Friedmann equations for deformed entropic gravity

Salih Kibaroğlu\textsuperscript{1} and Mustafa Senay\textsuperscript{2}

\textsuperscript{1}Department of Physics, Kocaeli University, 41380 Kocaeli, Turkey and
\textsuperscript{2}Naval Academy, National Defence University, 34940 Istanbul, Turkey

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In this study, we investigate the effects of the one and two-parameters deformed systems on the Friedmann equations of the Friedmann-Robertson-Walker (FRW) universe in the context of the entropic gravity approach. We give simplified forms for the deformed Unruh temperature and Einstein field equations for three different deformed systems. Based on these compact equations, we derive the Friedmann equations with the effective gravitational and cosmological constant.

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I. INTRODUCTION

Developments on the astronomical observation in the last century show that our universe has a form with homogeneous and isotropic characteristic in a large scale ($10^8$ light years and more). This is one of the most important properties of the universe which is called as the cosmological principle. By the help of this principle, we suppose that the universe has a symmetry which the curvature of the space to be same in everywhere. In this condition, we can use the Friedmann equations, which is derived by using Einstein’s general theory of relativity, to explain the dynamical evolution of the universe. Furthermore, the expansion rate of the universe is accelerating which was discovered in 1998 [1, 2]. In order to formulate this acceleration, one can add the cosmological constant and/or put an extra energy source term, which can be viewed as the dark energy [3, 4], to the Einstein field equations. This generalization leads to generalize the standard Friedmann equations with extra driving terms. From this point of view, various cosmological models were proposed to explain the acceleration [5–8]. Thus, one can say that the extended gravity theories may take a crucial role to explain this feature of the universe.

Recently, it has been proposed by Verlinde [9] (see also [10]) that gravity may have a thermodynamics origin and it can be considered as an entropic force which is called the entropic gravity (further developments on the thermodynamical description of gravity can also be found in [11–14]). Verlinde obtained the time-time components of Einstein’s field equations by motivating from the Bekenstein-Hawking entropy-area relation for black holes [15–17] and the holographic principles. The entropic gravity also provided a useful background to obtain generalized versions of gravity theory. Because if one uses the generalized forms of entropy, temperature or active gravitational mass functions instead of the standard ones, these idea may lead to generalize the gravitational field equations. From these ideas, many authors have extensively studied the extended gravity theories based on the entropic gravity proposal such as [18–28].

In cosmological context, the standard Friedmann equations were first derived in [29] by the help of Verlinde’s entropic gravity approximation and then various cosmological applications were studied in [30–35]. Besides, in studies [36, 37], the authors tried to explain the accelerating universe by adding extra entropic terms, instead of the cosmological constant, to the Friedmann equations. They also supposed that the horizon of the universe has an entropy and temperature. This model was named as the entropic cosmology and it has been studied extensively in [38–42].

It has been suggested that by Strominger [43] that quantum black holes obey deformed Bose or Fermi statistics instead of standard Bose or Fermi statistics. According to Strominger’s suggestion, the quantum black holes can be assumed $q$-deformed bosons or fermions. Thermodynamics and statistical properties of such $q$-deformed bosons and fermions have been extensively studied by several authors in the literature [44–48]. Moreover, these $q$-deformed bosons and fermions have a variety of applications in many branches of physics. For instance, in the Ref. [23–26], one and two-parameters deformed Einstein equations were obtained by considering the quantum black holes as deformed bosons or fermions.

With the above motivations, in this paper, we aim to obtain deformed Friedmann equations by using the results of three different deformed models and we will work in a frame where $c = k_B = 1$ where $c$ is the speed of light and $k_B$
is Boltzman’s constant. The paper is organized as follows. In Sec. 2, we review the useful results for our calculations of three different deformed models. We also give deformed temperature function and Einstein’s field equations in a compact form. In Sec. 3, the modified Friedmann equations and the density parameter are obtained. The last section concludes the paper and contains some discussion on our results and possible future works.

