Pricing and Energy Trading in Peer-to-peer Zero Marginal-cost Microgrids

Jonathan Lee, Rodrigo Henriquez-Auba, Bala Kameshwar Poolla, and Duncan S. Callaway

Abstract—Efforts to utilize 100% renewable energy in community microgrids require new approaches to energy markets and transactions to efficiently address periods of scarce energy supply. In this paper we contribute to the promising approach of peer-to-peer (P2P) energy trading in two main ways: analysis of a centralized, welfare-maximizing economic dispatch that characterizes optimal price and allocations, and a novel P2P system for negotiating energy trades that yields physically feasible and at least weakly Pareto-optimal outcomes. Our main results are 1) that optimal pricing is insufficient to induce agents with batteries to take optimal actions, 2) a novel P2P algorithm to address this while keeping private information, 3) a formal proof that this algorithm converges to the centralized solution in the case of two agents negotiating for a single period, and 4) numerical simulations of the P2P algorithm performance with up to 10 agents and 24 periods that show it converges on average to total welfare within 0.1% of the social optimum in the order of 10s to 100s of iterations, increasing with the number of agents, time periods, and total storage capacity.

Index Terms—microgrids, power system economics, transactive energy, energy storage, distributed power generation, power generation dispatch, smart grids, batteries, energy management systems, solar energy

NOMENCLATURE

- Sets and indices
  - C: Set of agents/consumers indexed by n.
  - U: Subset of agents that propose quantities (q-agents), indexed by k.
  - X: Subset of q-agents that have exited the negotiation.
  - Y: Subset of q-agents that are still negotiating.
  - υ: Subset of agents that respond with price (π-agents), indexed by v.
  - T: Set of time periods, indexed by t.
  - B: Set of batteries, indexed by i.

- Variables
  - G: Set of generators, indexed by g.
  - d_n,t: Local power consumption of agent n at time t.
  - p_g,t: PV power production from generation g at time t.
  - p_i,t: Discharge power from battery i at time t.
  - s_i,t: State of charge of battery i at time t.
  - q_k,t: Proposed quantity by q-agent k to receive from the π-agent at time t.
  - q̃_k,t: Projection of q_k,t to the feasible quantity set by the π-agent at time t.
  - β: Auxiliary variable used to project q̃_k,t → q_k,t.
  - α_n: Binary state, true iff agent n prefers current offer to no trade at all.
  - o_k,t: Binary state, true iff a proposed quantity from q-agent k at time t is “oscillating”, i.e., not monotonically increasing or decreasing over 3 iterations.

- Parameters
  - P^s_g,t: Max. PV power from generation g at time t.
  - P^b_i,t: Max. rate of charge/discharge of battery i at time t.
  - S^b_i,t: Max. energy capacity of battery i at time t.
  - δ_k^(0): Initial size of step-limiting constraint on a q-agent k.
  - γ: Shrinking rate of step-limiting constraint ∈ (0, 1).

Notation: The utility functions of the agents with respect to their local demand are denoted by U_n(d_n,t), marginal utility (∂U_n/∂d_n) by, g_n(d_n), and its inverse g_n^(-1)(d_n):=h_n(π_n). Bold symbols represent a vector or a collection of points, e.g., \{q_k\}_t∈T. The power and energy units are kW and kWh, and ΔT is the time step duration in hours. The symbols ¬, ∨, and ∧ denote logical negation, OR, and AND respectively.

I. INTRODUCTION

Due to declining technology costs and a drive to reduce carbon emissions, 100% renewable electricity grids systems are receiving increasing attention. California’s 2018 Senate Bill 100, for example, sets a large-scale 100% renewables target for 2045. At community scales, 100% renewable microgrids for resilience and energy access in rural areas have become competitive with hybrid solutions with fuel-based generators, and can be preferable in cases where emissions, fuel logistics, or generator maintenance are strong concerns.

Novel pricing mechanisms for 100% renewable systems are not yet well-developed, but we contend they will become increasingly important for policy-makers and practitioners. For
example, extending the current paradigm where load serving entities procure electricity at the lowest cost to meet inflexible demand, implies that zero (short-run) energy costs would lead to a zero (short-run) price [1]. However, Fripp et al. [2] have shown by including demand-side participation in a capacity expansion model for Hawaii that dynamic electricity pricing to consumers results in non-zero prices and is increasingly important for maximizing welfare in 100% renewable systems.

In this paper we contribute a general theoretical analysis of pricing in 100% renewable plus storage systems, characterizing optimal price dynamics and the challenges energy storage presents for standard bidding mechanisms. We focus on zero marginal cost renewables, with a solar generation case in simulations. We then propose an approach for community microgrids where individual “prosumers” with solar and storage could interact informally in a “peer-to-peer” (P2P) system to negotiate energy trades and form dynamic electricity prices and allocations that approximate optimal outcomes.

In the next two paragraphs we briefly review the relevant literature in this space, arguing specifically that the case of peer-to-peer trading in 100% renewable plus storage systems requires more analysis and innovation. P2P systems are valuable for grid resiliency, renewables integration, electricity access in less developed regions, and individual participation in electricity systems [3–7]. Early work proposed centrally coordinated energy trading between distributed energy resources (DERs) where the generation and battery storage are fully controllable [8, 9]. In [10], the authors lay the foundation for defining the physical and virtual layers required to consumers results in non-zero prices and is increasingly important for maximizing welfare in 100% renewable systems.

II. CENTRALIZED WELFARE MAXIMIZATION APPROACH

In this section, we define a model for optimal energy dispatch over a finite time horizon, analyze the solution for relevant insights into P2P electricity markets, and illustrate its dependence on energy storage through example. The model applies a utility maximization framework. For analysis of the optimal dispatch, we take the perspective of a benevolent central operator and assume knowledge of the individual utility functions. In practice this could be the perspective of a DER aggregator or a distribution system operator; however, it is difficult to know utility functions in practice, and this issue is a fundamental motivation for exploring peer-to-peer markets in the first place. We use a deterministic approach, where decisions are made off of a single, expected forecast of solar generation without hedging for uncertainty, and note this can be suboptimal to stochastic approaches. We also present an idealized battery model for simplicity, but show in Appendix A that the key insights still hold when we incorporate constant charge and discharge inefficiencies, self-discharge, and asymmetric power constraints. We also assume the battery is not required to achieve a final state-of-charge, but the model can easily include this constraint without loss of generality as long as the final state-of-charge is feasible. This model provides a baseline for comparing decentralized approaches, and could be extended to other DERs such as electric vehicles in the context where the generation is zero marginal-cost and energy constraints are relevant. In the case where the DERs introduce additional costs or utility functions, such as fuel costs or non-concave utility functions, the theoretical results may not hold.

The key theoretical insight we provide is that in the presence of energy storage, the dispatch cannot be controlled by price alone. Specifically, we show that if individuals act independently to maximize their utility in the presence of an
optimal price, there is no guarantee that their corresponding target power injections will be feasible and satisfy power balance. This highlights that ensuring feasibility is an important requirement of decentralized mechanisms. We describe why this is not trivial in the presence of storage, and also derive equations describing the optimal power and price trajectories.

