μSR measurement of the fundamental length scales in the vortex state of YBa$_2$Cu$_3$O$_6.60$

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The internal field distribution in the vortex state of YBa$_2$Cu$_3$O$_6.60$ is shown to be a sensitive measure of both the magnetic penetration depth $\lambda_{ab}$ and the vortex-core radius $\rho_0$. The temperature dependence of $\rho_0$ is found to be weaker than in the conventional superconductor NbSe$_2$ and much weaker than theoretical predictions for an isolated vortex. The effective vortex-core radius decreases sharply with increasing $H$, whereas $\lambda_{ab}(H)$ is found to be much stronger than in NbSe$_2$.

The magnetic penetration depth $\lambda$ and the coherence length $\xi$ are the two fundamental length scales in a superconductor. $\lambda$ is directly related to the superfluid density $\rho_s$, whereas $\xi$ is the length scale for spatial variations in the superconducting order parameter. In recent years, measurements of $\lambda$ have provided strong evidence for unconventional pairing in the high-$T_c$ materials in that there are line nodes in the superconducting energy gap. In particular, in both the Meissner and vortex states, $\lambda$ has been observed to increase linearly with temperature $T$ and magnetic field $H$ at low $T$ in single crystals of YBa$_2$Cu$_3$O$_6.95$.

Much less is known about the behaviour of $\xi$. In fact until now there has been no measurement of $\xi$ deep in the superconducting state of a high-$T_c$ superconductor. The magnitude of $\xi$ near the phase boundary has been estimated from measurements of the upper critical field $H_{c2}$ using Ginzburg-Landau (GL) theory. For a type-II superconductor for $H$ near $H_{c2}$, the coherence length is related to $H_{c2}(T)$ in GL theory by,

$$\xi(T) = \sqrt{\frac{\Phi_0}{2\pi H_{c2}(T)}}. \quad (1)$$

Reliable estimates of $\xi$ in YBa$_2$Cu$_3$O$_{7-\delta}$ made in this way are extremely difficult because the value of $H_{c2}$ at $T=0$ is so large (i.e. $\sim 100$ T). Consequently, $H_{c2}$ measurements are limited to high temperatures above $T/T_c \sim 0.85$, so that the low $T$ behaviour of $\xi(T)$ can only be determined by extrapolation. The situation is further complicated in the high-$T_c$ compounds by strong thermal fluctuations over a sizeable region near $T_c$, which result in broad transitions and poor estimates of $H_{c2}(T)$. Furthermore, Eq. (1) may not be valid for an unconventional superconductor.

It is desirable to have direct measurements of the coherence length, which in the vortex state is related to the size of the vortex cores. In particular, for a conventional superconductor $\xi \sim \rho_0$, whereas $\xi$ is the vortex-core radius $\lambda$. In principle both STM and μSR can be used to characterize the size of vortex cores and thereby determine $\xi$ directly deep in the superconducting state. Recent STM and μSR experiments on NbSe$_2$ show that $\rho_0$ decreases with increasing $H$, as a result of the increased interaction between vortices. Similar studies on high-$T_c$ materials have not yet been performed with STM. Pinning of the vortices due to surface roughness and oxygen vacancies eliminates the long-range order in the vortex lattice and results in variations in the electronic structure of the cores. Furthermore, the effect of the discontinuity in the quasiparticle excitation spectrum at the surface is still not understood.

On the other hand, μSR provides information on the vortex cores in the bulk of the sample. As explained in Ref. 6, in a μSR experiment $\rho_0$ is related to the high-field cutoff of the measured internal field distribution. The spectral weight at the cutoff grows as the density of the vortices increases. A well defined cutoff was observed in NbSe$_2$ (Ref. 6), where $\rho_0$ is several times larger than in YBa$_2$Cu$_3$O$_{6.95}$ (Refs. 2, 3). In YBa$_2$Cu$_3$O$_{6.95}$ no clear signal from the vortex cores was visible below 3 T. The temperature dependence of $\xi(T)$ was investigated in an earlier μSR study of YBa$_2$Cu$_3$O$_{6.95}$ at higher magnetic fields 6. Unfortunately the signal-to-noise ratio was poor due to the influence of the large magnetic field on the positron orbits and timing resolution. In addition, demagnetization effects likely contributed significantly to the measured field distribution since the sample consisted of nineteen small crystals, and $\lambda$ and $\xi$ were assumed independent of $H$ in the fitting procedure.

