Flavor changing neutral current processes in $B$ and $K$ decays in the supergravity model

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Abstract

Flavor changing neutral current processes such as $b \rightarrow s \gamma$, $b \rightarrow s l^+ l^-$, $b \rightarrow s \nu \bar{\nu}$, $\epsilon_K$, $\Delta m_B$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are calculated in the supersymmetric standard model based on supergravity. We consider two assumptions for the soft supersymmetry breaking terms. In the minimal case soft breaking terms for all scalar fields are taken to be universal at the GUT scale whereas those terms are different for the squark/slepton sector and the Higgs sector in the nonminimal case. In the calculation we have taken into account the next-to-leading order QCD correction to the $b \rightarrow s \gamma$ branching ratio, the results from the LEP II superparticles search, and the condition of the radiative electroweak symmetry breaking. We show that $\Delta m_B$ and $\epsilon_K$ can be enhanced up to 40% compared to the Standard Model values in the nonminimal case. In the same parameter region the $b \rightarrow s \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ branching ratios are reduced up to 10%. The corresponding deviation in the minimal case is 20% for $\Delta m_B$ and $\epsilon_K$ and within 3% for the $b \rightarrow s \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. For the $b \rightarrow s l^+ l^-$ process the significant deviation from the Standard Model is realized only when the $b \rightarrow s \gamma$ amplitude has an opposite sign to the Standard Model prediction. Significance on these results from possible future improvements of the $b \rightarrow s \gamma$ branching ratio measurement and top squark search is discussed.

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I Introduction

Rare processes such as flavor changing neutral current (FCNC) processes have been useful probes for the physics beyond the energy scale directly accessible in collider experiments. Among new physics beyond the standard model (SM), supersymmetry (SUSY) is considered to be the most promising candidate. Since FCNC is absent at the tree level in the minimal supersymmetric standard model (MSSM) as in the SM, these rare processes can give useful constraints on the masses and mixings of the SUSY particles through loop diagrams.

Although squark masses are free parameters within the framework of the MSSM, it is known that too large FCNC’s are induced if we allow arbitrary mass splittings and mixings among the squarks with the same quantum numbers [1]. This suggests that the SUSY breaking in the MSSM sector is induced from a generation-independent interaction. A simple realization of the generation-independent SUSY breaking is the minimal supergravity model. In this case the SUSY breaking in the hidden sector is transmitted to the MSSM sector by the gravitational interaction which does not distinguish the generation nor other gauge quantum numbers. As a result, induced soft SUSY breaking masses are equal at the Planck scale for all scalar fields in the MSSM sector. FCNC processes have been studied extensively in the supergravity model as well as more general context of the SUSY models for the $K^0-\overline{K}^0$ and the $B^0-\overline{B}^0$ mixings [2–4], $b \rightarrow s \gamma$ [3–5], $b \rightarrow s l^+ l^-$ [4–5] and $K \rightarrow \pi \nu \nu$ [11–13]. In Ref. [6] the $B^0-\overline{B}^0$ mixing and $\epsilon_K$ (CP violating parameter in the $K^0-\overline{K}^0$ mixing) were calculated in the minimal supergravity model under the LEP constraints and it was shown that these quantities can be larger than the SM values by 20%. Rare $b$ decay processes such as $b \rightarrow s \gamma$, $b \rightarrow s l^+ l^-$, and $b \rightarrow s \nu \nu$ are considered in Ref. [4] and it was pointed out that, taking account of the LEP 1.5 constraints, there is a parameter region where the $b \rightarrow s l^+ l^-$ branching ratio can be enhanced by 50% compared to the SM value. Also the $b \rightarrow s \nu \nu$ branching ratio is shown to be reduced at most by 10% from the SM prediction.

In this way effects of SUSY particles and the charged Higgs boson vary from a few % to several ten’s % depending on various FCNC processes. Since future experiments on $B$ and $K$ decays may reveal new physics effects of this magnitude it is important to make quantitative predictions using updated constraints on SUSY.
parameter space. Recent important theoretical improvement in this aspect is that
the complete next-to-leading order formula of the QCD correction to the branching
ratio of $b \to s \gamma$ becomes available for the SM \cite{12} and the two Higgs doublet models
\cite{13}. As a result, the theoretical uncertainty in the calculation of $B(b \to s \gamma)$ has
been reduced to $\lesssim 10\%$ level.

In this paper we study the SUSY contributions to FCNC processes under the
updated constraints. We take account of the next-to-leading order QCD corrections
for the evaluation of $B(b \to s \gamma)$ as well as the bounds on SUSY particle masses from
the recent LEP II results \cite{14} in order to obtain the allowed region in the SUSY
parameter space. Then we evaluate various FCNC quantities such as $b \to s l^+ l^−$, $b \to s \nu \nu$, $B^0 - \overline{B}^0$ mixing amplitude, $\epsilon_K$, $K^+ \to \pi^+ \nu \nu$ and $K_L \to \pi^0 \nu \nu$ within the
allowed parameter region. The numerical results depend on assumption of SUSY
breaking terms at the GUT scale. In particular, in the minimal supergravity model
soft SUSY breaking terms for all scalar fields are assumed to be the same at the
GUT scale. If we would like to suppress too large SUSY contributions to the $K^0 - \overline{K}^0$
mixing it is sufficient to require the degeneracy of the soft SUSY breaking masses
only in the squark sector. Because the strict universality for all scalar masses is
not necessarily required in the context of the supergravity model, we study how the
allowed deviations of the FCNC quantities change when the universality condition
is relaxed. For this purpose we take the soft SUSY breaking term for the Higgs
masses as a parameter independent of the universal squark/slepton mass. This kind
of assumption was considered in Ref. \cite{15} in a different context. We will see that the
SUSY effects are considerably enhanced in a parameter space which is excluded in
the minimal case from the condition of the proper electroweak symmetry breaking.
In the nonminimal case, the branching ratios of $K \to \pi \nu \nu$ can be smaller than the
SM values by 10\%, and at the same time, $\epsilon_K$ and the $B^0 - \overline{B}^0$ mixing become larger
than the SM values by 40\% for $\tan \beta = 2$. The corresponding values in the minimal
case are given by 3\% and 20\%, respectively. For $b \to s l^+ l^−$, the result does not
significantly differ from the minimal case: there is a parameter space where the
branching ratio becomes larger by 50\% than the SM value for a large $\tan \beta$. We
analyze the correlation between the SUSY contributions to the FCNC processes
and the $b \to s \gamma$ branching ratio. It turns out that the maximal deviation occurs
in the case that the $b \to s \gamma$ branching ratio is away from the central value of the
SM prediction. We also show that the large deviation occurs in a parameter region where the top squark is relatively light. Therefore the improvement in the $b \to s \gamma$ branching ratio measurement and the top squark mass bound will give great impacts on the SUSY search through the various FCNC processes.

The rest of this paper is organized as follows. In the next section, we introduce the supergravity model. In Sec. II we describe the calculation of each FCNC quantity. In Sec. IV our results of numerical analyses are presented. Sec. V is devoted for discussions and conclusions.

II Supergravity model

In this section we briefly outline calculations of the SUSY particles’ masses and the mixing parameters in the supergravity model for the minimal and the nonminimal cases. The actual procedure is the same as those discussed in Ref. [6, 8, 10] except for the choice of the initial soft SUSY breaking parameters for the nonminimal case.

