An Enhanced Particle Filter for Detection and Tracking Maneuvering Dim Target

Zhi-chao BAO\textsuperscript{1,*}, Qiu-xi JIANG\textsuperscript{1}, Fang-zheng LIU\textsuperscript{1} and Deng-rong XU\textsuperscript{2}

\textsuperscript{1}National University of Defense Technology, Number 460, Huangshan road, Hefei, Anhui Province, China
\textsuperscript{2}93046 Army, Qingdao, China
*Corresponding author

Keywords: Particle filter, Track-before-detect, Target tracking.

Abstract. Track-before-detect (TBD) is a powerful technique for detection and tracking dim targets. To solve the problem of target maneuvering, the basic idea of multiple model (MM) algorithm is introduced into the basic multiple model particle filter TBD, a novel and enhanced particle filter algorithm is proposed in this paper. The algorithm proposed formulates the probability of existence calculation respectively, avoiding the need for hybrid estimation. This simulation results show that the algorithm proposed is efficient in dim maneuvering target tracking.

Introduction

Due to the low signal-to-noise ratio of dim targets, traditional detection methods are difficult to detect such targets, and some maneuvering dim targets, such as stealth fighters, increase the difficulty of detection and tracking. In recent years, scholars in various countries have proposed the idea of continuously accumulating target information in multi-frame observation data according to the continuity of target motion and the relevance of the target in the frame. In order to distinguish it from the traditional detection method, this type of method is generally called track-before-detect (TBD).

There are many methods to implement the TBD, including dynamic programming\cite{1,2}, Hough Transform\cite{3}, maximum likelihood particle filter\cite{4,5,6,7}. Among this methods, particle filter related method can approach the optimal estimation and easy to implement, thus, particle filter based TBD has become a hotspot in corresponding research.

In view of this, this paper combines the multiple model algorithm idea with the PF-TBD algorithm, and proposes an enhanced particle filter TBD, and the simulation results show that the algorithm proposed by this paper can detect and track dim and maneuvering target efficiently.

Target State Transition and Measurement Model

Target State Transition Model

In this paper, we consider a single target to do complex motion in a two-dimensional plane. The general equation for target state transition is

\[ x_{k+1} = f(x_k, d_k, w_k), \quad k \in N \]  

in which \( x_k \) presents the target state, composed by

\[ x_k = [x_k \quad \dot{x}_k \quad y_k \quad \dot{y}_k \quad I_k]^T \]  

where \( x_k, \dot{x}_k \) and \( y_k, \dot{y}_k \) stands for corresponding axis’ position and speed, respectively, \( I_k \) represents the target signal intensity received by the sensor. \( d_k \) stands for target moving style, \( w_k \) is the white noise.
We assume that the target only does two kinds of movements at any time: uniform motion and
coordination turning motion, the corresponding state transition matrix is

\[
F^1 = \begin{bmatrix}
1 & T & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & T & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
F^2 = \begin{bmatrix}
1 & \frac{\sin(\omega T)}{\omega} & \frac{(1-\cos(\omega T))}{\omega} & 0 & 0 \\
0 & \cos(\omega T) & 0 & \sin(\omega T) & 0 \\
0 & 1-\cos(\omega T) & 1 & \frac{\sin(\omega T)}{\omega} & 0 \\
0 & \sin(\omega T) & 0 & \cos(\omega T) & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

in which \(T\) stands for sampling interval and \(\omega\) is the turning rate, both of which is assumed to be a
constant, generally.

As the evolution of \(d_k\) subjects to the first-order Markov process, define its Markov transition
matrix as

\[
\pi = \begin{bmatrix}
\pi_{11} & \pi_{12} \\
\pi_{21} & \pi_{22}
\end{bmatrix}
\]

Define target existence state which stands for if target is present or not as \(E_k\), the evolution of \(E_k\)
subjects to the first-order Markov process. \(E_k = 1\) represents the target is present and \(E_k = 0\) means
the target is lost. Define the Markov transition matrix as

\[
\Pi = \begin{bmatrix}
1 - P_b & P_b \\
P_d & 1 - P_d
\end{bmatrix}
\]

in which \(P_b\) represents the target “birth” probability and \(P_d\) stands for the target “death” probability.

