Meromorphic Continuations of Finite Gap Herglotz Functions

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Abstract. Let $\mu$ be a finite measure on the real line whose essential support $E$ is a finite number of intervals (with equal equilibrium measures). Then the associated Herglotz function (m-function) $m$ is meromorphic on $\mathbb{C} \setminus E$. $\mathbb{C} \setminus E$ can be viewed as one of the two sheets of a natural Riemann surface. I find a necessary and sufficient condition for the function $m$ to have a meromorphic continuation to some region on the second sheet of this surface. The domain of meromorphicity can be described explicitly in terms of the distance of the associated Jacobi matrix from the isospectral torus (of periodic Jacobi matrices) corresponding to the set $E$. 