Quantum dense coding over Bloch channels

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Abstract

Dynamics of coded information over Bloch channels is investigated for different values of the channel’s parameters. We show that, the suppressing of the travelling coded information over Bloch channel can be increased by decreasing the equilibrium absolute value of information carrier and consequently decreasing the distilled information by eavesdropper. The amount of decoded information can be improved by increasing the equilibrium values of the two qubits and decreasing the ratio between longitudinal and transverse relaxation times. The robustness of coded information in maximum and partial entangled states is discussed. It is shown that the maximum entangled states are more robust than the partial entangled state over this type of channels.

Keywords: Dense coding, Channels, Local and non-local information.

1 Introduction

Decoherence represents the most difficult obstacles in quantum information processing. This unavoidable phenomena can be seen in different pictures such as, the undesirable interactions between systems and their surroundings [1], device imperfections [2, 3], decay due to spontaneous, emission and noisy channel [4]. These interactions corrupt the information stored in the system and consequently cause errors in the transferred information. Therefore, investigating the dynamics of information in the presence of decoherence is one of the most important tasks in quantum computation and information.

Quantum coding is one of techniques that has been used to transfer information between two users [5]. To achieve quantum coding protocol with high efficiency, one needs maximum entangled state and ideal channels. There are some protocols which have been presented different treatments of quantum coding over noseless channels theoretically [6, 7] and experimentally [8]. In real word, it is very difficulty to keep systems which are used for quantum coding isolate. Therefore, it is important to introduce quantum coding protocols over noisy channels. Recently, Shadman and et.al., [9] have investigated super dense coding over noise, where they consider the case of Pauli channels in arbitrary dimension and derive the super dense coding capacity.

In the present work, we introduce different type of quantum channels called Bloch channels. The decoherence effect of these channels on the entanglement and information has been investigated by Ban and et. al. [10, 11]. They considered one qubit passing through the Bloch channel. Metwally [12] have investigated the effect of the Bloch channel on the fidelity of the teleported state, where the two qubits are pass through the channel. This motivated us to investigate the effect of the Bloch channels on the dynamics of coded information. Also, in this context the behavior of the local and non-local information is studied.

The paper is organized as follows: In Sec.2, we examine the evolution of a general two-qubits state passes through Bloch channel. The quantum dense coding is discussed in Sec.3. The dynamics of the local and non-local information is investigated in Sec.4. Finally, we discuss our results in Sec. 5.
2 Model and its solution

The characterization of the 2-qubit states produced by some sources requires experimental determination of 15 real parameters. Each qubit is determined by 3 parameters, representing the Bloch vectors, and the other 9 parameters represent the correlation tensor. Analogs of Pauli’s spin operators are used for the description of the individual qubits; the set \( \sigma_{1x}, \sigma_{1y}, \sigma_{1z} \) for the first qubit and \( \sigma_{2x}, \sigma_{2y}, \sigma_{2z} \) for the second qubit. Any two qubits state is described by \[13, 14, 15\],

\[
\rho_{ab} = \frac{1}{4}(1 + \vec{S} \cdot \sigma_{1}^1 + \vec{R} \cdot \sigma_{2}^1 + \vec{\sigma}_{1} \cdot \vec{Q} \cdot \sigma_{2}^1),
\]

where \( \vec{\sigma}_{1} \) and \( \vec{\sigma}_{2} \) are the Pauli’s spin vectors of the first and the second qubit respectively. The statistical operators for the individual qubits are specified by their Bloch vectors, \( \vec{A} = \langle \vec{\sigma}_{1} \rangle \) and \( \vec{R} = \langle \vec{\sigma}_{2} \rangle \). The cross dyadic, \( \vec{Q} \) is represented by a 3 \( \times \) 3 matrix, it describes the correlation between the first qubit, \( \rho_{a} = tr_{a}\{\rho_{ab}\} = \frac{1}{2}(1 + \vec{S} \cdot \sigma_{1}^1) \) and the second qubit, \( \rho_{b} = tr_{b}\{\rho_{ab}\} = \frac{1}{2}(1 + \vec{R} \cdot \sigma_{2}^1) \). The Bloch vectors and the cross dyadic are given by

\[
\vec{S} = (s_{x}, s_{y}, s_{z}), \quad \vec{R} = (r_{x}, r_{y}, r_{z}), \quad \text{and} \quad \vec{Q} = \begin{pmatrix}
q_{11} & q_{12} & q_{13} \\
q_{21} & q_{22} & q_{23} \\
q_{31} & q_{32} & q_{33}
\end{pmatrix}.
\]

