Analysis of student’s mathematical connection ability in linear equation system with two variables

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Abstract. Mathematical connection ability is very important for students, especially in solving problems that require connections between mathematical concepts, mathematical concepts with other sciences, and mathematical concepts with daily life. This research aims to analyze and categorize students' mathematical connection ability in linear equation system two variables. This research was descriptive qualitative with 30 students of SMA Negeri 1 Jogjalanan as the subject of the research. The variables of this research were indicators of mathematical connection ability, consisted of: connections between mathematical concepts, connections between mathematical concepts and other sciences, and connections between mathematical concepts with daily life. The data were collected using test and interview methods. Results of the research showed that the students' mathematical connection ability was categorized as very low. The number of students who answered correctly on the indicator of connection between mathematical concepts is 21.11%, on the indicator of connection between mathematical concepts with other sciences is 13.33%, and on the indicators of connections between mathematical concepts with daily life is 32.22%. Most students were not able to connect mathematical concepts with other sciences.

1. Introduction
Mathematics is all around us, although sometimes it is not realized. Mathematics everywhere is a collection of presentations on the role of mathematics in daily life, through science fields such as physics, engineering and statistic beside technology, and culture fields [1, 2]. Thus, it can be said that mathematics is the basic knowledge of individuals who have an important in the development of various fields.

The development of information and communication technology is increasingly advanced in the digital era, making problems of life more complex. The education system is always required to be harmony with development and challenges of the times. Therefore, one of Indonesian government’s efforts encountered in the current digital era is to apply HOTS (Higher Order Thinking Skills) problem in the National Examination. The minister of Education and Culture, Muhadjir Efendy stated that the percentage of HOTS in the National Examination was 10%. This was done to prepare students encountered of 4.0 Industrial Revolution.

According to [3], higher level thinking is thinking on a level that is higher than memorizing facts or telling something back to someone exactly the way it was told to you. Higher level thinking requires that we do something with the facts. We must understand them, infer from them, connect them to other facts and concepts, categorize them, manipulate them, put them together in new or novel ways, and apply them as we seek new solutions to new problems. This agrees with [4] which states that through HOTS problems, students can connect find problems and explore various views.
In other words, through the HOTS problem, students are able to develop their mathematical connection ability.

Focusing on 2013 curriculum, mathematical connection ability is one of the aspects competences that are expected to be achieved in mathematics learning. According to [5], mathematical connection is a cognitive process in which a person connects two or more ideas, concepts, definitions, theorems, procedures, representations and meanings between mathematics itself, with other disciplines or with real life. This is in line with the opinion of [6] which states that mathematical connections involve the relation of mathematics to the real world, with other sciences and other concepts in mathematics. If students have not been able to connect concepts in mathematics, students will have difficulty in understanding mathematics. However, if students can connect mathematical ideas, their understanding about mathematics will be deeper and longer [7]. Through mathematical connections, students can rebuild understanding of previous knowledge [8, 9]. Furthermore, mathematical connections are more significant because they support students to improve their understanding of the relationship between mathematical concepts and every other concept of science [10]. Thus, mathematical connections can also help and support students in learning mathematics [11].

The indicator of mathematical connection ability that used in this research is in line with [12], namely the connection between mathematical concepts, connections between mathematical concepts and other sciences, and connections between mathematical concepts with daily life.

This research aims to analyze students' mathematical connection ability in two variables linear equation system. Analysis of mathematical connection ability in this research based on three indicators of mathematical connection ability as said above. This research was used to analyze and categorize students' mathematical connection abilities in solving problems of two variables linear equation system, which can be used as a basis to consider improvement of mathematics learning process.

2. Method
This research was descriptive qualitative. The subjects of this research were 30 students of 11th grade high school from SMA Negeri 1 Jogoralan which was located in Klaten, Central Java, Indonesia. Subject’s schools were selected based on the absorption capacity of algebra material in the UN SMA 2016/2017 so that subject’s ability was assumed to represent the average ability of students in Klaten. The students used as research subjects were students with heterogeneous learning abilities who were selected by random sampling technique. Data were collected using tests and interview methods. Development of test about two variables linear equation system was based on indicators of mathematical connection ability was in line with [12], namely connection between mathematical concepts, connections between mathematical concepts and other sciences, connections between mathematical concepts with daily life. Mathematical connection ability test was given to students who have studied the two variables linear equation system, then the results were analyzing to knowing mathematical connection ability. The interview was used to finding out the steps students make in making connections to solve the problems. A description of scoring for mathematical connection ability test used in this research was shown in Table 1.

