NEW TESTS OF INFLATION

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Slow-roll inflation generically makes several predictions: a flat Universe, primordial adiabatic density perturbations, and a stochastic gravity-wave background. Each inflation model will further predict specific relations between the amplitudes and shapes of the spectrum of density perturbations and gravity waves. There are now excellent prospects for testing precisely these predictions with forthcoming cosmic microwave background (CMB) temperature and polarization maps.

1 Introduction

Although the physics responsible for slow-roll inflation is still not well understood, inflation generically predicts (1) a flat Universe; (2) that primordial adiabatic (scalar metric) perturbations are responsible for the large-scale structure (LSS) in the Universe today; and (3) a stochastic gravity-wave background (tensor metric perturbations). Furthermore, each inflationary model predicts (4) specific relations between the “inflationary observables,” the amplitudes and spectral indices of the scalar and tensor perturbations. The amplitude of the gravity-wave background is proportional to the height of the inflaton potential. Therefore, the height of the inflaton potential, $V(\phi)$, can be fixed by the tensor contribution to the CMB quadrupole moment, $C^{TT}_2$:

$$\mathcal{T} \equiv 6 C^{TT}_{2, \text{tensor}} = 9.2 V/m_{Pl}^4.$$  (1)

The predictions for the scalar amplitude and the spectral indices follow immediately from the shape of the inflaton potential. Therefore, determination of the inflationary observables would illuminate the physics responsible for inflation.

Until recently, none of these predictions could be tested with precision. Measured values for the density of the Universe span almost an order of magnitude. Furthermore, most measurements do not probe the possible contribution of a cosmological constant (or some other diffuse matter component), so they

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do not address the geometry of the Universe. The only observable effects of a stochastic gravity-wave background are in the CMB. COBE observations do in fact provide an upper limit to the tensor amplitude, and therefore an inflaton potential, \( (V/m_{Pl}^4) < 5 \times 10^{-12} \). However, there is no way to disentangle the scalar and tensor contributions to the COBE anisotropy.

In recent years, it has become increasingly likely that adiabatic perturbations are responsible for the origin of structure. Before COBE, there were numerous plausible models for structure formation: e.g., isocurvature perturbations both with and without cold dark matter, late-time or slow phase transitions, topological defects (cosmic strings or textures), superconducting cosmic strings, explosive or seed models, a “loitering” Universe, etc. However, the amplitude of the COBE anisotropy makes all these alternative models unlikely. With adiabatic perturbations, hotter regions at the surface of last scatter are embedded in deeper potential wells, so the reddening due to the the gravitational redshift of the photons from these regions partially cancels the higher intrinsic temperatures. Thus, other models will generically produce more anisotropy for the same density perturbation. When normalized to the density fluctuations indicated by galaxy surveys, alternative models thus generically produce a larger temperature fluctuation than that measured by COBE. In the past year, some leading proponents of topological defects have conceded that these models have difficulty accounting for the origin of large-scale structure.

We are now entering an exciting new era, driven by new theoretical ideas and developments in detector technology, in which the predictions of inflation will be tested with unprecedented precision. It is even conceivable that early in the next century, we will move from verification of inflation to direct investigation of the high-energy physics responsible for inflation.

The purpose of this talk is to review how forthcoming CMB experiments will test several of these predictions. I will first discuss how CMB temperature anisotropies will test the inflationary predictions of a flat Universe and a primordial spectrum of density fluctuations. I will then review how a CMB polarization map may be used to isolate the gravity waves and briefly review how detection of these tensor modes may be used to learn about the physics responsible for inflation.

2 Temperature Anisotropies

The primary goal of CMB experiments that map the temperature as a function of position on the sky is recovery of the temperature autocorrelation function or angular power spectrum of the CMB. The fractional temperature perturbation
\( \frac{\Delta T(\hat{n})}{T} \) in a given direction \( \hat{n} \) can be expanded in spherical harmonics,

\[
\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a^T_{(lm)} Y_{(lm)}(\hat{n}),
\]

where the multipole coefficients are given by

\[
a^T_{(lm)} = \int d\hat{n} Y^*_{(lm)}(\hat{n}) \frac{\Delta T(\hat{n})}{T}.
\]

Statistical isotropy and homogeneity of the Universe imply that these coefficients have expectation values

\[
\langle (a^T_{(lm)})^* a^T_{(l'm')} \rangle = C^{TT}_{l} \delta_{ll'} \delta_{mm'}
\]

when averaged over the sky. Roughly speaking, the multipole moments \( C^{TT}_l \) measure the mean-square temperature difference between two points separated by an angle \( (\theta/1^\circ) \sim 200/l \).

