Abstract—An l₀-regularized linear regression for a sparse signal reconstruction is implemented based on the quadratic unconstrained binary optimization (QUBO) formulation. In this method, the sparse signals are quantized and expressed as bit sequences. By transforming l₀-norm to a quadratic form of these bits, the quadratic objective function is provided to be optimized by the solver specialized for QUBO, such as the quantum annealer. Numerical experiments with a commercially available quantum annealer show the proposed method demonstrates the reconstruction success rate close to conventional methods based on the orthogonal matching pursuit (OMP) and the least absolute shrinkage and selection operator (LASSO).

I. INTRODUCTION

Sparse signal reconstruction is a technique that recovers the original signals from relatively few observed signals. Although this problem is usually ill-posed, there are several algorithms that perfectly recover the original signals with sufficient sparsity [1,2]. These algorithms can be widely applied to signal processing that requires large number of observations with few sensors. For example, direction-of-arrival (DOA) estimation, that detects and estimates the directions of the multiple signal sources with few sensors such as microphones and radars is suitable for these algorithms [3]. In these algorithms, the reconstruction is usually formulated as l₁ -regularized linear regression [1], an unconstrained extension of l₁-norm minimization that minimizes the sum of absolute values of the sparse signals. When the order of the regularization is generalized as p, the objective function of the l_p-regularized linear regression is described as,

\[ L_p(z) = \frac{1}{2\gamma_p} ||x - Az||_p^2 + ||z||_p, \]  

where \( z \in \mathbb{C}^M \) is a vector for the sparse signals, \( x \in \mathbb{C}^N \) is a vector for the observed signals, \( A \in \mathbb{C}^{N \times M} \) with \( (N < M) \) is an observation matrix, \( \gamma_p \) is a regularization parameter and \( ||z||_p \) represents p-norm of z. For \( p = 0 \), (1) is the objective function for the l₀-norm minimization that minimizes the number of nonzero values of the sparse signals. The l₁-norm minimization requires more conditions for success reconstruction than l₀-norm minimizations [4], thus the latter one is expected to work more stable than the former one. However the implementation is practically difficult due to the potential threat of the combinatorial explosion, which forces us to use approximate approaches based on greedy policy, such as orthogonal matching pursuit (OMP) [5].

II. SPARSE SIGNAL RECONSTRUCTION

A. Conventional techniques

To solve a QUBO by quantum annealing, the binary variables to be optimized need to be expressed as quantized spins as \( s \in \{-1,1\}^n \). When \( b = (1 - s)/2 \) is substituted into (2), the objective function for the quantum annealing is expressed as

\[ H(s) = -\sum_{i=1}^{n} \sum_{j<i} J_{ij}s_is_j - \sum_{k=1}^{n} h_k s_k, \]  

where \( J \in \mathbb{R}^{n \times n} \) is a matrix for the coupling constants and \( h \in \mathbb{R}^n \) is a vector for the local constants. When \( \{J, h\} \) in \( H(s) \) are given to the quantum annealer, the solution \( s^* \in \{-1,1\}^n \) is provided and the solution \( b^* \in \{0,1\}^n \) can be obtained by \( b^* = (1 - s^*)/2 \). The function \( H(s) \) is known as the energy function for the n spin system of the Ising model. Thus, the minimum of \( H(s) \) can be acquired when the spin system achieves the ground state of the model. To achieve the ground state effectively, the quantum annealer uses an annealing process that controls quantum fluctuation appropriately to explore the solution space. The annealing process consists of discrete-time steps, giving the system enough time to converge to the ground state of the following Hamiltonian:

\[ H_t(\sigma) = (1 - \Gamma_t)H_p(\sigma^z) - \Gamma_t \sum_{k=1}^{n} \sigma_k^x, \]  

where \( \sigma \) is the Pauli spin operators for n spins, and \( \sigma^z, \sigma^x \) are z and x elements of \( \sigma \), respectively. In (4), the second term represents quantum fluctuation, and \( \Gamma_t \) is a parameter that controls the balance between two terms at step t. While the first term provides the system with the convergence to the ground

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Sparse Signal Reconstruction with QUBO Formulation in \( l_0 \)-regularized Linear Regression

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Quantum annealing [6] is a method that can tackle such laborious computations. The method is specialized to solve the QUBO. The objective function of QUBO is expressed as,

\[ L(b) = b^T Qb, \]  

where \( b \in \{0,1\}^n \) is a vector of n binary variables, and \( Q \in \mathbb{R}^{n \times n} \) is a matrix of quadratic coefficients.

