{obs}_\gamma(E_k) = -Q^{(n)}_{\text{dm}} \left[\frac{\rho_0(\gamma)}{m_{\text{dm}}}\right]^n \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{\sigma_T}{b_0} \times f(E) g_0(p, E_k) \int E' \chi^{\gamma, p}(E').$$

where we assume for convenience that the losses can be well approximated by a constant $b_0$ times a function of the energy $f(E)$.

If we denote by

$$\mathcal{D}_{n\gamma} = \sigma_T \times Q^{(n)}_{\text{dm}} \left[\frac{\rho_0(\gamma)}{m_{\text{dm}}}\right]^n,$$

the quantity that only depends on the DM properties, by

$$O_{n\gamma}(b_{\text{res}}) = \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{\sigma_T}{b_0} \times f(E) g_0(p, E_k) \int E' \chi^{\gamma, p}(E'),$$
the integral of the DM halo profile along the line of sight and the experimental angular resolution and, finally, by
\[ J^{\text{norm}}(E_k) = \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{1}{p E} \frac{1}{f(E)} \times G_0(p, E_k) \times \int E' \alpha^{*}\rho(E), \]
the deviation to the blackbody spectrum that results from inverse Compton interactions, we can rewrite the above expression of equation (4) in a more compact form as
\[ \Delta I^{\text{obs}}_\gamma(E_k) = - \frac{\sigma_T}{b_0} \times \frac{d_{ny} \times O_{ny}(b_{\text{res}})}{\Delta \psi_{\text{res}}} \times J^{\text{norm}}(E_k) \]
where the DM, halo, electron spectrum properties are clearly separated and \( \tau_{ny} \) represents the optical depth. It is now very easy to estimate the RSZ effect for any type of candidate.

5.5.3. Computing \( D_{ny} \) and \( O_{ny}(b_{\text{res}}) \). Using the above expression for \( D_{ny} \), one obtains
\[ D_{ny} = 10^{-26} \text{ cm}^{-3} \text{ s}^{-1} \times \left( \frac{Q_{\text{dm}}^{(n)}}{Q_{0}^{(n)}} \right) \times \left( \frac{\rho_0(\gamma)}{\text{GeV/cm}^3} \right)^n \times \left( \frac{m_{\text{dm}}}{\text{GeV}} \right)^{-n}, \]
where \( Q_{0}^{(n)} \) is a normalization equal to \( Q_{0}^{(2)} = 10^{-26} \text{ cm}^{-3} \text{ s}^{-1} \) and \( Q_{0}^{(1)} = 10^{-26} \text{ s}^{-1} \).

The integration of the profile along the line of sight is problematic when \( \gamma \gtrsim 1 \) since the DM energy density is divergent toward the centre. Numerical integration of such profiles can be found in DarkSUSY or MicrOmega codes. However, in this section, we provide analytical expressions of the line-of-sight integration so that one can quickly estimate the effect that is expected for each profile, depending on whether the DM is annihilating or decaying.

Unlike numerical calculations where it is convenient to express the radius \( r \) as a function of the line of sight, namely
\[ r = \sqrt{l^2 + d^2 - 2dl \cos \psi}, \]
here we shall use the relation
\[ l_{\pm} = d \cos \psi \pm \sqrt{r^2 - d^2 \sin^2 \psi}, \]
where \( a = \sqrt{r^2 - d^2 \sin^2 \psi} \) and \( l_+ = l_- + 2a \). Since the profiles are divergent at small radii and the inner profile can be well approximated by a function \( g_{\text{dm}}(r) = (r_0 / r)\gamma \) for \( r < r_0 \), the line-of-sight integration can be rewritten as
\[ \int dl [g_{\text{dm}}(r)]^n = \int_b^{r_m} dr \left( r_{\text{yd}}^{-1} \right) \frac{1}{\sqrt{r^2 - b^2}}, \]
where \( r_m < r_0 \) and \( b = d \sin \psi \). Finally using \( v^2 = (r^2 - b^2) / b^2 \), we find that
\[ \int dl [g_{\text{dm}}(r)]^n = \left( \frac{r_0}{b} \right)^{\gamma} b \int_0^{\sqrt{(r_m^2 - b^2)/b}} \frac{dv}{(1 + v^2)^{\gamma/2}} \]
\[ = \left( \frac{r_0}{b} \right)^{\gamma} b \left[ I_{\gamma/2}(v) \right]_0^{\sqrt{(r_m^2 - b^2)/b}}. \]
The specific values of the product $n \gamma/2$ are summarized in the following table:

| Nature versus profile | NFW | Moore | Jaffe |
|-----------------------|-----|-------|-------|
| Decaying              | $I_{1/2}$ | $I_{3/4}$ | $I_1$ |
| Annihilating          | $I_1$ | $I_{3/2}$ | $I_2$ |

where, we do not consider the case of cored profile since the integration is trivial and the expressions for $I_{\infty}$ are given below.

