Dynamics and Sizes of the Fireball at Freeze-out

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Analyzing the \( m_\perp \)-spectrum and two-particle correlations of negative hadrons from 158 \( A \) GeV/c \( \text{Pb}+\text{Pb} \) collisions at slightly forward rapidities we find a (thermal) freeze-out temperature of about 100 MeV and transverse flow with \( \bar{v}_\perp \approx 0.55c \). The \( M_\perp \)-dependence of the correlation radii prefers a box-like transverse density profile over a Gaussian. From an analysis of the pion phase-space density we find \( \mu_\pi \approx 60 \text{ MeV} \) at thermal freeze-out.

1. MOTIVATION

In relativistic heavy-ion collisions the momenta of the escaping hadrons are fixed at the point of “thermal freeze-out”. They carry information about the final state of the fireball which one can use to reconstruct this state and thus obtain boundary conditions for back-extrapolation into earlier stages of its evolution. In this way one can examine the question whether or not a quark-gluon plasma was created.

Assuming thermalization and collective transverse flow at freeze-out, these two features can be extracted separately by combining single-particle \( m_\perp \)-spectra and two-particle Bose-Einstein correlations. Particles of non-zero transverse momentum are emitted from a fireball region which moves outwards transversely; in the local comoving frame their transverse momentum is lower, resulting in an enhanced Boltzmann factor. Transverse flow thus flattens the \( m_\perp \)-spectrum. A given spectral slope can, however, be reproduced by different combinations of flow and temperature, because larger flow can be compensated for by lower temperature and vice versa. On the other hand, the sizes of the regions emitting particles of given momentum (homogeneity regions, measured by correlation radii) are controlled by the expansion velocity gradients. The natural velocity measure is given by the chaotic thermal motion; increasing \( T \) and the transverse flow (velocity gradients) together at the proper rate will thus not change the resulting correlation radii. In summary, \( T \) and \( v_\perp \) are anticorrelated by a given spectral slope, but correlated by the measured homogeneity sizes. Exploiting both types of correlations their values can be determined unambiguously. This method was used for analyzing the data from central \( \text{Pb}+\text{Pb} \) collisions at 158 \( A \) GeV/c taken by the NA49 collaboration.

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2. THE MODEL

It is impossible to extract from Bose-Einstein correlations the global fireball features in a model-independent way. We therefore parameterize the fireball by an emission function, i.e. by a model for the (Wigner) phase-space density of the particles at freeze-out [1,5,6]:

\[
S(x, K) d^4x = \frac{m_\perp \cosh(y - \eta)}{(2\pi)^3} \exp \left( -\frac{K \cdot u(x) - \mu}{T} \right) \exp \left( -\frac{(\eta - \eta_0)^2}{2 (\Delta \eta)^2} \right) \\
x G(r) \frac{d\tau}{\sqrt{2\pi(\Delta \tau)^2}} \exp \left( -\frac{(\tau - \tau_0)^2}{2 (\Delta \tau)^2} \right) \tau d\eta r dr d\varphi. 
\] (1)

The main assumptions here are the following: (i) Approximate thermal equilibrium at temperature \( T \). (ii) Boost-invariant longitudinal flow combined with a tunable transverse expansion profile with a transverse flow rapidity given by \( \eta_t(r) = \eta_f(r/r_{\text{rms}}) \); this expansion is encoded in the velocity field \( u(x) \). (iii) For the transverse density profile \( G(r) \) two different forms were studied:

- box-like: \( G(r) = \theta(R_B - r) \),
- Gaussian: \( G(r) = \exp \left( -\frac{r^2}{2 R_G^2} \right) \). (2)

If the parameters \( T, \eta_f, \) and \( R_B \) or \( R_G \) are independent of space-time rapidity \( \eta \), the model cannot reproduce the rapidity dependence of the data. We therefore concentrate here on a single rapidity window \( 3.9 < Y < 4.4 \). The time parameters \( \tau_0, \Delta \tau \) were fixed together with the other parameters from the correlation radii, whereas the longitudinal width \( \Delta \eta = 1.3 \) of the density profile was fixed by adjusting the width of \( dN/dy \).

