Optimization of chip repose angle in dry hobbing machine considering minimum thermal accumulation on workbench

Bo Li · Xiao Yang · Yanbin Du · Lang He

Received: 25 June 2022 / Accepted: 21 August 2022 / Published online: 27 August 2022
© The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2022

Abstract
The thermal deformation problem is quite serious for the workbench of the environmental dry hobbing machine due to the thermal accumulation of hot chips. A better understanding of the quantity and optimization of the chip repose angle is quite necessary to reduce the workbench thermal deformation. Hence, this paper focuses on the chip repose angle modeling and optimization for the workbench of the dry hobbing machine. Firstly, the dynamic flow characteristics of chips are clarified by analyzing the chip generation mechanism and the motion trajectory of dry hobbing. Then, the chip repose angle is mathematically characterized with consideration of the chip rolling characteristics, with which a chip repose angle optimization model is established by minimizing the thermal accumulation on the workbench, and a chip repose angle optimization method is proposed based on the intelligent optimization algorithm. Finally, the effectiveness of the proposed method is analyzed through the simulation experiment on the dry hobbing of automotive reverse gear. The results show that the maximum overall thermal deformation is reduced by 8 μm for the workbench, and the skewness is reduced by 32% for the workbench spindle with the help of the proposed optimization method. The study provides an effective solution for the quantitative analysis and optimal design of chip repose angle on the workbench of the dry hobbing machine.

Keywords Green manufacturing · Dry hobbing · Thermal accumulation · Chip repose angle · Optimization method

Abbreviations

\( c^* \) Clearance coefficient
\( d_a \) Gear diameter
\( e \) Contact width of chip and shield
\( f \) Axial feed
\( g \) Gravity
\( h \) Tooth depth
\( h_1 \) Height of workbench upper part
\( h_2 \) Height of workbench lower part
\( h^* \) Addendum coefficient
\( l_1 \) Set distance from shield to workbench
\( l_2 \) Distance from shield to isolation baffle
\( l_3 \) Initial distance from shield to isolation baffle
\( l_b \) Regulation interval length of \( x_b \)
\( l_t \) Length of the chip rolling down
\( l_w \) Width of chip passing through shield
\( m \) Modulus
\( n \) Number of chips generated by machining one gear
\( n_b \) Hob rotation speed
\( Q_c \) Heat transfer of each chip to shield
\( Q_g \) Heat transfer from chip of one gear to shield
\( Q_{total} \) Heat transfer of all chips to the shield
\( r_h \) Radius of the hob
\( R_1 \) Structural parameter of workbench
\( R_2 \) Structural parameter of workbench
\( R_b \) Distance of shield bottom to the workbench center axis
\( R_1 \) Distance of shield top to the workbench center axis
\( t_f \) Fall time of chip
\( t_p \) Hobbing time for machining one gear
\( t_r \) Retention time of chips on shield
\( T_c \) Chip temperature
\( T_s \) Shield temperature
\( V \) Velocity of chip when it touches shield
\( V_0 \) The initial velocity of chip
$V_{ox}$ Chip’s initial velocity components in the horizontal directions

$V_{oz}$ Chip’s initial velocity components in the vertical directions

$x_0$ Bottom adjustment distance

$x_t$ Top adjustment distance

$\alpha$ Chip repose angle

$\alpha_{\text{man}}$ Original chip repose angle

$\alpha_{\text{opt}}$ Optimized chip repose angle

$\beta$ Helix angle

$\gamma$ Chip flow angle

$\delta$ Installation angle

$\delta_t$ Shield thickness

$\delta_{b,m}(x)$ Thermal displacement of the workbench axis bottom in X-direction

$\delta_{b,m}(y)$ Thermal displacement of the workbench axis bottom in Y-direction

$\delta_{t,m}(x)$ Thermal displacement of the workbench axis top in X-direction

$\delta_{t,m}(y)$ Thermal displacement of the workbench axis top in Y-direction

$\delta_{t,m}(z)$ Thermal displacement of the workbench

$\varepsilon$ Thermal conductivity of shield

$\lambda$ Tooth lead angle

$\Phi$ Heat flux

$\mu$ Surface friction coefficient of shield

$\omega_1$ Rotational angular velocity of gear

$\omega_2$ Rotational angular velocity of hob

which reduced the uncertainty of thermal deformation measurement. Ibaraki et al. [6] proposed a method to analyze the heat effect of the machine tool motion trajectory, which realized a quantitative analysis of the relationship between the machine tool motion trajectory and spindle heat generation. Blaser et al. [7] proposed an adaptive compensation method for the thermal error of the machine tool, which could follow the load change to adjust the center point of the tool. Shimoike et al. [8] proposed a method to evaluate and visualize the relationship between temperature variation and thermal deformation in the cutting space. Shi et al. [9] investigated the thermal deformation model of ball screw for precision boring machine and the relationship between the temperature rise and thermal error of the ball screw. Mori et al. [10] investigated the effect of the linear expansion coefficient difference between a machine tool and workpieces on the thermal deformation induced by room temperature change. Liu et al. [11] established a heat flow boundary model for machine tool spindle, and Liu et al. [12] proposed a thermal-fluid-solid coupling method for resolving thermal errors in machine tool electric spindle, all of which effectively improved the machining accuracy of the workpiece. However, as new types of gear manufacturing equipment, dry hobbing machines have specific structure and heat source differences compared to traditional machine tools. Therefore, the above research is more valuable and feasible for mitigating the heat deformation effect of other machine tools.

