Cosmic 21 cm fluctuations as a probe of fundamental physics

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Abstract. Fluctuations in high-redshift cosmic 21 cm radiation provide a new window for observing unconventional effects of high-energy physics in the primordial spectrum of density perturbations. In scenarios for which the initial state prior to inflation is modified at short distances, or for which deviations from scale invariance arise during the course of inflation, the cosmic 21 cm power spectrum can in principle provide more precise measurements of exotic effects on fundamentally different scales than corresponding observations of cosmic microwave background anisotropies.

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1. Introduction

The primary obstacle to studying fundamental physics experimentally is the difficulty of achieving sufficiently high energies in the laboratory. Conventional models of particle physics and string theory predict fundamentally new phenomena at energy scales of order $10^{16}$ GeV and above. A few observations accessible at low energies provide indirect clues about the underlying physics at these high scales, such as the long lifetime of the proton, small neutrino masses and, possibly, the cosmological constant. An additional rich source of data arises from observations of large-scale structure in the Universe.

In the standard model of inflationary cosmology, structure is seeded by primordial quantum fluctuations in the inflaton field that are stretched to super-horizon scales during $\gtrsim 60$ e-foldings of inflation, and subsequently collapse into the structure we observe in the Universe today. During inflation, while these perturbations were forming, the characteristic energy scale may have been as high as $H_i \sim 10^{14}$ GeV, which is far beyond any scale accessible to laboratory experiments. The cosmic evolution of these small perturbations is linear, and, if observed near the linear regime, they constitute a relatively direct window on physics at this extraordinarily high energy scale.

In addition to allowing a direct probe of the physics of inflation (see, e.g., [1]–[4]), this observation has led to the suggestion that the spectrum of temperature and polarization anisotropies in the cosmic microwave background (CMB) could be used as a probe of unconventional physics at scales at or above $H_i$ (see [5]–[14], and references therein). This approach is limited by several factors: the inflationary scale $H_i$, while high, is at best 1% of the fiducial scale of $M \sim 10^{16}$ GeV, and cosmic variance, coupled with the Silk damping of anisotropies due to photon diffusion, limits the theoretical precision of data taken from the CMB to approximately that same level.

Although its potential as a cosmological probe has long been known [15]–[18], there has recently been renewed interest in using the 21 cm hyperfine ‘spin-flip’ transition of
Figure 1. The scales probed by cosmic microwave background anisotropies (solid line) and cosmic 21 cm fluctuations (dashed line). The two power spectra have been aligned using the small-scale relation $k \simeq l/d_A(z_{CMB})$, where $d_A(z_{CMB}) \simeq 13.6$ Gpc is the comoving angular diameter distance at the surface of recombination in the standard cosmological model.

neutral hydrogen as a probe of the cosmic dark ages [19]–[35]5. As we will see, these 21 cm observations have the potential to sidestep the limitations of the CMB and provide a powerful technique for studying the Universe during and before inflation. There are two reasons for this. First, the data probes a different and complementary range of scales relative to the CMB at an epoch when these scales are essentially linear (see figure 1). Second, the quantity of data available is so large (cosmic 21 cm fluctuations probe a volume rather than a surface, down to much smaller scales than CMB anisotropies) that the in-principle cosmic variance limit on the precision improves dramatically (see section 3).

5 See [35] for a comprehensive review of 21 cm physics, astrophysics, and phenomenology.
Here, we identify two major categories of fundamental physics effects that could change the power spectrum of perturbations observable via cosmic 21 cm fluctuations. The first class is \textit{initial state effects}. At the beginning of inflation, when the expansion of the Universe first began to accelerate, there is no reason to expect that the initial Hubble patch was close to homogeneous; instead, the details of this initial configuration may provide important clues for understanding the origin of the Universe, the nature of the big bang, and perhaps even cosmological quantum gravity. However, significant periods of inflation greatly reduce the signatures of the initial state, producing a flat and uniform Universe in the present. The effects of a non-homogeneous initial state are therefore most significant shortly after the beginning of inflation, meaning that they are most easily visible today on very large scales. On the other hand, it is on the largest scales that cosmic variance places the most significant restrictions on our ability to determine (even in principle) the spectrum of perturbations. As we will see, this trade-off means that certain classes of initial states are more easily observed (or in some cases are only observable) at the shorter scales accessible in 21 cm data.

