Path-following enhancement of an 8 × 8 combat vehicle using active rear axles steering strategies

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Abstract
Active rear steering has been used in many research work to enhance ground vehicles’ lateral stability. However, there is a shortage in the published research studies that consider the incorporation of active rear steering for autonomous vehicles applications, especially in case of multi-axle combat vehicles. In this paper, various $H_\infty$ controllers are developed to actively steer rear axles of a multi-axle combat vehicle using a linearized bicycle model. The proposed controllers are incorporated with a 22 degrees of Freedom nonlinear Trucksim full vehicle model to study and compare the developed controllers’ performance on a hard surface. Moreover, a frequency-domain analysis is conducted to investigate the influence of the active rear steering on the path-following controllers’ robustness in terms of stability and performance. Three path-following controllers are designed, where the first controller is applied on the front two axles of the vehicle, while the rear two axles are fixed. The second is applied to all-wheel steering vehicle. The third controller is an integration between the designed front steering path-following controller and a developed lateral stability active rear steering controller. Eventually, a series of virtual maneuvers are performed to evaluate the effectiveness of the intended controllers to present the advantages and limitations of each controller at different driving conditions.

Keywords
Multi-wheeled vehicles, active rear steering, vehicle lateral stability, robust control, path-following control

Date received: 3 May 2021; accepted: 16 June 2021

Introduction
In the past two decades, safety demand has been increased in automotive engineering. Therefore, several safety controllers have been introduced to meet this requirement. Examples of these systems are anti-lock braking systems, electronic stability control, lane-keeping assist, active suspension, and torque vectoring control. Generally, the vehicle’s lateral, longitudinal, and vertical dynamics are controlled by these systems. However, in case of avoiding a crash or performing severe maneuvers, controlling the lateral stability is more beneficial. There are two strategies are used to enhance the vehicle lateral stability. The first is direct yaw moment control, while the second is active steering control, which can be performed using active front steering and/or active rear steering (ARS) systems.

In general, ARS has more effectiveness in lessening the vehicle sideslip than other methods and increases the vehicle stability. However, ARS is still not common in passenger vehicles because the continuous use of it delivers noise as driving with flat or snow tires beside a feeling that the vehicle points out from turning curvature. Meanwhile, ARS seems to be more efficient for military combat vehicles as it works in a harsh environment, so noise will not be an issue, and it requires a trained driver to operate it. Despite that, limited studies considered ARS to improve lateral stability of combat vehicles as in and for a 6x6 and 8 × 8 combat vehicles, respectively.

Recently, the research related to autonomous vehicles has great attention to prevent human errors and risking human lives in dangerous applications as in the military field. For developing an autonomous vehicle, path-following is an essential task to execute the vehicle’s motion on a desired path or trajectory. As for the past decade, path-following was limited to the front steering by actively steer the front axle to track the desired path as in. Recently, it was found that constraining the vehicle yaw rate and sideslip can enhance the tracking performance at high speed as in. Also, it was found that including the control of the wheels’ driving torque inside the path-following controller is beneficial in controlling the...
vehicle dynamics and consequently improve the high-speed tracking performance as in\textsuperscript{26–31}.

Still, there is a gap in the literature regarding the consideration of ARS in path-following applications, especially for multi-axle vehicles. Moreover, there is a lack in introducing a path-following controller for ground multi-axle vehicles in limit handling conditions for severe maneuvers. Therefore, the main contribution of this study can be summarized as follows:

(i) Include the steering of the rear axles in the path-following controller’s design in two different forms and compare their performance; all-wheel steering (AWS) path-following controller and front wheel-steer path following controller combined with a lateral stability ARS controller.

(ii) Investigates the effect of including the steering of the rear axles on tightening the controller’s design requirements and robustness against the velocity variation.

(iii) Fill the gap in the literature and introduce a path-following controller based on ARS for a multi-axle combat vehicle.

(iv) Enriches the shortage in the published research work related to automation of multi-axle combat vehicles.