**Observation Model**

In this paper, the complex signal form is used to establish the observation model, which is more
consistent with the actual sensor observation and reduces the error of the traditional point diffusion
model. Let the data received by the sensor at each moment be a two-dimensional image data. The
image data is the target intensity information in each image resolution unit, and can be expressed as

\[
z_k = \{z_{k}^{(i,j)} : i = 1, ..., n, j = 1, ..., m\}
\]

in which \(i\) stands for the resolution index in \(x\) axis and its total number is \(n\), similarly, \(j\) stands for the
resolution index in \(y\) axis and its total number is \(m\). Define

\[
z_k^{(i,j)} = |z_{A,k}^{(i,j)}|^2
\]

in which \(z_{A,k}^{(i,j)}\) is the complex amplitude of the target and

\[
z_{A,k}^{(i,j)} = \begin{cases}
A_k h_{A,k}^{(i,j)}(x_k) + v_k^{(i,j)} & E_k = 1 \\
v_k^{(i,j)} & E_k = 0
\end{cases}
\]

where \(A_k\) is the target complex amplitude, \(h_{A,k}^{(i,j)}(x_k)\) represents the degree of target influence on
adjacent resolution units and
\[ A_k = \hat{A}_k e^{i\phi_k}, \phi_k \in (0, 2\pi) \]

\[ h^{(i,j)}_A(x_k) = \exp \left\{ -\frac{(x_i - x_k)^2}{2\Delta x} - \frac{(y_j - y_k)^2}{2\Delta y} \right\} \]  

(9)

In which \( \Sigma_x, \Sigma_y \) stands for the degree of scattering of target influence in two axis, respectively.

\( x_k, y_k \) represents the true target position, \( \Delta x, \Delta y \) is the length of resolution units. Referring to the research in [5], achieving the likelihood function of the observation data is in accordance with the exponential distribution

\[ p(z^{(i,j)}_k | x_k) = \frac{1}{\mu_0^{(i,j)}} \exp\left\{ -\frac{1}{\mu_0^{(i,j)}} z^{(i,j)}_k \right\} \]  

(10)

In which

\[ \mu_0^{(i,j)} = E_{v_x, v_y}[z^{(i,j)}_k] \]

\[ = \begin{cases} \left[ P_k h^{(i,j)}_A(x_k) \right] + 2\sigma^2 & E_k = 1 \\ 2\sigma^2 & E_k = 0 \end{cases} \]  

(11)

\[ h^{(i,j)}_p(x_k) = (h^{(i,j)}_A(x_k))^2 \]

and the likelihood can be approximated as

\[ l(z_k | x_k, E_k = 1) = \frac{p(z_k | x_k, E_k = 1)}{p(z_k | E_k = 0)} = \prod_{i \in C_x(x_k)} \prod_{j \in C_y(x_k)} l(z^{(i,j)}_k | x_k, E_k = 1) \]

\[ l(z_k | x_k, E_k = 0) = 1 \]

(12)

In which \( C_x(x_k), C_y(x_k) \) represents the set of resolution units affected by the target in two axis respectively. Generally, the number of resolution units affected by target is presented as \( p \).

