Study of Generalized Second Law of Thermodynamics in Loop Quantum Cosmology with the Effect of Non-Linear Electrodynamics

Tanwi Bandyopadhyay and Ujjal Debnath

1Department of Mathematics, Shri Shikshayatan College, 11, Lord Sinha Road, Kolkata-71, India.
2Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711103, India.

In this work, we have discussed the Maxwell’s electrodynamics in non-linear forms in FRW universe. The energy density and pressure for non-linear electrodynamics have been written in the electro-magnetic universe. The Einstein’s field equations for flat FRW model in loop quantum cosmology have been considered if the universe is filled with the matter and electro-magnetic field. We separately assumed the magnetic universe and electric universe. The interaction between matter and magnetic field have been considered in one section and for some particular form of interaction term, we have found the solutions of magnetic field and the energy density of matter. We have also considered the interaction between the matter and electric field and another form of interaction term has been chosen to solve the field equations. The validity of generalized second law of thermodynamics has been investigated on apparent and event horizons using Gibb’s law and the first law of thermodynamics for magnetic and electric universe separately.

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I. INTRODUCTION

Observations of the redshift of supernovae type Ia [1, 2] and cosmic microwave background [3, 4] show that our universe is expanding with acceleration and lead to the search for a new type of matter which violates the strong energy condition, known as dark energy. The dark energy has the property that the energy component produce sufficient negative pressure, which drives the cosmic acceleration. There are many candidates supporting this behavior. Cosmological constant $\Lambda$ is the most popular candidate of dark energy satisfying the equation of state (EoS) $p_\Lambda = -\rho_\Lambda$. There are other strong favored candidates like quintessence, which is composed by a scalar field $\phi$ with self-interacting potential. The EoS of the fluid distribution of the universe is given by $p = \omega \rho$, where $\omega$ is known as the EoS parameter. The universe will accelerate if $\omega < -1/3$. $\omega = -1$ represents $\Lambda$CDM model and $\omega < -1$ corresponds to the phantom dominated model [7]. Recently, many candidates with variable EoS play the crucial role of dark energy to drive the acceleration of the universe namely, Chaplygin gas [8, 9], Tachyonic field [10], holographic dark energy [11], agegraphic dark...
energy [12], Ricci dark energy [13], Hessence [14], DBI-essence [15], K-essence [16], dilaton dark energy [17] etc.

At present, we live in an epoch where the dark energy and the dark matter are comparable. For this purpose, the interacting dark energy models have been studied to explain the cosmic coincidence problem [18]. Till now, many works have been proposed for this interacting dark energy [19, 20]. In recent years, the model of interacting dark energy has been explored in the framework of loop quantum cosmology (LQC). The LQC is the application in the cosmological context of loop quantum gravity (LQG) [21–23], which is a theory trying to quantize the gravity with a non-perturbative and background independent method. By studying the early universe inflation and the fate of future singularity in LQC, it is found that the big bang singularity, the big rip singularity and other future singularities can be avoided [24]. It has been verified that, the cosmological evolution in LQC for quintessence model is same as that in classical Einstein cosmology, whereas for the phantom dark energy, the loop quantum effect significantly reduce the parameter spacetime required by stability. Recently, the dynamics of phantom, quintom and hessence dark energy models in LQC have been studied [25].

On the other hand, a new approach [26] has recently been taken to avoid the cosmic singularity through a non-linear extension of the Maxwell’s electromagnetic theory. The associated Lagrangian and the resulting electrodynamics can theoretically be justified, based on different arguments. The homogeneous and isotropic non-singular FRW solutions can be obtained [26] by considering a generalized model of Maxwell’s electrodynamics, where local covariant and gauge-invariant Lagrangian depend on the field invariants up to the second order, as a source of classical Einstein’s equations. Exact solutions of the Einstein’s field equations coupled to non-linear electrodynamics (NLED) may hint at the relevance of the non-linear effects in strong gravitational and magnetic fields. An inhomogeneous and anisotropic non-singular model for the early universe filled with Born-Infeld type non-linear electromagnetic field was studied [27]. Recently, there are several works on NLED in various situations [28–31].

