Particles, Fields, Pomerons and Beyond

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Abstract

Summary

This paper is a set of musings on what particles really are – something one all too often as a particle physicist assumes is pretty well-established. The initial motivation for these thoughts comes from a question that I always ask Alberto Santoro whenever I see him which is “What exactly is a pomeron?” I argue that the concept of a particle that we normally have is really quite far from reality and that there could be deep physics in reconsidering very carefully exactly what we mean by particles. Perhaps one of the great coming challenges is not simply to “find more particles and measure their couplings” but to revisit the very concept itself of a particle, and that a good place to do this may well be very strongly interacting theories like QCD and in very forward scattering and the study of objects like pomerons.

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I. PARTICLES AND SYMMETRY

When I tell students about particles I tell them that they should start by thinking of
classical objects that they know, for surely the notion of a “particle” in physics should be
guided by this sort of intuition. For example, consider an equilateral triangle. I draw one
on the board and then draw a second one some distance away and ask if they’re the same.
Certainly they’re not, for no matter how exactly I draw them, even in a hypothetically
perfect way, one is here while the other is there. Quickly we decide that we want to consider
the triangles to be the same even if they are in different places, and also that we want
them to be the same even if they are rotated relative to one another. The requirement
that these objects here vs. there and rotated or not all be considered the same stems from
the apparently independence of the laws of physics under translations and rotations, or the
Euclidean group ISO(2) which is the semidirect product of SO(2) and the translation group.
We think of the presence of one triangle as not affecting another (they are non-interacting)
so we look for linear representations of this group and now identify objects (on which we
base our ideas of particles) with representations of ISO(2). Single objects are identified
with irreducible representations of ISO(2) and if we were to think of ISO(2) as acting on a
Hilbert space and conserving probability (rotating or translating a triangle should not make
one disappear!) then one has to find unitary irreducible representations of ISO(2).

Going to 3 space dimensions and 1 of time the same sort of argument leads us to consider a
particle like an electron as a unitary irreducible representation of the inhomogenous Lorentz
group ISO(1, 3) or “Poincaré group” – after all, an electron here is also an electron if it’s
there and rotating an electron or boosting it should not make it into a different particle.
Here the group theory is not trivial to work out, but leads to a classification which we all
know which is that there are several different types of representation possible depending on
what group preserves the momentum of a particle and particles are essentially classified by
mass and spin. At the level of group theory one then writes down wave equations which are
supposed to correspond to particles, but the actual route from wave equation to particle is
quite nontrivial.

Consider the simplest case of a scalar (spin $s = 0$) field of mass $m$. The relativistic
constraint on energy is $p^2 - m^2 = 0$. Interpreting this as an operator equation with $p_\mu =
-ih\frac{\partial}{\partial x_\mu}$ we get the Klein Gordon equation $(\Box^2 - m^2)\phi(x) = 0$. This is a wave equation
and certainly does not describe particles as it stands. It is also important to note that \( x \) does not refer to any sort of “location” for a particle. Rather, it is an infinite-dimensional (continuous) index on which the Poincaré group can act. (Noncompact groups such as the Poincaré group have no nontrivial unitary finite-dimensional representations.) Perhaps nowhere more clearly does one see this when one considers the action functional for \( \phi \) from which the wave equation can be derived, in which \( x \) is a dummy index integrated over just as much as a contracted Lorentz index are summed over. Whatever \( x \) is, by this stage it certainly is not the spacetime coordinates of anything at all!

The textbooks then go on to expand \( \phi \) in terms of plane waves of the form

\[
\phi = \sum_k [a_k \exp(i \vec{k} \cdot \vec{r} - i\omega t) + a_k^\dagger \exp(-i \vec{k} \cdot \vec{r} + i\omega t)]
\]

where the sum runs over all the allowed wavenumbers \( k \) which satisfy whatever boundary conditions are imposed on the field and \( a \) and \( a^\dagger \) have the usual commutation relations which come from quantizing a simple harmonic oscillator. Note that if we want to give the particle some definite energy and momentum (as might seem reasonable), the Heisenberg uncertainty principle forces us to give up any idea of where it actually is. Later we will look more at the issue of localizability and just how much you can try to capture the notion of where a particle is, but for now let’s stay close to the textbook treatments and see what happens.

One introduces a vacuum state \( |0 \rangle \) defined by

\[
a_k \langle 0 | = 0
\]

for any \( \vec{k} \) and from this constructs states

\[
|n_k \rangle = \frac{1}{\sqrt{n}} (a_k^\dagger)^n |0 \rangle
\]

which contain \( n \) quanta of \( \phi \) with wavevector \( \vec{k} \) with a total energy given by the sum of energies of individual excitations.

Note that the distinctive feature of the quantum simple harmonic oscillator is that energies are equally spaced which means that \( n \) particles of energy \( E \) correspond to a total energy \( nE \). There is no possibility of interaction here, since there is no possibility of an interaction energy. As an approximation, one could only hope that this might work consistently for very
weakly coupled particles, but there are severe logical problems in trying to make this make sense with any degree of rigour. Perhaps the simplest way to see this is that noninteracting particles would, by definition, never be detectable. You could also never make any kind of bound state. Bound states are, by definition, nonperturbative objects so it’s clear that even if one wants to base a theory on “almost noninteracting” particles, one cannot hope for perturbation theory to capture important qualitative elements of physics – even for the electromagnetic interaction there are no hydrogen atoms in perturbation theory with plane waves!

There are even more problems with this picture right from the outset. First of all, one could well ask what motivated the expansion in equation 1 instead of some other modes $\psi_i$ instead of plane waves – why would

$$\phi = \sum_i a_i \psi_i + a_i^\dagger \psi_i^*$$

(4)

and

$$a_i |0 >= 0 \quad \forall i$$

(5)

not be acceptable? The fact is, there’s nothing in principle wrong with it and the expansion in 1 is really motivated by the belief that the chosen basis is singled out by the fact that the background spacetime is usually, to a very good degree, flat Minkowski space. In the general case of a curved spacetime this argument does not apply and very different notions of particles can appear [1], but there are already problems if one simply removes the restriction of comparing measurements in different inertial frames.