In this Letter we present a μSR study of the oxygen deficient high-$T_c$ superconductor YBa$_2$Cu$_3$O$_{6.60}$.
Compared to the optimally-oxygenated compound, YBa$_2$Cu$_3$O$_{6.60}$ has a smaller carrier concentration in the CuO$_2$ planes, a reduced $T_c$ and $H_{c2}$, and a correspondingly larger $\xi$. This allows us now to report the first detailed study of the fundamental length scales $\lambda$ and $\xi$ in a high-$T_c$ superconductor.

We studied two different YBa$_2$Cu$_3$O$_{6.60}$ samples with identical transition temperatures (59 K). The first sample (S1) was obtained by deoxygenating the three-crystal mosaic of YBa$_2$Cu$_3$O$_{6.95}$ used in Refs. [3]. The twin boundary spacing was on the order of 1 $\mu$m in the bulk. The second sample (S2) was grown from a separate batch and consisted of two large single crystals with a total $a-b$ plane surface area of 30 mm$^2$. S2 was mechanically detwinned, such that the twin boundary density was about an order of magnitude smaller than in S1. Measurements were performed on field-cooled samples using the M15 and M20 surface muon beamlines at TRIUMF and the same apparatus as that used in Refs. [2,3].

The experimental muon spin precession signal was fit assuming the local field due to the vortex lattice at any point in the $a-b$ is given by

$$B(\rho) = B_0(1 - b^4) \sum_{G} e^{-iG\cdot\rho_0} K_1(u) / \lambda_{ab} G^2, \quad (2a)$$

with

$$u^2 = 2 \xi_{ab}^2 G^2 (1 + b^4)[1 - 2b(1 - b)^2]. \quad (2b)$$

where $B_0$ is the average magnetic field, $b = B/B_{c2}$, $\xi_{ab}$ is the GL coherence length and $K_1(u)$ is a modified Bessel function. This analytical model of the field profile agrees extremely well with the exact numerical solutions of the GL equations [10] at low reduced fields $b < 0.02$ for our measurements. The term $u K_1(u)$ cuts off the summation thereby removing the logarithmic divergence of field at the vortex cores in the conventional London model. The cutoff is done in a way which preserves circular symmetry around the vortex cores. We note that the results herein are qualitatively similar to that obtained using a modified London model for $B(\rho)$ with a Gaussian cutoff factor (as in Ref. [3]) and nearly identical to that using a Lorentzian cutoff—where only the latter is strictly valid at low fields. The internal field distribution $n(B)$ was convoluted with a Gaussian of width $\sigma$ to account for vortex-lattice disorder and nuclear dipolar fields.

The summation in Eq. (3) is taken over all reciprocal lattice vectors $G$ of a triangular vortex lattice—the structure which minimizes the free energy for a conventional superconductor. Infrared reflectance measurements of $\lambda$ in zero-field [11] show a small anisotropy in the $a-b$ plane ($\lambda_a/\lambda_b \approx 1.3$) which will stretch the triangular lattice, leading to elliptical cores in which $\xi_b/\xi_a = \lambda_a/\lambda_b$. It was shown in Ref. [4] through a scaling argument, that the corresponding magnetic field distribution is identical to the isotropic case.

![FIG. 1. The Fourier transforms of the muon spin precession signal in YBa$_2$Cu$_3$O$_{6.60}$ after field cooling in a magnetic field $H \sim 0.75$ T down to $T = 0.04$ and 0.84 $T_c$ (inset). The dashed curve is the Fourier transform of the simulated muon polarization function which best fits the data and the shaded region is the residual background signal.](image)

In a $d_{x2-y2}$-wave superconductor the magnetic field distribution in the core region can be fourfold symmetric [2,3] and twofold symmetric with $a-b$ anisotropy [4]. However, theoretical models for a $d_{x2-y2}$-wave vortex core contain too many parameters to be useful in fitting experimental data. Modelling the core region with circular (or elliptical) symmetry should be sufficient to characterize the changes in core size with field and temperature. In general one expects the symmetry of the $d_{x2-y2}$-wave vortex cores to distort the lattice from triangular symmetry, but only at high fields where the intervortex spacing is small. So far no experiments have imaged the vortex lattice in YBa$_2$Cu$_3$O$_{6.60}$.

