Tension of the $E_G$ statistic and RSD data with Planck/$\Lambda$CDM and implications for weakening gravity.

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The $E_G$ statistic is a powerful probe for detecting deviations from GR by combining weak lensing (WL), real-space clustering and redshift space distortion (RSD) measurements thus probing both the lensing and the growth effective Newton constants ($G_L$ and $G_{eff}$). We construct an up to date compilation of $E_G$ statistic data including both redshift and scale dependence ($E_G(R, z)$). We combine this $E_G$ data compilation with an up to date compilation of $f\sigma_8$ data from RSD observations to identify the current level of tension between the Planck/$\Lambda$CDM standard model based on general relativity and a general model independent redshift evolution parametrization of $G_L$ and $G_{eff}$. Each $f\sigma_8$ datapoint considered has been published separately in the context of independent analyses of distinct galaxy samples. However, there are correlations among the datapoints considered due to overlap of the analyzed galaxy samples. Due to these correlations the derived levels of tension of the best fit parameters with Planck/$\Lambda$CDM are somewhat overestimated but this is the price to pay for maximizing the information encoded in the compilation considered. We find that the level of tension increases from about 3.5σ for the $f\sigma_8$ data compilation alone to about 6σ when the $E_G$ data are also included in the analysis. The direction of the tension is the same as implied by the $f\sigma_8$ RSD growth data alone (lower $\Omega_m$ and/or weaker effective Newton constant at low redshifts for both the lensing and the growth effective Newton constants ($G_L$ and $G_{eff}$)). These results further amplify the hints for weakening modified gravity discussed in other recent analyses [1–4].

I. INTRODUCTION

The theory of general relativity (GR) and the standard $\Lambda$ cold dark matter ($\Lambda$CDM) [5] cosmological model have been remarkably successful in explaining a wide array of observations [6] including the observed accelerating expansion of the universe [7, 8]. Despite of its successes and simplicity, the validity of the cosmological standard model $\Lambda$CDM is currently under intense investigation. This is motivated by a range of profound theoretical and observational difficulties of the model. The most important theoretical difficulties of the $\Lambda$CDM model are the fine tuning [9–11] and coincidence problems [12, 13]. The first of these problems corresponds to the large discrepancy between observations and quantum field theoretical predictions on the value of the cosmological constant $\Lambda$ while the second is associated with the coincidence between the observed vacuum energy density $\Omega_\Lambda$ and the matter density $\Omega_m$ which in the present epoch are of the same order of magnitude despite of their very different evolution during the cosmic history.

A well known observational difficulty corresponds to the tension between the cosmic microwave background (CMB) measured value of the Hubble parameter $H_0$ [14, 15] in the context of the $\Lambda$CDM model and the local measurements from supernovae [16, 17] and lensing time delay indicators [18], with local measurements suggesting a higher value. Another observational puzzle for $\Lambda$CDM involves persisting indications from observational probes measuring the growth of matter perturbations that the observed growth is weaker than the growth predicted by the standard Planck/$\Lambda$CDM parameter values [15]. Modified gravity (MG) models constitute a prime theoretical candidate to explain this tension.

The combination of cosmological observational probes is a powerful tool for the identification of signatures of MG [19–25]. Such observational probes may be divided in two classes: geometric and dynamical (or structure formation) probes [26–29]. Geometric observations measure cosmological distances using standard candles (e.g. Type Ia supernovae) and standard rulers (e.g. the horizon at the time of recombination probed through Baryon Acoustic Oscillations) and thus probe directly the cosmic metric, independent of the underlying theory of gravity. Dynamical observations probe the growth rate of cosmological perturbations and thus the gravitational laws and the consistency of GR with data provided the background geometry is known.

Dynamical probes include cluster counts (CC) [29–32], weak lensing (WL) [25, 33–39] and redshift-space distortions (RSD) [1, 2, 40–42]. These probes are consistent with each other pointing either to a lower value of the matter density parameter $\Omega_m$ in the context of GR or to weaker gravitational growth power than the growth indicated by GR in the context of a Planck18/$\Lambda$CDM background geometry at about $2−3\sigma$ level [1, 2, 41]. Such weak growth may be quantified by the parameter $\sigma_8$ which is the matter density rms fluctuations within spheres of radius $8h^{-1}\text{Mpc}$ and is determined by the amplitude of the primordial fluctuations power spectrum and by the growth rate of cosmological fluctuation.

Various possible mechanisms have been proposed to
slow down growth at low redshifts and thus reduce the above tension (see e.g. [4]). Such mechanisms may be divided in two categories: non-gravitational and gravitational. The former includes the effects of interacting dark energy models [43–46], dynamical dark energy models [47, 48], running vacuum models [49, 50] and the effects of massive neutrinos [51]. The latter includes the effects of MG theories with a reduced (compared to GR) evolving effective Newton’s constant $G_{eff}$ at low redshifts [1, 2].

The effects of MG [52–61] models are indistinguishable from GR at the geometric cosmological background level [26, 62, 63]. Signatures of MG can only be obtained by investigating the dynamics of cosmological perturbations [64, 65] using specific statistics obtained through dynamical probe observables such as the two-point correlation of and power spectrum of the galaxy distribution, the RSD and WL. A useful bias free statistic is the $f\sigma_8$ product of the rate of growth of matter density perturbations $f$ times $\sigma_8$ discussed in more detail in what follows. An alternative observable statistic is the $E_G$ which was constructed to be independent of both the clustering bias factor $b$ and the parameter $\sigma_8$ on linear scales. This statistic was proposed in 2007 [66] and thereafter has been used several times to test MG theories [67, 68].

The expectation value of $E_G$ is equal to the ratio of the Laplacian of the sum of the Bardeen potentials $\Psi$ (the Newtonian potential) and $\Phi$ (the spatial curvature potential) $\nabla^2(\Psi + \Phi)$ over the peculiar velocity divergence $\theta = \nabla \cdot \vec{\nu}$ (where $\vec{\nu}$ is the peculiar velocity and $H(z)$ is the Hubble parameter in terms of the redshift $z$).

The $E_G$ statistic has been proposed as a model independent test of any MG theory [70] and is constructed from three different probes of large scale structure (LSS): the galaxy-galaxy lensing (GGL), the galaxy clustering and the galaxy velocity field which leads to galaxy redshift distortions. Alternatively, $E_G$ may be constructed from galaxy-CMB lensing [71] instead of galaxy-galaxy lensing as a more robust tracer of the lensing field at higher redshifts [72, 73].

The first probe, the GGL (a special type of WL), is the slight distortion of shapes of source galaxies in the background of a lens galaxy, which arises from the gravitational deflection of light due to the gravitational potential of the lens galaxy along the line of sight (see for example [74–77]). This WL probe is sensitive to $\nabla^2(\Psi + \Phi)$, since relativistic particles collect equal contributions from the two Bardeen potentials which appear in the scalar perturbed Friedmann-Lemaitre-Robertson-Walker (FLRW) metric in the Newtonian gauge [78–80]

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)d\vec{x}^2$$  \hspace{1cm} (1.1)

where $a$ is the scale factor that is related to the redshift $z$ through $a = \frac{1}{1+z}$.

The second probe, the galaxy clustering arises from the gravitational attraction of matter and is sensitive only to the potential $\Psi$. Similarly, the third probe, the galaxy velocity field, is quantified by measuring redshift space distortions (RSD) [81–84] (an illusory anisotropy that distorts the distribution of galaxies in redshift space generated by their peculiar motions falling towards overdense regions). This important probe of LSS is sensitive to the rate of growth of matter density perturbations $f$ which depends on the theory of gravity and provides measurements of $f\sigma_8$ that depends on the potential $\Psi$.

In most MG theories the potentials $\Phi$ and $\Psi$ obey generalized Poisson equations like the GR Newtonian potential where the MG effects are encoded in generalized space-time dependent effective Newton constants. These generalized Newton constants for the potential $\Psi$ and for the lensing combination $\Psi + \Phi$ are usually described by two parameters: the effective Newton’s constant parameter $\mu$ and the light deflection parameter $\Sigma$. In the modified Poisson equations [85] the $\mu$ and $\Sigma$ are connected with the potentials $\Psi$ and $\Psi + \Phi$ respectively. In GR the value of $\mu$ and $\Sigma$ coincides with unity while in a MG model $\mu$ and $\Sigma$ can be in general functions of both time and scale [19, 86]. Using $f\sigma_8$ and $E_G$ datasets constraints can be imposed on the parameters $\mu$ and $\Sigma$ [23, 87–92]). Such analyses have revealed various levels of tension of the best fit forms of $\mu$ and $\Sigma$ with the GR prediction of unity showing hints that these parameters may be less than unity implying weaker growth of perturbations than that predicted in GR. The goal of the present analysis is to extend these studies and use an updated data compilation for both the $f\sigma_8$ and $E_G$ statistics to identify the current level of tension with GR implied by these data compilations.

