Modelling, Simulation, and Stabilization of a Two-Wheeled Inverted Pendulum Robot Using Hybrid Fuzzy Control

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Abstract
Two wheels inverted pendulum robot has the same characteristics as inverted pendulum, which are unstable and nonlinear. Nonlinear systems can often be linearized by approximating them by a linear system obtained by expanding the nonlinear solution in a series, and then linear techniques can be used. Fuzzy logic control is the famous nonlinear controller that has been used by researchers to analyze the performance of a system due to the easiness to understand the nature of the controller. This research discusses about two wheels inverted pendulum robot design using hybrid fuzzy control. There are two types of fuzzy control, namely Fuzzy Balanced Standing Control (FBSC) to maintain stability and Fuzzy Traveling and Position Control (FTPC) to maintain position. Based on Takagi-Sugeno (T-S) fuzzy model on two wheels inverted pendulum robot, FBSC control used Parallel Distributed Compensation (PDC) with pole placement technic. Based on two wheels inverted pendulum robot movement characteristics, FTPC was designed using Mamdani Fuzzy architecture. FTPC control is used to help FBSC to maintain robot stability and to adjust to the desired position. Simulation result shows that controller for two wheels inverted pendulum robot can stabilize pendulum angle in 0 radian and close to the desired position.

Keywords:
Fuzzy T-S, Fuzzy Mamdani, Parallel Distributed Compensation (PDC), Pole Placement, Two Wheels Inverted Pendulum

I. INTRODUCTION
Two wheels inverted pendulum robot is a robot that has 2 (two) wheels that work like reversed pendulum. This robot has the same characteristics as inverted pendulum, which are unstable and nonlinear. The main goal of this research is to stabilize the robot to its reversed position and back to its previous position or desired position.

In recent years, fuzzy control design based on Takagi-Sugeno (T-S) model had been widely used on wider applications such as manipulator robot [1], trailer [2], hovercraft [3], and helicopter [4]. Fuzzy control had been well implemented in so many nonlinear systems. However, systematically, fuzzy control design for system stability analysis is still an interesting challenge, especially in nonlinear system control design. This is because nonlinearity is always interfering real system control since it is only partially known, it is hard to described, and there are only a few measured states [5].

There have been several studies on two-wheeled inverted pendulum robots with fuzzy control, among others, those conducted by Almeshal et al. who used robust hybrid fuzzy logic control with movable loads [6], in addition, Kharola's research made a control design using the fuzzy method. Logic control strategy to control and stabilize the robot [7], then [8] the author developed a fuzzy logic controller for the two-wheeled LEGO EV3 robot and finally the research conducted by Huang et al designed a type-2 interval fuzzy logic modeling and inverted pendulum control two-wheel moving [9].

This research proposed a design and simulation of two wheels inverted pendulum robot using hybrid fuzzy control.
There two fuzzy controls in the controller design, namely fuzzy balanced standing control (FBSC) and fuzzy traveling and position control (FTPC). FBSC uses parallel distributed compensation (PDC) concept with pole placement technic to stabilize the robot based on fuzzy Takagi-Sugeno. While FTPC uses heuristic rule Mamdani architecture to design movement and position. The robot is a nonlinear model and need to be changed to linear model via linearization process.

This paper organized into five sections. Section I contains introduction of this research. The next section discusses the mathematical model of two wheels inverted pendulum robot and hybrid fuzzy control. Section III explains about the result of the two wheels inverted pendulum robot while section IV contains discussion about the results, limitation and compare the results with others. Lastly, conclusion for this research is stated in section V.

II. METHOD AND MODELLING

A. Mathematical Model of Two Wheels Inverted Pendulum Robot

There are two steps to obtain mathematical model of two wheels inverted pendulum robot. First is knowing the coordinate system used by two wheels inverted pendulum robot with geometric parameters shown in Fig. 1. \( \phi \) is pendulum tilt angle and \( \theta \) is the average value of motor rotary angle, described as follows \( \theta = (\theta_l + \theta_r) / 2 \). \( \theta_l \) and \( \theta_r \) is rotary angle of left motor and right motor respectively. The geometric parameters and values of a two-wheeled inverted pendulum robot is presented on Table I [10]. Based on concept of physic, force resultant of x-axis described in equation (1).

\[
(M_p + M_c)r \dot{\theta} + M_p r \dot{\phi} \cos \phi = f_x
\]  

(1)

where \( f_x \) is force given by motor rim along x-axis, described as \( f_x = f_R + f_L \), where \( f_R \) and \( f_L \) describes the forces of rim of right motor and left motor respectively, the momentum on the pendulum circle point describes equation (2).

\[
M_p r \dot{\theta} \cos \phi + M_p l^2 \ddot{\phi} - M_p g l = 0
\]

\[
\ddot{\theta} = \frac{M_p g \sin \phi - M_p l^2 \ddot{\phi}}{M_p r \cos \phi} = \frac{\sin \phi - \dot{\phi}}{r \cos \phi}
\]  

