Research Article

A Novel TODIM with Probabilistic Hesitant Fuzzy Information and Its Application in Green Supplier Selection

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TODIM is a well-known multiple-criteria decision-making (MCDM) which considers the bounded rationality of decision makers (DMs) based on prospect theory (PT). However, in the classical TODIM, the perceived probability weighting function and the difference of the risk attitudes for gains and losses are not consistent with the original idea of PT. Moreover, probabilistic hesitant fuzzy information shows its superiority in handling the situation that the DMs hesitate among several possible values with different possibilities. Hence, a novel TODIM with probabilistic hesitant fuzzy information is proposed in this paper to simulate the perceptions of the DMs in PT. To show the advantages of the proposed method, a novel TODIM is combined with hesitant fuzzy information. Finally, a case study is carried out to demonstrate the feasibility of the proposed method, and a series of comparative analyses and the sensitivity analyses are used to show the stability of the proposed method.

1. Introduction

Decision makers (DMs) are considered to be completely rational among the existing multiple-criteria decision-making (MCDM) methods based on expected utility theory. However, the DMs are naturally bounded rational in real world. They are not able to obtain every detail of decision-making alternatives and are limited by their cognitions. Therefore, TODIM (TOmada deDecisão Iterativa Multicritério), a well-known MCDM method considering the bounded rational behaviors based on prospect theory (PT) [1], was proposed by Gomes and Lima [2]. It handles the vagueness and bounded rationality of the DMs to make the optimal choices based on multiple criteria.

However, the classical TODIM is based on crisp number which makes it restricted to express the vague perceptions of the DMs. Thus, the fuzzy sets (FSs) were introduced, and the TODIM had been extended to various FSs to provide more accurate and detailed information. The existing extensions of the TODIM are summarized in Table 1.

The TODIM has been not only extended to various fuzzy circumstances but also applied to various ranges of applications. After analyzing the existing TODIM, the application fields are summarized from the following aspects: supplier selection [8, 10, 12, 15, 17, 18, 27, 29], manufacture [3, 7, 13, 19, 30, 31], investment problem [5, 16, 25, 26, 32], service evaluation [23, 24], personnel selection [6, 33], emergency plan selection [9, 14], site selection [20, 22], air quality [4], and power sources [21]. Undoubtedly, the TODIM has demonstrated its unparalleled advantages in solving the MCDM problems by considering the psychological factors of the DMs. However, according to our review, we find that most of the existing TODIMs ignore the importance of the transformed probability weight in the original PT. What is more, the risk attitudes shown in the classical TODIM are not inconsistent with PT which only works on the gains and losses. That is, the classical TODIM should be adjusted according to the original PT which permits a more scientific result in its application. Meanwhile, the DMs may be hesitant between several possible evaluation information under the highly uncertain circumstance. Hence, a hesitant fuzzy set (HFS) [34] is an effective tool to express the hesitant situation in decision-
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Table 1: Extensions of TODIM with various FSs.

| FSs                                           | References |
|-----------------------------------------------|------------|
| Interval number                               | [3, 4]     |
| Intuitionistic fuzzy set                       | [5]        |
| Pythagorean fuzzy set                          | [6, 7]     |
| Q-rung orthopair fuzzy set                     | [8]        |
| Trapezoidal fuzzy set                          | [4, 9]     |
| Trapezoidal intuitionistic fuzzy set           | [10]       |
| Probabilistic interval-valued hesitant fuzzy set| [11]       |
| Multiset hesitant fuzzy set                    | [12]       |
| Probabilistic dual hesitant fuzzy set          | [13]       |
| Intervalvalued Pythagorean fuzzy linguistic term set | [14]   |
| Unbalanced hesitant fuzzy linguistic term set  | [15]       |
| Neutrosophic number                           | [16]       |
| Interval type-2 fuzzy set                     | [17, 18]   |
| Interval-valued intuitionistic fuzzy set       | [19, 20]   |
| Triangular fuzzy set                           | [3, 9]     |
| Triangular intuitionistic fuzzy set            | [21, 22]   |
| Hesitant fuzzy set                             | [23]       |
| Hesitant trapezoidal fuzzy set                 | [24]       |
| Probabilistic hesitant fuzzy set               | [25]       |
| Intuitionistic linguistic term set            | [26]       |
| Hesitant fuzzy linguistic term set             | [27, 28]   |
| Multiset hesitant fuzzy linguistic term set   | [29]       |
| Single-valued neutrosophic set                 | [30]       |

Table 2: Transformed probability weights.

| Weights  | Criteria |
|----------|----------|
| $c_1$    | $c_2$    | $c_3$    | $c_4$    |
| $\pi_{12}$ (A) | 0.389 | 0.183 | 0.275 | 0.306 |
| $\pi_{13}$ (A) | 0.389 | 0.183 | 0.275 | 0.306 |
| $\pi_{14}$ (A) | 0.389 | 0.183 | 0.275 | 0.306 |
| $\pi_{21}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{23}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{24}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{31}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{32}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{34}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{41}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{42}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |
| $\pi_{43}$ (A) | 0.368 | 0.197 | 0.275 | 0.301 |

Table 3: Relative weights.

| Criteria | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|----------|-------|-------|-------|-------|
| $\pi_{12}$ | 1     | 0.47  | 0.707 | 0.789 |
| $\pi_{13}$ | 1     | 0.47  | 0.707 | 0.789 |
| $\pi_{14}$ | 1     | 0.537 | 0.749 | 0.82  |
| $\pi_{21}$ | 1     | 0.537 | 0.749 | 0.82  |
| $\pi_{23}$ | 1     | 0.537 | 0.749 | 0.82  |
| $\pi_{24}$ | 1     | 0.47  | 0.707 | 0.789 |
| $\pi_{31}$ | 1     | 0.508 | 0.708 | 0.775 |
| $\pi_{32}$ | 1     | 0.537 | 0.749 | 0.82  |
| $\pi_{34}$ | 1     | 0.47  | 0.707 | 0.789 |
| $\pi_{41}$ | 1     | 0.47  | 0.707 | 0.789 |
| $\pi_{42}$ | 1     | 0.47  | 0.707 | 0.789 |
| $\pi_{43}$ | 1     | 0.497 | 0.747 | 0.834 |

Table 4: Relative prospect dominance degrees under each criterion.

| Criterion | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|----------|-------|-------|-------|-------|
| $\psi_{1}$ (A, A) | -37.23 | -139.33 | -36.21 | -33.35 |
| $\psi_{1}$ (A, A) | -9.25  | -55.75  | -32.14 | -50.83 |
| $\psi_{1}$ (A, A) | -36.46 | -63.05  | -25.14 | -41.38 |
| $\psi_{2}$ (A, A) | 1.80   | 1.70    | 0.92   | 1.04  |
| $\psi_{2}$ (A, A) | 2.12   | 1.16    | 1.49   | 2.23  |
| $\psi_{2}$ (A, A) | 2.58   | 1.07    | 0.98   | 1.78  |
| $\psi_{1}$ (A, A) | 0.45   | 0.68    | 0.82   | 1.59  |
| $\psi_{1}$ (A, A) | -43.89 | -94.96  | -58.35 | -71.57 |
| $\psi_{1}$ (A, A) | -40.28 | 0.40    | 0.62   | 0.84  |
| $\psi_{1}$ (A, A) | 1.76   | 0.77    | 0.64   | 1.29  |
| $\psi_{1}$ (A, A) | -53.35 | -88.08  | -38.39 | -57.11 |
| $\psi_{1}$ (A, A) | 1.95   | -32.87  | -24.37 | -26.89 |

2. Some Concepts

In this section, some fundamental concepts are presented, including PT, TODIM, and the probabilistic hesitant fuzzy information. They are the essential parts of this paper.
Table 5: Prospect dominance degrees.

| $\psi(A_1, A_2)$ | $\psi(A_2, A_1)$ | $\psi(A_1, A_3)$ | $\psi(A_3, A_1)$ | $\psi(A_1, A_4)$ | $\psi(A_4, A_1)$ |
|------------------|------------------|------------------|------------------|------------------|------------------|
| 246.12           | 5.46             | 7.00             | 3.53             | 4.46             | 147.93           |
| 147.97           | 268.77           | 38.42            | 236.93           | 82.19            |                  |
| 166.03           |                  |                  |                  |                  |                  |

Table 6: Overall prospect dominance degrees.

| $\Omega(A_i)$ | $\Omega(A_j)$ | $\Omega(A_k)$ | $\Omega(A_l)$ |
|---------------|---------------|---------------|---------------|
| 0             | 1             | 0.44          | 0.42          |

Table 7: Relative weights.

| $\omega_{1r}$ | $\omega_{2r}$ | $\omega_{3r}$ | $\omega_{4r}$ |
|----------------|---------------|---------------|---------------|
| 1              | 0.28          | 0.57          | 0.68          |

Table 8: Relative prospect dominance degrees.

