Gasdynamic Structure of Dispersed Flows in Two-Stage Air Purifiers

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Abstract. The work deals with numerical modelling of four-phase flows in a nozzle of two-stage air purifier. The aerodynamic structure of a gas-dispersed flow is studied in a wide range of determining parameters. The flow regimes are considered in the first stage nozzle with a high dust concentration.

1. Introduction
Protecting the environment from industrial emissions is a priority mission of the world. In this regard, improving the design of dust collecting equipment is one of the urgent tasks of modern science and technology. Among the existing methods for cleaning the emissions from suspended particles, the most common is the method of dry cleaning which can be successfully used for processing dust and gas flows with a high concentration of particles.

Despite the successes achieved in the theory of multiphase flows, there is still no single approach to adequately describe the complex gas-dynamic flows in dust separators in a wide range of the determining parameters. This work is aimed at solving this problem. In this paper, gas-particle flows are considered on the basis of the interpenetrating continua (Euler-Euler) approach [1]. Within this approach, the general system of equations is considered without any simplifying assumptions about the flow structure in the region under consideration. Analytical solution is only possible to get for a narrow range of problems that do not describe the whole picture. Therefore, the general problem can only be solved numerically. In the paper, the method of large particles [2] is used for numerical simulations.

2. Problem statement
Let us consider a multiphase flow of a gas and mono dispersed solid particles in a nozzle of the first stage of a two-stage inertial separator with an input section of radius $R$ (Figure 1).

As the primary phase, we consider the carrier gas. The secondary phase is the particle fraction supplied at the inlet section of the nozzle of the first stage. The third phase is the fraction of particles reflected by the side nozzle surface and the fourth phase is the fraction of particles reflected by the internal separator baffles located along the nozzle axis of the first and second stages. Collisions take place between particles of different fractions causing the interphase momentum exchange, so it is necessary to take into account the effective interaction force between particles from different phases. It should be noted that particle collisions can lead to randomization of particle motion, and hence the additional terms appear in the conservation law equations for particle momentum and energy. We suppose that the particle randomization effect is not significant for this type of flow, so corresponding term is not included in the governing equations.
Figure 1. The operation scheme of two-stage inertial separator with a central dust extraction.  
1 is the building of the first stage, 2 is the building of the second stage,  
3 is the dust-collecting bag and 4 is the partition.

3. Mathematical model

The problem is considered in the cylindrical coordinate system \((x, Y, \varphi)\), the direction of \(x\)-axis \(OX\) coincides with the nozzle axis. The governing equations within the framework of the interpenetrating continua (Euler-Euler approach) are as follows:

\[
\begin{align*}
\frac{\partial \rho_i}{\partial t} + \text{div} \rho_i \vec{v}_i &= 0 \\
\frac{\partial \rho_i \vec{v}_i}{\partial t} + \nabla (\rho_i \vec{v}_i \vec{v}_i) &= (\delta - 1)\nabla p + \vec{f}_i \\
\sum_{i=1}^{4} \left[ \frac{\partial \rho_i E_i}{\partial t} + \text{div}(\rho_i E_i + (1-\delta)\rho_i \vec{v}_i) \right] &= 0 \\
\frac{\partial \rho_i e_i}{\partial t} + \text{div} \rho_i e_i \vec{v}_i &= q_{ij} + \frac{\vec{f}_i}{\rho_i} \left( \vec{v}_i - \vec{v}_j \right)
\end{align*}
\]

where \(\delta = \begin{cases} 0, i = 1 \\ 1, i \neq 1 \end{cases}\), the subscript \(i, j = 1, 2, 3, 4\), \(i \neq j\) refers to the parameters of the gas and the corresponding fractions of particles, respectively; \(\rho_i, \vec{v}_i, e_i, E_i\) are the effective density, velocity, inner and the total energy of the \(i\)-th phase, \(p\) is the pressure in the gas, \(\vec{f}_{ij}\) is the power intensity of interaction between phases, and \(q_{ij}\) is the heat transfer between gas and particles of different fractions.

