Quantum Origin of the Primordial Fluctuation Spectrum and its Statistics

Gabriel León,1,* Susana Landau,2,† and Daniel Sudarsky,3,§

1Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, México D.F. 04510, México
2Instituto de Física de Buenos Aires, Ciudad Universitaria - Pab. 1, 1428 Buenos Aires, Argentina
3Instituto de Astronomía y Física del Espacio, Casilla de Correos 67, Sucursal 28, 1428 Buenos Aires, Argentina

Abstract

The account of the origin of cosmic structure, as provided by the standard inflationary paradigm, is not fully satisfactory, as has been argued in [A. Perez, H. Sahlmann, and D. Sudarsky, Class. Quantum Grav., 23, 2317, (2006)]. The central point of that work is to point out the need to discuss and explore the physical mechanism that is capable of generating the inhomogeneity and anisotropy of our Universe, starting from an exactly homogeneous and isotropic initial state associated with the early inflationary regime. We review this issue briefly here together with the proposal to address this shortcoming in terms of a dynamical collapse of the vacuum state of the inflaton field. We also briefly indicate how this issues might be connected to other questions being faced in the study of the quantum/gravity interface, and their relevance to the investigations concerning the statistical characterization of the primordial spectrum.

* On Sabbatical leave from Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. Postal 70-543, México D.F. 04510, México.
* gabriel.leon@nucleares.unam.mx
† slandau@df.uba.ar
§ sudarsky@nucleares.unam.mx
I. INTRODUCTION

At what point in the cosmic evolution do the actual primordial inhomogeneities arise? I.e. when does our universe depart from the homogeneity and isotropy that results from inflation. This is a question that one might expect should be addressed, at least in principle, by any theory which deals with the emergence of cosmic structure. Yet, in the standard inflationary account \[1\], which is nowadays regarded as a remarkable success, the context where such issues can be addressed seems to be simply absent \[2\]. That is, within the orthodox accounts, one can not identify the physical process responsible for the generation of those features in our Universe. In fact, according to the inflationary paradigm, from a relatively wide initial set of possibilities marking the end of the mysterious quantum gravity era, the accelerated inflationary burst leads to a homogenous and isotropic (H&I) Universe where the quantum fields are all characterized by the equally homogeneous and isotropic vacuum states (usually taken specifically to be the so called Bunch-Davies vacuum). From these conditions, it is usually argued, in a rather unclear \[1\] although strongly image-evoking manner, that the quantum fluctuations present in such quantum state morph into the seeds of anisotropies and inhomogeneities that characterize our late Universe. This issue is sometimes characterized as the “transition from the quantum regime to the classical regime”, but we find this a bit misleading: most people would agree that there exist no distinct and separated classical and quantum regimes. The fundamental description ought to be always quantum mechanical; the so called \textit{classical regimes} are those where certain quantities can be described to a sufficient accuracy by their classical counterparts representing the corresponding quantum expectation values. The paradigmatic example here is provided by the coherent states of an harmonic oscillator which correspond to minimal wave-packets with expectation values of position and momentum following the classical equations of motion. In any case, it seems clear that from a situation corresponding to a H&I background and a quantum aspect characterized by a H&I state, one can not end up \[2\] in a situation which is characterized, at the classical level, as containing actual inhomogeneities and anisotropies. It is clear that, in terms of the dynamics, such transition can not be accounted for by the gravity/inflaton action which

---

1 Acknowledgments that this is an unclear aspect can be seen for instance in Weinberg’s Cosmology \[3\] where the author explicitly states his view on the subject.

2 In the absence of something else, which in other circumstances would be identified as a measurement, but which clearly can not be invoked in the present setting. Observers and measuring apparatuses are only possible well after the H&I has been broken, so these can hardly be part of the cause of the breakdown.
preserves such symmetries. Simply put, if the initial state is H&I and the Schrödinger evolution is tied to a Hamiltonian that preserves these symmetries, the resulting state can not be anything but H&I. Nonetheless, various types of arguments are often put forward in attempts to bypass the above conclusion. The discussion of the conceptual problems and those associated have been discussed in previous works by some of us and by others in [4–6]. We will not reproduce those arguments here as the objective of the present work is to focus in the statistical aspects that emerge tied to what we consider as a more conceptually transparent picture, and which is based on a modified version of the standard inflationary paradigm, that we have been advocating in previous works [4, 7, 8].

We should note however, that we can not escape from the related problems even if we choose to adopt a very pragmatic position. Lets assume that one chooses to ignore the shortcomings of the more standard accounts, and accepts that, say decoherence, addresses somehow, the issue at hand, and that the mystery lies only in the question concerning the precise mechanism that lies behind the fact that, from the options exhibited in those analyses (i.e. those displayed in the reduced density matrix), one single particular option seems to be selected [9]. Within this point of view, one would be assuming that the initial symmetry has been lost –at least for practical purposes, as presumably one would be advocating when adopting such position– in association with that particular realization or actualization (represented by a particular element in the density matrix). Thus, it seems clear that for the sake of self consistency, one should consider, when studying aspects of the inhomogeneity and anisotropies in the CMB, the state corresponding to such “selected option” and not the entire vacuum state which describes the H&I state of affairs previous to the “selection”.

In following such views, the discussion that we are presenting in this paper, would have to be taken to represent the effective description corresponding to “our perceived Universe” (in a context where one puts together something like the many-worlds interpretation, with the arguments based on decoherence). Although we definitely do not adhere such view for the reasons explained in [5], it is clear that an effective description such as the one presented here is what would have to be contemplated when dealing with the issues within any view.

---

3 The selection of course refers to the fact that according to the standard arguments the resulting density matrix, after becoming essentially diagonal due to decoherence, represents an ensemble of universes and our particular one corresponds to one of them. That one can be considered as selected by nature to become realized. Alternatively, one might take the view that these other universes are also realized, and thus they also exits in realms completely inaccessible to us. In that case the selection corresponds to that universe in which we happen to exist.
which pretends to allow one deal with the details characterizing the inhomogeneities and anisotropies in the cosmic structure and its imprints in the CMB that we do observe.

In fact, it is perhaps worth noting, that in the usual accounts, it is hard to pinpoint where exactly the statistical aspects come into play and which kind of the statistics one is dealing with. That is, in the standard approach, the specific Universe is not described in any sense (not even in terms of any unknown, yet explicitly identified quantities), and the randomness which characterizes it lies hidden in unspecified aspects associated with the multiple interpretations. In other words, one can not identify the random variables; one does not know how many there are; one can not say how the various elements of the ensemble differ from each other, etc.

In order to fully and satisfactorily address the problem, it seems paramount to be able to point out what exactly is wrong with the argument leading to the conclusion of the theorem drawn above, i.e. where does nature deviate from the theory leading to the erroneous conclusion that our Universe is even today, at the fundamental quantum level, perfectly homogeneous and isotropic?. It follows that such explanation must indicate where, the ordinary \( U \)-evolution –with the symmetry preserving Hamiltonian– does break down. We can easily see that none of the proposals to deal with the issue, based on the standard paradigms, can point to any reason where that breakdown might occur, and much less, point to a physical reason for that departure from standard Quantum Theory. This has led us to take a view to tie this problem with the ideas advocated by Roger Penrose, who argues [10] that quantum theory should itself suffer modifications as a result of its combination with the fundamental theory of space-time structure. Among the aspects of the theory that would be substantially affected according to Penrose’s views, are those related to the reduction postulate (or R process, as he calls it), and its contraposition –and to some degree its contradiction– with the unitary evolution (or U process, as labeled in his works) controlled by Schrödinger’s equation. In fact, the issue of dynamical quantum reduction has received a lot of attention within the community working in foundational aspects of quantum theory, and there are in the existing literature several well defined proposals such as those in [11–13].

The proposal behind our work is based on the inclusion of the hypothesis that a dynamical

\[4\] This is what is often though as quantum gravity. We did not use that term because that often presupposes that one is considering the relevant theory to be simply the adaptation of general relativity to the standard quantum theory, while what one has in mind when following Penrose’s ideas is something much more distant from known physics, involving as indicated, modifications of quantum theory itself.
collapse of the wave function lies behind the breakdown of the initial homogeneity and isotropy. In other words, that a non-unitary jump in the quantum state plays a role in transforming the inflaton vacuum into a quantum state that lacks the translational and rotational symmetries of the former state.

It goes without saying that we can not at this stage try or hope to point out the physical origin of such dynamical collapse. However, once one has accepted that something of this sort is occurring, one can parametrize its basic characteristics, and use the relevant observational data to infer something about the nature of the new physics that lies behind the phenomena. This has been the basic attitude behind the program stared in [4].

In this paper our aim is to briefly explore for the first time some of the basic differences associated with the statistical considerations, between the usual account and the proposal we have been working with.

In order to be a bit more explicit lets start by reminding the reader that in the standard approaches, the study of the statistical nature of the problem is based on the study of the \( n \)-point functions of the Newtonian potential, \( \Psi(x_1) \ldots \Psi(x_n) \), with the overline denoting the average over an ensemble of Universes. There one needs to face the issue of what is the relationship between the quantum \( n \)-point functions and the quantities we actually measure. The usual approach is based on the identification of

\[
\langle 0 | \hat{\Psi}(x_1) \ldots \hat{\Psi}(x_n) | 0 \rangle = \Psi(x_1) \ldots \Psi(x_n),
\]

that is, based on the identification of quantum and statistical \( n \)-point functions. The latter are naturally associated with ensembles of Universes, all of which, even if real, are unaccessible to us. Therefore, invoking ergodic arguments, a further connection is made between ensemble averages and time –and in our case– spatial averages.

At this point we must express some caution regarding such identifications and to critically explore the validity of many of underlying assumptions, and also to unearth the places where the issues at hand can have important statistical/observational consequences.

In order to further motivate this work it is perhaps worth pointing out the simplest place where we can appreciate the problematic aspects of unquestioningly accepting such identifications in a very simple situation concerning the one point function: Consider the
standard treatments based in the so called Mukhanov-Sasaki variable:

\[ u \equiv \frac{a\Psi}{4\pi G\dot{\phi}_0}, \quad v \equiv a \left( \delta\phi + \frac{\dot{\phi}_0}{\mathcal{H}}\Psi \right), \quad (2) \]

where \( \Psi \) is the metric perturbation known as the Newtonian potential, \( \dot{\phi}_0 \) is the derivative of the background inflaton with respect to conformal time \( \eta \), \( \delta\phi \) is the perturbation in the inflaton field, \( a \) is the scale factor and \( \mathcal{H} = \frac{\dot{a}}{a} \) (related to the standard Hubble parameter \( H \) through \( \mathcal{H} = aH \)). The Einstein equations then lead to \( \Delta u = z\left(\frac{v}{z}\right) \) and \( v = \frac{1}{z}(zu) \) where \( z \equiv \frac{a\dot{\phi}_0}{\mathcal{H}} \). Given the equations of motion, the Newtonian potential can thus be expressed in terms of the field \( v(\vec{x}, \eta) \) and its momentum conjugate \( \pi_v(\vec{x}, \eta) = \dot{v}(\vec{x}, \eta) \). The expression for the corresponding Fourier components is

\[ \Psi_{\vec{k}}(\eta) = -\frac{4\pi G\epsilon H}{k^2} \left( \pi_{v\vec{k}}(\eta) - \frac{\dot{z}}{z} v_{\vec{k}}(\eta) \right), \quad (3) \]

where we \( \epsilon \) is the so-called slow-roll parameter \( \epsilon \equiv 1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \).