The dry hobbing machine has its own unique characteristics in heat generation and heat dissipation, its thermal deformation is a critical technical problem that needs to be solved for industrial development [13]. Zhu et al. [14] established a heat source model for a high-speed dry hobbing machine and described the heat energy accumulation characteristics of the dry hobbing machine. Liu et al. [15] revealed the temperature characteristics of the hob cutter assembly by quantifying the heat source of the dry hobbing machine. Kadashevich et al. [16] proposed an improved DEXEL model capable of visualizing the temperature field and geometric characteristics of gears, which effectively reduces the thermal errors during gear machining. Li et al. [17] analyzed the thermal-force coupling effect on the hob cutter spindle and proposed a method to calculate the thermal deformation error of the hob electric spindle. Yang et al. [18] regulated the thermal balance state of the dry hobbing machine by enhancing the heat dissipation capability of the machine tool. Li et al. [19] analyzed the heat transfer process, thermal deformation, and the time to reach the thermal balance of the workbench during dry hobbing, and found that there is significant thermal deformation at the fixture of the workbench assembly. The above researches focus on the heat source characteristics, thermal error compensation, and thermal balance control of the dry hobbing machines. There is still a gap in the research

1 Introduction

Dry hobbing machine is a green and efficient gear manufacturing equipment, which has the advantages of high production efficiency, low cost, and low pollution [1, 2]. However, the thermal deformation problem of dry hobbing machine is quite serious due to its multiply heat sources and poor heat dissipation conditions, especially for the workbench that tends to accumulate hot chips [3]. The cause is that the chips are responsible for carrying away more than 80% of the cutting heat in dry hobbing. When a large number of hot chips are retained on the workbench surface, it will trigger thermal accumulation and cause thermal deformation of the workbench [4]. It is worth noting that the chip retention time is directly interrelated with the chip repose angle. Therefore, it is of great theoretical significance and practical value to study the chip repose angle modeling and optimization method of the workbench for the dry hobbing machine to improve the gear machining accuracy.

The problem of thermal deformation for machine tools has been a hot topic of academic research. Brecher et al. [5] proposed a method for measuring the thermal deformation of machine tools based on integral deformation sensors,
on controlling the thermal effects of dry hobbing machines from the perspective of structural optimization. At present, the author team has conducted some studies on the thermal problems of dry hobbing machines, such as the generation of cutting heat during dry hobbing [20] and the heat flow characteristics of the dry hobbing machine [21].

As a component directly contacting the workpiece, the research on the thermal deformation of the workbench is of great significance. It is worth paying attention to the hot chip retention and accumulation on the workbench will directly affect the workbench thermal deformation, and the time chips remain on the workbench is closely related to the chip repose angle. On the one hand, the hot chips could not be quickly discharged to the outside of the machine after leaving the workpiece, which will cause thermal deformation of the machine bed and other structural parts, thus affecting the accuracy of gear machining. On the other hand, the current chip repose angle of the workbench for the dry hobbing machine is designed by subjective experience, lacking a systematic and quantitative theoretical basis, and the thermal effect of hot chips on the workbench is not considered. Therefore, this paper focuses on developing the modeling and optimization of the chip repose angle of the dry hobbing machine for minimizing the thermal accumulation on the workbench of the dry hobbing machine, and the main research structure is as follows: Sect. 2 reveals the dynamic flow mechanism of chips in dry hobbing; Sect. 3 establishes the optimization model of chip repose angle for dry hobbing machine; Sect. 4 conducts applied research on the optimization model of chip repose angle; Sect. 5 concludes the entire paper and plans subsequent research.

2 Chip flow mechanism of dry hobbing

2.1 Chip generation process

In the dry hobbing process, each cutting edge of the dry hob cutter removes the workpiece material one after another in thin slices. The gear is shaped by a series of transient position envelopes of the cutting edges. As shown in Fig. 1, according to the helix angle ($\beta$) of the gear, the installation angle ($\delta$) and the tooth lead angle ($\lambda$) of the dry hob cutter together determine the relative position of the dry hob cutter and the workpiece. The dry hob cutter and the workpiece according to the strict transmission ratio ($\omega_1/\omega_2$) to do the spreading movement, while the dry hob cutter along the axial direction of the workpiece to do the feeding movement ($f$) to complete the gear tooth shape processing on the entire width of the gear.

According to the spatial motion mechanism of the dry hobbing process, gear tooth formation is achieved through the synergistic removal of workpiece material by the top and side edges of the cutter teeth. The chip is the excess material of the gear blank removed by the spatial trajectory surface of the cutting edge during the gear forming process (Fig. 2). It is worth emphasizing that dry hobbing has the processing characteristics of multi-edge intermittent cutting. The position and range of contact between each cutter tooth and the workpiece vary. Therefore, the force, heat, and force-thermal deformation of the chips generated simultaneously are also different, resulting in some variability in the geometry of each chip. As shown in Fig. 2, the real chips produced in the hobbing process are different in shape and size.

