Bulk waves excited by a laser line pulse in a bi-layer cylinder

Y. Pan\textsuperscript{a}, N. Chigarev\textsuperscript{b}, and B. Audoin\textsuperscript{b}

\textsuperscript{a}Institute of Acoustics, Tongji University, 20092, Shanghai, China
\textsuperscript{b}Laboratoire de Mécanique Physique, UMR CNRS 8469, Université Bordeaux 1, 33405 Talence, France

ypan@tongji.edu.cn

Abstract. In this paper, the transient displacement excited by a laser line source is calculated for a bi-layer cylinder made of homogeneous and isotropic materials. A sample made by welding tin in a copper tube and a sample of cooper rod are considered. Experimental displacements are observed by the laser ultrasonic technique, and corresponding theoretical waveforms are calculated. Good agreement is found in the arrival time, shape and relative amplitude of various longitudinal and shear bulk waves propagating through the sample or reflected by the interface.

1. Introduction

Cylinders with bi-layer structure are extensively applied in machine assemblies using shaft bearing for instance. Fatigue cracks are likely to grow in such a structure, for this reason, there is a need for the nondestructive evaluation (NDE) to avoid any possible failure. The understanding of the acoustic wave propagation in such a geometry is thus of great interest for the NDE community.

The propagation of harmonic waves in a bi-layer cylinder was studied early in 1971 by Lai et al.\textsuperscript{1}. For possible submarine applications, the acoustic scattering of an immersed bi-layer cylindrical structure in water was studied. For instance, Liu and Qu\textsuperscript{2} have calculated the transient wave propagation along the circumferential direction by applying normal mode expansion (NME) method, and they have analyzed the influence of the incident angle of a PZT transducer for both generation and detection. However, since water is necessary for coupling, the potential application is limited. Here, a laser ultrasonic technique\textsuperscript{3} is better suited, since the transient waves can be generated and detected by lasers at a distance, without any contact to the sample.

With the optical non-contact technique, Kawald et al.\textsuperscript{4} observed the surface wave dispersion for a bi-layer cylinder in 1996. And Kley et al.\textsuperscript{5} observed the dispersion of transient waves propagating along the circumferential direction for a bi-layer cylinder in 1999. Since the thickness of the outer layer was very thin compared to the inner solid for the tested samples, only guided waves were experimentally observed and studied, and bulk waves were neither observed nor analyzed. To the best of author’s knowledge, no work has been published on the bulk wave propagation in a bi-layer cylinder.
2. Theoretical model and solution

Instead of a homogeneous or single-layered cylinder in Ref. 6, a bi-layer cylinder of infinite length is considered. It is made of a sheath tube with external radius \( a \) and an internal core cylinder with radius \( b \). A system of cylindrical coordinates \((r, \theta, z)\) is chosen with the \( z \)-axis coinciding with the symmetry axis of the bi-layer cylinder. A pulsed laser beam, focused along the \( z \)-axis by a cylindrical lens, impacts the cylinder to excite elastic waves. To model the laser generation at the surface \((r=a)\) of the external sheath tube, the same boundary conditions as that in Ref. 6 are chosen for the two components \( \sigma^1_{rr} \) and \( \sigma^1_{r\theta} \) of the stress tensor as

\[
\sigma^1_{rr}
\bigg|_{r=a} = -\sigma_A \delta(t) \delta(\theta),
\sigma^1_{r\theta}
\bigg|_{r=a} = 0
\]

for the ablation generation, where \( \sigma_A \) stands for the normal force magnitude in N·m⁻². Delta functions of time \( \delta(t) \) and of space \( \delta(\theta) \) are used. Additional boundary conditions are related to the continuity of the two non-zero components of displacement and stress, due to the welded interface between the outer layer and inner core of the cylinder. They can be written as

\[
\begin{align*}
\sigma^1_{rr}
\bigg|_{r=b} &= \sigma^2_{rr}
\bigg|_{r=b}, \\
\sigma^1_{r\theta}
\bigg|_{r=b} &= \sigma^2_{r\theta}
\bigg|_{r=b}, \\
U^1_r
\bigg|_{r=b} &= U^2_r
\bigg|_{r=b}, \\
U^1_\theta
\bigg|_{r=b} &= U^2_\theta
\bigg|_{r=b}.
\end{align*}
\]

Here superscripts 1 and 2 denote the external and internal solids of the bi-layer cylinder, respectively.