The MSSM Lagrangian is specified by the superpotential and the soft SUSY breaking terms. The superpotential is given by

$$W_{\text{MSSM}} = f^i_j Q_i D_j H_1 + f^i_j U_i H_2 + f^i_j E_i L_j H_1 + \mu H_1 H_2,$$

where the chiral superfields $Q, D, U, L, E, H_1$ and $H_2$ transform under $SU(3) \times SU(2) \times U(1)$ group as following representations:

$$Q_i = (3, 2, \frac{1}{6}), \quad U_i = (\overline{3}, 1, -\frac{2}{3}), \quad D_i = (\overline{3}, 1, \frac{1}{3}),$$

$$L_i = (1, 2, -\frac{1}{2}), \quad E_i = (1, 1, 1),$$

$$H_1 = (1, 2, -\frac{1}{2}), \quad H_2 = (1, 2, \frac{1}{2}).$$

The suffices $i, j = 1, 2, 3$ are generation indices. $SU(3)$ and $SU(2)$ indices are suppressed for simplicity. A general form of the soft SUSY breaking terms is given by

$$-L_{\text{soft}} = (m_Q^2)_i^j \tilde{q}_i \tilde{q}_j + (m_D^2)_i^j \bar{d}_i \bar{d}_j + (m_U^2)_i^j \bar{u}_i \bar{u}_j$$

$$+ (m_E^2)_i^j \bar{e}_i \bar{e}_j + (m_L^2)_i^j \bar{l}_i \bar{l}_j$$

$$+ (m_{\tilde{Q}}^2)_i^j \tilde{Q}_i \tilde{Q}_j + (m_{\tilde{D}}^2)_i^j \tilde{D}_i \tilde{D}_j + (m_{\tilde{U}}^2)_i^j \tilde{U}_i \tilde{U}_j + (m_{\tilde{E}}^2)_i^j \tilde{E}_i \tilde{E}_j + (m_{\tilde{L}}^2)_i^j \tilde{L}_i \tilde{L}_j.$$
\[ + \Delta_1^2 h_1^1 h_1 + \Delta_2^2 h_2^1 h_2 - (B \mu h_1 h_2 + \text{h.c.}) \]
\[ + \left( A_D^{ij} \bar{q}_i \bar{d}_j h_1 + A_U^{ij} \bar{q}_i \bar{u}_j h_2 + A_L^{ij} \bar{e}_i \bar{\ell}_j h_1 + \text{h.c.} \right) \]
\[ + \left( \frac{M_1}{2} \bar{B} \bar{B} + \frac{M_2}{2} \bar{W} \bar{W} + \frac{M_3}{2} \bar{G} \bar{G} + \text{h.c.} \right), \tag{2.3} \]

where \(\bar{q}_i, \bar{u}_i, \bar{d}_i, \bar{\ell}_i, \bar{e}_i, h_1\) and \(h_2\) are scalar components of chiral superfields \(Q_i, U_i, D_i, L_i, E_i, H_1\) and \(H_2\), respectively, and \(\bar{B}, \bar{W}\) and \(\bar{G}\) are \(U(1), \text{SU}(2)\) and \(\text{SU}(3)\) gauge fermions, respectively.

In the framework of the supergravity model, the soft SUSY breaking parameters are assumed to have a simple structure at the Planck scale. In the present analysis, we take the following initial conditions at the GUT scale \(M_{\text{GUT}} \sim 2 \times 10^{16}\) GeV. We neglect the difference between the Planck and the GUT scales:

\[
(m_Q^2)^i_j = (m_E^2)^i_j = m_0^2 \delta^i_j , \\
(m_D^2)^i_j = (m_U^2)^i_j = (m_L^2)^i_j = m_0^2 \delta^i_j , \tag{2.4a} \\
\Delta_1^2 = \Delta_2^2 = \Delta_0^2 , \tag{2.4b} \\
A_D^{ij} = f_D^{ij} A_X m_0 , \quad A_U^{ij} = f_U^{ij} A_X m_0 , \quad A_L^{ij} = f_L^{ij} A_X m_0 , \tag{2.4c} \\
M_1 = M_2 = M_3 = M_{\text{gX}} . \tag{2.4d} 
\]

In the minimal case \(m_0\) and \(\Delta_0\) are assumed to be equal, whereas in the nonminimal case we take \(m_0\) and \(\Delta_0\) as independent parameters. We also assume that \(A_X, M_{\text{gX}}\) and \(\mu\) are all real parameters to avoid a large electric dipole moment of the neutron \[17\]. Therefore, no new CP violating complex phase (other than that in the Cabibbo-Kobayashi-Maskawa (CKM) matrix) is introduced in the present analysis.

The soft SUSY breaking parameters at the electroweak scale are calculated by solving the renormalization group equations (RGEs) of the MSSM \[18\] and we also impose the radiative electroweak symmetry breaking condition \[19\]. Taking the quark masses, the CKM matrix and \(\tan \beta = \langle h_2^0 \rangle / \langle h_1^0 \rangle\) as inputs, we first solve one-loop RGEs for the gauge and Yukawa coupling constants to calculate the values at the GUT scale. Then we solve the RGEs for all MSSM parameters downward with initial conditions Eq. \[2.4\] for each set of the universal soft SUSY breaking
parameters \((m_0, \Delta_0, A_X, M_{gX})\). We include all generation mixings in the RGEs for both Yukawa coupling constants and the soft SUSY breaking parameters. Next, we evaluate the Higgs potential at \(m_Z\) scale including the one-loop corrections induced by the Yukawa couplings constants of the third generation \([20]\), and require that the minimum of the potential gives correct vacuum expectation values of the neutral Higgs fields as \(\langle h_1^0 \rangle = v \cos \beta \) and \(\langle h_2^0 \rangle = v \sin \beta\) where \(v = 174\, \text{GeV}\). The requirement of the radiative electroweak symmetry breaking fixes the magnitude of the SUSY Higgs mass parameter \(\mu\) and the soft SUSY breaking parameter \(B\). At this stage, all MSSM parameters at the electroweak scale are determined as functions of the input parameters \((\tan \beta, m_0, \Delta_0, A_X, M_{gX}, \text{sign}(\mu))\). With use of the MSSM parameters at the electroweak scale, we obtain the masses and the mixing parameters (both angles and phases) of all the SUSY particles by diagonalizing the mass matrices. We impose the following phenomenological constraints on the obtained particle spectra.

1. \(b \to s \gamma\) constraint from CLEO, \(i.e., 1.0 \times 10^{-4} < B(b \to s \gamma) < 4.2 \times 10^{-4}\) \([21]\).

2. The chargino mass is larger than 91 GeV, and all other charged SUSY particle masses should be larger than 80 GeV \([14]\).

3. All sneutrino masses are larger than 41 GeV \([22]\).

4. The gluino and squark mass bounds from TEVATRON experiments \([23]\). The precise bounds on the gluino and squark masses depend on various SUSY parameters. Here we impose the constraint reported in Ref. \([23]\) on the parameter space of the gluino mass and the averaged squark mass except for the top squark. Actually the gluino mass and the squark masses are more strictly constrained in this model from the chargino mass bound and the GUT relation of the gaugino masses, so that these masses are restricted to be larger than about 200 GeV except for the lighter top squark. For the light top squark, the experimental bound is obtained at LEP and TEVATRON experiments \([24]\) which was already taken into account in 2.

5. From the LEP neutralino search \([25]\), \(\Gamma(Z \to \chi \chi) < 8.4\, \text{MeV}\) and \(B(Z \to \chi \chi'),\)
\(B(Z \to \chi' \chi') < 2 \times 10^{-5}\) where \(\chi\) is the lightest neutralino and \(\chi'\) denotes other neutralinos.
6. The lightest SUSY particle is neutral.

7. The condition for not having a charge or color symmetry breaking minimum \[26\].

In the next section we calculate the FCNC and/or CP violating quantities such as the branching ratios for \(b \to s l^+ l^-\), \(b \to s \nu \bar{\nu}\), \(K^+ \to \pi^+ \nu \bar{\nu}\), \(K_L \to \pi^0 \nu \bar{\nu}\) and the \(B^0 - \bar{B}^0\) mixing and \(\epsilon_K\) in the allowed parameter region.