| Relative dominance degrees | Criteria |
|----------------------------|----------|
| $\varphi_1(A_1, A_2)$     | $c_1$    | $c_2$    | $c_3$    | $c_4$    |
| $\varphi_2(A_1, A_3)$     | $-1.88$  | $-4.85$  | $-2.02$  | $-1.87$  |
| $\varphi_3(A_1, A_4)$     | $-0.85$  | $-2.88$  | $-1.89$  | $-2.38$  |
| $\varphi_4(A_1, A_5)$     | $-1.86$  | $-3.09$  | $-1.64$  | $-2.11$  |
| $\varphi_5(A_1, A_6)$     | $1.67$   | $1.23$   | $1.01$   | $1.13$   |
| $\varphi_6(A_1, A_7)$     | $1.83$   | $0.99$   | $1.33$   | $1.74$   |
| $\varphi_7(A_1, A_8)$     | $2.05$   | $0.95$   | $1.05$   | $1.53$   |
| $\varphi_8(A_1, A_9)$     | $0.76$   | $0.73$   | $0.95$   | $1.44$   |
| $\varphi_9(A_1, A_{10})$  | $-2.06$  | $-3.90$  | $-2.65$  | $-2.88$  |
| $\varphi_{10}(A_1, A_{11})$ | $-1.95$  | $0.55$   | $0.82$   | $1.01$   |
| $\varphi_{11}(A_1, A_{12})$ | $1.65$   | $0.78$   | $0.82$   | $1.28$   |
| $\varphi_{12}(A_1, A_{13})$ | $-2.30$  | $-3.74$  | $-2.09$  | $-2.54$  |
| $\varphi_{13}(A_1, A_{14})$ | $1.74$   | $-2.16$  | $-1.63$  | $-1.67$  |

2.1. Prospect Theory. PT is a major innovation in describing the bounded behavior of the DMs. It makes choices by the prospect value $V(x_i)$, which is calculated by multiplying the values of the value function $v(x_{ij})$ and the weight function $w(p_j)$. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a finite set of alternatives, $C = \{c_1, c_2, \ldots, c_m\}$ be a finite set of criteria, and $N = \{1, 2, \ldots, n\}, M = \{1, 2, \ldots, m\}$ be a finite set of decision makers, $i \in N, j \in M$. The prospect value is obtained by the following equations:

$$V(x_i) = \sum_{j=1}^{m} v(x_{ij}) w(p_j),$$

$$v(x_{ij}) = \begin{cases} -\lambda (x_0 - x_{ij})^\beta & x_{ij} - x_0 < 0, \\ (x_{ij} - x_0)^\alpha & x_{ij} - x_0 \geq 0, \end{cases}$$

where $x_{ij}$ denotes the evaluation value of the alternative $A_i$ over $c_j$; $x_0$ represents the reference point; $p_j$ is the weight of $c_j$; and $\alpha, \beta, \lambda, \delta, \delta$ are the corresponding parameters acquired from the experiments. According to the experiment in the classical PT [31], $\alpha = \beta = 0.88, \lambda = 2.26, \delta = 0.69, \delta = 0.61$.

2.2. TODIM. The TODIM [2] is an effective MCDM method to simulate the behaviors of the DMs. It considers the risk attitudes of DMs during the decision-making processes and measures the alternative by comparing the relative dominance with other alternatives. The procedure of the classical TODIM is shown as follows:

Step 1: obtain the original decision-making information including the evaluation information $X = (x_{ij})_{nm}$ of the alternative $A_i$ regarding the criterion $c_j$ and the weighting vector of the criterion $\omega$:

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix} = (x_{ij})_{nm},$$

$$\omega = (\omega_1, \omega_2, \ldots, \omega_m),$$

$$\sum_{j=1}^{m} \omega_j = 1.$$

Step 2: normalize the decision matrix $X = (x_{ij})_{nm}$ into $X = (\tilde{x}_{ij})_{nm}$ according to the cost criterion and benefit criterion:

$$\tilde{x}_{ij} = \begin{cases} x_{ij}, & c_j \text{ is the benefit criterion}, \\ -x_{ij}, & c_j \text{ is the cost criterion}. \end{cases}$$

Step 3: obtain the relative weights $\omega_j (j = 1, 2, \ldots, m)$ of the criterion $c_j$ ($j = 1, 2, \ldots, m$):

$$\omega_j = \frac{\omega_j}{\omega_p},$$

where $r, j \in M, \omega_p = \max (\omega_j | j \in M)$ and $c_r$ is called the reference criterion.
Step 4: acquire the prospect dominance degree \(\psi(A_i, A_k)\) of each alternative \(A_i\) over the rest of the alternatives \(A_k\) \((k = 1, 2, \ldots, n, k \neq i)\):

\[
\psi(A_i, A_k) = \sum_{j=1}^{m} \varphi_j(A_i, A_k), \quad i, k \in N, \tag{7}
\]

where the relative dominance degree \(\varphi_j(A_i, A_k)\) over \(c_i\) is calculated by the following equation, and the parameter \(\lambda\) denotes the attenuation factor of the losses:

\[
\varphi_j(A_i, A_k) = \begin{cases} 
\frac{\omega_{jr}}{\sum_{j=1}^{m} \omega_{jr}} (\bar{x}_{ij} - \bar{x}_{kj}), & \bar{x}_{ij} - \bar{x}_{kj} > 0, \\
0, & \bar{x}_{ij} - \bar{x}_{kj} = 0, \\
\frac{1}{\lambda} \frac{1}{\sum_{j=1}^{m} \omega_{jr}} (\bar{x}_{ij} - \bar{x}_{kj}), & \bar{x}_{ij} - \bar{x}_{kj} < 0.
\end{cases}
\tag{8}
\]
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Table 16: The dominance degrees.

| \(\psi_j(A_1, A_2)\) | \(\psi_j(A_2, A_1)\) | \(\psi_j(A_3, A_1)\) | \(\psi_j(A_3, A_2)\) |
|----------------------|----------------------|----------------------|----------------------|
| -11.39              | 5.67                 | -4.67                | 3.97                 |
| -9.35               | 0.78                 | 4.05                 | -9.65                |
| -7.47               | 4.80                 | 1.05                 | -4.15                |

Table 17: Overall prospect dominance degrees.

| \(\Omega(A_1)\) | \(\Omega(A_2)\) | \(\Omega(A_3)\) | \(\Omega(A_4)\) |
|------------------|------------------|------------------|------------------|
| 0                | 1                | 0.74             | 0.47             |

Table 18: The results of 4 methods.

| Methods                               | Overall prospect dominance degrees | Overall prospect dominance degrees |
|---------------------------------------|-----------------------------------|-----------------------------------|
| Novel TODIM with probabilistic hesitant fuzzy information | \(\Omega(A_1)\) | \(\Omega(A_2)\) | \(\Omega(A_3)\) | \(\Omega(A_4)\) |
| Extended TODIM with probabilistic hesitant fuzzy information | 0 | 1 | 0.44 | 0.42 |
| Novel TODIM with hesitant fuzzy information | 0 | 1 | 0.40 | 0.39 |
| Extended TODIM with hesitant fuzzy information | 0 | 1 | 0.86 | 0.56 |

Step 5: calculate the overall prospect dominance degree \(\Omega(A_i)\):

\[
\Omega(A_i) = \frac{\sum_{k=1}^{n} \psi(A_i, A_k) - \min_{k} \left\{ \sum_{k=1}^{n} \psi(A_i, A_k) \right\}} {\max_{k} \left\{ \sum_{k=1}^{n} \psi(A_i, A_k) - \min_{k} \left\{ \sum_{k=1}^{n} \psi(A_i, A_k) \right\} \right\}}.
\]  
(9)

Step 6: rank the alternatives according to the overall dominance degree of each alternative \(\Omega(A_i)\). The bigger \(\Omega(A_i)\) is, the better the alternative \(A_i\) will be:

\[A_i > A_j \iff \Omega(A_i) > \Omega(A_j).\]

2.3. Probabilistic Hesitant Fuzzy Information. Let \(X\) be a fixed set, and a probabilistic hesitant fuzzy set (P-HFS) on \(X\) is expressed by

\[H = \left\{ <x_i, h_{x_i}(p_{x_i})> \mid x_i \in X \right\},\]

where \(h_{x_i}()\) is called the probabilistic hesitant fuzzy element (P-HFE). It represents all the possible membership degrees of \(x_i \in X\) in \([0, 1]\). \(p_{x_i}\) is a set of probabilities associated with \(h_{x_i}()\) and \(\sum p_{x_i} = 1\). To be more concise, we denote the P-HFE \(h_{x_i}(p_{x_i})\) as \(h(p) = \{h^{(p)}|t = 1, 2, \ldots, \#h(p)\}\), where \(\#h(p)\) is the number of all possible membership degrees, and \(\sum_{t=1}^{\#h(p)} p^t = 1\). If \(\sum_{t=1}^{\#h(p)} p^t < 1\) for a P-HFE \(h(p)\), it can be transformed into \(h(p)\), which is defined as \(h(p) = \{h^{(p)}|t = 1, 2, \ldots, \#h(p)\}\), where \(\sum_{t=1}^{\#h(p)} p^t = 1\) and \(p^t = \frac{p^t}{\sum_{t=1}^{\#h(p)} p^t}, (t = 1, 2, \ldots, \#h(p))\) [35]. To compare two pieces of probabilistic hesitant fuzzy information, the score function \(\sigma(h(p))\) and the deviation function \(\rho(h(p))\) are defined as

\[
\rho(h(p)) = \frac{\sum_{t=1}^{\#h(p)} p^t \times h^{(p)}}{\sum_{t=1}^{\#h(p)} p^t},
\]

(12)

\[
\sigma(h(p)) = \frac{\sum_{t=1}^{\#h(p)} \left( p^t \times (h^{(p)} - \rho(h(p)))^2 \right)}{\sum_{t=1}^{\#h(p)} p^t}.
\]

(13)