We are interested in the temperature anisotropies of the CMB observed today on the celestial two-sphere, which are related to the inhomogeneities in the Newtonian potential on the last scattering surface,

\[ \frac{\delta T}{T_0}(\theta, \varphi) = \frac{1}{3} \Psi(\eta_D, \vec{x}_D). \quad (4) \]

The data is described in terms of the coefficients \( \alpha_{lm} \) of the multipolar series expansion

\[ \frac{\delta T}{T_0}(\theta, \varphi) = \sum_{lm} \alpha_{lm} Y_{lm}(\theta, \varphi), \quad \alpha_{lm} = \int \frac{\delta T}{T_0}(\theta, \varphi) Y_{lm}^*(\theta, \varphi) d\Omega. \quad (5) \]

Here \( \theta \) and \( \varphi \) are the coordinates on the celestial two-sphere, with \( Y_{lm}(\theta, \varphi) \) the spherical harmonics.

The quantities \( \alpha_{lm} \) are then given by

\[ \alpha_{lm} = \frac{4\pi i^l}{3} \int \frac{d^3k}{(2\pi)^3} j_l(kR_D) Y_{lm}^*(\hat{k}) \Delta(k) \Psi_{\vec{k}}(\eta_R), \quad (6) \]

with \( j_l(kR_D) \) the spherical Bessel function of order \( l \); \( \eta_R \) is the conformal time of reheating which can be associated with the end of the inflationary regime and \( R_D \) the comoving radius of the last-scattering surface. We have explicitly included the modifications associated with late time physics encoded in the transfer functions \( \Delta(k) \).

Now, the problem is that if we compute the expectation value of the left hand side (i.e. identifying \( \langle \hat{\Psi} \rangle = \Psi \)) in the vacuum state \( |0\rangle \) we obtain 0, while it is clear that for any given
the measured value of this quantity is not 0. That is, if we rely in this case on the one-point function and the standard identification, we find a large conflict between expectation and observation. We might even be tempted to say that evidence of non-Gaussianity has already been observed in each measurement of a particular $\alpha_{lm}$. This is, of course, not what one wants, however this makes clear that disentangling the various statistical aspects (ensemble statistics; space and time statistics, including orientation statistics and finally the nature of the assumed connection of quantum and statistical aspects) and to make explicit the assumptions underlying the identifications, as well as the expected limitations, is paramount to avoid the dangers of confusion.

As a matter of fact, what exactly is wrong with the above argument?. Normally one would have to argue that the relevant prediction concerns the ensemble averages and thus identifying the result

$$\alpha_{lm} = \frac{4\pi l}{3} \int \frac{d^3k}{(2\pi)^3} j_l(kR_D)Y^*_l(\hat{k})\Delta(k)\langle 0|\hat{\Psi}_k(\eta_R)|0 \rangle = 0, \quad (7)$$

with the estimate of the actually measured quantity $\alpha_{lm}$ would be incorrect. One could however wonder why would it be incorrect to say that we might rely on ergodic arguments and thus expect that spatial averages would correspond to the ensemble averages, and note that the measured quantity corresponding to the expression above is after all already a weighted average over the celestial two sphere. The answer would need to be that we still must carry out a further average: the average over orientations in order to have any confidence that our estimates would be reliable. That is, we would have to compute:

$$\bar{\alpha}_l = \frac{1}{2l + 1} \sum_m \alpha_{lm}, \quad (8)$$

and is this quantity that could be expected to be zero. At this point two issues become apparent:

i) Why is that so? why can this average be expected to yield zero but not each individual $\alpha_{lm}$ as in (7).

ii) Empirically, does this hold? In other words, is the actual average of observed complex quantities in (8), in fact, zero or is it not?.
We will explore issues connected to these questions within the approach pioneered in [4] and which seems to have more potential to dealing with such questions than the standard one.

The paper is organized as follows: In section II we review the standard picture for primordial non-Gaussianities, in section III we review the collapse models description for the inflationary origin of the seeds of the cosmic structure. In section IV we focus on the statistical aspects of the primordial non-Gaussianities and propose new characterizations of the quantities associated with the bispectrum. Finally, in section V we discuss our findings. The conventions we will be using include a $(-, +, +, +)$ signature for the space-time metric. We will use units where $c = 1$ but will keep the gravitational constant $G$ and $\hbar$ explicit throughout the paper.

II. THE STANDARD PICTURE FOR THE PRIMORDIAL NON-GAUSSIANITIES

This section will briefly review the standard accounts on the primordial non-Gaussianities following closely references [14–16]. There is absolutely no original work in this section; we simply present here the usual treatment on the subject following what is commonly found in the literature in order to compare with our own approach and discuss the main differences. For more details and derivations we refer the reader to the comprehensive review by Komatsu [17], Bartolo et al. [18] and the references cited therein.

Historically, non-Gaussianity as a test of the accuracy of perturbation theory was first suggested by Allen et al. [19]. However most of its importance to date relies on the premise that it will play a leading role in furthering our understanding of two fundamental aspects of cosmology and astrophysics [20]:

- The physics of the very early Universe that created the primordial seeds for large-scale structures, and
- The subsequent growth of structures via gravitational instability and gas physics at later times.

Within the standard approach, by non-Gaussianity, people refer to any small deviations in the observed fluctuations from the random field of linear, Gaussian, curvature perturbations. The curvature perturbations, $\Psi$, generate the CMB anisotropy, $\delta T/T$. The linear
perturbation theory gives a linear relation between \( \Psi \) and \( \delta T/T \) on large-scales (where the Sachs-Wolfe effect dominates) at the decoupling epoch, i.e. \( \delta T/T \sim (1/3)\Psi \). It follows from the relation, \( \delta T \propto \Psi \), that if \( \Psi \) is Gaussian, then \( \delta T \) is Gaussian, but what exactly does one mean by Gaussian at the observational level?

One of the most important results of the inflationary paradigm is that the CMB anisotropy arises due to curvature perturbations which in turn are produced by quantum fluctuations. In the standard single field slow-roll scenario, these fluctuations are due to fluctuations of the inflaton field itself when it slowly rolls down its potential \( V(\phi) \). Within this approach, the primordial perturbation is Gaussian; in other words its Fourier components are uncorrelated and have random phases. When inflation ends, the inflaton \( \phi \) oscillates about the minimum of its potential and decays, thereby reheating the Universe.

In the inflationary paradigm, the perturbations of the field \( \delta \phi \) and the perturbations of the curvature \( \Psi \) are treated as standard quantum fields evolving in a classical quasi-De Sitter background space-time. The quantity of observational interest is called the power spectrum of the curvature perturbation \( P_\Psi(k, \eta) \). The power spectrum is obtained from

\[
\langle 0| \hat{\Psi}(\vec{x}, \eta) \hat{\Psi}(\vec{y}, \eta) |0 \rangle,
\]

where \( |0 \rangle \) is called the Bunch-Davies vacuum and represents the initial state of the field \( \hat{\nu} \), which is the Mukhanov-Sasaki field variable defined in (2).

It is precisely at this step where a subtle issue arises, namely that in the standard picture one is given various and distinct arguments (e.g. decoherence, horizon crossing, many-worlds interpretation of QM, etc.) to accept the identification

\[
\langle 0| \hat{\Psi}(\vec{x}, \eta) \hat{\Psi}(\vec{y}, \eta) |0 \rangle = \overline{\Psi(\vec{x}, \eta) \Psi(\vec{y}, \eta)},
\]

where \( \Psi(\vec{x}, \eta) \) now stands as a classical stochastic field and the overline denotes the average over an ensemble of Universes. In other words, the value of the field \( \Psi \) in each point \( (\vec{x}, \eta) \) varies from each one of the members of the ensemble of “Universes”, with a variance \( \overline{\Psi^2} \).

Therefore, the power spectrum \( P_\Psi(k, \eta) \) is defined in terms of the Fourier components of \( \Psi(\vec{x}, \eta) \) by

---

5 In fact they are both part of a unified field \( \nu \).
\[ \Psi_\kappa(\eta) \Psi_{\kappa'}(\eta) \equiv (2\pi)^3 \delta(\vec{k} + \vec{k}') P_\Psi(k, \eta). \] (11)

Consequently, the power spectrum is related to the 2-point function through

\[ \overline{\Psi(\vec{x}, \eta) \Psi(\vec{y}, \eta)} = \int_0^\infty \frac{dk}{k} P_\Psi(k, \eta) \frac{\sin kr}{kr}, \] (12)

with \( r \equiv |\vec{x} - \vec{y}| \) and we also used the definition of the dimensionless power spectrum \( P_\Psi(k, \eta) \equiv P_\Psi(k, \eta) k^3 / 2\pi^2 \). From the above expression one can find the variance \( \overline{\Psi^2} \)

\[ \overline{\Psi^2(\vec{x}, \eta)} = \int_0^\infty \frac{dk}{k} P_\Psi(k, \eta). \] (13)

The expression (13) diverges generically. In particular, we know that the spectrum of the primordial curvature perturbation is roughly \( P_\Psi(k, \eta) \propto k^{-3} \). That is, \( P_\Psi(k, \eta) \) is nearly constant (i.e. independent of \( k \)); therefore (13) diverges in a logarithm fashion for \( k \to 0 \) and \( k \to \infty \). The way the standard pictures deals with this issue is to establish a \( k_{\text{max}} \) equal to the “horizon”, and work in a cubic box of physical size \( aL \) much larger than the Hubble radius. Thus,

\[ \overline{\Psi^2(\vec{x}, \eta)} \simeq P_\Psi(\eta) \int_{L^{-1}}^{aH} \frac{dk}{k} = P_\Psi(\eta) \ln \frac{aL}{H^{-1}}. \] (14)

That is, in order to avoid the divergence in \( \overline{\Psi^2} \) one is forced to introduce some particular values of \( k \) as cut-offs.

The question that arises now is how can we do an average over an ensemble of Universes if we have observational access to just one –our own– Universe. In the next subsection, we will show how the standard approach deals with this issue. In the following we will accept the validity of (10).

If \( \Psi(\vec{x}, \eta) \) is Gaussian, then the two-point correlation function (9) specifies all the statistical properties of \( \Psi(\vec{x}, \eta) \), for the two-point correlation function is the only parameter in a Gaussian distribution. If it is not Gaussian, then we need higher-order correlation functions to determine the statistical properties.

For instance, a non-vanishing three-point function.

---

6 That is, there exist some physical mechanism for which the quantum variable \( \hat{\Psi}(\vec{x}, \eta) \) becomes a classical stochastic field \( \Psi(\vec{x}, \eta) \) with Gaussian distribution.

7 Similarly as the 2 point correlation function, the standard approach relies on the identification

\[ \langle 0 | \hat{\Psi}(\vec{x}, \eta) \hat{\Psi}(\vec{y}, \eta) \hat{\Psi}(\vec{z}, \eta) | 0 \rangle = \Psi(\vec{x}, \eta) \Psi(\vec{y}, \eta) \Psi(\vec{z}, \eta). \] (15)
\[ \Psi(\vec{x}, \eta) \Psi(\vec{y}, \eta) \Psi(\vec{z}, \eta), \]  

(16)

is an indicator of non-Gaussian features in the cosmological perturbations. The Fourier transform of the three-point function is called the bispectrum\(^8\) and is defined as

\[ \Psi_{k_1} \Psi_{k_2} \Psi_{k_3} \equiv (2\pi)^3 \delta(k_1 + k_2 + k_3) B_{\Psi}(k_1, k_2, k_3). \]  

(17)

The importance of the bispectrum comes from the fact that it represents the lowest order statistics able to distinguish non-Gaussian from Gaussian perturbations.

The delta function in (17) enforces the triangle condition, that is, the constraint that the wavevectors in Fourier space must close to form a closed triangle, i.e. \( \vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0 \). Different inflationary models predict maximal signal for different triangle configurations.

The standard approach of the study of the structure of the bispectrum is usually done by plotting the magnitude of \( B_{\Psi}(k_1, k_2, k_3)(k_2/k_1)^2(k_3/k_1)^2 \) (with \( |\vec{k}_i| \equiv k_i \)) as a function of \( k_2/k_1 \) and \( k_3/k_1 \) for a given \( k_1 \), with a condition that \( k_1 \geq k_2 \geq k_3 \) is satisfied. The usual classification of various shapes of the triangles uses the following names: squeezed (\( k_1 \approx k_2 \gg k_3 \)), elongated (\( k_1 = k_2 + k_3 \)), folded (\( k_1 = 2k_2 = 2k_3 \)), isosceles (\( k_2 = k_3 \)) and equilateral (\( k_1 = k_2 = k_3 \)). Within the cosmology community \([22, 24]\), these shapes of non-Gaussianity are potentially a powerful probe of the mechanism that creates the primordial perturbations.