The second category addresses effects that occur \textit{dynamically} throughout, or at some point during, the period of inflation. These effects do not ‘infl ate away’, and may occur at any time during inflation (and therefore affect any scale in the present). Examples in this category include quantum corrections to inflaton dynamics (such as wavefunction renormalizations from integrating out heavy fields) \cite{12}, a field going on resonance and producing particles \cite{36}, a sharp feature in the inflaton potential \cite{37,38}, and a myriad of other exotic possibilities. Observations of cosmic 21 cm fluctuations are generally superior to CMB data for probing this class of effects: they can provide much greater precision due to the greater amount of data available, and they can probe a longer ‘lever arm’ of data over scales inaccessible to CMB observations (or other probes of the matter power spectrum, such as galaxy surveys).

Our paper is structured as follows. We first briefly review potential effects of high-scale physics on the spectrum of density perturbations in the Universe today. We then review the physics of 21 cm fluctuations from the perspective of dynamics in the early Universe. Finally, we connect the two and discuss possible modifications to the predicted power spectrum of high-redshift cosmic 21 cm radiation due to new physics at high scales.

\section{2. High-scale physics and inflation}

As discussed in the introduction, we group the effects of high-scale physics into two categories: initial state modifications and corrections to inflaton dynamics.

\subsection{2.1. Initial state}

Given a scalar field evolving in a sufficiently flat potential, a region of space will begin to inflate if it is close to homogeneous over a region roughly the size of a few Hubble volumes \cite{39}. The state of the inflaton and any other relevant fields during this time constitutes the initial state for inflation. As the expansion proceeds, spatial gradients in these fields will stretch and inflate away, particles and other impurities will dilute, and the space will rapidly approach an approximate de Sitter phase, a state well described by the Bunch–Davies vacuum \cite{40}. Density perturbations in the initial state will be stretched by
inflation to scales that are large in the present, so the effects of initial inhomogeneities will be most apparent on the largest scales. However, if for some reason the perturbations in the initial state had a ‘blue’ spectrum (one with increasing power at large $k$) with significant power on small scales (relative to the inflationary Hubble length), the imprint could extend down to scales significantly smaller than the Hubble length today.

To make this more quantitative, we will follow the treatment of [41]. We define the Bunch–Davies vacuum state $|0\rangle$ by $\hat{a}_k|0\rangle = 0$, where $\hat{a}_k$ annihilates a mode $u_k$ with momentum $k$. An arbitrary initial state $|0\rangle_b$ in the same Fock space can be defined as

$$\hat{b}_k|0\rangle_b = 0,$$

where $\hat{b}_k$ is the annihilation operator for modes $v_k$ defined by

$$v_k = \alpha_k u_k + \beta_k u_k^*$$.  

As usual, $\alpha_k$ and $\beta_k$ satisfy $|\alpha_k|^2 = |\beta_k|^2 + 1$.

The two-point function for the inflaton in the late-time limit is easily computed [41]:

$$\langle 0_b|\phi_k|^2|0_b\rangle = \frac{H^2}{2k^3}(1 + 2|\beta_k|^2 + 2|\beta_k|\sqrt{1 + |\beta_k|^2}\cos \varphi_k),$$

where $\varphi_k = \arg(\alpha_k) - \arg(\beta_k)$.

