Clusters of resource consuming nodes in transportation networks

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Abstract. We study the problem of optimal source location in transportation networks. When the installation cost of source nodes is comparable to the transportation cost, regimes characterized by discrete fractions of resource consuming nodes emerge and resemble the Devil’s staircase, corresponding to different clusters of consumer nodes. When the fraction of deficient nodes increases, frustration arises in choices of locating extra source nodes and numerous sub-optimal configurations emerge, resembling typical glassy behavior.

1. Introduction
The physics of spin glasses is highly relevant to the study of optimization problems. Formally, objective functions in optimization problems can be mapped to spin glass Hamiltonians, enabling the analysis of optimization problems using statistical physical techniques. Moreover, many optimization problems are associated with algorithmic hardness, resembling the thermodynamics of spin glasses in the glassy phase. These similarities further enhance the application of techniques including the replica method [1, 2] and Bethe approximation on optimization problems. Successful examples are found in the MAX-SAT or satisfiability problem [3] and the minimal vertex cover problem [4, 5, 6].

Recently, resource allocation problems in transportation networks have been considered from these approaches [9, 10, 11]. These problems are important in load balancing in computer networks [7] and network flow of commodities [8]. In this paper, we address the problem of optimal source location in transportation networks. In this context, transportation networks consist of either surplus or deficient nodes, and the optimization task is to locate those deficient nodes that need to be converted to source nodes, so that networkwide satisfaction with minimal costs is achieved by flow of resources along the links. This problem has wide applications. In wireless sensor networks, for example, signals collected by the population of sensors are transmitted to base stations through the network formed by the sensors. However, due to the limited power of the sensors, it is advantageous to reduce the signal traffic by installing extra access points so that the service lifetimes of the sensors can be prolonged. These access points can be considered as source nodes which provide service to the sensors. Since the installation of access points requires various overheads in terms of hardware, software, management, finance, and so on, the optimal locations of access points involves balancing the installation cost of the extra access points and the transportation cost of the signal traffic.
In general, the installation cost required to convert a deficient node into a source node is step-like. This high nonlinearity gives rise to unique behaviors and a physical picture absent in the previous models [12]. When the installation cost is comparable to the transportation cost, the total cost is optimized either by saving the transportation cost (while increasing the installation cost), or decreasing the installation cost (while increasing the transportation cost). Frustration arises and numerous sub-optimal configurations of source nodes emerge, resembling typical glassy behavior. When the installation cost changes, regimes with different clustering patterns of resource consuming nodes are observed, resembling the Devil’s staircase observed in the circle map and other dynamical systems [14].

We analyze the problem through the cavity approach [1, 2], which is a tool developed in the context of spin glasses. In contrast to problems with discrete variables, the cavity fields in the present problem are in the form of functions. Such functional fields result in recursions of functions, complicating the analyses. We report here how these functional recursions are converted to simple recursions of probabilities, which allow us to derive the emergence condition for typical glassy behavior. The underlying physical picture of this model has some resemblance with the random field Ising model [15, 16, 17], the vertex cover problems [4, 5], and the Bethe lattice glass model [18, 19, 20], but it also has its own distinctive features. We refer interested readers to [12] for more details about these connections. More details of our study can be found in [13].

2. Model Formulation

We consider a network of $N$ nodes, labelled $i = 1 \ldots N$. Each node $i$ is connected randomly to a set $L_i$ of $K$ neighbors. Each node $i$ is either a surplus and deficient node of resources, as specified by its capacity $\Lambda_i$; positive and negative values of $\Lambda_i$ correspond to excess and shortage of resources respectively. The capacities $\Lambda_i$ are randomly drawn from a distribution $\rho(\Lambda_i)$. With network applications in mind, we consider a bimodal distribution in which $\Lambda_i = A(1)$ with probability $s$, and $\Lambda_i = -1$ with probability $d = 1 - s$. Surplus nodes are inherently source nodes, providing resources to other nodes. By default, the deficient nodes are consumer nodes receiving resources. However, to minimize cost functions that include transportation costs, it is often desirable to convert some deficient nodes into source nodes as well. Hence in general, the task is to optimally locate these extra source nodes.

