Numerical evaluation of rock strength with different sizes of circular opening

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Abstract. The presence of opening in the rock may be natural or artificial. In such circumstances, the stability should be examined through studying the strength of the rock. In the present study, the effect of size of the circular opening on the compressive and flexural strengths of limestone rocks has been studied. Both, the compressive and flexural strengths under confining pressure have been examined numerically. Two types of 2D-rock models have been studied are (200×200 mm$^2$) and (200×700 mm$^2$), respectively, for compression and flexural states with three sizes of openings having diameters of 10, 25, and 40 mm. The results of compression state showed that the strength reduces with existing opening and the reduction increases with the size of opening. The unconfined rock exhibited higher reduction in the strength than that of confined one with exists of opening. In case of flexural state, both compressive and tensile strengths increase with the increasing confining pressure, while they decrease with the opening size. At higher confining pressure, the size of opening has insignificant effect on the tensile strength. The tensile strength of the unconfined rock remains unchanged with increase in the size of opening beyond certain limit.

Key words
Rock strength; Openings; Compressive strength; Flexural strength; Numerical analysis.

1. Introduction
The presence of openings in rock mass may cause problem to the stability of the structures through the change in behavior of the rock. The openings may either be natural type like cavities or shear zones, or artificial type like underground structures. Al-Obaydi et al. [1] conducted an experimental triaxial and unconfined compression tests to study the effect of opening size on the strength of limestone, sandstone and gypsum rocks. It has been found that the strength decreases with increase in the opening size. The larger effect of the opening is observed when the ratio of its diameter to the diameter of the sample ranges between (0.2-0.25). The maximum effect of opening size was observed on the sandstone rock. Others also investigated the unconfined and triaxial stress states on the behavior of a thick-walled hollow cylinder [2-5]. Pan et al. [6] used a numerical analysis to study the effect of normal stress on circular tunnel within homogenous rock media. The results showed that the normal stress has a significant effect on the stability of tunnel. The amount of stresses around the tunnel is independent of its deformation. Labuz et al. [7] indicated a drop-in strength when an open included in
prismatic sample of sandstone rock. Also, Xu and Yu [8] stated a reduction in the strength of hard rock media when tunnel considered in the numerical analysis. Sagong et al. [9] used experimental and numerical analyses for the biaxial and compression states of rock-like material with a circular opening. The tension cracks that developed around the opening and type of failure are depending on the attitude of the rock joints. Similar observations have been reported by others [10-11].

Based on Brazilian test, Chen et al. [12] studied experimentally and numerically the effect of opening size on tensile characteristics of rock-like material. It has been concluded that the ratio of opening diameter to sample diameter as well as the attitude of joints has a considerable effect on cracks distribution and strength. The rate of reduction in the tensile strength of rock increased with increasing the size of opening especially in gypsum and limestone rocks by about 6 times compared to the solid one [1]. Li et al. [13] used DEM to simulate a hollow cylindrical torsional shear testing under different stress conditions that triaxial, torsional, compression and pure-compression. By testing a cylindrical hollow rock-like material, Hashemi et al. [14] observed a reduction in the ductility and failure strength of the specimen with increase in the opening size. With increase in the confining pressure, the peak strength increased. Yang [15] applied triaxial test and found that varying pattern of crack appeared with the confining pressure.

2. Numerical Analysis
2.1 Material Properties
A sedimentary rock of limestone type located in the Mosul district has been considered in the present numerical analysis. The engineering properties of such rock that used in the analysis are presented in Table 1. The values adopted here are representing the actual properties of limestone rock in the region. It is obtained from experimental tests of the earlier work [1].

| Properties | Units | Materials          |
|------------|-------|--------------------|
| γd         | kN/m³ | Mohr-Coulomb and   |
|            |       | Linear Elastic 18.4|
| γsat       | kN/m³ | 20                 |
| E          | MPa   | 5000               |
| µ          | -     | 0.25               |
| C          | MPa   | 6.5                |
| Ø          | degree| 36                 |
| Ψ          | degree| 0                  |

2.2 Modeling Procedures
A rock model was prepared according to the simulation requirements. Both solid rock and rock with circular opening have been modeled. A circular opening proposed at the center of the rock with different diameters (10, 25, and 40 mm) designed as (C10, C25, and C40) respectively. Three types of loading conditions have been considered to represent the unconfined compression, biaxial compression, and flexural (4-points). Three confining pressures of (1, 2, and 3 MPa) have been applied in addition to the unconfined case.

2.3 Simulation Modeling
Three models have been proposed in the present study using 2D-Plaxis package:
1. Model with dimension (200×200 mm$^2$) to represent an unconfined plane strain condition as shown in Figure 1.
2. Model with dimension (200×200 mm$^2$) to represent a biaxial plane strain condition as shown in Figure 2.
3. Model with dimension (200×700 mm$^2$) to represent a flexural condition as shown in Figure 3.

