Quality Parameter Index Estimation for Compressive Sensing Based Sparse Audio Signal Reconstruction

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Abstract. As we, all know that the size of data is increasing tremendously day by day. In a recent project, several petabytes were used to save an image of the Black Hole. Therefore, it is very crucial to develop a method that can reduce the size of data for transmission & storage purposes. The Traditional method for data compression & reconstruction requires so much data space, due to this problem another technique is proposed for the compression and recovery purpose. This method is termed Compressive Sensing (CS). As per the Nyquist sampling theorem, for proper reconstruction of the signal, we have to do sampling at the double rate of maximum data rate available in the signal. As a result, the storage requirement increased as well as the cost of the system was also enhanced. While on the other hand in Compressive Sensing, little samples are required for the reconstruction of the signal. So here in this paper, we have considered three music signals which are single tone, instrumental and vocal song. Values of Mean Square Error, Root Mean Square Error and Signal to Noise Ratio for different compression ratios mentioned in the tables and plots. By analyzing these values we can easily investigate the effectiveness of compressive sensing.

Keywords: Compressive Sensing (CS); Signal to Noise Ratio (SNR); Mean Square Error (MSE); Root Mean Square Error (RMSE); Restricted Isometry Property (RIP); Basis Pursuit (BP); Compression for Music Signals.

1. Introduction
Some standards form like JPEG, JPEG2000, MPEG, WAV, and MP3 associated with sparsity and compressibility give benefits to the transform coding schemes. Transform coding is a Leveraging but compressive sensing is the new combined groundwork for signal acquisition and sensor design. CS permits to reduce the sample of the signal which we want to sparse and compressed representation and evaluation cost of the signal. The states of the Nyquist Shannon sampling theorem need a few sample measurements to perfectly recover a band-limited signal but in the second case if the signal is sparse that’s known as a basis, we extremely deduced the measurement sample signal which we seriously needed for recovery. As the result, when the signal is sparse it can be better regarding performances. So the compressive sensing is not used as a firstly sampled the signal at a high rate and after that compressed the data. We would like to search for a path that compressed data should be sense directly with a low sampling rate. Emmanuel Candès, Justin Romberg, and Terence Tao, and David Donoho, Who work on the CS in their research work trying the large-scaled with sparse and compressible represented signal easily reconstructed from the small number of linear and nonadaptive measurement sets [1][2][3]. The most challenging task in the CS field is designing the measurement method. The
design of these measurement schemes and their dimension implement the practical data module and acquisition.

Many traditional and practical crises go along with the principle of the Nyquist sampling theorem for reconstruction and acquisitions of the image, video, audio from high dimensional data. To remove the conventional limits of sampling theory, compressed sensing (CS) recently days used in many fields of applied mathematics, computer science, and electrical engineering. The purpose of this paper gives a summary of the basic theory of compressive sensing. After the discussion of the overall introduction of the CS, let we start again discursion of the sparsity and other different low dimensional signal model. After that many questions were generated regarding this scheme that how to recover the complete signal from the few low dimensional signal accurately and give a high reliability guaranteed signal for different types of sparse recovery algorithms. We make a final result with the discussion of some sparse recovery framework. In the following work of the paper, we will take action to the fundamental for move on to some interesting and exploring directions. In which some new sensing design techniques with more advanced recovery conclusions are combined to describe the analog and discrete-time signal structure.

The paper is organized in such a manner that in section I the introduction about the techniques mentioned, in section II extensive literature review is given. Section III & IV depicts the problem statement and the objective of the paper. When we proceed to section V various properties & fundamentals are related to compressive sensing given. In section VI, the result and analysis mentioned followed by a conclusion in the next section.

2. Literature Survey

As we know, that audio processing is a very vast area, we can consider music processing as a tributary of audio processing. Various techniques evolved for speech processing but can also be used for music processing and provide good efficiency for the music signals. Because this paper contains a music signal for compression & recovery so we have to understand the basic parameter of the music signal. We can enhance the quality of the reconstructed signal by keeping these parameters in mind.