Denoting as $y_{ij} \equiv -y_{ji}$ the current of resources from node $j$ to node $i$, we consider the minimization of the cost function

$$E = \frac{u^2}{2} \sum_i \Theta(-\xi_i) + \sum_{(ij)} \frac{y_{ij}^2}{2},$$  

(1)

$\xi_i \equiv \Lambda_i + \sum_{j \in L_i} y_{ij}$ is the final resource of node $i$, and $\Theta(x) = 1$ when $x > 0$, and 0 otherwise. The link connecting nodes $i$ and $j$ is denoted as $(ij)$. The first term corresponds to the installation cost of an extra source node at deficient node $i$, since the condition $\Theta(-\xi_i) = 1$ indicates that the deficient node needs to be converted to a source node to satisfy the demand on resources from its neighbors. The most unique feature of this cost function is the discontinuity in its magnitude when the final resource changes from zero to negative, representing various installation overheads. In a more general context, one may envisage installation costs whose magnitudes depend on the shortages of resource $-\xi_i$, which is beyond the scope of the present study. As we shall see, the step-like installation cost already generates an extremely rich picture of network behavior.

Note that the cost function in Eq. (1) is identical to that in [12], but the present setting is far more relevant to network applications. The previous results in [12] have direct correspondence to the present case if the unsatisfied nodes are considered to be the extra source nodes. For

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example, the single-sat regime studied in [12] corresponds to the case that each consumer is surrounded by source nodes, since the installation cost is low compared with the transportation cost. When the installation cost is gradually raised, resource provision is achieved with less source nodes, but optimization requires the consumers to be located in clusters surrounded by source nodes, forming the clusters observed in [12].

3. Major Simulation Results and the Consumer Clusters

We first look for phase transitions in the model by numerical simulations. To formulate an algorithm, we introduce a variable \( s_i = \pm 1 \) for each node \( i \) and restrict its resources by 

\[
s_i (\Lambda_i + \sum_{j \in \mathcal{L}_i} y_{ij}) \geq 0.
\]

Introducing Lagrange multipliers \( \mu_i \) for the resource constraint, we minimize the Lagrangian

\[
L = \frac{u^2}{2} \sum_i s_i + \frac{1}{2} + \sum_{(ij)} \frac{y_{ij}^2}{2} + \sum_i \mu_i s_i \xi_i
\]

with the Kühn-Tucker conditions \( \mu_i s_i \xi_i = 0 \) and \( \mu_i \leq 0 \). Optimizing \( L \) with respect to \( y_{ij} \), one obtains

\[
y_{ij} = \mu_j s_j - \mu_i s_i, \quad (3)
\]

\[
\mu_i = \min_0 \left \{ \frac{1}{s_i K} \left ( \Lambda_i + \sum_{j \in \mathcal{L}_i} \mu_j s_j \right ) \right \}. \quad (4)
\]

Given a particular set of \( \{ s_i \} \), we iterate these equations to find the corresponding set of \( \{ \mu_i \} \). The set of optimal \( \{ s_i \} \) is found by an approach similar to the the GSAT algorithm [21], by comparing the Lagrangian in Eq. (2) for each choice of \( \{ s_i \} \). In each step of this algorithm, a cluster of \( N_{flip} \) nodes is randomly selected. The network energies involving the different configurations of this cluster are compared, and the cluster configuration is updated to the one that yields the lowest network energy.

As shown in Fig. 1 for \( \phi_d = 0.9 \) and \( K = 3 \), two phases can be identified: (1) all-source phase for \( u^{-1} \geq \sqrt{3} \), in which all nodes are assigned to be source nodes due to the very high transportation cost; (2) partial-source phase for \( 0 < u^{-1} < \sqrt{3} \), in which only some nodes are assigned to be source nodes. (In [12] we also identified a phase transition \( u^{-1} = 0 \) to an all-consumer phase.)

The fraction of source nodes is a discontinuous function of \( u^{-1} \), showing abrupt jumps at threshold values of \( u^{-1} \). The step size of the curve decreases as \( u^{-1} \) increases, and gradually becomes unresolvable by the numerical experiments. This resembles the Devil’s staircase observed in the circle map and other dynamical systems [14]. These threshold values of \( u^{-1} \) mark the positions at which certain configurations of the source and consumer nodes become energetically stable. Similar features are observed in RFIM due to the formation of ferromagnetic clusters resultant from the competition between the strengths of couplings and random fields [16, 17].