2.2 Chip flow characteristics

The chip flow behavior of dry hobbing is influenced by a combination of factors such as hob handed direction, hob tooth lead angle, hob installation angle, workpiece handed direction, and workpiece helix angle. During the chip formation process, the chip is driven by the rank face of the cutter tooth in the hob helix direction to make a rotational motion. To ensure the machining accuracy of the tooth shape, the helix direction of the contact point between the dry hob cutter and the workpiece should be consistent with the workpiece tooth space direction. Therefore, the instantaneous velocity of the chip leaving the workpiece could be approximated as the linear velocity of the hob cutter, i.e., Eq. (1), and the velocity direction is the common tangent of the contact point between the excircle of the hob cutter and the workpiece surface, as shown in Fig. 3.

$$V_0 = \frac{\pi n_h r_h}{30}$$  \hspace{1cm} (1)

where $V_0$ is the initial velocity of the chip, m/s, $n_h$ is the hob speed, r/min; $r_h$ is the radius of the hob, m.
After the chip is separated from the workpiece, it will be affected by gravity and air resistance in the cutting space to do falling body movement and fall to the workbench surface. This paper defines the angle between the shield of the workbench and the horizontal surface as the chip repose angle (α), as shown in Fig. 4 (see Sect. 3 for the meaning of parameters $h_1$, $h_2$, $R_t$, etc., in the figure). Considering that the chips could take away more than 80% of the cutting heat in dry hobbing, the chip repose angle of the workbench should be greater than the critical angle of the chips rolling down on the shield, thus avoiding the accumulation of hot chips on the surface of the workbench. According to the heat transfer theory, the contact time between chips and the workbench directly affects the heat exchange quantity. Therefore, the optimized design of chip repose angle combined with chip flow characteristics could reduce the accumulation number and retention time of chips on the workbench surface, thereby suppressing the thermal deformation of the workbench and ensuring the machining accuracy of the machine.

### 3 Chip repose angle optimization model

#### 3.1 Mathematical model

The structural model of chip repose angle is established according to the dry hobbing machine’s geometric structure and spatial layout relationship, as shown in Fig. 5a. Where $x_t$, $x_b$ denote the top adjustment distance (the horizontal movement distance of the shield top end relative to the original position) and the bottom adjustment distance (the horizontal movement distance of the shield bottom end relative to the original position), respectively, i.e., $x_t$, $x_b$ are the two regulation variables controlling the size of the chip repose angle. $l_r$ is the length of the chip rolling down on the shield. Therefore, when $x_t = 0$ and $x_b = 0$, the chip repose angle $\alpha_{\text{man}}$ adopted for manufacturing the dry hobbing machine could be obtained.

The mathematical representation model of chip repose angle is established based on the spatial kinematics theory of objects, i.e., Eq. (2).

$$\alpha = \arctan \frac{h_2}{R_b + x_b - R_t - x_t}$$  \hspace{1cm} (2)
where $h_2$ is the height of the workbench lower part, m. $R_t$ and $R_b$ are the distance of the top and bottom ends of the shield from the workbench center axis, respectively, m.

Take the dry hobbing of cylindrical spur gears as an example. The initial flow direction of chips after generation is shown in Fig. 5b. The vast majority of chips fly out along the direction of the chip flow angle $\gamma$. The chip flow angle is determined by the structural parameters of the hob cutter and gear, which could be calculated using Eq. (3).

$$
\gamma = \arctan \frac{r_h - h}{\sqrt{2hr_h - h^2}}
$$

By observing the actual production of the workshop, it is found that after the chips are separated from the workpiece,
the vast majority of them first fall onto the workbench surface and then enter the chip conveyor. Just a tiny part of the chips bounced or rolled down to the remaining parts surface of the machine tool, which is ignored in this paper.

Therefore, the fall time $t_f$ for the chip to fall from the workpiece to the workbench surface could be calculated by Eq. (4).

$$t_f = \sqrt{\left[V_{0x} \tan \alpha - V_{0y} \right]^2 - 2g^{-1}(h + h_0 + R_a - \frac{d_x}{2}) \tan \alpha - (h_1 + h_2)}$$

\[+(V_{0x} \tan \alpha - V_{0y})g^{-1} \]

(4)

where $V_{0x}$ and $V_{0y}$ are the chip’s initial velocity components in the horizontal and vertical directions, respectively, m/s. $g$ is gravity, 10 m/s$^2$. $d_x$ is workpiece diameter, m. $h_1$ is the height of the workbench upper part, m. $h$ is workpiece tooth depth, m. In addition, $V_{0x} = V_0 \cos \gamma$, $V_{0y} = V_0 \sin \gamma$, $h = \frac{(2\eta^2 + 7)100}{1000}$. Where $\eta$ is the addendum coefficient, $c^2$ is the clearance coefficient, and $m$ is the modulus, mm.

When the chip touches the shield, its velocity $V$ along the bevel direction could be expressed by Eq. (5). Since the chip falls to the shield and then moves with variable speed by gravity and friction, its retention time $t_r$ on the shield could be calculated by Eq. (6).

$$V = V_{0x} \cos \alpha + (V_{0x} + gt_f) \sin \alpha$$

(5)

$$t_r = \frac{V^2 + g(1 - \mu \tan \alpha^{-1}) \cdot [2(h_1 + h_2 - V_{ox}t_f) - gt_f^2] - V}{g(\sin \alpha - \mu \cos \alpha)}$$

(6)

where $t_r$ is the retention time of chips on the shield, $s$; $\mu$ is the surface friction coefficient of the shield.

