Collision damping in the $\pi^3He \to d'N$ reaction near the threshold

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We present a simple quantum mechanical model exploiting the optical potential approach for the description of collision damping in the reaction $\pi^3He \to d'N$ near the threshold, which recently has been measured at TRIUMF. The influence of the open $d'N \to NNN$ channel is taken into account. It leads to a suppression factor of about ten in the $d'$ survival probability. Applications of the method to other reactions are outlined.

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I. INTRODUCTION

In this paper we consider inelastic final state interaction (FSI) between the neutron and the hypothetical $d'$ resonance with the quantum numbers $T = 0, J^P = 0^−$ produced near threshold in the reaction $\pi^3He \to d'N$. This resonance has been suggested to explain the observed peculiar resonance-like behaviour \cite{3} of the pionic double charge exchange (DCX) on nuclei, $\pi^+ + A \to A' + \pi^−$, ranging from $^7Li$ to $^{93}Nb$ at $T_\pi \approx 50MeV$ (see \cite{1, 2, 3} and references therein). Such a $NN$-decoupled dibaryon had been predicted by QCD-inspired
models \[4, 5\]. In a sequence of recent papers this behaviour for the DCX reaction, on \(Ca\) isotopes \(^{42,44,48}Ca\), was qualitatively reproduced by Nuseirat et al. \[7\] and, for the medium and heavy nuclei, in the generalized seniority model, by Gibbs and Wu \[8\]. Still there is no conventional model, which could demonstrate to reproduce these low-energy phenomena consistently for all nuclei. An independent test of the idea would be investigation of the \(pp\pi^−\) invariant mass spectrum in the double pion production, \(pp \to pp\pi^−\pi^+\), since the quantum numbers of the resonance forbid strong coupling to \(np\). Recent measurements at CELSIUS \[9\] do not show an evidence of the resonance at the predicted level in contrast to data from ITEP taken at a higher proton energy \[10\]. Thus the situation appears not yet fully settled. Another way of searching for the \(d'\) resonance is DCX on Helium isotopes, where \(d'\) should manifest itself as a threshold phenomenon. At energies around 100\(MeV\) the total cross section of the reaction \(^4He(\pi^+,\pi^-)\) shows an excess over conventional calculations which could be ascribed to the contribution of the \(d'\) resonance \[11\]. At the same time, the contribution of this resonance for both, \(^4He(\pi^+,\pi^-)\) \[11\] and \(^3He(\pi^−,\pi^+)\) \[12\], occurred almost an order of magnitude smaller than expected. The reason for the suppression of the production cross sections near threshold can be due to the collision damping \(d'N \to NNN\), and in the present paper we study this hypothesis using a simple quantum mechanical model to describe the propagation of an unstable resonance in the nuclear medium. Although singularities near production threshold always attracted much attention and are described in many textbooks (see, \(e.g., [13, 14]\) and references therein), we are not aware of similar approaches to the final state interaction suggested before.

The paper is organized as follows. In the second section we present a simple quantum mechanical model exploiting the optical potential approach to account for the effect of collision damping. In the third section we estimate the spreading width of the \(d'\) in the nuclear medium, namely, the virtual pion exchange contribution to the spreading is calculated in detail. In the next, fourth, section we give our numerical estimates of collision damping and of the \(d'\) survival probability in the nuclear medium, and compare the latter to the experimental data. The fifth section contains our conclusions and outlook.
II. A SIMPLE QUANTUM MECHANICAL MODEL

In this section we present a simple quantum mechanical model to describe the propagation of a resonance in the nuclear medium. As an example, we consider the behaviour of the hypothetical resonance $d'$ which can contribute to the pionic DCX on $^3\text{He}$.

Due to a very short interaction time of the pion with the Helium nucleus it is the sudden influence to be the appropriate mechanism for considering the $\pi^3\text{He} \rightarrow d'N$ reaction [13]. Thus the $d'N$ system is created at the initial moment, $t = 0$, with the wave function of Helium $\psi_{\text{He}}$ and then separates with time as

$$\psi_{d'N}(\mathbf{r}, t) = \psi(\mathbf{r} - v t, 0) = \psi_{\text{He}}(\mathbf{r} - v t),$$

with the velocity $v$ defined by the energy excess over the threshold $\varepsilon$,

$$v = \sqrt{\frac{2\varepsilon}{\mu}},$$

where $\mu \approx \frac{3}{2}m_N$ is the reduced mass of the $d'N$ system.