| No | Indicators | Description | Score |
|----|------------|-------------|-------|
| 1  | Connection between mathematical concepts | a) No Answer | 0     |
|    |            | b) Can connect between mathematical concepts correctly, but steps to solving problems are not right. | 1     |
|    |            | c) Can connect between mathematical concepts correctly and steps to solve problems that are almost right. | 2     |
|    |            | d) Can connect between mathematical concepts correctly and steps to solve problems are right. | 3     |
The triangulation was used to confirm the findings of data. The confirmed data process was done by comparing information or data of test results and interviews. The data analysis phase in this research involve: (1) giving test mathematical connection ability about two variables linear equation system; (2) analyzing the results of test; (3) conducting interviews with students who make mistakes in each indicator; (4) analyzing the results of interview.

3. Results and Discussion

In this research, subjects were 30 students obtained from the random sampling technique. The analysis of the test result of 30 students showed that students' mathematical connection ability were still very low. Most students cannot connect mathematical concepts with other sciences. This can be seen in the second indicator which only reached 13.33% with student’s scores is 12, with the following details: the number of students who get a score of 3 is 0 student, the number of students who get a score of 2 is 1 student, the number of students who get a score of 1 is 9 students, the number of students who get a score of 0 is 20 students. As a whole, the summary of each indicator of mathematical connection ability on 3 problems of test mathematical connection ability in two variables linear equation system can be seen in Table 2.

| No | Indicators | Description | Score |
|----|-------------|-------------|-------|
| 2  | Connection between mathematical concepts with other sciences | a) No Answer | 0     |
|    |             | b) Can connect mathematical concepts with other sciences correctly, but steps to solving problems are not right. | 1     |
|    |             | c) Can connect mathematical concepts with other sciences correctly and steps to solve problems that are almost right. | 2     |
|    |             | d) Can connect mathematical concepts with other sciences correctly and steps to solve problems that are right. | 3     |
| 3  | Connections between mathematical concepts with daily life | a) No Answer | 0     |
|    |             | b) Can connect mathematical concepts with daily life correctly, but steps to solving problems are not right. | 1     |
|    |             | c) Can connect mathematical concepts with daily life correctly and steps to solve problems that are almost right. | 2     |
|    |             | d) Can connect mathematical concepts with daily life correctly and steps to solve problems that are right. | 3     |

Based on the percentage of mathematical connection ability above, a description of the analysis of the results of student answers and interviews related to the question of the mathematical connections ability on each indicator will be discussed in more detail, as follows.

Table 2. Percentage of students’ mathematical connection ability.

| Mathematical Connection Ability Indicators | Analysis |
|-------------------------------------------|----------|
|                                            | Student’s Score | Total Score | Percentage | Category |
| Connection between mathematical concepts   | 19        | 90          | 21.11%     | Very Low  |
| and other sciences                         | 12        | 90          | 13.33%     | Very Low  |
| Connection between mathematical concepts   | 29        | 90          | 32.22%     | Very Low  |
| and daily life                             |           |             |            |           |


3.1 Connection between mathematical concepts

The following is a test problem of mathematical connection ability with the indicators of connection between mathematical concepts. In this problem, the mathematical concept that connected is the concept of two variables linear equation system with concept of similarity gradient, and the concept of parallel lines. The problem 1 can be seen in figure 1.

**Figure 1.** The problem 1.

![Figure 1](image1)

The results of analyzing students test answers based on the indicators of connection between mathematical concepts shows that the percentage of students reached 21.11% and categorized is very low. The examples of the results of student test answers in problem 1 related to indicators of connection between mathematical concepts can be seen in figure 2 and figure 3.

**Figure 2.** Tests result of TYS on problem 1.

**Figure 3.** Tests result of MVW on problem 1.

![Figure 2 and Figure 3](image2)

Figure 2 shows that student (TYS) is not precise in connecting between the mathematics concepts on the problem 1. It means student (TYS) does not understand about the meaning of the relationship of line position. Student (TYS) uses substitution method to determining relationship of the line, and because the results obtained are not same, student (TYS) concluded that the relationship of line position on this problem don’t have a relationship. Meanwhile, in figure 3, student (MVW) can connect the concept of two variables linear equation system with the concept of similarity gradients, but the steps to solve the problems are not appropriate. Student (MVW) makes mistakes in determining the gradient in equation 2 with answer of the gradient on equation is -2.