The comparison rules of two P-HFEs are expressed as

1. If \(\rho(h_1(p)) > \rho(h_2(p))\), then \(h_1(p) > h_2(p)\)
2. If \(\rho(h_1(p)) < \rho(h_2(p))\), then \(h_1(p) < h_2(p)\)
3. If \(\rho(h_1(p)) = \rho(h_2(p))\), then \(h_1(p) = h_2(p)\)

Distance is also an important way to measure the relationship between two pieces of fuzzy information, and the same length of two P-HFEs is the premise for distance measurement. Therefore, probabilistic hesitant fuzzy values should be added to the shorter P-HFE. For example, let \(h_1(p)\) and \(h_2(p)\) be the two P-HFEs; if \(\#h_1(p) < \#h_2(p)\), \(\#h_2(p) - \#h_1(p)\) number of probabilistic hesitant fuzzy values should be added to \(h_1(p)\). In this paper, the largest possible probabilistic hesitant fuzzy value is added to \(h_1(p)\), and the corresponding probability is zero. In fact, there is no effect on the score function and the deviation function of the original P-HFE by adding a term with probability to be zero. Besides, the ordered P-HFE satisfies the following conditions:

1. For an ascending ordered P-HFE, \(p^t h^{(p)} \leq p^{t+1} h^{(p)} + 1\) (\(p^{t+1}\))
2. For a descending ordered P-HFE, \(p^t h^{(p)} \geq p^{t+1} h^{(p)} + 1\) (\(p^{t+1}\))
3. If \(p^t h^{(p)} = p^{t+1} h^{(p)} + 1\) and the orders are determined by \(p^t\) and \(p^{t+1}\), then

1. For an ascending ordered P-HFE, \(p^{t+1} < p^{t+1}\)
2. For a descending ordered P-HFE, \(p^{t+1} > p^{t+1}\)
3. If \(p^t = p^{t+1}\), the sequence of those two P-HFEs is random for both ascending ordered P-HFE and descending ordered P-HFE
Based on the ordered probabilistic hesitant fuzzy information, the Hamming distance is referred to [25] which is presented in the following equation:

\[
d(h_1, h_2) = \frac{1}{\#h_1(p)} \sum_{i=1}^{\#h_1(p)} |p_i^1 h_1^i(p_i^1) - p_i^2 h_2^i(p_i^2)|.
\] (14)

For convenience, the below probabilistic hesitant fuzzy information satisfies: \(\sum_{t=1}^{\#h_t(p')} p'^t = 1\), and it is ordered and standardized.

### 3. A Novel TODIM with Probabilistic Hesitant Fuzzy Information

This section firstly goes through the existing researches of the TODIM based on various kinds of fuzzy information. From the perspective of dominance function, the necessity of improving TODIM is also presented. According to the detailed analysis, we figure out that the probabilistic hesitant fuzzy information has great superiority in expressing the different hesitation degrees of the DMs, and the novel TODIM is more reasonable which derives from the original PT considering the importance of the transformed probability weight function during the decision-making process. Subsequently, a novel TODIM with probabilistic hesitant fuzzy information is proposed in this section. For the sake of comparison, the novel TODIM with hesitant fuzzy information is also given in the following part.

#### 3.1. Analysis of the Existing Researches about the TODIM with Fuzzy Information

The TODIM is known as an effective way to deal with the MCDM problems derived from PT, and it has advantages in expressing the behaviors of the DMs by using gains and losses. Actually, the crisp number is usually hard to access in the real world. Under this circumstance, the TODIM is applied to various FSs as analyzed in Table 1.

According to the review of extensions of the TODIM with fuzzy information, we find that most extensions are based on the classical TODIM, as shown in Section 2.2. That is, the risk attitudes work on the product of relative weight and the perceived gains or losses through the square root in the dominance function (equation (8)) which is inconsistent with the original PT (equation (2)). Besides, the existing TODIM calculates the relative weight by using objective probability instead of using the transformed probability weight function shown in equation (3). Actually, the dominance function is the main part to express the idea of PT. Hence, this section will show the model of the dominance function with fuzzy information.

Krohling and Souza [9] developed a TODIM by adjusting dominance function with trapezoidal fuzzy number as shown in the following equation:

\[
\varphi_j(A_i, A_k) = \begin{cases} 
\frac{\omega_j k}{\sum_{j=1}^{m} \omega_j k} d(r_{ij}, r_{kj}), & r_{ij} > r_{kj}, \\
0, & r_{ij} = r_{kj}, \\
-\frac{1}{\lambda} \sum_{j=1}^{m} \omega_j k d(r_{ij}, r_{kj}), & r_{ij} < r_{kj},
\end{cases}
\] (15)

where \(\omega_j k\) is the relative weight calculated from the original weight; \(\lambda\) is the attenuation factor of the losses; \(r_{ij}\) and \(r_{kj}\) are two FSs; and \(d(\cdot)\) is the distance between \(r_{ij}\) and \(r_{kj}\).

According to equation (15), the dominance function excludes the distance outside the square root. However, some researchers hold the view that the above dominance function (equation (15)) is far more deviating from the original PT. Hence, there is another progress proposed by Peng et al. [12] through adjusting the square number \(k\) as shown in equation (16). The distance of fuzzy evaluation information is included in the square root, which is similar to the classical TODIM:

\[
\varphi_j(A_i, A_k) = \begin{cases} 
\frac{\omega_j r}{\sum_{j=1}^{m} \omega_j r} d(r_{ij}, r_{kj}), & r_{ij} > r_{kj}, \\
0, & r_{ij} = r_{kj}, \\
-\frac{1}{\lambda} \sum_{j=1}^{m} \omega_j r d(r_{ij}, r_{kj}), & r_{ij} < r_{kj},
\end{cases}
\] (16)

where \(k\) is the regulating variable that is determined by the preference of the DMs, and when \(k = 2\), the dominance function perfectly agrees with the classical TODIM.

Tan et al. [36] thought that the square root or \(k\) used in the former dominance functions does not reflect the core idea of PT. The parameters could be different, which is shown in the experiments, while the square root or \(k\) is the same value all the time. Based on this, the dominance function was modified as follows where risk attitudes work on the product of the relative weight and distances:
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function was adjusted as and do not work on the weight. Hence, the dominance

et al. [32] thought that the risk attitudes only work on the

weight. Hence, the original weight was replaced with the

be the form of weight function rather than the original

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sively explains the idea of PT, and combining it with

the framework of this novel TODIM, which comprehen-

sively explains the idea of PT, and combining it with

probabilistic hesitant fuzzy information.

3.2. Procedure of the Novel TODIM with Probabilistic Hesitant Fuzzy Information. Based on the above analysis, this section presents a new procedure of the novel TODIM with probabilistic hesitant fuzzy information, which is based on the idea of the original PT. The procedure is given as follows:

Step 1: obtain the original evaluation information matrix \( Y = (h_{ij}(p_{ij}))_{n \times m} \) according to equation (4) and the weight of the corresponding criterion. Both the evaluation information and the weight satisfy the characteristic of the P-HFE:

\[
Y = \begin{pmatrix}
  h_{11}(p_{11}) & \cdots & h_{1m}(p_{1m}) \\
  \vdots & \ddots & \vdots \\
  h_{n1}(p_{n1}) & \cdots & h_{nm}(p_{nm}) \\
\end{pmatrix}_{n \times m},
\]

(19)

\[
\omega = (\omega_w(p_{w1}), \omega_w(p_{w2}), \ldots, \omega_w(p_{wm})),
\]

(20)

where \( i \in N, j \in M; h_{ij}(p_{ij}) \) is the evaluation information of the alternative \( A_i \) over the criterion \( c_j \); and \( h_{w}(p_{w}) \) is the weighting information of \( c_j \).

Step 2: normalize the evaluation information matrix according to Section 2.3:

\[
Y' = \begin{pmatrix}
  h'_{11}(p'_{11}) & \cdots & h'_{1m}(p'_{1m}) \\
  \vdots & \ddots & \vdots \\
  h'_{n1}(p'_{n1}) & \cdots & h'_{nm}(p'_{nm}) \\
\end{pmatrix}_{n \times m},
\]

(21)

\[
\omega' = (\omega'_1, \omega'_2, \ldots, \omega'_m),
\]

(22)

where \( \sum_{i=1}^{m} p'_{ij} = 1 \) (\( i \in N \)), \( \sum_{j=1}^{n} \omega'_j = 1 \); \( \omega'_j = \omega_j / \sum_{i=1}^{m} \omega_{ij} \); and \( \omega_j = \sum_{i=1}^{m} \omega_{ij} \).

Step 3: work out the transformed probability weight function \( \pi_{ikj}(\omega'_j) \) according to the weighting function of PT:

\[
\pi_{ikj}(\omega'_j) = \begin{cases}
  \frac{\omega'_j}{\left(\omega'_j + (1 - \omega'_j)^\gamma\right)^\gamma}, & h'_{ikj}(p'_{ij}) \geq h'_{ikj}(p'_{kj}) \\
  \frac{\omega'_j}{\left(\omega'_j + (1 - \omega'_j)^\gamma\right)^\gamma}, & h'_{ikj}(p'_{ij}) < h'_{ikj}(p'_{kj})
\end{cases}
\]

(23)
where the comparison between $h'_{ij}(p_{ij}')$ and $h'_{kj}(p_{kj}')$ is determined by using equations (12) and (13).