For an experiment pointing towards the cluster centre with a resolution $\psi_{\text{res}}$ (assuming that $\sin \psi \simeq \psi$), the average integrated profile along the line of sight and the angular resolution $O_{n\gamma}$ is given by

$$O_{n\gamma}(b_{\text{res}}) = \frac{r_{0}^{n\gamma}}{d^2} \int_{0}^{b_{\text{res}}} db \ b^{2-n\gamma} \left( I_{n\gamma/2} \right)_{0} \sqrt{r_{m}^{2}-b^{2}}/b,$$

where the product $n \gamma$ (with $n = 1$ corresponding to the decaying DM, $n = 2$ to the annihilating DM and $\gamma = 0, 1, 3/2$ and 2 describes the slope of the inner DM halo profile) ranges from 0 to 4 (for the profiles considered here):

| Nature versus profile | Core | NFW | Moore | Jaffe |
|-----------------------|------|-----|-------|-------|
| Decaying              | $O_0$ | $O_1$ | $O_{3/2}$ | $O_2$ |
| Annihilating          | $O_0$ | $O_2$ | $O_3$ | $O_4$ |

The expressions of $O_{(0,1,3/2,2,3,4)}$ are given below.

Note that, in principle, one should also consider the case where experiments are pointing off centre but the integration can be easily done numerically in this case since there is no divergence.

Computing at first the integral along the line-of-sight $I_{n\gamma/2} = \int du (1 + u^2)^{-n\gamma/2}$, we obtain the following for the different profiles ($\gamma = 1, 3/2, 2$) and different types of DM:

$$I_{1/2} = \ln \left( \frac{r_{m} + \sqrt{r_{m}^{2}-b^{2}}}{b} \right),$$

$$I_{3/4} = \sqrt{2} F \left( \arccos \left( \frac{b}{r_{m}} \sqrt{\frac{1}{2}} \right) \right),$$

$$I_1 = \arctan \left( \frac{\sqrt{r_{m}^{2}-b^{2}}}{b} \right),$$

$$I_2 = \frac{1}{2} \left[ \frac{\sqrt{r_{m}^{2}-b^{2}}}{b} + \arctan \left( \frac{\sqrt{r_{m}^{2}-b^{2}}}{b} \right) \right].$$
Averaging now over the angular resolution, we finally obtain the expressions for the different \( O_{ny}(b_{res}) \):

\[
O_1 \simeq \frac{r_0}{2d^2} \left[ b^2 \ln \left( \frac{r_m + \sqrt{r_m^2 - b^2}}{b} \right) - r_m \sqrt{r_m^2 - b^2} \right]^{b_{res}}_\varepsilon ,
\]

\[
O_2 = \frac{r_0^2}{d^2} \left[ b \arctan \left( \frac{\sqrt{r_m^2 - b^2}}{r_m} \right) - \sqrt{r_m^2 - b^2} \right]^{b_{res}}_\varepsilon ,
\]

\[
O_3 = \frac{r_0^3}{d^2} \left[ \frac{\sqrt{r_m^2 - b^2}}{r_m} + \ln \left( \frac{b}{r_m(r_m + \sqrt{r_m^2 - b^2})} \right) \right]^{b_{res}}_\varepsilon ,
\]

\[
O_4 = \frac{r_0^4}{d^2} \left[ r_m \ln \left( \frac{b}{r_m(r_m + \sqrt{r_m^2 - b^2})} \right) + b \arctan \left( \frac{\sqrt{r_m^2 - b^2}}{b} \right) \right]^{b_{res}}_\varepsilon ,
\]

where \( \varepsilon \to 0 \) describes the ability for an experiment to point exactly at the centre. Our calculations actually assume that the slope of the profile remains exactly the same at any \( \varepsilon < r < r_0 \). If the profile is cored at a distance \( r_c \) from the centre, \( \varepsilon \) then should be equal to \( r_c \). Here, we do not take into account the effect of galaxies inside the cluster DM halo. However, for experiments with very good resolution, this may represent a possible way of constraining the DM as one should see SZ point sources on top of the average signal from the cluster halo.