If the potential in the effective Schrödinger equation for the wave function (1) contains an imaginary part,

$$U(\mathbf{r}) = U_0(\mathbf{r}) + iU_1(\mathbf{r}), \quad U_1(\mathbf{r}) < 0,$$

then the probability to find the system in its initial state, $d'N$, decreases with time approaching a finite limit with the survival probability being simply

$$w_{\text{surv}} = \frac{|\psi_{d'N}(t \to \infty)|^2}{|\psi_{d'N}(t \to \infty)|^2_{U_1=0}}.$$

Let us, for simplicity, consider the step-like form of the potential $U_1(\mathbf{r})$,

$$U_1(\mathbf{r}) = \begin{cases} 
U_1 = \text{const}, & |\mathbf{r}| \leq R_1 \\
0, & |\mathbf{r}| > R_1,
\end{cases}$$

where the potential strength $U_1$ can be related to the elastic s-wave $d'N$ zero-angle amplitude using Born approximation:

$$\text{Im}F(0) \approx -\frac{\mu}{2\pi} \int U_1(\mathbf{r}) d^3r = -\frac{2}{3} \mu R_1^3 U_1,$$

whereas, according to the optical theorem,

$$\text{Im}F(0) = \frac{\mu v}{4\pi} \sigma_{\text{tot}} \approx \frac{\mu v}{4\pi} \sigma_{\text{in}}.$$
The problem of separation of the elastic and inelastic parts of the cross section in such kind of reactions is a very subtle question. One of the relevant worries concerns the possible broadening of the ground state, so that a slight change in the resonance position could have been erroneously arcsibed to the inelastic part. Bearing in mind that, in general case, such an effect could lead to an overestimate of the damping in Eq. (3), we still do not expect it to change our estimates dramatically. Indeed, from a posteriori estimate (see Eq. (44) below) we find the elastic part of the cross section to be an order of magnitude smaller than the corresponding inelastic part, and thus it appears beyond the accuracy of the present paper.

From Eqs. (3), (4) one finds

$$U_1 = -\frac{3}{8\pi} \frac{\sigma_{in} v}{R^3_1}.$$  

(7)

Notice that we do not meet any constraints on the wave length of the resonance since, in Born approximation, the zero-angle amplitude (3) does not depend on the energy of the $d'$. As a direct consequence of the Schrödinger equation one has the probability conservation law at any moment of time in the form:

$$\frac{d}{dt} \int_V \rho(r, t)d^3r + \int_S j(r, t)ds = 2 \int_V U_1(r)\rho(r, t)d^3r,$$

(8)

where

$$\rho(r, t) = |\psi_{d'N}(r, t)|^2, \quad j(r, t) = \frac{i}{2\mu}\psi_{d'N}(r, t)\nabla\psi^*_{d'N}(r, t) + c.c.$$

(9)

are the probability density and the probability flux through the corresponding surface $S$, respectively. The term on the r.h.s. of Eq. (8) describes absorption of particles. In the absence of this term the density of particles would evolve in space and time, according to Eq. (1), as

$$\rho_0(r, t) = |\psi_{d'N}(r - vt, 0)|^2 = |\psi_{He}(r - vt)|^2 = \rho_0(r - vt, 0),$$

(10)

if quantum spreading of the initial wave packet is neglected.

Eq. (8) becomes especially simple if the size of the wave packet, hereinafter called $R_2$, is small compared to the radius of the optical potential, $R_1$. Indeed, one can split the entire evolution time into two periods, $0 \leq t \leq T$ and $t > T$, where $T = R_1/v$ is the moment of time when $d'$ and $N$ leave the region of interaction. Then the solution to Eq. (8) reads

$$\rho(r, t) = \begin{cases} \rho_0(r, t)e^{-\Gamma t}, & t \leq T \\ \rho_0(r, t)e^{-\Gamma T}, & t > T, \end{cases}$$

(11)
with
\[ \Gamma = 2|U_1|. \] (12)

Hence the survival probability is
\[ w_{\text{surv}} = \exp \left( \frac{2U_1 R_1}{v} \right) = \exp \left( -\frac{3\sigma_{in}}{4\pi R_1^2} \right), \] (13)
where we also used the relation (7).