A. Utility maximization model

The model (1) is similar in structure to a standard discrete-time, centralized energy management system. The central constraint is matching supply and demand on the time scale of hours, while we assume that droop-like control of power converters is necessary and sufficient to adjust any power imbalance in the short-term. We include operational constraints on energy storage, but not the network constraints and assume strictly concave utility functions $U_{n,t}$.

\[
\min \ p.d.a \ - \ \sum_{i \in T} \ \sum_{n \in C} U_{n,i}(d_{n,t}) \quad \text{(1a)}
\]

subject to

\[
\pi_t : \ \sum_{n \in C} d_{n,t} = \sum_{i \in B} p^{b}_{i,t} + \sum_{g \in \mathcal{G}} p^{g}_{g,t}, \ \forall t \in T \quad \text{(1b)}
\]

\[
\lambda^{s}_{n,t} : 0 \leq p^{s}_{g,t} \leq \bar{P}^{s}_{g,t}, \ \forall t \in T \quad \text{(1c)}
\]

\[
\lambda^{d}_{n,t} : -d_{n,t} \leq 0, \ \forall n \in C, \forall t \in T \quad \text{(1d)}
\]

\[
\lambda^{b}_{i,t} : -P^{b}_{i,t} \leq p^{b}_{i,t} \leq \bar{P}^{b}_{i,t}, \ \forall i \in B, \forall t \in T \quad \text{(1e)}
\]

\[
\lambda^{c}_{i,t} : 0 \leq s_{i,t} \leq \bar{S}_{i,t}, \ \forall i \in B, \forall t \in T \quad \text{(1f)}
\]

\[
s_{i,t} = s_{i,t-1} - \Delta T \sum_{\tau \leq t} p^{b}_{i,\tau}, \ \forall i \in B, \forall t \in T \quad \text{(1g)}
\]

This allows battery constraints to be time-varying but typically $\bar{P}^b$ and $\bar{S}$ are static. The dual variables of the respective constraints are indicated before the colon. For compactness, we use a single variable to represent the difference in upper and lower bound duals, $\lambda := \lambda^+ - \lambda^-$. The initial state of charge $s_{i,0}$ is a parameter. We eliminate the constraint (1g) and decision variables $s_{i,t}$ by solving for it as $s_{i,t} = s_{i,0} - \Delta T \sum_{\tau \leq t} p^{b}_{i,\tau}$, and substituting this into (1).

B. Theoretical analysis

Firstly, note that all constraints in (1) are affine, thereby satisfying the linearity constraint qualification (LCQ). This implies that for a locally optimal primal solution, there exists a set of dual variables satisfying the Karush-Kuhn-Tucker (KKT) conditions. Secondly, as all $U_{n,t}$ are concave, the problem is convex. Any point satisfying the KKT conditions is thus globally optimal and strong duality holds.

Remark 1 (Dual decomposition into private decisions): The Lagrangian dual of the centralized problem (1) is separable and equivalent to the sums of Lagrangian duals for constrained individual welfare maximization for a price equal to $\pi_t$. This allows interpretation of $\pi_t$ as the electricity price.

Assuming the utility functions are concave, the Lagrangian dual problem gives the optimal price and total welfare. The Lagrangian of (1) can be written as:

\[
L(d, p^s, p^b, \pi, \lambda) = \sum_{t \in T} \left( -U_{n,t}(d_{n,t}) + (\pi_t - \lambda^{d}_{n,t}) d_{n,t} \right) + \sum_{g \in \mathcal{G}} \left( (\lambda^{s}_{g,t} - \pi_t) p^{s}_{g,t} + \lambda^{c}_{g,t} \bar{P}^{s}_{g,t} \right) + \sum_{i \in B} \left( (\lambda^{b}_{i,t} - \pi_t) p^{b}_{i,t} + \lambda^{c}_{i,t} \bar{P}^{b}_{i,t} \right)
\]

\[
+ \lambda^{c}_{i,t} \left( s_{i,0} - \Delta T \sum_{\tau \leq t} p^{b}_{i,\tau} \right) - (\lambda^{b}_{i,t} + \lambda^{c}_{i,t}) \bar{P}^{b}_{i,t} - \lambda^{s}_{i,t} \bar{S}_{i,t}.
\]

We define individual utility/profit-maximization problems for each of the consumers, PV, and battery operators for an electricity price as in (2)-(4).

\[
W_n(\pi) := \min_{d_{n,t} \geq 0} \ -U_{n,t}(d_{n,t}) + \pi_t d_{n,t}, \quad (2)
\]

\[
W_p(g, \pi) := \min_{\lambda^{s}_{g,t}, \pi_t} \ -\pi_t p^{s}_{g,t} \ \text{s.t.} \quad (1c), \quad (3)
\]

\[
W_b(i, \pi) := \min_{\lambda^{b}_{i,t}, \lambda^{c}_{i,t}} \ -\pi_t p^{b}_{i,t} \ \text{s.t.} \quad (1c) - (1g). \quad (4)
\]

Denoting their Lagrangians by $\mathcal{L}_n, \mathcal{L}_g, \mathcal{L}_i$, one can show that

\[
L(d, p^s, p^b, \pi, \lambda) = \sum_{n \in C} \mathcal{L}_n(d_{n,t}, \pi) + \sum_{i \in B} \mathcal{L}_g(p^{s}_{g,t}, \lambda^{s}_{g,t}, \pi) + \sum_{i \in B} \mathcal{L}_i(p^{b}_{i,t}, \lambda^{b}_{i,t}, \lambda^{c}_{i,t}). \quad (5)
\]

As $W_g$ and $W_i$ are linear programs, strong duality holds for these subproblems, and the Lagrangian dual problem is

\[
\max_{\pi, \lambda} \ i_{d^b, p^s} \mathcal{L}(d, p^s, p^b, \pi, \lambda) = \max_{\pi} \ \sum_{n \in C} W_n(\pi) + \sum_{i \in B} W_g(\pi) + \sum_{i \in B} W_i(\pi). \quad (6)
\]

By strong duality (6) gives the optimal objective value with its maximizer $\pi^*$ equal to the optimal price. However, as we establish later, the optimal $p^{b}_{i,t}$ for (4) is not necessarily unique, meaning that broadcasting an optimal price to individual agents does not necessarily satisfy constraint (1g) and clear the market; i.e., primal feasibility is not guaranteed.

Remark 2 For all $t \in T$, the following relations hold true at optimum and characterize the optimal price

\[
\pi_t^* = \partial U_{n,t}(d_{n,t})/\partial d_{n,t} + \lambda^{*,d}_{n,t}, \ \forall n \in C \quad (7a)
\]

\[
= \lambda^{*,b}_{i,t} - \Delta T \sum_{\tau \leq t} \lambda^{*,c}_{i,\tau}, \ \forall i \in B \quad (7b)
\]

\[
= \lambda^{*,s}_{i,t}, \ \forall g \in \mathcal{G}. \quad (7c)
\]

Each of the equalities follow from the stationarity conditions of (1). We interpret the dual variable $\pi_t^*$ as the price by Remark 1 and note from (7b) that it depends on the cumulative future shadow prices of the storage capacity constraint. Eq. (7a) requires $U_{n,t}$ to be differentiable for equality but can be replaced by the subdifferential of $U_{n,t}$ otherwise.

Remark 3 If at time $t$, a utility function for at least one customer is differentiable and strictly increasing on $\mathbb{R}^+$, then at optimum, the price is strictly positive and solar production is at its maximum.

This follows from Remark 2 and the properties of strictly
increasing functions:

\[ \exists \eta \in \partial U_{n,t}(\pi^*_t) / \partial d_{n,t} > 0 \forall d_{n,t} \Rightarrow \pi^*_t > 0 \Rightarrow \lambda^*_t > 0. \]

By complementary slackness, \( \lambda^*_t > 0 \Rightarrow p^*_t = P_t^c \). This is intuitive as it is better to supply any benefitting consumer than curtail available solar. This also implies that solar generation can be removed as a decision variable and set to the available resource in this case.

**Remark 4** The optimal price evolves as

\[ \pi^*_{t+1} - \pi^*_t = \lambda^*_i,t_{t+1} - \lambda^*_i,t + \Delta T \lambda^*_i,c. \]  

(8)

This follows from **Remark 2** by expanding the expression \( \pi^*_{t+1} - \pi^*_t \). This captures the price trajectory, from which price volatility can be analyzed. Note that both \( \lambda^*_i,t \), \( \lambda^*_i,c \) can be less than 0. We will use (8) for our analysis in **Remark 5**.

**Remark 5** (Non-uniqueness of decentralized battery dispatch): There are non-trivial optimal prices \( \pi^* \) such that the optimal individual battery dispatch \( W_i(\pi^*) \) is not-unique.