Next, interviews were conducted with student (MVW) to confirm the student’s answer.

| Question 1a | Answer(MVW) |
|-------------|-------------|
| What the concept do you use in determining the relationship of the position of the line? | Similarity gradient. |

| Question 1b | Answer(MVW) |
|-------------|-------------|
| What do you think about gradient? | Gradient is the number owned by variables. |

| Question 1c | Answer(MVW) |
|-------------|-------------|
| Are there other concepts that can be used to solve the problem 1? | No, I think that is all. |

| Question 1d | Answer(MVW) |
|-------------|-------------|
| So, are you sure with your answer? | Yes, I am sure. |

Based on the results of analysis of answers interviews with student (MVW), it was found that student was not fully able to connect between mathematics concepts in solving the problem 1. Furthermore, the student did not understand about the concept of gradient. According to him, gradient is a number owned by variables. So, the student is wrong in determining the gradient value from equation 2, namely -2. This causes student being wrong in determining the relationship between the positions of two lines that should be parallel to be perpendicular.

![Image of equations](image3)
3.2 Connection between mathematical concepts and other sciences

The following is a test problem of mathematical connection ability that contains indicators of connections between mathematical concepts and other sciences. In this problem, the mathematical concept of two variables linear equation system is connected with the concept of speed in physics. The problem 2 can be seen in figure 4.

A boat moves in the direction of the river can reach a distance of 46 km in 2 hours. If the boat moves in the opposite direction of the river, it can reach a distance of 51 km in 3 hours.

a. What are the speed of the boat and the speed of river water?

b. Write down the concepts used to solve the problem!

Figure 4. The problem 2.

The results of analyzing students test answers based on the indicators of connection between mathematical concepts and other sciences shows that the percentage of students reached 13.33% and categorized is very low. The examples of the results of student test answer in problem 2 related to indicators of connection between mathematical concepts and other sciences, can be seen in figure 5 and figure 6.

Figure 5. Tests result of MK on problem 2.

Suppose:

\[ V_1 = \text{the speed boat that is in the direction of the current} \]
\[ V_2 = \text{the speed of the boat that is not in the current direction} \]

**Answer:**
- Speed boat
  \[ V_1 = \frac{46}{2} = 23 \text{ km/hour} \]
  \[ V_2 = \frac{51}{3} = 17 \text{ km/hour} \]
- Speed of river water
  \[ 23 - 17 = 6 \text{ km/hour} \]

Figure 6. Tests result of QSK on problem 2.

Suppose:

\[ s_1 = \text{the speed boat that is in the direction of the current} \]
\[ s_2 = \text{the speed of the boat that is not in the current direction} \]

**Answer:**

a. \[ s_1 = \frac{v}{t} = \frac{46}{2} = 23 \text{ km/hour} \]
   \[ s_2 = \frac{v}{t} = \frac{51}{3} = 17 \text{ km/hour} \]

b. The concept used in this problem is speed in physics.

Figure 5 shows that student (MK) has been able to connect problems with the concept of speed in physics material using the formula of speed \( V = s / t \). However, the completion step given by the student (MK) in determining the speed of the boat and the speed of the river flow is not appropriate. This is because the student has not been able to connect with the concept of two variables linear equation system. Student (MK) also does not answer question (b) from problem 2. Meanwhile, from figure 6, the answers written by student (QSK) indicate that he wants to connect the problem with the concept of speed in physics material, but there is an error in writing the formula of speed. Student (QSK) also cannot solve the question (a) on problem 2 correctly. However, he was able to answer the question (b) related to the concept used in solving the problem.

Next, the interview was conducted on student (MK) to confirm student’s answers.

**Question 2a** : What steps did you use to solve problem 2?

**Answer (MK)** : I use \[ V_1 = \text{the speed boat that is in the direction of the current} \]
\[ V_2 = \text{the speed of the boat that is not in the current direction} \]

Then I put both of them in formula \( V = s / t \) to find out speed of boat. Whereas to know the speed of current, I just need to reduce the speed of the boat in the same direction as the opposite \( (V_1 - V_2) \).