One of the first (and most popular) ways to parameterize non-Gaussianity phenomenologically was via a small non-linear correction to the linear Gaussian perturbation \([25, 26]\),

\[ \Psi(\vec{x}, \eta) = \Psi_L(\vec{x}, \eta) + \Psi_{NL}(\vec{x}, \eta) \]

\[ \equiv \Psi_L(\vec{x}, \eta) + f_{\text{loc}}^{\text{NL}}[\bar{\Psi}_L^2(\vec{x}, \eta) - \bar{\Psi}_L^2(\vec{x}, \eta)], \]  

(18)

where \( \Psi_L(\vec{x}, \eta) \) denotes a linear Gaussian part of the perturbation, and the variance \( \bar{\Psi}_L^2(\vec{x}, \eta) \), is implemented in the same sense as presented in (14). Henceforth, we call \( f_{\text{loc}}^{\text{NL}}(\vec{x}, \eta) \) the \textit{local non-linear coupling parameter} which determines the “strength” of the primordial non-Gaussianity. This parametrization of non-Gaussianity is local in real space and therefore is

---

\(^8\) In the following we will not write the explicit dependance of the conformal time \( \eta \) unless it leads to possible confusion.
called \textit{local non-Gaussianity}. In this \textit{local model} the contributions from ‘squeezed’ triangles are dominant, that is, with e.g. $k_3 \ll k_1, k_2$. Using (18) and (17) the bispectrum of local non-Gaussianity may be derived

$$B_\Psi(k_1, k_2, k_3) = 2 f^{loc}_\text{NL}[P_\Psi(k_1)P_\Psi(k_2) + P_\Psi(k_2)P_\Psi(k_3) + P_\Psi(k_3)P_\Psi(k_1)].$$ (19)

In the standard picture, the non-Gaussianity produced by many single field slow-roll models, is considered small and likely unobservable. However, a large detectable amount of non-Gaussianity can be detected when any of the following conditions are violated \cite{18, 20, 27, 28}:

- **Single Field.** There was only one quantum field responsible for driving inflation.
- **Canonical Kinetic Energy.** The kinetic energy of the quantum field is such that the speed of propagation of fluctuations is equal to the speed of light.
- **Slow Roll.** The evolution of the field was always very slow compared to the Hubble time during inflation.
- **Initial Vacuum State.** The quantum field was in the preferred “Bunch-Davies vacuum” state.

It is expected that the primordial non-Gaussianity produced by inflation is very small, i.e. undetectable only when all of the above conditions are satisfied.

\section{Non-Gaussianity in the CMB}

In this subsection, we present the standard connection between the primordial bispectrum at the end of inflation and the observed bispectrum of CMB anisotropies.

\subsection{Theoretical predictions for the CMB bispectrum from inflation}

As we mentioned in section \textsection{II} the temperature anisotropies are represented using the $\alpha_{lm}$ coefficients of a spherical harmonic decomposition of the celestial sphere,
\[ \frac{\delta T}{T_0}(\theta, \varphi) = \sum_{lm} \alpha_{lm} Y_{lm}(\theta, \varphi), \quad (20) \]

and the curvature perturbation \( \Psi \) is imprinted on the CMB multipoles \( \alpha_{lm} \) by a convolution involving the called transfer functions \( \Delta(k) \) representing the linear perturbation evolution, through equation (6):

\[
\alpha_{lm} = \frac{4\pi i^l}{3} \int \frac{d^3k}{(2\pi)^3} j_l(k R_D) Y_{lm}^*(\hat{k}) \Delta(k) \Psi_{\vec{k}}(\eta_R).
\]

The CMB bispectrum also called the angular bispectrum is defined as the three point correlator of the \( \alpha_{lm} \)

\[
B_{m_1 m_2 m_3}^{l_1 l_2 l_3} \equiv \langle \alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3} \rangle. \quad (21)
\]

At this point, the standard picture lead us to another subtle issue, that is, the overline in (21) denotes in principle an average over an ensemble of Universes. In reality, we cannot measure the ensemble average of the angular harmonic spectrum, but one realization such as \( \{\alpha_{l_1 m_1}, \alpha_{l_2 m_2}, \ldots, \alpha_{l_n m_n}\} \). To overcome this issue, the standard approach relies on the ergodic assumption [21]. The ergodicity of a system refers to that property of process by which the average value of a process characteristic measured over time is the same as the average value measured over the ensemble. If one accepts the common supposition that

the inflationary perturbation is indeed ergodic, then one expects the volume average of the fluctuations to behave like the ensemble average: the Universe may contain regions where the fluctuation is atypical, but with high probability most regions contain fluctuations with root-mean-square amplitude close to \( \sigma \). Therefore the probability distribution on the ensemble, which is encoded in (21), translate to a probability distribution on smoothed regions of a determined size within our own Universe.

After the above analysis, we continue with the calculation relating the primordial bispectrum with the angular bispectrum. By substituting (6) in (21), one obtains

\[
13
\]
where in the last line we have integrated over the angular parts of the three \( k_i \) and used the exponential integral form for the delta function that appears in the bispectrum definition \((17)\). The last integral over the angular part of \( \bar{x} \) is known as the Gaunt integral, which can be expressed in terms of Wigner 3-\( j \) symbols as

\[
\mathcal{G}_{l_1l_2l_3}^{m_1m_2m_3} = \sqrt{(2l_1+1)(2l_2+1)(2l_3+1)} \frac{4\pi}{\sqrt{(2m_1+1)(2m_2+1)(2m_3+1)}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} m_1 & m_2 & m_3 \end{pmatrix}.
\]

The fact that the bispectrum \( B_{m_1m_2m_3}^{l_1l_2l_3} \) consists of the Gaunt integral, \( \mathcal{G}_{l_1l_2l_3}^{m_1m_2m_3} \), implies that the bispectrum satisfies the triangle conditions and parity invariance: \( m_1 + m_2 + m_3 = 0 \), \( l_1 + l_2 + l_3 = \text{even} \), and \( |l_i - l_j| \leq l_k \leq l_i + l_j \) for all permutations of indices.

One thus can write

\[
B_{m_1m_2m_3}^{l_1l_2l_3} = \mathcal{G}_{l_1l_2l_3}^{m_1m_2m_3} b_{l_1l_2l_3}, \quad (24)
\]

where \( b_{l_1l_2l_3} \) is an arbitrary real symmetric function of \( l_1, l_2 \) and \( l_3 \). This form, \((24)\), is necessary and sufficient to construct generic \( B_{m_1m_2m_3}^{l_1l_2l_3} \) satisfying rotational invariance; thus in the literature one encounters \( b_{l_1l_2l_3} \) more frequently than \( B_{m_1m_2m_3}^{l_1l_2l_3} \). The quantity \( b_{l_1l_2l_3} \) is called the reduced bispectrum as it contains all the physical information in \( B_{m_1m_2m_3}^{l_1l_2l_3} \). Since the reduced bispectrum does not contain the Wigner 3-\( j \) symbol, which merely ensures the triangle conditions and parity invariance, it is easier to calculate physical properties of the theoretical bispectrum.
In the standard picture one assumes that if there is a non-trivial bispectrum then it has arisen through a physical process which is statistically isotropic, so we can employ the angle-averaged bispectrum \( B_{l_1 l_2 l_3} \) without loss of information, that is \([17, 18]\),

\[
B_{l_1 l_2 l_3} = \sum_{m} \frac{\alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3}}{m_1 m_2 m_3}, \tag{25}
\]

We now can obtain a relation between the averaged bispectrum, \( B_{l_1 l_2 l_3} \) and the reduced bispectrum \( b_{l_1 l_2 l_3} \) by substituting (24) into (25),

\[
B_{l_1 l_2 l_3} = \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3} \end{pmatrix}, \tag{26}
\]

where the identity,

\[
\sum_{m} \frac{l_1 l_2 l_3}{m_1 m_2 m_3} g^{m_1 m_2 m_3}_{l_1 l_2 l_3} = \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}, \tag{27}
\]

was used. The reduced bispectrum obtained from (22) then takes the much simpler form

\[
b_{l_1 l_2 l_3} = \left( \frac{2}{\pi} \right)^3 \int dk_1 dk_2 dk_3 (k_1k_2k_3)^2 B_{\Psi}(k_1, k_2, k_3) \Delta(k_1) \Delta(k_2) \Delta(k_3) \times j_{l_1}(k_1R_D)j_{l_2}(k_2R_D)j_{l_3}(k_3R_D) \int_0^\infty dx x^2 j_{l_1}(k_1x)j_{l_2}(k_2x)j_{l_3}(k_3x). \tag{28}
\]

This is the main equation for this section, since it explicitly relates the primordial bispectrum, predicted by the standard inflationary theories, to the averaged bispectrum (through (26)) obtained from the CMB angular bispectrum \( \alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3} \). This formula is entirely analogous to the well known relation linking the primordial power spectrum \( P_{\Psi}(k) \) and the CMB angular power spectrum \( C_l \), i.e.,

\[
C_l = \frac{2}{\pi} \int k^2 P_{\Psi}(k) \Delta^2(k) j_l^2(kR_D)dk. \tag{29}
\]

\(^9\) Although it would be interesting, and possibly a more realistic approach to the problem, to proceed in the analysis without this assumption.
2. Measuring primordial non-Gaussianity from the CMB

As we mentioned before, in most inflationary models, the parameter characterizing primordial non-Gaussianity is $f_{NL}$. Thus the next task within the standard picture is to estimate $f_{NL}$ from the CMB data-set. That is, one chooses the primordial model that one wants to test, characterizing it through its bispectrum shape. One then proceeds to estimate the corresponding amplitude $f_{NL}^{\text{model}}$ from the data. If the final estimate is consistent with $f_{NL}^{\text{model}} = 0$, one concludes that no significant detection of the given shape is produced by the data, but one still determines important constraints on the allowed range of $f_{NL}^{\text{model}}$. Note that ideally one would like to do more than just constrain the overall amplitude, and reconstruct the entire shape from the data by measuring configurations of the bispectrum. However, the expected primordial signal is too small to allow the signal from a single bispectrum triangle to emerge over the noise. For this reason one studies the cumulative signal from all the configurations that are sensitive to $f_{NL}^{\text{model}}$.