This can be observed in a simple example. Suppose \( T = 5 \), \( \Delta T = 1 \), \( P^b_t = 3 \), \( S^t = 10 \), \( s^i = 5 \), and \( \pi^* = [1, 1, 2, 3, 1] \). One can verify that \( p^b_t = [-0.5, -0.5, 3, 3, 0] \), \( P^b_t = [0, -1, 3, 3, 0] \), and \( p^b_t = [-1, -1, 3, 3, 1] \) are all optimal solutions with a net cost of -14. Here, equal prices imply there is no change in cost to shift energy from one period to another and the constraints allow this shift. More formally, if an optimal solution is not on any of the constraint boundaries (13) at time and \( t + 1 \), then it will not be unique because not being on the boundary implies 1) that \( \lambda^*_i,t \), \( \lambda^*_i,t_{t+1} \), \( \lambda^*_i,c \), and \( \lambda^*_i,c_{t+1} \) are all 0, so \( \pi^*_{t+1} = \pi^*_t \) by (8), therefore \( p^b_t = p^*_{t+1} + \varepsilon \) and \( p^b_t = p^*_{t+1} - \varepsilon \) have equivalent net cost \( \forall \varepsilon \) without affecting the solution at other times; and 2) that this perturbation is feasible for sufficiently small \( |\varepsilon| > 0 \). Note that this condition is overly restrictive and not necessary for non-uniqueness; in particular energy may be shifted between non-consecutive time periods, and only particular combinations of constraints between the time periods need to be non-binding rather than all constraints. Equal prices between time periods may indicate non-uniqueness, but the optimal solution may still be unique if the constraints do not allow a perturbation to remain feasible.

In Appendix [A] we show that this applies with a variation in the optimal price profile even when battery inefficiencies and self-discharge are considered.

The consequence of **Remark 5** is that in general, an optimal price is not sufficient to yield individual battery dispatch decisions with the optimal quantity, meaning that a system operator cannot control dispatch outcomes solely by broadcasting a price signal or adequately forecast the decentralized response to price. Even when all utility functions are strictly concave and there are (likely common) conditions whereby an individual battery operator’s decision in response to \( \pi^* \) does not satisfy the constraint (15). Intuitively, if the price is constant between two successive periods, a battery operator would be indifferent to selling more energy in one period versus the next, so their dispatch is not unique and there is no guarantee that the dispatch will meet demand.

The previous observation implies that extending standard centralized market mechanisms to systems with energy storage faces limitations. If a centralized energy market limits entities with storage to submitting a single curve of price and quantity for each time-period, it is likely to result in suboptimal outcomes to the utility maximization problem even in infeasibility. Although not shown here, we expect this result to extend to load that can be shifted without cost, and to storage models with constant charge or discharge inefficiencies. P2P approaches where agents explicitly agree on quantity are a potential opportunity for addressing this challenge.

C. Example optimal trajectories and the effects of storage

To show how the PV profile and storage capacity affect the optimal trajectories of (1), we simulate scenarios with total storage capacity varying \( \bar{S}_s \in \{1, 5, 15, 300\} \) kWh and distributed evenly to batteries collocated with consumers. We sample hourly load and PV profiles from the 2017 Pecan Street...
data set \cite{30} over a random 66 hour interval and construct example utility functions by assuming a quasi-constant price-elasticity demand function and centering it at the observed load with a constructed time-varying price profile\cite{30} (see Appendix \cite{3}). We model 10 consumers and randomly select elasticities $\in [-3, -2]$.

Figure \ref{fig:trajectory} shows the optimal trajectories for each storage scenario. The solar output is identical across scenarios (a). As the storage capacity is increased, the consumption shifts to evening peaks from daytime peaks coincident with solar (b). Increasing storage reduces the swing between high and low price periods (c). The instances when the price changes, correspond to when the battery constraints are binding, as predicted by \cite{3}, which also explains how higher storage capacities lead to a flat price by reducing $\lambda_{t,t}$ to a negligible value. A flatter price means the conditions of Remark \ref{remark:static-convergence} are less likely to be met, highlighting the increasing need to coordinate battery dispatch as capacity increases. In contrast, a smaller capacity induces cyclical price fluctuations through peak-to-peak cycling. This also illustrates how the marginal value of storage in arbitraging high and low price periods depends on the existing capacity. These phenomena are explained analytically by the model; extending the model to derive optimal investment and planning decisions is a promising area for future work.

### III. Peer-to-Peer Negotiation

In this section we analyze how a decentralized, peer-to-peer energy market can arrive at a near-optimal dispatch solution using an intuitive negotiation approach. We model a process of exchanging price and quantity offers after the classic "cobweb" model of dynamic markets \cite{30} and observe that classical results show the process can diverge. We therefore, consider an additional dynamic step-limiting constraint on the process to ensure convergence, which could be thought of as a behavioral tendency of agents or an explicit rule to be imposed by a bidding platform. We assume agents are matched \textit{a priori} and that offers are synchronized so as to simplify the analysis and presentation, but posit that the process can be generalized to capture more informal interaction between agents.

As a starting point, consider an interaction between two agents who are \textit{"prosumers"} with private solar and storage systems and who individually derive private value from energy use. Most likely, there exists a trade that makes both agents better off. An intuitive way for the agents to find such a trade is for one to start by proposing a quantity (either positive or negative) and for the other to respond with a price. The first agent would likely reassess the quantity they would seek at that price, propose a new quantity, and so on. This iterative process is described by the cobweb model illustrated in Fig. \ref{fig:trajectory}. The equilibrium is the intersection of supply and demand curves arising from the utility functions. This is the optimum of the utility maximization model but the process converges to this point if and only if the magnitude of the slope of the demand curve exceeds that of the supply curve at the equilibrium \cite{30}.

We modify the cobweb model to ensure convergence even when this condition is not met by including a step-limiting constraint, illustrated in Fig. \ref{fig:step-limiting}. This constraint assumes (or enforces) that agents will not adjust their quantity offers by more than some threshold each iteration, and that this threshold shrinks if the quantity is "oscillating". We generalize to consider multiple agents proposing quantities (called $q$-agents) to agents who respond with price (called $\pi$-agents). The agents exchange vectors of quantity and price for each period over a finite time horizon. To simplify the analysis, we assume a single $\pi$-agent interacts with multiple $q$-agents. In practice, there would likely be multiple $\pi$-agents, and $q$-agents would select one or more $\pi$-agents to negotiate with, based on their expectation of the outcome of the negotiation, but this matching problem is beyond the scope of this paper.

We present formal decision models for the $q$-agents and the $\pi$-agents, and define an iterative process that guarantees physically feasible and at least weakly Pareto-optimal outcomes (i.e., no agents are worse off). We prove theoretically that the process converges to within a tolerance of the centralized solution for the 2-agent, single time step case, and demonstrate convergence using simulations for the general case in the next section. These results show that an informal, decentralized, peer-to-peer negotiation process is capable of approximating

30.10$/kWh between 21:00-11:00, 0.15$/kWh between 11:00-16:00, and 0.30$/kWh between 16:00-21:00.
the centralized welfare maximization problem, and offers a specific approach that could be implemented on a software platform and evaluated in practice. In addition, and contrary to the centralized approach, this negotiation process does not require full information exchange between agents, since private utility functions are hidden and only trading of quantities and prices are required for the execution of the algorithm.

We denote the set of $\pi$-agents with $\mathcal{V}$ and $q$-agents with $\mathcal{U}$, such that $\mathcal{C} = \mathcal{U} \cup \mathcal{V}$ and $\mathcal{U} \cap \mathcal{V} = \emptyset$. We index the $q$-agents by $k \in \mathcal{U}$ and the single $\pi$-agent as $v$, $\mathcal{V} = \{v\}$. The $q$-agents may exit the process early, which we track by partitioning $\mathcal{V} = U \cup V$.

We maintain that $\pi$ makes the decision to include a network feasibility validation as constraints over $d_{k,t}$, where $\beta = 0$ yields the requested $q$ and $\beta = 1$ is the closest point to $q$. Thus, minimizing $\beta \geq 0$ selects the closest point to $q$ satisfying the constraints:

$$\min_{d_v, p_{v,t}^v, s_v, q, \beta} \beta$$

s.t. $d_v + \sum_{k \in \mathcal{V}} q_{k,t} - \sum_{k \in \mathcal{X}} p_{v,t} = p_{v,t}^s + p_{v,t}^b$  \hspace{1cm} (9a)

$0 \leq \beta \leq 1$ \hspace{1cm} (9b)

$q_{k,t} = \beta \hat{q}_{k,t} + (1 - \beta) q_{k,t}$ \hspace{1cm} (9c)

and constraints (1d) - (1g). \hspace{1cm} (9d)

We maintain that $\hat{q}$ is feasible for all agents. Before any agents exit, $\mathcal{X} = \emptyset$ and $\hat{q} = 0$ is feasible, so we initialize with $\hat{q} = 0$ and update $\hat{q}$ as agents exit at feasible points. As shown below, $q$ is necessarily feasible for each $q$-agent, and their constraints are convex, so any point connecting two feasible points is feasible, and in particular $q'$. It is also possible to include additional constraints in this optimization problem, for example, to ensure power flow feasibility. Extending this model to include a network feasibility validation as constraints over $d_{k,t}$ is a promising direction for future work.