3.2 Objective function

The fallen chips will transfer partial heat to the shield by heat conduction, and then the shield will transfer partial heat to the workbench by heat radiation. Therefore, the workbench thermal accumulation could be decreased by reducing the heat transfer from the chips to the shield. Considering that the heat transfer accompanies the whole dry hobbing process and the short retention time of individual chips on the shield, this paper assumes that the chip temperature before and after heat transfer is constant, and the heat transfer from individual chips to the shield is considered a one-dimensional steady-state heat transfer. Therefore, the heat transfer $Q_i$ of each chip to the shield could be calculated by Eq. (7) [22], where the chip roll-down length $l_i$ on the shield could be expressed as Eq. (8).

$$Q_i = \varepsilon \rho \theta \frac{T_s - T_c}{\delta} t_r$$

(7)

$$l_i = \frac{h_1 + h_2 - V_{ox}t_f - 0.5gt_f^2}{\sin \alpha}$$

(8)

where $Q_i$ is the heat transfer from each chip to the shield, J. $\varepsilon$ is the thermal conductivity of the shield, W/(m·℃). $T_s$ is the chip temperature, ℃. $T_c$ is the shield temperature, ℃. $\delta$ is the shield thickness, m. $\varepsilon$ is the contact width between the chip and the shield, m. $l_i$ is the length of the chip rolling down on the shield, m.

During the machining of one gear, the heat $Q_g$ transferred from the generated chips to the shield could be calculated by Eq. (9). During a machining period, the heat $Q_{total}$ transferred from the generated chips to the shield could be calculated by Eq. (10).

$$Q_g = \sum_{i=1}^{n} (Q_{c1} + Q_{c2} + Q_{c3} \cdots + Q_{c_n})$$

(9)

$$Q_{total} = \sum_{j=1}^{k} (Q_{g1} + Q_{g2} + Q_{g3} \cdots + Q_{g_j})$$

(10)

where $Q_{c1}i, Q_{c2}, Q_{c3}, \cdots Q_{c_n}$ denotes the heat transferred from each chip to the shield, J. $n$ denotes the number of chips generated by machining a gear. i denotes the ith chip generated when machining a gear. $Q_{g1}, Q_{g2}, Q_{g3}, \cdots Q_{g_j}$ denotes the heat transferred from the chips of one gear to the shield, J. $k$ denotes the number of gears machined by the machine in a period. j denotes the jth gear machined by the machine in a period.

According to the previous analysis of the chip forming mechanism, calculating the number of chips generated during the hobbing process is complicated. Therefore, this paper adopts the averaging method to obtain the number of chips produced by machining a single gear. First, the mass change after machining the gear and the average mass of individual chips are calculated. Then, the number of chips produced by machining one gear is determined through the ratio of the former to the latter. It is assumed that the number of chips produced by machining each gear is the same, and the number is equal to $n$, and the heat transfer from the chips produced by machining each gear to the shield is the same. Thus, according to Eqs. (1)–(10), $Q_{total}$ could be further expressed as Eq. (11). To reduce the machine thermal deformation caused by the accumulation of hot chips on the workbench surface, the heat $Q_{total}$ transferred from chips to the shield should be minimized. Therefore, the objective function could be expressed as Eq. (12).
\[ Q_{\text{total}} = n k \varepsilon I e \frac{T_c - T_i}{\partial_t} t_r \]  

(11)

Optimize \( Q_{\text{total}} = \min Q_{\text{total}} \)  

(12)

The optimal values of the decision variables \( x_i \) and \( x_b \) could be obtained by solving Eq. (12) and then substituting them into Eq. (2) to get the optimal values of the chip repose angle.

3.3 Control constraints

The constraints for chip repose angle optimization include the clearance between the workbench and the shield and the distance between the shield and the isolation baffle. For the clearance constraint between the workbench and shield, considering that the various heat transfer properties of metal materials are stronger than air, the shield should not be in direct contact with the workbench to minimize the thermal accumulation on the workbench. Therefore, the point \( P_2 \) should be kept at a certain distance \( l_1 \) from the point \( P_1 \), i.e., \( l_1 > 0 \), as shown in Fig. 6. The constraint could be further expressed as Eq. (13) based on the spatial geometric relationship.

\[
h_3 x_i + (h_2 - h_3) x_b \geq (R_b - R) h_3 + (R_2 + l_1 - R_b) h_2
\]

(13)

\[
\frac{h_2 (l_1 + R_2 - R_b) - h_3 (l_1 + R_1 - R_b)}{h_2 - h_3} \leq x_b \leq \frac{h_2 (l_1 + l_b + R_2 - R_b) - h_3 (l_1 + l_b + R_1 - R_b)}{h_2 - h_3}
\]

(14)

where \( l_1 \) is the set distance between the shield and the workbench, \( m; R_2 \) is the structural parameter of the workbench, \( m. \)

