Dynamic Interaction Between Asset Prices and Bank Behavior: A Systemic Risk Perspective

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Abstract. Systemic risk in banking systems remains a crucial issue that it has not been completely understood. In our toy model, banks are exposed to two sources of risks, namely, market risk from their investments in assets external to the banking system and credit risk from their lending in the interbank market. By and large, both risks increase during severe financial turmoil. Under this scenario, the paper shows the conditions under which both the individual and the systemic default tend to coincide.

Key words: banking system; capital adequacy ratio (CAR); procyclicality; agent-based model; financial market

1 Introduction

Understanding of the interplay among banks through several channels is a crucial issue in the globalized world economy \cite{1,2}. In general, banks obtain their profits from the difference between deposit interest rates and interest rates in the interbank markets, stock markets, credit markets, and so on. Of course, money flows and interest rates are deeply interrelated to international economic conditions.

It is recognized that systemic risks are created by interconnection among banks. Helbing argues that systemic failures and extreme events are consequences of the highly interconnected systems and networked risks. He proposes a general theory to analyze, understand and manage systemic failures for various types of fields in socioeconomic-technological-environmental systems under a framework of Global Systems Science \cite{3}. He further addresses a list of common drivers of systemic instabilities that may destabilize anthropomorphic systems over time. These following drivers are to be considerably significant: (1) increasing system sizes, (2) reduced redundancies due to attempts to save resources (implying a loss of safety margins), (3) denser networks (creating increasing interdependencies between critical parts of the network), and (4) a high pace of innovation (producing uncertainties or "unknown unknowns").
These systemic risks probably stem from the positive feedback loop among financial markets and banks’ interactions. They can be formed by several reasons such as leverage trading, trend follower’s trading, loss cut trading, bankruptcy of banks, bunched sales, housing market and real economy, employment, excess concentration of wealth, a trade imbalance, political power, extreme low interest rates, excessive or lack of regulation and so on.

Several types of nonlinear positive feedback mechanisms can concurrently trigger serious crashes damaging all the financial systems. This study collocates with that stream of works that aim to estimate systemic risks among banks under exposures of risky assets traded in the stock market [4].

Some existing studies concern risk propagation through lending and borrowing networks. This approach focuses on interactions of debt and credit among banks.

For example, Iori et al. analyze the systemic risk in interbank money market by simulating the banks’ lending activities [5]. Their simulation model assumes that banks carry liquidity risk caused by the maturity gap between funding and investment activities. The model introduces a feature according to which banks pool this risk and further creates the potential for one bank’s crisis to propagate through the system.

Furthermore, the heterogeneity of banks is analyzed, which can cause knock-on effects in shock propagation, while the interbank market comprised of homogeneous banks tends to stabilize absorbing shocks. This model settings, two types of banks are assumed: those with positive cash and those with negative cash. Accordingly, they are classified as potential lenders and potential borrowers. Banks invest their money first and lend the remaining part as lending. The total demand in the market does not always match the total supply. Hence, there exists default risk of banks due to the shortage of liquidity, and shocks can be propagated through the system.

As for the performance of the interbank market in its role as a safety net, the insurance role of interbank lending prevails when banks are homogeneous; higher reserve requirements can lead to a higher incidence of bank failures. When banks are heterogeneous in average liquidity or average size, contagion effects may arise. They found that such effects can be of both a direct (i.e., knock-on from a failing bank to its direct creditors) and an indirect (i.e., causing criticality in the system as a whole) nature.

Despite the potential to create contagion, they insist that inter-bank lending always seems to stabilize the system: the elapsed time before the first failure is always observed to increase with connectivity. Their simulation results also indicate that heterogeneity alone can contribute to instability.

Gai and Kapadia study that probability and potential impact of contagion, which are influenced by aggregate and idiosyncratic shocks, change in network structure, and asset market liquidity [6].

A model of contagion in arbitrary financial networks is developed by using directed and weighted network model to express the widespread transmission of shocks through numerical simulation about shock transmission.
The banks are linked together by their financial claims on each other, including through interbank markets and payment systems. They model contagion stemming from unexpected shocks in complex financial networks with arbitrary structure and then use numerical simulations to illustrate and clarify the intuition underpinning our analytic results.

The result of simulation analysis suggests that financial systems exhibit a robust-yet-fragile tendency: while the probability of contagion may be low, the effects can be extremely widespread when problems occur. Adverse aggregate shocks and liquidity risk amplify the likelihood and extent of contagion.

It is also clarified why the resilience of the system in withstanding fairly large shocks prior to 2007 should not have been taken as a reliable guide to its future robustness. It means that we need more flexible assumption when building network based model.

The approach provides a first step towards modelling contagion risk when true linkages are unknown. They suggest a further extension of the analysis by relaxing the assumption that the defaulting bank is randomly selected and, considering the implications of targeted failure affecting big or highly connected interbank borrowers. As mentioned, added realism could also be incorporated into the model by using real balance sheets for each bank or endogenizing the formation of the network. Such extension of the model would be beneficial from a systemic risk research viewpoint.

Haldane and May study possible effects of risk optimization by individual financial institutions on the stability of the system as a whole [7]. They explore the interplay between complexity and stability in deliberately simplified models of financial networks to find some policy lessons with the explicit aim of minimizing systemic risk. They claim that the network dynamics of what might be called 'financial ecosystems' has parallels with ecology, where too much complexity implies instability. The well-known arbitrage pricing theories (APT) as well as other derivative pricing theories often assume perfect competition, market liquidity, no-arbitrage and market completeness. Crucially, these conditions are not always satisfied; therefore, trading activities that assume these conditions can destabilize markets, having possible effect on the dynamical behavior, while such activities seem to be successful at an initial stage.

