Process Parameters Optimization for an Injection-Molded Plastic Wheel by Uniform Design of Experiment and Kriging Interpolation

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ABSTRACT

This paper has presented a systematic optimization procedure which integrates uniform design of experiment, Kriging interpolation, compromise programming, and generalized reduced gradient nonlinear optimization to optimize the process parameters of an injection-molded plastic wheel which has two optimization objectives. Firstly, uniform design of experiment is applied to plan a set of experiments. Secondly, injection molding simulation software Moldflow is employed to analyze the maximum warpage and ejection time of each experiment. Thirdly, Kriging interpolation is used to create the surrogate model of maximum warpage and the surrogate model of ejection time. Fourthly, compromise programming (CP) is applied to integrate two objectives into one compromise objective. Finally, generalized reduced gradient (GRG) nonlinear optimization is applied to optimize the compromise objective. With the systematic procedure presented in this paper, maximum warpage and ejection time of the plastic wheel are improved by 7.7% and 11.91%, respectively.

INTRODUCTION

Injecting molten plastic materials into mold cavities, plastics injection molding is a very efficient way for manufacturing plastic parts. The geometry of injection molded part is usually different from the ideal geometry of part due to shrinkage and warpage effects. Much research (Hwang et al. [1]; Liao et al. [2]; Liao et al. [3]; Erzurumlu and Ozcelik [4]; Lee and Lin [5]) has addressed the warpage problems; however, little information is available on the integration of uniform design of experiment, Kriging interpolation, compromise programming, and generalized reduced gradient (GRG) nonlinear optimization to simultaneously reduce warpage and ejection time. In this paper, the injection molded part to be studied is a plastic wheel used in electrical curtain mechanisms. To predict warpage and ejection time of

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the part, the plastic injection molding simulation software Moldflow was applied. The geometric model of the part was built in Solidworks and was imported into Moldflow in IGES format. The uniform design of experiment method, proposed by Fang and Wang [6], was adopted to plan a set of experiments. The results of experiments were obtained by using the injection molding simulation software Moldflow. The surrogate models of warpage and ejection time were created based on Kriging interpolation. Kriging, named after the South African mining engineer D.C. Krige, is a geo-statistical interpolation technique that considers both the distance and the degree of variation between known data points when estimating values in unknown area. In this paper, the unknown coefficients in Kriging surrogate model were determined by DACE, which is a free-of-charge Matlab Kriging toolbox provided by Lophaven et al. [7]. The compromise programming (CP) method [8] was applied to integrate the two objectives, reducing warpage and reducing ejection time, into one single compromise objective. The compromise objective was optimized by the generalized reduced gradient (GRG) nonlinear optimization method [9]. After optimization, warpage and ejection time were improved simultaneously.

**ANALYZING WARPAGE AND EJECTION TIME BY INJECTION MOLDING SIMULATION SOFTWARE MOLDFLOW**

The plastic wheel manufactured by plastic injection molding technology is shown in Figure 1. The structure of mold is shown in Figure 2. The mold has four cavities, a sprue, a runner system and a cooling channel. In Moldflow, sprue, runner system and cooling channel are meshed by beam element. Cavities are meshed by surface element since the part is a thin-shell part. Mesh quality diagnosis and mesh modification are performed before performing simulation. For qualified mesh, the maximum aspect ratio is smaller than 6 and the match percentage is larger than 85%. After performing mesh modification, the maximum aspect ratio is 1.7. The match percentage is 93.8%. The polymer of the part is POM, the trade name of which is Delrin 500 and the manufacturing company of which is DuPont. Suggested by DuPont, the melt temperature of Delrin 500 is between 180°C to 235°C. The maximum allowable shear rate is 40000.

![Figure 1. The solid model of plastic wheel to be injection molded is created in Solidworks.](image)
The original condition for manufacturing the plastic wheel is as follows. The melt temperature is set to be 185°C. The fill time is set to be 0.8 second. The V/P switchover is set to be 99% volume filled. The packing pressure is set to be 85% of maximum pressure. The temperature of cooling water is set to be 20°C. By applying Moldflow to simulate the injection molding process, warpage distribution of the plastic wheel is obtained as shown in Figure 3. The maximum warpage is 0.1247mm. The ejection time, which is defined as the time to reach ejection temperature, is 10.76 seconds.

