Phenomenology of $B_s$ decays

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ABSTRACT

Using the QCD sum rules technique we study several aspects of the phenomenology of the b-flavoured strange meson $\overline{B}_s^0$. In particular, we evaluate the mass of the particle, the leptonic constant and the form factors of the decays $\overline{B}_s^0 \to D_s^+ \ell^- \bar{\nu}$, $\overline{B}_s^0 \to D_s^{*+} \ell^- \bar{\nu}$, $\overline{B}_s^0 \to K^{*+} \ell^- \bar{\nu}$. We also calculate, in the factorization approximation, a number of two-body non leptonic $\overline{B}_s^0$ decays. Finally, we compare our evaluation of the $SU(3)_F$ breaking effects in the $\overline{B}_s^0$ channel to other estimates.

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1 Introduction

The interest for the $b$-flavoured strange meson $\bar{B}_s^0 (bs)$ has been recently prompted by the reported evidence for the production of this particle in the hadronic $Z^0$ decays at LEP [1, 2, 3]. A signal of correlated $D_s^+\ell^-$ pairs ($\ell = \mu, e$) has been observed [4], with the lepton having a large momentum and a large momentum component with respect to the $b$ quark direction; this signal can be attributed to the semileptonic process

$$\bar{B}_s^0 \rightarrow D_s^+\ell^-\bar{\nu}_\ell X$$

which, by analogy with the $B_{u,d}$ case, is expected to occur at the 10% level. The indication of $B_s$ mesons is confirmed by the observation of an excess of inclusive $D_s^+$ production, whose measured value is larger than the expected production from $B_{u,d}$.

Evidence for the $B_s$ production at $\Upsilon(5S)$ was already reported by the CUSB Collaboration at CESR [5]. Moreover, indications for $B_s$ have been deduced from the measurement of the rate of same sign dileptons at the hadron $p\bar{p}$ colliders [1] and at LEP [4]: since this rate is larger than the corresponding quantity measured at $\Upsilon(4S)$, the difference can be attributed to the presence of $\bar{B}_s^0, B_s^0$ mesons with a (nearly) maximal mixing.

Ongoing measurements will soon provide us with a value for the mass difference $m_{B_s} - m_{B_d}$ by reconstructing non leptonic decay channels; as for the lifetime $\tau_{B_s}$, the measured value [4]

$$\tau_{B_s} = 1.1 \pm 0.5 \text{ ps}$$

is still dominated by the statistical error, so that no information on the possible role of non spectator effects in this channel is available yet.

From the theoretical standpoint, the interest for the $\bar{B}_s^0$ meson stems from the possibility of clarifying the size of the light flavour $SU(3)_F$ breaking effects in the $b$ quark sector. In the charm sector some hints on such effects can be obtained by comparing $D^+$ and $D_s$; the difference [10]

$$m_{D_s} - m_{D^+} = (99.5 \pm 0.6) \text{ MeV}$$

shows that these effects are of the order of 5% for the mass of the particles. In the $b$ system the $SU(3)_F$ breaking terms, which account for the deviations from unity of the ratios $m_{B_s}/m_{B_d}$, $f_{B_s}/f_{B_d}$, etc., play a significant role in the possibility of constraining the Cabibbo-Kobayashi-Maskawa matrix and, consequently, the quark sector of the Standard
Model. As a matter of fact, within the Standard Model the mixing between $\bar{B}_s^0$ and $B_s^0$ occurs with the parameter $x_s = (\Delta M/\Gamma)_{B_s}$ given by

$$x_s = \frac{G_F^2}{6\pi^2} m_{B_s}^2 m_{B_s} (f_{B_s}^2 B_{B_s}) \eta_{B_s} |V_{ts}^* V_{td}|^2 y_t f_2(y_t).$$

Eq. (4) shows that the ratio $x_s/x_d$ is independent of the (still unknown) top quark mass $m_t$; the experimental determination of this ratio implies a measurement of $|V_{ts}/V_{td}|$ once $(f_{B_s}^2 B_{B_s})/(f_{B_d}^2 B_{B_d})$, $m_{B_s}/m_{B_d}$ and $\tau_{B_s}/\tau_{B_d}$ have been calculated and (or) measured.

The ratios $m_{B_s}/m_{B_d}$ and $f_{B_s}/f_{B_d}$ are available presently from potential models for the quark-antiquark systems [13]. The quantity $(f_{B_s}^2 B_{B_s})/(f_{B_d}^2 B_{B_d})$ has been estimated also by using the Heavy Quark Effective Chiral Perturbation Theory [14].

In this paper we calculate $m_{B_s}$, $f_{B_s}$, and the ratios $m_{B_s}/m_{B_d}$, $f_{B_s}/f_{B_d}$ by QCD sum rules [13]. This method is deeply rooted in the QCD framework of the strong interactions, and has been successfully applied to different aspects of the light [16, 17] and heavy hadrons [18]. It avoids the notion of wave-function for a system of constituent quarks, and directly relates hadronic properties (masses, leptonic constants, etc.) to fundamental QCD quantities like current quark masses, $\alpha_s$, and a set of parameters, the "condensates", which describe the deviations from the asymptotically free behaviour at short distances by allowing the inclusion of a series of power corrections.

In the QCD sum rules approach the $SU(3)_F$ breaking effects in the static parameters of the heavy mesons can be systematically taken into account. Moreover, this technique permits the calculation of a number of dynamical heavy system properties, e.g. the form factors that describe the semileptonic decays $\bar{B}_s^0 \to D_s^+ (D_{s*}^+) \ell^{-} \nu$ and their deviations from the analogous quantities related to $\bar{B}_d^0 \to D^+ (D_{s*}^+) \ell^{-} \nu$.