Studying the path-following performance of combat vehicles on soft soil (off-road operation) requires a detailed tire-soil interactions model\textsuperscript{32–35}. In addition, the lateral dynamics of the vehicle will not only depend on the vehicle parameters, but also the soil mechanical properties\textsuperscript{36,37}. This study is focusing on the path-following performance on a hard surface (on-road). The off-road operation analysis on soil will be included in future work.

In this paper, three cases of the path-following controller are developed and compared for an 8 × 8 combat vehicle on a hard surface. At first, an H\textsubscript{\infty} path-following controller is designed based on the steering of the front axles and fixed rear axles. In the second case, the developed H\textsubscript{\infty} path-following controller includes the rear axles’ steering with modifying the controller’s weight functions and adding constraints to tighten the design requirements. The third case integrates the developed front steering path-following controller side-by-side with a separate H\textsubscript{\infty} ARS controller that is developed to enhance the vehicle’s lateral stability by exploiting the independent steering of the rear two axles. Eventually, virtual testing is conducted in TruckSim software in collaboration with Simulink using a series of double lane change (DLC) maneuvers. The testing is performed at different speeds and on various road’s friction coefficients.

**Mathematical model**

The bicycle model showed its effectiveness to design a lateral stability controller in previous studies as introduced by\textsuperscript{4,9,38,5,13,39,40}. Besides, it is also used for vehicles’ automation and path-following applications as presented in\textsuperscript{31,25,29,24}. Therefore, the state-space model required to design the controllers can be derived using a linearized 2 degree of freedom (DoF) bicycle model for the 8 × 8 combat vehicle, as presented in Figure 1. On the other hand, for evaluation and simulation purposes, the developed controllers are incorporated with the highly nonlinear 22 DoF full vehicle model. This model includes the pitch and roll dynamics and highly nonlinear characteristics for measured tires data at different operating conditions.

The state-space model is developed to control the vehicle’s heading angle, lateral position and speed, and yaw rate. The model is linearized based on the following assumptions:

1. Negligible longitudinal and lateral load transfer.
2. A small angle approximation is used.
3. All tires are identical.
4. Linear tires characteristics on hard surface (no off-road analysis).
5. Wheels’ camber, tow, and caster angles are neglected.
6. Constant longitudinal speed.

The tires’ slip angles \( \alpha_i \) can be defined as in equation (1), where \( \delta_i \) is the \( i \)-th axle average steering angle, \( V \) is the vehicle’s lateral speed, while the distance from the center of gravity to the \( i \)-th axle is \( a_i \), the yaw rate is \( \dot{r} \), and the longitudinal or forward velocity is \( U \).

\[
\alpha_i = \delta_i - \tan^{-1}\left(\frac{V + a_i \dot{r}}{U}\right) \tag{1}
\]

Using small angles approximation, equation (1) can be written as follows:

\[
\alpha_i = \delta_i - \left(\frac{V + a_i \dot{r}}{U}\right) \tag{2}
\]

By assuming linear tire characteristics, the \( \text{th} \)-th tire lateral force \( F_{yi} \) can be described as in equation (3), where \( C_{wi} \) is the tire lateral stiffness.

\[
F_{yi} = C_{wi} \alpha_i \tag{3}
\]

The steering angles of the front two axles are linked with a fixed ratio \( c_{21} \) as follows:

\[
\delta_2 = c_{21} \delta_1 \tag{4}
\]

The equation of motion in the lateral direction can be
described as in equation (5), where \( m \) is the vehicle mass.

\[
m(\dot{V} + Ur) = F_{y1} + F_{y2} + F_{y3} + F_{y4}
\]

(5)

The yaw equation of motion about the Z-axis can be presented as in equation (6), where \( I_{Z_e} \) is the mass moment of inertia around the Z-axis, and the rate of change of the yaw rate \( \dot{r} \).

\[
I_{Z_e}\dot{r} = a_1F_{y1} + a_2F_{y2} + a_3F_{y3} + a_4F_{y4}
\]

(6)

By rearranging the equations from (2) to (6) and substituting \( V \) with the rate of change in the Y-axis coordinate \( \dot{Y} \), and \( r \) with the rate of change in the vehicle heading angle \( \dot{\theta} \), the path-following state-space model can be described as in equation (7). This equation can represent a front steering vehicle by substituting \( a_2 \) and \( a_3 \) by zero, which represent fixed rear axles.