As per the paper, the author wants to propose a method by which the music signal processing or we can say audio signal processing will also depict the characteristics of the music or audio signal. The characteristic which is of interest is like pitch, melody, and timbre of the music signal. Here the consideration of music signal is like a very small part of the speech signal processing, and the method which is proposed by the author can give proper results like for the speech signal processing. All the qualities and characteristics which pertain to the speech signal also pertain to the music signal too [4].

In the paper of Masoumeh Azghani, Mostafa Karini proposed a multi hypothesis reestablishment scheme for a video sequence. In which it has a not only simple encoder but also a less complicated decoder [5]. MH technique involves the sparsity constraint and Tikhonov regularization. The author introduced the latest iteration algorithm which is based on MH. This iteration gives the best result for the performance during reconstruction. The computation process of Elasticnet is faster than the Thikhonov.

Compressed video sensing is focused on the paper for compression of a video sequence. In the single Hypothesis method, every block of the frame is defined and predicts the valve with the last resolving frame by this method. This is known as block matching because the encoder finds out the block of a particular frame with the highest matching. It has a drawback that it implicit the transmission at the transmitter side of the encoder to send the block motion vector that introduced the complexity at the transmitter side. So following issue is removing by the use of the multi hypothesis technique. In which the motion estimation task transmits from the encoder to decoder to be easy. MH- MC method gives a good efficient recovery production regarding cost and complexity at the receiver side. Further, it has some CVS problems Thikhonov regularization is offered with high PSNR. This is a combination of SH MH reconstruction strategies. Elasticnet based MH-HC was offered whose performance is good as compared to Tikonov regularization reconstruction.
As per the paper written by Mads Græsbøll Christensen, they applied a compressive sensing approach to the audio & speech signals. They used compressive sensing for the sparse decomposition of the speech signal with the help of window-based complex sinusoidal functions [6]. As the results discussed by the author the compressive sensing approach is valid for the sparse decomposition for audio & speech signals. But the problem they found is that the speech and audio signal have non-stationary components and complex too when the variation in sparsity is taken into the picture.

Azad M. Madni discussed how the artificial thinking of the system can improve the compressive sensing process and can also reduce the problems that arise in the sparse reconstruction of a signal [7]. He also discussed the sparse reconstruction algorithm. He found that the sparse recovery problem is a problem based on reverse engineering and is also distributed in various fields of information technology, like remote sensing & social networking.

As per the paper written by Ping-Keng Jao, Chiu-Chia Michael Yeh, [8] they proposed two modifications in the LASSO technique which is used for the assignment of different code words used in the compressive sensing technique. The two modifications which are proposed by them, the first one make use of the repetitive nature of the music signal and the second thing is to optimize the screening constraint for signal recovery. The experiments done by them show clearly that the runtime which is required for the 10,000 codewords reduces up to the level of runtime required for 1000 code words. The conclusion which was proposed by them is that the larger dictionary size can improve the value of mean average precision.

In this paper, three music signals are considered as input signals on which compressive sensing is applied. As through the theory of Compressive Sensing, there are various parameters on which the reconstruction is dependent. The various parameters are like basis and sensing matrices and the type of reconstruction algorithm, which is used. So to see the effects of basis and sensing matrices on the reconstruction of music signal various types of Basis and Sensing matrices are taken and the results are discussed in this paper. Two parameters one is the MSE & the other is the SNR (Signal to Noise Ratio) considered for the quality detection purpose of the music signal.

3. Problem Statement

It is planned that this ultimate objective may be achieved by studying the following concerns: As we know that various areas like medical, error correction, RADAR signal processing require a huge amount of memory to store the data, it is the major problem with all these applications. Therefore, we aim to find out the solution to this problem. From the literature survey, we found that traditional sampling theory (Nyquist Rate) not able to sort out this problem because we have to require more than twice samples for the proper reconstruction of the signal. To resolve this storage scarcity, we have to take a few samples from the whole sample space and then reconstruct the original sample by some means. We have gone through the various approaches given in the literature and find out that there is a solution that can be able to solve this problem and that solution is called Compressive Sensing (CS). Now the work, which we want to do, is to study various parameters of Compressive sensing and find out the significance of those parameters. Our proposed work is to enhance the level of compression and then find out a good reconstruction of the original signal. We want to find out the relationship between the different parameters, which are the basic need for compressive sensing. The difference between traditional data compression and compressive sensing can be easily understood using the below-given diagrams.
4. **Objective of Paper**

