Universal geometrical scaling of the elliptic flow

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The presence of scaling variables in experimental observables provide very valuable indications of the dynamics underlying a given physical process. In the last years, the search for geometric scaling, that is the presence of a scaling variable which encodes all geometrical information of the collision as well as other external quantities as the total energy, has been very active. This is motivated, in part, for being one of the genuine predictions of the Color Glass Condensate formalism for saturation of partonic densities. Here we extend these previous findings to the case of experimental data on elliptic flow. We find an excellent scaling for all centralities and energies, from RHIC to LHC, with a simple generalization of the scaling previously found for other observables and systems. Interestingly the case of the photons, difficult to reconcile in most formalisms, nicely fit the scaling curve. We discuss on the possible interpretations of this finding in terms of initial or final state effects.

The discovery of a sizable elliptic flow in AA collisions, first observed at RHIC [1, 2] and later at LHC [3], turned up as an experimental major breakthrough. The observed anisotropic flow can exclusively be understood if the measured particles in the final state depend not only on the physical conditions realized locally at their production point, but also on the global geometry of the event. This non-local information can solely emerge as a collective effect, requiring strong interaction among the relevant degrees of freedom, i.e., quarks and gluons. The study of higher harmonics has also shown very interesting features, including the ridge structure seen in AA collisions [4–7], pPb collisions [8, 9] and also in high multiplicity pp collisions [10]. The conventional understanding of the ridge is simply related to flow harmonics in a hydrodynamic scenario, where the description of the pPb ridge and, specially, the high multiplicity pp ridge is a challenge. The question is whether an initial state effect could determine the ridge structure. Or, in other words, if the elliptic flow is an initial state effect or, on the contrary, a final state effect amenable to a hydrodynamic description [11–20]. Along these lines, we study the possibility that an initial state property, as geometrical scaling due to gluon saturation, was preserved in a similar scaling in the elliptic flow. Similar questions related to how hydrodynamic descriptions could fit scaling laws observed in $v_2$ have already been raised [21]. Indeed, we show that the experimental data for $v_2$ at RHIC and LHC energies normalized to the saturation momentum, eccentricity and radius of the collision area satisfy geometrical scaling:

$$v_2(p_T)/\epsilon_1 Q_s^4 L = f(\tau),$$  \hspace{1cm} (1)

where

$$\epsilon_1 = 2 \int_0^{\pi/2} d\varphi \cos 2\varphi \frac{R^2 - R_\varphi^2}{R^2}, \quad R_\varphi = \frac{R_A \sin(\varphi - \alpha)}{\sin \varphi},$$  \hspace{1cm} (2)

$$\alpha = \arcsin\left(\frac{b}{2R_A \sin \varphi}\right), \quad R^2 = (R_\varphi^2) = \frac{2}{\pi} \int_0^{\pi/2} d\varphi R_\varphi^2$$  \hspace{1cm} (3)

and

$$\tau = \frac{p_T^2}{(Q_A^4)^2},$$  \hspace{1cm} (4)

being $Q_A^4$ the saturation momentum, $R_A$ the radius of the nucleus and $L$ the length associated to the size of the collision area at a given impact parameter and energy. Indeed, the product $Q_A^4 L$ is the inverse of the Knudsen number, i.e., the mean free path normalized to the length measured as the number of scattering centers.

$\epsilon_1$ is a measure of the eccentricity of the collision. It does not depend on the distribution of scattering centers (partons or nucleons) in the transverse plane and it is determined only by the almond shape of the collision at a given impact parameter.

