Reflecting star, regular scalar field and induced scalar hair

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Abstract

The existence of regular non-trivial scalar fields in the background of asymptotically flat reflecting stars and static black holes is discussed. The scalar field is assumed to be conformally coupled to the matter. The scalar charge adoption by the reflecting star is investigated and the required conditions to have regular field solution around the black hole and the reflecting star are obtained. The results show that the reflecting star, in contrast to the black hole, becomes polarized and adopts an induced scalar charge in the presence of an external matter.

1 Introduction

The only scalar field in the standard model of particles is the Higgs boson. But scalar fields are well motivated in the beyond standard models of particles and cosmology, especially in describing inflation, dark energy and dark matter. The study of scalar fields in a space-time with non-trivial geometry or topology, or with different boundary conditions, has been the subject of many studies. One of these interesting backgrounds is the black hole, whose horizon absorbs irreversibly radiation and matter. A question that naturally arises is whether there can be non-trivial (non-constant) regular scalar fields outside the horizon of a black hole. This may be studied by solving the scalar field equation. For a sole black hole, the absence of such a field is an outcome of the no-hair theorem \([1-5]\). Considering additional matter around the black hole may permit to have no trivial regular scalar field. This issue was studied in the context of screening model of dark energy in \([6]\), where the scalar field solution was numerically obtained in the presence of an accretion disc, and in \([7]\), where a spherically symmetric matter distribution was considered.

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In the presence of matter around the black hole we are interested in two problems: I- What are the constraints that must be imposed on the model parameters (such as the external matter distribution and the black hole parameters), to obtain a nontrivial regular scalar field solution? II) Does the matter induce a scalar charge into the black hole (induced charge)? In this situation, the black hole is polarized and gains a scalar induced hair due to the external matter. A simple example of gaining hair in the presence of an external matter, is an electrically grounded black hole which despite having zero total charge, gets electric hair and becomes polarized in the vicinity of an external charge [8], such a polarization as long as is restricted to electric charge is in agreement with the uniqueness theorem [1].

To see if the no-existence theorem for a regular scalar field is a unique characteristic of black-holes, the same problem was considered in the background of a compact object with a reflecting (repulsing) surface (instead of the absorbing event horizon) [9]. The domain of the scalar field is bounded by a boundary where the field vanishes and the spatial infinity where the field tends to zero. A physical compact object which on its surface the external field vanishes and Dirichlet boundary condition holds, was dubbed reflecting star in [9]. We use this terminology in this paper. In [9], and subsequent sequel papers [10–12], it was shown that the reflecting star does not support canonical massive scalar field, outside its surface. So, interestingly it seems that this space-time satisfies the no-scalar hair theorem similar to an asymptotically flat black hole. Note that there is a crucial difference between black holes and reflecting stars. In the black hole background, scalar and electric fields behave completely different, so although regularly a black hole does not support canonical scalar fields but may admit electric field. This is not true for reflecting stars, i.e. there is no scalar field and no electric field outside the reflecting star. This may be easily verified by solving the Maxwell equations by considering the imposed boundary condition. This situation is somehow similar to an artificial grounded black hole, studied in [8].

In this paper, by concentrating only on the scalar field equations, we aim to study the existence of a regular nontrivial massless scalar field in the background of an asymptotically flat reflecting star and compare the results with the black hole case. Inspired by screening models [6, 7, 13], we perform our study in the presence of matter which is conformally coupled to the scalar field and try to study the problems I and II (for reflecting star also) proposed in the first paragraph.

The scheme of the paper is as follows: In the second section, following the same steps as [14] in the study of scalarization, we obtain necessary conditions for the existence of a nontrivial scalar field without using a specific form for matter distribution. To continue the subject through analytic solutions, we consider a spherically symmetric Dirac delta matter distribution and obtain regular scalar field solutions.
We discuss the reflecting star polarization and its ability to adopt induced hair. We elucidate our results by choosing some specific conformal factors and compare our results with those we obtain for the black hole.