Next, the $\pi$-agent solves their utility maximization problem to obtain $\pi = \{\pi_t\}$ and their utility from these proposed trades. A key assumption is that they set $\pi$ at their marginal utility; i.e., they bid according to a competitive market strategy and cannot exercise market power. This is likely to hold in practice if there are sufficiently many $\pi$-agents the $q$-agents can access; however, we recommend a more careful analysis of market power in the scope of a “many-to-many” extension to this work. The maximization problem is:

$$\min_{d_v, p_{v,t}^v, s_v} \sum_{t \in \mathcal{T}} U_{v,t}(d_{v,t})$$

s.t. $\pi_t : d_v + \sum_{k \in \mathcal{V}} q_{k,t} - \sum_{k \in \mathcal{X}} p_{v,t} = p_{v,t}^s + p_{v,t}^b$ \hspace{1cm} (10a)

and constraints (1d) - (1g). \hspace{1cm} (10b)

Lastly, the $\pi$-agent checks whether its utility from this potential trade is at least as high as its optimal utility from no trade (specifically solving the same problem with $q' = 0$), and sets a binary variable $\alpha_v$ true if so, and false otherwise. This $\alpha_v$ signals whether $v$ would prefer $q'$ to no trade. We denote the entire decision as $P^v_\pi : (q, \pi) \mapsto (q', \pi_v, \alpha_v)$.

2) Optimization problem for $q$-agents: The $k$-th $q$-agent makes the decision $P^k_q : (\pi, q_k, \delta_k) \mapsto (q'_k, \alpha_k, \pi)$, where $\alpha_k$ carries the analogous meaning to $\alpha_v$, $\pi_k$ signals whether they are “satisfied”, $q'_k$ is the subset of $q'$ for $k$, and $\delta_k$ is the step-limiting constraint restricting the $q$-agent to select something close to the offer $q'$. The decision is:

$$\begin{align*}
\min_{d_k, q_k, p^s_k, p^b_k} & \sum_{t \in \mathcal{T}} -U_{k,t}(d_{k,t}) + \pi_k q_{k,t} \\
\text{s.t.} & \quad d_{k,t} - p^s_k - p^b_k - q_{k,t} = 0 \\
& \quad q_{k,t} - q'_{k,t} \leq \delta_{k,t} \\
& \quad \text{and constraints (1d) - (1g)}.
\end{align*}$$

Agent $k$ requests to finalize the trade and exit if their (not necessarily unique) optimal $q_k$ is close enough to the offer $q'_k$, where the distance is determined by a small $\varepsilon$:

$$\eta_k = \begin{cases} 
\text{True} & \text{if } |q_{k,t} - q'_{k,t}| \leq \gamma \varepsilon \\
\text{False} & \text{otherwise.}
\end{cases}$$

The exit condition includes the constant $\gamma \in (0, 1)$ to simplify the statement of Theorem 6, but could be modified with an update to the bound in the theorem. An alternative criterion based on whether the utilities from these offers are close enough could also be used but would affect the bound.

Algorithm 1: Bounded cobweb iteration for a single $\pi$-agent and multiple $q$-agents.

Result: Energy trades ($\pi_k, q'_k$) for each agent $k \in \mathcal{C}$.

Initialization: Define the $\pi$-agent $v \in \mathcal{C}$ and the parameters $\gamma \in (0, 1)$, $\varepsilon > 0$, initial step-limit $\delta^0 > \gamma \varepsilon$, and max iterations $M$.

Set $i \leftarrow 1$, $(q^{(1)}, \pi^{(1)}, \alpha_v^{(1)}) \leftarrow P^v_\pi(q^{(1)}, \hat{\pi})$.

while $\mathcal{Y} \neq \emptyset$ and $i \leq M$ do:

$q^{(i)}, \pi^{(i)}, \alpha_v^{(i)} \leftarrow P^v_\pi(q^{(i)}, \hat{\pi});$

for $k \in \mathcal{Y}$ do:

if $\eta_k$ then $\delta_k^{(i+1)} \leftarrow \delta_k^{(i)}$ else $\delta_k^{(i+1)} \leftarrow \delta_k^{(i)} - f(\delta_k^{(i)});$

end if $\alpha_v \forall j \in \mathcal{Y} \cup \{v\}$ then

$q \leftarrow q^{(i)};
$

for $k \in \mathcal{Y}$ where $\eta_k$ do

$\mathcal{Y} \leftarrow \mathcal{Y} \setminus \{k\}; \pi \leftarrow \pi \cup \{k\};$

end

end

$i \leftarrow i + 1$
3) Iterative Algorithm: The negotiation algorithm is presented in Algorithm 1. At each iteration, $q$-agents submit their energy quantity requests to the $\pi$-agent based on the last price and quantity offered by the $\pi$-agent. The $q$-agents are allowed to exit only when all have declared $(\pi, q')$ preferable to no trade through $\alpha$ (i.e., $\alpha_k = True \ \forall k \in U$), guaranteeing that trades are at least weak-Pareto improvements. Importantly, the step-limit $\delta$ is shrunk by $\gamma \in (0,1)$ if the quantity is “oscillating” (see Fig. 2), defined by the binary state $o^q(i)$ as the quantity not monotonically increasing or decreasing over 3 iterations, with $o^q(i) = 1$ and update maps $f^0$ and $f^3$: 

$$
 f^0: o^q(i) = \neg (q_{k,t}^{i+1} > q_{k,t}^{i-1} \lor q_{k,t}^{i+1} < q_{k,t}^{i-1}), \\
 f^3: \delta_{k,t}^{(i+1)} = (1 - o^q(i)) \delta_{k,t}^i + o^q(i) \gamma \delta_{k,t}^i.
$$

This shrinking step-limit prevents the divergent case of the cobweb model [30].

4) Optimality of the two-agent, single time step case: In this subsection we prove that Algorithm 1 converges within an $\varepsilon$ tolerance in finite iterations to the socially optimal quantity in the case of only two agents with single time horizon. We ignore storage in this case, as it can equivalently be treated as solar production for $T = \{1\}$, and drop the time index $t$ for brevity. We assume the the solar production is greater than zero for at least one agent, and that each agent’s marginal utility of consumption $\partial U_n(d_n)/\partial d_n$ is strictly monotonically decreasing on $(0, \infty)$ and decreasing asymptotically to zero.

Note that $q = d_k - p_k^n = -d_v + p_v^n$, and the unconstrained demand and supply curves are defined as $g_k \equiv \partial U_k(q)/\partial d_k$ and $g_v \equiv \partial U_v(q)/\partial d_v$. Thus, $g_k$ is monotonically decreasing and $g_v$ is monotonically increasing. Without the step-limiting constraint (Eq. 11c), the problem $P_k$ for the $q$-agent has a closed form solution:

$$
 q^i = g_k^{-1}(min(g_k(-p_k^n), \pi)) = h_k(\pi),
$$

where $g_k^{-1}$ denotes the inverse of $g$ with domain $(0, g_k(-p_k^n))$. With the step-limiting constraint, the solution is

$$
 q = \begin{cases} 
 q^i & \text{if } |q^i - q^i| < \delta \\
 q^i + \delta & \text{if } q^i > q^i + \delta \\
 q^i - \delta & \text{if } q^i < q^i - \delta
\end{cases},
$$

The projection step reduces to $q^i = min(p_v^n, q^i)$, and the $\pi$-agent’s price is given by $\pi = g_v(q^i)$.

