A Nonlocal Magneto-curvature Instability in a Differentially Rotating Disk

Fatima Ebrahimi1,2,3 and Matthew Pharr1,3

1 Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543, USA; ebrahimi@princeton.edu
2 Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA
3 Applied Physics and Applied Mathematics, Columbia University, New York, NY 10027, USA

Received 2022 June 18; revised 2022 August 9; accepted 2022 August 10; published 2022 September 9

Abstract

A global mode is shown to be unstable to nonaxisymmetric perturbations in a differentially rotating Keplerian disk containing either vertical or azimuthal magnetic fields. In an unstratified cylindrical disk model, using both global eigenvalue stability analysis and linear global initial-value simulations, it is demonstrated that this instability dominates at strong magnetic fields where local standard magnetorotational instability (MRI) becomes stable. Unlike the standard MRI mode, which is concentrated in the high flow shear region, these distinct global modes (with low azimuthal mode numbers) are extended in the global domain and are Alfvén-continuum-driven unstable modes. As its mode structure and relative dominance over MRI are inherently determined by the global spatial curvature as well as the flow shear in the presence of a magnetic field, we call it the magneto-curvature (magneto-spatial-curvature) instability. Consistent with the linear analysis, as the field strength is increased in the nonlinear simulations, a transition from MRI-driven turbulence to a state dominated by global nonaxisymmetric modes is obtained. This global instability could therefore be a source of nonlinear transport in accretion disks at a higher magnetic field than predicted by local models.

Unified Astronomy Thesaurus concepts: Magnetohydrodynamics (164); Accretion (14); Magnetic fields (994)

Supporting material: animation

1. Introduction

The search for instabilities to explain the accretion rates in flow-dominated astrophysical settings with massive central objects (such as protostars, neutron stars, and black holes) has led to great interest in magnetorotational instability (MRI) (Velikhov 1959; Chandrasekhar 1960; Balbus & Hawley 1991). It was recognized that weak magnetic fields could trigger an MHD instability in astrophysical disks, which could potentially cause turbulent-enhanced viscosity to account for the rapid angular momentum transport (Balbus & Hawley 1998). Analytical and numerical studies using both local shearing box (Hawley et al. 1995; Stone et al. 1996; Fromang et al. 2007a, 2007b; Lesur & Longaretti 2007; Pessah et al. 2007; Bodo et al. 2008) and global (Hawley 2000; Machida et al. 2000; Goodman & Ji 2002) simulations have demonstrated a 3D MHD turbulence state, as well as dynamo sustainment (Brandenburg et al. 1995; Rincon et al. 2007; Lesur & Ogilvie 2008; Ebrahimi et al. 2009) by the MRI. Most of these studies are either limited to the shearing-box approximations or are too complex in full global models (Beckwith et al. 2008) to clarify the underlying physical nature of the global instabilities. In this paper, via a first-principles, intermediate approach, i.e., in a real global domain with a spatial curvature, we examine the onset of global instabilities in rotating systems as well as their nonlinear evolution. We uncover globally extended MHD flow-driven modes, which are absent in the local simulations.

Global axisymmetric MRI (radially uniform channel modes in the shearing-box approximation) and nonaxisymmetric modes are the primary modes in a turbulent-driven MRI state. In particular nonaxisymmetric MRI modes are rather locally concentrated (Matsumoto & Tajima 1995; Ogilvie & Pringle 1996) in the region of the flow shear, while exponentially growing axisymmetric modes (Goodman & Xu 1994) could be the primary driver. Localized nonaxisymmetric MRI mode structures were found to be confined between the Alfvénic resonant points (where the magnitude of the Doppler-shifted wave frequency is equal to the Alfvén frequency; Matsumoto & Tajima 1995). In cylindrical shear flows (Ogilvie & Pringle 1996) and in the compressible limit (Goedbloed & Keppens 2022), it was shown that the forward and backward overlapping of these Alfvén continua results in discrete localized nonaxisymmetric modes at large axial and azimuthal mode numbers. The question arises whether, in a real domain with spatial curvature, global nonaxisymmetric modes with real frequencies can persist. Here, we find distinct global (i.e., large-scale) modes, which are Alfvén-continuum-driven unstable modes due to global differential rotation and magnetic curvature.

In this paper, using a hierarchy of models, we examine the global stability of differentially rotating disk systems threaded with either vertical or azimuthal magnetic fields. We find that due to the spatial curvature, globally extended nonaxisymmetric modes (with a low azimuthal number) are unstable at a wider range of $V_A/V_0$, beyond the MRI stability limit. For a differentially rotating Keplerian disk, unlike the standard MRI mode, which is concentrated in the high-shear region, it is shown that these global modes are extended in the domain (and with two Alfvénic resonant points). We find that due to the inherent presence of global flow shear and curvature, Alfvén continua can provide the free energy for a distinct global mode (even at low axial and azimuthal mode numbers, $k_z$ and $m$). These modes remain unstable at stronger magnetic fields where standard MRI modes are stable. Furthermore, consistent with the linear global analysis, as $V_A/V_0$ is increased, a transition
from MRI-driven turbulence to a much more laminar state, dominated by a global nonaxisymmetric mode, is also observed in the nonlinear global simulations.

We start by introducing our models in Section 1.1. In flow-dominated astrophysical systems, local approximations, due to their simplicity, have been the most commonly used analysis. Here we also first employ the local stability analysis with a general magnetic field in Section 2 and discuss the effect of magnetic and spatial curvature. In Section 3, we present the global sets of equations and the resulting ordinary differential equation (ODE) with both vertical and azimuthal fields. Unlike general global pressure or current-driven instabilities, where an integrated linearized self-adjoint force operator (Friberg 1980) is used to realize the free energy in the flowless MHD, a modified energy principle is required (Hameiri & Chun 1990) with flows (due to the non-self-adjoint property of the MHD force operator as per Frieman & Rotenberg 1960). Using the modified energy principle, we examine the additional energy terms in the differentially rotating system in Section 3. We then present the eigenvalue global solutions for vertical and azimuthal fields. The initial-value nonlinear extended MHD code NIMROD is used to verify the eigenmode solutions in the linear limit. In Section 4, direct nonlinear simulations are presented. We summarize our new findings in Section 5.