Amazingly, even equation 3 is in general no longer true if one simply moves to an accelerated reference frame. For a very long time it seems that most people simply assumed that the concept of a particle being present or not was an invariant one – surely one either has a particle or one does not and simply changing one’s system of reference should have no effect. In fact, if one takes the modes in equation 4 to be those of equation 1 after moving to an accelerated frame, then one finds that the creation and annihilation operators in the two expansions get mixed (via a “Bogoliubov transformation”) and what was the vacuum for the initial inertial observer is seen by the accelerated one as having a thermal distribution of particles with a temperate which is proportional to the acceleration! An appeal the the
equivalence principle also makes this a good way to get an intuitive feeling for the origin of Hawking radiation.

Before one claims that this may well be true, but is really just academic since an acceleration of $10^{20} \text{m/s}^2$ is needed just to get to a temperature of about 1K, let us make an estimate of the acceleration suffered by a proton during diffraction in the LHC – the sort of process which I know is close to Alberto’s heart. On dimensional grounds with $\ell$ an length and $t$ a time we could put $\ell/t^2 = (\ell/t)^2 (1/\ell) \approx c^2 / 1 \text{fermi}$ which corresponds to $9 \times 10^{16} / 10^{-15} \text{m/s}$ or about $10^{32} \text{m/s}^2$! It could well be that there’s some interesting physics to be seen by looking at the extreme accelerations of protons that don’t shatter! Note also that these issues do not arise very clearly in the traditional pictures of perturbative quantum field theory where a proton in some momentum state is “annihilated” while another in some other momentum state is “created”, with strict Lorentz invariance (only inertial frames) in place at all times.

Indeed, even the mass of a particle is different when it is accelerated. Ritus\cite{2} has shown that an electron has a shifted mass in a homogeneous constant electric field by an amount proportional to the field. The mass shift is larger for smaller mass particles, but in the next section we will see an example where large mass shifts can be obtained for heavy particles due to Higgs field (not particle!) effects.

II. FIELDS WITHOUT PARTICLES

I think it’s important to keep in mind that the particle concept in quantum field theory is really attached to a particular mode expansion of the field – but sometimes particles aren’t the right way to think about a field. One of my favourite recent results\cite{3} came about after a long time thinking about whether or not one could detect the Higgs field without actually making a Higgs particle. The intuition was based on realizing that one can detect electric fields without actually being able to detect individual photons – just run a comb through your hair on a dry day and you can pick up bits of paper with it. In this way, you detect the electric field, but not photons \textit{per se}. Similarly, could one look for the Higgs field without making a Higgs particle? If so, you could infer that there would be “particle” states, but you wouldn’t have to actually \textit{make} them.

What does the Higgs field do? It gives mass to things. Massive things get their masses from Yukawa couplings to a background Higgs field, but they are also \textit{sources} for the Higgs
field. If you put something next to a big mass, would you see its mass shift now that it interacts with a Higgs field which is that in the vacuum plus that due to another particle nearby?

A couple of years (and rather large quantity of beer) thinking about this finally led to the realization that you could use a heavy particle as the source of a Higgs field and another heavy particle ("heavy" means strongly coupled to the Higgs field) as a detector, revealing the Higgs field due to the first particle by having it mass shifted. The calculation is hard to do in momentum space since typical states are taken to be particles with well-defined masses that get out to "infinity", but if you consider a pair of Z bosons, say, next to each other, you can quickly find that as long as one decays while still near the other, its mass can be significantly shifted – a fact which can be revealed by the invariant mass reconstructed from its decay products which do get to "infinity".

Here we see a very nice example of how the mass of a particle isn’t even an intrinsic property, but can be shifted by the presence of another massive object due to its Higgs field. No Higgs “particle” need be involved, and in fact to a leading approximation the mass shift turns out to be independent of the Higgs mass! This is easy to understand intuitively: the Higgs field around a massive particle falls off as $\exp(-m_H r)$ where $r$ is distance and $m_H$ is the Higgs mass. For small $r$ one essentially gets $\exp(0) = \frac{1}{r}$ so one can actually test for the presence or not of a Higgs field independently of the Higgs mass!

This is quite important, both as a matter of principle, and because if the Higgs boson is massive enough its width (which grows as $m_H^3$) will eventually be so large (around 1.2 TeV in the Standard Model) that its lineshape will no longer look like a “mass bump” at all. Would one still be able to call it a “particle”?

Will we see the Higgs first as a particle or as a field? I’m betting on field first!

III. PROPAGATORS

To describe the propagation of a particle (again of definite momentum $p$) one takes the Fourier transform of $(\Box^2 - m^2)\phi(x)$ and comes to a propagator of the form

$$\frac{1}{p^2 - m^2}$$

Of course this picture starts to break down quickly in a number of interesting ways.
Suppose for example, that \( \phi \) describes a charged pion \( \pi^- \). As is well known, pions don’t last forever (see figure 1), which means we can’t really follow the initial logic that led us to consider unitary irreducible representations of the Lorentz group – clearly we can’t have time-translation symmetry on a one-particle \( \pi^- \) state since \( \pi^- \) decays, mainly via \( \pi^- \rightarrow \mu^- \bar{\nu}_\mu \).

How can we describe this lifetime?