### III FCNC processes in \(B\) and \(K\) decays

#### III.1 \(b \to s \gamma\), \(b \to s l^+ l^-\) and \(b \to s \nu \bar{\nu}\)

We basically follow Ref. \[9\] for the calculations of \(b \to s \gamma\), \(b \to s l^+ l^-\) and \(b \to s \nu \bar{\nu}\) branching ratios, but we improve the calculation taking into account the next-to-leading order QCD corrections.

The effective Hamiltonian for the \(b \to s\) transition processes is given as \[9, 27, 28\].

\[
\mathcal{H}_{1}^{\text{eff}} = \sum_{i=1}^{11} C_i(Q) \mathcal{O}_i(Q) + \text{h.c.} , \quad (3.1)
\]

where \(Q\) is the renormalization point. The operators relevant to the present study are

\[
\mathcal{O}_7 = \frac{e}{(4\pi)^2} m_b (\overline{s} \sigma^{\mu \nu} b_R) F_{\mu \nu} , \quad (3.2a)
\]

\[
\mathcal{O}_9 = \frac{e^2}{(4\pi)^2} (\overline{s} \gamma^\mu b_L)(\overline{\ell} \gamma_\mu \ell) , \quad (3.2b)
\]

\[
\mathcal{O}_{10} = \frac{e^2}{(4\pi)^2} (\overline{s} \gamma^\mu b_L)(\overline{\ell} \gamma_\mu \gamma_5 \ell) , \quad (3.2c)
\]

for \(b \to s \gamma\) and \(b \to s l^+ l^-\), and

\[
\mathcal{O}_{11} = \frac{e^2}{(4\pi)^2} (\overline{s} \gamma^\mu b_L)(\overline{\nu} \gamma_\mu (1 - \gamma_5) \nu) , \quad (3.3)
\]

for \(b \to s \nu \bar{\nu}\). Other operators (the four-quark operators \(\mathcal{O}_{1,2,\ldots,6}\) and the chromo-magnetic operator \(\mathcal{O}_8\)) contribute through the QCD corrections. We first calculate
the Wilson coefficients \( C_i \) at the electroweak scale with use of the masses and the mixings of SUSY particles as well as the SM ones. Then we evaluate \( C_i \) at \( m_b \) scale including the QCD corrections below the electroweak scale in order to obtain the branching ratios of \( b \to s \) decays.

As for the next-to-leading order QCD correction in the calculation of \( \mathcal{B}(b \to s \gamma) \), we follow Ref. [12, 29–32] for the SM contribution and Ref. [13] for the charged Higgs boson contribution. The QCD correction consists of the \( O(\alpha_s) \) matching at the electroweak scale \([13, 29, 30]\), the next-to-leading order anomalous dimension \([12]\), two-loop matrix elements \([31]\) and the Bremsstrahlung corrections \([32]\). In Ref. \([30]\), the SM value is given as \( \mathcal{B}(b \to s \gamma)^{\text{NLO}}_{\text{SM}} = (3.60 \pm 0.33) \times 10^{-4} \) compared to the leading order result \( \mathcal{B}(b \to s \gamma)^{\text{LO}}_{\text{SM}} = (2.8 \pm 0.8) \times 10^{-4} \). \( O(\alpha_s) \) matching conditions for the SUSY loop corrections have not been completed. In Ref. \([33]\), these corrections are given for the case that the ratio of the chargino mass and the top squark mass is large. Since we are mainly interested in the case that both particles are relatively light, we do not include these corrections. Recently electroweak radiative corrections to \( \mathcal{B}(b \to s \gamma) \) is considered in Ref. \([34]\). We will discuss these effects on the numerical results later although we have not included them explicitly in the calculation. For the next-to-leading order QCD corrections to \( b \to s l^+ l^- \) and \( b \to s \nu \bar{\nu} \) we follow Ref. \([28, 35]\).

The main SM contributions to the \( b \to s \) decays come from the loop diagrams involving the top quark and the relevant CKM matrix element is \( V_{ts}^* V_{tb} \), which is approximately written as \( V_{ts}^* V_{tb} \approx -V_{cs}^* V_{cb} \) because of the unitarity and the smallness of \( V_{us}^* V_{ub} \). Also the charm quark loop contribution has the CKM factor \( V_{cs}^* V_{cb} \). Consequently, unlike \( B^0 - \bar{B}^0 \) mixing, \( \epsilon_K \) and \( K \to \pi \nu \bar{\nu} \) the SM values of the branching ratios for above processes are calculable without much uncertainty since the relevant CKM factors are known in a good accuracy.

The SM predictions of the branching ratios for these processes are \( \mathcal{B}(b \to s l^+ l^-) \approx 0.8(0.6) \times 10^{-5} \) for \( l = e (\mu) \) and \( \mathcal{B}(b \to s \nu \bar{\nu}) \approx 4.2 \times 10^{-5} \). These processes have not yet observed experimentally and only upper bounds are given by \( \mathcal{B}(b \to s l^+ l^-) < 5.7(5.8) \times 10^{-5} \) for \( l = e (\mu) \) \([30]\) and \( \mathcal{B}(b \to s \nu \bar{\nu}) < 3.9 \times 10^{-4} \) \([37]\). The \( b \to s l^+ l^- \) process is expected to be observed in the near future at the \( B \) factories and hadron machines.
III.2 $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

The branching ratios of $K \to \pi \nu\bar{\nu}$ processes are calculated by evaluating the Wilson coefficient $C^d_{11}$ in the effective Hamiltonian

$$H^d_{\text{eff}} = C^d_{11}O^d_{11} + \text{h.c.},$$

$$O^d_{11} = \frac{e^2}{(4\pi)^2} (\bar{s}\gamma^\mu d_L)(\bar{\nu}\gamma_\mu (1 - \gamma_5)\nu),$$

in a similar way as $b \to s \nu\bar{\nu}$. The branching ratios normalized to that of the $K_{e3}$ decay are written as

$$\frac{B(K^+ \to \pi^+ \nu\bar{\nu})}{B(K^+ \to \pi^0 e^+ \nu_e)} = \left( \frac{\alpha}{4\pi} \right)^2 \frac{\sum_\nu \left| C^d_{11} \right|^2}{|V_{us}|^2 G_F^2} r_{K^+},$$

$$\frac{B(K_L \to \pi^0 \nu\bar{\nu})}{B(K^+ \to \pi^0 e^+ \nu_e)} = \left( \frac{\alpha}{4\pi} \right)^2 \frac{\sum_\nu \left| \text{Im} \ C^d_{11} \right|^2}{|V_{us}|^2 G_F^2} \frac{\tau_{K_L}}{\tau_{K^+}} r_{K_L},$$

where $\tau_{K_L}(\tau_{K^+})$ denotes the lifetime for $K_L(K^+)$ and $r_{K^+}$ and $r_{K_L}$ are isospin breaking factors.

The SM contributions to $C^d_{11}$ come from both the top and the charm loops with CKM factors $V_{ts}^* V_{td}$ and $V_{cs}^* V_{cd}$, respectively. The dependencies on $V_{td}$ (or $\rho$ and $\eta$ in the Wolfenstein’s parametrization) are different in $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ since only the $V_{ts}^* V_{td}$ term contributes to $K_L \to \pi^0 \nu\bar{\nu}$ while the sum of both terms contributes to $K^+ \to \pi^+ \nu\bar{\nu}$. The details of the calculation of $K \to \pi \nu\bar{\nu}$ processes in the SM are available in Ref. [28]. Following this reference, we have taken into account the next-to-leading order QCD correction to the SM contribution.