1.1. Hierarchy of Models

Here, we employ a hierarchy of physics models and computational techniques for a Keplerian disk, including local and global linear stability analysis, as well as nonlinear simulations. We first examine the local stability analysis using the WKB approximation and obtain a general MHD dispersion relation in a cylinder. Second, the linear global stability of Keplerian flows is studied in the incompressible regime by obtaining an ordinary differential equation (ODE) of the system. Numerical solutions of the global study are presented by both solving the ODE using the shooting method and performing direct linear simulations using the NIMROD code. For completeness and verification, the linear results from shooting and NIMROD simulations are compared in the case with the pure azimuthal magnetic field. We finally present nonlinear results by performing global nonlinear simulations using the NIMROD code.

For our direct numerical simulations of a Keplerian disk, we employ the NIMROD code. The NIMROD (Non-Ideal Magnetohydrodynamics with Rotation, an Open Discussion project) code (Sovinec et al. 2004) solves the 3D, nonlinear, time-dependent, compressible, extended MHD equations. The discrete form of the equations in NIMROD uses high-order finite elements to represent the poloidal plane (r-z) and is pseudo-spectral with Fast Fourier Transforms (FFT) for the periodic direction (φ). The basis functions of the finite elements are polynomials. The finite element spatial discretization allows arbitrary poloidal cross sections and flexibility in the geometry. The full extended MHD model in NIMROD includes resistive, two-fluid, and Finite Larmor Radius (FLR) effects (Zhu et al. 2008; Ebrahimi et al. 2011). In this paper, we model a differentially rotating system with an initial magnetic field in an unstratified Keplerian cylinder (r, z, φ), where a finite element discretization in the poloidal plane (r-z) and a spectral representation in the azimuthal (φ) direction are employed. We use a similar setup as described in Ebrahimi et al. (2011) in both linear and nonlinear MHD simulations. As NIMROD is an initial-value code, in the linear NIMROD computations, the initial conditions consist of an equilibrium (f(r)) plus a mode with a single m perturbation m(1) exp(iφ). Only the mode with a specific m is then evolved; in particular, the equilibrium (f(1)) is not evolved and remains fixed in time.

In fully nonlinear computations, all modes (axisymmetric m = 0 and nonaxisymmetric m ≠ 0) are initialized with small random amplitudes and are evolved in time, including the full nonlinear terms. An impenetrable radial boundary condition is used, while the azimuthal and vertical (z) directions are periodic. Starting with a current-free equilibrium (by applying a uniform magnetic field or a pure azimuthal field), the disk rotates azimuthally with a mean Keplerian flow V(φ(r)) ∝ r⁻¹/². The set of NIMROD simulations with the relevant dimensionless parameters used in this paper, namely the magnetic Reynolds, Lundquist number, and magnetic Prandtl numbers, are given in Table 1.

2. Local Stability Analysis

We begin with the momentum and induction equations:

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \eta \nabla \times \mathbf{J}, \quad (1) \]

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \frac{\mathbf{B}^2}{\mu_0} + \rho \nu \nabla^2 \mathbf{v}, \quad (2) \]

where v, B, J, ρ, η, and ν are the fluid velocity, magnetic field, current, pressure, and electric and kinetic diffusivities, respectively. We consider perturbed quantities in the form of \( \mathbf{V}(r, t) = \mathbf{V}_0 \exp \{i(k \cdot r - \omega t)\} \), defining \( \omega = \omega_r + i\gamma \) and \( k \cdot r = -ik_z r + m\phi + k_z z \). We have chosen this construction for k so that for real k_r, the solutions will be evanescent in the radially outward direction and to eliminate what would otherwise be complex terms in Equations (8)–(11). We assume an equilibrium with \( \mathbf{B}_0 = B_0(r) \hat{\mathbf{e}}_\phi + B_z \hat{\mathbf{e}}_z \) and \( \mathbf{v}_0 = v_0(r) \hat{\mathbf{e}}_\phi \), where the radially dependent quantities are constrained by an assumption of a currentless magnetic field and a Keplerian flow profile. Additionally, we chose to assume a constant radial density for the sake of simplicity in investigating the global behaviors of Alfvénic resonance instabilities; much complexity could be added by considering different density profiles, so we leave it as a topic for future study:

\[ i\omega \mathbf{B}_0 + 0\mathbf{B}_0 + (iF)\mathbf{v}_0 + 0\mathbf{v}_0 = 0, \quad (3) \]

\[ \left( \frac{\partial \Omega}{\partial \ln r} \right) \mathbf{B}_0 + i\omega \mathbf{B}_0 + \left( \frac{2B_0^2}{r} \right) \mathbf{v}_0 + (iF)\mathbf{v}_0 = 0, \quad (4) \]

\[ iF \rho \left( 1 - \frac{k_z^2}{\rho \rho_0} \right) \mathbf{B}_0 + \left( \frac{Fk_z k_z}{\rho_0^2} - \frac{1}{r} \right) \mathbf{B}_0 \hat{\mathbf{e}}_\phi \]

\[ + i\omega \left( 1 - \frac{k_z^2}{\rho \rho_0} \right) \mathbf{v}_0 + \left( 2\Omega + \frac{2k_z k_z}{k_z^2} \right) \mathbf{v}_0 = 0, \quad (5) \]
The Astrophysical Journal, 936:145 (12pp), 2022 September 10

Ebrahimi & Pharr

Table 1
Parameters Used in Global NIMROD Simulations

| Model                        | $Rm = VL/\eta$ | $Pm = \nu/\eta$ | $S = V_\phi L/\eta$ | Resolution | $L/R$ |
|------------------------------|---------------|----------------|---------------------|------------|-------|
| Nonlinear global (VNF-MRI)   | 12,500        | 1             | 2500                | 40 × 40 deg. | 4 (43 modes) | 2   |
| Nonlinear global (VNF-C)     | 25,000        | 1             | 14,600              | 40 × 40 deg. | 43 modes)    | 2   |
| Nonlinear global (PAF-MRI)   | 12,500        | 1             | 6,500               | 40 × 40 deg. | 86 modes)    | 2   |
| Nonlinear global (PAF-C)     | 15,500        | 1             | 17,000              | 80 × 80 deg. | 4 (22 modes)  | 2   |
| Linear global                | 25,000        | 1             | 12,000–20,000       | 80 × 80 (160 × 160) deg | 4 (m = 1) | 1.2 |

Note. VNF = Vertical net flux, PAF = pure azimuthal (toroidal) field, and here C denotes curvature modes.

