Generalized Quantum Theory in Evaporating Black Hole Spacetimes

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Abstract

Quantum mechanics for matter fields moving in an evaporating black hole spacetime is formulated in fully four-dimensional form according to the principles of generalized quantum theory. The resulting quantum theory cannot be expressed in a $3+1$ form in terms of a state evolving unitarily or by reduction through a foliating family of spacelike surfaces. That is because evaporating black hole geometries cannot be foliated by a non-singular family of spacelike surfaces. A four-dimensional notion of information is reviewed. Although complete information may not be available on every spacelike surface, information is not lost in a spacetime sense in an evaporating black hole spacetime. Rather complete information is distributed about the four-dimensional spacetime. Black hole evaporation is thus not in conflict with the principles of quantum mechanics when suitably generally stated.

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I. INTRODUCTION

The early '80s were a memorable time to be at Chicago when Chandra was writing *The Mathematical Theory of Black Holes* [1]. His method was that of an explorer in many ways — voyaging through the complex landscape of equations that the classical theory of black holes presents — discovering novel perspectives, relationships, and hidden symmetries. We discussed these many times in long walks near the lake front in Hyde Park on often very cold Sunday afternoons. It is therefore a special pleasure for me to contribute to this commemoration of Chandra’s work.

In the prologue to the *Mathematical Theory*, Chandra sums up his views on black holes in a sentence: “The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time.” I note that Chandra used the word “macroscopic” to qualify the black holes that exhibit perfection. That was either prudent or prescient, for now in theoretical physics we are engaged with the question of whether black holes are or are not a blot on the perfection of quantum theory — the organizing principle of *microscopic* physics. This essay puts forward the thesis that black hole evaporation is not inconsistent with the principles of quantum mechanics provided those principles are suitably generalized from their usual flat space form.

II. BLACK HOLES AND QUANTUM THEORY

A. Fixed Background Spacetimes

The usual formulations of quantum theory rely on a fixed, globally hyperbolic background spacetime geometry as illustrated in Figure 1. In these usual formulations, complete information about a physical system is available on any spacelike (Cauchy) surface and is summarized by a state vector associated with that surface. This state vector evolves through a foliating family of spacelike surfaces \( \{ \sigma \} \), either unitarily between spacelike surfaces

\[
|\Psi (\sigma')\rangle = U|\Psi (\sigma')\rangle ,
\]

or by reduction on them

\[
|\Psi (\sigma)\rangle \rightarrow \frac{P|\Psi (\sigma)\rangle}{||P|\Psi (\sigma)\rangle||} .
\]

A spacetime is *globally hyperbolic* if it admits a surface \( \Sigma \), no two points of which can be connected by a timelike curve, but such that every inextendible timelike curve in the spacetime intersects \( \Sigma \). Such a surface is called a *Cauchy surface*. Classically data on a Cauchy surface determine the entire future and past evolution of the spacetime. Globally hyperbolic spacetimes have topology \( \mathbb{R} \times \Sigma \) and can be foliated by a one-parameter family of Cauchy surfaces defining a notion of time. For more details see [2].
Figure 1: The Penrose diagram for a globally hyperbolic spacetime like flat Minkowski space. A globally hyperbolic spacetime may be smoothly foliated by a family of spacelike surfaces a few of which are shown. When a spacetime can be so foliated quantum evolution can be described by a state vector that evolves unitarily between surfaces or by reduction on them.

Usual formulations of quantum theory thus depend crucially on a background spacetime exhibiting non-singular foliations by spacelike surfaces to define the notion of quantum state and its evolution.

Nowhere does the close connection between spacetime structure and quantum theory emerge so strikingly as in the process of black hole evaporation. Black holes and quantum theory have been inextricably linked since Hawking’s 1974 discovery of the tunneling radiation from black holes that bears his name. That radiation when analyzed in the approximation that its back reaction on the black hole can be neglected requires nothing new of quantum theory.

The familiar story is summarized in the Penrose diagram in Figure 2. The entire region of spacetime outside the horizon is foliable by a non-singular family of spacelike surfaces. States of matter fields defined on a spacelike, pre-collapse surface $\sigma'$ in the far past evolve unitarily to states on later spacelike surfaces like $\sigma''$. The complete information represented by the state is available on any spacelike surface. The evolution defines a pure state $|\Psi(H, I^+)\rangle$.

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1Spacetime has been rescaled so that it is contained in the interior of the triangle and so that radial light rays move on 45° lines. The dotted line is radius = 0. The remaining boundaries are the various parts of infinity. Future null infinity, $I^+$, consists of the endpoints of light rays that escape to infinity. The points $i_0, i_+,$ and $i_-$ are respectively spacelike infinity — where spacelike surfaces that reach infinity end — and future and past timelike infinity which are the endpoints of timelike curves. We shall use similar conventions in other illustrations in this paper. For more on the construction of Penrose diagrams see [2].
**Figure 2:** The Penrose diagram for a black hole when the back reaction of the Hawking radiation is neglected. The shaded region in this diagram represents spherically symmetric matter collapsing to form a black hole with horizon $H$ and eventually a spacelike singularity represented by the jagged horizontal line. Light rays emitted from a point inside the horizon end move on $45^\circ$ lines, end at the singularity, and never escape to infinity.