Derivation of Particle Filter Proposed and Implement Steps

Bayesian method has been widely used in various fields in recent years due to its superiority in state estimation of nonlinear systems. In this paper, we use Bayesian method to give a cohesive derivation of the particle filter proposed. Let \( Z_k \) be the set of all previous measurement till \( k \) instant, so that

\[ Z_k = \{ z_1, z_2, \ldots, z_k \} \]  

(13)

Using Bayesian method, the posterior probability density in the case of multiple models can be expressed as

\[ p(x_k, d_k, E_k = 1 | Z_k) = p(x_k, d_k | E_k = 1, Z_k) P_k \]  

(14)

In which \( P_k = P(E_k = 1 | Z_k) \) is the probability of target existence estimation. In this way, we can separate the target state estimate from the target existence estimate. In the case of multiple models, the model is defined only if the target exists. Therefore, the target state can be expanded, and the target moving style variable \( d_k \) is regarded as one dimension of the target state, and the target state after the expansion is recorded as \( x^A_k \)

\[ x^A_k = \begin{bmatrix} x_k^T \ d_k \end{bmatrix}^T \]  

(15)

After the target state vector is expanded, the derivation will be converted into a single model condition.
\[ p(x_k^A, E_k = 1 | Z_k) = p(x_k^A | E_k = 1, Z_k) P_k \]  

The derivation process of the algorithm for a single model is discussed in detail in [5], and will not be represented here.

The implementation steps of the particle filter algorithm proposed are summarized as follows:

**step 1:** generate particles based on the prior state probability distribution of the target state and the prior distribution of the target moving style model

\[ x_k^{(b)} \sim q(x_k^A \mid E_k = 1, E_{k-1} = 0, z_k) \]  

**Step 2:** calculate the weight of the new born particle and normalize it

\[ \tilde{w}_k^{(b)} = \frac{l(z_k \mid x_k^{(b)}, E_k^{(b)} = 1) p(x_k^{(b)} \mid E_k^{(b)} = 1, E_{k-1}^{(b)} = 0)}{N_b q(x_k^{(b)} \mid E_k^{(b)} = 1, E_{k-1}^{(b)} = 0, z_k)} \]  

\[ w_k^{(b)} = \frac{\tilde{w}_k^{(b)}}{\sum_{j=1}^{N_k} \tilde{w}_k^{(bj)}} \]  

**Step 3:** estimate the state of the remaining particles based on the state transition probability density and the moving style Markov transition matrix

\[ x_k^{(c)} \sim q(x_k^A \mid x_{k-1}^A, E_{k-1} = 1, E_k = 1, z_k) \]  

**Step 4:** calculate the continued existence of particle weights and normalize it

\[ \tilde{w}_k^{(c)} = \frac{1}{N_c} l(z_k \mid x_k^{(c)}, E_k^{(c)} = 1) \]  

\[ w_k^{(c)} = \frac{\tilde{w}_k^{(c)}}{\sum_{j=1}^{N_k} \tilde{w}_k^{(cj)}} \]  

**Step 5:** calculate the mixing probability using non-normalized weights and normalized again

\[ \tilde{M}_b = P_b [1 - \tilde{P}_{k-1}] \sum_{i=1}^{N_b} \tilde{w}_k^{(bi)} \quad \tilde{M}_c = [1 - P_d] \tilde{P}_{k-1} \sum_{i=1}^{N_c} \tilde{w}_k^{(ci)} \]  

\[ M_b = \frac{\tilde{M}_b}{\tilde{M}_b + \tilde{M}_c} \quad M_c = \frac{\tilde{M}_c}{\tilde{M}_b + \tilde{M}_c} \]  

**Step 6:** target detection probability estimation

\[ \hat{P}_k = \frac{\tilde{M}_b + \tilde{M}_c}{\tilde{M}_b + \tilde{M}_c + P_d \tilde{P}_{k-1} + [1 - P_d] [1 - \tilde{P}_{k-1}]} \]  

**Step 7:** weighted combination of the state of the new born and continuing particles

\[ \hat{w}_k^{(b)} = M_b w_k^{(b)} \]  

\[ \hat{w}_k^{(c)} = M_c w_k^{(c)} \]  