In particular, we address the following questions:

- What are efficient phenomenological redshift dependent parametrizations of the generalized normalized Newton constants $\mu(z)$ and $\Sigma(z)$ that are consistent with solar system and nucleosynthesis constraints that indicate that GR is restored at high $z$ and at the present time in the solar system?

- What are the constraints imposed by the $E_G$ and $f\sigma_8$ updated data compilations on the parameters of the above parametrizations and do these constraints amplify the hints for weakening gravity at low $z$ implied by the $f\sigma_8$ data alone as indicated by previous studies?

The plan of this paper is the following: In the next Section II we present a brief review of the theoretical expression for $E_G$. We also present phenomenologically motivated parametrizations for $\mu$ and $\Sigma$ and describe how we use them in order to probe possible deviations from GR on cosmological scales. In Section III we use compilations of $f\sigma_8$ and $E_G$ data along with the theoretical expressions for $f\sigma_8$ and $E_G$ which involve $\mu$ and $\Sigma$ to derive constraints on these parameters and to identify the tension level between the Planck/ΛCDM parameter values favoured by Planck 2018 [15] shown in Table I and the corresponding parameter values favored by the two datasets. Finally in Section IV we conclude, summarize and discuss the implications and possible
II. THEORETICAL BACKGROUND

II.1. $E_G$ statistic

The $E_G$ statistic [66, 70] is designed as a probe of the ratio of the Bardeen potentials of the perturbed FRW metric (1.1) in such a way as to be independent of the effects of galaxy bias at linear order. It is defined as the ratio of the cross correlation power spectrum $P_{g\nabla z}(\Phi + \Psi)$ between lensing maps (cosmic shear or CMB) and galaxy positions, over the the cross-correlation power spectrum $P_{g\theta}$ between galaxies and velocity divergence field $\theta$

$$E_G \equiv \frac{P_{g\nabla z}(\Phi + \Psi)}{P_{g\theta}}$$

(2.1)

In Fourier space the $E_G$ statistic may also be expressed as [66]

$$E_G(l, \Delta l) = \frac{C_{\kappa g}(l, \Delta l)}{3H_0^2a^{-1}\sum \alpha \sigma_{\alpha}(l, \Delta l)P_{\nabla g}^\alpha}$$

(2.2)

where $H_0$ is the Hubble parameter today, $l$ is the magnitude of two-dimensional wavenumber of the on-sky Fourier space, $C_{\kappa g}(l, \Delta l)$ is the galaxy-galaxy lensing cross correlation power spectrum in bins of $\Delta l$, $P_{\nabla g}^\alpha$ is the galaxy-velocity cross correlations power spectrum between $k_\alpha$ and $k_{\alpha+1}$ (where $k$ three-dimensional wavenumber of the on-sky Fourier transform with $k_1 < k_2 < ... < k_{\alpha} < ...$) and $q_{\alpha}(l, \Delta l)$ is the weighting function defined accordingly.

The corresponding expectation value of $E_G$, averaged over $l$ is the ratio of the Laplacian of the gravitational scalar potentials $\Psi$ and $\Phi$ which appear in the scalar perturbed Friedmann-Lemaître-Robertson-Walker (FLRW) metric Eq. (1.1) over the peculiar velocity divergence [67]

$$\langle E_G \rangle = \left[ \frac{\nabla^2(\Psi + \Phi)}{3H_0^2a^{-1}\theta} \right]_{k=1/\bar{\chi}, \bar{z}}$$

(2.3)

where $\bar{\chi}$ is the comoving mean distance corresponding to the mean redshift $\bar{z}$.

In $\Lambda$CDM cosmology and assuming that the velocity field is generated under linear perturbation theory, the peculiar velocity divergence is connected to the growth rate $f$ as $\theta = f \delta$ [93] where $\delta = \frac{\delta\rho}{\rho}$ is the matter overdensity field, $\rho$ is the matter density of the background, $f(a) \equiv \frac{d\ln D(a)}{d\ln a}$ is the linear growth rate of structure and $D(a) \equiv \frac{\delta(a)}{\delta(a=1)}$ the growth factor.

In the case of GR and in the absence of any anisotropic stress the Bardeen potentials are equal ($\Psi = \Phi$) and the gravitational field equations reduce to Poisson equations of the form

$$\nabla^2\Phi = \nabla^2\Psi = 4\pi G a^2 \rho \delta = \frac{3}{2}H_0^2\Omega_{0m} a^{-1} \delta$$

(2.4)

where $\Omega_{0m} = \Omega_m(z = 0)$ is the matter density parameter today and the second equality is straightforwardly derived assuming non-relativistic matter species and using the equations $H_0^2 = \frac{8\pi G \rho_c}{3}$, $\rho = \rho_0 a^{-3}$ and $\Omega_{0m} = \frac{\rho_0}{\rho_c}$ (with $\rho_0$ the matter density today and $\rho_{c,0}$ the critical density today).

Therefore within GR Eq.(2.4), the Eq.(2.3) reduce to

$$E_G = \frac{\Omega_{0m}}{f(z)}$$

(2.5)

where $f$ is well approximated as $f(z) \simeq \Omega_m(z)$ with the growth index $\gamma$ in a narrow range near 0.55, for a wide variety of dark-energy models in GR [94–102]. Note that $E_G$ in GR is scale independent (Eq.(2.5)). This is not necessarily the case in the context of MG theories where the growth rate $f$ may be strongly scale dependent even on subhorizon scales.

II.2. The effective Newton’s constant parameter $\mu$ and the light deflection parameter $\Sigma$

The gravitational slip parameter $\eta$ describes the possible inequality [103, 104] of the two Bardeen potentials that may occur in MG theories. It is defined as

$$\eta(a, k) = \frac{\Phi(a, k)}{\Psi(a, k)}$$

(2.6)
Clearly an observation of $\eta \neq 1$ would indicate physics beyond GR. In this case the gravitational field equations at linear level take the form of Poisson equations that generalize Eqs. (2.4). At linear level, in MG models, using the perturbed metric (1.1) and the gravitational field equations the following phenomenological equations emerge [42, 86, 105–110] for the scalar perturbation potentials

\begin{equation}
 k^2(\Psi + \Phi) = -8\pi G_N \Sigma(a,k)a^2\rho \Delta \tag{2.7}
 \end{equation}

\begin{equation}
 k^2\Psi = -4\pi G_N \mu(a,k)a^2\rho \Delta \tag{2.8}
 \end{equation}

where $\rho$ is the matter density of the background, $\Delta$ the comoving matter density contrast defined as $\Delta \equiv \delta + 3H a (1 + w)v/k$ which is gauge-invariant [106], $w = p/\rho$ is the equation-of-state parameter and $v^i = -\nabla^i u$ is the irrotational component of the velocity field. Also $\mu$ and $\Sigma$ are the generalized growth and lensing effective Newton constants. They are in general functions of time and scale encoding the possible modifications of General Relativity constants. They are in general functions of time and scale encoding the possible modifications of General Relativity defined as

\begin{align*}
 \mu(a,k) &= \frac{G_{eff}(a,k)}{G_N} \\
 \Sigma(a,k) &= \frac{G_L(a,k)}{G_N}
\end{align*}

with $G_N$ is the Newton’s constant as measured by local experiments, $G_{eff}$ is the effective Newton’s constant which is related to the growth of matter perturbation and $G_L$ is related to the lensing of light (the propagation of relativistic particles, such as photons when they traverse equal regions of space and time along null geodesics experiencing gravitational lensing collecting equal contributions from two gravitational potentials $\Psi$ and $\Phi$). Using the gravitational slip Eq.(2.6) and the ratios of the Poisson equations (2.7), (2.8) defined above the two LSS functions $\mu$ and $\Sigma$ are related via

\begin{equation}
 \Sigma(a,k) = \frac{1}{2} \mu(a,k) [1 + \eta(a,k)] \tag{2.9}
 \end{equation}

In GR which predicts a constant homogeneous $G_{eff} = G_N$, we obtain $\mu = 1$, $\eta = 1$ and $\Sigma = 1$.