As shown in [1] for \(\frac{\rho_1^0}{\rho_2^0} \ll 1\), the main contribution to the expression for the force interaction of the phases is made by the friction force between the gas and the particles, having the form:

\[
\vec{f}_{ij} = 0.75 \rho_1^0 \rho_2 C_{di} \frac{|\vec{v}_i - \vec{v}_j| (\vec{v}_i - \vec{v}_j)}{\rho_2^0 d^2} \psi_{\alpha_i} \cdot C_{di} = \frac{24}{\Re_{ii}} + \frac{4}{\sqrt{\Re_{ii}}} + 0.4 ,
\]

\[
\Re_{ii} = \frac{\rho_1^0 |\vec{v}_i - \vec{v}_j| d}{\mu_i} , \quad \psi_{\alpha_i} = (1 - \alpha_i)^{-2.7} , \quad \alpha_i = \frac{\rho_2}{\rho_1^0} .
\]
Here $\rho_i^0$ is the true density phase; $C_{di}$, $Re_{ui}$, $Nu_{ui}$ are the aerodynamic drag coefficient, Reynolds number and Nusselt number of relative flow around the particle of the $i$-th phase, respectively; $Pr$ is the Prandtl number, $\mu_l$ and $d$ are the dynamic viscosity coefficient of gas and particle diameter. The expression for the effective interaction force between particles of different factions of $\vec{f}_{ui}$ the same, as in [3.4]:

$$\vec{f}_{ui} = \frac{k^{(f)} \rho_i \rho_j (\vec{v}_i - \vec{v}_j) |\vec{v}_i - \vec{v}_j|}{\beta^v}, \beta^v = \frac{\rho_l^0 d}{\rho_i^0 R}$$

Here the $k^{(f)}$ stands for the intensity of the force interaction between particles of different phases, an $\beta^v$ is the degree of inertia of the particle. In [4] where multiphase flows in pipes are studied, the dependence of this coefficient on the phase velocity difference is defined. It was shown that $k^{(f)} \approx 0.1$ for gas velocity $v_1 \approx 10$ m/s. As closing relations for system (1), the equations of state of the phases were used:

$$p = \rho_i^0 (\gamma - 1) e_i, e_1 = c_{v1} T_1, e_2 = c_{v2} T_2,$$

where $\gamma$ is gas specific heats ratios; $c_{v1}, c_{v2}$ the specific heat of the gas at constant volume and specific heat of particles; $T_i$ is the gas temperature.

To integrate the system (1), it is necessary to set boundary and initial conditions. It is assumed that the outlet boundary of the computational domain is located far enough from the nozzle. Then $x \rightarrow \infty$ a flow is realized without dynamic (in speed) and thermal (in temperature) particle lag, with vertical components of gas and particle velocities equal to zero. In this case fulfillment of the flow homogeneity condition [5, 6] can be considered fair, i.e. assume that the condition is satisfied:

$$\frac{\partial V_{i}^{(s)}}{\partial x} = 0$$

It was also assumed that the mixture flow in this section is isentropic and isenthalpic, i.e., $H_2 = const, S = const$. It was also believed that the reduced density of the second phase in the input section is a given value. In the calculations, these boundary conditions were carried into $x = 2$ section. On the side walls of the nozzle, the symmetry condition is fulfilled for gas guaranteeing the absence of flow through this boundary. For particles, the condition of normal reflection is set

$$V_{s}^{(s)} = V^{(s)}, V_{l}^{(s)} = -k^{(n)} V_{l}^{(s)}$$

where $k^{n}$ is a reflection coefficient. In the nozzle outlet section, the relations obtained for the one-dimensional isentropic gas dynamic theory are used for the gas [7]. As the initial conditions, the parameters of the undisturbed flow in the cross section $x = -2$, with $F_{ui} = 0$ and $q_{ui} = 0$ are prescribed.

To integrate the system (1) in the curvilinear region, the finite-difference grid is constructed in the computational domain. A direct replacement leads to the appearance of irregular nodes (or calculated cells) near the boundaries of the layer region. In this case, to set the boundary conditions in the layer of irregular calculated cells in a method of large particles, fractional cells are introduced into consideration [8]. The practice of performing calculations using fractional cells has shown that this algorithm is rather cumbersome, especially in the case of low Mach numbers $M_0 \ll 1$. Therefore, it is more practical to introduce new variables $\xi = \xi(x, y), \eta = \eta(x, y)$, in which the curvilinear region becomes rectangular. It was shown in [5, 6] that if, under such transformation, the Jacobian of the transformation $I = D(\xi, \eta)/D(x, y)$ exists and does not vanish at any point in the region, then the divergent form of equations (1) is preserved. Using new independent variables $(x, \xi)$, where
\[ \xi = \frac{y - G(x)}{F(x) - G(x)} \]; \( F(x) \) and \( G(x) \) are the equations of the upper and lower boundaries of the nozzle of the first stage of the separator, we get the rectangular computational domain \( N: 0 \leq x \leq 1, 0 \leq \xi \leq 1 \).