A state of a field in this background on an initial surface $\sigma'$ can evolve unitarily through an interpolating family of spacelike surfaces to a surface like $\sigma''$. By pushing $\sigma''$ forward the state can be evolved unitarily to the boundary of the region exterior to the black hole consisting of $I^+$ and the horizon $H$. The state $|\Psi(I^+, H)\rangle$ on $I^+ \cup H$ exhibits correlations (entanglements) between $I^+$ and $H$. Thus complete information cannot be recovered far from the black hole on $I^+$. Indeed for late times the predictions of $|\Psi(I^+, H)\rangle$ for observations on $I^+$ are indistinguishable from those of a thermal density matrix at the Hawking temperature. This does not mean that information is lost in quantum evolution; it is fully recoverable on the whole surface $I^+ \cup H$.

on the surface $H \cup I^+$ consisting of the horizon and future null-infinity. The Hilbert space of states is the product $\mathcal{H}(H) \otimes \mathcal{H}(I^+)$ of field states on the horizon and on future null-infinity. Measurements of observers at infinity probe only $\mathcal{H}(I^+)$. The probabilities of their outcomes may therefore be predicted from the density matrix

$$\rho(I^+) = tr_H \left[ |\Psi(H, I^+)\rangle\langle \Psi(H, I^+) | \right]$$

that results from tracing over all the degrees of freedom on the horizon. For observations at late times this represents disordered, thermal radiation.

Of course, information is missing from $\rho(I^+)$ so that complete information is not available on $I^+$. Specifically, the information in $|\Psi(H, I^+)\rangle$ concerning correlations between observables on $H$ and $I^+$ is missing. However, it is only necessary to consider observables on
Figure 3: The Penrose diagram for an evaporating black hole spacetime. The black hole is assumed here to evaporate completely, giving rise to a naked singularity $N$, and leaving behind a nearly flat spacetime region (lightly shaded above). A spacelike surface like $A$ is complete and data on this surface completely determine the evolution of fields to its future. Yet complete information about a quantum matter field moving in this spacetime is not available on $A$. Even if the initial state of the matter field on a surface like $\sigma'$ is pure, the state of the disordered Hawking radiation on $A$ would be represented by a density matrix. A pure state cannot evolve unitarily into a density matrix, so the usual formulation of quantum evolution in terms of states evolving through a foliating family of spacelike surfaces breaks down. The geometry of evaporating black hole spacetimes suggests why. There is no smooth family of spacelike surfaces interpolating between $\sigma'$ and $A$ and even classically there is not a well defined notion of evolution of initial data on $\sigma'$ to $A$. The usual notion of quantum evolution must therefore be generalized to apply in spacetime geometries such as this.

Both $H$ and $I^+$ to recover it. Usual quantum mechanics with its notion of a state carrying complete information evolving unitarily through families of spacelike surfaces is adequate to discuss the Hawking radiation when its back reaction is neglected.

Only in the complete evaporation of a black hole does one find a hint that the usual framework may need to be modified. Spacetime geometries representing a process in which a black hole forms and evaporates completely have a causal structure summarized by the Penrose diagram in Figure 3. Let us consider for a moment the problem of quantum mechanics of matter fields in a fixed geometry with this causal structure. (We shall return later to the fluctuations in spacetime geometry that must occur in a quantum theory of gravity.
Figure 4: A spacetime with a compact region of closed timelike curves (CTC’s).

As a consequence of the CTC’s there is no family of spacelike surfaces connecting an initial spacelike surface $\sigma'$ with a final one $\sigma''$, both outside the CTC region. The quantum evolution of a matter field therefore cannot be described by a state evolving through a foliating family of spacelike surfaces. The notion of quantum evolution must be generalized to apply to spacetimes such as this.

that is necessary to fully describe the evaporation process.) Any pure initial state $|\Psi(\sigma')\rangle$ leads to a state of disordered radiation on a hypersurface $A$ after the black hole has evaporated, so we have

$$|\Psi(\sigma')\rangle\langle \Psi(\sigma')| \rightarrow \rho(A),$$

(2.3)

where $\rho(A)$ is the mixed density matrix describing the radiation. This cannot be achieved by unitary evolution. Indeed, it is difficult to conceive of any law for the evolution of a density matrix $\rho(\sigma)$ through a family of spacelike surfaces that would result in (2.3) because there is no non-singular family of spacelike surfaces that interpolate between $\sigma'$ and $A$. Even classically there is no well defined notion of evolution of initial data on $\sigma'$ to the surface $A$ because of the naked singularity $N$.

I shall not attempt to review the discussion this situation has provoked.$^3$ What is clear is that a generalization of quantum mechanics is needed for a quantum theory of fields in geometries such as this.

Evaporating black hole geometries are not the only backgrounds whose causal structure requires a generalization of usual quantum mechanics for a quantum theory of matter fields. Consider a spacetime with a compact region of closed timelike curves such as occur in certain geometries.

$^3$For reviews of this discussion see 6.
Figure 5: A spacetime with a simple change in spatial topology. There is no non-singular family of spacelike surfaces between an “initial” spacelike surface \( \sigma' \) and “final” spacelike surfaces \( A \) and \( B \). The usual notion of quantum evolution must therefore be generalized to apply to spacetimes such as this. Complete information about the initial state is plausibly not available on spacelike surfaces \( A \) and \( B \) separately, but only surfaces \( A \) and \( B \) together.

wormhole geometries [7]. (See Figure 4.) Given a state on a spacelike surface \( \sigma' \) before the non-chronal region of closed timelike curves, how do we calculate the probabilities of field measurements inside the non-chronal region or indeed on any spacelike surface like \( \sigma'' \) such that the non-chronal region is contained between it and \( \sigma'' \)? Certainly not by evolving a state or density matrix through an interpolating family of spacelike surfaces, whether by a unitary or non-unitary rule of evolution. No such foliating family of spacelike surfaces exists! A generalization of usual quantum mechanics is required.