Haldane and May also delve into the shock propagation mechanism, applying network system approach originally developed in ecology. A financial system is expressed as a network in which many banks are connected with credit linkage, forming an inter-bank money market. An initial shock that arose in some node are transmitted to other connected nodes when their shock absorbing capacity of a node is insufficient to the incoming shock.

In addition the liquidity factor plays a major role in the shock propagation. The losses in the value of bank external assets, caused by a generalized fall in market prices, such liquidity shocks amplify as more banks fail accordingly. Thus, relatively small initial liquidity shocks have the potential to make strong contributions to systemic risk. Iori et al. also emphasize the cascading effect of shock propagation, in which diminished availability of interbank loans caused serial
failure of liquidity funding by banks. These complicated interactions between nodes in a network cannot be clarified by the traditional economic theory.

The traditional rationale for setting regulatory capital/liquidity ratios is that idiosyncratic risks are reduced to the balance sheets of individual banks. Prudential regulation is following in the footsteps of ecology, which has increasingly drawn on a system-wide perspective when promoting and managing ecosystem resilience.

The scientists also listed two policy implications from their topological network analysis: the diversity across the financial system and the modularity within the financial system. They warn that banks' balance sheets and risk management systems became increasingly homogeneous; homogeneity bred fragility. As for the modularity within the financial system, it protects the systemic resilience of both natural and constructed networks by limiting the potential for cascades. They emphasized the importance of encouraging modularity and diversity in banking ecosystems as a means of buttressing systemic resilience.

One of the authors (P.T.) proposed DebtRank to measure default risk of banks. The paper assumes that bankruptcy may influence the balance sheet situation of other banks and that contagion effect may happen. DebtRank is one of the indicators to calculate the risk of the contagion effect for each bank.

Reducing procyclicality and promoting countercyclical buffers is one of the most important issues in the Basel III Accords. It provides a message that “One of the most destabilizing elements of the crisis has been the procyclical amplification of financial shocks throughout the banking system, financial markets and the broader economy.”

In this study, we focus on correlations among financial assets. This may play a role of common factors and create procyclicality. To do so, we study interactions between banking systems and the financial market. Financial prices fluctuate and show volatility clustering and volatility synchronization. In fact, fluctuations of financial prices seem to be random, however, volatilities of financial assets are sometimes synchronous. If many banks have positions in financial assets, then their capital adequacy ratios sometimes may vary synchronously. This is a common factor effect of financial assets in balance sheet. In order to investigate this scenario, we construct an agent-based model consisting of banks and the financial market.

The linkage via underlying common factor is widely observed in the financial market. Shocks prevail to the whole market in a short period of time through arbitrary transactions by market participants. Individual assets return, therefore they tend to synchronize in terms of volatility fluctuation: a large scale of volatility fluctuations are observed at specific timing, resulting in a market shock event.

Asset price fluctuation in the financial markets have been studied by many researchers. Stochastic models as well as agent-based models are well-studied. Some researchers pay a significant attention to the network effects in the financial markets. From empirical studies on financial time series, asset price fluctuations follow fat-tailed distributions. This is often mod-
eled as a random number drawn from a student $t$ distribution \[1,16,17\]. However, since asset price fluctuations are generated from trading by market participants, high volatility regimes are not independent of banks behavior.

How does banks behavior affect financial markets and how do the asset price fluctuations influence banks’ behavior? This forms a circular causality. This is a main question of this study. To do so, we consider a toy model of interaction among banks by means of an agent-based model. We assume that banks have lending and borrowing relationships with other banks and invest their money to an asset. We focus on two viewpoints: capital adequacy ratio and interaction between lending and borrowing network and financial markets.

One of the authors (T.I.) analyzed the Japanese stock market by applying GARCH model to individual stock returns separately \[18\]. The result shows the market-wide synchronization of extreme volatilities (larger than the 95th percentile of the empirical distribution of individual volatilities), which occurred mostly at crisis periods.

The purpose of this paper is to clarify influence of procyclicility. We assume that financial assets in the balance sheet play a role as a common factor in a banking system. From an agent-based model consisting a banking system and financial market, we construct a model and measure correlation of capital adequacy ratio influenced by financial markets.

The rest of the paper is organized as follows: Section 2 defines a banking system; Section 3 defines behavior of market participants; Section 4 shows a model of chain-reaction bankruptcy; Section 5 exhibits simulation results obtained from the proposed agent-based model; Section 6 tells some limitations of the model; Section 7 is devoted to draw our conclusions.

2 Banks

Let us consider $N(t)$ banks which interplay one another under lending-borrowing relationships and a financial market at time $t$.

Suppose a balance sheet consisting of liabilities and assets in bank $i$ (See Fig. 1). Equity $E_i(t)$, Deposit $D_i$, and Debt $L_i(t)$ are categorized as liabilities and Cash $C_i(t)$, Credit $K_i(t)$ and Financial assets $J_i(t)$ as assets. We assume that the bank holds $n_i(t)$ units of risky assets with their market price $S(t)$ and cash $C_i(t)$ at time $t$. Therefore, the value of risky asset is estimated as $J_i(t) = n_i(t)S(t)$.

Since the bank is initially financed by their bank depositors, we assume that $n_i(0) > 0$ and $C_i(0) > 0$. The bank deposit is described as $D_i$, which is a constant value. If the $i$-th bank buys $V_i(t)$ units of risky asset then $n_i(t + \Delta t) = n_i(t) + V_i(t)$ and $C_i(t + \Delta t) = C_i(t) - V_i(t)S(t)$. If the $i$-th bank sells then $n_i(t + \Delta t) = n_i(t) - V_i(t)$ and $C_i(t + \Delta t) = C_i(t) + V_i(t)S(t)$. $V_i(t)$ represents volume traded by the bank $i$ at time $t$.