Let us consider that each qubit is forced to pass through Bloch channel. This type of channels is defined by the Bloch equations \[10\], for the first qubit,

\[
\frac{d}{dt}\langle \sigma_{1x} \rangle_{t} = -\frac{1}{T_{2a}}\langle \sigma_{1x} \rangle_{t}, \quad \frac{d}{dt}\langle \sigma_{1y} \rangle_{t} = -\frac{1}{T_{2a}}\langle \sigma_{1y} \rangle_{t},
\]

\[
\frac{d}{dt}\langle \sigma_{1z} \rangle_{t} = -\frac{1}{T_{1a}}\langle \sigma_{1z} \rangle_{t} - \langle \sigma_{1z} \rangle_{eq},
\]

while for the second qubit, they are given by

\[
\frac{d}{dt}\langle \sigma_{2x} \rangle_{t} = -\frac{1}{T_{2b}}\langle \sigma_{2x} \rangle_{t}, \quad \frac{d}{dt}\langle \sigma_{2y} \rangle_{t} = -\frac{1}{T_{2b}}\langle \sigma_{2y} \rangle_{t},
\]

\[
\frac{d}{dt}\langle \sigma_{2z} \rangle_{t} = -\frac{1}{T_{1b}}\langle \sigma_{2z} \rangle_{t} - \langle \sigma_{2z} \rangle_{eq},
\]

where \( T_{1i} \) and \( T_{2i}, \quad i = a, b \) are the longitudinal and transverse relaxation times for the first and the second qubit, and \( \langle \sigma_{1z} \rangle_{eq}, \quad \langle \sigma_{2z} \rangle_{eq} \) are the equilibrium values of \( \langle \sigma_{1z} \rangle_{t} \) and \( \langle \sigma_{2z} \rangle_{t} \) respectively. Now, we assume that the two qubits pass through the channels (3), and (1). Then the output state is given by \[12\],

\[
\rho_{q}(t) = \frac{1}{4}\left(1 + \vec{S}(t) \cdot \sigma_{1}^1 + \vec{R}(t) \cdot \sigma_{2}^1 + \vec{\sigma}_{1} \cdot \vec{Q}(t) \cdot \sigma_{2}^1\right),
\]
where,

\[
\begin{align*}
\dot{S}(t) &= (s_x \beta_1, - \beta_1 s_y, \gamma_1 s_z + (1 - \gamma_1) \langle \sigma_{1z} \rangle_q), \\
\dot{R}(t) &= (\beta_2 r_x, - \beta_2 r_y, \gamma_2 r_z + (1 - \gamma_2) \langle \sigma_{2z} \rangle_q), \\
Q_{xx}(t) &= \beta_1 \beta_2 q_{11}, \quad Q_{xy}(t) = -q_{12} \beta_1 \beta_2, \\
Q_{xz}(t) &= \beta_1 \gamma_2 q_{13} + \beta_1 (1 - \gamma_2) \langle \sigma_{z1} \rangle_q s_x, \\
Q_{yz}(t) &= -q_{12} \beta_1 \beta_2, \quad Q_{yy}(t) = q_{22} \beta_1 \beta_2, \\
Q_{yz}(t) &= -\beta_1 \gamma_2 q_{23} - \beta_1 (1 - \gamma_2) \langle \sigma_{2z} \rangle_q s_y, \\
Q_{xz}(t) &= \beta_2 \gamma_1 q_{31} + \beta_2 (1 - \gamma_1) \langle \sigma_{z1} \rangle_q r_x, \\
Q_{zy}(t) &= -\beta_2 \gamma_1 q_{32} - \beta_2 (1 - \gamma_1) \langle \sigma_{1z} \rangle_q r_y, \\
Q_{zz}(t) &= \gamma_1 \gamma_2 q_{33} + (1 - \gamma_1)(1 - \gamma_2) \langle \sigma_{z1} \rangle_q \langle \sigma_{2z} \rangle_q \\
&\quad + \gamma_1 (1 - \gamma_2) \langle \sigma_{2z} \rangle_q s_3 + \gamma_2 (1 - \gamma_1) \langle \sigma_{2z} \rangle_q r_z,
\end{align*}
\]

and \(\gamma_i = \exp\{-\frac{t}{\tau_{z_i}}\}\), \(\beta_i = \exp\{-\frac{t}{\tau_{z_i}}\}\), and \(i = a, b\).