Given the above analysis, the standard picture then makes use of estimation theory to extract an estimate for $f_{NL}$. An unbiased bispectrum-based minimum variance estimator for the nonlinearity parameter can be written as [29, 30]

$$
\hat{f}_{NL} = \frac{1}{N} \sum_{l_1, m_1} \left( \sum_{l_2, m_2} B_{l_1 l_2 l_3}^{th} C_{l_1} C_{l_2} C_{l_3} \right) \left( C_{l_1} C_{l_2} C_{l_3} \right)_{\text{obs}} (\alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3})_{\text{obs}},
$$

(30)

where $B_{l_1 l_2 l_3}^{th}$ is the angle averaged theoretical CMB bispectrum for the model in consideration with $f_{NL}^{th} = 1$, $C_l$ is the observed angular spectrum and $\alpha_{lm}$ are the multipoles of the observed CMB temperature fluctuations. The normalization $N$ is calculated requiring that the estimator to be “unbiased”, i.e. the averaged value is equal to the “true” value of the parameter, $\langle \hat{f}_{NL} \rangle = f_{NL}$. If the bispectrum $B_{l_1 l_2 l_3}$ is calculated for $f_{NL} = 1$ then the normalization takes the following form

$$
N = \sum_{l_i} \frac{(B_{l_1 l_2 l_3})^2}{C_{l_1} C_{l_2} C_{l_3}}.
$$

(31)

The estimator for non-Gaussianity (30) is then simplified using (28) and (26) to yield
\[
\hat{f}_{\text{NL}} = \frac{1}{N} \sum_{l,m_1} \int d\Omega \bar{Y}_{l,m_1}(\hat{x}) Y_{l,m_2}(\hat{x}) Y_{l,m_3}(\hat{x}) \int_0^\infty x^2 dx j_{l_1}(k_1 x) j_{l_2}(k_2 x) j_{l_3}(k_3 x) (C_{l_1}^{-1} C_{l_2}^{-1} C_{l_3}^{-1})_{\text{obs}} \times 
\left( \frac{2}{\pi} \right)^3 \int dk_1 dk_2 dk_3 (k_1 k_2 k_3)^2 B(k_1, k_2, k_3) \Delta(k_1) \Delta(k_2) \Delta(k_3) j_{l_1}(k_1 R_D) j_{l_2}(k_2 R_D) j_{l_3}(k_3 R_D) \times 
(\alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3})_{\text{obs}},
\]

where \( B(k_1, k_2, k_3) \) is the primordial bispectrum obtained from the 3-point function as defined in [17]. In this manner the sought constraints are obtained. The best results, corresponding to the so called, local, equilateral and orthogonal shape of non-Gaussianities using the WMAP 7-year data [31] yield \( f_{\text{local}}^{\text{NL}} = 32 \pm 21 \) (1σ), \( f_{\text{equil}}^{\text{NL}} = 26 \pm 140 \) (1σ) and \( f_{\text{orthog}}^{\text{NL}} = -202 \pm 104 \) (1σ).

### III. THE COLLAPSE MODEL ACCOUNT FOR THE INFLATIONARY ORIGIN OF COSMIC STRUCTURE

Before proceeding it seems worthwhile to briefly explain the view we take regarding quantum physics and Einstein’s gravity. The framework we adopt is based on a description of the problem that allows, at the same time, the quantum treatment of other fields and a classical treatment of gravitation. That is the realm of semi-classical gravity together with quantum field theory in curved space-time. We will assume to be a valid approximation for most of the time, with the exception associated precisely with the dynamical collapse as we will explain below. Such description of gravitation in interaction with quantum fields is reflected in the semi-classical Einstein’s equation: \( R_{\mu\nu} - (1/2) g_{\mu\nu} R = 8\pi G \langle \hat{T}_{\mu\nu} \rangle \), whereas the other fields, including the inflaton, are treated in the standard quantum field theory fashion. It seems clear that this approximated description would break down in association with the quantum mechanical collapses or state jumps, that we consider to be part of the underlying quantum theory containing gravitation. The reason for this breakdown is simply that the left hand side of the previous equation does not contain any divergences, while the right hand side, and in particular its divergence \( \nabla_\nu \langle \hat{T}^{\mu\nu} \rangle \), will have discontinuities associated with the quantum jumps.

In this setting we start from the assumption that, in accordance with the standard inflationary accounts, and as mentioned before, the state of the Universe before the time at which
the seeds of structure emerge is described by the H&I Bunch-Davies vacuum state for the
matter degrees of freedom (D.O.F.) and the corresponding H&I classical Robertson-Walker
space-time.

Then, we assume that at a latter stage, the quantum state of the matter fields reaches
a stage whereby the corresponding state for the gravitational D.O.F. is forbidden, and a
quantum collapse of the matter field wave function is triggered by some unknown physical
mechanism. In this manner the state resulting from the collapse of the quantum state of the
matter fields needs not to share the symmetries of the initial state. After the collapse, the
gravitational D.O.F. are assumed to be, once more, accurately described by Einstein’s semi-
classical equation. However as \( \langle \hat{T}_{\mu \nu} \rangle \) for the new state, needs not have the symmetries of the
pre-collapse state, we are led to a geometry that generically will no longer be homogeneous
and isotropic.

The starting point of the specific analysis is the same as the standard picture, i.e. the
action of a scalar field coupled to gravity:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R[g] - \frac{1}{2} \nabla_a \phi \nabla_b \phi g^{ab} - V(\phi) \right],
\]

(33)

where \( \phi \) stands for the inflaton and \( V \) for the inflaton’s potential. One then splits both, met-
ic and scalar field into a spatially homogeneous (‘background’) part and an in-homogeneous
part (‘fluctuation’), i.e. \( g = g_0 + \delta g, \phi = \phi_0 + \delta \phi \).

The background is taken to be the spatially flat Friedmann-Robertson Universe with
line element \( ds^2 = a(\eta)^2 \left[ -d\eta^2 + \delta_{ij}dx^i dx^j \right] \), and the homogeneous scalar field \( \phi_0(\eta) \). The
evolution equations for this background are scalar field equations,

\[
\ddot{\phi}_0 + 2\frac{\dot{a}}{a} \dot{\phi}_0 + a^2 \partial_\phi V[\phi] = 0, \quad 3\frac{\dot{a}^2}{a^2} = 4\pi G (\dot{\phi}_0^2 + 2a^2 V[\phi_0]).
\]

(34)

The scale factor solution corresponding to the inflationary era of standard inflationary cos-
mology, written using a conformal time, is: \( a(\eta) = -1/[H_I^2(1-\epsilon)\eta] \) with \( H_I^2 \simeq (8\pi/3)GV \),
\( \epsilon \equiv 1 - \dot{H}/H^2 \) is the slow-roll parameter which during the inflationary stage is considered
to be very small \( \epsilon \ll 1 \), thus \( H_I \simeq \) constant, and with the scalar \( \phi_0 \) field in the slow roll
regime, i.e. \( \dot{\phi}_0 = -(a^3/3\dot{a})V' \). According to the standard inflationary scenario, this era is
followed by a reheating period in which the Universe is repopulated with ordinary matter
fields, a regime that then evolves towards a standard hot big bang cosmology regime leading
up to the present cosmological time. The functional form of $a(\eta)$ during these latter periods changes, but we will ignore those details because most of the change in the value of $a$ occurs during the inflationary regime. We will set $a = 1$ at the “present cosmological time”, and assume that inflationary regime ends at a value of $\eta = \eta_0$, negative and very small in absolute terms ($\eta_0 \simeq -10^{-22}$ Mpc).

Next we consider the perturbations. We shall focus on the scalar perturbations and ignore for simplicity the gravitational waves. Working in the so called longitudinal gauge, the perturbed metric is written as:

$$ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi) d\eta^2 + (1 - 2\Psi) \delta_{ij} dx^i dx^j \right], \quad (35)$$

where $\Psi$ stands for the scalar perturbation usually known as the Newtonian potential.

The perturbation of the scalar field is related to a perturbation of the energy momentum tensor, and reflected into Einstein’s equations which at lowest order lead to the following constraint equation for the Newtonian potential:

$$\nabla^2 \Psi = 4\pi G \dot{\phi}_0 \delta \dot{\phi} = s \delta \dot{\phi}, \quad (36)$$

where we introduced the abbreviation $s \equiv 4\pi G \dot{\phi}_0$.

Now we consider in some detail the quantum theory of the field $\delta \phi$. It is convenient to work with the rescaled field variable $y = a \delta \phi$ and its conjugate momentum $\pi = \delta \dot{\phi} / a$. For simplicity we set the problem in a finite box of side $L$, which can be taken to $\infty$ at the end of all calculations. We decompose the field and momentum operators as

$$\hat{y}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} \hat{y}_\vec{k}(\eta), \quad \hat{\pi}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} \hat{\pi}_\vec{k}(\eta), \quad (37)$$

where the sum is over the wave vectors $\vec{k}$ satisfying $k_i L = 2\pi n_i$ for $i = 1, 2, 3$ with $n_i$ integer and where $\hat{y}_\vec{k}(\eta) \equiv y_k(\eta) \hat{a}_{\vec{k}} + y_k^*(\eta) \hat{a}_{\vec{k}}^\dagger$ and $\hat{\pi}_\vec{k}(\eta) \equiv g_k(\eta) \hat{a}_{\vec{k}} + g_k^*(\eta) \hat{a}_{\vec{k}}^\dagger$ with the usual choice of modes:

$$y_k(\eta) = \frac{1}{\sqrt{2k}} \left( 1 - \frac{i}{\eta k} \right) \exp(-ik\eta), \quad g_k(\eta) = -i \sqrt{\frac{k}{2}} \exp(-ik\eta), \quad (38)$$

which leads to what is known as the Bunch-Davies vacuum.

Note that according to the point of view we discussed at the beginning of this section, and having at this point the quantum theory for the relevant matter fields, the effects of
the quantum fields on the geometrical variables are codified in the semiclassical Einstein’s equations. Thus equation (36) must be replaced by

$$\nabla^2 \Psi = 4\pi G \dot{\phi}_0 \delta \dot{\phi} = s \langle \delta \dot{\phi} \rangle = (s/a) \langle \hat{\pi} \rangle.$$  (39)

At this point one can clearly observe that if the state of the quantum field is in the vacuum state, the metric perturbations vanish and thus the space-time is homogeneous and isotropic.

Our proposal is based on consideration of a self induced collapse which we take to operate in close analogy with a “measurement” (but evidently, with no external measuring apparatus or observer involved). This lead us to want to work with Hermitian operators, which in ordinary quantum mechanics are the ones susceptible of direct measurement. Therefore we must separate both $\hat{y}_{\vec{k}}(\eta)$ and $\hat{\pi}_{\vec{k}}(\eta)$ into their real and imaginary parts $\hat{y}_{\vec{k}}(\eta) = \hat{y}_{\vec{k}}^R(\eta) + i\hat{y}_{\vec{k}}^I(\eta)$ and $\hat{\pi}_{\vec{k}}(\eta) = \hat{\pi}_{\vec{k}}^R(\eta) + i\hat{\pi}_{\vec{k}}^I(\eta)$ so that the operators $\hat{y}_{\vec{k}}^{R,I}(\eta)$ and $\hat{\pi}_{\vec{k}}^{R,I}(\eta)$ are hermitian operators.

So far we have proceeded in a manner similar to the standard one, except in that we are treating at the quantum level only the scalar field and not the metric fluctuation. At this point it is worthwhile to emphasize that the vacuum state defined by $\hat{a}_{\vec{k}}^{R,I}|0\rangle = 0$ is 100% translational and rotationally invariant. That is, under spatial translations $\hat{T}(d_i) = \exp[i\hat{P}_i d_i]$ and rotations $\hat{R}_x(\theta_i) = \exp[i\hat{L}(x)_i \theta_i]$, we have $\hat{T}(d_i)|0\rangle = 0$ and $\hat{R}_x(\theta_i)|0\rangle = 0$, with $\hat{P}_i$ and $\hat{L}(x)_i$ the linear and the angular momentum operators, and $d_i$ and $\theta_i$ parameters labeling the transformations.

For the next step we must specify in more detail the modeling of the collapse. Then, take into account that after the collapse has taken place, we must consider the continuing evolution of the expectation values of the field variables until the end of inflation and eventually up to the last scattering hypersurface (in fact, if we want to actually compare our analysis with the detailed observations, we must evolve also through the reheating period and through the decoupling era up to today’s Universe. This, however, is normally taken into account through the use of appropriate transfer functions, and we will assume that the same procedure can be implemented after the present analysis).

We will further assume that the collapse is somewhat analogous to an imprecise measurement of the operators $\hat{y}_{\vec{k}}^{R,I}(\eta)$ and $\hat{\pi}_{\vec{k}}^{R,I}(\eta)$. Now we will specify the rules according to

---

10 An imprecise measurement of an observable is one in which one does not end with an exact eigenstate
which collapse happens. Again, at this point our criteria will be simplicity and naturalness. What we have to describe is the state $|\Theta\rangle$ after the collapse. It turns out that, for the goals at hand, all we need to specify is the quantity $d_{k}^{R.I} \equiv \langle \Theta | \hat{a}_{k}^{R.I} | \Theta \rangle$, as this determines the expectation value of the field and momentum operator for the mode $\vec{k}$ at all times after the collapse.