The scaling variable $\tau$ is known from the geometrical scaling verified in deep inelastic scattering, pp, pA and AA collisions [22–26], namely,

$$\frac{1}{N_A} \frac{dN_{ch}}{dp_T^2} = \frac{1}{Q_0^2} F(\tau).$$  \hspace{1cm} (5)

In Fig. 1 this scaling is shown for pp, pA and AA collisions at different centralities and energies and for $\tau < 1$, using for $(Q_A^4)^2$ the following parametrization,

$$(Q_s^4)^2 = (Q_s^4)^2 A^{\alpha(s)/2} N_A^{1/6},$$  \hspace{1cm} (6)

being $N_A$ the number of wounded nucleons. $\alpha(s)$ and the proton saturation momentum are given, respectively, by the equations,

$$\alpha(s) = \frac{1}{3} \left(1 - \frac{1}{1 + \ln\left(\sqrt{s/s_0} + 1\right)}\right)$$  \hspace{1cm} (7)
ticles via fragmentation. Even at moderate high energies
in the multiparticle production process. In gluon
available energy is
to create at least a couple of hadrons. However, the total
average, the gluon density is larger for smaller
energies \[31, 32\].

In order to relate the geometrical saling of the trans-
verse momentum distribution, equation \[57\], with the
scaling of the elliptic flow of equation \[1\], we define an
azimuthal angle saturation momentum, \(Q^A_{s\varphi}\). As, on
average, the gluon density is larger for smaller \(R\) (small
\(\varphi\), we assume \((Q^A_{s\varphi})^2 \sim 1/R^2_{\varphi}\). In addition, \((Q^A_{s\varphi})^2\)
should be proportional to the inverse of the mean free
path, \(\lambda_{mfp}\), normalized to the length size of the scatter-
ing, \(L\), i.e. inverse proportional to the Knudsen number
\(k_n = \frac{\lambda_{mfp}}{L}\). Therefore, we can write:

\[
(Q^A_{s\varphi})^2 \equiv \frac{L}{\lambda_{mfp}} \frac{1}{R^2_{\varphi}} = \frac{1}{k_n R^2_{\varphi}} = \frac{Q^A_{s\varphi}}{R^2_{\varphi}}.
\]  

(9)

We assume that the azimuthal dependence of the transverse momentum distribution can be encoded in the
saturation momentum, \(Q^A_{s\varphi}\), in such a way that we can write,

\[
v_2 = \int_0^{\pi/2} d\varphi \cos 2\varphi \frac{dN}{d^2p_{\varphi}} = \int_0^{\pi/2} d\varphi \cos 2\varphi F(\tau_{\varphi}),
\]

(10)

where

\[
\tau_{\varphi} = \frac{p^2_{T}}{(Q^A_{s\varphi})^2} = \frac{p^2_{T}}{(Q^A_{s\varphi})^2} \frac{R^2_{\varphi}}{L} = \frac{R^2_{\varphi}}{L} Q^A_{s\varphi}.
\]  

(11)

Expanding \(F(\tau_{\varphi})\) in powers of \(R^2_{\varphi} - R^2\) and retaining
the first non-vanishing term, we have,

\[
v_2 = 2 \frac{2}{\pi} \int_0^{\pi/2} d\varphi \cos 2\varphi \frac{R^2 - R^2_{\varphi}}{R^2} \frac{1}{2\pi F(\tau)} \frac{dF}{d\tau} Q^A_{s\varphi} L,
\]

(12)

where we approximate \(L \approx R\).

Denoting by

\[
\frac{1}{2\pi F(\tau)} \frac{dF}{d\tau} = \varphi(\tau),
\]

(13)

we have

\[
v_2 = \epsilon_1 Q^A_{s\varphi} L \tau \varphi(\tau)
\]

(14)
or

\[
v_2 k_n = \epsilon_1 \tau \varphi(\tau),
\]

(15)

which is the scaling law \[1\].