Equation (6), represents the time evaluation of any two qubits state passes through the Bloch channels Eqs. (3) and (4). Assume that the users, Alice and Bob share one maximum entangled state of Bell’s states, \(|\psi^\pm\rangle\) or \(|\phi^\pm\rangle\). The dynamics of these states can be obtained from (5) by setting \(s_x(0) = s_y(0) = s_z(0) = 0\), for the first qubit and \(r_x(0) = r_y(0) = r_z(0) = 0\) for the second qubit, \(Q_{ij}(0) = 0\) for \(i \neq j\) and \(Q_{xx}(0) = Q_{yy}(0) = Q_{zz}(0) = -1\) for \(|\psi^+\rangle\), \(|\phi^+\rangle\), \(|\psi^-\rangle\), and so on. On the other hand, the users can use an initial pure partial entangled state. This class of states is characteristic by one parameter \(|\psi\rangle\). It is defined by :

\[
\dot{S} = (0, 0, p), \quad \dot{R} = (0, 0, -p) \text{ and } Q_{ij} = 0 \text{ for } i \neq j, Q_{xx} = Q_{yy} = -\sqrt{1 - p^2} \text{ and } Q_{zz} = -1.
\]

### 3 Quantum coding

Let us consider that the partners Alice and Bob share maximum or partial entangled state. The aim of Alice is sending the coded information to Bob. But for some reasons the carrier of these coded information is forced to pass through the Bloch channel. We quantify the amount of information which decoded by Bob and investigating the effect of the channel parameters and the type of the initial carrier on the accuracy of the decoded information. To show our idea, we implement the original dense coding protocol which has been proposed by Bennett and Wienser [5]. This protocol is described as follows:

1. Alice encodes two classical bits by using one of local unitary operators.

2. If Alice applies these unitary operators randomly with probability \(\eta_i\), then she codes the information in the state,

\[
\rho_c = \sum_{j=0}^{3} \left\{ \eta_j U_j \otimes I_2 \rho^\text{out}_j U_j^\dagger \otimes I_2 \right\},
\]

where \(\rho^\text{out}\) is given by (5) and \(U_j = I_1, \sigma_{1x}, \sigma_{1y}, \sigma_{1z}\), are the unitary operators for the first qubit and \(I_2\) is the identity operator for the second qubit.
3. Alice sends her qubit to Bob, who makes joint measurements on the two qubits. The maximum amount of information which Bob can extract from Alice’s message is Bounded by,

\[ I_d = S\left(\sum_{j=0}^{3} \eta_j \rho_j^{\text{out}}\right) - \sum_{j=0}^{3} \eta_j S(\rho_j^{\text{out}}). \]  

(9)

Fig.(1a) displays the effect of the equilibrium absolute values of the first qubit, \(\langle \sigma_{1z} \rangle_{eq}\) on the decoded information. It is clear that, before the interaction is switched on, the decoded information is very large and and goes down quickly once the interaction is devoloped. This decay of the decoded information increases as \(\langle \sigma_{1z} \rangle_{eq}\) decreases. However as time increases, the decoded information increases gradually and reaches its upper bound.

The effect of the ratio between longitudinal and transverse relaxation times, \(\alpha_i\) is depicted in Fig.(1b). As \(\alpha_i\) increases and the equilibrium absolute values of the two qubits are large, the decay of the decoded information becomes faster. For \(t > 50\), the decoded information increases faster for small values of \(\alpha_i\). In Fig.(2), we consider that Alice coded her information in partial entangled state, where we set \(p = 0.5\) in \((7)\). The dynamics of the decoded information is similar to that shown in Fig.(1). However the maximum amount of the decoded information for the PES is smaller than that for MES.

From Figs.(1 & 2), it is clear that the travelling coded information in a state prepared initially in maximum entangled states, MES is much better than using partial entangled states, PES. This means that MES are more robust than PES for this type of channels.

4 Dynamics of Local and non Local Information

Suppose we have a source supplies each user with a qubit to code their own information. In this case, one says that these information are local information. If the qubits are forced to pass through environment (say Bloch channels), then the two qubits will entangled with

![Figure 1: The dynamics of the decoded information in a state initially prepared in maximum entangled state (a) the solid, dash-dot and dot curves are for \(\langle \sigma_{1z} \rangle_{eq} = 1, 0.9, 0.8\) respectively and \(\langle \sigma_{2z} \rangle_{eq} = 0.9, \alpha = 0.5\). (b) the solid, dash-dot and dot lines for \(\alpha = 0.7, 0.6, 0.5\) respectively and \(\langle \sigma_{1z} \rangle_{eq} = 1, \langle \sigma_{2z} \rangle_{eq} = 0.9\).]
each other and interact with the environment. As a resultant of this interaction the local information will be transferred between the two qubits and called non-local information.