Step 4: acquire the relative weight $\pi_{ikj}'$ of $A_i$ over $A_k$:

$$\pi_{ikj}' = \frac{\pi_{ikj}(\omega_j')}{\pi_{ikr}(\omega_r')}, \quad r, j \in M, \forall (i, k), \quad (24)$$

where $\pi_{ikr}(\omega_r') = \max(\{\pi_{ikj}(\omega_j') | j \in M\})$ and $\pi_{ikr}(\omega_r')$ is named as the reference criterion.

Step 5: calculate the relative prospect dominance degrees $\varphi_{ij}^B(A_i, A_k)$ of the alternative $A_i$ over $A_k$ under the criterion $c_j$ as follows:

When $c_j$ is the benefit criterion, the relative prospect dominance degree is $\varphi_{ij}^B(A_i, A_k)$:

$$\varphi_{ij}^B(A_i, A_k) = \begin{cases} \frac{\pi_{ikj'}((d(h'_{ij'}(p_{ij'}'), h'_{kj'}(p_{kj'}')))^{\alpha}}{\sum_{j'=1}^{m}\pi_{ikj'}}, & h'_{ij'}(p_{ij'}) > h'_{kj'}(p_{kj'}), \\ 0, & h'_{ij'}(p_{ij'}) = h'_{kj'}(p_{kj'}), \\ -\lambda\left(\sum_{j'=1}^{m}\pi_{ikj'}\right)\left(d(h'_{ij'}(p_{ij'}'), h'_{kj'}(p_{kj'}'))\right)^{\beta}, & h'_{ij'}(p_{ij'}) < h'_{kj'}(p_{kj'}), \end{cases} \quad (25)$$

When $c_j$ is the cost criterion, the relative prospect dominance degree is $\varphi_{ij}^C(A_i, A_k)$:

$$\varphi_{ij}^C(A_i, A_k) = \begin{cases} \frac{-\lambda\left(\sum_{j'=1}^{m}\pi_{ikj'}\right)\left(d(h'_{ij'}(p_{ij'}'), h'_{kj'}(p_{kj'}'))\right)^{\beta}}{\pi_{ikj'}}, & h'_{ij'}(p_{ij'}) > h'_{kj'}(p_{kj'}), \\ 0, & h'_{ij'}(p_{ij'}) = h'_{kj'}(p_{kj'}), \\ \pi_{ikj'}\left(d(h'_{ij'}(p_{ij'}'), h'_{kj'}(p_{kj'}'))\right)^{\alpha}, & h'_{ij'}(p_{ij'}) < h'_{kj'}(p_{kj'}), \end{cases} \quad (26)$$

where $\alpha$, $\beta$, and $\lambda$ are the parameters of PT; $d(h'_{ij'}(p_{ij'}'), h'_{kj'}(p_{kj'}'))$ is the corresponding distance calculated by using equation (14).

Step 6: obtain the prospect dominance degrees based on equation (7):

$$\psi(A_i, A_k) = \frac{\sum_{j'=1}^{m}\varphi_{ij}^*(A_i, A_k)}{\sum_{k=1}^{n}\psi(A_i, A_k)}, \quad \forall (i, k). \quad (27)$$

Step 7: calculate the overall prospect dominance degrees from equation (9):

$$\Omega(A_i) = \frac{\sum_{k=1}^{n}\psi(A_i, A_k) - \min\{\sum_{k=1}^{n}\psi(A_i, A_k)\}}{\max\{\sum_{k=1}^{n}\psi(A_i, A_k)\} - \min\{\sum_{k=1}^{n}\psi(A_i, A_k)\}}, \quad \forall i, k \in N. \quad (28)$$
The bigger the $\Omega(A_j)$ is, the better the alternative $A_i$ will be.

3.3. Procedure of the Novel TODIM with Hesitant Fuzzy Information. Hesitant fuzzy information is represented by HFS [34] and is used to describe the situation that the DMs hesitate between several different values, and each hesitation value is equally important. In fact, it also can be expressed as a special form of probabilistic hesitant fuzzy information. When the probabilities are equal, probabilistic hesitant fuzzy information turns into hesitant fuzzy information. The HFS can be denoted as $H = \{x_t | x_t \in X\}$, and $h_x = \{h_{x,t}^{t} | t = 1, 2, \ldots, \#h_x\}$ is called a hesitant fuzzy element (HFE). To demonstrate the effectiveness of the proposed method in Section 3.2, we further combine the novel TODIM with hesitant fuzzy information in this section. The process is shown as follows:

Step 1: obtain the original information matrix and weight information:

$$Y = \begin{pmatrix} h_{11} & \cdots & h_{1m} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nm} \end{pmatrix} = (h_{ij})_{nm \times m},$$

$$\omega = (h_{w_1}, h_{w_2}, \ldots, h_{w_n}),$$

where $i \in N, j \in M; h_{ij}$ is the evaluation information of the alternative $A_j$ over the criterion $c_i$; and $h_{w_j}$ is the weighting information of $c_i$.

Step 2: normalize the weight information based on the following equation:

$$\omega_j' = \frac{\omega_j}{\sum_{j=1}^{m} \omega_j},$$

where $\omega_j = \rho(h_{w_j})$ and $\rho(h_{w_j})$ is the score function (32) of the HFE $h_{w_j}$:

$$\rho(h) = \frac{1}{\#h} \sum_{i=1}^{\#h} h_i.$$  (32)

Step 3: obtain the transformed probability weight function $\pi_{ikj}(\omega_j')$ according to the following equation:

$$\pi_{ikj}(\omega_j') = \begin{cases} \frac{\omega_j'}{\omega_j' + (1 - \omega_j')^{\frac{1}{\delta}}}, & h_{ij} > h_{kj}' \\
\frac{\omega_j'}{\omega_j' + (1 - \omega_j')^{\frac{1}{\delta}}}, & h_{ij} < h_{kj}' \end{cases}$$  (33)

where the comparison of the HFEs $h_{ij}'$ and $h_{kj}'$ is decided by the score function (equation (32)) and the deviation function (the following equation) of the HFEs:

$$\sigma(h) = \frac{1}{\#h} \sum_{h_j \in h} (h_j' - \rho(h))^2.$$  (34)

The detailed rules are represented as follows:

(1) If $\rho(h_{ij}') > \rho(h_{kj}')$, then $h_{ij}' > h_{kj}'$

(2) If $\rho(h_{ij}') < \rho(h_{kj}')$, then $h_{ij}' < h_{kj}'$

(3) If $\rho(h_{ij}') = \rho(h_{kj}')$, then

(1) If $\sigma(h_{ij}') > \sigma(h_{kj}')$, then $h_{ij}' < h_{kj}'$

(2) If $\sigma(h_{ij}') < \sigma(h_{kj}')$, then $h_{ij}' > h_{kj}'$

(3) If $\sigma(h_{ij}') = \sigma(h_{kj}')$, then $h_{ij}' = h_{kj}'$

Step 4: calculate the relative weight $\pi_{ikj}'$ based on equation (24) and the transformed probability weight $\pi_{ikj}(\omega_j')$

Step 5: work out the relative prospect dominance degrees $\phi_j^\beta(A_i, A_k)$ of the alternatives $A_i$ over $A_k$ under the criterion $c_j$ as follows:

When $c_j$ is the benefit criterion, the relative prospect dominance degree is $\phi_j^\beta(A_i, A_k)$:

$$\phi_j^\beta(A_i, A_k) = \begin{cases} \frac{\pi_{ikj}'(d(h_{ij}', h_{kj}'))}{\sum_{j=1}^{m} \pi_{ikj}'}, & h_{ij}' > h_{kj}' \\
0, & h_{ij}' = h_{kj}' \\
\lambda \left(\frac{\sum_{j=1}^{m} \pi_{ikj}'}{\pi_{ikj}'(d(h_{ij}', h_{kj}'))} \right)^{\beta}, & h_{ij}' < h_{kj}' \end{cases}$$  (35)
When $c_j$ is the cost criterion, the relative prospect dominance degree is $\varphi^c_j (A_i, A_k)$:

$$
\varphi^c_j (A_i, A_k) = \begin{cases} 
\frac{\lambda(\sum_{j=1}^m \pi_{ikj}) (d(h'_j, h_{kj}))^\beta}{\pi_{ikj}}, & h'_j > h_{kj}, \\
0, & h'_j = h_{kj}, \\
\frac{\pi_{ikj} (d(h'_j, h_{kj}))^\beta}{\sum_{j=1}^m \pi_{ikj}}, & h'_j < h_{kj},
\end{cases}
$$

(36)

where $\lambda$ denotes the attenuation factor of the losses and $d(h'_j, h_{kj})$ is the corresponding distance calculated by

$$
d(h_{ij}, h_{kj}) = \frac{1}{\#h_{ij}} \sum_{i=1}^{\#h_{ij}} |h'_{ij} - h'_{kj}|, \quad \#h_{ij} = \#h_{kj}.
$$

(37)

Step 5: acquire the prospect dominance degrees based on equation (27).

Step 6: calculate the overall dominance degrees according to equation (28), and the bigger the $\Omega (A_i)$ is, the better the alternative $A_i$ will be.