The usual fix is to include the lifetime via a complex shift in the mass: \( m \rightarrow m + i \frac{1}{2} \Gamma \) where \( \Gamma = 1/\tau \) with \( \tau \) the negative pion lifetime.

\[
\frac{1}{p^2 - (m + i\Gamma/2)^2}
\]  

Naively, if one considers the wavefunction for a pion as having a time-dependent factor of the form \( e^{iEt/\hbar} = e^{imc^2t/\hbar} \) then this corresponds to \( e^{iEt/\hbar} = e^{imc^2t/\hbar} e^{-\Gamma t} \) which would seem to be a neat fix, but at best can only be an approximation. Going from time to energy via a Fourier transform, exponential decay in time corresponds to a Lorentzian lineshape in energy – a distribution which is nonzero for all value of energy and not even normalizable. The need to cut off the Lorentzian leads to the often-ignored fact that strictly exponential decay is just not physically possible, and survival probability as a function of time has deviations from exponential behaviour at both large and short times. Interestingly enough, the first time most experimentally-oriented physicists realize this is when they go to fit a “mass bump” and discover that the Lorentzian line shape can’t be normalized. This often leads to a lot of fiddling about arguing, about how it should be “cut off” or “normalized” before any real deep appreciation starts to kick in. I blame the textbooks – this sort of thing really has to stop!

The more correct way to deal with this is via spectral functions and the Källen-Lehmann representation which includes the fact that not only can a \( \pi^- \) decay into \( \mu^- \bar{\nu}_\mu \) but also that the decay products can be accompanied by an arbitrary number of (soft) photons (see figure 2). In fact there are also decays into \( e^- \bar{\nu}_e + n\gamma \) and one can write propagators (for the scalar case at least) quite generally of the form:

\[
\frac{1}{p^2 - (m + i\Gamma/2)^2}
\]  

FIG. 1: One of the diagrams contributing to a negative pion (incoming and outgoing lines) propagator due to its ability to decay into a muon and antimuon neutrino (solid and dashed lines in the bubble).
\[ \int \frac{\rho(s)ds}{s - m^2 + i\epsilon} \]  

(8)

Clearly whatever a particle is, if it can decay the structure of the propagator is far more complex than just a mass (and a spin, which I’m leaving out here for simplicity) and a width, however much we like to think of them in that way.

In fact, even if a particle can’t decay, if it can interact there are already interesting effects in the propagator. In the infrared limit one can work out what the corrections are the propagator of a “free” electron due to its electromagnetic interactions with itself and one finds[4] an expression proportional to

\[ \frac{p + m}{(p^2 - m^2)^{1+\gamma}} \]  

(9)

where \( \gamma = 1 + \frac{2}{\pi} \) in the quenched approximation.

There is a nice physical interpretation of this (see figure 3). As the electron propagates, it does so in its own electric field. In fact the expression is easiest derived in x-space by looking at the one-loop correction due to the electron at point \( x_1 \) exchanging a photon with itself at point \( x_2 \) and integrating over all these positions. Note that the propagator now has a fundamentally different singularity structure, with the original simple pole now replaced by a branch cut. The expression is nonlocal (fractional powers of momentum correspond to fractional derivatives[5] in x-space, which are mixed integro-differential operators and thus nonlocal) since it now considers the electron not as a non-interacting particle, but as one which carries its electric field with it. In no way is this a perturbative result. An electron with an electric charge is fundamentally different from one without one! For example, its presence or not with an arbitrarily large volume can be determined from measurements made arbitrarily far away using Gauss’ law and this ability to know about a charge at infinite distances is due to the masslessness of the photon and the \( 1/r^2 \) nature of the electrostatic
force. It also makes no sense arguing that in some sense the charge can be considered as small as one wants: in the real world charge is quantized!

An additional point can be made with respect to acceleration. While it was noted earlier that the vacuum is expected to change under acceleration, here one clearly sees that an accelerated electron will radiate real photons itself, considerations of the vacuum aside. A charged particle, which in an inertial frame carries a cloud of virtual photons that make up its electric field, will, in an accelerated frame radiate real photons!

There is a nice interpretation of the non-integer exponent in terms of what sorts of \textit{paths} a particle takes. Classically, the dimension of a particle path is 1, but in quantum mechanics this goes up \cite{6,11} to 2, with path integrals dominated by paths of infinite action. These are highly jagged paths with derivatives nowhere. This lack of derivatives is yet another manifestation of the Heisenberg uncertainty principle which does not allow one to specify the momentum (mass times the time derivative of position on the path) and the position simultaneously.

Intuitively one can understand the dimension 2 result for the nonrelativistic quantum mechanical case by thinking of the Schrödinger equation as a diffusion equation in imaginary time\cite{10}. For diffusion one has the distance a particle covers in time \( t \) satisfying a relationship of the form \( t \propto r^d \) where \( d \) is the fractal dimension of the “path”. Of course \( t \propto r^2 \) in the diffusion limit, and \( t \propto r \) in the ballistic (simple path) limit\cite{11}.

Here we have a very similar situation but with a 4-dimensional Hamiltonian \( \mathcal{H} \) and with fractal diffusion in proper time, and as was shown in \cite{4}, one has

\begin{equation}
    d = 2(1 + \gamma) \approx 2 + \frac{2\alpha}{\pi} + \ldots ,
\end{equation}

In a sense, an electron repels itself from where it was (or will be!), roughening the paths. Not only is an interacting electron not free in the sense that it interacts with other charges – it’s not even really free if there’s just one of them in an otherwise empty universe! So much

\[ \text{FIG. 3: One of the diagrams contributing to the fractal infrared propagator of an electron. To obtain the fractional exponent one sums an infinity of these, with photon lines starting and ending anywhere on the line.} \]
for free particles!

As discussed in [12], the corresponding corrections due gravity can also be calculated in the Newtonian limit and one finds a similar, though much smaller correction. Nevertheless, as a matter of principle, any particle coupled to an infinite-range field cannot reasonably be expected to have a propagator with simple poles.

An extremely interesting point raised in [12] is that if one considers a “red” (say) quark, even propagating in empty space (probably a bad approximation, as we will see later) will get corrections with a $\gamma$ of opposite sign which could be large enough to force the exponent to which the propagator is raised to zero and drop the path dimension also to zero - the quark would be unable to propagate in the IR limit due to interactions with its own glue – it would be confined! It has not been possible to make this argument rigorous yet, but it certainly is an interesting way to think about confinement and how a particle could be effectively unable to propagate in the IR limit. It also raises the point that a “particle” may behave quite differently not just on whether the frame from which it is viewed is inertial or not, but it may also depend on the energy scale of the probe used to observe it – a point to which we will return later.