We use units $\hbar = c = G = 1$.

2 Non-trivial conformally coupled scalar fields and the no-hair theorem

We consider the Lagrangian

$$L = \frac{1}{16\pi} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \mathcal{L}_m (\psi_m^{(i)}, g_{\mu\nu}^{(i)}),$$  \hspace{1cm} (1)

describing a massless scalar field, interacting with matter ingredient $\psi_m^{(i)}$, through the conformal coupling

$$g_{\mu\nu}^{(i)} = A^2(\phi) g_{\mu\nu}.$$  \hspace{1cm} (2)

The equation of motion is obtained as

$$\Box \phi = V_{,\phi} - (\ln A)_{,\phi} T_{(m)},$$  \hspace{1cm} (3)

where the trace of matter energy momentum tensor is $T_{(m)} = T_{\mu\nu}^{(m)} g_{\mu\nu}$. It is convenient to define $\rho = -A^{-1} T_{(m)}$ which is the conserved density for non relativistic distribution of matter. So for a static scalar field and pressureless (non relativistic) matter distribution we have

$$\Box \phi = A_{,\phi} \rho,$$  \hspace{1cm} (4)

where $\rho$ is matter density. If $A_{,\phi} = cte$, (4) reduces the field equation in the presence of the source $\rho$, and for $A_{,\phi} \propto \phi$, (1) describes a static scalar field with an squared effective mass term proportional to $\rho$. In the absence of $A_{,\phi} \rho$, we are left with $\Box \phi = V_{,\phi}$. This equation has not regular non-trivial (by trivial we mean constant) static solution in the background of a black hole. Based on no hair theorem, static black holes do not admit scalar hair in the minimal case in general theory of relativity [15]. The same occurs for reflecting stars: $\Box \phi = V_{,\phi}$ has not static nontrivial solution satisfying $\phi(r \to \infty) = 0$ and $\phi(r_s) = 0$ where $r = r_s$ is the star surface [9]. In our study, if (1) has an horizon located at $r_h$, we take $r_s > r_h$. In the following we aim to study whether the source term $A_{,\phi} \rho$, arisen from the conformal coupling, alerts these no existence theorems.

By using (4), one obtains

$$\int_{\partial V} \phi \nabla^\mu \phi \sqrt{h} n_\mu d^3 \sigma - \int_V \nabla_\mu \phi \nabla^\mu \phi \sqrt{g} d^4 x = \int_V \phi A_{,\phi} \rho \sqrt{g} d^4 x,$$  \hspace{1cm} (5)
where the region under study, $\mathcal{V}$, is bounded by the reflecting surface and spatial infinity (and in time, by two partial Cauchy surfaces [16] which cancel each other [5]). The contribution of the reflecting surface, where the scalar field is zero, vanishes and the field is assumed to fall off sufficiently fast at infinity. The same is true for scalar field in the asymptotically flat black hole background where the reflecting surface boundary is replaced by a Killing horizon [17]. So we have

$$-\int_{\mathcal{V}} \nabla_\mu \phi \nabla^\mu \phi \sqrt{-g} d^4 x = \int_{\mathcal{V}} \phi A_{,\phi} \rho \sqrt{-g} d^4 x. \quad (6)$$

In the absence of the source term, the right-hand side would be zero implying $\phi = cte$. For reflecting stars we have $\phi(r_s) = 0$, therefore $\phi(r > r_s) = 0$. This in agreement with [9], where this theorem was proved in another way. It is clear that in the presence of the source term in the right hand side of (6), alters this statement. The left hand side of (6) is negative, hence a necessary condition to have nontrivial scalar field is

$$\int_{\mathcal{V}} \phi A_{,\phi} \rho \sqrt{-g} d^4 x < 0. \quad (7)$$

Now let us multiply both sides of (4) by $A_{,\phi}$, to obtain

$$\int_{\partial \mathcal{V}} (A_{,\phi} \nabla_\mu \phi) n^\mu \sqrt{h} d^3 x - \int_{\mathcal{V}} (A_{,\phi} \nabla^\mu \phi \nabla_\mu \phi + A_{,\phi}^2 \rho) \sqrt{-g} d^4 x = 0 \quad (8)$$

