Negative refraction by quantum vacuum

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Abstract

The phase velocity of light is co–parallel to the direction of energy flow in classical vacuum. However, in certain uncommon materials, these two vectors can be oppositely directed, in which case the phase velocity is termed ‘negative’. This negative phase velocity (NPV) gives rise to many exotic phenomenons, such as negative refraction, inverse Doppler shift and inverse Čerenkov radiation, and has technological allure. According to quantum electrodynamics, the presence of a magnetostatic field makes vacuum an anisotropic medium for the passage of light. Under the influence of a sufficiently strong magnetostatic field, vacuum supports NPV. Such ultrastrong magnetic fields are believed to arise due to dynamo action in newborn neutron stars and in binary neutron star mergers, for examples. In view of the possible occurrence of negative refraction, the influence of ultrastrong magnetostatic fields must be carefully taken into account in astronomical observations relating to neutron stars and associated gamma–ray bursts.

Key words: negative–phase–velocity propagation, quantum electrodynamics, vacuum birefringence

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In the usual descriptions of light propagation, as learned by generations of students from standard textbooks on optics [8] and electromagnetics [9], the phase velocity casts a positive projection onto the direction of energy flow, as provided by the time–averaged Poynting vector. That is, the phase velocity is *positive*. However, in certain special circumstances, it is possible for the phase velocity to cast a negative projection onto the time–averaged Poynting vector; i.e., the phase velocity can be *negative*.

The transition from positive phase velocity (PPV) to negative phase velocity (NPV) is of particular interest because NPV underpins the much–heralded phenomenon of negative refraction [1, 2], as well as many other exotic phenomena such as inverse Doppler shift and inverse Čerenkov radiation [3]. The scientific and technological possibilities offered by negative refraction — such as the construction of near–perfect lenses from planar slabs of NPV–supporting materials — have been widely reported on [2, 10]. Much of the research effort has been directed towards the realization of metamaterials which can be used to achieve negative refraction. These metamaterials are artificial composite materials, often having complex micromorphologies [1, 11, 12]. Metamaterials with simple micromorphologies can also support NPV, as is demonstrated by certain homogenized composite materials [13]. A significant milestone was reached recently by experimentalists through the fabrication of metamaterials which support negative refraction at the optical wavelengths [11, 12].

We report here on the manifestation of NPV in a quite different context, namely in vacuum under the influence of a magnetostatic field $\mathbf{B}_s = |\mathbf{B}_s| \hat{\mathbf{B}}_s$. In classical vacuum, the phase velocity is positive and the passage of light is unaffected by $\mathbf{B}_s$, as reported by an inertial observer. However, this is not the case for the quantum electrodynamical (QED) vacuum. The QED vacuum is a nonlinear medium which can be linearized for rapidly time–varying plane waves. Thereby, for propagation of light, QED vacuum is represented by the anisotropic dielectric–magnetic constitutive relations [4]

$$
\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu_0 \mu \mathbf{H},
$$

with $\varepsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}$ and $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$ being the permittivity and permeability of classical vacuum, respectively. The relative permittivity and relative permeability dyadics of QED vacuum have the uniaxial forms

$$
\varepsilon = \begin{cases} 
1 - 8a|\mathbf{B}_s|^2 & \mathbf{I} - \mathbf{B}_s \mathbf{B}_s \\
1 + 20a|\mathbf{B}_s|^2 & \mathbf{B}_s \mathbf{B}_s
\end{cases}
$$

$$
\mu = \begin{cases} 
1 & \mathbf{I} - 8a|\mathbf{B}_s|^2 \\
1 - 24a|\mathbf{B}_s|^2 & \mathbf{B}_s \mathbf{B}_s
\end{cases}
$$

with $a = 8.854 \times 10^{-12} \text{ F m}^{-1}$ and $\mathbf{I}$ the identity matrix.
respectively, where $I$ is the $3 \times 3$ identity dyadic and the constant $a = 6.623 \times 10^{-26} \text{H}^{-1} \text{kg}^{-1} \text{m}^2 \text{s}^2$. The constitutive dyadics (2) were derived by Adler [4] from the Heisenberg–Euler effective Lagrangian of the electromagnetic field [14, 15].

Suppose that plane waves with field phasors

$$E(r) = E_0 \exp (ik \cdot r), \quad H(r) = H_0 \exp (ik \cdot r)$$

(3)

and wavevector $\hat{k} = |k| \hat{k}$ propagate through the QED vacuum described by (1) and (2). For simplicity, let $\hat{B}_s$ and $\hat{k}$ both be aligned with the Cartesian $z$ axis. In this case there is only one wavenumber, namely $|k| = \omega \sqrt{\varepsilon_0 \mu_0}$ (unlike the $\hat{B}_s \neq \hat{k}$ case where there are two wavenumbers for each propagation direction, as is described in the Supplementary Information section). The time–averaged Poynting vector $\mathbf{P}$ and phase velocity $v_p$ are straightforwardly derived by combining (1), (2) and (3) with the Maxwell curl postulates [16]. Thus, the projection of the phase velocity onto the time–averaged Poynting vector emerges as

$$v_p \cdot \mathbf{P} = \frac{1}{2\mu_0} \left( 1 - 8a |\mathbf{B}_s|^2 \right) |E_0|^2.$$  

(4)

Clearly, the phase velocity is positive for $|\mathbf{B}_s| < 1/\sqrt{8a}$ but negative for $|\mathbf{B}_s| > 1/\sqrt{8a}$. In fact, the same inequalities for PPV and NPV hold when the direction of planewave propagation is perpendicular to the direction of the magnetostatic field, as is described in the Supplementary Information section.

