A New Method for Estimating High Frequency Attenuation Parameter of Kappa

Yuan Ji1,a, Wang Haiyun2,b

1 Institute of Engineering Mechanics, China Earthquake Administration, Harbin, 150080, China
2 Institute of Engineering Mechanics, China Earthquake Administration, Harbin, 150080, China
a-e-mail: yuanji18@mails.ucas.ac.cn, b-e-mail: haiyun@iem.ac.cn

Abstract: The Fourier amplitude spectrum of acceleration will drop rapidly in the high frequency range. Anderson and Hough quantitatively studied the attenuation law of the acceleration Fourier spectrum above the corner frequency, and proposed the high-frequency attenuation parameter Kappa (κ) to describe this phenomenon. It has important applications in the field of engineering earthquakes, and is widely used by seismologists and engineers in the work of ground motion simulation and attenuation relations. Based on the Anderson method, this paper proposed a method of first smoothing the Fourier acceleration spectrum with a 0.4 Hz Parzen window and then fitting the solution value. The Fukushima earthquake data from Japan's KiK-net seismic network on February 13, 2021 was used to verify the reliability of the method. The results showed that when the width of the Parzen window was set to 0.4 Hz, the calculation accuracy of the value was effectively improved.

1. Introduction
Kappa proposed by Anderson and Hough (1984) is one of the important parameters to simulate ground motion, which is of great significance to correctly evaluate the seismic risk of an area [1]. In the early 1980s, with the improvement of random ground motion simulation methods, seismologists observed that the distance attenuation term of seismic wave propagation in the crust was insufficient to explain the attenuation in the high frequency band of Fourier acceleration spectrum [2][3]. According to Brune’s $\omega^{-2}$ source spectrum model [3], the high-frequency part of the Fourier acceleration spectrum should be flat, while Hanks observed that when the frequency was greater than a certain value, the Fourier acceleration spectrum would decline rapidly [4]. In 1984, Anderson and Hough proposed the high frequency attenuation parameter Kappa (κ) model to explain the attenuation of Fourier acceleration amplitude spectrum above the corner frequency [1]:

$$A(f) = A_0 \exp(-\pi \kappa f), \quad f > f_E$$

In the equation, $A_0$ is a factor depending on the source characteristics and propagation distance; $\kappa$ represents the frequency spectrum attenuation parameter Kappa; $f_E$ is the corresponding frequency of the starting point of the approximate linear decline of the Fourier spectrum in the semi-logarithmic coordinates. The attenuation of the frequency spectrum exceeding this frequency corresponds to the $f$
approximate linear in the linear logarithmic diagram, that is, \( \kappa \) can be calculated by fitting the high frequency part of the Fourier spectrum in the semi-logarithmic coordinates:

\[
\kappa = -\frac{d \ln A(f)}{\pi df}, f_1 \leq f \leq f_2
\]

(2)

In the equation, \( A(f) \) is the Fourier acceleration amplitude spectrum; \( f_1 \) is the starting frequency of high frequency attenuation; \( f_2 \) is the end frequency of high frequency attenuation. This parameter in essence describes the deviation between the observed Fourier acceleration amplitude spectrum and Brune’s \( \omega^{-2} \) source model in the high frequency part [5]. After more than 30 years of development, it has been widely used in random synthetic earthquake. However, there is an inherent defect of subjectivity in Anderson’s method of determining \( \kappa \), that is, \( f_1 \) and \( f_2 \) determined by different researchers are often different, which leads to great differences in the estimated \( \kappa \) values of the same data. For instance, when calculating the \( \kappa \) parameters of Wenchuan Earthquake in China, the results of Fu et al. (2017) and Sun et al. (2013) were quite different [6][7].

Based on Anderson’s method, this paper proposes a new method to reduce human error and improve calculation accuracy. By smoothing the Fourier acceleration spectrum with the Parzen window, the influence of different window widths on the \( \kappa \) value is studied. Combined with the variation trend of goodness of fit and variance with the window widths, the most favorable Parzen window width for improving the \( \kappa \) value accuracy is analyzed, which is applied to the original method as a new method to calculate the \( \kappa \) value.