This modification can naively be made arbitrarily large by increasing $|\beta_k|$. However, if the causal region is to inflate at all, the energy density in the perturbations must be subdominant compared to the vacuum energy contribution from the scalar potential. This leads to an integrated constraint on the occupation numbers $|\beta_k|^2$ that must be satisfied at the beginning of inflation.

The expectation value of the energy density (normalized with respect to the Bunch–Davies vacuum and time-averaged) is approximately [41]

$$\langle 0_b|(-T_0^0)|0_b\rangle = (H_i\eta)^4 \int \frac{d^3k}{(2\pi)^3}k|\beta_k|^2,$$

$$\langle 0_b|(-T_0^0)|0_b\rangle = \int \frac{d^3p}{(2\pi)^3}p n_p,$$

where $\eta$ is the conformal time, $p = k/a$ is the physical momentum, and the occupation number $n_p$ is given by $n_p = |\beta_k|^2$. We thus find that this energy density must be less than the vacuum energy $3M_{Pl}^2H_i^2$ for the patch to begin inflating. However, as this is an integral constraint, there are a large set of initial states for which it is satisfied, and it is apparent that some of the energy density can reside in short wavelength modes. Such initial states may be visible in the fluctuation spectrum of cosmic 21 cm radiation. We emphasize, however, that inflation must have been relatively short for these effects to be potentially visible. Longer inflationary periods will push the effects out to very large scales, meaning that only very short wavelength initial perturbations will be inside our horizon today. However, larger $k$ requires smaller $|\beta_k|$ to satisfy equation (4), implying a smaller overall effect on the spectrum (see equation (3)).

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6 Here, $M_{Pl} = \sqrt{\hbar c/8\pi G_N}$ is the reduced Planck mass.
If the phases in the initial state are random, the cosine term in equation (3) will average to zero, and the effect on the perturbation spectrum will be $O(\beta^2_k)$. However, the effect can be enhanced if the phases of the $k$-modes in the initial state are chosen such that $\varphi_k$ in equation (3) are either constant or slowly varying with $k$, in which case the effect on the perturbation spectrum will be $O(\beta_k)$ (for $\beta_k \ll 1$). Such carefully chosen phases can imprint characteristic oscillation patterns on the perturbation spectrum (see, e.g., [14]).

It is important to stress that, in general, $\beta_k$ and $\varphi_k$ are functions of the vector $k$, and not just $k = |k|$. In other words, because it is fixed by physical processes prior to inflation, the initial state need not be isotropic or random. The potential anisotropy or structure in the initial state may be an important signature of the underlying physics, and this should be kept in mind when considering how such signatures arise in the (otherwise isotropic and random) power spectrum of inflationary fluctuations.

At present we have no compelling reason to claim that any particular initial state is preferred; we simply aim to point out the possibility of studying the initial state via cosmic 21 cm observations. It is worth noting here that there are many models in the literature that predict both special initial conditions and short inflation. In the string theoretical scenarios [42, 43], long inflation requires fine-tuning beyond that required for vacuum selection. The generic expectation is therefore that inflation will be relatively short, although determining precisely how short may be difficult. Furthermore, if the landscape is populated by tunneling, the initial conditions at the big bang are quite special, and precision observations at large scales may even contain some information about neighboring minima [44, 45].

### 2.2. Dynamics

There are many possible effects on inflaton physics that can create interesting features in the spectrum of density perturbations during inflation. Some examples include abrupt changes in the inflaton potential [37], and couplings to other particles that result in resonant production [36]. Another example involves the direct effect of high-scale physics on inflaton dynamics. Such effects occur when integrating out massive fields coupled to the inflaton gives rise to wavefunction renormalizations [12]. Because these features can occur at any time during inflation, the CMB alone provides a limited observational window, and is hindered by the constraints of cosmic variance. Cosmic 21 cm fluctuations would therefore provide access to a different part of the history of inflation, and with much greater precision. This is illustrated in figure 1, where both the CMB and cosmic 21 cm power spectra are shown on the same figure. While the power in the CMB anisotropies drops off precipitously above $l \approx 1000$ ($k \approx 0.07 \text{ Mpc}^{-1}$), the cosmic 21 cm fluctuations continue to grow, peaking near $k \approx 100 \text{ Mpc}^{-1}$.