Besides, a lending and borrowing relationship exists among banks. Such a relationship can be described as an asymmetric weighted matrix. Let $W_{ij}(t)$ be expressed as a lending amount from the bank $i$ to the bank $j$ at time $t$. The
Fig. 1. A model of balance sheet. $E_i(t)$ represents Equity, $D_i(t)$ Deposit, $L_i(t)$ Debt, $C_i(t)$ cash, $K_i(t)$ Credit and $J_i(t)$ financial assets.

debt of the $i$-th bank at time $t$ towards other banks is estimated as $L_i(t) = \sum_{j=1}^{N} W_{ji}(t)$, and the bank credit of the $i$-th bank at time $t$ is estimated as $K_i(t) = \sum_{j=1}^{N} W_{ij}(t)$. Therefore, the survival condition of the $i$-th bank is given by

$$C_i(t) + J_i(t) + K_i(t) > L_i(t) + D_i \tag{1}$$

Namely, if $C_i(t) + J_i(t) + K_i(t) \leq L_i(t) + D_i$ then the $i$-th bank goes bankruptcy. We assume that the counterparties of the $i$-th bank can receive $\rho \times 100$ percent ($0 < \rho < 1$) of their exposure. This can be expressed as temporal development of interconnection among banks. The update rule of $W_{ji}(t)$ is given for all $i$ as

$$W_{ji}(t + \Delta t) = \begin{cases} W_{ji}(t) \text{(If } C_i(t) + J_i(t) + K_i(t) \geq L_i(t) + D_i) \\ \rho W_{ji}(t) \text{(Otherwise)} \end{cases} \tag{2}$$

If we can simulate the agent-based model repeatedly, then from the relative frequency we can estimate the default probability of the $i$-th bank as

$$\Pr[C_i(t) + J_i(t) \leq D_i + L_i(t) - K_i(t)] \approx \frac{M[C_i(t) + J_i(t) \leq D_i + L_i(t) - K_i(t)]}{M_{sim}} \tag{3}$$

where $M_{sim}$ is the number of simulations, $M[C_i(t) + J_i(t) < D_i + L_i(t) - K_i(t)]$ is the number of defaults for the $i$-th bank.
The bank $i$ must pay money to both depositors with deposit interest rates and lenders with interest rates in the interbank market every step. Such payments write

\[ C_i(t + \Delta t) = C_i(t) - \lambda_D D_i - \lambda_I L_i + \lambda_I K_i, \quad (4) \]

where $\lambda_D$ represents the deposit interest rate, and $\lambda_I$ the interest rate in the interbank market. We assume that the interest rate is given by

\[ \lambda = \left(1 + \text{(annual interest rate)}\right)^{\frac{\Delta t}{365}} - 1, \quad (5) \]

where $\Delta t$ is measured daily. In the case that the annual interest rate is 1% and $\Delta t = 1 \text{[day]}$, the daily interest rate is estimated as $2.67262 \times 10^{-5}$.

Capital requirements are designed to ensure that banks hold enough resources to absorb shocks to their balance sheets. A standard measure of the health of individual banks is their capital adequacy ratio (CAR). Introduced in 1988 with the Basel I Accords, the CAR is calculated as the total regulatory capital of a bank divided by its risk-weighted assets. The Basel II revision refined the calculation of risk weights and incorporated three major components of risk: credit, operational, and market risk. The Basel II revision also set the minimum CAR at 8 percent for international banks and at 4 percent for domestic banks. Conservatively-run banks tend to have high CARs to cushion against higher losses. In addition, Basel III revision introduced that a Total Capital Ratio to total Risk Weighted Assets should be larger than 8% and that the common Equity Tier 1 to risky asset ratio is greater than 4.5%.

Bank capital to assets is the ratio of bank capital and reserves to total assets. Capital and reserves include funds contributed by owners, retained earnings, general and special reserves, provisions, and valuation adjustments. Capital includes tier 1 capital (paid-up shares and common stock), which is a common feature in the banking systems all over the world and total regulatory capital, which includes several specified types of subordinated debt instruments that need not to be repaid if the funds are required to maintain minimum capital levels (these comprise tier 2 and tier 3 capital). Total assets include all non-financial and financial assets. CAR is defined as

\[ CAR = \frac{(\text{Tier 1 capital}) + (\text{Tier 2 capital})}{(\text{Risk weighted assets})}, \quad (6) \]

where Tier 1 capital $T_1$ is defined as

\[
\begin{align*}
(\text{Tier 1 capital}) &= (\text{paid up capital}) + (\text{statutory reserves}) \\
&\quad + (\text{disclosed free reserves}) \\
&\quad - (\text{equity investment in subsidiary}) \\
&\quad - (\text{intangible assets}) \\
&\quad - (\text{current and b/f losses}),
\end{align*}
\]

and Tier 2 capital $T_2$ as

\[
(\text{Tier 2 capital}) = (\text{Undisclosed Reserves})
\]
The risk-weight depends on kinds of assets. In the case of cash and government securities, the weight is 0%. Mortgage loans have 50% (Basel I) weight or 35% (Basel II). Other loans and assets have 100% weight (Basel I) or 75% to 150% (Basel II). Stocks have 100% weight (Basel I and II).