The optimal quantity of the centralized problem $q^*$ is the unique fixed point of the iteration if $q^* < p_v^n$ or if $q^* = p_v^n$ and $g_v(p_v^n) = g_v(q^*)$. Indeed, note that $-p_v^n \leq q^* \leq p_v^n$ by the constraints, and that $h_k(\pi^*) = q^*$. If $q^*(i) = q^*$, then $q^* = \pi^*$ and $\pi^*(i) = g_v(q^*) = \pi^* - \lambda_*^{d,v}$ by (7a). When $q^* < p_v^n$, or $g_v(p_v^n) = g_v(q^*)$, then we have $\lambda_*^{d,v} = 0$, so $q^*(i) < \pi^*$, and $\pi^*(i) < \pi^*$, and hence $q^*(i) = q^*$ with $q^*(i+1) = q^*$. Otherwise, $\lambda_*^{d,v} > 0$ and $q^*(i) < \pi^*$, so $q^*(i) > \pi^*$ by the strict monotonicity of $h_k$, and $q^*(i+1) > q^*$, so it is not a fixed point. In other words, the fixed point is the intersection of the curves $g_v$ and $g_v$, as shown in Fig. 2. Since both curves are strictly monotonic, this fixed point is unique. If they do not intersect on $[-p_k^n, p_v^n]$, then $q^*$ is only a fixed point if it is $-p_k^n$.

Lemma 1 (Movement towards equilibrium) At any iteration $i$, if $q^* < p_v^n$, then $q^i \leq q^* \iff q^{(i+)} \geq q^*$ and $q^i \geq q^* \iff q^{(i+1)} \leq q^*$. Moreover, $q^i \leq q^* \iff q^{(i+1)} \geq q^(*)$ and $q^i \geq q^* \iff q^{(i+1)} \leq q^(*)$.

Proof: We prove this by showing the forward direction of the first set of statements $q^i \leq q^* \Rightarrow q^{(i+1)} \geq q^*$ and $q^i \geq q^* \Rightarrow q^{(i+1)} \leq q^*$. Each of these statements implies the converse of the other is true, establishing the reverse direction. We use the same approach for the second set of statements.

Let $\pi = g_v(q^*)$. If $q^* < p_v^n$, then $g_v(q^*) = \pi$ and if $q^* \leq q^* \Rightarrow \pi \leq q^* \Rightarrow h_k(\pi) \geq h_k(\pi^*) \Rightarrow q^i \geq q^*$ because $g_v$ is monotonically increasing and $h_k$ is monotonically decreasing. Thus, $q^i \geq q^* \Rightarrow q^i \leq q^*$. Moreover, $q^i < q^*$ implies $q^i \leq q^i < p_v^n$, therefore, $q^{(i+1)} \geq q^{(i+1)} \geq q^i$. Finally, because $q^i = \min(p_v^n, q^i) \leq q^i$, and by showing that $q^i \geq q^* \Rightarrow q^{(i+1)} \leq q^*$, it follows that $q^i \geq q^* \Rightarrow q^{(i+1)} \leq q^i$.

Lemma 2 (Entry to the oscillatory mode) If the system is not in the oscillatory mode at iteration $i$, then $\exists \gamma > 0$ such that if the algorithm does not terminate at iteration $i < l+1$, it will be in the oscillatory mode at $i+1$.

Proof: First, consider the case $q^i < p_v^n$. We will show the case when $q^{(i+1)} < q^*$, since the other case is analogous. By Lemma 1 $q^{(i+1)}$ moves towards the equilibrium and $\delta^{(i+1)}$ is not reduced when moving in the same direction. Thus, for some $j > i$, $q^{(j)} \geq q^*$ (with $q^{(j-1)} < q^*$); hence, by Lemma 1 $q^{(j-1)} \leq q^{(j)}$ and we enter the oscillatory mode at $j = i+1$.

Second, we consider the case of $q^* = p_v^n$. Observe that when eventually $q^{(j)} \geq p_v^n$, it will be projected back to $q^j = \min(q^{(j)}, p_v^n) = p_v^n$ to ensure feasibility for the $\pi$-agent. Then, since there is no intersection of marginal utility curves in the interior, it implies that $g_v(q^j) \geq g_v(q^*) = q^*$, and hence, the $\pi$-agent again requests $q^{(i+1)} \geq p_v^n$, that gets projected back to $q^i = p_v^n$. Thus, since repeated values of $q$ are received, it will enter the oscillatory mode (and eventually converge to $p_v^n$).

Lemma 3 (Boundedness of distance from the equilibrium) Assume $q^* < p_v^n$ and suppose the system is in the oscillatory mode at iteration $i$. Then, $|q^i - q^*| < \gamma^{-1}\delta^{(i)}$.

Proof: Let $q^{(i-1)}$ denote the offer from the $\pi$-agent at the previous iteration. We first prove the case when $q^i \geq q^*$ by contradiction. To this end, assume $q^i - q^* < \gamma^{-1}\delta^{(i)}$. This implies $q^{(i)} - q^* < \gamma^{-1}\delta^{(i)}$ (because $q^{(i)} \geq q^*$ by $q^i = \min(q^{(i)}, p_v^n)$). This in turn implies $q^{(i)} \geq q^*$ by the step-limiting constraint (11c) at the previous iteration (observe $\delta^{(i-1)} \leq \gamma^{-1}\delta^{(i)}$). Then, $q^{(i-1)} \geq q^*$, $q^{(i)} \leq q^{(i-1)} \leq q^{(i-1)}$ by Lemma 1.

For the system to be oscillating with $q^{(i)} \leq q^{(i-1)}$, either $q^{(i)} > q^{(i)}$ (which contradicts $q^{(i)} \geq q^*$ by Lemma 1), or we have equality at $q^{(i-1)} = q^{(i)}$ or $q^{(i)} = q^{(i-1)}$ (which implies $q^* = q^*$ by the unique fixed point, contradicting $q^* - q^* < \gamma^{-1}\delta^{(i)}$).

We show the second case, $q^i < q^*$, directly. We have $q^{(i+1)} > q^*$ by Lemma 1 and $q^{(i)} = q^*$ because $q^i < p_v^n$. Thus, the oscillating mode implies $q^{(i)} \leq q^{(i-1)}$. It holds that $q^{(i)} \geq q^*$: if $q^{(i)} > q^*$, then $q^{(i+1)} = p_v^n \geq q^*$. Alternatively, if $q^{(i)} \leq q^*$, then $q^{(i+1)} = q^{(i)} \geq q^*$ and $q^{(i+1)} \leq q^*$. Thus, $q^{(i-1)} = q^{(i)} \leq q^*$ by Lemma 1. This implies $q^{(i)} \geq q^* - \delta^{(i)}$ by the step-limiting constraint at $i-1$. Since $q^i = q^{(i)}$ and $\delta^{(i-1)} \leq \gamma^{-1}\delta^{(i)}$, this proves the lemma.
there exists $K$ indicating the number of finite iterations, such that $\delta(K) \leq \varepsilon$.

**Proof:** Let $m^{(i)}$ denote the cumulative number of times the system has been in the oscillatory mode at iteration $i$, with $m^{(1)} = 0$ and $m^{(i+1)} = m^{(i)} + q^{(i)}$. Thus $\delta^{(i)} = \delta^{(0)} \gamma^{m^{(i)}}$. Following Lemma 2 for any $m > 0$, if $m^{(i)} = m$, there exists $l > 0$ such that $m^{(i+l)} = m + 1$. Thus, we can make $m^{(i)}$ arbitrarily large with sufficient iterations, and therefore, $\delta^{(i)} = \delta^{(0)} \gamma^{m^{(i)}}$ can be made arbitrarily small. □

**Lemma 5 (Termination)** If the algorithm terminates at iteration $i$ due to the stopping criterion, then $|q^{(i)} - q^{*}| < \varepsilon$.