There are many more exotic possibilities that have appeared in the literature that roughly fit into this category of effects. For instance, if the constraint in equation (4) can be avoided (for example, by using a mechanism along the lines of [46, 47]), there is the possibility of extending the initial state perturbations up to arbitrarily high frequency scales, rendering them visible throughout inflation. Examples of this are the de Sitter $\alpha$-vacua [48, 49], which, due to the fact they are de Sitter invariant, do not inflate away (and cannot be regarded as a perturbation above the Bunch–Davies state). While we do
not regard this possibility as plausible per se [48, 50, 51], we point out that cosmic 21 cm fluctuations could significantly constrain these and other related ideas.

3. Cosmic 21 cm fluctuations

Following the formation of the first atoms at \( z \sim 1000 \), the Universe became essentially neutral and transparent to photons. The tiny residual population of free electrons was able, via Compton scattering, to couple the temperature of cosmic gas \( T_g \) to the CMB temperature \( T_\gamma \) until redshift \( z \sim 200 \). At this point the cosmic gas began to cool adiabatically as \( T_g \propto (1 + z)^2 \) relative to the CMB, which redshifts as \( T_\gamma \propto (1 + z) \).

During this epoch, the hyperfine spin state of the gas relevant to the 21 cm transition is determined by two competing processes: radiative interactions with CMB photons at \( \lambda_{21} = 21.1 \) cm, and spin-changing atomic collisions. Conventionally, the fraction of atoms in the excited (triplet) state versus the ground (singlet) state,

\[
\frac{n_1}{n_0} = 3e^{-T_\gamma/T_*},
\]

is characterized by the spin temperature \( T_s \).\(^7\) Here, \( T_* = hc/\lambda_{21}k_B = 68.2 \) mK is the energy of the 21 cm transition in temperature units, and the factor of \( g_1/g_0 = 3 \) accounts for the degeneracy of the triplet state.

Spin-changing collisions efficiently couple \( T_s \) to \( T_g \) until \( z \sim 80 \), at which point \( T_s \) rises relative to \( T_g \), becoming equal to \( T_\gamma \) by \( z \sim 20 \). There is thus a window between \( z \sim 200 \) and \( z \sim 20 \) where \( T_s < T_\gamma \), and fluctuations in the density and bulk velocity of neutral hydrogen may be seen in the absorption of redshifted 21 cm photons.

Measurements of these fluctuations may be a powerful probe of the matter power spectrum [19]. The observable quantity is the brightness temperature

\[
T_b(\hat{r},z) = \frac{3\lambda_{21}^3AT_*}{32\pi(1+z)^2(\partial V_r/\partial r)^n_H} \left( 1 - \frac{T_\gamma}{T_s} \right),
\]

measured in a radial direction \( \hat{r} \) at redshift \( z \) (corresponding to 21 cm radiation observed at frequency \( \nu = c/\lambda_{21}(1+z) \)). Here, \( A \) is the Einstein spontaneous emission coefficient for the 21 cm transition, \( V_r \) is the physical velocity in the radial direction (including both the Hubble flow and the peculiar velocity of the gas \( v \)), and \( \partial V_r/\partial r \) is the velocity gradient in the radial direction. Explicitly, we have

\[
\frac{\partial V_r}{\partial r} = \frac{H(z)}{1+z} + \frac{\partial(v \cdot \hat{r})}{\partial r}.
\]

Combining equations (7) and (8) and expanding to linear order, we find

\[
\delta T_b = -T_b \frac{1+z}{H(z)} \frac{\partial v_r}{\partial r} + \frac{\partial T_b}{\partial \delta},
\]

where \( \overline{T}_b \) is the mean brightness temperature, \( v_r = v \cdot \hat{r} \) is the peculiar velocity in the radial direction, and \( \delta = (n_H - \overline{n}_H)/\overline{n}_H \) is the overdensity of the gas.

