Towards the field theory of the Standard Model on fractional D6-branes on $T^6/\mathbb{Z}_6'$: Yukawa couplings and masses

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We present the perturbative Yukawa couplings of the Standard Model on fractional intersecting D6-branes on $T^6/\mathbb{Z}_6'$ and discuss two mechanisms of creating mass terms for the vector-like particles in the matter spectrum, through perturbative three-point couplings and through continuous D6-brane displacements.

1 Introduction

While the massless spectra of D6-brane models in type IIA string theory have been discussed in a variety of cases, see e.g. the statistics for various orbifold backgrounds in [1, 2, 3], the inspection of the low-energy field theory limit has to a large extent focussed on the gauge couplings, see e.g. [4, 5, 6] and [7] for a discussion within the string landscape. For interaction terms such as Yukawa couplings, exact string theoretic results are only known on the six-torus, e.g. [8, 9], and its $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold without discrete torsion, and consequently a treatment of Standard Model (SM) vacua in globally consistent supersymmetric D6-brane models on $T^6/\mathbb{Z}_6'$ [10, 11, 12, 13, 3, 7] or other $T^6/\mathbb{Z}_2^{N}$ [14, 15, 16] and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2^{M}$ backgrounds with discrete torsion [17, 18] has not been performed. To go one step further in the understanding of the field theory of intersecting fractional D6-brane SM vacua, we assume here that the formula for Yukawa couplings on the six-torus directly carries over to its orbifolds and extends to the new case with chiral matter at one vanishing angle, i.e. ‘triangular’ worldsheets with an acute angle zero. Our discussion focusses on the dominant worldsheet contribution to each three-point coupling.

1.1 Geometry of a D6-brane Standard Model vacuum with ‘hidden’ $USp(6)$

The D6-brane configuration of a particular SM vacuum on the ABa lattice orientation of $T^6/(\mathbb{Z}_6' \times \Omega R)$, which is generated by $\theta : z_k \rightarrow e^{2\pi i n_k} z_k$ with $\vec{v} = \frac{1}{6}(1, 2, -3)$ and $\mathcal{R} : z_k \rightarrow \bar{z}_k$, is given in table 1. The resulting massless open string spectrum consists of a ‘chiral’ part computed from intersection numbers (i.e. including the anomalous $U(1)_b \subset U(2)_c$ charge of otherwise vector-like matter), which contains the SM quarks and leptons and nine Higgs generations [3, 7].

\[
[C] = 3 \times \left[ (3, 2)_{\frac{1}{6}} + (3, 1)_{\frac{1}{3}} + (3, 1)_{-\frac{2}{3}} + (1, 1)_{1} + (1, 1)_{0} + 2 \times (1, 2)_{-\frac{1}{2}} + (1, 2)_{\frac{1}{2}} \right] + 9 \times \left[ (1, 2)_{-\frac{1}{2}} + (1, 2)_{\frac{1}{2}} \right]
\]

\[\equiv 3 \times [Q_L + d_R + u_R + e_R + \nu_R + 2 \times L + T] + 9 \times [H_d + H_u],\]

where we already used the breaking $USp(2)_c \rightarrow U(1)_c$ by a continuous displacement $\sigma_2^2$. Besides the lower case index of the hypercharge, $Q_Y = Q_{\frac{1}{3}}^u + Q_{-\frac{1}{3}}^d + Q_{\frac{1}{2}}^e$, the baryon minus lepton number

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To investigate the interactions among the massless open string states, it is necessary to know their origin from a given intersection sector \(x(\theta^k y)\) of a D6-brane \(x\) with the orbifold images \(\phi(x(\theta^k y))\) of \(y\), which can e.g. be computed using the beta function coefficients of gauge threshold amplitudes [7]. In addition, the localisations of the intersection points along \(T^6/\mathbb{Z}_6\) are needed. As examples, the localisations of leptons and of quarks and Higgses are displayed in table 2 and 3 respectively (for more details see [19]). The situation is depicted in figure 1 for D6-branes \(b\), \(c\) and \(bd\) supporting the left-handed leptons \(L_i\), right-handed leptons \(E_j\) and six of the Higgs generations \(H_k\).