Notice that Eqs. (2.7) and (2.8) indicate that a possible observation of reduced gravitational growth of the Bardeen potentials may be interpreted either as reduced strength of gravitational interaction (reduced $\mu$ and/or $\Sigma$) or due to reduced matter density $\rho$ (or $\Omega_m$). In the context of a fixed value of matter density determined by geometric probes of the cosmological background, the reduced gravitational growth could be either interpreted as a tension within the $\Lambda$CDM parameter value for the matter density or as a hint for weakening gravity. Indeed, such hints of weaker than expected gravitational growth of the Bardeen potentials has been observed at low redshifts by a wide range of dynamical probes including RSD observations [1, 2, 41, 42], WL [25, 34, 36–39] and CC data [29–32]. In most cases this weak growth has been interpreted as a tension for the parameters $\sigma_8$ and $\Omega_m$ which are found by dynamical probes to be smaller than the values indicated by geometric probes in the context of $\Lambda$CDM.

The observables $f\sigma_8(a,k)$ and $E_G(a,k)$ can probe directly the gravitational strength functions $\mu(a,k)$ and $\Sigma(a,k)$. In particular $f\sigma_8$ is easily expressed in terms of the amplitude $\sigma_8$ and the matter overdensity $\delta$ using the matter overdensity evolution equation (see e.g. [80])

\begin{equation}
 \ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \mu(a,k) k^2 \rho \delta = 0 \tag{2.10}
 \end{equation}

where the dot denotes differentiation with respect to cosmic time $t$. In terms of redshift Eq. (2.10) takes the form [1, 80]

\begin{equation}
 \delta''(z) + \left( \frac{H(z)^2}{2} - \frac{1}{1 + z} \right) \delta'(z) - \frac{3 (1 + z) \Omega_{om} \mu(z,k)}{H(z)^2/H_0^2} \delta(z) = 0 \tag{2.11}
 \end{equation}

where primes denote differentiation with respect to the redshift. While in terms of the scale factor we have [52, 101, 111]

\begin{equation}
 \delta''(a) + \left( \frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta'(a) - \frac{3 \Omega_{om} \mu(a,k)}{2 a^2 H(a)^2/H_0^2} \delta(a) = 0 \tag{2.12}
 \end{equation}

Here primes denote differentiation with respect to the scale factor. In Eqs. (2.11), (2.12) possible deviations from GR are expressed by allowing for a scale and redshift-dependent $\mu = \mu(z,k)$. In the present section and in section III.1 we ignore scale dependence due to the lack of good quality scale dependent $f\sigma_8$ and $E_G$ data. However, in section III.2 we discuss the scale dependence of $E_G$ data.

For a given parametrization of $\mu(a)$ and initial conditions deep in the matter era where GR is assumed to be valid leading to $\delta \sim a$ equations (2.11), (2.12) may be

\[ \text{Note: in the literature $\mu$ and $\Sigma$ are also referred to as $G_M$ and $G_L$ (e.g. in Refs. [1, 109]) or as $G_{\text{matter}}$ and $G_{\text{light}}$ (e.g. in Refs. [42, 107]).} \]
easily solved numerically leading to a predicted form of \( \delta(a) \) for a given \( \Omega_m \) and background expansion \( H(z) \). In the context of the present analysis we assume a \( \Lambda \)CDM background \( H(z) \)

\[
H^2(z) = H_0^2 \left[ \Omega_{m0} (1 + z)^3 + (1 - \Omega_{m0}) \right]
\]  

(2.13)

Once the evolution of \( \delta \) is known, the observable product \( f \sigma_8(a) = f(a) \cdot \sigma(a) \) can be obtained using the definitions

\[
f(a) \equiv \frac{d \ln \delta(a)}{d \ln a}
\]  

(2.14)

\[
\sigma(a) \equiv \sigma_8 \frac{\delta(a)}{\delta(a = 1)}
\]  

(2.15)

where \( \sigma(a) \) is the redshift dependent rms fluctuations of the linear density field within spheres of radius \( R = 8h^{-1}\text{Mpc} \) and \( \sigma_8 \) is its value today. Thus, we have

\[
f \sigma_8(a, \sigma_8, \Omega_{m0}, \mu) = \frac{\sigma_8}{\delta(a = 1)} a \delta'(a, \Omega_{m0}, \mu)
\]  

(2.16)

This theoretical prediction may now be used to compare with the observed \( f \sigma_8 \) data and obtain fits for the parameters \( \Omega_{m0}, \sigma_8 \) and \( \mu(z) \) (assuming a specific parametrization of \( \mu(z) \)).

The lensing gravity parameter \( \Sigma(z) \) can be fit in the context of specific parametrizations using its connection with the \( E_G(a) \) observable as [112–114]

\[
E_G(a, \Omega_{m0}, \mu, \Sigma) = \frac{\Omega_{m0} \Sigma(a)}{f(a, \Omega_{m0}, \mu)}
\]  

(2.17)

This equation assumes that the redshift of the lens galaxies can be approximated by a single value while \( E_G \) corresponds to average value along the line of sight [114]. In the context of Eq. (2.17) and assuming a specific parametrization for \( \mu \) and \( \Sigma \), the theoretical prediction for \( E_G \) may be used to compare with the observed \( E_G \) datapoints and lead to constraints on \( \Omega_{m0}, \mu, \Sigma \). These constraints may be considered either separately from those of the \( f \sigma_8 \) data or jointly by combining the \( E_G \) and \( f \sigma_8 \) datasets. The allowed range of these parameters may then be compared with the standard Planck/\( \Lambda \)CDM parameter values \( \mu = 1, \Sigma = 1, \Omega_{m0} = 0.315 \pm 0.0073, \sigma_8 = 0.811 \pm 0.006 \) to identify the likelihood of Planck/\( \Lambda \)CDM in the context of the dynamical probe data \( E_G \) and \( f \sigma_8 \). This plan is implemented in what follows in the context of specific parametrizations describing the possible evolution of \( \mu \) and \( \Sigma \).

On scales much smaller than the Hubble scale for most modified gravity models the scale dependence of \( \mu \) and \( \Sigma \) is weak. For example in scalar-tensor (ST) model (for \( k \gg aH \)) \( \mu \) is independent of the scale [115]. Thus, we start by considering scale independent parametrizations for \( \mu \) and \( \Sigma \) which reduce to the GR value at early times and at the present time as indicated by solar system (ignoring possible screening effects) and Big Bang Nucleosynthesis constraints (\( \mu = 1 \) and \( \mu' = 0 \) for \( a = 1 \) and \( \mu = 1 \) for \( a \ll 1 \)) [116–118]. Such parametrizations are of the form [1, 2, 119]

\[
\mu = 1 + g_a (1 - a)^n - g_a (1 - a)^2m = 1 + g_a \left( \frac{z}{1 + z} \right)^n - g_a \left( \frac{z}{1 + z} \right)^2m
\]  

(2.18)

\[
\Sigma = 1 + g_b (1 - a)^m - g_b (1 - a)^2m = 1 + g_b \left( \frac{z}{1 + z} \right)^m - g_b \left( \frac{z}{1 + z} \right)^2m
\]  

(2.19)

where \( g_a \) and \( g_b \) are parameters to be fit and \( n \) and \( m \) are integer parameters with \( n \geq 2 \) and \( m \geq 2 \) which we set equal to 2 in the present analysis.

### III. OBSERVATIONAL CONSTRAINTS

#### III.1. Scale Independent Analysis

The \( f \sigma_8(z) \) and \( E_G(z) \) updated data compilations used in our analysis are shown in Tables VI and VII of the Appendix B along with the references where each datapoint was originally published. The datapoints are also shown in Figs. 1 and 2 along with curves corresponding to the Planck/\( \Lambda \)CDM prediction and the best fit parameter values. As it can be seen the datapoints from the various surveys are consistent with each other at any given redshift and at 1σ level. Clearly, in both cases the data appear to favor lower values of \( f \sigma_8 \) and \( E_G \) than the values corresponding to the Planck/\( \Lambda \)CDM parameters. This trend combined with the indications for a Planck/\( \Lambda \)CDM background from geometric probes may be interpreted as a need for a new degree of freedom which in our approach is coming from MG. In addition, we see that there is no tension between different \( f \sigma_8 \) datapoints. Instead, there is a combined trend of the datapoints to be in tension with the Planck/\( \Lambda \)CDM prediction. This tension disappears when we keep the same \( \Lambda \)CDM background but allow for a MG evolution of the effective Newton’s constant. In fact, this trend may be shown to be translated.
into a trend for lower values for the gravitational parameters $\mu$ and $\Sigma$ and is quantified through a detailed maximum likelihood analysis.