Figure 1 shows the real amplitude of the Fourier transform of the muon precession signal in YBa$_2$Cu$_3$O$_{6.60}$ for $T = 0.04 \ T_c$ and $H = 0.75 \ T$ (solid curve) and of the simulated muon polarization function which best fits the data (dashed curve). Above $T_c$ the lineshape is nearly a perfect Gaussian with a width entirely due to the nuclear-dipolar fields. Well below $T_c$ the internal field distribution is very similar to that previously observed in NbSe$_2$, where the vortex lattice is known to be triangular. A small peak due to a residual background signal is also visible. At 0.84 $T_c$ the field distribution is no longer asymmetric (Fig. 1 inset). We attribute this qualitative change in the lineshape to melting of the vortex lattice, i.e.: a transition from continuous 3D vortex lines to 2D
vortex "pancakes" which are uncorrelated between planes. Such a transition has been observed in µSR studies of highly anisotropic Bi$_{2}$Sr$_{2}$CaCu$_{2}$O$_{8+δ}$ (BSCCO) \[15\] but does not occur in YBa$_{2}$Cu$_{3}$O$_{6}$ in similar fields because of stronger interplane coupling. We estimate the variation of the crossover temperature $T_m$ with magnetic field for 0.5 < $H$ < 1.5 T to be $T_m \approx T_c - \alpha H$, where $\alpha = 17(1)$ K/T.

Figure 2 shows the temperature dependence of $\lambda_{ab}^{-2}(T)$ below $T_m$ for three magnetic fields. As previously observed in YBa$_{2}$Cu$_{3}$O$_{6.95}$ \[1\] and La$_{1.65}$Sr$_{0.15}$CuO$_{4}$ \[14\], there is a strong linear-$T$ dependence at low temperature which is independent of $H$. The inset of Fig. 2 shows a comparison between the $T$-dependence of $\lambda_{ab}(T)$ at 0.5 T and microwave cavity measurements \[17\] of $\Delta \lambda_{ab}(T)$ in zero magnetic field. The excellent agreement confirms that the fitting procedure which assumes a triangular vortex lattice, introduces at most only a small systematic error in the absolute value of $\lambda$. This is reasonable since it has been shown theoretically that including additional terms in the free energy of the vortex state produce only minor changes in the internal field distribution \[18\].

As in Ref. \[8\] we define $\rho_0$ to be the radius at which the supercurrent density $J_s(\rho) = \nabla \times B(\rho)$ reaches its maximum value. $J_s(\rho)$ profiles were generated from fits of the data to the field profile of Eq. \[6\]. Figure 3 shows the $T$-dependence of $\rho_0(T)$ for the same fields as in Fig. 2. The solid lines are fits to the linear relation $\rho_0(T) = \rho_0(0)[1 + \beta T/T_c]$, where $\beta \sim 0.23$ is essentially independent of field. The inset of Fig. 3 shows the $T$-dependence of $\rho_0$ at 0.5 T normalized to $\rho_0$ at $T = 0$. The linear term is much weaker than that found in NbSe$_2$ at 0.19 T \[14\], where $\beta \sim 1.2$. Better agreement (see Fig. 3) is obtained if $T$ is normalized to $T^* = 250$ K, the temperature below which a pseudogap opens in the spectrum of low-energy excitations in YBa$_{2}$Cu$_{3}$O$_{6.6}$ (Ref. \[20\]). One possible interpretation is that the pairing amplitude is established prior to the onset of long range phase order at $Tc$ \[21\]. In both materials the size of the vortex core does not decrease as steeply with temperature as expected in theoretical predictions for a $s$-wave \[22\] or $d_{x^2-y^2}$-wave \[13\] superconductor. However, these theories pertain to a single isolated vortex and do not account for vortex lattice effects. Thermal fluctuations of the vortices about their average positions result in a premature truncation of the high-field tail in the µSR lineshape—which results in an overestimate of $\rho_0$ that increases with $T$. However, thermal fluctuations are expected to be most important at high magnetic fields \[23\] or near $T_m$, and do not account for the weaker $T$-dependence relative to NbSe$_2$.