Predictions for the \( C_l \)'s can be made given a theory for structure formation and the values of several cosmological parameters. The curves in Fig. 1 show these predictions for several values of \( \Omega \). The bumps come from oscillations in the photon-baryon fluid at the surface of last scatter. As the Figure shows, these small-angle CMB anisotropies can be used to determine the geometry of the Universe. The angle subtended by the horizon at the surface of last scatter is \( \theta_H \sim \Omega^{1/2} 1^\circ \), and the peaks in the CMB spectrum are due to causal processes at the surface of last scatter. Therefore, the angles (or values of \( l \)) at which the peaks occur determine the geometry of the Universe. Detailed calculations also show that the angular position of the first peak is relatively insensitive to the values of other undetermined (or still imprecisely determined) cosmological parameters such as the baryon density, the Hubble constant, and the cosmological constant (as well as several others). Therefore, determination of the location of this first acoustic peak should provide a robust measure of the geometry of the Universe.

The precision attainable is ultimately limited by cosmic variance and practically by the finite angular resolution, instrumental noise, and partial sky coverage in a realistic CMB mapping experiment. Taking these considerations into account, it can be shown that future satellite missions may potentially determine \( \Omega \) to better than 10% after marginalizing over all other undetermined parameters, and better than 1% if the other parameters can be fixed by independent observations or assumption. This would be far more accurate than any traditional determinations of the geometry.

It can similarly be shown that the CMB should provide determinations of the cosmological constant and baryon density far more precise than those from
Figure 1: Theoretical predictions and current and simulated data for CMB spectra as a function of multipole moment $l$ for models with primordial adiabatic perturbations. The curves are for three different values of the total density $\Omega$. Simulated MAP data points are shown.
traditional observations. If there is more nonrelativistic matter in the Universe than baryons can account for—as suggested by current observations—it will become increasingly clear with future CMB measurements. Subsequent analyses have confirmed these estimates with more precise numerical calculations.

Although these forecasts relied on the assumptions that adiabatic perturbations were responsible for structure formation and that reionization would not erase CMB anisotropies, these assumptions have become increasingly justifiable in the past few years. As discussed above, the leading alternative theories for structure formation now appear to be in trouble, and recent detections of CMB anisotropy at degree angular separations show that the effects of reionization are small.

NASA has recently approved the flight of a satellite mission, the Microwave Anisotropy Probe (MAP), in the year 2000 to carry out these measurements, and ESA has approved the flight of a subsequent more precise experiment, the Planck Surveyor. Therefore, it appears increasingly likely that the inflationary prediction of a flat Universe will be carried out precisely in the near future.

The predictions of a nearly scale-free spectrum of primordial adiabatic perturbations will also be further tested with measurements of small-angle CMB anisotropies. The existence and structure of the acoustic peaks will provide an unmistakable signature of adiabatic perturbations and the spectral index $n_s$ can be determined from fitting the theoretical curves to the data in the same way that the density, cosmological constant, baryon density, and Hubble constant are also fit.

Temperature anisotropies produced by a stochastic gravity-wave background would affect the shape of the angular CMB spectrum, but there is no way to disentangle the scalar and tensor contributions to the CMB anisotropy in a model-independent way. Unless the tensor signal is large, the cosmic variance from the dominant scalar modes will provide an irreducible limit to the sensitivity of a temperature map to a tensor signal.