The SM predictions for above branching ratios are given by $B(K^+ \to \pi^+ \nu\bar{\nu}) = (0.6-1.5) \times 10^{-10}$ and $B(K_L \to \pi^0 \nu\bar{\nu}) = (1.1-5.0) \times 10^{-11}$ taking into account the ambiguity of unknown CKM parameters [28]. Recently one candidate event of $K^+ \to \pi^+ \nu\bar{\nu}$ is reported and the branching ratio derived from this observation corresponds to $4.2^{+9.7}_{-3.5} \times 10^{-10}$ [39]. On the other hand for $K_L \to \pi^0 \nu\bar{\nu}$ only the upper bound is given by $B(K_L \to \pi^0 \nu\bar{\nu}) < 1.8 \times 10^{-6}$ [10]. Although the upper bound is still $10^5$ larger than the SM prediction, dedicated searches for $K_L \to \pi^0 \nu\bar{\nu}$ are planned at KEK [11], BNL [12] and Fermilab [13]. The $K \to \pi \nu\bar{\nu}$ processes are theoretically very clean and the theoretical errors, such as QCD corrections, are expected to be $\lesssim 10\%$ for $K^+ \to \pi^+ \nu\bar{\nu}$ and a few $\%$ for $K_L \to \pi^0 \nu\bar{\nu}$ [28]. Therefore
$K \rightarrow \pi \nu \bar{\nu}$ processes may give useful information on the SUSY parameters if the branching ratios are measured at 10% level.

III.3 $B^0-\bar{B}^0$ mixing and $\epsilon_K$

The $B^0-\bar{B}^0$ mixing matrix element $M_{12}(B)$ is calculated from the effective Hamiltonian

$$\mathcal{H}_2^{\text{eff}} = \frac{1}{128\pi^2} A(B)(\bar{d}\gamma^\mu b_L)(\bar{d}\gamma^\mu b_L) + \text{h.c.}, \quad (3.6)$$

with

$$M_{12}(B) = \frac{1}{2m_B}\langle B^0|\mathcal{H}_2^{\text{eff}}|\bar{B}^0 \rangle = \frac{\hat{B}_B\eta_B f_B m_B}{384\pi^2} A(B), \quad (3.7)$$

where $m_B, f_B, \hat{B}_B$ and $\eta_B$ are the $B$-meson mass, decay constant, bag parameter and QCD correction factor, respectively. The $K^0-\bar{K}^0$ mixing matrix element $M_{12}(K)$ is obtained in the same way by replacing the external bottom quark with the strange quark and the $\epsilon_K$ is proportional to $\text{Im} M_{12}(K)$. We calculate the coefficient $A(B)$ and $A(K)$ as described in Ref. [6] with the inclusion of the next-to-leading order QCD corrections given in Ref. [44]. The experimental values for the $B^0-\bar{B}^0$ and $K^0-\bar{K}^0$ mixings are given as $\Delta m_B = 2|M_{12}(B)| = (0.474 \pm 0.031) \text{ ps}^{-1}$ [22, 45] and $|\epsilon_K| = (2.280 \pm 0.013) \times 10^{-3}$ [22]. At present these observables do not constrain the SUSY parameters very strongly because the CKM parameters relevant to these quantities are not well-determined and considerable hadronic uncertainties still exist in $\hat{B}_K, \hat{B}_B$ and $f_B$.

IV Numerical Results

In this section we show our numerical results. We scan the soft SUSY breaking parameter space in the range of $m_0 \leq 600 \text{ GeV}, \Delta_0 \leq 600 \text{ GeV}, M_{pX} \leq 600 \text{ GeV}$ and $|A_X| \leq 5$ for each fixed value of $\tan \beta$. For the CKM matrix, we use the ‘standard’ phase convention of the Particle Data Group [22], taking $V_{us} = 0.2205$, $V_{cb} = 0.041$, $|V_{ub}/V_{cb}| = 0.08$ and $\delta_{13} = 90^\circ$ as input parameters. We also change
the value of $\delta_{13}$ and comment on the results if necessary. We fix the pole masses of the top, bottom and charm quarks as 175 GeV, 4.8 GeV and 1.4 GeV, respectively. We also take $\alpha_s(m_Z) = 0.118$.

Let us first discuss general features of the mass spectrum and the generation mixings of squarks determined by RGEs.

1. The first and second generation squarks with the same gauge quantum numbers remain highly degenerate in masses but the third generation squarks, especially the top squark can be significantly lighter due to the renormalization effect of the top Yukawa coupling constant.

2. The squark flavor mixing matrix which diagonalize the squark mass matrix is approximately the same as corresponding CKM matrix apart from the left-right mixing of the top squarks.

As a result, SUSY contributions to the $b \rightarrow s \ (s \rightarrow d)$ transition amplitudes and $M_{12}(B) \ (M_{12}(K))$ are proportional to $V_{tb} V_{ts}^* (V_{ts}V_{td}^*)$ and $(V_{tb} V_{td}^*)^2 ((V_{ts}V_{td}^*)^2)$, respectively. Therefore CP violating phase of $M_{12}(B(K))$ is equal to that in the SM. These features are the same as those in the minimal case [3, 6, 16].

The quantitative difference between the minimal and the nonminimal choices of the soft SUSY breaking parameters appears in the mass spectrum. In Fig. 1 we show the allowed region in the space of the lighter chargino and the lighter top squark masses for a different assumption on $m_0$ and $\Delta_0$ for $\tan \beta = 2$ and 30. We present the allowed region for the full parameter space, and the minimal case ($m_0 = \Delta_0$). Contrary to the minimal case we see that a relatively light top squark and chargino with masses $m_{\tilde{t}_1} \sim 100$ GeV and $m_{\tilde{\chi}^+_1} \sim 100$ GeV are simultaneously realized especially for $\tan \beta = 2$. This difference of the allowed mass spectrum leads to a quantitative change in the prediction of the FCNC observables for the minimal and the nonminimal cases.

**IV.1 $b \rightarrow s \gamma$, $b \rightarrow s l^+ l^-$ and $b \rightarrow s \nu \overline{\nu}$**

As discussed above the SUSY contribution to the $b \rightarrow s$ transition amplitudes is proportional to the $V_{tb} V_{ts}^*$ element just as the SM and the charged Higgs boson contributions. As discussed in the subsection III.1, the $V_{tb} V_{ts}^*$ element is well-constrained
from the unitarity of the CKM matrix so that there is little ambiguity associated with this input parameter. The Wilson coefficients $C_7$, $C_9$ and $C_{10}$ are relevant to the $b \to s \gamma$ and $b \to s l^+ l^-$ decays. The values of $C_7$, $C_9$ and $C_{10}$ in the supergravity model are shown in Fig. 2. Each coefficient is evaluated at the bottom mass scale and is normalized by the corresponding SM value. The SUSY contribution to $C_7$ can be as large as or even larger than the SM contribution especially for a large $\tan \beta$. We can see that the sign of $C_7$ can be opposite to that of the SM prediction. On the other hand the SUSY contributions to $C_9$ and $C_{10}$ are relatively small and interfere constructively in $C_9$ and destructively in $C_{10}$. These features are the same as those in the minimal case discussed in Ref. [9].

In Fig. 3 we show the branching ratio of $b \to s \gamma$ as a function of the charged Higgs boson mass for $\tan \beta = 2$ (minimal and nonminimal cases) and $\tan \beta = 30$ (nonminimal case). For $\tan \beta = 30$, the plot looks the same even if the parameter space is restricted to the minimal case. Here we fix the renormalization point $\mu_b$ as $\mu_b = m_b$. In the calculation of $B(b \to s \gamma)$ we use the electromagnetic coupling constant $\alpha_{\text{EM}}$ at $m_b$ scale which is given by $\alpha_{\text{EM}}^{-1}(m_b) \simeq 132$. Considering that the next-to-leading order formulas still contain theoretical ambiguities due to the $\mu_b$ dependence and the choice of the various input parameters, we should allow theoretical uncertainty at 10% level for each point. It is interesting to notice that for the minimal case with $\tan \beta = 2$ there are two branches for $B(b \to s \gamma)$. In one branch the branching ratio is close to the two Higgs doublet model (type II) prediction, therefore the contributions from SUSY particles are small. In the other branch it is consistent with the SM value, so that the charged Higgs boson contribution is canceled by the SUSY contributions.