\[ |V| = \left[(3, 2)_{\frac{1}{2}} + 3 \times (3, 1)_{\frac{1}{2}} + 3 \times (\bar{3}, 1)_{\frac{1}{2}} + 3 \times (\bar{3}, \text{Antihybrid})_{1} + 6 \times (1, 3_{\text{Sym}})_0 + c.c. \right] \]

\[ + \left[1 \times (1, \bar{3})_{\frac{1}{2}} + 1 \times (\bar{1}, \bar{3})_{\frac{1}{2}} + 2 \times (1, \bar{1}_{\frac{1}{2}})_{\frac{1}{2}} + 1 \times (1, 2)_{\frac{1}{2}} + 4 \times (1, 1_{\text{Antihybrid}})_0 + c.c. \right] \]

\[ + \left[2 \times (1, 1)_{\frac{1}{2}} + 1 \times (1, 1)_{0} + c.c. \right] + 2 \times (1, 1)_{\frac{1}{2}} + 10 \times (1, 3_{\text{Adj}})_0 + 26 \times (1, 1)_0 \]

(2)

\[ |H| = 2 \times (1, 1; 15_{\text{Antihybrid}})_0 + (1, 2; 6)_0 + (1, \bar{2}; 6)_0 + 2 \times (1, 1; 6)_{\frac{1}{2}} + (1, 1; 6)_{\frac{1}{2}} \]

(3)

Table 1: Five supersymmetric stacks of D6-branes, which cancel all RR tadpoles on the ABa lattice of \(T^6/\mathbb{Z}_6\) with SM spectrum and a ‘hidden’ \(USp(6)_h\) gauge factor. The enhancements \(U(N) \rightarrow USp(2N)\) arise for D6-branes parallel or perpendicular to the O6-planes. Only in the first case, the enhancement can be cancelled by a continuous displacement \(x^A\).

| D6-brane | \(\mathcal{T}^2\) | angle on \((T^2_1, T^2_2, T^2_3)\) w.r.t. \(\OmegaK\)-plane | displacement \((\sigma^1, \sigma^3)\) on \((T^2_1, T^2_2, T^2_3)\) | orientation on \(T^2_2\) vs. \(\OmegaK\)-plane | gauge group |
|----------|-----------------|---------------------------------------------|-----------------------------|---------------------------------|-------------|
| Baryonic | I \(a\) | \(\pi \left(-\frac{1,1}{2}, -\frac{1}{2}, 0\right)\) | \((1,1)\) | \(\perp\) | \(U(3)_a\) |
| Left     | I \(b\) | \(\pi \left(-\frac{1,1}{2}, -\frac{1}{2}, 0\right)\) | \((1,0)\) | \(\perp\) | \(U(2)_b\) |
| Right    | I \(c\) | \(\pi \left(-\frac{1,1}{2}, 0\right)\) | \((1,1)\) | \(\perp\) | \(U(3)_c\) |
| Leptonic | I \(d\) | \(\pi \left(\frac{1}{2}, 0, -\frac{1}{2}\right)\) | \((0,1)\) | \(\perp\) | \(USp(2)_e\) |
| Hidden   | I \(h\) | \(\pi \left(-\frac{1,1}{2}, -\frac{1}{2}, 0\right)\) | \((0,0)\) | \(\perp\) | \(USp(6)_h\) |

Table 2: Assignment of left- and right-handed leptons to Table 3. Assignment and chirality \(x(\theta^k y)\) or non-chiral \(z(\theta^k y)\) of a given intersection sector \(x(\theta^k y)\) with counting of chiral- counting \(\varphi (\theta^k y)\) of left- and right-handed quarks and Higgses \(x(\theta^k y)\) or multiplicities of the non-chiral matter con-
ges in the gauge enhanced phase \(U(1)_c \rightarrow USp(2)_c\) tent \(\varphi (\theta^k y)\). The notation \(E_i = (c^i_L, c^i_R)\) or \(\tau^i_L\) or \(\tau^i_R\) is tailored for with \(U_i = (u^i_L, d^i_L)\) and \(H_i = (H^i_u, H^i_d)\). While family replication in the Higgs sector is related to the intersection points along \(T^2_2\), the quarks display a more intricate pattern on \(T^2_1 \times T^2_2\).
2 Yukawa couplings

2.1 Holomorphic three-point couplings

Three-point couplings arise in perturbation theory from triangular worldsheets with edges given by the three D6-branes under which matter is charged. For example, three chiral multiplets $\phi^i_{xy}$, $\phi^j_{yz}$ and $\phi^k_{zx}$ with respective charges $(N_x, N_y)$, $(N_y, N_z)$ and $(N_z, N_x)$ under $U(N_x) \times U(N_y) \times U(N_z)$ lead to a superpotential term of the form

$$W = W_{ijk} \phi^i_{xy} \phi^j_{yz} \phi^k_{zx} \quad \text{with} \quad W_{ijk} = \prod_{m=1}^{3} \left( \sum_{A_{ijk,(m)}} e^{-A_{ijk,(m)}} \right).$$