The plan of the paper is as follows. In section 2 we evaluate the mass and the leptonic constant of the $\bar{B}_s^0$ meson by two-point function QCD sum rules. An analysis of the ratios $m_{B_s}/m_{B_d}$ and $f_{B_s}/f_{B_d}$ allows us to estimate the size of $SU(3)_F$ breaking in these quantities. Since the calculation can be extended in a straightforward way to the $D_s$ meson, we calculate $f_{D_s}$ and compare our findings with a number of recent experimental and theoretical determinations. By using three point function QCD sum rules we calculate in section 3 the hadronic matrix elements that describe the semileptonic decays $\bar{B}_s^0 \to D_s (D_{s*}^+) \ell^{-} \nu$ and $\bar{B}_s^0 \to K^* \ell^{-} \nu$. Also in this case we evaluate the light flavour symmetry breaking effects. In section 4 we estimate, in the factorization hypothesis, the width of several two-body non leptonic $B_s$ decays.
2  $B_s$ mass and leptonic constant

A number of estimates of the leptonic constants for the heavy-light quark mesonic systems can be found in the literature. In particular, QCD sum rules have been used to evaluate the B meson leptonic constant $f_B$ both for a finite $[19, 20]$ and an infinite heavy quark mass $m_b [21, 22]$. Here we apply this method to the calculation of $f_{B_s}$ defined by the matrix element

$$
\langle 0 | \bar{b} i \gamma_5 s | B_s^0 \rangle = \frac{f_{B_s} m_{B_s}^2}{m_b + m_s} \tag{5}
$$

($m_b$ and $m_s$ are the $b$ and $s$ quark masses). As usual in the QCD sum rules approach, the starting point is the correlator of quark currents:

$$
\Pi(q^2) = i \int dx e^{iqx} \langle 0 | T(J_5(x) J_5^\dagger(0)) | 0 \rangle \tag{6}
$$

with $J_5 = \bar{b} i \gamma_5 s$. This correlator can be evaluated in two different ways. First, by a short-distance operator product expansion in QCD ($q^2 \to -\infty$), which gives the perturbative (P) contribution, written through a dispersion relation

$$
\Pi(q^2) = \frac{1}{\pi} \int ds \frac{\rho_P(s)}{s-q^2}, \tag{7}
$$

and non perturbative (NP) power corrections parameterized by vacuum matrix elements of quark and gluon field operators. These terms are ordered according to the dimension; they represent the breaking of asymptotic freedom. Therefore, the QCD form of the correlator reads:

$$
\Pi_{QCD}(q^2) = \Pi^P(q^2) + \Pi^{NP}(q^2) = \Pi^P(q^2) + C_3(q^2) < \bar{s}s > + C_4(q^2) < \frac{\alpha_s}{\pi} G^2 > + C_5(q^2) < \bar{s}g\sigma Gs > + ... \tag{8}
$$

The perturbative spectral function $\rho_P(s)$ is given to the lowest order in $\alpha_s$ by

$$
\rho_P(s) = \frac{3}{8\pi} \sqrt{\lambda(s, m_b^2, m_s^2)} \frac{\sqrt{s - (m_b - m_s)^2}}{s} \Theta[s - (m_b + m_s)^2] \tag{9}
$$

where $\lambda$ is the triangular function; the $O(\alpha_s)$ corrections can be found in Ref. [17]. The coefficients $C_3$, $C_4$ and $C_5$ in eq.(8) can be calculated using the fixed point technique [23].
with the result:

\[
C_3 = \frac{m_b}{q^2 - m_b^2} - \frac{m_s q^2 - 2m_b^2}{2 (q^2 - m_b^2)^2} + \frac{m_s^2 m_b^2}{(q^2 - m_b^2)^3} \tag{10}
\]

\[
C_4 = \frac{1}{12} \frac{1}{(q^2 - m_b^2)^2} \left[ -1 - 6 \frac{m_s m_b q^2}{(q^2 - m_b^2)^2} \ln \frac{q^2 - m_b^2}{m_s m_b} + \right.
\]
\[
+ \left. \frac{m_s}{m_b} \left( 1 + \frac{8m_b^2}{(q^2 - m_b^2)} + \frac{6m_b^4}{(q^2 - m_b^2)^2} \right) \right] \tag{11}
\]

\[
C_5 = -\frac{1}{2} \left[ \frac{m_b}{(q^2 - m_b^2)^2} + \frac{m_b^3}{(q^2 - m_b^2)^3} \right]. \tag{12}
\]

Actually, the main contribution comes from the \(D = 3\) and \(D = 5\) terms.

The second evaluation of the correlator is obtained by writing the spectral function \(\rho(s)\) in terms of hadronic (H) resonances and of a continuum of states; assuming the dominance of the lowest lying resonance, one writes:

\[
\rho_H(s) = \pi \left( \frac{f_{B_s} m_{B_s}^2}{m_b + m_s} \right)^2 \delta (s - m_{B_s}^2) + \rho_{\text{CONT}}(s) \Theta (s - s_0), \tag{13}
\]

where \(s_0\) is an effective threshold which separates the contribution of the resonance from the continuum. According to duality, the continuum spectral function can be modeled as in perturbative QCD: therefore in (13) \(\rho_{\text{CONT}}(s) = \rho_P(s)\).