In the case of our model, we assume that the total capital adequacy ratio is approximated as

\[ CAR_i(t) = \frac{C_i(t) + J_i(t) + K_i(t) - L_i(t) - D_i}{J_i(t) + K_i(t)} \times 100(\%), \tag{7} \]

and that the Tier1 common equity adequacy ratio is approximated by a ratio of Tier1 common equity (Cash and Common Stocks) to risk weighted assets and operational risk;

\[ CEAR_i(t) = \frac{C_i(t)}{J_i(t) + (1 + \lambda_I)C_i(t) + c|y_i(t)|V_i(t)S(t)} \times 100(\%), \tag{8} \]

where \( c \) is a positive constant less than 1. The Basel II Accords forced the requirement such that \( CAR_i(t) \geq 8 \) (%). The Basel III Accords have required \( CEAR_i(t) \geq 4.5 \) (%) since 3Q in 2013 in addition to this.

This is a model of a banking system. The variables in this model are listed in Tab. 1.

| variables | items |
|-----------|-------|
| \( t \)   | time  |
| \( N(t) \) | The number of market participants |
| \( N_{tf}(t) \) | The number of trend followers |
| \( S(t) \) | Market price of risky assets |
| \( E_i(t) \) | Equity |
| \( D_i \) | Deposit |
| \( L_i(t) \) | Debt |
| \( C_i(t) \) | Cash |
| \( K_i(t) \) | Credit |
| \( J_i(t) \) | Financial assets |
| \( W_{ij}(t) \) | Lending amount from bank \( i \) to bank \( j \) |
| \( n_i(t) \) | Holding volume of financial assets |
| \( V_i(t) \) | Traded volumes |
| \( y_i(t) \) | investment attitude |

Fig. 2 shows averaged CAR in 8 countries from year 2000, where data can be downloaded from Worldbank’s DataBank [19]. The graph tells us that the averaged CAR varies in time and that it fluctuates in a range from 4.0 to 12.0.
The average CAR of United States maintains more than 8%. France, Germany and Japan are less than 6% from 2000 to 2010. Canada and South Korea take more than 6%.

However, these values are averages over the country. A specific bank takes larger values than the average. For example, although the Japanese averages are less than 5%, Japanese three mega banks (Mizuho Financial Group, Mitsubishi Tokyo Financial Group and Sumitomo Mitsui Financial Group) take larger values; Mizuho Financial Group shows 15.09% total capital ratio as of December 31, 2014 [20] (See Tab. 2). Mitsubishi Tokyo Financial Group also shows 15.39% total capital ratio as of September 31, 2014 [21] (See Tab. 3). Sumitomo Mitsui Financial Group shows 16.79% total capital ratio as of December 2014 [22] (See Tab. 4). The current capital-to-asset ratios reported by Worldbank are averaged over banks belonging to countries not regarding total amounts of banks equity and assets. It may be necessary to compare the capital adequacy ratios in accordance with sizes of banks’ capital and risk weighted assets.

![Graph showing averaged bank capital adequacy ratio in 8 countries](image)

**Fig. 2.** COLOR ONLINE. Averaged bank capital adequacy ratio in 8 countries (Japan, United States, Germany, Italy, France, Switzerland, Korea Rep and Canada) for a period from 2000 to 2013.

### 3 Market mechanism

The risky assets are traded through a common market. The bank traders buy and sell their risky assets. For the sake of simplicity, we assume that the investment attitude in the financial market is determined on the basis of the last change in the market price.
Table 2. Mizuho Financial Group, as of December 31, 2014.

| Items                        | Amount          | Basel III Template No. |
|------------------------------|-----------------|------------------------|
| Tier1 capital                | 7,481,242M JPY  | 45                     |
| Tier2 capital                | 2,181,862M JPY  | 58                     |
| Total capital                | 9,663,105M JPY  | 59                     |
| Risk weighted asset          | 64,023,907M JPY | 60                     |
| Tier1 CAR                    | 11.68%          | 62                     |
| Tier1 CAR (Common Equity Tier 1) | 9.25%      | 61                     |
| Total CAR                    | 15.09%          | 63                     |

Table 3. Mitsubishi Tokyo Financial Group, as of September 31, 2014.

| Items                        | Amount          | Basel III Template No. |
|------------------------------|-----------------|------------------------|
| Tier1 capital                | 12,726,118M JPY | 45                     |
| Tier2 capital                | 3,313,073M JPY  | 58                     |
| Total capital                | 16,039,191M JPY | 59                     |
| Risk weighted asset          | 104,160,164M JPY| 60                     |
| Tier1 CAR                    | 12.21%          | 62                     |
| Tier1 CAR (common Equity Tier 1) | 10.97%     | 61                     |
| Total CAR                    | 15.39%          | 63                     |

Table 4. Sumitomo Mitsui Financial Group, as of December 31, 2014.

| Items                        | Amount          | Basel III Template No. |
|------------------------------|-----------------|------------------------|
| Tier1 capital                | 8,366,228M JPY  | 45                     |
| Tier2 capital                | 2,547,949M JPY  | 58                     |
| Total capital                | 10,914,178M JPY | 59                     |
| Risk weighted asset          | 64,992,642M JPY | 60                     |
| Tier1 CAR                    | 12.87%          | 62                     |
| Tier1 CAR (common Equity Tier 1) | 11.17%     | 61                     |
| Total CAR                    | 16.79%          | 63                     |

The market participants are classified into two types; trend followers and contrarians. The trend followers are traders who want to buy (sell) assets when their price goes up (down). The contrarians are traders who want to buy (sell) assets when their price goes down (up). Suppose that $N(t)$ banks trade a single asset at time $t$. It is assumed that the banks can take three investment attitudes coded as three states (buying: 1, selling: -1, and waiting: 0):

$$y_i(t + Δt) = \begin{cases} 
1 & \text{with probability } p^{(+)}_i(R(t)) \\
0 & \text{with probability } 1 - p^{(+)}_i(R(t)) - p^{(-)}_i(R(t)) \\
-1 & \text{with probability } p^{(-)}_i(R(t)) 
\end{cases}, \quad (9)$$

where

$$p^{(+)}(R) = \frac{1}{2} \text{erfc} \left( \frac{\theta_2 - a_1 R}{\sqrt{2}\sigma} \right), \quad (10)$$
$$p^{-}(R) = \frac{1}{2} \text{erfc} \left( \frac{a_i R - \theta_{1i}}{\sqrt{2} \sigma} \right).$$  \hspace{1cm} (11)$$

$a_i$ is a parameter which determines behavior of banks. If $a_i > 0$, then the $i$-th bank is a trend follower. If $a_i < 0$, then the $i$-th bank is a contrarian. $\theta_{1i}$ and $\theta_{2i}$ are parameters of the $i$-th bank ($\theta_{1i} < \theta_{2i}$). The value of $\sigma (> 0)$ represents the uncertainty of decision.