II. ONE AND TWO-PARAMETERS DEFORMED ENTROPIC GRAVITY

According to the Hawking radiation [16, 17], the mass of charged black holes decreases until it reaches a critical value proportional to its charge. Therefore, resulting structure of the charged black holes can be named as extremal (quantum) black holes which appears to be a quantum mechanically stable object [43]. In addition, such black holes behave like point particles and obey the deformed statistics. So, these black holes can be seen as deformed bosons or deformed fermions. Furthermore, Verlinde showed that the gravity occurs as an entropic force associated with the information on the holographic screen. From this idea, Verlinde succeeded to derive the time-time component of the Einstein field equation by using the equipartition rule and Tolman-Komar mass. This consideration allows to generalize the gravity by changing the thermodynamical quantities such as the entropy, temperature or energy functions.

In recent years, the study [23] showed us that the deformed Einstein equation can be derived by using deformed statistics rather than standard one to describe gravitational behaviors of the quantum black holes in the framework of the entropic gravity suggestion. From this motivation, the different deformation of Einstein’s field equations have been obtained by considering the extremal quantum black holes as \( q \)-deformed fermions [25], \( q \)-deformed bosons and fermions [26], and \((q, p)\)-deformed fermions [24] for the high-temperature limits. In this section, we briefly review the useful results of these three models.

In the first model [25], the extremal quantum black holes were considered as deformed fermions. The oscillators algebra of these deformed fermions was introduced in [49–51] and some of the high-temperature thermostatistical properties of these deformed fermions were investigated in [52]. The deformed entropy function of them is defined [25] as

\[
S = \frac{(2\pi m)^{3/2} V}{T \hbar^3} E^{5/2} \tilde{F}(z, q),
\]

(1)

where

\[
\tilde{F}(z, q) = \frac{5}{2} f_{5/2}(z, q) - f_{3/2}(z, q) \ln z,
\]

(2)

\[
f_n(z, q) = \frac{1}{|\ln q|} \left[ \sum_{l=1}^{\infty} (-1)^{l-1} \left( \frac{zq}{l+1} \right)^l - \sum_{l=1}^{\infty} (-1)^{l-1} \left( \frac{z}{l+1} \right)^l \right].
\]

(3)

Also, \( m \) and \( T \) indicate mass and temperature of deformed particles, respectively, and \( q \) is real deformation parameter in the interval \( 0 < q < 1 \). In the statistical equilibrium, the total entropy of the system should be constant and extremal and so the variation of the entropy goes to zero such as

\[
\frac{d}{dx^a} S(E, x^a) = 0.
\]

(4)

Eq.(4) can also be reexpressed

\[
\frac{\partial S}{\partial E} \frac{\partial E}{\partial x^a} + \frac{\partial S}{\partial x^a} = 0.
\]

(5)

where \( \frac{\partial E}{\partial x^a} = -F_a \) and \( \frac{\partial S}{\partial x^a} = \nabla_a S = (-2\pi m N_a) / \hbar \). Considering a generalized Newton potential as a function of killing vectors \( \phi = \ln (-\xi^a \xi_a) \) in general relativity, we can write the entropic force as \( F_a = T \nabla_a S = -me^{\phi} \nabla_a \phi \).

Using these relations and the entropy function in Eq.(1), the deformed temperature function on the holographic screen can be derived as follows
where $e^\phi$ is the redshift factor and $N^a$ is the unit outward pointing vector. When taking into account the Unruh temperature,

$$
T_U = \frac{\hbar}{2\pi} e^\phi N^a \nabla_a \phi.
$$

the deformed temperature function can be written as

$$
T^{(1)} = \alpha^{(1)} T_U,
$$

where the parameter $\alpha^{(1)}$ is deformation contribution of the model to the temperature function which defined as