In this paper, we propose a method for the sparse signal reconstruction that solves the l₀-regularized linear regressions with quantum annealing. In the following sections, we explain the QUBO-based technique, and compare the performance of the proposed method with that of the conventional methods.
state, the second term provides the system with the superposition of all possible vectors to explore the solution space. Typically, \( I_n \) starts from 1 for the initial superposition, and it gradually decreases to 0 to converge the system to the ground state. At the last step, the annealer provides a solution \( s^* \) as the measurement value of \( \sigma_z \). The mechanism of quantum annealing is similar to simulated annealing, which numerically simulates thermal fluctuation. However, the quantum fluctuation generates tunneling effect, that enables state transition over a relatively large energy barrier in (3). Theoretical analysis has proved the superiority of the quantum annealing on the computation time over the simulated annealing [7]. Note that the proof is not the theoretical guarantee for drastic acceleration of computation time like factorization in quantum gate computing.

The quantum annealing hardware system produced by D-Wave Systems [8] has demonstrated quantum annealing at the device level. In this system, the variables \( s \) and the coefficients \( \{J, h\} \) are embedded on the qubit chips, and their hardware parameters. Currently, the number of the available qubits (spins) is approximately 5000. In the implementation of the spin system, the physical network of qubits devices is automatically generated with the program codes using the dedicated software libraries. In addition, there is also software to extract the input parameters \( \{J, h\} \) from program codes about an objective function instead of an explicit expression of parameters. Currently, the number of the available qubits (spins) is approximately 5000. In the implementation of the spin system, the physical network of qubits devices is automatically generated with the program codes using the dedicated software libraries. In addition, there is also software to extract the input parameters \( \{J, h\} \) from program codes about an objective function instead of an explicit expression of parameters.

While the first term is obviously quadratic, the second term is not apparent. We formulate the second term with binary variables, and transform it into a quadratic form. When a number \( z \) is expressed by a bit sequence as \( z = (b_1, b_2, ..., b_K) \), the product of \( 1 - b_k (k = 1, ..., K) \) takes 1 for \( z = 0 = (0,0,0,0) \) and otherwise takes 0 for nonzero \( z \). Thus, the \( l_0 \)-norm (number of nonzero elements) in (6) is expressed as:

\[
\|B w\|_0 = \sum_{i=1}^{M} \left( 1 - \prod_{k=1}^{K} (1 - b_{ik}) \right), \tag{7}
\]

where \( b_{ik} \) is the \( k \)-th bit for the \( i \)-th representation entry. The value inside the large parenthesis takes 1 if any of \( b_{ik} (k = 1, ..., K) \) takes 1, which means \( z_i \) is nonzero. Although (7) is a binary form, it is not quadratic for \( 2 < K \) and needs to reduce the dimension for the QUBO formulation. For this reduction, it is effective to replace products of two binary variables with another binary variable, because the product also takes 0 or 1.

