One-loop determination of mass dependent $O(a)$ improvement coefficients for the heavy-light vector and axial-vector currents with relativistic heavy and domain-wall light quarks

Norikazu Yamada*, Sinya Aoki* and Yoshinobu Kuramashi

a RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA
b Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan

We present the one-loop results of the mass dependent $O(a)$ improvement coefficients for the heavy-light vector and axial-vector currents consisting of the relativistic heavy and the domain-wall light quarks. The calculations are performed with the plaquette, Iwasaki and DBW2 gauge actions. The heavy quark mass and domain-wall height dependence is investigated. We point out that the exact chiral symmetry held by the lattice light quark action leads to an exact relation between the improvement coefficients for the vector and axial-vector currents without regard to the lattice heavy quark action.

1. Introduction

Lattice QCD has played an essential role in the nonperturbative study of the heavy-light physics [1], and further progress, especially in precision, is expected. In the lattice study of heavy-light physics, the scaling violations due to the large heavy quark mass and the uncertainty arising from long extrapolations to the physical light quark mass have been significant sources of uncertainty, and toward more precision studies it is indispensable to reduce these systematic errors. Combining the relativistic heavy quark [2] and the domain-wall (DW) light quark [4] is a promising choice to simulate heavy-light system. In the following, we present the one-loop calculation of the $O(a)$ improvement coefficients for the heavy-light current consisting of these quarks. The detailed descriptions are available in Refs. [7].

2. Quark actions

The relativistic heavy quark action [2],

\[ S_Q = \sum_x \bar{Q}(x) \left[ m_0 + \gamma_0 D_0 + \nu_Q \sum_i \gamma_i D_i \right] Q(x), \]

is obtained by pushing the on-shell improvement program [8] to the massive Wilson fermions, and is equal to the Fermilab action [9] with a special choice of parameters. The four parameters, $\nu_Q, r_s, c_E, c_B$, have been known to the one-loop level [3].

In this work, we employ the following version of the DW action,

\[ S_{dw} = \sum_{x,y,s,t} \bar{\psi}_s(x) D_{dw}(x,s; y,t) \psi_t(y), \]

\[ D_{dw}(x,s; y,t) = D_w(x,y) \delta_{s,t} + \delta_{s,y} D_5(s,t), \]

\[ D_w(x,y) = \gamma_0 D_0 + \nu_q \sum_i \gamma_i D_i \]

\[ -\frac{\alpha}{2} \sum_{\mu} \bar{u} \gamma_\mu u - M_5 \delta_{x,y}, \]

where $P_{R/L} = (1 \pm \gamma_5)/2$, and $s$ and $t$ label the coordinate of the fifth dimension running 1 to $N_5$. **
In the action, we have introduced two new parameters \( \nu_q \) and \( R_s \) according to the following reason. It was found that some of the improvement coefficients for the heavy-light currents can be determined only if the light quark mass is finite \(^{10}\), therefore we need to consider the case that \( m_f \) is finite. However, once the quark acquires a finite mass, \( m_f \rho \) corrections to the dispersion relation causes unwanted infrared divergences in the loop integrations. The new parameters \( \nu_q \) and \( R_s \), where \( \nu_q \rightarrow 1 \) and \( R_s \rightarrow 0 \) in the massless limit, are to adjust the dispersion relation and eliminate the infrared divergences. Perturbative technique for the DW quarks is described in Ref. \(^{11}\). It should be noted that the both actions restore the exact space-time rotational symmetry in the massless limit.

3. Properties of the coefficients

We define the improvement coefficients, \( \Delta_{V_\mu} \), \( c_{V_\mu}^{L,R,L} \), for the vector current as follows,

\[
V_{\mu}^{MS} = Z_{V_{\mu}}^{MS-latt} V_{\mu}^{lat,imp},
\]

\[
Z_{V_{\mu}}^{MS-latt} = Z_{wf} [1 - g^2 \Delta_{V_{\mu}}],
\]

\[
V_{\mu}^{lat,imp} = \bar{q} \gamma_\mu Q - g^2 c_{V_{\mu}}^{L,R} \{ \bar{q} Q \}
- g^2 c_{V_{\mu}}^{L,R} \partial_\mu \{ \bar{q} Q \}
- g^2 c_{V_{\mu}}^{L,R} \partial_\mu \{ \bar{q} Q \} + O(g^4),
\]

where \( \partial_\pm = \partial_\mu \pm i \partial_\mu \) and \( Q \) are lattice light and heavy quark fields, respectively, and \( Z_{wf} \) is a known factor relating the wave function renormalizations for both quarks. While, in general, the improvement coefficients for the temporal and spacial components have different values, in the massless limit they agree because of the restoration of space-time rotational symmetry. The equation of motion always allows us to set \( c_{V_{\mu}}^{L} = c_{V_{\mu}}^{R} = 0 \).