\(^7\) The single spin temperature description outlined here is only an approximation and, in fact, the full spin-resolved distribution function of hydrogen atoms computed in [32] should be used when computing 21 cm fluctuations in detail.
Moving to Fourier space, we find
\[ \delta \tilde{T}_b = -T_b \frac{1+z}{H(z)} \mu^2 (ik\tilde{v}) + \frac{\partial T_b}{\partial \delta} \delta, \]
(10)
\[ \delta \tilde{T}_b = \overline{T}_b [\mu^2 + \xi] \delta, \]
(11)
where \( \mu = \hat{k} \cdot \hat{r} = \cos \theta_k \) is the cosine of the angle between the radial direction and the direction of the wavevector \( k \), and \( \xi \) is defined by \( \xi \equiv (1/\overline{T}_b)(\partial T_b/\partial \delta) \). The second line, equation (11), uses the additional relation \( \delta = -(ik\tilde{v})(1+z)/H \), which is, strictly speaking, valid on scales larger than the Jean’s length during the matter dominated epoch. The total brightness–temperature power spectrum is thus \([32,52]\)
\[ \langle \delta \tilde{T}_b(k) \delta \tilde{T}_b(k') \rangle = (2\pi)^3 \delta^{(3)}(k + k') P_{\tilde{T}_b}(k), \]
(12)
where
\[ P_{\tilde{T}_b}(k) = (\overline{T}_b)^2 [\xi^2 + 2 \xi \mu^2 + \mu^4] \left| \Phi(k\mu) \right|^2 P_{\delta\delta}(k). \]
(13)
Here, \( \Phi(k\mu) \) is a cut-off in the magnitude of the radial wavevector \( |k| = k\mu \), and is given in detail by the Fourier transform of the 21 cm line profile \([32]\). For our estimates we use a simple Gaussian form \( \Phi(k\mu) = e^{-\mu^2 k^2/(2k_\sigma^2)} \), with \( k_\sigma \approx 500 \text{ Mpc}^{-1} \) to approximate the small-scale cut-off due to the atomic velocity distribution found in \([32]\). After averaging over \( \mu \), we find
\[ P_{\tilb}(k) = (\overline{T}_b)^2 [\xi^2 \mathcal{E}_0(k) + 2 \xi \mathcal{E}_2(k) + \mathcal{E}_4(k)] P_{\delta\delta}(k), \]
(14)
where \( \mathcal{E}_j \to 1/(j+1) \) as \( k \to 0 \), and \( \mathcal{E}_j \to 0 \) for \( k \ll k_\sigma \) (see the appendix for further details). For \( k < k_\sigma \), we see that the power spectrum of fluctuations in \( T_b \) is related in a simple way to the power spectrum of fluctuations of the hydrogen density field.

Since primordial fluctuations in the inflaton field ultimately result in the fluctuations in baryon density and velocity that generate high-redshift cosmic 21 cm fluctuations, we can use the spectra of cosmic 21 cm fluctuations to study the types of high-scale modifications discussed above. To account for these effects in the present context, we apply the corrections to the primordial power spectrum from, for example, equation (3) to the power spectrum of the cosmic hydrogen field \( P_{\delta\delta}(k) \).