In Einstein’s gravity, the connection between black hole thermodynamics and Einstein’s equations was first discovered in [32] by deriving the Einstein’s equations from the proportionality of entropy and horizon area together with the first law of thermodynamics. The thermodynamical laws also have been applied in the cosmological context, considering universe as a thermodynamical system bounded by the apparent horizon. At the apparent horizon, the first law of thermodynamics is equivalent to the Friedmann equations and generalized second law (GSL) is obeyed at the horizon. There are several studies in thermodynamics for dark energy filled universe on apparent and event horizons [33–37]. Thermodynamical properties in non-linear electrodynamics has extensively studied in [38]. In our previous work [39], we have assumed the apparent, event, Hubble and particle horizons of the FRW universe and have studied the first law and GSL of thermodynamics in non-linear electrodynamics with magnetic field only. In the present work, we extend the previous work in LQC by considering both the electric and magnetic fields and examine the validity of GSL in apparent and event horizons of the magnetic universe and electric universe.

The investigation has been done in the following way: We have briefly discussed the Maxwell’s electrodynamics in non-linear forms in section II. The energy density and pressure for non-linear electrodynamics have been written in the electro-magnetic universe. The Einstein’s field equations for flat FRW model in loop quantum cosmology have been considered if the universe is filled with the matter and electro-magnetic field. We separately assumed the magnetic universe ($E = 0$) and electric universe ($B = 0$). The interaction between matter and magnetic field have been considered in section III and for some particular form of interaction term, we have found the solutions of magnetic field and the energy density of matter. In section IV, we have considered the interaction between the matter and electric field only and another form of interaction term has been chosen to solve the field equations. In section V, the validity of generalized second law of thermodynamics have been investigated on apparent and event horizons using Gibbs’ law and the first law of thermodynamics for magnetic universe and electric universe separately. Finally, some conclusions are drawn in section IV.

## II. BASIC EQUATIONS IN NON-LINEAR ELECTRODYNAMICS

The Lagrangian density in Maxwell’s electrodynamics (linear) can be written as [31]

$$\mathcal{L} = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} = -\frac{1}{4\mu_0} F$$  \hspace{1cm} (1)
where $F^{\mu \nu}$ is the electromagnetic field strength tensor and $\mu_0$ is the magnetic permeability. The generalization of Maxwell’s electro-magnetic Lagrangian (non-linear) up to the second order terms of the fields is given by \[31\]

$$\mathcal{L} = -\frac{1}{4\mu_0} F + \omega F^2 + \eta F^{*2}$$

(2)

where $\omega$ and $\eta$ are arbitrary constants and

$$F^* = F^{*\mu \nu} F_{\mu \nu}$$

(3)

where $F^{*\mu \nu}$ is the dual of $F_{\mu \nu}$. Here we consider the FRW model of our universe. Since the spatial section of FRW geometry are isotropic, electromagnetic field can generate such a universe only if an averaging procedure is performed \[40, 41\]. In this situation, the energy density and the pressure of the NLED field should be evaluated by averaging over volume. So the corresponding energy-momentum tensor for non-linear electro-magnetic theory has the form

$$T_{\mu \nu} = -4 \frac{\partial \mathcal{L}}{\partial F} F^a_{\mu \alpha} F_{\alpha \nu} + \left( \frac{\partial \mathcal{L}}{\partial F^*} F^* - \mathcal{L} \right) g_{\mu \nu}$$

(4)

The modified Lagrangian in non-linear electrodynamics for accelerated universe is considered as \[42\]

$$\mathcal{L} = -\frac{1}{4} F + \alpha F^2 + \beta F^{-1}$$

(5)

where $\alpha$ and $\beta$ are arbitrary (constant) parameters. As seen this Lagrangian contains both positive and negative powers of $F$. The second (quadratic) term dominates during very early epochs of the cosmic dynamics, while the Maxwell term (first term above) dominates in the radiation era. The last term is responsible for the accelerated phase of the cosmic evolution \[43\]. The above Lagrangian density yields a unified scenario to describe both the acceleration of the universe (for weak fields) and the avoidance of the initial singularity, as a consequence of its properties in the strong-field regime.