The opposite limit, \( R_2 \ll R_1 \), leads to the expression similar to (13) with \( R_1 \) interchanged with \( R_2 \):
\[ w_{\text{surv}} = \exp \left( -\frac{3\sigma_{in}}{4\pi R_2^2} \right). \] (14)

To see this let us consider the following ansatz for the probability density:
\[ \rho(r, t) = \rho_0(r, t)w(t), \] (15)
so that the survival probability defined above is just \( w_{\text{surv}} = w(\infty) \). After integration over an infinitely large volume \( V \) (bounded by an infinitely remote surface \( S \)) in Eq. (8) one finds the following equation for the function \( w(t) \):
\[ \frac{d}{dt}w(t) = -w(t) \int_V 2|U_1(r)|\rho_0(r - vt, 0)d^3r, \quad w(0) = 1, \] (16)
which can be easily solved with the result
\[ w(t) = \exp \left( -\int_0^t dt' \int_V 2|U_1(r)|\rho_0(r - vt', 0)d^3r \right). \] (17)

We further simplify our estimates and consider the step-like form for both functions, \( U_1(r) \) and \( \rho_0(r, t) \):
\[ U_1(r) = -\frac{\sigma_{in} v}{2V_1}\Theta(R_1 - |r|), \quad \rho_0(r, t) = \frac{1}{V_2}\Theta(R_2 - |r - vt|), \quad V_{1,2} = \frac{4}{3}\pi R_{1,2}^3. \] (18)

For the two limiting cases, \( R_1 \gg R_2 \) and \( R_1 \ll R_2 \), the integral \( \int 2|U_1|\rho_0 d^3r \) entering Eq. (16) equals
\[ \frac{\sigma_{in} v}{V_1 V_2} \int \Theta(R_1 - |r|)\Theta(R_2 - |r - vt'|)d^3r = \frac{\sigma_{in} v}{V_{\text{big}}}\Theta(R_{\text{big}} - vt'), \] (19)
where
\[ R_{\text{big}} = \max \{ R_1, R_2 \}, \quad V_{\text{big}} = \max \{ V_1, V_2 \}, \] (20)
and we have neglected the edge phenomena.
FIG. 1: The diagram for the virtual pion exchange contribution to the spreading width of the $d'$ resonance in the nuclear medium.

Now the integral over $t'$ in Eq. (17) can be done trivially that gives for the survival probability:

$$w_{\text{surv}} = \exp \left( -\sigma_{\text{in}} R_{\text{big}} \right),$$  \hspace{1cm} (21)

which coincides with Eqs. (13), (14).

In general case, $R_1 \sim R_2$, the survival probability can be presented as

$$w_{\text{surv}} = \exp \left( -\frac{\sigma_{\text{in}} \tilde{R}}{\tilde{V}} F(R_1/R_2) \right), \hspace{1cm} \tilde{R} = \sqrt{R_1^2 + R_2^2}, \hspace{1cm} \tilde{V} = \frac{4}{3} \pi \tilde{R}^3,$$

with $F$ being a smooth function of $R_1/R_2$, $F(0) = F(\infty) = 1$.

III. VIRTUAL PION EXCHANGE CONTRIBUTION TO THE SPREADING WIDTH OF THE $d'$ RESONANCE IN NUCLEAR MEDIUM

In this section we estimate the spreading width of the $d'$ resonance in the nuclear medium. Let us start from the virtual pion contribution described by the diagram in Fig. 1.

