Power corrections in charmless $B$ decays

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ABSTRACT
Power corrections seem to play an important role in charmless $B$ decays as indicated by recent analysis using QCD Factorization. In this talk, I would like to report on a recent work on power corrections in charmless $B$ decays. By using the ratio of the branching fraction of $B^+ \rightarrow \pi^+ K^{*0}$ to that of $B^0 \rightarrow \pi^- \rho^+$, for which the theoretical uncertainties are greatly reduced, it is shown in a transparent manner that power corrections in charmless $B$ decays are probably large and that the $B^0 \rightarrow K^- \rho^+$ decay could be explained with the annihilation term included. For ratios of direct CP asymmetries, QCD Factorisation with the annihilation terms included would predict the direct CP asymmetry of $B \rightarrow \pi^+ \pi^-$ to be about 3 times larger than that of $B \rightarrow \pi^\pm K^\mp$, with opposite sign. In particular, the large measured value for $B \rightarrow \pi^\pm K^\mp$ CP asymmetry implies naturally a corresponding large $B \rightarrow \pi^+ \pi^-$ CP asymmetry as observed by Belle. Experimentally any significant deviation from this prediction would suggest either new physics or possibly the importance of long-distance rescattering effects.
1 Introduction

In QCD Factorization (QCDF)[1], the $O(1/m_b)$ power corrections in penguin matrix elements and other chirally enhanced corrections could make important contributions to the penguin-dominated charmless $B$ decays as in $B \to \pi K$ decays. Other power corrections terms such as annihilation contributions may also be present in PP and PV decays as first noticed in the perturbative QCD method for charmless $B$ decays[2] and indicated by recent analysis of charmless two-body non-leptonic $B$ decays[3, 4, 5]. In a recent work[6], we have shown that in QCDF, it is possible to consider certain ratios of the $B \to PV$ amplitudes which depend only on the Wilson coefficients and the known hadronic parameters. The discrepancy between prediction and experiment for the ratio would be a clear evidence for annihilation or other non factorisable contributions. We find that annihilation topology likely plays an indispensable role at least for penguin-dominated PV channels. Including the annihilation terms in QCDF, we find that the direct CP asymmetry of $B \to \pi^+\pi^-$ to be about 3 times larger than that of $B \to K^\mp\pi^\pm$, with opposite sign, in agreement with experiment.

2 QCD factorization for charmless $B$ decays

The effective Lagrangian for non-leptonic $B$ decays can be obtained from operator product expansion and renormalization group equation, in which short-distance effects involving large virtual momenta of the loop corrections from the scale $M_W$ down to $\mu = O(m_b)$ are integrated into the Wilson coefficients. The amplitude for the decay $B \to M_1M_2$ can be expressed as

$$A(B \to M_1M_2) = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{6} \sum_{q=u,c} \lambda_q C_i(\mu) \langle M_1M_2|O_i(\mu)|B \rangle$$

(1)

$\lambda_q$ is a CKM factor, $C_i(\mu)$ are the Wilson coefficients perturbatively calculable from first principles and $O_i$ are the tree and penguin operators given by(neglecting other operators):

$$O_1 = \langle \bar{s}u \rangle_L (\bar{u}b)_L \quad , \quad O_4 = \sum_q \langle \bar{s}q \rangle_L (\bar{q}b)_L$$

$$O_6 = -2 \sum_q \langle \bar{s}_L q_R \rangle (\bar{q}_R b_L)$$

(2)

The hadronic matrix elements : $\langle M_1M_2|O_i(\mu)|B \rangle$ contains the physics effects from the scale $\mu = O(m_b)$ down to $\Lambda_{QCD}$. In the heavy quark limit, QCD Factorisation [1] allows the decay amplitude $\langle M_1M_2|O_i(\mu)|B \rangle$ to be factorized into hard radiative corrections and non perturbative matrix elements which can be parametrized by the semi-leptonic decays form factors and meson light-cone distribution amplitudes (LCDAs).

Power corrections in $1/m_b$ come from penguin matrix elements, chirally enhanced corrections and annihilation contributions. For example, in the $B \to \pi K$ amplitude, the matrix element of $O_6$ is of the order $O(1/m_b)$ compared to the $(V - A) \times (V - A)$ $O_1$ and $O_4$ matrix.
elements, since $<K|\bar{s}_L d_R|0>$ is proportional to $m_K^2/m_s \approx 2.5\text{GeV}$ while $<K|\bar{s}_L d_L|0>$ is proportional to $K$ momentum which is $O(m_b)$, thus numerically, the matrix element of $O_6$ which has a factor

$$r^K_\chi = \frac{2m_K^2}{m_b(m_s + m_d)} \approx O(1)$$

(3)
is comparable to that of $O_4$. For penguin-dominated, decays, the $O_4$ and $O_6$ matrix element are of the same sign in PP channel, while in PV channel they are of opposite sign. Thus in QCDF one expects a small $B \to K\rho$ branching ratio relative to $B \to \pi K$. Because of cancellation between the $O_4$ and $O_6$ contributions, the $B \to K\rho$ decay is more sensitive to other power corrections and non factorisable contributions. Including the chirally-enhanced corrections in terms of two quantities $X_{A,H}$ and a strong phase, the $B \to M_1M_2$ decay amplitudes in QCDF can be thus be written as[7, 8]:

$$A(B \to M_1M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V_{ps}^* \left( -\sum_{i=1}^6 a_i^c(M_1M_2|O_i|B)_f + \sum_j f_B f_{M_1} f_{M_2} b_j \right) ,$$