\[
\begin{bmatrix}
\dot{\bar{Y}} \\
\dot{\bar{r}} \\
\dot{\bar{\theta}} \\
\dot{\tau}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & a_{11} & 0 & a_{12} \\
0 & 0 & 0 & 1 \\
0 & a_{21} & 0 & a_{22}
\end{bmatrix}
\begin{bmatrix}
\bar{Y} \\
\bar{r} \\
\bar{\theta} \\
\tau
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 \\
\frac{C_{a1}+C_{a2}}{m} & \frac{C_{a3}}{m} & \frac{C_{a4}}{m} & 0 \\
0 & \frac{a_1C_{a1}+a_2C_{a2}}{I_{Z_e}} & \frac{a_3C_{a3}}{I_{Z_e}} & \frac{a_4C_{a4}}{I_{Z_e}}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_3 \\
\delta_4
\end{bmatrix}
\]

(7)

where

\[
\begin{align*}
a_{11} &= -\frac{C_{a1} - C_{a2} - C_{a3} - C_{a4}}{mU} \\
a_{12} &= -\frac{a_1C_{a1} - a_2C_{a2} - a_3C_{a3} - a_4C_{a4} - mU^2}{mU} \\
a_{21} &= -\frac{C_{a1} - C_{a2} - C_{a3} - C_{a4}}{I_{Z_e}U} \\
a_{22} &= -\frac{a_1^2C_{a1} - a_2^2C_{a2} - a_3^2C_{a3} - a_4^2C_{a4}}{I_{Z_e}U}
\end{align*}
\]

The yaw model that is used for lateral stability controller’s design can be derived by isolating the \( \bar{Y} \) and \( \bar{r} \) from equation (7) as follows:

\[
\begin{bmatrix}
\bar{Y} \\
\bar{r}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\bar{Y} \\
\bar{r}
\end{bmatrix} + \begin{bmatrix}
\frac{C_{a1}+C_{a2}}{m} & \frac{C_{a3}}{m} & \frac{C_{a4}}{m} & 0 \\
\frac{a_1C_{a1}+a_2C_{a2}}{I_{Z_e}} & \frac{a_3C_{a3}}{I_{Z_e}} & \frac{a_4C_{a4}}{I_{Z_e}} & 0
\end{bmatrix}
\begin{bmatrix}
\delta_3 \\
\delta_4
\end{bmatrix}
\times
\begin{bmatrix}
\delta_1
\end{bmatrix}
\]

(8)

Now, equations (7) and (8) are represented as in the state-space equation (9).

\[
\dot{x} = Ax + Bu
\]

(9)

The reference model considered the desired lateral coordinate of the desired path \( Y_d \), a zero desired lateral velocity to minimize the vehicle sideslip angle \( \beta \) that equal to \( \dot{Y}/U \), the path desired heading angle \( \theta \) as in equation (10), where \( X \) is the X-axis coordinates of the desired path, and the desired yaw rate \( \dot{\theta}_d \) as in equation (11), where \( \rho \) is the path curvature, which can be determined from equation (12).

\[
\theta_d = \tan^{-1}\frac{dY}{dX}
\]

(10)

\[
\dot{\theta}_d = \rho U
\]

(11)

\[
\rho = \frac{d^2Y/dX^2}{(1 + (dY/dX)^2)^{3/2}}
\]

(12)

Eventually, the reference model is filtered with a low-pass filter (LPF) transfer function to smooth the desired signal to match the vehicle dynamics and response as in equation (13), where \( \tau \) is a small delay time.

\[
LPF = \frac{1}{\tau s + 1}
\]

(13)

**Controllers design**

This section introduces the procedures that are used for developing the controllers using \( H_{\infty} \) control theory. The section starts with a brief about the control theory followed by a structure of the weighted and augmented plant.