Measuring the average maximum cluster size of the consumers in the samples, we observe abrupt jumps of the cluster size at the same threshold values. This indicates that new types of clusters are formed at each jump, as sketched in the top of Fig. 1. The observed threshold values can be calculated by considering the energies of consumer clusters surrounded by source nodes as shown in Fig. 2, obtained by the minimization of Eq. (1). By comparing the energy of different configurations, we have

\[
\mathcal{E}_0 \geq \mathcal{E}_1 \quad \text{(singlet),}
\]
Figure 1. Simulation results of average energy per node and the fraction of consumer nodes. Parameters: $K = 3$, $\phi_d = 0.9$, $N = 100$, 100 samples, 1000N steps and $N_{flip} = 4$. New clusters formed on increasing $u^{-1}$ are sketched at the top, with filled and unfilled circles representing consumer and source nodes respectively.

Figure 2. Cluster of (a) source node, (b) singlet consumer, (c) doublet consumers, (d) triplet consumers and (e) quadruplet consumers.
\[ E_0 + \mathcal{E}_1 \geq \mathcal{E}_2 \] (doublet),

\[ E_0 + 2\mathcal{E}_1 \geq \mathcal{E}_3 \] (triplet),

\[ E_0 + \mathcal{E}_1 + \mathcal{E}_2 \geq \mathcal{E}_4 \] (quadruplet),

resulting in the threshold values

\[ u^{-1} \leq \sqrt{K} \] (singlet),

\[ u^{-1} \leq \sqrt{\frac{K(K-1)}{K+1}} \] (doublet),

\[ u^{-1} \leq \sqrt{\frac{K(K^2-2)}{(K+2)^2}} \] (triplet),

\[ u^{-1} \leq \sqrt{\frac{K(K-1)(K^2-K-1)}{(K+1)(K^2+K-1)}} \] (quadruplet).

These results agree with those obtained through the cavity approach in Section 4. Further analyses of larger consumer clusters lead to additional regimes as \( u^{-1} \) increases. These threshold values are closer to one another when \( u^{-1} \) increases, leading to a decrease of step size in the fraction of source nodes in Fig. 1, resembling the Devil’s staircase [14].

The threshold values are shown in Fig. 1 for \( K = 3 \). We call the regime \( \sqrt{3/2} < u^{-1} < \sqrt{3} \) with isolated consumer the singlet regime, and \( \sqrt{21/25} < u^{-1} < \sqrt{3/2} \) the doublet regime.

4. The Cavity Approach

We start the analysis in the simple case with only deficient nodes in the networks (i.e. \( \Lambda_i = -1 \) for all \( i \)). Since the network has a tree-like structure locally, we carry out the analysis using the Bethe approximation. As shown in Fig. 3, we identify node \( i \) to be the ancestor of node \( j \), and nodes labelled by \( k \) to be the descendents of \( j \). We call this a sub-tree terminated at vertex node \( j \) in the absence of \( i \). A new sub-tree is constructed by connecting \( K - 1 \) sub-trees to a newly added vertex. We describe the physical properties of a node in the absence of its ancestor, leading to a cavity on the node. Energy functions of the sub-trees are passed from the vertices of the sub-trees to the next layer, resulting in their recursion relations. We note the equivalence of this cavity approach with message passing algorithms [22], in which a message passed from a vertex describes the conditional probability of the vertex states in the absence of the receiving node. In the cavity approach for discrete variables [3, 23], these messages are usually the local fields in the absence of the ancestors of the vertices, known as the cavity fields. Here, since the variables in the cost function of Eq. (1) are continuous, the messages become energy functions [9, 10]. Let \( E_j(y_{ij}) \) be the energy of the tree terminated at a link from vertex \( j \) to its ancestor \( i \), when a current \( y_{ij} \) is drawn from \( j \) to \( i \). One can express \( E_j(y_{ij}) \) in terms of the energies of its descendents \( k = 1, \ldots, K - 1 \),

\[ E_j(y_{ij}) = \min_{(y_{jk})} \left[ \sum_{k=1}^{K-1} E_k(y_{jk}) + \frac{u^2}{2} \Theta(-\xi_j(y_{ij})) + \frac{y_{ij}^2}{2} \right] , \tag{5} \]

where \( \xi_j(y_{ij}) = \Lambda_j + \sum_k y_{jk} - y_{ij} \). We call \( E_k(y_{jk}) \) the cavity energy functions which is sent from the vertices to the next layer as shown in Fig. 3.