(4)

3 Power corrections in $B \to \rho \pi$ decays

Consider the ratio of $A(B^+ \to \pi^+ K^{*0})$ to $A(B^0 \to \rho^+ \pi^-)$ amplitudes. If the power corrections were negligible, this ratio would be theoretically very clean where the form factors cancel out, furthermore it is almost independent of the CKM angle $\gamma$ and the strange-quark mass:

$$\left| \frac{A(B^+ \to \pi^+ K^{*0})}{A(B^0 \to \rho^+ \pi^-)} \right| \simeq \frac{V_{ub}V_{cs}}{V_{ub}V_{ud}} \left| \frac{f_{K^*}}{f_{\rho}} \right| \left| \frac{a_4^c(\pi K^*) + r^K_\chi a_6^c(\pi K^*)}{a_4^u} \right| .$$

(5)

\[ (a_4^c(\pi K^*) + r^K_\chi a_6^c(\pi K^*))/a_4^u \] should be about or less than 0.04 in QCDF. ($f_{K^*}/f_{\rho} \approx 1$). The ratio $|V_{ub}/V_{cb}|$ is not very well determined experimentally, but a stringent lower limit can be obtained from the unitarity of the CKM matrix. Since [9, 10] :

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \lambda \sin \beta \sqrt{1 + \frac{\cos^2 \alpha}{\sin^2 \alpha}} \geq \lambda \sin \beta .$$

(6)

and from the current Babar and Belle measured values $\sin 2\beta = 0.736 \pm 0.049$ [11], we have

$$\left| \frac{V_{ub}}{V_{cb}} \right| \geq \lambda \sin \beta = 0.090 \pm 0.007 > 0.078$$

(7)

Eq.(5) implies the following inequality:

$$0.53 > \left| \frac{A(B^+ \to \pi^+ K^{*0})}{A(B^0 \to \rho^+ \pi^-)} \right| = 0.77 \pm 0.09 ,$$

(8)

where the number on the rhs is from the measured branching ratios [12, 13]. The lhs would be reduced further to $0.46 \pm 0.04$, if one neglects a small $\cos^2 \alpha$ term in Eq.(6).

Since the chirally enhanced corrections for penguin-dominated decays are not expected to be large, this large discrepancy is strong indication that annihilation topology and/or
Figure 1: The ratio $\mathcal{B}(B^+ \to \pi^+ K^{*0})/\mathcal{B}(B^0 \to \rho^+ \pi^-)$ versus the weak annihilation phase $\phi_A$. The default parameters are used but letting the annihilation parameter $\rho_A = 1$. The dashed lines show the ratios without weak annihilation contributions. The gray areas denote the experimental measurements with $1\sigma$ error.

Figure 2: The ratio $\mathcal{B}(B^0 \to K^+ \rho^-)/\mathcal{B}(B^0 \to \rho^- \pi^+)$ versus the weak annihilation phase $\phi_A$. The default parameters are as in Fig. 1.

other sources of power corrections might play an important role at least in $B \to PV$ decays. There is similar disagreement between theory and experiment in another ratio, the branching fraction of $B^0 \to K^+ \rho^-$ to that of $B^0 \to \rho^- \pi^+$, though with large theoretical uncertainties. For $\gamma = 70^\circ$, $V_{ub}/V_{cb} = 0.09$, $a_4^r(\rho K) - r_K^2 a_6^r(\rho K) = 0.037 + 0.003i$, $m_s = 90$ MeV, we find

$$\frac{\mathcal{B}(B^0 \to K^+ \rho^-)}{\mathcal{B}(B^0 \to \rho^- \pi^+)} = 0.38$$

(9)

far below the measured value of $1.01 \pm 0.34$, though, this ratio could be increased to 0.69, if $m_s$ is lowered to 70 MeV.

Taken together, these results indicate that the penguin-dominated $B \to PV$ decay amplitudes are consistently underestimated without annihilation contributions. Including the annihilation terms, from Eq. (4), we have
\[ A(B^+ \rightarrow \pi^+K^0) = f_K^* F B \pi m_B^2 a_4 + b_3(V, P) \]
\[ A(B^0 \rightarrow K^+\rho^-) = f_K A_0^B m_B^2 (a_4 - r^K a_6) + b_3(P, V) \]

\[ b_3(M_1, M_2) = \frac{C_F}{N_c} \left\{ C_3 A_1^3(M_1, M_2) + C_5 A_3^3(M_1, M_2) + (C_5 + N_c C_6) A_3^f(M_1, M_2) \right\} \]

With the penguin terms \( a_4 \simeq -0.03 \) and \( a_4 - r^K a_6 \simeq 0.037 \) having opposite sign, the key observation is that \( b_3(V, P) \) and \( b_3(P, V) \), which get most of the contribution from \((C_5 + N_c C_6) A_3^f\) term, are also roughly of the opposite sign since \( A_3^f(P, V) = -A_3^f(V, P) \). Thus QCDF can easily enhance both ratios without fine tuning (no large strong phase) as can be seen in Fig. 2.