\[
\frac{F}{\mu_0 \rho} \left( \frac{k_x \bar{k}_r}{k_z^2} \right) \bar{B}_r + \frac{if}{\mu_0 \rho} \left( 1 + \frac{k_z^2}{k_c^2} \right) \bar{B}_\phi + \left( \omega - \frac{k_x^2}{k_c^2} \right) \bar{B}_r + \frac{i \omega}{k_c} \frac{k_z^2}{k_c^2} + 1 \bar{B}_\phi = 0, \tag{6}
\]

where $k_\phi \equiv m/r$, $k_r \equiv (k_r + \delta_r/r)$, $F \equiv k \cdot B = k_x B_x(r) + k_y B_y$, and $\kappa$ is the epicyclic frequency, where $\kappa^2 \equiv 4\Omega^2 + \frac{\partial^2 \Omega}{\partial \ln r}$. Notably, for quasi-Keplerian disks being examined in this paper, $\kappa = \Omega$. Using the curvilinear definition of $\nabla$ in all derivatives gives rise to the curvature term for the toroidal magnetic field, $2 B_z/r$, accompanied by $\delta_r$. The notation $\delta_r$ is placed to denote terms that disappear in the local Cartesian approximation, $\delta_r \to 0$. We note that due to the current-free condition, $-\frac{\partial B_r}{\partial r} = B_\phi/r$. We have let the dissipative terms go to zero and denoted $\bar{\omega} = \omega - m \Omega(r)$, the Doppler-shifted frequency. For nonzero perturbations, it is clear that the determinant of the coefficients of Equations (3)–(6) must be zero, giving us a fourth-order polynomial equation as our dispersion relation, relating $k$ and $\bar{\omega}$,

\[
\bar{\omega}^4 + C_3 \bar{\omega}^3 + C_2 \bar{\omega}^2 + C_1 \bar{\omega} + C_0 = 0, \tag{7}
\]

where

\[
C_3 = \frac{2 \kappa k_x \bar{k}_r \Omega}{k^2} - \frac{\kappa^2 k_x k_r}{2 \Omega}, \tag{8}
\]

\[
C_2 = -\kappa^2 \frac{k_x^2}{k_c^2} - 2 \omega_\Lambda^2 - \frac{1}{2} \omega_\Lambda \omega_c \delta_c \frac{k_x k_r}{k_c}, \tag{9}
\]

\[
C_1 = \frac{1}{2} \omega_\Lambda \omega_c \delta_c \frac{\partial \Omega}{\partial \ln r} \delta_c \frac{k_x^2}{k_c^2} + k_x \bar{k}_r + \frac{\kappa^2}{2 \Omega} \left( \omega_\Lambda^2 \frac{k_x^2}{k_c^2} \right) + \frac{1}{2} \omega_\Lambda \omega_c \delta_c \frac{k_w k_r^2}{k_c^2} \delta_c \frac{k_x k_r}{k_c}, \tag{10}
\]

\[
C_0 = \omega_\Lambda^4 + \frac{\partial \Omega^2}{\partial \ln r} \omega_\Lambda^2 \frac{k_x^2}{k_c^2} + k_x \bar{k}_r + \frac{1}{2} \omega_\Lambda^2 \omega_c \delta_c \frac{k_x k_r}{k_c}, \tag{11}
\]

and

\[
\omega_\Lambda \equiv \frac{k \cdot B_0}{\sqrt{\mu_0 \rho}} = \frac{1}{\sqrt{\mu_0 \rho}} \left( k_x B_x + \frac{m}{r} B_\phi \right), \quad \omega_c \equiv \frac{2 B_\phi}{r \sqrt{\mu_0 \rho}} \left( k_x^2 + k_\phi^2 - k_r \bar{k}_r \right) \tag{12}
\]

In the limit of $k_\phi$, $\delta_r$, $\bar{k}_r \to 0$, the local dispersion relation for axisymmetric MRI (Balbus & Hawley 1991) is obtained. As we are interested in the largest-scale solutions, we will make no assumptions regarding the magnitudes of $k$ or $B$. Figure 1 shows the WKB solutions for growth rates in the $k-\nu$ plane for both pure toroidal (Figures 1(a) and (b)) and vertical (Figures 1(c) and (d)) magnetic fields. The pronounced effect of the curvature terms is observed best in the toroidal field case, where there are additional differences that arise from the curvature of the magnetic field, but is also subtly observable in the vertical field case. It is important to note that for real disks, both $m$ and $k_r$ are quantized by boundary conditions. For the sake of matching with simulations we will later present, we assume the boundary conditions are periodic both toroidally and vertically, and the disk has height equal to its width $r_2 - r_1$, where $r_2$ and $r_1$ are the outer and inner radii. This gives us a minimum value for $k_z$ of $k_z \equiv 2\pi/2(r_2 - r_1)$. Solutions matching the vertical periodic boundary condition are integer multiples $k_z = nk_z^0$.

Due to the variance of equilibrium quantities in the radial direction, $k_r$ is a very poor choice for a quantum number, as radially sinusoidal eigenfunctions do not portray the fact that flow shear and field-line bending change as a function of radius. Additionally, one would expect waves propagating in a disk to move with the disk, but such eigenfunctions would be broken up by flow shear. This motivates the construction of a better model for this system that utilizes an ordinary differential equation to find the eigenfunctions and complex frequencies through a shooting method.

3. Global Stability Analysis and the Energy Principle

Now we examine the linear global stability of our system by allowing the radial variation of the linear perturbations. In the ideal limit (in the absence of magnetic and fluid diffusivities), the velocity and magnetic perturbations are expressed in terms of the displacement vector $\xi(r, \phi, z, t) = (\xi_x(r), \xi_y(r), \xi_z(r)) \exp(i m \phi + k_z t - \omega t)$ (Chandrasekhar 2006) as $\bar{v} = -i \bar{\omega} \xi - r \frac{\partial \Omega}{\partial \ln r} \bar{\xi}_r + \bar{B} = i(k \cdot B) \xi + \frac{2k_r}{r} \bar{\xi}_r e_\phi$, respectively.

Combining with incompressibility, $\nabla \cdot \xi = \frac{1}{r^{\partial \Omega/\partial \ln r}} (r \xi_r) + \frac{ik}{r} \bar{\xi}_r + \frac{2k_r}{r} \bar{\xi}_r e_\phi$, respectively.
The Astrophysical Journal, 936:145 (12pp), 2022 September 10

Ebrahimi & Pharr

(a) \( \psi \) vs. \( \psi \). (b) \( \psi \) vs. \( \psi \). (c) \( \psi \) vs. \( \psi \). (d) \( \psi \) vs. \( \psi \).