In Fig. 4 we show the correlation between the branching ratios of $b \to s \gamma$ and $b \to s \mu^+ \mu^-$. In this figure in order to avoid the $J/\psi$ resonance we use the branching ratio for $b \to s \mu^+ \mu^-$ integrated in the region $2m_{\mu} < \sqrt{s} < m_{J/\psi} - 100$ MeV where $\sqrt{s}$ is the invariant mass of $\mu^+ \mu^-$ pair. As discussed in Ref. [3], the branching ratio in this region depends on the phase of the $b-s-J/\psi$ coupling $\kappa$ through the interference effect. Although the branching ratio can change by $\pm 15\%$, this ambiguity will be reduced if we can measure the lepton invariant mass spectrum near the $J/\psi$ resonance region. As an example we take $\kappa$ as $+1$ here. We can see that a strong correlation between the two branching ratios since only $C_7$ receives
the large SUSY contribution. In the present supergravity model therefore a large deviation of $B(b \to s l^+ l^-)$ from the SM prediction is expected only when the sign of $C_7$ is opposite to that in the SM, which is realized for a large $\tan \beta$. This situation is similar to the minimal case \cite{9}.

The amplitude of $b \to s \nu \overline{\nu}$ is determined by the Wilson coefficient $C_{11}$. Apart from the CKM matrix element the SUSY contribution to $C_{11}$ is the same as the SUSY contribution to $C_{d11}$. The branching ratio for $b \to s \nu \overline{\nu}$ normalized by the SM prediction ($B(b \to s \nu \overline{\nu}) / B(b \to s \nu \overline{\nu})_{SM}$) is practically the same as a similar ratio for $K_L \to \pi^0 \nu \overline{\nu}$ ($B(K_L \to \pi^0 \nu \overline{\nu}) / B(K_L \to \pi^0 \nu \overline{\nu})_{SM}$), which is discussed in the next subsection.

**IV.2 $K^+ \to \pi^+ \nu \overline{\nu}$ and $K_L \to \pi^0 \nu \overline{\nu}$**

As shown in Eq. (4.5) the branching ratios for $K^+ \to \pi^+ \nu \overline{\nu}$ and $K_L \to \pi^0 \nu \overline{\nu}$ are proportional to $|C_{11}^d|^2$ and $|\text{Im} C_{11}^d|^2$, respectively. In the SM $C_{11}^d$ is divided into two parts according to the relevant CKM matrix elements as follows:

$$C_{11}^d = V_{td} V_{ts}^* C_{11}^d \text{(top)} + V_{cd} V_{cs}^* C_{11}^d \text{(charm)}.$$  \hspace{1cm} (4.1)

As discussed before the SUSY contribution is proportional to $V_{td} V_{ts}^*$ therefore we can write

$$C_{11}^d \approx V_{td} V_{ts}^* (C_{11}^d \text{(top)} + C_{11}^d \text{(SUSY)}) + V_{cd} V_{cs}^* C_{11}^d \text{(charm)},$$  \hspace{1cm} (4.2)

where $C_{11}^d \text{(SUSY)}$ is the SUSY contribution including the charged Higgs boson contribution. This kind of parametrization for $K \to \pi \nu \overline{\nu}$ is considered in Ref. \cite{11}.

In Fig. 5 we show the branching ratio for $K_L \to \pi^0 \nu \overline{\nu}$ normalized by the SM prediction as a function of the lighter chargino mass and the lighter top squark mass for $\tan \beta = 2$. Also the correlation with the $B(b \to s \gamma)$ is shown. In Fig. 5(a) and Fig. 5(b), we use the CLEO bound on $B(b \to s \gamma)$ as a constraint on the SUSY parameter space. In order to take into account the theoretical ambiguity in a simple way, we allow 10% uncertainty in the branching ratio and use $(1.0 \times 10^{-4}) \times 0.9$ and $(4.2 \times 10^{-4}) \times 1.1$ as lower and upper bounds, respectively. Note that the ratio $B(K_L \to \pi^0 \nu \overline{\nu}) / B(K_L \to \pi^0 \nu \overline{\nu})_{SM}$ does not depend on the CKM parameters because only the first term in Eq. (4.1) contributes to this process. We see that the branching ratio for $K_L \to \pi^0 \nu \overline{\nu}$ becomes smaller than the SM prediction by
10%. In the minimal case the maximal deviation is within 3%. We investigated in which parameter region the maximal deviation is realized. We found that the large deviation occurs in the $m_0 \simeq 150$ GeV and $\Delta_0 \simeq 400$ GeV region which corresponds to the parameter region with $m_{\chi^\pm_1}, m_{\tilde{t}_1} \simeq 100$ GeV shown in Fig. 1. From Fig. 2(c) we can see that a sizable reduction of $B(K_L \to \pi^0 \nu \bar{\nu})$ occurs when $B(b \to s \gamma)$ becomes larger than the SM value. We also calculate $B(K_L \to \pi^0 \nu \bar{\nu})$ for different $\tan \beta$ and found that the deviation becomes smaller for a large $\tan \beta$. For example the maximal deviation is about 5% for $\tan \beta = 30$. As we can see in Eq. (4.2), the branching ratio of $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ have a strong correlation. We show the correlation for three different values of $\delta_{13}$ in Fig. 6. In this figure we fix $m_0 = 150$ GeV, but the correlation does not depend on the value of $m_0$. The deviation from the SM value for $B(K^+ \to \pi^+ \nu \bar{\nu})$ is about 20% smaller than that for $B(K_L \to \pi^0 \nu \bar{\nu})$.

**IV.3 $B^0 - \bar{B}^0$ mixing and $\epsilon_K$**

Just as in the $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ case, the $B^0 - \bar{B}^0$ mass splitting $\Delta m_B$ and $\epsilon_K$ normalized to SM values are linearly correlated with each other as noted in [5, 6]. We show the correlation for $\delta_{13} = 30^\circ, 90^\circ$ and $150^\circ$ in Fig. 7. We see that the deviation from SM in $\epsilon_K$ is about 80% of that in $\Delta m_B$. In the following, we only show the results for $\Delta m_B$, but the corresponding results on $\epsilon_K$ can be easily obtained from Fig. 6. In Fig. 8 we show $\Delta m_B$ normalized by the SM value as a function of the lighter chargino mass, the lighter top squark mass and $B(b \to s \gamma)$ for $\tan \beta = 2$. The deviation can be as large as 40% in the nonminimal case whereas 20% in the minimal case. From Fig. 8(b) we can see that the deviation larger than 20% is realized only in the nonminimal case when the top squark mass is smaller than 200 GeV. In this region $B(b \to s \gamma)$ also deviates from the SM value significantly as shown in Fig. 8(c). This result indicates the importance of the further improvement of the $B(b \to s \gamma)$ measurement and the top squark search. If the lower bound for the top squark mass is raised to 200 GeV, the maximal deviation of $\Delta m_B$ is reduced to 25%. On the other hand, if the $b \to s \gamma$ branching ratio turns out to be close to the present upper or lower bound, $\Delta m_B$ and $\epsilon_K$ might be significantly enhanced. We should notice that because the theoretical uncertainty is already reduced to 10% level the experimental determination of $B(b \to s \gamma)$ at that level will put a strong
constraint on the SUSY parameter space. We also calculated \( \Delta m_B \) for \( \tan \beta = 30 \) and found that the deviation from the SM value is less than 10%. In Fig. 9 we show the correlation between \( B(K_L \to \pi^0 \nu \overline{\nu}) \) and \( \Delta m_B \). For \( \tan \beta = 2 \) we see a strong correlation between these two quantities: \( B(K_L \to \pi^0 \nu \overline{\nu}) \) is reduced by 10% when \( \Delta m_B \) is enhanced by 40%. We can also see the correlation for \( \tan \beta = 30 \). In this case \( \Delta m_B \) can be enhanced by 10% in the region where \( B(K_L \to \pi^0 \nu \overline{\nu}) \) is reduced by 5%.