The energy density and pressure for electro-magnetic (EM) field are given by \[42\]

$$\rho_F = -\mathcal{L} - 4E^2 \mathcal{L}_F$$

(6)

and

$$p_F = \mathcal{L} - \frac{4}{3} (2B^2 - E^2) \mathcal{L}_F$$

(7)

where, $B$ and $E$ are respectively magnetic field and electric field. Now, the electro-magnetic field has the expression $F = 2(B^2 - E^2)$, so the explicit forms of the energy density and the pressure for electro-magnetic field will be \[42, 44\]

$$\rho_F = \frac{1}{2} (B^2 + E^2) - 4\alpha (B^2 - E^2)(B^2 + 3E^2) - \frac{\beta (B^2 - 3E^2)}{2(B^2 - E^2)^2}$$

(8)

and

$$p_F = \frac{1}{6} (B^2 + E^2) - \frac{4\alpha}{3} (B^2 - E^2)(5B^2 - E^2) + \frac{\beta (7B^2 - 5E^2)}{6(B^2 - E^2)^2}$$

(9)

Due to the loop quantum effect, the standard Friedmann equation in LQC can be modified by adding a correction term as (for flat model) \[45, 46\].
\[ H^2 = \frac{8\pi G}{3} \rho_{\text{total}} \left( 1 - \frac{\rho_{\text{total}}}{\rho_1} \right) \]  

(10)

and

\[ \dot{H} = -4\pi G (\rho_{\text{total}} + p_{\text{total}}) \left( 1 - \frac{2\rho_{\text{total}}}{\rho_1} \right) \]  

(11)

Also the energy-conservation equation is given by

\[ \dot{\rho}_{\text{total}} + 3H (\rho_{\text{total}} + p_{\text{total}}) = 0 \]  

(12)

with, \( \rho_{\text{total}} = \rho_m + \rho_F \) and \( p_{\text{total}} = p_m + p_F \) where, \( \rho_m \) and \( p_m \) are energy density and pressure for matter obeying the equation of state \( p_m = w_m \rho_m \), \( \rho_1 = \sqrt{3\pi^2 \gamma^3 G^2 h} \) is called the critical loop quantum density, \( \gamma \) is the dimensionless Barbero-Immirzi parameter and \( H = \frac{\dot{a}}{a} \) is the Hubble parameter.

Now if we consider the homogeneous electric field \( E \) in plasma, it gives rise to an electric current of charged particles and then rapidly decays. So the squared magnetic field \( B^2 \) dominates over \( E^2 \), i.e., in this case, the average value \( < E^2 > \approx 0 \) and hence \( F = 2B^2 \). So \( F \) is now only the function of magnetic field (vanishing electric component) and hence the FRW universe may be called the magnetic universe. Similarly, if the average value \( < B^2 > \approx 0 \), then \( F = -2E^2 \). So \( F \) is now only the function of electric field (vanishing magnetic component) and hence the FRW universe may be called the electric universe. In the following two sections, we shall assume the magnetic and electric universes separately.

### III. INTERACTION BETWEEN MATTER AND MAGNETIC FIELD

In the magnetic universe, we assume \( E = 0 \). Therefore the energy density and pressure are (using equations (8) and (9))

\[ \rho_B = \frac{1}{2} B^2 - 4\alpha B^4 - \frac{\beta}{2B^2} \]  

(13)

\[ p_B = \frac{1}{6} B^2 - \frac{20\alpha}{3} B^4 + \frac{7\beta}{6B^2} \]  

(14)

and subsequently

\[ \rho_{\text{total}} = \rho_m + \frac{1}{2} B^2 - 4\alpha B^4 - \frac{\beta}{2B^2} \]  

(15)

and

\[ p_{\text{total}} = \omega_m \rho_m + \frac{1}{6} B^2 - \frac{20\alpha}{3} B^4 + \frac{7\beta}{6B^2} \]  

(16)

Now if we consider the interaction between matter and magnetic field, then the conservation equation (12) becomes

\[ \dot{\rho}_m + 3H (1 + w_m) \rho_m = Q \]  

(17)

and
\[ \dot{\rho}_B + 3H(\rho_B + p_B) = -Q \]  

where \( Q \) is the interaction term. Now let us take the interaction component \( Q \) as

\[ Q = 2\delta B \left( 1 - 16\alpha B^2 + \frac{\beta}{B^4} \right) H \]