In the QCD Sum Rules approach, a region in \(q^2\) (duality window) has to be found where the hadronic and the QCD expressions for the correlator match with each other. The matching can be improved by a Borel transformation defined by the operator

\[
\mathcal{B} = \left( \frac{-Q^2}{n-1} \right)^n \left( \frac{d}{dQ^2} \right)^n \tag{14}
\]

in the limit \(Q^2 \to \infty\) (\(Q^2 = -q^2\)), \(n \to \infty\) and \(Q^2/n = M^2\) fixed, applied to both the hadronic and QCD sides of the rule. One obtains:

\[
\frac{1}{\pi} \int ds \rho(s) \frac{e^{-s/M^2}}{M^2} = \Pi^P(M^2) + \Pi^{NP}(M^2) \tag{15}
\]

and a daughter sum rule for the mass of the meson by differentiating eq.(15) with respect to \(1/M^2\).

Let us discuss the values of the parameters appearing in the sum rule eq.(15). The strange quark mass \(m_s\) and the strange quark condensate \(< \bar{s}s >\) are responsible for the deviation of eq.(15) from the analogous expression for the \(B\) meson. Both these parameters
are fixed by the analysis of the baryonic states given in ref.\[24\]: \(m_s = 0.14 - 0.15 \text{ GeV}\) and \(< \bar{s}s > = 0.8 < \bar{d}d > \) with \(\bar{d}d > = (-0.23 \text{ GeV})^3\); the mixed \(D = 5\) condensate can be expressed in terms of \(< \bar{s}s >: \bar{s}g\sigma Gs > = m_0^2 < \bar{s}s >\) with \(m_0^2 = 0.8 \text{ GeV}^2\).

The (pole) mass of the \(b\) quark plays a crucial role in the sum rule. We use the value fixed in ref.\[19\] by analyzing the \(\Upsilon\) system (see also \[25\]): \(m_b = 4.6 - 4.7 \text{ GeV}\). \[26\].

The last QCD input parameter is \(\alpha_s\); we use the value obtained at the scale \(m_b\) with \(\Lambda_{QCD} = 150 - 200 \text{ MeV}\).

There are now two quantities that must be fixed: the effective threshold \(s_0\) and the duality window in the Borel parameter \(M^2\). The range of acceptable \(M^2\) values can be fixed by requiring a hierarchical structure in the contributions of the OPE and in the resonance-continuum hadronic side. On the other hand, the value of \(s_0\) can be changed in a small interval: we use \(s_0 = 33 \div 36 \text{ GeV}^2\). The typical curves are depicted in fig.1, where the duality region is also shown. Our result is:

\[
\begin{align*}
m_{B_s} &= (5.4 \pm 0.1) \text{ GeV} \\
f_{B_s} &= (190 \pm 20) \text{ MeV}
\end{align*}
\]

where the uncertainties are due to the variation of the parameters in their allowed intervals.

Before discussing these results let us observe that the same calculation can be performed for the \(D_s^+ (c\bar{s})\) meson. Using \(m_c = 1.35 \text{ GeV}\), \(s_0 = 6 - 7 \text{ GeV}^2\) and \(\alpha_s\) at the scale \(m_c\) we get:

\[
\begin{align*}
m_{D_s} &= (2.0 \pm 0.1) \text{ GeV} \\
f_{D_s} &= (195 \pm 20) \text{ MeV}
\end{align*}
\]

Within the uncertainties the result for the leptonic constant is compatible with the value obtained in ref.\[27\] by a numerical calculation on the lattice: \(f_{D_s} = (230 \pm 50) \text{ MeV}\). Moreover, it is in agreement with the measurement of the WA75 Collaboration \[28\]

\[
f_{D_s} = (232 \pm 45 \pm 20 \pm 48) \text{ MeV}
\]

obtained by the observation of leptonic decays \(D_s^+ \rightarrow \mu^+\nu\) in emulsion. Another estimate of \(f_{D_s}\) has been given in \[29, 30\] using the non leptonic decay channel \(B \rightarrow D(D^*) D_s^+\) and the factorization hypothesis, with a similar result.
As stated above, the uncertainties in eqs. (16,17) are due to the variation of $s_0$ and $M^2$ in the stability window. Trying to reduce this error (mainly in the prediction of $m_{B_s}$) we have studied the ratios $\frac{m_{B_s}}{m_{B_d}}$ and $\frac{f_{B_s}}{f_{B_d}}$ by writing the ratios of the corresponding rules with two different continuum thresholds $s_0$ (33 ÷ 36 GeV$^2$ for $B_s$ and 32 ÷ 35 GeV$^2$ for $B$). These quantities display a softer dependence on the parameter $s$ and are remarkably stable in $M^2$ as shown in fig. 2. This allows us to predict:

$$\frac{m_{B_s}}{m_{B_d}} = 1.005 \pm 0.002$$  \hspace{1cm} (19)

$$\frac{f_{B_s}}{f_{B_d}} = 1.09 \pm 0.03$$  \hspace{1cm} (20)

with the uncertainty reduced by a factor of 2 with respect to eqs. (16). The conclusion is that the size of $SU(3)_F$ breaking effects are of 0.5% for the $B_s$ mass and less than 10% for the leptonic constant; these effects mainly come from the value of the $<\bar{s}s>$ condensate.