$$
\alpha^{(1)} := \frac{5V (2\pi mE)^{3/2}}{2h^3} \tilde{F} (z, q).
$$

Now, if we define $\Psi^{(1)} := \tilde{F} (z, q)$ and

$$
f (E, m, V) = \frac{5V (2\pi mE)^{3/2}}{2h^3},
$$

the parameter $\alpha^{(1)}$ takes the compact form as follows

$$
\alpha^{(1)} := f (E, m, V) \Psi^{(1)}.
$$

In the second model $[26]$, the quantum algebraic structures of deformed bosons and fermions were introduced in $[51, 53, 54]$ and some of the high and low-temperature thermostatistical properties of these deformed bosons and fermions were examined in $[55]$. The deformed entropy function of them is defined $[26]$ as

$$
S = \frac{(2\pi m)^{3/2} V}{Th^3} E^{5/2} H^\kappa (z, q),
$$

where

$$
H^\kappa (z, q) = \frac{5}{2} h^\kappa_{5/2} (z, q) - h^\kappa_{3/2} (z, q) \ln z,
$$

$$
h^\kappa_n (z, q) = \frac{1}{q - q^{-1}} \left[ \sum_{l=1}^{\infty} (\kappa q^l)^l l^{n+1} - \sum_{l=1}^{\infty} (\kappa q^{-l})^l l^{n+1} \right].
$$

and $q$ is real deformation parameter in the interval $0 < q < 1$. Similar to the derivation of Eq.(10), the deformed temperature function can be obtained as,

$$
T^{(2)} = \alpha^{(2)} T_U,
$$

where

$$
\alpha^{(2)} := f (E, m, V) H^\kappa (z, q) = f (E, m, V) \Psi^{(2)}.
$$
In the third model [24], both of the quantum algebraic and some of the high-temperature thermostatistical properties of deformed fermions were studied in [56]. The deformed entropy function of them is given [24] as

\[ S = \frac{(2\pi m)^{3/2}V}{Th^3}E^{5/2}F(z, q, p), \]  \hspace{1cm} (17)

where,

\[ F(z, q, p) = \frac{5}{2}f_{5/2}(z, q, p) - f_{3/2}(z, q, p) \ln z, \]  \hspace{1cm} (18)

\[ f_n(z, q, p) = \frac{1}{|\ln(q^2/p^2)|} \left[ \sum_{l=1}^{\infty} (-1)^{l-1} \frac{(q^2z)^l}{l^{n+1}} - \sum_{l=1}^{\infty} (-1)^{l-1} \frac{(p^2z)^l}{l^{n+1}} \right]. \]  \hspace{1cm} (19)

and \((q, p)\) is real deformation parameter in the interval \(0 < (q, p) < 1\). Therefore, the temperature function for this model can be written as the following form,

\[ T^{(3)} = \alpha^{(3)}T_U, \]  \hspace{1cm} (20)

where,

\[ \alpha^{(3)} := f(E, m, V)F(z, q, p) = f(E, m, V)\Psi^{(3)}. \]  \hspace{1cm} (21)

Now we can define the general form of the temperature function for these three models as,

\[ T = T^{(i)} = \alpha^{(i)}T_U, \]  \hspace{1cm} (22)

where the upper indice \(i\) represents the corresponding model and takes the values as \(i = 1, 2, 3\).

On the other hand, the studies [23–26] show that the deformed Einstein field equations can be derived as follows by using Verlinde’s entropic gravity proposal [9],

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_{\text{eff}}T_{\mu\nu}. \]  \hspace{1cm} (23)

In addition to this, one can introduce the effective cosmological constant to the Einstein field equations by changing the active gravitational mass (for more detail see [25]),

