The influence of strong connections on dynamics of neuron population with long-tailed synaptic weight distribution

E A Chizhkova¹, A V Chizhov¹²
¹Ioffe Physical-Technical Institute of the Russian Academy of Sciences, Saint-Petersburg, Russia
²Sechenov Institute of Evolutionary Physiology and Biochemistry of the Russian Academy of Sciences, Saint-Petersburg, Russia

Abstract. This work is devoted to exploring the influence of long-tailed synaptic weights distribution on neural population dynamics. We show numerically that both strong sparse and very weak connections pay crucial role for spontaneous activity whereas they almost do not influence on driven population activity. Moreover, according to obtained results, we suggest that neurons with strong connections form multiple chains, which are the core of spontaneous activity, whereas neurons with weak connections pay little role of partially enhancing of population activity.

1. Introduction

Recently, cortical synaptic weights have been shown experimentally to obey a long-tailed unimodal distribution peaking near the lowest values which can be fitted with lognormal law [1]. These results gives birth to series of theoretical and experimental works dedicated to finding a shape of synaptic weights distribution in different neural brain structures [2,3,4], genesis of such distribution [5,6], features of networks dynamics with such connection strengths distribution [7,8,9] and numerical description of feed-forward network with different, including long-tailed, synaptic weights distribution [10].

In work of J. Teramae, et al. [7] – which was an inspiration for this study – some features of network with lognormal weights distribution were theoretically obtained, such as producing of spontaneous activity and conditions of the fidelity of spike transmission along the strongest synapses. The results were obtained using numerical simulations of system constituted from integrate-and-fire neurons with lognormal coupling strengths distribution, which form were determined according to experimental results. All connections were divided into weak-dense and strong-sparse and it was concluded that strong-sparse synapses form multiple unidirectional synaptic pathways while weak-dense connections redistribute excitatory activity routed reliably on strong connections over the network as optimal noise sources to sustain spontaneous firing of recurrent networks.

In this work we follow the trend of studying the features of such network, more exactly the influence of neurons with strong connections and with only weak once and the influence of strong and weak connections on population activity. In the next section we will demonstrate the influence of presence of sparse strong connections and weak ones on current-driven activity and on spontaneous activity via eliminating them from the system, then we will do something similar to see role of neurons having strong connections and not in network.
2. The influence of strong and weak connections on neuron population activity.

In order to exploring the influence of sparse strong and weak dense connections on population activity we firstly reproduced the network system and repeated numerical simulation presented in work Teramae et al, specifically we obtained steady low-frequency irregular spontaneous activity (fig.1A). Simulations were performed for population with lognormal coupling strengths distribution constructed from 10000 excitatory and 2000 inhibitory integrate-and-fire randomly connected neurons, to trigger a spontaneous firing Poisson spike trains were applied to all neurons during 100 ms (see Appendices for more details).

After that we wanted to see behavior of the system driven with step of current after spontaneous activity. Then we eliminated strong connections (> 10 mV) just before driven state to see how strong connections influence on this activity. In another case strong connections were eliminated from the start. Figure 1 shows that current-driven activity of population after spontaneous activity differ from such activity after silent state with less amplitude followed by faster setting of stationary state. The last two figures are almost equivalent to first ones and show only little difference in firing rate of stationary phase that leads to conclusion that the strong connections almost do not influence on driven activity, but have crucial role for spontaneous activity (silent state in second figure is result of deleting strong connections, not of turning off initial Poisson spike train).

Figure 1. Population activity (rate of firing neurons) under different cases. A: initial system, B system with eliminated strong connections (>10 mV) just before driven activity (left graph) and from the start (right graph), C system with eliminated weak connections (<0.3 mV) from the start. Arrows point the time of current supply.
Similarly, we eliminated weak connections (<0.3 mV) to see their role in spontaneous activity. Spontaneous activity having disappeared in this case either (fig. 1C), weak connections pay crucial role for such activity as well.