Figure 1. Contour plots of normalized WKB growth rates from (Equations (3)–(6)) in the \( k_{l}/k_{t} \) and \( V_{A}/r_{S} \) plane for \( k_{l} = 0 \) (a) without \( \delta_{r} \) (in the Cartesian limit) (b) with \( \delta_{r} = 1 \) with a purely toroidal magnetic field. (c) and (d) are equivalent instead with a purely vertical magnetic field.

The equation:

\[
(-\omega^2 + \omega_A^2 + \omega_c^2 + \omega_{\xi}^2)\xi_r + [i\omega_A \omega_c \delta_r + 2i\omega \Omega(r)]\xi_\phi = -\frac{\partial P}{\partial r},
\]

(13)

\[
(-\omega^2 + \omega_A^2)\xi_\phi - i[2\omega \Omega(r) + \omega_A \omega_c \delta_r] \xi_r = \frac{i m P}{r},
\]

(14)

\[
(-\omega^2 + \omega_A^2)\xi_c = -i k_c P,
\]

(15)

where \( P \equiv \frac{1}{\rho} \left( p + \frac{1}{\rho} \mathbf{B} \cdot \mathbf{B} \right) \) and \( \omega_r^2 \equiv \frac{\partial \Omega^2}{\partial \ln r} \). By combining Equations (13)–(15), an ordinary differential equation is then obtained:

\[
\left[ \frac{(\omega - \omega_A^2)}{k^2 x + m^2} \xi_r \right]'' + \left[ m \left( \frac{\Omega \omega_c + \frac{1}{2} \omega \omega_A}{k^2 x + m^2} \right) \right] \xi_r
\]

\[
- \frac{(\omega - \omega_A^2 - \omega_c^2 - \omega_{\xi}^2)}{4 x} + \left( \frac{\Omega \omega_c + \frac{1}{2} \omega \omega_A}{k^2 x + m^2} \right)^2 \frac{k^2}{\omega^2 - \omega_A^2}
\]

\[= 0,
\]

(16)

where we have defined \( u \equiv r \xi_r \) and \( x \equiv \xi_r \). In the limit of \( \omega_c \to 0 \), Equation (16) is reduced to the ordinary differential equation found in Khalozov et al. (2006) with only a vertical magnetic field. The variable substitutions here were made for compactness and simplicity, though the equation in terms of the original variables is easily recovered for comparison with other work. It should be mentioned that other forms of differential equations for rotating plasmas (Hameiri 1981), local pressure-driven instabilities (Bondeson et al. 1987), current-carrying disks (Ebrahimi & Prager 2011), and more general astrophysical disk equilibrium (Keppens et al. 2002; Blokland et al. 2005) were also previously obtained. Here, we solve this equation using a complex shooting method, employing a zero-finding algorithm to obtain the eigenfunctions and their eigenvalues \( \tilde{\omega} \) (the sign of whose imaginary part signifies instability) as a function of \( k \), flow shear, \( m \), and \( B_0 \). In this paper, we will focus mostly on the lowest nonaxisymmetric values of \( k \) and \( m \), though this method could enable the study of higher modes, different configurations of mixed fields, and different flow profiles in the future. Before we present the numerical global solutions of Equation (16), we first revisit the energy principle for a general global stability criterion.

The stability of static ideal plasmas can generally be determined using the ideal MHD energy principle (Bernstein et al. 1958; Freidberg 1980; Schnack 2009) without solving differential equations. The stability criterion is constructed based on the sign of the energy integral, the volume integral \( \int \xi \cdot F_0(\xi)dx^3 \), where \( F_0(\xi) \) is the self-adjoint static force operator obtained from the linearized momentum equation. However, in the presence of an equilibrium mean flow, self-adjointness of the linear stability problem is lost, and the stability criterion was reconstructed with nonzero flows by Frieman & Rotenberg (1960). In flowing plasma, the total force operator in terms of Lagrangian displacement (i.e., the displacement of a fluid element moving with the equilibrium flow) was given as \( F(\xi) = F_0(\xi) + F_1(\xi) = F_0(\xi) + \nabla \cdot [\rho \xi(\hat{\mathbf{r}} \cdot \nabla) \hat{\mathbf{r}} - \rho \hat{\mathbf{r}} \cdot \nabla \xi] \), where the modification due to flows has been introduced through \( F_1(\xi) \) with the two additional flow-dependent terms. The sufficient stability condition (for \( \xi \) not to have a positive growth) was found to be \( \int \xi \cdot F(\xi)dx^3 < 0 \) (Frieman & Rotenberg 1960). For our differentially rotating system here, we expand \( F_1(\xi) \).

\[
F_1(\xi) = \nabla \cdot \Gamma = \nabla \cdot [\rho \xi(\hat{\mathbf{r}} \cdot \nabla) \hat{\mathbf{r}} - \rho \hat{\mathbf{r}} \cdot \nabla \xi] \quad (17)
\]

where in the curvilinear cylindrical coordinate with only an azimuthal flow, it reduces to

\[
F_1(\xi) = \nabla \cdot \Gamma = \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_{\phi}) - \frac{\Gamma_{\phi}}{r} \right] \hat{r}
\]

\[+ \left[ \frac{1}{r} \frac{\partial}{\partial \phi} (\Gamma_{\phi}) \right] \hat{\phi}
\]

\[= \rho \left[ -d \frac{\Omega^2}{d \ln r} \xi_r + \frac{\omega_c^2}{r^2} (r \xi_r)' - i k_c \xi_c \frac{v_r^2}{r} \xi_r \right]
\]

\[\hat{r} - \rho \frac{v_r^2}{r^2} (im) \xi_r \xi_c \hat{\phi} \quad (18)
\]

Combining Equation (18) with incompressibility, \( \nabla \cdot \xi = \frac{1}{r} \frac{\partial}{\partial r} (r \xi_r) + \frac{im}{r} \xi_{\phi} + ik \xi_c = 0 \), the total energy integral of Frieman & Rotenberg (1960) due to flow-dependent terms becomes

\[
\int \xi^* F_1(\xi) dx^3 = \delta w_t + \delta w_H
\]

\[= \int \rho \left( -\frac{\partial \Omega^2}{\partial \ln r} |\xi_r|^2 + im \Omega^2 |\xi_r\xi_{\phi}^* - \xi_{\phi}\xi_r^*| \right) dx^3 \quad (19)
\]