To reduce the dimension of (7), the product in (7) is replaced with auxiliary variables \( C \in \{0,1\}^{M \times (K-2)} \) as following way:

\[
\prod_{k=1}^{K} (1 - b_{ik}) = c_{i1} (1 - b_{i3}) \cdots (1 - b_{iK}) = c_{iK-2} (1 - b_{iK}), \tag{8}
\]

where \( c_{ik} (k = 1, ..., K - 2) \) is the \( i, k \) element of \( C \). Although (8) is quadratic, the optimization of (8) requires the following constraints for the variables \( c_{ik-1}, c_{ik} \), and \( b_{ik+1} \):

\[
c_{ik} = c_{i(k-1)} (1 - b_{i(k+1)}), \forall k = 1, 2, ..., K - 2, \tag{9}
\]

with \( c_{i0} = 1 - b_{i1} \). To formulate the constraint (9) in an unconstrained optimization, it is usual to add a penalty term, which takes 0 only when the constraints are satisfied otherwise takes a positive value. When a constraint \( c = ab \) is required for binary variables \( a, b, c \), a possible scheme that provides appropriate penalty is \((c - ab)^2 = c + ab - 2abc\). However, it still contains a triple factor term, \( abc \). To remove \( abc \), we further add (1) \((1 - a)(1 - b)c \), which also takes 0 when \( c = ab \). Then, the final penalty term is

\[
p(a, b, c) = 3c + ab - 2ac - 2bc, \tag{10}
\]

With Equation (8), (10) and a positive real parameter \( \lambda_c \), the objective function is now described:

\[
L_p(B,C) = \frac{1}{2\gamma_0} \| x - ABw \|_2^2 + \sum_{i=1}^{M} (1 - c_{ik-2} (1 - b_{ik})) + \sum_{i=1}^{M} \sum_{k=1}^{K-2} \lambda_c p(1 - b_{ik+1}, c_{ik-1}, c_{ik}). \tag{11}
\]

The objective function becomes fully quadratic in terms of \( B \) and \( C \). From this expression, the software [9] provides the parameters \( \{J, h\} \), thus the optimization of (11) can be evaluated using the quantum annealer.

C. Extension for multiple observation

In this setting, the sparse signals are recovered from the multiple observations. For each observation, the observed signals are generated from the sparse signals with some
fluctuation. This fluctuation is assumed to be added only to the nonzero values of the sparse signals.

The $l_1$-SVD (singular value decomposition) [3] is a method that handles these observations by group $l_1$-regularized linear regression (group LASSO). In this method, when $L$ observations $X = (x_1...x_L) \in \mathbb{C}^{N \times L}$ are obtained, the method optimizes $L$ sparse vectors $Z = (z_1...z_L) \in \mathbb{C}^{M \times L}$, with assigning a group for each row of $Z$. The objective function is regularized by the sum of $l_2$-norm of these groups and expressed as,

$$ L(Z) = \frac{1}{2Y_1} \|X - AZ\|_2^2 + \|\text{diag} \left( \sqrt{ZZ^T} \right)\|_1,$$

(12) where diag($X$) is a vector for diagonal elements of $X$. To accommodate the scenario for the multiple observation, we modify (5) to the following form:

$$ Z = Bw,$$

(13) where $B \in \{0,1\}^{M \times L \times K}$ is the $M \times L \times K$ tensor with binary variables to be optimized, $L$ is the number of the observations and $w \in \mathbb{R}^L$ is the weight vector as same in (5). The matrix $Z \in \mathbb{C}^{M \times L}$ consists of the sparse vectors $z_1, z_2 ... z_L \in \mathbb{C}^M$ whose nonzero positions are also the nonzero position in the original sparse vector $\zeta \in \mathbb{C}^M$. From this regulation, the number of the nonzero values in $\zeta$ is inferred greater or equal to the number of the nonzero rows in $Z$, which allows approximation of $\|\zeta\|_0$ with $\|\text{diag}(ZZ^T)\|_0$. Thus, the objective function is written as,

$$ L(B) = \frac{1}{2Y_0} \|X - ABw\|_2^2 + \|\text{diag} \left( \sqrt{ZZ^T} \right)\|_0,$$

(14) Each diagonal element in the second term takes nonzero value only when all the columns of the corresponding row takes nonzero values. Thus, similar to (8), the $l_0$-norm group can be obtained as:

$$ \|\text{diag} \left( \sqrt{ZZ^T} \right)\|_0 = \sum_{i=1}^{M} \left( 1 - \prod_{l=1}^{L} (1 - c_{i,l,k-}) \right).$$