### 3 Semileptonic form factors

The hadronic matrix elements of the transitions $\bar{B}_s^0 \rightarrow P^+ e^- \bar{\nu}_e$ and $\bar{B}_s^0 \rightarrow V^+ e^- \bar{\nu}_e$ ($P$ and $V$ are strange pseudoscalar and vector mesons, respectively) can be written in terms of form factors using the decomposition in Ref. [31]:

$$\langle P^+(p_P) | V_{\mu} | \bar{B}_s^0(p_{B_s}) \rangle = F_1(q^2) (p_{B_s} + p_P) + \frac{m_{B_s}^2 - m_P^2}{q^2} q_{\mu} \left[ F_0(q^2) - F_1(q^2) \right]$$  \hspace{1cm} (21)

$$\langle V^+(p_V) | J_{\mu} | \bar{B}_s^0(p_{B_s}) \rangle = \frac{2V(q^2)}{m_{B_s} + m_V} \epsilon_{\mu
u\rho\sigma} \epsilon^{*\alpha} p_{B_s}^{\rho} p_{V}^{\sigma} - i[(m_{B_s} + m_V)A_1(q^2)\epsilon^*_\mu - \frac{A_2(q^2)}{m_{B_s} + m_V}(\epsilon^* \cdot p_{B_s})(p_{B_s} + p_V)_\mu - \frac{m_V}{q^2} q_\mu (A_3(q^2) - A_0(q^2))]$$  \hspace{1cm} (22)

where $q^2 = (p_{B_s} - p_{P,V})^2$ and $J_\mu = \bar{q} \gamma_{\mu}(1 - \gamma_5)b$ ($q = c, u$); $\epsilon$ is the $V^+$ meson polarization vector. The conditions

$$F_1(0) = F_0(0)$$

$$A_3(0) = A_0(0)$$  \hspace{1cm} (23)

must be implemented in eqs. (21,22) in order to avoid unphysical poles at $q^2 = 0$; $A_3$ can be expressed in terms of $A_1$ and $A_2$:

$$A_3(q^2) = \frac{m_{B_s} + m_V}{2m_V} A_1(q^2) - \frac{m_{B_s} - m_V}{2m_V} A_2(q^2)$$  \hspace{1cm} (24)
In the limit of massless charged leptons the relevant form factors are \( F_1, V, A_1 \) and \( A_2 \). Their calculation by QCD sum rules \(^3\) can be done by considering the three-point correlators

\[
\Pi_\mu(p_{B_s}, p_P, q) = (i)^2 \int d\mathbf{x} \, d\mathbf{y} \, e^{i(p_{B_s} - p_P, y)} \langle 0 \mid T(J_P(x) V_\mu(0) J_B^{\dagger}(y)) \mid 0 \rangle \tag{25}
\]

and

\[
\Pi^\nu A_{\mu\nu} = (i)^2 \int d\mathbf{x} \, d\mathbf{y} \, e^{i(p_{\mu\nu} - p_{B_s}, y)} \langle 0 \mid T(J_V^\nu(x) J^\nu_{\mu A}(0) J_B^{\dagger}(y)) \mid 0 \rangle \tag{26}
\]

where \( J_B^\nu(y) = \bar{s}(y)i\gamma_5 b(y), \) \( J^P(x) = \bar{s}(x)i\gamma_5 q(x), \) \( J^V_{\nu}(x) = \bar{s}(x)\gamma_\nu q(x) \). For \( q = c \) the last two currents interpolate the \( D_s^+ \) and \( D_s^{++} \) meson respectively, whereas for \( q = u, J^V_{\nu} \) interpolates \( K^{*+} \).

The correlators in \(^2\) can be decomposed in Lorentz invariant structures:

\[
\Pi_\mu(p_{B_s}, p_P, q) = (p_{B_s} + p_P)_\mu \Pi + (p_{B_s} - p_P)_\mu \Pi'
\tag{27}
\]

\[
\Pi^\nu_{\mu\nu}(p_{B_s}, p_{V}, q) = \epsilon_{\mu\nu\rho\sigma} p^\rho_F p^\sigma_{B_s} \Pi_V
\tag{28}
\]

\[
\Pi^A_{\mu\nu}(p_{B_s}, p_{V}, q) = i[g_{\mu\nu} \Pi_1 - (p_{B_s} + p_{V})_\mu p_{B_s \nu} \Pi_2 - (p_{B_s} - p_{V})_\mu p_{B_s \nu} \Pi_3 - p_{V \mu}(p_{B_s} + p_{V})_\nu \Pi_4 - p_{V \mu}(p_{B_s} - p_{V})_\nu \Pi_5]
\tag{29}
\]

The saturation of the \( p_{B_s} \) and \( p_{P,V} \) channels by hadronic states provides the hadronic side of the sum rules. For the invariant structures \( \Pi, \Pi_V, \Pi_1 \) and \( \Pi_2 \) the following expressions can be written, keeping the contribution of the lowest lying resonances only:

\[
\Pi^H = \left( \frac{f_B m_{B_s}^2}{m_b + m_s} \right) \left( \frac{f_{P} m_P^2}{m_q + m_s} \right) F_1(q^2) \frac{1}{p_{B_s}^2 - m_{B_s}^2 + i\epsilon} \frac{1}{p_{P}^2 - m_P^2 + i\epsilon}
\tag{30}
\]

\[
\Pi^{H}{V} = \left( \frac{f_B m_{B_s}^2}{m_b + m_s} \right) \frac{m_v^2}{g_v} \frac{2V(q^2)}{m_{B_s} + m_{\nu}} \frac{1}{p_{B_s}^2 - m_{B_s}^2 + i\epsilon} \frac{1}{p_{V}^2 - m_V^2 + i\epsilon}
\tag{31}
\]

