1. Introduction

Warps of disk galaxies are a common phenomenon (as common as spiral structure), yet they are still not fully understood. In this review I will try to summarize their observed properties, including preliminary results of a new HI survey we have carried out with Westerbork, and try to relate these to several proposed mechanisms for explaining warps. I will not attempt a historic review of the theoretical work on this subject; such reviews can be found in Binney (1992), or Kuijken (1998), for example.

2. Observed Properties

The global observational understanding of galaxy warps has been summarized in two pairs of Laws.

- **Bosma’s Laws** (Bosma 1991, on overall statistics of warps)
  1. At least half of all galaxies are warped
     **Implication:** Warps are long-lived or continuously generated
  2. Galaxies with small dark halo core radii (as determined from a rotation curve decomposition) are less likely to be warped
     **Implication:** Link between warps and the dark halo potential

- **Briggs’s Laws** (Briggs 1990, on structure of individual warps)
  1. Disks are generally flat inside radius $R_{25}$. Out to radius $R_{26.5}$ the line of nodes of a warp is straight
     **Implication:** Self-gravity of the disk is important (it keeps the different parts of the disk precessing synchronously and hence the line of nodes straight—cf. the winding problem of spiral waves)
  2. The outer line of nodes advances in the direction of galactic rotation
     **Implication:** Warps are not quite in equilibrium at large radii. This points to a link to the environment, or to very long timescales

Warps are primarily observed in HI. Significant warps (misalignments between inner and outer parts of the disk of more than a degree or two) in stellar disks are rarely observed (Reshetnikov & Combes 1999), though recently the warp of the Milky Way has also been seen in the stellar distribution (Alard 2000). This may indicate that warps are a phenomenon which affects only
the cold ISM, or that only the very outskirts of galaxy disks, which are only observable in HI, are involved.

The Galaxy warp illustrates the general point that, analogously to spiral structure, warps come in several varieties: ‘grand-design’, nice integral-sign shaped bi-symmetric warps; ‘irregular’ warps which are only visible on one side, or in which one side of the galaxy is more warped than the other; and ‘feeble’ (weak or absent) warps. To determine the relative frequency of these classes is important, as it points the way for attempts at an explanation: should we be looking for mechanisms which naturally produce beautiful integral-sign warps, or for—possibly more chaotic—ways to make irregular ones?

In order to address this question, we have performed a blind survey of edge-on (as judged from optical images) galaxies. These galaxies form part of the WHISP sample, which is basically an HI flux-density limited sample (> 200 mJy) selected from the UGC catalogue. We chose galaxies with blue major diameters larger than 2', and inclination class 6 or 7. DSS images of these galaxies were inspected to filter out obviously less-inclined galaxies, to leave us with a sample of galaxies which, as judged from optical images, have inclinations at least ∼ 80°. We chose edge-on galaxies for this survey so that the warps can be studied purely morphologically, without the need for interpretation or modelling of the velocity field (e.g., by means of tilted ring models).

Our sample clearly shows that warps occur in all types. About 1/3 of our galaxies show a nice integral-sign warp, about 1/3 are flat, and the final 3rd of the sample are of the irregular (one-sided or asymmetric) types (Table 1).

Table 1. The classification of warps in a sample of 28 edge-on galaxies, observed in HI with Westerbork (García-Ruiz et al., in preparation). Note the large fraction of galaxies which do not fall in the classical, integral-sign warp category.

| Warp type   | Number |
|-------------|--------|
| Flat        | 7      |
| Integral-sign| 9     |
| Asymmetric  | 8      |
| U-shaped    | 2      |
| Total       | 26     |

3. Models

Though there have been suggestions that magnetic fields coupling to a slightly ionized ISM could explain the observed warps (Battaner et al. 1991) we will concentrate here on models invoking gravitational processes. The fact that regular disk galaxies show no major discrepancies between stellar and gaseous rotation curves (beyond the well-understood asymmetric drift) indicates that magnetic fields are in any case not a dominant contributor to large-scale galaxy dynamics. Moreover, in the Milky Way, recently Alard (2000) has shown that the stellar disk shares in the warp seen in the HI.
I will briefly discuss three scenarios for warps: the normal mode models, accretion of angular momentum, and interaction with satellites.

### 3.1. Normal Modes

The normal mode picture of warps (Toomre 1983; Dekel & Shlosman 1983; Sparke & Casertano 1988) envisions the disks as embedded in a flattened dark halo. If the disk is misaligned with the equator of the halo, the resulting torque combined with the disk’s spin will cause it to precess about the halo minor axis. This precession rate is easily calculated (Kuijken 1991) as the ratio of the torque from the halo (determined from the circular and vertical frequencies $\Omega_H$ and $\nu_H$ in the potential of the halo) to the spin of the disk (governed by the total circular frequency $\Omega_c$):

$$
\Omega_{\text{pre}} = \frac{\int \Sigma R^3(\Omega_H^2 - \nu_H^2) dR}{\int 2\Sigma R^3 \Omega_c dR} \quad (1)
$$

where $\Sigma$ is the surface density of the disk at radius $R$.