(15) where $C \in \{0,1\}^{M \times L \times (K-1)}$ is the auxiliary binary variables. Similar to (11), the quadratic objective function of the $l_0$-norm-group-regularized linear regression is expressed as,

$$ L_B(B, C, D) = \frac{1}{2Y_0} \|X - ABw\|_2^2 + \sum_{i=1}^{M} (1 - d_{i,l-1})$$

$$ + \sum_{l=1}^{M} \sum_{k=1}^{L-1} \lambda_c p(1 - c_{i,l-1}, d_{i,l-1} - d_{il-1})$$

$$ + \sum_{i=1}^{M} \sum_{k=1}^{L-1} \sum_{k=1}^{L-1} \lambda_d p(1 - b_{i,lk+1}, c_{i,lk+1} - c_{i,lk}),$$

(16) where $D \in \{0,1\}^{M \times (L-2)}$, and $\lambda_d$ is a positive real parameter. This objective function has the quadratic and linear terms, which consist of the binary variables, $B, C$ and $D$.

Following the $l_1$-SVD method, the method can adopt an SVD-based preprocess to reduce computational costs. The process replaces $X$ in (14) with a matrix $X_t = (u_1 ... u_t) \in \mathbb{C}^{N \times t}$ from the decomposed form $X = U\Sigma V$. The substitution varies the sparse signals different from using the original observations. However, the nonzero positions in $Z$ are preserved regardless of this replacement.

To evaluate this method, we extract parameters $\{J, h\}$ from the observations $X$ and reconstruct $Z$ from obtained $B$ with (13).

III. EXPERIMENT WITH THE QUANTUM ANNEALER
A. Experimental Setup
All the quantum annealing process was performed on D-Wave Advantage Systems 4.1. The program codes for the proposed method were executed remotely through the cloud interface. We used a Python library, dwave-ocean-sdk 3.1.0, installed on a Windows 10 machine with an Intel Core i7-CPU and 64 GB of RAM. For the comparison of these results, simulated annealing computation was performed using dwave-neal as a reference for the same QUBO problems. We also used pyqubo 1.0.0 to extract $\{J, h\}$ from the program codes. For LASSO, OMP, SVD, and group-LASSO, we utilized publicly available libraries, including sklearn, numpy, and group-lasso.

In the evaluation, the original sparse vectors were sampled with random values for randomly selected nonzero entries. The observation vectors were simulated with the sampled sparse vectors and given observation matrices explained later. The reconstruction methods recovered the original sparse signals from the given observation, and the accuracy on the nonzero position recovery was evaluated for each method. Specifically, the method that provided the reconstruction whose nonzero positions matched exactly for the original representations were given the score 1, and 0 otherwise. The evaluation procedures were repeated typically 100 times and the average scores were used as reconstruction success rates. In addition, we monitored the achieved values of (6) for the original signals $z$, quantized signals $q$, and reconstructed signals $s$ and those of (11) for $s$.

The observation matrix $A \in \mathbb{C}^{N \times M}$ for DOA estimation is composed of the steering vectors and the $n,m$ element is expressed as

$$ a_{nm} = \exp \left( 2\pi n \frac{d}{\lambda} \sin(\phi_m) \right),$$

(17) where $n = 1, 2 ... N$ is the $n$-th sensor, $m = 1, 2 ... M$ is the $m$-th azimuth grid, $\lambda$ is the wavelength of the signal and $d$ is the distance of each array elements, typically, $d = 0.5\lambda$ is chosen. Here, we use $\phi_m = \arcsin(2m/M - 1), m = 0, 1 ... M - 1$, instead of the monospaced grid and obtain,

$$ a_{nm} = (-1)^n \exp \left( 2\pi n \frac{m}{M} \right).$$

(18) This expression is equivalent to the top $N$ rows of the $M \times M$ discrete Fourier transformation matrix, except that its sign depends on row indices. Although the arrangement varies the grid width non-uniform, it is more useful than the observation matrix based on steering vector. In addition, (18) provides a higher resolution than in (17) in the middle of the azimuth angles. For instance, when $M=160$, it is calculated that the resolution inside angle $\pm 45^\circ$ becomes less than $1^\circ$, which is finer than in the uniform spacing with $M=180$. 