In Fig. 4, $\lambda_{ab}$, $\rho_0$ and $\kappa$ extrapolated to $T = 0$ are plotted as a function of $H$. The magnitude of $\lambda_{ab}$ determined in S1 is significantly lower than that in S2. The difference is likely a result of vortex lattice distortions in S1 due to twin boundary pinning. This introduces a systematic uncertainty in the determination of $\lambda_{ab}$. The RMS deviation of the vortices from their positions in a perfect...
The microscopic theory for a conventional superconductor predicts such behaviour \[23\]. It is conceivable, the result may be attributed to quantum fluctuations at low temperatures which become important for large \( H \) \[29\]. However, quantum fluctuations are likely to have a small effect given the observed field dependence of \( \rho_0 \) in NbSe\(_2\) (Refs. \[4,5\]). The essential point is that quantum fluctuations are expected to be negligible in a conventional superconductor where \( \xi \) is large \[23\]. It is important to realize that our measurements in YBa\(_2\)Cu\(_3\)O\(_6.6\) extend over a very narrow range of \( H/H_c2 \) relative to those for NbSe\(_2\). We were limited to this field range by the melting transition for higher \( H \) and the small amplitude of the high-field cutoff for lower \( H \). Consequently, no reduction in the rate of change of \( \rho_0 \) with \( H \) (expected for larger fields) was observed. A linear fit to the data yields an intercept \( \rho_0(0,0) = 89.5 \, \text{Å} \) and slope \(-28.2 \, \text{Å}/T\).

Figure 4(c) shows the \( H \)-dependence of \( \kappa = \lambda_{ab}/\xi_{ab} \). A linear fit gives an intercept \( \kappa(0,0) = 17.0 \) and slope \( 15.3 \, \text{T}^{-1} \). We also find that \( \kappa \) is essentially independent of \( T \) over the entire temperature range. This agrees not only with magnetization measurements in the vortex state of BSCCO for \( T/T_c > 0.43 \) \[27\], but also with our previous measurements in NbSe\(_2\).

In conclusion, we have measured the \( T \) and \( H \)-dependences of \( \lambda_{ab} \) and \( \rho_0 \) in YBa\(_2\)Cu\(_3\)O\(_6.6\) for \( H < H_c2 \). We find that \( \lambda_{ab} \) and \( \rho_0 \) both vary linearly with \( T \) at low temperatures. The \( T \)-dependence for \( \rho_0 \) is considerably smaller than that found in NbSe\(_2\). Also, \( \lambda_{ab} \) increases while \( \rho_0 \) decreases with increasing magnetic field. We attribute the field dependence of \( \rho_0 \) to the compression of the vortices due to vortex-vortex interactions.

We would like to thank Alain Yaouanc, Ian Affleck, John Berlinsky, Katherine Kallin, and Marcel Franz for many helpful discussions, and Syd Kreitzman, Curtis Ballard and Mel Good for technical assistance. This work is supported by NSERC of Canada and by the U.S. Department of Energy through grant DE-FG05-88ER45353.

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**FIG. 4.** The field dependence of (a) \( \lambda_{ab} \), (b) \( \rho_0 \) and (c) \( \kappa_{ab} = \lambda_{ab}/\xi_{ab} \) extrapolated to \( T = 0 \) in YBa\(_2\)Cu\(_3\)O\(_6.6\). The data for S1 (twinned) are shown as open circles whereas the solid circles designate S2 (detwinned).

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The triangular lattice (estimated from the fitted values of \( \sigma \)) was found to be \( \sim 8\% \) and \( 5\% \) of the intervortex spacing for S1 and S2, respectively. This disorder was independent of \( H \). The lines in Fig. 4(a) are fits to the linear relation, \( \lambda_{ab}(0,0) = \lambda_{ab}(0,0)[1 + \gamma H/H_c2] \). Assuming \( H_c2 = 70 \, \text{T} \), \( \lambda_{ab}(0,0) = 1586 \, \text{Å} \) and \( \gamma = 6.6 \) in S1, and \( \lambda_{ab}(0,0) = 1699 \, \text{Å} \) and \( \gamma = 5.0 \) in S2. The increase in \( \lambda_{ab} \) with \( H \) is comparable to that observed recently in YBa\(_2\)Cu\(_3\)O\(_{6.95}\) \[4\] and is considerably stronger than that reported in NbSe\(_2\) where \( \gamma = 1.6 \) \[3\]. This difference can be attributed to an enhancement in pair breaking caused by the applied field for an energy gap function with nodes—alike to the nonlinear Meissner effect \[24\].

Figure 4(b) shows how \( \rho_0 \) decreases with increasing magnetic field. Twin boundaries appear to have a negligible effect on the core size since there is good agreement between S1 and S2. A physical interpretation is that the increased interaction between vortices with field “squeezes” the vortex cores causing a reduction in \( \rho_0 \). The microscopic theory for a conventional superconduc-
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