**Question 2b** : Do you think the steps you took to solve the problem 3 were correct?

**Answer (MK)** : Yes, I think.
Question 2c : Is this problem quite difficult for you?
Answer (MK) : Yes, because to solve this problem must connect with physics material.
Question 2d : Why is the question (b) on problem 3 not your answered?
Answer (MK) : Because I don’t understand what the concept is. It feels strange to hear it.

Based on the results of analysis of answers interviews with student (MK), it was found that the student was not fully able to connect mathematical concepts with other sciences (the concept of speed in physics) to solve the problem 3. This can be seen from the steps to solve the problem not yet right, so that the answers obtained are also incorrect. Student (MK) also does not understand what the meaning about the concept. According to him, it feels strange and rarely hears it. So, he was unable to write down what concepts were used in solving the problem 2.

3.3 Connection between mathematical concepts and daily life
The following is a matter of mathematical connection ability that contains indicators of connection between mathematical concepts and daily life. In this problem, the concept of two variables linear equation system is connected to daily life by looking for variations in money withdrawals at different nominal ATMs. The problem 3 can be seen in figure 7.

Sari needs money to pay school fees. The amount she has to pay is 500,000,00 IDR. At a minimarket there are two ATMs that Sari can use, include 100,000,00 IDR and 50,000,00 IDR.

a. How do you write the variation in taking money that Sari can do?
b. Write down the concepts used in the problem!

**Figure 7.** The problem 3.

The results of analyzing students test answers based on the indicator of connection between mathematical concepts and daily life shows that the percentage of students reached 32.22% and categorized is very low. The examples of the results of student test answer in problem 3 related to indicators of connection between mathematical concepts and daily life can be seen in figure 8 and figure 9.

**Figure 8.** Tests result of ACK on problem 3.

- a. Known:
  - The money needed Sari is 500,000,00 IDR.
  - Asked: variation on taking money?
  - Answer:
    - If sari takes money at an ATM with a nominal value of 100,000,00 IDR it is obtained
      \[
      \frac{100,000,00}{500,000,00} = 5 \text{ sheets.}
      \]
    - If sari takes money at an ATM with a nominal value of 50,000,00 IDR, it is obtained
      \[
      \frac{50,000,00}{100,000,00} = 10 \text{ sheets.}
      \]
  - b. The concept used is logic.

**Figure 9.** Tests result of ANS on problem 3.

- Known:
  - Sari has to pay is 500,000,00 IDR. At a minimarket there are two ATMs include 100,000,00 IDR and 50,000,00 IDR.
  - Asked: (a) variation on taking money?
  - (b) concepts used in the problem?
  - Answer:
    - a. Suppose:
      - ATM with 100,000,00 IDR is \( x \)
      - ATM with 50,000,00 IDR is \( y \)
      - So it can be written,
        \[
        100,000x + 50,000y = 500,000
        \]
    - b. The concept used in the problem is equation.

Figure 8 shows that student (ACK) has not been able to connect mathematical concepts (concept of linear equation system two variables) to daily life. This can be seen from the answers of student in determining the variation in taking money, student (ACK) only divides the amount of money to be taken with the currency value that comes out at each ATM. Student (ACK) writes the answer to
problem 3 about the concepts used in solving problems with logic. This ensures that student (ACK) has not been able to solve problem with mathematical connection ability. Meanwhile, from figure9, student (ANS) has been able to connect the concept of two variables linear equation system with daily life, but the problem solving steps taken by the student in writing variations on taking of money are not complete. Student (ANS) also writes the concepts used are equation. Even though it is not fully appropriate, mathematical connection ability to student (ANS) can be seen. This is in line with the opinion [13] that mathematical connections cannot be seen directly because they occur in the brain, but the results of mathematical connections can be seen in the form of processes or steps in solving problems.

Next, interviews were conducted with student (ACK) to confirm student’s answers.

Question 3a : Try to retell the meaning of the problem 3 with your own words?
Answer (ACK) : So, Sari wants to take money at ATM to pay for school fees. Meanwhile, in the minimarket there are two types of ATMs with nominal money coming out, each of which is 100,000,00 IDR and 50,000,00 IDR. Then, the question is how the variation in money taking made by Sari is?

Question 3b : Do you think the steps you took to solve the problem were correct?
Answer (ACK) : Yes, logically ATM only has one type of nominal money. So if Sari wants to take 500,000,00 IDR in an ATM that has a nominal value of 100,000,00 IDR, the ATM will expand 5 sheets.