### 3 CMB Polarization and Gravitational Waves

Although a CMB temperature map cannot unambiguously distinguish between the density-perturbation and gravity-wave contributions to the CMB, the two can be decomposed in a model-independent fashion with a map of the CMB polarization. Suppose we measure the linear-polarization “vector” $P(\hat{n})$...
at every point \( \hat{n} \) on the sky. Such a vector field can be written as the gradient of a scalar function \( A \) plus the curl of a vector field \( \vec{B} \),

\[
\vec{P}(\hat{n}) = \nabla A + \nabla \times \vec{B}.
\]  

(4)
The gradient (i.e., curl-free) and curl components can be decomposed by taking the divergence or curl of \( \vec{P}(\hat{n}) \) respectively. Density perturbations are scalar metric perturbations, so they have no handedness. They can therefore produce no curl. On the other hand, gravitational waves do have a handedness so they can (and we have shown that they do) produce a curl. This therefore provides a way to detect the inflationary stochastic gravity-wave background and thereby test the relations between the inflationary observables. It should also allow one to determine (or at least constrain in the case of a nondetection) the height of the inflaton potential.

As with a temperature map, the sensitivity of a polarization map to gravity waves will be determined by the instrumental noise and fraction of sky covered, and by the angular resolution. Suppose the detector sensitivity is \( s \) and the experiment lasts for \( t_{yr} \) years with an angular resolution better than 1°. Suppose further that we consider only the curl component of the polarization in our analysis. Then the smallest tensor amplitude \( T_{min} \) to which the experiment will be sensitive at 1\( \sigma \) is

\[
\frac{T_{min}}{6 C_2^T} \simeq 5 \times 10^{-4} \left( \frac{s}{\mu K \sqrt{sec}} \right)^2 t_{yr}^{-1}.
\]

(5)

Thus, the curl component of a full-sky polarization map is sensitive to inflaton potentials \( (V/m_P^4) \gtrsim 5 \times 10^{-15} t_{yr}^{-1} (s/\mu K \sqrt{sec})^2 \). Improvement on current constraints with only the curl polarization component requires a detector sensitivity \( s \lesssim 40 t_{yr}^{1/2} \mu K \sqrt{sec} \). For comparison, the detector sensitivity of MAP will be \( s = \mathcal{O}(100 \mu K \sqrt{sec}) \). However, Planck may conceivably get sensitivities around \( s = 25 \mu K \sqrt{sec} \).

Even a small amount of reionization will significantly increase the polarization signal at low \( l \). For example, suppose the optical depth to the surface of last scatter is \( \tau = 0.1 \). With such a level of reionization, the sensitivity to the tensor amplitude is increased by more than a factor of 5 over that in Eq. (5). This level of reionization (if not more) is expected in cold-dark-matter models, so if anything, Eq. (5) provides a conservative estimate.

\footnote{Strictly speaking, the linear polarization does not transform as a vector, but the argument given here generalizes when one describes the polarization state properly as a symmetric trace-free 2 \( \times \) 2 tensor.}
Furthermore, the estimate in Eq. (5) takes into account only the information provided by the curl polarization moments. A complete likelihood analysis will fit the temperature-polarization map to the temperature moments, the gradient component, and the temperature-polarization cross-correlation, and this will improve the sensitivity significantly over that given in Eq. (5).

4 Discussion

If MAP and Planck find a CMB temperature-anisotropy spectrum consistent with a flat Universe and nearly-scale-free primordial adiabatic perturbations, then the next step will be to isolate the gravity waves with the polarization of the CMB. If inflation has something to do with grand unification, then it is possible that Planck’s polarization sensitivity will be sufficient to see the polarization signature of gravity waves. However, it is also quite plausible that the height of the inflaton potential may be low enough to elude detection by Planck. If so, then a subsequent experiment with better sensitivity to polarization will need to be done.

Inflation also predicts that the distribution of primordial density perturbations is gaussian, and this can be tested with CMB temperature maps and with the study of the large-scale distribution of galaxies. Since big-bang nucleosynthesis predicts that the baryon density is $\Omega_b \lesssim 0.1$ and inflation predicts $\Omega = 1$, another prediction of inflation is a significant component of nonbaryonic dark matter. This can be either in the form of vacuum energy (i.e., a cosmological constant), and/or some new elementary particle. Therefore, discovery of particle dark matter could be interpreted as evidence for inflation.

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