As shown in Ref. [16], at low energies there is only one Lorenz invariant structure describing the $d'NN\pi$ vertex and, therefore, the amplitude of the decay $d' \rightarrow NN\pi$ can be written as

$$\mathcal{M}_{d' \rightarrow NN\pi} = \frac{f}{2m_N} \bar{u}_1 C \gamma_5 (i \tau_2 \vec{\tau}) u_2^T \vec{\pi},$$ \hspace{1cm} (23)

where $u_{1,2}$ are bispinors and $C = \gamma_2 \gamma_0$. The coupling constant $f$ can be expressed through
the $NN\pi$ decay width $\Gamma = \Gamma_{pp\pi^-} + \Gamma_{nn\pi^+} + \Gamma_{np\pi^0}$ as

$$f \approx \sqrt{\frac{128\pi^2\sqrt{2M}\Gamma}{3\eta_0(M - 2m_N - m_\pi)^2}} \sqrt{\frac{m_N}{m_\pi}},$$ \hspace{1cm} (24)$$

where $m_N$, $m_\pi$ and $M$ are the nucleon, pion and $d'$ masses, respectively; $\eta_0$ being the enhancement factor due to the $NN$ FSI in this decay, $\eta_0 \approx 4 \div 5$. For $\Gamma \approx 0.5 MeV$ (as deduced from the data on the DCX reactions to discrete levels \[1\]) $f \approx 14$.

The invariant matrix element of the process $d'N \rightarrow NNN$, when all initial and outgoing particles are on mass shell, can be written in the form

$$\mathcal{M}^{(0)} = \sum_{ijk} \varepsilon_{ijk} \mathcal{M}_{ijk}^{(0)} = \frac{fg}{4m_N} \sum_{ijk} \bar{u}_i C_\gamma_5 (\tau_2 \bar{\tau}) \bar{u}^T_j \frac{\varepsilon_{ijk}}{(P - p_k)^2 - m_\pi^2} \bar{u}_k \gamma_5 \bar{\tau} u, \hspace{1cm} (25)$$

$$\sum_{i=1}^3 p_i^\mu = P^\mu + Q^\mu,$$

where $g$ is the pseudoscalar $NN\pi$ coupling, $g^2/4\pi \approx 14.3$, $\varepsilon_{ijk}$ is the totally anti-symmetrical tensor, and indices $i, j, k = 1, 2, 3$ numerate the outgoing nucleons. As a result, the amplitude $\mathcal{M}^{(0)}$ is totally antisymmetric under permutation of the final nucleons. See also Fig. 1 where the notations are explained.

The matrix element $\mathcal{M}^{(0)}$ contains three different contributions, hence, when squared, it produces three different diagonal terms and three cross terms. After integration over the three-particle phase space all diagonal terms equally contribute to the cross section. The same holds true for the cross terms. Hence, it is sufficient to consider only one diagonal term squared (e.g., $\{ijk\} = \{123\}$) and one of the cross terms (e.g., $\{ijk\} \times \{i'j'k'\} = \{123\} \times \{321\}$).

The diagonal term $\{123\}$ squared and averaged/summed over spins and isospins of the initial/final nucleons equals to:

$$\langle |\mathcal{M}_{123}^{(0)}|^2 \rangle = 12 \frac{(fg)^2}{m_N^2} (p_1 p_2 + m_N^2) \frac{(P p_3 - m_N^2)}{[(P - p_3)^2 - m_\pi^2]^2}. \hspace{1cm} (26)$$

Integration over the phase space is essentially simplified if the initial particles are at rest, i.e., $P^\mu = (m_N, 0, 0, 0)$ and $Q^\mu = (M, 0, 0, 0)$, which means the limit $v \to 0$. In this limit $\sigma_{in} = \sigma_{d'N \rightarrow NNN} \propto 1/v$, so that the product $\sigma_{in} v$ remains constant. Thus, in this limit,

$$\langle |\mathcal{M}_{123}^{(0)}|^2 \rangle = 24 \frac{(fg)^2}{m_N} (E_3 - m_N) \frac{(M + m_N)^2 + m_N^2 - 2(M + m_N)E_3}{(m_N^2 - 2m_N^2 + 2m_N E_3)^2}, \hspace{1cm} (27)$$
\[ \langle |M_{123}^{(0)}|^2 \rangle \text{ depends only on the energy of one of the nucleons, } E_3, \text{ which ranges between } m_N \text{ and } \]
\[ E_{\text{max}} = \frac{(M + m_N)^2 - 3m_N^2}{2(M + m_N)} . \tag{28} \]