Spacetimes exhibiting spatial topology change, as in the “trousers” spacetime of Figure 5, are another class of backgrounds for which quantum field theory requires a generalization of usual quantum mechanics. Given an initial state on a spacelike surface \( \sigma' \), one could think of calculating the probabilities of alternatives on spacelike surfaces \( A \) or \( B \). But because such spacetimes are necessarily singular [8], there is no smooth family of surfaces interpolating between \( \sigma' \) and \( A \) and \( B \). A generalization of usual quantum dynamics is again required.\(^4\)

\(^4\)Field theory in such spacetimes has been studied by a number of authors [9][10].
B. Quantum Gravity

The evaporating black hole spacetimes illustrated in Figure 3 are singular. Spacetimes that are initially free from closed timelike curves but evolve them later must be singular or violate a positive energy condition [11]. Spatial topology change implies either a singularity or closed timelike curves [8]. These pathologies suggest a breakdown in a purely classical description of spacetime geometry. One might therefore hope that the difficulties with usual formulations of quantum theory in such backgrounds could be resolved in a quantum theory of gravity. The recent successful calculation of black hole entropy in string theory [12] raises several questions related to this hope. First, there is the question of whether there are general principles mandating a connection between the entropy and the logarithm of a number of states in any sensible quantum theory of gravity. More important for the present discussion, however, is the question of whether these calculations mean that black hole evaporation can be described within usual quantum mechanics in string theory. It is possible that string theory will yield a unitary $S$-matrix between asymptotic pre-collapse and evaporated states. However, as with any quantum theory of gravity, the need for a generalization of the idea of unitary evolution of states through spacelike surfaces is only more acute than it is for field theory in fixed non-globally-hyperbolic spacetimes for the following reason:

We have seen how the usual quantum mechanics of fields with evolution defined through states on spacelike surfaces relies heavily on a fixed, globally hyperbolic, background geometry to define those surfaces. But in any quantum theory of gravity spacetime geometry is not fixed. Geometry is a quantum variable — generally fluctuating and without definite value. Quantum dynamics cannot be defined by a state evolving in a given spacetime; no spacetime is given. A generalization of usual quantum mechanics is thus needed. This need for generalization becomes even clearer if one accepts the hint from string theory that spacetime geometry is not fundamental.

In the rest of this paper we shall discuss some generalizations of quantum theory that are applicable to the process of black hole evaporation. We shall discuss these primarily for the case of field theory in a fixed background evaporating black hole spacetime. There we can hope to achieve a concreteness not yet available in quantum theories of gravity. However, there is every reason to believe that the principles of the main generalization we shall describe are implementable in quantum gravity as well [13].

III. THE $\$-MATRIX

Hawking [14] suggested one way that the principles of quantum mechanics could be generalized to apply to an evaporating black hole. For transitions between asymptotic states in the far past and far future employ, not a unitary $S$-matrix mapping initial states to final states, but rather a $\$-matrix mapping initial density matrices to final density matrices:

$$\rho_f = \$\rho_i.$$  \hfill (3.1)

That way an initial pure state could evolve into a mixed density matrix as the evaporation scenario represented in Figure 3 suggests.

Hawking gave a specific prescription for calculating the $\$-matrix: Use Euclidean sums over histories to calculate Euclidean Green’s functions for the metric and matter fields.
Continue these back to Lorentzian signature in asymptotic regions where the continuation is well defined because the geometry is flat. Extract the $-$matrix elements from these Green’s functions as one would for the product of two $S$-matrices. When the topology of the Euclidean geometries is trivial, the resulting $-$matrix factors into the product of two $S$-matrices and (3.1) implies that pure states evolve unitarily into pure states. However, the $-$matrix generally does not factor if the topology the Euclidean geometries is non-trivial and then pure states can evolve into mixed states. It remains an open question whether this prescription in fact yields a $-$matrix. It is not evident, in particular, whether the mapping that results preserves the positivity and trace of the density matrices on which it acts.

However constructed, a $-$matrix connects only asymptotic initial and final density matrices. A full generalization of quantum theory would predict the probabilities of non-asymptotic observables “inside” the spacetime. To this end, various interpolating equations that will evolve a density matrix through a family of spacelike surfaces have been investigated [15,16]. These typically have the form

\[ i\hbar \frac{\partial \rho}{\partial t} = [H, \rho] + \begin{pmatrix} \text{additional terms} \\ \text{linear in } \rho \end{pmatrix}, \quad (3.2) \]

where additional terms give rise to non-unitary evolution. Perhaps these could be adjusted to yield the $-$matrix of Hawking’s construction when evolved between the far past and far future [17]. Such equations display serious problems with conservation of energy and charge when the additional terms in (3.2) are local, although Unruh and Wald [16] have demonstrated that only a little non-locality is enough to suppress this difficulty at energies below the Planck scale.

From the perspective of this paper, conservation or the lack of it is not the main problem with such modifications of the quantum evolutionary law. Rather, it is that the “$t$” in the equations is not defined. As we have argued in Section II, we do not expect to have a non-singular, foliating family of spacelike surfaces in evaporating black-hole spacetimes through which to evolve an equation like (3.2). A generalization of quantum mechanics even beyond such equations seems still to be required.