In fact, there are some categories of banks such as investment, retail and central banks. Central banks sometimes trade stocks for non-profit making but policy-oriented purposes (e.g., asset purchase for monetary easing). Investment banks and retail banks have different risk preference to financial markets. These differences can be tuned by parameters $a_i$, $\theta_{1i}$ and $\theta_{2i}$. The parameters strongly depend on market conditions and banks’ risk appetite. However, we do not have enough information about banks’ risk appetite in general. We need to infer model parameters from available information about macro economic conditions, bank balance sheet, and market sentiments. This problem arises other problems that are not meant to be investigated in this paper. Parameter estimations should be seen as an important future tasks.

These are models of response curves between perception (price change) and three types of investment attitudes. We use these curves that relate a price change to investment attitude. Fig. shows the probabilities for three investment attitude. The probabilities can be adjusted by using $\theta_{1i}$, $\theta_{2i}$, $a$ and $\sigma$. Specifically, $\theta_{1i}$ and $\theta_{2i}$ are parameters to describe a range of unresponsiveness to the price change. While $\theta_{1i}$ normally takes negative values, $\theta_{2i}$ takes positive values. The mode of $p^{(0)}(R)$ is equivalent to $(\theta_{1i} + \theta_{2i})/2$. If we differ $|\theta_{1i}|$ from $|\theta_{2i}|$, then we can express asymmetric response of price changes to investment attitudes. This is also understood from probit-logit reasoning.

Furthermore, it is assumed that excess demand for a risky asset $\sum_{i=1}^{N} V_i(t) y_i(t)$ drives the market price. The volume traded by the bank $i$ is assumed to be proportional to its amount of equity:

$$V_i(t) = \eta \left( C_i(t) + J_i(t) + K_i(t) - L_i(t) - D_i \right), \hspace{1cm} (12)$$

where $\eta$ is an investment ratio taking from 0 to 1. There is a general strategic framework to control risk from price fluctuations in banking based on an amount of bank’s risk-based capital. In general, banks should trade risky assets under the limit of its economic capital. This is a fundamental requirement in economic capital management [23]. The risk-based capital is basically linked with the amount of equity, although it is complicated to calculate risk-based capital exactly from balance sheet data. Therefore, we use an amount of equity to determine the volume traded by the bank.

To guarantee positive market prices, the following log return is chosen:

$$R(t) = \log S(t + \Delta t) - \log S(t), \hspace{1cm} (13)$$
and the log returns are proportional to the excess demand,

$$R(t) = \gamma \sum_{i=1}^{N} V_i(t) y_i(t), \quad (14)$$

where $\gamma$ is a positive constant to represent the response of the return to the excess demand. This constant is associated with price elasticity.

From Eqs. (13) and (14), we have

$$S(t + \Delta t) = S(t) \exp\left(\gamma \sum_{i=1}^{N} V_i(t) y_i(t)\right). \quad (15)$$
After the bank trades stocks, the bank’s amount of cash and the holding number of stocks is updated. For the buyer side, if \( C_i(t) \geq V_i(t)S(t) \) then
\[
C_i(t + \Delta t) = C_i(t) - V_i(t)S(t), \quad n_i(t + \Delta t) = n_i(t) + V_i(t).
\]
For the seller side, if \( n_i(t) \geq V_i(t) \) then
\[
C_i(t + \Delta t) = C_i(t) + V_i(t)S(t), \quad n_i(t + \Delta t) = n_i(t) - V_i(t).
\]

4 Debt exposure

The \( i \)-th bank has equity \( E_i(t) = C_i(t) + J_i(t) + K_i(t) - L_i(t) - D_i(t) \) at time \( t \). If \( E_i(t) \) is less than zero, we define that the \( i \)-th bank goes bankrupt. Thus, it would be useful to formalize the Default Event (DE) such that
\[
DE := E = 0 := \frac{CAR(J + K)}{100\%},
\]
which can be also seen as a function in terms of \( CAR \).

Let us assume that a symmetric weighted matrix \( W_{ij}(t) \) describes a lending and borrowing relationship among banks at time \( t \). The debt of the \( j \)-th bank is estimated as \( L_j(t) = \sum_{i=1}^{N} W_{ij}(t) \). The default impact of the bank \( i \) at time \( t \) is denoted as \( W_{ij}(t) \).

In order to estimate the worst case scenario, we assume that \( \rho \) in Eq. (2) is set as zero (\( \rho = 0 \)). If we use a nonzero value for \( \rho \), then we can simulate the case where some percentage of the debt of \( j \)-th bank to \( i \)-th bank can be collectible.