Let \( \rho_i, \lambda_i, \) and \( \mu_i \) \((i=1, 2)\) stand for the density and the two independent Lamé coefficients of the corresponding external and internal solids of the bi-layer cylinder, respectively. The wave number of the longitudinal and shear waves for the outer layer material are given by:

\[
k_L = \frac{\omega V_L}{\sqrt{\rho_i (\lambda_i + 2\mu_i)}}, \quad k_S = \frac{\omega V_S}{\sqrt{\rho_i \mu_i}},
\]

with \( V_L \) and \( V_S \), the corresponding longitudinal and shear wave velocities. And the wave number of the longitudinal and shear waves for the solid of the internal core cylinder are denoted as

\[
k_L = \frac{\omega V_L}{\sqrt{\rho_i (\lambda_i + 2\mu_i)}}, \quad k_S = \frac{\omega V_S}{\sqrt{\rho_i \mu_i}},
\]

with \( V_L \) and \( V_S \), the corresponding longitudinal and shear wave velocities of the solid. Following the same calculation scheme as in Ref. 6, at any position on the bi-layer cylinder surface, the transformed displacement components \( U^1_n(r, \nu, \omega) \) \((n=r, \theta)\) are then obtained from Eqs. (1) and (2) for the ablation generation as

\[
U_n^1 (a, \nu, \omega) = \frac{\sigma_A^1}{2 \mu D(\nu, \omega)} \left\{ A_n^1 B_{\nu L}^a + A_n^1 Y_{\nu L}^a - j \nu A_L - j \nu A_\nu \right\},
\]

and parameters \( b_{\nu W}^a, \) and \( Y_{\nu W}^a \) \((W=L, S)\) are determined by

\[
\begin{align*}
B_{\nu W}^a &= k_w a J_{\nu W}^a (k_w a) / J_{\nu W} (k_w a), \\
Y_{\nu W}^a &= k_w a Y_{\nu W}^a (k_w a) / Y_{\nu W} (k_w a).
\end{align*}
\]

Here, \( J_{\nu W}(x) \) is the derivative of \( J_{\nu}(x) \), the Bessel function of first kind; \( Y_{\nu W}(x) \) is the derivative of \( Y_{\nu}(x) \), the Bessel function of second kind. Coefficients \( A_n \) in Eq. (3) are given by

\[
A_1 = d_{11}, A_2 = d_{12}, A_3 = d_{13}, A_4 = d_{14}
\]

where \( \{d_{ij}\} \) are the cofactors of the 6×6 matrix
\begin{equation}
\{m_{ij}\} = \begin{bmatrix}
\nu^2 - k^2\beta^2/2k^2 & \nu^2 - k^2\beta^2/2k^2 & \nu \{1 - R_S\} & \nu \{1 - R_L\} & 0 & 0 \\
\nu \{1 - R_L\} & \nu \{1 - R_L\} & k^2\alpha/2k^2 + iR_L & k^2\alpha/2k^2 + iR_L & 0 & 0 \\
(\nu^2 - k^2\beta^2/2k^2)R_L & (\nu^2 - k^2\beta^2/2k^2)R_L & (\nu \{1 - R_L\})k^2 & (\nu \{1 - R_L\})k^2 & 0 & (\nu \{1 - R_L\})k^2 \beta - \nu \{1 - R_L\} \beta \\
\nu \{1 - R_L\}R_L & \nu \{1 - R_L\}R_L & (\nu \{1 - R_L\})k^2 & (\nu \{1 - R_L\})k^2 & -\nu \{1 - R_L\} \beta - (\nu \{1 - R_L\})k^2 \beta & \nu \\
-\nu \{1 - R_L\} & -\nu \{1 - R_L\} & -\nu \{1 - R_L\} & -\nu \{1 - R_L\} & 0 & \nu \\
\end{bmatrix}
\end{equation}

Parameters $B^a_S$ and $y^a_S$ are obtained by replacing $L$ with $S$ in Eq. (4), and parameters $B^c_W$ and $y^c_W$ (W=L, S) are determined by

\begin{equation}
B^c_W = J_v(k_w b)/J_v(k_w a), \quad Y^c_W = Y_v(k_w b)/Y_v(k_w a).
\end{equation}

And parameters $B^b_W$ and $y^b_W$ (W=L, S) can be obtained by replacing $a$ with $b$ in Eq. (4). And parameters $B^b_W$ (W=l, s) can also be obtained by replacing $a$ with $b$, $k_l$ with $k_s$, and $k_s$ with $k_l$ in Eq. (4). The parameter $\beta$ is defined by $\beta = \mu^b/\mu = \rho_0 \nu^2/\rho v^2$, i.e., the ratio of the lame constants of the two solids.

Note that in Eq. (3), $D(\nu, \omega) = \det(m_{ij})$, the determinant of the matrix $\{m_{ij}\}$. The roots of this determinant reveal the dispersion behaviour of the bi-layer cylinder. Numerical results together with the comparison to the experimental displacements are going to demonstrate the theoretical solutions in the next section.