Due to the quadratic form of transportation cost in Eq. (1), we have demonstrated that the energy \( E_k(y_{jk}) \) is a continuous piecewise-quadratic function [12]. Each segment of the function has the quadratic form

\[ J_{nk}^k(y_{jk}) = a_{nk}^k (y_{jk} - y_{nk}^k)^2 + d_{nk}^k \], \tag{6}
Figure 3. The tree-like assumption in cavity approaches. Node $i$ and nodes $k$ are respectively the ancestor and descendents of node $j$. $y_{ij}$ denotes the flow of resources from $j$ to $i$. Negative $y_{ij}$ and $y_{jk}$ correspond to flows in the opposite direction.

Figure 4. An example of $E_k^j(y)$ composed of three quadratic functions $f_{nk}^k(y)$ labelled by $n_k = 0, 1, 2$. Each composite function is characterized by its minimum position $\tilde{y}_n^k$, minimum value $d_n^k$, and curvature $a_n^k$.

| Descendants $n_k$ | $\tilde{y}_{n_k}^l$ | $a_{n_k}^l$ | $d_{n_k}^l$ | $n_j$ |
|-------------------|---------------------|------------|------------|------|
| $n_k = 0$, $n_l = 0$ | $-\frac{1}{3}$ | $\frac{3}{4}$ | $\frac{1}{6}$ | $1$ |
| $n_k = 1$, $n_l = 0$ | $-\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{1}{3}$ | $2$ |

Table 1. Generation of the composite functions labelled by $n_j = 1$ and $2$ from descendents $k$ and $l$ for $K = 3$. $\Lambda_i = -1$ for deficient nodes.

where $n_k = 0, 1, 2, \ldots$ is the label of the $n_k$-th composite function of $E_k(y_{jk})$, and is characterized by the coefficients $a_{n_k}^l$, $\tilde{y}_{n_k}^l$, and $d_{n_k}^l$. An example of the cavity energy function $E_k(y_{jk})$ is shown in Fig. 4, composing of the three quadratic functions labelled by $n_k = 0, 1, 2$. The composite function with $n_k = 0$ is given by $\tilde{y}_0^k = 0$, $a_0^k = 1/2$, and corresponds to the case of resource provision. Other composite functions can be generated by plugging the composite functions into Eq. (5) recursively. The first two resultant composite functions, for the case $K = 3$ and obtained with the constraint $y_j = \Lambda_j + y_k + y_l$ active, are given in Table 1.

To determine the composite functions selected by the minimization in Eq. (5), we also need to compare the functions in Table 1 with the composite function in case of resource provision, labelled by $n_j = 0$ and obtained from Eq. (5) with $\tilde{y}_0^k = 0$, $a_0^k = 1/2$ and $d_0^k = u^2/2 + d_{\min}^k + d_{\min}^k$ where $d_{\min}^k = \min_{n_k} d_{n_k}^k$. (For negative $\Lambda$, the composite function with $y_j < \Lambda_j + y_k + y_l$ does not contribute.) Since the network behavior only depends on the energy differences among the composite functions, we consider recursion relations for the energy differences $\epsilon_{n_j}^k \equiv d_{n_j}^k - d_0^k$. In particular, in the singlet regime where the composite functions with $n_j = 0$ and $1$ have the
The recursion relations for the deficient nodes follow the close-packing rule of Eq. (8), whereas composite functions with \( n_k = 0 \) and 1 only. Indeed, since \( \gamma \geq 0 \) in this regime, the values \( c^k_1 = -\gamma, 0, \gamma \) form a closed set under the iteration of Eq. (7). Furthermore, higher composite functions do not contribute in this regime.