3. Results and Discussion
3.1 Results of Compression State
3.1.1 Effect of opening size on failure stress. The effect of circular opening size on normal stress ($\sigma_1$) at failure has been studied. Different size of opening has been considered as (C10, C25 and C40). The results presented in Figure 4 showed that the solid rock has a compressive strength at failure higher than those with opening. Moreover, the rock under confining pressure exhibited a higher strength than the unconfined one.

The strength of rock decreased with increase in the size of opening. In case of unconfined rocks, these reductions are (40.4, 55.0 and 66.2 %) for rocks with opening size (C10, C25 and C40), respectively, upon solid one. The reductions in the strength with the opening size calculated from
Table 2 under the confining pressures of (1, 2 and 3 MPa), respectively, are (4.9, 3.5, 0.05%) for (C10); (41.9, 30.5 and 8.7 %) for (C25); and (53.5, 31.1 and 20.2 %) for (C40). It is clear that the negative effect of opening on the strength of rock reduces with increase in the confining pressure.

Table 2. Results of the numerical analyses of uniaxial and biaxial states.

| Model Type | Opening size (mm) | $\sigma_3$ (MPa) | $\sigma_1$ (MPa) | deformation (mm) |
|------------|-------------------|------------------|------------------|------------------|
| Solid      |                   |                  |                  |                  |
|            | ---               | 0                | 47.20            | 0.037            |
|            | 1                 | 1                | 52.20            | 0.186            |
|            | 2                 | 1                | 57.93            | 0.186            |
|            | 3                 | 1                | 63.69            | 0.260            |
| Circle (C10) | 10           | 0                | 28.12            | 0.372            |
|            | 1                 | 1                | 49.64            | 0.440            |
|            | 2                 | 1                | 55.91            | 0.522            |
|            | 3                 | 1                | 63.66            | 0.548            |
| Circle (C25) | 25           | 0                | 15.94            | 0.215            |
|            | 1                 | 1                | 30.31            | 0.317            |
|            | 2                 | 1                | 40.24            | 0.354            |
|            | 3                 | 1                | 58.18            | 0.484            |
| Circle (C40) | 40           | 0                | 24.25            | 0.258            |
|            | 1                 | 1                | 39.93            | 0.267            |
|            | 3                 | 1                | 50.80            | 0.450            |

Figure 4. Effect of opening size on failure normal stress ($\sigma_1$).

Deformed shape. A typical deformed shape of rock with openings under a confining pressure of (3 MPa) has been illustrated in Figure 5. It is obvious that the deformation increases around the opening. This increase is particularly at the crown of the opening as a result of stress concentration there. Consequently, the tangential stress increased to its highest value, while the radial stress is almost zero at the boundary of the opening. Fukhimi et al. [16] reported a closure of opening due to the stress concentration.
The smallest opening (C10) show a higher deformation ratio with respect to its radius. All openings exhibited about similar deformation for each value of confining pressure (see Table 2).

![Figure 5. Typical deformation shape of biaxial with opening size under ($\sigma_3= 3$ MPa)](image)

3.1.3 Stress Re-distribution. The rock with opening of (C10) has higher strength and lower stress concentration around the opening. In contrast, lower strength and higher stress concentration when rock with (C40) opening. Under the confining pressure, a tensile zone initiated at the crown of the opening. This zone enlarges with the size of opening. This is can be attributed to the higher re-distribution of stresses under uniaxial and biaxial stress states as the size of the opening increased.

3.2 Results of Flexural State
3.2.1 Effect of circular opening on failure stresses. The effect of opening size on the flexural strength of the rock has been examined through the normal failure stress ($\sigma_1$), compressive stress ($\sigma_3$) and tensile stress ($\sigma_t$) that developed in the rock under flexural loading state.

The results are presented in Figure 6 for the three sizes of opening (C10, C25 and C40), in addition to the solid one. It is obvious that the normal stress ($\sigma_1$) at failure decreases with increase in opening size. A higher reduction is occurred in the rock with (40 mm) opening. The reductions in ($\sigma_1$) under $\sigma_3$ of (1, 2, and 3 MPa), respectively, are (11.5, 14.4, and 14.0 %) for (C10); (18.5, 19.1, and 19.7 %) for (C25); and (20.4, 23.5, and 36.1 %) for (C40) (see Table 3). Rate of reduction in ($\sigma_1$) is highest at ($\sigma_3= 3$ MPa). Both ($\sigma_3= 1$ and 2 MPa) have similar behavior. In unconfined rock ($\sigma_3= 0$), the opening size has a negligible effect on normal stress ($\sigma_1$) as the opening is located out of re-distribution stress zone.

![Figure 6. Effect of opening size on normal stress ($\sigma_1$) at failure under flexural state.](image)

Figure 7 shows the variation in compressive stress ($\sigma_3$) with the confining pressure ($\sigma_3$). Generally, the rock shows an increase in the ($\sigma_3$) with ($\sigma_3$), while it decreased with increase in the opening size. Under all the confining pressure ($\sigma_3$), the lowest ($\sigma_3$) is associated with the (C40) opening due to
increased stress concentration around the opening. The values of \((\sigma_3 = 1, 2,\) and 3 MPa) give almost same trends. At \((\sigma_3 = 3 \text{ MPa})\) the values of \((\sigma_c)\) for (C10, C25, and C40) are, respectively, (32, 26, and 22 MPa), while for solid it is (38 MPa) as presented in Table (3).