(2)

While the torque is described in equation (3).

\[
u_f(t) = f_x(t) \cdot r
\]

\[
(M_p + M_c) \left[ \frac{g \sin \phi - \dot{\phi}}{\cos \phi} \right] + M_p \dot{\phi} \cos \phi = \frac{\nu_f(t)}{r}
\]  

(3)

Two wheels inverted pendulum robot consist of 4 states that describe as a vector \( x(t) = [\phi \ \dot{\phi} \ \theta \ \dot{\theta}]^T \) where \( x_1 = \phi \) declares pendulum tilt angle, \( x_2 = \dot{\phi} \) declares pendulum angle speed which is the first derivative of pendulum tilt angle \( x_2 \) = \( d \ x_1 / dt \), \( x_3 = \dot{\theta} \) declares motor rotational angle speed which is the first derivative of motor rotational angle \( d\theta / dt \), and \( x_4 = \theta \) declares motor rotational angle. The following is two wheels inverted pendulum robot state equation (4).

\[
\dot{x}_1 = x_2
\]

\[
\dot{x}_2 = \frac{(M_p + M_c) g a x_1}{l(1-\beta^2)M_p + M_c} - \frac{\nu_f(t) \beta}{r l(1-\beta^2)M_p + M_c}
\]

\[
\dot{x}_3 = \frac{\nu_f(t)}{[(1-\beta^2)M_p + M_c] \beta} - \frac{M_p g a x_1 \beta}{[(1-\beta^2)M_p + M_c] r}
\]

\[
\dot{x}_4 = x_3
\]

Where;

\[
\alpha = \frac{\sin \phi}{\phi}
\]

\[
\beta = \cos \phi
\]  

(4)

Fig. 1. Dynamic Model of Two Wheels Inverted Pendulum Robot [6].

TABLE I. GEOMETRIC PARAMETERS AND VALUES OF A TWO-WHEELED INVERTED PENDULUM ROBOT [10].

| Parameter                          | Symbol | Value | Unit  |
|-----------------------------------|--------|-------|-------|
| Pendulum mass                     | \( M_p \) | 9.1   | [kg]  |
| Train Mass                        | \( M_c \) | 25.2  | [kg]  |
| The length between the wheel shaft and the center of gravity of the pendulum | \( l \) | 0.5   | [m]   |
| Wheel radius                      | \( r \) | 0.1   | [m]   |
| Distance between left and right wheels | \( D \) | 0.44  | [m]   |
| Gravitational acceleration        | \( g \) | 9.8   | [m/s²]|

B. Hybrid Fuzzy Control on Two Wheels Inverted Pendulum Robot

1) Mamdani Fuzzy Controller Design

FTPC controller for position of the two wheels inverted pendulum robot is using Mamdani Fuzzy model with 2 premises and 49 if-then rules. This control design is used not only
to help FBSC controller to maintain robot stability but also to
guide to the desired position. At the initial condition, positive or
negative direction value is assigned to the pendulum angle, then
the two wheels inverted pendulum robot will move forward or
backward preventing pendulum from falling and keeping the
robot walking through to the desired position. If an error occurs,
the two wheels inverted pendulum robot position describes in
equation (5).

\[ e(t) = p(t) - p_d \]  \hspace{1cm} (5)

where \( p_d \) declares the two wheels inverted pendulum robot
desired position and \( p(t) \) declares the two wheels inverted
pendulum robot current position, so the desired state will be
\( x_d(t) = [\phi_d(t) \ 0 \ 0] \). While state error \( z(t) \) describes
equation (6).

\[ z(t) = x(t) - x_d(t) \]

\[ z(t) = [\phi(t) - \phi_d(t) \ \dot{\phi}(t) \ \dot{\theta}(t)] \]

\[ = [e_d(t) \ \phi(t) \ \dot{\phi}(t)] \]  \hspace{1cm} (6)

where \( e_d(t) \) declares pendulum angle error.

Membership function used in \( e_p, \dot{\theta}, \) and \( \phi_d \) are triangle and
describes as follows (7):

\[ \mu(x) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & c \leq x 
\end{cases} \]  \hspace{1cm} (7)

There are 7 (seven) fuzzy sets used in \( e_p, \dot{\theta}, \) and \( \phi_d, \) namely
negative big (NB), negative medium (NM), negative small
(NS), zero (ZO), positive small (PS), positive medium (PM),
and positive big (PB). If consequence “\( u \) is \( \phi_d \)” is a conclusion
obtained when premises “\( x \) is \( e_p \)” and “\( x \) is \( \dot{\theta} \)” are fulfilled, then
there are 49 fuzzy rules for robot control position shown in
Table 2. Fig. 2 depicts membership function with two inputs
(premises), i.e., position error \( e_p \) and wheel rotation speed \( \dot{\theta} \)
output (consequence) shown as pendulum desired angle
\( \phi_d \).