The motive of this work is to provide a better approach to apply compressive sensing. Different parameter estimation approaches given in the literature. Our objective is to find out the essence of all these things and provide a better result to show the significance of this approach. As we know that the Basis matrix and Sensing matrix are two essential backbones of compressive sensing theory. One major thing is the reconstruction algorithm. Hence, to solve our purpose we have taken different sensing and measurement matrices and then applied reconstruction algorithms to them. Different signals are considered in which the very first signal, which is considered, is a Non-stationary single tone signal then we further move towards instrumental and vocal song signal. Then after enough iteration on the sound signal, we have gone through the change in basis & sensing matrix. The overall analysis of these signals provides different results for MSE, RMSE, SNR, etc because these are the quality parameters for any signal.

Throughout observation, we have certain conclusions and different statements, which show the significance of results. All significant quantities like SNR, MSE & RMSE are shown in tables for distinct random functions, and various plots associated with them are also shown in this paper. This whole analysis is very helpful to understand various results and the parameters shown in the results.

5. **Reconstruction Properties**

Some important properties related to compressive sensing stated below.
5.1. Restricted Isometry Property
If we consider it then the prime motive of encoding is to convert Nx1 K-sparse signal x to M x 1 measurement by incorporating an appropriate measurement matrix ϕ. The measurement matrix ϕ should project 2 distinct signals to 2 distinct sets of observations, so all the column sub-matrices of ϕ defined prominently [9].

Candès & Tao suggested a case for matrix ϕ which is known as the measurement matrix. For a K-sparse vector, a matrix kept the K-RIP if

$$\|x_1 - x_2\|^2 \leq \|\Phi x_1 - \Phi x_2\|^2 \leq (1 + \delta_{K}) \|x_1 - x_2\|^2$$  \hspace{1cm} (1)

When the value of $\delta_{K} < 1$. Equation (1) signifies that all the submatrices of ϕ with K columns are well defined and show a good Isometry property. If $\delta_{K} \ll 1$ measurement matrix ϕ has a massive probability to recover the (K/2) - sparse signal with a stable value of x. This condition is termed as Restricted Isometry Property (RIP) [9]. The relation between RIP & CS is well behaved if $\delta_{K}$ is adequately less than 1. All pairwise distances between K-Sparse signals must be sustained in the measurement space.

$$\|x_1 - x_2\|^2 \leq \|\Phi x_1 - \Phi x_2\|^2 \leq (1 + \delta_{K}) \|x_1 - x_2\|^2$$  \hspace{1cm} (2)

This implies that the above expression holds for all K-sparse vectors $x_1$ and $x_2$. Because $x_1$ and $x_2$ are, two distinct vectors and $\|x_1 - x_2\|^2$ are invariably greater than zero. The value $\|\Phi x_1 - \Phi x_2\|^2$ will never be zero. Therefore, to change the stability of sampling theory & get a K-sparse signal, the RIP constant $\delta_{2K}$ must be as small as possible. However, it is quite difficult to ensure that whether ϕ satisfies the equation (1) or not. As we go through the literature, we find out that luckily many random matrices fulfil the requirements of RIP condition the best suitable example is the Gaussian measurement matrix ϕ. An MxN i.i.d. Gaussian matrix will show RIP with great probability if $M \geq CK \log(N/K)$ where C is a constant [10-11] termed as RIP constant. The conclusion is that the N number of samples can be easily reconstructed using an M number of samples. This paper contains various basis & sensing matrices. The random matrices that are taken into consideration show nice RIP behavior if the signals have fair stability & high accuracy.

5.2. $l_1$ Minimization
As the discussion made in the last if the RIP holds, then l1 reconstructs K-sparse signals & does a decent calculation to assume the compressible signals with excellent probability using.