![Graph showing the effect of opening size on compressive stress at failure](image)

**Figure 7.** Effect of opening size on compressive stress \((\sigma_c)\) at failure under flexural state.

**Table 3.** Results of numerical analysis of flexural state.

| Model Type | Opening Size (mm) | \(\sigma_3\) (MPa) | \(\sigma_1\) (MPa) | \(\sigma_c\) (MPa) | \(\sigma_t\) (MPa) | Deflection (mm) |
|------------|------------------|-------------------|-------------------|-------------------|-------------------|-----------------|
| Solid      |                  | 0                 | 22.10             | 22.00             | 2.007             | 0.181           |
|            |                  | 1                 | 41.82             | 28.00             | 2.005             | 0.217           |
|            |                  | 2                 | 43.51             | 31.00             | 2.005             | 0.227           |
|            |                  | 3                 | 57.27             | 38.00             | 2.004             | 0.230           |
| Circle (C10) | 10               | 0                 | 22.07             | 14.00             | 1.006             | 0.135           |
|            |                  | 1                 | 37.01             | 24.00             | 1.807             | 0.227           |
|            |                  | 2                 | 37.25             | 27.00             | 1.905             | 0.230           |
|            |                  | 3                 | 49.25             | 32.00             | 2.004             | 0.304           |
| Circle (C25) | 25               | 0                 | 20.56             | 11.00             | 1.003             | 0.132           |
|            |                  | 1                 | 34.08             | 19.00             | 1.506             | 0.221           |
|            |                  | 2                 | 35.19             | 20.00             | 1.803             | 0.231           |
|            |                  | 3                 | 45.96             | 26.00             | 2.002             | 0.302           |
| Circle (C40) | 40               | 0                 | 19.96             | 11.00             | 1.002             | 0.137           |
|            |                  | 1                 | 33.27             | 18.00             | 1.301             | 0.231           |
|            |                  | 2                 | 33.29             | 19.00             | 1.400             | 0.249           |
|            |                  | 3                 | 36.62             | 22.00             | 2.002             | 0.249           |

The variations in the tensile stress \((\sigma_t)\) with the confining pressure \((\sigma_3)\) at failure are shown in Figure 8. With increasing opening size, the value of \((\sigma_t)\) reduces. As the \((\sigma_t)\) reaches a value of (3 MPa), the effect of the opening size on \((\sigma_t)\) becomes negligible. In addition, the unconfined rock exhibited minor changes in \((\sigma_t)\) with increasing opening size beyond (C10), however, the reduction upon solid one is 50%. The higher \((\sigma_3)\) may provide sufficient lateral support and in turn higher confinement pressure around the opening and therefore, the effect of such opening is vanished. In the unconfined rock, the opening may locate out of the tensile zone, therefore, the size becomes ineffective.
3.2.2 Deformed shape with opening size. Figure 9 shows a typical deformed shape around the openings under a flexural loading state. Again, the large opening (C40) exhibited less deformation comparison with small one (C10) which exhibited higher deformations.

The deformed shape characterized by lateral compression. This behavior is conversely of that occurred in biaxial state condition. This means that the sidewall of the opening is subjected to compression and accordingly the extension cracks like Griffith’s crack extended from the bottom of opening to the boundary of the model. As illustrated in Table 3 the opening size has insignificant effect on deformation, while the ($\sigma_3$) shows some effect.

3.2.3 Stress distribution
The stress distribution varies with the size of the opening. As known, the concentration of the stresses occurred around the opening. The zone of compressive stress appeared at the top and the bottom of the opening and increased with increase in its size. Zones of tensile stress may appear due to application of confining pressure.
4. Conclusions

1. The compressive strength reduced as a result of the presence of opening in the rock.
2. The strength of unconfined rock decreased with increase in the opening size by (40.4, 55.0, and 66.2 %) for rock with (C10, C25, and C40) openings, respectively.
3. In biaxial state, the compressive strength increased with increasing confining pressure upon unconfined one. With increasing confined pressure, the effect of opening reduces. The reductions in the strength of confined rock under (σc=3 MPa), are (0.05, 8.7 and 20.2%) for (C10, C25 and C40) openings, respectively.
4. The normal stress (σn) of the flexural state decreases with increase in the opening size and rate of reduction is higher at larger confining pressure. The unconfined rock shows no effect of opening size. Also, the compressive strength (σc) increased with increasing confining pressure (σn), while decreases with opening size under flexural load condition.
5. In flexural state, the tensile strength (σt) decreases with the opening size. At higher confining pressure (σc= 3 MPa), the size of opening has insignificant effect. Unconfined rock shows no effect of the opening size beyond (C10).

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