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B. Results for Single Observation

Here, we show the experimental results for an observation matrix $A$ defined by (18) for $(N, M)$ = (8, 32) and (128, 512). For the original sparse signals, nonzero values are sampled from a uniformed distribution on [0, 1]. As the reference methods, we used LASSO and OMP. The parameters for these methods were determined by grid searches over the datasets prepared for the parameter search. For LASSO, we set $\gamma_1=0.0005$ for $M=32$ and $\gamma_1=0.00006$ for $M=512$. For the proposed method, we set $K=4$, $\gamma_0=0.001$ and $\lambda_1=1.5$. To avoid the false positive detection from the small nonzero values, a threshold of 0.02 was used for the nonzero decision in the evaluation.

Fig. 1 shows the reconstruction success rates (left) and the loss values defined above (right) for $(N, M)$ = (8, 32). In the left graph, the proposed method shows better success rates over other methods except for leftmost 2 plots. The degradation in the two leftmost plots was caused by rounding small values to zero. The similar results were observed for $N=4, 12$ and $M=32, 64$, but not for $N=16$ and $M=128, 256$. The right graph shows that the plots for $L(s)$, the loss for recovered signals $s$, overlap those for $L_\rho(s)$, the loss with penalties, thus the constraints (9) were satisfied for all plots. It also shows that the plots for $L(s)$ space evenly from those for $L(z)$, the loss for the original signals $z$ and plots overlap those of $L(q)$, the loss for the quantized original signals up to the leftmost 4 plots. Therefore, the dominant error is the quantization error up to the leftmost 4 plots which has higher success probability. The rightmost 2 plots indicate that the original signals are not the minimum of the loss function and the remained 3 plots indicate that the optimization was insufficient to provide even the upper bound of the optimum.

Fig. 2 shows the results for $(N, M)$ = (8, 32) and 8-bit quantization ($K=8$) for the proposed method. The left graph shows the degradation from Fig. 1 in the proposed method. We observed the degradation gradually increased when we increased $K$ from 4. The right graph shows the overlapping plots of $L(q)$ and $L(z)$, which is reasonable due to the increase of resolution. It also shows the increase of the difference between $L(s)$ and $L(q)$, which indicates increase in the insufficiency of the optimization.

Fig. 3 shows the results for $(N, M)$ = (128, 512) and 4-bit quantization ($K=4$) for the proposed method. In the left graph, the proposed method degrades from the reference methods. However the degradation of the proposed methods is not as large as in Fig. 2. The right graph shows that $L(q)$ gets close to $L(s)$, which is reasonable when $M$ increases. It also shows the increase of the difference between $L(s)$ and $L(q)$, however not as large as that in Fig. 2.

We observed that the proposed method achieved nearly equivalent results of the reference methods for $(N, M, K)$ = (32, 128, 4) and when the problem size is smaller than that size, the performance can be expected to be improve by adopting the proposed method. We also evaluated the proposed method with the simulated annealing algorithm instead of the quantum annealer. For $(M, K)$ = (32, 4) the success recovery rates were slightly lower than those of the quantum annealer in some cases. For $(M, K)$ = (64, 4), the solver response time for a single problem was 150 seconds on average, while that of the quantum annealing, including the communication time to the cloud machine, was less than 180 seconds even for $(M, K)$ = (512, 4). The QPU process time was within 300 milli seconds.
C. Results for Multiple Observations