Besides the field theoretic consistency of charge neutrality, the string theoretic selection rule states that $W_{ijk} \neq 0$ is only possible if $x$, $y$ and $z$ form a closed triangle with area $A_{ijk,(m)}$ along $T^2_{(m)}$, which can also be shrunken to a point or, for fractional or rigid D6-branes, have one apex with vanishing angle. The infinite sums in (4) are due to the periodicity of the underlying six-torus. If e.g. only the two-torus $T^2_{(2)}$ is considered with vanishing Wilson lines $\tau^2_{x} \equiv 0$ for all D6-branes $x \in \{a, b, c, d, h\}$, the sum over areas can be compactly written as [8, 9],

$$\frac{\delta_2}{2\pi} \left( \frac{t_2 A_{(2)}}{2\pi} \right) = \sum_{t_2 \in \mathbb{Z}} e^{-t_2 A_{(2)}(\delta_2 + t_2)^2} \delta_2 = 0, t_2 = 3 \quad 1 + 2e^{-3A_{(2)}} + 2e^{-12A_{(2)}} + \ldots,$$

with $t_2 \in \mathbb{N}$ the (absolute value of the) product of torus intersection numbers, $\delta_2$ a linear function of the location of intersections and the corresponding displacements $\sigma_x^2$ and $A_{(2)} > 1$ the two-torus volume in units of $2\pi \alpha'$. The sum in (5) is dominated by the term $l = 0$, as can be seen in the example of the family diagonal lepton Yukawa couplings at $b$, $c$ and $(\theta d)$ intersections with $\delta_2 = 0$ and $t_2 = 3$. The sum on the r.h.s. of (5) converges very fast since already the first non-trivial contribution $2e^{-3A_{(2)}} < 2e^{-3} < 0.1$ is tiny for areas $A_{(2)} > 1$ in the geometric regime of type IIA string compactifications. Non-diagonal matter couplings arise typically from worldsheets with a non-vanishing triangular area, which can be parameterised as a fraction $\delta_3 \in \mathbb{Q}$ of the two-torus area $A_{(2)}$. Typical numerical examples of such suppression factors are given in table 4. A hierarchical structure of Yukawa couplings for lepton and quark families can thus be generated by a suitable choice of matter localisations and corresponding triangular areas. Two ways of tuning arise by varying the ratios $A_{i(i)} / A_{j(j)}$ of two-torus volumes ($i \neq j$) and by choosing different continuous displacement parameters $\sigma_x^2$ for each D6-brane $x$ along $T^2_{(2)}$ as displayed in figure 2.

![Figure 1](image-url)
While the Kähler potential $K$ normalised one-cycle volume of D6-branes $V$ all right-handed leptons $E_l$, $E_{L_f}$, $E_{L_f'}$ is formed by one of the $bc$ intersections, at which six of the Higgs families $H_k$ are localised. The dominant terms provide diagonal Yukawa couplings with one Higgs generation per lepton family, whereas all area suppressed couplings providing lepton flavour mixing couplings, 

$$W_{E_i, L_k H_{H_k}} \sim O(1)$$

$$W_{E_i, L_k H_{H_{H_k}}'} \sim O(e^{-A_{(i)}/12})$$

$$W_{E_i, L_k H_{H_{H_k}}'} \sim O(e^{-A_{(i)}/12})$$

with $i = 1, 2, 3,$

$$W_{E_i, L_k H_k} \sim O(e^{-A_{(i)}}/6)$$

with $(i, j, k)$ permutations of $(1, 2, 3)$,
Since all worldsheets are spanned by triangles with edges $b$, $c$ and $(\theta d)$, the non-holomorphic prefactor 
\[ (K_{bc} K_{c(\theta d)} K_{b(\theta d)})^{-1/2} = g(S, U, A) \frac{1}{2(50)^{1/4}} \] with $g(S, U, A) \equiv \frac{(A_{i}(1) A_{i}(1) A_{i}(1))^{3/4}}{f(S, U)}$ and $\frac{1}{2(50)^{1/4}} \approx 0.188$

is universal in the lepton sector.