The exact chiral symmetry of the DW fermions brings an exact relation between the improvement coefficients for the vector and axial-vector currents (for the proof, see Ref. \(^{12}\)). Thanks to this relation, once \( Z_{V_{\mu}}^{MS-latt} \) and \( c_{V_{\mu}}^{L,R,L} \) are known, with these coefficients the continuum (improved) axial-vector current is immediately given by

\[
A_{\mu}^{MS} = Z_{A_{\mu}}^{MS-latt} A_{\mu}^{lat,imp},
\]

\[
A_{\mu}^{lat,imp} = \bar{q} \gamma_\mu \gamma_5 Q + g^2 c_{V_{\mu}} \partial_\mu \{ \bar{q} \gamma_5 Q \}
+ g^2 c_{V_{\mu}} \partial_\mu \{ \bar{q} \gamma_5 Q \} + g^2 c_{V_{\mu}} \{ \bar{q} \gamma_5 \gamma_\mu Q
- g^2 c_{V_{\mu}} \gamma_5 \gamma_\mu Q \} + O(g^4).
\]

This property holds irrelevance to heavy quark action, and this statement can be extended to any pair of operators which belong to the same chiral multiplet.

The relation \( Z_{V_{\mu}}^{MS-latt} = Z_{A_{\mu}}^{MS-latt} \) is especially useful for the test of the soft pion relation \( f^0(q_{max}^2) = f_B / f_\pi \) \(^{13}\), because the unknown two loop coefficient in the ratio has prevented the test from being more precise.

4. Results

The loop integrations on the lattice are performed by a mode sum and BASES. The dependence of the coefficients on the tree level pole mass of heavy quark, \( m_Q^{(0)} \), and the domain-wall height, \( M_5 \), are searched by choosing several values for both parameters. We checked that the two methods give consistent values if \( m_Q^{(0)} < 7.0 \) and \( 0.1 < M_5 < 1.9 \), and found that in that region the discrepancy is less than 0.001, which can only affect physical quantities by 0.5\%, at most. The continuum part is evaluated in the naive dimensional regularization (NDR) with the modified minimal subtraction scheme (\textsc{ms})

In Fig. 4 the \( m_Q^{(0)} \) dependence of \( \Delta_{V_{\nu}} \) (circles) with three different gauge actions, the plaquette, Iwasaki \(^{14}\) and DBW2 \(^{14}\), is shown as an example. The results with \( M_5 = 0.5, 1.1 \) and 1.7 are shown as examples to give some idea about the \( M_5 \) dependence, which is which turned out to be smaller than the \( m_Q \) dependence in the most cases. The solid lines are obtained by fitting the numerical data to a certain functional form. The fit reproduces the data very well over the whole region of \( m_Q^{(0)} \) and \( M_5 \) as seen in the figures.
5. Improving perturbation series

We examined two ways to improve the perturbation series. One is the mean field improvement (MF) using the mean plaquette value, and another is involving the nonperturbative renormalization factor for the light-light vector current consisting of the DW fermions in addition to the mean field improvement. The latter which we call QNP+MF has an advantage because the wave function renormalization for the DW fermion is taken care of in all orders.

Reorganizing $\Delta_{V_0}$ with two ways, we worked out two coefficients, $\Delta_{V_0}^{MF}$ (triangle-up) and $\Delta_{V_0}^{QNP+MF}$ (triangle-down), which are plotted in Fig. 1 for the three different gauge actions. From the figure, it is seen that in the non-improved case the size of the one-loop coefficient largely depends on the gauge action while once one introduced the quasi non-perturbative and mean field improvements together the dependence becomes less transparent. It is also interesting to see that the mean field improvement makes the $m_Q$ dependence smaller. The similar findings are made for the improvements of $\Delta_{V_k}$ as well.

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