In Fig. 2(a) we plot the measured values of \(v_2(p_T)\) for
Au-Au collisions for different centralities at RHIC \[32\]
and for PbPb collisions at LHC \[34\] divided by the product \(\epsilon_1 Q^A_{s\varphi} L\) computed for each centrality and energy. We take the usual values of \(b\) and \(N_A\) for each centrality to compute \(\epsilon_1\) and \(Q^A_{s\varphi}\) using the equations \[2\], \[3\] and \[9\]
respectively. \(L\) is a measure of the number of longitudinal
scatterings, which in the Glauber model is proportional
to \(N^{1/3}_A\). Nevertheless, we use \((1 + N^{1/3}_A)/2\), which is
used by most of the strings models as dual parton model
\[35, 36\], quark gluon string model \[37\], Venus \[38\] or
EPOS \[39\]. The solid black line corresponds to a fit to
these data, given by

\[
v_2 = \frac{v_2}{\epsilon_1 Q^A_{s\varphi} L} = a e^b;
\]

(16)

where \(a = 0.1264 \pm 0.0076\) and \(b = 0.404 \pm 0.025\.

The Fig. 2 shows that this scaling is satisfied.
In order to see the quality of this scaling we show in Fig. 2 (b) the ratio of Pb-Pb 10-20% at 2.76 TeV, Pb-Pb 40-50% at 2.76 TeV, Au-Au 20-30% at 200 GeV and Au-Au 30-40% at 200 GeV over Pb-Pb 30-40% at 2.76 TeV as a function of $\tau$. All the ratios lie in the range 0.8 – 1.15 for the whole $\tau$ considered, showing that the scaling is quite good (most of the experimental error data are of the order of 10%).

Changing the eccentricity, $\epsilon_1$, by the usual eccentricity, $\epsilon = \langle y^2 - x^2 \rangle / \langle y^2 + x^2 \rangle$, or by the participant eccentricity, the scaling is not satisfied for both Monte-Carlo Glauber and Color Glass distributions.

Assuming that the $v_2$ scaling can be extended to pp collisions, we compute the elliptic flow as a function of the transverse momentum, $v_2(p_T)$, for $\tau < 1$. In Fig. 3, we show our predictions for $\sqrt{s} = 14$ TeV and for impact parameters values of $b = 0.5$ fm and $b = 0.7$ fm. The $v_2(p_T)$ obtained is much smaller than the computed one using hot spots inside the proton and only slightly smaller than the one found considering usual impact parameter distributions.

The scaling law obtained for $v_2$ is based exclusively on two ingredients: the geometrical scaling of transverse momentum distributions for $\tau < 1$ (initial state effect) and the assumption for the azimuthal saturation momentum which encodes all the angle dependence. The main question is whether the assumption can be considered as a natural consequence of the structure of the initial state or, on the contrary, it is a final state effect. On the one hand, the possibility of domains of color flux tubes or clusters of strings having different azimuthal angles, has
been pointed out in several approaches\textsuperscript{15,16,10}. This would be the origin of the ridge structure. In this initial state approach the equation (9) is a natural assumption. On the other hand, the mean free path or the Knudsen number of equation (9) can be regarded as a measure of the path needed to way out the collision and, consequently, as a measure of the energy lost by the partons produced in the fragmentation of a color flux tube (or in a cluster of strings) interacting with the color field of other color flux tubes\textsuperscript{20}.

We have not included in our analysis the $v_2$ data on pPb collisions due to the uncertainties in the values of $N_A$ at a given impact parameter. Moreover, in this stage of our research, we have not studied the scaling for specified particles. As far as geometrical scaling is satisfied for $\pi$, $\kappa$ and $p$\textsuperscript{24}, we expect that there will also be a $v_2$ scaling for identified particles, using $m_T - m$ instead of $p_T$. In this way, we could compute $v_2(p_T)$ of any particle whose momentum distribution verifies the scaling.

In addition, since direct photon production satisfies geometrical scaling\textsuperscript{24}, its elliptic flow may be of the same size and $p_T$ shape of the rest of particles. In order to check this point, in Fig. 11 we plot the ALICE preliminary data\textsuperscript{44} and the PHENIX data\textsuperscript{15} at different centralities. PHENIX collaboration quote two different points at the same $p_T$ and centrality obtained by different analysis methods (BBC and RXN detectors). In any case, we observe that the data are close to the scaling curve.

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