**IV. THINK FOUR-DIMENSIONALLY**

In the previous section we have argued that quantum theory needs to be generalized to apply to physical situations such as black hole evaporation in which quantum fluctuations in the geometry of spacetime can be expected or situations such as field theory in evaporating black hole backgrounds where geometry is fixed, but not globally hyperbolic. What features of usual quantum mechanics must be given up in order to achieve this generalization? In this section, we argue that one feature to be jettisoned is the notion of a state on spacelike surface and quantum evolution described in terms of the change in such states from one spacelike surface to another.

The basic argument for giving up on evolution by states through a foliating family of spacelike surfaces in spacetime has already been given: When geometry is not fixed, or even when fixed but without an appropriate causal structure, a foliating family of spacelike surfaces is not available to define states and their evolution.
Figure 6: The Two-Slit Experiment. An electron gun at left emits an electron traveling towards a screen with two slits and then on to detection at a further screen. Two histories are possible for electrons detected at a particular point on the right-hand screen defined by whether they went through slit A or slit B. Probabilities cannot be consistently assigned to this set of two alternative histories because they interfere quantum mechanically.

This basic point is already evident classically. There is a fully four-dimensional description of any spacetime geometry in terms of four-dimensional manifold, metric, and field configurations. However, for globally hyperbolic geometries that four-dimensional information can be compressed into initial data on a spacelike surface. That initial data is the classical notion of state. By writing the Einstein equation in 3 + 1 form the four dimensional description can be recovered by evolving the state through a family of spacelike surfaces. However such compression is not possible in spacetimes like the evaporating black hole spacetime illustrated in Figure 3. Only a four-dimensional description is possible.

Similarly there is a fully four-dimensional formulation of quantum field theory in background spacetime geometries in terms of Feynman’s sum-over-field-histories. Transition amplitudes between spacelike surfaces are specified directly from the four-dimensional action $S$ by sums over field histories of the form

$$\sum_{\text{histories}} \exp[iS(\text{history})/\hbar].$$

(4.1)

When the background geometry is globally hyperbolic, these transition amplitudes between spacelike surfaces can be equivalently calculated by evolving a quantum state through an interpolating family of spacelike surfaces. However, if the geometry is not globally hyperbolic, we cannot expect such a 3 + 1 formulation of quantum dynamics any more than we can for the classical theory. Following Feynman, however, we expect fully four-dimensional
spacetime formulation of quantum theory to supply the necessary generalization applicable to field theory in evaporating black hole spacetimes and to the other examples we have mentioned. We shall describe this generalization and its consequences in this article. Our motto is: “Think four-dimensionally.” When we do there is no necessary conflict between quantum mechanics and black hole evaporation.

V. GENERALIZED QUANTUM THEORY

To generalize usual quantum theory it is just as necessary to decide which features of the usual framework to retain as it is to decide which to discard. Generalized quantum theory is a comprehensive framework incorporating the essential features of a broad class of generalizations of the usual theory. The full apparatus of generalized quantum theory is not essential to reach the conclusion that black-hole evaporation is consistent with the principles of quantum mechanics sufficiently generally stated. Indeed, similar conclusions have been reached without invoking these general principles. (See, e.g. [18].) What is necessary is a generalization of the usual notion of unitary evolution. However, generalized quantum theory is a useful setting to consider generalizations because it provides basic principles and a framework to compare to compare different generalizations. In this section we give a brief and informal exposition of these principles.

The most general objective of a quantum theory is to predict the probabilities of the individual members of a set of alternative histories of a closed system, most generally the universe. The set of alternative orbits of the earth around the sun is an example. These are sequences of positions of the earth’s center of mass at a series of times. Another example are the four-dimensional histories of matter fields in an evaporating black hole spacetime.

The characteristic feature of a quantum theory is that not every set of alternative histories can be consistently assigned probabilities because of quantum mechanical interference. This is clearly illustrated in the famous two-slit experiment shown in Figure 6. There are two possible histories for an electron which proceeds from the source to a point \( y \) at the detecting screen. They are defined by which of the two slits (A or B) it passes through. It is not possible to assign probabilities to these two histories. It would be inconsistent to do so

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5There are other ideas for the necessary generalization. Wald has reached similar conclusions in the algebraic approach to field theory in curved spacetime [18]. The most developed is the idea from canonical quantum gravity that states evolve, not through surfaces in spacetime, but rather through surfaces in the superspace of possible three-dimensional geometries. For lucid and, by now, classic reviews of the various aspects and difficulties with this approach see [19]. These ideas are not obviously applicable to the cases of fixed spacetime geometry discussed in this paper. For another that might be, see [20] where states are defined in local regions later patched together.

6The application of sum-over-histories methods to gravity has a considerable history that cannot be recapitulated here. Some of the more notable early references are [21].

7For expositions see, e.g. [22,13].
because the probability to arrive at $y$ is not the sum of the probabilities to arrive at $y$ via the two possible histories:

$$p(y) \neq p_A(y) + p_B(y).$$

That is because in quantum mechanics probabilities are squares of amplitudes and

$$|\psi_A(y) + \psi_B(y)|^2 \neq |\psi_A(y)|^2 + |\psi_B(y)|^2$$

when there is interference.