$M = O(L \log N / K)$ i.i.d. Gaussian measurements. Then the equation for recovery for l0 norm will change to be:

$$x = \arg \min_x \|\bar{x}\|_1, \text{ subjected to } y = \Phi \bar{x}$$ \hspace{1cm} (3)

Eq. (3) is equivalent to the linear problem

$$\min \sum_{j=1}^{2N} v_j, \text{ subjected to } v \geq 0, \text{ } y = (\Phi x - \Phi \bar{x})v$$ \hspace{1cm} (4)

According to the theory, we can see that v has positive real coefficients & the size of v is 2N. The signal x is obtained from the solution $v^*$ of (4) via $x = (I - \Phi) v^*$. So there the conclusion is that $l_1$ minimization can easily be deduced using LPP. Other different methods like Interior-point, projected gradient [12] & iterative thresholding [13] can also be taken into consideration to deduce Eq. (4).
5.3. Greedy Pursuit

Greedy pursuit is the better option to recover the signal. It is a ceaseless signal reconstruction algorithm, to deduce the backing of the signal & it will provide an optimal choice of solution, which will fulfill the criterion, and it repeats until the condition is verified. The signal $S$ recovered easily $x = (\Phi')^+ y$. Here $\Phi$ is the measurement matrix with the values of coefficients given by $S$ & $(\Phi')^+$ is the pseudo inverse of the measurement matrix $\Phi$. If we define the pseudo- inverse of a matrix $\Phi$, which is a full rank in nature, then it is given $\Phi^+ = (\Phi^* \Phi)^{-1} \Phi^*$. Greedy Pursuit has very less response time while it is not so much stable & does not provide uniformity.

Compressive sensing is much favorable on an assumption that the $l_1$ minimization approach provides a better way for music signal compression & the way, which is utilized optimum in nature. Further work on this way is going on to find an algorithm, which is much faster & has very less response time with great recovery performance. Now here we provide a brief discussion about the recovery of signal $x$ from the measured signal $y = \Phi_3 x + e$ (here $e$ denotes error) by using existing algorithms. Here we describe two types of algorithms first one is a variation on matching pursuit methodology and the others are Thresholding algorithms.

6. Result and Analysis

Here we apply the compressive sensing theory to the music signal. Basis Pursuit (BP) algorithm is used for recovery purposes. A MATLAB code is prepared for the application of the algorithm on the music signal. First of all, the signal is compressed using compressive sensing and then using the Minimum-$l_1$ algorithm the reconstruction takes place. This basis pursuit algorithm has higher efficiency for the sparse recovery in the random domain. All three signals are considered in the form of a “.wav” file. The length of the music signal is 7 sec for all three domains i.e., in a single tone, instrumental & music. In the below section we are going to describe different values of MSE’s for different signals and different measurement & basis matrices. Because due to this observation we can find out how much error will be there in the recovered signal concerning the value of the compression ratio (CR).

Table.1 CR v/s MSE values for Single-tone signal with different Basis and Sensing Matrices

| CR (M/N) | MSE | Gaussian | Poisson | Exponential | Rayleigh |
|----------|-----|---------|---------|-------------|----------|
|          |     | DCT | DST | DCT | DST | DCT | DST | DCT | DST |
| 0.2      |     | 0.0033 | 0.0037 | 0.0034 | 0.0036 | 0.0034 | 0.0036 | 0.0035 | 0.0037 |
| 0.4      |     | 0.0007 | 0.0006 | 0.00079 | 0.00077 | 0.00079 | 0.00077 | 0.00076 | 0.00079 |
| 0.6      |     | 0.00021 | 0.00020 | 0.000249 | 0.00018 | 0.000249 | 0.00018 | 0.000241 | 0.00016 |
| 0.8      |     | 0.000049 | 0.000045 | 0.0000404 | 0.000045 | 0.0000404 | 0.000045 | 0.0000399 | 0.000040 |