From the procedures above, the novel TODIM with probabilistic hesitant fuzzy information and the novel TODIM with hesitant fuzzy information are given in Sections 3.2 and 3.3, respectively. It is worth noting that in the proposed methods, the original weight information is represented by P-HFE and HFE according to (20) and (30) respectively. Moreover, the score function is used to represent the weight information of (22) and (31), which is inspired from [37]. Indeed, the score function is an excellent tool that reflects the comprehensive information of a piece of the evaluation for the alternative, and it is also good at grasping the basic information. Therefore, we also use the score function to complete the weight transformation in this paper. Besides, both these methods conform to the original PT by modifying the perceived probability weighting function and the difference of the risk attitudes for gains and losses. Concerning the ability to express the information of DMs, the novel TODIM with probabilistic hesitant fuzzy information has more advantages in describing different hesitant degrees of the hesitant values by using possibilities. The novel TODIM with hesitant fuzzy information is used to carry out a series of convincing comparisons. Actually, it is regarded as a particular form of the former method when the probability is equal. Hence, the novel TODIM with probabilistic hesitant fuzzy information is our main focus in this paper, and we believe that it can reflect more evaluation information than the novel TODIM with hesitant fuzzy information. Then, a case study is carried out to show the application of the proposed methods.

### 3.4. Theoretical Analysis of the Proposed Method

It is critical to know the advantages of the proposed method which helps us to understand the MCDM process and at the same time contributes to analyzing the ranking results reasonably. The theoretical superiority of combining the novel TODIM with probabilistic hesitant fuzzy information can be concluded from the two aspects: information distortion and information attenuation. In terms of information distortion, the proposed method has modified three inconsistencies of classical TODIM with PT. First, this method reflects the actual meaning of parameters compared with the method proposed by Krohling and Souza [9] and Peng et al. [12], and their dominance functions are equations (15) and (16), respectively. Although the values of $\alpha$ and $\beta$ are equal under this circumstance, their meanings are completely different, and their values may be different in different experiments. $\alpha$ indicates the concavity of the power function for gains, while $\beta$ represents the convexity case for losses. It is improper to depict those two different states with only one uniform parameter $k$, which may lead to information distortion. The proposed method has also made the second measure to avoid information distortion, and it is the use of weight function compared with the method proposed in [36] and its dominance function shown as equation (17). The weight function is important in PT because it modifies an easily overlooked situation that people tend to overestimate low probability events and underestimate high probability events. Third, the most important point revealed by the proposed method and neglected by most existing studies is that the $\alpha$ and $\beta$ only appear in the value function and work on gains and losses according to the original PT.

The novel TODIM does compensate for some shortcomings of the traditional TODIM by reflecting the actual meaning of parameters, considering the transformed weight function and modifying its core idea referring to the original PT. However, the information attenuation is inevitable when the novel TODIM is explained by crisp number [32]. In some practical situations, the DMs could not give an accurate assessment and usually hesitant in several assessments. This situation can be well simulated by hesitant fuzzy information. But hesitant fuzzy information could not reflect the different preferences for every possible value. Probabilistic hesitant fuzzy information can describe different preferences for each possible value with probabilities, so the performance of the novel TODIM with probabilistic hesitant fuzzy information will be superior to the one with scrip number and the one with hesitant fuzzy information. For example, if a person is invited to evaluate a suitable supplier of electric vehicle charging piles in the urban planning, he/she is not very sure about the score and hesitates between several values 81, 85, and 90. Furthermore, among those three values, he/she prefers 81, and he/she thinks there is 0.7 probability of 81 and 0.2 and 0.1 probabilities of 85 and 90, respectively. In this situation, the evaluation information can be interpreted as {81(0.7), 85(0.2), 90(0.1)} by probabilistic hesitant fuzzy information. While using hesitant fuzzy information, this situation is only interpreted as {81, 85, 90}, which could not reflect the preference of DM. In addition, probabilistic hesitant fuzzy information can reflect the opinions of DMs in group decision-making. For instance, when five experts were invited, they need to give their opinions. If one expert gives 70, the other three of them assign 83, and only one expert gives 91; the evaluation information will be expressed as {70(0.2), 83(0.6), 91(0.2)} in the form of probabilistic hesitant fuzzy information. If hesitant fuzzy information is used, the
evaluation information will be \([70, 83, 91]\). Obviously, the use of hesitant fuzzy information sometimes leads to information loss. Furthermore, the information attenuation will be amplified when the gap between these evaluation values is large. Hence, considering the limitations of hesitant fuzzy information in expressing the idea of individuals and in collecting ideas of a group, probabilistic hesitant fuzzy information is more suitable to describe the uncertainties of DMs.

From the theoretical analysis above, the novel TODIM with probabilistic hesitant fuzzy information has more advantages than the one with crisp number or the one with hesitant fuzzy information. It eases information distortion by adjusting itself to the original PT and avoids information attenuation by using probabilistic hesitant fuzzy information. The superiority of the proposed method is theoretically illustrated. Then, an illustrative example is given to show its advantages in further detail and to enhance the understanding of the proposed method.

### 4. Illustrative Example

This section presents an electric bus bid case with four different methods. They are the extension of two types of different fuzzy information including probabilistic hesitant fuzzy information and hesitant fuzzy information.

#### 4.1. Background of the Case

With the development of new energy technologies, electric buses have become one of the most mature areas of new energy vehicle applications. Electric bus is clean, low noise, and environmental protection, which greatly enhances the user’s experience. It has been reported by Bloomberg that the U.S. has a fleet of 300 electric public transportation buses in September 6, 2019 with the development of new energy technologies, electric buses have become one of the most mature areas of new energy vehicle applications. Electric bus is clean, low noise, and environmental protection, which greatly enhances the user’s experience. It has been reported by Bloomberg that the U.S. has a fleet of 300 electric public transportation buses in September 6, 2019.

**Hesitant Fuzzy Information**

**Step 1:** to better distinguish the probability in the evaluation information from the membership degree, it is magnified by 100 times. Then, the evaluation information is given as follows:

\[
\omega = \{0.34(0.68), 0.40(0.32)], [0.09(0.39), 0.11(0.61)], [0.19(0.56), 0.22(0.44)], [0.21(0.43), 0.27(0.57)]\}.
\]

**Step 2:** normalize the evaluation matrix of the four green suppliers and get the normalized weight information at the same time:

\[
\begin{align*}
\epsilon_1 & \quad \epsilon_2 & \quad \epsilon_3 & \quad \epsilon_4 \\
A_1 & \quad \{55(0.22), 70(0.16), 68(0.51), 73(0.27)] \quad [60(0.45), 66(0.39), 70(0.16)] & \quad [60(0.69), 68(0.21), 71(0.1)] & \quad [64(0.66), 72(0.32), 77(0.02)] \\
A_2 & \quad [62(0.28), 79(0.09), 77(0.63), 79(0.09)] & \quad [68(0.29), 77(0.68), 80(0.03)] & \quad [60(0.18), 73(0.21), 85(0.61)] & \quad [77(0.6), 88(0.36), 80(0.04)] \\
A_3 & \quad [63(0.32), 71(0.48), 77(0.2)] & \quad [66(0.39), 71(0.52), 77(0.09)] & \quad [68(0.59), 74(0.32), 79(0.09)] & \quad [71(0.53), 78(0.22), 81(0.25)] \\
A_4 & \quad [67(0.49), 72(0.44), 75(0.07)] & \quad [62(0.58), 69(0.3), 74(0.12)] & \quad [67(0.61), 71(0.26), 78(0.13)] & \quad [68(0.36), 73(0.49), 79(0.15)] \\
\end{align*}
\]

\[\omega' = (0.395, 0.112, 0.224, 0.269)\]

By adjusting itself to the original PT and avoids information attenuation by using probabilistic hesitant fuzzy information. It eases information distortion with probabilistic hesitant fuzzy information has more advantages in further detail and to enhance the understanding of the proposed method.

#### 4.2. Screening Process of the Novel TODIM with Probabilistic Hesitant Fuzzy Information

**Step 1:** to better distinguish the probability in the evaluation information from the membership degree, it is magnified by 100 times. Then, the evaluation information is given as follows:

\[
\begin{align*}
\epsilon_1 & \quad \epsilon_2 & \quad \epsilon_3 & \quad \epsilon_4 \\
A_1 & \quad \{55(0.22), 70(0.16), 68(0.51), 73(0.27)] \quad [70(0.16), 66(0.39), 70(0.45)] & \quad [71(0.1), 68(0.21), 62(0.69)] & \quad [71(0.1), 68(0.21), 62(0.69)] & \quad [70(0.16), 66(0.39), 70(0.45)] \\
A_2 & \quad [79(0.09), 62(0.28), 77(0.63), 79(0.09)] & \quad [80(0.03), 68(0.29), 77(0.68)] & \quad [60(0.18), 73(0.21), 85(0.61)] & \quad [80(0.04), 88(0.36), 77(0.60)] \\
A_3 & \quad [77(0.02), 63(0.32), 71(0.48)] & \quad [77(0.09), 66(0.39), 71(0.52)] & \quad [79(0.09), 74(0.32), 68(0.59)] & \quad [78(0.22), 81(0.25), 71(0.53)] \\
A_4 & \quad [75(0.07), 72(0.44), 67(0.49)] & \quad [74(0.12), 69(0.3), 62(0.58)] & \quad [78(0.13), 71(0.26), 67(0.61)] & \quad [79(0.15), 68(0.36), 73(0.49)] \\
\end{align*}
\]

\[\omega' = (0.395, 0.112, 0.224, 0.269)\]
Step 3: calculate the transformed probability weights according to equation (23), and the results are shown in Table 2.

Step 4: obtain the relative weights according to equation (24), and the results are shown in Table 3.