In a quantum theory a rule is needed to specify which sets of alternative histories may be assigned probabilities and which may not. The rule in usual quantum mechanics is that probabilities can be assigned to the histories of the outcomes of measurements and not in general otherwise. Interference between histories is destroyed by the measurement process, and probabilities may be consistently assigned. However, this rule is too special to apply in the most general situations and certainly insufficiently general for cosmology. Measurements and observers, for example, were not present in the early universe when we would like to assign probabilities to histories of density fluctuations in matter fields or the evolution of spacetime geometry.

The quantum mechanics of closed systems\footnote{See, e.g. \cite{23} for an elementary review and references to earlier literature.} relies on a more general rule whose essential idea is easily stated: A closed system is in some initial quantum state $|\Psi\rangle$. Probabilities can be assigned to just those sets of histories for which there is vanishing interference between individual histories as a consequence of the state $|\Psi\rangle$ the system is in. Such sets of histories are said to decohere. Histories of measurements decohere as a consequence of the interaction between the apparatus and measured subsystem. Decoherence thus contains the rule of usual quantum mechanics as a special case. But decoherence is more general. It permits assignment of probabilities to alternative orbits of the moon or alternative histories of density fluctuations in the early universe when the initial state is such that these alternatives decohere whether or not the moon or the density fluctuations are receiving the attention of observers making measurements.

The central element in a quantum theory based on this rule is the measure of interference between the individual histories $c_\alpha$, $\alpha = 1, 2, \cdots$ in a set of alternative histories. This measure is called the decoherence functional, $D(\alpha', \alpha)$. A set of histories decoheres when $D(\alpha', \alpha) \approx 0$ for all pairs $(\alpha', \alpha)$ of distinct histories.

The decoherence functional for usual quantum mechanics is defined as follows: A set of alternative histories can be specified by giving sequences of sets of yes/no alternatives at a series of times $t_1, \cdots, t_n$. For example, alternative orbits may be specified by saying whether a particle is or is not in certain position ranges at a series of times. Each yes/no alternative in an exhaustive set of exclusive alternatives at time $t_k$ is represented by a Heisenberg picture projection operator $P_{\alpha_k}^k(t_k)$, $\alpha_k = 1, 2, \cdots$ where $\alpha$ labels the different alternatives in the set. An individual history $\alpha$ is a particular sequence of alternatives $\alpha \equiv (\alpha_1, \cdots, \alpha_n)$ and is represented by the corresponding chain of Heisenberg picture projections:

$$C_\alpha = P_{\alpha_n}^n(t_n) \cdots P_{\alpha_1}^1(t_1).$$
If the initial state vector of the closed system is $|\Psi\rangle$, (non-normalized) branch state vectors corresponding to the individual histories $\alpha$ may be defined by
\begin{equation}
C_\alpha |\Psi\rangle , \tag{5.4}
\end{equation}
and the decoherence functional for usual quantum mechanics is:
\begin{equation}
D(\alpha', \alpha) = \langle \Psi | C_\alpha^\dagger C_{\alpha'} |\Psi\rangle . \tag{5.5}
\end{equation}
This is the usual measure of interference between different histories represented in the form (5.4).

The essential properties of a decoherence functional that are necessary for quantum mechanics may be characterized abstractly \cite{22} and (5.5) is only one of many other ways of satisfying these properties. Therein lie the possibilities for generalizations of usual quantum mechanics.

VI. SPACETIME GENERALIZED QUANTUM MECHANICS

Feynman’s sum-over-histories ideas may be used with the concepts of generalized quantum theory to construct a fully four-dimensional formulation of the quantum mechanics of a matter field $\phi(x)$ in a fixed background spacetime. We shall sketch this spacetime quantum mechanics in what follows. We take the dynamics of the field to be summarized by an action $S[\phi(x)]$ and denote the initial state of the closed field system by $|\Psi\rangle$.

The basic (fine-grained) histories are the alternative, four-dimensional field configurations on the spacetime. These may be restricted to satisfy physically appropriate conditions at infinity and at the singularities. Sets of alternative (course-grained) histories to which the theory assigns probabilities if decoherent are partitions of these field configurations into exclusive classes $\{c_\alpha\}, \alpha = 1, 2, \ldots$. For example, the alternative that the field configuration on a spacelike surface $\sigma$ has the value $\chi(x)$ corresponds to the class of four-dimensional field configurations which take this value on $\sigma$. In flat spacetime the probability of this alternative is the probability for the initial state $|\Psi\rangle$ to evolve to a state $|\chi(x), \sigma\rangle$ of definite field on $\sigma$. The history where the field takes the value $\chi'(x)$ on surface $\sigma'$ and $\chi''(x)$ on a later surface $\sigma''$ corresponds to the class of four-dimensional field configurations which take these values on the respective surfaces, and so on.

The examples we have just given correspond to the usual quantum mechanical notion of alternatives at a definite moment of time or a sequence of such moments. However, more general partitions of four-dimensional field configurations are possible which are not at any definite moment of time or series of such moments. For example, the four-dimensional field configurations could be partitioned by ranges of values of their averages over a region extending over both space and time. Partition of four-dimensional histories into exclusive classes is thus a fully four-dimensional notion of alternative for quantum mechanics.