The quark Yukawa couplings display a different pattern since left- and right-handed quarks arise each in two sectors, $a b'$ and $a (\theta b')$ and $a c$ and $a (\theta^2 c)$, respectively. The $a c$ sector is exemplary for the exceptional situation that on fractional D6-branes chiral matter states can arise at one vanishing intersection angle along $T^3_{(1)}$. We therefore include the option of a triangle with zero angle and vanishing area, for which

\[ \frac{1}{2(50)^{1/4}} \] may require modification. The dominant Yukawa interactions involve only two right- and three left-handed quark generations $U_1$, $U_3$ and $Q_1$, $Q_3$, $Q_4$ plus three Higgs generations $H_1$, $H_4$, $H_7$, one of which also couples to the first lepton generation,

\[ W_{U_i Q_j H_k} \sim \left\{ \begin{array}{l}
\mathcal{O}(1) \\
\mathcal{O}(e^{-A_i(1)/4}) \\
\mathcal{O}(e^{-A_i(1)/12}) \\
\mathcal{O}(e^{-A_i(2)/6}) \\
\mathcal{O}(e^{-A_i(1)/4 - A_i(2)/12}) \\
\mathcal{O}(e^{-A_i(1)/4 - A_i(2)/6})
\end{array} \right. \quad \text{with} \quad (i, j, k) = (3, 1, 1), (3, 4, 7), (1, 3, 1), (1, 4, 4), (3, 1, 4), (3, 3, 7), (3, 2, l), (2, 3, 5 - l), (2, 4, 8 - l), (3, 4, 6 + l), (3, 2, 3 + l), (3, 3, 6 + l), \quad \text{with} \quad l \in \{2, 3\}. \]

Since quark Yukawa couplings arise from three different types of triangular worldsheets, distinct nonholomorphic prefactors arise, 
\[ (K_{a b'} K_{a(\theta b') c} K_{b(\theta c)})^{-1/2} = g(S, U, A) \frac{1}{2(50)^{1/4}} \] with $\frac{1}{2(50)^{1/4}} \approx 0.178$, 
\[ (K_{a(\theta b')} K_{a(\theta c)} K_{b(\theta c)})^{-1/2} = g(S, U, A) \frac{1}{2(50)^{1/4}} \] with $\frac{1}{2(50)^{1/4}} \approx 0.168$ and 
\[ (K_{a(\theta b')} K_{a c} K_{b(\theta c)})^{-1/2} = g(S, U, A) \frac{1}{2(50)^{1/4}} \] with $\frac{1}{2(50)^{1/4}} \approx 0.193$, which, however, are numerically all of the same order of magnitude as the one for the leptons, cf. also the last line of table for the very mild additional suppression by $A^{-1/4}$ in the last case.

### 2.4  Masses for vector-like matter

The vector-like matter spectrum can be decomposed into three types with different string theoretic origin and consequently distinct mechanisms by which they acquire masses \[19\]. The first kind consists of $\mathcal{N} = 2$ supersymmetric sectors with two D6-branes parallel along $T^3_{(2)}$, for which masses are provided by separations of the D6-branes. The matter states in brackets on the second and third line of equation \[2\] are of this type with $\sigma_6^2 \neq 0$ and $\sigma_7^2 \neq 0$, respectively, providing the masses.

A second type of vector-like matter arises at two distinct $\mathcal{N} = 1$ supersymmetric intersections such as the left-handed quarks $Q_j$ at $a(\theta^2 b')_{k \in \{0,1\}}$ and the conjugate $\bar{Q}$ at an $a(\theta^2 b')$ intersection in table \[3\]. The masses arise via couplings to SM singlets, e.g. those inside the adjoint representations $A_i$ of $U(3)_a$, \[a_{U(3)_a} = (8_{SU(3)_a})_0 + (1)_0,\]

\[ W_{\bar{Q}_i A_2} \sim \left\{ \begin{array}{l}
\mathcal{O}(1) \\
(e^{-A_i(1)/8}) \\
(e^{-A_i(1)/4}) \\
(e^{-A_i(1)/8 - A_i(2)/4})
\end{array} \right. \quad \text{with} \quad i = 4 \\
W_{\frac{1}{2} W_j A_2} \sim \left\{ \begin{array}{l}
\mathcal{O}(1) \\
(e^{-A_i(1)/8}) \\
(e^{-A_i(1)/4}) \\
(e^{-A_i(1)/8 - A_i(2)/4})
\end{array} \right. \quad \text{with} \quad j \in \{2, 3\} \quad \text{and} \quad (\hat{i}, j) = (1, 1), \]

which provide mass terms if $A_2$ receives a vev along some flat direction of the (trivial D-term) and F-term potential, see \[19\] for more details. The example on the r.h.s. shows the same mechanism for the
\((3_\text{Anti}, 1)_{1/3}\) representations \(W_j\) and their conjugates \(\overline{W_j}\). By combining several vevs of singlets inside the adjoints of \(U(3)_a\) and \(U(2)_b\), all states on the first line and the remaining charged states on the last line of (2) are rendered massive. The matter states with \(SU(2)_b \times U(1)_Y\) charge in the ‘hidden’ spectrum (3) acquire masses by a similar mechanism, namely if the \((15_\text{Anti})_0\) of \(USp(6)\) receives some vev.