Table.2 CR v/s RMSE values for Single-tone signal with different Basis and Sensing Matrices

| CR(M/N) | RMSE | Gaussian | Poisson | Exponential | Rayleigh |
|---------|------|---------|---------|-------------|----------|
|         |      | DCT | DST | DCT | DST | DCT | DST | DCT | DST |
| 0.2     |      | 0.0573 | 0.0605 | 0.0594 | 0.0607 | 0.0593 | 0.0603 | 0.0589 | 0.606 |
| 0.4     |      | 0.0276 | 0.0264 | 0.0279 | 0.0259 | 0.0281 | 0.0289 | 0.0248 | 0.0290 |
| 0.6     |      | 0.0147 | 0.0142 | 0.0157 | 0.0133 | 0.0158 | 0.0129 | 0.0149 | 0.0127 |
| 0.8     |      | 0.0065 | 0.0067 | 0.0064 | 0.0069 | 0.0064 | 0.0067 | 0.0062 | 0.0063 |
If we analyze tables 1, & 2 then we can see in most of the cases the value of MSE & RMSE is less in DCT than a similar case in DST. The question arises here that why there is a difference in the MSE & RMSE values for different measurement matrices. The answer to this question is very simple as we see that every random matrix has its PDF value. If we consider the Gaussian random matrix at a lower compression ratio then the value of MSE is lesser than other random matrices. According to the properties of random matrices, we can easily conclude that Gaussian provides the best results for compression. However, if we see the PDF curve of the Gaussian then we can analyze that when we increase the mean value then the PDF curve is much wider. That is why due to that the signal, which is to be applied here for transformation, interacts at a wider area so the MSE, which we get, has a lesser value than other random matrices. However, if we consider Poisson, Exponential & Rayleigh random matrices then for single tone signal the value of MSE is large in all these matrices because of their properties & PDF curve nature.

Table 3: Compression Ratio (CR) v/s MSE values for Instrumental Song signal with different Basis and Sensing Matrices

| CR (M/N) | MSE For Instrumental Song Signal | Gaussian | Poisson | Exponential | Rayleigh |
|----------|----------------------------------|----------|---------|-------------|---------|
|          |                                  | DCT | DST | DCT | DST | DCT | DST | DCT | DST |
| 0.2      |                                  | 0.0195 | 0.0201 | 0.0180 | 0.0191 | 0.0196 | 0.0198 | 0.0195 | 0.0197 |
| 0.4      |                                  | 0.0011 | 0.00135 | 0.00106 | 0.00119 | 0.00126 | 0.0013 | 0.0012 | 0.001256 |
| 0.6      |                                  | 0.0009 | 0.00099 | 0.00089 | 0.00095 | 0.00099 | 0.0010 | 0.00097 | 0.000985 |
| 0.8      |                                  | 0.000764 | 0.000812 | 0.00079 | 0.00084 | 0.000749 | 0.00083 | 0.00080 | 0.000821 |

Table 4: Compression Ratio (CR) v/s RMSE values for Instrumental Song signal with different Basis and Sensing Matrices

| CR (M/N) | RMSE For Instrumental Song Signal | Gaussian | Poisson | Exponential | Rayleigh |
|----------|----------------------------------|----------|---------|-------------|---------|
|          |                                  | DCT | DST | DCT | DST | DCT | DST | DCT | DST |
| 0.2      |                                  | 0.1396 | 0.1458 | 0.1341 | 0.1477 | 0.1452 | 0.1498 | 0.1364 | 0.1503 |
| 0.4      |                                  | 0.0761 | 0.0779 | 0.0698 | 0.0788 | 0.0742 | 0.0728 | 0.0726 | 0.0795 |
| 0.6      |                                  | 0.0389 | 0.0399 | 0.0345 | 0.0392 | 0.0323 | 0.0389 | 0.0312 | 0.0402 |
| 0.8      |                                  | 0.0277 | 0.0286 | 0.0281 | 0.0291 | 0.0274 | 0.0286 | 0.0283 | 0.0297 |

Now here we are analyzing the Instrumental song signal. The reason for the variation is the same as given in the previous section but here we can see that the values of MSE are much larger in comparison to single tone signal. The reason for this is that the instrumental song has higher frequency components than the single tone signal, and these all components are of different frequencies. So, these frequency components provide much more disturbance for each other. Therefore, reconstructing the original signal here is much more complicated. Therefore, it has larger error values than a single tone signal.
Table 5: Compression Ratio (CR) vs MSE values for Vocal Song signal with different Basis and Sensing Matrices