\[
\Pi^{I}_1 = \left( \frac{f_B m_{B_s}^2}{m_b + m_s} \right) \frac{m_v^2}{g_v} \frac{(m_{B_s} + m_{\nu}) A_1(q^2)}{m_{B_s} + m_{\nu}} \frac{1}{p_{B_s}^2 - m_{B_s}^2 + i\epsilon} \frac{1}{p_{V}^2 - m_V^2 + i\epsilon}
\tag{32}
\]

\[
\Pi^{I}_2 = \left( \frac{f_B m_{B_s}^2}{m_b + m_s} \right) \frac{m_v^2}{g_v} \frac{A_2(q^2)}{m_{B_s} + m_{\nu}} \frac{1}{p_{B_s}^2 - m_{B_s}^2 + i\epsilon} \frac{1}{p_{V}^2 - m_V^2 + i\epsilon}
\tag{33}
\]

where \( < 0 | J_\mu^V | V(p_{\nu}, \epsilon) > = (m_{V}^2 / g_v) \epsilon_{\mu} \). On the other hand, the correlators can be computed, for \( p_{B_s}^2, p_{P,V}^2 \rightarrow -\infty \), by an operator product expansion in QCD in terms of a perturbative contribution and non perturbative power corrections. For example, the perturbative contribution to \( \Pi \) in eq. \(^2\) reads:

\[
\Pi^P(p_{B_s}^2, p_P^2, q^2) = \frac{1}{\pi^2} \int ds \, ds' \, \frac{\rho_P(s', s, q^2)}{(s' - p_{B_s}^2)(s - p_P^2)}
\tag{34}
\]
by performing a double Borel transform to the variables $M^2$ in the hadronic side. By requiring stability in the variables $M^2$ the sum rule by factorials, and enhances the contribution of the lowest lying resonances the effective thresholds are the same as in the previous section (for the form factors at $Q^2$ the power corrections and in the resonance-continuum contributions, a prediction for $s$ with $\Delta = V$ and the power corrections to $\Pi$ from the relation $f_{D_s}/f_D = [m_{D_s}/g_{D_s}]/[m_{D^*}/g_{D^*}]$, with $g_{D^*} = 7.8$) and $g_{K^*} = 4.3$.

where:

$$\rho_P(s, s', q^2) = \frac{3}{2\chi^2} \left\{ \frac{\chi}{2}(\Delta + \Delta') - \chi m_s (2m_s - m_b - m_u) \right\}$$  \hspace{1cm} (35)

$$\left[ 2(s\Delta' + s\Delta) - u(\Delta + \Delta') \right] \times \left[ m_s^2 - \frac{u}{2} + m_b m_q - m_q m_s - m_b m_s \right]$$  \hspace{1cm} (36)

$$s(s')_+ = \frac{2s'(m_b^2 - m_s^2) - s'(m_s^2 - m_b^2)}{2s' + (m_b^2 - m_s^2 - s')^{\frac{3}{2}} - 4s'm_s^2}$$  \hspace{1cm} (38)

The power corrections to $\Pi$ [33], given in terms of quark and gluon condensates, read:

$$\Pi^{NP} = - \frac{<\bar{s}s >}{2rr'} (m_b + m_q) + \frac{m_b}{4r^2r'} + \frac{(m_b + m_q)(m_b^2 + m_q^2 + Q^2)}{2rr'^3} - \frac{m_q}{2rr'^3} + \frac{m_b}{4r^2r'^2} + \frac{m_{q'}}{4r^2r'^2} + \frac{(m_b + m_q)(m_b - m_q)^2 + Q^2}{2r^2r'^2}$$  \hspace{1cm} (39)

where $r = p_b^2 - m_q^2$ and $r' = p_{B_s}^2 - m_b^2$. The perturbative spectral densities $\rho_V$, $\rho_1$ and $\rho_2$ and the power corrections to $\Pi_V$, $\Pi_1$ and $\Pi_2$ can be found in the appendix.

We improve the matching between the hadronic side and the QCD side of the sum rule

$$\Pi^H = \Pi^P + \Pi^{NP}$$  \hspace{1cm} (40)

by performing a double Borel transform to the variables $M^2$ and $M'^2$ (conjugated to $-p_{P,V}^2$ and $-p_{B_s}^2$). This suppresses the higher order power corrections in the QCD side of the sum rule by factorials, and enhances the contribution of the lowest lying resonances in the hadronic side. By requiring stability in the variables $M^2$, $M'^2$ and hierarchy in the power corrections and in the resonance-continuum contributions, a prediction for the form factors at $Q^2 = 0$ can be obtained. The quark masses, the condensates and the effective thresholds are the same as in the previous section (for $B_s \rightarrow K^*$ we use $s_0 = 1.2 \pm 1.3 \text{GeV}^2$); as for the leptonic constants of the vector mesons, we use $g_{D_s} = 8.3$ (from the relation $f_{D_s}/f_D = [m_{D_s}/g_{D_s}]/[m_{D^*}/g_{D^*}]$, with $g_{D^*} = 7.8$) and $g_{K^*} = 4.3$.  