## 4 Direct CP Violations

We now turn to the CP asymmetries in QCDF with annihilation terms included. Because of the CKM factor and \( SU(3) \) symmetry for the tree and penguin matrix elements in \( B^0 \rightarrow \pi^+\pi^- \) and \( B^0 \rightarrow K^+\pi^- \) decays, one can derive a relation between direct CP asymmetries in these two channels. With the CP asymmetry given as:

\[ A_{\pi\pi} = \frac{4|V_{ub} V_{ud} V_{cb} V_{cd}| T_{\pi\pi} P_{\pi\pi}| \sin \gamma \sin \delta \cdot 2B(B \rightarrow \pi^+\pi^-)}{2} \]
\[ A_{\pi K} = -\frac{4|V_{ub} V_{ud} V_{cb} V_{cs}| T_{\pi K} P_{\pi K}| \sin \gamma \sin \delta \cdot 2B(B \rightarrow \pi^+K^-)}{2} \]

\( \delta = \delta_T - \delta_T \) = strong phases difference between the penguin and tree amplitudes), we find

\[ \frac{A_{\pi\pi}}{A_{\pi K}} = -\frac{f_+^2 B(B \rightarrow \pi^+K^-)}{f_K^2 B(B \rightarrow \pi^+\pi^-)} \frac{|T_{\pi\pi} P_{\pi\pi}|}{|T_{\pi K} P_{\pi K}|} \frac{\sin \delta}{\sin \delta} \]

\[ \simeq (-2.7 \pm 0.3) \frac{\sin \delta}{\sin \delta} \]

a consequence of the fact that \( T_{\pi\pi} P_{\pi\pi}/T_{\pi K} P_{\pi K} \) is close to 1, a reasonable approximation in QCDF, at about 10 percent level uncertainty. A previous derivation of this relation is given in [16]. Belle has claimed large direct CP asymmetry observed in \( B^0 \rightarrow \pi^+\pi^- \) decay while BaBar has not confirmed it yet, but both of them are close to a measurement on \( A_{CP}(\pi^-K^+) \) [14, 15]

\[ A_{\pi\pi} = \begin{cases} 0.58 \pm 0.15 \pm 0.07 \ (\text{Belle}) \\ 0.19 \pm 0.19 \pm 0.05 \ (\text{BaBar}) \end{cases} \]

\[ A_{\pi K} = \begin{cases} (-8.8 \pm 3.5 \pm 1.8)\% \ (\text{Belle}) \\ (-13.3 \pm 3.0 \pm 0.9)\% \ (\text{BaBar}) \end{cases} \]

We thus expect very naturally a larger direct CP violation for \( \pi^+\pi^- \) decay compared with \( \pi^-K^+ \) decay, since the \( \pi^+\pi^- \) decay rate is smaller than the \( \pi^-K^+ \) decay rate by factor 3 – 4.
Experimentally,
\[
\frac{A_{\pi\pi}}{A_{\pi K}} = \frac{0.42 \pm 0.13}{-0.11 \pm 0.03} = -4.0 \pm 1.8 ,
\]
still consistent with the theoretical estimation of $-2.7 \pm 0.3$.

Similar relation between CP asymmetries for the $B \to PV$ decays for which the CP-violating interference terms are essentially of the same magnitude, but with opposite sign:

\[
\frac{A_{\text{CP}}(B^0 \to \rho^+ \pi^-)}{A_{\text{CP}}(B^0 \to K^{*+} \pi^-)} \simeq \frac{B(B^0 \to K^{*-} \pi^-) f_{\rho}^2 \sin \delta_{\pi\rho}}{B(B^0 \to \rho^+ \pi^-) f_{K^*}^2 \sin \delta_{\pi K^*}}
\]
\[
\frac{A_{\text{CP}}(B^0 \to \rho^- \pi^+)}{A_{\text{CP}}(B^0 \to \rho^- K^+)} \simeq \frac{B(B^0 \to \rho^- K^+) f_{\pi}^2 \sin \delta_{\rho\pi}}{B(B^0 \to \rho^- \pi^+) f_{K}^2 \sin \delta_{\rho K}}
\]

5 Conclusion

Power corrections in charmless B decays are probably large, at least for the penguin-dominated PV channel. The key observation is that QCDF predicts the annihilation terms for $B^+ \to \pi^+ K^{*0}$ and $B^0 \to K^+ \rho^-$ to be almost equal in magnitude but opposite in sign and thus enhance the decay rates for these two modes to accommodate the experimental data.

The relation for the direct CP asymmetry would naturally implies a large CP asymmetry for $B \to \pi^+ \pi^-$, about 3 times larger than that of $B \to \pi^\pm K^\mp$ with opposite sign.

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