The governing equations (1) in the variables \((x, \xi)\) take the form:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x} + \frac{\partial \rho U_i}{\partial \xi} &= -\frac{\partial U_i}{\partial x} \\
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i^2}{\partial x} + \frac{\partial \rho u_i U_i}{\partial \xi} &= \frac{\delta}{\varepsilon} (-\varepsilon \frac{\partial p}{\partial x} + \xi \varepsilon \frac{\partial p}{\partial \xi}) - \frac{\partial U_i}{\partial \xi} + f_{ij}^x \\
\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_i u_i}{\partial x} + \frac{\partial \rho v_i U_i}{\partial \xi} &= \frac{\delta}{\varepsilon} (-\varepsilon \frac{\partial p}{\partial x} - \frac{\partial U_i}{\partial \xi} + f_{ij}^y \\
\sum_{i=1}^{4} \left( \frac{\partial \rho E_i}{\partial t} + \frac{\partial \rho E_i u_i}{\partial x} + \frac{\partial \rho E_i U_i}{\partial \xi} \right) + \frac{\partial p u_i}{\partial x} + \frac{\partial p U_i}{\partial \xi} &= -\frac{\partial p U_i}{\partial \xi} - \sum_{i=1}^{4} \frac{\partial \rho E_i U_i}{\partial \xi} \\
\frac{\partial \rho e_{ij}}{\partial t} + \frac{\partial \rho e_{ij} u_i}{\partial x} + \frac{\partial \rho e_{ij} U_i}{\partial \xi} &= -\rho \xi \frac{\partial U_i}{\partial \xi} + q_{ij} + \frac{1}{2} f_{ij}^y (\hat{v}_i - \hat{v}_j) \\
U_i &= v_i - u_i \xi e', U_i^c = v_i + u_i \xi e', \; \varepsilon = F(x) - G(x)
\end{align*}
\]

The system (2) can be written as:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x} + \frac{\partial \rho U_i}{\partial \xi} &= 0 \\
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i^2}{\partial x} + \frac{\partial \rho u_i U_i}{\partial \xi} &= (1 - \delta) \left( -\frac{\partial p}{\partial x} + \frac{\partial \rho e}{\partial \xi} \right) + \varepsilon^2 \frac{\partial f_{ij}^x}{\partial \xi} \\
\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_i u_i}{\partial x} + \frac{\partial \rho v_i U_i}{\partial \xi} &= (\delta - 1) \xi \frac{\partial p}{\partial \xi} + \varepsilon^2 \frac{\partial f_{ij}^y}{\partial \xi} \\
\sum_{i=1}^{4} \left( \frac{\partial \rho E_i}{\partial t} + \frac{\partial \rho E_i u_i}{\partial x} + \frac{\partial \rho E_i U_i}{\partial \xi} \right) + \frac{\partial p u_i}{\partial x} + \frac{\partial p U_i}{\partial \xi} &= -\frac{\partial p U_i}{\partial \xi} - \sum_{i=1}^{4} \frac{\partial \rho E_i U_i}{\partial \xi} \\
\frac{\partial \rho e_{ij}}{\partial t} + \frac{\partial \rho e_{ij} u_i}{\partial x} + \frac{\partial \rho e_{ij} U_i}{\partial \xi} &= \varepsilon^2 \xi (q_{ij} + 0.5 f_{ij}^y (\hat{v}_i - \hat{v}_j)), \; \alpha = \gamma M_0^2
\end{align*}
\]