After some calculation we find

$$\int_{\partial \mathcal{V}} (A_{,\phi} \nabla_\mu \phi) n^\mu \sqrt{h} d^3 x - \int_{\mathcal{V}} (A_{,\phi} \nabla^\mu \phi \nabla_\mu \phi + A_{,\phi}^2 \rho) \sqrt{-g} d^4 x = 0 \quad (8)$$

The region and its boundary is the same as pointed above. If $A_{,\phi} = 0$ on $r = r_s$ (e.g for a power law $A$), and for sufficiently fast decaying scalar field at spatial infinity, we obtain

$$\int_{\mathcal{V}} (A_{,\phi} \nabla^\mu \phi \nabla_\mu \phi + A_{,\phi}^2 \rho) \sqrt{-g} d^4 x = 0. \quad (9)$$

Hence if everywhere $\rho A_{,\phi} > 0$, only a trivial $\phi$ satisfying $A_{,\phi} = 0$ is allowed. Conversely if only a trivial $\phi$ is allowed, then no hair theorem requires $A_{,\phi} = 0$. This can also be verified in another way: If there is no solution for $A_{,\phi} = 0$ (e.g. $A(\phi) \propto \phi$ or $A(\phi) \propto \exp(\beta \phi)$), there is no trivial solution for (4). $\rho A_{,\phi}$ is the effective mass squared corresponding to the field $\phi$, when is linearly perturbed around the vacuum, i.e. by inserting $\phi = \phi_0 + \delta \phi$ in (4) we find

$$\square \delta \phi = \rho A_{,\phi} (\phi_0) \delta \phi \quad (10)$$

So we can evade from no hair theorem when this squared mass is negative. The same argument was used in [14], to explain scalarization. Additional couplings of the scalar field, permitting evading no existence theorem, is studied also in [18].
In the following, we try to solve (4). As for a general \( \rho \), obtaining an analytic solution for (4) is not possible, we consider the matter density as a Dirac delta-like test charge. Such a density outside the black hole horizon has been considered extensively for deriving forces on particles outside the horizon [19], and also examining the no-hair theorem [20], without engaging in complicated equations which have not analytical solutions. In this context, by only solving the field equation and without referring to the full Einstein equations and back-reaction effects, one may study the black hole ability to admit charges (free or induced) and to acquire hair [8].

3 Scalar test charge, no-hair theorem, and regular static scalar field

In the spherically symmetric space-time

\[
 ds^2 = -e^{\mu(r)}dt^2 + e^{\nu(r)}dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2),
\]

we consider a spherically symmetric scalar test charge (a shell with radius \( R \)). The density is given by [21]

\[
 \rho_s = \frac{\lambda \delta(r - R)}{4\pi r^2 \sqrt{e^\mu(r) e^\nu(r)}}.
\]

With this source, in the static case, and with the metric (11), (4) becomes

\[
 \frac{d}{dr}f(r) \frac{d}{dr}\phi = \lambda A_{\phi} \delta(r - R),
\]

where

\[
 f(r) = 4\pi e^{-\frac{\nu(r)}{2}} e^{\frac{\mu(r)}{2}} r^2.
\]

In the absence of the coupling, the zeroes of \( f(r) \) correspond to singularities of the solution. For example, solving \( \frac{d}{dr}f(r) \frac{d}{dr}\phi = 0 \), in a black hole background gives a singular scalar field at the horizon. This situation, as we will see, changes when the source term is considered.