This expression is used to simplify the calculation procedure. Here \( \delta \) may be treated as interaction parameter which is a small positive quantity. Using the expressions of \( \rho_B, p_B \) from (15) and (16) and the relation between \( \rho_m \) and \( p_m \), the above two equations (17) and (18) give the solutions of \( B \) and \( \rho_m \) in the following forms:

\[ B = -\delta + \frac{B_0}{a^2} \]  

and

\[ \rho_m = \rho_0 a^{-3(1+\omega_m)} + \frac{2\delta}{a^6} \left[ -\frac{a^6(\beta + \delta^4 - 16\alpha \delta^6)}{3\delta^3(1 + \omega_m)} + \frac{16\alpha B_0^3}{3(1 - \omega_m)} + \frac{48\alpha^2 \alpha \delta B_0^2}{3\omega_m - 1} \right] 
+ \frac{2B_0}{a^2 \delta^3(1 + 3\omega_m)} \left( -3\beta + \delta^4 - 48\alpha \delta^6 + 6\beta \right) \left[ \frac{1}{2}(1 + 3\omega_m), 1, \frac{3(1 + \omega_m)}{2}, \frac{a^2 \delta}{B_0} \right] 
- 4\beta \left[ \frac{1}{2}(1 + 3\omega_m), 2, \frac{3(1 + \omega_m)}{2}, \frac{a^2 \delta}{B_0} \right] + \beta \left[ \frac{1}{2}(1 + 3\omega_m), 3, \frac{3(1 + \omega_m)}{2}, \frac{a^2 \delta}{B_0} \right] \]  

where \( B_0 \) and \( \rho_0 \) are positive integration constants and \( _2F_1 \) is the hypergeometric function.

**IV. INTERACTION BETWEEN MATTER AND ELECTRIC FIELD**

This section deals with the case \( B = 0 \) i.e., the universe is filled with matter and electric field. Thus the expressions of the energy density and pressure for the electric field become (using equations (8) and (9))

\[ \rho_E = \frac{1}{2}E^2 + 12\alpha E^4 + \frac{3\beta}{2E^2} \]  

\[ p_E = \frac{1}{6}E^2 - \frac{4\alpha}{3}E^4 - \frac{5\beta}{6E^2} \]

and therefore

\[ \rho_{total} = \rho_m + \frac{1}{2}E^2 + 12\alpha E^4 + \frac{3\beta}{3E^2} \]

and

\[ p_{total} = \omega_m \rho_m + \frac{1}{6}E^2 - \frac{4\alpha}{3}E^4 - \frac{5\beta}{6E^2} \]

In this case, we consider the interaction between matter and electric field. So the conservation equation (12) takes the form
\[ \dot{\rho}_m + 3H(1 + w_m)\rho_m = \dot{Q} \]  

(26)

and

\[ \dot{\rho}_E + 3H(\rho_E + p_E) = -\dot{Q} \]  

(27)

To simplify the calculation, the interaction component \( \dot{Q} \) is taken as

\[ \dot{Q} = 3\dot{\delta}E^2 \left( 1 + 16\alpha E^2 + \frac{\beta}{E^4} \right)H \]  

(28)

where \( \dot{\delta} \) is a small positive quantity. In this case, exact solutions for \( \rho_m \) and \( E \) from the above two equations can only be obtained if they are solved numerically using the expressions of \( \rho_E \) and \( p_E \) together with the relation between \( \rho_m \) and \( p_m \).

V. GENERALIZED SECOND LAW OF THERMODYNAMICS

In this section, the validity of the generalized second law of thermodynamics is studied. It states that, the sum of entropy of total matter enclosed by the horizon and the entropy of the horizon does not decrease with time. In the following, we consider apparent and event horizons only. The variation of entropy inside the horizon will be calculated via Gibb’s equation and the variation of entropy on the horizon will be calculated using first law of thermodynamics. Hence we shall examine the validity of GSL of thermodynamics of the magnetic universe and electric universe separately bounded by the above mentioned horizons.