In this section, we investigate the dynamics of local information $I_A$ which coded in Alice’s qubit $\rho_a = tr_b\{\rho_c\}$, $I_B$ is the local information which is coded in Bob’s qubit, $\rho_b = tr_a\{\rho_c\}$ and the non-local information between Alice and Bob, $I_{ab}$ which is coded in the state $\rho_c$. Due to the undesirable interactions there are some information lose. These interactions can be considered as another person (Eve), who tries to distill information from the travelling state between Alice to Bob. Mathematically, this information is defined as:

$$I_{AE} = F\log\{F\} + (1 - F)\log(1 - F),$$

(10)

where $\mathcal{F}$ is the fidelity that Bob decoded the information.

Figs.(3 & 4), describe the effect of the channel’s parameters on the dynamics of the local information $I_A, I_B$, and the non-local information between Alice and Bob, $I_{AB}$, and between

Figure 3: The dynamics of the local and non-local information for $\alpha = 0.7, \langle \sigma_{2z} \rangle_{eq} = 0.9$. The solid, dash-dot,long-dash and dot curves for $I_A, I_B, I_{AB}$ and $I_{AE}$ respectively. (a)$\langle \sigma_{1z} \rangle_{eq} = 1$(b) $\langle \sigma_{1z} \rangle_{eq} = 0.3$. 

Figure 2: The same as Fig.(1), but for the partial entangled state.
Alice and Eve, $I_{AE}$. In Fig.(3), we investigate the effect of the absolute equilibrium values on the dynamics of the travelling coded information where we assume that Alice has coded her information in maximum entangled state. It is clear that, at the beginning the non-local information between Alice and Bob, $I_{AB}$ and between Alice and Eve, $I_{AE}$ are zero, while $I_A$ and $I_B$ are non-zero. As soon as the interaction times goes on, $I_{AB}$ and $I_{AE}$ increase on the expance of the local information owned by Alice and Bob. As time increases more, Eve distill more information from Alice and $I_{AE}$ is much larger than $I_A$. On the other hand, due to the lose of the information from Alice side, the non-local information between Alice and Bob decreases. The dynamical behavior of these different types of the information are depicted in Fig.(3a). Fig.(3b) describes the dynamics of the local and non-local information for small value of $\langle \sigma_{1z} \rangle_{eq}$ (say $\approx 0.3$). In this case, the amount of information which is distilled by Eve is smaller than that shown in Fig.(3a). However, Alice’s information is slightly affected. Therefore decreasing the absolute equilibrium value of one qubit, maximize the non-local-information between the two qubits.

Fig.(4) displays the dynamics of information, where Alice has coded her information in partial entangled state. In this case, for large values of $\langle \sigma_{1z} \rangle_{eq}$ and $\langle \sigma_{2z} \rangle_{eq}$ the information which is gained by Eve, increases abruptly on the expance of Alice’s information and for $t > 100$, $I_{AE} > I_{AB}$. However as one of the absolute equilibrium values is decreased, the non-local information between Alice and Bob, $I_{AB}$ is increased very fast and its maximum value is always larger than that depicted in Fig.(3). As time goes on, $I_{AB}$ decreases slowly and its minimum value is always larger than that depicted in Fig.(3b). Although Eve’s information increases fast, but $I_{AE} < I_{AB}$. In a very small range of time $I_{AE} > I_A$. So for this choice of the channel’s parameters, Alice and Bob can communicate safely for long range of time.

The effect of the ratio between longitudinal and transverse relaxation times, $\alpha_i$ is shown in Fig.(5), where we set $\alpha_1 = \alpha_2 = 0.3$, $\langle \sigma_{1z} \rangle_{eq} = 1$ and $\langle \sigma_{2z} \rangle_{eq} = 0.9$. It is clear that, from Fig.(5a), (we assume that Alice coded her information in MES), $I_A$ decreases very fast and Alice’s state turns into a completely mixed state for $t > 200$ and consequently Eve’s information increases very fast and reaches its maximum value faster than that shown in Fig.(4a), in which $\alpha_i = 0.7$. As soon as Alice loses her information completely, $I_{AB}$ and $I_B$ have asymptotically the same values $t > 200$, which is much earlier than that displayed in Fig.(3a). In Fig.(5b), we assume that the information is initially coded in PES. In general, the

![Figure 4: The same as Fig.(3) but for the partial entangled state.](image-url)
Figure 5: $\langle \sigma_{1z} \rangle_{eq} = 0.9$ and $\alpha = 0.3$ (a) for the maximum entangled state (b) For the partial entangled state.

dynamics of information is similar to that shown in Fig.(5a), but from Fig.(4a) and Fig.(5b), we can see that the safely communicate time decreases and the non-local information between Alice and Bob, $I_{AB}$ decreases very fast.

