Researches on the distribution of stresses in a wall girder with geometrical discontinuities

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Abstract. The paper presents the results of the numerical and experimental analysis of the stress distribution in a wall beam with geometrical discontinuities subjected to flexural bending. These discontinuities are represented by five circular holes arranged symmetrically with respect to the longitudinal axis of the beam. The numerical analysis was performed with the finite element method and the experimental investigation by electrical strain gauge. The finite element method applied in 2D and 3D for the case of hollow wall beams confirms that Bernoulli's assumption and the hypothesis regarding the variation of stresses on beam height made in the strength of the materials in the beam calculation cannot be used in this case. The results obtained with the finite element method and the experimental ones with the electrical strain gauge method are presented and their comparison is made. The contribution of the experimental research to the selection of a model as close as possible to the practical reality is emphasized. The results of the research allow to draw conclusions about how to position the electrical transducers on the wall beam at the chosen analysis points and its influence on the stress distribution.

1. Introduction

The wall beam is an element of construction with a flat surface, of small width, at which the ratio of height to opening is larger than 0.5. The wall beam is required to bend in its plan. The stress state in the full beam is a Boussinesq and Flamant type problem [1], [2], [3] in the theory of elasticity. P. P. Teodorescu [4] theoretically addresses this problem using for the determination of the stresses and displacements, in the case of the plane state in a wall beam, a function of biharmonic stress in the form of double Fourier series. In another papers [5], [6] referring to recessed wall beams the boundary conditions are set in both displacement and stress. In the case of wall beams with static discontinuities (supports, concentrated forces) or geometrical, the planar section or Bernoulli's assumption [4], [5] is no longer applicable. In addition, in this case the specific deformations and normal stress along the axis OY do not vary linearly and the stress state in the beam can be considered as plane stress state (\(\sigma_x \neq 0, \sigma_y \neq 0, \tau_{xy} \neq 0\)).

A calculation procedure for such hollow wall beams for static bar models is determined [7]. The elements of straight bars are chosen according to the direction of the main stress so that they do not form a mechanism. The paper [8] presents the experimental study carried out by photoelasticity of a large wall beam, which has a rectangular shaped door gap. The beam is simply supported and loaded by a concentrated force applied to the middle of the opening. The study finds strong concentrations of stresses in the corners of the hollow for the door. The disruptive influence of the door hollow,
however, extends only a short distance along the beam, the rest behaving like a simple full beam without a hollow, which verifies the hypothesis of B. de Saint Venant [1].

The wall beam investigated in this work, made of aluminum (Young's modulus $E = 70$ GPa and Poisson's ratio $\nu = 0.33$), has five circular holes with a diameter of 18 mm arranged symmetrically against the longitudinal axis of the beam. Figure 1, a and b shows the wall beam, its geometry, dimensions and load.

In the case of the studied beam (figure 1), the ratio between height and opening is equal to 0.72, so that it can be considered as a wall beam [2], [7]. The existence of holes, which are stress concentrators [9], [10], makes the analysis of the distribution of stresses produced by loads particularly important for these structures. The paper is a numerical and experimental investigation on the determination and distribution of stresses. The studied wall beam is simply supported and vertically loaded on consoles ($c = 85$mm) with the forces $F = 500$ N, as in figure 1,a. In this loading situation, the beam portion between the supports ($l = 140$ mm) will be flexural loading through a constant bending moment $M_y = Fc = 42500$ Nmm. In the full section of the beam the modulus of resistance is $W_y = 8501$ mm$^3$ and in the section weakened by two holes it results $W_y = 6177$ mm$^3$. The maximum stress, calculated in the structure considering the Navier relation valid, are in these two cases: $\sigma_x = 5$ MPa and respectively $\sigma_x = 6.88$ MPa.

![Figure 1. Wall beam with five holes.](image)

**2. Numerical analysis of the distribution of stresses in the beam wall with holes**

In the case studied, the thickness of the beam is relatively small compared to the dimensions of the plane and for this reason, a 2D model was developed and was discretized with eight noded quadrilateral elements (Plane183 in Ansys) in plane stress state conditions [11], [12]. At the same time, a 3D quarter model, due to the double reflective symmetry (in plane and in the thickness direction) of the beam was studied using a uniform discretization with Brick 20 noded elements (Solid186) of 1 mm size, finer than the 2D model discretized with 1.5 mm elements. The numerical analysis was performed with a 3D discretization, although this was not required, for a thin beam with a thickness of 5 mm, being found extremely small differences of the stress compared to the case of the analysis with a discretization 2D. As the results of the finite element analysis on 2D and 3D models are very close, in this paper only the results obtained for the 3D model are presented and commented on.