Before discussing this, we wish to estimate the theoretical precision of such data. As emphasized in \([19]\), cosmic 21 cm data correspond to a three-dimensional volume rather than the two-dimensional last scattering surface of the CMB\(^8\). Furthermore, the scales involved are much smaller, implying a vastly greater number of potential data points. If measurements are limited by cosmic variance, the minimum relative precision using all available modes would be \( N_{21}^{-1/2} \approx (\Delta r_{\text{com}}/r_{\text{com}})^{-1/2} l_{\text{max}}^{-3/2} \sim 10^{-8} \) for \( \Delta r_{\text{com}}/r_{\text{com}} \sim 1/20 \) and \( l_{\text{max}} \approx k_{\max} d_L \sim 10^6 \) \([19]\). This should be contrasted with the corresponding cosmic variance limit for the CMB, which is roughly \( N_{\text{CMB}}^{-1/2} \approx 2^{1/2} l_{\text{max}}^{-1} \sim 2 \times 10^{-4} \) for \( l_{\text{max}} \sim 3000 \). We note that, while high-redshift 21 cm fluctuations are essentially described by linear physics, nonlinear corrections due to mode coupling must be included

\(^8\) See \([53]\) for another interesting method to probe our observable volume and partially circumvent cosmic variance.
if the relative precision of measurements is pushed beyond the accuracy of linear theory.\(^9\)

The presence of foreground signals is unavoidable, and will reduce the number of modes that can be measured with a high signal-to-noise ratio. If they are smooth in frequency space, however, such nuisance signals can likely be removed using techniques that will not inhibit measurements of the small-scale 21 cm fluctuations [55].

A particularly simple type of high-scale modification of the primordial spectrum arises in the effective field theory that results from integrating out massive fields that couple to the inflaton. This will affect the power spectrum throughout inflation, so it belongs to the second category of effects discussed in section 2. The size of the effects will be \(\mathcal{O}(H_i^2/M^2)\), where \(M\) is the mass scale of the heavy fields [50]. Due to the increased precision mentioned above, such effects are much easier to see (and the range of masses \(M\) that can be probed is much larger) using 21 cm data rather than corresponding observations of the CMB. More exotic models (which cannot be described by effective field theory) predict modifications of order \(H_i/M\) (see, e.g., [9,11,56] and references therein), allowing an even greater range of \(M\).

We note that, absent an independent measurement of the inflationary Hubble scale or direct knowledge of the inflaton potential [50], this type of effect is difficult or impossible to distinguish from a modification of the inflaton potential itself. However, if additional information about the inflaton potential can be determined, for example, by the detection of inflationary gravitational waves, then it may be possible to use cosmic 21 cm fluctuations to probe the particle spectrum above the inflationary energy scale.

Another possibility is the observation of initial state effects. Naively, we expect such effects to be most detectable on the largest scales today, and the possibility of any detection at all assumes a short inflationary period. However, depending on the spectrum of perturbations in the initial state, the effects may be more visible either in the CMB or in cosmic 21 cm fluctuations.

As a simple example, consider an initial state with occupation numbers \(n_p = \beta^2\), defined to be constant up to a physical cut-off \(p_{\text{max}}\), and \(n_p = 0\) for \(p > p_{\text{max}}\). For this state, the constraint in equation (4) implies that \(p_{\text{max}}^4|\beta|^2 \ll 24\pi^2 H_i^2 M_{\text{Pl}}^2\). For purposes of illustration, we demand here (and for the other initial states we consider) that the energy density in the initial state at the start of inflation (which redshifts as \(a^{-4}\)) is 10% of the energy density in the inflaton potential. With minimal inflation, such that \(p_{\text{max}}/H_i = k_{\text{max}}/H_0\), and choosing \(k_{\text{max}} = 4\) Mpc\(^{-1}\), we find that \(\beta^2 \approx 2.7 \times 10^{-4}\) for \(H_i/M_{\text{Pl}} \sim 10^{-6}\).