2. Methods and Data

2.1. Methods

Anderson’s method is to obtain \( \kappa \) by fitting the high frequency part of Fourier spectrum of acceleration in semi-logarithmic coordinates. The key of this method lies in the range of fitting interval, that is, the selection of \( f_1 \) and \( f_2 \) in formula (2). In the calculation process, \( f_1 \) and \( f_2 \) are generally selected by visual inspection [1]. In order to reduce the influence of source effect on \( \kappa \), \( f_1 \) must be greater than the corner frequency \( f_0 \) [8]. Anderson suggested that \( f_1 \) should be greater than or equal to \( 1.5f_0 \) \((f_1 \geq 1.5f_0)\) to retain the buffer range. Given the influence of instrument response and noise level, each record in the general database is assigned a minimum and maximum available frequency. The effective frequency band range of Japan KiK-net seismic network is generally 0 ~ 100Hz, and the measured \( \kappa \) value should also be in this range. In the calculation, in order to ensure the stability of the slope of the estimated spectrum and reduce the influence of the local anomaly of the Fourier acceleration spectrum on the results, the measurement range of \( \kappa \) must be wide enough, usually greater than the minimum width of 8 ~ 10 Hz [9].

Different researchers’ estimation of \( \kappa \) varies greatly, mainly due to the significant difference in the interval range of determining \( f_1 \) and \( f_2 \). Figure 1a is the unsmoothed original Fourier spectrum, and Figure 1b is the Fourier spectrum smoothed with a 0.4 Hz Parzen window. From Fig.1a, it can be seen that the boundary of the linear downward trend of the Fourier spectrum is not clear. If the Parzen window is not used to smooth the spectrum, the fitting interval may be extended to 30 Hz, which would lead to
a larger error of the resultant $\kappa$ value. In Fig.1b, it is obvious that the downward trend of Fourier spectrum changes when the frequency is greater than 20 Hz, so the upper limit of fitting interval should be 20 Hz. This study first proposes to smooth the recorded Fourier acceleration spectrum with a 0.4 Hz Parzen window. Then $f_1$ and $f_2$ are selected, and the data points between $f_1$ and $f_2$ are fitted to obtain the $\kappa$ value (Fig. 1b). The range of fitting interval determined in this way is more accurate. The $\kappa$ values obtained by different researchers using this method should also be consistent.

![Figure 1](image1.png)

**Figure 1.** Schematic Diagram of Determining Kappa Parameter Based on Fourier Acceleration Spectrum

(a) original Fourier spectrum; (b) Fourier spectrum smoothed with a 0.4 Hz Parzen window

### 2.2. Data

In this paper, the data of Fukushima earthquake ($M_W= 7.3$) on February 13, 2021 in Japan’s KiK-net seismic network are used, and the records of 15 stations are selected to verify the new method. The information of the MYGH14 station is not provided. As for the other 14 stations, according to the site classification standards of National Earthquake Hazard Reduction Program (NEHRP) [10], the dominant site is the D-type site, accounting for 57%; according to the site classification standards of Code for Seismic Design of Buildings (2010) [11], the Class II site is the dominant site, accounting for 86%. The specific station information is shown in Table 1.

| Number | Station Code | Longitude (°E) | Latitude (°N) | $V_{s30}$ (m/s) | Type of Sites (USA) | $V_{s20}$ or $V_{s500}$ (m/s) | Type of Sites (China) |
|--------|--------------|---------------|---------------|----------------|---------------------|------------------------------|---------------------|
| 1      | AKTH03       | 39.2223       | 140.1283      | 320.23         | D                   | 278                          | II                  |
| 2      | AOMH12       | 40.5846       | 141.1547      | 281            | D                   | 235.29                       | II                  |
| 3      | FKSH09       | 37.353        | 140.4264      | 584.62         | C                   | 244.19                       | II                  |
| 4      | FKSH11       | 37.2006       | 140.3386      | 239.83         | D                   | 235.04                       | II                  |
| 5      | FKSH14       | 37.0264       | 140.9702      | 236.56         | D                   | 219.53                       | III                 |
| 6      | FKSH18       | 37.4894       | 140.538       | 307.18         | D                   | 184.8                        | II                  |
| 7      | FKSH19       | 37.4703       | 140.7227      | 338.06         | D                   | 255                          | II                  |
| 8      | FKSH20       | 37.4911       | 140.9871      | 403.85         | C                   | 437.5                        | II                  |
| 9      | IBRH19       | 36.2137       | 140.0893      | 692.31         | C                   | 100                          | II                  |
| 10     | MYGH07       | 38.1802       | 140.6405      | 365.63         | C                   | 334.29                       | II                  |
| 11     | MYGH09       | 38.0091       | 140.6027      | 358.25         | D                   | 315.79                       | II                  |
| 12     | MYGH10       | 37.9411       | 140.8924      | 347.54         | D                   | 329.59                       | II                  |
| 13     | MYGH13       | 38.6993       | 141.4176      | 570.59         | C                   | 250                          | II                  |
| 14     | MYGH14       | 38.34         | 140.9551      | \             | \                   | \                            | \                  |
| 15     | YMTH11       | 38.7132       | 140.5544      | 411.97         | C                   | 336.21                       | II                  |