It is natural to assume that after the collapse, the expectation values of the field and momentum operators in each mode, will be related to the uncertainties of the pre-collapse state (recall that the expectation values in the vacuum state are zero). In the vacuum state, $\hat{y}_{k}$ and $\hat{\pi}_{k}$ individually are distributed according to Gaussian wave functions centered at 0 with spread $(\Delta \hat{y}_{k})_{0}^{2}$ and $(\Delta \hat{\pi}_{k})_{0}^{2}$, respectively.

We might consider various possibilities for the detailed form of this collapse. Thus, for their generic form, associated with the ideas above, we write:

$$\langle \hat{y}_{k}^{R.I}(\eta_{k}^{c}) \rangle_{\Theta} = \lambda_{1} x_{k,1}^{R.I} \sqrt{(\Delta \hat{y}_{k}^{R.I})_{0}^{2}} = \lambda_{1} x_{k,1}^{R.I} |y_{k}(\eta_{k}^{c})| \sqrt{\hbar L^{3}/2},$$  \hspace{1cm} (40)

$$\langle \hat{\pi}_{k}^{R.I}(\eta_{k}^{c}) \rangle_{\Theta} = \lambda_{2} x_{k,2}^{R.I} \sqrt{(\Delta \hat{\pi}_{k}^{R.I})_{0}^{2}} = \lambda_{2} x_{k,2}^{R.I} |g_{k}(\eta_{k}^{c})| \sqrt{\hbar L^{3}/2},$$  \hspace{1cm} (41)

where $x_{k,1}^{R.I}, x_{k,2}^{R.I}$ are selected randomly from within a Gaussian distribution centered at zero with spread one and $\eta_{k}^{c}$ represents the time of collapse for each mode. Here $\lambda_{1}$ and $\lambda_{2}$ are two real numbers (usually 0 or 1) that allow us to specify the collapse proposal we want to consider. At this point, we must emphasize that our Universe corresponds to a single realization of these random variables, and thus each of these quantities $x_{k,1}^{R.I}, x_{k,2}^{R.I}$ has a single specific value. On the other hand, we will be using the values for $\lambda_{1}$ and $\lambda_{2}$ to characterize the different collapse schemes, e.g.: i) $\lambda_{1} = 0, \lambda_{2} = 1$, ii) $\lambda_{1} = \lambda_{2} = 1$ (which we call the “symmetric scheme”). It is clear that one can devise many other models of collapse, a good fraction of them can be described within in the scheme above, while others require a slightly modified treatment \[7\]. Still, there are surely many other possibilities which we have not even thought about and which might require drastically modified formalisms.

Finally for each model we obtain the information giving the relevant expectation values of the field operators in the post collapse state $|\Theta\rangle$. That is, from the equations above, one can consider measuring a certain particle’s position and momentum so as to end up with a state that is a wave packet with both position and momentum defined to a limited extent, and which, of course, does not entail a conflict with Heisenberg’s uncertainty bound.

---

\[21\]
solve in each case for the quantities \( d_k^{R,I} \), and using the result in the evolution equations for the expectation values (i.e. using Ehrenfest’s Theorem) one obtains \( \langle \hat{a}_k^{R,I}(\eta) \rangle \) and \( \langle \hat{\pi}_k^{R,I}(\eta) \rangle \) for the state that resulted from the collapse, for all later times. The explicit expressions for the \( \langle \hat{y}_k^{R,I}(\eta) \rangle \) and \( \langle \hat{\pi}_k^{R,I}(\eta) \rangle \) are

\[
\langle \hat{y}_k^{R,I}(\eta) \rangle = \left[ \cos D_k \left( \frac{1}{k\eta} - \frac{1}{z_k} \right) + \sin D_k \left( \frac{1}{k\eta z_k} + 1 \right) \right] \langle \hat{\pi}_k^{R,I}(\eta^c) \rangle \Theta + \left( \cos D_k - \sin D_k \frac{k\eta}{k\eta} \right) \langle \hat{y}_k^{R,I}(\eta^c) \rangle \Theta, \tag{42}
\]

\[
\langle \hat{\pi}_k^{R,I}(\eta) \rangle = \left( \cos D_k + \frac{\sin D_k}{z_k} \right) \langle \hat{\pi}_k^{R,I}(\eta^c) \rangle \Theta - k \sin D_k \langle \hat{y}_k^{R,I}(\eta^c) \rangle \Theta, \tag{43}
\]

where \( D_k \equiv k\eta - z_k \) and \( z_k \equiv k\eta^c_k \).

With this information at hand we can now compute the perturbations of the metric after the collapse of all the modes [11].

### A. Connection to Observations

Now, we must put together our semi-classical description of the gravitational D.O.F. and the quantum mechanics description of the inflaton field. We recall that this entails the semi-classical version of the perturbed Einstein’s equation that, in our case, leads to equation [39]. The Fourier components at the conformal time \( \eta \) are given by:

\[
\Psi_k(\eta) = -\sqrt{\frac{\epsilon}{2M_Pk^2}} \langle \hat{\pi}_k^{R,I}(\eta) \rangle, \tag{44}
\]

where we used that during inflation \( s \equiv 4\pi G \dot{\phi}_0 = \sqrt{\epsilon/2aH_I/M_P} \), with \( M_P \) the reduced Planck’s mass \( M_P^2 \equiv \hbar^2/(8\pi G) \). The expectation value depends on the state of the quantum field, therefore, as we already noted, prior to the collapse, we have \( \Psi_k(\eta) = 0 \), and the space-time is still homogeneous and isotropic at the corresponding scale. However after the collapse takes place, the state of the field is a different state with new expectation values which generically will not vanish, indicating that after this time the Universe becomes

[11] In fact, we need only be concerned with the relevant modes, those that affect the observational quantities in a relevant way. Modes that have wavelengths that are either too large or too small are irrelevant in this sense.
anisotropic and in-homogeneous at the corresponding scale. We now can reconstruct the space-time value of the Newtonian potential using

$$\Psi(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} \Psi_{\vec{k}}(\eta),$$  \hspace{1cm} (45)$$

to extract the quantities of observational interest.

In order to make contact with the observations we shall relate the expression (44) for the evolution of the Newtonian potential during the early phase of accelerated expansion, to the small anisotropies observed in the temperature of the cosmic microwave background radiation, $\delta T(\theta, \varphi)/T_0$ with $T_0 \approx 2.725K$ the temperature average. They are considered as the fingerprints of the small perturbations pervading the Universe at the time of decoupling, and undoubtedly any model for the origin of the seeds of cosmic structure should account for them. As already mentioned in section [1] these data can be described in terms of the coefficients $\alpha_{lm}$ of the multipolar series expansion, i.e. equation (5). The different multipole numbers $l$ correspond to different angular scales; low $l$ to large scales and high $l$ to small scales. At large angular scales ($l \lesssim 20$) the anisotropies in the CMB arise due to the Sachs-Wolfe effect. That effect relates the anisotropies in the temperature observed today on the celestial sphere to the inhomogeneities in the Newtonian potential on the last scattering surface,

$$\frac{\delta T}{T_0}(\theta, \varphi) = \frac{1}{3} \Psi(\eta_D, \vec{x}_D).$$  \hspace{1cm} (46)$$

Here $\eta_D$ is the conformal time of decoupling which lies in the matter-dominated epoch, and $\vec{x}_D = R_D(\sin \theta \sin \varphi, \sin \theta \cos \varphi, \cos \theta)$, with $R_D$ the radius of the last scattering surface. Furthermore, using (45) and $e^{i\vec{k} \cdot \vec{x}_D} = 4\pi \sum_{lm} i^l j_l(kR_D)Y_{lm}(\theta, \varphi)Y_{lm}^*(\hat{k})$, the expression (5) for $\alpha_{lm}$ can be rewritten in the form (6). The transfer function $\Delta(k)$ represents the evolution of the Newtonian potential from the end of inflation $\eta_R$ to the last scattering surface at the time of decoupling $\eta_D$, i.e. $\Psi_{\vec{k}}(\eta_D) = \Delta(k)\Psi_{\vec{k}}(\eta_R)$.

Substituting (43) in (44) and using (40), (41) gives

$$\Psi_{\vec{k}}(\eta_R) = \frac{-(L\hbar)^{3/2} \sqrt{\epsilon H_I}}{2\sqrt{2} M_P k^{3/2}} \left[ \lambda_2 \left( \cos D_k + \frac{D_k}{z_k} \right)(x_{k,2}^R + ix_{k,2}^I) - \lambda_1 \sin D_k \left( 1 + \frac{1}{z_k^2} \right)^{1/2} \left( x_{k,1}^R + ix_{k,1}^I \right) \right].$$  \hspace{1cm} (47)$$

Finally using, (47) in (6) yields
\[ \alpha_{lm} = -\frac{\pi^4 \hbar^3/2 \sqrt{2} e H_I \Delta(k)}{3(Lk)3/2 M_P} \sum_{\mathbf{k}} \Delta(k) j_l(kR_D)Y^*_m(\hat{k}) \left[ \lambda_2 \left( \cos D_k + \frac{\sin D_k}{z_k} \right) \left( x_{k,2} + i x_{k,2}^I \right) \right. \\
- \lambda_1 \sin D_k \left( 1 + \frac{1}{z_k^2} \right)^{1/2} \left( x_{k,1}^R + i x_{k,1}^I \right) \right], \quad (48) \]

note that in (47) and (48), \( D_k \) is evaluated at \( \eta_R \), i.e. \( D_k(\eta_R) = k \eta_R - z_k \).

It is worthwhile to mention that the relation for \( \alpha_{lm} \) with the Newtonian potential, as obtained in (48) within the collapse framework has no analogue in the usual treatments of the subject. It provides us with a clear identification of the aspects of the analysis where the “randomness” is located. In this case, it resides in the randomly selected values \( x_{k,1}^R, x_{k,2}^I \) that appear in the expressions of the collapses associated with each modes. Here, we also find a clarification of how, in spite of the intrinsic randomness, can we make any prediction at all.

The individual complex quantities \( \alpha_{lm} \) correspond to large sums of complex contributions, each one having a certain randomness but leading in combination to a characteristic value in just the same way as a random walk made of multiple steps. In other words, the justification for the use of statistics in our approach is: The quantity \( \alpha_{lm} \) is the sum of contributions from the collection of modes, each contribution being a random number leading to what is in effect a sort of “two-dimensional random walk” whose total displacement corresponds to the observational quantity. Nothing like this can be found in the most popular accounts, in which the issues we have been focussing on, are hidden in a maze of often unspecified assumptions and unjustified identifications [5].

Thus, according to (48) all the modes contribute to \( \alpha_{lm} \), with a complex number. If we had the outcomes characterizing each of the individual collapses we could of course predict the value of this quantities. However, we have at this point no other access to such information than the observational quantities \( \alpha_{lm} \) themselves.

We hope to be able to say something about these but doing so requires the consideration of further hypothesis regarding the statistical aspects of the physics behind the collapse as well as the conditions previous to them.

As is in general the case with random walks, one can not hope to estimate the direction of the final displacement, however one might say something about its estimated magnitude. It is for that reason we will be focussing on estimating the most likely value of the magnitude:
We can progress for instance by making some assumption allowing us to regard the specific outcomes characterizing our Universe as a typical member of some hypothetical ensemble of Universes.

For example, we are interested in estimating the most likely value of the magnitude of \(|\alpha_{lm}|^2\) above, and in such hypothetical ensemble we might hope that it comes very close to our single sample. It is worthwhile emphasizing that for each \(l\) and \(m\) we have one single complex number characterizing the actual observations (and thus the real Universe we inhabit). For a given \(l\) for instance, we should avoid confusing ensemble averages with averages of such quantities over the \(2l + 1\) values of \(m\). The ensemble are figments of our imagination and there is nothing in our theories that would indicate that they are real.