So in conclusion to this section, whatever an interacting particle is, it is certainly not characterizable by a single mass (even complex to allow for it not being stable) and simple pole structure in its propagator. Clearly there’s a lot more to a particle than the (newer) textbooks would have us believe.

**IV. COMPOSITE PARTICLES, STRUCTURE FUNCTIONS, AND BOOTSTRAPS**

There are several interesting ways in which one can think of a particle as being composite and the hydrogen atom is a good place to start.

The hydrogen atom is composite in the sense that it’s made of things which can be separated from each other, but which have formed a bound state with mass less (in the case of a hydrogen atom, 13.6 eV less) than the sum of the masses of its constituents.

A different notion of compositeness applies to particles like the proton, which we think of as made of 3 valence quarks (uud), but if we’re honest about it, we can’t really see these at all without putting substantial energy into whatever we use as a probe. We might, however,
infer their presence from the proton’s failure to satisfy the Dirac equation for a pointlike particle due to its (very far from 2) value for its magnetic moment.

You could argue similarly for a hydrogen atom, since it itself has a magnetic moment (due largely to the proton) which would seem quite inconsistent with a pointlike charged spinless object. But while enough energy will pull a proton and an electron apart, it seems that the quarks can’t be removed. Indeed, if enough energy is put in one finds that in fact there seem to be more and more quarks (!), as well as antiquarks and some other things called gluons. Finally one gives up thinking of assigning sensible fixed numbers to how many are present and starts talking about parton distribution functions and structure functions. At high energies a proton is no longer a particle, but an energy-dependent variable number of increasingly loosely-bound constituents. This is certainly a far cry from any intuitive notion of a particle that one might have had when one started physics, and also a very good time to really push the idea that it’s not enough to specify a Lorentz frame and coordinate axes to describe physics, but also the energy scale of what’s being used to probe the “particles” involved.

Indeed, one might well ask how it could be that quarks are bound in a proton and yet act as free particles simply by being boosted! This is another point to which little deep thought is usually given, other than to wave one’s hands and say that when one does a high energy scattering experiment the energy scales are well above any sort of binding energy so one can just neglect it. But surely if the quarks are bound, they would act in a correlated way, although experimentally a proton flying by at high speed seems to look like a gas of weakly interacting quarks or partons. This issue is resolved in a very nice way by Kim[13] who shows that time dilation slows characteristic oscillations of the bound quarks so much that the interaction time in a high energy collision is short compared to it and an external probe simply has not time to see anything of the boundstate dynamics – yet another subtle aspect of how composite particle looks different from different Lorentz frames. Various pictures of what goes on all hold together, but one is really dealing with covariance of the concepts rather than invariance!

Finally we come to the electron which seems to not to be “made” of smaller things, satisfying the Dirac equation for a pointlike particle very well, but the small discrepancies ultimately point out the electron is interacting with its own field which extends out to infinity! For one photon exchange in perturbation theory this gives the famous $g-2$ of QED.
In a sense, as discussed above in terms of fractal propagators, the electron is a composite object made of a pointlike charge and an infinite range electric field. Observation of the electron with sufficiently high energy probes can see these photons directly and within the photons electrons and positrons. Ultimately even electrons, which we think of as pointlike and stable, have to be thought of as having their own structure functions (see figure 4).

At the end of the day, everything is coupled to everything and there’s a little bit of everything inside everything else. I know this is all very reminiscent of old bootstrap[14] ideas, but it’s hard to say that there’s no truth to them at all!

FIG. 4: One of the diagrams contributing to the propagator of an electron with a photon shown fluctuating into and $e^+e^-$ pair.

V. WIGNER’S FORGOTTEN REPRESENTATIONS

In a tour-de-force paper[15] of 1939, Eugene Wigner classified all the unitary irreducible representations of the Poincaré group. They fall into 6 classes depending on their momentum $p^\mu$, whose square is a Lorentz invariant:

1) $p^2 = m^2 > 0, p^0 > 0$

2) $p^2 = m^2 > 0, p^0 < 0$

3) $p^2 = 0, p^0 > 0$

4) $p^2 = 0, p^0 < 0$

5) $p^\mu = 0$

6) $p^2 = m^2 < 0$

In the spirit of section[1] these should represent types of “things” – that is “particles”.

The usual textbook argument is that the first and third correspond to massive and massless particles. The sixth corresponds to tachyons, which normally one might consider unphysical except that they’re used to good advantage to indicate instability of a mode, and, in
the Standard Model, electroweak symmetry breaking. The other classes are often considered unphysical, but I will suggest here that they may correspond to something interesting.

The classification proceeds by considering the unitary irreducible representations of the “little group”, which is a subgroup of the Poincaré group that leaves a particular choice of $p^\mu$, compatible with the conditions above, invariant. For massive particles one can go to the rest frame of the particle and look at $p^\mu = (m, 0, 0, 0)$ and the little group is $SU(2)$ so massive particles correspond to representations labelled by a mass, and a representation of $SU(2)$. For $p^2 = 0$ we can pick $p^\mu = (k, 0, 0, k)$ and the little group is $ISO(2)$, which we already saw above. In this case states turn out to be labelled by helicity.

Case 5 is a rather interesting one. In this case, the little group is the entire Lorentz group. The unitary irreducible representations were not found by Wigner, but only much later. They are labelled by two real numbers $\ell_0$ and $\ell_1$ and there are two cases:

a) $\ell_0$ is an arbitrary integer or half-integer and $\ell_1$ is imaginary: the “main series”

b) $\ell_0 = 0$ and $\ell_1$ is real and $|\ell_1| \leq 1$: the “supplementary series”

The only finite dimensional case is for $\ell_0 = 0$ and $\ell_1 = 1$. What are all these representations good for?