Interestingly, the first term on the right-hand side (RHS) of Equation (19) is the differential rotation term, which similarly to the local WKB dispersion relation \( \omega^2 < 0 \) in Equation (11) results in MRI instability. The first term causes instability in systems with angular velocity a globally decreasing function of radius. We note that the additional second term on the RHS
however, only arises due to the effect of spatial curvature (here in cylindrical geometry) and can be obtained in the boundary of the mean equilibrium flows. By combining Equations (14) and (15) and the incompressibility, we obtain

\[
\xi_\phi = \frac{r^2 k_z^2}{k_0^2} \left[ \frac{2i\Omega(r)\omega}{(\omega_A^2 - \omega^2)}\xi_r + \frac{i\omega_k \omega_d \xi_r}{(\omega_A^2 - \omega^2)} \xi_r \right. \\
+ \left. \frac{im}{r^2 k_z^2} (r\xi_r)' \right],
\]

(20)

where \( k_0^2 = (r^2 k_z^2 + m^2) \). Inserting this equation into Equation (19), we find

\[
\delta W_H = \int \left[ m\Omega^2(r) - \frac{2p^2 r^2 k_z^2}{k_0^2} \frac{2m\Omega(r)\omega + \omega_k \omega_d \xi_r}{(\omega_A^2 - \omega^2)} \right] |\xi_r|^2 \\
- \rho m^2 \left\{ (r\xi_r)'\xi_r^* + \xi_r (r\xi_r')^* \right\} dx^3.
\]

(21)

This introduces yet another source of free energy for the instability due to nonzero flows, i.e., causes a positive contribution to \( \int \xi \cdot F(\xi) dx^3 \) when \( \omega_A^2 < \omega^2 \). This term (Equation (21)) is destabilizing due to (1) nonzero nonaxisymmetric modes (i.e., \( m = 0 \)) and (2) nonzero averaged global angular velocity and magnetic curvature.

Alternatively, in addition to considering the Frieman & Rotenberg energy term (Equation (19)), we can directly calculate the energy terms from the sets of momentum equations (Equations (13)–(15)). Multiplying Equations (13)–(15) by \( \xi_r^* \), \( \xi_r \), and \( \xi_r^* \), respectively, and integrating over the volume, we obtain

\[
\int \left[ (\omega_A^2 - \omega^2) |\xi_r|^2 + \omega_A^2 |\xi_r|^2 \right. \\
+ \left. 2i\Omega(r) (\xi_r \xi_r^* - \xi_r^* \xi_r) \right] rdr = 0.
\]

(22)

In the absence of flow, using the incompressibility condition and Equation (20), from this equation, the general \( \delta W \) condition for a current-free cylinder with magnetic field is obtained \( \int \omega_A^2 |(r\xi_r)'|^2/ k_0^2 + |\xi_r|^2 \right\} rdr \) (Schnack 2009). With flow and nonaxisymmetric perturbations, additional terms from Equation (19) contribute to the energy. The contributions of the individual terms in the energy equation will be examined in a future study.

3.1. Global Solutions with a Vertical Magnetic Field

First, we present the solutions obtained from Equation (16) with only the vertical magnetic field and Keplerian flow (see equilibrium profiles in Figure 2). We use a complex shooting method in which we consider the solution at the boundary (which we refer to as the complex and imaginary “tails”) as a function of complex frequency \( \omega \) (see Figure 3). The modes are obtained when \( \xi \) goes to zero at the right-hand boundary, enforcing the boundary condition. The growth rates and frequencies of several unstable \( m = 0 \) and \( m = 1 \) modes with positive \( \gamma \) versus the normalized Alfvén velocity \( (V_A/r_1)\Omega_1 \) are shown in Figure 4. As expected with only the vertical magnetic field, the most unstable MRI modes are the axisymmetric modes, which span from weak to strong fields. However, we find two sets of distinct nonaxisymmetric \( m = 1 \) instabilities with different mode structures. The first, which resemble usual localized MRI modes (inner modes), centered about the point of maximum flow shear, we refer to as such; the second, which can feature more global structure at lower magnetic fields, we refer to as curvature modes (outer modes). The MRI and curvature modes for \( m = 1, k = k_1 \), at small magnetic fields are shown in Figures 5(a) and (b), respectively.

As seen in Figure 4, we also uncover how the new global curvature mode remains unstable to much higher magnetic fields. The nonaxisymmetric modes, as discussed in Tajima & Shibata (1997) and Ogilvie & Pringle (1996), are localized between the two Alfvén resonance points, where

\[
\Re(\bar{\omega})^2 - \omega_A^2 = (\omega_A - m\Omega)^2 - \omega_A^2 \\
= (\omega_A - m\Omega - \omega_A)(\omega_A - m\Omega + \omega_A) = 0.
\]

(23)

These are the points where the magnitude of the Doppler-shifted frequency of the mode is equal to its Alfvén frequency and are always centered about the point of corotation, where \( \omega = \Omega(r) \). In particular, localized MRI and curvature modes in systems with weak magnetic fields (i.e., super-Alfvénic flows) feature structures localized in the inner and outer parts of the system as also obtained by Ogilvie & Pringle (1996) and Goedbloed & Keppens (2022), due to the differing locations of the Alfvén singularities. Inner (MRI) modes are located between the conducting boundary at \( r_1 \) and the singularity \( r_{\text{out}} \); outer (curvature) modes are between the singularity \( r_{\text{in}} \) and the conducting boundary at \( r_2 \) (Figures 5(a) and (b)). We find that increasing the magnetic field results in both modes taking on more global properties, though this is especially and more rapidly true for the curvature modes. Global versions of the curvature mode can be seen in Figures 5(e) and (f) with increasing normalized Alfvén flows. The comparison of mode structures for MRI and the curvature mode with exactly the same sets of parameters \( (k = 2k_1, V_A = 0.193) \) can also be seen in Figures 5(d) and (e). The frequencies of these two modes also exhibit differing properties. For low fields, each mode’s frequency reflects the angular velocity of the system at the mode’s local position and converges on a moderate frequency as fields grow. This can be seen in Figure 4(b), where the MRI mode’s frequency in the low-\( B \) limit converges to \( \Omega_0 \), the system’s angular velocity at the inner wall, and the
curvature mode’s frequency similarly converges to $\Omega_0 (r_1/r_2)^3/2$ (0.0894$\Omega_0$ in our system), the angular velocity at the outer wall.