The function \( \mathcal{J}^{\text{norm}}(E_k) \) was given in [74, 75]. It does not fix the magnitude of the effect but gives the shape of the distortion. To estimate the magnitude of the distortion for different DM candidates and profiles, one therefore has to estimate the product \( D_{ny} O_{ny}(b_{res})/\Delta \psi_{res} \).

### 5.5.4. Expected distortion.

In the previous subsections, we saw how to compute the optical depth that is basically the magnitude of the distortion.

To give numerical estimates, we shall now consider four specific cases, namely: (i) decaying DM with an NFW \((n \times \gamma = 1)\); (ii) annihilating DM and an NFW \((n \times \gamma = 2)\) that is basically equivalent to the decaying DM with a very spiky (Jaffe) profile \((n \times \gamma = 2)\), (iii) annihilating DM with a Moore profile \((n \times \gamma = 3)\) and (iv) annihilating DM with a Jaffe profile.

We find:

\[
\tau_{\gamma n=2} \simeq 8 \times 10^{-19} \left( \frac{m_{dm}}{\text{GeV}} \right)^{-n} \left( \frac{b_0}{10^{-16} \text{s}^{-1}} \right)^{-1} \left( \frac{\rho_0}{10^{-3} \text{GeV cm}^{-3}} \right)^{n} \left( \frac{Q_{dm}}{10^{-26}} \right)^n \frac{O_{ny}/\text{kpc}}{\Delta \psi_{res}}.
\]

The \( O_{ny}/\Delta \psi_{res} \) are plotted in figure 1 for a resolution in between 60 and 0.1 arcsec (which is the range that is expected to be covered by experiments such as PLANCK [76]—for arcminute resolution—and ALMA—for arcsecond resolution—[77]).

One can now estimate the SZ effect from light annihilating DM particles. For MeV particles, the mass term \( (m_{dm}^{-2}) \) increases the signal by six orders of magnitude. However, as we saw, the annihilation cross section into electron and positron is suppressed by four orders of magnitudes with respect to the standard value. Hence, overall, the SZ signal expected with light DM is only 100 times larger than that from very heavy particles. In addition,
the number of $e^+e^-$ generated by light DM annihilation is only equal to two since, unlike heavy DM particles, annihilations into quarks or heavy vector boson are impossible in this scenario.

Results are plotted in figure 2 assuming $\rho_0 = 0.01$ GeV cm$^{-3}$ and losses of $b_0 = 10^{-17}$ s$^{-1}$.

As one can see, even for arcsec resolution, annihilating MeV particles do not lead to any SZ effect signature unless the DM halo profile behaves as $\rho(r) \propto 1/r^\gamma$ with $\gamma > 1$ in the very inner part of the cluster, which seems unlikely. However, if a dark mater halo profile in a cluster is found to be as spiky as a Moore profile or a Jaffe profile in the very inner part (corresponding to sub-arcsecond resolution), then one could constrain the MeV DM scenario by using the SZ effect. Indeed, in this case, there should be a visible displacement of the minimum in the function $G(p, E_k)$ [74, 78, 79].

This may become of interest if the presence of a non-thermal SZ effect, as discussed in [80], was confirmed and if the SZ effect measurement continues to be in conflict with x-ray data [81].

6. Experimental tests for MeV particles

As discussed in the appendix of [47], MeV particles are compatible with particle physics constraints but the question of their detectability is of crucial importance. In [47], it was noticed that ASP (an experiment at PEP which was triggering single photon events with the aim of determining the number of neutrinos) could in principle constrain the $e^+e^- \rightarrow Z'\gamma$ process (with $Z'$ decaying into two DM particles) [47]. PEP luminosity being insufficient, the authors concluded that the light DM scenario is in fact compatible with the constraints obtained from past experiments.

Figure 1. $(\Theta_{ny}/kpc)/\Delta\psi_{res}$. 