IMPROVING WARPAGE AND EJECTION TIME BY UNIFORM DESIGN OF EXPERIMENT

The definitions, units, ranges and original conditions of process parameters are shown in Table 1. Because all process parameters are continuous, the uniform design of experiment method is applied to plan a set of experiments. Table 2 shows the set of experiments planed by using the uniform table $U_{16}(16^7)$. For each experiment, Moldflow is employed to analyze maximum warpage and ejection time. The values in parentheses are improved rates. Only experiments 3, 4 and 15 have positive improved rates of maximum warpage and ejection time. If maximum warpage is the
objective, experiment 3 is better than experiment 4. If ejection time is the objective, experiment 4 is better than experiment 3.

**TABLE 1. DEFINITIONS, UNITS, RANGES, AND ORIGINAL CONDITIONS OF PROCESS PARAMETERS.**

| Parameter | Definition                                      | Unit | Range         | Original condition |
|-----------|------------------------------------------------|------|---------------|--------------------|
| A         | Diameter of sprue in the lower end              | mm   | 4.5 - 8.5     | 6.5                |
| B         | Diameter of gate in the small end               | mm   | 3 - 5         | 4                  |
| C         | Diameter of runner                              | mm   | 4.8 - 6.8     | 5.8                |
| D         | Temperature of cooling water                    | °C   | 15 - 35       | 20                 |
| E         | Melt temperature                               | °C   | 170 - 200     | 185                |
| F         | Fill time                                      | s    | 0.8 - 1       | 0.8                |
| G         | Height of the horizontal packing pressure curve | %    | 80 - 90       | 85                 |

**TABLE 2. THE PLANNED EXPERIMENTS AND THE RESULTS OF EXPERIMENTS.**

| Exp. | A [mm] | B [mm] | C [mm] | D [°C] | E [°C] | F [s] | G [%] | Maximum warpage [mm] | Ejection time [s] |
|------|--------|--------|--------|--------|--------|-------|-------|----------------------|-----------------|
| 1    | 4.50   | 3.13   | 5.20   | 24.33  | 194    | 0.99  | 90.00 | 0.1127 (9.62%)       | 12.07 (-12.17%) |
| 2    | 4.77   | 3.40   | 5.73   | 35.00  | 186    | 0.96  | 89.33 | 0.1184 (5.05%)       | 23.1 (-114.68%) |
| 3    | 5.03   | 3.67   | 6.27   | 23.00  | 178    | 0.93  | 88.67 | 0.1188 (4.73%)       | 10.36 (3.72%)   |
| 4    | 5.30   | 3.93   | 6.80   | 33.67  | 170    | 0.91  | 88.00 | 0.1229 (1.44%)       | 9.949 (7.53%)   |
| 5    | 5.57   | 4.20   | 5.07   | 21.67  | 196    | 0.88  | 87.33 | 0.1147 (8.02%)       | 12.18 (-13.2%)  |
| 6    | 5.83   | 4.47   | 5.60   | 32.33  | 188    | 0.85  | 86.67 | 0.1244 (0.24%)       | 11.98 (-11.34%) |
| 7    | 6.10   | 4.73   | 6.13   | 20.33  | 180    | 0.83  | 86.00 | 0.1178 (5.53%)       | 16.95 (-57.53%) |
| 8    | 6.37   | 5.00   | 6.67   | 31.00  | 172    | 0.80  | 85.33 | 0.1228 (1.52%)       | 13.79 (-28.16%) |
| 9    | 6.63   | 3.00   | 4.93   | 19.00  | 198    | 1.00  | 84.67 | 0.1120 (3.16%)       | 12.25 (-13.85%) |
| 10   | 6.90   | 3.27   | 5.47   | 29.67  | 190    | 0.97  | 84.00 | 0.1252 (-0.4%)       | 12.02 (-11.17%) |
| 11   | 7.17   | 3.53   | 6.00   | 17.67  | 182    | 0.95  | 83.33 | 0.1251 (-0.32%)      | 10.58 (1.67%)   |
| 12   | 7.43   | 3.80   | 6.53   | 28.33  | 174    | 0.92  | 82.67 | 0.1320 (-5.85%)      | 10.22 (5.02%)   |
| 13   | 7.70   | 4.07   | 4.80   | 16.33  | 200    | 0.89  | 82.00 | 0.1204 (3.45%)       | 12.35 (-14.78%) |
| 14   | 7.97   | 4.33   | 5.33   | 27.00  | 192    | 0.87  | 81.33 | 0.1257 (-0.8%)       | 13.58 (-26.2%)  |
| 15   | 8.23   | 4.60   | 5.87   | 15.00  | 184    | 0.84  | 80.67 | 0.1230 (1.36%)       | 10.72 (0.37%)   |
| 16   | 8.50   | 4.87   | 6.40   | 25.67  | 176    | 0.81  | 80.00 | 0.1280 (-2.65%)      | 19.7 (-83.09%)  |