We also search in the limit of super-Alfvénic flows where $\omega_A \ll m\Omega$ as studied by Ogilvie & Pringle (1996) and Goedbloed & Keppens (2022). In this region, the high-$m$, high-$k$ nonaxisymmetric Alfvénic resonance instability is extremely localized due to being confined between two very close-by Alfvénic resonances. This can be seen in Figure 6 with $m = 10$ and $k = 80k_1$. The complex eigenvalues are similar to their approximate counterparts in Goedbloed & Keppens (2022). Regardless of the use of a global treatment, these extremely localized instabilities only occupy a narrow region of parameter space, as they are only unstable for very weak magnetic fields (i.e., super-Alfvénic flows). For the remainder of this paper, we will focus on the most global nonaxisymmetric instabilities that are unstable in a far wider range of magnetic fields, from small fields to fields with corresponding Alfvén velocities on the order of $r_1\Omega_0$.

To further elucidate the distinct nature of the MRI and curvature modes, we also present the complex solutions in a 2D plane in Figure 3, where they can be seen as two discrete families of solutions: a low-frequency (outer) curvature mode, and the right-hand family of solutions is the high-frequency (inner) MRI mode. Both figures use a solely vertical background field and $m = 1$. Numbers show $n_r$. For $m = 0$, these modes are characterized by the number of times they cross the $x$-axis ($n_r$); however, nonaxisymmetric modes are necessarily complex, and therefore, this definition loses its meaning, though we may keep this labeling convention defined by the value of $n_r$ each mode takes on when quasi-continuously changed from $m = 1$ to $m = 0$. We
may additionally verify the applicability of Equation (23); for the low-field case in Figure 3(a), the Alfvénic points are calculated based on the inner and outer angular velocities. In the MRI (inner) case, the two Alfvénic points are located at $\omega_{RMS(inner)} = 0.68, 1.31$. In the curvature (outer) case, the two Alfvénic points are $\omega_{RMS(outter)} = -0.22, 0.40$. This gives two distinct ranges for the frequencies of inner and outer modes, which can be seen in the two fractal structures in Figure 3(a). This also holds for the higher-field case in Figure 3(b), though care must be taken as all modes in the high-field case are more global than their lower field counterparts and therefore $\Omega$ is an intermediate as opposed to a boundary value. Therefore, the range of real frequencies in these modes is consistent with Equation (23), as modes with different frequencies are located at different radial points with different values of $\Omega$.

The eigenfunctions and eigenfrequencies exist in a phase space defined by considering $m, k,$ and $V_A$ as continuous quantities, though the first two are discrete in a real system. To further explore the main differences between the nonaxisymmetric MRI and curvature modes, we perform an exercise by gradually varying $m = 1 \rightarrow m = 0$ starting from the same state (i.e., $m = 1, k = 2k_1, V_A = 0.193r_1\Omega_0$). This is shown in Figure 7. We find that both modes will transition to a global axisymmetric state as shown in Figure 5(c). This shows that one mode can be obtained from the other by continuously varying azimuthal wavevector. However, there is only a unique path for the local $m = 1$ MRI mode (point 7) to transition to the $m = 0$ MRI mode (as shown in red in Figure 7), while the transition from the nonaxisymmetric curvature mode (point 1) is nontrivial. Immediately varying $m$ from point 1 results in the mode becoming stable in between $m = 1$ and $m = 0$ (this trajectory is shown in blue). This is avoided by first changing $k$ and $V_A$ as detailed in the table in Figure 7(b). This path, as shown in green, also results in an axisymmetric MRI mode, which, upon varying $k$ and $V_A$ back to their original values, converges with the mode obtained from point 7. This point of convergence is at point 6.

To further corroborate the hypothesis that the curvature mode is driven by the global derivative terms in the WKB analysis, we will now examine what happens as we change the size of our system (preserving the cross-sectional dimensions $h/(r_2 - r_1)$). As the aspect ratio $r_1/(r_2 - r_1)$ of our system grows, the radially dependent terms vary increasingly little across our system and the average curvature of motion goes down with $1/r_1$, leading to a quasi-Cartesian approximation. Conversely, upon shrinking the system aspect ratio, radially dependent terms vary increasingly more and the average curvature of the flow grows asymptotically, leading to a system dominated by the curvature terms.

Figure 5. Mode structures for the vertical magnetic field from the shooting method. (a) and (b) $m = 1$ mode at low field showing MRI and curvature mode localization between Alfvénic points at the inside/outside of the disk, respectively; (c) and (d), (e) at intermediate field in Figure 7 at (c) point 6, the point of convergence, (d) point 7, the $m = 1$ MRI mode, and (e) point 1, the $m = 1$ curvature mode (outer Alfvénic resonance not pictured at $r_{out} = 20.3r_1$). (f) at high field for the most global curvature mode for $m = 1 k = k_1$ ($\gamma = 0.021, \omega_r = 0.485$ and Alfvénic resonances at $r_m = 1.04r_1, r_{out} = 10.3r_1$).

Figure 6. (a) high-field and (b) low-field modes shown here in the super-Alfvénic region, i.e., high $m (m = 10)$. We find that at a high toroidal mode number and high $k (k = 80k_1)$, the range of magnetic fields to which the system is linearly unstable to a perturbation is narrowed to the low-field region. This can be seen in comparison with Figures 4, 8, and 9 varying $V_A$ for $m = 1$.
The high-aspect-ratio case is seen in Figure 8(a). It is apparent that as the system approaches quasi-Cartesian geometry, the curvature mode is dominated by an everywhere-more-unstable MRI mode. The low-aspect-ratio case follows in Figure 8(b), where it can be seen that the range of magnetic fields where the MRI mode is dominant shrinks and the curvature mode dominates for most field values. This justifies our labeling of this mode as one caused by curvature effects.

### 3.2. Global Solutions with a Purely Azimuthal Magnetic Field

In the presence of a purely azimuthal magnetic field, the nonaxisymmetric modes are the only accretion disk modes unstable (Terquem & Papaloizou 1996) to Keplerian shear flows. The global curvature effect of the system and magnetic field introduces additional radially dependent terms as shown in the WKB approximation in Section 2. Here, we numerically search for the global structures of nonaxisymmetric modes through both the shooting method and linear simulations with the NIMROD code (Sovinec et al. 2004). This would provide confidence in the results, as we would verify our solutions with both numerical techniques (shooting and NIMROD simulations). We start the simulations with a Keplerian flow and azimuthal magnetic field in both eigenvalue shooting and initial-value NIMROD simulations.