In my opinion, this is a very interesting bit of math which is far from having been used to its fullest. These are representations of the symmetries of the vacuum! Could the vacuum correspond to a “particle” of some sort? And, if so, what sort might it be? I’ll return to this after a brief digression on why we think about particles at all, when really what we have might better be described as processes.

VI. THINGS VS. PROCESSES

As Paul Davies so eloquently put it, “Particles are what particle detectors are designed to detect”. There is a sense in which we may have got caught up too much in the “thing” interpretation of particles rather than in the processes which they mediate.

A simple question that often comes up with students (they still worry about things we learn to stop thinking about!) is how you can tell a virtual particle from a real one. The usual textbook answer is that a real particle is one whose 4-momentum $p^\mu$ satisfies $p^\mu p_\mu = m^2$ where $m$ is the particle’s mass (as defined by a simple pole in its propagator - a concept
which I hope section III helped you to lose some confidence in). In fact, the only possible way a particle could be strictly on-shell would be to have lived an infinite time (in order to have zero uncertainty in its energy) and covered an infinite distance (in order to have zero uncertainty in its momentum) so that one could truly say $E^2 - p^2 = m^2$ (where, of course $c = 1$). The only particles you ever really detect are, by definition, virtual!

Again, particles are what particle detectors detect. They’re really inferred concepts. If the thing you built and labelled “detector” goes “click” then you say a particle hit it, otherwise not. Particles correspond to particular states of fields, and it may not always be necessary or even useful to think of particles, as we saw in section III “We really need to think of particles!” says the particle detector builder, but as Robert Anton Wilson said “What the Thinker thinks, the Prover proves.”[18], or, as the common expression in English goes “If all you’ve got is a hammer, everything starts to look like a nail.”

It’s not really clear how essential the picture of a little almost non-interacting ball with a somehow fairly well-defined position and momentum in a beam is, other than as a way of thinking about processes.

It would be interesting to see whether a change of viewpoint might lead to useful insights. Perhaps languages of other cultures might also offer a few hints. As a native English speaker, I continue to be amazed that Slavic languages like Russian seem to get by fine without definite and indefinite articles, and Latin-derived languages like Portuguese (ola Alberto! – esta ainda lendo?) assign genders to every object!

I’m not entirely alone in thinking that this is worth some consideration and I’ve long been interested in how the languages we speak may shape the ways we think.

It has even been suggested by many of the founders of quantum mechanics, including Bohr and Bohm that we could be at least partially caught in a linguistic trap due to the fact that European languages seem to very accurately mirror the concepts of classical physics. For example, "the bat hits the ball" makes good sense classically, but "the electron hits the proton" is clearly a much more complicated notion. In 1992 David Bohm and David Peat [19] met with native American elders including members of the Micmac, Blackfoot, and Ojibwa tribes who all speak languages in the Algonquian family. Apparently these languages have very sophisticated classes of verbs which do not correspond in any precise way to our own, but a rather reduced degree of division of the world into categories such as “fish” or “trees” (and, I would wager, “particles”). A quote from David Peat[19] may get the idea
across:

“Take, for example, the phrase in the Montagnais language Hipiskapigoka iagusit. In a 1729 dictionary, this was translated as the magician/sorcerer sings a sick man. According to Alan Ford, an expert in Algonquian languages at the University of Montreal, Canada, this deeply distorts the nature of the thinking processes of the Montagnais people, for the translator had tried to transform a verb-based concept into a European language dominated by nouns and object categories. Rather than there being a medicine person who is doing something to a sick patient, there is an activity of singing, a process. In this world view, songs are alive, singing is going on, and within the process is a medicine person and a sick man.”

He goes on to describe the Algonquian-speaker’s world-view as one of “flux and change, of objects emerging and folding back into the flux of the world. There is not even a sense of fixed identity - even a person’s name will change during their life.” Is this not something like the way real particles act?

With that in mind, I’d like to offer a few thoughts on what a pomeron might be (thought of as).

VII. POMERONS AND THE VACUUM

A pomeron is, without a doubt, one of the weirdest concepts of something particle-like that one could imagine[20]. Protons scatter off protons with a cross section that goes like $s^{\alpha(0)}$ at small $t$ with $\alpha(0) = 1.04$ and with the exponent rising linearly with increasing $t$ and people say this is due to the exchange of a “pomeron”.

Presumably if one sees a proton scatter off another proton with a cross section that looks like a power of centre-of-mass energy squared $s$ of the form $s^{\alpha}$ one imagines that some “thing” is being exchanged to carry energy and momentum between the two. Even more amazingly, that “thing” is supposed to carry the quantum numbers of the vacuum[20].

If someone claims to have discovered a new particle, the first thing that most people ask is what it’s mass and spin are. Ask anyone what a pomeron’s mass or spin (or lifetime) is and you get a blank look.... or maybe you get told that it’s a “trajectory” (a collection of objects with angular momentum $J$ with $J = \alpha(0) + \alpha'(t)$ where $t$ represent a squared 4-momentum transfer and $M$ rising with $J$ much as one might expect for a bit of relativistic
string). You might get told it’s a sort of glueball (colour-singlet collection of gluons)\cite{21} or a trajectory that corresponds to it – like a meson Regge trajectory but without any quarks.

If one goes back to the arguments of section I, then this “thing” should fall in some representation of the Poincaré group. If we look back now at section V to see where we should put it, might one not consider putting it in one of the representations of the vacuum? The relevant little group is now the Lorentz group, and since it is noncompact all its unitary representations are infinite-dimensional (and thus could accommodate the infinite number of states that lie on a trajectory).

The idea I’m suggesting here is that exchange of a soft pomeron actually corresponds to the exchange of a piece of vacuum and should lie in an infinite-dimensional representation of the Lorentz group. Let me now carry this a bit further and argue that there is already quite a convincing argument for this in the literature, although the suggestion of the Lorentz properties I am making here is new.