The correlation radii were calculated numerically from the “model-independent expressions” [4]. When calculating the single-particle spectrum resonances were taken into account as in [4]; since we fitted the \( h^- \) spectrum we also added negative kaons and antiprotons. For the resonances baryonic and strangeness chemical potentials, \( \mu_B \) and \( \mu_S \), were taken into account as required by baryon number and strangeness conservation [5].

3. THE FIT

Fig. 1 shows the \( \chi^2 \) contours of our fit to the data. The valley extending from the upper left to the lower right corner shows the ambiguity of the extraction of temperature and transverse flow from the spectrum. Moreover, the left and right figure show also a model ambiguity: at fixed temperature different density profiles lead to slightly different transverse expansion velocities. The results of the fit to the correlation radii (width parameters in a Gaussian parametrization of the correlation function) indicate that a box-like density fits the data better than a Gaussian one, although the latter cannot be excluded. Strictly speaking, only models with too weak transverse flow seem to be completely wrong. With the box-model we find \( T \approx 80 - 110 \text{ MeV} \) and \( \dot{v}_\perp \approx 0.47c - 0.62c \). The radius of the transverse density distribution \( R_B = 12.1 \text{ fm} \), corresponding to \( r_{\text{rms}} = 8.6 \text{ fm} \), is about twice as large as the initial transverse overlap region of the two Pb nuclei, consistent with the strong transverse flow.

Both models fit the \( m_\perp \)-spectrum well but, as shown in Fig. 2, they differ in the quality of the fit to the correlation radii. The \( M_\perp \)-dependence of transverse correlation radii \( R_\alpha \),
and $R_s$ is better captured by the box-model. This can be traced back to the fact that the box-model has a rigid radial density cutoff; for increasing $K_{\perp}$ the homogeneity region is thus squeezed towards that edge, making it narrower in the direction of $K_{\perp}$. As a result $R_o$ decreases more strongly with $K_{\perp}$ than for the Gaussian model where the homogeneity region expands into the tail of the density distribution (see Fig. 2b). We stress that it is the generic feature shown in the upper half of Fig. 2b (and not the box-like transverse density distribution itself) which appears to be required by the data.

4. MULTIPLICITY

From the fitted models we can calculate the total pion multiplicity, assuming chemical equilibrium at thermal freeze-out with $\mu_B \approx 360 \text{ MeV}$, corresponding to an entropy per baryon of about $S/A=38$ \cite{6,9}, and $\mu_S$ fixed by strangeness neutrality. This implies $\mu_\pi=0$ and results in a predicted multiplicity which is more than a factor 3 below the measured value. By investigating the pion phase-space density \cite{6} we inferred a chemical potential for the direct pions of $\mu_\pi \approx 60 \text{ MeV}$ at $T \approx 100 \text{ MeV}$. This implies that at thermal freeze-out the pions are out of chemical equilibrium. This is expected if chemical freeze-out happens in equilibrium near the confinement phase transition at $T_{\text{cr}} \approx 170 \text{ MeV}$ \cite{9,10}.

5. CONCLUSIONS

A combined analysis of single-particle spectra and two-particle correlations avoids the ambiguities and strongly reduces the model-dependencies resulting from an analysis of the $m_{\perp}$-spectrum of only a single particle species \cite{11}. It leads to the unavoidable conclusion that thermal freeze-out of the fireball happens at a very low temperature ($T_{\text{therm}} \approx 80 - 110 \text{ MeV}$) and is accompanied by very strong transverse flow ($\vec{v}_{\perp} \approx 0.47 \text{c} - 0.62 \text{c}$). The $M_{\perp}$-dependence of the homogeneity lengths can be somewhat better reproduced by a box-like than by a Gaussian transverse density profile at freeze-out.
At this low temperature the fireball is no longer in chemical equilibrium. The pion abundance appears to be fixed by chemical freeze-out already at $T_{\text{chem}} \approx T_{\text{cr}} \approx 170$ MeV. We find that at $T_{\text{therm}}=100$ MeV this leads to a pion chemical potential of $\mu_{\pi} \approx 60$ MeV.

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