Each $f_{\sigma_8}(z)$ and $E_G(z)$ datapoint of the compilations of Tables VI and VII has been published separately in the context of independent analyses of distinct galaxy samples and lensing data. However, the correlations among the datapoints considered due to overlap of the analyzed galaxy samples may lead to an amplification of the existing trends indicated by the data and an amplification of the existing tension of the best fit parameters with Planck/ΛCDM. Despite of this fact we have chosen to keep the relatively large number of distinct published datapoints in order to maximize the information encoded in the compilations considered keeping in mind that this may lead to an artificial amplification of the trends that already exist in the data.

An additional motivation for keeping the full set of published datapoints is that it is not always clear which one of the correlated points is more suitable to keep. Ignoring one of the correlated points arbitrarily or simply based on time of publication criteria could lead to loss of useful information or selection bias.

Keeping the full set of points does not significantly change the results and the level of tension between the growth data best fit parameter values corresponding to MG and Planck/ΛCDM best fit in the context of GR. In order to demonstrate the validity of the above reasons we have repeated our analysis for a subset of the $f_{\sigma_8}$ and $E_G$ data where we have removed most earlier data that were subject to correlations with more recent data as indicated with bold font in the index of the Tables VI and VII and as shown in Figs. 1 and 2 with dark red. The result was a data compilation of about half the $f_{\sigma_8}$ and $E_G$ datapoints with significantly less correlation. The results of the statistical analysis of this dataset are presented in Appendix A and indicate a minor reduction of the overall tension.

For the construction of the likelihood contours of the model parameters in the context of the $f_{\sigma_8}$ and $E_G$ datasets we construct $\chi^2_{f_{\sigma_8}}$ and $\chi^2_{E_G}$. For the construction of $\chi^2_{f_{\sigma_8}}$ we use the vector $[2]$

$$V_{f_{\sigma_8}}(z_i,p) \equiv f_{\sigma_8}^{obs}(z_i,p) - f_{\sigma_8}^{th}(z_i,p)q(z_i,\Omega_{0m},\Omega_{0m}^{fid})$$

where $f_{\sigma_8}^{obs}$ is the the value of the $i$th datapoint, with $i = 1, \ldots, N_{f_{\sigma_8}}$ (where $N_{f_{\sigma_8}} = 66$ corresponds to the total number of datapoints of Table VI) and $f_{\sigma_8}^{th}(z_i,p)$ is the theoretical prediction, both at redshift $z_i$. The parameter vector $p$ corresponds to the parameters $\sigma_8, \Omega_{0m}, g_a$ of Eq. (2.16) with the parametrization (2.18). The fiducial Alcock-Paczynski correction factor $q$ [1, 2, 41] is defined as

$$q(z_i,\Omega_{0m},\Omega_{0m}^{fid}) = \frac{H(z_i)d_A(z_i)}{H^{fid}(z_i)dz_A^{fid}(z_i)}$$

where $H(z)$, $d_A(z)$ correspond to the Hubble parameter and the angular diameter distance of the true cosmology and the superscript $^{fid}$ indicates the fiducial cosmology used in each survey to convert angles and redshift to distances for evaluating the correlation function. As shown in Table II, the effects of this correction factor are less than about 10% in the derived best fit parameter values.
Thus we obtain $\chi_{f\sigma_8}^2$ as
\[ \chi_{f\sigma_8}^2(\Omega_{0m}, \sigma_8, g_b) = V_{f\sigma_8}^j F_{f\sigma_8,ij} V_{f\sigma_8}^i \] (3.3)

where $F_{f\sigma_8,ij}$ is the Fisher matrix (the inverse of the covariance matrix $C_{f\sigma_8,ij}$ of the data) which is assumed to be diagonal with the exception of the 3 × 3 WiggleZ subspace (see [2] for more details on this compilation).

Similarly, for the construction of $\chi_{E_G}^2$, we consider the vector
\[ V_{E_G}^i (z_i, p) = E_{G, i}^{\text{obs}} - E_{G}^{\text{th}} (z_i, p) \] (3.4)

where $E_{G, i}^{\text{obs}}$ is the the value of the $i$th datapoint, with $i = 1, \ldots, N_{E_G}$ (where $N_{E_G} = 16$ corresponds to the total number of datapoints of Table VII), while $E_{G}^{\text{th}} (z_i, p)$ is the theoretical prediction (Eq. (2.17)), both at redshift $z_i$. The parameter vector $p$ corresponds to the parameters of Eq. (2.17) with the parametrization (2.18) namely $\Omega_{0m}$, $\sigma_8$, $g_b$.

Thus we obtain $\chi_{E_G}^2$ as
\[ \chi_{E_G}^2(\Omega_{0m}, g_a, g_b) = V_{E_G}^j F_{E_G,ij} V_{E_G}^i \] (3.5)

where $F_{E_G,ij}$ is the Fisher matrix also assumed to be diagonal.

By minimizing $\chi_{f\sigma_8}^2$, $\chi_{E_G}^2$ separately and combined as $\chi_{\text{flat}}^2 = \chi_{f\sigma_8}^2 + \chi_{E_G}^2$, we obtain the constraints on the parameters $\Omega_{0m}$, $\sigma_8$, $g_a$, $g_b$ shown in Figs. 3, 4 and 5 respectively. Each one of these Figures corresponds to a 2D projection that goes through the best fit parameter point in parameter space of the full three or four dimensional contour plot in each case. The full number of parameters (three or four) was assumed.
FIG. 5. The six 1σ - 7σ confidence contours in 2D projected parameter spaces of the parameter space $(\Omega_m, \sigma_8, g_a, g_b)$ in the context of parametrizations Eqs. (2.18) and (2.19) with $n = 2$ and $m = 2$ including the fiducial correction factor Eq. (3.2). The data $E_G(z)$ and $f\sigma_8(z)$ from Tables VII and VI of the Appendix B was used. The third and the forth parameter in each contour were fixed to the best fit values. The red and green dots describe the Planck/ΛCDM best fit and the best-fit values from data.

when constructing the contour 2D projections. Previous studies [1, 2] have considered similar 2D projections that go through the Planck/ΛCDM best fit parameter point in the higher dimensional parameter space. This later choice tends to change somewhat (in most projections it is increased) the apparent tension between the best fit MG parameter values and the best fit Planck/ΛCDM parameters in the 2D projection parameter subspaces. This 2D tension may be in some cases misleading due to projection effects and thus in Table (III) we stress the tension in the full 3D or 4D parameter space.

The tension level between the best fit MG parameter values and the Planck/ΛCDM best fit parameter values is significant in both the 2D projection parameter spaces shown in Figs 3, 4 and 5 and in the higher 3D parameter space likelihood surfaces shown in Fig. 6. The best fit parameter values obtained in the context of the datasets considered and the tension levels in both the 2D projections and in the full 3D-4D parameter spaces are shown in Tables II and III respectively. In these Tables we also show the cases corresponding to fits without including the correction factor (3.2) in the $f\sigma_8$ data demonstrating that there is a small change in the best fit parameter values.

The following comments can be made on the results shown in Figs. 3, 4 and 5 and Tables II and III:

- The left part of Table III shows the tension level in the full 3D or 4D parameter space. The tension level between Planck/ΛCDM and best fit MG model parametrizations (2.18) and (2.19) in the context of the $f\sigma_8$ data is significant (about $3.5\sigma$) but is is less than the corresponding tension ob-
FIG. 6. Left: The 1σ - 2σ confidence contour of the parameter space \((\Omega_0, \sigma_8, g_a)\) in the context of parametrization Eq. (2.18) with \(n = 2\) including the fiducial correction factor Eq. (3.2). The RSD data \(f\sigma_8(z)\) from Table VI of the Appendix B was used. The red and green dots describe the Planck18/ΛCDM best fit and the best-fit values from data. Right: The 1σ - 2σ confidence contour of the parameter space \((\Omega_0, g_a, g_b)\) in the context of parametrizations Eqs. (2.18) and (2.19) with \(n = 2\). The data \(E_G(z)\) from Table VII of the Appendix B was used. The red and green dots describe the Planck18/ΛCDM best fit and the best-fit values from data. The 3D contours include only the surfaces in 3D while the intermediate space is not filled. Thus, the white gaps that appear in the right figure between the surfaces, simply correspond to the white background seen from behind.