According to the chip flow regularity in hobbing processing, to ensure that the shield could play a guiding role in the chip flow and make the chips discharged quickly from the machine tool, it should make \( l_1 > 0 \). Combined with Fig. 6, i.e., there is a gap between points \( A_1 \) and \( A_2 \).

\[
F(x_i, x_b) = \min Q_{\text{total}}
\]

s.t.

\[
\begin{align*}
& h_3 x_i + (h_2 - h_3) x_b \geq (R_b - R) h_3 + (R_2 + l_1 - R_b) h_2 \\
& R_1 - R_i \leq x_i \leq l_1 + R_1 - R_i \\
& \frac{h_2 (l_1 + l_b + R_2 - R_b) - h_3 (l_1 + l_b + R_1 - R_b)}{h_2 - h_3} \leq x_b \\
& \frac{h_2 (l_1 + l_b + R_2 - R_b) - h_3 (l_1 + l_b + R_1 - R_b)}{h_2 - h_3} \leq x_b
\end{align*}
\]

(15)

where \( h_3 \) is the workbench structural parameter, \( m; l_3 \) is the initial distance between the shield and the isolation baffle, \( m. \)

In summary, the optimization model of the chip repose angle with the minimization of workbench thermal accumulation as the optimization objective, \( x_i \) and \( x_b \) as the decision variables, and the workbench structural limitations as the constraints is established, i.e., Eq. (18).

Fig. 6 Regulated interval of chip repose angle
4 Analysis of results and discussion

4.1 Intelligent optimization method for chip repose angle

Genetic algorithm is a method for global exploration of optimal solutions, which is simple, general, efficient, and robust [23]. This paper will use a genetic algorithm to solve the optimal chip repose angle of the dry hobbing machine. The specific optimization search process is shown in Fig. 7.

Step 1: Coding and initialization of populations. According to the range of values for the regulatory variables $x_t, x_b$, obtained in Sect. 3.3, taking the values of $x_t, x_b$ in the form of an equivariant series with all the terms $N$, set a matrix of size $2\times N$ with only one “1” and $N-1$ “0” in each row. Meanwhile, the positions of $x_t$ and $x_b$ in their value ranges correspond to the positions of “1” in the matrix.

Step 2: Generate initial populations. The $2\times N$ matrices generated in step 1 represent the range of the chip repose angle. Each initialized chip repose angle corresponds to a randomly generated “1” and “0” matrix of size $2\times N$. The generated $R$ non-repeating $2\times N$ matrices constitute a population (in Fig. 7, $w$ is the number of populations; $R$ is the population size).

Step 3: Determine whether the initial population reaches the iteration termination condition. The iteration termination moment of the algorithm is when the set number of iterations is completed or when the change rate of the average fitness of two adjacent generations is extremely low.

Step 4: Population screening is implemented by calculating each individual’s fitness. The adaptation degree of each individual could be calculated and ranked by Eq. (11). Since the thermal impact of chips on the workbench is smaller when the $Q_{\text{total}}$ is smaller in this paper, individuals with smaller $Q_{\text{total}}$ should be retained in the population selection process.

Step 5: Generate new populations. The matrices screened by step 4 will generate new populations by duplication, crossover, and variation until the optimal result is found from the population.

Step 6: Determine whether the population reaches the iteration termination condition. If the population does not satisfy the iteration termination condition, go to step 4. Otherwise, implement the next step.

Fig. 7 Solving process of chip repose angle
Step 7: Determine the optimal chip repose angle of the workbench. Substitute the values of $x_t$ and $x_b$ obtained from the solution into Eq. (2) to get the optimal chip repose angle.

### 4.2 Optimization effect analysis

This paper takes the example of automotive reverse gear machining on a dry hobbing machine for 2 h. The structural parameters of the workbench are obtained by consulting the machine design manual (Table 1). Hob spindle speed equals 670 r/min, and axial feed equals 1.6 mm/r. The hob diameter equals 80 mm, and the number of hob threads equals 2. The modulus of the workpiece is 2 mm, and the number of teeth is 36. The machining time of one gear is 63 s (hobbing time equals 48 s, auxiliary time equals 15 s). The ambient temperature of the workshop is 22 °C, i.e., the initial temperature $T_s$ of the shield is 22 °C.

The optimization model of chip repose angle takes Eq. (12) as the objective function and Eqs. (13)–(17) as the constraints. With the intelligent search algorithm for the chip repose angle in Sect. 4.1, the relationship between the chip heat transfer $Q_c$ to the shield and the regulation variables $x_t, x_b$ could be obtained, as shown in Fig. 8. The results show that the optimal values of $x_t, x_b$ are $-10.0$ and $5.7$ mm, respectively, and the optimized chip repose angle $\alpha_{opt}$ could be obtained as 55.31° by substituting them into Eq. (2).

To verify the effectiveness of the chip repose angle optimization model, 3D modeling and temperature field simulation of the workbench are carried out in this paper, and the thermal deformation of the workbench before and after the chip repose angle optimization is analyzed. First, the workbench structure is modeled using 3D modeling software based on the values before and after chip repose angle optimization (Fig. 9a). Secondly, the hexahedron dominant meshing technique is used to mesh the workbench 3D model. Then, the properties are set according to the thermal physical parameters of the material (Table 2), where the workbench’s material is HT250, and the shield’s material is carbon steel. Finally, the temperature field simulation of the workbench is completed by imposing boundary conditions, solving the simulation, and post-processing the results [24].