We can simplify things even further by taking the ensemble average \(|\alpha_{lm}|^2\) (the bar will from now on means that we are taking the ensemble average) and identifying it with the most likely value of the quantity, and needless is to say that these two notions are different in many types of ensembles. However, let us for the moment ignore this issue and assume the identity of those two values and look at the ensemble average of the quantity \(|\alpha_{lm}|^2\) which is given by

\[
|\alpha_{lm}|^2 = \frac{16\pi^2}{9L^6} \sum_{\vec{k}, \vec{k}'} \Delta(k)\Delta(k') j_l(kR_D)j_l(k'R_D)Y^*_{lm}(\hat{k})Y_{lm}(\hat{k}')\bar{\Psi}_{\vec{k}}(\eta_R)\bar{\Psi}_{\vec{k}'}(\eta_R). \tag{49}
\]

One can for instance assume that the collapsing events are all uncorrelated, and then consider estimating the most likely value thus

\[
|\alpha_{lm}|^2_{ML} = \frac{16\pi^2}{9L^6} \sum_{\vec{k}, \vec{k}'} \Delta(k)\Delta(k') j_l(kR_D)j_l(k'R_D)Y^*_{lm}(\hat{k})Y_{lm}(\hat{k}')\bar{\Psi}_{\vec{k}}(\eta_R)\bar{\Psi}_{\vec{k}'}(\eta_R). \tag{50}
\]

Under the assumption of the validity of such approximation and the additional assumption that the random variables \(x_{k,1}^R, x_{k,1}^I, x_{k,2}^R, x_{k,2}^I\) are all uncorrelated, we obtain that all the information regarding the “self-collapsing” model will be codified in the quantity:

\[
\bar{\Psi}_{\vec{k}}(\eta_R)\bar{\Psi}_{\vec{k}'}(\eta_R). \tag{52}
\]
Furthermore, we can take the limit $-k\eta_R \rightarrow 0$ in (52), which can be expected to be appropriate when restricting interested on the modes that are “outside the horizon” at the end of inflation, since these are the modes that give a major contribution to the observationally relevant quantities.

Under those conditions, and with the help of (47), after taking the continuum limit ($L \rightarrow \infty$), the quantity of interest becomes:

\[ |\alpha_{lm}|^2_{ML} = \frac{\hbar^3 \epsilon H^2}{36\pi M_p^5} \int \frac{dk}{k} \Delta^2(k) j_l^2(kR_D)C(k), \]  

(53)

where some of the information regarding that a collapse has occurred is contained in the function $C(k)$. The explicit form of $C(k)$ is

\[ C(k) = \lambda_1^2 \left( 1 + \frac{1}{z_k^2} \right) \sin^2 z_k + \lambda_2^2 \left( \cos z_k - \frac{\sin z_k}{z_k} \right)^2. \]  

(54)

As we have noted in previous works, this quantity becomes a simple constant if the collapse time happens to follow a particular pattern in which the time of collapse of the mode $\vec{k}$ is given by $\eta_k = Z/k$ with $Z$ a constant. In fact, the standard answer would correspond to $C(k) = \text{constant}$ (which can be thought as an equivalent “nearly scale invariant power spectrum”). Thus, the result obtained for the relation between the time of collapse and the mode’s frequency, i.e. $\eta_k^2 k= \text{constant}$ is a rather strong conclusion which could represent relevant information about whatever the mechanism of collapse is. A preliminary study of the effects of small deviations from such pattern for the “symmetric scheme” ($\lambda_1 = \lambda_2 = 1$); for the “Newtonian scheme” ($\lambda_1 = 0$, $\lambda_2 = 1$) and for a third scheme that falls outside the category contemplated here, have been carried out in [7].

It is quite clear that if the time of collapse of each mode do not adjust exactly to the pattern $k\eta_k^2 = Z$, then the various schemes of collapse, characterized by the values of $\lambda_1, \lambda_2$ or some other function $C(k)$, would lead to different predictions for the exact form of the

\[ P\zeta(k)^2 \propto \frac{H^2(M^2_P)}{P_{\zeta} / \epsilon} \propto V / (\epsilon \lambda^4), \]  

26

This quantity is constant for modes “outside the horizon” (irrespectively of the cosmological epoch), thus it avoids the use of the transfer function $\Delta(k)$. In the standard literature it is common to find the power spectrum for the quantity $\zeta(x)$, a field representing the curvature perturbation in the co-moving gauge. This quantity is constant for modes “outside the horizon” (irrespectively of the cosmological epoch), thus it avoids the use of the transfer function $\Delta(k)$. The quantity $\zeta$ can be defined in terms of the Newtonian potential as $\zeta \equiv \Psi + (2/3)(H^{-1}\dot{\Psi} + \Psi)/(1 + \omega)$, with $\omega \equiv p/\rho$. For large-scale modes $\zeta \simeq \Psi[(2/3)(1 + \omega)^{-1} + 1]$, and during inflation $1 + \omega = (2/3)\epsilon$. For these modes $\zeta \simeq \Psi / \epsilon$ and the power spectrum is $P_\zeta(k) = \mathcal{P}_\Psi(k) / \epsilon^2 \propto H^2(M^2_P \epsilon) \propto V / (\epsilon \lambda^4)$, which contains the correct amplitude. For a detailed discussion regarding the
spectrum, and comparing these predictions with the observations can help us to discriminate between the distinct collapse schemes.

We end this section by noting that the treatment of the statistical aspects in the collapse proposal is quite different from the standard inflationary paradigm. We will deepen this discussion in the next section. However, at this point the differences should be evident. In the standard accounts, one is going from quantum correlation functions to classical $n$-point functions averaged over an ensemble of Universes; then one goes to $n$-point correlation functions averaged over different regions of our own Universe, and finally one relates this last quantity with the observable $|\alpha_{lm}|^2$. These series of steps are not at all direct and they involve a lot of subtle issues that the standard picture does not provide in a transparent way.

On the other hand, in the collapse proposal, the observable $|\alpha_{lm}|^2$ is related to the random variables $x_k$’s through a “random walk”, as we mentioned, the value of $|\alpha_{lm}|^2$ corresponds to the “length” of the random walk. This random walk is associated to a particular realization of a physical quantum process (i.e. the collapse of the inflaton’s wave-function), since we have only access to one realization—the random walk corresponding to our own Universe—the most natural assumption (but certainly not the only one) is that the average value of the length of the possible random walks, which corresponds to $|\alpha_{lm}|^2$, is equal to the most likely value, i.e. to $|\alpha_{lm}|_{ML}^2$, and this in turn, is associated with the $|\alpha_{lm}|^2$ of our observable Universe.

IV. STATISTICAL ASPECTS

The first thing we should now note is that there are several statistical issues at play, and that within our approach, various novel ones emerge. One aspect is the exact nature of the state previous to all collapses, i.e. the state characterizing the first stages of the inflationary regime and normally taken to be the Bunch-Davies vacuum. There are various aspects that might affect and modify the nature of that state: For instance, if the field is not truly a free field and self interactions are important one might find correlations between the various modes of the field. These effects could be manifest, for instance by non vanishing values of quantities like $\langle 0|\hat{y}_k \hat{y}_{k'}|0 \rangle$. However, we should be aware, not only of the inherent problems of accessing these associated with the fact that we have at our disposal a single Universe, but also that our Universe, including the relevant perturbations is not characterized by the
vacuum state, but rather by the state that results after the collapses of all the modes, and, it is quite clear that the collapse process itself can be a source of unexpected correlations. These would manifest themselves, for example, in correlations between the values taken by the $x_k$’s appearing in the collapse process and which we have so far assumed were different and independent quantities for each mode.

Moreover, we have to note that the quantities that are more or less directly accessible to observational investigation are not the $\langle \Theta | \hat{y}_k | \Theta \rangle$ and the $n$-point functions in general, for the post-collapse state, but the various $\alpha_{lm}$’s, and the latter are related to the former, as can be seen in (48) in a nontrivial way. In fact as we saw each $\alpha_{lm}$ corresponds to a sort of two dimensional random walk (i.e. a sum of complex quantities) and each of the steps is related to $\langle \Theta | \hat{y}_k | \Theta \rangle$. It is thus clear that there might be correlations between the various $\alpha_{lm}$’s simply due to the fact that they arise from different combinations of the same random variables. Of course, we should note that the particular analysis behind our analysis was based on the assumption that the elementary process was associated with the collapse of the observables $\hat{y}_k$ and their conjugate momenta according to (40), (41). It is clearly conceivable that the elementary process might have been associated instead with other observables. One simple possibility for those alternative observables, are the various options offered by linear combinations of the former.

A. The new outlook on non-Gaussianities

In this section we discuss the aspects that need modification in the study of primordial non-Gaussianities in view of the approach we have been discussing to the origin of the primordial fluctuations.

The first point we should stress is that from the two aspects of cosmology mentioned in section I we have seen that we have had to modified the first, namely the nature of the quantum state in order to be compatible with the existence, at the fundamental (quantum) level, of the inhomogeneities and anisotropies that are behind the emergence of structure and thus of everything –including observers– in our Universe.

Namely, the standard physics of the very early Universe had to be supplemented with the collapse hypothesis in order to fully account for the process that created the primordial seeds for large-scale structure. Otherwise, we could not really identify the process by which
the inhomogeneity and anisotropies emerged from the initial vacuum.

As in the standard approach, we take the curvature perturbations $\Psi$ to be the generators of the CMB anisotropy, $\delta T/T$, however, in our approach, the observed fluctuations are determined not just by the initial vacuum state, which is and remains homogeneous and isotropic, but also by the characteristics of the collapse process, besides, of course the effects of the late time physics.

In this more precise and detailed approach, it is clear that even if the primordial state can be consider as Gaussian; in the sense that the corresponding $n$-point functions are completely determined by the 2-point functions, and thus the odd $n$, $n$-point functions vanish—it might still be possible for collapse process to drastically affect and modify this. In other words there exist in principle the possibility that the collapse process itself introduces non-Gaussian characteristics into the state. We will not discuss this possibility here but only point it out as something to have in mind, and as a topic for future research.

As we have argued, the quantity of observational interest is not really $\langle 0|\hat{\Psi}(\vec{x}, \eta)\hat{\Psi}(\vec{y}, \eta)|0\rangle$ as the argument to justify that in the standard approach, depends not only on accepting the identification $\langle 0|\hat{\Psi}(\vec{x}, \eta)\hat{\Psi}(\vec{y}, \eta)|0\rangle = \Psi(\vec{x}, \eta)\Psi(\vec{y}, \eta)$, where $\Psi(\vec{x}, \eta)$ is taken to be a classical stochastic field and the overline denoting the average over an ensemble of Universes, but also, on a series of arguments indicating one can replace the ensemble averages with suitable spatial averages of quantities in our Universe.

As a matter of fact, a clear example of how a careless approach to the statistics at hand can lead to wrong conclusions, is brought by the variance $\overline{\Psi^2}$. We mentioned in section [II] that $\overline{\Psi^2}$ diverges generically if we do not introduce an ad-hoc cut-off for $k$. Therefore, if we consider the temperature fluctuations in a particular point $\vec{x}_0$ of the CMB 2-sphere and we estimate it in terms of $\langle 0|\hat{\Psi}^2(\vec{x}_0, \eta)|0\rangle$, we obtain a divergent quantity, but clearly, from the observational data, we know that these fluctuations of the mean temperature in a particular point are rather small $\sim 10^{-5}$. Meanwhile, in the collapse proposal, we do not need to deal with these issues because the variables subjected to the collapse are not $\hat{y}(\vec{x}, \eta)$, $\hat{\pi}(\vec{x}, \eta)$ but the field modes $\hat{y}_k(\eta)$, $\hat{\pi}_k(\eta)$, i.e. the collapse does not occur in the position space, instead we consider it as a collapse for each mode $\vec{k}$. Therefore, the quantities of observational interest, namely the $|\alpha_{lm}|^2$’s depend on the expectation values $\langle \hat{y}_k(\eta) \rangle_{\Theta}$, $\langle \hat{\pi}_k(\eta) \rangle_{\Theta}$ in the state $|\Theta\rangle$ after the collapse.