From Figs.(4 & 5), one concludes that the absolute equilibrium values and the ratio between longitudinal and transverse relaxation times can be considered as control parameters. One can improve the local information for one qubit by decreasing the equilibrium values of the other qubit. Also, the eavesdropper information can be minimized be decreasing the absolute equilibrium of one qubit and increasing the ratio between longitudinal and transverse relaxation times. In this case, the information lose is always smaller than the information between the sender and receiver. Therefore, the users can increase the safety communication time and improve the non-local information.

Also, the initial state which is used to code information plays an important role on the secure communication. It is clear that, coding information in maximum entangled state is much better than using partial entangled states, where for the first the users can increase the safe time of communication by controlling on the channel parameters.

5 Conclusion

The time evaluation of a system consists of two qubits passes through Bloch channel is investigated. The quantum dense coding protocol is implemented by using two different initial states setting:maximum and partial entangled states. The coded information is send with high accuracy by increasing the absolute equilibrium values of the two qubits and decreasing the ratio of the longitudinal and transverse relaxation times. However, if the absolute equilibrium value of one qubit decreases, the decoded information decreases. It is shown that, using maximum entangled state for coding information is much better than using partial entangled state. This means that, the maximum entangled states are more robust then partial entangled states when they travel through Bloch channels.

The local and non-local information are quantified for different values of the channel parameters. There are some cases, where the eavesdropper can distill more information on the expanse of the travelling coded information. However the partners can communicate
safely when the non-local information between the two users is larger than that distilled from the travelling coded information. Also, the absolute equilibrium values and the ratio of the longitudinal and transverse relaxation times can be considered as a control parameters. It is clear that, for large values of the equilibrium absolute parameters for both qubit, the local information of both qubit decreases faster and consequently the information gained by eavesdropper increases. However, if the equilibrium absolute value of one qubit decreases, its corresponding local information is slightly affected. Therefore, to send the coded information from the sender to the receiver safely, one has to decrease the absolute equilibrium value. Also, as one increases the ratio of the longitudinal and transverse relaxation times, the survival time of the local and non-local information increases.

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In the present work, we introduce different type of quantum channels called Bloch channels. The decoherence effect of these channels on the entanglement and information has been investigated by Ban and et. al. [10] [11]. They considered one qubit passing through the Bloch channel. Metwally [12] have investigated the effect of the Bloch channel on the fidelity of the teleported state, where the two qubits are pass through the channel. This motivated us to investigate the effect of the Bloch channels on the dynamics of coded information. Also, in this context the behavior of the local and non-local information is studied.

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$$\rho_{ab} = \frac{1}{4} (1 + \vec{S} \cdot \sigma_{1}^{\dagger} + \vec{R} \cdot \sigma_{2}^{\dagger} + \vec{Q} \cdot \sigma_{2}^{\dagger}),$$

(1)

where $\vec{\sigma}_{1}$ and $\vec{\sigma}_{2}$ are the Pauli’s spin vectors of the first and the second qubit respectively. The statistical operators for the individual qubits are specified by their Bloch vectors, $\vec{A} = \langle \vec{\sigma}_{1} \rangle$ and $\vec{R} = \langle \vec{\sigma}_{2} \rangle$. The cross dyadic, $\vec{Q}$ is represented by a $3 \times 3$ matrix, it describes the correlation between the first qubit, $\rho_{a} = tr_{b}\{\rho_{ab}\} = \frac{1}{2} (1 + \vec{S} \cdot \sigma_{1}^{\dagger})$ and the second qubit, $\rho_{b} = tr_{a}\{\rho_{ab}\} = \frac{1}{2} (1 + \vec{R} \cdot \sigma_{2}^{\dagger})$. The Bloch vectors and the cross dyadic are given by

$$\vec{S} = (s_{x}, s_{y}, s_{z}), \quad \vec{R} = (r_{x}, r_{y}, r_{z}), \quad \text{and} \quad \vec{Q} = \begin{pmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{pmatrix}.$$ (2)