Let us consider now the FSI between the nucleons in the relative \( s \)-state. Since there are three identical nucleons in the exit channel, only two of them can be in the relative \( s \)-wave with the third one being in the relative \( p \)-wave. It is easy to see that the matrix element \( M_{ijk}^{(0)} \) (nucleon number \( k \) being in the \( NN\pi \) vertex) is proportional to the three-momentum \( |p_k| \), therefore this nucleon is in \( p \)-wave in the rest frame of the initial \( d' \) and \( N \) and, hence, in \( p \)-wave relative to the nucleons \( i \) and \( j \) in the \( d'NN\pi \) vertex (see Fig. 1). At the same time nucleons from the \( d'NN\pi \) vertex are in relative \( s \)-wave \[ 16 \]. It means that only FSI between the nucleons in the \( NN\pi \) vertex is important. It can be taken into account by multiplying \( M_{ijk}^{(0)} \) by \( \psi_q(0) \), where \( \psi_q(r) \) is the continuum \( NN \) wave function \( (q \approx (p_i - p_j)/2) \) containing the \( s \)-wave \( NN \) scattering amplitude, if the \( d'NN\pi \) vertex is assumed to be point-like:
\[ M_{ijk} \approx M_{ijk}^{(0)} \left( 1 + \frac{R^{-1}}{-a_s^{-1} - iq_{ij}} \right) , \tag{29} \]
where \( R \approx 0.8 \text{ fm} \[ 16 \], \( a_s \) is the \( 1S \) scattering length, and \( q_{ij} \) is the three-momentum of either nucleon, \( i \) or \( j \), in their centre-of-mass frame,
\[ q_{ij} = \sqrt{ \frac{1}{4} \left[ (M + m_N)^2 + m_N^2 - 2E_k(M + m_N) \right] - m_N^2 } . \tag{30} \]

The Coulomb effects, which are important for very small invariant masses in the \( pp \)-subsystem, can be neglected in the integrated cross section.

With the FSI taken into account, the diagonal term has to be replaced by
\[ \langle |M_{123}^{(0)}|^2 \rangle \rightarrow \langle |M_{123}|^2 \rangle = \langle |M_{123}^{(0)}|^2 \rangle \left( 1 + \frac{R^{-1}}{-a_s^{-1} - iq_{ij}} \right) . \tag{31} \]

Finally, the summary contribution of all diagonal terms to the differential cross section is
\[ d\sigma_{\text{diag}} = \frac{3}{(2\pi)^3} \frac{1}{3!} \frac{1}{4m_N M v} \langle |M_{123}|^2 \rangle d^3p_1 d^3p_2 d^3p_3 \delta^4(Q + P - p_1 - p_2 - p_3) , \tag{32} \]
where 3! accounts for the three identical particles in the final state.
As already mentioned, in the limit \( v \to 0 \) the integration over the phase space is simplified since there is no angular dependence, and

\[
\int \langle |M_{123}|^2 \rangle \frac{d^3p_1 d^3p_2 d^3p_3}{2E_1 2E_2 2E_3} \delta^4(Q + P - p_1 - p_2 - p_3) = \pi^2 \int \langle |M_{123}|^2 \rangle dE_2 dE_3. \tag{33}
\]

Now let us consider contributions of the cross terms (e.g., \( \{123\} \times \{321\} \)):

\[
- \langle M_{123}^{(0)} M_{321}^{(0)*} + M_{321}^{(0)} M_{123}^{(0)*} \rangle = -3 \left( \frac{fg}{m} \right)^2 \tag{34}
\]

\[
x \left( p_2 p_3 (P - m_N) + (p_1 p_2) (P p_1 - m_N) + m_N^2 (P \sum p_i) - (p_1 p_3) (P p_2 + m_N) - m_4^2 \right). \]

As \( v \to 0 \) this expression depends only on energies of the outgoing nucleons,

\[
- \langle M_{123}^{(0)} M_{321}^{(0)*} + M_{321}^{(0)} M_{123}^{(0)*} \rangle = -3 \left( \frac{fg}{m} \right)^2 \tag{35}
\]

\[
x \left\{ (E_1 - m_N) [E_3 (M + m_N) - m_N^2] + (E_3 - m_N) [(E_3 - m_N) [E_1 (M + m_N) - m_N^2] - M \left[ \frac{1}{2} (M^2 - 4m_N^2) - (M - E_1 - E_3) (M + m_N) \right] \right\} \frac{1}{||(P - p_k) - m_4^2|| (P - p_i)^2 - m_4^2}],
\]

and the FSI can be taken into account in the way described above, so that each \( M_{ijk}^{(0)} \) has to be replaced by \( M_{ijk} \), as in Eq. (29).