Note: In the table, $V_{s30}$ refers to the equivalent shear wave velocity at the depth of 30 m; $V_{s20}$ refers to the equivalent shear wave velocity at the depth of 20 m; $V_{s500}$ refers to the equivalent shear wave velocity of soil layer corresponding to 500 m/s.
3. Method Validation

In order to make the calculated value as accurate as possible, in the process of fitting, this paper takes the width of the Parzen window to increase from 0.1 Hz to 1.0 Hz at a step length of 0.1 Hz, and describes the variation trend of the value, goodness of fit and variance with the increase of the width of the Parzen window. Fig. 2 evaluates the accuracy of fitting results from the perspective of goodness of fit and variance. It can be seen from Fig. 2a that the goodness of fit is in a rapid growth stage when the window width is 0.1 ~ 0.2 Hz, and in a slow growth stage when the window width is 0.2 ~ 0.4 Hz. When the window width is increased to 0.4 Hz, the goodness of fit is basically unchanged. Similarly, in Fig. 2b, the variance is in the rapid decline stage when the window width is 0.1 ~ 0.2 Hz, and in the slow decline stage when the window width is 0.2 ~ 0.4 Hz. When the window width is increased to 0.4 Hz, the variance is basically unchanged. From the size of the values, with the increase of the window width, the goodness of fit tends to approach 1 and the variance tends to approach 0.

Fig. 3 shows the trend of the value. It can be seen from the figure that with the increase of the window width, the value first increases gradually, and reaches the peak value when the window width is 0.4 Hz. When the window width increases gradually, the value decreases gradually and tends to be stable. When the width of the Parzen window increases to more than 0.4 Hz, the smoothing degree is too high, and the trend of over-fitting appears, so the value gradually decreases. Combined with Fig. 2 and Fig. 3, it can be concluded that when the width of the Parzen window is 0.4 Hz, the calculation accuracy of the value is effectively improved.
4. Conclusion

Based on Anderson’s classical Fourier acceleration spectrum method, this paper presented a method of smoothing the Fourier spectrum by using the Parzen window to improve the calculation accuracy. By using the Fukushima earthquake data of February 13, 2021 from Japan’s KiK-net seismic network, the variation trend of the kappa value with the widths of the Parzen window was compared, and the Parzen window width with the most accurate estimation of the kappa value was obtained. The conclusion was verified from the angle of goodness of fit and variance. The results showed that the most appropriate width of the Parzen window was 0.4 Hz. The kappa value estimated by this method can effectively improve the calculation accuracy of Anderson’s classical method, which indicates that the method is feasible.

Reference

[1] Anderson J G, Hough S E. A model for the shape of the Fourier amplitude spectrum of acceleration at high frequencies [J]. Bulletin of the Seismological Society of America, 1984, 74(5): 1969-1993.
[2] Aki K. Scaling law of seismic spectrum [J]. Journal of geophysical research, 1967, 72(4): 1217-1231.
[3] Brune J N. Tectonic stress and the spectra of seismic shear waves from earthquakes [J]. Journal of geophysical research, 1970, 75(26): 4997-5009.
[4] Hanks T C. f max [J]. Bulletin of the Seismological Society of America, 1982, 72(6A): 1867-1879.
[5] Ktenidou O J, Abrahamson N A, Darragh R B, et al. A methodology for the estimation of kappa (κ) for large datasets. Example application to rock sites in the NGA-East database [J]. 2016.
[6] Fu L, Li X J. 2017. The kappa (κ) model of the Longmenshan region and its application to simulation of strong ground-motion by the Wenchuan Ms8.0 earthquake. Chinese J. Geophys. (in Chinese), 060(008): 2935-2947.
[7] Sun X, Tao X, Duan S, et al. Kappa (κ) derived from accelerograms recorded in the 2008 Wenchuan mainshock, Sichuan, China[J]. Journal of Asian Earth Sciences, 2013, 73: 306-316.
[8] Anderson J G. Implication of attenuation for studies of the earthquake source [J]. Earthquake Source Mechanics, 1986, 37: 311-318.
[9] Anderson J G. Physical processes that control strong ground motion [J]. 2015.
[10] NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures (2003 edition) [S]. Washington DC: Building Seismic Safety Council, 2004.
[11] Ministry of Housing and Urban-Rural Construction of the People’s Republic of China Code for seismic design for buildings (GB 50011-2010) [S]. Beijing: China Building Industry Press, 2010. (in Chinese)