We solve the system (3) in dimensionless variables introduced as:

\[ \bar{\rho}_i = \frac{\rho_i}{\rho_{i0}}, \bar{u}_i = \frac{u_i}{u_0}, \hat{v}_i = \frac{v_i}{u_0}, \hat{E} = \frac{E_i}{u_0^2}, \hat{e}_i = \frac{e_i}{u_0^2}, \hat{t} = \frac{t}{t_0}, \hat{x} = \frac{x}{R}, \hat{E} = \frac{E}{R}, \]

where subscript "0" refers to the value of the corresponding parameters at the initial time, \( R \) is the characteristic length scale of the problem, in this case this is the input separator section radius.
In dimensionless variables, the system (3) has the following form (the upper line of dimensionless variables is omitted):

\[
\frac{\partial \rho \varepsilon^2 \xi}{\partial t} + \frac{\partial \rho u \varepsilon^2 \xi}{\partial x} + \frac{\partial \rho U \varepsilon}{\partial \xi} = 0
\]

\[
\frac{\partial \rho u \varepsilon^2 \xi}{\partial t} + \frac{\partial \rho u^2 \varepsilon^2 \xi}{\partial x} + \frac{\partial \rho u U \varepsilon}{\partial \xi} = \frac{1}{\alpha} \left( 1 - \delta \right) \left( - \frac{\partial \rho e^2 \xi}{\partial x} + \frac{\partial (\rho e^2 \xi)}{\partial \xi} \right) + \varepsilon^2 \gamma_{ij}^2
\]

\[
\frac{\partial \rho v \varepsilon^2 \xi}{\partial t} + \frac{\partial \rho u v \varepsilon^2 \xi}{\partial x} + \frac{\partial \rho v U \varepsilon}{\partial \xi} = \left( \delta - 1 \right) \frac{\varepsilon}{\alpha} \frac{\partial p}{\partial \xi} + \varepsilon^2 \gamma_{ij}^2
\]

(4)

\[
\sum_{i=1}^{4} \left\{ \frac{\partial \rho_i \varepsilon^2 \xi}{\partial t} + \frac{\partial \left[ \rho_i E_i + \frac{1}{\alpha} p \right] u \varepsilon^2 \xi}{\partial x} + \frac{\partial \left[ \rho_i E_i + \frac{1}{\alpha} p \right] U \varepsilon}{\partial \xi} \right\} = 0
\]

\[
\frac{\partial \rho j \varepsilon^2 \xi}{\partial t} + \frac{\partial \rho j u \varepsilon^2 \xi}{\partial x} + \frac{\partial \rho j U \varepsilon}{\partial \xi} = \varepsilon^2 \gamma \left[ q_{ij} + \frac{1}{2} f_j \left( \bar{v}_i - \bar{v}_j \right) \right]
\]

\[
\alpha = \gamma M_0^2
\]

The system (4) is integrated numerically by the method of large particles using an implicit Euler scheme [8]. As it is shown in this paper, for flows in rectangular region with \( M_0 \ll 1 \) at the Euler stage, it is advisable to use a time-implicit difference scheme for calculating pressure. In [3], a generalization of the method of large particles with implicit Euler stage is presented for the case of regions of a complex shape.

4. Results
The results of the calculations presented in Figures 2 and 3 show that since the mass concentration of dispersed phase in the input section of the separator is high \( m_2 = 0.2 \), the dispersed phase not only affects the gas flow, but also significantly influences the flow of various particles fractions. So, for example, with an increase in the intensity of the interaction force with particles of the second phase, particles of the third phase are more intensively drifted towards the exit section of the nozzle, thereby reducing the region in which the fourth phase are located.

5. Conclusion
Based on the theory of interpenetrating continua, mathematical model and calculation method have been created that allow determining the characteristics of the gas-dispersed flows in areas of complex shapes. It was revealed that at high contents of the disperse phase in the input section of the inertial separator, particles that bounced off the side walls of the nozzle of the first stage significantly affect the flow structure. In this case, the streamline of both gas and dispersed fractions undergo significant changes due to the force interaction between particles of different phases.
Figure 2. The flow lines gas and particles in the first stage of the first stage of the inertial separator for the case of $m_{20} = 0.2; M_0 = 0.05; d = 200 \mu m$.

Fig. 3. The gas and particles streamlines in the first stage of the inertial separator for the case $m_{20} = 0.5, M_0 = 0.05, d = 200 \mu m$, solid lines are for gas; dotted lines are for the second phase; circles are for the third phase; asterisks are for the fourth phase; dashed lines are the sonic lines.

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