During the dry hobbing process, convective heat exchange occurs between the air and the shield. Air’s natural convection heat transfer coefficient is 10 W/(m²·K). Meanwhile, the hot chips will transfer heat to the shield. Considering the difficulty of the actual measurement and the requirement for mechanism analysis, the theoretical

| No. | Parameters | Value | Unit | Number | Parameters | Value | Unit |
|-----|------------|-------|------|--------|------------|-------|------|
| 1   | $h_1$      | 363   | mm   | 7      | $R_1$      | 146   | mm   |
| 2   | $h_2$      | 342   | mm   | 8      | $R_2$      | 287   | mm   |
| 3   | $h_3$      | 124   | mm   | 9      | $R_b$      | 377   | mm   |
| 4   | $l_1$      | 10    | mm   | 10     | $R_t$      | 156   | mm   |
| 5   | $l_2$      | 30    | mm   | 11     | $l_j$      | 112   | mm   |
| 6   | $\delta_t$ | 1     | mm   | 12     | $\mu$      | 0.35  | -    |

Table 2 Thermal physical parameters

| No. | Properties          | Unit | HT250  | Carbon steel |
|-----|---------------------|------|--------|--------------|
| 1   | Density             | kg/m³| 7150.00| 7850.00      |
| 2   | Thermal conductivity| W/(m·K)| 53.00   | 52.00        |
| 3   | Heat emissivity     | -    | 0.63   | 0.79         |
| 4   | Specific heat capacity| J/(kg·K)| 500.00 | 450.00      |
| 5   | CTE                 | $10^{-6}$ K$^{-1}$| 11.00 | 12.32       |
| 6   | Elastic modulus     | GPa  | 130.00 | 206.00       |
| 7   | Poisson’s ratio     | -    | 0.27   | 0.30         |
analysis method is employed to investigate the relationship among heated area, chip repose angle, and machining parameters. The main heat transfer area could be determined by the length of the hob-workpiece contact line in conjunction with the length of the chip roll down on the shield (Fig. 9b). The machining process of one gear includes hobbing time and auxiliary time, and cutting heat is generated only during the hobbing time. According to the heat transfer theory, the heat flux $\Phi$ is added to the shield heated area, the magnitude of which is determined by Eq. (19). The value of $n$ is determined by the ratio of the average mass change for the gear after machining to the chip average mass. Determine the value of $e$ in combination with the contact characteristics of the chip and the shield. The chip temperature $T_c$ is the average value obtained from multiple shot acquisitions using Fluke Ti32 infrared thermal imaging.

$$\Phi = \frac{n e e(T_c - T_s) h_w}{l_w \delta t_p}$$  \hspace{1cm} (19)$$

where $\Phi$ denotes the heat flux, W/m$^2$; $l_w$ is the width of the area over which the chips pass on the shield, m; $t_p$ is the hobbing time for machining a gear, s. The value of $l_w$ and the shield heated area is shown in Fig. 9b.

The workbench’s temperature field and thermal deformation are simulated before and after the optimization of chip repose angle, respectively. According to the temperature field clouds before and after optimization (Fig. 10), the higher temperature on the side of the workbench near the chip flow is that the hot chips transfer heat to the workbench. It is found by comparing the temperature fields that the workbench’s maximum temperature before the optimization is 33.5 °C, and the workbench’s maximum temperature after optimization is 31 °C. It could be seen that the workbench’s maximum temperature is reduced by using the optimized chip repose angle. Meanwhile, the high-temperature area of the workbench is obviously reduced, and the temperature field in the adjacent part of the workbench spindle is effectively improved, which is conducive to improving the machining accuracy of the machine.
Figure 11 shows the thermal deformation of the workbench before and after optimization. It is seen that the thermal deformation of the workbench is significant and is an essential factor affecting the gear machining accuracy. The maximum thermal deformation of the workbench in \(X\), \(Y\), and \(Z\)-directions is further obtained by post-processing the Ansys finite element simulation (Table 3). The thermal deformation in the \(X\)-direction causes the relative position of the workpiece and the hob to shift, thus producing radial deviation, affecting the actual tooth profile shape and leading to tooth shape error and tooth pitch error. The thermal deformation in \(X\)- and \(Y\)-directions will lead to the skewness of the workbench axis, resulting in tooth direction error. Since the hob will take the feed motion along the workpiece axial direction, the thermal deformation of the workbench spindle in the \(Z\)-direction has less influence on the tooth shape error and tooth pitch error of the gear. The maximum thermal deformation of the workbench in the \(X\)-, \(Y\)-, and \(Z\)-directions is reduced by 8 μm, 6 μm, and 7 μm, respectively, by optimizing the chip repose angle, which helps to reduce various errors in gear production.

The workbench spindle is the rotation center and the positioning reference for the workpiece in dry hobbing. In this paper, the thermal displacements of the top and bottom end of the workbench spindle are measured separately to investigate the thermal deformation regularity (Table 4). The workbench bottom surface (\(X_bO_bY_b\) plane in Fig. 12) is supported by the machine bed. Therefore, the simulation experiment sets the workbench bottom surface as frictionless support, i.e., the bottom end of the workbench spindle has no displacement in the \(Z\)-direction.