Kharzeev and Levin \cite{22} have made what to my mind is a beautiful argument that what one might naively think of as the exchange of a colour singlet ladder of gluons between two protons can sensibly be broken into perturbative and nonperturbative pieces. A cross section like $s^\epsilon$ is found with a nonperturbative piece connected via a spectral function and a low energy theorem to the trace of the energy-momentum tensor of the vacuum. The key insight here is that this is not zero in QCD. Classically, for a massless theory, $T^{\mu\mu}_\mu = 0$, but quantum effects spoil this leading to the so-called trace anomaly. Physically, of course, what we have is an instance of scale covariance (expressed by the renormalization group and in a rather subtle way) rather than scale-invariance and we see a connection back to the earlier discussions of the fractal nature of quantum corrections. In QCD there are nonperturbative corrections to the vacuum which can be parametrized in terms of colour-singlet condensates which make contributions to $T^{\mu\mu}_\mu$ and which can be measured from low energy hadronic physics \cite{23}. There is a classical contribution already from quark-antiquark condensates of the form $<0|\bar{q}q|0>$ but the main contribution is from a colour-singlet “gluon” condensate with $<0|g^2F^2|0>$ of about \(1.22\text{Gev})^4$ and the net result, remarkably, is a non-perturbative contribution to the cross section of $s^{1.04}$. Note that the term “gluon condensate” refers to the field rather than to a collection of weakly-coupled “gluon” degrees of freedom, which brings us back to earlier discussions about particles and fields. The fields are always “there” in some absolute sense, but the particles themselves are not. Particles are only particular field
configurations. Again we see some indication of language driving thought. We say “gluon” condensate even though we don’t really think there are any “gluon” particles there in the sense that we think of a gluon at high momentum transfer.

Physically, I would take this as a confirmation of the picture that the soft Pomeron is essentially an interaction mediated by the QCD vacuum – which, if you want to think of it as a “thing” would be a rather unconventional sort of “particle” falling into one of Wigner’s forgotten representations. It is also interesting to keep in mind that the little group for a zero-momentum gluon would also be the Lorentz group, and zero-momentum gluons have been recently considered with regard to the total $pp$ and $p\bar{p}$ cross section[24].

There is also a hard pomeron with an $\alpha(0) \approx 1.4$ which might better correspond to something like an exchange of gluons thought of as perturbative particles. There is an interesting fact, which I don’t think has been noticed often and to which I hope to return in a later paper, which is that 1.4/1.08 is strikingly close to 4/3 which is a very familiar factor in QCD. I’ll leave it at that for now, but I think there may be some nice physics in there...I’m working on it!

There may be many ways to think about confinement beyond simply getting an interquark potential that grows with distance!

VIII. SCALE SYMMETRY

One topic to which I think insufficient attention has been paid is the physical meaning of scale symmetry. We tend to think of objects as having well-defined positive integer dimensionality, but this is really a prejudice based on Euclidean geometry and, I would argue, classical thinking. Classical objects trace out straight 1-dimensional lines as they move and that 1-dimensionality is completely scale invariant: you can zoom in as close to, or as far from, a classical line as you want and it’s still a line. I remember as a child in elementary school insisting that real one-dimensional lines could not possibly exist. If you zoom in on a pencil line eventually you see that it’s really a rectangle of graphite on a sheet of paper and however much the teacher insisted that the line was infinitely thin, I could clearly see that it wasn’t – nor could it ever be due to the finite size of a carbon atom. Infinitely thin lines, like free particles, are idealizations and neither really corresponds to the physical world.
In the extreme IR limit, we saw earlier that an electron has a fractional dimension path due to self-interaction. In QED, the interaction goes to a constant at large distances so after getting some ways out (many electron Compton wavelengths) the electron path has an essentially constant fractal dimension. If one were to zoom in, however, one would start to resolve fluctuations in the individual photons as they split into pairs of charged particles, one would see not only photons exchanged along the the worldline but also $Z^0$’s and all sorts of things and a vastly complicated froth would appear. Not only is an electron not like a classical particle in terms of the paths it takes, but those very paths look different depending on the scale at which they are observed.

Even in simple cross sections one has the usual tree level $e^+e^- \rightarrow \mu^+\mu^-$ cross section proportional to $1/s$. Since $s$ has units of energy squared, this simply says that the cross section scales like length squared – which is area. Now consider the first corrections which are typically of the form $1 + k\alpha \ln(s)$. What does this really mean? Using $x^\epsilon = 1 + \epsilon \ln(x)$ this means the dimension has shifted by $k\alpha$. In low orders of perturbation theory, changes of scale dimension must appear as logarithms and an energy scale must be introduced so that one can take the logarithm of a quantity with dimensions. The fact that predictions of physical quantities should be independent of that scale is the meaning of the renormalization group, but it also reflects the fundamentally fractal nature of quantum processes.

It may well be that a future approach to doing calculations in quantum field theory would show that all propagators “Reggeize” with low order logarithmic corrections all becoming changes in powers. Indeed most quantum corrections can be thought of as due to “anomalous dimensions” even though terms like “beta function” tend to obscure this. It’s interesting to think that the early hadronic physics that motivated string theory may yet have lessons which are important in nonperturbative dynamics involving the ultimate Reggeization of everything.

Consider the pomeron again. What does it mean for a cross section to go like $s$ raised to some noninteger power? It means that the cross section scales in a fractal way...in this case due to the breaking of naive scale invariance of the QCD vacuum.

Quantum effects break not only scale invariance, but also conformal invariance (symmetry under local rescalings of the spacetime metric), and one might want to think more carefully about implications of the conformal group. In the absence of masses, Yang-Mills theories are conformally invariant – a remarkable feature of living in 4-dimensions and perhaps a clue...
to important things that we have only begun to glimpse. In some sense, breaking of that symmetry seems to be at the root of radiative corrections – that is, of allowing particles to interact and thus have a chance of being observed!