**\( H_{\infty} \) path-following controller**

In general, \( H_{\infty} \) control design is based on minimizing the \( H_{\infty} \) norm of the lower fractional transformation \( F_1 \) of a closed-looped system (CLS) formed from a plant and controller \( G \) and \( K \), respectively. The \( H_{\infty} \) norm can be represented with the singular value (\( \sigma \)) of \( F_1 \) for the CLS as in the following equation:

\[
\|F_1(G, K)\|_{\infty} = \sup_{\omega \in \mathbb{R}} \sigma(F_1(G, K)(j\omega))
\]

(14)

This problem can be presented as a minimization problem as in equation (15), which is called the mixed sensitivity problem. In this equation, \( K \) denotes the desired controller that stabilizes the system \( G \) and minimizes the \( H_{\infty} \) norm of the sensitivity functions \( S \) and \( KS \), which are penalized by the performance and control input weight functions \( W_p \) and \( W_u \), to achieve good tracking performance and less control effort, respectively.

\[
K = \min_{} \text{stabilizing} \| \begin{bmatrix} W_pS \\ W_pKS \end{bmatrix} \|_{\infty}
\]

(15)

These weight functions can be absorbed in the interconnected or augmented system \( G_{\text{aug}} \) as in equation (16) and the \( H_{\infty} \) norm of the system in equation (15) can be represented as in equation (17) to include the constraints on the system.

\[
G_{\text{aug}} = \begin{bmatrix} W_p & -W_pG \\ 0 & W_u \\ I & -G \end{bmatrix}
\]

(16)
Considering the introduced weighting functions, the whole plant can be augmented for designing the $H\infty$ controller. In this figure, $\Delta$ is the model parameters uncertainty. The performance weight functions matrix is shown in equation (18), where $W_p \in \mathbb{R}^{5 \times 5}$. $W_{p1}, \ldots, W_{p4}$ are the weights that are used to penalize the tracking errors $(err_1, \ldots, err_4)$ between the reference signals $(ref_1, \ldots, ref_4)$ and the vehicle’s states presented by the lateral position, lateral velocity, heading angle, and the rate of change in the yaw motion. Meanwhile, $W_{p5}$ is used to penalize the error’s integration of the lateral position and ensure the convergence to zero at steady state. $W_u \in \mathbb{R}^{m \times m}$ is synthesized to penalize the control effort at high frequencies, where $m$ is the number of control inputs.

\[
W_p = \begin{bmatrix}
W_{p11} & 0 & 0 & 0 & 0 \\
0 & W_{p22} & 0 & 0 & 0 \\
0 & 0 & W_{p33} & 0 & 0 \\
0 & 0 & 0 & W_{p44} & 0 \\
0 & 0 & 0 & 0 & W_{p55}
\end{bmatrix}
\]  

(18)

The weighting functions in the frequency domain can be derived by converting the performance requirements of the CLS from the time domain, as can be presented in the following equations.

Equation (19) present the performance weight function $M_p$ is the maximum sensitivity gain, $\omega_{p}$ is the difference between upper and lower cutoff frequencies, $\epsilon_{p}$ is a parameter that presents the maximum desired error, and $n_{pu}$ is tuned to control the weight function steep41.

\[
W_{pu} = \left( \frac{S/\sqrt{M_p} + \omega_p}{S + \omega_p \sqrt{\epsilon_p} n_{pu}} \right)^{n_{pu}}
\]  

(19)

On the other hand, the control input weight functions $W_{uc}$ are formulated and tuned as a high-pass filter in the following equation:

\[
W_{uc} = \left( \frac{c_1 S + c_2}{S + c_3} \right)^{n_{uc}}
\]  

(20)

$K$ is the stabilizing $F_1(G_{aug}, K)$

\[
\min \left\{ \| F_1(G_{aug}, K) \|_\infty \right\}
\]  

(17)
Eventually, the control system block diagram for the augmented plant can be represented in Figure 2, where \( w \) denotes the plant inputs (\( \text{ref}_{1:4} \)), \( z \) is plant output (\( z_p \) and \( z_u \)), \( e \) is the measured outputs and controller inputs (\( \text{err}_{1:4} \)), and \( u \) is the controller output (\( \delta_{1:4} \)). The \( H_\infty \) control problem can be solved based on the state-space representation augmented plant as in equation (21). Moreover, the solution satisfies the condition \( \|T(S)\|_\infty < \gamma \), where \( T \) is the CLS formed by the augmented plant \( G_{aug} \) and the controller \( K \), and \( \gamma \) is constrained to be between [0.1, 5] to achieve robustness\(^4\).