Because different parts of the disk have a different free precession frequency in the potential of the halo, without disk self-gravity the warp would wind up and disappear, analogously to spiral perturbations in massless disks. However, the disk does have self-gravity, and Sparke and Casertano were able to show that solutions exist in which the disk precesses as a single, warped, unit in the halo. When the precession frequency difference across the disk is too large, or the disk is too lightweight, these solutions do not exist.

The problem with normal modes was already foreseen by Toomre (1983), who pointed out that a crucial assumption in this picture is the neglect of back-reaction from the halo to the precession of a massive disk in its center. This effect, analogous to dynamical friction, was investigated by Dubinski & Kuijken (1995), Nelson & Tremaine (1995), and Binney et al. (1998) after earlier investigation in terms of WKB density waves by Bertin and Mark (1980). It turns out to be very important: the precession frequencies implied by equation (1) are so low compared to the dynamical time of the inner halo that there is ample time for the inner halo to adjust itself to the reorientation of the disk. This aligned inner halo results in a diminished torque, which in turn further reduces the precession speed. The overall effect is an alignment wave which propagates outwards through the disk, as the precession grinds to a halt. The alignment between disk and halo is rather fast, on the order of three local orbital times.

Under some conditions, the close coupling between halo and disk can instead lead to a fast excitation. This can happen, for example, if the halo rotates retrograde with respect to the disk, or if it is prolate (Nelson & Tremaine 1995).

### 3.2. Accretion of Angular Momentum

There is evidence that galaxies from time to time acquire material with significantly different alignment of angular momentum. Polar rings and counterrotating components in disks are clear examples.

If this such accretion continuously alters the orientation of the dark halo angular momentum axis, the disk finds itself in a slowly changing potential, in which it is continuously trying to align itself with the dark halo symmetry plane. Damping is now not such a problem, as the halo is continuously changing...
Figure 1. $N$-body simulation of the evolution of an inclined disk in a live dark halo (Dubinski & Kuijken 1995). The azimuth from which the disk is viewed is indicated in each panel. Note that the precession very quickly comes to a halt, before even a quarter of a precession orbit has been completed.
in response to outside influences, rather than only to the disk. As shown by Jiang & Binney (1999), this process can generate realistic warp amplitudes in disks. If this mechanism dominates, one might expect this slowly changing tidal fields to result in rather symmetric warps, though no detailed simulations have yet been performed.

3.3. Forcing by Satellites

One of the first possible explanations for the warp of the Galaxy was the tidal perturbation by the Magellanic Clouds. Hunter & Toomre (1969) however showed that these are too weak to have a strong effect. Lately this idea has been revived, however, by the work of Weinberg (1995, 1998). He presented a model for which he calculated the self-consistent response (wake) of the Galactic dark halo to the orbiting Clouds. This wake turns out to be rather massive, and to be strongest at a radius inside that of the Cloud orbit; both help to boost the tidal effect of the wake to dominate that of the Cloud itself. With such halo-enhanced tides, he was able to reproduce the observed Galactic warp amplitude.

Weinberg’s model requires some tuning in order to achieve the amplitudes observed. It also makes several assumptions:

1. The Cloud orbit is modelled as a quasi-periodic orbit, i.e., the dynamical friction-induced decay of the orbit is ignored. This means that the model disk is perturbed at a constant set of frequencies, and it has time to build up its response to these perturbations. In reality, however, the frequencies at which the disk is perturbed are continuously increasing.

2. The halo’s back-reaction to the disk precession is not included in the calculations. In fact, in these models the halo is assumed to be spherically symmetric (before the LMC perturbs it). We have seen above, however, that the halo couples very effectively to a precessing misaligned disk.
These caveats suggest that satellites may not generally be responsible for causing warps in their hosts.

Some extra evidence against the LMC as cause comes from studying the orientation of the warp and of the LMC orbit. Garcia-Ruiz et al (in prep.) made a very simple disk/halo/satellite system. The model consists of a rigid spinning disk, a flattened dark halo, and a satellite orbiting on a circular orbit. The equations of motion for the polar coordinates \((\theta, \phi)\) of the disk axis (halo axis \(\theta = 0\)) can then be linearized in terms of \((x, y) = (\theta \cos \phi, \theta \sin \phi)\) to

\[
\begin{align*}
I_1 \ddot{x} + S \dot{y} + V_H x &= V_S \sin 2\theta_S(t) \cos \phi_S(t) \\
I_1 \ddot{y} - S \dot{x} + V_H y &= V_S \sin 2\theta_S(t) \sin \phi_S(t)
\end{align*}
\]