As we saw in the Introduction, if we really took $\Psi(\vec{x}, \eta)$ to be Gaussian and allowed the
identification of its $n$-point functions with the observations, we would have to accept that such identification holds in particular for the 1-point function, and that would lead us to a clear conflict between theory and observation.

Similarly we must be careful when we consider the three-point function:

$$\Psi(\vec{x}, \eta)\Psi(\vec{y}, \eta)\Psi(\vec{z}, \eta),$$

(55)

and use the identification with ensemble averages with the measured quantities as an indicator of non-Gaussian features in the cosmological perturbations.

As we saw the bispectrum $\Psi_{\vec{k}_1}\Psi_{\vec{k}_2}\Psi_{\vec{k}_3} = (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)B_\Psi(k_1, k_2, k_3)$ is usually said to represent the lowest order statistics able to distinguish non-Gaussian from Gaussian perturbations, because Gaussianity is identified with the requirement that all statistical information is contained in the 2-point functions, and thus implicitly with the vanishing of all $n$-point functions with $n$ odd. However, the lowest odd integer is 1 not 3, and as we already saw the serious issue that arises with the 1-point function, forces us to question the standard arguments.

In fact looking anew at the quantities normally associated with the 1-point function we see that we have at our disposal not only the average quantities $C_l$, but also, for every value of $l$ and $m$, the individual quantities $\alpha_{lm}$. Each one of those correspond, in our approach, to different random walks. It could prove very interesting to study the distribution of these numbers themselves: namely we can look at the plot of, say the magnitudes $|\alpha_{lm}|$ for a given value of $l$. This set of $2l+1$ numbers can naturally be expected to display a Gaussian shape.

This seems to be a particularly relevant analysis and it is not clear to us if something like that has been studied in the literature. It seems to us, that the traditional approach would have naturally lead to the consideration of that issue. Looking at the distribution of the corresponding phases should be equally enlightening. Moreover, as we mentioned in the discussion around the equation (55), it would be interesting to evaluate the quantity $\pi_l$ defined there, and compare the result with the any of the natural estimates for its value, particularly the expected ensemble average of its magnitude.

Another point worth to note is, as we have mentioned already, it is usually believed that a large detectable amount of non-Gaussianity is to be expected when the initial state of the quantum field is not the preferred Bunch-Davies vacuum state. Nevertheless, in the collapse proposal we are considering, the quantum state of the field after the collapse
is $|\Theta\rangle \neq |0\rangle$ (the analysis of a particular characterization of the post-collapse state has been done in \cite{32}). Therefore, the curvature perturbation responsible for the temperature anisotropies in the CMB, is due to the expectation values $\langle \hat{y}_k \rangle_\Theta$ and $\langle \hat{\pi}_k \rangle_\Theta$ which in principle could generate detectable non-Gaussianities, as these quantities are never considered in the standard accounts. Of course, a further exploration of these ideas is required.

The other delicate issue related to the statistical aspects of the traditional approach is encoded in the ergodicity assumption which we will discuss in the following. For instance, as we already saw in section \ref{sec:ergodicity} the CMB bispectrum was defined as the 3-point correlator of the $\alpha_{lm}$ through $B_{l_1l_2l_3}^{m_1m_2m_3} \equiv \bar{\alpha}_{l_1m_1}\alpha_{l_2m_2}\alpha_{l_3m_3}$. The standard picture forces one to deal with the issue that the RHS represents an average over an ensemble of Universes, while we have but one realization $\{\alpha_{l_1m_1}, \alpha_{l_2m_2}, \ldots, \alpha_{l_n m_n}\}$. To overcome this issue, the standard approach relies on an ergodicity assumption, and which identifies the average value of a process characteristic measured over time to be the same as the average value measured over the ensemble.

There are various issues that lead one to be concerned about this assumption and the application to the situation at hand: The first thing we must be aware is that ergodicity is a property of systems in equilibrium and it is rather unclear why this should be valid regarding the conditions associated with the inflationary regime.

Next, as we mentioned, the ergodicity assumption is translated in the case at hand into the notion that the volume average of the fluctuations behave like the ensemble average “the Universe may contain regions where the fluctuation is atypical, but with high probability most regions contain fluctuations with root-mean-square amplitude close to $\sigma$”, and thus one argues that the probability distribution on the ensemble, translates to a probability distribution on smoothed regions of a determined size within our own Universe.

There are three issues here:

i) How do we go from the arguments supporting ergodicity in time averages, to corresponding arguments for spatial averages?.

ii) Regarding the CMB, we in fact do not have access to the spatial sections that would allow us to investigate the space averages. We only have access to the particular intersection of our past light come with the 3-d hypersurface of last scattering; that is to a 2-sphere that we see as the source of the CMB photons that reach us today. Let
call it from now on the CMB 2-sphere. \footnote{We note in relation to this point that there are intrinsic problems in considering ergodicity of processes within a two sphere as discussed in \cite{33}.}

iii) The third problem is that each one of the quantities of interest $\alpha_{lm}$ is itself already a weighted average over the CMB 2-sphere (with the weight function given by the corresponding $Y_{lm}(\theta, \varphi)$). Therefore, what would be this new average we would be talking about in the arguments above?. In other words, if one is willing to accept that the ensemble averages should coincide with averages over the 2-sphere, why would one not also accept that the weighted average over the two sphere, should coincide with the equally weighted average over ensembles?. The latter would be zero so this is clearly unacceptable.

It seems clear that we are dealing here with an orientation average: The different $\alpha_{lm}$ would mix among themselves if we were to redefine the orientation of the coordinate chart used to describe the CMB 2-sphere. When we look at the averages that are actually performed in connection with the study of the primordial spectrum we see these are indeed orientation averages. For instance, the observational quantity $C_{l}^{\text{obs}} = \frac{1}{2l+1} \sum_m |\alpha_{lm}|^2$ is just the orientation average value of the magnitude of the $\alpha_{lm}$’s for a fixed value of $l$. In the same fashion, we see that the angle-averaged bispectrum $B_{l_1,l_2,l_3}$ is an orientation average for fixed $l$’s and, as for the same reason as the 1-point function, it is quite unclear how to identify orientation averages with ensemble averages. Thus, the statistical analysis would be more transparent if one would focus on the distribution of the quantities $B_{m_1m_2m_3}^{l_1l_2l_3}$.

As we saw, it is customary to take as an estimator for the nonlinearity parameter the quantity $\hat{f}_{\text{NL}}$ defined in \cite{32}. This seems a bit problematic as it involves a mixture of theoretical and observational quantities. Ideally one would like to have the two aspects rather well separated. In fact even within the standard approach, for the case of the 2-point functions we have on the one hand: the theoretical quantity

$$C_{l}^{\text{th}} = \frac{2}{\pi} \int k^2 P_{\Psi}(k) \Delta^2(k) j_l^2(kR_D) dk,$$

and on the other hand the observational quantity,

$$C_{l}^{\text{obs}} = \frac{1}{2l+1} \sum_m |\alpha_{lm}|^2.$$

\cite{32}
This independence of the definitions allows one to cleanly compare theory and observation. It thus seems that one would want to consider studying the aspects tied to non-Gaussianity, using a quantity that can be equally susceptible to theoretical and observational determination. Here we would like to propose, based on the considerations we have been discussing, the option we discuss below.

First, motivated by the quantity defined in (25) let us introduce the definition of the observed bispectrum as the orientation average

$$B_{l_1 l_2 l_3}^{\text{obs}} \equiv \sum_{m_i} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} (\alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3})_{\text{obs}}, \quad (58)$$

and the definition of the normalized observational reduced bispectrum as the quantity:

$$\tilde{b}_{l_1 l_2 l_3}^{\text{obs}} \equiv \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}^{-1} B_{l_1 l_2 l_3}^{\text{obs}}, \quad (59)$$

and finally let us define the magnitude of the bispectral fluctuations as:

$$F_{l_1 l_2 l_3}^{\text{obs}} \equiv \frac{1}{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)} \sum_{m_i} \left[ (\alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3})_{\text{obs}} - G_{l_1 l_2 l_3}^{m_1 m_2 m_3} \tilde{b}_{l_1 l_2 l_3}^{\text{obs}} \right]^2. \quad (60)$$

One can then compare this pure observational quantity with the corresponding theoretical estimation corresponding to suitable ensemble average where each element of the ensemble is characterized by the specific random choice of the numbers $x_{\vec{k}}$ that characterize the collapses.

This seems to offer an approach to the issue at hand that indeed has the advantage of allowing a direct comparison between the purely observational quantities untainted by theoretical models, and the quantities that are purely defined in terms of such theoretical analysis. This in fact seems to share some of the spirit of the analysis made in references \[14, 15\], although our proposal provides a clear potion to compute the observational and theoretical quantities in complete separation, and that seems not to be available in the former. The reason for this seems easy to understand: The fact that we maintain a clear distinction between ensemble averages and orientation averages avoids the possibility of the confusion associated with the fact that the ensemble average of the quantity

$$(\alpha_{l_1 m_1} \alpha_{l_2 m_2} \alpha_{l_3 m_3})_{\text{obs}} - G_{l_1 l_2 l_3}^{m_1 m_2 m_3} \tilde{b}_{l_1 l_2 l_3}^{\text{obs}}, \quad (61)$$
appearing in (60) vanishes identically.

The detailed analysis of estimators like this will be carried out in future works, but we wanted to present it as an example of the type of analysis that is motivated by our approach to the whole question of emergence of structure from quantum fluctuations in the inflationary early Universe.

V. PREDICTIONS AND DISCUSSION

Focussing in trying to understand the essence of the emergence of inhomogeneous and anisotropic features from a quantum state that is homogeneous and isotropic and in the absence of a measurement process, has lead us to consider modifying the standard approach through the incorporation of the collapse hypothesis.

We have seen in previous works that despite the fact that the motivation for such considerations seems to be purely philosophical and tied to issues like the measurement problem in quantum mechanics, the analysis have led to expect certain departures that could potentially be of observational significance.

In previous works, we have focussed on two main observationally related issues: The shape of the spectrum and the question of tensor modes. We have argued previously that it would be very unlikely that one could find a scheme in which the function \( C(k) \) would be exactly a constant, and that some dependence of \( k \) will remain in any reasonable collapse scheme, simply because we do not expect those collapses to follow exactly the \( \eta_k = Z/k \) rule for the time of collapse for each mode. These dependences lead to slight deviations from the standard form of the spectrum. In fact a preliminary analysis of this issue has been carried, confirming these expectations. A more detailed analysis of the exact form of the spectrum incorporating the late time physics is needed, and under way, to study the traces left by such effects in the observational CMB spectrum, and as we have indicated, these can be searched for observationally.

We have also stated, in previous works, that the most clear prediction of the novel paradigm we have been proposing, is the absence of tensor modes, or at least their very strong suppression. The reason for this can be understood by considering the semiclassical version

\[ 14 \] In the early Universe there are no observers or measuring devices, and in fact, the conditions for their emergence is the result from the breakdown of such symmetries, so it would seem very odd if one took a view that they are part of the cause of that breakdown.
of Einstein’s equations and its role in describing the manner in which the inhomogeneities and anisotropies in the metric arise. As we have explained in our approach, the metric is taken to be an effective description of the gravitational D.O.F., in the classical regime, and not as the fundamental D.O.F. susceptible to be described at the quantum level. It is thus the matter degrees of freedom (which in the present context are represented by the inflaton field) the ones that are described quantum mechanically and which, as a result of a fundamental aspect of gravitation at the quantum level, undergo effective quantum collapse (the reader should recall that our point of view is that gravitation at the quantum level will be drastically different from standard quantum theories, and that, in particular, it will not involve universal unitary evolution). This collapse of the quantum state of the inflaton field leads to a nontrivial value for $\langle T_{\mu\nu} \rangle$, which then generates the metric fluctuations. The point is that the energy momentum tensor contains linear and quadratic terms in the expectation values of the quantum matter field fluctuations, which are the source terms determining the geometric perturbations. In the case of the scalar perturbations, we have first order contributions proportional to $\dot{\phi}_0 \langle \delta \dot{\phi} \rangle$ while no similar first order terms appear as source of the tensor perturbations (i.e. of the gravitational waves). Of course, it is possible that the collapse scheme works at the level of the simultaneously quantized matter and metric fluctuations as has been presented in [34] although, as explained there, we would find much harder to reconcile that with the broad general picture that underlies of our current understanding of physics.