**TABLE II. FUZZY RULE TABLE FOR FTPC CONTROL**

| Consequence \( \phi_d \) | Premise \( \theta \) |
|------------------------|-----------------|
| NB | PB | PB | PB | PB | PM | PM | PB |
| NM | PB | PB | PB | PM | PS | PM | PB |
| NS | PB | PB | PM | PS | ZO | NS | NM |
| ZO | PS | PS | ZO | NS | NM | NB | NB |
| PS | PS | ZO | NS | NM | NB | NB | NB |
| PM | ZO | NS | NM | NB | NB | NB | NB |

2) Fuzzy Takagi-Sugeno Controller Design

Fuzzy T-S model design for balancing the two wheels
inverted pendulum robot is using 2 rules. Membership function
in this fuzzy rule only implemented for pendulum angle output
\( \phi \). Membership function in this research is triangle
membership function describes in equation (8).

\[ \mu(x) = f(x; a, b, c) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & c \leq x 
\end{cases} \]  \hspace{1cm} (8)

where \( a \) is the left limit parameter, \( b \) is the midpoint and \( c \) is
the right limit, while \( x \) is a variable.
This research is conducted using 2 rules. The first rule is used when the angle is 0 radian and the second is when the angle ±0.2 radian, so the membership grade can be depicted in Fig. 3.

![Fig. 3. Pendulum Angle Membership Function.](image)

Henceforth, Parallel Distributed Compensation (PDC) is used to specify controller parameter state $z(t)$ in equation (6). Control rules in PDC designed based on corresponding plant rule in equation (9) are specified as follows:

1. **First rule for controller:**
   
   $\nu = -K_1z(t)$

2. **Second rule for controller:**
   
   $\nu = -K_2z(t)$

Result from all control signal written as an equation

$$u = -(\mu_1K_1 + \mu_2K_2)z(t)$$

where $\mu_1$ is the weight of rule 1 and $\mu_2$ is the weight of rule 2.

Block diagram of two wheels inverted pendulum robot using hybrid fuzzy control is depicted in Fig. 4.

![Fig. 4. Two Wheels Inverted Pendulum Robot System Block Diagram.](image)
III. RESULTS

In this part, the designed controller is simulated on two wheels inverted pendulum robot plant. Based on equation 4 Linearization, robot system state-space equation is obtained in equation (12).

\[ \dot{x}(t) = A_1 x(t) + B_1 u(t), \]  
\[ i = 1, 2 \]  
\[ (12) \]

With working point \( x_1 \) is 0; 0, 2 \( A_1 \) dan \( B_1 \) are then obtained as follows respectively:

\[
A_1 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
26.6778 & 0 & 0 & 0 \\
-35.3898 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[ \text{and } B_1 = \begin{bmatrix}
-0.7937 \\
3.9683 \\
0
\end{bmatrix} \]

\[
A_2 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
25.0541 & 0 & 0 & 0 \\
-31.1953 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[ \text{and } B_2 = \begin{bmatrix}
-0.7669 \\
3.9125 \\
0
\end{bmatrix} \]

With pole placement areas at -2.7; -2.72; -2.9; -2.91 and 2.8; -2.81; -3.0; -3.1, then feedback gain values are:

\[
K_1 = [-97.1624 \ - 19.8332 \ - 1.1367 \ - 0.7968] \]

\[
K_2 = [-104.7158 \ - 22.1657 \ - 1.3518 \ - 0.9875] \]

**Fig. 5** is the simulation result based on state feedback gain control algorithm for initial angle 0.2 radian and initial angle 0.3 radian with pendulum angle speed, pendulum rotary angle, motor rotary angle, and robot position is 0.

Peak time used to measure robot position performance and pendulum angle is determined when reached the highest point. **Fig. 5** shows performance measure for robot position compared to when initial angle 0.2 radian is around 0.61 second with maximum overshoot is around 1.465 meters, and settling time is around 5.91 seconds, while peak time at initial angle 0.3 radian is around 0.72 second with maximum overshoot is around 2.053 meters, and settling time is around 6.29 seconds.

**Fig. 5** also shows that pendulum robot is experiencing deviation before moving back to its original position. At initial angle 0.3 radian, robot experiencing the largest deviation compare to when the initial angle is 0.2 radian. To be concluded that the larger the given initial angle, the larger the peak time likely to be and the maximum overshoot as well.

**Fig. 6** depicts control signal performance measurement with comparison between initial angle 0.2 radian against initial angle 0.3 radian. The generated control signal is 20 Newton meter.