Numerical research considered three loading situations:
1. The wall beam with the dimensions ($l = 140$ mm, $c = 85$ mm) and the load $F_1 = 500$ N as in figure 1. Figure 2,a shows the symmetry properties, the loading and the support mode for this beam.
2. The beam in figure 1, in which the distance between the supports l was increased to $l = 210$ mm in order to be able to observe the influence of the position of the reactions on the distribution of stresses in the weakened area of holes. In order to maintain the same value of
the bending moment on the beam opening, the concentrated forces had the value $F_2 = 850$ N (figure 2,b).

3. The beam works in pure bending by a bending moment equivalent to the one acting on the middle part of the beam. Figure 2,c shows the symmetry properties and the load for the studied beam. In this case in the full section the maximum stress calculated with Navier's relation is $p = \sigma_x = 5$ MPa.

![Figure 2. Different load cases which give the same bending moment in the central zone.](image)

As it was found that the load cases 2 and 3 lead to results, for stresses and displacements, which differ very little from those obtained in case 1, only the results for this first load case will be retained and commented on.

Structural percentage error in energy norm [12] in the square central zone is only 0.175, indicating a very good mesh for this load case, however, a convergence study was performed before considering coarser meshes.

The distribution of the displacements in the wall beam for load case 1 (figure 3) shows that for the beams with holes the Bernoulli’s assumption is no longer applicable. The fields of the normal stresses $\sigma_x$ and $\sigma_z$, the shear stresses $\tau_{xy}$ and the equivalent von Mises stresses $\sigma_{eq}$ obtained using Ansys program, are given for the central area of the wall beam in the figures 4,a–d. The normal stress $\sigma_x$ and also the shearing stresses $\tau_{xz}$ and $\tau_{yz}$ have very small values, confirming the state plane condition in the central zone, and were omitted in the presentation. The shear stresses $\tau_{xy}$ have important non zero values (figure 4,c), but they do not appear at the points where the normal stresses have important values, but in the area of application of the concentrated forces (not shown here) and around the holes.

The variation of the principal stresses ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) in the central area of the perforated wall beam subjected to bending by the two concentrated forces $F_1 = 500$ N is shown in figure 5.

![Figure 3. Distribution of displacements USUM (a) and horizontal displacements UX (b) to the perforated wall beam.](image)
Figure 4. Stresses distribution in perforated wall beam: $\sigma_x$ (a), $\sigma_y$ (b), $\tau_{xy}$ (c) and $\sigma_{eqv}$ (d).

Figure 5. Distribution of the principal stresses $\sigma_1$, $\sigma_2$ and $\sigma_3$.

Based on the obtained results, the variations of the normal stress $\sigma_x$ at the wall beam in the weakened sections through a hole and two holes are shown in figure 6. The results of the numerical analysis for points located on the contour of the holes are shown in table 1.

Figure 6. Variation of normal stresses $\sigma_x$ for load case 1, where $F = 500$ N.
From the figure 4,a it is found that at the beam wall with holes subjected to flexural loading the
distribution of the normal stress $\sigma_x$ per section is not linear and its maximum value is not located
on the contour exterior of the beam. The maximum values of the normal stress $\sigma_x$ is on the outline of the
holes. Once again, the conclusion can be drawn that the hypothesis of the Bernoulli’s assumption is no
longer applicable in the case of the wall beams with holes.

In the section weakened by two holes the maximum stress resulting is $\sigma_x = 10.5$ MPa, obtaining for
stress concentration factor [10] a value $k = 1.53$.

The distribution of stresses in the central area of the wall beam for the load cases 2 and 3 is very
close to the result for load case 1, and for this reason they are no longer presented in the paper.

For the same plate with geometric discontinuities subjected to traction [9] along the OX axis, with
the force $F = 10$ kN (uniformly distributed load $p = 19.81$ MPa) through the finite element analysis,
the normal stresses distribution was established, and it is shown in figure 7.

![Figure 7](image-url)

**Figure 7.** Normal stresses $\sigma_x$ distribution – tensile loading, $F = 10$ kN.

Based on this analysis, the values of the normal stresses in the sections of the plate with
discontinuities were determined and their variation was plotted (figure 8). The resulting stress
concentration coefficient $k = 2.44$ is larger than in bending.