New Journal of Physics 11 (2009) 105009 (http://www.njp.org/)
Figure 2. Optical depth expected for decaying and annihilating particles for MeV particle with $\sigma v = 2 \times 10^{-30} \text{ cm}^3 \text{s}^{-1}$ and $\Gamma = 2 \times 10^{-26}$ and for various profiles. We consider a cluster at 50 Mpc, with $r_0 = 200 \text{kpc}$ and a mean density of $10^{-2} \text{ GeV cm}^{-3}$ and losses equal to $b_0 = 10^{-17} \text{s}^{-1}$. As one can see, unless the DM halo profile is very spiky ($\gamma > 1$), the optical depth is extremely suppressed above an arcsecond resolution and it will be extremely difficult to detect an SZ signal from the MeV DM.

Nevertheless, as was pointed out in [82], present low-energy experiments (such as BaBar, BELLE in B-factories and a DAΦNE at Φ-factory) have the required luminosity to test the existence of a new light gauge boson decaying into invisible particles. In particular, they noticed that, unlike the signal (characterized by the $e^+e^- \rightarrow Z'\gamma$ process), the number of background events (dominated by the $e^+e^- \rightarrow e^+e^-\gamma$ contributions) was very suppressed if one performs an angular cut corresponding to the condition that the final state particles must not go to the beam pipe. They therefore concluded that it may be possible to detect light DM particles using present low-energy particle physics experiments. The author of [83] came to the same conclusion by considering charm factories. Since these papers, there are ongoing efforts to search for such DM particles in the data already accumulated by low-energy experiments. In some cases, this meant, in fact, adding a single-photon event trigger.

Another signature of light DM is expected in an electron anomalous magnetic moment. Indeed, if MeV particles are at the origin of the 511 keV line, the electron $g - 2$ is directly proportional to the pair DM annihilation cross section into electron–positron. Thus, by using the value of the cross section that fits SPI data, one can predict the value of the deviation in the measurement of the electron $g - 2$ with respect to the standard model. This contribution is about

$$\delta a_e = 5 \times 10^{-12} m_{dm} \text{ MeV}^{-1}.$$
This deviation actually translates into a deviation in the measurement of the fine structure constant (with respect to the standard model expectation) [49, 70]. Surprisingly enough, this deviation is large enough [84] to be measured by the ongoing experiments [85]. Hence, the present electron $g - 2$ experiment in Harvard should be able to rule out MeV DM as a possible source of 511 keV photons very soon.

Signatures of light particles could also show up in rare pion/meson decays, e.g. [86, 87]. Finally a new experimental set up, dedicated to light DM, has been proposed by [88]. Unless predominantly coupled to neutrino [87], one expects that light DM is coupled to a new gauge boson to obtain an acceptable relic density [47]. The observational consequence is that one should observe missing energy in low-energy electron–proton scattering.

7. Conclusion

The introduction of MeV particles has opened new perspectives, including a possible (yet unexpected) explanation of the 511 keV line. In [49], it was demonstrated that light (<7 MeV) scalar particles with a velocity-independent annihilation cross section could fit SPI data. By predicting the right amount of positrons and the right spatial and energy distributions, such particles (introduced initially to illustrate the notion of a warm collisional DM candidate) provide an unexpected explanation to the 511 keV emission line, which is observed at the centre of the galaxy. On the contrary, decaying candidates, particles with velocity-dependent annihilation cross section or fermionic particles cannot explain the morphology of the 511 keV line that was measured by the SPI experiment.

The requirement of a small DM mass was also derived using inflight annihilations (assuming that light DM is at the sole origin of the 511 keV emission). This confirms that only a narrow range of the DM mass could be a viable explanation to the 511 keV line, if it is not due to astrophysical sources. Yet, this was basically the range that was first favoured when MeV particles were introduced, before SPI data [47].

Particle physics experiments such as the electron $g - 2$, rare pion/meson decay or ep scattering could provide a way of detecting such particles or could falsify this scenario. However, there are ongoing searches for light particles coupled to new gauge boson in present low-energy experiments (including B-factories). Unfortunately, from the astrophysical point of view, there are not so many possibilities to probe or dismiss the light DM scenario. Both the signals from the RSZ effect generated by the DM or the monochromatic line are too small to be detected. Yet a possible asymmetry found in SPI data [89] may favour hard low mass x-ray binaries, if a very large fraction (up to a few times $10^{41}$) of positrons can indeed escape per second from these objects. This nevertheless remains uncertain.

If new neutral particles were to be discovered with a mass of a few MeV, there is no doubt that our understanding of particle physics would be drastically affected. The existence of a dark sector at low-energy, difficult to access experimentally so far, would put in question many of the fundamental principles acquired in the field until now.

Acknowledgments

I thank the referees for their useful comments.
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