| Param. Planck18/ΛCDM | Dataset \(f\sigma_8(z)\) | Dataset \(f\sigma_8(z)\) | Dataset \(E_G(z)\) | Datasets \(f\sigma_8(z) + E_G(z)\) | Datasets \(f\sigma_8(z) + E_G(z)\) |
|----------------------|-------------------------|-------------------------|------------------|-------------------------|-------------------------|
| \(\Omega_0\)       | 0.3153 ± 0.0073         | 0.272 ± 0.019           | 0.263 ± 0.015    | 0.313 ± 0.024           | 0.275 ± 0.015           | 0.264 ± 0.012           |
| \(\sigma_8\)       | 0.8111 ± 0.0060         | 0.886 ± 0.015           | 0.90 ± 0.016     | 0.848 ± 0.015           | 0.879 ± 0.015           |                     |
| \(g_a\)            | -1.306 ± 0.140          | -1.331 ± 0.138          | -0.129 ± 0.490   | -0.957 ± 0.144          | -1.115 ± 0.137          |                     |
| \(g_b\)            | 0                       | 0                       | -2.308 ± 0.423   | -2.448 ± 0.414          | -2.422 ± 0.416          |                     |

TABLE II. Planck18/ΛCDM based on TT,TE,EE+lowE+ lensing likelihoods best fit [15] and the best-fit values from data.

tained using the \(E_G\) statistic data (more than 4σ). In fact for the combined \(f\sigma_8 + E_G\) dataset the tension level increases to close to 6σ! This significant tension level comes independently from both the \(f\sigma_8\) and \(E_G\) data and hints towards weaker gravity \((\mu\) and \(\Sigma\) lower than 1) compared to the predictions of GR at low \(z\). We stress however that this extreme level of tension is partly due to correlations among the considered datapoints which necessarily exist in our compilations.

- The weaker than expected gravitational growth indicated by the data is expressed as both a lower best fit \(\Omega_0\) than expected from ΛCDM and as negative best fit values for the gravitational strength evolution parameters \(g_a\) and \(g_b\) (see e.g. Fig. 5).

- Ignoring the fiducial model correction factor of Eq. (3.2) in most cases tends to slightly increase the tension level (compare e.g. the last two lines of Table III). Thus the consideration of this correction in our analysis is a conservative approach.

The trend for weaker gravity at low redshifts is also evident in Fig. 7 which shows the best fit form of \(\mu(a)\) and \(\Sigma(a)\) in the context of each dataset.

Also, the likelihood contours in the \(\sigma_8-\Omega_m\) parameter
space obtained using the growth data in the presence of the MG parameter $g_a$ and in the context of GR ($g_a = 0$) are shown in Fig. 8. We have considered both the case of a marginalized MG parameter value and the case of setting $g_a$ to its best fit value. Clearly the tension level between the best fit parameter values and Planck/ΛCDM decreases significantly in the presence of the MG parameter $g_a$.

The introduction of additional parameters of any type would in general widen the likelihood contours and thus reduce the tension between growth data and geometric/CMB data. In general a faster expansion rate ($w < -1$) would tend to reduce the growth rate of perturbations in agreement with dynamical observables. However, geometric observables (SnIa, BAO etc.) do not allow significant deviations of the expansion rate from ΛCDM. Thus the most efficient way to produce a weaker growth of perturbations is the introduction of evolution of the MG parameters $\mu$ and $\Sigma$. In Fig 9 we have demonstrated this effect by fixing $g_a = 0$, $g_b = 0$ and constructing the $\sigma_8$-$\Omega_m$ contours with $w = -1$ and $w$ free to vary in a range $[-1.5, -0.5]$ consistent with geometric probes. The reduction of the tension in this case is significantly smaller compared to the introduction of MG degrees of freedom.

### III.2. Scale Dependent Data Compilations

Scale dependent parametrizations for $\mu$ and $\eta$ can describe a large class of MG models [86, 106]. For example a scale dependent class of parametrizations predicted by scalar-tensor theories for $\mu$ and $\eta$ is of the form [120, 121]

$$
\mu(a, k) = 1 + f_1(a) \frac{1 + c_1(\lambda H/k)^2}{1 + (\lambda H/k)^2} \quad (3.6)
$$

$$
\eta(a, k) = 1 + f_2(a) \frac{1 + c_2(\lambda H/k)^2}{1 + (\lambda H/k)^2} \quad (3.7)
$$

where $f_1$ and $f_2$ are properly chosen functions that depend on the scale factor. Thus a physically motivated scale dependent generalization of the parametrizations (2.18) and (2.19) for $\mu$ and $\Sigma$ may be written as

$$
\mu(R, z) = 1 + \left[ g_a \left( \frac{z}{1+z} \right)^n - g_a \left( \frac{z}{1+z} \right)_{2n} \right] \frac{1 + s_a(\lambda H R)^2}{1 + (\lambda H R)^2} \quad (3.8)
$$

$$
\Sigma(R, z) = 1 + \left[ g_b \left( \frac{z}{1+z} \right)^m - g_b \left( \frac{z}{1+z} \right)_{2m} \right] \frac{1 + s_b(\lambda H R)^2}{1 + (\lambda H R)^2} \quad (3.9)
$$

![FIG. 7. Evolution of $\mu$ and $\Sigma$ as functions of the scale factor $a$ considering the best fit values for $g_a$ and $g_b$ in the context of parametrizations Eqs. (2.18) and (2.19) with $n = 2, m = 2$. The data $E_G(z)$ and $f\sigma_8(z)$ from Tables VII and VI of the Appendix B was used. The dashed curves correspond to 1σ deviations of the parameters $\mu$ and $\Sigma$. The red lines correspond to the GR-ΛCDM model.](image_url)
where \( s_a, s_b \) and \( \lambda \) are parameters to be determined from a proper scale dependent dataset. Such a scale dependent data compilation for the statistic \( E_G \) in two redshift ranges is shown in Fig. 10 and in Tables VIII and IX for low and high \( z \) respectively in the Appendix B. The analysis of this compilation may be performed in the context of the scale dependent parametrizations (3.8) and (3.9). Clearly as shown in Fig. 10, for both low and high \( z \) the scale independent MG parametrizations of Eqs. (2.18) and (2.19) at \( z = 0.3 \) and at \( z = 0.7 \), lead to a best fit value of \( E_G \) that is lower compared to the Planck/ΛCDM prediction. The full scale dependent analysis leads to similar levels of tension as those indicated in Table III for the scale independent case and will be presented in detail elsewhere.

IV. CONCLUSIONS-DISCUSSION

We have used up to date compilations of \( E_G \) and \( f\sigma_8 \) data (Tables VI and VII) based on WL and RSD observations to obtain updated estimates of the tension between the Planck/ΛCDM best fit parameter values and the best fit parameter values obtained in the context of an effective MG gravity model allowing for properly parametrized evolution of the growth and lensing gravitational constants \( \mu \) and \( \Sigma \). The scale independent parametrizations (Eqs. (2.18), (2.19)) of \( \mu \) and \( \Sigma \) depend on the parameters \( a_g \) and \( b_g \) respectively and are by construction consistent with GR at early times and at present as indicated by nucleosynthesis and solar system constraints assuming no screening is present. We have assumed a flat ΛCDM expansion background and we thus fit the parameters \( (\Omega_{0m}, \sigma_8, a_g, b_g) \).

We find that the \( E_G \) data amplify the previously well known indications for low \( \Omega_{0m} \) and/or weaker gravity (\( \mu < 1 \)) at low \( z \) and favor weaker gravity for both the growth and the lensing gravitational constants (\( \mu < 1 \) and \( \Sigma < 1 \)). The tension level between the Planck/ΛCDM parameter values \( (\Omega_{0m}, \sigma_8, a_g, b_g) = (0.31, 0.81, 0, 0) \) and the best fit parameter values obtained using the combined \( E_G + f\sigma_8 \) dataset \( (\Omega_{0m}, \sigma_8, a_g, b_g) = (0.28, 0.85, -0.96, -2.45) \) is 6σ which is significantly larger compared to the tension obtained when only the \( f\sigma_8 \) dataset is used (3.7σ as shown in Table III). Even though the absolute magnitude of the derived tension is overestimated due to the correlations among the datapoints the amplified trend for weaker gravity at low \( z \) is clearly indicated by both the \( f\sigma_8 \) and \( E_G \) data compilations and appears to be stronger for the case of the \( E_G \) data.