Pendulum angle performance is shown in **Fig. 7**. Peak time at initial angle 0.2 radian is around 0.61 second with maximum overshoot is around 0.008755 radian, and settling time is around 1.83 seconds, while peak time at initial angle 0.3 radian is around 0.81 second with maximum undershoot is around 0.01215 radian, and settling time is around 1.86 second. Based on the responses, it can be concluded that the larger the initial angle the larger the undershoot value.
Pendulum angle performance is shown in Fig. 11 with initial angle 0.2 radian, peak time when initial angle 0.2 radian is around 0.62 second with maximum undershoot is around 0.01269 radian, and settling time is around 2.14 seconds.

Based on the available data and compare it with the error obtained from simulation result against the desired position, it can be concluded that the greater the desired position (\(p_d\)), the greater the position error. When \(p_d = 0.2\) meter, position error is 7.65%, while when \(p_d = 0.4\) meter, position error is 8.02%.

Fig. 8. Result of Robot Position Response Simulation with Initial Angle 0.2 radian and \(p_d = 0.2\) meter.

Fig. 9. Result of Pendulum Angle Response Simulation with Initial Angle 0.2 radian and \(p_d = 0.2\) meter.

Fig. 10. Result of Robot Position Response Simulation with Initial Angle 0.2 radian and \(p_d = 0.4\) meter.

Fig. 11. Result of Pendulum Response Simulation with Initial Angle 0.2 radian and \(p_d = 0.4\) meter.

**IV. DISCUSSION**

Simulation result shows that controller for two wheels inverted pendulum robot can stabilize pendulum angle in 0 radian and close to the desired position. The simulation results on fuzzy traveling and position control (FTPC) have an error of about 8% to control and stabilize the two-wheeled inverted pendulum robot in the desired position. Huang et-al developed design and implementation of fuzzy control on a two-wheeled inverted pendulum. The combination of FTPC and FBSC works very well. The proposed FTPC can enhance the performance of the FBSC. For the purpose of the FBSC, the TWIP moves backward quickly to avoid falling down. Then, the TWIP travels forward to reach the desired position due to the effort of the FTPC.

**V. CONCLUSION**

Based on testing result of Mamdani and Takagi-Sugeno fuzzy control system simulation using Parallel Distributed Compensation (PDC) concept and pole placement technic in this research, it can be concluded that the simulation result of the FBSC controller is that the two wheels inverted pendulum robot system is able to maintain the upright pendulum position at angle 0 radian. Meanwhile the result of the FTPC controller simulation is that the two wheels inverted pendulum robot system can approach to the desired position with an error of about 8%. Research can be developed by implementing hybrid fuzzy control on a two-wheeled pendulum robot.

**REFERENCES**

[1] Y.-W Liang, S.-D. Xu, D.-C Liaw, and C.-C Chen, “A Study of T-S model based SMC scheme with application to robot control”, IEEE Trans. Ind Electron., vol. 55, no.11, page 3964-3971, November 2008.

[2] Tanaka, K and T. Kosaki, “Design of a stable fuzzy controller for an articulated vehicle”, IEEE Trans. Syst., Man, Cybern B, Cybern., vol.27, no. 3, page 552-558, June 1997.

[3] Tanaka, K, M. Iwasaki and H. O. Wang, “Switcing control of an R/C hovercraft: stabilization and smooth Switching”, IEEE Trans. Syst., Man, Cybern B, Cybern., vol.31, no. 6, page 853-863, December 2001.

[4] Tanaka, K, H. Ohtakeaki dan H. O. Wang, “A practical design approach to stabilization of a 3-DOF RC helicopter “, IEEE Trans. Control Syst. Technol., vol. 12, no. 2, page 315-325, March 2004.

[5] Ogata, Katsuhiko., “Teknik Kontrol Automatik”, translated by Ir. Edi Leksono, Erlangga, Jakarta, 1993.

[6] A-M Almeshal, K-M Goher, “Robust hybrid fuzzy logic controller for a novel two wheeled robotic vehicle with a movable payload under various operating conditions” IEEE, page 747-752, September 2012.

[7] Kharola, K “The control of Two-wheeled Inverted Pendulum Robot (TWIPR) using fuzzy logic”, IEEE, December 2015.

[8] M-A Akmal, N-F Jamin and N-M Ghani, “Fuzzy Logic Controller for Two Wheeled EV3 LEGO Robot” IEEE, page 134-138, December 2017.

[9] Huang, J, Ri, M, and Wu, D, “Interval Type-2 Fuzzy Logic Modeling and Control of a Mobile Two-Wheeled Inverted Pendulum” IEEE. Trans on Fuzzy System, October 2017.

[10] C.-H Huang, W-J Wang, and C-H Chiu, “Design and implementation of fuzzy control on a two wheel inverted pendulum”, IEEE Trans. Ind. Electron., vol. 58, no.7, July 2011.