Step 5: work out the relative prospect dominance degrees of the alternative $A_i$ over the others under each criterion, which is determined by using equations (25) and (26), shown in Table 4.

Step 6: obtain the prospect dominance degrees of the alternative $A_i$ over the others by using equation (27), shown in Table 5.

Step 7: the overall prospect dominance degrees of each alternative is calculated by using equation (28), and the results are exhibited in Table 6.

Step 8: since $\Omega(A_2) > \Omega(A_3) > \Omega(A_4) > \Omega(A_1)$, there exists $A_2 \succ A_3 \succ A_4 \succ A_1$. The company $A_2$ should be selected in this bid.

4.3. Screening Process of the Extended TODIM with Probabilistic Hesitant Fuzzy Information

Step 1: the normalized evaluation matrix is transformed in the same way as shown in Step 1 and Step 2 in Section 4.2.

Step 2: calculate the relative weight of each criterion based on (6), shown in Table 7.

Step 3: obtain the relative dominance degrees $\phi_j (A_i, A_k)$ of the alternative $A_i$ over $A_k$ under the criterion $c_j$ as follows, and the result is exhibited in Table 8, and $\phi_j (A_i, A_i) = 0$ is not shown in this table:

When $c_j$ is the cost criterion, the relative prospect dominance degree is

\[
\phi_j^c (A_i, A_k) = \begin{cases} 
1 - \frac{1}{\lambda} \sqrt{\sum_{j=1}^{m} \omega_{jr}^{f}} d(h_{i,j}^l(p_{i,j}), h_{k,j}^l(p_{k,j})), & h_{i,j}^l(p_{i,j}) > h_{k,j}^l(p_{k,j}) , \\
0, & h_{i,j}^l(p_{i,j}) = h_{k,j}^l(p_{k,j}) , \\
-\frac{1}{\lambda} \sqrt{\sum_{j=1}^{m} \omega_{jr}^{f}} d(h_{i,j}^l(p_{i,j}), h_{k,j}^l(p_{k,j})), & h_{i,j}^l(p_{i,j}) < h_{k,j}^l(p_{k,j}) .
\end{cases}
\] (41)

where $d(h_{i,j}^l(p_{i,j}), h_{k,j}^l(p_{k,j}))$ is the distance of $h_{i,j}^l(p_{i,j})$ and $h_{k,j}^l(p_{k,j})$.  

Step 4: the prospect dominance degrees of the alternative $A_i$ over the others under each criterion are
4.4. Screening Process of the Novel TODIM with Hesitant Fuzzy Information

Step 1: obtain the evaluation matrix and the weight information as hesitant fuzzy information:

\[
Y = \begin{bmatrix}
55, 68, 73 & 60, 66, 70 & 62, 68, 71 & 64, 72, 77 \\
62, 77, 80 & 70, 73, 85 & 77, 80, 88  \\
63, 71, 77 & 66, 71, 77 & 68, 74, 79 & 71, 78, 81 \\
67, 72, 75 & 62, 69, 74 & 67, 71, 78 & 68, 73, 79
\end{bmatrix},
\]

\[
\omega = \{(0.34, 0.40), (0.09, 0.11), (0.19, 0.22), (0.21, 0.27)\}.
\]

Step 2: normalize the evaluation matrix. The normalized evaluation information matrix is the same as (42). Besides, the normalized weight information is based on equation (31).

Step 3: obtain the transformed probability weights by using equation (33), and the results are shown in Table 11. The comparison of the two HFEs \(h_{ij}^r\) and \(h_{kj}\) is determined by the score function (equation (32)) and the deviation function (equation (34)).

Step 4: obtain the relative weight of \(A_i\) over \(A_k\) based on equation (24).

Step 5: calculate the relative prospect dominance degrees \(\varphi_j^B(A_i, A_k)\) of the alternative \(A_i\) over \(A_k\) under the criterion \(c_j\). When \(c_j\) is the benefit criterion, the relative prospect dominance degree is calculated by using equation (35). Otherwise, the relative prospect dominance degree is calculated by using equation (36). The results are exhibited in Table 12.

Step 6: obtain the dominance degrees of the alternative \(A_i\) over the others by using equation (27).

Step 7: obtain the overall dominance degrees according to equation (28), shown in Table 13.

Step 8: since \(\Omega(A_3) > \Omega(A_2) > \Omega(A_4) > \Omega(A_1)\), we can get \(A_2 > A_3 > A_4 > A_1\). Thus, the company \(A_2\) should be selected in this bid.

4.5. Screening Process of the Extended TODIM with Hesitant Fuzzy Information

Step 1: obtain the evaluation matrix and the weight information as hesitant fuzzy information. They are the same as the information shown in (42) and (43), respectively.

Step 2: calculate relative weights, and the results are shown in Table 14:

\[
\omega_{jr} = \frac{\omega_j}{\omega_r} = \frac{\rho(h_{ij})}{\rho(h_{kj})}, \quad h_{ij} > h_{kj},
\]

where \(j, r \in M\), \(\rho(h_{w})\) is the score function shown in (32).

Step 3: calculate the relative dominance degrees \(\varphi_j^B(A_i, A_k)\) of the alternative \(A_i\) over \(A_k\) under the criterion \(c_j\) as follows, and the results are exhibited in Table 15:

\[
\varphi_j^B(A_i, A_k) = \begin{cases} 
0, & h_{ij} = h_{kj} \\
\frac{1}{\lambda} \sum_{j=1}^{m} \omega_{jr} d(h_{ij}, h_{kj}), & h_{ij} < h_{kj}
\end{cases}
\]

When \(c_j\) is the cost criterion, the relative dominance degree is

\[
\varphi_j^C(A_i, A_k) = \begin{cases} 
0, & h_{ij} = h_{kj} \\
\frac{1}{\lambda} \sum_{j=1}^{m} \omega_{jr} d(h_{ij}, h_{kj}), & h_{ij} < h_{kj}
\end{cases}
\]

where \(d(h_{ij}, h_{kj})\) is the distance of hesitant fuzzy information \(h_{ij}\) and \(h_{kj}\).

Step 4: obtain the dominance degrees of the alternative \(A_i\) over the others by using equation (27), shown in Table 16.

Step 5: obtain the overall dominance degrees by using equation (28), shown in Table 17.

Step 6: since \(\Omega(A_4) > \Omega(A_3) > \Omega(A_2) > \Omega(A_1)\), we can get \(A_2 > A_3 > A_4 > A_1\). Thus, the company \(A_2\) should be selected in this bid.
4.6. Analysis. In this section, we summarize the results of the above decision-making processes and display them in Table 18. By comparing the results of these four methods, the preponderance of combining the novel TODIM with probabilistic hesitant fuzzy information is fully illustrated.

From Table 18, the same ranking results \((A_2 > A_3 > A_4 > A_1)\) are presented from the four kinds of methods. Obviously, the company \(A_2\) is considered to be the optimal choice, and the company \(A_1\) is the worst choice. However, there are huge differences in the value of overall prospect dominance degrees obtained by the four methods.

Based on Table 18 and Figure 1, compared with Method 1 and Method 2, as well as Method 3 and Method 4, the difference of overall prospect dominance degrees between the alternatives \(A_1\) and \(A_3\), which use the novel TODIM based on PT, is greater than the extended one. We attribute this phenomenon to the different risk attitudes of the DMs concerning about gains and losses which are considered in PT. Compared with Method 1 and Method 3, as well as Method 2 and Method 4, we can discover that the overall prospect dominance degrees obtained from the methods, which adopt probabilistic hesitant fuzzy information and concern different preference degrees of hesitant values, are smaller than those obtained from hesitant fuzzy information. We contribute this phenomenon in reflecting more details of DMs are produced by using the probabilistic fuzzy information. The result also illustrates that probabilistic hesitant fuzzy information is good at expressing the DMs’ evaluation information and different preferences for hesitant fuzzy values. Therefore, we believe the novel TODIM with probabilistic hesitant fuzzy information is more comprehensive and effective in decision-making.

5. Comparative Analysis

To better illustrate the effectiveness of the proposed method, we carry out the comparative analysis with TOPSIS, sensitivity analysis, and simulation analysis. The results of the analyses strongly support the superiority of the developed method.

5.1. Comparative Analysis with TOPSIS. Analysis in Section 4.6 focuses on comparing the different extensions of TODIM with different fuzzy information. In this section, to illustrate the advantages of TODIM with probabilistic hesitant fuzzy information, we compare it with TOPSIS under probabilistic hesitant fuzzy environment. It offers a more convincing analysis because this does not focus on the psychological factor of DMs. Motivated by He and Xu [43] and Dagdeviren et al. [44], probabilistic hesitant fuzzy information is extended to TOPSIS. By applying the example in Section 4, we use this method to obtain the best alternative.