**Proof:** We will prove the case when $q^{*} < p^{*}_{c}$; First, consider the case when $q^{(i-1)} < q^{*}$ (and hence $q^{(i-1)} = q^{(i-1)}$), then it follows from Lemma 1 that $q^{(i)} \geq q^{(i-1)}$. There are two cases, if (i) $q^{(i)} > q^{*}$, then $q^{(i)} - q^{(i-1)} \leq \delta^{(i-1)}$, and it follows directly that $|q^{(i)} - q^{*}| \leq |q^{(i)} - q^{(i-1)}| \leq \gamma^{i} \varepsilon < \varepsilon$, since the algorithm terminated at iteration $i$. Alternatively, if (ii) $q^{(i)} \leq q^{*}$, we have $q^{(i)} = q^{(i)}$. From (14) and the intersection of $g_{k}$ and $g_{v}$, in the case when $q^{(i)} - q^{(i-1)} = \delta^{(i-1)} \leq \gamma^{i} \varepsilon$. Now, we have two cases: (a) if the system was oscillating, we have that $q^{(i-1)} > q^{*}$ and so $q^{(i-2)} < q^{(i-1)}$ from Lemma 1. It is also true that $\delta^{(i-1)} = \gamma \delta^{(i-2)}$ and that $q^{(i-1)} - q^{(i-2)} \geq \delta^{(i-2)}$, therefore, $|q^{(i)} - q^{(i-1)}| |q^{(i)} - q^{(i-2)}| \leq (1 - \gamma) \delta^{(i-1)} \leq (1 - \gamma) \gamma^{i-1} \varepsilon = (1 - \gamma) \varepsilon < \varepsilon$. Or (b), if the system was not oscillating, this implies that $\delta^{(i-1)} = \gamma \delta^{(i-2)}$, and from the same argument as before satisfying that $|q^{(i)} - q^{(i-1)}| \leq \gamma \varepsilon$. This is a contradiction, since that would also imply that $|q^{(i-1)} - q^{(i-2)}| \leq \delta^{(i-2)} = \delta^{(i-1)} \leq \gamma^{i} \varepsilon$, hence terminating before $i$. Second, for the case when $q^{(i)} > q^{*}$, the proof is equivalent to the first case, but considering the special instance that if $q^{(i-1)} > p^{*}_{c}$, then $q^{(i-1)} = p^{*}_{c}$, but is still larger than $q^{*}$ so the same idea holds by invoking Lemma 1.

Finally, we address the case when $q^{(i)} = p^{*}_{c}$. As described in Lemma 2, the algorithm will get stuck at $q^{(i)} = p^{*}_{c}$ for more than two iterations, terminating the algorithm. Since $p^{*}_{c}$ is $\varepsilon$ of the centralised optimum $q^{*}$, then $|q^{(i)} - q^{(i-1)}| = |q^{(i)} - q^{*}| = 0 < \varepsilon$. □

**Theorem 6 (Optimality of Algorithm 1)** For 2 agents with strictly concave utility functions, $T = 1$, and with sufficiently large max iterations $M$, Algorithm 1 returns a quantity within $\varepsilon$ of the centralised optimum $q^{*}$.

**Proof:** By Lemma 4, if we set $M \geq K$, then the algorithm will terminate due to the stopping criterion in at most $K$ iterations, and by Lemma 5 the quantity is within $\varepsilon$ of $q^{*}$. □

In Fig. 3 we depict a case that converges to the centralized solution via the step-limiting constraint. This case otherwise diverges based on the classic result [30] without the step-limiting constraint. This theoretical analysis provides the foundation for extending the algorithm to multiple agents $|C| > 2$ with finite time horizon $T > 1$. We explore the behavior of the algorithm numerically for such cases in the next section.

**IV. COMPUTATIONAL EXPERIMENTS AND SIMULATIONS**

To provide additional insight into the algorithm performance, we perform two simulation-based computational experiments following the methodology and nomenclature in [31]. The simulation flowchart for both experiments is summarized in Fig. 4. The first examines how the two algorithm parameters $\gamma$, $\delta^{(0)}$ affect the rate of convergence. The second, tests convergence for the unproven cases for $|C| > 2$ and $T > 1$, and studies the effect of battery energy and power capacity on convergence and explores welfare differences between the centralized and P2P approaches. In all experiments we use hourly load and PV profiles from Pecan Street [29], and constant pricing utility functions fit to the baseline load with elasticities random on $[-1.5, -0.5]$ as in section II-C.

**A. Effect of parameters $\gamma$ and $\delta^{(0)}$ on convergence**

In this experiment, we study the convergence rate for the 2-agent, single period case. We systematically vary $\gamma \in \{0.05, 0.1, \ldots, 0.95\}$, $\delta^{(0)} \in \{0.1, 0.2, \ldots, 2\}$ kWh as independent variables, generating 380 unique pairs of $(\gamma, \delta^{(0)})$. For each pair, we execute 100 trials with randomly generated confounding variables (the two load profiles, hour of the year, price elasticities, and solar power between zero and twice the load) and compute the iterations to convergence. We use a stopping tolerance $\varepsilon = 10^{-3}$ for all trials.

The results in Fig. 5 show that $\gamma$ has a strong effect on the convergence rate and exhibits a minimum for $\gamma \in [0.3, 0.5]$ that is consistent across the different ranges of $\delta^{(0)}$, and that the algorithm converges in on the order of 10-20 iterations on average for $\gamma$ in the middle range. We found that $\delta^{(0)}$ was not very significant in influencing the number of iterations except for causing an increase at especially small values, suggesting the parameter ought to be set to a relatively large value. A possible intuition behind the effect of $\gamma$ is that especially small values shrink the box too quickly away from the equilibrium, while large values do not shrink rapidly enough.

**B. Performance for unproven cases**

To study the performance in the general (multi-agent) case, we vary the total battery capacity $S_{\text{tot}} \in \{15, 25, 40, 80, 300\}$ kWh and the maximum rate of charge/discharge of the battery $P_{b} \in \{1, 2, 4, 8\}$ kW as independent variables, yielding 20 distinct pairs. Similar to section IV-A for each pair $(S_{\text{tot}}, P_{b})$ we execute 60 trials ($60 \times 20 = 1200$ simulations), randomly selecting PV and load profiles, price elasticities, an hour of the year, $T \in \{1, 12, 24\}$ hours, and number of agents $N \in \{2, 10\}$. A battery capacity fraction is assigned uniformly to each agent (and then normalized) from the total battery capacity. The PV profiles are scaled so the total PV energy equals the total baseline load energy, and $(\gamma, \delta^{(0)}) = (0.5, 0.5)$.

1) **Convergence performance:** All of the 1200 treatments converge to a solution. The average iterations required to convergence is 112.5, with a standard deviation of 257.3 and a median of 61. We observe that larger time horizons with more agents require more iterations for the algorithm to converge.

4The experiment parameters, data files, and MATLAB code to reproduce the experiments can all be found at https://github.com/Energy-MAC/TSG-P2P-Pricing.
Experimental Data:
PV, Loads, Storage, Prices, Elasticities, etc.

Select Trial Data
Randomly choose confounding variables

Test Set 1
... For each trial, generate a test set for each combination of independent variables.
... Test Set n

Simulation
Centralized Model
+ P2P Algorithm

Simulation Output 1
... Simulation Output n

Metrics for the Trial
• Convergence
• # of iterations
• Differences: welfare and price

2) Effect of battery parameters: The effect of battery capacity on convergence is illustrated via boxplots in Fig. 6 depicting the distribution of the number of iterations for convergence against battery capacity (with outliers omitted). In general, a higher battery capacity requires more iterations to converge. The intuition being that with higher battery availability, the flexibility for each agent to adapt to successive trades increases, thus requiring more iterations. This highlights the importance of storage in a P2P setting and the effect on the implementation of energy trading algorithms. In contrast, the maximum charge/discharge rate of the battery does not significantly affect the number of iterations. This is expected, because given the demand profiles, a maximum rate of 1 kW is usually enough to achieve a trade.

![Fig. 6: Number of iterations to converge with varying battery capacity.](image)

3) Welfare comparison: In order to compare the total welfare of all agents for the centralized and the iterative P2P algorithm, we classify the trials by grouping the time horizon. The statistics of welfare difference percentages $\Delta W_p$ and absolute welfare differences $\Delta W$ are presented in Table I. We note that most of the entries for $\Delta W_p$ are lower than 0.1%, i.e., in the range of numerical tolerance used for MATLAB based optimizers. These results indicate that in most cases the centralized welfare is close to that of the proposed algorithm. However, there exist cases when $T > 1$, for which although the algorithm converges, the welfare is significantly different from the centralized solution.