where \(I_1\) is the moment of inertia of the disk about its diameter, \(S\) is the spin of the disk, \(V_H\) is the torque per unit angle exterted by the halo, \(V_S\) parameterized the satellite tidal field strength and the satellite orbit follows \(\theta = \theta_S(t), \phi = \phi_S(t)\). This equation is easily solved: for the case of a polar orbit with \(\phi_S = 90^\circ\), \(\theta_S = \Omega_S t\) the solution is (with \(\Delta = (V_H - 4I_1 \Omega_S^2)^2 - (2\Omega_S S)^2\))

\[
\begin{align*}
x &= -\frac{2\Omega_S S}{\Delta} V_S \cos 2\Omega_S t \\
y &= \frac{V_H - 4I_1 \Omega_S^2}{\Delta} V_S \sin 2\Omega_S t
\end{align*}
\]

The precession path of such a rigid disk, for Galaxy/LMC parameters, is shown in Fig 3. The dominant effect of the satellite perturbation is to make the disk axis nod perpendicular to the satellite orbital plane, a consequence of the slow satellite orbital frequency compared to the natural precession frequency of the disk. The orientation is independent of the satellite mass, or of the strength of the tidal field; it is governed only by the frequencies of the satellite and disk.

An extension of this calculation to a disk consisting of many rigid, precessing rings all anchored to a common center shows the same trend (Fig 3): the warp orientation (defined as the direction along which the inner and outer disk orientations differ maximally) is perpendicular to the satellite orbital plane.

By contrast, the LMC orbit lies in the plane along which the Galactic warp is strongest.

A full N-body simulation of a disk/galaxy/LMC system, including halo back-reaction on the disk, shows the same effects. It appears that the wake of the satellite in the halo basically acts in phase with the satellite, and so mostly changes the amplitude of the tides but not their orientation. The backreaction of the halo on the precessing disk is more difficult to predict in these analytical models, as it can act both as a damping or as a destabilizing factor (see discussion on normal modes above); our numerical experiments in simulating the Galaxy/LMC situation have in not turned up any cases where the disk response was as strong as observed in the Milky Way, or oriented as observed.

Interestingly, the Sagittarius dwarf has an orbit almost at right angles to that of the LMC. It could therefore in principle produce a response in the disk of the right phase (but see Binney 2000). However, at this moment the mass of the Sgr dwarf is still rather uncertain, and it is not clear whether it could raise sufficiently strong tides. It is also in a sufficiently small orbit that the tides will be very asymmetric (Ibata & Razoumov 1998).
Figure 3. Left: precession path of a rigid disk in a flattened halo, perturbed by an orbiting satellite. The dots indicate the position corresponding to the current phase of the LMC in its orbit, which is in the $yz$ plane for halo potential flattenings of 0 (solid), 0.05, 0.1, 0.15, 0.2. The direction of the observed tilt of the axis of the outer disk of the Milky Way is indicated by the arrow. Right: The warp direction (direction of the average alignment of the outer disk with reference to the inner disk plane) for an exponential disk consisting of centrally-pivoted concentric spinning rings. In both cases, the response of the disk is primarily to tilt in the plane perpendicular to the satellite orbit, unlike what is observed in the Galaxy/LMC system.

All in all, therefore, it appears that satellites as an explanation of the warp of the Galaxy, and by extension as a generic model for warps in other galaxies, pose some difficulties.

4. Summary

In spite of being known now for many decades, warps are still a puzzle. The observational situation and theoretical models to explain the data are still evolving. In this talk I have tried to summarize the situation as follows.

1. Warps are a very common phenomenon. However, in addition to the classic ‘grand-design’ integral-sign warps, in many cases the warp is asymmetric or even one-sided. Models for warps should therefore not only address the symmetric, regular ones. Quite possibly there is a good analogy here to spiral structure, which also displays varying degrees of regularity.

2. Normal modes (equilibrium configurations of a tilted, warped disk precessing about the symmetry axis of a flattened halo) interact strongly with the dark halo. As a consequence they are easily and strongly damped, or, in special circumstances, excited.

3. Satellite tides are generally too weak to produce warps of the amplitudes observed. However the dark halo can under certain circumstances respond
to the satellite in a way that significantly adds to the tidal perturbation on
the disk. A useful diagnostic for satellite tides as explanation for a warp is
the orientation of the warp with respect to the satellite orbit. In the case
of the LMC/Galaxy system, this orientation appears to be different from
predictions.

4. Accretion of material can generate warps through a continual change in the
orientation of the halo symmetry plane, to which different parts of the disk
respond on different timescales. Such models are yet to be investigated in
full detail; in particular the question of whether the observed frequency
of asymmetric warps can be reproduced by this kind of model may be a
useful avenue to explore.

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