\[
G_{aug} = \begin{cases} \dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) \\ z(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t) \\ e(t) = C_2x(t) + D_{21}w(t) \end{cases} \tag{21}
\]

### \( H_\infty \)-ARS lateral stability controller

The same theory and structure of the path-following controller are applied for the lateral stability controller. However, the controlled states and consequently the performance weights are decreased into two (\( W_{p11} \) and \( W_{p44} \)). In addition, the control inputs for this controller are the steering angle of the 3\(^{rd} \) and 4\(^{th} \), \( \delta_3 \) and \( \delta_4 \), respectively.

### Controllers’ implementation

This section introduces the design procedure and design requirement limitations for each control case. The design includes the performance and control weighted
functions that are selected and tuned to satisfy an accepted range for stability and performance. Moreover, a frequency domain analysis is conducted to demonstrate the performance and stability robustness against the velocity uncertainty.

**Front steering path-following controller**

Equation (7) is used to design the controller, where the steering of the 3rd and 4th axles $\delta_3$ and $\delta_4$ are substituted by zero to demonstrate fixed rear axles. The performance weight functions are selected as in Table 1, and the $H_\infty$ problem is solved using Matlab software toolbox with constraining the performance index $\gamma$ to be between 0.1, 5]. At first, the optimizer failed to find a solution that satisfies the performance requirements. So, the problem is solved at a fixed nominal speed 20 km/h, with releasing the constrain on the control input.

Figure 3 presents the frequency analysis of the uncertain CLS for the determined controller. Figure 3(a) shows that the output lateral position satisfies the designed performance weighted function $W_{p_{11}}$. The function $W_{p_{11}}$ is chosen to result in an acceptable lateral error at high frequencies. In addition, the performance weighted function $W_{p_{55}}$ penalizes the lateral error integration at low frequencies, to ensure that the steady-state lateral error will reach zero, as presented in Figure 3(b). The effect of $W_{p_{55}}$ is reflected in the error at Figure 3(a) as the gain at low frequencies are very small (below $-100$ dB). Figure 3(c) shows the control input ($\delta_1$) response to the input reference, and it can be seen that there is almost no constrain on the control input at high frequency. Figure 4(a) shows the

![Figure 6. CLS frequency response of the control input. (a) $\delta_1$ to the reference inputs, (b) $\delta_3$ to the reference inputs, and (c) $\delta_4$ to the reference inputs.](image)

![Figure 7. Robustness of the front steering CLS. (a) Stability robustness and (b) performance robustness.](image)

| Design parameters | ARS          |
|-------------------|-------------|
| $W_{p_1}$         | $s^2 + 80s + 1600$ |
| $W_{p_3}$         | $4s^2 + 120s + 900$ |
| $W_{u_1}$         | 0.01        |
| $W_{u_{11}}$      | $0.05s + 11.18$ |
| $W_{u_2}$         | $0.0001s + 10$ |
| $W_{u_{23}}$      | $0.05s + 5.97$ |
| $\gamma$          | 0.53        |

Table 2. Lateral stability controller’s design parameters.
stability performance of the uncertain CLS and shows that the system attains robust stability as the structured singular value bounds $\mu$ are $<1$. However, the system does not achieve robust performance at high frequencies as it violates the controller performance weighted function $W_{11}$, as shown in Figure 4(b).