IX. SOME THOUGHTS ON NON-COMPACT GROUPS AND INFINITE-DIMENSIONAL REPRESENTATIONS

There is a marked tendency in particle physics to pay relatively little attention to non-compact groups and for good reason. The representation theory of compact groups is quite well studied and the fact that they have finite dimensional unitary irreducible representations connects well with intuitions about particle having a finite number of states, or at least a finite dimensional state space. For the rotation group, we have finite dimensional unitary representations of dimension \(2s + 1\) where \(s\) is a positive integer or half integer and we have come to feel comfortable thinking of a particle having a finite dimensional space of states (neglecting the “where” and “when” \(x\) variables). Similarly for \(SU(3)\) we’re used to 3 colour states for a quark and 8 for a gluon and the fact that the indices labelling a representation come from a finite set makes everything seem much more manageable.

Of course this is a bit of a con. Even in the usual textbook treatments of quantum field theory, particles fall into infinite dimensional representations of the Poincaré group with the position coordinates “\(x\)” being essentially infinite component (continuous) indices. The fields only look like they’re finite-component objects since their Lorentz indices (if they have any) come from a finite dimensional set induced from finite dimensional representations of the rotation subgroup of the Poincaré group. Perhaps nowhere does one see this more clearly then when one writes down the action for a field. Indices are summed over for Lorentz invariance, but full Poincaré invariance requires integration over \(d^4x\).

We may say that we have a finite-dimensional gauge group \(SU(3)\), but in reality we have \(SU(3)\) valued fields which correspond to groups of maps of spacetime into \(SU(3)\) which, except in 0-dimensional spacetime, have infinite dimension. Even if we stick to Wilson loops as observables we’re really looking at the loop group \(\Omega SU(3)\) of \(SU(3)\), which is parametrized by 8 parameters for \(SU(3)\) times an infinity of points on a line.

The Poincaré group and Lorentz group themselves are contained in, or can be obtained by contraction from, larger (also noncompact) groups including \(O(4,1)\) and \(O(3,2)\) which
represent the symmetries of de Sitter and anti de Sitter space. These invariably involve the introduction of a length scale, which is usually taken to be cosmological and the argument is made that Poincaré is a good approximation as a large radius parameter is taken to infinity, just as a sphere of very large radius looks very much like a plane in the vicinity of any point. An even larger and very interesting potential spacetime symmetry group is the conformal group, which turns out to be isomorphic to $SO(4,2)$, and contains both the de Sitter and anti de Sitter groups. It is also intimately connected with acceleration, all of which might make one suspect that it might have roles to play in connection with radiation and in general with scale changes. Might these groups (perhaps realized in strange ways) be relevant for particle physics? Matters of principle are important. For example, there is a sense in which you could argue that the existence of kitchen magnets clearly attests to the importance of the Poincaré group instead of the Galileo group – if $c$ is taken to infinity so that Poincaré goes to Galileo, magnets simply don’t exist (nor, for that matter, does light!). Note that this is sort of insight does not require any fancy accelerators or the need to get anywhere near relativistic speeds.

I’d like to close this section with yet another speculation. In general relativity one imagines that the vacuum is just empty, and with $T_{\mu}^{\mu} = 0$ one just has flat spacetime. But as we have seen the vacuum is populated by all sorts of condensates and while colourless particles would not be expected to see the gluon condensate, it does seem to be critical for QCD. Perhaps the effective spacetime seen by quarks (dynamically their vacuum isn’t ours!) and perhaps old ideas like “strong gravity” [29] are worth more attention than they have had in recent times. Certainly a black hole is suggestive of some notion of confinement, and a pair of gluons in a spin-2 or spin-0 state might well look like tensor or scalar gravity – at least to coloured objects.

X. LOCALIZATION

Any intuitive notion of particle which should somehow be like a classical ball runs into serious problems in terms of localization. First of all, as we have seen, the Heisenberg uncertainty principle only lets us say exactly where a particle is if we lose all idea of what its momentum is – certainly you can never think of a particle at rest in a well-defined place. Noninteracting particles (which are a fiction) travel on highly jagged paths of dimension 2
and if they are coupled to long-range fields (and everything couples to gravity!) that 2 gets shifted to a non-integer value.

Colosi and Rovelli [32] make the very interesting observation that there are really two distinct notions of “particle” in physics. One is in terms of globally defined \textit{n-particle Fock states} (the ones one gets from mode expansions and creation and annihilation operators) and \textit{local particle states} which are detected by finite-sized (real-life) detectors. In the limit of large detectors, these concepts can be shown to coincide, but only in a rather subtle way, and in a weak topology which is not given by norms.

When the effects of quantum field theory are included, you find you can’t localize a particle to within its Compton wavelength without making more particles of the same kind. If you try to even localize it to within an electron Compton wavelength you risk making $e^+e^-$ pairs and no matter what you do you will always make some massless photons or gravitons if you do anything at all!

Even worse, as Newton and Wigner [30, 31] showed, the position operators for particles with spin don’t even commute among themselves except in the case of spin-zero particles and with the exception of the yet-to-be-discovered Higgs boson, there don’t seem to be any fundamental ones in nature. Even the concept of a location with well-defined $x,y,$ and $z$ seems not to be tenable! Clearly things are a lot subtler than we’re usually comfortable thinking about.

\section*{XI. \textbf{Spin and Statistics}}

One of the most basic attributes of a particle is whether it is a boson or a fermion. This is usually not explicitly stated, due to the spin-statistics theorem [33] which states that half-integer spin particles are fermions and integer-spin particles are bosons.

This theorem is derived using a lot of explicitly Minkowski-space concepts and assumptions, in addition, of course, to the nontrivial step from field to particle (see section I). Might it sometimes not hold?

I argued in [34] that this connection might break down (see also [35] for arguments that this might be expected in string theory) in theories with gravity. I suggested that it was in fact generic to expect exotic statistics for ordinary particles in quantum gravity and gave a concrete picture for how this could happen in loop quantum gravity. A scale-dependent
change of symmetry is in fact not entirely unknown, with “quasibosons” such as those that
occur in superfluid helium-4 appearing to be bosons until pushed hard enough that their
fermionic constituents start to be become visible.