**Step 8:** resampling

**Step 9:** target state, detection probability estimation and moving style probability estimation update
The resampled particle group is \( \{x_i^o \mid i = 1, \ldots, N_r\} \), at this point, the extended state of the particle is restored to the target state \( \{[x_i', d_i'] \mid i = 1, \ldots, N_r\} \) and the style state \( \{d_i, k = 1, 2, \ldots, N_r\} \). Thus, the target state estimation is

\[
\hat{x}_k = \frac{\sum_{i=1}^{N_r} x_i^o}{N_c}
\]  

(25)

The estimation probability of style 1, style 2 is represented as \( \hat{p}_{d_i=1}, \hat{p}_{d_i=2} \)

\[
\hat{p}_{d_i=1} = \frac{\sum (d_i = 1)}{N_c}
\]

\[
\hat{p}_{d_i=2} = \frac{\sum (d_i = 2)}{N_c}
\]  

(26)

Simulation

The target performs complex movements in the plane for a total of 40 seconds. The target appears in the 7th second, first, doing 8 seconds of uniform linear motion, then 8 seconds of coordinated turning motion, turning rate \( \omega = 0.2 \), and then the target continues to do 8 seconds of uniform linear motion and disappear. Sampling time interval \( T = 1 \) seconds, the number of distance resolution units of the axis is 30, 35, the size of each distance resolution unit \( \Delta_x = \Delta_y = 1 \), the process noise is zero mean white Gaussian noise, and its standard deviation \( \sigma = 0.01 \), the moving style and target existence Markov matrix are

\[
\Pi = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}, \quad \pi = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}
\]  

(1)

Set the particle number \( N = 10000 \), the prior probability of the new born particles is evenly distributed over the whole plane, particle velocity is evenly sampled in the interval \([-2, 2]\), target signal intensity is evenly sampled in the interval \([10, 30]\), the signal to noise ratio is \( 6dB \), the simulation results are as follows:

![Simulation results](image)

(a) the probability of target existence; (b) trace estimation

Figure 1. The probability of target existence and trace estimation.
Figure 1 shows that the probability of target existence and trace estimation is consistent with the real situation, however, because of the target signal-to-noise ratio is low, the detection has one instant delay. Figure 2(a) shows that when maneuvering happens, the corresponding style probability has changed along, which means that the particle filter works well even the dim target is maneuvering. Figure 2(a) shows the tracking RMSE of algorithm proposed with 100 Monte Carlo tests is low, which means the algorithm’s accuracy is high.

Summary

In this paper, a new particle filter algorithm is proposed for the detection and tracking of the dim target. The algorithm uses Bayesian inference to separate the target existence probability estimation from the target state estimation, which improves the utilization efficiency of the particles. The simulation results show that the algorithm works well and has higher precision and better stability.

References

[1] Grossi E, Lops M, Venturino L. A Novel Dynamic Programming Algorithm for Track-Before-Detect in Radar Systems, J. IEEE Transactions on Signal Processing, 2013, 61(10):2608-2619.

[2] Cho H, Chun J. A new TBD-DP algorithm using multiple IR sensors to locate the target launch point, J. Proceedings of SPIE - The International Society for Optical Engineering, 2011, 8185(4):461-510.

[3] Garvanov I, Kabakchiev C. Radar detection and track determination with a transform analogous to the hough transform, C. International Radar Symposium. IEEE, 2006.10.

[4] Rutten M G, Gordon N J, Maskell S. Recursive track-before-detect with target amplitude fluctuations[J]. IEE Proceedings-Radar, Sonar and Navigation, 2005, 152(5):345-0.

[5] Rutten M G, Gordon N J, Maskell S. Efficient particle-based track-before-detect in Rayleigh noise, C. International Conference on Information Fusion. 2004: 693-700.

[6] Rutten M G, Ristic B, Gordon N J. A comparison of particle filters for recursive track-before-detect, C. Information Fusion, 2005 8th International Conference on. IEEE, 2005, 1-7.

[7] Garcia F J I, Mandal P K, Bocquel M, et al. Riemann-Langevin Particle Filtering in Track-Before-Detect, J. IEEE Signal Processing Letters, 2018.