Let us consider that each qubit is forced to pass through Bloch channel. This type of channels is defined by the Bloch equations [10], for the first qubit,

$$\frac{d}{dt} \langle \sigma_{1x} \rangle_{t} = -\frac{1}{T_{2a}} \langle \sigma_{1x} \rangle_{t}, \quad \frac{d}{dt} \langle \sigma_{1y} \rangle_{t} = -\frac{1}{T_{2a}} \langle \sigma_{1y} \rangle_{t},$$

$$\frac{d}{dt} \langle \sigma_{1z} \rangle_{t} = -\frac{1}{T_{1a}} (\langle \sigma_{1z} \rangle_{t} - \langle \sigma_{1z} \rangle_{eq}),$$

(3)

while for the second qubit, they are given by

$$\frac{d}{dt} \langle \sigma_{2x} \rangle_{t} = -\frac{1}{T_{2b}} \langle \sigma_{2x} \rangle_{t}, \quad \frac{d}{dt} \langle \sigma_{2y} \rangle_{t} = -\frac{1}{T_{2b}} \langle \sigma_{2y} \rangle_{t},$$

$$\frac{d}{dt} \langle \sigma_{2z} \rangle_{t} = -\frac{1}{T_{1b}} (\langle \sigma_{2z} \rangle_{t} - \langle \sigma_{2z} \rangle_{eq}),$$

(4)

where $T_{1i}$ and $T_{2i}, \ i = a, b$ are the longitudinal and transverse relaxation times for the first and the second qubit, and $\langle \sigma_{1z} \rangle_{eq}, \langle \sigma_{2z} \rangle_{eq}$ are the equilibrium values of $\langle \sigma_{1z} \rangle_{t}$ and $\langle \sigma_{2z} \rangle_{t}$ respectively. Now, we assume that the two qubits pass through the channels (3), and (1). Then the output state is given by [12],

$$\rho_{q}(t) = \frac{1}{4} \left(1 + \vec{S}(t) \cdot \sigma_{1}^{\dagger} + \vec{R}(t) \cdot \sigma_{2}^{\dagger} + \vec{Q}(t) \cdot \sigma_{2}^{\dagger} \right),$$

(5)
and $\gamma$ by Bennett and Wienser [5]. This protocol is described as follows:

To show our idea, we implement the original dense coding protocol which has been proposed parameters and the type of the initial carrier on the accuracy of the decoded information. The aim of Alice is sending the coded information to Bob. But for some reasons the carrier

Let us consider that the partners Alice and Bob share maximum or partial entangled state. This class of states is characteristic by one parameter [14]. It is defined by:

$$
\gamma_i = \exp\{-\frac{t}{\tau_i}\}, \quad \beta_i = \exp\{-\frac{t}{\tau_{2i}}\}, \quad i = a, b.
$$

Equation (5), represents the time evaluation of any two qubits state passes through the Bloch channels Eqs. (3) and (4). Assume that the users, Alice and Bob share one maximum entangled state of Bell’s states, $|\psi^+\rangle$ or $|\phi^+\rangle$. The dynamics of these states can be obtained from (5) by setting $s_x(0) = s_y(0) = s_z(0) = 0$, for the first qubit and $r_x(0) = r_y(0) = r_z(0) = 0$ for the second qubit, $Q_{ij}(0) = 0$ for $i \neq j$ and $Q_{xx}(0) = Q_{yy}(0) = Q_{zz}(0) = -1$ for $|\psi^+\rangle$, and $Q_{xx}(0) = Q_{yy}(0) = Q_{zz}(0) = 1$ for $|\phi^+\rangle$, $Q_{xx}(0) = Q_{zz}(0) = 1, Q_{yy}(0) = -1$ for $|\psi^-\rangle$ and so on. On the other hand, the users can use an initial pure partial entangled state. This class of states is characteristic by one parameter [14]. It is defined by:

$$
\vec{S} = (0, 0, p), \quad \vec{R} = (0, 0, -p) \quad \text{and} \quad Q_{ij} = 0 \quad \text{for} \quad i \neq j, Q_{xx} = Q_{yy} = -\sqrt{1-p^2} \quad \text{and} \quad Q_{zz} = -1.
$$

3 Quantum coding

Let us consider that the partners Alice and Bob share maximum or partial entangled state. The aim of Alice is sending the coded information to Bob. But for some reasons the carrier of these coded information is forced to pass through the Bloch channel. We quantify the amount of information which decoded by Bob and investigating the effect of the channel parameters and the type of the initial carrier on the accuracy of the decoded information. To show our idea, we implement the original dense coding protocol which has been proposed by Bennett and Wienser [5]. This protocol is described as follows:

1. Alice encodes two classical bits by using one of local unitary operators.

2. If Alice applies these unitary operators randomly with probability $\eta_i$, then she codes the information in the state,

$$
\rho_c = \sum_{j=0}^{3}\{\eta_j U_j \otimes I_2 \rho^{out}_j U_j^\dagger \otimes I_2\},
$$

where $\rho^{out}$ is given by (5) and $U_j = I_1, \sigma_{1x}, \sigma_{1y}, \sigma_{1z}$, are the unitary operators for the first qubit and $I_2$ is the identity operator for the second qubit.
3. Alice sends her qubit to Bob, who makes joint measurements on the two qubits. The maximum amount of information which Bob can extract from Alice’s message is Bounded by,

$$I_d = S\left(\sum_{j=0}^{j=3} \eta_j \rho_{j}^{out}\right) - \sum_{j=0}^{j=3} \eta_j S(\rho_{j}^{out}).$$

(9)

Fig.(1a) displays the effect of the equilibrium absolute values of the first qubit, $\langle \sigma_{1z} \rangle_{eq}$ on the decoded information. It is clear that, before the interaction is switched on, the decoded information is very large and goes down quickly once the interaction is devolped. This decay of the decoded information increases as $\langle \sigma_{1z} \rangle_{eq}$ decreases. However as time increases, the decoded information increases gradually and reaches its upper bound.

The effect of the ratio between longitudinal and transverse relaxation times, $\alpha_i$ is depicted in Fig.(1b). As $\alpha_i$ increases and the equilibrium absolute values of the two qubits are large, the decay of the decoded information becomes faster. For $t > 50$, the decoded information increases faster for small values of $\alpha_i$. In Fig.(2), we consider that Alice coded her information in partial entangled state, where we set $p = 0.5$ in (7). The dynamics of the decoded information is similar to that shown in Fig.(1). However the maximum amount of the decoded information for the PES is smaller than that for MES.

From Figs.(1 & 2), it is clear that the travelling coded information in a state prepared initially in maximum entangled states, MES is much better than using partial entangled states, PES. This means that MES are more robust than PES for this type of channels.

4 Dynamics of Local and non Local Information

Suppose we have a source supplies each user with a qubit to code their own information. In this case, one says that these information are local information. If the qubits are forced to pass through environment (say Bloch channels), then the two qubits will entangled with
each other and interact with the environment. As a resultant of this interaction the local information will be transferred between the two qubits and called non-local information.

In this section, we investigate the dynamics of local information $I_A$ which coded in Alice’s qubit $\rho_a = \text{tr}_b\{\rho_c\}$, $I_B$ is the local information which is coded in Bob’s qubit, $\rho_b = \text{tr}_a\{\rho_c\}$ and the non-local information between Alice and Bob, $I_{ab}$ which is coded in the state $\rho_c$. Due to the undesirable interactions there are some information lose. These interactions can be considered as another person (Eve), who tries to distill information from the travelling state between Alice to Bob. Mathematically, this information is defined as:

$$I_{AE} = F \log \{F\} + (1 - F) \log (1 - F),$$

where $F$ is the fidelity that Bob decoded the information.

Figs.(3 & 4), describe the effect of the channel’s parameters on the dynamics of the local information $I_A, I_B$, and the non-local information between Alice and Bob, $I_{AB}$, and between

![Figure 2: The same as Fig.(1), but for the partial entangled state.](image1)

![Figure 3: The dynamics of the local and non-local information for $\alpha = 0.7, \langle \sigma_{2z} \rangle_{eq} = 0.9$. The solid, dash-dot,long-dash and dot curves for $I_A, I_B, I_{AB}$ and $I_{AE}$ respectively. (a)$\langle \sigma_{1z} \rangle_{eq} = 1$(b)$\langle \sigma_{1z} \rangle_{eq} = 0.3$.](image2)
Alice and Eve, $I_{AE}$. In Fig.(3), we investigate the effect of the absolute equilibrium values on the dynamics of the travelling coded information where we assume that Alice has coded her information in maximum entangled state. It is clear that, at the beginning the non-local information between Alice and Bob, $I_{AB}$ and between Alice and Eve, $I_{AE}$ are zero, while $I_A$ and $I_B$ are non-zero. As soon as the interaction times goes on, $I_{AB}$ and $I_{AE}$ increase on the expanse of the local information owned by Alice and Bob. As time increases more, Eve distill more information from Alice and $I_{AE}$ is much larger than $I_A$. On the other hand, due to the lose of the information from Alice side, the non-local information between Alice and Bob decreases. The dynamical behavior of these different types of the information are depicted in Fig.(3a). Fig.(3b) describes the dynamics of the local and non-local information for small value of $\langle \sigma_{1z} \rangle_{eq}$ (say $\approx 0.3$). In this case, the amount of information which is distilled by Eve is smaller than that shown in Fig.(3a). However, Alice’s information is slightly affected. Therefore decreasing the absolute equilibrium value of one qubit, maximize the non-local-information between the two qubits.