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The results for the form factors of the transitions \( B_s \to D_s, D_s^* \) at \( Q^2 = 0 \) are collected in Table I (\( F_0(0), A_0(0) \) and \( A_3(0) \) are obtained by eqs. (23, 24)). One can see that these values qualitatively agree with the predictions of the BSW model [31]. As for the \( Q^2 \) dependence, it can be obtained in principle by QCD sum rules. However, to avoid the relevant numerical uncertainties we prefer to assume a polar dependence dominated by the nearest resonance. These resonances are \( \bar{b}c \) mesons whose mass, for the lowest lying states, have been estimated in [34]; the \( 0^+ \) and \( 1^+ \) states are 500 MeV above \( 0^- \) and \( 1^- \), as suggested by the splitting between \( S \) and \( P \) states in the \( D \) channel. In any case, the results for the semileptonic widths, as well as for the non leptonic widths calculated in the following section, are quite insensitive to the exact position of the poles. As for the Cabibbo suppressed transition \( B_s \to K^* \), the results for the form factors at \( Q^2 = 0 \) are:

\[
V(0) = 0.12 \pm 0.02, \quad A_1(0) = 0.3 \pm 0.1 \quad \text{and} \quad A_2(0) \simeq 0; \quad \text{however, a test of the predictions based on these form factors is difficult.}
\]

Using the form factors in Table I we predict, for \( V_{cb} = 0.045 \):

\[
\Gamma(B_s^0 \to D_s^+ \ell^- \bar{\nu}) = (1.35 \pm 0.21) \cdot 10^{-14} \text{ GeV}, \quad (41)
\]

\[
\Gamma(B_s^0 \to D_s^{*+} \ell^- \bar{\nu}) = (2.5 \pm 0.1) \cdot 10^{-14} \text{ GeV}. \quad (42)
\]

An estimate of the \( SU(3)_F \) breaking effects can be obtained by studying the ratios \( F_1(B_s \to D_s)/F_1(B \to D) \), etc., with the result:

\[
\frac{F_1(B_s \to D_s)}{F_1(B \to D)} = 1.12 \pm 0.04 \quad (43)
\]

\[
\frac{V(B_s \to D_s^*)}{V(B \to D^*)} = 1.3 \pm 0.1 \quad (44)
\]

\[
\frac{A_1(B_s \to D_s^*)}{A_1(B \to D^*)} = 0.9 \pm 0.1 \quad (45)
\]

\[
\frac{A_2(B_s \to D_s^*)}{A_2(B \to D^*)} = 1.3 \pm 0.1. \quad (46)
\]

### 4 Two-body non leptonic \( B_s \) decays

We consider the two-body non leptonic \( \bar{B}_s^0 \) decays induced by the effective weak Hamiltonian

\[
H_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_2 q_1}^{*} \left[ c_1 \left( \bar{c}b \right)_L (\bar{q}_1 q_2)_L + c_2 \left( \bar{c}q_2 \right)_L (\bar{q}_1 b)_L \right]. \quad (47)
\]
The Wilson coefficients \(c_1\) and \(c_2\), evaluated at the \(b\)-quark mass scale \(m_b \simeq 5\) GeV, are given by [35]:

\[
c_1(m_b) = 1.1, \quad c_2(m_b) = -0.24.
\] (48)

The usual way to evaluate the matrix element of the operator (47) between the \(\bar{B}_s^0\) state and, e.g., the \(D^+\pi^-\) state is to assume a factorization in the product of the \(\langle D^+_s | (\bar{c}b)_L | \bar{B}_s^0 \rangle\) matrix element and the \(\langle \pi^- | (\bar{d}u)_L | 0 \rangle\) matrix element. One obtains

\[
\langle D^+_s \pi^- \rangle H_W \langle \bar{B}_s^0 \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{q2q1}^* a_1 \langle D^+_s | (\bar{c}b)_L | \bar{B}_s^0 \rangle \langle \pi^- | (\bar{d}u)_L | 0 \rangle,
\] (49)

with \(a_1 = c_1 + c_2/N_c\) (\(N_c\) is the number of colours). In this way the non leptonic amplitude is given in terms of the semileptonic matrix element parameterized in eq. (21) and of the pion leptonic constant \(f_\pi = 132\) MeV. As for the coefficient \(a_1\), the analysis of non leptonic \(B_{u,d}\) decays shows that the rule of discarding \(1/N_c\) corrections should be adopted [34]; we follow this rule and use \(a_1 = c_1 = 1.1\). The relevant leptonic constants are the same as in section 2, or they are fixed from the experimental data. The resulting non leptonic widths for several two body \(\bar{B}_s^0\) decays are collected in Table II; the branching ratios in the same Table are obtained using \(\tau_{B_s} = 1.2\) ps [37].

It is worth observing that the channels with largest branching ratio, e.g. \(\bar{B}_s^0 \to D^+_s \pi^-\) or \(\bar{B}_s^0 \to D^+_s D^-_s\), could be revealed in the LEP experiments [38].

\section*{5 Conclusions}

We have studied several aspects of the \(B_s\) meson phenomenology by QCD sum rules. Our main result concerns the possibility of obtaining the size of the \(SU(3)_F\) breaking effects in this channel; we have shown that the method is sensitive to such effects and can predict them carefully.

The deviation of \(f_{B_s}\) from the leptonic constant of the \(B_d\) meson is around 10\%; such deviation is of the same order as predicted by the Heavy Quark Effective Chiral Theory [14] but its origin is different since in the QCD sum rules approach it must be ascribed to the finite strange quark mass and to the value of the strange quark condensate, whereas in [14] the deviation is connected to chiral loops. \(SU(3)_F\) breaking effects are at 10 – 20\% level in the semileptonic form factors; it should be interesting to compare this result with the prediction of the Heavy Quark Effective Chiral Theory.