First, we obtain the normalized evaluation matrix which is the same as the matrix of Step 2 in Section 4.2. Second, we find the positive ideal alternative \(A^+\) which is the alternative with the closest distance to ideal solution and the negative ideal alternative \(A^-\) which is farthest to the ideal solution by using equations (49) and (50) based on each criterion. The results are presented in Table 19:

For Table 18, \(A_1\) and \(A_2\) are the two best choices. However, the distance measure of \(A_2\) to the positive ideal solution is smaller than that of \(A_1\) which is the alternative \(A_1\) is the worst choice. Therefore, we believe the novel TODIM with probabilistic hesitant fuzzy information is fully illustrated.

\[
A^+ = \{h_i^+, h_2^+, \ldots, h_n^+\} = \{\max_i h_{ij} \mid i = 1, 2, \ldots, n\}, \quad (47)
\]

\[
A^- = \{h_1^-, h_2^-, \ldots, h_n^-\} = \{\max_i h_{ij} \mid i = 1, 2, \ldots, n\}. \quad (48)
\]

Then, we use equations (49) and (50) to compute the distances between the alternative and the ideal solution, where the distance measures are obtained by using equation (14), and the results are shown in Table 20:

\[
D_i^* = \sum_{j=1}^{m} w_j d_i^j(h_{ij}(p_{ij}), h_i^+). \quad (49)
\]

\[
D_i^{-} = \sum_{j=1}^{m} w_j d_i^- j(h_{ij}(p_{ij}), h_i^-). \quad (50)
\]

We can easily obtain the relative closeness coefficients of each alternative by using equation (51). They are listed in Table 21. Hence, the ranking of the alternatives obtained from TOPSIS with probabilistic hesitant fuzzy information is \(A_1 > A_4 > A_3 > A_2\):

\[
C_i^* = \frac{D_i^*}{D_i^{-} + D_i^+}, \quad i = 1, 2, \ldots, n. \quad (51)
\]

The superiority of the proposed method can be seen from the results shown in Tables 18 and 21. The ranking result obtained from the TOPSIS method is different from the one obtained from the novel TODIM with probabilistic hesitant fuzzy information, and the ranking result of the middle two alternatives \((A_3\) and \(A_4\) is different in those two methods. This distinction can be attributed to the following reasons, which are also the advantages of the proposed method. First, the novel TODIM has identified more information on DMs. It not only involves the transformed probability weight but also considers the difference between every two alternatives, instead of focusing on the difference between the alternative and the positive ideal solution or the negative ideal solution shown in TOPSIS. In addition, TOPSIS with probabilistic hesitant fuzzy information does not reflect the psychological factors of DMs, while probabilistic hesitant fuzzy information describes the uncertain evaluation information, and all participants usually are bounded rational in real decision-making situations. This defect is fully compensated in the TODIM by considering the risk attitudes for gains and losses, which makes the results more accurate, objective, and more consistent with practical experience.

By addressing the comparison of the existing method, the demand for combining probabilistic hesitant fuzzy information with TODIM has also been fully demonstrated. The overall prospect dominance degrees of TODIM and the relative closeness coefficients of the TOPSIS with probabilistic hesitant fuzzy environment are much smaller than the results of TODIM with hesitant fuzzy information. It is strongly proven that the probabilistic hesitant fuzzy information has discerned and reflected more information in the decision-making process.
In the beginning, we analyze the difference of sensitivity between the novel TODIM with the same hesitant fuzzy information and the hesitant fuzzy information in a fixed method. Moreover, this paper presents the comparative analyses to illustrate the advantages of the novel TODIM based hesitant fuzzy information, respectively. Both comparative analyses fully illustrate the advantages of the novel TODIM based on PT. Therefore, we use the overall prospect dominance degrees to show the strength of the novel TODIM.

5.2. Sensitivity Analysis Based on the Parameter Values.
To better illustrate the advantages of the novel TODIM with probabilistic hesitant fuzzy information, this part conducts a sensitivity analysis of the novel TODIM and the extended TODIM with probabilistic hesitant fuzzy information and hesitant fuzzy information, respectively. Both comparative analyses fully illustrate the advantages of the novel TODIM based on PT. Moreover, this paper presents the comparative analysis to illustrate the superiority of probabilistic hesitant fuzzy information in reflecting more evaluation information of DMs by comparing it with hesitant fuzzy information in a fixed method.

5.2.1. Sensitivity Analysis of the Novel TODIM and the Extended TODIM with the Same Fuzzy Information. In the beginning, we analyze the difference of sensitivity between the novel TODIM and the extended TODIM with the same fuzzy information. According to the results, we recognize that no matter how the parameter λ changes, there are no significant changes for the ranking results from each method. Therefore, we use the overall prospect dominance degrees to show the strength of the novel TODIM.

(1) Sensitivity Analysis of the Novel TODIM and the Extended TODIM with Probabilistic Hesitant Fuzzy Information. Since λ is the only common parameter both in the novel TODIM and in the extended TODIM with probabilistic hesitant fuzzy information, the fluctuation of the overall prospect dominance degree can be easily observed by changing the value of λ (1.25 ≤ λ ≤ 2.25) which is shown in Figure 2.

The overall prospect dominance degrees of the first and the last alternative remain unchanged which is naturally determined by the TODIM itself, and they are 1 and 0 separately when the ranking result is unchanged. Subsequently, making the alternatives A4 as an analysis group, when λ varies, the fluctuation range of the overall prospect dominance degree from the novel TODIM is smaller than the one obtained from the extended TODIM, which indicates that the novel TODIM is stable. Besides, the overall prospect dominance value obtained from the two methods shows a reverse trend. The main reason is that λ is proportional to the dominance function in the novel TODIM, and it is also proportional to the overall prospect dominance degree. In the extended TODIM, the dominance function is affected by the reciprocal form of λ, so λ is inversely proportional to the dominant function. This kind of reverse trend also can be found in the following analysis under the hesitant fuzzy environment.

Table 19: The positive ideal solution and negative ideal solution.

|      | c₁       | c₂       | c₃       | c₄       |
|------|----------|----------|----------|----------|
| A⁺   | [79 (0.09), 62 (0.28), 77 (0.63)] | [80 (0.03), 68 (0.29), 77 (0.68)] | [60 (0.18), 73 (0.21), 85 (0.61)] | [80 (0.04), 88 (0.36), 77 (0.60)] |
| A⁻   | [55 (0.22), 73 (0.27), 68 (0.51)] | [70 (0.16), 66 (0.39), 60 (0.45)] | [71 (0.11), 68 (0.21), 62 (0.69)] | [77 (0.02), 72 (0.32), 64 (0.66)] |

Table 20: Distances between the alternatives and the idea solution.

|      | A₁   | A₂   | A₃   | A₄   |
|------|------|------|------|------|
| D⁺   | 6.5980 | 0.0000 | 9.1503 | 8.5449 |
| D⁻   | 0.0000 | 6.5980 | 4.0679 | 5.6463 |

Table 21: The relative closeness coefficients.

|      | A₁   | A₂   | A₃   | A₄   |
|------|------|------|------|------|
| C⁺   | 0.00 | 1.00 | 0.31 | 0.40 |
Sensitivity Analysis of the Novel TODIM and the Extended TODIM with Hesitant Fuzzy Information. The changes of the overall prospect dominance degree in Figure 3 are obtained by altering the value of $\lambda$ ($1.25 \leq \lambda \leq 2.25$) in the novel TODIM and the extended TODIM with hesitant fuzzy information. According to Figure 3, it is apparent that the overall prospect dominance degrees of the alternatives $A_1$ and $A_2$ stay constant when the parameter $\lambda$ varies. At the same time, the fluctuation of the overall prospect dominance degree from the novel TODIM is smaller than that from the extended TODIM which also demonstrates that the novel TODIM is stable.

5.2.2. Sensitivity Analysis of the Novel TODIM and the Extended TODIM Based on Different Types of Fuzzy Information. This section presents two sets of comparative analyses to illustrate the advantages of probabilistic hesitant fuzzy information in expressing the perceptions of the DMs. The first one is the novel TODIM with probabilistic hesitant fuzzy information and with hesitant fuzzy information. The second one is the extended TODIM with probabilistic hesitant fuzzy information and with hesitant fuzzy information. We find that the ranking results of each alternative keep unchanged when the parameters change. Subsequently, the overall prospect dominance degrees are used to show the advantages of probabilistic hesitant fuzzy information.

(1) Sensitivity Analysis of the Novel TODIM with Probabilistic Hesitant Fuzzy Information and Compared with Hesitant Fuzzy Information. Since many parameters are used in the novel TODIM, this part presents the changes of overall prospect dominance degree of each alternative when the parameters change, which are shown in Figures 4–8.

Figure 2: Sensitivity analysis with probabilistic hesitant fuzzy information. $1.25 \leq \lambda \leq 2.25$, $\alpha = \beta = 0.88$, $\delta = 0.69$, and $\gamma = 0.61$. (a) $A_1$; (b) $A_2$; (c) $A_3$; (d) $A_4$. 

Figure 4 presents the fluctuation of the overall prospect dominance degree from the novel TODIM with probabilistic hesitant fuzzy information and with hesitant fuzzy information separately by changing the parameter $\lambda$ ($1.25 \leq \lambda \leq 2.25$). We can clearly see that for the alternatives $A_3$ and $A_4$, the changes of overall prospect dominance degree obtained by probabilistic hesitant fuzzy information are smaller than the one obtained by hesitant fuzzy information.
information, which indicates the stability of probabilistic hesitant fuzzy information.

Figure 5 presents the fluctuation of the overall prospect dominance degree from the novel TODIM with probabilistic hesitant fuzzy information and with hesitant fuzzy information separately by changing the parameter $\alpha$ ($0.68 \leq \alpha \leq 1.21$). It is obvious that for the alternatives $A_3$ and $A_4$, the changes of overall prospect dominance degree obtained by probabilistic hesitant fuzzy information are smaller than the one obtained by hesitant fuzzy information, which also indicates that the probabilistic hesitant fuzzy information is stable.