Question 3c : Then, is it possible if Sari took the money at the 2 ATMs, with the nominal amount taken still amounting to 500,000,00 IDR?
Answer (ACK) : Maybe, but I don't think that.

Based on the results of the analyzing the answers interviews with student (ACK), it was found that the student has not been able to connect mathematical concepts with daily life to determine variations in taking money. When the student (ACK) was asked to retell the meaning of problem 3, she was able to tell it correctly. However, the steps to solve the problem taken by student (ACK) are not right, so the answers obtained are also not correct. Student (ACK) uses logic to solve the problem 3, without connecting with the mathematical concept of two variables linear equation system.

Based on the results of the analysis, related to the analysis of test answers and the analysis of interviews with several students, the low mathematical connection ability is caused by various influencing factors. One of the factors that influence is the most students have not been able to connect mathematical concepts, both in mathematics itself, with other sciences and in daily life. This is line with [14] who stated that in solving mathematical problems, one must be able to connect one concept to another because mathematics has a connection with one another. Thus, efforts can be made to improve student’s mathematical connection ability are mathematics teachers should be teach mathematics through real concepts in students' personal lives [15]. If mathematical concepts are given abstractly without being connected to real life, students with low mathematical connection ability will encounter many problems. Furthermore, according to [16], to improve mathematical connection ability, learning process must have representative media that can connect students' knowledge in real situations and learning materials. In addition, there must be a learning model that can develop old and new knowledge from students and facilitate their mathematical connection ability. In order to fulfill all competencies, the teacher must be able to be a facilitator and mediator in meeting the needs of students related to 21st century competence.

4. Conclusion
The results of analysis illustrate that student’s mathematical connection ability of students in SMA Negeri I Jogonalan is still very low and not as expected yet. Description of student’s mathematical connection ability can be seen in the percentage of students' mathematical connection ability on the indicator connection between mathematical concepts is 21.11% and classified as very low category, the percentage of connection between mathematical concepts and other sciences is 13.33% and classified as very low, and the percentage of connection between mathematical concepts with daily life is 32.22% and classified as very low category. Most students cannot connect mathematical
concepts with other sciences. Mathematics teachers are expected be able to design learning that can improve students' mathematical connection ability, including by choosing the right learning model and being able to connect students' knowledge with the real world, able to contextualize mathematical concepts with other sciences. In addition, teachers must be able to be facilitators and mediators in facilitating students' mathematical connections ability with provide challenging problems, which is expected to improve mathematical connection ability. So that later, the students are able to work on any problem such as HOTS.

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References
[1] Muijs D and Reynold D 2011 Effective teaching: Evidence and practice (3rd ed.) 256 London: SAGE
[2] Martin A 2010 Mathematics Everywhere (America: American Mathematical Society)
[3] Thomas A and Thorne G 2009 How to Increase Higher Order Thinking (Los Angeles: Center for Development and Learning)
[4] Barahal S 2008 Sage Journal Centre for Development and Learning 4 90
[5] Garcia J G and Flores C D 2018 International Journal of Mathematical Education in Science and Technology 49 229
[6] Ozgen K 2013 Journal of International Education Research 9 pp 305-316
[7] National Council of Teacher of Mathematics 2000 Principles and Standards for School Mathematics (Reston: VA)
[8] Olkun S 2003 International Journal of Mathematics Teaching and Learning 1 1
[9] Mousley J 2004 An Aspect of Mathematical Understanding: The Notion of Connected Knowing (Deakin University: Australia)
[10] Hendriana H, Slamet U R & Sumarmo U 2014 International Journal of Education 8 pp 1-11
[11] Wilburne J M & Napoli M 2008 Journal Issues in the Undergraduate Mathematics Preparation of School Teachers 2 1
[12] Noto M S, Hartono W and Sundawan D 2016 Infinity Journal 5 pp 99-108
[13] Rismawati D, Edy B I and Heri S 2017 Jurnal Pendidikan: Teori, Penelitian, dan Pengembangan 2 pp 465-49
[14] Hurst C 2007 Numeracy in Action: Students Connecting Mathematical Knowledge to a Range of Contexts Mathematics: Essential Research, Essential Practice 1 pp 440-449
[15] Salout S S, Behzadi M H, Shahvarani A and Manuchehri M 2013 Journal Mathematics Education Trends and Research 2013 1
[16] Abidin Z and Jupri A 2017 International E-Journal of Advances in Education 3 604