Let \( Q_i^{(n)} \) be a cumulative loss of the bank \( i \) in the \( n \)-th iteration. If we introduce a binary variable \( h_j^{(n)} \) such that \( h_j^{(n)} = 0 \) (normal) and \( h_j^{(n)} = 1 \) (default), then the cumulative losses of the bank \( i \) can be calculated as
\[
Q_i^{(n+1)} = Q_i^{(n)} + \sum_{j=1}^{N} W_{ij}^{(n)} h_j^{(n)},
\]
where \( W_{ij}^{(n)} \) is updated as
\[
W_{ij}^{(n)} = \begin{cases} 
  W_{ij}^{(n-1)} & \text{if } h_j^{(n-1)} = 0 \\
  \rho W_{ij}^{(n-1)} & \text{if } h_j^{(n-1)} = 1 
\end{cases}.
\]

with the initial conditions given by
\[
Q_i^{(0)} = 0, \quad W_{ij}^{(0)} = W_{ij}(t).
\]

If the cumulative loss \( Q_i^{(n)} \) of the bank \( i \) becomes greater than its equity \( E_i(t) \)
\[
Q_i^{(n)} > E_i(t),
\]
then we recognize that the bank $i$ goes bankrupt and set $h_i^{(n+1)} = 1$ (it becomes default) otherwise $h_i^{(n+1)} = h_i^{(n)}$. We obtain the new matrix at time $t + \Delta t$ as $W_{ij}(t + \Delta t) = W_{ij}^{(\infty)}$.

Our interest is to understand the total losses over all the banks, which is denoted as $H(t)$, triggered by the default of the bank $j$, which is estimated as the total losses

$$H_i(t) = \sum_{j \neq i} h_j^{(\infty)} \left( C_j(t) + J_j(t) + K_j(t) \right),$$  

$$H(t) = \sum_{i=1}^{N} H_i(t),$$

under the initial condition $h_i^{(0)} = 0$ $(i \neq j)$. $C_j(t) + J_j(t) + K_j(t)$ is assumed to be the economic value of the bank $j$.

5 Simulation

In order to compute this algorithm, we set parameters as shown in Tab. We sample parameters from a uniform distribution. In fact, some relationships between parameters do exist and they are time-dependent. However, the purpose of this numerical simulation is to show the interplay between financial markets and the trend followers and how it can play a role of the positive feedback mechanism and procyclical motion of market prices. If we have detailed data on banks, we can set more realistic parameters. Moreover, it is difficult to obtain information about the behavior of all the banks, as mentioned above. We should consider a way to set reliable parameters from available information about banks balance sheets, however, such an extension makes a problem more complicated. The is an important issue to be analyzed and discussed in future studies.

In our numerical simulation, we assume that the interest rate (5%) is higher than the deposit interest rate (1%) but one is not much higher than the other. The current parameter model setting is the toughest case scenario. If we set interest rates higher than the deposit rates, then we can simulate a safer case scenario than the assumptions given by the model presented.

We also selected the total cash possessed by banks at time $t$ defined as

$$C_{total} = \sum_{i=1}^{N} C_i(t),$$

and the total number of stocks held by banks at time $t$ computed by

$$n_{total} = \sum_{i=1}^{N} n_i(t),$$
Table 5. Model parameters

| Parameter                         | Description                                                                 |
|----------------------------------|-----------------------------------------------------------------------------|
| \( N(0) \)                       | Initial number of banks \( N(0) = 100 \)                                    |
| \( S(0) \)                       | Initial market price \( S(0) = 1 \)                                         |
| \( \theta_1 \)                   | Sampled from the uniform distribution \( U(-1.0, -0.3) \)                   |
| \( \theta_2 \)                   | Sampled from the uniform distribution \( U(0, 1.0) \)                       |
| \( \bar{a} \)                    | Sampled from the uniform distribution \( U(a_0, a_0 + 2.1) \)               |
| \( a_i(t) \)                     | Sampled from the uniform distribution \( U(-\sigma_a, \sigma_a) \)          |
| \( \sigma \)                     | Sampled from the uniform distribution \( U(3.0, 4.0) \)                     |
| \( C_i(0) \)                     | Sampled from the uniform distribution \( U(2000, 3000) \)                   |
| \( n_i(0) \)                     | Sampled from the uniform distribution \( U(2000, 3000) \)                   |
| \( W_{ij}(0) \)                  | Sampled from the uniform distribution \( U(100, 500) \)                    |
| Deposit interest rate \( \lambda_D \) | \( 2.7202 \times 10^{-5} \) (1%)  |
| Interbank interest rate \( \lambda_I \) | \( 1.3368 \times 10^{-4} \) (5%)  |
| Investment ratio to equity \( \eta \) | 0.001                        |
| Price elasticity constant \( \gamma \) | 0.1                          |
| Randomness of \( a_i(t) \) \( \sigma_a \) | 0.3                          |
| Recovery ratio \( \rho \)        | 0.0                          |
| Averaged number of links \( L \) | 6                            |
| Lending money \( W_{ij} \)       | Sampled from the uniform distribution \( U(100, 500) \)                    |

as representative quantities describing conditions of banks. The ratio of the trend followers to the total traders

\[
\alpha(t) = \frac{N_{tf}(t)}{N(t)} ,
\]

where \( N_{tf}(t) \) is the number of trend followers. Fraction \( \alpha(t) \) is used as a parameter to characterize the financial market at time \( t \). We can control \( \alpha(t) \) by changing \( a_i(t) \) for every bank \( i \) at time \( t \). Usually, the fraction of the number of trend followers to the total number of market participants varies in time due to the evolution of the mechanism of participants’ trading strategies. To consider the temporal dependence of fraction \( \alpha(t) \), we introduced some randomness to \( a_i(t) = \bar{a}_i + a_i'(t) \), where \( a_i'(t) \) is sampled from the uniform distribution \( U(-\sigma_a, \sigma_a) \). Here \( \sigma_a \) is a positive constant.