#### 3.2.1. Solutions from Global Eigenvalue and Linear Simulations

Figure 9 shows the growth rates and frequencies of the $m = 1$ modes obtained from shooting and the NIMROD simulations. The solid curves show the results from the shooting method, and the numerical points are obtained by performing linear NIMROD simulations with the same initial state. By varying the normalized Alfvén velocity (through changing $B$) we observe that the growth rates of the MRI mode show a two-hump curve similar to what was obtained via the dispersion relation shown in Figure 1(b). This is similar to what is shown in the growth rates of the NIMROD points. However, the mode
structures of the first 10 points (at low-field $V_A/V_0 < 0.8$) resemble a system with $k = 8k_1$ as shown in Figure 10(b). which is consistent with the localized mode structure obtained from the shooting solutions at $k = 8k_1$ (Figure 10(a)). The 11th point ($V_A/V_0 \sim 0.86$) is transitional. Figure 10(e) shows a local mode structure near the interior of the disk and a global mode structure at the edge of the disk, with each growing at similar magnitudes. This can be understood as the Alfvén velocity at which the MRI and curvature modes have growth rates of the same order. The points at Alfvén velocities above this show a dominant global mode, as seen in Figures 10(c) and 10(d) for $V_A/V_0 \sim 1.157$. This would indicate that for sufficiently high magnetic fields, the MRI mode is subject to higher dissipation in NIMROD simulations due to its local structure and higher vertical wavenumber (and loses more of its free energy at high fields from field-line bending) and therefore the $k = k_1$ curvature mode dominates.

Similar to the instability with a vertical magnetic field in Section 3.1, here with only a purely azimuthal $B$ we also observe distinct global mode structure at high $B$. This is due to the curvature of the system (including the magnetic field). The most global mode structure is obtained with the curvature mode as shown in Figure 10(c). The equivalent contour plots of the $m = 1$ solutions obtained from NIMROD simulations are also shown in Figure 10. Similarly, the transition from a local MRI (Figure 10(b)) to a global curvature mode (Figure 10(d)) is observed. Interestingly, unlike eigenvalue shooting, NIMROD also captures the most unstable solutions in the 2D $R$–$Z$ plane.
for a specific azimuthal number (here for \( m = 1 \)), we observe a mixed solution of local MRI and global curvature at the transition point (\( V_A = 0.868r_1\Omega_0 \)) shown in Figure 10(e). We therefore find that both the global shooting method and NIMROD solutions provide the same eigenvalues (growth rates and frequencies) as well as mode structures. A transition from a more standard MRI mode to a very global mode structure (curvature mode) is obtained in both analyses.

Here, we have not only verified the NIMROD solutions with the solutions obtained from Equation (16), but both models have also produced new physics. This means that in many circumstances, the curvature mode is newly found to be dominant and therefore contributes to momentum transport in differentially rotating systems.

4. Full Nonlinear Simulations

To further investigate the transition from MRI-like modes to global curvature modes, we have performed 3D nonlinear MHD simulations. In contrast to the quasi-linear simulations where only one specific azimuthal mode number is linearly evolved (Section 3), here we evolve the global domain in the azimuthal direction by including many nonzero Fourier azimuthal modes (43 or 22 Fourier modes). Figures 11(a)–(d) show the evolution of the total volume-averaged magnetic energies for simulations started with a vertical net flux (Figures 11(a) and (b)) and a purely azimuthal field (Figures 11(c) and (d)). Consistent with the linear \( m = 1 \) calculations (Figure 4), in the simulations started at a low field where the localized MRI modes are dominant, we obtain several nonaxisymmetric MRI modes that grow and saturate to form a more turbulent state (Figure 11(a)). A poloidal cross section of the turbulent structure during the saturation is shown in Figure 11(e). This nonlinear simulation with a weak vertical nonzero flux (VNF-MRI in Table 1), with a dominant primary \( m = 0 \) growth and the subsequent nonaxisymmetric modes, was extensively studied in Rosenberg & Ebrahimi (2021). However, for simulations started at a strong toroidal field (at \( V_A = 0.58r_1\Omega_0 \), VNF-C in Table 1 and shown with the star marker in Figure 4), the global \( m = 1 \) curvature mode grows linearly first and saturates to form a more laminar state (Figure 11(b)). The cross section of the modal structure with a dominant global \( m = 1 \) mode late during the saturation is shown in Figure 11(f).

Similarly, for a full nonlinear simulation with a purely toroidal magnetic field, we obtain a transition from a turbulent to a laminar state as normalized magnetic field is increased. Figures 11(c) shows the magnetic energy of several localized MRI nonaxisymmetric modes growing to large amplitudes, while for simulations at a high field the magnetic field is dominated by a global \( m = 1 \) curvature mode (Figure 11(d)). Similarly, a transition from a turbulent to a laminar saturated state dominated by the \( m = 1 \) curvature mode is also clearly seen through the cross section of modal structures in Figures 11(g) and (h), respectively. The growth and the structure of the global \( m = 1 \) mode in the nonlinear simulation is consistent with the linear curvature mode obtained with the toroidal field in Figures 10(c) and (d).

![Figure 11](image)

**Figure 11.** Evolution of the magnetic energy for several azimuthal modes \((m = 1, 2, 3,...)\) from nonlinear simulations with (a) low \((V_A = 0.2r_1\Omega_0)\) and (b) high \((V_A = 0.58r_1\Omega_0)\) vertical field; and (c) low \((V_A = 0.52r_1\Omega_0)\) and (d) high \((V_A = 1.08r_1\Omega_0)\) toroidal field; nonlinear mode structures \(v_c\) in a system with a (e) low and (f) high vertical seed field and a (g) low and (h) high toroidal seed field. An animated version of (g) and (h) for time-dependent solutions is available. The animation runs from 0 to 773.319 \(\mu s\) in (g) and from 585 to 1220 \(\mu s\) in (h). The real-time durations of (g) and (h) are 13 and 9 \(s\), respectively. As the toroidal field strength is increased (from \( V_A = 0.52r_1\Omega_0 \) to \( V_A = 1.08r_1\Omega_0 \)) a transition from an MRI-driven turbulence (g) to a state dominated by a global \( m = 1 \) magneto-curvature mode (h) is observed.

(An animation of this figure is available.)

5. Summary and Conclusions

The global stability of accretion flows in a differentially rotating Keplerian cylindrical disk is examined. Using a variety of linear local and global eigenvalue stability analyses, as well as linear and 3D nonlinear initial-value global simulations using the extended MHD NIMROD code, we uncover a global nonaxisymmetric instability. Although this global mode coexists with the local nonaxisymmetric MRI modes, it persists at stronger magnetic fields as \( V_A/V_0 \) is increased, where the local MRI is stable. As the mode structure and relative
dominance over MRI of this mode is inherently determined by the global spatial curvature as well as the flow shear in the presence of magnetic field, we call it the magneto-spatial-curvature instability.