General solution of (13) is

\[
 \phi = \begin{cases} 
  C_2 F(r) + C_1 & \text{if } r \geq R \\
  C_4 F(r) + C_3 & \text{if } r \leq R,
\end{cases}
\]

where

\[
 \int_{r}^{r'} \frac{dr'}{f(r')} \equiv F(r).
\]

Asymptotically flatness requires \( \lim_{r \to \infty} e^{\mu(r)} = 1 \) and \( \lim_{r \to \infty} e^{\nu(r)} = 1 \). Therefore by counting superficial degree of (16), we obtain \( \lim_{r \to \infty} F(r) = 0 \).

We demand also \( \lim_{r \to \infty} \phi(r) = 0 \), hence \( C_1 = 0 \). Integrating (13)

\[
 \lim_{\epsilon \to 0} \int_{R-\epsilon}^{R+\epsilon} \frac{d}{dr}f(r) \frac{d}{dr}\phi = \lambda A_{\phi}(R),
\]

5
gives \( C_2 - C_4 = \lambda A_\phi(R) \). Continuity of \( \phi \) at \( R \) necessitates \( C_2 F(R) = C_4 F(R) + C_3 \). By applying the boundary condition \( \phi(r_s) = 0 \), where \( r_s < R \), we obtain \( C_3 = -C_4 F(r_s) \). Gathering all together, we obtain the final result for the reflecting star as

\[
\phi = \begin{cases} 
\lambda A_\phi(R) F(r) - \frac{F(R)}{F(r_s)} A_\phi(R) F(r) & \text{if } r \geq R \\
\lambda A_\phi(R) F(r) - \frac{F(R)}{F(r_s)} A_\phi(R) F(r) & \text{if } r_s \leq r \leq R
\end{cases}
\]  

This looks like the potential of the original test charge shell, plus the potential of an induced spherical charge located at \( r < r_s \) (because the only charge outside \( r_s \) is the original charge \( \lambda \))

\[
\lambda' = -\frac{F(R)}{F(r_s)} \lambda
\]

In other words \( \phi \) is originated both from the source shell and an induced charge adopted by the reflecting star (so the reflecting star gets scalar hair). For \( R = r_s \), we have \( \lambda' = -\lambda \), i.e. the sum of the scalar charges (free+induce) become zero, leading to \( \phi(r) = 0 \) (no hair) in agreement with [9]. This shows that the star adopted hair in the presence of the test external matter.

The condition for existence of nontrivial solution is

\[
\phi(R) = \lambda A_\phi(R) F(R) - \frac{F(R)}{F(r_s)} A_\phi(R) F(R),
\]

which is derived by setting \( r = R \) in (18). Depending on the form of \( A(\phi) \) this may put constraint on the parameters.

Without the boundary condition on \( r = r_s \), the solution would be

\[
\phi = \begin{cases} 
\lambda A_\phi(R) F(r) + C_4 F(r) & \text{if } r \geq R \\
\lambda A_\phi(R) F(r) + C_4 F(r) & \text{if } r \leq R
\end{cases}
\]  

For a black hole with horizon located as \( r_h < R \), where \( F(r_h) \) is singular, nonsingular solution is obtained by setting \( C_4 = 0 \):

\[
\phi = \begin{cases} 
\lambda A_\phi(R) F(r) & \text{if } r \geq R \\
\lambda A_\phi(R) F(r) & \text{if } r \leq R
\end{cases}
\]

One important feature of this solution is that it is regular on the horizon. In this case, there are not nontrivial regular solution in the absence of the source, i.e if we set \( \lambda = 0 \) we obtain \( \phi = 0 \). In (22) there is no trace of an induced charge and the black hole has not gain hair. The necessary condition ti have a regular solution is

\[
\phi(R) = \lambda A_\phi(R) F(r)
\]

In the above study, the back reaction of the test charge on the metric in solving the scalar field equation would give a higher-order term of \( \lambda (O(\lambda^\alpha)) \), with \( \alpha > 1 \) which has been ignored in the small test charge limit analysis.
We conclude that in the presence of the external matter (12), the nonexistence theorems change:

1- Reflecting star gets induced hair and admits an induced scalar charge. This means that the reflecting star contributes as a source of the scalar field.