V Conclusions and discussions

In this paper we have studied the FCNC processes of \( B \) and \( K \) mesons in the minimal supergravity model and in the supergravity model with an extended parameter space of the soft SUSY breaking parameters. We take into account the recent mass bounds for SUSY particles at LEP II and the next-to-leading order QCD corrections to various processes including \( b \to s \gamma \).

We find that the branching ratio for \( b \to s l^+ l^- \) can be enhanced by about 50% compared to the SM value for a large \( \tan \beta \) when the sign of \( C_7 \) becomes opposite to that of SM. For \( \tan \beta = 2 \), the \( b \to s \nu \overline{\nu}, K^+ \to \pi^+ \nu \overline{\nu} \) and \( K_L \to \pi^0 \nu \overline{\nu} \) processes have similar SUSY contributions and it turns out that these branching ratios are reduced at most 10% in the nonminimal case whereas less than 3% in the minimal case. The \( B^0 - \overline{B}^0 \) mixing and \( \epsilon_K \) are enhanced up to 40% from the SUSY contributions in the nonminimal case whereas 20% in the minimal case. We investigate the correlation among \( \Delta m_B, \epsilon_K \) and \( B(K \to \pi \nu \overline{\nu}) \), and found that the large deviation occurs when the chargino is lighter than 150 GeV and the top squark is lighter than 200 GeV. In the same parameter region \( B(b \to s \gamma) \) is close to the upper or lower bound of the presently allowed region. For a large \( \tan \beta \), the deviations of \( \Delta m_B, \epsilon_K \) and \( B(K \to \pi \nu \overline{\nu}) \) are smaller. In the minimal case these deviations are somewhat smaller than the previous calculation especially for \( b \to s \nu \overline{\nu} \). This is because the mass bounds for chargino \( \text{etc.} \) have been improved by the LEP II experiments.

We note that the maximal deviation depends on the light top squark mass bound. Therefore the light top squark search in TEVATRON experiments can reduce a possible parameter space where a large deviation from the SM value in FCNC processes is realized.
In this paper we extend the minimal supergravity model by introducing an additional parameter for the soft SUSY breaking term in the Higgs sector. This is not the unique way to extend the soft SUSY breaking terms. In order to avoid too large FCNCs, we only require that the squarks/sleptons in the same quantum numbers should have the common mass term at the Planck scale. Since the main difference is the change of the SUSY mass spectrum, a deviation with a similar magnitude is expected to be realized in a more general case as long as a light top squark and light chargino mass region is allowed.

In Ref. [34] electroweak radiative corrections to $B(b \to s \gamma)$ is computed. They found that the fermion and the photonic loop effects reduce the branching ratio by $9 \pm 2\%$. It is argued that the dominant contribution is due to the electric charge renormalization, and as a result the electromagnetic coupling constant should be evaluated at $q^2 = 0$, i.e., $\alpha_{EM}(0) = 137.036$. Since we use $\alpha_{EM}(m_b)$, this correction reduces $B(b \to s \gamma)$ by 3%.

Let us finally discuss the implications of these results when various information is obtained in future $B$ and $K$ decay experiments. Firstly, since no new phase appears in $M_{12}(B)$, the CP asymmetry measured in the $B^0(\bar{B}^0) \to J/\psi K_S$ decay is directly related to the angle $\phi_1 = \arg \left(\frac{-V_{tb}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$ of the unitarity triangle. CP asymmetries in other $B$ decay modes and the ratio of the $\Delta m_B$’s for $B_s$ and $B_d$ also provide information on the CKM matrix elements as in the SM. On the other hand, $|V_{td}|$ obtained from $\Delta m_B$ and $\epsilon_K$ may be different from that obtained above if we assume the SM analysis. In the same way $|V_{td}|$ from the branching ratios of $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ may be different. As shown in Fig. 9, the SUSY contributions are constructive to the SM contribution in $\Delta m_B$ ($\epsilon_K$) and destructive in $B(K \to \pi\nu\bar{\nu})$ so that the deviations of $|V_{td}|$ from the true value become opposite. Therefore combining CP asymmetry in $B$ decay, $\Delta m_{B_s}$ and various FCNC observables in $B$ and $K$ decays, we may obtain a hint on the existence of SUSY particles.

Acknowledgments

The work of Y. O. was supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan. The work of T. G.
was supported in part by the Soryushi Shogakukai.
References

[1] J. Ellis and D.V. Nanopoulos, *Phys. Lett.* **110B**, 44 (1982);
R. Barbieri and R. Gatto, *Phys. Lett.* **110B**, 211 (1982);
T. Inami and C.S. Lim, *Nucl. Phys.* **B207**, 533 (1982).

[2] L.J. Hall, V.A. Kostelecky and S. Raby, *Nucl. Phys.* **B267**, 415 (1986);
T. Kurimoto, *Phys. Rev.* **D 39**, 3447 (1989); *Mod. Phys. Lett.* **A 10**, 1577 (1995);
J.S. Hagelin, S. Kelley and T. Tanaka, *Mod. Phys. Lett.* **A 8**, 2737 (1993);
*Nucl. Phys.* **B415**, 293 (1994);
M.P. Worah, *Phys. Rev.* **D 54**, 2198 (1996);
G.T. Park and S.K. Kang, *Phys. Rev.* **D 54**, 4687 (1996);
M. Ciuchini, E. Franco, G. Martinelli and A. Masiero, *Phys. Rev. Lett.* **79**, 978 (1997);
J.A. Bagger, K.T. Matchev and R.J. Zhang, *Phys. Lett.* **B 412**, 77 (1997);
R. Barbieri and A. Strumia, *Nucl. Phys.* **B508**, 3 (1997);
T. Nihei, *Prog. Theor. Phys.* **98**, 1157 (1997).

[3] F. Gabbiani and A. Masiero, *Nucl. Phys.* **B322**, 235 (1989);
I.I. Bigi and F. Gabbiani, *Nucl. Phys.* **B352**, 309 (1991).

[4] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Nucl. Phys.* **B353**, 591 (1991).

[5] G.C. Branco and G.C. Cho, Y. Kizukuri and N. Oshimo, *Phys. Lett.* **B 337**, 316 (1994); *Nucl. Phys.* **B449**, 483 (1995).

[6] T. Goto, T. Nihei and Y. Okada, *Phys. Rev.* **D 53**, 5233 (1996); erratum, *ibid.* **54**, 5904 (1996).