3. **The influence of neurons with strong connections and without on neurons population activity.**

Similarly, to explore the role of neurons having strong connections and those having only weak ones we made simulations with eliminating such neurons. Although we set connection probability to only 14%, it appears that near 97% of neurons have at least 1 strong connection with EPSP more than 10 mV due to huge number of neurons and therefore huge number of connections – 1000 connections per neuron. There is no surprise that system continued to produce spontaneous activity without 3% of neurons with only week connections.

Then we eliminated neurons without as input thus as output strong connections that represented near 50% of all neurons. After that, system continued to produce spontaneous activity and even without dramatic casualties in frequency (fig. 2A). More surprising result was keeping spontaneous generation by system constituted from neurons having strong connections only in one directions or not having ones, but with less frequency (fig. 2B).

![Figure 2. Population activity (spiking neurons rate) under different cases. A: system with eliminated neurons without as input thus as output strong connections >10 mV (left graph) and without strong connections >16 mV (right graph), B system with neurons having strong connections only in one directions or not having ones (left graph) and without strong connections >16 mV (right graph). Arrows point the time of current supply.](image)

It seems that the main role in forming spontaneous activity belongs to chains consisted of neurons with strong connections, serving as sources of input stimulus via weak connections for each other and for the rest of neurons. This looks more likely for real systems that were shown experimentally and theoretically to have correlated strengths of synaptic connections, implying that strong connections are even more clustered [1,8]. Nevertheless, spontaneous activity can be formed without chains made of strong connections, as in figure 2B (left), or by only weak connections (<10 mV). Actually, we have
even met parameters with which system is keeping to generate low-frequency spontaneous activity without strong connections (fig. 3) Such activity has less frequency than original system and thus has secondary role in forming spontaneous activity and should have rather undirected propagation of excitation.

Figure 3. System with eliminated strong connections $>10$ mV.

After eliminating neurons without connections with EPSP more than 16 mV (connections are suggested to be formed by more than one synapse), the rest part (30%) kept to produce activity. It was the least found part of system capable to produce spontaneous activity without dramatic casualties in frequency (fig. 2A). Without this part system kept silence before driven activity (fig. 2B).

4. Discussion

In this study, we have explored role of neurons having strong connections and those having only weak ones and role of strong and weak connections themselves in spontaneous and driven population activity. Results are briefly summarized in table 1, presenting changes in driven and spontaneous activity under different cases. We found that strong connections almost don’t have influence on driven activity, which complements the result obtained in [10] for feed-forward networks with lognormal and Gaussian synaptic current distributions, and described some features of driven activity following after spontaneous one. On the other hand, strong and weak connections are crucial for producing of spontaneous activity, what was illustrated in [7] as well. Then we showed neurons with strong connections to pay main role in forming spontaneous activity and found the least part that could be derived with our methods reproducing spontaneous activity with frequency close to original system frequency.

Table 1. Influence of strong and weak connections and neurons with and without strong connections on population activity. Here, sc, wc are strong and weak connections, sn, wn are neurons with and without strong connections, respectively.

|          | spontaneous activity | driven activity     |
|----------|----------------------|---------------------|
| sc (>10 mV) | crucial              | ~ no influence      |
| wc (<0.3 mV) | crucial              | ~ no influence      |
| sn (>16 mV) | crucial              | ~ no influence      |
| wn (<16 mV) | little enhancing of frequency | ~ no influence      |

These results help to understand which model we can use for different cases for mathematical description of population activity. For example, it seems appropriate using statistical models for
current-driven state of system whereas spontaneous activity should be described by direct numerical simulation at least that part that forms such activity or by more sophisticated approach.

It’s notable that presented system is rather instable in respect of system parameters. This implies that there is rather little range of parameters values when system produces spontaneous activity and does not present epileptic-like activity. Instability can be a result of using simple conductance-base model for neuron without adaptive currents which could prevent too high activity and expand range parameters presenting stable spontaneous activity.