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda_{\text{eff}} = 8\pi G_{\text{eff}}T_{\mu\nu}, \]  \hspace{1cm} (24)

where \(R_{\mu\nu}, R, g_{\mu\nu}\) and \(T_{\mu\nu}\) are the Ricci tensor, Ricci scalar, metric tensor and energy-stress tensors, respectively. Besides, the effective forms of the cosmological constant and gravitational constants are defined, respectively,

\[ \Lambda_{\text{eff}} = \frac{\Lambda}{\alpha^{(i)}}, \hspace{1cm} G_{\text{eff}} = \frac{G}{\alpha^{(i)}}. \]  \hspace{1cm} (25)

In this approach, we just need the deformation parameter \(\alpha^{(i)}\) of the corresponding deformation model to obtain the effects on Einstein’s field equations in the framework of the entropic gravity proposal. Therefore, we can say that this simplification can be applied to similar deformed gas models given in [57, 58].
III. MODIFIED FRIEDMANN EQUATIONS

The Friedmann equations describe the dynamical evolution of our universe in isotropic and homogeneous spacetime in large scales within the framework of Einstein’s theory of general relativity [7]. It can be said that if we use generalized versions of Einstein’s theory of gravity, it may lead to modify the Friedmann equations. From this motivation, we consider the deformed Einstein equations, as we discussed in the previous section, to derive new contributions to the Friedmann equations. For this purpose, we follow the procedure of [29] where the standard Friedmann equations were obtained from the entropic force together with the Unruh temperature and equipartition law of energy. We first begin with the Friedmann-Robertson-Walker universe with the metric,

$$ds^2 = dt^2 - a^2(t) \left( dr^2 + r^2 d\Omega^2 \right),$$

(26)

where \( a(t) \) is a dimensionless arbitrary function of time, known as the scale factor, which is related to the expansion of the universe, and \( \Omega \) denotes the line element of a unit sphere. Considering Verlinde’s paper [9], there is a spherical holographic screen \( S \) with a compact spatial region \( V \) and a compact boundary \( \partial V \) which have following physical radius,

$$\tilde{r} = a(t) r.$$  

(27)

The number of bits on the holographic screen is defined as [29]

$$N = \frac{A}{G\hbar},$$

(28)

where \( A = 4\pi\tilde{r}^2 \) is the area of the screen, and considering the equipartition law of energy, the total energy on the screen is given as

$$E = \frac{1}{2} NT,$$

(29)

where \( T \) is the temperature on the screen. Besides, we need energy-mass relation,

$$E = M,$$

(30)

where \( M \) corresponds to the mass in the spatial region \( V \). Due to supposing our universe have homogeneity and isotropic form in a large scale, the matter content of the universe can be interpreted as a perfect fluid with following stress-energy tensor,

$$T_{\mu\nu} = (\rho + p) u_{\mu}u_{\nu} - pg_{\mu\nu},$$

(31)

$$T = T^{\mu}_{\mu} = \rho - 3p,$$

(32)

where \( \rho(t) \) and \( p(t) \) are energy density and the pressure of cosmological fluids, respectively. Also, \( u^\mu = (1, 0, 0, 0) \) represents four velocity and satisfies \( g_{\mu\nu}u^\mu u^\nu = 1 \). Now, we can write the continuity equation from the conservation of the energy stress tensor, \( \nabla_\mu T^{\mu\nu} = 0 \), as follows

$$\dot{\rho} + 3H (\rho + p) = 0,$$

(33)

where \( H(t) = \dot{a}/a \) is the Hubble parameter which describes the expansion rate of the universe. We also note that a dot over any quantity, such as \( \dot{\rho} \), denotes the time derivative of that quantity. The total mass in the spatial volume \( V \) can be written as

$$M = \int_V dV (T_{\mu\nu}u^{\mu}u^{\nu}),$$

(34)
considering Eq. (30), the term $T_{\mu \nu} u^\mu u^\nu$ corresponds to the energy density and it can be easily found by using Eq. (31) and the definition of the four velocity. Furthermore, the acceleration of radial observer, which caused by the matter in the spatial region, with respect to fixed $r$ at the place of the screen is

$$a_r = -\frac{d^2 \tilde{r}}{dt^2} = -\ddot{\tilde{r}},$$

and by using this acceleration, the Unruh temperature takes the following form

$$T_U = \frac{\hbar a_r}{2\pi}.$$ (36)