The three functions with \( c^k_1 = \gamma, -\gamma \) and 0 are referred to as the \( s \)-, \( c \)- and \( b \)-states, as shown in Fig. 5(a-c). Respectively, they correspond to states favoring resource provision (as a source), consumption, and bistability, and have absolute minima at \( y = 0, y = -1/3 \), and both \( y = 0 \) and \( -1/3 \). Furthermore, numerical iterations of Eq. (5) starting with random \( E_k(y) \) show that this set of solutions is stable. For general values of \( K \), the closed set consists of more states having absolute minima at \( y = 0 \), and local minima at different energies at \( y = -K^{-1} \), similar to the \( c \)-state in Fig. 5.

Considering the recursion relations among the \( K \) states, we find that all states with absolute minima at \( y_k = 0 \) (i.e., \( c^k_1 \geq 0 \)) behave in the same way. Thus, we denote the \( s \)- and \( b \)-states by the \( S \)-state, and the \( c \)-state by the \( C \)-state. Their recursion relations are given by

\[
S + \cdots + S \rightarrow C,
\]

all other combinations \( \rightarrow S \). \hfill (8)

The closure property of the \( K \) states greatly simplifies the calculation. The physical states of a node and its links can be obtained by feeding \( K \) vertices to a central node. The recursion rules imply that optimization is achieved by the close packing of consumers in the network, with the constraint that they do not form clusters. This reduces the problem to the vertex cover problem [4, 5, 6]. Since there is at most one consumer at the end of each link, the maximization of the number of consumers is equivalent to the minimization of the covered set size in the vertex cover problem. Alternatively, the model can be considered as the close packing limit of a lattice glass model [18]. Both the vertex cover problem and the lattice glass model exhibit glassy behavior, and both phases with single ground state and multiple metastable states are found therein, corresponding to the computationally easy and hard phases respectively.

5. Networks with Both Surplus and Deficient Nodes

Next, we consider the cases where both surplus and deficient nodes are present in the networks. The recursion relations for the deficient nodes follow the close-packing rule of Eq. (8), whereas
the surplus nodes are always in the $S$-state. We let $\psi_{s}^{k-j}$ be the probability that node $k$ is in the $S$-state with the absence of node $j$. Its recursion relation can be written as

$$
\psi_{s}^{j-i} = \delta_{\Lambda_{s}-1} \left( 1 - \prod_{k=1}^{K-1} \psi_{s}^{k-j} \right) + \delta_{\Lambda_{s} \Lambda_{A}}. \tag{9}
$$

This implies that the probability $\eta_{s}$ of finding a cavity source node can be obtained from the equation $\eta_{s} = \phi_{s} + \phi_{d} (1 - \eta_{s}^{K-1})$. At low $\phi_{d}$, a stable fixed point solution with all $\psi^{k-j} = 0$ or 1 exists. From the perspectives of the replica analysis [1], this corresponds to the so-called replica symmetric (RS) ansatz, in which the network behavior is dominated by a single ground state. In terms of algorithms, the optimal network states in this regime can be obtained by the BP algorithm, initializing the messages to 0 or 1.

The stability of the RS solution can be studied by considering the propagation of fluctuations $\langle (\delta \psi_{s}^{k-j})^{2} \rangle$ under the recursion relation Eq. (9) [25]. This leads to the Almeida-Thouless (AT) stability condition, which reads

$$
(K - 1) \phi_{d} \langle \psi_{s}^{2} \rangle^{K-2} \leq 1. \tag{10}
$$

In the RS regime, $\langle \psi_{s}^{2} \rangle = \langle \psi_{s} \rangle = \eta_{s}$ for $\psi_{s}^{k-j} = 0$ or 1. Simple algebra leads us to the AT line

$$
\phi_{d}^{AT} = \frac{K^{K-2}}{(K-1)^{K-1}} \tag{11}
$$

which separates the RS and the RSB phases in the space of $K$ and $\phi_{d}$ as shown in Fig. 6(a).