Starting with the local WKB analysis of the nonaxisymmetric modes in Keplerian accretion flows, we first observed the pronounced effect of the curvature terms due to the toroidal field curvature, or just spatial curvature terms in the vertical field case. Although WKB solutions provided some guidance, it clearly lacked the spatial radial variation of the flow shear and field-line bending. We therefore moved to the global eigenvalue shooting method and linear NIMROD simulations to further investigate the effect of global curvature. Through global eigenvalue analysis (i.e., shooting solutions of the ODE; Equation (16)), we first found two distinct nonaxisymmetric modes: a localized MRI concentrated in the high-shear region and a new radially globally extended curvature mode. The latter is absent in the local limit and persists at stronger magnetic fields (with vertical or azimuthal fields). The distinct nature of the MRI and curvature modes is further demonstrated in the 2D plane of complex frequencies, where the MRI and curvature modes occupy different regions (curvature at a lower frequency and MRI at a high frequency), and coexist for a wide range of values of $V_A/V_0$ until at larger values of $V_A/V_0$ (around unity), where only curvature modes are present. Curvature modes are unstable Alfvén continua, which become global due to the inherent global shear flow and curvature. The modified energy principle (Frieman & Rotenberg 1960) was also examined, and we recovered the global differential rotation term (similar to $C_0$ in WKB) as the destabilizing effect. Additional terms due to the spatial curvature were also obtained. As we quasi-continuously change $m = 1$ to $m = 0$, we observe a bifurcation of nonaxisymmetric solutions (Figure 7). We then inspected the modes’ relative dominance in the low- and high-curvature regimes. By increasing the aspect ratio $r_1/(r_2 - r_1)$, i.e., by moving the outer boundary closer in dimensionless quantities and therefore lowering the average spatial curvature, we approach a Cartesian approximation and see the inner MRI mode dominate, whereas in the low-aspect-ratio limit, the global curvature mode dominates (Figure 8).

Second, we also compared the global eigenvalue solutions with the direct linear simulations from NIMROD (i.e., the solution found by only evolving one axisymmetric Fourier mode, $m = 1$, for example) and found similar mode structures with the same growth rates (and real frequencies). In particular, NIMROD single-mode ($m = 1$) growth rates with pure azimuthal field versus $V_A/V_0$ also exhibit two humps, where the second hump occurs at stronger field due to curvature modes.  

Third, we finally performed full nonlinear simulations (where all Fourier modes $m = 0$, 1, 2...43 are evolved in the azimuthal direction) with the same initial condition as for the linear simulations. Consistent with the linear global solutions, a similar nonaxisymmetric mode structure is found in the early phase of the nonlinear simulations. In addition, two distinct nonlinear saturated states are obtained. Full nonlinear simulations at stronger magnetic fields (as the ratio of $S/Rm = V_A/V_0$ approaches 1 and above) showed a global laminar nonlinear state, while a turbulent state is obtained in the simulations at weaker fields.

In summary, in addition to the high-frequency and localized nonaxisymmetric MRI modes (at high and $k_z$) (Matsumoto & Tajima 1995; Ogilvie & Pringle 1996; Goedbloed & Keppens 2022), we find a previously unidentified distinct global curvature mode (at low $m$ and $k_z$), which could be unstable at stronger fields. Both local MRI and the curvature mode structures remain confined between the regions of Alfvénic points (where the magnitude of the Doppler-shifted wave frequency is equal to the Alfvén frequency). However, due to the inherent presence of global flow shear and spatial curvature, the curvature mode extends throughout the domain. The instability covers a wide range of parameter regions from weak to strong magnetic fields (i.e., from super-Alfvénic to sub-Alfvénic flows). Magneto-curvature instability is an Alfvénic eigenmode instability, which is nonlocal due to global spatial and magnetic field curvature. The nonlinear dynamics in the presence of curvature modes (or the combination of local MRI and curvature modes) including the momentum transport and dynamo in accretion flows will be topics of further investigations. In this paper, the cylindrical geometry with real spatial curvature provided a rich family of unstable nonaxisymmetric modes. The interplay between these modes and more classical local MRI modes is critical in understanding the process of momentum transport and the dynamo field generation (Brandenburg et al. 1995; Rincon et al. 2007; Ebrahimi et al. 2009) and the destruction of fields via magnetic reconnection (Rosenberg & Ebrahimi 2021) in disks. Due to the less dissipative nature of the global curvature modes, our results could therefore be relevant also to the experimental observations of different flow-driven modes in search of MRI in the laboratory (Goodman & Ji 2002; Stefani et al. 2006; Seilmayer et al. 2014; Wei et al. 2016; Mishra et al. 2022; Wang et al. 2022) and require further investigations.

Simulations were conducted on the cluster at the Princeton Plasma Physics Laboratory and the National Energy Research Scientific Computing Center (NERSC). This work was supported in part by funding from the Department of Energy for the Summer Undergraduate Laboratory Internship (SULI) program. F. E. acknowledges support by the US DOE contract No. DE-AC02-09CH11466, DE-SC0010565 and the Max-Planck-Princeton Center for Plasma Physics (MPPC).

ORCID iDs
Fatima Ebrahimi https://orcid.org/0000-0003-3109-5367
Matthew Pharr https://orcid.org/0000-0001-9741-4071

References
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A., & Hawley, J. F. 1998, RvMP, 70, 1
Beckwith, K., Hawley, J. F., & Kroll, J. H. 2008, ApJ, 678, 1180
Bernstein, I. B., Frieman, E., Kruskal, M. D., & Kulsrud, R. 1958, RSJ, 244, 17
Blokland, J., Van der Swaluw, E., Keppens, R., & Goedbloed, J. 2005, A&A, 444, 337
Bodo, G., Mignone, A., Cattaneo, F., Rossi, P., & Ferrari, A. 2008, A&A, 487, 1
Bondeson, A., Iacono, R., & Bhattacharjee, A. 1987, PhFl, 30, 2167
Brandenburg, A., Nordlund, A., Stein, R. F., & Torkelsson, U. 1995, ApJ, 446, 741
Chandrasekhar, S. 1960, PNAS, 46, 253
Chandrasekhar, S. 2006, Hydrodynamic and Hydromagnetic Stability (Oxford: Clarendon), 382