In this section, we show the results for the comparison between the proposed $l_0$-SVD with the conventional $l_1$-SVD. In this comparison, we observed the experimental results for the matrix $X$ in (18) with a size of $(N, M) = (16, 64)$ and $(40, 160)$. We generated an original sparse vector, choosing the nonzero positions randomly and sampling nonzero values from a uniformed distribution on $[0, 1]$. We generated a matrix $Z$ from the 16 sparse signals generated from the original sparse signal with random values sampled from a normal distribution $N(0, 0.1\sigma)$. Finally, we obtain the matrix $X$ for 16 observation vectors with $X = AZ$. Following to the $l_1$-SVD, we decomposed $X$ as $U\Sigma V^T$ and provided the methods with the leftmost 2 columns in $U$ as $X$ in (14). In the evaluation phase, we evaluated the accuracy of the nonzero detection in the leftmost column of recovered $Z$ in (13). For the proposed method, we chose $K = 4$, $\gamma_0 = 0.0005$ supposed to be $0.5^{d}$ and $\lambda_c, \lambda_d$=1.5 supposed to be $0.5d (d = 0, 1, \ldots)$. As reference methods, we used the $l_1$-SVD and a modified LASSO, that processes the averaged values of the multiple observations. We chose $\gamma_1 = 0.00006$ for both the $l_1$-SVD and the LASSO. Fig. 4 shows the results for $M = 64$ (left) and $160$ (right) with $N / M = 1/4$, and $K = 4$. In the left graph, the proposed method performs nearly equivalent to the reference method, but in the graph on the right, the proposed method degrades from the reference method.

IV. DISCUSSION

The proposed method showed advantages over the reference methods for small $M, K$. In the following subsection, we discuss the mechanism of the advantages in the signal reconstruction and the cause of the degradation in the larger scale problems.

A. Effect of Quantization

As in the experiment section, we analyzed the difference between the loss in (1) for the estimated signals $s$ and original signals $z$. When the estimation error and the observation noise are defined by $e = s - z$ and $n = x - Az$, respectively, the difference $\delta(e) = l_0(s) - l_0(z)$ is expressed as,

$$
\delta(e) = \frac{1}{2\gamma_0} \| Ae \|_2^2 + (\| s \|_0 - \| z \|_0) - \frac{1}{\gamma_0} An. \quad (19)
$$

The condition of success recovery is $\delta(e) > 0$ for all $e$ except for $e = 0$. The quantization of $z$ replaces $n$ with $n + Ar$, where $r$ is the quantization error in $z$ and increases both the bias and the variance of (19) in proportion to $\Delta^2/\gamma_0$, where $\Delta = 2^{-k}$, and the additional bias provides the margin to solve the solution at $s = z$. The ratio of the standard deviation and bias is roughly estimated as $\Delta^2/\gamma_0$, thus, the coarser approximation of $z$ with larger $\Delta$ provides the more stable performance as in the experiment. Similarly, the smaller $\gamma_0$ is expected to provide the more margin to improve the performance, however it was not observed in the experiments.

B. Dependency on the problem size

As shown in the graphs for the loss values, the optimization did not achieve the minimum for large $M$ and $K$. Theoretically, the quantum annealing can provide the global optimum in high probability [7]. However, the annealing time required to achieve that probability increases exponentially with the number of the variables. In other words, if the annealing time is insufficient, the probability of obtaining the optimum decreases as the number of variables increases. A similar discussion to the annealing time can apply to the simulated annealing algorithm. Thus, it is inevitable to reduce the number of the binary variables to avoid the insufficient annealing time. Another issue is the insufficient resolution that causes by embedding the coefficients $(f, h)$ on the physical parameters of the hardware. From the loss function (6), to acquire the advantage of $l_0$-norm, the necessary resolution is roughly estimated as $\sim \gamma_0$. However the resolution is limited by the hardware and thus it is possible that smaller $\gamma_0$ did not work well. We will confirm the dominant cause for the larger size degradation by increasing the size and the resolution for future.

V. CONCLUSION

The proposed $l_0$-regularized linear regression based on the QUBO formulation was evaluated for the performance on the sparse signal reconstruction with a commercially available quantum annealer. The method demonstrated the recovery success rates nearly equivalent to those of the conventional methods under the several conditions.

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