The integration over the three-particle phase space is similar to the integration performed in Eq. (33), so that for the total inelastic cross section one finds

\[
\sigma_{in} = \frac{4.5}{v} \text{mb}, \tag{36}
\]

where the contribution of the cross terms is positive and does not exceed 15% when \( v \to 0 \).

The product \( \sigma_{in} v \) is a smooth function of the energy over the threshold, \( \varepsilon \), so that, e.g., for \( \varepsilon = 20\text{MeV} \) (which corresponds to the kinetic energy of the nucleon of about 30\text{MeV} in the \( d' \) rest frame) the contribution of the diagonal terms is only 10% larger than for \( v \to 0 \).

To proceed further we consider the \( d' \) propagation in the nuclear medium, which we assume infinite, for simplicity. Then, similarly to (12), one has for the spreading width:

\[
\Gamma_s = 2 |\text{Im} U_A|. \tag{37}
\]

Here \( U_A \) is the summary effective potential created by nucleons in the nuclear matter,

\[
U_A = \sum_{i=1}^{N_1} U_i \approx \rho_A V_1 (U_0 + iU_1), \tag{38}
\]
where $\rho_A = \frac{1}{2} m_n^3$ is the density of the nuclear matter, $N_1 = \rho_A V_1$ being the number of nucleons in the interaction region of the volume $V_1$. Using Eqs. (7) and (37) one easily finds:

$$\Gamma_s = 2\rho_A V_1 |U_1| = \rho_A \sigma_{in} v,$$

(39)

or, if the Pauli blocking factor, $\eta \approx 2.2$, is taken into account$^1$, then

$$\Gamma_s = \rho_A (\sigma_{in}/\eta) v.$$

(40)

With the help of the result (39) one can estimate the spreading width to be about $7\text{MeV}$. Notice, however, that the pion exchange (the diagram in Fig. 1) contributes only $30 \div 40\%$ of the observed spreading width in nuclei, i.e., there must be an additional mechanism for the reaction $d'N \rightarrow NNN$, the total spreading width being

$$\Gamma_s \approx 10 \div 20\text{MeV}.$$

(41)

Such a mechanism could be $\sigma$-exchange as discussed in Ref. [3].

Now we can check how well the approximation of the infinite nuclear medium works. To this end we have to ensure that the typical free-path length of the $d'$ due to collision damping, $L_{d'} \sim v/\Gamma_s$, does not exceed the radius of the nucleus $R_A \left(\frac{4}{3}\pi R_A^3 \rho_A \equiv N_A\right)$. For DCX to discrete levels this condition is fulfilled since for $\Gamma_s$ given by (41) and $v \sim 1/10$, which corresponds to the energy $\varepsilon \approx 3\text{MeV}$ above the threshold, both lengths appear to be of the same order of magnitude, and $L_{d'} \lesssim R_A$.

Besides we can perform a posteriori check of the validity of the approximations made in Section II. Namely, we can justify Born approximation used in Eq. (5):

$$\mu R_1^2 |U_{0,1}| \approx \frac{\eta}{4\pi} \frac{m_N \Gamma_s}{\rho_A R_1} \approx \frac{1}{3},$$

(42)

and the one used in Eq. (3) when the elastic part of the total cross section was neglected. Indeed, for the scattering off the step-like potential (2) one has

$$\sigma_{el} = \frac{16}{9} \pi \mu^2 R_1^2 (U_0^2 + U_1^2), \quad \sigma_{in} = \frac{8}{3} \pi R_1^3 U_1 \frac{1}{v},$$

(43)

$^1$To estimate Pauli blocking in the reaction $d'N \rightarrow NNN$ we simply limit the phase space for the final nucleons by the constraint $p_N > P_F$, where $P_F$ is the Fermi momentum.