Finally, we have calculated the width of several non leptonic \(B_s\) decays; some of them are in the LEP discovery potential.
Appendix

The perturbative spectral densities $\rho$ in eqs. (34) can be computed by applying the Cutkosky rule

$$\frac{i}{k^2 - m^2} \rightarrow 2\pi\delta_+(k^2 - m^2) \quad (1)$$

to the triangle diagrams corresponding to the three point functions in eqs. (25) and (26). For the vector and axial current correlators these spectral densities are as follows:

$$\rho_V(s, s', q^2) = \frac{3}{\chi^2} \{(2s'\Delta - u\Delta')(m_s - m_q) + (2s\Delta' - u\Delta)(m_s - m_b) + m_s\chi\} \quad (2)$$

$$\rho_1(s, s', q^2) = \frac{3}{\chi^2} \{m_b - m_s\}[m_s^2 + \frac{1}{\chi}(s'\Delta^2 + s\Delta'^2 - u\Delta\Delta')] - m_q(m_s^2 - \frac{\Delta'}{2}) - m_b(m_s^2 - \frac{\Delta}{2}) + m_s[m_s^2 - \frac{1}{2}(\Delta + \Delta' - u) + m_bm_q]\} \quad (3)$$

$$\rho_2(s, s', q^2) = \frac{3}{2\chi^2} \{m_b[2s\Delta' - u\Delta + 4\Delta\Delta' + 2\Delta^2] + m_bm_s^2(4s - 2u) + m_q(2s\Delta' - u\Delta') - m_s[2(3s\Delta' + s'\Delta) - u(3\Delta + \Delta')] + \chi + 4\Delta\Delta' + 2\Delta^2 + m_s^2(4s - 2u)] + \frac{6}{\chi}(m_b - m_3)[4ss'\Delta\Delta' - u(2s\Delta\Delta' + s'\Delta^2 + s\Delta'^2)] + 2s(s'\Delta^2 + s\Delta'^2)]\} \quad (4)$$

The non perturbative power corrections can be computed by applying the fixed point technique [23]. The result is:

$$\Pi^{<\bar{s}s>}_{V} = -<\bar{s}s> \{\frac{1}{r^2} - \frac{2m_b^2m_s^2}{r^2r'^3} - \frac{2m_q^2m_s^2}{r^2r'^3} + \frac{m_s^2(m_b^2 + m_q^2 + Q^2)}{r^2r'^2}\} \quad (5)$$

$$\Pi^{<\bar{s}sGs>}_{V} = \frac{1}{6} <\bar{s}s \ g \ Gs> \{\frac{3m_q^2}{r^3r'} + \frac{3m_b^2}{r^3r'} - \frac{2}{r^2r'^2} + \frac{1}{r^2r'^2}(2m_q^2 + 2m_b^2 - m_qm_b + 2Q^2)\} \quad (6)$$
\[ \Pi_{1}^{<\bar{s}s>} = -<\bar{q}q g s> \left\{ \frac{1}{2r} \left[ \frac{1}{2} (m_{b} + m_{q})^{2} + Q^{2} \right] - \frac{m_{s}^{2}}{2} \right\} + \frac{1}{2r} + \frac{m_{s}^{2}}{4} \frac{1}{r^{2}r'^{2}} (m_{b} + m_{q})^{2} + Q^{2} + \frac{m_{s}^{2}m_{q}^{2}}{r^{2}r'^{2}} (m_{b} m_{q} + m_{b}^{2} + m_{q}^{2} + Q^{2}) + \frac{m_{b}^{2}m_{q}^{2}}{r^{2}r'^{2}} (m_{b} m_{q} + m_{b}^{2} + m_{q}^{2} + Q^{2}) + \frac{m_{b}^{2}m_{q}^{2} + Q^{2}}{r^{2}r'^{2}} ((m_{b} + m_{q})^{2} + Q^{2} - \frac{m_{s}^{2}}{2} - \frac{m_{b}^{2}m_{q}^{2}}{2r'^{3}}) \] (7)

\[ \Pi_{1}^{<\bar{s}g G s>} = \frac{1}{12} <\bar{s} g G s> \left\{ \frac{3m_{q}^{2}}{r^{2}r'^{2}} (m_{q}^{2} + m_{b}^{2} + 2m_{b}m_{q} + Q^{2}) + \frac{m_{b}^{2}m_{q}^{2}}{r^{2}r'^{2}} (m_{b} m_{q} + m_{b}^{2} + m_{q}^{2} + Q^{2}) + 2((m_{b}^{2} + m_{q}^{2} + Q^{2})^{2} - m_{b} m_{q}) + \frac{1}{r^{2}r'^{2}} [3m_{b} m_{q} (m_{b}^{2} + m_{q}^{2} + Q^{2})] + \frac{1}{r^{2}r'^{2}} [3m_{b} m_{q} (m_{b} + m_{q}) + 2(m_{b}^{2} + Q^{2})] + \frac{3m_{b}^{2} + 3m_{q}^{2} + 4m_{q}^{2} + Q^{2})] - \frac{2}{r^{2}r'} + \frac{3m_{b}^{2} + 3m_{q}^{2}}{r^{3}r'^{2}} + \frac{3m_{b}^{2} + 3m_{q}^{2}}{r^{3}r'^{2}} + \frac{2}{r^{2}r'^{3}} \right\} \] (8)

\[ \Pi_{2}^{<\bar{s}s>} = -\frac{1}{2} <\bar{s}s> \left\{ \frac{1}{r^{2}r'^{2}} + \frac{m_{b}^{2}m_{q}^{2}}{r^{2}r'^{2}} + \frac{m_{b}^{2}m_{q}^{2}}{r^{2}r'^{2}} \right\} + \frac{2m_{b}^{2} + 2Q^{2}}{r^{2}r'^{2} - m_{b} m_{q})} \] (9)

\[ \Pi_{2}^{<\bar{s}g G s>} = \frac{1}{12} <\bar{s} g G s> \left\{ \frac{3m_{q}^{2}}{r^{2}r'^{2}} + \frac{3m_{b}^{2}}{r^{2}r'^{2}} + \frac{2m_{b}^{2} + 2Q^{2} + Q^{2})m_{s}^{2}}{r^{2}r'^{2}} - \frac{m_{s}^{2}}{r^{2}r'^{2}} \right\} + \frac{2m_{b}^{2} + 2Q^{2} - m_{b} m_{q})} \] (10)
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Table captions