A. Apparent Horizon

We know that radius of apparent horizon in the FRW universe,

\[ R_A = \frac{1}{H} \]  

(29)

Therefore,

\[ \dot{R}_A = -\frac{\dot{H}}{H^2} \]  

(30)

Case-I: \( E = 0 \): Using equations (10), (11), (15) and (16)

\[ \dot{R}_A = \frac{B^2 \left( 1 - 16\alpha B^2 + \frac{\beta}{B^2} \right) + \frac{3}{2}(1 + w_m)\rho_m}{\rho_m + \frac{B^2}{2} - 4\alpha B^2 - \frac{\beta}{2B^2}} \]  

(31)

Considering the net amount of energy crossing through the apparent horizon in time \( dt \) as [47]

\[ -dE = 4\pi R_A^3 H(\rho_{total} + p_{total})dt \]  

(32)

and assuming the validity of first law of thermodynamics on the apparent horizon, i.e.,

\[ -dE = T_A dS_A \]  

(33)
Fig. 1 represents rate of change of total entropy of apparent horizon i.e., $\dot{S}_A + \dot{S}_I$ against the scale factor $a$ for $B = 0$ with interaction for $w_m = 1/3$ (solid line), $w_m = 0$ (dotted line) and $w_m = -0.5$ (dashed line).

where, $S_A$ and $T_A$ are the entropy and temperature on the apparent horizon, we have

$$\frac{dS_A}{dt} = \frac{4\pi R_A^3 H}{T_A} \left[ \frac{2B^2}{3} \left( 1 - 16\alpha B^2 + \frac{\beta}{B^4} \right) + (1 + w_m)\rho_m \right]$$

(34)

Again from the Gibb’s eqn

$$T_A dS_I = dE_I + p_{total} dV$$

(35)

where, $S_I$ is the internal entropy, $V$ is the volume and $E_I = \rho V$ is the internal energy, we have

$$\frac{dS_I}{dt} = \frac{4\pi R_A^2}{T_A} \left[ \frac{2B^2}{3} \left( 1 - 16\alpha B^2 + \frac{\beta}{B^4} \right) + (1 + w_m)\rho_m \right] \left( \dot{R}_A - HR_A \right)$$

(36)

From eqns (34) and (36), the rate of change of the total entropy becomes

$$\frac{d}{dt}(S_A + S_I) = \frac{4\pi R_A^2}{T_A} \left[ \frac{2B^2}{3} \left( 1 - 16\alpha B^2 + \frac{\beta}{B^4} \right) + (1 + w_m)\rho_m \right] \dot{R}_A$$

(37)

Substituting the expression (31) in eqn (37) and using the expressions of $B$ and $\rho_m$ from eqns (20) and (21) respectively, we plot the rate of change of total entropy of the apparent horizon, i.e., $\dot{S}_A + \dot{S}_I$ against the scale factor in figure 1 with interaction ($\delta = 0.0001$) for different matter components i.e., $w_m = 1/3$ (solid line), $w_m = 0$ (dotted line) and $w_m = -0.5$ (dashed line). From the figure, we see that the rate of change of total entropy for apparent horizon is initially positive but as $a$ increases, it decreases and in late time, it becomes negative. So we may conclude that, under the loop quantum cosmological effects, the GSL is valid initially, but after certain stage of the evolution of the universe, the GSL will not valid for apparent horizon for interacting scenarios in the magnetic universe.

Case-II : $B = 0$:

Proceeding in the same way as in the previous case, the expression for the rate of change of the total entropy becomes
Fig. 2 represents rate of change of total entropy of apparent horizon i.e., $\dot{S}_A + \dot{S}_I$ against the scale factor $a$ for $E = 0$ with interaction for $w_m = 1/3$ (solid line), $w_m = 0$ (dotted line) and $w_m = -0.5$ (dashed line).

Substituting the expression of $\dot{R}_A$ from eqn (30) and using the numerical solutions of $E$ and $\rho_m$, we plot the rate of change of total entropy of the apparent horizon, i.e., $\dot{S}_A + \dot{S}_I$ against the scale factor in figure 2 with interaction ($\delta = 0.0001$) for different matter components i.e., $w_m = 1/3$ (solid line), $w_m = 0$ (dotted line) and $w_m = -0.5$ (dashed line). From the figure, we see that the rate of change of total entropy for apparent horizon is always positive. Thus it may be concluded that even under the loop quantum cosmological effects, the GSL is always satisfied in late time for apparent horizon for interacting scenarios in the electric universe.