[7] N. Oshimo, *Nucl. Phys.* **B404**, 20 (1993);
J. Hewett, *Phys. Rev. Lett.* **70**, 1045 (1993);
V. Barger, M. Berger and R.J.N. Phillips, *Phys. Rev. Lett.* **70**, 1368 (1993);
R. Barbieri and G.F. Giudice, *Phys. Lett.* **B 309**, 86 (1993);
J.L. Lopez, D.V. Nanopoulos and G.T. Park, *Phys. Rev.* **D 48**, 974 (1993);
J.L. Lopez, D.V. Nanopoulos, G.T. Park and A. Zichichi, Phys. Rev. D 49, 355 (1994);
Y. Okada, Phys. Lett. B 315, 119 (1993);
R. Garisto and J.N. Ng, Phys. Lett. B 315, 372 (1993);
M.A. Diaz, Phys. Lett. B 322, 207 (1994);
F.M. Borzumati, Z. Phys. C 63, 291 (1994);
S. Bertolini and F. Vissani, Z. Phys. C 67, 513 (1995);
J. Wu, P. Nath and R. Arnowitt, Phys. Rev. D 51, 1371 (1995);
P. Nath and R. Arnowitt, Phys. Lett. B 336, 395 (1994); Phys. Rev. Lett. 74, 4592 (1995);
F.M. Borzumati, M. Drees and M.M. Nojiri, Phys. Rev. D 51, 341 (1995);
J.L. Lopez, D.V. Nanopoulos, X. Wang and A. Zichichi, Phys. Rev. D 51, 147 (1995);
G. Kane, C. Kolda, L. Roszkowski and D. Wells, Phys. Rev. D 49, 6173 (1994);
M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B426, 269 (1994);
C. Kolda, L. Roszkowski, D. Wells and G. Kane, Phys. Rev. D 50, 3498 (1994);
V. Barge, M.S. Berger, P. Ohmann and R.J.N. Phillips, Phys. Rev. D 51, 2438 (1995);
B. de Carlos and J.A. Casas, Phys. Lett. B 349, 300 (1995); erratum, ibid. B351, 604 (1995);
T. Goto and Y. Okada, Prog. Theor. Phys. 94, 407 (1995); Prog. Theor. Phys. Suppl. 123, 213 (1996);
G.T. Park, Mod. Phys. Lett. A 11, 1187 (1996);
H. Baer and M. Brhlik, Phys. Rev. D 55, 3201 (1997);
H. Baer, M. Brhlik D. Castano and X. Tata, hep-ph/9712305.

[8] A. Ali, G. Giudice and T. Mannel, Z. Phys. C 67, 417 (1995);
P. Cho, M. Misiak and D. Wyler, Phys. Rev. D 54, 3329 (1996);
J. Hewett, J.D. Wells, Phys. Rev. D 55, 5549 (1997);
C.S. Huang and Q.S. Yan, hep-ph/9803366;
C.S. Huang, W. Liao and Q.S. Yan, hep-ph/9803460.

[9] T. Goto, Y. Okada, Y. Shimizu and M. Tanaka, Phys. Rev. D 55, 4273 (1997).
[10] S. Bertolini and A. Masiero, *Phys. Lett.* B 174, 343 (1986);  
B. Mukhopadhyaya and A. Raychaudhuri, *Phys. Lett.* B 189, 203 (1987);  
I.I. Bigi and F. Gabbiani, *Nucl. Phys.* B367, 3 (1991);  
G. Couture and H. König, *Z. Phys.* C 69, 174 (1995);  
Y. Nir and M.P. Worah, *Phys. Lett.* B 423, 319 (1998).  

[11] A.J. Buras, A. Romanino and L. Silvestrini, *Nucl. Phys.* B 520, 3 (1998);  
G.C. Cho, hep-ph/9801406.  

[12] K. Chetyrkin, M. Misiak and M. Münz, *Phys. Lett.* B 400, 206 (1997).  

[13] M. Ciuchini, G. Degrassi, P. Gambini and G.F. Giudice, hep-ph/9710335;  
P. Ciafaloni, A. Romanino and A. Strumia, *Nucl. Phys.* B 524, 361 (1998);  
F. Borzumati and G. Greub, hep-ph/9802391;  
T.M. Aliev and Mersin and E.O. Itlan, hep-ph/9803459.  

[14] P. Janot, talk given at the E.P.S '97, Jerusalem, Aug. 20-26, 1997.  

[15] D. Matalliotakis and H.P. Nilles, *Nucl. Phys.* B435, 115 (1995).  

[16] T. Goto, T. Nihei and J. Arafune, *Phys. Rev.* D 52, 505 (1995).  

[17] J. Ellis, S. Ferrara and D.V. Nanopoulos, *Phys. Lett.* B 114, 231 (1982);  
W. Buchmüller and D. Wyler, *Phys. Lett.* B 121, 393 (1983);  
J. Polchinski and M. Wise, *Phys. Lett.* B 125, 393 (1983);  
F. del Aguila, M. Gavela, J. Grifols and A. Mendez, *Phys. Lett.* B 126, 71 (1983);  
D.V. Nanopoulos and M. Srednicki, *Phys. Lett.* B 128, 61 (1983);  
M. Dugan, B. Grinstein and L. Hall, *Nucl. Phys.* B255, 413 (1985);  
Y. Kizukuri and N. Oshimo, *Phys. Rev.* D 45, 1806 (1992);  
*Phys. Rev.* D 46, 3025 (1992);  
S. Bertolini and F. Vissani, *Phys. Lett.* B 324, 164 (1994);  
T. Inui, Y. Mimura, N. Sakai and T. Sasaki, *Nucl. Phys.* B 449, 49 (1995);  
T. Kobayashi, M. Konmura, D. Suematsu, K. Yamada and Y. Yamagishi, *Prog. Theor. Phys.* 94, 413 (1995);  
T. Falk, K. Olive, M. Srednicki, *Phys. Lett.* B 354, 99 (1995);
R. Barbieri, A. Romanino, A. Strumia, Phys. Lett. B 369, 283 (1996); S.A. Abel, W.N. Cottingham, I.B. Whittingham, Phys. Lett. B 370, 106 (1996); T. Falk, K.A. Olive, Phys. Lett. B 375, 196 (1996); T. Ibrahim, P. Nath, Phys. Lett. B 418, 98 (1998); Phys. Rev. D 57, 478 (1998).

[18] A. Bouquet, J. Kaplan and C.A. Savoy, Phys. Lett. 148B, 69 (1984); Nucl. Phys. B262, 299 (1985); J.F. Donoghue, H.P. Nilles and D. Wyler, Phys. Lett. 128B, 55 (1983); L.E. Ibáñez and C. López, Nucl. Phys. B233, 511 (1984); L.E. Ibáñez and C. López and C. Muñoz, ibid. B256, 218 (1985).

[19] K. Inoue, A. Kakuto, H. Komatsu and S. Takeda, Prog. Theor. Phys. 68, 927 (1982); ibid. 71, 413 (1984); L. Ibáñez and G.G. Ross, Phys. Lett. 110B, 215 (1982); L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. B221, 495 (1983); J. Ellis, J.S. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 125B, 275 (1983).

[20] Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85, 1 (1991); Phys. Lett. B 262, 54 (1991); J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B 257, 83 (1991); Phys. Lett. B 262, 477 (1991); H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66, 1815 (1991).

[21] CLEO Collaboration, M.S. Alam et al., Phys. Rev. Lett. 74, 2885 (1995).

[22] Particle Data Group, R.M. Barnett et al., Phys. Rev. D 54, 1 (1996).

[23] CDF Collaboration, F. Abe et al., Phys. Rev. D 56, 1357 (1997); D0 Collaboration, S. Abachi et al., Phys. Rev. Lett. 75, 618 (1995).

[24] D0 Collaboration, S. Abachi et al., Phys. Rev. Lett. 76, 2222 (1996); D0 Collaboration, S. Abachi et al., Phys. Rev. D 57, 589 (1998); P. Azzi, (CDF), Proceedings of XXXII Rencontres de Moriond, QCD and High Energy Hadronic Interactions, Les Arcs, France, 1997, FERMILAB-CONF-97-241-E;
P.J. Wilson, (CDF), Proceedings of Les Rencotres de Physique de La Vallee D’Aosta, La Thuile, Italy, 1997, FERMILAB-CONF-97-241-E;
R.L. Culbertson, (CDF and D0), Proceedings of the 5th International Conference on Supersymmetries in Physics (SUSY ’97), Philadelphia, PA, 1997, FERMILAB-CONF-97-277-E.

[25] L3 Collaboration, F. Abe et al., Phys. Lett. B 350, 109 (1995);
     ALEPH Collaboration, D. Decamp et al., Phys. Rep. 216, 253 (1992);
     DELPHI Collaboration, P. Abreu et al., Phys. Lett. B 247, 157 (1990);
     OPAL Collaboration, M.Z. Acrawy et al., ibid. B 248, 211 (1990).