2- A regular scalar field may exist in the black hole background. This field is originated only from the external matter and the black hole does not gain scalar hair.

To more elucidate these points, let us use some specific forms for the conformal factor $A$ and the metric.

3.1 $A = M_0 + \frac{\phi}{M}$

As a first example we take the simplest case: $A = M_0 + \frac{\phi}{M}$, where $M_0$ and $M$ are two real constants. Based on our discussion after (9), for $\lambda \neq 0$, only nontrivial solutions exist. The equation (13) becomes

$$M \nabla^2 \phi = \rho,$$

which for the metric (11), and the density (12) reduces to

$$M \frac{d}{dr} f(r) \frac{d}{dr} \phi = \lambda \delta(r - R).$$

The solution is

$$\phi = \left\{ \begin{array}{ll}
\frac{\lambda F(r)}{M} - \frac{\lambda F(R)}{M F(r_s)} F(r) & r \geq R \\
\frac{\lambda F(R)}{M} - \frac{\lambda F(r_s)}{M F(r_s)} F(r) & r_s \leq r \leq R,
\end{array} \right.$$ (26)

which may be rewritten as $\phi = \phi_I + \phi_{II}$, where $\phi_I$ is the potential due to the test charge,

$$\phi_I = \left\{ \begin{array}{ll}
\frac{\lambda F(r)}{M} & r \geq R \\
\frac{\lambda F(r_s)}{M F(r_s)} & r_s \leq r \leq R,
\end{array} \right.$$ (27)

and the potential due to the induced charge $\lambda' = -\frac{F(R)}{F(r_s)} \lambda$ is

$$\phi_{II} = \frac{\lambda'}{M} F(r), \ r \geq r_s.$$ (28)

For the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{r_h}{r}\right) dt^2 + \left(1 - \frac{r_h}{r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$ (29)

we have

$$\phi = \phi_I + \phi_{II} = \left\{ \begin{array}{ll}
\frac{\lambda}{4 \pi M r_h} \ln \frac{r - r_h}{r} + \frac{\lambda'}{4 \pi M r_h} \ln \frac{r - r_h}{r} & r \geq R \\
\frac{\lambda}{4 \pi M r_h} \ln \frac{R - r_s}{R} + \frac{\lambda'}{4 \pi M r_h} \ln \frac{r - r_s}{r} & r_s \leq r \leq R
\end{array} \right.$$ (30)
\[ \lambda' = \frac{-\ln \frac{R-r_h}{r_s}}{\ln \frac{r-r_h}{r_s}} \lambda. \]  

(31)

Note that \( r_s > r_h \). As explained in the general case, (30) shows that we can have a regular nontrivial scalar field whose sources are the test scalar charge \( \lambda \) and an induced scalar charge hair \( \lambda' \) inside the reflecting star. For \( R = r_s \), the reflecting star loses its hair.

Without the \( r_s \) surface, the induced charge is removed and the solution reduces to

\[ \phi(r) = \begin{cases} \frac{\lambda}{4\pi M r_h} \ln \frac{r-r_h}{r}, & r \geq R \\ \frac{\lambda}{4\pi M r_h} \ln \frac{R-r_h}{R}, & r \leq R \end{cases} \]

(32)

which is regular on the horizon, \( r = r_h \), but in contrast to (30) there is no any trace of black hole polarization and induced charge. By putting \( R = r_h \), we obtain

\[ \phi = \frac{\lambda}{4\pi M r_h} \ln \frac{r-r_h}{r}, \quad r \geq r_h, \]

(33)

which is the same as the solution obtained for a test scalar field in [22] with \( \frac{\lambda}{M} \ll r_h \) (to be allowed to ignore the backreaction). This solution, in contrast to (32) is singular at \( r = r_h \).