As shown in Fig. 12, the workbench spindle has the largest thermal deformation in the \(Z\)-direction, followed by the \(X\)-direction, and almost no change in the \(Y\)-direction. The reason is that the workbench has the largest size in the \(Z\)-direction, which causes the largest thermal deformation under the influence of thermal expansion and contraction. Due to the uneven heating on the left and right sides of the workbench, a large thermal deformation is generated in the \(X\)-direction. Since the temperature field of the workbench is symmetrically distributed, there is little change in thermal deformation in the \(Y\) direction. On the other hand, the top thermal displacement \(\delta_{t,m}(x)\) and the bottom thermal displacement \(\delta_{b,m}(x)\) of the workbench spindle in the \(X\)-direction are 5.2 and 4.7 μm, respectively, before the chip repose angle is optimized. When the optimized chip repose angle is adopted for the workbench, the top thermal displacement \(\delta_{t,o}(x)\) and the bottom thermal displacement \(\delta_{b,o}(x)\) of the workbench spindle are reduced to 3.2 and 3.5 μm in the \(X\)-direction, respectively. The inconsistent thermal displacement of the top and bottom ends of the workbench spindle will cause the workbench axis to become skewed, resulting in phenomena such as crowned tooth (Fig. 12), which affects gear machining.

**Table 3** Maximum thermal deformation of the workbench

|         | \(X\) (μm) | \(Y\) (μm) | \(Z\) (μm) | Total deformation (μm) |
|---------|------------|------------|------------|------------------------|
| Pre-optimization | 29         | 18         | 18         | 29                     |
| Optimized | 21         | 12         | 11         | 21                     |
| Effects  | ↓8         | ↓6         | ↓7         | ↓8                     |

**Table 4** Thermal displacement of the workbench axis

|         | Thermal displacement (μm) | Optimized Thermal displacement (μm) | Effects  |
|---------|---------------------------|------------------------------------|---------|
| \(\delta_{t,m}(x)\) | 5.2                      | \(\delta_{t,o}(x)\) | 3.2 | ↓38.5% |
| \(\delta_{t,m}(y)\) | –0.003                   | \(\delta_{t,o}(y)\) | –0.0002 | – |
| \(\delta_{t,m}(z)\) | 17.4                     | \(\delta_{t,o}(z)\) | 10.6 | ↓39.7% |
| \(\delta_{b,m}(x)\) | –4.7                     | \(\delta_{b,o}(x)\) | –3.5 | ↓25.5% |
| \(\delta_{b,m}(y)\) | –0.26                    | \(\delta_{b,o}(y)\) | –0.14 | ↓34.6% |
accuracy. Optimizing the chip repose angle resulted in a reduction in thermal displacement at both the top and bottom of the workbench spindle and a 32% reduction in its skewness compared to the pre-optimization period, which is conducive to improving the machining accuracy of gears.

5 Conclusions

The chip repose angle is an essential factor affecting the retention and accumulation of hot chips on the workbench surface, which is directly related to the processing accuracy of dry hobbing. This paper focuses on the mathematical modeling and optimization methods for the chip repose angle of the workbench, and the main research results are as follows:

1. The chip flow mechanism of dry hobbing is clarified. The chip generation process and flow behavior are analyzed based on dry hobbing’s spatial motion characteristics. The intrinsic relationship between the initial velocity of chip fall and hob speed and tooth space direction is clarified by mathematical modeling of chip flow angle;
2. A optimization model of chip repose angle for dry hobbing is developed. The optimization model of chip repose angle takes the minimum influence of the workbench by chip heat transfer as the objective, the adjustment distance ($x_t$ and $x_b$) between the top and bottom of the shield as the decision variables, and the clearance between the workbench and the shield as well as the distance between the shield and the isolation baffle as the constraints;
3. The thermal deformation regularity of the workbench before and after chip repose angle optimization is revealed. The optimum chip repose angle is 55.31°, obtained by an intelligent search. The thermal characteristics of the workbench before and after chip repose angle optimization are analyzed, and it was found that the workbench’s maximum temperature is reduced by 2.5 °C, the total thermal deformation is reduced by 8 μm, and the workbench spindle skewness is reduced by 32% after adopting the optimized chip repose angle, which is conducive to improving gear machining accuracy;

In this paper, a specific dry hobbing machine is used as an example to quantitatively analyze and optimize the chip repose angle of the workbench. The following work will conduct a more systematic and in-depth study on the optimized design of the chip repose angle around different types of dry machines and different machining parameters.

Author contribution Bo Li contributed to the conception of the study; Bo Li and Xiao Yang contributed significantly to the analysis and manuscript preparation; Yanbin Du and Lang He helped perform the analysis with constructive discussions.

Funding This work was supported by the National Natural Science Foundation of China (NSFC) (grant no. 51905059), the Innovative Research Group of Universities in Chongqing (CXTD20124), the Natural Science Foundation of Chongqing, China (grant no. cstc2019jcyj-mxmX0205), the Special Funding for Postdoctoral Research Projects in Chongqing (grant no. 2021XM2020), the Science and Technology
Research Program of Chongqing Municipal Education Commission (grant no. KJQN201800839), and the Graduate Research Innovation Project (grant no. CYS22621).