If this trend has some physical origin and is not due only to data systematics or physical effects in the context of GR, there are significant implications for theoretical models. In particular \( f(R) \) theories generically predict stronger gravity at low \( z \) compared to its present time\([122]\) (thus the prediction is \( \mu(z) > 1, g_a > 0 \)) and therefore if the identified tension has physical origin this can not be attributed to an \( f(R) \) MG gravity theory for any expansion background. Similarly minimal scalar tensor theories \([3, 122]\), Horndeski theories \([123, 124]\) and beyond Horndeski Gleyzes-Langlois-Piazza-Vernizzi (GLPV) theories \([125]\) can only produce weaker gravity at low \( z \) under very specific and in some cases unnatural conditions\([52]\). For example minimal scalar-tensor theories would require the existence of a phantom cosmological background expansion (equation of state parameter \( w < -1 \))\([3, 122]\). In fact, a very large class of MG
FIG. 9. The confidence contours of the parameter space ($\sigma_8$-$\Omega_m$) in the context of GR (left panels) and in the presence of the $w$ parameter (fixing $g_a = 0$ and $g_b = 0$). We have considered both the case of a marginalized $w$ ($[-1.5,-0.5]$) parameter value (right panels) and the case of setting $w$ to its best fit value ($-0.94$ and $-1.29$ from $f\sigma_8(z)$ and $f\sigma_8(z) + E_G(z)$ data respectively) (middle panels). The red and green dots describe the Planck18/$(\Lambda$CDM) best fit and the best-fit values from data. The $E_G(z)$ and $f\sigma_8(z)$ data compilations of datapoints with less correlation from Tables VII and VI of the Appendix B was used. Notice that the reduction of tension between the best fit parameter values and Planck/$(\Lambda$CDM) is less efficient when the $w$ degree of freedom (modified background expansion rate) is introduced compared to the MG degree of freedom $g_a$ shown in Fig. 8.

FIG. 10. Measurements of $E_G$ as a function of scale $R$ in the range $0.15 < z < 0.43$ (left panel) and $0.43 < z < 1.2$ (right panel). The data $E_G(R)$ from Tables VIII and IX of the Appendix B was used. The dashed black line shows the Planck18/$(\Lambda$CDM) prediction at $z = 0.3$, the dotted black line shows the Planck18/$(\Lambda$CDM) prediction at $z = 0.7$, while the dot-dashed black line and the large dashed black line shows the best fit of the $E_G$ in the context of parametrizations Eqs. (2.18) and (2.19) at $z = 0.3$ and at $z = 0.7$ respectively.
models, the scalar-tensor Horndeski models, are not consistent with the observational indications of weakening gravity. In fact as stated in Ref. [126] (p. 12), $\mu$ for stable Horndeski models is always larger than, or equal to, 1 so that matter perturbations in viable Horndeski models always grow faster than the corresponding GR models with the same backgrounds. Thus these MG models (which include $f(R)$ gravities) are unable to account for the weakening and would provide a worse fit than GR to the $f\sigma_8/E_G$ data. The search for MG models that can account for the observed indications for weakening gravity is thus an interesting extension of the present analysis.

A partial cause of the $E_G$ data tension with Planck/ΛCDM is lensing magnification. As shown in [127, 128] the effects of lensing magnification modify the galaxy-galaxy lensing correlations as well as galaxy-galaxy correlations and as a consequence introduce systematic errors in the estimate of $E_G$ while making it bias dependent. The effect is small for redshifts smaller than 1 (about 5 – 10%) but it can become as large as 20 – 40% for redshifts $z \approx 1.5$. Thus, this systematic contribution can be relevant already for Dark Energy Survey (DES) [38, 129–132] and certainly for higher redshift surveys. However, the magnitude of lensing contribution at the redshifts of the data compilation we are using ($z < 1$) is not large enough to significantly reduce the identified tension which exists even at the level of the RSD data alone. The systematic effect discussed in [127, 128] is important especially for upcoming surveys like Euclid [133] which probe higher redshifts even though even in that case it may not be large enough to be the only source the observed tension. An interesting feature of our compilation is the scale dependence of the $E_G(R, z)$ data. This may be used to probe the parameters of scale dependent MG $\mu$ and $\Sigma$ parametrizations which are well motivated physically. We plan to present such constraints elsewhere using upcoming and more extensive data able to constrain the required larger parameter space that appears in scale dependent $\mu$ and $\Sigma$ parametrizations. A key question to address is whether the addition of scale dependence in the parametrizations can improve significantly the overall fit. No such indications are currently known [120] but this may well change using more extensive and accurate scale dependent $E_G$ and $f\sigma_8$ data.

The introduction of the MG parameters $\mu$ and $\Sigma$ along with the variation of the parameters $\Omega_m$ and $\sigma_8$ leads to a model (MG-ΛCDM) that is a much better fit to the growth $f\sigma_8$ and $E_G$ data than the Planck/ΛCDM model in the context of GR. We have called this effect a ‘tension’ between the new best fit parameter values (MG-ΛCDM) and the GR-Planck/ΛCDM parameter values (from Planck18 fit) which are $5 – 6\sigma$ away from the new best fit parameter values. On the other hand, the MG parameters do not seem to change significantly the fit of the Planck data as indicated in Ref. [4] and in Planck18 [15] which indicate that pure CMB data appear to favor GR. Thus, the particular parametrization we have used does not seem to significantly reduce the tension between CMB and growth/weak lensing data since MG gravity appears to be favored by growth/weak lensing but not by the CMB. This is an issue we plan to investigate in more detail in the future by considering e.g. different MG parametrizations for the evolution of the $\mu$ and $\Sigma$ parameters that will not only improve the fit to the $f\sigma_8/E_G$ data but also improve the fit to the CMB data where some tensions are already evident (e.g. the lensing anomaly discussed in Planck18 [15]).

**Supplemental Material:** The Mathematica file used for the numerical analysis and for construction of the figures can be found in [134].

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Appendix A: ANALYSIS OF SUBSETS OF DATAPoints WITH LESS CORRELATION

In this Appendix we present the results of the statistical analysis of the $f \sigma_8(z)$ and $E_G(z)$ data compilations of datapoints with less correlation. These subsets of the data are indicated with bold font in the index of the Tables VI and VII of the Appendix B. Using these subsets of the data and repeating our analysis we obtain the best fit parameter values and the tension levels in both the 2D projections and in the full 3D-4D parameter spaces as shown in Tables IV and V respectively.

These results indicate that even though the tension level for the combined ($E_G + f \sigma_8$) reduces somewhat (from 6σ to about 5.5σ) it remains high enough to cause concerns for the self consistency of the Planck/ΛCDM model and indications for the presence of weakening gravity.

Appendix B: DATA USED IN THE ANALYSIS

In this appendix we present the data used in the analysis.