4) Special instance: In this section we explore one instance where there is a considerable mismatch ($\Delta W = \$30.71$) between the welfare values obtained from the two approaches. This occurs for $T = 24, N = 6$, and low total battery capacity of $S_{tot} = 15$ kWh. The key difference is that the prices for the agents in the algorithm are significantly different than those obtained in the centralized solution, as observed in Fig. 7.

This simulation converges in 59 iterations, when agent-1 (ag-1) exits the algorithm. However, at iteration 32, agent-2 exits based on its stopping criteria, while the remaining agents continue trading, before exiting at iterations 59, 58, 58, and 55 respectively with similar price profiles, as indicated in Fig. 7. The consumption profiles and hence the individual welfare of

![Fig. 5: Effect of $\gamma$ on the number of iterations to convergence. The solid lines show the mean over all trials where $\delta^{(0)}$ lies in the interval specified in the legend. The dashed lines show the maximum.](image)

![Fig. 4: Trial procedure flowchart.](image)

TABLE I: Welfare difference statistics for different time horizons.

| $T$ | 1   | 12  | 24  |
|-----|-----|-----|-----|
| #Simulations | 300 | 420 | 476 |
| $\Delta W_p$ | Mean [%] | 0.023 | 0.001 | 0.072 |
| Std [%] | 0.079 | 0.004 | 0.706 |
| Mean [%] | 0.558 | 0.034 | 7.758 |
| $\Delta W$ | Mean [$] | 0.004 | 0.002 | 0.319 |
| Std [$] | 0.015 | 0.007 | 3.338 |
| Max [$] | 0.086 | 0.057 | 36.717 |

TABLE II: Welfare difference statistics for the special instance considered in Section IV-B.4.

| $W_{so}$ [$] | $W_{cen}$ [$] | $W_{lep}$ [$] | $\Delta W$ [$] | $\Delta W_p$ [%] |
|-------------|---------------|---------------|----------------|----------------|
| ag-1        | 11.923        | 19.804        | 14.292         | 5.512          | 27.833         |
| ag-2        | 6.617         | 16.785        | 9.079          | 7.706          | 45.910         |
| ag-3        | 2.784         | 2.933         | 3.516          | -0.583         | -19.877        |
| ag-4        | 202.124       | 202.906       | 202.920        | -0.014         | -0.007         |
| ag-5        | 164.184       | 229.159       | 203.345        | 25.814         | 11.265         |
| $\pi$-ag    | 1.633         | 1.711         | 3.429          | -1.718         | -100.409       |
| Total       | 389.265       | 473.298       | 436.581        | 36.717         | 7.758          |
Fig. 7: Centralized and P2P algorithm price profiles for the special instance of considerable difference in welfare.

each agent are thus significantly different from the centralized solution. Table II summarizes the total welfare (consumption + trading) of each agent using the centralized and P2P algorithm. The welfare for the no-trading case $W_{no}$, is also presented for comparison. A closer inspection reveals that while that agent-3 and the $\pi$-agent are better off in the P2P case, agents 1, 2, and 5 are well placed in the centralized case. Furthermore, for this particular simulation, exiting earlier is not optimal for agent-2, although the price is lower than the other $q$-agents.

Summary: The simulation results in this section highlight the main contributions of this work:

1. The P2P algorithm achieves similar welfare results as the centralized approach in most of the cases (Table I), with the caveat that an early exit by some agents may introduce sub-optimality, in which case different agents end up as winners and losers relative to the social optimum (Table I), but all agents are better off than no trading.

2. More flexibility for the agents via larger storage or longer time horizons increases the number of iterations (Fig. 6).

3. The expected number of iterations is minimized by setting the shrinking rate of the step-size $\gamma$ around 0.4 (Fig. 5).

4. Real-time prices in a zero marginal-cost system arise from the marginal utility of consumption under scarcity.

V. CONCLUSIONS

In this paper, we address the question of optimal pricing and mechanisms for achieving optimal dispatch in microgrids with scarce, zero marginal-cost energy resources. We contribute a novel analysis of a centralized economic dispatch with welfare maximization that uses a Lagrangian dual decomposition to state the equilibrium optimal price conditions and show a previously unstated result that although optimal prices can induce unique and optimal consumption profiles and generator output, they do not yield unique or power-balanced battery dispatch decisions except in particular circumstances. Next, we propose a P2P algorithm where agents keep utility functions private and iteratively interact by exchanging price and quantity offers to arrive at mutually agreeable and weakly Pareto-optimal trades. We theoretically prove this outcome converges to the social optimum within a specified tolerance for the 2-agent case, and show via numerical experiments that the P2P algorithm converges in the multi-agent case, but we do not derive specific bounds. Although we find that the P2P algorithm obtains total welfare on average within 0.1% of the centralized solution for a wide range of parameters, significant differences in welfare and allocation can arise for longer time horizons and larger numbers of agents. We also find from simulations that the number of iterations for the P2P algorithm to converge increases with the total storage capacity, and that the P2P algorithm shrinking parameter $\gamma$ impacts the number of iterations, and should be set to the neighborhood of 0.3-0.5 to minimize iterations in contexts similar to our simulations, while the initial maximum step-size $\delta^{(0)}$ is not significant.

The proposed P2P algorithm was designed to resemble an informal decentralized trading process where prices arise from the value of electricity consumption under scarcity. We envision it is feasible to implement such an interaction in practice via a software platform that defines the rules and aids in the iteration, or even with informal negotiation between neighbors in a community. However, there are several limitations that need to be addressed for this approach to be useful in practice. First, we do not study the impact of strategic gaming between agents, which could be significant in small markets, nor the equity of outcomes. Conducting this analysis likely requires removing the assumption that $\pi$-agents offer prices equal to their dual variables and considering their profit maximizing strategy, given expectations of $q$-agents’ demand curves. Second, our analysis only considers strictly concave utility functions. This is a common assumption, but may not capture the discrete nature of decisions to use particular loads at small time-scales. We expect it will be difficult for researchers to derive useful theoretical insights with non-concave utility functions, but the construction of realistic utility functions and consumption decision models for use in simulation would be of tremendous value to this and related work. Third, we do not include network constraints or validate power flow. While it is relatively straightforward to validate whether a particular negotiated dispatch is feasible given a network model, the impact of binding constraints on pricing and negotiation is non-trivial and warrants further study. Fourth, a system for matching agents into smaller negotiation pools based on expected outcomes may be necessary to handle large numbers of agents, e.g., hundreds. Here, a challenge is to design suitable exit strategies for satisfied agents without compromising the inviolability of agreements, and should also account for network constraints in creating market power (see [17]). This introduces significant complexity, where methods to certify optimality or bound the outcome are important theoretical directions for future work. Lastly, we suggest the inclusion of uncertainty via scenarios in a stochastic programming framework to deal with uncertainty in solar forecasts and load estimation. The inclusion of power flow feasibility and network validation in the P2P algorithm, and extensions to a broader class of DERs are next steps in this research.

APPENDIX A

NON-IDEAL BATTERY MODELING

In this appendix, we extend the analysis in Section II to a more realistic model of the battery and show that the results hold when battery inefficiencies and asymmetric charge and discharge curves. Second, our analysis only considers strictly concave utility functions, but the construction of realistic utility functions and consumption decision models for use in simulation would be of tremendous value to this and related work. Third, we do not include network constraints or validate power flow. While it is relatively straightforward to validate whether a particular negotiated dispatch is feasible given a network model, the impact of binding constraints on pricing and negotiation is non-trivial and warrants further study. Fourth, a system for matching agents into smaller negotiation pools based on expected outcomes may be necessary to handle large numbers of agents, e.g., hundreds. Here, a challenge is to design suitable exit strategies for satisfied agents without compromising the inviolability of agreements, and should also account for network constraints in creating market power (see [17]). This introduces significant complexity, where methods to certify optimality or bound the outcome are important theoretical directions for future work. Lastly, we suggest the inclusion of uncertainty via scenarios in a stochastic programming framework to deal with uncertainty in solar forecasts and load estimation. The inclusion of power flow feasibility and network validation in the P2P algorithm, and extensions to a broader class of DERs are next steps in this research.
discharge power constraints are accounted for. We emphasize that we assume a linear model and the results may not hold for nonlinear models. However, agents may use decision models of varying complexity in practice, so understanding the implications of simplified models remains highly relevant.