In the present work we have focused on the modified statistical considerations associated with this novel paradigm. We have argued that the collapse process itself could be the source of non-Gaussian features. We discussed some difficulties associated with the usual identification of measuring quantities with the quantum $n$-point functions and particularly found that extending the standard arguments to the 1-point functions lead to disastrous disagreements with observations.

We have shown that our approaches provides expressions which have no parallel in the standard formulations and which allow a precise identification of the location of the randomness, as exemplified by our theoretical formula (48) for $\alpha_{lm}$ in terms of the random numbers characterizing the collapses, namely the quantities $x_{R,1,2}^{R,1,2}$. This kind of expression facilitates all resulting statistical considerations and in particular, it is the basis for the theoretical estimation of the quantity (60).
We have proposed various novel ways to look into the statistical aspects of the problem:

i) We indicate the importance of exploring the true nature of the one point function by studying the degree of deviation from zero from the complex quantity $\bar{\alpha}_{l}^{\text{obs}} = \frac{1}{2l+1} \sum_{l} \alpha_{lm}^{\text{obs}}$.

ii) We have argued that it is worthwhile to study the specific form of the distribution of the values of the observed quantities $|\alpha_{lm}^{\text{obs}}|$ for each fixed $l$.

iii) We have proposed new characterizations of the quantities normally associated with the bispectrum and the quantum 3-point functions which can be computed both, in purely theoretically, and in a completely observational fashion. This is the quantity defined in (60).

It is clear that this work represents only the first step in the study of the statistical aspects of the cosmic structure and its generating process during inflation, within the context of the new paradigm which centers on the collapse hypothesis. Much more work remains to be done, but we hope this can become a research avenue of great richness, and one which would lead to important insights, with possible implications not only for the generation of structure itself but for the modification of quantum theory, which would underly the collapse mechanism and which, as has been argued before, might have deeper origins at the quantum/gravity interface [5, 10, 12, 35].

ACKNOWLEDGMENTS

The work of GL and DS is supported in part by the CONACyT grant No 101712. DS was supported in part by sabbatical fellowships from CONACyT and DGAPA-UNAM and the hospitality of the IAFE. The work of SJL is supported by PICT 2007-02184 from Agencia Nacional de Promoción Científica y Tecnológica, Argentina and by PIP N 11220090100152 from Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina.

[1] A. Guth and S.-Y. Pi, “Quantum Mechanics of the scalar field in the new inflationary Universe”, Phys. Rev. D 32, 1899 (1985);
S. W. Hawking “Fluctuations in the Inflationary Universe”, Nucl. Phys. B 224, 180 (1983); J.J. Halliwell and S. W. Hawking, “Origin of Structure in the Universe”, Phys. Rev. D 31, 1777 (1985).

[2] J.J. Halliwell, “Decoherence in Quantum Cosmology”, Phys. Rev. D 39, 2912 (1989); C. Kiefer “Origin of Classical Structure From Inflation”, Nucl. Phys. Proc. Suppl. 88, 255 (2000) [arXiv:astro-ph/0006252]; D. Polarski and A. A. Starobinsky, “Semiclassicality and decoherence of Cosmological perturbations”, Class. Quantum Grav. 13, 377 (1996) [arXiv:gr-qc/9504030]; W.H. Zurek, “Environment Induced Superselection In Cosmology”, Environment Induced Superselection In Cosmology in Moscow 1990, Proceedings, Quantum gravity (QC178:S4:1990), 456-472;

R. Branderberger H. Feldman and V. Mukhavov, “Theory of Cosmological Perturbations”, Phys. Rep. 215, 203 (1992);

R. Laflamme and A. Matacz “Decoherence Funtional and Inhomogeneities in the Early Universe”, Int. J. Mod. Phys. D 2, 171 (1993) [arXiv:gr-qc/9303036]; M. Castagnino and O. Lombardi, “The self-induced approach to decoherence in cosmology”, Int. J. Theor. Phys. 42, 1281 (2003) [arXiv:quant-ph/0211163]; F. C. Lombardo and D. Lopez Nacir, “Decoherence during inflation: The generation of classical inhomogeneities”, Phys. Rev. D 72, 063506 (2005) [arXiv:gr-qc/0506051]; J. Martin, “Inflationary Cosmological Perturbations of Quantum Mechanical Origin”, Lect. Notes Phys. 669, 199 (2005) [arXiv:hep-th/0406011]; J. B. Hartle, “Quantum cosmology: Problems for the 21st century”, [arXiv:gr-qc/9701022] (1997).

J. B. Hartle, “Generalizing quantum mechanics for quantum gravity”, Int. J. Theor. Phys. 45, 1390 (2006) [arXiv:gr-qc/0510126].

A. O. Barvinsky, A. Y. Kamenshchik, C. Kiefer, and I. V. Mishakov, “Decoherence in Quantum Cosmology at the onset of Inflation”, Nucl. Phys. B 551, 374 (1999) [arXiv:gr-qc/9812043].

[3] S. Weinberg, “Cosmology”, Oxford University Press, U.S.A. (2008) 593 p.

[4] A. Perez, H. Sahlmann and D. Sudarsky, “On the quantum origin of the seeds of cosmic structure”, Class. Quantum Grav. 23, 2317 (2006) [arXiv:gr-qc/0508100].
[5] D. Sudarsky, “Shortcomings in the Understanding of Why Cosmological Perturbations Look Classical”, Int. J. Mod. Phys. D 20, 509 (2011) [arXiv:0906.0315].

[6] P. M. Pearle, “Dynamical wave function collapse: Could it have cosmological consequences?”, [arXiv:0710.0567] (2007).

[7] A. De Unanue and D. Sudarsky, “Phenomenological analysis of quantum collapse as source of the seeds of cosmic structure”, Phys. Rev. D 78, 043510 (2008) [arXiv:0801.4702].

[8] G. León and D. Sudarsky, “The Slow roll condition and the amplitude of the primordial spectrum of cosmic fluctuations: Contrasts and similarities of standard account and the 'collapse scheme' ”, Class. Quantum Grav. 27, 225017 (2010) [arXiv:1003.5950].

[9] R. M. Wald (private communication).

[10] R. Penrose, “The Emperor's New Mind”, Oxford University Press, U.K. (1989) 480 p;

R. Penrose, “On Gravity’s Role in Quantum State Reduction”, Gen. Rel. Grav. 28, 581 (1996).

[11] G. C. Ghirardi, A. Rimini, and T. Weber, “A Unified Dynamics For Micro And Macro Systems”, Phys. Rev. D 34, 470 (1986).

[12] L. Diosi, “A universal master equation for the gravitational violation of quantum mechanics”, Phys. Lett. A 120, 377 (1987);

L. Diosi, “Models for universal reduction of macroscopic quantum fluctuations”, Phys. Lett. A 40, 1165 (1989).

[13] P. Pearle, “Combining stochastic dynamical state-vector reduction with spontaneous localization”, Phys. Rev. A 39, 2277 (1989).

[14] A. P. S. Yadav and B. D. Wandelt, “Primordial Non-Gaussianity in the Cosmic Microwave Background”, (2010) Adv. Astron. 2010, 565248 (2010) [arXiv:1006.0275].

[15] M. Liguori, E. Sefusatti, J. R. Fergusson and E. P. S. Shellard, “Primordial non-Gaussianity and Bispectrum Measurements in the Cosmic Microwave Background and Large-Scale Structure”, Adv. Astron. 2010, 980523 (2010) [arXiv:1001.4707].

[16] E. Komatsu, “The pursuit of non-gaussian fluctuations in the cosmic microwave background”, PhD thesis Tohoku University (2001) [arXiv:astro-ph/0206039].

[17] E. Komatsu, “Hunting for primordial non-Gaussianity in the cosmic microwave background”, Class. Quant. Grav. 27, 124010 (2010) [arXiv:1003.6097].

[18] N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto “Non-Gaussianity from Inflation: Theory and Observations”, Phys. Rept 402, 1003 (2004) [arXiv:astro-ph/0406398].
[19] T. J. Allen, B. Grinstein and M. B. Wise, “Non-gaussian density perturbations in inflationary cosmologies”, Phys. Lett. B 197, 66 (1987).

[20] E. Komatsu and others, “Non-Gaussianity as a Probe of the Physics of the Primordial Universe and the Astrophysics of the Low Redshift Universe”, (2009) [arxiv:0902.4759].

[21] D. H. Lyth and A. R. Liddle, “The primordial density perturbation: cosmology, inflation and the origin of structure,” Cambridge University Press, U.K. (2009) 516 p.

[22] D. Babich, P. Creminelli and M. Zaldarriaga, “The shape of non-Gaussianities”, JCAP 0408, 009 (2004) [arXiv:astro-ph/0405356].

[23] P. Creminelli and M. Zaldarriaga, “Single field consistency relation for the 3-point function”, JCAP 0410, 006 (2004) [arXiv:astro-ph/0407059].

[24] L. Senatore, K. M. Smith and M. Zaldarriaga, “Non-Gaussianities in Single Field Inflation and their Optimal Limits from the WMAP 5-year Data”, JCAP 1001, 028 (2010) [arXiv:0905.3746].

[25] D. S. Salopek and J. R. Bond, “Nonlinear evolution of long wavelength metric fluctuations in inflationary models”, Phys. Rev. D 42, 3936 (1990).

[26] D. S. Salopek and J. R. Bond, “Stochastic inflation and nonlinear gravity”, Phys. Rev. D 43, 1005 (1991).

[27] J. L. Lehners, “Ekpyrotic Nongaussianity: A Review”, Adv. Astron. 2010, 903907 (2010).

[28] C. T. Byrnes, and K. Y. Choi, “Review of Local Non-Gaussianity from Multifield Inflation”, Adv. Astron. 2010, 724525 (2010).

[29] A. P. S. Yadav, E. Komatsu, B. D. Wandelt, M. Liguori, F. K. Hansen and S. Matarrese, “Fast Estimator of Primordial Non-Gaussianity from Temperature and Polarization Anisotropies in the Cosmic Microwave Background II: Partial Sky Coverage and Inhomogeneous Noise”, Astrophys. J. 678, 578 (2008) [arXiv:0711.4933].

[30] P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark and M. Zaldarriaga, “Limits on non-gaussianities from wmap data”, JCAP 0605, 004 (2006) [arXiv:astro-ph/0509029].

[31] E. Komatsu and others, “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation”, ApJs 192, 18 (2011) [arXiv:1001.4538].

[32] G. León, A. De Unanue and D. Sudarsky, “Multiple quantum collapse of the inflaton field and its implications on the birth of cosmic structure”, Class. Quantum Grav. 28, 155010 (2011) [arXiv:1012.2419].
[33] L. P. Grishchuk and J. Martin, “Best unbiased estimates for the microwave background anisotropies”, Phys. Rev. D 56, 1924 (1997) [arXiv:gr-qc/9702018].

[34] A. Diez-Tejedor, G. León and D. Sudarsky, “The collapse of the wave function in the joint metric-matter quantization for inflation”, [arXiv:1106.1176] (2011).

[35] D. Sudarsky, “Can we learn something about the quantum/gravity interface from the primordial fluctuation spectrum?”, Int. J. Mod. Phys. D 20, 821 (2011).