When $\phi_{d} > \phi_{d}^{AT}$, nodes with $0 < \psi_{s}^{k-j} < 1$ exist, as shown in Fig. 6(b). The fraction of $\psi_{s}^{j-i}$ not converging to 0 and 1 can be deduced from Eq. (9), and simulation results of non-converging BP messages have an excellent agreement with it, as shown in Fig. 6(b). This implies that nodes with free states start to exist, analogous to the free states in the non-trivial solution of vertex cover [4, 5] and graph coloring [23, 26]. This characterizes the region with multiple solutions of optimal source locations, and the network behavior is described by the so-called full replica symmetry-breaking (RSB) ansatz, which assumes an infinite hierarchy of states. We have considered the one-step RSB approximation, assuming that the number $\Gamma(E)$ of states with energy $E$ is distributed according to $\Gamma(E) \propto \exp[\Gamma \Sigma(e)]$, where $e = E/N$ [12]. $\Sigma(e)$ is referred
Figure 7. (a) The distribution $Q(\psi_s)$ in the singlet regime for $K = 3$ and $x = 0$. (b) The 1RSB complexity $\Sigma(e)$ for $K = 3$ and $\phi_d = 0.96, 0.98, 1$.

to as the complexity or the configurational entropy [24]. For small changes in the average energy, we can write $\Sigma(e) = x(e - e^K)$, where $e^K$ is the reference energy. The recursion relation Eq. (9) is then replaced by

$$
\psi_s^{j-i} = \delta_{\Lambda_j,-1} \left( 1 - \frac{1}{Z^{j-i}} \prod_{k \in \mathcal{L}_j \setminus \{i\}} \psi_s^{k-j} \right) + \delta_{\Lambda_j,A},
$$

$$
Z^{j-i} = \prod_{k \in \mathcal{L}_j \setminus \{i\}} \left[ e^{e^K} - (e^{e^K} - 1) \psi_s^{k-j} \right].
$$

The stable distribution $Q(\psi_s)$ of $\psi_s$ for $K = 3$ and $x = 0$ is shown in Fig. 7(a). When $\phi_d \leq \phi_d^{AT}$, there are no fractional components of $\psi_s$, and $Q(\psi_s) = (\psi_s) \delta(\psi_s - 1) + (1 - (\psi_s)) \delta(\psi_s)$. When $\phi_d > \phi_d^{AT}$, non-zero components of $0 < \psi_s < 1$ exist. When $\phi_d = 1$, there is no disorder in the capacities. All vertices are identical and $Q(\psi_s) = \delta(\psi_s - \eta_s)$. This means that among the different states of the system, all vertices are equally probable to be in the $S$ state with probability $\eta_s$.

The complexity $\Sigma(e)$ is shown in Fig. 7(b). The abscissa $f_s$ is the fraction of source nodes related to the average energy $e$ through $f_s = (e - 1/2K) / \gamma$. The physical segments of $\Sigma$ are indicated by the solid segments with $\Sigma \geq 0$ in Fig. 7(b). The energy $e_s$ with $\Sigma(e_s) = 0$ corresponds to the lowest energy among the states with non-vanishing complexity, which is considered as the typical ground state in the picture of 1RSB. The typical behaviors of the system is thus described by Eq. (12) with the value of $x$ satisfying $\Sigma(e_s) = 0$. The energy $e_d$ with the highest complexity is believed to be the point where search algorithms get trapped, giving rise to dynamical transitions. (However, recent work on the coloring problem shows that the efficacy of search algorithms is not affected by the dynamical transition [27]. We leave this issue for future studies.)

We have verified that the 1RSB ansatz yields better energy estimates than RS. We have done simulations at $\phi_d = 1$ using a decimated BP algorithm [12]. The fractions $f_s$ of source nodes are found to be 0.551 (simulation), 0.551 (1RSB at $e = e_d$), 0.549 (1RSB at $e = e_s$) and 0.545 (RS) at $K = 3$ and $N = 5000$. 

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6. Conclusion
We have studied the problem of optimal source location on transportation networks with step-like source node installation cost. As the installation cost increases, the number of source nodes decreases. Regimes characterized by discrete fractions of source nodes emerge and resemble the Devil’s staircase, revealing the optimal flow pattern of resources supplying different consumer clusters. In the singlet regime, which has a correspondence with the vertex cover problem and the lattice glass models, a closed set of only a few cavity energy functions is able to describe the physics of the system. In the case with surplus and deficient nodes, an increase in the fraction of deficient nodes induces a glassy phase transition incorporating the picture of numerous suboptimal solutions. The one-step replica symmetry-breaking ansatz yields better agreement with simulations than the replica symmetric ansatz.

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