| Dataset | $z$ | $f \sigma_8(z)$ | Refs. | Year | Fiducial Cosmology |
|---------|-----|----------------|-------|------|-------------------|
| SDSS-LRG | 0.35 | 0.440 ± 0.050 | [135] | 30 October 2006 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.756)$[136] |
| VVDS | 0.77 | 0.490 ± 0.18 | [135] | 6 October 2009 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.78)$ |
| 2dFGRS | 0.17 | 0.510 ± 0.060 | [135] | 6 October 2009 | $(\Omega_m, \Omega_K) = (0.3, 0.0, 0.9)$ |
| 2MRs | 0.02 | 0.314 ± 0.048 | [137], [138] | 13 November 2010 | $(\Omega_m, \Omega_K, \sigma_8) = (0.266, 0.0, 0.65)$ |
| SnIa+RAS | 0.02 | 0.398 ± 0.065 | [139], [138] | 20 October 2011 | $(\Omega_m, \Omega_K, \sigma_8) = (0.3, 0.0, 0.814)$ |
| SDSS-LRG-200 | 0.25 | 0.3512 ± 0.0583 | [140] | 9 December 2011 | $(\Omega_m, \Omega_K, \sigma_8) = (0.276, 0.0, 0.8)$ |
| SDSS-LRG-200 | 0.37 | 0.4602 ± 0.0378 | [140] | 9 December 2011 | $(\Omega_m, \Omega_K, \sigma_8) = (0.276, 0.0, 0.8)$ |
| SDSS-LRG-60 | 0.25 | 0.3665 ± 0.0601 | [140] | 9 December 2011 | $(\Omega_m, \Omega_K, \sigma_8) = (0.276, 0.0, 0.8)$ |
| SDSS-LRG-60 | 0.37 | 0.4031 ± 0.0586 | [140] | 9 December 2011 | $(\Omega_m, \Omega_K, \sigma_8) = (0.276, 0.0, 0.8)$ |
| WiggleZ | 0.44 | 0.413 ± 0.080 | [141] | 12 June 2012 | $(\Omega_m, h, \sigma_8) = (0.27, 0.71, 0.8)$ |
| WiggleZ | 0.60 | 0.390 ± 0.063 | [141] | 12 June 2012 | $(\Omega_m, h, \sigma_8) = (0.27, 0.71, 0.8)$ |
| WiggleZ | 0.73 | 0.437 ± 0.072 | [141] | 12 June 2012 | $(\Omega_m, h, \sigma_8) = (0.27, 0.71, 0.8)$ |
| 6dFGS | 0.067 | 0.423 ± 0.055 | [84] | 4 July 2012 | $(\Omega_m, \Omega_K, \sigma_8) = (0.27, 0.0, 0.76)$ |
| SDSS-BOSS | 0.30 | 0.407 ± 0.055 | [142] | 11 August 2012 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.804)$ |
| SDSS-BOSS | 0.40 | 0.419 ± 0.041 | [142] | 11 August 2012 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.82)$ |
| SDSS-BOSS | 0.50 | 0.427 ± 0.043 | [142] | 11 August 2012 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.82)$ |
| SDSS-BOSS | 0.60 | 0.433 ± 0.067 | [142] | 11 August 2012 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.82)$ |
| VIPERs | 0.80 | 0.470 ± 0.080 | [143] | 9 July 2013 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.82)$ |
| SDSS-DR7-LRG | 0.35 | 0.429 ± 0.089 | [144] | 8 August 2013 | $(\Omega_m, \Omega_K, \sigma_8) = (0.25, 0.0, 0.890)$[145] |
| GAMA | 0.18 | 0.360 ± 0.090 | [146] | 22 September 2013 | $(\Omega_m, \Omega_K, \sigma_8) = (0.27, 0.0, 0.8)$ |
| GAMA | 0.38 | 0.440 ± 0.060 | [146] | 22 September 2013 | $(\Omega_m, \Omega_K, \sigma_8) = (0.27, 0.0, 0.8)$ |
| BOSS-LOWZ | 0.32 | 0.384 ± 0.095 | [147] | 17 December 2013 | $(\Omega_m, \Omega_K, \sigma_8) = (0.274, 0.0, 0.8)$ |
| SDSS DR10 and DR11 | 0.32 | 0.48 ± 0.10 | [147] | 17 December 2013 | $(\Omega_m, \Omega_K, \sigma_8) = (0.274, 0.0, 0.8)$[148] |
| Index | Dataset | $z$ | $E_G(z)$ | $\sigma_{E_G}$ | Scale [Mpc/h] | Reference |
|-------|---------|-----|-----------|---------------|-------------|-----------|
| 1     | KiDS GAMMA | 0.207 | 0.43 | 0.13 | $5 < R < 40$ | [174] |
| 2     | SDSS BOSS LOWZ | 0.27 | 0.40 | 0.05 | $25 < R < 150$ | [175] |
| 3     | CMB lens BOSS LOWZ | 0.27 | 0.46 | 0.085 | $25 < R < 150$ | [175] |
| 4     | KiDS 2dFLEN S BOSS LOWZ 2DFLOZ | 0.305 | 0.27 | 0.08 | $5 < R < 60$ | [174] |
| 5     | RCSLenS CFHTLenS WiggleZ BOSS WIZLoZ LOWZ | 0.32 | 0.40 | 0.09 | $R > 3$ | [176] |
| 6     | RCSLenS CFHTLenS WiggleZ BOSS WIZLoZ LOWZ | 0.32 | 0.48 | 0.10 | $R > 10$ | [176] |
| 7     | SDSS | 0.32 | 0.39 | 0.06 | $10 < R_p < 50$ | [70] |
| 8     | KiDS 2dFLEN S BOSS CMASS 2dFHIZ | 0.554 | 0.26 | 0.07 | $5 < R < 60$ | [174] |
| 9     | RCSLenS CFHTLenS WiggleZ BOSS WIZHiZ CMASS | 0.57 | 0.31 | 0.06 | $R > 3$ | [176] |
| 10    | RCSLenS CFHTLenS WiggleZ BOSS WIZHiZ CMASS | 0.57 | 0.30 | 0.07 | $R > 10$ | [176] |
| 11    | SDSS-III BOSS CMB lens CMASS | 0.57 | 0.24 | 0.06 | $R > 150$ | [73] |
| 12    | CFHTLenS SDSS-III BOSS CMASS | 0.57 | 0.42 | 0.056 | $5 < R < 26$ | [177] |
| 13    | CMB lens BOSS CMASS | 0.57 | 0.39 | 0.05 | $25 < R < 150$ | [175] |
| 14    | CFHTLenS BOSS CMASS | 0.57 | 0.43 | 0.10 | $10 < R < 60$ | [178] |

| Index | Dataset | $z$ | $E_G(z)$ | $\sigma_{E_G}$ | Scale [Mpc/h] | Reference |
|-------|---------|-----|-----------|---------------|-------------|-----------|
| 1      | KiDS GAMMA | 0.207 | 0.43 | 0.13 | $5 < R < 40$ | [174] |
| 2      | SDSS BOSS LOWZ | 0.27 | 0.40 | 0.05 | $25 < R < 150$ | [175] |
| 3      | CMB lens BOSS LOWZ | 0.27 | 0.46 | 0.085 | $25 < R < 150$ | [175] |
| 4      | KiDS 2dFLEN S BOSS LOWZ 2DFLOZ | 0.305 | 0.27 | 0.08 | $5 < R < 60$ | [174] |
| 5      | RCSLenS CFHTLenS WiggleZ BOSS WIZLoZ LOWZ | 0.32 | 0.40 | 0.09 | $R > 3$ | [176] |
| 6      | RCSLenS CFHTLenS WiggleZ BOSS WIZLoZ LOWZ | 0.32 | 0.48 | 0.10 | $R > 10$ | [176] |
| 7      | SDSS | 0.32 | 0.39 | 0.06 | $10 < R_p < 50$ | [70] |
| 8      | KiDS 2dFLEN S BOSS CMASS 2dFHIZ | 0.554 | 0.26 | 0.07 | $5 < R < 60$ | [174] |
| 9      | RCSLenS CFHTLenS WiggleZ BOSS WIZHiZ CMASS | 0.57 | 0.31 | 0.06 | $R > 3$ | [176] |
| 10     | RCSLenS CFHTLenS WiggleZ BOSS WIZHiZ CMASS | 0.57 | 0.30 | 0.07 | $R > 10$ | [176] |
| 11     | SDSS-III BOSS CMB lens CMASS | 0.57 | 0.24 | 0.06 | $R > 150$ | [73] |
| 12     | CFHTLenS SDSS-III BOSS CMASS | 0.57 | 0.42 | 0.056 | $5 < R < 26$ | [177] |
| 13     | CMB lens BOSS CMASS | 0.57 | 0.39 | 0.05 | $25 < R < 150$ | [175] |
| 14     | CFHTLenS BOSS CMASS | 0.57 | 0.43 | 0.10 | $10 < R < 60$ | [178] |

TABLE VII: The $E_G(z)$ data compilation used in the present analysis.
The subset of the datapoints with less correlation is indicated with bold
font in the index.
TABLE VIII: The $E_G(R)$ data compilation in the range $0.15 < z < 0.43$ used in the present analysis.