Using Eqs (22), (28), (29), (30), (34), (35), (36), the area $A = 4\pi \tilde{r}^2$ and the volume $V = \frac{4}{3} \pi \tilde{r}^3$ we get,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{eff}}{3} \rho.$$ (37)

This equation corresponds to the dynamical equations for the Newtonian cosmology [6] in the deformed case. To obtain the Friedmann equations of the FRW universe for the deformed general relativity, we use active gravitational mass $M$ rather than total mass $M$ in the spatial region $V$. In our context, the active gravitational mass, Tolman-Komar mass, is defined as [59]

$$M = 2 \int_V dV \left( T_{\mu \nu} - \frac{1}{2} T g_{\mu \nu} + \frac{\Lambda}{8\pi G} g_{\mu \nu} \right) u^\mu u^\nu,$$ (38)

where $\Lambda$ is the cosmological constant. After some calculations, the active gravitational mass takes the following form

$$M = \left( \rho + 3p + \frac{\Lambda}{4\pi G} \right) V.$$ (39)

Then, using similar calculations to the derivation of Eq. (37), we obtain the following expression,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_{eff}}{3} (\rho + 3p) - \frac{\Lambda_{eff}}{3}.$$ (40)

Thus, we obtained the deformed version of the first Friedmann equation (or the acceleration equation) with the cosmological constant for the dynamical evolution of the FRW universe. After multiplying $\ddot{a}a$ on both sides of the last equation and using Eq. (33), and integrating, we obtain,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{eff}}{3} \rho - \frac{\Lambda_{eff}}{3}.$$ (41)

This is the deformed form of the second Friedmann equation which controls the time evolution of the FRW universe. Here, $k$ corresponds to the integration constant and it can be interpreted as the spatial curvature of the region $V$ in Einstein’s theory of gravity. In addition, $k = 1, 0$ and $-1$ correspond to a closed, flat, and open FRW universe, respectively (for more detail see [8]). The last equation can be written as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{eff}}{3} (\rho - \rho_\Lambda),$$ (42)

where $\rho_\Lambda$ can be seen an additional energy density to the universe which is associated with the cosmological constant,

$$\rho_\Lambda := \frac{\Lambda}{8\pi G}.$$ (43)
Therefore, we can easily say that the deformation does not contribute to the energy density of the universe. Considering the continuity equation Eq.(33), one can write the pressure, \( p_\Lambda \), which is associated with the cosmological constant, as follows

\[
p_\Lambda = -\rho_\Lambda = -\frac{\Lambda}{8\pi G}.
\]

In addition, using the relation \( \dot{H} = \frac{\ddot{a}}{a} - H^2 \) together with Eq.(40) and Eq.(41), we obtain the following expression:

\[
\dot{H} = -4\pi G_{\text{eff}} (\rho + p) + \frac{k}{a^2}.
\]

This equation is another Friedmann equation for the deformed case. Also, by the help of Eq.(41), the energy density \( \rho(t) \) can be written as follows

\[
\rho(t) = \rho_c(t) + \frac{3k}{8\pi G_{\text{eff}} a^2} + \frac{\Lambda}{8\pi G},
\]

where \( \rho_c(t) \) is the effective critical density and defined as

\[
\rho_c(t) = \frac{3H^2}{8\pi G_{\text{eff}}},
\]

it depends on the effective gravitational constant and the given value of the Hubble parameter. In general, we use present value of the Hubble parameter which is named as Hubble constant \( H_0 = H(t_0) = \frac{\ddot{a}}{a} \big|_{t=t_0} \), and the subscript \( 0 \) represents present values. Thus, Eq.(47) takes following form

\[
\rho_c(t) \rightarrow \rho_c(t_0) = \frac{3H_0^2}{8\pi G_{\text{eff}}}.
\]