Fig. 4 shows the market prices at (a) \( \alpha(0) = 0.31 \), (b) \( \alpha(0) = 0.48 \), (c) \( \alpha(0) = 0.56 \), and (d) \( \alpha(0) = 0.75 \). \( \alpha(0) = 0.31 \) (contrarians-dominant market) represents a case where contrarians are dominant in the market. \( \alpha(0) = 0.48 \) (contrarians-predominant-market) corresponds to a case where contrarians are still dominant but trend followers are more in number than in the case of \( \alpha(0) = 0.31 \). In the case of \( \alpha(0) = 0.56 \) (trend-followers-predominant market), trend followers are more than contrarians in the market. \( \alpha(0) = 0.75 \) (trend-followers-dominant market) shows a case where trend followers are dominant in the market. We compare the four cases and examine dependency of \( \alpha(t) \) on the market condition.

Tab. 6 shows descriptive statistics of the market prices for the four cases. As shown in Fig. 5, we confirmed that \( \alpha(t) \) fluctuates with some variance and slightly varies in time due to a bankruptcy of banks. Namely, if a bank goes bankrupt
at time $t$, then $N(t+1) = N(t) - 1$. If it is further a trend follower, then $N_{tf}(t+1) = N_{tf}(t) - 1$.

The duration of these plots corresponds to 100 years (36,500 days). Fig. 6 shows the total amount of cash and Fig. 7 shows the total amount of risky assets. Tabs. 7 and 8 show descriptive statistics of the total amount of cash and risky assets.

When the market price goes down, the total amount of risky assets decreases and the total amount of losses increases. This situation can be confirmed at $\alpha(0) = 0.75$.

### Table 6. Descriptive statistics of the market prices for four cases.

| $\alpha(0)$ | Mean  | Median | Var   | Min.  | Max.     |
|-------------|-------|--------|-------|-------|----------|
| 0.31        | 0.396906 | 0.388796 | 0.002148 | 0.288575 | 0.925797 |
| 0.48        | 0.422041 | 0.419790 | 0.003362 | 0.276761 | 0.890119 |
| 0.56        | 0.268880 | 0.260136 | 0.015824 | 0.025530 | 0.876253 |
| 0.75        | 0.024344 | 0.001470 | 0.004728 | 0.000013 | 0.838115 |

### Table 7. Descriptive statistics of the total amount of cash for four cases.

| $\alpha(0)$ | Mean  | Median | Var   | Min.  | Max.     |
|-------------|-------|--------|-------|-------|----------|
| 0.31        | 221025.65 | 216784.15 | 434426403.00 | 193298.04 | 258798.49 |
| 0.48        | 222981.91 | 221224.41 | 471846434.54 | 190538.46 | 262366.22 |
| 0.56        | 200522.00 | 193584.94 | 686616124.42 | 163129.25 | 271543.81 |
| 0.75        | 262675.93 | 265049.78 | 959870096.96 | 179773.39 | 329123.67 |

### Table 8. Descriptive statistics of the total amount of risk assets for four cases.

| $\alpha(0)$ | Mean  | Median | Var   | Min.  | Max.     |
|-------------|-------|--------|-------|-------|----------|
| 0.31        | 199288.73 | 193253.00 | 71126678.75 | 162122.00 | 253637.00 |
| 0.48        | 198077.09 | 199944.00 | 82768672.91 | 150323.00 | 253288.00 |
| 0.56        | 173392.31 | 167072.00 | 710520661.60 | 132277.00 | 253255.00 |
| 0.75        | 137052.78 | 128987.00 | 1839165861.20 | 68731.00 | 253252.00 |

If the ratio of the contraians to the total traders increases, then price movement seems to be mean-reverting. As shown in Tab. 6, the variance of the contrarians-dominant-market at $\alpha(0) = 0.31$ is the smallest in the four cases. Other cases ($\alpha(0) = 0.56$ and 0.75) show higher volatilities and correspond procyclical market behavior between bubbles and crashes. As shown in Tabs. 7 and
it is shown that the total amounts of cash and risky assets at $\alpha(0) = 0.56$ become less volatile than other cases. This implies that the total amounts of cash and risky assets in the banking system of the trend-followers-predominant market at $\alpha(0) = 0.56$ change less drastically than other cases ($\alpha(0) = 0.75$). Namely, if trend followers are dominant in the market, then the procyclical behavior of the market becomes harmful for market participants and it may make banking systems more unstable.

The cumulative losses $H(t)$ is shown in Fig. 8. It is said that if market participants are homogeneous, the total amount of losses is less than more heterogeneous cases. In both the trend-followers-predominant and contrarians-predominant markets, the number of bankruptcy is larger than the two other cases such as the trend-followers-dominant and contrarians-dominant markets. Contrarians may obtain profit from the market when the market price is mean-reverting. Trend followers may obtain profit from the market when the market price is procyclical.

Figs. 9 and 10 show scatter plots between the market prices and the total amount of cash and those between the market prices and the total losses. We found that mean-reverting price movements may be less harmful to banks than high volatile price movements.

Fig. 11 shows time series capital adequacy ratio (CAR) at (a) $\alpha(0) = 0.31$, (b) $\alpha(0) = 0.48$, (c) $\alpha(0) = 0.56$, and (d) $\alpha(0) = 0.75$. It is confirmed that the banks
which went bankrupt have small CAR. Before the banks went bankrupt, value of CAR dropped steeply. Therefore, the default probability should be a function of CAR, which is not homogeneous. In general, a market where contrarians are dominant shows mean-reverting price movements, and a market where trend followers are dominant makes the market price volatile. However, the CAR of the contrarians tends to decrease and the CAR of the trend followers tends to increase.