Now let us see what would happen if we considered an electrostatic field in the background of the reflecting star? In the usual black hole models, a black hole may support electric charge whereas it has not scalar hair. Is this the same for the reflecting star? or the boundary condition forces \( A_0 \) to vanish too. To see this, let us consider an electrostatic field \( A_0 \) and a Schwarzschild like black hole space-time, \( \frac{e^{\frac{\phi(r)}{r}}} {e^{\frac{\phi(r)}{r}}} = 1 \). By considering the metric (29), the corresponding equation, without an external source outside the horizon, is

\[ \frac{d}{dr} 4\pi r^2 \frac{d}{dr} A_0 = 0 \]

(34)

The nontrivial solution vanishing at \( r \to \infty \) is \( A(r) = \frac{q}{4\pi r} \), and the solution which vanishes both at \( r = r_s \) and \( r \to \infty \) is \( A_0 = 0 \). So it seems that no-hair theorem for the reflecting star, in contrast to black holes, includes also the electric charge. This situation is similar to a grounded black hole studied in [8], i.e. a black hole whose horizon has zero potential and zero net electric charge. Note that this does not mean that the grounded black does not accept electric charges, rather it indicates that the sum of the charges (positive+negative) is zero. The ability to accept an electric charge by a grounded black hole can be investigated by inserting an electric test charge outside the horizon [ ]. To do the same for a reflecting star, as before, we consider a spherically symmetric charge density \( \rho = \frac{q\delta(r-R)}{4\pi r} \). The electrostatic equation is now

\[ \frac{d}{dr} f_e \frac{d}{dr} A_0 = q\delta(r-R) \]

(35)
where \( f_e = 4\pi e^{-\frac{\mu(r)}{2}} e^{-\frac{\nu(r)}{2}} r^2 \). By taking \( e^{\mu(r)} = e^{-\nu(r)} = \frac{q}{4\pi r^2} \), we find \( f(r) = r^2 \) (so unlike the scalar field, \( A_0 \) is only singular at \( r = 0 \)). The solution of (35), satisfying the boundary condition at \( r_s \) is

\[
A_0 = \begin{cases} 
-\frac{q}{4\pi r^2} + \frac{q'}{4\pi r_s^2}, & r \geq R \\
-\frac{q}{4\pi r_s^2} + \frac{q'}{4\pi r_r^2}, & r_s \leq r \leq R 
\end{cases}
\]  

(36)

where \( q' = -\frac{r}{R} q \). This shows that the potential has two sources: the test charge \( q \), and a charge \( q' \) induced in the reflecting star. So in the presence of an external charge, the star has gained hair and adopted electric charge. By approaching \( R \) to \( r_s \), we have \( q' \to -q \), and for \( R = r_s \) we find \( A_0 = 0 \) as a result of total zero net charge of the star.

Hence we conclude that in the absence of any source, the asymptotically flat reflecting star has a null field in its background, while by adding some densities in the background it may adopt induced charge. The important point is that the situation is the same for both the scalar and electric fields.

### 3.2 \( A(\phi) = M_0 + \frac{\phi^2}{2M} \)

As a second example we choose \( A(\phi) = M_0 + \frac{\phi^2}{2M} \), where \( M_0 \) and \( M \) are two real nonzero constants. \( \frac{\phi^2}{2M} \) behaves as an effective mass squared. The solution is now

\[
\phi(r) = \begin{cases} 
\frac{\lambda}{M} \phi(R) \left( 1 - \frac{F(r)}{F(r_s)} \right) F(r), & r \geq R \\
\frac{\lambda}{M} \phi(R) \left( 1 - \frac{F(r)}{F(r_s)} \right) F(R), & r \leq R 
\end{cases}
\]  

(37)