Would any effect be suppressed by powers of the Planck mass? Frankly, I don’t know,
but while I’m not a big fan of theories of TeV-scale quantum gravity, might such theories
show themselves in pp scattering at the LHC? Notably, some nonperturbative topological
effects which could be relevant are not suppressed by powers of the Planck scale\[36\].

For \(pp \rightarrow pp\) there are two amplitudes, a direct one and an exchanged one which should
appear multiplied by -1, just as occurs in Bhabha scattering. It’s not trivial to figure out how
to extract the relative phase of the two terms in elastic pp scattering, but if we understood
pomeron better, it might be easier to make some concrete predictions. Certainly whether
or not protons obey the Pauli principle at all energy scales is an interesting question worthy
of experimental study.

XII. WHAT ELSE COULD WE LEARN FROM FORWARD PP SCATTERING?

I’d like to make a few more points on the value of very forward and “diffractive” scattering,
which I think really show that one can learn very interesting physics from these sorts of
measurements.

It is a remarkable fact\[37\] that if there is a fundamental length scale \(R\) characterizing new
physics, and \(a(s,t)\) is the spin-independent amplitude for \(pp\) scattering (or \(p\bar{p}\) scattering)
and one defines \(\rho\) as the ratio of real to imaginary forward scattering amplitudes:

\[
\rho(s) = \frac{Re(a(s, t = 0))}{Im(a(s, t = 0))}
\]

this quantity can be very sensitive to the new length scale and suffer large changes even
when \(\sqrt{s}R \approx 0.1\) – quite a reach in energy! The point is that hard and soft physics are mixed
together in hadronic interactions and it is by no means clear (despite the strong tendency
to think differently) that new physics corresponding to high energies might not be seen first
in what people usually think of as “soft” interaction.

As a concrete example, work\[38\] I did with colleagues looking at the consequences of
large extra dimensions is of interest. The standard calculations of cross sections involve
squared matrix elements integrated over the available phase space. If there are extra large
dimensions that open up at some energy scale, then past that energy scale the integral over
the extra phase space variables typically gives rise to power law growth with $\sqrt{s}$. This
is faster than the $\ln(s)^2$ allowed by the Froissart bound and could appear dramatically in
total cross sections without the need to look for specific exclusive final states with exotic
particles, missing energy, or black holes. The apparent failure of unitarity is simply the
result of tracing over the degrees of freedom in the extra dimensions to describe the effective
physics of the usual $3+1$. We find no evidence of extra dimensions out to about 100 TeV,
with the most important data coming from cosmic ray physics which might otherwise be
regarded as “boring forward stuff” by some!

In the coming years it’s perhaps a good idea to keep in mind that “interesting physics”
used to be forward physics until high-$P_T$ events captured everyone’s imagination[39]. It’s
by no means obvious that there isn’t a lot of good stuff in the forward region at the LHC.

XIII. CONCLUSION

The main point of this essay, written by a particle physicist in honour of another particle
physicist, is to suggest that we don’t really know what a particle is yet, except for when
we try to have the notion conform very closely to classical ideas of invariance. With just
rotation and translation invariance and invariance under boosts, one get the usual Wigner
classification, but even then we may have disregarded a class of representations relevant for
things like the pomeron.

If more general changes are made to how one observes nature, including going to non-
inertial frames (which surely one must admit for general coordinate invariance – note how
general relativity is carefully excluded from all the standard textbook Poincaré invariant
treatments of field theories) one can even find observers disagreeing as to the presence or
absence of particles!

Consider scale changes represented by the momentum transferred in an interaction, and
particles reveal dramatically different properties, apparently transforming into all sorts of
collections of other “particles” which were apparently not really there until the observation
was made.

At constant couplings this would happen anyway, as soon as one tried to localize a particle
inside its Compton wavelength (or the Compton wavelength of a light particle – and, pun
intended, photons are light so there is inevitably some radiation in any measurement where charged particles are observed), but with running coupling constants there is a rich structure which is revealed as any particle is probed with increasingly fine resolution.

Real, strictly on-shell particles are never observed - we only see the virtual ones. Particles are supposed to be localizable things and yet we usually represent them by well-defined momentum states which correspond to infinite uncertainty in where they are. For particles with spin, the x,y,z coordinates of “where they are” don’t even commute! Some effects, like the Higgs-induced mass shifts of section [11] can really only be easily seen a field picture. The counterintuitive features go on and on!

It seems to me that there’s still a lot yet to be learned about the very concept of a particle (or how we should think of one). Many issues become easier to see in the case of strongly interacting theories like QCD, so it may be that while small angle pp scattering is not a good way to make new particles (which is what many particle physicists seem to think they should be doing) it may ultimately do something of equal or even greater importance which is to force on to rethink the notion of “particle” itself. Arguably, this is already happening with the pomeron, which challenges all the usual interpretations of what a particle should be.

XIV. CLOSING

The foregoing text was all about physics, and included, I hope, a vigorous defense of the importance of forward physics and things like pomerons to particle physics in a very profound sense – in terms of making us rethink the very notions of particles and their absences (the vacuum!). I hope Alberto finds something of interest in all this small offering of thanks for his kindesses over the years. I’m always happy to see him (and not just because that’s usually in Brazil – Eu sempre falo que Brasil é o melhor país do mundo!), and always impressed at his endless energy and enthusiasm to get things done!

There is more to life though than just physics, so I’d like to finish up with a quote from Eugene Wigner[40], who, I would argue, is the person who really formalized what most of us mean when we say “particle”, but who I am sure, would also have agreed that this was just a first start:

“It has been said that the only occupations which bring true joy and satisfaction are those
of poets, artists and scientists, and, of these, the scientists are the happiest.”

“Parabéns Alberto!” with my best wishes for many years of true joy, satisfaction, and happiness...and maybe to really figure out what on earth a pomeron really is!

XV. ACKNOWLEDGEMENTS

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