Figure 6 presents the fluctuation of the overall prospect dominance degree from the novel TODIM with probabilistic hesitant fuzzy information and with hesitant fuzzy information separately by changing the parameter $\beta$ ($0.68 \leq \beta \leq 1.02$). We can see that for the alternatives $A_3$ and $A_4$, the changes of overall prospect dominance degree obtained by probabilistic hesitant fuzzy information and by hesitant fuzzy information have obvious differences. For the alternative $A_4$, the change trend of the methods with two different types of information goes in the same direction; however, the greater fluctuation occurs in the method with hesitant fuzzy information. For the alternative $A_3$, the change trend of the two methods goes in the opposite direction.

Figure 7 presents the fluctuation of the overall prospect dominance degree from the novel TODIM with probabilistic hesitant fuzzy information and with hesitant fuzzy information separately by changing the parameter $\delta$ ($0.36 \leq \delta \leq 0.84$). For the alternatives $A_3$ and $A_4$, significant fluctuation can be observed in the overall prospect dominance degree which is obtained by probabilistic hesitant fuzzy information, while small changes happen to the one obtained from hesitant fuzzy information. For both the alternatives $A_3$ and $A_4$, the overall prospect dominance degrees obtained from the two methods tend to change in the same direction.

Figure 8 presents the fluctuation of the overall prospect dominance degree obtained from the novel TODIM with
Figure 4: Sensitivity analysis of the novel TODIM by changing $\lambda$. $1.25 \leq \lambda \leq 2.25$, $\alpha = \beta = 0.88$, $\delta = 0.69$, and $\gamma = 0.61$.
(a) $A_1$; (b) $A_2$; (c) $A_3$; (d) $A_4$.

Figure 5: Continued.
Figure 5: Sensitivity analysis of novel TODIM by changing $\alpha$. $0.68 \leq \alpha \leq 1.21$, $\beta = 0.88$, $\delta = 0.69$, $\gamma = 0.61$, and $\lambda = 2.25$. (a) $A_1$; (b) $A_2$; (c) $A_3$; (d) $A_4$.

Figure 6: Sensitivity analysis of novel TODIM by changing $\beta$. $0.68 \leq \beta \leq 1.02$, $\alpha = 0.88$, $\delta = 0.69$, $\gamma = 0.61$, and $\lambda = 2.25$. (a) $A_1$; (b) $A_2$; (c) $A_3$; (d) $A_4$. 
probabilistic hesitant fuzzy information and with hesitant fuzzy information separately by changing the parameter \( \gamma \) (0.55 \leq \gamma \leq 0.721). For the alternatives \( A_3 \) and \( A_4 \), the overall prospect dominance degrees obtained from these two kinds of methods are almost unchanged. No matter how \( A_3 \) or \( A_4 \) alters, the overall prospects tend to change in the same direction.

(2) Sensitivity Analysis of the Extended TODIM with Probabilistic Hesitant Fuzzy Information and Compared with Hesitant Fuzzy Information. Since there is only one mutual parameter \( \lambda \) in the extended TODIM, this section considers the changes of the overall prospect dominance degree by changing it.

Figure 9 presents the fluctuation of the overall prospect dominance degree by changing the parameter \( \lambda \) (1.25 \leq \lambda \leq 2.25) in the extended TODIM with probabilistic hesitant fuzzy information and with hesitant fuzzy information. For the alternatives \( A_3 \) and \( A_4 \), the overall prospect dominance degree obtained by hesitant fuzzy information changes significantly, while the one obtained by probabilistic hesitant fuzzy information is nearly unchanged, and it also continues to decrease when increasing the value of the parameter \( \lambda \). Besides, regardless of the alternative \( A_3 \) or \( A_4 \), the overall prospect dominance degrees obtained from the two methods tend to change in the same direction.

In summary, the novel TODIM is stable and effective (Figures 2 and 3) with the same type of fuzzy information. The overall prospect dominance degree obtained by probabilistic hesitant fuzzy information changes slightly than the one obtained by hesitant fuzzy information (Figures 4 and 9).
The results illustrate that the probabilistic hesitant fuzzy information is steadier and contains more information from DMs. Figures 5–9 present the changes in the overall prospect dominance degree when changing other parameters with different types of fuzzy information from the novel TODIM.

5.3. Simulation Analysis. After sensitivity analysis of the parameters based on one sample, we present the analysis results of 1000 sets of data which are randomly generated by MATLAB software. The ranking results are shown in Table 22 and Figures 10–13.

From Table 22, the ranking results of 1000 sets of random data by using different kinds of methods are presented. 393 sets of data have the same ranking results in these four methods. With probabilistic hesitant fuzzy information, 633 sets of data have the same ranking results by using the novel TODIM and the extended TODIM. However, with hesitant fuzzy information, 654 sets of data are observed to have the same ranking results by using the novel TODIM and the extended TODIM. The number of ranking results with hesitant fuzzy information is bigger than that of the ranking results with probabilistic hesitant fuzzy information because the latter one includes more information and it is more difficult to get the same ranking result. 622 sets of data have the same ranking results by using the novel TODIM with probabilistic hesitant fuzzy information and the novel TODIM with hesitant fuzzy information.

According to the results, there are large numbers of random data with the same ranking results, which shows the feasibility and applicability of the proposed method. On
Extended TODIM with probabilistic hesitant fuzzy information
Extended TODIM with hesitant fuzzy information

Figure 9: Sensitivity analysis of extended TODIM by changing $\lambda$. $1.25 \leq \lambda \leq 2.25$, $\alpha = \beta = 0.88$, $\gamma = 0.61$, and $\delta = 0.69$. (a) $A_1$; (b) $A_2$; (c) $A_3$; (d) $A_4$.

Figure 10: Ranking results of the alternative $A_1$ (50 sets of random samples).
the contrary, there are still some existing sets of data samples with different ranking results which indicates the differences between these methods. We attribute this difference to the following two points: (1) Compared with hesitant fuzzy information, probabilistic hesitant fuzzy information contains more original decision information. The former one is just a special form of probabilistic hesitant fuzzy information when the probability is equal, and the latter one is more general. (2) Compared with the extended TODIM, the novel TODIM based on PT rewrites the dominance function of TODIM and makes it more in line with the actual decision-making environment, which contains more details about risk attitudes for gains and losses of DMs. Based on Table 22, the ranking results are presented in the form of numbers. To observe the ranking results of each alternative more intuitively, Figures 10–13 present the ranking results of the first 50 sets of random data in detail.

Table 22: Ranking results of each method with 1000 sets of random samples.

| Methods                                      | Results                                      | |
|----------------------------------------------|----------------------------------------------|---|
| Novel TODIM with probabilistic hesitant fuzzy information | The number of the same ranking result | The number of the different ranking result |
| Extended TODIM with probabilistic hesitant fuzzy information | 393 | 607 |
| Novel TODIM with hesitant fuzzy information | 633 | 367 |
| Extended TODIM with hesitant fuzzy information | 654 | 346 |
| Extended TODIM with probabilistic hesitant fuzzy information | 622 | 378 |
| Extended TODIM with hesitant fuzzy information | 679 | 321 |

Figure 11: Ranking results of the alternative $A_2$ (50 sets of random samples).

Figure 12: Ranking results of the alternative $A_1$ (50 sets of random samples).
6. Conclusions

The TODIM is a MCDM method based on PT which shows the risk aversion attitude through the dominance function. Based on equation (6) of the classical TODIM, the relative weight is calculated by the one-dimensional probability weight; however, according to equation (3), the original PT considers that the DMs adopt the nonlinear transformed probability weight function in the decision-making process. Without a doubt, the classical TODIM ignores the effect of the transformed probability weighting function on decision-making results. Besides, it is easy to recognize from equation (8) that the multiply value of relative probability weight and the perceived gain or loss value are regarded as the overall preferences of the DMs in the classic TODIM. However, the different risk attitudes for gains and losses are mainly reflected by the value function in the classic PT according to equation (2), and it takes the product of the value function and the weight function as a decision reference. Such a phenomenon has not been well proclaimed in the classical TODIM.

At the same time, it is considered that the DMs are more likely to express their perceptions as the form of probabilistic hesitant fuzzy information under the highly uncertain circumstance because they can express their preference for the hesitant fuzzy values by probabilistic hesitant fuzzy information. That is the reason why we propose a probabilistic hesitant fuzzy TODIM based on a new perspective of PT. To illustrate the feasibility and effectiveness of the proposed method, this paper also presents the novel TODIM with hesitant fuzzy information and TOPSIS with probabilistic hesitant fuzzy information.

The most important innovation of this paper is that an improved TODIM based on PT with probabilistic hesitant fuzzy information is proposed. This paper realizes the reconstruction of the relative dominance function of the classical TODIM based on PT and integrates probabilistic hesitant fuzzy information into the improved TODIM. Moreover, this paper combines the novel TODIM with the hesitant fuzzy information. Furthermore, a case study, parameter sensitivity analysis, and simulation analysis are all carried out to show the advantages of the proposed methods and the differences between these methods and the existing ones.

The proposed method has some certain advantages in expressing fuzzy information of the DMs. For example, probabilistic hesitant fuzzy information can express the degree of hesitation by using different probabilities for hesitant values. On the contrary, probability can also represent the proportion that the DMs give the same hesitation value in group decision-making. There is no doubt that group decision-making has become an effective way to solve complicated problems and consensus is the precondition to make a reasonable decision. Hence, more concentration should be put into consensus problems based on PT in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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