| CR (M/N) | MSE  | MSE  | MSE  | MSE  | MSE  |
|----------|------|------|------|------|------|
|          | Gaussian | Poisson | Exponential | Rayleigh |
|          | DCT | DST | DCT | DST | DCT | DST | DCT | DST |
| 0.2      | 0.0817 | 0.0896 | 0.0803 | 0.0835 | 0.0859 | 0.0896 | 0.0834 | 0.0865 |
| 0.4      | 0.0336 | 0.0357 | 0.0324 | 0.0341 | 0.0359 | 0.0384 | 0.0371 | 0.0389 |
| 0.6      | 0.0141 | 0.0153 | 0.0138 | 0.0143 | 0.0148 | 0.0151 | 0.0141 | 0.0147 |
| 0.8      | 0.0044 | 0.0046 | 0.0045 | 0.00465 | 0.00426 | 0.00436 | 0.0042 | 0.00441 |

Table 6: Compression Ratio (CR) vs RMSE values for Vocal Song signal with different Basis and Sensing Matrices

| CR (M/N) | RMSE  | RMSE  | RMSE  | RMSE  |
|----------|-------|-------|-------|-------|
|          | Gaussian | Poisson | Exponential | Rayleigh |
|          | DCT | DST | DCT | DST | DCT | DST | DCT | DST |
| 0.2      | 0.2859 | 0.3012 | 0.2834 | 0.2969 | 0.2931 | 0.2962 | 0.2888 | 0.2995 |
| 0.4      | 0.1833 | 0.1944 | 0.1824 | 0.1912 | 0.1869 | 0.1956 | 0.1865 | 0.1895 |
| 0.6      | 0.1187 | 0.1359 | 0.1126 | 0.1089 | 0.1029 | 0.1139 | 0.1069 | 0.1078 |
| 0.8      | 0.0663 | 0.0785 | 0.0661 | 0.0728 | 0.0648 | 0.0712 | 0.0651 | 0.0689 |

Here above in tables 5 & 6, we can easily observe that the values of Mean Square Error & Root Mean Square Error are larger than both single tone & instrumental song because the song signal has voice signal and other frequency components of different instruments. It will provide much more noise than the single tone & instrumental song signal. So here, it is too typical to reconstruct the original signal.

Different plots for Compression Ratio vs SNR

Different plots that describe the relationship between the compression ratio and the SNR given below. For plotting these graphs different Basis and measurement matrices are taken into the consideration. The Basis matrices are DCT and DST, the sensing matrices which are taken are Gaussian, Poisson, exponential, and Rayleigh. We can see the difference between all the graphs significantly, which shows that SNR & CR are the parameters, which are directly proportional to each other.

![Plot Compression Ratio vs SNR(For DCT & Gaussian)](image)

Figure 3: Plot for CR vs SNR value For DCT as a basis & Gaussian as a sensing matrix for 3 Non-stationary music signals
Figure 4: Plot for CR v/s SNR value for DST as a basis & Gaussian as a sensing matrix for 3 Non-stationary music signals.

Figure 5: Plot for CR v/s SNR value for DCT as a basis & Poisson as a sensing matrix for 3 Non-stationary music signals.

Figure 6: Plot for CR v/s SNR value for DST as a basis & Poisson as a sensing matrix for 3 Non-stationary music signals.

Figure 7: Plot for CR v/s SNR value for DCT as a basis & Exponential as a sensing matrix for 3 Non-stationary music signals.
7. Conclusion

It is a well-known fact that memory is a limited resource for any storage device. It is understood from the result and analysis section that we can recover the best quality of the actual music signal with very few samples. From the plots mentioned in the previous section, we can deduce that if we increase the compression ratio or we can say the number of measurements then the value of the signal to noise ratio also increases it means the quality of the reconstructed signal increases. From the result and analysis section, we can conclude that the value of the signal-to-noise ratio for the single tone signal is quite good than the instrumental and the vocal signal. It means that the complexity of the signal reduces the quality of reconstruction. Different parameters like MSE and RMSE are listed in Tables 1-6 in the
previous section. From the tables, one point is very clear that with the variation in basis & sensing matrices & the type of signal values of SNR, MSE & RMSE also varies. The variations in the values of MSE & RMSE are due to the random distribution nature of Basis & Sensing matrices.

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