![Figure 8](image-url)

**Figure 8.** Variation of normal stresses $\sigma_x$ in the three sections of the plate in tensile loading, when $F = 10$ kN.

### 3. Experimental analysis of the stress distribution in the wall beam with geometric
discontinuities

The determination of the experimental state of tension in the beam wall with holes, for load case 1,
was performed also experimentally by the method of electrical strain gauge analysis [13].

The transducers, with base on 3 mm, were applied, at the points of analysis of the stress state on the
contour of the holes, both on the outer surface of the beam and on the inner surface of the holes.
Except for the transducers 1 and 2 the other transducers were placed around the holes. The wall beam was flexural loading produced by the forces \( F_1 = 500 \) N symmetrically applied to the consoles. Figure 9 shows the diagram of the placement of the electrical transducers on the wall beam.

For the perforated wall beam subjected to bending by the forces \( F_1 = 500 \) N, in a test machine, the specific deformations at the points where the transducers were applied were determined by electrical strain gauge. With the connection relations between the strains and stresses corresponding to the plane state of tension in the theory of elasticity [1], the values of the normal stresses \( \sigma_x \) at the measurement points were determined.

![Figure 9. Location of the transducers on the perforated wall beam.](image)

In the numerical analysis with finite elements the transducers were modeled with Link180 elements having the negligible section in order to not influence the structure locally. In this analysis, the numbering of the transducers from the experimental monitoring was kept. The values of the experimentally determined stress were shown in table 1. In the section weakened by two holes the maximum stress resulted was \( \sigma_x = 9.1 \) MPa, obtaining a coefficient of concentration of stresses \( k = 1.32 \). The results of the measurements made with the 1...6 transducers that were located in the beam axis or in the immediate vicinity of the beam were not introduced in the table, leading to insignificant values for stresses, anyhow they are important for axial loads for example.

In the situation of tensile loading the same traction plates along the OX axis [9] with the axial force \( F = 10 \) kN (uniformly distributed load \( p = 19.81 \) MPa), based on the experimental results, the coefficient of stress concentration value is \( k = 2.05 \).

| Stress | Transducer location |
|--------|---------------------|
| \( \sigma_{FEM} \) [MPa] | 7 | 8 | 9 | 10 | 11 | 12 |
| 10.5 | -10.5 | -8.2 | -8.2 | 8 | 8 |
| \( \sigma_{exp} \) [MPa] | 8.6 | -9.1 | -7.1 | -6.9 | 6.8 | 6.4 |

4. Conclusions
The research carried out and the results obtained for the stress distribution, shown in table 1, allow some conclusions to be drawn:
- the strains and the normal stress \( \sigma_x \) in the section of the wall beam with holes do not have a linear variation. The hypothesis of the Bernoulli's assumption is not valid in this case;
- the maximum stress is \( \sigma_x = 10.5 \) MPa determined by FEM and \( \sigma_x = 9.1 \) MPa determined experimentally and does not appear on the external contour of the beam, but on the contour of the holes closer to the supports. The position of the supports with respect to the holes and the application of the loads on the consoles influences to a very small extent the values and the distribution of the stress in the central area of the wall beam;
- the choice of the points and the way of placing the transducers on the contour of the plate holes in which the stress [9] will be calculated, experimentally or by the finite element method of the for the case of the bending request is important for the correctness of the results. At the same point, there are large differences in the stress obtained, as for example at the point where the transducers no. 8 and 10 were applied, the stresses determined experimentally have the value of $\sigma_x = 9.1$ MPa and respectively $\sigma_y = 6.9$ MPa. These jumps are also validated by results obtained from the FEM application, i.e. $\sigma_x = 10.5$ MPa respectively $\sigma_y = 8.2$ MPa;

- finite element analysis proves to be comparable, in regard to the values of stresses, with the experimental analysis. For the wall beam studied the stress concentration factor by FEM resulted $k = 1.52$ and determined by electrical strain gauge $k = 1.32$.

At the tensile loading [9] the coefficient of concentration of stresses determined by FEM was $k = 2.44$ and the value of the same coefficient obtained experimentally was $k = 2.05$. It is found that at the stretching load, for the same studied problem, the stress concentration factor have higher values than in the bending load.

The research carried out proves the necessity of using in parallel the numerical and experimental analysis for the realization of the calculation model that will be the basis for establishing methodologies for evaluating the distribution of stresses in engineering structures.

5. References

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