Branch state vectors corresponding to individual classes $c_\alpha$ in a partition of the fine-grained field configurations $\phi(x)$ can be constructed from the sum over fields in the class $c_\alpha$. We write schematically
\begin{equation}
C_\alpha |\Psi\rangle = \int_{c_\alpha} \delta\phi \exp\left(iS[\phi(x)]/\hbar\right)|\Psi\rangle . \tag{6.1}
\end{equation}
Usual
Quantum Mechanics | Generalized
Quantum Mechanics
---|---
Dynamics | \( e^{-iHt} | \Psi \rangle \) | \( \sum_{\text{histories} \in c_\alpha} e^{iS(\text{history})/\hbar} | \Psi \rangle \)
Alternatives | On spacelike surfaces or sequences of surfaces | Arbitrary partitions of fine-grained histories
Probabilities assigned to | Histories of measurement outcomes | Decohering sets of histories

It is fair to say that the definition of such integrals has been little studied in interesting background spacetimes, but we proceed assuming a careful definition can be given even in singular spacetimes such as those discussed in Section II. Even then some discussion is needed to explain what (6.1) means as a formal expression. In a globally hyperbolic spacetime we can define an operator \( C_\alpha \) corresponding to the class of histories \( c_\alpha \) by specifying the matrix elements

\[
\langle \chi''(x), \sigma'' | C_\alpha | \chi'(x), \sigma' \rangle = \int_{[\chi' c_\alpha \chi'']} \delta \phi \exp \left( iS[\phi(x)]/\hbar \right) .
\]

(6.2)

The sum is over all fields in the class \( c_\alpha \) that match \( \chi'(x) \) and \( \chi''(x) \) on the surfaces \( \sigma' \) and \( \sigma'' \) respectively. This operator can act on \( |\Psi\rangle \) by taking the inner product with its field representative \( \langle \chi'(x), \sigma' | \Psi \rangle \) on a spacelike surface far in the past. By pushing \( \sigma'' \) forward to late times we arrive at the definition of \( C_\alpha |\Psi\rangle \). The same procedure could be used to define branch state vectors in spacetimes with closed timelike curves (Figure 4), in spacetimes with spatial topology change (Figure 5), and in evaporating black hole spacetimes (Figure 3). The only novelty in the latter two cases is that \( C_\alpha |\Psi\rangle \) lives on the product of two Hilbert spaces. There are the Hilbert spaces on the two legs of the trousers in the spatial topology change case and, in the black hole case, there are the Hilbert space of states inside the horizon and the Hilbert space of states on late time surfaces after the black hole has evaporated.

The decoherence functional is then

\[
D(\alpha', \alpha) = \mathcal{N} \langle \Psi | C_\alpha^\dagger C_{\alpha'} | \Psi \rangle ,
\]

(6.3)

where \( \mathcal{N} \) is a constant to ensure the normalization condition

\[
\sum_{\alpha'} D(\alpha', \alpha) = 1 .
\]

(6.4)

A set of alternative histories decoheres when the off-diagonal elements of \( D(\alpha', \alpha) \) are negligible. The probabilities of the individual histories are

\[
p(\alpha) = D(\alpha, \alpha) = \mathcal{N} \| C_\alpha |\Psi\rangle \|^2 .
\]

(6.5)

There is no issue of “conservation of probability” for these \( p(\alpha) \); they are not defined in terms of an evolving state vector. As a consequence of decoherence, the \( p(\alpha) \) defined by (6.5) obey the most general probability sum rules including, for instance, the elementary normalization condition \( \sum_\alpha p(\alpha) = 1 \) which follows from (6.4).
Figure 7: Recovery of the unitary evolution of states through spacelike surfaces from sum-over-histories quantum mechanics in a globally hyperbolic spacetime. A sum-over-field-configurations between $\mathcal{I}^-$ and a spacelike surface $\sigma$ defines a state on that spacelike surface. This sum-over-fields specifies the evolution of this state as $\sigma$ is pushed forward in time. In a globally hyperbolic spacetime this evolution is unitary until a surface like $\sigma_1$ is encountered where there is a restriction on the spatial field configurations to a class $F$. This restriction is equivalent to the action of a projection on the state. Thus the two standard laws of quantum evolution in a “3 + 1” formulation are recovered from the more general four-dimensional framework. That recovery, however, requires a spacetime geometry that is smoothly foliable by spacelike surfaces.

This spacetime generalized quantum theory is only a modest generalization of usual quantum mechanics in globally hyperbolic backgrounds as the above table shows. The two laws of evolution (2.1) have been unified in a single sum-over-histories expression. The alternatives potentially assigned probabilities have been generalized to include ones that extend in time and are not simply the outcomes of a measurement process. These generalizations put the theory in fully four-dimensional form.