We compare sensitivity of the CAR and that of the CEAR for some cases. Figure 12 shows temporal developments of CAR and CEAR for 100 banks ($\alpha(0) = 0.79$). When banks go bankrupt, their CAR decreases eventually, however, their CEAR does not change. This implies that the CAR can be used as an indicator to measure banks condition but the CEAR might not be used as an indicator for such a purpose. The sensitivity of CAR is extremely better than CEAR. We should recognize the difference of characteristics between CAR and CEAR.

6 Discussion

Our approach has several limitations. The first problem is related to relationships between simple and complex agent-based model. Usually, simple agent models tend to be too simple to apply actual risk estimation. The main purpose of simple
agent-based models is to specify roles and components assumed in phenomena which we want to draw. Through a modeling process, we eventually understand the structure of these problems and we identify roles and components. In our case, we use simple agent-based model to identify fundamental relationships among agents and roles of financial assets in balance sheets.

Meanwhile, the main purpose of complex agent-based models is to capture, reproduce and simulate phenomena. To do so, we also consider how to calibrate model parameters. Generally speaking, it is not easy to estimate all the parameters under a reliable procedure. We sometimes face parameters’ ambiguity to turn out similar results. Namely, similar results can be created by different sets of parameters. This problem is related to nonlinearity of parameters.

It is known that a simple agent-based model tends to become a complex agent-based model through a process of improvement. If we improve our model, eventually the purpose of our model changes from structure specification to risk estimation. In this case, we will also suffer from calibration problems.

The second limitation of our approach is parameter calibration. We do not have sufficient knowledge on actual banking systems. In fact, partial data of financial and banking systems can be used; however, we do not have all the data to calibrate model parameters. Furthermore, our agent-based model is too simple to apply risk assessment of actual banking networks. Even though it is simple, we can understand interplay between banks and to use it for developing and testing

Fig. 6. COLOR ONLINE. The total amount of cash; (a) contrarians-dominant market, (b) contrarians-predominant-market, (c) trend-followers-predominant market, and (d) trend-followers-dominant market.
indicators. For example, we can recognize that financial assets have a potential to play a role of a common factor and cash adequacy ratio and capital adequacy
ratio can be used as measures to estimate. Furthermore, if we can calculate their correlations in terms of banks, we can quantify intensity of common factors.

The third limitation is related to the expressions of the model. We used a matrix to describe relationships between banks. However, banks sometimes appear and disappear due to new launch, bankruptcy and M&A. The matrix representation cannot express such things.

However, our aim is to identify important components in the bank balance sheet and have deeper understanding among them. Although we model a simplified version of the banking and financial systems, we established a useful benchmark reference model that clarifies how the feedback loop between bank behaviors and how asset prices impact of default risk.

According to BIS consolidated banking statistics in 2015Q1 [24], total assets for all bank nationalities is 70,082.7 (billions of USD), the total amount of loans and deposits is 65,919.9 (billions of USD) and the total amount of debt securities is 7,862.0 (billions of USD) (see Table [1]). We found that the total amount of assets is larger than liabilities for debt securities. Namely, we may justify our hypothesis that the exposure to financial markets is larger than amounts lent and borrowed in an interbank network.

Fig. 9. Scatter plots between the market prices and the total amount of cash; (a) contrarians-dominant market, (b) contrarians-predominant-market, (c) trend-followers-predominant market, and (d) trend-followers-dominant market.
Fig. 10. Scatter plots between the market prices and the total losses; (a) contrarians-dominant market, (b) contrarians-predominant market, (c) trend-followers-predominant market, and (d) trend-followers-dominant market.

Table 9. BIS consolidated banking statistics in 2015Q1. The amount is totaled over all CSB-reporting banks. Amounts outstanding, in billions of US dollars.

| Items                                | amount   |
|--------------------------------------|----------|
| Foreign claims (Immediate counterparty)| 27,077.7 |
| Foreign claims (Ultimate risk)       | 24,231.7 |
| Domestic claims (Immediate counterparty)| 46,882.2 |
| Domestic claims (Ultimate risk)      | 46,627.4 |
| Total assets                         | 70,082.7 |
| Liabilities (Total)                  | 65,919.9 |
| Liabilities (Of which Loans and deposits)| 45,095.7 |
| Liabilities (Of which Debt securities)| 7,862.0  |
| Liabilities (Of which Derivatives)   | 6,336.0  |
| Total equity                         | 4,809.1  |

7 Conclusion and future work

We emphasized the fact that in our suggested model we were able to capture the relation between banks behavior and asset prices. We described the positive feedback-loop between banks default probability and asset price dynamics. The results showed that the procyclical banks’ behaviors (i.e., feedback loop between default probability and asset prices) can explain the realization of asset price bubbles and their burst. The characteristic of the interbank market plays in this
context a minor role if exposures in financial markets are larger than capital in an interbank network. From this view, the interbank market simply allows us to condense the individual default probabilities into the systemic default probability. The capital adequacy ratio (Leverage ratio) is a useful indicator to monitor the default probability.

As for the future, we need to check our hypothesis against detailed data and estimate probability of procyclicality. The model parameters depend on the macroscopic behavior of financial markets and bank default probability obviously. However, the parameters strongly depend also on market conditions and banks risk appetite. Unfortunately, we do not have enough information about bank risk appetite in general. We need to infer macro economic conditions, bank balance sheets, and market sentiment. Our paper paves the way to discuss this important issue in further studies. Therefore, parameter estimations from available information about actual banks’ behavior are crucial future tasks.

Conflict of interests

The authors declare no conflict of interest associated with this manuscript. Views expressed are those of authors and do not necessarily reflect those of the Bank (Bundesbank, Bank of Japan).
Fig. 12. COLOR ONLINE. Temporal development of (a) CAR and (b) CEAR at $\alpha(0) = 0.79$.

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