By the help of \( \rho(t) \) and \( \rho_c(t) \), we can write the following expression

\[
\Omega(t) = \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{k}{H^2 a^2} + \frac{\Lambda_{\text{eff}}}{3H^2},
\]

where \( \Omega(t) \) represents the density parameter (or the cosmological parameter \( \Omega \)). Furthermore, Eq.(49) can alternatively be written as

\[
\Omega(t) + \Omega_k(t) + \Omega_\Lambda(t) = 1,
\]

where \( \Omega_k(t) \) and \( \Omega_\Lambda(t) \) represent density parameters with respect to the integral constant and the cosmological constant, respectively, and these parameters are defined as

\[
\Omega_k = -\frac{k}{H^2 a^2}, \quad \Omega_\Lambda = -\frac{\Lambda_{\text{eff}}}{3H^2}.
\]

The density parameter contains the information about the shape of our universe. For instance, if we take \( \Omega = 1 \), this model describes a flat universe. Besides, the condition \( \Omega < 1 \) corresponds to an open universe and \( \Omega > 1 \) corresponds to a closed universe. Currently, the present value of the density parameter is close to one \( \Omega_0 \approx 1 \). Considering the present-day value of the density parameter, a relation occurs as \( \Omega_k \approx -\Omega_\Lambda \) and thus the effective cosmological constant can be found as

\[
\Lambda_{\text{eff}} \approx -\frac{3k}{a^2}.
\]
IV. CONCLUSION

The Friedmann equations govern the evolution of our universe for a homogeneous and isotropic space geometry. We know that our universe obeys this geometry condition on a large scale. So, if we modify the Friedmann equation, this may lead to important results in the cosmology. In this work, we studied the possible effects of the one and two parameters deformation on the Friedmann equations. For this purpose, we considered three different deformed gas models in the high-temperature limits. First two models 25, 26 are related one parameter $q$-deformed fermion and fermion-boson gas system, respectively, and the third one 24 corresponds to two-parameters ($q, p$)-deformed of fermion gas system. We wrote the temperature function in a compact form depending on these models in Eq.(22). We also gave the deformed form of the Einstein field equations for given deformed gas systems with the effective gravitational and cosmological constant in Eq.(24) which is derived from the Verlinde’s entropic gravity proposal. According to Strominger’s idea, the charged extremal quantum black holes obey deformed statistics. Thus, one can say that the gravitational behaviors of Strominger’s black holes can be described by the deformed Einstein field equations in Eq.(24).

Moreover, we found the deformed Friedmann equations of the FRW universe in Eqs.(40)-(45). In addition, we also derived the density parameter in Eq.(49) with the effective cosmological term which depends on corresponding deformation parameter $\alpha^{(i)}$. The value of the density parameter gives important information about the shape of our universe. So, we can say that the deformed system may take a role to explain the shape of the universe.

According to these results, we can say that there are exact contributions come from the deformed gas models to the Friedmann equations. In a special limit, for instance, $q \rightarrow 1$ or $(q, p) \rightarrow 1$, the models return into their ideal form in which there are no interactions between related particles 60. But, since the function $\alpha^{(i)}$ in Eq.(22) does not equal to one, our deformed equations cannot reduce to their standard forms.

Moreover, taking into account of Eqs.(40) and (41), our results may be interpreted as the generalization of $\Lambda$CDM model 5, 7, 8 which is used to explain the accelerated expansion of the late universe by including extra driving terms. In standard $\Lambda$CDM model, the Friedmann equations are given as follows

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3p \right) + \frac{\Lambda}{3}, \quad (53)$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3}, \quad (54)$$

where $\Lambda$ is the cosmological constant and the driving term $\frac{\Lambda}{3}$ is responsible for the acceleration. Therefore, the deformed $\Lambda$CDM model may help to improve the explanation of the expansion of our universe.

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