For \( R = r_s \), we have \( \phi = 0 \), a sign of no hair theorem for a sole reflecting star without external matter. To have a non-trivial solution \( (\phi(r) \neq 0) \) it is necessary to have \( R \neq r_s \) and

\[
\frac{\lambda}{M} \left( 1 - \frac{F(R)}{F(r_s)} \right) F(R) = 1. 
\]  

(38)

For the metric (29), this gives

\[
\frac{\lambda}{16\pi Mr_h} \ln \left( \frac{R - r_h}{R} \right) \left( 1 - \frac{\ln \left( \frac{R - r_h}{R} \right)}{\ln \left( \frac{r_s - r_h}{r_s} \right)} \right) = 1.
\]  

(39)

As \( R > r_s > r_h \), we must have \( \frac{\lambda}{M} < 0 \). This is in agreement with our general results in the second section and can be immediately obtained by setting \( A_0 = \frac{\phi}{M} \) and \( A_{\phi \phi} = \frac{1}{M} \), in (13) or (9). Also by multiplying both sides of (13) by \( \phi \), and integrating from \( r_s \) to infinity we obtain

\[
- \int_{r_s}^{\infty} f(r) \left( \frac{d\phi}{dr} \right)^2 dr = \lambda \phi(R) A_{\phi}(R) = \frac{\lambda}{M} \phi^2(R),
\]  

(40)
indicating that $\frac{\lambda}{M} < 0$.

In the absence of the surface $r_s$, the non-trivial solution, regular at a horizon $r_h < R$, is

$$\phi(r) = \left\{ \begin{array}{ll}
\frac{\lambda}{M} \phi(R) F(r), & r \geq R \\
\frac{\lambda}{M} \phi(R) F(R), & r \leq R
\end{array} \right. \quad (41)$$

This solution is regular on the horizon, as long as $R \neq r_h$. Again, the condition to have this nontrivial solution is $\frac{\lambda}{M} F(R) = 1$ which for (29) becomes

$$\frac{\lambda}{4\pi M r_h} \ln \left( 1 - \frac{r_h}{R} \right) = 1. \quad (42)$$

By assuming that $R$ is close to $r_h$ ($R \simeq r_h$), (42) may hold only when $\frac{\lambda}{M} \simeq 0$, indicating the disappearance of the scalar charge confirming the no scalar hair theorem for a sole black hole.

4 Conclusion

The existence of scalar fields in the background of an asymptotically flat reflecting star was discussed. Both the reflecting star and the black hole obey no scalar field existence theorem in their background, but there is an essential difference between them: the reflecting star does not support electric field either and has a null electric charge (see section 3.1).

Although black holes do not support scalar fields in their canonical minimally coupled form, this is not the case where the scalar field is non-minimally coupled to the geometry the matter surrounding a black hole. Is the same true for the reflecting star? Inspired also by the previous works in the screening models (see (3)), this motivated us to consider a conformal coupling between the scalar field and the matter around the star. We derived some general results, which are very similar to the black hole case, showing that the presence of matter and the new coupling allow a regular no trivial scalar field solution, provided that some conditions be satisfied (see discussions after Eqs. (5) and (7)).

Analytical solutions can be very enlightening in this regard, but unfortunately obtaining an analytic solution is not feasible for a realistic form of matter density. Use of a test scalar charge (see (12)), by allowing us to get an analytical answer, can be very promising. The solutions derived by choosing such a probe showed that the reflecting star behaves rather like an electrically grounded black hole: in the presence of the conformally coupled scalar test charge, a continuous non-trivial scalar field appears. This field consists of two parts: one originated from the test charge located outside the star, and the other one originated from an induced charge adopted by the star (see (18)). To have such a solution, a constraint on the parameters of our model must be satisfied (see (20)).
We showed also that in the background of a black hole, surrounded by a conformably coupled test charge, we may have a regular non-trivial scalar field, provided that the black hole and the test charge parameters satisfy some conditions \(^{23}\). In contrast to the reflecting star, the black hole does not adopt an additional primitive scalar charge related to the regular scalar field and so does not get hair.

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