| Index | $R [\text{Mpc}/h]$ | $E_G(R)$ | $\sigma_{E_G}$ | $z$ | Reference |
|-------|---------------------|----------|----------------|----|-----------|
| 1     | 3.61                | 0.37     | 0.10           | 0.27 | [175]     |
| 2     | 4.91                | 0.42     | 0.08           | 0.27 | [175]     |
| 3     | 6.60                | 0.50     | 0.07           | 0.27 | [175]     |
| 4     | 9.07                | 0.39     | 0.07           | 0.27 | [175]     |
| 5     | 12.20               | 0.37     | 0.06           | 0.27 | [175]     |
| 6     | 16.58               | 0.45     | 0.06           | 0.27 | [175]     |
| 7     | 22.54               | 0.32     | 0.04           | 0.27 | [175]     |
| 8     | 30.30               | 0.39     | 0.05           | 0.27 | [175]     |
| 9     | 41.19               | 0.44     | 0.06           | 0.27 | [175]     |
| 10    | 55.99               | 0.45     | 0.08           | 0.27 | [175]     |
| 11    | 76.98               | 0.34     | 0.10           | 0.27 | [175]     |
| 12    | 103.47              | 0.28     | 0.15           | 0.27 | [175]     |
| 13    | 2.45                | 0.28     | 0.23           | 0.32 | [70]      |
| 14    | 3.41                | 0.49     | 0.16           | 0.32 | [70]      |
| 15    | 4.64                | 0.50     | 0.12           | 0.32 | [70]      |
| 16    | 6.62                | 0.32     | 0.09           | 0.32 | [70]      |
| 17    | 9.85                | 0.34     | 0.07           | 0.32 | [70]      |
| 18    | 14.83               | 0.45     | 0.08           | 0.32 | [70]      |
| 19    | 22.10               | 0.43     | 0.09           | 0.32 | [70]      |
| 20    | 45.87               | 0.32     | 0.10           | 0.32 | [70]      |
| 21    | 1.76                | 0.74     | 0.21           | 0.15-0.43 | [176] |
| 22    | 2.23                | 0.71     | 0.15           | 0.15-0.43 | [176] |
| 23    | 2.85                | 0.35     | 0.14           | 0.15-0.43 | [176] |
| 24    | 3.56                | 0.30     | 0.11           | 0.15-0.43 | [176] |
| 25    | 4.45                | 0.35     | 0.11           | 0.15-0.43 | [176] |
| 26    | 5.65                | 0.28     | 0.10           | 0.15-0.43 | [176] |
| 27    | 7.059               | 0.43     | 0.11           | 0.15-0.43 | [176] |
| 28    | 8.94                | 0.45     | 0.11           | 0.15-0.43 | [176] |
| 29    | 11.33               | 0.47     | 0.12           | 0.15-0.43 | [176] |
| 30    | 14.34               | 0.55     | 0.12           | 0.15-0.43 | [176] |
| 31    | 17.98               | 0.40     | 0.12           | 0.15-0.43 | [176] |
| 32    | 22.21               | 0.37     | 0.14           | 0.15-0.43 | [176] |
| 33    | 28.88               | 0.39     | 0.18           | 0.15-0.43 | [176] |
| 34    | 36.15               | 0.35     | 0.19           | 0.15-0.43 | [176] |
| 35    | 45.26               | 0.30     | 0.30           | 0.15-0.43 | [176] |
| 36    | 5.01                | 0.25     | 0.16           | 0.15-0.43 | [174] |
| 37    | 5.37                | 0.39     | 0.16           | 0.15-0.43 | [174] |
| 38    | 5.58                | 0.094    | 0.18           | 0.15-0.43 | [174] |
| 39    | 8.15                | 0.30     | 0.14           | 0.15-0.43 | [174] |
| 40    | 8.57                | 0.41     | 0.14           | 0.15-0.43 | [174] |
| 41    | 9.02                | 0.41     | 0.24           | 0.15-0.43 | [174] |
| 42    | 13.23               | 0.49     | 0.16           | 0.15-0.43 | [174] |
| 43    | 13.95               | 0.43     | 0.16           | 0.15-0.43 | [174] |
| 44    | 14.76               | 0.15     | 0.17           | 0.15-0.43 | [174] |
| 45    | 21.08               | 0.51     | 0.23           | 0.15-0.43 | [174] |
| 46    | 22.75               | 0.33     | 0.23           | 0.15-0.43 | [174] |
| 47    | 23.96               | 0.32     | 0.32           | 0.15-0.43 | [174] |
| 48    | 35.52               | 0.33     | 0.29           | 0.15-0.43 | [174] |
| 49    | 36.98               | 0.40     | 0.33           | 0.15-0.43 | [174] |
| 50    | 39.00               | 0.32     | 0.38           | 0.15-0.43 | [174] |
| 51    | 56.60               | 0.37     | 0.80           | 0.15-0.43 | [174] |

TABLE IX: The $E_G(R)$ data compilation in the range $0.43 < z < 1.2$ used in the present analysis.

| Index | $R [\text{Mpc}/h]$ | $E_G(R)$ | $\sigma_{E_G}$ | $z$ | Reference |
|-------|---------------------|----------|----------------|----|-----------|


|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 5.13 | 0.23 | 0.14 | 0.43-0.7 |
| 2 | 5.69 | 0.19 | 0.19 | 0.43-0.7 |
| 3 | 8.28 | 0.32 | 0.12 | 0.43-0.7 |
| 4 | 9.19 | 0.27 | 0.17 | 0.43-0.7 |
| 5 | 13.69 | 0.21 | 0.12 | 0.43-0.7 |
| 6 | 14.98 | 0.46 | 0.25 | 0.43-0.7 |
| 7 | 22.02 | 0.22 | 0.13 | 0.43-0.7 |
| 8 | 36.28 | 0.48 | 0.18 | 0.43-0.7 |
| 9 | 39.84 | 0.84 | 0.57 | 0.43-0.7 |
| 10 | 59.78 | 0.54 | 0.45 | 0.43-0.7 |
| 11 | 1.74 | 0.34 | 0.29 | 0.43-0.7 |
| 12 | 2.25 | 0.31 | 0.17 | 0.43-0.7 |
| 13 | 2.74 | 0.57 | 0.14 | 0.43-0.7 |
| 14 | 3.46 | 0.43 | 0.11 | 0.43-0.7 |
| 15 | 4.45 | 0.35 | 0.10 | 0.43-0.7 |
| 16 | 5.56 | 0.30 | 0.09 | 0.43-0.7 |
| 17 | 6.92 | 0.24 | 0.09 | 0.43-0.7 |
| 18 | 8.84 | 0.28 | 0.08 | 0.43-0.7 |
| 19 | 9.19 | 0.27 | 0.17 | 0.43-0.7 |
| 20 | 10.42 | 0.01 | 0.16 | 0.7-1.2 |
| 21 | 14.13 | 0.64 | 0.22 | 0.7-1.2 |
| 22 | 19.75 | 0.34 | 0.12 | 0.5-0.7 |
| 23 | 26.21 | 0.31 | 0.15 | 0.5-0.7 |
| 24 | 35.44 | 0.22 | 0.18 | 0.5-0.7 |
| 25 | 4.60 | 0.36 | 0.09 | 0.57 |
| 26 | 6.12 | 0.40 | 0.08 | 0.57 |
| 27 | 8.20 | 0.33 | 0.07 | 0.57 |
| 28 | 10.00 | 0.43 | 0.09 | 0.57 |
| 29 | 14.87 | 0.03 | 0.07 | 0.57 |
| 30 | 19.75 | 0.34 | 0.12 | 0.57 |
| 31 | 26.21 | 0.31 | 0.15 | 0.57 |
| 32 | 35.44 | 0.22 | 0.18 | 0.57 |
| 33 | 4.60 | 0.36 | 0.09 | 0.57 |
| 34 | 6.12 | 0.40 | 0.08 | 0.57 |
| 35 | 8.20 | 0.33 | 0.07 | 0.57 |
| 36 | 10.00 | 0.43 | 0.09 | 0.57 |
| 37 | 14.87 | 0.03 | 0.07 | 0.57 |
| 38 | 19.75 | 0.34 | 0.12 | 0.57 |
| 39 | 26.21 | 0.31 | 0.15 | 0.57 |
| 40 | 35.44 | 0.22 | 0.18 | 0.57 |
| 41 | 4.60 | 0.36 | 0.09 | 0.57 |
| 42 | 6.12 | 0.40 | 0.08 | 0.57 |
| 43 | 8.20 | 0.33 | 0.07 | 0.57 |
| 44 | 10.00 | 0.43 | 0.09 | 0.57 |
| 45 | 14.87 | 0.03 | 0.07 | 0.57 |
| 46 | 19.75 | 0.34 | 0.12 | 0.57 |
| 47 | 26.21 | 0.31 | 0.15 | 0.57 |
| 48 | 35.44 | 0.22 | 0.18 | 0.57 |
| 49 | 4.60 | 0.36 | 0.09 | 0.57 |
| 50 | 6.12 | 0.40 | 0.08 | 0.57 |
| 51 | 8.20 | 0.33 | 0.07 | 0.57 |
| 52 | 10.00 | 0.43 | 0.09 | 0.57 |
| 53 | 14.87 | 0.03 | 0.07 | 0.57 |
| 54 | 19.75 | 0.34 | 0.12 | 0.57 |
| 55 | 26.21 | 0.31 | 0.15 | 0.57 |
| 56 | 35.44 | 0.22 | 0.18 | 0.57 |
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