AWS path-following controller

Equation (7) is used with no modifications to design the AWS path-following controller that utilizes the steering of the first, 3rd, and 4th axles, and the selected weighted functions are illustrated Table 1. Including the steering of the rear axles in the model reflects several advantages on the tightening procedures as follows:

- The optimizer was able to find a solution considering the velocity uncertainty at nominal forward velocity $60$ km/h.
- Tightening the constrains on the lateral error by increasing the gain of $W_{p11}$ at low frequencies, as can be noticed in Figure 5(a).
- Adding constrain on the heading angle tracking error at low frequency ($W_{p22}$), as presented in Figure 5(b), to ensure faster stabilization of the vehicle and less swing.
- Increasing the constrain on the lateral error integration weight function $W_{p55}$ for a higher frequency to ensure that the steady-state lateral error will reach zero much faster, as observed in Figure 5(c).
- Penalizing the heading angle tracking error integration at low frequency, as shown in Figure 5(d), to ensure a zero steady-state heading angle at the end of the maneuver by adding the weighted function $W_{p66}$.
- Penalize the control inputs maximum steering angle at high frequencies, as shown in Figure 6.

Figure 8. CLS frequency response. (a) Response of the lateral velocity, (b) response of the control input $\delta_3$ to the reference inputs, and (c) response of the control input $\delta_4$ to the reference inputs.

Figure 9. Robustness of the front steering CLS. (a) Stability robustness and (b) performance robustness.

Figure 10. NATO AVTP 03-160 W DLC setup$^{47,19}$. 
Despite the advantages of adding the steering of the rear axles on the design procedures, the CLS achieves robust stability but lacks performance robustness, as can be concluded from Figure 7(a) and (b), respectively. However, by observing Figures 5 and 6, it can be noticed that the violation of the performance requirements is in an acceptable range and still, can achieve good performance.

ARS lateral stability controller

In this section, an $H_{\infty}$-ARS lateral stability controller is developed to be integrated with the front steering path-following controller. The controller is developed to penalize the vehicle’s lateral velocity and consequently limits its sideslip and improves the stability while performing steering. The performance and control weighted functions of the developed ARS are presented in Table 2.

Figure 8 presents the frequency response of the CLS, where Figure 8(a) shows the lateral velocity response and Figure 8(b) and (c) shows the response of the control inputs $\delta_3$ and $\delta_4$ to the reference input, respectively. It can be seen that the response of the uncertain CLS meets the required weighted functions. Furthermore, the control weight functions $W_{u1}$ and $W_{u2}$ are chosen to ensure that the steering angle of the $4^{th}$ axle is higher than the $3^{rd}$ axle at high frequencies. Besides, it ensures satisfying the desired performance with minimum use of the rear axles’ steering angles. The CLS stability and performance robustness are illustrated in Figure 9(a) and (b), respectively. The figures show that the controlled system attains robust stability and performance, which allows binding the design requirements if needed.

Evaluation and simulation results

A nonlinear model for an $8 \times 8$ combat vehicle, with a 22 DoF model, was developed on TruckSim software and used in the evaluation. The vehicle is powered by a diesel engine and each axle has one mechanical differential (four in total), and independent suspensions. The model was validated in43,44 and exploited in various research studies and showed high fidelity45,19,13.

The controllers are compared and evaluated by performing the NATO DLC as in Figure 10 at various

![Image](image_url)
speeds (40 and 100 km/h) and different friction coefficient (0.5 and 0.85). These coefficients of friction (CoF) demonstrate two driving scenarios. The value 0.5 presents driving the vehicle on wet asphalt, while the 0.85 presents driving on dry asphalt.

**Low-speed evaluation using DLC maneuver**

In this section, the controllers will be evaluated at low speed ($U = 40$ km/h) and on roads with various CoF (0.85, and 0.5).

At friction coefficient = 0.85 and forward speed = 40 km/h.

Figure 11(a) and (b) shows the vehicle trajectory and the corresponding lateral error when driving at speed 40 km/h over a surface with CoF 0.85. It can be seen that all cases have finished the maneuver and the AWS results in a minimum error, followed by other cases. However, the error in the case of front steering with and without ARS is generated due to a phase shift from the desired trajectory. Besides, the front steering cases can stabilize the vehicle faster than the AWS case.

The first, third, and fourth axle’ average steering angles are illustrated in figure 12(a), (b), and (c), respectively. It is noticed that there is no huge difference between front steering-based controllers (front steering and front-ARS), while the front steering angle of the AWS is slightly higher. In contrast, there is a big difference in the 3rd and 4th steering angles between the AWS and
front-ARS controllers, where the AWS exhibits higher angles.