$^2$Let us remind the reader that Born approximation is valid for $|U_{0,1}| \ll 1/(\mu R_1^2)$. 
with the ratio
\[
\frac{\sigma_{el}}{\sigma_{in}} = \frac{2}{3} \pi \frac{\mu^2 R_1^3 U_0^2 + U_1^2}{|U_1|} v \approx \frac{\eta}{9 \pi} \frac{m_N^2 \Gamma_s}{\rho_A} v \approx \frac{1}{10},
\]
where we put $|U_0| \sim |U_1|$ and substituted the velocity $v \sim 1/10$.

### IV. NUMERICAL ESTIMATES

In this section we present the results of numerical calculations using Eqs. (22), (40), and the estimate (41). Thus for the survival probability one, finally, has
\[
w_{\text{surv}} = \exp \left( -\frac{\eta \bar{R} \Gamma_s}{\rho_A V v} \right),
\]
where we put the function $F(R_1/R_2)$ equal to unity everywhere for the sake of simplicity.

In Fig. 2 we depict the survival probability (45) as function of the excess energy of the initial pion over the threshold for $d'$ production using for the spreading width $\Gamma_s = 20\text{MeV}$ and for the radius $\bar{R}$ the values $1.4\text{fm}$ and $1.6\text{fm}$, respectively. For the excess energy of $3\text{MeV}$ above the threshold we find that only about 10% of all created resonances survive, so that the suppression factor is of order ten. Finally, in Fig. 3, we give the “bare” total cross section of the $d'$ production in the reaction $\pi^3\text{He} \rightarrow d'\text{N}$ as well as the same cross section multiplied by the factor of the survival probability. We find our theoretical predictions to comply reasonably well with the experimental data given in Ref. [12] (dots with error bars in Fig. 3), where already the effects of collision damping had been discussed briefly.

### V. CONCLUSIONS AND OUTLOOK

Using an optical potential approach we have shown that collision damping strongly decreases the survival probability of $d'$ in presence of a nuclear medium. Even for $^3\text{He}$ it reduces the $d'$ production cross section near threshold by an order of magnitude, thereby leading to a reasonable agreement with the data. Another way to approach the problem of collision damping of the $d'$ resonance near its production threshold would be a full coupled-channel treatment. This requires the consideration of (at least) three channels: $\pi^3\text{He}, d'\text{N}$ and $\text{NNN}$ (one could even neglect the vacuum width of the $d'$ considering it as a stable particle). However, solving the coupled-channel problem requires a rather accurate knowledge of many reaction amplitudes (with $J^P = 1/2^-$) involved in this problem. In principle, this would
allow us to find the correction to the $d'$ production amplitude by summing up all possible contributions to the amplitude $\pi^3He \rightarrow d'N$ including the route $\pi^3He \rightarrow NNN \rightarrow d'N$.

Let us note that such effects should equally renormalize the amplitude of the $d'$ formation in heavier nuclei. However, it is just this amplitude (deduced from DCX transitions to discrete levels \[1\]) that was taken as an input to predict the $d'$ production off $^3He$ \[17\]. The difference between these phenomena is, therefore, due to different details of the $d'$ propagation following its initial production by the incoming pion. In this respect it appears noteworthy to compare with the situation of $\Lambda$ and $\Sigma$ production in the reaction $pp \rightarrow KNY$ near threshold, where the observed \[18\] surprisingly small $\Sigma$ production cross section is interpreted as being due to $pp \rightarrow K^+p\Sigma^0 \rightarrow K^+p\Lambda$, \textit{i.e.}, due to a strong FSI between $p$ and $\Sigma^0$, which transfers $\Sigma^0$ immediately into the energetically much more likely $\Lambda$ — a situation very similar to that of $d'N \rightarrow NNN$ discussed above for the DCX on Helium isotopes.
FIG. 3: The cross section for the $d'$ production in the reaction $\pi^3He \rightarrow d'N$ (in $\mu$ barn) versus the kinetic energy of the pion $T_\pi$ (in MeV). The curve 1 gives the “bare” result, without the damping effect taken into account [17]; the curves 2 and 3 represent the same cross section multiplied by the survival probability, Eq. (45), for $\Gamma_s = 20MeV$ and $\bar{R}$ taking the values 1.6$fm$ (the curve 2) and 1.4$fm$ (the curve 3). Experimental data [12] are shown by the dots with error bars.

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