If the phases of the initial state are correlated in such a way that \(\cos \varphi_k \sim 1\) in equation (3), the resulting effect on the perturbation spectrum appears at the level \(2\beta \sim 0.03\) for all \(k \lesssim 4\) Mpc\(^{-1}\). Alternatively, if the phases of the initial state are completely random (such that \(\cos \varphi_k \to 0\)), then an identical effect on the power spectrum is produced if the Hubble scale during inflation is lowered to \(\tilde{H}_i\), such that \(\tilde{H}_i/M_{\text{Pl}} \sim 1.3 \times 10^{-7}\) and \(2\beta^2 \sim 0.03\). Cases with equivalent and constant \(|\beta_k|\) are shown in figure 2. While such an effect would be difficult or impossible to see in the CMB alone, it should be apparent with a clean measurement of cosmic 21 cm fluctuations.\(^10\)

\(^9\) A proposal for calculating nonlinear corrections to the statistics of the dark-matter density field in a controlled manner was discussed in [54].

\(^10\) Relative to this spectrum of initial perturbations, a blue spectrum would be easier to see using the 21 cm data, while a red spectrum would be more difficult.
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Figure 2. Left panel: 21 cm power spectra with high-energy modifications due to a remnant feature from the initial state of the Universe before inflation. Shown are the effects from an initial state with a sharp feature centered around $k = 4 \, \text{Mpc}^{-1}$ (solid line), and an initial state with all modes $k < 4 \, \text{Mpc}^{-1}$ populated with a small amplitude (dotted line). The energy density in the initial state at the start of inflation is assumed to be $10\%$ of the energy density in the inflaton potential. For correlated $\varphi_k$, these effects would occur at the level shown if $H_i/M_{\text{Pl}} \sim 10^{-6}$, while for random $\varphi_k$ they would occur at the level shown if $H_i/M_{\text{Pl}} \sim 1.3 \times 10^{-7}$. Right panel: the same curves enlarged to magnify the region around the spectral features.

Another example, also shown in figure 2, is an initial state with a narrow spectral line centered at a comoving wavevector $k \approx 4 \, \text{Mpc}^{-1}$. This effect is shown with $H_i/M_{\text{Pl}} \sim 10^{-6}$, and with correlated phases $\varphi_k$, where the modification occurs at $O(\beta)$. A similar effect can occur with random phases $\varphi_k$, with $H_i/M_{\text{Pl}} \sim 1.3 \times 10^{-7}$, were the modification appears at $O(\beta^2)$. For simplicity, we have focused on initial states that are isotropic in $k$ (depending only on $k = |k|$), which amount to simple modifications to homogeneous and isotropic power spectra. We stress that this condition can be relaxed, and doing so may provide an additional avenue for studying initial state effects. In all the cases discussed here, modifications to the standard spectrum of 21 cm fluctuations appear in a range inaccessible to observations of the CMB.

4. Conclusions

There is a large window in momentum space over which potential signals of fundamental high-energy (perhaps quantum-gravitational) effects are invisible in CMB temperature anisotropies, but should be apparent in the spectrum of 21 cm fluctuations. From the analysis presented here, one concludes that this range runs from roughly $k \sim 0.1 \, \text{Mpc}^{-1}$, where the CMB spectrum becomes strongly suppressed, to $k \sim 1000 \, \text{Mpc}^{-1}$, where the cosmic 21 cm spectrum is suppressed (due to atomic velocities and the inhibition of growth below the Jean’s length of the primordial gas).
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At the moment, the study of high-scale physics effects on the primordial perturbation spectrum is in its infancy. Although tantalizing hints have emerged, we are unable to say with confidence what a generic initial state is, or how high-scale physics might affect inflaton dynamics. As our knowledge improves, it will be valuable to keep in mind that there may be effects that lie outside the regime of CMB observations, but where detection using 21 cm observations may be possible.

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Appendix. The small-scale cut-off

Cosmic 21 cm fluctuations in the radial direction will have an unavoidable small-scale cut-off due to the finite velocity distribution of hydrogen atoms [32]. In this appendix we calculate the anisotropy-averaged power spectrum under the approximation that the radial window function \( \tilde{\Phi}(k\mu) \) is a Gaussian.