**OPTIMIZING WARPAGE AND EJECTION TIME BY KRIGING INTERPOLATION, CP AND GRG**

Based on Kriging interpolation and the data points shown in table 2, the first generation of surrogate model of maximum warpage is obtained and denoted by $\hat{S}_1$. 

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The first generation of surrogate model of ejection time is obtained and denoted by $\hat{T}_1$. Because minimizing $\hat{S}_i$ and minimizing $\hat{T}_i$ are two objectives, the compromise programming (CP) method is employed to transform the two objectives into one single objective $F_i$. The next step is to apply the generalized reduced gradient (GRG) nonlinear optimization method to minimize $F_i$. The solution of process parameters obtained by minimizing $F_i$ is added into the existed data points shown in Table 2. The number of data points is increased by 1. The updated data points are used to create the next generation of Kriging surrogate models of maximum warpage and ejection time. According to the above-mentioned iterative procedure, 10 optimal solutions of process parameters are obtained as shown in Table 3. From the 10 optimal solutions, solution 9 is selected as the final optimal solution for manufacturing the plastic wheel since its improved rate of maximum warpage is 7.7% and its improved rate of ejection time is 11.91%.

| Sol. | A [mm] | B [mm] | C [mm] | D [°C] | E [°C] | F [s] | G [%] | Maximum warpage [mm] | Ejection time [s] |
|------|--------|--------|--------|--------|--------|-------|------|----------------------|------------------|
| 1    | 4.5    | 3.89   | 6.72   | 32.05  | 171    | 0.91  | 0.9  | 0.118 (5.37%)        | 9.972 (7.32%)    |
| 2    | 4.5    | 4.55   | 5.75   | 15.00  | 185    | 0.84  | 0.9  | 0.1088 (12.75%)      | 10.76 (0%)       |
| 3    | 4.5    | 4.12   | 6.80   | 32.23  | 170    | 0.90  | 0.9  | 0.1175 (5.77%)       | 9.872 (8.25%)    |
| 4    | 4.5    | 3.51   | 5.26   | 25.95  | 182    | 0.95  | 0.9  | 0.1101 (11.71%)      | 10.86 (-0.93%)   |
| 5    | 4.5    | 3.46   | 4.80   | 31.06  | 170    | 0.91  | 0.9  | 0.1105 (11.39%)      | 15.16 (-40.89%)  |
| 6    | 4.5    | 4.44   | 6.00   | 26.72  | 170    | 0.95  | 0.9  | 0.1163 (6.74%)       | 9.602 (10.76%)   |
| 7    | 4.5    | 3.03   | 5.99   | 26.90  | 183    | 0.95  | 0.9  | 0.1162 (6.82%)       | 11.08 (-2.97%)   |
| 8    | 4.5    | 5.00   | 5.74   | 26.42  | 170    | 0.95  | 0.9  | 0.1105 (11.39%)      | 15.8 (-46.84%)   |
| 9    | 4.5    | 4.28   | 4.80   | 25.98  | 170    | 0.99  | 0.9  | 0.1151 (7.7%)        | 9.478 (11.91%)   |
| 10   | 4.5    | 4.39   | 4.80   | 26.88  | 170    | 0.98  | 0.9  | 0.1171 (6.09%)       | 9.551 (11.42%)   |

**SUMMARY**

This paper concludes that the presented systematic optimization procedure, which integrates uniform design of experiment, plastics injection molding simulation, Kriging interpolation, compromise programming (CP) and generalized reduced gradient (GRG) nonlinear optimization, can effectively reduce maximum warpage and ejection time for the plastic injection-molded wheel. Since the procedure can also be applied to improve other injection molding problems that have multiple objectives, it is deserved to be promoted.
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