Table I. Values at $q^2 = 0$ of the form factors appearing in the matrix elements of the decays $\bar{B}^0_s \to D^+_s(D^{*+}_s)\ell^-\bar{\nu}_\ell$. The quantum numbers and the mass of the poles which determine the $q^2$ dependence of the form factors is also shown.

Table II. Two-body non leptonic $\bar{B}^0_s$ decay widths. The branching ratios are obtained for $\tau_{B_s} = 1.2 \text{ ps}$. 
Figure captions

**Fig. 1.** Stability analysis for the mass and the leptonic constant of the $B_s$ meson. The solid line corresponds to $s_0 = 33 \text{ GeV}^2$, the dashed line to $s_0 = 34 \text{ GeV}^2$, the dotted line to $s_0 = 35 \text{ GeV}^2$ and the dashed-dotted line to $s_0 = 36 \text{ GeV}^2$. $M$, $f_{B_s}$ and $m_{B_s}$ are in GeV.

**Fig. 2.** Stability analysis for the ratios $\frac{m_{B_s}}{m_{B_d}}$ and $\frac{f_{B_s}}{f_{B_d}}$. The symbols are the same as in fig. 1.
### Table I

$B_s \to D_s, D_s^*$ value at $q^2 = 0$ \quad $J^P$ of the pole \quad pole mass (GeV)

|   |    |    |    |
|---|----|----|----|
| $F_1$ | $0.7 \pm 0.1$ | $1^-$ | 6.3 |
| $F_0$ | $0.7 \pm 0.1$ | $0^+$ | 6.8 |
| $V$   | $0.63 \pm 0.05$ | $1^-$ | 6.3 |
| $A_0$ | $0.52 \pm 0.06$ | $0^-$ | 6.3 |
| $A_1$ | $0.62 \pm 0.01$ | $1^+$ | 6.8 |
| $A_2$ | $0.75 \pm 0.07$ | $1^+$ | 6.8 |
| $A_3$ | $0.52 \pm 0.06$ | $1^+$ | 6.8 |
| Decay mode                  | Width $\times (\frac{V_{cb}}{0.045})^2$ (GeV) | Branching ratio |
|----------------------------|-----------------------------------------------|-----------------|
| $\bar{B}_s^0 \to D_s^{*+}D_s^{-}$ | $9 \times 10^{-15}$                           | $1.6 \times 10^{-2}$ |
| $\bar{B}_s^0 \to D_s^{*+}\rho^-$   | $7 \times 10^{-15}$                           | $1.3 \times 10^{-2}$ |
| $\bar{B}_s^0 \to D_s^{*+}D_s^+$    | $5 \times 10^{-16}$                           | $8 \times 10^{-4}$  |
| $\bar{B}_s^0 \to D_s^{*+}K_s^-$   | $4 \times 10^{-16}$                           | $6 \times 10^{-4}$  |
| $\bar{B}_s^0 \to D_s^+\rho^-$     | $7 \times 10^{-15}$                           | $1.3 \times 10^{-2}$ |
| $\bar{B}_s^0 \to D_s^+a_1$      | $6 \times 10^{-15}$                           | $1.1 \times 10^{-2}$ |
| $\bar{B}_s^0 \to D_s^+D_s^-$     | $6 \times 10^{-15}$                           | $1 \times 10^{-2}$  |
| $\bar{B}_s^0 \to D_s^+D_s^{-}$    | $4 \times 10^{-15}$                           | $8 \times 10^{-3}$  |
| $\bar{B}_s^0 \to D_s^+\pi^-$     | $3 \times 10^{-15}$                           | $5 \times 10^{-3}$  |
| $\bar{B}_s^0 \to D_s^{*+}\pi^-$  | $1 \times 10^{-15}$                           | $2 \times 10^{-3}$  |
| $\bar{B}_s^0 \to D_s^{*+}D_s^-$  | $2 \times 10^{-15}$                           | $4 \times 10^{-3}$  |
| $\bar{B}_s^0 \to D_s^{*+}K^-$    | $1 \times 10^{-16}$                           | $2 \times 10^{-4}$  |
| $\bar{B}_s^0 \to D_s^{*+}D^-$    | $1 \times 10^{-16}$                           | $2 \times 10^{-4}$  |
| $\bar{B}_s^0 \to D_s^+D^-$       | $3 \times 10^{-16}$                           | $5 \times 10^{-4}$  |
| $\bar{B}_s^0 \to D_s^+K^-$       | $2 \times 10^{-16}$                           | $4 \times 10^{-4}$  |
| $\bar{B}_s^0 \to D_s^+D^{*-}$    | $2 \times 10^{-16}$                           | $4 \times 10^{-4}$  |
| $\bar{B}_s^0 \to D_s^+K^{*-}$    | $4 \times 10^{-16}$                           | $6 \times 10^{-4}$  |