The yaw rate response is shown in Figure 13(a), where there is almost no difference between the three cases. Figure 13(b) presents the vehicle sideslip angle for each case, it is notable that for almost the same yaw rate, the integrated front-ARS generates the least sideslip. Meanwhile, the AWS reaches the same sideslip as the front steering controller. However, due to the added weight for tracking the yaw angle at low frequencies, AWS generates a uniform sideslip that has the same shape and phase as the yaw rate signal.

At friction coefficient $= 0.5$ and forward velocity $= 40$ km/h. The vehicle trajectory when driving on a surface with friction coefficient 0.5 with speed $= 40$ km/h is illustrated in Figure 14(a), while the corresponding error is presented in Figure 14(b). The result is almost the same as maneuvering with the same speed on a hard surface with CoF 0.85, and it has also the same yaw rate and sideslip response.

### High-speed evaluation using DLC maneuver

In this section, all cases are evaluated at the maximum speed of each test on roads with various CoF (0.85 and 0.5). This speed is determined based on a pass/fail criteria, where the vehicle should finish the maneuver without stepping on the cones and without losing its stability.

At friction coefficient $= 0.85$ and forward velocity $= 100$ km/h. The vehicle trajectory is presented in Figure 15(a) when performing the maneuver with speed 100 km/h and all cases pass the maneuver successfully. The integrated front-ARS results in less mean square error and
stabilize the trajectory faster than other cases, as can be observed from Figure 15(b). However, it has a higher steady-state error due to the tracking limitations of the front-steering controller that was integrated with the ARS.

The front steering controller resulting in the highest average front steering angle among all cases as shown in Figure 16(a). The front-ARS results in a higher 3rd axle average steering than the AWS as presented in Figure 16(b), meanwhile, they reach the same 4th axle average steering angle as illustrated in Figure 16(c). In addition, it can be noticed that the front-ARS results in parallel steering, which means that the rear and front axles steer in the same direction. On contrary, AWS results in a random steering pattern, which is sometimes parallel and others counter.

The dynamic response of all cases is illustrated in Figure 17. The less reduction in the longitudinal velocity is achieved by the integrated front-ARS, followed by the AWS, then the front steering that produces the highest reduction as presented in Figure 17(a). Meanwhile, the front-ARS results in the least yaw rate because of the very low sideslip as observed in Figure 17(b) and (c), respectively. In addition, the AWS and the front-ARS stabilize almost at the same time and faster than the standalone front steering.

At friction coefficient = 0.5 and forward velocity = 100 km/h. In this section, the controllers are evaluated at high speed ($U = 100$ km/h) and low-CoF (0.5), respectively. This test shows the importance of integrating a lateral stability controller in autonomous vehicles for crash mitigation applications.

It is shown in Figure 18(a) that all other cases fail to stabilize the vehicle and finish the maneuver except for the integrated front-ARS controllers, which pass the maneuver despite the generated high lateral error, as presented in Figure 18(b). As can be observed in Figure 19(a), there is a saturation in the first 1st axle’s steering angle. Despite that, the ARS controller can stabilize the vehicle by exhibiting small steering angles on the 3rd and 4th axles, as presented in Figure 19(b) and (c), respectively.

The yaw rate and sideslip responses show the effectiveness of the ARS in limiting the yaw rate and sideslip as shown in Figure 20(a) and (b), respectively and regain the vehicle stability.
Conclusions

In this paper, three H\textsubscript{\infty} controllers are developed to control the vehicle path. The first is a path-following controller that utilizes the steering of the front axles only. The second is also a path-following controller, which exploits the steering of all axles. The third controller integrates the front steering path-following controller with a lateral stability ARS controller.

Frequency analysis is conducted for the closed-loop control systems. The analysis shows that including the steering of the rear axles enhances the path-following performance by tightening the constraints on the lateral position tracking, heading angle tracking, and control inputs responses. Furthermore, the frequency response analysis of the developed ARS-stability controller shows the effectiveness of the rear axles in constraining the vehicle sideslip with minimum utilization of the rear axles.