When the more general framework is restricted to globally hyperbolic backgrounds, to histories of alternatives on spacelike surfaces, and to the outcomes of measurements, usual quantum theory is recovered as a special case. In particular, one recovers the notion of a state evolving either unitarily or by reduction through a foliating family of spacelike surfaces. To see how this works consider the globally hyperbolic spacetime shown in Figure 7 and a single alternative where the field is restricted to some set $F$ of spatial field configurations on a spacelike surface $\sigma_1$. The class operator $C_\alpha$ is defined through (6.1) by integrating $\exp(iS[\phi(x)]/\hbar)$ over all field configurations between $\mathcal{I}^-$ and $\mathcal{I}^+$ restricted to $F$ on $\sigma_1$. We can do the integral first only over fields between $\mathcal{I}^-$ and a spacelike surface $\sigma$, and then push $\sigma$ to $\mathcal{I}^+$. (See Figure 7.) The integral over fields between $\mathcal{I}^-$ and a spacelike surface $\sigma$ on which a given spatial field configuration $\chi(x)$ is specified defines the field representative...
\[
\langle \chi(x) | \Psi(\sigma) \rangle \text{ of a state } |\Psi(\sigma)\rangle \text{ on } \sigma. \text{ As } \sigma \text{ is pushed forward this integral defines the evolution of } |\Psi(\sigma)\rangle. \text{ As Feynman showed, this evolution is unitary between infinitesimally separated surfaces. As a consequence } |\Psi(\sigma)\rangle \text{ evolves unitarily up to } \sigma_1. \text{ There, the restriction of the sum over fields to a set } F \text{ is equivalent to the action of the projection on } F \text{ on } |\Psi(\sigma)\rangle. \text{ The state is reduced as in (2.11). Thus, a state evolving unitarily or by reduction is recovered from the more general sum-over-histories formulation in globally hyperbolic spacetimes.}
\]

In an evaporating black hole spacetime (Figure 8), the probabilities for evolution — for decoherent alternatives on \( A \) given an initial state on \( \sigma' \) for instance — are generally defined four-dimensionally through (6.3) and (6.5). However, that evolution cannot be reproduced by the unitary evolution of a state on spacelike surfaces. If one attempts to define a state \( |\Psi(\sigma)\rangle \) by the procedure described above for globally hyperbolic spacetimes, one finds that the surface \( \sigma \) cannot remain spacelike and be pushed smoothly into the lightly shaded region to the future of the naked singularity.

The best that can be done is to push the surface \( \sigma \) so that integration is over fields with support in the region bounded by \( I^- \) and the spacelike surfaces \( A \) and \( B \) shown in Figure 8. That defines a kind of two component “state” with one component on \( A \) and the other on \( B \). By tracing products of this object over the degrees of freedom on \( B \) and normalizing the result, a density matrix may be constructed that is sufficient for predictions of alternatives on \( A \). However, that density matrix does not evolve by anything like an equation of the form (3.2). Indeed it evolves unitarily as the surface \( A \) is pushed forward in time. There is no problem with conservation of energy or charge because of the general arguments given in [24]. For alternatives that refer to field configurations in the interior of the spacetime even this kind of density matrix construction is in general unavailable and the general expressions (6.3) and (6.5) must be used.

Similar statements also could be made for other spacetimes that are not globally hyperbolic, such as the spacetimes with closed timelike curves or spatial topology change mentioned in Section II, and for quantum gravity itself.

The process of black hole evaporation is thus not in conflict with the principles of quantum mechanics suitably generally stated. It is not in conflict with quantum evolution described four-dimensionally. It is only in conflict with the narrow idea that this evolution be reproduced by evolution of a state vector through a family of spacelike surfaces.

### VII. INFORMATION — WHERE IS IT?

A spacetime formulation of quantum mechanics requires a spacetime notion of information that is also in fully four-dimensional form. In this section we describe a notion of the

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9It should be noted that a generalized quantum theory formulated four-dimensionally in terms of histories that extend over the whole of a background spacetime is not necessarily causal in the sense that predictions on a spacelike surface are independent of the background geometry to its future. See [25, 27] for further discussion.
**Figure 8:** Quantum evolution in an evaporating black hole spacetime can be described four-dimensionally using a Feynman sum-over-histories. However, that evolution is not expressible in $3 + 1$ terms by the smooth evolution of a state through spacelike surfaces.

Complete information is not recoverable on surface $A$ because of correlations (entanglements) between the field on $A$ and the field on a surface like $B$. Even though information is not necessarily available on any one spacelike surface, it is not lost in an evaporating black hole geometry but it is distributed over four-dimensional spacetime.

Information available in *histories* and not just in alternatives on a single spacelike surface. We then apply this to discuss the question of whether information is lost in an evaporating black-hole spacetime.

In quantum mechanics, a statistical distribution of states is described by a density matrix. For the forthcoming discussion it is therefore necessary to generalize the previous considerations a bit and treat mixed density matrices $\rho$ as initial conditions for the closed system as well as pure states $|\Psi\rangle$. To do this it is only necessary to replace (5.5) with

$$D(\alpha', \alpha) = Tr \left( C_{\alpha'} \rho C_{\alpha}^\dagger \right).$$

(7.1)

A generalization of the standard Jaynes construction [30] gives a natural definition of the missing information in a set of histories $\{c_{\alpha}\}$. We begin by defining the entropy functional on density matrices:

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10 The particular construction we use is due to Gell-Mann and the author [25]. There are a number of other ideas, e.g. [28, 29] with which the same points about information in a evaporating black-hole spacetime could be made.
\[ S(\tilde{\rho}) \equiv -Tr (\tilde{\rho} \log \tilde{\rho}) . \] (7.2)

With this we define the missing information \( S(\{c_\alpha\}) \) in a set of histories \( \{c_\alpha\} \) as the maximum of \( S(\tilde{\rho}) \) over all density matrices \( \tilde{\rho} \) that preserve the predictions of the true density matrix \( \rho \) for the decoherence and probabilities of the set of histories \( \{c_\alpha\} \). Put differently, we maximize \( S(\tilde{\rho}) \) over \( \tilde{\rho} \) that preserve the decoherence functional of \( \rho \) defined in terms of the corresponding class operators \( \{C_\alpha\} \). Thus, the missing information in a set of histories \( \{c_\alpha\} \) is given explicitly by:

\[
S(\{c_\alpha\}) = \max_{\tilde{\rho}} \left[ S(\tilde{\rho}) \right]_{\{Tr(C_\alpha \tilde{\rho} C_\alpha^\dagger) = Tr(C_\alpha \rho C_\alpha^\dagger)\}} .
\] (7.3)