[26] J.P. Derendinger and C.A. Savoy, Nucl. Phys. B237, 307 (1984).

[27] B. Grinstein, M.J. Savage and M.B. Wise, Nucl. Phys. B319, 271 (1989).

[28] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996), and references therein.

[29] K. Adel and Y.-P. Yao, Phys. Rev. D 49, 4945 (1994);
     A.J. Buras, A. Kwiatkowski and N. Pott, Nucl. Phys. B 517, 353 (1998).

[30] A.J. Buras, A. Kwiatkowski and N. Pott, Phys. Lett. B 414, 157 (1997); update hep-ph/9707482 v3.

[31] C. Greub, T. Hurth and D. Wyler, Phys. Lett. B 380, 380 (1996); Phys. Rev. D 54, 3350 (1996).

[32] A. Ali and C. Greub, Z. Phys. C 49, 431 (1991); Phys. Lett. B 259, 182 (1991);
     Phys. Lett. B 361, 146 (1995);
     N. Pott, Phys. Rev. D 54, 938 (1996).

[33] H. Anlauf, Nucl. Phys. B430, 245 (1994).

[34] A. Czarnecki and W.J. Marciano, Phys. Rev. Lett. 81, 277 (1998).

[35] M. Misiak, Nucl. Phys. B393, 23 (1993); erratum,ibid. B439, 461 (1995);
     B. Buchalla and A.J. Buras, Nucl. Phys. B400, 225 (1993).

[36] CLEO Collaboration, S. Glenn et al., Phys. Rev. Lett. 80, 2289 (1998).
[37] Y. Grossman, Z. Ligeti, E. Nardi, *Nucl. Phys.* B465, 369 (1996).

[38] W.J. Marciano and Z. Parsa, *Phys. Rev.* D 53, R1 (1996).

[39] E787 Collaboration, S. Adler *et al.*, *Phys. Rev. Lett.* 79, 2204 (1997).

[40] KTeV

Collaboration, R. Ben-David for the collaboration, *Nucl. Phys. Proc. Suppl.* 66, 473 (1998)

[41] T. Inagaki *et al.*, “Measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay”, KEK proposal, (June 1996).

[42] I-H. Chiang *et al.*, “Measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$”, (September 1996).

[43] KAMI Collaboration, E. Cheu *et al.*, hep-ex/9709026.

[44] S. Herrlich and U. Nierste, *Nucl. Phys.* B476, 27 (1996); *ibid.* B419, 292 (1994);

*Phys. Rev.* D 52, 6505 (1995);

A.J. Buras, M. Jamin and P.H. Weisz, *Nucl. Phys.* B347, 491 (1990).

[45] ALEPH Collaboration, D. Buskulic *et al.*, *Z. Phys.* C 75, 397 (1997);

OPAL Collaboration, K. Ackerstaff *et al.*, *Z. Phys.* C 76, 417 (1997);

L3 Collaboration, M. Acciarri *et al.*, CERN-EP/98-28 (1998);

CDF Collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* 80, 2057 (1998).
Figure Captions

FIG. 1 Allowed regions in the space of the lighter chargino mass $m_{\tilde{\chi}_1^\pm}$ and the lighter top squark mass $m_{\tilde{t}_1}$ for (a) tan $\beta = 2$ and (b) tan $\beta = 30$. The dots represent the allowed region for the full parameter space and the squares show the allowed region for the minimal case ($m_0 = \Delta_0$).

FIG. 2 $C_7$, $C_9$ and $C_{10}$ normalized to the SM values for (a) the full parameter space with tan $\beta = 2$; (b) the minimal case with tan $\beta = 2$; (c) the full parameter space with tan $\beta = 30$; and (d) the minimal case with tan $\beta = 30$.

FIG. 3 $B(b \to s \gamma)$ in the supergravity model as a function of the charged Higgs mass for (a) tan $\beta = 2$ and (b) tan $\beta = 30$. Each solid line shows the branching ratio in the two Higgs doublet model (type II). Each dashed line shows the branching ratio in the SM. Dotted lines denote the upper and lower bounds on the branching ratio given by CLEO. For tan $\beta = 2$ the values in the minimal case is also plotted with circles.

FIG. 4 Branching ratios of $b \to s \gamma$ and $b \to s \mu^+ \mu^-$ for (a) tan $\beta = 2$; and (b) tan $\beta = 30$. Here, $B(b \to s \mu^+ \mu^-)$ is obtained by integrating in the range $2m_\mu < \sqrt{s} < m_{J/\psi} - 100$ MeV where $\sqrt{s}$ is the invariant mass of $\mu^+ \mu^-$ pair. The dots show the values in the full parameter space, the squares show those in the minimal case and the circle represents the SM value. The vertical dotted lines show the upper and lower bounds on $B(b \to s \gamma)$ given by CLEO.

FIG. 5 The branching ratio for $K_L \to \pi^0 \nu \bar{\nu}$ normalized to the SM value for tan $\beta = 2$ (a) as a function of the lighter chargino mass; (b) as a function of the lighter top squark mass; and (c) as a function of $B(b \to s \gamma)$. Each dot represents the value in the full parameter space and each square shows the value for the minimal case. The vertical dotted lines in (c) show the upper and lower bounds on $B(b \to s \gamma)$ given by CLEO. In (a) and (b) the CLEO bound is imposed (see text).

FIG. 6 Correlation between $B(K^+ \to \pi^+ \nu \bar{\nu})/B(K^+ \to \pi^+ \nu \bar{\nu})_{SM}$ and $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ for tan $\beta = 2$. Here, $m_0$ is fixed to 150 GeV and $\delta_{13}$ is taken as 30°, 90° and 150°.
FIG. 7 Correlation between $\epsilon_K/(\epsilon_K)_{SM}$ and $\Delta m_B/(\Delta m_B)_{SM}$ for $\tan \beta = 2$. Here, $m_0$ is fixed to 150 GeV and $\delta_{13}$ is taken as $30^\circ$, $90^\circ$ and $150^\circ$.

FIG. 8 $\Delta m_B$ normalized by the SM value for $\tan \beta = 2$ (a) as a function of the lighter chargino mass; (b) as a function of the lighter top squark mass; and (c) as a function of $B(b \to s \gamma)$. Each dot represents the value in the full parameter space and each square shows the value for the minimal case. The vertical dotted lines in (c) show the upper and lower bounds on $B(b \to s \gamma)$ given by CLEO. In (a) and (b) the CLEO bound is imposed.

FIG. 9 Correlation between $B(K_L \to \pi^0 \nu \bar{\nu})/B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ and $\Delta m_B/(\Delta m_B)_{SM}$ for (a) $\tan \beta = 2$; and (b) $\tan \beta = 30$. 

24
Figures

Fig. 1(a)

$m(\tilde{\tau}_1) \ [\text{GeV}]$

$m(\tilde{\chi}_1^\pm) \ [\text{GeV}]$

$\tan \beta = 2$

- all
- minimal
Fig. 1(b)
Fig. 2(a)

$\tan \beta = 2$
Fig. 2(b)
Fig. 2(c)
Fig. 2(d)
Fig. 3(b)
Fig. 4(a)
Fig. 4(b)
Fig. 5(a)
Fig. 5(b)
Fig. 5(c)
$\tan \beta = 2$

$m_0 = 150$ GeV

$\delta_{13} = 150^\circ$

$\delta_{13} = 30^\circ$

$\delta_{13} = 90^\circ$
$\tan \beta = 2$

$m_0 = 150 \text{ GeV}$

$\delta_{13} = 30^\circ$

$\delta_{13} = 90^\circ$

$\delta_{13} = 150^\circ$

**Fig. 7**
Fig. 8(a)
Fig. 8(b)
Fig. 8(c)
Fig. 9(a)
Fig. 9(b)