5. Appendices
We use the following integrate-and-fire model for describing activity of one neuron:

$$\frac{dV}{dt} = -\frac{1}{\tau_m} (V - V_L) - g_E (V - V_E) - g_I (V - V_I),$$

where $V$ is the membrane potential, $\tau_m$ is the membrane time constant, $V_{L,E,I}$ are reversal potentials for leak, excitatory and inhibitory postsynaptic currents, $g_{E,I}$ are the excitatory and inhibitory synaptic conductances normalized by the membrane capacitance ([$\text{ms}^{-1}$]) and described by

$$\frac{dg_X}{dt} = -\frac{g_X}{\tau_s} + \sum_j G_{X,j} \sum_{s_j} \delta(t - s_j - d_j), \quad X = E, I$$

where $\delta(t)$ is the delta function, $G_j$, $d_j$, $s_j$ are the weight, delay and spike timing of synaptic input from the $j$-th neuron, respectively, $\tau_s$ is the decay constant. The values of $G_j$ for excitatory-to-excitatory connections are distributed such that the amplitude of EPSPs $x$ measured from the resting potential obey a lognormal distribution

$$p(x) = \frac{\exp\left[-\frac{(\log x - \mu)^2}{2\sigma^2}\right]}{\sqrt{2\pi}\sigma x},$$

where $\sigma=1.0$ and $\mu^2=\log(0.2)$, the latter corresponds the mode=0.2. Excitatory-to-inhibitory, inhibitory-to-excitatory and inhibitory-to-inhibitory synapses’ weights have constant values of $G_E=0.018$, $G_I=0.002$ and $G_{I,I}=0.0025$, respectively. The probabilities of inhibitory and excitatory connections are 0.5 and 0.14, respectively. Excitatory-to-excitatory synaptic transmissions fail with probability $p_F=a/(a+EPSP)$, where $a=0.1$ [mV].

Picked the other parameters values are the following: $V_L$, $V_E$, $V_I$ are -70[mV], 0[mV] and -80 [mV], respectively, spike threshold is -50 mV and V is -60 mV after spiking, the refractory period is 1 ms, $\tau_m$ is 20 ms for excitatory and 10 ms for inhibitory neurons, $\tau_s$ is 4.7 for excitatory and 3.9 for inhibitory neurons and these values are picked in order to system didn’t oscillate, synaptic delays are chosen randomly between $d_0-1$ to $d_0+1$ ms, where $d_0=2$ for excitatory-to-excitatory connections and $d_0=1$ for other connection types. For initiation driven activity we use step of current different for different neurons with mean value 4[nA/nF] and variance 2[nA/nF] (current is normalized by the membrane capacitance).

6. References
[1] Song S, Sjostrom P, Reihl M, Nelson S, Chklovskii D 2005 PLoS Biology 3 e68
[2] Hromadka T, Deweese MR, Zador AM 2008 PLoS Biology 6 e16
[3] Lefort S, Tomm C, Floyd, et al 2009 Neuron 61 301
[4] Ikegaya Y, Sasaki T, Ishikawa D, Honma N, Tao K, et al 2012 Cereb Cortex 23 293
[5] Zheng P, Dimitrakakis C, Triesch J 2013 PLoS Comput Biol 9 e1002848
[6] Gilson M, Fukai T 2011 PLoS One 6 e25339
[7] Teramae J, Tsubo Y, Fukai T 2012 Sci Rep 2 485
[8] Koulakov A, Hromádka T, Zador A 2009 J Neurosci 29(12) 3685
[9] Klinshov V, Teramae J, Nekorkin V, Fukai T 2014 PLoS One 9 e94292
[10] Iyer R, Menon V, Buice M, Koch C, Mihalas S 2013 PLoS Comput Biol 9 e1003248