Complete information, \( S_{\text{compl}} \) — the most one can have about the initial \( \rho \) — is found in the decoherent set of histories with the least missing information:

\[
S_{\text{compl}} = \min_{\text{decoherent}} \left[ S(\{c_\alpha\}) \right] .
\] (7.4)

In usual quantum mechanics it is not difficult to show that \( S_{\text{compl}} \) defined in this way is exactly the missing information in the initial density matrix \( \rho \)

\[
S_{\text{compl}} = S(\rho) = -Tr(\rho \log \rho) .
\] (7.5)

Information in a set of histories and complete information are spacetime notions of information whose construction makes no reference to states or alternatives on a spacelike surfaces. Rather the constructions are four-dimensional making use of histories. Thus for example, with these notions one can capture the idea of information in entanglements in time as well as information in entanglements in space.

One can find where information is located in spacetime by asking what information is available from alternatives restricted to fields in various spacetime regions. For example, alternative values of a field average over a region \( R \) refer only to fields inside \( R \). The missing information in a region \( R \) is

\[
S(R) = \min_{\text{decohering } \{c_\alpha\} \text{ referring to } R} S(\{c_\alpha\}) .
\] (7.6)

This region could be part of a spacelike surface or could extend in time; it could be connected or disconnected. When \( R \) is extended over the whole of the spacetime the missing information is complete. But it is an interesting question what smaller regions \( R \) contain complete information, \( S(R) = S_{\text{compl}} \).

When spacetime is globally hyperbolic and quantum dynamics can be described by states unitarily evolving through a foliating family of spacelike surfaces, it is a consequence of the definitions (7.3) and (7.4) that complete information is available on each and every spacelike surface

\[
S(\sigma) = S_{\text{compl}} = -Tr(\rho \log \rho) .
\] (7.7)

However, in more general cases, such as the evaporating black hole spacetimes under consideration in this paper, only incomplete information may be available on any spacelike surface, and complete information may be distributed about the spacetime. We now show why.
There is no reason to expect to recover complete information on a surface like $A$ in Figure 8. The analysis of the black-hole spacetime without back reaction (Figure 2) shows that much of the information on the surface $(\mathcal{I}^+, H)$ consists of correlations, or entanglements, between alternatives on $\mathcal{I}^+$ and alternatives on $H$. One should rather expect complete information to be available in the spacetime region which is the union of $A$ and a surface like $B$. Even though this region is not a spacelike surface there are still entanglements “in time” between alternatives on $A$ and alternatives on $B$ that must be considered to completely account for missing information.

The situation is not so very different from that of the “trousers” spacetime sketched in Figure 5. Complete information about an initial state on $\sigma'$ is plausibly not available on surfaces $A$ or $B$ separately. That is because there will generally be correlations (entanglements) between alternatives on $A$ and alternatives on $B$. Complete information is thus available in spacetime even if not available on any one spacelike surface like $A$. The surfaces $A$ and $B$ of this example are similar in this respect to the surfaces $A$ and $B$ of the evaporating black hole spacetime in Figure 8.

Thus even though it is not completely available on every spacelike surface like $A$, information is not lost in evaporating black-hole spacetimes. Complete information is distributed about the spacetime.

**VIII. CONCLUSIONS**

This paper has made five points:

- For quantum dynamics to be formulated in terms of a state vector evolving unitarily or by reduction through a family of spacelike surfaces, spacetime geometry must be fixed and foliable by a non-singular family of spacelike surfaces.

- When spacetime geometry is fixed but does not admit a foliating family of spacelike surfaces or where, as in quantum gravity, spacetime geometry is not fixed, quantum evolution cannot be defined in terms of states on spacelike surfaces.

- But quantum mechanics can be generalized so that it is fully four-dimensional, spacetime form, free from the need for states on spacelike surfaces.

- The complete evaporation of a black hole is not in conflict with the principles of quantum mechanics stated suitably generally in four-dimensional form.

- Even though complete information may not be available on every spacelike surface, four-dimensional information is not lost in an evaporating black hole spacetime. Rather, complete information is distributed over spacetime.

We have argued that, whether or not a quantum theory of gravity exhibits a unitary $S$-matrix between pre-collapse and evaporated states, some generalization of quantum mechanics will be necessary when spacetime geometry is not fixed. Black hole evaporation is not in conflict with the principles of quantum mechanics suitably generally stated. Whatever the outcome of the evaporation process generalized quantum theory is ready to describe them.
IX. EPILOGUE

When I was a much younger scientist working on the physics of neutron stars, I remember reporting to Chandra how I had been criticized after a colloquium for daring to extrapolate the theory of matter in its ground state to the densities of nuclear matter and beyond. He emphatically advised me to pay no attention, as if worried that I might, saying people such as my critics were never right. He might, I imagine, have been thinking of his own extrapolation of the properties of stellar matter to the densities of white dwarfs. In this article I have been engaged in extrapolating the principles of quantum mechanics to the domains of quantum gravity and black holes. One hopes Chandra would have approved.

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