3. Bulk waves excited in a single-layer cylinder

A Nd:YAG laser was used for ultrasonic wave generation in the ablation regime. The pulse duration was 20 ns and the infrared light emission was obtained at 1064 nm with a maximum burst energy output of 340 mJ. The collimated optical beam was focused by means of a cylindrical lens (focus length is 150 mm). The line length and width were about 4 cm and 0.1 mm, respectively. An interferometer$^2$ of a bandwidth 120 MHz, a power 100 mW and sensitivity $10^{-14} m/\sqrt{Hz}$ was used to measure the radial displacement at the surface for an observation angle of $\theta = 180^\circ$. The laser energy for the generation was about 50 mJ, and the signal was averaged by 15 shots.

Before investigating the bulk-wave propagation in a bi-layer cylinder, it is necessary to understand the bulk wave excited in a uniform bulk cylinder. A copper rod of diameter $2a=16$ mm is chosen for the investigation. To avoid numerical problems in the calculation, the longitudinal and shear wave velocities of the external layer is chosen to be slightly (here 1%) different from the core to simulate the single cylinder. The found values for the copper rod parameters are: density $\rho_l=8900$ kg/m$^3$, and $V_L=4.70$ km/s, $V_S=2.26$ km/s, velocities of longitudinal wave shear wave.$^5$ As shown in Figure 1, the measured and calculated radial displacements are in quite good agreement regarding the time, shape and relative amplitude of various bulk waves. They are direct longitudinal wave (L), reflected longitudinal waves ($2P$), reflected transverse wave ($2S$), the conversion of the longitudinal and transverse waves ($PS$) and head wave ($H$). This agreement illustrates the effectiveness of the current bi-layer modeling on a single-layered cylinder.

In Figure 1, the time scale is made dimensionless by dividing the time $t$ by a factor $t_s$, the transit time of the direct longitudinal wave propagating along the diameter of the single-layered cylinder ($t_s=2a/V_L$, $V_L$ the longitudinal wave velocity of the copper rod). The calculated waveform has been scaled vertically by a constant corresponding to the source strength to bring the amplitudes of the two signals into the same scale. Sparse and dense dash lines are repeatedly displayed to indicate the arrival times of waves that may propagate in the bulk of the cylinder or along its surface. The arrival time of
each wave is calculated based on its corresponding ray trajectory. Symbol at the top or bottom of each
dash line respectively represents the name of the wave marked by the dash line. For the details of the
ray trajectory and uncommented symbols please refer to Ref. 6.

Figure 1. Displacements calculated (bottom) and measured (top) at an observation angle \( \theta = 180^\circ \) in a copper rod for the ablation generation.

4. Bulk waves excited in a bi-layer cylinder

To demonstrate the current modeling on bi-layer cylinders, a sample was built by melting tin in the
copper tube. The experimental waveform was obtained by the same laser ultrasonic technique as
described in the previous section. The sample size is \( 2a=10 \) mm with the ratio of external to core
radius \( b/a=0.86 \). Velocities of longitudinal wave \( V_L=2.75 \) km/s, shear wave \( V_S=1.16 \) km/s, and density
\( \rho_2=8400 \) kg/m\(^3\) were used for the tin core\(^4\). A similar data processing procedure as for Figure 1 was
applied here for better comparison. As shown in Figure 2, agreement can be found for the calculated
and experimental waveforms in the time arrival, shape and relative amplitude of direct longitudinal (\( L \)),
the once (\( I \)) and twice (\( 2I \)) reflected longitudinal wave by the interface, and circularly reflected shear
bulk waves (\( 4S \)). Since the ray paths for circularly reflected longitudinal (\( 2P, 3P \)) and shear (\( 2S, 3S \))
bulk waves encounter the tin core for this ratio, these waves are hardly observable. However, with the
thin sheath tube, the echoes induced by the interface are most likely visible as waves \( I \) and \( 2I \) or more.
The appearance of these bulk wave echoes in both theoretical modeling and experimental results
emphasizes possible new applications of the laser ultrasonic technique for the non destructive control
and evaluation of two layer cylindrical structures. The agreement indicates the effectiveness of the
proposed modeling on a bi-layer cylinder.

5. Conclusion

A physical model and corresponding solution are presented to predict bulk waves propagating in a
bi-layer cylinder excited by a laser line pulse. Experimental radial displacements were obtained and
compared for a single copper rod and a copper sheath tube with tin core. Agreements are observed in
the arrival time, shape, and relative amplitude of the various longitudinal and transverse bulk waves
propagating through the core or reflected by the solid interface and at the free external surface.
Figure 2. Displacements calculated (bottom) and measured (top) at an observation angle $\theta = 180^\circ$ in a copper-tin cylinder of $b/a$ ratio 0.86 for ablation generation.

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