Data availability The datasets used or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Yes.

Competing interests The authors declare no competing interests.

References

1. Fratila D (2009) Evaluation of near-dry machining effects on gear milling process efficiency. J Clean Prod 17:839–845
2. Goindi G, Sarkar P (2017) Dry machining: a step towards sustainable machining – challenges and future directions. J Clean Prod 165:1557–1571
3. Gupta K, Laubsher R, Davim J, Jain N (2016) Recent developments in sustainable manufacturing of gears: a review. J Clean Prod 112:3320–3330
4. Yang X, Cao H, Chen Y, Zhu L, Li B (2017) An analytical model of chip heat-carrying capacity for high-speed dry hobbing based on 3D chip geometry. Int J Precis Eng Manuf 18:245–256
5. Brecher C, Klatte M, Lee T, Tzanetos F (2018) Metrological analysis of a mechatronic system based on novel deformation sensors for thermal issues in machine tools. Procedia CIRP 77:517–520
6. Ibaraki S, Blaser P, Shimoike M, Takayama N, Nakaminami M, Ido Y (2016) Measurement of thermal influence on a two-dimensional motion trajectory using a tracking interferometer. CIRP Ann 65:483–486
7. Blaser P, Pavliček F, Mori K, Mayr J, Weikert S, Wegener K (2017) Adaptive learning control for thermal error compensation of 5-axis machine tools. J Manuf Syst 44:302–309
8. Mori M, Iriso N, Shimoike M (2019) A new measurement method for machine tool thermal deformation on a two-dimensional trajectory using a tracking interferometer. CIRP Ann 68:551–554
9. Shi H, Ma C, Yang J, Zhao L, Mei X, Gong G (2015) Investigation into effect of thermal expansion on thermally induced error of ball screw feed drive system of precision machine tools. Int J Mach Tools Manuf 97:60–71
10. Mori K, Kono D, Matsubara A (2021) Effect of expansion coefficient difference between machine tool and workpiece to the thermal deformation induced by room temperature change. Procedia CIRP 101:318–321
11. Liu J, Ma C, Wang S, Wang S, Yang B, Shi H (2019) Thermal-structure interaction characteristics of a high-speed spindle-bearing system. Int J Mach Tools Manuf 137:42–57
12. Liu T, Gao W, Zhang D, Zhang Y, Chang W, Liang C, Tian Y (2017) Analytical modeling for thermal errors of motorized spindle unit. Int J Mach Tools Manuf 112:53–70
13. Cao H, Zhu L, Li X, Chen P, Chen Y (2016) Thermal error compensation of dry hobbing machine tool considering workpiece thermal deformation. Int J Adv Manuf Technol 86:1739–1751
14. Zhu L, Cao H, Zeng D, Yang X, Li B (2017) Multi-variable driving thermal energy control model of dry hobbing machine tool. Int J Adv Manuf Technol 92:259–275
15. Liu Z, Tang Q, Li X, Zou Z, Yang Y (2019) A method for thermal characteristics modelling of hob assembly on dry hobbing machine. Proc Inst Mech Eng Part C J Mech Eng Sci 233:2262–2274
16. Kadashchevich I, Beutner M, Karpuschewski B, Halle T (2015) A novel simulation approach to determine thermally induced geometric deviations in dry gear hobbing. Procedia CIRP 31:483–488
17. Li Y, Zhang Y, Zhao Y, Shi X (2021) Thermal-mechanical coupling calculation method for deformation error of motorized spindle of machine tool. Eng Fail Anal 128:105597
18. Yang X, Cao H, Li B, Jafar S, Zhu L (2018) A thermal energy balancing optimization model of cutting space enabling environmentally benign dry hobbing. J Clean Prod 172:2323–2335
19. Li X, Yang Y, Zou Z, Liu Z, Wang L, Tang Q (2019) Critical study on the thermal-structural characteristics of worktable assembly of a dry hobbing machine. Int J Adv Manuf Technol 100:179–188
20. Yang X, Chen P (2022) Heat transfer enhancement strategies for eco-friendly dry hobbing considering the heat exchange capacity of chips. Case Stud Therm Eng 29:101716
21. Yang X, Zeng L, Chen P, Du Y, Li B (2022) Complex characteristics and multi-dimensional control strategies of heat flow in dry gear hobbing machines. China Mech Eng 33:623–629
22. Alftar K, Tariq A, Ahmad S, Hussain G, Ratlamwala T, Ali H (2022) Thermal and hydraulic analysis of slotted plate fins heat sinks using numerical and experimental techniques. Case Stud Therm Eng 35:102109
23. Dang Q, Diessen T, Martagan T, Adan I (2021) A matheuristic for parallel machine scheduling with tool replacements. EUR J Oper Res 291:640–660
24. Chen B, Guan X, Cai D, Li H (2022) Simulation on thermal characteristics of high-speed motorized spindle. Case Stud Therm Eng 35:102144

Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor holds exclusive rights to this article under SPRINGERNATURE COPYRIGHT NOTICE and may not be reproduced, copied, or distributed without the consent of Springer Nature.