This is approximately the power spectrum that would be seen if cosmic 21 cm fluctuations in a redshift range \( \Delta z \) (or range of comoving radius \( \Delta r_{\text{com}} = (\partial r_{\text{com}}/\partial z)\Delta z \)) about a central redshift \( z^* \) were observed. In more detailed calculations, one should use the actual radial window functions found in [32], based on the non-Gaussian shape of the 21 cm line profile, and corrections for the evolution of 21 cm brightness–temperature fluctuations (\( T_b(z), \xi(z), \) and \( \tilde{\delta} \propto 1/(1 + z) \)) across the redshift range \( \Delta z \) should be included.

Starting from equation (13), we want to calculate the anisotropy-averaged power spectrum

\[
P_{\text{T}_b}(k) = \int_{-1}^{1} \frac{d\mu}{2} P_{\text{T}_b}(k).
\]

(A.1)

Defining the moments

\[
E_{2m}(k) = \int_{-1}^{1} \frac{d\mu}{2} \mu^{2m} |\tilde{\Phi}(k\mu)|^2,
\]

(A.2)

we directly obtain equation (14) shown above. Specializing to the Gaussian case

\[
\tilde{\Phi}(k\mu) = e^{-\mu^2 k^2/(2 k^2\sigma)},
\]

(A.3)

(with \( k\sigma \simeq 500 \text{ Mpc}^{-1} \)) we have

\[
E_{2m}(k) = \int_{-1}^{1} \frac{d\mu}{2} \mu^{2m} e^{-\mu^2 k^2/k^2\sigma}.
\]

(A.4)
In this case we have
\[ \mathcal{E}_{2m}(k) = \left( -\frac{k^2}{2k} \frac{\partial}{\partial k} \right)^m \mathcal{E}_0(k), \]  
(A.5)
where
\[ \mathcal{E}_0(k) = \frac{\sqrt{\pi k_\sigma}}{2k} \text{erf} \left( \frac{k}{k_\sigma} \right) \]  
(A.6)
is evaluated in terms of the error function erf(z). Explicitly, \( \mathcal{E}_2(k) \) and \( \mathcal{E}_4(k) \) appear as
\[ \mathcal{E}_2(k) = \frac{k_\sigma^2}{2k^2} \mathcal{E}_0(k) - \frac{k_\sigma^2}{2k^2} e^{-k^2/k_\sigma^2}, \]  
(A.7)
and
\[ \mathcal{E}_4(k) = \frac{3k_\sigma^4}{4k^4} \mathcal{E}_0(k) - \left( \frac{3k_\sigma^4}{4k^4} + \frac{k_\sigma^2}{2k^2} \right) e^{-k^2/k_\sigma^2}. \]  
(A.8)

On large scales \( k \ll k_\sigma \) we have
\[ \mathcal{E}_0(k) \simeq 1 - \frac{k^2}{3k_\sigma^2} + \frac{k^4}{10k_\sigma^4} + o(k^6), \]  
(A.9)
\[ \mathcal{E}_2(k) \simeq \frac{1}{3} - \frac{k^2}{5k_\sigma^2} + \frac{k^4}{14k_\sigma^4} + o(k^6), \]  
(A.10)
\[ \mathcal{E}_4(k) \simeq \frac{1}{5} - \frac{k^2}{7k_\sigma^2} + \frac{k^4}{18k_\sigma^4} + o(k^6). \]  
(A.11)

On small scales \( k \gg 3k_\sigma \) we have
\[ \mathcal{E}_0(k) \simeq \frac{\sqrt{\pi k_\sigma}}{2k}, \]  
(A.12)
\[ \mathcal{E}_2(k) \simeq \frac{\sqrt{\pi k_\sigma^3}}{4k^3}, \]  
(A.13)
\[ \mathcal{E}_4(k) \simeq \frac{3\sqrt{\pi k_\sigma^5}}{8k^5}. \]  
(A.14)

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