The extended model replaces the net battery discharge $p_{i,t}^b$ with its positive discharge and charge components $p_{i,t}^{b,+}$ and $p_{i,t}^{b,-}$. We allow different discharge and charge power constraints $P_{i,t}^{b,+} > 0$ and $P_{i,t}^{b,-} > 0$ and assume that power is converted to and from the stored energy with charge efficiency $\sigma_+ \in (0, 1]$ and discharge efficiency $\sigma_- \in (0, 1]$, and that a battery self-discharges at a rate $(1-\theta_t)$ proportional to the state-of-charge, with $\theta_t \in [0, 1]$. Thus, with the extended model, the problem formulation can be stated as:

$$\min_{p,d,s} \sum_{t \in T} \sum_{n \in C} U_{n,t}(d_{n,t})$$

subject to

$$\pi_t: \sum_{n \in C} d_{n,t} = \pi_{i,t}^{b,+} - \pi_{i,t}^{b,-} + \sum_{g \in G} p_{g,t}^{b,\pi}, \forall t \in T$$

$$\lambda_{i,t}^{a,-} : 0 \leq \pi_{i,t}^{a,-} - \bar{P}_{i,t}^{a,-} \geq 0, \forall g \in G, \forall t \in T$$

$$\lambda_{i,t}^{d,-} : -d_{i,t} \leq 0, \forall n \in C, \forall t \in T$$

$$\lambda_{i,t}^{b,-} : 0 \leq \pi_{i,t}^{b,-} - \bar{P}_{i,t}^{b,-}, \forall i \in B, \forall t \in T$$

$$\pi_{i,t}^{b,+}, \pi_{i,t}^{b,-} = 0, \forall i \in B, \forall t \in T$$

$$\lambda_{i,t}^{c,-} : 0 \leq s_{i,t} \leq \bar{s}_{i,t}, \forall i \in B, \forall t \in T$$

$$s_{i,t} = \theta_{i,t} s_{i,t-1} + \sigma^{a,-} \pi_{i,t}^{b,-} - (\sigma_-)^{-1} \pi_{i,t}^{b,+} \Delta T, \forall i \in B, \forall t \in T.$$

The analogous form of Remark 2 follows from the stationarity conditions of (15), and is

$$\pi_t^\star = \partial U_{n,t}(d_{n,t}) / \partial d_{n,t} + \lambda_{n,t}^{d,-}, \forall n \in C$$

$$= \lambda_{n,t}^{b,+} - \Delta T (\sigma_+)^{-1} \sum_{T \geq t} \theta^{T-t} \lambda_{i,T}^{c,-}, \forall i \in B$$

$$- \lambda_{n,t}^{b,-} - \Delta T \sigma_- \sum_{T \geq t} \theta^{T-t} \lambda_{i,T}^{c,-}, \forall i \in B$$

$$= \lambda_{n,t}^{b,+}, \forall g \in G.$$

Remark 4 again follows from Remark 2, and takes the form

$$\theta \pi_{i+1}^{a,+} - \theta \pi_{i+1}^{a,-} = \theta \lambda_{i+1}^{b,+} - \lambda_{i+1}^{b,+} + \Delta T (\sigma_+)^{-1} \lambda_{i+1}^{c,-}$$

$$= \lambda_{i+1}^{b,+} - \theta \lambda_{i+1}^{b,+} + \Delta T \sigma_- \lambda_{i+1}^{c,-}.$$ 

These dynamics imply that the equilibrium optimal price, and equivalently the marginal value of consumption at optimum, will evolve depending on the battery inefficiencies and whether battery constraints are active at the optimum.

For Remark 5, consider again the example from the main text with $T = 5, \Delta T = 1, \bar{P}_{i,t}^b = 3, \bar{s}_{i,t} = 10, s_{i,t} = 5$. Take $\sigma_+ = 0.95, \sigma_- = 0.9, \theta_1 = 0.98, P_{i,t}^{b,+} = 2$, and a modified price $p^\star = [1, 1.0204, 2.3, 1.2680]$. Analogous to the example in the main text, solutions $\bar{p}^b = [-0.9587, -0.9587, 3.3, 3.0], P^b = [0, -1.8983, 3.3, 3.0], p^b = [-1.5863, -1.5863, 3.3, 3.1]$ are all optimal solutions with net benefit $-13.063$.

In this example, $\pi_2$ was constructed by noting that if an optimal dispatch has the battery charging and unconstrained in power and stored energy at both $t=1$ and $t=2$, then $\pi_2^{a,+} = \theta^{-1} \pi_1$, i.e., the equilibrium price trajectory must have these dynamics when the storage is charging between successive periods and is not constrained. It then follows by the same principle as described in the main text that charge power can be feasibly shifted from one period to another without affecting the cost, thus the storage dispatch is not unique. Here $\pi_5$ was chosen as $\pi_5 = (\sigma^+ \sigma_-)^{-1} \theta^{-4}$ to show equilibrium conditions where a higher future price exactly compensates for the lost energy from charging. Additional discussion about the impact of storage inefficiencies on optimal pricing can be found in [32].
APPENDIX B
CONSTANT PRICE-ELASTICITY UTILITY FUNCTIONS

Here we describe the procedure used for developing sample utility functions from data, assuming constant price-elasticities, which is one of two common simple assumptions in pricing theory (the other being a linear demand curve / quadratic utility function). We emphasize that this was chosen for example purposes only, and that all of the analysis only assumes that utility functions are strictly concave, and would apply to logarithmic or quadratic utility functions as well.

Let the marginal utility of consumption be denoted by $g(d) = \partial U(d)/\partial d$. As described in the main text, the equilibrium price is equal to the marginal utility of consumption, i.e., $\pi^* = g(d^*)$. Then the demand function of price is given by $h$ (as the inverse of $g$), where $d = h(\pi) := g^{-1}(\pi)$. The price-elasticity, which we define as $r(\pi)$, is defined as the ratio of the percentage change in quantity to the percentage change in price, and in general depends on the price

$$r(\pi) = \frac{\pi h'(\pi) h(\pi)}{d \pi^2}.$$  \hspace{1cm} (19)

A constant price-elasticity implies $r(\pi) \equiv \hat{r}$. The general family of demand functions with this property has the form $h(\pi) = a \pi^{\hat{r}}$ for some constant $a$. This can be fit to an empirical price and consumption pair $(\pi_0, d_0)$ by setting $h(\pi_0) = d_0$ and obtaining $a = d_0 \pi_0^{-\hat{r}}$. Inverting this to marginal utility and integrating to utility, one obtains

$$g(d) = \pi_0 \left( \frac{d}{d_0} \right)^{\frac{1}{\hat{r} + 1}},$$ \hspace{1cm} (20a)

$$U(d) = \frac{\hat{r} \pi_0 d^{\hat{r} + 1}}{(\hat{r} + 1) d_0},$$ \hspace{1cm} (20b)

However, general downward-sloping demand curves imply $r < 0$, thus $\lim_{d \to 0^+} g(d) = \infty$ and $\lim_{d \to 0^+} U(d) = -\infty$ which can be problematic for optimization solvers and is also an unrealistic extreme in practice. Thus, we modify the function to have “quasi-constant” price elasticity, by shifting the marginal utility curve to the left by a small $\delta > 0$ and compensating the exponent for the shift so that $r(\pi_0) = \hat{r}$. We also choose a $c$ such that $U(0) = 0$. The resulting marginal utility and utility functions are

$$g(d) = \pi_0 \left( \frac{d + \delta}{d_0 + \delta} \right)^{\frac{1}{\hat{r} + 1}},$$ \hspace{1cm} (21a)

$$U(d) = \frac{\hat{r} \pi_0 ((d + \delta)^{\frac{1}{\hat{r} + 1}} - \delta^{\frac{1}{\hat{r} + 1}})}{(\hat{r} + 1)(d_0 + \delta)^{\frac{1}{\hat{r} + 1}}},$$ \hspace{1cm} (21b)

$$r' = \hat{r} \left( 1 + \frac{\delta}{d_0} \right)^{-1}.$$ \hspace{1cm} (21c)

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