Fig.(4) displays the dynamics of information, where Alice has coded her information in partial entangled state. In this case, for large values of $\langle \sigma_{1z} \rangle_{eq}$ and $\langle \sigma_{2z} \rangle_{eq}$ the information which is gained by Eve, increases abruptly on the expanse of Alice’s information and for $t > 100$, $I_{AE} > I_{AB}$. However as one of the absolute equilibrium values is decreased, the non-local information between Alice and Bob, $I_{AB}$ is increased very fast and its maximum value is always larger than that depicted in Fig.(3). As time goes on, $I_{AB}$, decreases slowly and its minimum value is always larger than that depicted in Fig.(3b). Although Eve’s information increases fast, but $I_{AE} < I_{AB}$. In a very small range of time $I_{AE} > I_A$. So for this choice of the channel’s parameters, Alice and Bob can communicate safely for long range of time.

The effect of the ratio between longitudinal and transverse relaxation times, $\alpha_i$ is shown in Fig.(5), where we set $\alpha_1 = \alpha_2 = 0.3$, $\langle \sigma_{1z} \rangle_{eq} = 1$ and $\langle \sigma_{2z} \rangle_{eq} = 0.9$. It is clear that, from Fig.(5a), ( we assume that Alice coded her information in MES), $I_A$ decreases very fast and Alice’s state turns into a completely mixed state for $t > 200$ and consequently Eve’s information increases very fast and reaches its maximum value faster than that shown in Fig.(4a), in which $\alpha_i = 0.7$. As soon as Alice loses her information completely, $I_{AB}$ and $I_B$ have asymptotically the same values $t > 200$, which is much earlier than that displayed in Fig.(3a). In Fig.(5b), we assume that the information is initially coded in PES. In general, the
Figure 5: $\langle \sigma_{1z} \rangle_{eq} = 0.9$ and $\alpha = 0.3$ (a) for the maximum entangled state (b) For the partial entangled state.

dynamics of information is similar to that shown in Fig.(5a), but from Fig.(4a) and Fig.(5b), we can see that the safely communicate time decreases and the non-local information between Alice and Bob, $I_{AB}$ decreases very fast.

From Figs.(4 & 5), one concludes that the absolute equilibrium values and the ratio between longitudinal and transverse relaxation times can be considered as control parameters. One can improve the local information for one qubit by decreasing the equilibrium values of the other qubit. Also, the eavesdropper information can be minimized be decreasing the absolute equilibrium of one qubit and increasing the ratio between longitudinal and transverse relaxation times. In this case, the information lose is always smaller than the information between the sender and receiver. Therefore, the users can increase the safety communication time and improve the non-local information.

Also, the initial state which is used to code information plays an important role on the secure communication. It is clear that, coding information in maximum entangled state is much better than using partial entangled states, where for the first the users can increase the safe time of communication by controlling on the channel parameters.

5 Conclusion

The time evaluation of a system consists of two qubits passes through Bloch channel is investigated. The quantum dense coding protocol is implemented by using two different initial states setting:maximum and partial entangled states. The coded information is send with high accuracy by increasing the absolute equilibrium values of the two qubits and decreasing the ratio of the longitudinal and transverse relaxation times. However, if the absolute equilibrium value of one qubit decreases, the decoded information decreases. It is shown that, using maximum entangled state for coding information is much better than using partial entangled state. This means that, the maximum entangled states are more robust then partial entangled states when they travel through Bloch channels.

The local and non-local information are quantified for different values of the channel parameters. There are some cases, where the eavesdropper can distill more information on the expanse of the travelling coded information. However the partners can communicate
safely when the non-local information between the two users is larger than that distilled from the travelling coded information. Also, the absolute equilibrium values and the ratio of the longitudinal and transverse relaxation times can be considered as a control parameters. It is clear that, for large values of the equilibrium absolute parameters for both qubit, the local information of both qubit decreases faster and consequently the information gained by eavesdropper increases. However, if the equilibrium absolute value of one qubit decreases, its corresponding local information is slightly affected. Therefore, to send the coded information from the sender to the receiver safely, one has to decrease the absolute equilibrium value. Also, as one increases the ratio of the longitudinal and transverse relaxation times, the survival time of the local and non-local information increases.

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