The three cases were compared and evaluated by performing series of DLC maneuvers. The results show that AWS system generates the minimum lateral error. In addition, penalizing the heading angle tracking error and its integration adds the benefits of fast stabilization at a relatively high-speed maneuver. It is also observed that the AWS generates a sideslip with the same phase as the yaw rate at low speeds, which increases the vehicle maneuverability, and a counter phase at high speed, which reduces the vehicle sideslip and enhances the stability. On the other hand, the ARS generates a counter-phase sideslip in all driving conditions resulted in a reduction of the sideslip to enhance stability. This behavior shows the superiority of ARS in performing maneuvers at the limited-handling conditions, in which the front steering and AWS path-following controllers failed.

It is more effective to design a path-following controller utilizing all-axles steering than including an ARS to the front steering at low speeds. In contrast, it is more beneficial to integrate a lateral stability-ARS with a front steering path-following controller at high speeds. Therefore, it is recommended to design a parametric variable or a switching controller that alters the objective of steering the rear axles considering the road friction coefficient and vehicle’s driving speed.

Finally, combat vehicles are required to be operated in different driving conditions, including on-roads and off-roads. Therefore, the off-road analysis will be performed in a future study.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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Figure 20. Vehicle’s dynamic response (CoF = 0.5, speed = 100 km/h). (a) Yaw rate and (b) Sideslip.
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**Appendix 1**

**Notation**

| Symbol | Description |
|--------|-------------|
| \(a_i\) | distance between vehicle center of gravity \(i\)th axle |
| \(A\) | system states matrix |
| \(B\) | system input matrix |
| \(c_i\) | positive constant value |
| \(c_{ij}\) | steering ratio between the average steering angle of the \(i\)th to the \(j\)th axles |
| \(C\) | state-space output matrix |
| \(C_{ni}\) | cornering stiffness of the \(i\)th tire |
| \(dX\) | change in the \(X\)-coordinate |
| \(dY\) | change in the \(Y\)-coordinate |
| \(D\) | state-space feed-forward matrix |
| \(e(t)\) | augmented plant controller input |
| \(F_1\) | lower fractional transformation |
| \(F_{yi}\) | lateral forces on the \(i\)th tire |
| \(G\) | system’s transfer function |
| \(G_{aug}\) | augmented/interconnected plant |
| \(I_{zz}\) | vehicle’s mass moment of inertia about the Z-axis |
| \(K\) | controller gain |
| \(M_{pi}\) | system maximum sensitivity |
| \(m\) | vehicle mass |
| \(n_{pi}\) | performance weight function |
| \(n_{ui}\) | controller weight function |
| \(r\) | vehicle yaw rate |
| \(T(S)\) | closed-loop system |
| \(u\) | controller output or system control input |
| \(U\) | vehicle longitudinal velocity |
| \(V\) | vehicle lateral velocity |
| \(W_p\) | performance weight function |
| \(W_u\) | control weight function |
| \(x\) | system state variables |
| \(X\) | desired path \(X\)-coordinates |
| \(x\) | state vector |
| \(Y\) | desired path \(Y\)-coordinates |
| \(z(t)\) | augmented plant output |
| \(\alpha_i\) | slip angle of the \(i\)th tire |
| \(\beta\) | vehicle sideslip |
| \(\gamma\) | performance index |
| \(\delta_i\) | average steering angle of the \(i\)th tire |
| \(\epsilon_{pi}\) | tuning parameter |
| \(\tau\) | positive constant |
| \(\rho\) | path curvature |
| \(\theta\) | vehicle heading angle |
| \(\theta_d\) | desired heading angle |
| \(\sigma\) | singular value |
| \(\omega(t)\) | system disturbances |
| \(\omega_{pi}\) | system bandwidth |

**Abbreviations**

- AFS: active front steer
- ARS: active rear steer
- AWS: all wheel steer
- CLS: closed-loop system
- CoF: coefficient of friction
- DLC: double lane change
- DoF: degree of freedom
- HPF: high-pass filter
- TVC: torque vectoring control
- LPF: low-pass filter