The last type of vector-like matter consists of three lepton \((L^{3+i}, L^i)\) and nine Higgs \((H^u_i, H^d_i)\) pairs in equation (1), which are ‘chiral’ w.r.t. the anomalous \(U(1)_b\) symmetry. These states couple perturbatively to the symmetric representations \(b_i\) of \(U(2)_b\) or its conjugate \(b_i\) instead of the adjoints for the above mentioned vector-like quark pairs, e.g. the non-suppressed couplings read \(W_{\overline{b_i} L^3+i L^3}, W_{H^u_i b_i H^d_j} \sim O(1)\). Any vev will then lead to a gauge symmetry breaking \(SU(2)_b \rightarrow U(1)_b\), which might happen at an intermediary energy scale between \(M_{\text{weak}}\) and \(M_{\text{string}}\) [19]. Alternatively, D-brane instantons or higher order \(n\)-point couplings might provide mass terms without gauge symmetry breaking.

3 Conclusions

We presented an estimation of the relative order of magnitude of Yukawa couplings in the lepton and quark sector for the SM on \(T^6/\mathbb{Z}_6\) based on a scrutiny of the worldsheets with minimal areas. While the dominant terms for the leptons are flavour diagonal, mixings already occur at leading order in the quark sector. Our derivation of perturbative three-point couplings, which also includes masses for the vector-like matter states through Higgs-like couplings to Standard Model singlets, relies on methods developed for the six-torus. A thorough string theoretic derivation is needed to clarify if further stringy selection rules such as discrete symmetries restrict the sums over worldsheet areas and if a similar lattice sum occurs as well for the special case of one vanishing angle. Last but not least, higher order and D-instanton couplings are expected to modify the subleading behaviour.

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References

[1] F. Gmeiner, R. Blumenhagen, G. Honecker, D. Lüst and T. Weigand, JHEP 0601 (2006) 004
[2] F. Gmeiner, D. Lüst and M. Stein, JHEP 0705 (2007) 018
[3] F. Gmeiner and G. Honecker, JHEP 0807 (2008) 052
[4] D. Lüst and S. Stieberger, Fortsch. Phys. 55 (2007) 427
[5] N. Akerblom, R. Blumenhagen, D. Lüst and M. Schmidt-Sommerfeld, Phys. Lett. B 652 (2007) 53
[6] R. Blumenhagen and M. Schmidt-Sommerfeld, JHEP 0712 (2007) 072
[7] F. Gmeiner and G. Honecker, Nucl. Phys. B 829 (2010) 225
[8] D. Cremades, L. E. Ibáñez and F. Marchesano, JHEP 0307 (2003) 038
[9] D. Cremades, L. E. Ibáñez and F. Marchesano, JHEP 0405 (2004) 079
[10] D. Bailin and A. Love, Nucl. Phys. B 755 (2006) 79 [Nucl. Phys. B 783 (2007) 176]
[11] D. Bailin and A. Love, Phys. Lett. B 651 (2007) 324 [Erratum-ibid. B 658 (2008) 292]
[12] F. Gmeiner and G. Honecker, JHEP 0709 (2007) 128
[13] D. Bailin and A. Love, Nucl. Phys. B 809 (2009) 64
[14] R. Blumenhagen, L. Görlich and T. Ott, JHEP 0301 (2003) 021
[15] G. Honecker and T. Ott, Phys. Rev. D 70 (2004) 126010 [Erratum-ibid. D 71 (2005) 069902]
[16] G. Honecker, Mod. Phys. Lett. A 19 (2004) 1863
[17] R. Blumenhagen, M. Cvetič, F. Marchesano and G. Shiu, JHEP 0503 (2005) 050
[18] S. Förste and G. Honecker, JHEP 1101 (2011) 091
[19] G. Honecker and J. Vanhoof, arXiv:1201.3604 [hep-th].
[20] G. Honecker, arXiv:1109.3192 [hep-th].
[21] G. Honecker, arXiv:1109.6533 [hep-th].