Magnetic fields of magnitude greater than $1/\sqrt{8a} = 1.374 \times 10^{12} \text{ Tesla}$ are needed for QED vacuum to support NPV propagation. To find fields of this magnitude we turn to astrophysical environments. Ultrastrong magnetic fields, developed by dynamo action, are associated with certain neutron stars. For example, fields of the order of $10^{10}–10^{12} \text{ Tesla}$, have been estimated for newborn neutron stars, such as soft gamma repeaters [5, 6]. Considerably stronger fields, of the order of $10^{11}–10^{14} \text{ Tesla}$, are predicted to arise during the merger of a binary neutron star system [7]. Accordingly, NPV propagation may be expected to occur in these neutron–star environments. Across boundaries between two regions, one of which experiences a magnetostatic field of magnitude $1/\sqrt{8a} < 1.374 \times 10^{12} \text{ Tesla}$ and the other a magnetostatic field of magnitude $1/\sqrt{8a} > 1.374 \times 10^{12} \text{ Tesla}$, negative refraction of light will occur.

Our results have far–reaching implications for observational astronomy and theoretical astrophysics. The possibility of negative refraction arising from ultrastrong magnetic fields should be taken into consideration in estimating astronomical positions, particularly if the light path from
the detector to the point of observation traverses regions in the vicinity of neutron stars, for example. Furthermore, negative refraction may well influence the propagation of gamma–ray bursts and other electromagnetic radiation emitted by neutron stars.

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**Supplementary Information**

In our Letter we demonstrate that electromagnetic plane waves can propagate in a quantum electrodynamical (QED) vacuum with negative phase velocity (NPV), provided that a sufficiently strong magnetostatic field is acting. Here we provide further details of the planewave analysis. As in our Letter, suppose that plane waves with field phasors (3) propagate through the QED vacuum described by (1) and (2). The wavevector \( \mathbf{k} = |\mathbf{k}| \hat{\mathbf{k}} \) and corresponding phasor amplitude \( \mathbf{E}_0 = (E_{0x}, E_{0y}, E_{0z}) \) (and similarly \( \mathbf{H}_0 \)) are straightforwardly deduced by combining the frequency–domain constitutive relations (1) with the source–free Maxwell curl postulates

\[
\begin{align*}
\nabla \times \mathbf{H}(\mathbf{r}) + i\omega \mathbf{D}(\mathbf{r}) &= \mathbf{0} \\
\nabla \times \mathbf{E}(\mathbf{r}) - i\omega \mathbf{B}(\mathbf{r}) &= \mathbf{0}
\end{align*}
\]

For our purpose, it suffices to consider only two cases: (i) propagation parallel to \( \hat{\mathbf{B}} \), and (ii) propagation perpendicular to \( \hat{\mathbf{B}} \). Without loss of generality, our coordinate system is oriented such that \( \hat{\mathbf{B}} = \hat{\mathbf{z}} \).
For the case $\hat{k} = \hat{z}$, the dispersion relation yields one wavenumber, namely $|k| = \omega \sqrt{\epsilon_0 \mu_0}$. The electric field phasor lies in the Cartesian $xy$ plane but is otherwise arbitrary. The scalar product of the phase velocity and time–averaged Poynting vector is given as

$$v_p \cdot P = \frac{1}{2\mu_0} (1 - 8a|B_\perp|^2) \left(|E_0^x|^2 + |E_0^y|^2\right).$$

(6)

The case where $\hat{k} = \hat{x}$ is more complicated as there are two possible wavenumbers, namely $|k| = k_{1,2}$ where

$$k_1 = \omega \sqrt{\epsilon_0 \mu_0} \sqrt{\frac{1 - 20a|B_\perp|^2}{1 - 8a|B_\perp|^2}}, \quad k_2 = \omega \sqrt{\epsilon_0 \mu_0} \sqrt{\frac{1 - 8a|B_\perp|^2}{1 - 24a|B_\perp|^2}}.$$

(7)

The wavenumber $k_1$ corresponds to a propagating mode for $|B_\perp| < 1/\sqrt{20a}$ or $|B_\perp| > 1/\sqrt{8a}$; otherwise it is an evanescent mode. Similarly, the wavenumber $k_2$ corresponds to a propagating mode for $|B_\perp| < 1/\sqrt{24a}$ or $|B_\perp| > 1/\sqrt{8a}$; otherwise it is an evanescent mode. The electric field phasor is directed along the Cartesian $z$ axis for the wavenumber $k_1$, whereas it is directed along the Cartesian $y$ axis for the wavenumber $k_2$. The scalar product of the phase velocity and time–averaged Poynting vector is given as

$$v_p \cdot P = \begin{cases} 
\frac{1}{2\mu_0} (1 - 8a|B_\perp|^2) |E_0^x|^2 & \text{for } |k| = k_1 \\
\frac{1}{2\mu_0} (1 - 24a|B_\perp|^2) |E_0^y|^2 & \text{for } |k| = k_2
\end{cases}.$$

(8)

Therefore, plane waves propagate with NPV in QED vacuum provided that $|B_\perp| > 1/\sqrt{8a}$, regardless of whether the propagation direction is parallel or perpendicular to $B_\perp$. 