### B. Event Horizon

The event horizon radius is given by

$$R_E = a \int_a^\infty \frac{da}{Ha^2}$$

The differential eqn of which can be written as

$$\dot{R}_E = HR_E - 1$$

Considering the net amount of energy crossing through the event horizon in time $dt$ as

$$-dE = 4\pi R_E^3 H(\rho_{total} + p_{total})dt$$

and assuming the validity of first law of thermodynamics on the event horizon, i.e,

$$-dE = T_E dS_E$$

(42)
Case-I : $E=0$: Fig.3 represents rate of change of total entropy of event horizon i.e., $\dot{S}_E + \dot{S}_I$ against the scale factor $a$ for $B = 0$ with interaction for $w_m = 1/3$ (solid line), $w_m = 0$ (dotted line) and $w_m = -0.5$ (dashed line).

we have the rate of change of the total entropy as in the following cases:

Case-I : $E = 0$:

$$\frac{d}{dt}(S_E + S_I) = \frac{4\pi R_E^2}{T_E} \left[ \frac{2B^2}{3} \left( 1 - 16\alpha B^2 + \frac{\beta}{B^4} \right) + (1 + w_m) \rho_m \right] (HR_E - 1) \quad (43)$$

Case-II : $B = 0$:

$$\frac{d}{dt}(S_E + S_I) = \frac{4\pi R_E^2}{T_E} \left[ \frac{2E^2}{3} \left( 1 + 16\alpha E^2 + \frac{\beta}{E^4} \right) + (1 + w_m) \rho_m \right] (HR_E - 1) \quad (44)$$

Substituting the expressions of $R_E$, $H$, $B$, $E$ and $\rho_m$ in eqns (43) and (44), the rate of change of total entropy of the event horizon, i.e, $\dot{S}_E + \dot{S}_I$ is plotted against the scale factor in figures 3 and 4 for the above two cases with interaction ($\delta = 0.0001$) for different matter components i.e., $w_m = 1/3$ (solid line), $w_m = 0$ (dotted line) and $w_m = -0.5$ (dashed line). From the figures, we see that the rate of change of total entropy for event horizon is always positive for both magnetic and electric universes and hence the GSL is always satisfied for event horizon for interacting scenarios of the magnetic and electric universes.

VI. DISCUSSIONS

In this work, we have briefly discussed Maxwell’s electrodynamics in non-linear forms for accelerating universe. The energy density and pressure for non-linear electrodynamics have been written in magnetic universe in one section and for universe with electric field only in another section. The Einstein’s field equations for loop quantum cosmological model have been considered for FRW model of the universe. The interaction between matter and electric and magnetic fields have been incorporated separately and for particular forms of interaction terms, we have found the solutions for both electric as well as magnetic fields and the energy density of matter.
Case-II : B=0  Fig.4 represents rate of change of total entropy of event horizon i.e., $\dot{S}_E + \dot{S}_I$ against the scale factor $a$ for $E = 0$ with interaction for $w_m = 1/3$ (solid line), $w_m = 0$ (dotted line) and $w_m = -0.5$ (dashed line).

In addition to this, our endeavor was to investigate the validity of the generalized second law of thermodynamics of the universe bounded by the apparent and event horizons. The variation of entropy has been calculated inside the horizon using Gibb's equation and that on the horizon using the first law of thermodynamics. After that, we have studied the GSL of the universe bounded by the above mentioned horizons.

In figures 1 - 4, the variation of total entropy on the apparent and event horizons have been drawn against the scale factor $a$ for interacting scenarios ($\delta = 0.0001$) of magnetic universe as well as universe with electric field only, for $w_m = 0, 1/3, -0.5$. From figure 1, we see that the rate of change of total entropy was initially positive but in late epoch, it becomes negative and thus the GSL is not valid for the magnetic universe on the apparent horizon. Whereas on the event horizon for the magnetic universe, the GSL remains always valid. This can be clearly seen from figure 3. Figure 2 represents the rate of change of total entropy on the apparent horizon for the universe with electric field only, which again shows the validity of the GSL throughout the evolution of the universe for all the cases. Finally, from figure 4, we see that GSL was initially not valid on the event horizon, but in late time it is satisfied for all types of matter.

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